Robust Backstepping Super-Twisting Sliding Mode Control for Autonomous Vehicle Path Following

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ABSTRACT

This paper focuses on the path following control problems for autonomous driving vehicles. Aiming at enhancing the robustness and attenuating the chattering phenomenon, a super-twisting sliding mode control algorithm (STA) is developed based on Lyapunov theory, where the proof of the stability of the control system is presented by applying the backstepping technique. Moreover, co-simulation between Matlab/Simulink and Carsim is carried out to verify the path following control performance. In this research, Stanley controller, conventional sliding mode control (SMC), and model predictive control (MPC) are used as the benchmark controllers for evaluating the proposed STA performance. Two driving scenarios are considered in the simulations, including normal driving and fierce driving. To comprehensively assess the control performance and control effort (i.e., magnitude of steering), an integrated and weighted performance evaluation index (IWPEI) is novelly proposed. Simulation results show that the IWPEI of the proposed STA can be reduced by 40.5\%, 25.8\%, 10.9\% in the normal driving scenario; and 62.5\%, 24\%, 6.8\% in the fierce driving scenario as compared with Stanley controller, conventional SMC, and MPC, respectively. The results also indicate that the proposed STA outperforms the conventional SMC in terms of the chattering attenuation, resulting in a smoother front steering wheel angle input and a smoother yaw rate performance. As compared with MPC, the advantage of the proposed STA lies in its much lower computational complexity. Furthermore, the robustness of the controllers is verified by changing the vehicle mass and tire parameters. The proposed STA can reduce the fluctuation of the IWPEI by 22.6\%, 22.3\%, and 5.9\% compared with the benchmark approaches. These results imply that the consideration of system perturbations is very critical in the design of the super-twisting sliding mode controller which can improve the robustness of the autonomous vehicle path following system.

INDEX TERMS

Path following control, super-twisting sliding mode, backstepping, robustness, perturbations.

I. INTRODUCTION

Autonomous vehicles receive more and more attention nowadays with the ever-increasing safety transportation requirements. The famous “DAPRA Grand Challenge” and “DAPRA Urban Challenge” began in the U.S. more than ten years ago [1] to foster the development of autonomous driving (AD) technology. It is well known that AD is a hard and complex technology for putting it into practical use and mass production, especially for the trajectory tracking problems under high velocity [2]. It demands the steering system automatically steers the vehicle to follow the desired path. Besides, it should also maintain driving safety, lateral stability, and comfort simultaneously [3]. However, the path following control is very challenging due to high system nonlinearity, perturbations, and influence on other vehicle performances [4]–[9].

The goal of the trajectory tracking control is to minimize the lateral deviation as much as possible and maintain an acceptable yaw rate simultaneously [10], [11]. The existing
researches generally adopt three types of vehicle models for the path following controller design, including geometric-based model, kinematic-based model, and dynamic model. The three different models describe the geometrical configuration of the steering system (mainly the Ackerman steering system), the kinematic relationship of the vehicle motion, and the vehicle dynamic motion with internal states, respectively [5]. As reported, many vehicles in the DAPRA competition use the geometric-based model to design the steering controller. Meanwhile, many researchers use the kinematic model for controller development [1], [12]. However, due to the negligence of the internal dynamics of the model, the geometric and kinematic-based controllers have some limitations for practical applications [13]. Therefore, the dynamic vehicle model-based controllers are mostly used in trajectory following controller design recently [10], [11], [14]–[18].

Linear, nonlinear, and optimization control methods were employed by many studies on model-based controllers for trajectory following [3], [19]–[30] The famous Stanley steering control law was designed based on the nonlinear feedback function of lateral deviation, and exponential convergence is obtained under some assumptions in [1]. However, it is unsuitable for passenger vehicles as it is mainly designed for lower longitudinal vehicle speed. In [31], a robust $H_{\infty}$ controller based on a linear parameter varying (LPV) model is developed to improve the path tracking performance. The internal parameter uncertainties are incorporated into the polytopic model, which consists of eight sub-models. However, the external disturbances are not discussed in [32], [33], the authors developed and compared three types of trajectory tracking controllers, namely, super twisting sliding mode controller (ST-SMC), immersion and invariance controller (I&I), and adaptive passivity-based controller (APBC). These controllers were regarded as nonlinear proportional and integral control methods. The control performances were verified through a driver simulator SCANeR Studio, and the results showed all three controllers had good tracking performance. However, the external disturbances and internal parameter variations (e.g. vehicle mass variation, change of cornering stiffness, etc.) are not sufficiently considered during the controller design. Hence the I&I controller is sensitive to the parameter uncertainties and the APBC has a high possibility to request high control gains during a large curvature variation. The high gains are prone to cause more energy consumption and input saturation. For the ST-SMC controller in [32], it combines a square-root correction term and the integral of the high gain switch function to the steering input angle. Even though the control law is the simplest one among the three controllers, the proposed ST-SMC is sensitive to disturbances or parametric uncertainties resulting in significant peaks during the rapid variation of curvature. Another form of SMC entitled integral sliding mode (ISM) was proposed in [4] to improve the chattering phenomenon and control performance. The ISM is combined with composite nonlinear feedback (CNF) controller for controlling the front wheel steering angle to track the desired path, which has smaller overshoots and lateral deviation as compared with the pure CNF controller [34]. However, the longitudinal speed variation and the unmodeled dynamics (i.e. perturbations) are not covered in this research. Besides, linear and nonlinear model predictive control (MPC) was used in [35], [36] to design the path following controller. In [35], the changing road conditions and the small-angle simplification for the vehicle sideslip angle are cast into a measurable disturbance. The discrete previewed path points are used to construct the objective function to improve the performance under lower acceleration in [36]. To improve the computational efficiency in addressing the non-linear optimization problem, [37] adopted a continuous-time finite-horizon approximate dynamic programming method. Nevertheless, the relatively heavy computation burden is still the major problem with using the MPC for practical vehicles.

