Looking for New Physics via Semi-leptonic and Leptonic rare decays of $D$ and $D_s$

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Abstract

It is well recognized that looking for new physics at lower energy colliders is a tendency which is complementary to high energy machines such as LHC. Based on large database of BESIII, we may have a unique opportunity to do a good job. In this paper we calculate the branching ratios of semi-leptonic processes $D_s^+ \to K^+ e^- e^+$, $D_s^+ \to K^+ e^- \mu^+$ and leptonic processes $D^0 \to e^- e^+$, $D^0 \to e^- \mu^+$ in the frames of $U(1)'$ model, 2HDM and unparticle separately. It is found that both the $U(1)'$ and 2HDM may influence the semi-leptonic decay rates, but only the $U(1)'$ offers substantial contributions to the pure leptonic decays and the resultant branching ratio of $D^0 \to e^- \mu^+$ can be as large as $10^{-7} \sim 10^{-8}$ which might be observed at the future super $\tau$-charm factory.
I. INTRODUCTION

One of the tasks of the colliders with high-intensity but lower-energy is to find traces of new physics beyond the Standard Model (SM) through measuring the rare decays with high accuracy, namely look for deviations of the measured values from the SM predictions. Generally, it is believed that new physics scale may exist at several hundreds of GeV to a few TeV whereas for lower energies, the contributions from new physics might be drowned out in the SM background. However, in some rare decays, contributions from SM are highly suppressed or even forbidden, then the new physics beyond SM (BSM) might emerge and play the leading role. If such processes are observed in high precision experiments, a trace of BSM could be pinned down. Concretely, the processes where the flavor-changing-neutral-current (FCNC) is involved, are the goal of our studies. Even though such results may not determine what kind of new physics, it may offer valuable information about new physics to the high energy colliders such as LHC. In SM, FCNC and lepton flavor violation (LFV) processes can only occur via loop diagrams so would suffer a suppression. Thus study on the FCNC/LFV transitions would compose a key for the BSM search.

The rare decays of D and B mesons provide a favorable area because they are produced at $e^+e^-$ colliders, where the background is much cleaner than that at hadron colliders. The newest measurements set upper bounds for the branching ratios of $D_s^+ \to K^+e^-e^+$ and $D_s^+ \to K^+e^-\mu^+$ as $3.7 \times 10^{-6}$ and $9.7 \times 10^{-6}$ respectively \cite{1}, and the upper bounds for $D^0 \to e^-e^+$ and $D^0 \to e^-\mu^+$ are $7.9 \times 10^{-8}$ and $2.6 \times 10^{-7}$ \cite{1}. Theoretically, those decay processes receive contributions from both short and long distance effects of SM \cite{2}. Especially, for $D_s^+ \to K^+e^-e^+$, its rate mainly is determined by the long distance effect and the SM predicted value is $1.8 \times 10^{-6}$, which is higher than the short distance contribution ($2 \times 10^{-8}$) by two orders. For other concerned processes, the contributions from SM are so small that can be neglected.

As indicated, at lower energy experiments, one can notice the new physics trace, but cannot determine what it is, thus in collaboration, theorists would offer possible scheme(s) to experimentalists and help them to extract information from the data. That is the main idea of this work.

There are many new physics models (BSM) constructed by numerous theorists, for example, the fourth generation \cite{3}, the non-universal $Z'$ boson \cite{4}, \cite{7}, the 2 Higgs doublet
model (2HDM) and the unparticle etc., in their framework, FCNC/LFV processes occur at tree level. Thus once such rare decays involving FCNC/LFV processes are experimentally observed, one may claim existence of BSM, then comparing the values predicted by different models with the data, he would gain a hint about what BSM may play role which is valuable for high energy colliders.

In Refs. [14, 15], based on several BSM models the authors derived the formulas and evaluated the decay rates of semi-leptonic and leptonic decays of $D$ mesons while the model parameters are constrained mainly by the data of $D^0 - \bar{D}^0$ mixing. The result obtained by them was pessimistic that these decay rates cannot provide any trace of the concerned models. In this work we choose three new physics models: $U'(1)$ model, 2HDM type III and unparticle but relax the constraint from $D^0 - \bar{D}^0$ mixing by supposing there were some unknown reasons to suppress the rate if the present measurements are sufficiently accurate, instead we consider the constraints obtained by fitting the experimental data for $\tau \to 3l[1, 7]$. Then we calculate the branching ratios of $D^+_s \to K^+ e^- e^+$, $D^+_s \to K^+ e^- \mu^+$, $D^0 \to e^- e^+$ and $D^0 \to e^- \mu^+$ in the framework of those models respectively. Our numerical results show that only $Z'$ which is from a broken extra $U'(1)$ gauge symmetry and 2HDM of type III can result in substantial enhancement to the branching ratios of $D^+_s \to K^+ e^- e^+$ and $D^+_s \to K^+ e^- \mu^+$ up to $10^{-6} \sim 10^{-7}$. Those results will be tested in future BES III experiment. Indeed, we lay our hope on the huge database of BES III, without which we cannot go any further to search for new physics after all.

We, in this work, also try to set schemes for analyzing the data on those decays based on the BES III data and extract information about new physics BSM.

This paper is organized as follows. In Sections 2 and 3, we first briefly review the SM results for the semi-leptonic and pure leptonic rare decays and then derive corresponding contributions induced by new physics models: extra $U'(1)$, 2HDM of type III and unparticle one by one. In fact some of them had been deduced by other authors and here we only probe their formulation, moreover add those which were not derived before. We obtain the corresponding Feynman amplitudes and decay widths for $D^+_s \to K^+ e^- e^+$, $D^+_s \to K^+ e^- \mu^+$, $D^0 \to e^- e^+$ and $D^0 \to e^- \mu^+$. In section 4, we present our numerical results along with the constraints on the model parameters obtained by fitting previous experimental data except the $D^0 - \bar{D}^0$ mixing. In section 5, we set an experimental scheme for analyzing the data which will be achieved by the BES III collaborations in the near future. In the last section,
we present a brief discussion and draw our conclusion.

