Spherically Symmetric, Metrically Static, Isolated Systems in Quasi-Metric Gravity

Dag Østvang

Department of Physics, Norwegian University of Science and Technology (NTNU)
N-7491 Trondheim, Norway

Abstract

Working within the quasi-metric framework (QMF) described elsewhere, we examine the gravitational field exterior respectively interior to a spherically symmetric, isolated body made of perfect fluid. By construction, the system is “metrically static”, meaning that its associated gravitational field is static except for the effects of the global cosmic expansion on the spatial geometry. To ensure that the global cosmic expansion will not induce instabilities in the fluid source and thus violating the metrically static condition, the equation of state of the fluid is required to take a particular form (fulfilled for, e.g., an ideal gas).

We set up dynamical equations for the gravitational field and give an exact solution for the exterior part. Furthermore, we find equations of motion to be applied to inertial test particles moving in the exterior gravitational field. The metrically static condition implies that the radius of the source increases and that distances between circular orbits increase according to the Hubble law, but such that circle orbit velocities are unaffected. This means that the dynamically measured mass of the source increases linearly with cosmic scale. We show that, if this model of an expanding gravitational field is taken to represent the gravitational field of the solar system, this has no serious consequences for observational aspects of planetary motion. On the contrary, some observational facts of the Earth-Moon system are naturally explained within the QMF. Finally, the QMF predicts different secular increases for two different gravitational coupling parameters. But such secular changes are neither present in the Newtonian limit of the quasi-metric equations of motion nor in the Newtonian limit of the quasi-metric field equations valid inside metrically static sources. Thus standard interpretations of space experiments, testing the secular variation of the gravitational “constant”, are explicitly theory-dependent and do not apply to the QMF.

1 Introduction

The idea that the cosmic expansion may possibly be relevant for local systems came up many years ago; see, e.g., [1] and references therein. More recently, there has been renewed interest in this idea; in part because it has become clear that there is no compelling
observational evidence showing the expected deviations from the global Hubble law for galaxies in the vicinity of or even within the local group of galaxies. See, e.g., [2] and references therein.

Even if there are in principle no direct observations ruling out the relevance of the global Hubble law on local scales, the generally accepted view is that for all practical purposes, local systems may be treated as decoupled from the cosmic expansion. This view reflects predictions coming from the standard framework of metric gravity. That is, it is well-known that metric theory predicts that realistic local systems are hardly affected at all by the cosmological expansion (its effect should at best be totally negligible, see, e.g., [1] and references therein). The reason for this prediction is basically that in metric theory, the cosmological expansion must be modelled within a mathematical framework where space-time is postulated to be a pseudo-Riemannian manifold. However, when analyzing the influence of the cosmological expansion on local systems, there should be no reason to expect that predictions made within the metric framework should continue to hold in a theory where the structure of space-time is non-metric.

Recently, a review of a new type of non-metric space-time framework, the so-called quasi-metric framework (QMF), was presented in [3]. Also presented was an alternative relativistic theory of gravity formulated within this framework. (A more detailed presentation can be found in [4].) This theory correctly predicts the results of the “classical” solar system tests in the so-called “metric approximation” case where an asymptotically Minkowski background is invoked and the non-metric features of the theory (and thus the cosmological expansion) can be neglected. However, for reasons explained in [3], the theory is based on a $S^3 \times \mathbb{R}$-background rather than a Minkowski background as the global basic (“prior”) geometry of the Universe. As long as the cosmological expansion is neglected, the choice of cosmic background geometry does not matter for the predictions of said solar system tests, though.

Moreover, since it represents the non-metric sector of the theory, in quasi-metric theory the nature of the cosmological expansion is described as fundamentally different from its counterpart in metric theory. One consequence of the quasi-metric description of the cosmological expansion is that the expansion applies to all systems where gravitational dynamics dominates (hereafter called “gravitational systems”), regardless of scale. That is, in quasi-metric gravity the mathematical modelling of the Hubble expansion and thus its physical interpretation are different from their counterparts in metric theory, and as a consequence, the Hubble expansion is predicted to influence local, gravitationally bound systems sufficiently that its effects should be observable in experiments. On the other hand, since quantum-mechanical states should be unaffected by the expansion,
quasi-metric theory allows that the global cosmic expansion does not apply to quantum-mechanical systems bound by non-gravitational forces where gravitational interactions are negligible (hereafter called “atomic systems”) [5].

To find more exactly how the cosmological expansion affects local gravitational systems according to the quasi-metric theory, one must first calculate the spherically symmetric gravitational field with the $\mathbf{S}^3 \times \mathbf{R}$-background, both interior and exterior to the source. Then one should use the quasi-metric equations of motion (with their non-metric terms included) to calculate how test particles move in the exterior gravitational field. We show in section 3.3 of this paper that the quasi-metric theory predicts that the exterior gravitational field should expand according to the Hubble law. This also applies to the interior gravitational field if potential instabilities induced by the global cosmic expansion can be neglected (see section 3.2). In particular, for a source made of ideal gas, the cosmic expansion induces no instabilities so the radius of a body made of ideal gas is predicted to expand. This result may support an interpretation of geological data indicating that the Earth is expanding according to the Hubble law, see reference [6] and references cited therein. (It is difficult to measure such a small expansion rate directly due to the existence of larger local displacements of the Earth’s surface.) Besides, an expanding Earth should cause changes in its spin rate; we show in section 4.2 of this paper that the secular spin-down of the Earth as inferred from historical astronomical observations may in fact be of cosmological origin and only about half of the currently accepted value. Quasi-metric theory also predicts a cosmological origin of and different values for the recession of the Moon and its mean acceleration, other than those inferred from lunar laser ranging (LLR) experiments using standard theory. However, these differences are due to model-dependence since the LLR data yields that the recession of the Moon follows Hubble’s law when analyzed within the QMF, and the quasi-metric predictions are consistent with a modern lunar ephemeris.

Finally, it is shown that the predicted cosmic expansion of the solar system’s gravitational field does not lead to easily detected perturbations in the observed motion of the planets. However, some less easily detected effects should be measurable; in fact a newly discovered secular increase of the astronomical unit may be explained by cosmic expansion. Also active mass is predicted to show a secular increase; in section 4.3 we argue that the predicted value of this increase is not in conflict with current test experiments. All these results are very different from their counterparts in metric theory. On the other hand, predictions of and data analyses based on metric theory are the reasons why it is generally believed that observations confirm that the solar system is decoupled from the cosmic expansion when it is in fact the other way around.
2 Quasi-metric relativity in brief

2.1 General formulae

In this section we summarize the main features of the QMF and a quasi-metric theory of gravity. A considerably more extensive discussion can be found in [3] or [4].

The mathematical foundation of the QMF can be described by first considering a 5-dimensional product manifold $\mathcal{M} \times \mathbb{R}_1$, where $\mathcal{M} = S \times \mathbb{R}_2$ is a (globally hyperbolic) Lorentzian space-time manifold, $\mathbb{R}_1$ and $\mathbb{R}_2$ are two copies of the real line and $S$ is a compact Riemannian 3-dimensional manifold (without boundaries). Then the global time function $t$ representing the extra (degenerate) time dimension $\mathbb{R}_1$ is introduced as a coordinate on $\mathbb{R}_1$. Moreover, for $t$ given it is convenient to use a coordinate system $\{x^\mu\}$ ($\mu$ taking integer values in the range $0 - 3$) where the ordinary time coordinate $x^0$ on $\mathcal{M}$ scales like $ct$; this ensures that $x^0$ is in some sense a mirror of $t$ and thus a “preferred” global time coordinate. A coordinate system with a global time coordinate of this type we call a global time coordinate system (GTCS). Hence, expressed in a GTCS $\{x^\mu\}$, $x^0$ is interpreted as a global coordinate on $\mathbb{R}_2$ and $\{x^j\}$ ($j$ taking integer values in the range $1 - 3$) as spatial coordinates on $S$. The class of GTCSs is a set of preferred coordinate systems inasmuch as the equations of quasi-metric relativity take special forms when expressed in a GTCS. Note that there exist infinitely many GTCSs.

The 4-dimensional quasi-metric space-time manifold $\mathcal{N}$ can now be defined by slicing the sub-manifold $x^0 = ct$ (using a GTCS) out of the initial 5-dimensional space-time manifold. Furthermore, $\mathcal{N}$ is equipped with two families of Lorentzian space-time metric tensor fields $\bar{g}_t$ and $g_t$. The “dynamical” metric family $\bar{g}_t$ represents a solution of field equations, and from $\bar{g}_t$ one can construct the “physical” metric family $g_t$ which is used when comparing predictions to observations (involving the equations of motion). It is convenient to think of the metric families as single degenerate metrics on (a subset of) $\mathcal{M} \times \mathbb{R}_1$, where the degeneracy manifests itself via the conditions $\bar{g}_t(\frac{\partial}{\partial t}, \cdot) \equiv 0$ and $g_t(\frac{\partial}{\partial t}, \cdot) \equiv 0$. Finally, note that $\mathcal{N}$ differs from a Lorentzian manifold and that this becomes evident only when it is equipped with an affine connection (see below).

From the above description we see that within the QMF, the canonical description of space-time is taken as fundamental. That is, quasi-metric space-time is constructed as consisting of two mutually orthogonal foliations: on the one hand space-time can be sliced up globally into a family of 3-dimensional space-like hypersurfaces (called the fundamental hypersurfaces (FHSs)) by the global time function $t$, on the other hand space-time can be foliated into a family of time-like curves everywhere orthogonal to the
FHSs. These curves represent the world lines of a family of hypothetical observers called the fundamental observers (FOs). There exists a unique relationship between $t$ and the proper time as measured by any FO.

Now one characteristic property of quasi-metric theory is that it postulates the existence of systematic scale changes between gravitational and atomic systems (and the main role of $t$ is to describe the global aspects of such changes). This means that gravitational quantities are postulated to exhibit an extra variation when measured in atomic units (and vice versa). One may think of this as if fixed operationally defined atomic units vary formally in space-time. Moreover, since $c$ and Planck’s constant $\hbar$ by definition are not formally variable, the formal variation of time units is equal to that of length units and inverse to that of mass units. We now postulate that this formal variation of atomic length (or time) units can be defined from a particular geometric feature of the FHSs in $(N, \bar{g}_t)$. That is, said formal variation is defined in terms of the variation of the spatial scale factor $\bar{F}_t$ of the FHSs being a distinctive geometric feature of $\bar{g}_t$. Thus by definition, measured in atomic units, the formal variability of gravitational quantities with the dimension of time or length goes as $\bar{F}_t$, whereas the the formal variability of gravitational quantities with the dimension of mass goes as $\bar{F}_t^{-1}$ (gravitational quantities with the dimension of charge have no formal variability).

To determine the form of $\bar{F}_t$, we require that no extra arbitrary scale or parameter should be introduced (i.e., no characteristic scale should be associated with $\bar{F}_t$). This yields the (rather unique) choice $\bar{F}_t = c \bar{N}_t$, where $\bar{N}_t$ is the lapse function field family of the FOs in $(N, \bar{g}_t)$. With $\bar{F}_t$ given, together with the requirement that the FHSs should be compact and have a trivial topology, it is now straightforward to set up the general form of $\bar{g}_t$. Thus it can be argued [3, 4] that, expressed in component notation in a suitable GTCS, the most general form allowed for the family $\bar{g}_t$ may be represented by the family of line elements (we use the metric signature $(-+++)$ and Einstein’s summation convention throughout)

$$d\bar{s}_t^2 = \bar{N}_t^2 \left\{ [\bar{N}^k_{(t)} \bar{N}^s_{(t)} \bar{h}_{(t)ks} - 1] (dx^0)^2 + 2 \frac{t}{t_0} \bar{N}^k_{(t)} \bar{h}_{(t)ks} dx^s dx^0 + \frac{t^2}{t_0^2} \bar{h}_{(t)ks} dx^k dx^s \right\}. \tag{1}$$

Here, $t_0$ is some arbitrary reference epoch (usually chosen to be the present epoch) setting the scale of the spatial coordinates and $\bar{N}^k_{(t)}$ are the components of the shift vector family of the FOs in $(N, \bar{g}_t)$. Also, $d\bar{s}_t^2 = \bar{h}_{(t)ks} dx^k dx^s \equiv \frac{t^2}{t_0^2} \bar{N}_t^2 \bar{h}_{(t)ks} dx^k dx^s$ is the spatial line element family corresponding to the spatial metric family $\bar{h}_t$ intrinsic to the FHSs. Note that there are prior-geometric restrictions on $\bar{h}_{(t)ks}$ (see equations (15) and (16) below). However, these restrictions do not show up explicitly in equation (1); rather, they are
expressed via a certain term in one of the field equations (see equation (14) or (17) below). Also note that equation (1) may be taken as a postulate.

