Astrophysical Implication of Low $E(2^{+}_1)$ in Neutron-rich Sn Isotopes

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Abstract. The observation and prediction of unusually depressed first excited $2^{+}_1$ states in even-A neutron-rich isotopes of semi-magic Sn above $^{132}$Sn provide motivations for reviewing the problems related to the nuclear astrophysics in general. In the present work, the $\beta$-decay rates of the exotic even Sn isotopes ($^{134,136}$Sn) above the $^{132}$Sn core have been calculated as a function of temperature (T). In order to get the necessary $f_{t}$ values, B(GT) values corresponding to allowed Gamow Teller (GT−) $\beta$-decay have been theoretically calculated using shell model. The total decay rate shows decrease with increasing temperature as the ground state population is depleted and population of excited states with slower decay rates increases. The abundance at each Z value is inversely proportional to the decay constant of the waiting point nucleus for that particular Z. So the increase in half-life of isotopes of Sn, like $^{136}$Sn, might have substantial impact on the r-process nucleosynthesis.

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INTRODUCTION

Nuclei with $50 \leq Z \leq 56$ and $82 \leq N \leq 88$ in the $\pi(gdsh) \otimes \nu(hfpi)$ valence space above the $^{132}$Sn core lie on or close to the path of astrophysical r-process flow. Their structure, particularly the binding energy (BE), low-lying excited states and beta decay rates at finite temperatures are important ingredients for nucleosynthesis calculations. Sn isotopes are of particular importance. Even Sn isotopes, say $^{136}$Sn, is known [1] to be the classical "waiting point" nucleus in A=130 solar system abundance peak under typical r-process condition. Spectroscopic information, such as BE and low lying spectrum, is known experimentally only for $^{134}$Sn [2]. Half-lives of $^{135-137}$Sn have been measured through $\beta^- n$ decay process [3]. No other information exists. Lifetimes of these nuclei are very small and production rate very low presenting challenges to spectroscopic studies. Reliable theoretical results are therefore necessary and useful.

In general, the $\beta^-$-decay rates ($\lambda$) relevant in the astrophysical scenario depend on the density ($\rho$), electron fraction ($Y_e$) and the prevalent temperature ($T$) in the environment [4]. In the present work, the decay rates of the exotic Sn isotopes ($^{134,136}$Sn) above the $^{132}$Sn core have been calculated as a function of temperature (T) only. The necessary $ft$ values have been obtained from the B(GT)s, corresponding to allowed Gamow Teller (GT−) $\beta^-$-decay using untruncated shell model calculations.
Untruncated shell model calculations in the valence space consisting of $\pi(1g_{7/2}, 2d_{5/2}, 2d_{3/2}, 3s_{1/2}, 1h_{11/2})$ and $\nu(1h_{9/2}, 2f_{7/2}, 2f_{5/2}, 3p_{3/2}, 3p_{1/2}, 1i_{13/2})$ orbitals with the empirical SMPN [5] and effective CWG [6] (1+2) - body Hamiltonians show very good agreement with the available experimental data. But it is remarkable that for $^{136,138}$Sn, where experimental level schemes are not known, the theoretical predictions differ dramatically [7]. For $^{134-138}$Sn (Fig. 1a), CWG interaction predicts nearly constant energies of $2^+_1$ states, normally expected for semi-magic nuclei. But SMPN predicts a remarkable new feature: decreasing $E(2^+_1)$ energies with increasing neutron number. Casten and Sherrill [8] have pointed out that, although $[E(2^+_1)_{Sn} - E(2^+_1)_{Te}] \approx 400$ keV (Fig.1b) for a given neutron number over most of the $N = 50 - 82$ shell, the difference $[E(2^+_1)_{Sn} - E(2^+_1)_{Te}]$ is only $119$ keV for $N = 84$. It is indeed remarkable that the difference $[E(2^+_1)_{^{136}Sn} - E(2^+_1)_{^{138}Te}]$ for $N = 86$ is $108$ keV with SMPN. So it is consistent with the trend discussed by Casten and Sherrill. For CWG, this difference is $733 - 356 = 377$ keV for $N = 86$, which deviates from the trend.

This observation of depressed first excited states in even-$A$ Sn isotopes provides very useful ingredient for reviewing the problems related to the nuclear astrophysics in general. In the present work an estimation of the effect of the depressed energies on the $\beta$- decay rates of the exotic even Sn isotopes ($^{134,136}$Sn) above the $^{132}$Sn core have been calculated as a function of temperature (T).

