Improved Measurements of the Branching Fractions for $B^0 \to \pi^+\pi^-$ and $B^0 \to K^+\pi^-$, and a Search for $B^0 \to K^+K^-$

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We present measurements of the branching fractions for the charmless two-body decays $B^0 \rightarrow \pi^+ \pi^-$ and $B^0 \rightarrow K^+ \pi^-$, and a search for the decay $B^0 \rightarrow K^+ K^-$. We include the effects of final-state radiation from the daughter mesons for the first time, and quote branching fractions for the
inclusive processes \(B^0 \rightarrow h^+ h^- \gamma\), where \(h\) and \(h'\) are pions or kaons. The maximum value of the sum of the energies of the \(n\) undetected photons, \(E_{\gamma}^{\text{max}}\), is mode-dependent. Using a data sample of approximately 227 million \(\Upsilon(4S) \rightarrow B \overline{B}\) decays collected with the BABAR detector at the PEP-II asymmetric-energy \(e^+ e^-\) collider at SLAC, we measure:

\[
\begin{align*}
\mathcal{B}(B^0 \rightarrow \pi^+ \pi^- n\gamma; E_{\gamma}^{\text{max}} = 150 \text{ MeV}) &= (5.1 \pm 0.4 \pm 0.2) \times 10^{-6}, \\
\mathcal{B}(B^0 \rightarrow K^+ \pi^- n\gamma; E_{\gamma}^{\text{max}} = 105 \text{ MeV}) &= (18.1 \pm 0.6 \pm 0.6) \times 10^{-6}, \\
\mathcal{B}(B^0 \rightarrow K^+ K^- n\gamma; E_{\gamma}^{\text{max}} = 59 \text{ MeV}) &< 0.5 \times 10^{-6} \text{ (90\% confidence level)},
\end{align*}
\]

where the first uncertainty is statistical and the second is systematic. Theoretical calculations can be used to extrapolate from the above measurements the non-radiative branching fractions, \(\mathcal{B}^0\).

Using one such calculation, we find:

\[
\begin{align*}
\mathcal{B}^0(B^0 \rightarrow \pi^+ \pi^-) &= (5.5 \pm 0.4 \pm 0.3) \times 10^{-6}, \\
\mathcal{B}^0(B^0 \rightarrow K^+ \pi^-) &= (19.1 \pm 0.6 \pm 0.6) \times 10^{-6}, \\
\mathcal{B}^0(B^0 \rightarrow K^+ K^-) &< 0.5 \times 10^{-6} \text{ (90\% confidence level)}.
\end{align*}
\]

Meaningful comparison between theory and experiment, as well as combination of measurements from different experiments, can be performed only in terms of these non-radiative quantities.

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Charmless hadronic two-body \(B\) decays to pions and kaons provide a wealth of information on \(CP\) violation in the \(B\) system, including all angles of the unitarity triangle. The time-dependent \(CP\) asymmetries in the \(\pi\pi\) system can be used to estimate the angle \(\alpha\); the decay rates for the \(K\pi\) channels provide information on \(\gamma\). Recently, direct \(CP\) violation in decay was established in the \(B\) system through observation of a significant rate asymmetry between \(B^0 \rightarrow K^+ \pi^-\) and \(\overline{B}^0 \rightarrow K^- \pi^+\) decays. As \(B\) physics experiments accumulate much larger data sets, charmless two-body \(B\) decays will continue to play a fundamental role in testing the standard model description of \(CP\) violation. Measurements of branching fractions for all the charmless two-body decays are invaluable in testing the various theoretical approaches to the underlying hadron dynamics.

We present measurements of branching fractions for the decays \(B^0 \rightarrow \pi^+ \pi^-\) and \(K^+ \pi^-\), and a search for the decay \(B^0 \rightarrow K^+ K^-\) using a data sample about 2.5 times larger than that used for the most precise, previously published measurements of these quantities.

