Nonleptonic charmed meson decays:
Quark diagrams and final-state interactions

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Abstract

Effects of final-state interactions in nonleptonic decays of charmed mesons are studied in the framework of quark-diagram approach. For the case of $u$-$d$-$s$ flavour symmetry we discuss how the inelastic coupled-channel rescattering effects (and, in particular, resonance formation in the final state) modify the input quark-diagram weak amplitudes. It is shown that such inelastic effects lead to the appearance of nonzero relative phases between various quark diagrams, thus invalidating some of the conclusions drawn in the past within the diagrammatical approach. The case of SU(3) symmetry-breaking in Cabibbo once-forbidden $D^0$ decays is also studied in some detail.
1 Introduction

Various theoretical models of nonleptonic decays of charmed mesons have been developed over the years. The most general and complete one is the diagrammatical approach of Chau and Cheng [1, 2, 3]. The factorization method [4] is just a special case of this approach. Another subset of diagrammatical approach is singled out by large-$N_c$ arguments [5]. The basic problem with the diagrammatical approach is the way in which final-state interactions (FSI) and SU(3) breaking are treated. The importance of FSI has been stressed by Lipkin [6], Sorensen [7], Kamal and Cooper [8], Donoghue [9], Chau [10], Chau and Cheng [11], Hinchliffe and Kaeding [12], and others.

Complete descriptions of nonleptonic decays must take into account final-state interactions. Since full dynamical calculations of these effects are not possible at present, a meaningful comparison of theoretical models with experiment requires at least a phenomenological estimate of FSI. Such an approximate estimate may be obtained using e.g. unitarity constraints [13]. Alternatively, one may consider approaches based on approximate flavour-symmetry groups. Their predictions include automatically all effects of those FSI which are invariant under these symmetries [6]. The diagrammatical methods of Chau and Cheng provide an approach complementary to that based on flavour-symmetry group [3, 12]. It has been argued [3] that ”it is folly to proceed with the quark-diagram approach without considering these (i.e. rescattering) effects” since ”rescattering can mix up the classification of diagrams”. Although the latter statement is obviously true, one should realize that the quark-diagram approach - being complementary to that based on flavour symmetry groups - ”deals with effective quark diagrams with all FSI included” [4].
The aim of this paper is

1) to examine in some detail in what way the introduction of flavour-symmetric FSI in the form of coupled-channel rescattering effects renormalizes the input quark-diagram weak amplitudes (with particular emphasis on resonance formation in the final state) and to compare the results thus obtained with the treatment of FSI adopted in the diagrammatic approach so far (Section 3), and

2) to discuss some aspects of flavour-symmetry breaking (especially $D^0$ decays into $\pi\pi$ and $K\bar{K}$, (Section 4)).

2 General

In this paper we will consider how weak decay amplitudes are changed when final-state rescattering effects are added. When there is only one possible final state, the answer is well-known and given by Watson’s theorem [14]. For the coupled-channel case the situation was discussed in [15] where the generalized Watson’s theorem was introduced. For our purposes we will use the $K$-matrix parametrization of the time-reversal-invariant $S$-matrix [3]:

$$ S = \frac{1 + iK}{1 - iK} $$

with $K$ real and symmetric. Let the charmed meson $D$ decay weakly into $n$ two-body (in general many-body) coupled channels $j$. The $S$-matrix may then be written as

$$ S = \begin{bmatrix} 1 + O(W^2) & iW^T \\ iW & S_0 \end{bmatrix} $$

where $S_0$ is an $n \times n$ submatrix describing purely strong interaction in the coupled channel system, while $iW$ is an $n$-dimensional vector describing weak $D$-decays into
these two-body channels. Since $S$ is unitary, $W$ contains all final-state strong effects as well. The corresponding reaction matrix $K$ may be written as

$$
K = \begin{bmatrix}
0 & w^T \\
w & K_0
\end{bmatrix}
$$

(3)

where $K_0$ is the $K$-matrix for the $n \times n$ purely strong submatrix $S_0 = (1 + iK_0)/(1 - iK_0)$, while $w$ is an $n$-dimensional vector describing FSI-unmodified weak decays of $D$ mesons.

The relation between the input and the full weak decay amplitudes is then

$$
W = (1 + S_0)w = \frac{2}{1 - iK_0}w
$$

(4)

i.e. in the case of vanishing FSI we have

$$
W = 2w
$$

(5)

Matrix equation (4) admixes into a given decay amplitude contributions from all coupled channels. For the $n = 1$ case one has $S_0 = \exp(2i\delta_0)$, $(K_0 = \tan \delta_0)$ and Eq.(4) reproduces Watson’s theorem:

$$
W = 2w \cos \delta_0 \exp(i\delta_0)
$$

(6)

For the coupled-channel case Eq.(4) constitutes a generalized case of Watson’s theorem. It includes all final-state interactions: both elastic scattering and all inelastic rescattering effects. As we shall see in this paper explicitly, such inelastic rescattering effects have not been included in the diagrammatical approach of refs. [1, 2, 3] so far. For reasons related to the introduction of FSI the diagrammatical approach has been criticised by Donoghue [4], and, very recently, by Hinchcliffe and Kaeding [12]. The question of how the inelastic effects change the overall picture of quark-line approach was first examined in ref.[4]. Here we will show how the problem gets simplified conceptually, and - in the most important case of final-state $q\bar{q}$ resonance contribution -
also computationally, through diagonalization of the $K_0$ ($S_0$) matrix and application of Watson’s theorem in the diagonalizing basis. Diagonalization will be discussed on the example of Cabibbo-allowed parity-conserving decays of $D^0$ and $D_s^+$ into the $PV$ (pseudoscalar meson + vector meson) final states. Results of a similar treatment of the Cabibbo-allowed parity-violating decays $D^0$, $D_s^+ \rightarrow PP$ will also be presented. Finally, we will consider the interesting case of Cabibbo-forbidden parity-violating decays of $D^0$ into the $\pi\pi$ and $K\overline{K}$ channels where SU(3) breaking is expected to play a significant role. In all of our examples we will neglect complications due to possible presence of coupled channels other than those listed above.

