The study on Reliability Design of Knotter Cam Gear of D-knotter

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Abstract. The outer gear of knotter cam gear of D-knotter and pinion of knotter hook to achieve coordination and cooperation is important guarantee for the movement of "winding rope - biting rope". This paper analyzes the damage mechanism of the outer gear of knotter cam gear. By application the probability theory for equal effect accuracy synthesis, the outer gear of knotter cam gear and pinion of knotter hook selected from 8 grade precision, and modification of teeth top along the normal direction of the measurement of 50.04 μm for pinion of knotter hook, can control the additional impact dynamic load less than 0.085 times of the calculated load. Two methods of traditional fatigue strength calculation and finite element analysis are used to check the strength of the incomplete outer teeth. And the results obtained by the finite element simulation described in this paper are closer to the real value. This study is of practical value in improving the reliability of the knotter cam gear and promoting the localization of D-knotter.

Keywords: D-knotter; knotter cam gear; reliability design; finite element analysis

1. Introduction
Knotter is the key component of square baler. Its space structure is complex, and the manufacture of key parts is difficult [1,2]. The outer gear of knotter cam gear of D-knotter and pinion of knotter hook to achieve coordination and cooperation is important guarantee for the movement of "winding rope - biting rope". If the gear pair's gear wear is serious or the gear precision is low, it will result in the serious consequences of breakage of the knotter hook and the knife arm or even the breaking of the lock surface of the pinion of knotter hook. According to the precision of the outer tooth of the knotter cam gear higher and easy to wear characteristics, this paper analyzes the damage mechanism of the gear tooth plate. By application the probability theory for equal effect accuracy synthesis, the design of precision and gear top modification of the outer gear and pinion of knotter hook are calculated. Finally, two methods of traditional fatigue strength calculation and finite element analysis are used to check the strength of the incomplete outer teeth.

2. Failure mechanism of the outer gear of knotter cam gear
The movement of the clamps is driven by a pair of by a pair of incomplete straight bevel gear. If the gear pair's gear wear is serious or the gear precision is low, it will result in two serious consequences. One is that it is unable to complete the “winding rope” correctly, because of the shock of gear too big, causing the rope to be entangled on the knotter hook. It can result in the breaking of the knotter hook and the knife arm(see figure 1). Two is the pinion of knotter hook no correctly return, resulting in
crushing the locking surface of the pinion of knotter hook [2] (see figure 2). Figure 3 is a magnified picture of the wear of the outer gear of knotter cam gear. It can be seen from the figure that the 2 short teeth wear are very serious at the end of the outer gear.

There are three main reasons for this phenomenon. One is that the four teeth at both ends of the outer teeth are incomplete because of realizing the motion characteristics of the pinion of knotter hook. It makes the tooth stress concentrated and aggravates the wear of the outer teeth. Two is that the design space of the knotter frame is limited. In order to avoid knotter cam gear and the frame body interference, the outer gear design of teeth width is also limited, and the teeth width can not be increased. It makes the additional impact dynamic load of the unit tooth width on the outer gear difficult to be reduced. Three is that the knotter cam gear is obtained by precision casting. The casting parts have the disadvantages of low dimensional precision, loose casting organization and poor bearing capacity inevitably. As shown in figure 4, sand inclusion can be seen on the outer gear because of improper casting.

The data is the statistics by Inner Mongolia Huade forage Machinery Co. Ltd in figure 5. It is the statistical data of the number of maintenance of imported RS3770 D-knotter in the last two years. There are 233 pieces in all. The fault caused by the wear of the outer gear occupies 30.9% of the total failure rate. According to the statistics obtained by domestic factories, the D-knotter obtained by reverse engineering is not only difficult to reach the life requirement of the imported knotter, but also the outer gear fault rate accounts for 70% of the total fault rate. This difference is mainly due to the low precision
of the outer gear obtained through reverse engineering, and it is impossible to reach its original design standard.
In order to reduce the wear of the outer gear, we should redesign the outer gear of knotter cam gear and pinion of knotter hook.

