MASS AND GAUGE INVARIANCE IV
(HOLOGRAPHY FOR THE KARCH-RANDALL MODEL)

M. Porrati

Department of Physics, NYU, 4 Washington Pl, New York NY 10003

Abstract

We argue that the Karch-Randall compactification is holographically dual to a 4-d conformal field theory coupled to gravity on Anti de Sitter space. Using this interpretation we recover the mass spectrum of the model. In particular, we find no massless spin-2 states. By giving a purely 4-d interpretation to the compactification we make clear that it represents the first example of a local 4-d field theory in which general covariance does not imply the existence of a massless graviton. We also discuss some variations of the Karch-Randall model discussed in the literature, and we examine whether its properties are generic to all conformal field theory.
1 Introduction

Nowadays, it is commonly known that gauge invariance does not guarantee the existence of massless spin-one particles. The W and Z particles of the Standard Model are the best known example of this fact, but it was noticed first by Julian Schwinger in his seminal article [1] echoed by our title.

General covariance is a different story. Until recently no example was known of a theory invariant under general coordinate transformations in four dimensions that did not contain a massless spin-2 particle. String theory and Kaluza-Klein compactifications contain many massive states of spin-2 (and higher), but they always have a unique massless graviton.

The first example of a consistent generally covariant theory without any massless spin-2 state has been given only recently by Karch and Randall [2], who studied a compactification similar to the Randall-Sundrum II model (RSII). In RSII [3], pure 5-d Einstein gravity with a negative cosmological constant is “compactified” on an Anti de Sitter (AdS) space cut off near its boundary by a flat 4-d brane, i.e. a brane with flat induced metric. In spite of the fact that the regularized AdS space has an unbounded transverse coordinate, RSII is a compactification, since the 5-d graviton has a normalizable zero mode, localized near the 4-d brane, that gives rise to the standard 4-d Einstein gravity up to small corrections, due to the exchange of massive spin-2 states.

The Karch-Randall model (KR) is a variation of RSII, in that the induced metric on the 4-d brane is itself AdS. The 4-d cosmological constant of the induced metric is an additional free parameter, besides the 5-d cosmological constant previously mentioned. The 4-d cosmological constant can be set to zero, and the KR model is continuous in that limit. In our opinion, the greatest surprise of the KR compactification is that it has no massless spin-2 states; instead, its spectrum consists of a tower of massive spin-2 states, of mass $O(\lambda)$, and a lighter state of mass $O(\lambda^2/M^2_{Pl})$. Here $\lambda$ is the 4-d “cosmologist’s” cosmological constant, i.e. the vacuum energy $V$ divided by the 4-d Planck constant: $\lambda = V/M^2_{Pl}$. Yet, this model is generally covariant in 4-d, as shown in [2] and more extensively in [4]. Thus, the KR model is the first example in which general covariance does not imply the existence of a massless spin-2 particle.

Before proceeding further, let us solve an apparent puzzle. In the limit $\lambda \to 0$, the KR model goes smoothly into the RSII model, which does possess a massless graviton. The smoothness of the limit is not in contradiction with the well-known van Dam-Veltman-Zakharov (vDVZ) discontinuity [3], or other discontinuities, for two reasons. First of all, the vDVZ discontinuity does not exist in AdS space. More accurately, the tree-level, one-particle exchange amplitude of a massive spin-2 field in 4-d AdS space is continuous in the limit $m^2/\lambda \to 0$ [6] (see also [4]). A discontinuity in quantum loops involving the massive graviton was claimed to exist in ref. [8]. Ref. [8] works in a theory with a single
massive spin-2, so its results may not apply to KR, which has a much richer spectrum. Anyway, that discontinuity is not relevant here, as throughout our paper we work in the weak-gravity regime, where the semiclassical approximation for 4-d gravity is valid. In that approximation all loops involving the 4-d graviton, hence their discontinuities, are negligible. Finally, KR may be altogether free from quantum discontinuities as it may be realizable as a string theory background, continuous in the limit \( \lambda \to 0 \), as argued in [9].

Randall-Sundrum compactifications admit a dual interpretation as four-dimensional local field theories coupled to gravity [10, 11, 12]. In the dual theory, the field theory is strongly interacting, but gravity is weakly interacting at all scales up to the cutoff, which must be, therefore, smaller than the 4-d Planck scale. This duality is a consequence of the holographic duality [13, 14, 15], as most clearly pointed out in [16]. The arguments used in [16] to justify the duality between RS compactifications and 4-d field theories coupled to standard gravity, hereafter called “gauged holography,” [11] do not require that the metric on the 4-d brane is flat. Indeed, the brane metric used in [16] is that of a Euclidean 4-sphere. If gauged holography works for curved branes as well as Minkowsky branes, we are naturally led to conjecture that even the KR compactification must admit a holographic dual. Clearly stated, we propose that

The Karch-Randall compactification is dual to a four-dimensional conformal field theory on a four-dimensional AdS space, coupled to (regularized) Einstein gravity

As demanded by the holographic duality, the 4-d conformal field theory (CFT) is strongly interacting when the five-dimensional description is semiclassical. Four-dimensional gravity is weak. The rest of this paper is devoted to prove the claim made here.

In section 2, after briefly reviewing the KR compactification, we adapt and expand the treatment of gauged holography given in [11, 16] to cover the case of arbitrary 4-d induced metrics on the brane. We exhibit a very simple argument showing that gauged holography, both for Minkowsky 4-d metrics and for curved 4-d metrics, is not an independent conjecture, but that it follows instead from the “rigid” holographic duality. We also specialize the discussion of gauged holography to the case that the metric on the brane is a small fluctuation around a conformally flat background. In that case, we will be able to show that if a RS compactification has a holographic dual, then the KR compactification related to it must have a holographic dual too, and that the dual CFT is the same in both cases.

Section 3 treats the “universal” part of the effective action of a CFT on a curved background, namely the term that comes from integrating the Weyl anomaly: the Riegert action [17]. Unlike two-dimensional CFTs, CFTs in four dimensions cannot be solved completely by integrating the Weyl anomaly, since 4-d metrics admit infinitely many
different conformal classes. This implies that the effective action of a 4-d CFT contains a model-dependent, Weyl-invariant piece, besides the Riegert action. In section 3, we will show that the Riegert action does not give a mass to the graviton \(^1\). The graviton mass found in the KR compactification is thus a highly non-trivial effect due to the model-dependent term in the effective action. This term, it is worth repeating, cannot be determined by Weyl invariance alone.

Section 4 uses the holographic duality to compute the effective action of gravity coupled to a CFT on an AdS background, expanded to quadratic order in the metric fluctuations. This is the term that gives the two-point correlator of the stress-energy tensor. In section 4 we show that the same term also gives a mass \(O(\lambda^2)\) to the 4-d graviton. Finally, section 4 shows that the two-point correlator so computed has the correct flat-space limit, proportional to \(p^4 \log p^2/\mu^2\) \[^4\].

Section 5 contains our conclusions and a brief discussion of whether other behaviors for the two-point function of the graviton are possible for generic CFTs on AdS spaces. We also discuss the case, studied in \[^{18}\], of 5-d AdS space bound by two positive-tension AdS branes. In particular, we show how, in that case, the holographic duality implies immediately that a massless graviton must exist, as shown in \[^{18}\].

A discussion of Weyl transformations, diffeomorphisms, and the counting of degrees of freedom for the KR model is given in appendix A. Appendix B presents an explicit, simple change of coordinates for AdS\(_d\) that maps its Poincaré coordinates into new coordinates in which AdS\(_d\) is sliced by AdS\(_{d-1}\) surfaces.

## 2 KR and Gauged Holography in Curved Space

### 2.1 KR (Abridged)

To describe the Karch-Randall compactification \[^2\], let us starts with the Einstein-Hilbert action in five dimensions, with a negative 5-d cosmological constant \(\Lambda\),

\[
S_{EH} = \frac{1}{16\pi G} \int d^5x \sqrt{-g}(R - 2\Lambda).
\]

A solution of the Einstein equations derived from this action is the Anti de Sitter space. Among its many equivalent metrics, we choose one in terms of a space-like radial coordinate \(z\), ranging from \(-\pi L/2\) to \(+\pi L/2\) and four other coordinates \(x^\mu, \mu = 0, 1, 2, 3\)

\[
ds^2 = \frac{1}{\cos^2(z/L)}(dz^2 + ds^2_4), \quad \Lambda \equiv -6/L^2.
\]

\[^1\]The Riegert action is ill-defined in the infrared. In this paper, that name always denotes an appropriately regularized version of that action.

\[^2\]Appendix B exhibits a reparametrization that transforms the Poincaré coordinates into those of eq. \((\text{2})\).
The 4-d section, with line element $ds^2_4$, is an AdS$_4$ space with radius $L$.

The AdS space can be truncated by restricting $z$ to the range $-\pi L/2 \leq z \leq +\pi L/2 - \epsilon$, $\epsilon \ll L$, together with appropriate (e.g. Neumann) boundary conditions at $z = +\pi L/2 - \epsilon$. This is the Karch-Randall compactification\textsuperscript{3}. Notice that the induced metric on the boundary is $ds^2 = \sin^{-2}(\epsilon/L)ds^2_4$. Because of the scaling factor in front of $ds^2_4$, the induced metric has a negative 4-d cosmological constant $\lambda = -3\sin^2(\epsilon/L)/L^2 \approx -3\epsilon^2/L^4$.