II. $D^+_s$ SEMI-LEPTONIC DECAY

For the decay processes $D^+_s \rightarrow K^+e^-e^+$ and $D^+_s \rightarrow K^+e^-\mu^+$, the contributions of SM to these FCNC processes are realized via electromagnetic penguin diagrams and suppressed. However, besides the short-distance effects, there exist a long-distance contribution which is larger. Moreover, because of smallness of the direct SM process, any new physics model whose Hamiltonian includes FCNC interactions, may induce the semi-leptonic and leptonic decays of $D^+_s$ and $D^0$ at tree level. In this section we only explore three possible models: $U(1)'$ model, 2HDM of type III and unparticle. Since those models have been studied by many authors from various aspects, here we only give a brief review.

A. the SM contribution

The authors of Refs.\[2, 15\] gave the amplitudes for $D^+_s \rightarrow K^+e^-e^+$, here we only list the formulas for readers’ convenience. The Feynman amplitude of decay $D^+_s \rightarrow K^+e^-e^+$ in the framework of SM is

$$M_{SM} = \frac{4G_F}{\sqrt{2}} \frac{e^2m_c}{16\pi^2} C_7 \langle e^- \bar{\nu}_L \sigma^{\alpha\beta} c_R \rangle F_{\alpha\beta} \langle \gamma K^+ | O_7 | D^+_s \rangle + C_9 \langle e^- K^+ | O_9 | D^+_s \rangle$$

where

$$O_7 = \frac{e^2}{16\pi^2} m_c \bar{\nu}_L \gamma^{\alpha\beta} c_R$$

$$O_9 = \frac{e^2}{16\pi^2} (\bar{\nu}_L \gamma^{\alpha} c_L) \bar{l}_\gamma l$$

After some simple reductions, $M_{SM}$ is transited to

$$M_{SM} = \frac{4G_F}{\sqrt{2}} \frac{e^2m_c}{16\pi^2} \frac{C_7}{2\sqrt{2}} [\bar{u}(p_2)(\gamma_\beta q_\alpha - \gamma_\alpha q_\beta)\bar{v}(p_1)] \frac{f_{\gamma}(q^2)}{m_{D_s}} [(p + p')^\alpha q^\beta - (p + p')^\beta q^\alpha + i e^{\alpha\beta\rho\sigma} (p + p')_\rho q_\sigma]

+ \frac{e^2}{32\pi^2} C_9 \bar{u}(p_2) \gamma^\delta v(p_1) \{f_+(q^2)[(p + p')^\delta - \frac{m_{D_s}^2 - m_K^2}{q^2} q_\delta] + f_0(q^2) \frac{m_{D_s}^2 - m_K^2}{q^2} q_\delta\}$$

where $q = p_1 + p_2$, $C_7 = 4.7 \times 10^{-3}$ \[17\]. Following Refs.\[2, 15\], we also consider the resonance processes $D^+_s \rightarrow K^+V_i \rightarrow K^+e^-e^+$ with $i = \rho, \omega, \phi$ which are accounted as long distance contributions and the corresponding Feynman diagrams are shown in Fig.\[1\].
FIG. 1: The Feynman diagrams of process $D_s^+ \to K^+ e^- e^+$ through SM long distance.

Thus $C_9$ can be written as

$$C_9 = (0.012 + \frac{3\pi}{\epsilon^2} \sum_{i=\rho,\omega,\phi} \kappa_i \frac{m_{V_i} \Gamma_{V_i \to e^+ e^-}}{m_{V_i}^2 - q^2 - i m_{V_i} \Gamma_{V_i}})(V_{ud} V_{cd} + V_{us} V_{cs})$$  \hspace{1cm} (4)$$

with $\kappa_\rho = 0.7$, $\kappa_\omega = 3.1$ and $\kappa_\phi = 3.6$. The second part in the parenthesis corresponds to the long-distance contributions.

Following Ref.\[14, 17\], the hadronic form factors are written as

$$f_T(q^2) = \frac{f_{D_s K}(0)}{(1-q^2/m_{D_s}^2)(1-a_T q^2/m_{D_s}^2)}$$
$$f_+(q^2) = \frac{f_{D_s K}(0)}{(1-q^2/m_{D_s}^2)(1-\alpha_{D_s K} q^2/m_{D_s}^2)}$$
$$f_0(q^2) = \frac{f_{D_s K}(0)}{1-q^2/\beta_{D_s K} m_{D_s}^2}$$  \hspace{1cm} (5)$$

where $f_{D_s K}(0) = 0.46$, $a_T = 0.18$, $f_{D_s K}(0) = 0.75 \pm 0.08$, $\alpha_{D_s K} = 0.30 \pm 0.03$ and $\beta_{D_s K} = 1.3 \pm 0.07$.

The long-distance contribution is of an order of $10^{-6}$ \[2\]. Thus the contribution from SM may be close or even larger than that of BSM, so they would interfere among each other. We will discuss it in section 4.

B. Contributions of $Z'$ in the $U(1)'$ model

The $U(1)'$ model was proposed and applied by many authors \[4, 5, 18, 19\], and the corresponding lagrangian is

$$\mathcal{L}_{Z'} = \sum_{i,j} [\bar{l}_i \gamma^\mu (\omega_{ij}^L P_L + \omega_{ij}^R P_R) l_j Z'_\mu + \bar{q}_i \gamma^\mu (\varepsilon_{ij}^L P_L + \varepsilon_{ij}^R P_R) q_j Z'_\mu] + h.c.$$  \hspace{1cm} (6)$$

where $P_{L(R)} = \frac{1-(+)^\gamma}{2}$, $\omega_{ij}$ ( $\varepsilon_{ij}$) denote the chiral couplings between the new gauge boson $Z'$ and various leptons (quarks). Whether it can be applied to solve some phenomenological
anomalies, the key point is the intensity of the coupling and the mass of $Z'$ gauge boson which would be fixed by fitting available data.