The time evolution of the scale factor \( \bar{F}_t \equiv c \bar{N}_t t \) of the FHSs in the hypersurface-orthogonal direction may conveniently be split up into different terms. Using the notation where a comma denotes a partial derivative, the symbol ‘\( \bar{\perp} \)’ denotes a scalar product with the negative unit normal vector field \(-\bar{n}_t\) of the FHSs, and where \( \mathcal{L}_{\bar{n}_t} \) denotes a Lie derivative in the direction normal to the FHSs holding \( t \) constant, we define

\[
\bar{F}_t^{-1} \mathcal{L}_{\bar{n}_t} \bar{F}_t \equiv \bar{F}_t^{-1} \left( (c \bar{N}_t)^{-1} \bar{F}_{t,tt} + \mathcal{L}_{\bar{n}_t} \bar{F}_t \right) = \frac{1}{c \bar{N}_t t} + \frac{\bar{N}_{t,tt}}{c \bar{N}_t^2} - \frac{\bar{N}_{t,\bar{\perp}}}{\bar{N}_t} \equiv c^{-2} \bar{x}_t + c^{-1} \bar{H}_t. \tag{2}
\]

Here, \( c^{-2} \bar{x}_t \) represents the kinematical contribution to the evolution of the spatial scale factor and \( c^{-1} \bar{H}_t \) represents the so-called non-kinematical contribution defined by

\[
\bar{H}_t = \frac{1}{\bar{N}_t t} + \bar{y}_t, \quad \bar{y}_t = c^{-1} \sqrt{\bar{a}_{F k} \bar{a}_F^k}, \quad c^{-2} \bar{a}_{F j} \equiv \bar{N}_{t \bar{j}}, \tag{3}
\]

We see from equation (3) that the non-kinematical evolution (NKE) of \( \bar{F}_t \) takes the form of an “expansion”. Furthermore the NKE consists of two terms; the first term \( \frac{1}{\bar{N}_t} \) represents the global NKE of the FHSs, whereas the second term \( \bar{y}_t \) represents the local NKE coming from the gravitational field. This second term is not “realized” globally since it is absent in equation (2). Besides, we see from equation (2) that the evolution of \( \bar{N}_t \) with time may also be written as a sum of one kinematical and one non-kinematical term, i.e.,

\[
\frac{\bar{N}_{t,tt}}{c \bar{N}_t^2} - \frac{\bar{N}_{t,\bar{\perp}}}{\bar{N}_t} = c^{-2} \bar{x}_t + c^{-1} \bar{y}_t. \tag{4}
\]

The split-ups defined in equations (2), (3) and (4) are necessary to be able to construct \( \bar{g}_t \) from \( \bar{g}_t \) [3]. Note that the kinematical evolution (KE) of the spatial scale factor may be positive or negative.

Next we define two linear, symmetric “degenerate” connections \( \hat{\nabla} \) and \( \hat{\nabla} \) on the quasi-metric space-time manifold \( \mathcal{N} \). These connections are called degenerate due to the fact that they are essentially connections compatible with the 5-dimensional degenerate metrics \( \bar{g}_t \) and \( g_t \), respectively, on \( \mathcal{M} \times \mathbb{R}_1 \) and then just restricted to \( \mathcal{N} \). In the following, we describe the connection \( \hat{\nabla} \) since this connection yields the quasi-metric equations of motion in \( (\mathcal{N}, \bar{g}_t) \). That is, we introduce a torsion-free, metric-compatible 5-dimensional connection \( \hat{\nabla} \) with the property that

\[
\hat{\nabla}_{\bar{n}_t} \bar{g}_t = 0, \quad \hat{\nabla}_{\bar{n}_t} n_t = 0, \quad \hat{\nabla}_{\bar{n}_t} h_t = 0, \tag{5}
\]
on $\mathcal{M} \times \mathbb{R}_1$ and consider the restriction of $\hat{\nabla}$ to $\mathcal{N}$. Here, $\mathbf{h}_t$ is the spatial metric family intrinsic to FHSSs and $\mathbf{n}_t$ is the unit vector family normal to the FHSSs in $(\mathcal{N}, \mathbf{g}_t)$. It can be shown [4] that, expressed in a GTCS, the components which do not vanish identically of the degenerate connection field are given by

$$\hat{\Gamma}_i^j = \frac{1}{2} h^i_{(t)j}(t)_{s,t}, \quad \hat{\Gamma}_\nu^\mu = \frac{1}{2} g^\sigma_{(t)} \left( g_{(t)\sigma\mu} + g_{(t)\nu\sigma} - g_{(t)\nu\mu} \right) \equiv \Gamma^\alpha_{(t)\nu\mu}. \quad (6)$$

The general equations of motion for test particles are identical to the geodesic equation obtained from $\hat{\nabla}$. In a GTCS they take the form (see [4] for a derivation)

$$\frac{d^2 x^\mu}{d\lambda^2} + \left( \hat{\Gamma}_\nu^\mu \frac{dt}{d\lambda} + \Gamma_{(t)\beta\nu} \frac{dx^\beta}{d\lambda} \right) \frac{dx^\nu}{d\lambda} = \left( \frac{cd\tau_t}{d\lambda} \right)^2 c^{-2} a^\mu_{(t)}, \quad (7)$$

where $d\tau_t$ is the proper time as measured along the curve, $\lambda$ is some general affine parameter and $a_t$ is the 4-acceleration as measured along the curve. From equations (6) and (7), we see that quasi-metric theory cannot be identified with any metric theory since the affine connection compatible with a general metric family is non-metric.

As mentioned above, a basic property of the QMF is that gravitational quantities will be formally variable when measured in atomic units. In particular, this applies to the “bare” gravitational coupling parameter $G^B_t$, formally varying like length squared when measured in atomic units (i.e., like $\bar{F}_t^2$). Now $G^B_t$ couples to charge squared, or more generally to the electromagnetic stress-energy tensor [5]. On the other hand, for material sources, masses formally vary as $\bar{F}_t^{-1}$, but this is not measurable in non-gravitational experiments. This means that the “screened” gravitational parameter $G^S_t$ measured for material sources effectively varies as $\bar{F}_t$. Consequently, local gravitational experiments designed to measure gravitational coupling parameters should depend on source composition, so that it will be necessary to distinguish between $G^B_t$ and $G^S_t$.

However, it is convenient to define constants $G^B$ and $G^S$ as the values of $G^B_t$ and $G^S_t$, respectively, measured in (hypothetical) local gravitational experiments at the arbitrary reference epoch $t_0$, such that the formal variabilities of $G^B_t$ and $G^S_t$ are transferred to mass (and charge, if any). Thus, we have to distinguish between active mass, which is a scalar field, and passive mass (passive gravitational mass and inertial mass). (Similarly one must distinguish between active charge and passive charge [5].) For a material particle, the above discussion implies that active mass $m_t$ varies formally as $\bar{F}_t$ measured in atomic units (but passive mass does of course not vary). That is, for a material particle we have that

$$m_{t,t} = \left( \frac{1}{t} + \frac{\bar{N}_{t,t}}{N_t} \right) m_t, \quad m_{t,\perp} = \frac{\bar{N}_{t,\perp}}{N_t} m_t, \quad m_{t,j} = c^{-2} a_{Fj} m_t. \quad (8)$$
where $\vec{a}_F$ is the 4-acceleration of the FOs in the family $\vec{g}_t$. On the other hand, for a local electromagnetic source, active mass $m_t$ (or active energy) varies formally as $\overline{F}_{\mu}^2$ measured in atomic units. (For extended electromagnetic sources, one must also take into account a secular attenuation (not noticeable locally) of the electromagnetic field [3, 5]). For the rest of the present paper we will assume no net charge but that photons as a gravitational source cannot always be neglected.

Taking into the account said variation of active mass in quasi-metric space-time, it is possible to find local conservation laws. These local conservation laws are valid for fixed $t$ and involve the metric covariant divergence $\nabla_t \cdot \mathbf{T}_t$ (using the Levi-Civita connection $\nabla_t$) of the active stress-energy tensor $\mathbf{T}_t$. They take the form (in component notation) [4]

$$T^\nu_{(t)\mu;\nu} = 2\frac{\bar{N}_t}{N_t} T^\nu_{(t)\mu}.$$  \hspace{1cm} (9)

If the dependence on $t$ of $\mathbf{T}_t$ is entirely due to the above mentioned formal variability, $\mathbf{T}_t$ is locally conserved when $t$ varies as well. These local conservation laws are valid independent of the nature of the gravitating source, i.e., they are valid for material sources as well as for electromagnetic sources. Note that local conservation of $\mathbf{T}_t$ implies that inertial observers move along geodesics of $\hat{\nabla}_t$ in $(\mathcal{N}, \bar{g}_t)$, and that this guarantees that inertial observers move along geodesics of of $\hat{\nabla}$ in $(\mathcal{N}, g_t)$ as well [3, 4]. This means that the equations of motion (7) are consistent with the local conservation laws (9).

It is useful to project these local conservation laws with respect to the FHSs. We then get the equations (in content equivalent to equation (9))

$$\mathcal{L}_{\bar{n}_t} T_{(t)\perp\perp} = \left( \bar{K}_t - 2\frac{\bar{N}_t}{N_t} \right) T_{(t)\perp\perp} + \bar{K}_{(t)ik} \hat{T}_{(t)ik} - \hat{T}_{(t)\perp\perp},$$ \hspace{1cm} (10)

$$\frac{1}{N_t} \mathcal{L}_{\bar{n}_t} T_{(t)\perp\perp} = \left( \bar{K}_t - 2\frac{\bar{N}_t}{N_t} \right) T_{(t)\perp\perp} - c^{-2}a_{\bar{F}j} T_{(t)\perp\perp} + c^{-2}a_{\bar{F}i} \hat{T}_{(t)ij} - \hat{T}_{(t)\perp\perp},$$ \hspace{1cm} (11)

where $\mathcal{L}_{\bar{n}_t}$ denotes a Lie derivative of spatial objects in the direction normal to the FHSs (with $t$ fixed). (A “hat” denotes an object intrinsic to the FHSs.) See [4] for a derivation of these equations. Also postulated in [4] are the quasi-metric field equations involving the geometric tensor family $\bar{Q}_t$, the active electromagnetic stress-energy tensor $\mathbf{T}_{(t)}^{(EM)}$ and the active stress-energy-tensor for material sources $\mathbf{T}_{(t)}^{(MA)}$ (where ‘$|$’ denotes a space covariant derivative and where $\kappa^B \equiv \frac{8\pi G^B}{c^4}$, $\kappa^S \equiv \frac{8\pi G^S}{c^4}$)

$$\bar{Q}_{(t)\perp\perp} \equiv 2\bar{R}_{(t)\perp\perp} = 2\left( c^{-4}a_{\bar{F}k}a_{\bar{F}k} + c^{-2}a_{\bar{F}jk} - \bar{K}_{(t)ik} \bar{K}_{(t)ik} + \mathcal{L}_{\bar{n}_t} \bar{K}_t \right) = \kappa^B (T_{(t)\perp\perp}^{(EM)} + \hat{T}_{(t)\perp\perp}^{(EM)}) + \kappa^S (T_{(t)\perp\perp}^{(MA)} + \hat{T}_{(t)\perp\perp}^{(MA)}),$ \hspace{1cm} (12)
\[ Q_{(ij)\parallel \perp} \equiv \tilde{R}^i_{(ij)\parallel \perp} - \tilde{K}^i_{(ij)\parallel \perp} + \left( \tilde{h}^i_{(ik)} \frac{\partial}{\partial x^k} \tilde{h}_{(ij)} \right)_{\parallel \perp} - \left( \tilde{h}^i_{(ik)} \frac{\partial}{\partial x^k} \tilde{h}_{(ij)k} \right)_{\parallel \perp} = \kappa^{B \mathcal{T}}_{(t)j\parallel \perp} + \kappa^{S \mathcal{T}}_{(t)j\parallel \perp}. \] (13)

