Production of four heavy quarks and $B_c$-mesons at the $Z^0$-boson pole

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Abstract

The cross-section for the production of $b\bar{b}c\bar{c}$ quarks in $e^+e^-$ annihilation, that proves to be at a level of $\sigma(e^+e^- \rightarrow b\bar{b}c\bar{c})/\sigma(e^+e^- \rightarrow b\bar{b}) \sim 10^{-2}$ for $\sqrt{s} = M_Z$ is calculated within the frames of the QCD perturbation theory. The cross sections for the associated production of $1S$- and $2S$-wave states of $B_c$-meson in the reaction $e^+e^- \rightarrow B_c b\bar{c}$ were calculated in the nonrelativistic model of a heavy quarkonium. The fragmentation function of $b \rightarrow B_c^{(*)}c$ is analysed in the scaling limit. The number of $\Lambda_{bc}$-hyperons to be expected at LEP is estimated on the basis of the assumption on quark-hadron duality.
Introduction.

The study of heavy quark production processes is of great interest because this can give an additional check of the applicability of the QCD theory to the description of both free and bound states. The properties of 2- and 3-jet events have been studied fairly thoroughly at the PETRA, KEK and LEP colliders. In the light of a high luminosity expected at the LEP collider, $\sim 10^7 Z^0$, the study of other rarer processes is also interesting. Among such processes is the production of four quarks, $b\bar{b}c\bar{c}$, in $e^+e^-$ annihilation. We are treating this process, on the one hand, as verification of the QCD theory in higher orders of perturbation theory, $O(\alpha_s^2)$, and, on the other hand, as a further step in our understanding of, for example, bound states $b\bar{c}$ ($bc$) not yet discovered experimentally. The bound states of $b$- and $\bar{c}$-quarks -- $B_c$-mesons occupy in the mass spectrum an intermediate state between the families of $J/\psi$- and $\Upsilon$-mesons. In this respect the predictions of the nonrelativistic quark model describing the $J/\psi$- and $\Upsilon$-families very well can also be applicable for $B_c$-mesons. Therefore the discovery and study of $B_c$-mesons will become a good test of QCD and QCD-inspired nonrelativistic potential model. The processes of $B_c$-meson production in $e^+e^-$, $p\bar{p}$- and also neutrino-hadron collisions have been studied in papers [1] - [6]. As to the $B_c$-meson spectroscopy, it was investigated in [7]. Various aspects related to $B_c$-meson decays were considered in [8] - [10].

Using the exact formulas of QCD perturbation theory we calculate the cross-section $\sigma (e^+e^- \rightarrow b\bar{b}c\bar{c})$ and also the cross section for $B_c$-meson production in the process $e^+e^- \rightarrow B_c\bar{b}c$ in nonrelativistic approximation for $B_c$. We treat these two processes in one work because they are closely related to each other. The very convenient method used to calculate the helicity amplitudes for the process $e^- \rightarrow b\bar{b}c\bar{c}$ can also be applied actually in the same form for calculating the process $e^+e^- \rightarrow B_c\bar{b}c$. If one considers the approximate quark-hadron duality to be valid then the cross section for the process $^+e^- \rightarrow b\bar{b}c\bar{c}$ can be related in the region of small invariant masses of a $(b\bar{c})$-pair with that for the production of bound states of $B_c$-mesons in the process $e^+e^- \rightarrow B_c\bar{b}c$. Using the same assumption on quark-hadron duality one can also make certain conclusions on the specific value of the cross-section for the production of $\Lambda_{bc}$-hyperons -- a bound system of two heavy quarks and one light valent quark of different flavours.

The paper is arranged as follows. The method of calculating the cross section for the process $e^+e^- \rightarrow b\bar{b}c\bar{c}$ is presented in Section 1. Section 2 analyzes the most interesting distributions for this process. The cross-section for the $B_c$-mesons production in the process $e^+e^- \rightarrow B_c\bar{b}c$ is discussed in Section 3. Section 4 presents a brief review of the present experimental situation regarding the possibility of search for bound states with heavy quarks of different flavours.
1 Calculational technique

At the tree level, 8 Feynmann diagrams, 4 with photon and 4 with Z°-boson exchange in the s-channel (see Fig.1) give contribution into the process of the production of two pairs of heavy quarks, $b\bar{b}$ and $c\bar{c}$:

$$e^+(q_1) + e^-(q_2) \rightarrow b(p_1) + \bar{b}(p_2) + c(p_3) + \bar{c}(p_4) \quad (1)$$

The traditional techniques of calculating these diagrams by the analytical quadrature of the amplitude result in too cumbersome expressions. But still, there are some papers devoted to calculations of the cross-sections with four-quark final states. For example, the authors of [11] have studied within the frames of QCD perturbation theory the cross-section for the process $gg \rightarrow q\bar{q}q'\bar{q}'$ for the case of massless quarks and those of [12] have studied it also for massive quarks. In [13] the cross-section of the production of four quarks in $e^+e^-$annihilation, $e^+e^- \rightarrow q\bar{q}q'\bar{q'}$, for the case of massless quarks has been calculated with the method of spinor products [14]. As for our paper, to analyze the cross-section of the process $e^+e^- \rightarrow b\bar{b}c\bar{c}$, where the consideration for the masses is obligatory, we apply a more convenient, from our viewpoint, technique of paper [12], i.e. the technique of direct numeric calculation of the amplitude.

