Testing the phenomenological interacting dark energy with observational \(H(z)\) data

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2 June 2011

ABSTRACT

In order to test the possible interaction between dark energy and dark matter, we investigate observational constraints on a phenomenological scenario, in which the ratio between the dark energy and matter densities is proportional to the power law case of the scale factor, \(r = (\rho_X/\rho_m) \propto a^\xi\). By using the Markov chain Monte Carlo method, we constrain the phenomenological interacting dark energy model with the newly revised \(H(z)\) data, as well as the cosmic microwave background (CMB) observation from the 7-year Wilkinson Microwave Anisotropy Probe (WMAP7) results, the baryonic acoustic oscillation (BAO) observation from the spectroscopic Sloan Digital Sky Survey (SDSS) data release 7 (DR7) galaxy sample and the type Ia supernovae (SNe Ia) from Union2 set. The best-fit values of the model parameters are \(\Omega_m^0 = 0.27^{+0.02}_{-0.02}(1\sigma)\), \(\xi = 3.15_{-0.50}^{+0.48}(1\sigma)\), and \(w_X = -1.05_{-0.11}^{+0.07}(1\sigma)\), which are more stringent than previous results. These results show that the standard \(\Lambda\)CDM model without any interaction remains a good fit to the recent observational data; however, the interaction that the energy transferring from dark matter to dark energy is slightly favored over the interaction from dark energy to dark matter. It is also shown that the \(H(z)\) data can give more stringent constraints on the phenomenological interacting scenario when combined to CMB and BAO observations, and the confidence regions of \(H(z)\) alone, \(H(z)+\text{BAO+CMB}\), \(\text{SNe+BAO+CMB}\), and \(H(z)+\text{SNe+BAO+CMB}\) combinations are consistent with each other.

Key words: (cosmology:) cosmological parameters — cosmology: observations

1 INTRODUCTION

The fact that the universe is undergoing an accelerating expansion has been supported and confirmed by many cosmological observations, such as the luminosity distances of Type Ia Supernovae [SNe Ia, (Riess et al. 1998; Perlmutter et al. 1998; Astier et al. 2006; Hicken et al. 2009, 2004; Amanullah et al. 2010)], cosmic microwave background (CMB) from Wilkinson Microwave Anisotropy Probe [WMAP, (Spergel et al. 2003, 2007; Komatsu et al. 2002, 2010)], and the large scale structure from Sloan Digital Sky Survey [SDSS, (Tegmark et al. 2004; Eisenstein et al. 2005)]. In order to explain this mysterious phenomenon, the existence of dark energy with negative pressure, which dominates the total energy density and causes an accelerating expansion of our universe at late times, has been widely proposed. The most simple candidate of dark energy models is considered to be in the form of vacuum energy density or cosmological constant (\(\Lambda\)), with a equation of state (EoS): \(w_\Lambda = p_\Lambda/\rho_\Lambda \equiv -1\). However, the corresponding \(\Lambda\)CDM model is always entangled with the coincidence problem: The density of the cosmic component decreases with \(a^{-3(1+w_\Lambda)}\) during the expansion of our universe; therefore the matter density \(\rho_m\) decreases with \(a^{-3}\), and the cosmological constant density \(\rho_\Lambda\) do not change in the cosmic expansion; however, the dark energy density is comparable with the dark matter density today.

In order to alleviate the coincidence problem, many alternative models include the scalar field models with dynamical EoS (e.g., the quintessence [Ratra & Peebles 1988; Caldwell et al. 1998], phantom [Caldwell et al. 2002, 2003], k-essence [Armendariz-Picon et al. 2001; Chiba 2002], as well as quintom model [Feng et al. 2009] have been proposed, however, the nature of dark energy is still unknown. It is natural to consider the possibility of exchanging energy between dark energy and dark matter. In the interacting scenario, \(\rho_m\) could decrease slower than \(a^{-3}\) during the cosmic expansion to alleviate the coincidence problem. Considering that the dark energy \(\rho_X\), assuming a constant EoS, \(w_X \equiv \text{const}\) and the dust matter including the baryon and dark matter component \((\rho_m = \rho_b + \rho_{DM})\) exchange energy
through an interaction term \( Q \),
\[
\begin{align*}
\dot{\rho}_X + 3H\rho_X(1+w_X) &= -Q, \\
\dot{\rho}_m + 3H\rho_m &= Q,
\end{align*}
\]
which preserves the total energy conservation equation \( \dot{\rho}_{\text{tot}} + 3H(\rho_{\text{tot}} + p_{\text{tot}}) = 0 \), where \( \rho_{\text{tot}} = \rho_X + \rho_m \). Various interaction theoretical models have been put forward and studied \cite{Amendola_2000, Zimdahl_2001, Chimento_2003, Guo_2003, Guo_2005, Wei_2006, Wei_2007a, Zhang_2007, Chen_2009, Baldi_2011, Cervantes-Cota_2010, Zhang_2011, Cai_2011}, but the format of interaction term \( Q \) still can not be determined from fundamental physics.

