Nonlinear Multifunctional Sensor Signal Reconstruction Based on Total Least Squares

X Liu, J W Sun and D Liu
Dept. of Automatic Measurement and Control, Harbin Institute of Technology, Harbin 150001, China
E-mail: xinliu@hit.edu.cn, jwsun@hit.edu.cn, liudan@hit.edu.cn

Abstract. The least squares method is often used to estimate the parameters in multi-functional sensor signal reconstruction. If the data has been contaminated, the computational result of the method turns out to be insignificant. Two methods presented in this paper are suitable for different nonlinear conditions, which are based on the combination of the total least squares algorithm with the local linearization strategy and Stone-Weierstrass theorem. The two methods evaluate both the sensor output bias and its input error. The results of emulation and theory analysis indicate that the proposed algorithms are more accurate and reliable for signal reconstruction.

1. Introduction
In the last decades, people pay more attention to the development of multifunctional sensors, which is a new direction of modern sensor technology. And it has been applied into the field of environmental perception [1,2] and industry measurement [3,4]. In general, multifunctional sensor can simultaneously detect several different non-electrical signals, greatly reduce the size and consumption of the measurement system. The schematic structure of general multifunctional sensor is shown in Figure 1, and the multi-variable transfer function model can be described as:

\[ Y(t) = F(X(t)) \]  \hspace{1cm} (1)

Figure 1. Schematic structure of multifunctional sensor.

where \( X(t) = [x_1(t), \ldots, x_n(t)]^T \) is the input signal vector, \( Y(t) = [y_1(t), \ldots, y_n(t)]^T \) is the output signal vector, and \( f_i \) describes the transfer function of sensitive component \( i \), which is nonlinear in general. The estimation of the input signal vector \( \hat{X} = [\hat{x}_1, \ldots, \hat{x}_n]^T \) can be obtained through the signal reconstruction algorithm, and this process is called signal reconstruction.
By now, many signal reconstruction algorithms have been proposed: the literature [5] uses look-up table method to solve signal reconstruction problem in two dimensions, without considering the measurement matrix is ill-conditioned, literature [6] uses Moore-Penrose generalized inverse to solve the ill-conditioned measurement matrix problem and extend this method to three dimensions, while this two methods ignore the condition that the input signals and output signals both contain noise, with respect to this condition, literature [7] uses total least squares method [8] to realize linear multifunctional sensor signal reconstruction. This paper present two different method to approximate the nonlinear sensor transfer function, and reconstruct the measurement signal under the condition that input signal and out signal are contaminated by noise.

2. Theory and Algorithm

2.1. Total Least Squares
Consider the overdetermined equation:
\[ Ax = b \]  
where, \( A \in \mathbb{R}^{m \times n}, x \in \mathbb{R}^n, b \in \mathbb{R}^m \), and \( m \geq n \). To solve (2) by means of least squares (LS) strategy is to find \( x_{LS} \) satisfying \( \| e \|_2 = \| b - Ax_{LS} \|_2 = \min_x \| b - Ax \|_2 \). It is obviously that only the vector \( b \) is supposed to have an error \( \Delta b \) in least squares sense, thus equation (2) can be rewritten as:
\[ Ax = b + \Delta b \]  
and define \( e \):
\[ e = \| \Delta b \|_2^2 = (\Delta b)^T \Delta b = (b - Ax)^T (b - Ax) \]  
then we can obtain the least-squares solution by minimizing \( e \). From the above, it can be seen that least squares method only think about the error of vector \( b \), unfortunately, in practice, the matrix \( A \) and vector \( b \) are consist of measurement signals, which are inevitably contaminated by noise. Therefore, the total least squares (TLS) method is much more reasonable in the case of noise data, which considers the bias of both \( A \) and \( b \). Here, we suppose \( E \) and \( e \) are respectively the errors of \( A \) and \( b \), thus (2) is depicted as:
\[ (A + E)x = b + e \]  
Obviously it can be transformed as follows:
\[ (B + D)z = 0 \]  
Where \( B=[-b:A], D=[-e:E], z=[1 \ x]^T \). The total least squares solution of (6) can be formulated as a constrained optimization problem:
\[ \min_{D, x} \| D \|_F \quad \text{s.t.} \quad (b + e) \in \text{range}(A + E) \]  
Where \( \| D \|_F \) denotes the Frobenius norm of matrix \( D \), and the solution can be calculated by singular value decomposition (SVD).

2.2. Algorithm
For a multifunctional sensor, any output signal should represent the unique input signal, which can be called one-to-one, otherwise it is impossible to distinguish a input signal from another. Thus, the inverse mapping of multifunctional transfer function is unique based on inverse mapping theorem, then the equation (1) can be rewritten as:
\[ \begin{align*}
x_1(t) &= g_1(y_1(t),...,y_n(t)) \\
\vdots \\
x_n(t) &= g_n(y_1(t),...,y_n(t))
\end{align*} \]  
where \( g_i \) is the inverse function of \( f_i \). \( N \) groups input and output signals can be obtained from the sensor system as follows:
then two signal reconstruction methods can be adopted for different conditions.

