On Symmetric Lepton Mixing Matrices

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Abstract

Contrary to the quark mixing matrix, the lepton mixing matrix could be symmetric. We study the phenomenological consequences of this possibility. In particular, we find that symmetry would imply that $|U_{e3}|$ is larger than 0.16, i.e., above its current 2σ limit. The other mixing angles are also constrained and CP violating effects in neutrino oscillations are suppressed, even though $|U_{e3}|$ is sizable. Maximal atmospheric mixing is only allowed if the other observables are outside their current 3σ ranges, and $\sin^2 \theta_{23}$ lies typically below 0.5. The Majorana phases are not affected, but the implied values of the solar neutrino mixing angle have some effect on the predictions for neutrinoless double beta decay. We further discuss some formal properties of a symmetric mixing matrix.

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Low energy neutrino physics \[1\] is described by the neutrino mass matrix

\[ m_\nu = U m_\nu^{\text{diag}} U^T, \]

where \( U \) is the leptonic mixing, or Pontecorvo-Maki-Nakagawa-Sakata (PMNS) \[2\], matrix in the basis in which the charged lepton mass matrix \( m_\ell \) is real and diagonal. The three neutrino masses are contained in \( m_\nu^{\text{diag}} = \text{diag}(m_1, m_2, m_3) \). A useful parameterization for the unitary PMNS matrix is

\[
U = \begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu1} & U_{\mu2} & U_{\mu3} \\
U_{\tau1} & U_{\tau2} & U_{\tau3}
\end{pmatrix}
\]

\[
= \begin{pmatrix}
c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\
-s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & s_{23} c_{13} \\
s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta} & c_{23} c_{13}
\end{pmatrix}
\text{diag}(1, e^{i\alpha}, e^{i(\beta+\delta)}) ,
\]

where we have used the usual notations \( c_{ij} = \cos \theta_{ij} \), \( s_{ij} = \sin \theta_{ij} \) and introduced the Dirac CP-violating phase \( \delta \). There are two independent Majorana CP-violating phases \( \alpha \) and \( \beta \) \[3\] contained in the diagonal phase matrix \( P = \text{diag}(1, e^{i\alpha}, e^{i(\beta+\delta)}) \). Various experiments and their analyzes revealed the following allowed 2, 3 and 4\( \sigma \) ranges of the mixing angles \[4\]:

\[
\begin{align*}
\sin^2 \theta_{12} & = 0.30^{+0.06,0.10,0.14}_{-0.04,0.06,0.08} , \\
\sin^2 \theta_{23} & = 0.50^{+0.13,0.18,0.21}_{-0.12,0.16,0.19} , \\
\sin^2 \theta_{13} & < 0.025 \ (0.041, 0.058)
\end{align*}
\]

Note that the present best-fit value for \( \sin^2 \theta_{13} \) is zero, and that there is no information on any of the phases.

Obviously, the precise form of the PMNS matrix will shed some light on the underlying theory of lepton flavor. One interesting possible property of the PMNS matrix is that it might be symmetric\[5\], \( U = U^T \). In this letter we study in detail the consequences of this possibility. Recently, a symmetric PMNS matrix has been shown to follow from certain classes of models in which the very same unitary matrix is associated with the diagonalization of all fermion mass matrices \[6\]. To conduct a more detailed phenomenological analysis of a symmetric PMNS matrix than the one performed in Ref. \[6\] is one of the motivations of this letter. However, to put the discussion on a broader basis, let us first comment on the formal properties of a symmetric PMNS matrix, which are similar to the properties of a symmetric CKM matrix\[2\] \[7,8,9\]:

\[^1\text{Symmetry of the PMNS matrix around its } U_{e3}-U_{\mu2}-U_{\tau1}\text{-axis has been studied in }[5].\]

\[^2\text{This possibility has been ruled out, see below.}\]
(i) first recall that in general $U = U_3^U U_\nu$ holds, where $U_\ell$ is associated with the diagonalization of the charged lepton mass matrix via $m_\ell m_\ell^\dagger = U_\ell (m_\ell^\dagger)^2 U_\ell^\dagger$, with $m_\ell^\dagger = (m_e, m_\mu, m_\tau)$. Hence, if