3. Determine the outer gear and pinion of knotter hook parameters
At the time of design, the gear material is 40Cr. This material has good impact resistance and mechanical properties. The speed of the knotter cam gear is set \( n_1 = 90 \text{r/min} \). The simulation of the D-knotter model obtained by reverse engineering can be obtained, the maximum torque of the outer gear of knotter cam gear \( T_1 = 55 \text{N·m} \) (see figure 6), the speed of pinion of knotter hook \( n_2 = 605 \text{r/min} \) (see figure 7). Table 1 is the basic dimension obtained by reverse engineering of the outer gear and pinion of knotter hook, and the basic parameters obtained by the gear design formula in reference [3]. At the time of calculation, set \( h_4^* = 1 \) and \( c^* = 0.25 \).

![Figure 6. The outer gear - pinion of knotter hook torque diagram.](image)

![Figure 7. The speed of pinion of knotter hook.](image)

| Parameter name                                      | The outer gear of knotter cam gear | The pinion of knotter hook |
|-----------------------------------------------------|-----------------------------------|----------------------------|
| Theoretical teeth number                           | \( z_1 = 54 \)                   | \( z_2 = 8 \)               |
| Tooth width                                         | \( b_1 = b_2 = 26 \text{mm} \)   |                            |
| Modulus                                             | \( m_1 = m_2 = 4 \text{mm} \)    |                            |
| Pressure angle                                      | \( \alpha = 25^\circ \)          |                            |
| Shaft angle                                         | \( \Sigma = 98^\circ \)          |                            |
| Reference cone angle                                | \( \delta_1 = 89.48^\circ \)    | \( \delta_2 = 8.52^\circ \) |
| Addendum modification coefficient                   | \( x_1 = -0.4594 \)             | \( x_2 = 0.4594 \)         |
| Outer cone distance                                 | \( R_e = 107.9953 \text{mm} \)  |                            |
| Main aspects pitch diameter                         | \( d_1 = 216 \text{mm} \)       | \( d_2 = 32 \text{mm} \)   |
| Midpoint pitch diameter                             | \( d_{m1} = 190 \text{mm} \)    | \( d_{m2} = 28.15 \text{mm} \) |
| Tooth addendum                                      | \( h_{a1} = 2.16 \text{mm} \)   | \( h_{a2} = 5.84 \text{mm} \) |
| Tooth dedendum                                      | \( h_{f1} = 6.64 \text{mm} \)   | \( h_{f2} = 2.96 \text{mm} \) |
| Top clearance                                       | \( c_1 = c_2 = 1 \text{mm} \)   |                            |
| Main aspects pitch diameter tooth thickness         | \( s_1 = 4.57 \text{mm} \)      | \( s_2 = 7.99 \text{mm} \)  |
| Space width                                         | \( e_1 = 7.99 \text{mm} \)      | \( e_2 = 4.57 \text{mm} \)  |
| Equivalent gear rolling circle teeth distance       | \( p_{vb} = 10.02 \text{mm} \)  |                            |
| Length of line of action                            | \( g_{vb} = 14.44 \text{mm} \)  |                            |
| Contact ratio and overlap ratio                     | \( \varepsilon_{vb} = 1.44 \)   |                            |

4. The outer gear and pinion of knotter hook reliability design
The impact load will affect the transmission accuracy of the external gear meshing pair of the knotted hook. In order to reduce the impact load, it is necessary to gear top modification for pinion of knotter hook. The influence of the additional dynamic load on the gear transmission precision is the dynamic
load coefficient $k_v$. In order to ensure the lower transmission error and reach the "precise transmission", the dynamic load coefficient is usually 1 ~ 1.1 in the bevel gear transmission [4]. But the designed gear are the nonstandard bevel gear. So the reliability design will be based on the impact dynamic load theory, using probability method to calculate the pitch deviation and gear top modification amount of bevel gear, determining the precision range of the meshing gear. This way can reduce the additional dynamic load, decrease its vibration and enhance the service life of the outer gear.

