Quark stars and quantum-magnetically induced collapse

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Quark matter is expected to exist in the interior of compact stellar objects as neutron stars or even the more exotic strange stars, based on the Bodmer-Witten conjecture. Bare strange quark stars and (normal) strange quark-matter stars, those possessing a baryon (electron-supported) crust, are hypothesized as good candidates to explain the properties of a set of peculiar stellar sources as the enigmatic X-ray source RX J1856.5-3754, some pulsars as PSR B1828-11 and PSR B1642-03, and the anomalous X-ray pulsars and soft gamma-ray repeaters. In the MIT bag model, quarks are treated as a degenerate Fermi gas confined to a region of space having a vacuum energy density $B_{bag}$ (the Bag constant). In this note, we modify the MIT Bag Model by including the electromagnetic interaction. We also show that this version of the MIT model implies the anisotropy of the Bag pressure due to the presence of the magnetic field. The equations of state of degenerate quarks gases are studied in the presence of ultra strong magnetic fields. The behavior of a system made-up of quarks having (or not) anomalous magnetic moment is reviewed. A structural instability is found, which is related to the anisotropic nature of the pressures in this highly magnetized matter. The conditions for the collapse of this system are obtained and compared to a previous model of neutron stars build-up on a
neutron gas having anomalous magnetic moment.

**Keywords**: magnetic field; MIT Bag Model; instabilities

1. Introduction

The relation between strong magnetic fields and dense matter is a subject that has attracted so much attention recently, especially after the observations of peculiar X-ray emission from anomalous X-pulsars (AXPs) and low energy γ-ray radiation from soft gamma-ray repeaters (SGRs). The central engine of these radiations is believed to be a neutron star or 'quark star' endowed with a magnetic field larger than $10^{13.5}$ G. The x-ray binaries dubbed as Galactic Black Hole Candidates have also been recently suggested as possessing a strange star as its primary Ref. 4,5,6.

The existence of quark stars was proposed in 1969, about five years after than the Gell-Man prediction of quarks. Bodmer Ref. 1 also advanced the idea of strange quark matter in 1971. By quark matter is meant here a Fermi gas of 3A quarks which together constitute a single color-singlet baryon with baryon number A. It is known two types of 'quark matter': the first one is called 'non-strange matter', which consists only of two flavor $u$ and $d$ (the 2SC model), while the second one, dubbed 'strange matter', is made-up of 3 flavors $u$, $d$ and $s$. Strange matter has the particularity that it guarantees the equilibrium with weak interactions. In 1984 Witten Ref. 2, and later Farhi and Jaffe Ref. 3, showed that for strange matter, the binding energy could be lower than for Fe over a rather wide range of QCD parameters. Witten Ref. 2 has pointed out two possible pathways to form strange matter: the quark hadron phase transition in the early Universe and the conversion of neutron stars into strange ones at extremely high densities. In this way the most stable compact star might be a strange star rather than a neutron star. Neutrons and protons are both constituted of quarks, hence as a highly dense system of nucleons, a neutron star, especially its core, might have been deconfined to a quark (plus gluons) system under the extraordinary conditions imposed by a a very dense nuclear matter and supercritical magnetic field. Because of its extremely high density ($\geq 10^{14}$ g/cm$^3$), a neutron star might actually be a pure 'quark star', or alternatively a neutron star might contain a 'quark core'.

The equation of state (EOS) of both strange and non-strange matter, and the structure and stability of stars constituted of each of these types of matter have been studied using simplified models such as the MIT Bag Model Ref. 3 (see also Lugones & Horvath 2003, and the important set of actualized references therein). The MIT Bag Model is the most important hypothesis introduced to study the properties of quark matter for quark stars. As such, it considers the balance quarks as a relativistic free-particle system confined in an impenetrable bag whose equilibrium rests on the bag constant $B_{bag}$, i.e.,

$$B_{bag} = -\frac{1}{2} \frac{\partial}{\partial r} (\bar{\psi} \psi),$$

(1)

where $\psi$ defines the quark field. In other words, the balance pressure on the bag
surface stems from the outward Fermi gas pressure counterbalanced by the inward (vacuum) bag pressure, which mimics the strong interaction that hold the quarks together.

