Preventing collusion created by tie-breaking rules

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Abstract

Games played in the last round of a round-robin tournament inspire match-fixing or tacit collusion if the two opposing teams can benefit from a particular result at the expense of other teams. In the case of four teams, the current study identifies all these situations caused by using head-to-head records as the primary tie-breaking principle. Simulations based on the 2016 UEFA European Football Championship reveal that the official tie-breaking policy substantially increases the risk of collusion, but it can be mitigated by choosing an optimal order of matches. Following the proposed schedule improves the competitiveness of the two games played in the last round and requires no reform on the format of the competition.

Keywords: football; match-fixing; OR in sports; simulation; tournament design

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1 "What here seems so prolix in the explanation is often decided in the concrete case at first sight; but still, the tact of a practised judgment is required for that, and person must have thought over every one of the cases now developed." (Source: Carl von Clausewitz: On War, Book 6, Chapter 24 [Operating Against a Flank]. Translated by Colonel James John Graham, London, N. Trübner, 1873. http://clausewitz.com/readings/OnWar1873/TOC.htm)
1 Introduction

A round-robin tournament is a contest where each contestant plays against every other participant. Most association football (henceforth football) leagues are organised in a double round-robin format, that is, each team meets all the others once at home and once away. The system is also used in major football tournaments and their qualifications, such as the FIFA World Cup and the UEFA European Football Championship (shortly UEFA Euro).

The competitors in a round-robin tournament are usually ranked by converting wins, draws, and losses into points, which are then added up. However, two teams can score the same number of points, when further rules need to be applied to resolve the ties. In football, two main concepts are used for this purpose (Berker, 2014; Csató, 2021; Pakaslahti, 2019): (1) head-to-head records; and (2) goal difference.

The choice of tie-breaking criteria has non-negligible sporting effects as the following historical cases make clear.

Table 1: Ranking in Group C of the 2004 UEFA European Football Championship before the last matchday

| Pos | Team     | W | D | L | GF | GA | GD | Pts |
|-----|----------|---|---|---|----|----|----|-----|
| 1   | Sweden   | 1 | 1 | 0 | 6  | 1  | +5 | 4   |
| 2   | Denmark  | 1 | 1 | 0 | 2  | 0  | +2 | 4   |
| 3   | Italy    | 0 | 2 | 0 | 1  | 1  | 0  | 2   |
| 4   | Bulgaria | 0 | 0 | 2 | 0  | 7  | −7 | 0   |

Pos = Position; W = Won; D = Drawn; L = Lost; GF = Goals for; GA = Goals against; GD = Goal difference; Pts = Points. All teams have played two matches.

Example 1. (Kendall and Lenten, 2017, Section 3.9.3) Table 1 shows the standing of Group C in the 2004 UEFA European Football Championship after two matchdays, with the games Italy vs. Bulgaria and Denmark vs. Sweden still to be played.

The top two teams advanced to the quarterfinals. If two or more teams in the group were equal on points on completion, the following tie-breaking criteria were applied in the order given:

1. Higher number of points obtained in the matches played among the teams in question;
2. Superior goal difference in the matches among the teams in question;
3. Number of goals scored in the matches among the teams in question;

Further tie-breaking rules are not important for our discussion.

Bulgaria could have finished with at most three points. Italy would have five points by defeating Bulgaria, thus, Italy might have qualified if Denmark and Sweden did not play a draw. If Denmark and Sweden would have drawn and the score had at least 2-2, then both of them would have qualified independently of the result of Italy vs. Bulgaria. Therefore, the two Scandinavian teams had an opportunity to collude because of using head-to-head records as the primary tie-breaking principle. However, if the order of teams with equal points would have been decided by goal difference, such a collusion would be impossible.
Table 2: Ranking in Group C of the 2017 UEFA European
Under-21 Championship before the last matchday

| Pos | Team           | W | D | L | GF | GA | GD | Pts |
|-----|----------------|---|---|---|----|----|----|-----|
| 1   | Germany        | 2 | 0 | 0 | 5  | 0  | +5 | 6   |
| 2   | Czech Republic | 1 | 0 | 1 | 3  | 3  | 0  | 3   |
| 3   | Italy          | 1 | 0 | 1 | 3  | 3  | 0  | 3   |
| 4   | Denmark        | 0 | 0 | 2 | 0  | 5  | −5 | 0   |

Pos = Position; W = Won; D = Drawn; L = Lost; GF = Goals for; GA = Goals against; GD = Goal difference; Pts = Points. All teams have played two matches.

