Hopf Galois Extension in Braided Tensor Categories

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Abstract

The relation between crossed product and $H$-Galois extension in braided tensor category $C$ with equivalisers and coequivalisers is established. That is, it is shown that if there exist an equivaliser and a coequivaliser for any two morphisms in $C$, then $A = B#_\sigma H$ is a crossed product algebra if and only if the extension $A/B$ is Galois, the canonical epic $q : A \otimes A \to A \otimes_B A$ is split and $A$ is isomorphic as left $B$-modules and right $H$-comodules to $B \otimes H$ in $C$.

Keywords: braided Hopf algebra, crossed product algebra, $H$-Galois extension.

0 Introduction

The Hopf Galois extension has its roots in the work of Chase-Harrison-Rosenberg [6] and Chase-Sweedler [7]. The general definition about Hopf Galois extension appeared in [14] and the relation between crossed product and $H$-Galois extension was obtained in [5][10][11] for ordinary Hopf algebras. See also the books by Montgomery [18] and Dascalescu-Nastasecu-Raianu [9] for reviews about the main results in this topic.

On the other hand, braided Hopf algebras have attracted much attention in both mathematics and mathematical physics (see [1][13][15][17][20][21]). In particular, braided Hopf algebras play an important role in the classification of finite-dimensional pointed Hopf algebras (see [21]). So it is desirable to generalize the above results to the case of braided tensor categories. In this paper we show that if there exist an equivaliser and a coequivaliser for any two morphisms in the braided tensor category $C$, then $A = B#_\sigma H$ is a crossed product algebra if and only if the extension $A/B$ is Galois, the canonical epic $q : A \otimes A \to A \otimes_B A$ is split and $A$ is isomorphic as left $B$-modules and right $H$-comodules to $B \otimes H$ in $C$. 

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This paper is organized as follows. In section 1 since it is possible that the category is not concrete, we define the coinvariants $B = A^{coH}$ and the tensor product $A \otimes_B A$ over algebra $B$ by equivaliser and coequivaliser for right $H$-comodule algebra $(A, \psi)$. Our main result in this section is given in theorem [18]. In section 2, we apply the conclusion in section 1 to the Yetter-Drinfeld category $\mathcal{D}$ and give an example to explain our result.

1 Hopf Galois extension in braided tensor categories

In this section we give the relation between crossed product and $H$-Galois extension in braided tensor categories.

Let $(C, \otimes, I, C)$ be a braided tensor category with equivalisers and coequivalisers, where $I$ is the identity object and $C$ is the braiding. We denote by equivaliser and coequivaliser for right $H$-module algebra $(A, \psi)$, respectively. Let $(A, \psi)$ be a braided Hopf algebra in $C$. We called the equivaliser $(\psi, id_A \otimes \eta_H)$ the coinvariants of $A$, written $(A^{coH}, p)$. We called the coequivaliser $((m_A \otimes A)(A \otimes p \otimes A), (A \otimes m_A)(A \otimes p \otimes A))$ the tensor product of $A$ and $A$ over $A^{coH}$, written $(A \otimes A^{coH}, A, g)$. Note that $p$ is monic from $A^{coH}$ to $A$ and $q$ is epic from $A \otimes A$ to $A \otimes A^{coH}$ $A$ (see [12]).

Definition 1.1 Let $(A, \psi)$ be a right $H$-comodule algebra in $C$ and $can' =: (m_A \otimes H)(A \otimes \psi)$ a morphism from $A \otimes A$ to $A \otimes H$. If there exists an equivalence can in $C$ from $A \otimes A^{coH}$ A to $A \otimes H$ such that $can \circ q = can'$, then we say that $A$ is a right $H$-Galois, or the extension $A/A^{coH}$ is Galois.

We first recall the crossed product $B\#_\sigma B$ of $B$ of $H$ in $C$ similar to [18] Definition 7.1.1. $(H, \alpha)$ is said to act weakly on algebra $B$ if the following conditions are satisfied:

(WA): \( \alpha(H \otimes m_B) = m_B(\alpha \otimes \alpha)(H \otimes C \otimes B)(\Delta_H \otimes B \otimes B) \) and \( \alpha(H \otimes \eta_B) = \eta_B \epsilon_H \).

