The UV sensitivity of the Higgs potential in Gauge-Higgs Unification

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Abstract

In this paper, we discuss the UV sensitivity of the Higgs effective potential in a Gauge-Higgs Unification (GHU) model. We consider an $SU(N)$ GHU on $M^4 \times S^1$ spacetime with a massless Dirac fermion. In this model, we evaluate the four-Fermi diagrams at the two-loop level and find them to be logarithmically divergent in the dimensional regularization scheme. Moreover, we confirm that their counter terms contribute to the Higgs effective potential at the four-loop level. This result means that the Higgs effective potential in the GHU depends on UV theories as well as in other non-renormalizable theories.
I. INTRODUCTION

The standard model (SM) of particle physics is a Yang-Mills theory symmetric under $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge transformations. In the SM, the gauge symmetry is spontaneously broken via the Higgs mechanism, which is caused by the nonzero vacuum expectation value (VEV) of the Higgs boson. As a result, physical quantities such as the mass of a particle include the Higgs VEV. By measuring the physical parameters including the Higgs VEV, the SM is confirmed to be consistent with phenomena at Large Hadron Collider \cite{1, 2}

While the phenomenology below the electroweak (EW) scale is understandable by the SM, it is concerned that the SM has difficulty explaining the scale hierarchy between the EW and UV theories such as the grand unified theory (GUT) or quantum gravity because of the dangerous quadratic divergences derived from the Higgs boson. In a model with supersymmetry (SUSY) \cite{3-7}, there is a superpartner for each particle to cancel the quadratic divergences consequently. Instead of the SUSY scenarios, we can invoke gauge symmetry in a non-SUSY theory defined on extra-dimensional spacetime to protect the Higgs mass term. This higher-dimensional gauge theory is called the gauge-Higgs unification (GHU) \cite{8-13}.

In the GHU models, the Higgs bosons are identified with the Yang-Mills Aharonov-Bohm (AB) phases. Therefore, the Higgs boson has no potential at the tree level. Meanwhile, at the loop level, the Higgs potential is generated by the AB effect due to the non-simplicity of spacetime. This symmetry breaking mechanism is called the Hosotani mechanism \cite{11, 13}.

It has been found that the Higgs potential is finite, i.e. independent of any UV cutoff at the one- and two-loop levels in some GHU models \cite{11, 13, 19}, although higher-dimensional gauge theories are generally non-renormalizable. Moreover, the finiteness at all orders has been conjectured \cite{20, 22}.

In the previous work \cite{18}, we confirmed that the Higgs potential does not suffer from the divergence at the two-loop level in a non-Abelian gauge theory defined on $M^4 \times S^1$ spacetime, where $M^n$ is the $n$-dimensional Minkowski spacetime with $n \geq 1$ and $S^1$ is a circle. Besides, we proceeded to discuss the finiteness of the Higgs potential on $M^5 \times S^1$ spacetime, which is related to this paper. Evaluating the four-Fermi diagrams at the one-loop level, we obtained the logarithmic divergences contributing to the Higgs potential. Hence, the Higgs potential is UV sensitive on the six-dimensional spacetime. This is consistent
with the non-renormalizability of higher-dimensional gauge theory. Based on this result, we concluded that the Higgs potential would be also suffering from the divergence in the five-dimensional spacetime.

In this paper, we go back to $M^4 \times S^1$ spacetime again and explicitly show that the Higgs potential depends on UV theories in an $SU(N)$ GHU model. Since there are no logarithmic divergences at the one-loop level in odd-dimensional theories, we consider divergences at the two-loop level in the dimensional regularization scheme. The four-Fermi diagrams, in practice, are evaluated at the two-loop level. We find that they are indeed logarithmically divergent and their counter terms contribute to the Higgs potential at the four-loop level. This fact means the UV sensitivity of the Higgs potential in this GHU model. Since we use a simple setup, the Higgs potential would generically be UV sensitive in other GHU models.

The remainder of this paper is organized as follows. In Sect. II, we briefly describe the Hosotani mechanism along with the theoretical setup. In Sect. III, we explain how to evaluate the divergence of the Higgs potential and show that the Higgs potential receives a contribution from counter terms to the divergences that cannot be subtracted by the renormalization to the gauge coupling. Finally, we summarize the results obtained in this paper in Sect. IV.

II. HOSOTANI MECHANISM

In this section, we describe a theoretical setup used in this paper and review the Hosotani mechanism.

Since the Hosotani mechanism is a quantum effect related to the global structure of spacetime, let us consider $M^4 \times S^1$ as an example of non-simply connected spacetime, where $M^4$ and $S^1$ are the four-dimensional Minkowski spacetime and a circle with radius $R$ respectively. We use coordinates $x^\mu$ with $\mu \in \{0, 1, 2, 3\}$ for $M^4$ and $y \in [0, 2\pi R)$ for $S^1$. As mentioned above, on a spacetime with a hole, the fifth component of the gauge boson, $A_5^a$, has its VEV expressed by

$$\langle A_5^a \rangle = \frac{\theta^a}{2\pi R g},$$

where $\theta^a$'s are the Yang-Mills AB phases around $S^1$ and $g$ is the coupling constant. Here, $a$ denotes the group index.

Through this paper, we use the background field method [23] for calculating the contri-
Contributions to the Higgs effective potential and $A_5^a$'s are shifted by its VEV:

$$A_5^a \rightarrow A_5^a + \frac{\theta^a}{2\pi R g}. \quad (2)$$

Due to compactified extra-dimension, the boundary conditions on field functions are introduced. We consider a massless Dirac fermion, $\psi$, as only one type of matter field and suppose $A_M^a$ and $\psi$ satisfy

$$A_M^a(x^\mu, y + 2\pi R) = A_M^a(x^\mu, y), \quad (3)$$

$$\psi(x^\mu, y + 2\pi R) = e^{i\beta} \psi(x^\mu, y), \quad (4)$$

where $M \in \{0, 1, 2, 3, 5\}$ and $\beta \in [0, 2\pi)$.

The Lagrangian we consider with an $SU(N)$ gauge symmetry is

$$\mathcal{L} = -\frac{1}{4} F_{MN}^a F^{aMN} + \bar{\psi} i\gamma^M D_M \psi + \mathcal{L}_{GF} + \mathcal{L}_{\text{ghost}}, \quad (5)$$

where the gauge fixing terms, $\mathcal{L}_{GF}$, and the Faddeev-Popov ghost terms, $\mathcal{L}_{\text{ghost}}$, are given by

$$\mathcal{L}_{GF} = -\frac{1}{2} \mathcal{F}^a \mathcal{F}^a, \quad \mathcal{F}^a \equiv \partial^M A_M^a + \frac{f^{abc}}{2\pi R} A_5^b \theta^c, \quad (6)$$

$$\mathcal{L}_{\text{ghost}} = -\bar{\psi} \left[ \partial^M D_M^{ab} \frac{f^{ace} f^{bed}}{2\pi R} \theta^c \left( \frac{\theta^d}{2\pi R} + g A_5^d \right) \right] \psi. \quad (7)$$

Here, $f^{abc}$ denotes the structure constant of an $SU(N)$ and the covariant derivative for each field is defined by

$$D_M^c \equiv \left( \partial_M - ig A_M^a T^a - i \frac{\theta^a T^a}{2\pi R} \delta^c_N \right) c, \quad (8)$$

$$D_M \psi \equiv \left( \partial_M - ig A_M^a \tau^a - i \frac{\theta^a \tau^a}{2\pi R} \delta^c_N \right) \psi, \quad (9)$$

where $[T^a]^{bc} = -if^{abc}$ and $\tau^a$'s are representation matrices of $\psi$.

