Root square mean labelling of some graphs obtained from path

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Abstract. Let $G$ be a graph with $q$ edges. A labelling $f$ of $G$ is said to be root square mean labelling if $f : V(G) \cup E(G) \rightarrow \{1, 2, \ldots, q + 1\}$ such that when each edge $e = uv$ is labelled with $f(e) = \sqrt{\frac{f(u)^2 + f(v)^2}{2}}$ or $f(e) = \sqrt{\frac{f(u)^2 + f(v)^2}{2}}$, then the resulting edge labels are distinct. A graph $G$ is called a root square mean graph if $G$ can be labelled by a root square mean labelling. In this paper we determine a root mean square labelling of two graphs obtained from path, which are corona product of ladder and complete graph with order 1, and a graph obtained from triangular snake by join one vertex with degree 2 in each triangle to a new vertex. The method of labelling construction is we need to do labeling to the vertices of the graph with label 1, 2, 3, …, $q+1$. The labels of the vertices are not necessarily different. The next step is we need to do labeling to the edges with the certain formula by using the vertex labeling. The edge labels must be different. By the labelling we construct, we prove that the two graphs are root square mean graphs.

1. Introduction

Root square mean labelling was introduced by Sandhya et al. [1]. A labelling $f$ of a graph $G$ with $q$ edges is said to be root square mean labelling if $f : V(G) \cup E(G) \rightarrow \{1, 2, \ldots, q + 1\}$ such that when each edge $e = uv$ is labeled with $f(e) = \sqrt{\frac{f(u)^2 + f(v)^2}{2}}$ or $f(e) = \sqrt{\frac{f(u)^2 + f(v)^2}{2}}$, then the resulting edge labels are distinct. A graph $G$ is called a root square mean graph if it is possible to label the vertices of $G$ by root square mean labeling. In the same paper, they prove that path, cycle, comb, ladder, triangular snake, quadrilateral snake, star, and complete graph, are root square mean graphs. Other results about root square mean labelling was given by Sandhya, Somasundaram, and Anusa [2]. Sandhya continue the before research about root square mean graph and gave some new disconnected root square mean graphs, one of them is $C_m \cup (P_n \odot K_1)$ [3]. Sandhya et al., gave some root square mean graphs, among others are kite graphs, crown graphs, and a graph obtained by attaching a pendent edge to both sides of each vertex of a path $P_n$ [4]. Given root square mean labelling of new crown graphs [5].

Another variation of root square mean graph is super-root square mean labelling. This labelling was introduce by Thirugnanasambandam and Venkatesan [6].
Let $G$ be a graph with $p$ vertices and $q$ edges. If $f: V(G) \rightarrow \{1,2,\ldots,p+q\}$ is an injective function. For a vertex labelling $f$, the induced edge labelling $f^*(e = uv)$ is defined by $f^*(uv) = \left\lfloor \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rfloor$ or $f^*(uv) = \left\lfloor \sqrt{\frac{f(u)^3 + f(v)^3}{2}} \right\rfloor$ then $f$ is called a super root square mean if $f: V(G) \cup \{f(e) / e \in E(G)\} \rightarrow \{1,2,\ldots,p+q\}$. A graph which admits super root square mean labelling is called super root square mean graph. In this paper, we investigate super root square mean labelling of some graphs.

Gopi proved that the slanting ladder is a super root square mean graph [7]. Sandhya proved that double triangular snake, alternate double triangular snake, double quadrilateral snake and alternate double quadrilateral snake graphs are super root square mean graphs [8]. Devi and Kumar proved that $m$ copies of path $P_n$, some copies of complete graph, corona product of path and complement of complete graph $K_2$, middle graph of path, and dragon graph are super root square mean graphs [9].

Gowri and Vembarasi introduced root cube mean labelling of graphs [10]. A graph $G = (V, E)$ with $p$ vertices and $q$ edges is said to be a root cube mean graph if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from $1,2,\ldots,q+1$ in such a way that when each edge $e = uv$ is labelled with $f(e = uv) = \left\lfloor \sqrt{\frac{f(u)^3 + f(v)^3}{2}} \right\rfloor$ or $f(uv) = \left\lfloor \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rfloor$, then the resulting edge labels are distinct. Here $f$ is called a root cube mean labelling of $G$.

Next, we will write the definition of some graphs used in this research.

