(LACK OF) LENSGING CONSTRAINTS ON CLUSTER DARK MATTER PROFILES

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ABSTRACT

Using stellar dynamics and strong gravitational lensing as complementary probes, Sand et al. (2002, 2003) have recently claimed strong evidence for shallow dark matter density profiles in several lensing clusters, which may conflict with predictions of the Cold Dark Matter paradigm. However, systematic uncertainties in the analysis weaken the constraints. By re-analyzing their data, we argue that the tight constraints claimed by Sand et al. were driven by prior assumptions. Relaxing the assumptions, we find that no strong constraints may be derived on the dark matter inner profile from the Sand et al. data; we find satisfactory fits (with reasonable parameters) for a wide range of inner slopes $\rho \propto r^{-\beta}$ with $0 < \beta < 1.4$. Useful constraints on the mass distributions of lensing clusters can still be obtained, but they require moving beyond mere measurements of lensing critical radii into the realm of detailed lens modeling.

Subject headings: gravitational lensing — dark matter — galaxies: clusters — galaxies: clusters: individual (Abell 383, MS 2137–23)

1. COLD DARK MATTER OR CORED DARK MATTER?

The central density profile of dark matter halos has been the subject of considerable debate in recent years. On the theoretical side, numerous groups have claimed that dissipationless N-body simulations in the Cold Dark Matter (CDM) model produce universal halo profiles. Despite vigorous debate over the exact shape of the inner profile, there is general agreement that predicted CDM halos have central density cusps, $\rho \propto r^{-\beta}$ with $\beta \sim 1–1.5$ (e.g., Moore et al. 1998; Navarro et al. 1997, 2003). On the observational side, similar controversy has raged over the question of whether such cusps are present in real galaxies, and whether their absence would challenge the CDM paradigm (e.g., Dutton et al. 2003; Simon et al. 2003, and references therein). The most difficult issue is accounting for the effects of baryonic matter, which contributes directly to the gravitational potential and may also modify the dark matter distribution (e.g., Blumenthal et al. 1986; Elmaz et al. 2003; Loeb & Peebles 2003).

Strongly lensed arcs in galaxy clusters probe the gravitational potential on scales ($r \sim 50–100$ kpc) large enough to avoid significant baryonic contamination, and hence can provide a relatively clean test of the predictions of dissipationless N-body simulations. Several groups have constrained the density profiles of individual clusters by studying their lensing properties (Athreya et al. 2002; Clowe & Schneider 2004; Dahle 2003), and in some cases have obtained stringent limits from combined strong- and weak-lensing analyses (e.g., Gavazzi et al. 2003; Kneib et al. 2003).

Another approach is to disentangle the baryonic and dark matter contributions to the net potential on small scales, similar to studies of galaxy rotation curves. Kelson et al. (2002) have used the velocity dispersion profile of the cD galaxy in the cluster Abell 2199 together with the kinematics of the cluster members to decompose the stellar and dark matter components. They found that a $\beta = 1.5$ dark matter cusp is ruled out by the velocity dispersion data, while a $\beta = 1$ cusp is difficult to reconcile with a reasonable mass-to-light ratio for the cD galaxy. Sand et al. (2002, 2003) have recently claimed strong evidence for shallow dark matter density profiles in several lensing clusters, which may conflict with predictions of the Cold Dark Matter paradigm. However, systematic uncertainties in the analysis weaken the constraints. By re-analyzing their data, we argue that the tight constraints claimed by Sand et al. were driven by prior assumptions. Relaxing the assumptions, we find that no strong constraints may be derived on the dark matter inner profile from the Sand et al. data; we find satisfactory fits (with reasonable parameters) for a wide range of inner slopes $\rho \propto r^{-\beta}$ with $0 < \beta < 1.4$. Useful constraints on the mass distributions of lensing clusters can still be obtained, but they require moving beyond mere measurements of lensing critical radii into the realm of detailed lens modeling.

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constraints.

After the completion of this paper, we became aware of independent work by Bartelmann & Meneghetti (2003), who reach similar conclusions.

