Simulation-Informed Revenue Extrapolation with Confidence Estimate for Scaleup Companies Using Scarce Time-Series Data

Lele Cao
Motherbrain, EQT
Stockholm, Sweden
lele.cao@eqtpartners.com

Sonja Horn
Motherbrain, EQT
Stockholm, Sweden
sonja.horn@eqtpartners.com

Vilhelm von Ehrenheim
Motherbrain, EQT
Stockholm, Sweden
vilhelm.vonehrenheim@eqtpartners.com

Richard Anselmo Stahl
Motherbrain, EQT
Stockholm, Sweden
richard.stahl@eqtpartners.com

Henrik Landgren
Ark Kapital, Stockholm, Sweden
henrik@arkkapital.com
Motherbrain, EQT
Stockholm, Sweden

ABSTRACT

Investment professionals rely on extrapolating company revenue into the future (i.e., revenue forecast) to approximate the valuation of scaleups (private companies in a high-growth stage) and inform their investment decision. This task is manual and empirical, leaving the forecast quality heavily dependent on the investment professionals’ experiences and insights. Furthermore, financial data on scaleups is typically proprietary, costly and scarce, ruling out the wide adoption of data-driven approaches. To this end, we propose a simulation-informed revenue extrapolation (SiRE) algorithm that generates fine-grained long-term revenue predictions on small datasets and short time-series. SiRE models the revenue dynamics as a linear dynamical system (LDS), which is solved using the EM algorithm. The main innovation lies in how the noisy revenue measurements are obtained during training and inferencing. SiRE works for scaleups that operate in various sectors and provides confidence estimates. The quantitative experiments on two practical tasks show that SiRE significantly surpasses the baseline methods by a large margin. We also observe high performance when SiRE extrapolates long-term predictions from short time-series. The performance-efficiency balance and result explainability of SiRE are also validated empirically. Evaluated from the perspective of investment professionals, SiRE can precisely locate the scaleups that have a great potential return in 2 to 5 years. Furthermore, our qualitative inspection illustrates some advantageous attributes of the SiRE revenue forecasts.

CCS CONCEPTS

• Mathematics of computing → Maximum likelihood estimation; Expectation maximization; Time series analysis; Kalman filters and hidden Markov models; • Computing methodologies → Modeling methodologies; Uncertainty quantification; • General and reference → Measurement; • Applied computing → Business process management.

KEYWORDS

revenue forecast; growth company; scaleup; time series extrapolation; confidence estimation; measurement; linear dynamical system; Kalman filter; expectation maximization; simulation; private capital; investment; company valuation; accounting; financial data

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1 INTRODUCTION

Revenue is a company’s total income generated from its main business activities including sales of goods and services online and offline and is commonly regarded as one of the most important performance metrics when evaluating companies. Revenue is commonly perceived as a good approximation of the company’s worth, and its prediction is therefore widely considered an essential yet-challenging component of investment analysis. In the public markets, the availability of large amounts of data on publicly listed companies enables investors to adopt a data-driven approach to their investment analysis. In contrast, data on private companies has traditionally been difficult to gather and share due to confidentiality, while data on public companies has low relevance in the private setting. These conditions have formed a huge obstacle for directly applying data-driven methodologies to the private capital (PC) sector, especially for smaller high-growth companies such as startups and scaleups. Startups are usually in the early stages of operations with high costs and limited revenue, meaning revenue forecast is not a must-have activity during business evaluation. A startup moves into scaleup territory when proving the scalability and viability of its business model and experiencing an accelerated cycle of revenue growth. This transition is usually accompanied
by the fundraising of outside capital [6]. Consequently, revenue becomes a highly relevant metric for scaleup companies where historical values are usually available to the potential investor, even though such data remains proprietary, costly, and scarce. These conditions require PC investment professionals to manually and empirically forecast scaleup revenue taking some factors like business model, competitor landscape, market trends and go-to-market efficiency into consideration. The task is essential to evaluate the attractiveness of an investment, as it informs the change in valuation during the ownership period. However, the level of automation, objectiveness, consistency and adaptability is far from optimal.

To that end, it is highly desirable for PC investment professionals to have a data-driven method that performs scaleup revenue extrapolation on scarce data in an automated way. There are two usecases for such a method: (1) A quick way to assess a companies’ revenue potential with little information needed; (2) benchmarking generality, we assume contains monthly revenue time-series for a number of scaleups. Without loss of proprietary data forming a small dataset 

$U = \{(u_1^{(1)}, b_1^{(1)}, \bar{z}_1^{(1)}, \bar{z}_1^{(3)}) \ldots, (u_1^{(1)}, b_1^{(1)}, \bar{z}_1^{(1)}, \bar{z}_1^{(3)}) \} \ldots \{(u_1^{(3)}, b_1^{(3)}, \bar{z}_1^{(3)}, \bar{z}_1^{(3)}) \}$

(1)

The tuple $(u_1^{(i)}, b_1^{(i)}, \bar{z}_1^{(i)}, \bar{z}_1^{(i)})$ describes the revenue state for the $c$-th scaleup at the $t$-th month\(^4\), where the four elements represent the book revenue (i.e. the revenues recorded in the financial records of the company), the calendar date, the current YoY (Year-over-Year) revenue growth $z_{t+1}$, and the next YoY revenue growth $z_{t+1} = u_{t+1} / u_{t+1}$, respectively. For simplicity, we will omit “YoY” in the rest of this paper. Variable $c$ is the total number of scaleups, and $T$ is the time-series length of the $c$-th scaleup.

