"Pseudo" pseudo forces in gauge synchronization theories

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Abstract

The role of extra terms in Newton’s second law that arise as a result of non-Einstein synchronization is explored. Although such extra terms have been interpreted as pseudo forces that constrain physical theory to a unique method of synchronization (the Einstein, or standard, synchronization), a fully relativistic analysis indicates that such extra terms do not invalidate the conventionalist thesis of synchronization.

1 Introduction

In a recent article Ohanian[1] contends that synchronizations other than the standard Einstein synchronization (E synchronization) result in pseudo force terms arising in Newton’s second law, and that because pseudo forces are not observed in inertial frames, non-standard synchronizations of the Reichenbach[2] type (R synchronizations) can not represent the true state of inertial frames in the physical world. That is, synchronization is not simply a physically meaningless gauge to be applied to clock settings, but is constrained by nature to be unique for inertial frames.

As a counterpoint, one could argue that if clocks in various places have different settings, and one is measuring the motion of a body on which a force is applied, the body will seem to take a different amount of time to get from point A to point B, depending on the settings on the different sets of clocks. One measures acceleration with clocks and rods, and having different sets of clocks means having different numerical measurements of acceleration $a$, for the same particle motion. So $\mathbf{F} = \mathbf{m} \cdot \mathbf{a}$ would not be expected to describe the motion caused by a force $\mathbf{F}$ for any but the standard synchronization, and we would need additional term(s)/factor(s) to correct for the difference between $\mathbf{F}$ and $\mathbf{ma}$ that arises.

If we simply transformed the standard 4D relativistic form of the 2nd law to new synchronization coordinates, and took the low speed limit, we would get the appropriate non-standard coordinate form of the 2nd law, which would have extra term(s)/factor(s) that could be interpreted as pseudo force(s). We would then have a whole (different) set of dynamical laws that would be consistent internally and in agreement with experiment (provided the same synchronization were used on the experimental clocks.) So does this, or does this not, mean that $\mathbf{R}$ synchronizations are invalid? Does the presence of apparent pseudo forces in the equation of motion doom the conventionalist interpretation of synchronization? This article addresses
these questions, and concludes that the conventionalist thesis is not only still alive, but also still kicking.

In Section 2 as a reference example of pseudo forces, Coriolis and centrifugal forces in rotation are derived using relativity theory. The same procedure is then applied in Section 3 to coordinate frames with R synchronization. The results are compared in Section 4 with Ohanian’s derivation and in Section 5 with various definitions for “inertial frame”. Philosophical implications are addressed in Section 6, and conclusions drawn in Section 7.

2 Psuedo Forces Example: Rotation

Pseudo forces in accelerating frames arise (mathematically) from the non-Lorentzian metric. The following is well known and presented for background, as well as for later comparison to the non-standard synchronization case.

For rotation, with familiar symbols for cylindrical coordinates and the coordinate transformation

\[
\begin{align*}
    cT &= ct \\
    R &= r \\
    \Phi &= \phi + \omega t \\
    Z &= z
\end{align*}
\]

where upper case refer to the lab frame, and lower case to the rotating frame, the metric and its inverse\(^3\) are

\[
\begin{align*}
    g_{\alpha\beta} &= \begin{bmatrix}
        -\left(1 - \frac{r^2\omega^2}{c^2}\right) & 0 & \frac{r^2\omega}{c} & 0 \\
        0 & 1 & 0 & 0 \\
        \frac{r^2\omega}{c} & 0 & r^2 & 0 \\
        0 & 0 & 0 & 1
    \end{bmatrix}, &
    g^{\alpha\beta} &= \begin{bmatrix}
        -1 & 0 & \frac{\omega}{c} & 0 \\
        0 & 1 & 0 & 0 \\
        \frac{\omega}{c} & 0 & \left(1 - \frac{r^2\omega^2}{c^2}\right)/r^2 & 0 \\
        0 & 0 & 0 & 1
    \end{bmatrix}.
\end{align*}
\]

