A Stringy Solution to the FCNC Problem*

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Abstract

The solution to the supersymmetric FCNC problem may not have a simple field theory interpretation. Here I advocate a possibility based on string selection rules. The requirement of non–trivial Yukawa structures restricts the form of the soft terms such that the FCNC problem often does not arise.

1. Introduction

Generically, supersymmetric contributions to flavor changing neutral currents (FCNC) are orders of magnitude too large, which is known as the supersymmetric FCNC problem [1]. This problem appears in models with a light (< 1 TeV) SUSY spectrum and arbitrary flavor structure of the soft terms. Simple field theory arguments suggest the following common remedies for the problem:

- universality
- decoupling
- alignment

The first possibility implies that the soft terms are flavor–blind, the second – that the first two sfermion generations are very heavy, and the third – that the

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soft and SM flavor structures are aligned. Any of these options leads to sufficient suppression of the unwanted effects, yet it is quite nontrivial to embed these mechanisms in string theory. For example, universality of the soft terms would imply a special point in moduli space and one would have to argue that the Universe rests precisely there. The decoupling option assumes a rather peculiar SUSY spectrum and so on. This is not to say that these solutions cannot be realized, rather they leave one wondering if there could be options suggested by string theory itself and which may not have a simple bottom–up interpretation. I will try to argue that such an option exists and it is based on

- string selection rules

The basic idea is that all flavor effects are interrelated in string theory and the FCNC problem should be analyzed in settings which allow for realistic Yukawa matrices,

\[
\text{Yukawa couplings} \leftrightarrow \text{FCNC}
\]

In this framework, the form of the soft terms is restricted and the FCNC problem often does not arise.

### 2. String Yukawa couplings

Replication of chiral fermion families and generation of non–trivial flavor structures have been successfully realized in heterotic and intersecting brane models. The fermion mass hierarchy appears naturally if the matter fields are twisted, i.e. localized at special points in the compactified 6D space. The Yukawa coupling among three fields located at three different points is exponentially suppressed by the area of the triangle formed by these points [2]. Then order one variations in the area translate into a large hierarchy among the Yukawa couplings.

In heterotic orbifold compactifications, twisted states are localized at orbifold fixed points. Not all fixed points can couple together, they have to satisfy string selection rules [2]. Consider three fields \( S_a \) \((a=1,2,3)\) belonging to twisted sectors \( \theta_a \) and localized at fixed points \( f_a \). That is, the boundary conditions for the corresponding closed string coordinates are given by

\[
X_a(\tau, \sigma = \pi) = \theta_a X_a(\tau, \sigma = 0) + l_a
\]

with \( l_a \) being a torus lattice vector in the conjugacy class associated with the fixed point \( f_a \). The Yukawa couplings \( S_1 S_2 S_3 \) are allowed only if (see e.g. [3])

\[
\theta_1 \theta_2 \theta_3 = 1
\]
and
\[(1 - \theta_1)f_1 + (1 - \theta_2)f_2 + (1 - \theta_3)f_3 = 0.\]

This is required by the existence of classical solutions to the string equations of motion with proper boundary conditions (instantons), which are responsible for correlators of the twist fields. These constraints can be derived from the monodromy condition \(\int_C dz \frac{\partial X}{\partial z} + \int_C d\bar{z} \frac{\partial X}{\partial \bar{z}} = 0\) with \(z = \exp(\tau + i\sigma)\), \(X(z, \bar{z})\) being a classical solution to the string equations of motion in the presence of the twist fields, and \(C\) is a contour encircling the positions of all three twist fields. If this condition is not satisfied, the correlator of the twist fields vanishes and the corresponding Yukawa coupling is zero. Physically, the reason for this selection rule is that the strings should have proper boundary conditions to be able to join or split, i.e. participate in interactions.

For the allowed Yukawa couplings, the result depends on the T-modulus:
\[Y_{123} \sim e^{-\alpha_{123} T}\]
with \(\alpha_{123} \leq O(1)\). If all \(S_a\) are located at the same fixed point, \(\alpha_{123} = 0\) and the coupling is order one, otherwise it is exponentially suppressed. This is the origin of the fermion mass hierarchy. As explicit calculations show, semi–realistic Yukawa matrices can be produced if different generations of one matter field belong to the same twisted sector. To illustrate this point, let us consider a few examples.