When a new scaleup (noted as $c’$) with a historical time-series 

$(u_t^{(c’)}, b_t^{(c’)}, \bar{z}_t^{(c’)}, \bar{z}_t^{(c’)}) \in \mathbb{Z} \cap [1, T_c’ - 1]$ \n
is evaluated, our objective is to extrapolate the revenue for $T’$ steps, obtaining $\hat{z}^{(c’)}$: 

$\hat{z}^{(c’)} = \left[ \hat{X}_{T_c’+1} \pm \hat{\beta}_{T_c’+1}, \hat{X}_{T_c’+2} \pm \hat{\beta}_{T_c’+2}, \ldots, \hat{X}_{T_c’+T’} \pm \hat{\beta}_{T_c’+T’} \right]$ (2)

where $\hat{X}_{T_c’+1}$ and $\hat{\beta}_{T_c’+T’}$ are respectively the predicted revenue mean and error margin for time step $T_c’ + t’$.

In this paper, we propose an algorithm SiRE (Simulation-informed Revenue Extrapolation) that achieves the objective defined above. Compared to the existing methods, SiRE fulfills eight important practical requirements: (1) it is sector-agnostic, which permits PC investment professionals to apply it across multiple investment cases; (2) it works on small datasets with only a few hundred scaleups; (3) the extrapolation can commence from short revenue time-series, enabling revenue predictions even without granular historical data; (4) it should produce a fine-grained time-series of at least three-year length, a typical investment period in private markets; (5) each predicted revenue point should come with a confidence estimation, providing PC investment professionals with guidance on the certainty of the outcome; (6) it does not require any alternative data other than sector information; (7) the model can be timely and effortlessly adapted to data change, making it seamless to integrate newly received data over time. (8) the prediction is explainable, promoting transparency to build trust and capture feedback. According to our thorough literature survey (cf. Section 2), this is the first work that meets all practical requirements simultaneously.

2 RELATED WORK

Revenue time-series extrapolation for scaleups shares many commonalities with general time-series forecasting methodologies. Existing methods for time-series forecasting can be roughly grouped into two categories: classical approaches serving as the backbone (e.g. [3, 23, 28]), and deep learning techniques following an encoder-decoder paradigm (e.g. [12, 17, 35, 38]). Recently, the second category has become dominant, especially the Transformer-based [31]
models such as [16, 38]. However, the general-purpose approaches usually require a lot of data to prevent overfitting, and they typically produce point prediction without confidence estimation. Moreover, they are not designed to exploit the common dynamics among companies in similar sectors, stages and so on.

**Financial time-series** describe the performance of companies from many different financial aspects such as sales, earnings, and stock price. Financial time-series forecasting is widely adopted by finance researchers in both academia and industry to benchmark the performance of companies. Many machine learning (ML) based methods (e.g. [21, 29]) have been proposed, reporting relatively better performances compared to traditional approaches using analytical (e.g. [2]) and signal processing (e.g. [7]) techniques. In recent years, deep learning (DL) based methods have become dominant due to the greatly improved availability of data and DL frameworks. Sezer et al. [24] carried out a thorough review (2005–2019) of DL-based financial time-series forecasting methods. During the past two years, the state-of-the-art DL-based methods, such as [9, 22, 33], mainly adopt either RNN (Recurrent Neural Network) or Attention-based architectures [31], obtaining superior results compared to ML-based baselines. However, the existing financial time-series forecasting methods often have one or more of the three problems: (1) dependent on alternative data beyond financial and economic sources, such as news and patent data (2) incapable of predicting long-term series, and (3) ignorant of prediction uncertainty.

**Revenue** is a specific type of financial data. An accurate forecast of future revenues relies on capturing the unique dynamics of revenue development in different business sectors. Instead of using revenue time-series as input, some trials [13, 15] only use aggregated financial and alternative information to predict the revenue of the very next period. ARIMA (Autoregressive Integrated Moving Average) [1, 3] used to be a popular choice to extrapolate revenue time-series (e.g. [10, 30]), but it is univariate and mainly tested on single sector datasets. Rahman et al. [20] attempted to produce long-term probability estimation using an exponential smoothing approach. In [11], a linear dynamical system (LDS) was used to impute and denoise the revenue signal. We have also seen some ML-based approaches such as [13, 36] that usually do not provide fine-grained predictions. The DL-based methods [18, 19, 34, 37] have started getting traction recently, illustrating strength in forecasting aggregated future revenues. Based on our literature survey to date, there has not been any work that simultaneously fulfills all practical requirements stated previously in Section 1.

### 3 THE PROPOSED APPROACH: SIRE

To perform revenue extrapolation with confidence estimate for scaleup companies using scarce time-series, we propose the SIRE approach which is visualized in Figure 1. The key components and steps will be introduced in the sections that follows.

#### 3.1 Revenue Model

We assume that any revenue observed at time $t$ (i.e. $y_t$) is made up of a continuous and twice-differential latent component $x_t$ and a noise component $\omega_t$:  
$$y_t = x_t + \omega_t,$$  
(3)

where $x_t$ is a fundamental component that captures a smooth trend of the revenue, $\omega_t$ inherits the non-differentiable external/internal noises. Inspired by [7], we can conveniently apply Taylor expansion (from time point $t$ to $t + \Delta t$):  
$$x_{t+\Delta t} = x_t + \frac{\partial x_t}{\partial t} |_{t=0} \cdot \Delta t + \frac{1}{2} \frac{\partial^2 x_t}{\partial t^2} |_{t=0} \cdot \Delta t^2 + \cdots.$$  
(4)

When the value of $\Delta t$ is close to zero, we can assume the last term in Equation (4) is approximately zero, therefore  
$$x_{t+\Delta t} \approx x_t + \frac{\partial x_t}{\partial t} |_{t=0} \cdot \Delta t + \frac{1}{2} \frac{\partial^2 x_t}{\partial t^2} |_{t=0} \cdot \Delta t^2,$$  
(5)

which essentially implies that the revenue at time $t + \Delta t$ can be largely calculated from the revenue at time $t$. For the sake of conciseness, we use $v_t$ and $a_t$ to denote $\frac{\partial x_t}{\partial t} |_{t=0}$ and $\frac{\partial^2 x_t}{\partial t^2} |_{t=0}$ respectively, and obtain  
$$x_{t+\Delta t} \approx x_t + v_t \cdot \Delta t + \frac{1}{2} a_t \cdot \Delta t^2.$$  
(6)