The 4-d cosmological constant vanishes with $\epsilon$, and as expected the RS compactification is indeed recovered in the limit $\epsilon \to 0$. In particular, with the change of variables $\hat{z} = (L/\epsilon)(\pi L/2 - z)$, $\hat{x}^\mu = (L/\epsilon)x^\mu$, the 5-d metric assumes in the limit the standard RS form (see for instance [11])

$$ds^2 = \frac{L^2}{\hat{z}^2}(d\hat{z}^2 + \eta_{\mu\nu}d\hat{x}^\mu d\hat{x}^\nu), \quad \hat{z} \geq L. \quad (3)$$

### 2.2 Rigid and Gauged Holography I

The holographic duality states that the generating functional of a 4-d conformal field theory is the partition function of quantum gravity on a 5-d manifold $X$. In particular, given a 5-d field $\phi(x,z)$, with 5-d 1PI action $\Gamma[\phi]$, its boundary value $\phi(x,0)$ is the source for a gauge-invariant operator $O$ in the CFT, and the partition function is given by

$$\langle \exp[-\int_M d^4x\phi(x)O(x)] \rangle_{CFT} = \exp(-\Gamma[\phi]). \quad (4)$$

The manifold $X$ is a solution of the 5-d Einstein equations with cosmological constant $\Lambda$. $M$, the boundary of $X$, is the space on which the CFT lives, and near the boundary $z = 0$ the $X$ metric is

$$ds^2 = \frac{L^2}{z^2}[dz^2 + g_{\mu\nu}(x,z)dx^\mu dx^\nu], \quad (5)$$

$$g_{\mu\nu}(x,z) = g^0_{\mu\nu}(x) + z^2 g^1_{\mu\nu}(x) + z^4 \log z^2 g^2_{\mu\nu}(x) + O(z^4). \quad (6)$$

The metric on $M$ is $g^0_{\mu\nu}(x)$ and both $g^1_{\mu\nu}(x)$ and $g^2_{\mu\nu}(x)$ are local function of $g^0_{\mu\nu}(x)$ and its derivatives. Eqs. (5,6) define the metric up to diffeomorphism that act on $g^0_{\mu\nu}(x)$ as conformal transformations. This property will be very useful later on.

To compute $\Gamma[\phi]$, we use the semiclassical approximation, where $\Gamma[\phi]$ is the classical action on shell. Even in this approximation $\Gamma[\phi]$ must be regularized.

Let us consider in particular the case where $\phi$ is the 5-d metric, and $\Gamma[\phi]$ is the 5-d action of pure gravity with negative cosmological constant. The action is $\Gamma = S_{EH} + S_{GH}$. The Einstein-Hilber action $S_{EH}$ has been given in eq. (1), while the Gibbons-Hawking

\textsuperscript{3}A physical but by no means unique way to truncate the space is to place a 4-d brane of appropriate tension at the boundary.
boundary term, necessary to have an action that depends only on the first derivative of the metric \[19\] is

\[ S_{GH} = \frac{1}{8\pi G} \int d^4x \sqrt{-h}K. \] (7)

\( h \) is the determinant of the induced metric on the boundary and \( K \) is the trace of the extrinsic curvature of the boundary. When computed on-shell, the action \( S_{EH} + S_{GH} \) diverges. For instance, the Einstein-Hilbert piece is

\[ S_{EH} = \frac{1}{16\pi G} \int_0^\infty dz \int d^4x \frac{L^5}{z^3} \left[ \frac{4\Lambda}{3} \sqrt{-g^0(x)} + O(z^2) \right] = \infty. \] (8)

To regularize it the integral in \( z \) is cut off at some positive value \( \epsilon \). The regularized action \( \Gamma_\epsilon[g^0_{\mu\nu}(x)] \) has the following expansion in powers of \( \epsilon \) \[20, 21\]

\[ \Gamma_\epsilon = \epsilon^{-4}a_0 + \epsilon^{-2}a_2 + \log(\epsilon^2)a_4 + \Gamma_\epsilon^F. \] (9)

In this equation, \( a_0 \) is proportional to the 4-d cosmological constant term, \( \int_M d^4x \sqrt{-g^0} \), \( a_2 \) is proportional to the 4-d Einstein-Hilbert action, \( \int_M d^4x \sqrt{-g^0}R(g^0) \), and \( a_4 \) is a linear combination of the Euler curvature and the square of the Weyl tensor

\[ a_4 = \frac{L^3}{128\pi G} \int d^4x \sqrt{-g^0} \left( \frac{1}{3} R^2(g^0) - R_{\mu\nu}(g^0)R^{\mu\nu}(g^0) \right). \] (10)

The holographic duality can be resumed in one equation now, namely:

\[ \lim_{\epsilon\to 0} \Gamma_\epsilon^F = W_{CFT}[g]. \] (11)

Here \( W_{CFT}[g] \) is the generating functional of the (connected) correlators of the stress-energy tensor. Eq. (11) makes explicit a point that is often hidden in the literature on the holographic duality. Namely, that the regularized generating functional of the CFT is still given by a 5-d holographic dual even when the cutoff \( \epsilon \) is small but nonzero. Notice that when \( g_{\mu\nu} \) is expanded around flat space, \( g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \), one obtains the “usual” correlators of \( T_{\mu\nu} \) in Minkowsky space. On the other hand, we could have expanded eq. (11) around any background. When expanded around a curved background, \( W_{CFT} \) gives the stress-energy tensor correlators of the CFT on that curved background. This fact has been used time and again in the literature on the holographic duality. After all, ref. \[15\] already studies holography not only on \( R^4 \) but also on \( S_4 \) and \( S_3 \times S_1 \). The local, divergent terms \( a_0, a_2, a_4 \) are harmless in the standard, “rigid” holographic duality, since they give rise only to contact terms.

In “rigid” holography, the boundary value of the 5-d metric, \( g^0_{\mu\nu} \), is simply an external, fixed quantity, that acts as the source for the stress-energy tensor. On the other hand, holography was at first derived in string theory, where the graviton is dynamical. This
is not in contradiction with the previous assumption that $g_{\mu\nu}^0$ is an external source, as long as the 4-d Newton constant vanishes. This is the case in the limit $\epsilon \to 0$, since eq. (9) implies $G_4 \propto G \epsilon^2$. In other words, the induced 4-d Newton constant vanishes when the cutoff is removed. This result is consistent with the holographic interpretation. After all, in a 4-d CFT with ultraviolet cutoff $\epsilon$ the induced Newton constant is indeed proportional to $\epsilon^2$.

If $\epsilon$ is kept finite, then it is natural to promote $g_{\mu\nu}^0$ to a dynamical field. For instance, in string theory, when deriving the holographic duality using, say, D3 branes, $g_{\mu\nu}^0$ is just the value of a dynamical field at some arbitrary boundary in between an AdS “throat” and an asymptotically flat region in the string background. In that case, the background is found by solving the equations of motion of $g_{\mu\nu}^0$. Moreover, it is natural, and allowed by the symmetries of our system, to add to the action $\Gamma_\epsilon$ local 4-d terms, proportional to $a_0$, $a_2$, and $a_4$. The coefficient of the $a_0$ term is the best known in the literature as it is the tension of the Randall-Sundrum “Planck” brane [3, 11]. It acts as a counterterm that cancels part of the induced cosmological constant in eq. (9). The term proportional to $a_2$ was discussed in [11] and its interpretation is reviewed in the next paragraph. The third term is largely irrelevant when studying 4-d gravity at low energy. It can be set equal to $-\log(\epsilon^2)a_4$ to simplify eq. (9), without loss of generality.

Let us examine again the conclusion of the previous paragraph. We noticed that if $\epsilon$ is nonzero, and if we assume the standard “rigid” holographic duality, the on-shell 5-d Einstein action with cosmological constant $\Lambda = -6/L^2$, computed on the manifold $X$ with boundary condition $g_{\mu\nu}^4 \equiv (L/\epsilon)^2 g_{\mu\nu}^0$ on $\partial X = M$ is

$$\Gamma_H[g_{\mu\nu}^4] = \frac{1}{16\pi G_4} \int_M d^4x \sqrt{-g}^4(R - 2\lambda) + W_{\text{CFT}}[g_{\mu\nu}^4] + ...,$$

(12)

$$\frac{1}{16\pi G_4} = \frac{L}{32\pi G} + \frac{1}{16\pi G_4^{\text{bare}}}, \quad \frac{1}{16\pi G_4} \lambda = -\frac{3\Lambda L}{16\pi G} - T.$$  

(13)

$T$ is the coefficient of the $a_0$ term; we have already interpreted it as the tension of a brane placed at the boundary [4]. $\frac{1}{16\pi G_4^{\text{bare}}}$ is the coefficient of the $a_2$ term. As implied by our label, it does appear as a bare Newton constant. The other coefficients given in eq. (13) are those of eq. (9); they have been computed, for instance, in [21, 24].

