A Bisimulation-based Method for Proving the Validity of Equations in GSOS Languages*

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This paper presents a bisimulation-based method for establishing the soundness of equations between terms constructed using operations whose semantics is specified by rules in the GSOS format of Bloom, Istrail and Meyer. The method is inspired by de Simone’s FH-bisimilarity and uses transition rules as schematic transitions in a bisimulation-like relation between open terms. The soundness of the method is proven and examples showing its applicability are provided. The proposed bisimulation-based proof method is incomplete, but the article offers some completeness results for restricted classes of GSOS specifications.

1 Introduction

Equations play a fundamental role in the development of the theory and practice of process calculi and programming languages since they offer a mathematically appealing and concise way of stating the ‘laws of programming’ (to borrow the title of a paper by Hoare et al. [22]) that apply to the language at hand. In the setting of process calculi, the study of equational axiomatizations of behavioural relations has been a classic area of investigation since, e.g., the early work of Hennessy and Milner [20, 25], who offered complete axiom systems for bisimilarity [30] over the finite and regular fragments of Milner’s CCS [26]. Such axiomatizations capture the essence of bisimilarity over those fragments of CCS in a syntactic, and often revealing, way and potentially pave the way for the verification of equivalences between processes by means of theorem proving techniques. Despite these early achievements, the search for axiomatizations of process equivalences that are powerful enough to establish all the valid equations between open process terms—that is, terms possibly containing variables—has proven to be a very difficult research problem; see [3] for a survey of results in this area. For instance, to the best of our knowledge, there is no known axiomatization of bisimilarity over recursion-free CCS that is complete over open terms. Stepping stones towards such a result are offered in, e.g., [4, 6].

The most basic property of any equation is that it be sound with respect to the chosen notion of semantics. Soundness proofs are often lengthy, work-intensive and need to be carried out for many equations and languages. It is therefore not surprising that the development of general methods for proving equivalences between open terms in expressive process calculi has received some attention since the early developments of the algebraic theory of processes—see, e.g., the references [12, 23, 32, 34, 35] for some of the work in this area over a period of over 20 years. This article offers a contribution to this line of research by developing a bisimulation-based method, which we call rule-matching bisimilarity, for establishing the soundness of equations between terms constructed using operations whose semantics

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is specified by rules in the GSOS format of Bloom, Istrail and Meyer [11]. Rule-matching bisimilarity is inspired by de Simone’s FH-bisimilarity [34] and uses transition rules as transition schemas in a bisimulation-like relation between open terms. We prove that rule-matching bisimilarity is a sound proof method for showing the validity of equations with respect to bisimilarity and exhibit examples witnessing its incompleteness.

The incompleteness of rule-matching bisimilarity is not unexpected and raises the question whether the method is powerful enough to prove the soundness of ‘interesting’ equations. In order to offer a partial answer to this question, we provide examples showing the applicability of our proof method. In particular, our method does not only apply to a more expressive rule format than the one proposed by de Simone in [34], but is also a sharpening of de Simone’s FH-bisimilarity over de Simone languages. See Section 6 where we apply rule-matching bisimilarity to prove the soundness of the equations in de Simone’s ‘clock example’. (This example was discussed by de Simone in [34] to highlight the incompleteness of FH-bisimilarity.) On the theoretical side, we also offer some completeness results for restricted classes of GSOS specifications.

Overall, we believe that, while our conditions are neither necessary nor in general can they be checked algorithmically, they frequently hold, and they are more accessible to machine support than a direct proof of soundness.

The paper is organized as follows. Sections 2 and 3 introduce the necessary preliminaries on the GSOS rule format that are needed in the reminder of the paper. In particular, Section 3 recalls the notion of ruloid, which plays a key role in the technical developments to follow. In Section 4 we introduce a simple logic of transition formulae and establish a decidability result for the validity of implications between formulae. Implication between certain kinds of transition formulae that are naturally associated with the premises of (sets of) ruloids is used in the definition of rule-matching bisimilarity in Section 5. In that section, we prove that rule-matching bisimilarity is a sound method for showing the validity of equations in GSOS languages modulo bisimilarity and exhibit examples witnessing its incompleteness. We apply rule-matching bisimilarity to show the validity of some sample equations from the literature on process algebra in Section 6. We then offer some partial completeness results for rule-matching bisimilarity (Section 7). The paper concludes with a discussion of related and future work (Section 8).

2 Preliminaries

We assume familiarity with the basic notation of process algebra and structural operational semantics; see e.g. [5, 8, 11, 18, 19, 21, 26, 27, 31] for more details.

Let Var be a countably infinite set of process variables with typical elements \( x,y \). A signature \( \Sigma \) consists of a set of operation symbols, disjoint from Var, together with a function arity that assigns a natural number to each operation symbol. The set \( \mathcal{T}(\Sigma) \) of terms built from the operations in \( \Sigma \) and the variables in Var is defined in the standard way. We use \( P, Q, \ldots \) to range over terms and the symbol \( \equiv \) for the relation of syntactic equality on terms. We denote by \( \mathcal{T}(\Sigma) \) the set of closed terms over \( \Sigma \), i.e., terms that do not contain variables, and will use \( p, q, \ldots \) to range over it. An operation symbol \( f \) of arity 0 will be often called a constant symbol, and the term \( f() \) will be abbreviated as \( f \).

Besides terms we have actions, elements of some given nonempty, finite set Act, which is ranged over by \( a, b, c, d \). A positive transition formula is a triple of two terms and an action, written \( P \xrightarrow{a} P' \). A negative transition formula is a pair of a term and an action, written \( P \xleftarrow{a} \).

A (closed) \( \Sigma \)-substitution is a function \( \sigma \) from variables to (closed) terms over the signature \( \Sigma \). For \( t \) a term or a transition formula, we write \( t\sigma \) for the result of substituting \( \sigma(x) \) for each \( x \) occurring in \( t \), and
vars(\(t\)) for the set of variables occurring in \(t\). A \(\Sigma\)-context \(C[\vec{x}]\) is a term in which at most the variables \(\vec{x}\) appear. \(C[\vec{P}]\) is \(C[\vec{x}]\) with \(x_i\) replaced by \(P_i\) wherever it occurs.