On the other hand, the interacting dark energy can be investigated in a phenomenological way with minimal underlying theoretical assumptions. \cite{Dalal_2001} proposed a simple phenomenological scenario in which the ratio between the dark energy and matter densities is proportional to the power law case of the scale factor,
\[
r \equiv \frac{\rho_X}{\rho_m} \propto a^{-\xi},
\]
where \( \xi \) is a key parameter, which quantifies the severity of the coincidence problem. The special cases \( \xi = 3 \) and \( \xi = 0 \) correspond to the \( \Lambda \)CDM model and the self-similar solution without coincidence problem respectively. Hence, any solution with a scaling parameter \( 0 < \xi < 3 \) makes the coincidence problem less severe \cite{Pavon_2004}. Considering a flat FRW universe with \( \Omega_X + \Omega_{m0} = 1 \) (where \( \Omega_X, \Omega_{m0} \) are the present value of density parameter of the dark energy and dust matter, respectively), and setting \( r = a^2\Omega_X/\Omega_{m0} \), we can obtain the corresponding interaction term \( \frac{Q}{\Omega_X} = \frac{H p_m}{\Omega_{m0}} \) \cite{Guo_2007, Wei_2007b},
\[
Q = -H p_m(\xi + 3w_X)\Omega_X,
\]
where \( \Omega_X = (1 - \Omega_{m0})/(1 - \Omega_{m0} + \Omega_{m0}(1 + z)\xi) \). In this case, the standard cosmology without interaction between the dark sector is characterized by \( \xi = 3w_X = 0 \), while \( \xi + 3w_X \neq 0 \) denotes non-standard cosmology. The case \( \xi + 3w_X < 0 \) denotes that the energy is transferred from dark energy to dark matter (\( Q > 0 \)), which can alleviate the coincidence problem; whereas the case \( \xi + 3w_X > 0 \) indicates that the energy is transferred from dark matter to dark energy (\( Q < 0 \)), in which the coincidence problem is more severe.

The Friedmann equation of the phenomenological scenario with a constant EoS of dark energy can be expressed as
\[
E(z)^2 = \frac{H^2}{H_0^2} = (1 + z)^3 \left[ \Omega_{m0}(1 + \Omega_{m0})(1 + z)^{-\xi}\right]^{-3w_X/\xi},
\]
From Eq. (4), the simple phenomenological interacting scenario has been constrained from many cosmological observations. \cite{Guo_2007} considered the cosmological constraints on this phenomenological scenario with the luminosity distances \( d_A \) of SNe Ia data from the Supernova Legacy Survey [SNLS, \cite{Astier_2006}], the shift parameter \( \Delta \) from the 3-year WMAP [WMAP3, \cite{Spergel_2007}] results, and the distance parameter \( A \) of the baryon acoustic oscillation [BAO, \cite{Eisenstein_2005}] observations. Recently, \cite{Chen_2010} tested this phenomenological form by combining the 397 SNe Ia from the Constitute Set \cite{Hicken_2009}, the \( \Delta \) parameter from the 5-year WMAP [WMAP5, \cite{Komatsu_2009}] results and \( A \) parameter, to show that the \( \Lambda \)CDM model still remains a good fit to the recent observational data, as well as, the coincidence problem indeed exists and is quite severe. More recently, \cite{Wei_2010a} constrained the general type which is characterized by \( \rho_X/\rho_m = f(a) \) (where \( f(a) \) can be any function of scale factor \( a \)) from the latest observational data including the 397 SNe Ia data set, the 7-year WMAP [WMAP7, \cite{Komatsu_2010}] results and \( A \) parameter. Some relevant works in other phenomenological way can be found in \cite{Wang_2003, Wei_2010b, Costa_2011}.