Notice that eq. (12) is exactly the effective action of gravity coupled to a CFT with ultraviolet cutoff $L$, up to terms vanishing with the cutoff. Eq. (12) proves by itself that “rigid” holography implies, without any further assumption, “gauged” holography, namely holography in the presence of dynamical gravity.

A few comments are now appropriate.

4The “brane” can be sometimes just an effective description of a more complicated mechanism see [22] for an early example of this possibility, and [23] for a recent discussion.
1. We have rescaled the boundary metric in eq. (12) so that at $z = \epsilon$, $ds^2 = g^4_{\mu\nu}dx^\mu dx^\nu$. In this fashion, the 4-d length is given by the metric $g^4_{\mu\nu}$ without the need of any further rescaling. The definition of $g^4_{\mu\nu}$ also makes evident that we can always set the cutoff at $L$, instead of $\epsilon$, by appropriately rescaling the Newton and cosmological constants of the 4-d theory.

2. Eq. (12) is evidently the effective action obtained after integrating out the CFT, but before taking into account graviton loops. That is the appropriate effective action whenever the CFT is strongly interacting but all true quantum gravity effects are negligible, i.e. when the CFT is cut off at a scale well below both the 5-d and 4-d Planck scales. The condition that the cut-off is below the 5-d Planck scale $M \equiv (16\pi G)^{-1/3}$ translates into $ML \gg 1$. This is the regime where holographic duality is computationally effective. In this regime the AdS$_5$ curvature is small compared with the 5-d Planck scale and one is thus justified in neglecting all higher-curvature terms in the 5-d gravitational action, and in equating the latter to the Einstein-Hilbert action with cosmological constant. In physical examples, $G^4_{\text{bare}} > 0$, thus $ML \gg 1$ is also sufficient to ensure that the momentum cutoff, $1/L$, is well below the 4-d Planck scale $M_4 \equiv (16\pi G_4)^{-1/2}$.

3. In eq. (12) we omitted terms that vanish with the curvature faster than $\int_M d^4x \sqrt{-\hat{g}^4(R - 2\lambda)}$ or $W_{\text{CFT}}$.

4. If we denote by $K_{\mu\nu,\rho\sigma}$ the bare kinetic term of the 4-d graviton, and if we expand $\Gamma_H[g^4_{\mu\nu}]$ to quadratic order around a stationary point, obeying $\delta \Gamma_H/\delta g^4_{\mu\nu} = 0$, we obtain the self-energy $\Sigma_{\mu\nu,\rho\sigma}$ as

$$\frac{1}{2} \frac{\delta^2 \Gamma_H}{\delta g^4_{\mu\nu} \delta g^4_{\rho\sigma}} = K_{\mu\nu,\rho\sigma} + \Sigma_{\mu\nu,\rho\sigma}. \quad (14)$$

2.3 Rigid and Gauged Holography II

When the metric $g^4_{\mu\nu}$ can be expanded as

$$g^4_{\mu\nu} = \exp(2\sigma)(\eta_{\mu\nu} + h_{\mu\nu}), \quad (15)$$
i.e. when the 4-d background metric is conformally flat, we can show that flat-space holography implies curved-space holography in yet another way, particularly tailored to our background. This new proof is given here not only to convince the skeptic, but also to introduce some formulas that will be useful in the rest of this paper.

We assume that holography holds perturbatively around a flat background, i.e. that

$$\Gamma^F_\epsilon[\eta_{\mu\nu} + h_{\mu\nu}] = W_{\text{CFT}}[\eta_{\mu\nu} + h_{\mu\nu}] + o(\epsilon), \quad (16)$$
whenever both sides of this equation are expanded in powers of $h_{\mu\nu}$.

$W_{\text{CFT}}$ transforms as follows under the Weyl rescaling $g_{\mu\nu} = \exp(2\omega)\bar{g}_{\mu\nu}$:

$$
\frac{\delta W_{\text{CFT}}}{\delta \omega} = c\{ A[\bar{g}] - 4\Box_4 \omega \}, \quad A[g] = 2R_{\mu\nu}R^{\mu\nu} - \frac{2}{3}R^2 - \frac{2}{3}\Box R, \quad c = \text{constant}. \quad (17)
$$

The computation of Henningson and Skenderis [20] gives $c = L^3/128\pi G$ (see eqs. (9,10)); $\Box_4$ is the operator \cite{17}

$$
\Box_4 = \Box^2 + 2R^{\mu\nu}D_\mu D_\nu - \frac{2}{3}R \Box + \frac{1}{3}(\partial_\mu R)D^\mu. \quad (18)
$$

Hereafter, an overbar will denote quantities computed with respect to the metric $\bar{g}_{\mu\nu}$. $D_\mu$ is the covariant derivative and $\Box \equiv D_\mu D^\mu$. $\Box_4$ maps scalars of conformal weight zero into scalars of conformal weight four.

As shown explicitly in ref. [20], $\delta W_{\text{CFT}}/\delta \omega = \delta \Gamma^F / \delta \omega$. This equation, together with eq. (16) is sufficient to ensure that

$$
\Gamma^F [\exp(2\sigma)(\eta_{\mu\nu} + h_{\mu\nu})] = W_{\text{CFT}}[\exp(2\sigma)(\eta_{\mu\nu} + h_{\mu\nu})] + o(\epsilon). \quad (19)
$$

This equation states that if holography is valid perturbatively around flat space, then it is also valid perturbatively around any conformally flat background.

Notice that the conformal transformation may map flat space into components joined only at their boundary. This phenomenon is manifest, for instance, when the flat metric $ds^2 = -dt^2 + \sum_{i=1}^3 dx^i dx^i$ is scaled by the Weyl transformation $\exp(\omega) = 1/|x^3|$. The Weyl scaling given here is induced by a 5-d diffeomorphism, as shown in appendix B. As we will see in section 4, in our background, the 4-d boundary of AdS$_5$ has two components, joined only at their edge. This does not contradict the theorem of Witten and Yau [25], as the 4-d boundary has negative curvature. The very fact that the 4-d space we need, made of two AdS$_4$ components, can be connected to flat (Minkowsky) space by a Weyl transformation suggests that holography must hold also for our background.

The transformation property of the generating function, given in eq. (17) gives us additional information on $W_{\text{CFT}}$ when we expand $W_{\text{CFT}}$ to quadratic order in $h_{\mu\nu}$. By using known properties of $W_{\text{CFT}}$ (see for instance [20]) it is easy to see that

$$
W_{\text{CFT}}[\eta_{\mu\nu} + h_{\mu\nu}] = -\frac{1}{2}c \bar{C}^\rho_{\mu\nu} \log(\Box/\mu^2)\bar{C}^\mu_{\rho\sigma} + O(h^3). \quad (20)
$$

Here and below we omit the sign of integration in $d^4x$ in our formulae whenever unambiguous. $C^\rho_{\mu\nu}$ is the Weyl tensor, and $\mu$ is a mass scale introduced for dimensional reasons. Its arbitrariness reflects the fact that $W_{\text{CFT}}$ is defined up to local Weyl-invariant terms.
We want to find the analog of eq. (20), but now when the metric is expanded to quadratic order in the fluctuations around an AdS background. To do this we must promote \( \log(\bar{\Box}/\mu^2) \) to a linear operator \( F_{\rho\sigma\gamma\delta}^{\mu\nu\alpha\beta}(x, y) \), acting on tensors of same conformal weight and symmetries as \( C_{\rho\sigma}^{\mu\nu} \). \( F \) must satisfy two properties

\[
(\delta F/\delta \omega)_{\mu\nu\alpha\beta}^{\rho\sigma\gamma\delta}(x, y) = \delta^4(x, y) \delta_{\mu\nu}^{\gamma\delta} \delta_{\alpha\beta}^{\rho\sigma},
\]

\[
\lim_{g_{\mu\nu} \to \eta_{\mu\nu}} F_{\mu\nu}^{\rho\sigma\gamma\delta} = -\frac{1}{2} \delta_{\mu\nu}^{\gamma\delta} \delta_{\rho\sigma}^{\alpha\beta} \log(\bar{\Box}/\mu^2).
\]

The first equation generalizes the transformation property of \( \log(\bar{\Box}/\mu^2) \) under constant conformal transformations to arbitrary Weyl rescaling. The second equation is obviously necessary to reproduce the expansion around flat space given in eq. (20). That an \( F \) obeying eqs. (21,22) exists has been argued in [27], using the results in [28]. The form of \( F \) is not uniquely fixed by the Weyl anomaly, since one can always add to \( F \) Weyl-invariant terms. We will compute \( F \) using the holographic correspondence in section 4. Another possible \( F \) is \(-1/4) \log \Delta/\mu^2\), where \( \Delta \) is an operator that respects the symmetry properties of \( C_{\rho\sigma}^{\mu\nu} \), and maps the Weyl tensor into a tensor of conformal weight 6 [28].

