Analysis of the spanwise extent and time persistence of uniform momentum zones in zero pressure gradient and adverse pressure gradient turbulent boundary layers

Shevarjun Senthil\textsuperscript{1}, Callum Atkinson\textsuperscript{1}, and Julio Soria\textsuperscript{1,2}
\textsuperscript{1}Laboratory for Turbulence Research in Aerospace and Combustion, Department of Mechanical and Aerospace Engineering, Monash University, Melbourne 3800, Australia
\textsuperscript{2}Department of Aeronautical Engineering, King Abdulaziz University, Jeddah 21589, Kingdom of Saudi Arabia
E-mail: shevarjun.senthil@monash.edu

Abstract. Time-resolved velocity fields from direct numerical simulations (DNS) are used to investigate the spanwise extent and the time persistence of uniform momentum zones (UMZs) in a zero pressure gradient turbulent boundary layer (ZPG-TBL) and a self-similar adverse pressure gradient turbulent boundary layer (APG-TBL) at the verge of separation. The instantaneous detection methodology introduced by Adrian et al.\textsuperscript{[1]} is used to detect the UMZs and is extended to take into account the spanwise domain length and the temporal evolution of the UMZs. The Reynolds number based on friction velocity ($Re_\tau$) ranges from 1176 to 1277 for the ZPG-TBL and from 1652 to 1745 for the self-similar APG-TBL within the domain of interest. For both the TBL cases, probability density functions (PDFs) of the number of UMZs are computed as a function of the streamwise extent, spanwise extent and time extent. For the ZPG-TBL, when the streamwise length of the domain is greater than or equal to 3 boundary layer thickness, the probability of finding 4 UMZs becomes almost negligible. For the APG-TBL, even when the streamwise domain length is taken as large as 1.3 boundary layer thickness, the probability of finding 4 UMZs is still significant. The spanwise extent of the UMZs is found to be shorter than their streamwise extent regardless of the pressure gradient in the flow. In the ZPG-TBL, the majority of the UMZs have a spanwise extent of the order of one-tenth of a boundary layer thickness while for the APG-TBL, it is found to be on the order of one-hundredth of a boundary layer thickness. In the ZPG-TBL, the probability of finding 2 UMZs that persist over a time period of 2 integral time scale is around 50%. Similarly, for the APG-TBL, the probability of finding 2 UMZs with a time persistence of 0.4 integral time scale is over 50%. In the case of the ZPG-TBL, it is observed that some of the UMZs with higher persistence in time have higher streamwise momentum and are found to be closer to the free-stream in general. This result is consistent with the previous observations by Laskari et al.\textsuperscript{[2]}. In contrast, for the APG-TBL, UMZs with longer time persistence are found closer to the wall with lower streamwise momentum.

1. Introduction
Wall-bounded flows have different types of coherent structures like low-speed streaks, sweeps and ejections, and hairpin vortices\textsuperscript{[3,5]}. One of the many coherent structures in wall-bounded
flows are uniform momentum zones (UMZs), which are uneven regions in the flow with similar streamwise momentum and varying shape with time. Meinhart & Adrian [6] were the first to report the existence of these zones. The UMZs are separated from each other by layers which have high values of the local wall-normal gradient of the streamwise velocity with spanwise vorticity clustered along these boundaries [6]. The interfaces between the UMZs are similar to a shear layer. Adrian et al. [1] proposed a method to identify the instantaneous UMZs based on the probability density function (PDF) of the instantaneous streamwise velocity. Kwon et al. [7] identified the presence of a large core with uniform velocity and low turbulence levels in a turbulent channel flow. Similar experimental studies on turbulent boundary layers using particle image velocimetry have also revealed regions of relatively uniform streamwise velocity [8, 9]. More recently, Laskari et al. [2] investigated the UMZs in a streamwise wall-normal plane of a turbulent boundary layer using time-resolved particle image velocimetry. Laskari et al. [2] found that the presence of higher number of UMZs is linked with the large-scale ejection events, whereas the lower number of UMZs is related to large-scale sweep events. The focus of the present study is to investigate the spanwise extent and time persistence of the UMZs in a zero pressure gradient and an adverse pressure gradient turbulent boundary layer. To the best of the authors’ knowledge, the present analysis is the first to investigate the time persistence of the UMZs as well as the spanwise extent using three dimensional (3D) velocity fields to construct the PDFs.