For cosmological observations, it is well known that SNe Ia, CMB and BAO use the distance scale \( (e.g., d_L, R, A) \) measurement to determine the cosmological parameters. However, we need to integrate the Hubble parameter to get the distance scale. The integral cannot take the fine structure of \( H(z) \) into consideration and lose some important information compiled in it. Therefore, it is more rewarding to investigate the observational \( H(z) \) data directly \cite{Cao_2011}.

The observational Hubble parameter depends on the differential age as a function of redshift \( z \) in the form
\[
H(z) = -\frac{1}{1 + z} \frac{dz}{dt}.
\]
\cite{Jimenez_2003} demonstrated the feasibility of the method by applying it to a \( z \sim 0 \) sample. \cite{Simon_2005} determined nine \( H(z) \) data in the range \( 0 \leq z \leq 1.8 \) by using the differential ages of passively evolving galaxies determined from the Gemini Deep Deep Survey [GDDS, \cite{Abraham_2004}] and archival data \cite{Treu_2001, Nolan_2003a, Wei_2007b} used the nine observational \( H(z) \) data to constrain the interacting dark energy models, and some other relevant works using these \( H(z) \) data for cosmological constraint include \cite{Samushia_2006, Lazkoz_2007, Yi_2007, Wu_2007a, Kurek_2008, Sen_2008, Zhang_2008, Xu_2008, Lin_2009, Zhai_2010}. Recently, \cite{Stern_2010} revisited the \( H(z) \) data at 11 different redshifts from the differential ages of red-envelope galaxies. On the other hand, \cite{Gaztañaga_2009} determined other two Hubble parameter data at \( z = 0.24 \) and \( z = 0.43 \) from observations of BAO peaks. \cite{Cao_2011} used these newly observational \( H(z) \) data to constrain the Interacting Dark Matter (IDM) scenario, and some other works for cosmological constraint can be found in \cite{Wang_2009, Gongs_2010, Liang_2010, Liang_2011, Xu_2010, Ma_2011}. For recent review of the observational \( H(z) \) data, see e.g. \cite{Zhang_2011}.

In this paper, we investigate observational constraints on the simple phenomenological interacting scenario by performing a Markov Chain Monte Carlo (MCMC) analysis. For cosmological observations, we focus on the newly \( H(z) \) data from the differential ages of red-envelope galaxies \cite{Stern_2010} and observations of BAO peaks
In order to break the degeneracy of model parameters, we combine the \( H(z) \) data with the CMB observation from the WMAP7 results \cite{Komatsu:2010fb}, and the BAO distance ratio \( (d_z) \) from the spectroscopic Sloan Digital Sky Survey (SDSS) data release 7 (DR7) galaxy sample \cite{Percival:2010ca}. For examining the role of the \( H(z) \) data played in cosmological constraints, we also add the newly revised Union2 set which consists of 557 SNe Ia \cite{Amanullah:2010vv, Gaztañaga:2009wd}. This paper is organized as follows: In section 2 we introduce the observational data including \( H(z) \), BAO, CMB, as well as SNe Ia. In section 3 we perform a Markov Chain Monte Carlo analysis spanning the full parameter space of the model using different data sets to constrain the phenomenological interacting model. Finally, we summarize the main conclusions in Section 4.

## 2 Observational Data

In order to break the degeneracy of model parameters, we combine the \( H(z) \) data with the CMB observation from the WMAP7 results \cite{Komatsu:2010fb}, the BAO observation from the SDSS DR7 galaxy sample \cite{Percival:2010ca}, the CMB data sets to constrain the phenomenological interacting model. We also add the newly revised Union2 set which consists of 557 SNe Ia \cite{Amanullah:2010vv, Gaztañaga:2009wd} to examine the role of the \( H(z) \) data played in cosmological constraints.

For the observational \( H(z) \) data, we adopt the 11 data obtained from the differential ages of red-envelope galaxies \cite{Stern:2010iz}, and two data at \( H(z = 0.24) = 76.69 \pm 3.61 \text{Mpc}^{-1} \), and \( H(z = 0.43) = 86.45 \pm 4.96 \text{Mpc}^{-1} \) determined from observations of BAO peaks \cite{Gaztañaga:2009wd}. The \( \chi^2 \) value of the \( H(z) \) data can be given by

\[
\chi^2_H = \sum_{i=1}^{13} \frac{[H_i(z_i) - H_{obs}(z_i)]^2}{\sigma_{H,i}^2},
\]

where \( \sigma_{H,i} \) is the 1σ uncertainty in the \( H(z) \) data.

