A Parameters Identification Method of the Equivalent Circuit Model of the Supercapacitor Cell Module Based on Segmentation Optimization

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ABSTRACT In practical settings, the supercapacitor is often used as the storage battery, which is composed of several supercapacitor cells in series. In order to accurately estimate the State of Charge (SoC) in the supercapacitor cell module, an equivalent model of supercapacitor cell module is invoked, which is expected to reflect the characteristics of supercapacitor cell module, especially the self-discharge characteristics during standing. The results of parameter identification directly affect the model accuracy. Hitherto, most supercapacitor equivalent models have been proposed for supercapacitor cells, but if the module equivalent model is characterized by connecting many equivalent models of supercapacitor cells in series, it would lead to the cumulative errors and the additional errors, which would incur errors in the parameter identification, and directly affect the model accuracy. The paper aims to obtain the accurate equivalent model parameters, the supercapacitor cell module is regarded as the object, the three-branch equivalent circuit model is established for the supercapacitor cell module, a discussion is given on the parameter identification methods about Circuit Analysis Method (CA) and Recursive Least Squares Method (RLS). This paper establishes the Simulink simulation model for the multi-method parameter identification of supercapacitor cell module, the simulation and analysis are performed to illustrate the advantages and disadvantages of CA and Circuit Analysis-Recursive Least Squares Method (CA-RLS). Then, it proposes a parameters identification method of the equivalent circuit model of supercapacitor cell module based on Segmentation Optimization (SO). The effectiveness of SO is verified by simulation and error analysis, the results indicate that SO can more effectively reflect the charging characteristics and self-discharge characteristics of the supercapacitor cell module. In particular, the comprehensive error in the static self-discharge phase is 0.28%, which is 6.83% and 0.64% lower than CA and CA-RLS, respectively.

INDEX TERMS Supercapacitor cell module, equivalent circuit model, parameter identification, segmentation optimization.

I. INTRODUCTION

The pitch system is one of the core parts of the megawatt wind turbine control system, which plays an important role in the safe, stable and efficient operation of the wind turbine [1]. The backup power of the pitch system usually uses a traditional lead-acid battery or a Lithium-ion battery [2]. The supercapacitors have the advantages of high-power density, fast charge and discharge speed, long cycle life and green environmental protection in scrap treatment [3]–[6]. They began to gradually replace traditional batteries as the backup power of wind turbine pitch systems. In the case of a sudden drop in the grid voltage or a power failure in the ultra-high wind speed grid, the pitch system will enable the backup power to control the emergency feathering for ensuring the safe operation of the wind turbine. However, the backup power is in a static state under normal conditions, its working process is typically intermittent. At present, the wind turbine only monitors the voltage of the supercapacitor cell module in the Supervisory Control and Data Acquisition (SCADA)
systems, which are equipped with the supercapacitor cell module as the backup power, if the State of Charge (SoC) of the supercapacitor cell module is not estimated, the SoC and the real-time performance of the backup power would not be known in time, which leads to a potential risk to the emergency feathering of wind turbines.

In order to accurately estimate the SoC of the supercapacitor cell module, an equivalent circuit model of the supercapacitor cell module should be established to accurately reflect the charge and discharge characteristics of the module, especially the static self-discharge characteristics. Since the selection of its identification method directly affects the model identification results, the equivalent circuit model parameters must be identified with a high accuracy, and it is important to choose an appropriate method to identify the model parameters. Many scholars have done a lot of research on the supercapacitor cell model. In [7], several models based on impedance characteristics are analyzed and summarized. A multi-scale impedance model is developed for evaluating the charge storage capacity and ion transfer rate of supercapacitor porous electrodes in [8]. Furthermore, in [9], the paper focuses on supercapacitors and Lithium-ion batteries as energy storage systems, and presents an analysis of fractional modeling based on equivalent circuits. Starting with the basic concepts and technical tools from fractional-order calculus, the modeling principles for these energy systems include supercapacitor are presented by identifying the dynamic response processes and using electrochemical impedance spectroscopy in [10]. In [11], the authors present a set of calculation methods for supercapacitor metrics using fractional-order calculus when such devices are operated under the commonly used (i) sinusoidal excitation, (ii) step current input and (iii) linear voltage input. A new approach of using the discrete time fractional order artificial neural network controller is proposed to control charging and discharging process for system with supercapacitor in [12]. Furthermore, in [13], the electrical model based on parameter optimization of passive electrical circuit is proposed. The existing parameter identification methods of the supercapacitor cell model include circuit analysis method [14], binary quadratic equation fitting method [15], least squares method [16], particle swarm algorithm [17], [18], recursive least squares method [19], etc. It is relatively simple to identify model parameters by the circuit analysis method, which is of clear physical meaning, and meets the actual engineering application, and requires less experimental equipment, but the identification accuracy is not high. The binary quadratic fitting method solves the result as a specific solution, but the identification result is often inaccurate due to the virtual root problem. The least squares method is simple and easy-to-use, although it is of good statistical effects on nonlinear model parameters, but its accuracy is not high. The particle swarm optimization algorithm is of high recognition accuracy, but its calculation process is complex [20].

The paper aims to obtain the accurate equivalent model parameters. It takes the supercapacitor cell module as the object, and establishes the three-branch equivalent circuit model of the supercapacitor cell module. The Simulink simulation model for multi-method parameter identification of supercapacitor cell modules is constructed. In view of the disadvantages of Circuit Analysis Method and Circuit Analysis-Recursive Least Squares Method, the paper puts forward a parameter identification method of the equivalent circuit model of supercapacitor cell module based on segmentation optimization.

