FAIRNESS PREFERENCE BASED DECISION-MAKING MODEL FOR CONCESSION PERIOD IN PPP PROJECTS

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ABSTRACT. Both government and private sector have the characteristic of fairness preference when deciding a suitable concession period for infrastructure projects. The appropriate concession period is helpful to construct the Public-Private-Partnership (PPP) project, to alleviate governments financial burden, and to boast the economic growth. Therefore, this paper aims to develop a decision-making model of concession period with fairness preference based on the two sides equitable utilities. To better describe decision makers fair psychology, the Nash bargaining game solution was adopted as a fair reference point. The results show that the concession period with fairness preference will become longer than that without fairness preference. Furthermore, the longer the concession period is, the better construction quality of the infrastructure project (highway) is. So, decision makers with fairness preference tend to make good decisions. In conclusion, the developed decision-making model renders useful references for both government and private sector in negotiating the concession period for infrastructure projects.

1. Introduction. The demand for infrastructure projects is necessary to uphold the development of the economy. This circumstance pressures the government to expand infrastructure expenditure. In order to alleviate the financial burden, the government has commonly used the PPP scheme to attract the private sector to invest in infrastructure projects [6, 30]. In PPP contract, a concession period is
one of the key variables to be determined [18]. The concession period is the agreed period during which the private investor is authorized to develop, construct and operate the infrastructure project before being transferred back to the government [28]. Generally, a fair and reasonable concession period is helpful to spread risks and share interests between the government and the private [35]. If concession period is prolonged, it may result in the loss of the public interests. On the other hand, if concession period is too short, it will result in failure of the cooperation or a higher toll regime, which will make the risks shifted to people who use the project [26, 28]. Consequently, how to decide a reasonable concession period will become an important issue for decision-making of PPP projects.

To determine the reasonable concession period, current studies have developed various models. These methods tend to fall into three categories: technical and economic evaluation method, emulation technology, and game theory. Net Present Value (NPV) [15] analysis had been widely applied to identify the reasonable concession period [28, 32, 38]. To satisfy the decision makers demands of investment, Shen et al. [27, 28] used the NPV method to establish the build-operate-transfer concession model (BOTCcM). With this objective, Ng et al. [21] and Engel et al. [7] utilized International Rate of Return (IRR) or Least Present Value of Revenue (LPVR) to compute the concession period. To mitigate the effect of risks and uncertainties, emulation technologies like Monte Carlo simulation and System Dynamic technology had been adopted to determine the concession period appropriately. For example, Zhang [36] developed a web-based concession period analysis system (WCPAS) based on the Monte Carlo simulation technique. Mostafa et al. [18] proposed a Fuzzy-Delphi technique to calculate the reasonable concession period with considering uncertainties. As a useful tool to study the behavior of rational people, the game theory can be applied to assist the decision makers on the negotiations [23, 29]. Wu et al. [33] established the Stackelberg game model to explore the impacts of the cost on the concession period. Shen et al. [27] modelled the negotiation process of the concession period based on the bargaining game theory. Yang et al. [34] proposed a two stage game model to build the optimal decision model of concession period through backtracking method. Bao et al. [1] developed a model to identify the reasonable concession period based on the incomplete information bargaining game. Undoubtedly, these methods offer research foundation for this paper, however, they are all based on pure self-serving hypothesis, which hold that people only pursue to maximize their own interests.

