Compton scattering of blackbody photons by relativistic electrons

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ABSTRACT

We present simple and accurate analytical formulas for the rates of Compton scattering by relativistic electrons integrated over the energy distribution of blackbody seed photons. Both anisotropic scattering, in which blackbody photons arriving from one direction are scattered by an anisotropic electron distribution into another direction, and scattering of isotropic seed photons are considered. Compton scattering by relativistic electrons off blackbody photons from either stars or cosmic microwave background takes place, in particular, in microquasars, colliding-wind binaries, supernova remnants, interstellar medium and the vicinity of the Sun.

Key words: acceleration of particles – radiation mechanisms: non-thermal – relativistic processes – scattering – gamma-rays: general – gamma-rays: stars.

1 INTRODUCTION

Compton scattering by relativistic electrons is a very common astrophysical process. Electrons are accelerated to relativistic energies in shocks or reconnection in a variety of cosmic sources, e.g. jets, colliding winds in binaries, supernova remnants or coronae of accreting sources. Relativistic electrons are also a component of cosmic rays. The seed photons are often blackbody, usually from either the cosmic microwave background (CMB) or stars. The blackbody field is anisotropic if it comes from a star or isotropic, e.g. in the CMB. Some sites where this process takes place are listed below.

Black hole X-ray binaries in some of their states have jets, which contain relativistic electrons. In case of binaries with massive donors, e.g. Cyg X-1 or Cyg X-3, the electrons significantly Compton up-scatter stellar radiation (see, e.g., Jackson 1972; Georganopoulos, Aharonian & Kirk 2002; Bosch-Ramon, Romero & Paredes 2006; Abd et al. 2009; Dubus, Cerutti & Henri 2010a; Zdziarski, Lubiński & Sikora 2012a; Zdziarski et al. 2012b; Malyshev, Zdziarski & Chernyakova 2013; Zdziarski, Pjanka & Sikora 2013). The blackbody radiation field coming from the surface of the donor star arriving at the jet is strongly anisotropic. The probability of scattering is highest for head-on collisions, and relativistic electrons radiate in a beam along their direction. This leads to a pronounced anisotropy of the Compton-scattered radiation, e.g. Jackson (1972), Dubus, Cerutti & Henri (2008, 2010a,b), Dubus (2013), with the observed Compton flux peaking around the superior conjunction. Assuming the star to be a point source, calculations of the phase-averaged jet emission require four-dimensional integration, over the electron and photon distributions, the jet length and the orbital phase. Taking into account the finite size of the star adds two more dimensions to the integration.

The distribution of the relativistic electrons is, in general, not a power law, in particular when energy losses and a high-energy cut-off are considered (e.g. Zdziarski et al. 2013). Then Compton rates averaged over power-law electrons cannot be used. Such calculations are thus very computationally intensive, and integrating analytically the Compton rate over the blackbody distribution significantly facilitates them.

The same Compton anisotropy takes place in binary systems with pulsar wind colliding with wind of a massive star, so-called γ-ray binaries (see Dubus 2013 for a review). In those systems, interaction of the two winds leads to acceleration of relativistic electrons, which then radiate, in particular, Compton up-scattered blackbody photons from the massive star. That radiation will then be orbitally modulated, e.g. in PSR B1259–63, LS I +61° 303 and LS 5039 (Kirk, Ball & Skjærnaes 1999; Dubus et al. 2008; Cerutti et al. 2010).

Related systems are so-called colliding wind binaries, comprising of two massive stars with interacting winds. Electrons are accelerated in the interaction region, and they Compton up-scatter the stellar radiation. Such emission appears to be observed from η Car (Bednarek & Paciβ 2011).

In the case of pulsar winds, stellar photons may also be Compton up-scattered by cold electrons in the wind, which is ultrarelativistic and can have the bulk Lorentz factor as high as γ ≥ 105. This process takes place in γ-ray binaries (Cerutti, Dubus & Henri 2008), but it appears to be seen also in systems with pulsars with low-mass companions, e.g. PSR B1957+20, the black widow pulsar (Wu et al. 2012). In these cases, both the electron distribution and the blackbody radiation from the star are anisotropic. A related process is Compton up-scattering by relativistic bulk motion of electrons in a jet, considered by Begelman & Sikora (1987).