2. Details of the direct numerical simulation

The turbulent boundary layer (TBL) datasets were computed by solving the incompressible Navier-Stokes equation for velocity and pressure fields [10, 11]. The TBL flows are solved in a Cartesian coordinate system with the flow directions as streamwise \((x)\), wall-normal \((y)\) and spanwise \((z)\). The mean velocity components are represented by \((\langle u \rangle, \langle v \rangle, \langle w \rangle)\) while the corresponding fluctuating components are represented by \((u', v', w')\).

The first version of the code was developed by Simens et al. [10, 11] which was subsequently optimized by Borrell et al. [12] by adding OpenMP (Open Multi-Processing) to the MPI Parallelization. The current version of the code is the one presented in Kitsios et al. [13, 14] modified to enable the simulation of adverse pressure gradient turbulent boundary layer flow. The governing equations are solved using the fractional step method [15, 16]. The grid is staggered only in the streamwise and the wall-normal directions. The spanwise direction is periodic while compact finite difference is used for spatial discretization in the \(x\) and \(y\) directions [17]. Time stepping is achieved using a 3-step Runge Kutta method [11]. The fluid density \((\rho = 1)\) and kinematic viscosity \((\nu)\) are taken as constants. Further details on the DNS code and the parallelisation techniques used in it can be found in Sillero [18] and Borrell et al. [12]. The desired pressure gradient is applied via the far-field boundary condition using the methodology developed by Kitsios et al. [13, 14].

The non-dimensional pressure gradient \((\beta)\) is given by

\[
\beta = \frac{\delta_1 P_{e,x}}{u_T^2 \rho} = \frac{\delta_1 P_{e,x}}{\tau_w},
\]  

(1)

where \(u_T = \sqrt{\tau_w / \rho}\) is the friction velocity, \(\tau_w\) is the mean wall shear stress, \(\rho\) is the fluid density, \(P_{e,x}\) is the far-field streamwise pressure gradient and \(\delta_1\) is the displacement thickness.

The displacement thickness \((\delta_1)\), based on the definition of Spalart & Watmuff [19], is given by

\[
\delta_1(x) = \frac{-1}{U_e} \int_{0}^{\delta_1} y(\Omega_z)(x, y)dy,
\]  

(2)

where \(U_e\) is the outer reference velocity, \((\Omega_z)\) is the mean spanwise vorticity, and \(\delta_1\) is the wall-normal position or the boundary layer thickness at which \((\Omega_z)\) has decayed to 0.2% of the mean
Table 1: Numerical details of the ZPG and the APG-TBLs in their respective domain of interest (DoI). $\delta_{\Omega}$ is $\delta_{\Omega}$ at the start of the DoI and $\langle u \rangle_{\infty}$ is the far-field mean streamwise velocity at the start of the DoI. The integral time scale is defined as $\delta_{\Omega}/\langle u \rangle_{\infty}$.

