Optimization of Convergent –Divergent Taper Angle with Combustion Chamber of Rocket Engine through Numerical Analysis

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ABSTRACT: Present days speed of vehicle is concerned in both sub -sonic vehicle( like car, bus and truck etc) super -sonic vehicles (like rockets). Convergent divergent component is the main part which decide the speed of any vehicle. A attempt is made to find the optimal convergent angle (inlet taper angle [β]) and divergent angle(outlet taper angle [α]). with the help of Numerical analysis through ANSYS 19.0(R3). Deciding parameter for greater thrust is angles i.e.; [β], and [α]. So, Convergent divergent nozzles with combustion chamber is designed, modelled and analysed numerical for getting optimal values of convergent and divergent angles . The various angles [β,α] used are [35,15] [35,20] [35,25] [40,15] [40,20] [40,25] [45,15] [45,20] [45,25]. At these different angle the parameters such as velocity, temperature, pressure and Mach numbers are estimated with the help of steady state fluent with carbon dioxide as inlet to the combustion chamber.

Key words: Sub-sonic, Super-sonic, Convergent- Divergent chamber, Inlet taper angle, outlet taper angle

INTRODUCTION

The nozzle is the basic structure that which converts the generated thermal energy into kinetic energy . The nozzle is a device that which is used to transform the low velocity, high pressure, high temperature gas in the combustion chamber into high velocity gas of low pressure and temperature.

The normal existing range of exhaust velocity of rocket nozzle is 2 to 4.5 kilo meters per second. velocity must be in above mentioned range then it results expected thrust value. The Mach number is the main consideration in this design.

Fig.1: Combustion chamber with convergent divergent nozzle

The inlet Mach number is less than one at the nozzle inlet(the velocity of the fluid is less than sound velocity of fluid ), it is equal to one at the throat(the velocity of the fluid is equal to sound velocity of fluid) and it is greater than one at outlet(the velocity of the fluid is greater than sound velocity of fluid). Therefore the velocity of fluid is increased from subsonic condition from nozzle to supersonic condition at nozzle outlet. To the prefix of this convergent divergent nozzle, combustion chamber is emerged. When the high pressure oxidizer combines with the high pressurised fuel in the combustion chamber they get burn and it will be convert into high pressure, high temperature gas and then enter to convergent divergent nozzle. In the rocket engines, there are some different engines, which are with good combination of components to give output that which consist of solid and liquid or gaseous propellants. The propellant introduce into the chamber by using both liquid and hybrid injectors. In this project the designing and analysis of combustion chamber with CD nozzle geometries optimized by varying the inlet taper (β )and outlet tapers (α)is done by using ANSYS fluent 19.0 (R3).CD nozzle with combustion chamber is designed with compressible flow relations. The simulation of rocket engine combustion with Multi-physics is a 2D axisymmetric representation of the experiment, which cannot simulate the true mixing process of the shear layer of the oxidizer injection. For future study full 3 dimensional simulation in transient process will be modelled[1]. A fluid dynamic numerical simulation of fibre nozzle to evaluate its potential in rocket motors enabled to investigate the flow conditions and the temperature distributions during the experiment. A maximum Mach number of 2.5 and temperature of 2730K were achieved in the flow filed inside the nozzle [2]. Nozzles come in variety of shapes and sizes in analysis of CD nozzle using computational fluid dynamics is pending on the mission of the rocket this is very important for the understanding of the performance characteristics of rocket[3]. The advantages of liquid propellant rocket engines are high performance compared to any other conventional chemical engine and their controllability in terms of thrust modulation. Undeniably, the most important component of these engines is the thrust chamber which generates thrust by providing a volume for combustion and converting thermal energy to kinetic energy[4].
The nozzle converts the low velocity, high pressure, high temperature gas in the combustion chamber into high velocity gas of lower pressure and low temperature in the analysis of fluid flow [5]. The interaction of environmental conditions together with the requirement that dimensional stability of the nozzle throat must be maintained, makes the selection of suitable materials extremely challenging in aeronautical space applications. c/c, c/sic and sic/sic are the materials currently in use[6]. The major advantages of hybrid rockets are safe without explosive concern, propellant versatility, temperature insensitivity to operating chamber pressure, flexibility in throttling and termination[7]. For complete description of the thermal environment in the combustion chamber a radiative heat transfer model with a finite volume integration method is also employed in it. In the combustion chamber, the main participating species in the radiation model are carbon dioxide[8]. The best size of rocket engine nozzle to be used within the atmosphere is when the exit pressure matches ambient (atmosphere) pressure, which diminishes with altitude. For rockets travelling from the earth to orbit, a simple nozzle design is only optimal at one altitude losing efficiency and wasting fuel at other altitudes[9]. If the pressure of jet leaving the nozzle exit is still above ambient pressure then a nozzle is said to be under expanded. If the jet is below ambient pressure then it is over expanded[10]. Minor over expanded causes a slight reduction in the efficiency but other wise does little harm however, if the exit pressure is less than approximately 40% that of ambient then flow separation occurs[11]. At the exit section, the Mach number is found to increase with rise in divergent angle. Similarly at the throat section also, the Mach number goes on increasing with rise in divergent angle. The static pressure falls with increased divergent angle[12]. In supersonic flow, both the density and velocity are changing as we change the area in order to conserve mass but in case of a subsonic flows, the density remains fairly constant. In supersonic flows, there are two changes, the velocity and the density. And the variation of density is very drastic[13]. The change in length of nozzle increase more drastically when the required exit Mach number is pushed on to higher values. There fore as Mach number increases the expansion has to be larger and thus longer straightening length resulting in longer length of nozzle[14]. The obtained contours were results obtained by the simulation in fluent and values of Mach number at the exit obtained through computational simulation was found to be close to the theoretical values and the small discrepancy in values of solution for the viscous flows can be explained by the isentropic assumptions in the theoretical values[15].