4.1. Determine the dynamic load coefficient of bevel gear
The general expression of the dynamic load coefficient is [5]

$$k_v = \frac{w_t + w_d}{w_t} = 1 + \frac{w_d b}{F_{mt} K_A}$$

(1)

In the formula: $w_d$ — The additional dynamic load on the tooth width of the gear tooth caused by the impact, N/mm; $w_t$ — The calculated load on the unit tooth width of the bevel gear, N/mm; $w_t = F_{mt} K_A / b$; $F_{mt}$ — The circumferential force at the middle point of the tooth width of the active bevel gear, N; $F_{mt} = 2000 T_1 / d_{m1}$; $T_1$ — Torque transmission of active bevel gear, N · m; $d_{m1}$ — The pitch diameter at the middle point of the tooth width of the active bevel gear, mm; $b$ — Tooth width of bevel gear, mm; $K_A$ — Application coefficient, because there is a slight vibration during the meshing of the knob gear, taking $K_A = 1.35$.

According to the theory of impact, the derivation of reference [6] can obtain:

$$k_v = 1 + N \sqrt{(1.8 f_{ptx} - C_a)}$$

(2)

In the formula:

$$N = \psi V (1 + i) b \sqrt{C_f m_{red} \times 10^2}$$

$$\sqrt{\frac{2(1.8 f_{ptx} - C_a)d_{2a} \sin \alpha_v}{a_v d_{1a}}} F_{mt} K_A$$

By querying the relevant formulas of reference [5] and [7], can know Impact attenuation coefficient $\psi=0.54$, The linear velocity of the equivalent gear pitch diameter of the driven bevel gear, $V = 0.9014$m/s, Gear ratio, $i=6.75$, Curvature radius of the circular tooth profile of the small bevel gear equivalent gear tooth top $d_{2a} = 15.37mm$, Curvature radius of the circular tooth profile of the large bevel gear equivalent gear tooth root $d_{1a} = 264.59mm$, End face pressure angle of equivalent gear $\alpha_v = 25^\circ$, Center distance of the equivalent gears drive $a_v = 662.49mm$. Meshing stiffness of bevel gear teeth $C_f = 6.012N/(mm \cdot \mu m)$. Induction quality of straight bevel gear pair $m_{red} = 2.91kg / mm$. Finally, we can get it, $N = 0.01429$.

4.2. Estimation of the pitch deviation and gear top modification by probability theory
The so-called “equal effect accuracy synthesis” is a method to determine the offset of every variable quantities in the function according to the allowable output deviation of a function when all variables in the function have the same influence on the output deviation of the function. Suppose that there is an output function $y = f(x_1, x_2, ..., x_n)$, $x_1, x_2, ..., x_n$ of which are independent random variables. Then it is known by probability theory that the relation between the deviations of the variables in the function and the output deviation value of the function is shown in the following formula [6]:

$$\Delta x_i = \frac{\Delta y}{\sqrt{2|\Delta y / \Delta x|}}$$

(3)

Where $\Delta y / \Delta x$ - Error transfer function

First, the function of the Y is constructed. The formula can be obtained by the formula (2).

$$(k_v - 1)^2 = N^2(1.8 f_{ptx} - C_a)$$

Therefore,

$$y = (k_v - 1)^2 = N^2(1.8 f_{ptx} - C_a)$$

(4)
Then, it is determined that the allowable deviation of $y$ is $\Delta y$. According to the literature [8] and the probability theory, the coefficient of variation of the dynamic load coefficient is:

$$\tau_{k_y} = \frac{(k_y - 1)}{3k_y}$$

(5)

Then the standard deviation of the dynamic load coefficient $k_y$ is:

$$\sigma_{k_y} = k_y \tau_{k_y}$$

(6)