Let us introduce the following notations for momenta:

$q_{12} = q_1 + q_2$, $p_{12} = -p_1 - p_2$, $p_{34} = -p_3 - p_4$,
$p_{124} = p_{12} - p_4$, $p_{123} = -p_{12} + p_3$, $p_{342} = p_{34} - p_2$, $p_{341} = -p_{34} + p_1$,

for currents

$$J^\mu_\gamma = Q^e \frac{\bar{v}(q_2)\gamma^\mu u(q_1)}{q_{12}^2}, \quad (2)$$

$$J^\mu_Z = \frac{\bar{v}(q_2)\gamma^\mu(v_Z^e - a_Z^e\gamma^5)u(q_1)}{q_{12}^2 - M_Z^2 + iM_Z\Gamma_Z}, \quad (3)$$

$$J^\mu_b = \frac{\bar{u}(p_1)\gamma^\mu v(p_2)}{p_{12}^2}, \quad (4)$$

$$J^\mu_c = \frac{\bar{u}(p_3)\gamma^\mu v(p_4)}{p_{34}^2}, \quad (5)$$

and also the following subsidiary spinors

$$v_{124} = \frac{(p_{124} + m_c)}{p_{124}^2 - m_c^2} \bar{J}_b v(p_4), \quad (6)$$
\[ v_{342} = \frac{(\hat{p}_{342} + m_b)}{p_{342}^2 - m_b^2} \hat{j}_e v(p_2), \quad (7) \]

\[ \bar{u}_{123} = \bar{u}(p_3) \hat{j}_b \frac{(\hat{p}_{123} + m_c)}{p_{123}^2 - m_c^2}, \quad (8) \]

\[ \bar{u}_{341} = \bar{u}(p_1) \hat{j}_c \frac{(\hat{p}_{341} + m_b)}{p_{341}^2 - m_b^2}, \quad (9) \]

In terms of the quantities introduced above the amplitude of the process \( e^+e^- \rightarrow b\bar{b}c\bar{c} \) has the form

\[ M = \sum_{i=1}^{4} M_i, \]

where

\[ M_1 = \bar{u}(p_3)[\hat{j}_c Q_c + \hat{j}_Z (v_c^* - a_c^* \gamma^5)] v_{124} \quad (10) \]

\[ M_2 = \bar{u}_{123}[\hat{j}_c Q_c + \hat{j}_Z (v_c^* - a_c^* \gamma^5)] v(p_4) \quad (11) \]

\[ M_3 = \bar{u}(p_1)[\hat{j}_b Q_c + \hat{j}_Z (v_b^* - a_b^* \gamma^5)] v_{342} \quad (12) \]

\[ M_4 = \bar{u}_{341}[\hat{j}_c Q_b + \hat{j}_Z (v_b^* - a_b^* \gamma^5)] v(p_2) \quad (13) \]

Here \( Q_e, Q_b, Q_c \) and \( v_Z^e, a_Z^e, v_Z^b, a_Z^b, v_Z^c, a_Z^c \) are electromagnetic charges and vector and axial coupling constants of electron, \( b \)- and \( c \)-quark with \( Z^0 \)-boson.

The amplitude squared should be calculated as a sum over 2\(^6\) = 64 independent states of fermions. We choose the eigenstates of the helicity operator \( \vec{\Sigma} \vec{p} \) as two independent states of the spinor \( \psi \):

\[ \langle \vec{\Sigma} \vec{p} \rangle \psi(p, \lambda) = \lambda \psi(p, \lambda). \quad (14) \]

To calculate spinor with the specified value of 4-momentum \( p \) and helicity \( \lambda = \pm 1 \) it is necessary to choose a specific representation. In our case we choose a spinor (Weyl) representation:

\[ \gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \vec{\gamma} = \begin{pmatrix} 0 & -\vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}, \quad \vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix} \quad (15) \]

where

\[ \vec{\sigma} = \{\sigma_x, \sigma_y, \sigma_z\} = \left\{ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right\} \quad (16) \]

In this representation the spinors of particles, \( u(p, \pm) \), have the form

\[ u(p, +) = \frac{1}{\sqrt{2|p|(|p| + p_z)}} \begin{pmatrix} \sqrt{E + |p|} (|p| + p_z) \\ \sqrt{E + |p|} (p_x + ip_y) \\ \sqrt{E - |p|} (|p| + p_z) \\ \sqrt{E - |p|} (p_x + ip_y) \end{pmatrix} \quad (17) \]
The spinors of antiparticles, \( v(p, \pm) \), are defined as

\[
v(p, \pm) = C u^*(p, \mp)
\]

where \( C = i \gamma^2 \) is a charge conjugation matrix.

