Superradiant instability of extremal brane-world Reissner-Nordström black holes to charged scalar perturbations

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(Dated: May 7, 2014)

We examine the stability of extremal braneworld Reissner-Nordström black holes under massive charged scalar perturbations. We show that similar to the four-dimensional case, the superradiant amplification can occur in braneworld charged holes. More interestingly we find that when the spacetime dimension is higher than four, a trapping potential well can emerge outside the braneworld black hole. This potential well is the extra dimensional effect and does not exist in four dimensions. It triggers the superradiant instability of the extremal braneworld charged holes.

PACS numbers: 04.70.Bw, 42.50.Nn, 04.30.Nk

Starting from the influential study by Regge and Wheeler [1], the stability of black holes has been investigated over half a century. It has been demonstrated that most black holes are stable under various types of perturbations (for a recent review see for example [2]), which shows that the black hole is realizable in practice and is not just a mathematical curiosity.

In Einstein gravity, the Kerr family exhausts the black hole solutions of the Einstein equations in the vacuum. The Kerr black hole is rotating, which is a realistic model to describe astrophysics. The Kerr solution was shown to be stable under dynamical perturbations [3–18]. However, such a conclusion will be changed due to the superradiance effect [19]. Considering a wave of the form $e^{-i\omega t + ik\phi}$ incident upon a Kerr black hole with the angular velocity $\Omega$, if the frequency of the incident wave satisfies $\omega < k\Omega$, the scattered wave will be amplified. This means that the energy radiated away to infinity can exceed the energy of the initial perturbation by extracting rotational energy of the black hole. This superradiance can affect the classical fields and influence the stability of the background configuration. If the Kerr black hole is enclosed inside a spherical mirror, the
initial perturbation can get successfully amplified near the black hole event horizon and reflected back at the mirror, thus creating an instability. This is the black hole bomb devised in [19]. Besides the artificial mirror, one can devise a natural wall if one considers massive fields [17, 18, 20–28]. For a massive field, the field amplified by superradiance gets trapped in the effective potential well, thus triggering the instability. The bound state due to the potential well plays the same role as the artificial mirror, what results in the instability.

The Reissner-Nordström (RN) black holes share many common features with the Kerr black holes. The stability of the RN black hole under neutral perturbations was disclosed long time ago [29, 30]. The analogous superradiant phenomena was also observed in the RN black hole [31]. Considering the charged perturbation of a bosonic field, if the frequency of the incident wave $\omega$ satisfies the relation

$$\omega < e\Phi,$$ \hspace{1cm} (1)

where $e$ is the charge of the incident field and $\Phi$ is the electric potential of the RN black hole, then superradiance occurs by extracting the Coulomb energy and the electric charge from the RN black hole. Can this superradiance trigger the instability as disclosed in the Kerr black hole? Recently, this problem was examined in [32, 33]. It was found that for massive charged scalar perturbations, there does not exist a trapping potential well separated from the black hole horizon by a potential barrier outside the black hole. This shows that for the RN black hole there is no bound state to mimic the artificial mirror as in the Kerr black hole case to trigger the superradiant instability.

The stability analysis has been performed mainly in the four-dimensional black hole backgrounds. Recently, extensive studies of d-dimensional cases led to the fact that black branes and strings are generally unstable against a certain sector of gravitational perturbations [34]. More interestingly, it was observed that the usual stability result does not hold in the RN de Sitter black holes for spacetime dimensions larger than six [35, 36]. It is of great interest to examine how is the extra dimensional effect on the superradiant instability. This is the main purpose of the present work. We extend the discussion in usual four-dimensional RN black holes [32, 33] to the brane-world black holes [37–40]. Notice that these are black holes in the brane, that is, we do not consider perturbations that run into the extra dimensions. The $(4+n)$-dimensional RN black hole was described in [41] in the form