It is also known by the formula (4):

$$\frac{dy}{dk_y} = 2(k_y - 1)$$

$$dy = 2(k_y - 1)dk_y$$

Hypothesis,

$$dy = \Delta y, \quad dk_y = \Delta k_y = k_y - 1$$

Therefore,

$$\Delta y = 2(k_y - 1)\Delta k_y = 2(k_y - 1)^2$$

(7)

It is assumed that the pitch deviation $f_{pt\Sigma}$ and modification of teeth top along the normal direction $C_a$ of the straight bevel gear are random variables. Then the dynamic load coefficient $k_y$ and the function $y$ are also random variables, and the variables are independent of each other. So $y$ is a function of $f_{pt\Sigma}$ and $C_a$, that is to say, $y = f(\Sigma{f_{pt\Sigma}}, \Sigma{C_a})$.

According to the formula of equal influence precision (3), we know that:

$$\Delta f_{pt\Sigma} = \frac{\Delta y}{\sqrt{2|\partial y/\partial f_{pt\Sigma}|}}$$

(8)

$$\Delta C_a = \frac{\Delta y}{\sqrt{2|\partial y/\partial C_a|}}$$

(9)

The transfer functions of each deviation are:

$$\frac{\partial y}{\partial f_{pt\Sigma}} = 1.8N^2$$

$$\frac{\partial y}{\partial C_a} = -N^2$$

By the formula (8), (9) to calculate the $\Delta f_{pt\Sigma}$ and $\Delta C_a$, make $C_a \approx \Delta C_a$, $f_{pt\Sigma} \approx \Delta f_{pt\Sigma}$. It is known from the literature [3] that the difference of the limit deviation of the tooth distance between the large bevel gear and the small bevel gear is $1 \sim 3 \mu m$. According to this formula [6],

$$f_{pt\Sigma} = \sqrt{f_{pt1}^2 + f_{pt2}^2}$$

(10)

The $f_{pt1}$ of the outer gear and the $f_{pt2}$ of pinion of knotter hook can be determined.

Under guaranteeing the transmission precision, the machining cost is reduced as much as possible, and the dynamic load coefficient of the calculation is $k_y=1.085$.

By making the known parameters into the formula, we can know, $\partial y/\partial f_{pt\Sigma}=3.6757 \times 10^{-4}$, $\partial y/\partial C_a=2.0421 \times 10^{-4}$, $\Delta y=0.01445$, $f_{pt\Sigma} \approx 27.79\mu m$, $C_a \approx 50.04\mu m$.

Through formula (10), we can see that the value of $f_{pt1}$ for the limit deviation of the tooth distance of the outer gear is between (20~21) $\mu m$, and the value of the $f_{pt2}$ of the pinion of knotter hook is between (18~19) $\mu m$. Reference documentation [3] shows that the bevel gear corresponding to the precision level between 7 grade and 8 grade. To reduce processing costs, will use 8 grade precision. And modified of teeth top along the normal direction of the measurement of 50.04 $\mu m$ for pinion of knotter hook. Reference documentation [3] shows that the modification amount of the normal direction of the 8 level precision is 60 $\mu m$, which is different from the calculated value. The amount of modification obtained by calculation is used in the gear design.
4.3. Precision grade calculation of bevel gear considering vibration factor

In this section, the method of considering the vibration factors will be adopted to calculate the dynamic load coefficient \( k_p \), and the precision grade of the gear will be obtained.