$$U_\ell = S U_\nu^* ,$$

where $S$ is symmetric and unitary, then $U$ is also symmetric \[8\]. It holds that $S = U_\ell U_\nu^T$. If $S = 1$ and $m_\ell$ is hermitian we recover the model from Ref. \[6\]. In this scenario we have $m_\nu = U_\nu m_\nu^\dagger U_\nu^T$ and $m_\ell = U_\nu^* m_\ell^\dagger U_\nu^T$. Another special case occurs if $S = 1$ and $m_\ell$ is symmetric. Hereby we obtain $m_\nu^* = U_\nu^* m_\nu^\dagger U_\nu^T$ and $m_\ell = U_\nu^* m_\ell^\dagger U_\nu^T$, i.e., $m_\nu^*$ and $m_\ell$ are diagonalized by the same matrix;

(ii) another formal aspect is the following \[7\]: we can write any unitary matrix, in particular the PMNS matrix, as $U = X U_3^\text{diag} X^\dagger$, where $X$ is unitary and $U_3^\text{diag}$ contains the eigenvalues of $U$: $U_3^\text{diag} = \text{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3})$. At first we assume non-degenerate eigenvalues. If $X$ is real, then $U$ is obviously symmetric. To turn the argument around, note that from $U = X U_3^\text{diag} X^\dagger$ one can obtain $U_3 = X U_3^\text{diag}$ and in case of a symmetric $U$ – that $U X^* = X U_3^\text{diag}$. Thus, the columns of $X$ and $X^\dagger$ are eigenvectors of $U$ with identical eigenvalues. Consequently, they only differ by a phase and we can write $X = O Q$, where $O$ is real and orthogonal and $Q = \text{diag}(e^{i\omega_1}, e^{i\omega_2}, e^{i\omega_3})$. However, from $U = X U_3^\text{diag} X^\dagger$ it follows that multiplying $X$ with $Q^\dagger$ will lead to the same $U$ and hence the phases in $Q$ are unphysical. Thus, we have shown that a symmetric PMNS matrix with non-degenerate eigenvalues implies that its eigenvectors are real \[7\]. Suppose now that two of the eigenvalues of $U$ are degenerate: in this case, without loss of generality, $U_3^\text{diag} = \text{diag}(e^{i\phi_1}, e^{i\phi_1}, e^{i\phi_3})$, which can be written as $U_3^\text{diag} = e^{i\phi_1} (1 + \text{diag}(0, 0, e^{i(\phi_3 - \phi_1)} - 1))$. Simply evaluating $U = X U_3^\text{diag} X^\dagger$ shows that $|U|$ is symmetric\[3\]. We will show next that, if $|U|$ is symmetric, rephasing of the lepton fields allows to make $U$ symmetric;

(iii) it is a special feature of three fermion generations that a mixing matrix having symmetric moduli ($|U_{e3}| = |U_{\tau 1}|$, $|U_{e2}| = |U_{\mu 1}|$ and $|U_{\mu 3}| = |U_{\tau 2}|$) can be rephased in a way such that $\arg(U_{e3}) = \arg(U_{\tau 1})$, $\arg(U_{e2}) = \arg(U_{\mu 1})$ and $\arg(U_{\mu 3}) = \arg(U_{\tau 2})$ \[7\]. To see this, consider a rephasing of the neutrino and charged lepton fields via $\nu_i \rightarrow \nu_i e^{i\sigma_i}$ and $\ell_j \rightarrow \ell_j e^{i\rho_j}$ with $i, j = e, \mu, \tau$ or 1, 2, 3. As a consequence, the PMNS matrix element $U_{ij}$ is changed to $U_{ij} e^{i(\sigma_i - \rho_j)}$ and in addition the Majorana phases are modified: $\text{diag}(1, e^{i\alpha}, e^{i(\beta + \delta)}) \rightarrow \text{diag}(e^{i\sigma_1}, e^{i(\alpha + \sigma_2)}, e^{i(\beta + \delta + \sigma_3)})$. Suppose the arguments of $U_{ij}$ before rephasing are $\phi_{ij}$. In order to have $\arg(U_{ij}) = \arg(U_{ji})$ after rephasing, the parameters with which we rephase the lepton fields have to submit to the condition $\phi_{ij} - \phi_{ji} = \sigma_i - \sigma_j + \rho_i - \rho_j \mod(2\pi)$. There is a solution for this condition if

$$\text{Im}\{U_{e2} U_{e3}^* U_{\tau 2} U_{\mu 3} U_{\tau 1} U_{\mu 1}^*\} = 0 .$$