3  Cabibbo-allowed decays of $D^0$ and $D_s^+$ into PV and PP final states

3.1 Parity conserving PV decays

3.1.1. $D^0 \rightarrow PV$. The parity-conserving part of weak interactions induces $D^0$ decays into eight possible final p-wave $PV$ channels: $K^-\rho^+$, $\overline{K}\rho^0$, $\overline{K}\omega$, $\eta_s\overline{K}^\circ$ and $K^*\pi^+$, $\overline{K}^\circ\pi^0$, $\overline{K}^\circ\eta_{ns}$, $\phi\overline{K}^0$.

In the first four decay channels (below called $PV$) the strange quark from the weak decay of the charmed quark ends up in the pseudoscalar meson $P$, while in the latter four decay channels (called $VP$) this strange quark ends up in the vector meson. The decays proceed through diagrams (a), (b), (c) from Fig. 1.
Fig. 1 Quark-line diagrams for weak meson decays
(a), (b) - factorization; (c) - W-exchange; (d) - annihilation;
(e) - ”horizontal penguin”; (f) - ”vertical penguin”

Let us first consider the case when there are no final-state strong interactions
\(S_0 = 1\). By \(a, b, c, a', b', c'\) we denote matrix elements corresponding to diagrams
(a), (b), (c), for PV (\(\bar{V}P\)) channels respectively. Evaluation of contributions from
these diagrams yields the following weak decay amplitudes

\[
2 < (\bar{K}\rho)_{3/2} | w | D^0 > = -\frac{1}{\sqrt{3}} (a + b)
\]

\[
2 < (\bar{K}\rho)_{1/2} | w | D^0 > = \frac{1}{\sqrt{6}} (b - 2a - 3c)
\]

\[
2 < \bar{K}^0 \omega | w | D^0 > = -\frac{1}{\sqrt{2}} (b + c)
\]

\[
2 < \eta_s \bar{K}^{*0} | w | D^0 > = -c
\]  \hspace{1cm} (7)

and

\[
2 < (\bar{K}^* \pi)_{3/2} | w | D^0 > = -\frac{1}{\sqrt{3}} (a' + b')
\]

\[
2 < (\bar{K}^* \pi)_{1/2} | w | D^0 > = \frac{1}{\sqrt{6}} (b' - 2a' - 3c')
\]

\[
2 < \bar{K}^{*0} \eta_{ns} | w | D^0 > = -\frac{1}{\sqrt{2}} (b' + c')
\]

\[
2 < \phi \bar{K}^0 | w | D^0 > = -c'
\]  \hspace{1cm} (8)

Subscripts 1/2 and 3/2 denote total isospin of \( \bar{K}\rho \) and \( \bar{K}^* \pi \) states. Dependence on Cabibbo factors is suppressed both in Eqs (7, 8) and elsewhere in this paper. That is, all expressions on the right-hand side of Eqs (7, 8) are to be multiplied by the Cabibbo factor of \( \cos^2 \Theta_C : -(a + b)/\sqrt{3} \rightarrow -\cos^2 \Theta_C \cdot (a + b)/\sqrt{3} \) etc. The normalisation is such that matrix elements \( a, b, c \) etc. are identical with those used in [1, 2, 3] (weak decay amplitudes in the case of vanishing FSI are given by Eq.(5)).

Final-state interactions may include quark-exchange [4], resonance formation [4], elastic scattering etc. Of these it is resonance formation that is in general expected to affect naive approaches most significantly [4, 5, 6, 7]. In the following we will consider FSI through \( q\bar{q} \) resonance formation only. Later, it should become clear that our general conclusions remain valid also when other FSI are included. The final-state coupled-channel processes to be considered are visualised in Fig.2
Fig. 2 Resonance contribution to final-state interactions

Since the intermediate state is a pseudoscalar state (with kaon flavour quantum numbers), the flavour structure of the $K_0$ matrix may be easily calculated from the product of two $F$-type ($VP \to P'$) couplings. (In general, one chooses symmetric(antisymmetric) D(F) coupling $Tr(M_1\{M_2, M_3\})$ ($Tr(M_1[M_2, M_3])$) when the product of charge conjugation parities of the three mesons is positive (negative).)

This gives the following strong $K_0$ matrix for the $PV$ subsector:

$$K_0 = \begin{bmatrix} \frac{3}{2} & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \sqrt{\frac{3}{2}} & \frac{1}{\sqrt{2}} & 1 \end{bmatrix} \cdot \kappa^2$$

(9)

where the channels are ordered $(K\rho)_{1/2}$, $K^0\omega$, $\eta_sK^{*0}$ and "$\kappa$" is the flavour-symmetric strength factor of the $VP \to P'$ coupling. (In the $(K\rho)_{3/2}$ sector the $K_0$ matrix obviously vanishes).

The eigenvalues and eigenvectors of matrix (9) are:

$$\lambda_1 = 3\kappa^2 \equiv \tan \delta_1 \quad |1> = \frac{1}{\sqrt{6}}(\sqrt{3}(K\rho)_{1/2} + (K^0\omega) + \sqrt{2}(\eta_sK^{*0}))$$

$$\lambda_2 = 0 \equiv \tan \delta_2 \quad |2> = \frac{1}{\sqrt{6}}(-\sqrt{3}(K\rho)_{1/2} + (K^0\omega) + \sqrt{2}(\eta_sK^{*0}))$$

$$\lambda_3 = 0 \equiv \tan \delta_3 \quad |3> = \frac{1}{\sqrt{3}}(\sqrt{2}(K^0\omega) - (\eta_sK^{*0}))$$

(10)
Equations (7) may be easily rewritten in the new basis (for the $I = 1/2$ sector):

\[
2 < 1|w|D^0 > = -\frac{1}{\sqrt{3}}(a + 3c) \\
2 < 2|w|D^0 > = \frac{1}{\sqrt{3}}(a - b) \\
2 < 3|w|D^0 > = -\frac{1}{\sqrt{3}}b
\]  

(11)

In this basis Eq.(4) reads

\[
<j|W|D^0 > = 2 \cos \delta_j \exp(i\delta_j) < j|w|D^0 >
\]  

(12)

with $j = 1, 2, 3$. Eq.(12) is as simple as the standard one-channel case of Watson’s theorem (Eq.(6)). Going back from Eqs.(12) to the old basis (and adding the expression for the $I = 3/2$ decay amplitude) one obtains

\[
< (K\rho)_{3/2}|W|D^0 > = -\frac{1}{\sqrt{3}}(A + B) \\
< (K\rho)_{1/2}|W|D^0 > = \frac{1}{\sqrt{6}}(B - 2A - 3C) \\
< K\omega|W|D^0 > = -\frac{1}{\sqrt{2}}(B + C) \\
< \eta s K^0|W|D^0 > = -C
\]  

(13)

with

\[
A = a \\
B = b \\
C = c + \left(c + \frac{a}{3}\right)(\cos \delta_1 \exp i\delta_1 - 1)
\]  

(14)

where vanishing of $\delta_2$ and $\delta_3$ has been taken into account.