On the other hand, the matter in this condition, extremely dense, exhibits novel properties which are worthwhile to study. Several authors have studied equations of state and thermodynamical properties of these systems and all have concluded that the physical behavior change Ref. 2-7: Thermodynamical properties, dynamics, etc. In particular, in the paper Ref. 8 we study the neutron gas in presence of a strong magnetic field with the aim to describe neutron stars. We unveiled the anisotropic behavior of the pressures perpendicular and parallel to the magnetic field direction. This result implies that the 'shape' of some astrophysical object endowed with extremely high magnetic fields may become prolate once quantum effects are taken into account. Other important conclusions of this paper suggest that we can determine the physical conditions for which the pressure perpendicular to the (dipole) magnetic field could vanish, and thus for the system to be unstable to collapse pulled down by its own gravity. It is then timely to emphasize that when these systems are described within a classical approach the conclusions are quite opposite, in the sense that the pressure parallel to the $B$-field turns out to be smaller than that perpendicular to it Ref. 9. The star then takes an oblate shape and the collapse may take place in the parallel direction to the field. The shape the astrophysical object would adopt would be flat or toroidal, i.e., similar to a disk or torus perpendicular to the $B$-field Ref. 10.

Strong magnetic fields are known to exist in the interior of compact stars. Notwithstanding, only a few attempts to study the behavior of quark matter permeated by a magnetic field have been performed so far Ref. 11.

The scope of this paper is to show that for 'quark matter' acted upon by a strong magnetic field the same anisotropic behavior obtained earlier in Ref. 7-8 holds. For the sake of simplicity, we focus here on the case of non-strange quark matter, as a degenerate Fermi gas of quarks. Nonetheless, our treatment and conclusions prove to be independent of the specific model being used. As could be seen later, it is possible (self-consistently) to use the MIT bag model in presence of a magnetic field and the result leads to obtain an anisotropic Bag-pressure that depends on the direction of the magnetic field. The equation (1) is modified because spinors interact with the magnetic field.

To study the degenerate quark gas in presence of a magnetic field we follow two approaches: in the first one, one considers the quark field interacting with magnetic field via its charge. In the second one, it is considered that quarks have anomalous magnetic moment. At the end, we obtain little differences between both approaches which are related to the stability of the system. When we study a model which takes into account the anomalous magnetic moment, then the system becomes more stable; in the sense that for a fixed density it is necessary to have a more stronger magnetic field in order to bring the system into instability.

Chakrabarty et al. Ref. 11 analyzed the conditions for stability of nucleonic bags,
e.g., neutrons and protons, and obtained the instability condition for these particles inasmuch the same way as we do here for a quark gas. Although we agree with their statement that nucleons immerse in a huge magnetic field become unstable, we have some criticisms to their approach to the issue. The first one is related to the use of our thermodynamical arguments for nucleons since these are 3 particle systems (it is not completely satisfactory to use quantum statistical physics to study a few body system). The second one is that in the extremely degenerate case the anisotropic behavior of pressures obtained in this paper Eq.(5)-(6) is such that the transverse pressure goes to zero when the magnetic field grows while the parallel pressure grows with the field strength. Counter to this view, those authors claimed just the opposite (such a behavior only occurs in the classical case).

This paper is organized in the following way. In section 2 we describe shortly the general conditions of two-flavor, non-strange quark matter of the MIT model in presence of a magnetic field. We also rewrite the energy-momentum tensor and state its relation to the EOS of the system. In section 3 we study the system of a degenerate quark gas by firstly including the anomalous magnetic moment, and after by ignoring it. We also include a comparison of our results here with the model of the neutron gas with anomalous magnetic moment as a toy model of neutron stars Ref. 8. Section 4 gives the conclusions.