In practice, decision makers not only had the self-interest preference, but also had the fairness preference [14], which had been demonstrated repeatedly by ultimatum game [11], public good game [17], dictator game [10], and gift exchange game [8]. Samuelson [24] and Sen [25] thought fairness preference was that people were not only concerned about their own interests, but also the others interests. Further, Rabin [22] built the concept of fairness on the cognition that people like to help those who help them and hurt those who hurt them. Fehr and Schmidt [9] modelled fairness preference as inequity aversion, namely the people were willing to give up some monetary payoff when the distribution was unfairness. Charness and Rabin [4] established a linear utility function model which including the decision makers fairness preference. This utility function model of fairness preference on making decisions was different from that of pure self-serving. Consequently, fairness had become an important factor in bargaining games [2]. Some researchers have already
found both the government and the private concerned about whether the concession period was fair or not. Wang and Liu [31] proposed an excess revenue-sharing model to assure the interest one received aligning with risks one taken. Carbonara et al. [3] built a win-win decision model to determine the reasonable concession period which allowed for a fair risk sharing between the government and the private. In determining the fair concession period, Zhang [37] established a workshop price mechanism to ensure the equity in benefit distribution. These literatures were all concerned about fair, however, peoples fair reference point in these model were others interests, rather than his own expect fair interests. Although, Nash bargaining game solution had been deduced by Hanaoka and Palapus [12] to depict the decision makers expected fair value. However, the fairness preference degree in these literatures was unchanged, and this ignored the influence of different fairness preference degree on fair concession period. The aim of this work is to apply the fairness preference theory and the Nash bargaining game theory in making the concession period decision. Firstly, we add the fairness preference into the concession period decision to study whether the decision can be influenced by the peoples fair psychology or not. Furthermore, we introduce the disadvantageous inequality parameter and the advantageous inequality parameter to describe the fair utility. That is, our inequality parameters are variables, so we can study the effect of different fair preference degree on the reasonable concession period. Secondly, to be more in line with fair psychology, we represent Nash bargaining game solutions as the decision makers fair reference point. Nash bargaining game solution not only overcomes the defect of absolute fairness but also takes into account the strength of the parties.

The outline of the paper is as follows. Section 2 gives the concession period decision model with fairness preference. The modeling is divided into two-step: constructing the Nash bargaining fairness analysis framework and building the fair concession period decision model. Section 3 analyzes the effects of fairness preference on model results. This section considers three cases: both sides have fairness preference, one side has fairness preference, and neither side has fairness preference. Section 4 gives a numerical analysis. Section 5 concludes the paper.

2. The fairness utility, the Nash bargaining game solution and the fair concession period. Suppose that the government plans to build a toll highway between two places. To raise money, the government uses the PPP scheme to attract private investors. In project investment, the total cost which is predicted by the government is divided into the construction cost \( c \) and the operational cost \( v(c) \). The operational cost \( v(c) \) is strictly decreasing in construction cost \( c \), namely \( \partial v/\partial c < 0 \) [34]. That is because the increased construction cost leads to the improved quality of highway construction, and then it causes the decreased operational cost. So, the function relationship [16] is

\[
v(c) = kc^{-\alpha}, (k > 0, c > 0)
\]

With considering the direct economic income of the highway, the governments interest \( \pi_G \), the privates interest \( \pi_P \), and the total interest are displayed in formula respectively as

\[
\pi_G = R(T_f - T_c) - \int_{T_c}^{T_f} kc^{-\alpha} dt
\]

\[
\pi_P = R(T_c - T_b) - \int_{T_b}^{T_c} kc^{-\alpha} dt - c
\]
\[ \pi = \pi_G + \pi_P = R(T_f - T_b) - \int_{T_b}^{T_f} kc^{-\alpha} dt - c \]  

(4)

Where \( T_f \) denotes the highways economic life, \( T_c \) is the concession period, and \( T_b \) is the construction period. Also, we use \( R(R > kc^{-\alpha}) \) as the highways average annual return.