From Eqs (13,14) we see that the case with resonance-induced coupled-channel effects differs from the no-FSI case by a change in the size and phase of the $C$ parameter only:

\[
C = c \rightarrow C = c + \left(c + \frac{a}{3}\right)(\cos \delta_1 \exp i\delta_1 - 1)
\]  

(15)
Thus, after including resonance-induced coupled-channel effects, the reduced matrix elements corresponding to diagrams (a) and (b) remain unchanged and real, while the matrix element of diagram (c) acquires new size and nonvanishing phase. The coupled-channel effects may generate a sizable nonvanishing effective (c)-type diagram even if the original (c)-type amplitude was negligible as assumed in many papers [9, 4, 7]. This is shown in diagrammatic terms in Fig.3. Eq.(15) is a mathematical representation of how FSI-modified quark-diagram amplitudes are obtained. When rescattering reactions proceed by quark exchange, the corresponding $K_0$ matrix has three nonzero eigenvalues. The counterparts of Eqs (14) are then less transparent and more complicated leading in particular to the appearance of Zweig-rule violating "hairpin" diagrams [16]. In this paper we will not consider these quark-exchange contributions in detail.

In ref. [9] a simple case of flavour symmetry breaking was considered. While the couplings were still assumed to be $SU(3)$-symmetric, symmetry breaking was introduced through flavour-symmetry-breaking phase-space factor. Such breaking modifies the reaction matrix $K_0$ in the same way, independently of whether the rescattering is due to quark exchange or resonance formation. In our case Eq.(9) is replaced by

$$K_0 = \left[ \begin{array}{ccc} \frac{3}{2} & \frac{\sqrt{3}}{2} & \frac{3}{2} \epsilon \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \epsilon & \frac{1}{\sqrt{2}} \\ \sqrt{\frac{3}{2}} \epsilon & \frac{1}{\sqrt{2}} \epsilon & \epsilon^2 \end{array} \right] \cdot \kappa^2$$

(16)

where $\epsilon$ is a measure of the phase-space-induced suppression of the contribution from the $\eta_\kappa K^{*0}$ channel. By writing Eq.(16) with a universal $\kappa$ we assume an artificial situation in which the (pseudoscalar) resonances $P'$ themselves are still $SU(3)$-symmetric. Although this assumption is certainly not expected in an approach in which pseudoscalars from the intermediate $PV$ states exhibit $SU(3)$ symmetry breaking, it is not incorrect on general grounds and - for illustrative purposes - may be used as a
The eigenvalues of the $K_0$ matrix are now $\lambda_{1,2,3} = (2 + \epsilon^2)\kappa^2, 0, 0$. Proceeding as before we obtain formulae (13) again, where the only change with respect to Eqs (14,15) is

$$C = c \rightarrow C = c + \left( c + \frac{a}{2 + \epsilon^2} \right) (\cos \delta_1 \exp i\delta_1 - 1)$$  \hspace{1cm} (17)

As before, only the size and phase of the $(c)$-type amplitude is affected which should be obvious from Fig.3.

![Fig. 3 Resonance-induced generation of $(c)$-type (exchange) amplitude from $(a)$-type (factorization) diagram](image)

The procedure applied above to the $PV$ sector may be repeated in the $VP$ sector. In this sector the $K_0$ matrix has the form given in Eq.(9) (with decay channels ordered $(K^\pi)_1/2, \overline{K}^{\ast 0}\eta_{ns}, \phi\bar{K}^0$) and the same is true for the off-diagonal $(PV-VP)$ part of the total $K_0$ matrix. Introducing three eigenvectors $|j'>(j' = 1, 2, 3)$ of the $VP$ sector (given by formulae (10) with $(K\rho)_1/2, \overline{K}^0\omega, \eta_s\overline{K}^{*0}$ replaced by $(K^\pi)_1/2,$
respectively one obtains the total $K_0$ matrix:

$$K_0 = \begin{pmatrix}
3 & 3 & 0 & \ldots \\
3 & 3 & 0 & \ldots \\
0 & 0 & 0 & \ldots \\
\vdots & \vdots & \vdots & \ddots
\end{pmatrix} \cdot \kappa^2 \quad (18)
$$

with rows and columns ordered $|1>, |1'>, |2>, |2' > \ldots$. Diagonalizing the total $K_0$ matrix and repeating the procedure outlined earlier one obtains - in addition to Eqs (13) - the following expressions for the new FSI-modified $\overline{V}P$ decays:

$$< (\overline{K}^0 \pi)_{3/2} | W | D^0 > = -\frac{1}{\sqrt{3}} (A' + B')$$

$$< (\overline{K}^0 \pi)_{1/2} | W | D^0 > = \frac{1}{\sqrt{6}} (B' - 2A' - 3C')$$

$$< \overline{K}^{*0} \eta_{ns} | W | D^0 > = -\frac{1}{\sqrt{2}} (B' + C')$$

$$< \overline{\phi K}^0 | W | D^0 > = -C' \quad (19)$$

The reduced matrix elements of Eqs (13,19) are now given by

$$A = a$$

$$A' = a'$$

$$B = b$$

$$B' = b'$$

$$C - C' = c - c'$$

$$C + C' = c + c' + \left( \cos \delta_{PV} \exp i \delta_{PV} - 1 \right) \left( c + c' + \frac{a + a'}{3} \right) \quad (20)$$

where $\tan \delta_{PV} = 6\kappa^2$. In conclusion, including resonance-induced coupled-channel effects in the SU(3)-symmetric case results in a change of size and phase of matrix elements of diagrams (c) only. Since in the diagrammatical approach the size of (c)-
type amplitudes was treated as a free parameter anyway, the only observable effect of FSI is the appearance of (in general different) nonzero phases of $C$ and $C'$.

