Influence of the Mass and Line Stiffness on the Dynamic Line Tension of a Floating Offshore Wind Turbine Stabilized by a Suspended Counterweight

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Abstract. As the offshore wind industry moves towards deeper waters, developers are adapting classical floating platform designs to optimize performance and cost. One such concept is a multi-body system consisting of a counterweight suspended from a buoyant hull using flexible cables. If the suspension lines remain in tension, the system may be assumed to behave as a rigid body. However, if a suspension line loses tension, relative motion between the bodies will occur which may destabilize the system. In this work, the hanging-mass floating offshore wind turbine is idealized as a forced spring pendulum. A parametric study is conducted by varying the line stiffness and counterweight mass to determine their effect on the dynamic line tension and global response of the system.

1. Introduction

Over the past decade, the global offshore wind industry has matured to a point where renewable energy is increasingly economically competitive with traditional electricity generation methods [1]. European developers have taken advantage of relatively shallow seas [2] to evolve into world leaders in offshore wind energy. Shallow seas allow for the use of fixed-base wind turbines utilizing support structures that are embedded directly in the sea floor. As the water depth increases, the required support structure becomes too large to be economical. Thus, floating platforms have been developed to access the wind resources located where the water depth exceeds 50 meters [3]. This is the case in the United States, where the majority of coastal seas are too deep for fixed-base turbines [4].

Currently, the design of floating offshore wind turbines (FOWTs) is centered around three classical concepts adopted from the oil and gas industry; the spar, the tension leg platform (TLP), and the semi-submersible. Today examples of semi-submersible FOWTs include Maine Aqua Ventus’s VolturnUS [5] and Principle Power’s WindFloat [6] while Equinor’s Hywind [7] is an example of a spar FOWT. To further reduce the levelized cost of energy (LCOE) hybrid designs that utilize the strengths of these three classical designs are being developed. One such concept is a multi-body system consisting of a buoyant hull stabilized by a counterweight suspended by a system of cables. In theory, the counterweight would either be towed alongside or beneath the hull and subsequently lowered during installation to a design depth for operation. Thus, this “hanging-mass” concept combines the low center of gravity attributed to spars with the low tow-out draft seen in semi-submersibles resulting in a system with favorable stability.
and construction characteristics. FOWTs based on this concept, which including the Stiesdal Offshore Technologies A/S TetraSpar [8] and the Saipem Hexafloat [9], are scheduled for prototype testing in the near future. A simple general arrangement of this concept is provided in Figure 1(a).

In addition to performance benefits, many believe that hanging-mass FOWTs will be cheaper to build than other designs due to the relatively little material required to achieve a low center of gravity. However, these suggested benefits are achievable only if one assumes there is negligible relative motion between the hull and the counterweight (i.e. the system behaves as a rigid body). This assumption requires that suspension lines remain in tension at all times. The suspension lines cannot support compressive loads and will therefore become slack when tension is lost. The subsequent re-tensioning of the lines can lead to large snap loads which may cause damage to the lines and are potentially destabilizing to the entire system. The governing equations for this system are difficult to linearize with out sacrificing valuable information. Thus, phenomena associated with nonlinear vibration problems such as multiple equilibrium points, instability, nonlinear resonance, and chaos, are all scenarios that need to be examined before the system is assumed to act as a rigid-body.

The nonlinear response of offshore mooring systems has been extensively studied, particularly in TLP tendons [10; 11] and top-tensioned risers [12–14]. These studies have shown that the response can be described by Mathieu’s equation, which takes the form

\[ \ddot{x} + (a + b \cos t) x = 0, \]  

where \( x \) is the degree of freedom of interest, \( t \) is time, and \( a \) and \( b \) are system parameters. The harmonic restoring force in equation 1 leads to unstable regions that are a function of \( a \) and \( b \) which must be accounted for during the design process. However, these studies involve the motion of a single body moored to a stationary reference frame. For the problem discussed in this paper the response of two elastically coupled bodies must be assessed.

