Growth after the streaming instability
From planetesimal accretion to pebble accretion

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ABSTRACT

Context. Streaming instability is a key mechanism in planet formation, clustering pebbles into planetesimals with the help of self-gravity. It is triggered at a particular disk location where the local volume density of solids exceeds that of the gas. After their formation, planetesimals can grow into protoplanets by feeding from other planetesimals in the birth ring as well as by accreting inwardly drifting pebbles from the outer disk.

Aims. To investigate the growth of planetesimals into protoplanets at a single location by the streaming instability. For a solar-mass star, we test the conditions under which super-Earths are able to form within the lifetime of the gaseous disk.

Methods. We modify the Mercury N-body code to trace the growth and dynamical evolution of a swarm of planetesimals at a distance of 2.7 AU from the star. The code simulates gravitational interactions and collisions among planetesimals, gas drag, type I torque, and pebble accretion. Three distributions of planetesimal sizes are investigated: (i) a mono-dispersed population of 400 km radius planetesimals, (ii) a poly-dispersed populations of planetesimals from 200 km up to 1000 km, (iii) a bimodal distribution with a single runaway body and a swarm of smaller, 100 km size planetesimals.

Results. The mono-disperse population of 400 km size planetesimals cannot form ≥ Earth mass protoplanets. Their eccentricities and inclinations are quickly excited, which suppresses both planetesimal accretion and pebble accretion. Planets can form from the poly-dispersed and bimodal distributions. In these circumstances, it is the two-component nature that damps the random velocity of the large embryo by small planetesimals’ dynamical friction, allowing the embryo to accrete pebbles efficiently when it approaches 10$^{-3}$ $M_\oplus$. Accounting for migration, close-in super-Earth planets form. Super-Earth planets are preferred to form when the pebble mass flux is higher, the disk turbulence is lower, or the Stokes number of the pebbles is higher.

Conclusions. For the single site planetesimal formation scenario, a two-component mass distribution with a large embryo and small planetesimals promotes planet growth, first by planetesimal accretion and then by pebble accretion of the most massive protoplanet. Planetesimal formation at single locations such as ice lines naturally leads to super-Earth planets by the combined mechanisms of planetesimal accretion and pebble accretion.

Key words. methods: numerical planets and satellites: formation

1. Introduction

In protoplanetary disks, micron-sized dust grains coagulate into pebbles of mm-cm sizes (Dominik & Tielens 1997, Birnstiel et al. 2012, Knijt et al. 2016, Pérez et al. 2015, Tazzari et al. 2016). But further growth is suppressed by bouncing or fragmentation due to the increasing compaction in collisions (Güttler et al. 2010, Zsom et al. 2010). In addition, these pebbles also drift too fast compared to their growth (Weidenschilling 1977), so that the particles cannot cross the meter size barrier even if they would stick perfectly (Birnstiel et al. 2012, Lambrechts & Johansen 2014), unless the pebbles can remain fluffy during the growth (Okuzumi et al. 2012, Kataoka et al. 2015). The subsequent growth of these pebbles to planetesimals is still not well understood in planet formation theory (see Johansen et al. 2014 for a review).

The streaming instability mechanism provides a promising solution by concentrating drifting pebbles due to a locally enhanced solid-to-gas ratio. Once the threshold of solid-to-gas ratio is satisfied, the pebble clumps can directly collapse into planetesimals (Youdin & Goodman 2005, Johansen et al. 2007, 2009, Bai & Stone 2010). The characteristic size of these planetesimals approximate a few hundred kilometres (Johansen et al. 2012, 2015, Simon et al. 2016, Schäfer et al. 2017, Simon et al. 2017, Abod et al. 2018).

The subsequent growth after planetesimal formation by streaming instability has not been well studied. These newly born planetesimals would interact with each other. In the classical planetesimal accretion scenario (see Raymond et al. 2014, Izidoro & Raymond 2018) for reviews), the gravitational interactions among these planetesimals lead to orbital crossings, scatterings and collisions. Initially, the velocity dispersions of planetesimals are not strongly excited and remain modest. In this stage the accretion is super linear ($\frac{dm_p}{dt} \propto m_p^\gamma$, with $\gamma > 1$), which is termed ‘runaway growth’ (Greenberg et al. 1978, Wetherill & Stewart 1989, Ida & Makino 1993, Kokubo & Ida 1996, Rafikov 2004, Ida & Lin 2004). It means that the massive body has a faster accretion rate and therefore will get more massive quickly. However, this stage cannot last forever. Since the growing massive bodies would stir the random velocities of neighbouring small planetesimals, the accretion rates of the massive bodies slow down and turn into a self-limiting mode. This phase is called ‘oligarchic growth’ where $\frac{dm_p}{dt} \propto m_p^\gamma$ with $\gamma < 1$ (Kokubo & Ida 1998). This phase is characterized...
by a decreasing mass ratio of two adjacent massive runaway bodies (Lissauer 1987; Kokubo & Ida 2000; Thommes et al. 2003; Ormel et al. 2010).

In addition to planetesimal accretion, planetesimals formed by streaming instability can accrete inward drifting pebbles. A planetesimal may capture a fraction of the pebbles which cross its orbit (Ormel & Klahr 2010; Lambrechts & Johansen 2012). This is known as pebble accretion (see recent reviews by Johansen & Lambrechts 2017; Ormel 2017). Even if only a fraction of pebbles are able to be accreted by planets, the pebble accretion rate can still be high for two reasons. First, the accretion cross section is significantly enhanced by gas drag (Ormel & Klahr 2010); and second, a large flux of pebbles grow and drift inward from the outer regions of disks (Birnstiel et al. 2012; Lambrechts & Johansen 2014). Pebble accretion can be classified into 2D/3D regimes (Ormel & Klahr 2010; Morbidelli et al. 2015). When the pebble accretion radius is larger than the pebble scale height, the accretion is in the 2D regime, where \( \frac{dm_p}{dt} \propto m_p^{2/3} \) (Lambrechts & Johansen 2014 Hill regime). On the other hand, when the pebble scale height exceeds the pebble accretion radius, only pebbles with heights smaller than the accretion radius can be accreted. Therefore, the accretion rate in this 3D regime is reduced compared to 2D, and \( \frac{dm_p}{dt} \propto m_p \) (Ida et al. 2016).

In general, the efficacy of the pebble accretion mechanism to grow planetesimals depends on many variables related to the properties of the disk, pebble, and planet (the eccentricity, the inclination, and the mass of the planet, the pebble size, the disk turbulence, etc.). A key quantity is the pebble accretion efficiency \( \varepsilon_{PA} \) (Guillot et al. 2014; Lambrechts & Johansen 2014), defined as the number of pebbles that are accreted divided by the total number of pebbles that the disk supplies. Recently we have computed \( \varepsilon_{PA} \) under general circumstances (Li & Ormel 2018; Ormel & Liu 2018). For instance, when the eccentricity and inclination of the planet become high, \( \varepsilon_{PA} \) drops significantly compared to planets on coplanar and circular orbits, because pebbles are approaching at too high velocity to the planet.

Progress in planet formation requires an improved understanding under which conditions the mass growth is dominated by accreting pebbles or planetesimals. In this work, our goal is to investigate the growth of planetesimals after their formation by streaming instability at a single disk location (e.g., the H_2O ice line) (Hansen 2009), already proposed that the architecture of the Solar system’s terrestrial planets can be explained when planetesimals grow in a narrow annulus. In his model the width of the annulus is 0.3 AU, much wider than our planetesimal forming zone (see Sect. 2). Furthermore, Hansen (2009) focused on the planetesimal accretion in a gas-free environment. Our work instead considers the growth just after the streaming instability in gas-rich disk phase.

Constrained by the size distribution of planetesimals in the asteroid belt, Morbidelli et al. (2009) concluded that their born size should be large (>100 km), while Weidenschilling (2011) argued that planetesimals starting from sub-km-sized still cannot be ruled out. Kenyon & Bromley (2010) studied the formation of ice planets beyond 30 AU and found that protoplanets grow more efficiently with smaller planetesimal sizes. Motivated by streaming instability simulations, our adopted initial sizes are typical >100 km. In the context of combined planetesimal and pebble accretion, Johansen et al. (2015) studied the growth of asteroids using a statistical approach, and concluded that massive protoplanets or even super-Earths can form by a combination of pebble accretion, planetesimal accretion and giant impacts.

In order to study planet formation from a narrow ring of planetesimals, we employ direct N-body techniques. Growth can be classified into two phases: (A) the planetesimal accretion dominated phase and (B) the pebble accretion dominated phase. The N-body approach is necessary to treat phase A and the transition from phase A to phase B. The Mercury N-body code has been modified to include gas drag, type I torque and pebble accretion. Three different types of initial planetesimal size distributions are investigated. We find that a two-component mass distribution (large embryo + small planetesimals) is needed to grow a massive planet. This condition could arise either from the high mass tail distribution of planetesimals formed by streaming instability or be the result of runaway growth of a population of small planetesimals.

The paper is structured as follows. In Sect. 2, we outline our model and the implementation of the N-body code. In Sect. 3, three initial size distributions of planetesimals are investigated, including a mono-dispersed population in Sect. 3.1, a poly-dispersed population in Sect. 3.2 and a single runaway body plus a swarm of small planetesimals in Sect. 3.3. In Sect. 4, we investigate the influence of different parameters in the pebble accretion dominated growth phase (phase B). The key results are summarized in Sect. 5.