**Definition 2.1** [GSOS Rule] Suppose \(\Sigma\) is a signature. A GSOS rule \(\rho\) over \(\Sigma\) is a rule of the form:

\[
\bigcup_{j=1}^{f} \left\{ x_i \xrightarrow{a_{ij}} y_{ij} \mid 1 \leq j \leq m_i \right\} \cup \bigcup_{i=1}^{f} \left\{ x_i \xrightarrow{b_{ik}} \mid 1 \leq k \leq n_i \right\}
\]

\(f(x_1, \ldots, x_l) \xrightarrow{c} C[\vec{x}, \vec{y}]\)  

where all the variables are distinct, \(m_i, n_i \geq 0, a_{ij}, b_{ik}\), and \(c\) are actions, \(f\) is an operation symbol from \(\Sigma\) with arity \(l\), and \(C[\vec{x}, \vec{y}]\) is a \(\Sigma\)-context.

It is useful to name components of rules. The operation symbol \(f\) is the principal operation of the rule, and the term \(f(\vec{x})\) is the source. \(C[\vec{x}, \vec{y}]\) is the target; \(c\) is the action; the formulae above the line are the antecedents (sometimes denoted by \(\text{ante}(\rho)\)); and the formula below the line is the consequent (sometimes denoted by \(\text{cons}(\rho)\)).

For a GSOS rule \(\rho\), \(SV(\rho)\) and \(TV(\rho)\) are the sets of source and target variables of \(\rho\); that is, \(SV(\rho)\) is the set of variables in the source of \(\rho\), and \(TV(\rho)\) is the set of \(y\)'s for antecedents \(x \xrightarrow{a} y\).

**Definition 2.2** A GSOS language is a pair \(G = (\Sigma_G, R_G)\) where \(\Sigma_G\) is a finite signature and \(R_G\) is a finite set of GSOS rules over \(\Sigma_G\).

Informally, the intent of a GSOS rule is as follows. Suppose that we are wondering whether \(f(\vec{P})\) is capable of taking a \(c\)-step. We look at each rule with principal operation \(f\) and action \(c\) in turn. We inspect each positive antecedent \(x_i \xrightarrow{a_{ij}} y_{ij}\), checking if \(P_i\) is capable of taking an \(a_{ij}\)-step for each \(j\) and if so calling the \(a_{ij}\)-children \(Q_{ij}\). We also check the negative antecedents; if \(P_i\) is incapable of taking a \(b_{ik}\)-step for each \(k\). If so, then the rule fires and \(f(\vec{P}) \xrightarrow{c} C[\vec{P}, \vec{Q}]\). This means that the transition relation \(\xrightarrow{G}\) associated with a GSOS language \(G\) is the one defined by the rules using structural induction over closed \(\Sigma_G\)-terms. This transition relation is the unique sound and supported transition relation. Here sound means that whenever a closed substitution \(\sigma\) ‘satisfies’ the antecedents of a rule of the form [1], written \(\xrightarrow{G, \sigma}\), then \(f(x_1, \ldots, x_l) \sigma \xrightarrow{G} C[\vec{x}, \vec{y}] \sigma\). On the other hand, supported means that any transition \(p \xrightarrow{c} q\) can be obtained by instantiating the conclusion of a rule \(\rho\) of the form [1] with a substitution that satisfies its premises. In that case, we say that \(p \xrightarrow{G} q\) is supported by \(\rho\). A rule \(\rho\) is junk in \(G\) if it does not support any transition in \(\xrightarrow{G}\). We refer the interested reader to [11] for the precise definition of \(\xrightarrow{G}\) and much more information on GSOS languages.

For each closed term \(p\), we define \(\text{init}(p) = \{a \in \text{Act} \mid \exists q : p \xrightarrow{a} G q\}\). For a GSOS language \(G\), we let \(\text{init}(T(\Sigma_G)) = \{\text{init}(p) \mid p \in T(\Sigma_G)\}\).

The basic notion of equivalence among terms of a GSOS language we will consider in this paper is bisimulation equivalence [26, 30].

**Definition 2.3** Suppose \(G\) is a GSOS language. A binary relation \(\sim \subseteq T(\Sigma_G) \times T(\Sigma_G)\) over closed terms is a bisimulation if it is symmetric and \(p \sim q\) implies, for all \(a \in \text{Act}\),

\[p \xrightarrow{a} G p' \text{ then, for some } q', q \xrightarrow{a} G q' \text{ and } p' \sim q'.\]

We write \(p \xrightarrow{G} q\) if there exists a bisimulation \(\sim\) relating \(p\) and \(q\). The subscript \(G\) is omitted when it is clear from the context.
It is well known that $\leftrightarrow_G$ is a congruence for all operation symbols $f$ of $G$ [11]. Let $\text{Bisim}(G)$ denote the quotient algebra of closed $\Sigma_G$-terms modulo bisimulation. Then, for $P, Q \in \mathcal{T}(\Sigma_G)$,

$$\text{Bisim}(G) \models P = Q \iff (\forall \text{closed } \Sigma_G\text{-substitutions } \sigma : P\sigma \leftrightarrow_G Q\sigma).$$

In what follows, we shall sometimes consider equations that hold over all GSOS languages that extend a GSOS language $G$ with new operation symbols and rules for the new operations. The following notions from [2] put these extensions on a formal footing.

**Definition 2.4** A GSOS language $G'$ is a disjoint extension of a GSOS language $G$ if the signature and rules of $G'$ include those of $G$, and $G'$ introduces no new rules for operations of $G$. If $G'$ disjointly extends $G$ then $G'$ introduces no new outgoing transitions for the closed terms of $G$. This means in particular that $P \leftrightarrow_G Q$ iff $P \leftrightarrow_{G'} Q$, for $P, Q \in \mathcal{T}(\Sigma_G)$. (More general conservative extension results are discussed in, e.g., [15, 29].)

For $G$ a GSOS language, let $\text{BISIM}(G)$ stand for the class of all algebras $\text{Bisim}(G')$, for $G'$ a disjoint extension of $G$. Thus we have, for $P, Q \in \mathcal{T}(\Sigma_G)$,

$$\text{BISIM}(G) \models P = Q \iff (\forall G' : G' \text{ a disjoint extension of } G \implies \text{Bisim}(G') \models P = Q).$$

Checking the validity of a statement of the form $\text{Bisim}(G) \models P = Q$ or $\text{BISIM}(G) \models P = Q$ according to the above definition is at best very impractical, as it involves establishing bisimilarity of all closed instantiations of the terms $P$ and $Q$. It would thus be helpful to have techniques that use only information obtainable from these terms and that can be used to this end. The development of one such technique will be the subject of the remainder of this paper.