Much of the current literature on elastically coupled bodies operating in marine environment focuses on marine lifting applications. The problem of payload suspended from a shipboard crane has been studied by many authors with varying degrees of complexity. Early studies [15] of the maximum line tensions in mooring lines or crane lines were studied numerically using lumped mass models. The cables were excited using base excitation derived from the predetermined motion of the ship or platform. The motions of a payload supported by a shipboard crane were studied by [16] where the cable and payload was assumed to act as a rigid pendulum undergoing base excitation. The nonlinear equations of motion were solved analytically to determine the effects of damping and lateral displacements of the boom tip on dynamic instability. Parametric resonance in ship-based crane cables was further studied by [17] where a singular cable was modeled as a pinned-pinned beam using large deformation beam theory. The cable was subjected to fluid forces from the ocean current in addition to base excitation in the form of a time-varying axial tension. Parametric resonance was predicted using the solution to the Mathieu equation leading to the conclusion that both dynamic tension variation and payload can both cause dynamic instability of the system, although the addition of damping bounds these extreme motions. These studies provide a wealth of knowledge into the dynamics of suspended masses where undergoing base excitation. The use of base excitations assumes that the motion of the payload does not influence the motion of the support structure [16]. In other words, there is one-way coupling between the two bodies. This assumption is valid for the previous examples since the mass of the payload is assumed to be orders of magnitude less than the supporting structure. In the case of a hanging-mass FOWT, the mass of the counterweight is on the same order of magnitude as the hull, meaning that this assumption is not valid for the application presented in this work. Thus, the motion of the counterweight cannot be decoupled from the motion of the hull, adding another degree of complexity to the problem.
Thus far, there has been relatively little research published on hanging-mass FOWTs. Previous studies have shown that there is a static pitch limit where a suspension line will become slack [8] which is function of geometry alone and easily calculated analytically. However, dynamic variations in line tension may reduce this maximum pitch limit and should be thoroughly investigated. This study aims to supplement the existing literature by evaluating the response of elastically-coupled bodies with masses on the same order of magnitude in regards to a hanging-mass FOWT. As a first step, the FOWT will be modeled as a forced, planar, spring-pendulum with three degrees of freedom. The effects of line stiffness and counterweight mass on the global performance of the system and on the rigid-body assumption will be evaluated. In the future, these results will be used to develop a more thorough investigation into the physics of a hanging-mass FOWT using experiments and multi-physics numerical tools.

2. Methodology
The hull is modeled as a floating mass, \( m_{Hull} \), with inertia, \( I_{Hull} \), and is free to translate along the \( z \)-axis (heave) and rotate about the \( y \)-axis (pitch) as shown in Figure 1(b). The displacements in these degrees of freedom are denoted \( \eta_3 \) (positive upwards) and \( \eta_5 \) (positive clockwise) in accordance with naval architecture nomenclature. The counterweight is idealized as a point mass, \( m_{CW} \), and is attached to the hull via a single tension-only spring. There is assumed to be no bending/transverse displacement in the spring meaning there is only an axial displacement, \( r \), and the counterweight rotates through the same angular displacement as the hull. Furthermore, the spring is assumed to have an unstretched length denoted as \( l \). The instantaneous positions of the hull and counterweight in the \( xz \)-plane are given by