For the decay processes $D_+^{s} \rightarrow K^{+}e^{-}e^{+}$ and $D_+^{s} \rightarrow K^{+}e^{-}\mu^{+}$, corresponding Feynman diagrams are shown in Fig. 2.

The corresponding Feynman amplitude with $Z'$ as the mediate particle was derived by the authors of [4, 5, 18, 19] as

$$M_{Z'}(D_+^{s} \rightarrow K^{+}l_i\bar{l}_j) = \left\{ f_+(q^2)[(p + p')_\sigma - \frac{m_{l_2}^2 - m_{l_1}^2}{q^2}q_\sigma] + f_0(q^2)\frac{m_{l_2}^2 - m_{l_1}^2}{q^2}q_\sigma \right\}$$

$$\frac{e^L + e^R}{g/\sqrt{2}} \frac{1}{q^2 - m_{Z'}^2} \bar{u}(p_2)(\omega^L_{ij}P_L + \omega^R_{ij}P_R)\gamma^\sigma v(p_1)$$

where $\omega_{ij} = \omega_{ee}$ for $D_+^{s} \rightarrow K^{+}e^{-}e^{+}$ and $\omega_{ij} = \omega_{e\mu}$ for $D_+^{s} \rightarrow K^{+}e^{-}\mu^{+}$ respectively.

The contributions of SM (indeed from the long-distance part) and $Z'$ might be of the same order depending on the model parameters thus we should consider their interference. So we have

$$|M|^2 = |M_{SM} + M_{Z'}e^{i\phi}|^2$$

$$= |M_{SM}|^2 + |M_{Z'}|^2 + 2|M_{SM}M_{Z'}|\cos\phi.$$  

Averaging initial spin and summing over final spin polarizations, the decay width $\Gamma(D_+^{s} \rightarrow$...
$K^+ e^- e^+$ is

$$\frac{d\Gamma}{dq^2} = \left[ \frac{G^2 F^2 \alpha^2}{1536\pi^2 m_{D_s}^2} |C_9 f_+(q^2)|^2 + 2C_7 f_T(q^2) \frac{m_{c u}}{m_{D_s}} \right]^2 + \left( \frac{\epsilon_{D_s}^L + \epsilon_{D_s}^R}{192\pi^2 g^2 m_{D_s}^2 m_{D_s}^2} \right) f_+(q^2) \frac{1}{384\pi^2 \sqrt{m_{D_s}^2 m_{D_s}^2}} f_+(q^2) \right] \lambda^{1/2}(q^2, m_{D_s}^2, m_K^2)$$

(9)

where $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ca$ is the Kallen function. We can obtain the total decay width by integrating over $dq^2$, as

$$\Gamma = \int_{4m_c^2 - m_K^2}^{m_D^2 - m_K^2} \frac{d\Gamma}{dq^2} dq^2$$

(10)

C. Contributions of heavy neutral Higgs in the two-Higgs-Doublet Model of type III

In 2HDM of type III, there are two neutral CP even Higgs bosons, one is the Higgs boson in SM and another is a heavy Higgs boson, the corresponding Lagrangian for the heavy Higgs boson is

$$\mathcal{L}_{Yukawa} = \sum_{i,j} \left[ \tilde{l}_i \left( \frac{m_i}{v} \cos \alpha \delta_{ij} - \frac{\rho_{ij}^E}{\sqrt{2}} \cos \phi H \right) \right] \bar{q}_j (\frac{m_j}{v} \cos \alpha \delta_{ij} - \frac{\rho_{ij}^U}{\sqrt{2}} \sin \alpha q_j H) + h.c.$$ (11)

where $\rho_{ij}^E$ and $\rho_{ij}^U$ stand for effective coupling constants for leptons and quarks respectively, $\cos \alpha$ is the mixing angle between light and heavy Higgs bosons. Following Refs. [10, 20], we take $\cos \alpha \rightarrow 0$ and do not adopt the so-called ChengCSherr ansatz for $\rho_{ij}^f$ which was discussed in Ref. [8]. Instead, we take a range of $\rho_{ij}^f$ to 0.1 ~ 0.3 as suggested in Ref. [20].

The Feynman amplitude corresponding to contributions through exchanging a heavy Higgs boson is

$$\mathcal{M}_{hh}(D_s^+ \rightarrow K^+ l^{-1} l^-) = \left\{ 2 f_{D_s K}^+(q^2) \frac{p^+ \cdot p^+}{m_{D_s}} + [f_{D_s K}^+(q^2) + f_{D_s K}^-(q^2)] \frac{p^- \cdot p^-}{m_{D_s}} \right\}$$

(12)

$$\rho_{ij}^E \frac{1}{\sqrt{m_{h}^2 - m_{D_s}^2}} \bar{u}(p_1) v(p_2) \rho_{ij}^U$$

where $\rho_{ij} = \rho_{ee}, \rho_{ij} = \rho_{em}$ stand for $D_s^+ \rightarrow K^+ e^- e^-$ and $D_s^+ \rightarrow K^+ e^- \mu^+$ respectively.