|                         | ZPG       | APG       |
|-------------------------|-----------|-----------|
| Nominal non-dimensional pressure gradient | $\beta$   | 0         | 39        |
| Streamwise data points  | $n_x$     | 1035      | 1001      |
| Wall-normal data points | $n_y$     | 315       | 1000      |
| Spanwise data points    | $n_z$     | 2048      | 2048      |
| Streamwise domain size  | $l_x/\delta_{\Omega}$ | 9.32      | 1.28      |
| Wall-normal domain size | $l_y/\delta_{\Omega}$ | 3.49      | 2.55      |
| Spanwise domain size    | $l_z/\delta_{\Omega}$ | 12.34     | 1.76      |
| Friction velocity based Reynolds number | $Re_{\tau}$ | 1176 $\rightarrow$ 1277 | 1652 $\rightarrow$ 1745 |
| Displacement thickness based Reynolds number | $Re_{\delta_1}$ | 4678 $\rightarrow$ 5098 | 22182 $\rightarrow$ 28789 |
| Momentum thickness based Reynolds number | $Re_{\delta_2}$ | 3360 $\rightarrow$ 3679 | 9857 $\rightarrow$ 12101 |
| Mean boundary layer thickness | $\overline{\delta_{\Omega}}/\delta_{\Omega}$ | 1.05      | 1.15      |
| Mean friction velocity  | $\overline{\tau}$ | 0.039     | 0.007     |
| Total time period for 200 fields | $t\langle u \rangle_{\infty}/\delta_{\Omega}$ | 3.2       | 0.47      |

Figure 1: Profiles of (a) mean streamwise velocity ($\langle u \rangle$), and (b) Reynolds shear stress ($\langle u'v' \rangle$) for both the TBL cases. The profiles are averaged in streamwise direction within DoI and are non-dimensionalised by $\delta_1$.

The outer velocity ($U_e$), based on the definition of Lighthill [20], is given by

$$U_e(x) = U_\Omega(x, \delta_{\Omega}),$$

(3) vorticity at the wall.
where

\[ U_{\Omega}(x, y) = - \int_0^y \langle \Omega_z \rangle(x, \tilde{y}) \, d\tilde{y}. \]  

(4)

In the present study of the UMZs, a time-resolved DNS of a zero pressure gradient turbulent boundary layer (ZPG-TBL) and a self-similar adverse pressure gradient turbulent boundary layer (APG-TBL) at the verge of separation are considered. For the APG-TBL, a self-similar domain is considered to minimise the influence of the history effects and \( \beta \) has an average value of 39 within the domain of interest (DoI). Profiles of the mean streamwise velocity \( \langle u \rangle \) and the Reynolds shear stress \( \langle u'v' \rangle \) for both the TBL cases are compared in Figure 1. For the ZPG-TBL, the Reynolds shear stress has a broader profile whereas its profile is confined to a much narrower region in the case of the APG-TBL. For the APG-TBL, the peak of the Reynolds shear stress in Figure 1b and the inflection point of the mean streamwise velocity in Figure 1a coincide at an approximate height of the displacement thickness \( y/\delta_1 = 1 \). Profiles of the kinetic energy budgets and the momentum balances for both the TBLs can be found in Kitsios et al. [13, 14].

Numerical details of the two TBL cases in their respective DoI are given in Table 1. \( \delta_{\Omega}^\ast \) is \( \delta_{\Omega} \) at the start of the DoI and \( \langle u \rangle_{\infty}^\ast \) is the far-field mean streamwise velocity at the start of the DoI. For the APG-TBL, the available streamwise domain size relative to the boundary layer thickness \( l_x/\delta_{\Omega} \) is shorter because of the higher boundary layer thickness. The profiles of the boundary layer thickness \( \delta_{\Omega}(x_I) \) for both the TBLs are given in Figure 2, where \( x_I \) is the position of the inlet plane. 200 time-resolved velocity fields are used in the investigation of the UMZs for both the TBL cases. The integral time scale is defined as \( \delta_{\Omega}/\langle u \rangle_{\infty} \). The wall-normal domain size \( l_y \) used in all the analyses is fixed as \( 1.3\delta_{\Omega} \) and \( 0.7\delta_{\Omega} \) for the ZPG-TBL and the APG-TBL respectively. All the PDFs related to the ZPG case are in green while the ones corresponding to the APG-TBL are in red colour.