Likewise, variable $v_t$ can be approximated with a Taylor series up to the first degree:  
$$v_{t+\Delta t} \approx v_t + a_t \cdot \Delta t,$$  
(7)

where $d_t$ is the unit error at time $t$ which is brought into the system by the measurement process at time $t$. We assume $d_t$ largely stays the same if $\Delta t$ is sufficiently small, thus the unit error could be scaled by $\Delta t$ and added to the latent component, yielding the measured revenue. If the same approach is utilized for measuring revenue at any time point, $d_t$ could be regarded as a relatively static term:  
$$d_{t+\Delta t} \approx d_t.$$  
(10)

Since revenues are usually measured periodically (e.g. monthly and quarterly), we choose to discretize equations (6) to (10) by setting $\Delta t=1$, obtaining a linear dynamical system (LDS):  
$$x_{t+1} \approx Ax_t \text{ and } y_t \approx cx_t,$$  
(11)

where $x_t = [y_t, x_t, v_t, a_t, d_t]^T$.  

Vector $x_t$ is the latent state vector for the $t$-th time step. Vector $c$ is responsible for transforming the latent state to the measurable revenue, hence $c$ is called measurement vector. Matrix $A$ transforms the current latent state to the next one. According to equations (6) to (10), matrices $A$ and $c$ should take the values of  
$$A = \begin{bmatrix} 0 & 1 & 1/2 & 1 \\ 0 & 0 & 1/2 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } c = [1, 0, 0, 0].$$  
(12)

To make the right-hand-side terms exactly equivalent to the left-hand-side ones in equations (11), we add time-dependent noise terms $\omega_t$ and $\epsilon_t$ to both equations, getting a more general form of  
$$x_{t+1} = Ax_t + \omega_t \text{ and } y_t = cx_t + \epsilon_t,$$  
(13)
where $\omega_t$ and $\epsilon_t$ are zero-mean normally-distributed random variables with covariance matrices $Q$ and $R$, i.e., $\omega_t \sim N(0, Q)$ and $\epsilon_t \sim N(0, R)$. As a result, the company revenue can be modeled as a linear time-variant dynamical system, also known as linear Gaussian state-space models. The measured revenue $y_t$ is a linear function of the state, $x_t$, and the state at any time step depends linearly on the previous state.

### 3.2 Model Optimization

The revenue model unfolds recursively from the initial state $x_1$, which is a normal random vector with mean vector $\mu \in \mathbb{R}^3$ and a $5 \times 5$ covariance matrix $\Omega$. The optimization parameters of the model are $Q, R, \mu,$ and $\Omega$. The goal is to maximize $\mathcal{D} = \mathbb{E} \log p((x_t, \{y_t\}) | \{y\})$, which is the joint log likelihood of $(y_t = (y_1, y_2, \ldots, y_T) \text{ and } x_t = (x_1, x_2, \ldots, x_T),$ conditioned on the observable $\{y\}$:

$$
\mathcal{D} = \mathbb{E} \log p((x_t, \{y_t\}) | \{y\}) = \frac{1}{2} \log |\mathbf{R}| + \frac{1}{2} \sum_{t=1}^{T} \left[ (y_t - \mathbf{cx}_t^T)^2 + \mathbf{c}[\mathbf{x}_t^T - \mathbf{P}_t^2] \mathbf{c}^T \right] - \frac{1}{2} \log |\mathbf{Q}| - \frac{1}{2} \mathbf{c}[\mathbf{P}_t^2 + \mathbf{x}_t^T - \mu] (\mathbf{x}_t^T - \mu)^T - \frac{1}{2} \sum_{t=1}^{T} \log |\mathbf{Q}|.
$$

(14)

where $E = \sum_{t=2}^{T} \left[ \mathbf{P}_{t-1}^2 + \mathbf{x}_{t-1}^T (\mathbf{x}_{t-1})^T \right]$, $G = \sum_{t=1}^{T} \left[ \mathbf{P}_t^2 + \mathbf{x}_t^T (\mathbf{x}_t)^T \right]$ and $F = \sum_{t=1}^{T} \left[ \mathbf{P}_{t-1}^2 + \mathbf{x}_{t-1}^T (\mathbf{x}_{t-1})^T \right]$.

The operation $\mathbf{Tr}(\cdot)$ denotes the trace calculation, $(y_t)^{t_n}$ represents a sub-sequence of $\{y_t\}$, i.e. $(y_t)^{t_n} = (y_{t_0}, y_{t_1}, \ldots, y_{t_n})$, and $\mathbf{x}_t^2$ defines the conditional mean $\mathbb{E}(x_t | \{y_t\})$. The terms $\mathbf{P}_t^2$ and $\mathbf{P}_{t-1}^2$ define the covariances $\operatorname{Cov}(x_t | \{y_t\})$ and $\operatorname{Cov}(x_{t-1} | \{y_t\})$, respectively. The derivation of (14) can be found in Appendix A of [5]. To maximize (14), we adopt the EM (Expectation Maximization) algorithm [4, 8] explained in Appendix B of [5]. The convergence properties of EM are discussed in [32].