\[
\begin{align*}
    x_{Hull} &= 0, \\
    z_{Hull} &= \eta_3, \\
    x_{CW} &= -(l + r) \sin \eta_5, \\
    z_{CW} &= \eta_3 - (l + r) \cos \eta_5.
\end{align*}
\]
Thus, the kinetic energy in the system is
\[ E_K = \frac{1}{2} m_{Hull} \dot{\eta}_3^2 + \frac{1}{2} I_{Hull} \dot{\eta}_5^2 + \cdots + \frac{1}{2} m_{CW} \left[ \dot{\eta}_3^2 + (l + r)^2 \dot{\eta}_5^2 + r^2 + 2 (l + r) \dot{\eta}_3 \dot{\eta}_5 \sin \eta_5 - 2 \dot{\eta}_3 r \cos \eta_5 \right] \] (6)
and the potential energy is
\[ E_P = \frac{1}{2} \left( k_1 \eta_3^2 + k_3 r^2 \right) + k_2 \cos \eta_5 - T_0 (l + r) \cos \eta_5 \] (7)
where \( k_1 \) and \( k_2 \) are the hydrostatic heave and pitch stiffnesses of the hull, respectively, and \( k_3 \) is the effective vertical stiffness of the system of suspension lines. The suspension line is modeled as a tension only spring meaning
\[ k_3 = \begin{cases} 0 & r \leq 0 \\ \frac{n E A}{l} \cos^2 \psi_z & r > 0 \end{cases} \] (8)
where \( n \) is the number of lines, \( E A \) is the axial stiffness of a line, and \( \psi_z \) is the angle a line makes with the positive \( z \)-axis as shown in Figure 1(a). Furthermore, \( T_0 \) is the pretension in the spring and defined as
\[ T_0 = m_{CW} g - \rho_w g \nabla_{CW} \] (9)
where \( \rho_w \) is the water density and \( \nabla_{CW} \) is the displaced volume of the counterweight. In accordance with Archimedes principle, the weight of the total displaced water must equal the total weight for the system to float. Consequently, the sum of the masses and the sum of the displaced volumes cancel each other out and are not included equation 7.

Defining three generalized coordinates as \( q_1 = \eta_3 \), \( q_2 = \eta_5 \), and \( q_3 = r \) the equations of motion can be derived using the Euler-Lagrange equations
\[ \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = Q_i \quad i = 1, 2, 3 \] (10)
where \( L = E_K - E_P \) is Lagrangian function and \( Q_i \) is the generalized force associated with the generalized coordinate \( q_i \) including hydrodynamic and dissipative forces. The counterweight is assumed to be well below the free-surface, meaning the wave forces significantly smaller than those on the hull due to the exponential decay of wave pressure with depth. Thus, only the hull experiences external wave forces. Furthermore, damping is assumed to be linear. Thus, generalized forces are defined as
\[ Q_1 = F_1 \sin (\omega t - \phi_1) - A_{11} \dot{q}_1 - A_{12} \ddot{q}_2 - A_{33} \left[ \ddot{q}_1 + (l + q_3) \dddot{q}_2 \sin q_2 - \dddot{q}_3 \cos q_2 \right] - c_1 \dot{q}_1 \] (11)
\[ Q_2 = F_2 \sin (\omega t - \phi_2) - A_{12} \ddot{q}_1 - A_{22} \ddot{q}_2 - A_{33} \left[ \dddot{q}_2 \sin q_2 + (l + q_3) \dddot{q}_2 \right] - c_2 \dot{q}_2 \] (12)
\[ Q_3 = -A_{33} \left( \dddot{q}_3 - \dddot{q}_1 \cos q_2 - c_3 \dddot{q}_3 \right) \] (13)
where \( F_i \) and \( \phi_i \) are the magnitude and phase of the wave induced force/moment; \( A_{ij} \) is the added mass in the \( i \)th degree of freedom due to acceleration in the \( j \)th degree of freedom, \( c_i \) is the linear damping coefficient in the \( i \)th degree of freedom, and \( \omega \) is the wave frequency. Taking \( M_1 = m_{Hull} + A_{11} \), \( M_2 = I_{Hull} + A_{22} \), and \( M_3 = m_{CW} + A_{33} \), the equations of motion are
\[ (M_1 + M_3) \ddot{q}_1 + \left[ A_{12} + M_3 (l + q_3) \sin q_2 \right] \dddot{q}_2 = M_3 \dddot{q}_3 \cos q_2 \] (14)
\[ = F_1 \sin (\omega t - \phi_1) - M_3 \left[ 2 \dddot{q}_3 \sin q_2 + (l + q_3) \dddot{q}_2 \right] - c_1 \dot{q}_1 - k_1 q_1, \]
\[ \left[ A_{12} + M_3 (l + q_3) \sin q_2 \right] \ddot{q}_1 + \left[ M_2 + M_3 (l + q_3)^2 \right] \dddot{q}_2 \] (15)
\[ = F_2 \sin (\omega t - \phi_2) - 2M_3 (l + q_3) \dddot{q}_3 - T_0 (l + q_3) \sin q_2 - c_2 \dot{q}_2 - k_2 \sin q_2, \]
\[ -M_3 \dddot{q}_1 \cos q_2 + M_3 \dddot{q}_3 = (l + q_3) \dddot{q}_2 - T_0 \cos q_2 - c_3 \dddot{q}_3 - k_3 q_3. \] (16)
Next, non-dimensional coordinates $\ddot{q}_1 = q_1/L_0$, $\ddot{q}_2 = q_2$, and $\ddot{q}_3 = q_3/L_0$ and time $\hat{t} = \omega_1 t$ are introduced. Here, $L_0 = l + q_2(0)$ is a characteristic length defined as the static distance between the hull and counterweight centers of gravity. This distance is effectively the length of the pendulum. The characteristic frequency $\omega_1$ is chosen as the linear heave frequency and will be defined later. The non-dimensional equations of motion are