In refs. [1, 2, 3] an attempt to include the effects of FSI has been made. To permit easy comparison with expressions derived above we rewrite below a few formulae from Table 1 of ref. [3] ($SU(3)$-symmetry case):

$$
< (K \rho)_{3/2} | W | D^0 > = \frac{1}{\sqrt{3}} (A + B) \exp i\delta_{3/2}^{K \rho}
$$

$$
< (K \rho)_{1/2} | W | D^0 > = \frac{1}{\sqrt{6}} (2A - B + 3C) \exp i\delta_{1/2}^{K \rho}
$$

$$
< (K^* \pi)_{3/2} | W | D^0 > = \frac{1}{\sqrt{3}} (A' + B') \exp i\delta_{3/2}^{K^* \pi}
$$

$$
< (K^* \pi)_{1/2} | W | D^0 > = \frac{1}{\sqrt{6}} (2A' - B' + 3C') \exp i\delta_{1/2}^{K^* \pi}
$$

$$
< \phi K^0 | W | D^0 > = C' \exp i\delta^\phi
$$

$$
< \overline{K^0} \eta_{ns} | W | D^0 > = (mixture)
$$

In papers [1, 2, 3] amplitudes $A$, $B$, $C$, etc. (Eq. (21)) were real while the phase factors were allowed both real and imaginary parts in the hope of taking into account all inelasticities and phase shifts possible. After comparing Eq. (21) with Eqs (13, 19, 20) we see that resonance contribution does not follow the ansatz of [2, 3]: In Eq. (21) the relative phases of contributions from diagrams (a), (b), (c) are zero, while proper consideration of resonance-induced inelastic coupled-channel effects leaves relative phases of $A$, $B$, $A'$, $B'$ zero, but adds an important nonzero phase to $C$ and $C'$. Thus, analysis of ref. [2, 3] does not take into account effects due to the possible formation of resonances in the final state even in the case of $SU(3)$-symmetry (though elastic scattering is taken care of). (See also the paper of Hinchcliffe and Kaeding [12] for a general comment on the inclusion of FSI in the diagrammatic approach.)
3.1.2. $D_s^+ \rightarrow PV$. In this case one obtains the following FSI-modified expressions:

\[
\begin{align*}
\langle (\rho\pi)_2|W|D_s^+ \rangle & = 0 \\
\langle (\rho\pi)_1|W|D_s^+ \rangle & = D - D' \\
\langle \omega\pi^+|W|D_s^+ \rangle & = -\frac{1}{\sqrt{2}}(D + D') \\
\langle \phi\pi^+|W|D_s^+ \rangle & = -A' \\
\langle \rho^+\eta_{ns}|W|D_s^+ \rangle & = -\frac{1}{\sqrt{2}}(D + D') \\
\langle \rho^+\eta_{s}|W|D_s^+ \rangle & = -A \\
\langle K^*K^0|W|D_s^+ \rangle & = -B - D' \\
\langle K^{*0}K^+|W|D_s^+ \rangle & = -B' - D
\end{align*}
\]  

(22)

with

\[
\begin{align*}
A & = a \\
A' & = a' \\
B & = b \\
B' & = b' \\
D + D' & = d + d' \\
D - D' & = d - d' + (\cos \delta_{PV}^+ \exp i\delta_{PV}^* - 1)(d - d' - \frac{1}{3}(b - b'))
\end{align*}
\]  

(23)

From Eqs(22,23) we see that, as before, matrix elements corresponding to diagrams (a), (b) are not affected by FSI, while for (d)-type diagrams it is only the difference $D - D'$ that is modified. Even if one starts with $d - d' = 0$ (expected on the basis of flavour $u \leftrightarrow \bar{d}$ symmetry, see Fig. 1), the coupled channel effects generate a nonvanishing effective $D - D'$ proportional to $b - b'$.
3.2 Parity-violating PP decays

In the parity-violating $s$-wave decays $D^0, D^+_s \to PP$ the final-state interactions are most probably dominated by formation of scalar resonances [17]. We accept here that in the $S = -1$ sector under discussion the properties of these resonances are susceptible to a simple $q\bar{q}$ description (see next section for discussion of this assumption) so that the treatment of the previous section may be applied. Proceeding as before one can then derive:

\[
\begin{align*}
< (K\pi)_{3/2}|W|D^0 > & = -\frac{1}{\sqrt{3}}(A + B) \\
< (K\pi)_{1/2}|W|D^0 > & = \frac{1}{\sqrt{6}}(B - 2A - 3C) \\
< K^0_{\eta ns}|W|D^0 > & = -\frac{1}{\sqrt{2}}(B + C) \\
< \eta_s K^0|W|D^0 > & = -C
\end{align*}
\]

with $A = a$, $B = b$ and $C = c + (c + a/3)(\cos \delta_{PP}^0 \exp i\delta_{PP}^0 - 1)$. Even if the input quark-model ($c$)-type amplitude is negligible [7], the effective $W$-exchange amplitude may be significant.

Similarly, for $D^+_s$ decays we get

\[
\begin{align*}
< \pi^+\pi^0|W|D^+_s > & = 0 \\
< \pi^+\eta_{\eta s}|W|D^+_s > & = -\sqrt{2}D \\
< \pi^+\eta_s|W|D^+_s > & = -A \\
< K^+K^0|W|D^+_s > & = -(B + D)
\end{align*}
\]

with $A = a$, $B = b$ and $D = d + (\cos \delta_{PP}^{+-s} \exp \delta_{PP}^{+-s} - 1)(d + b/3)$.

Examples of Cabibbo-allowed $D^0$ and $D^+_s$ decays studied in this and previous subsections show how quark-line-diagram approach is affected when $q\bar{q}$ resonance
formation in the final state is taken into account. In phenomenological approaches in which sizes of matrix elements constitute free parameters, the only observable effect of such resonances in the final state is the appearance of non-zero phases of the \((c)\) and \((d)\) - type amplitudes (when compared with the FSI-unaffected amplitudes \((a)\) and \((b)\)). This is a part of the quark-diagram version of the general statement that "predictions based on flavour-symmetry groups automatically include all effects of those FSI which are invariant under these symmetries" [8]. It should be obvious now that in the most general case the amplitudes corresponding to individual diagrams should be allowed independent nonzero phases. Depending on what types of FSI are considered one can then have various conditions imposed upon these phases.