### 3.3 Revenue Measurements

Obtaining measurements of the revenue signal is crucial (cf. formulations in 3.2). One might argue to simply use the data from financial bookings as the measurement of revenue, but this type of revenue measurement is not available in the future. Appendix C in [5] conducts an extensive discussion on why booked revenue should not be directly used as measurement. We propose a simulation-informed approach, illustrated in Figure 2, that enables measuring the revenue in both past and future time horizons. Specifically, to measure the revenue for a company-in-focus $c' \in [1, C]$ at time point $t' + 1$, we assume the approximated revenue signal from the previous time point $t' > 0$ is known: the book revenue $u_{(c')_{t'_+1}}$ (for the past known period: left part of Figure 2) or latent state $x_{(c')_{t'_+1}}$ (for the future unknown period: right part of Figure 2). In this section, we mainly focus on measuring revenues in the past, while the measurement process for the future will be detailed in Section 3.5. The objective here is to measure the noisy value of the current revenue $u_{(c')_{t'_+}}$. The overall approach is to (1) construct a measuring dataset

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2EM algorithm is used due to its universality and popularity. There are analytical solutions (e.g. [14]) that may give more stable parameter estimations.
\( \gamma(t) \) values that appear in \( U_t^{(c)} \):

\[
\tilde{Z} = \left\{ y_t^{(c)} \mid \exists \left( u_t^{(c)}, b_t^{(c)}, z_t^{(c)}, c_t^{(c)} \right) \in U_t^{(c)} \right\}.
\]

(16)

We divide \( \tilde{Z} \) evenly into \( n \) quantiles, obtaining \( n+1 \) quantile boundaries \( q_0, q_1, \ldots, q_n \), where \( q_0 = \min(\tilde{Z}) \) and \( q_n = \max(\tilde{Z}) \). Assume \( z_t^{(c)} \) falls in the \( k \)-th quantile, i.e. \( z_t^{(c)} \in (q_{k-1}, q_k) \), we gather the tuples (from set \( U_t^{(c)} \)) with \( z_t^{(c)} \) in the same range, forming the measuring dataset \( \tilde{U} \):

\[
\tilde{U} = \left\{ \left( u_t^{(c)}, b_t^{(c)}, z_t^{(c)}, x_t^{(c)} \right) \mid \left( u_t^{(c)}, b_t^{(c)}, z_t^{(c)}, x_t^{(c)} \right) \in U_t^{(c)} \wedge z_t^{(c)} \in (q_{k-1}, q_k) \right\}
\]

(17)

s.t. \( z_t^{(c)} \in [q_{k-1}, q_k], k \in \mathbb{Z} \cap [1, n] \).

where the tuples \((u_t^{(c)}, b_t^{(c)}, z_t^{(c)}, x_t^{(c)})\) can be viewed as known benchmarking snapshots of revenue dynamics that are comparable to the scale-up-in-focus \( c' \) at date \( b_t^{(c')} \). The total number of quantiles \( n \) needs to be searched, and we empirically find that \( n = 4 \) gives reasonable revenue measurements.

3.3.2 Measure revenue by sampling a growth. Recall that we want to measure the revenue \( y_t^{(c)} \), which is equivalent to calculating the revenue growth \( z_t^{(c)} \) since \( y_t^{(c)} = u_t^{(c)} \cdot z_t^{(c)} \). (1/12) where \( u_t^{(c)} \) is the known previous revenue. Because \( \tilde{U} \) contains similar states to company \( c' \) at time \( t' \), we may sample the last element (i.e. \( z_t^{(c)} \)) of tuples from \( U_t^{(c)} \). As a result, we extract all \( z_t^{(c)} \) from \( \tilde{U} \):

\[
\tilde{Z} = \left\{ z_t^{(c)} \mid \exists \left( u_t^{(c)}, b_t^{(c)}, z_t^{(c)}, x_t^{(c)} \right) \in \tilde{U} \right\},
\]

(18)

where \( \tilde{Z} \) contains the future revenue growth rates that once occurred to scaleups in a state comparable to scaleup \( c' \) at time \( t' \). Now, we sample one value from the underlying distribution of \( Z \) using a two-step procedure:

first \( \tilde{z} \sim U(\tilde{Z}) \), then \( \tilde{z} \sim N\left( \tilde{z}, \frac{\sigma(Z)^2}{|Z|} \right) \),

(19)

where \( |Z| \) denotes the number of elements in set \( Z \) and \( \sigma(\tilde{Z}) \) represents the standard deviation over \( \tilde{Z} \). Equation (19) first samples a value \( \tilde{z} \) uniformly from set \( Z \), then it samples a value \( \tilde{z} \) from a \( \tilde{z} \)-mean Normal distribution with a variance calculated following the Silverman’s rule of thumb [26]. For revenue growth definition, we approximate \( y_t^{(c)} \) with

\[
y_t^{(c)} = u_t^{(c)} \cdot \tilde{z}.
\]

(20)

3.4 Parameter Initialization

As discussed previously, the parameters that need to be optimized are \( Q, R, \mu \), and \( \Omega \), hence we need to assign reasonable initial values to them. For \( Q, R, \) and \( \Omega \), we simply set

\[
Q = \Omega = I_5 \quad \text{and} \quad R = I_1.
\]

(21)

For any company, we know its booked revenue \( u_0, u_1, \ldots, u_T \). Following the measuring procedure described in Section 3.3, we can obtain \( y_1, y_2, \ldots, y_T, y_{r+1} \), and for any \( t \in \mathbb{Z} \cap [1, T] \) we define \( d_t = y_t - u_t \), obtaining \( d_1, d_2, \ldots, d_T \). Recalling that \( \mu \) is the mean vector of the initial state \( x_1 = [y_1, x_1, a_1, d_1]^T \), we can intuitively define \( \mu \) as

\[
\mu = \begin{bmatrix} u_0 + \frac{1}{T} \sum_{i=1}^{T} d_t \\ (u_1 - u_0)/\Delta t \\ ((u_2 - u_1) - (u_1 - u_0))/(2\Delta t) \\ 1/\Delta T \sum_{i=1}^{T} d_t \end{bmatrix}, \quad \text{where} \ \Delta t = 1.
\]

(22)

3.5 Revenue Extrapolation

After we have trained our revenue model (Section 3.1) with historical measurements (Section 3.3) and initial parameters (Section 3.4) for the \( c' \)-th company, we can start extrapolating the revenue into the future using the measurements \( y_t^{(c')}, \) where \( T_c' \) is the total number of available book revenues for company \( c' \). Forecasting \( T' \) revenues into the future is equivalent to estimating \( x_t^{(c')} \), where \( t \in \mathbb{Z} \cap [T_c' + 1, T_c' + T'] \). Notice that we already calculated \( y_t^{(c')} \) in Section 3.3, so the first predicted revenue \( x_t^{(c')} \), can be extracted from the vector of \( x_t^{(c')} \), i.e. \( x_t^{(c')} \) obtained via forward filtering following Equation (38) in Appendix B of [5]. Afterwards, we repeat a two-step operation (i.e. 3.5.1 measuring and 3.5.2 filtering) for \( T' - 1 \) times so that we obtain \( T' \) predicted revenues. Figure 2 gives an illustration of this process. In the end, we apply a global smoothing (Section 3.5.3) to nudge the revenues using all measurements (historical and future).

