Evidence for a Bilayer Quantum Wigner Solid

H. C. Manoharan, Y. W. Suen, M. B. Santos, and M. Shayegan

Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544

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Abstract

As the electronic charge distribution in a wide quantum well is tuned from a single-layer through an interacting bilayer configuration to weakly-coupled parallel layers, we observe an insulating phase concurrently manifesting a dramatic evolution. The data reveal that interlayer interactions, playing a crucial role, are able to stabilize a correlated bilayer electron insulator, thus providing tantalizing evidence of a pinned bilayer Wigner solid phase crystallizing at total filling factor $\nu$ as large as 0.54 ($\nu > \frac{1}{4}$ in each layer).

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It is possible to fundamentally alter the many-body ground states of a two-dimensional electron system (2D ES) at high magnetic fields \( B \) through the introduction of an additional degree-of-freedom. For example, the addition of a spin degree-of-freedom enables the formation of particular spin-unpolarized fractional quantum Hall (FQH) states observed at lower \( B \) [1,2], while radically increasing the perpendicular spatial degree-of-freedom leads to a weakening and eventual collapse of the FQH effect [3,4]. Here we report magnetotransport measurements on an interacting bilayer ES confined in a wide quantum well. In this system, the additional layer degree-of-freedom stabilizes new FQH states such as the even-denominator incompressible liquids at Landau level filling factors \( \nu = \frac{1}{2} \) [5–7] and \( \nu = \frac{3}{2} \) [6], which have no counterpart in standard single-layer 2D ESs. Our data reveal an intriguing interplay between the FQH effect and an insulating phase (IP) that displays behavior profoundly different from any observed in a standard 2D ES. In this paper we focus on this IP, which we demonstrate evolves into a correlated bilayer electron insulator with intralayer and interlayer interactions holding equal significance, thus providing unique evidence for a pinned bilayer Wigner crystal (WC).

The sample, grown by molecular beam epitaxy, consists of a 750 Å GaAs quantum well flanked by Al\(_{0.35}\)Ga\(_{0.65}\)As spacers and Si \( \delta \)-doped layers. When cooled to low temperature \( (T) \) in a dilution refrigerator, this sample has typical dark density \( n \simeq 1 \times 10^{11} \) cm\(^{-2} \) and mobility \( \mu \simeq 1 \times 10^6 \) cm\(^2\)/Vs. Both \( n \) and the charge distribution symmetry are controlled via front- and back-side gates [5,6,8]. In the density range spanned by our experiments, \( 3.7 \times 10^{10} < n < 19.0 \times 10^{10} \) cm\(^{-2} \), at most two subbands are occupied. As electrons are added to the wide quantum well, their electrostatic repulsion causes them to pile up near the sides of the well, and the resulting electron distribution appears increasingly bilayer-like as \( n \) grows [Fig. 1(inset)] [8]. For symmetric charge distributions, i.e. “balanced” states, two relevant parameters that quantify this evolution are the symmetric-to-antisymmetric energy gap \( \Delta_{SAS} \), which is a measure of the coupling between the two layers, and the interlayer distance \( d \) [Fig. 1(inset)]. Experimentally, \( \Delta_{SAS} \) is deduced from Fourier transforms of the low-\( B \) Shubnikov-de Haas oscillations at various \( n \). We then perform self-consistent
Hartree-Fock calculations at zero $B$, with the well width as a single fitting parameter, to match the calculated $\Delta_{\text{SAS}}$ with the measured values. In general, the agreement between the calculated and measured $\Delta_{\text{SAS}}$ is excellent [5,6], and $d$ may then be reliably deduced from the calculated charge distributions [4]. A crucial property of the ES in a wide quantum well is that, for a given well width, both $\Delta_{\text{SAS}}$ and $d$ depend on $n$: increasing $n$ makes $d$ larger and $\Delta_{\text{SAS}}$ smaller so that the system can essentially be tuned from a bilayer ES at high $n$ to a (thick) single-layer-like system by decreasing $n$ [Fig. 1(inset)]. This evolution with $n$ plays a critical role in the properties of the correlated electron states.