Here, \( \tilde{R}_t \) is the Ricci tensor family, \( \tilde{G}_t \) is the Einstein tensor family and \( \tilde{K}_t \) is the extrinsic curvature tensor family of the FHSs corresponding to the metric family (1). (\( \tilde{K}_t \) is the trace of \( \tilde{K}_t \).) The set of quasi-metric field equations is completed with the traceless quantity

\[ Q_{(ij)} \equiv \frac{1}{N_t} \mathcal{L}_{\bar{N}_t} \tilde{R}_{(ij)} + \frac{1}{3} \left[ 2 \tilde{K}_{(t)ks} \tilde{K}^{ks}_{(t)} - \tilde{K}^2_{(t)} - \mathcal{L}_{\bar{N}_t} \tilde{K}_{(t)} \right] \tilde{h}_{(ij)} + \tilde{K}_t \tilde{R}_{(t)ij} \]

\[ - c^{-2} \tilde{a}_f \tilde{a}_j + c^{-4} \tilde{a}_f \tilde{a}_j + \left[ c^{-2} \tilde{a}_f \tilde{a}_j + \frac{1}{(ct \bar{N}_t)^2} \right] \tilde{h}_{(ij)} - \tilde{H}_{(ij)} = 0, \] (14)

where the prior-geometric requirement on the spatial Ricci curvature scalar family \( \tilde{P}_t \),

\[ \tilde{P}_t = -4 c^{-2} \tilde{a}_f \tilde{a}_j + 2 c^{-4} \tilde{a}_f \tilde{a}_j + \frac{6}{(ct \bar{N}_t)^2}. \] (15)

ensures that equation (14) is indeed manifestly traceless. Besides, the components of the spatial Einstein tensor family \( \tilde{H}_t \) is given by

\[ \tilde{H}_{(ij)} = - c^{-2} \tilde{a}_f \tilde{a}_j - c^{-4} \tilde{a}_f \tilde{a}_j + c^{-2} \tilde{a}_f \tilde{a}_j \tilde{h}_{(ij)} + \tilde{H}_{(ij)}, \] (16)

where \( \tilde{H}_t \) is the spatial Einstein tensor family calculated from the spatial metric family \( \tilde{h}_t = \frac{\bar{a}}{c t} N_t^{-2} \tilde{h}_t \) in \( (\mathcal{N}, \tilde{g}_t) \). Note that, while equation (16) implies that \( \tilde{P}_t = \frac{\bar{a}}{(ct)^2} \) is fixed by the prior 3-geometry, we have that \( \tilde{H}_{(ij)} \) is not necessarily equal to the prior-geometric quantity \( - \frac{1}{(ct \bar{N}_t)^2} \tilde{h}_{(ij)} \). This shows that, while there is prior 3-geometry, there is still sufficient dynamical freedom left associated with the metric family \( \tilde{h}_t \). This is further illustrated by writing equation (14) in the form (using equations (12) and (16))

\[ \frac{1}{N_t} \mathcal{L}_{\bar{N}_t} \tilde{R}_{(ij)} + \tilde{K}_t \tilde{R}_{(t)ij} - \tilde{H}_{(t)ij} \]

\[ = \frac{1}{3} \left[ \tilde{R}_{(t)\parallel \perp} + \tilde{K}^2_t - \tilde{K}_{(t)ks} \tilde{K}^{ks}_{(t)} - c^{-2} \tilde{a}_f \tilde{a}_j - c^{-4} \tilde{a}_f \tilde{a}_j + \frac{3}{(ct \bar{N}_t)^2} \right] \tilde{h}_{(t)ij}. \] (17)

We notice that by taking the trace of equation (17), we recover the (general) expression for \( \tilde{R}_{(t)\parallel \perp} \) given in equation (12). Besides, note that equation (17) is only partially coupled to matter sources via the scalar quantity \( \tilde{R}_{(t)\parallel \perp} \) and equation (12). That is, equation (17) (or equation (14)) is not fully coupled to the corresponding projection of \( T_t \), so the above field equations only represent a partial coupling of \( \tilde{Q}_t \) to \( T_t \). Nevertheless, due to the form of equation (17) (or equation (14)), just as for General Relativity, the quasi-metric field equations yield two independent propagating dynamical degrees of freedom.
Also note that $\bar{Q}_t$ is not a “genuine” space-time tensor family since it is defined from its projections with respect to a particular foliation of quasi-metric space-time into spatial hypersurfaces (i.e., the FHSs). See [3, 4] for a further discussion. Finally we notice that equation (14) follows from the criterion [3, 4]

$$\bar{C}_{(t)\perp_i\perp_j} = \bar{H}_{(t)ij} + \frac{1}{(ctN_t)^2}\bar{h}_{(t)ij},$$

(18)

involving a particular projection of the Weyl tensor family $\bar{C}_t$.

It should be emphasized that, although the field equations are postulated rather than derived, they are by no means arbitrary; equation (12) for example, follows naturally from a geometrical correspondence with Newton-Cartan theory and has a similar (apart from the composition-dependent coupling) counterpart valid for General Relativity. Besides, equation (15) implies that the temporal evolution of the FHSs does not have any gauge freedom associated with it, so that as opposed to canonical General Relativity, there is no freedom to choose lapse and shift. This means that the quasi-metric field equations determine the FHSs uniquely as well as the metric family $\bar{g}_t$, and that equations (12)-(17) are exactly valid only for projections with respect to the FHSs.

The uniqueness of $t$ and thus the foliation of $\bar{g}_t$ into a unique set of FHSs is a result of the topology of quasi-metric space-time where space is by definition compact and with positive global curvature scalar $\bar{P}_t$. Said topology makes it natural to identify the FOs with observers being approximately at rest with respect to the cosmic rest frame associated with the smeared-out motion of the galaxies (i.e., the frame where the cosmic relic microwave background is observed to be isotropic on average). This identification is practical when doing quasi-metric cosmology but not when doing gravitational physics for isolated systems. However, for a sufficiently small isolated system we may set $\bar{P}_t \approx 0$ so that the FHSs may be treated as approximately asymptotically flat. Then an alternative (approximately global) time function $t'$ foliating $\bar{g}_{t'}$ into an alternative set of hypersurfaces (also being asymptotically flat) may be defined. That is, an alternative class of observers always moving orthogonally to the alternative hypersurfaces may be defined such that said observers are at rest with respect to the barycentre of the isolated system. Moreover, the field equations (with $\bar{P}_{t'} = 0$) may then be transformed with respect to this new set of hypersurfaces. However, the field equations will not be invariant under said transformation; rather they will depend on the velocity of the isolated system with respect to the cosmic rest frame. But in practice, the “preferred frame”-effects introduced by said procedure should be small (at most of post-Newtonian order), if the size of the isolated system is small compared to $ct_0$ and its local speed with respect to the cosmic rest frame is much smaller than the speed of light.
In what follows, we shall do calculations applied to some so-called “metrically static” gravitational systems (see the next section for an explanation). Then it is convenient to have a specific expression for the geometry intrinsic to the FHSs obtained from equation (16). That is, for these cases $\bar{K}_i$ vanishes identically, so using equation (14) we find that

$$\bar{H}_{(t)ij} = c^{-2}(\dot{a}_k^k - \frac{1}{(\bar{N}_t)^2})\bar{h}_{(t)ij} - c^{-1}\bar{a}_k\bar{a}_{kj} - c^{-2}\bar{a}_k\bar{a}_{ij}. \quad (19)$$

We notice that also this expression includes a prior-geometric term.

### 2.2 Special equations of motion

In this paper, we analyse the equations of motion (7) in the case of a uniformly expanding, isotropic gravitational field in vacuum exterior to an isolated, spherically symmetric source in an isotropic, compact spatial background. We also require that the source is at rest with respect to some GTCS. Furthermore, we require that $\bar{N}_t$ and $\bar{h}_{(t)ks}$ are independent of $x^0$ and $t$; i.e., that the only explicit time dependence is via $t$ in the spatial scale factor (using the chosen GTCS). Then it turns out that also the FOs must be at rest with respect to the chosen GTCS and consequently the shift vector field vanishes (this is strictly true only if the isolated system is at rest with respect to the cosmic rest frame, but see the previous section for a discussion of why this is not a crucial issue). We denote this a “metrically static” case. This scenario may be taken as a generalization of the analogous case with a Minkowski background (that case is analysed in [4]) and is more realistic since the Minkowski background is not a part of our theory but rather invoked as an approximation being useful in particular cases.

We start by making a specific ansatz for the form of $\bar{g}_t$. Introducing a spherical GTCS $\{x^0, r, \theta, \phi\}$, where $r$ is a Schwarzschild radial coordinate, we assume that the metric families $\bar{g}_t$ and $g_t$ can be written in a form compatible with equations (1) and (19) (using the notation $' \equiv \frac{\partial}{\partial r}$), i.e.,

$$c^2d\tau^2_t = \bar{B}(r)(dx^0)^2 - \left(\frac{t}{t_0}\right)^2\left(\bar{A}(r)dr^2 + r^2d\Omega^2\right),$$

$$c^2d\tau^2_t = B(r)(dx^0)^2 - \left(\frac{t}{t_0}\right)^2\left(A(r)dr^2 + r^2d\Omega^2\right), \quad (20)$$

$$\bar{A}(r) \equiv \left[1 - \frac{r\bar{B}'(r)}{2\bar{B}(r)}\right]^2, \quad (21)$$

where $\bar{B}(r) = \bar{N}_t^2(r)$, $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ and $\Xi_0 \equiv ct_0$. We notice that for metrically static cases, we have from equation (19) that $\bar{h}_{(t)ks} = S_{ks}$, where $S_{ks}dx^kdx^s$ is the metric of the
3-sphere $S^3$ (with radius equal to $ct_0$). This is why the function $\bar{A}(r)$ takes the form (21). Moreover, we notice that the spatial coordinate system covers only half of $S^3$, thus the range of the radial coordinate is $r \leq \Xi_0$ only. The function $\bar{B}(r)$ and thus the function $\bar{A}(r)$ may be calculated from the field equations; we treat this problem in the next section. The functions $A(r)$ and $B(r)$ may then be found from $\bar{g}_t$ and $\bar{y}_t$.

We now calculate the metric connection coefficients from the metric family $g_t$ given in equation (20). A straightforward calculation yields

\[
\begin{align*}
\Gamma^r_{(t)rr} &= \frac{A'(r)}{2A(r)}, & \Gamma^r_{(t)\theta\theta} &= -\frac{r}{A(r)}, & \Gamma^r_{(t)\phi\phi} &= \Gamma^r_{(t)\theta\theta}\sin^2 \theta, \\
\Gamma^r_{(t)00} &= \left(\frac{t_0}{t}\right)^2 \frac{B'(r)}{2A(r)}, & \Gamma^\theta_{(t)r\theta} &= \frac{\theta}{r}, & \Gamma^\theta_{(t)\phi\phi} &= -\sin \theta \cos \theta, \\
\Gamma^\phi_{(t)r\phi} &= \Gamma^\phi_{(t)\phi r} = \frac{1}{r}, & \Gamma^\phi_{(t)\phi \theta} &= \Gamma^\phi_{(t)\theta \phi} = \cot \theta, & \Gamma^0_{(t)0r} &= \Gamma^0_{(t)r0} = \frac{B'(r)}{2B(r)}. \tag{22}
\end{align*}
\]

In the following we use the equations of motion (7) to find the paths of inertial test particles moving in the metric family $g_t$. Since $a_t$ vanishes for inertial test particles, we get the relevant equations by using equation (6) and inserting the expressions (22) into equation (7). This yields (making explicit use of the fact that $cdt = dx_0$ in a GTCS)

\[
\begin{align*}
\frac{d^2 r}{d\lambda^2} + \frac{A'(r)}{2A(r)} \left(\frac{dr}{d\lambda}\right)^2 - \frac{r}{A(r)} \left(\left(\frac{d\theta}{d\lambda}\right)^2 + \sin^2 \theta \left(\frac{d\phi}{d\lambda}\right)^2\right) + \left(\frac{t_0}{t}\right)^2 \frac{B'(r)}{2A(r)} \left(\frac{dx_0}{d\lambda}\right)^2 + \frac{1}{ct} \frac{dr}{d\lambda} \frac{dx_0}{d\lambda} &= 0, \\
\frac{d^2 \theta}{d\lambda^2} + \frac{2}{r} \frac{d\theta}{d\lambda} \frac{dr}{d\lambda} - \sin \theta \cos \theta \left(\frac{d\phi}{d\lambda}\right)^2 + \frac{1}{ct} \frac{d\theta}{d\lambda} \frac{dx_0}{d\lambda} &= 0, \tag{23}
\end{align*}
\]

\[
\begin{align*}
\frac{d^2 \phi}{d\lambda^2} + \frac{2}{r} \frac{d\phi}{d\lambda} \frac{dr}{d\lambda} + 2\cot \theta \frac{d\phi}{d\lambda} \frac{d\theta}{d\lambda} + \frac{1}{ct} \frac{d\phi}{d\lambda} \frac{dx_0}{d\lambda} &= 0, \tag{24}
\end{align*}
\]

\[
\frac{d^2 x_0}{d\lambda^2} + \frac{B'(r)}{B(r)} \frac{dx_0}{d\lambda} \frac{dr}{d\lambda} = 0. \tag{25}
\]