4 Cabibbo-forbidden \(D^0\) decays and \(SU(3)\) symmetry breaking

In this section we will analyze in some detail the case of \(SU(3)\) symmetry breaking in Cabibbo-once-forbidden \(D^0\) decays (in particular, the long-standing problem of the \(\Gamma(D^0 \rightarrow K^+K^-)/\Gamma(D^0 \rightarrow \pi^+\pi^-)\) ratio). In these decays there are 9 possible s-wave \(PP\) final states: \(\pi^+\pi^-, \pi^0\pi^0, K^-K^+, K^0\overline{K}^0, \eta_8\pi^0, \eta_1\pi^0, \eta_8\eta_8, \eta_8\eta_1, \eta_1\eta_1\). Each of these can be reached from the initial \(D^0\) state by appropriate linear combination of six reduced matrix elements corresponding to diagrams \((a)-(f)\) of Fig. 1. Omitting the Cabibbo factor of \(\sin \Theta_C \cos \Theta_C\) (see comment after Eqs (7,8)) we obtain the following expressions for FSI-uncorrected weak decays of \(D^0\):

\[
2 < \pi^+\pi^-|w|D^0 > = -(a + c - e - 2f)
\]

\[
2 < \pi^0\pi^0|w|D^0 > = -\frac{1}{\sqrt{2}}(b - c + e + 2f)
\]

\[
2 < K^-K^+|w|D^0 > = -(\tilde{a} + \tilde{c} + e + 2f)
\]
In Eq. (26) matrix elements with (without) a tilde correspond to a strange (non-strange) pair emitted in the original weak interaction process.

In standard approaches the contribution from diagrams (e) and (f) is negligible [11] (it vanishes in the SU(3) limit). Accordingly, we will neglect (e), (f)-type amplitudes in FSI-uncorrected $D^0$ decays. In the $SU(3)$ limit we also have $\tilde{a} = a, \tilde{b} = b, \tilde{c} = c$. Our aim is to study if and how final-state interactions reintroduce (e)- and (f)-type amplitudes and lift equalities $\tilde{a} = a, \tilde{b} = b, \tilde{c} = c$.

We will consider the contribution of PP coupled-channel effects only. The two-meson s-wave PP state may interact strongly through formation of neutral scalar resonances $S$. Assuming these belong to a $q\bar{q}$ nonet (see discussion later on) we consider three resonances with flavour quantum numbers of $\pi^0$, $\eta_8$, and $\eta_1$. Their couplings to the PP-state are of $D$-type.

The $K_0$-matrix splits block-diagonally into three submatrices in the isospin $I = 2, 1, 0$ sectors respectively:

1. sector $I = 2$: $K_0(I = 2) = 0$

   for the $(\pi\pi)_{I=2}$ state only.
2. sector $I = 1$

$$K_0(I = 1) = 2 \begin{bmatrix} 1 & \sqrt{\frac{3}{2}} & \frac{2}{\sqrt{3}} \\ \sqrt{\frac{3}{2}} & 2 & \frac{2\sqrt{2}}{3} \\ \frac{2}{\sqrt{3}} & \frac{2\sqrt{2}}{3} & \frac{4}{3} \end{bmatrix}\cdot(\kappa_S^{I=1})^2$$ \hspace{1cm} (27)

with rows (columns) corresponding to states $(K\bar{K})_{I=1}$, $\pi^0\eta_8$, $\pi^0\eta_1$.

The eigenvalues and their corresponding eigenvectors are:

$$\lambda_1^{I=1} = 6\kappa_S^2 \equiv \tan \delta_1^{I=1}$$
$$|1^{I=1}> = \frac{1}{3}(\sqrt{3}(K\bar{K})_{I=1} + \sqrt{2}(\pi^0\eta_8) + 2(\pi^0\eta_1))$$

$$\lambda_2^{I=1} = 0 \equiv \tan \delta_2^{I=1}$$
$$|2^{I=1}> = \frac{1}{3}(\sqrt{6}(K\bar{K})_{I=1} - (\pi^0\eta_8) - \sqrt{2}(\pi^0\eta_1))$$

$$\lambda_3^{I=1} = 0 \equiv \tan \delta_3^{I=1}$$
$$|3^{I=1}> = -\frac{1}{\sqrt{3}}(\sqrt{2}(\pi^0\eta_8) - (\pi^0\eta_1))$$ \hspace{1cm} (28)

Vanishing of $\lambda_2^{I=1}$ results from the assumed nonet symmetry of couplings, which relates the couplings of $(K\bar{K})_{I=1}$, $(\pi^0\eta_8)$, and $(\pi^0\eta_1)$ in such a way that linear combination $|2^{I=1}>$ decouples from the scalar $I = 1$, $I_z = 0$ meson (hereafter denoted $\pi^0_S$). State $|2^{I=1}>$ would couple to $\pi^0_S$ if $SU(3)$-breaking coupling $Tr(M_{\pi^0_S}\{M_2\lambda_8 M_3 + M_3\lambda_8 M_2\})$ were introduced. Vanishing of $\lambda_3^{I=1}$ corresponds to the absence of hairpin diagrams ($|3^{I=1}> = \pi^0_S$).

3. sector $I = 0$

$$K_0(I = 0) = 2 \begin{bmatrix} 3 & -\sqrt{3} & 1 & 2 & 0 \\ -\sqrt{3} & 3 & -\sqrt{3} & 0 & 0 \\ 1 & -\sqrt{3} & 1 & 0 & 0 \\ 2 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}\cdot(\kappa_S^{I=0})^2$$ \hspace{1cm} (29)
with rows (columns) corresponding to states (in that order): \((K\bar{K})_{I=0}, (\pi\pi)_{I=0}, \eta_{ns}\eta_{ns}, \eta_{ns}\eta_{s}, \eta_{ns}\eta_{s}\).

