Strangeness production in high-multiplicity events

Marat Siddikov, Iván Schmidt
Departamento de Física, Universidad Técnica Federico Santa María,
y Centro Científico - Tecnológico de Valparaíso, Casilla 110-V, Valparaíso, Chile
(Dated: January 5, 2021)

In this paper we analyze in detail the production of strangeness in proton-proton collisions in the kinematics of large transverse momenta $p_T$ of produced hadrons. Using the color dipole framework, we estimated the production cross-sections for kaons and demonstrated that the shapes of the $p_T$-dependence are in agreement with available experimental data. We also analyzed the self-normalized yields of strange hadrons as a function of multiplicity of co-produced hadrons, and found that the predictions are in agreement with the faster-than-linear growth seen in experimental data. Our description is largely parameter-free and complements our previous studies dedicated to the explanation of multiplicity enhancement of quarkonia, as well as $D$- and $B$-mesons.

Keywords: CGC approach, strangeness production, multiplicity dependence.

I. INTRODUCTION

Since the early experiments at RHIC and SPS [1-6], the production of hadrons containing strange quarks has been used as one of the probes of Quark-Gluon Plasma (QGP) formation, described in the framework of QGP-inspired hydrodynamic models [7-21]. The QGP manifests itself in different ways in experimental observables. For example, it might lead to an enhancement of strange particle yields in heavy ion collisions, compared to $pA$ and $pp$ production of the same strange hadrons. Another possibility to observe the effects of QGP is via the enhancement of strangeness in events with large multiplicity of co-produced hadrons. This observable might be studied independently in $pp$, $pA$ or $AA$ collisions. While early experiments confirmed the enhancement of strangeness in heavy ion collisions, similar multiplicity enhancements has been recently observed at the LHC not only in heavy ion, but also in $pA$ [22, 23] and even in $pp$ collisions [24], where QGP formation in significant amounts is highly unlikely even at TeV-range collision energies. For this reason it makes sense to understand better the microscopic mechanisms of this phenomenon, at least in $pp$ collisions. In general, application of perturbation theory for strangeness production is challenging due to lack of the hard scale (like e.g. heavy mass of the quark). Therefore the phenomenological description of strangeness production has been mostly limited to studies in the framework of Monte-Carlo generators [25, 26], and inevitably includes additional model-dependent assumptions.

Recently detailed studies of heavy quarkonia [27, 28] and open heavy flavor mesons [29] in $pp$ collisions have discovered that similar enhancement with multiplicity also happens for the production of heavier quarks, charm and bottom. This enhancement has a quite complicated dependence on the rapidity separation of the bins used to collect quarkonia and light particles, on the existence of rapidity gaps between the heavy hadrons and colliding protons [30], as well as (possibly) on the quantum numbers of the produced quarkonia states [31]. While the production of these mesons can be described in the two-pomeron fusion picture [32, 33], as was pointed out in [25], the description of the multiplicity dependence presents challenges for the established two-pomeron paradigm. For this reason a number of new mechanisms have been suggested for its description: e.g. the percolation approach [34], a modification of the slope of the elastic amplitude [35] or contributions of multipomeron diagrams [36, 37].

In view of the similarity of multiplicity enhancements observed in strange, charm and bottom sectors, it is very desirable to describe the phenomenon for all flavors in the same framework. Since the strange quarks have very light mass, in general it is very challenging to apply the theoretical tools which rely on the perturbative QCD or heavy quark mass limit for their justification. Nevertheless, in the kinematics of very large transverse momenta $p_T$ of produced strange hadrons, the latter variable effectively plays the role of hard scale which partially justifies the use of such perturbative tools.

In what follows we will use the color dipole (CGC/Saturation) approach [38-59] which was previously applied to $D$- and $B$-meson production in [60, 61]. The generalization of this framework to high-multiplicity events is well-known from the literature [62, 63] and allows to explain the multiplicity dependence observed in both the charm and bottom sectors. Our analysis will be mostly focused on the production of kaons and $\Lambda$-baryons, due to lack of information about fragmentation functions of other strange hadrons.