As radiative corrections have already proved to be important in precise determinations of interesting quantities in the context of kaon physics, we account for them in this analysis as well. We can relate the observable decay rates \(\Gamma_{hh'}(E_{\gamma}^{\text{max}})\) for \(B^0 \rightarrow h^+ h^- \gamma\) (and thus the branching fractions) to the theoretical non-radiative widths \(\Gamma_{hh'}^0\), using the energy-dependent correction factors \(G_{hh'}(E_{\gamma}^{\text{max}}; \mu)\) and \(\Gamma_{hh'}(E_{\gamma}^{\text{max}}) = \Gamma(B^0 \rightarrow h^+ h^- \gamma) \sum_{E_{\gamma} < E_{\gamma}^{\text{max}}} \sigma_{\gamma}, G_{hh'}(E_{\gamma}^{\text{max}}; \mu) = \Gamma_{hh'}^0(\mu) G_{hh'}(E_{\gamma}^{\text{max}}; \mu),\)

where \(E_{\gamma}^{\text{max}}\) is the maximum value allowed for the sum of the undetected photon energies and \(\mu\) is the renormalization scale at which \(\Gamma_{hh'}^0\) and \(G_{hh'}(E_{\gamma}^{\text{max}})\) are calculated (the product being independent of \(\mu\)). Extracting \(\Gamma_{hh'}^0\), allows a more meaningful comparison with theoretical calculations and also between different experimental results. Additionally, for \(E_{\gamma}^{\text{max}}\) at the kinematic limit, \(G\) approaches unity (to order \(\alpha_{\text{QED}}/\pi\), so that the \(\Gamma_{hh'}^0\), and the corresponding branching fractions, can be interpreted theoretically in a cleaner way.

The data sample used for this analysis contains \((226.6 \pm 2.5) \times 10^6\) \(\Upsilon(4S) \rightarrow B \overline{B}\) decays collected by the BABAR detector at the SLAC PEP-II \(e^+ e^-\) asymmetric-energy storage ring. The primary detector components used in the analysis are a charged-particle tracking system consisting of a five-layer silicon vertex detector and a 40-layer drift chamber surrounded by a 1.5-T solenoidal magnet, an electromagnetic calorimeter comprising 6580 CsI(Tl) crystals, and a dedicated particle-identification system consisting of a detector of internally reflected Cherenkov light providing at least 3 \(\sigma\) \(K\)–\(\pi\) separation over the range of laboratory momentum relevant for this study (1.5–4.5 GeV/c).

The data sample used in this analysis is similar to that used in the BABAR measurements of direct \(CP\) violation in \(B^0 \rightarrow K^+ \pi^-\) and time-dependent \(CP\)-violating asymmetry amplitudes \(S_{\pi\pi}\) and \(C_{\pi\pi}\) in \(B^0 \rightarrow \pi^+ \pi^-\) (the reader is referred to those references for further details of the analysis technique). Event selection criteria are identical to those used in the \(CP\) analyses, except that we remove the requirement on the difference in the decay times \((\Delta t)\) between the two \(B\) mesons in order to mini-
TABLE I: Percentage of events with $|\Delta E| < 150$ MeV and photon energy below the cut-off (2.6 MeV) in the Monte Carlo simulation, as given by the (see Tab. I) simulation and by the QED calculation described in the text.

| Mode   | MC     | QED calculation |
|--------|--------|-----------------|
| $\pi^+\pi^-$ | 89.8 ± 0.1 | 88.8 ± 0.5     |
| $K^+\pi^-$  | 92.4 ± 0.1 | 91.7 ± 0.5     |
| $K^+K^-$    | 94.7 ± 0.1 | 94.7 ± 0.5     |

lowed total energy of the photons, $E_{\text{max}}$. The chosen $\Delta E$ window allows for the presence of radiated photons with total energy up to 150 MeV + $\langle \Delta E \rangle$, where the average value of $\Delta E$, $\langle \Delta E \rangle$, differs for each mode, due to the pion mass hypothesis being assigned to all tracks. As the $\pi^+\pi^-$ events are centered at $\Delta E \sim 0$ MeV, while the $K^+\pi^-$ and $K^+K^-$ distributions are shifted by $-45$ MeV and $-91$ MeV, respectively, the corresponding energy requirements on the radiated photons are $E_{\text{max}} = 150$, 105 and 59 MeV for $\pi^+\pi^-$, $K^+\pi^-$, and $K^+K^-$, respectively. The smearing of $\Delta E$ due to finite momentum resolution leads to a small difference between the number of events that satisfy the $\Delta E$ requirement and the number of events that satisfy an equivalent $E_{\text{max}}$ requirement. We use the MC simulation to evaluate the associated systematic uncertainty on the branching fraction measurements.