Path with order $n$, denoted by $P_n$, is a graph with $2$ vertices of degree $1$ and $n - 2$ vertices of degree $2$. Complete graph with order $n$, denoted by $K_n$, is a graph with the two vertices are adjacent. Complete graph with order $1$, denoted by $K_1$, is a graph with one vertex and has no edge.

Let $G$ be a graph with order $n$ and $H$ be a graph with order $m$. The corona product $G \odot H$ is obtained by taking one copy of $G$ and $n$ copies of $H$; and by joining each vertex of the $i$-th copy of $H$ to the $i$-th vertex of $G$, where $1 \leq i \leq n$.

The Cartesian product $G \square H$ of graphs $G$ and $H$ is a graph such that the vertex set of $G \square H$ is the Cartesian product $V(G) \times V(H)$ and any two vertices $(u, u')$ and $(v, v')$ are adjacent in $G \square H$ if and only if either $u = v$ and $u'$ is adjacent with $v'$ in $H$, or $u' = v'$ and $u$ is adjacent with $v$ in $G$.

Ladder $L_n$ is the Cartesian product of path with order $n$ and path with order $2$.

A Triangular Snake $T_n$ is obtained from a path $u_1u_2 \cdots u_n$, by joining $u_i$ and $u_{i+1}$ to a new vertex $v_i$ for $1 \leq i \leq n - 1$. That is every edge of a path is replaced by a triangle $C_3$.

Aims of this research are as follows:

- To prove that the corona product of ladder and complete graph with order $1$ is a root square mean graphs.
- To prove that the graph obtained from triangular snake by join one vertex with degree $2$ in each triangle to a new vertex is a root square mean graphs.

2. Methodology

The method we use in this research are literature study and analyzing. In the literature study, we check some research about this topic. Then, we continue the research. We choose the graphs for analyzing. Then, we construct a root square mean labeling of the graphs and proof that the graphs are root square mean graphs. To proof that a graph is a root square mean graph, we need to construct a root square mean labeling of the graph. The first step of the labeling is we need to do labeling to the vertices of the graph with label $1, 2, 3, \ldots, q + 1$, where $q$ is the number of edges of the graph. The labels of the vertices are not necessary different. The next step is we need to do labeling to the edges with the certain formula by using the vertex labeling. The edge labels must be different.
3. Results and discussion
In this research, we determine root square mean labelling that introduced by Sandhya in [1]. We use the similar method with proof method [1-4], for some different graphs. We construct the original formula of root square mean labelling of other graphs.

In the first theorem, we will proof that the corona product of ladder and complete graph with order 1 is a root square mean graphs.

**Theorem 1.** If $L_n$ denote ladder with order $2n$ and $K_1$ denote complete graph with order 1, then $L_n \odot K_1$ is a root square mean graphs.

**Proof.** $L_n$ denote ladder with order $2n$ and $K_1$ denote complete graph with one vertex. Let the vertex set of $L_n \odot K_1$ is

$$V(L_n \odot K_1) = \{u_i | 1 \leq i \leq n\} \cup \{v_i | 1 \leq i \leq n\} \cup \{w_i | 1 \leq i \leq n\} \cup \{x_i | 1 \leq i \leq n\}$$

and the edge set of $L_n \odot K_1$ is

$$E(L_n \odot K_1) = \{u_i v_i | 1 \leq i \leq n\} \cup \{v_i w_i | 1 \leq i \leq n\} \cup \{w_i x_i | 1 \leq i \leq n\} \cup \{v_i v_{i+1} | 1 \leq i \leq n-1\}$$

An illustration of $L_n \odot K_1$ can be seen in the Figure 1.

![Figure 1](image)

From the definition of $E(L_n \odot K_1)$, we can see that $L_n \odot K_1$ has $5n - 2$ edges. Define a labelling $f$ from $V(L_n \odot K_1)$ to $\{1, 2, \cdots, 5n - 1\}$as follows:

$$f(u_i) = 5i - 2 \text{ for } 1 \leq i \leq n;$$

$$f(v_i) = 5i - 4 \text{ for } 1 \leq i \leq n;$$

$$f(w_i) = 5i - 3 \text{ for } 1 \leq i \leq n;$$

$$f(x_i) = 5i - 1 \text{ for } 1 \leq i \leq n.$$ 

From the vertex labelling, we have the edge labelling as follows:

$$f(u_i v_i) = \sqrt{\frac{f(u_i)^2 + f(v_i)^2}{2}} = \sqrt{(5i - 3)^2 + 1} = 5i - 3;$$