The eigenvalues and eigenvectors are:

\[
\begin{align*}
\lambda_{I=0}^1 &= 12\kappa_2^2 \equiv \tan\delta_{I=0}^1 \\
|1_{I=0}^f> &= \frac{1}{3}(-2(K\bar{K})_{I=0} - (\eta_s\eta_s) + \sqrt{3}(\pi\pi)_{I=0} - (\eta_{ns}\eta_{ns})), \\
\lambda_{I=0}^2 &= 6\kappa_2^2 \equiv \tan\delta_{I=0}^2 \\
|2_{I=0}^f> &= \frac{1}{3}((K\bar{K})_{I=0} + 2(\eta_s\eta_s) + \sqrt{3}(\pi\pi)_{I=0} - (\eta_{ns}\eta_{ns})), \\
\lambda_{I=0}^3 &= 0 \equiv \tan\delta_{I=0}^3 \\
|3_{I=0}^f> &= \frac{1}{3}(2(K\bar{K})_{I=0} - 2(\eta_s\eta_s) + \frac{\sqrt{3}}{2}(\pi\pi)_{I=0} - \frac{1}{2}(\eta_{ns}\eta_{ns})), \\
\lambda_{I=0}^4 &= 0 \equiv \tan\delta_{I=0}^4 \\
|4_{I=0}^f> &= \frac{1}{2}((\pi\pi)_{I=0} + \sqrt{3}(\eta_{ns}\eta_{ns})), \\
\lambda_{I=0}^5 &= 0 \equiv \tan\delta_{I=0}^5 \\
|5_{I=0}^f> &= \eta_{ns}\eta_{s} \\
\end{align*}
\]

State \(|1_{I=0}^f>\) is \(SU(3)\) singlet, while \(|2_{I=0}^f>\) is \(SU(3)\) octet. The couplings of the remaining three states are zero as a result of the quark-level nonet symmetry of D-type coupling. For example, \(\lambda_{I=0}^4\) may be nonzero if \((\pi\pi)_{I=0}\) and \((\eta_{ns}\eta_{ns})\) couplings to \(\sigma_{ns}\) (i.e. to \((u\bar{u} + d\bar{d})/\sqrt{2}\)) are not related as specified by D-type coupling. Similarly, deviation of the scale of couplings involving strange quarks from those in which strange quarks are absent would result in nonvanishing of the coupling between the \(|3_{I=0}^f>\) state and the \(I = 0\) mesons. Coupling of state \(|5_{I=0}^f>\) to \(q\bar{q}\) mesons would be nonzero if hairpin diagrams were allowed.

Let us reexpress the general FSI-modified formulae of type (26) in the \(K_0\)-matrix approach:
a) sector $I = 2$

\[
< (\pi\pi)_{I=2}|W|D^0 > = -\frac{1}{\sqrt{3}} (A + B) = \\
= 2 < (\pi\pi)_{I=2}|w|D^0 > = (-\frac{1}{\sqrt{3}})(a + b) \quad (31)
\]

\[
< (K\bar{K})_{I=1}|W|D^0 > = -\frac{1}{\sqrt{2}} (\hat{A} + C + E) = \\
= \frac{2}{\sqrt{3}} T^{(1)}_1 < 1^{I=1}|w|D^0 > + 2 < (K\bar{K})_{I=1}|w|D^0 >
\]

\[
< \pi^0\eta_8|W|D^0 > = \frac{1}{\sqrt{3}} (\hat{B} - C - E) = \\
= \frac{2\sqrt{2}}{3} T^{(1)}_1 < 1^{I=1}|w|D^0 > + 2 < \pi^0\eta_8|w|D^0 >
\]

\[
< \pi^0\eta_1|W|D^0 > = -\frac{1}{\sqrt{6}} (\hat{B} + 2C + 2E) = \\
= \frac{4}{3} T^{(1)}_1 < 1^{I=1}|w|D^0 > + 2 < \pi^0\eta_1|w|D^0 > \quad (32)
\]

where

\[
2 < 1^{I=1}|w|D^0 > = -\frac{1}{\sqrt{6}} (\hat{a} + 3c + 3e) \rightarrow SU(3) - \frac{1}{\sqrt{6}} (a + 3c) \quad (33)
\]

\[
2 < (K\bar{K})_{I=1}|w|D^0 > = -\frac{1}{\sqrt{2}} (\hat{a} + c + e) \rightarrow SU(3) - \frac{1}{\sqrt{2}} (a + c) \quad (34)
\]

and

\[
T^{(1)}_1 \equiv \cos \delta^{I=1}_1 \exp i\delta^{I=1}_1 - 1 \quad (35)
\]

Solution of Eqs (32) for SU(3)-symmetric input amplitudes is

\[
\hat{A} = a \\
\hat{B} = b \\
C + E = c + T^{(1)}_1 (c + \frac{a}{3}) \quad (36)
\]
c) sector $I = 0$

\[ < (K\bar{K})_{I=0}|W|D^0 > = \frac{1}{\sqrt{2}}(-\tilde{A} + C - 2\tilde{C} - E - 4F) = \]
\[ = \frac{2}{3}(-2T_1^{(0)} < 1^{I=0}|w|D^0 > + T_2^{(0)} < 2^{I=0}|w|D^0 >) + 2 < (K\bar{K})_{I=0}|w|D^0 > \]
\[ < (\pi\pi)_{I=0}|W|D^0 > = \sqrt{\frac{2}{3}}(-A + \frac{1}{2}B - \frac{3}{2}C + \frac{3}{2}E + 3F) = \]
\[ = \frac{2}{\sqrt{3}}(T_1^{(0)} < 1^{I=0}|w|D^0 > + T_2^{(0)} < 2^{I=0}|w|D^0 >) + 2 < (\pi\pi)_{I=0}|w|D^0 > \]
\[ < \eta_8\eta_8|W|D^0 > = \frac{1}{\sqrt{2}}(B - C - \frac{1}{3}E - 2F) + \frac{2}{3\sqrt{2}}(2(C - \tilde{C}) + (\tilde{B} - B)) = \]
\[ = \frac{2}{3}(T_1^{(0)} < 1^{I=0}|w|D^0 > + T_2^{(0)} < 2^{I=0}|w|D^0 >) + 2 < \eta_8\eta_8|w|D^0 > \]
\[ < \eta_1\eta_1|W|D^0 > = -\frac{\sqrt{2}}{3}(-C + \tilde{C} - B + \tilde{B} + E + 3F) = \]
\[ = -\frac{2}{3}T_1^{(0)} < 1^{I=0}|w|D^0 > + 2 < \eta_1\eta_1|w|D^0 > \]
\[ < \eta_1\eta_8|W|D^0 > = \frac{1}{\sqrt{2}}(B + 2C - \frac{2}{3}E + \frac{4}{3}(\tilde{C} - C) + \frac{1}{3}(\tilde{B} - B)) = \]
\[ = \frac{4}{3}T_2^{(0)} < 2^{I=0}|w|D^0 > + 2 < \eta_1\eta_8|w|D^0 > \]  
\[ \text{(37)} \]

where

\[ 2 < 1^{I=0}|w|D^0 > = -\frac{\sqrt{2}}{3}(a - \tilde{a} + 3(c - \tilde{c}) - 3e - 9f) \]
\[ 2 < 2^{I=0}|w|D^0 > = -\frac{1}{\sqrt{2}}(\frac{1}{3}(\tilde{a} + 2a) + c + 2\tilde{c} - e) \]  
\[ \text{(38)} \]

and

\[ T_j^{(0)} = \cos \delta_j^{I=0} \exp \delta_j^{I=0} - 1 \]  
\[ \text{(39)} \]

with $j = 1, \ldots, 5$ (in Eq.(37) we have used $T_{3,4,5}^{(0)} = 0$).