\( \sigma \) is called a 2-cocycle from $H \otimes H$ to $B$ if the following conditions are satisfied:

(2-COC): \[ m_B(\alpha \otimes \sigma)(H \otimes C \otimes H)(H \otimes H \otimes \sigma \otimes m)(H \otimes H \otimes C \otimes H)(\Delta_H \otimes \Delta_H \otimes \Delta_H) = m_B(B \otimes \sigma)(\sigma \otimes m \otimes H)(H \otimes C \otimes H \otimes H)(\Delta_H \otimes \Delta_H \otimes \Delta_H) \] and \( \sigma(H \otimes \eta_H) = \sigma(\eta_H \otimes H) = \eta_B \epsilon_H \).

$(B, \alpha)$ is called a twisted $H$-module if the following conditions are satisfied:
is clear that the multiplications in $B$ twisted algebra with unity element $\eta$ and unit $\eta$ (2.2). Consequently, it is sufficient to show that [3, Relation (2.3)] holds if and only if

(i) the relation (2.3) in [3, Proposition 2.2] by the following:

(ii) $\phi_{2,1} = (\alpha \otimes H)(H \otimes C)(\Delta_B \otimes B)$. It is clear that the multiplications in $B #_{\sigma} H$ and $B #_{\phi_{2,1}} H$ are the same (see [3, Proposition 2.2]). Consequently, it is sufficient to show that [3, Relation (2.3)] holds if and only if $(H, \alpha)$ acts weakly on $B$ and $(B, \alpha)$ is a twisted $H$-module with 2-cocycle $\sigma$. Let us denote the relation (2.3) in [3, Proposition 2.2] by the following:

(i) $(\alpha \otimes H)(H \otimes C)(\Delta_B \otimes B)(\eta_B \otimes B) = (B \otimes \eta_B)$. 

(ii) $(\alpha \otimes H)(H \otimes C)(\Delta_B \otimes B)(\eta_B \otimes B) = (B \otimes \eta_B)$. 

(iii) $(m_B \otimes H)(B \otimes \sigma \otimes m_B)(B \otimes H \otimes C \otimes H)(\Delta_B \otimes B \otimes B)$. 

Lemma 1.2 If $B$ is an algebra and $H$ is a bialgebra in $C$, then $A = B #_{\sigma} H$ is an algebra with unity element $\eta_A = \eta_B \otimes \eta_H$ iff $(H, \alpha)$ acts weakly on $B$ and $(B, \alpha)$ is a twisted $H$-module with 2-cocycle $\sigma$. Let us denote

(i) $(\alpha \otimes H)(H \otimes C)(\Delta_B \otimes B)(\eta_B \otimes B) = (B \otimes \eta_B)$. 

(ii) $(\alpha \otimes H)(H \otimes C)(\Delta_B \otimes B)(\eta_B \otimes B) = (B \otimes \eta_B)$. 

(iii) $(m_B \otimes H)(B \otimes \sigma \otimes m_B)(B \otimes H \otimes C \otimes H)(\Delta_B \otimes B \otimes B)$. 

Let us denote the relation (2.3) in [3, Proposition 2.2] by the following:

(i) $(\alpha \otimes H)(H \otimes C)(\Delta_B \otimes B)(\eta_B \otimes B) = (B \otimes \eta_B)$. 

(ii) $(\alpha \otimes H)(H \otimes C)(\Delta_B \otimes B)(\eta_B \otimes B) = (B \otimes \eta_B)$. 

(iii) $(m_B \otimes H)(B \otimes \sigma \otimes m_B)(B \otimes H \otimes C \otimes H)(\Delta_B \otimes B \otimes B)$. 