2.1. The fair utility. The F-S model which was established by Fehr and Schmidt [9] had been widely used to portray people’s fair psychology [5]. According to the F-S model, we hold that if the actual interest is less than the expected fair interest, people will get disadvantageous inequality due to jealousy, if not, people will also get advantageous inequality due to sympathy. With fairness preference, peoples fair utilities can be written as:

\[ u_i = \pi_i - \gamma_i(\bar{\pi}_i - \pi_i) - \xi_i(\bar{\pi}_i - \pi_i)(i = G, P) \]  

(5)

In equation (5), \( \gamma_i \geq 0(i = G, P) \) is peoples disadvantageous inequality parameter, and \( 0 \leq \xi_i < 1(i = G, P) \) is peoples advantageous inequality parameter. The larger the inequality parameter is, the stronger the degree on fairness is. \( \bar{\pi}_i(i = G, P) \) is peoples expect fair interests. Actually, people payed more attention to disadvantageous inequality and less concern about advantageous inequality, namely \( \gamma_i > \xi_i(i = G, P) \). This accorded with the idea of “Man struggles up wards” [2, 13]. Hence, in the case of only considering the disadvantageous inequality, the governments and the privates fair utilities can be expressed as

\[ u_G = \pi_G - \gamma_G(\bar{\pi}_G - \pi_G) \]  

(6)

\[ u_P = \pi_P - \gamma_P(\bar{\pi}_P - \pi_P) \]  

(7)

2.2. Nash bargaining game solution. In solving the Nash bargaining problem, Nash [19, 20] existed four axioms: Pareto efficiency, Symmetry, Invariance of Linear Transformations, and Independence of Irrelevant Alternatives. Based on the four axioms, Nash [19, 20] deduced the solution of the two-person bargaining problem, namely the unique bargaining game solution was to maximize the Nash product.

Based on the analysis of the Nash bargaining game solution, we can drive the Nash product as

\[ u_G u_P(\pi_G, \pi_P) = [(1 + \gamma_G)(\pi_G - \gamma_G\bar{\pi}_G)][(1 + \gamma_P)(\pi - \pi_G) - \gamma_P(\pi - \bar{\pi}_G)] \]  

(8)

Hence, the decision model can be written as

\[ \max_{\pi_G, \pi_P} u_G u_P(\pi_G, \pi_P) \]

\[ \text{s.t.} \]  

\[ \bar{\pi}_G + \bar{\pi}_P \leq \pi \]

\[ u_P = \pi_P - \gamma_P(\bar{\pi}_P - \pi_P) \]

\[ u_G = \pi_G - \gamma_G(\bar{\pi}_G - \pi_G) \]

\[ \pi = \pi_G + \pi_P \]

\[ 0 \leq \pi_G \leq \pi; 0 \leq \pi_P \leq \pi \]  

(9)

By seeking second derivative of \( u_G u_P(\pi_G, \pi_P) \), we have

\[ \frac{\partial^2 u_G u_P(\pi_G, \pi_P)}{\partial \pi_G^2} = -2(1 + \gamma_G)(1 + \gamma_P) < 0 \]

It means there is the only optimum solution \( \pi_G^* \) for the objective function, namely \( \max u_G u_P(\pi_G, \pi_P) \). Based on
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\[
\frac{\partial u_G}{\partial \pi_G} = \frac{(1 + \gamma_G)[(1 + \gamma_P)(\pi - \pi_G^*) - \gamma_P(\pi - \pi_G)] - (1 + \gamma_P)[(1 + \gamma_G)\pi_G^* - \gamma_G\pi_G]}{\gamma_G\pi_G} = 0
\]

we have

\[
\pi_G^* = \frac{1 + \gamma_G}{2 + \gamma_G + \gamma_P}
\]

Due to the fixed point theorem, the Nash bargaining game optimum solution is the expected fair solution, namely \(\pi_G^* = \bar{\pi}_G\). Therefore, we have

\[
\bar{\pi}_G = \frac{1 + \gamma_G}{2 + \gamma_G + \gamma_P}
\]  

(10)

Then, we can easily find out

\[
\bar{\pi}_p = \pi - \bar{\pi}_G = \frac{1 + \gamma_P}{2 + \gamma_G + \gamma_P}
\]  

(11)