To summarise, Denmark and Sweden knew that both teams progress with a draw of 2-2. And the final result was 2-2, although both teams denied any collusion.

**Example 2.** Table 2 presents the standing of Group C in the 2017 UEFA European Under-21 Championship after two matchdays, with the games Italy vs. Germany and the Czech Republic vs. Denmark still to be played. The first tie-breaking criteria were analogous to the 2004 UEFA European Football Championship, see Example 1.

Besides the three group winners, the best runner-up also advanced to the semifinals. Obviously, the runners-up were evaluated based on the number of points, followed by goal difference. The other two groups were already finished, and the better second-placed team was Slovakia, which scored 6 and conceded 3 goals, implying a goal difference of +3.

In this situation, Italy and Germany had an opportunity to collude. If Italy defeats Germany by 4-2, then two cases should be distinguished:

1. If the Czech Republic wins against Denmark, three teams have 6 points. The head-to-head goal differences are 3-3 (Czech Republic), 5-5 (Italy), 4-4 (Germany). Therefore, Italy qualifies as the group winner and Germany as the runner-up due to scoring more goals than Slovakia.

2. If the Czech Republic does not win against Denmark, Italy qualifies as the group winner and Germany as the runner-up.

Finally, Italy beat Germany 1-0 and Denmark defeated the Czech Republic, thus Italy and Germany progressed at the expense of Slovakia. The prime minister of Slovakia wrote an open letter to UEFA, emphasising that “I believe that you will have the whole issue investigated and UEFA will make rules for the next tournament which will give precedence to a sportsmanlike performance instead of unfair agreements” (BBC, 2017).

According to Examples 1 and 2, preferring head-to-head results to goal difference may threaten the competitiveness of games played in the last round of a round-robin contest by creating a match-fixing opportunity. Consequently, if the final result lies in the appropriate “collusion zone”, suspicions will immediately arise even if the two opposing teams have exerted full effort to score. It is important to emphasise that, similar to Chater et al. (2021), we call a match to be vulnerable to collusion if two teams playing against each other can secure high-ranked positions for them such that they can be ranked lower than these places in some situations. For instance, Germany had at least some incentives to be satisfied with a loss of 1-0 in Example 2 since a loss of 3-0 would have led to elimination.

The current paper will attempt to quantify the probability of collusion opportunities caused by using head-to-head records as the primary tie-breaking principle. In particular,
our case study will be the UEFA European Football Championship, a tournament where this rule is applied. We will provide a straightforward recommendation for the Union of European Football Associations (UEFA) on how to schedule group matches in order to minimise the threat of match-fixing scandals.

Our work is a close companion to Chater et al. (2021) and Guyon (2020) that analyse different formats of the FIFA World Cup to increase the stakes of the games played in the last round of the group stage. However, there is a crucial difference to both studies: FIFA favours goal difference in tie-breaking, which, as demonstrated by Examples 1 and 2, eliminates a number of collusion opportunities compared to the UEFA policy of preferring head-to-head results.

Hence, we also contribute to the relatively limited literature on tie-breaking rules for sports tournaments. Berker (2014) calculates how often the relative ranking of two teams depends on the outcome of a game played by other teams under the two main tie-breaking concepts. According to the philosophical arguments of Pakaslahti (2019), tie-breaking systems of round-robin football competitions should give more importance to overall goal difference than to head-to-head records. Csató (2021, Chapter 1.3) demonstrates the lack of consensus around tie-breaking criteria in the top tier football leagues in Europe.