2. Bag Model in presence of a Magnetic Field

To study quark matter we have to use QCD. However, for our purpose here is enough to use the MIT-Bag Model Ref. 3, which in a phenomenologically way mimics the strong interaction. We start by introducing a couple of basic parameters: $\alpha_c$ the coupling constant, and $B_{bag}$ as the vacuum pressure in the MIT Bag Model.

Nevertheless, in this paper it is enough for us to work with a two-flavor quark matter despite it is not in equilibrium with the weak interaction. We essentially want to show that the presence of a magnetic field in a quark matter ensemble forces the appearance of an instability which is of similar nature to that we have previously found for both the electron and the neutron gas Ref. 7-8.

The Lagrangian density including the MIT $B_{bag}$ Model in presence of a magnetic field should be written as

$$L_{Bag} = \bar{\psi}(i\gamma_\mu(\partial^\mu - ie_q A^\mu) - m_q)\psi - 1/4F_{\mu\nu}F^{\mu\nu} - B_{Bag}\theta_\nu(x) - \frac{1}{2}\bar{\psi}\Delta_\theta$$  (2)

where $\psi$ is the wave function of quarks $m_q$ represents the current quark masses and $e_q$ corresponds to the quark charges, $B_{bag}$ is the bag constant, and the parameter $\theta_\nu$ taking the values: $\theta_\nu = 1$ inside the Bag, while outside it takes on $\theta_\nu = 0$. $\partial\theta_\nu/\partial x^\nu = n_\nu\Delta_\delta$, with $\Delta_\delta$ being the surface $\delta$-function and $n_\nu$ is a space-like unit vector normal to the surface, and $F_{\mu\nu}$ defines the electromagnetic (Maxwell) tensor.

The total energy-stress tensor has the form

$$T_{\mu\nu} = \frac{\partial L_{Bag}}{\partial a_{i,\mu}}a_{i,\nu} - \delta_{\mu\nu}L_{Bag}. \quad (3)$$
where the index $i$ denotes the fields (either fermions or vector components). In this case $a_i$ refers to the $\psi$ and $A_\mu$ fields Ref. 13.

In that way $T_{\mu\nu}$ has the form

$$T_{\mu\nu} = \left( \frac{1}{2} i [\bar{\psi} \gamma^\mu \partial^\nu \psi - i \partial^\mu \bar{\psi} \gamma^\nu \psi] - \frac{\partial L_{\text{Bag}}}{\partial A_{\mu}} A_\nu - B_{\text{Bag}} \right) \theta^\nu - g^{\mu\nu} L_{\text{Bag}}. \quad (4)$$

Using the energy-momentum conservation written as $T^\beta_{\beta\nu} = 0$ we obtain

$$B_{\text{Bag}} \Delta s_n^\nu + \left( \frac{i}{2} [\bar{\psi} \gamma^\mu \partial^\nu \psi - \partial^\nu \bar{\psi} \gamma^\mu \psi] - \frac{\partial L_{\text{Bag}}}{\partial A_{\mu}} A_\nu \right) n^\mu A_\nu = 0, \quad (5)$$

and

$$B_{\text{Bag}} n^\nu \frac{1}{2} \partial_{x^\nu} \bar{\psi} \psi + n^\mu \frac{\partial L_{\text{Bag}}}{\partial A_{\mu}} A_\nu = 0. \quad (6)$$

This equation is the pressure balance equation. Here we take the normalization condition $n^\mu n_\mu = -1$ and considering a pure constant magnetic field with $A_\mu = B_2 \left[ -x_2, x_1, 0, 0 \right]$, and the gauge derivative $\partial_\mu \rightarrow \partial_\mu - ie_q A_\mu$.

In Ref. 14 was proved that the last term in Eq.(6) is proportional to $B^2$ and it is absorbed by a renormalization process. Thus, only the first term of (6) remains. More detailed calculations will appear in Ref. 12. If we take $n^\mu = (0, x_i), i = 1, 2, 3$ then $B_{\text{Bag}}$ takes the form

$$B_{\text{Bag}} = \frac{1}{2} n_i \left( \frac{\partial}{\partial x_i} \bar{\psi}(x) \psi(x) \right) \quad (7)$$

and therefore $B_{\text{Bag}}$ in the presence of a magnetic field has an anisotropic form which depends on the $B$-direction in space, here related to the $n_i$ direction.

