Isotopic dependence of the fragments’ internal temperatures observed in multifragment emission

S.R. Souza$^{1,2}$ and R. Donangelo$^{1,3}$

$^1$Instituto de Física, Universidade Federal do Rio de Janeiro Cidade Universitária, CP 68528, 21941-972, Rio de Janeiro, Brazil
$^2$Instituto de Física, Universidade Federal da Bahia, Campus Universitário de Ondina, 40210-340, Salvador, Brazil
$^3$Instituto de Física, Facultad de Ingeniería, Universidad de la República, Julio Herrera y Reissig 565, 11.300 Montevideo, Uruguay

(Dated: July 11, 2018)

The internal temperatures of fragments produced by an excited nuclear source are investigated using the microcanonical version of the Statistical Multifragmentation Model, with discrete energy. We focus on the fragments’ properties at the breakup stage, before they have time to deexcite by particle emission. Since the adopted model provides the excitation energy distribution of these primordial fragments, it allows one to calculate the temperatures of different isotope families and infer on the sensitivity to their isospin composition. It is found that, due to the functional form of the nuclear density of states and the excitation energy distribution of the fragments, proton rich isotopes are hotter than neutron rich ones. This property has been taken to be an indication of earlier emission of the former from a source that cools down as it expands and emits fragments. Although this scenario is incompatible with the prompt breakup of a thermally equilibrated source, our results reveal that the latter framework also provides the same qualitative features just mentioned. Therefore they suggest that this property cannot be taken as evidence for non-equilibrium emission. We also found that this sensitivity to the isotopic composition of the fragments depends on the isospin composition of the source, and that it is weakened as the excitation energy of the source increases.

PACS numbers: 25.70.Pq,24.60.-k

I. INTRODUCTION

When nuclei collide at not too peripheral impact parameters, with incident energies per nucleon above a few tens of MeV, the overlap region reaches an excitation energy of several MeV per nucleon and also has an appreciable increase of its nuclear density. The succeeding fast expansion may lead the system to configurations of dynamical instabilities, during which many fragments are created. Nevertheless, some dynamical calculations also suggest that the fragment composition is determined early in the expansion stage of the system, before low densities have been attained. In some studies, it is also found that the properties of the system at the breakup stage assumed in the statistical models are compatible with the configurations found in these dynamical calculations. Although it gives support to a scenario in which the fragments are emitted statistically, simulations based on the Antisymmetrized Molecular Dynamics Model suggest that a two stage model, in which fragments are formed in a prompt breakup and subsequently decay by particle emission, may not be the best representation of the actual process. Indeed, it has been reported in Ref. that fragment deexcitation and fragment formation may take place concomitantly during the process. This scenario is, therefore, incompatible with the traditional hybrid treatments which separate the multifragment production in the two stages just mentioned.

Efforts have been made to experimentally determine the properties of the fragments created in the breakup stage, right after the most violent stages of the collision. Some of these experimental observations have been compared to Statistical Models, which adequately reproduced many experimental features. However, other characteristics reported in different experimental studies, such as the saturation of the primary fragments’ average excitation energies as a function of their atomic number, have not been satisfactorily accounted for by the statistical treatment employed in the analysis. Similar conclusions may be drawn from the average excitation energies reported in Ref., which yield the same results for different isotope families, as a function of their mass number. The separation between the average excitation energies associated with different isotope families has not been reproduced by the statistical calculations, which predict a very weak isotopic dependence.

In this work we examine the primary fragments’ temperatures and focus on their isotopic dependence. We employ the version of the Statistical Multifragmentation Model (SMM) presented in Ref., which is built on the recurrence formulas developed in Ref., where the energy is treated as a discrete quantity. In order to distinguish this version from the traditional SMM, we label it SMM-DE, emphasizing the discretization of the energy. This SMM-DE is particularly useful to the present purpose as it provides the energy distribution of the primary fragments, rather than its average value as...
The results are presented and discussed in Sect. III. We conclude in Sect. IV with a brief summary.