Now we are ready to compute \( W_{CFT}[\exp(2\sigma)(\eta_{\mu\nu} + h_{\mu\nu})] \), expanded to quadratic order in \( h \). As before, we set \( g_{\mu\nu} = \exp(2\sigma)(\eta_{\mu\nu} + h_{\mu\nu}) \), \( \bar{g} = \eta_{\mu\nu} + h_{\mu\nu} \). We use eq. (17) to find

\[
W_{CFT}[\bar{g}] = -c\{\sigma A[g] + 2\sigma \bar{\Box} 4\sigma\} + W_{CFT}[g].
\]

Using the properties of \( F \) given in eqs. (21,22), we also have

\[
-\frac{1}{2} \bar{C} \log(\bar{\Box}/\mu^2) \bar{C} = CFC - \sigma C^2 + O(h^3).
\]

Combining eqs. (20,23,24) we find an expression for \( W_{CFT}[g] \):

\[
W_{CFT}[g] = c\{CFC + \sigma (A[g] - C^2) + 2\sigma \bar{\Box} 4\sigma\} + O(h^3).
\]

The explicit \( \sigma \) dependence in this expression can be canceled by adding to it the trivially Weyl-invariant term \(-c/8)(\bar{A} - \bar{C}^2) \bar{\Box} 4^{-1}(A - C^2)\). We finally find

\[
W_{CFT}[g] = c \left\{ CFC - \frac{1}{8} (A - C^2) \bar{\Box} 4^{-1}(A - C^2) \right\} + O(h^3).
\]

This equation gives the generating functional of the conformal field theory, expanded to quadratic order around a conformally flat background. This equation also provides us with a useful decomposition of \( W_{CFT}[g] \) into the “universal” Riegert [17] term, \(-c/8)(A - C^2) \bar{\Box} 4^{-1}(A - C^2)\) obtained form integrating the conformal anomaly, and a model-dependent term, \( c CFC \). In the next section we will show that the Riegert term does not give a mass to the the graviton.
3 Analysis of the Riegert Term

The upshot of this section is that the Riegert term does not give rise to a mass term for the graviton. Readers not interested in the details of the computation can skip ahead to section 4, or to the end of this section for an alternative proof of the statement.

The Riegert term can be rendered local by introducing an auxiliary field $\zeta$, in terms of which the term reads

$$W_R[g] \equiv -\frac{c}{8}(A[g] - C^2) \Box_4^{-1}(A[g] - C^2) = -\frac{c}{8} \int d^4x \sqrt{-g}[-\zeta \Box_4 \zeta + 2\zeta (A[g] - C^2)], \quad (27)$$

where $\zeta$ obeys its own Euler-Lagrange equation: $\Box_4 \zeta = A[g] - C^2$. Notice that the combination $C^2 - A$ is $E_4 + (2/3) \Box R$ and $E_4$ is the Euler density $E_4 = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2$.

We want to show that the Riegert term does not give a mass to the graviton on $\text{AdS}_4$. To prove this statement, we will show that an Einstein space is always a solution of the equations of motion that follow from the Einstein-Hilbert action modified by the addition of the Riegert term: $S = S_{EH} + W_R$. An Einstein space satisfies $R_{\mu\nu} = g_{\mu\nu} R/4$.

When linearized around an $\text{AdS}_4$ background, this equation describes the propagation of the standard massless graviton. Recall that in the KR model, there is no massless graviton excitation propagating on $\text{AdS}_4$. This means that in the KR compactification, no Einstein space except $\text{AdS}_4$ can solve the equations of motion.

To compute the first variation of the action $S = S_{EH} + W_R$ around an Einstein space, we decompose the variation of the metric into a trace part, $\phi$, a diffeomorphism part $\epsilon_\mu$, and a transverse-traceless part $\psi_{\mu\nu}$:

$$\delta g_{\mu\nu} = g_{\mu\nu} \phi + D_{(\mu} \epsilon_{\nu)} + \psi_{\mu\nu}, \quad D^\mu \psi_{\mu\nu} = \psi^\mu_{\mu} = 0. \quad (28)$$

The variation of $S$ with respect to $\epsilon_\mu$ is the simplest to compute: by diffeomorphism invariance $\delta S/\delta \epsilon_\mu = 0$ around any background.

The variation of $S$ with respect to $\phi$ is computed using eq. (17) and the definition of $W_R$

$$\frac{\delta S}{\delta \phi} = \frac{1}{16\pi G_4} (R - 4\lambda) + \frac{c}{2} (A - C^2). \quad (29)$$

We can further simplify the calculation, now and later, by noticing that we are looking for the quadratic part of the effective action around an $\text{AdS}$ background. This means that we need to compute the variation of $S$ only to linear order in the fluctuation. Now, the Weyl tensor vanishes on $\text{AdS}$, since $\text{AdS}$ is conformally flat. Moreover, by denoting with $\bar{R}$ the curvature of the $\text{AdS}$ background, we find $A = -(1/6) \bar{R}^2 + O(h^2)$. Thus,

$$\frac{\delta S}{\delta \phi} = \frac{1}{16\pi G_4} (\bar{R} - 4\lambda) - \frac{c}{12} \bar{R}^2 + O(h^2). \quad (30)$$
To the order in $h$ we are interested in, this equation just defines the scalar curvature $\bar{R}$ in terms of the “bare” cosmological constant $\lambda$.

The variation of the Einstein-Hilbert action with respect to $\psi_{\mu\nu}$ is relatively simple. From the variation of the Christoffel connection

$$\delta \Gamma_{\rho}^{\mu\nu} = \frac{1}{2} g^{\rho\lambda}(D_{\mu} \delta g_{\nu\lambda} + D_{\nu} \delta g_{\mu\lambda} - D_{\lambda} \delta g_{\mu\nu}),$$

we have

$$\delta R = -\Box \delta g + D^{\mu} D^{\nu} \delta g_{\mu\nu} - \delta g_{\mu\nu} R_{\mu\nu}$$

Using the definition of $\psi_{\mu\nu}$ we find $\delta S_{EH}/\delta \psi_{\mu\nu} = 0$ on an Einstein background (which obeys $R_{\mu\nu} = (1/4) g_{\mu\nu} R$).

The variation of $W_R$ is less straightforward. First of all, we must notice that the variation $\delta W_{R}/\delta \zeta$ vanishes, since $\zeta$ satisfies its own Euler-Lagrange equations. Thus, in eq. (27) we must only compute the explicit variation of the metric.

Let us compute now the variation of the term $2 \int d^{4}x \sqrt{-g} \zeta (A[g] - C^{2})$ in eq. (27). Using

$$R_{\mu\nu\rho} = D_{\mu} \delta \Gamma_{\nu\lambda}^{\rho} - \mu \leftrightarrow \nu,$$

we find, after a short calculation

$$\delta \int d^{4}x \sqrt{-g} \zeta (A[g] - C^{2}) = -4 \int d^{4}x \sqrt{-g} D_{\lambda} \zeta R_{\mu\nu\lambda}^{\rho} \psi_{\rho\mu}$$

up to terms that vanish on Einstein backgrounds. For the purpose of our calculation, in this variation we need to keep only terms linear in the fluctuation around the AdS background. We use the AdS$_{4}$ metric $ds^{2} = dr^{2} + \exp(2r/l)(-dt^{2} + d\vec{x}^{2})$ and the expansion

$$\dot{\zeta} \equiv \frac{d\zeta}{dr} = -\frac{4}{l} + O(h^{2}).$$

This definition implies a particular choice for the asymptotic behavior of $\zeta$. That choice is an infrared regularization that we adopt as part of the definition of the Riegert action.

Notice that the variation $\psi_{\mu\nu}$ is traceless with respect to the metric $g_{\mu\nu} + h_{\mu\nu}$. Denoting by $\bar{\psi}_{\mu}$ etc. quantities with indices raised and lowered with the background metric $\bar{g}_{\mu\nu}$, after another short calculation, we arrive to

$$4 D_{\lambda} D_{\mu} \zeta R_{\mu\nu\lambda}^{\rho} \psi_{\rho\mu} = 2 \frac{\tilde{\zeta}}{l} \tilde{\Delta} h_{\mu\nu} \bar{\psi}_{\mu\nu} + O(h^{2}) = -\frac{8}{l^{2}} \Delta h_{\mu\nu} \bar{\psi}_{\mu\nu} + O(h^{2}).$$

---

5The $\phi$ variation of the complete effective action, $\Gamma_{H}[g]$, which includes the term $CFC$, is $\delta \Gamma_{H}/\delta \phi = (1/16\pi G_{4})(R - 4\lambda) - (c/12) R^{2}$ on any Einstein metric. Eq. (30) is thus valid beyond the quadratic approximation used in the text.
Here, $\bar{\Delta}$ is the 3-d background Laplacian.

To compute the variation of the term $\int d^4x \sqrt{-g} \zeta \Box_4 \zeta$ we first integrate by part to find

$$\int d^4x \sqrt{-g} \Box_4 \zeta = \int d^4x \sqrt{-g} \left[ \frac{\Box^2 \zeta}{2} + \frac{2}{3} R \partial_\mu \zeta D^\mu \zeta - 2 R^\mu_\nu \partial_\mu \zeta \partial_\nu \zeta \right].$$

(37)

We further simplify the problem by the choice

$$\psi_{33} = h_{33} = 0 \, (x^3 \equiv r, \, i = 0, 1, 2).$$

Likewise, we choose $\psi_{33} = \psi_{3i} = 0$. The rationale for this choice is that if a mass term is generated by the Riegert action, it would make its variation nonzero to linear order in $h$ even with respect to this restricted class of variations. This choice dramatically simplifies our calculations. Indeed, the only term in eq. (37) that does not manifestly vanish is

$$-2 \delta R^{33} \partial_3 \zeta \partial_3 \zeta = [\Box \psi^{33} - 2D^3 D_\mu \psi^{3\mu} + 2 R^{33}_\chi \psi^{\lambda \rho} - 2 R^{3(3)} \psi^{3^3}] \zeta^2.$$  

(38)

The computation of the various terms in the variation eq. (38) is tedious but standard.