The paper is structured as follows. In the next Section II we describe a framework for strangeness production in the CGC/Sat approach. In Section III we make numerical estimates for the cross-sections and compare with available experimental data. In Section IV we discuss the multiplicity dependence of strange hadrons in the large-$p_T$ kinematics and demonstrate that our approach can describe the experimentally observed dependence for kaons and $\Lambda$-baryons. Finally, in Section V we draw conclusions.
Figure 1: Left plot: The leading order two-pomeron mechanism of quark pair production. The diagram includes two cut pomerons (upper and lower gluon ladders). Right plot: Example of 3-pomeron mechanisms which might be relevant in the small-\(p_T\) kinematics (additional reggeons shown with gray color, not considered in this paper for the reasons discussed in the text). In all plots the vertical dashed line stands for unitarity cuts. A summation over all possible permutations of gluon vertices in the quark line (loop) is implied.

II. PRODUCTION OF STRANGE HADRONS VIA FRAGMENTATION

We assume that all strange hadrons are produced via a fragmentation mechanism, and we will perform our evaluations within the framework developed earlier in [60–64]. In this approach the cross-section is related to the quark pair \(\bar{Q}Q\) production cross-section by

\[
\frac{d\sigma_{pp \to M+X}}{dy d^2p_T} = \sum_i \int_{x_1}^{1} \frac{dx_Q(y)}{z^2} D_i \left(\frac{x_Q(y)}{z}\right) \frac{d\sigma_{pp \to \bar{Q}Q_i+X}}{dy^* d^2p_{T}^*}.
\]

where \(y\) is the rapidity of the produced strange hadron, \(y^* = y - \ln z\) is the rapidity of the quark, \(p_T\) is the transverse momentum of the produced strange hadron, \(D_i(z)\) is the fragmentation function which describes the formation of a given final state from a parton of flavor \(i\), and \(d\sigma_{pp \to \bar{Q}Q_i+X}/dy^*\) is the cross-section of quark pair production with quark rapidity \(y^*\) and transverse momentum \(p_T^* = p_T/z\). For the fragmentation functions of kaons and \(\Lambda\)-baryons we will use the expressions available from the literature (see the Appendix [A] for details). Up to the best of our knowledge, currently there is no data for the fragmentation functions for \(\Omega\), \(\Xi\)-baryons and \(\phi\)-mesons, for this reason we will not consider them in what follows. Naturally, the dominant contribution in the strange sector stems from the strange quarks, although there are also contributions from other flavors. In what follows we will focus on the evaluation of the cross-section \(d\sigma_{pp \to \bar{Q}Q_i+X}/dy^* d^2p_{T}^*\) which appears in the integrand of (1).

In high energy kinematics, the inclusive production gets its dominant contribution from the fusion of two pomerons, which for heavy quarkonia production is given by the diagram shown in the left panel of Figure 1. In the rest frame of one of the protons, this process might be viewed as a fluctuation of the incoming virtual gluon into a heavy \(\bar{Q}Q\) pair, with subsequent scattering of the \(\bar{Q}Q\) dipole on the target proton. In the kinematics of LHC experiments the average light-cone momentum fractions \(x_{1,2}\) carried by gluons are very small (\(\ll 1\)), and the gluon densities are enhanced. This enhancement implies that there could be sizable corrections from multiple pomeron exchanges between the heavy dipole and the target, which are formally suppressed for small dipoles. For this reason, instead of a hard process on individual partons it is more appropriate to use the color dipole framework (also known as CGC/Sat) [51–59]. At high energies, the color dipoles are eigenstates of interaction, and therefore they can be used as universal elementary building blocks, automatically accumulating both the hard and soft fluctuations [72]. In fact, the light-cone color dipole framework has been successfully applied to phenomenological descriptions of both hadron-hadron and lepton-hadron collisions [73–80]. Another advantage of the CGC/Sat framework is that it allows a relatively straightforward extension for the description of high-multiplicity events, as discussed in [51–71].
In the dipole approach, the quark production cross-section is given by [63–64]

