Study the Acousto-Optic Spatial Filters Diffracted Light at Zeroth-Order Using MATLAB

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Abstract. This paper introduces the essentials of acousto-optics with the theory and applications using MATLAB. There are some software used to process acousto-optics filter and the implementation details of any acousto-optics algorithm are inaccessible. Our work is focused on software used to solving the coupled equations in Bragg diffraction by creating MATLAB function for use in Bragg regime and acoustic-optic spatial filtering and employing mainly the MATLAB program for solving this problem. We review the role of acousto-optic and spatial filtering for high-pass spatial filtering. Classically, acousto-optic interactions comprise scattering of photons by energetic phonons into higher and lower orders. Standard weak interaction theory describes diffraction in the Bragg regime as the propagation of a uniform plane wave of light through a uniform plane wave of sound, resulting in the well known first- and zeroth-order diffraction. We investigate Raman-Nath diffraction corresponding to near-Bragg diffraction. A result shows the dependence of various scattered orders on the incident angle/Bragg angle illustrating the Bessel function dependence.

1. Introduction
Acousto-optics devices are used in laser equipment for electronic control of the intensity and position of the laser beam. In this paper, we will explain the theory and application of acousto-optics filter. Acousto-optics interaction occurs in all optical mediums when an acoustic wave and a laser beam are present in the medium. When an acoustic wave is launched into the optical medium, it generates a refractive index wave that behaves like a sinusoidal grating. An incident laser beam passing through this granting will diffract the laser beam into several orders. With appropriate design, the first order beam has the highest efficiency. Its angular position is linearly proportional to the acoustic frequency, so that the higher the frequency, the larger the diffracted angle [1].

Currently, an acousto-optics method of light beams regulation find wide applications in many areas of science and technology. The interest in the diffraction of light by ultrasonic waves may be explained by the advantages of acousto-optics in regulation of non-coherent optical beams as well as of rays generated by lasers. In modern optics, optical engineering, and laser technology, the acousto-optics methods have been successfully applied for the control of the amplitude, frequency, phase, and polarization of coherent optical signals. The diffraction of optic beams by acoustic waves has also been used for the regulation of direction of light propagation. Due to high reliability, quick action, efficiency of operation, and simplicity of design, the acousto-optics instruments can be recommended for use in modern systems of intelligent optics and in novel generations of devices providing processing of optical information in real time [2].

In general, good operation parameters of the acousto-optics cells have made it possible to use them as spatial frequency filters in optical information processing systems. Such filters possess a number of advantages in comparison with processing instruments of other types. For example, acousto-optics
filters are not critical in terms of the place of their installation in an optical system. Moreover, they can be electronically tuned in real time by direct modulation of the amplitude and frequency of acoustic waves resulting in the desired quick and reliable processing of optical beams. However, a number of acute problems exist in the field of electronic control of optical signals that are not yet solved by modern science and technology. One of the problems is the necessity to find a simple way to quickly and efficiently regulate parameters of non-collimated optical rays and optical beams forming images in coherent light [2].

2. Acousto-Optics Effect

Acousto-optic effect, also known in the scientific literature as acousto-optic interaction or diffraction of light by acoustic waves, was first predicted by Brillouin in 1921 and experimentally revealed by Lucas, Biquard and Debye, Sears in 1932 [3]. The acousto-optics effect is a specific case of photoelasticity, where there is a change of a material's permittivity, $\varepsilon$, due to a mechanical strain. Photoelasticity is the variation of the optical indicatrix coefficients $B_i$ caused by the strain $a_j$ given by [4]:

$$\Delta B_i = p_{ij} a_j$$

(1)

Where $p_{ij}$ is the photoelastic tensor with components, $i,j = 1,2,...,6$ specifically in the acousto-optics effect, the strains $a_j$ is a result of the acoustic wave which has been excited within a transparent medium. This then gives rise to the variation of the refractive index. For a plane acoustic wave propagating along the $z$ axis, the change in the refractive index can be expressed as [4]:

$$n(z,t) = n + \Delta n \cos (\omega t - k z)$$

(2)

where, $n$ is the undisturbed refractive index, $\omega$ is the angular frequency, $k$ is the wave number, and $\Delta n$ is the amplitude of variation in the refractive index generated by the acoustic wave, and is given as [4]:

$$\Delta n = -\frac{1}{2} n^3 p_{ij} a_j$$

(3)