II. THEORITICAL DESIGN CALCULATIONS

Nozzle Design:

Mach number ratio = \( \frac{\text{object speed}}{\text{speed of sound}} = \text{mach number} \)

Conservation of mass: \( m = \rho VA = \text{constant} \)

\[ \frac{dV}{V} + \frac{dA}{A} = 0 \]

Conservation of momentum: \( \rho V dV = -dp \)

Isentropic flow: \( \frac{dp}{p} = \gamma \frac{dp}{\rho} \) \( dp = a^2 dp \)

Combine with momentum: \( -M^2 \frac{dV}{V} = \frac{dp}{\rho} \)

Combine with mass: \( (1 - M^2) \frac{dV}{V} = -\frac{dA}{A} \)

\( m = \text{Mass Flow rate}, V = \text{Velocity}, a = \text{speed of sound}, \) \( \gamma = \text{Specific heat ratio} \)

Isentropic Flow Relations:

\[ M = \frac{V}{a}, \quad a = \sqrt{\gamma \rho RT}; \quad \frac{p}{\rho^\gamma} = \text{constant} = \frac{p_t}{\rho_t}, \quad \frac{p}{p_t} = \left( \frac{p_t}{\rho_t} \right) ^{\frac{1}{\gamma - 1}}; \]

\[ q = \frac{1}{2} \rho V^2 = \frac{\gamma}{2} \rho M^2; \quad M = \left( 1 + \frac{\gamma - 1}{2} M^2 \right)^{\frac{\gamma}{\gamma - 1}}; \quad T = \left( 1 + \frac{\gamma - 1}{2} M^2 \right)^{-1} \]

