Cosmological constraints on parameters of one-brane models with extra dimension

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Abstract

We study some aspects of cosmologies in 5D models with one infinite extra dimension. Matter is confined to the brane, gravity extends to the bulk. Models with positive and negative tension of the brane are considered. Cosmological evolution of the 4D world is described by warped solutions of the generalized Friedmann equation. Cosmological solutions on the brane are obtained with the input of the present-time observational cosmological parameters. We estimate the age of the Universe and abundance of $^4$He produced in primordial nucleosynthesis in different models. Using these estimates we find constraints on dimensionless combinations of the 5D gravitational scale, scale of the warp factor and coupling at the 4D curvature term in the action.

1 Introduction

Cosmological models with extra dimensions provide an extension of the standard cosmological model and naturally appear in microscopic theories unifying gravity with other interactions. As models of the real world they must be tested as cosmological theories. In this work we study some aspects of cosmologies in 5D models with one infinite extra dimension. Matter is confined to the brane, gravity extends to the bulk.

The action of the model is

$$I_5 = \int_\Sigma \sqrt{-g^{(5)}} \left( \frac{R^{(5)}}{2\kappa^2} + \Lambda \right) + \int_{\partial \Sigma} \sqrt{-g^{(4)}} \left( \frac{R^{(4)}}{2\kappa_1^2} - \sigma \right) - \int_{\partial \Sigma} \sqrt{-g^{(4)}} L_m, \quad (1)$$

where $\kappa^2 = 8\pi/M^3$, $\kappa_1^2 = 8\pi/r_cM^3$. $L_m$ is the Lagrangian of matter on the brane. $M$ is the gravitational scale of the 5D gravity, parameter $r_c$ defines the strength of gravity in the 4D term.

Cosmological evolution on the brane is described by solutions of the non-standard Friedmann equation [1234]. In the Gaussian normal frame the metric in the bulk is found as a warped solution of the Einstein equations, satisfying Israel junction conditions on the brane with matter.

We consider models with positive and negative tensions of the brane. The latter case is suggested by models with two branes, which allow for a possibility to reconsider the hierarchy problem. In these models the visible brane should have negative tension [8].

In this work we do not make a fit of observational cosmological data, but taking as the input the set of present-time cosmological parameters (Hubble parameter, fractions of total energy density of cold matter and radiation and deceleration parameter) we look for constraints on dimensionless...
combinations of scales of the models which follow from the requirement that the models reproduce the age of the Universe and abundance of $^4$He produced in primordial nucleosynthesis.

There are no strong constraints on parameters of the models with extra dimensions. Models with the fundamental scale varying in a very broad range of values were considered.

In the framework of the ADD models with large extra dimensions [9, 10] the fundamental scale of theory was taken at the scale of the standard model $M_{SM}$ thus evading the hierarchy problem.

A model with the action [11]

$$S = M^3 \int d^4x \int_0^R dy \sqrt{-g^{(5)}} R^{(5)} + M^3 r_c \int d^4x \sqrt{-g^{(4)}} R^{(4)},$$

with compactified extra dimension with the fundamental scale $M$ in the $TeV$ range, compactification radius $R \geq 10^{16}m$ ($R \sim 10^{32}GeV^{-1}$) and $R/r_c \sim 10^{-4}$ was argued to be compatible with measurements of the Newton law and cosmological data.

In a number of papers (for a list of refs. see [12]) cosmology was investigated in different variants of the DGP model [13]. In these models the fundamental scale was taken well below the standard model scale.

In the model proposed in [14] with the action of the form (1) with infinite flat extra dimension the fundamental scale is renormalized by the standard model interactions to the level $\sim 10^{-3}eV$. The model was argued to be consistent with collider experiments, cosmology and gravity measurements.

In papers [15, 16, 17] for asymptotically flat metrics and in [18] for dS metrics (in the static coordinate system) it was found that with the fundamental scale $M$ in the range $\sim 10^{-10^{2}}MeV$ and $r_c \sim H_0^{-1}$ the models are consistent with observational data. In [16] this was verified with the input of parameters of the standard cosmological model, in [17] was performed more precise comparison of the model with the SN1 and CMB data with the independent fit of parameters of the model (however in [19] difficulties in confronting the DGP model with SN data were reported).

In IIB string theory the RS scenario can be modeled as a stack of branes at the orbifold fixed point with the warping scale $\mu \sim M_{st}/(4\pi g N)^{1/4}$, where $M_{st}$ and $g$ are the string scale and the string coupling [20]. The scale $\mu$ can be made arbitrarily small for large enough number $N$ of stacked branes. Identifying the scale $\mu$ with the inverse radius $R$ of extra dimension, a constraint $\mu \geq 10^{-3}eV$ was obtained from estimation of corrections to the Newton law in the RSII model [21].

Although collider physics predictions were mostly discussed within the ADD-like models with the number of extra dimensions $n \geq 2$ and with the fundamental scale in the $TeV$ range (for example, [22, 24, 23]), it was noted that many results remain qualitatively relevant also for models with warped compactifications with the fundamental scale of the same order as in the ADD model.

In the present paper, in models with the action (1), instead of asymptotically flat solutions we consider warped metrics (in the Gaussian normal frame). The fundamental scale $M$ and the scale $\mu$ of the exponent in the warp factor of the metric $a’priory$ are practically unconstrained. In this situation we assume that $\mu$ is in the range $10^{-12}GeV - 10^6GeV$. The lower limit is suggested by precision of measurements of the Newton law, the upper limit is of order of the Standard Model scale.

The non-standard Friedmann equation in the models with extra dimension contains terms quadratic in total energy density and the dark radiation term [3, 4, 5, 6]. To obtain the estimates of the age of the Universe and of abundance of Helium produced in primordial nucleosynthesis it is sufficient to consider ”late” cosmology, i.e. times at which the terms quadratic in energy densities are small as compared to the term linear in energy densities.

Comparing predictions of the models with experimental data, we obtain a number of relations...
connecting dimensionless combinations of parameters of the models. In particular, we find relations
\[ \frac{\mu M^2_{pl}}{M^2} \sim \frac{2(1 + q_0)}{3\Omega_m} (\mu r_c \pm 1), \]
where \( q_0 \) and \( \Omega_m \) are present-time acceleration parameter and the fraction of cold matter in the total energy density and the sign corresponds to the sign of tension of the brane.

The paper is organized as follows. After briefly reviewing in Sect.1 the one-brane model, in Sect.2 we discuss the model with positive tension of the brane. Two cases are considered separately: those with and without 4D curvature term present in the 5D action. We calculate the age of the Universe and find constraints on the scales of the models which follow from the requirement that the lifetime of the Universe is within the observational bounds.

In Sect.4 the same program is carried out for models with negative tension of the brane. In this case, in distinction to the models with positive tension, for different signs of the combination \( \mu r_c - 1 \) the pictures are different. The model with \( 1 > \mu r_c \) does not yield correct age of the Universe, the model with \( \mu r_c > 1 \) can reproduce the observational data.

In Sect.5 we consider the model without dark radiation term in Friedmann equation.