2. Analysis

2.1. Data and models

The observables modeled by S03 are the lensing critical radii (basically the radii of the tangential and radial arcs) and the velocity dispersion profiles of the brightest cluster galaxies; the data are given in their Tables 4–5. For a given mass model, the critical radii are determined by projecting the density distribution and locating the singular points of the lens mapping (see e.g. Schneider et al. 1992), while the velocity dispersion profile is computed by solving the spherical Jeans equation (see appendix).

We use a generalization of the two-component mass model used by S03. For the stellar component, we use the η model (Tremaine et al. 1994),

\[ \rho_{\text{gal}} = \frac{(3 - \beta_g) M_g}{4 \pi r_g^3} \left[ \left( \frac{r}{r_g} \right)^{\beta_g} \left( 1 + \frac{r}{r_g} \right)^{4 - \beta_g} \right]^{-1} \]

S03 mainly used a Jaffe model, which is a particular case of the η model with \( \beta_g = 2 \). Following S03, we describe the dark matter with a generalized NFW-type profile,

\[ \rho_{\text{DM}} = \frac{M_h}{4 \pi r_h^3 f(c_{\text{vir}})} \left[ \left( \frac{r}{r_h} \right)^{\beta_h} \left( 1 + \frac{r}{r_h} \right)^{3 - \beta_h} \right]^{-1} \]

where \( f(c) = \int_{c/c_{\text{vir}}}^{c} x^{-2\beta_h} / (1 + x)^{3 - \beta_h} \, dx \).

2.2. A case study: Abell 383

In the S03 results Abell 383 provided the strongest evidence that the dark matter slope is shallower than \( r^{-1} \), so we use this cluster to study important systematic uncertainties in the analysis. If we center both the stellar and dark matter models on the observed central galaxy position, orient them along the observed galaxy position angle, and fix the stellar model’s half-light radius to the observed galaxy effective radius (13.75″), then the remaining model parameters are: the galaxy mass, inner slope, and ellipticity (\( M_g, \beta_g, e_g \)); and the halo mass, scale radius, inner slope, and ellipticity (\( M_h, r_h, \beta_h, e_h \)).

S03 used a spherical Jaffe model for the stellar component (\( \beta_g = 2 \) and \( e_g = 0 \)), and a spherical halo (\( e_h = 0 \)) with scale radius \( r_h = 400 \) kpc. With the same assumptions, we reproduce their constraints on the dark matter inner slope \( \beta_h \).

The surface brightness profile of the central galaxy, shown by Smith et al. (2001), is steeper than expected for a Jaffe model and so we adopt \( \beta_g \approx 2.2 \), however we note that this steep slope has been disputed (R. Ellis 2003, private communication). We find that with this stellar component the models can accommodate a somewhat steeper dark matter profile: with a \( \beta_g = 2 \) galaxy the best-fit halo has \( \beta_h \approx 0.38 \), while with \( \beta_g = 2.2 \) it shifts to \( \beta_h \approx 0.45 \). The shift of \( \Delta \beta_h \lesssim 0.1 \) is comparable to various systematic effects considered by S03.

The model with a \( \beta_g = 2.2 \) galaxy fits better than the S03 model (with \( \Delta \chi^2 = -1.5 \)), so henceforth we focus on it.

A larger effect is associated with the dark matter scale radius. The S03 constraints on \( \beta_h \) depend crucially on their assumption of \( r_h = 400 \) kpc, because there is a degeneracy between \( r_h \) and the dark matter slope \( \beta_h \). For example, assuming \( r_h = 200 \) kpc would yield \( \beta_h \approx 0.18 \), while assuming \( r_h = 800 \) kpc would yield \( \beta_h \approx 0.66 \) (for spherical models). The latter model fits considerably better than the S03 model (\( \Delta \chi^2 = -2.6 \)), even though S03 claimed that \( \beta_h > 0.55 \) was excluded at 99% confidence. For spherical models, larger \( \beta_h \) do require large scale radii \( r_h \); for example, the best-fitting model with \( \beta_h = 0.8 \) has \( r_h > 1 \) Mpc! Such implausibly large scale radii (or small concentrations) would suggest that steep
The use of more detailed lensing data and modeling can strongly constrain the dark matter slope. Fortunately, the lensing critical radii with dynamical data cannot without an unphysically large scale radius.)