If we restrict the motion to the equatorial plane, equation (24) becomes vacant, and equation (25) reduces to

\[
\frac{d^2 \phi}{d\lambda^2} + \frac{2}{r} \frac{d\phi}{d\lambda} \frac{dr}{d\lambda} + \frac{1}{ct} \frac{d\phi}{d\lambda} \frac{dx_0}{d\lambda} = 0. \tag{27}
\]
Dividing equation (27) by $\frac{d\phi}{d\lambda}$ we find (assuming $\frac{d\phi}{d\lambda} \neq 0$)

$$\frac{d}{d\lambda} \left[ \ln \left( \frac{d\phi}{d\lambda} \right) + \ln \left( \frac{r^2 t}{t_0} \right) \right] = 0. \quad (28)$$

We thus have a constant of the motion, namely

$$J \equiv \frac{t}{t_0} r^2 \frac{d\phi}{d\lambda}. \quad (29)$$

Dividing equation (26) by $\frac{dx^0}{d\lambda}$ yields

$$\frac{d}{d\lambda} \left[ \ln \left( \frac{dx^0}{d\lambda} \right) + \ln B(r) \right] = 0. \quad (30)$$

Equation (30) yields a constant of the motion which we can absorb into the definition of $\lambda$ such that a solution of equation (30) is [7]

$$\frac{dx^0}{d\lambda} = \frac{1}{B(r)}. \quad (31)$$

Multiplying equation (23) by $\frac{2 r^2 A(r)}{t_0^2} \frac{dr}{d\lambda}$ and using the expressions (29), (31) we find that

$$\frac{d}{d\lambda} \left[ \frac{t^2 A(r)}{t_0^2} \left( \frac{dr}{d\lambda} \right)^2 - \frac{1}{B(r)} + \frac{J^2}{r^2} \right] = 0, \quad (32)$$

thus a constant $E$ of the motion is defined by

$$\frac{t^2 A(r)}{t_0^2} \left( \frac{dr}{d\lambda} \right)^2 - \frac{1}{B(r)} + \frac{J^2}{r^2} \equiv -E. \quad (33)$$

Equation (33) may be compared to an analogous expression obtained for the spherically symmetric, static gravitational field in the metric framework [7]. Inserting the formulae (29), (31) and (33) into equation (20) and using the fact that in a GTCS we can formally write $dx^0 = c dt$ when traversing the family of metrics, we find that

$$c^2 d\tau^2 = Ed\lambda^2. \quad (34)$$

Thus our equations of motion (7) force $d\tau / d\lambda$ to be constant, quite similarly to the case when the total connection is metric, as in the metric framework. From equation (34) we see that we must have $E = 0$ for photons and $E > 0$ for material particles.

We may eliminate the parameter $\lambda$ from equations (29), (31), (33) and (34) and alternatively use $t$ as a time parameter. This yields

$$\frac{t}{t_0} r^2 \frac{d\phi}{c dt} = B(r)J. \quad (35)$$
\[
\left(\frac{t}{t_0}\right)^2 A(r) B^{-2}(r) \left(\frac{dr}{c dt}\right)^2 - \frac{1}{B(r)} + \frac{J^2}{r^2} \equiv -E,
\]
\[
dr^2 = EB^2(r) dt^2.
\]

We may integrate equations (35) and (36) to find the time history \((r(t), \phi(t))\) along the curve if the functions \(A(r)\) and \(B(r)\) are known.

For the spherically symmetric, static vacuum metric case with no global NKE, one can solve the geodesic equation for particles orbiting in circles with different radii, and from this find the asymptotically Keplerian nature of the corresponding rotational curve [7]. In our case we see from equations (35) and (36) that we can find circle orbits as solutions; such orbits have the property that the orbital speed \(w(r) = B^{-1/2}(r) \frac{T_{t0} r \partial \phi}{\partial t}\) is independent of \(t\). (Here \(w(r)\) is the norm of the 3-velocity \(w_t = \sqrt{1 - \frac{w^2}{c^2} \frac{\partial \phi}{\partial t}}\).

### 3 Metrically static, spherically symmetric systems

#### 3.1 Perfect fluid sources

We now seek general solutions of the type (20) of the field equations, and where the source is modelled as a perfect fluid. Then any active stress-energy tensor \(T_t\) takes the form (valid for both a photon fluid and material perfect fluid sources)

\[
T_t = (\bar{\rho}_m + \bar{p}/c^2) \bar{u}_t \otimes \bar{u}_t + \bar{p} \bar{g}_t,
\]

where \(\bar{\rho}_m\) is the density of active mass-energy and \(\bar{p}\) is the active pressure seen in the local rest frame of the fluid. Moreover, \(\bar{u}_t\) is the 4-velocity of the fluid in \((\mathcal{N}, \bar{g}_t)\). But what can be measured locally is not \(T_t\) but the passive stress-energy tensor \(\bar{T}_t\) in \((\mathcal{N}, \bar{g}_t)\), given by

\[
\bar{T}_t = (\bar{\rho}_m + p/c^2) \bar{u}_t \otimes \bar{u}_t + p \bar{g}_t,
\]

where \(\rho_m\) is the passive mass-energy as measured in the local rest frame of the fluid and \(p\) is the associated passive pressure. Observe that the counterpart \(T_t\) in \((\mathcal{N}, g_t)\) to \(\bar{T}_t\) is given by

\[
T_t = \sqrt{\frac{h_t}{\bar{h}_t}} \left[ (\rho_m + p/c^2) u_t \otimes u_t + p g_t \right];
\]
where $\tilde{h}_t$ and $h_t$ are the determinants of the spatial metrics $\tilde{h}_t$ and $h_t$, respectively. Since electromagnetic and material active mass-energy have different formal variability in quasi-metric space-time, we have that the relationship between $\tilde{\rho}_m$ and $\dot{\rho}_m$ is given by

$$
\tilde{\rho}_m = \begin{cases} 
\frac{\rho}{t_0} \tilde{N}_t \dot{\rho}_m & \text{for a fluid of material particles,} \\
\frac{\rho^2}{t_0} \tilde{N}_t^2 \dot{\rho}_m & \text{for the electromagnetic field,}
\end{cases}
$$

and similarly for the relationship between $\tilde{p}$ and $p$. In the following sections, we set up the relevant equations both for the interior and the exterior gravitational field. As we shall see, it is possible to find an implicit exact solution inside the source but this solution is not very useful. Therefore, to find explicit solutions for the interior field, the equations should be solved numerically. However, we have not performed any numerical calculations. On the other hand an explicit exact solution may be found for the exterior field.

3.2 The interior field

In this section we analyse the gravitational field inside the source. That is, we do the necessary analytical calculations in order to write the relevant equations in a form appropriate for numerical treatment. For this purpose, it is convenient to rewrite the line element $ds_t^2$ given in equation (20) by changing to a new radial coordinate $\rho \equiv r/\sqrt{B}$.

Then we find, using equation (21), that

$$
\overline{ds}_t^2 = \overline{B} \left[ - (dx_0)^2 + \left( \frac{t}{t_0} \rho^2 \right) \left( \frac{d\rho}{1 - \frac{\rho^2}{\Xi^2_0}} + \rho^2 d\Omega^2 \right) \right].
$$

Furthermore, from equation (42) and the definitions, we find that

$$
c^{-2} a_{\tau \rho} = \frac{\overline{B}_{t \rho}}{2B}, \quad c^{-2} a_{\tau \rho \rho} = \frac{\overline{B}_{t \rho \rho}}{2B} - \frac{3}{4} \left( \frac{\overline{B}_{t \rho}}{B} \right)^2 - \frac{\rho}{\Xi^2_0 (1 - \frac{\rho^2}{\Xi^2_0})} \frac{\overline{B}_{t \rho}}{2B},
$$

$$
c^{-2} \sin^{-2} \theta a_{\phi \phi} = c^{-2} a_{\phi \phi \theta} = \frac{\rho^2}{2} \left( 1 - \frac{\rho^2}{\Xi^2_0} \right) \left[ \frac{1}{2} \left( \frac{\overline{B}_{\phi \theta}}{B} \right)^2 + \frac{1}{\rho} \frac{\overline{B}_{\phi \theta}}{B} \right],
$$

$$
c^{-2} a_{k k} = \left( \frac{t_0}{t} \right)^2 \frac{1}{B} \left\{ \left( 1 - \frac{\rho^2}{\Xi^2_0} \right) \left[ \frac{\overline{B}_{\phi \phi}}{2B} - \frac{1}{4} \left( \frac{\overline{B}_{\phi \phi}}{B} \right)^2 \right] + \frac{1}{\rho} \left( 1 - \frac{3 \rho^2}{2 \Xi^2_0} \right) \frac{\overline{B}_{\phi \phi}}{B} \right\}.
$$

For a fluid of material particles, active mass density varies formally as $F_t^{-2}$ whereas for electromagnetic field energy (e.g., photon energy), it varies as $F_t^{-1}$ according to equation (41). However, the metrically static condition requires that $\overline{B}$ must be independent of $t$, implying that the time variability of source densities must cancel out in the field.
equations. This means that we must require a cosmic redshift of photon energy, yielding an extra factor $\frac{\tilde{t}}{t}$ in the source photon energy density. Besides, gravitational spectral shifts of photon energy must yield an extra factor $\tilde{N}^{-1}_t$, so that one effectively gets an extra factor $\tilde{F}^{-1}_t$ for photon sources. Thus, for a metrically static source, we may treat material particle sources and photons equally, as if active mass of both formally vary as $\tilde{F}^{-2}_t$ (this approximation is only valid if the net energy transfer between photons and material particles is negligible).

For reasons of convenience, we choose to extract this formal variability explicitly. What is left after separating out the formal variability from the active mass density, is by definition the properly scaled density of active mass $\tilde{\rho}_m = \tilde{\rho}_{m}^{(EM)} + \tilde{\rho}_{m}^{(MA)}$. (The corresponding pressure is $\tilde{p}$.) For the metrically static case $T_{(t)\perp j} = 0$, and besides we find from equation (38) that

\[ T_{(t)\perp} = \tilde{\rho}_m c^2 = \frac{\tilde{t}_0^2 \tilde{\rho}_m c^2}{B} = \frac{\tilde{t}_0^2}{t^2 B} [\tilde{\rho}_{m}^{(EM)} c^2 + \tilde{\rho}_{m}^{(MA)} c^2], \]

\[ T_{(t)\rho} = T_{(t)\phi} = \tilde{p} = \frac{\tilde{t}_0^2 \tilde{p}}{t^2 B} = \frac{\tilde{t}_0^2}{t^2 B} [\tilde{p}_{m}^{(EM)} + \tilde{p}_{m}^{(MA)}], \]

where $\tilde{\rho}_m$ and $\tilde{p}$ do not depend on $t$ since the direct effects of the cosmic expansion have been scaled out. We thus have, by using equations (44) and the local conservation laws (10), (11) applied to the metrically static case, that (with $\dot{\equiv} \frac{\partial}{\partial t}$)

\[ \dot{\tilde{\rho}}_m = \dot{\tilde{p}} = 0, \quad \tilde{p}_{\rho}^{(EM)} = \frac{\tilde{\rho}_{m}^{(EM)}}{\tilde{p}} \tilde{p}_{\rho}, \quad \tilde{p}_{\rho}^{(MA)} = \frac{\tilde{\rho}_{m}^{(MA)}}{\tilde{p}} \tilde{p}_{\rho}, \]

\[ \tilde{p}_{\rho} = -c^{-2} \tilde{a}_{F\rho} (\tilde{\rho}_m c^2 - 3\tilde{p}) = - (\tilde{\rho}_{m}^{(MA)} c^2 - 3\tilde{p}_{m}^{(MA)}) \frac{\tilde{B}_{\rho}}{2B}, \]

where the last equation holds since the only contribution to $T_{(t)}^{(EM)}$ comes from electromagnetic radiation. This means that radiation does not contribute to $\tilde{p}_{\rho}$.