For the evaluation of path following control performance, most of the researches [7], [8], [12], [13] only took the mean absolute error (MAE) of lateral deviation and heading angle with respect to the desired trajectory. Honestly, the evaluation indexes are probably not fair because the control effort and the product of error and time are not taken into account. These factors are pretty critical to objectively assess the control performance. Hence, it is recommended to propose a novel integrated and weighted index for a comprehensive evaluation of the controller.

To this end, this paper aims to develop a path following controller which can overcome the chattering problem of the front steering angle input with good robustness in response to external or internal perturbations under the premise of taking less control effort and computation time. Based on these requirements, a novel supertwisting sliding mode control method with consideration of perturbations is developed in this work. The contributions of this study are summarized as follows: 1) A novel super-twisting sliding mode control algorithm (STA) considering the external and internal perturbations is proposed for path following. Due to the consideration of the perturbations, the proposed super-twisting sliding mode controller can improve the accuracy and robustness in the path following problem for autonomous vehicles; 2) The stability of the proposed controllers is proved using the Lyapunov method, in which the backstepping technique is applied to simplify the proof of the controller stability; 3) A novel integrated and weighted performance evaluation index (IWPEI) is proposed to more fairly assess the control performance under the same conditions.

The remainder of this paper is organized as follows. Section II describes vehicle dynamic modeling, control problem formulation, and steering system constraints. Stanley controller, conventional SMC MPC, and the proposed STA controllers are presented in Section III. The simulation results and discussion are shown in Section IV. Finally, Section V gives a conclusion.
II. AUTONOMOUS GROUND VEHICLE MODELING AND CONTROL PROBLEM FORMULATION

The path following control for an autonomous vehicle is requested to track the reference path as accurately as possible. It is assumed that the longitudinal speed is changed slowly \( V_x = 0 \). The planar motion is mainly considered during the path following process. The following parts concern the vehicle dynamic model and control problem formulation.

A. SINGLE-TRACK MODEL

A schematic diagram for a single-track vehicle model is shown in Figure 1. Generally, the front steering angle is assumed to be a small angle [36]. The planar motion of the autonomous vehicle can be formulated to represent the lateral dynamic behavior (sideslip angle \( \beta \), yaw rate \( \dot{\phi} \)). The side slip angles of tires on the same axle are assumed as the same. The roll and pitch dynamics are neglected for simplification. The front steering wheel angle \( \delta_f \) and sideslip angle \( \beta \) are considered as small angles under normal driving conditions. Then a linear tire lateral force region can be obtained. Based on these assumptions, a linear parameter varying (LPV) model can be considered as small angles under normal driving conditions. Then a linear tire lateral force region can be obtained. Based on these assumptions, a linear parameter varying (LPV) model can be constructed, where the longitudinal velocity \( V_x \) is regarded as a varying parameter [38]. The dynamic single-track model in terms of sideslip angle and yaw rate \( \dot{\phi} \) is shown as below:

\[
\begin{align*}
\dot{\beta} &= -\frac{\mu}{m V_x} (C_f + C_r) \beta - \left( 1 + \frac{\mu}{m V_x^2} \right) \beta - \frac{\mu l_f C_f - l_r C_r}{m V_x^2} \dot{\phi} + \frac{\mu l_f C_f}{m V_x} \delta_f \\
\dot{\phi} &= -\frac{\mu}{l_z} (l_f C_f - l_r C_r) \beta - \frac{\mu l_f^2 C_f + l_r^2 C_r}{l_z V_x} \dot{\phi} + \frac{\mu l_f C_f}{l_z} \delta_f
\end{align*}
\]

where \( \mu \) is the road friction coefficient, \( m \) is the vehicle mass, \( l_f \) and \( l_r \) are the distances of Center of Gravity (CG) to front axle and rear axle, \( l_z \) is the yaw moment of inertia, \( C_f \) and \( C_r \) are cornering stiffness of front and rear tires. Here, the cornering stiffness is considered twice over a single tire [39]. The following part introduces the control problem formulation.

B. CONTROL PROBLEM FORMULATION

As shown in Figure 2, the path following problem can be considered as minimizing the lateral deviation of the autonomous vehicle with respect to the predefined reference path. The second-order deviation of the lateral error is denoted as

\[
\ddot{e} = a_y - a_{\text{ref}},
\]

where \( a_y \) represents the actual lateral acceleration of the vehicle; \( a_{\text{ref}} \) denotes the desired lateral acceleration on the designed path.