Eqs (37) are much simplified when one accepts the $SU(3)$-limit for input weak
decays:

\[ 2 < 1^I=0 |w|D^0 > \xrightarrow{SU(3)} 0 \]
\[ 2 < 2^I=0 |w|D^0 > \xrightarrow{SU(3)} -\frac{1}{\sqrt{2}}(a + 3c) \quad (40) \]

From Eqs (40) it follows that \(SU(3)\)-singlet resonances do not affect the final formulae since in the \(SU(3)\) limit the \(SU(3)\)-singlet state is not produced through an FSI-unmodified weak process. Solution of Eqs (37) for \(SU(3)\)-symmetric input amplitudes is

\[
\begin{align*}
\tilde{B} &= b \\
A - 2B &= a - 2b \\
\tilde{A} + B &= a + 2b \\
B + C - \tilde{C} - E - 3F &= b \\
\tilde{C} + F &= c + T_2^{(0)}(c + \frac{a}{3}) 
\end{align*}
\quad (41)
\]

A look at how resonance-induced FSI (Fig. 3 and its counterparts) generate FSI-modified diagrams from the input \((a), (b), (c)\) amplitudes confirms that diagrams of type \((a)\) and \((b)\) cannot actually be generated. Thus, we must have

\[
\begin{align*}
A = \tilde{A} &= a \\
B = \tilde{B} &= b \\
C - E - 2F &= \tilde{C} + F 
\end{align*}
\quad (42)
\]

From Eqs (36,41,42) we see that in the case when strong interactions exhibit \(SU(3)\) symmetry, i.e. when the intermediate \(I = 1\) and \(I = 0\) octet scalar resonances are
degenerate and couple to $PP$ with the same strength $\kappa_S^I = \kappa_S^{I=1} = \kappa_S$ so that

$$\delta_2^{I=0} = \delta_1^{I=1} \equiv \delta$$

$$T_2^{(0)} = T_1^{(1)}$$

one obtains

$$E + F = \frac{1}{2}(T_1^{(1)} - T_2^{(0)})(c + \frac{a}{3}) = 0$$

$$C + E = \tilde{C} + F = c + (\cos \delta \exp i\delta - 1)(c + \frac{a}{3})$$

Thus, physically measurable amplitudes may be described with vanishing effective penguin amplitudes $E$, $F$ and $SU(3)$-symmetric exchange amplitudes $\tilde{C} = C$.

In reality, in the energy region below 1500 $MeV$ the scalar resonance sector exhibits peculiar $SU(3)$-symmetry breaking \cite{18, 19}. The masses and couplings of physical resonances are known to be different from those expected when these resonances are assigned a simple $qq$ structure. Descriptions of these resonances as $qqq\bar{q}$ states have been proposed. Despite years of intensive efforts the questions related to the nature of these states have not been settled as yet. The strangeness $S = 0$ sector under consideration in this section is particularly troublesome as exhibited by conflicting interpretations of the $a_0(980)$, $f_0(980)$, and $f_0(1300)$ states \cite{20}. One should therefore expect that in the $D$-mass region around 1870 $MeV$ the situation is also complicated (see also \cite{17}). A detailed analysis of the nature of resonances affecting two-meson interactions at this energy is clearly far beyond the scope of this paper. For our purposes the crucial point is the relative size of amplitudes corresponding to diagrams of Fig. 2 and Fig. 4.
If diagrams of Fig. 4 do not contribute significantly to the mechanism of physical resonance formation, as is the case in the unitarised quark model (UQM) of Törnqvist [18, 19], one should expect that the values of amplitudes $A, B$ may be taken from the $S = -1$ sector of $D^0$ decays or from $D^+$ decays, and subsequently used in the $S = 0$ sector. On the other hand, the remaining parameters of the quark-line approach needed for the description of the $D^0 \rightarrow \pi\pi, K\overline{K}$ etc. decays may exhibit significant $SU(3)$-breaking within and between the $I=0$ and $I=1$ subsectors of the $S=0$ sector (and, of course, between the $S=0$ and $S=-1$ sectors) [18]. In the analysis of Törnqvist [19] the scalar meson sector exhibits energy-dependent mixing of the two $I = 0$ resonances. The value of the mixing angle undergoes a fairly rapid change in the vicinity of the $K\overline{K}$ threshold. Below $900 \text{ MeV}$ the mixing is nearly ideal while above $1.1 \text{ GeV}$ one has nearly pure $SU(3)$ eigenstates. The $f_0(1300)$ appears then as a near-octet resonance. Although the analysis of [19] stops at $1.6 \text{ GeV}$ there are no reasons to expect a qualitative change in the mixing angle when energy changes from 1.6 to 1.87 $\text{GeV}$: The majority of $PP$ thresholds lie well below 1.6 $\text{GeV}$. Despite significant $SU(3)$ breaking incorporated into the UQM (i.e. through realistic positions of thresholds) this model confirms essentially that our use of pure $SU(3)$ eigenstates at $D^0$ energy is justified. $SU(3)$-breaking enters the $K_0$ matrix through the difference in the real parts of the vacuum polarization functions $\Pi(s)$ [19] which are different.
for each of the $I = 1$ and two $I = 0$ states. Consequently, the $I = 0$ and $I = 1$ octet channels are affected differently corresponding to strong $SU(3)$ breaking between $a_0(980)$ and $f_0(1300)$. In our simplified treatment this means that the sizes of effective couplings in the $I = 1$ and $I = 0$ octet channels are different. As a result we should treat the isospin amplitudes in the $I=0$ and $I=1$ sectors as independent free parameters.