The explicit form of spinors \( v(p, \pm) \) is as follows:

\[
v(p, +) = \frac{1}{\sqrt{2|\vec{p}|(|\vec{p}| + p_z)}} \begin{pmatrix}
-\sqrt{E + |\vec{p}|} (|\vec{p}| + p_z) \\
-\sqrt{E + |\vec{p}|} (p_x + ip_y) \\
\sqrt{E - |\vec{p}|} (|\vec{p}| + p_z) \\
\sqrt{E - |\vec{p}|} (p_x + ip_y)
\end{pmatrix}
\]

\[
v(p, -) = \frac{1}{\sqrt{2|\vec{p}|(|\vec{p}| + p_z)}} \begin{pmatrix}
\sqrt{E - |\vec{p}|} (p_x + ip_y) \\
\sqrt{E - |\vec{p}|} (|\vec{p}| + p_z) \\
-\sqrt{E + |\vec{p}|} (p_x + ip_y) \\
-\sqrt{E + |\vec{p}|} (|\vec{p}| + p_z)
\end{pmatrix}
\]

But if \( p_z = -|\vec{p}| \) we put

\[
u(p, +) = \begin{pmatrix}
0 \\
\sqrt{E + |p|} \\
0 \\
\sqrt{E - |p|}
\end{pmatrix} \quad u(p, -) = \begin{pmatrix}
-\sqrt{E - |p|} \\
0 \\
-\sqrt{E + |p|} \\
0
\end{pmatrix}
\]

\[
v(p, +) = \begin{pmatrix}
0 \\
-\sqrt{E + |p|} \\
0 \\
\sqrt{E - |p|}
\end{pmatrix} \quad v(p, -) = \begin{pmatrix}
-\sqrt{E - |p|} \\
0 \\
\sqrt{E + |p|} \\
0
\end{pmatrix}
\]

The colour structure of the process \( e^+e^- \rightarrow b\bar{b}c\bar{c} \) is very simple. The colour factor \( F \) corresponding to this process is \( F = (N^2 - 1)/4 \), where \( N=3 \) is the number of colours.

The amplitude of the process \( e^+e^- \rightarrow b\bar{b}c\bar{c} \) presented by the above formulas can easily be written in terms of FORTRAN codes. The resultant program
is compact and runs fairly fast: it takes 0.03s of a VAX/VMS CPU time to compute the square of the amplitude summed over all polarization states.

We have verified that our two independent programs written for calculating the square of the amplitude, summed over all helicity states, hold the test for Lorentz-invariance (boost along the beam axis) and test for azimuthal invariance ($p_x \rightarrow p_y$ and $p_y \rightarrow -p_x$).

The integration over phase space of final particles was carried out by Monte Carlo methods with the help of programs written for this process, that minimized the spread in weights. The results of both programs coincide to an accuracy of the computational errors, caused by Monte Carlo methods.

2 The cross-section for the production of $b\bar{b}c\bar{c}$ at the $Z^0$-peak in QCD perturbation theory

In our numeric calculations we choose the standard set of the parameters of electroweak theory: $M_Z = 91.17$ GeV, $\sin^2 \theta_W = 0.23$, $m_b = 4.7$ GeV, $m_c = 1.4$ GeV. The QCD coupling constant $\alpha_s$ is set equal to

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2n_f)\ln(Q^2/\Lambda^2)},$$

where $\Lambda = 100$ MeV and for $Q^2 = M_Z^2, n_f = 5$ we have $\alpha_s(M_Z) = 0.12$ in accordance with [15].

It is clear from the form of diagrams on Figs.1a-d that the main contribution into the cross section for the process $e^+e^- \rightarrow b\bar{b}c\bar{c}$ will come from the diagrams of Figs.1c,d because for these diagrams the virtual gluon, due to the small value of the $c$-quark mass, approaches closer the mass shell. A noticeable peak in the region $M_{c\bar{c}} = 2 \div 4m_c$, related to the contribution from the diagrams of Figs.1c,d, is seen on Fig.2a in the distribution over the invariant mass of $c\bar{c}$-quarks, $M_{c\bar{c}} = \sqrt{(p_c + p_{\bar{c}})^2}$. However, the distribution over the invariant mass of $b\bar{b}$-quarks, $M_{b\bar{b}} = \sqrt{(p_b + p_{\bar{b}})^2}$, has a completely different form (see Fig.2b). A small peak in the region of small masses $M_{b\bar{b}}$ is due to the virtual gluon approaching the mass shell for those diagrams in which the $b\bar{b}$-pair is coupled to the $c(\bar{c})$-quark (see diagrams of Fig.1a,b). But if the mass $M_{b\bar{b}}$ of $b(\bar{b})$-quarks is large, then the mass $M_{c\bar{c}}$ of $c(\bar{c})$-quarks is, as a rule, small and only diagrams of Figs.1c,d ”work”, leading to a pronounced peak in the region of large masses $M_{b\bar{b}}$.

The process $e^+e^- \rightarrow b\bar{b}c\bar{c}$ takes place in the second order of the QCD coupling constant $\alpha_s(Q^2)$. Therefore it is important to make the right choice of the specific square of the momentum transferred $Q^2$ in the argument $\alpha_s(Q^2)$. As it has been mentioned above, the characteristic scale of the running $\alpha_s(Q^2)$ constant may be defined by the typical gluon virtuality ($M_{c\bar{c}}^2 \sim 4m_c^2$), however,
Table 1: The cross-section for the process $e^+e^- \rightarrow b\bar{b}c\bar{c}$ versus the total beam energy $\sqrt{s}$ for the two values of the square of the momentum transferred: $Q^2 = s$ and $Q^2 = 4m_c^2$. Bracketed is the error (one standard deviation) for the last digital, caused by Monte Carlo method.

| $\sqrt{s}$, GeV | 30     | 50     | 70     | 91.17  | 200    | 500    |
|-----------------|--------|--------|--------|--------|--------|--------|
| $\sigma(\alpha_s(s))$, pb | 0.0552(4) | 0.0782(9) | 0.192(3) | 69.7(2) | 0.061(2) | 0.014(1) |
| $\sigma(\alpha_s(4m_c^2))$, pb | 0.1294(9) | 0.217(3) | 0.593(9) | 234.2(7) | 0.254(8) | 0.073(5) |

the b-quark virtuality may be more greater and of the order of $s$, so that we present the results for two different choices of $Q^2 = \{4m_c^2, s\}$. Table 1 presents the cross-sections for the process $e^+e^- \rightarrow b\bar{b}c\bar{c}$ versus the total beam energy $\sqrt{s}$ for these two cases.