II. THEORETICAL FRAMEWORK

In Refs. [36–38], efficient recurrence relations have been developed for the canonical ensemble which impose baryon number and charge conservation in each fragmentation mode. Treating the system energy $E$ as a discrete quantity, in Ref. [30], this formalism has been extended so that $E$ is also kept fixed in each fragmentation mode. This allows its application to the microcanonical ensemble and an implementation based on the SMM has been developed in Ref. [29].

In the framework of the SMM-DE, the total energy is then written as $E = Q\Delta Q$, where $Q$ is an integer number and $\Delta Q$ is the granularity of the discretization. The average fragment multiplicity, with mass and atomic numbers $a$ and $z$, respectively, and energy $q\Delta Q$, is given by [30]

$$\overline{m}_{a,z} = \frac{\omega_{a,z,q}}{\Omega_{A_0, Z_0}} \Omega_{A_0-a, Z_0-z, Q-q} ,$$  \hspace{1cm} (1)

where $A_0$ and $Z_0$ respectively represent the mass and atomic numbers of the decaying source. The quantity $\Omega_{A, Z, Q}$ represents the number of states corresponding to the breakup of a nucleus $(A, Z)$ with energy $Q\Delta Q$. In Ref. [30], it is shown that $\Omega_{A, Z, Q}$ can be calculated through the following recurrence relation:

$$\Omega_{A, Z, Q} = \sum_{\alpha, \rho, q} \frac{a_A}{A} \omega_{\alpha, \rho, q} \Omega_{A_0-a, Z_0-z, Q-q} .$$  \hspace{1cm} (2)

The number of states of a nucleus $(A, Z)$ with energy $Q\Delta Q$ is calculated through

$$\omega_{A, Z, Q} = \gamma_A \int_{0}^{\epsilon_{A, Z, Q}} dK \sqrt{K} \rho(\epsilon_{A, Z, Q} - K) ,$$  \hspace{1cm} (3)

where

$$\gamma_A = \Delta_Q \frac{V_f(2m_n A)^{3/2}}{4\pi^2 \hbar^3} ,$$  \hspace{1cm} (4)

$m_n$ denotes the nucleon mass, $V_f$ is the free volume, and $\epsilon_{A, Z, Q}$ represents the sum of the fragment’s kinetic and excitation energies. The density of states $\rho(\epsilon^*)$ is described in Refs. [29, 39] and is built in such a way that it reproduces the behavior of the standard SMM’s Helmholtz free energy at high temperatures [40] and describes the experimental $\rho(\epsilon^*)$ at low excitation energies [41].

Thus, the above relations allow us to calculate the primary fragment distribution for the breakup of a source at a fixed excitation energy $E$. We will not provide further details on the model formulation in this work and, instead, refer the reader to Refs. [29, 39], where a detailed presentation is made.

III. RESULTS

The breakup of the $^{112}$Sn and $^{112}$Ba nuclei at density equal to $\rho_0/3$, where $\rho_0$ corresponds to its saturation value, is discussed below. We focus on the temperatures
of the primary isotopes, which can be calculated through the standard thermodynamical relation

\[
\frac{1}{T_{a,z,q}} = \frac{\partial \ln[\rho(\epsilon^*)]}{\partial \epsilon^*} \bigg|_{\epsilon^* = \epsilon_{a,z,q}} ,
\]

from which the average value is readily obtained with the help of Eq. (1)

\[
T_{a,z,q} = \frac{\sum_q \rho_{a,z,q} T_{a,z,q}}{\sum_q \rho_{a,z,q}} .
\]

The average excitation energy \(\bar{\epsilon}_{a,z,q}\) is calculated through

\[
\bar{\epsilon}_{a,z,q} = \frac{\gamma_a}{\omega_{a,z,q}} \int_0^{\epsilon_{a,z,q}} dK \left( \epsilon_{a,z,q} - K \right) \sqrt{K} \rho(\epsilon_{a,z,q} - K) .
\]

