The two-dimensional fretting contact with a bulk stress. Part II – Loading history and dissimilar elastic materials

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Abstract. The contact model for similarly elastic materials advanced in the companion paper is enhanced to account for the more general case of contacting bodies with different elastic properties. The latter contact model, involving dissimilarly elastic bodies under partial slip, is particularly difficult to solve due to the mutual interaction between the normal and the shear tractions. As opposed to the contact of similar elastic materials, pressure is affected by the shear tractions and vice versa. Therefore, the contact submodels in the normal and in the tangential direction are not independent. The connection can only be overcome in an iterative manner, by adding a new external loop to the contact algorithm, in which the solutions of the systems in the normal and in the tangential directions are mutually adjusted by successively solving both systems until convergence is reached. Moreover, with friction being a dissipative process, any state depends upon the process path, i.e., on all previous states attained during the loading history. The contact model is solved as a series of quasi-static states that are expected to replicate the loading path. Two levels of iterations are added to the model of similarly elastic materials, described in the companion paper, to account for the dissimilarity in the elastic properties. The contact process is reproduced on a prolonged loading history, consisting in the first fretting loops. The distributions of contact tractions are obtained using a technique for the fast computation of convolutions involving the fast Fourier transform. The conducted simulations prove the method robustness and its application in the modelling of the fretting processes.

1. Introduction

A fretting contact solver for similar elastic materials proposed in the companion paper is extended in this work to account for an oscillating tangential force and a discrepancy in the elastic properties of the contacting materials. These conditions are specific to fretting processes and require numerical treatment, as the existing analytic solutions cannot handle the connection between the normal and the shear contact tractions. Semi-analytical methods, relying on fundamental solutions and iterative approaches, provide the needed computational efficiency and precision, especially when coupled with spectral methods for efficient calculation of the convolutions arising in displacement computation [1,2].

Chen and Wang [3] advanced an algorithm for the point contact of dissimilar elastic bodies considering tangential tractions, and presented results for increasing tangential loading without reproduction of the loading history. A contact model allowing the modelling and simulation of fretting wear under gross slip and partial slip conditions was later advanced by Gallego and Nélías [4], and...
then extended [5] to an efficient algorithm for fretting contacts with application to fretting modes I, II and III. The importance of the loading history simulation was clearly validated through numerical experimentations. Spinu and Glovnea [6] later advanced a numerical solution for the three-dimensional fretting contact between dissimilarly elastic materials, whereas Spinu and Frunza [7] studied the hysteretic behaviour of partial slip elastic contacts undergoing a fretting loop. They found that the fretting contact between dissimilar elastic materials exhibits a unique path in the first two loading cycles, and then stabilizes to a fixed trajectory, as in case of similarly elastic materials.

An iterative solution to the 2D slip-stick contact of dissimilar elastic bodies is promoted in this paper, aiming to a better understanding of the fretting contact process in the presence of a supplementary bulk stress. The analysed contact scenario fits perfectly experimental rigs used in the study of fretting, and therefore can be a useful tool for advancing the understanding of the fretting phenomena.

2. Coupling of normal and tangential effects
The starting point in the solution of the fretting contact between dissimilar elastic materials is the algorithm advanced in the companion paper for similar elastic materials. In the latter case, the decoupling between the submodels in the normal and in the tangential direction results in vanishing contribution of one type of contact tractions to the perpendicular displacement: shear tractions does not affect the relative normal displacement, whereas pressure in its turn does not change the relative tangential displacement. As a result, the pressure distribution can be calculated without knowing the shear stresses, and, more important, the pressure distribution is not affected by changes in the shear tractions as long as the normal force is kept constant. Consequently, the pressure distribution in a spherical contact subjected constant normal load and an oscillating tangential force, as in the case of fretting, follows the Hertz framework and is not affected by the frictional coefficient or by the level of the tangential force. The problem in the tangential direction is also simplified, as pressure is needed only to assess shear tractions in the slip region, where Coulombian friction is assumed. The size of the stick region and the associated shear tractions distribution depend on the relative displacement field in the tangential direction, whose magnitude varies with the level of the tangential force and is independent of the applied pressure. This decoupling between the normal and the shear tractions in displacement calculation is best illustrated by the mathematical model. Let $K^{(i)}_{ij}$, with i, j = x, z and $\ell = 1, 2$, denote the influence coefficients for the contribution of the contact traction along direction j to the displacement in the i direction, calculated for the $\ell$ contacting body. It follows that the relative displacement field $u$ in the frictional contact results as:

$$
\begin{bmatrix}
  u_x \\
  u_z 
\end{bmatrix}
= \begin{bmatrix}
  u^{(2)}_x \\
  u^{(2)}_z 
\end{bmatrix}
- \begin{bmatrix}
  u^{(1)}_x \\
  u^{(1)}_z 
\end{bmatrix}
= \begin{bmatrix}
  K^{(2)}_{xx} & K^{(2)}_{xz} \\
  K^{(2)}_{zx} & K^{(2)}_{zz} 
\end{bmatrix}
\otimes
\begin{bmatrix}
  q^{(2)}_x \\
  q^{(2)}_z 
\end{bmatrix}
- \begin{bmatrix}
  K^{(1)}_{xx} & K^{(1)}_{xz} \\
  K^{(1)}_{zx} & K^{(1)}_{zz} 
\end{bmatrix}
\otimes
\begin{bmatrix}
  q^{(1)}_x \\
  q^{(1)}_z 
\end{bmatrix}.
$$

With $q$ and $p$ the contact tractions, shear and pressure, and symbol $\otimes$ denotes convolution product. Considering the relations between the contact tractions at the contact interface, i.e. $p^{(1)} = p^{(2)}$ and $q^{(1)} = -q^{(2)}$, equation (1) simplifies to:

$$
\begin{bmatrix}
  u_x \\
  u_z 
\end{bmatrix}
= \begin{bmatrix}
  K^{(2)}_{xx} + K^{(1)}_{xx} & K^{(2)}_{xz} - K^{(1)}_{xz} \\
  K^{(2)}_{zx} - K^{(1)}_{zx} & K^{(2)}_{zz} + K^{(1)}_{zz} 
\end{bmatrix}
\otimes
\begin{bmatrix}
  q^{(2)}_x \\
  q^{(2)}_z 
\end{bmatrix}.
$$

As the formulas for the influence coefficient depend on the elastic properties of the material (i.e., Young modulus and Poisson’s ratio), the anti-diagonal terms in the influence coefficients matrix from equation (2) vanish only for similar elastic materials. Equation (2) proves that all contact tractions are needed to perform calculation of displacement in any of the two directions. This leads to the coupling of the contact subproblems: shear stress, a priori unknown, is needed to find the pressure distribution, and vice versa. An iterative level is implemented in the numerical solution to overcome this dependence.
3. Loading history
The model equations are developed under the assumption of a quasi-static contact. However, slip is a process that is essentially history-dependent. The solution of the Cattaneo-Mindlin [8,9] problem proves that, when the tangential force is increased, the slip region expands from the outer boundary of the contact area into the central stick zone. However, when the tangential force is decreased, the process is not reversed, i.e., the slip annulus does not gradually retract, and the previously attained boundaries between slip and stick are not followed. Instead, instantaneous stick occurs everywhere on the contact area, followed by slip in the opposite direction beginning from the contact area boundary. If the tangential force is further decreased until its amplitude is reached with a changed sign, the reversed slip region covers gradually the original (i.e., during loading) slip annulus, and the shear stress distribution is a reversal (i.e., equal, but of opposite sign) of that corresponding to the maximum level of the tangential force during loading. Consequently, the states attained during unloading are different from those achieved during loading, which is the feature of an irreversible process. This is to be expected, considering that friction is a dissipative process.

The loading history depicted in figure 1 is considered in the simulations described in this paper. The time span of the loading window is denoted by \( \tau \), and dimensionless times are defined as ratio to the latter parameter. It should be noted, however, that the contact is assumed quasi-static and studies of dynamic effects are beyond the point of this paper. The normal force is initially increased from zero until its nominal value \( W_{\text{max}} \) is reached, and subsequently the tangential force magnitude is oscillated with an amplitude lower than the one inducing full slip: \( W_{\text{max}}/(\mu W_{\text{max}}) < 1 \). In the initial normal indentation, i.e. \( 0 \leq t/\tau < 0.1 \), radial slip (fretting mode II) may occur only if there is a mismatch in the elastic properties of the contacting bodies. For the subsequent interval \( 0.1 \leq t/\tau < 1 \), tangential slip (fretting mode I) is expected regardless of the elastic mismatch. In the case of similar elastic materials, pressure is unaltered by the shear tractions and therefore is dictated by the Hertz framework. Once the final level \( W_{\text{max}} \) is achieved, the final pressure profile is attained, and further oscillation in the tangential force has no effect on pressure. The resulting contact Hertz parameters, i.e. the Hertz pressure \( p_H \) and the contact radius \( a_H \), are used as normalizers. A fretting loop can thus be simulated by calculating only the first loop. The assessment of any state relies on the calculation of intermediate states in which the increment of the tangential force changes sign. The normalized normal and shear tractions are depicted in figure 2. Due to this periodicity, the profiles in figure 2 characterize any fretting loop achieved during the oscillation of the tangential force. A good agreement with the theoretical framework [10,11] is found.