In Sect.6 we calculate abundance of Helium produced in the BBN in the models with positive and negative tensions. More precise constraints on parameters of the models are obtained.

In Conclusions we summarize constraints on parameters obtained in different models.

## 2 One-brane model

We start with a brief introduction of 5D models with the action (1). It is assumed that matter is confined to the spatially flat brane and gravity is 5-dimensional. To have homogeneous cosmology on the brane the energy-momentum tensor of matter on the brane is taken in a phenomenological form
\[ T^\mu_\nu = \text{diag}\{-\hat{\rho}, \hat{\rho}, \hat{\rho}, \hat{\rho}\}, \]
(2)
where \( \hat{\rho}(t) \) and \( \hat{\rho}(t) \) are the sums of energy densities and pressures of cold matter and radiation. For the following it is convenient to introduce the normalized expressions for bulk cosmological constant, energy density, pressure and cosmological constant on the brane which all have the same dimensionality GeV
\[ \mu = \sqrt{\frac{\kappa^2}{6}} \rho, \quad \sigma = \frac{\kappa^2}{6} \sigma, \quad p = \frac{\kappa^2}{6} p. \]
(3)
The Einstein equations are solved in a class of metrics of the form
\[ ds^2 = -n^2(y, t)dt^2 + a^2(y, t)\gamma_{ij}dx^idx^j + dy^2. \]
(4)
The brane is located at the fixed position \( y = 0 \). Using the freedom in parametrization of time, the component of the metric \( n(y, t) \) is normalized so that \( n(0, t) = 1 \).

We study cosmologies on the brane by solving the non-standard Friedmann equation [3, 4, 6, 7] which follows from the system of the Einstein equations and junction (Israel) conditions on the brane (in the following we assume the symmetry \( y \rightarrow -y \))
\[ \begin{align*}
\frac{a'(0, t)}{a(0, t)} &= -\sigma - \sum p_i(t) + \frac{r_c \dot{a}^2(0, t)}{2 a^2(0, t)}, \\
n'(0, t) &= -\sigma + \sum (2p_i + 3p_i) + \frac{r_c}{2} \left( -\frac{\dot{a}^2(0, t)}{a^2(0, t)} + 2 \frac{\ddot{a}(0, t)}{a(0, t)} \right),
\end{align*} \]
(5)
where \( \rho_i, p_i \) \( i = r, m \) are densities and pressures of the cold matter and radiation. Equation for the scale factor \( a(0, t) \) can be written in the form with the second-order derivatives of the scale factor [3],

\[
\frac{\ddot{a}(0, t)}{a(0, t)} + \frac{\dot{a}^2(0, t)}{a^2(0, t)} = -2\mu^2 \left( \sigma + \sum \rho_i - \frac{r_c \dot{a}^2(0, t)}{2 a^2(0, t)} \right) \left( -2\sigma + \sum \rho_i + 3 \sum p_i + r_c \dot{a}(0, t) \right),
\]

(6)
or, starting from the partially integrated system of the Einstein equations, in the form with the first-order derivatives [4, 5, 6]

\[
\frac{\dot{a}^2(0, t)}{a^2(0, t)} = -\mu^2 + \left( \sigma + \sum \rho_i - \frac{r_c \dot{a}^2(0, t)}{2 a^2(0, t)} \right)^2 + C \frac{a^4(0, t)}{a^4(0, t)},
\]

(7)
or, equivalently,

\[
(1 + \sigma r_c) H^2 = \sigma^2 - \mu^2 + 2\sigma (\rho_m + \rho_r) + \left( \rho_m + \rho_r - \frac{r_c H^2}{2} \right)^2 + C(z + 1)^4
\]

(8)

where

\[
H(t) = \frac{\dot{a}(0, t)}{a(0, t)}, \quad z = \frac{a(0, t_0)}{a(0, t)} - 1
\]

and

\[
\rho_m(z) = \rho_{m0}(1 + z)^3, \quad \rho_r(z) = \rho_{r0}r(z)(1 + z)^4.
\]

(9)
The function \( r(z) \) is a slow function of \( z \) which counts, as the function \( g_*(T) \) in the expression for radiation energy density as the function of temperature, \( \hat{\rho}_r = \pi^2 g_*(T)T^4/30 \), the number of relativistic degrees of freedom.

\( C \) is the integration constant of dimension mass\(^2\), the term

\[
C(z + 1)^4 \equiv \mu \rho_w(z)
\]

is interpreted as dark radiation [25-7].

\[
q_0 = -\frac{\ddot{a}(t_0)}{a(t_0)H_0^2}
\]

is the present-time deceleration parameter. Equation (8) can be obtained from (7) by differentiation over \( t \) and the use of conservation equation for energy densities of cold matter and radiation.

### 3 Model with positive tension of the brane

#### 3.1 Model without 4D curvature term

In this model the term \( R^{(4)}/2\kappa_4^2 \) in the action [11] is absent. Setting in (7) and (6) \( t = t_0 \) we have [3]

\[
H_0^2 = \sigma^2 - \mu^2 + 2\sigma (\rho_{m0} + \rho_{r0}) + (\rho_{m0} + \rho_{r0})^2 + C.
\]

(10)

1 Substituting the explicit \( z \)-dependent expressions for energy densities and pressures in Eq.(11) and integrating it we obtain

\[
H^2 = (\sigma^2 - \mu^2) + 2\sigma \rho_{m0}(z + 1)^3 + (z + 1)^6 (\rho_{m0} + \rho_{r0}(z + 1))^2 + \tilde{C}(z + 1)^4,
\]

where \( \tilde{C} \) is the integration constant. Comparing this equation with (11), we obtain the relation between the constants \( C \) and \( C' \) : \( C = \tilde{C} - 2\sigma \rho_{r0} \).
\[(1 - q_0)H_0^2 = 2(\sigma^2 - \mu^2) + \sigma\rho_m + (\rho_m^2 + 3\rho_m\rho_0 + \rho_0^2).\]  \hspace{1cm} (11)

We shall consider the period of "late cosmology", when the terms linear in energy densities are dominant in Friedmann equation
\[
\sigma > \rho_m(z) + \rho_r(z). \hspace{1cm} (12)
\]

From relations (10) and (11), neglecting the terms quadratic in energy densities, we obtain
\[
\sigma^2 - \mu^2 = \frac{H_0^2}{2p^2}(-1 + (1 - q_0)p^2), \hspace{1cm} (13)
\]
and
\[
C \simeq \frac{H_0^2}{2p^2}(-3 + (1 + q_0)p^2 - 4\Omega_r/\Omega_m). \hspace{1cm} (14)
\]

Here we introduced the dimensionless ratio
\[
p^2 = \frac{H_0^2}{\sigma\rho_m}. \hspace{1cm} (15)
\]

Using (13) and (14) we present Eq.(7) in a form
\[
H^2 \simeq (z + 1)^{4} \frac{H_0^2}{2p^2} [(-1 + (1 - q_0)p^2)(z + 1)^{-4} + 4(z + 1)^{-1}
+ \frac{H_0^2}{p^2\mu^2}(z + 1)^{2} \left( 1 + \frac{\Omega_r}{\Omega_m}r(z)(z + 1) \right)^2 - 3 + (1 + q_0)p^2 + \frac{\Omega_r}{\Omega_m}(r(z) - 1) \right].
\hspace{1cm} (16)
\]