The generated refractive index, gives a diffraction grating moving with the velocity given by the speed of the sound wave in the medium. Light which then passes through the transparent material, is diffracted due to this generated refraction index, forming a prominent diffraction pattern. This diffraction pattern corresponds with a conventional diffraction grating at angles $\Phi_m$ from the original direction, and is given by [4]:

$$\Lambda \sin \Phi_m = m \lambda$$

(4)

Where, $\lambda$ is the wavelength of the optical wave, $\Lambda$ is the wavelength of the acoustic wave and $m$ is the integer order maximum. Light diffracted by an acoustic wave of a single frequency produces two distinct diffraction types. These are Raman-Nath diffraction and Bragg diffraction [4].

Raman-Nath diffraction is observed with relatively low acoustic frequencies, typically less than 10 MHz, and with a small acousto-optics interaction length, $\ell$, which is typically less than 1cm. This type of diffraction occurs at an arbitrary angle of incidence, $\Phi_{inc}$. In contrast, Bragg diffraction occurs at higher acoustic frequencies, usually exceeding 100 MHz. The observed diffraction pattern generally consists of two diffraction maxima; these are the zeroth and the first orders. However, even these two maxima only appear at definite incidence angles close to the Bragg angle, $\Phi_B$. The first order maximum or the Bragg maximum is formed due to a selective reflection of the light from the wave fronts of ultrasonic wave. The Bragg angle is given by the expression [4]:

$$\sin \Phi_B = -\frac{\lambda f}{2 n d} \left[ 1 + \frac{v^2}{\lambda^2 f^2} (n_i^2 - n_d^2) \right]$$

(5)

where $\lambda$ is the wavelength of the incident light wave (in a vacuum), $f$ is the acoustic frequency, $v$ is the velocity of the acoustic wave, $n_i$ is the refractive index for the incident optical wave, and $n_d$ is the refractive index for the diffracted optical waves.
In general, there is no point at which Bragg diffraction takes over from Raman-Nath diffraction. It is simply a fact that as the acoustic frequency increases, the number of observed maxima is gradually reduced due to the angular selectivity of the acousto-optics interaction. Traditionally, the type of diffraction, Bragg or Raman-Nath, is determined by the conditions \( Q \gg 1 \) and \( Q \ll 1 \) respectively, where \( Q \) is given by [5]:

\[
Q = \frac{2\pi \ell f^2}{n v^2} = \frac{2\pi \lambda_0 L}{n \lambda^2}
\] (6)

Which \( Q \) is known as the Klein-Cook parameter, where \( \lambda_0 \) is the wavelength of the laser beam, \( n \) is the refractive index of the crystal, \( L \) is the distance the laser beam travels through the acoustic wave and \( \lambda \) is the acoustic wavelength [6].

\( Q \ll 1 \), this is the Raman-Nath regime (figure 1 (a)). The laser beam is incident roughly normal to the acoustic beam and there are several diffraction orders \( \ldots -2 -1 0 1 2 3 \ldots \) with intensities given by Bessel functions [6].

\( Q \gg 1 \), this is the Bragg regime (figure 1 (b)). At one particular incidence angle, only one diffraction order is produced the others are annihilated by destructive interference.

In the intermediate situation, an analytical treatment isn’t possible and a numerical analysis would need to be performed by computer. Most acousto-optics devices operate in the Bragg regime, the common exception being acousto-optics mode lockers and \( Q \)-switches [6].

Since, in general, only the first order diffraction maximum is used in acousto-optics devices, Bragg diffraction is preferable due to the lower optical losses. However, the acousto-optics requirements for Bragg diffraction limit the frequency range of acousto-optics interaction. As a consequence, the speed of operation of acousto-optics devices is also limited [1].