![Figure 2: Profiles of the boundary layer thickness (δΩ) in the streamwise direction for both the TBL cases, where x_I is the position of the inlet plane.](image)

3. UMZ detection methodology

The instantaneous UMZs and the boundaries that demarcate them are identified based on the method introduced by Adrian et al. [1]. This method is extended to consider the instantaneous three dimensional velocity fields and the temporal evolution of the UMZs [2]. In this method, the local maxima (peaks) and the local minima (troughs) in the probability density function (PDF) of the instantaneous streamwise velocity fields are detected. The modal velocity is defined as the velocity that corresponds to a local peak in the PDF. Similarly, the edge velocity is defined as
Figure 3: For the ZPG-TBL with no thresholds \((T_h = 0, T_p = 0, \text{ and } T_d = 0)\): (a) The PDF of \(u/U_e\) for an instantaneous 2D velocity field in the xy plane. The triangles represent all the detected peaks while the dashed lines refer to the edge velocities; (b) Corresponding contour plot of the instantaneous velocity field with the solid lines representing the contour lines of the edge velocities. \(\delta_{\Omega}\) is the boundary layer thickness at the start of the DoI. Non-dominant peaks with lower streamwise velocity have been detected as no thresholds are used.

the velocity associated with a local minimum in the PDF. The modal velocity can be considered as the characteristic velocity of each of the UMZs [1, 2, 8] and the contour lines of the edge velocities refer to the boundaries between the UMZs in physical space.

Laskari et al. [2] used different thresholds in their peak detection algorithm. In a similar way, three thresholds are defined for the current peak detection method. They are the minimum height required for a peak to be considered detectable \((T_h)\), the minimum prominence of a peak compared to its troughs \((T_p)\), and the minimum allowed distance between two peaks in terms of number of bins \((T_d)\). These thresholds are used to reject non-dominant peaks. \(T_h\) is given by

\[
T_h = \frac{P_i}{P_{NFS_{\text{max}}}},
\]

where \(P_i\) is any given peak in the PDF and \(P_{NFS_{\text{max}}}\) is the maximum among the detected non-freestream (NFS) peaks in the PDF. \(P_i\) is normalised by \(P_{NFS_{\text{max}}}\) to allow comparison of the peaks of the UMZs relative to each other and to ensure that the presence of the freestream peak in the PDF does not influence the detection methodology. \(T_p\) is given by

\[
T_p = \frac{P_i - (E_i + E_{i+1})/2}{P_i},
\]

where \(E_i\) and \(E_{i+1}\) are the troughs in the PDF (the PDF values corresponding to the edge velocities) on both the sides of any given peak \(P_i\) in the PDF.

The number of bins \((N_{\text{bins}})\) used to construct the PDF is 50 for both the TBL cases. For the ZPG-TBL, \(u/U_e \in [0.1, 1.1]\) with the bin width approximately equal to 0.5\(u_\tau\). For the APG-TBL, the size of each bin is approximately equal to 3.1\(u_\tau\) with \(u/U_e \in [0.02, 1.1]\). The range of \(u/U_e\) is started slightly above zero to avoid the peak close to zero because of the no slip boundary condition. Figure 3 shows an example of a PDF and the identified UMZs in the
Figure 4: For the ZPG-TBL: (a) The PDF of \( u/U_e \) for an instantaneous 2D velocity field in the \( xy \) plane. The triangles represent all the detected peaks after applying the thresholds while the dashed lines refer to the edge velocities; (b) Corresponding contour plot of the instantaneous velocity field with the solid lines representing the edges between the UMZs. Similarly, for the APG-TBL: (c) and (d). The thresholds used are \( T_h = 0.2, T_p = 0.2, \) and \( T_d = 2 \) bins. \( \delta_{\Omega^*} \) is the boundary layer thickness at the start of the DoI.

ZPG-TBL when no thresholds are used. Non-dominant peaks with lower streamwise velocity are detected as all the thresholds are taken as zero.