The rest of the paper is arranged as follows. In Section II, it introduces the existing model of the supercapacitor cell briefly, and proposes a physical model for the supercapacitor cell module. In Section III, the discussion is given on the parameter identification methods of two module equivalent models about CA and RLS on the basis of some existing parameter identification methods. A parameter identification method of the equivalent circuit model of the supercapacitor cell module based on segmentation optimization is put forward. Besides it also establishes the simulink simulation model of the multi-method parameter identification of supercapacitor cell module. In Section IV, the simulation and analysis are carried out. The simulation and error analysis are performed to verify the effectiveness of the SO. In Section V, the conclusions are put forward by summarizing the paper.

II. EQUIVALENT CIRCUIT MODEL OF SUPERCAPACITOR CELL MODULE

The rated voltage of the supercapacitor cell is generally between 2.7V and 3V, which is a small voltage. The equipment usually requires a higher voltage on practical applications, especially in the backup power of the wind turbine pitch system, where the pitch motor driven by the actuator drives the blade to rotate. The input voltage of the actuator is 450V. Since the supercapacitor cell need to be usually connected in series to form a module, such that output voltage can meet the requirements.

Supercapacitor are often modelled using equivalent circuits composed of resistors and capacitors, including inductive elements which may become important at higher frequencies, both in the time and frequency domains [21]–[23]. Many literatures have studied the model of the supercapacitor cell, which are summarized in Fig. 1. Fig. 1(a) is a model based on the physical structure [24], which can show the energy storage principle of the supercapacitors in an intuitive and simple way, but the parameters of this model is too many, it is difficult to identify these parameters, and this model is not suitable for practical applications [25]. Fig. 1(b) describes a model parameters of the trapezoidal mode [26], [27], which model parameters need to be determined by the impedance spectrum analysis. More experimental equipment required and the parameters that need to be determined would increase with the increase of its order, which is difficult to select the parameters correctly. Fig. 1(c) shows a classical equivalent circuit model [28], which circuit is simple and easy to identify. However, there are large errors in long-term charging, discharging and
standing, which cannot well characterize nonlinear characteristics. Fig. 1(d) is a supercapacitor cell model based on impedance characteristics [29], which has a better frequency adaptability, and gives a better representation for the impedance characteristics of supercapacitors. Representing a supercapacitor in terms of an equivalent circuit composed of passive elements is not only a convenient and computationally efficient method to describe the supercapacitor [30]. But the parameters also need to be determined by the impedance spectrum analysis, more experimental equipment is needed, and the experiment is more complicated. In order to characterize the relationship between the terminal voltage and the capacitance, a fixed capacitor is used in parallel with a variable capacitor that varies with voltage in the multi-branch model. The capacitance of the supercapacitor is roughly linear with its terminal voltage [31]. The other RC branches characterize the supercapacitor dynamic response at different time constants. The resistance characterizes the self-discharge effect of the supercapacitor under static conditions, and the number of branches can be selected according to the accuracy presented in [32].

In practical engineering applications, the supercapacitor cells are used in series to form a module. Due to the difference among cells, the error of the module increases as the number of cells increases. Besides, the additional errors such as additional resistance would be introduced when the supercapacitor cells are connected in series, and it would lead to serious model deviations of the module, output parameter and the actual parameter of the module. Hence, it is necessary to study the modeling of the supercapacitor cell module in the practical production applications.

The paper takes the supercapacitor cell module as the research object, which is composed of supercapacitor cells in series. Because there are differences in the supercapacitor cells, the cells are evenly pressurized during the charging and discharging process, and the module with super capacitors connected in series can be approximated as a model, where a supercapacitor cell is connected in series with the additional resistor \( R_{ad} \), which is shown in Fig. 2. As shown in Fig. 3, this paper describes the characteristics of the supercapacitor cell module by using the three-branch equivalent model, such that the dynamic characteristics of the supercapacitor cell module can well characterized during charge and discharge, internal charge redistribution after charge and discharge, and self-discharge during standing.

The Charge and Discharge Branch (CDB) consists of a fixed resistor \( R_f \) and a variable capacitor \( C_f \), the fixed capacitor \( R_f \) is the equivalent series internal resistance of the supercapacitor module, the variable capacitor \( C_f \) is formed by connecting a fixed capacitor \( C_{f0} \) and a variable capacitor \( C_{f1} \) in parallel, and the \( C_{f1} \) that varies with the voltage. The branch represents the dynamic process of the module during charging and discharging.

As shown in Eqs. (1) and (2), it has

\[
C_f = C_{f0} + C_{f1}(U)
\]  

(1)

\[
C_{f1}(U) = k \times U
\]  

(2)

where \( k \) is the coefficient of the variable capacitance \( C_{f1} \) that varies with the terminal voltage \( U \).

As the terminal voltage \( U \) of the supercapacitor cell module changes, so does the total capacitance \( C_f \) of the supercapacitor cell module, as shown in Eq. (3).

\[
C_f(U) = \frac{dQ}{dU} \bigg|_U
\]  

(3)
where \( Q \) is the charge that stores in the supercapacitor cell module.

The Charge Balance Branch (CBB) consists of a fixed resistor \( R_f \) and a fixed capacitor \( C_f \), which characterizes the redistribution of the internal charge of the supercapacitor cell module after the end of charge and discharge [33], [34], where \( R_f \ll R_l \).

