A COMPREHENSIVE STUDY OF LUMINOSITY FUNCTIONS AND EVENT RATE DENSITIES OF LONG GAMMA-RAY BURSTS WITH NON-PARAMETRIC METHOD

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ABSTRACT

The current event rate estimates of long gamma-ray bursts based on distinct methods or samples especially at lower redshift are largely debated, which motivates us to re-study the dependence of luminosity function and event rates for different burst samples on the criteria of sample selection and threshold effect in this letter. To ensure the sample completeness as possible, we have chosen two samples including 88 and 118 long bright bursts with known redshift and peak flux over 2.6 ph cm⁻² s⁻¹. It is found that the evolution of luminosity with redshift can be expressed by $L \propto (1 + z)^k$ with a diverse $k$ relied more on the sample selection. Interestingly, the cumulative distributions of either non-evolving luminosities or redshifts are found to be also determined by the sample selection rather the instrumental sensitivity. Nevertheless, the non-evolving luminosities of our samples are similarly distributed with a comparable break luminosity of $L_0 \sim 10^{51}$ erg s⁻¹. Importantly, we verify with a K-S test that three cases of event rates for the two burst samples evolve with redshift similarly except a small discrepancy due to sampling differences at low-redshift of $z < 1$, in which all event rates show an excess of gaussian profile instead of monotonous decline. Most importantly, it is found that the low-redshift burst event rates violate the star formation rates, while both of them are good in agreement with each other in the higher-redshift regions as many authors discovered previously. Consequently, we predict that two types of long gamma-ray bursts should be expected on the basis of whether they match the star formation or not.

Keywords: gamma-ray burst: general—galaxies: star formation—stars: luminosity function—methods: data analysis

1. INTRODUCTION

Gamma-ray bursts (GRBs) are the most energetic explosions found ever in the universe and produce huge amounts of energy in gamma-rays over a short time period ranging from a few milliseconds to thousands of seconds (Kouveliotou et al. 1993; Zhang & Choi 2008; Zhang et al. 2020). They can even be detected at much higher redshifts than supernovae (SNe) that are generated from a stellar death. In theory, long GRBs (lGRBs) with a duration $T_{90} > 2$ s are believed to produce from core-collapsed massive stars (e.g., Woosley 1993; Paczyński 1998; Woosley & Bloom 2006) which is evidently supported by observations of some GRBs associated with SNe, such as GRB 980425/SN 1998bw and GRB 030329/SN 2003dh (e.g., Hjorth et al. 2003; Stanek et al. 2003). The collapsar model implies that the GRB event rate should in principle trace the cosmic star formation rate (SFR; Totani 1997; Wijers et al. 1998; Lamb & Reichart 2000; Porciani & Madau 2001; Piran 2004; Zhang & Mészáros 2004; Zhang 2007). It can be interestingly found that the low isotropic energy ($E_{\gamma, iso}$) SN/GRBs are relatively brighter in radio band compared to other long GRBs on a whole. According to Wijers & Galama (1999), one can infer that the observed radio spectral peak luminosity ($L_{peak}$) of the SN/GRBs with smaller $E_{\gamma, iso}$ needs larger magnetic field (B) or larger number density ($n$) as $L_{peak} \sim n^{1/2}B^{1/2}$ in theory.

