Microvibration Attenuation based on $H_\infty$/LPV Theory for High Stability Space Missions

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Abstract. This paper presents a LPV (Linear Parameter Varying) solution for a mixed passive-active architecture used to mitigate the microvibrations generated by reaction wheels in satellites. In particular, $H_\infty$/LPV theory is used to mitigate low frequency disturbances, current baseline for high frequency microvibration mitigation being based on elastomer materials. The issue of multiple harmonic microvibrations is also investigated. Simulation results from a test benchmark provided by Airbus Defence and Space demonstrate the potential of the proposed method.

1. Introduction

1.1. Motivations and learnt lessons from space experiments

As line-of-sight stability requirements get tighter with the increasing resolution, microvibrations become a key contributor to the performance of an increasing number of Earth observation and space science missions. The micro-dynamic behaviour of observation satellites has been studied as a potential contributor to the performance since the development of the SPOT/HELIOS satellite family. In the 80s and 90s, Airbus Defence and Space started gathering flight observations of microvibration effects, e.g. Doppler images on the SOHO satellite and the MICROMEDY experiment dedicated to the observation of SPOT 4 microvibrations, to name a few. Microvibrations degrade the image in two different ways: “high frequency” disturbances that cause image blurring, and “low frequency” vibrations that degrade the geometry inducing image distortions. In some cases, image distortions can be corrected on-ground by dedicated algorithms, but image blurring is definitely not rectifiable. This high/low frequency limit separation depends on the integration time: it is usually roughly equal to 0.1 times the inverse of the integration time. Hence, long integration time instruments like the ones used in scientific missions are more affected by high frequency microvibrations than short integration time ones as in LEO (Low Earth Orbit) observation missions. Any on-board equipment including one or several mobile devices shall then be considered as a potential source of microvibrations. This includes equipments such as Reaction Wheel Assemblies (RWAs) used for attitude control, gyroscopes, infrared-earth sensors, solar array drive mechanisms, thrusters, or payload elements such as pointing mechanisms, cryo-coolers, scan mechanisms, etc. However, the most critical
sources are RWAs that generate multi-frequency and time-varying harmonic disturbances. Mass imbalances and ball bearing imperfections are considered as the main causes of microvibrations.

Two harmonic disturbances are generally considered as the main effects of microvibrations: The first harmonic, denoted $H_{11}$ by the space industry, is due to unbalance: the force and torques are proportional to the square of the wheel rate. The second harmonic, denoted $H_{0,6}$ generates forces and torques in the same order of magnitude as $H_{11}$ but at lower frequency (below 60 Hz for a wheel rate below 100 Hz).

Airbus Defence and Space flight proven isolation solutions consists of passive isolator which provides good rejection at high frequencies. A passive isolator can be roughly compared to a second order filter with amplification at a resonance frequency, and a rejection factor increasing with the disturbance frequency. Current baseline for micro vibration mitigation is based on elastomer design initially developed on Earth observation missions (Theos, Pleiades). Although apparently simple this concept requires a deep understanding of the elastomer materials and their physical properties when developing passive isolators taking into account all the space-related constraints (thermal, radiations, lifetime, launch stresses), the needs of compatibility with AOCS (Attitude and Orbit Control System), the desired isolation performance, etc. However, passive solutions are insufficient for future demanding Earth and Science Observation programmes. Usually, the order of magnitude of the best performances that can be reached when using passive isolators only set at the base of typically 3 wheels is about 1 $\mu$rad for wheel rates varying from 0 to typically 6000 rpm; the major contributors are roughly in the 10 – 100 Hz range; performances below 10 Hz are usually better than 0.1 $\mu$rad, thanks to the naturally low wheel disturbances at these frequencies; performances above 100 Hz are also generally below 0.1 $\mu$rad, thanks to the passive isolator filtering. 20 dB of disturbances rejection above 10 Hz is then required to meet the 0.1 $\mu$rad objective and two solutions can be envisaged, i.e. i) the use of micro-propulsion system instead of wheels or ii) the use of mixed active/passive microvibration mitigation system. Micro-propulsion system allow to reach very high stability performances (< 0.01 $\mu$rad) but is very expensive, massive (> 140 kg without propellant) and complex whereas mixed active/passive microvibration mitigation provide lower cost/complexity and mass solution (< 30 kg with the worst case assumption) while allowing using the wheels over the full operating range and during all the mission phases. Missions for which stability requirements evolve between 0.01 and 0.1 $\mu$rad may be efficiently handled using such hybrid active/passive isolation solution.

The work presented in this paper should be understood in this context. It concerns the development of the active part of a mixed passive/active microvibrations mitigation solution.

