Hybrid-Adaptive Differential Evolution with Decay Function (HyDE-DF) Applied to the 100-Digit Challenge Competition on Single Objective Numerical Optimization

Fernando Lezama, João Soares, Ricardo Faia, and Zita Vale
GECAD-Research Group on Intelligent Engineering and Computing for Advanced Innovation and Development
Polytechnic of Porto, Portugal

ABSTRACT
In this paper, a hybrid-adaptive differential evolution with a decay function (HyDE-DF) is proposed for numerical function optimization. The proposed HyDE-DF is applied to the 100-Digit Challenge in a set of 10 benchmark functions. Results show that HyDE-DF can achieve a 93/100 score, proving its effectiveness for numerical optimization.

CCS CONCEPTS
• Computing methodologies → Search methodologies; • Applied computing → Engineering.

KEYWORDS
Evolutionary computation, differential evolution, numerical optimization

1 INTRODUCTION
Classical optimization approaches have been struggling to match the growing complexity presented in different scientific and industry domains. In such a context, alternative methods, such as evolutionary computation (EC), represent a viable option for solving complex problems. EC is a branch of computational intelligence that encompasses a set of algorithms for global optimization mostly inspired by biological and evolutionary processes [1].

In this paper, we propose a hybrid-adaptive differential evolution with a decay function (HyDE-DF) for numerical optimization. The proposed algorithms is a modified version of HyDE [2], including a decay function and an extra re-initialization step. HyDE-DF integrates ideas from different EAs and has already demonstrated good performance in solving a real-world application in the energy domain [2].

2 BENCHMARK FUNCTIONS
In this study, we use the set of ten benchmark functions from the 100-digit challenge [3]. Functions are listed in Table 1. The complete details of the used functions are given in [3].

Table 1: The 100-Digit Challenge Basic Test Functions

| No. | Function                  | Min | D  | Range        |
|-----|--------------------------|-----|----|--------------|
| F1  | Storn’s Chebyshev        | 1   | 9  | [8192,8192]  |
| F2  | Inverse Hilbert          | 1   | 16 | [-16384,16384] |
| F3  | Lennard-Jones            | 1   | 18 | [-4,-4]      |
| F4  | Rastrigin’s Function     | 1   | 10 | [-100,-100]  |
| F5  | Griewangk’s Function     | 1   | 10 | [-100,-100]  |
| F6  | Weierstrass Function     | 1   | 10 | [-100,-100]  |
| F7  | Modified Schwefel’s      | 1   | 10 | [-100,-100]  |
| F8  | Expanded Schaffer’s Fr   | 1   | 10 | [-100,-100]  |
| F9  | Happy Cat                | 1   | 10 | [-100,-100]  |
| F10 | Ackley Function          | 1   | 10 | [-100,-100]  |

3 HYDE WITH DECAY FUNCTION AND RE-INITIALIZATION STEP
HyDE-DF combines ideas from different evolutionary algorithms being an improved version of the HyDE algorithm proposed in [2]. HyDE-DF uses the so-called “DE/target + perturbed_best/1” mutation strategy of HyDE with a decay factor δG, which is a function that decreases gradually from 1 to 0 in a period of DF iterations. The operator is as follows:

\[
\bar{m}_iG = \bar{x}_{i,G} + \delta_G \cdot [F_i1(\epsilon \cdot \bar{x}_{best} - \bar{x}_{l,G})] + F_i2(\bar{x}_{r1,G} - \bar{x}_{r2,G})
\]

(1)

where \(\bar{x}_{r1,G}\) and \(\bar{x}_{r2,G}\) are two random individuals from the population (Pop), mutually different and also different from the current target vector \(\bar{x}_{i,G}\), while \(\bar{x}_{best}\) is the best found solution. \(F_i1, F_i2,\) and \(F_i3,\) are scale factors in the range \([0,1]\) independent for each individual \(i,\) and updated at each iteration following the self-adaptive parameter mechanism of jDE algorithm (see [4]). \(\epsilon = N(F_i1,1)\) is a random perturbation factor taken from a normal distribution with mean \(F_i1\) and standard deviation 1. The factor \(\delta_G\) is used to gradually decrease the influence of the term \(F_i1(\epsilon \cdot \bar{x}_{best} - \bar{x}_{l,G})\) responsible for the fast convergence towards the best individual in the population.