As is seen from this table, the difference in the definition of $\alpha_s$ leads to the difference in the cross-section by some times. For the most important energy range close to the $Z^0$-boson peak this difference attains a factor of 3.4. Such a strong dependence on $\alpha_s$ is related to the fact that the cross-section for the process $e^+e^- \rightarrow b\bar{b}c\bar{c}$ is proportional to $\alpha_s^2$. Therefore the accurate experimental measurement of the cross-section for the process $e^+e^- \rightarrow b\bar{b}c\bar{c}$ will allow us to draw a conclusion on the value of $\alpha_s^2$ in this process.

Let us define the ratio

$$ R_{c\bar{c}} = \frac{\sigma(e^+e^- \rightarrow b\bar{b}c\bar{c})}{\sigma(e^+e^- \rightarrow bb)} \quad (25) $$

According to our calculations, the value of $R_{c\bar{c}}$ is $R_{c\bar{c}} = 0.8 \cdot 10^{-2}$ for $Q^2 = s$ and $R_{c\bar{c}} = 2.6 \cdot 10^{-2}$ for $Q^2 = 4m_c^2$. These values obtained from the exact formulas of QCD perturbation theory should be compared with the estimate obtained from the Monte Carlo program HERWIG (version 5.0), $R_{c\bar{c}} = (0.8 \div 1.8) \cdot 10^{-2}$.

Figures 3a,b present the distributions over the variables $x = 2|\vec{p}|/\sqrt{s}$, where $|\vec{p}|$ is the quark momentum, for the b- and c-quark, respectively. One should pay attention to a completely different nature of the spectra for the b- and c-quarks. The b-quarks mainly have a large momentum, whereas c-quarks a small one, pointing once again to the dominating nature of diagrams with fragmentation of b-quarks.

Of course, the free b- and c-quarks are not observed experimentally because of confinement. In fact they catch the light quark and transform into $B$- and $D$-mesons. To estimate the total and differential inclusive cross-section of $B_c$-meson production in $p\bar{p}$-collisions the authors of [3] have used phenomenological model of the heavy quark fragmentation into hadrons. This model describes quite accurately $B$- and $D$-meson production. In this approach the fragmenta-
tion function of the \(i\)-th quark into meson, consisting of \(i\)- and \(j\)-quarks, has the form \[17\]

\[
D_{ij}(x) = N x^{-\alpha_i} (1 - x)^{-\alpha_j},
\]

where \(\alpha_i\) is the leading Regge trajectory intercept, connected with the quark of the \(i\)-type, \(\gamma = (1 \div 1.5)\) and \(N\) is the normalization coefficient:

\[
N = \frac{\Gamma(2 + \gamma - \alpha_i - \alpha_j)}{\Gamma(1 + \gamma - \alpha_j)\Gamma(1 - \alpha_i)}.
\]

(27)

The values of \(\alpha_c\) and \(\alpha_b\) lie in the range: \(\alpha_c = -(2 \div 2.5)\), \(\alpha_b = -(8 \div 9)\).

Let us take \(\alpha_c = -2.2\), \(\alpha_b = -8.5\) and \((\gamma - \alpha_j) = 1\). Then

\[
D_{b\rightarrow B_c}(x) = 99.75 x^{8.5} (1 - x),
\]

(28)

\[
D_{c\rightarrow B_c}(x) = 13.44 x^{2.2} (1 - x).
\]

(29)

Figures 3a,b present the spectra of \(B\)- and \(D\)-mesons, which were obtained by means of convolution of \(b\)- and \(c\)-quarks distributions with the fragmentation functions of \(b\)- and \(c\)-quarks into \(B\)- and \(D\)-mesons, respectively.

The analysis of angular correlations also discovers a different behaviour for \(b\bar{b}\) and \(c\bar{c}\)-pairs (see Fig.4a,b). \(b\)- and \(c\)-quarks emit mainly in opposite directions whereas \(c\)- and \(\bar{c}\)-quarks tend to be collinear. This can also be explained by the aforementioned behaviour of the gluon propagator for small virtualities.

### 3 The \(B_c\)-meson production cross-section

In this Section we discuss the production of the \(S\)-wave states of \(B_c\)-mesons. As to the study of the \(P\)-wave levels, we plan to make it in our next paper.

The diagrams describing the production of \(B_c\)-mesons are obtained from the diagrams of Fig.1a-d by combining two quark lines into mesonic one (see Fig.5a-d).

The underlying assumption of our calculations is that the binding energy for two quarks, \(b, c\), is much less than their masses and, hence, heavy quarks in the bound state \(B_c\) are actually on the mass shell. In this case the 4-momenta \(p_b\) and \(p_c\) of the quarks-constituents of \(B_c\)-meson are related with 4-momentum \(P\) of \(B_c\)-meson as follows:

\[
p_b = \frac{m_b}{M} P, \quad p_c = \frac{m_c}{M} P,
\]

(30)

where \(M = m_b + m_c\) is the \(B_c\)-meson mass. This assumption corresponds to the leading order in the effective theory of heavy quarks \[18\].