1.2. Related work and antecedents
Mixed passive/active microvibrations attenuation solutions for satellites have been reported in a number of published works. In [1], a multi-purpose active isolation system based on six active struts (co-located piezo sensor and actuator), arranged in hexapod configuration, is considered. Active vibration isolation interface using piezoelectric ceramic actuators is discussed in [2]. A six-degree of freedom Stewart platform based on six local/decentralized integral force feedback controllers is proposed as a microvibration isolator in [3]. A complementary active control strategy is too discussed recently in [4].

Generally speaking, it could be argued that the aforementioned solutions suffer from a lack of formal proof, both in terms of stability and performances. More advanced control methodologies has to be investigated to provide formal stability and performance guarantees while taking into account the time-varying spectrum of microvibrations.

1 The terminology $H_{1}/H_{0,6}$ is used by the space industry to outline that the frequency of the microvibration is equal to $1/0.6\Omega$, $\Omega$ being the speed of the reaction wheel.
Linear Parameter Varying (LPV) control strategies offer a solution to this problem. LPV framework is appealing because it provides stability and performance guarantees over a wide range of changing parameters, see for instance the none exhaustive list of papers [5–8]. In our case, noting that the frequencies of the microvibrations generated by RWAs have a known dependency on the angular velocity of the reaction wheel [9], it seems clear that it can be used as a scheduling parameter for the active control strategy, leading the philosophy of LPV (Linear Parameter Varying) theory to be a suitable framework.

Note that LPV solutions have been successfully used for active vibration control in a number of other applications such as active magnetic bearing systems [10], noise canceling headsets [11] or car engines [12], to name a few.

1.3. Contribution
The main contribution of this paper is the development of a LPV solution to mitigate low frequency microvibrations. The issue of multiple harmonic (i.e. $H_1$ and $H_{0.6}$) microvibrations is also investigated. The work is an extension of previous research results developed by the authors, see [13] and [14]. It is shown how the proposed LPV controller is capable of attenuating time-varying harmonic disturbances. This is achieved by introducing a parameter dependent weighting function within the $H_{\infty}$/LPV synthesis procedure. This weighting function represents an extension of the notch filter used in [10]. This strategy is applied on a test benchmark provided by Airbus Defence and Space, that is representative of a flexible spacecraft structure.

2. Background on $H_{\infty}$/LPV theory
2.1. Disturbance rejection for time-varying frequencies
Consider a LTI (Linear Time Invariant) plant described by the following system:

$$G(s) : = \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Ed(\theta, t) \\ y(t) = Cx(t) + v(t) \end{cases}$$ (1)

with the state vector $x \in \mathbb{R}^{n_x}$, control signal $u \in \mathbb{R}^{n_u}$, disturbance $d \in \mathbb{R}^{n_d}$ and measurement vector $y \in \mathbb{R}^{n_y}$ with measurement noise $v \in \mathbb{R}^{n_v}$. It is assumed that the spectrum of the disturbance $d$ depends on a known $\theta$ parameter.

The aim is to design a controller $K(\theta)$ to attenuate as much as possible the disturbances $d$ on the measurements $y$, in the $L_2$-norm sense. Emphasis is put around some known harmonics that are linearly dependent on the central frequency $\theta$. This $\theta$ parameter is considered adjustable, bounded and time-varying i.e.

$$\theta(t) \in D_\theta = [\underline{\theta}, \bar{\theta}]$$ (2)

The control law is of course constrained by stability and robustness margins requirements. Let $T_{w \rightarrow z}$ represent the transfer between the signals $w$ and $z$. We define $S_y = T_{r \rightarrow e}$, $KS_y = T_{r \rightarrow w}$, $S_yG = T_{d \rightarrow y}$ and $T_u = T_{d \rightarrow u}$ as the output, input, disturbance and input complementary sensitivity functions respectively.

To carry out this problem, the mixed sensitivity approach of the $H_{\infty}$ framework is used. To proceed, let us introduce the reference signal $r$ and input disturbance $d_n$ as illustrated on Fig. 1. It follows that the disturbance and performance channels are defined as $w = (r^T, d_n^T)^T$ and $z = (z_1^T, z_2^T)^T$ together with the weighting functions $W_1$, $W_2$ and $W_3$. It is assumed that the weight $W_3$ depends on a bounded and measured parameter $\theta$ so that $|\theta| \leq 1$. This can be done without loss of generality since the problem can always be scaled.