In the past two decades, many authors had focused on the study of relationship between the GRB event rate and the SFR in terms of different methods and samples, of which the direct fitting procedure with a specific function (e.g., Liang et al. 2007; Yüksel et al. 2008; Nakar & Sari 2012; Wanderman & Piran 2015) and the non-parametric method (e.g., Wu et al. 2012; Petrosian et al. 2015; Yu et al. 2015) have been popularly adopted. However, parts of the results of GRB event rate especially at low-redshift are contradictory with each other even though the same non-parametric method has been applied. There are several algorithms to derive the luminosity function and event rate of GRBs for a specific kind of astronomical sources. In fact, the observed GRB data are truncated in that the observational flux sensitivity of the satellite is limited. It is thus difficult to obtain a uniformly distributed GRB sample unless the selection effect is corrected. Lynden-Bell’s $e^{-}$ method (Lynden-Bell 1971; Efron & Petrosian 1992) is one of the non-parametric and non-binning data processing techniques. It can readily combine samples
with varied selection processes and is thus more powerful than the traditional fitting methods. (e.g. Wu et al. 2012; Yu et al. 2015; Petrosian et al. 2015; Pescalli et al. 2016; Tsvetkova et al. 2017; Zhang & Wang 2018; Lloyd-Ronning et al. 2019). For example, Yu et al. (2015) (hereafter Y15) adopted the non-parametric method for 127 GRBs and found that the event rate of GRBs decreases with the increase of redshift. While the SFR increases with redshift before \( z \sim 1 \) and decreases with redshift after \( z \sim 1 \), so that they claimed an excess of GRB event rate at low-redshift of \( z < 1 \) (see also Petrosian et al. 2015; Zhang & Wang 2018; Lloyd-Ronning et al. 2019). On the contrary, Pescalli et al. (2016) (hereafter P16) utilized the same non-parametric method with a special sample selection to a sample of 81 Swift IGRBs and found no more excess of the GRB event rate than the SFR in the range of low-redshifts (see also Wu et al. 2012). Very interestingly, Deng et al. (2019) applied the non-parametric method to 38 fast radio bursts (FRBs) and found that these FRBs with relatively lower redshifts match the SFRs well too. Recently, Lan et al. (2019) employed a maximum likelihood method to re-examine the luminosity function and event rate of 81 IGRBs used in P16. They concluded that the GRB event rate may be consistent with the SFRs at \( z < 2 \), but shows a discrepancy between them at \( z > 2 \). Therefore, the relation of GRB event rate and the SFR especially at low-redshift end is still an open question. Unfortunately, such contradictions still can not be explained in theory reasonably. It is noticeable that the number of low-redshift GRBs in previous works is too limited, which may cause the estimate of event rate at lower redshift to be significantly biased.

To disclose the real evolution of GRB event rate with redshift, we will consider the same IGRB samples but with different sample selection criteria and see how the GRB event rates evolve with redshifts diversely. Furthermore, we will expand the sample size of low-redshift GRBs in order to perform more reliable tests on the excessive component in statistics. In Section 2, we describe how to build two IGRB samples for three cases. In Section 3, we illustrate the non-parametric method and the data processing. Our results are presented in Section 4. Lastly, we end with conclusions in Section 5.

2. DATA

Since the launch of Swift (Gehrels et al. 2004) and Fermi (Meegan et al. 2009) satellites, more and more GRBs with measured redshift are available recently which is very helpful to investigate the evolution of luminosity with redshift completely. P16 pointed out that incompleteness of GRB samples will inevitably cause an excessive GRB event rate at low-redshifts because of the observational biases. To avoid the negative influences, they had chosen 81 Swift IGRBs with known redshift and higher peak photon flux than 2.6 \( \text{ph cm}^{-2} \text{s}^{-1} \) to re-constrain the GRB event rates at different redshifts. Strangely, they did not find the excessive components compared with the SFRs at lower redshift. In addition, Bryant et al. (2020) recently argued that an underestimation of detection threshold will also lead to severely-incomplete IGRB samples which eventually affects the inferred event rates.

Undoubtedly, the estimate of GRB event rate significantly depends on the sampling methods and/or the energy range of a detector in a certain sense. To check whether the excess of GRB event rate at low-redshift is biased by the effects of sample selection and threshold, it is necessary to give a comparative study for a united sample complied with distinct sensitivities. For this purpose, we also adopt the lower flux limit of 2.6 \( \text{ph cm}^{-2} \text{s}^{-1} \) as our basic sampling criterion.

Firstly, we pick 88 bright bursts out of 127 IGRBs with both redshift and good spectral parameter from Y15 to comprise our sample I. Secondly, we add 30 low-redshift (0 < \( z < 1 \)) IGRBs published in (Zhang et al. 2018) to build our sample II (\( N = 118 \)) in order to compensate the number deficiency of low-redshift IGRBs of previous works. We caution that all IGRBs have been strictly chosen by taking into account the P16 sample selection standard. Consequently, we have selected 25 Swift/BAT, 62 Fermi/GBM and 31 Konus-wind IGRBs with well-measured spectra and reshifts from Yu et al. (2015) and Zhang et al. (2018). Note that the fraction of low-redshift bursts with \( z < 1 \) is as high as about 42 percent in our whole sample, which guarantees that the GRB event rates at lower redshift can be really reproduced.