We identify $B^0 \to h^+h'^- (h, h' = \pi$ or $K)$ candidates with selection requirements on track and Cherenkov angle ($\theta_c$) quality, $B$ decay kinematic variables, and event topology. The final sample contains 69264 events and is defined by requirements on two kinematic variables: (1) the difference $\Delta E = E_B^2 - \sqrt{s}/2$ between the reconstructed energy of the $B$ candidate in the $e^+e^-$ center-of-mass (CM) frame and $\sqrt{s}/2$; and (2) the beam-energy substituted mass $m_{\text{ES}} = \sqrt{(s/2 + 4)E_i^2 - \vec{p}_B^2}$. Here, $\sqrt{s}$ is the total CM energy, and the $B$ momentum $\vec{p}_B$ and the four-momentum $(E_i, \vec{p}_i)$ of the $e^+e^-$ initial state are defined in the laboratory frame. To simplify the analysis, we use the pion mass for all tracks in the track reconstruction and the calculation of the kinematic variables. We select those $B$ candidates with $|\Delta E| < 150$ MeV, and $5.20 < m_{\text{ES}} < 5.29$ GeV/c$^2$.

The efficiencies of the selection criteria are determined in samples of GEANT-4 based [14] Monte Carlo (MC) simulated signal decays, where we include the effects of electromagnetic radiation from the final-state charged particles using the PHOTOS simulation package [15].

We compare the performance of our simulation with a scalar QED calculation [11] resummed to all orders of $\alpha_{\text{QED}} E_{\text{max}}$. Among events selected by the $|\Delta E| < 150$ MeV requirement, the MC simulation and Ref. [11] predict different fractions of events with photons with energy below 2.6 MeV, the soft photon energy cut-off used in our simulation (see Tab. I). We therefore reweight the $\Delta E$ distributions for each mode to account for this different fraction of radiating events and use these reweighted distributions in the final maximum likelihood fit. The difference in event yields obtained with the original distributions and with the reweighted ones is used to evaluate the associated systematic error, and it is found to be negligible.

As explained in Ref. [11], while taking into account radiative corrections, one needs to be careful to quote the results in such a way that the radiation effects can be disentangled. In principle, it would be necessary to select $B$ candidates with a specified maximum amount of $O(100 \text{ MeV})$ photon energy in the final state, a quantity that is difficult to reconstruct with the BABAR detector. Instead, we define our data sample by selecting on $\Delta E$, an observable that can be related to the maximum al-
TABLE II: Summary of results from the ML fit for the yields. The subscript $b$ refers to background. For the nominal fit, we use a double Gaussian for the signal $\Delta E$ PDF, as described in the text. We also show, for comparison purposes, the results using a single Gaussian, which corresponds to an analysis that ignores FSR effects.

| Parameter | Nominal Fit | Ignoring FSR |
|-----------|-------------|--------------|
| $N_{\pi\pi}$ | 485 ± 35 | 469 ± 34 |
| $N_{K\pi}$ | 1656 ± 52 | 1634 ± 52 |
| $A_{K\pi}$ | -0.136 ± 0.030 | -0.135 ± 0.030 |
| $N_{KK}$ | 3 ± 13 | 5 ± 13 |
| $N_{b\pi\pi}$ | 32983 ± 194 | 32998 ± 194 |
| $N_{bK\pi}$ | 20778 ± 160 | 20801 ± 160 |
| $A_{bK\pi}$ | 0.002 ± 0.008 | 0.002 ± 0.008 |
| $N_{bKK}$ | 13358 ± 126 | 13356 ± 126 |

TABLE III: Summary of relative systematic uncertainties on signal yields. For the $K^+K^-$ yield we show the absolute uncertainty. The total uncertainties are calculated as the sum in quadrature of the individual contributions.