In this study, for both the TBL cases, the values of the thresholds used to reject the non-dominant peaks in the PDF are \( T_h = 0.2, T_p = 0.2, \) and \( T_d = 2 \) bins. Peaks in the PDF are considered detectable if they have values above these thresholds. Figure 4 shows a representative example of a constructed PDF and the corresponding identified UMZs using the described detection methodology for both the TBL cases. Two dimensional (2D) velocity fields in the \( xy \) planes are used to generate the PDFs in the section 4 while three dimensional (3D) velocity fields are used to form the PDFs in all the other following sections.
4. Streamwise extent of UMZs

For a given streamwise domain length \( l_x \), the instantaneous PDFs of the streamwise velocity \((u/U_e)\) are constructed using the various 2D xy planes available in all the fields. Using these velocity PDFs, the number of UMZs \( N_{UMZs} \) for each of the 2D xy planes is calculated. Then, the PDF of \( N_{UMZs} \) is computed for that streamwise extent. This process is repeated for different streamwise lengths. Following this approach, the PDF of \( N_{UMZs} \) as a function of extent in the streamwise direction is obtained, which is illustrated in Figure 5 for both the cases.

Figure 5: (a) PDFs of the number of UMZs \( N_{UMZs} \) as a function of the streamwise extent \( l_x \) for the ZPG-TBL. The selected length is \( l_x = 2\delta_{\Omega^*} \). (b) Similarly, for the APG-TBL upto \( l_x = 1.28\delta_{\Omega^*} \). The selected length is \( l_x = 1.28\delta_{\Omega^*} \).
Figure 6: An example of “super-structures” identified in the ZPG-TBL with the streamwise extent \(l_x\) as long as 6 boundary layer thickness. 3 UMZs are detected in this \(xy\) plane. \(\delta_{\Omega^*}\) is the boundary layer thickness at the start of the DoI.

In Figure 5, the number of samples used to construct each of the PDFs is 409,600. The aim of this approach is to select a domain length which will maximise the probability of finding more number of zones. The streamwise extent is varied up to 6.01\(\delta_{\Omega^*}\) and 1.28\(\delta_{\Omega^*}\) for the ZPG-TBL and the APG-TBL respectively. As the streamwise extent is increased, the probability of finding more number of zones reduces for both the TBLs.

In the case of the ZPG-TBL in Figure 5a, for \(l_x = 0.39\delta_{\Omega^*}\), the probability of finding 4 UMZs is over 20%, while it drops down to less than 10% and becomes insignificant for \(l_x \geq 3.01\delta_{\Omega^*}\). Hence, an extent of \(l_x = 2\delta_{\Omega^*}\) is chosen, which has a probability of over 10% in finding 4 UMZs. Therefore, for the ZPG-TBL, the streamwise extent is fixed as \(l_x = 2\delta_{\Omega^*}\) for all the subsequent analysis, which is the streamwise length that has also been used in previous studies [8]. It is also worth mentioning that there are few UMZs which have a streamwise extent as long as 6\(\delta_{\Omega^*}\). An example of such “super-structures” is shown in Figure 6.

For the APG-TBL in Figure 5b, the probability of finding 4 UMZs is over 30% for \(l_x = 0.25\delta_{\Omega^*}\). When the streamwise extent is increased to \(l_x = 1.28\delta_{\Omega^*}\), the probability of finding 4 UMZs is still significant and over 10%. Therefore, for the APG-TBL, the entire available streamwise extent \(l_x = 1.28\delta_{\Omega^*}\) is chosen to be used in all the analyses.

5. Spanwise extent of UMZs

The spanwise extent of the UMZs are investigated by considering 3D velocity fields with the streamwise length \(l_x = 2\delta_{\Omega^*}\) for the ZPG-TBL and \(l_x = 1.28\delta_{\Omega^*}\) for the APG-TBL. For a given spanwise length (\(l_z\)), the number of UMZs in different 3D sub-domains is computed by varying the location of the domain in the spanwise direction of an instantaneous field. In a similar way, \(N_{UMZs}\) can be computed for all the sub-domains in the available 200 fields, which results in the PDF of \(N_{UMZs}\) for that spanwise length. This process is repeated for different spanwise lengths to obtain the PDF of \(N_{UMZs}\) as a function of the spanwise extent. As shown in Figure 7, the spanwise extent is varied up to 1.6\(\delta_{\Omega^*}\) for both the TBLs. The probability of finding higher number of UMZs decreases with increasing spanwise length. It is apparent right away that the spanwise extent of the UMZs are much shorter than their streamwise extent for both the TBL cases as the results in Figure 7 show.