### 2.2.1. Local linearization method.

According to local linearization [9], in the neighborhood of point \( P_0(x_1^0, \ldots, x_n^0, y_1^0, \ldots, y_n^0) \), equation (8) can be replaced by a set of linear equations as follows:

\[
\begin{align*}
    x_1 &= a_{11} y_1 + a_{12} y_2 + \cdots + a_{1n} y_n \\
    & \vdots \\
    x_n &= a_{21} y_1 + a_{22} y_2 + \cdots + a_{2n} y_n
\end{align*}
\]  

(10)

where \((x_1, \ldots, x_n)\) is the input signal, \((y_1, \ldots, y_n)\) is the output signal and \(A = (a_{ij})_{n \times n}\) is the linear transfer matrix. For a given output signal \((y_1(i), \ldots, y_n(i))\) in the neighborhood of point \( P_0 \), which is used to reconstruct the relative input signal, this method can be implemented in two steps.

- Estimate the transfer matrix based on TLS method
  
  From the \(N\) groups of datum, \(M\) groups of datum in the neighborhood of point \( P_0 \) can be picked out:

\[
\begin{align*}
    (x_1(j), \ldots, x_n(j), y_1(j), \ldots, y_n(j)) & \quad (j = 1, \ldots, M)
\end{align*}
\]  

(11)

Firstly, we estimate the first row of matrix \(A\), inserting the \(M\) groups of datum into the first function of equation (10), and obtain the equation:

\[
\begin{bmatrix}
    x_1(1) \\
    \vdots \\
    x_1(k)
\end{bmatrix} =
\begin{bmatrix}
    y_1(1) & \cdots & y_n(1) \\
    \vdots & \vdots & \vdots \\
    y_1(k) & \cdots & y_n(k)
\end{bmatrix}
\begin{bmatrix}
    a_{11} \\
    \vdots \\
    a_{1n}
\end{bmatrix}
\]  

(12)

Because the matrix in equation (12) consist of output signals, which are contaminated by noise in general, it is suitable to solve by TLS method. Then we can get the TLS estimation of matrix \(A\).

- Reconstruct input signal
  
  Since the estimation of transfer matrix in equation (10) has been obtained, the reconstructed input signal can be computed by fitting the relative output signal into the equation (10).

### 2.2.2. Polynomial approximate method.

It can be concluded from the Stone-Weierstrass Theorem [10]that any real valued continuous function can be uniformly approximated by a polynomial function within a given accuracy, therefore equation (8) can be transformed as follows:

\[
\begin{align*}
    x_1 &= a_0^1 + \sum_{h=1}^{n} a_h^1 y_h + \sum_{h=1}^{n} \sum_{i=1}^{n} a_{h,i}^1 y_h y_i + \cdots + \sum_{h=1}^{n} \sum_{i_1=1}^{n} \cdots \sum_{i_n=1}^{n} a_{h,i_1,\ldots,i_n}^1 y_h y_{i_1} \cdots y_{i_n} \\
    & \vdots \\
    x_n &= a_0^n + \sum_{h=1}^{n} a_h^n y_h + \sum_{h=1}^{n} \sum_{i=1}^{n} a_{h,i}^n y_h y_i + \cdots + \sum_{h=1}^{n} \sum_{i_1=1}^{n} \cdots \sum_{i_n=1}^{n} a_{h,i_1,\ldots,i_n}^n y_h y_{i_1} \cdots y_{i_n}
\end{align*}
\]  

(13)

To get the reconstructed input signal, we firstly estimate the real coefficients in equation (13) using the datum in (9) and TLS method, which is similar to estimating transfer matrix in local linearization method. Then we can get the reconstructed input signal by inserting the relative output signal into the equation (13). From the theory analysis mentioned above, it can be observed that the two methods fit different conditions, local linearization method is suitable for unknown system transfer function, and polynomial approximate method is suitable for continuous system transfer function.

### 3. Result and Analysis

To verify the feasibility of the proposed methods, a physical model of the two-input/output multifunctional sensor used in the experiment has been built up and shown in Figure 2. The input
signals are $x$ and $y$, which represent the rate of the slide resistor lower side resistances to the entire resistances.

The output signals are $u$ and $v$ (Voltage). According to KCL, the system transfer function can be described as follows:

$$
\begin{align*}
  u &= \frac{5[2x + y + axy(2 - x - y)]}{3 + a[y(1 - y) + x(1 - x)]} \\
  v &= \frac{5[2y + x + axy(2 - x - y)]}{3 + a[y(1 - y) + x(1 - x)]}
\end{align*}
$$

(14)

$$
\begin{align*}
  &\text{Figure 2. Circuit model of two input/output sensor.}
\end{align*}
$$

The ideal input signals are shown in Figure 3, where $o$ represents the training input signals and $*$ represents the test input signals. Bring the data in Figure 3 into the equation (14) and the circuit, where the parameter $a$ is set to 2, the relative output signals can be obtained, where $u_{t}$ and $v_{t}$ are ideal training output signals, $u_{c}$ and $v_{c}$ are ideal test output signals, which are calculated from the transfer function, $u_{fx}$ and $v_{fx}$ are simulation training output signals, $u_{fc}$ and $v_{fc}$ are simulation test output signals which are measured from the circuit, and ideal training output signal $u_{tx}$ is shown in Figure 4.

