Computational Advantages of the Local Strain Energy Density for Fracture and Fatigue Design

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Abstract. This field of brittle fracture has engaged researchers from various fields of engineering from the early days until today. There are nowadays different approaches for the fracture and fatigue design. A high potential has been found in the strain energy density (SED) approach recently applied under different fracture and fatigue loading conditions. This approach has strong numerical advantages especially if applied to advanced materials and complex structures. Different problems and applications are presented in this paper, considering some recent outcomes at different scale levels, showing the possibility of considering a coarse mesh without any definition of a control volume. The potential of the method fits well with the application to connections of any type and in particular with welded joints. The main aim of this work is to compare the SED when applied to traditional welding processes as friction stir welding, and advanced technologies as hybrid bonding.

1. Introduction

In steel structures welding is a widely used and accepted joining technique. A weld line represents a geometrical discontinuity (a notch) and both weld toes and weld roots act as stress concentrators resulting in severe localized stresses, which strongly affect the static and the fatigue strength of a structure. The structural integrity of welded joints is further worsened by the manufacturing process, which may introduce defects, distortions and residual stresses that can be very detrimental for the fatigue and fracture behavior as shown by Fricke [1]. In this complex scenario, another challenge is the multiaxial stress state that usually is present in complex structures [2–4]. The ability to perform an accurate fatigue assessment of welded connections is an issue which must be properly addressed in order to avoid in-service unexpected failures. The criteria, which can be found in the current force standards and in the literature, vary depending on the parameters to be considered meaningful for the fatigue resistance. These criteria can be divided into different categories depending on the complexity of the stress analysis performed on structural details for the final calculation, as widely discussed by Radaj [5]. It is possible to distinguish criteria based on nominal stress, structural stress, local stress or other well-established methodologies concerning residual life assessments carried out on the basis of real defects/cracks/flaws or assumed crack-like defects, according to linear elastic fracture mechanics, as shown by Maddox [6] and Gurney [7]. Basically, according to Radaj et al. [8], it is possible to distinguish between global and local approaches. The former ones are less time consuming methods, whereas the latter are more advanced and powerful methods which require a large computational effort and high professional skills and competence. Despite this scenario, current standards are lacking in giving a real
guidance on how to perform a reliable fatigue assessment. Still, most of them do not refer to any local concept and lead to use the nominal stress method. Even so, no recommendations exist on how to derive the nominal stress from a finite element model, so that it is left to the engineering assessment of a designer to establish which nominal stress is the suitable for performing a fatigue check, as recently pointed out by Hobbacher [9]: "At this point no clear definition of nominal stress does exist, nor a recommendation, how to derive it from the finite element stress plot. It is more than astonishing that several new design codes do not refer to any local concepts, leaving the designer alone with his modern computational equipment."

![Figure 1](image)

**Figure 1.** Fatigue strength of fillet welded joints made of steels as a function of the averaged local strain energy density; scatter band defined by mean value ± 2 standard deviations; fatigue crack initiation at weld toe or weld root; diagram taken from [15].

The strain energy density (SED) approach has been formulated in [10,11]. According to this criterion, the total strain energy density, averaged over a control volume surrounding the weld root or weld toe, is the parameter which controls the fatigue resistance of a joint. Failure occurs when the SED reaches a critical value, dependent on the material but independent of the weld geometry, which corresponds to the Beltrami criterion [12]. The underlying approach to which this idea can be related is Neuber's concept of elementary structural volume [13,14]. The critical volume is represented by a portion of a circular sector at the weld toe and a full circular sector at the weld root. In case of steel welded joints the radius of the control volume is equal to 0.28 mm (see Figure 1). Thereafter, it has been proved that SED can successfully predict brittle and high cycle fatigue failures of pre-cracked U- or V-notched specimens made out of several materials, including metals and ceramics, trying to propose SED as an all-around method [15-17]. This method keeps the robustness of the notch stress intensity approach, as shown by Livieri and Lazzarin [18], because closed form relations between the strain energy density and the relevant NSIFs exist and at the same time it does not require extremely refined meshes which is an impressive advantage dealing with complex structures [19, 20]. The advantage of the coarse mesh is surely amplified if the control volume has not been modelled in the real structure and this could play a fundamental role also in the automatic on time topology optimization of a complex component or
structure subjected to in service loadings. Aim of this paper is to briefly discuss some advantages of the SED approach considering some recent forefront applications and emerging technologies: simulation based design, fracture and fatigue property assessment. All fields together will enable novel advanced design of complex parts, their direct conversion into products as well as the guarantee of their compliance through properly defined and tailored failure criterions. This interdisciplinary and integrative research will streamline the manufacturing workflow all the way from the idea to the final product and will enable designers and engineers to better transform their ideas into reality.