It is trivial to show that this equation is automatically fulfilled due to unitarity of the PMNS matrix in the case of symmetric moduli: consider the unitarity relation

\[3\]The case of all three eigenvalues being identical is trivial.
for the second and third row and column of $U$: $U_{e2} U_{e3}^* + U_{\mu 2} U_{\mu 3}^* + U_{\tau 2} U_{\tau 3}^* = 0$ and $U_{\mu 1} U_{\mu 1}^* + U_{\mu 2} U_{\mu 2}^* + U_{\mu 3} U_{\mu 3}^* = 0$. Multiplying the first expression with $U_{\mu 1}^*$ and the second with $U_{\mu 3}^*$ and subtracting the two resulting equations while assuming $|U_{\mu 3}| = |U_{\tau 2}|$ yields $U_{e2} U_{e3}^* U_{\tau 2}^* = U_{\mu 1} U_{\mu 1}^* U_{\mu 3}^*$. This is again the condition in Eq. (5). One can show that for more than three fermion generations symmetric moduli are not automatically equivalent to a complete symmetric $U$ \[7\]. This would be of importance if the LSND result was confirmed;

(iv) a related question is the number of constraints the assumption of a symmetric PMNS matrix imposes on the observables. Even though there are in principle three symmetry conditions, $|U_{e3}| = |U_{\tau 1}|$, $|U_{e2}| = |U_{\mu 1}|$ and $|U_{\mu 3}| = |U_{\tau 2}|$, it is easy to see that as a consequence of unitarity

$$|U_{e3}|^2 - |U_{\tau 1}|^2 = |U_{e2}|^2 - |U_{\mu 1}|^2 = |U_{\mu 3}|^2 - |U_{\tau 2}|^2.$$

(6)

Therefore, only one constraint is inflicted on the neutrino mixing observables.

Hence the message delivered by the last two points is that in order to investigate the phenomenological consequences of a symmetric PMNS matrix it is obviously sufficient to consider symmetric moduli. This implies in particular that the Majorana phases are generally not subject to any constraint. Only the Dirac phase and the three mixing angles will be affected. Moreover, the same result for the observables will be obtained for all three symmetry conditions.

Note that a symmetric CKM matrix $V$ is ruled out with current data. For instance, one finds that \[10\] $|V_{ub}| = (3.82^{+0.40}_{-0.44}) \cdot 10^{-3}$ yet $|V_{td}| = (8.28^{+1.38}_{-0.86}) \cdot 10^{-3}$, where the errors are at the 3$\sigma$ level. In terms of the Wolfenstein parameterization \[11\], one has $|V_{ub}| = A \lambda^3 (\rho^2 + \eta^2)$ and $|V_{td}| = A \lambda^3 ((1 - \rho)^2 + \eta^2)$. Since $\rho \neq \frac{1}{2}$ the two elements can not be equal.

From now on we will focus on the phenomenological consequences of a symmetric PMNS matrix. Though one could use a parameterization suitable for the study of symmetric matrices and identify the elements of this parameterization with the usual mixing angles and $CP$ phases from Eq. (2)\[4\], we will focus in this letter on the usual parameterization of Eq. (2) and directly obtain correlations between the neutrino mixing observables $\theta_{12}, \theta_{23}, \theta_{13}$ and $\delta$. First, let us obtain the ranges of the individual elements of the PMNS matrix: we vary the mixing angles in their allowed ranges given in Eq. (3) and the phase $\delta$ between

\[4\]Such a parameterization will be useful if the PMNS matrix turns out to be indeed (close to) symmetric.
We have also given the maximal possible value of the Jarlskog invariant $J_{CP}$ to which any $CP$ violating effect in neutrino oscillations is proportional \cite{12}:

$$J_{CP} = \text{Im} \left\{ U_{e3} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \right\} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta .$$

(8)

Note that the rephasing of the PMNS matrix elements as discussed before Eq. (5) leaves $J_{CP}$ invariant.