Another astrophysical case of anisotropic Compton scattering is that of cosmic ray electrons and the interstellar starlight, which contributes to the Galactic diffuse emission. That starlight field is...
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2 INTEGRATION OF THE COMPTON CROSS-SECTION OVER A BLACKBODY SPECTRUM

2.1 Anisotropic seed photons

The rate of Compton scattering (in both Thomson and Klein–Nishina regimes) of a photon beam from direction $\mathbf{\Omega}$ into direction $\mathbf{\Omega}'$ by a cloud of relativistic electrons with Lorentz factors $\gamma \gg 1$ and in the Thomson, $\epsilon / \Omega \ll 1$ any $y \ll \epsilon \gamma N_1$ emit in a narrow beam along its $\gamma N_1$, the rate (1) is given by the corresponding rates of Compton scattering by monoenergetic electrons over the blackbody distribution, obtaining simple and highly accurate formulæ in close form. In the isotropic case, we compare our results with the approximation of Petruk (2009), as well as with calculation in the electron rest frame, see Appendix A. Here we study both anisotropic and isotropic scattering. We integrate the corresponding rates of Compton scattering by relativistic electrons over the blackbody distribution, obtaining simple and highly accurate formulæ in close form. In the isotropic case, we compare our results with the approximation of Petruk (2009), as well as with calculation in the electron rest frame, see Appendix A.

Our results give Compton photon redistribution functions for highly relativistic electrons at a single energy, i.e. the distribution of the energies and directions of the scattered photon. At low energies, the electrons have usually a Maxwellian distribution, for which the corresponding redistribution function has recently been presented by Suleimanov, Poutanen & Werner (2012).

\begin{equation}
\frac{d\hat{n}}{d\epsilon_1 d\epsilon_2 d\Omega_1 d\Omega_2} = \frac{3\sigma_T}{4\epsilon_1^2} \frac{\epsilon_1}{\epsilon} \frac{dN(\gamma)}{d\Omega_1} \frac{dN_0(\epsilon)}{d\Omega_2} \left[ 1 + \frac{\epsilon_1 \epsilon_2 \gamma}{\epsilon} + \frac{2\epsilon_1 \epsilon_2}{\epsilon^2} \right],
\end{equation}

where $\epsilon_1$, $\epsilon_2$ and $\gamma$ are the energy in units of $m_e c^2$ and the direction of the incoming and scattered photon, respectively, $m_e$ is the electron mass, $dN(\gamma)/d\Omega_1$ is the electron number per unit $\gamma$ and per unit solid angle, and $dN_0/d\Omega_2$ is the density of photons coming from direction of $\Omega$. Then, the range of kinematically allowed values of $\epsilon_1$, $\epsilon_2$ and $\gamma$ can be expressed in three equivalent forms:

\begin{align}
[\epsilon_1 \geq \epsilon_m, \quad \epsilon_1 < \gamma], \\
\epsilon_1 \leq \frac{2\epsilon \gamma}{1 + 2\epsilon \gamma}, \\
\gamma \geq \frac{\epsilon_1}{2} \left( 1 + \sqrt{1 + \frac{2}{\epsilon_1 \gamma}} \right),
\end{align}

as well as $\epsilon_1 \leq \epsilon_2$ is required. The first of the above conditions is used for integration over $\epsilon$. Note that equation (1) assumes the directions of the electron and scattered photons to be the same. This is because electrons with $\gamma \gg 1$ emit in a narrow beam along its direction of motion, and thus, in the adopted approximation, only electrons moving along the scattered photon direction contribute to the spectrum. Consequently, the same equation applies for either anisotropic or isotropic $dN(\gamma)/d\Omega_1$ (although Aharonian & Atoyan 1981 considered it for isotropic electrons only). Indeed, equation (1) agrees, in the limits of $\gamma \gg 1$ and $\epsilon \ll \epsilon_1$, with equation 20 of Fargion, Konoplich & Salis (1997), which gives a formula for emission by an electron beam scattering off a photon beam valid at any $\gamma$ and $\epsilon$. For an isotropic electron distribution, $dN(\gamma)/d\Omega_1 = N(\gamma)/(4\pi)$.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{The scattering rate on seed photons at a single energy, equation (1), plotted per unit $\ln \epsilon_1$ (red solid curve) compared to the Thomson-limit rate (black dashed curve) for $y = 10^{-4}$, $\gamma = 10^6$, $\epsilon_1 = 1$ and in the Thomson units. At low $\epsilon_1$, the rate per unit $\epsilon_1$ is constant.}
\end{figure}

In the Thomson limit, in which the photon energy in the electron rest frame, $\epsilon_1 = \epsilon \gamma$, is $\ll 1$, the rate (1) is given by the corresponding expression without the term $\epsilon \epsilon_1 m\gamma$ and with $\epsilon_1 = \epsilon_1/(2y\gamma^2)$. On the other hand, the low-energy, $\epsilon_1 \ll 2\epsilon \gamma/(1 + 2\epsilon \gamma)$, limit of the scattering rate is different, and given by equation (1) with the factor in brackets replaced by 1, which is valid for all $\epsilon_1$. The distinction between these two limits is illustrated in Fig. 1.