Let us also make use of the fact that the projection operators

\[
\frac{1}{\sqrt{2}} \left\{ v(p, +)\bar{u}(p, +) - v(p, -)\bar{u}(p, -) \right\} = \frac{1}{\sqrt{2}}(\hat{p} - M)\gamma^5
\]

(31)
\[
\frac{1}{\sqrt{2}} \{v(p, +)\bar{u}(p, +) + v(p, -)\bar{u}(p, -)\} = \frac{1}{\sqrt{2}}(\hat{p} - M)\bar{\varepsilon}^*(p, 0)
\]  \tag{32}
\[
v(p, -)\bar{u}(p, +) = \frac{1}{\sqrt{2}}(\hat{p} - M)\bar{\varepsilon}^*(p, +)
\]  \tag{33}
\[
v(p, +)\bar{u}(p, -) = \frac{1}{\sqrt{2}}(\hat{p} - M)\bar{\varepsilon}^*(p, -)
\]  \tag{34}

where
\[
\varepsilon^\mu(p, 0) = \frac{E}{M|\vec{p}|} \left\{ \frac{|\vec{p}|^2}{E}, p_x, p_y, p_z \right\}
\]  \tag{35}
\[
\varepsilon^\mu(p, +) = N \left\{ 0, \varepsilon_x, \varepsilon_y, \varepsilon_z \right\}
\]  \tag{36}
\[
\varepsilon^\mu(p, -) = N \left\{ 0, -\varepsilon_x^*, -\varepsilon_y^*, -\varepsilon_z^* \right\}
\]  \tag{37}

and
\[
N = \frac{1}{2\sqrt{2}|\vec{p}|(|\vec{p}| + p_z)}
\]  \tag{38}
\[
\varepsilon_x = -(|\vec{p}| + p_z)^2 + (p_x + ip_y)^2
\]  \tag{39}
\[
\varepsilon_y = -[(|\vec{p}| + p_z)^2 + (p_x + ip_y)^2]i
\]  \tag{40}
\[
\varepsilon_z = 2(|\vec{p}| + p_z)(p_x + ip_y)
\]  \tag{41}

separate the states with the specified value of the total spin \(S\) and its projection \(S_z\) on axis \(z\) of the system described by the spinors \(\bar{u}(p)\) and \(v(p)\) (\(\varepsilon^\mu(p, \lambda)\) is the polarization vector of the vector state with momentum \(p\) and helicity \(\lambda\)). Then the amplitude of the process \(e^+e^- \rightarrow b\bar{b}c\bar{c}\) corresponding to the diagrams of Fig.5 can be expressed through these projection operators and the helicity amplitudes \(M_{h\bar{h}}(\lambda_i)\) found in Section 1 as follows:

\[
M(\lambda_i) = \sqrt{\frac{2M}{2m_b\sqrt{2m_c}}} \Psi(0) \sum_{h, \bar{h}} P_{h, \bar{h}} M_{h, \bar{h}}(\lambda_i)
\]  \tag{42}

where summation is made over the helicity states \(h, \bar{h}\) of the quark and antiquark producing \(B_c\)-meson. The helicities of the remaining fermions are symbolically noted through \(\lambda_i\). The projection operators \(P_{h, \bar{h}}\) have the following explicit form (\(H = h - \bar{h}\)) \(^{19}\):

\[
P_{h, \bar{h}} = \frac{1}{\sqrt{2}} (-1)^{\bar{h}-1/2} \delta_{H,0}
\]  \tag{43}
Table 2: The cross section for the production of $B_c$ ($B_c^*$)-mesons for $\sqrt{s} = M_Z$ and $\alpha_s(Q^2 = M_Z^2) = 0.12$. Bracketed is the error (one standard deviation) in the last digital, caused by Monte Carlo method

| $n^{25}\pm L_J$ | $^1S_0$ | $^1S_1$ | $^2S_0$ | $^2S_1$ |
|-----------------|---------|---------|---------|---------|
| $\sigma$, pb    | 0.924(1)| 1.285(1)| 0.237(3)| 0.3168(3)|

for the $^1S_0$-state,

$$P_{h,\bar{h}} = |H| + \frac{1}{\sqrt{2}}\delta_{H,0}$$

(44)

for the $^3S_1$-state.