The only non-zero Christoffel symbols, found from

\[
\Gamma_{\alpha\beta\gamma} = \frac{1}{2} (g_{\alpha\beta,\gamma} + g_{\alpha\gamma,\beta} - g_{\beta\gamma,\alpha});
\Gamma^\alpha_{\beta\gamma} = g^{\alpha\mu} \Gamma_{\mu\beta\gamma},
\]

are

\[
\Gamma^r_{tt} = \Gamma^1_{00} = -\frac{\omega^2 r}{c^2}; \quad \Gamma^\theta_{tr} = \Gamma^2_{01} = \frac{\omega}{cr}; \quad \Gamma^r_{t\theta} = \Gamma^1_{02} = -\frac{\omega r}{c}.
\]

The equation of motion for a geodesic particle is

\[
\frac{d^2x^\alpha}{d\tau^2} + \Gamma^\alpha_{\beta\gamma} \frac{dx^\beta}{d\tau} \frac{dx^\gamma}{d\tau} = 0.
\]

The relevant 4-velocities are

\[
\begin{align*}
    u^0 &= \frac{dx^0}{d\tau} = c \frac{dt}{d\tau} = \frac{c}{\sqrt{1 - \omega^2 r^2/c^2}} \\
    u^1 &= \frac{dx^1}{d\tau} = \frac{dr}{d\tau} \\
    u^2 &= \frac{dx^2}{d\tau} = \frac{d\theta}{d\tau}.
\end{align*}
\]
2.1 Radial Direction

For the \( x^1 = r \) direction, the equation of motion (5) becomes

\[
a^r = a^1 = \frac{\omega^2 r}{(1 - \omega^2 r^2/c^2)} + \frac{\omega r \, d\theta}{\sqrt{1 - \omega^2 r^2/c^2}} d\tau = \frac{\omega^2 r}{(1 - \omega^2 r^2/c^2)} + \frac{\omega u^\theta}{\sqrt{1 - \omega^2 r^2/c^2}} \tag{7}
\]

where \( u^\theta = \sqrt{g_{\theta \theta}} d\theta/d\tau = r d\theta/d\tau \) is the physical velocity (i.e., measured in m/s using standard meter sticks) of the particle in the \( \theta \) direction relative to the rotating frame. Since the particle is undergoing geodesic motion, as seen from the rotating frame there is acceleration relative to the rotating frame coordinates. For a particle fixed at constant radius \( r \) in the rotating frame, centrifugal and Coriolis pseudo forces equal to the mass times the terms on the RH side of (7) would appear to arise.

2.2 Tangential Direction

For the \( x^2 = \theta \) direction, the equation of motion (5) becomes

\[
a^\theta = a^2 = \frac{d^2 \theta}{d\tau^2} = -\frac{\omega}{r\sqrt{1 - \omega^2 r^2/c^2}} \frac{dr}{d\tau} = -\frac{\omega}{r\sqrt{1 - \omega^2 r^2/c^2}} u^r \tag{8}
\]

where \( u^r = \sqrt{g_{rr}} dr/d\tau = dr/d\tau \) is the physical velocity in the radial direction relative to the rotating frame.

The physical (measured in m/s\(^2\)) value for the tangential acceleration is

\[
a^\theta = \sqrt{g_{\theta \theta}} a^\theta = ra^\theta = -\frac{\omega}{\sqrt{1 - \omega^2 r^2/c^2}} u^r. \tag{9}
\]

3 Pseudo Forces and Non-standard Synchronization Gauges

3.1 Relativity, Geodesics, and Pseudo Accelerations

From eqs (5) or (6) of Anderson et al[4], the transformation from Lorentz coordinates (with standard, Einstein synchronization) to non-standard R synchronization coordinates is

\[
\begin{align*}
\tilde{c}t &= ct - kx \\
\tilde{x} &= x \\
\tilde{y} &= y \\
\tilde{z} &= z 
\end{align*} \quad \begin{align*}
(a) \\
(b) \\
(c) \\
(d),
\end{align*} \tag{10}
\]

where \( k \) is a constant and \(-1 < k < 1\). One should note that where we and Anderson et al employ \( k \), Ohanian uses \( ck \). From (10), one can derive the metric for the R synchronization and its inverse,

\[
\tilde{g}_{\alpha\beta} = \begin{bmatrix}
-1 & -k & 0 & 0 \\
-k & 1 - k^2 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}, \quad \tilde{g}^{\alpha\beta} = \begin{bmatrix}
-(1 - k^2) & -k & 0 & 0 \\
-k & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}. \tag{11}
\]
Repeating the procedure used in rotation to find pseudo forces, one finds, because the metric is constant in spacetime, that all Christoffel symbols are zero, i.e.,

\[ \Gamma^\alpha_{\beta\gamma} = 0 \quad \text{for all } \alpha, \beta, \gamma. \] (12)

Thus, from (6) we must have

\[ \frac{d^2 \tilde{x}^\alpha}{d\tau^2} = 0 \quad \text{for all } \alpha, \] (13)

there are no pseudo accelerations, and there can be no pseudo forces, in apparent contradiction to Ohanian’s claim.