### 2.1. Quark matter EOS in presence of magnetic field

This section is devoted to obtain the equation of state of magnetized quark matter which starting point is the statistical average of energy-momentum tensor, Eq.(4). To do that we use standard methods of finite temperature quantum field theory Ref. 15.

The calculation of $T_{\mu\nu} = < T_{\mu\nu} >$ formally means to replace the Lagrangian by the thermodynamical potential $\Omega$, which then is given by the expression

$$\Omega = \frac{1}{\beta} \ln \left< e^{\beta H} d^4 x \delta^3 \mathbf{x} L_{\text{Bag}}(x_4, \mathbf{x}) \right> \quad (8)$$

where the four-components of $x_\mu$ have the form $x_\mu \equiv (x_4, \mathbf{x}) \equiv (ict, \mathbf{x})$, and $\beta = \frac{1}{kT}$, with $k$ the Boltzman constant and $T$ is the temperature.

Let us note that in the context of the modified MIT Bag model to describe magnetized quark matter, only what happens inside the bag is interesting, so we will put $\theta^\nu = 1$ in both the Lagrangian and thermodynamical potential.

The calculation of the thermodynamical potential is presented in next section. Starting from it we can calculate the energy momentum tensor. As we see below,
the structure of $T_{\mu\nu}$ remains model independent. Similar structure was obtained in Ref. 14, 8, 7 where the energy-momentum tensor was derived from QED and electroweak theory. The thermodynamical potential contains information about the matter and electromagnetic interactions, and the electromagnetic tensor also appears in the structure.

Let us note that these steps have been followed in details in Ref. 7-8 for degenerate gases of electrons and neutrons. For that reason in this note we do not devote time in deriving the energy momentum tensor, which can be deduced doing a careful extrapolation of the previous papers in Ref. 7-8. The expression of $T_{\mu\nu}$ has the form

$$
T_{\mu\nu} = \left( T \frac{\partial \Omega}{\partial T} + \sum_\mu \mu_i \frac{\partial \Omega}{\partial \mu_i} \right) \delta_{\mu\nu} + 4F_\mu^\lambda F^\nu_\lambda \frac{\partial \Omega}{\partial F^2} - \delta_{\mu\nu} \Omega,
$$

where for the sake of simplicity we assume

$$
\Omega = \Omega_q - B_{bag},
$$

and $\Omega_q$ is the thermodynamical potential of the magnetized degenerate quark gas, (obviously independent of the $B_{bag}$ constant), and $q$ denotes the species of quarks.

Let us recall that the expression of $T_{\mu\nu}$ is general and would be used to take into account the electric and magnetic interactions. The scope of this paper is to analyze only magnetic effects, so in what follow we consider the magnetic field pointing in the direction $x_3$.

All off-diagonal components of this tensor vanish. The diagonal components of the tensor $T_{\mu\nu}$ corresponds to the energy density and the pressures, albeit the last ones are anisotropic due to the magnetic field. The tensor can be written as follows

$$
T_{\mu\nu} = \begin{pmatrix}
U & 0 & 0 & 0 \\
0 & P_\perp & 0 & 0 \\
0 & 0 & P_\perp & 0 \\
0 & 0 & 0 & P_3
\end{pmatrix}, \quad P_\perp = P_3 - MB, \quad P_3 = -\Omega.
$$

Equations (11) show that $P_\perp \leq P_3$ if the magnetization is a positive quantity, and thus the behavior of the gas is paramagnetic. In the next section we calculate explicitly the magnetization and the pressures. We can prove that the magnetization is a positive quantity.

