Spacetime Brout–Englert–Higgs effect in General Relativity interacting with p-brane matter

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Abstract. We review the manifestation of the Brout-Englert-Higgs effect in general relativity interacting with point-like and extended objects (p-branes including string for p=1 and membrane for p = 2), which manifests itself in the appearance of the brane source in the Einstein equation while the graviton remains massless [4]–[8], and discuss briefly its relation and differences with the model for massive spin 2 field proposed recently by G. t’Hooft [3].

Brout-Englert-Higgs effect [1], which was also known as Higgs effect, consists in that the gauge fields of an internal symmetry group acquires the mass when the gauge symmetry is spontaneously broken. The increasing of the number of polarizations is explained by that the gauge fields “eat” Goldstone fields of the spontaneously broken gauge symmetry and incorporates their degrees of freedom as additional polarizations.

To be more specific, let us consider, following [3], the Lagrangian
\[ L = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{1}{2} D_{\mu} \phi^a D^{\mu} \phi^a + V(\phi^2) \]
for the gauge field \( A_\mu = A_\mu^a T^a \) and the scalars \( \phi = \psi^a T^a \) in the adjoint representation of the gauge group. If the potential \( V(\phi) \) forces the scalar field to have a non-vanishing vacuum expectation value \( \langle \phi \rangle \), one can fix the ‘preference gauge’ [3] \( \phi = \langle \phi \rangle \). In this gauge the kinetic term for the scalar field, Goldstone field for the spontaneously broken gauge symmetry, gives rise to the mass term of the gauge field,
\[ \frac{1}{2} m^2 A_\mu A^\mu \propto \langle \phi^2 \rangle, \]
and the scalar field equations are reduced to \( \partial_\nu A_\nu = 0 \). These are dependent: they can be obtained from selfconsistency conditions for the gauge field equations \( D_\mu F_{\mu \nu} = m A_\nu \).

General Relativity is invariant under the spacetime diffeomorphisms (general coordinate transformations) \( Diff_4 \) (\( Diff_D \) in \( D \)-dimensional case)
\[ \delta x^\mu := x'^\mu - x^\mu = -a^\mu(x), \quad \delta g_{\mu \nu} := g'_{\mu \nu}(x) - g_{\mu \nu}(x) = \partial_\mu a^\rho g_{\nu \rho} + \partial_\nu a^\rho g_{\mu \rho} + a^\rho \partial_\rho g_{\mu \nu}. \] (1)
and graviton can be described by gauge field for the \( Diff_4 \) gauge symmetry, which is identified with vielbein \( e^a = dx^\mu e_\mu^a(x) \) or with metric \( g_{\mu \nu} = e_\mu^a(x) \eta_{ab} e_\nu^b(x) \). Then it was natural to expect that the spontaneous breaking of diffeomorphism symmetry should result in that the graviton becomes massive, see [2] and refs. in [3, 4].

On the other hand, the spontaneous breaking of diffeomorphism symmetry occurs in the presence of material objects: particles (0-branes), strings (1-branes), membranes (2-branes) and p-branes with \( p > 2 \) for higher dimensional cases \( D \geq 4 \). The corresponding spacetime Higgs effect, its supersymmetric generalization and consequences were the subjects of study in [4, 5, 6, 7] and in [8] (where some statements of previous papers on brane degrees of freedom...
were refined/improved). This spacetime Higgs effect manifests itself in the modification of the ‘free’ Einstein equations by the p-brane sources, i.e. by the energy-momentum tensor localized on the p-brane worldvolume \( W^{p+1} \),

\[
G^{\mu \nu}(g) := \sqrt{|g|}(R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R) = \frac{T_p}{\kappa} \sqrt{|g|} (g^{p+1})^{mn} (\xi, 0) \delta^{\alpha}_{\mu} \delta^{\alpha}_{\nu} \delta^{(D-p-1)} (x^I) 
\]  

(see below for the notation). However, although such an energy-momentum tensor is a counterpart of the cosmological constant contribution \([4]-[7]\). The above statements of \([4]-[7]\) might seem to be in contradiction with \([2]\) and particularly with \([3]\), where a model looking similar to brane interacting with gravity is discussed. The aims of the present contribution is to review the pure bosonic results of the study in \([4]-[7]\) and to point out the differences between gravity interacting with branes and the model considered in \([3]\) which result in different properties of spin 2 fields in these dynamical systems.

1. Spacetime Higgs effect in the presence of p-brane matter.

Material objects, p-branes (particles for \( p=0 \), strings for \( p=1 \)) can be described by the coordinate functions \( \hat{x}^\mu(\xi^m) \) defining parametrically their worldvolume \( W^{p+1} \) as a surface in spacetime \( M^D \),

\[
W^{p+1} \subset M^D : \quad x^\mu = \hat{x}^\mu(\xi^m) = \hat{x}^\mu(\tau, \vec{\sigma}) , \quad \{ \mu = 0,1,\ldots,(D-1) \}, \quad \{ m = 0,1,\ldots,p \} .
\]  

Their transformations under \( SDiff_D \)

\[
\delta \hat{x}^\mu(\xi) := \hat{x}^\mu(\xi) - \hat{x}^\mu(\xi) = a^{\mu}(\hat{x}(\xi)) - \delta \xi^m \partial_m \hat{x}^\mu(\xi) \quad ( \iff \quad \phi = < \phi > ) , \quad \{ \mu = 0,1,\ldots,(D-1) \}, \quad \{ m = 0,1,\ldots,p \} .
\]  

this is to say by choosing a local frame where \( W^{p+1} \) looks locally as a flat hyperplane. In this gauge the p-brane degrees of freedom are carried (are ‘eaten’) by the metric gauge field.