The average temperatures of different primary isotopes predicted by the SMM-DE as a function of their atomic numbers are shown in Fig. (a) for the breakup of a \(^{112}\)Sn source with excitation energy \(E^*/A = 4\) MeV. The results exhibit a clear \(A\) dependence, which weakens as \(Z\) increases, and the proton rich isotopes are hotter than the neutron rich ones. The average breakup temperature of the source is depicted in this figure by the dashed horizontal line and is obtained from

\[
\frac{1}{T} = \frac{\partial \ln(\Omega_{A_0,z_0,q})}{\partial (Q\Delta Q)} \approx \frac{\ln(\Omega_{A_0,z_0,q}) - \ln(\Omega_{A_0,z_0,q-1})}{\Delta Q} .
\]

One sees that important deviations from the average breakup temperature occur for neutron-deficient and neutron-rich isotopes. In order to examine the sensitivity of the isospin composition of the source, Fig. (b) also displays \(T_{a,z,q}\) for isotopes produced in the breakup of the \(^{112}\)Ba nucleus at the same excitation energy per nucleon and density as the \(^{112}\)Sn nucleus. The same features are once more observed but the magnitude of the effects is amplified.

To understand the behavior of the average temperatures just presented, we show, in Fig. (a), the distribution of the average excitation energy of \(^{11}\)C and \(^{13}\)C isotopes produced in the breakup of a \(^{112}\)Sn nucleus at \(E^*/A = 4\) MeV. Both distributions are qualitatively similar, exhibiting a bell shape with the peak of the heavier isotope occurring at a slightly higher excitation energy value than the one associated with the lighter one. The density of states \(\rho(\epsilon^*)\) for both fragments are also shown in this figure. One sees that it exhibits a larger slope in the case of the \(^{13}\)C isotope in the region where the excitation energy distribution is non-negligible. It thus leads to a smaller temperature value than in the case of the \(^{11}\)C isotope. This explains the isotopic dependence of the fragments’ temperatures observed in Fig. (b).

\[\text{FIG. 2. (Color online) Excitation energy distribution (left scale) and density of states (right scale) of the }^{11}\text{C and }^{13}\text{C isotopes produced in the breakup of the }^{112}\text{Sn nucleus at (a) }E^*/A = 4\text{ MeV and (b) }E^*/A = 10\text{ MeV. For details see the text.}\]

As the excitation energy of the source increases, the fragments’ energy distributions is expected to broaden and shift towards higher excitation energy values. This is indeed observed in Fig. (b) where the energy distributions are shown for \(E^*/A = 10\) MeV. Since the difference between the slopes of the density of states of these carbon isotopes becomes smaller as the energy increases, one should expect the mass dependence of the isotope temperature to weaken as the excitation energy of the source increases. This is indeed observed in Fig. (b) which exhibits the ratio between the temperatures of different carbon isotopes to that of the \(^{8}\)C for the source’s excitation energy \(E^*/A = 4\) and \(10\) MeV. Thus we expect that the isotopic dependence of the temperature reported in this work should become negligible as the excitation energy of the source becomes large.

It is important to emphasize that, in the framework of the SMM-DE, all the primary fragments are produced simultaneously. The decrease of the temperature of a given isotope with the increase of its neutron number is thus explained, in the framework of this model, through the behavior of the fragments’ excitation energy distributions and their densities of states. As a consequence, our results suggest that this feature is not a signature of non-equilibrium process.

\[\text{IV. CONCLUDING REMARKS}\]

We have examined the isotopic dependence of the fragments’ temperature in the framework of the SMM-DE. This version of the model furnishes the excitation en-
FIG. 3. (Color online) Ratio between the temperatures of the Carbon isotopes to that of the $^{12}$C isotope. The source is the $^{112}$Sn at $E'/A = 4$ and 10 MeV. For details see the text.

energy distribution, rather than the average value, of each species. It thus allows one to calculate the internal temperatures of the primary fragments. We found a fairly strong mass dependence of the temperature within each isotope family, neutron poor isotopes being hotter than neutron rich ones. This characteristic has been previously considered to be an indication of the existence of non-equilibrium effects, in particular that the proton rich isotopes were emitted earlier than the neutron rich ones. Since, in the framework of the SMM, all of the primary fragments are produced simultaneously by a thermally equilibrated source, our results offer an alternative interpretation to this effect. We also found that this sensitivity of the temperature to the isospin composition of the fragments becomes weaker as the excitation energy of the source increases, a prediction we consider could be interesting to verify experimentally.