An interesting feature of the rate (1) in the Klein–Nishina limit, $\epsilon_1 \gg 1$, is it showing a sharp peak at the maximum allowed $\epsilon_1$, above which it is cut off. This is illustrated in Fig. 1, comparing that rate to its Thomson limit. The total, integrated over $\epsilon_1$, rate is lower, but the rate per unit $\epsilon_1$ is much higher than that in the Thomson limit just before the cut-off. The presence of the peak is due to a compression of the allowed range of $\epsilon_1$ close to $\gamma$ in the Klein–Nishina limit, since the condition of $\epsilon_1 < \gamma$ has to be satisfied for any scattering. Mathematically, the peak appears due to the presence of the $(\gamma - \epsilon_1)$ factor in the denominator of the definition of $\epsilon_m$.
The total scattering rate, i.e. that of equation (1) integrated over \( \epsilon_1 \), is directly related to the total Klein–Nishina cross-section (e.g. Rybicki & Lightman 1979), \( \sigma_{KN} \),

\[
\frac{dn}{de} dy \frac{d\Omega}{d\Omega} = \frac{A(\Omega) \epsilon^2}{\exp(\epsilon/\theta) - 1},
\]

where \( \theta = kT/m_e c^2 \), \( T \) is the temperature, \( A(\Omega) \) is direction-dependent normalization, and \( T \) and \( \epsilon \) are given in the frame of the scattering gas. Note that the relation to the specific intensity, \( I \), is \( I(\epsilon, \Omega) = \epsilon c^2 n(\epsilon) / d\Omega \). For undiluted blackbody, \( A = 2(m_e c^2/h)^3 \), where \( h \) is the Planck constant. The case of photons arriving from a single direction, \( \Omega_s \), corresponds to the appearance of \( \delta(\Omega - \Omega_s) \) in \( A(\Omega) \). Then, the scattering rate can be integrated over \( \Omega \). In particular, for photons arriving from a spherical star with radius \( R_s \), at a distance from its centre of \( R > R_s \), in which case the arriving photons can be approximated by a mono-directional beam, we have

\[
A(\Omega) = 2 \pi D^2 \left( \frac{m_e c^2}{h} \right) \left( \frac{R}{R_s} \right)^2 \delta(\Omega - \Omega_s),
\]

where \( D \) is the Doppler factor of stellar photons seen in the electron gas frame, \( \theta \) in equation (4) equals \( \theta_s / D \), and \( \theta_s \) is the stellar temperature. This formula follows from the fact that the flux, \( F \), from a uniformly emitting sphere is given by \( F = \pi I(R_s/R)^2 \) (Rybicki & Lightman 1979), and the photon density for a mono-directional beam is \( F / (c \epsilon m_e c^2) \).

The scattering rate can be integrated over the photon distribution,

\[
\frac{dn}{de} dy \frac{d\Omega}{d\Omega} = \int_{\epsilon_1}^{\infty} \frac{dn(\epsilon)}{d\epsilon} \frac{d\Omega}{d\Omega} d\epsilon.
\]

Here, we have set the upper limits of integration to infinity for the sake of simplicity, in spite of the condition of \( \epsilon \ll \epsilon_1 \) assumed in equation (1). This has a negligible effect on the results as long as \( \theta \ll \epsilon_1 \).

Comparing equations (1) and (4), we see that the rate (6) involves three integrals with different powers of \( \epsilon \). We can perform two of them analytically,

\[
\int_{\epsilon_1}^{\infty} \frac{dx'}{\exp x' - 1} \equiv f_1(x) = \frac{\pi^2}{6} + \frac{x^2}{2} + \text{Li}_2(1 - \exp x)
\]

\[
\int_{\epsilon_1}^{\infty} \frac{dx'}{\exp x' - 1} \equiv f_0(x) = -\ln \left[ 1 - \exp(-x) \right],
\]

where \( \text{Li}_2(x) \) is the dilogarithm \((-\int x' \ln(1 - x') / x' \); note that Petruk 2009 uses a different definition).