![Figure 1. The loading history.](image1)

![Figure 2. Shear tractions \( q/(\mu p_H) \) in the fretting contact of similar elastic materials.](image2)
The presence of a constant tensile bulk stress $\sigma$ in one of the contacting bodies, applied simultaneously with the tangential force, perturbs the shear stress distribution in the latter body as shown in figure 3. However, reproduction of the loading history only implies calculation of the states when the tangential force changes sign, i.e. $t/\tau = 0.2; 0.4; 0.6; 0.8$. A periodicity in stress distribution is also attained, as suggested by the overlapping of the curves corresponding to time moments separated by a $t/\tau = 0.4$ gap.

Figure 3. Shear tractions $q/(\mu p_H)$ in the presence of a bulk stress: (a) $\sigma = 0.5 \mu p_H$; (b) $\sigma = 2 \mu p_H$.

4. Dissimilar elastic materials

The case of dissimilar elastic materials is more complicated, requiring the reproduction of the entire loading history as proved by Spinu and Frunza [7] for the 3D fretting contact. This replication can be achieved by applying the load in small increments, and by computing a series of contact states corresponding to each new loading increment. The starting point for the simulation is not, however, the application of the tangential force. Due to dissimilarity in the elastic properties of the materials, shear tractions also arise during the initial normal indentation (i.e., for $0 \leq t/\tau < 0.1$), perturbing the pressure distribution, which is no longer follows the Hertz framework. An additional algorithm loop is thus needed for the loading history simulation.

The previous sections suggest that the case of dissimilar materials thus requires additional iterative level compared to the contact of similar elastic materials. A three-level iteration scheme is thus proposed for the solution of the 2D fretting contact. As described in the companion paper, the inner iterative level is charged with the solution of the normal or the tangential contact problems, when the contact tractions in the tangential and in the normal directions, respectively, are known or can be neglected. The pressure or shear tractions solution is thus obtained with the algorithm proposed by Polonsky and Keer [12], by taking as input the tractions in the conjugated direction:

$$p = p(q) \quad \text{and} \quad q = q(p) .$$  \quad (3)

However, when first solving equation (3) with the pressure distribution as unknown, there is no available estimate for the shear tractions. This coupling between the contact tractions suggested by equation (3), makes the slip-stick contact problem between dissimilar elastic materials unsolvable by analytical techniques [10]. Numerical iteration can be employed, consisting in solving the two dependencies in equation (3) successively, until convergence of pressure is reached. By denoting the iteration number with an upper index, the algorithm proposed for the intermediate loop can be summarized as:

1. Adopt the initial guess for the shear tractions: $q^{(0)} = 0$. 

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2. Solve the contact problem in the normal direction with known shear tractions: 
\[ p^{(k)} = p(q^{(k-1)}) \] at the \( k \)th iteration.
3. Solve the contact in the tangential direction with known pressure: 
\[ q^{(k)} = q(p^{(k)}) \] at the \( k \)th iteration.
4. Solve again the contact problem in the normal direction with known shear tractions, but with
the newly obtained, more precise, shear tractions: 
\[ p^{(k+1)} = p(q^{(k)}) \].
5. Compare \( p^{(k)} \) with \( p^{(k+1)} \), if convergence not reached, return to step 3 with incremented \( k \).