Neglecting the terms quadratic in energy densities and setting \(r(z) = 1\), we obtain the Friedmann equation as
\[
H^2 \simeq \frac{H_0^2}{2} \left[ 1 - q_0 + (1 + q_0)(z + 1)^{4} + \frac{1}{p^2}(-1 + 4(z + 1)^3 - 3(z + 1)^4) \right].
\hspace{1cm} (17)
\]

The rhs of (17) is positive for all \(z\) if
\[
(1 + q_0)p^2 - 3 > 0. \hspace{1cm} (18)
\]

From (13) we obtain that
\[
\frac{\sigma}{\mu} = 1 + O \left( \frac{\mu r_c H_0^2}{\mu^2} \right). \hspace{1cm} (19)
\]

Below we assume that \((\mu r_c)H_0^2/\mu^2 \ll 1\). Because \(\sigma/\mu \simeq 1\), we can write approximately
\[
p^2 \simeq \frac{H_0^2}{\mu \rho_m}. \hspace{1cm}
\]

Introducing
\[
\hat{\Omega}_V = \frac{\sigma^2 - \mu^2}{H_0^2}, \quad \hat{\Omega}_m = \frac{2\mu \rho_m}{H_0^2} = \frac{\mu M_{pl}^2}{M^3\Omega_m}, \quad \hat{\Omega}_r = \frac{2\mu r_0}{H_0^2} = \frac{\mu M_{pl}^2}{M^3\Omega_r}, \quad \hat{\Omega}_d = \frac{C}{H_0^2},
\]

\[\footnote{Although above we neglected the terms quadratic in matter(radiation) densities, it is verified that the bound remains valid with inclusion of these terms.}
where $\Omega_{m,r} = \hat{\rho}_{m,r0}/\hat{\rho}_c$, and $\hat{\rho}_c = 3H_0^2M_{pl}^2/8\pi$ is the critical density of the Universe approximately equal to the total present-time energy density, we can present Eq. (10) without the quadratic terms in a form

$$1 = \hat{\Omega}_V + \hat{\Omega}_m + \hat{\Omega}_r + \hat{\Omega}_d.$$ 

If we set $\mu M_{pl}^2/M^3 = 1$ and $C = 0$, we obtain an equation similar to that in the standard 4D cosmology with $\sigma^2 - \mu^2$ interpreted as the dark energy term. However, in the present case neither $C$, nor $\mu$ and $\sigma$ are fixed a\textquoteleft priori.

To have a qualitative picture let us consider the form of the Friedmann equation in different regions of $z$.

(i) Cold matter-dominated region $1 < z + 1 < (\Omega_m/\Omega_r) \sim 10^4$, where Eq.(16) is approximated by (17).

(ii) Radiation-dominated region where all $z$-dependent terms are small as compared to the radiation term $-3 + (1 + q_0)p^2 = O(\Omega_r)$

$$\frac{H_0^2}{p^2\mu^2}z^2 \left(1 + \frac{\Omega_r}{\Omega_m}z\right)^2 \ll (1 + q_0)p^2 - 3,$$

$$4z^{-1} \ll (1 + q_0)p^2 - 3.$$

Conditions (20) are fulfilled for

$$\Omega_r^{-1} \ll z < \left(\frac{p^4\Omega_m^2\mu^2}{H_0^2\Omega_r}\right)^{1/4} \simeq 10^{23}(\mu/GeV)^{1/2}. \quad (21)$$

In this region Eq.(16) approximately is

$$H^2 = \frac{\dot{z}^2}{z^2} \simeq z^4\frac{H_0^2}{2p^2}((1 + q_0)p^2 - 3). \quad (22)$$

Condition (12) defining the period of late cosmology can be written as

$$z < \left(\frac{p^2\Omega_m\mu^2}{H_0^2\Omega_r}\right)^{1/4}, \quad (23)$$

which is approximately the same as (21).

(iii) Radiation-dominated high-energy region $z > 10^{25}(-3 + (1 + q_0)p^2)^{1/4}p^{1/4}$, where the term quadratic in densities is dominant, and Friedmann equation takes the form

$$\dot{z}^2 \simeq \rho_{r0}r^2(z)z^{10} \quad (24)$$

In both regions (ii) and (iii) one can use an approximate equation

$$\dot{z}^2 \simeq z^6 \left[\frac{(1 + q_0)p^2 - 3}{2} - \mu \rho_m + \rho_{r0}r^2(z)z^4\right] = z^6\frac{H_0^2}{2p^2} \left[-3 + (1 + q_0)p^2 + \frac{H_0^2}{p^2\mu^2} \left(\frac{\Omega_r}{\Omega_m}r(z)^2\right)^2\right]. \quad (25)$$
3.2 The age of the Universe

Integrating Eq. (17), we obtain

\[
H_0(t_0 - t) = \frac{p}{\sqrt{2}} \int_{(z(t)+1)^{-2}}^{1} \frac{du}{\left[(-1 + (1 - q_0)p^2)u^2 + 4u^{1/2} - 3 + (1 + q_0)p^2\right]^{1/2}},
\]

(26)

where the bound (18) is understood. To estimate of the age of the Universe we set \( t = 0 \). For \(|q_0| > 0.5\) the results are presented in Fig. 1.

Figure 1: \( t_0H_0 \) as a function of \( p^2 \) for \(|q_0| = 0.5, 0.55, 0.6, 0.7\), curves 1-4.

From analysis of experimental data it follows that the normalized age of the Universe \( t_0H_0 \) is close to unity (\( t_0H_0 = 0.95 \pm 0.08 \), for a review and refs. see [27], recent processing of data [28, 29, 30, 31], related discussion [32]). From Fig. 1 it is seen that for \( -q_0 \) in the range 0.5 \( \pm 0.6 \) and for \( p^2 \approx 3/(q_0 + 1) \), one obtains \( t_0H_0 \approx 0.94 \div 0.96 \). For larger \(|q_0|\) to obtain \( t_0H_0 \sim 1 \) one must take larger larger \( p^2 \).

Because \( \rho_{m0} \approx \rho_c\Omega_m \approx H_0^2M_{pl}^2\Omega_m/2M^3 \), we can write

\[
p^2 \approx \frac{2M^3}{\mu M_{pl}^2\Omega_m}.
\]

Relation \( p^2 \approx 3/(1 + q_0) \) can be equivalently written as

\[
\frac{\mu M_{pl}^2}{M^3} \approx \frac{2(1 + q_0)}{3\Omega_m}.
\]

(27)

The rhs of (27) is close to unity. This is a natural result, because with \( \mu M_{pl}^2/M^3 \approx 1 \) the model is close to the standard cosmological model (cf. [26]). For \( \mu \sim 10^3 \text{ GeV} \) and \( \mu \sim 10^{-12} \text{ GeV} \) we obtain 5D scales \( M \sim 10^{14} \text{ GeV} \) and \( M \sim 5 \cdot 10^8 \text{ GeV} \).