2.1. Luminosity Limit

The peak luminosity of the GRBs is calculated by \( L = 4\pi d_L^2(z) F K \), where \( F \) is the observed peak flux within a certain energy range and \( K \) denotes the factor of K-correction factor (Zhang et al. 2018). The luminosity distance \( d_L(z) \) at a redshift \( z \) (Hogg 1999) is written as

\[
d_L(z) = \frac{c}{H_0} (1 + z) \int_0^z \frac{dz}{\sqrt{1 - \Omega_m + \Omega_m (1 + z)^3}}. \tag{1}\]

Throughout the paper, a flat ΛCDM universe with \( \Omega_m = 0.27 \), \( \Omega_\Lambda = 0.73 \) and \( H_0 = 70 \text{ km s}^{-1}\text{Mpc}^{-1} \) has been assumed.

It is known that the threshold of Swift/BAT is \( F_{lim,1} = 2.0 \times 10^{-8} \text{ erg cm}^{-2}\text{s}^{-1} \) (Gehrels et al. 2004). The luminosity limit at a certain redshift \( z \) can be given as \( L_{lim} = 4\pi d_L^2(z) F_{lim} \) in that the K-correction parameter is narrowly distributed around 1 in view of precious investigations (see e.g. Bloom et al. 2003; Zhang et al. 2018). On the other hand, Bryant et al. (2020) pointed out that the detection threshold effect will be underestimated in a sense and should be given a conservative estimation of \( F_{lim,2} = 1.0 \times 10^{-7} \text{ erg cm}^{-2}\text{s}^{-1} \). The difference between \( F_{lim,1} \) and \( F_{lim,2} \) could play an un-negligible role on
modeling the observed luminosity-redshift relation when a significant fraction of GRBs reside below the $L_{lim}$ for a GRB sample. Therefore, we will pay more attention to the influence of not only the sample selection but also the threshold effect on the GRB luminosity evolving with redshift for the above two refined GRBs samples.

3. METHOD

The Lynden-Bell’s $e^-$ method adopted here requires the luminosity $L$ is independent of the redshift $z$ in advance (Lynden-Bell 1971; Efron & Petrosian 1992), so that the luminosity function and the GRB event rate can be accurately determined. Therefore, we need to reduce the redshift evolution effect of $L$ on $z$ with a non-parametric $\tau$ test method firstly.

3.1. The method of $\tau$ statistics

If $L$ and $z$ are independent of each other then one can write the joint distribution as $\Psi(L, z) = \psi(L)\phi(z)$, in which $\psi(L)$ is a luminosity function of the GRBs and $\phi(z)$ represents the cumulative redshift distribution (Efron & Petrosian 1992). Observationally, the luminosity is positively correlated with the redshift for our samples as shown in Figure 1. As usual, $\Psi(L, z)$ can be decomposed into the form of $\Psi(L, z) = \psi(L/g(z))\phi(z)$, where $g(z)$ describes the evolutionary relationship between $L$ and $z$. And if letting $L_0 = L/g(z)$, we then get $\Psi(L_0, z) = \psi(L_0)\phi(z)$, of which the redshift $z$ and the modified luminosity $L_0$ are independent and already satisfy the requirement of the non-parametric $\tau$ test.

We continuously adopt the power-law form of $g(z) = (1 + z)^k$ that has been used in many literatures (e.g. Lloyd-Rowning et al. 2002; Yonetoku et al. 2004; Dainotti et al. 2015; Petrosian et al. 2015; Yu et al. 2015; Zhang & Wang 2018). First, suppose that we get a definite value of $k$ and after removal, each data point changes from $(z_i, L_i)$ to $(z_i, L_0,i)$. For the $i$th data in the $(z_i, L_0,i)$ data set, we can define $J_i$ as

$$J_i = \{j|L_{0,j} \geq L_{0,i}, z_j \leq z_i^{max}\} ,$$

where $L_{0,i}$ is the $i$th GBR luminosity without redshift evolution and $z_i^{max}$ is the maximum redshift at which a GBR with luminosity $L_{0,i}$ can be observed. The number of GRBs contained in this region is $n_i$. The number of GRBs with redshift $z$ less than or equal to $z_i$ in this region is defined as $R_i$. The $\tau$ test statistic is defined to be