The differential decay width $d\Gamma(D_s^+ \rightarrow K^+ e^- e^-)$ is

$$\frac{d\Gamma}{dq^2} = \left[ \frac{G^2 F^2 \alpha^2}{1536\pi^2 m_{D_s}^2} |C_9 f_+(q^2)|^2 + 2C_7 f_T(q^2) \frac{m_c^2}{m_{D_s}^2} \right] \lambda(q^2, m_{D_s}^2, m_K^2)$$

$$+ \left( \frac{\rho_{uu}^E}{192\pi^2 g^2 m_{D_s}^2 m_{D_s}^2} \right) f_+(q^2) \left[ (\epsilon_{D_s}^L + \epsilon_{D_s}^R)(q^2 m_{D_s}^2 - m_K^2) (m_{D_s}^2 - m_K^2 + s_{12}) - f_{D_s K}^+(q^2) (m_{D_s}^2 + (m_{D_s}^2 + s_{12})^2 - 2m_{D_s}^2 (m_{D_s}^2 + s_{12})) \right]$$

$$\frac{1}{64\pi^2 m_{D_s}^4 m_{h}^4 \pi^4 s_{12}}$$

$$\lambda^{1/2}(q^2, m_{D_s}^2, m_K^2)$$

(13)

Then we obtain the total decay width by integrating over $dq^2$ as done in Eqn[10].
D. contribution from unparticle

The idea of unparticle was proposed by Georgi\cite{11} a while ago. Then many authors followed him to explore relevant phenomenology and study the basic theory. In the scenario of unparticle, flavor changing term exists in the basic Lagrangian, so that the FCNC can occur at tree level. One is naturally tempted to conjecture that the unparticle mechanism may contribute to \( D_s^+ \to K^+e^-e^+ \) and \( D_s^+ \to K^+\mu^-\mu^+ \). Following Ref.\cite{12,21,22}, we only consider the interactions between fermions and scalar unparticle. The corresponding effective interaction is:

\[
\mathcal{L} = \sum_{f',f} \frac{c_{sf}c_{sf'}}{\Lambda_{uf}} \bar{f} \gamma_\mu (1-\gamma_5) f \partial^\mu \mathcal{O}_u + h.c. \tag{14}
\]

where \( c_{sf} \) stands for coupling constants between unparticle and fermions, \( \mathcal{O}_u \) is the scalar unparticle field, \( d_{uf} \) is a nontrivial scale dimension and \( \Lambda_{uf} \) is an energy scale at order of TeV.

The propagator of the scalar unparticle is\cite{13,22,23}

\[
\int d^4x e^{iP.x} \langle 0 | \mathcal{O}_u(x) \mathcal{O}_u(0) | 0 \rangle = i \frac{A_{d_{uf}}}{2 \sin(d_{uf})} (P^2)^{2-d_{uf}} e^{-i(d_{uf}-2)\pi}, \tag{15}
\]

with \( A_{d_{uf}} \) is

\[
A_{d_{uf}} = \frac{16 \pi^{5/2}}{(2\pi)^{d_{uf}}} \frac{\Gamma(d_{uf}+1/2)}{\Gamma(d_{uf}-1/2)(2d_{uf})}. \tag{16}
\]

Supposing \( D_s^+ \to K^+e^-e^+ \) and \( D_s^+ \to K^+\mu^-\mu^+ \) occur via exchanging a scalar unparticle, the corresponding Feynman amplitude is

\[
\mathcal{M}(D_s^+ \to K^+l_1\bar{l}_2) = \left\{ 2f_{D_sK}^+(q^2)p' \cdot q + [f_{D_sK}^+(q^2) + f_{D_sK}^-(q^2)]q^2 \right\} \frac{c_{e^+e^-}^+}{\Lambda_{uf}^2} (q^2)^{2-d_{uf}} e^{-i(d_{uf}-2)\pi} \bar{u}(p_1)\gamma_\mu(1-\gamma_5)v(p_2) \frac{c^{ij}}{\Lambda_{uf}^2}, \tag{17}
\]

where \( c^e \), \( c^{ij} = c^{e\mu} \) correspond to \( D_s^+ \to K^+e^-e^+ \) and \( D_s^+ \to K^+\mu^-\mu^+ \) respectively.

Since numerically the unparticle contribution to \( D_s^+ \to K^+e^-e^+ \) and \( D_s^+ \to K^+\mu^-\mu^+ \) is much smaller than that from SM and other models BSM, we list the formula involving unparticle, and for completeness, we include the numerical results of the unparticle contribution in the corresponding tables. The differential decay width \( \Gamma(D_s^+ \to K^+e^-e^+) \) is

\[
\frac{d\Gamma}{dq^2} = \frac{1}{2\pi\sin^2m_{D_s}} \frac{1}{2^{12-4d_{uf}}m_{D_s}m_{K}^{2\gamma-4d_{uf}}(2m_{D_s}^2+s_{12})} \frac{\Gamma^2[1/2+d_{uf}]}{\Lambda_{uf}^{2d_{uf}} \sin^2d_{uf} \pi} \frac{f^{0}_{D_sK}(q^2)(m_{D_s}^2-m_{K}^2)(2m_{D_s}^2+s_{12})^{2-2d_{uf}}}{m_{D_s}^2m_{K}^2} \lambda^{1/2}(q^2, m_{D_s}^2, m_{K}^2). \tag{18}
\]
E. Semi-leptonic decay of $D^+$

Decays of $D^+ \rightarrow \pi^+ e^- e^+$ and $D^+ \rightarrow \pi^+ e^- \mu^+$ are similar to $D^+_s \rightarrow K^+ e^- e^+$ and $D^+_s \rightarrow K^+ e^- \mu^+$, only difference is the species of the spectators. Therefore all the formulas of $D^+_s \rightarrow K^+ l_i \overline{l}_j$ can be transferred to $D^+ \rightarrow K^+ l_i \overline{l}_j$ by an $SU(3)$ symmetry.