For the CMB observation, we use the data set including the acoustic scale \( (l_0) \), the shift parameter \( (R) \), and the redshift of recombination \( (z_r) \). From the WMAP7 measurement, the best-fit values of the data set are \cite{Komatsu:2009kd}

\[
\begin{pmatrix}
\bar{P}_{CMB} & = & \begin{pmatrix}
l_0 \\
R \\
z_r
\end{pmatrix}
= & \begin{pmatrix}
302.09 \pm 0.76 \\
1.725 \pm 0.018 \\
1091.3 \pm 0.91
\end{pmatrix}.
\end{pmatrix}
\]

The 2σ uncertainty of the CMB observation can be expressed as \cite{Komatsu:2009kd}

\[
\chi^2_{CMB} = \Delta \bar{P}_{CMB}^T C_{CMB}^{-1} \Delta \bar{P}_{CMB},
\]

where \( \Delta \bar{P}_{CMB} = \bar{P}_{CMB} - \bar{P}_{CMB} \), and the corresponding inverse covariance matrix is

\[
C_{CMB}^{-1} = \begin{pmatrix}
2.305 & 29.698 & -1.333 \\
29.698 & 6825.270 & -113.180 \\
-1.333 & -113.180 & 3.414
\end{pmatrix}.
\]

The acoustic scale can be expressed as

\[
l_a = \frac{\Omega_k^{-1/2} \sin \theta_k \int_0^{z_+} \frac{dz}{E(z)} / H_0}{r_s(z_+)}
\]

where \( r_s(z_+) = \frac{H_0}{\Omega_k^{-1/2} \sin \theta_k \int_0^{z_+} \frac{dz}{E(z)} / H_0} \), is the comoving sound horizon at photo-dissolving epoch. The shift parameters can be expressed as

\[
R = \Omega_m^{1/2} \Omega_k^{-1/2} \sin \theta_k \int_0^{z_*} \frac{dz}{E(z)}
\]

The redshift of recombination is \( z_* = 1048 [1 + 0.00124(\Omega_m h^2)^{-0.738} (1 + g_1(\Omega_m h^2)^{0.92})] \), where \( g_1 = 0.0783(\Omega_m h^2)^{-0.238} [1 + 39.5(\Omega_m h^2)^{-0.763}]^{-1} \) and \( g_2 = 0.560 (1 + 21.1(\Omega_m h^2)^{1.81})^{-1} \) \cite{Hu:1998aw}.

For the BAO observation, we use the measurement of the BAO distance ratio \( (d_z) \) at \( z = 0.2 \) and \( z = 0.35 \) \cite{Percival:2010ca}. From SDSS data release 7 (DR7) galaxy sample, the best-fit values of the data set \( (d_{0.2}, d_{0.35}) \) are \cite{Percival:2010ca}

\[
P_{BAO} = \begin{pmatrix}
d_{0.2} \\
d_{0.35}
\end{pmatrix} = \begin{pmatrix}
0.1905 \pm 0.0061 \\
0.1097 \pm 0.0036
\end{pmatrix}.
\]

The 2σ value of the BAO observation from SDSS DR7 can be expressed as \cite{Percival:2010ca}

\[
\chi^2_{BAO} = \Delta P_{BAO}^T C_{BAO}^{-1} \Delta P_{BAO},
\]

where the corresponding inverse covariance matrix is

\[
C_{BAO}^{-1} = \begin{pmatrix}
30124 & -17227 \\
-17227 & 86977
\end{pmatrix}.
\]

The BAO distance ratio can be expressed as

\[
d_z = \frac{r_s(z_d)}{D_V(z_{BAO})},
\]

where the distance scale \( D_V \) is given by \cite{Eisenstein:2005su}

\[
D_V(z_{BAO}) = \frac{1}{H_0} \left[ \frac{z_{BAO}}{E(z_{BAO})} \left( \int_0^{z_{BAO}} \frac{dz}{E(z)} \right)^2 \right]^{1/3},
\]

and \( r_s(z_d) \) is the comoving sound horizon at the drag epoch, where \( z_d = \{1291(\Omega_m h^2)^{0.251} / [1 + 0.659(\Omega_m h^2)^{0.825}] \} \left[ 1 + b_1(\Omega_m h^2)^{0.52} \right] \), where \( b_1 = 0.313(\Omega_m h^2)^{-0.419} [1 + 0.607(\Omega_m h^2)^{0.674}]^{-1} \) and \( b_2 = 0.238(\Omega_m h^2)^{0.223} \) \cite{Eisenstein:1998hr}.