The Self-Discharge Branch (SDB) consists of a large resistor \( R_{sd} \), which characterizes the self-discharge phenomenon of the supercapacitor cell module during the stationary process after charging and discharging, where \( R_f \ll R_l \ll R_{sd} \).

### III. PARAMETERS IDENTIFICATION METHOD OF MODULE EQUIVALENT MODEL

In view of the above three-branch equivalent circuit model of the supercapacitor cell module, the paper uses the Circuit Analysis Method (CA) and the Least Squares Method (LS) to identify the unknown parameters.

The circuit analysis method is a parameter identification method of the equivalent circuit model by analyzing the circuit relations. The method is relatively simple to identify the parameters of the equivalent circuit model. It is of clear physical meaning, and doesn’t need a lot of experimental equipment.

LS is an easy-to-use algorithm, it is of clear principle and fast convergence, and is widely used in parameter identification. However, as the number of observation data increases, the computer memory required by LS also increases. Moreover, when the parameter matrix \( \theta \) changes, the parameter estimation \( \hat{\theta}_{LS} \) cannot automatically track its change, which make the real-time performance worse. However, the Recursive Least Squares Method (RLS) is of good real-time performance, it is usually used for online parameter identification.

#### A. CIRCUIT ANALYSIS METHOD

The resistance and capacitance of the three branches of the above model are different, so the time constants are obviously different. Therefore, CA is used to identify the model parameters, and the transient process of each branch can be independently analyzed by the time-varying terminal voltage data.

1) CHARGE AND DISCHARGE BRANCH

The initial voltage \( U_0 \) of the supercapacitor cell module is small. The constant current charging of the supercapacitor cell module is started from the time \( t_0 \), let the constant current be \( I \). After the small time \( t_1 \), \( U_1 \) represents the voltage after the terminal voltage of the module increases \( \Delta U \). Due to the time is short, the voltage of the variable capacitor \( C_f \) is approximately zero. At this time, it can be considered that the terminal voltage of the module is equal to the voltage across the resistor \( R_f \), then it has:

\[
R_f = \frac{U_1 - U_0}{I} \tag{4}
\]

When the supercapacitor cell module continues to charge for a small time to \( t_2 \), the terminal voltage of the module becomes \( U_2 \), which is still small. The variable capacitor \( C_f(U) \) can be approximated to zero, and the terminal voltage of the supercapacitor cell module is shared by the resistance \( R_f \) and the fixed capacitor \( C_f_0 \), the current \( I \) is expressed as

\[
I = \frac{dQ}{dt} = \frac{d(C_f * U)}{dt} = C_f * \frac{dU}{dt} \tag{5}
\]

where \( Q \) is the amount of charge accumulated by the variable capacitor \( C_f \).

The variable capacitance \( C_f(U) \) can be approximated to zero because the voltage is small. From Eqs. (1) and (5), \( C_f_0 \) can be derived, which is expressed as

\[
C_f_0 = I * \frac{dt}{dU} = I * \frac{t_2 - t_1}{\Delta U} \tag{6}
\]

\( U_3 \) represents the rated voltage of the supercapacitor cell module at the time \( t_3 \), the constant current charging stops, and the current drops from \( I \) to 0 rapidly. At this time, \( U_4 \) represents the voltage at the time \( t_4 \), CDB has a charge amount \( Q \), that is the amount of charge accumulated by the variable capacitor \( C_f \), which is expressed as

\[
Q = I * (t_4 - t_1) \tag{7}
\]

From Eqs. (1), (2), (3), and (7), the coefficient \( k \) is obtained, which is expressed as

\[
k = \frac{2}{U_4} * (\frac{I * (t_4 - t_1)}{U_4} - C_f_0) \tag{8}
\]

2) CHARGE BALANCE BRANCH

When the supercapacitor cell module has fully been charged and the external power supply is disconnected, the capacitor \( C_f \) of CBB has stored the charge \( Q \), and \( C_f \) is used as the power source to supply power to the latter two branches.

At this time, an equivalent current generated inside the module flows to CBB, and an internal charge redistribution process occurs. The voltage changes \( \Delta U \) from \( U_4 \) to \( U_5 \). When the time \( t_5 \), the internal equivalent current \( I_0 \) can be expressed as [14]

\[
I_0 = \frac{U_4 - \Delta U}{2 \cdot R_l} \tag{9}
\]

The equivalent current \( I_0 \) can also be expressed as

\[
I_0 = \frac{C_f * \Delta U}{t_5 - t_4} \tag{10}
\]

From Eqs. (9) and (10), \( R_l \) is deduced, which is expressed as

\[
R_l = \frac{(U_4 - \Delta U) * (t_5 - t_4)}{C_f * \Delta U} \tag{11}
\]

Let the time be \( t_6 \), the internal charge redistribution has ended, the voltage is \( U_6 \) at this time, and the time of the charge
balance process from CDB to CBB should be three times the CBB time constant [14], which is expressed as
\[ t_6 = t_5 + 3(R_l * C_l) \] (12)

From Eqs. (1), (2), (3), and according to the principle of conservation of charge, \( Q \) is obtained, which is expressed as
\[ Q = C_i * U_6 + U_6 * (C_f0 + k \star \frac{U_6}{2}) \] (13)

