MECHANICAL ENGINEERING | RESEARCH ARTICLE

About the comfort of house-buildings with earth walls in San Carlos, Tamaulipas

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Abstract: Earth construction is a revisited topic due to the low-cost self-production employed in communities and locations where conventional materials (concrete and steel) are not available. One of the properties that are popularly recognized for earth-based buildings is the pleasant indoor thermal sensation. This article presents the development of a model to characterize the temperature of the indoor air applied to adobe houses located in San Carlos, Tamaulipas, Mexico. The model is developed from the heat transfer equations in an ideal case where the temperature is modified by conduction through the wall, and the interior air is heated by convection. In the ideal case, it is considered that there are no doors or windows open because it is what commonly occurs in the earth-based houses during the warmest period of the year. It was found that for a thickness greater than 10 cm there is no significant increase in temperature assuming an outside temperature above 40 °C and an initial indoor temperature of 25 °C. The model characterizes and predicts a strong relation between the physical properties of the air and the wall, the thickness of the wall, the dimensions and shape of the room, and the roughness of the outer wall. This consideration makes it possible to quantitatively estimate comfortability based on indoor temperature and wall thickness, which is an expected behavior according to literature. This is important because not only has an impact on the construction of new buildings but also in the critical maintenance of existing houses.

Subjects: Mechanical Engineering; Surface Engineering-Materials Science; Structure, Materials and Detailing; Construction Materials

Keywords: Heat transfer in adobe; clay soil-sand wall; caloric conduction; caloric convection; comfortability

1. Introduction
Heat transfer analysis is an important parameter to adjust indoor thermal comfort in living spaces, for both cooling and heating. The heat exchange process has an impact on the location of the equipment when ventilation is forced (Vazquez-Ruiz et al., 2021) as in the cases of passive ventilation to reduce energy consumption, from natural convection (Anthony & Nath Verma, 2021). There are several computational methods for the analysis of the thermal behavior of buildings (Yamamoto et al., 2021) obtaining simulations from the numerical solution of various systems of differential equations, although other analog methods can also be used from temperature transients (Duan et al., 2022).
The study of the impact of materials on comfort, as well as the modeling of temporary temperature changes, allows a better use of comfortable architectural designs. If heat transfer mechanisms are considered, it is also possible to create desirable interior environments (Li & Zhu, 2022; Xue et al., 2021) promoting the use of passive strategies that reduce energy consumption (Resano et al., 2021).

In several studies, different wall configurations have been studied form the point of view of the material’s thermal performance. Kumar et al. (2020) determined the life-cycling cost of different materials through the amount of heat gain through the wall:

$$Q = \frac{86400CDD}{R}$$

where Q is the heat gain, CDD represents the cooling degree-days calculated from the number of cooling and heating days in a month, and R is the thermal resistance of the wall (Kumar et al., 2020). Other studies analyzed the materials heat-storage performance or the hygrothermal behavior for several applications (Giada et al., 2019; Zhang et al., 2019). Most of the studies involve computational modeling to determine the ideal material of the wall according to the local or regional conditions (Dabaieh & Serageldin, 2020; Liu et al., 2019); however, this is extremely specific and not always it is possible to apply these methodologies to other regions in the world.

It is common to find earth constructions in various communities of the world. For example, in some African regions it is common to find compressed earth buildings, and these are studied under the conduction transfer functions (CTF) recommended in literature for the next equations (Neya et al. 2021; Yang et al., 2015):

$$q_k^i(t) = -Z_0T_{it} - \sum_{j=1}^{n_z} Z_j T_{i,t-j0} + Y_0 T_{0t} + \sum_{j=1}^{n_x} Y_j T_{0,t-j0} + \sum_{j=1}^{n_q} \Phi_j q_{k0,t-j0}$$

$$q_0^i(t) = -Y_0T_{it} - \sum_{j=1}^{n_y} Y_j T_{i,t-j0} + X_0 T_{0t} + \sum_{j=1}^{n_x} X_j T_{0,t-j0} + \sum_{j=1}^{n_q} \Phi_j q_{k0,t-j0}$$