$$\tau = \frac{\sum_i (R_i - E_i)}{\sqrt{\sum_i V_i}},$$

where $E_i = \frac{1 + n_i}{2}$, $V_i = \frac{(n_i - 1)^2}{12}$ are the expected mean and the variance of $R_i$, respectively. As known from the $\tau$ test statistic, if $R_i$ is exactly uniformly distributed between 1 and $n_i$ then the sample number of $R_i \leq E_i$ and $R_i \geq E_i$ should be nearly equal and the value of $\tau$ will be nearly 0, then $L_0$ and $z$ become independent of each other after removing the evolution with $g(z) = (1 + z)^k$. Based on this, we have to adjust the value of $k$ until the $\tau$ is equal to 0 from which we can get the expected value of $k$ in $g(z)$.

Subsequently, we constrain the $k$ values for the distinct samples I and II. When the $F_{lim,1}$ is used, the $k$ values are roughly equal to 2.88 and 3.92 for samples I and II. The power-law index of $k$ for the sample II will be about 3.62 once the $F_{lim,2}$ is applied. It demonstrates that the deduced $k$ values depend more on the sample selection but less on the instrumental effect, which is somewhat different from what mentioned by Bryant et al. (2020). The reason is that all bursts in our samples are located above the lower limits of luminosities. Moreover, Wu et al. (2012) found that the value of $k$ to be 2.3$^{+0.26}_{-0.31}$, Tsvetkova et al. (2017) got a smaller value of $k \sim 1.7$, Tsvetkova et al. (2021) got a value of $k$ as 1.2 and P16 found $k \sim 2.5$ and so on. The diverse $k$ values reported in many literatures vary obviously from sample to sample and confirm again that the sample selection effect does influence the determination of non-evolving luminosites and GRB event rates in evidence.

For simplification, we define cases 1, 2 and 3 to represent sample I with $F_{lim,1}$, sample II with $F_{lim,1}$ and sample II with $F_{lim,2}$, respectively. Figure 2 shows the relationships between $z$ and $L_0$ after the redshift evolution of $g(z)$ was removed by $L_0 = L/(1 + z)^k$ for the above three cases. It can be clearly seen that the non-evolving luminosity $L_0$ of two above samples is already independently with redshift. In the following, we will utilize the data of $L_0$ and $z$ to derive
3.2. Luminosity Function and Event Rate of GRBs

The Lynden-Bell’s $c^-$ method is an effective way to determine the redshift distribution and luminosity function of astronomical objects using the truncated samples. Let

$$N_i = n_i - 1$$

represent the number of GRBs contained in $J_i$, which can be understood as the minus one count (taking the $i$th point out) called as the Lynden-Bell’s $c^-$ method (Lynden-Bell 1971). And we then set

$$J_i' = \{ j | L_{0,j} \geq L_{0,i} |_{lim}^{lim}, z_j < z_i \},$$

and let $M_i$ to be the number of IGRBs contained in $J_i'$. The cumulative luminosity function can be derived from the following formula by the non-parametric method (Lynden-Bell 1971; Efron & Petrosian 1992):

$$\psi(L_{0,i}) = \prod_{j<i}(1 + \frac{1}{N_j}),$$

where $j < i$ means that GRB has luminosity $L_{0,j}$ larger than $L_{0,i}$. The cumulative redshift distribution $\phi(z)$ can be obtained from

$$\phi(z_i) = \prod_{j<i}(1 + \frac{1}{M_j}),$$

where $j < i$ means that GRB has redshift $z_j$ less than $z_i$. The event rate of GRBs can be written as

$$\rho(z) = \frac{d\phi(z)}{dz}(1 + z)(\frac{dV(z)}{dz})^{-1},$$

where $(1 + z)$ results from the cosmological time dilation and $dV(z)/dz$ is the differential comoving volume which can be expressed as (Khokhriakova et al. 2019)

$$\frac{dV(z)}{dz} = \frac{c}{H_0 (1 + z)^2} \frac{1}{\sqrt{1 - \Omega_m + \Omega_m(1 + z)^3}},$$

where the comoving volume at a redshift of $z$ is $V = 4\pi d_M^3/3$ with the comoving distance of $D_M = d_i/(1 + z)$ (Hogg 1999).