(1). \(Z_3\). This orbifold has nine moduli associated with the sizes of the \(T^2\)-tori and angles between them. If the matter fields are twisted, the Yukawa couplings are functions of these moduli which provides enough freedom to fit the fermion masses\(^1\) [4]. There is only one twisted sector \(\theta\) and the allowed coupling is of the type \(\theta \theta \theta\). Note that if one assumes that some of the fields are untwisted, the Yukawa couplings would essentially be zero or one. Thus, the desired moduli dependence will be lost.

(2). \(Z_4\). There are two twisted sectors \(\theta, \theta^2\) and 4 moduli which enter the Yukawa couplings \(\theta \theta \theta^2\). A satisfactory fit to the quark masses can be obtained if different generations belong to the same twisted sector, e.g. the quark, lepton and Higgs doublets are in \(\theta\), while the singlets are in \(\theta^2\) [6]. If one were to place, for instance, one generation of the quarks doublets in \(\theta^2\), the corresponding Yukawa coupling \(\theta \theta^2 \theta^2\) would be prohibited and the success of the fit would be lost.

(3). \(Z_6\). This orbifold has three twisted sectors \(\theta, \theta^2, \theta^3\) and allows for flavor non–diagonal Yukawa couplings \(\theta \theta^2 \theta^3\). A satisfactory fit to the quark masses and mixings is obtained for the Higgses in \(\theta\), the doublets in \(\theta^2\) and the singlets in \(\theta^3\) [7]. This is also required by the presence of physical CP violation at the renormalizable level, i.e. a non–zero Jarlskog invariant [8]. If some of the generations

\(^1\)The quark mixings can be generated by non–renormalizable operators [4] or by introducing 3 generations of Higgses [5].
Figure 1: Family replication in intersecting brane models. The picture represents a $T^2$ torus with multiple brane intersections corresponding to three generations of the quark doublets.

were reassigned to a different twisted sector, the Yukawa matrices would contain many zeros and the Jarlskog invariant as well as some mixing angles would vanish.

There are, of course, many more similar examples. The main message is that a satisfactory fit to the quark masses or mixings at the renormalizable level appears to require that different fermion generations belong to the same twisted sector\(^2\).

This result also applies to semirealistic intersecting brane models [11]. The chiral fermions are localized at intersections of the branes which support the Standard Model gauge groups. Different generations correspond to different intersections of the same branes (Fig.1). Their Yukawa couplings are again exponentially suppressed by the area of the triangle formed by the Higgs, the left–handed and the right–handed quark vertices\(^3\). Realistic quark masses and mixings can be obtained in such settings [13]. What is important is that the existing mechanism of family replication implies that different generations differ only in their distances to the location of the Higgs, but are the same in other aspects. In particular, the angles between the branes are the same at different intersections (Fig.1) leading to the same Kähler potential. Thus we reach the same conclusion as in the heterotic string case.

3. Soft terms and FCNC

What are the implications of the above conclusion for the soft terms? To answer this question, one has to analyze the Kähler potential,

$$K = \hat{K} + K_{\bar{\alpha} \bar{\beta}} \phi^{\bar{\alpha}} \phi^{\bar{\beta}} + \ldots, \quad (1)$$

where $\phi^\alpha$ are matter fields. In the heterotic string, twisted states have the following Kähler metric,

$$K_{\bar{\alpha} \bar{\beta}} = \delta_{\bar{\alpha} \bar{\beta}} (T + \bar{T})^{n_n}, \quad (2)$$

\(^2\)This is also favored by the mechanisms of family replication in the heterotic string (see e.g. [9], [10]).

\(^3\)For a discussion of the selection rules, see [12].
where $T$ is an overall modulus and $n_\alpha$ is a modular weight. Diagonality of the Kähler metric is enforced by the space group selection rules since different fixed points have different quantum numbers with respect to this group. The modular weights are determined by the twisted sector the state belongs to. For non–oscillator states, the modular weight only depends on whether the corresponding twist rotates all three complex planes in the compactified 6D space or just two of them [14]:

$$n = -1, \text{ untwisted}$$
$$n = -2, \text{ twisted with 3 planes rotated}$$
$$n = -1, \text{ twisted with 2 planes rotated}.$$  \hspace{1cm} (3)

In semirealistic models, quark fields usually correspond to non–oscillator states, so oscillators will not be discussed further.

