Shot Noise by Quantum Scattering in Chaotic Cavities

S. Oberholzer, E. V. Sukhorukov, C. Strunk, and C. Schönenberger
Institut für Physik, Universität Basel
Klingelbergstr. 82, CH-4056 Basel, Switzerland

T. Heinzel and K. Ensslin
Solid State Physics Laboratory, ETH-Zürich
CH-8093 Zürich, Switzerland

M. Holland
Department of Electronics, University of Glasgow
Glasgow G12 8QQ, United Kingdom
(November 9, 2018)

We have experimentally studied shot noise of chaotic cavities defined by two quantum point contacts in series. The cavity noise is determined as $1/4 \cdot 2e/|I|$ in agreement with theory and can be well distinguished from other contributions to noise generated at the contacts. Subsequently, we have found that cavity noise decreases if one of the contacts is further opened and reaches nearly zero for a highly asymmetric cavity.

The non-equilibrium time dependent fluctuations of the electrical current, known as shot noise, are caused by the randomness of charge transfer in units of $e/|I|$. If the electron transfer can be described by a Poissonian process, the spectral density $S$ of the current fluctuations is $S_{\text{Poisson}} = 2e/|I|$. Correlations imposed by fermionic statistics as well as Coulomb interaction may change shot noise from $S_{\text{Poisson}}$. A quantum wire with an intermediate barrier with energy-positive transmission probability is

\[ S = 2G \int dE f_C(1 - f_C). \]  \hspace{1cm} (1)

Here, $f_C(E)$ denotes the distribution function inside the cavity, which is homogeneous and isotropic. The total conductance $G = G_0(N_L N_R)/(N_L + N_R)$ with $G_0 = 2e^2/h$ is equal to the series conductance of the left and right contact with $N_L$ ($N_R$) open channels (i.e. $\Gamma_{1\ldots N_L, R} = 1$, $\Gamma_{N_L, R'} = 0$). For non-interacting electrons the distribution function in the cavity $f_C$ just equals the weighted average of the distribution functions $f_L$ and $f_R$ in the left and right reservoirs.
In the symmetric case $N_L = N_R$, i.e. $f_C = \frac{1}{2}(f_L + f_R)$, and Eq. (1) yields a Fano factor of $1/4$. For very asymmetric contacts ($N_L \gg N_R$) shot noise approaches zero, since the system can then be regarded as a single contact with $N_R$ open and therefore noiseless channels. The general Fano factor $F = S/I^2$ for cavity noise is
\[
F(\eta) = \frac{N_L N_R}{(N_L + N_R)^2} = \frac{\eta}{(1 + \eta)^2},
\]
where we introduce the parameter $\eta = N_L/N_R$ measuring the symmetry of the cavity.

Experimentally, we have realized chaotic cavities by two quantum point contacts (QPC) in series. These are electrostatically defined in a two dimensional electron gas (2 DEG) by metallic split gates on top (see Fig. 1 on page 2). The opening of the contacts can be individually tuned by varying the applied gate voltages independently. The 2 DEG forms 80 nm below the surface at the interface of a standard GaAs/Al0.3Ga0.7As heterojunction. Magnetoresistance measurements yield a carrier density of $2.7 \times 10^{15}$ m$^{-2}$, corresponding to a Fermi energy of $\simeq 100$ K and a mobility of $83$ V s/m$^{-1}$ resulting in a mean free path of $\simeq 7\mu$m comparable to the size of the cavity. Three QPCs in series as shown in Fig. 1 enable to define two cavities of different size: either the outer gates A and C with the middle gate B kept completely open can be used to define a relatively large cavity of $\simeq 11 \times 8 \mu$m, or 2 of the inner gates (A, B or B, C) creating a smaller cavity of $\simeq 5 \times 8 \mu$m. The conductance of the QPCs is quantized according to the Landauer formula
\[
G = G_0 \sum_n \Gamma_n
\]
(iset (b) of Fig. 1). An open cavity is defined when both QPCs are adjusted to a conductance plateau, where $N$ modes are fully transmitted ($\Gamma = 1$) and the others are totally reflected ($\Gamma = 0$). The two-terminal conductance $G$ is experimentally found to correspond to the series conductance of the two contacts $G_L G_R/(G_L + G_R)$ with an accuracy of less than $1\%$ [13]. Therefore, direct transmission of electrons from the left to the right contact can be excluded, as well as quantum corrections [14].