**Remark 1** In this work, it is assumed that $\theta$ is a scalar since, as it is explained later, $\theta$ plays the role of the central frequency of a notch filter. However, it is well known from the $H_{\infty}$/LPV community that the LPV theory remains valid if $\theta$ is a vector, see for instance [6, 15].
Then, the synthesis problem turns out to be the design of the controller $K(\theta)$ that (intrinsically) stabilizes the closed-loop and solves the following optimization problem

$$
\min_{K(\theta)} \gamma \ 	ext{s.t.} \ \left\| \mathcal{F}_1(P(\theta), K(\theta)) \right\|_\infty < \gamma, \ |\theta| \leq 1
$$

(3)

where $\mathcal{F}_1$ defines the lower linear fractional transformation. The performance $z$ is given by (4).

$$
\begin{bmatrix}
    z_1 \\
    z_2 \\
    z
\end{bmatrix} =
\begin{bmatrix}
    W_1(\theta)S_y - W_1(\theta)S_yGW_3 \\
    W_2KS_y - W_2T_uW_3 \\
    \mathcal{F}_1(P(\theta), K(\theta))
\end{bmatrix}
\begin{bmatrix}
    r \\
    d_n \\
    w
\end{bmatrix}
$$

(4)

**Figure 1.** Augmented plant model

### 2.1.1. Problem formulation within the polytopic setting:

By proper choice of $W_1(\theta), W_2, W_3$, the scheduled controller $K(\theta)$ can achieve the rejection of the disturbance signal $d$ together with bounds on the different sensibility functions. Stability margin requirements and disturbance rejection performance can be managed through $W_1(\theta)$. Note that this filter shapes both the output $S_y$ and disturbance $S_yG$ sensitivity functions. The following expression of $W_1(\theta)$ is also retained:

$$
W_1(\theta) = W_sW_p(\theta)I_{ny}
$$

(5)

In this equation, the function $W_s$ is used to shape $S_y$ and introduce a modulus margin stability requirement, whereas $W_p(\theta)$ is chosen as an inverted notch filter to provide attenuation around $\theta$. Here, the formulation introduced in [10] is used for $W_p(\theta)$, i.e.

$$
W_p(\theta) = \begin{pmatrix}
-2\theta\xi_2 & -\theta \\
\theta & 0
\end{pmatrix} \begin{pmatrix}
20(\xi_1 - \xi_2) \\
0 & 0
\end{pmatrix} + \theta \begin{pmatrix}
0 & 0 \\
C_p & 0
\end{pmatrix}
$$

(6)

where (6) is affine in $\theta$. The damping ratio $\xi = \xi_2/\xi_1$ sets the notch depth corresponding to the disturbance attenuation requirement over $\Omega$. For a fixed damping ratio, the $\xi_2$ parameter controls the width of the notch filter around the center frequency $\theta$.

$W_2$ is also chosen to decrease the actuation effort and improve the controller roll-off. $W_3$ is used to bound the maximum value of the disturbance sensitivity function $S_yG$. Adjusting $W_3$ also influences the controller effort by shaping $T_u = T_{d\rightarrow u}$.

With these weight definitions, the augmented plant $P(\theta)$, has the following affine structure:

$$
\begin{bmatrix}
    \dot{x} \\
    z \\
    e
\end{bmatrix} =
\begin{bmatrix}
    A(\theta) & B_1(\theta) & B_2 \\
    C_1(\theta) & D_{11} & D_{12} \\
    C_2 & D_{12} & 0
\end{bmatrix}
\begin{bmatrix}
    x \\
    w
\end{bmatrix}
$$

with

$$
\begin{align*}
A(\theta) &= A_0 + \theta A_1 \\
B_1(\theta) &= B_{10} + \theta B_{11} \\
C_1(\theta) &= C_{10} + \theta C_{11}
\end{align*}
$$

(7)

### 2.1.2. Problem formulation within the LFR setting:

Direct application of the LFR formalism enables the following alternative formulation for the $W_p(\theta)$ weight (6):

$$
W_p(\theta) = \mathcal{F}_u \begin{pmatrix}
\frac{1}{s}A_p & B_p \\
\frac{1}{s}C_p & 1
\end{pmatrix}, \Delta = \begin{bmatrix}
\theta & 0 \\
0 & \theta
\end{bmatrix}
$$

(8)
where $\mathcal{F}_u$ represents the upper linear fractional transformation.

With this expression of $W_p(\theta)$ and incorporating $W_1(\theta)$, $W_2$ and $W_3$ into the augmented plant model (7), the following LFR can be derived,

$$P(\theta) = \mathcal{F}_u \begin{pmatrix} A & B_0 & B_1 & B_2 \\ C_0 & D_{00} & D_{01} & D_{02} \\ C_1 & D_{10} & D_{11} & D_{12} \\ C_2 & D_{20} & D_{21} & 0 \end{pmatrix}, \Theta(t) \text{ with } \Theta(t) = -\tilde{\theta}(t)I_{12} \text{ and } \tilde{\theta}(t) = \frac{2(\theta(t) - \hat{\theta})}{\bar{\theta} - \hat{\theta}} - 1$$

(9)

which is then suitable for the LPV controller synthesis technique presented in [5,7]. This aspect is addressed in the next section.

