High-Momentum Correlated Nucleons, Tensor Blocking, and Nuclear Shell Structure

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Abstract. Studies of light neutron-rich nuclei revealed many new phenomena in nuclear structures and reactions. One of the most important recent discoveries is the change of magic numbers. Traditional magic numbers (2, 8, 20, 28, 50, 128) have been understood by the spin-orbit coupling in the mean-field potential and they provide the base of nuclear shell model. Recently, however, it was found that some of those magic numbers \((N=8, 20)\) disappear in neutron-rich nuclei. On the other hand, new magic numbers \((N=6, 14, 16, 32\) and 34) emerge in neutron-rich nuclei. In the present paper, it is shown that tensor correlations give notable effects to such new structures. In nuclei a large amount of binding energy is gained by high-momentum correlated pairs of nucleons due to the tensor interaction. Such tensor correlations strongly depend on the configuration space available for exciting the 2p-2h states. When additional neutrons occupy a new orbital, the previously available 2p-2h configurations may be blocked, resulting in a sudden loss of binding energy otherwise gained by the 2p-2h excitations. Such tensor blocking effects enlarge the energy gaps at all the observed new magic numbers.

1. Introduction

Structures of neutron-rich nuclei are being studied after beams of radioactive nuclei became available [1]. Many new properties of nuclei have been observed such as neutron halos, neutron skins, and others [2]. Among important findings are the magic numbers and stabilities of neutron rich nuclei as listed below.

1. Why asymmetric "doubly magic" \(^{10}\text{He}\) and \(^{28}\text{O}\) are not bound. On the other hand, why \(^{48}\text{Ca}\) is doubly magic?
2. Is there any common mechanism on emergence of new \(N=6, 14, 16, 32, \) and 34 magic numbers? [3,4,5,6]
3. Why magic numbers \(N=8\) and \(N=20\) disappear in neutron rich nuclei?
4. Why the neutron dripline suddenly extends very much in F isotopes? [7,8]

Each of the phenomena above has been studied extensively and some suggested models could reproduce each phenomon individually. However, no unified mechanism for understanding these phenomena has been discussed.

Recent ab-initio type calculations by Green-function Monte Carlo method shows the importance of pions in binding nuclei and indicates that the 80% of binding energy comes from pion exchanges [9,10]. It is well known that the strong tensor interaction is caused by pion exchange between nucleons due to the pseudo-scalar nature of the pion. The large amount of binding energy comes from tensor-correlated pairs in nuclei, which has large relative momentum that could not be introduced by mean field models.

In addition to S-wave, the lightest nucleus deuteron has a \(~10\%\) D-wave component which is caused by the tensor interaction. The tensor interaction provides a large amount of potential energy and thus deuteron is bound [11]. It should be noted that this large binding come from the cross term of S- and D-wave. It provides much more binding energy than S-wave diagonal term. (SS term gives 3.96 MeV and SD term gives 16.64 MeV of potential energies) The most important is that this D-wave includes a high-momentum component that is not included in usual shell model wave functions.
The $^4\text{He}$ nucleus has a large binding energy of 28.3 MeV. A large fraction of the binding is also caused by the tensor interaction [12]. The total potential energy gained by the tensor interaction is 54.6 MeV using the AV8’ potential in the tensor optimized shell model [13]. Two-particle two-hole (2p-2h) excitations by the tensor interaction include excitations to higher orbitals that include high-momentum nucleons. The tensor optimized shell model (TOSM) calculations showed that the excitations to $l=1$ orbitals only already provide a large potential energy, and among them the largest contribution comes from the 2p-2h excitation from the $(1s\frac{1}{2})^2$ to the $(1p\frac{1}{2})^2$ orbitals (see Fig. 1) providing 8.4 MeV of potential energy [13]. More general, the largest contribution comes from the excitation of $S=1$ pn pairs from filled $(nlj)^2$ configuration to open $(n+1l\pm lj)^2$ or $(n+1l-lj)^2$ orbitals.