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2d\Omega_{n+2}^2,$$

$$f(r) = 1 - \frac{2\mu}{r^{n+1}} + \frac{q^2}{r^{2(n+1)}},$$ \hspace{1cm} (2)
where \( n \) denotes the number of compact extra dimensions, the parameters \( \mu, q \) are related to the mass \( M \) and charge \( Q \) of the black hole through the relations

\[
\begin{align*}
\mu &= \frac{8\pi GM}{(n + 2)V_{n+2}}, \\
q^2 &= \frac{8\pi G Q^2}{(n + 2)(n + 1)},
\end{align*}
\]

and the volume of the \((n + 2)\)-dimensional sphere is \( V_{n+2} = \frac{2\pi^{(n+3)/2}}{\Gamma(\frac{n+3}{2})} \). \( d\Omega^2_{n+2} \) describes the corresponding line-elements of the \((n + 2)\)-dimensional unit sphere,

\[
d\Omega^2_{n+2} = d\theta_{n+1}^2 + \sin^2\theta_{n+1} \left( d\theta_n^2 + \sin^2\theta_n \left( \ldots + \sin^2\theta_2 \left( d\theta_1^2 + \sin^2\theta_1 d\phi^2 \right) \ldots \right) \right).
\]

In the above, \( \phi \in [0, 2\pi] \) and \( \theta_i \in [0, \pi] \) \((i = 1, \ldots, n + 1)\). We use \( n \) additional azimuthal coordinates \( \theta_i \) \((i = 2, \ldots, n + 1)\) to denote the \( n \) compact extra dimensions. The Maxwell field is

\[
A = h(r)dt, \quad h(r) = -\sqrt{\frac{1}{8\pi G} \frac{n + 2}{n + 1} \frac{q}{r^{n+1}}},
\]

We concentrate on the charged perturbation of a bosonic field. Considering that the standard model fields live on the brane while only gravity can travel into the extra dimensions in the braneworld scenario, we focus on the braneworld metric induced by a \((4 + n)\)-dimensional RN extreme spacetime. This simply follows by fixing the values of the extra angular azimuthal coordinates, i.e. \( \theta_i = \frac{\pi}{2} \), for \( i = 2, \ldots, n + 1 \), which leads to the projection of the \((4 + n)\)-dimensional RN metric \((2)\) on a four-dimensional slice playing the role of our four-dimensional world. The induced metric then has the form

\[
ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2),
\]

where the subscript “1” from the remaining azimuthal coordinate has been dropped. Note that the metric function \( f(r) \) remains unchanged during the projection and is still given by \((2)\). Since the Maxwell field \((5)\) totally lives on the brane, it remains the same under the projection. We show that with the extra dimensional influence, the effective potential out of the black hole can have a potential well to act as the natural mirror to trap the superradiant wave and thus trigger the instability.

In this paper, we only consider the extremal case, satisfying the condition \( \mu = q \). Thus the metric coefficient function \( f(r) \) is given by the expression

\[
f(r) = \left( 1 - \frac{\mu}{r^{n+1}} \right)^2,
\]