Two independent low-noise amplifiers (EG&G 5184) operating at room temperature are used to detect the voltage fluctuations across the cavity. A spectrum analyzer (HP 89410A) calculates the cross-correlation spectrum of the two amplified signals. This technique allows to reduce uncorrelated noise contributions which do not originate from the sample itself. Experimental details can be found in [8,15]. Furthermore, the whole setup is filtered against RF-interference at low temperatures. From the Nyquist-relation \( S_V = 4k_B RT \) the voltage gain as well as the offset in the voltage noise $S_{V}^{0,I}$ caused by the finite current noise $S_{I}^{0,I}$ of the amplifiers can be determined with high accuracy. Although shot noise is a non-equilibrium phenomenon observed in its purest form in the limit $eV \gg k_B T$, in this experiment bias voltages are limited to $\simeq 8k_BT/e$, only. This is to avoid non-linearities of the current-voltage characteristics of the QPCs [16] and 1/f-noise contributions occurring at larger currents [15].

Within this limit, the differential resistance, recorded for all noise measurements, changes by $\lesssim 2.5\%$. The current noise is finally obtained from the measured voltage fluctuations by $S_1 = S_C/(dV/dI)^2 - S_{V}^{0,I}$. Fig. 2 shows shot noise measurements of a cavity defined by gates A and B with a size of $\simeq 5 \times 8 \mu$m for different symmetry parameters $\eta = G_L/G_R$. The solid curves describe the crossover from thermal to shot noise for the measured value of $\eta$ given by [6].

\[
S = S_{eq} \left\{ 1 + F(\eta) \cdot \left[ \frac{eV}{2k_B T} \coth \left( \frac{eV}{2k_B T} \right) - 1 \right] \right\}.
\]

$S_{eq} = 4k_BTG$ denotes the equilibrium noise and $F(\eta)$ the Fano factor (Eq. 3). In the symmetric case ($\eta = 1$) with $N_L = N_R = 5$ we obtain a very good agreement between the experimental data and the theoretical prediction of $1/4 \cdot 2e|I|$. When the right contact is further opened ($G_R > G_L$) $\eta$ increases from 1 (symmetric) to $\simeq 41$ (asymmetric). Thereby, shot noise gradually disappears for larger values of $\eta$ as expected from Eq. 2. For partial transmission in the contacts shot noise is larger than $1/4 \cdot 2e|I|$ because additional noise is generated at the contacts. This is shown in the inset of Fig. 2 where the first mode in the contacts is fully transmitted ($\Gamma_1 = 1$) while the second one is partially reflected ($\Gamma_2 = 0.16$). The curves are numerical calculations for no
mode mixing (dotted) and for slight mode mixing of \( \simeq 10\% \) (solid) with \( \Gamma_1 = 0.90 \) and \( \Gamma_2 = 0.26 \).

Up to now we have assumed that inelastic electron scattering inside the cavity can be neglected. In general, heating caused by electron-electron interaction enhances shot noise [1]. The Fano factor of a diffusive wire, for example, changes from 1/3 for non-interacting (cold) electrons to 3/4 for interacting (hot) electrons [17]. Heating also affects the shot noise of a chaotic cavity. The Fano factor is modified to [18]:

\[
F(\eta) = \frac{\sqrt{3N_L N_R}}{\pi(N_L + N_R)} = \frac{\sqrt{3\eta}}{\pi(1 + \eta)},
\]

and the crossover from thermal to shot noise is described by

\[
S = \frac{S_{eq}}{2} \left\{ 1 + \sqrt{1 + F(\eta)^2 \cdot \left( \frac{eV}{k_BT} \right)^2} \right\}.
\]

For a symmetric cavity \( F(\eta=1) \simeq 0.276 \) for hot electrons, which is only slightly larger than \( F(\eta=1) = 0.25 \) for cold electrons. The inset of Fig. 3 compares \( S(\epsilon V / k_BT) \) in the hot and cold electron regime for a diffusive wire and a cavity. As is evident, the differences are very small, in particular in case of a cavity where even a crossing at \( \epsilon V / k_BT \sim 15 \) occurs. In Fig. 3 the measured noise for \( \eta = 1 \) of Fig. 2 is replotted and compared to the prediction for cold (solid) and for hot electrons (dashed). Although the data points lie clearly closer to the prediction for cold electrons, this alone is not sufficient to decide which regime is realized in the cavity, because of the finite experimental accuracy. An additional criterion is needed.