2.2. Extension to multiple harmonics

The aforementioned formulated problem can be extended to take into account the first $n$ harmonics of $d$, assuming that all harmonic frequencies denoted $\theta_i$, depend linearly on $\theta$, i.e.

$$\theta_i(t) = \alpha_i + \beta_i \theta(t), \quad i = 1...n$$

(10)

Using the affine property in $\theta$ of $W_p(\theta)$, see Eq. (6), a set of $n$ weighting functions $W_{p_i}(\theta), i = 1...n$ can be defined according to

$$W_{p_i}(\theta) = \begin{pmatrix} 0 & 0 \\ C_{p_i} & 1 \end{pmatrix} + \theta_i \begin{pmatrix} A_{p_i} & B_{p_i} \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \alpha_i A_{p_i} & \alpha_i B_{p_i} \\ C_{p_i} & 1 \end{pmatrix} + \theta(t) \begin{pmatrix} \beta_i A_{p_i} & \beta_i B_{p_i} \\ 0 & 0 \end{pmatrix}, \quad i = 1...n$$

(11)

so that each filter $W_{p_i}(\theta), i = 1...n$ can be independently parametrized to have a different width and depth. Combining all these filters leads to the following expression for $W_p(\theta)$:

$$W_p(\theta) = \begin{pmatrix} \alpha_1 A_{p_1} & \alpha_1 B_{p_1} \\ \cdot & \cdot \\ \cdot & \cdot \\ \alpha_n A_{p_n} & \alpha_n B_{p_n} \\ C_{p_1} \cdots C_{p_n} & 1 \end{pmatrix} + \theta \begin{pmatrix} \beta_1 A_{p_1} & \beta_1 B_{p_1} \\ \cdot & \cdot \\ \cdot & \cdot \\ \beta_n A_{p_n} & \beta_n B_{p_n} \\ 0 & 0 \end{pmatrix}$$

(12)

3. Design of the LPV controller $K(\theta)$

This section addresses different LPV synthesis techniques to design $K(\theta)$. Both the LFR and polytopic approaches are investigated. Due to space limitations, only brief descriptions will be provided. The reader can refer to the original papers for additional details.

For an LPV plant described either by (7) or by (9), the $H_\infty$ LPV control problem consists of finding a dynamic output feedback controller $K(\theta)$ such that the closed-loop is internally stable and the $L_2$-gain $\gamma$ of the closed-loop operator between $w$ and $z$ is minimized, for all $\theta(t)$.

3.1. The LFR approach

The LPV synthesis approach introduced in [5] considers the LFR description (9) for the plant. A combination of symmetric and skew-symmetric matrices leads to a scaled version of the Bounded Real Lemma. The scalings commute with the $\Theta$ block and account for the realness of the scheduling parameter. The following theorem [5,7] solves the problem:
Theorem 1 Let the LPV plant be described by (9) with \( N_X = \ker[C_2, D_{20}, D_{21}, 0] \) and \( N_Y = \ker[B_2^T, D_{20}^T, D_{21}^T, 0] \). There exists an LPV controller guaranteeing \( L_2 \)-performance level \( \gamma \) for the scaled Bounded Real Lemma, if and only if there exists the pairs of symmetric matrices \((X, Y), (S, \Sigma)\) and a pair of skew-symmetric matrices \((T, \Gamma)\) such that the following LMIs are feasible:

\[
\begin{align*}
N_X^T & \begin{bmatrix} A^T X + XA & * & * & * \\
B_\theta^T X + TC_\theta & -S + TD_{\theta 0} + D_{\theta 0}^T T^T & * & * \\
B_\theta^T X & D_{\theta 0}^T T^T & -\gamma I & * \\
SC_\theta & SD_{\theta 0} & -S & * \\
C_1 & D_{\theta 0} & D_{11} & 0 & -\gamma I \\
\end{bmatrix} N_X < 0, \\
N_Y^T & \begin{bmatrix} AY + YA^T & * & * & * \\
C_\theta Y + \Gamma B_\theta^T & -S + \Gamma D_{\theta 0}^T + D_{\theta 0} \Gamma^T & * & * \\
SC_\theta & SD_{\theta 0} & -\Sigma & * \\
B_1^T & D_{\theta 0} & 0 & -\gamma I \\
\end{bmatrix} N_Y < 0,
\end{align*}
\]

(13)

3.2. The polytopic approach

Another analysed LPV synthesis technique is the one used in [16]. This method applies to systems which admit a polytopic representation and uses the Projection Lemma to find a Single Quadratic Lyapunov Function (SQLF), valid for all scheduling domain (2). One issue with these techniques is that they sometimes lead to controllers with very fast poles. The poles in this case are calculated for fixed values of the scheduling parameter \( \theta(t) \). To overcome this problem, one can use the concept of LMI regions introduced by [17]. Using a synthesis technique based on a change in the controller variables [18] allows the introduction of LMI regions [17]. These regions are new LMI constraints that restrict the closed-loop poles to subsets of the complex plane.