| Source | $\pi^+\pi^-$ (%) | $K^+\pi^-$ (%) | $K^+K^-$ (events) |
|--------|-----------------|----------------|------------------|
| $m_{ES}$ | 0.2 | 0.4 | 1.3 |
| $\Delta E$ | 0.1 | 0.0 | 0.3 |
| signal $F$ | 2.9 | 1.5 | 2.8 |
| bkgd $F$ | 0.5 | 0.2 | 5.9 |
| $\theta_c$ quality | 0.2 | 0.1 | 0.4 |
| Fit bias | 2.2 | 0.9 | 1.3 |
| $B$ bkgd | 0.8 | 0.2 | < 0.1 |
| Total | 3.8 | 1.8 | 6.8 |

The parameters are determined separately for each channel and fixed in the fit. For the signal shape in $m_{ES}$, we use a double Gaussian for the $\Delta E$ PDF, as described in the text. We also show, for comparison purposes, the results using a single Gaussian, which corresponds to an analysis that ignores FSR effects, and a sum of two Gaussian distributions for $F$ (five parameters). For the signal shape in $m_{ES}$, we use a single Gaussian distribution to describe all three channels and allow the mean and width to vary in the fit. For $\Delta E$, we use the sum of two Gaussian distributions (core + tail), where the core parameters are common to all channels and are allowed to vary freely, and the tail parameters are determined separately for each channel from the reweighted MC simulation (explained above), and fixed in the fit. For the signal shape in $F$, we use an asymmetric Gaussian function with different widths below and above the mean. All three parameters are determined from MC simulation and fixed in the maximum-likelihood fit. The $\theta_c$ PDFs are obtained from a sample of approximately 430000 $D^{*+} \rightarrow D^0\pi^+(D^0 \rightarrow K^-\pi^+)$ decays reconstructed in data, where $K^-/\pi^+$ tracks are identified through the charge correlation with the $\pi^+$ from the $D^{*+}$ decay. We construct the PDFs separately for $K^+, K^-, \pi^+$, and $\pi^-$ tracks as a function of momentum and polar angle using the measured and expected values of $\theta_c$, and its uncertainty. We use the same PDFs for tracks in signal and background events.

Table II summarizes the fitted signal and background yields, and $K\pi$ charge asymmetries. We find a value of $A_{K\pi}$ consistent with our previously published result $^{[2]}$, and a background asymmetry consistent with zero. The signal yields are slightly higher than the values reported in Ref. $^{[2]}$ due to the removal of the $\Delta t$ selection requirement and the addition of the radiative tail in the signal $\Delta E$ PDF. In order to quantify the effect of FSR on the fitted yields, we perform a second fit using a single Gaussian for the $\Delta E$ PDF, allowing the mean and width to vary. The results are shown in the second column of Table II, where we find that ignoring FSR lowers the $\pi\pi$ yield by 3.4% and the $K\pi$ yield by 1.3%.

As a crosscheck, in Fig. I we compare the PDF shapes (solid curves) to the data using the event-weighting technique described in Ref. $^{[14]}$. For each plot, we perform a fit excluding the variable being plotted and use the fitted yields and covariance matrix to determine the relative probability that an event is signal or background. The distribution is normalized to the yield for the given component and can be compared directly to the assumed PDF shape. We find excellent agreement between the data and the PDFs. Figure 2 shows the likelihood ratio $L_S/L_I$ for all 69264 events in the fitted sample, where $L_S$ is the likelihood for a given signal hypothesis, and the summation in the denominator is over all signal and background components in the fit. We find good agreement between data (points with error bars) and the distributions obtained by directly generating events from the PDFs (histograms).

Systematic uncertainties on the branching fractions arise from uncertainties on the selection efficiency, signal yield, and number of $B\bar{B}$ events in the sample. Uncertainty on the efficiency is dominated by track reconstruction efficiency (1.6%) and by the uncertainty on FSR (1.3% for $\pi\pi$, 1.4% for $K\pi$ and 2.9% for $KK$), which is evaluated assuming 100% uncertainty on the smearing effect on $\Delta E$.

