This paper proposes distributed adaptive algorithms based on the conjugate gradient (CG) method and the diffusion strategy for parameter estimation over sensor networks. We present sparsity-aware conventional and modified distributed CG algorithms using $l_1$ and log-sum penalty functions. The proposed sparsity-aware diffusion distributed CG algorithms have an improved performance in terms of mean square deviation (MSD) and convergence as compared with the consensus least-mean square (Diffusion-LMS) algorithm, the diffusion CG algorithms and a close performance to the diffusion distributed recursive least squares (Consensus-RLS) algorithm. Numerical results show that the proposed algorithms are reliable and can be applied in several scenarios.

**Index Terms**— Distributed Processing, Diffusion Strategy, Conjugate Gradient, Sparsity Aware.

1. INTRODUCTION

Distributed processing has become a very common and useful approach to extract information in a network by performing estimation of the desired parameters. The efficiency of the network depends on the communication protocol used to exchange information between the nodes, as well as the algorithm to obtain the parameters. Another important aspect is to prevent a failure in any agent that may affect the operation and the performance of the network. Distributed schemes can offer better estimation performance of the parameters as compared with the centralized approach, based on the principle that each node communicates with several other nodes and exploits the spatial diversity in the networks [1], [36], [37], [38], [39], [40].

The main strategies for communication in distributed processing are incremental, consensus and diffusion. In the incremental protocol, the communication flows cyclically and the information is exchanged from one node to the adjacent nodes. In this strategy the flow of information must be preset at the initialization [2]. The consensus strategy is an elegant procedure to enforce agreement among cooperating nodes [3]. In the diffusion mechanism, each node communicates with the rest of the nodes [2] without any enforcement constraint.

...
The main task of parameter estimation is to adjust the unknown \( M \times 1 \) weight vector \( \omega_{k,i} \) of each node, where \( M \) is the length of the filter [3]. The desired signal \( d_{k,i} \) at each time \( i \) is drawn from a random process and given by

\[
d_{k,i} = \omega_0^H x_{k,i} + n_{k,i}
\]  

(1)

where \( \omega_0 \) is the \( M \times 1 \) system weight vector, \( x_{k,i} \) is the \( M \times 1 \) input signal vector and \( n_{k,i} \) is the measurement noise. The output estimate is given by

\[
y_{k,i} = \omega_{k,i}^H x_{k,i}
\]  

(2)

The main goal of the network is to minimize the following cost function:

\[
C(\omega_{k,i}) = \sum_{k=1}^{N} E[|d_{k,i} - \omega_{k,i}^H x_{k,i}|^2]
\]  

(3)

By solving this minimization problem it is possible to obtain the optimum solution of the weight vector at each node. The optimum solution for the cost function is given by

\[
\omega_{k,i} = R_{k,i}^{-1} b_{k,i}
\]  

(4)

where \( R_{k,i} = E[x_{k,i} x_{k,i}^H] \) is the \( M \times M \) correlation matrix of the input data vector \( x_{k,i} \), and \( b_{k,i} = E[d_{k,i}^* x_{k,i}] \) is the \( M \times 1 \) cross-correlation vector between the input data and \( d_{k,i} \).

### 2.2. Problem Statement

We consider a diffusion algorithm for a network where each agent \( k \) has access at each time instant to the realization \( \{d_{k,i}, x_{k,i}\} \) of zero-mean spatial data \( \{d_{k,i}, x_{k,i}\} \) [32][40]. For a network with possibly sparse parameter vectors, the cost function also involves a penalty function which exploits sparsity. In this case the network needs to solve the following optimization problem:

\[
\min C(\omega_{k,i}) = \sum_{k=1}^{N} E[|d_{k,i} - \omega_{k,i}^H x_{k,i}|^2] + f(\omega_{k,i})
\]  

(5)

where \( f(\omega_{k,i}) \) is a penalty function that exploits the sparsity in the parameter vector \( \omega_{k,i} \). In the following sections we focus on distributed diffusion CG algorithms to solve (5).

### 3. PROPOSED DISTRIBUTED DIFFUSION CG ALGORITHM

In this section, we present the proposed distributed CG algorithm using the diffusion strategy with a penalty function that is equal to zero. This corresponds to the diffusion strategy without the exploitation of sparsity. We first derive the CG algorithm and then consider the diffusion protocol.