The third integral has no closed form. We thus use the following series expansions:

\[
\frac{1}{\exp x - 1} = x^{-1} - \frac{2}{x} - \sum_{k=1}^{N} \frac{x^{2k-1} B_{2k}}{(2k)!}, \quad x < 2\pi,
\]

where \( B_k \) is the Bernoulli number (Gradshteyn & Ryzhik 1980), and

\[
\frac{1}{\exp x - 1} = \sum_{k=1}^{N} \exp(-kx), \quad x > 0.
\]

Since \( B_{2k}(2k)! \) decreases very fast, in particular, it is equal to 1/12, \(-1/720\) and 1/30 240 for \( k = 1, 2 \) and 3, respectively, and the second series is a sum of exponentials of negative numbers, both series are rapidly converging. We can integrate the series (9) times \( \epsilon^{-1} \) term by term for \( x < 2\pi \), which yields

\[
\int_{\epsilon_1}^{\infty} \frac{dx}{\exp x - 1} \equiv g_0(x, N) = -x^{-1} - \ln x + \sum_{k=1}^{N} \frac{x^{2k-1} B_{2k}}{(2k-1)(2k)!},
\]

which is exact for \( N \to \infty \). We then calculate the definite integral of \( g_0(q, N) - g_0(x, N) + g_0 \) with \( q = 2.257, N = 3 \), where

\[
g_0 = \int_{\epsilon_1}^{\infty} \frac{dx}{\exp x - 1} \approx 0.0366377.
\]

We can then integrate the series (10) times \( \epsilon^{-1} \) term by term,

\[
\int_{\epsilon_1}^{\infty} \frac{d\epsilon}{\exp \epsilon - 1} \equiv g_0(x, N) = \sum_{k=1}^{N} E_k(\epsilon x), \quad x > 0,
\]

which is exact for \( N \to \infty \), and where \( E_k(\epsilon) = \int_{\epsilon_1}^{\infty} d\epsilon \epsilon^{-1} / \epsilon^{2k-1} \) is the exponential integral. Our approximation to the definite integral, equation (13), is then

\[
f_{v_0}(\epsilon) = \begin{cases} g_0(q, 3) - g_0(x, 3) + g_0, & \epsilon \leq q; \\ g_0(x, 3), & \epsilon > q. \end{cases}
\]

The maximum fractional error of this expression for any value of \( \epsilon \) is \( \sim 3.6 \times 10^{-4} \), which error is reached around \( \epsilon = q \). We chose the value of \( q \) to minimize the maximum error of our approximation.

The resulting rate integrated over blackbody is

\[
\frac{dn}{de} dy \frac{d\Omega}{d\Omega} = \frac{3 \pi c A}{4 \gamma^2} \frac{dN(\gamma)}{d\Omega} \left[ 1 + \epsilon m_\gamma \theta \right] \theta^2 f_1 \left( \frac{\epsilon m_\gamma \theta}{\theta} \right) + 2\epsilon^2 m_\gamma f_0 \left( \frac{\epsilon m_\gamma \theta}{\theta} \right).\]

In the Thomson regime of \( 2.7 \theta_\gamma \ll 1 \) (where \( 2.7 \theta_\gamma \) is the average blackbody photon energy), the rate is as above but without the \( \epsilon \gamma m_\gamma \) term and with \( \epsilon m_\gamma = \epsilon_{1}/(2\gamma^2 \theta) \). The \( f \)-function argument, \( \epsilon m_\gamma / \theta \), can still be \( \geq 1 \). On the other hand, in the low-energy limit of \( \epsilon \ll 2\gamma^2 \theta / (1 + 2\gamma^2 \theta) \), \( \dot{n} \) is constant with \( \epsilon_{1} \),

\[
\frac{dn}{de} dy \frac{d\Omega}{d\Omega} = \frac{\pi c A}{4 \gamma^2} \frac{dN(\gamma)}{d\Omega} \left[ 1 + \epsilon m_\gamma \theta \right] \theta^2 \frac{8}{\gamma^2}.
\]

for any value of \( \theta_\gamma \), from the Thomson to extreme Klein–Nishina regimes. This constant rate is seen in Figs 2(a) and (b), which show examples of \( \dot{n} \) in the Klein–Nishina and Thomson regimes. It also shows the corresponding Thomson-limit dependences. In the Klein–Nishina regime, Fig. 2(a), we see a sharp peak around the maximum allowed \( \epsilon_{1} \), similar to that in Fig. 1.