\footnote{For estimates we take \( H_0 = 10^{-42} \text{ GeV}^{-1} \), \( q_0 = -0.57 \), \( \Omega_m = 0.24 \), \( \Omega_r = 4.6 \cdot 10^{-5} \).}

\footnote{For example, [27] give \( -q_0 = 0.58 \pm 0.01 \).}

\footnote{If \( q_0 \approx -1 + 3\Omega_m/2 \) [33, 34, 37], one obtains \( 3\Omega_m/(2(1 + q_0)) \approx 1 \).}
3.3 Model with 4D curvature term in the 5D action

In this subsection we discuss solutions of the Friedmann equation in the model with curvature term on the brane included in the 5D action (1).

As in the model with \( r_c = 0 \), in the present model it is useful to define the period of late cosmology. In the most straightforward way "late cosmology" can be defined as a period in which in the Friedmann Eq.(8) the terms linear in matter (radiation) energy densities are dominant

\[
\sigma \rho (z) > \rho^2(z), \quad \sigma \rho (z) > r_c H^2 \rho (z), \quad \sigma \rho (z) > (r_c H^2)^2.
\]  

(28)

Additionally we assume that parameter \( r_c \) is constrained so that

\[
(\mu r_c) \frac{H_0^2}{\mu^2} \ll 1.
\]  

(29)

In the most stringent case, for \( \mu \sim 10^{-12} \text{GeV} \), this condition is valid for \( \mu r_c < 10^{60} \). From Eqs. (6) and (7) taken at present time, neglecting quadratic terms, we have

\[
\sigma^2 - \mu^2 \simeq \frac{H_0^2}{2p^2} \left[ (1 + r_c \sigma)(1 - q_0)p^2 - 1 \right],
\]  

(30)

and

\[
C \simeq \frac{H_0^2}{2p^2} \left[ -3 - 4 \frac{\Omega_r}{\Omega_m} + (1 + r_c \sigma)(1 + q_0)p^2 \right].
\]  

(31)

Below from the estimates of the age of the Universe it will be shown that

\[ p^2(1 + \mu r_c)(1 + q_0) - 3 \ll 1, \]

and thus \( p^2 \simeq 3/(1 + \mu r_c)(1 + q_0) \). Using in (30) this relation, we have

\[
\left( \frac{\sigma}{\mu} \right)^2 - 1 \simeq \frac{H_0^2}{6\mu^2} \left[ (3(1 - q_0) - (1 + q_0)) + \mu r_c \left( 3(1 - q_0) \frac{\sigma}{\mu} - (1 + q_0) \right) \right].
\]

Taking into account the bound (29), we obtain that

\[
\frac{\sigma}{\mu} = 1 + O \left( \mu r_c \frac{H_0^2}{\mu^2} \right).
\]  

(32)

In the following, in expressions containing the factor \( H_0^2/\mu^2 \), we set \( \sigma = \mu \).

Provided conditions (28) and (29) are satisfied, Friedmann Eq. (8) in the period of late cosmology can be approximately written as

\[
(1 + r_c \mu) H^2 \simeq \sigma^2 - \mu^2 + 2 \mu (\rho_m(z) + \rho_r(z)) + \mu \rho_w(z).
\]  

(33)

The first condition (28), \( \sigma \simeq \mu > \rho(z) \), is valid for

\[
z^4 \ll \frac{\mu^2 p^2 \Omega_m}{\Omega_r H_0^2},
\]  

(34)

which is the same as condition (21). The second and the third conditions (28) are satisfied, if the first condition is valid.
Substituting in (33) expressions (30) and (31), we obtain

\[ H^2 \simeq \frac{H_0^2}{2} \left[ 1 - q_0 + (1 + q_0)(z + 1)^4 + \frac{1}{p^2(1 + \mu r_c)} (-1 + 4(z + 1)^3 - 3(z + 1)^4) \right]. \] (35)

which is the same as (17) in the case without 4D curvature term up to the substitution

\[ p^2 \rightarrow p^2(1 + \mu r_c). \]

The rhs of (35) is positive if

\[ p^2(1 + \mu r_c)(1 + q_0) > 3. \] (36)

Making in the expression for the age of the Universe obtained in the model with \( r_c > 0 \) redefinition \( p^2 \rightarrow p^2(\mu r_c + 1) \), we obtain the corresponding expression in the model with \( r_c > 0 \).

The normalized age of the Universe as a function of \( |q_0| \) in the region \( 0.5 - 0.6 \) and with \( p^2 = 3/(1 + q_0) \) is shown in Fig 2.

![Figure 2: \( t_0H_0 \) as a function of \( |q_0| \) with \( p^2 = 3/(1 + q_0)(1 + \mu r_c) \)](image)

It is seen that the normalized age of the Universe \( t_0H_0 \) is of order unity, if \( p^2(\mu r_c + 1) \simeq 3/(1 + q_0) \).

Substituting \( p^2 \simeq 2M^3/\mu M_{pl}^2 \Omega_m \), we obtain

\[ \frac{\mu M_{pl}^2}{M^3} \simeq \frac{2(1 + q_0)}{3\Omega_m} (\mu r_c + 1), \] (37)

where the combination \( 3\Omega_m/2(1 + q_0) \) is of order unity.

4 Model with negative tension of the brane

4.1 Model without 4D curvature term in the 5D action

In the model with negative tension on the brane, using Eqs. (10) and (11) with \( \sigma = -|\sigma| \), we have

\[ (1 - q_0)H_0^2 = 2(\sigma^2 - \mu^2) - |\sigma|\rho_m - (\rho_{r0}^2 + 3\rho_{r0}\rho_m + \rho_{r0}^2), \] (38)

\[ lp^2aH_0^2 = \sigma^2 - \mu^2 - 2|\sigma|(\rho_{r0} + \rho_m) + (\rho_m + \rho_{r0})^2 + C. \] (39)
Neglecting the terms quadratic in energy densities from these equations we express $\sigma^2 - \mu^2$ and $C$ as

$$\sigma^2 - \mu^2 \simeq \frac{H_0^2}{2p^2}(1 + (1 - q_0)p^2), \quad (40)$$

$$C \simeq \frac{H_0^2}{2p^2} \left[3 + 4 \frac{\Omega_r}{\Omega_m} + (1 + q_0)p^2\right]. \quad (41)$$

Substituting (40) and (41), we obtain the Friedmann equation as

$$H^2 \simeq (z + 1)^4 \frac{H_0^2}{2p^2} \left[ (1 + (1 - q_0)p^2)(z + 1)^{-4} - 4(z + 1)^{-1}
+ \frac{H_0^2}{p^2 \mu^2} (z + 1)^2 \left( 1 + \frac{\Omega_r}{\Omega_m} r(z)(z + 1) \right)^2 + 3 + (1 + q_0)p^2 \right]. \quad (42)$$

We neglected the term $(r(z) - 1)\Omega_r/\Omega_m$ as compared with $3 + (1 + q_0)p^2$. In contrast to the model with positive tension of the brane, in the present case we cannot make direct comparison of Eq. (42) with the corresponding relation in the standard cosmological model. However, comparison of the age of the Universe and production of Helium in BBN with the standard cosmological model is possible.

