In a semiclassically quantized two-dimensional cosmological model, it can be shown that the parameter of the equation of state for the accelerating universe can be positive due to the negative energy density and the negative pressure, which is a little different from the conventional wisdom that the parameter is negative with the positivity of the energy density. Furthermore, we show that the full parameter composed of the classical and the quantum-mechanical contributions is positive and finite even though the partial state parameter from the quantum-mechanical contribution is not positive definite, which means that the state parameter is not perturbatively additive in this model.

Keywords: 2D Gravity, Models of quantum gravity, Non-commutative geometry, Cosmology of theories beyond the SM

I. INTRODUCTION

Recently, much attention has been paid to the accelerating expansion of the universe[1, 2, 3], which is essentially related to the equation-of-state parameter[4, 5, 6]. In the Einstein gravity, the decelerating universe satisfying the positive energy conditions appears, and the energy density and the pressure are naturally positive definite. However, the dark energy related to the accelerating expansion of the universe is defined by the negative state parameter[7, 8, 9, 10, 11, 12], which means that the pressure is negative.

In the two-dimensional gravity[13, 14, 15, 16, 17, 18, 19, 20, 21], it is a little easier to treat some unsolved cosmological problems[22, 23, 24, 25, 26, 27, 28], so the cosmological solutions describing the phase change from the decelerating universe to the accelerating universe are obtained[29] by assuming noncommutativity among the relevant fields[30, 31, 32, 33]. In this toy model, the equation of state is based on the matter part having been solved in a self-consistent manner. However, in the semi-classical quantized region, the fields are redefined, and the newly defined fields are a combination of the metric and dilaton, so that, contrary to the conventional Friedman equation, it is not straightforward to see the relation between the acceleration for the scale factor and the energy density along with the pressure. Therefore, it would be interesting to study whether the energy density and the pressure affect the equation-of-state parameter or not because the quantum-mechanically-induced energy density and pressure may modify the signature for the state parameter.

In this paper, we would like to calculate the classical and the quantum-mechanically-induced energy-momentum tensors separately in comoving coordinates in order to find the signature of the equation-of-state parameter in this phase changing from the decelerating universe to the accelerating one. In Sec. II the stress-energy-momentum tensors are formally calculated in this dilaton cosmology, and their expressions for the energy density and the pressure are written in the form of a perfect fluid. In Sec. III using the noncommutative algebra during finite time, we obtain the cosmological solution describing the accelerating universe from the decelerating phase; then, this geometry is patched up by regular geometry to avoid the future curvature singularity. In fact, if noncommutativity between fields is applied to all cosmic time, then the future curvature singularity appears in a finite proper time. Next, the energy density and the pressure as a perfect fluid are investigated in this geometry. Then, we obtain the equation-of-state parameters for the classical and the quantum-mechanical cases separately to clarify their roles and their signatures. Consequently, the total energy density and the pressure derived from the classical and the quantum mechanical energy densities and the pressures give the so-called total state parameter, which is curiously always positive definite, because the total energy density is negative, as is total pressure. Finally, a summary and discussion are given in Sec. IV.
II. ENERGY-MOMENTUM TENSORS IN THE DILATON COSMOLOGY

We now start with the following Callan-Giddings-Harvey-Strominger (CGHS) model\[13\], which is split into two pieces in order to regard the kinetic part of the dilaton as matter. Thus, the action can be written as

$$S = S_G + S_{Cl} + S_{Qt}. \quad (1)$$

The gravitational action is defined by

$$S_G = \frac{1}{2\pi} \int d^2x\sqrt{-g} e^{-2\phi} R, \quad (2)$$

which is trivial in its form without additional terms because the spacetime is flat. The remaining classical matter and its quantum correction are given in the form

$$S_{Cl} = \frac{1}{2\pi} \int d^2x\sqrt{-g} \left[ e^{-2\phi} \left( 4(\nabla\phi)^2 + 4\lambda^2 \right) - \frac{1}{2} \sum_i (\nabla f_i)^2 \right], \quad (3)$$

$$S_{Qt} = \frac{\kappa}{2\pi} \int \sqrt{-g} \left[ -\frac{1}{4} R \frac{1}{2} R + (\nabla\phi)^2 - \phi R \right], \quad (4)$$

where \( \kappa = (N - 24)/12 \) and the cosmological constant \( \lambda^2 \) will be set to zero. The nonlocal form of the action in Eq. (1) is written, by introducing an auxiliary field \( \psi \) for later convenience, as

$$S_{Qt} = \frac{\kappa}{2\pi} \int d^2x\sqrt{-g} \left[ \frac{1}{4} R \psi - \frac{1}{16} (\nabla\psi)^2 + (\nabla\phi)^2 - \phi R \right]. \quad (5)$$