Parameter dependent Lyapunov Functions (PDLF) were used to reduce the conservatism associated with using a SQLF for the whole scheduling interval [6]. Considering bounds on \( \theta \) so that \( \theta(t) \in [\tilde{\theta}, \bar{\theta}] \), leads to the following theorem:

Theorem 2 For an LPV plant described by (7), there exists an LPV controller that guarantees closed-loop global asymptotic stability, \( L_2 \)-gain smaller than \( \gamma \) and closed-loop poles constrained to the circle of rayon \( r \) around the origin, whenever there exists parameter-dependent symmetric matrices \( X(\theta) \) and \( Y(\theta) \) and parameter-dependent matrices \( (\hat{A}(\theta), \hat{B}(\theta), \hat{C}(\theta), D(\theta)) \) that satisfy for every \( (\theta, \dot{\theta}) \in [\tilde{\theta}, \bar{\theta}] \times [\tilde{\theta}, \bar{\theta}] \) the following infinite dimensional matrix inequalities:

\[
\begin{align*}
[\dot{X} + XA + BC_2 + (\ast) & , \hat{A}^T + A + B_2 DC_2 & -Y + AY + B_2 \hat{C} + (\ast) & * & * \\
(\overline{X}B_1 + \overline{B}D_{21})^T & (B_1 + B_2 DC_{21})^T & -\gamma I & * \\
C_1 + D_{12} DC_2 & C_1 Y + D_{12} \hat{C} & D_{11} + D_{12} DC_{21} & -\gamma I \\
\end{bmatrix} & < 0, \\
\begin{bmatrix} \mathcal{M}_{11} & * \\
\mathcal{M}_{21} & \mathcal{M}_{11} \end{bmatrix} & < 0, \\
\mathcal{M}_{11} = \begin{bmatrix} -rY & -rI \\
-rI & -rX \end{bmatrix}, \quad \mathcal{M}_{21} = \begin{bmatrix} AY + B_2 \hat{C} & \hat{A}^T & \\
A + B_2 DC_2 & XA + \hat{B}C_2 \end{bmatrix}
\end{align*}
\]

(14)

where the dependencies on \( \theta \) are dropped for simplicity.

To reduce the optimization problem to a finite-dimensional case, the Lyapunov function variables \( X, Y \) and the transformed controller variables \( (\hat{A}, \hat{B}, \hat{C}, \hat{D}) \) can be chosen to have the same affine dependence on the scheduling parameter \( \theta \) as the plant model. This results in the following new expressions for the optimization variables:

\[
\hat{A}(\theta) = \hat{A}_0 + \theta \hat{A}_1 \\
\hat{B}(\theta) = \hat{B}_0 + \theta \hat{B}_1 \\
\hat{C}(\theta) = \hat{C}_0 + \theta \hat{C}_1 \\
\hat{D}(\theta) = \hat{D}_0 + \theta \hat{D}_1
\]

and \( X(\theta) = X_0 + \theta X_1 \quad Y(\theta) = Y_0 + \theta Y_1 \)

(15)
The problem is still not convex but can be approached by gridding the parameter space $[\theta, \theta] \times [v, v]$. The LMI constraints can be checked afterwards across a denser grid. However, stability and performances are not guaranteed between gridding points.

Another way to convexify the LMI problem (2), assuming (3.2), is by adding the following multiconvexity constraint, as in [19]:

$$\frac{\partial^2 F}{\partial \theta^2} \geq 0 \quad \forall \theta \in [\theta, \theta]$$ (16)

Thanks to the new convex nature of the problem, it is sufficient for the LMI constraints (2), (14) to hold at the bounding vertices of the domain $(\theta, \dot{\theta})$ $\in [\theta, \theta] \times [v, v]$.

The controller synthesis problem (3) is thus reduced to the following LMI optimization:

$$\begin{align*}
\min_{A_i, B_i, C_i, D_i, X_i, Y_i} & \quad \gamma \\
\text{subject to} & \quad (2), (14) \text{ and } (16).
\end{align*}$$ (17)

For a known $(\theta(t), \dot{\theta}(t))$ pair, the controller matrices can be reconstructed by undoing the initial controller transformation using the formulas (18), as described in [6].

$$\begin{align*}
A_K &= N^{-1}(X \dot{Y} + NM^T + \hat{A} - X(A - B_2DC_2)Y - \hat{B}C_2Y - XB_2\hat{C})M^{-T} \\
B_K &= N^{-1}(\hat{B} - XB_2D) \\
C_K &= (\hat{C} - DC_2Y)M^{-T} \\
D_K &= D
\end{align*}$$ (18)

where $N, M$ solve the factorization problem, $I - XY = NM^T$. To avoid the need for the scheduling parameter derivative $\dot{\theta}$ during controller reconstruction, either $X$ or $Y$ can be fixed to a constant value.

In the case of PDLF, the new LMI constraints have to be valid across the polytopic domain defined by $\theta \times \dot{\theta}$. This leads to an increase in the number of polytope vertices, compared to the synthesis based on SQLF. On-line computational burden is also increased because for every new value of the parameter $\theta(t)$, a new controller reconstruction is required based on the new values of $X(\theta(t))$ or $Y(\theta(t))$. This can be a critical problem to implement the LPV controller, which is the case here for micro-vibrations mitigation.

