We present recent preliminary results from five decay channels. From the $K_L \rightarrow \pi^+\pi^-\gamma$ channel, we extract form factors for the CP violating M1 direct photon emission amplitude and the fraction of the total decay amplitude that is due to direct emission. We have placed an upper limit on the $K_L \rightarrow \pi^0\pi^0\gamma$ branching ratio, and preliminary measurements of the $K_L \rightarrow \pi^\pm\nu\nu\pi^\mp\gamma$ and $\pi^0 \rightarrow e^+e^-\gamma$ branching ratios are presented. Finally, we report measurements of both the branching ratio and the form factor parameters for the decay $K_L \rightarrow e^+e^-\gamma$.

1 The KTeV Detector

The KTeV detector was used by two Fermilab experiments. The first, E799-II, was designed to measure rare kaon decays such as $K_L \rightarrow \pi^0\pi^0\gamma$ and $K_L \rightarrow \pi^\pm\nu\nu\pi^\mp\gamma$. The second, E832, was designed to measure the CP violating parameter $Re(\epsilon'/\epsilon)$.

The two neutral kaon beams used by KTeV passed through a 60m vacuum decay volume. Downstream of the decay volume was a charged spectrometer, which consisted of four drift chambers and a dipole magnet. The spectrometer determined charged particle momenta with a resolution of $\sigma(P)/P = 0.38\% \oplus 0.16\% \ast P$(GeV/c). Beyond the spectrometer was a transition radiation detector (TRD), which was used to separate electrons and pions. Immediately following the TRD was a 3100 crystal CsI calorimeter. The energy resolution achieved on clusters in the calorimeter was $\sigma(E)/E = 0.45\% \oplus 2%/\sqrt{E}$(GeV). Behind the calorimeter were alternating layers of steel and scintillator used to reject backgrounds with muons in the final state.
2 The Decay $K_L \to \pi^+\pi^-\gamma$

2.1 Motivation

The decay $K_L \to \pi^+\pi^-\gamma$ can proceed through two channels. The inner Bremsstrahlung (IB) channel is identical to the CP violating $K_L \to \pi^+\pi^-$ decay, except that an additional photon is internally radiated by one of the charged pions. In the direct emission (DE) channel, all three decay particles emerge directly from the $K_L$ decay vertex. The DE process usually occurs via a CP conserving M1 term. However, there can also be a small contribution from a CP violating E1 term. The E1 DE amplitude can contribute via interference with the IB amplitude as well.

The expressions for each of these decay amplitudes are given in Eqs. 1-3. Using these expressions, the values of the parameters $|\tilde{g}_{M1}|$, $a_1/a_2$, and $|g_{E1}|$ can be extracted from the measured amplitudes.

\begin{align*}
E_{IB}(K_L) & = \left( \frac{2M_K}{E_\gamma} \right)^2 \frac{|\eta_{+-}|e^{i\Phi_{+-}}e^{i\delta_0}}{1 - \left(1 - \frac{4m_{\pi}^2}{M_{\pi+\pi-}^2}\right)\cos^2(\theta)} \\
M_{DE}(K_L) & = i\tilde{g}_{M1} \left( 1 + \frac{a_1/a_2}{(M_\rho^2 - M_K^2) + 2M_KE_\gamma} \right) e^{i\delta_1} \\
E_{DE}(K_L) & = |g_{E1}|e^{i\delta_1}
\end{align*}

In the above expressions, $M_K$, $M_\rho$, and $M_{\pi+\pi-}$ are the kaon mass, rho mass, and the mass of the $\pi^+\pi^-$ system, respectively. $E_\gamma$ is the photon energy in the kaon rest frame and $\theta$ is the angle between the $\gamma$ and $\pi^+$ in the $\pi^+\pi^-$ rest frame. $\delta_0(s = M_K^2)$ and $\delta_1(s = M_{\pi+\pi-}^2)$ are the isospin 0 and 1 strong phase shifts, and $|\eta_{+-}|e^{i\Phi_{+-}}$ is the $K_L \to \pi^+\pi^-$ amplitude.

2.2 $K_L \to \pi^+\pi^-\gamma$ Analysis

The IB and M1 DE decay amplitudes can be separated in the $E_\gamma$ and $\cos(\theta)$ variables described in Sec. 2.1. The Monte Carlo prediction for both of these components is compared to data as shown in Figs. 1 and 2. A two-dimensional likelihood fit is performed in $E_\gamma$ and $\cos(\theta)$ to extract the values of $|\tilde{g}_{M1}|$ and $a_1/a_2$. Despite isolating $111.4 \times 10^3$ signal events over a 0.4% background, the E1 DE amplitude is still too small to be measured.