The total stress-energy-momentum tensor is given as

$$T_{\mu\nu}^M = T_{\mu\nu}^{Cl} + T_{\mu\nu}^{Qt}, \quad (6)$$

where the energy-momentum tensors of the classical and the quantum matter are defined as

$$T_{\mu\nu}^{Cl} = -\frac{2\pi}{\sqrt{-g}} \frac{\delta S_{Cl}}{\delta g^{\mu\nu}},$$

$$T_{\mu\nu}^{Qt} = -\frac{2\pi}{\sqrt{-g}} \frac{\delta S_{Qt}}{\delta g^{\mu\nu}}$$

respectively. We would like to investigate the effect of quantum matter on the expansion of universe. Thus, for simplicity, we assume that there are no conformal matter fields, i.e., \( f_i = 0 \). Then, the equation of motion for \( \psi \) is easily obtained as \( \Box\psi = -2R \). In the conformal gauge, \( ds^2 = -e^{2\phi} dx^+ dx^- = -e^{2\phi(t)} (dt^2 - dx^2) \), the stress-energy-momentum tensors in Eqs. (7) and (8) are written as

$$T_{\pm\pm}^{Cl} = -4e^{-2\phi} (\partial_+ \phi)^2,$$

$$T_{\mp\mp}^{Cl} = -2e^{-2\phi} \partial_+ \phi \partial_- \phi,$$

$$T_{\mp\pm}^{Qt} = -\kappa \left[ \partial_+ \phi \partial_\pm \phi - \partial_" \phi \partial_- \phi \right] - \kappa e^{-2\phi} \partial_\pm \phi^2,$$

$$T_{\mp\mp}^{Qt} = -\kappa \partial_+ \partial_- \phi + \kappa \partial_+ \partial_\pm \phi,$$

$$T_{\pm\mp}^{Qt} = \frac{\kappa}{4} (\partial_+ \phi - \partial_- \phi), \quad (9)$$

$$T_{\pm\pm}^{Qt} = \frac{\kappa}{4} (\partial_\pm \phi), \quad (10)$$

$$T_{\mp\mp}^{Qt} = \frac{\kappa}{4} (\partial_\pm \phi), \quad (11)$$

$$T_{\mp\pm}^{Qt} = \frac{\kappa}{4} (\partial_\pm \phi), \quad (12)$$
where \( t_\pm \) reflects the nonlocality of the induced gravity of the conformal anomaly and the overdot denotes the derivative with respect to \( t \). We obtained these expressions by substituting the solution for the auxiliary field \( \psi \) from the equation of motion for \( \psi \).

In a comoving coordinate system, \( ds^2 = -d\tau^2 + a^2(\tau)dx^2 \), the stress-energy-momentum tensors of a perfect fluid are given as

\[
\hat{T}_{\mu\nu}^M = p g_{\mu\nu} + (p + \rho)u_\mu u_\nu,
\]

where \( u_\mu = (1, 0) \) and \( p \) and \( \rho \) are the pressure and the energy density, respectively. Then, we obtain the expressions for the energy density and the pressure as

\[
\rho = \hat{T}_{\tau\tau}^M, \quad \quad \rho = \frac{1}{a^2}\hat{T}_{xx}^M,
\]

which are composed of the classical and the quantum matter as follows:

\[
\rho = \rho_{\text{Cl}} + \rho_{\text{Qt}}, \quad \quad p = p_{\text{Cl}} + p_{\text{Qt}}.
\]

By a coordinate transformation, the relations between the stress-energy-momentum tensors are obtained as

\[
\hat{T}_{\tau\tau}^M(\tau) = e^{-2\tilde{\phi}}(T_{++}^M + 2T_{+-}^M + T_{--}^M),
\]

\[
\hat{T}_{\tau x}^M(\tau) = e^{-\tilde{\phi}}(T_{++}^M - T_{--}^M),
\]

\[
\hat{T}_{xx}^M = T_{++}^M - 2T_{+-}^M + T_{--}^M,
\]

with the comoving time \( \tau \), where \( \tau = \int e^{\tilde{\phi}(t)} dt \).