Let us see the Hosotani mechanism with the setup above. In our model, the gauge boson propagator is given by

$$S_A(p^\mu, n) = \frac{-i\eta_{MN}^{MN}}{p^\mu p_\mu - \left( \frac{n}{R} + \frac{\phi^a T^a}{2\pi R} \right)^2}, \quad (10)$$
where \( n \in \mathbb{Z} \) denotes the Kaluza-Klein (KK) modes. To shift the fifth component of a momentum, we consider the following gauge transformations:

\[
A_5(x^\mu, y) \to e^{-i\frac{\theta^a y}{2\pi R}} A_5(x^\mu, y) e^{i\frac{\theta^a y}{2\pi R}} - \frac{\theta^a T^a}{2\pi R} y,
\]

\[
\psi_\ell(x^\mu, y) \to e^{-i\frac{\theta^a y}{2\pi R} y} \psi_\ell(x^\mu, y),
\]

(11)

(12)

where \( A_M = A^a_M T^a \). Without any boundary conditions, arbitrary \( \theta^a \) can be gauged away by the above gauge transformations. With Eqs. (3) and (4), in contrast, the gauge transformations are restricted to be periodic on \( S^1 \) in order to keep the boundary conditions invariant. Namely, removable \( \theta^a \)'s satisfy

\[
e^{i\theta^a T^a} = \mathbb{I},
\]

(13)

where \( \mathbb{I} \) is the identity matrix. Remaining \( \theta^a \)'s become physical degrees of freedom. Due to the gauge symmetry, \( \theta^a \) has no potential at the tree level. Under the boundary conditions which characterize the non-simplicity of spacetime, however, its potential is generated by loop corrections as shown in \([11, 13–19]\). When the minimum point of the potential is nonzero, the gauge symmetry is dynamically broken and the gauge bosons become massive. This is how the Hosotani mechanism works.

III. HIGGS POTENTIAL DIVERGENCE AT THE FOUR-LOOP LEVEL

Up to the two-loop level, we have found no divergence of the Higgs potential in the previous works \([11, 13, 19]\). However, since the higher-dimensional gauge theory is non-renormalizable, it is natural that the Higgs potential receives \( \theta \)-dependent contributions from the infinite number of counter terms. Indeed, we have proven that the four-Fermi diagrams contribute to the Higgs potential on \( M^5 \times S^1 \). On the five-dimensional spacetime, it is expected that they are logarithmically divergent at the two-loop level and their counter terms contribute to the Higgs potential. In this section, we show those divergences and the UV dependence of the Higgs potential. For automating calculations, we have used a Mathematica package, \textit{FeynCalc} \([24, 26]\).
Let us consider a diagram shown below;

which is logarithmically divergent. To obtain its divergent part, we concentrate on loop momenta going around the UV region. Ignoring momenta lying external lines, we have

\[
\frac{1}{2\pi R} \sum_{n_1} \int \frac{d^4 p}{(2\pi)^4} \frac{1}{2\pi R} \sum_{n_2} \int \frac{d^4 k}{(2\pi)^4} \left[ \frac{i g \gamma^M}{\gamma^\mu k^\mu - \gamma^5 \frac{n_1}{R}} \frac{i g \gamma^N}{\gamma^\mu p^\mu - \gamma^5 \frac{n_2}{R}} \frac{i g \gamma^L}{\gamma^\mu k^\mu - \gamma^5 \frac{n_2}{R}} \right]_{\alpha\beta} \left[ \tau^a \tau^b \tau^c \right]_{ij} \\
\times \left[ \gamma^\delta \right]_{kl}
\]

where \( \alpha \) and \( \gamma \) are spin indices of \( \bar{\psi} \), and \( \beta \) and \( \delta \) are those of \( \psi \). \( i, j, k, l \) represent indices of \( \tau^a \)'s. Note that all summation indices in this paper run all integers. In the previous work [18], we derived the following formula[1]

\[
\frac{1}{2\pi R} \sum_n S \left( \frac{n}{R} + \frac{\Theta}{2\pi R} \right) = \sum_m e^{i\Theta m} \int_{-\infty}^{\infty} \frac{dk_5}{2\pi} e^{-i2\pi Rk_5m} S(k_5),
\]

where \( \Theta \) is an arbitrary Hermitian matrix. Here, \( S(\cdot) \) denotes an analytic function and its

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1 This transformation has been introduced in [27] and used in [28, 31] for Abelian theory. For non-Abelian cases, the previous calculations [11, 13, 14] have adopted essentially the same way, while we use it before the four-dimensional integration to obtain the five-dimensional one.
generalization to a matrix-valued one. Using Eq. (15) for $\Theta = 0$, we get
\[
\nabla \cdot \left( i g^6 \sum_{m_1, m_2} \int \frac{d^5 p}{(2\pi)^5} \int \frac{d^5 k}{(2\pi)^5} e^{-i 2\pi R (m_1 p_5 + m_2 k_5)} \right.
\]
\[
\times \frac{p^P k^Q p^R k^S}{(p^2)^3 (p + k)^2 (k^2)^3}
\times \left[ \gamma^M \gamma_P \gamma^N \gamma_Q \gamma^L \right]_{\alpha \beta} \left[ \gamma^M \gamma_R \gamma^N \gamma_S \gamma_L \right]_{\gamma \delta}
\times \left[ \tau^a \tau^b \tau^c \right]_{ij} \left[ \tau^a \tau^b \tau^c \right]_{kl}.
\]
\begin{equation}
(16)
\end{equation}

Let us define $I$ by
\[
I = I(x, y) \equiv -\int \frac{d^5 p}{(2\pi)^5} \int \frac{d^5 k}{(2\pi)^5} \frac{1}{(p^2)^3 (p + k)^2 (k^2)^3} e^{-i 2(p \cdot x + k \cdot y)},
\]
\begin{equation}
(17)
\end{equation}
where $x$ and $y$ are space-like vectors. For the spacetime dimension $D = 5 - 2\epsilon$ with $\epsilon > 0$, $I$ is calculated using a formula deduced in Appendix A. Integrals in Eq. (16) can be rewritten as a derivative of $I$;
\[
\int \frac{d^5 p}{(2\pi)^5} \int \frac{d^5 k}{(2\pi)^5} \frac{p^P k^Q p^R k^S}{(p^2)^3 (p + k)^2 (k^2)^3} e^{-i 2\pi R (m_1 p_5 + m_2 k_5)} = -2^{-4} \partial_x^P \partial_y^Q \partial_x^R \partial_y^S I |_{x^M = \delta^M_{m_1} \pi R, \ y^M = \delta^M_{m_2} \pi R}
\]
\begin{equation}
(18)
\end{equation}
The behavior of the integrand in the UV region is shown in Eq. (A18); after integration over $k$ and angular variables, the remaining (radial) integral has the form,
\[
\mathcal{I} \propto \int_0^\infty d|p_E| |p_E|^{a-1} K_r(2b\sqrt{-y^2}|p_E|) \, _0F_1 \left( \frac{a + r}{2}; (x - \beta y)^2 |p_E|^2 \right),
\]
\begin{equation}
(19)
\end{equation}
where $a$, $b$, $r$ and $\beta$ are independent of $x$ and $y$. Here, $K_r(z)$ is the modified Bessel function of the second kind and $\, _0F_1(a; z)$ is a generalized hypergeometric function. Note that $\, _0F_1(a; z)$ is expressed by the Bessel function of the first kind, $J_\alpha(z)$;
\[
J_\alpha(2z) = \frac{(z)^\alpha}{\Gamma(\alpha + 1)} \, _0F_1(\alpha + 1; -z^2).
\]
\begin{equation}
(20)
\end{equation}
Plugging Eq. (20) into Eq. (19), we see that the integrand is a multiplication of $J_\alpha$, $K_r$, and the power of $|p_E|$. Therefore, the UV divergence of $\mathcal{I}$ is suppressed by $K_r$ for its exponential dumping when $y^M = \delta^M_5 m_2 \pi R \neq 0$. Because $\mathcal{I}(x, y) = \mathcal{I}(y, x)$, there is also no
UV divergence when $x^M = \delta_0^M m_1 \pi R \neq 0$.

