Reconstruction of surface impedance of an object located over a planar PEC surface

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Abstract. A method for the determination of inhomogeneous surface impedance of an arbitrary shaped cylindrical object located over a perfectly conducting (PEC) plane is presented. The problem is reduced to the solution of an ill-posed integral equation by the use of single layer representation which is handled by Truncated Singular Value Decomposition (TSVD). The total field and its normal derivative on the boundary of the object which are required for the evaluation of the surface impedance are obtained through Nyström method. The method can also be used in shape reconstruction by using the relation between the shape of a PEC object and its equivalent one in terms of the surface impedance. The numerical implementations yield quite satisfactory results.

1. Introduction

One of the effective approaches used in the solution of electromagnetic scattering problems is to establish an equivalent problem to the original one through the impedance boundary condition (IBC), which aims to reduce its mathematical and numerical complexities. IBC gives a relation between the electric and magnetic field vectors on a certain boundary in terms of a coefficient called surface impedance. IBC can directly be defined either on the surface of the actual scatterer or on a fictitious boundary [1–4]. On the other hand, due to the equivalence principle the surface impedance is related to the geometrical and physical properties of the actual scatterer. In [5] it is shown that there is an explicit relation between the geometrical variations of an arbitrary perfectly conducting object and equivalent surface impedance placed on a circular one covering the original scatterer. Thus as long as the surface impedance is known, one can extract the geometrical properties of the PEC scatterer from this information. This property can be used in the inverse scattering problems whose aim is to determine the shape of an inaccessible PEC object from the measured scattered field. For that reason, the reconstruction of the inhomogeneous surface impedance of a known boundary is of importance from both theoretical and application points of view.

The main objective of this paper is to extend the method developed in [6] to the reconstruction of the surface impedance of a cylindrical object located above a perfectly conducting planar surface and its use in the shape reconstruction of PEC targets. The results of such problems may have applications in the inverse scattering problems related to objects located in the atmosphere. In such a case, the earth surface can be modeled by the perfectly conducting plane. In the present study, the scattered field in the half space above the PEC plane is represented by a single-layer potential and the density of the single layer potential is obtained by solving the resulting Fredholm integral equation of the first kind.
Since the latter one is ill posed, a regularized solution is given via the truncated singular value decomposition scheme (TSVD). The use of the jump relations for single-layer potential leads to the explicit expressions of the scattered field and its normal derivative on the impedance surface. Then the least-squares reconstruction of the surface impedance is achieved by using the SIBC itself. The surface impedance reconstruction algorithm mentioned above is also extended to the shape reconstruction of the perfectly conducting objects. To this aim, the unknown object is equivalently represented in terms of a known fictitious surface having a surface impedance on its boundary. Then the surface impedance is reconstructed via the method described above from the measured values of the scattered field due to the object to be reconstructed. The determination of the surface impedance is achieved using the explicit expression between the surface impedance and the shape of the actual object given in [5] the determination of the shape is achieved. The method is very effective for the reconstruction of smooth and slightly varying impedances. Similar observation is valid for the application of the method to the shape reconstruction problems, i.e it is capable of determining the shape of the PEC objects whose surfaces are relatively simple.

In section 2 the general formulation of the inverse impedance problem is given and a solution is presented. Section 3 is devoted to the application of the method for the shape reconstruction of PEC objects while in section 4 the numerical simulations are given. Finally, conclusions and concluding remarks are given in section 5.

A time factor \( \exp(-\omega t) \) is omitted throughout the paper.

2. Solution of the Inverse Problem

Consider the two-dimensional (2D) electromagnetic scattering problem illustrated in figure 1. In this configuration a body \( D \) having an inhomogeneous surface impedance \( Z(x) \) on its boundary \( \partial D \) is located in a homogeneous half space bounded by the perfectly conducting plane \( x = 0 \), where \( x = (x_1, x_2) \) is the position vector in \( \mathbb{R}^2 \). It is assumed that \( \partial D \) is a smooth boundary and can also be represented in the parametric form \( \partial D : \{(x_1(t), x_2(t)); t \in (0, 2\pi)\} \). The electromagnetic constitutive parameters of the background medium are \( \varepsilon, \mu \) and \( \sigma = 0 \). On the boundary \( \partial D \) the applicable boundary condition is the standard impedance boundary condition given by

\[
-n \times (n \times E) = Z(x)n \times H \text{ on } \partial D, \tag{2.1}
\]

where \( E \) and \( H \) are the total electric and magnetic field vectors, respectively and \( n \) is the outward unit normal vector of \( \partial D \). The inverse scattering problem considered here is to reconstruct the inhomogeneous surface impedance \( Z(x) \) through the measured values of the scattered field on a certain limited domain denoted by \( \Gamma \) (See Figure 1).