By inserting equations (10) and (11) into equations (6) and (7), we readily obtain

\[
u_G = (1 + \gamma_G)\pi_G - \frac{\gamma_G(1 + \gamma_G)}{2 + \gamma_G + \gamma_P} \pi
\]

\[
= (1 + \gamma_G)[R(T_f - T_c) - \int_{T_c}^{T_f} kc^{-\alpha} dt] - \frac{\gamma_G(1 + \gamma_G)}{2 + \gamma_G + \gamma_P} [R(T_f - T_b) - \int_{T_b}^{T_f} kc^{-\alpha} dt - c]
\]  

(12)

\[
u_P = (1 + \gamma_P)\pi_P - \frac{\gamma_P(1 + \gamma_P)}{2 + \gamma_G + \gamma_P} \pi
\]

\[
= (1 + \gamma_P)[R(T_c - T_b) - \int_{T_b}^{T_c} kc^{-\alpha} dt - c] - \frac{\gamma_P(1 + \gamma_P)}{2 + \gamma_G + \gamma_P} [R(T_f - T_b) - \int_{T_b}^{T_f} kc^{-\alpha} dt - c]
\]  

(13)

**Proposition 1.** If one side has strong fairness preference, his expected fair interest will be increased and the other ones expected fair interest will be decreased.

**Proof.** By equations (10) and (11), we can get

\[
\frac{\partial \bar{\pi}_G}{\partial \gamma_G} = \frac{1 + \gamma_P}{(2 + \gamma_G + \gamma_P)^2} \pi > 0
\]

and

\[
\frac{\partial \bar{\pi}_P}{\partial \gamma_G} = -\frac{1 + \gamma_P}{(2 + \gamma_G + \gamma_P)^2} \pi < 0
\]

It means that the governments expected fair interest \(\bar{\pi}_G\) increases with his disadvantageous inequality parameter \(\gamma_G\), while the private expected fair interest \(\bar{\pi}_P\) decreases with the \(\gamma_G\). Similarly, the \(\bar{\pi}_P\) increases with \(\gamma_P\), while the \(\gamma_G\) decreases with the \(\gamma_P\).
2.3. The fair concession period. According to equations (6) and (7), the decision model in the first stage can be written as

\[
\max_{T_c} \left(1 + \gamma_G \pi_G - \frac{\gamma_G(1 + \gamma_G)}{2 + \gamma_G + \gamma_P} \pi \right) \tag{14}
\]

subject to

\[
\begin{align*}
(1 + \gamma_P)\pi_P - \frac{\gamma_P(1 + \gamma_P)}{2 + \gamma_G + \gamma_P} \pi & \geq \mu \\
\pi_G &= R(T_f - T_c) - \int_{T_c}^{T_f} kc^{-\alpha} dt \\
\pi_P &= R(T_c - T_b) - \int_{T_b}^{T_f} kc^{-\alpha} dt - c \\
\pi &= R(T_f - T_b) - \int_{T_b}^{T_f} kc^{-\alpha} dt - c \\
0 &< \mu \leq \bar{\pi}_p \\
R &> kc^{-\alpha} > 0 \\
k, c, T_b, T_f &> 0
\end{align*} \tag{15}
\]

Equation (14) lies in the maximization of the governments utility \(u_G\). In Equation(15), \((1 + \gamma_P)\pi_P - \frac{\gamma_P(1 + \gamma_P)}{2 + \gamma_G + \gamma_P} \pi \geq \mu\) means the privates utility \(u_p\) is not less than his opportunity cost. Combining equations(14) and(15), the Lagrange function is constructed. \(\lambda\) is a Lagrange multiplier. Then we have

\[
L(T_c, \lambda) = \left(1 + \gamma_G \pi_G - \frac{\gamma_G(1 + \gamma_G)}{2 + \gamma_G + \gamma_P} \pi + \lambda\left[(1 + \gamma_P)\pi_P - \frac{\gamma_P(1 + \gamma_P)}{2 + \gamma_G + \gamma_P} \pi - \mu\right) \tag{16}
\]