The evolution of the FQH states in this system has been studied recently [6]. As this evolution provides a context for understanding the IP, we briefly summarize now the overall behavior. At the lowest $n$ the data exhibit the usual FQH effect at odd-denominator fillings, while at the highest $n$ the strongest FQH states are those with even numerators, as expected for a system of two 2D layers in parallel. For intermediate $n$, even-denominator FQH states at $\nu = \frac{1}{2}$ and $\frac{3}{2}$, which are stabilized by both interlayer and intralayer correlations, are observed. Concurrent with this evolution of the FQH states, we observe an IP which moves to higher $\nu$ as $n$ is increased.

Before discussing the FQH state evolution in more detail, it is instructive to first examine the spectacular evolution of this IP. Its behavior is summarized in Fig. 1, where the diagonal resistivity $\rho_{xx}$ at base $T$ ($\simeq 25$ mK) is plotted vs $\nu^{-1} \propto B$ for several representative $n$. Experimentally, the IP is identified by a $\rho_{xx}$ that is both large ($> h/e^2 \simeq 26$ kΩ/□, the quantum unit of resistance) as well as strongly $T$-dependent. For very low $n$, the IP appears near $\nu = \frac{1}{5}$ (trace A), while at the highest $n$ there is an IP for $\nu \lesssim \frac{1}{2}$. The IP observed in the intermediate density range ($10 \times 10^{10} \lesssim n \lesssim 14 \times 10^{10}$ cm$^{-2}$) is most remarkable as it very quickly moves to larger $\nu$ with small increases in $n$ (see, e.g., traces B, C, and D); along the way, it also shows reentrant behavior around well-developed FQH states at $\nu = \frac{2}{7}$ (trace B), $\nu = \frac{1}{3}$ (traces C and D), and $\nu = \frac{1}{2}$ (bold trace E). Then, as $n$ increases past this point, the IP begins to move in the opposite direction to lower $\nu$ (trace F).

The data in Fig. 1 for $n = 12.6 \times 10^{10}$ cm$^{-2}$ are plotted in more detail in Fig. 2, along
with the associated Hall resistivity and the $T$-dependence of both the reentrant IP peak (showing a diverging $\rho_{xx}$ as $T \to 0$) and the $\nu = \frac{1}{2}$ minimum (exhibiting activated behavior characteristic of a FQH liquid state with finite energy gap). As a whole, the data of Fig. 2 bear a striking resemblance to the IP observed reentrant around $\nu = \frac{1}{5}$ in low-disorder, single-layer 2D ESs, generally interpreted as a pinned Wigner solid \cite{10}; here, however, the IP is reentrant around the bilayer $\nu = \frac{1}{2}$ FQH state, with the reentrant peak reaching the prominently high filling of $\nu = 0.54$!

The IPs presented in Fig. 1 cannot be explained by single-particle localization. First, in the case of standard, single-layer 2D ESs it is well known that as $n$ is lowered, the quality of the 2D ES deteriorates and the sample shows a disorder-induced IP at progressively larger $\nu$ \cite{11}. This is opposite the behavior observed here: as $n$ decreases from $19.0 \times 10^{10}$ to $3.7 \times 10^{10}$ cm$^{-2}$, the quality worsens somewhat as expected (e.g. mobility decreases monotonically from $1.4 \times 10^6$ to $5.3 \times 10^5$ cm$^2$/Vs) but the IP moves to smaller $\nu$. Second, the observation of IPs which are reentrant around correlated FQH states, and particularly around the very fragile and disorder-sensitive $\nu = \frac{1}{2}$ state \cite{6}, strongly suggests that electron interactions are also essential for stabilizing the IP.