**Eliminating Junk Rules** Note that the definition of a GSOS language given above does not exclude junk rules, i.e., rules that support no transition in $\rightarrow_G$. For example, the rule

$$\frac{x \rightarrow_G y, \quad x \not\rightarrow_G}{f(x) \not\rightarrow_G f(y)}$$

has contradictory antecedents and can never fire. Also it can be the case that a (seemingly innocuous) rule like

$$\frac{x \rightarrow_G y}{f(x) \not\rightarrow_G f(y)}$$

does not support any transition if $\rightarrow_G$ contains no $a$-transitions. The possible presence of junk rules does not create any problems in the development of the theory of GSOS languages as presented in [2, 11] and the authors of those papers saw no reason to deal with these rules explicitly.

Our aim in this paper is to develop a test for the validity of equalities between open terms in GSOS languages. The test we shall present in later sections is based upon the idea of using GSOS rules as ‘abstract transitions’ in a bisimulation-like equivalence between open terms. In order to ease the applicability of this method, it is thus desirable, albeit not strictly necessary, to eliminate junk rules from GSOS languages, as these rules would be interpreted as ‘potential transitions’ from a term which, however, cannot be realized.

Consider, for example, the trivial GSOS language TRIV with unary operations $f$ and $g$, and rule

$$f(x) \rightarrow_G f(x).$$
It is immediate to see that Bisim(TRIV) \models f(x) = g(y) as the set of closed terms in TRIV is empty. However, if we considered the rule for \( f \) as a transition from \( f(x) \) in a simple-minded way, we would be led to distinguish \( f(x) \) and \( g(y) \) as the former has a transition while the latter does not. Obviously, the rule for \( f \) given above is junk.

Clearly junk rules can be removed from a GSOS language \( G \) without altering the associated transition relation. Of course, in order to be able to remove junk rules from a GSOS language, we need to be able to discover effectively what rules are junk. This is indeed possible, as the following theorem, due to Aceto, Bloom and Vaandrager [Theorem 5.22][5], shows.

**Theorem 2.1** Let \( G = (\Sigma, R_G) \) be a GSOS language. Suppose that \( \rho \in R_G \). Then it is decidable whether \( \rho \) is junk in \( G \).

As a consequence of the above theorem, all the junk rules in a GSOS language can be effectively removed in a pre-processing step before applying the techniques described in the subsequent sections. Thus we will henceforth restrict ourselves to GSOS languages without junk rules.

### 3 Ruloids and the Operational Specification of Contexts

As mentioned above, the essence of our method for checking the validity of equations in GSOS languages is to devise a variation on bisimulation equivalence between contexts which considers GSOS rules as transitions. For primitive operations in a GSOS language \( G \), the rules in \( R_G \) will be viewed as abstract transitions from terms of the form \( f(\vec{x}) \). However, in general, we will be dealing with complex contexts in \( \mathcal{T}(\Sigma) \). In order to apply our ideas to general open terms, we will thus need to associate with arbitrary contexts a set of derived rules (referred to as *ruloids* [11]) describing their behaviour.

A *ruloid* for a context \( D[\vec{x}] \), with \( \vec{x} = (x_1, \ldots, x_l) \), takes the form:

\[
\bigcup_{i=1}^{l} \left\{ x_i \xrightarrow{a_{ij}} y_{ij} \mid 1 \leq j \leq m_i \right\} \cup \bigcup_{i=1}^{l} \left\{ x_i \xrightarrow{b_{ik}} \mid 1 \leq k \leq n_i \right\}
\]

\[
D[\vec{x}] \xrightarrow{c} C[\vec{x}, \vec{y}]
\]

(2)

where the variables are distinct, \( m_i, n_i \geq 0, a_{ij}, b_{ik} \), and \( c \) are actions, and \( C[\vec{x}, \vec{y}] \) is a \( \Sigma \)-context.

**Definition 3.1** A set of ruloids \( R \) is *supporting* for a context \( D[\vec{x}] \) and action \( c \) iff all the consequents of ruloids in \( R \) are of the form \( D[\vec{x}] \xrightarrow{c} C[\vec{x}, \vec{y}] \), and whenever \( D[\vec{P}] \xrightarrow{c} p \), there are a ruloid \( \rho \in R \) and a closed substitution \( \sigma \) such that \( \text{cons(}\rho\text{)}\sigma = D[\vec{P}] \xrightarrow{c} p \) and \( \rightarrow_G, \sigma \mid \text{ante(}\rho\text{)} \).

The following theorem is a slightly sharpened version of the Ruloid Theorem in [11].

**Theorem 3.1** [Ruloid Theorem] Let \( G \) be a GSOS language and \( X \subseteq \text{Var} \) be a finite set of variables. For each \( D[\vec{x}] \in \mathcal{T}(\Sigma) \) and action \( c \), there exists a finite set \( R_{D,c} \) of ruloids of the form (2) such that:

1. the ruloids in \( R_{D,c} \) are sound and supporting for \( D[\vec{x}] \), and
2. for every \( \rho \in R_{D,c} \), \( \text{TV}(\rho) \cap X = \emptyset \).

\[1\]Our terminology departs slightly from that of [11]. Bloom, Istrail and Meyer use ‘specifically witnessing’ in lieu of ‘supporting’.
Proof: A straightforward adaptation of the proof of the corresponding result in [11], where we take care in choosing the target variables in ruloids so that condition 2 in the statement of the theorem is met. □

Definition 3.2 Let $G$ be a GSOS language. For each $D[\vec{x}] \in \mathcal{T}(\Sigma_G)$, the ruloid set of $D[\vec{x}]$, notation $R_G(D[\vec{x}])$, is the union of the sets $R_{D,c}$ given by Theorem 3.1.

The import of the Ruloid Theorem is that the operational semantics of an open term $P$ can be described by a finite set $R_G(P)$ of derived GSOS-like rules. Examples of versions of the above result for more expressive formats of operational rules may be found in, e.g., the references [10, 16].