### 2.2 Isotropic seed photons

In the case of isotropic seed photons, \( n_0(\epsilon) = 4\pi d\epsilon n_0(\epsilon) / d\Omega \), the scattering rate has been derived by Jones (1968), see also equation 2.48 of Blumenthal & Gould (1970) or equation 22 of Aharanion &
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The total scattering rate, i.e. integrated over $\epsilon_1$, is given by (Aharonian, Kirillov-Ugryumov & Vardanian 1985; Protheroe 1986; Zdziarski 1988)

$$\frac{dn}{d\gamma d\Omega} = \frac{3\sigma T c n_0(\epsilon)}{4\epsilon^2} \frac{dN(\gamma)}{d\Omega_1} \left[ 1 + 2\epsilon_1 \epsilon_n + \epsilon_1 (1 - 2\epsilon_1 \epsilon_n) \right]$$

As required, an integral of equation (1) over $\Omega$ (i.e. over $\gamma$ and times $2\pi$) equals the above formula. Note that since relativistic electrons emit mostly along their direction of motion, the above formula applies for anisotropic electrons. If the electrons are isotropic, we can integrate over $\Omega_1$ (i.e. multiply by $4\pi$) and substitute $dN(\gamma)/d\Omega_1 = N(\gamma)/(4\pi)$ in equations (17), (29)–(30). (Thus, only removal of $\Omega_1$ on the left-hand side and replacement of $dN(\gamma)/d\Omega_1$ by $N(\gamma)$ is required.) The ranges of kinematically allowed values of $\epsilon$, $\epsilon_1$ and $\gamma$ are given by $\epsilon \ll \epsilon_1$ and

$$[\epsilon \geq \epsilon_n, \epsilon_1 < \gamma], \epsilon_1 \leq \frac{4\epsilon \gamma^2}{1 + 4\epsilon \gamma}, \gamma \geq \frac{\epsilon_1}{2} \left(1 + \sqrt{1 + \frac{1}{\epsilon_1}}\right).$$

The rate in the Thomson limit, $\epsilon \ll 1$, corresponds to the rate (17) without the terms of $2\epsilon_1 \epsilon_n$ and with $\epsilon_1 = \epsilon_1/(4\gamma^2)$. The low-energy rate is constant, analogously to the case of anisotropic scattering, with the term in brackets set to 1. In the Klein–Nishina limit, the profile of $n$ shows a sharp peak very similar to that shown in Fig. 1.

The blackbody-integrated scattering rate of equation (15) per unit $\ln \epsilon_1$ (red solid curves) compared to the Thomson-limit rate (black dashed curves) for $\theta = 3 \times 10^{-5}, y = 1$ and (a) $\gamma = 1.6$ and (b) $\gamma = 10^3$. The units are Thomson and $A_0 = 1$. The maximum values of $\epsilon_1$ corresponding to the average blackbody energy, $\epsilon \approx 2.7\theta$, given by equation (2), are $\approx 10^6$ and $\approx 140$ for (a) and (b), respectively.

Figure 2. The blackbody-integrated scattering rate into a given direction, $\Omega_1$, is

$$\frac{dn}{d\Omega_1 d\gamma d\Omega_2} = \frac{3\sigma T c n_0(\epsilon)}{4\epsilon^2} \frac{dN(\gamma)}{d\Omega_1} \left[ 1 + 2\epsilon_1 \epsilon_n + \epsilon_1 (1 - 2\epsilon_1 \epsilon_n) \right]$$

As required, an integral of equation (1) over $\Omega$ (i.e. over $\gamma$ and times $2\pi$) equals the above formula. Note that since relativistic electrons emit mostly along their direction of motion, the above formula applies for anisotropic electrons. If the electrons are isotropic, we can integrate over $\Omega_1$ (i.e. multiply by $4\pi$) and substitute $dN(\gamma)/d\Omega_1 = N(\gamma)/(4\pi)$ in equations (17), (29)–(30). (Thus, only removal of $\Omega_1$ on the left-hand side and replacement of $dN(\gamma)/d\Omega_1$ by $N(\gamma)$ is required.) The ranges of kinematically allowed values of $\epsilon$, $\epsilon_1$ and $\gamma$ are given by $\epsilon \ll \epsilon_1$ and

$$[\epsilon \geq \epsilon_n, \epsilon_1 < \gamma], \epsilon_1 \leq \frac{4\epsilon \gamma^2}{1 + 4\epsilon \gamma}, \gamma \geq \frac{\epsilon_1}{2} \left(1 + \sqrt{1 + \frac{1}{\epsilon_1}}\right).$$