### III. THE DISTRIBUTION OF THE ENERGY DENSITY AND THE PRESSURE

In the conformal gauge, \( ds^2 = -e^{2\phi}dx^+dx^- \), without conformal fields, \( f_i = 0 \), the total action and the constraint equations are written as

\[
S = \frac{1}{\pi} \int d^2x \left[ e^{-2\phi} (2\partial_+ \partial_- \rho - 4\partial_+ \phi \partial_- \phi) - \kappa (\partial_+ \rho \partial_- \rho + 2\phi \partial_+ \partial_- \rho + \partial_+ \phi \partial_- \phi) \right]
\]

and

\[
e^{-2\phi} \left[ 4\partial_+ \rho \partial_- \phi - 2\partial_+^2 \phi \right] + \kappa \left[ \partial_+^2 \rho - (\partial_+ \rho)^2 \right]
\]

\[
- \kappa (\partial_+^2 \phi - 2\partial_+ \rho \partial_- \phi) - \kappa (\partial_+ \phi)^2 - \kappa \tau = 0.
\]

Defining new fields as \( \Omega = e^{-2\phi} \) and \( \chi = \kappa (\rho - \phi) + e^{-2\phi} \) \[17, 19\], the gauge fixed action is obtained in the simplest form of

\[
S = \frac{1}{\pi} \int d^2x \left[ \frac{1}{\kappa} \partial_+ \Omega \partial_- \Omega - \frac{1}{\kappa} \partial_+ \chi \partial_- \chi \right]
\]

and the constraints are given by

\[
\kappa \tau_\pm = \frac{1}{\kappa} (\partial_+ \Omega)^2 - \frac{1}{\kappa} (\partial_+ \chi)^2 + \partial_\pm^2 \chi.
\]

In a homogeneous spacetime, the Lagrangian and the constraints are obtained as

\[
L = \frac{1}{4\kappa} \dot{\Omega}^2 - \frac{1}{4\kappa} \dot{\chi}^2,
\]

\[
\frac{1}{4\kappa} \dot{\Omega}^2 - \frac{1}{4\kappa} \dot{\chi}^2 + \frac{1}{4\kappa} \chi^2 - \kappa \tau_\pm = 0,
\]
where the action is redefined by \( S/L_0 = \frac{1}{2} \int dt L \) and \( L_0 = \int dx \), and the overdot denotes the derivative with respect to the cosmic time \( t \). Then, the Hamiltonian becomes

\[
H = \kappa P_\Omega^2 - \kappa P_\chi^2
\]  

(27)
in terms of the canonical momenta \( P_\chi = -\frac{1}{2\kappa} \dot{\chi} \) and \( P_\Omega = \frac{1}{2\kappa} \dot{\Omega} \).

We consider the modified Poisson brackets corresponding to the noncommutative algebra in the quantized theory\cite{30,31}:

\[
\{\Omega, P_\Omega\}_{\text{MPB}} = \{\chi, P_\chi\}_{\text{MPB}} = 1, \\
\{\chi, \Omega\}_{\text{MPB}} = \theta_1 [\epsilon(t - t_1) - \epsilon(t - t_2)], \\
\{P_\chi, P_\Omega\}_{\text{MPB}} = \theta_2 [\epsilon(t - t_1) - \epsilon(t - t_2)], \\
others = 0,
\]  

(28)

where \( \theta_1 \) and \( \theta_2 \) are two independent positive constants, and \( \epsilon(t) \) is a step function, 1 for \( t > 0 \) and 0 for \( t < 0 \). Thus, these are nontrivial and \( \theta \)-dependent for the finite time interval \( t_1 < t < t_2 \). With the Hamiltonian in Eq. (27), the equations of motion are obtained as

\[
\dot{\chi} = \{\chi, H\}_{\text{MPB}} = -2\kappa P_\chi, \\
\dot{\Omega} = \{\Omega, H\}_{\text{MPB}} = 2\kappa P_\Omega, \\
\dot{P}_\chi = \{P_\chi, H\}_{\text{MPB}} = 2\kappa \theta_2 P_\Omega, \\
\dot{P}_\Omega = \{P_\Omega, H\}_{\text{MPB}} = 2\kappa \theta_2 P_\chi.
\]  

