Composite (pseudo) scalar contributions to muon $g - 2$

Deog Ki Hong$^{\dagger}$ and Du Hwan Kim

Department of Physics, Pusan National University, Busan 46241, Korea

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Abstract

We have calculated the composite (pseudo) scalar contributions to the anomalous magnetic moment of muons in models of walking technicolor. By the axial or scale anomaly the light scalars such as techni-dilaton, techni-pions or techni-eta have anomalous couplings to two-photons, which make them natural candidates for the recent 750 GeV resonance excess, observed at LHC. Due to the anomalous couplings, their contributions to muon $(g - 2)$ are less suppressed and might explain the current deviation in muon $(g - 2)$ measurements from theory.

Keywords: 750 GeV resonance, walking technicolor, muon g-2
Introduction

After discovery of Higgs boson [1, 2], signals for new physics beyond standard model (BSM) have been intensively searched at LHC. Very recently both the ATLAS and CMS group have observed an excess at 750 GeV with the local significance by about 3 $\sigma$ in the diphoton channel at the 13 TeV LHC [3, 4], which, if confirmed, will be a genuine direct signal for new physics at colliders. There have been since proposed numerous models of BSM to explain this single resonance. In establishing correct models of BSM, it will be therefore desired to constrain those proposed models, if possible, from the precision measurement of low energy physics [5–7], which often severely constrain BSM models, otherwise difficult to be ruled out at colliders.

It is well known that the standard model (SM) estimation of the anomalous magnetic moment of the muon has quite a significant deviation from the experiments, which might be due to a new physics beyond standard model. Recent measurements of the anomalous magnetic moment of the muon [8], performed at the Brookhaven National Laboratory (BNL), find

$$a_\mu = 11659208.0(5.4)(3.3) \times 10^{-10},$$

which deviates by 3.2 $\sigma$ above the current SM estimate, based on $e^+e^-$ hadronic cross sections [9, 10]. An improved muon ($g - 2$) experiment is approved and under construction at the Fermilab to achieve a precision of 0.14 ppm [11, 12], which will move the deviation, if persistent, to 5 $\sigma$.

In this paper, we estimate the new physics contributions to the anomalous magnetic moment of muons to see if it naturally fixes the current deviation, provided that the recent excess at 750 GeV at ATLAS and CMS is due to the scalar or pseudo-scalar resonances, predicted in the models of new strong dynamics such as walking technicolor [13, 14] or models of composite axions [15].

Candidates for 750 GeV resonance

The anomalous magnetic moment of muons is one of a few physical observables that are measured so precisely, with an accuracy of parts per million (ppm). It is therefore quite sensitive to new physics at TeV, since generically the new physics contribution to the
anomalous magnetic moment of muons is given by the dimensional analysis as

\[ a_\mu^{\text{NP}} \sim \frac{\alpha_{\text{em}}}{\pi} \left( \frac{m}{M_{\text{NP}}} \right)^2 \sim 10^{-10} \left( \frac{1 \text{ TeV}}{M_{\text{NP}}} \right)^2, \]  

(2)

where \( m \) is the muon mass and \( M_{\text{NP}} \) is a typical scale of new physics. Generically the new physics contribution is well within the experimental accuracy and thus might explain the current 3.2 \( \sigma \) deviation [10] if \( M_{\text{NP}} \) is not too higher than 1 TeV. Indeed the new physics contribution is well studied up to two-loops for the weakly interacting new particles to exclude certain parameter regions in some extension of the standard model such as MSSM or simplified models [16, 17]. In this paper we focus on strong dynamics extension of the standard model, especially the walking technicolor (WTC) models which break dynamically not only the electroweak symmetry but also the approximate scale symmetry, introduced to accommodate the constraints from the electroweak precision measurements [18–20].

By the hypothesis of partially conserved dilatation currents (PCDC) among the spin-0 excitations of WTC the lightest one should be the techni-dilaton, which is a pseudo Nambu-Goldstone boson, associated with the spontaneous broken scale symmetry [19, 21–23]. Since PCDC assumes the techni-dilaton saturates the matrix elements of dilatation current at low energy, we have from the trace anomaly [22, 23]

\[ F_D^2 M_D^2 \sim m_{\text{TC}}^4, \]  

(3)