As in the model with positive tension, we distinguish the following regions of $z$.

(i) Cold matter-dominated region $1 < z + 1 < (\Omega_m/\Omega_r) \sim 10^4$, where Eq. (42) can be approximated as

$$H^2 \simeq \frac{H_0^2}{2p^2} \left[ (1 + (1 - q_0)p^2) - 4(z + 1)^3 + (3 + (1 + q_0)p^2)(z + 1)^4 \right] \quad (43)$$

For $q_0$ in the physically interesting region $|q_0| < 1$ the rhs of (43) is a positive increasing function for all $z > 0$, and there are no constraints on $p^2$.

(ii) Radiation-dominated region, in which $z > \Omega_m/\Omega_r$ and

$$3 + (1 + q_0)p^2 > \frac{H_0^2 \Omega_r^2}{p^2 \mu^2 \Omega_m^2} z^4.$$ 

In this region Eq. (42) can be written as

$$H^2 \simeq z^4 \frac{H_0^2}{2p^2} \left[ 3 + (1 + q_0)p^2 \right]. \quad (44)$$
(iii) Radiation-dominated high-energy region \( z > 10^{25} p^{1/2} \), where the term quadratic in densities is dominant, and equation is

\[
\dot{z}^2 \simeq \rho_r^2 r^2(z) z^{10}. \tag{45}
\]

### 4.2 The age of the Universe

Integrating Eq. (43), we obtain

\[
H_0(t_0 - t) = \frac{p}{\sqrt{2}} \int_{(z(t)+1)^{-2}}^{1} \frac{du}{(1 + (1 - q_0)p^2)u^2 - 4u^{1/2} + 3 + (1 + q_0)p^2]^{1/2}}. \tag{46}
\]

For an estimate we have set \( t = 0 \) and the lower limit in (46) to zero. For \( p^2 \simeq 2M^3/\mu M_{pl}^2 \Omega_m \ll 1 \),

or equivalently, for \( \mu M_{pl}^2/M^3 \gg 1 \), the normalized age of the Universe

\[
H_0 t_0 \simeq \frac{p}{\sqrt{3}} \left[ \ln \frac{2\sqrt{3}}{p} - 1 \right], \tag{47}
\]

is much below the currently accepted estimate \( H_0 t_0 \simeq 1 \).

As \( p \) increases, \( t_0 H_0 \) also increases. For \( p^2 \gg 1 \) we find that \( H_0 t_0 \) is approximately independent of \( p \). For \( |q_0| \sim 0.5 - 0.6 \) the age of the Universe is below the observational bounds and smaller than in \( \Lambda \)CDM model with \( \Omega_m \simeq 0.3 \) and \( \Omega_m + \Omega_\Lambda = 1 \). To obtain \( t_0 H_0 \simeq 1 \) one must take \( |q_0| > 0.9 \) what is outside the allowed region of \( q_0 \) \( [27, 34] \).

### 4.3 Model with 4D curvature term in the 5D action

Taking Eqs. (10) and (11) at present time and omitting the terms of higher order in energy densities, we have

\[
H_0^2(1 - r_c|\sigma|)(1 - q_0) \simeq 2(\sigma^2 - \mu^2) - |\sigma| \rho_{m0}, \tag{48}
\]

\[
H_0^2(1 - r_c|\sigma|) \simeq -\mu^2 + \sigma^2 - 2|\sigma|(\rho_{m0} + \rho_{r0}) + C. \tag{49}
\]
From these relations we express $\sigma^2 - \mu^2$ and $C$.

\[
\sigma^2 - \mu^2 \simeq \frac{H_0^2}{2p^2} \left( 1 + (1 - q_0)(1 - \sigma r_c)p^2 \right),
\]

\[
C \simeq \frac{H_0^2}{2p^2} \left[ 3 + 4 \frac{\Omega_r}{\Omega_m} + (1 - \sigma r_c)(1 + q_0)p^2 \right].
\]

Conditions of late cosmology are the same as (28) with the substitution $\sigma \rightarrow |\sigma|$. Constraining $r_c$ so that $(\mu r_c)H_0^2/\mu^2 \ll 1$, we show that with this accuracy we can substitute $\mu$ for $\sigma$.

If $1 - \mu r_c > 0$, Friedmann equation in the period of late cosmology is

\[
H^2 \simeq \frac{H_0^2}{2} \left[ (1 - q_0) + (1 + q_0)(1 + z)^4 + \frac{1}{p^2(1 - \mu r_c)} \left( 1 - 4(z + 1)^3 + 3(z + 1)^4 \right) \right],
\]

which is the the same as (43) with substitution $p^2 \rightarrow p^2(1 - \mu r_c)$. As it was discussed in the previous subsection, this model does not yield correct age of the Universe.

Let us consider the model with $\mu r_c - 1 > 0$. In this case we obtain the Friedmann equation in a form similar to Eq. (35) in the model with positive tension of the brane

\[
H^2 \simeq \frac{H_0^2}{2} \left[ 1 - q_0 + (1 + q_0)(z + 1)^4 + \frac{1}{p^2(\mu r_c - 1)} \left( -1 + 4(z + 1)^3 - 3(z + 1)^4 \right) \right].
\]

The rhs of (53) is positive for

\[
p^2(\mu r_c - 1)(1 + q_0) > 3.
\]

In this model, as in the model with positive tension of the brane, we obtain the normalized age of the Universe $t_0H_0 \simeq 1$ if $p^2(\mu r_c - 1) \simeq 3/(1 + q_0)$. This condition can be expressed as

\[
\frac{\mu M_{pl}^2}{M^3} \simeq \frac{2(1 + q_0)}{3\Omega_m}(\mu r_c - 1).
\]

\section{Friedmann equation without the dark radiation term}

In the model with curvature term on the brane included in the 5D action, the Friedmann equation without the dark radiation term $C/a^4(0, t)$ takes the form

\[
H^2 = \left( \rho + \sigma - \frac{r_c H^2}{2} \right)^2 - \mu^2.
\]

From this equation written at the current-time we determine the tension of the brane $\sigma$ as a function of $H_0^2$ and $\rho_0$, or equivalently, of $p^2$. The deceleration parameter will be discussed later.