where $X_j$ is the outdoor CTF coefficient, $Y_j$ is the cross CTF coefficient, $Z_j$ is the indoor CTF coefficient, $\Phi_j$ is the flux CTF coefficient, $T_i$ the indoor face temperature, $T_0$ the outdoor face temperature, and $q_{k0}$ and $q_{k1}$ are the conduction heat flux on outdoor and indoor face. However, these coefficients are suitable for specific materials, therefore the materials in every region should be studied to quantify their properties. In Mexico, researchers attempt to resume the use of traditional or vernacular materials that reduce the impact on the environment, such as rammed earth (Avila et al., 2022) or adobe, recently finding adaptation of technologies to local contexts including the transformation of mixtures based on cement or modified concrete (Giada Giuffrida et al., 2021; Lee et al., 2021).

Comfort is associated with the interior temperature of the room, and the capacity for it to remain at an adequate value for as long as possible despite the variations that exist in the ambient temperature. From the thermodynamic point of view, the temperature of a room tends irreversibly to take the value of the ambient temperature because the air inside the room exchanges heats with the air of the outside environment through the walls. The speed of this exchange determines the temporal behavior of the interior temperature, being the room more comfortable to the extent that the speed of variation of the interior temperature is lower. This behavior is likely to the fluid heat exchange and the thermodynamic aspect applied (Alhamid & Al-Obaidi, 2021; Al-Obaidi, 2022; Al-Obaidi & Sharif, 2021). The objective of this work was to obtain a mathematical model to analyze how the type of material, the thickness of the walls, and the dimensions of a room

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influence the temporal behavior of the interior temperature before a change in the ambient temperature.

2. Method

2.1. Cases in the san carlos area, tamaulipas

The Municipality of San Carlos, in Mexico, is located at 24°31’ north latitude and at an altitude of 432 meters above sea level. In May, there are temperatures during the day that exceed 40 °C but in the afternoon-night, the record is below 25 °C. Even when these temperature variations, it has been observed that the houses built from mixtures of sand with clay soil, keep a qualitatively pleasant interior temperature. As for the construction systems, most of the walls exceed 25 cm thickness and denote the absence of windows or the location of very small windows in the walls. However, there is no quantitative explanation of the impact of the composition of the wall concerning temperature changes.

2.2. Model obtention and development

To obtain the model it was considered that the process of heat transport in the wall takes place by conduction, while inside the room the predominant mechanism is the natural convection of air. The behavior of temperature in the wall is determined from the equations of exchange temperature in a solid material, and is given by the partial differential equation:

\[
C_1 \frac{\partial T_1(x, t)}{\partial t} = \kappa_1 \frac{\partial^2 T_1(x, t)}{\partial x^2}
\]

(1)

The temporal behavior of the air temperature in the room \(T_2\), which for simplicity will be considered independent of the spatial position, is obtained from the macroscopic energy balance (Staniec & Nowak, 2016), and is described through the differential equation:

\[
V C_2 \frac{dT_2(t)}{dt} = UA(T_1(h, t) - T_2(t))
\]

(2)

where

\(T_1(°C)\), is the temperature on the wall

\(t \text{ (h)},\) is the period of the thermal analysis

\(x(\text{m}),\) is the distance measured taking as a reference the outer side of the wall

\(C_1\) and \(C_2 \left(\frac{\text{J}}{\text{m}^2 \cdot \text{K}}\right),\) are the caloric capacity

\(\kappa_1 \left(\frac{\text{W}}{\text{m} \cdot \text{K}}\right),\) is the coefficient of thermal conductivity of the wall material.

\(V(\text{m}^3),\) is the volume of the air contained in the room

\(A(\text{m}^2),\) is the total area of heat exchange (sum of the areas of the walls and ceiling)

\(h(\text{m}),\) is the thickness of the wall

\(U \left(\frac{\text{W}}{\text{m}^2 \cdot \text{K}}\right),\) is the coefficient of heat transport by natural convection of the air

To solve equation 2, the following initial and boundary conditions are considered:
\[
\begin{align*}
\left( \frac{\partial T_1(x, t)}{\partial t} \right)_{t=0} &= 0 \\
\left( \kappa_1 \frac{\partial T_1(x, t)}{\partial x} \right)_{t=\infty} &= 0 \\
T_1(h, 0) &= T_{2.0} \\
T_2(0) &= T_{2.0}
\end{align*}
\] (3)

from the established conditions, it is obtained that for \( t = 0 \):

\[
\kappa_1 \frac{\partial^2 T_1(x, t)}{\partial x^2} = 0
\] (4)

\[
T_1(x, 0) = T_{1.0} + \frac{(T_{2.0} - T_{1.0})}{h} x
\] (5)

