Local CP-violation in quark-gluon plasma: a lattice study.

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Topological charge density

\[ G^{a}_{\mu \nu} \tilde{G}^{a}_{\mu \nu} \]

\[ \rho_R \neq \rho_L \]
Chiral Magnetic Effect

$\mathbf{J}_\mu = \kappa_B \mathbf{B}_\mu$

Excess of positive charge

Excess of negative charge

Abelev et al. (2009-2010), Kharzeev, McLerran, Warringa (2007)
Step 1: Lattice action

\[ S = -\beta \sum_{x, \mu > \nu} \left\{ \frac{5}{3} \frac{P_{\mu\nu}}{u_0^4} - r_g \frac{R_{\mu\nu} + R_{\nu\mu}}{12 u_0^6} \right\} + c_g \beta \sum_{x, \mu > \nu > \sigma} \frac{C_{\mu\nu\sigma}}{u_0^6}, \]

\[ R_{\mu\nu} = \frac{1}{3} \text{Re} \text{ Tr} \]

\[ C_{\mu\nu\sigma} \equiv \frac{1}{3} \text{Re} \text{ Tr} \]

\[ r_g = 1 + 0.48 \alpha_s(\pi/a) \]

\[ c_g = 0.055 \alpha_s(\pi/a) \]

Lüscher and Weisz (1985), see also Lepage hep-lat/9607076
Step 2: Monte Carlo

• Heat bath for SU(2)

• Using the standard algorithm for each subgroup. Cabibbo & Marinari (1982)

\[
\begin{align*}
    a_1 &= \begin{pmatrix} \alpha_1 & \\ 1 & \end{pmatrix} &
    a_2 &= \begin{pmatrix} 1 & \\ \alpha_2 & \end{pmatrix} &
    a_3 &= \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} & \end{pmatrix}
\end{align*}
\]

• Overrelaxation. Adler (1981)

• Cooling (smearing) for some particular cases. DeGrand, Hasenfratz, Kovács (1997)
Step 3: Fermions & B-field

\[
D_{ov}(0) = \frac{1}{a} \left(1 - A(A^+ A)^{-1/2}\right)
\]

\[
A = 1 - a D_W(0)
\]

Neuberger overlap operator (1998)

\[
\langle \bar{\Psi} \hat{\Gamma} \Psi \rangle \sim Tr \left[ \hat{\Gamma} D_{ov}^{-1} \right]
\]

\[
\hat{\Gamma} \in \{ 1, \gamma^5, \gamma^\mu, \sigma_{\mu \nu} \ldots \}
\]

Buvidovich, Chernodub, Luschevskaya, Polikarpov (2009)
Fluctuations of chirality

\[ \rho_5(x) = \bar{\Psi} \gamma^5 \Psi \]

\[ \langle \rho_5^2 \rangle_{\text{IR GeV}^6} \]

For $14^4$, $T < T_c$: $a = 0.105$ fm, $m_q = 0$

For $14^4$, $T < T_c$: $a = 0.105$ fm, $m_q = 20$ MeV
Current fluctuations

\[ j^\mu (x) = \overline{\Psi} \gamma^\mu \Psi \]
Current-current correlator

\[ G_{ij}(\tau) = \int d^3 \vec{x} \langle J_i(\vec{0}, 0) J_j(\vec{x}, \tau) \rangle \]
... and its spectral function

\[ G_{ij}(\tau) = \int_0^\infty \frac{d\omega}{2\pi} \left[ \frac{\omega}{2T} \frac{\cosh(\omega(\tau - \frac{1}{2T}))}{\sinh(\omega/2T)} \right] \rho_{ij}(\omega) \]

Conductivity

Meson masses

\[ \int_0^\infty d^\tau \rho_{ij}(\tau) = \int_0^\infty d\omega \rho_{ij}(\omega) \]

\[ \rho_{ij}(\tau) = \frac{\omega}{2T} \frac{\cosh(\omega(\tau - \frac{1}{2T}))}{\sinh(\omega/2T)} \]

\[ \rho_{zz}, T = 0, qB = 0 \]

\[ \rho_{zz}, T = 0, qB = (0.45 \text{ GeV})^2 \]

\[ \rho_{xx}, T = 0, qB = (0.45 \text{ GeV})^2 \]

\[ \rho_{zz}, T > T_c, qB = (0.19 \text{ GeV})^2 \]

\[ \rho_{ij}(\omega), \text{GeV}^2 \]

\[ \omega, \text{GeV} \]
Electrical conductivity

\[ \sigma_{ij} = \lim_{\omega \to 0} \frac{\rho_{ij}(\omega)}{4T} \]

P.V. Buividovich, M.N. Chernodub, D.E. Kharzeev, T.K., E.V. Luschevskaia, M.I. Polikarpov (2010)
What does it mean?

- There are similar effects for $T > T_c$ and thus the local CP-violation is present in the both confinement and deconfinement phases.
- Above $T_c$ vacuum is a conductor.
- Below $T_c$ vacuum is either an insulator (for $B = 0$) or an anisotropic conductor (for strong $B$).
- This supports the idea of the CME as a macroscopic current.
Where is it localized?

Negative topological charge density

Positive topological charge density
Inverse Participation Ratio

Observables:

\[ \rho_\lambda(x) = \psi_\lambda^*(x) \psi_\lambda(x) \]  

„Chiral condensate“ for eigenvalue \( \lambda \)

\[ \rho_\lambda^5(x) = \left(1 - \frac{\lambda}{2}\right) \psi_\lambda^* \gamma_5 \psi_\lambda(x) \]  

„Chirality“ = Topological charge density

Inverse Participation Ratio (inverse volume of the distribution):

\[ IPR = N \sum_x \rho_i^2(x) \]

Unlocalized: \( \rho(x) = \text{const}, \) \( IPR = 1 \)
Localized on a site: \( IPR = N \)
Localized on fraction \( f \) of sites: \( IPR = 1/f \)

Fractal dimension (performing the number of measurements with finite lattice spacing):

\[ IPR(a) = \frac{\text{const}}{a^d} \]
Localization of zero-modes

Definition:

$$\text{IPR}_0 = V \left[ \frac{\int_V \left( \rho_0(x) \right)^2}{\left( \int_V \rho_0(x) \right)^2} \right]_{\lambda=0},$$

$$\rho_\lambda(x) = \psi_\lambda^{*\alpha}(x) \psi_\lambda^\alpha(x)$$

$$\rho_5^\lambda(x) = \left(1 - \frac{\lambda}{2}\right) \psi_\lambda^{*\alpha}(x) \gamma_{\alpha\beta}^5 \psi_\lambda^\beta(x)$$
Definition 1:

\[ \text{IPR}^5_0 = V \left[ \frac{\int_V d^4x \left| \rho_0^5(x) \right|^2}{\left( \int_V d^4x \rho_0^5(x) \right)^2} \right]_{\lambda=0}, \]

Definition 2:

\[ \text{IPR}^5_0 = V \left[ \frac{\int_V d^4x \left( \rho_0^5(x) \right)^2}{\left( \int_V d^4x \rho_0(x) \right)^2} \right]_{\lambda=0}, \]
Fractal dimension

Our result: $d = 2 \div 3$

- $d = 0$: instantons
- $d = 1$: percolating monopoles
- $d = 2$: percolating vortices
- $d = 3$: low-dim. domains

About low-dimensional defects in QCD see also V.I. Zakharov, Phys.Atom.Nucl. 68 (2005) 573 [hep-ph/0410034]
Thank you for the attention!

and

Have a good time!