### 3.2 Proper Time, Applied and Pseudo Forces

However, to be precise, Ohanian claimed only that motion of a non-goedesic (accelerated as seen in a Lorentz coordinate frame by a force \( F \)) would be modified by additional pseudo forces (as seen in the R synchronization coordinate frame.) Thus, consider the more general form of the 4D equation (5) when forces are present,

\[ m \left( \frac{d^2 x^\alpha}{d\tau^2} + \Gamma^\alpha_{\beta\gamma} \frac{dx^\beta}{d\tau} \frac{dx^\gamma}{d\tau} \right) = F^\alpha, \] (14)

where \( F^\alpha \) is the 4D generalized covariant force. For the Lorentz coordinate frame, the Christoffel symbols in (14) are zero, and we have

\[ ma^\alpha_\tau = m \frac{d^2 x^\alpha}{d\tau^2} = F^\alpha. \] (15)

Transform (15) to the R synchronization coordinates via the matrix inherent in (10),

\[
\begin{bmatrix}
\tilde{a}^0_\tau \\
\tilde{a}^1_\tau \\
\tilde{a}^2_\tau \\
\tilde{a}^3_\tau
\end{bmatrix} =
\begin{bmatrix}
1 & -k & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
a^0_\tau \\
a^1_\tau \\
a^2_\tau \\
a^3_\tau
\end{bmatrix} =
\begin{bmatrix}
1 & -k & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
F^0 \\
F^1 \\
F^2 \\
F^3
\end{bmatrix} =
\begin{bmatrix}
\tilde{F}^0 \\
\tilde{F}^1 \\
\tilde{F}^2 \\
\tilde{F}^3
\end{bmatrix}.
\] (16)

Thus, in 3D, nothing is really changed (the same is true for the \( 0^{\text{th}} \) component as the \( k \) terms drop out), and we have

\[ m\tilde{a}^i_\tau = m \frac{d^2 \tilde{x}^i}{d\tau^2} = \tilde{F}^i = F^i \quad (i = 1, 2, 3), \] (17)

i.e., there are no new pseudo forces arising in the R synchronization coordinate system.

### 3.3 Coordinate Time, Applied and Pseudo Forces

Note, however, that the time variable in (17) is the proper time on the particle, which, of course, is invariant, i.e., the same in any coordinate system with any time coordinate synchronization. One could (and in the present context, should) ask what the equation of motion for a geodesic would be in terms of the coordinate time \( \tilde{t} \) of the R synchronization of (10).
From (11)
\[ c^2 dr^2 = -\bar{g}_{\alpha\beta} dx^\alpha dx^\beta = c^2 dl^2 + 2ckdx^1 dl - (1 - k^2) \left( \frac{dx^1}{c} \right)^2 - \left( \frac{dx^2}{c} \right)^2 - \left( \frac{dx^3}{c} \right)^2. \] (18)

Dividing (18) by \( \bar{d}t \) and solving for \( d\tau \) one finds
\[ d\tau = \sqrt{\left( 1 + \frac{k}{c} \bar{v}^1 \right)^2 - \frac{\bar{v}^2}{c^2} \bar{d}t} \approx \left( 1 + \frac{k}{c} \bar{v}^1 \right) \bar{d}t \] (19)
with the approximation on the RH to first order and
\[ \bar{v}^1 = \frac{d\bar{x}^1}{dt}, \quad \bar{v}^2 = \left( \frac{d\bar{x}^1}{dt} \right)^2 + \left( \frac{d\bar{x}^2}{dt} \right)^2 + \left( \frac{d\bar{x}^3}{dt} \right)^2. \] (20)