The first order condition of equation (16) is then given by

\[
\frac{\partial L(T_c, \lambda)}{\partial T_c} = (\lambda - 1 - \gamma_G)(R - kc^{-\alpha}) = 0 \tag{17}
\]

\[
\frac{\partial L(T_c, \lambda)}{\partial \lambda} = (1 + \gamma_P)[R(T_c - T_b) - \int_{T_b}^{T_c} kc^{-\alpha} dt - c] - \frac{\gamma_P(1 + \gamma_P)}{2 + \gamma_G + \gamma_P} \pi - \mu = 0 \tag{18}
\]

Through solving equations (17) and (18), we get the fair concession period \(T_c\) as follow

\[
T_c = T_b + \frac{\mu}{1 + \gamma_P} + \frac{\gamma_P}{2 + \gamma_G + \gamma_P} + c
\]

In the second stage, the private should determine the actual construction cost \(c(T_c)\) to maximize his utility, namely

\[
\max_{c(T_c)} \left(1 + \gamma_P \pi_P - \frac{\gamma_P(1 + \gamma_P)}{2 + \gamma_G + \gamma_P} \pi \right)
\]

subject to

\[
\begin{align*}
\pi_P &= R(T_c - T_b) - \int_{T_b}^{T_c} kc^{-\alpha} dt - c \\
\pi &= R(T_f - T_b) - \int_{T_b}^{T_f} kc^{-\alpha} dt - c \\
u_G &= \pi_G - \gamma_G(\bar{\pi}_G - \pi_G) \\
R &> kc^{-\alpha} > 0 \\
k, c, T_b, T_f &> 0
\end{align*} \tag{20}
\]
By calculating the first order condition
\[
\frac{\partial u_T}{\partial c(T_c)} = \alpha c(T_c - T_h) k c^{-\alpha - 1} - \frac{\alpha \gamma P}{2 + \gamma G + \gamma P} k c^{-\alpha - 1} (T_f - T_h) + \frac{\gamma P}{2 + \gamma G + \gamma P} = 0
\]
Then, the actual construction period is
\[
c(T_c) = \left( \frac{2 + \gamma G + \gamma P}{2 + \gamma G} \right)^{\frac{1}{\alpha}}
\]
(21)

Proposition 2. Concession period and actual construction cost have positive correlation relationship.

Proof. The actual construction cost is viewed as a function of the concession period, and the formula is \(c(T_c) = \left[ \frac{(2 + \gamma G + \gamma P) k \alpha (T_c - T_h) - k \alpha \gamma P (T_f - T_h)}{2 + \gamma G} \right]^{\frac{1}{\alpha}}\). Taking derivative regarding \(T_c\) on \(c(T_c)\), we then have
\[
\frac{\partial c(T_c)}{\partial T_c} = - (\alpha + 1) \left( \frac{(2 + \gamma G + \gamma P) k \alpha (T_c - T_h) - k \alpha \gamma P (T_f - T_h)}{2 + \gamma G} \right)^{\frac{1}{\alpha} - 1} < 0
\]
Hence, concession period and construction cost have positive correlation relationship.

Proposition 2 shows that if the concession period which is decided in the first stage is too short, the private will decrease the actual construction cost to maximize his interest or fair utility. Too short concession will reduce the quality of the highway construction and increase the governments operational cost. Conversely, prolong concession period will encourage the private to increase the highway construction investment, and this contributes to advance the equality of highway construction. Then, form Proposition 1, we know the excessively long concession period will harm the governments interest. So, a reasonable concession period is important for the construction equality of the highway.