III. RARE LEPTONIC DECAYS OF $D^0$

The rare leptonic decays of $D^0$ refer to $D^0 \rightarrow l \overline{l}$ and $D^0 \rightarrow l_i \overline{l}_j$ with $i \neq j$ which is not only a FCNC, but also a lepton-flavor violation (LFV) process. In SM, in $D^0 \rightarrow l \overline{l}$, charm-quark and $\bar{u}$ annihilate into a virtual photon via an electromagnetic penguin which suppresses the reaction rate. For the LFV process, not only at the initial part, $c$ and $\bar{u}$ need annihilating into a $Z$ virtual meson which later turns into a pair of neutrinos, then via a weak scattering the neutrinos eventually end with two leptons with different flavors. Because neutrinos are very light, this process is much suppressed than $D^0 \rightarrow l \overline{l}$. In fact, if there does not exist new physics BSM, such LFV processes can never be experimentally measured. Therefore, search for such LFV processes composes a trustworthy probe of BSM. Actually, contribution to the leptonic decays (both lepton-flavor conserving and lepton-flavor violating processes) of SM is too small to be observed [2], thus we only consider contribution from new physics. Since $D^0$ is a pseudo-scalar meson and heavy Higgs is scalar boson, processes $D^0 \rightarrow e^- e^+$ and $D^0 \rightarrow e^- \mu^+$ cannot occur through exchanging heavy Higgs boson. In the $Z'$ and unparticle scenarios $D^0 \rightarrow e^- e^+$ and $D^0 \rightarrow e^- \mu^+$ might be induced to result in sizable rates.

A. The $Z'$ gauge boson from $U(1)'$ model

For the decay processes $D^0 \rightarrow e^- e^+$ and $D^0 \rightarrow e^- \mu^+$, corresponding Feynman diagrams are shown in Fig.3.
IV. NUMERICAL RESULTS

The corresponding Feynman amplitude with $Z'$ as the mediate particle is written as

$$
\mathcal{M}(D^0 \to l_i l_j) = Tr[\bar{v}(q_2)(\varepsilon_{ei} P_L + \varepsilon_{et} P_R)\gamma^\mu u(q_1)] \frac{1}{m_{D^0} - m_{Z'}} \bar{u}(p_1)(\omega_{ij} P_L + \omega_{ij} P_R)\gamma^\mu v(p_2)
$$

(19)

where $\omega_{ij} = \omega_{ee}$ for $D^+_s \to K^+ e^- e^+$ and $\omega_{ij} = \omega_{e\mu}$ for $D^+_s \to K^+ e^- \mu^+$. Following Ref. [24], we have

$$
u(q_1)\bar{v}(q_2) \to f_D(\bar{\nu} + m_D)\gamma_5.
$$

(20)

The decay width $\Gamma(D^0 \to e^- e^+)$ is

$$
\Gamma = \frac{(\varepsilon_{ei} - \varepsilon_{et})^2(\omega_{ee} - \omega_{e\mu})^2 f_D^2 m_\pi^2 \sqrt{m_D^2 - 4m_\pi^2}}{2\pi(m_{D^0}^2 - m_{Z'}^2)^2}.
$$

(21)

B. contribution from unparticle

$D^0 \to e^- e^+$ and $D^0 \to e^- \mu^+$ could also be realized via exchanging a scalar unparticle, and the corresponding Feynman amplitude is

$$
\mathcal{M} = Tr[\bar{v}(q_2)\bar{\nu}(1 - \gamma_5)u(q_1)] \frac{c_{ei}^u}{\Lambda_{D^0}^2} \frac{1}{m_{D^0}^2 - m_{\pi}^2} e^{i(m_{D^0} - 2\pi)\nu(p_2)}(1 - \gamma_5)v(p_1) \frac{c_{et}^e}{\Lambda_{D^0}^2},
$$

(22)

where $c_{ij}^e = c_{ei}^e$ for $D^0 \to e^- e^+$ and $c_{ij}^e = c_{e\mu}^e$ for $D^0 \to e^- \mu^+$.

The decay width $\Gamma(D^0 \to e^- e^+)$ is

$$
\Gamma = \frac{(c_{ei}^u c_{et}^e)^2 f_D^2 \sqrt{m_{D^0}^2 - 4m_\pi^2} m_\pi^2 \sin^2 d_{\pi} \gamma_{D^0}^2 \Gamma^2[1/2 + d_{\pi}] \Gamma^2[1/2 + d_{\pi}]\Gamma^2[1/2 + d_{\pi}]}{m_{D^0}^4 - 4m_\pi^4 \gamma_{D^0}^2 \Gamma^2[1/2 + d_{\pi}]\Gamma^2[1/2 + d_{\pi}]\Gamma^2[1/2 + d_{\pi}]}.
$$

(23)

IV. NUMERICAL RESULTS

For $D^+_s \to K^+ e^- e^+$ and $D^+_s \to K^+ e^- \mu^+$ where a $Z'$ boson is exchanged at s-channel, we follow the authors of Ref. [7, 18] and set the ranges of $\varepsilon_{ei}^L, \varepsilon_{et}^R, \omega_{ee(L)}^L$ and $\omega_{e\mu(L)}^R$ to $-0.5 \sim 0.5$ accordingly.
We plot the branching ratios of $D_s^+ \rightarrow K^+e^-e^+$ and $D_s^+ \rightarrow K^+e^-\mu^+$ versus the mixing angle between SM $Z$ and $Z'$ of $U(1)'$ $\theta$ in Fig.4.

FIG. 4: The branching ratios of processes $D_s^+ \rightarrow K^+e^-e^+$ and $D_s^+ \rightarrow K^+e^-\mu^+$ versus the mixing angle $\theta$ between SM and $U(1)'$ with $\varepsilon'_{cu} = \varepsilon'_{eL} = \omega'_{ee} = \omega'_{e\mu} = 0.2$ and $m_{Z'} = 2000$GeV. The theoretical uncertainty comes from the form factors.