To evaluate the UV divergence of $\mathcal{I}$, we set $m_1 = m_2 = 0$. Substituting Eq. (A20) into Eq. (18), we obtain

$$
\int \frac{d^D p}{(2\pi)^D} \int \frac{d^D k}{(2\pi)^D} \frac{p^P k^Q p^R k^S}{(p^2)^3 (p + k)^2 (k^2)^3} \bigg|_{\text{div}}
= -\frac{1}{2^4 (4\pi)^5} \cdot \frac{1}{16\epsilon} \int_0^1 d\alpha \int_0^1 d\beta \alpha^2 (1 - \alpha)^{-4} (1 - \beta)^2 \left[ \frac{\alpha}{1 - \alpha} + \beta (1 - \beta) \right]^{-2}
\times \partial_x^P \partial_y^Q \partial_x^R \partial_y^S \left[ -(x - \beta y)^2 - y^2 \left( \frac{\alpha}{1 - \alpha} + \beta (1 - \beta) \right)^2 \right] \bigg|_{x = y = 0}. \quad (21)
$$

The derivatives are given by

$$
\partial_x^P \partial_y^Q \partial_x^R \partial_y^S \left[ -(x - \beta y)^2 - y^2 \left( \frac{\alpha}{1 - \alpha} + \beta (1 - \beta) \right)^2 \right]
= 8\beta^2 (\eta^P Q \eta^R S + \eta^P S \eta^Q R) + 8\eta^P R \eta^Q S \left( \frac{\alpha}{1 - \alpha} + \beta \right). \quad (22)
$$

In Appendix A, it is shown that

$$
\int_0^1 d\alpha \int_0^1 d\beta \alpha^{s-1} (1 - \alpha)^{s-1} (1 - \beta)^{t-1} \left[ \frac{\alpha}{1 - \alpha} + \beta (1 - \beta) \right]^v
= B(s, -s - v)B(s + t + v, s + u + v) \quad (23)
$$

with

$$
B(x, y) \equiv \int_0^1 dt t^{x-1} (1 - t)^{y-1} = \frac{\Gamma(x) \Gamma(y)}{\Gamma(x + y)}. \quad (24)
$$

Using this formula, we have

$$
\begin{array}{c}
\includegraphics{diagram}\end{array}
\quad \bigg|_{\text{div}}
= \frac{ig^6}{2^{14} \cdot 105\pi^4 \epsilon} (\eta^P Q \eta^R S + \eta^P S \eta^Q R + 22\eta^P R \eta^Q S)
\times [\gamma^M \gamma^N \gamma^Q \gamma^L]_{\alpha \beta} [\gamma_M \gamma_R \gamma_N \gamma_S \gamma_L]_{\gamma \delta} [\tau^a \tau^b \tau^c]_{ij} [\tau^a \tau^b \tau^c]_{kl}. \quad (25)
$$

Repeating the above procedure for the other two-loop four-Fermi diagrams, we find the $\epsilon$-poles at the two-loop level. They are shown explicitly in Appendix B. The divergence has
the form,
\[-\frac{ig^6}{\epsilon} \sum_{\mathcal{X}} C_{\mathcal{X}} [G^{(1)}_{\mathcal{X}}]_{\alpha\beta} [G^{(2)}_{\mathcal{X}}]_{\gamma\delta} [T^{(1)}_{\mathcal{X}}]_{ij} [T^{(2)}_{\mathcal{X}}]_{kl} - (\alpha \leftrightarrow \gamma, i \leftrightarrow k),
\]
(26)
where $\mathcal{X}$ denotes diagrams with four fermion legs and $C_{\mathcal{X}}$ is a constant. Here, $G^{(1,2)}_{\mathcal{X}}$ and $T^{(1,2)}_{\mathcal{X}}$ are products of $\gamma^M$'s and $\tau^a$'s respectively. The above example corresponds to
\[\mathcal{X} = \begin{array}{cccc}
\hline
\hline
\hline
\hline
\end{array},
\]
(27)
\[G^{(1)}_{\mathcal{X}} = \gamma^M \gamma^P \gamma^N \gamma^Q \gamma^L,
\]
(28)
\[G^{(2)}_{\mathcal{X}} = \gamma^M \gamma^R \gamma^N \gamma^S \gamma^L,
\]
(29)
\[T^{(1)}_{\mathcal{X}} = T^{(2)}_{\mathcal{X}} = \tau^a \tau^b \tau^c.
\]
(30)
The following counter term is introduced to cancel the above divergence\(^2\)
\[\mathcal{L}_{CT} = \frac{\delta_{4F}}{2} \sum_{\mathcal{X}} C_{\mathcal{X}} (\bar{\psi} G^{(1)}_{\mathcal{X}} T^{(1)}_{\mathcal{X}} \psi) (\bar{\psi} G^{(2)}_{\mathcal{X}} T^{(2)}_{\mathcal{X}} \psi),
\]
(31)
where $\delta_{4F} = \delta_{4F}^{\text{div}} + \delta_{4F}^{\text{fin}}$. Here, $\delta_{4F}^{\text{fin}}$ is an arbitrary constant and $\delta_{4F}^{\text{div}}$ is defined as
\[\delta_{4F}^{\text{div}} = \frac{\delta^6}{\epsilon},\]
(32)
to subtract the $\epsilon$-pole.