Example 3.1 Consider a GSOS language $G$ containing the sequencing operation specified by the following rules (one such pair of rules for each $a \in \text{Act}$).

$$
\begin{align*}
& x \xrightarrow{a} z \\
& x;y \xrightarrow{a} z;y \\
& x \xrightarrow{b} (\forall b \in \text{Act}), y \xrightarrow{a} z \\
& x;y \xrightarrow{a} z
\end{align*}
$$

(3)

Let $R[x,y,z] = x;(y;z)$ and $L[x,y,z] = (x;y)z$. The ruloids for $L$ and $R$ are:

$$
\begin{align*}
& x \xrightarrow{a} x' \\
& L \xrightarrow{a} (x';y);z \\
& R \xrightarrow{a} x';(y;z)
\end{align*}
\quad
\begin{align*}
& x \xrightarrow{a} x' \\
& L \xrightarrow{a} y';z \\
& R \xrightarrow{a} y';z
\end{align*}
\quad
\begin{align*}
& x \xrightarrow{a} x', y \xrightarrow{b} y' \\
& L \xrightarrow{a} z' \\
& R \xrightarrow{a} z'
\end{align*}
(4)

where we write $x \xrightarrow{a}$ in the antecedents of ruloids as a shorthand for $x \xrightarrow{b} (\forall b \in \text{Act})$.

Remark 3.1 Note that the set $R_G(D[\vec{x}])$ of ruloids for a context $D[\vec{x}]$ in a GSOS language $G$ may contain junk ruloids even when $G$ has no junk rule. For example, consider the GSOS language with constants $a$ and $0$, unary operation $g$ and binary operation $f$ with the following rules.

$$
\begin{align*}
& a \xrightarrow{a} 0 \\
& x \xrightarrow{a} x', y \xrightarrow{b} y' \\
& f(x,y) \xrightarrow{a} 0 \\
& g(x) \xrightarrow{b} 0
\end{align*}
$$

None of the above rules is junk. However, the only ruloid for the context $f(x,g(x))$ is

$$
\begin{align*}
& x \xrightarrow{a} x', x \xrightarrow{a'} \\
& f(x,g(x)) \xrightarrow{a} 0
\end{align*}
$$

which is junk. However, junk ruloids can be removed from the set of ruloids for a context using Theorem 2.1 In what follows, we shall assume that the set of ruloids we consider have no junk ruloids.

In the standard theory on GSOS, it was not necessary to pay much attention to the variables in rules and ruloids, as one was only interested in the transition relation they induced over closed terms. (In the terminology of [18], all the variables occurring in a GSOS rule/ruloid are not free.) Here, however, we intend to use ruloids as abstract transitions between open terms. In this framework it becomes desirable to give a more reasoned account of the role played by variables in ruloids, as the following example shows.
Example 3.2 Consider a GSOS language \( G \) containing the unary operations \( f \) and \( g \) with the following rules:

\[
\begin{align*}
x & \xrightarrow{a} y \\
f(x) & \xrightarrow{a} y \\
x & \xrightarrow{a} z \\
g(x) & \xrightarrow{a} z
\end{align*}
\]

It is easy to see that \( \text{Bisim}(G) \models f(x) = g(x) \), regardless of the precise description of \( G \). However, in order to prove this equality, any bisimulation-like equivalence relating open terms in \( \Omega(\Sigma_G) \) would have to relate the variables \( y \) and \( z \) in some way. Of course, this will have to be done carefully, as \( y \) and \( z \) are obviously not equivalent in any nontrivial language.

As the above-given example shows, in order to be able to prove many simple equalities between open terms, it is necessary to develop techniques which allow us to deal with the target variables in ruloids in a reasonable way. In particular, we should not give too much importance to the names of target variables in ruloids.

Definition 3.3 [Valid Ruloids] Let \( G \) be a GSOS language and \( P \in \Omega(\Sigma_G) \). We say that a ruloid \( \rho = \frac{H}{P \xrightarrow{a} P'} \) is valid for \( P \) iff there exist \( \rho' \in R_G(P) \) and an injective map \( \sigma : \text{TV}(\rho') \rightarrow (\text{Var} - \text{SV}(\rho)) \) such that \( \rho \) is identical to \( \rho' \sigma \).

For example, it is immediate to notice that the rules

\[
\begin{align*}
x & \xrightarrow{a} z \\
f(x) & \xrightarrow{a} z \\
x & \xrightarrow{a} y \\
g(x) & \xrightarrow{a} y
\end{align*}
\]

are valid for the contexts \( f(x) \) and \( g(x) \) in the above-given example. Note, moreover, that each ruloid in \( R_G(P) \) is a valid ruloid for \( P \).

The following lemma states that, if \( \rho' \) is obtained from \( \rho \) as in Definition 3.3 then \( \rho \) and \( \rho' \sigma \) are, in a sense, semi-equivalent ruloids.

Lemma 3.1 Let \( G = (\Sigma_G, R_G) \) be a GSOS language and \( P \in \Omega(\Sigma_G) \). Assume that \( \rho \) is a valid ruloid for \( P \) because \( \rho = \rho' \sigma \) for some \( \rho' \in R_G(P) \) and injective \( \sigma : \text{TV}(\rho') \rightarrow (\text{Var} - \text{SV}(\rho)) \). Then:

1. \( \rho \) is sound for \( \rightarrow_G \);
2. \( \text{Supp}(\rho) = \text{Supp}(\rho') \), where, for a GSOS rule/ruloid \( \hat{\rho} \), \( \text{Supp}(\hat{\rho}) \) denotes the set of transitions supported by \( \hat{\rho} \).

The set of valid ruloids for a context \( P \) is infinite. However, by Theorem 3.1 we can always select a finite set of valid ruloids for \( P \) which is sound and supporting for it. We will often make use of this observation in what follows.

4 A Logic of Transition Formulae

The set of ruloids associated with an open term \( P \) in a GSOS language characterizes its behaviour in much the same way as GSOS rules give the behaviour of GSOS operations. In fact, by Theorem 3.1 every transition from a closed term of the form \( P\sigma \) can be inferred from a ruloid in \( R_G(P) \).
The antecedents of ruloids give the precise conditions under which ruloids fire. When matching ruloids in the definition of the bisimulation-like relation between open terms that we aim at defining, we will let a ruloid $\rho$ be matched by a set of ruloids $J$ only if the antecedents of $\rho$ are stronger than those of the ruloids in $J$, i.e., if whenever $\rho$ can fire under a substitution $\sigma$, then at least one of the ruloids in $J$ can. In order to formalize this idea, we will make use of a simple propositional logic of initial transition formulae.

We define the language of initial transition formulae to be propositional logic with propositions of the form $x \xrightarrow{a}$. Formally, the formulae of such a logic are given by the following grammar:

$$F ::= \text{True} \mid x \xrightarrow{a} \mid \neg F \mid F \land F.$$ 

As usual, we write $\neg \text{True}$ for $\neg \text{True}$, and $F \lor F'$ for $\neg(\neg F \land \neg F')$.