Figure 2. Optical micrograph showing the macrostructure of the HYB joint.

2. Application of the Strain Energy Density to a new generation of butt welded joints
Solid state joining processes produce sound joints at a temperature below the melting temperature of the parent material. Bonding between the components is obtained using extreme deformation and diffusion under the action of mechanical, electrical or thermal energy. There is a diversity of solid state joining processes available for commercial use. For instance, forge welding, ultrasonic welding, cut welding, explosion welding, cold pressure welding, friction welding and friction stir welding. The process of choice is dependent on the final product to be joined. Hybrid metal extrusion & bonding (HYB) is a novel solid state joining technique that has the potential to drastically reduce detrimental characteristics of traditional welding [20, 21]. By operating below the base metal melting temperature, it eliminates the emergence of a fusion zone (FZ), reduces residual stresses as well as the chances of intermetallic compounds (IMCs) compared to traditional welding. A typical geometry is shown in Figure 2.

Here, the HYB geometry has been reanalyzed in terms of the volume-free SED approach, under plane strain condition. Two different finite element software have been used: Ansys® and Straus7®. Linear plane finite elements have been adopted, namely Plane182 and Quad4 in Ansys® and Straus7®, respectively. In both software default element formulation and default free meshing criteria have been set. The finite element models are shown in Figure 2, where only a half of the geometry has been considered taking advantage of the symmetry conditions. It has been applied a nominal stress equal to 102 MPa, the Young’s modulus and the Poisson’s ratio equal to 70000 MPa and 0.3, respectively.

The stress singularity, as well as the mode 1 NSIF, have been computed in Ansys® by using a very refined mesh with the smallest element having size of the order of $10^{-5}$ mm and are shown in Figure 3. Afterwards, the volume-free SED approach has been used adopting free-generated meshes. The notch opening angle $2\alpha$ is equal to 135°. The values of the strain energy density and of the NSIFs obtained by means of the volume-free approach (in both Ansys® and Straus7®) are reported in Table 1 and are compared to those obtained by means of the direct approach. The maximum error between the $K_1$ value accurately determined from the whole local stress field and by using the volume-free approach is 0.8% and 1.0% in Ansys® and in Straus7®, respectively (see Table 1). Obtained results show a robustness of the volume-free approach with respect to different software. This will open the doors to the SED application for topology optimization of complex structures.
Figure 3. Butt welded joint. Very refined mesh generated in Ansys® to determine $K_I$ from local stress field (b). Coarse mesh free-generated in Ansys® to determine $K_I$ from the volume-free SED and Coarse mesh free-generated in Straus7® to determine $K_I$ from the volume-free SED (c)

Table 1. Comparison between the values of the NSIFs calculated: from local stress, from the direct SED and from the volume-free SED, with different software. $R_0=0.28$ mm is considered.

| Software       | $K_I$ [MPa mm$^{0.326}$] | $\Delta\%$ |
|----------------|--------------------------|------------|
| Local stress   | Ansys®                   | 151.2      |
| Direct SED     | Ansys®                   | 150.3      | -0.6 |
| SED Volume-free| Ansys®                   | 152.3      | 0.8  |
|                | Straus7®                 | 152.6      | 1.0  |
3. Conclusions
The fatigue assessment and service life of welded joints is an ambitious task, for which global and local approaches are available. Global approaches, which comprise the nominal stress and the hot spot stress methods, are generally recognized by the national and international design codes and are under permanent amendment. Local approaches, which include the strain energy density based approaches, are very powerful and if well applied very advantageous. In fact, they are useful for the design development in industry in respect of satisfactory reliability, higher strength and longer life. Additionally, they are important as a means of assessing strength and life of structural elements without reference to the global approach. This must take place if global approach data are not available in cases of unconventional design and service conditions.

4. References
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