Proof. Let $\hat{\sigma} = (\sigma \otimes m_B)(H \otimes C \otimes H)(\Delta_H \otimes \Delta_B)$ and $\phi_{2,1} = (\alpha \otimes H)(H \otimes C)(\Delta_B \otimes B)$. It is clear that the multiplications in $B #_{\sigma} H$ and $B #_{\phi_{2,1}} H$ are the same (see [3, Proposition 2.2]). Consequently, it is sufficient to show that [3, Relation (2.3)] holds if and only if $(H, \alpha)$ acts weakly on $B$ and $(B, \alpha)$ is a twisted $H$-module with 2-cocycle $\sigma$. Let us denote the relation (2.3) in [3, Proposition 2.2] by the following:
(iv) 

\[
\begin{align*}
(m \otimes H)(B \otimes \sigma \otimes m_H)(B \otimes H \otimes C \otimes H)(\alpha \otimes \Delta_H \otimes \Delta_H)
&= (m_B \otimes H)(B \otimes \alpha \otimes H)(B \otimes H \otimes C)(B \otimes \Delta_H \otimes B)(\sigma \otimes m_H \otimes B) \\
&\quad (H \otimes C \otimes H \otimes B)(\Delta_H \otimes \Delta_H \otimes B).
\end{align*}
\]

Obviously, if (WA), (2-COC) and (TM) hold then relation (i) holds. Therefore, we assume that relation (i) holds. Applying \((\text{id}_B \otimes \epsilon_H)\) on relation (iii), we can obtain (2-COC). It is straightforward that \((ii) \Leftrightarrow (WA), (iii) \Leftrightarrow (2-COC)\) and \((iv) \Leftrightarrow (TM)\). Consequently, using \([3, \text{ Proposition 2.2}]\), we complete the proof. \(\square\)

**Theorem 1.3** Let \(C\) be a braided tensor category with equivalisers and coequivalisers for any two morphisms in \(C\). Then the following assertions are equivalent:

(i) There exists an invertible 2-cocycle \(\sigma : H \otimes H \to B\), and a week action \(\alpha\) of \(H\) on \(B\) such that \(A = B \#_\sigma H\) is a crossed product algebra.

(ii) \((A, \psi)\) is a right \(H\)-comodule algebra with \(B = A^{\text{co}H}\), the extension \(A/B\) is Galois, the canonical epic \(q : A \otimes A \to A \otimes B\) is split and \(A\) is isomorphic as left \(B\)-modules and right \(H\)-comodules to \(B \otimes H\) in \(C\), where the \(H\)-comodule operation and \(B\)-module operation of \(B \otimes H\) are \(\psi_{B \otimes H} = \text{id}_B \otimes \Delta_H\) and \(\alpha_{B \otimes H} = m_B \otimes \text{id}_H\) respectively.

**Proof.** \((i) \Rightarrow (ii)\). It is clear that \((A, \psi)\) is a right \(B\)-comodule algebra under \(H\)-comodule operation \(\psi = (B \otimes \Delta_H)\). Let \(p : = \text{id}_B \otimes \eta_H\). For object \(D\) and any morphism \(f : D \to B \#_\sigma H\) in \(C\) with \(\psi f = (id_A \otimes \eta_H)f\), set \(\bar{f} : = (id_B \otimes \epsilon)f\). Thus \(p \circ \bar{f} = f\), which implies that \((B, p)\) is the coinvariants of \((A, \psi)\). Set \(\Theta' : = (m_A \otimes \eta_B \otimes H)(A \otimes C_{H,B} \otimes H)(A \otimes H \otimes \sigma^{-1} \otimes H)(A \otimes S \otimes S \otimes \Delta)(A \otimes \Delta^2_H) : A \otimes H \to A \otimes A\) and \(\Theta : = q \circ \Theta' : A \otimes H \to A \otimes B\).

It is remain to show that \(\text{can} \circ \Theta = \text{id}\) and \(\Theta \circ \text{can} = \text{id}\). For this we first give the relation between the module operation and the 2-cocycle. Let \(\overline{\sigma}\) denote the convolution-inverse of \(\sigma\) and \(\sigma^{-1}\). We denote the multiplication, comultiplication, antipode, braiding and inverse braiding by 

\[
\begin{align*}
\hat{\otimes} &\quad , \\
\hat{\otimes} &\quad , \\
\hat{\bullet} &\quad , \\
\hat{\otimes} &\quad and \\
\hat{\otimes} &\quad ,
\end{align*}
\]

respectively.
We first give the relations between the module operation and the 2-cocycle.

\[
\begin{array}{c}
H \\
\downarrow \\
H \\
\downarrow \\
\cdots \cdots (*)
\end{array}
\]