SNe Ia provide the most direct indication of the accelerated expansion of the universe. We add SNe Ia data to examine the role of the \( H(z) \) data played in cosmological constraints. Recently, the Supernova Cosmology Project (SCP) collaboration have released their Union2 compilation which consists of 557 SNe Ia \cite{Amanullah:2010vv}, which have been used to constrain cosmological models in \cite{Wong:2010cw, Xu:2010vu, Liang:2011}). The distance modulus of SN Ia can be given by

\[
\mu = 5 \log(d_L/\text{Mpc}) + 25,
\]

where the luminosity distance can be calculated as \( d_L = [c(1 + z) / H_0] \int_0^z dz / E(z) \). In the calculation of the likelihood from SNe Ia, we have marginalized the nuisance parameter \( \Omega_k = 0.273 \pm 0.020 \) \cite{DiPietro:2009}. The SN Ia data can be given by

\[
\chi^2_{SN} = A - B^2 \frac{C}{2} + \frac{\ln(C)}{2},
\]

where \( A = \sum_i (\mu_{\text{data}} - \mu_{\text{th}})^2 / \sigma_{\mu,i}^2 \), \( B = \sum_i (\mu_{\text{data}} - \mu_{\text{th}})^2 / \sigma_{\mu,i}^2 \), \( C = \sum_i 1 / \sigma_{\mu,i}^2 \), and \( \sigma_{\mu,i} \) is the 1σ uncertainty of the SNe data.
3 CONSTRAINT ON THE PHENOMENOLOGICAL INTERACTING SCENARIO

The model parameters are determined by applying the maximum likelihood method of $\chi^2$ fitting by using the Markov Chain Monte Carlo (MCMC) method. The total $\chi^2$ with the joint data of $H(z)$+CMB+BAO+SNe can be given by

$$\chi^2 = \chi^2_H + \chi^2_{BAO} + \chi^2_{CMB} + \chi^2_{SNe}.$$  \hspace{1cm} (19)

In adopting the MCMC approach, we generate a chain of sample points distributed in the parameter space according to the posterior probability by using the Metropolis-Hastings algorithm with uniform prior probability distribution, and then repeat this process until the established convergence accuracy can be satisfied. Our code is based on CosmoMC (Lewis & Bridle 2002).

We show the 1-D probability distribution of each parameter ($w_X$, $\xi$, $\Omega X_0$, $\Omega m_0$, $H_0$) and 2-D plots for parameters between each other for the phenomenological interacting scenario with $H(z)$ in Fig. 1. The best-fit values of the model parameters are $\Omega m_0 = 0.32^{+0.12}_{-0.19}$, $w_X = -1.34^{+0.58}_{-0.66}$ and $\xi = 3.72^{+1.27}_{-1.42}$, which show that the $H(z)$ data only can not tightly constrain the model parameters. Fitting results from the joint data of $H(z)$+CMB+BAO are given in Fig 2 with the best-fit values $\Omega m_0 = 0.26^{+0.04}_{-0.03}$, $w_X = -1.11^{+0.27}_{-0.27}$ and $\xi = 3.35^{+0.68}_{-0.92}$, which show that when combined to CMB+BAO data, the $H(z)$ data can give more stringent constraints. For comparison, fitting results from SNe+CMB+BAO without $H(z)$ are given in Fig 3 with the best-fit values $\Omega m_0 = 0.27^{+0.03}_{-0.03}$, $w_X = -1.03^{+0.16}_{-0.14}$ and $\xi = 3.10^{+0.54}_{-0.92}$, which are in good agreement with those of $H(z)$+BAO+CMB data. The contours constrained with $H(z)+$SNe+BAO+CMB are shown in Fig 4 and the best-fit values are $\Omega m_0 = 0.27^{+0.02}_{-0.02}$, $w_X = -1.05^{+0.15}_{-0.14}$ and $\xi = 3.15^{+0.48}_{-0.50}$, which are in good agreement with those of $H(z)+$BAO+CMB and SNe+BAO+CMB. We present the best-fit values of parameters with 1-$\sigma$ and 2-$\sigma$ uncertainties of the phenomenological interacting scenario in Table I.