From Eq. (13), \( C_i \) is obtained, which is expressed as
\[ C_i = \frac{Q}{U_6} - (C_f0 + \frac{k}{2} \star U_6) \] (14)

3) SELF-DISCHARGE BRANCH

SDB characterizes the self-discharge phenomenon of the supercapacitor cell module in the case of standing. After the module is fully rested, until its terminal voltage changes very slowly, the moment \( t_r \) is recorded, and \( U_T \) represents the terminal voltage of the module. After the first two branches are stabilized, the terminal voltage is \( U_6 \). The terminal voltage is \( U_T \) after the full rest. The voltage across the resistor \( R_l \) in CBB is obtained, which is expressed as
\[ U_{R_l} = U_6 - U_7 \] (15)

The self-discharge current \( I_{sd} \) is expressed as
\[ I_{sd} = \frac{U_{R_l}}{R_{sd}} \] (16)

Therefore, the self-discharge resistor \( R_{sd} \) can be introduced, which is expressed as
\[ R_{sd} = \frac{U_7}{I_{sd}} \] (17)

B. RECURSIVE LEAST SQUARES METHOD

The basic idea of RLS is
\[ \hat{\theta}(k + 1) = \hat{\theta}(k) + \Delta \hat{\theta}(k) \] (18)

That is, the newly generated parameter estimation value \( \hat{\theta}(k + 1) \) is obtained by correcting the previous parameter estimation value \( \hat{\theta}(k) \) with the new observation data, where \( \Delta \hat{\theta}(k) \) is the correction value of the new observation data to the previous parameter estimation value.

The input-output model of the system to be identified is expressed as
\[ y(k) = -\sum_{i=1}^{n} a_i y(k-i) + \sum_{i=0}^{n} b_i u(k-i) + \epsilon(k) \]
\[ \psi^T(k)\theta + \epsilon(k) \] (19)

where \( \psi^T(k) = [-y(k-1), -y(k-2), \ldots, -y(k-n), u(k), u(k-1), \ldots, u(k-n)] \)

\[ \theta^T = [a_1, a_2, \ldots, a_n, b_0, b_1, \ldots, b_n] \]

\( y(k) \) is the \( k \)-th observation value of the system output, \( y(k-i) \) is the \( (k-i) \)-th observation value of the system output, \( u(k-i) \) is the \( (k-i) \)-th input value of the system input, \( \epsilon(k) \) is the measurement noise of the system, \( a_i \) and \( b_i \) are the coefficients of the input and output of the system that need to be solved, \( \psi^T(k) \) is the observation matrix of the system input and output, \( \theta \) is parameter vector to be identified.

The recursive formula of the least squares estimate vector \( \hat{\theta} \) of the unknown parameter vector \( \theta \) is
\[ \hat{\theta}(k + 1) = \hat{\theta}(k) + K(k + 1)[y(k+1) - \psi^T(k+1)\hat{\theta}(k)] \] (20)

where
\[ K(k + 1) = \frac{P(k)\psi(k+1)}{\lambda + \psi^T(k+1)P(k)\psi(k+1)}, \]
\[ \lambda = \frac{\lambda_1(k)}{\lambda_2(k)} \]
\[ P(k + 1) = \frac{1}{\lambda_1(k)}[[I - K(k+1)\psi^T(k+1)]P(k), \]
\[ \Phi^T = [\psi(1), \psi(2), \ldots, \psi(k)], P(k) = (\Phi^T \Phi)^{-1} \]

\( K(k + 1) \) is the correction gain vector of the new data, \( \lambda \) is a forgetting factor, usually the range of values is \( 0.95 \leq \lambda \leq 0.99 \) [35]. The weights \( \lambda_1, \lambda_2 \) are chosen to correspond to the variation profile of the adaptation gain \( P(k+1) \) which minimizes the error obtained via this algorithm. We discuss the case of a variable forgetting factor for \( \lambda = 1 \) when \( \lambda_1 = \lambda_2 = 1 \) in this paper. \( P(k) \) is the intermediate vector in the algorithm, \( \hat{\theta}(k) \) and \( \hat{\theta}(k + 1) \) are the identification results of the \( k \)-th and \( (k + 1) \)-th, respectively. The initial states \( \hat{\theta}(k) \) and \( P(k) \) are usually taken as [33].

\[ \begin{cases} \hat{\theta}(0)=0 \\ P(0)=\delta E \delta = 10^6 \sim 10^{10}, \ E \text{ is Unit matrix} \end{cases} \] (21)

It can be seen that the constant current \( I(t) \) is used as the input \( I(t) \) of the equivalent circuit system of the supercapacitor cell module from Fig. 2. The terminal voltage of the supercapacitor cell module is the output \( U(t) \) of the equivalent circuit system. Since the three branches are connected in parallel, the voltages at both ends of each branch are equal, the sum of the currents of the branches is equal to the total current, which can be expressed as
\[ U(t) = R_l * I_f(t) + \frac{1}{C_f} \int_0^t I dt \] (22)
\[ U(t) = R_l * I_f(t) + \frac{1}{C_l} \int_0^t I dt \] (23)
\[ U(t) = R_{sd} * I_{sd}(t) \] (24)
\[ I(t) = I_f(t) + I_{sd}(t) \] (25)