When \( t>0 \), the behavior for the partial differential equation is solved, for which the variable is defined:

\[
\theta_1 = T_1(x, t) - T_1(x, 0)
\] (6)

and the parameter:

\[
\Phi_1 = \frac{\kappa_1}{C_1}
\] (7)

in such a way that the partial differential equation and boundary conditions are rewritten:

\[
\begin{align*}
C_1 \frac{\partial \theta_1(x, t)}{\partial t} &= \kappa_1 \frac{\partial^2 \theta_1(x, t)}{\partial x^2} \\
\left( \frac{\partial \theta_1(x, t)}{\partial x} \right)_{t=\infty} &= 0 \\
\theta_1(0, h) &= 0
\end{align*}
\] (8)

Applying the method of separation of variables and substituting appropriately it is obtained:

\[
T_1(x, t) = \left( T_{1.0} + \frac{(T_{2.0} - T_{1.0})}{h} x - T_{1.0} \right) \exp \left( -\frac{t}{\tau_1} \right) + T_{1.0}
\] (9)

being the relaxation time or time constant corresponding to the wall \( \tau_1 \), and its value is:

\[
\tau_1 = \frac{h^2}{\kappa_1 C_1}
\] (10)

The behavior of the temperature on the inner side of the wall is given by:

\[
T_1(h, t) = (T_{2.0} - T_{1.0}) \exp \left( -\frac{t}{\tau_1} \right) + T_{1.0}
\] (11)

To determine the behavior of the temperature inside the room \( T_2 \) it is then necessary to solve the ordinary differential equation:
\[
\tau_2 \frac{dT_2(t)}{dt} + T_2(t) = (T_{2.0} - T_{1.0}) \exp\left(-\frac{t}{\tau_1}\right) + T_{1.0}
\]

\[T_2(0) = T_{2.0}\] 

(12)

where \(\tau_2\) is the relaxation coefficient or time constant corresponding to the air:

\[
\tau_2 = \frac{C_2 V}{U A}
\]

(13)

Solving equation 12 gives the equation that describes the behavior of the temperature inside the room:

\[
T_2(t) = \frac{(T_{2.0} - T_{1.0})}{(r_1 - \tau_2)} \left( r_1 \exp\left(-\frac{t}{r_1}\right) - r_2 \exp\left(-\frac{t}{r_2}\right) \right) + T_{1.0}
\]

(14)

Figures 1 and 2 show the temporal behavior predicted by the model considering as parameters the sum of the time constants \(T = r_2 + r_1\) and the relation \(b = \frac{r_1}{r_2 + r_1}\), where \(q = \frac{\tau_2}{(r_1 + r_2)}\). In this case, it can be seen that the value of the sum of the time constants influences more significantly the time required for the system to reach the final steady-state (equality of the interior and exterior temperatures), while for a constant value of this sum it is appreciated that the change of the time constant of the wall keeping the value of constant \(T\) does not significantly influence the time in reach steady state.

2.3. Temporal comfort analysis

For an ideal living space and its surroundings, it is considered that 25 °C is the optimal temperature of comfort in the region. However, during summer months (when ambient temperatures are about 40 °C) sunny surfaces can reach temperatures between 55 and 60 °C. The increase in temperature on an interior face of walls that constitute a house, leads to an increase in the interior temperature of the wall, which later, under ideal conditions will increase the internal ambient temperature of the room. In this sense, starting at 25°C on average, for this work it is defined the comfort parameter \(\Theta\) for the room, which is characterized through the sum of the time constants, in such a way that an increase in this value is associated with a greater comfort:

\[
\Theta = \frac{h^2}{k_1} C_1 + \frac{C_2 V}{U A}
\]

(15)

Figure 1. Predicted temporal behavior considering as a parameter the sum of the time constants \(T\), keeping the time constant of the wall fixed 
\(b = \frac{\tau_1}{r_1 + \tau_2} = 0.1\).
In this way, the parameter predicts that the comfort increases when:

- decreases the outside area of the room per unit of the interior volume
- the caloric capacity of the material increases (the caloric capacity of the air can be considered constant)
- decreases the coefficient of thermal conductivity of the material (the coefficient of heat transfer of the air can be considered constant)
- the wall thickness is increased

3. Results and discussion

3.1. Model predictions

Solid surfaces are characterized by a certain roughness, associated with a random variation of the height of the surface for a reference scale, which has originated from the stochastic nature of the microscopic processes that occur during the formation of the solid and due to the interaction of this with the environment. This roughness causes, on the one hand, the actual surface area of the solid is greater than that corresponding to a flat surface, while on the other hand, its random nature makes it impossible for its value to be determined exactly by Euclidean geometry.

For this reason, it is necessary to apply physicochemical experimental methods or the application of fractal geometry, based on a morphological analysis of the observed surface. In this work, the method based on fractal geometry was adopted to consider the effect of the roughness of the exterior wall of the rooms on comfort. There are different methods to quantitatively determine the value of roughness (Nayak et al., 2019), all based on the observation of the surface height profile, which is equivalent to the irregular line observed because of the intersection between the surface and a plane perpendicular to it. This line can be obtained through a rough meter or through a photograph of the surface and an appropriate program for image processing (Mandelbrot, 1977), where the height is identified with the average intensity value of the pixels. The length $l$ of the irregular line is greater than the length $L$ corresponding to the segment that joins its ends, and according to fractal geometry this is estimated as:

$$l = kl^f$$

where $k$ is a constant that depends on the accuracy of the measurement and $f$ is the fractal dimension, a parameter that is greater than or equal to 1 and less than 1.5. For Euclidean lines $k = 1$ and $f = 1$. If it is assumed that the fractal dimension is equal at all sites on the surface and
that the area is the product of two perpendicular lines, then the surface fractal area \( a \), proportional to the actual area:

\[
a = L^{2f}
\]

(17)

\( 2f \) is the estimation of the fractal surface dimension. The fractal dimension is determined by the box-counting method (Vandenbeng & Osborne, 1992):

\[
f = \lim_{y \to 0} \frac{\ln n(y)}{\ln N(y)}
\]

(18)

It is assumed that the surface area \( a \) can be estimated according to the relationship:

\[
a = A^f
\]

(19)

where \( A \) is the area corresponding to the equivalent flat surface.

The value of the flat surface area of a room depends on its shape. Table 1 presents the area and volume values for cubic, conical, pyramidal, and spherical rooms. These forms were selected as a possible room shape and the combinations between them. For each case it has been considered that the sides of the room and the height are equal (width = length = height = \( L \)) and the effect of the roughness of the surface quantified through the fractal dimension of the height profile.

To analyze the model predictions, the parameters \( C_1, \ k_1, \ U, \) and \( C_2 \) were determined through the theory of the Linear Transient Heat Source, which is useful for studying thermal properties in soils, adobes, and CEB. It consists of the measurement and correlation of the temperature-induced within a material and its transfer through a thermal needle, for which a KD2 Pro Thermal Conductivity Meter was used with the SH-1 Sensor, with 30 minutes readings. The material of the room was a mixture of sand and clay, in such a way that the physical properties of the wall and air are \( C_1 = 1.26 \frac{\text{MJ}}{\text{m}^2\text{K}}, \ k_1 = 0.283 \frac{\text{W}}{\text{mK}}, \ U = 5 \frac{\text{W}}{\text{mK}}, \) and \( C_2 = 1.2142 \times 10^{-3} \frac{\text{MJ}}{\text{m}^2\text{K}} \).

Figure 3 shows the comfort values concerning the fractal dimension of the surface height profile predicted for dwellings of different shapes. In this case, it is predicted that the increase in the roughness of the surface leads to a decrease in comfort, while for the shape of the room the order obtained from the greatest comfort to that of the least comfort are spherical, conical, cubic and pyramidal. However, the variation observed is very small, so the comfort values obtained are in the range between 12.35 and 12.45. Therefore, from a practical point of view, it can be established that neither the shape of the house nor the roughness of the wall, determine the comfort.