The condition of quantum-magnetically-induced transverse collapse: $P_\perp = 0$, discussed in Ref. 7-8, implies for the magnetically modified MIT Bag model, the relation

$$
B_{bag}^\perp = -\Omega_q - MB.
$$

However, this condition would be enforced also for the Bag collapse, together with

$$
B_{bag}^\parallel = -\Omega_q.
$$
This would mean an anisotropic bag pressure (which is otherwise expected from (7) since the nucleon is deformed by the magnetic field), and in place of (8) we would have $\Omega \delta_{\mu\nu} \rightarrow \delta_{\mu\nu} \Omega_q - B_{\mu\nu}^{Bag}$. Notice, nonetheless, that in the absence of the magnetic field the pressure becomes isotropic and we recover the condition of stability (or instability) of the Bag model given by $B_{bag} = -\Omega_q$.

Thus, we conclude that a magnetic field brings in an instability to the system, but it is compatible with the Bag Model if we consider that the Bag is not isotropic and the nucleons are deformed in a prolate-shape.

3. Degenerate quark gas with/without anomalous magnetic moment (AMM)

Our aim in this section is to discuss the collapse described in the previous section. We derive the thermodynamical quantities of the degenerate quark gas in presence of an ultra strong magnetic field. In particular we calculate the thermodynamical potential and magnetization to evaluate the anisotropic pressures Eq. (11).

In general, the quark spectrum in the external field $B$ has the form

$$E_q = \sqrt{p_q^2 + \left(\sqrt{2e_q B n + m_q^2 + \eta Q_q B}\right)^2},$$

(14)

where $e_q$ is the quark’s charge, $m_q$ represents the quark masses, $\eta = \pm$ are the eigenvalues corresponding to two orientations of magnetic moment (parallel and antiparallel), and $Q_q$ anomalous magnetic moment of quarks. We also define $y_q = Q_q/m_q$, $b_q = 2e_q/m_q^2$ as relative quantities and $x_q = \mu_q/m_q$, $g_q(x_q, B, n) = \sqrt{x_q^2 - h_q(B, n)^2}$ and $h_q(B, n) = b_q B n + 1 + \eta y_q B$ as dimensionless ones.\(^a\)

For the sake of simplicity we study here nonstrange matter but the conclusions for strange matter are essentially the same. In this case: $Q_u = 1.82\mu_N$, $Q_d = -0.9\mu_N$, being $\mu_N$ the nuclear magneton $\mu_N = hc/m_n$. For quarks the critical $B$-field at which the coupling energy of its magnetic moment equals the rest-energy is for $u$-quarks $B_u^c = 6.3 \times 10^{15}$ G and for $d$ $B_d^c = 1.2 \times 10^{16}$ G.

Thus, for the quarks thermodynamical potential Eq. (8) we get

$$\Omega = \sum \Omega_q,$$

(15)

with

$$\Omega_q = -\Omega_q^0 B \sum_{n} \sum_{\pm\eta} \left[ x_q g_q - h_q^2 \ln \frac{x_q + g_q}{h_q} \right],$$

(16)

where $\Omega_q^0 = e_q m_q^2 B / 4\pi^2 (hc)^2$, and the sum over $n$ represents the sum over Landau levels up to $n_{max}$ given by the expression

$$n_{max} = I \left( \frac{(x_q - \eta y_q B)^2 - 1}{h_q B} \right),$$

(17)

\(^a\)Thermodynamical quantities have been written here in the cgs system and $m_q$ has energy dimension.
where \( I \) stands for the integer part function. The magnetization is then given as
\[
\mathcal{M} = \sum \mathcal{M}_q, \tag{18}
\]
with
\[
\mathcal{M}_q = \mathcal{M}_q^0 \sum_n \sum_{\pm \eta} \left\{ g_q x_q - \left( x_q^2 - 2 h_q B \left[ \frac{b_q n}{2 \sqrt{b_q B_n + 1}} + \eta y_q \right] \right) \ln \frac{x_q + g_q}{h_q} \right\}, \tag{19}
\]
with \( \mathcal{M}_q^0 = \frac{e_q m_q^2}{4 \pi^2 (\hbar c)^2} \). The expression (19) is always a positive quantity because the first term \( g_q x_q \) is greater than the second one.