The results for $K_L \to \pi^+\pi^-\gamma$ have been submitted to Phys. Rev. Lett. The fit results for
the M1 direct emission parameters are

\[ |g_{M1}| = 1.198 \pm 0.035 \text{(stat)} \pm 0.086 \text{(syst)} \] (4)

\[ a_1/a_2 = -0.738 \pm 0.007 \text{(stat)} \pm 0.018 \text{(syst)} \text{GeV}^2/c^2 \] (5)

These measurements are the most precise to date. We also measure the direct emission fraction and an upper limit on the CP violating E1 direct emission parameter \( g_{E1} \):

\[ \frac{DE}{DE + IB} = 0.689 \pm 0.021 \quad (E_\gamma > 20 \text{MeV}) \] (6)

\[ g_{E1} < 0.21 \quad (90\% \text{ confidence}) \] (7)

3 The Decay \( K_L \to \pi^0\pi^0\gamma \)

3.1 Motivation

The decay \( K_L \to \pi^0\pi^0\gamma \) can only proceed via direct emission from the \( K_L\pi^0\pi^0 \) vertex. Since the two pions in the decay are identical, the lowest contributing multipole moment is \( L=2 \). The E2 direct emission term is CP conserving, while the M2 direct emission term is CP violating.

The decay amplitude for this mode vanishes to \( O(p^4) \) in chiral perturbation theory (ChPT), and therefore provides a direct probe of \( O(p^6) \) ChPT. The predicted branching ratio from ChPT is \( (7 \times 10^{-11}) \). However, extrapolations from the \( K_L \to \pi^+\pi^-\gamma \) branching ratio can give predictions as high as \( (1 \times 10^{-8}) \). The current upper limit from the NA31 experiment is \( (5.6 \times 10^{-6}) \).

3.2 \( K_L \to \pi^0\pi^0\gamma \) Analysis

The search for \( K_L \to \pi^0\pi^0\gamma \) is performed using a blind analysis. We isolate \( K_L\pi^0\pi^0_D \pi \to e^+e^-\gamma \) to utilize our sensitive charged particle trigger. There is a large background due to \( K_L \to \pi^0\pi^0\pi^0_D \) events with a missing photon, however most of these events can be eliminated by requiring that the \( e^+e^- \) mass lie within 3 MeV/c² of the kaon mass and that the square of the momentum transverse to the kaon direction be less than 0.00015 GeV²/c². The analysis cuts retain 80% of the signal Monte Carlo.

The preliminary result presented here is based on 40% of the total data set. The Monte Carlo gives a prediction of \( 1.66 \pm 0.59 \) background events in the signal region. No events were found in the signal region. We set an upper limit on the branching ratio value of

\[ BR(K_L \to \pi^0\pi^0\gamma) < 2.52 \times 10^{-7} \quad (90\% \text{ confidence}) \] (8)

This measurement represents a factor of 22 improvement on the NA31 result.

4 The Decay \( K_L \to \pi^\pm\pi^\mp\nu\bar{\nu} \)

4.1 Motivation

The CKM matrix element \( V_{us} \) can be extracted from \( K_L \to \pi^\pm l^\mp\nu \) decays via

\[ \Gamma_{K_L \to \pi^\pm l^\mp\nu} = \frac{G_F^2M_K^5}{192\pi^3}S_{EW} \left( 1 + \delta_K \right) C^2|V_{us}|^2f_+(0)I_K \] (9)

where \( l \) refers to either \( e \) or \( \mu \), \( G_F \) is the Fermi constant, \( M_K \) is the kaon mass, \( S_{EW} \) is the short-distance radiative correction, \( \delta_K \) is the long-distance radiative correction, \( f_+(0) \) is the form factor at zero momentum transfer to the \( l\nu \) system, and \( I_K \) is the phase-space integral,
which depends on the measured semileptonic form factors. The value of $C^2$ is 1 for neutral kaon decays and 1/2 for charged kaon decays.

The uncertainty in the $K\pi W$ vertex parameters is currently larger than the experimental uncertainty in $\Gamma_{K_L \rightarrow \pi^0 e^+ e^-}$. The direct emission $K_L \rightarrow \pi^\pm e^\mp \nu \gamma$ decay is sensitive to the structure of the $K\pi W$ vertex, but unfortunately this decay is dominated by inner Bremsstrahlung. If the radiated photon in $K_L \rightarrow \pi^\pm e^\mp \nu \gamma$ is virtual, there is an enhancement in the DE/IB ratio, therefore the decay $K_L \rightarrow \pi^\pm e^\mp \nu e^\pm e^-$ is a more sensitive probe of the $K\pi W$ vertex.

