Instantons and Monopoles for Nonperturbative QCD

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Abstract

We study the nonperturbative QCD in terms of topological objects, i.e., QCD-monopoles and instantons. In the 't Hooft abelian gauge, QCD is reduced into an abelian gauge theory with QCD-monopoles, and the confinement mechanism can be understood with the dual superconductor picture. Regarding the flux-tube ring as the glueball, we study the glueball properties by combining the dual Ginzburg-Landau theory and the Nambu-Goto action. We find the strong correlations between instantons and monopole world-lines both in the continuum theory and in the lattice QCD. The nonperturbative QCD vacuum is filled with instantons and monopoles, and dense instantons generate a macroscopic network of the monopole world-line, which would be responsible for the confinement force in the infrared region.

1 Introduction

Quantum chromodynamics (QCD) is the fundamental theory of the strong interaction \([1]-[3]\). In spite of the simple form of the QCD lagrangian,

\[ L_{\text{QCD}} = -\frac{1}{2} \text{tr} G_{\mu\nu} G^{\mu\nu} + \bar{q} (D - m_q) q, \]

it miraculously provides quite various phenomena like color confinement, dynamical chiral-symmetry breaking, non-trivial topologies, quantum anomalies and so on (Fig.1). It would be interesting to compare QCD with the history of the Universe, because a quite simple 'Big Bang' also created various things including galaxies, stars, lives and thinking reeds. Therefore, QCD can be regarded as an interesting miniature of the history of the Universe. This is the most attractive point of the QCD physics.

Since it is quite difficult to understand the various QCD phenomena and their underlying mechanism at the same time, many methods and models have been proposed to understand each phenomenon (Table 1). We show in Fig.2 a brief sketch on the history of QCD and typical QCD effective models \([0]-[3]\). In '80s, chiral symmetry breaking was the central issue. The chiral bag model, the NJL model and the \(\sigma\) model were formulated with referring chiral symmetry. In '90s, on the other hand, the confinement physics is providing an important current of the hadron physics, since recent lattice QCD studies \([4]\) shed a light on the confinement mechanism. The key word for the understanding of confinement is the "duality", which is recently paid attention by many theoretical particle physicists after Seiberg-Witten's discovery on the essential role of monopole condensation for the confinement in a supersymmetric version of QCD \([5]\). The origin of color confinement can be recognized as the dual Higgs mechanism by monopole condensation, and the nonperturbative QCD vacuum is regarded as the dual superconductor \([6]\). The dual Ginzburg-Landau (DGL) theory \([7]-[15]\) was formulated from QCD with this picture, and provides a useful framework for the study of the nonperturbative QCD.

2 Color Confinement and Dual Higgs Mechanism

We briefly show the modern current of the confinement physics. The QCD vacuum can be regarded as the dual version of the superconductor, which was firstly pointed out by Nambu,
Here, the “dual version” means the interchange between the electric and magnetic sectors. With referring Table 2, we compare the ordinary electromagnetic system, the superconductor and the nonperturbative QCD vacuum regarded as the dual superconductor.

In the ordinary electromagnetism in the Coulomb phase, both electric flux and magnetic flux are conserved, respectively. The electric-flux conservation is guaranteed by the ordinary gauge symmetry. On the other hand, the magnetic-flux conservation is originated from the dual gauge symmetry \[8\]-\[14\], which is the generalized version of the Bianchi identity. As for the inter-charge potential in the Coulomb phase, both electric and magnetic potentials are Coulomb-type.

The superconductor in the Higgs phase is characterized by electric-charge condensation, which leads to the Higgs mechanism or spontaneous breaking of the ordinary gauge symmetry, and therefore the electric flux is no more conserved. In such a system obeying the London equation, the electric inter-charge potential becomes short-range Yukawa-type similarly in the electro-weak unified theory. On the other hand, the dual symmetry is not broken, so that the magnetic flux is conserved, but is squeezed like a one-dimensional flux tube due to the Meissner effect. As the result, the magnetic inter-charge potential becomes linearly rising like a condenser.

The nonperturbative QCD vacuum regarded as the dual Higgs phase is characterized by color-magnetic monopole condensation, which leads to the spontaneous breaking of the dual gauge symmetry. Therefore, color-magnetic flux is not conserved, and the magnetic inter-change potential becomes short-range Yukawa-type. Note that the ordinary gauge symmetry is not broken by such monopole condensation. Therefore, color-electric flux is conserved, but is squeezed like a one-dimensional flux-tube or a string as a result of the dual Meissner effect. Thus, the hadron flux-tube is formed in the monopole-condensed QCD vacuum, and the electric inter-charge potential becomes linearly rising, which confines the color-electric charges \[8\]-\[15\].