To this aim the body is illuminated by a time-harmonic TM polarized plane wave whose electric field vector is

\[
E(x) = (0, 0, u^i(x)), \quad u^i = e^{-ik(x_1 \cos \varphi_0 + x_2 \sin \varphi_0)}
\]

where \( \varphi_0 \in (0, \pi) \) is the incidence angle and \( k = \omega \sqrt{\varepsilon \mu} \) stands for the wave number of the background medium. Note that in such a case the total electric vector will be in the form of \( E = (0, 0, u) \) and hence the problem can be formulated in terms of the scalar field function \( u \).
In order to formulate the problem in an appropriate way, we first introduce the field \( u^0 \) which is the total field in the half space in the absence of the body \( D \) and its explicit expression can be found in any ordinary textbook,

\[
u^0 = e^{-ik(x_1 \cos \phi_1 + x_2 \sin \phi_1)} - e^{-ik(x_1 \cos \phi_2 - x_2 \sin \phi_2)}
\]

Then the difference \( u' = u - u^0 \) corresponds to the scattered field due to the body \( D \) and satisfies the reduced wave equation

\[
\Delta u^s + k^2 u^s = 0 \quad \text{in} \quad \mathbb{R}^2 \setminus \overline{D} \quad \text{and} \quad x_2 > 0
\]

under the boundary conditions

\[
u + \frac{\eta}{ik} \frac{\partial u}{\partial n} = 0 \quad \text{on} \quad \partial D
\]

and

\[
u' = 0 \quad \text{on} \quad x_2 = 0
\]

with the Sommerfeld radiation condition

\[
\lim_{r \to \infty} \sqrt{r} \left( \frac{\partial u'}{\partial r} (x) - i k u' (x) \right) = 0, \quad r = |x|
\]

In (2.3) \( \eta \) is the normalized surface impedance defined by

\[
\eta(x) := \frac{Z(x)}{Z_0}, \quad x \in \partial D
\]
where $Z_0 = \sqrt{\mu / \varepsilon}$ denotes the intrinsic impedance of the background medium.

In view of (2.3) the surface impedance can be obtained from the values of the total field $u$ and its normal derivative $\partial u / \partial n$ on $\partial D$ via

$$\eta(x) = -ik \frac{u(x)}{\partial u / \partial n(x)}, \quad x \in \partial D. \quad (2.6)$$

In what follows a method similar to one given in [6] is described for reconstructing the required field values on the boundary $\partial D$ from the measured scattered field data. To this aim by the use of image theorem the scattered field is represented as a single-layer potential of the form

$$A \varphi(x) = i \int_{\partial D} H^{(1)}_0(k|x-y|)\varphi(y)ds(y) + i \int_{\partial D'} H^{(1)}_0(k|x-y'|)\varphi(y')ds(y') \quad (2.7)$$

with an unknown density function $\varphi$. Here the point $y' = (y_1, -y_2)$ is the image of the point $y$ and $H^{(1)}_0$ is the Hankel function of the first kind with zero order. We also note that the boundary $\partial D'$ is the image of $\partial D$ with respect to the plane $x = 0$ (see figure 2.) and as a result of the image theorem $\varphi(y') = -\varphi(y)$.

Figure 2. Image of the object $D$

Once the single-layer density $\varphi$ is known after solving the Fredholm integral equation of the first kind
\[ A\phi(x) = \frac{i}{4} \int_{\partial D} H_0^{(1)}(k|x-y|)\varphi(y)\,ds(y) + \frac{i}{4} \int_{\partial D} H_0^{(1)}(k|x-y'|)\varphi(y')\,ds(y') \]

the values \( u \) and \( \partial u/\partial n \) of the total field on the boundary \( \partial D \) can be recovered through the jump relations for the single-layer potential [7], that is, by

\[
u(x) = u^0(x) + \frac{i}{4} \int_{\partial D} H_0^{(1)}(k|x-y|)\varphi(y)\,ds(y) \]
\[ + \frac{i}{4} \int_{\partial D} H_0^{(1)}(k|x-y'|)\varphi(y')\,ds(y'), \quad x \in \partial D \]

and

\[
\frac{\partial u}{\partial n}(x) = \frac{\partial u^0}{\partial n}(x) + \frac{i}{4} \int_{\partial D} \frac{\partial H_0^{(1)}(k|x-y|)}{\partial n(x)}\varphi(y)\,ds(y) \]
\[ + \frac{i}{4} \int_{\partial D} \frac{\partial H_0^{(1)}(k|x-y'|)}{\partial n(x)}\varphi(y')\,ds(y') \frac{1}{2} \varphi(x), \quad x \in \partial D\]

For the numerical evaluation of these singular integrals over \( \partial D \), we make use of the same quadrature formulae in connection with the Nyström method [8].