Fixing both $X$ and $Y$ to constant values (19), leads to a SQLF problem with additional pole constraints.

$$\begin{align*}
X(\theta) &= X_0 \quad \text{and} \quad Y(\theta) &= Y_0
\end{align*}$$ (19)

In this case, the derivatives $\dot{X}$ and $\dot{Y}$ are equal to zero and can be dropped from (2) and (18) together with the constraint (16). The controller can thus be scheduled, for a known $\theta$, by performing the interpolation (20) between the controller matrices calculated at $\theta \in \{\theta, \theta\}$.

$$K(\theta) = (1 - \rho)K(\theta) + \rho K(\theta) \quad \text{with} \quad \rho(\theta) = \frac{\theta(t) - \theta}{\dot{\theta}} \in [0, 1]$$ (20)

4. Application to microvibration mitigation

The method described in Section 3 is applied to the problem of microvibration mitigation. The application support is a physical system jointly developed by the European Space Agency (ESA) and Airbus Defence and Space. The final issue is to propose a solution for the active part of a mixed passive-active control architecture to mitigate the microvibrations generated by reaction wheels in satellites. In this sense, it is assumed that the application support is representative of a flexible satellite panel put in space.
4.1. System description
The hardware architecture of the system is presented in Fig. 2. The platform consists of a breadboard suspended by four springs. For this system, a shaker is installed on top of a dummy reaction wheel and is used to simulate the multi-harmonic microvibrations induced by a real reaction wheel.

The dummy wheel is connected to an active plate that can vibrate to compensate for the vibrations introduced by the shaker. Actuation of the active plate is provided using a set of six Proof Mass Actuators (PMA) mounted on three cubes; three along the vertical axis and three in the tangential direction. These actuators generate reactive forces by accelerating a small mass connected to a spring-damper system using a magnetic field. The field is produced by passing an electric current through a voice coil. The current in turn, is provided via amplifiers that convert the voltage control signal vector \( u(t) \in \mathbb{R}^6 \).

The active plate, in turn, is connected to an interface plate using four elastomer isolator modules. These passive isolators are used to attenuate the high frequency disturbances transmitted from the shaker to the interface plate. On the bottom of each isolator, a tri-axis force cell sensor is installed. When strained, piezoelectric elements inside these cells generate electric charges that are converted into voltages using charge amplifiers. The transmitted forces and moments are obtained by projecting the force cell measurements in the Force and Torques space and forming the output vector \( y(t) \in \mathbb{R}^6 \).

4.2. System model
Model identification was performed by Airbus Defence and Space using real measurement data. The identification procedure was done by considering a null wheel rate and ignores gyroscopic effects. The resulting model is a 36 order nominal LTI model for \( G(s) \), see Eq. (1). The estimated dynamics contains multiple bending modes with natural frequencies \( \omega_n \in [1.92, 42.26] \) Hz and damping ratios \( \xi_n \in [3.63 \cdot 10^{-2}, 5.96 \cdot 10^{-2}] \). Additionally, the presence of strong coupling effect between axes is evident. Therefore, it is not possible to decouple the control laws for each axis and a full MIMO control law is judged necessary.

4.3. Objectives and the \( H_\infty \) criterion
In this paper, the focus is put on mitigating two harmonics denoted \( H_1 \) and \( H_{0.6} = 0.6H_1 \), so that \( H_1 \) corresponds to the speed of the reaction wheel. Since the speed of the reaction wheel is measured, \( H_1 \) is chosen as the LPV scheduling parameter \( \theta \in \mathcal{D}_\theta \). The interval \( \mathcal{D}_\theta \) is fixed according to:

\[
\theta(t) \in \mathcal{D}_\theta = [15, 40] \text{ Hz}
\]

The harmonics \( H_1 \) and \( H_{0.6} \) are chosen based on the fact that, for a wheel speed in the interval (21), most of the disturbance energy is concentrated in these two harmonics, see [20] if necessary.

Fig. 2. Hardware configuration.

Additional details about the plant model cannot be supplied due to industrial confidentiality.
So the aim is to achieve around 20 dB disturbance attenuation for $H_1$ and 10 dB for $H_{0.6}$ on the force channels. Additionally, a $||S_y||_\infty < 6.5$ dB modulus margin is desired as a stability indicator. Robustness against additive and multiplicative output uncertainty is achieved by aiming for the bounds $||KS_y||_\infty < 7.5$ dB and $||T_y||_\infty < 6.5$ dB.