As a remarkable fact in the duality physics, these are two “see-saw relations” \[13\] between the electric and magnetic sectors.

(1) There appears the Dirac condition \(eq = 4\pi\) \[8\] in QCD. Here, unit electric charge \(e\) is the gauge coupling constant, and unit magnetic charge \(g\) is the dual gauge coupling constant. Therefore, a strong-coupling system in one sector corresponds to a weak-coupling system in the other sector.

(2) The long-range confinement system in one sector corresponds to a short-range (Yukawa-type) interaction system in the other sector.

Let us consider usefulness of the latter “see-saw relation”. One faces highly non-local properties among the color-electric charges in the QCD vacuum because of the long-range linear confinement potential. Then, the direct formulation among the electric-charged variables would be difficult due to the non-locality, which seems to be a destiny in the long-distance confinement physics. However, one finds a short-range Yukawa potential in the magnetic sector, and therefore the electric-confinement system can be approximated by a local formulation among magnetic-charged variables. Thus, the confinement system, which seems highly non-local, can be described by a short-range interaction theory using the dual variables. This is the most attractive point in the dual Higgs theory.

Color-magnetic monopole condensation is necessary for color confinement in the dual Higgs theory. As for the appearance of color-magnetic monopoles in QCD, ‘t Hooft proposed an interesting idea of the abelian gauge fixing \[17\], \[18\], which is defined by the diagonalization of a gauge-dependent variable. In this gauge, QCD is reduced into an abelian gauge theory with the color-magnetic monopole \[8\], \[17\], which will be called as QCD-monopoles hereafter. Similar to the ‘t Hooft-Polyakov monopole \[13\] in the Grand Unified theory (GUT), the QCD-monopole appears from a hedgehog configuration corresponding to the non-trivial homotopy
group $\pi_2(\text{SU}(N_c)/\text{U}(1)^{N_c-1}) = \mathbb{Z}_{N_c-1}$ on the nonabelian manifold.

Many recent studies based on the lattice QCD in the 't Hooft abelian gauge show QCD-monopole condensation in the confinement phase, and strongly support abelian dominance and monopole dominance for the nonperturbative QCD (NP-QCD), e.g., linear confinement potential, dynamical chiral-symmetry breaking (DχSB) and instantons [4, 13, 14, 19-22]. Here, abelian dominance means that QCD phenomena is described only by abelian variables in the abelian gauge. Monopole dominance is more strict, and means that the essence of NP-QCD is described only by the singular monopole part of abelian variables [4, 13, 14, 19-22].

Figure 3 is the schematic explanation on abelian dominance and monopole dominance observed in the lattice QCD.

(a) Without gauge fixing, it is difficult to extract relevant degrees of freedom in NP-QCD.
(b) In the 't Hooft abelian gauge, QCD is reduced into an abelian gauge theory including the electric current $j_\mu$ and the magnetic current $k_\mu$. Particularly in the maximally abelian (MA) gauge [4, 19-22], only U(1) gauge degrees of freedom including monopole is relevant for NP-QCD, while off-diagonal components do not contribute to NP-QCD: abelian dominance. Here, the very long monopole world-line appears in the confinement phase. We show the lattice data of the monopole world-line in Fig.4.
(c) The U(1)-variable can be decomposed into the regular photon part and the singular monopole part [19-22, 23], which corresponds to the separation of $j_\mu$ and $k_\mu$. The monopole part leads to NP-QCD (confinement, DχSB, instanton): monopole dominance. On the other hand, the photon part is almost trivial similar to the QED system. Thus, monopoles in the MA gauge can be regarded as the relevant collective mode for NP-QCD, and therefore the NP-QCD vacuum can be identified as the dual-superconductor in a realistic sense.

3 Glueballs in the Dual Ginzburg-Landau Theory

In the dual Higgs theory, the monopole appears as a scalar glueball with $J^{PC} = 0^{++}$ like the Higgs particle in the standard model [3]. Here, the glueball is an elementary excitation without valence quarks in the NP-QCD vacuum, and is considered as the flux-tube ring in the DGL theory, while ordinary hadrons are described as open flux-tubes with the valence quarks at the ends. In this chapter, we study the glueball properties by analyzing the flux-tube ring solution in the DGL theory.