The density of particles has the form
\[
N = \sum_q N_q \tag{20}
\]
with
\[
N_q = N_q^0 \left( \frac{B}{B_c^q} \right) \sum_n \sum_{\pm \eta} g_q(x_q, B, n), \tag{21}
\]
where \( N_q^0 = \frac{m_q^3}{(4 \pi^2 (\hbar c)^2)^{\frac{3}{2}}} \), and \( B_c^q = \frac{m_q^2}{e_q (\hbar c)} \).

The charge neutrality in this case is given by the expression
\[
N_d = 2 N_u. \tag{22}
\]

The presence of the magnetic field introduces restrictions to given values of the chemical potential of the species of quarks. Using Eq. (17) and Eq. (22) we can compute numerically the relationship between the chemical potentials \( \mu_d \) and \( \mu_u \) and their corresponding dimensionless quantity \( x_q \).

By bringing the relations for the pressures given in (11) to the expressions for the Bag pressures we obtain for the anisotropic pressures the following expressions
\[
B_{\| \text{bag}} = P_{\|} = -\frac{e_q m_q^2 B}{4 \pi^2 \hbar c} \sum_n \sum_{\pm \eta} \left[ x_q g_q - h_q^2 \ln \frac{x_q + g_q}{h_q} \right], \tag{23}
\]
\[
B_{\perp \text{bag}} = P_{\perp} = \frac{2 e_q m_q^2 B^2}{\pi^2 (\hbar c)^2} \sum_n \sum_{\pm \eta} \left( 2 h_q \left[ \frac{b_q n}{2 \sqrt{b_q B_n + 1}} + \eta y_q \right] \ln \frac{x_q + g_q}{h_q} \right). \tag{24}
\]

Let us remark that we can also study a model of a degenerate quark gas without taking into account the quark magnetic moment. This means to make \( y_q = 0 \) in all the equations above. We recover in that case the expressions for all the thermodynamical quantities.

In Fig.1 is shown the phenomenon of anisotropy of the pressures. In this case, we observe that the limiting case \( P_{\perp} = 0 \) is possible to achieve for typical values of the density \( N = 10^{39} \text{ cm}^{-3} \) and \( B \)-field typical of the interior of millisecond spinning just-born neutron stars \( B \sim 10^{17} \text{ G} \).
No AMM

AMM

--Ppar

++Ppar

- Pper

..Ppar

0

1

2

3

4

5

6

7

8

P[(eV/cm^3)*10^35]

1e+18 5e+18 1e+19 5e+19 1e+20

B[Gauss]

Fig. 1. Anisotropy of pressures for the two cases studied above: a gas of quarks \((u, d)\) having anomalous magnetic moment, and without it. The first being more stable as the pressure \(P_{\perp}\) goes to zero for a large magnetic field.

The relation between the particle density and magnetic field strength that fulfills the condition \(P_{\perp} = 0\) is the following

\[
N_q(B) = \sqrt{2} N_q^0 y_q \left( \frac{B^2}{B_q^2} \right). \tag{25}
\]

In Fig.2 we present our result for the degenerate quark gas with and without anomalous magnetic moment. We also plot the curve obtained by using a model of a neutron gas having anomalous magnetic moment. We conclude that the most stable configuration can be reached for models of quark stars. In other words, the condition of \(P_{\perp} = 0\) is obtained for a lower value of the magnetic field in the case of a neutron gas as compared to the quark gas. In the same figure, one can also see that a model of a degenerate quark gas with anomalous magnetic moment gives a more wide region of stability than both the neutron gas and the quark gas models which have no anomalous magnetic moment. In other words, and as expected, the degenerate quark gas with anomalous magnetic moment is the more stable configuration.
4. Conclusions

We have explored the behavior of a quark gas in the presence of extremely large magnetic fields $B \sim m_q^2/e_q$. We used a version of the MIT Bag Model which includes the electromagnetic interaction between quarks. From it we verify that the $B_{bag}$ can be replaced by an anisotropic tensor. We confirm that the instability due to the strong magnetic field discussed in Ref. 7-8 is present also in this case. Finally, we show that the degenerate quark gas with anomalous magnetic moment is more stable than the quark gas without it.

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