From Eqs. (22)-(25), the transfer function of the system is obtained by Laplace transform, which is expressed as
\[ G(s) = \frac{U(s)}{I(s)} = \frac{(R_l R_i R_{sd} C_s s^2 + (R_l C_f R_{sd} + R_l C_i R_{sd})s + R_{sd})/A}{s^2 + (C_f R_{sd} + C_i R_{sd} + R_l C_j + R_l C_i)s + 1/A} \] (26)

where \( A = (R_{sd} R_l + R_{sd} R_f + R_f R_l)C_j C_l \).
The result is obtained by simplifying Eq. (26), which is expressed as
\[
G(s) = \frac{U(s)}{I(s)} = \frac{a_0 s^2 + a_1 s + a_2}{s^2 + \beta_0 s + \beta_1} \tag{27}
\]
where
\[
\begin{align*}
    a_0 &= \frac{R_f R_l C_f C_l}{A} \\
    a_1 &= \frac{(R_f C_f R_l + R_l C_l R_f)}{A} \\
    a_2 &= \frac{R_{sd}}{A} \\
    \beta_0 &= \frac{(C_j R_{sd} + C_l R_{sd} + R_l C_l + R_f C_f)}{A} \\
    \beta_1 &= \frac{1}{A}
\end{align*}
\tag{28}
\]

Since the model structure of RLS is the form of the difference equation that is expressed as Eq. (19), it is necessary to discretize the transfer function of Eq. (26). The first-order backward difference method is used in the paper, \( s \) is the complex variable,
\[
s = \frac{1 - z^{-1}}{T} \tag{29}
\]
where \( z \) is a complex variable, \( T \) is the sampling period.

Substituting Eq. (29) into Eq. (27), The discrete transfer function is obtained, which is expressed as
\[
G(z) = \frac{a_0 (1 - z^{-1})^2 + a_1 T (1 - z^{-1}) + T^2 a_2}{(1 - z^{-1})^2 + \beta_1 T (1 - z^{-1}) + T^2 \beta_2} \tag{30}
\]

Then the discrete transfer function can also be expressed as
\[
G(z) = \frac{b_3 z^{-2} + b_2 z^{-1} + b_1}{a_2 z^{-2} + a_1 z^{-1} + 1} \tag{31}
\]

The corresponding difference equation is expressed as
\[
U(z) = a_1 U(z - 1) + a_2 U(z - 2) + b_1 I(z) + b_2 I(z - 1) + b_3 I(z - 2) \tag{32}
\]
where \( U(z) \) is the \( z \)-th output of the system, \( U(z-i) \) is the \( (z-i) \)-th output of the system, \( I(z) \) is the \( z \)-th input of the system, \( I(z-i) \) is the \( (z-i) \)-th input of the system, \( i = 1, 2, 3 \ldots, a_1, a_2, b_1, b_2 \) and \( b_3 \) are the coefficients of the discrete transfer function.

From Eqs. (30) and (31), \( a_1, a_2, b_1, b_2 \) and \( b_3 \) are obtained, which are expressed as
\[
\begin{align*}
    a_1 &= \frac{2\beta_0 + 2}{4\beta_1 + 2\beta_0 + 1} \\
    a_2 &= \frac{4\beta_1 + 2\beta_0 + 1}{4\beta_2 + 2\alpha_1 + \alpha_0} \\
    b_1 &= \frac{4\beta_1 + 2\beta_0 + 1}{-2\alpha_0 - 2\alpha_1} \\
    b_2 &= \frac{4\beta_1 + 2\beta_0 + 1}{\alpha_0} \\
    b_3 &= \frac{4\beta_1 + 2\beta_0 + 1}{4\beta_1 + 2\beta_0 + 1}
\end{align*}
\tag{33}
\]

\[ \phi^T(z) \] and \( \hat{\theta} \) are obtained by discretizing the experimental input and output observation matrix \( \phi^T(k) \) in Eq. (19) and the parameter matrix \( \theta \) of the equivalent circuit to be identified in the supercapacitor cell module, which are expressed as
\[
\phi^T(z) = [-U(z - 1), -U(z - 2), I(z), I(z - 1), I(z - 2)] \tag{34}
\]
\[
\hat{\theta} = [-a_1, -a_2, b_1, b_2, b_3] \tag{35}
\]

The values of \( a_1, a_2, b_1, b_2 \) and \( b_3 \) can be identified by RLS, then substituting the identified values into Eq. (33), \( \beta_0, \beta_1, a_0, a_1 \) and \( a_2 \) are obtained. Substituting the values into Eq. (28), the values of the equivalent circuit parameters \( R_f, C_f, R_l, C_l \) and \( R_{sd} \) are obtained. In practical engineering applications, the equivalent circuit of the supercapacitor cell module must be of physical meaning, so each parameter is required to be greater than zero.

### C. SEGMENTATION OPTIMIZATION METHOD

In order to reduce the parameter identification error, and to improve the model accuracy, this paper proposes a parameter optimization method for the equivalent circuit model of the supercapacitor cell module based on segmentation optimization (SO). In this new method, the RLS method is used to identify the parameters of equivalent circuit model, but the CA identification results are taken as its recursive initial values, and the experimental test data in the static self-discharge phase are used as its recursive input data. As shown in Fig. 4, SO can not only ensure the accuracy of identification in the charging phase, but also can further reduce the error in the static self-discharge phase.