In what follows, one could use the exact expression of (19) and take the first order limit of the final result, though for simplicity we shall from the beginning simply use the approximation on the RH of (19). Noting that
\[ \bar{u}^i = \frac{d\bar{x}^i}{d\tau} \equiv \frac{1}{\left( 1 + \frac{k}{c} \bar{v}^1 \right)} \left( \frac{d\bar{x}^i}{dt} \right) = \frac{\bar{v}^i}{\left( 1 + \frac{k}{c} \bar{v}^1 \right)}, \] (21)
we can re-express (17) as
\[ m \frac{d^2 \bar{x}^i}{d\tau^2} = F^i = m \frac{d\bar{u}^i}{d\tau} \approx \frac{m}{\left( 1 + \frac{k}{c} \bar{v}^1 \right)} \frac{d}{dt} \left( \frac{\bar{v}^i}{\left( 1 + \frac{k}{c} \bar{v}^1 \right)} \right). \] (22)

Carrying out the derivative above, one gets
\[ F^i \approx \frac{m \bar{a}^i}{\left( 1 + \frac{k}{c} \bar{v}^1 \right)^2} - \frac{m \bar{v}^i}{\left( 1 + \frac{k}{c} \bar{v}^1 \right)^2} \left( \frac{k\bar{a}^1}{c} \right). \] (23)
Rearranged and expressed in vector notation, this becomes the same as Ohanian’s equation (21),
\[ m\bar{a} = \left( 1 + \frac{k\cdot\bar{v}}{c} \right)^2 \bar{F} + \frac{k\bar{a}}{1 + \frac{k}{c} \bar{v}} m\bar{v}, \] (24)
where here and from henceforth we drop the approximate symbol on the equal signs, and we remind the reader that Ohanian’s notation has \( k \), where we have \( k/c \) (conforming with Anderson et al). Performing an inner product of (24) with \( k \), solving for \( k \cdot \bar{a} \), and substituting the result back into (24), one obtains Ohanian’s equation (10),
\[ m\bar{a} = \left( 1 + \frac{k}{c} \cdot \bar{v} \right)^2 \left( \bar{F} + \frac{k}{c} \cdot \bar{F} \bar{v} \right). \] (25)

At first blush, (25) appears to contain extra terms not found in the more familiar form of the second law of Newton’s dynamics, and one might be tempted to interpret these terms as pseudo forces. However, all the unfamiliar terms and factors in (25) are due solely to the relationship between proper and coordinate standard times being different in the E and R systems. The non-standard expression (19) for proper time in terms of R system clock times lies at the root of (25). Since proper times are the same in both systems, the difference is due to the difference in coordinate clocks, i.e., to the system of synchronization chosen. The apparent dynamic effect in (25) is thus not “real”, being based wholly in the definitions chosen for clock settings.
4 Alternative Synchronization and Newton’s Laws

4.1 Pseudo Forces and Acceleration

To gain a feeling for the physical implications of the relation (25), we break that relation into components of acceleration parallel, and perpendicular, to the direction of alternative synchronization (the $k$ direction.)

4.1.1 Acceleration Parallel to $k$

For the parallel direction,

$$m\ddot{a}_k = \left(1 + \frac{k\tilde{v}_k}{c}\right)^2 \left(F_k + \frac{kF_k}{c} \tilde{v}_k\right).$$

(26)

Consider the case where velocity is in the $k$ direction. The equation of motion (26) then becomes

$$m\ddot{a}_k = \left(1 + \frac{k\tilde{v}_k}{c}\right)^3 F_k,$$

(27)

and the unusual factor in parentheses is simply a correction to the acceleration due to the difference in clock readings as the object moves in the $k$ direction. That is, the E synchronization acceleration $a_k$ takes on the value of $\ddot{a}_k$ in the R synchronization scheme as

$$a_k \rightarrow \ddot{a}_k = \left(1 + \frac{k\tilde{v}_k}{c}\right)^3 a_k = \left(1 + \frac{k\tilde{v}_k}{c}\right)^3 \frac{F_k}{m},$$

(28)

and no actual change in force, i.e. no pseudo force, exists, either in the parallel or transverse direction.