For the ZPG-TBL, when \(l_z = 0.05\delta_{\Omega^*}\), the probability of finding 3 UMZs is over 25% and it becomes almost negligible for \(l_z \geq 0.2\delta_{\Omega^*}\). This shows that most of the UMZs have a spanwise extent of the order of one-tenth of a boundary layer thickness. Therefore, for the ZPG-TBL, the
spanwise extent is chosen as \( l_z = 0.1\delta_\Omega^* \) for the subsequent analysis. In the case of the APG-TBL, the probability of finding 3 UMZs is over 25\% for \( l_z = 0.01\delta_\Omega^* \), and it becomes insignificant for \( l_z \geq 0.1\delta_\Omega^* \). The spanwise length of most of the UMZs is around the order of one-hundredth of a boundary layer thickness. Hence, the spanwise extent is selected as \( l_z = 0.05\delta_\Omega^* \) for the APG-TBL. It is important to note that when the pressure gradient increases from the ZPG case to the point of verge of separation in the APG case, the spanwise extent of the majority of the UMZs decreases.

\[
\begin{align*}
\text{(a)} & \quad l_z = 0.05\delta_\Omega^* \\
\text{PDFs of the number of UMZs (} N_{UMZs} \text{) as a function of the spanwise extent upto} & \quad l_z = 1.6\delta_\Omega^* \text{ for the ZPG-TBL.} \\
\text{(b)} & \quad l_z = 0.01\delta_\Omega^* \\
\text{Similarly, for the APG-TBL upto} & \quad l_z = 1.6\delta_\Omega^* .
\end{align*}
\]

Figure 7: (a) PDFs of the number of UMZs (\( N_{UMZs} \)) as a function of the spanwise extent upto \( l_z = 1.6\delta_\Omega^* \) for the ZPG-TBL. (b) Similarly, for the APG-TBL upto \( l_z = 1.6\delta_\Omega^* \).
6. Time persistence and time evolution of UMZs

For the ZPG-TBL, the time extent ($l_t$) over which the UMZs persist are investigated by using 3D velocity fields with the domain lengths $l_z = 0.1\delta_{\Omega^*}$, $l_y = 1.3\delta_{\Omega^*}$ and $l_x = 2\delta_{\Omega^*}$, chosen based on the results of sections 4 and 5. Similarly, for the APG-TBL, the selected domain lengths are $l_z = 0.05\delta_{\Omega^*}$, $l_y = 0.7\delta_{\Omega^*}$ and $l_x = 1.28\delta_{\Omega^*}$. 200 time-resolved velocity fields are used in this analysis. For a particular time extent ($l_t$), the total time period is divided into different time subsets. For a particular z sub-domain in a time subset, velocity PDF is constructed to calculate the number of UMZs ($N_{UMZs}$) in that sub-domain. In a similar way, $N_{UMZs}$ are computed for the same z sub-domain in the other time subsets. This process can be repeated for all the z sub-domains in all the time subsets to get the PDF of $N_{UMZs}$ for that particular time extent ($l_t$). Following this approach, the PDF of $N_{UMZs}$ are computed for different time extents as illustrated in Figure 9. The time extent is varied up to $2\delta_{\Omega^*}/\langle u \rangle_{\infty}$ and $0.4\delta_{\Omega^*}/\langle u \rangle_{\infty}$ for the ZPG-TBL and the APG-TBL respectively. In the case of the ZPG-TBL, the probability of finding 2 UMZs is around 50% for all the time extents. This shows that most of the UMZs in the ZPG-TBL persist for a time period of 2 integral time scale. Similarly, for the APG-TBL, the probability of finding 2 UMZs is around 50% for all the time extents considered. This shows that most of the UMZs in the APG-TBL persist over the entire available time period of 0.4 integral time scale.