### IV. EXPERIMENTAL TESTING AND ANALYSIS

#### A. TEST PLATFORM CONSTRUCTION AND DATA ACQUISITION

The research object of the paper is a supercapacitor cell module composed of eight supercapacitor cells connected in series. The supercapacitor cells are manufactured by Maxwell, which product model is BCAP0350 E270 T11 350F. Its rated voltage is 2.7V. The rated voltage and the rated capacitance of the supercapacitor cell module are 21.6V and 43.75F, respectively. In order to measure the parameter identification data, an experimental test platform is set up,
as shown in Figure 5, the tester is a battery tester, whose product model is EBC-A10H. The supercapacitor cell module is charged at a constant current ($I = 1\text{A}$), and allowed to stand for a long time after the charging is completed. In the whole process, the experimental test platform is used for data acquisition. The collected data includes the total sampling time, charging current, voltage across the module, etc. The experimental environment is $23^\circ\text{C}$, the sampling period is 2 seconds.

**B. PARAMETER IDENTIFICATION SIMULATION MODEL ESTABLISHMENT**

After identifying the equivalent circuit parameters, the multi-method parameter identification simulation model of the supercapacitor cell module is built in Matlab/Simulink environment, which is shown in Fig. 6. The accuracy of the identification parameters is verified by simulation. The Simulink simulation sub-model of the equivalent circuit of the supercapacitor cell module is shown in Fig. 7. The Simulink simulation sub-model of the variable capacitance module is shown in Fig. 8, which reflects the variation of the variable capacitance $C_f$ with the terminal voltage.

From Eq. (3) and the equivalent circuit of the supercapacitor, all charges are accumulated in the equivalent capacitance during the constant current charging process of the supercapacitor cell module. The charging ends when the charging current $I$ drops to zero, and the total charge $Q$ accumulated in the equivalent capacitance is expressed as

$$Q = \int_0^t Idt \quad (36)$$

From Eqs. (3) and (36), the relationship between the equivalent capacitance of the supercapacitor and the terminal voltage can be revealed by fitting the experimental data, which is

$$C_f = 0.9199U + 35.91 \quad (37)$$

The experimental test data of the voltage is input into the voltage data block diagram in Fig. 6. The experimental test data of the current is input into the current data block diagram shown in Fig. 6. The variable capacitance parameter $C_f$ identified by CA is input into the Simulink simulation sub-model in Fig. 8. The other parameters are input into the Simulink simulation sub-model of the equivalent circuit of the supercapacitor cell module in Fig. 7. The simulation graph of the identification results by CA is obtained by simulation operation.

The parameter input and simulation of CA-RLS and SO are the same as CA.

**C. SIMULATION AND ANALYSIS**

Based on the above parameter identification simulation models, the parameter of the equivalent circuit of the supercapacitor cell module is identified by CA, CA-RLS and SO, respectively. The data used for the above parameter identification is the test data measured by the experimental platform. The model output voltage simulation results and the experimental test voltage are compared, and the errors are analyzed to verify the accuracy of the identified parameters. The relative error is calculated as shown in Eq. (38). The absolute value of the relative error is shown in Eq. (39). The comprehensive error is shown in Eq. (40). Fig. 9 summarizes
The schematic process of the parameter identification simulation and error analysis.

\[ r_m = \frac{U_{Sim} - U_{EX}}{U_{EX}} \times 100\%, \ m = 1, 2, 3 \ldots \] (38)

\[ |r_m| = \left| \frac{U_{Sim} - U_{EX}}{U_{EX}} \right| \times 100\%, \ m = 1, 2, 3 \ldots \] (39)

\[ r_c = \frac{1}{m} \sum_{1}^{m} |r_m| \] (40)

where \( r_m \) is the relative error between the simulation results of the methods and the experimental test data, \(|r_m|\) is the absolute value of the relative error between the simulation results of the methods and the experimental test data, \( r_c \) is the comprehensive error of the simulation results and the experimental test data, \( U_{Sim} \) is the voltage of the simulation output, \( U_{EX} \) is the experimental test voltage, \( m \) is the number of samples.

1) SIMULATION AND ERROR ANALYSIS OF CA IDENTIFICATION RESULTS

The parameters of the equivalent model of supercapacitor cell module are identified by CA. According to the experimental test data of the whole process from charging to static self-discharge, the experimental test data are summarized in Tab.1 and from Eqs. (1), (2), (4)-(16), the parameter identification results of the three-branch equivalent model are calculated. The results are summarized in Tab.2. The identified parameters are input into the simulation sub-models in Fig. 7 and Fig. 8, respectively, and the simulation is carried out in Matlab/Simulink environment. The simulation curve is shown in Fig. 10, and the curve of relative error (\( r_m \)) is shown in Fig.11.

| voltage /V  | Time /s |
|-------------|---------|
| \( U_0 \)  | \( t_1 \)  | 2       |
| \( U_1 \)  | \( t_2 \)  | 918     |
| \( U_2 \)  | \( t_3 \)  | 940     |
| \( U_3 \)  | \( t_4 \)  | 992     |
| \( U_4 \)  | \( t_5 \)  | 2698    |
| \( U_5 \)  | \( t_6 \)  | 8998    |

| \( R_f/\Omega \) | \( C_f/\mu F \) | \( R_i/\Omega \) | \( C_i/\mu F \) | \( R_o/\Omega \) |
|-----------------|---------------|---------------|---------------|---------------|
| 0.275           | 47.1904       | 475.5388      | 1.2007        | 17200         |
It can be seen that the simulation curve of CA is basically in accordance with the trend of the experimental curve in Fig. 10. As shown in Fig. 11, the error \( r_m \) is [-5%, 5%] in the charging phase, that is, 0 < \( t < 938s \). In the static stage after the charging is completed, that is, \( t > 938s \), the error increases with the extension of the standing time. When \( t = 9000s \), the error \( r_m \) is 8.33%. According to the experimental results and the error of the simulation results, the comprehensive error \( r_c \) can be obtained, which is 6.63%. The comprehensive error \( r_c \) during the charging phase of CA is 2.52%, and the comprehensive error \( r_c \) during the static self-discharge phase of CA is 7.11%. Therefore, the simulation results reveal that the model parameters identified by CA can effectively simulate the dynamic characteristics of the supercapacitor cell module during the charging phase, and that the ability to identify the parameters of the supercapacitor cell module is insufficient during the stationary phase.