Let $G$ be a GSOS language. A $G$-model for initial transition formulae is a substitution $\sigma$ of processes (closed $\Sigma_G$-terms) for variables. We write $\rightarrow_G, \sigma \models F$ if the closed substitution $\sigma$ is a model of the initial transition formula $F$. The satisfaction relation $\models$ is defined by structural recursion on $F$ in the obvious way. In particular,

$$\rightarrow_G, \sigma \models x \xrightarrow{a} \text{ iff } \sigma(x) \xrightarrow{a_G} p, \text{ for some } p.$$ 

The reader familiar with Hennessy-Milner logic \[20\] will have noticed that the propositions of the form $x \xrightarrow{a}$ correspond to Hennessy-Milner formulae of the form $\langle a \rangle \text{True}$. If $H$ is a set of positive or negative transition formulae (e.g., the hypotheses of a rule or ruloid), then $\text{hyps}(H)$ is the conjunction of the corresponding initial transition formulae. For example, $\text{hyps}\left(\{x \xrightarrow{a} y, z \xrightarrow{b}\}\right) = (x \xrightarrow{a}) \land \neg(z \xrightarrow{b})$. If $J$ is a finite set of ruloids, we overload $\text{hyps}(\cdot)$ and write:

$$\text{hyps}(J) \triangleq \bigvee_{\rho' \in J} \text{hyps}(\text{ante}(\rho')) . \quad (5)$$

The semantic entailment preorder between initial transition formulae may be now defined in the standard way; for formulae $F, F'$, we have $\models_G F \Rightarrow F'$ iff every substitution that satisfies $F$ must also satisfy $F'$.

In the remainder of this paper, we will use the semantic entailment preorder between transition formulae in our test for equivalence of open terms to characterize the fact that if one ruloid may fire, then some other may do so too. Of course, in order to be able to use the entailment preorder between transition formulae in our test for open equalities, we need to able to check effectively when $\models_G F \Rightarrow F'$ holds. Fortunately, the semantic entailment preorder between formulae is decidable, as the following theorem shows.

**Theorem 4.1** Let $G$ be a GSOS language. Then for all formulae $F$ and $F'$, it is decidable whether $\models_G F \Rightarrow F'$ holds.

Theorem 4.1 tells us that we can safely use semantic entailment between formulae in our simple propositional language in the test for the validity of open equations in GSOS languages which we will present in what follows.

### 5 Rule-matching Bisimulation

We will now give a method to check the validity of equations in the algebra $\text{Bisim}(G)$ based on a variation on the bisimulation technique. Our approach has strong similarities with, and is a sharpening of,
FH-bisimulation, as proposed by de Simone in [33, 34]. (We remark, in passing, that FH-bisimilarity checking has been implemented in the tool ECRINS [14, 24].)

**Definition 5.1** [Rule-matching Bisimulation] Let $G$ be a GSOS language. A relation $\approx \subseteq \mathbb{T}(\Sigma_G) \times \mathbb{T}(\Sigma_G)$ is a **rule-matching bisimulation** if it is symmetric and $P \approx Q$ implies

for each ruloid $\frac{H}{P \xrightarrow{a} P'}$ in the ruloid set of $P$, there exists a finite set $J$ of valid ruloids for $Q$ such that:

1. For every $P' = \frac{H'}{Q \xrightarrow{a} Q'} \in J$, we have:
   (a) $a' = a$,  
   (b) $P' \approx Q'$,
   (c) $(\text{TV}(P') \cup \text{TV}(P)) \cap (\text{SV}(P) \cup \text{SV}(P')) = \emptyset$ and
   (d) if $y \in \text{TV}(P) \cap \text{TV}(P')$, then $x \xrightarrow{h} y \in H \cap H'$ for some source variable $x \in \text{SV}(P) \cap \text{SV}(P')$ and action $h$.

2. $\models_G \text{hyps}(P) \Rightarrow \text{hyps}(J)$.

We write $P \leftrightarrow^\text{RM}_G Q$ if there exists a rule-matching bisimulation $\approx$ relating $P$ and $Q$. We sometimes refer to the relation $\leftrightarrow^\text{RM}_G$ as **rule-matching bisimilarity**.

Note that, as the source and target variables of GSOS rules and ruloids are distinct, condition (1c) is equivalent to $\text{TV}(P) \cap \text{SV}(P') = \emptyset$ and $\text{TV}(P') \cap \text{SV}(P) = \emptyset$. Moreover, $\leftrightarrow^\text{RM}_G$ is just standard bisimilarity over closed terms.

Of course, the notion of rule-matching bisimulation is reasonable only if we can prove that it is sound with respect to the standard extension of bisimulation equivalence to open terms. This is the import of the following theorem.

**Theorem 5.1** [Soundness] Let $G$ be a GSOS language. Then, for all $P, Q \in \mathbb{T}(\Sigma_G)$, $P \leftrightarrow^\text{RM}_G Q$ implies $\text{Bisim}(G) \models P = Q$.

The import of the above theorem is that, when trying to establish the equivalence of two contexts $P$ and $Q$ in a GSOS language $G$, it is sufficient to exhibit a rule-matching bisimulation relating them. A natural question to ask is whether the notion of rule-matching bisimulation is **complete** with respect to equality in $\text{Bisim}(G)$, i.e. whether $\text{Bisim}(G) \models P = Q$ implies $P \leftrightarrow^\text{RM}_G Q$, for all $P, Q \in \mathbb{T}(\Sigma_G)$. Below, we shall provide a counter-example to the above statement.