In practice, the reaction wheel acceleration is of small magnitude and thus the scheduling parameter $\theta$ is slow-varying. This fact motivated the use of the following method to translate the design requirements into different values for $W_1$, $W_2$ and $W_3$:

i) Choose $W_1(\theta)$ as in Eq. (5). To incorporate stability requirements, $W_2^{-1} = 6$ dB is set as the desired modulus margin. Two harmonics are to be attenuated therefore $W_p(\theta)$ contains two filters, $W_{p1}$ and $W_{p2}$, defined for the $H_1$ and $H_{0.6}$ harmonics. The parameters used for these specifications are included in Table 4.3. The singular values of the $W_1^{-1}(\theta)$ weight are illustrated in Fig. 4.3 for 30 values of $\theta \in D_\theta$.

![Figure 3. Singular values of $W_1(\theta)$ for $\theta \in [15, 40]$ Hz.](image)

Table 1. Parameters for the $w_p(\theta)$ weights

| Frequency $\theta$ | Depth $\xi_2/\xi_1$ | Width $\xi_2$ |
|-------------------|-------------------|--------------|
| $w_{p1}$          | $\theta$          | 20 dB        | 0.1          |
| $w_{p2}$          | $0.6 \cdot \theta$ | 10 dB        | 0.1          |


ii) Parametrize $W_2$ and $W_3$ as in (22). The possible value intervals for $w_2$ and $w_3$ were chosen based on the fact that they are used to bound the $KS_y$ and $GS_y$ sensitivities.

$$W_2(s) = w_2^{-1}I_6 \quad W_3(s) = w_3^{-1}I_6 \quad \text{with} \quad w_2 \in [0, 28] \text{ dB} \quad w_3 \in [-5, 22] \text{ dB} \quad (22)$$

iii) Grid the parameter space defined by $w_2$ and $w_3$ and construct a different controller at every grid point using the PDLF-LPV synthesis detailed in Section 3.2.

iv) For every controller, evaluate the frequency domain performance by computing the maximum bounds of $||S_y(\theta)||_\infty$, $||T_y(\theta)||_\infty$, $||KS_y(\theta)||_\infty$ and $||GS_y(\theta)||_\infty$ across multiple fixed $\theta \in D_\theta$.

v) Reduce the possible values for $w_2$ and $w_3$ to subsets by only considering values such that $||S_y(\theta)||_\infty$, $||T_y(\theta)||_\infty$ and $||KS_y(\theta)||_\infty$ are below certain values.

Following this approach, the weights (22) are fixed according to:

$$w_2 = 0 \text{ dB} \quad w_3 = 16 \text{ dB} \quad (23)$$

For this choice, the optimal value of the optimisation problem (2) is found to be $\gamma = 5.77$.

This slightly high performance value can be explained due to the fact that some of the weights are too demanding and the $H_\infty$ constraints can’t be satisfied simultaneously. However, for this application, the rate of change of the scheduling variable $\theta$ is sufficiently low in practice. As such, the original $H_\infty$ constraints can be validated a posteriori for pointwise values of $\theta$. The bounds associated to the different closed-loop sensitivity functions, evaluated at 500 different fixed values of $\theta \in D_\theta$, are found to be:

$$||S_y(\theta)||_\infty \leq 6.02 \text{ dB} \quad ||T_y(\theta)||_\infty \leq 6.23 \text{ dB}$$

$$||KS_y(\theta)||_\infty \leq 7.06 \text{ dB} \quad ||S_yG(\theta)||_\infty \leq 18.5 \text{ dB} \quad (24)$$
Figure 4. Singular value plots for the closed-loop sensitivity functions.

Figure 4 illustrates the singular value plots of these sensitivities together with the singular values of the controller $K(\theta)$, for three different $\theta \in \mathcal{D}_\theta$. The adaptive behaviour of the controller is evident, with the peaks in the singular values coinciding with harmonics to be attenuated. The harmonic attenuation performance was further analysed for multiple values of $\theta \in \mathcal{D}_\theta$ by measuring the gap between the singular values of the closed and open-loop transfer $T_d \rightarrow y$ (i.e. $|G| - |SyG|$) at the $H_1$ and $H_{0.6}$ frequencies. The attenuation levels in terms of statistics criteria (mean and standard deviations) are presented in Table 2 for each direction $x, y, z$. Based on this analysis, it can be concluded that the LPV controller achieves good pointwise multi-harmonic attenuation performances.