The reference lateral acceleration can be written as

\[
a_{\text{ref}} = \rho(S) V_x^2,
\]

where \( \rho(S) \) is the curvature of the reference path, and \( S \) is the displacement. Assume the actual lateral acceleration is given by \( a_y = V_x (\dot{\beta} + \dot{\phi}) \) then

\[
\ddot{e} = V_x (\dot{\beta} + \dot{\phi}) - \rho(S) V_x^2.
\]

Substituting (1) into (4), the following error dynamics can be obtained:

\[
\ddot{e} = -\frac{\mu}{m} (C_f + C_r) \beta - \frac{\mu l_f C_f - l_r C_r}{m V_x} \dot{\phi} - \rho(S) V_x^2 + \frac{\mu l_f \delta_f}{m} \dot{\phi}
\]

By defining the state vector as \( x = (\beta, \dot{\phi}, \dot{e}, \dot{\phi})^T \) which corresponds to vehicle sideslip angle, yaw rate, the first derivative of lateral error and lateral error, the control input as front steering angle \( \delta_f \) the reference curvature \( \rho(S) \) as external disturbance, the following system equation can be obtained:

\[
\dot{x} = Ax + B_1 u + B_2 d,
\]
where

\[
A = \begin{bmatrix}
\frac{\mu (C_f + C_r)}{mV_x} & -1 - \frac{\mu (l_f C_f - l_r C_r)}{mV_x^2} & 0 & 0 \\
\frac{\mu (l_f C_f - l_r C_r)}{I_z} & -\frac{\mu (l_f^2 C_f + l_r^2 C_r)}{mV_x^2} & 0 & 0 \\
0 & \frac{\mu (C_f - C_r)}{m} & 0 & 0 \\
0 & \frac{mV_x}{m} & 0 & 1 \\
\end{bmatrix},
\]

\[
B_1 = \begin{bmatrix}
\frac{\mu C_f }{mV_x} \\
\frac{\mu l_f C_f }{l_f V_x} \\
\frac{m}{\mu C_f} \\
0 \\
\end{bmatrix}, 
B_2 = \begin{bmatrix}
0 \\
0 \\
-\frac{V_x^2}{m} \\
0 \\
\end{bmatrix}, 
\]

\[u = \delta_f = \rho(S).\]

Under the predefined curvature and longitudinal velocity for the desired trajectory, the desired control target is \( \dot{e} = e = 0 \).

The desired sideslip angle and yaw rate for a linear vehicle model is

\[
\begin{align*}
\beta_{ref} &= \left( l_f - \frac{l_f m V_x^2}{\mu C_f (l_f + l_r)} \right) \rho(S), \\
\dot{\beta}_{ref} &= V_x \rho(S) .
\end{align*}
\]

The equilibrium point of the state is given by \( x_{eq} = (\beta, \dot{\beta}, \rho, e)^T = (\beta_{ref}, \dot{\beta}_{ref}, 0, 0)^T \). The control input at the equilibrium point is obtained as follows:

\[
\delta_{ref} = (l_f + l_r) \rho(S) + \frac{m V_x^2 (l_r C_r - l_f C_f)}{\mu C_f (l_f + l_r)} \rho(S). 
\]

The sideslip angle error, yaw rate error, and front steering angle are derived as

\[
\begin{align*}
\dot{\beta} &= \beta - \beta_{ref} \\
\dot{\psi} &= \dot{\beta} - \dot{\psi}_{ref} \\
\delta &= \delta_f - \delta_{ref}.
\end{align*}
\]

As previously mentioned, the steady-state solution to the single-track model can be found in (7) and (8). Substituting (9) into the control system represented in (6), the following system is generated as

\[\dot{X} = AX + B_1 \delta_f.\]

The new equilibrium point becomes \( X_{eq} = (\dot{\beta}, \ddot{\beta}, \rho, e)^T = (0, 0, 0, 0)^T \). The target system of Model (9) under \( \dot{e} = e = 0 \) can be expressed as

\[
\begin{align*}
\dot{\beta} &= -\dot{\beta} \\
\ddot{\beta} &= \frac{\mu C_f (l_f + l_r)}{I_z} \dot{\beta} - \frac{\mu l_f C_r (l_f + l_r)}{I_z V_x} \dot{\psi} \\
\dot{\psi} &= \frac{\mu C_r (l_f + l_r) \dot{\beta} - \mu l_f C_r (l_f + l_r) \dot{\psi}}{I_z V_x}.
\end{align*}
\]

It is easily verified that the equilibrium origin of System (10) is at the \( (0, 0, 0, 0) \). Let \( \Xi = [\dot{\beta}, \ddot{\beta}, \rho, e]^T \), then (11) can be rewritten as \( \dot{\Xi} = \Theta \Xi \) with

\[
\Theta = \begin{bmatrix}
0 & 0 & -1 \\
\frac{\mu C_r (l_f + l_r)}{I_z} & -\frac{\mu l_f C_r (l_f + l_r)}{I_z V_x} & 0 \\
\frac{\mu C_f (l_f + l_r)}{I_z} & -\frac{\mu l_f C_r (l_f + l_r)}{I_z V_x} & 0 \\
\frac{\mu C_r (l_f + l_r)}{I_z} & -\frac{\mu l_f C_r (l_f + l_r)}{I_z V_x} & 0 \\
\end{bmatrix}. \]

By taking the Laplace transformation, the characteristic polynomial is obtained as

\[
\text{det}(s I - \Theta) = s^2 + \frac{\mu C_f (l_f + l_r)}{I_z V_x} s + \frac{\mu C_r (l_f + l_r)}{I_z V_x}. \]

where \( s \) is the Laplace operator. In (13), the following parameters \( C_r, l_f, l_r, I_z, V_x \) are all considered as positive values. Then, Matrix \( \Theta \) can meet the Routh–Hurwitz stability conditions with longitudinal velocity large than zero, which means the global equilibrium origin of System (10) is \( (0, 0) \).