Equations (45) are valid for any metrically static perfect fluid. But to have experimental input, we need to specify an equation of state $p = p(\rho_m)$ consistent with the metrically static condition. That is, the metrically static condition holds only when the explicit dependence $p(\rho_m)$ is linear, i.e., essentially of the form $p \propto \rho_m$, since otherwise the equation of state will not be consistent with the given time evolution. Once a suitable equation of state is given, it is necessary to use the expressions (41) and (44) relating $\tilde{\rho}_m$ to the passive mass density $\rho_m$ and similarly for a relationship between $\tilde{p}$ and the passive pressure $p$. As mentioned above, the relationship between $\tilde{\rho}_m$ and $\rho_m$ will be the same for photons and material particles when taking into account the cosmic redshift of photons.

To find how the active mass $m_\gamma$ varies in space-time, note that we are free to choose the background value of the active mass far from the source to be $m_0$. We use this to
define $G^B$ and $G^S$ as the constants measured in local gravitational experiments performed far from the source at epoch $t_0$. Then, using the metrically static condition and equations (8) and (43), we find that

$$m_t(\rho, t) = \tilde{B}^{1/2}(\rho) \frac{t}{t_0} m_0.$$  \hspace{1cm} (46)

Now we can insert equations (43) and (44) into the field equation (12). Since $\bar{K}_t$ vanishes identically for the metrically static case [4], equation (13) becomes vacant and equation (14) only confirms the form (42) of the metric family. On the other hand, equation (12) yields

$$(1 - \frac{\rho^2}{\Xi_0^2}) \frac{\bar{B}_{\rho\rho}}{B} + \frac{2}{\rho} (1 - \frac{3 \rho^2}{2 \Xi_0^2}) \frac{\bar{B}_{\rho}}{B} = \left[ \kappa^B (\bar{e}^{(EM)}_m c^2 + 3 \bar{p}^{(EM)}) + \kappa^S (\bar{e}^{(MA)}_m c^2 + 3 \bar{p}^{(MA)}) \right].$$  \hspace{1cm} (47)

From equation (21) we easily see that $\bar{A}(0) = 1$. Moreover, $\bar{B}_{\rho\rho}(0)$ must be finite and we must also have $\bar{B}_{,\rho}(0) = \bar{p}_{,\rho}(0) = 0$, yielding $\bar{A}_{,\rho}(0) = 0$. Furthermore, noticing that $\rho^{-1} \bar{B}_{,\rho}$ must be stationary near the centre of the body, we must have

$$\bar{B}_{,\rho}(0) = \lim_{\rho \to 0} \left[ \rho^{-1} \bar{B}_{,\rho}(\rho) \right], \quad \Rightarrow$$

$$\frac{\bar{B}_{,\rho}(0)}{\bar{B}(0)} = \frac{\kappa^B}{3} \left( \bar{e}^{(EM)}_m(0) c^2 + 3 \bar{p}^{(EM)}(0) \right) + \frac{\kappa^S}{3} \left( \bar{e}^{(MA)}_m(0) c^2 + 3 \bar{p}^{(MA)}(0) \right).$$  \hspace{1cm} (48)

where the implication follows from equation (47). Next it is straightforward to show that equation (47) can be once integrated to yield

$$\bar{B}_{,\rho}(\rho) = \frac{2 [G^B \bar{M}^{(EM)}_{t_0}(\rho) + G^S \bar{M}^{(MA)}_{t_0}(\rho)]} {c^2 \rho^2 \left( 1 - \frac{\rho^2}{\Xi_0^2} \right)^{1/2}},$$

$$\bar{M}^{(MA)}_{t_0}(\rho) \equiv \frac{4 \pi}{c^2} \int_0^\rho \frac{\bar{B}(\rho') \left( \bar{e}^{(MA)}_m(\rho') c^2 + 3 \bar{p}^{(MA)}(\rho') \right) \rho'^2 d\rho'} {\sqrt{1 - \frac{\rho'^2}{\Xi_0^2}}},$$  \hspace{1cm} (49)

where a similar definition can be made for $\bar{M}^{(EM)}_{t_0}(\rho)$. Integrating equation (49), using integration by parts, then yields the implicit solution

$$\bar{B}(\rho) = \bar{B}(0) - \frac{2 [G^B \bar{M}^{(EM)}_{t_0}(\rho) + G^S \bar{M}^{(MA)}_{t_0}(\rho)]} {c^2 \rho \left( 1 - \frac{\rho^2}{\Xi_0^2} \right)^{1/2}}$$

$$+ \frac{8 \pi}{c^4} \int_0^\rho \bar{B}(\rho') \left( G^B \bar{e}^{(EM)}_m(\rho') c^2 + 3 \bar{p}^{(EM)}(\rho') \right) \rho' d\rho', \quad \rho \leq \rho_{sf}. \hspace{1cm} (50)$$

To match the exterior solution (see the next section) at the surface $\rho = \rho_{sf}$ of the body, we must have (setting $\bar{M}^{(EM)}_{t_0} \equiv \bar{M}^{(EM)}_{t_0}(\rho_{sf})$ and $\bar{M}^{(MA)}_{t_0} \equiv \bar{M}^{(MA)}_{t_0}(\rho_{sf})$)

$$\bar{B}(\rho_{sf}) = 1 - \frac{r_{s_0}}{\rho_{sf}} \sqrt{1 - \frac{\rho_{sf}^2}{\Xi_0^2}}, \quad r_{s_0} \equiv \frac{2 [G^B \bar{M}^{(EM)}_{t_0}(\rho_{sf}) + G^S \bar{M}^{(MA)}_{t_0}(\rho_{sf})]} {c^2}.$$  \hspace{1cm} (51)
(where $r_{s0}$ is the generalized Schwarzschild radius at epoch $t_0$ of the body), so that equation (50) yields the “normalizing” condition

$$\bar{B}(0) + \frac{8\pi}{c^4} \int_0^{r_{sf}} \bar{B}(\rho') \left( G^B \bar{\rho}_{m}^{(EM)} \rho' c^2 + 3 \bar{p}^{(EM)} \right) + G^S \left[ \bar{\rho}_{m}^{(MA)} \rho' c^2 + 3 \bar{p}^{(MA)} \right] \rho' d\rho' = 1. \quad (52)$$

Inserting the condition (52) into equation (50) then yields the final form of the implicit solution, i.e.,

$$\bar{B}(\rho) = 1 - \frac{2 [G^B \bar{M}_{t_0}^{(EM)}(\rho) + G^S \bar{M}_{t_0}^{(MA)}(\rho)]}{c^2 \rho} \sqrt{1 - \rho^2 \Xi_0} - \frac{8\pi}{c^4} \int_\rho^{r_{sf}} \bar{B}(\rho') \left( G^B \bar{\rho}_{m}^{(EM)} \rho' c^2 + 3 \bar{p}^{(EM)} \right) + G^S \left[ \bar{\rho}_{m}^{(MA)} \rho' c^2 + 3 \bar{p}^{(MA)} \right] \rho' d\rho', \quad \rho \leq \rho_{sf}. \quad (53)$$

The implicit solution (53) is not very useful, so one should rather try to solve equations (45) and (47) numerically for $\bar{p}(\rho)$ and $\bar{B}(\rho)$ using equation (49). One may proceed as follows. Given a suitable equation of state, specify the boundary conditions $\bar{p}^{(EM)}(0)$ and $\bar{p}^{(MA)}(0)$ at the centre of the body for some arbitrary time in addition to some value for $\bar{B}(0)$ chosen as an initial value for iteration. Now integrate equation (45) outwards from $\rho = 0$ using equation (49) until the pressure vanishes. The surface of the body $\rho_{sf}$ is then reached. But the condition (52) must be satisfied. If it is not, add a constant to $\bar{B}(\rho)$ everywhere such that equation (52) holds. Then repeat the calculations with the new value of $\bar{B}(0)$. Iterate until a value for $\bar{B}(0)$ is found that satisfies equation (52) (to desired accuracy) without any further adjustments. Once we have finished these calculations for an arbitrary time, we know the time evolution of the system from equation (45) and we have found the family $\bar{g}_t$ numerically inside the body. To find the corresponding family $g_t$ one uses the method described in [3, 4].

As already mentioned; to have a metrically static system, it is necessary to specify an equation of state of the type $p \propto \rho_{m}$ (potential implicit dependences not included) since this ensures that $\rho_{m}$ and $p$ are independent of $t$. This implies that a spherical gravitationally bound body made of perfect fluid obeying an equation of state of type $p \propto \rho_{m}$ will expand according to the Hubble law. But for bodies made of perfect fluid obeying other equations of state (degenerate star matter for example), the expansion may induce instabilities; mass currents will be set up and such systems cannot be metrically static. However, for the metrically static case, the gravitational field interior to the body will expand along with the fluid; this is similar to the expansion of the exterior gravitational field found in the next section. Since the equation of state for an ideal gas has the required form and since non-degenerate star matter is reasonably well approximated by an ideal gas, it should not be too unrealistic to apply the metrically static condition to main-sequence
stars. (The metrically static condition holds in general for an ideal gas even if the gas is not isothermal; i.e., even if the temperature depends on the pressure so that the equation of state takes a polytropic form.)

We finish this section by estimating how the cosmic expansion will affect a spherically symmetric body made of perfect fluid obeying an equation of state of the form \( p \propto \rho^{\gamma} \) (e.g., a polytrope made of degenerate matter). To do that, we assume that the fluid-dynamical effects on the gravitational field coming from instabilities can be neglected; i.e., we assume that the body can be treated as being approximately in hydrostatic equilibrium for each epoch \( t \). (Effects coming from gravitational heating of the body due to contraction are also neglected.) To justify this approximation we work in the Newtonian limit, so that we can use the approximations \( \rho \approx r, \bar{\rho}_m \approx \rho_m, \bar{\rho} \approx \rho \) and \( G^B \bar{g}_m^{(EM)} + G^S \bar{g}_m^{(MA)} \approx G_N \bar{g}_m \), where \( G_N \) is Newton's constant. Applying these approximations, we take the Newtonian limits of equations (45) and (47), getting

\[
\frac{d}{dr} \frac{r^2 dp}{\bar{\rho}_m} = -4\pi G_N r^2 \bar{\rho}_{m0},
\]

where \( \bar{\rho}_{m0} \) is the density field \( \rho_m \) at the present epoch \( t_0 \). As it stands, equation (54) is valid only for epoch \( t_0 \) if \( \gamma \neq 1 \). However, by making the substitution \( r \to (\frac{t}{t_0})^{\frac{1}{\gamma-4}} \bar{r}, \bar{\rho}_{m0} \to (\frac{t}{t_0})^{\frac{3}{\gamma-4}} \bar{\rho}_{m0} \equiv \rho_m, G_N \to \frac{t}{t_0} G_N \equiv G_t \), equation (54) may effectively be transformed from epoch \( t_0 \) to epoch \( t \) and it may then be applied to the body at each fixed epoch \( t \) even if \( \gamma \neq 1 \). Equation (54) then becomes equivalent to its counterpart in Newtonian theory except for a variable \( G_t \). Thus the usual Newtonian analysis of polytropes [7] applies, but with \( G_N \) variable. And as consequences of this we see that the physical radius of a polytrope will actually shrink with epoch (if \( \gamma > \frac{4}{3} \)), and that the Chandrasekhar mass limit will decrease with epoch. Thus any white dwarf made of degenerate matter is predicted to shrink with epoch and eventually explode as a type Ia supernova when the Chandrasekhar mass gets close to the mass of the white dwarf. In particular this should happen to isolated white dwarfs, so according to quasi-metric theory it is not necessary to invoke mass accretion from exterior sources to ignite type Ia supernovae.

### 3.3 The exterior field

To find the function \( \bar{B}(\rho) \) for the exterior field, we must solve equation (47) without sources, i.e.,

\[
(1 - \frac{\rho^2}{\Xi_0}) \frac{\bar{B}_{\rho\rho}}{B} + \frac{2}{\rho} (1 - \frac{3 \rho^2}{2 \Xi_0}) \frac{\bar{B}_{\rho}}{B} = 0.
\]
However, before we try to solve equation (55), it is important to notice that no solution of it can exist on a whole FHS (except for the trivial solution $\overline{B}=$constant), according to the maximum principle applied to a closed Riemannian 3-manifold. The reason for this is the particular form of equation (55), see reference [4] and references therein for justification. This means that in quasi-metric theory, isolated systems cannot exist except as an approximation. But even if a non-trivial solution of equation (55) does not exist on a whole FHS, we may try to find a solution valid in some finite region of a FHS. That is, we want to find a solution of equation (55) in the region $\rho_{sf}\leq \rho \leq \Xi_0$, with the chosen boundary condition $\overline{B}(\Xi_0) = 1$ to have a correspondence with the limiting case where the mass of the central source goes to zero. The limited region of validity of such a solution is not of much concern since the approximation made by assuming an isolated system is physically reliable only if $\frac{\rho}{\Xi_0} \ll 1$.