For the case where velocity is solely (and instantaneously) perpendicular to $k$, the equation of motion (26) becomes

$$m\ddot{a}_\perp = \left(1 + \frac{k\tilde{v}_k}{c}\right)^2 \left(F_\perp + \frac{kF_k}{c} \tilde{v}_\perp\right).$$

(30)

For velocity solely in the $k$ direction, this becomes

$$m\ddot{a}_\perp = \left(1 + \frac{k\tilde{v}_k}{c}\right)^2 F_\perp$$

(31)

for which the bracketed quantity is once again a correction to the acceleration due to the particle velocity in the $k$ direction and the concomitant change in measured values of acceleration using different clocks.
For velocity solely perpendicular to \( \mathbf{k} \), (30) becomes
\[
m\ddot{a}_\perp = F_\perp + \frac{kF_k}{c} \ddot{v}_\perp,
\]
which seems to contain the very unusual (and certainly non-Newtonian) characteristic of having force in the \( \mathbf{k} \) direction \( F_k \) contribute to acceleration in the direction transverse to \( \mathbf{k} \). However, one has to recognize that the force \( F_k \) will accelerate the object in the \( \mathbf{k} \) direction via (29), thereby giving it motion in that direction. So, when subsequent measurements of time are taken, which are used to calculate acceleration \( \ddot{a}_\perp \) in the transverse direction, the latter measurements for time will be on clocks that are further out along the \( \mathbf{k} \) direction than earlier clocks. In fact, by substituting (29) into (32), one sees the purely kinematic dependence of \( \ddot{a}_\perp \) on \( \ddot{a}_k \). And hence, the settings on the clocks in a non-E synchronization will modify what one would otherwise expect the measurement of \( \ddot{a}_\perp \) to be. Certainly, we would expect this modification to be dependent on the magnitude of the R synchronization \( k \) and acceleration \( \ddot{a}_k \) in the \( \mathbf{k} \) direction, as is found on the RH side of (32). Further, \( \ddot{a}_\perp \) is measured with meter sticks as well as clocks, and the more meter sticks the object passes in the transverse direction for a given change in the readings on the clocks the object passes, the greater the value of \( \ddot{a}_\perp \). Thus, we would also expect \( \ddot{a}_\perp \) to depend on velocity \( \ddot{v}_\perp \), as it indeed does in (32).

4.2 Clocks, Acceleration, and Pseudo Forces

We have seen there are no real pseudo forces, as the term is usually used, in the R system equations of motion. Though one might be tempted to conclude that the extra terms/factors in (25) represent such pseudo forces, we see that a correction in the equation of motion arises to account for the different numbers measured for acceleration with different clocks. As the particle moves in the direction of non-Einstein synchronization, the different readings on the clocks along that direction change our numerical value for acceleration. The correction is found in the extra terms and factors of (25), and thus, it seems more appropriate to call the function on the RH of (25) a “synchronization function”, rather than a “force function”. There are no “true” pseudo forces, as in rotation, but only a clock correction, or “pseudo” pseudo force.

4.3 Observation from the R System

If we merely change the settings on the clocks – similar to merely changing the settings (numbers, or labels) on the spatial coordinate grid - observers of any process (single particle motion, collision of particles, etc.), would see no intrinsic visual difference in behavior. Coordinate values would change, but not observed behavior. No strange pseudo forces would seem to push objects in unexpected directions. Colliding billiard balls would look no different to our eyes (unlike in rotation where they would.)

Further, the first order length changes calculated[7] for the R system would also not manifest visually. This is because Lorentz contraction and other length changes, such as that considered here, are calculated in relativity by assuming the 3D endpoints of a given rod exist at the same moment in time[8] as seen by a given observer.

Consider a rod moving to the right as seen by both E and R observers. At \( t = \tilde{t} = 0 \) the LH end of the rod is at \( x = \tilde{x} = 0 \) For the E observer, the RH end is at \( x = L \) for \( t = 0 \). But the RH end according to the R observer (though she sees the same thing visually as the E observer) when it is at \( \tilde{x} = L \) is not at \( \tilde{t} = 0 \), but at some earlier time, according to the R clock
at \( \tilde{x} = L \). A short time later the RH end will have moved further out along the \( x \) axis, and then the R clock where the RH end is located would read \( \tilde{t} = 0 \). Taking the endpoints of the rod as simultaneous (existing at the same time) means the distance between the LH and RH ends of the rod according to the R system, at time \( \tilde{t} = 0 \), is greater than \( L \). But this difference is simply an artifact of arbitrarily setting the R clocks differently than the E clocks. Physically, an R observer actually perceives no visual difference in rod length from the E observer.