**Example 5.1** Consider a GSOS language $G$ consisting of a constant $(a + b)\omega$ with rules

$$
\frac{}{(a + b)\omega \xrightarrow{a} (a + b)\omega} \quad \frac{}{(a + b)\omega \xrightarrow{b} (a + b)\omega}
$$

and unary function symbols $f$, $g$, $h$ and $i$ with rules

$$
\frac{x \xrightarrow{a} y_1, x \xrightarrow{b} y_2}{h(x) \xrightarrow{a} f(x)} \quad \frac{x \xrightarrow{a} y_1, x \xrightarrow{b} y_2}{i(x) \xrightarrow{a} g(x)} \quad \frac{x \xrightarrow{a} y_1, x \xrightarrow{b} y_2}{f(x) \xrightarrow{a} f(x)} \quad \frac{x \xrightarrow{a} y_1}{g(x) \xrightarrow{a} g(x)}
$$

First of all, note that no rule in $G$ is junk as the hypotheses of each of the above rules are satisfiable.
We claim that \( \text{Bisim}(G) \models h(x) = i(x) \). To see this, it is sufficient to note that, for all \( p \in T(\Sigma_G) \),

\[
\begin{align*}
  h(p) \xrightarrow{c} r & \iff p \xrightarrow{a}, \ p \xrightarrow{b}, \ c = a \text{ and } r \equiv f(p) \quad \text{and} \\
  i(p) \xrightarrow{c} r & \iff p \xrightarrow{a}, \ p \xrightarrow{b}, \ c = a \text{ and } r \equiv g(p) .
\end{align*}
\]

Moreover, for a term \( p \) such that \( a, b \in \text{init}(p) \), it is immediate to see that \( f(p) \leftrightarrow g(p) \) as both these terms can only perform action \( a \) indefinitely.

However, \( h(x) \) and \( i(x) \) are not rule-matching bisimilar. In fact, in order for \( h(x) \xleftrightarrow{\text{RM}} i(x) \) to hold, it must be the case that \( f(x) \xleftrightarrow{\text{RM}} g(x) \). This does not hold as the unique rule for \( g(x) \) cannot be matched by the rule for \( f(x) \) because \( \not\models (x \xrightarrow{a} \implies (x \xrightarrow{a} \land x \xrightarrow{b} )) \). Take, \( e.g. \), a closed substitution \( \sigma \) such that \( \sigma(x) \equiv h((a + b)^a) \).

Intuitively, the failure of rule-matching bisimulation in the above example is due to the fact that, in order for \( \text{Bisim}(G) \models h(x) = i(x) \) to hold, it is sufficient that \( f(p) \) and \( g(p) \) be bisimilar for those terms \( p \) which enable transitions from \( h(p) \) and \( i(p) \), rather than for arbitrary instantiations.

Note that the equation discussed in Example 5.1 is valid in each disjoint extension of the GSOS language considered there. In the following section we will provide examples that will, hopefully, convince our readers that rule-matching bisimulation is a tool which, albeit not complete, can be used to check the validity of many interesting equations.

It is natural to ask oneself at this point whether rule-matching bisimilarity is preserved by taking disjoint extensions, \( \text{i.e.} \), whether an equation that has been proven to hold in a language \( G \) using rule-matching bisimilarity remains sound for each disjoint extension of \( G \). The following example shows that this is not the case.

**Example 5.2** Consider a GSOS language \( G \) consisting of a constant \( a^0 \) with rule \( a^0 \xrightarrow{a} a^0 \) and unary operations \( f \) and \( g \) with the following rules.

\[
\begin{align*}
  x^a & \xrightarrow{c} x' \\
  f(x) & \xrightarrow{a} f(x) \\
  y^a & \xrightarrow{c} y' \\
  g(y) & \xrightarrow{a} g(y)
\end{align*}
\]

First of all, note that no rule in \( G \) is junk as the hypotheses of each of the above rules are satisfiable.

We claim that \( \text{Bisim}(G) \models f(x) = g(y) \). To see this, it is sufficient to note that each closed term in the language is bisimilar to \( a^0 \). Moreover, \( f(x) \xleftrightarrow{\text{RM}} g(y) \) holds because the formulae \( x \xrightarrow{a} \) and \( y \xrightarrow{a} \) are logically equivalent in \( G \). On the other hand, consider the disjoint extension \( G' \) of \( G \) obtained by adding the constant \( 0 \) with no rules to \( G \). In this disjoint extension, \( f(x) \xleftrightarrow{\text{RM}}_{G'} g(y) \) does not hold because \( x \xrightarrow{a} \) does not entail \( y \xrightarrow{a} \).

However, rule-matching bisimilarity in language \( G \) is preserved by taking disjoint extensions if the language \( G \) is sufficiently expressive in the sense formalized by the following result.

**Theorem 5.2** Let \( G \) be a GSOS language such that \( \text{init}(T(\Sigma_G)) = 2^{\text{Act}} \). Then, for all \( P, Q \in \mathcal{T}(\Sigma_G) \), \( P \xleftrightarrow{\text{RM}}_{G} Q \) implies \( \text{BISIM}(G) \models P = Q \).

**Proof:** The proof of Theorem 5.1 can be replayed, making use of the observations that for each disjoint extension \( G' \) of \( G \), the collection of ruloids in \( G' \) for a \( \Sigma_G \)-term \( P \) coincides with the collection of ruloids for \( P \) in \( G \). Moreover, in light of the proviso of the theorem, \( \models_{G} F \Rightarrow F' \text{ iff } \models_{G'} F \Rightarrow F' \), for all formulae \( F \) and \( F' \). \( \Box \)
A conceptually interesting consequence of the above result is that, when applied to a sufficiently expressive GSOS language $G$, rule-matching bisimilarity is a proof method that is, in some sense, \textit{monotonic with respect to taking disjoint extensions of the original language}. This means that rule-matching bisimilarity can only prove the validity of equations in $G$ that remain true in all its disjoint extensions. A similar limitation applies to the proof methods presented in, \textit{e.g.}, [34, 35].

6 Examples

We shall now present some examples of applications of the ‘rule-matching bisimulation technique’. In particular, we shall show how some well known equations found in the literature on process algebra can be verified using it.

 Associativity of Sequencing  Let $G$ be any GSOS language containing the sequencing operation specified by (3). Let $R[x,y,z] = x; (y;z)$ and $L[x,y,z] = (x;y); z$. The ruloids for these two contexts were given in (4).

Consider the symmetric closure of the relation

$$\approx \overset{\Delta}{=} \{(R[x,y,z], L[x,y,z]) \mid x,y,z \in \text{Var}\} \cup I$$

where $I$ denotes the identity relation over $\Sigma_G$. By Theorem 5.1, to show that the contexts $L$ and $R$ are equivalent, it is sufficient to check that what we have just defined is a rule-matching bisimulation. In particular, we need to check the correspondence between the ruloids for these contexts (which is the one given in (4)), and then check that the targets are related by $\approx$. The verification of these facts is trivial. Thus we have shown that sequencing is associative in any GSOS language that contains the sequencing operation.

The associativity proofs for the standard parallel composition operators found in \textit{e.g.} ACP, CCS, SCCS and MEIJE, and for the choice operators in those calculi follow similar lines.