The main novelties of our research can be summarised as follows:

• In the last round of a round-robin contest with four teams, all match-fixing opportunities arising from preferring head-to-head records for tie-breaking are identified.

• Scheduling options are compared with respect to the probability of these unwanted situations in the 2016 UEFA Football European Championship via simulations.

• The probability of collusion opportunities is uncovered to increase by multiple times if head-to-head results are favoured over goal difference.

Fortunately, the optimal match schedule is found to be wholly in line with the previous recommendations from the literature (Chater et al., 2021; Stronka, 2020): the strongest team should play against the weakest team in the last round. Therefore, while tournament organisers create several fairness issues by choosing head-to-head results for tie-breaking (see also Berker (2014) and Pakaslahti (2019)), at least the competitiveness of the games can be maximised with the same policy. Since the proposed schedule does not affect other aspects of the competition (number of teams, games played, qualification rules, etc.), UEFA is strongly encouraged to follow it in the future.

The paper is structured as follows. Section 2 presents the methodology of the study. The design of the UEFA Euro 2016 is discussed in Section 2.1, the match-fixing opportunities arising from the preference of head-to-head results to goal difference are characterised in Section 2.2, and the simulation model is provided in Section 2.3. Section 3 is devoted to estimating the risk of match-fixing for the groups of the UEFA Euro 2016. We end with some discussions and conclusions in Section 4.

2 Methodology

In the following, the theoretical background of the study will be discussed. In particular, 2.1 presents the design of the recent and future UEFA Euros, the competition that provides our case study. The collusion opportunities implied by the ranking rules are explored in Section 2.2. Finally, the simulation model is detailed in Section 2.3.
Table 3: The groups of the 2016 UEFA European Football Championship

| Team       | Elo  | Team       | Elo  | Team       | Elo  |
|------------|------|------------|------|------------|------|
| France     | 1954 | England    | 1959 | Germany    | 2069 |
| Switzerland| 1810 | Russia     | 1774 | Ukraine    | 1817 |
| Romania    | 1764 | Slovakia   | 1744 | Poland     | 1773 |
| Albania    | 1608 | Wales      | 1673 | Northern Ireland | 1594 |

| Team       | Elo  | Team       | Elo  | Team       | Elo  |
|------------|------|------------|------|------------|------|
| Spain      | 2003 | Belgium    | 1928 | Portugal   | 1919 |
| Croatia    | 1814 | Italy      | 1871 | Austria    | 1801 |
| Czech Republic | 1766 | Sweden     | 1767 | Hungary    | 1700 |
| Turkey     | 1805 | Rep. of Ireland | 1770 | Iceland   | 1673 |

**Abbreviations:** Rep. of Ireland = Republic of Ireland.

The strengths of the teams are measured by their World Football Elo Ratings on 11 December 2015 (the date of the group draw), see [https://www.international-football.net/elo-ratings-table?year=2015&month=12&day=11&confed=UEFA](https://www.international-football.net/elo-ratings-table?year=2015&month=12&day=11&confed=UEFA).

2.1 The format of the UEFA Euro 2016

The 2016 UEFA European Football Championship has been the first European Championship contested by 24 teams, which have been divided into six round-robin groups of four teams each. The tournament is chosen for further analysis because the European Championship is a quadrennial competition and the 2020 edition was unusually organised in 11 cities around the continent—but the UEFA Euro 2024 will be hosted by one particular nation (Germany), similar to the UEFA Euro 2016.

The composition of the groups is shown in Table 3. The group ranking is based on the number of points, followed by head-to-head records as described in Example 1.

The organiser can decide the order of group matches. Denote the four teams by $T_1$–$T_4$ according to their strength. Since each team plays three games in the group, three scheduling alternatives exist for the last round, where potential match-fixing may arise:

- **Schedule A:** $T_1$ plays against $T_2$ and $T_3$ plays against $T_4$;
- **Schedule B:** $T_1$ plays against $T_3$ and $T_2$ plays against $T_4$;
- **Schedule C:** $T_1$ plays against $T_4$ and $T_2$ plays against $T_3$.