Figure 4 shows the behavior of comfort for the thickness of the wall for different shapes of the room, considering that the exterior walls are smooth. In this case, the comfort is independent of the shape of the room, as previously proposed, but it depends on the thickness of the wall. This is

| Shape      | Air volume | Surface area | Comfort equation |
|------------|------------|--------------|------------------|
| Pyramidal  | \( \frac{1}{3}(L-h)^3 \) | \( \sqrt[3]{6L^2} \)^f | \( \frac{C_1 h^2}{k_1} + \frac{P h^2}{\rho C_2 (L-h)^3} (1.5651 L)^{-2f} \) |
| Cubic      | \( (L-h)^3 \) | \( 5L^3 \)^f | \( \frac{C_1 h^2}{k_1} + \frac{P h^2}{\rho C_2 (L-h)^3} (2.2361 L)^{-2f} \) |
| Conic      | \( \frac{1}{3}\pi(L-h)^3 \) | \( \pi \sqrt{2L} \)^f | \( \frac{C_1 h^2}{k_1} + \frac{P h^2}{\rho C_2 (L-h)^3} (2.1078 L)^{-2f} \) |
| Spherical  | \( \frac{2}{3}\pi(L-h)^3 \) | \( 2\pi L^2 \)^f | \( \frac{C_1 h^2}{k_1} + \frac{P h^2}{\rho C_2 (L-h)^3} (2.5066 L)^{-2f} \) |
because the ratio of the caloric capacity to the heat transfer coefficient of the air is significantly lower than the ratio of the caloric capacity to the thermal conductivity coefficient of the wall so the effect of natural air convection can be disregarded. Figure 4 shows the behavior of comfort when the transport of heat inside the room is disregarded, in such a way that comfort can be predicted through the relationship:

\[ \Theta \approx \frac{C_1}{\kappa_1} h^2 \]  

(20)

From these results, it can be understood why the earth elements present a better performance in terms of comfort, from a minimum change of temperature in the inner face of the same. For the houses that are in San Carlos, Tamaulipas, based on earth materials, there is a thermal conductivity of almost half of a common industrial according to (Suarez-Dominguez et al., 2015a) and (Suárez-Domínguez et al., 2015b). In this way, it is explained why a wall of material based on a concrete block of 15 cm common in the area, heats up completely much faster in hot climates for a house, while a wall of earth of more than 25 cm thick does not detect high changes in the interior temperature of the same throughout the year. Additionally, these results are compared to those obtained for other regions in the world for different materials based on concrete and earth (Avila et al., 2021; Beckett et al., 2018; Fabbri et al., 2018; G. Giuffrida et al., 2019; Gomes et al., 2018; Rosti et al., 2020). The relevance of this work is that the interventions for maintenance in
rural houses can be modulate and studied prior taking actions, and most important, selection of materials can be provisioned according to the requirements of users, improving the use of local materials for the low-cost self-production.

4. Conclusions
The present work aims with the indoor comfortability of adobe houses in San Carlos, Tamaulipas, Mexico. Considering that the comfort of a room is related to the time in which indoor temperature becomes equal to the external temperature, when the outside temperature is much higher than interior, a model is proposed that describes the heat exchange behavior. Such a behavior occurs through conduction as the predominant mechanism in the wall, while inside the room is the natural convection of air. From the temporal behavior of the temperature inside the room predicted it was obtained that the comfortability can be quantified from the sum of the relaxation times of the wall and air, respectively. In this case, the order of comfort is obtained, from highest to lowest, according to the shape of the room: cylindrical, conical, cubic, and pyramidal, and that comfort decrease with the increase in roughness. However, these effects are negligible for the earth-based material compared to the wall thickness due to the comfort is significantly increased when this parameter increases. The positive impact of this work is that the interventions for maintenance and construction of living spaces in rural regions can be set up along with a correct selection of local materials promoting the low-cost self-production.

This study can be replicated for other material types in other regions in the world if the material properties: caloric capacity and the coefficient of thermal conductivity, can be determined by direct measure in existing houses. However, there are some limitations due to the shapes considered for the model development and the roughness of the surfaces. It should be pointed that for construction techniques as rammed earth or poured earth, would require the analysis of the fractal dimension to achieve a more reliable result.

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