where \( F_D \) and \( M_D \) are the dilaton decay constant and mass, respectively, and the trace anomaly is given mostly by the dynamical mass of techni-fermions [24], \( m_{\text{TC}} \), which characterizes the IR scale of WTC, about 1 TeV. Having the theory very near the quasi infrared (IR) fixed point, one can separate widely the ultra-violet (UV) scale from the IR scale of WTC or \( F_D \gg m_{\text{TC}} \) to have a light dilaton, \( M_D \sim m_{\text{TC}}^2 / F_D \ll m_{\text{TC}} \sim 1 \text{ TeV} \) [22, 23]. It is therefore quite natural to interpret the 125 GeV boson, discovered at LHC [1, 2] as the techni-dilaton, if WTC is responsible for electroweak symmetry breaking that describes the BSM physics. Compared to the standard model Higgs, the techni-dilaton couples to gluons more strongly but to SM fermions more weakly. Hence, with current LHC data the techni-dilaton is still a viable interpretation of the 125 GeV boson [25]. On the other hand, if WTC is not so extremely conformal, the IR and UV scales are not widely separated and the techni-dilaton mass will not be much smaller than the typical IR scale of WTC, \( m_{\text{TC}} \sim 1 \text{ TeV} \), but it should be still the lightest one in the spectrum by PCDC, though it
lies close to other scalar excitations such as the composite Higgs. In this case the 125 GeV boson may be interpreted as the composite Higgs of WTC [20], since it can be very light due to the top-quark loop corrections [26], and then the 750 GeV resonance may be interpreted as the techni-dilaton of WTC that decays into two photons by the trace anomaly.

Being Nambu-Goldstone bosons, pseudo scalars that are associated with spontaneously broken chiral symmetry of techni-fermions in WTC are generically also light, though they are strongly coupled. If the 750 GeV resonance is a pseudo scalar, it may be interpreted as either techni-pion [14] or techni-eta in WTC [27] or a composite axion [15], which decay into two photons by the Adler-Bell-Jackiw anomaly [28, 29].

New scalars contributions to muon $(g - 2)$

The low energy interaction Lagrangian density, relevant for our discussions on the muon $(g - 2)$, is given as

$$\mathcal{L}_{\text{int}} = -\bar{\psi} (g_D \varphi + i \gamma_5 g_A \mathcal{P}) \psi + e^2 \frac{c_D}{4F_D} \varphi F_{\mu\nu} F^{\mu\nu} + e^2 \frac{c_A}{4F_A} \mathcal{P} F_\mu \tilde{F}^{\mu\nu},$$

(4)

where the techni-dilaton ($\varphi$) coupling to the muon field, denoted as $\psi$, $g_D = (3 - \gamma_m) m / F_D$ with the anomalous dimension of the techni-fermion bilinear, $\gamma_m \approx 1$. The pseudo-scalar coupling $g_A$, induced by the extended technicolor (ETC) is given as $\sqrt{3} m / (2 F_D) [30]$. $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, the field strength tensor of the photon field $A_\mu$ with the electric charge $e$ and $\tilde{F}_{\mu\nu}$ is its dual. The two-photon coupling of techni-dilaton, $c_D$, or pseudo-scalar ($\mathcal{P}$), $c_A$, is in general the momentum-dependent anomalous form factor but can be regarded as a constant in the effective theory, which is determined by the UV physics anomaly.

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1 The couplings of composite (pseudo) scalars with other standard model particles besides muons and photons will be relevant only for two or higher loop contributions to muon $g - 2$. For instance the techni-dilaton coupling to two-photons through the W boson loop will contribute to muon $g - 2$ at two-loop, as in the case of Higgs contributions, but further suppressed by $(v_{\text{ew}} / F_D)^2$ and thus much smaller than our one-loop results.

2 The exact form of the anomalous form factor is difficult to calculate due to its non-perturbative nature. As in the QCD corrections to the light-by-light contribution to muon $(g-2)$, one may approximate, to correctly reproduce its asymptotic UV behavior similar to the Lepage-Brodsky formula in QCD, the anomalous form factor by a single (techni) vector-meson $F_{\gamma\gamma}(q^2, Q^2) \approx C_1 M_V^2 / (Q^2 + M_V^2) [10]$ or an infinite tower of (techni) vector mesons in holographic models [31]. However, since the form factor will significantly differ from the constant approximation only for the internal momentum bigger than the vector meson mass, $Q^2 \gtrsim M_V^2$ and the loop diagram Fig. 1(b) is dominant by the momentum smaller than the (pseudo) scalar.
At one-loop the (pseudo) scalar contributions to the anomalous magnetic moment of muons consists of two pieces (see Fig. 1). The diagram in Fig. 1(a), which is same as the one-loop Higgs contribution except the couplings and mass, gives

$$a_{NP(a)}^\mu \simeq \frac{g_i^2}{8\pi^2} \frac{m^2}{M_i^2} \ln \left( \frac{M_i^2}{m^2} \right),$$

where $i$ denotes either $D$ for the techni-dilaton or $A$ for the pseudo scalar fields. From the anomalous coupling diagram, Fig. 1(b), we find with $\bar{g}_i = g_i \cdot F_i / m$

$$a_{NP(b)}^\mu \simeq \frac{\alpha_{em}}{2\pi} \bar{g}_i c_i \frac{m^2}{F_i^2} \ln \left( \frac{16\pi^2 F_i^2}{M_i^2} \right) \sim 10^{-9} \left( \frac{\bar{g}_i c_i}{2.5} \right) \left( \frac{0.5 \text{ TeV}}{F_i} \right)^2. \tag{6}$$

where we have taken $4\pi F_i$ as the UV cutoff of the effective interactions in Eq. (4), following the naive dimensional analysis \[32\].