If one calculated the times of arrival of light rays from rod endpoints to an E and an R observer co-existent at the same 4D location, the calculations in the two systems would be different, but the conclusions as to what each would see visually would be the same.

And so it goes with all other ostensible physical differences between the R and E systems. They do not manifest as any difference to physical witnesses (unlike rotation, for example), but only in the equations to describe phenomena those witnesses would employ, based on their disparate choices for clock settings.

5 Inertial or Non-inertial Frame?

Ohanian contends that, due to the arising of “pseudo forces”, R synchronization coordinates constitute non-inertial frames. He does not discredit conventionality of synchronization completely, but posits that R systems cannot therefore be valid representations of inertial frames. Indeed, Ohanian says, “... the R frame is a possible reference frame for describing physics. [But because] Newton’s laws of dynamics are not valid in their standard form ... the R frame is not an inertial frame.”

Yet, one could argue that the most fundamental definition of a “frame” is a set of continuous 3D points, each of which keeps a constant spatial distance from every other such point. This is certainly true for the R coordinates. Further, \( x = \tilde{x} = \) constant for all points for all time, be it \( t \) or \( \tilde{t} \) time. Hence, there is never any motion between the two frames, and they must therefore be the same frame. They have different coordinates, specifically the time coordinates, but the frame is the same. This is analogous to a purely spatial coordinate change (such as \( x \rightarrow x + 3 \)) for which the underlying frame stays the same. Thus, if the E system represents an inertial frame, so must the R system.

Additionally, we must draw a distinction between \( \mathbf{F} = m\mathbf{a} \) acting on a particle seen in an inertial frame, and the force felt by an observer fixed to a frame that is non-inertial. Although the form of the second law describing particle motion changes in going from E to R, the force felt by any observer fixed to a given spatial coordinate point stays the same, i.e., zero. If one feels no force on one’s own body, one is in an inertial frame.

Though, from this logic, the R system and the E system appear to be different coordinate systems, yet constitute the same frame, one’s conclusion in this regard seems to depend on one’s choice of definition for inertial frame.

Traditionally, physics employs several heretofore seemingly equivalent ways to define an inertial frame. These are listed in Table 1. The third method in the list is not usually found in texts, though I submit most physicists would agree that it is a valid means. Although the last column is for a rotating frame, any non-inertial (accelerating or gravitational) frame would do.
Table 1: Inertial Frame Definition Comparison

| Inertial Frame Definition Method | E Synch Sys | R Synch Sys | Rotating Frame |
|----------------------------------|------------|------------|----------------|
| 1 Fixed observers feel no body force? | Y          | Y          | N              |
| 2 Geodesics look like straight lines? | Y          | Y          | N              |
| 3 Fixed 3D points are permanently stationary relative to a known inertial frame? | Y          | Y          | N              |
| 4 $m \frac{d^2 x^i}{d \tau^2} = F^i$?* | Y          | Y          | N              |
| 5 $m \frac{d^2 x^i}{dt^2} = F^i$ to 1st order?* | Y          | N          | N              |

* Assumes Cartesian spatial grid

Four of the five criteria directly imply the R system is inertial. This includes the fully relativistic equation of motion (fourth method), though not what that typically reduces to at first order, i.e., Newton’s second law for inertial frames (fifth method). Thus, though these two methods have usually been considered more or less equivalent, when R synchronization systems are considered, they are not. Of the two, most would consider the fully relativistic (4th method) to be the more fundamental.

The difference arises in the determination of coordinate standard clock time $t$ from the proper time on the particle $\tau$. In the R synchronization this has first order dependence on the velocity component in the direction of the non-Einstein clock synchronization. Again, I would argue that this is merely a coordinate difference between the R and E systems, and does not imply they are different frames. Thus, by any measure, the R system constitutes an inertial frame.

6 Philosophy, Semantics, and Synchronization

Virtually everyone agrees that the “.. in the R [system] physics is consistent and complete.” [10] Calculations, based on theory, can be made that predict observed phenomena. Further, everyone
admits that the R system is not as “pretty”, or economical, as the E system. After that, the arguments seem to drift to the philosophical, even semantic, rather than the scientific.