From Fig. 1 and Table I, it is shown that our results are more stringent and consistent with the constraint results obtained by combining previous SNe Ia data to BAO+CMB data (Guo et al. 2007; Chen et al. 2010; Wei 2010a). And the special case ($\xi = 3$, $w_X = -1$, corresponding to the ACDM with no interaction) is included at 1-$\sigma$ confidence level with the recent observational data; however, it is also shown that the constraints favor $\xi + 3w_X > 0$ for the phenomenological scenario, and indicate that the energy is transferred from dark matter to dark energy and the coincidence problem is quite severe, which is consistent with those obtained in Guo et al. (2007); Chen et al. (2010). We also find that the $H(z)$ data can give more stringent constraints on the phenomenological interacting scenario when combined to CMB and BAO observations. Comparing the SNe Ia data in the same way, we can find the confidence regions of $H(z)+BAO+CMB$ data are in good agreement with those of SNeIa+BAO+CMB data; this situation has also been noted in Zhai, Wan & Zhang (2010) for constraining on the ACDM and XCDM scenario. We also find the confidence regions of $H(z)+BAO+CMB$, SNe+BAO+CMB, and $H(z)+$SNe+BAO+CMB are consistent with each other. This situation is similar to that obtained in Cao, Zhu & Liang (2011) for constraining on the Interacting Dark Matter (IDM) scenario with $H(z)+$SNe data.

4 CONCLUSIONS

In this paper, we test the interacting dark energy scenario with a phenomenological scaling solution $\rho_X \propto \rho_m a^\xi$, which is proposed as a candidate to ease the coincidence problem of the concordance ACDM model. With the newly revised observational $H(z)$ data, the CMB observation from the WMAP7 results, the BAO observation from the SDSS Data Release and the Union2 SNeIa set, we obtain the best-fit values of the model parameters in the phenomenological interacting scenario, $\Omega m_0 = 0.27^{+0.02}_{-0.02}(1\sigma)^{+0.03}_{-0.03}(2\sigma)$, $\xi = 3.15^{+0.48}_{-0.50}(1\sigma)^{+0.72}_{-0.72}(2\sigma)$, and $w_X = -1.05^{+0.15}_{-0.14}(1\sigma)^{+0.21}_{-0.21}(2\sigma)$, which are more stringent and consistent with the previous constraint results (Guo et al. 2007; Chen et al. 2010; Wei 2010a).

Our results show that the ACDM model still remains a good fit to the recent observational data. However, the interaction that the energy transferring from dark matter to dark energy is slightly favored over the interaction from dark energy to dark matter, which is consistent with that obtained in Guo et al. (2007); Chen et al. (2010), therefore, the coincidence problem is quite severe in the phenomenological scenarios. When combined the $H(z)$ data with CMB and BAO observations, it is shown that the $H(z)$ data can give more stringent constraints on the phenomenological interacting scenario. In order to examine the role of
the $H(z)$ data played in cosmological constraints, we compared the SNe Ia data in the same way and find the constraints with $H(z)$+BAO+CMB, SNeIa+BAO+CMB, and $H(z)$+SNe+BAO+CMB combinations are consistent with each other. With a large amount of the observational $H(z)$ data in the future, it is reasonable to expect that the observational $H(z)$ data will play an increasingly important role in cosmological researches (Zhai, Wan & Zhang [2011: Ma & Zhang [2011]).

**ACKNOWLEDGMENTS**

We thank Lixin Xu for introducing the powerful program cosmoMCMC and Yu Pan for some calculations. We are also grateful to Yun Chen, Hao Wang, Xiaolong Gong, Xijing Zhu, Yan Dai, Fang Huang, Jing Ming, Kai Liao, Yubo Ma, Huihua Zhao and Dr. Yi Zhang for helpful discussions. This work was supported by the National Natural Science Foundation of China under the Distinguished Young Scholar Grant 10825313 and Grant 11073005, the Ministry of Science and Technology national basic science Program (Project 973) under Grant No.2007CB815401, the Fundamental Research Funds for the Central Universities and Scientific Research Foundation of Beijing Normal University.

**REFERENCES**

Abraham, R. G., et al. 2004, ApJ, 127, 2455

Aamannullah, R., et al. 2010, ApJ, 716, 712

Amedola, L. 2000, PRD, 62, 043511

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Table 1. The best-fit values of parameters $w_X$, $\xi$, $\Omega_{X0}$, $\Omega_{m0}$, and $H_0$ for the phenomenological scenario with the 1-σ and 2-σ uncertainties, as well as $\chi^2_{min}$, for the data sets $H(z)$, $H(z)$+BAO+CMB, SNe+BAO+CMB, and $H(z)$+SNe+BAO+CMB, respectively.