The rate in the Thomson limit, $\epsilon \ll 1$, corresponds to the rate (17) without the terms of $2\epsilon_1 \epsilon_n$ and with $\epsilon_1 = \epsilon_1/(4\gamma^2)$. The low-energy rate is constant, analogously to the case of anisotropic scattering, with the term in brackets set to 1. In the Klein–Nishina limit, the profile of $n$ shows a sharp peak very similar to that shown in Fig. 1.

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Atoyan (1981). The scattering rate into a given direction, $\Omega_1$, is

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As required, an integral of equation (1) over $\Omega$ (i.e. over $\gamma$ and times $2\pi$) equals the above formula. Note that since relativistic electrons emit mostly along their direction of motion, the above formula applies for anisotropic electrons. If the electrons are isotropic, we can integrate over $\Omega_1$ (i.e. multiply by $4\pi$) and substitute $dN(\gamma)/d\Omega_1 = N(\gamma)/(4\pi)$ in equations (17), (29)–(30). (Thus, only removal of $\Omega_1$ on the left-hand side and replacement of $dN(\gamma)/d\Omega_1$ by $N(\gamma)$ is required.) The ranges of kinematically allowed values of $\epsilon$, $\epsilon_1$ and $\gamma$ are given by $\epsilon \ll \epsilon_1$ and

$$[\epsilon \geq \epsilon_n, \epsilon_1 < \gamma], \epsilon_1 \leq \frac{4\epsilon \gamma^2}{1 + 4\epsilon \gamma}, \gamma \geq \frac{\epsilon_1}{2} \left(1 + \sqrt{1 + \frac{1}{\epsilon_1}}\right).$$

The rate in the Thomson limit, $\epsilon \ll 1$, corresponds to the rate (17) without the terms of $2\epsilon_1 \epsilon_n$ and with $\epsilon_1 = \epsilon_1/(4\gamma^2)$. The low-energy rate is constant, analogously to the case of anisotropic scattering, with the term in brackets set to 1. In the Klein–Nishina limit, the profile of $n$ shows a sharp peak very similar to that shown in Fig. 1.
The resulting blackbody-integrated rate is

\[
\frac{d\dot{n}}{d\epsilon d\gamma d\Omega_1} = \frac{3\pi\sigma T c A}{\gamma^2} \frac{dN(\gamma)}{d\Omega_1} \left[ \left( 1 - 2\epsilon_1 \epsilon_n + 2 \ln \frac{\epsilon_n}{\theta} \right) \epsilon_n \theta f_0 \left( \frac{\epsilon_n}{\theta} \right) \right.
+ \left. \left( 1 + 2\epsilon_1 \epsilon_n \right) \theta^2 f_1 \left( \frac{\epsilon_n}{\theta} \right) - 2\epsilon_1^2 f_{-1} \left( \frac{\epsilon_n}{\theta} \right) \right]
- 2\epsilon_n \theta f_0 \left( \frac{\epsilon_n}{\theta} \right). \tag{29}
\]

The Thomson regime, $2.7\theta \gamma \ll 1$, has the rate as above but without the terms of $2\epsilon_1 \epsilon_n$ and with $\epsilon_n = \epsilon_1/(4\gamma^2)$. On the other hand, the low-energy, $\epsilon_1 \ll 4\gamma^2 \theta/(1 + 4\gamma^2)$, limit has a constant rate in any regime, where the factor in brackets above is replaced by $\theta^2 \pi^2 / 6$, similar to that of equation (16), see Figs 3(a) and (b). We note that the integration over the blackbody spectrum can also be done in the electron rest frame, see Appendix A, which gives identical results to the numerical integration (22).

The fractional error of our equations (15) and (29) is <10^{-5} over most of the parameter space. The highest error of $\approx 6 \times 10^{-4}$ for equation (15) is reached around $\epsilon_n/\theta \approx q$ in the Thomson regime, and the error is much lower in the Klein–Nishina regime. Equation (29) has the highest error around $\epsilon_n/\theta \approx q$, reaching $\approx 3 \times 10^{-3}$ in the Thomson regime.