On shell ($R_{\mu}^\nu = (1/4) \delta_{\mu}^\nu R$) we have

$$R_{\rho}^{(3)} \delta g^{3^3} = (1/4) R \psi_{\rho}^{\mu} = 0.$$  

(39)

To linear order in the fluctuations around the AdS background, we find

$$R_{i}^{33} \psi_{i}^{ij} = \left(-\frac{1}{2} \tilde{h}^i_i \tilde{g}_{ij} - \frac{1}{2} \tilde{h}_i^i \tilde{g}_{ij} \right) \varepsilon^{ij} + O(h^2).$$  

(40)

As earlier, an overbar indicates that indices are raised and lowered with the background metric $\bar{g}_{\mu\nu}$. Also, in the same approximation,

$$\Box \psi^{33} = \frac{2}{\tilde{h}^i_i} \varepsilon^{ij} \varepsilon^{ij},$$  

$$2D^3 D_\mu \psi^{3\mu} = -\partial_r (h_j \varepsilon^{ij}).$$  

(41)

(42)

This is the last result we need, since by combining eqs. (39,40,41,42) with eq. (36) and the definition of $W_R$ in eq. (27) we conclude

$$\delta W_R = -\frac{c}{l^2} \int d^4x \sqrt{-\bar{g}} \left( \tilde{h}^i_i + \frac{3}{l^2} \tilde{h}^i_i + \bar{\Delta} \tilde{h}^i_i \right) \psi^{ij} + O(h^2).$$  

(43)

In this equation we have integrated by part the term $2D^3 D_\mu \psi^{3\mu}$. Eq (43) is just the standard equation of motion of the massless graviton in AdS; therefore, the mass term can not be generated by $W_R$.

An alternative proof of this statement can be obtained by using the results of Mazur and Mottola [31]. In that paper, Mazur and Mottola find an expression for the variation

\[\text{Eq (43)}\] Most of the computations have been already done for the de Sitter background in [30].
of the Riegert action by extending the computation of the conformal anomaly to $4 + \epsilon$ dimensions, and taking the limit $\epsilon \to 0$. The expression, given in their eq. (5.12), contains an auxiliary tensor, called by them $C_{\mu\nu}$. Ref. [31] contains in its appendix C an explicit formula for $C_{\mu\nu}$. Using that formula, and recalling that in our background the square of the Weyl tensor vanishes up to quadratic order in the fluctuation $h_{\mu\nu}$, we arrive again at eq. (43).

4 The Holographic Computation

Since the mass term found in [2] cannot come from the Riegert term, we have to look for it in the model-dependent term of the action, whose quadratic part is proportional to $CFC$. To compute the operator $F^{\rho\sigma\gamma\delta}_{\mu\nu\alpha\beta}(x, y)$ we use the “gauged” holographic duality introduced in section 2.

The generating functional $W_{CFT}$ computed holographically automatically obeys all properties expected in field theory, in particular it obeys the Ward identities of conformal symmetry. The reason behind this is that conformal transformations in the 4-d boundary theory are just special general coordinate transformations of the 5-d theory, that obeys general covariance (see for instance [32] and appendix A).

In the rest of this section, we will study only the transverse-traceless part of the metric fluctuation. To determine the true value of the mass term, as opposed to effects due to a trivial renormalization of the cosmological constant we must treat more precisely the Gibbons-Hawking term.

4.1 The Gibbons-Hawking Term and the Effective Action $\Gamma_H$

Let us write the 5-d metric $g_{mn}$ ($m, n = 0, 1, 2, 3, 4$) in eq. (2) as

$$ds^2 = \exp(2A)[dz^2 + (\bar{g}_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu], \quad A = -\log \cos z, \quad \mu, \nu = 0, 1, 2, 3. \quad (44)$$

In this section we set $L = 1$. In this metric, the Gibbons-Hawking term, given in eq. (7), can be written as

$$S_{GH} = \frac{1}{8\pi G} \int d^4x \sqrt{-\text{det}(\bar{g} + h)} \exp(4A) \frac{1}{\sqrt{-\bar{g}}} \partial_z(\sqrt{-\bar{g}n^z}), \quad n^z = \exp(-A). \quad (45)$$

Expanding this equation to quadratic order in $h$, after a short computation we arrive at

$$S_{GH} = -\frac{1}{8\pi G} \int d^4x \sqrt{-\bar{g}} \exp(3A) \left[ \frac{1}{2} h^\nu_{\mu} h^\mu_{\nu} \bar{A} + \dot{h}^\nu_{\mu} h^\mu_{\nu} \right], \quad \cdot \equiv \partial / \partial z. \quad (46)$$

\footnote{The computation is standard and can be found, for instance in [16].}
Here and elsewhere in this section indices are raised and lowered with the 4-d metric \( \bar{g}_{\mu\nu} \). The quadratic part of the Einstein-Hilbert action, computed on-shell, reduces to a boundary term

\[
S_{EH} = \frac{1}{16\pi G} \int d^4x \sqrt{-\bar{g}} \exp(3A) \left[ \frac{3}{4} \bar{h}h + \frac{1}{2} \dot{A} h^2 \right], \quad h^2 \equiv \bar{h}_\mu^\nu h_\nu^\mu \text{ etc.} \tag{47}
\]

We put the boundary at \( \cos z = \varepsilon \).

Here, it is convenient to keep \( h_{\mu\nu} \) as our 4-d metric instead of performing the rescaling of subsection 2.2. The sum of the Einstein-Hilbert action and the Gibbons-Hawking term gives the regularized action \( \Gamma_\varepsilon \). Expanded to quadratic order in \( h_{\mu\nu} \) it reads

\[
\Gamma_\varepsilon = -\frac{1}{16\pi G} \int d^4x \sqrt{-\bar{g}} \exp(3A) \left[ \frac{1}{4} \bar{h}h + \frac{3}{2} \dot{A} h^2 \right] \bigg|_{\cos z = \varepsilon}. \tag{48}
\]

Before jumping to the conclusion that \( (3/2)\dot{A} h^2 \) is a mass term we must recall that 4-d counterterms can be added to \( \Gamma_\varepsilon \). As we mentioned earlier, these counterterms cancel the \( \Gamma_\varepsilon \) divergences in the limit \( \varepsilon \to 0 \). Here, \( \varepsilon \) is kept finite, but the counterterms can still be added. The mass term is an artifact if it can be canceled by appropriately choosing them.

To properly identifying the meaning of the term \( (3/2)\dot{A} h^2 \), we notice that

\[
\dot{A} = \tan z = \frac{1}{\cos z} - \frac{1}{2} \cos z - \frac{1}{8} (\cos z)^3 + O[(\cos z)^5]. \tag{49}
\]

Using this expansion we can write eq. (48) as

\[
\Gamma_\varepsilon = \frac{1}{16\pi G} \int d^4x \sqrt{-\bar{g}} \left[ -\frac{1}{4\varepsilon^2} \bar{h}h + \left( -\frac{3}{2\varepsilon^4} + \frac{3}{4\varepsilon^2} + \frac{3}{16} \right) h^2 \right] + O(\varepsilon^2). \tag{50}
\]

It is now easy to identify the various terms in this equation. Terms that diverge in the \( \varepsilon \to 0 \) limit are local; they do not give a mass term, instead, they renormalize the cosmological constant and Einstein term in the 4-d action. In particular, the term \(- (3/2)\varepsilon^{-4}h^2 \) renormalizes the cosmological constant as \( \delta[(16\pi G_4)^{-1}\lambda] = -3(16\pi G)^{-1}\varepsilon^{-4} \).

The term \( (3/4)\varepsilon^{-2}h^2 \) renormalizes the Newton constant as \( \delta[(16\pi G_4)^{-1}] = (32\pi G)^{-1}\varepsilon^{-2} \).

The finite term \( (3/16)h^2 \) is not necessarily a true mass term, since it may arise from a finite renormalization of the 4-d cosmological constant. To find if this is the case, let us compute the variation of the holographic effective action \( \Gamma_H \) under a scale transformation. Using the definition in eq. (12) and eq. (17) we find

\[
1\sqrt{-\bar{g}} g_{\mu\nu} \frac{\delta \Gamma_H}{\delta g_{\mu\nu}} = \frac{1}{16\pi G_4} (-4\lambda + \bar{R}) - \frac{c}{12} \bar{R}^2, \quad c = \frac{1}{128\pi G}. \tag{51}
\]

The background metric \( \bar{g}_{\mu\nu} \) solves by construction the equations of motion \( \delta \Gamma_H / \delta g_{\mu\nu} = 0 \). Our metric \( \bar{g}_{\mu\nu} \) has scalar curvature \( \bar{R} = -12 \). Recall from the discussion at the end of
subsection 2.2 that to guarantee the validity of the holographic computation, both the 4-d and 5-d curvatures must be smaller than their Newton constants. In our units this means \(|cR^2/16| \ll |\bar{R}/16\pi G_4| \ll 1\). It means also that we can expand the cosmological constant \(\lambda\) as \(\lambda = \bar{\lambda} + \delta \lambda\); \(\bar{\lambda} = -3\), \(\delta \lambda \ll 1\). Substituting in eq. (51) we find

\[
\frac{1}{16\pi G_4} \delta \lambda = -\frac{3}{16\pi G 8}.
\]

This finite renormalization of \(\lambda\) generates an apparent mass term equal to

\[
\frac{1}{32\pi G_4} \delta \lambda \tilde{h}^2 = -\frac{1}{16\pi G 16} h^2.
\]

This is exactly the term necessary to cancel the \(\varepsilon\)-independent term in eq. (50).