Since equation (55) is linear, it is easy to solve and the unique solution, given said boundary condition, is

$$\overline{B}(\rho) = 1 - \frac{r_{s0}}{\rho} \sqrt{1 - \frac{\rho^2}{\Xi_0^2}} \quad \rho_{sf}\leq \rho \leq \Xi_0. \quad (56)$$

The solution (56) will be more useful if we rather express $\overline{B}$ as a function of $r = \sqrt{B}\rho$, since that radial coordinate was used in section 2.2. We then find that

$$\overline{B}(r) = \left( \sqrt{1 + \left(\frac{r_{s0}}{2r}\right)^2 - \frac{r^2}{\Xi_0^2} - \frac{r_{s0}}{2r} \right)^2 + \frac{r^2}{\Xi_0^2}, \quad r_{sf} = \sqrt{\overline{B}(\rho_{sf})\rho_{sf}} \leq r \leq \Xi_0. \quad (57)$$

Moreover, from equations (57) and (21) we find that

$$\overline{A}(r) = \left[1 + \left(\frac{r_{s0}}{2r}\right)^2 - \frac{r^2}{\Xi_0^2}\right]^{-1} \overline{B}(r). \quad (58)$$

For small $r$, we may write expressions (57) and (58) as series expansions, i.e., as perturbations around the analogous problem in a Minkowski background. But in contrast to the analogous case with a Minkowski background, there exists the extra scale $\Xi_0$ in addition to the generalized Schwarzschild radius $r_{s0}$ defined in equation (51). To begin with, we try to model the gravitational field exterior to galactic-sized objects, so we may assume that the typical scales involved are determined by $\frac{1}{\Xi_0} > \frac{\rho}{r}$; this criterion tells how to compare the importance of the different terms of the series expansion. One may straightforwardly show that series expansions of equations (57) and (58) yield

$$\overline{B}(r) = 1 - \frac{r_{s0}}{r} + \frac{r_{s0}^2}{2r^2} + \frac{r_{s0}r}{2\Xi_0} - \frac{r_{s0}^3}{8r^3} + \cdots, \quad \Rightarrow$$

$$\overline{A}(r) = 1 - \frac{r_{s0}}{r} + \frac{r_{s0}^2}{4r^2} + \frac{r^2}{\Xi_0^2} + \cdots. \quad (59)$$
To construct the family $g_t$ as described in [3], we need the quantity $v(r)$, which for spherically symmetric systems takes the form [4]

$$v(r) = \frac{c}{2} B'(r) = \frac{r_s}{2r} \sqrt{\frac{c}{1 + \left(\frac{r_s}{2r}\right)^2 - \frac{r^2}{\Xi_0}^2}} = \frac{r_s c}{2r} \left[1 + O\left(\frac{r^2}{r^2}\right)\right].$$  \hspace{1cm} (60)$$

We notice that $v(r)$ does not depend on $t$. The functions $A(r)$ and $B(r)$ are found from the relations (valid for spherically symmetric systems [4])

$$A(r) = \left(\frac{1 + \frac{v(r)}{c}}{1 - \frac{v(r)}{c}}\right)^2 \tilde{A}(r), \quad B(r) = \left(1 - \frac{v^2(r)}{c^2}\right)^2 \tilde{B}(r).$$  \hspace{1cm} (61)$$

From equations (57), (58) and (61) we then get

$$B(r) = \frac{(1 - \frac{r^2}{\Xi_0})^2}{(1 + \left(\frac{r_s}{2r}\right)^2 - \frac{r^2}{\Xi_0})^2} \left[\left(1 + \left(\frac{r_s}{2r}\right)^2 - \frac{r^2}{\Xi_0}ight)^2 + \frac{r^2}{\Xi_0}^2\right],$$  \hspace{1cm} (62)$$

$$A(r) = \frac{\left(1 + \left(\frac{r_s}{2r}\right)^2 - \frac{r^2}{\Xi_0} + \frac{r_s}{2r}\right)^2}{1 + \left(\frac{r_s}{2r}\right)^2 - \frac{r^2}{\Xi_0}} \left\{1 + \left(\frac{1 + \left(\frac{r_s}{2r}\right)^2 - \frac{r^2}{\Xi_0} + \frac{r_s}{2r}}{\left(1 - \frac{r^2}{\Xi_0}\right)^2 - \frac{r^2}{\Xi_0}^2}\right)^2 \frac{r^2}{\Xi_0}\right\}. (63)$$

Note that, although $B(r)$ increases for small $r$, for large $r$ it eventually reaches a maximum value and then decreases towards zero when $r \to \Xi_0$. This is merely a curious effect due to the global curvature of space and the unrealistic assumption that an isolated source determines the gravitational field at cosmological distances. That is, it is utterly unrealistic to assume that an isolated source dominates the gravitational field over cosmological scales and that this source has been present since the beginning of time. Thus the from equation (62) inferred gravitational repulsion on cosmological scales is nothing but an unrealistic model artefact.

It is useful to have series expansions for $B(r)$ and $A(r)$. Putting these into a family of line elements we find

$$ds^2_t = -\left(1 - \frac{r_s}{r} + \frac{r_s r}{2\Xi_0^2} + \frac{3r_s^3}{8r^3} + \cdots\right)(dx^0)^2$$

$$(\frac{t}{t_0})^2 \left\{1 + \frac{r_s}{r} + \frac{r^2}{\Xi_0} + \frac{r_s^3}{4r^2} + \cdots\right\}dr^2 + r^2 d\Omega^2. \hspace{1cm} (64)$$

This expression represents the wanted metric family as a series expansion. Note in particular the fact that all spatial dimensions expand whereas the corresponding Newtonian potential $-U = -\frac{c^2 r_s}{2r}$ (to Newtonian order) remains constant for a fixed FO. This means
that the physical radius of any circle orbit (i.e., with \( r \) constant) increases but such that the orbital speed remains constant. That is, the (active) mass of the central object as measured by distant orbiters increases to exactly balance the effect on circle orbit velocities of expanding circle radii. This is not as outrageous as it may seem due to the extra formal variation of atomic units built into our theory. So this result is merely a consequence of the fact that the coupling between matter and geometry depends directly on the formal variation via the field equations.

What is measured by means of distant orbiters is not the “bare” mass \( M_{(MA)}^{t} + M_{(EM)}^{t} \) itself but rather the combination \( M_{(MA)}^{t} G_{S} + M_{(EM)}^{t} G_{B} \). We have, however, defined \( G_{S} \) and \( G_{B} \) to be constants. And as might be expected, it turns out that the variation of \( M_{(MA)}^{t} G_{S} + M_{(EM)}^{t} G_{B} \) with \( t \) as inferred from equation (64) is exactly that found directly from the formal variation of the active masses \( M_{(MA)}^{t} = \frac{c}{t_0} M_{(MA)}^{t_0} \) and \( M_{(EM)}^{t} = \frac{c}{t_0} M_{(EM)}^{t_0} \) with \( t \) by using equation (8). This means that the dynamically measured mass increase should not be taken as an indication of actual particle creation but that the general dynamically measured mass scale should be taken to change via a linear increase of \( M_{(MA)}^{t} \) and \( M_{(EM)}^{t} \) with \( t \), and that this is directly reflected in the gravitational field of the source. That is, measured in atomic units, active mass increases linearly with epoch in accordance with equation (8) (for photon energy this only works when including the cosmic redshift of photon energy in an expanding source).

Any dynamical measurement of the mass of a central object by means of distant orbiters does not represent a local test experiment. Nevertheless, the dynamically measured mass increase thus found is just as “real” as the expansion in the sense that neither should be neglected on extended scales. This must be so since in quasi-metric relativity, the global scale increase and the dynamically measured mass increase are two different aspects of the same basic phenomenon.

4 The effects of cosmic expansion on gravitation

4.1 Shapes of orbits and rotational curves

We now explore which kinds of free-fall orbits we get from equation (64) and the equations of motion. To begin with, we find the shape of the rotational curve as defined from the 3-velocities \( w_{(t)} \) of the circle orbits. (The 4-velocities \( u_{(t)} \) may be split up into pieces respectively orthogonal to and intrinsic to the FHSs according to the formula \( \sqrt{1 - \frac{w^{2}}{c^{2}}} u_{(t)} = c n_{(t)} + w_{(t)} \).) Since equation (36) has no time dependence for such orbits we
can do a standard calculation [7], and the result is that orbital speed $w$ varies as

$$w(r) = \frac{t}{t_0} r \frac{d\phi}{dt} \sqrt{1 - \frac{u^2}{c^2} \frac{dt}{d\tau}} = B^{-1/2}(r) \frac{t}{t_0} r \frac{d\phi}{dt} = \sqrt{\frac{B'(r)}{2B(r)} c},$$  (65)

where the second step follows from the formula $\frac{d\tau}{dt} = \sqrt{B(r) - \frac{t^2}{t_0^2} r^2 \left(\frac{d\phi}{c dt}\right)^2}$ (obtained from equation (20) for circular motion) together with a consistency requirement. However, when we apply equation (65) to the metric family (64), we get a result essentially identical to the standard Keplerian rotational curve; the only effect of the dynamically measured mass increase and the non-kinematical expansion is to increase the scale but such that the shape of the rotational curve remains unaffected. It is true that $B(r)$ as found from equation (64) contains a term linear in $r$ in addition to terms falling off with increasing $r$; in reference [8] it is shown that such a linear term may be successfully used to model the asymptotically non-Keplerian rotational curves of spiral galaxies. But the numerical value of the linear term found from equation (64) is too small by a factor of order $10^{-10}$ to be able to match the data. So at least the simple model considered in this paper is unable to explain the asymptotically non-Keplerian rotational curves of spiral galaxies from first principles.

Another matter is how the time dependence in the equations of motion will affect the time histories and shapes of more general orbits than the circle orbits. Clearly, time histories will be affected as can be seen directly from equation (36). However, to see if this is valid for shapes as well, we may insert equation (35) into equation (36) to obtain $r$ as a function of $\phi$. This yields the equation

$$A(r) \left(\frac{dr}{d\phi}\right)^2 + \frac{1}{r^2} - \frac{1}{J^2 B(r)} = \frac{E}{J^2},$$  (66)

and this is identical to the equation valid for the case of a single spherically symmetric static metric [7]. Thus the shapes of free-fall orbits are unaffected by the global non-kinematical expansion present in the metric family (64).

### 4.2 Expanding space and the solar system

One may try to apply the metric family (64) to the solar system by using it to describe the gravitational field of the Sun (when gravitational fields of other solar system bodies than the Sun are neglected). That is, as a good approximation, we may neglect the gravitational effects of the galaxy and treat the solar system as an isolated system. But the solar system is not at rest with respect to the cosmic rest frame; this follows from
the observed dipole in the cosmic microwave background radiation. However, as long as the solar system can be treated as approximately isolated, its velocity with respect to the cosmic rest frame is not crucial when solving the field equations (see the discussion at the end of section 2.1). So as a good approximation, we may neglect the solar system’s motion with respect to the cosmic rest frame and use the metric family (64) to describe the gravitational field of the Sun. Also, since the solar system is small, we can neglect any dependence on $\Xi_0$ (and thus the global curvature of space). The errors made by neglecting terms depending on $\Xi_0$ in equation (64) are insignificant since the typical scales involved for the solar system are determined by $\frac{r}{\Xi_0} \sim \frac{r_0^3}{r^3}$. Equation (64) then takes the form

$$ds_i^2 = -\left(1 - \frac{r_0}{r} + O\left(\frac{r_0^3}{r^3}\right)\right)(dx^0)^2 + \left(\frac{t}{t_0}\right)^2 \left\{1 + \frac{r_0}{r} + O\left(\frac{r_0^2}{r^2}\right)\right\} dr^2 + r^2 d\Omega^2.$$  (67)

Equation (66) shows that the shapes of orbits are unaffected by the expansion; this means that all the classical solar system tests come in just as for the analogous case of a Minkowski background [4]. However, we get at least one extra prediction (irrespective of whether or not the galactic gravitational field can be neglected); from equation (67) we see that the effective distance between the Sun and any planet is predicted to have been smaller in the past. That is, the spatial coordinates are co-moving rather than static, thus average distances (measured in atomic units) between bodies within the solar system are predicted to show a secular increase as a consequence of the cosmic expansion. For example, the distance between the Sun and the Earth at the time of its formation may have been almost 50% smaller than today. But since main-sequence stars are predicted to expand according to quasi-metric theory, a small Earth-Sun distance should not be incompatible with paleo-climatic data, since the Sun is expected to have been smaller and thus dimmer in the past. Actually, since neither the temperature at the centre of the Sun (as estimated from the virial theorem), nor the radiation energy gradient times the mean free path length of a photon depend on $t$, the cosmic luminosity evolution of the Sun should be determined from the cosmic expansion of its surface area as long as the ideal gas approximation is sufficient. And this luminosity evolution exactly balances the effects of an increasing Earth-Sun distance on the effective solar radiation received at the Earth.