As above, we consider the period of late cosmology, when conditions (28) are valid. Eq.(56) can be approximately written as

\[
\left| \rho + \sigma - \frac{r_c H^2}{2} \right| \simeq \mu + \frac{H^2}{2\mu}
\]

In the case of positive and negative tensions we have \footnote{To obtain cosmological solutions we must have $\mu r_c > 1$.}

\[
\sigma \simeq \mu - \rho_0 + \frac{H_0^2}{2\mu}(\mu r_c + 1)
\]

\[
|\sigma| \simeq \mu + \rho_0 - \frac{H_0^2}{2\mu}(\mu r_c - 1).
\]
With expressions (58) and (59) we obtain the Friedmann equations in both models as

\[ H^2 = H_0^2 \left[ 1 + w \left[ ((z+1)^3 - 1) + \Omega_r/\Omega_m \left( (z+1)^4 - 1 \right) \right] \right], \quad (60) \]

where

\[ w = \frac{2}{p^2(\mu r_c + 1)}, \quad \sigma > 0, \quad (61) \]
\[ w = \frac{2}{p^2(\mu r_c - 1)}, \quad \sigma < 0. \]

For large \( z \), such that \( z^4 \Omega_r/\Omega_m \gg 1 \), condition \( H^2/\mu^2 \ll 1 \) is satisfied if

\[ z^4 \frac{H_0^2 \Omega_r}{\mu^2 p^2 \Omega_m} \ll 1. \]

Integrating Eq. (60), we obtain the age of the Universe\(^7\)

| \( w \) | 0.26 | 0.28 | 0.3 | 0.31 | 0.32 | 0.35 | 0.5 |
|---|---|---|---|---|---|---|---|
| \( t_0 H_0 \) | 1.00 | 0.98 | 0.96 | 0.96 | 0.95 | 0.92 | 0.83 |

From Friedmann Eq. (60) we can obtain relation between \( w \) and \( q_0 \). Taking the time derivative of this equation,

\[ 2\dot{H}H = H_0^2 w \dot{z} [3(z+1)^2 + 4(z+1)^3 \Omega_r/\Omega_m], \]

and evaluating it at present time \( t = t_0 \), we obtain

\[ w \simeq \frac{2(1 + q_0)}{3}, \]

where we neglected the small second term. From the table it is seen that for \( w = 0.3 \pm 0.05 \) the age of the Universe is within the observational bounds \( t_0 H_0 = 0.96 \pm 0.06 \). For \( w = 0.3 \) the deceleration parameter is \( q_0 = -0.55 \).

From (61) we obtain a relation between parameters of the model

\[ \frac{\mu M_{pl}^2}{M^2} \simeq \frac{2(1 + q_0)}{3 \Omega_m (\mu r_c \pm 1)}. \quad (62) \]

6 Primordial nucleosynthesis

6.1 Model with positive tension of the brane

In this section, comparing predictions of BBN in the standard and non-standard cosmologies, we obtain constraints on parameters of the non-standard model. First, we study the model with positive tension of the brane without the 4D curvature term in the action.

\(^7\)In the cold matter dominated region the the Friedmann equation \( H^2 = H_0^2 \left[ 1 + w ((z+1)^3 - 1) \right] \) is integrated analytically and yields \( H_0 t_0 = 2/(3\sqrt{1-w}) \ln \left[ \sqrt{1-w} + 1)/\sqrt{w} \right] \). Numerical results which follow from this formula are very close to those in the table which include contribution from the radiation period.
In the standard cosmology, the expressions for the radiation energy density as functions of $t(z)$ and of temperature of the Universe $T$ are

\[ \dot{\rho}(t) = \dot{\rho}_0 r(z) z^4 = \frac{\dot{\rho}_0}{4(H_0 t)^2}, \]

\[ \dot{\rho}(T) = \frac{\pi^2}{30} g_*(T) T^4. \]

In non-standard model, in the period of late cosmology we have

\[ \ddot{\rho}(t) \simeq \frac{\dot{\rho}_0}{2(\dot{H}_0 t)^2} \left[ (1 + q_0) p^2 - 3 + 4 \left( r(z) - 1 \right) \Omega_r / \Omega_m \right], \]

\[ \ddot{\rho}(\tilde{T}) = \frac{\pi^2}{30} g_*(\tilde{T}) \tilde{T}^4 \]

where by tilde we distinguish the non-standard case. The freezing temperature $T_F$ of the reaction $n \leftrightarrow p$ is estimated as a temperature at which the Hubble parameter $H$ is of order of the reaction rate $G_2^2 T_F^5$. Time dependence of the Hubble parameter in both the standard and non-standard cosmologies is the same $H = 1/(2t)$. Using (63) and (64), we obtain the ratio of freezing temperatures in the standard and non-standard cosmologies

\[ \frac{\tilde{T}_F}{T_F} = \left( \frac{g_*(\tilde{T}_F) \left[ (1 + q_0) p^2 - 3 + 4 \left( r(z) - 1 \right) \Omega_r / \Omega_m \right]}{g_*(T_F) 2p^2 r(z) \Omega_r} \right)^{1/6}. \]

The mass fraction of $^4He$ produced in nucleosynthesis of the total baryon mass is

\[ X_4 = \frac{2(n/p)_f}{(n/p)_f + 1}, \]

where the subscript ”f” indicates that the ratio is taken at the end of primordial nucleosynthesis, $(n/p)_f \simeq 1/7$. The equilibrium value of the neutron-proton ratio $(n/p)_T = \exp \left[ -(m_n - m_p)/(T_F) \right]$ is very sensitive to the value of $T_F$. The value of $X_4$ calculated in the standard cosmological model fits well the cosmological data $X_4 = 0.25 \pm 0.01$. We constrain parameters of the model requiring that the difference between the calculated values of $X_4$ in the standard and non-standard models is within the experimental errors. Under variation of freezing temperature variation of $X_4$ is

\[ \delta X_4 \simeq \frac{2}{((n/p)_f + 1)^2 (n/p)_f \ln(p/n)_f} \frac{\delta T_F}{T_F}. \]

Variation of $X_4$ is within the experimental errors for variations of freezing temperature in the range

\[ \frac{\delta T_F}{T_F} < 0.0255. \]

Comparing $T_F$ and $\tilde{T}_F$ in (65), we find that variation of $X_4$ is within the experimental bounds if

\[ \left| \frac{g_*(\tilde{T}_F) \left[ (1 + q_0) p^2 - 3 + 4 \left( r(z) - 1 \right) \Omega_r / \Omega_m \right]}{g_*(T_F) 2p^2 r(z) \Omega_r} - 1 \right| < \varepsilon, \]

\[ \delta T_F < 0.0255. \]
where \( \varepsilon = 0.164 \). Characteristic temperatures of nucleosynthesis are below the neutrino decoupling temperature \( T \sim (0.8 \div 1) MeV \) and \( e^\pm \) annihilation temperature \( \sim 0.5 MeV \). At these temperatures the number of effective degrees of freedom in the expression of radiation energy density is constant \( g_*(T) = 3.36 \). Since in this range the number of effective degrees of freedom does not change, parameter \( r \) is also constant and equal the present-time value \( r = 1 \).

Substituting in (68) \( p^2 \approx 2 M^3 / \mu M_{pl}^3 \Omega_m \) and setting \( r = 1 \), we obtain

\[
\frac{4 \Omega_r}{3 \Omega_m} (1 - \varepsilon) < \frac{\mu M_{pl}^2}{M^3} - \frac{2(1 + q_0)}{3 \Omega_m} \leq \frac{4 \Omega_r}{3 \Omega_m} (1 + \varepsilon). \tag{69}
\]

which is specification of the relation (27) obtained from the estimate of the age of the Universe.