We have concluded at last that \(\Gamma_H\) expanded to quadratic order in the transverse-traceless fluctuation \(h_{\mu\nu}\) is:

\[
\Gamma_H = \frac{1}{16\pi G_4} \int d^4x \sqrt{-g} \frac{1}{4} [h(-\Delta_L^{(2)} + 2\bar{\lambda})h] - \frac{1}{16\pi G} \int d^4x \sqrt{-g} \frac{1}{4\varepsilon^3} \dot{h}h
\]

Here, \(\Delta_L^{(2)}\) is the Lichnerowicz operator on symmetric tensors [33]. On our AdS background, and on transverse-traceless tensors it reads

\[
\Delta_L^{(2)} h_{\mu\nu} = -\Box h_{\mu\nu} + \frac{8\bar{\lambda}}{3} h_{\mu\nu}.
\]

We can perform an easy check on our result: substituting into it the non-normalizable graviton zero mode, eq. (54) must give the action of a massless spin 2. This is obviously correct as the non-normalizable zero mode is independent of \(z\), and thus it obeys \(\dot{h}_{\mu\nu} = 0\).

### 4.2 The Dressed Graviton Propagator

We are at last ready to compute the self-energy \(\Sigma_{\mu\nu,\rho\sigma}\) introduced in eq. (14). We use the holographic duality so that the self-energy we are looking for is given by

\[
h_{\mu\nu} \Sigma_{\mu\nu,\rho\sigma} h^{\rho\sigma} = -\frac{1}{16\pi G} \int d^4x \sqrt{-g} \frac{1}{4\varepsilon^3} \dot{h}h.
\]

The equation obeyed by \(h_{\mu\nu}\) is [2, 34]

\[
\left[-\partial_z (\cos z)^{-3} \partial_z - (\cos z)^{-3} \left(\Box - \frac{2\bar{\lambda}}{3}\right)\right] h_{\mu\nu} = 0.
\]

Let us decompose \(h_{\mu\nu}\) into eigenstates of \(\Box\), denoted by \(h_{\mu\nu}^m(x)\):

\[
h_{\mu\nu}(x, z) = \sum_m h_{\mu\nu}^m(x) H^m(z), \quad [\Box - (2\bar{\lambda}/3)] h_{\mu\nu}^m(x) = m^2 h_{\mu\nu}^m(x).
\]
The differential equation for \( H^m(z) \) can be transformed into a standard hypergeometric form by the change of variable \( y = (\cos z)^2 \). In terms of the new variable, the equation reads

\[
(1 - y) y \frac{d^2}{dy^2} + \left( -1 + \frac{1}{2} y \right) \frac{d}{dy} + \frac{m^2}{4} \right] H^m(z) = 0. \tag{59}
\]

Its two independent solutions are

\[
\begin{align*}
H^m_1 &= \left[ \psi(1) + \psi(3) - \psi(a + 2) - \psi(b + 2) \right] y^2 F(a + 2, b + 2; 3; y), \\
H^m_2 &= y^2 F(a + 2, b + 2; 3; y) - \frac{2}{ab(a + 1)(b + 1)} + \frac{2}{(a + 1)(b + 1)} y + \\
&\quad + y^2 \sum_{n=1}^{\infty} \frac{(-1)^n}{\Gamma(a + 2 + n) \Gamma(b + 2 + n) \Gamma(3 + n) \Gamma(n + 1)} \left[ \psi(a + 2 + n) - \psi(a + 2) + \psi(b + 2 + n) - \psi(b + 2) - \psi(3 + n) + \psi(3) - \psi(n + 1) + \psi(1) \right], \tag{61}
\end{align*}
\]

\[
ab = -\frac{m^2}{4}, \quad a + b + 1 = -\frac{1}{2}. \tag{62}
\]

See [35] for notations. Since our change of variables \( y = (\cos z)^2 \) is 2-to-1, the interval \( 0 \geq y \geq 1 \) actually covers two distinct domains, \([-\pi/2, 0]\) and \([0, \pi/2]\). The solution we are looking for has two different expansions in the two domains; namely

\[
\begin{align*}
H^m &= \alpha H^m_1 + \beta H^m_2 \quad \text{in } [-\pi/2, 0], \tag{63} \\
H^m &= \alpha' H^m_1 + \beta' H^m_2 \quad \text{in } [0, \pi/2]. \tag{64}
\end{align*}
\]

For the holographic computation we impose the following boundary conditions.

1. At “our end” of the AdS\(_5\) space, \( z = \pi/2 \) \((y = 0)\) we set \( H^m(\pi/2) = 1 \). By this choice, the boundary value of the fluctuation \( h_{\mu\nu}(x, z) \) becomes a sum over 4-d free fields of AdS mass \( m \)

\[
h_{\mu\nu}(x, \pi/2) = \sum_m h^m_{\mu\nu}(x). \tag{65}
\]

2. At the “other end” of AdS\(_5\), \( z = -\pi/2 \) (again \( y = 0 \)) we set \( H^m(-\pi/2) = 0 \). With this boundary condition no field can leak out of the AdS\(_5\) space. This boundary condition also ensures that the \( H^m(z) \) are normalizable, and it removes from the spectrum the zero mode \( H^0(z) = 1 \). This is the crucial choice that gives rise to a graviton mass.

\^8We have been sloppy here, as the proper normalization condition is \( H^m = 1 \) at \( \cos z = \epsilon \). The proper normalization condition can be obtained from that used in the text by rescaling \( H^m \) as follows

\[
H^m(y) \to H^m(y)/H^m(\sqrt{\epsilon}).
\]

This rescaling does not significantly affect our computations and results.
3. At $z = 0$ ($y = 1$) we have to match the two expansions for $H^m(z)$. This is done by matching $H^m(z)$ and its first derivative $dH^m(z)/dz$. Notice that in the interval $[-\pi/2, 0]$ we have

$$\partial_z = 2\sqrt{y(1-y)}\partial_y,$$

while in $[0, \pi/2]$ we have

$$\partial_z = -2\sqrt{y(1-y)}\partial_y.$$  

This means that the matching conditions at $y = 1$ are

$$\alpha H^m_1(1) + \beta H^m_2(1) = \alpha' H^m_1(1) + \beta' H^m_2(1),$$

$$\lim_{y \to 1} \sqrt{(1-y)} \left( \alpha \frac{dH^m_1}{dy} + \beta \frac{dH^m_2}{dy} + \alpha' \frac{dH^m_1}{dy} + \beta' \frac{dH^m_2}{dy} \right)_{y=1} = 0.$$  

The boundary condition at $z = \pi/2$ sets

$$\beta' = -\frac{ab(a + 1)(b + 1)}{2}.$$  

The boundary condition at $z = -\pi/2$ implies

$$\beta = 0.$$  

To analyze the matching conditions at $y = 1$ we use standard identities among hypergeometric functions to find

$$H^m_1(y) = A + \sqrt{1-y}B + O(1-y), \quad H^m_2(y) = C + \sqrt{1-y}D + O(1-y),$$

$$A = [\psi(1) + \psi(3) - \psi(a+2) - \psi(b+2)] \frac{\Gamma(3)\Gamma(1/2)}{\Gamma(1-a)\Gamma(1-b)},$$

$$C = -2F(a, b; a + b + 2; 1) \frac{\Gamma(a)\Gamma(b)}{\Gamma(1/2)},$$

$$B = D = [\psi(1) + \psi(3) - \psi(a+2) - \psi(b+2)] \frac{\Gamma(3)\Gamma(-1/2)}{\Gamma(2+a)\Gamma(2+b)}.$$  

Since $B = D$, matching the $z$-derivative gives $\alpha + \alpha' + \beta' = 0$. Finally, eq. (68) gives

$$\alpha' = \frac{ab(a + 1)(b + 1)}{4} \left(1 + \frac{C}{A}\right).$$

After applying some further hypergeometric identities we find that $H^m(z)$ near $z = \pi/2$ can be written as

$$H^m(y) = 1 - aby - \frac{ab(a + 1)(b + 1)}{2} y^2 \log(y/\sqrt{e}) + y^2 F(m^2) + O(y^3),$$

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\[ F(m^2) = \frac{ab(a+1)(b+1)}{4} \left\{ [\psi(1) + \psi(3) - \psi(a+2) - \psi(b+2)] + \right. \\
\left. - \frac{\Gamma(a)\Gamma(b)(1-a)\Gamma(1-b)}{\Gamma(1/2-a)\Gamma(1/2-b)\Gamma(3)\Gamma(1/2)} \right\}. \] (78)

Three important checks can be performed on this formula.