In intersecting brane models, a similar expression holds and the role of the modular weights is played by the angles between the relevant branes $\pi \nu_\alpha$ [15]:

$$K_{\tilde{\alpha} \tilde{\beta}} \propto \delta_{\tilde{\alpha} \tilde{\beta}} (T + \bar{T})^{-\nu_\alpha}.$$  \hspace{1cm} (4)

What these expressions tell us is that (1) the Kähler potential is flavor–diagonal, (2) the diagonal entries are equal if different generations belong to the same twisted sector. This is intuitively clear from the intersecting brane picture (Fig.1): localized fields couple only to themselves in the Kähler potential (to leading order) and these couplings are generation–independent since the angles between the branes are the same.

The soft terms relevant to the FCNC problem depend on the Kähler potential and the Yukawa couplings $Y_{\alpha \beta \gamma} \equiv Y_{Q_L, Q_R, H}$ [16],

$$m^2_\alpha = (m^2_{3/2} + V_0) - \bar{F}_m^m F^m \partial_m \partial_n \log K_\alpha,$$

$$A_{\alpha \beta \gamma} = F^m \left[ \hat{K}_m + \partial_m \log Y_{\alpha \beta \gamma} - \partial_m \log (K_\alpha K_\beta K_\gamma) \right],$$  \hspace{1cm} (5)

where a diagonal Kähler metric has been assumed, $K_{\tilde{\alpha} \tilde{\beta}} = \delta_{\tilde{\alpha} \tilde{\beta}} K_{\beta}$. $m^2_\alpha$ are the squark masses and $A_{\alpha \beta \gamma}$ are the trilinear soft couplings. $F^m$ is an F–term associated with a hidden superfield $h^n$ and subscripts denote differentiation with respect to these fields. The gravitino mass and the “vacuum energy” are denoted by $m_{3/2}$ and $V_0$, respectively.

Eqs.(2),(4) and (5) tell us that the squark masses are generation–independent as long as different generations belong to the same twisted sector. This means that the FCNC problem is absent. Note that the Yukawa and the A–term flavor structures can still be complicated [17]. The FCNC induced by the A–terms are small [18] since the corresponding SUSY contributions are suppressed by the quark masses. This is illustrated in Fig.2.

Thus, the correlation between the Yukawa structure and the Kähler potential in string models suppresses FCNC to a desired level. The above considerations apply
mostly to the quark sector. In the lepton sector, FCNC constraints require the charged slepton masses to be universal and the A–terms to be diagonal [18]. The latter condition is specific to the lepton sector and implies that the corresponding Yukawa matrix is diagonal. This can be implemented in many orbifold models with diagonal selection rules for renormalizable couplings, however the analysis is obscured by the unknown origin of the neutrino masses and mixings (see e.g. [19]).

4. Relaxing the assumptions

The conclusion that the FCNC are suppressed is based on the analysis of renormalizable couplings in the superpotential. One may wonder if non–renormalizable couplings can affect it. There are two types of such contributions:

(i) non–renormalizable couplings are only perturbations over the renormalizable Yukawas

(ii) non–renormalizable couplings lead to qualitative changes such as the twisted sector assignment

In the former case the above discussion applies, whereas in the latter the conclusions may change.

In general, some of the Yukawa entries may only be allowed at the non–renormalizable level. For instance, off–diagonal entries of the Yukawa matrices in $Z_3$ are forbidden at the renormalizable level (see, however, [5]) and are induced by higher dimensional operators involving singlet fields, e.g. of the type $\theta\theta\theta(\theta)^9$ [4]. In many cases, the presence of such operators does not change the conclusion that different generations should belong to the same twisted sector in order to get a good fit to the quark masses and mixings. For instance, in $Z_3$ and $Z_7$ there is only one twisted sector, so this condition will be satisfied as long as the fields are twisted. Yet, it is also possible to evade this conclusion in some cases.