ACKNOWLEDGMENTS

This work was supported in part by the Brazilian agencies Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) and Fundao Carlos Chagas Filho de Amparo Pesquisa do Estado do Rio de Janeiro (FAPERJ), a BBP grant from the latter. We also thank the Programa de Desarrollos de las Ciencias Basicas (PEDECIBA) and the Agencia Nacional de Investigación e Innovación (ANII) for partial financial support.

[1] W. Bauer, J. P. Bondorf, R. Donangelo, R. Elmér, B. Jakobsson, H. Schulz, F. Schussler, and K. Sneppen, Phys. Rev. C 47, R1838 (1993).
[2] S. Hudan, R. T. de Souza, and A. Ono, Phys. Rev. C 73, 054602 (2006).
[3] S. R. Souza and C. Ngo, Phys. Rev. C 48, R2555 (1993).
[4] J. P. Bondorf, A. S. Botvina, I. N. Mishustin, and S. R. Souza, Phys. Rev. Lett. 73, 628 (1994).
[5] J. Aichelin, Phys. Rep. 202, 233 (1991).
[6] L. G. Moretto and G. J. Wozniak, Annu. Rev. Nucl. Part. Sci. 43, 379 (1993).
[7] P. Danielewicz, R. A. Lacey, P.-B. Gossiaux, C. Pinkenburg, P. Chung, J. M. Alexander, and R. L. McGrath, Phys. Rev. Lett. 81, 2438 (1998).
[8] A. Bonasera, F. Guiminnelli, and J. Molitoris, Physics Reports 243, 1 (1994).
[9] A. Ono and H. Horiuchi, Progress in Particle and Nuclear Physics 53, 501 (2004).
[10] B. Borderie and M. F. Rivet, Progress in Particle and Nuclear Physics 61, 551 (2008).
[11] G. F. Burgio, M. Baldo, and A. Rapisarda, Physics Letters B 321, 307 (1994).
[12] R. Donangelo and S. R. Souza, Phys. Rev. C 58, R2659 (1998).
[13] J. Bondorf, D. Idier, and I. Mishustin, Physics Letters B 359, 261 (1995).
[14] C. Dorso and J. Randrup, Physics Letters B 301, 328 (1993).
[15] C. Dorso and J. Aichelin, Physics Letters B 345, 197 (1995).
[16] J. P. Bondorf, A. S. Botvina, A. S. Iljinov, I. N. Mishustin, and K. Sneppen, Phys. Rep. 257, 133 (1995).
[17] D. H. E. Gross, Rep. Prog. Phys. 53, 605 (1990).
[18] C. B. Das, S. Das Gupta, W. G. Lynch, A. Z. Mekjian, and M. B. Tsang, Phys. Rep. 406, 1 (2005).
[19] S. Das Gupta, A. Z. Mekjian, and M. B. Tsang, Adv. Nucl. Phys. 26, 89 (2001).
[20] S. R. Souza, R. Donangelo, M. B. Tsang, and W. G. Lynch, ArXiv e-prints (2017), arXiv:1711.03347 [nucl-th].
[21] S. Piantelli, B. Borderie, E. Bonnet, N. L. Neindre, A. Raduta, M. Rivet, R. Bougault, A. Chibihi, R. Dayras, J. Frankland, E. Galichet, F. Gagnon-Moisan, D. Guinet, P. Lautesse, G. Lehnart, O. Lopez, D. Mercier, J. Moisan, M. Prlog, E. Rosato, R. Roy, B. Tamaï, E. Vient, M. Vigilante, and J. Wieleczko, Nuclear Physics A 809, 111 (2008).
J. B. Natowitz, E. J. Kim, T. Materna, L. Qin, P. K. Sahu, K. J. Schmidt, S. Wiesenschel, and H. Zheng, Phys. Rev. C 89, 021601 (2014).