To illustrate that the behavior of this IP is indeed consistent with the WC picture, it is important to elaborate on the evolution of FQH states in an ES confined in a wide quantum well \cite{6}. This evolution can be understood by examining the competition between (1) $\Delta_{\text{SAS}}$, (2) the in-plane correlation energy $Ce^2/\ell_B$ [where $C$ is a constant $\sim 0.1$ and $\ell_B \equiv (\hbar/eB)^{1/2}$ is the magnetic length], and (3) the interlayer Coulomb interaction ($\sim e^2/\epsilon d$). To quantify behavior it is useful to construct the ratios $\gamma \equiv (e^2/\ell_B)/\Delta_{\text{SAS}}$ and $(e^2/\ell_B)/(e^2/\epsilon d) = d/\ell_B$. As $n$ is increased, $\gamma$ increases since both $\Delta_{\text{SAS}}$ and $\ell_B$ (for a FQH state at a given $\nu$) decrease, and $d/\ell_B$ increases. Experiments show that when $\gamma$ is small, the ES exhibits only “one-component” (1C) FQH states (standard single-layer odd-denominator states) constructed solely from the symmetric subband, while for large $\gamma$ the in-plane Coulomb energy becomes sufficiently strong to allow the antisymmetric subband to mix into the correlated ground state to lower its energy, and a “two-component” (2C) state ensues. These 2C states,
constructed out of the now nearly degenerate symmetric and antisymmetric basis states, come in two classes. For large $d/\ell_B$, the ES behaves as two independent layers in parallel, each with density $n/2$; FQH states in this regime therefore have even numerator and odd denominator. For small enough $d/\ell_B$, the interlayer interaction can become comparable to the in-plane interaction and a fundamentally new kind of FQH state becomes possible. Such a state has strong interlayer correlation and can be at even-denominator $\nu$. A special example is the $\Psi_{331}$ state associated with the $\nu = \frac{1}{2}$ FQH state observed in bilayer ESs with appropriate parameters [5–7].

We have determined the quasiparticle excitation gaps $\Delta_\nu$ of several FQH states in the current system for several $n$ via thermal activation measurements; these gaps are plotted vs $\gamma$ in Fig. 3(a). As expected, we observe that increasing $\gamma$ suppresses 1C states and enhances 2C states. Two states, $\nu = \frac{2}{3}$ and $\nu = \frac{4}{3}$, undergo a 1C to 2C phase transition as $\gamma$ is increased [3], as evidenced by sharp minima in their gaps. The critical point for this transition, $\gamma \simeq 13.5$, matches the point where the gaps of other 1C and 2C states emerge from zero. Surrounding this point is a region where the $\nu = \frac{1}{2}$ FQH liquid stabilizes. Note that since this 2C state also possesses interlayer correlation (the 2C $\nu = \frac{2}{3}$ and $\frac{4}{3}$ states are simply $\frac{1}{3}$ and $\frac{2}{3}$ states in parallel layers), it exists only within a finite range of $\gamma$. The relevance of this plot to the IP is immediately highlighted by examining the three main reentrant peaks in Fig. 1 (from traces B, D, and E), which appear at $\nu = 0.30, 0.39,$ and 0.54 for the IPs surrounding the $\nu = \frac{2}{7}, \frac{1}{3},$ and $\frac{1}{2}$ FQH states, respectively. The values of $\gamma$ at these reentrant peaks are respectively 16.9, 16.3, and 16.5. The peak positions span a large range of $\nu$, and yet the associated $\gamma$ are remarkably similar. Moreover, at this value of $\gamma \simeq 16.5$, interlayer interactions are clearly important as this point is straddled by the 2C $\frac{1}{2}$ state in Fig. 3(a).