$$
\begin{align*}
  &\text{Figure 3. Ideal input signals.}
\end{align*}
$$

$$
\begin{align*}
  &\text{Figure 4. Ideal test output signal $u_{tx}$ ($a=2$).}
\end{align*}
$$

For the simulation training point $(x_{fx}, y_{fx}, u_{fx}, v_{fx})$ and simulation test point $(x_{fc}, y_{fc}, u_{fc}, v_{fc})$, the reconstructed simulation test input signals can be computed respectively by local linearization method and polynomial approximate method. Let $x_{j}$ and $y_{j}$ be the reconstructed signal calculated by local linearization method and $x_{d}$ and $y_{d}$ be the reconstructed signal calculated by polynomial approximate method, then the relative error of reconstructed signal $x_{j}$ and $y_{j}$ can be described as:

$$
\begin{align*}
  E_{x} &= \frac{x_{j} - x_{fc}}{x_{fc}} \times 100\%; & E_{y} &= \frac{y_{j} - y_{fc}}{y_{fc}} \times 100\% \\
\end{align*}
$$

(15)

which is similar to the reconstructed signal $x_{d}$ and $y_{d}$, and Figure 5 is the relative error of $x_{j}$, Figure 6 is the relative error of $x_{d}$, Figure 7 is the relative error of $y_{j}$, Fig.8 is the relative error of $y_{j}$.
It can be seen from the above figures that the accuracy of both methods are maintained in a reasonable range, while the performance of polynomial approximate method is better than local linearization method in the experiment. To approximate the sensor transfer function, local linearization method is to replace the unknown system transfer function by a linear function in the neighborhood of reconstructed signal point, which has a simple formula, reasonable accuracy and low calculation, while polynomial approximate method is to replace the transfer function by a polynomial in the whole input signals space, which has a complex formula, high accuracy and high calculation.

4. Conclusion
A wide range of problems in estimation, identification and reconstruction can be transformed into seeking the solution of an overdetermined equation. In solving the overdetermined equation, the least squares is a popular method, which only take into account the error of observe vector, while total least squares consider the errors of both transfer matrix and observer vector, therefore, total least squares has higher precision in the parameter estimation, and the theory analysis implies that the estimation accuracy of TLS solution is better than LS method in the case of input signals and output signals are all contaminated by noise [11]. The methods presented in this paper enhance the precision and stability of signal reconstruction by using the total least squares and it is suitable for the multivariable condition.

References
[1] K.D. Schierbaum, U. Weimer and W. Gopel 1990 Multicomponent Gas Analysis: An Analytical Chemistry Approach Applied to Modified SnO2 Sensors Sensors and Actuators B: Chemical 2 71–78
[2] J. Sun and K. Shida 2002 Multilayer Sensing and Aggregation Approach to Environmental Perception with One Multifunctional Sensor IEEE Sensors Journal 2 62–72
[3] J. Sun and K. Shida 2001 A New Multifunctional Sensor for Measuring Oil/Water Two-phase State in Pipelines Japanese Journal of Applied Physics 40 1487–92
[4] Yuji J and Shida K 2000 New multifunctional tactile sensing technique by selective data processing IEEE Transactions on Instrumentation and Measurement 49 1091–94
[5] Alessandra F, Daniele M. Andrea T. 1999 Application of an Optimal Look-Up Table to Sensor Data Processing. IEEE Transactions on Instrumentation and Measurement 48: 813–816

[6] Sun Jinwei, Zheng Yungang and 2004 Liu Dan. Look-up Table Based Approach to Data Reconstruction in Multi-Functional Sensing. TRANSACTIONS OF CHINA ELECTROTECHNICAL SOCIETY 19: 76–80

[7] Sun Jinwei, Liu Xin and Sun Shenghe 2004 TLS Algorithm-Based Study on Multifunctional Sensor Data Reconstruction. ACTA ELECTRONICA SINICA 32: 391–394

[8] Zhang Xianda 1993 Modern Signal Processing (Beijing: Tsinghua University Press)

[9] Qiang Gan and Chris J. Harris 1999 Linearization and State Estimation of Unknown Discrete-Time Nonlinear Dynamic Systems Using Recurrent Neurofuzzy Networks. IEEE Transactions on Systems, Man, AND Cybernetics—PART B: CYBERNETICS 29: 802–817

[10] Zengjun Xiang, Guangguo Bi and Tho Le-Ngoc 1994 Polynomial Perceptrons and Their Applications to Fading Channel Equalization and CO-Channel Interference Suppression. IEEE Transactions on Signal Processing 42: 2470–80

[11] Zhang Hongyue, Huang Jingdong and Fan Wenlei 1995 Total Least Square Method and Its Application to Parameter Estimation. ACTA AUTOMATICA SINICA 21: 40–47