3. Acousto-Optics Filters

There are two types of the acousto-optics filters, collinear and non-collinear filters depending on geometry of acousto-optics interaction. Here we consider only non-collinear filters based on the TeO\(_2\) single crystal [7]. The polarization of the incident light can be either ordinary or extraordinary. For the definition, we assume ordinary polarization. By tuning the frequency of the acoustic wave, the desired wavelength of the optical wave can be diffracted acousto-optically. Input light need not be polarized for a non-collinear design. Un-polarized input light is scattered into orthogonally polarized beams separated by the scattering angle for the particular design and wavelength. If the optical design provides an appropriate beam block for the un scattered light, then two beams (images) are formed in an optical pass-band that is nearly equivalent in both orthogonally linearly polarized output beams (differing by the Stokes and Anti-Stokes scattering parameter) [1].

4. Brief Description

The refractive index of an optical medium is altered by the presence of sound. Sound therefore modifies the effect of the medium on light; i.e., sound can control light (figure 2). Many useful devices make use of this acousto-optics effect; these include optical modulators, switches, deflectors, filters,
isolators, frequency shifters, and spectrum analyzers. Sound is a dynamic strain involving molecular vibrations that take the form of waves which travel at a velocity characteristic of the medium (the velocity of sound) [8].

As an example, a harmonic plane wave of compressions and rarefactions in a gas is pictured in figure 3.

In those regions where the medium is compressed, the density is higher and the refractive index is larger; where the medium is rarefied, its density and refractive index are smaller. In solids, sound involves vibrations of the molecules about their equilibrium positions, which alter the optical polarizability and consequently the refractive index.

An acoustic wave creates a perturbation of the refractive index in the form of a wave. The medium becomes a dynamic graded-index medium an inhomogeneous medium with a time-varying stratified refractive index. The theory of acousto-optics deals with the perturbation of the refractive index caused by sound, and with the propagation of light through this perturbed time-varying inhomogeneous medium [8].

Since optical frequencies are much greater than acoustic frequencies, the variations of the refractive index in a medium perturbed by sound are usually very slow in comparison with an optical period. There are therefore two significantly different time scales for light and sound. As a consequence, it is possible to use an adiabatic approach in which the optical propagation problem is solved separately at every instant of time during the relatively slow course of the acoustic cycle, always treating the material as if it were a static (frozen) inhomogeneous medium. In this quasi-stationary approximation, acousto-optics becomes the optics of an inhomogeneous medium (usually periodic) that is controlled by sound [8].
The simplest form of interaction of light and sound is the partial reflection of an optical plane wave from the stratified parallel planes representing the refractive-index variations created by an acoustic plane wave (figure 4) [8].

Figure 4. Bragg diffraction: an acoustic plane wave acts as a partial reflector of light (a beam splitter) when the angle of incidence $\theta$ satisfies the Bragg condition [8].

A set of parallel reflectors separated by the wavelength of sound $\Lambda$ will reflect light if the angle of incidence $\theta$ satisfies the Bragg condition for constructive interference [8]: $\sin \theta = \frac{\lambda}{2\Lambda}$, where $\lambda$ is the wavelength of light in the medium [10]. This form of light-sound interaction is known as Bragg diffraction, Bragg reflection, or Bragg scattering. The device that effects it is known as a Bragg reflector, a Bragg deflector, or a Bragg cell [8].

The principle of operation of the filters is based on the dependence of the diffracted light wavelength on the acoustic frequency. Conventional applications using acousto-optics interactions have been extensively confined to signal. The best way to understand image processing using acousto-optics is through the use of spatial transfer function, which describes acousto-optics interaction between the sound and the incident optical image decomposed in terms of the angular plane wave spectrum of the light field.

We now define $\xi$ which is the normalized distance inside the acousto-optics cell, and $\xi = 1$ signifies the exit plane of the cell [11]. We define one more important variable called the $Q$ parameter or the Klien-Cook parameter in acousto-optics. $\phi_B$, Bragg angle and $\delta$ represent the deviation of the incident plane wave away from the Bragg angle. By limiting case to two diffracted orders and the case of zeroth-order light, we decompose the incident field into plane waves with different amplitudes propagating in direction defined by: $\Phi' = \delta \times \Phi_B$.

