Large Margin Nearest Neighbor Embedding for Knowledge Representation

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Abstract—Traditional way of storing facts in triplets (head_entity, relation, tail_entity), abbreviated as \((h, r, t)\), makes the knowledge intuitively displayed and easily acquired by mankind, but hardly computed or even reasoned by AI machines. Inspired by the success in applying Distributed Representations to AI-related fields, recent studies expect to represent each entity and relation with a unique low-dimensional embedding, which is different from the symbolic and atomic framework of displaying knowledge in triplets. In this way, the knowledge computing and reasoning can be essentially facilitated by means of a simple calculation, i.e. \(h + r \approx t\). We thus contribute an effective model to learn better embeddings satisfying the formula by pulling the positive tail entities \(t^+\) to get together and close to \(h + r\) (Nearest Neighbor), and simultaneously pushing the negatives \(t^-\) away from the positives \(t^+\) via keeping a Large Margin.

We also design a corresponding learning algorithm to efficiently find the optimal solution based on Stochastic Gradient Descent in iterative fashion. Quantitative experiments illustrate that our approach can achieve the state-of-the-art performance, compared with several latest methods on some benchmark datasets for two classical applications, i.e. Link prediction and Triplet classification. Moreover, we analyze the parameter complexities among all the evaluated models, and analytical results indicate that our model needs fewer computational resources on outperforming the other methods.

I. INTRODUCTION

Owing to the long-term efforts on distilling the explosive information on the Web, several large scale Knowledge Bases (KBs), such as WordNet[1], OpenCyc[2], YAGO[3] and Freebase[4], have already been built. Despite the different domains these KBs serve for, nearly all of them concentrate on enriching entities and relations. To facilitate storing, displaying and even retrieving knowledge, we use the highly structured form, i.e. \((head\_entity, relation, tail\_entity)\), for knowledge representation. Each triplet is called a fact. So far, some general domain KBs, such as Freebase[4] and YAGO[3] have contained millions of entities and billions of facts, and they should have led a huge leap forward for some canonical AI-related tasks, e.g. Question Answering Systems. However, it is realised that this symbolic and atomic framework of representing knowledge makes it difficult to be utilized by most of AI-machines, especially of those which are dominated by statistical approaches.

Recently, many AI-related fields, such as Image Classification [5], Natural Language Understanding [6] and Speech Recognition [7], have made significant progress by means of learning Distributed Representations. Taking an example of distributed representations of words applied to Statistical Language Modeling [8], it has achieved considerable success by grouping similar words which are close to each other in low-dimensional vector spaces. Moreover, Mikolov et al. [9], discovered somewhat surprising patterns that the learnt word embeddings, to some extent, implicitly capture syntactic and semantic regularities in language. For example, the result of vector calculation \(v_{Madrid} - v_{Spain} + v_{France}\) is closer to \(v_{Paris}\) than to any other words [11].

Inspired by the idea of word vector calculation, we look forward to making knowledge computable. If we ideally consider the example mentioned above, the most probable reason \(v_{Madrid} - v_{Spain} + v_{France}\approx v_{Paris}\), is that capital_city_of is the relation between Madrid and Spain, and so is Paris and France. In other words, we can conclude that:

- There are two facts/triplets, i.e. \((Madrid, capital\_city\_of, Spain)\) and \((Paris, capital\_city\_of, France)\).
- We also derive the approximate equation that \(v_{Spain} - v_{Madrid} \approx v_{France} - v_{Paris}\) from \(v_{Madrid} - v_{Spain} + v_{France} \approx v_{Paris}\).

The shared relation capital_city_of may help establish the approximate equation due to certain implicit connection. If we assume that the relation capital_city_of can also be explicitly represented by a vector, i.e. \(v_{capital\_city\_of}\), the connection will be \(v_{Spain} - v_{Madrid} \approx v_{France} - v_{Paris} \approx v_{capital\_city\_of}\). Therefore, the fact \((Madrid, capital\_city\_of, Spain)\) can be modeled in another way, i.e. \(v_{Madrid} + v_{capital\_city\_of} \approx v_{Spain}\).