and the degenerate horizon is located at \( r_H = \mu^{1/(n+1)} \).
We consider a massive charged scalar living on the brane. Its dynamics in the braneworld RN extreme black hole is governed by the Klein-Gordon equation
\[
[(\nabla_\mu - ieA_\mu)(\nabla^\mu - ieA^\mu) - m^2] \Psi = 0, \tag{8}
\]
where \( e \) and \( m \) are the charge and mass of the field, respectively. We do the decomposition of the field as
\[
\Psi_{lk}(t, r, \theta, \phi) = e^{-i\omega t} R_{lk}(r) S_{lk}(\theta)e^{-ik\phi}, \tag{9}
\]
where \( \omega \) is the conserved energy of the mode, \( l \) is spherical harmonic index, and \( k \) is the azimuthal harmonic index with \(-l \leq k \leq l\). In the following, for brevity, we will omit the indices \( l \) and \( k \). With the decomposition, we can obtain the radial Klein-Gordon equation
\[
r^2 f(r) \frac{d}{dr} \left( r^2 f(r) \frac{dR}{dr} \right) + U R = 0, \tag{10}
\]
with
\[
U = (\omega + eh(r))^2 r^4 - f(r) \left[ m^2 r^4 + l(l + 1)r^2 \right]. \tag{11}
\]
It is convenient to define a new radial function \( \psi \) by
\[
\psi \equiv r f(r)^{1/2} R. \tag{12}
\]
Then the radial equation (10) can be written in the form of a Schrodinger-like wave equation,
\[
\frac{d^2 \psi}{dr^2} + (\omega^2 - V_{\text{eff}}) \psi = 0, \tag{13}
\]
where the effective potential is
\[
V_{\text{eff}} = \omega^2 - \frac{1}{f(r)^2} \left[ \frac{U}{r^4} + \frac{1}{4} \left( \frac{df(r)}{dr} \right)^2 - \frac{f(r) df(r)}{r} \frac{df(r)}{dr} - \frac{f(r) d^2 f(r)}{2} \right]. \tag{14}
\]
In the following, we set \( 8\pi G = c = \hbar = 1 \). Substituting the expressions of \( f(r) \) and \( h(r) \) from (7) and (5) into the above equation, we get
\[
V_{\text{eff}} = \omega^2 + \frac{1}{(\rho^{n+1} - \mu)^4} \left[ (m_\rho^2 - \omega^2) \rho^{4n+4} + l(l + 1)\rho^{4n+2} + 2 \left( \frac{n + 2}{n + 1} \omega - m^2 \right) \rho^{3n+3} \right. \\
- \left. [n(n + 1) + 2l(l + 1)] \mu \rho^{3n+1} + \left( \frac{m^2 - \frac{n + 2}{n + 1} e^2}{\rho^{2n+2}} \right) \mu^2 \rho^{2n+2} \\
+ [3n(n + 1) + l(l + 1)] \mu^2 r^{2n} - 3n(n + 1) \mu^3 r^{n-1} \\
+ n(n + 1) \mu^4 r^{-2}. \tag{15}
\]
As $r \to r_H$, we have $V_{\text{eff}} \to -\frac{r_H^4(\omega - e\Phi)^2}{(n+1)^2(r - r_H)^4}$, where $\Phi = -\hbar(r_H)$ is the electric potential at the horizon. When $r \to \infty$, $V_{\text{eff}} \to m^2$.

In a scattering experiment, (13) has the following asymptotic behavior with $\omega^2 > m^2$

$$\psi \sim \begin{cases} Te^{-i\sigma(r - r_H)^{-1}} & \text{as } r \to r_H, \\ Re^{i\sqrt{\omega^2 - m^2}\sigma} + e^{-i\sqrt{\omega^2 - m^2}\sigma} & \text{as } r \to \infty, \end{cases}$$

The boundary conditions correspond to an incident wave of unit amplitude, $e^{-i\sqrt{\omega^2 - m^2}r}$, coming from $+\infty$ and giving rise to a reflected wave of amplitude $R$ going back to $+\infty$ and a transmitted wave of amplitude $T$ at the horizon. We also define the constant $\sigma \equiv \frac{(\epsilon \Phi - \omega) r_H^2}{(n+1)^2}$. At the black hole horizon, the boundary behavior of the radial equation (16) is that of a purely ingoing wave.

Considering that the effective potential is real, the complex conjugate of the solution $\psi$ satisfying the boundary conditions (16) will satisfy the complex-conjugate boundary conditions:

$$\psi^* \sim \begin{cases} T^* e^{i\sigma(r - r_H)^{-1}} & \text{as } r \to r_H, \\ R^* e^{-i\sqrt{\omega^2 - m^2}\sigma} + e^{i\sqrt{\omega^2 - m^2}\sigma} & \text{as } r \to \infty. \end{cases}$$

Because the two solutions $\psi$ and $\psi^*$ are linearly independent, then their Wronskian, $W(\psi, \psi^*) \equiv \psi \frac{d}{dr} \psi^* - \psi^* \frac{d}{dr} \psi$, is a constant independent of $r$. Evaluating the Wronskian at the horizon and infinity respectively, we get

$$W(r \to r_H) = -\frac{2i\sigma}{(r - r_H)^2}|T|^2,$$

$$W(r \to \infty) = -2i\sqrt{\omega^2 - m^2}(|T|^2 - 1).$$

By equating the two values, we get

$$|R|^2 = 1 + \frac{\sigma}{(r - r_H)^2\sqrt{\omega^2 - m^2}}|T|^2.$$

Now, we can see that if $\sigma > 0$, we have $|R|^2 > 1$. This means that one gets back more than one threw in, and superradiant phenomena occurs. So we get the condition to occur the superradiance,

$$\omega < e\Phi,$$

which has the same form as in the four-dimensional case, and does not change in the braneworld picture. However, the dimensional influence hides in the electric potential.