4. RESULTS

In this section, we give our results of luminosity functions and event rate of IGRBs constrained by the non-parametric method. Simultaneously, we compare the evolutionary history of distinct IGRB event rate with that of star formations.

4.1. Luminosity functions of different IGRB samples

Using the Lynden-Bell $c^-$ method in Eq.(6), we now obtain the cumulative luminosity functions of two different IGRB samples with distinct sensitivities. Figure 3 depicts that the normalized cumulative luminosity functions decrease gradually with the increase of non-envolving luminosity, which is similar to some previous studies (Yu et al. 2015; Pescalli et al. 2016; Tsvetkova et al. 2017; Lan et al. 2019). It is noticeable that the luminosity function in case 1 is significantly different from those in both cases 2 and 3, while the cases 2 and 3 are largely consistent with each other. This indicates that the derived luminosity function is indeed sensitive to the sample selection other than the sensitivity dramatically provided that no GRBs appear below $L_{lim}$ in Figure 1.

We now fit the cumulative luminosity distribution in each case with a smoothly broken power-law function defined by

$$\psi(L_0) = \psi_s([\frac{L_0}{L_b}]^{\alpha}\omega + \frac{L_0}{L_b}^{\beta}\omega]^{-\frac{1}{\omega}},$$

in which $L_b$ is the break luminosity and $\psi_s$ is a normalization factor. $\alpha$ and $\beta$ are two power-law indexes characterizing the decay of luminosity function before and after the $L_b$, and $\omega$ is a smoothness parameter assigned to be 0.18 empirically in this study. As a result, we get the break luminosity $L_b$ in three cases as $L_{b,1} = 8.5 \times 10^{50}$ erg s$^{-1}$, $L_{b,2} = 2.5 \times 10^{51}$ erg s$^{-1}$ and $L_{b,3} = 1.8 \times 10^{51}$ erg s$^{-1}$, respectively. Interestingly, the three break luminosities are very close although the luminosity distribution of case 1 is obviously different from those of both cases 2 and 3. It is worthy to emphasize that the break
luminosities are not affected by the threshold effect and the sample selection remarkably.

4.2. Event rate densities of IGRBs

4.2.1. Comparison between different samples

Figure 4 displays the normalized cumulative redshift distributions of IGRBs for samples I and II from Eq.(7). It can be seen that \( \phi(z) \) increases gradually with redshift, which is consistent with some previous studies (e.g., Wu et al. 2012; Petrosian et al. 2015; Yu et al. 2015). Especially, the cumulative \( \phi(z) \) functions of cases 2 and 3 are found to evolve with redshift in a similar way. In contrast, the cumulative \( \phi(z) \) function in case 1 behaves smoother at lower redshift and steeper at higher redshift. Figure 4 also demonstrates that the cumulative redshift distributions are almost unaffected by the sensitivity of detectors while evidently biased by the effect of sample selection which is perfectly consistent with what illustrated in Figure 3.

Noticeably, a slope transformation of the redshift distribution function in Figure 4 will lead to distinct evolutions of GRB event rate with the cosmological redshift according to Eq.(8). Three ladder lines in Figure 5 correspond to the three cases of GRB event rates \( \rho(z) \) evolving with redshift. It is worthy to point out that all GRB event rates derived from our samples show an excess of gaussian profile compared with the SFRs at lower redshift of \( z < 1 \), which vastly differs from either the monotonous decline proposed by many authors (e.g., Yu et al. 2015; Petrosian et al. 2015; Lloyd-Ronning et al. 2019; Tsvetkova et al. 2021) or the monotonous rise presented by (Yonetoku et al. 2004; Wu et al. 2012; Pescalli et al. 2016; Lan et al. 2019). It is because the number of low-redshift bursts in previous papers is too limited to manifest the real evolitional profile of GRB event rates. Again, one can find that the GRB event rates are not affected by the threshold significantly but depend more on the sample selection instead. A Kolmogorov-Smirnov (K-S) test to any two kinds of distributions of GRB event rates returns \( D_{12} \simeq 0.31 (p = 0.53) \), \( D_{23} \simeq 0.23 (p = 0.86) \) and \( D_{13} \simeq 0.27 (p = 0.70) \), respectively. If adopting the critical value \( D_{0.01} \simeq 0.64 \) at a significance level of \( \alpha = 0.01 \), we can therefore conclude that the three groups of GRB event rates are surprisingly taken from the same distribution.