Closing the fermion lines, we get a contribution to the Higgs potential from $\mathcal{L}_{CT}$ with
\[\text{We do not confirm } \mathcal{L}_{CT} \text{ nonzero by computing it directly. When } \mathcal{L}_{CT} = 0, \text{ it does not contribute to the Higgs potential. As a proof of the existence of } \mathcal{L}_{CT} \text{ in a specific case, FIG. } \ref{fig:example} \text{ shows numerical results of its contributions to the Higgs potential.}\]
Therefore, it is concluded that the nontrivial $\theta$-dependence;

\[
V_{\text{CT}}(\theta) = \frac{i}{2} \frac{1}{2\pi R} \sum_{m_1, m_2} \int \frac{d^4 p}{(2\pi)^4} \frac{1}{2\pi R} \sum_{n_1} \int \frac{d^4 k}{(2\pi)^4} i \delta_{4F}^{\text{fin}} \sum_{\chi} C_{\chi} 
\times \left\{ \text{tr} \left[ G_X^{(1)} T^{(1)}_{\chi} \gamma_{\mu p^\mu} - \gamma_5 \left( \frac{n_1}{R} + \frac{\theta a - \beta}{2\pi R} \right) \right] \text{tr} \left[ G_X^{(2)} T^{(2)}_{\chi} \gamma_{\mu k^\mu} - \gamma_5 \left( \frac{n_2}{R} + \frac{\theta a - \beta}{2\pi R} \right) \right] \right\},
\]

(33)

where we have traced out matrix indices of both $\gamma^M$'s and $\tau^a$'s. Using Eq. (15), we get

\[
V_{\text{CT}}(\theta) = \frac{\delta_{4F}^{\text{fin}}}{2} \sum_{m_1, m_2} \int \frac{d^4 p}{(2\pi)^4} p^A \int \frac{d^4 k}{(2\pi)^4} k^B e^{-i2\pi R(m_1 p_5 + m_2 k_5)} \sum_{\chi} C_{\chi} 
\times \left\{ \text{tr} \left[ G_X^{(1)} T^{(1)}_{\chi} \gamma_{A} e^{i(\theta a - \beta) m_1} \right] \text{tr} \left[ G_X^{(2)} T^{(2)}_{\chi} \gamma_{B} e^{i(\theta a - \beta) m_2} \right] \right\}.
\]

(34)

In the previous work [18], we have derived that

\[
\int \frac{dDk}{(2\pi)^D} (-k^2 - i\epsilon)^{-s} e^{-i2k \cdot x} = \frac{i}{(4\pi)^{D/2}} \frac{\Gamma(s)}{\Gamma(s)} (-x^M x_M)^{s-D/2}.
\]

(35)

From this formula, we obtain

\[
V_{\text{CT}}(\theta) = \frac{9\delta_{4F}^{\text{fin}}}{2^{9} \pi^{12} R^8} \sum_{m_1, m_2} \sum_{m_1 \neq 0, m_2 \neq 0} \frac{m_1 m_2}{|m_1|^5 |m_2|^5} \sum_{\chi} C_{\chi} 
\times \left\{ \text{tr} \left[ G_X^{(1)} T^{(1)}_{\chi} \gamma_5 e^{i(\theta a - \beta) m_1} \right] \text{tr} \left[ G_X^{(2)} T^{(2)}_{\chi} \gamma_5 e^{i(\theta a - \beta) m_2} \right] \right\}.
\]

(36)

The contributions from each diagram are explicitly written down in Appendix B. To get $V_{\text{CT}}(\theta)$ in an Abelian gauge theory, we replace $\tau^a$'s with $Q$, the $U(1)$-charge of $\psi$. We have computed $V_{\text{CT}}(\theta)$ in an $SU(2)$ gauge theory with a fermion in the fundamental representation and an Abelian case with a fermion having the $U(1)$-charge $Q = 1$, which is shown in FIG. 1. Therefore, it is concluded that the $\theta$-dependent part of the Higgs potential is UV sensitive.
FIG. 1. Solid line: $V_{CT}(\theta)$ in an $SU(2)$ gauge theory with a fermion in the fundamental representation. By $SU(2)$ transformations, the VEV of the Higgs boson can be written as a diagonal matrix; $\langle A_5 \rangle = \text{diag}(\theta, -\theta)/2\pi Rg$. We have set $\beta$ to be zero. The contribution from the counter terms with the $\theta$-dependence results in the UV sensitivity of the Higgs potential. Dashed line: $V_{CT}(\theta)$ in an Abelian gauge theory with a fermion whose $U(1)$-charge equals to one. $\beta$ is specified to be zero. In the previous work [18], it was shown that contributions to the Higgs potential from the one-loop four-Fermi diagrams vanished in the Abelian gauge theory. Based on this numerical calculation, we reject the all-order finiteness of the Higgs potential in an Abelian gauge theory.

IV. SUMMARY

In this paper, we investigate the finiteness of the Higgs potential beyond the two-loop level in the GHU by evaluating the loop corrections explicitly. While the Higgs potential was found finite at the one- or two-loop levels on many non-simply connected manifolds, its finiteness at higher-order had been unclear. As suggested in the previous work [18], it is shown that the Higgs potential receives the nontrivial $\theta$-dependent contributions from the counter terms for the four-Fermi diagrams on $M^4 \times S^1$ at the four-loop level and, thus, it is UV sensitive.

For logarithmic divergences found in this paper, when we impose a UV cutoff on the GHU to make sense as an effective field theory, the maximum value of the cutoff, denoted as $\Lambda_{\text{max}}$, satisfies $\ln(R\Lambda_{\text{max}}) \sim g^2 \Lambda_{\text{max}}$. Hence, in the GHU, the perturbation is valid at most around the compactification scale.
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Appendix A: Two-loop integrals

This appendix is dedicated to evaluating a following integral:

\[ I \equiv \int \frac{d^D p}{(2\pi)^D} \int \frac{d^D k}{(2\pi)^D} \left[ -p^2 - i\epsilon \right]^{-s} \left[ -(p + k)^2 - i\epsilon \right]^{-t} \left[ -k^2 - i\epsilon \right]^{-u} e^{-i2(p \cdot k)} , \quad (A1) \]

where \( s, t, u \) are positive constants satisfying \( s + t + u > D \) and \( x, y \) are space-like vectors independent of \( p \) and \( k \).

Introducing the Feynman parameters, we get

\[ I = \int \frac{d^D p}{(2\pi)^D} \frac{d^D k}{(2\pi)^D} \int_0^1 d\alpha \int_{-\alpha}^1 d\beta \frac{\alpha^{s-1}\beta^{t-1}(1-\alpha-\beta)^{u-1}}{[-\alpha p^2 - \beta(p + k)^2 - (1-\alpha-\beta)k^2 - i\epsilon]^{s+t+u}} \frac{\Gamma(s+t+u)}{\Gamma(s)\Gamma(t)\Gamma(u)} e^{-i2(p \cdot k)} , \quad (A2) \]

In the previous work [18], we showed

\[ \int \frac{d^D k}{(2\pi)^D} (-k^2 + 2p \cdot k + m^2 - i\epsilon)^{-s} e^{-i2k \cdot x} = \frac{2i}{(4\pi)^{D/2}\Gamma(s)} \frac{e^{-i2p \cdot x}(-x^2)^{s/2-D/4}}{(p^2 + m^2 - i\epsilon)^{s/2-D/4}} K_{s-D/2}(2\sqrt{(-x^2)(p^2 + m^2 - i\epsilon)}), \quad (A3) \]

where \( \text{Re}(s) > 0, p^2 + m^2 \neq 0 \). Here, \( K_v(z) \) is the modified Bessel function of the second
kind. Using this formula, we have

$$I = \frac{2i(-y^2)^{s+t+u-\frac{D}{2}}}{(4\pi)^D/2 \Gamma(s) \Gamma(t) \Gamma(u)} \int_0^1 d\alpha \int_0^1 d\beta \int \frac{d^Dp}{(2\pi)^D}$$

$$\times \alpha^{s-1}(1-\alpha)^{-s-1} \beta^{t-1}(1-\beta)^{u-1} e^{-2ip(x-\beta y)}$$

$$\times \left\{ -\left[ \frac{\alpha}{1-\alpha} + \beta(1-\beta) \right] p^2 - \frac{i\epsilon}{1-\alpha} \right\}^{-s+t+u+\frac{D}{2}}$$

$$\times K_{s+t+u-\frac{D}{2}} \left( 2\sqrt{(-y^2)} \left\{ -\left[ \frac{\alpha}{1-\alpha} + \beta(1-\beta) \right] p^2 - \frac{i\epsilon}{1-\alpha} \right\} \right), \quad (A4)$$

where we have scaled $\beta$ to $(1-\alpha)\beta$.