\[
\frac{d\sigma_{pp\to Q\bar{Q}+X}}{dy \, d^2p_T} = \int d^2k_T \, x_1 \, g(x_1, p_T - k_T) \int_0^1 \, dz \int_0^1 \, dz' \times \frac{d^2z}{4\pi} \frac{d^2r_2}{4\pi} \, e^{i(r_1 - r_2) \cdot k_T} \Psi_{Q\bar{Q}}^\dagger(r_2, z, p_T) \Psi_{Q\bar{Q}}^\dagger(r_1, z, p_T) \times N_M(x_2(y); \bar{r}_1, \bar{r}_2),
\]

\[
x_{1,2} \approx \frac{\sqrt{m_M^2 + (p_{1,2}^T)^2}}{s} \, e^{\mp y}
\]

where \( y \) and \( p_T \) are the rapidity and transverse momenta of the produced strange quark in the center-of-mass frame of the colliding protons; \( k_T \) is the transverse momentum of the strange quark with respect to incident gluon \( g(x_1, p_T) \) in the first line of (2) is the unintegrated gluon PDF; \( \Psi_{g\to Q\bar{Q}}(r, z) \) is the light-cone wave function of the \( QQ \) pair with transverse separation between quarks \( r \) and the light-cone fraction carried by the quark \( z \). In general this is a nonperturbative object, and there is no model-independent way to evaluate it [101]. For this reason, in what follows we will restrict our consideration to the kinematics of large transverse momenta of produced hadrons, which in view of (2) implies that typical sizes of the dipoles are also small, \( \sim 1/p_T \). In this kinematics we may use standard perturbative expressions [81–82]

\[
\Psi_T^\dagger(r_2, z, Q^2) \, \Psi_T(r_1, z, Q^2) = \frac{\alpha_s N_c}{2 \pi^2} \left\{ \frac{e^2}{2} K_1(\epsilon_f r_1) K_1(\epsilon_f r_2) \left[ e^{i\theta_1 z} + e^{-i\theta_1 (1-z)^2} \right] \right. + m_f^2 K_0(\epsilon_f r_1) K_0(\epsilon_f r_2) \right\},
\]

\[
\Psi_L^\dagger(r_2, z, Q^2) \, \Psi_L(r_1, z, Q^2) = \frac{\alpha_s N_c}{2 \pi^2} \left\{ 4Q^2 z^2(1-z)^2 K_0(\epsilon_f r_1) K_0(\epsilon_f r_2) \right\},
\]

\[
e^2 = z(1-z) Q^2 + m_f^2
\]

\[
\left| \Psi^{(f)}(r, z, Q^2) \right|^2 = \left| \Psi_T^{(f)}(r, z, Q^2) \right|^2 + \left| \Psi_L^{(f)}(r, z, Q^2) \right|^2
\]

The meson production amplitude \( N_M \) depends on the mechanism of \( \bar{Q}Q \) pair formation. For the case of the two-pomeron fusion, in leading order it is given by [60–63]

\[
N_M(x, \bar{r}_1, \bar{r}_2) = -\frac{1}{2} N(x, \bar{r}_1 - \bar{r}_2) - \frac{1}{16} [N(x, \bar{r}_1) + N(x, \bar{r}_2)] - \frac{9}{8} N(x, \bar{z}(\bar{r}_1 - \bar{r}_2))
\]

\[
+ \frac{9}{16} [N(x, \bar{z} \bar{r}_1 - \bar{r}_2) + N(x, \bar{z} \bar{r}_2 - \bar{r}_1) + N(x, \bar{z} \bar{r}_1) + N(x, \bar{z} \bar{r}_2)],
\]

where \( N(x, \vec{r}) \) is the color singlet dipole scattering amplitude. In the LHC kinematics at large transverse momenta (our principal interest) the natural choice of the saturation scale is \( \mu_F \sim p_T \), which significantly exceeds the saturation scale \( Q_s(x) \). This finding justifies the use of two-pomeron approximation. However, in the kinematics of smaller \( p_T \), potentially there could be multipomeron contributions, like those shown in the right panel of Figure 1. We will not consider such contributions since, as we mentioned earlier, we can’t describe the small-\( p_T \) region, due to lack of the nonperturbative photon wave function \( \Psi_{QQ} \).