However, an obvious question is if the predicted effect of the expansion on the time histories of non-relativistic orbits is compatible with the observed motions of the planets. In order to try to answer this question, it is illustrating to calculate how the orbit period of any planet depends on $t$. For simplicity, consider a circular orbit $r = R =$constant.
Equation (35) then yields

\[ \frac{d\phi}{dt} = \frac{t_0}{t} B(R) R^{-2} J. \]  

(68)

Now integrate equation (68) one orbit period \( T << t \) (i.e., from \( t \) to \( t + T \)). The result is

\[ T(t) = \frac{t}{t_0} T(t_0) \exp \left[ \frac{T_{GR}}{t_0} \right] - 1 = \frac{t}{t_0} T_{GR}(1 + \frac{T_{GR}}{2t_0} + \cdots), \quad T_{GR} = \frac{2\pi R^2}{cJB(R)}. \]  

(69)

where \( T_{GR} \) is the orbit period as predicted from General Relativity. From equation (69), we see that (sidereal) orbit periods are predicted to increase linearly with cosmic scale, i.e.,

\[ T(t) = \frac{t}{t_0} T(t_0), \quad \frac{dT}{dt} = \frac{T(t_0)}{t_0}, \]  

(70)

and such that any ratio between periods of different orbits remains constant. In particular, equation (70) predicts that the (sidereal) year \( T_E \) should be increasing with about 2.5 ms/yr and the martian year \( T_M \) should be increasing by about 4.7 ms per martian year at the present epoch. This should be consistent with observations since the observed difference in the synodical periods of Mars and the Earth is accurate to about 5 ms.

To compare predictions coming from equation (67) against timekeeping data, one must also take into account the predicted cosmological contribution to the spin-down of the Earth. If one assumes that the gravitational source of the exterior field (67) is stable with respect to internal collapse (as for a source made of ideal gas), i.e., that possible instabilities generated by the expansion can be neglected, one may model this source as a uniformly expanding sphere. Due to the increase with time of active mass, the angular momenta of test particles moving in the exterior field (67) increase linearly with cosmic scale. This also applies to the angular momentum \( L_s \) of a spinning source made of ideal gas [9], that is

\[ L_s(t) = \frac{t}{t_0} L_s(t_0), \quad \frac{dL_s}{dt} = \frac{1}{t} L_s = (1 + O(2)) H L_s, \]  

(71)

where the term \( O(2) \) is of post-Newtonian order and where the locally measured Hubble parameter \( H \) is defined by \( H \equiv \frac{1}{Nt} \), or equivalently \( (\tau_F \text{ is the proper time of the local FO}) \)

\[ H \equiv \frac{t_0}{t} \frac{d}{d\tau_F} \left( \frac{t}{t_0} \right) = \frac{ct_0}{t} \left( \sqrt{B(r)} \right)^{-1} \left( \frac{d}{dx} \right)^0 \left( \frac{t}{t_0} \right) = \left( \sqrt{B(r)} \right)^{-1}. \]  

(72)

Since the moment of inertia \( I \propto M R_s^2 \), where \( M \) is the passive mass and \( R_s \) is the measured radius of the sphere, we must have (neglecting terms of post-Newtonian order)

\[ \frac{dR_s}{dt} = H R_s, \quad \frac{d\omega_s}{dt} = -H \omega_s, \quad \frac{dT_s}{dt} = HT_s, \]  

(73)
where $\omega_s$ is the spin circle frequency and $T_s$ is the spin period of the sphere. (To show equation (73), use the definition $L_s = I\omega_s$.) This means that the spin period of a sphere made of ideal gas increases linearly with $t$ due to the cosmic expansion. Does this apply to the Earth as well? The Earth is not made of ideal gas, so the cosmic expansion may induce instabilities, affecting its (sidereal) spin period $T_{sE}$. However, here we assume that the Earth’s mantle is made of a material which may be approximately modelled as a perfect fluid obeying an equation of state close to linear. Then, if this assumption holds, the Earth should be expanding close to the Hubble rate according to the discussion following equation (54). Moreover, averaged over long time spans, shorter timescale effects of instabilities on $T_{sE}$ should average out to a good approximation. We may also assume that there is no significant tidal friction since given the cosmic contribution, this would be inconsistent with the observed so-called mean acceleration $\dot{n}_m$ of the Moon (see below). We then get

$$\frac{dT_{sE}}{dt} = HT_{sE}, \quad \Rightarrow \quad T_{sE}(t) = \frac{t}{t_0}T_{sE}(t_0). \quad (74)$$

From equation (74), we may estimate a cosmic spin-down of the Earth at the present epoch to be about $0.68$ ms/cy (using $H\sim 2.5 \times 10^{-18}$ s$^{-1}$). To see if this is consistent with the assumption that the dominant contribution is due to cosmic effects, we may compare to results obtained from historical observations of eclipses from AD 1000 and onwards. These observations can be used to infer a lengthening of the day of about $1.4$ ms/cy [10], whereas an average over the last 2700 years shows a value of about $1.70$ ms/cy [11]. But the interpretation of the historical data depends on an assumed value of $-26''/cy^2$ for the tidal contribution $\dot{n}_{tid}$ to the mean acceleration $\dot{n}_m$ of the Moon (moreover, other significant (theory-dependent) contributions to $\dot{n}_m$ are neglected without justification, see below). This value of $\dot{n}_{tid}$ corresponds to a calculated lengthening of the day (using standard theory) of about $2.3$ ms/cy [11]; thus the agreement with the values inferred from the historical data is not very good without invoking a secular shortening of the length of the day of non-tidal origin. On the other hand, the QMF yields a value of about $-13.6''/cy^2$ for $\dot{n}_m$ (see below). Reinterpreting the historical data using this value, yields a correction to the lengthening of the day of about $-0.62$ ms/cy, i.e., the observations could indicate a lengthening of the day of about $0.78$ ms/cy and $1.08$ ms/cy, respectively, rather than the values given above. This means that the values obtained from the historical observations are theory-dependent and that the secular spin-down of the Earth may be only about half of the currently accepted value. Note that such a theory-dependence also affects the comparison of equation (74) to results obtained from sedimentary tidal rhythms [12].
Another quantity that can be calculated from equation (74) is the number of days $N_y$ in one (sidereal) year $T_E$. This is found to be constant since

$$T_E = N_y T_{sE} = \frac{t}{t_0} T_E(t_0), \quad \Rightarrow \quad \frac{dN_y}{dt} = 0.$$  

(75)

Moreover, the number of the days $N_m$ in one (sidereal) month $T_m$ can be calculated similarly. We then get

$$T_m = N_m T_{sE} = \frac{t}{t_0} T_m(t_0), \quad \Rightarrow \quad \frac{dN_m}{dt} = 0.$$  

(76)

That $N_y$ and $N_m$ are predicted to be constant is not in agreement with standard (model-dependent) interpretations of paleo-geological data [6, 12]. (The predicted constancy of the ratio $N_y/N_m$ agrees well with a standard interpretation of the data, though.) In addition to the assumption that active masses do not vary with time, the assumption that $T_E$ is constant is routinely used in the interpretation of tidal rhythmities and fossil coral growth data; in particular this applies to [12], where one has explicitly used this assumption when calculating $N_y$ from the data. However, values determined directly from the rhythmite record presented in [12] and the predictions given here, usually agree within two standard deviations when the predicted variable length of the year is taken into account.

As mentioned above, the mean acceleration of the Moon, $\dot{n}_m \equiv \frac{d}{dt} n_m$ (where $n_m$ is the mean geocentric angular velocity of the Moon as observed from its motion) is a very important quantity for calculating the evolution of the Earth-Moon system. From equation (76) we find the quasi-metric prediction

$$n_m(t) = \frac{d\phi_m}{dt} = \frac{t_0}{t} n_m(t_0), \quad \Rightarrow \quad \dot{n}_m \equiv \frac{d}{dt} n_m = -H n_m,$$  

(77)

and inserting the observed value 0.549′′/s for $n_m$ at the present epoch, we get the corresponding cosmological contribution to $\dot{n}_m$, namely about $-13.6''/cy^2$. This value may be compared to the value $-13.74''/cy^2$ obtained from fitting LLR data to a model based on the lunar theory ELP [13]. Note that this second value is the total mean acceleration, wherein other modelled (positive) contributions are included. These other contributions amount to about 12.12''/cy^2 and are mainly attributed to the secular variation of the solar eccentricity due to (indirect) planetary perturbations [13]. When these contributions are removed, one deduces a tidal contribution $\dot{n}_{tid}$ of about $-25.86''/cy^2$ to $\dot{n}_m$ [13]. Similar values for $\dot{n}_{tid}$ as inferred from LLR data have been found in, e.g., [14] ($-25.9''/cy^2$). We see that in absolute values, the quasi-metric result is smaller than the tidal term inferred from LLR data using standard theory. But the non-tidal secular contributions to $\dot{n}_m$ can
be treated as model-dependent since they are calculated from the ELP theory based on an inferred secular acceleration of the mean longitude of the Earth’s perihelion \( \dot{n}_{\text{PE}} \) of about 1.06"/cy\(^2\) [13]. However, said inferred value is not valid in quasi-metric theory since this value most likely follows from mismodelling the observational data; an alternative explanation of the data can be found from the predicted cosmic expansion of the Earth’s orbit (see below). So it is in principle possible to omit both tidal and the traditional non-tidal secular contributions to \( \dot{n}_m \) and construct a quasi-metric model containing only the cosmic contribution. And as shown above, such a model fits the data well.

We may also use Hubble’s law directly to calculate the secular recession \( \dot{a}_{\text{qmr}} \) of the Moon due to the global cosmic expansion; this yields about 3.0 cm/yr whereas the value \( \dot{a}_{\text{tid}} \) inferred from LLR using standard theory is \((3.82 \pm 0.07)\) cm/yr [14]. To see if the difference between these results can be easily explained in terms of model-dependence, we notice that in standard theory, \( \dot{n}_{\text{tid}} \) represents the value \( \dot{n}_m \) would have had if the Earth-Moon system were isolated. Therefore \( \dot{n}_{\text{tid}} \) enters into an expression found by taking the time derivative of Kepler’s third law. On the other hand, the quasi-metric model includes the cosmic contribution \( \dot{n}_{\text{qmr}} \) only, so that quantity enters into a similar expression. Taking into account the fact that active masses increase linearly with time according to quasi-metric theory, we find the relationship

\[
\dot{a}_{\text{tid}} = \frac{2}{3} \frac{\dot{n}_{\text{tid}}}{\dot{n}_{\text{qmr}}} \dot{a}_{\text{qmr}},
\]

which is quite consistent with the numerical values given above. It thus seems that there is a simple explanation of the fact that \( \dot{a}_{\text{tid}} \) as inferred from LLR data using standard theory differs from \( \dot{a}_{\text{qmr}} \) as found from Hubble’s law. In other words, analysing the LLR data within the QMF yields, to within one standard deviation, that the recession of the Moon follows Hubble’s law.

Note once more that, whereas the secular recession of the Moon and its mean acceleration have traditional explanations based on tidal friction, these explanations are not confirmed by direct evidence. That is, tidal friction is of nature a mesoscopic phenomenon and it should in principle be possible to measure the tidal energy dissipated in the Earth’s oceans. But since no mesoscopic measurements confirming the tidal friction scenario exist so far [15], there are no restrictions on interpreting the secular evolution of the Earth-Moon system as due to cosmological effects.

The apparent constancy of the sidereal year (as indicated by astronomical observations of the Sun and Mercury since about AD 1680) represents the observational basis for adopting the notion that ephemeris time (i.e., the time scale obtained from the observed motion of the Sun) is equal to atomic time (plus a conventional constant), but different
from so-called universal time (any time scale based on the rotation of the Earth). But from equation (70) we see that ephemeris time should be scaled with a factor $\frac{1}{t_0}$ compared to atomic time according to quasi-metric relativity. From equation (74) we see that averaged over long time spans, universal time should also be scaled with a factor $\frac{1}{t_0}$ compared to atomic time, as should any conventional constant difference between ephemeris time and universal time. Thus the predicted effect of the spin-down of the Earth and the expansion of the Earth’s orbit is a seemingly inconsistency between gravitationally measured time and time measured by an atomic clock. But within the Newtonian framework, any secular changes in the Earth-Moon system are explained in terms of tidal friction (and external perturbations), so seemingly secular inconsistencies between different time scales may be blamed on the variable rotation of the Earth. In practice this means introducing leap seconds. Given the fact that leap seconds are routinely used to adjust the length of the year, the predicted differences between gravitational time and atomic time should be consistent with observations. In particular, the extra time corresponding to an increasing year as predicted from the quasi-metric model may easily be hidden into the declining number of days in a year as predicted from standard theory.