$$f(v_i w_i) = \sqrt{\frac{f(v_i)^2 + f(w_i)^2 + 1}{2}} = \sqrt{(5i - 4)^2 + 5i - 4} = 5i - 4;$$
\[ f(w_i) = \left\lfloor \frac{f(w_i)^2 + f(x_i)^2}{2} \right\rfloor = \left\lfloor \sqrt{(5i - 2)^2 + 1} \right\rfloor = 5i - 2; \]
\[ f(v_i v_{i+1}) = \left\lfloor \frac{f(v_i)^2 + f(v_{i+1})^2 + 1}{2} \right\rfloor = \left\lfloor \sqrt{(5i - 1)^2 - 5i + 8} \right\rfloor = 5i - 1; \]
\[ f(w_i v_{i+1}) = \left\lfloor \frac{f(w_i)^2 + f(w_{i+1})^2 + 1}{2} \right\rfloor = \left\lfloor \sqrt{(5i)^2 - 5i + 7} \right\rfloor = 5i \]

for \( 1 \leq i \leq n \).

It can be seen that there are no two edges with the same label. So that, \( f \) is a root square mean labeling of \( L_n \circ K_1 \). Furthermore, we can conclude that \( L_n \circ K_1 \) is a root square mean graph.

In the second theorem, we will proof that the graphs obtained from path \( u_1 u_2 \cdots u_n \) by joining \( u_i \) and \( u_{i+1} \) to a new vertex \( v_i \) and joining \( v_i \) to new vertex \( w_i \), for \( 1 \leq i \leq n - 1 \), is a root square mean graph.

**Theorem 2.** Let \( G \) be a graph graphs obtained from path \( u_1 u_2 \cdots u_n \) by joining \( u_i \) and \( u_{i+1} \) to a new vertex \( v_i \) and joining \( v_i \) to new vertex \( w_i \), for \( 1 \leq i \leq n - 1 \). Then \( G \) is a root square mean graph.

**Proof.** An illustration of \( G \) can be seen in Figure 2.

![Figure 2. An illustration of G.](image)

From the vertex labelling, we have the edge labelling as follows:

\[ f(w_i v_i) = \left\lfloor \frac{f(w_i)^2 + f(v_i)^2 + 1}{2} \right\rfloor = \left\lfloor \sqrt{(4i - 2)^2 + 4i + 1} \right\rfloor = 4i - 2 \]

for \( 1 \leq i \leq n \), also

\[ f(v_i u_i) = \left\lfloor \frac{f(v_i)^2 + f(u_i)^2 + 1}{2} \right\rfloor = \left\lfloor \sqrt{(4i - 3)^2 + 4i - 2} \right\rfloor = 4i - 3; \]
\[
f(v_iu_{i+1}) = \left\lfloor \frac{f(v_i)^2 + f(u_{i+1})^2}{2} \right\rfloor = \left\lfloor \frac{(4i)^2 + 4}{2} \right\rfloor = 4i;
\]
\[
f(u_iu_{i+1}) = \left\lfloor \frac{f(u_i)^2 + f(u_{i+1})^2 + 1}{2} \right\rfloor = \left\lfloor \frac{(4i - 1)^2 + 4}{2} \right\rfloor = 4i - 1
\]
\[
f(w_iw_{i+1}) = \left\lfloor \frac{f(w_i)^2 + f(w_{i+1})^2 + 1}{2} \right\rfloor = \left\lfloor \frac{(5i)^2 - 5i + 7}{2} \right\rfloor = 5i
\]
for \(1 \leq i \leq n - 1\).

It can be seen that there are no two edges with the same label. So that, \(f\) is a root square mean labelling of \(G\). Futhermore, we can conclude that \(G\) is a root square mean graph.

4. Conclusion

From Theorem 1, we can see that there are a root square mean labeling of the corona product of ladder and complete graph with order 1. Also from Theorem 2, we can check that we can construct a root square mean labeling of the graphs obtained from path \(u_1u_2 \cdots u_n\) by joining \(u_i\) and \(u_{i+1}\) to a new vertex \(v_i\) and joining \(v_i\) to new vertex \(w_{i+1}\) for \(1 \leq i \leq n - 1\). So that, we can conclude that the corona product of ladder and complete graph with order 1 and the graphs obtained from path \(u_1u_2 \cdots u_n\) by joining \(u_i\) and \(u_{i+1}\) to a new vertex \(v_i\) and joining \(v_i\) to new vertex \(w_i\), for \(1 \leq i \leq n - 1\) are root square mean graphs.

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