$$\ddot{q}_1 + [\alpha_{12} + \mu (\lambda + \ddot{q}_3)] \ddot{q}_2 - \mu \ddot{q}_3 \cos \dot{q}_2 = \hat{F}_1 \sin (\hat{t} - \phi_1) - \mu \left[2\ddot{q}_2 \dot{q}_3 \sin \ddot{q}_2 + (\lambda + \ddot{q}_3) \ddot{q}_2^2\right] - \left[2\zeta_1 \dot{q}_1 + \dot{q}_1\right],$$

$$[\alpha_{12} + \mu (\lambda + \ddot{q}_3)] \ddot{q}_1 + \left[\nu + \mu (\lambda + \ddot{q}_3)^2\right] \ddot{q}_2 = \hat{F}_2 \sin (\hat{t} - \phi_2) - 2\mu (\lambda + \ddot{q}_3) \ddot{q}_2 \dot{q}_3 + \hat{T}_0 (\lambda + \ddot{q}_3 - 1) \sin \ddot{q}_2$$

$$- (\nu + \mu) \left(\frac{\omega_2}{\omega_1}\right) \left[2\dot{q}_2 \dot{q}_3 + \left(\frac{\omega_2}{\omega_1}\right) \sin \ddot{q}_2\right],$$

$$-\mu \ddot{q}_1 \cos \ddot{q}_2 + \mu \ddot{q}_3 = \hat{T}_0 \cos \ddot{q}_2 - \mu (\lambda + \ddot{q}_3) \ddot{q}_2^2 - \mu \left(\frac{\omega_3}{\omega_1}\right) \left[2\dot{q}_3 \dot{q}_2 + \left(\frac{\omega_3}{\omega_1}\right) \ddot{q}_3\right],$$

where $\alpha_{12} = A_{12}/(M_1 + M_3) L_0$, $\lambda = l/L_0$, $\mu = M_3/(M_1 + M_3)$, and $\nu = M_2/(M_1 + M_3) L_0^2$. The non-dimensional force and moment amplitudes are defined as $\hat{F}_1 = F_1 \omega^{-2}/(M_1 + M_3) L_0$ and $\hat{F}_2 = F_2 \omega^{-2}/(M_1 + M_3) L_0^2$, respectively, and the pretension is normalized as $\hat{T}_0 = T_0 \omega^{-2}/(M_1 + M_3) L_0$. The linear frequencies for each degree of freedom are defined as

$$\omega_1 = \sqrt{\frac{k_1}{M_1 + M_3}},$$

$$\omega_2 = \sqrt{\frac{k_2 + T_0 L_0}{M_2 + M_3 L_0^2}}$$

$$\omega_3 = \sqrt{\frac{k_3}{M_3}}.$$

Finally, $\zeta_i$ is the damping ratio in the $i$th coordinate. Equations 17–19 can now be written as a system of first order differential equations and solved numerically.