Generally speaking, this paper explores a better approach on knowledge representation by means of learning a unique low-dimensional embedding for each entity and relation in KBs, so that each fact \((head\_entity, relation, tail\_entity)\) can

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1 http://www.princeton.edu/~wordnet
2 http://www.cyc.com/platform/opencyc
3 www.mpi-inf.mpg.de/yago-naga/yago
4 http://www.freebase.com
5 According to the statistics released by the official site, Freebase contains 43 million entities and 1.9 billion triplets for now.
6 As of 2012, YAGO2s has knowledge of more than 10 million entities and contains more than 120 million facts about these entities.
be represented by a simple vector calculation \( h + r \approx t \). To achieve this goal, we contribute a generic model named Large Margin Nearest Neighbor Embedding (LMNNE). As intuitively shown by Figure 1, LMNNE follows the principle of pulling the positive tail entities \( t^+ \) close to \( h + r \), and simultaneously pushing the negative tail entities \( t^- \) a large margin \( \gamma \) away from the positives \( t^+ \). The details about model formulation are described in Section 3. Section 4 presents the algorithm that solves LMNNE efficiently based on Stochastic Gradient Descent (SGD) in iterative fashion. To prove both effectiveness and efficiency of the proposed model, we conduct quantitative experiments in Section 5, i.e. evaluating the performance of Link prediction and Triplet classification on several benchmark datasets. We also perform qualitative analysis via comparing model complexity among all related approaches in Section 6. Results demonstrate that LMNNE can achieve the state-of-the-art performance and demand for fewer computational resources compared with many prior arts.

II. RELATED WORK

All the related studies work on studying better way of representing a fact/triplet. Usually, they design various scoring functions \( f_r(h,t) \) to measure the plausibility of a triplet \((h,r,t)\). The lower dissimilarity of the scoring function \( f_r(h,t) \) is, the higher compatibility of the triplet \((h,r,t)\) will be.

Unstructured [12] is a naive model which just exploits the occurrence information of the head and the tail entities without considering the relation between them. It defines a scoring function \( ||h - t||\), and this model obviously can not discriminate entity-pairs with different relations. Therefore, Unstructured is commonly regarded as the baseline approach.

Distance Model (SE) [13] uses a pair of matrices \( W_{rh}, W_{rt} \), to characterize a relation \( r \). The dissimilarity of a triplet \((h,r,t)\) is calculate by the \( L_1 \) norm of \( ||W_{rh}h - W_{rt}t||\). As pointed out by Socher et al. [14], the separating matrices \( W_{rh} \) and \( W_{rt} \) weaken the capability of capturing correlations between entities and corresponding relations, even though the model takes the relations into consideration.

Single Layer Model proposed by Socher et al. [14] aims at alleviating the shortcomings of Distance Model by means of the nonlinearity of a single layer neural network \( g(W_{rh}h + W_{rt}t + b_r) \), in which \( g = \tanh \). Then the linear output layer gives the scoring function: \( u^Tg(W_{rh}h + W_{rt}t + b_r) \).

Bilinear Model [15], [16] is another model that tries to fix the issue of weak interaction between the head and tail entities caused by Distance Model with a relation-specific bilinear form: \( f_r(h,t) = h^T W_r t \).

Neural Tensor Network (NTN) [14] proposes a general scoring function: \( f_r(h,t) = u^T g(h^T W_r h + W_{rh} h + W_{rt} t + b_r) \), which combines the Single Layer Model and the Bilinear Model. This model is more expressive as the second-order correlations are also considered into the nonlinear transformation function, but the computational complexity is rather high.

TransE [12] is the state-of-the-art method so far. Differing from all the other prior arts, this approach embeds relations into the same vector space of entities by regarding the relation \( r \) as a translation from \( h \) to \( t \), i.e. \( h + r = t \). This model works well on the facts with ONE-TO-ONE mapping property, as minimizing the global loss function will impose \( h + r \) equaling to \( t \). However, the facts with multi-mapping properties, i.e MANY-TO-MANY, MANY-TO-ONE and ONE-TO-MANY, impact the performance of the model. Given a series of facts associated with a ONE-TO-MANY relation \( r \), e.g. \((h,r,t_1),(h,r,t_2),...,(h,r,t_m)\), TransE tends to represent the embeddings of entities on MANY-side extremely the same with each other and hardly to be discriminated. Moreover, the learning algorithm of TransE only considers the randomly built negative triplets, which may bring in bias for embedding entities and relations.