1. In the limit \( m^2 \to 0, \ a \to 0 \) and \( b \to -3/2 \) (see the definition of \( a, b \) in eq. (62)). The formula for \( H^0(y) \) simplifies dramatically:

\[ H^0(y) = 1 - \frac{3}{16} y^2 + O(y^3). \] (79)

This equation must be compared with the explicit, elementary solution of eq. (57) for \( m^2 = 0 \)

\[ H^0(z) = \frac{1}{2} + \frac{3}{4} \sin z - \frac{1}{4} (\sin z)^3 = \frac{1}{2} + \frac{3}{4} (1-y)^{1/2} - \frac{1}{4} (1-y)^{3/2} = 1 - \frac{3}{16} y^2 + O(y^3). \] (80)

2. In the far Euclidean region eq. (78) must reproduce the flat space result. Setting \( -m^2 = p^2 \) we have \( b^* = a = -(3/4) + ip + O(p^{-1}) \). Using the asymptotic formula

\[ \left| \frac{\Gamma[-(3/4)+ip]\Gamma[(7/4)+ip]}{\Gamma[(5/4)+ip]} \right|^2 = 2\pi \exp(-\pi |p|)[|p|^{1/2} + O(|p|^{-1/2})], \] (81)

we find

\[ H^{ip}(y) = 1 + \frac{p^2}{4} y - \frac{p^2}{8} \left( \frac{p^2}{4} - \frac{1}{2} \right) y^2 \log(y/\sqrt{e}) + \\
- \left[ \frac{p^2}{8} \left( \frac{p^2}{4} - \frac{1}{2} \right) \log |p| + P(p^2) \right] y^2 + O(y^3). \] (82)

Here \( P(p^2) \) is a polynomial in \( p^2 \). For \( p^2 \gg 1 \) this formula does indeed coincide with its flat space analog (see \[14\]).

3. The pure AdS4 mass spectrum is recovered when we ask that \( H^m \) vanishes also at \( z = \pi/2 \). This give the conditions

\[ \hat{A} \equiv A/[\psi(1) + \psi(3) - \psi(a+2) - \psi(b+2)] = \frac{\Gamma(3)\Gamma(1/2)}{\Gamma(1-a)\Gamma(1-b)} = 0, \] (83)

or

\[ \hat{B} \equiv B/[\psi(1) + \psi(3) - \psi(a+2) - \psi(b+2)] = \frac{\Gamma(3)\Gamma(-1/2)}{\Gamma(2+a)\Gamma(2+b)} = 0. \] (84)
A zeros of either $\hat{A}$ or $\hat{B}$ arises only when the denominator in either eq. (83) or eq. (84) has a pole. This happens for $2 + b = 0, -1, -2, \ldots$ and $1 - a = 0, -1, -2, \ldots$

Recalling the definition of $a$ and $b$ we arrive at the equation

$$m^2 = n(n + 3), \quad n = 1, 2, 3, \ldots$$

These are precisely the masses of the 4-d spin-2 excitations that make up the spectrum of the 5-d massless graviton.

Now we are ready to compute the self energy defined in eq. (56). We use the expansion eq. (58) and the formula for $H^m(z)$ given in eqs. (77,78). After using

$$\frac{\partial H^m}{\partial z} = -4y^{3/2}F(m^2) + P(m^2) + O(y^{5/2}), \quad P(m^2) = \text{polynomial in } m^2; \quad (86)$$

we find

$$h^{m\mu\nu}\Sigma_{\mu\nu,\rho\sigma}h^{m\rho\sigma} = \frac{1}{16\pi G} \int d^4x \sqrt{-g}h^{m\mu\nu}(x)F(m^2)h^{m\mu\nu}(x) + Q(m^2) + O(\varepsilon^2). \quad (87)$$

$Q(m^2)$ is another polynomial in $m^2$. It is not difficult to show that it vanishes at $m^2 = 0$, so that it effects only a renormalization of the Newton constant and the coupling constants for higher-derivative local terms, irrelevant at low energies.

Now we come to the central result of this paper. As predicted by Karch and Randall, we find that:

The 4-d graviton acquires a nonzero mass $O(\lambda^2)$.

To prove this we recall the definition of the effective action $\Gamma_H$, eq. (54). On the transverse-traceless field $h^m_{\mu\nu}$ it reads

$$\Gamma_H = \frac{1}{16\pi G} \int d^4x \sqrt{-g}h^{m\mu\nu}m^2h^{m\mu\nu} + \frac{1}{16\pi G} \int d^4x \sqrt{-g}h^{m\mu\nu}F(m^2)h^{m\mu\nu}. \quad (88)$$

For small $m^2$, $F(m^2) \approx -(3/16)$, so that the pole of the propagator is shifted to a nonzero (positive) value. In other words, the graviton gets a mass $m^2 = (3G_4/4G)!$

Let us conclude this section with a few comments

1. Recall that in this section we measured $G$ in units such that $L = 1$, while $G_4$, the 4-d Newton constant, was measured in units such that $l = 1$. By re-introducing the 5-d AdS radius, the 5-d Newton constant scales as $G \to G/L^3$. Likewise, by re-introducing the the 4-d radius, the 4-d Newton constant scales as $G_4 \to G_4/l^2$.

9A pellucid introduction to mass spectra in AdS$_4$ can be found in [38]; [37] contains an up to date discussion of harmonic analysis for AdS spaces in various dimensions.
After these rescalings, recalling that the 4-d cosmological constant is \( \lambda = -3/l^2 \), we find that the graviton mass assumes the more familiar form

\[
m^2 = \frac{1}{12} \frac{G_4 L^3}{G} \lambda^2. 
\]  

(89)

Notice that when the Newton constant is completely induced by the CFT (\( G_4^{\text{bare}} = \infty \) in eq. (13)) we find \( m^2 = L^2 \lambda^2 / 6 \).

2. With our holographic calculation, we have found an explicit formula for the term CFC: thanks to the asymptotic expansion eq. (82) we see that \( \mathcal{F}(m^2) \) has all the right properties of the operator \( F \); namely, it obeys eqs. (21,22). Of course, \( \mathcal{F}(m^2) \) is a gauge-fixed version of \( F \). In fact, the boundary condition \( H^m(-\pi/2) = 0 \), together with the metric choice in eq. (44), \( g_{\mu4} = 0, g_{44} = \exp(2A) \), completely fixes the gauge for \( h_{\mu\nu} \), as no diffeomorphism leaves both the metric and the boundary condition invariant. Invariance under diffeomorphisms and Weyl transformations of the non-gauge-fixed version of \( \Gamma_H \) is nevertheless guaranteed because they both come from 5-d diffeomorphisms [32]. A more complete treatment of gauge fixing and the counting of degrees of freedom is given in appendix A.

3. Eq. (88) gives not only the almost-massless graviton but also the tower of massive Kaluza-Klein states found in [4]. The easiest way to see this is to expand \( \mathcal{F}(m^2) \) near one of its massive poles as

\[
\mathcal{F}(m^2) \approx \frac{F_i}{m^2 - m_i^2}. 
\]  

(90)

the rescaling introduced above tells us that \( m_i^2 \) and \( F_i \) are \( O(\lambda) \). To find the Kaluza-Klein states we must set

\[
\frac{1}{16\pi G_4} \frac{m^2 l^4}{4} + \frac{L^3}{16\pi G} \frac{F_i}{m^2 - m_i^2} = 0. 
\]  

(91)

By writing \( m^2 = m_i^2 + \delta m^2, \delta m^2 \ll m_i^2 \) we solve eq. (91) as

\[
\delta m^2 \approx -\frac{4G_4 L^3 F_i}{l^4 G m_i^2}. 
\]  

(92)

To ensure the consistency of the approximation used here we must have \( G_4 \lambda \ll GL^{-3} \). In physical models where \( G_4^{\text{bare}} > 0, G_4 < 2GL^{-1} \) so that the consistency condition is always satisfied, since in that case it becomes \( \lambda \ll L^{-2} \), and the 4-d cosmological constant is always much smaller than the 5-d one.
Variations on the KR compactification have been considered in the literature. In particular, [18] considers the AdS equivalent of RSI, in which another (positive-tension) brane is set at $z = -\pi (\ell/2) L + \epsilon'$. In that case the graviton zero mode is normalizable, so that the spectrum contains two spin-2 states much lighter than $\lambda$: one massless, the other with mass $O(L^2 \lambda^2)$. We propose to interpret the “far” brane as an effective description of an infrared cutoff, as in the holographic interpretation of the RSI model [11]. While there are some aspects of this identification that are somewhat puzzling—e.g. what does it mean that the far brane has positive tension?—one by-product of this identification is satisfying. Namely, with an infrared cutoff $\mu$, the generating functional of any 4-d field theory can always be expanded at low energies in terms of local functions of the 4-d metric:

$$W[g] = \int d^4x \sqrt{-g} \sum_{n=0}^{\infty} \mu^{4-2n} O^{(2n)}(g),$$

(93)

where $O^{(2n)}$ denotes local operators of dimension $2n$. This expansion guarantees that in an AdS background there always exists a massless graviton [4].