2.2 Types of match-fixing

Two teams $A$ and $B$ can exploit the use of head-to-head records as primary tie-breaking criteria to collude if: (a) at least three teams, including $A$ and $B$, can be equal on points on completion before the last two games are played; and (b) they play against each other in the last round. It is straightforward to check that these two teams might finish with 2, 3, 4, 5, or 6 points. Based on this observation, all match-fixing opportunities can be derived by “backward induction”:
1. Teams $A$ and $B$ finish with 6 points
Before the last round, $A$ has had 6 and $B$ has had 3 points such that the goal difference of the other team with 3 points has not been positive.
The scenario is called Robust collusion 1 (RC1) because the game $A$ vs. $B$ has an outcome which ensures that teams $A$ and $B$ will be the top two teams, independently of the result of the other match played in the last round.
This situation is analogous to Example 2.

2. Teams $A$ and $B$ finish with 5 points
Before the last round, both $A$ and $B$ have had 4 points, and a third team has had 2 points.\(^2\)
The scenario is called Robust collusion 2 (RC2) because the game $A$ vs. $B$ has an outcome which ensures that teams $A$ and $B$ will be the top two teams, independently of the result of the other match played in the last round.
This situation is analogous to Example 1.

3. Teams $A$ and $B$ finish with 4 points
Before the last round, $A$ has had 4 points and $B$ has had 1 point such that the goal difference of the team with 3 points has not been positive.
The scenario is called Conditional collusion 1 (CC1) because the game $A$ vs. $B$ has an outcome which ensures that teams $A$ and $B$ will be the top two teams if the other match played in the last round is a draw.

4. Teams $A$ and $B$ finish with 3 points
Before the last round, $A$ has had 3 and $B$ has had 0 points such that the goal difference of the other team with 3 points has not been positive.
The scenario is called Conditional collusion 2 (CC2) because the game $A$ vs. $B$ has an outcome which ensures that teams $A$ and $B$ will be the top two teams if the other match played in the last round is won by the team which has had 6 points after two rounds due to defeating both $A$ and $B$.

5. Teams $A$ and $B$ finish with 2 points
Before the last round, both $A$ and $B$ have had 1 point.
The scenario is called Conditional collusion 3 (CC3) because the game $A$ vs. $B$ has an outcome which ensures that teams $A$ and $B$ will be the runner-up and the third-placed team if the other match played in the last round is won by the team which has had 6 points after two rounds due to defeating both $A$ and $B$.

Finally, note that the conditions of RC1 and CC2 might hold simultaneously if the team having 6 points plays against a team having 3 points in the last round (consequently, the other team having 3 points plays against the team having 0 points) and the goal differences of both teams with 3 points are not positive. This scenario is called Dual collusion (DC).

The conditions for all types of match-fixing opportunities are summarised in Table 4.

### 2.3 The simulation model

Tournament designs—in our case, the three scheduling options (see Section 2.1)—are usually compared by simulating the matches on the basis of a reasonable statistical model

\(^2\) If $A$ and $B$ have had 4 points after two rounds, the other two teams might have 1 point each. Then teams $A$ and $B$ can secure the first two positions for them by playing a draw, independently of the tie-breaking criteria.
Table 4: Conditions for different types of match-fixing before the last round

| Type of collusion | Point distribution | Games to be played | Goal difference |
|-------------------|-------------------|--------------------|----------------|
| RC1               | [6, 3, 3, 0]      | (6 vs 3); (3 vs 0) | ≤ 0 for 3 in (3 vs 0) |
| RC2               | [4, 4, 2, 0]      | (4 vs 4); (2 vs 0) | —               |
| CC1               | [4, 3, 2, 1]      | (4 vs 1); (3 vs 2) | ≤ 0 for 3       |
| CC2               | [6, 3, 3, 0]      | (6 vs 3); (3 vs 0) | ≤ 0 for 3 in (6 vs 3) |
| CC3               | [6, 2, 1, 1]      | (6 vs 2); (1 vs 1) | —               |
| DC                | [6, 3, 3, 0]      | (6 vs 3); (3 vs 0) | ≤ 0 for 3 in (3 vs 0) |

In the last two columns, the teams are denoted by their number of points scored. Goal difference is irrelevant for types RC2 and CC3 because head-to-head results serve as the primary tie-breaking criteria.