Taking into account that the colour part of the $B_c$-meson wave function is $\delta_{ij}/\sqrt{N}$ the colour factor $F$, corresponding to the process $e^+e^- \rightarrow B_c\bar{B}_c$, is $F = (N^2 - 1)/4N^2$. The value of the wave function at zero, $\Psi(0)$, is calculated in nonrelativistic potential model and also in the QCD sum rules [10] and is related with the decay constant $f_{B_c}$ of the pseudoscalar $(0^-)$ $B_c$-meson and the constant $f_{B_c^*}$ of the vector $(1^-)$ $B_c^*$-meson as follows:

$$\Psi(0) = \sqrt{\frac{M}{12}} f_{B_c},$$

(45)

where

$$f_{B_c} = f_{B_c^*} = 570.$$  

(46)

The potentials of various types yield actually the same values of masses of lower states of $B_c$-mesons. For example, the mass of pseudoscalar $0^-$-meson ($1S$-state) is $M = 6.3$ GeV [7]. In this connection, when calculating the bound states of $B_c$-mesons of $1S$ levels we take the values of the masses of $b$- and $c$-quarks somewhat larger than those obtained during the production of free $bbcc\bar{c}$-quarks and equal $m_b = 4.8$ GeV and $m_c = 1.5$ GeV. The mass of $2S$ levels is predicted to be $M = 6.9$ GeV [4]. In this case the masses of $b$- and $c$-quarks are taken to be still larger, $m_b = 5.1$ GeV and $m_c = 1.8$ GeV. We would like to note that the authors of paper [3] do not take into account the effective increase of the masses of the $b$- and $c$-quarks, constituents of $B_c$-mesons, during the transition to higher excited states of $B_c$-meson. However, this must be done within the frames of the “on mass shell” formalism described above. The value of the wave function at zero, $\Psi(0)$ for $2S$-states is $\Psi(0) = 275$ MeV$^{3/2}$ [3].

Table 2 presents the calculated cross sections for the production of $B_c$- and $B_c^*$-mesons including the first excited levels.

By analogy to what has been said in Section 2 regarding the dependence of the cross-section for the process $e^+e^- \rightarrow bbcc\bar{c}$ on the square of the momentum transferred $Q^2$ in the argument of the function $\alpha_s(Q^2)$, similar statement for the
process $e^+e^- \to B_c \bar{b}c$ also leads to an essential uncertainty of the predictions regarding the cross-section for the $B_c$-mesons production. In this connection, the cross-sections for the process $e^+e^- \to B_c \bar{b}c$ calculated for $Q^2 = s = M_Z^2$ can be interpreted as pessimistic. More optimistic predictions for the number of $B_c$-mesons to be produced can be obtained if as the square of the momentum transferred $Q^2$ we take $Q^2 = 4m_{c}^2$. In this case the cross section for the production of $B_c$-mesons is $[\alpha_s(4m_{c}^2)/\alpha_s(s)]^2 = 3.4$ times larger.

The above total cross section of $1S$- and $2S$-states, that is equal to 2.76 pb, is clearly a lower estimate in these calculations. Really, the total number of the bound states of $B_c$-mesons lying below the production threshold for $B$- and $D$-mesons is about 15. All these excited $B_c$-states with probability equal to 1 transform into $B_c$-meson due to cascade radiative decays. Therefore higher excited states, not only $1S$ and $2S$, can make an essential contribution at least due to their large number. If we take into account the production of $\bar{B}_c$-mesons, the above cross-section for the production of $B_c$-mesons should be doubled.

Let us define the ratio of the number of the produced $B_c(\bar{B}_c)$-mesons, including also the $B'_c(\bar{B}'_c)$-states calculated above:

$$R_{B_c} = \frac{\sigma(e^+e^- \to B_c \bar{b}c) + \sigma(e^+e^- \to \bar{B}_c bc)}{\sigma(e^+e^- \to bb)} . \quad (47)$$

According to our calculations, the value of $R_{B_c}$ is $R_{B_c} = 0.6 \cdot 10^{-3}$ for $Q^2 = s$ and $R_{B_c} = 2.0 \cdot 10^{-3}$ for $Q^2 = 4m_{c}^2$. These values obtained from the formulas of QCD perturbation theory should also be compared with the estimate derived with the help of the Monte Carlo program HERWIG, $R_{B_c} = (0.1 \div 1.0) \cdot 10^{-3}$.

The authors of [5] have recently considered single production of $B_c$-mesons in $e^+e^-$-annihilation and obtained the analytical form for the fragmentation function $D(x)$ of $b$-quark into $B'_c$-meson. Really, in the limit of $s \to \infty$, one can neglect the terms of the order of $M_2/s$ and higher. Then the differential cross section is equal to

$$\frac{d\sigma(B_{c}^{(s)})}{dz} = N^{(s)} \frac{z(1-z)^2}{(1-rz)^6} \left( f_1^{(P,V)}(z) + f_2^{(P,V)}(z) + f_3^{(P,V)}(z) \right) , \quad (48)$$

where for the pseudoscalar state one has

$$f_1^{(P)}(z) = 2 + (2 - 12r)z + \left( \frac{2}{3}r - \frac{14}{3}r^2 - 12r^3 \right)z^3 + \left( r^2 + 2r^3 + 2r^4 \right)z^4 ,$$

$$f_2^{(P)}(z) = 2z^2(1 - rz)^2 ,$$

$$f_3^{(P)}(z) = 2z(1 - rz) \left( 2 + (1 - 6r)z + (r + 2r^2)z^2 \right) , \quad (49)$$

and for the vector state one gets

$$f_1^{(V)}(z) = 2 + (-2 - 4r)z + (3 - 6r + 12r^2)z^2 +$$
\[(2r - 6r^2 - 4r^3)z^3 + (3r^2 - 2r^3 + 2r^4)z^4,
\]
\[f_2^{(V)}(z) = 6z^2(1 - rz)^2,
\]
\[f_3^{(V)}(z) = 6z^2(1 - rz)(1 - 2r + rz),
\]

where \(r = m_b/M_{BC}^1\) and the calculations have been performed in the covariant gauge, so that the \(f_{1,2}^{(P,V)}(z)\) functions correspond to the squared diagrams shown on Figs.1c, 1d, and the \(f_3^{(P,V)}(z)\) function corresponds to the interference of those diagrams. In the paper [5], the expression for the fragmentation of the \(b\)-quark into the vector \(B_c^+\) state coincides with our one. However, the fragmentation function into the pseudoscalar meson is not correct in ref. [5].