There exist at most two mutually different solutions $\beta \in (0, 1)$ for $(x-\beta y)^2 = 0$. When the number of such solutions is denoted by $n$, we divide the integration interval over $\beta$ into $n+1$ parts where $(x-\beta y)^2$ is positive or negative entirely:

$$\int_0^1 d\beta \beta^{t-1}(1-\beta)^{u-1} F = \sum_{i=0}^{n} \int_{\beta_i}^{\beta_{i+1}} d\beta \beta^{t-1}(1-\beta)^{u-1} F, \quad (A5)$$

where $\beta_0 \equiv 0$, $\beta_{n+1} \equiv 1$, and $\beta_i \in (0, 1)$ satisfies $(x-\beta_i y)^2 = 0$ for $0 < i < n+1$. Here, we have defined

$$F = \int \frac{d^Dp}{(2\pi)^D} e^{-2ip(x-\beta y)} \left\{ -\left[ \frac{\alpha}{1-\alpha} + \beta(1-\beta) \right] p^2 - \frac{i\epsilon}{1-\alpha} \right\}^{-s+t+u+\frac{D}{2}}$$

$$\times K_{s+t+u-\frac{D}{2}} \left( 2\sqrt{(-y^2)} \left\{ -\left[ \frac{\alpha}{1-\alpha} + \beta(1-\beta) \right] p^2 - \frac{i\epsilon}{1-\alpha} \right\} \right), \quad (A6)$$

When $x-\beta y$ is space-like, we set $x^0-\beta y^0$ to be zero via the Lorentz transformations. Using the Wick rotation,

$$p^0 \rightarrow ip_E^0, \quad p \equiv p_E, \quad (A7)$$

$$x \equiv x_E, \quad y \equiv y_E, \quad (A8)$$
we get
\[
\mathcal{F} = \frac{i}{(2\pi)^D} \int_0^\infty d|p_E| |p_E|^{D-1} \left\{ \left[ \frac{\alpha}{1-\alpha} + \beta(1-\beta) \right] p_E^2 \right\}^{-\frac{s+t+u+D}{2}}
\times K_{s+t+u-\frac{D}{2}} \left( 2|y_E| \sqrt{\left[ \frac{\alpha}{1-\alpha} + \beta(1-\beta) \right] p_E^2} \right) \int d^\Omega_D e^{i^2 p_E \cdot (x_E - \beta y_E)}. \quad (A9)
\]

Carrying out the integral over all angles except \(\theta\), the angle between \(p_E = (p_E^0, \mathbf{p}_E)\) and \(x_E - \beta y_E = (0, \mathbf{x}_E - \beta \mathbf{y}_E)\), we get
\[
\int d^\Omega_D e^{i^2 p_E \cdot (x_E - \beta y_E)} = \Omega_{D-1} \int_0^\pi d\theta \sin^{D-2} \theta e^{i^2 |p_E||x_E-\beta y_E| \cos \theta}, \quad (A10)
\]
where \(\Omega_D\) is the area of the unit sphere in the \(D\)-dimensional space;
\[
\Omega_D = \frac{2\pi^{D/2}}{\Gamma(D/2)}. \quad (A11)
\]

The integral over \(\theta\) is evaluated as
\[
\int_0^\pi d\theta \sin^{a-1} \theta e^{i^2b \cos \theta} = \frac{\sqrt{\pi} \Gamma \left( \frac{a}{2} \right)}{\Gamma \left( \frac{a+1}{2} \right)} {}_0F_1 \left( \frac{a+1}{2} ; -b^2 \right) \quad (A12)
\]
for \(\text{Re}(a) > 0\) and \(b \in \mathbb{R}\). Here, \( {}_0F_1(a; z) \) is a generalized hypergeometric function;
\[
{}_0F_1(a; z) \equiv \sum_{n=0}^{\infty} \frac{1}{(a)_n n!} z^n, \quad (A13)
\]
where
\[
(a)_0 = 1, \quad (a)_n = \prod_{m=0}^{n-1} (a + m), \quad n \in \mathbb{Z}^+. \quad (A14)
\]

When \(x - \beta y\) is time-like, on the other hand, \(x - \beta y\) is set to be zero via the Lorentz transformations. After the Wick rotation, \(\mathcal{F}\) becomes
\[
\mathcal{F} = \frac{i}{(2\pi)^D} \int_0^\infty d|p_E| |p_E|^{D-1} \left\{ \left[ \frac{\alpha}{1-\alpha} + \beta(1-\beta) \right] p_E^2 \right\}^{-\frac{s+t+u+D}{2}}
\times K_{s+t+u-\frac{D}{2}} \left( 2|y_E| \sqrt{\left[ \frac{\alpha}{1-\alpha} + \beta(1-\beta) \right] p_E^2} \right) \int d^\Omega_D e^{i^2 p_E^0 (x_E^0 - \beta y_E^0)}. \quad (A15)
\]
where \( x_E^0 \equiv x^0 \) and \( y_E^0 \equiv y^0 \). We evaluate angle integrals with a similar way to Eqs. (A10) and (A12):

\[
\int d\Omega_D e^{2p_E^0(x_E^0-\beta y_E^0)} = \Omega_{D-1} \int_0^{\pi/2} d\theta \sin^{D-2}\theta e^{2|p_E||x_E-\beta y_E|\cos\theta} \tag{A16}
\]

and

\[
\int_0^{\pi} d\theta \sin^{a-1}\theta e^{2b\cos\theta} = \frac{\sqrt{\pi}\Gamma\left(\frac{a}{2}\right)}{\Gamma\left(\frac{a+1}{2}\right)} _0F_1\left(\frac{a+1}{2}; b^2\right) \tag{A17}
\]

for \( \text{Re}(a) > 0 \) and \( b \in \mathbb{R} \).

The integral over \( |p_E| \) is given by

\[
\int_0^\infty d|p_E| |p_E|^{a-1} K_r(2b|p_E|) _0F_1\left(\frac{a+r}{2}; c|p_E|^2\right) = \frac{1}{4} \Gamma\left(\frac{a-r}{2}\right) \Gamma\left(\frac{a+r}{2}\right) b^{-r}(b^2-c)^{-a/r} \tag{A18}
\]

for \( b > 0, c < 0, \text{Re}(a-r) > 0, \) and \( \text{Re}(a+r) > 0 \).

