Calculation of the foundation with allowance for the shear modulus at small deformations

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Abstract. Continuous improvement of mathematical models of soil and software complexes implementing numerical simulation methods allows taking into account the effects characteristic of the ground environment, such as nonlinearity of deformations, creep, hardening and softening. These models require a large number of parameters to be determined during laboratory tests. The methods for determining these parameters are for the most part not represented in modern normative literature. A typical example of such parameters is the shear modulus for small deformations.

The study of small deformations is of great importance in the calculation of underground structures. When designing such structures it is necessary to take into account the nonlinearity of unloading of the ground base. The currently widely used model of hardening soil [HS] does not take into account the nonlinearity of the unloading, while the improved model of hardening soil in view of small deformations [HSsmall] allows to take into account the hysteretic behavior of soils.

The account of the hysteretic behavior of the soil during the cycles of unloading and repeated loading is carried out under the condition that the shear modulus at unloading is equal to the initial tangent module for the initial loading curve. In this case, the unloading and reloading curve has the same shape as the initial loading curve, but in the doubled size.

Since elastoplastic soil models are not able to describe the hysteretic behavior of soils, the solution of the range of problems associated with the accumulation of deformations under vibrational and cyclic influences with the use of these models is impossible. In such cases, the forecasting of the stressed-deformed state of soil bases under the action of dynamic loads of a different nature can be performed only on the basis of the use of an elastoplastic model of a toughening ground with allowance for ultra-small deformations.

In the present work, the results of laboratory determination of soil stiffness for small deformations are presented and the use of these data in numerical modelling of the problem of sediments of a foundation of finite width on a base in the PLAXIS software complex.

1. Introduction

Hardening soil model for small deformations [HSsmall] it is based on the elastic-plastic model hardening soil [HS], and uses almost the same set of input parameters of soils. In fact, to describe the change in stiffness from the amount of deformation, only two additional parameters are required [1-6]:

- initial shear modulus or shear modulus for small deformations \(G_0\);
- the level of shear deformation \(\gamma_{0.7}\), at which the secant shear modulus \(G_s\) decreases to approximately 70% of the value of \(G_0\).
These additional parameters, like the basic parameters of the HS model [7-8], can be determined from the results of soil tests in a triaxial device (Fig. 1). At the same time, to increase the accuracy of measurements of vertical and horizontal deformations (with an accuracy of $10^{-6} [-]$), the device should be equipped with measurement sensors on a local base (Fig. 2).

**Figure 1.** Equipment for triaxial testing of soils

**Figure 2.** Sample sand to conduct triaxial tests to established local deformation sensors and measurement radius
2. Laboratory research

2.1. Test device

A servo-hydraulic load frame with a maximum axial force of 63 kN is used to perform the laboratory tests. A triaxial compression chamber of the "A" type is used. The unit includes a servo-hydraulic drive control unit, a processing unit for data from pressure sensors, forces and displacements, and a pneumatic pressure control unit. The test is controlled by a personal computer, through which the initial data for the test are given and information on the progress of the test and its results is displayed [9-12].

Measuring channels of the device include monitoring the following parameters:

- vertical axial force (force sensor 10 kN)
- vertical displacement of the stamp (LVDT-sensor) - measuring range of movement: 0-25 mm, measuring accuracy: 10^{-3} mm);
- vertical movement on the local sample base (two LVDT-sensors, which are fixed diametrically opposite on the sample at a distance of half its height - the range of displacement measurements: 0-10 mm, the strain measurement accuracy is 10^{-5} mm);
- circumferential length measuring sample at half its height (LVDT-sensor is attached to a chain - measuring range 0-10 mm, 10^{-5} mm strain measurement accuracy);
- liquid pressure in the working chamber (electronic control unit for pneumatic pressure - measuring range: 0-1.6 MPa, measuring accuracy: 10^{-5} MPa);
- the pore fluid pressure in the soil sample (the pressure sensor installed in the down stamp)
- back pressure applied to the soil sample (controlled via the electronic pneumatic pressure control unit).

The purpose of laboratory studies of soils is to construct the dependence of the shear stresses $\tau$ [kPa] on the shear strains $\gamma$ [-] (Fig. 3). In the triaxial regime, after preliminary consolidation of the samples at natural pressure, a stage of cyclic loading with various amplitudes of shear stresses is carried out [13-15]. The results obtained make it possible to establish the dependence of the change in the shear modulus for different values of shear deformations (Fig. 4).

2.2. Analysis of test results

Determination of vertical deformation based on the results of triaxial tests is carried out in accordance with the following equation:

$$\varepsilon_i = \frac{u_i}{u_0},$$  \hspace{1cm} (1)

where $u_i$ is the current reading of the displacement sensor on the local base; $u_0$ is the value of the initial local measurement base; $\varepsilon_i$ - current axial deformation.