Note, the scaling results (48-50) are in a good agreement with the exact perturbative calculations at \(\sqrt{s} = m_Z\) (see Figs.6a, 6b). However, the nonscaling contributions of the order of \(M^2/s\) may be essential for the both low energies and high mass of the meson like in the case of the \(b\)-quark fragmentation into the \(\Upsilon\)-particle at the \(Z\)-pole energy.

Figures 6a,b show our fragmentation functions \(D(x) = \frac{1}{\sigma} \frac{d\sigma}{dx}\) of \(b\)-quark into \(B_c\)-mesons, where \(x = 2|p|/\sqrt{s}\) and \(|p|\) is the \(B_c\)-mesons momentum, for the production of pseudoscalar (0\(^-\)) and vector (1\(^-\)) \(B_c\)-mesons. The fragmentation function for vector mesons (see Fig.6b) as compared with that for pseudoscalar ones (see Fig.6a) has a somewhat sharper peak in the region \(x = 0.8\) and a slight dip on the left of the peak.

Using our computer program it is also easy to obtain some other distributions. Some of them, the most interesting ones, are presented below.

Figures 7a,b show the momentum distributions of the associative \(c\)-quark for the case of the production of pseudoscalar and vector \(B_c\)-mesons. It is easy to understand that they are related with the distributions of Fig.6a,b by a simple relationship: \(D(x_{B_c}) \approx D(1 - x_c)\).

Figure 8 presents the distributions in the emission angle between the pseudoscalar \(B_c\)-meson and \(c\)-quark. As is seen from this figure, the overwhelming number of quarks emit at small angles (\(\leq 30 - 40^o\)) with respect to the direction of the motion of \(B_c\)-mesons, or, as it follows from Fig.9, at small transverse momenta, \(\leq 5 - 10\) GeV, with respect to the same direction.

The distribution over the emission angle of \(B_c(B_c^\ast)\)-mesons with respect to the beam axis (see Fig.10) can be interesting because of the effect of the acceptance of real detector on the number of \(B_c(B_c^\ast)\)-mesons to be produced.

Figs.8-10 are shown for the case of the production of pseudoscalar (0\(^-\)) \(B_c\)-mesons. The distributions for the case of the production of vector (1\(^-\)) \(B_c^\ast\)-mesons have the same form, differing from the previous cases in the normalization only.

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\(^1\) In the symmetric \(cc\bar{c}\bar{c}\) and \(bb\bar{b}\bar{b}\) case, when the unflavoured mesons are produced (\(\psi, \eta_c, \Upsilon, \eta_b\)), the fragmentation function may be obtained from eqs. (48-50) by the substitution \(r = 1/2\).
4 Discussions

So, we have calculated in our paper the cross-section for the production of four quarks of different flavours, $b\bar{b}c\bar{c}$, in $e^+e^-$-annihilation. At the $Z^0$ peak it is $70 \div 234 \text{ pb}$ and depends upon the choice of the square of the transferred momentum in the argument $\alpha_s(Q^2)$. When recalculated for the number of events, it makes up $R_{c\bar{c}} = (0.8 \div 2.6) \cdot 10^{-2}$ from the production of $b\bar{b}$ pairs or $(1.2 \div 3.9) \cdot 10^{-3}$ from the total number of $Z^0$-bosons.

Since the total number of $Z^0$-boson expected from all experiments by the end of 1994 is about $2 \cdot 10^7$, this corresponds to $(2.4 \div 7.8) \cdot 10^4$ events with the production of $b\bar{b}c\bar{c}$-quarks. The experimental study of this process may be considered as a way to verify the QCD predictions in higher orders of perturbation theory.

However, the process $e^+e^- \rightarrow b\bar{b}c\bar{c}$ is also interesting from another viewpoint. The bound states of $b$- and $c$-quarks, i.e. $B_{c^-}$-mesons, predicted by the theory have not yet been discovered experimentally. Meanwhile, the discovery and study of $B_{c^-}$-mesons may yield valuable information on the behaviour of the quark potential in the region intermediate with respect to the families of $J/\psi$- and $\Upsilon$-mesons. Assuming the existence of quark-hadron duality, one may relate the cross-section for the production of the $(b\bar{c})$-system singlet in colour (the colour factor is $F = (N_c^2 - 1)^2/4N_c^2$) in the region of small invariant masses, $M_{b\bar{c}}$, with the cross-sections for the production of the bound states of $B_{c^-}$-mesons as follows:

$$
\int_{m_0^2}^{M_{c^-}^2\text{thresh}} \frac{d\sigma(e^+e^- \rightarrow b\bar{b}c\bar{c})_{(b\bar{c})-\text{sing}}}{dM_{b\bar{c}}^2} dM_{b\bar{c}} = \sum \sigma(e^+e^- \rightarrow B_c\bar{b}c),
$$