The prediction that the year increases with about 2.5 ms at the present epoch due to the expansion of the Earth’s orbit, corresponds to a heliocentric mean angular acceleration $\dot{n}_E$ of about $-1.0''/cy^2$. At first glance, this seems wildly inconsistent with the orbital motion of the Earth-Moon barycentre as inferred from the ELP theory and LLR data since the published value of $\dot{n}_E$ is only about $-0.040''/cy^2$ [13]. On the other hand, a value for $\dot{n}_{pE}$ of about $1.06''/cy^2$ is also inferred from the ELP theory and the LLR data [13]. But since this value is fitted and thus model-dependent, in addition to having the right size and sign, it is very possible that it has been used to hide a large (negative) value of $\dot{n}_E$ of about $-1.0''/cy^2$. Thus the quasi-metric prediction of $\dot{n}_E$ is not in conflict with the LLR data if the value of $\dot{n}_{pE}$ is much smaller than that inferred from using the ELP theory.

It is not clear if the small value $-0.040''/cy^2$ of $\dot{n}_E$ as inferred from the ELP theory can be attributed entirely to an expansion of the Earth’s mean orbital radius, or if some part of said value is due to external orbit perturbations. We shall assume the former to estimate the expansion $\dot{a}_E$ of the Earth’s orbit radius corresponding to the ELP value of $\dot{n}_E$. We then compare this estimate to an independent result. We do the estimate by comparing the ELP and the quasi-metric models using an equation similar to equation (78). We also assume that external perturbations of the Earth’s orbit can be neglected in the quasi-metric model. From Kepler’s third law, we find an equation similar to equation (78) relating the expansion of the Earth’s orbit radius $\dot{a}_E$ and $\dot{n}_E$ as calculated from ELP
theory to their counterparts as calculated from quasi-metric theory. Using Hubble’s law to calculate $\dot{a}_E$ we find a value of about $1.2 \times 10^3$ m/cy from quasi-metric theory. Then, using the values for $n_E$ mentioned above, we estimate $\dot{a}_E$ as calculated from ELP theory to be about $32$ m/cy from the relationship between said quantities. Interestingly, an analysis of all available radiometric measurements of distances between the Earth and the major planets, where radiometric data are compared to calculated distances using planetary ephemerides and standard theory, yields a value of $15 \pm 4$ m/cy for $\dot{a}_E$ [16], i.e., about half of the value estimated above. So, contrary to what is asserted in [16], an explanation of this result based on the cosmic expansion is not at all shown to be inadequate since most, if not all, of the substantial difference between $\dot{a}_E$ as calculated from Hubble’s law on the one hand and that inferred from radiometric data on the other hand, could be due to gross modelling errors. Further evidence for the existence of modelling errors due to local cosmic expansion comes from optical observations of the Sun, indicating an inconsistency in modern ephemerides which may be interpreted as an error of about $1''$/cy in $n_E$; see, e.g., references [17, 18].

In this section, we have seen that the predicted effects of the cosmic expansion on the Earth-Moon and solar system gravitational fields have a number of observable consequences, none of which is shown to be in conflict with observations so far, even though superficially, it would seem that some are. That is, in every case where there is an apparent conflict between quasi-metric predictions and observations, the discrepancies can be explained in terms of model-dependent assumptions made when analysing the data. In the next section we will see that a similar situation exists for the predicted versus the observationally inferred time variation of the gravitational “constant”.

### 4.3 The secular increase of active mass

In quasi-metric relativity, active mass varies throughout space-time (but not in the Newtonian limit of the QMF, since this variation is defined in terms of a varying scale factor). In particular, for material particles, there is a secular increase linear in $t$ as seen from equation (8). This is equivalent to a secular increase of the gravitational “constant” $G^S_t$. On the other hand, the secular increase of active electromagnetic field energy has an extra factor $\frac{1}{t_0}$ corresponding to a secular increase going as $t^2$ for the second gravitational “constant” $G^B_t$. This means that $G^B_t$ and $G^S_t$ will be equal for some particular cosmic epoch, but the possibility that this is close to the present epoch is very unlikely. Therefore, $G^B_t$ and $G^S_t$ are probably very different today. With two different coupling parameters, what is measured in a local gravitational test experiment where the gravitational sources do
not follow the cosmic expansion (e.g., a Cavendish experiment), will depend on source composition. That is, although electromagnetic field energy does not contribute much to source mass, one may in principle measure $G_{t_0}^S$ and $G_{t_0}^B$ at the present epoch by varying source composition. For the rest of this paper we will assume that electromagnetic field energy contribution to source masses is negligible so that we can set $M_{t_0}^{(EM)} \approx 0$. With this approximation, we get the predicted time variation of $G_t^S$ from equation (8), i.e.,

$$\frac{G_{t_0}^S}{G_t^S} = \frac{1}{t} = (1 + O(2))H \approx 8 \times 10^{-11} \text{yr}^{-1}, \tag{79}$$

for the present epoch. However, laboratory gravitational experiments are nowhere near the experimental accuracy needed to test this prediction. On the other hand, space experiments in the solar system (e.g., ranging measurements) and observational constraints on solar models from helioseismology are claimed to rule out any possible fractional time variation of $G_t^S$ larger than about $10^{-12} \text{yr}^{-1}$. See, e.g., references [19-21] and references therein. It thus may appear as the prediction (79) is in conflict with experiment. But as we shall see in what follows, this is not the case.

To illustrate the difference between metric and quasi-metric theory when it comes to the effects of a varying $G_t^S$ on the equations of motion, we note that in the weak field limit of metric theory we may set $G_t^S = G_{t_0}^S + \frac{\dot{G}_{t_0}^S}{G_{t_0}^S} (t - t_0) + \cdots$ directly into the Newtonian equation of motion (with $G_{t_0}^S = G_N$). For an inertial test particle this yields (using a Cartesian coordinate system)

$$\frac{d^2x^j}{dt^2} = U \left[ t, x^k, G_{t_0}^S \left( 1 + \frac{\dot{G}_{t_0}^S}{G_{t_0}^S} (t - t_0) + \cdots \right) \right], \tag{80}$$

leading to an extra, time-dependent term in the coordinate acceleration of objects. It is the presence of such an extra term which is ruled out to a high degree of accuracy according to the space experiments testing the temporal variation of $G_t^S$. That is, one tests a combination of the predicted changes of the solar system scale and orbit periods $T$ which are predicted to vary as $\frac{\dot{T}}{T} = -2 \frac{G_{t_0}^S}{G_t^S}$. This follows from Kepler’s third law since one requires that the conservation of angular momentum takes the form $\dot{n} = -2 \frac{\dot{a}}{a} n$ for any object with mean heliocentric angular velocity $n$, mean angular acceleration $\dot{n}$ and fractional change of orbit radius $\frac{\dot{a}}{a}$. But this requirement is inconsistent with quasi-metric gravity, since we see from equation (35) that the conserved quantity is given by $\frac{\dot{\ell}}{t_0} \ell^2 n$ (where $\ell \equiv \frac{\ell}{t_0} r$ and where corrections of post-Newtonian order have been neglected), and not by $r^2 n$. This yields $\dot{n} = -\frac{\dot{a}}{a} n$ and thus equation (79) when applying Kepler’s third law.
Contrary to metric theory, no such extra term as shown in equation (80) is present in the weak field limit of quasi-metric theory since \( U \) does not depend on \( t \). An example of this can be seen from equations (64) and (67), where \( U \approx M^{(MA)}_{t_0} G^S_{t_0} \) does not depend on \( t \). (On the other hand, \( U \) may depend on \( x^0 \), but any variation of \( M^{(MA)}_{t_0} G^S_{t_0} \) with \( x^0 \) is not (directly) due to cosmology.) However, from equations (70) and (79) we see that in quasi-metric theory, we have \( \dot{T} = \frac{G^S}{G^S} \). But as we have seen in section 4.2, in combination with the predicted scale changes due to the cosmic expansion, this is not inconsistent with observations.

In the weak-field limit of metric theory, one may calculate the effects on stellar structure coming from a possible variation of \( G^S_t \). Such effects are found by putting a variable \( G^S_t \) directly into the Poisson equation. That is, a change in \( G^S_t \) directly induces a change in the Newtonian potential yielding a change in star luminosity. Such calculated changes in luminosity are tightly constrained from their effects on star models, which can be compared to observations, e.g., data obtained from helioseismology. On the other hand, in quasi-metric gravity the effect of the secular increase of active mass cannot be separated from the cosmic expansion, so their total effect is to decrease the density (of a body made of ideal gas) with cosmic epoch but such that the Newtonian potential is unchanged. (To see how this works, recall how equation (12) reduces to equation (47) and take its Newtonian limit. Discover that the resulting Poisson equation is unaffected by the combination of expansion and increasing \( G^S_t \).) Thus for a main-sequence star there will be approximately no change in luminosity except for that due to the increase of scale (i.e., the increase in luminosity due to the increasing surface area of the expanding star).

We conclude that all space experimental tests of the secular variation of \( G^S_t \) are based on the assumption that this variation is present explicitly in the Newtonian potential. (This underlying assumption is also made when analysing tests based on stellar structure and in particular restrictions coming from helioseismology.) However, said assumption (and in particular equation (80)) does not hold in quasi-metric theory. Hence, the interpretations of these tests are explicitly theory-dependent and the prediction made in equation (79) has not been shown to be in conflict with current experimental results, despite the variety of tests apparently showing otherwise. Finally, note that any cosmological constraints on the secular variation of \( G^S_t \) found within the metric framework are utterly irrelevant for quasi-metric theory.
5 Conclusion

In this paper, we have shown that according to the QMF, average distances within gravitationally bound, metrically static systems are predicted to expand according to the Hubble law. Interior to sources the metrically static condition applies whenever the equation of state is of the form $p \propto \rho_m$ (fulfilled, e.g., for an ideal gas). When it is not, the global cosmic expansion is predicted to induce instabilities violating hydrostatic equilibrium. For any such source mass, currents are set up to compensate and the system cannot be metrically static. (An example of this is a body made of degenerate star matter, e.g. a white dwarf, which is predicted to shrink with epoch.)

According to the QMF, the predicted effects on gravitationally bound systems of the global cosmic expansion have a number of observable consequences, none of which has been shown to be in conflict with observations. That is, it seems that at this time no model-independent evidence exists that may rule out the possibility that the size of the solar system (measured in atomic units) expands according to the Hubble law; on the contrary the quasi-metric model fits some observational data more naturally than traditional models do. (But note that predictions coming from the QMF fit these data naturally only as long as the data are analysed and interpreted in a manner consistent with the QMF.)

Some examples of observations being naturally explained within the non-metric sector of the QMF have been discussed in this paper; e.g., the spin-down of the Earth [10, 11], the recession of the Moon and its mean acceleration [13, 14], and the newly discovered secular increase of the astronomical unit [16]. Also the so-called “Pioneer effect” has a natural explanation within the QMF [22]. Thus the non-metric sector of the QMF has considerable predictive power in the solar system, since it makes it possible to explain from first principles a number of seemingly unrelated phenomena as different aspects of the same model. On the other hand, explanations of these phenomena coming from standard theory are invariably ad hoc; such explanations always involve free parameters and mechanisms invented to explain each phenomenon separately. Such an approach is untenable according to Occam’s razor.

So, fact is that several observations in the solar system represent evidence that space-time is quasi-metric. Moreover, metric gravity (and General Relativity in particular) fails to address the challenge represented by these observations. And the main reason that this challenge has not been recognized as important, is that experimental gravity in the solar system is analysed within a weak-field formalism (the so-called PPN-formalism) where it is inherently assumed that space-time must be modelled as a pseudo-Riemannian
manifold, and consequently that the cosmic expansion should be unmeasurably small at the scale of the solar system. This situation may change in the future, when solar system gravitational experiments reach a precision level where the quasi-metric effects can no longer reasonably be “explained” by adding \textit{ad hoc} hypotheses to metric theory.

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