Other systematic uncertainties on selection efficiency are those due to requirements on the quality of the $\theta_c$ measurement (1.0% for $\pi\pi$, 0.8% for $K\pi$ and 0.5% for $KK$) and on event topology (1.1%). Uncertainty on the fitted signal yields is dominated by the shape of the signal PDF for $F$ (2.9% for $\pi\pi$, 1.5% for $K\pi$) and potential bias (2.2% for $\pi\pi$, 0.9% for $K\pi$) in the fitting technique, as determined from large samples of MC-simulated signal events and a large ensemble of pseudo-experiments generated from the PDF shapes. Uncertainties due to imperfect knowledge of the PDF shapes for $m_{ES}$, $\Delta E$, and $\theta_c$ are all less than 1%. Tables II and IV summarize the uncertainties on the signal yields and branching fractions, respectively.

Table IV summarizes the results for the charge-averaged branching fractions. For comparison, we use the efficiencies and signal yields determined under the assumption of no FSR and find $B(B^0 \rightarrow \pi^+\pi^-) = 5.0 \times 10^{-6}$ and $B(B^0 \rightarrow K^+\pi^-) = 18.0 \times 10^{-6}$, which are consistent with our previously published results $^{[7]}$. We determine the upper limit for the signal yield for $K^+K^-$ using a
FIG. 1: Data distributions (points with error bars) of \( m_{ES} \), \( \Delta E \), and \( F \) for signal \( \pi^+\pi^- \) (a,b,c), signal \( K^+\pi^- \) (d,e,f) and background for the three channels (g,h,i), using the weighting technique described in the text. Solid curves represent the corresponding PDFs used in the fit. The distribution of \( \Delta E \) for signal \( K^+\pi^- \) events is shifted due to the assignment of the pion mass for all tracks.

TABLE IV: Summary of relative systematic uncertainties on yields, efficiencies, and number of \( B\bar{B} \) pairs. For the \( K^+K^- \) yield we show the absolute uncertainty. The total uncertainties are calculated as the sum in quadrature of the individual contributions.

| Source | \( \pi^+\pi^- \) | \( K^+\pi^- \) | \( K^+K^- \) |
|--------|-----------------|----------------|--------------|
| yields | 3.8%            | 1.8%           | 6.8 events   |
| efficiency | 2.5%           | 2.5%           | 3.5%         |
| \( N_{B\bar{B}} \) | 1.1%           | 1.1%           | 1.1%         |
| Total  | 4.7%            | 3.3%           | see text     |

Bayesian procedure that assumes a flat prior on the number of events. The upper limit is given by the value of \( N_0 \) for which \( \int_0^{N_0} \mathcal{L}_{\text{max}} \, dN / \int_0^\infty \mathcal{L}_{\text{max}} \, dN = 0.90 \), corresponding to a one-sided 90% confidence interval. Here, \( \mathcal{L}_{\text{max}} \) is the likelihood as a function of the \( K^+K^- \) yield \( N \), maximized with respect to the remaining fit parameters. We find \( N_0 = 25.4 \), and the upper limit on the branching fraction is calculated by increasing the signal yield upper limit and reducing the efficiency by their respective total errors (Table IV). For the purpose of combining with measurements by other experiments, we also evaluate the central value for the branching fraction and find \( \mathcal{B}(B^0 \rightarrow K^+K^-\nu\gamma) = (4 \pm 15 \pm 8) \times 10^{-8} \).

Although we cannot directly measure the non-
In summary, we have presented updated measurements of charge-averaged branching fractions for the decays $B^0 \to \pi^+\pi^-$ and $B^0 \to K^+\pi^-$, with FSR effects taken into account. We find that the branching fractions are a few percent higher when the effect of FSR is included in the calculation of the efficiency and signal yield determination. This difference should be taken into account when comparing with previous measurements of these quantities [7, 8, 9, 18] that do not include these effects. In order to perform the most meaningful comparison, we also evaluated the bare branching fractions for the three channels, as explained in Ref. [11]. Our results are consistent with current theoretical estimates from different models [2]. We find no evidence for the decay $B^0 \to K^+K^-$ and set an upper limit of $5.0 \times 10^{-7}$ at the 90% confidence level.

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