The simplified formula for calculating \( k_p \) is [3]:

\[
K_p = \left( \frac{A}{A + \sqrt{200v_{et}}} \right)^B
\]

(11)

Where \( A = 50 + 56 \times (1 - B) \) \( B = 0.25 \times (C - 5)^{0.667} \)

In the formula, \( C \)-Transmission precision coefficient, it can be used as a bevel gear designed precision grade; \( v_{et} \)-The linear velocity of bevel gear Main aspects node, m/s, \( v_{et} = \pi d t n_s / 60 / 1000 \)

When \( K_p \) is 1.085, C is 7.74. Therefore, the precision of the bevel gear is 8 grade. Therefore, when the the dynamic load coefficient \( K_p \) is determined, considering the impact factor or the vibration factor, the precision grade of the bevel gear is basically the same. At the same time, it is proved that the way is correct and feasible by probability theory to calculate the precision grade of bevel gear.

4.4. Traditional theory checking of the outer gear

This check is mainly to check the 4 incomplete teeth of the outer gear which are usually serious wear. Through the reverse engineering survey, the tooth width of 4 incomplete teeth is 14mm, that is to say, \( b_\theta = 14mm \).

The length of meshing line in the middle of the tooth [3] :

\[
l_{bm} = \frac{2b_\theta \sqrt{e_{va} - 1}}{e_{va}} = 12.9mm
\]

According to reference [4], the basic value of contact stress on the tooth surface \( \sigma_{H0} \) is known.

\[
\sigma_{H0} = Z_M Z_H Z_E Z_L S Z_B Z_K \left( \frac{F_{mt}}{d_m l_{bm}} \right) \sqrt{\frac{l^2 + 1}{l}}
\]

(12)

The formula of contact stress of tooth surface is known.

\[
\sigma_H = \sigma_{H0} \sqrt{K_A K_V K_{H\beta} K_{Ha}}
\]

(13)

For open gear drive, the formula of permissible contact stress:

\[
\sigma_{hp} = 1.05 \sigma_{Hlimmin}
\]

(14)

By querying the relevant formulas of reference [3] and [7], can know Application coefficient \( K_A = 1.35 \), Dynamic load coefficient \( K_V = 1.085 \), Longitudinal load distribution \( K_{H\beta} = 1.65 \), Transverse load coefficient \( K_{Ha} = 1.2 \), Coefficient of node region \( Z_H = 2.28 \), Coefficient of middle point region \( Z_M = 1.0502 \), Coefficient of elasticity \( Z_E = 189.8 \sqrt{MPa} \), Helix angle coefficient \( Z_B = 1 \), Bevel gear coefficient \( Z_B = 0.8 \), Load distribution coefficient \( Z_L = 1 \), Contact fatigue limit of test gear \( \sigma_{Hlimmin} = 1000MPa \).

Finally, we can know \( \sigma_{H0} = 441.32MPa \), \( \sigma_H = 751.14MPa < \sigma_{hp} = 1050MPa \). The bending fatigue strength of the tooth root can also be known by the formula of reference [3]. The basic value of the bending stress of the root of the outer gear tooth is \( \sigma_{F0} = 45.17MPa \). The calculation value of the bending stress of the outer gear tooth root is \( \sigma_F = 130.5MPa \). The wear coefficient is \( K_m = 1.25 \). The checking value of the tooth root bending fatigue strength is \( K_m \sigma_F = 163.1MPa < \sigma_{fp} = 210MPa \).

5. Finite element contact analysis of the outer gear

In order to see more clearly the stress change of the designed outer gear, ANAYS Workbench finite element analysis software is applied to conduct finite element contact analysis of the outer gear in this paper. The process includes three major steps [9].
5.1. The preprocessing stage of contact analysis

Set up the three-dimensional model of gear. In the three-dimensional modeling software, the two gear shaft angle of 98 degrees finish assembling. Then the file is saved as “*.STEP” and directly into the ANAYS Workbench.

Set the basic parameters of the model. Young’s modulus of material $E = 2.06 \times 10^{11} Pa$, Density of materials $\rho = 7.86 \times 10^3 kg/m^3$, Poisson ratio $\mu = 0.29$.

Mesh division of gear model. An automatic mesh is used for the whole model and the mesh is refined at the contact area, as shown in figure 8.