hot (with the virial masses and concentrations expected for radius \( r_c \) with moderate ellipticities it is possible to have mass \( M_{\text{halo}} \) on small ellipticity we find successful models over the range of spherical symmetry we recover the tight constraints of the halo. Figure 2 shows likelihood contours in the ellipticity of the halo. Figure 2 shows likelihood contours in the plane of \( \beta_h \) and \( \epsilon_h \), optimizing over the other parameters \( (M_g, M_h, \text{and} r_h) \). We find that a broad range of dark matter slopes are consistent with the data. In the limit of spherical symmetry we recover the tight constraints on \( \beta_h \) found by S03, but if we allow even a relatively small ellipticity we find successful models over the range \( 0 < \beta_h < 1.4 \). Thus, the limits found by S03 appear to be an artifact of their prior assumptions for the mass model, notably the assumption of spherical symmetry. Note that the models we find with steeper inner slopes \((\beta \approx 1)\) have perfectly sensible parameters from a theoretical standpoint. For example, model b in Table 1 with a \( \beta_h = 1 \) cusp, has galaxy mass \( M_g = 10^{12} M_\odot \), halo mass \( M_h = 1.9 \times 10^{15} M_\odot \), and a halo scale radius \( r_h = 610 \) kpc, corresponding to a concentration of \( c_{\text{vir}} = 4.7 \). These halo parameters are fully consistent with the virial masses and concentrations expected for hot \((T = 7.1 \text{ keV})\) X-ray luminous clusters. (Note that with moderate ellipticities it is possible to have \( \beta_h = 1 \) without an unphysically large scale radius.)

2.3. Additional constraints: MS 2137–23

The case of Abell 383 suggests that simply combining the lensing critical radii with dynamical data cannot strongly constrain the dark matter slope. Fortunately, the use of more detailed lensing data and modeling can provide constraints on the ellipticity that significantly improve the constraints on the dark matter slope. To illustrate, we consider MS 2137–23. While S03 used only the lensing critical radii, Gavazzi et al. (2003) identified multiple images of 26 distinct sources in the arcs produced by this cluster, enabling much more detailed modeling. Additionally, the X-ray temperature has been measured to be \( T = 5.56 \) keV (Allen et al. 2001), which can provide constraints on the halo mass via the \( M–T \) relation determined by Allen et al. (2001) for relaxed lensing clusters. We combine these data with the velocity dispersions from S03, but we inflate the positional error bars on the multiply imaged knots reported by Gavazzi et al. to 1"., and inflate the error on the X-ray temperature to 1 keV, in order to maximize the impact of the velocity dispersions on the fit. (We neglect the 5th image claimed by Gavazzi et al. as its detection is tentative.)

We again use two-component mass models, assuming the galaxy and dark matter halo to be concentric. We use the observed values of the galaxy half-light radius \((5.02")\), ellipticity \((\epsilon_g = 0.17)\), and position angle. If we force the halo to be spherical and to have scale radius \( r_h = 400 \) kpc we recover the same constraints on \( \beta_h \) as S03. However, if we allow the dark matter halo parameters to vary and optimize over them (and also over the galaxy mass \( M_g \)), we find the likelihood as a function of the dark matter slope \( \beta_h \) shown in Figure 3. The dark matter slope is constrained to be \( \beta_h = 1 \pm 0.35 \) (95% confidence). This is quite consistent with the results found by Gavazzi et al. which perhaps is not too surprising: since we found earlier that the velocity dispersion data are not terribly restrictive, it is reasonable that the joint constraints from dynamics and detailed lens modeling are similar to those obtained from lens modeling alone. Typical halo virial masses and concentrations obtained were roughly \( M_h \sim 7 \times 10^{14} M_\odot \) and \( c_{\text{vir}} \sim 7 \). Incidentally, we find that mild halo ellipticities \((\epsilon_h \sim 0.2)\) and mild misalignment between halo and galaxy \((\Delta \theta \lesssim 10^\circ)\) are favored by the fits.

