Numerical simulation of fluid flow in a saturated fractured porous media based on the linear poroelasticity model

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Abstract. Field development processes change the stress-strain state of the formation, which can especially affect on the state of fractured reservoir. In this paper, the coupled flow and geomechanics model was used to simulate a fractured-porous reservoir. To describe fractures in the fluid flow problem the discrete fracture model was used; the fracture was considered as an internal boundary for the geomechanics problem. The control volume method, which is widely used in computational fluid dynamics, was used for both flow and mechanics problems. Based on the developed coupled flow and geomechanics model, simulations of an fractured poroelastic system was carried out.

1. Introduction
The existence of fractures has a significant effect on the fluid flow in reservoir systems. Traditional reservoir flow simulation takes into account the dependence of the fracture permeability only on pore pressure. At the same time, laboratory studies show that the permeability of fractures depends on normal stresses [1]. The paper [2] states that shear stresses have a decisive influence on the hydrodynamic characteristics of reservoir faults: the critically-stressed-fault hypothesis is introduced, according to which mechanically active faults, determined by the Coulomb-Mohr criterion, are hydraulically conductive. Thus, to obtain a complete picture of the fluid flows in a fractured-porous medium, it is necessary to use coupled flow-geomechanical modeling.

Approaches to the coupled fluid flow and geomechanical reservoir simulation are based on the Biot theory [3, 4]. One of the methods for modeling flow in fractured-porous media is the discrete fracture model (DFM) [5]. A fracture is considered as an internal boundary for geomechanical problem, for which it is additionally necessary to set the boundary conditions that describe the mechanical contact between the fracture sides. In reservoir fluid flow simulations, the control volume method (FVM) [6] is widely known. The finite element method (FEM) is the traditional approach of continuum solid mechanics, however, recently FVM has become an alternative in solid mechanics field [7].

In this study, our aim is to explore the fractured poroelastic system. To achieve this, we have developed coupled fluid flow and geomechanical model for fractured reservoirs simulations. Developed coupled model was verified by comparison with Terzaghi and Mandel poroelasticity problems.
2. Mathematical model

2.1. Poroelasticity

The governing equations for coupled flow and geomechanics come from mass and linear-momentum conservation laws. Linear momentum balance equations for poroelastic body under quasi-static condition and neglecting the body force can be written as

$$\nabla \cdot [\mu \nabla \mathbf{u} + \mu \nabla \mathbf{u}^T + \lambda \text{tr}(\varepsilon) \mathbf{I} - \frac{bp}{\text{coupling}} \mathbf{I}] = 0 \tag{1}$$

where $\mathbf{u}$ is the displacement vector, $\varepsilon$ is infinitesimal strain tensor, $\lambda$ and $\mu$ are Lame’s coefficients, $b$ is the Biot coefficient, $\mathbf{I}$ is identity matrix.

The single-phase flow equation in a deforming porous medium is

$$\frac{1}{M} \frac{\partial p}{\partial t} + b \frac{\partial \varepsilon_v}{\partial t} - \nabla \cdot \left( \frac{k}{\mu_f} \nabla p \right) = q \tag{2}$$

where $p$ is fluid pressure, $\varepsilon_v = \text{tr}(\varepsilon)$ is volume strain, $k$ is rock permeability, $\mu_f$ is fluid viscosity, $q$ is a source term, $M = \phi c_f + \frac{\phi - b}{K_s}$ is the Biot modulus, $\phi$ is rock porosity, $c_f$ is fluid compressibility, $K_s$ is is the bulk modulus of the solid grain.

The coupled poroelastic system consists of equations (1) and (2).

2.2. Fracture representation

The high aspect ratio of the fracture allows us to consider them as zero thickness inclusions for geomechanical problem. Normal $\sigma_n$ and shear $\tau$ stresses acting on a fracture depend on the relative normal $du_n$ and shear $du_s$ displacements between fracture boundaries. For the fracture under compressive stress we adopt following linear constitutive law:

$$\begin{bmatrix} d\sigma_n \\ d\tau \end{bmatrix} = \begin{bmatrix} k_n & 0 \\ 0 & k_s \end{bmatrix} \begin{bmatrix} du_n \\ du_s \end{bmatrix} \tag{3}$$

where $k_n$ and $k_s$ are normal and shear stiffness respectively.