3. Can the degree on fairness preference effects the fair concession period.

3.1. Both sides have fairness preference. If both the government and the private have fairness preference, we make \(\gamma_G > 0\) and \(\gamma_P > 0\). The fair concession period is \(T_c = T_h + \frac{\mu}{R - k c^{\alpha}} + \frac{\gamma_G \pi + \gamma_P \pi + c}{R - k c^{\alpha}}\) \((\gamma_G > 0, \gamma_P > 0)\), and the actual construction cost is \(c(T_c) = \left[ \frac{(2 + \gamma G + \gamma P) k \alpha (T_c - T_h) - k \alpha \gamma P (T_f - T_h)}{2 + \gamma G} \right]^{\frac{1}{\alpha}}\) \((\gamma_G > 0, \gamma_P > 0)\).

Proposition 3. The fair concession period \(T_c\) decreases with the governments fairness preference parameter \(\gamma_G\).

Proof. Due to the fair concession period \(T_c = T_h + \frac{\mu}{R - k c^{\alpha}} + \frac{\gamma_G \pi + \gamma_P \pi + c}{R - k c^{\alpha}}\) \((\gamma_G > 0, \gamma_P > 0)\), we get the derivative of the fair concession period with respect to the governments fairness preference parameter \(\gamma_G\), namely \(\frac{\partial T_c}{\partial \gamma_G} = \frac{-\gamma_G \pi}{(R - k c^{\alpha})(2 + \gamma G + \gamma P)}\). It is easy to get \(\frac{\partial T_c}{\partial \gamma_G} < 0\).

Proposition 4. The fair concession period \(T_c\) increases with the privates fairness preference parameter \(\gamma_P\).

Proof. Because of the fair concession period \(T_c = T_h + \frac{\mu}{R - k c^{\alpha}} + \frac{\gamma_G \pi + \gamma_P \pi + c}{R - k c^{\alpha}}\) \((\gamma_G > 0, \gamma_P > 0)\) the derivative of the \(T_c\) with respect to the \(\gamma_P\) is can be written as \(\frac{\partial T_c}{\partial \gamma_P} = \frac{\gamma_P \pi}{(R - k c^{\alpha})(2 + \gamma G + \gamma P)}\).
Only when the government has no fairness preference, the fair concession period will also be better. The fair concession period is given by

\[
T_c(T_e) = \frac{(1+\gamma_p)\pi}{(2+\gamma_p+\gamma_G)^2} + \frac{1}{(1+\gamma_p)^2}\frac{\gamma_G}{2+\gamma_p+\gamma_G}. \tag{22}
\]

Let \( f(\mu) \) be \( f(\mu) = -\frac{\mu}{(1+\gamma_p)^2} + \frac{(1+\gamma_p)^2}{(2+\gamma_p+\gamma_G)^2} \). Because \( f'(\mu) = -\frac{1}{(1+\gamma_p)^2} < 0 \) and \( \mu \in (0, \bar{\pi}_p) \), we get \( f(\bar{\pi}_p) = \frac{\gamma_p(1+\gamma_p)^2}{(2+\gamma_p+\gamma_G)^2} > 0 \). This means when \( \gamma_p > 0 \), \( \frac{\partial T_c}{\partial \gamma_p} > 0 \).

Proposition 3 shows that the stronger the governments fairness preference is, the shorter the fair concession period is. From Proposition 2, we know the longer concession period is, the more actual construction cost is. Hence, the governments fairness preference is not good for the construction quality of the highway. Proposition 4 shows that the stronger the privates fairness preference is, the longer the fair concession period is, further the better the construction quality of the highway is.