3.5.1 The measuring step. When extrapolating for scaleup company \( c' \), we know the filtered revenues \( x_t^{(c')} \) but not the booked ones \( u_t^{(c')} \). Therefore, as shown in the right part of Figure 2, we use \( x_t^{(c')} \) as an approximation of \( u_t^{(c')} \). At the \( t+1 \) prediction step, we know the values of \( x_t^{(c')} \), and the corresponding revenue growth \( z_t^{(c')} \) can be simply estimated following their definition:

\[
z_t^{(c')} = \begin{cases} x_t^{(c')}/x_{t-12}^{(c')} & \text{if } u_t^{(c')} \text{ is not available} \\ x_t^{(c')}/u_{t-12}^{(c')} & \text{if } u_t^{(c')} \text{ is available} \end{cases},
\]

(23)

where \( t \in \mathbb{Z} \cap [T_c' + 1, T_c' + T'] \).

The measuring operation aims at obtaining \( y_t^{(c')} \) using an approach described in Section 3.3 with a few minor differences. Like Equation (15), we first obtain set \( U_t^{(c')} \):

\[
U_t^{(c')} := \left\{ \left( u_t^{(c)}, b_t^{(c)}, x_t^{(c)}, z_t^{(c)}, c_t^{(c)} \right) \mid \left( u_t^{(c)}, b_t^{(c)}, x_t^{(c)}, z_t^{(c)}, c_t^{(c)} \right) \in U_t^{(c)} \wedge z_t^{(c)} \in (q_{k-1}, q_k) \right\}
\]

(24)

s.t. \( z_t^{(c')} \in [q_{k-1}, q_k], k \in \mathbb{Z} \cap [1, n] \).

where parameter \( r \) should use exactly the same value as in Equation (17). Thereafter, we sample a revenue growth \( \tilde{z} \) by strictly following Equation (18) and (19). Finally, we compute \( y_t^{(c')} \) with

\[
y_t^{(c')} = x_t^{(c')}/\tilde{z}, \quad \text{where } t \in \mathbb{Z} \cap [T_c' + 1, T_c' + T' - 1].
\]

(26)
We summarize the procedure of generating revenue forecasts, with minor changes. Notations $y_{t+1}, \tilde{x}_{t+1}^{c}, \tilde{x}_{t+1}^{c'}$, and $\tilde{x}_{t+1}^{c''}$ are respectively equivalent to $y_{t+1}, \tilde{x}_{t}^{c'}, \tilde{x}_{t}^{c''}$ and $\tilde{x}_{t}^{c''}$ when the company identifier $c'$ is neglected. By definition, we can take the second element (denoted $\tilde{x}_{t+1}^{c''}$) from $\tilde{x}_{t+1}^{c'}$ in order to calculate $\tilde{x}_{t+1}^{c''}$ when starting to measure the next revenue in Equation (23).

3.5.3 The global smoothing. We iterate the above measuring and filtering steps $T' - 1$ times, obtaining $x_{t+1}^{c''}, p_{t+1}^{c'}$, and $p_{t+1}^{c''}$, where $t \in Z \cap [T_{c'}+1, T_{c'}+T'-1]$. In the beginning of forecasting, we obtained $\tilde{x}_{t+1}^{c''}, \tilde{x}_{t+1}^{c'}$, and $\tilde{x}_{t+1}^{c''}$. When the model was optimized in Section 3.2 (cf. Appendix B in [5]), we also obtained $x_{t}, p_{t-1}^{c'}$, and $p_{t}^{c''}$, where $t \in Z \cap [1, T_{c'}]$. To unify, if we define $t \in Z \cap [1, T_{c'}+T']$, then we know all the values of $x_{t}^{c'}, p_{t}^{c'}$, and $p_{t}^{c''}$. It is viable to smooth our predictions using all measurements so that the resulting revenue predictions are correlated better among one another. Concretely, we apply the following recursively from $i=T_{c'}+T'$ all the way till $i = T_{c'}+1$:

$$J_{i-1} = p_{i-1}^{c''}A^T(p_{i-1}^{c''})^{-1},$$

$$x_{i-1}^{c''} = x_{i-1}^{c''} + J_{i-1}(x_{i}^{c''} - Ax_{i-1}^{c''}),$$

$$p_{i}^{c''} = p_{i-1}^{c''} + J_{i-1}(p_{i}^{c''} - p_{i-1}^{c''})J_{i-1}^{-1}.$$  

To this point, we manage to obtain $x_{T_{c'}+1}^{c''}, x_{T_{c'}+2}^{c''}, \ldots, x_{T_{c'}+T'}^{c''}$. According to the definition in Equation (11), the final revenue forecasts $x_{T_{c'}+1}^{c''}, x_{T_{c'}+2}^{c''}, \ldots, x_{T_{c'}+T'}^{c''}$ (i.e., $x_{T_{c'}+1}, x_{T_{c'}+2}, \ldots, x_{T_{c'}+T'}$) are simply the two elements in those corresponding state vectors:

$$[x_{T_{c'}+1}, \ldots, x_{T_{c'}+T'}] = \left[\begin{array}{c} x_{T_{c'}+1}^{c''} \\ \vdots \\ x_{T_{c'}+T'}^{c''} \end{array}\right] = \left[\begin{array}{c} \hat{x}_{T_{c'}+1}^{c'} \\ \vdots \\ \hat{x}_{T_{c'}+T'}^{c'} \end{array}\right].$$  

where operation $(\cdot)_1$ returns the second element (with index starting from zero) from a vector.