In order to decide whether the cold or hot electron theory is appropriate for the comparison with the measurements, the electron-electron scattering time \( \tau_{ee} \) is compared with the dwell time for electrons inside the cavity. We argue that thermalization is present if \( \tau_D \gg \tau_{ee} \). The average dwell time is the product of the ballistic flight time across the cavity \( \tau_F \simeq L / v_F \) with the number of scattering events inside the cavity given by the ratio of the cavity size \( L \) and the width of the contacts \( W = W_L + W_R = \frac{L}{2} (N_L + N_R) \):

\[
\tau_D = \frac{2\pi\hbar}{E_F} \left( \frac{L}{\lambda_F} \right)^2 \frac{1}{(N_L + N_R)}.
\]

The electron-electron scattering rate \( \tau_{ee}^{-1} \) in a two dimensional electron system is given by [19]:

\[
\tau_{ee}^{-1} = \frac{E_F}{2\pi\hbar} \left( \frac{k_BT_e}{E_F} \right)^2 \left[ \ln \left( \frac{E_F}{k_BT_e} \right) + \ln \left( \frac{2q}{k_F} \right) + 1 \right]
\]

with the Thomas-Fermi screening wave vector \( q = 2mc^2 / \epsilon_0\hbar^2 \). Because the system is out of equilibrium the temperature \( T_e \) in Eq. (7) has to be replaced by the effective electron temperature \( T_{eff} \) given by \( T_{eff} = (1/k_B) \int \text{d}f_C(1 - f_C) \) [20]. The ratio \( \tau_D / \tau_{ee} \) is plotted in the inset of Fig. 4 as a function of \( \eta = G_L / G_R \)

![FIG. 3. Shot noise of a symmetric cavity and theoretical predictions for cold (solid) and hot electrons (dashed). Inset: comparison of the noise of a chaotic cavity (1/4 and \( \sqrt{3}/2\pi \)) with a diffusive wire (1/3 and \( \sqrt{3}/4 \)) for cold and hot electrons.](image)

For the two different types of cavities taking \( \tau_{ee} \) from Eq. (7) for \( T_{eff} \) corresponding to the largest applied voltage \( V \) in the experiment. The upper curve belongs to the large cavity (~11x8 \( \mu \)m), where the right contact is nearly closed (\( G_R \) fixed to \( G_0 \)). In this case, \( \tau_D \gg \tau_{ee} \). The lower curve corresponds to the smaller cavity (~5x8 \( \mu \)m) with a 5 times larger opening of the right contact. For this type of cavity we find \( \tau_D < \tau_{ee} \).

According to this argument we use Eq. (6) valid for hot electrons to fit the noise data obtained for chaotic cavities with \( \tau_D / \tau_{ee} > 1 \). The Fano factor \( F \) is the only fitting parameter. On the other hand, we use Eq. (5) valid for cold electrons if \( \tau_D / \tau_{ee} < 1 \). The Fano factors \( F = S / 2e|I| \) obtained according to this procedure are plotted as a function of the measured \( \eta \) for the two different cavities described above. For the black squares, which belong to the large cavity with nearly closed contacts (large dwell time), we find good agreement with the theoretical Fano factor for hot electrons given by Eq. (6) (dashed). The open circles are results for the small cavity with wider opened contacts (small dwell time) which are consistent with the prediction for non-interacting electrons described by Eq. (5). If we use the formula for cold electrons instead of the one for hot-electrons to fit the data obtained for the larger cavity, the black squares move only slightly downwards by \( \simeq 0.02 - 0.03 \). They still lie clearly above the open circles, demonstrating that heating is indeed important for the larger cavity. Good agreement between theory and experiment is found for both regimes with the exception of very asymmetric contacts, i.e. \( \eta \gg 1 \). Here, we attribute the deviations to slight mode mixing within the QPCs, which is difficult to avoid [15]. Let us assume, as an example for the data point at \( \eta = 180 \), that two modes instead of one participate in the left
contact transmitting respectively with $\Gamma_1 = 0.97$ and $\Gamma_2 = 0.03$ instead of $\Gamma_1 = 1.00$ and $\Gamma_2 = 0$. This yields a Fano factor of $\approx 0.06$ in agreement to what is experimentally observed.

In conclusion, we have experimentally studied shot noise of open chaotic cavities defined by two QPCs in series. In the regime of non-interacting electrons a Fano factor $F = S/2e|I|$ of 1/4 has been measured as theoretically predicted for symmetric cavities. The origin of this shot noise is partitioning of the electron wave function by quantum-mechanical diffraction inside the cavity. The contacts themselves, which actually define the resistance of the system, do not contribute to noise. In addition, we have also investigated heating effects due to inelastic electron-electron scattering by changing the opening of the contacts as well as the size of the cavity. Similar to other mesoscopic systems heating increases shot noise in agreement with theory. Shot noise in chaotic cavities is a purely quantum phenomenon. It would be interesting to study the crossover from “quantum chaos” to “classical chaos”, where shot noise is predicted to be absent [21].

The authors would like to thank Ya. M. Blanter for valuable comments. This work was supported by the Swiss National Science Foundation.

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