The SUSY Flavor Problem, Proton Decay 
and 
Discrete Family Symmetry *

Etsuko Itou\textsuperscript{a}, Yuji Kajiyama\textsuperscript{b} and Jisuke Kubo\textsuperscript{c}

\textsuperscript{a} Department of Physics, Graduate School of Science, Osaka University, Osaka 560-0043, Japan
\textsuperscript{b} National Institute of Chemical Physics and Biophysics, Tallinn 10143, Estonia
\textsuperscript{c} Institute for Theoretical Physics, Kanazawa University Kanazawa 920-1192, Japan

Abstract

We consider a supersymmetric extension of the standard model, which possess a family symmetry based on a binary dihedral group $Q_6$, and investigate the consequences of the family symmetry on the mixing of fermions, FCNCs and the stability of proton.

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The classification of finite groups has been completed 1981 by Gorenstein, about 100 years later than the case of the continues group. Therefore, we believe it is worthwhile to look at finite groups more in detail and find applications into particle physics. In fact there are renewed interests [1, 2] in finite groups as such as $S_3$ or $A_4$ to explain the large mixing of neutrinos.

In [3] it has been motivated to obtain a mass matrix of the nearest neighbor type [4] from a non-abelian discrete family symmetry. We found that this is in fact possible and that the smallest group is the binary dihedral group $Q_6$, which is the covering group of the smallest non-abelian group $S_3$. There are two two-dimensional irreps of $Q_6$: $2_1$ is pseudo-real and $2_2$ is a real irrep. There are also two real one-dimensional irreps. $1_{+0}, 1_{+2}$, and two complex one-dimensional irreps. $1_{-1}, 1_{-3}$, while $1_{+0}$ is the true singlet [5]. Table I shows the $Q_6$ assignment. This is an alternative assignment to the one given in [3]. We consider this assignment, because it can explain the maximal mixing of the atmospheric neutrinos. (The leptonic sector is basically the same as that of [6].) Further, we assume that CP is spontaneously broken. It is possible to construct a Higgs superpotential for which CP can be spontaneously broken.

In the quark sector we have 9 independent real parameters to describe 6 quark masses and 4 parameters of the CKM matrix. So there is one real prediction, which can be displayed in different planes. The absolute value of $V_{td}$ over $V_{ts}$, for instance, is predicted to be $0.23 \pm 0.02$ by the model. This can be directly compared with the experimental values $0.16 \pm 0.04$ [8] and $0.208 \pm 0.07$ [7] because the oscillation frequency in the $B_s - \bar{B}_s$ system has been measured at Tevatron this year [9]. Fig. 1 shows the prediction of the model in the $V_{ub} - \sin 2\phi_1(\beta)$ plane.

In the leptonic sector, there are only 7 independent real parameters, of which one is a CP phase, to describe 6 masses of the charged leptons and neutrinos, three angles of the

![Figure 1](attachment:image.png)
neutrino mixing matrix along with one Dirac CP phase and three Majorana phases. So there are 12-6=6 predictions. First: the model predicts the neutrino mass spectrum is inverted. Second: there exist only one independent CP phase, and the absolute scale of the neutrino mass depends on the independent phase. Third: $|U_{e3}| = m_e / \sqrt{2} m_\mu + O(10^{-5}) \approx 0.0034$, and $|U_{\mu 3}| = 1 / \sqrt{2} + O(10^{-5})$. Fourth: since there exits only one independent phase, the average neutrino mass appearing in neutrinoless double beta decays can be predicted as a function of the independent phase. For a wide range of the independent phase, the average mass stays at the minimum $(0.034 - 0.069)$ eV [6].

Now we come to proton decay [10]. As we know, the lowest dimension of the proton-decay-leading operators is five, if we assume $R$ parity [11]. With $R$ parity, there are two types of operators, the left-handed and right-handed types [11]. If there are no further constraints, there will be 27 independent operators for each. These operators can be generated by GUTs or by some unknown Planck scale physics. Here we assume that unknown Planck scale physics generates baryon number violating operators and respects the $Q_6$ family symmetry. Then it turns out that the number of the independent left-handed operators reduces to ONE and in the case of the right-handed operators to TWO. Moreover, the left-handed operator

$$\sum_{I=1,2} Q_I Q_I Q_3 L_3$$

(see Table 1) gives the dominant contribution, so that in the first approximation there is only one coupling constant. The reason that the left-handed operator gives the dominant contribution to proton decay is basically the same as the in the usual case [12] without family symmetry. The usual argument [12] is based on the fact that the diagrams with the neutral gaugino loops are negligibly suppressed if all the squarks masses are degenerate. The degeneracy of the squark masses is needed to suppress FCNCs. This cancellation mechanism does not work for the diagrams with wino loops. Therefore, the left-handed operators, which can be dressed with the wino loops, are dominant over the right-handed operators. As we will see later on, the soft scalar mass matrices are diagonal, and the first and second elements are equal by the family symmetry. So, the almost degeneracy of the squaks masses is automatic thanks to the family symmetry.