4.2.2. GRB event rate versus star formation rate

Porciani & Madau (2001) pointed out that the event rate of GRB traces the global star formation history of the universe. It was usually assumed that the GRB event rates is proportional to the SFRs in literatures (e.g., Porciani & Madau 2001; Lloyd-Ronning et al. 2019; Lan et al. 2019; Palmerio et al. 2020), which enables us to compare our newly-built GRB event rates with the SFRs constrained by the largest sample of stars observed within a wider redshift range ever in Figure 5, where we find that all three GRB event rates trace the SFRs well at higher redshift of \( z > 1 \) and exhibit an obvious excess with a gaussian-like shape at \( z < 1 \). This strongly indicates that the IGRBs should be classified into two groups depending on whether they match the SFRs or not. Hence we can optimistically propose that the low-redshift IGRBs with higher event rates are not associated with the SFRs and should stand for a separate subclass. Notably, the majority of these low-redshift IGRBs are less luminous as displayed in Figure 1.
Regarding the SFRs themselves, we use the common form of \( \dot{\rho}_* = (a + bz)/(1 + (z/c)^d) \) with \( h = 0.7 \) (Hopkins & Beacom 2006) to get the best fitting parameter set \( a = 0.014 \pm 0.009, b = 0.140 \pm 0.018, c = 2.98 \pm 0.22 \) and \( d = 4.55 \pm 0.54 \). The best fit to the updated SFR data in the work has been highlighted with solid line in Figure 5, from which one can find our results are coincident with those previous ones (for example Hopkins 2004; Thompson et al. 2006; Li 2008).

5. CONCLUSIONS

We have carefully studied the effects of sample selection and threshold on the luminosity functions and event rates of distinct bright lGRB samples and compared the GRB event rates with the SFRs in a more robust way. The following conclusions can be drawn:

1) The observed luminosity of lGRBs in our samples evolves with the cosmological redshift as \( L \propto (1+z)^k \) with an index \( k \) varying from 2.88 to 3.92 that is marginally consistent with previous values. The \( k \) parameter is more sensitive to the effects of sample selection instead of threshold of a detector if all bursts reside above the lower limits of luminosities.

2) It is found for the first time that a gaussian-like component of lGRB even rates always exceeds the SFRs at lower redshift of \( z < 1 \) no matter what kinds of GRB samples are considered. On the contrary, those high-redshift IGRBs are perfectly associated with star formations, which is good in agreement with some previous conclusions. This directly demonstrates that two types of IGRBs are evidently expected.

3) It proves that the sample selection effect would play more important roles than the instrumental effect on calculating the cumulative luminosity functions, redshift distributions together with event rates of the complete IGRB samples. This is almost always right since the bursts located below the luminosity limits are very rare.

4) It is worthy of addressing that the gaussian-like excess of IGRB event rates at \( z < 1 \) in all three cases is largely different from the monotonous rise or drop patterns found before, which implies that the low-redshift IGRBs might originate from some special progenitors unconnected with the SFRs at all. On the other hand, the high-redshift IGRBs matching the SFRs ideally provide a convincing evidence supporting their physical origins from the core-collapse of massive stars.

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Figure 5. Comparison of GRB event rate with SFR. The green, red and blue ladder lines represent the evolutions of GRB event rate changing with redshift for the case 1, 2 and 3, respectively. Gray dots, triangles, pentagons and stars represent the observed SFRs recorded in Hopkins (2004), Thompson et al. (2006), Bouwens et al. (2011) and Li (2008), respectively. The pink dashed, red double-dot dashed, and magenta dashed lines correspond to the theoretical lines of the SFR evolving with the redshift in Li (2008), Hopkins & Beacom (2006), Madau & Dickinson (2014), respectively, while the blue solid line represents our best fit to all the SFR data with a 99.7% confidence level.