2. Method

The hypothesis of this paper is that planetesimals only form at a specific location by streaming instability, which requires a locally enhanced pebble density (Carrera et al. 2015; Yang & Johansen 2014; Li et al. 2017, 2018). For instance, Ros & Johansen (2013); Schoonenberg & Ormel (2017); Drążkowska & Alibert (2017) have proposed that the ice line could be such a place since the water vapor inside the ice line will diffuse back and re-condense on ice pebbles, enriching the solid to gas density ratio. We focus on planetesimals that form at the ice line \( r_{\text{ice}} = 2.7 \) AU based on the disk model in Sect. 2.1. But the following results and applications can be scaled to other locations where the streaming instability condition is realized, e.g., the inner edge of the zone (Chatterjee & Tan 2014; Hu et al. 2018) or a distant location due to the FUV photoevaporation (Carrera et al. 2017). In order to generalize our results, we did not include the specific ice line effects such as a reduced pebble size and pebble flux inside of the ice line due to sublimation.

This scenario is illustrated in Figure 1. We consider the situation where streaming instability operates to quickly spawn planetesimals at the initial time of our simulations. A population of planetesimals has formed in a narrow annulus of relative width equal to the normalized pressure gradient (\( \Delta P = \eta \), see Fig. 1 and further discussion in Sect. 2.3). These planetesimals can grow further in two ways: by coagulation among themselves (planetesimal accretion) or by sweeping up pebbles that drift in from the outer disk (pebble accretion).

2.1. Disk model

The surface density and the aspect ratio of the gas disk are assumed to be

\[
\Sigma_{\text{gas}} = \Sigma_{\text{gas,0}} \left( \frac{r}{1 \text{AU}} \right)^{-1},
\]

\[
h_{\text{gas}} = h_{\text{gas,0}} \left( \frac{r}{1 \text{AU}} \right)^{1/4},
\]
where \( \Sigma_{\text{gas0}} \) and \( h_{\text{gas0}} \) are the gas surface density and the aspect ratio at 1 AU and \( r \) is the distance to the central star. The aspect ratio index of 1/4 assumes an optically thin stellar irradiated disk. For simplicity, we neglect that in the early stages disks might be hotter due to viscous accretion (Garaud & Lin 2007; Bitsch et al. 2015). Here \( \Sigma_{\text{gas0}} = 500 \, \text{g cm}^{-2} \) and \( h_{\text{gas0}} = 0.033 \) (Hayashi 1981) are adopted as the default values in this paper. Therefore, the disk temperature \( T \) and \( \eta \) are

\[
T = \frac{\mu \Sigma_{\text{gas}}GM_\star}{r R_\star} = T_0 \left( \frac{r}{1 \, \text{AU}} \right)^{-1/2},
\]

and

\[
\eta = -\frac{h_{\text{gas}}^2}{2} \frac{\partial \Omega_{\text{gas}}}{\partial \log r} = \eta_0 \left( \frac{r}{1 \, \text{AU}} \right)^{1/2},
\]

where \( G \) is the gravitational constant, \( R_\star = 8.31 \times 10^9 \, \text{erg m}^{-3} \, \text{K}^{-1} \) is the gas constant, \( M_\star \) is the stellar mass, \( \mu = 2.34 \) is the mean molecular weight and \( F_{\text{gas}} \) is the gas pressure in the disk. In the above equations \( T_0 = 280 \, \text{K} \) and \( \eta_0 = 1.5 \times 10^{-5} \) are the values at 1 AU.

Although the adopted surface density power-law index is shallower than minimum mass solar nebula model, this profile \( (\Sigma_{\text{gas}} \propto r^{-1}) \) is more consistent with disk observations (Andrews et al. 2009). The fiducial value of \( \Sigma_{\text{gas0}} \) is chosen to be the same as Lambrechts & Johansen (2014), and the pebble flux is calibrated accordingly in Sect. 2.2.2. Based on the above disk model, the ice line \( (T \sim 170\text{K}) \) is located at \( r_{\text{ice}} = 2.7 \, \text{AU} \).

### 2.2. Simulation setup

We use numerical N-body simulations to study the planetesimal growth after the streaming instability. We have adopted the Mercury code (Chambers 1999) and used the Bulirsch-Stoer integrator. An initial timestep of 3 days and the integration accuracy parameter of \( 10^{-12} \) are chosen.

For the N-body part, collisions between bodies are treated as inelastic mergers that conserve the linear momentum. The fragmentation/restitution (Leinhardt & Stewart 2012; Mustill et al. 2018) is not taken into account in this work. Since eccentricities are initially low, the perfect merger assumption is appropriate. For instance, the impact velocity of 400-km-sized planetesimals is lower than the escape velocity when their eccentricities are lower than a few times \( 10^{-2} \). Fragmentation among planetesimals will become important after embryos form and stir the planetesimals more vigorously. However, by then pebble accretion is already expected to commence, rendering planetesimal fragmentation irrelevant.

The effects of the disk gas on the planetesimals/planets, such as gas drag, type I torque, eccentricity and inclination damping are taken into account by applying effective forces in Sect. 2.2.1. In Sect. 2.2.2 we implement the pebble accretion prescriptions by Liu & Ormel (2018) and Ormel & Liu (2018), which accounts for the effects of the planet’s eccentricity, inclination and the disk turbulence. The modified code uniquely simulates planet-planet, planet-disk and planet-pebble interactions.

#### 2.2.1. Gas drag and type I migration torque

Small embryos and planetesimals experience the aerodynamic gas drag (Adachi et al. 1976),

\[
a_{\text{drag}} = -\frac{3C_D \rho_{\text{gas}}}{8 \pi R_p a^2} v_{rel} v_{rel},
\]

where the drag coefficient \( C_D = 0.5 \), \( v_{rel} \) is the relative velocity between the planetesimal and the gas, \( v_{rel} = v - v_{\text{gas}} \), where \( v_{\text{gas}} \) equals \( v_K (1 - \eta) \) in the azimuthal direction, \( v_K \) is the Keplerian velocity, \( \rho_{\text{gas}} \) is the local gas density, \( \rho_a \) and \( R_p \) are the internal density (assumed to be \( 1.5 \, \text{g cm}^{-3} \), with half water and half silicate rock) and the physical radius of the planetesimal.

Large embryos and low-mass planets feel the gravitational torques from the disk gas (called type I migration, Goldreich & Tremaine 1979; Kley & Nelson 2012; Baruteau et al. 2014). The characteristic migration timescale for a planet on a circular orbit is (Cresswell & Nelson 2008)

\[
t_m \approx 0.5 \left( \frac{M_\star}{m_p} \right) \left( \frac{M_\star}{\Sigma_{\text{gas0}} a_p^2} \right)^{1/2} \left( \frac{h_{\text{gas}}^2}{\Sigma_{\text{gas0}}} \right)^{-1/2} \left( \frac{h_{\text{gas0}}}{3.3 \times 10^{-2}} \right)^2 \left( \frac{a_p}{2.7 \, \text{AU}} \right)^{5/2} \text{yr},
\]

where \( m_p \) and \( a_p \) are the mass and the semimajor axis of the planet, respectively, and \( v_K \) is the Keplerian angular velocity at the planet location.

The accelerations acting on planets due to type I migration torque are

\[
a_m = -\frac{\mathbf{v}}{t_m}, \quad a_e = -\frac{2(\mathbf{v} \cdot \mathbf{r})\mathbf{r}}{r^2a_e}, \quad a_i = -\frac{v_e}{t_i},
\]

where \( \mathbf{r} = (x, y, z) \), \( \mathbf{v} = (v_x, v_y, v_z) \) are the position and the velocity vectors of the planet. In the above expression, \( t_m, t_e, t_i \) are the characteristic type I migration, eccentricity and inclination damping timescales from Eqs. (13), (11) and (12) of Cresswell & Nelson (2008). We note that the torque prescription is based on an isothermal disk and planets always migrate inward. The effect of the unsaturated corotation torque due to the thermal diffusion in a radiative disk (Paardekooper et al. 2010, 2011; Bitsch et al. 2015; Brasser et al. 2017), the dynamical torque (Paardekooper 2014; McNally et al. 2018), and the heating torque from planet gas accretion (Benitez-Llambay et al. 2018) are taken into account by applying effective forces in Sect. 2.2.1.
2.2.2. Pebble accretion

The pebble-sized particles drift inwards across the protoplanetary disk. The radial drift velocity is \( v_r = -\frac{2\eta v_r \tau_s}{1 + \tau_s^2} \) (Weidenschilling 1977) where \( \tau_s \) is the dimensionless stopping time (Stokes number). The drift speed is determined by the aedynamical size of the pebble \( \tau_a \) and \( \eta \). Based on Lambrechts & Johansen (2014) and Schoonenberg et al. (2018), the pebble mass flux \( \dot{M}_{\text{peb}} = 2\pi r_v \Sigma_{\text{peb}} \) is proportional to the disk pebble surface density, but weakly dependent on time. In the default model we adopt a constant pebble flux of \( \dot{M}_{\text{peb}} = 100 M_\oplus/\text{Myr} \) (consistent with Eq. (14) of Lambrechts & Johansen (2014)) and neglect the time dependence for simplicity. A lower \( \dot{M}_{\text{peb}} \) means an intrinsic mass-deficient/less massive disk and the planetesimals/planets take longer time to grow by pebble accretion. On the other hand, the pebble flux cannot be too high as the den-

The pebble flux therefore needs to be smaller than \( 2\pi r_v \Sigma_{\text{gas}} H_{\text{peb}} / H_{\text{gas}} \). Our default value is well below this limit. We also explore two additional pebble fluxes \( \dot{M}_{\text{peb}} = 200 M_\oplus/\text{Myr}^{-1} \) and \( 50 M_\oplus/\text{Myr}^{-1} \) in Sect. 4.