3.6 Confidence Estimation

We summarize the procedure of generating $T'$ revenues for company $c'$ in Algorithm 1 of [5], which outputs a $T'$-dimensional vector $[x_{T_{c'}+1}, x_{T_{c'}+2}, \ldots, x_{T_{c'}+T'}]$ representing one possible revenue trajectory. Because of the stochasticity of our prediction procedure, we might get a slightly different revenue trajectory if we run the same algorithm again. As illustrated in Figure 3, this difference is insignificant (i.e., for $x_{T_{c'}+1}$), but it gets accumulated and amplified along with the extrapolation roll-out. Intuitively, the uncertainty of the predicted revenue is reflected in these differences. In order to capture that uncertainty in the predicted revenue trajectory, we choose to carry out $M \geq 10$ trials of revenue forecasting (i.e., Algorithm 1 in [5]) with random seeds. It produces $M$ extrapolated revenue trajectories (for scaleup company $c'$), organized in a matrix $\hat{X}(c')$:

$$\hat{X}(c') = \left[\begin{array}{cccc} x_{T_{c'}+1}^{(1)} & x_{T_{c'}+2}^{(1)} & \cdots & x_{T_{c'}+T'}^{(1)} \\ x_{T_{c'}+1}^{(2)} & x_{T_{c'}+2}^{(2)} & \cdots & x_{T_{c'}+T'}^{(2)} \\ \vdots & \vdots & \ddots & \vdots \\ x_{T_{c'}+1}^{M} & x_{T_{c'}+2}^{M} & \cdots & x_{T_{c'}+T'}^{M} \end{array}\right].$$  

where $m$-th row $X_{(m,c')} = (x_{T_{c'}+1}^{(m)}, x_{T_{c'}+2}^{(m)}, \ldots, x_{T_{c'}+T'}^{(m)})$ represents the $m$-th predicted revenue trajectory, and the $t$-th column $x_{T_{c'}+t}^{(j)} = (x_{T_{c'}+1}^{(j)}, x_{T_{c'}+2}^{(j)}, \ldots, x_{T_{c'}+T'}^{(j)})^T$ contains a collection of predicted revenues for time point $T_{c'}+t$ representing a revenue distribution for that period, as illustrated in Figure 3. The expected revenue at $T_{c'}+t'$ is the mean of $\hat{X}(c')$, noted as $\hat{x}_{T_{c'}+t'}$. To capture uncertainty, we calculate the 95% confidence interval (CI) $\hat{x}_{T_{c'}+t'} \pm \beta_{T_{c'}+t'}$ over $\hat{X}(c')$ assuming a normal distribution:

$$\hat{x}_{T_{c'}+t'} \pm \beta_{T_{c'}+t'} = 2 \cdot \frac{\gamma_{M-1} \left( x_{T_{c'}+t'} - \hat{x}_{T_{c'}+t'} \right)^2}{M(M-1)},$$  

where $\beta_{T_{c'}+t'}$ is the margin of error for the revenue prediction at time $T_{c'}+t'$. The constant $\gamma \approx 1.96$ is the Z value corresponding to a 95% confidence interval. Similarly, we should set $\gamma \approx 1.645$ if we look for the 90% confidence interval. Therefore, the prediction for the $c'$-th company $\hat{X}(c')$ is noted as $\hat{X}(c')$ in the form of (2).

Algorithm 2 in the Appendix of [5] contains the pseudo code to produce the final resulting vector $\hat{x}(c')$.

3.7 Complexity and Limitation

Algorithm 1 in [5]3 is the pseudo code to output a $T'$-dimensional vector $[x_{T_{c'}+1}, x_{T_{c'}+2}, \ldots, x_{T_{c'}+T'}]$, formalized in Equation (29), representing one possible revenue trajectory. The computational complexity of Algorithm 1 is approximately $O(C \times T_{c'}^2)$. Algorithm 2 describes the logic to produce the final resulting vector $\hat{x}(c')$ formalized in Equation (2). To generate the confidence estimates, Algorithm 2 runs Algorithm 1 $M$ times, followed by an $O(T')$ confidence estimation step. Since $M$ is a constant and $O(T')$ has a lower order than $O(C \times T_{c'}^2)$, the overall computational complexity of SIRE is
still $O(C \times T^2)$. Figure 1 depicts the overall SiRE approach outlining Algorithm 1 and 2, where it only requires revenue and sector information, thus fulfilling the 6th practical requirement in Section 1.

The standards (e.g. IFRS and GAAP) that govern financial reporting and accounting vary from country to country. SiRE implicitly assumes consistency of accounting standards, because how revenue is recognized is likely going to contribute to the applicability of SiRE. To that end, we recommend readers to ensure the consistency of the accounting standard adopted when creating the dataset. Finally, the authors would also like to point out that recurring revenue is not the only way to derive valuations for scaleups, rather a key aspect for many business models such as a subscription based one.

### 4 EXPERIMENTS

The revenue time-series of scaleups are usually considered sensitive and typically not shared externally. Therefore, there has not been any public dataset available for benchmarking SiRE. We have the advantage to gain access to such data from EQT Group\(^6\) and Standard & Poor’s (S&P) Capital IQ\(^6\), forming two multi-sector datasets ARR129 and SapiQ respectively. ARR129 contains 1,485 monthly ARR (Annual Recurring Revenue) data points from 129 SaaS (Software as a Service) companies in growth stage. SapiQ comprises 766 yearly revenues from 158 scaleups headquartered in North America or Europe. ARR129 and SapiQ represent the common kind of revenue data possessed by PC firms. For parameter \(r\) in Equations (15) and (24), we search \(\{1, 2, 3, 4\}\) and empirically discover that \(r = 3\) turns out to be a balanced choice. In confidence estimation (Section 3.6), we use of \(M = 10\) for fast inference. Because the scale of the datasets are small, we carry out the experiments on a single machine: Intel Quad-Core i7, 1.7-GHz CPU with 16-GB, 2133-MHz RAM.

