Multi-scale Finite Element Method for Members for Pipe Frames

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Abstract. To consider the local buckling of the members in reticulated frames, based on the multi-scale simulation, the part of member may be collapsed were divided by the shell elements as a micro-model, and the other part of the member was simulated by beam elements as a macro-model. The incremental displacement constraint equations for the nodes on the section between the two models are established based on the plane section premise of classical beam theory. By constraint variational principle, the tangent stiffness matrixes and the nodal load vectors of the micro-model and the outside structure are combined, and then the equations are solved. The location of the collapsed part is predicted by the deflection function of the beam, and the length of the collapsed part is estimated. Two case studies about a single beam and a single-layer reticulated dome are presented to show the feasibility and the validity of this method.

1. Introduction

The elastoplastic analysis of the conventional pipe pole structure does not take into account the local buckling of the rod piece, assuming that the rod piece will have a full-section plastic yield and form an ideal plastic hinge [1-2]. The plastic hinge can stably transmit the bending moment, and if the structure is not degraded into a mechanism, the axial force can be stably transmitted. However, the diameter-thickness ratio of the rods commonly used in reticulated shell engineering is between 15 to 80. M.Elchalakni [3-5] and other scholars have found that, when such thin-walled rods are bent, the rod pieces will be locally buckling where the bending moment is maximum, forming a bulge or depression, resulting in a rapid decrease in the bearing capacity of the rod pieces. Zhang Ailin also found this phenomenon in the ultimate bearing capacity test of the suspendome [6]. In order to solve this problem, Liew [6-7] and other scholars have proposed advanced plastic hinge method, which saves computational complexity, but assumes too many conditions, and cannot directly obtain the stress and strain information of the local buckling of the rod piece. This information is very important for the study of structural fracture and damage [8-9].

According to the idea of multi-scale finite element [10-11], the part of the rod that may have local buckling is taken as the micro-scale model and the shell element discretization is adopted, and the rest of the rod is taken as a macro-scale model and the beam element discretization is adopted, then the displacement increment constraint equations at the interface of the two scale models are derived based on the flat-section assumption of the classical beam theory. According to the constrained variational principle, the tangent stiffness matrix and the load increment vector of the micro-scale model and the macro-scale model are combined into the tangent stiffness matrix and the load increment vector of the whole structure, and then solved. The local buckling position of the rod piece is determined by the deflection line equation of the beam, and the estimated length of local buckling is given. In this paper,
the analysis of the ultimate bearing capacity of single-layer spherical reticulated shell is taken as an example, and the effectiveness and feasibility of the method are illustrated.

2. Derivation of interface constraint equations for different scale models

There are two difficulties in multi-scale finite element calculation: one is the selection of different scale finite element models; the other is to ensure the coupling of the degrees of freedom of each node on the interface of different scale finite element models. The existing buckling test of thin-walled steel pipe clearly shows that the failure process of the steel pipe can be accurately simulated by using the discrete element of the shell element. Therefore, the part of the rod that may undergo local buckling can be discretized as a micro-scale model using a shell element, while the rest is used as a macro-scale model with beam element discretization, as show in Figure 1.

![Figure 1 Multi-scale model](image)

According to the Saint-Venant principle, the surface forces distributed on the upper boundary of the elastic body are transformed into the surface forces with different distributions but static equivalents. The stress distribution in the vicinity will change significantly, and the influence on the distant stress can be neglected. Where the rod piece is buckling, the stress-strain relationship is complex and does not conform to the stress-strain relationship of the beam. At the interface A-A’ where the distance from the weld is S, if S is large enough, the stress-strain relationship on the section is assumed to be in accordance with the plane cross-section assumption. The plane cross-section assumption assume: section A-A’ is perpendicular to the beam centerline before deformation, will remain flat after deformation, and is still perpendicular to the centerline. Therefore, a displacement coordination equation between the nodes of the solid element on the rod end section of the microscopic model and the nodes of the corresponding beam unit can be established.

In the local coordinate system shown in Figure 2, the beam element has a node O at the center of the section A-A’, and the displacements are: $u_b$, $v_b$, $w_b$, $\phi_{bx}$, $\phi_{by}$, $\phi_{bz}$; the shell element has n nodes on section A-A’, the displacement is:

![Figure 2 Local coordinate system for constraint equations](image)

When the beam section A-A’ is translated in the z-axis direction, the displacement increment of any node $h$ on the section A-A’ can be expressed as:
When the section $A$-$A'$ is rotated $\varphi_z$ about the $z$-axis in the $x$-$y$ plane, as shown in Figure 3, the displacement increment of the node $h$ in the $x$ and $y$ directions is:

$$
\begin{align*}
\Delta u_{sx} &= -r_h [\cos(\alpha + \phi_{hx}) - \cos \alpha] \\
\Delta v_{sx} &= r_h [\sin(\alpha + \phi_{hx}) - \sin \alpha] \\
\Delta \phi_{sx} &= \phi_h
\end{align*}
$$