3. Parametric Study

The non-dimensional equations can be readily solved using a numerical method such as a variable-step, variable-order solver (e.g., ode15s in MATLAB) to evaluate the response for a variety of parameters. This solver is well-suited for stiff problems as opposed to more common solvers such as those employing Runge-Kutta methods.

This particular study will investigate the influence of counterweight mass and line stiffness on the response of a conceptual hanging-mass FOWT in the early stages of design. The system was initially sized using hydrostatic principles to obtain global properties such as mass and hydrostatic stiffness. The added mass and wave force magnitudes acting on the hull were determined using the potential flow solver ANSYS AQWA and are assumed to be independent of the pretension and line stiffness. For simplicity, the counterweight is assumed to be a homogeneous sphere appropriately sized to displace the correct amount of water for the system to be at equilibrium. The total system mass is held constant throughout the study while appropriately allocating mass between the hull and counterweight to obtain the desired mass ratio. Furthermore, the infinite frequency limit of the added mass is used in all cases. Finally, the pendulum length $L_0$ is held constant throughout the study. The non-dimensional values of
Table 1. Non-dimensional parameter values held constant during the parametric study of the hanging-mass FOWT.

| Parameter                                      | Non-dimensional Form                           | Value   |
|------------------------------------------------|-----------------------------------------------|---------|
| Total System Mass                              | \( (M_{\text{Hull}} + M_{\text{CW}})/(M_1 + M_3) \) | 0.460   |
| Hull Moment of Inertia                         | \( I_{\text{Hull}}/(M_1 + M_3)L_0^2 \)        | 0.046   |
| Displaced Volume                               | \( \nabla/L_0^3 \)                             | 0.027   |
| Hull Heave Stiffness                           | \( k_1/(M_1 + M_3)\omega_1^2 \)               | 0.950   |
| Hull Pitch Stiffness                           | \( k_2/(M_1 + M_3)L_0^2\omega_1^2 \)           | -0.385  |
| Hull Heave Added Mass                          | \( A_{11}/(M_1 + M_3) \)                      | 0.490   |
| Hull Heave-Pitch Added Mass                    | \( A_{11}/(M_1 + M_3)L_0 \)                    | -0.003  |
| Hull Pitch Added Mass                          | \( A_{2}/(M_1 + M_3)L_0^2 \)                   | 8.847 \times 10^{-5} |
| Counterweight Added Mass                       | \( A_{33}/(M_1 + M_3) \)                      | 0.050   |

Figure 2. (a) Non-dimensional heave force and (b) and non-dimensional pitch moment per unit wave amplitude.

the parameters that are held constant throughout the study are provided in Table 1. Note that the hull’s pitch stiffness is negative indicating that hull alone is unstable at the design draft.

As previously mentioned, the mass of each body is on the same order of magnitude, so the motions cannot be decoupled from each other. Therefore, the hull is subjected to forces induced by an irregular wave generated by a white noise spectrum as opposed to user-defined base-excitation. The non-dimensional wave force amplitudes per unit wave amplitude are provided in Figure 2. The wave spectrum and a 40 second sample of the wave elevation are given in Figure 3. In order to account for viscous damping effects, 5% of critical damping is assumed for each degree of freedom. Following the simulation, a response amplitude operator (RAO) for each degree of freedom was generated using Welch’s averaged periodogram method [18] using the wave elevation as an input and the response of each degree of as output. Keeping in mind that there are multiple nonlinearities in the equations of motion, the RAO is only valid for the white
Figure 3. (a) The white noise spectrum $S(\omega)$ used to generate the irregular wave time-series and (b) a 40 second sample of the wave elevation $\xi(t)$.