Where,

\( M = \text{Mach} \), \( \gamma = \text{Speed of Sound} \), \( R = \text{Gas Constant} \), \( Y = \text{Specific heat ratio} \), \( T = \text{total condition} \), \( \gamma = \text{sonic condition} \), \( V = \text{Velocity} \), \( P = \text{pressure} \), \( T = \text{Temperature} \), \( P = \text{Density} \), \( A = \text{Area} \), \( q = \text{Dynamic Pressure} \)

Thrust:

\[ F = \frac{\text{change in momentum with time}}{t_f}; \quad F = \frac{[mv1]_t - [mv1]_0}{t_2 - t_1} \]

\( M = \text{mass flow rate} = \text{mass/time} \), \( M = \text{r} \times v \times A \) Where, \( r = \text{density}, v = \text{velocity}, A = \text{area} \)

Convergent-Divergent Nozzle:

1. Exhaust Velocity is given by \( V_e = \sqrt{\frac{2y}{\gamma - 1} R T_0 (1 - \frac{P_e}{P_0})^{\gamma - 1}} \)

2. Convergence area in nozzle = \( Ac = 3 A_t \)

3. Radius of throat is given by \( r_t = \sqrt{\frac{Ac}{\pi}} \)

4. Length of Diverging Nozzle is \( L_{dn} = \sqrt{\frac{Ac}{\pi}} \times \frac{1}{\tan \theta} \)

5. Length of Converging Nozzle is \( L_{cn} = \sqrt{\frac{Ac}{\pi}} \times \frac{1}{\tan \beta} \)

6. Diameter of combustion \( D_{cn} = \sqrt{\frac{4Ac}{\pi}} \)

7. Diameter of Throat Dthr = \( \sqrt{\frac{4Ac}{\pi}} \)

8. Throat Pressure = \( P_t = P_c [1 + (\gamma - 1)/2] \times \frac{T_{atm}}{T_{atm} - 1} \)

9. Mach Number of nozzle exist \( = Me^2 = \frac{\gamma - 1}{2} \times \frac{P_c}{P_{atm}} - 1 \)
10. Area of convergent is three to five times to area of throat
\[ A_c = 3 A_t; \quad A_t = \frac{A_c}{3} \]

11. Exit area
\[ A_e = \frac{A_c}{M_e} \left( \frac{1 + \frac{\gamma - 1}{2} M_e^2}{\gamma} \right)^{\frac{\gamma + 1}{\gamma - 1}} \]

12. Exit Temperature
\[ T_e = \left[ 1 + \frac{\gamma - 1}{2} M_e^2 \right]^{\frac{\gamma}{\gamma - 1}} \]

13. Exit Pressure
\[ P_e = \frac{P_t}{\gamma} \left( \frac{T_e}{T_t} \right)^{\frac{\gamma - 1}{\gamma}} \]

CALCULATIONS

Diameter of combustion chamber: \( D_c = 100m = 0.1m \)
Length of combustion chamber: \( L = 40mm = 0.04m \)
\[ D_e = \sqrt[\gamma]{\frac{4A_c}{\pi}}; \quad 100 = \sqrt{\frac{4A_c}{\pi}} \]
\[ A_c = 7853.981 mm^2 \]

Diameter of throat: \( D_{th} = 31mm, D_{th} = \sqrt[\gamma]{\frac{A_c}{\pi}} \)

Throat area \( A_t = 754.76 mm^2 \)

Throat temperature: \( T_t = T_c \left[ \frac{1}{1 + \frac{2}{\gamma}} \right] \)
\[ T_t = 2800(0.909) = 2545.2 K \]

Throat temperature: \( P_t = P_c \left[ 1 + \frac{\gamma - 1}{2} M_t^2 \right]^{\frac{\gamma}{\gamma - 1}} = 2039000 \left[ 1 + \frac{1.2-1}{2} \right] = 1150811.6 \]

\[ M = \frac{816.4966}{2.54} + 2.54 \] pascals

\[ \sqrt{\frac{e}{\pi}} \]

\[ Throat area: \quad A_e = \frac{A_c}{M_e} \left( \frac{1 + \frac{\gamma - 1}{2} M_e^2}{\gamma} \right)^{\frac{\gamma + 1}{\gamma - 1}} \]
\[ = 754.76 \left( \frac{1 + \frac{1.2 - 1}{2} M_e^2}{\gamma} \right)^{\frac{\gamma + 1}{\gamma - 1}} \]
\[ = 296.2056[9.27814] = 2748.23 mm^2 \]