 Commutativity of Interleaving Parallel Composition  Many standard axiomatizations of behavioural equivalences in the literature cannot be used to show that, \textit{e.g.}, parallel composition is commutative and associative. We will now show how this can be easily done using the rule-matching bisimulation technique. We will exemplify the methods by showing that the interleaving parallel composition operation $\parallel$ [21] is commutative.

We recall that the rules for $\parallel$ are (one pair of rules for each $a \in \text{Act}$):

$$
\begin{align*}
  &\frac{x \overset{a}{\rightarrow} x'}{x \parallel y \overset{a}{\rightarrow} x' \parallel y} & \quad &\frac{y \overset{a}{\rightarrow} y'}{x \parallel y \overset{a}{\rightarrow} x \parallel y'}.
\end{align*}
$$

The ruloids for the contexts $x \parallel y$ and $y \parallel x$ given by Theorem 3.1 are the following ones (one pair of ruloids for each $a \in \text{Act}$).

$$
\begin{align*}
&\frac{x \overset{a}{\rightarrow} x'}{x \parallel y \overset{a}{\rightarrow} x' \parallel y} & \quad &\frac{y \overset{a}{\rightarrow} y'}{x \parallel y \overset{a}{\rightarrow} x \parallel y'}.
\end{align*}
$$

It is now immediate to see that the relation $\{(x \parallel y, y \parallel x) \mid x,y \in \text{Var}\}$ is a rule-matching bisimulation in any GSOS language that includes the interleaving operator. In fact, the correspondence between the ruloids is trivial and the targets are related by the above relation.
Fix a partial, commutative and associative function \( \gamma : \text{Act} \times \text{Act} \rightarrow \text{Act} \), which describes the synchronization between actions. The \(|\|\) operation can be described by the rules (for all \(a, b, c \in \text{Act}\)):

\[
\begin{align*}
\frac{x \xrightarrow{a} x'}{x\|y \xrightarrow{a} x'\|y} & \quad \frac{y \xrightarrow{a} y'}{x\|y \xrightarrow{a} x'\|y'} & \quad \frac{x \xrightarrow{a} x', y \xrightarrow{b} y'}{x\|y \xrightarrow{a} x'\|y'} \quad \gamma(a, b) = c
\end{align*}
\]

Figure 1: The rules for \(||\)

**De Simone’s Clock Example** In his seminal paper [34], de Simone presents a bisimulation based technique useful for proving open equations between contexts specified using the so-called de Simone format of operational rules. On page 260 of that paper, de Simone discusses two examples showing that there are valid open equalities between contexts that his technique cannot handle. Below, we shall discuss a variation on one of his examples, the **clock example**, which maintains all the characteristics of the original one in [34], showing how rule-matching bisimulations can be used to check the relevant equalities.

Suppose we have a GSOS language which includes parallel composition with synchronization, \(||\), described by the rules (6), the interleaving operation, \(\|\), described by the rules (6), and a constant \(\Omega_{\text{Act}}\) (the **clock over the whole set of actions** in de Simone’s terminology) with rules

\[
\Omega_{\text{Act}} \xrightarrow{a} \Omega_{\text{Act}} \quad (a \in \text{Act})
\]

Consider the contexts \(C[x] \equiv x\|\Omega_{\text{Act}}\) and \(D[x] \equiv x\|\Omega_{\text{Act}}\). We do have that, regardless of the precise description of \(G\), the terms \(C[x], D[x]\) and \(\Omega_{\text{Act}}\) are all equal in \(\text{Bisim}(G)\). This can be easily shown by establishing that the symmetric closures of the relations \(\{(C[p], \Omega_{\text{Act}}) \mid p \in \text{T}(\Sigma_G)\}\) and \(\{(D[p], \Omega_{\text{Act}}) \mid p \in \text{T}(\Sigma_G)\}\) are bisimulations. However, as argued in [34], de Simone’s techniques based on FH-bisimilarity cannot be used to establish these equalities. We can instead show their validity using our rule-matching bisimulation technique as follows.

First of all, we compute the ruloids for the contexts \(C[x]\) and \(D[x]\). These are, respectively,

\[
\begin{align*}
\frac{C[x] \xrightarrow{a} C[x]}{(a \in \text{Act})} & \quad \frac{x \xrightarrow{a} x'}{C[x] \xrightarrow{a} C[x']} (a \in \text{Act}) & \quad \frac{x \xrightarrow{a} x'}{C[x] \xrightarrow{b} C[x']} & \quad \exists c \in \text{Act} : \gamma(a, c) = b
\end{align*}
\]

and

\[
\begin{align*}
\frac{D[x] \xrightarrow{a} D[x]}{(a \in \text{Act})} & \quad \frac{x \xrightarrow{a} x'}{D[x] \xrightarrow{a} D[x']} (a \in \text{Act})
\end{align*}
\]

Now, it can be easily checked that the symmetric closure of the relation

\[
\{(C[x], D[z]), (C[x], \Omega_{\text{Act}}), (D[x], \Omega_{\text{Act}}) \mid x, z \in \text{Var}\}
\]

is a rule-matching bisimulation. The point is that any ruloid for \(C[x]\) can be matched by an axiom for \(D[z]\), and, vice versa, any ruloid for \(D[z]\) can be matched by an axiom for \(C[x]\). This is because it is always the case that \(\models (x \xrightarrow{a}) \Rightarrow \text{True}\) for \(x \in \text{Var}\).
7 Partial Completeness Results

In previous sections, we showed that the rule-matching bisimulation technique, albeit not complete in general, can be used to prove several important equations found in the literature on process algebras. In particular, the soundness of all the equations generated by the methods in [2] can be proven by exhibiting appropriate rule-matching bisimulations. A natural question to ask is whether there are some classes of contexts for which rule-matching bisimulations give us a complete proof technique for establishing equality between contexts. One such class of contexts is, of course, that of closed terms, as rule-matching bisimilarity coincides with bisimilarity over processes.

Below we will present another partial completeness result, this time with respect to a class of contexts that we call ‘persistent’.

Definition 7.1 Let $G$ be a GSOS language and $P \in \mathcal{P}(\Sigma_G)$. We say that $P$ is persistent iff each ruloid in $R_G(P)$ is of the form $\frac{H}{P \cong P}$ for some $a \in \text{Act}$.

Thus persistent contexts are terms that test their arguments, perform actions according to the results of these tests, and then remain unchanged.