[24] W. Lin, X. Liu, M. R. D. Rodrigues, S. Kowalski, R. Wada, M. Huang, S. Zhang, Z. Chen, J. Wang, G. Q. Xiao, R. Han, Z. Jin, J. Liu, P. Ren, F. Shi, T. Keutgen, K. Hagel, M. Barbui, C. Bottosso, A. Bonasera, J. B. Natowitz, T. Materna, L. Qin, P. K. Sahu, and H. Zheng, Phys. Rev. C 90, 044603 (2014).

[25] M. R. D. Rodrigues, W. Lin, X. Liu, M. Huang, S. Zhang, Z. Chen, J. Wang, R. Wada, S. Kowalski, T. Keutgen, K. Hagel, M. Barbui, C. Bottosso, A. Bonasera, J. B. Natowitz, T. Materna, L. Qin, P. K. Sahu, and K. J. Schmidt, Phys. Rev. C 88, 034605 (2013).

[26] X. Liu, W. Lin, R. Wada, M. Huang, P. Ren, Z. Chen, J. Wang, G. Xiao, S. Zhang, R. Han, J. Liu, F. Shi, M. Rodrigues, S. Kowalski, T. Keutgen, K. Hagel, M. Barbui, A. Bonasera, J. Natowitz, and H. Zheng, Nuclear Physics A 933, 290 (2015).

[27] A. H. Raduta and A. R. Raduta, Phys. Rev. C 55, 1344 (1997).

[28] A. H. Raduta and A. R. Raduta, Phys. Rev. C 65, 054610 (2002).

[29] S. R. Souza, B. V. Carlson, R. Donangelo, W. G. Lynch, and M. B. Tsang, Phys. Rev. C 88, 014607 (2013).

[30] S. Pratt and S. Das Gupta, Phys. Rev. C 62, 044603 (2000).

[31] J. P. Bondorf, R. Donangelo, I. N. Mishustin, C. Pethick, H. Schulz, and K. Sneppen, Nucl. Phys. A443, 321 (1985).

[32] J. P. Bondorf, R. Donangelo, I. N. Mishustin, and H. Schulz, Nucl. Phys. A444, 460 (1985).

[33] K. Sneppen, Nucl. Phys. A470, 213 (1987).

[34] T. X. Liu, W. G. Lynch, M. J. van Goethem, X. D. Liu, R. Shomin, W. P. Tan, M. B. Tsang, G. Verde, A. Wagner, H. F. Xi, H. S. Xu, W. A. Friedman, S. R. Souza, R. Donangelo, L. Beaulieu, B. Davin, Y. Larochelle, T. Lefort, R. T. de Souza, R. Yanez, V. E. Viola, R. J. Charity, and L. G. Sobotka, Europhys. Lett. 74, 806 (2006).

[35] W. A. Friedman and W. G. Lynch, Phys. Rev. C 28, 16 (1983).

[36] K. C. Chase and A. Z. Mekjian, Phys. Rev. C 52, R2339 (1995).

[37] S. Das Gupta and A. Z. Mekjian, Phys. Rev. C 57, 1361 (1998).

[38] P. Bhattacharyya, S. Das Gupta, and A. Z. Mekjian, Phys. Rev. C 60, 054616 (1999).

[39] S. R. Souza, B. V. Carlson, R. Donangelo, W. G. Lynch, and M. B. Tsang, Phys. Rev. C 92, 024612 (2015).

[40] W. P. Tan, S. R. Souza, R. J. Charity, R. Donangelo, W. G. Lynch, and M. B. Tsang, Phys. Rev. C 68, 034609 (2003).

[41] A. Gilbert and A. G. W. Cameron, Canadian Journal of Physics 43, 1446 (1965). http://dx.doi.org/10.1139/p65-139.