(51)

where $m_0 = m_b + m_c \leq M_{b\bar{c}} \leq M_B + M_D + \Delta M = M_{c^-}\text{thresh}(\Delta M \simeq 0.5 \div 1 \text{ GeV})$. Taking $m_0 = 6.1 \text{ GeV}$ as the boundary values of invariant masses and, for example, $M_{c^-}\text{thresh} = 8 \text{ GeV}$, we obtain the estimate for the cross-section of the $B_{c^-}$-meson production, equal to $2.07(4) \div 6.9(1) \text{ pb}$. This estimate should be compared with a more accurate calculation of the cross-section for the production of $B_{c^-}$-mesons obtained on the basis of the formalism expounded in Section 3. The cross-section for the production of $B_{c^-}$-meson and of its first excitations, 1S- and 2S-states, without consideration for other, (about 10), states, whose contribution, according to our estimates, is suppressed as compared with the contribution from 1S- and 2S-states, is $2.76 \div 9.3 \text{ pb}$. This is in a reasonable agreement with the value following from relationship (48) assuming quark-hadron duality.

The share of events with the production of $B_{c^-}$-mesons is $R_{B_c} = (0.6 \div 2.0) \cdot 10^{-3}$ from the production of $b\bar{b}$ pairs or $(0.9 \div 3.0) \cdot 10^{-4}$ from the total number of events. As expected, $2 \cdot 10^7 Z^0$-bosons will be produced at LEP. This means that the number of events with the production of $B_{c^-}$-bosons, also including
those with the production of $B_c$-mesons, will be $(1.8 \div 6.0) \cdot 10^3$. Of course, the real number of experimentally reconstructed events will be less because one should take into account the branchings of $B_c$-meson decays into specific modes. As differed from $J/\psi$- and $\Upsilon$-mesons, $B_c$-mesons do not have decays of annihilation type into 2 or 3 gluons and decay into a weak channel. The decay $B_c \rightarrow J/\psi + X$ with a branching of $\sim 30\%$ and further decay of $J/\psi$ into two muons with a branching of $6\%$ is the most promising in terms of experimental identification of the final $30 \div 110$ reconstructed events with the production of $B_c(\bar{B}_c)$-mesons. $B_c$-mesons have quite a long lifetime and, hence, a long decay length, that may be helpful for identifying the vertex of $B_c$-meson decay. The availability of vertex detectors and separation of hard leptons in the jets from the decays of $b$- and $c$-quarks in the process $e^+e^- \rightarrow B_c\bar{B}_c$ might also be helpful for identifying the final state, specific for the process with the production of $B_c$-mesons. However, one should take into account that this channel of $B_c$-meson decay has a noticeable background coming from the decays of ordinary (noncharmed) rather than charmed $B$-mesons into $J/\psi$-mesons. Therefore a thorough analysis of this background is required for separation of the events with the production of $B_c$-mesons.

It is interesting to note that the cross section of the production of a $(bc)$-pair with an invariant mass of a $M_{bc}$ pair in the region $m_0 \leq M_{bc} \leq M_{thresh}$ (see formula (51)) is also a few $pb$. It is natural to assume that such a pair of $bc$-quarks, catching a light quark can fragment with a high probability into a colourless object, $\Lambda_{bc}$-hyperon, decaying further in a cascade way. According to our estimates, there should be some thousands of such events per $2 \cdot 10^7 Z^0$-bosons. Observation of new hyperon $\Lambda_{bc}$ and of $B_c$-meson is an interesting and quite realistic problem of the nearer future.

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Figure 1: Feynmann diagrams for the process $e^+e^- \rightarrow b\bar{b}c\bar{c}$.

Figure 2: a) Distribution over invariant mass $M_{c\bar{c}} = \sqrt{(p_c + p_{\bar{c}})^2}$ of $c\bar{c}$-pair. b) the same as in a) but for $M_{b\bar{b}} = \sqrt{(p_b + p_{\bar{b}})^2}$ of $b\bar{b}$-pair.

Figure 3: a) Distributions over $x = 2|\vec{p}|/\sqrt{s}$, where $|\vec{p}|$ is the $b$-quark momentum (solid line) or $B$-meson momentum (dotted line). b) The same as in a), where $|\vec{p}|$ is the $c$-quark momentum (solid line) or $D$-meson momentum (dotted line).

Figure 4: a) Distribution over the angle between the directions of motions of $b$- and $\bar{b}$-quarks. b) The same as in a) but for $c$- and $\bar{c}$-quarks.

Figure 5: Feynmann diagrams for the process $e^+e^- \rightarrow B_c\bar{b}c$.

Figure 6: a) Fragmentation function of pseudoscalar $B_c$-meson vs. the variable $x = 2|\vec{p}|/\sqrt{s}$, where $|\vec{p}|$ is $B_c$-meson momentum. b) The same as in a) but for vector $B_c^*$-meson. The curves are the scaling expressions (48-50).

Figure 7: a) Distribution over variable $x = 2|\vec{p}|/\sqrt{s}$, where $|\vec{p}|$ is $c$-quark momentum for the case of the pseudoscalar $B_c$-meson production. b) The same as in a) but for the vector $B_c^*$-meson production.

Figure 8: Distribution over the angle between the directions of motions of $B_c$-meson and $c$-quark.

Figure 9: Distribution over the transverse momentum of $c$-quark relatively to the direction of motion of $B_c$-meson.

Figure 10: Distribution over the angle of $B_c$-meson with respect to the $e^-$-beam direction.