| Parameter | $H(z)$ | $H(z)$+BAO+CMB | SNe+BAO+CMB | $H(z)$+SNe+BAO+CMB |
|-----------|--------|----------------|-------------|---------------------|
| $w_X$     | $-1.34^{+0.58}_{-0.68}$ (1σ) $-0.79$ (2σ) | $-1.11^{+0.27}_{-0.32}$ (1σ) $-0.63$ (2σ) | $-1.03^{+0.19}_{-0.16}$ (1σ) $-0.24$ (2σ) | $-1.05^{+0.15}_{-0.21}$ (1σ) $-0.21$ (2σ) |
| $\xi$     | $3.87^{+0.52}_{-0.88}$ (1σ) $-1.98$ (2σ) | $3.32^{+0.88}_{-1.37}$ (1σ) $-1.32$ (2σ) | $3.10^{+0.54}_{-0.79}$ (1σ) $-0.79$ (2σ) | $3.15^{+0.48}_{-1.01}$ (1σ) $-0.72$ (2σ) |
| $\Omega_{X0}$ | $0.68^{+0.15}_{-0.15}$ (1σ) $-0.92$ (2σ) | $0.74^{+0.04}_{-0.04}$ (1σ) $-0.06$ (2σ) | $0.73^{+0.03}_{-0.04}$ (1σ) $-0.04$ (2σ) | $0.73^{+0.02}_{-0.04}$ (1σ) $-0.04$ (2σ) |
| $\Omega_{m0}$ | $0.32^{+0.15}_{-0.26}$ (1σ) $-0.26$ (2σ) | $0.28^{+0.03}_{-0.05}$ (1σ) $-0.05$ (2σ) | $0.27^{+0.03}_{-0.04}$ (1σ) $-0.04$ (2σ) | $0.37^{+0.02}_{-0.04}$ (1σ) $-0.03$ (2σ) |
| $H_0$     | $73.10^{+6.40}_{-5.84}$ (1σ) $7.83$ (2σ) | $72.09^{+5.90}_{-7.04}$ (1σ) $7.83$ (2σ) | $70.44^{+3.12}_{-4.43}$ (1σ) $7.83$ (2σ) | $70.96^{+2.52}_{-4.05}$ (1σ) $7.83$ (2σ) |

$\chi^2_{min}$ | 9.851 | 10.935 | 544.117 | 555.762 |

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Figure 2. The 2-D regions and 1-D marginalized distribution with the 1-σ and 2-σ contours of parameters $w_X$, $\xi$, $\Omega_{X0}$, $\Omega_{m0}$, and $H_0$ in the phenomenological interacting scenario, for the data sets $H(z)$, $H(z)$+BAO+CMB, SNe+BAO+CMB, and $H(z)$+SNe+BAO+CMB, respectively.

Figure 3. The 2-D regions and 1-D marginalized distribution with the 1-σ and 2-σ contours of parameters $w_X$, $\xi$, $\Omega_{X0}$, $\Omega_{m0}$, and $H_0$ in the phenomenological interacting scenario, for the data sets SNe+BAO+CMB.
Figure 4. The 2-D regions and 1-D marginalized distribution with 1- and 2-$\sigma$ contours of parameters $w_X$, $\xi$, $\Omega_X$, $\Omega_m$, and $H_0$ in the phenomenological interacting scenario, for the data sets $H(z)$+SN+BAO+CMB.