**Results and Discussion**

For the contribution from the diagram in Fig. 1(a), Eq. (5), which is doubly suppressed by $(m/F_i)^2$ and $(M_H/M_i)^2$, is more suppressed than the one-loop SM Higgs contribution, $a_{\mu}^{(2)EW(H)} < 5 \times 10^{-14}$.

$$a_{\mu}^{NP(a)} \approx a_{\mu}^{(2)EW(H)} \cdot \left( \frac{M_H}{M_i} \right)^2 \cdot \left( \frac{v_{EW}}{F_i} \right)^2, \tag{7}$$

where the vacuum expectation value of Higgs, $v_{EW} = 246$ GeV. $M_H = 125$ GeV is the Higgs mass and $M_i (F_i)$ is the mass (decay constant) of either the techni-dilaton or a pseudo scalar. On the other hand, since the one-loop contribution, Eq. (6), from the anomalous coupling, the diagram in Fig. 1(b) is singly suppressed by $(m/F_i)^2$, compared to the one-loop QED contribution, it may be comparable to the current $3.2\sigma$ deviation, $\Delta a_{\mu} = a_{\mu}^{exp} - a_{\mu}^{th} \approx (290 \pm 90) \times 10^{-11}$ \[10\]. Indeed, for $\bar{g}_i c_i = 2.5$ and $F_i = 0.5 \text{ TeV}$, we find that the new physics contribution Eq. (6) is of the order of $\Delta a_{\mu}$. However, if $F_i$ is much bigger than 0.5 TeV or the product of the Yukawa and diphotons couplings, $\bar{g}_i c_i$, of the (pseudo) scalar is too small, the muon anomaly may not be explained in models of WTC.

In models of WTC, the anomalous couplings $c_i$'s are generically too small to account for the muon anomaly, since they are suppressed by the techni-fermion loop factors \[^3\]. One

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\[^3\] In the case of one-family WTC model, the neutral techni-pion is conjectured to the 750 GeV resonance \[14\].

In this model, the number of technicolor $N_{TC} = 3$, the techni-pion decay constant $F_\pi = 123$ GeV, and $c_A \bar{g}_A = 1/(2\pi)^2$, which gives $a_{\mu}^{NP} \approx 6 \times 10^{-11}$. 

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mass, $M_i^2$, the error that we are making is about $M_i^2 / M_V^2 \approx (M_i / 4\pi F_i)^2 \approx 0.25$, if $M_V \gtrsim 1.5 \text{ TeV}$. 

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FIG. 1: One-loop corrections to the anomalous magnetic moment of muons. The dotted line denotes either the techni-dilaton field ($\varphi$) or a pseudo scalar $P$, the neutral techni-pion or techni-eta (or composite axion): (a) A diagram similar to the one-loop SM Higgs contribution. (b) One-loop diagram due to anomalous couplings of (pseudo) scalar to photons, denoted as a blob.

could add to this model an extra techni-lepton of an electric charge $qe$, which is electrically charged but QCD-color neutral, to enhance the diphoton coupling. For a minimal WTC model [20, 33], which has one techni-fermion doublet of the symmetric second-rank tensor with the electric charge $(q+1, q)$ will give the anomalous coupling $c_D = N_{TC}(N_{TC}+1)(2q^2 + 2q+1)/(12\pi^2)$ from the one-loop QED beta function. For $q = 3$ we get $c_D = 2.53$, if $N_{TC} = 3$, but the beta function is still perturbative, $\beta_{QED}(e)/e < 1$ [7].

4 However, if we introduce too many electrically-charged techni-fermions or a techni-lepton with too large electric charge, QED or $U(1)_Y$ might develop a Landau pole at much below the Planck scale.
with large axial charges.

To conclude, we have calculated the one-loop contributions to the anomalous magnetic moment of muons in models of walking technicolor, which contain generically a light techni-dilaton, techni-pions or techni-eta. At one-loop the diagrams that involve the anomalous coupling of (pseudo) scalar to two-photons is suppressed only by a single power of muon mass squared, \((m/F_i)^2\), compared the one-loop QED contribution, where \(F_i\) is the decay constant of either techni-dilaton or techni-pion (eta), roughly of the order of the ultra-violet scale of the effective interaction, Eq. [4]. We find for \(F_i \sim 0.5\) TeV and the anomalous coupling to diphoton \(c_i = \mathcal{O}(1)\) the one-loop contribution of WTC is comparable to the current 3.2\(\sigma\) deviation and thus may explain the deviation in the anomalous magnetic moment of muon. However, the anomalous couplings are generically small in models of WTC, since they are suppressed by the loop factors, unless one introduces extra techni-fermions with large electric charges or axial charges. In this case the 125 GeV Higgs is the composite Higgs and the 750 GeV resonance is the techni-dilaton or the techni-pion or techni-eta in the scenario of WTC.

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* dkhong@pusan.ac.kr

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Note added: After this paper is finished, it appeared the paper [34] which studies a similar problem but for the weakly interacting new (pseudo) scalars.