The pebble mass can be measured from (sub)millimeter dust continuum emission from the young protoplanetary disks. The inferred values are from a few Earth mass to a few hundreds of Earth mass (Reyes et al. 2010; Andrews et al. 2013) and in Andrews et al. (2013), which is correlated with the gas disk accretion rates (Manara et al. 2016). For a typical Taurus star with a disk accretion rate of \( 10^{-7} M_{\odot} \text{yr}^{-1} \), the dust mass is \( \sim 100 M_\oplus \) (Fig. 1 of Manara et al. 2016), consistent with our adopted fiducial value.

In Eq. (8), \( \alpha_{\text{turb}} \) is the coefficient of turbulent gas diffusivity, which is different from the concept of turbulent viscosity \( \alpha_v \). We nevertheless note that the above two quantities are approximated the same when the turbulence is driven by magnetorotational instability (Johansen & Klahr 2005; Zhu et al. 2015; Yang et al. 2018). The value of \( \alpha_{\text{turb}} \) can be constrained from the molecule line broadening measurements (Flaherty et al. 2015, 2017) and the level of dust settling (Pinte et al. 2016). We adopt a fiducial value of \( \alpha_{\text{turb}} = 10^{-3} \) and test the influence of a lower turbulent disk \( \alpha_v = 10^{-4} \) in Sect. 4.

For pebbles we adopt a fiducial aedynamical size of \( \tau_a = 0.1 \). This chosen value is consistent with the sophisticated dust coagulation model (Birnstiel et al. 2012) and disk observations (Tazzari et al. 2016). A lower \( \tau_a = 0.03 \) is also explored in Sect. 4.

A fraction of pebbles will be accreted onto a planetesimal when pebbles drift through its orbit. The pebble accretion efficiency (\( \varepsilon_{\text{PA}} = \dot{M}_{\text{PA}} / \dot{M}_{\text{peb}} \)) is taken from Liu & Ormel (2018) and Ormel & Liu (2018). This quantity depends on the disk properties (\( \tau_a, \eta, \alpha_v \)) and the planet properties (\( M_p, a, e, i \)). In the limit of 2D and 3D pebble accretion these are given by

\[
\varepsilon_{2D} = \frac{0.32}{\eta} \sqrt{\frac{\dot{M}_p \Delta v}{M_* v_K \tau_s}} f_{\text{set}} \tag{9}
\]

and

\[
\varepsilon_{3D} = \frac{0.39}{\eta} \frac{\dot{M}_p}{h_{\text{peb}} M_*} f_{\text{set}} \tag{10}
\]

respectively, where \( \Delta v \) is the relative velocity between the planet and the pebble, \( h_{\text{peb}} = H_{\text{peb}} / r \) is the aspect ratio of the pebble disk. In Eq. (10) we have already assumed that \( i < h_{\text{peb}} \). The above expressions include a modulation factor,

\[
f_{\text{set}} = \exp \left[ -0.5(\Delta v / v_*)^2 \right] \tag{11}
\]

where \( v_* = (M_p / \Sigma_{\text{gas}})^{1/3} v_K \). Physically, when the pebble-planet encounter is too fast compared to the coupling time between the gas and the pebble, gas drag is no longer effective to aid the planet to capture pebbles. Therefore, \( f_{\text{set}} \ll 1 \) and pebble accretion fails (Visser & Ormel 2016).

In the multi-planetesimal system we consider the filtering of flux when pebbles drift through these planetesimals. Therefore, the pebble flux of the body \( i \) is given by

\[
\dot{M}_{i, \text{peb}} = \begin{cases} 
\dot{M}_{\text{peb}} & i = 1 \\
\dot{M}_{\text{peb}} \prod_{k=1}^{i-1} (1 - \varepsilon_{k, \text{PA}}), & i \geq 2
\end{cases} \tag{12}
\]

where \( i \) is ordered for bodies from the furthest to closest in terms of semimajor axis and \( \varepsilon_{i, \text{PA}} \) is the pebble accretion efficiency of the \( i^{th} \) body.

We focus on the formation of protoplanets with \( m_p \gtrsim 1 M_\oplus \) (progenitors of super-Earths). In our simulations the forming planets are still not massive enough to inverse the local gas pressure gradient and truncate the inward drift of pebbles (Lambrechts et al. 2014, Bitsch et al. 2018, Aitaie et al. 2018). Therefore, we do not implement the termination of pebble accretion.

2.3. Planetesimal initial condition

In the streaming instability mechanism, pebbles accumulate into dense filaments. The typical width of the filament is \( \Delta r \sim \eta_{r, \text{ice}} \) (Yang & Johansen 2016; Li et al. 2018). The threshold condition to trigger gravitational collapse requires \( \rho_{\text{peb}} \sim \rho_{\text{gas}} \). The total solid mass available to build planetesimals is therefore

\[
2\pi r_{\text{ice}} \Sigma_{\text{peb}}(r_{\text{ice}}) = 2\pi r_{\text{ice}} \Delta r \Sigma_{\text{gas}}(r_{\text{ice}}) H_{\text{peb}} / H_{\text{gas}} \tag{11}
\]

find that the planetesimal generation efficiency approximates 50% (half of the pebbles convert into planetesimals, see their Fig. 10a). We note that this value might be also dependent on the local metallicity, Stokes number of the pebbles, and the disk turbulence. However, due to a lack of detailed streaming instability simulations exploration, we still use this number to approximate the total mass in planetesimals. With our adopted disk parameters, this amounts to 0.039 \( M_\oplus \).
3. Scenarios with different initial sizes

Many works have simulated streaming instability numerically, finding that it spawns planetesimals of typical size of several hundreds kilometers, albeit with a considerable uncertainty regarding the precise shape of the size distribution (Johansen et al. 2015; Simon et al. 2016; Schäfer et al. 2017; Abod et al. 2018). However, this initial distribution affects the planet growth, as it will determine the duration of the planetesimal-dominated growth phase (phase A). Here we consider three scenarios for the initial planetesimal distribution in the following subsections: (i) a mono-dispersed population of big planetesimals; (ii) a population of planetesimals of various sizes; and (iii) a two-component population of one large body among many small planetesimals. The first and third scenario reflect standard practice where planetesimals typically start from a fixed size, whereas the second more closely follows the streaming instability results. While the first and second case can be integrated directly, the third scenario employs a superparticle approach to handle the large number of small bodies with N-body techniques.

In this section we investigate the question that provided in total 100 $M_{\oplus}$ of pebbles (100 $M_{\oplus}$ yr$^{-1}$ of $\dot{M}_{\text{pel}}$ over 1 Myr), what is the mass growth of planetesimals for the above three different initial size distributions.

3.1. Mono-dispersed initial conditions

In this section we consider that all planetesimals start with an equal size (400 km in radius). The mass growth and dynamical evolution are simulated after the formation at a single site location. We conduct two simulations when the planetesimals orbits are coplanar and inclined. In the former case planetesimals are all initiated at $z = 0$ and, by construction, remain in the disk midplane. This (as we will see) is not realistic, but comparing the results between these two gives us important insights on key factors (e.g., planet inclination) shaping the planetesimal + pebble accretion process.

As mentioned in Sect. 2.3, the streaming instability produces in total 0.039 $M_{\oplus}$ planetesimals at $r_{\text{ice}}$. In the mono-dispersed initial conditions, we assume that all planetesimals are 400 km in radius. (6.7 x 10$^{-5}$ $M_{\oplus}$ in mass). Therefore, $N'$ = 580 planetesimals are generated in this compact ring belt ($\Delta r = \eta r$ in width). These bodies are injected into the simulation one by one after each 0.1 orbital period. Their initial semimajor axes are uniformly distributed from $r_{\text{ice}} - \Delta r/2$ to $r_{\text{ice}} + \Delta r/2$. The initial eccentricities are assumed to follow a Rayleigh distribution, $p(e) = e/\epsilon_0 \exp[-e^2/2\eta^2]$.

We conduct two sets of simulations. The first idealised set considers that planetesimals are in coplanar orbits ($i_0 = 0$, run_md_test in Table 1). However, this configuration ($i_0 = 0$) is an unphysical case. Realistically, although the initial inclinations are tiny, they are not zero due to the stochastic fluctuation driven by the disk turbulence (Ida & Lin 2008; Gressel et al. 2011; Yang et al. 2012; Okuzumi & Ormel 2013). The planetesimals would be lifted out of the midplane and acquire an inclination distribution.