To see whether the superradiance will cause the instability of the braneworld black hole spacetime, we need to check whether there exists a potential well outside the horizon to trap the reflected wave. If the potential well exists, the superradiant instability will occur and the wave will grow exponentially over time near the black hole to make the background braneworld black hole unstable.
Now, we analyze the behavior of the effective potential. From Eq. (14), we can get the derivative of the effective potential as

\[
V'_{\text{eff}}(r; \mu, m, e, \omega, l, n) = \frac{1}{(r^{n+1} - \mu)} \left[ -2l(l + 1)r^{5n+2} - 2(n + 1) \left( m^2 + \left( e \sqrt{\frac{n+2}{n+1}} - 2\omega \right) \omega \right) \mu r^{4n+3} \right.
\]

\[
+ [n(n+1)(n+3) - 2(n-2)(l+1)] \mu^4 r^{4n+1} + 2(2n-1)(l+1) - n(n+1)(3n+11) \mu^2 r^{3n} + 2(2(n+1)m^2 + (n+2)e^2 - 3 \sqrt{(n+1)(n+2)e\omega}) \mu^2 r^{3n+2}
\]

\[
+ 2 \left[ (n+2)e^2 - (n+1)m^2 \right] \mu^3 r^{2n+1} + n [3(n+1)(n+5) - 2l(l+1)] \mu^3 r^{2n-1} - n(n+1)(n+9) \mu^4 r^{n-2} + 2n(n+1) \mu^5 r^{-3} \right].
\] (21)

We denote the roots of \(V'_{\text{eff}}(r) = 0\) which are larger than \(r_H\) by \(\{r_1, r_2, \cdots, r_N\}\) with \(r_H < r_1 \leq r_2 \leq \cdots \leq r_N\). As \(r \to r_H\), we have

\[
V'_{\text{eff}}(r \to r_H) = \frac{4(n+1)\mu^{(5n+4)/(n+1)}}{(r^{n+1} - \mu)^5} (\omega - e\Phi)^2.
\] (22)

As long as \(\omega \neq e\Phi\), which includes the superradiant regime [20] we considered, the derivative of the potential near the horizon is always positive, which means that the first zero point \(r_1\) of \(V'_{\text{eff}}(r)\) corresponds to the maximum of the effective potential \(V_{\text{eff}}\). This can be easily checked by calculating the sign of \(V''_{\text{eff}}(r)\). In [33], it was shown analytically that for the \(n = 0\) case there only exists one root of \(V'_{\text{eff}}(r) = 0\) which is larger than \(r_H\). Thus in four dimensions, there is only a potential barrier (and no potential well) outside the black hole so that there is no superradiant instability. However, when \(n \geq 1\), the situation becomes very different. From (21), we can see that when \(n \geq 1\), the equation becomes a rather high order equation. It is hard to analyze its roots analytically. We will count on numerical analysis.

In table I, we list the numerical roots of \(V'_{\text{eff}}(r) = 0\) and the corresponding \(V_{\text{eff}}\) for \(1 \leq n \leq 6\). We fix the parameters \(\mu = 1\) and \(l = 0\), so that the horizon locates at \(r_H = 1\). We are interested in solutions of the radial equation [13] with the physical boundary conditions of purely ingoing waves at the horizon and a decaying solution at spatial infinity. A bound state decaying exponentially at spatial infinity is characterized by \(\omega^2 < m^2\). We choose the parameters \((e\Phi, m\) and \(\omega\) to meet this condition together with the super-radiant condition (20). We observe that there exists a negative minima of the potential well. If we choose other parameters, we can also obtain a positive minima of the potential well. From the table, we can see that there may exist more than one root for \(n \geq 1\). The first root \(r_1\) corresponds to the location of a potential barrier, the second root \(r_2\) corresponds to the position of a potential well, and the third one \(r_3\) is the place of a potential barrier again.
This can be confirmed by checking the sign of \( V''_{\text{eff}} \) at these points. These results imply that, when \( n \geq 1 \), there may exist a potential well which is separated from the horizon by a potential barrier. Then the two conditions to trigger superradiant instability can be satisfied simultaneously. This is very different from that in the four-dimensional case \([33]\).