#### 4.1 Quantitative Benchmarking

We compare SiRE with five baseline methods that are either state-of-the-art in time-series prediction or widely adopted in revenue forecasting: ARIMA [1, 3], Prophet [28], LSTM [12], DeepAR [22], and Informer [38]. During evaluation, it is difficult to use classical cross validation because the observations are scarce and not exchangeable. We largely follow [27, 28] to perform a “rolling origin” evaluation procedure by producing one extrapolation trajectory at each predefined cutoff point along the timeline. Since the feasible cutoff points and the length of the extrapolated trajectory can vary among different methods, we only include the common cutoff points and trajectories in metrics calculations. Since ARR129 and SapiQ are monthly and yearly datasets, we extrapolate 36 months and 12 years respectively. For yearly data, Equation (20) becomes \(y_{t+1} = w_{t+1} \cdot z\). For LSTM and Informer, a confidence estimate is not available, hence we apply dropout during both training and inference to get 10 samples for each step. For monthly data, we report results from 10 samples for each extrapolated step to approximate probability distributions. Dropout can be viewed as a form of ensemble and regularization, which might have contradicting effect: ensemble boosts the performance while regularization might make the convergence harder on small datasets. We adopt three metrics (cf. [9, 22, 28]) computed using the mean prediction:

- root-mean-square error $\text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x - \hat{y}_i)^2}$
- mean absolute percentage error $\text{MAPE} = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{x - \hat{y}_i}{x} \right|$
- Pearson correlation coefficient (PCC) that measures whether the actual ($u$) and predicted ($\hat{u}$) time-series move in the same (+1) or opposite direction (-1).

We also compute two metrics (cf. [36]) using the prediction distribution (the confidence is estimated from this distribution):

- negative log-likelihood $\text{NLL} = -\frac{1}{n} \sum_{i=1}^{n} \log \hat{p}_u(y|x)$, where $\hat{p}_u$ is the probability density function.
- 95% CI accuracy (ACC) indicating if the booked revenue $u$ lies in the predicted 95% CI.

In Table 1, the best results are emphasized in bold format. On both datasets, SiRE performs better than all other methods by a large margin over all five metrics. Despite of the simplicity and popularity of statistical methods (ARIMA and Prophet), they suffered severely from ignoring the time-series of other scaleups in similar sectors and stages, obtaining the worst result. The RNN based approaches (LSTM and DeepAR) are greatly relieved from that problem by learning from all time-series up till the cutoff date, coinciding the findings in [25]. The Transformer based method, Informer, achieves the second best performance of all probably due to its effective attention mechanism. These experiments confirm that SiRE meets the 1st and 2nd practical requirements mentioned in Section 1.

#### 4.2 Qualitative Inspection

In Figure 4, we visualize the extrapolated revenue trajectories for the same scaleup as a qualitative example. Besides SiRE, the inspected methods are ARIMA, DeepAR and Informer, which represent statistical, RNN-based and Transformer-based methodologies respectively. As a result, we will mainly report results from those methods in the rest of this paper. Seen from the predicted mean

| Metrics: | ARR129 | SapiQ | ARR129 | SapiQ | ARR129 | SapiQ | ARR129 | SapiQ | ARR129 | SapiQ | ARR129 | SapiQ |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Dataset: | | | | | | | | | | | | |
| RMSE | 9.6917 | 57.8620 | 0.0480 | 0.6571 | 0.8284 | 0.6049 | 7.0578 | 8.5866 | 0.7102 | 0.3539 |
| MAPE | 15.4210 | 10.4210 | 0.5233 | 0.3203 | 0.5721 | 0.4544 | 11.0396 | 10.4210 | 0.4985 | 0.3407 |
| ACC | 9.5108 | 0.7102 | 0.5539 | 0.8630 | 0.5621 | 0.5091 | 10.1357 | 10.6687 | 0.6238 | 0.4009 |
| NLL | 26.9251 | 88.0435 | 0.1894 | 0.9504 | 0.5721 | 0.4544 | 11.0396 | 10.4210 | 0.6300 | 0.4095 |

\(^1\) Since probability distributions can not be directly inferred from LSTM or Informer models, we use dropout during both training and inference to get 10 samples for each step.

\(^2\) To enable a fair comparison, the information on sector and customer focus is one-hot encoded and concatenated with the decoder output.
We calculate five metrics (cf. Table 1 and 2) using the predicted revenue data that has an average span of 12 months (i.e. 12 data points). The ability of obtaining reasonable revenue forecasts from short time-series is valued by investment professionals. In Table 2, we measure the quality of extrapolations based on time-series with only three (e.g. “RMSE@3m”) and six (e.g. “RMSE@6m”) data points. Because ARR129 contains monthly revenue data that has an average span of 12 months (i.e. 12 data points), ARR129 is better suited for this evaluation. Table 2 shows that SiRE performs the best (indicated in boldface) in most of the cases, thereby satisfying the 3rd practical requirement in Section 1; (4) the 95% CI starts much narrower and grows much slower than the compared methods.

### 4.3 Performance on Short-Time Series

According to (22), SiRE can start generating reasonable predictions using as few as three data points. The ability of obtaining reasonable revenue forecasts from short time-series is valued by investment professionals. In Table 2, we measure the quality of extrapolations based on time-series with only three (e.g. “RMSE@3m”) and six (e.g. “RMSE@6m”) data points. Because ARR129 contains monthly revenue data that has an average span of 12 months (i.e. 12 data points), ARR129 is better suited for this evaluation. Table 2 shows that SiRE performs the best (indicated in boldface) in most of the cases, thereby satisfying the 3rd practical requirement in Section 1. Compared to the overall results in Table 1, all methods suffered greatly when dealing with short time-series, and their performance generally improves when the length of the time-series increases.