In hydrodynamic simulations (Simon et al. 2016; Schäfer et al. 2017) the planetesimals are generated from pebble filaments, their random fluctuation is at least smaller compared to the size of the filament ($\epsilon_{\text{fil}}$, $\epsilon_0 \ll \eta$). Therefore, for the second set we assume that their inclinations also follow a Rayleigh distribution with $i_0 = \epsilon_0/2$. Two different initial values for $\epsilon_0$ are numerically tested, 10$^{-5}$ and 10$^{-6}$. We find that since the orbit of planetesimals are readily excited, the results are insensitive to the initial values. In this non-coplanar configuration, three different simulations are performed with the randomized initial semi-major axes and orbital phase angles (run_md_1 to run_md_3 in Table 1).

Figures 2a and 2b show the mass growth of planetesimals when they are in co-planar orbits (panel a) and inclined orbits (panel b). We clearly find that mergers (red dots) occur more frequently when the orbits of the planetesimals are coplanar. In Fig. 2a, the mass growth is dominated by planetesimal-planetesimal collision at the beginning (the sudden jump in mass of grey lines in Fig. 2a). Since the pebble accretion rate is an increasing function of the planet mass, when the mass approaches 10$^{-2}$ $M_{\oplus}$, the growth is driven by pebble accretion (smooth growth in mass in Fig. 2a).

Only a few bodies survive after 1 Myr, and the mass of the dominant body is 4 $M_{\oplus}$. Even though the accretion timescale in this
configuration is artificially short, it clearly illustrates that the growth initially proceeds in a planetesimal accretion-dominated phase (phase A) and transitions to a rapid pebble accretion-dominated phase (phase B) when a massive body of $\sim 10^{-3} M_\oplus$ to $10^{-2} M_\oplus$ emerges. For the realistic case when planetesimals are on inclined orbits (Fig. 3), however, much fewer mergers occur and the mass growth remains modest. The mass of the largest body remains $< 10^{-2} M_\oplus$, and there is no efficient pebble accretion at the end of the simulation.

**3.1.2. Excitation of eccentricities and inclinations**

Fig. 3 shows the time evolution of the root-mean-square (rms) of the isotropic, dispersion-dominated regime. In that case the rms of eccentricities get excited to values larger than the Hill velocity ($h_{\text{gas}} \ll R_\text{H}$). Tidal interactions (type I torque) between an embryo and the gas also damps its eccentricity and inclination. The damping terms is expressed as (Artymowicz 1993)

$$\tau_{\text{tidal}} \simeq \left( \frac{M_\star}{m_p} \right) \left( \frac{M_\star}{\Sigma_{\text{gas}} a_p^2} \right) \left( \frac{h_{\text{gas}}^4}{\Omega_K} \right) \approx 7 \times 10^7 \left( \frac{R_p}{400 \text{ km}} \right)^{-3} \left( \frac{500 \text{ g cm}^{-2}}{\Sigma_{\text{gas}}} \right)^{-1} \left( \frac{0.033}{h_{\text{gas}}} \right)^4 \left( \frac{\rho_\star}{1.5 \text{ g cm}^{-3}} \right)^{-1} \left( \frac{\alpha}{2.7 \text{ AU}} \right)^{3/2} \text{ yr.} \quad (15)$$

The type I damping is inversely proportional to the mass of the embryo, and is negligible when the size of the body is smaller than $1000 \text{ km} (10^{-3} M_\oplus)$. From the above equations we obtain $\tau_{\text{tidal}} \ll \tau_{\text{drag}}$ (9) and (10), less massive bodies accrete pebbles more slowly. Second, pebble accretion is quenched due to a high eccentricity and inclination. Specifically, from Eq. (11) it follows that pebble accretion requires encounters to have a sufficiently low relative velocity

$$\Delta v \lesssim v_s = \left( \frac{M_p/M_\star}{\tau_s} \right)^{1/3} v_K. \quad (16)$$

Inserting $\Delta v \approx e v_K$ and $e \sim 10^{-2}$ we find that pebble accretion becomes only significant when the mass of the planet approaches $e^3 \tau_s M_\star \approx 10^{-2} M_\oplus$. But from the above discussion we know that gas drag and type I damping are ineffective to reduce the random motions of 400-km-sized planetesimals. In addition, a high planetesimal inclination further reduces pebble accretion when it becomes larger than the aspect ratio of the pebble disk ($i_{\text{rms}} > h_{\text{peb}}$) (Levison et al. 2015). In Fig. 4 we find that this happens at $t \approx 5 \times 10^5$ yr. This means after that the planetesimals exceed the pebble layer during part of its orbits, reducing the amount of pebbles they can "eat".

**3.1.3. Spreading of the planetesimal belt**

In Fig. 4 we plot the time evolution of the planetesimals’ semi-major axes in run,mid_J. We define the full width of the planetesimal belt as $\Delta W = 2 \sqrt{\sum_i (a_i - a_{\text{rms}})^2} / N$. The initial width

| Name          | initial planetesimal size | inclination | $m_p$ ($M_\oplus$) | $a_p$ (AU) |
|---------------|---------------------------|-------------|--------------------|------------|
| run,mid_Jest  | mono-size                 | No          | 4.0                | 0.1        |
| run,mid_I     | mono-size                 | Yes         | 0.004              | 2.7        |
| run,mid_2     | mono-size                 | Yes         | 0.009              | 2.7        |
| run,mid_3     | mono-size                 | Yes         | 0.02               | 2.6        |
| run_0         | poly-size                 | Yes         | 1.3                | 1.7        |
| run_0         | poly-size                 | Yes         | 1.6                | 1.5        |
| run_sp_1      | one embryo + small planetesimals | Yes      | 1.3                | 1.8        |
| run_sp_2      | one embryo + small planetesimals | Yes      | 1.4                | 1.6        |
| run_sp_3      | one embryo + small planetesimals | Yes      | 1.3                | 1.7        |

**Table 1.** Simulations set-up in Sect. 3.1 Sect. 3.2 and Sect. 3.3. The mass and semi-major axis of the most massive planet are given at $t = 1 \text{ Myr.}$
is $\eta_r$ (blue area). After their injection, planetesimals in this compact zone experience mutual gravitational interactions; scatterings excite their eccentricities and, in order to conserve the angular momentum, the range of the semi-major axes also expands. In Fig. 4 we find that the width increases steadily with time and becomes 0.05 AU after 10^5 yr.

This orbital expansion can be described by a diffusion process. Ohtsuki & Tanaka (2003) obtain an analytical expression of the viscosity due to the mutual gravitational scattering based on equal size planetesimals. Substituting their viscosity ($\nu$ in their Eq. (18)) into $\Delta W = \sqrt{\nu t}$, we have

$$\Delta W = C_w^3 \frac{\sqrt{\beta N \Omega \kappa \Gamma}}{a^2} R_p^2 / \alpha,$$

(17)

where the prefactor $C_w^3 = C_{\text{fit}} / 24! (\beta / \pi) / [2 \ln (\Lambda^2 + 1)]$, $I(\beta) = 0.2$, $t$ is the time, $\Lambda = \dot{\varpi}_{\text{rms}} (\dot{q}_{\text{rms}} + \dot{e}_{\text{rms}}) / 3$, and $\dot{e}_{\text{rms}} = a \dot{e}_{\text{rms}} / R_H$ and $\dot{\varpi}_{\text{rms}} = a \dot{\varpi}_{\text{rms}} / R_H$. The numerical factor of $C_{\text{fit}} = 30$ is calibrated with our numerical simulations and is \(\approx\) 3 times larger than Ohtsuki & Tanaka (2003)’s result. This formula is obtained under the assumption that planetesimals are in the dispersion-dominated accretion regime. We show in Fig. 2 that our analytical expression (Eq. (17)) agrees well with the simulation (black line). For fixed total mass $N m_p$ and $a$, $\Delta W$ increases with time ($\propto t^{1/2}$), the initial size of the planetesimal ($\propto R_0$), and decreases with the eccentricity ($\propto e^{-2/3}$).

Since the ring belt expands over time, the surface density of the planetesimal also decreases. Therefore, the accretion of the planetesimals formed from a narrow ring will be longer than the classical/oligarchic accretion that assumes an ‘infinite’ width of the planetesimal disk (Kokubo & Ida 2000).

It is clearly seen from Fig. 2 that the growth of planetesimals is slower than Fig. 1 when considering the non-zero inclinations. We find that the runs in Table 1 with different initial randomness have a similar growth trend. The masses of the most massive planetesimals from run _nomd_1 to run _nomd_3 are all far below an Earth mass.

In conclusion, when starting from a narrow ring belt of 400-km-sized planetesimals, the growth of planetesimals is suppressed by the self-excitation of the planetesimals. Planetesimal accretion is slow and pebble accretion is inefficient. Formation of planetary embryos, let alone super-Earth planets, is impossible in this scenario, even though 100 M$_\oplus$ of pebbles drift through.