The unintegrated gluon PDF, which appears in the prefactor of (2), can be related to the integrated PDF \( x g(x, \mu_F) \) as [63]

\[
x g(x, k^2) = \frac{\partial}{\partial \mu_F^2} xG(x, \mu_F) \bigg|_{\mu_F^2 = k^2},
\]

and the latter is closely related to the dipole scattering amplitude \( N(y, r) = \int d^2b \, N(y, r, b) \) via a set of identities [64–64]

\[
\frac{C_F}{2\pi^2 \alpha_s} N(y, r) = \int \frac{d^2k_T}{k_T^2} \phi(y, k_T) \left( 1 - e^{i\vec{r} \cdot \vec{x}} \right); \quad xG(x, \mu_F) = \int_0^{\mu_F^2} \frac{d^2k_T}{k_T^2} \phi(x, k_T),
\]
where \( y = \ln(1/x) \). The Eq. (10) can be inverted and gives the gluon uPDF in terms of the dipole amplitude,

\[
xG(x, \mu_F) = \frac{C_F \alpha_s}{2 \pi^2 \alpha_s} \int d^2 r \frac{J_1(r \mu_F)}{r} \nabla_r^2 N(y, \vec{r}).
\]

This allows to rewrite the result in a symmetric and self-consistent form, which allows straightforward generalization for high-multiplicity events.

### III. NUMERICAL RESULTS

For the sake of definiteness, in our numerical evaluations we will take the “CGC” parametrization of the dipole cross-section \( [85][87] \),

\[
N(x, \vec{r}) = \sigma_0 \times \left\{ \begin{array}{ll}
N_0 \left( \frac{r Q_s(x)}{2} \right)^{2 \gamma_{\text{eff}}(r)}, & r \leq \frac{2}{Q_s(x)} \\
1 - \exp(-A \ln(Br Q_s)), & r > \frac{2}{Q_s(x)}
\end{array} \right.,
\]

\[
A = \frac{N_0^2 \gamma_s}{(1 - N_0)^2 \ln(1 - N_0)}, \quad B = \frac{1}{2} (1 - N_0)^{-\frac{1}{2} N_0},
\]

\[
Q_s(x) = \left( \frac{x_0}{x} \right)^{\lambda/2}, \quad \gamma_{\text{eff}}(r) = \gamma_s + \frac{1}{\kappa \chi} \ln \left( \frac{2}{r Q_s(x)} \right),
\]

\[
\gamma_s = 0.762, \quad \lambda = 0.2319, \quad \sigma_0 = 21.85 \text{mb}, \quad x_0 = 6.2 \times 10^{-5}
\]

which is widely used in the literature, and in what follows we will also focus on \( K^0 \)-meson production. The fragmentation functions for \( K^\pm \) and \( K^0 \) are constrained by the isospin symmetry relation \( [88] \),

\[
D^i_{K^0} (z, \mu^2) = \frac{1}{2} D^i_{K^\pm} (z, \mu^2),
\]

and for this reason the cross-sections for \( K^0 \) and \( K^\pm \) production are proportional to each other with very good precision. For \( \Lambda \)-baryons we found only one parametrization for the fragmentation function only \( [88] \), and as shown in Appendix (A), at large-\( z \) its strange flavor component is approximately proportional to that of kaons; therefore the \( \Lambda \)-baryon cross-sections are proportional to that of kaons in the large-\( p_T \) kinematics.

In the left panel of Figure 2 we show the results for \( K^0 \) production cross-section, in the kinematics of ongoing and planned experiments. As can be seen in the literature, all the experimental papers contain results for the self-normalized yields \( N_{ev}^{-1} dN/dy dp_T \), instead of cross-sections. The normalization parameter \( N_{ev} \) is chosen as the total number of events \( N_{ev} \sim \int dp_T dN/dp_T \) or the number of non-single-diffractive events \( N_{NSD} \). The self-normalized yields are difficult to describe in our approach, because the normalization coefficient gets its dominant contribution from the small-\( p_T \) region and thus cannot be evaluated reliably. This introduces a normalization uncertainty in our evaluations of such self-normalized yields. In Figure 3 we show the calculated yields in comparison with available experimental data from ALICE \( [89] \), CMS \( [90] \), CDF \( [91] \) and STAR \( [92] \) collaborations \( [102] \). The STAR data \( [92] \) were included by authors of \( [88] \) in their global fit of fragmentation functions, for this reason the description of these data is nearly perfect. For data from LHC and Tevatron the model provides a very reasonable description of the shape, although, as expected, there is a mismatch in the normalization by a factor of two. The theoretical shape of the \( p_T \)-dependence starts deviating from the experimental data in the region \( p_T \lesssim 2 \text{GeV} \), where nonperturbative effects become pronounced.