### C. STEERING SYSTEM CONSTRAINTS

The controller designed in this research directly controls the front steering wheel angle. Hence, a set of constraints on the magnitudes of steering and the rate of steering are considered to achieve a smooth transition. The following inequalities are used to saturate the steering system output:

\[
-20 \text{deg} \leq \delta_f \leq 20 \text{deg}
\]

\[
-25 \text{deg/s} \leq \dot{\delta}_f \leq 25 \text{deg/s}.
\]

### III. PATH FOLLOWING CONTROLLER DESIGN

The variable structure sliding control method has been widely used due to its high robustness with respect to model uncertainty and disturbance. The control objective is to force the system states eventually to stay on a pre-designed sliding surface. Figure 3 is a phase-plane graphical representation for sliding mode control, which shows the control objective for tracking error and its derivative. The state path can converge and stay along the pre-designed sliding surface. In this section, a standard Stanley controller and a non-linear MPC in the existing research are introduced as the benchmark controllers for comparison. Besides, a conventional SMC controller is developed without considering the perturbations, whereas a novel STA controller is...
FIGURE 4. Structure of proposed path following controller.

The proposed STA controller structure is depicted in Figure 4. It receives upper-level path information as well as the vehicle position and states. Based on the formulated tracking error dynamics, the desired steering angle can be computed accordingly.

A. BENCHMARK I — STANLEY CONTROLLER
Stanley controller is firstly proposed in [1] by Stanford University for DARPA Challenge. The steering angle is denoted as below:

$$\delta_f = \varphi + \arctan\left(\frac{ke}{V_x}\right), \quad (15)$$

where $\varphi$ denotes the heading error with respect to an ideal trajectory. It can be calculated according to the Serret-Frenet frame. $k$ represents the positive controller gain which determines the decay rate. The stability proof is omitted here for simplification, but it can be found in [1]. For this research, the Stanley controller is introduced as a benchmark controller for comparison.

B. BENCHMARK II — CONVENTIONAL SMC
This subsection shows a conventional SMC controller from existing literature. The lateral deviation concerning a reference is minimized by adopting the algorithm. The details refer to [32], [33].

A general form for the sliding surface is designed as

$$s = \dot{e} + \lambda e, \quad (16)$$

where $\lambda$ is a positive real number. By taking the derivative of (16), the following equation can be obtained:

$$\dot{s} = \dot{\varepsilon} + \lambda \dot{\varepsilon}. \quad (17)$$

The relative degree between the sliding surface and control input equals one, as illustrated by (18). Then the control law can be derived in a general rule. It is known that the common structure of SMC control input consists of an equivalent command $\delta_{EQ}$ and a switching function $\delta_{SW}$ of a discontinuous signal. The equivalent command $\delta_{EQ}$ acts as an auxiliary control command added to the control input. $\delta_{EQ}$ is derived from the equation $\dot{s} = 0$. The equivalent command aims to drive the system to the sliding surface. It is given by

$$\left\{ \begin{array}{l}
\delta_{EQ} = C_f + C_r \beta + \frac{l_r C_f - l_r C_r}{C_f V_x} \dot{\varphi} + \frac{m v_x^2}{\mu C_f} \rho(S) \\
\delta_{SW} = -\frac{m \alpha_1}{\mu C_f} \text{sign}(s),
\end{array} \right. \quad (19)$$

where $\alpha_1$ is a positive coefficient. Generally, a big value is assigned to $\alpha_1$ for the finite-time convergence and robustness improvement.

**Proposition 1:** If the total control input is defined as the sum of equivalent and switching functions:

$$\delta_f = \delta_{SW} + \delta_{EQ}, \quad (20)$$

the conventional SMC controller performs the finite-time convergence and the proof of controller stability is followed by Lyapunov theory.

**Proof:** A Lyapunov candidate is designed as

$$V_0(t) = \frac{1}{2} s^2. \quad (21)$$
The first derivative of the Lyapunov function is obtained as:

\[ \dot{V}_0(t) = s \cdot \dot{s} = s \cdot \left( -\frac{\mu (C_f + C_r)}{m} \beta - \frac{\mu (l_f C_f - l_r C_r)}{mV_s} \phi - \rho(S) V_s^2 \right. \\
\left. + \frac{\mu C_f \delta_f + \lambda \dot{e}}{m} \right). \] (22)

Substituting (19) and (20) into (22), the following expression can be formulated:

\[ \dot{V}_0(t) = s \cdot \dot{s} = s \cdot \left[ -\frac{\mu (C_f + C_r)}{m} \beta \\
- \frac{\mu (l_f C_f - l_r C_r)}{mV_s} \phi - \rho(S) V_s^2 \\
+ \frac{\mu C_f}{m} \left( \frac{C_f + C_r}{C_f} \right) \beta \\
+ \frac{l_f C_f - l_r C_r}{C_f V_s} \phi + \frac{m V_s^2}{\mu C_f} \rho(S) \\
- \frac{m \lambda}{\mu C_f} \dot{e} - \frac{m \dot{\alpha}_1}{\mu C_f} \text{sign}(s) \right] + \lambda \dot{e} \]

\[ = s \cdot (-\alpha_1 \text{sign}(s)) = -\alpha_1 |s| \leq 0. \] (23)

On the basis of the above analysis, it is seen that the conventional SMC controller will converge to zero in a limited time. Indeed, the conventional SMC control law drives the state trajectories of the system to a pre-designed sliding surface by using the inconsecutive signum function. The next part of this section presents the design of the proposed STA by considering the perturbations.