3.2. Poly-dispersed initial conditions with large planetesimals

In the previous section we found that when the sizes of planetesimals are all 400 km, their growth is inefficient at the ice line. Then the question is: what is the realistic sizes of the planetesimals? In this section we consider a poly-dispersed population of planetesimals from the streaming instability simulations. In this case, planetesimals evolve into two-component mass distribution (one large embryo + small planetesimals). The large body is able to start rapid pebble accretion to eventually grow into a massive planet.
Following this distribution, we adopt their parameters: \( p = 0.6, \beta = 0.35 \). The minimum mass and exponential characteristic mass are fitting parameters, which are given as \( m_{\text{min}} = 1 \times 10^{-6} \, M_\oplus \) (100 km), \( m_e = 1.6 \times 10^{-5} \, M_\oplus \) (250 km), respectively. Figure 5 illustrates the mass distribution of planetesimals for all mass range (red + blue) and for the simulated mass branch (blue). There are in total 0.039 \( M_\oplus \) planetesimals. Here we simulate the most massive 470 bodies from this population, corresponding to the two-thirds of the total mass (blue brach in Fig. 5). The simulated bodies span two order of magnitude in mass and a factor of 5 in size, from 200 km to 1000 km. For simplicity, we neglect the lower one-thirds of small planetesimals. These bodies, if presented, would only modestly contribute to the planetesimal accretion and dynamical friction.

Other initial conditions are chosen to be the same as the simulation of equal 400-km-sized planetesimals (Table 1). The mass growth of all planetesimals is shown in Fig. 6c, while the semi-major axis of the largest body is shown in Fig. 6b. Figure 6 illustrates the mass growth of all planetesimals (Fig. 4 in Schäfer et al. (2017)). From Schäfer et al. (2017) simulations, the number fraction of planetesimals with \( m_p > m \) is given by their Eq.(19),

\[
\frac{N_>(m)}{N_{\text{tot}}} = \left( \frac{m}{m_{\text{min}}} \right)^{-p} \exp \left[ \left( \frac{m_{\text{min}}}{m_e} \right)^{\beta} - \left( \frac{m}{m_e} \right)^{\beta} \right].
\]

Fig. 6. Panel (a): mass growth of the planetesimals for a poly-dispersed size population of planetesimals based on the blue branch of Fig. 5. The black line represent the growth of the final most massive body and grey lines are for other planetesimals. Red dots represents the merging of two bodies, marked at the position of the less massive body. The sudden growth is caused by planetesimal accretion and the smooth growth by pebble accretion. Note that \( f_{\text{fic}} \) approaches 1 when the planet is 0.01 \( M_\oplus \). Panel (b): semi-major axis evolution of the final most massive body. Panel (c): Eccentricity (solid) and Inclination (dashed) evolution where black lines represent the final most massive body and the red lines indicate rms values of the planetesimals. The simulation is from \( \text{run}_p/d_1 \).

The initial mass function of planetesimals by the streaming instability has been investigated by numerous authors in detail. Simon et al. (2016, 2017) find that the initial size can be described by a single power law. Recently Schäfer et al. (2017) suggests that this initial mass distribution is better fitted by a power law plus an shallow exponential decay. The power law is for low and intermediate size planetesimals, while the exponential decay tail represents the high mass cutoff of planetesimals (Fig. 4 in Schäfer et al. 2017). From Schäfer et al. (2017) simulations, the number fraction of planetesimals with \( m_p > m \) is
would be a consequence of forming planetesimals directly from the streaming instability. Generally, a lower pebble surface density yields smaller planetesimals in streaming instability simulations (Johansen et al. 2015; Simon et al. 2016). For the single-site planetesimal formation scenario, the pebble density is locally enhanced at a particular disk location (e.g., the ice line). But how strong the enhancement is determined by complicated physical processes such as water vapor diffusion and condensation (Schoonenberg & Ormel 2017). Taken these factors and uncertainties into account, we also consider the case of an initially small planetesimal size. Nevertheless, we argue in Sect. 3.3.1 that in this circumstance runaway growth of planetesimals proceeds fast, producing the desired two-component mass distribution that promotes the formation of massive planets.

### 3.3.1. Runaway growth in a narrow planetesimal belt

The characteristic runaway planetesimal accretion timescale is (Ormel et al. 2010)

\[
\tau_{\text{rg}} = C_{\text{rg}} \rho_k R_0 \frac{R_0}{100 \text{ km}} \left( \frac{\Sigma}{0.1 \text{ g cm}^{-2}} \right)^{-1} \left( \frac{a}{2.7 \text{ AU}} \right)^{3/2} \text{ yr.}
\]

(19)

where \( C_{\text{rg}} \approx 0.1 \) is a numerical-corrected prefactor from Ormel et al. (2010), and \( \rho_k = 1.5 \text{ g cm}^{-3} \) is the internal density of the planetesimal. Small planetesimals have three advantages in terms of mass growth. First, based on Eq. (19) the runaway growth timescale is proportional to the initial size of the planetesimals \( (R_0) \). It takes less time to form a runaway body (before the onset of the oligarchic growth) when starting with smaller planetesimals. Second, the eccentricity and inclination excitation by self-stirring is less severe for smaller planetesimals (Equation (13)), which also boosts the planetesimal and pebble accretion. Third, as shown by in Sect. 3.1.3 the spreading of their semimajor axes increases with the size of the planetesimal \( (R_0 \propto R_{\text{Hi}} \text{ in Eq. (17)}) \). For the same total planetesimal mass, smaller planetesimals expands their orbits less, resulting in a higher surface density. Therefore, the subsequent accretion also becomes faster.

Accounting for these effects, we assume that the planetesimals are all 100 km in size (\( \lesssim 10^{-6} M_{\oplus} \) in mass). These planetesimals would undergo a faster runaway growth and less orbital expansion compared to 400-km-sized bodies. Since the runaway growth will end up with a steep mass distribution \((dN/dm_{\text{sp}} \propto m_{\text{sp}}^{-2.5})\) (Weberhill & Stewart 1993, Kokubo & Ida 1996, 2000, Ormel et al. 2010), Morishima et al. 2008, most of the mass remains in small planetesimals. As discussed in Sect. 3.3, the inclination and eccentricity of the large body could therefore be damped by small planetesimals through dynamical friction. The planet can accrete more pebbles when the orbit remains nearly coplanar and circular.

The key difference between the classical runaway and oligarchic growth scenario and our scenario is that we here consider planetesimal formation at a single location. In this study we propose a clean and simplistic physical picture. The hypothesis is that after the planetesimal runaway growth phase, the system can be well-described by a two components: a single big embryo of radius \( \sim 10^4 \) km and a swarm of 100 km size planetesimals, which dominate the total mass. We reasonably neglect bodies in between these two are dynamically insignificant and cannot compete with the runaway body in terms of growth (also shown in Fig. 6a). Altogether, the situation resembles the classical runaway/oligarchy transition, except that, in our case, there is only one single “oligarch”.

Why only one big embryo instead of multiple oligarchs? The transition from the runaway to the oligarchic growth takes place when the viscous stirring timescale \( \tau_{\text{vs}} \) (Eq. (13)) equals the runaway timescale \( \tau_{\text{rg}} \) (Eq. (19)) \( \text{[2]} \). When \( \tau_{\text{vs}} > \tau_{\text{rg}} \), the stirring of the random velocities is smaller compared to the accretion, which is the key feature of runaway growth. When \( \tau_{\text{vs}} < \tau_{\text{rg}} \), the eccentricity growth is faster than the accretion, and the growth transitions to the oligarchic regime. The transition size of the planet between two regimes is given by Eq. (11) of Ormel et al. (2010).

\[
R_{\text{rg/oli}} \approx 850 \left( \frac{C_{\text{rg}}}{0.1} \right)^{3/7} \left( \frac{R_0}{100 \text{ km}} \right)^{3/7} \left( \frac{a}{2.7 \text{ AU}} \right)^{5/7} \left( \frac{\Sigma}{1 \text{ g cm}^{-2}} \right)^{2/7}
\]

(20)

Substituting Eq. (19) into Eq. (17), we obtain that when the massive body grows into a transition radius, the planetesimal belt width is

\[
\Delta W = \sqrt{\frac{C_{\text{rg}} C_{\text{rg}}^2 R_{\text{HI}}^3}{2R_0}}
\]

(21)

We note that \( R_{\text{HI}} \) is the Hill radius of the planetesimal with size of \( R_0 \). From \( \Sigma_{\text{sp}} = M_{\text{tot,pl}}/2\pi a \Delta W \) and Eq. (21), we calculate that the belt width extends to \( \Delta W = 0.029 \text{ AU} \) and the planetesimal surface density reduces to \( \Sigma_{\text{sp}} = 2 \text{ g cm}^{-2} \) after the runaway growth (\( \sim 10^5 \) yr). Substituting the above values into Eq. (20), we obtain,

\[
R_{\text{rg/oli}} \approx 1000 \left( \frac{C_{\text{rg}}}{0.1} \right)^{3/7} \left( \frac{R_0}{100 \text{ km}} \right)^{1/7} \left( \frac{M_{\text{tot,pl}}}{0.039 M_{\oplus}} \right)^{2/7} \text{ km.}
\]

(22)

We therefore conclude that the feeding zone of this embryo (\( \sim 10 R_{\text{Hi}}, \text{Kokubo & Ida (1998)} \)) is similar to the width of the planetesimal belt, supporting one single embryo assumption.