### IV. MULTIPLICITY DEPENDENCE

Since the dipole approach \( [8][8][12] \) provides a reasonable description of the strangeness production in the large-\( p_T \) kinematics, we can apply it to the study of the dependence on the number of charged particles \( N_{ch} \) co-produced together with a given strange hadron. The extension of the color dipole (CGC/Sat) framework to the description of high-multiplicity events is quite straightforward, as was discussed in \( [30][45][60][64][71] \). In what follows we will briefly summarize the main results which will be used for the phenomenological estimates of the multiplicity dependence. In view of the Local Parton Hadron Duality (LPHD) hypothesis \( [93][94] \), the multiplicity of produced hadrons in a given event is directly proportional to the number of partons produced in a collision. For this reason, the study of high
multiplicity events in different channels allows to understand better the onset of the saturation regime in high energy collisions.

The probability of multiplicity fluctuations decreases rapidly as function of the number of produced charged particles \(N_{\text{ch}}\) [96], for this reason in the study of the multiplicity dependence it is more common to use a self-normalized ratio [89]

\[
\frac{dN_M}{dy} = \frac{d\sigma_M(y, \eta, \sqrt{s}, n)}{d\sigma_M(y, \eta, \sqrt{s}, \langle n \rangle = 1)} / \frac{d\sigma_{\text{ch}}(\eta, \sqrt{s}, Q^2, n)}{d\sigma_{\text{ch}}(\eta, \sqrt{s}, Q^2, \langle n \rangle = 1)}
\]

where \(n = N_{\text{ch}}/\langle N_{\text{ch}} \rangle\) is the relative enhancement of the number of charged particles in a given observation window (e.g. pseudorapidity bin \((\eta - \Delta \eta/2, \eta + \Delta \eta/2)\)); \(d\sigma_M(y, \sqrt{s}, n)\) is the strange hadron \(M\) production cross-section, with rapidity \(y\) and \(N_{\text{ch}} = n \langle N_{\text{ch}} \rangle\) charged particles; \(d\sigma_{\text{ch}}(\eta, \sqrt{s}, n)\) is the total production cross-section for \(N_{\text{ch}} = n \langle N_{\text{ch}} \rangle\) charged particles in the same observation window. Since the cross-sections are proportional to the probability to produce a given final state, the ratio (17) might be interpreted as a conditional probability to produce a strange hadron \(M\) in a \(pp\) collision in which \(N_{\text{ch}}\) charged particles are produced.

We expect that even in high multiplicity events each gluon cascade (“pomeron”) should satisfy the nonlinear Balitsky-Kovchegov equation, and therefore the dipole amplitude (12) should keep its form, although the value of the saturation scale \(Q_s\) might be modified. As was demonstrated in [65-67], the observed number of charged multiplicity \(dN_{\text{ch}}/dy\) of soft hadrons in \(pp\) collisions is proportional to the saturation scale \(Q^2_s\) (modulo logarithmic corrections), and therefore in the dipole framework the events with large multiplicity might be described by simply rescaling \(Q^2_s\) as a function of \(n\) [65-71],

\[
Q^2_s(x, b; n) \approx n Q^2_s(x, b).
\]

The accuracy of the approximation (18) was tested in [64], and it was found that its error does not exceed 10 percent in the region of interest \((n \lesssim 10)\), on par with the precision of current evaluations. Therefore, in what follows we
will use \(^\text{18}\) for our estimates. While at LHC energies it is expected that the typical values of the saturation scale \(Q_s(x, b)\) fall into the range 0.5-1 GeV, from \(^\text{18}\), we can see that in events with enhanced multiplicity this parameter might lead to an interplay of the large-\(Q_s\) and large-\(p_T\) limits. Since increasing multiplicity and increasing energy (decreasing \(x\)) affect \(Q_s^2\) in a similar way, the study of the high-multiplicity events allows to study a deeply saturated regime, which determines the dynamics of all processes at significantly higher energies.