To see this, we notice that the effective action

$$\Gamma[g] = \frac{1}{16\pi G_4} \int_M d^4x \sqrt{-g} (R - 2\lambda) + W[g],$$

(94)

is built with polynomials in the scalar curvature, $R$, the tensor $R_{\mu\nu} - g_{\mu\nu} R/4$, and the Weyl tensor $C^{\mu\nu}$. Expanding to quadratic order around an AdS solution of the equations of motion of $\Gamma$, $\delta \Gamma/\delta g_{\mu\nu} = 0$, one finds

$$\Gamma[g] = \frac{1}{16\pi G_4} \int_M d^4x \sqrt{-g} h_{\mu\nu} L(\Delta_L^{(2)})(\Delta_L^{(2)} - \bar{R}/2) h_{\mu\nu} + O(h^3).$$

(95)

As before, the AdS background is $\bar{g}_{\mu\nu}$ and the fluctuation is $h_{\mu\nu}$; $L(\Delta_L^{(2)})$ is a polynomial in $\Delta_L^{(2)}$. Eq. (94) makes it manifest that the massless graviton—obeying $(\Delta_L^{(2)} - \bar{R}/2) h_{\mu\nu} = 0$—still solves the linearized equations of motion.

Alternatively, the far brane could be interpreted as a CFT on another AdS$_4$ space, joined with “our” AdS$_4$ at its boundary, $S_2 \times R$. In this case, the challenge is to understand whether the presence of two light gravitons can be seen as due to a peculiar boundary interaction between the two universes. The very possibility of this effect is probably due to the fact, peculiar to AdS spaces, that null rays take a finite coordinate time to complete the round trip from the interior to the boundary and back.

At this point we need to repeat that in this paper we have argued that holography works even when the boundary is made of disconnected components, provided that we give appropriate boundary conditions on the metric. If this is the case, the peculiar
phenomena described in this paper (i.e. the massive graviton) should be interpretable, as suggested in the previous paragraph, as due to a non-standard behavior of the 4-d fields at the boundary of the 4-d space.

Another interesting question is whether the function $F$ we found in section 4 is generic in CFTs. In particular, is the graviton mass a universal feature of CFTs coupled to gravity in AdS or is it an accident of our holographic computation?

To answer this question, it would be interesting to compute $F$ in a theory as far removed as possible from the strongly-coupled CFT studied here using its holographic dual. For instance, a conformally-coupled free scalar could be an excellent test-ground. The free scalar computation would also clarify the effect of the AdS$_4$ boundary conditions on the mass spectrum. Anyway, even before any computation, it is easy to see that we cannot rule out the possibility that the mass term is peculiar to holographic models. Indeed, one can exhibit other operators that obey eqs. (21,22) besides $F$. One such example was mentioned in section 2: $F = - (1/4) \log \Delta$. The operator $\Delta$ is the conformally covariant completion of $\Box^2$ that maps conformal tensors with the symmetries of the Weyl tensor into conformal tensors with the same symmetries and weight 6 [27, 28].

Note Added in Proof

After this paper was accepted for publication, it was pointed out to us that in 3 dimensions a local modification of the Einstein-Hilbert action exists, that gives a nonzero mass to the graviton while preserving general covariance [38]. The mechanism of ref. [38] is peculiar to 3 dimensions.

Acknowledgments

We would like to thank LBL, where part of this research has been done, for its hospitality and support; L. Randall for many discussions on KR, related and unrelated topics and S. Deser for, among other things, bringing to light ref. [28]. This work is supported in part by NSF grant PHY-0070787.

Appendix A: Diffeomorphisms and Gauge Fixing

In the holographic setting, 4-d diffeomorphisms and conformal transformations both come from 5-d diffeomorphisms that keep the 5-d metric $g_{mn}$ in the gauge

$$g_{44} = \exp(2A), \quad g_{\mu 4} = 0, \quad \mu, \nu = 0,.., 3.$$  \hfill (A.1)

The definition of the 5-d metric is

$$ds^2 = \exp[2A(z)]g_{mn}dx^m dx^n, \quad m, n = 0,.., 4, \quad x^4 \equiv z.$$  \hfill (A.2)
In the KR model \( A(z) = -\cos(z/L) \), in RSII \( A(z) = -\log(z/L) \). The gauge-preserving diffeomorphisms act on the 4-d metric as

\[
\delta g_{\mu\nu}(x, z) = D_\mu \zeta_\nu(x, z) + D_\nu \zeta_\mu(x, z) + 2 \dot{A} g_{\mu\nu} \zeta_5(x, z). \tag{A.3}
\]

The gauge choice eq. (A.1) gives (see e.g. [32, 2, 4])

\[
\zeta_5 = \omega(x), \tag{A.4}
\]
\[
\zeta_\mu = G(z) D_\mu \omega(x) + \epsilon_\mu(x), \quad G = \int dz \exp(-2A), \tag{A.5}
\]

where both \( \omega(x) \) and \( \epsilon_\mu(x) \) are independent of \( z \). From its action on \( g_{\mu\nu} \), it is clear that \( \omega \) is a 4-d Weyl transformation and \( \epsilon_\mu \) a 4-d diffeomorphism. As explained in [2, 4] the general solution of the equations of motion for the metric \( g_{\mu\nu} = \exp(2A)(\bar{g}_{\mu\nu} + h_{\mu\nu}) \) is, to linear order in \( h_{\mu\nu} \),

\[
h_{\mu\nu}(x, z) = h^{TT}_{\mu\nu}(x, z) + 2GD_\mu D_\nu \Phi(x) + 2\dot{A}\bar{g}_{\mu\nu} \Phi(x), \tag{A.6}
\]
\[
\Box + \frac{4}{3} \lambda \Phi = 0 \tag{A.7}
\]

The field \( \Phi(x) \), independent of \( z \), can be canceled by setting \( \omega = -\Phi \). [2].

The transverse-traceless field \( h^{TT}_{\mu\nu}(x, z) \) can be further decomposed as

\[
h^{TT}_{\mu\nu}(x, z) = \sum_m h^m_{\mu\nu}(x) H^m(z) + D_\mu A_\nu(x) + D_\nu A_\mu(x), \tag{A.8}
\]
\[
\Box A_\mu + D^\nu D_\nu A_\mu = 0, \quad D_\mu A^\mu = 0. \tag{A.9}
\]

Notice that \( A_\mu(x) \) does not respect the boundary condition \( h^{TT}_{\mu\nu}(x, \pi/2) = 0 \), so that it cannot be decomposed as \( A_\mu = \sum_m A^m_\mu H^m \).

To bring \( h^{TT}_{\mu\nu} \) into the form given in the text, \( h^{TT}_{\mu\nu} = \sum_m h^m_{\mu\nu} H^m \), we use a 4-d diffeomorphism: \( \epsilon_\mu = -A_\mu \). After the diffeomorphism, we are left with a tower of massive spin-2 fields, each one carrying 5 degrees of freedom and without any further gauge invariance. The role of \( A_\mu \) identifies it as the Stückelberg field of 4-d diffeomorphisms.

**Appendix B: a Change of Coordinates**

Here we exhibit an explicit change of coordinates that maps the Poincaré parametrization of AdS\(_d\) into the parametrization used in the text, with slice AdS\(_{d-1}\).

In Poincaré coordinates the line element of AdS\(_d\) is

\[
ds^2 = \frac{L^2}{z^2}(dz^2 + dw^2 + ds^2), \tag{B.1}
\]
\[
ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu, \quad \mu, \nu = 0, ..., d-3. \tag{B.2}
\]
We are looking for a change of variables that puts the metric in the form

\[ ds^2 = \exp[2A(z)]\left[dz^2 + w^{-2}(dw^2 + ds^2)\right]. \]  \hfill (B.3)

We use the ansatz

\[ z \to wz/L, \quad w \to F(w, z). \]  \hfill (B.4)

The metric eq. (B.1) is transformed into

\[ ds^2 = \frac{L^4}{z^2w^2} \left[ \left( F_z^2 + \frac{w^2}{L^2} \right) dz^2 + \left( F_w^2 + \frac{z^2}{L^2} \right) dw^2 + ds^2 \right] \]  \hfill (B.5)

To have an AdS\(_{d-1}\) slice we need \( F_w^2 + \frac{z^2}{L^2} = 1 \), whose solution is

\[ F(z, w) = \pm w \sqrt{1 - \frac{z^2}{L^2}} + f(z). \]  \hfill (B.6)

The second condition we need is that the \( dz^2 \) term in the line element depends only on \( z \). It gives the equation

\[ F_z^2 + \left( \frac{w}{L} \right)^2 = g(z)w^2, \]  \hfill (B.7)

where \( g(z) \) is an arbitrary function of \( z \) only. This equation is solved by \( f(z) = \text{constant} \). At this point, the metric can be cast in the form given in eq. (2) with a redefinition of \( z \): \( z \to h(z) \). The equation for \( h \) is

\[ h_z^2 = \left( 1 - \frac{h^2}{L^2} \right), \]  \hfill (B.8)

which is solved by \( h(z) = L \cos(z/L) \).

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