(29)\,(30)

Note that the momenta are no longer constants of the motion because of the nonvanishing \( \theta_2 \); hence, a new set of equations of motion are obtained from Eqs. (29) and (30):

\[
\ddot{\chi} = -2\kappa \theta_2 \dot{\Omega}, \\
\ddot{\Omega} = -2\kappa \theta_2 \dot{\chi}.
\]  

(31)

The solutions for the above coupled equations of motion are easily obtained as

\[
\Omega = e^{2\kappa \theta_2 t} + \beta e^{-2\kappa \theta_2 t} + A, \\
\chi = e^{2\kappa \theta_2 t} - \beta e^{-2\kappa \theta_2 t} + B,
\]  

(32)\,(33)

where \( \alpha, \beta, A, \) and \( B \) are constants, and they should satisfy the constraint equation in Eq. (26),

\[
\kappa t_\pm = \kappa^2 \theta_2^2 (\alpha e^{2\kappa \theta_2 t} - \beta e^{-2\kappa \theta_2 t}) - 4\kappa \theta_2^2 \alpha \beta,
\]  

(34)

which determines the unknown time-dependent function \( t_\pm \).

Now, for \( \beta = -\alpha > 0 \), the solutions and the constraints are written as

\[
\Omega = e^{-2\phi} = 2\beta \sinh(2\kappa \theta_2 t) + A, \\
\chi = \kappa (\rho - \phi) + e^{-2\phi} = -2\beta \cosh(2\kappa \theta_2 t) + B,
\]  

(35)\,(36)

and

\[
\kappa t_\pm = -2\beta \kappa^2 \theta_2^2 \cosh(2\kappa \theta_2 t) + 4\kappa \beta \theta_2^2.
\]  

(37)

Note that among the two positive constants, only \( \theta_2 \) plays an important role in our analysis. Furthermore, \( \Omega = e^{-2\phi} \) in Eq. (35) is positive definite, so the initial time should be restricted to \( t_1 > -1/(2\kappa \theta_2) \sinh^{-1}[A/(2\beta)] \). Especially, for \( A = 0 \), the time interval becomes \( 0 < t_1 < t < t_2 \). Hereafter, we regard \( t_1 \) as the initial time of the beginning of the universe in our model. Since \( da(\tau)/d\tau = d\tilde{\rho}(t)/dt \), we obtain the expanding velocity of the universe as

\[
\frac{da(\tau)}{d\tau} = -2\beta \theta_2 \left[ \cosh(2\kappa \theta_2 t) + \sinh(2\kappa \theta_2 t) + \frac{\kappa \cosh(2\kappa \theta_2 t)}{A + 2\beta \sinh(2\kappa \theta_2 t)} \right],
\]  

(38)

which is always negative since \( \cosh(2\kappa \theta_2 t) > |\sinh(2\kappa \theta_2 t)| \) and \( \Omega = A + 2\beta \sinh(2\kappa \theta_2 t) > 0 \) for any time \( t \).

At this juncture, from Eqs. (35) and (36), we calculate the curvature scalar related to the acceleration and the deceleration in terms of \( R = 2\ddot{a}/a \) in comoving coordinates; then,

\[
R_\theta = -8\beta \kappa^2 \theta_2^2 \exp(4\beta \cosh(2\kappa \theta_2 t) - 2B) \left[ \cosh(2\kappa \theta_2 t) (2\beta e^{2\kappa \theta_2 t} + A) - \frac{2}{\kappa} e^{2\kappa \theta_2 t} (2\beta \sinh(2\kappa \theta_2 t) + A) \left( 2\beta \sinh(2\kappa \theta_2 t) + A + \frac{\kappa}{2} \right) \right].
\]  