Since at high energies each pomeron hadronizes independently \(^\text{97}\), the observed enhancement of multiplicity in the whole process must be shared between all pomeron which might contribute in a given rapidity window. For this reason, for phenomenological estimates of the multiplicity dependence it is important if the rapidity bin used to collect charged particles \(N_{\text{ch}}\) overlaps with the bin used for the observation of strange large-\(p_T\) hadrons, as explained in Figure \(^\text{4}\).

In the limit of large-\(p_T\) the typical sizes of the dipoles are small, \(r \sim 1/p_T\), so we may expect from \(^\text{2} \, \text{12} \, \text{18}\) that the contribution of each cut pomeron to the multiplicity dependence is given by the factor \(\sim n_i^{(\gamma_{\text{eff}})}\), where \(n_i\) is the relative enhancement of multiplicity assigned to a given pomeron, and the value of the parameter \(\gamma_{\text{eff}}\) is given in \(^\text{14} \, \text{15}\). For the configuration when the bins used to collect charged and strange particles are separated by rapidity (see the left panel of the Figure \(^\text{4}\)) the multiplicity is assigned to one of the pomeron, so the expected multiplicity dependence of the cross-section is \(\sim n^{(\gamma_{\text{eff}})}\). For the case when the strange and charged particle bins partially overlap (as shown in the right panel of the Figure \(^\text{4}\)), we should average over all possible partitions of the observed number of charged particles. This evaluation technically is quite complicated, although we can assume with good precision that the multiplicity enhancement is shared equally between both pomeron \(^\text{46}\). For this reason in this case we expect that the multiplicity cross-section would be \(\sim (n/2)^{2(\gamma_{\text{eff}})}\).
Figure 4: (color online) Demonstration that sharing of the enhanced multiplicity of the whole process between individual pomeron depends on the experimental setup. Left plot: The experimental setup in which the rapidity bin used for collection of strange hadrons (blue box) does not overlap with the bin used for the collection of charged particles (red box). The elevated multiplicity in this case should be unambiguously attributed to the lower pomeron. Right plot: The experimental setup when the bins partially overlap. For the partons in the intersection region (magenta color) the assignment to upper or lower pomeron depends on the position of large-$p_T$ quark inside the bin $(y - \Delta y/2, y + \Delta y/2)$. In the final result we should average over all possible rapidities of strange quark inside the bin.

Currently the data on multiplicity dependence of strange hadrons are available from the ALICE experiment [89]. As we can see from Figure 5, the theoretical curves can describe reasonably well the slope of the experimentally observed $n$-dependence (in logarithmic coordinates), although apparently overestimate all the experimental points by the same normalization factor $\sim 1.2$. We would like to stress that by definition at the point $n = 1$ the self-normalized ratio (17) equals one. This condition is fulfilled in our theoretical curves, and therefore we believe that the normalization of our curves is correct.

V. CONCLUSIONS

In this paper we studied the production of strange hadrons in the color dipole approach. We found that the CGC/Sat approach can describe the shapes of $p_T$ distributions in the large-$p_T$ kinematics, although it might not be very reliable for smaller $p_T$. The latter restriction implies that the suggested approach cannot be applied to $p_T$-integrated observables, which get its dominant contribution from the nonperturbative small-$p_T$ region. As a consequence, our predictions for experimentally measurable self-normalized yields suffer from a global normalization uncertainty, which complicates the direct comparison of model predictions with data, even for large $p_T$. For this reason we call experimentalists to publish also the cross-sections, for which there is unambiguous separation of small-$p_T$ and large-$p_T$ physics. We made predictions for such cross-sections in the kinematics of ongoing and future experiments, and can provide further predictions on demand.

We also applied the CGC/Sat approach to the description of the multiplicity dependence measured by ALICE [89]. Fortunately, all kaons and $\Lambda$-baryons were collected with sufficiently large transverse momenta $p_T \gtrsim 4$ GeV, where our approach is well justified. We found that the theoretical predictions are in reasonable agreement with experimental data. Our evaluation is largely parameter-free and relies only on the choice of the parametrization for the dipole cross-section (12) and fragmentation functions of strange hadrons.