C. SUPER-TWISTING ALGORITHM WITH PERTURBATIONS

This part develops an STA control law considering the unmodelled dynamics and disturbances. The error dynamics is given by

\[ \dot{e} = -\frac{\mu (C_f + C_r)}{m} \beta - \frac{\mu (l_f C_f - l_r C_r)}{mV_s} \phi \\
- \rho(S) V_s^2 + \frac{\mu C_f}{m} \delta_f + \xi(t). \] (24)

where \( \xi(t) \) is considered as the perturbations containing the external disturbance and unmodelled dynamics. Here a new state vector \( z = [z_1, z_2]^T \) is introduced with \( z_1 = e \) and \( z_2 = \dot{e} \). Rewriting (24) as a state-space function, the following equation is obtained:

\[
\begin{aligned}
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= -\frac{\mu (C_f + C_r)}{mV_s} \beta - \frac{\mu (l_f C_f - l_r C_r)}{mV_s^2} \phi \\
&\quad - \rho(S) V_s^2 + \frac{\mu C_f}{m} \delta_f + \xi(t, z).
\end{aligned}
\] (25)

Assuming that the two desired tracking references are given as \( z_{1d} \) and \( z_{2d} \), respectively. In this research, the desired state values are set to zero. The corresponding errors are given as below:

\[
\begin{aligned}
e &= z_1 - z_{1d} \\
\dot{e} &= z_2 - z_{2d} \\
\end{aligned}
\] (26)

**Proposition 2:** If the control input is designed as (27), which yields the finite-time convergence of the designed sliding surface to zero. The introduced new states \( z_1 \) and \( z_2 \) converge to zero in finite time.

\[
\begin{aligned}
\delta_f &= \frac{C_f + C_r}{C_f} \beta + \frac{l_f C_f - l_r C_r}{C_f V_s} \phi + \frac{m V_s^2}{\mu C_f} \rho(S) \\
&\quad - \frac{m \lambda}{\mu C_f} \dot{e} - \frac{m \dot{\alpha}_1}{\mu C_f} \text{sign}(s) - \frac{k_1 m |\dot{s}|}{C_f} \text{sign}(s) - \frac{k_2 m}{C_f} \int_0^t \text{sign}(s) \, dt \\\nk_1 &> \frac{(\lambda_1 + \lambda_2^2 - 2 \xi^+)}{4 \lambda_1 \lambda_2} > 0, \quad k_2 = \frac{1}{2} \lambda_2 k_1.
\end{aligned}
\] (27)

where \( \xi^+ > 0 \) is considered as the upper bound of \( \dot{\xi}(t, z) \), and hence \( |\dot{\xi}(t, z)| \leq \dot{\xi}^+ \) The details for proof are shown below.

**Proof:** To prove the closed-loop stability of the controller, the first derivative of the sliding surface is given as

\[ \dot{s} = (z_2 - z_{1d}) + \lambda \dot{e}. \] (28)

Substituting the predesigned control law (27) and (25) into (28), the following equation can be generated:

\[ \dot{s} = -k_1 m |\dot{s}| \frac{1}{2} \text{sign}(s) - k_2 \int_0^t \text{sign}(s) \, dt + \xi(t, z). \] (29)

Equation (29) also shows the dynamic requirement of the sliding surface. To simplify the proof of stability, the backstepping technique is used. The new variables \( \eta_1 \) and \( \eta_2 \) are put forward accordingly. A new tuple \( [\eta_1, \eta_2] \) is introduced to rewrite (29) for further simplification suggested by [40], [41]:

\[
\begin{aligned}
\eta_1 &= s \\
\eta_2 &= -k_2 \int_0^t \text{sign}(s) \, dt + \xi(t, z).
\end{aligned}
\] (30)

By taking the first derivatives of \( \eta_1 \) and \( \eta_2 \), and rearranging the terms, the sliding surface differential inclusion is given as

\[
\begin{aligned}
\dot{\eta}_1 &= -k_1 |\eta_1| \frac{1}{2} \text{sign}(\eta_1) + \eta_2 \\
\dot{\eta}_2 &= -k_2 \text{sign}(\eta_1) + \dot{\xi}(t, z).
\end{aligned}
\] (31)

Equation (31) can be reformulated as a new state-space model by introducing a new state vector \( \rho = [\rho_1 \rho_2]^T = [|\eta_1| \frac{1}{2} \text{sign}(\eta_1) \eta_2]^T \). Then, (31) can be
written in terms of \( \varrho_1 \) and \( \varrho_2 \),
\[
\begin{align*}
\dot{\varrho}_1 &= \frac{1}{2|\varrho_1|} (-k_1\varrho_1 + \varrho_2), \\
\dot{\varrho}_2 &= \frac{1}{2|\varrho_1|} (-2k_1\varrho_1) + \dot{\xi}(t, z).
\end{align*}
\]

As aforementioned, \( \dot{\xi}(t, z) \) is a bounded signal, then it can be expressed as
\[
\dot{\xi}(t, z) = \sigma(t, z) \text{sign}(\eta_1) = \sigma(t, z) \frac{\varrho_1}{|\varrho_1|},
\]
where \( \sigma(t, z) \) is a bounded function satisfying \( 0 < \sigma(t, z) < \dot{\xi}^+ \). Equation (32) can be rewritten as
\[
\begin{bmatrix}
\dot{\varrho}_1 \\
\dot{\varrho}_2
\end{bmatrix} = \frac{1}{2|\varrho_1|} \begin{bmatrix}
-k_1 \\
-2k_1 + 2\sigma
\end{bmatrix} \begin{bmatrix}
\varrho_1 \\
\varrho_2
\end{bmatrix} = F_{\varrho_1} \begin{bmatrix}
\varrho_1 \\
\varrho_2
\end{bmatrix}.
\]

**Remark:** If the new state vector \( \varrho = [\varrho_1 \varrho_2]^T \) converges to zero within a limited time, \( \eta_1 \) and \( \eta_2 \) converge to zero within a limited time.