In fact,

the right side of \((*)\) by \(2\text{-coc} \)
Indeed, it is clear that $\alpha(H \otimes \sigma^{-1})$ is the convolution inverse of $\alpha(H \otimes \sigma)$. Therefore, it is sufficient to show that the right side of (***) is also the convolution inverse of $\alpha(H \otimes \sigma)$. See
\[
\begin{array}{c}
H \quad H \quad H \\
\Downarrow \quad \Downarrow \quad \Downarrow \\
\sigma \quad \sigma \quad \sigma \\
\Downarrow \quad \Downarrow \quad \Downarrow \\
B \\
\end{array}
\]

\[
\begin{array}{c}
H \quad H \quad H \\
\Downarrow \quad \Downarrow \quad \Downarrow \\
\sigma \quad \sigma \quad \sigma \\
\Downarrow \quad \Downarrow \quad \Downarrow \\
B \\
\end{array}
\]
Now we see that

\[ \text{can} \circ \Theta = \text{can'} \circ \Theta' = \]

by \((2-\text{coc})\)
by \((WA)\) = \(B H H\)
\[ \Theta' \circ \text{can'} = B \]

Diagram: 

\[ A \quad B \quad H \]

\[ B \quad H \quad B \quad H \]

\[ \Theta' \circ \text{can'} = B \]

Diagram: 

\[ A \quad B \quad H \]

\[ B \quad H \quad B \quad H \]
and

by \( (** \) \)
Consequently,

$$\Theta' \circ \text{can}' = \begin{array}{llll}
B & H & B & H \\
\text{can}' & \text{can}' & \text{can}' & \text{can}'
\end{array}$$

See that

$$\Theta \circ \text{can} \circ q = q \circ \Theta' \circ \text{can}'$$

$$= q(m_A \otimes A)(A \otimes p \otimes A)(A \otimes B \otimes \eta_B \otimes H)$$

$$= q(A \otimes m_A)(A \otimes p \otimes A)(A \otimes B \otimes \eta_B \otimes H)$$

$$= q .$$

Thus $$\Theta \circ \text{can} = id_{A \otimes B A}$$. Furthermore, since $$\Theta = q \circ \Theta'$$ we have $$q \circ (\Theta' \circ \Theta^{-1}) = id$$, which implies $$q$$ is a split epic morphism.

(ii) $$\Rightarrow$$ (i). Since $$q$$ is split, there exists morphism $$w : A \otimes_B A \rightarrow A \otimes A$$ such that $$q \circ w = id_{A \otimes_B A}$$. Let $$\Theta' =: w \circ \text{can}^{-1} : A \otimes H \rightarrow A \otimes A$$, $$\text{can}' =: \text{can} \circ q : A \otimes A \rightarrow A \otimes H$$
and \( \gamma =: \Phi(\eta_A \otimes H) : H \to A, \ u = \text{can}^{-1}(\eta_A \otimes H) : H \to A \otimes_B A, \ u' = \Theta'(\eta_A \otimes H) : H \to A \otimes A \) and \( \mu = m_A(A \otimes p \otimes \epsilon_H)(A \otimes \Phi^{-1})u' : H \to A \). Obviously, \( \text{can}^{-1} = q \circ \Theta' \), \ann(\gamma) = \text{can} \circ q \circ \Theta'(\eta_A \otimes H) = \text{can} \circ \text{can}^{-1}(\eta_A \otimes H) = (\eta_A \otimes H) \) and \( \psi \circ \gamma = (\gamma \otimes H)\Delta \).

\[
(A \otimes \Theta)u' = (u' \otimes H)\Delta
\]  
(1)

Since \( (\text{can} \otimes H)(A \otimes \psi)u' = (m_A \otimes H \otimes H)(A \otimes \psi \otimes H)(A \otimes \psi)u' = (m_A \otimes \Delta)(A \otimes \psi)u' = (\eta_A \otimes \Delta) \) and \( (\text{can} \otimes H)(u' \otimes H)\Delta = (\text{can} \otimes \Delta)(u' \otimes H)\Delta = (\eta_A \otimes \Delta) \).