(1)

When the section $A$-$A'$ is rotated $\varphi_z$ about the $z$-axis in the $x$-$y$ plane, as shown in Figure 3, the displacement increment of the node $h$ in the $x$ and $y$ directions is:

$$
\begin{align*}
\Delta w_x &= r_h \sin \phi_{hx} \\
\Delta v_x &= -r_h (1 - \cos \phi_{hx}) \\
\Delta \phi_x &= \phi_h
\end{align*}
$$

(2)

Where: $r_h$ is the distance from the solid element node $h$ to the beam node $O$, and $h$ is the node number.

When the section $A$-$A'$ is rotated about the $x$-axis in the $y$-$z$ plane, as shown in Figure 4, the displacement increment of the node $h$ in the $z$ and $y$ directions is:

$$
\begin{align*}
\Delta w_x &= -r_h \sin \phi_{by} \\
\Delta u_x &= -r_h (1 - \cos \phi_{by}) \\
\Delta \phi_y &= \phi_h
\end{align*}
$$

(3)

Where: $r_h$ is the projection distance of $r_h$ on the $y$-axis, $r_{by} = r_h \sin \alpha$.

When the section $A$-$A'$ is rotated $\varphi_y$ about the $y$-axis in the $x$-$z$ plane, as shown in Figure 5, the displacement increment of the node $h$ in the $x$ and $z$ directions is:

$$
\begin{align*}
\Delta w_x &= -r_h \sin \phi_{bx} \\
\Delta u_x &= -r_h (1 - \cos \phi_{bx}) \\
\Delta \phi_z &= \phi_h
\end{align*}
$$

(4)

Where: $r_h$ is the projection distance of $r$ on the $x$-axis, $r_{bx} = r_h \cos \alpha$.

These constraint equations are nonlinear constraint equations. Since the large rotation effect of the rod in the reticulated shell cannot be ignored, the differential equation between the displacement increments is obtained by differentiating the equation (5):
\[
\begin{align*}
\Delta u &= \Delta u_0 - r_z \sin(\alpha + \phi_z) \Delta \phi_z - r_\phi \sin \phi_\phi \Delta \phi_\phi \\
\Delta v &= \Delta v_0 + r_z \cos(\alpha + \phi_z) \Delta \phi_z - r_\phi \sin \phi_\phi \Delta \phi_\phi \\
\Delta w &= \Delta w_0 + r_\phi \cos \phi_\phi \Delta \phi_\phi - r_\phi \cos \phi_\phi \Delta \phi_\phi \\
\Delta \phi_\phi &= \Delta \phi_\phi \\
\Delta \phi_\phi &= \Delta \phi_\phi \\
\Delta \phi_\phi &= \Delta \phi_\phi
\end{align*}
\]

(5)

Where: the increments of the displacements \( u, v, w, \phi_\phi, \phi_\phi, \phi_\phi \), respectively.

3. Numerical test
The grid structure used is as follows:

Figure 6 Orthotropic reinforced single layer reticulated dome

Figure 7 load displacement cure for node \( v \)

Figure 6 shows that after considering the local buckling of the rod piece, the ultimate bearing capacity of the reticulated shell is reduced by 17.6%, which is quite significant. This is because after the local buckling of the No.1 rod piece, the ability to withstand bending moments and axial forces is quickly lost, resulting in instability of the entire structure.

In the process of analyzing the ultimate bearing capacity of the reticulated shell, it is only necessary to set the plastic hinges for the middle of No.5 rod piece and the No.3 rod piece. This is because most of the types of reticulated shells are still elastic in case of instability failure, and only a few of the rods will be plastically damaged, as shown in a previous research[15]. Therefore, the calculation load brought by this method is affordable.
4. Conclusion
(1) The local buckling of the rod has a great influence on the performance of the structure. When the rod piece is partially buckling, the bulge or depression will be formed at the maximum bending moment, resulting in the rod piece cannot transmit bending moment and axis force stably like the ideal plastic hinge.

(2) The position of local buckling of the beam is determined according to the deflection differential equation of the beam; the length of local buckling is determined according to the local buckling theory of thin-walled pipe, which is 4 times radius; and a method to check whether the length and position of local buckling are reasonable is given.

(3) Discrete each micro-scale model requires only less than 500 nodes. At the same time, when the grid structure is unstable or collapsed, only a few rod pieces will usually have local buckling. Therefore, with the rapid development of computer technology, the increased computational load of this method is affordable.

Acknowledgments
The authors gratefully acknowledge the financial supports from National Grid (GCB17201700026)

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