**Proof:** The Lyapunov candidate function for the proof of the convergence of \( \varrho_1 \) and \( \varrho_2 \) is selected as
\[
V_1(\varrho) = \varrho^TP\varrho.
\]
where the symmetric matrix \( P \) is defined as
\[
P = P^T = \begin{bmatrix}
\lambda_1 + \lambda_2^2 & -\lambda_2 \\
-\lambda_2 & 1
\end{bmatrix}.
\]

In (36), \( P \) is a positive definite matrix since \( \lambda_1 > 0 \) and \( \lambda_2 \) is an arbitrary real number. Therefore, the Lyapunov candidate function \( V_1(\varrho) \) is a positive definite function. Taking the first derivative of the Lyapunov candidate function, the following equation is obtained:
\[
\dot{V}_1(\varrho) = \varrho^T [PA(\varrho_1) + A^T(\varrho_1)P] \varrho.
\]

Considering the upper bound of \( \dot{\xi}(t, z) \), (37) can be rewritten as
\[
\dot{V}_1(\varrho) \leq -\frac{1}{2|\varrho_1|} \varrho^T Q \varrho.
\]
where \( Q = Q^T \); and \( Q \) is expressed as
\[
Q = \begin{bmatrix}
2\lambda_1 + 2\lambda_2^2k_1 - 4\lambda_2k_2 + 4\lambda_2\xi^+ & * \\
2k_1 - \lambda_2k_1 - \lambda_1 - \lambda_2^2 - 2\xi^+ & 2\lambda_2
\end{bmatrix}.
\]
where * is the symmetric expression of a matrix.

By setting the gain \( k_2 = 0.5\lambda_2k_1 \), (39) can be expressed as
\[
Q = \begin{bmatrix}
2\lambda_1k_1 + 4\lambda_2\xi^+ & * \\
- (\lambda_1 + \lambda_2^2 + 2\xi^+) & 2\lambda_2
\end{bmatrix}.
\]

To make \( \dot{V}_1(\varrho) \) as a semi-definite function, the matrix \( Q \) should be positive definite. Therefore, the controller gain \( k_1 \) is chosen as
\[
k_1 > (\lambda_1 + \lambda_2^2 - 2\xi^+)/4\lambda_1\lambda_2.
\]

As the matrix \( P \) is dependent on the values of \( \lambda_1 \) and \( \lambda_2 \) (both of them are real numbers), the following inequality holds:
\[
\begin{bmatrix}
\lambda_{\text{min}}(P) \| \varrho \|^2 \\
\| \varrho_1 \|^2 \\
\| \varrho_2 \|^2
\end{bmatrix} \leq \begin{bmatrix}
V_1(\varrho) \\
\varrho_1 \\
\varrho_2
\end{bmatrix} \leq \begin{bmatrix}
V_{\text{min}}(\varrho) \\
\lambda_{\text{max}}(P) \| \varrho \|^2
\end{bmatrix}.
\]

The derivative of the Lyapunov candidate function (35) can be rewritten as
\[
\dot{V}_1(\varrho) \leq -\frac{1}{2\|\varrho_1\|} \lambda_{\text{min}}(Q) \| \varrho \|^2
\]
\[
\leq -\frac{1}{2\|\varrho_1\|} \lambda_{\text{min}}(Q) \frac{V_1(\varrho)}{\lambda_{\text{max}}(P)}.
\]

where \( \| \varrho \|^2 = \| \varrho_1 \| + \| \varrho_2 \| \). By replacing (38) with (43), the following equation can be derived as
\[
\dot{V}_1(\varrho) \leq -\epsilon V^{0.5}(\varrho)
\]
\[
\text{with } \epsilon = \frac{\lambda_{\text{min}}(P) \lambda_{\text{min}}(Q)}{2\lambda_{\text{max}}(P)}.
\]

Since the solution of the equation is \( \dot{v} = -v^{0.5} \) and \( v(0) = v_0 \geq 0, v(t) \) is given as
\[
v(t) = \left(v_0^{0.5} - 0.5\epsilon t\right)^2,
\]
where \( t \) denotes the time. It conforms to the comparison principle in [42] that \( V_1(\varrho) \leq v(t) \) when \( V_1(\varrho_0) \leq v_0 \), in which \( \varrho_0 \) is the initial state. Equation (44) implies that \( V_1(\varrho) \) and \( \varrho \) can converge to zero within finite time. Meanwhile, (45) indicates that \( V_1(\varrho) \) and \( \varrho \) can converge to zero after \( t = 2V^{0.5}(\varrho_0) \epsilon^{-1} \) unit of time. If the control gains \( k_1, k_2 \) and \( \lambda_2 \) meet the conditions mentioned in (27), the sliding surface \( s = \dot{\varrho} + \lambda \dot{\varrho} e \) also converges to zero in finite time. Thus, it can be concluded that the lateral tracking error can achieve asymptotic convergence, that
\[
\lim_{t \to \infty} e = 0, \text{ and } \lim_{t \to \infty} \dot{e} = 0.
\]

The above steps complete Proposition 2 and Remark.