(39)
By substituting the solutions in Eqs. (35) and (36) into Eqs. (9), (10), (11), and (12), the stress-energy-momentum tensors are calculated in the conformal gauge as

\[
T_{\pm \pm}^{CL} = \frac{-4\beta^2\kappa^2\theta_0^2 \cosh^2(2\kappa \theta_0 t)}{2\beta \sinh(2\kappa \theta_0 t) + A},
\]

\[
T_{\pm -}^{CL} = \frac{-2\beta^2\kappa^2\theta_0^2 \cosh^2(2\kappa \theta_0 t)}{2\beta \sinh(2\kappa \theta_0 t) + A},
\]

\[
T_{\pm \mp}^{CL} = -\kappa t \pm -2\beta \kappa^2 \theta_0^2 \epsilon^{2\kappa \theta_0 t} - 4\beta^2 \kappa \theta_0^2 e^{4\kappa \theta_0 t},
\]

\[
T_{\mp \pm}^{CL} = 2\beta \kappa^2 \theta_0^2 e^{2\kappa \theta_0 t}.
\]

Then, by using Eqs. (14) and (15), we obtain the energy densities and the pressures of the classical and the quantum matter as

\[
\rho_{CL} = -12\beta^2\kappa^2\theta_0^2 \cosh^2(2\kappa \theta_0 t) \exp \left[\frac{2}{\kappa} (A - B) + \frac{4\beta}{\kappa} e^{2\kappa \theta_0 t}\right], \quad (44)
\]

\[
\rho_{QT} = (2\beta \sinh(2\kappa \theta_0 t) + A) \exp \left[\frac{2}{\kappa} (A - B) + \frac{4\beta}{\kappa} e^{2\kappa \theta_0 t}\right]
\]

\[
\times \left((-8\kappa \beta^2 \theta_0^2 e^{4\kappa \theta_0 t} - \kappa(t_+ + t_-))\right), \quad (45)
\]

\[
p_{CL} = -4\beta^2\kappa^2\theta_0^2 \cosh^2(2\kappa \theta_0 t) \exp \left[\frac{2}{\kappa} (A - B) + \frac{4\beta}{\kappa} e^{2\kappa \theta_0 t}\right]
\]

\[
\times \left((-8\kappa \beta^2 \theta_0^2 e^{2\kappa \theta_0 t} - \kappa + \beta e^{2\kappa \theta_0 t}) - \kappa(t_+ + t_-)\right). \quad (46)
\]

\[
p_{QT} = (2\beta \sinh(2\kappa \theta_0 t) + A) \exp \left[\frac{2}{\kappa} (A - B) + \frac{4\beta}{\kappa} e^{2\kappa \theta_0 t}\right]
\]

\[
\times \left((-8\kappa \beta^2 \theta_0^2 e^{2\kappa \theta_0 t} (\kappa + \beta e^{2\kappa \theta_0 t}) - \kappa(t_+ + t_-))\right). \quad (47)
\]

For \(t > t_2\), the conventional Poisson brackets are recovered as follows:

\[
\{\Omega, P_\Omega\}_{PB} = \{\chi, P_\chi\}_{PB} = 1, \quad \text{others} = 0, \quad (48)
\]

and the Hamiltonian equations of the motion in Ref. [20] are given by \(\dot{O} = \{O, H\}_{PB}\), where \(O\) represents fields and corresponding momenta. They are explicitly written as

\[
\dot{\chi} = -2\kappa P_\chi, \quad \dot{\Omega} = 2\kappa P_\Omega, \quad (49)
\]

\[
P_\chi = 0, \quad P_\Omega = 0. \quad (50)
\]

Since the momenta \(P_\Omega\) and \(P_\chi\) are constants of the motion as seen from Eq. (50), we easily obtain the solutions as

\[
\Omega = 2\kappa P_\Omega t + A_0, \quad (51)
\]

\[
\chi = -2\kappa P_\chi t + B_0, \quad (52)
\]

where \(P_\Omega = P_{\Omega_0}\), \(P_\chi = P_{\chi_0}\), and \(A_0\) and \(B_0\) are arbitrary constants. Next, the dynamical solutions in Eqs. (51) and (52) should satisfy the constraint in Eq. (26),

\[
\kappa t_\pm = \kappa(P_{\Omega_0}^2 - P_{\chi_0}^2), \quad (53)
\]

On the other hand, by using Eqs. (51) and (52), the curvature scalar is calculated as

\[
R = 4\kappa^2 P_{\Omega_0}^2 e^{-2\rho + 4\phi} = 4\kappa^2 T_{\Omega_0}^2 e^{-2B_0 + 4\kappa P_{\chi_0} t} \left|A_0 + 2\kappa P_{\Omega_0} t\right|. \quad (54)
\]

Since \(\Omega = e^{-2\phi}\) in Eq. (51) should be positive definite, there are two types of branches: The first one is \(t > -A_0/(2\kappa P_{\Omega_0})\) for the positive charge of \(P_{\Omega_0} > 0\), and the second is \(t < A_0/(2\kappa P_{\Omega_0})\) for the negative charge of \(P_{\Omega_0} < 0\). Note that the universe is always accelerating irrespective of the vacuum energy density.