(Scarf et al., 2009). Since the number of goals scored and conceded is of high importance for us, the traditional Poisson model is used to obtain these values (Maher, 1982; Van Eetvelde and Ley, 2019). If the match is played on field $f$ (home, neutral, or away), team $i$ scores $k$ goals against team $j$ with a probability of

$$P_{ij}(k) = \frac{\left(\lambda_{ij}^{(f)}\right)^k \exp\left(-\lambda_{ij}^{(f)}\right)}{k!},$$

(1)

where $\lambda_{ij}^{(f)}$ is the expected number of goals scored by team $i$ in this particular match.

The strength of a team is measured by its World Football Elo Rating as it is widely accepted in the literature (Cea et al., 2020; Gámez and Royuela, 2016; Hvattum and Arntzen, 2010; Lasek et al., 2013). The Elo ratings of the teams on the day of the UEFA Euro 2016 draw are shown in Table 3. The implied win expectancy of team $i$ against team $j$ is:

$$W_{ij} = \frac{1}{1 + 10^{-\left(E_i - E_j\right)/400}},$$

where $E_i$ and $E_j$ are the Elo ratings of teams $i$ and $j$, respectively, such that the rating of the home team is increased by 100 points.

Football rankings (2020) has estimated the parameter $\lambda_{ij}^{(f)}$ in (1) as the function of win expectancy $W_{ij}$ based on more than 40 thousand matches played by national football teams. We have recently applied this approach to quantify the unfairness of the 2018 FIFA World Cup qualification (Csató, 2022b) and to analyse the incentive incompatibility of the European Qualifiers for the 2022 FIFA World Cup (Csató, 2022a). Since the novelty of our paper does not reside in the simulation model, readers are directed to these articles for further details.

The competitiveness of any group format can be assessed by simulating the first two rounds of the games to obtain the probability of each type of collusion opportunity. The number of iterations $N$ affects the reliability of the results: when a probability $p$ of an event is estimated by its relative frequency in the simulation, the margin of error required for 99% confidence is approximately $2.6\sqrt{p(1-p)/\sqrt{N}}$. Our choice of $N = 1$ million essentially means that it is sufficient to focus on relative frequencies because the margin of error is at most 0.13%.
Figure 1: The probability of match-fixing opportunities before the last round
3 The relative frequencies of collusion opportunities

Figure 1 conveys the main message of the paper by presenting the probability of different types of match-fixing under all possible schedules in a group of four. The risk of robust collusion—when two teams can secure the first two positions for them, independently of the outcome of the other match played in the last round—is minimised if the strongest and the weakest team play against each other in the last round (Schedule C). This can be easily explained: robust collusion is possible between two teams having the most points after two rounds, which rarely occurs if the weakest team has already played against the strongest.

Note also that RC1 is more than five times more likely compared to RC2 because a team should play two draws out of two matches for RC2. Furthermore, both types of collusion have a much higher probability than the chance of collusive behaviour found by Chater et al. (2021) in the case of the FIFA World Cup (between 0.96% and 1.76% for the three settings). The crucial difference is caused by the tie-breaking rule, which is goal difference in the FIFA World Cup. Therefore, the role of tie-breaking cannot be neglected in the investigation of competitiveness.

Conditional collusion is a less serious threat as match-fixing will be successful only if the result of the other game played in the last round is appropriate. Again, CC1 and CC3 are substantially less frequent than CC2 since both CC1 and CC3 require a team to play two draws out of two matches. From the perspective of conditional collusion, Schedule A is a better option than Schedule C: the latter minimises only CC3, which is more than compensated by the smaller chance of CC1 and, especially, CC2 under Schedule A. Finally, dual collusion (when the conditions of RC1 and CC2 hold simultaneously) has essentially the same probability according to Schedules A and C.