For example, if prior well-used, successful, and heretofore interchangeable definitions of “inertial” seem in conflict with one another, contending that one is more correct than another seems to miss the point. The old definitions must be re-thought and refined, in the context of the new knowledge that has arisen.

And whether we consider violation of a long-held sacrosanct definition, principle, or philosophical position to be enough to “defeat” the conventionalist thesis seems to be largely a matter of personal predilection. For example, Poincare’ invariance has held prominence for decades as a seeming inviolate bedrock of natural law. When it, or its Newtonian sibling $F = ma$, are controverted by a new theory, one can feel a certain historical justification in rejecting such a theory on principle. However, there is no a priori reason, no reason other than past experience with standard synchronization, upon which to base such a judgment. As it does not appear that the conventionalists have ever made any claim other than that their thesis is consistent, internally and with experiment, if we are to invalidate that thesis, then it seems that claim, and that claim alone, is where we should start.

7 Conclusion

With regard to non-standard Reichenbach synchronization gauges, dynamics is no different from kinematics. Kinematically, we know we are in a non-Einstein synched system because the measured one-way speed of light is anisotropic. Dynamically, we know because $F = ma$ is not isotropic (or more precisely, the first order relativistic generalization of Newton’s second law where time is local standard clock time, not proper time on the particle.)

Ohanian’s discrediting of non-standard synchronization is right if we demand that nature’s symmetry extend beyond the fully covariant tensor form (using proper time) of physical laws to that of our humanly chosen systems of clocks and rods. Then we must have Einstein synchronization. But that demand seems decidedly artificial (though more esthetic and simple, to be sure.) Using tensor notation or generalized coordinates, no new pseudo forces arise. Using non-standard synchronized clocks, the coordinate equations of motion change form, but that must be expected. In any case, the R system, as judged from almost any perspective, remains an inertial frame. And in either the E or R system, provided we use the same clock synchronization for both analysis and test, we will get theoretical predictions that match experiment. And in the end, that is all we can really ask of physics.

I love symmetry, beauty, and simplicity in my physics, and the conventionalist view of synchronization, lacking, in my opinion, those qualities, is not something I am particularly enamored of. I would, in fact, be quite pleased if someone could find a way to do away with it, emphatically and finally. Thus, I initially welcomed, whole-heartedly, the seeming refutation of the conventionalist’s thesis by Professor Ohanian, for whom I have long held considerable admiration. However, upon further reflection, I reluctantly concluded that, once again, an attempt to invalidate the theory of gauge synchronization seems to have come up short.
References

[1] H.C. Ohanian, “The role of dynamics in the synchronization problem,” Am J. Phys. 72, 141-148 (Feb 2002).

[2] H. Reichenbach, The Philosophy of Space and Time (Dover, New York, 1957). First published in German under the title Philosophie der Raum-Teit-Lehre in 1927.

[3] Found in many places. For example, see R.D. Klauber, “New Perspectives on the Relativistically Rotating Disk and Non-time-orthogonal Reference Frames,” Found. Phys. Lett., 11(5), 405-443. qc-gr/0103076. See Section 4.1.

[4] R. Anderson, I. Vetharaniam, G.E. Stedman, “Conventionality of Synchronization, Gauge Dependence, and Test Theories of Relativity,” Phys. Rep., 295, 3&4, 93-180 (1998).

[5] Ref. [4], unnumbered equation in Section 5.1, pg. 111.

[6] One might wonder why we seem to ignore subsequent time measurements in (29) in the \( k \) direction, but not in (32). The reason is that in the former case, though \( \tilde{a}_k \) would be modified by later velocity changes in the \( k \) direction, the instantaneous velocity in the \( k \) direction is zero. In the latter case, \( \tilde{a}_\perp \) is modified instantaneously by an acceleration in the \( k \) direction, which is instantaneously non-zero.

[7] Ref. [1], Section III, 145-146

[8] It is straightforward to show, in Lorentzian relativity theory, that if the 3D endpoints of a rod are simultaneous (exist at the same time) as seen in two different systems, then there is no difference in length (no Lorentz contraction) for the rod between the two systems. Lorentz contraction is intimately related to differences in simultaneity (and thus to synchronization).

[9] Ref. [1], Section III, p. 146.

[10] Ref. [1], Section IV., p. 146.