Armendariz-Picon, C., et al. 2001, PRD, 63, 103510
Astier, P. et al. 2006, A&A, 447, 31
Baldi, M. & Viel, M. 2010, MNRASL, 409, 1
Cai, R. G., & Su, Q. P. 2010, PRD, 81,103514
Caldwell, R., & Dave, R., & Steinhardt, P. J. 1998, PRL, 81,1582
Caldwell, R. R. 2002, PLB, 545, 23
Caldwell, R. R., Kamionkowski, M., & Weinberg, N. N. 2003, PRL, 91, 071301
Cao, S., Zhu, Z.-H., & Liang, N. 2011, A&A, 529, A61 [arXiv:1011.4848]
Cervantes-Cota, J. L., Putter, R. D., & Linder, E. V. 2010, JCAP, 12, 019
Chen, Y., Zhu, Z.-H., Alcaniz, J. S., & Gong, Y. G. 2010, ApJ, 711, 439
Chen, X. M., Gong, Y. G., & Saridakis, E. N. 2009, JCAP, 0904, 001
Chiba, T. 2002, PRD, 66, 063514
Chimento, L. P., Jakubi, A. S., Pavon, D., & Zimdahl, W. 2003, PRD, 67, 083513
Costa, F. E. M., & Alcaniz, J. S. 2010, PRD, 81, 043506
Dalal, N., et al. 2001, PRL, 87, 141302
Di Pietro, E., & Claeskens, J. F. 2003, MNRAS, 341, 1299
Eisenstein, D., & Hu, W. 1998, ApJ, 496, 605
Eisenstein, D. J., et al. 2005, ApJ, 633, 560
Feng, B., Wang, X., & Zhang, X. 2005, PLB, 607, 35
Gaztañaga, E., Cabré, A., & Hui, L. 2009, MNRAS, 399, 1663
Gong, Y. G., et al. 2010, JCAP, 01, 019
Guo, Z. K., Cai, R. G., & Zhang, Y. Z. 2005, JCAP, 05, 002
Guo, Z. K., & Zhang, Y. Z. 2005, PRD, 71, 023501
Guo, Z. K., et al. 2007, PRD, 76, 023508
Hicken, M., et al. 2009, ApJ, 700, 1097 [arXiv:0901.4804]
Hu, W., & Sugiyama, N. 1996, ApJ, 471, 542
Jimenez, R., Verde, L., Treu, T., & Stern., D. 2003, ApJ, 593, 622
Komatsu, E. et al. 2009, ApJS, 180, 330
Komatsu, E., et al. [arXiv:1001.4538]
Kurek, A., & Szydlowski, M. 2008, ApJ, 675, 1
Lazkoz, R., & Majorotto, E., [arXiv:0704.2699]
Lewis, A., & Bridle, S. 2002, PRD, 66, 103
Liang, N., Wu, P., & Zhang, S. N. 2010, PRD, 81, 083518
Liang, N., Wu, P., & Zhu, Z.-H. 2011, RAA, in press [arXiv:1006.1105]
Liang, N., & Zhu, Z.-H. 2011, RAA, 11, 497
Liang, N., Xu, L. X., & Zhu, Z.-H. 2011, A&A, 527, A11
Lin, H., et al. 2009, MPLA, 24, 1699
Ma, C., & Zhang, T. J. 2011, ApJ, 730, 74
Nolan, L. A., Dunlop, J. S., Jimenez, R., & Heavens, A. F. 2003, MNRAS, 341, 464
Nolan, P. L., Tompkins, W. F., Grenier, I. A., & Michelson, P. F. 2003, ApJ, 597, 615
Pavon, D., Sen, S., & Zimdahl, W. 2004, JCAP, 0405, 009
Percival, W. J. et al. 2010, MNRAS, 401, 2148
Perlmutter, S., et al. 1999, ApJ, 517, 565
Ratra, B., & Peebles, P. E. J. 1988, PRD, 37, 3406
Riess, A. G. et al. 1998, AJ, 116, 1009
Samushia, L., & Ratra, B. 2006, ApJ, 650, L5
Sen, A. A., & Scherrer, R. J. 2008, PLB, 659, 457
Simon, J., Verde, L., & Jimenez, R. 2005, PRD, 71, 123001
Spergel, D. N., et al. 2003, ApJS, 148, 175
Spergel, D. N., et al. 2007, ApJS, 170, 377
Stern, D., et al. 2010, JCAP, 02, 008
Tegmark, M. et al. 2004, ApJ, 606, 702
Treu, T., et al. 2001, MNRAS, 326, 221
Treu, T., et al. 2002, ApJL, 564, L13
Wang, P., & Meng, X. H. 2005, Class. Quant. Grav. 22, 283
Wang, T. S. & Wu, P. 2009, PLB, 678, 32
Wei, H., & Cai, R. G. 2006, PRD, 73, 083002
Wei, H., & Zhang, S.-N. 2007a, PLB, 644, 7
Wei, H., & Zhang, S.-N. 2007b, PLB, 654, 139
Wei, H. 2010a, PLB, 691, 173
Wei, H. 2010b, Nucl. Phys. B in press [arXiv:1008.4968]
Wei, H. 2010c, JCAP, 08, 020
Wu, P., & Yu, H. W. 2007, PLB, 644, 16
Wu, P. X., & Yu, H. W. 2007, JCAP, 0703, 015
Xu, L. X., Chang, B. R., & Liu, H. Y., 2008, MPLA, 23, 193
Xu, L. X., Zhang, C. W., Chang, B. R., & Liu, H. Y., 2008, MNRAS, 184284
Zimdahl, W., Pavon, D., & Chimento, L. P. 2001, PLB, 521, 133

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