In astrophysical environments, thermally populated excited states in the mother nucleus (namely, Sn isotope in the present situation) can have an equilibrium population. The excited levels undergo $\beta$-decay transitions to the ground state or to excited states in the daughter nucleus (corresponding Sb isotope). These additional $\beta$-decay transitions may alter the astrophysical half-lives of the Sn isotopes compared to the laboratory values. The change in the half-life can in turn have an impact on the r process nucleosynthesis and generation of more exotic neutron rich species.

In order to get the necessary $ft$ values corresponding to the decay of the thermally populated excited states of the mother to the excited states of the daughter nucleus only allowed Gamow -Teller (GT-) transitions have been considered. B(GT) values have been calculated using OXBASH code [9] with both SMPN and CWG Hamiltonians. It is
evident that the thermal population of excited nuclear levels becomes more important with increasing temperature and lower excitation energy. The beta decay rate for a nucleus in astrophysical environment at a temperature $T$ is given by

$$\lambda = \sum_i (2I_i + 1)e^{-E_i/kT} \sum_j \lambda_{ij}/G$$

where $E_i$ is the energy of the state of the mother nucleus and $G (=\sum_i (2I_i + 1)e^{-E_i/kT})$ is the partition function of the mother nucleus. Index $j$ sums over states of the daughter nucleus to which transitions are allowed. The rate from the parent nuclear state $i$ to the daughter nuclear state $j$ is given by

$$\lambda_{ij} = \ln 2 \frac{f_{ij}}{(ft)_{ij}}$$

$f_{ij}$ is the phase-space factor for beta decay. The $(ft)_{ij}$ value of an allowed beta decay is given by

$$\frac{6250s}{B(F)_{ij} + (g_A/g_V)^2 B(GT)_{ij}}$$

where $g_A$, $g_V$ are vector and axial-vector coupling constants. $B(F)_{ij}$ and $B(GT)_{ij}$ denote the Fermi (F) and Gamow-Teller (GT) transition probabilities from $i$th mother state to $j$th daughter state.

**RESULTS AND DISCUSSIONS**

For both the isotopes of Sn, ground state to ground state beta decays are forbidden transitions. But they are quite fast. The relevant information for these isotopes incorporated for modifications in the decay rates that result from inclusion of excited states due to thermal excitations are shown in Table 1. The selection rules for GT transitions only allow transitions from single particle $\nu 1h_{9/2}$ orbital to $\pi 1h_{11/2}$ orbital in this model space. But the wavefunction compositions of the relevant low lying states in these isotopes of Sn and Sb have very small contribution from the shell model configurations involving these orbitals. So the calculated allowed GT strengths are generally very small.

For SMPN interaction, as discussed in Ref. [5], the excitation energies of lowest yrast excited states of $^{134}Sn, Sb$ have been used for adjusting the two body neutron-neutron and neutron-proton matrix elements. So the energies of the $2^+_1$ states for $^{134}Sn$ are almost similar with the two interactions. But wavefunctions and energies predicted by CWG and SMPN are different for some of the states in $^{134}Sb$ resulting in different values of calculated $B(GT)$s.

With both the interactions the effective half-life increases with increasing temperature for $^{134,136}Sn$ (Fig.2). Since the decay rate from the ground state is quite fast and those from the first and second excited $2^+$ states are two orders of magnitude slower (for $kT=1$ MeV), so the total decay rate decreases with increasing temperature as the ground state population is depleted and population of excited states with slower decay rates increases. For each case the extent and rate of increase depend on the details of the wavefunctions.
At around 0.1 MeV temperature (≃ 10⁹ K), deviation from the laboratory value of half life is initiated. In ¹³⁴Sn (Fig.2a) with both interactions, the deviation starts at same temperature and the saturation is reached above 10 MeV. The saturated values of half lives with two interactions are different, ≃ 5 s for SMPN and ≃ 7.5 s for CWG. For ¹³⁶Sn (Fig.2b), the relatively lower E(2⁺) value (578 keV) with SMPN compared to 733 keV with CWG is manifested by the faster rate of change of T₁/₂ with SMPN. But both saturates at ≃ 2.2 s.

The abundance at each Z value is inversely proportional to the decay constant of the waiting point nucleus for that particular Z. So the increase in half-life of isotopes of Sn, like, ¹³⁶Sn will definitely have substantial impact on the r-process nucleosynthesis.

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