By substituting the solutions in Eqs. (51) and (52) into Eqs. (9), (10), (11), and (12), we obtain the stress-energy-momentum tensor as

\[
T_{\pm \pm}^{CL} = -\frac{\kappa^2 P_{\Omega_0}^2}{2\kappa P_{\Omega_0} t + A_0}. \quad (55)
\]
Then, by using Eqs. (14), (15), (16), and (17), we obtain the energy densities and the pressures as

\[
\rho_{\text{Cl}} = -3\kappa^2 P_{\Theta 0}^2 \exp \left[ \frac{2}{\kappa} (A_0 - B_0) + 4(P_{\Theta 0} + P_{\chi 0}^2)t \right],
\]

\[
\rho_{\text{Qt}} = (2\kappa P_{\Theta 0} t + A_0) \exp \left[ \frac{2}{\kappa} (A_0 - B_0) + 4(P_{\Theta 0} + P_{\chi 0}^2)t \right]
\times \left( -2\kappa(P_{\Theta 0} + P_{\chi 0})^2 - \kappa(t_+ + t_-) \right),
\]

\[
p_{\text{Cl}} = -\kappa^2 P_{\Theta 0}^2 \exp \left[ \frac{2}{\kappa} (A_0 - B_0) + 4(P_{\Theta 0} + P_{\chi 0}^2)t \right],
\]

\[
p_{\text{Qt}} = (2\kappa P_{\Theta 0} t + A_0) \exp \left[ \frac{2}{\kappa} (A_0 - B_0) + 4(P_{\Theta 0} + P_{\chi 0}^2)t \right]
\times \left( -2\kappa(P_{\Theta 0} + P_{\chi 0})^2 - \kappa(t_+ + t_-) \right).
\]

Now, in order to describe the geometry from the decelerating phase to the accelerating universe, which finally ends up with a vanishing curvature scalar corresponding to the zero acceleration in Ref. [29], one should patch two different solutions at \( t = t_2 \). Thus, matching the solutions in Eqs. (35) and (36) with Eqs. (51) and (52) up to their time derivatives at \( t = t_2 \) yields the following conditions:

\[
\beta = \frac{P_{\Theta 0}}{2\theta_2} \coth (2\kappa \theta_2 t_2),
\]

\[
A = A_0 + \frac{P_{\Theta 0}}{\theta_2} [2\kappa \theta_2 t_2 - \tanh(2\kappa \theta_2 t_2)],
\]

\[
B = B_0 - \frac{P_{\chi 0}}{\theta_2} [2\kappa \theta_2 t_2 - \coth(2\kappa \theta_2 t_2)],
\]

\[
\frac{P_{\chi 0}}{P_{\Theta 0}} = \tanh(2\kappa \theta_2 t_2).
\]

The above conditions in Eqs. (63) and (64) are rewritten as

\[
P_{\Theta 0} = 2\beta \theta_2 \cosh(2\kappa \theta_2 t_2),
\]

\[
P_{\chi 0} = 2\beta \theta_2 \sinh(2\kappa \theta_2 t_2).
\]

Since \( da(\tau)/d\tau = d\bar{\rho}(t)/dt \), we obtain the expanding velocity of the universe as

\[
\frac{da(\tau)}{d\tau} = -2(P_{\Theta 0} + P_{\chi 0}) - \frac{\kappa P_{\Theta 0}}{A_0 + 2\kappa P_{\Theta 0} t},
\]

which is always negative because \( P_{\Theta 0} > |P_{\chi 0}| \) from Eqs. (57) and (58) and \( \Omega_0 = A_0 + 2\kappa P_{\Theta 0} t > 0 \) for any time \( t \).