Consider splitting the third family from the other two. For example, in $Z_6$ one can place the first two families in the $\theta$ twisted sector and the third family in the untwisted sector [20]. In this case, the Kähler metric remains diagonal, yet the modular weights for the light and heavy families become different (Eq.(3)). That means, the squark masses at the GUT scale have the structure

$$m_2^2 = \text{diag}(m^2, m^2, M^2),$$

with $m^2 - M^2 \sim m^2$. What is relevant for the FCNC is off–diagonal elements of the squark mass matrix at low energies in the physical (sCKM) basis, i.e. the basis in which the quark masses are diagonal. To translate the above GUT boundary condition into these quantities, one needs to (1) RG–run $m_2^2$ to low energies, (2) rotate $m_2^2$ to the sCKM basis. Both of these effects are very important. First, the gluino renormalization effect increases $m_2^2$ by about an order of magnitude, $m_2^2 \rightarrow m_2^2 + 8m_2^2$ [21]. Second, the rotation to the sCKM basis induces the off
diagonal elements \((m^2 - M^2)\epsilon\), where \(\epsilon\) is the rotation angle. The quantity governing
the size of SUSY contributions to the FCNC is
\[
\delta \sim \frac{m^2 - M^2}{m^2 + 8m^2 \tilde{g}} \epsilon \leq 10^{-3},
\]
with \(m^2 \sim m^2_\tilde{g}\) and \(\epsilon \leq 10^{-2}\). For hierarchical Yukawa textures and the gluino
mass larger or similar to the squark masses, it is usually small enough to satisfy
all experimental FCNC constraints [18]. Thus, we see that this scenario does not
suffer any significant FCNC problem (Fig.3).

One could also consider the possibility that all three generations are split,
\[
m^2_i = \text{diag}(m^2, M^2, M^2).
\]
(It would be difficult however to find convincing examples.) In this case, the FCNC
constraints are more severe, yet the RG–dilution mechanism is at work. One finds
that the FCNC constraints are satisfied as long as the rotation matrices to the
physical basis are similar to the CKM matrix [18].

Thus, some of our assumptions can be relaxed without encountering a severe
FCNC problem. What is important in this analysis is that the Kähler metric stays
diagonal\(^4\), which is true in essentially all reasonable models (see e.g. [22]). Finally,
note that string theory restricts not only the form of the Kähler potential but also
the “numbers” entering into it: the modular weights in semirealistic models are
either -1 or -2 (Eq.(3)), so it is likely that the first two generations have the same
weights purely on the statistical basis.

5. Conclusion

Correlations between the structure of the Yukawa couplings and that of the
Kähler potential suppress SUSY–induced FCNC in string models. This conclusion
is valid for a standard SUSY spectrum with no peculiarities, nor does it rely
on family symmetries or universality. It is based on string selection rules which
stem from properties of the compactified space in heterotic and intersecting brane
models.

The question that remains is what part of the analysis will survive in a truly
realistic model. Perhaps the details will change entirely, but, as the above examples
show, string theory may be clever enough to avoid automatically the problems that
bug phenomenologists.

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\(^4\)Higher dimensional operators may lead to off–diagonal terms in the Kähler metric. These however
are significantly suppressed.
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Figure 2: Flavor violating mass insertions due to non-universal A–terms [18]. The SUSY parameters are chosen as $m = 2A$, $M_{1/2} = 200$ GeV, $	an \beta = 15$ and order one $\tilde{A}_{u,d}^{ij}$ are generated randomly. All experimental bounds (marked by a line) on FCNC observables are satisfied. (The nEDM, which is a flavor–conserving observable, is problematic as shown in the two top left blocks. This represents the SUSY CP problem which is not solved by the field theory mechanisms mentioned in the introduction.)
Figure 3: Flavor violating mass insertions due to non–universal soft scalar masses, $m_1 = m_2 \neq m_3$ [18]. The SUSY parameters are chosen as $A = 0$, $M_{1/2} = 200$ GeV, $\tan \beta = 15$ and $m_3$ is varied randomly in the range $m_1/2 \div m_1$. The Yukawa matrices are assumed to be diagonalized by matrices similar to the CKM one. All experimental bounds (marked by a line) on FCNC observables are satisfied. (The $B$-physics observables are not shown.)