Theorem 7.1 [Completeness for Persistent Contexts] Let $G$ be a GSOS language. Then $\text{Bisim}(G) \models P = Q$ iff $P \leftrightarrow_{RM} Q$, for all persistent $P, Q \in \mathcal{P}(\Sigma_G)$.

We now proceed to introduce another class of operations for which rule-matching bisimilarity yields a complete proof method.

Definition 7.2 [Non-inheriting Rule] A GSOS rule of the form (1) is non-inheriting if none of the variables in $\vec{x}$, namely the source variables in the rule, occurs in the target of the conclusion of the rule $C[\vec{x}, \vec{y}]$. A GSOS language is non-inheriting if so is each of its rules. Non-inheriting de Simone rules and languages are defined similarly.

Theorem 7.2 Let $G$ be a non-inheriting GSOS language that, for each $P \in \mathcal{P}(\Sigma_G)$ and $c \in \text{Act}$, contains at most one ruloid for $P$ having $c \in \text{Act}$ as action. Let $G'$ be the disjoint extension of $G$ obtained by adding to $G$ the operations and rules of the language BCCSP [17, 26] with Act as set of actions. Let $P$ and $Q$ be terms over $\Sigma_G$. Then $\text{Bisim}(G') \models P = Q$ implies $P \leftrightarrow_{G'} Q$.

A minor modification of the proof for the above result yields a partial completeness result for a class of de Simone systems.

Theorem 7.3 Let $G$ be a non-inheriting de Simone language that, for each $f \in \Sigma_G$ and $c \in \text{Act}$, contains at most one rule having $f \in \Sigma_G$ as principal operation and $c \in \text{Act}$ as action. Let $G'$ be the disjoint extension of $G$ obtained by adding to $G$ the constant $0$ and the Act-labelled prefixing operations from the language BCCSP [17, 26]. Let $P$ and $Q$ be terms over $\Sigma_G$. Then $\text{Bisim}(G') \models P = Q$ implies $P \leftrightarrow_{G'} Q$.

For instance, the above theorem yields that rule-matching bisimilarity can prove all the sound equations between terms constructed using variables and the operations of restriction and injective relabelling from CCS [26] and synchronous parallel composition from CSP [21].
8 Related and Future Work

The development of general methods for proving equivalences between open terms in expressive process calculi is a challenging subject that has received some attention since the early developments of the algebraic theory of processes—see, e.g., the references \[12, 23, 32, 34, 35\] for some of the work in this area. De Simone’s FH-bisimilarity \[34\] represents an early meaningful step towards a general account of the problem, presenting for the first time a sound bisimulation method in place of the usual definition which involves the closure under all possible substitutions. Our method relies mainly on the concepts underlying FH-bisimilarity and it is a refinement of that notion in the more expressive setting of GSOS languages. (See de Simone’s ‘Clock Example’ discussed on page \[12\] where FH-bisimilarity fails while \(\leftrightarrow RM\) succeeds.)

Later Rensink addressed the problem of checking bisimilarity of open terms in \[32\], where he presented a natural sharpening of de Simone’s FH-bisimilarity. His extension of FH-bisimilarity is orthogonal to ours and provides another method to check equivalences between open terms that is more powerful than the original FH-bisimilarity. Rensink defined a new notion of bisimulation equivalence, called hypothesis preserving bisimilarity, that adds to FH-bisimilarity the capability to store some kind of information about the variable transitions during the computation.

To explain the import of hypothesis preserving bisimilarity we can look at Example 5.1. We note that \(\leftrightarrow RM\) fails to establish the sound equation \(h(x) = i(x)\) because at the second step of the computation some knowledge about the transitions of the closed term \(p\) substituted for \(x\) is already established (indeed, at that point we know that \(p\) performs a \(b\)-transition, since this has been tested at the first step). Nevertheless, when comparing \(f(x)\) and \(g(x)\), rule-matching bisimulation behaves in memoryless fashion and ignores this information. Rensink’s hypothesis preserving bisimilarity takes into account the history and this is enough to overcome the difficulties in that example and analogous scenarios. Adding this feature to \(\leftrightarrow RM\) would lead to a more powerful rule-matching equivalence; we leave this further sharpening for future work together with extensions of \(\leftrightarrow RM\) to more expressive rule formats.

Recently, van Weerdenburg addressed the automation of soundness proofs in \[35\]. His approach differs from the one in \[32, 34\] and ours since he translates the operational semantics into a logical framework. In such a framework, rules are encoded as logical formulae and the overall semantics turns out to be a logical theory, for which van Weerdenburg provides a sequent calculus style proof system. In the aforementioned paper, he offers some examples of equivalences from the literature that can be proved using his method in order to highlight its applicability. However, even though the ultimate aim of the research described in \[35\] is the automation of soundness proofs, van Weerdenburg’s system presents some drawbacks. The main point is that the user is not only required to provide the operational semantics and the equation to check (together with the standard encoding of bisimilarity), but he must also provide a candidate bisimulation relation that can be used to show the validity of the equation under consideration together with all the axioms that are needed to complete the proof. The user is supposed thus to have a clear understanding of what the proof is going to look like. This seems to be a general and inescapable drawback when approaching the problem of checking equations through a translation into a logical system.

Despite the aforementioned slight drawback, the approach proposed by van Weerdenburg is, however, very interesting and complements the proposals that are based on the ideas underlying de Simone’s FH-bisimilarity, including ours. We believe that an adequate solution to the problem of automating checks for the validity of equations in process calculi will be based on a combination of bisimulation-based and logical approaches.

A related line of work is the one pursued in, e.g., the papers \[1, 13, 28\]. Those papers present
rule formats that guarantee the soundness of certain algebraic laws over a process language ‘by design’, provided that the SOS rules giving the semantics of certain operators fit that format. This is an orthogonal line of investigation to the one reported in this article. As a test case for the applicability of our rule-based bisimilarity, we have checked that the soundness of all the equations guaranteed to hold by the commutativity format from [28] can be shown using $\leftrightarrow_{RM}$. We are carrying out similar investigations for the rule formats proposed in [1, 13].

Another avenue for future research we are actively pursuing is the search for more, and more general, examples of partial completeness results for rule-matching bisimulation over GSOS and de Simone languages. Indeed, the partial completeness results we present in Section 7 are just preliminary steps that leave substantial room for improvement. Last, but not least, we are about to start working on an implementation of a prototype checker for rule-matching bisimilarity.

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