\[
m_Au' = \eta_A \epsilon_H
\]  
(2)

Since \( mu' = (m_A \otimes \epsilon_H)(A \otimes \psi)u' = (A \otimes \epsilon)\text{can} \circ u' = (A \otimes \epsilon)(\eta_A \otimes H) = \eta_A \epsilon_H \).

\[
(m_A \otimes A)(A \otimes u')\psi = \eta_A \otimes A
\]  
(3)

Since \( \text{can}'(m_A \otimes A)(A \otimes u')\psi = (m_A \otimes H)(m_A \otimes \psi)(A \otimes u')\psi = (m_A \otimes H)(m_A \otimes A \otimes H)(A \otimes u' \otimes H)(A \otimes \Delta_H)\psi = \psi \) by (1), (2), and \( \text{can}'(\eta_A \otimes A) = \psi \).

Now we show that \( \mu \) is the convolution inverse of \( \gamma \). Indeed,

\[
\gamma * \mu = m(\Phi \otimes \mu)(\eta_B \otimes \Delta_H)
\]
\[
= m_A(A \otimes \mu)\psi\Phi(\eta_B \otimes H) \quad \text{since \( \Phi \) is an \( H \)-comodule morphism}
\]
\[
= (m_A \otimes \epsilon_H)(A \otimes m_A \otimes H)(A \otimes A \otimes p \otimes H)(A \otimes A \otimes \Phi^{-1})(A \otimes u')\psi\Phi(\eta_B \otimes H)
\]
\[
= (p \otimes \epsilon_H)(\eta_B \otimes H) \quad \text{by (3)}
\]
\[
= \eta_A \otimes \epsilon_H 
\]

and

\[
\mu * \gamma = m_A(m_A \otimes A)(A \otimes p \otimes \epsilon \otimes A)(A \otimes \Phi^{-1} \otimes \Phi)(u' \otimes \eta_A \otimes H)\Delta_H
\]
\[
= m_A(A \otimes \Phi)(A \otimes B \otimes \epsilon \otimes H)(A \otimes \Phi^{-1} \otimes H)(u' \otimes H)\Delta_H
\]

since \( \Phi \) is a \( B \)-module morphism

\[
= m_A(A \otimes \Phi)(A \otimes B \otimes \epsilon \otimes H)(A \otimes \Phi^{-1} \otimes H)(A \otimes \psi)u' \quad \text{by (1)}
\]
\[
= m_A(A \otimes \Phi)(A \otimes B \otimes \epsilon \otimes H)(A \otimes B \otimes \Delta_H)(A \otimes \Phi^{-1})u'
\]

since \( \Phi^{-1} \) is an \( B \)-comodule morphism

\[
= m_A(A \otimes \Phi)(A \otimes \Phi^{-1})u'
\]
\[
= m_Au'
\]
\[
= \epsilon_H \eta_A \quad \text{by (2)}.
\]

Thus \( \gamma \) has a convolution inverse \( \mu \). Since both \( \psi \gamma^{-1} \) and \( C_{H,A}(S \otimes \gamma^{-1})\Delta \) are the convol-
olution inverse of $\psi\gamma$, $\psi\gamma^{-1} = C_{H,A}(S \otimes \gamma^{-1})\Delta$. Set

$$\alpha' = m_A(A \otimes m_A)(\gamma \otimes p \otimes \gamma^{-1})(H \otimes C_{H,B})(\Delta \otimes B) : H \otimes B \to A,$$

$$\sigma' = m_A(m_A \otimes \gamma^{-1})(\gamma \otimes \gamma \otimes m)(H \otimes C \otimes H)(\Delta_H \otimes \Delta_H) : H \otimes H \to A.$$

$$\omega' = m_A(\gamma \otimes m_A)(m_H \otimes C_{A,A})(H \otimes H \otimes \gamma^{-1} \otimes \gamma^{-1})(H \otimes C \otimes H) \quad (\Delta_H \otimes \Delta_H) : H \otimes H \to A.$$

Since $\psi m_A(p \otimes p) = (A \otimes \eta_H)m_A(p \otimes p)$, there exists $m_B : B \otimes B \to B$ such that $pm_B = m_A(p \otimes p)$. Similarly, there exist $\eta_B : I \to B$, $\sigma : H \otimes H \to B$, $\alpha : H \otimes B \to B$ and $\omega : H \otimes H \to B$ such that $p\eta_B = \eta_A$, $p\sigma = \sigma'$, $p\alpha = \alpha'$ and $p\omega = \omega'$. It is easy to check that $\omega$ is the convolution inverse of $\sigma$. Furthermore, $(B, m_B, \eta_B)$ is an algebra in $C$.