Table 2. Peak response values for $\mu = 0.23$ and varying line stiffness.

| Line Stiffness ($\hat{k}_3$) | Heave ($\hat{q}_1$) | Pitch ($\hat{q}_2$) | Line Tension ($\hat{T}$) |
|-----------------------------|--------------------|--------------------|--------------------------|
| 5.98                        | 0.0219             | 0.2026             | 0.1187                   |
| 11.96                       | 0.0218             | 0.2082             | 0.0499                   |
| 59.79                       | 0.0217             | 0.2114             | 0.0295                   |

noise spectrum defined earlier. However, this is suitable for gaining a qualitative understanding of the influence the mass and stiffness have on system performance.

The first set of results was computed whilst holding the mass of counterweight constant at a value of $\mu = 0.23$, which corresponds to 50% of the total structural mass. The non-dimensional line stiffness, defined as

$$\hat{k}_3 = \frac{k_3}{(M_1 + M_3) \omega_1^2},$$

was varied between 5.98 and 59.8. These values correspond to unstretched lengths $l$ that are between 90% and 99% of the pendulum length $L_0$. To put these values in perspective, the lower effective stiffness values correspond to soft synthetics, such as polyester, and higher values correspond to steel chain. Intermediate values include other synthetics, such as HMPE, and wire rope. The RAO’s for heave, pitch, and tension are shown in Figure 4 and the peak responses are summarized in Table 2. Here, the wave frequency, $\omega$, has been normalized by the linear heave frequency $\omega_1$, as defined in Equation 20, since the net mass $M_1 + M_3$ and the heave stiffness $k_1$, remain constant throughout the study.

Several observations can be made from the results shown in Figure 4. First, as seen in Figure 4(a), the heave natural frequency does not appear to be heavily influenced by variation in line stiffness. The peak heave amplitude occurs at the linear heave natural frequency, $\omega_1$,
Figure 4. RAO for (a) heave, (b) pitch, and (c) line tension for varying line stiffness $\hat{k}_3$ for a white noise spectrum with $S(\omega) = 1$ and $\mu = 0.23$. The wave frequency $\omega$ has been normalized by the rigid-body heave frequency $\omega_1$.

and sees a negligible change in amplitude as the stiffness is varied. Further observation show that a secondary peak is located at a frequency greater than the linear line frequency, $\omega_3$, which decreases in amplitude and moves toward higher frequencies as the stiffness is increased. This is hypothesized to be due to a phase shift between the responses as the system becomes effectively rigid. There also appears to be a smaller peak around twice the linear heave frequency, indicating that a harmonic may be excited which could lead to parametric resonance.

Moving on to the pitch degree of freedom, the amplitude or natural frequency does not appear to be affected by variation in line stiffness. This is expected as the line stiffness does not appear in Equation 21. However, the pitch response at primary resonance is very high which is likely an artifact modeling assumptions. Currently, the pitch damping is applied at the center of rotation where, in reality, the majority of the viscous damping comes from drag on the counterweight. Thus, in future work, the pitch damping should be adjusted to account for distance between the counterweight and the center of rotation. Moreover, there appears to be no peak at the line extension frequency, $\omega_3$, as there was in the heave response.

Figure 4(c) shows that there are two primary peaks in the tension response. The first peak occurs at the linear line extension frequency, $\omega_3$, and the second occurs at the same frequency as the second peak in the heave response. For softer lines, this secondary peak is near $\omega_3$ causing the two responses to coalesce into a single large amplitude peak. This higher-frequency response decreases in amplitude and increases in frequency as the line becomes more stiff, eventually moving to zero as the system becomes effectively rigid. As with the heave degree of freedom, there is a peak near twice the rigid-body heave frequency, which should be investigated further.