In terms of amplitudes with definite isospin the amplitudes of the four measured $D^0 \to K\bar{K}, \pi\pi$ decays are given by

\[
\begin{align*}
<K^+K^-|W|D^0> &= \frac{1}{\sqrt{2}}(\mathcal{K}_1 + \mathcal{K}_0) \\
<K^0\bar{K}^0|W|D^0> &= \frac{1}{\sqrt{2}}(\mathcal{K}_1 - \mathcal{K}_0) \\
<\pi^+\pi^-|W|D^0> &= \frac{1}{\sqrt{3}}(\mathcal{P}_2 + \sqrt{2}\mathcal{P}_0) \\
<\pi^0\pi^0|W|D^0> &= \frac{1}{\sqrt{3}}(\sqrt{2}\mathcal{P}_2 - \mathcal{P}_0)
\end{align*}
\]

(45)

where the amplitudes of definite isospin (specified by subscript) can be expressed in terms of quark-line amplitudes as follows

\[
\begin{align*}
\mathcal{K}_1 &= -\frac{1}{\sqrt{2}}(\tilde{A} + C + E) \\
\mathcal{K}_0 &= -\frac{1}{\sqrt{2}}(\tilde{A} + \tilde{C} + F) \\
\mathcal{P}_2 &= -\frac{1}{\sqrt{3}}(A + B) \\
\mathcal{P}_0 &= \sqrt{\frac{2}{3}}(-A + \frac{B}{2} - \frac{3}{2}(\tilde{C} + F)) = \frac{1}{\sqrt{6}}(A + B) + \sqrt{3}\mathcal{K}_0
\end{align*}
\]

(46)

Note that (apart from the contribution of the FSI-unmodified $(a)$- and $(b)$-type amplitudes) the isospin $I = 0$ $(1)$ amplitude may be put in a one-to-one correspondence with the $\tilde{C} + F$ $(C + E)$ combination of quark-diagram amplitudes respectively. Defining $Y = \frac{1}{\sqrt{2}}(\mathcal{K}_1 - \mathcal{K}_0)$ and $X = \sqrt{2}\mathcal{K}_0$ the amplitudes of the four considered decays...
acquire simple form given in Table 1, where the corresponding experimental branching ratios taken from [21] are also displayed.

Table 1. Theoretical amplitudes and branching ratios of the four $D^0 \to KK, \pi\pi$ decays measured.

| decay       | amplitude | branching ratio in % |
|-------------|-----------|----------------------|
| $K^+ K^-$   | $X + Y$   | 0.454 ± 0.029        |
| $K^0 \bar{K}^0$ | $Y$      | 0.11 ± 0.04          |
| $\pi^+\pi^-$ | $X$      | 0.159 ± 0.012        |
| $\pi^0\pi^0$ | $-\frac{1}{\sqrt{2}}(A + B + X)$ | 0.088 ± 0.023        |

The data of Table 1 permit us to establish that

$$|X + Y| = (4.51 \pm 0.14) \times 10^{-6} \text{ GeV}$$
$$|Y| = (2.21 \pm 0.40) \times 10^{-6} \text{ GeV}$$
$$|X| = (2.47 \pm 0.09) \times 10^{-6} \text{ GeV}$$
$$|A + B + X| = (2.60 \pm 0.3) \times 10^{-6} \text{ GeV}$$

From the branching ratio of the $D^+ \to \pi^+\bar{K}^0$ decay (equal to $(2.74 \pm 0.29) \times 10^{-2}$ [21]) described by amplitude $A + B$ one infers that

$$|A + B| = (1.35 \pm 0.07) \times 10^{-6} \text{ GeV}$$

Equations (47) and (48) show that, contrary to the conclusions of ref.[11], it is still possible to keep $SU(3)$-symmetry in factorization amplitudes $A, B$ provided one correctly describes final state interactions ($SU(3)$-symmetry of factorization amplitudes was used when writing the last equality in Eq.(16)). In particular, even if the data
were consistent with \( Y = 0 \) (and \(|X + Y| = |X| \approx 2.5 \times 10^{-6} \text{GeV}\)), i.e. if the amplitudes were \( SU(3) \)-symmetric, we would still have to conclude from the values of \(|A + B|, |X|, \) and \(|A + B + X|\) that the relative phase of \( A + B \) and \( X \) must be close to 90°. Since in the quark-diagram approach \( X = -A - \tilde{C} - F \), it follows that the relative phase of \( \tilde{C} + F \) and \( A, B \) must be significant, in agreement with the message of this paper. Nonzero phase of \( \tilde{C} + F \) is a direct result of inelasticity in FSI.

The \( SU(3) \)-breaking \( Y \) amplitude is expressed through quark-diagram amplitudes as \( Y = -\frac{1}{2}(C + E - \tilde{C} - F) \). Large observed size of \( Y \) means that, when interpreted in terms of quark-diagram amplitudes, the data can be described either by a strong breaking of \( SU(3) \)-symmetry in \( W \)-exchange amplitude (\( C \neq \tilde{C} \) and \( E - F \approx 0 \)) or by a large contribution from effective long-range penguins (\( E - F \neq 0 \)), or both. One has to keep in mind, however, that in the \( SU(3) \)-symmetry breaking case the combination \( C + E \) (determined from the \( I = 1 \) sector) cannot be used in the \( I = 0 \) sector (i.e. in Eq.(42)): The diagrammatic approach suggests more symmetry between the \( I = 0 \) and \( I = 1 \) sectors than is actually present.

5 Conclusions

We have studied in some detail how inelastic coupled-channel rescattering effects (and, in particular, \( q\bar{q} \) resonance formation in the final state) modify the input weak amplitudes of the quark-line diagrammatical approach. Through an explicit calculation it has been demonstrated that such coupled-channel effects lead to the appearance of nonzero relative phases between various quark diagrams, thus invalidating the way in which final-state interactions were incorporated into the diagrammatical approach in the past. The case of \( SU(3) \)-symmetry breaking in Cabibbo once-forbidden \( D^0 \)
decays has been also discussed. It has been shown that data may be described when inelastic final-state interactions (which must be $SU(3)$-breaking as well) are introduced. On the other hand, contrary to statements in literature, the data do not require $SU(3)$-symmetry breaking in factorization amplitudes.

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