First, we calculate the energy $E_{cl}(R)$ of the classical ring solution with the radius $R$ in the DGL theory, and estimate the effective string tension $\sigma_{eff}(R)$ satisfying $E_{cl}(R) = 2\pi R\sigma_{eff}(R)$. Although the flux-tube ring seems to shrink at the classical level, this ring solution is stabilized against such a collapse by the quantum effect.

Second, regarding the flux-tube ring as a relativistic closed string with the effective string tension $\sigma_{eff}(R)$, we introduce the quantum effect to the ring solution using the Nambu-Goto (NG) action in the string theory. From the Hamiltonian of the NG action, we find the energy of the closed string as

$$E(R, P_R) = \sqrt{P_R^2 + (2\pi R\sigma_{eff}(R))^2},$$

where $P_R$ is the canonical conjugate momentum of the coordinate $R$.

Using the uncertainty relation $P_R \cdot R \geq 1$ as the quantum effect, the energy and radius of the ring solution are estimated as $M \approx 1.2$ GeV and $R \approx 0.22$ fm, respectively. Although the angular momentum projection related to the ring motion is needed to extract the physical glueball state with definite quantum numbers, the DGL theory can provide a useful method for the study of the glueball.
4 Correlation between Instanton and QCD-monopole

In the Euclidean metric, the “instanton” appears as the classical solution of QCD with the nontrivial topology corresponding to \( \pi_3(\text{SU}(N_c)) = \mathbb{Z}_\infty \), and is responsible for the \( \text{U}_A(1) \) anomaly or the large mass generation of \( \eta \)

Recent lattice studies \([13],[14],[20]-[22]\) indicate abelian dominance for nonperturbative quantities in the maximally abelian (MA) gauge. In the abelian-dominant system, the instanton seems to lose the topological basis for its existence, and therefore it seems unable to survive in the abelian manifold. However, even in the MA gauge, nonabelian components remain relatively large around the topological defect, i.e. monopoles, and therefore instantons are expected to survive only around the monopole world-lines in the abelian-dominant system \([13],[14],[20]-[22]\).

We have pointed out such a close relation between instantons and monopoles, and have demonstrated it in the continuum QCD with \( N_c = 2 \) using the MA gauge \([27]\) or the Polyakov-like gauge, where \( A_4(x) \) is diagonalized \([13],[14],[20]-[22]\).

We summarize our previous analytical works as follows. (1) Each instanton accompanies a monopole current. In other words, instantons only live along the monopole world-line. (2) The monopole world-line is unstable against a small fluctuation of the location or the size of instantons, although it is relatively stable inside the instanton profile. (3) In the multi-instanton system, monopole world-lines become highly complicated, and there appears a huge monopole clustering, which covers over the whole system, for the dense instanton system. (4) At a high temperature, the monopole world-lines are drastically changed, and become simple lines along the temporal direction.

We study also the correlation between instantons and monopoles in the MA gauge using the Monte Carlo simulation in the SU(2) lattice gauge theory \([13],[14],[20]-[22]\). The SU(2) link variable can be separated into the singular monopole part and the regular photon part as shown in Fig.3. Using the cooling method, we measure the topological quantities \( Q \) and \( I_Q \) in the monopole and photon sectors as well as in the ordinary SU(2) sector. Here, \( I_Q \equiv \int d^4x |\text{tr}(G_{\mu\nu}\tilde{G}_{\mu\nu})| \) corresponds to the total number \( N_{\text{tot}} \) of instantons and anti-instantons.

On the 16\(^4\) lattice with \( \beta = 2.4 \), we find that instantons exist only in the monopole part in the MA gauge, which means monopole dominance for the topological charge \([13],[22]\). Hence, monopole dominance is also expected for the \( \text{U}_A(1) \) anomaly and the large \( \eta \) mass.