Next, the effect of varying counterweight mass was assessed by generating the RAO’s with a counterweight mass weighing 40%, and 50%, and 60% of the total structural mass. The line stiffness was kept at a constant value of $\hat{k}_3 = 11.96$. The RAO’s are provided in Figure 5 and the peak values are summarized in Table 3. Similar observations are observed in the heave response when mass is varied compared to when stiffness is varied. As seen in figure 5(a), the peak value of the heave response occurs at the linear heave frequency, with secondary peaks occurring at
Figure 5. RAO for (a) heave, (b) pitch, and (c) line tension for varying counterweight mass $\mu$ for a white noise spectrum with $S(\omega) = 1$ and $k_3 = 11.96$. The wave frequency $\omega$ has been normalized by the rigid-body heave frequency $\omega_1$.

Table 3. Peak response values for $k_3 = 11.96$ and varying counterweight mass

| Counterweight Mass ($\mu$) | Heave ($\hat{q}_1$) | Pitch ($\hat{q}_2$) | Line Tension ($\hat{T}$) |
|---------------------------|--------------------|--------------------|--------------------------|
| 0.18                      | 0.0218             | 0.0008             | 0.0417                   |
| 0.23                      | 0.0218             | 0.2082             | 0.0499                   |
| 0.28                      | 0.0218             | 0.2708             | 0.0599                   |

twice that frequency and at a frequency greater than the linear extension frequency. There is a noticeable change in the pitch response in both magnitude and frequency. For $\mu = 0.18$ the linear pitch frequency, $\omega_2$, is outside of the range of wave frequencies and was not excited in this analysis. For the remaining two cases, the frequency decreases as the mass increases, while the inverse is true with regards to the amplitude. This is expected due to the presence of the counterweight mass in Equation 21. Finally, like heave, the tension response is similar to when the stiffness was varied with the peak located at the extension frequency, $\omega_3$ and a secondary high-frequency response.

4. Conclusions and Future Work
In this paper, a hanging-mass FOWT was idealized as a forced spring pendulum. A parametric study was conducted by keeping the hull properties constant and varying the counterweight mass and line stiffness. RAO’s were generated by numerically solving the equations of motion with the system subjected to an irregular wave train generated using a white-noise spectrum.

The peak heave response is unaffected by both counterweight mass and line stiffness. There is a secondary high-frequency response which can be extreme for softer line stiffnesses. The pitch response is largely unaffected by changes in line stiffness but changes in both magnitude
and frequency with changes in the mass ratio. An increase in counterweight mass will lower the pitch natural frequency and increase the amplitude. As expected, the most marked affect of mass and stiffness variation is seen in the line tension. Two peaks are present; one at the linear line frequency, $\omega_3$, and another high-frequency response. For soft lines this high-frequency response may coalesce with the primary response resulting in a single high-amplitude peak. Furthermore, there appears to be another response at approximately twice the rigid-body heave frequency in both heave and line tension. Further work is required to determine the cause of this phenomenon and explore the implications.

The effects of damping also require further investigation. The peak of the pitch response appears to be unrealistically high, indicating that more damping may be required to accurately model the system. This can be attributed to applying the pitch damping force at the center of rotation. Future work should determine if the inclusion of nonlinear hydrodynamic damping applied at the counterweight is required to achieve an accurate result. This inclusion may result in a pitch response that is more strongly coupled to the other two response.

Finally, the ability of this model to predict slack line events needs to be examined. No slack lines where induced for the cases shown which is likely a consequence of the range of parameters chosen. More extreme parameter ranges in addition to a higher energy wave spectrum would likely produce slack events. The results should also be compared with a higher-order model (e.g. ANSYS AQWA) that is capable of modeling each line a flexible body that under goes transverse displacements in addition to axial.

Overall, the results show that rigid body behavior of a hanging-mass FOWT cannot be initially assumed. A designer should first investigate whether the coupled response between the two bodies is important to the global response of the system. This simplified model presents an easy way of assessing the magnitude of that coupling before turning to higher order simulations.

For example, a designer may use this tool to ensure that the suspension lines are sufficiently stiff to ensure that the two primary modes in the tension response do not coalesce. Thus, this tool can be used in the early stages of design to produce acceptable ranges of design parameters before more complex optimization is attempted. More importantly, these results show that the response of a hanging-mass FOWT warrants further investigation.

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