Applying the above formulae to \( \mathcal{F} \), we obtain

\[
\mathcal{F} = \frac{i}{2} \frac{\Gamma(D-s-t-u)}{(4\pi)^{D/2}} \left(\sqrt{-y^2}\right)^{D-s-t-u} \left[\frac{\alpha}{1-\alpha} + \beta(1-\beta)\right]^{D-s-t-u}
\]

\[
\times \left[-(x-\beta y)^2 - y^2 \left(\frac{\alpha}{1-\alpha} + \beta(1-\beta)\right)\right]^{s+t+u-D} \tag{A19}
\]

regardless of whether \( x-\beta y \) is space-like or time-like. Therefore, after integration over \( k \) and \( p \), we get

\[
\mathcal{I} = -\frac{\Gamma(D-s-t-u)}{(4\pi)^D\Gamma(s)\Gamma(t)\Gamma(u)} \int_0^1 d\alpha \int_0^1 d\beta \alpha^{s-1}(1-\alpha)^{-s-1}\beta^{t-1}(1-\beta)^{u-1}
\]

\[
\times \left[\frac{\alpha}{1-\alpha} + \beta(1-\beta)\right]^{D-s-t-u} \left[-(x-\beta y)^2 - y^2 \left(\frac{\alpha}{1-\alpha} + \beta(1-\beta)\right)\right]^{s+t+u-D} \tag{A20}
\]

By expanding the last factor, finding the \( \epsilon \)-pole of \( \mathcal{I} \) comes down to calculating the following integral:

\[
\mathcal{J} \equiv \int_0^1 d\alpha \int_0^1 d\beta \alpha^{\sigma-1}(1-\alpha)^{-\sigma-1}\beta^{\tau-1}(1-\beta)^{\kappa-1} \left[\frac{\alpha}{1-\alpha} + \beta(1-\beta)\right]^\lambda \tag{A21}
\]

where \( \sigma, \tau, \kappa, \) and \( \lambda \) are arbitrary constants.
Setting \( \gamma \) to be
\[
\gamma \equiv \frac{\alpha}{1 - \alpha},
\]
we have
\[
\mathcal{J} = \int_{0}^{\infty} d\gamma \int_{0}^{1} d\beta \gamma^{\sigma-1}\beta^{\tau-1}(1 - \beta)^{\kappa-1}[\gamma + \beta(1 - \beta)]^\lambda
\]
\[
= B(\sigma, -\sigma - \lambda) \int_{0}^{1} d\beta \beta^{\sigma+\tau+\lambda-1}(1 - \beta)^{\sigma+\kappa+\lambda-1}
\]
\[
= B(\sigma, -\sigma - \lambda) B(\sigma + \tau + \lambda, \sigma + \kappa + \lambda),
\]
where \( B(x, y) \) is the beta function;
\[
B(x, y) \equiv \int_{0}^{1} dt t^{x-1}(1 - t)^{y-1} = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x + y)}.
\]

**Appendix B: Table of divergence and its contribution to the Higgs potential**

As shown in Sect. III at the two-loop level, the divergences of the four-Fermi diagrams have the form,
\[
-\frac{i\mathbf{g}^6}{\epsilon} C_X [G_X^{(1)}]_{\alpha\beta} [G_X^{(2)}]_{\gamma\delta} [T_X^{(1)}]_{ij} [T_X^{(2)}]_{kl} - (\alpha \leftrightarrow \gamma, i \leftrightarrow k),
\]
where \( G_X^{(1,2)} \) and \( T_X^{(1,2)} \) are products of \( \gamma^M \)'s and \( \tau^a \)'s respectively. Here, \( C_X \) is a constant and \( X \) denotes divergent diagrams without crossing of the fermion lines. The second term represents the same diagram with the fermion lines being crossed.

In this appendix, we summarize the divergent diagrams and their contributions to the Higgs potential in a table. For making appearance simple, we write down the following factors;
\[
-64(4\pi)^5 C_X [G_X^{(1)}]_{\alpha\beta} [G_X^{(2)}]_{\gamma\delta} [T_X^{(1)}]_{ij} [T_X^{(2)}]_{kl}
\]
and
\[
-64(4\pi)^5 C_X \left\{ \text{tr} \left[ G_X^{(1)} T_X^{(1)} \gamma_5 e^{i(\theta^a \tau^a - \beta)m_1} \right] \text{tr} \left[ G_X^{(2)} T_X^{(2)} \gamma_5 e^{i(\theta^a \tau^a - \beta)m_2} \right] 
- \text{tr} \left[ G_X^{(1)} T_X^{(1)} \gamma_5 e^{i(\theta^a \tau^a - \beta)m_1} C_X^{(2)} T_X^{(2)} \gamma_5 e^{i(\theta^a \tau^a - \beta)m_2} \right] \right\}.
\]
TABLE I: The divergent part of the four-Fermi diagrams at the two-loop level and its contribution to the Higgs potential. Each diagram is represented by $\mathcal{X}$. $G^{(1)}_X$ and $T^{(1)}_X$ are products of $\gamma^M$'s and $\tau^a$'s, respectively, on the upper fermion line. $G^{(2)}_X$ and $T^{(2)}_X$ are ones on the lower fermion line. $C_X$ is a collection of the other factors up to $-ig^6/\epsilon$, where $\epsilon = (5 - D)/2$ with the spacetime dimension $D$. Note that we have explicitly traced out with respect to $\gamma^M$'s in the expressions of the contributions to the Higgs potential. $c_r$ is a constant which depends on the representation of a fermion; $\text{tr}[\tau^a \tau^b] = c_r \delta^{ab}$.

For the fundamental representation, $c_r = 1/2$.

$$
-64(4\pi)^5 C_X \left[ G^{(1)}_X [G^{(2)}_X]_{\alpha\beta} [T^{(1)}_X]_{ij} [T^{(2)}_X]_{kl} \right].
$$

$$
\mathcal{X} = -64(4\pi)^5 C_X \left\{ \text{tr} \left[ G^{(1)}_X T^{(1)}_X \gamma^5 e^{i(\theta^a X a - \beta)} m_1 \right] \text{tr} \left[ G^{(2)}_X T^{(2)}_X \gamma^5 e^{i(\theta^a X a - \beta)} m_2 \right] - \text{tr} \left[ G^{(1)}_X T^{(1)}_X \gamma^5 e^{i(\theta^a X a - \beta)} m_1 G^{(2)}_X T^{(2)}_X \gamma^5 e^{i(\theta^a X a - \beta)} m_2 \right] \right\}
$$

$$
\frac{4}{105} \pi \left( \eta^{PS} \eta^{QR} + 22\eta^{PR} \eta^{QS} + \eta^{PQ} \eta^{RS} \right) \times \left[ \gamma^M \gamma^P \gamma^N \gamma^Q \gamma^L \right]_{\alpha\beta} \left[ \gamma^M \gamma^P \gamma^N \gamma^S \gamma^L \right]_{\gamma\delta} \left[ \tau^a \tau^b T^c \right]_{ij} \left[ \tau^a \tau^b T^c \right]_{kl},
$$

$$
- \frac{128}{15} \pi \left\{ 394 \text{tr} \left[ \tau^a \tau^b T^c e^{i(\theta^d X d - \beta)} m_1 \right] \text{tr} \left[ \tau^a \tau^b T^c e^{i(\theta^d X d - \beta)} m_2 \right] + 393 \text{tr} \left[ \tau^a \tau^b T^c e^{i(\theta^d X d - \beta)} m_1 \tau^a \tau^b T^c e^{i(\theta^d X d - \beta)} m_2 \right] \right\}
$$