### 3.3.2. The superparticle approach

Since the total planetesimal mass is 0.039 \( M_{\oplus} \) and small planetesimals dominate the mass, there are in total 35000 100-km-sized planetesimals (\( \sim 10^{-6} M_{\oplus} \)) in this case. It is prohibitively computationally to simulate the interactions among all these bodies with an N-body scheme. Therefore, we adopt the superparticle approach to mimic the dynamics of small bodies (see Levison et al. (2015), Raymond et al. (2016) and references therein), in which \( N_{\text{sp}} \), small planetesimals are clustered as one superparticle. The superparticle still feels the aerodynamic gas drag as if it was a single 100-km-sized planetesimal, but the mass of the particle is \( N_{\text{sp}} \times m_{\text{sp}} \). These superparticles gravitationally interact with the embryo but not with each other. This is based on the fact that the collision timescale among small planetesimals is much longer than the embryo-planetesimal collision. In principle, the approach requires \( m_0 \ll m_{\text{sp}} \ll m_{\text{p}} \), where \( m_{\text{p}} \) is the embryo's mass. The latter inequality is needed to ensure that dynamical friction operates correctly. The simulation would be very time-consuming for a too small \( N_{\text{sp}} \), while the dynamical evolution may become

\[ \text{foreign.} \]

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\[^{2}\] This criterion is different from Kokubo & Ida (1998). See discussion in Ormel et al. (2010).
artificially stochastic for a too large $N_{sp}$ (low number of superparticles).

We here assume that the velocity dispersion of small planetesimals is excited by the planetesimals themselves, and that it is close to their escape velocity ($\delta v \simeq v_{\text{esc}} = \sqrt{2Gm_0/R_0}$), i.e., $e \simeq e_{\text{esc}} = v_{\text{esc}}/v_K$. The eccentricities and inclinations are adopted to follow the Rayleigh distributions, where $e_0 = 2l_0 = e_{\text{esc}}$. Their semi-major axes are randomly initialized from $r_{\text{ice}}(1 - \eta/2)$ to $r_{\text{ice}}(1 + \eta/2)$. In addition, since pebble accretion is extremely inefficient for 100 km size planetesimals (the inequality (16) is far from satisfied), it is reasonable to neglect pebble accretion of the superparticles. The embryo is initially placed at $r_{\text{ice}}$.

### 3.3.3. Results

We have conducted simulations with different masses of the superparticle, $N_{sp} = 25$ and 50, respectively. The results are in good agreement with each other (see Appendix for details). The results presented in Sects. 3.3.3 and 3.3.4 are based on the $N_{sp} = 50$ run.

Fig. 7. Panel (a): Mass growth of the embryo for the two-component initial conditions in run_sp. Panel (b): Semi-major axis evolution of the embryo. The red thick line is for the averaged value and the light red is for the spreading among three different runs. The black dashed line is for the growth of a single embryo without small planetesimals. This comparison enables us to isolate the effect of N-body dynamics (planetesimal accretion). Simulations are terminated when $t = 1$ Myr.

Figure 7 shows the mass growth and orbital evolution of the embryos in panel (a) and (b), respectively. For the superparticle approach, the thick red lines represent the mean values averaging three runs (e.g., $m(t) = \sum_{i=1}^{3} m_i(t)/3$), whereas the light red areas represent the spreading among these runs. It can be seen that the difference in mass and semi-major axis among the three simulations is small. The black dashed line represents the growth of a single embryo (assuming it is on a circular and coplanar orbit) purely by pebble accretion. We find that when starting with the transition size embryo, the planetesimal accretion are already modest and the mass growth is mainly driven by pebble accretion.

When the mass of the embryo is beyond $0.1 M_\oplus$, the type I migration becomes important. The planet would take $\sim 0.5$ Myr to migrate into the inner region of the disk. Eventually, a $1.3 M_\oplus$ planet forms within 1 Myr.

### 3.3.4. Timescale analysis

In this section we analyse the growth timescale of the embryo. The pebble accretion timescale in the 2D regime is given by

$$
\tau_{PA,2D} = \frac{m_p}{m_p \, M_{\text{peb}}} \simeq 9 \times 10^4 \, f^{-1}_{\text{set}} \left( \frac{m_p}{0.05 \, M_\oplus} \right)^{1/3} \left( \frac{\tau_s}{0.1} \right)^{1/3} \left( \frac{\eta}{2.5 \times 10^{-3}} \right) \left( \frac{M_{\text{peb}}}{100 \, M_\oplus / \text{Myr}} \right)^{-1} \, \text{yr.}
$$

(23)

where we use Eq. (9) and $\Delta v$ is assumed to be dominated by the Keplerian shear since the eccentricity of the embryo is insignificant due to dynamical friction. From Eq. (11), the above $f_{\text{set}}$ evaluates as

$$
f_{\text{set}} = \exp \left[ -0.07 \left( \frac{\eta}{2.5 \times 10^{-3}} \right)^2 \left( \frac{m_p}{0.01 \, M_\oplus} \right)^{-2/3} \left( \frac{\tau_s}{0.1} \right)^{2/3} \right].
$$

(24)

When the planet is $\gtrsim 10^{-2} M_\oplus$, $f_{\text{set}} \simeq 1$. From Eq. (24), we clearly see that $f_{\text{set}}$ becomes smaller when a planet is less massive or their pebble–planet relative velocity is higher.

Similarly, in the 3D limit (Eq. (10)), the pebble accretion timescale becomes

$$
\tau_{PA,3D} = \frac{m_p}{m_p \, M_{\text{peb}}} \simeq 9 \times 10^4 \, f^{-2}_{\text{set}} \left( \frac{h_{\text{peb}}}{4.2 \times 10^{-3}} \right) \left( \frac{\eta}{2.5 \times 10^{-3}} \right) \left( \frac{M_{\text{peb}}}{100 \, M_\oplus / \text{Myr}} \right)^{-1} \, \text{yr.}
$$

(25)

From the planet mass dependence on Eqs. (23) and (25), we know that the pebble accretion is in 3D when the planet mass is low, and it transitions to 2D accretion when the planet becomes more massive. The transition mass between 2D and 3D is $\sim 0.05 M_\oplus$ for the adopted parameters.

In order to numerically capture the transition from the planetesimal dominated accretion to the pebble dominate accretion (phase A to phase B), we conduct the superparticle approach.

The disk and pebble parameters in this (run_sp) are taken to be identical to run_p in Sect. 3.2. The mass of the embryo is chosen to be the transition value ($m_p = 10^{-3} M_\oplus$ when $R_D = R_{\text{g/oli}}$ in Eq. (22)). Three individual simulations (run_sp_1 to run_sp_3) are conducted, varying only the initial positions and orbital phase angles of the planetesimals. In addition, we also conduct a separate simulation for the growth of a single embryo without small planetesimals. This comparison enables us to isolate the effect of N-body dynamics (planetesimal accretion). Simulations are terminated when $t = 1$ Myr.
In this section, we particularly focus on growth in the pebble accretion dominated regime. We start the mass of the embryo from $10^{-3} M_\oplus$. As illustrated in Fig. 7, the pebble accretion dominates the growth and the planetesimals’ contribution can be neglected after $m_p \gtrsim 10^{-3} M_\oplus$. The relative velocity of the massive embryo is very low due to the dynamical friction by small planetesimals and later on type I tidal damping when it grows larger. It is therefore justified to assume the embryo is on a circular and coplanar orbit. We conduct simulations for a single embryo and neglect all planetesimals, investigating the role of various disk and pebble parameters as shown in Table 2. Simulations are terminated when the planet has migrated inside of 0.1 AU.

In Table 2, the fiducial run ($run_{fid}$) is adopted to be the same disk and pebble values as in the previous sections. From $run_{pmig}$ to $run_{jpeb}$, only one parameter is varied compared to the fiducial $run_{fid}$. For instance, in $run_{pmig}$ we assume the planet is at the zero-torque location, where $\alpha/\alpha = 0$. In this case, the orbital expansion is less significant for smaller planetesimals ($\Delta W \sim R_0$ from Eq. 21) and $\Delta p \sim \Delta W^{-1}$, the runaway accretion timescale is strongly correlated with the initial size ($t_{rg} \sim R_0^2$ from Eq. 19). For the mono-dispersed population of 400-km-sized planetesimals, planet growth fails mainly because it takes too long to evolve into a two-component mass distribution. However, in the case of 100-km-sized planetesimals, this configuration can be realized within a fraction of the disk lifetime. In that case, the eccentricity and inclination of the large embryo remains low due to the dynamical friction of small planetesimals. It is therefore possible for the largest embryo to accrete pebbles efficiently and grow into an Earth-mass planet. As a result, massive planets could form from planetesimals with a smaller initial size.

The above discussed preferable size of the planetesimal ($R_0$) to grow protoplanets is dependent on the formation location of planetesimals. When the planetesimals form further out, from Eq. 19 the runaway planetesimal growth timescale increases. On the other hand, the starting size of the embryo for the efficient pebble accretion also increases with the distance (Visser & Ormel 2016). This is because $f_{set}$ becomes smaller when $r$ increases with $r$. Therefore, both two effects further limit the growth of the protoplanets from a large size planetesimal (e.g., 400 km) when the planetesimal formation occurs at a large distance.