Exit diameter: \( D_{exit} = \sqrt{\frac{A_e}{\pi}} = 59.15 mm \)

Exit temperature: \( T_{exit} = T_{th} \left[ 1 + \frac{\gamma - 1}{2} M_e^2 \right]^{-\frac{1}{\gamma}} = 1543.21 \)
\[ P_{exit} = \frac{P_t}{\gamma} \left[ \frac{T_{exit}}{T_{th}} \right]^{\frac{\gamma - 1}{\gamma}} = 57544.329 pascals \]

Throat Mach number:
\[ T_{th} = \left[ 1 + \frac{\gamma - 1}{2} M_e^2 \right]^{-\frac{1}{\gamma}} = \frac{2800}{2542.2} = \left[ 1 + \frac{1.2 - 1}{2} M_e^2 \right]^{-\frac{1}{\gamma}} = 1 + 0.1M_e^2 \]

\[ M = 1 \]

Exit sound speed: \( a_e = \sqrt{RT_e} = \sqrt{(1.2)(360)(1543.21)} = 816.4966 \)

Exit velocity: \( V_e = a_e M_e = 816.4966 \times 2.54 = 2080.51 m/s \)

Mass flow rate:
\[ m = \frac{\Delta x}{\Delta t} \left( \frac{2}{\sqrt{\gamma - 1}} \right) \left( \frac{1}{\gamma} \right) \left[ 1 + \frac{\gamma - 1}{2} M_e^2 \right]^{\frac{\gamma + 1}{(\gamma - 1)}} \]
\[ = \frac{2039000 \times 0.274823}{\sqrt{2800}} \left( \frac{1}{360} \right) (2.5480) \left( 1 + \frac{1.2 - 1}{2} \right)^{\frac{1.2 + 1}{(2 - 1)}} \]
\[ = \frac{560364.907}{599.9150} (0.05773)(2.5480)(1.32461)^{-5.5} = 293.124117 \]

Thrust: \( F = m \cdot V_e = (P_e - P_0) \)
\[ = 293.124117 \times 2080.51 + [57544.329 - 101325] \times 0.2748 \]
\[ F = 597816.69 N \]

III. GEOMETRIC MODELLING AND NUMERICAL ANALYSIS

ANSYS is a simulation software which is applied in various fields. In Aerospace Engineering it is used to understand the fluid mechanics and Aero Dynamics. It is a wind tunnel of the own. It is used to test any object condition with any environment. 2D Sketch is created with the convergent divergent nozzle with the length of 180mm, combustion chamber diameter of 100mm and length of 40mm with the inlet taper[\( \beta \)] and outlet taper[\( \alpha \)] of 35.15. The surface is generated with the help of concept, the generated surface area is 7997.3mm².

Fig-2.- Starting the sketch by using draw in sketching

Fig-3.- Giving symmetry to axis
In the mesh, the edge sizing is selected for combustion chamber with 4 edges and 50 divisions, later no of divisions with 30, 50 at convergent nozzle, throat and 100 divisions at divergent nozzle. And then these all are generated. Next Face mesh is applied by selecting all faces at get generated. After that the named selections are applied to inlet, outlet and axis. Finally the quadrilateral mesh is generated with nodes of 10302 and elements of 10050.