Create contact pairs. The potential contact surface of the outer gear is the contact surface, and the potential contact surface of pinion of knotter hook is the target surface. And set the relevant parameters, Then the contact type of the tooth surface is frictional, Frictional coefficient of gear tooth surface $f = 0.2$, Normal contact stiffness factor $FKN = 2$.

5.2. Loading and solving phase of contact analysis

The gear contact analysis is a statics analysis. The pinion of knotter hook is driven wheel. Therefore, the inner surface of the shaft of the gear and the end face of the gear are all constrained. The outer gear is the driving wheel. So, the teeth are defined as X and Y direction constraints, and the Z circumference direction is free. And the torque of $T1 = 55N \cdot m$ is applied to the rotation center of the active wheel. As is shown in figure 9, is the applied loads and constraints.

5.3. Post-processing stage

Post processing stage is mainly based on graphic display for structural analysis. figure 10 is contact stress cloud chart of “the outer teeth-the bevel pinion”. figure 11 is equivalent stress cloud chart of gear pair. figure 12 is equivalent stress cloud chart of outer teeth.

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Figure 8. Mesh division.

Figure 9. Applied loads and constraints.

Figure 10. Contact stress cloud chart of gear pair
Table 2 shows the contact stress of the engagement pair and the error values compared with the traditional checking results. Table 3 shows the bending stress of the outer gear tooth root and the error values compared with the traditional checking results. Among them, the formula of error value is: error value (%) = (traditional calculation value - simulation value) / traditional calculation value.

### Table 2. The result and comparison of finite element calculation of contact stress of teeth surfaces

| Gear pair name          | Traditional Calculation Value (Mpa) | Simulation Value (Mpa) | Error Value | Load coefficient | Check value (Mpa) | Allowable Stress (Mpa) |
|-------------------------|------------------------------------|------------------------|-------------|------------------|-------------------|------------------------|
| The outer teeth-        | 441.32                             | 295.5                  | 33.04%      | 1.71             | 505.31            | 1050                   |
| The bevel pinion        |                                    |                        |             |                  |                   |                        |

### Table 3. The result and comparison of the finite element calculation of bending stress of the root of teeth

| Gear name                | Traditional Calculation Value (Mpa) | Simulation Value (Mpa) | Error Value | Load Coefficient | Wear Coefficient | Check value (Mpa) | Allowable Stress (Mpa) |
|--------------------------|------------------------------------|------------------------|-------------|------------------|-------------------|-------------------|------------------------|
| The incomplete outer teeth| 45.17                              | 41.36                  | 8.4%        | 2.9              | 1.25              | 149.93            | 210                    |

From the data of table 2 and table 3, we can see that the strength checking of the incomplete outer teeth is qualified. The stress values obtained by finite element simulation are generally less than those obtained by traditional calculation. This is mainly to improve the safety level of checking, and the various calculation coefficients selected by traditional checking methods are generally large. It is also found that the simulation values of gear contact stress deviate greatly from the calculated values. This is mainly due to the larger experimental value of node area coefficient $Z_H$. Therefore, the results obtained by the finite element simulation described in this paper are closer to the real value.

### 6. Conclusions

According to the impact dynamic load theory and application the probability theory for equal effect accuracy synthesis, the outer gear of knitter cam gear and pinion of knitter hook selected from 8 grade precision, and modification of teeth top along the normal direction of the measurement of 50.04 μm for
pinion of knotter hook, can control the additional impact dynamic load less than 0.085 times of the calculated load.

According to the impact dynamic load theory and application the probability theory for equal effect accuracy synthesis, the approximate precision grade of bevel gear is in agreement with the precision grade (GB standard) which is now used in China to consider the vibration factors.

Two methods of traditional fatigue strength calculation and finite element analysis are used to check the strength of the "the outer teeth-the bevel pinion " obtained by the design. Finally, we know that the fatigue strength of the incomplete outer teeth is all qualified. And the results obtained by the finite element simulation described in this paper are closer to the real value.

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