3.2. One side has fairness preference. If the government has no fairness preference, his disadvantageous inequality parameter is equal to zero. \( \gamma_G=0 \) is substituted into equations (19) and (21) respectively, so the fair concession period \( T_c \) can be expressed as

\[
T_c(T_e) = T_b + \frac{1}{(R-\alpha)(1+\gamma_p)^2}(\gamma_p > 0, \gamma_p > 0) and the fair neural concession period is given by \( T_c(T_e) = T_b + \frac{1}{(R-\alpha)(1+\gamma_p)^2} \). It is clear that

\[
T_c(T_e) = \frac{1}{(R-\alpha)(1+\gamma_p)^2} < 0.
\]

If the private has no fairness preference, \( \gamma_p=0 \). Then the fair concession period \( T_c(T_e) \) can be expressed as

\[
T_c(T_e) = T_b + \frac{c+\mu}{R-\alpha}, \tag{22}
\]

and the actual construction cost \( c'(T_c'(T_e)) \) is

\[
c'(T_c'(T_e)) = \left[\frac{(2+\gamma_p)k\alpha(T_c',T_e) - k\alpha\gamma_p(T_c'-T_e)}{2}\right]^{\frac{1}{\gamma_G}}.
\]

Proposition 5. Only when the government has no fairness preference, the fair concession period \( T_c(T_e) \) is shorter than the fair neural concession period \( T_c'(T_e) \).

Proof. The fair concession period is \( T_c(T_e) = T_b + \frac{1}{(R-\alpha)(1+\gamma_p)^2}(\gamma_p > 0, \gamma_p > 0) \) and the fair neural concession period is given by \( T_c'(T_e) = T_b + \frac{1}{(R-\alpha)(1+\gamma_p)^2} \). It is clear that \( T_c(T_e) = \frac{1}{(R-\alpha)(1+\gamma_p)^2} < 0 \).

Form equation (22), we obtain \( T_c(T_e)' \) \( \begin{cases} \geq 0, \mu \leq \frac{1+\gamma_p}{2+\gamma_p+\gamma_G} \pi & \frac{1+\gamma_p}{2+\gamma_p+\gamma_G} \pi \leq \pi \left(\frac{1+\gamma_p}{2+\gamma_p+\gamma_G} \pi - \frac{1+\gamma_p}{2+\gamma_p+\gamma_G} \pi = 0\right. \\
< 0, \mu > \frac{1+\gamma_p}{2+\gamma_p+\gamma_G} \pi \end{cases} \). If \( \mu > \frac{1+\gamma_p}{2+\gamma_p+\gamma_G} \pi \), \( \mu - \bar{\pi}_C > \frac{1+\gamma_p}{2+\gamma_p+\gamma_G} \pi - \frac{1+\gamma_p}{2+\gamma_p+\gamma_G} \pi = 0 \). In this case, the privates opportunity cost is more than his expect fair interests, so the private will not invest in this highway. Therefore, \( T_c(T_e) \geq T_c'(T_e) \).

Propositions 5 and 6 show that compared with the government, the private have more impact on the fair concession period. If the privates fairness preference is very strong, the fair concession period will be longer, and the construction quality of the highway will also be better.
3.3. Neither side has fairness preference. If the government and the private have no fairness preference, their disadvantageous inequality parameters are equal to zero. Hence, putting $\gamma_G=0$ and $\gamma_P=0$ into equation (19), we can derive the concession period $T'_c$ as follows:

$$T'_c = T_b + \frac{c + \mu}{R - kc^{-\alpha}} \quad (23)$$

Similarly, we put $\gamma_G=0$ and $\gamma_P=0$ into equation (21), and the actual construction cost $c'(T'_c)$ is

$$c'(T'_c) = \left[ k\alpha(T'_c - T_b) \right]^{\frac{1}{\alpha+1}} \quad (24)$$

**Proposition 7.** When neither the government nor the private has fairness preference, the fair neural concession period $T'_c$ is shorter than the fair concession period $T_c$.