**IV. SIMULATION RESULTS AND ANALYSIS**

To avoid accidents in road tests, simulation tests are carried out to evaluate the performance of the proposed controller. It is noted that the conventional SMC without considering system perturbations is given by (19) and (20). The vehicle parameters are given in Table 1. The controller parameter for Stanley controller is set as \( k = 9 \), while for conventional SMC, the controller parameters are set to \( k = 9 \) and \( \omega_1 = 4.5 \). The control input for the proposed STA with considering system perturbations is given by (27). It can be regarded as a nonlinear PID control law with dynamic state feedback. The gains for this controller are set as \( \lambda = 9, k_1 = 4.5, k_2 = 5, \lambda_1 = 1.2, \lambda_2 = 1.2, \) and \( \xi^+ = 5 \). The related vehicle parameters can be found in Table 1. The details of
performance evaluation are introduced in the following subsections. Given that the sideslip angle is hard to acquire by vehicle onboard sensors, a sliding mode observer mentioned in [43] is used for observing the sideslip angle.

The validation platform is implemented based on the commercial software, Matlab/Simulink and Carsim. The Carsim software provides a full vehicle model, which can describe the coupled vehicle dynamic performance and behavior sophisticatedly.

A. NOVEL EVALUATION INDEX FOR CONTROLLER PERFORMANCE

As the path following control is highly related to vehicle lateral dynamics, the proposed evaluation index consists of yaw rate error $e_{\text{yawrate}} = \dot{\phi} - \dot{\phi}_{\text{ref}}$, control input front-steering angle, and lateral tracking error $e_{\text{trc}} = Y_{\text{real}} - Y_{\text{ref}}$, where $Y_{\text{real}}$ and $Y_{\text{ref}}$ are the $i_{th}$ coordinates for real lateral tracking point and desired path point in the world coordinate system respectively [44].

1) The integral absolute lateral tracking and yaw rate errors ($\text{IATE}$) is:

$$\text{IATE} = \int_0^t (|e_{\text{trc}}| + |e_{\text{yawrate}}|) \, dt.$$  

2) The integral time product of the absolute lateral tracking error and yaw rate error (i.e. $\text{IAPE}$) is:

$$\text{IAPE} = \int_0^t t (|e_{\text{trc}}| + |e_{\text{yawrate}}|) \, dt.$$  

3) The integral of the absolute value of the front steering angle (i.e. $\text{IASA}$) is:

$$\text{IASA} = \int_0^t |\delta_f| \, dt.$$  

where $t$ is control time duration.

The $\text{IAPE}$ acts as a complement for $\text{IATE}$ which does not consider the time duration. To make the above-mentioned indexes more generally and objectively, a dimensionless integrated and weighted performance evaluation index (i.e. $\text{IWPEI}$) is defined as

$$\text{IWPEI} = \frac{\omega_1}{\max (|e_{\text{trc}}|, |e_{\text{yawrate}}|)} \text{IATE} + \frac{\omega_2}{\max (|e_{\text{trc}}|, |e_{\text{yawrate}}|)} t \text{IAPE} + \frac{\omega_3}{|\delta_f|_{\max}} \text{IASA},$$

where $\omega_1, \omega_2$ and $\omega_3$ are user-defined weights for the three indexes respectively, and $\omega_1 + \omega_2 + \omega_3 = 1$. The value of $\max (|e_{\text{trc}}|, |e_{\text{yawrate}}|)$ can be set according to the experimental results. $|\delta_f|_{\max}$ represents the maximum front steering angle. Since the performances quantified by the three indexes are considered equally important with each other in the evaluation, the weights $\omega_1, \omega_2$ and $\omega_3$ are set as an equal value of 1/3 in this simulation test. The weights are selected to penalize the lateral tracking error and minimize the control input. The smaller the $\text{IWPEI}$, the better the performance.

B. PATH FOLLOWING SIMULATION TEST RESULTS WITH KNOWN PARAMETERS UNDER NORMAL DRIVING

In this normal driving scenario, conservative motion planning received from the upper planning layer is assumed. That is to
As shown in Figure 5, a variable curvature built-in handling course in Carsim is adopted to test the performance of the controllers. The corresponding curvatures and longitudinal speed profiles for this course are shown in Figure 6. The curvature ranges from $-0.032m^{-1}$ to $0.0458m^{-1}$. The longitudinal velocity $V_x$ varies between 32.5km/h and 113.6km/h. The simulation results are shown in Figure 7 to Figure 10 with the known nominal vehicle mass and the same tire parameters. It is noted that the path length of the built-in handling course in Carsim is 2273.848m, which requires a long travelling time of about 141 seconds to finish as shown in the x-axis of Figure 8 to Figure 10.

Figure 7 presents that both the benchmark controllers and proposed STA controller can maintain the vehicle driving along the desired trajectory. Their lateral deviations with respect to the desired path are small, in which the maximum value is less than 12m as shown in Figure 8. Compared with other benchmark controllers, the proposed STA can effectively restrain the lateral deviation even in tight turns.

Figure 9 gives the control inputs of the front steering wheel angles of both controllers. Compared with the benchmark controllers, the proposed STA gives a continuous and smooth control input without chattering and sharp changes. The control effort behaves smaller than the Stanley controller conventional SMC, and MPC.