Our findings are similar to Levison et al. (2015). They report that large core formation by pebble accretion is only feasible after a period of classical planetesimal runaway growth. The system then ends up with a few large embryos and a swarm of small planetesimals. The embryos stir the random velocities of small planetesimals. The embryos were born over the entire disk region and assumed to follow the same power-law as the disk gas, which is based on the classical planet formation scenario (e.g., Ida & Lin (2004) and Mordasini et al. (2009)). The key difference with this work is that we consider the planetesimals to form at a single site disk location with a narrow radial width. Nevertheless, the ‘common ground’ between both works is that a two-component mass distribution for planetesimals is essential to promote further growth from planetesimals into protoplanets.

4. Growth and migration in the pebble accretion dominated regime

In this section, we particularly focus on growth in the pebble accretion dominated regime (phase B). We start the mass of the embryo from $10^{-3} M_\oplus$. As illustrated in Fig. 7, the pebble accretion dominates the growth and the planetesimals’ contribution can be neglected after $m_p \gtrsim 10^{-3} M_\oplus$. The relative velocity of the massive embryo is very low due to the dynamical friction by small planetesimals and later on type I tidal damping when it grows larger. It is therefore justified to assume the embryo is on a circular and coplanar orbit. We conduct simulations for a single embryo and neglect all planetesimals, investigating the role of various disk and pebble parameters as shown in Table 2. Simulations are terminated when the planet has migrated inside of 0.1 AU.

In Table 2, the fiducial run ($run_{fid}$) is adopted to be the same disk and pebble values as in the previous sections. From $run_{pmig}$ to $run_{jpeb}$, only one parameter is varied compared to the fiducial $run_{fid}$. For instance, in $run_{pmig}$ we assume the planet is at the zero-torque location, where $\alpha/\alpha = 0$. In this case, the orbital expansion is less significant for smaller planetesimals ($\Delta W \sim R_0$ from Eq. 21) and $\Delta p \sim \Delta W^{-1}$, the runaway accretion timescale is strongly correlated with the initial size ($t_{rg} \sim R_0^2$ from Eq. 19). For the mono-dispersed population of 400-km-sized planetesimals, planet growth fails mainly because it takes too long to evolve into a two-component mass distribution. However, in the case of 100-km-sized planetesimals, this configuration can be realized within a fraction of the disk lifetime. In that case, the eccentricity and inclination of the large embryo remains low due to the dynamical friction of small planetesimals. It is therefore possible for the largest embryo to accrete pebbles efficiently and grow into an Earth-mass planet. As a result, massive planets could form from planetesimals with a smaller initial size.
case, the planet does not undergo type 1 migration. Therefore, the planet accretes materials in-situ at the ice line. For a comparison between run\_nnmig and run\_fid, we will gain a knowledge of the effect of migration on the growth of the planet. For this purpose, we stop run\_nnmig at the same time when the planet in run\_fid migrates inside of 0.1 AU. For a comparison between run\_fid and the other individual runs (run\_alpha, run\_tau, run\_lpeb, run\_hpeb), we can understand the effect of disk turbulence, pebble size and pebble mass flux on the planet growth.

We find in Fig. 9 that in a low turbulent disk (run\_alpha, magenta) and in a disk with high pebble flux (run\_hpeb, thick red), the mass growth is faster than the fiducial disk (red). In these two circumstances the planets reach 4.1 M_⊕ and 6.8 M_⊕, respectively, when they migrate inside of 0.1 AU within 1 Myr. We find the mass growth in run\_nnmig is very similar to the fiducial run but slightly less efficient. At t = 1.2 Myr, the final planet mass is 4.0 M_⊕ in run\_fid while in run\_nnmig the planet always stays at the ice line and attains 3.8 M_⊕. It is also clearly seen that when the pebble flux is lower (run\_lpeb, light red), or the Stokes number is lower (run\_tau, orange), the mass growth is slower than the fiducial run. The protoplanet with n_p \gtrsim 1 M_⊕ does not form within 1 Myr from these two configurations. In run\_tau the planet attains 4.8 M_⊕ when it arrives at the inner edge of the disk at t = 1.5 Myr, while in run\_lpeb it grows into a 2.3 M_⊕ planet at t = 2.1 Myr.

The planet grows much faster in a high pebble flux disk. The effect of pebble mass flux on planet growth is intuitive since pebble accretion benefits from a high pebble flux (a massive pebble effect of pebble mass flux on planet growth is intuitive since pebbles mean they are more tightly coupled to the gas, and the pebble scale height becomes larger. Therefore, from Eqs. (23) and (25) transition mass from 3D to 2D in run\_tau is 0.15 M_⊕, three times higher than the fiducial run. In this case starting with 10^{-3} M_⊕, the planet grows a significant fraction of its mass in the slow, 3D pebble accretion regime. On the other hand, when the planet enters 2D pebble accretion, the accretion is faster when the Stokes number is lower (Eq. (23)). Balancing these two effects, we find that the planet growth is slightly slower in run\_tau compared to the fiducial run.

The effect of pebble mass flux on the planet growth is super linear. A high pebble disk mass benefits the formation of a massive planet. A less turbulent disk and a large Stokes number pebbles also promotes pebble accretion and the formation of a massive planet.

### 5. Summary

Streaming instability is an important mechanism to convert pebbles into planetesimals. It occurs at the location in the disk where the pebble density is locally enhanced (e.g., the ice line). In this work, we have focused on the growth of the planetesimals after they have formed by this mechanism at a single site disk location.

This annular (ring) planetesimal formation scenario differs from the classical planetesimal accretion scenario from two aspects. First, streaming instability generates planetesimals in a narrow ring at a specific location. Most of the mass is in the largest planetesimals of a few hundred km in size. In contrast, in the classical scenario planetesimals form everywhere in the disk, e.g., typically following the surface density distribution.
of the gas. In simulations their planetesimal surface density remains constant by suppling material from the neighbouring region (Kokubo & Ida 2000). This means that the orbital spreading is unimportant, in contrast to the ring formation scenario. Second, the planetesimals in our scenario grow their mass by accreting surrounding planetesimals and inwardly drifting pebbles. The mass at which the planet transitions from accreting predominantly planetesimals to pebbles occurs at $\approx 10^{-3} \, M_\oplus$.

We have modified the Mercury N-body code to perform simulations of the mass growth and orbital evolution of these planetesimals (Sect. 2). The code includes the effects of gravitational interactions and collisions among planets/planetesimals, planet-disk interactions (gas drag and type I torque), pebble accretion based on the calculation of Liu & Ormel (2018) and Ormel & Liu (2018) that accounts for the disk parameters ($r_0$, $q$, $\alpha$ and $M_{\text{peb}}$) and planet properties ($m_p, a, e, i$). Simulations with different initial planetesimal sizes and disk parameters are investigated.

The key findings of this study are the following:

1. Protoplanets cannot emerge from a mono-dispersed population of 400 km size planetesimals, fuelled by a 100 $M_\oplus$ reservoir of pebbles in the outer disk. Although the initial eccentricities and inclinations are very tiny, they soon get excited through gravitational scatterings. Mechanisms such as gas drag, type I damping are not sufficient enough to damp their random velocities. Both planetesimal and pebble accretion are strongly suppressed when inclinations and eccentricities of planetesimals become moderate. In this circumstance, the growth of the planetesimals is mainly in the slow planetesimal accretion dominated phase (Sect. 3.1).

2. Protoplanets can form when streaming instability has in addition spawned a population of larger planetesimals. The largest body grows by planetesimal accretion. Soon after it approaches $10^{-2} \, M_\oplus$, the growth enters the rapid pebble accretion dominated regime. During this time the random velocity of the largest body remains low through the dynamical friction of small planetesimals. Finally a super-Earth planet can form within 1 Myr (Sect. 3.2).

3. Alternatively, protoplanets also form out of their birth ring when the initial size of the planetesimals is small (e.g., 100 km). These small planetesimals are expected to undergo a runaway planetesimal accretion to form a massive embryo rapidly. In this way, the two-component mass distribution is also achieved. We find that an Earth mass planet can form as well (Sect. 3.3).

4. Planets grow larger when the pebble mass flux is higher, the disk is less turbulent, the Stokes number of pebbles are larger. In particular, the growth of the planet mass increases super linearly with the disk pebble mass flux (Sect. 3.4).

Planetesimal accretion and pebble accretion are not two isolated processes in planet formation. From the streaming instability point of view, pebbles are converted into planetesimals, whereafter these planetesimals accrete nearby planetesimals and pebbles at the same time. The total amount of solids in disks is either in small pebbles, or in large planets/planetesimals. Therefore, a certain level of competition exist between planetesimal formation and planet growth by pebble accretion.

In addition, this work only considered the case of a single burst of planetesimal formation by streaming instability. In reality, even at the ice line the streaming instability may be triggered multiple times (episodic bursts) during the gas disk lifetime, because the planets migrate out of their birth ring. Protoplanets then emerge sequentially and a chain of multiple planets forms, as envisioned by Ormel et al. (2017).

To address these issues, a global disk model is needed. Using a novel Lagrangian approach, such a model has just been developed by Schoonenberg et al. (2018). Because of the flexibility of the Lagrangian (particle-oriented) model, it is straightforward to couple it to the N-body model presented in this paper. Such a model can then, potentially, simulate planet formation in its entirety – starting from dust coagulation and ending with a planetary architecture. It can be applied to model “complete” (as far as one can tell) planetary systems, such as those discovered around TRAPPIST-1 (Gillon et al. 2017).