3. Discussion

We have argued that the stringent constraints claimed by Sand et al. (2003) on the inner slope of the dark mat-

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**Fig. 2.**—Constraints on dark matter inner slope \( \beta_h \) and ellipticity \( \epsilon_h \) for Abell 383. Contours are drawn at the 68, 90, 95 and 99% confidence levels.

**Fig. 3.**—Likelihood as a function of the dark matter slope \( \beta_h \) in MS 2137–23, based on lensing, X-ray temperature, and dynamics.
\[ \rho(x = r/r_s) = \frac{\rho_s}{x^\beta(1 + x)^{n-\beta}} , \]

where \( \Sigma(\beta) \) is the stellar density, \( G \) is Newton's constant and \( \rho \) and \( M \) are the mass and luminosity profiles respectively, \( r \) is the radius, and \( F(x) = \frac{x}{\sqrt{1 - x^2}} + \sin^{-1}(x) - \frac{\pi}{2} \) for \( x < 1 \) and \( 0 \) for \( x \geq 1 \). The density profile is given by

\[ \rho(r) = \frac{GM(r)}{r^2} \]

where \( M(r) \) is the total mass enclosed within radius \( r \). The velocity dispersion is given by

\[ \sigma^2(r) = \frac{1}{\rho(r)} \int_0^\infty \rho(r') \frac{GM(r')}{r'^2} dr' , \]

where \( \Sigma(r) \) is the projected surface density. Averaging over a finite radial bin from \( R_1 \) to \( R_2 \) gives

\[ \sigma^2_{\text{bin}}(R_1, R_2) = \frac{\int_{R_1}^{R_2} \sigma^2(R) \Sigma(R) dR}{\int_{R_1}^{R_2} \Sigma(R) dR} = \frac{1}{2} \int_{R_1}^{R_2} \rho(r)GM(r) \left[ F\left( \frac{R}{R_s} \right) - F\left( \frac{R_1}{R_s} \right) \right] dr , \]

where \( A(x) = \left\{ \begin{array}{ll} \sin^{-1}(x) & x < 1 \\ \frac{\pi}{2} & x \geq 1 \end{array} \right. \) and \( F(x) = \left\{ \begin{array}{ll} x\sqrt{1-x^2} + \sin^{-1}(x) - \frac{\pi}{2} & x < 1 \\ 0 & x \geq 1 \end{array} \right. \)

For the density profiles we have employed, of the form

\[ \rho(x = r/r_s) = \frac{\rho_s}{x^\beta(1 + x)^{n-\beta}} , \]

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For the density profiles we have employed, of the form

\[ \rho(x = r/r_s) = \frac{\rho_s}{x^\beta(1 + x)^{n-\beta}} , \]
there are no closed-form expressions for the projected surface density in terms of elementary functions. The convergence \( \kappa = \Sigma / \Sigma_{\text{crit}} \) takes the form (e.g., Wyithe et al. [2001])

\[
\kappa(u = R/r_s) = 2\kappa_s u^{1-\beta} \int_0^{\pi/2} \frac{(\sin \theta)^{n-2}}{(u + \sin \theta)^{n-\beta}} d\theta,
\]

(5)

where \( \kappa_s = \rho_s r_s / \Sigma_{\text{crit}} \) and as usual \( \Sigma_{\text{crit}} \) is the lensing critical surface density. For elliptical surface density profiles with axis ratio \( q = 1 - e \), we use \( \kappa(\sqrt{x^2 + y^2}/q^2) \) with \( \kappa_s = \rho_s r_s / (q^2 \Sigma_{\text{crit}}) \). Keeton [2001] discusses how the deflection angle and distortion tensor may be expressed as one-dimensional integrals over \( \kappa \) and its derivatives. The ellipticities of interest to us are small enough (\( e \sim 0.1-0.2 \) in the density, even smaller in the potential) that we assume it is a reasonable approximation to use the spherical Jeans equations to compute the line-of-sight velocity dispersion (Kochanek [1994]).

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