The construction of a phase diagram for the observed IPs facilitates a lucid connection between the IP evolution, the 1C to 2C transition, and the development of the $\nu = \frac{1}{2}$ liquid. To this end, we first collected a $\rho_{xx}$ dataset for a fairly dense grid of points in the $n - B$ plane by incrementally changing $n$ and sweeping $B$ at base $T$. Next, $\rho_{xx}$ was mapped to
a color interpolating between blue \((\rho_{xx} = 0)\) and red \((\rho_{xx} \geq h/e^2)\). Finally, using known values for \(B\), \(n\), and \(\Delta_{\text{SAS}}\) at each point, the color-mapped \(\rho_{xx}\) dataset was plotted vs \(\nu\) and \(\gamma\) [Fig. 3(b)]. By utilizing \(h/e^2\) as a natural resistivity scale for demarcating the IP and non-insulating states [12], the result is a comprehensive phase diagram depicting incompressible phases (dark blue) together with compressible phases, both insulating (dark red) and metallic (all other colors).

Immediately obvious in the phase diagram are the various FQH transitions, manifested by the appearance or disappearance of dark blue FQH phases at several \(\nu\) (see, e.g., \(\frac{3}{5}, \frac{4}{5}\), and the \(\frac{1}{2}\) “island”), or by a change in vertical width of the FQH phase (see, e.g., \(\frac{2}{3}\)); these transitions correlate directly with the measured energy gaps [Fig. 3(a)]. Another striking feature is the wrinkling in the IP boundary: this is caused by the reentrance of the IP around several FQH states as discussed earlier. The limiting critical \(\nu\) at low \(n\) is close to \(\frac{1}{5}\), consistent with a low-disorder single-layer 2D ES, while for the highest \(n\), where the ES is effectively two layers in parallel, the IP is present for \(\nu \lesssim \frac{1}{2}\), i.e. \(\nu \lesssim \frac{1}{4}\) in each layer. This is reasonable considering that even at the largest \(n\), interlayer interactions are present in this ES as evidenced by the observation of a correlated \(\nu = 1\) QH state at high \(n\) in the same well [14]. Such interactions can move the WC ground state to \(\nu \simeq \frac{1}{4}\) (for each layer), somewhat larger than \(\frac{1}{5}\) expected if there are no interlayer interactions (see below). We note, however, that our measurements on wider quantum well samples indicate that in the high-\(n\) limit, the onset of the IP indeed approaches \(\nu \simeq \frac{2}{5}\), consistent with two high-quality parallel layers becoming insulating near \(\frac{1}{5}\) filling in each layer in the absence of interlayer interactions. This regime is evidently outside the density limits of our current sample.

We can examine in more detail the evolution of the IP as depicted in the phase diagram of Fig. 3(b) by making comparisons to Fig. 3(a). For intermediate \(n\), as \(\gamma\) increases, the IP first remains close to \(\nu = \frac{1}{5}\) but then begins to move to higher \(\nu\) in the range \(12 < \gamma < 15\). This range is precisely bisected by \(\gamma \simeq 13.5\) [Fig. 3(a)] where the 1C to 2C transition occurs. Then the IP moves very quickly to \(\nu \simeq \frac{1}{2}\) as evidenced by the nearly vertical phase boundary at \(\gamma \simeq 16\). As discussed earlier and as evident from Fig. 3, this \(\gamma\) is centrally located in
the parameter range in which the $\frac{1}{2}$ state stabilizes. A quick glance at the phase diagram underscores this central point: the $\gamma$-extent of the $\nu = \frac{1}{2}$ island coincides directly with the rapid $\nu$-shift in the phase boundary of the insulator.