Figure 10 depicts that the yaw rate performance is also better than other benchmark controllers with the smoother variation under a large change of steering angle. Table 3 shows that the IWPEI of the proposed STA is reduced by 40.5%.

### Figure 7. Trajectory comparison under different controllers for normal driving.

### Figure 8. Lateral deviation with respect to desired path for normal driving.

### Table 3. Evaluation index and computation time comparison in normal driving scenario.

| Index            | Stanley controller | Conventional SMC | MPC    | Proposed STA |
|------------------|--------------------|------------------|--------|--------------|
| IWPEI            | 87.1%              | 72.4%            | 57.5%  | 46.6%        |
| Computation time | 70.2ms             | 93.6ms           | 337ms  | 95ms         |
25.8%, and 10.9% respectively, as compared with the three benchmark controllers in the normal case (Tire A and $m = 1270$ kg).

In addition, the computer in the simulation equips with an Intel(R) Core(TM) i7-4600U CPU and 8.0 GB RAM. As shown in Table 3, the computation time of the MPC controller is nearly 337 milliseconds, which is slower than Stanley controller, conventional SMC, and the proposed STA by 4.8 times, 3.6 times, and 3.55 times, respectively. The reason is that the Stanley controller is the simplest one among the four types of control methods. However, its overall control performance is not so good compared with the other three controllers. Comparing Stanley controller and the MPC, the proposed STA gives a very good balance between the control performance and the computational cost.

**C. DOUBLE LANE CHANGE SCENARIO**

To evaluate the controller performance under the fierce driving mode, a double lane change (DLC) scenario recommended by ISO 3888:1 2018 is adopted here. In this scenario, the longitudinal velocity is set as $V_x = 120$ km/h. Simulation results are given in Figure 11 to Figure 14.

As shown in Figure 11, the Stanley controller cannot react quickly enough to follow the desired trajectory. The conventional SMC, MPC, and the proposed STA controllers present a good trajectory tracking performance under a high longitudinal velocity in the DLC scenario. The simulation results of conventional SMC present a chattering phenomenon as shown in Figure 12 to Figure 14. As presented in Table 4, the proposed STA performs the best with the smallest $IWPEI = 18.3\%$, followed by $IWPEI = 25.1\%$ for MPC, $IWPEI = 42.3\%$ for conventional SMC, and $IWPEI = 80.8\%$ for Stanley controller. Therefore, the proposed STA
controller has a better control performance compared with the other benchmark controllers.

In the DLC scenario, the MPC still needs more computation time compared to the other three controllers. Table 4 depicts that the MPC takes about 314 milliseconds for computation, which is slower than Stanley controller, conventional SMC, and the proposed STA by 4.1 times, 3.3 times, and 3.27 times, respectively. Given the good control performance and acceptable computation time, the proposed STA is more suitable for real-time path following control of autonomous vehicles.

D. ROBUSTNESS VERIFICATION FOR PROPOSED CONTROLLER

For the path following controller, the other major challenge is the robustness against vehicle model uncertainties, perturbations, and driving condition change.

This subsection demonstrates the robustness under a time-varying vehicle mass and two different tires. The simulation test is conducted at the co-simulation platform. Different tire parameters are described in Table 2. Generally, it is difficult to accurately estimate the cornering stiffness because it considerably varies with vertical load, road conditions, and suspension parameters, etc. In addition, the vehicle mass also affects controller performance significantly. The mass is also hard to estimate because of the varying number of persons in the car, weight of fuel, etc. Therefore, it is important to verify the robustness of the proposed controller.

The controller performance evaluation index $IWPEI$ is illustrated in Figure 15. By changing the vehicle mass and tire simultaneously, the fluctuation of the $IWPEI$ can be calculated by using the percentage of $IWPEI$ change:

$$Per_{Controller} = \frac{\max (IWPEI) - \min (IWPEI)}{\max (IWPEI)}$$

(51)

Table 5 shows that $Per_{ProposedSTA}$ is less than $Per_{Stanley}$, $Per_{Conventional SMC}$, and $Per_{MPC}$ by 22.6%, 22.3%, and 5.9%, respectively. Therefore, the proposed STA is insensitive to vehicle weight change and tire parameter variation. It should be emphasized that the proposed STA always gives a minimum $Per_{Controller}$ value regardless of varying vehicle mass and tire parameters, and produces small, continuous, and smooth control input compared with the benchmark controllers.

In a word, the proposed STA exhibits better performance than Stanley controller, conventional SMC, and MPC according to the simulation results and overall performance analysis.
V. CONCLUSION

In this work, a novel super-twisting sliding mode controller is designed with considering perturbations for autonomous vehicle path following problem. The control performance is evaluated based on a novel IWPEI proposed in this work, which consists of control input, lateral deviation, and yaw rate error. Comparing the benchmark Stanley controller, conventional SMC and MPC the simulation results show that the proposed STA generates a continuous control input without chattering problem, resulting in a smaller lateral deviation and smoother yaw rate. Besides, the robustness of the controller is tested with the varying vehicle mass and tire parameters under the same controller parameters. Simulation results also show that the robustness of the proposed STA outperforms the benchmark controllers with the smallest PerController. In summary, the overall performance of the proposed STA is better than the benchmark controllers because of the smooth front steering angle and yaw rate without chattering and small lateral error.

Future work will consider the learning-based path following controller design. Moreover, the hardware and real vehicle experiments will be considered in the upcoming path following research.

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