Petruk (2009) obtained an approximate formula for the Compton rate integrated over isotropic blackbody photons, given by their equation 32, which, in our notation, is

\[
\frac{d\dot{n}}{d\epsilon d\gamma d\Omega_1} = \frac{\sigma_T c A \theta^2 \pi^3}{2\gamma^2} \frac{dN(\gamma)}{d\Omega_1} \exp \left( -\frac{2\epsilon_n}{3\theta} \right)
\]

\[
\times \left\{ \exp \left[ -\frac{5}{4} \left( \frac{\epsilon_n}{\theta} \right)^{1/2} \right] + 2\epsilon_1 \epsilon_n \exp \left[ -\frac{5}{7} \left( \frac{\epsilon_n}{\theta} \right)^{7/10} \right] \right\}. \tag{30}
\]

Figs 3(a) and (b) shows the rate of equation (29) compared to its Thomson limit and to the approximation of Petruk (2009). We find the formula of Petruk (2009) to be generally quite accurate. Fig. 3(c) shows its accuracy in a Thomson-limit case. Equation (30) is fully accurate in the low-energy limit, and then it has the fractional error of $\lesssim 30$ per cent at all energies except the high-energy tail, where, however, the scattering rate itself is negligibly small, see the middle panel.

3 CONCLUSIONS

We have obtained simple and accurate formulae for the rates of Compton scattering by relativistic electrons of a given energy off blackbody photons. The blackbody-integrated rate for scattering from one direction into another is given by equation (15). The corresponding rate for scattering off isotropic blackbody seed photons is given by equation (29). These formulae apply to electrons with either isotropic or anisotropic angular distribution. We note that the isotropic calculations apply also to the case of a mono-directional photon beam irradiating isotropic electrons provided the spectrum of the scattered photons is integrated over all directions.

Our results involve two special functions, the dilogarithm and the exponential integral. Those functions can be easily calculated in any standard numerical package, e.g. Press et al. (1992), Wolfram (1996). Obviously, the required integrations can also be carried out numerically. However, formulas are always preferable to numerical results. In particular, using our formulas in multidimensional integration of jet emission (Zdziarski et al. 2013) instead of integrating the scattering rate over blackbody has resulted in a very substantial increase of computational efficiency, in some cases reducing the computing time by a factor of $\approx 100$.

Figure 3. The isotropic blackbody-integrated scattering rate of equation (29) per unit $\ln \epsilon_1$ (red solid curves) compared to the approximation of Petruk (2009) (blue dotted curves) and the Thomson-limit rate (black dashed curves) for $\theta = 3 \times 10^{-5}$, and (a) $\gamma = 10^6$ and (b) $\gamma = 10^9$. The units are Thomson and $A = 1/(4\pi)$. (c) The ratio of the approximation of Petruk (2009) to the actual rate for the case shown in the panel (b).

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APPENDIX A: SCATTERING IN THE ELECTRON REST FRAME

We note that Strong (1975) considered isotropic scattering by relativistic electrons on blackbody photons using an approach different from that adopted in Section 2.2. He calculated the spectrum of isotropic blackbody photons in the electron rest frame (see also Fargion & Salis 1998), assuming that all the photons arrive in the frame direction, which is approximately satisfied for the electron Lorentz factor of \( \gamma \gg 1 \). This yields (Strong 1975)

\[
\frac{n_0(\epsilon)}{\theta} = \left( \frac{m_e c}{\hbar} \right)^{3} \frac{4 \pi e^2 \theta}{\gamma^2} \ln \frac{1}{1 - \exp(-\epsilon/2\gamma\theta)},
\]

where \( \epsilon \) is the dimensionless photon energy in the electron rest frame, see Section 2.1.

Since \( n_0(\epsilon) \) is an invariant (e.g. Blumenthal & Gould 1970), the above distribution can be used for numerical integration of the rate of Compton scattering over \( \epsilon \), see equation 4.7 in Strong (1975). This is fully equivalent to integrating the angle-averaged rate (17) with blackbody photons over \( \epsilon \), see Section 2.2. An advantage of using the photon distribution in the particle rest frame is that it can be readily used for other reactions by highly relativistic particles on blackbody photons, e.g. \( e^\pm \) pair production by cosmic ray protons.

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