Up to now we have focused exclusively on symmetric (“balanced”) charge distributions. Intuitively, however, similar to the 2C FQH states [6], the strength of a bilayer WC should be diminished under unbalanced conditions. This is indeed observed quite prominently in our system, and can be highlighted by examining the high-$\nu$ reentrant IPs. Figure 4 shows the effect of asymmetry on the IP reentrant around $\nu = \frac{1}{3}$ at fixed $n = 11.0 \times 10^{10}$ cm$^{-2}$ and for varying $n_t$, where $n_t$ is the electron density transferred from the back layer to the front by proper gate biasing from the balanced condition. It is very clear that, while the 1C $\nu = \frac{1}{3}$ state is strengthened as expected, the IP is weakened by increasing imbalance $|n_t|$: the IP is most stable in a perfectly balanced state, and the IP peak at $\nu = 0.38$ drops dramatically even for a small imbalance $n_t = 4.6 \times 10^9$ cm$^{-2}$. As $n_t$ is increased past $\simeq 7 \times 10^9$ cm$^{-2}$, the reentrant IP is destroyed ($\rho_{xx} < h/e^2$) and the $\frac{1}{2}$ state disappears. For the IP reentrant around $\nu = \frac{1}{2}$ at $n = 12.6 \times 10^{10}$ cm$^{-2}$, the corresponding destabilization of the insulator (not shown here) occurs at an imbalance of less than 3% ($|n_t|/n \simeq 0.027$). In all cases, note that both the $\nu = \frac{1}{2}$ state and the reentrant IP are strongest in the balanced condition; asymmetry simultaneously destroys both the bilayer quantum liquid and the insulator.

Finally, it is plausible that interlayer interactions can modify the ground-state energies so that for appropriate parameters a crossing of the liquid and solid states occurs at the large fillings we identify (e.g., $\nu = 0.54$, i.e. $\nu > \frac{1}{4}$ in each layer). Calculations [14] indicate that the effect of interlayer Coulomb interaction can be particularly strong near the magnetoroton minimum and lead to the vanishing of the FQH liquid gap. This vanishing can be associated with an instability toward a ground state in which each of the layers condenses into a 2D WC [14].

Before concluding, it is beneficial to set this work in context with previously-reported reduced-dimensional insulators [10]. Our bilayer electron system provides a unique means of tuning the effective electron-electron interactions underpinning the formation of various
many-particle ground states. The crux of this reasoning is that this system possesses two vital “yardsticks” for gauging the relative importance of inter- and intralayer interactions: the 1C to 2C transition and the novel bilayer $\nu = \frac{1}{2}$ condensate. Utilizing these unimpeachable measuring sticks, we can connect the fascinating evolution of the IP with the significance and critical counterbalance of electron-electron interactions. In doing so, we believe this work transcends specificity to bilayer systems, and provides convincing evidence that electron-electron correlations are a compulsory component of IPs observed in all 2D systems possessing similarly low disorder.

In summary, our data conclusively indicates that the IP we observe for $\gamma \gtrsim 13$ is a collective 2C state with comparable interlayer and intralayer correlations. The characteristics of this bilayer electron insulator are remarkably consistent with the formation of a novel pinned bilayer-correlated Wigner solid, a unique 2D electron crystal stabilized through the introduction of an additional quantum degree-of-freedom.

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FIGURES

FIG. 1. Evolution of the IP at $T \simeq 25$ mK for varying $n$ (specified in units of $10^{10}$ cm$^{-2}$ within legend). Inset: Conduction band potentials (solid curves) and electron density profiles (dotted curves) from self-consistent Hartree-Fock simulations for increasing $n$.

FIG. 2. Diagonal and Hall resistivity vs $B$ at $n = 12.6 \times 10^{10}$ cm$^{-2}$, highlighting the reentrant IP around the bilayer $\nu = 1/2$ FQH liquid. Inset: $T$-dependence of the $\nu = 1/2$ minimum and the reentrant peak at $\nu = 0.54$.

FIG. 3. (a) Measured energy gaps $\Delta_\nu$ of several FQH states vs $\gamma$. The number of components (C) in each state is shown in parentheses. (b) Phase diagram showing $\rho_{xx}$, chromatically mapped according to the colorbar (right), vs $\nu$ and $\gamma$.

FIG. 4. Effect of asymmetry on the reentrant IP. Traces are at fixed total $n$, with varying amounts of charge $n_t$ transferred between layers. Corresponding traces for negative $n_t$ show equivalent behavior: slight imbalance $|n_t|$ destabilizes the IP.