Next, we assign one more condition of \( R(t_2) = R_0(t_2) \) in order to find the appropriate time “\( t_2 \)” that connects the respective scalar curvatures. This continuity requirement leads to

\[
P_{\Theta 0}^2 = 2\beta^2 \left[ \frac{2}{\kappa} e^{2\kappa \theta_2 t_2} (2\beta \sinh(2\kappa \theta_2 t_2) + A) \left( 2\beta \sinh(2\kappa \theta_2 t_2) + A + \frac{\kappa}{2} \right)
\right.
\]

\[ - \cosh(2\kappa \theta_2 t_2) \left( 2\beta e^{2\kappa \theta_2 t_2} + A \right) \left],
\]

which corresponds to the requirement that derivatives up to the second derivatives of the metric and the dilaton fields be continuous. From the beginning, we have considered that \( \kappa, \theta_2, \) and \( P_{\Theta 0} \) to be positive and \( P_{\chi 0} \) to be negative; then, from Eq. (65), a consistent patching appears at the negative value of \( t_2 \). Thus, we obtain the desired geometry connecting the decelerating universe to the accelerating universe, where its acceleration tends to vanish eventually.

Then, the behaviors of the energy densities and the pressures are shown in Figs. [2] and [3]. The total energy and pressure are negative, but the induced matter from the quantum back reaction changes from positive energy and pressure to negative energy and pressure. Note that \( w_{\text{Cl}} = p_{\text{Cl}}/\rho_{\text{Cl}} = 1/3 \) for \( t > t_1 \) and \( w_{\text{Qt}} = p_{\text{Qt}}/\rho_{\text{Qt}} = 1 \) only for \( t > t_2 \). The behavior of the state parameter \( w \) is shown in Fig. [4].
FIG. 1: Change of the $\theta_2$-dependent curvature scalar in Eq. (39) from the negative to the positive region. After $t = t_2$, it is always accelerating and converges at $t \to \infty$ by Eq. (54) for $P_\chi^0 < 0$. This figure is plotted for $\beta = \kappa = \theta_2 = 1$, $A = 10$, and $B = 3$, $t > t_1$. Then, $t_1 \approx -1.156$ and $t_2 \approx -0.632$. Consistent constants satisfying the continuity equations are chosen as $P_\Omega^0 \approx 3.826$, $P_\chi^0 \approx -3.262$, $A_0 \approx 11.579$, and $B_0 \approx 3.301$.

FIG. 2: The solid, the dashed, and the dotted lines denote the energy densities of the total, the quantum, and the classical matter, respectively. At $t = t_2$, the total energy density is discontinuous. This figure is plotted for the same constants used in Fig. 1.

IV. DISCUSSION

In summary, the classical energy density and the classical pressure start with a negative value that goes to the infinity so that the equation-of-state parameter is always positive constant. As for the quantum-mechanical energy density and pressure, they have some finite positive and negative values alternatively in the noncommutative region, and they jump down at the critical point $t_2$, and eventually end up as divergent quantities in the commutative region. Correspondingly, the state parameter is singular at the critical time $t_2$. Except near the point, it is mostly positive. Finally, the total energy density and the total pressure rely on the dilatonic contribution because the classical dilaton contribution is larger than the quantum-mechanical one, so that they have negative values both in the noncommutative and the commutative regions. However, the state parameter is remarkably positive; furthermore, it is regular every time. Note that the total state parameter from the total contributions is not the same for each contributions from the classical and the quantum-mechanical regions because $w \neq w_{C1} + w_{Q1}$; by definition, the exact form is easily written as $w = (w_{C1}\rho_{C1} + w_{Q1}\rho_{Q1})/(\rho_{C1} + \rho_{Q1})$, which is reminiscent of the center-of-mass coordinate in
FIG. 3: The solid, the dashed, and the dotted lines denote the pressures of the total, the quantum, and the classical matter, respectively. This figure is plotted for the same constants used in Fig. 1.

FIG. 4: The solid, the dashed, and the dotted lines denote the state parameters of the total, the quantum, and the classical matter, respectively. This figure is plotted for the same constants used in Fig. 1.

a mechanical system. Consequently, a positive definite value is given even though the universe is accelerating because the quantum-mechanically-induced energy is not positive definite.

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