To conclude, schedule C seems to be the best alternative to reduce collusion opportunities in a group of four teams owing to the following considerations:

- The advantage of Schedule C over Schedule A is between 4% and 8.3% in the probability of RC1, while the advantage of Schedule A over Schedule C is between 4% and 6% in the probability of CC2.
- Even though the superiority of Schedule C in RC2 and CC3 does not always offset numerically the dominance of Schedule A in CC1, the overall balance tends to favour Schedule C for the set of match-fixing types RC2, CC1, and CC3, after taking into account the different quality of robust and conditional collusion.

On the other hand, Schedule B is certainly the worst alternative.

4 Discussion

This study has investigated how preferring head-to-head records among the tie-breaking criteria creates collusion and match-fixing opportunities. Situations, when two opposing teams can secure certain positions for them with a particular outcome, need to be avoided because it is impossible to prove that the teams played competitively if the beneficial result occurs. Inspired by two real-world examples from European football, we have identified five types of potential match-fixing in the last round of a single round-robin tournament with four teams, all of them depending on the use of head-to-head results to break the ties among teams with the same number of points.
In particular, two teams can guarantee the first two positions for them in two types of tacit collusion (RC1 and RC2) before the last round, independently of the result of the simultaneous game played by the other two teams. Three further types of match-fixing are conditional on the result of the parallel game, one of them gives the top two positions for the conspiring teams (CC1), and the other two provide the second and third places in the group (CC2 and CC3). These instances of match-fixing might seem redundant at first glance as usually the group winner and the runner-up qualify, but they are relevant for the UEFA Euro where the four best third-placed teams also advance to the Round of 16.

Thus, the six groups of the UEFA Euro 2016 have been simulated in order to obtain the probability of each collusion opportunity as the function of the game schedule. Favouring head-to-head results over goal difference increases the threat of match-fixing by multiple times compared to the FIFA World Cup. However, an appropriate schedule offers a remedy to improve the competitiveness of the games played in the last round: the a priori strongest team should face the weakest one to minimize the risk of collusion. While the results of our simulation depend on the quality of the statistical model, the study of six different groups provides a kind of sensitivity analysis, and the findings are similar for all cases.

The practice of UEFA is worth discussing, too. Head-to-head results are used for tie-breaking since the 1996 UEFA European Football Championship (UEFA, 2003). In the 1996 edition, only four teams were seeded, thus, the match schedules are difficult to identify. The tournament contained four groups of four teams each until 2012 and is organised with six groups of four teams each since 2016. The UEFA Euro 2020 took place in 11 cities across 11 countries, meaning that scheduling has many aspects other than increasing competitiveness and minimising the threat of collusion. The order of group matches for the UEFA Euros from 2000 to 2016 is shown in Table 5. Seemingly, UEFA has not followed any pattern and decided randomly that often led to Schedule B, the worst possible option regarding collusion opportunities. The schedule of the UEFA Euro 2024 has also been announced recently (UEFA, 2022), but it depends on the order in the draw, which is yet to be made. Hence, it is still possible to follow our recommendation by using Schedule C across all groups.

The current paper reinforces a crucial message of the academic literature for tournament organisers: scheduling is a powerful tool to increase the competitiveness of the games played at the end of a contest. Since previous research has neglected the role of tie-breaking rules, we have verified that the optimal order of the games does not change in a single round-robin tournament with four teams if head-to-head results are the primary tie-breaking criteria instead of goal difference. Our main finding can be immediately used by decision-makers to guarantee more exciting sports competitions, including the next editions of the UEFA Euro. The recent modification of the knockout bracket in this tournament has been based...
on a research study (Guyon, 2018), which gives hope that UEFA is open to similar design changes in order to address fairness issues.

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