#### 3.1. Derivation of the CG algorithm

The CG method can be applied to adaptive filtering problems [30][32][41]. The main objective in this task is to solve (4). The cost function for one agent is given by

\[
C_{CG}(\omega) = \frac{1}{2} \omega^H R \omega - b^H \omega
\]  

(6)

For distributed processing over sensor networks, we present the following derivation. The CG algorithm does not need to compute the matrix inversion of \( R \), which is an advantage compared with RLS algorithms. It computes the weights \( \omega_{k,i} \) for each iteration \( j \) until convergence, i.e., \( \omega_{k,i}(j) \). The gradient of the method in the negative direction is obtained as follows [30]:

\[
g_{k,i}(j) = g_{k,i}(j) - R_{k,i}(j) \omega_{k,i}(j)
\]  

(7)

Calculating the Krylov subspace [32] through different operations, the recursion is given by

\[
\omega_{k,i}(j) = \omega_{k,i}(j-1) - \alpha(j) p_{k,i}(j)
\]  

(8)

where \( p \) is the conjugate direction vector of \( g \) and \( \alpha \) is the step size that minimizes the cost function in (6) by replacing (7) in (4). Both parameters are calculated as follows:

\[
\alpha(j) = \frac{g_{k,i}^H(j-1) g_{k,i}(j-1)}{p_{k,i}^H(j) R_{k,i}(j) p_{k,i}(j)}
\]  

(9)

\[
p_{k,i}(j) = g_{k,i}(j) + \beta(j) p_{k,i}(j)
\]  

(10)

The parameter \( \beta \) is calculated using the Gram-Schmidt orthogonalization procedure [32] as given by

\[
\beta(j) = \frac{g_{k,i}^H(j) g_{k,i}(j)}{g_{k,i}^H(j-1) g_{k,i}(j-1)}
\]  

(11)

Applying the CG method to a distributed network the cost function is expressed based on the information exchanged between all nodes \( k = 1, 2, ..., N \). Each of the equations presented so far takes place at each agent during the iterations of the CG algorithm. Therefore, we have the cost function:

\[
C_{CG}(\omega_{k,i}) = \frac{1}{2} \sum_{k=1}^{N} \omega_{k,i}^H R_{k,i} \omega_{k,i} - b_{k,i}^H \omega_{k,i}
\]  

(12)

Using the data window with an exponential decay, the resulting autocorrelation matrix and cross-correlation vector are defined using the forgetting factor \( \lambda \) as given by

\[
R_{k,i} = \lambda R_{k,i-1} + x_{k,i} x_{k,i}^H
\]  

(13)

\[
b_{k,i} = \lambda b_{k,i-1} + d_{k,i} x_{k,i}
\]  

(14)

In the diffusion strategy, all nodes interact with their neighbors sharing and updating the system parameter vector. Each node \( k \) is able to run its update simultaneously with the other agents [1][4]. Figure [4] illustrates the diffusion strategy.
In diffusion protocols there are two well-known variants that switch the order of the combination and adaptation steps, namely, Combine-then-Adapt (CTA) and Adapt-then-Combine (ATC), each one based on the connectivity among nodes. These mechanisms perform adaptation and learning at the same time [3][4].

### 3.2. CTA Diffusion Distributed CG algorithm

In the CTA diffusion strategy, the convex combination term is first evaluated into an intermediate state variable and subsequently used to perform the weight update [4]. The local estimation is given by

$$\varphi_{k,i} = \sum_{l \in N_k} a_{lk} \omega_{l,i-1}$$  \hspace{1cm} (15)

where $a_{lk}$ represents the combining coefficients of the data fusion which should comply with

$$\sum_{l \in N_k} a_{lk} = 1, \forall k.$$  \hspace{1cm} (16)

In this work the strategy adopted for the $a_{lk}$ combiner is the Metropolis rule [11] given by

$$c_{kl} = \begin{cases} \max \{0, |a_{lk}|/|a_{lk}| \}, & \text{if } k \neq l \text{ are linked} \\ 1 - \sum_{l \in N_k \setminus k} c_{kl}, & \text{for } k = l, \end{cases}$$  \hspace{1cm} (17)

The distributed CTA CG algorithm based on the derivation steps obtains the updated weight substituting (15) in (8), giving as result:

$$\omega_{k,i}(j) = \omega_{k,i}(j - 1) - \alpha(j) p_{k,i}(j)$$  \hspace{1cm} (18)

where $\omega_{k,i}(0) = \varphi_{k,i}$ [32]. The rest of the derivation is based on the solution presented in the previous section and the pseudo-code is detailed in Table I.