Fig 4-Mesh has generated and number of divisions are given

Fig 5-According to the count of divisions the mesh has updated

In the setup work bench, Models is selected to kept the energy On, Viscous to Realizable K-e, Standard Wall Function. In Materials select Fluid to carbon dioxide. In cell zone conditions, the fluid is selected to surface body fluid. In boundary conditions, zone select the inlet and zone type as pressure inlet, the gauge total pressure is 203900 and supersonic/initial gauge pressure is 2029000 and temperature is given as 2800 k, and in pressure outlet gauge pressure is zero and back flow temperature is 582.497. In velocity inlet, velocity magnitude is zero and outflow gauge pressure is 2029000. In the monitors, print to console and plot are selected and residual equations of continuity, x-velocity, y-velocity, energy are get selected with monitor and check convergence in on conditions. In solution Initialization set to hybrid initialization all initial values have given and then select initialize to it. Later in calculation the iterations are given up to 1500 and then calculations will complete and results have displayed in the contours. In the contours, by selecting different parameters like velocity, temperature, pressure and Mach numbers the results are displayed.

IV. RESULTS AND COMPARISON

After completion of setup by taking the different convergent and divergent angles at their respective geometries the following different parameters are obtained

Case-I: Convergent angle-35, Divergent angle -15,

Case-II: Convergent angle-35, Divergent angle -20,

Case-III: Convergent angle-35, Divergent angle -25,

Case-VI: Convergent angle-45, Divergent angle -15,

Case-V: Convergent angle-45, Divergent angle -20,
Case-VI: Convergent angle 45, Divergent angle -25

Case-VII: Convergent angle 40, Divergent angle -15

Case-VIII: Convergent angle 40, Divergent angle -20

Tab-1-Comparison parameters at different convergent and divergent angles:

| Angles          | Throat Pressure (Pa) | Exit Pressure (Pa) | Throat Temperature (k) | Exit Temperature (k) | Throat Velocity (m/s) | Exit Velocity (m/s) |
|-----------------|----------------------|--------------------|------------------------|----------------------|-----------------------|---------------------|
| Convergent      | Divergent            |                    |爱                        |                      |                       |                     |
| (in degrees)    | (in degrees)         |                    |                         |                      |                       |                     |
| 35              | 15                   | 1.1 754 e^6         | 7.00                    | 2660 .24             | 1.76 8.04 e^3         | 75.04 1.67 e^3      |
| 35              | 20                   | 1.2 589 e^6         | 5.67 e^4                | 2624 .31             | 1.714 70 3.65 7     | 1.71                |
| 35              | 25                   | 1.1 879 e^6         | 1.65 5 e^4              | 2603 .75             | 1.44 2.5 e^3         | 74.3.15 9           |
| 40              | 15                   | 1.2 369 3e^6        | 8.35                    | 2617 .93             | 1.81 2e^3            | 31.5.20 1           |
| 40              | 20                   | 1.0 23 e^6          | 3.76 3 e^4              | 2631 .38             | 1.554 66 3.09 1     | 1.77                |
The Pressure, Temperature, velocity chamber is designed, modeled and conducted numerical ratio’s. Among these three ratio’s also at 1.6 ratio’s 1.4, 1.6 and 1.8 giving best results apart from other and divergent angle to the exit velocity, it is shown that at 45°.

**Exit Velocity at different ratio of convergent divergent angle**

Graph is drawn between ratio of convergent angle and divergent angle to the exit velocity, it is shown that at ratio’s 1.4, 1.6 and 1.8 giving best results apart from other ratio’s. Among these three ratio’s also at 1.6 (β = 40°, α = 25°) gives best result as shown in the fig.42.

**V. CONCLUSION**

Convergent Divergent nozzle with combustion chamber is designed, modeled and conducted numerical analysis for different β and α values by Ansys 19.0(R3). The Pressure, Temperature, velocity and Mach numbers are get obtained by considering these different angles. At β = 40°, α = 25° optimum values of pressure =1.1518*10^6 Pa, temperature = 1.428e³ k, velocity = 1.925e³ m/s and Mach number = 2.3 are obtained.

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