The time evolution of the UMZs is investigated for both the TBL cases in a similar manner to Laskari et al. [2]. This is done by following a particular z sub-domain over consecutive time steps. Figure 8 shows an example of the instantaneous PDFs generated using 3D fields from a random domain of the chosen size for both the cases. Figure 10 shows the time evolution of the UMZs in that domain for both the cases in terms of the integral time scale. The contours represent the PDF of $u/U_e$ and the squares refer to the modal velocities of each of the detected UMZs. In case of the ZPG-TBL in Figure 10a, the results indicate that the UMZs having higher time persistence are closer to the free-stream and have higher streamwise momentum relative to the other detected UMZs. This behaviour of higher momentum zones having more persistence in time was also noted by Laskari et al. [2]. For the APG-TBL in Figure 10b, the important difference to be noted is that the UMZs with higher time persistence are found closer to the plate. When the flow reaches the point of the verge of separation in the APG-TBL, the results...
Figure 9: (a) PDFs of the number of UMZs ($N_{UMZs}$) as a function of the time extent up to $l_t = 0.13\delta_\Omega^*/\langle u \rangle_\infty^*$ for the ZPG-TBL. (b) Similarly, for the APG-TBL up to $l_t = 0.201\delta_\Omega^*/\langle u \rangle_\infty^*$.

indicate that the UMZs with lower streamwise momentum have more persistence in time relative to the higher momentum UMZs in the flow.

7. Concluding remarks
The 3D time persistence and evolution of the uniform momentum zones (UMZs) have been investigated in a ZPG-TBL and a self-similar APG-TBL at the verge of separation. 200 time-resolved velocity fields from two DNS were used in this study. The instantaneous detection methodology introduced by Adrian et al. [1], which is based on the PDFs of the streamwise
Figure 10: (a) Time evolution of the UMZs in the ZPG-TBL for 200 consecutive velocity fields. The squares refer to the modal velocities of each of the detected UMZs and the contour represents the PDF of $u/u_\infty$. The time ($t$) is expressed in terms of the integral time scale defined based on $\delta_\Omega^*$ and $\langle u \rangle^*_\infty$. (b) Similarly, for the APG-TBL. Modal velocities corresponding to the free stream is not shown here.

velocity is used in the present study and is extended to account for the spanwise extent and the temporal evolution of the UMZs [2]. For the ZPG-TBL, when the streamwise domain length ($l_x$) is greater than or equal to $3\delta_\Omega^*$, the probability of finding 4 UMZs become negligible. In the case of the APG-TBL, even when
the streamwise domain length is chosen as high as $1.28\delta_{\Omega^*}$, the probability of finding 4 UMZs does not become insignificant. The spanwise extent ($l_z$) of the predominant number of UMZs are shorter than their streamwise extent irrespective of the pressure gradient in the flow. For the ZPG case, the majority of the UMZs have a spanwise extent of the order of one-tenth of a boundary layer thickness ($\delta_{\Omega^*}$), whereas, for the APG case, the spanwise extent of most of the UMZs is found to be shorter with values of the order of one-hundredth of a boundary layer thickness.

In the ZPG-TBL, the probability of finding 2 UMZs of size $l_z = 0.1\delta_{\Omega^*}$, $l_y = 1.3\delta_{\Omega^*}$ and $l_x = 2\delta_{\Omega^*}$ with a time persistence of 2 integral time scale is around 50%. Similarly, for the APG-TBL, the probability of finding 2 UMZs of size $l_z = 0.05\delta_{\Omega^*}$, $l_y = 0.7\delta_{\Omega^*}$ and $l_x = 1.28\delta_{\Omega^*}$ with a time persistence of 0.4 integral time scale is over 50%. For the ZPG-TBL, based on the time evolution of a single sample, it is observed that some of the UMZs with larger time persistence are the zones with higher streamwise momentum and are found closer to the free stream. This result is also consistent with the previous observations made by Laskari et al. [2]. In contrast to the ZPG-TBL, for the APG-TBL at the verge of separation, the UMZs with higher time persistence are found to be the lower momentum zones that are closer to the wall.

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