Measurement of the shear modulus under small deformations was performed at the initial stage of the shear stress-shear strain diagram [16]. The range for determining the module is taken from the condition that more than five points of the shear strain versus shear stresses must be on the same straight line. The shear modulus for ultra-small deformations was calculated using the following formula

$$G_0 = \frac{\Delta \tau}{\Delta \gamma},$$  \hspace{1cm} (2)

where $\Delta \tau$ is the change in shear stresses; $\Delta \gamma$ is the change in the shear strain. In this case, $G_0 = G_0^{\text{ref}}$ when tested in a three-axis mode with a hydrostatic pressure corresponding to the depth of the soil sample in the array (reference).

These calculations were carried out at each stage of cyclic loading for different values of the amplitude of the vertical stress [17].
Using the LVDT sensor to change the circumference of the sample allows one to determine the radial deformations, and hence the volume deformations, which are determined in accordance with the following formula:

\[
\varepsilon_v = 1 - \left( \frac{d_i}{d_0} \right)^2 \left( 1 - \varepsilon_1 \right),
\]

where \(d_i\) is the current diameter; \(d_0\) is the initial diameter; \(\varepsilon_v\) and \(\varepsilon_1\) are the current volume and axial strains, respectively.

![Figure 3. The graph of the dependence of shear strain of \(\gamma\) [-] on the shear stresses \(\tau\) [kPa]](image1)

![Figure 4. A plot of the shear modulus \(G/G_0\) [-] of the shear strain \(\gamma\) [-] (logarithmic scale)](image2)

The measurement of volume deformations is necessary to calculate the small shear strains \(\gamma_0\) by the following formula

\[
\gamma_0 = \varepsilon_1 - \varepsilon_3,
\]
where $\varepsilon_3$ is the horizontal deformation determined in accordance with formula

$$
\varepsilon_3 = \frac{\varepsilon_1 - \varepsilon_2}{2}.
$$

(5)

The value of $\gamma_{0.7}$ is determined from the results of tests on the graph of the dependence of the change in the normalized shear modulus $G/G_0$ on the shear strains $\gamma$ when the condition [18-20]:

$$
G_x = 0.722G_0.
$$

(6)

3. Numerical simulation

3.1. Formulation of the problem

The experimental values of the shear modulus at small deformations $G_0$ and the level of shear strain $\gamma_{0.7}$ were used to solve the classical problem of the foundation sedimentation on the ground. The problem is solved by the finite element method in the PLAXIS 2D 2017.1 software. The formulation of the problem is axisymmetric (Fig. 5). Dimensions of the model: width 5.0 m, depth 5.0 m. A uniformly distributed vertical load of 0.1 MPa is transmitted through a rigid stamp. Soils are represented by one kind - sand. Soil model HSsmall [21-22].

The purpose of the numerical simulation is to evaluate the correctness of additional parameters of the HSsmall model obtained during the laboratory tests when solving geotechnical problems. The accuracy of the model, taking into account the rigidity of the soil at low strain values under load and repeated loading, and also the nonlinear dependence on the strain amplitude is estimated.

![Figure 5](image)

Figure 5. Geomechanical model with a mesh of finite elements

The soil parameters used to solve the present problem are presented in Table 1 and Table 2
### Table 1. Sandy soil parameters (model HS)

|                | Specific gravity of the soil [kN/m³] | Secant modulus [kN/m²] | Tangential odometric module with primary loading [kN/m²] | Stiffness during unloading / reloading [kN/m²] | Angle of internal friction [°] | Cohesion [kN/m²] |
|----------------|-------------------------------------|------------------------|--------------------------------------------------------|---------------------------------------------|-----------------------------|------------------|
| **Sand**       | 20                                  | 3.1*10⁴                | 3.601*10⁴                                             | 1.108*10⁴                                   | 28                          | 5                |

### Table 2. Sandy soil parameters (model HSsmall).

|                | Shear modulus for small deformations [kN/m²] | The exponent for the dependence of rigidity on the stress level [-] | Poisson's ratio [-] |
|----------------|----------------------------------------------|---------------------------------------------------------------|-------------------|
| **Sand**       | 1.0*10⁵                                      | 1.5*10⁻⁴                                                   | 0.5               | 0.2               |

3.2. **Results of numerical simulation**

As a result of the foundation calculations, isopoles of vertical displacements were obtained (Figure 6). In total, three cycles of "load-unloading" were performed. The graph of the dependence of the vertical displacements on the loading time is shown in Fig. 7.

![Figure 6](image-url) [Isolines vertical displacements at the last stage of application of the vertical load of 0.1 MPa]
4. Conclusion
The features of the equipment presented in this paper allow performing soil tests to determine the parameters of the HSsmall model. The present work contains a technique allowing, by applying additional equipment for standard triaxial devices, to obtain parameters for modern ground models (HS and HSsmall).

The solution of the problem of settling the foundation on sand with allowance for the shear modulus at small deformations shows that residual displacements appear during the realization of the cycles of unloading and re-loading. There is an accumulation of deformations, which is also observed in real conditions. The results of numerical simulation show that at different depths from the loaded foundation the nature of the accumulation of residual displacements is different. The closer to the base of the foundation, the more residual movements.

The solutions used in the present work allow geotechnical engineer specializing in numerical modeling to apply the soil properties obtained in accordance with the results of laboratory tests in calculations of the bases of buildings and structures.

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