**Proof.** The fair concession period is $T_c = T_b + \frac{\mu}{R - kc^{-\alpha}} + \frac{\gamma_P \pi}{R - kc^{-\alpha}} + \frac{c}{R - kc^{-\alpha}}$ ($\gamma_G > 0, \gamma_P > 0$) and the fair neural concession period is given by $T'_c = T_b + \frac{c + \mu}{R - kc^{-\alpha}}$. From Proposition 5, we know $T_c - T'_c = \frac{\gamma_P [(1+\gamma_P)(\pi-(2+\gamma_P)\mu)]}{(R - kc^{-\alpha})(1+\gamma_P)(2+\gamma_P+\gamma_G)} \geq 0$.

Proposition 7 expresses the concession period with both sides fairness preference is longer than that without fairness preference. Combining Proposition 2, we can infer that decision-making with fairness preference will be more conductive to construction quality of the highway than that without fairness preference.

4. Numerical analysis. In this section, a hypothetical PPP highway is presented to illustrate the application of the fair concession model. In this case, the data from Guang-Shen highway has been processed for the purpose of demonstration. The economic life $T_f = 35$ years, the construction period $T_b = 6$ years, the construction cost $c = 5.823$ billion RMBs, and the average annual return $R = 0.675$ billion RMBs. Because the operational cost is expressed by $v(c) = kc^{-\alpha}$, we assume the $k$ is equal to 475, and the $\alpha$ is equal to 1.2. Further, we accurately the average annual operational cost $v = 0.229$ billion RMBs.

To study the influence of the governments fair preference degree ($\lambda_G$) on the concession period, we make the privates fair preference degree be a constant ($\lambda_P=1$). In Figure 1, we find the stronger the governments fair preference degree is, the shorter the concession period is. That is exactly what the Proposition 3 shows. Further, the governments expected fair interest increases with its fair preference degree, while the privates expected fair interest decreases with it. It is the same to Proposition 1. Proposition 2 indicates the longer the concession period is, the more the actual construction cost is. Hence, the strong governments fairness preference is not good for the construction quality of the highway. Similarly, to study the influence of the privates fair preference degree ($\lambda_P$) on the concession period, we make the privates fair preference degree $\lambda_P=1$. Figure 2 shows the concession period increases with the privates fair preference degree, which has been demonstrated by Proposition 4. Combining the Proposition 2, we know the privates fairness preference is good to the construction quality of the highway. As Proposition 1 shows, the privates expected fair interest increases with its fair preference degree, while the governments expected fair interest decreases with it.

In Figure 3, we will study the influence of both sides fair preference degree on concession period. Figure 3 shows the concession period is shorter when $\lambda_G=0$ and
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Figure 1. The influence of governments fair preference degree on concession period and decision makers expected fair interest

Figure 2. The influence of privates fair preference degree on concession period and decision makers expected fair interest

$\lambda_P=0$ than that when $\lambda_G \neq 0$ and $\lambda_P \neq 0$. This represents the concession period with both sides fairness preference is longer than that without fairness preference. This, in turn, has exposed decision-making with fairness preference will be more conducive to construction quality of the highway than that without fairness preference. Proposition 7 also shows that. In addition, the concession period is shorter when $\lambda_G=0$ than that when $\lambda_G \neq 0$, and it is longer when $\lambda_P=0$ than that when $\lambda_P \neq 0$. This only when the government has no fairness preference, the fair concession period is shorter than the others, and only when the private has no fairness preference, the fair concession period is longer than the others. These results are found to be in good agreement with the Proposition 5 and Proposition 6.

5. Conclusion. In conclusion, this research has developed a concession period model with fairness preference. Making the Nash bargaining game solution as a
fair reference point, this model calculates the governments and the privates fairness utilities. This research has also discussed the effects of fairness preference degree on concession period. The result shows that the concession period decreases with the governments fairness preference degree, and it increases with the privates fairness preference degree. Research also shows that the concession period and the construction cost have positive correlation relationship, so the government with too strong fairness preference is not good for the highway construction. In addition, a comparative analysis has conducted between the concession period with fairness preference and that without fairness preference. It reveals that when both the private and the government have fairness preference, the concession period will become longer, and then the construction quality of the highway will be better.

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