Table 2. Average attenuation performance

|          | $F_x$          | $F_y$          | $F_z$          |
|----------|----------------|----------------|----------------|
| $H_1$ harmonic | $-30.32 \pm 5.78$ dB | $-24.43 \pm 2.84$ dB | $-21.57 \pm 2.77$ dB |
| $H_{0.6}$ harmonic | $-12.31 \pm 2.62$ dB | $-8.81 \pm 3.43$ dB | $-7.52 \pm 2.25$ dB |

4.4. Remarks on the choice of PDLF-LPV synthesis method

The weight tuning method uses a PDLF-Grid with pole constrains as the main LPV synthesis method. This was selected as the method of choice out of all other described in Section 3 after performing a comparison between the different methods as detailed in [13]. A summary of this comparison is presented in this section. The LPV comparison relies on the augmented plant model (4) using the following selection of weights:

$$W_1(\theta) = \frac{1}{6\text{ dB}} \begin{bmatrix} -0.2 \cdot \theta & -\theta & -0.19 \cdot \theta \\ \theta & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} I_6 \quad W_2(s) = 0.03 \quad W_3(s) = 0.5 \quad I_6 \quad (25)$$

After synthesising an LPV controller using each technique, the synthesis results are presented in Table 3. The methods are afterwards compared as follows: The controllers are reconstructed at 100 linearly spaced values for $\theta \in [15, 40]$. For every a priori fixed values, the achieved $H_\infty$ criterion (3) performance criteria $\gamma$ is computed, leading to a plot $\gamma(\theta)$. This function provides a pointwise indicator of the performance level for each LPV controller. A lower bound for the $\gamma(\theta)$ function can be obtained by performing a classical pointwise $H_\infty$ synthesis for the same $\theta$ values. The $\gamma(\theta)$ plots for the LPV controllers results are presented in Figure 5 together with
the pointwise $\mathcal{H}_\infty$ lower bound. As expected, all LPV synthesis methods introduce a degree of conservatism over the pointwise $\mathcal{H}_\infty$ synthesis.

| Controller | $\gamma$ | Fastest pole  |
|------------|---------|--------------|
| LFR        | 1.43    | 19.2 kHz     |
| SQLF       | 1.43    | 12.7 kHz     |
| PDLF       | 1.47    | 821 Hz       |
| SQLF*      | 1.47    | 821 Hz       |
| PDLF*      | 1.27    | 1392 Hz      |
| PDLF-Grid* | 1.00    | 531 Hz       |

*poles constrained via LMI regions [17]

Table 3. LPV controller synthesis results

The first employed synthesis methods are the LFR and Polytopic SQLF methods. As it can be seen from Table 3, these methods achieve the same performance level for this particular system and also lead to controllers with fast dynamics when evaluated for fixed values of $\theta$. The next synthesis approach is based on Polytopic SQLF with pole constraints via LMI regions. This is performed by fixing $X(\theta)$ and $Y(\theta)$ as in (19). The introduction of pole constraints leads to a slight performance penalty ($\gamma = 1.47$ vs $\gamma = 1.43$). This small drop in performance was considered an acceptable compromise for implementability reasons and motivates the usage of pole constraints for all the synthesis methods further analysed.

Two other LPV synthesis methods based on PDLF have also been considered, considering bounds on the variations of the scheduling parameter $\theta$: PDLF based on multiconvexity (16) and PDLF-Grid based on gridding the parameter space $[\theta, \theta'] \times [v, v']$ over 20 grid points. As expected, both PDLF methods outperform the SQLF approaches. For the PDLF-Grid controller, the stability and performance levels defined by (2), (14) have been verified a posteriori using a dense grid defined by $10^4$ points.

From Figure 5 and Table 2, it follows that the best performing LPV controller is the PDLF-Grid controller. This is why this method was selected during the weight optimization technique described in Section 4.3.

5. Simulations

The performances in the time-domain are evaluated using a Simulink-based simulator provided by Airbus Defence and Space. Noises acting on measurements ($\sigma^2_{n_m} = 10^{-5}$), plant inputs ($\sigma^2_{n_u} = 10^{-4}$) and on the scheduling variable $\theta$ ($\sigma^2_{n_{\theta}} = 10^{-3}$) are considered. The controller is rescheduled every 0.1s using the noisy measurement $\theta$ of the main disturbance frequency. The controller is discretized using a Tustin transformation with a $T_s = 10^{-3}s$ sample time. Fig. 6
presents the open and closed-loop transmitted force measurements \( (F_x, F_y, F_z) \) together with the wheel speed profile and the scheduling parameter \( \theta \). The wheel speed was selected to vary linearly between 15 and 40 Hz in 22 seconds. A side-by-side spectrogram comparison of these signals for the open and closed-loop case is shown in Fig. 7. The mitigation requirements are met for all \( \theta \in [15, 40] \) Hz despite the noises affecting the system. The control signals \( u \in \mathbb{R}^6 \) are illustrated in Fig. 5 and are bounded by \( |u(t)| < 0.3 \) corresponding to reasonable actuator signals. These results suggest the fact that the proposed controller is a viable solution.

**Figure 7.** Spectrograms of \( F_x, F_y \) and \( F_z \).

**Figure 8.** Control signals for the PMAs.

6. Conclusions
The study has shown how a mixed active-passive solution can be used for microvibration mitigation on-board satellite platforms. The control problem is formulated and managed in the \( \mathcal{H}_\infty \)/LPV framework. The performance and robustness of the proposed method was evaluated using simulations. Future works will concentrate on incorporating fault tolerant capabilities.

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