3. Application to shape reconstruction of conducting objects

In [5] it is shown that there is an explicit relation between the shape of a PEC object and its equivalent surface impedance defined on a fictitious boundary which is assumed to cover the actual one. By considering this property, the reconstructed surface impedance can be used for the shape determination of the PEC objects. To this aim, a circular domain is considered which covers the perfectly conducting object to be reconstructed and set an equivalent problem by imposing an inhomogeneous surface impedance \( Z(\varphi) \) on the new circular boundary (See figure 3).

![Figure 3. PEC object with arbitrary geometry and its equivalent one in terms of surface impedance](image)
If the object has a slightly varying and smooth boundary, it can be represented in terms of a standard impedance boundary condition \([5]\) and the following relation is valid between the surface function \(f(t) = \sqrt{x_1(t)^2 + x_2(t)^2}\) of the PEC object and parametric form of the equivalent surface impedance \(Z(t)\) on the circular boundary \(\rho = a\), namely,

\[
Z(t) = i\omega \mu_0 (f(t) - a), \quad t \in (0, 2\pi)
\]  

(3.1)

Then, for the given measured scattered data related to the actual object, we first reconstruct the surface impedance \(Z(t)\) via the method given in previous section then obtain the surface function.

4. Numerical Results
In this section, some illustrative examples are presented both for surface impedance determination and its use in the shape reconstruction. The data which should be collected by real measurements are created synthetically by solving the associated scattering problem through the mixed layer potential approach \([9]\). In all numerical examples, the frequency of the incident wave is chosen as \(f = 300\, \text{MHz}\) and the background as free space corresponding to a wavelength \(\lambda = 1\, \text{m}\). A random noise of level 1\% is added to the simulated data for each example. In particular, a random term \(n_i |u^e| e^{2i\pi \varphi} \) is added to each scattered field value \(u^e\), with \(n_i\) being the noise level and \(r_d\) a random number between 0 and 1. In the application of the least-squares solution, the basis functions are chosen as combinations of \(\cos(2\pi p \varphi)\) and \(\sin(2\pi p \varphi)\), \(p = 0, 1, \ldots, P\), and the number \(P\) is determined by trial and error.

The use of the reconstructed surface impedance in the shape reconstruction applications is tested by first considering a perfectly conducting object with the parametric equation,

\[
\partial D = \{(0.4 + 0.02 \cos(8t)) \cos(t), 2 + (0.4 + 0.02 \sin(8t)) \sin(t) : t \in [0, 2\pi]\}.
\]  

(4.1)

The scattered field measurements due to this object is assumed to be performed on the semi circle \(R = 5, \varphi \in (0, \pi)\). The equivalent impedance boundary is chosen as the circle with the radius \(a = 0.42\) and center \((0, 2)\) and the surface impedance is reconstructed on this circle from above given data. By using the relation (3.1) the shape of the object is reconstructed. In figure 4 the variation of the exact and reconstructed values of the surface impedance on the circle \(a=0.42\, \text{m}\) are given. The exact and reconstructed shapes of the object are illustrated in figure 5. As can easily be observed the reconstructed shape is very close to the actual one. Note that in this example the variation of the surface is small compared to wavelength.
5. Conclusions

The inverse scattering problem whose aim is to reconstruct the surface impedance of a cylindrical object of arbitrary shape over a PEC plane is solved through the extension of the method in [6]. On the other hand, by means of the equivalency theorem, it is possible to show that a PEC object can be represented in terms of a surface impedance defined on a known surface and the surface impedance is explicitly related to the shape of the actual object. By the use of this result the method can also be used in the shape reconstruction problems related to PEC bodies.

The method yields quite satisfactory surface impedance reconstructions for a single illumination even in the case of aspect limited data. This is due to the fact that the planar PEC boundary reflects all the field which carries the information about the non-illuminated part of the object. In other words, the
reflection of the incident field from the PEC plane behaves like a second excitation and interacts with the shadow part of the object. When a PEC object is equivalently represented by a surface impedance, it yields quite accurate shape reconstructions for object with slightly varying boundaries. This is the result of using only the standard impedance boundary condition for the equivalent problem. By using higher order IBC, it may be possible to reconstruct more complex shapes. Future studies are devoted in this direction.

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