2) SIMULATION AND ERROR ANALYSIS OF CA-RLS IDENTIFICATION RESULTS

According to theoretical analysis of RLS [24], \( P(0) = 10^{6}E \) and \( \theta(0) = [0 0 0 0 0] \) are generally chosen as the initial values of RLS, and the experimental test data of the whole process from charging to static self-discharge are used as the recursive input data. The effective model parameters are identified by RLS, the identification results are presented in Tab. 3. Since each parameter in the equivalent circuit of the supercapacitor cell module has its physical meaning, it must be greater than zero, but \( C_l = -0.10278 < 0 \) in the identification result. There is an error in the identification. This shows that the initial values \( P(0) = 10^{6}E, \theta(0) = [0 0 0 0 0] \) are not feasible when the parameters of the equivalent model of the supercapacitor cell module are identified by RLS.

Because the model parameters identified by CA can effectively represent the dynamic characteristics of the supercapacitor cell module during the charging stage, this paper proposes a Circuit Analysis-Recursive Least Squares Method (CA-RLS) by combining CA and RLS. The primary enhancement is that the parameters identified by CA are used as the initial value of RLS, as shown in Tab. 2. Substituting them into Eq. (28), the value of \( \beta_0, \beta_1, \alpha_0, \alpha_1 \) and \( \alpha_2 \) are obtained, then \( \hat{\theta}(0) \) is obtained by substituting them into Eq. (33). The \( \delta \) is determined as 0.22 after many debugging. The specific results are shown in Eq. (41).

\[
\begin{bmatrix}
-1.996405566351870 \\
0.996405574979998 \\
0.317149367712585 \\
-0.590849085786264 \\
0.273848121865746
\end{bmatrix}
\begin{bmatrix}
0.22
\end{bmatrix}
\]

Eq. (41) is used as the initial value of RLS, and the experimental test data of the whole process from charging to static self-discharge are used as the recursive input data, the model parameters are identified by CA-RLS, the identification results are shown in Tab. 4. Then, the identified parameters are input into the simulation model in Fig. 7 and Fig. 8. The simulation result is shown in Fig. 12. Fig. 13 shows the error curve of CA-RLS. Fig. 14 shows the error curve of CA-RLS after the errors taking the absolute value.

From Fig. 12, it can be seen that the simulation curve is very close to the experimental curve. When 0 < \( t < 4148s \), the simulated voltage is larger than the experimental test voltage. When \( t = 4148s \), the simulation voltage is closest to the experimental voltage. When \( t > 4148s \), the simulation voltage is smaller than the experimental voltage and decrease with time delay. In general, the simulation curve is basically consistent with the identification simulation curve of CA.
When $0 < t < 498s$, the simulation curve is above the identification simulation curve of CA. When $t = 498s$, the simulation curve is most consistent with the identification simulation curve of CA. When $t > 498s$, the simulation curve is below the method identification simulation curve of CA, which is of high error.

As indicated by Fig. 13, the error first increases, and then decreases in the charging phase of the supercapacitor cell module, that is, $0 < t < 938s$, the maximum of the error $r_m$ is 0.78%. However, the maximum of the error $r_m$ by CA in the same time period is 5%. Compared with CA, the identification accuracy has greatly improved in the charging phase. In the stationary phase after charging is completed, when $938s < t < 4148s$, the error first increases, and then decreases, the maximum of the error $r_m$ is 0.6%. When $t = 4148s$, the minimum of the error $r_m$ is obtained, which is 0.0017%. When $t > 4148s$, starting from the feature point A in Figure 14, the error is less than 0 and $|r_m|$ is increased gradually. When $t = 9000s$, the error $r_m$ is $-2.77\%$.

According to the experimental results and the error of the simulation results, the comprehensive error $r_c$ is obtained, it is 0.86%, which is 5.77% lower than the CA. The comprehensive error $r_c$ during the charging phase is 0.36%, which is 2.16% lower than the CA. The integrated error $r_c$ during the static self-discharge phase is 0.92%, which is 6.19% lower than CA.

By fitting the curve of the AB segment and the CD segment in Fig. 14, the slope of the AB segment is 2.088, and the slope of the CD segment is 0.972. The slope of the AB segment is 2.1 times that of the CD segment, that is, the growth rate of the AB segment is greater than the growth rate of the CD segment.

In summary, the CA-RLS inherits the advantages of CA, it can accurately describes the characteristics of the supercapacitor cell module in the charging phase, and further reduces the identification error. The results also indicate that the CA-RLS can accurately simulate the self-discharge characteristics of the supercapacitor cell module during the static stage after charging. However, the error changes greatly after the feature point A, which causes the reduced accuracy.