When the corresponding upper shell orbital is occupied by nucleons, tensor blocking occurs because the 2p-2h excitation to the orbital is blocked by the Pauli principle. In $^{12}\text{C}$ nucleus such blocking does not occur at all, because $1p\frac{1}{2}$ and $1d\frac{3}{2}$ orbitals are open for excitations of pn pairs from $1s\frac{1}{2}$ and $1p\frac{3}{2}$ orbitals, respectively. The $1p\frac{1}{2}$ orbital is occupied in $^{16}\text{O}$, then $(1s\frac{1}{2})^2$ to $(1p\frac{1}{2})^2$ excitation is blocked and the nucleus loses the potential energy gained otherwise. In $^{16}\text{O}$, however, the added proton and neutron pairs in the $1p\frac{1}{2}$ shell open new excitations to the $(2s\frac{1}{2})^2$ and gain potential energy. When the mass number increases along the line of stability, tensor blocking and tensor opening occur simultaneously. It thus makes the saturation property of the nuclear binding. This is exactly the same mechanism that produces the saturation property of nuclear matter explained by H. A. Bethe [14]. It should be noted however that tensor blocking occurs even if only neutron orbitals are filled. When only the neutron number increases, tensor blocking occurs but tensor opening does not occur. It therefore makes a difference in the effects of the tensor interactions between $Z-N$ nuclei and neutron-rich nuclei. As shown in the right panel of Fig. 2 for $^{28}\text{O}$, all $\Delta l=1$ 2p-2h transition is blocked loosing all binding by the tensor interactions from $\Delta l=1$.

The tensor blocking effect in nuclei has been demonstrated by Myo et al. in $^{11}\text{Li}$ [15]. To form $^{11}\text{Li}$, two-neutrons added to $^9\text{Li}$, in which the $1p\frac{3}{2}$ orbital is fully occupied, have to be filled in higher orbitals. Although the normal shell model predicts $1p\frac{1}{2}$ as the next lowest orbital, tensor blocking occurs if neutrons occupy the $1p\frac{1}{2}$ orbital and thus the occupation of the $1p\frac{1}{2}$ orbital is disfavored. The contribution of $2s\frac{1}{2}$, the next expected orbital, then become important because no tensor blocking occurs for that orbital. It is because $2s\frac{1}{2}$ orbital is not used for gaining the tensor energy.

Recently, high-momentum neutrons and tensor correlated high-momentum p–n pairs in $^{16}\text{O}$ ground state have been reported experimentally [16,17]. A clear effect of high-momentum correlated p-n pair have been observed by $^{16}\text{O}(p, pd)^{14}\text{N}$ reaction with incident proton energy of 400 MeV.
experiment shows that $T=0, S=1$ pairs are dominated compared with $T=1, S=0$ pairs when a high-momentum neutron is picked up to form a final state deuteron. It indicates the importance of tensor correlated pairs in the ground state of $^{16}\text{O}$.

2. The tensor blocking in shell model

To include the effect of the tensor interaction through high-momentum correlated nucleon-pairs, we introduce a simple model based on the TOSM [18,19], which includes 2p-2h excitations explicitly in the wave function, $\Psi = \psi_{Sh} + \psi_{2p-2h}$, where $\psi_{Sh}$ is the usual shell model wave function and $\psi_{2p-2h}$ expresses the 2p-2h states excited by the tensor interaction and include high-momentum nucleons. The Hamiltonian includes both the central and the tensor interactions written as, $H = T + H_c + H_T$, where $T$ is the kinetic energy of nucleons and

$$H_c = \sum v^c_l \quad \text{and} \quad H_T = \sum v^t_l \quad .$$

(1)

where $H_c$ contains the two-body central interactions including the spin–orbit interactions, and $H_T$ denotes the tensor interactions. The potential energy of a nucleus is calculated by,

$$\langle \Psi | H - T | \Psi \rangle = \langle \psi_{Sh} | H_c | \psi_{Sh} \rangle + \langle \psi_{2h} | H_T | \psi_{2h} \rangle + 2 \langle \psi_{Sh} | H_T | \psi_{2p-2h} \rangle + \langle \psi_{2p-2h} | H_T | \psi_{2p-2h} \rangle \quad .$$

(2)

The second term on the right-hand side is the monopole term arising from exchange interactions (Fock term) of the tensor interaction, and gives an important contribution to changes of $l s$ splitting in neutron-rich nuclei [20]. The last two terms are contributions from the 2p-2h excitations, which provide a large binding energy. The contribution from the last term (diagonal term) is very small so that it is ignored in the following discussion. A large energy gain comes from the third term (cross term). For example, in $^4\text{He}$ nucleus [13] 2p-2h excitations provide 55 MeV of attraction by the tensor interaction, which is comparable with the potential energy of 56 MeV from the central interactions in the first term in eq. (2). The wave function $\psi_{2p-2h}$ includes excitations to many higher $l$ orbitals. Among them, more than 10 MeV binding comes from 2p-2h excitations to $\Delta l=1$ alone. So we have to remember that the 2p-2h excitations of $\Delta l=1$ as providing about 10 MeV binding in a nucleus. Higher excitations ($\Delta l \geq 2$) provides remaining potential energy ~45 MeV. As seen in TOSM calculations, the inclusion of higher $l$ excitations, however, requires considerable computer power and it is time consuming in practice to complete the calculation for nuclei above B isotopes [18, 19].