We study the finite-temperature system using the 16\(^3\) \times 4 lattice with various \( \beta \) around \( \beta_c \approx 2.3 \) \([13],[22]\). We show in Fig.5 the correlation between \( I_Q(\text{SU}(2)) \) and \( I_Q(\text{Ds}) \), which are measured in the SU(2) and monopole sectors, respectively, after 50 cooling sweeps. The monopole part holds the dominant topological charge in the full SU(2) gauge configuration. On the other hand, \( I_Q(\text{Ph}) \), measured in the photon part, vanishes quickly by several cooling sweeps. Thus, monopole dominance for instantons is observed in the confinement phase even at finite temperatures \([13],[22]\). Near the critical temperature \( \beta_c \approx 2.3 \), a large reduction of \( I_Q \) is observed. In the deconfinement phase, \( I_Q \) vanishes quickly by several cooling sweeps, which means the absence of the instanton, in the SU(2) and monopole sectors as well as in the photon sector \([13],[22]\). Therefore, the gauge configuration becomes similar to the photon part in the deconfinement phase.

Finally, we study the correlation between the instanton number and the monopole loop length. Our analytical studies suggest the appearance of the highly complicated monopole world-line in the multi-instanton system \([13],[14],[20]-[22]\). We conjecture that the existence of instantons promotes monopole condensation, which is characterized by a long complicated monopole world-line covering over \( \mathbb{R}^4 \) observed in the lattice QCD \([13],[22]\). Using the lattice QCD, we measure the total monopole-loop length \( L \) and the integral of the absolute value of the topological density \( I_Q \), which corresponds to the total number \( N_{\text{tot}} \) of instantons and anti-instantons. We plot in Fig.6 the correlation between \( I_Q \) and \( L \) in the MA gauge after 3 cooling sweeps on the 16\(^3\) \times 4 lattice with various \( \beta \). A linear correlation is clearly found.
between $I_Q$ and $L$, although naive dimension counting suggests $I_Q^{1/4} = \text{const} \cdot L^{1/3}$.

In terms of the microscopic correlation, each instanton accompanies a small monopole loop nearby, whose length would be proportional to the instanton size $I_Q^{1/4} \cdot L^{1/3}$. Since instantons are dense in the confinement phase, monopole loops around them overlap each other, and there appears a macroscopic network of the monopole world-line, which bonds neighboring instantons [22,27-33]. In fact, the $NP$-QCD vacuum is filled with two sort of topological objects, instantons and monopoles, and dense instantons generate the macroscopic monopole clustering, which would be responsible for the confinement force in the infrared region.

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Fig. 1: A brief sketch of the QCD physics.
Fig. 2: A brief history QCD and QCD effective models.
Fig. 3: Abelian dominance and monopole dominance:
(a) Without gauge fixing, it is difficult to extract relevant degrees of freedom in NP-QCD.
(b) In the MA gauge, QCD is reduced into an abelian gauge theory including the electric current $j_\mu$ and the magnetic current $k_\mu$. Only U(1) gauge degrees of freedom including monopole is relevant for NP-QCD, while off-diagonal components do not contribute to NP-QCD: abelian dominance. Here, the very long monopole world-line appears in the confinement phase.
(c) The U(1)-variable can be decomposed into the regular photon part and the singular monopole part, which corresponds to the separation of $j_\mu$ and $k_\mu$. The monopole part leads to NP-QCD (confinement, $D\chi_{SB}$, instanton): monopole dominance. On the other hand, the photon part is almost trivial similar to the QED system.

Fig. 4: The monopole world-lines projected on the 3-dimensional space in the confinement phase at $\beta=2.2$ in the SU(2) lattice QCD with $16^3 \times 4$.
Fig. 5: The correlation between $I_Q(SU(2))$ and $I_Q(DS)$ in the MA gauge after 50 cooling sweeps on the $16^3 \times 4$ lattice; $\circ$ denotes in the confinement phase ($\beta=2.2$), $\times$ at the critical temperature ($\beta=2.3$) and $\triangle$ in the deconfinement phase ($\beta=2.35$). The monopole part holds instantons in the full SU(2) gauge configuration in the confinement phase, while there is no instanton in the SU(2) and monopole sectors in the deconfinement phase similarly in the photon sector.

Fig. 6: The SU(2) lattice QCD data for the correlation between the total monopole-loop length $L$ and $I_Q$ (the total number of instantons and anti-instantons) in the MA gauge. We plot the data at 3 cooling sweep on the $16^3 \times 4$ lattice with various $\beta$.

Table 1: Features of the lattice QCD and the QCD effective models

Table 2: The comparison among the ordinary electromagnetic system, the superconductor and the nonperturbative QCD vacuum regarded as the QCD-monopole condensed system.