$$
\frac{8}{105} \pi \left( \eta^{PS} \eta^{QR} - 6\eta^{PR} \eta^{QS} + \eta^{PQ} \eta^{RS} \right) \times \left[ \gamma^M \gamma^P \gamma^N \gamma^Q \gamma^L \right]_{\alpha\beta} \left[ \gamma^N \gamma^R \gamma^M \gamma^{S} \gamma^L \right]_{\gamma\delta} \left[ \tau^a \tau^b T^c \right]_{ij} \left[ \tau^b \tau^a T^c \right]_{kl},
$$

$$
\frac{128}{15} \pi \left\{ 80 \text{tr} \left[ \tau^a \tau^b T^c e^{i(\theta^d X d - \beta)} m_1 \right] \text{tr} \left[ \tau^b \tau^a T^c e^{i(\theta^d X d - \beta)} m_2 \right] + 81 \text{tr} \left[ \tau^a \tau^b T^c e^{i(\theta^d X d - \beta)} m_1 \tau^b \tau^a T^c e^{i(\theta^d X d - \beta)} m_2 \right] \right\}
$$
\[
\frac{8}{105} \pi \left( \eta^{PS} \eta^{QR} - 6 \eta^{PR} \eta^{QS} + \eta^{PQ} \eta^{RS} \right) \\
\times \left[ \gamma^M \gamma^P \gamma^N \gamma^Q \gamma^L \right] \alpha \beta \left[ \gamma^M \gamma^P \gamma^L \gamma^S \gamma^N \right] \gamma \delta \left[ \tau^a \tau^b \tau^c \right]_{ij} \left[ \tau^a \tau^b \tau^c \right]_{kl},
\]
\[
\frac{128}{15} \pi \left\{ 80 \text{tr} \left[ \tau^a \tau^b \tau^c e^{i(\theta^d - \beta)m_1} \right] \text{tr} \left[ \tau^a \tau^b \tau^c e^{i(\theta^d - \beta)m_2} \right] \right. \\
+ 81 \text{tr} \left[ \tau^a \tau^b \tau^c e^{i(\theta^d - \beta)m_1} \tau^a \tau^b \tau^c e^{i(\theta^d - \beta)m_2} \right] \right\}
\]
\[
\frac{8}{105} \pi \left( \eta^{PS} \eta^{QR} - \eta^{PR} \eta^{QS} - \eta^{PQ} \eta^{RS} \right) \\
\times \left[ \gamma^M \gamma^P \gamma^N \gamma^Q \gamma^L \right] \alpha \beta \left[ \gamma^N \gamma^R \gamma^L \gamma^S \gamma^M \right] \gamma \delta \left[ \tau^a \tau^b \tau^c \right]_{ij} \left[ \tau^b \tau^c \tau^a \right]_{kl},
\]
\[
\frac{128}{15} \pi \left\{ 80 \text{tr} \left[ \tau^a \tau^b \tau^c e^{i(\theta^d - \beta)m_1} \right] \text{tr} \left[ \tau^b \tau^c \tau^a e^{i(\theta^d - \beta)m_2} \right] \right. \\
+ 99 \text{tr} \left[ \tau^a \tau^b \tau^c e^{i(\theta^d - \beta)m_1} \tau^b \tau^c \tau^a e^{i(\theta^d - \beta)m_2} \right] \right\}
\]
\[
\frac{8}{105} \pi \left( 4 \eta^{PS} \eta^{QR} + \eta^{PR} \eta^{QS} + \eta^{PQ} \eta^{RS} \right) \\
\times \left[ \gamma^M \gamma^P \gamma^N \gamma^Q \gamma^L \right] \alpha \beta \left[ \gamma^L \gamma^R \gamma^M \gamma^S \gamma^N \right] \gamma \delta \left[ \tau^a \tau^b \tau^c \right]_{ij} \left[ \tau^c \tau^a \tau^b \right]_{kl},
\]
\[
\frac{256}{15} \pi \left\{ 197 \text{tr} \left[ \tau^a \tau^b \tau^c e^{i(\theta^d - \beta)m_1} \right] \text{tr} \left[ \tau^a \tau^c \tau^b e^{i(\theta^d - \beta)m_2} \right] \\
+ 54 \text{tr} \left[ \tau^a \tau^b \tau^c e^{i(\theta^d - \beta)m_1} \tau^c \tau^b \tau^a e^{i(\theta^d - \beta)m_2} \right] \right\}
\]
\[
\frac{4}{35} \pi \left( \eta^{PS} \eta^{QR} - 6 \eta^{PR} \eta^{QS} + \eta^{PQ} \eta^{RS} \right) \\
\times \left[ \gamma^M \gamma^P \gamma^N \gamma^Q \gamma^L \gamma^R \gamma^M \right] \alpha \beta \left[ \gamma^N \gamma^S \gamma^L \gamma^S \gamma^N \right] \gamma \delta \left[ \tau^a \tau^b \tau^c \tau^a \right]_{ij} \left[ \tau^b \tau^c \right]_{kl},
\]
\[
\frac{1728}{5} \pi \left\{ 4 \text{tr} \left[ \tau^a \tau^b \tau^c \tau^a e^{i(\theta^d - \beta)m_1} \right] \text{tr} \left[ \tau^b \tau^c e^{i(\theta^d - \beta)m_2} \right] \\
+ 3 \text{tr} \left[ \tau^a \tau^b \tau^c \tau^a e^{i(\theta^d - \beta)m_1} \tau^b \tau^c e^{i(\theta^d - \beta)m_2} \right] \right\}
\]
\[
\frac{4}{35} \pi \left( \eta^{PS} R^{QQR} - 6 \eta^{PR} R^{QQS} + \frac{5}{2} \eta^{PQ} R^{RS} \right) \\
\times \left[ \gamma^{N} \gamma^{N} \gamma^{L} \right]_{\alpha \beta} \left[ \gamma^{M} \gamma^{N} \gamma^{Q} \gamma^{R} \gamma^{M} \right]_{\gamma \delta} \left[ \tau^{b} \tau^{c} \right]_{ij} \left[ \tau^{a} \tau^{b} \tau^{c} \tau^{a} \right]_{kl},
\]

\[
\frac{1728}{5} \pi \left\{ 4 \text{ tr} \left[ \tau^{b} \tau^{c} \tau^{d} \tau^{e} \left( \eta^{d} \eta^{d} - \beta \right) m_{1} \right] \text{ tr} \left[ \tau^{a} \tau^{b} \tau^{c} \tau^{d} e^{i(\eta^{d} \eta^{d} - \beta) m_{2}} \right] + 3 \text{ tr} \left[ \tau^{c} \tau^{b} \tau^{d} \tau^{e} e^{i(\eta^{d} \eta^{d} - \beta) m_{1}} \tau^{a} \tau^{b} \tau^{c} \tau^{e} e^{i(\eta^{d} \eta^{d} - \beta) m_{2}} \right] \right\}
\]

\[
\frac{4}{35} \pi \left( \eta^{PS} R^{QQR} - 6 \eta^{PR} R^{QQS} + \frac{5}{2} \eta^{PQ} R^{RS} \right) \\
\times \left[ \gamma^{L} \gamma^{N} \gamma^{Q} \gamma^{R} \gamma^{M} \right]_{\alpha \beta} \left[ \gamma^{L} \gamma^{N} \gamma^{Q} \gamma^{R} \gamma^{M} \right]_{\gamma \delta} \left[ \tau^{c} \tau^{b} \right]_{ij} \left[ \tau^{a} \tau^{c} \tau^{b} \tau^{a} \right]_{kl},
\]