The modified CG (MCG) algorithm comes from the conventional CG algorithm previously presented and only requires one iteration per coefficient update. Specifically, the residual is calculated using (7), (8) and (13) [30].

$$g_{k,i} = b_{k,i} - R_{k,i} \varphi_{k,i}$$

$$= \lambda g_{k,i-1} - \alpha_{k,i} R_{k,i} p_{k,i-1}$$

$$+ x_{k,i} [d_{k,i} - \omega_{k,i-1} x_{k,i}]$$  \hspace{1cm} (19)

### Table 1. CTA CG ALGORITHM

| Parameters initialization: |
|-----------------------------|
| $\omega_{k,0} = 0$ |
| For each time instant $i > 0$ |
| For each agent $k=1,2,\ldots,N$ |
| $R_{k,i} = \lambda R_{k,i-1} + x_{k,i} x_{k,i}^H$ |
| $b_{k,i} = \lambda b_{k,i-1} + d_{k,i}^T x_{k,i}$ |
| $\varphi_{k,i} = \sum_{l \in N_k} a_{lk} \omega_{l,i-1}$ |
| $\omega_{k,i}(j) = \varphi_{k,i}$ |
| $g_{k,i} (0) = b_{k,i}(0) - R_{k,i}(0) \omega_{k,i-1}$ |
| $p_{k,i} (0) = g_{k,i}(0)$ |
| For each CG iteration $j = 1$ until convergence |
| $\alpha(j) = g_{k,i}^{H}(j-1) g_{k,i}(j-1)$ |
| $\omega_{k,i}(j) = \omega_{k,i}(j-1) - \alpha(j) p_{k,i}(j)$ |
| $g_{k,i}(j) = g_{k,i}(j-1) - \alpha(j) R_{k,i}(j) p_{k,i}(j-1)$ |
| $\beta(j) = g_{k,i}^{H}(j) g_{k,i}(j)$ |
| $p_{k,i}(j) = g_{k,i}(j) + \beta(j) p_{k,i}(j-1)$ |
| End For |
| $\omega_{k,i} = \omega_{k,i}(j_{last})$ |
| End for |

The previous equation [19] is multiplied for the search direction vector:

$$p_{k,i}^{H} g_{k,i} = \lambda p_{k,i}^{H} g_{k,i-1} - \alpha_{k,i} R_{k,i} p_{k,i-1}$$

$$+ p_{k,i}^{H} x_{k,i} [d_{k,i} - \omega_{k,i-1} x_{k,i}]$$  \hspace{1cm} (20)

Applying the expected value, considering $p_{k,i-1}$ uncorrelated with $x_{k,i}$, $d_{k,i}$, and $\varphi_{k,i}$, and that the algorithm converges the last term of (20) can be neglected. The line search to compute $\alpha$ has to satisfy the convergence bound [11] given by

$$\frac{(\lambda_f - 0.5) E[ p_{k,i-1}^{H} g_{k,i} ] }{ E[ p_{k,i-1}^{H} R_{k,i} p_{k,i-1} ] } \leq E[ \alpha_{k,i} ] \leq \frac{ E[ p_{k,i-1}^{H} g_{k,i} ] }{ E[ p_{k,i-1}^{H} R_{k,i} p_{k,i-1} ] }$$  \hspace{1cm} (21)

$$\alpha_{k,i} = \eta \frac{ p_{k,i}^{H} g_{k,i} }{ p_{k,i}^{H} R_{k,i} p_{k,i} },$$  \hspace{1cm} (22)

where $(\lambda_f - 0.5) \leq \eta \leq \lambda$. The Polak-Ribiere method [30,31,33] for the computation of $\beta$ is given by

$$\beta_{k,i} = \frac{(g_{k,i} - g_{k,i-1})^{H} g_{k,i}}{g_{k,i}^{H} g_{k,i}}$$  \hspace{1cm} (23)

### 3.3. ATC Distributed CG algorithm

Similarly to CTA, the ATC protocol switches the order of the operations. The difference lies in the variable chosen to update the weight $\omega_{k,i}$. In this case, the update estimate is the convex combination of the adaptation step. Table II shows the pseudo code of the ATC strategy. The MCG version for the
in the cost function, we have

When the logarithmic penalty function

Based on the previous development of distributed CG algorithms, this section presents the proposed distributed sparsity-aware diffusion CG algorithms. The Table 5 shows the computational complexity of all diffusion methods. It can be seen that the complexity of the modified versions is lower than the conventional methods.

4. PROPOSED SPARSITY-AWARE DIFFUSION CG

Based on the previous development of distributed CG algorithms, this section presents the proposed distributed sparsity-aware diffusion CG algorithms using $l_1$ (ZA) and log-sum (RZA) norm penalty functions.