Consider now in Eq. (7) the symmetry condition $|U_{e3}| = |U_{\tau 1}|$. Apparently, to fulfill this condition the current 2$\sigma$ ranges of the observables do not suffice. This can be easily understood qualitatively since $|U_{e3}|$ is given by $\sin \theta_{13}$ and is therefore small, whereas $|U_{\tau 1}| = |s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta}|$ is generally large. For a small $|U_{\tau 1}|$ of order $|U_{e3}|$ it is necessary that $\theta_{13}$ is large and that $\delta$ lies close to zero or $\pi$ in order to subtract the second term in $U_{\tau 1}$ from the first one. Moreover, $s_{12} s_{23} - c_{12} c_{23} s_{13}$ is smaller when $c_{23}$ is larger than $s_{23}$, i.e., atmospheric neutrino mixing will tend to be governed by $\sin^2 \theta_{23} < 1/2$. These statements can be made more precise: it is easy to proof that all three symmetry conditions, $|U_{e3}| = |U_{\tau 1}|$, $|U_{e2}| = |U_{\mu 1}|$ and $|U_{\mu 3}| = |U_{\tau 2}|$ are fulfilled for one single condition, namely

$$|U_{e3}| = \frac{\sin \theta_{12} \sin \theta_{23}}{\sqrt{1 - \sin^2 \delta \cos^2 \theta_{12} \cos^2 \theta_{23} + \cos \delta \cos \theta_{12} \cos \theta_{23}}}. \quad (9)$$

Interestingly, this constraint can also be derived by setting the real and imaginary parts of $U_{e3} = U_{\tau 1}$ equal. One gets then two relations, $s_{13} = s_{12} s_{23} / (c_\beta + c_\delta c_{12} c_{23})$ and $(s_\beta + c_{12} c_{23} s_\delta) s_{13} = 0$. From the first one it follows that vanishing of $s_{13}$ in not possible, since experiments show that $s_{12}$ and $s_{23}$ cannot be zero. Realizing this, the second relation is fulfilled when $\sin \beta = - \sin \delta \cos \theta_{12} \cos \theta_{23}$. Inserting this expression in the first condition yields Eq. (9). However, care has to be taken when working with real and imaginary parts of mixing matrix elements, since their individual phases have no physical meaning.

With the 2 (3 and 4)$\sigma$ ranges of $\theta_{12}$ and $\theta_{23}$, and with varying the phase $\delta$ between zero and $2\pi$, one obtains from Eq. (9) that $|U_{e3}|^2 \approx 0.035$ (0.028 and 0.023), which has to
be compared to the experimental upper limits of 0.025 (0.041 and 0.058). Therefore, a symmetric PMNS matrix predicts that $|U_{e3}|$ should be above its current 2σ limit. Consequently, the scenario is easily falsifiable, since the indicated value of $|U_{e3}|$ should be verified in upcoming measurements, in particular by the Double Chooz experiment 13 (see also 14): according to Ref. 13, data taking can start in 2008 and the 3σ limit on $|U_{e3}|$ will be improved from its current value 0.04 to 0.01 (0.006) after 2 (6) years of data taking. These numbers are well below the prediction of a symmetric PMNS matrix. Fixing $θ_{23}$ to $π/4$, gives $|U_{e3}|^2 ≳ 0.050$ (0.046 and 0.042). This means that maximal atmospheric neutrino mixing would be at roughly 3 standard deviations in conflict with a symmetric PMNS matrix. Fixing $θ_{12}$ to its best-fit point gives $|U_{e3}|^2 ≳ 0.041$ (0.036 and 0.033), which is hardly compatible with the current 3σ limit of $|U_{e3}|^2$. Moreover, maximal atmospheric mixing is not compatible with $\sin^2 θ_{23} = 0.30$. We conclude that the current best-fit values of the oscillation parameters are not compatible with a symmetric PMNS matrix. To be precise, the prediction of $\sin^2 θ_{12} = 0.30$ and $\sin^2 θ_{23} = 1/2$ would be $|U_{e3}|^2 ≳ 0.059$. To quantify the compatibility of a symmetric mixing matrix with current data we have also performed a simple $χ^2$ minimization: we introduce

$$\chi^2 = \sum_{ij=12,13,23} \frac{(s_{ij}^2 - (s_{ij})_{\text{best-fit}})^2}{σ^2_{ij}},$$

where $(s_{ij})_{\text{best-fit}}$ and $σ_{ij}$ are the best-fit values and errors from Eq. 3. Obeying the symmetry condition Eq. 9, one can find a minimum of $\chi^2 = 10.29$ for the parameters $\sin^2 θ_{12} ≃ 0.28$, $\sin^2 θ_{23} ≃ 0.36$, $|U_{e3}|^2 ≃ 0.035$ and $δ ≃ 0$. The corresponding pulls for $\sin^2 θ_{12}$, $\sin^2 θ_{23}$, and $|U_{e3}|^2$ are $-0.81$, $-1.78$ and 2.55, respectively.