Now we can verify that in the non-standard model BBN takes place at the period of late cosmology. In the non-standard model, in the radiation-dominated period, solving Eq. (25) which interpolates between the early and late radiation-dominated periods, we find

\[ z^{-4} \approx \frac{2[(1 + q_0)p^2 - 3]}{p^2(\mu r_c + 1)}(H_0 t)^2 + \frac{4 \Omega_r}{p^2 \mu \Omega_m} H_0^2 t. \tag{70} \]

From (70) follows that the transition time \( \bar{t} \) from \( \rho^2 \) to \( \rho \)-dominated law is at \( \bar{z} \sim (\mu^2/H_0^2 \Omega_r)^{1/4} \). In the most stringent case, for \( \mu \sim 10^{-12} GeV \), this yields \( \bar{z} \sim 10^{16} \), which is much larger than \( z_{BBN} \sim 10^{9 \div 10} \). From this estimate and (70) it follows that the transition time is \( \bar{t} \sim 1/\mu \). For the limiting values in the assumed interval of \( \mu \) transition times are \( \bar{t} \sim 10^{-3} GeV^{-1} \) and \( \bar{t} \sim 10^{12} GeV^{-1} \) which are much smaller than the characteristic time of nucleosynthesis \( 1 \div 10^{2} s \), or \( 10^{21 \div 26} GeV^{-1} \) estimated in the standard cosmology.

Let us turn to the model with 4D curvature term included in the action. In the radiation-dominated period of late cosmology we have

\[ z^{-4} \approx \frac{2[p^2(\mu r_c + 1)(1 + q_0) - 3]}{p^2(\mu r_c + 1)}(H_0 t)^2. \tag{71} \]

Following the same steps as in the model with \( r_c = 0 \), we obtain that production of Helium is within the experimental bounds if

\[ \frac{p^2(\mu r_c + 1)(1 + q_0) - 3}{2p^2(\mu r_c + 1) \Omega_r - 1} \leq \varepsilon, \tag{72} \]

where \( \varepsilon \approx 0.164 \). From (72) we find a constraint on \( p^2 \)

\[ \frac{6 \Omega_r (1 + \varepsilon)}{(1 + q_0)} < p^2(1 + \mu r_c)(1 + q_0) - 3 \leq \frac{6 \Omega_r (1 + \varepsilon)}{(1 + q_0)}, \tag{73} \]

and constraint on the scales of the model

\[ \frac{3 \Omega_r \Omega_m (1 + \varepsilon)}{(1 + q_0)^2} < \frac{M^3(\mu r_c + 1)}{\mu M_{pl}^2} - \frac{3 \Omega_m}{2(1 + q_0)} \leq \frac{3 \Omega_r \Omega_m (1 + \varepsilon)}{(1 + q_0)^2}. \tag{74} \]

Using the inequalities (73) we obtain the bounds on \( \rho_{w0} \)

\[ \frac{H_0^2}{\mu^2 \Omega_r(1 + \mu r_c)} \left( 1 - \varepsilon - \frac{2(1 + q_0)}{3 \Omega_m} \right) \leq \rho_{w0} \leq \frac{H_0^2}{\mu^2 \Omega_r(1 + \mu r_c)} \left( 1 + \varepsilon - \frac{2(1 + q_0)}{3 \Omega_m} \right). \tag{75} \]

\( ^8 \)In another context similar estimates were done in [36].

\( ^9 \)The same estimate follows from condition \( \rho(z) \sim \mu \).
The combination \(2(1 + q_0)/3\Omega_m\) is of order unity, and within the existing uncertainties of cosmological parameters the sign of \(\rho_{w0}\) is ambiguous. From (75) it follows that \(\rho_{r0}/\mu > \rho_{w0}/\mu\).

In the model with \(r_c \neq 0\) transition from the early to late cosmology takes place at \(\bar{z}\) such that \(\rho(\bar{z}) < \mu\) or

\[
\bar{z}^4 < \frac{\mu^2}{H_0^2 \Omega_r (\mu r_c + 1)}.\]

Requiring that \(z_{BBN} \sim 10^{9-10} \ll \bar{z}\), we obtain

\[
\mu r_c \ll \frac{1}{\Omega_r z_{BBN}^4} \frac{\mu^2}{H_0^2}. \tag{76}
\]

In the most stringent case, for \(\mu \sim 10^{-12} GeV\), this is satisfied if

\[
\mu r_c \ll 10^{25}. \tag{66}
\]

### 6.2 Model with negative tension of the brane

First, we consider the model without the 4D curvature term in the action. Solution of Friedmann equation in regions (ii) and (iii) of the radiation-dominated period is

\[
\bar{z}^{-4} \approx \frac{2(3 + (1 + q_0)p^2)}{p^2} (H_0 t)^2 + \frac{4\Omega_r}{p^2 \mu \Omega_m} H_0^2 t. \tag{77}
\]

At the characteristic times of nucleosynthesis the first term in (77) is dominant, and we can neglect the second one.

Following the same steps as in the model with positive tension of the brane, we obtain the ratio of the freezing temperatures in the standard and non-standard cosmologies

\[
\frac{\tilde{T}_F}{T_F} = \left( \frac{g_*(\bar{T}_F) [3 + (1 + q_0)p^2]}{2p^2 \Omega_r} \right)^{1/6}. \tag{78}
\]

Because \((3 + (1 + q_0)p^2)/2p^2 \Omega_r\) is not a small number, the abundance of \(He\) calculated in this model is outside the experimental bounds.

In the model with 4D curvature term included in the 5D action we consider two cases. If \(1 - \mu r_c > 0\), the model is similar to the case without 4D term. As it was discussed above, in this case the model does not describe nucleosynthesis correctly. If \(\mu r_c - 1 > 0\), the model is similar to the model with positive tension of the brane and can yield correct value for \(X_4\).

### 7 Conclusions

In this paper we discussed a class of warped cosmological solutions in 5D models with infinite extra dimension. We considered the models with positive and negative tensions of the brane. We studied separately the models with and without 4D curvature term on the brane included in the 5D action. We looked for constraints on parameters of the models which follow from consistency of predictions of the models with the main cosmological data - the age of the Universe and abundance of \(^4He\) produced in primordial nucleosynthesis.
For numerical estimates we used the present-time values of the Hubble parameter $H_0$, fractions of cold matter and radiation $\Omega_m, \Omega_r$, deceleration parameter $q_0$ and the mass fraction $X_4$ of the total barion mass. The warping scale $\mu$ was taken in the interval $10^{-12} - 10^3 GeV$.

In the models with positive tension of the brane in both cases with and without 4D curvature term on the brane the normalized age of the Universe $t_0H_0$ was obtained within the observational bound if parameters of the model are connected by the relation

$$\frac{\mu M^2_{pl}}{M^3} \simeq (\mu r_c + 1) \frac{3\Omega_m}{2(1 + q_0)}.$$  

Comparing abundance of $^4He$ in the standard and non-standard models, we obtained a further constraint on parameters

$$\left| \frac{M^3(\mu r_c + 1)}{\mu M^2_{pl}} - \frac{3\Omega_m}{2(1 + q_0)} \right| < C\Omega_r,$$

where $C = O(10)$. Because $3\Omega_m/2(1 + q_0) \sim 1$, from these relations it follows that

$$M^3 \simeq \frac{\mu M^2_{pl}}{\mu r_c + 1}.$$  

In the case of negative tension of the brane situation is different. With the input of the current-time cosmological parameters the model without 4D curvature term in the action does not yield correct age of the Universe and the abundance of $^4He$. In particular, this refers also to the RS-type two-brane models, in which the observable brane is that with negative tension[5].