4.1. ZA and RZA CG algorithms

The cost function in this case is given by

$$C_{CG}(\omega_{k,i}) = \frac{1}{2} \sum_{k=1}^{N} \omega_{k,i}^{H} R_{k,i} \omega_{k,i} - b_{k,i}^{H} \omega_{k,i} + f_1,$$

where $f_1$ denotes the $l_1$ penalty function and is defined by

$$f_1 = \rho \| \omega_{k,i} \|_1.$$  

Applying the partial derivative of the penalty function gives

$$\frac{\partial(f_1)}{\partial(\omega_{k,i})} = \text{sgn}(\omega_{k,i}).$$  

When the logarithmic penalty function $f_2$ instead $f_1$ is used in the cost function, we have

$$f_2 = \rho \sum_{i=1}^{M} \log(1 + \frac{\|\omega_{k,i}\|}{\varepsilon}).$$

The partial derivative of the penalty function applied with respect to $\omega_{k,i}$ is described by

$$\frac{\partial(f_1)}{\partial(\omega_{k,i})} = \frac{\text{sgn}(\omega_{k,i})}{1 + \varepsilon \| \omega_{k,i} \|_1}.$$  

In both cases these sparsity-aware algorithms attract to zero the values of the parameter vector which are very small or are not useful. This results in an algorithm with a faster convergence and lower MSD values as can be seen in the following sections. Using the penalty functions (25) and (28), we obtain the sparsity-aware algorithms with the CTA and ATC strategies. Table 5 shows the sparsity-aware method for the ACT protocol. In case of CTA, the same steps applied with the ZA or RZA penalty functions to the steps previously presented in Section III are carried out.

4.2. ZA and RZA Modified CG algorithms

The ATC and CTA MCG algorithms are very similar as presented in previous section, including the penalty functions. In the ATC strategy, the generation of the first state resulting from the adaptation step, is used in the final update.

4.3. Computational Complexity

The Table 5 shows the computational complexity of all diffusion distributed methods proposed in terms of additions and multiplications.

It can be seen that the complexity of the modified versions is lower than the conventional methods.
Table 4. Sparsity-aware ATC CG Algorithm

| Parameters initialization: | $\omega_{k,0} = 0$ |
|---------------------------|-------------------|
| For each time instant $i > 0$ | For each agent $k=1,2, \ldots, N$ |
| $R_{k,i} = \lambda R_{k,i-1} + x_k,i x_k,i^H$ | $b_{k,i} = b_{k,i-1} + d_k x_k,i$ |
| $g_k,i(0) = b_{k,i}(0) - R_{k,i}(0)\omega_{k,i-1}(0)$ | $p_k,i(0) = g_k,i(0)$ |
| $\omega_{k,i}(0) = \omega_{k,i-1}$ | For each CG iteration $j = 1$ until convergence |
| $\alpha(j) = g_k,i(j-1)g_k,i(j-1)^H$ | $\omega_{k,i}(j) = \omega_{k,i}(j-1) - \alpha(j)p_k,i(j)$ |
| $\omega_{k,i}(j) = \omega_{k,i}(j-1) - \alpha(j)p_k,i(j)$ | $g_k,i(j) = g_k,i(j-1) - \alpha(j)R_{k,i}(j)p_k,i(j-1)$ |
| $\beta(j) = g_k,i(j-1)g_k,i(j)$ | $p_k,i(j+1) = g_k,i(j) + \beta(j)p_k,i(j)$ |
| End For | End For |

End for

5. SIMULATIONS RESULTS

In this section, we evaluated the proposed distributed diffusion CG algorithms and compare them with existing algorithms. The results are based on the mean square deviation MSD of the network. We consider a network with 20 nodes and 1000 iterations per run. Each iteration corresponds to a time instant. The results are averaged over 100 experiments. The length of the filter is 10 and the variance of the input signal 1, which has been modeled as a complex Gaussian noise and the SNR is 30 dB.

5.1. Comparison between standard and sparsity-aware CTA distributed CG algorithms

For the standard CTA CG version, the system parameter vector was randomly set. In the case of the sparsity-aware algorithms it was set to two values equal to one and the remaining values were set to zero. After all the iterations, the performance of each algorithm in terms of MSD is shown in Fig. ???. The results show that the sparsity-aware versions outperform the standard versions and the best results are obtained for the RZA versions. At the same time the MCG algorithms have a better performance than the standard ones.

5.2. Comparison between sparsity-aware ATC distributed CG algorithms.

The same configuration used before was set for CTA strategy. Fig. ?? below shows the performance of the results in the simulations. Fig. ?? shows the comparison between the consensus and diffusion algorithms with RZA. It can be observed that the diffusion ATC CG algorithm has a faster convergence as compared to the CTA and consensus strategy, as well as the MSD value at steady state.

6. CONCLUSIONS

In this work we proposed distributed CG algorithms for parameter estimation over sensor networks as well as the modified versions of them. The proposed ATC diffusion CG algorithms have a faster convergence than the CTA. The ATC strategy outperforms both consensus and CTA protocols. In all cases, the modified versions obtained the low MSD values and faster convergence rate. Simulations have shown that the proposed distributed CG algorithms are suitable techniques for adaptive parameter estimation problems.

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