When we calculate the branching ratios of $D_s^+ \rightarrow K^+e^-e^+$ and $D_s^+ \rightarrow K^+e^-\mu^+$ via exchanging a heavy Higgs boson, we just follow Ref.[20] and take the range of $\rho'_{ij}$ within 0.01 ~ 0.3, other than adopting the so-called ChengCSher ansatz for the couplings $\rho'_{ij}$ which were done in Ref.[8]. We plot the branching ratios of $D_s^+ \rightarrow K^+e^-e^+$ and $D_s^+ \rightarrow K^+e^-\mu^+$ versus the mass of the heavy Higgs boson in Fig.5.
FIG. 5: The branching ratios of $D_s^+ \to K^+e^-e^+$ and $D_s^+ \to K^+e^-\mu^+$ versus the mass of the heavy Higgs boson with $\rho_{cu} = \rho_{ee} = \rho_{e\mu} = 0.15$. The theoretical uncertainty comes from the form factors.

Then, we calculate branching ratios of $D^+ \to K^+e^-e^+$ and $D_s^+ \to K^+e^-\mu^+$ via exchanging a scalar unparticle. Following Refs. [13, 21–23], we take $\Lambda_U = 1\text{TeV}$, $1 < d_U < 2$ and the range of $c_S$ to be $0.01 \sim 0.04$ with the relation

$$c_{sf}^{f'} = \begin{cases} 
  c_S & f \neq f' \\
  \kappa c_S & f = f'
\end{cases} \quad (24)$$

where $\kappa = 3$ [21]. Then we plot the branching ratio of decays $D_s^+ \to K^+e^-e^+$ and $D_s^+ \to K^+e^-\mu^+$ versus $\Lambda_U$ with different $d_U$ in Fig. 6.
FIG. 6: The branching ratio of $D^+_s \rightarrow K^+e^-e^+$ and $D^+_s \rightarrow K^+e^-\mu^+$ versus the energy scale $\Lambda_U$ with $c^e_S = c^{e\mu}_S = 0.04$, $c^{ee}_S = 0.12$, $d_U = 1.3$ and $1.5$.

We list the branching ratios of $D^+_s \rightarrow K^+e^-e^+$ and $D^+_s \rightarrow K^+e^-\mu^+$ predicted by various new physics models (BSM) in Tab. I and II separately. From those tables we notice that for the $U(1)'$ model [18] and 2HDM of type III [20], the branching ratios can be up to order of $10^{-6} \sim 10^{-7}$.

| model          | mass             | couplings constants | BR         |
|----------------|------------------|---------------------|------------|
| $U(1)'$ [18]   | $1000 \sim 2000$GeV | $-0.5 \sim 0.5$    | $10^{-8} \sim 10^{-6}$ |
| 2HDM type III [20] | $1000 \sim 1500$GeV | $0.05 \sim 0.3$    | $10^{-8} \sim 10^{-6}$ |
| unparticle     | $1000 \sim 2000$GeV | $0.02 \sim 0.04$   | $10^{-21} \sim 10^{-18}$ |

TABLE I: Branch ratios of process $D^+_s \rightarrow K^+e^-e^+$ in different kinds of new physics beyond SM.

We also list branching ratios of leptonic decays $D^0 \rightarrow e^-e^+$ and $D^0 \rightarrow e^-\mu^+$ predicted by various models of new physics beyond SM in Tab. III. Since $D^0$ cannot decay to $l_i\bar{l}_j$ through a scalar particle, only $Z'$ and unparticle could contribute to those leptonic decays.
TABLE II: Branching ratios of $D_s^+ \to K^+ e^- \mu^+$ predicted by various models of new physics beyond SM.

| model            | mass             | couplings constants | BR               |
|------------------|------------------|---------------------|------------------|
| $U(1)'$ [18]     | $1000 \sim 2000\text{GeV}$ | $-0.5 \sim 0.5$    | $10^{-8} \sim 10^{-6}$ |
| 2HDM type III[20]| $1000 \sim 1500\text{GeV}$ | $0.05 \sim 0.3$    | $10^{-8} \sim 10^{-6}$ |
| unparticle       | $1000 \sim 2000\text{GeV}$ | $0.02 \sim 0.04$   | $10^{-18} \sim 10^{-15}$ |

TABLE III: Branching ratios of $D^0 \to e^- e^+$ and $D^0 \to e^- \mu^+$ predicted by $U(1)'$ and unparticle models.

| decay          | model         | mass             | couplings constants | BR             |
|----------------|---------------|------------------|--------------------|----------------|
| $D^0 \to e^- e^+$ | $U(1)'$      | $1000 \sim 2000\text{GeV}$ | $-0.5 \sim 0.5$    | $10^{-13} \sim 10^{-10}$ |
|                | unparticle    | $1000 \sim 2000\text{GeV}$ | $0.02 \sim 0.04$   | $10^{-16} \sim 10^{-14}$ |
| $D^0 \to e^- \mu^+$ | $U(1)'$      | $1000 \sim 2000\text{GeV}$ | $-0.5 \sim 0.5$    | $10^{-9} \sim 10^{-7}$ |
|                | unparticle    | $1000 \sim 2000\text{GeV}$ | $0.02 \sim 0.04$   | $10^{-11} \sim 10^{-9}$ |

Our numerical results indicate that as the experimental bounds being taken into account and the corresponding coupling constants in $U(1)'$ model and 2HDM taking their maximum values, the branching ratios of $D_s^+ \to K^+ e^- e^+$ and $D_s^+ \to K^+ e^- \mu^+$ can be up to order of $10^{-6}$. Whereas the contribution of the scalar unparticle to the branching ratios can only reach an order of $10^{-18}(10^{-15})$.