The model with 4D curvature term included in the 5D action can meet the observational data provided $\mu r_c > 1$. The normalized age of the Universe $t_0H_0$ is obtained of order unity, if

$$\frac{\mu M^2_{pl}}{M^3} \simeq \frac{2(1 + q_0)}{3\Omega_m} (\mu r_c - 1).$$

From the estimate of the abundance of $^4He$ follows a more precise constraint

$$\left| \frac{M^3(\mu r_c - 1)}{\mu M^2_{pl}} - \frac{3\Omega_m}{2(1 + q_0)} \right| < C\Omega_r,$$

where $C \sim 10$. Further restrictions are obtained, if $M_{pl}$ is the largest scale of the theory, i.e. $M_{pl}^2 > r_cM^3$, and $3\Omega_m/2(1 + q_0) < 1$. In this case we have

$$\mu r_c < \frac{\mu M^2_{pl}}{M^3} < \left( \frac{2(1 + q_0)}{3\Omega_m} - C\Omega_r \right)^{-1} (\mu r_c - 1),$$  

or

$$\mu r_c \left( 1 - \frac{3\Omega_m}{2(1 + q_0)} + C\Omega_r \right) > 1.$$  

Because $3\Omega_m/2(1 + q_0) \sim 1$, it follows that in this model $\mu r_c \gg 1$.

Generally, numerical values of cosmological parameters depend on a model in which the experimental data were processed. Recently fits of parameters were performed in one-brane models with positive tension of the brane [37, 40, 38, 39]\textsuperscript{10}.

\textsuperscript{10}The dark radiation term was taken as $C/a^4 - \alpha$, where $\alpha = 0$ corresponds to the case of the present paper.
Let us compare the estimates of the present paper in which we used parameters of the standard cosmological model with those obtained in these fits. Identifying the corresponding terms in the Friedmann equation in the form employed in the present paper

\[ H^2 = \sigma^2 - \mu^2 + 2\sigma\rho + \rho^2 + Cz^4 \]

with those in [37, 38, 39]

\[ H^2 = \frac{\bar{\Lambda}}{3} + \frac{\bar{\kappa}^2 \rho}{6\lambda^2} + 2m_0 z^4 \]

(because of different normalizations, in the latter equation we use notations with a bar), we have

\[ \bar{\kappa}^2 = \kappa^2\sigma, \quad \lambda = \frac{6\sigma}{\kappa^2}, \quad \bar{\Lambda}/3 = \sigma^2 - \mu^2, \quad 2m_0 = C, \]

Introducing

\[ \bar{\Omega}_\rho = \frac{\bar{\kappa}^2 \rho_0}{3H_0^2}, \quad \bar{\Omega}_\Lambda = \frac{\bar{\kappa}^2 \rho_0^2}{6\Lambda H_0^2}, \quad \bar{\Omega}_{\bar{\Lambda}} = \frac{\bar{\Lambda}}{3H_0^2}, \quad \bar{\Omega}_d = \frac{2m_0}{H_0^2}, \]

we obtain the relations

\[ \bar{\Omega}_\rho = \Omega_m b^2 = \frac{2}{p^2}, \quad \bar{\Omega}_{\bar{\Lambda}} = -\frac{1 + (1 - q_0)p^2}{2p^2}, \quad \bar{\Omega}_d = \frac{C}{H_0^2} = -3 + \frac{(1 + q_0)p^2 - 4\Omega_\rho/\Omega_m}{2p^2} \]

(79)

Some examples of the best fits of parameters \( \bar{\Omega} \) are presented in the Table.

| ref. | \( \bar{\Omega}_\rho \) | \( \bar{\Omega}_\Lambda \) | \( \bar{\Omega}_d \) | \( \bar{\Omega}_{\bar{\Lambda}} \) | \(-q_0 \) derived |
|------|----------------|----------------|----------------|----------------|----------------|
| [39] | prior(0.15, 0.35) | 0.225 | 0.735 | prior(-0.03, 0.07) | 0.04 | 0.58 |
| [40] | 0.15 | 0.80 | prior (-0.1, 0.1) | 0.008 | 0.026 | 0.67 |
| [40] | prior (0.2, 0.4) | 0.29 | 0.78 | prior(-0.1, 0.1) | -0.09 | 0.02 | 0.68 |
| [37] | 0.25 | 0.73 | prior 0 | | 0 | 0.02 | 0.57 |

As an example, substituting parameters \( \bar{\Omega}_\rho \) and \( \bar{\Omega}_{\bar{\Lambda}} \) from the first line of the table in relations (79), we calculate \( p^2 = 8.89, b^2 = 0.999 \). With these numbers we find \( \bar{\Omega}_d = 0.04 \).

Let us estimate the BBN constraints on ratio of the dark radiation to photon energy density at \( T_F \sim (0.8 \div 1) MeV \) in the model with \( r_c = 0 \). Dark radiation and photon energy density scale with \( z \) in the same way. Photon energy density is [35]

\[ \rho_\gamma(z) = \frac{2}{3.36} \rho_{r0} z^4. \]

Here

\[ \rho_{r0} = \frac{\Omega_r H_0^2}{\Omega_m p^2 \mu}. \]

The bound (75) can be rewritten as

\[ \frac{H_0^2}{p^2 \mu^2 1 + q_0} \frac{3\Omega_r}{1 - \varepsilon - \frac{2(1 + q_0)}{3\Omega_m}} < \frac{\rho_w}{\mu} < \frac{H_0^2}{p^2 \mu^2 1 + q_0} \frac{3\Omega_r}{1 + \varepsilon - \frac{2(1 + q_0)}{3\Omega_m}}. \]
For the ratio of energy densities we obtain

\[
3.36 \frac{3 \Omega_m}{2(1 + q_0)} \left( 1 - \varepsilon - \frac{2(1 + q_0)}{3 \Omega_m} \right) < \frac{\rho_w}{\rho_\gamma} < 3.36 \frac{3 \Omega_m}{2(1 + q_0)} \left( 1 + \varepsilon - \frac{2(1 + q_0)}{3 \Omega_m} \right).
\]

With \( \varepsilon = 0.164 \) and \( 2(1 + q_0)/3 \Omega_m = 1.00 \) we have

\[
\left| \frac{\rho_w}{\rho_\gamma} \right| < 0.55,
\]

to compare with the bound \(-0.4 < \rho_w/\rho_\gamma < 0.1 \) of \[41\].

Using parameters of best fit of \[39\] (the first line of the table) we obtain \( 2(1 + q_0)/3 \Omega_m = 1.24 \). In this case \(-1.1 < \rho_w/\rho_\gamma < -0.2 \) to compare with the result of \[39\] \(-1.32 < \rho_w/\rho_\gamma < 0.34 \).

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