\[
\begin{array}{cccccc}
\text{ } & n=1 & n=2 & n=3 & n=4 & n=5 \\
\text{ } & e\Phi & 0.88 & 1 & 1 & 1 \\
\text{ } & m & 0.86 & 0.9 & 1.2 & 1.8 & 2 \\
\text{ } & \omega & 0.85 & 0.85 & 0.85 & 0.85 & 0.85 \\
\text{ } & r_1 & 1.01176 & 1.04196 & 1.02797 & 1.01674 & 1.01327 & 1.01214 \\
\text{ } & r_2 & 1.16282 & 1.12098 & 1.07759 & 1.08363 & 1.055 & 1.03202 \\
\text{ } & r_3 & 6.59754 & 4.75378 & 3.23743 & 2.27369 & 2.41529 & 2.8146 \\
\text{ } & V_{\text{eff}}(r_1) & 975.3594 & 12.4182 & 17.916 & 164.914 & 173.854 & 34.6291 \\
\text{ } & V_{\text{eff}}(r_2) & -1.19747 & -4.10825 & -10.0049 & -6.85324 & -20.1684 & -58.4737 \\
\text{ } & V_{\text{eff}}(r_3) & 0.740589 & 0.811597 & 1.44511 & 3.26462 & 4.00854 & 4.00108 \\
\end{array}
\]

**TABLE I:** Roots of \( V'_{\text{eff}}(r) = 0 \) and the corresponding \( V_{\text{eff}} \) for \( 1 \leq n \leq 6 \).

In table II, we fix the parameters: \( \mu = 1, l = 0, e\Phi = 1, m = 2 \) and \( \omega = 0.85 \), and list the roots and the corresponding effective potential for \( 1 \leq n \leq 6 \). From the table, we can see that as \( n \) increases, a potential well appears and becomes deeper and deeper, and the first and second potential barrier becomes lower and lower.

\[
\begin{array}{cccccccc}
\text{ } & n=0 & n=1 & n=2 & n=3 & n=4 & n=5 & n=6 \\
\text{ } & r_1 & 1.07367 & 1.03639 & 1.02441 & 1.01861 & 1.01529 & 1.01327 & 1.01214 \\
\text{ } & r_2 & - & - & - & - & 1.10994 & 1.055 & 1.03202 \\
\text{ } & r_3 & - & - & - & - & 1.98135 & 2.41529 & 2.8146 \\
\text{ } & V_{\text{eff}}(r_1) & 551.515 & 526.309 & 474.566 & 397.286 & 296.166 & 173.854 & 34.6291 \\
\text{ } & V_{\text{eff}}(r_2) & - & - & - & - & -1.1807 & -20.1684 & -58.4737 \\
\text{ } & V_{\text{eff}}(r_3) & - & - & - & - & 4.0624 & 4.00854 & 4.00108 \\
\end{array}
\]

**TABLE II:** Roots of \( V'_{\text{eff}}(r) = 0 \) and the corresponding \( V_{\text{eff}} \) for \( 1 \leq n \leq 6 \) with fixed parameters: \( e\Phi = 1, m = 2, \omega = 0.85 \). Here “-” means not exist.

In conclusion, we have investigated the possible existence of superradiant instability for extreme charged RN brane-world black hole due to the charged massive perturbations on the brane. We
have shown that in contrast with the four-dimensional charged black hole, there exists a trapping potential well when we consider the extra dimensional contribution. Thus, the wave can be trapped in the well and amplified by superradiance. This triggers the superradiant instability in the charged brane-world black hole.

Acknowledgments

This work is supported by the National Natural Science Foundation of China.

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