This study complements our previous analysis of the multiplicity dependence of heavier charm and bottom production [60] and demonstrates that at sufficiently large $p_T$ it is possible to describe all of them within the same framework.
Figure 5: Comparison of the theoretical multiplicity dependence for $K^0_S$ meson (production solid curve) and \( \Lambda \) baryons (dashed curve) with experimental data from ALICE [89]. For the sake of reference we have also shown a dotted line, which corresponds to a linear dependence. The charged particles and strange hadrons are collected at central rapidities.

Acknowledgements

We thank our colleagues at UTFSM University for encouraging discussions. This research was partially supported by Proyecto Basal FB 0821(Chile) and Fondecyt (Chile) grant 1180232. Also, we thank Yuri Ivanov for technical support of the USM HPC cluster, where part of the evaluations were performed.

Appendix A: Fragmentation functions

In this section we would like to summarize briefly the fragmentation functions used in our evaluations. These functions are nonperturbative objects, which cannot be evaluated from first principles. For this reason currently their parametrization is extracted from the phenomenological fits of experimental data. For the sake of definiteness, for our evaluations we used the fragmentation functions for kaons and \( \Lambda \) from [88] (so-called AKK08 parametrization). The fragmentation functions for $K^\pm$ and $K^0_S$ are constrained by the isospin symmetry relation

$$ D_i^{K^0_S} (z, \mu^2) = \frac{1}{2} D_i^{K^\pm} (z, \mu^2), $$

therefore in what follows we will consider only the fragmentation function of neutral kaons $K^0_S$. For kaons we checked that the alternative parametrizations of fragmentation functions DSS17 [98], NNPDF [99] and JAM [100] give similar results in the region of interest. We have not found parametrizations for fragmentation functions of strange baryons $\Omega$, $\Xi$, and neither of $K^0_S$ and $\phi$-mesons, and for this reason we do not consider them in this paper.
In the AKK08 parametrization [88] it is assumed that the fragmentation function is given by

$$D_{i/H}^j(z) = N_i z^{a_i}(1 - z)^{b_i} [1 + c_i (1 - z)^{d_i}], \quad (A2)$$

where $N_i, a_i, b_i, c_i, d_i$ are some numerical coefficients which depend on the hadron and quark flavor $i$. We expect that for strange hadrons the largest contribution comes from the fragmentation of the strange quark, thus we will discuss below the fragmentation function $D_{s/H}$. As we can see from Figure 6, the parametrizations for $K^\pm$ mesons and $\Lambda$-baryons differ quite substantially in the region of small $z \lesssim 0.3$, although become comparable for all hadrons at larger values of $z$.

In this paper we are mostly interested in the large-$p_T$ kinematics, and it is possible to show that this region has stronger sensitivity to the region of large $z$. Indeed, as we can see from the structure of (2), at large $p_T$ the cross-section $d\sigma_{\bar{Q}Q}/dp_{\bar{Q}Q}^T$ is suppressed as $\sim \left(1/p_{\bar{Q}Q}^T\right)^n$ with $n \gtrsim 5$. The momentum of the quark pair $p_{\bar{Q}Q}^T$ is related to the momentum of the strange hadron as $p_{\bar{Q}Q}^T = p_T/z$, so this implies that in the integral over the fragmentation fraction $z$ effectively we get an additional prefactor $\sim z^{n-2}$, which suppresses the contribution of the small-$z$ domain. As we can see from the right panel of Figure 6 the dominant contribution comes from the region $z \sim 0.6 - 0.8$, where the difference between fragmentation functions does not exceed a factor of two.
We would like to remind that in case of $D$- and $B$-meson production studied earlier in [60–64] the scale in the small-$p_T$ kinematics was set by the heavy quark mass, so the use of the framework was justified up to $p_T \approx 0$. In case of the strangeness production this is no longer true, and we have to restrict our consideration to the large-$p_T$ domain only.

We also would like to mention that the CMS and STAR data were not normalized to unity. For example, the CMS data instead of $N_e$ used the number of non-single-diffractive events $N_{NSD}$, so we also corrected the normalization of experimental data.