\[
\frac{1728}{5} \pi \left\{ 4 \text{ tr} \left[ \tau^{c} \tau^{b} \tau^{e} \tau^{a} e^{i(\eta^{d} \eta^{d} - \beta) m_{1}} \right] \text{ tr} \left[ \tau^{a} \tau^{b} \tau^{c} \tau^{e} e^{i(\eta^{d} \eta^{d} - \beta) m_{2}} \right] + 3 \text{ tr} \left[ \tau^{c} \tau^{b} \tau^{e} \tau^{a} e^{i(\eta^{d} \eta^{d} - \beta) m_{1}} \tau^{a} \tau^{b} \tau^{c} \tau^{e} e^{i(\eta^{d} \eta^{d} - \beta) m_{2}} \right] \right\}
\]

\[
\frac{8}{105} \pi \left( \eta^{PS} R^{QQR} + \frac{5}{2} \eta^{PR} R^{QQS} + 8 \eta^{PQ} R^{RS} \right) \\
\times \left[ \gamma^{L} \gamma^{N} \gamma^{Q} \gamma^{M} \gamma^{R} \gamma^{L} \right]_{\alpha \beta} \left[ \gamma^{N} \gamma^{S} \gamma^{L} \gamma^{R} \gamma^{M} \right]_{\gamma \delta} \left[ \tau^{b} \tau^{c} \right]_{ij} \left[ \tau^{a} \tau^{b} \tau^{c} \tau^{a} \right]_{kl},
\]

\[
\frac{192}{5} \pi \left\{ 28 \text{ tr} \left[ \tau^{a} \tau^{b} \tau^{c} \tau^{e} e^{i(\eta^{d} \eta^{d} - \beta) m_{1}} \right] \text{ tr} \left[ \tau^{b} \tau^{c} \tau^{e} \tau^{a} e^{i(\eta^{d} \eta^{d} - \beta) m_{2}} \right] + 29 \text{ tr} \left[ \tau^{a} \tau^{b} \tau^{c} \tau^{e} e^{i(\eta^{d} \eta^{d} - \beta) m_{1}} \tau^{b} \tau^{c} \tau^{e} \tau^{a} e^{i(\eta^{d} \eta^{d} - \beta) m_{2}} \right] \right\}
\]

\[
\frac{8}{105} \pi \left( \eta^{PS} R^{QQR} + \frac{5}{2} \eta^{PR} R^{QQS} + 8 \eta^{PQ} R^{RS} \right) \\
\times \left[ \gamma^{N} \gamma^{S} \gamma^{L} \right]_{\alpha \beta} \left[ \gamma^{M} \gamma^{N} \gamma^{Q} \gamma^{M} \gamma^{R} \gamma^{L} \right]_{\gamma \delta} \left[ \tau^{b} \tau^{c} \right]_{ij} \left[ \tau^{a} \tau^{b} \tau^{a} \tau^{c} \right]_{kl},
\]

\[
\frac{192}{5} \pi \left\{ 28 \text{ tr} \left[ \tau^{b} \tau^{c} \tau^{e} \tau^{a} e^{i(\eta^{d} \eta^{d} - \beta) m_{1}} \right] \text{ tr} \left[ \tau^{a} \tau^{b} \tau^{c} \tau^{e} e^{i(\eta^{d} \eta^{d} - \beta) m_{2}} \right] + 29 \text{ tr} \left[ \tau^{b} \tau^{c} \tau^{e} \tau^{a} e^{i(\eta^{d} \eta^{d} - \beta) m_{1}} \tau^{a} \tau^{b} \tau^{c} \tau^{e} e^{i(\eta^{d} \eta^{d} - \beta) m_{2}} \right] \right\}
\]
\[
\frac{2}{105} \pi c_r \left( 10 \eta^P S \eta^{QR} + 3 \eta^P R \eta^{QS} + 3 \eta^P Q \eta^{RS} \right) \\
\times \text{tr} \left[ \gamma^M \gamma^Q \gamma^R \right] \left[ \gamma^M \gamma^P \gamma^N \right] \alpha \beta \left[ \gamma^L \gamma^S \gamma^M \right] \gamma \delta \left[ \tau^a \tau^b \right] ij \left[ \tau^a \tau^b \right]_{kl},
\]

\[
\frac{256}{5} \pi c_r \left\{ 8 \text{tr} \left[ \tau^a \tau^b e^{i(\theta^a \tau^d \beta^a \beta^b)} \right] \text{tr} \left[ \tau^a \tau^b e^{i(\theta^a \tau^d \beta^a \beta^b)} \right] \\
+ 9 \text{tr} \left[ \tau^a \tau^b e^{i(\theta^a \tau^d \beta^a \beta^b)} \right] \tau^a \tau^b e^{i(\theta^a \tau^d \beta^a \beta^b)} \right\}
\]

\[
- \frac{2}{105} \pi c_r \left( 10 \eta^P S \eta^{QR} + 3 \eta^P R \eta^{QS} + 3 \eta^P Q \eta^{RS} \right) \\
\times \text{tr} \left[ \gamma^N \gamma^Q \gamma^R \right] \left[ \gamma^M \gamma^P \gamma^N \right] \alpha \beta \left[ \gamma^N \gamma^S \gamma^L \right] \gamma \delta \left[ \tau^a \tau^b \right] ij \left[ \tau^a \tau^b \right]_{kl},
\]

\[
- \frac{256}{5} \pi c_r \left\{ 8 \text{tr} \left[ \tau^a \tau^b e^{i(\theta^a \tau^d \beta^a \beta^b)} \right] \text{tr} \left[ \tau^a \tau^b e^{i(\theta^a \tau^d \beta^a \beta^b)} \right] \\
+ 9 \text{tr} \left[ \tau^a \tau^b e^{i(\theta^a \tau^d \beta^a \beta^b)} \right] \tau^a \tau^b e^{i(\theta^a \tau^d \beta^a \beta^b)} \right\}
\]

\[
- \frac{2}{105} \pi c_r \left( 10 \eta^P S \eta^{QR} + 3 \eta^P R \eta^{QS} + 3 \eta^P Q \eta^{RS} \right) \\
\times \text{tr} \left[ \gamma^N \gamma^Q \gamma^R \right] \left[ \gamma^M \gamma^P \gamma^N \right] \alpha \beta \left[ \gamma^N \gamma^S \gamma^L \right] \gamma \delta \left[ \tau^a \tau^b \right] ij \left[ \tau^a \tau^b \right]_{kl},
\]

\[
- \frac{256}{5} \pi c_r \left\{ 8 \text{tr} \left[ \tau^a \tau^b e^{i(\theta^a \tau^d \beta^a \beta^b)} \right] \text{tr} \left[ \tau^a \tau^b e^{i(\theta^a \tau^d \beta^a \beta^b)} \right] \\
+ 3 \text{tr} \left[ \tau^a \tau^b e^{i(\theta^a \tau^d \beta^a \beta^b)} \right] \tau^a \tau^b e^{i(\theta^a \tau^d \beta^a \beta^b)} \right\}
\]

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