The action describing the interaction of the gauge and Goldstone fields reads

\[
S = \frac{1}{\kappa} \int d^D x \sqrt{|g|} R + S_p \quad \iff \quad \mathcal{L} = -\frac{1}{2} F_{\mu \nu} F^{\mu \nu} - \frac{1}{2} D_\mu \varphi^a D^\mu \varphi^a - V(\varphi^2)
\]  

where \( \int d^D x \sqrt{|g|} R \) is the standard \((D\)-dimensional\) Einstein-Hilbert action and \( S_p \) is the p-brane action. In the simplest case of absence of the spacetime and worldvolume gauge fields (bulk and worldvolume fluxes) this reads ( \( S_0 = m \int d\tau \sqrt{|det(\hat{x}^\mu \hat{x}^\nu g_{\mu \nu}(\hat{x}(\tau)))|} \) for \( p = 0 \), \( T_0 = m \))

\[
S_p = T_p \int d^{p+1} \xi \sqrt{|det(\partial_m \hat{x}^\mu \partial_n \hat{x}^\nu g_{\mu \nu}(\hat{x}))|} \quad \iff \quad -\frac{1}{2} D_\mu \varphi^a D^\mu \varphi^a - V(\varphi^2)
\]  

Varying the action with respect to the spacetime metric we arrive at the Einstein equation

\[
G_{\mu \nu}(g) := \sqrt{|g|}(R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R) = \kappa T_{\mu \nu} , \quad T_{\mu \nu} = \frac{T_p}{\kappa} \int d^{p+1} \xi \sqrt{|det(\hat{x}^\mu \hat{x}^\nu g_{\mu \nu}(\hat{x}))|} \delta^D(x - \hat{x}(\xi)) \quad (8)
\]  

with \( R = g^{\mu \nu} R_{\mu \nu} , \quad R_{\mu \nu} = R_{\mu \rho \nu \sigma} \) and the energy momentum tensor \( T_{\mu \nu} \) having support on the p-brane worldvolume \( W^{p+1} \) (3). Varying (6) with respect to \( \delta \hat{x}^\mu(\xi^m) \) one obtains the so-called minimal surface equation. This is dependent: as it was shown in \([10]\) for the bosonic string and in \([4]\) for super–p–brane cases it can be obtained as a consequence of the Einstein equation.
Thus the $p$-brane source term given by energy momentum tensor in the r.h.s. of the Einstein eq. (2) is the counterpart of the gauge field mass term in $D^\mu F_{\mu
u} = m A_\nu$. However [4]–[7], this $p$-brane source term cannot be considered as a mass term for graviton, but rather as a counterpart of the cosmological constant term which cannot be identified with the graviton mass [7].

2. Spacetime filling brane ($p$-brane with $p = D - 1$) as cosmological constant.

In the $p=D$ case the $S_{p=(D-1)} = \propto T_{D-1} \frac{1}{\sqrt{\det(\eta_{\mu\nu} \partial_\mu \xi^\nu \eta_{\rho\sigma} g_{\rho\sigma}(\hat{x}))}}$ term in the interacting action (6), can be reduced (by gauge fixing $\xi^m = \delta^m_\mu x^\mu$) to cosmological constant term

$$S_{p=(D-1)} = \frac{\Lambda}{8} \int d^{D} x \sqrt{|g_{\mu\nu}(x)|}.$$  \hspace{1cm} (9)

We have to stress that the notion of spacetime filling brane becomes nontrivial in the String/M-theory context (the references can be found in [4]–[7]) in particular, because such branes (e.g. D9–branes of type IIB string theory) are carriers of the gauge fields and fermionic Goldstone fields corresponding to spontaneous breaking of bulk supersymmetry.

With this in mind, it is tempting to speculate on that this simple observation can be used as a basis to resolve the cosmological constant problem. Namely, the small value of cosmological constant with respect to a straightforward QFT estimation for vacuum energy, as well as the fact that its value is nonzero and positive while supersymmetry and supergravity prefer zero or negative cosmological constant, might be explained just by that the cosmological constant is determined by the tension of a spacetime filling brane, $\Lambda \propto T_{D-1}$. However, developing such a conjecture goes beyond the scope of this contribution.

