Using the Van Hiele theory to analyze primary school teachers’ written work on geometrical proof problems

A Jupri
Program Studi Pendidikan Matematika dan Program Studi Pendidikan Dasar, Sekolah Pascasarjana, Universitas Pendidikan Indonesia, Bandung, Indonesia

Corresponding author’s e-mail : aljupri@upi.edu

Abstract. The lack of ability of primary school teachers in deductive thinking, such as doing geometrical proof, is an indispensable issue to be dealt with. In this paper, we report on results of a three-step of the field document study. The study was part of a pilot study for improving deductive thinking ability of primary school teachers. First, we designed geometrical proof problems adapted from literature. Second, we administered an individual written test involving nine master students of primary education program, in which they are having experiences as primary school mathematics teachers. Finally, we analyzed the written work from the view of the Van Hiele theory. The results revealed that even if about the half of the teachers show ability in doing formal proof, still the rest provides inappropriate proving. For further investigation, we wonder whether primary school teachers would show better deductive thinking if the teaching of geometry is designed in a systematic and appropriate manner according to the Van Hiele theory.

1. Introduction
Geometry, an important topic in school mathematics, promotes the development of student deductive thinking [1,2]. This development should be nurtured by mathematics teachers from primary to secondary school levels as a preparation for further study or professional work. As such, mathematics teachers should have this ability to foster their student thinking development through the teaching of geometry. This ability should be mastered by the mathematics teachers, not only for the secondary school level, but also for the primary school level.

Previous research [3,4] shows, however, that primary school mathematics teachers encounter difficulties dealing with geometry problems that call for deductive and formal thinking. As an attempt to overcome this condition, we have conducted a pilot study to design geometry teaching materials for improving primary school mathematics teachers’ ability in deductive thinking, particularly in doing formal geometry proof. In this paper, we report part of the results of this study, namely analysing the written work of the primary school teachers dealing with geometrical proof problems. The primary school mathematics teachers involved in this study are master student of the primary mathematics education program.

To investigate primary school teachers’ ability in solving geometrical proof problems, we use the Van Hiele theory as the theoretical framework. According to this theory, student geometric thought can be classified into five levels [5-9]. In the level 1 (recognition), the student can name and recognize a geometric shape as a whole. For example, the student recognizes a rectangle as different shape from
a trapezoid. In the level 2 (analysis), the student can identify properties of geometric shapes. For example, a square has four right angles and all the sides have equal lengths. In the level 3 (abstraction), the student can logically relate between properties of geometric figures. For example, a rectangle is considered as a parallelogram because it has all properties of the parallelogram. In the level 4 (deduction), the student understand the significance of deduction and is able to comprehend the roles of postulates, definitions, theorems, and proof. For example, the student can prove that the opposite angles of a parallelogram have the same sizes. Finally, in the level 5 (rigor), the student understands axiomatic systems in geometry and is able to do abstract deductions. For example, the student is able to analyse the consequences of manipulating axioms and definitions.

2. Methods
To delve into primary school teachers’ ability in solving geometrical proof problems, we conducted a three-step of the field document study of nine master students of primary mathematics education program. The study was part of a pilot study to design learning materials for improving deductive thinking of primary school teachers. All the nine involved students have experienced as primary school teachers for four to five years. The documents for the purpose of analysis included all written solutions of the nine teachers on solving two geometrical proof problems shown in Figure 1.

First, we designed two geometrical proof problems. The problem A is adapted from [10], and problem B is adapted from [11]. We consider the two problems can be solved by the primary school teachers as they require the use of basic geometry knowledge, including the knowledge on the areas of a rectangle and a circle. As the problems ask for teachers to do proving, in the light of the Van Hiele theory, therefore they assess teacher deductive thinking ability. Second, we administered an individual written test, lasted for 40 minutes, and asked for teachers to write down reasons for their solutions. The written test was conducted after the teachers attended five lessons of a geometry course focused on problem solving and proving. In the third step, we analysed written teacher work from the perspective of the Van Hiele theory, and the deductive level of the theory in particular.

![Figure 1. Two geometrical proof problems used in the study](image-url)
3. Results and Discussion

A summary of written teacher work on Problems A and B is shown in Table 1. The first and second columns present the number of teachers who performed correct and incorrect proofs with reasoning respectively. The overall result shows that about half of the participated primary school teachers are able to do geometrical proofs. This indicates that the learning and teaching processes in five lessons seem not enough for improving the deductive thinking ability of all teachers.

Table 1. A summary of written teacher work on Problems A and B (N = 9)

|               | #Correct with reasoning (%) | #Incorrect with reasoning (%) |
|---------------|----------------------------|------------------------------|
| Problem A     | 4 (44.4)                   | 5 (55.6)                     |
| Problem B     | 5 (55.6)                   | 4 (44.4)                     |

Typical incorrect solutions for the Problem A, such as shown in Figure 2(a), include the use of specific numbers rather than general algebraic variables in the ‘proving’ process. From the perspective of the Van Hiele theory, this result suggests that these teachers still do not reach level 4 (deduction), and seem not be ready for doing geometrical proofs [9]. In other words, they are still in between informal and formal deductive thinking [3]. Figure 2(b) shows a representative example of written teacher work for correct solutions. In this case, algebraic manipulation skills—as part of vertical mathematization process [12]—are imperative for the success of the proving process.
Figure 2. Representative of written teacher work on problem A (a) The use of specific numbers in the ‘proving’ process, (b) A representative example of written teacher work for correct solutions]

For the case of Problem B, typical encountered difficulty includes mistakes in applying algebraic manipulations and in devising proving process. Again, in the light of the Van Hiele theory, this result shows that the teachers still do not reach the deduction level. For the case of correct proving, we found two different strategies. The first strategy is regenerated in Figure 3(a), and the second strategy is regenerated in Figure 3(b). In addition to have algebraic skills, these two different strategies shows that in doing geometrical proof problems the teachers also need problem solving competence [3].

Figure 3. (a) The first strategies in doing geometrical proof of problem B, (b) The second strategies in doing geometrical proof of problem B
4. Conclusion

From results described in the previous section, we draw the following two main conclusions. First, even if involved teachers have attended five lessons of the geometry teaching focused on problem solving and proving, about the half of the teachers still lack of ability in deductive thinking and in dealing with geometrical proof problems in particular. This result indicates that the teachers need more time to reach deductive level of geometric thought. Second, different strategies produced by the teachers when dealing with geometrical proof problems show that problem solving skills are important for the success of proving process. Also, algebraic manipulation skills play important role in both proving and problem solving processes. For further research, we wonder whether primary school teachers would show better ability in deductive thinking if the teaching of geometry is designed in a systematic and appropriate manner according to the Van Hiele theory.

5. References

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