The HyDE-DF also incorporates a reinitialization mechanism that is activated if \(R_N = 1e4\) successive iterations show no improvement in the objective function. In such case, the original population is replaced by generating new individuals around a given number of best solutions found so far (we used 10 in our experiments). The new individuals are generated using random numbers that follow a normal distribution with mean of those best solutions and standard deviation of 10e-4. The best individual in the population after reinitialization is kept to preserve memory. A pseudocode of the algorithm is given in Algorithm 1.
We can see that HyDE-DF was able to obtain a final score of 93. In fact, HyDE-DF obtained a score in 9 out of 10 functions. Only in the Function 9 (Happy Cat Function), HyDE-DF was not able of finding all 10 digits, then the score for that function is a perfect 10.

Table 3: Fifty runs sorted by the number of correct digits

| F | Number of correct digits | Score |
|---|--------------------------|-------|
| 1 | 0 0 0 0 0 0 0 0 0 50 | 10    |
| 2 | 0 0 0 0 0 0 0 0 0 50 | 10    |
| 3 | 0 7 0 0 0 0 0 0 0 43 | 10    |
| 4 | 0 0 0 0 0 0 0 0 0 50 | 10    |
| 5 | 0 0 0 0 0 0 0 0 0 50 | 10    |
| 6 | 0 0 0 0 0 0 0 0 0 50 | 10    |
| 7 | 0 1 16 0 0 0 0 0 0 33 | 10    |
| 8 | 0 0 4 0 0 0 0 0 0 46 | 10    |
| 9 | 0 0 4 46 0 0 0 0 0 0 | 3     |
| 10| 0 0 0 0 0 0 0 0 0 50 | 10    |

Total: | 93

5 CONCLUSIONS
In this paper, we applied a new HyDE-DF algorithm to the 100-digit challenge. HyDE-DF is an improved version of HyDE, a self-adaptive DE algorithm that also incorporates a perturbation of the best individual and a decay factor in its main operator. Being a self-adaptive version of DE, HyDE-DF eliminates the tedious tuning of parameters and yet achieved a good performance in the tested functions. Results showed that HyDE-DF can achieve a final score of 93, struggling only with the happy cat function in which only 3 digits of precision were found. Future work will analyze and compare the performance of HyDE-DF in a more complete and complex set of benchmark functions.

ACKNOWLEDGMENTS
This research has received funding from FEDER funds through the Operational Programme for Competitiveness and Internationalization (COMPETE 2020), under Project POCI-01-0145-FEDER-028983; by National Funds through the FCT Portuguese Foundation for Science and Technology, under Projects PTD/EEI-EEE/28983/2017 (CENERGETIC).

REFERENCES
[1] Joao Soares, Tiago Pinto, Fernando Lezama, and Hugo Morais. Survey on complex optimization and simulation for the new power systems paradigm. Complexity, 214(1), 2018.
[2] F. Lezama, J. Soares, R. Paia, T. Pinto, and Z. Vale. A new hybrid-adaptive differential evolution for a smart grid application under uncertainty. In 2018 IEEE Congress on Evolutionary Computation (CEC), pages 1–8, July 2018.
[3] K. V. Price, N. H. Awdal, M. Z. Ali, and P. N. Suganthan. Problem definitions and evaluation criteria for the 100-digit challenge special session and competition on single objective numerical optimization. Technical Report Nanyang Technological University, Singapore, 2018.
[4] J. Brest, A. Zuniga, B. Boskovic, M. S. Maucek, and V. Zumer. Dynamic optimization using self-adaptive differential evolution. In IEEE Congress on Evolutionary Computation, pages 415–422, May 2009.