This iterative process convergences in a few iterations to the solution of the slip-stick contact
between dissimilar elastic materials. This solution, however, is obtained without taking into
consideration the history of the contact process. As proved in [7], reproduction of the loading history
guarantees the solution accuracy. Fretting contact conditions may refer to: (1) tangential fretting –
constant normal force and oscillating tangential force; (2) radial fretting – variable normal force, or (3)
torsional fretting – constant normal force and oscillating torsional moment. Only the first two cases are
considered in this paper for brevity. The reproduction of the loading path is achieved by incremental
load application. The radial fretting scenario is solved first, during which the normal force is increased
from zero to the nominal value. During this loading path, although no tangential force is applied, the
discrepancy in the tangential displacements of the contacting bodies gives birth to shear tractions. The
contact state at the end of the normal indentation thus requires reproduction of the loading history, as
friction dictates the outcome. The tangential fretting process is subsequently replicated by
incrementing the tangential force using positive or negative increments. The scheme for the outer
iterative level can be outlined as:

1. Acquire the contact state at the previous iteration: get the contact tractions \( (p^{(k-1)}, q^{(k-1)}) \) that
verify all contact equations for the loading level \( (W^{(k-1)}, T^{(k-1)}) \).
2. Apply a new load increment: \( (\Delta W, 0) \) for radial fretting or \( (0, \Delta T) \) for tangential fretting.
Obtain the new loading level \( (W^{(k)}, T^{(k)}) \).
3. Obtain the contact tractions \( (p^{(k)}, q^{(k)}) \) that verify all contact equations for the current
loading level.
4. Repeat step 2 until the desired loading history is fulfilled.

This three level nested loop scheme can be employed to reproduce contact processes for prolonged
periods of time with a moderate computational effort.

A first simulation addresses the normal indentation scenario (fretting mode II), aiming to prove the
importance of the simulation of the loading history in the fretting contact between dissimilar elastic
materials. A spherical rigid indenter is pressed against an elastic half-space. As opposed to the contact of
similar elastic materials, a difference in the tangential displacements induced by pressure on the
contacting surfaces of the two bodies will lead to shear tractions, although no tangential force is applied.
The latter tractions will in their turn affect the pressure distribution, although the effect is generally
small. Figure 4 compares the contact tractions for \( t/\tau = 0.1 \), obtained in one step, with the one predicted
by the nested-loop scheme described in the previous section. The importance of the loading history
reproduction on the accurate calculations of shear tractions is clearly illustrated by this example. Figure
4(a) shows that the maximum central pressure is increased compared to the reference Hertz case.

The effect of a bulk stress in the contact of dissimilar materials, subjected to fretting mode I, is
depicted in figure 5. The employed loading history covers the time span presented in figure 1, and the
bulk stress is assumed to be applied in a step loading at \( t/\tau = 0.1 \), and then kept constant. The
predicted tractions distributions suggest that the bulk stress has a significant and complex contribution
to the contact process. In all cases, a periodicity is attained after the first loading, which is unique.
Figure 4. Fretting mode II of dissimilar materials: (a) pressure; (b) shear tractions.

Figure 5. The effect of a bulk stress in the fretting mode I of dissimilar materials:
(a) $\sigma = 0$; (b) $\sigma = 0.5\mu p_H$; (c) $\sigma = \mu p_H$; (d) $\sigma = 2\mu p_H$. 
5. Conclusions
A numerical solution to the slip-stick two-dimensional contact problem is achieved in this paper based on fundamental solutions for point forces acting on the half-plane boundary. Some limiting assumptions present in the analytical frameworks are relieved in this model: (1) the contacting bodies are allowed to have arbitrary yet known contact geometry, (2) the coupling between the normal and the shear tractions is accounted for, and (3) the bulk stress is allowed to have a magnitude large enough as to induce reversed slip.

The proposed algorithm is based on three nested loops: (1) the inner loop solves the uncoupled contact problem, in the normal or in the tangential direction, (2) the intermediate loop stabilises the normal and the shear tractions by mutual interaction, thus solving the contact state for a specific load level, and (3) the outer loop is charged with the replication of the loading history.

Numerical examples are presented for benchmarking purposes and to prove the method ability to predict the behaviour of fretting contact processes. The requirements for the loading history reproduction are discussed in detail. The contact of similar elastic materials can be simulated by computing only the contact states in which the tangential force changes sign, whereas for the case of dissimilar materials, the entire loading path must be incrementally replicated, including the initial normal indentation.

The considered scenarios also deal with the presence of a bulk stress in one of the contacting bodies, thus faithfully replicating the conditions present in classic setups for the experimental study of the fretting fatigue. The proposed numerical tool is expected to advance the understanding of the fretting processes.

6. References
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