In Fig. 4 we show plots of $\sin^2 θ_{12}$, $\sin^2 θ_{23}$ and $|J_{CP}|$ against $|U_{e3}|$, obtained from Eq. 9. When we simply vary the mixing angles $θ_{12,13,23}$ in their allowed 3 and 4σ ranges from Eq. 9 and require the PMNS matrix to be symmetric, the plots look identical. Note that the 3σ ranges of the oscillation parameters imply $\sin^2 θ_{23} < 1/2$. In the plot of $|U_{e3}|$ against $J_{CP}$, we also indicated the maximal $|J_{CP}|$ allowed by current data. This serves to illustrate that $CP$ violating effects are rather small when the PMNS matrix is symmetric, even though $|U_{e3}|$ is sizable. Indeed, the scenario is compatible (at 3σ) with $CP$ conservation, in which case $|U_{e3}|^2 ≳ 0.035$ (0.028 and 0.023). Note further that, for the 3σ ranges of the oscillation parameters, $\sin^2 θ_{12}$ takes values on the lower side of its allowed range. This has interesting implications for the effective mass $⟨m⟩ = |\sum U_{ei}^2 m_i|$ governing neutrinoless double beta decay. If neutrinos enjoy an inverted ordering, the minimal value of the effective mass is $⟨m⟩^{\text{min}}_{\text{IH}} = c_{13}^2 \sqrt{|Δm^2_{\text{AT}}|} \cos^2 θ_{12}$. Hence, the smaller $θ_{12}$, the larger the minimal value of $⟨m⟩$ in the inverted ordering. This in turn simplifies distinguishing the normal from the inverted hierarchy with neutrinoless double beta decay, or fully probing the inverted ordering regime 15. To quantify this statement, the lower limit in case of an inverted hierarchy is in general $⟨m⟩^{\text{min}}_{\text{IH}} ≃ 0.2 c_{13}^2 \sqrt{|Δm^2_{\text{AT}}|}$, where we have inserted the lowest possible value of $\cos^2 θ_{12}$ at 3σ. With the constraint stemming from a symmetric PMNS matrix we see from Fig. 4 that $\sin^2 θ_{12} ≲ 0.32$ and consequently $⟨m⟩^{\text{min}}_{\text{IH}} ≃ 0.36 c_{13}^2 \sqrt{|Δm^2_{\text{AT}}|}$.  

This is almost a factor of two larger than without the constraint from symmetry.

To summarize, we studied what consequences would arise from a symmetric PMNS matrix \( U \), a scenario which in contrast to a symmetric CKM matrix is still compatible with data. We noted that in this case either the eigenvectors of \( U \) are real or \( U \) has two degenerate eigenvalues. A symmetric \( U \) arises when \( U_\ell \) and \( U_\nu \) are connected by a symmetric and unitary matrix \( S \) via \( U_\ell = S U_\nu^* \). One simple example is when the neutrino mass matrix and the complex conjugate of a symmetric charged lepton mass matrix are diagonalized by the same matrix. Symmetry implies one constraint on the neutrino oscillation observables, not on the Majorana phases. The scenario is easily falsifiable in the near future, since it predicts that \( |U_{e3}| \) is larger than its current 2\( \sigma \) limit of about 0.16. Experiments like Double Chooz can therefore easily rule out a symmetric PMNS matrix. In addition, there are interesting and testable correlations between the observables, as given in Eq. (9) and illustrated in Fig. 1. If the 3\( \sigma \) ranges of the oscillation parameters are taken, \( \theta_{23} \) cannot be maximal and lies below \( \pi/4 \). Solar neutrino mixing is far away from its maximal allowed value, which affects predictions for neutrinoless double beta decay. In general, the \( CP \) phase \( \delta \) is close to a \( CP \) conserving value.

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Figure 1: Plots of the oscillation observables for a symmetric PMNS matrix. The correlations are a consequence of Eq. (9). We allowed the parameters to vary in their current 3 (left plots) and 4σ (right plots) ranges, which are indicated in the plot.