V. SEARCHING FOR SEMI-LEPTONIC AND LEPTONIC DECAYS BASED ON THE LARGE DATABASE OF BESIII

In this section, let us discuss possible constraints and the potential to observe the aforementioned rare semi-leptonic and leptonic decays of D mesons based on the large database of BES III. Unlike the hadron colliders, electron-positron colliders have much lower background which is well understood at present and helps to reduce contaminations from the measure-
ment circumstance. Thus controllable and small systematic uncertainties are expected.

The BES III experiment has accumulated large data samples at 3.773 and 4.18 GeV, which are just above the production thresholds of $D\bar{D}$ and $D_s^{*+}D_s^{-} + c.c.$ This provides an excellent opportunity to investigate the decays of these charmed mesons.

At these energies, the charmed mesons are produced in pairs. That is to say, if only one charmed meson is reconstructed in an event, which is defined as a single tag event, there must exist another charmed meson in the recoiling side. With the selected singly tagged events, the concerned rare charm decays can be well studied in the recoiling side of the reconstructed charmed meson.

This is named as the double-tag technology, which is firstly employed by the MARK-III Collaboration and now widely used in the BES III experiments. With this method, the two charmed mesons are both tagged in one event, one of the charmed mesons is reconstructed through a well measured hadronic channel, then the other one decays into the concerned signal process. Benefiting from the extremely clean background, the systematic uncertainties in double tag measurements can be reduced to a fully controlled level.

In principle, there are two ways to perform the search for rare/forbidden decays. One is based on the single tag method where one charmed meson is reconstructed for the signal process while no any constraint is set to the other. This method can provide larger statistics meanwhile a more complex and higher background might exist as the price to pay. Another way is using the double tag method which presents a simple and clean backgrounds but a relatively poorer statistics (see table IV). Whether employing the double-tag technique for studying the relevant processes depends on a balance between reducing background contaminations and expecting higher statistics.

| Method              | Statistics (charged/neutral) | Background | Sensitivity          |
|---------------------|------------------------------|------------|----------------------|
| Single Tag Method   | $1.7 \times 10^7/2.1 \times 10^7$ | not good | Bkg. vs Stat.       |
| Double Tag Method   | $1.6 \times 10^6/2.8 \times 10^6$ | clean     | Bkg. vs Stat.       |

**TABLE IV:** Two methods on searching for rare/forbidden $D$ decays.

In the following, we discuss the statistics of the measurements on the rare decays, which may compose the factor to restrict the ability of searching for new physics in most cases. For single tag method, the background analysis is severely mode dependent. Thus, to simplify
the estimation, we will focus our discussion on the result of double tag method. The BESIII experiment has accumulated huge threshold data samples of about $2.95 \text{ fb}^{-1}$ and $3.15 \text{ fb}^{-1}$ at the cms energies $\sqrt{s} = 3.773$ and $4.180 \text{ GeV}$, which are about 3.5 times and 5 times more than the previously accumulated database, respectively. According to the published papers of the BESIII experiments, there are more than $1.6 \times 10^6$ and $2.8 \times 10^6$ singly tagged charged and neutral $D \bar{D}$ mesons, respectively. These modes can be used as the tagging side for the double-tag method. Namely, because of the advantage of the double-tag method which may remarkably reduce the background and enhance the confidence level, we suggest to adopt the double-tag method for the analysis on the rare decay data while employing the well established modes as the tagging side. Then at the recoiling side, one can look for the expected signal. Omitting some technical details, we know that while adopting this double-tag method, the experimental sensitivity can reach about $10^{-6}$ at 90% Confidence Level (CL) if assuming zero-signal and zero-background events. In next 10 years, 4 to 6 times more charm data can be expected, we may have a better chance to detect such rare decays.

However unfortunately according to our predictions this sensitivity is still below the bound of observing the pure leptonic rare decays of $D^0$ (no matter lepton-flavor-conserving or lepton-flavor-violating processes). If the size of BESIII data sample can reach 20 fb$^{-1}$ in next 10 years, the sensitivity would be at $10^{-7}$ level which is almost touching the bottom line of our prediction on the rate of pure leptonic modes. The analysis is a little more complex at the 4.180 GeV even though the method is similar. The sensitivities for the rare semi-leptonic decays of $D_s^+$ or $D_s^{*+}$ mesons can be expected to reach $10^{-5}$ at 90% CL, however it is not enough to test our predictions for the rare $D_s^+$ semi-leptonic decays.

If the proposed super $\tau$-charm factory is launched in the near future, we would be able to collect at least 100 or 1000 times more data since the designed luminosity of STCF will be as high as $1 \times 10^{35} \text{ cm}^{-2}\text{s}^{-1}$ which is 100 times of the BEPC II. Then, the sensitivities of searching for the concerned signals in $D$ or $D_s^+$ decays can be greatly improved as $10^{-9}$~$10^{-10}$ or $10^{-7}$~$10^{-8}$ at 90% CL, respectively, may be expected. With this improved sensitivities, the rates of $D_s^+ \rightarrow K^+e^+e^-$ and $D_s^+ \rightarrow K^+\mu^+\mu^-$ predicted by the $U'(1)$ or 2HDM models become measurable. Then, the more challengeable lepton-flavor-violation modes $D^0 \rightarrow e^-\mu^+$ predicted by the $U'(1)$ and unparticle models can be possibly tested.
VI. DISCUSSION AND CONCLUSION

The rare decays of heavy flavored hadrons which are suppressed or even forbidden in SM can serve as probe portals for searching new physics BSM. Experimentally measured “anomalies” which obviously deviate from the SM predictions are considered as the candidate signals of BSM, at least provide hints to BSM for the experiments of high energy colliders, such as LHC. That is the common sense for experimentalists and theorists of high energy physics. However, how to design a new experiment which might lead to discovery of new physics is an art. As following the historical experience, beside the blind search in experiments, researchers tend to do measurements according to the prediction made by theorists based on the available and reasonable models.