In light neutron-rich nuclei, neutrons may occupy the next major shell ($\Delta l=1$) but $\Delta l \geq 2$ shells are always open. The blocking of 2p-2h excitations by the occupation of nucleons, thus, does not occur for $\Delta l \geq 2$ excitations. Blocking occurs only for $\Delta l=1$ 2p-2h excitations.

In usual shell models two-body interactions are truncated to one-body mean-field potential and two-body residual interactions,

$$H = T + V_{Sh}(r) + \sum \tilde{V}^0_l \quad ,$$

(3)

where $V_{Sh}(r)$ is the mean-field potential from which single particle orbitals are formed, $\tilde{V}^0_l$ is the residual two-body interactions. In general $V_{Sh}(r)$ comes from both central and tensor two-body interactions, $V_{Sh} = V_c + V_T$. $V_T$ comes from many excitations to higher $l$ orbitals and can be written as a sum of contributions from different $\Delta l$ components as done in TOSM calculations,

$$V_T = V^0_{T1} + \sum_{\Delta l \neq 1, 2} V^0_{T\Delta l} \quad ,$$

(4)

where $\Delta l$ terms are separated just for the purpose below. When symmetric closed shell nuclei are concerned, all orbitals above the closed shell are open for 2p-2h excitations. When nucleons are added to a closed shell nucleus, upper orbitals are occupied and blocking of 2p-2h excitations may occur. For known neutron rich light nuclei, excess nucleons exist only in orbitals of one major shell above, only $V_{T1}$ term changes when nucleon number changes. If we introduce $V^0_{T1}$ as the value of $V_{T1}$ when all upper orbitals are open, Eq. (4) can be written as,
\[ V_T = V_{T1} - V_{T1}^0 + (V_{T1}^0 + \sum_{M=2} V_{TM}) \]
\[ = (V_{T1} - V_{T1}^0) + V_T^0 \]  

where \( V_T \) is the potential energy given by the tensor interactions when all orbitals above the closed shell are open. Therefore \( V_c + V_T^0 \) is the shell model potential \( (V_{Sh}) \) used for closed shell nuclei and can effectively be replaced by an effective single particle potential such as Woods-Saxon potential with spin-orbit term. The remaining term \( (V_{T1} - V_{T1}^0) \) is the difference of potential energy when upper orbitals are occupied and the energy gain from 2p-2h excitations are blocked. We call this as the tensor blocking energy for the \( \Delta l=1 \) shell. In the following we see the behavior of this term when the number of nucleons increases in p, sd, and fp shells.

In the following discussion, the Woods–Saxon potential with standard spin–orbit coupling is used as the effective potential to obtain the starting single-particle orbitals. The starting single-particle orbitals at \( A/Z=3 \) are shown in Fig. 3 in which the potential parameters are taken from Bohr–Mottelson [21]. We consider that most of the potential energy obtained by the tensor interaction is effectively included in this Woods–Saxon potential near the stability line. It is noted that the order and spacing of the single-particle orbitals change depending on the binding energy. The low angular momentum orbitals gain binding energy under weakly bound conditions, which is a halo effect.

The tensor blocking shell model treats separately the sudden change of the binding energy of a nucleus due to tensor blocking: the \( (V_{T1} - V_{T1}^0) \) term. Important 2p-2h excitations and their potential energies are selected from the results of TOSM [13,15,18,19,22]. For \( \Delta l=1 \) excitations, the largest contribution comes from the excitation of \( S=1 \) p–n pairs from the \( (nlj)^2 \) configuration to \( (n+1l+1j)^2 \) or \( (n+1l-1j)^2 \). Blocking of such excitations reduces the binding energy considerably according to TOSM calculations. For example, an excitation from \( 1s_{1/2} \) to \( 1p_{1/2} \) gives about 7 MeV of binding energy. By this effect, the filling of weakly bound orbitals is drastically affected.

Part of the \( ls \) splitting is known to originate from the tensor interaction [23]. The present blocking effects affect the splitting between \( j_+ \) and \( j_- \) orbitals and would contribute to the \( ls \) splitting. However presently we do not know how to remove the effect of the tensor interaction in the shell model \( ls \) splitting. Therefore, we used the usual \( ls \) splitting without modification. In the following we discuss the ground state properties of light neutron-rich nuclei under the tensor-blocking shell model semi-quantitatively using the results of TOSM calculation. More quantitative calculations are in progress based on a phenomenological analysis.