3) SIMULATION AND ERROR ANALYSIS OF SEGMENTATION OPTIMIZATION IDENTIFICATION RESULTS

The results of the above two methods show that CA can accurately simulate the dynamic characteristics of the supercapacitor cell module during the charging phase, and that CA-RLS can more accurately simulate the self-discharge characteristics of the supercapacitor cell module in the static stage after it is fully charged.

Eq. (41) is used as the initial value of RLS, and the experimental test data of the static self-discharge are used as the recursive input data, then the effective model parameters are identified by SO, the results are given in Tab.5. The identified parameters are input into the simulation circuit in Fig. 7 and Fig. 8 to simulate in Matlab/Simulink environment. The simulation result is shown in Fig. 15. Fig. 16 is the error curve of SO. Fig. 17 is the error curve of SO after the errors take the absolute value.

Fig. 15 indicates that the simulation curve is in line with the experimental curve. When $0 < t < 4960s$, the simulation voltage is larger than the experimental voltage. When $t > 4960s$, the simulation voltage is smaller than the experimental voltage, and decrease with time delay. When $0 < t < 102s$, the curve is between the CA curve and the CA-RLS curve. When $102 < t < 1068s$, the curve is below the CA-RLS curve. When $1068 < t < 4960s$, the curve is between the CA curve and the CA-RLS curve, and is above the experimental curve. When $t > 4960s$, the curve is between the experimental curve and the CA-RLS curve.

From Fig. 16, it can be seen that the error first increases and then decreases in the charging phase of the supercapacitor cell module, that is $0 < t < 938s$, the maximum value of the error

| Table 5. The parameter identification results of SO. |
|-----------------|-----------------|-----------------|-----------------|
| $R_f/\Omega$     | $C_f/F$         | $R_s/\Omega$   | $C_s/F$        |
| 0.2752          | 47.1805         | 279.8238       | 2.0319         | 2964.01        |

Fig. 15: Identification results Simulation diagram of SO.
the error in the static self-discharge phase. It can effectively identify the parameters of the equivalent model of the super capacitor module.

V. CONCLUSION

The paper takes the supercapacitor cell module as the object, and establishes the three-branch equivalent circuit model that is suitable for the supercapacitor cell module. Besides it also constructs the Simulink simulation model for multi-method parameter identification of supercapacitor cell modules, and the simulation and analysis of CA, RLS, CA-RLS are carried out, respectively. CA has the disadvantage that the identification precision during the stationary phase is imprecise. As RLS adopts universal recursive initial values to identify parameters, the identification results do not meet the actual application requirements. CA-RLS uses the whole process data for identification, which is of the disadvantages of low identification precision during the stationary phase.

In view of the disadvantages of CA and CA-RLS, the paper puts forward a segmentation optimization based parameter identification method for the equivalent circuit model of the supercapacitor cell module. It takes CA identification result as the initial value of the identification parameter of RLS and the experimental test data in the static self-discharge phase are used as the recursive input data of CA-RLS to identify. The results of simulation and error analysis show that SO is of higher recognition accuracy than CA and CA-RLS. In particular, the comprehensive error in the static self-discharge phase is 0.28%, which is 6.83% lower than CA, and is 0.64% lower than CA-RLS. The comprehensive error in the whole process is 0.32%, which is reduced by 6.31% and 0.54% compared with CA and CA-RLS, respectively.

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\( r_m = 1.17\% \), which is greater than the error of CA-RLS in the charging phase (0.78%), but is much smaller than the error of CA in the charging phase (5%). In the stationary phase after the completed charging, that is, \( t > 938s \), the error curve first changes straight, then slopes down slightly. The rate of change is much less than the CA-RLS method. When \( 938s < t < 4714s \), the error \( r_m \) is \([0.01\%, 0.26\%]\). When \( 4714s < t < 4730s \), the error \( r_m \) decreases to 0.001% gradually. When \( 4732s < t < 4960s \), the error \( r_m \) is \([-0.069\%, 0.061\%]\). When \( t > 4960s \), starting from the feature point E in Figure 17, the error \( r_m \) is less than 0 and its \( |r_m| \) is increased gradually. When \( t = 9000s \), the error \( r_m \) is \(-1.03\%\).

According to the experimental and simulation results, the comprehensive error \( r_c \) is obtained, the \( r_c = 0.32\% \), which is 6.31% lower than CA, and is 0.54% lower than CA-RLS. The comprehensive error \( r_c \) in the charging phase is 0.66%, which is 1.85% lower than CA, and is 0.3% higher than CA-RLS. The comprehensive error \( r_c \) in the static self-discharge phase is 0.28%, which is 6.83% lower than CA, and is 0.64% lower than CA-RLS.

By fitting the curve of the EF segment and the GH segment in Fig. 17, the slope of the EF segment is 0.936, and the slope of the GH segment is 0.9. It can be seen that the slope of the EF segment is slightly larger than the slope of the GH segment, that is, the growth rate of the EF segment is basically the same as the growth rate of the GH segment.

Therefore, SO can not only ensure the accuracy of identification in the charging phase, but also can further reduce the error in the static self-discharge phase. It can effectively identify the parameters of the equivalent model of the super capacitor module.

\( FIGURE \ 16. \ Curve \ of \ relative \ error \ (r_m). \)

\( FIGURE \ 17. \ Curve \ of \ relative \ error \ took \ absolute \ value \ (*r_m*). \)
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