To simulate the flow in a fractured-porous medium, we used the discrete fracture model, which is applicable for modeling systems of long-scale fractures. It is assumed that the fracture has some effective hydrodynamic aperture, to describe the fluid flow inside the fracture in the equation (2), we use the permeability $k_f$ and porosity $\phi_f$ corresponding to the fracture, and volumetric strains $\varepsilon_v$ inside the fracture due to its small size is neglected. The transfer function between the matrix and the fractures determine by pressure difference and can be written as

$$q_{f-m} = \frac{T_{f-m}}{\mu_f} (p_f - p_m) \tag{4}$$

where $T_{f-m}$ is the transmissibility between the fracture and the matrix, $p_m$ and $p_f$ are pressure in matrix and fracture respectively.

3. Numerical Model

The fluid flow and geomechanical equations was discretized using the cell-centered FVM, which is up to second-order accuracy. Approaches of equations (1) and (2) discretization are discussed in detail in [7]. For the poroelasticity equations system implicit-explicit discretization was used, which leads to decoupled system of linear equations solved in the segregated manner. For solution of coupled flow-mechanics problem unconditionally stable fixed-stress split method was used [8].

Detail of using of the DFM in numerical method can be found in [5]. Using of the FVM for mechanical problem of fractured media is presented in [9].
4. Validation

Developed coupled model was verified by comparison with Terzaghi and Mandel poroelasticity problems (figure 1). Poroelastic parameters used for both problem are shown in table 1.

![Terzaghi and Mandel problems formulation](image)

**Figure 1.** Terzaghi (a) and Mandel (b) problems formulation.

**Table 1.** Parameters used in calculation of the Terzaghi and Mandel problems.

| Parameter               | Value | Unit   |
|-------------------------|-------|--------|
| Matrix porosity         | 0.2   | -      |
| Matrix permeability     | 10    | md     |
| Fluid viscosity         | 0.1   | Pa·s   |
| Fluid compressibility   | $8 \cdot 10^{-4}$ | MPa$^{-1}$ |
| Young’s modulus         | $2 \cdot 10^4$ | MPa   |
| Poisson’s ratio         | 0.2   | -      |
| Biot coefficient        | 1     | -      |

4.1. Terzaghi Problem

In the Terzaghi problem, a saturated column under load is considered. The column is laterally constrained, load $F = p_0$ is applied to the upper boundary. The no-flow boundary conditions are set at all boundaries except the upper one, where the pressure $p_{top} = 0$ MPa is set. The initial pore pressure $p_0 = 0.1$ MPa and initial deformations are equal to zero. The calculation domain is 10 m high ($h$) and 1 m thickness presented on figure 1a is divided on 99 grid blocks along the vertical-direction and on 9 in horizontal-direction.

A comparison for dimensionless pressure $\tilde{p} = \frac{p}{p_0}$ and vertical displacement $\tilde{v} = \frac{uK_{dr}}{p_0}$ for dimensionless coordinate $\tilde{z} = \frac{z}{h}$ at various characteristic times $\tilde{t} = \frac{kt}{\frac{\mu}{\mu_f} + \frac{K_{dr}^2}{h^2}}$, where $K_{dr} = 2\mu + \lambda$ is the bulk modulus, is showed in figure 2.

4.2. Mandel Problem

The main feature of the Mandel problem is the Mandel-Cryer effect[10]. A saturated poroelastic medium with long of $2a = 20$ m and height $2b = 20$ m sandwiched between two rigid impermeable plates (figure 1b) was considered. The pressure at lateral boundaries of the region is equal to zero.
and they are assumed traction free. At the initial time, the constant compressive force $2F = 2$ MPa immediately begins to act on the plates. The initial pore pressure $p_0$ and deformations are equal to zero. Because of the symmetry of this problem, the simulation was run only for one-quarter divided on 31 grid blocks in both directions.