4.2 $K_L \rightarrow \pi^\pm e^\mp \nu e^\pm e^-$ Analysis

The main background to $K_L \rightarrow \pi^\pm e^\mp \nu e^\pm e^-$ decays comes from $K_L \rightarrow \pi^0 e^0 \pi_D^0 (\pi_D^0 \rightarrow e^+ e^-)$ where the photon is missing and one of the pions is mis-reconstructed as an electron. This background can be greatly reduced by cutting on the square of the longitudinal momentum of the missing $\pi^0$ in the frame where the kaon momentum is perpendicular to the $\pi^+ \pi^-$ momentum. If this quantity is negative, it indicates that there is not enough phase space to form a $\pi^0$ if the missing energy is assumed to be a photon.

We have isolated 19466 signal events with a background of 4.95%. To measure the absolute branching ratio, the $K_L \rightarrow \pi^\pm e^\mp \nu e^\pm e^-$ rate is normalized to the $K_L \rightarrow \pi^+ e^- \pi_D^0 (\pi_D^0 \rightarrow e^+ e^-)$ rate. We find 300526 normalization mode events with a background of 2.23%. In addition, we define $K_L \rightarrow \pi^\pm e^\mp \nu e^\pm e^-$ events to have an $e^+ e^-$ mass greater than 5 MeV/c$^2$. The preliminary branching ratio measurement is

$$BR(K_L \rightarrow \pi^\pm e^\mp \nu e^\pm e^-, M_{e^+ e^-} > 5\text{MeV}/c^2) = (1.606 \pm 0.012(stat) \pm 0.026(syst)) \times 10^{-5}$$

The additional (ext.syst) term is due to the experimental uncertainty on the branching ratio of $K_L \rightarrow \pi^+ e^- \pi_D^0$.

5 The Decay $\pi^0 \rightarrow e^+ e^-$

5.1 Motivation

The decay $\pi^0 \rightarrow e^+ e^-$ proceeds through a loop process at lowest order. This mode provides a sensitive probe of theoretical models due to the precision with which it can be measured and the seemingly simple nature of the process. The contribution from on-shell photons sets a lower unitary bound on the rate:

$$\Gamma(\pi^0 \rightarrow e^+ e^-)/\Gamma(\pi^0 \rightarrow \gamma\gamma) \geq 4.75 \times 10^{-8}$$

Any additional contribution from off-shell photons will enhance the rate. Vector meson dominance (VMD) models predict a branching ratio of $6 - 6.4 \times 10^{-25}$, whereas predictions from chiral perturbation theory have a range of $6.2 - 8.3 \times 10^{-25}$.

5.2 The $\pi^0 \rightarrow e^+ e^-$ Measurement

The main background to $\pi^0 \rightarrow e^+ e^-$ is the decay $\pi^0 \rightarrow e^+ e^- \gamma$ where the photon is either lost or very soft. Fortunately, the $\pi^0 \rightarrow e^+ e^- \gamma e^+ e^-$ mass spectrum falls off very quickly as the $e^+ e^-$ mass approaches the $\pi^0$ mass, whereas the $\pi^0 \rightarrow e^+ e^-$ spectrum peaks sharply at high $e^+ e^-$ mass. Figure 3 shows the $e^+ e^-$ mass spectrum in data compared to the Monte Carlo background estimate. A clear $\pi^0 \rightarrow e^+ e^-$ peak can be seen at the pion mass. The main systematic uncertainty comes from the mismatch between the background level predicted by the Monte Carlo and the amount of background observed in the data.
There are 714 events in the signal region with a predicted background of $39.9 \pm 12.3$ events. The $\pi^0 \to e^+e^-$ rate is normalized to the $\pi^0 \to e^+e^-\gamma$ rate. The preliminary result for this ratio is

$$\frac{BR(\pi^0 \to e^+e^-, x > 0.95)}{BR(\pi^0 \to e^+e^-\gamma, x > 0.232)} = (1.721 \pm 0.068(stat) \pm 0.036(syst)) \times 10^{-4}$$

(12)

where $x$ is the squared ratio of the $e^+e^-$ mass to the $\pi^0$ mass. Using the $\pi^0 \to e^+e^-\gamma$ branching ratio and the full $e^+e^-$ spectrum, we find

$$BR(\pi^0 \to e^+e^-, x > 0.95) = (6.56 \pm 0.26(stat) \pm 0.23(syst)) \times 10^{-8}$$

(13)

The systematic error now includes the 2.7% uncertainty in the $\pi^0 \to e^+e^-\gamma$ branching ratio and the 0.5% uncertainty in the $\pi^0$ slope parameter.