FCNC/LFV processes provide a sensitive test for new physics BSM, which compose a complementary area to high energy collider physics. Definitely, those processes where SM substantially contributes, do not stand as candidates for seeking new physics BSM, because the new physics contributions would be drown in the SM background. Researchers are carefully looking for rare processes where SM contributions are very suppressed or even forbidden by some rules. The rare semi-leptonic and leptonic decays of B and D mesons are ideal places because they are caused by FCNC. Especially the lepton-flavor violation decays which cannot be resulted in by the SM because neutrino masses are too tiny to make any non-negligible contribution, are the goal which we have interests in.

Recently, most of researches focus on B decays. The reason is obvious, that B mesons are at least three times heavier than D mesons, so the processes involving B-mesons are closer to new physics scale and moreover, the coupling between b-quark and top-quark has a large CKM entry. Indeed, there are many research works concerning $B \rightarrow K(\ast)\ell\bar{\ell}$\cite{25,26} and $B^0(B_s) \rightarrow \ell\bar{\ell}$ have emerged\cite{27,28}. On another aspect, several authors have studied the case of D mosons, and drawn constraints on the free parameters in the proposed models by fitting available data. The model parameters can be compared with those obtained by fitting the data of B decays. In this work, based on the large database of the BESIII, we follow the trend to investigate possibilities of detecting the rare semi-leptonic and pure leptonic decays of D meson, and specially we pay more attention to the analysis of the lepton-flavor-violation processes.

In this work, we calculate the decay rates of $D_s^+ \rightarrow K^+e^-\bar{e}^+, D_s^+ \rightarrow K^+e^-\mu^+, D^0 \rightarrow e^-\bar{e}^+$.
and $\bar{D}^0 \rightarrow e^- \mu^+$ through exchanging a neutral particle in terms of three BSM new physics models: the extra $U'(1)$, 2HDM of type III and unparticle. The decay rate of $D_s^+ \rightarrow K^+ e^- e^+$ receives sizable contribution from SM whose branching ratio is up to orders of $10^{-6}$. It is noted that the branching ratio of direct decay process via penguin diagram is small at order of $10^{-8}$, while the long-distance reaction makes a larger contribution. Our numerical results show that $U'(1)$ and 2HDM of type III can make significant contributions to the process $D_s^+ \rightarrow K^+ e^- e^+$ as long as the model parameters which are obtained by fitting relevant data are adopted, but the unparticle model cannot make any substantial contribution. The recent researchers seem to be more tempted to use the extra $U'(1)$ model and we follow their trends. But here for fixing the model parameters, we deliberately relax the constraint set by the $D^0 - \bar{D}^0$ mixing as we discussed in the above text. If the constraints were taken into account, the predicted branching ratio of $D_s^+ \rightarrow K^+ e^- e^+$ would be reduced by two more orders as $10^{-8}$ which is much lower than the contribution of the SM long-distance effect. Thus the new physics contribution would be buried in the SM background. However, as we only consider the constraints on $U'(1)$ parameter taken by fitting the data of $\tau \rightarrow 3l$ other than $D^0 - \bar{D}^0$ mixing, the predicted branching ratio can be large to order of $10^{-6}$, thus the resultant amplitude might interfere with the SM long-distance contribution.

In future BES III experiment, the experimental sensitivity can be up to order of $10^{-6} \sim 10^{-7}$, thus the data on $D_s^+ \rightarrow K^+ e^- e^+$ might tell us some information of new physics.

Our numerical results show that the $U'(1)$ model and 2HDM of type III could make an observable branching ratio of $D_s^+ \rightarrow K^+ e^- e^+$ with the BES III data as its precision can reach orders of $10^{-6} \sim 10^{-7}$.

For processes $D^0 \rightarrow e^- e^+$ and $D^0 \rightarrow e^- \mu^+$, the theoretically predicted branching ratio of decay $D^0 \rightarrow e^- e^+$ is of the order of $10^{-10}$ since its width is proportional to $m_e^2$, such a small value is hard to be observed. While for decay $D^0 \rightarrow e^- \mu^+$, its branching ratio can be up to orders of $10^{-7}$, which may be observed in future super $\tau$-charm factory. Moreover, one can expect to watch $D^0 \rightarrow \mu^- \mu^+$, while unfortunately, $D^0 \rightarrow \mu^- \tau^+$ is forbidden by the phase space of final states because $m_{\mu} + m_\tau > m_{D^0}$.

According to the presently available new physics models, $U'(1)$, 2HDM and unparticle model, the data on D-mesons which will be collected in the future 10 years can marginally detect the new physics contributions to $D_s^+ \rightarrow K^+ e^- e^+$, $D_s^+ \rightarrow K^+ e^- \mu^+ + h.c.$, $D^0 \rightarrow e^- e^+$ and $D^0 \rightarrow e^- \mu^+ + h.c.$ as long as only the constraints set by some experiments are accounted,
but the data of $D^0 - \bar{D}^0$ mixing are relaxed. If the data of $D^0 - \bar{D}^0$ mixing are taken into account the BES III and even the planned high luminosity $\tau$-charm factory will not be able to “see” those rare decays as predicted by these models. However, it by no means forbids experimental search for these rare decays in the charm energy regions based on the huge data sample collected by BES and the future $\tau$-charm factory. Blind experimental search is not affected by the available theoretical prediction because the present BSM are only possible ones conjectured by theorists, while nature might suggest an alternative scenario. Once such new observation is made, we would be stunned and explore new models BSM to explain the phenomena, thus our theories would make new progress, and that is what we expected.

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