Figure 3 shows a comparison for dimensionless pressure $\bar{p} = \frac{3ap}{FB(1+\nu_u)}$ and vertical displacement $\bar{v} = \frac{2\mu u}{F\nu_u}$ for dimensionless coordinate $\bar{x} = \frac{x}{a}$ at various characteristic times $\bar{t} = \frac{kt}{\mu f(\frac{3}{2} + \frac{b^2}{K_o})} a^2$, where $B = \frac{bM}{K_o+b^2M}$ is the Skempton’s coefficient and $\nu_u = \frac{3\nu+bB(1-2\nu)}{3-bB(1-2\nu)}$ is the undrained Poisson’s ratio.

We obtain a good agreement with both Terzaghi and Mandel analytical solutions for pressure and displacement fields at all times steps.
5. Results
Based on the approaches to simulation fractured reservoirs considered above, a model for modeling fractured poroelastic media is developed. The pressure, the strain and the stress fields are considered, and special attention is paid to the fracture mechanical state. Reservoir and fracture geometry are shown in figure 4, the domain size is $500 \times 500 \times 5$ m. The fracture 300 m long was located in the center of the computational domain, three various angles of fracture inclination ($30^\circ$, $45^\circ$, $60^\circ$) are investigated. An unstructured computational grid was used to describe the geometry of the fracture.

![Figure 4. Reservoir geometry with different fracture orientation.](image)

For all boundaries of the region the no-flow boundary condition was applied. For the geomechanical problem boundary the maximum stress applied vertically $\sigma_{max} = 60$ MPa and the minimum stress applied horizontally $\sigma_{max} = 40$ MPa. Production well with bottomhole pressure of $P_{prod} = 22$ MPa and injection operating with constant flow rate $Q_{inj} = 10 m^3/day$ are located inside the computational domain. Initial strains are agree with boundary conditions and uniform initial pressure $p_{init} = 25$ MPa. Simulation time was 600 days.

The same parameters as in validation section was used, which are shown in table 1. Additionally, it is necessary to describe the properties of the fracture: $k_f = 10^7 md$, $\phi_f = 0.5$, $k_s = 2 \cdot 10^3$ MPa and $k_n$ is equal to matrix Young’s modulus.

The figure 6 shows the distribution of pore pressure for the case of inclination of the crack at 60 degrees. The effect of a fracture on the direction of the filtration flows is observed, especially near the wells, which is well demonstrated in the figure by arrows.

Let consider the mechanical state of the investigated fracture. The increase of of the average pressure was observed for the reservoir system, which leads to the fracture effective normal stresses decrease. To estimate fracture stress state it is convenient to use the Coulomb failure function (CFF):

$$CFF = \tau - \mu \sigma_n$$

where $\mu$ is the coefficient of friction.

Following the critically-stressed-faults hypothesis, faults that are mechanically active ($CFF > 0$) are hydraulically alive. As shown on Mohr–Coulomb diagram (figure 5), only for fracture with $60^\circ$ inclination angle are possible to reach a critical stress state. This condition is achieved after 440 days of reservoir simulation, it can serve as an indicator of the shear on a fracture plane.

6. Conclusions
A coupled fluid flow and geomechanical model for fractured reservoir has been developed using cell-centered FVM, DFM and fixed-stress split method. The model was validated against known
Figure 5. Mohr–Coulomb diagram represent the fracture stressed state. Solid line is initial fractures state, dashed line is fractures stressed state at 440 day.

Figure 6. Pressure field for the case of a fracture 60° inclination at 300 day. The arrows indicate the direction of fluid flow in the cells. Black line represent the fracture.

The fractures orientation has a decisive effect on their mechanical condition. Fractured reservoir modeling was carried out for various fracture orientations. In the studied reservoir only fractures with an orientation close to 60° can be critically stressed.

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