Here we would like to explain some structure of the left-handed operator (1). The quark superfield $Q_3$ which is singlet of $Q_6$ contains only little component of the u and d quarks, which can be read off from $|U_{uL}| \approx 0.0023$, where $U_{uL}$ is the mixing matrix of the left-handed quarks. This gives an overall suppression of $10^{-6}$ in the decay mode into a charged lepton. The rate of the decay mode into a charged lepton is controlled by $(U_{eL})_{\tau 1} = 1$ and $(U_{eL})_{\tau 2} = m_e / m_\mu$, where $U_{eL}$ is the mixing matrix of the left handed leptons. From this observation we conclude that the branching fraction for the decay into a muon is five orders of magnitude smaller than the decay into a positron. Moreover, this small number is nothing but $|U_{e3}|$.

Since there is basically only one coupling constant for the left-handed operator (1) in our model, the relative branching fractions can be fixed in the first approximation, and can be compared with the result of the minimal $SU(5)$ supergravity model [13, 14]. In the $SU(5)$ case, it is assumed that the up type quarks are mass eigenstates from the beginning. (If
this type of assumptions are removed, one can find different conclusions\cite{15}. We find that there are considerable differences\cite{13,14}.

We believe that the effect of supersymmetry breaking appears as soft-supersymmetry breaking terms in our 4D Lagrangian. Moreover, one has to highly fine tune these parameters so that they do not cause problems with experimental observations on the FCNC processes and CP-violation phenomena. There are several approaches to overcome this problem. Here we consider a mechanism which is based on the family symmetry $Q_6$. In the present model, the soft scalar mass matrices are diagonal by the family symmetry, and the first two entries are the same by the same symmetry. Further, the left-right soft mass matrices have the same structure as the fermion mass matrices. (This structure is the same as the $S_3$ invariant supersymmetric model of\cite{16}.) Since we assume that CP is spontaneously broken, these soft parameters are real. We found that the family symmetry and the assumption of the spontaneous CP violation interplay in such a way that the CP phases of the $\delta$’s cancel exactly, where $\delta$’s\cite{17} are dimensionless parameters measuring the deviation of the corresponding soft parameters from the universal ones. In this way we can satisfy the most stringent constraints coming from the EDMs. We have calculated the deltas $\delta$’s and compared with the experimental bounds given in\cite{18}. Table 2 shows some examples of the case of the

| $|\langle \delta e_1 \rangle_{LL}$ | Exp. bound | $Q_6$ Model |
|-----------------|------------|-------------|
| $(|\langle \delta e_2 \rangle_{LL}|, |\langle \delta e_3 \rangle_{LL}|)$ | $4.0 \times 10^{-5}$ $\tilde{m}_\ell^2$ | $4.9 \times 10^{-3}$ $\Delta a_L$ |
| $(|\langle \delta e_1 \rangle_{RR}|, |\langle \delta e_2 \rangle_{RR}|)$ | $9 \times 10^{-4}$ $\tilde{m}_\ell^2$ | $8.4 \times 10^{-8}$ $\Delta a_R$ |
| $(|\langle \delta e_1 \rangle_{LR}|, |\langle \delta e_2 \rangle_{LR}|)$ | $8.4 \times 10^{-7}$ $\tilde{m}_\ell^2$ | $5 \times 10^{-6}$ $\Delta a_L$ |
| $(|\langle \delta e_1 \rangle_{LL}|, |\langle \delta e_3 \rangle_{RR}|)$ | $2 \times 10^{-2}$ $\tilde{m}_\ell^2$ | $1.7 \times 10^{-5}$ $\Delta a_L$ |
| $(|\langle \delta e_1 \rangle_{RR}|, |\langle \delta e_3 \rangle_{RR}|)$ | $3 \times 10^{-1}$ $\tilde{m}_\ell^2$ | $5.9 \times 10^{-2}$ $\Delta a_R$ |
| $(|\langle \delta e_1 \rangle_{LR}|, |\langle \delta e_3 \rangle_{LR}|)$ | $1.7 \times 10^{-2}$ $\tilde{m}_\ell^2$ | $3 \times 10^{-7}$ $\Delta a_L$ |

TABLE II: Experimental bounds on $\delta$’s and the theoretical values in $Q_6$ model, where the parameter $\tilde{m}_\ell$ denote $m_\ell/100$ GeV. See\cite{10} for the quark sector.

leptonic sector. The capital deltas $\Delta$’s are free dimensionless parameters which can not be constrained by the family symmetry. The small numbers appearing in the right column have approximate analytic expressions. For instance, $4.9 \times 10^{-3}$ is $m_e/m_\mu$, which is $\sqrt{2}|U_{e3}|$. In the quark sector, apart from some cases, the soft supersymmetry breaking parameters need not be fine tuned in the present model to satisfy the experimental constraints coming from FCNCs and CP violations (see\cite{10} for more details).

To conclude, we could say that the smallness of the three apparently independent quantities has the same origin in the model; the smallness of $|U_{e3}|$, the suppression of $\mu \rightarrow e\gamma$, and the ratio of proton decay branching fractions into a $\mu$ and a $e^+$. This is a consequence
of a low energy flavor symmetry.

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