6 The Decay $K_L \to e^+e^-\gamma$

6.1 Motivation

The decay $K_L \to e^+e^-\gamma$ is interesting due to the implications it has on the interpretation of the $K_L \to \mu^+\mu^-$ rate. The decay $K_L \to \mu^+\mu^-$ contains short-distance contributions from which one can extract $|V_{td}|$. However, the $K_L \to \mu^+\mu^-$ rate is dominated by long distance contributions containing a $K_L\gamma^*\gamma^*$ vertex, which must first be subtracted. The $K_L\gamma^*\gamma^*$ vertex can be probed by various double and single Dalitz decays such as $K_L \to e^+e^-\gamma$.

6.2 $K_L\gamma^*\gamma^*$ Form Factor

Two form factor models were considered in this analysis. The D’Ambrosio, Isidori, and Portoles (DIP) model is a general two term model consistent with $O(p^6)$ chiral perturbation theory. The $K_L \to e^+e^-\gamma$ process is sensitive to the parameter $\alpha_{\text{DIP}}$. Bergstrom, Masso, and Singer (BMS) have proposed a vector meson dominance model that depends on the parameter $\alpha_{K^*}$ as shown:

$$f_{\text{BMS}}(x) = \frac{1}{1 - x \frac{M_{\rho}^2}{M_{K^*}^2}} + \frac{C \alpha_{K^*}}{1 - x \frac{M_{\rho}^2}{M_{K^*}^2}} \left(\frac{4}{3} \frac{1}{1 - x \frac{M_{\rho}^2}{M_{\pi}^2}} + \frac{1}{9} \frac{1}{1 - x \frac{M_{\rho}^2}{M_{\pi}^2}} - \frac{2}{9} \frac{1}{1 - x \frac{M_{\rho}^2}{M_{\pi}^2}}\right)$$

(14)

In this context, the variable $x$ is the squared ratio of the $e^+e^-$ mass to the kaon mass. The $M_i$ variables correspond to the mass of the indicated meson, and the constant $C$ is given by the following expression.

$$C = (8\pi\alpha_{\text{EM}})^{1/2}G_{\text{NL}}f_{K^*\gamma^*\gamma^*}m_{\rho}^2/(f_{K^*}f_{\rho}^2 A_{\gamma^*})$$

(15)
The value of $C$ used in past measurements has not been consistent despite the fact that measurements of $\alpha_{K^*}$ are often compared directly. In some cases, $C$ is recalculated using the most recent values for the physical inputs to Eq. 15, and in others, the value of $C$ from a previous publication is used. To avoid the issue of a time varying value for $C$, we fit for the combined quantity $C\alpha_{K^*}$.

The shape of the $e^+e^-$ mass distribution is very sensitive to the form factor. To extract a measurement of the form factor parameter, a bin-by-bin shape-$\chi^2$ fit is performed between the data and sets of Monte Carlo with differing values of $\alpha$. Figure 4 shows three comparisons of data to Monte Carlo and the fit to the shape-$\chi^2$ versus $C\alpha_{K^*}$ distribution.

![Figure 4](image_url)

Figure 4: The $K_L \to e^+e^-\gamma$ data (dots) is compared to three sets of Monte Carlo, each with a different value of $C\alpha_{K^*}$. The lower right shows the fit to the shape-$\chi^2$ versus $C\alpha_{K^*}$ distribution.

6.3 Preliminary Results

The measured branching ratio for $K_L \to e^+e^-\gamma$ is

$$BR(K_L \to e^+e^-\gamma) = (9.25 \pm 0.03(stat) \pm 0.07(syst) \pm 0.26(ext.syst)) \times 10^{-6}$$  \hspace{1cm} (16)

The $(ext.syst)$ term is due to the uncertainty in the branching ratio of the normalization mode, $K_L \to \pi^0\pi^0\pi_D^0$. The form factor fits yield the following values:

$$C\alpha_{K^*} = -0.517 \pm 0.030(\text{fit}) \pm 0.022(\text{syst})$$  \hspace{1cm} (17)
$$\alpha_{DIP} = -1.729 \pm 0.043(\text{fit}) \pm 0.028(\text{syst})$$  \hspace{1cm} (18)

References

1. M. Arenton et al, [hep-ex/0604035]
2. G. Ecker et al, Nucl. Phys. B 413, 321 (1994).
3. P. Heiliger and L. M. Sehgal, Phys. Lett. B 307, 182 (1993).
4. G. D. Barr et al, PLB 328, 528 (1994).
5. L. Bergström et al, PLB 126, 117 (1983)
6. Ll. Ametller et al, Phys. Rev. D 48, 3388 (1993).
7. M. Knecht et al, Phys. Rev. Lett. 83, 5230 (1999).
8. D. Gomez Dumm and A. Pich, Phys. Rev. Lett. 80, 4633 (1998).