### 4.4 Performance on Long-Term Forecast

Accurate long-term revenue forecast is one of the key requirements when evaluating a scaleup prior to making an investment decision. To benchmark SiRE’s performance on predicting long-term revenue, we calculate five metrics (cf. Table 1 and 2) using the predicted revenue for either the 2nd-to-3rd (e.g. “RMSE@2-3y”) or the 4th-to-5th (e.g. “RMSE@4-5y”) year in the future. As shown in Table 3, we report the results on the SapiQ dataset, where each scaleup has five annual revenue points on average. Because it is not viable to calculate PCC using only one or two predicted samples, we exclude PCC from the comparison. Generally speaking, the further into the future, the more challenging the prediction becomes. Nonetheless, SiRE achieves the best performance (in both cases of “>2-3y” and “>4-5y”) when predicting long-term revenue. This fulfills the 4th practical requirement in Section 1.

### 4.5 On Training and Inference Efficiency

Practically, the revenue dataset $U$ [cf. Equation (1)] changes frequently because of (1) incoming scaleup companies, (2) new revenue data points for existing scaleups, and (3) removal of incorrect or outdated data. The model hence needs to adapt to any data change timely and efficiently, which fulfills the 7th practical requirement mentioned in Section 1. To evaluate this, we measure the training and inference time for SiRE, ARIMA, DeepAR and Informer on both datasets. Table 4 shows that SiRE is largely on par with statistical methods like ARIMA, and more efficient than DL-based models, by a large margin. Specifically, SiRE and ARIMA can incorporate data change every second while other models need at least 5 minutes. Moreover, the superior performance of SiRE compared to ARIMA
**4.6 On Explainability of SiRE Forecasts**

When predicting each revenue point, our measuring approach puts together a measuring dataset \( \hat{U} \) [cf. Equation (25)] containing fragments of revenue time-series from other scaleups, which can provide direct explainability to SiRE results. Figure 5 demonstrates how one can interactively interpret the extrapolated revenue via our investment platform (Motherbrain)

**Figure 5: Explaining the extrapolated revenue from SiRE: a screenshot of our investment platform (EQT Motherbrain).**

(c.f. Tables 1-3) makes SiRE the best methodological choice for the benchmarking datasets.

**Table 4: The average training and inference time (measured in seconds) required by different methods on two datasets.**

| Methods       | SiRE         | ARIMA†       | DeepAR      | Informer     |
|---------------|--------------|--------------|-------------|--------------|
| ARR129    | Training (sec.) | 1.180        | 0.896       | 294.460      | 467.412      |
| (1,485)†   | Inference (sec.) | 1.206        | 0.763       | 1.139        | 1.439        |
| SapiQ      | Training (sec.) | 0.958        | 0.804       | 243.392      | 420.340      |
| (766)‡     | Inference (sec.) | 1.054        | 0.682       | 1.094        | 1.277        |

* The current implementation of SiRE does not strive for high performance.
† ARIMA training does not require traversing other scaleup companies.
‡ The number of revenue data points in each dataset.

**Table 5: The evaluation results from the perspective of investment professionals.**

| Methods                  | SiRE         | ARIMA†       | DeepAR      | Informer     |
|--------------------------|--------------|--------------|-------------|--------------|
| ARR129‡                  | TPR>2x_in_2/3y | 0.6938       | 0.5560      | 0.5904       | 0.5875       |
|                          | TPR>3x_in_2/3y | 0.6809       | 0.5100      | 0.6060       | 0.5723       |
| SapiQ                    | TPR>2x_in_2/3y | 0.5402       | 0.3903      | 0.4516       | 0.4126       |
|                          | TPR>3x_in_2/3y | 0.5402       | 0.3729      | 0.4040       | 0.3890       |
|                          | TPR>3x_in_4/5y | 0.5537       | 0.3505      | 0.3830       | 0.4057       |
|                          | TPR>4x_in_4/5y | 0.5290       | 0.3511      | 0.3714       | 0.3965       |

E.g. TPR>2x_in_2/3y is the true positive rate of scaleups that reach 2x revenue after 2 or 3 years.

4.7 Evaluation from the perspective of investment professionals

From the perspective of PC investment professionals, it is not critical to have each extrapolated revenue point close to the ground truth. To make an informed investment decision, it is of great importance to understand if the revenue will change significantly several years down the line. For instance, if the investment target demands that the revenue should be at least 3 times the value at the time of investment (so called 3x) after 4 or 5 years, then the model should be able to identify such scaleup companies (noted as “>3x_in_4/5y”) as accurately as possible. This is equivalent to computing the true positive rate (TPR) of “>3x_in_4/5y” cases. We test several such cases on both datasets, and present the results in Table 5. Although SiRE displays the best performance from the investors’ point of view, its TPR scores struggle to surpass 60% for SapiQ and 70% for ARR129. Those results might improve when incorporating more context features such as country and size of the company.

5 CONCLUSION

In this work, we propose the SiRE algorithm to unlock the possibility of automating long-term fine-grained revenue extrapolation using scarce data. SiRE models the revenue dynamics as a specific LDS which is solved using the EM algorithm, where the core innovation lies in how we choose to obtain noisy revenue measurements. By design, it works for scaleups that operate in various sectors and provides confidence estimates. The quantitative experiments on two datasets show that SiRE significantly surpasses the popular and state-of-the-art baselines on all carefully selected metrics. The qualitative analysis illustrates some advantageous attributes of SiRE, such as imputation of missing data, smoothness of trajectories and explainability of predictions. We also observed great performance when SiRE extrapolates long-term predictions from short time-series. SiRE is an agile algorithm that, due to its high training and inference efficiency, adapts effortlessly to data change. For investment professionals, SiRE can precisely find scaleup companies that have great potential of a revenue uplift in 2 to 5 years.

Future work includes studies on filter ablation and applicability to other relevant metrics such as customer churn, life-time value and conversion rate.

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