Fig. 3 Single-particle orbitals in a Woods–Saxon potential. Numbers in circles show the traditional magic numbers.
3. Loss of traditional magic numbers and appearance of new magic numbers

An example of an abrupt change of binding is seen in He isotopes. Until $^8$He, neutrons mostly occupy $p_{3/2}$ orbital, and the further addition of neutrons to form $^{10}$He is expected to fill the bound $p_{1/2}$ orbital of ~4MeV binding energy. However, the filling of the $p_{1/2}$ orbital blocks the 2p-2h excitation from the $1s_{1/2}$ orbital to the $1p_{1/2}$ orbital and thus a loss of binding energy occur. The loss of energy is larger than the binding energy of the $1p_{1/2}$ orbital and thus creates a large energy gap above $N=6$, making $N=6$ magic and $^{10}$He unbound.

A similar blocking effect occurs in O isotopes. Six neutrons in $^{22}$O nucleus mostly fill the $1d_{5/2}$ orbitals. The open $2s_{1/2}$ orbital is used for 2p-2h excitation from $1p_{1/2}$ to $2s_{1/2}$ orbitals. Another 2p-2h excitation from $1p_{3/2}$ to $1d_{3/2}$ is also open. Adding neutrons to either of the orbital blocks the tensor interaction and widens the energy gap. The energy gap thus produced makes $N=14$ a magic number. After filling two neutrons in $2s_{1/2}$ after the gap, the addition of neutrons to the $1d_{3/2}$ orbital again blocks the tensor interaction and thus another energy gap occurs at $N=16$, creating another magic number. This time the $1d_{3/2}$ orbital is weakly bound and the blocking makes both $^{24}$O and $^{28}$O unbound. The addition of protons (F isotopes), instead, opens new configurations for the 2p-2h excitations: the excitation of an sd-shell proton–neutron pair into the fp shell. It therefore suddenly increases the binding of nuclei compared with neutron-rich O isotopes, and thus makes the dripline of F isotope extend much more than that of O isotope.

The same mechanism works for Ca isotopes, as seen in the expected shell orbitals in Fig. 3. In Ca isotopes, the open shells $2p_{3/2}$, $2p_{1/2}$ and $1f_{5/2}$ are used for 2p-2h excitations from the sd shell. Therefore, tensor blocking occurs in those orbitals and larger energy gaps are created for neutron-rich isotopes of Ca at $N=28$, 32 and 34. $^{48}$Ca, the first asymmetric doubly closed shell nuclei in the neutron-rich region, is enhanced by 2p-2h excitations due to tensor interactions. Tensor blocking does not occur at the $1f_{7/2}$ orbital but occurs at the $2p_{3/2}$ orbital and enlarges the gap at $N=28$. Blocking occurs for $^{10}$He and $^{28}$O because they have $LS$ closed shells. However, blocking does not occur in $jj$ closed $^{48}$Ca.

4. Break down of the magic numbers in neutron-rich nuclei

As discussed above, the tensor blocking mechanism helps to mix the 1p shell and 2s1d shell when the numbers of protons and neutrons are very asymmetric. Before protons fill the $1p_{1/2}$ orbital significantly, a neutron in the $1p_{1/2}$ orbital exhibits tensor blocking, while the $1d_{5/2}$ and $2s_{1/2}$ orbitals do not cause tensor blocking. The tensor blocking helps to narrow or even close the shell gap between the p-shell and the sd-shell originally expected by the mean field potential.

A similar phenomenon is expected for $N=20$ neutron-rich nuclei just above O. While the $1d_{3/2}$ neutron orbital exhibits blocking, the upper $1f_{7/2}$ and $2p_{3/2}$ orbitals do not cause blocking. The tensor blocking helps to mix the fp shell orbitals into the sd shell and thus should contribute to making an island of inversion. It is suggestive to see that the limit of the island is at around Si where proton starts to fill $2s_{1/2}$ shells and open new 2p-2h excitations.

It should be noted, under present consideration, deformations of the nuclei may not be the origin of the break down of $N=20$ magic number, but the results of break down of the magic number. The breakdown of the magic number brings $1d_{3/2}$ up into fp shell. An increase of an orbital density, then drive the deformations.

5. Summary and conclusion

In summary, we examined recently discovered change of magic numbers in neutron-rich nuclei under a new paradigm of 2p-2h excitations (or high-momentum correlated p–n pairs) based on an explicit treatment of the tensor interaction in nuclei. All the new magic numbers $N=6$, 14, 16, 32 and 34 could be understood as being due to blocking of the tensor interaction. The blocking generates abrupt changes of binding energy and forms an additional energy gap at the neutron-rich region. It is understood that this blocking mechanism effectively works only for very asymmetric nuclei. It was, thus, found that high-momentum nucleons and tensor blocking play essential roles in the structure of asymmetric nuclei. Theoretical developments that include an explicit treatment of the tensor interaction through two-particle two-hole excitations of high-momentum proton–neutron pairs are anticipated for a more quantitative understanding.
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