Appendix A: Convergence test for superparticle simulations

We show the simulations with different mass of the superparticle ($m_{\text{sp}} = N_{\text{sp}} \, m_0$) for runs $\text{sp}$ starting with $m_0 = 3 \times 10^{-4} \, M_\oplus$ in Sect. 3.2. Three simulations with randomized initial conditions are performed for each set of $N_{\text{sp}}$. The results are shown in Fig. A.1 where red, green represent $N_{\text{sp}} = 25$ and $50$, respectively. The thick line represents the mean value averaged from three individual simulations, for instance, $\bar{m}(t) = \sum_{i=1}^{3} m_i(t)/3$ whereas the light region marks the range between the minimum and maximum values from the three simulations. Fig. A.1 shows the mass, semi-major axis evolution of the embryo and eccentricities of both planetesimals and the embryo. We find that the mass and semi-major axis converge quite well for the above two $N_{\text{sp}}$. In Fig. A.1, the difference between the mean mass for the above $N_{\text{sp}}$ is smaller compared to the spreading among individual runs (stochastic N-body effects). The excitation of planetesimals by the presence of the embryo is also agreed with each other in Fig. A.1.

To summarize, results from the above three tested $N_{\text{sp}}$ are in general agreement with each other.

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References

Abod, C. P., Simon, J. B., Li, R., et al. 2018, ArXiv e-prints
Adachi, I., Hayashi, C., & Nakazawa, K. 1976, Progress of Theoretical Physics, 56, 1756
Andrews, S. M., Rosenfeld, K. A., Kraus, A. L., & Wilner, D. J. 2013, ApJ, 771, 152
Andrews, S. M., Wilner, D. J., Hughes, A. M., Qi, C., & Dullemond, C. P. 2009, ApJ, 700, 1502
Ansdell, M., Williams, J. P., Manara, C. F., et al. 2017, AJ, 153, 240
Artymowicz, P. 1993, ApJ, 419, 155
Ataiee, S., Baruteau, C., Alibert, Y., & Benz, W. 2018, A&A, 615, A110
Bai, X.-N. & Stone, J. M. 2010, ApJ, 722, 1437
Baruteau, C., Crida, A., Paardekooper, S.-J., et al. 2014, Protostars and Planets VI, 667
Benítez-Llambay, P., Masset, F., Koenigsberger, G., & Szulágyi, J. 2015, Nature, 520, 63
Birnstiel, T., Klahr, H., & Ercolano, B. 2012, A&A, 539, A148
**Fig. A.1.** Convergence test for $N_{dp} = 25$ (red) and 50 (blue). Mass, semi-major axis and eccentricity evolution of the embryo are shown. The line represents the mean value of three individual runs while the area indicates the scattering from these runs. In the bottom panel the RMS eccentricity of the planetesimals are shown in the dashed line.

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Iida, S., Kokubo, E., & Makino, J. 1993, MNRAS, 263, 875
Iida, S. & Lin, D. N. C. 2004, ApJ, 604, 388
Iida, S. & Lin, D. N. C. 2008, ApJ, 673, 487
Iida, S. & Makino, J. 1993, Icarus, 106, 210
Izidoro, A. & Raymond, S. N. 2018, ArXiv e-prints:1803.08830
Johansen, A., Blum, J., Tanaka, H., et al. 2014, Protostars and Planets VI, 547
Johansen, A. & Klahr, H. 2005, ApJ, 634, 1353
Johansen, A. & Lambrechts, M. 2017, Annual Review of Earth and Planetary Sciences, 45, 359
Johansen, A., Mac Low, M.-M., Lacerda, P., & Bizzarro, M. 2015, Science Advances, 1, 1500109
Johansen, A., Oishi, J. S., Mac Low, M.-M., et al. 2007, Nature, 448, 1022
Johansen, A., Youdun, A., & Mac Low, M.-M. 2009, ApJ, 704, L75
Johansen, A., Youdin, A. N., & Lithwick, Y. 2012, A&A, 537, A125
Kataoka, A., Tanaka, H., Okuzumi, S., & Wada, K. 2013, A&A, 557, L4
Kenyon, S. J. & Bromley, B. C. 2010, ApJ, 188, 242
Kley, W. & Nelson, R. P. 2012, ARA&A, 50, 211
Kokubo, E. & Ida, S. 1996, Icarus, 123, 180
Kokubo, E. & Ida, S. 1998, Icarus, 131, 171
Kokubo, E. & Ida, S. 2000, Icarus, 143, 15
Krijt, S., Ormel, C. W., Dominik, C., & Tielens, A. G. M. G. 2016, A&A, 586, A20
Lambrechts, M. & Johansen, A. 2012, A&A, 544, A32
Lambrechts, M. & Johansen, A. 2014, A&A, 572, A107
Lambrechts, M., Johansen, A., & Morbidelli, A. 2014, A&A, 572, A35
Leinhardt, Z. M. & Stewart, S. T. 2012, ApJ, 745, 79
Levison, H. F., Kretke, K. A., & Duncan, M. J. 2015, Nature, 524, 322
Li, R., Youdin, A. N., & Simon, J. B. 2018, ApJ, 862, 14
Lissauer, J. J. 1987, Icarus, 69, 249
Liu, B. & Ormel, C. W. 2018, A&A, 615, A138
Manara, C. F., Rosotti, G., Testi, L., et al. 2016, A&A, 591, L3
Masset, F. S. 2017, MNRAS, 472, 4204
McNally, S. N., Raymond, S. N., & Paardekooper, S.-J. 2018, MNRAS, 477, 4596
Morbidelli, A., Bottke, W. F., Nesvorný, D., & Levison, H. F. 2009, Icarus, 204, 588
Morbidelli, A., Lambrechts, M., Jacobson, S., & Bitsch, B. 2015, Icarus, 258, 418
Mordasini, C., Alibert, Y., & Benz, W. 2009, A&A, 501, 1139
Morishima, S. & Ormel, C. W. 2013, ApJ, 771, 43
Okuzumi, S. & Ormel, C. W. 2013, A&A, 771, 43
Okuzumi, S., Tanaka, H., Kobayashi, H., & Wada, K. 2012, ApJ, 752, 106
Ormel, C. W. 2017, in Astrophysics and Space Science Library, Vol. 445, Astrophysics and Space Science Library, ed. M. Pessah & O. Gressel, 197
Ormel, C. W., Dullemond, C. P., & Spaans, M. 2010, ApJ, 714, L103
Ormel, C. W. & Klahr, H. H. 2010, A&A, 520, A43
Ormel, C. W. & Liu, B. 2018, A&A, 615, A178
Paardekooper, S.-J., Baruteau, C., & Kley, W. 2011, MNRAS, 410, 293
Paardekooper, S.-J., Baruteau, C., Crida, A., & Kley, W. 2010, MNRAS, 401, 1190
Paardekooper, S.-J., Baruteau, C., & Kley, W. 2011, MNRAS, 410, 293
Pérez, L. M., Chandler, C. J., Isella, A., et al. 2015, ApJ, 813, 41
Pinte, C., Dent, W. R. F., Mérand, F., et al. 2016, ApJ, 816, 25
Rafikov, R. R. 2004, AJ, 128, 1348
Raymond, S. N., Izidoro, A., Bitsch, B., & Jacobson, S. A. 2016, MNRAS, 458, 2962
Raymond, S. N., Kokubo, E., Morbidelli, A., Morishima, R., & Walsh, K. J. 2014, Protostars and Planets VI, 395
Ricci, L., Testi, L., Natta, A., et al. 2010, A&A, 512, A15
Ros, K. & Johansen, A. 2013, A&A, 552, A137
Schäfer, U., Yang, C.-C., & Johansen, A. 2017, A&A, 597, A69
Schöner, D. & Ormel, C. W. 2017, A&A, 602, A21
Schöner, D., Ormel, C. W., & Kruijssen, J. M. D. 2017, A&A, 598, A67
Simon, J. B., Armitage, P. J., Li, R., & Youdin, A. N. 2016, ApJ, 822, 55
Simon, J. B., Armitage, P. J., Youdin, A. N., & Li, R. 2017, ApJ, 847, L12
Tazzari, M., Testi, L., Ercolano, B., et al. 2016, A&A, 588, A53
Thommes, E. W., Duncan, M. J., & Levison, H. F. 2003, Icarus, 161, 431
Visser, R. G. & Ormel, C. W. 2016, A&A, 586, A66
Weidenschilling, S. J. 1977, MNRAS, 180, 57
Weidenschilling, S. J. 2011, Icarus, 214, 671
Wetherill, G. W. & Stewart, G. R. 1993, Icarus, 106, 190
Yang, C.-C., Mac Low, M.-M., & Johansen, A. 2018, ApJ, 868, 27
Yang, C.-C., Mac Low, M.-M., & Menou, K. 2012, ApJ, 748, 79
Youdin, A. N. & Goodman, J. 2005, ApJ, 620, 459
Youdin, A. N. & Lithwick, Y. 2007, Icarus, 192, 588
Zhu, Z., Stone, J. M., & Bai, X.-N. 2015, ApJ, 801, 81
Zsom, A., Ormel, C. W., Güttler, C., Blum, J., & Dullemond, C. P. 2010, A&A, 513, A57