Raman-Nath diffraction given as:

$$\frac{d\psi_m}{d\xi} = -j \frac{\alpha}{2} e^{-j\frac{\lambda}{2}\delta \xi} \left[\frac{\phi_{inc}+(2m-1)}{\phi_B}\right] \psi_{m-1} - j \frac{\alpha}{2} e^{j\frac{\lambda}{2}\delta \xi} \left[\frac{\phi_{inc}+(2m-1)}{\phi_B}\right] \psi_{m+1}$$

And the Raman-Nath solution is:

$$\psi_m = (-j)^m \psi_{inc} J_m(\alpha \xi)$$

The plane wave amplitude of the zeroth-order light when a plane wave $\psi_{inc}$ is incident at $\psi'$ away from the nominally Bragg angle of incidence is given in below equation:

$$\psi_0(\xi) = \psi_{inc} e^{-j\beta_0 \xi/4} \left\{ \cos \left( \frac{\delta \xi}{4} \right) \left[ \frac{\alpha}{2} \right]^2 + \left[ \frac{\alpha}{2} \right]^2 \right\} + j \frac{\delta_0 \sin \left[ \left( \frac{\delta \xi}{4} \right)^2 + \left( \frac{\alpha}{2} \right)^2 \right]^{1/2} \xi}{\left[ \frac{\delta_0}{4} \right] + \left( \frac{\alpha}{2} \right)^2 \xi^{1/2}}$$

Where $\alpha$ is the peak phase delay, hence we can define the so-called transfer function of the zeroth-order light as follows:
\[ H_0(\delta) = \frac{\psi_0(\xi)|_{\xi=1}}{\psi_{inc}} \]  

(10)

Where \( \psi_0(\xi)|_{\xi=1} \) is the result from equation (9) evaluated at the exit of the Bragg cell. This definition of the transfer function permits us to relate the input (incident) spectrum, \( \Psi_{inc}(k_x') \), to the output (zeroth-order) spectrum, \( \Psi_0(k_x') \), as [11]:

\[ \Psi_{inc}(k_x') = \Psi_{inc}(k_{x1})H_0(\delta) \]  

(11)

Where \( \Psi_{inc}(x') \) and \( \Psi_0(x') \) are the field distribution of the incident light and that of the zeroth-order light, respectively and \( k_{x1} \) representing the transform variables. Finally, the concept of spatial filtering becomes clear when we relate the spatial frequency \( k_{x1} \) along \( x' \) to \( \phi' \) by [11]:

\[ \Psi_0(k_x') = \Psi_{inc}(k_{n1})H_0(k_{x1}A/\pi) \]  

(12)

As \( \phi' = \delta \times \phi_B \) and \( \phi_B = \lambda_0/2A \) eq. (12) can now be written in terms of spatial frequency if we use the above derived relationship such that \( \delta = k_{x1}A/\pi \).

\[ k_{x1} = k_0 \sin(\phi') = k_0 \delta \phi_B = \pi \delta / A \]  

(13)

The spatial distribution \( \psi_0(x') \) is then given by:

\[ \psi_0(x') = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Psi_{inc}(k_{x1})H_0(k_{x1}A/\pi) \exp(-jk_{x1}x')dk_{x1} \]  

(14)

Equation (14) determines the profile of the scattered zeroth-order field, \( \psi_0(x') \), from any arbitrary incident field, \( \psi_{inc}(x') \), in the presence of the acoustic field. Using equation (12) and equation (14), the amplitude of the zeroth-order diffracted light written in a full 2D version, \( \psi_0(x',y) \), at the exit of the Bragg cell, can be approximately written as [12]:

\[ \psi_0(x,y) = \left( A - B \frac{\partial}{\partial x} \right) \psi_{inc}(x,y) \]  

(15)

5. Results and Discussion

We proceed with the acousto-optics spatial filter. In this paper results, we used the transfer function formalism to demonstrate some numerical results obtained by MATLAB where, after we calculate the diffracted intensities involving deferent diffracted orders in up shifted Bragg interaction, we have created a MATLAB function for deferent coupled differential equations, and then used equation (7) to investigate Raman-Nath diffraction and equation (8) to investigate Raman-Nath solution. Explain the phenomenon of high-pass filtering or edge enhancement in the zeroth-order diffracted light theoretically from the transfer function directly from equation (9) and then the zeroth-order beam has been spatially filtered by the transfer function. In the first, corresponding to near-Bragg diffraction. We used equation (7) to investigate Raman-Nath diffraction. Figure 5 shows the dependence of various scattered orders on the incident angle/Bragg angle illustrating the Bessel function dependence. Figure 6 illustrates non-ideal Raman-Nath diffraction for normal incidence.