A simple proof of that the cosmological constant cannot be considered as graviton mass has been presented in [7]. Schematically, the arguments are as follows. In the case of small but non-vanishing cosmological constant, $\Lambda \rightarrow 0$ but $\Lambda \neq 0$, one may consider decomposition over the flat spacetime $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ of the Einstein equation with cosmological constant,

$$G_{\mu\nu}(\eta + h) = G'_{\mu\nu}(h) + O(hh) = \Lambda \eta_{\mu\nu} + \Lambda h_{\mu\nu}.$$  \hspace{1cm} (10)

Here $G''_{\mu\nu}(h)$ contains the contributions of the first order in weak field $h$ and the terms of higher order in $h$ are denoted by $O(hh)$ (the Einstein tensor calculated with the flat metric vanishes $G_{\mu\nu}(\eta) = 0$). The first impression might be that $\Lambda \eta_{\mu\nu}$ provides the mass term for graviton $h_{\mu\nu}$. This, however, is not the case. Indeed, if the first order approximation to Eq. (10) is given by $G_{\mu\nu}(h) = \Lambda \eta_{\mu\nu}$, then zero order approximation should be $\Lambda \eta_{\mu\nu} = 0$ which implies the vanishing of the cosmological constant $\Lambda = 0$ (in contradiction with the original assumption). Thus the selfconsistent weak (h-) field approximation to (10) with $\Lambda \neq 0$ can be formulated only assuming that the cosmological constant is of the same order as $h$, $h \propto \Lambda$, so that the first order approximation is given by $G''_{\mu\nu}(h) - \Lambda \eta_{\mu\nu} = 0$; this equation does not contain mass term for $h$.

3. Discussion and conclusion

The deep reason beyond the fact that cosmological constant contribution to field equations cannot be considered as a mass term is that the equation possesses the gauge symmetry with the same number of parameters as in the case of vanishing cosmological constant, and, hence, the graviton maintains the same number of polarizations $D(D - 3)/2$ (which is 2 for $D = 4$) as in the case of massless spin 2 field. So the AdS graviton is also massless (like the Minkowski one).

Indeed, despite fixing the gauge (5), the functional (9) and the corresponding $p = D$ interacting action (6) is invariant under the spacetime diffeomorphism symmetry $Diff_{D}$. The reason for this is that the direct group of the gauge invariance of the interacting action (6) is the direct product $Diff_{D} \otimes Diff_{p+1}$ of the spacetime and worldvolume diffeomorphism transformations. These correspond to the parametric functions $a^m(x)$ and $\xi^m(\xi)$ in Eq. (4) which can be used to fix (on the worldvolume $W^{p+1}$) the gauge (5). This remains invariant under combined (spacetime and worldvolume) $Diff_{p+1}$ diffeomorphisms.
In the case of spacetime filling brane $p=D-1$ this residual $Diff_{p+1}$ invariance is the spacetime diffeomorphism invariance $Diff_D$ of the Einstein action with cosmological constant. Then the number of polarization of graviton remains the same as in the case of absence of cosmological constants. In the case of not-spacetime–filling brane, $p<D-1$, the gauge symmetry is reduced to $Diff_{p+1}$, but only on the worldvolume $W^{p+1}$, while outside $W^{p+1}$ the $Diff_D$ still acts on graviton. Thus after gauge fixing graviton acquires some additional degrees of freedom (counterparts of additional polarizations), but only on the constant $\delta$ of (11) differs from that by that it involves not one but two spacetime metrics: $g_{\mu\nu}(x)$ and the constant $\delta_{ab}$ used to contract the indices of the Goldstone fields $X^a(x)$. As a result, the (spontaneous) breaking of the diffeomorphism symmetry by the dynamical system described by the action (11) is complete, the gauge fixed action ($X^a = \delta^a_{\mu} x^\mu \cdot \frac{m_p}{2\kappa}$) does not possess gauge symmetry and hence its ‘graviton’ acquires additional polarizations [3].

Although seemingly similar to the spacetime filling brane action ((7) of (6)), the second term of (11) differs from that by that it involves not one but two spacetime metrics: $g_{\mu\nu}(x)$ and the constant $\delta_{ab}$ used to contract the indices of the Goldstone fields $X^a(x)$. As a result, the (spontaneous) breaking of the diffeomorphism symmetry by the dynamical system described by the action (11) is complete, the gauge fixed action ($X^a = \delta^a_{\mu} x^\mu \cdot \frac{m_p}{2\kappa}$)

$$S_{\text{t'Hooft}} = \frac{1}{2\kappa} \int d^Dx \sqrt{|g|} R + \int d^4x \sqrt{|g|} g_{\mu\nu}(x) \partial_\mu X^a(x) \partial_\nu X^a(x), \quad a = 1, \ldots, D$$

does not possess gauge symmetry and hence its ‘graviton’ acquires additional polarizations [3].

As far as the $p$-brane plus gravity interacting system described by Eqs. (6) and (7) is concerned, although the $p$-brane source terms play in it the same role as the mass terms of the gauge fields in the standard Higgs effect, they cannot be identified with the mass term, but rather with the counterpart of the cosmological constant term localized on the $p$-brane worldvolume [4]–[7]. The cosmological term itself can be associated with a spacetime feeling brane ($p = (D - 1)$) and, as we have discussed above, cannot be considered as a mass term.

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