Now we show that conditions (2-COC'), (WA) and (TM) hold.
Thus (2-COC), (TM) and (WA) hold. Finally, we show that \( \Phi \) is an algebra isomorphism from \( B \#_\sigma H \) onto \( A \).
Since Φ is an isomorphism we have Φ(η_B ⊗ η_H) = η_A. □
2 Hopf Galois extension in the Yetter-Drinfeld category

Using the conclusion in Theorem 1.3 in this section we give the relation between crossed product and $H$-Galois extension in the Yetter-Drinfeld category $D^{\mathcal{YD}}$ with Hopf algebra $D$ over field $k$.

It follows from [9, Corollary 2.2.8] that $D^{\mathcal{YD}}$ is an additive category. Therefore, by [12, Page 242], $\text{equiv}(f_1, f_2) = \ker(f_1 - f_2)$ and $\text{coequiv}(f_1, f_2) = A/\text{Im}(f_1 - f_2)$ for any two morphisms $f_1, f_2$ in $D^{\mathcal{YD}}$. Consequently we have

**Corollary 2.1** Let $\mathcal{C}$ be the Yetter-Drinfeld category $D^{\mathcal{YD}}$. Then the following assertions are equivalent:

(i) There exists an invertible 2-cocycle $\sigma : H \otimes H \to B$, and a week action $\alpha$ of $H$ on $B$ such that $A = B \#_{\sigma} H$ is a crossed product algebra.

(ii) $(A, \psi)$ is a right $H$-comodule algebra with $B = A^{coH}$, the extension $A/B$ is Galois, the canonical epic $q : A \otimes A \to A \otimes B \#^\alpha H$ is split and $A$ is isomorphic as left $B$-modules and right $H$-comodules to $B \otimes H$ in $\mathcal{C}$, where the $H$-comodule operation and $B$-module operation of $B \otimes H$ are $\psi_{B \otimes H} = \text{id}_B \otimes \Delta_H$ and $\alpha_{B \otimes H} = m_B \otimes \text{id}_H$ respectively.

Corollary 2.1 implies [9, Theorem 6.4.12] since $q$ is split in the category of vector spaces with trivial braiding.

**Example 2.2** Let $H$ be a braided Hopf algebra in $D^{\mathcal{YD}}$. Let $B = H$ and $H$ act on $B$ by adjoint action $\alpha = m(H \otimes m)(H \otimes C_{H,B})(H \otimes S \otimes B)(\Delta_H \otimes B)$. Thus $B \#_{\sigma} H = A$ is a smash product algebra in $D^{\mathcal{YD}}$. By Corollary 2.1 $A/B$ is an $H$-Galois extension in $D^{\mathcal{YD}}$.

Note that many braided Hopf algebras have been known. For example, by the Radford’s method in [19, Theorem 1 and Theorem 3], one can obtain a braided Hopf algebra $H$ in the Yetter-Drinfeld category $D^{\mathcal{YD}}$ for any graded Hopf algebra $A$ with $D = A_0$. One can also obtain a braided Hopf algebra $H$, the braided group analogue of $H$ in the Yetter-Drinfeld category $D^{\mathcal{YD}}$ for any (co)quasitriangular Hopf algebra $H$ with $D = H$ (see [17]). Furthermore, there exist many graded Hopf algebras as they can be constructed by Hopf quivers (see [8]). Otherwise, Nichols algebras also are braided Hopf algebras.

**Acknowledgement** : The first two authors were financially supported by the Australian Research Council. S.C.Z thanks the Department of Mathematics, University of Queensland and Hong Kong University of Science and Technology for hospitality. Thank Y.Bespalov and V.Lyubashenko for their t-angles.sty.
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