Note that, the true zeros, which are the characteristic of the Bessel functions for the ideal case, disappear for large value of incident angle/Bragg angle. When the interaction length \( L \) (distance the laser beam travels through the acoustic wave) is short enough so that the accumulated of phase mismatch is small. This is called the Raman-Nath or Debye-Sears region and is characterized by the simultaneous existence of many scattered orders. For a special condition on the angle of incidence and the angle of diffraction into the ±1 orders, namely \( \phi_{\pm1} = -\phi_0 \), there is a phase synchronism between the 0 and \( -1 \) orders, and between the 0 and \( +1 \) orders. When \( \phi_{\pm1} = -\phi_0 \), this means that the zeroth order and the diffracted order propagate symmetrically with respect to the sound wave fronts. The angle of incidence of the zeroth order in this case is according \( \phi_{\pm1} = -\phi_0 \).
The amplitude of the various diffracted orders, therefore, show the same functional dependence as the corresponding orders of the Bassel functions on the phase delay. This behavior is depicted in figure 5 and figure 6 for the first three lowest orders.

In Bragg region, the interaction length is long enough so that the accumulated phase mismatch for all diffracted orders other than the ±1 render their intensities negligible. The only constructive interference can occur for the +1 or the -1 orders. The criterion for this region is just the opposite of that defining the Raman-Nath region. As we discussed before, it is of great interest for practical applications of acousto-optics interaction theory to consider small deviations from the phase synchronism condition characterizing this bragg region. Such cases can be due to either in the sound
frequency or due to changes in the angle of incidence in such a way that it no longer satisfies the Bragg condition. Finally, in the near Bragg region we consider the transition region is difficult to treat analytically and, consequently, the condition for operation in the Bragg regime is rather vague. It is often possible, however, to define the Bragg region more accurately by specifying the maximum percentage of light diffracted into the first order in interaction of the kind carried out by Klein an Cook. They performed numerical calculations of the amount of light diffracted as a function of the parameter $Q$, so that for the ideal Bragg case, all the light would have been diffracted. Figures 7, 8 and 9 shows the incident, the zeroth-order beams and the pure differentiation results respectively. Figure 7 shows its magnitude of zeroth-order transfer function as a function of spatial frequency.

![Figure 7. The incident beams, magnitude of zeroth-order transfer function (length of square= 1mm).](image)

Figure 8 shows the profile of incident beam and the output zeroth-order beam profile and figure 9 shows that the left edge of the square has been emphasized. Both edges of the square image have been extracted equally along the x-direction. Selective edge extraction, that is, either only the left edge of the image or the right edge of the image, can be performed when we change the sound pressure through incidence angle.

![Figure 8. The zeroth-order beams, profile of incident beam (length of square= 1mm).](image)
6. Conclusions
This paper is, in general, devoted to the theoretical study of the acousto-optics selection of optical spatial frequencies. The Bragg diffraction occurs at higher acoustic frequencies and the observed diffraction pattern generally consists of two diffraction maxima; these are the zeroth and the first orders, these two maxima only appear at definite incidence angles close to the Bragg angle. The first order maximum or the Bragg maximum is formed due to a selective reflection of the light from the wave. An optical filter is completely described by its frequency response, which is information on how the amplitude and phase is acted for each frequency component. This is most often given in the form of a complex valued transfer function. The standard weak interaction theory describes diffraction in the Bragg regime as the propagation of a uniform plane wave of light through a uniform plane wave of sound, resulting in the well-known first- and zeroth-order diffraction.

7. References
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