Therefore, we propose a generic model (LMNNE) in the subsequent section to tackle the margin-based knowledge embedding problem. This model can fully take advantage of both positive and negative triplets, and in addition, be flexible enough when dealing with the multi-mapping properties.
III. PROPOSED MODEL

Given a triplet \((h, r, t)\), we use the following formula as the scoring function \(f_r(h, t)\) to measure its plausibility, i.e.

\[
f_r(h, t) = ||h + r - t||,
\]

where \(||\cdot||\) stands for the generic distance metrics (\(L_1\) norm or \(L_2\) norm) depending on the model selection.

Ideally, we look forward to learning a unique low-dimensional vector representation for each entity and relation, so that all the triplets in the KB will satisfy the equation \(h + r = t\). However, this can not be done perfectly because of the subsequent multi-mapping reasons,

- Not all entity pairs involve in only one relation. For example, \((Barack Obama, president_of, U.S.A)\) and \((Barack Obama, born_in, U.S.A)\) are both correct facts in the KB. However, we do not expect that the embedding of the relation \(president_of\) is the same as the relation \(born_in\), since those relations are not semantically related.

- Besides the multi-relation property above, we may also face the problem of multiple head entities or tail entities when the other two elements are given. For example, there are at least five correct tail entities, i.e. \(\text{Comedy film, Adventure film, Science fiction, Fantasy film, Satire}\), given \((\text{WALL-E}, \text{has_the_genre, ?})\). Likewise, we do not desire that those tail entities share the same embedding.

Therefore, we suggest a ‘soft’ way of modeling triplets in KBs. Suppose that \(\Delta\) is the set of facts in the KB. For each triplet \((h, r, t)\) in \(\Delta\), we build a set of constructed triplets \(\Delta_{(h,r,t)}^+\) by means of replacing the head or the tail with all the other entities in turn. \(\Delta_{(h,r,t)}^+, \Delta_{(h,r,t)}^-\) and \(\Delta_{(h,r,t)}\) is the positive set of triplets reconstructed from \((h, r, t)\) as \(\Delta_{(h,r,t)}^+ \subset \Delta\), and \(\Delta_{(h,r,t)}^-\) is the negative because \(\Delta_{(h,r,t)}^- \not\subset \Delta\). The intuitive goal of our model is to learn the embeddings of positive tail entities \(t^+\) closer to \(h + r\) than any other negative embeddings \(t^\cdot\). Therefore, the goal contains two objects, i.e. pulling the positive neighbors near (NN) each other while pushing the negatives a large margin (LM) away.

Specifically, for a pair of positive triplets, \((h, r, t)\) and \((h^+, r, t^+)\), we pull the head or the tail entities together (left panel, Figure 1) by minimizing the loss of variances in distance, i.e.

\[
\mathcal{L}_{\text{pull}} = \min \sum_{(h, r, t) \in \Delta} \sum_{(h^+, r, t^+) \in \Delta_{(h, r, t)}^+} (||h - h^+|| + ||t - t^+||).
\]

Simultaneously, we push the negative head or tail entities away (right panel, Figure 1) via keeping all the negative distances \(f_r(h^-, t^-)\) at least one margin \(\gamma\) farther than \(f_r(h^+, t^+)\). Therefore, the objective function is,

\[
\mathcal{L}_{\text{push}} = \min \sum_{(h, r, t) \in \Delta} \sum_{(h^-, r, t^-) \in \Delta_{(h, r, t)}^-} [\gamma + f_r(h, t) - f_r(h^-, t^-)] +
\]

IV. LEARNING ALGORITHM

To efficiently search the optimal solution of \(\text{LMNNE}\), we use \textit{Stochastic Gradient Descent (SGD)} to update the embeddings of entities and relations in iterative fashion. As shown in Algorithm 1, we firstly initial all the entities and relations following a uniform distribution. Each time we pick a triplet \((h, r, t)\) from \(\Delta\), an accompanied triplet \((h', r, t')\) is sampled at the same time by replacing the head or the tail with another entity from the entity set \(E\), i.e. \(\Delta_{(h', r, t)} = \{(h', r, t) \in E\} \cup \{(h, r, t) \mid t' \in E\}\). Then we choose one of the pair-wise SGD-based updating options depending on which camp \(\Delta_{(h', r, t)}^+\) or \(\Delta_{(h, r, t)}^-\) that \((h', r, t')\) belongs to.

\[
\mathcal{L} = \min \mu \mathcal{L}_{\text{pull}} + (1 - \mu) \mathcal{L}_{\text{push}}.
\]

V. QUANTITATIVE EXPERIMENTS

Embedding the knowledge into low-dimensional vector spaces makes it much easier for AI-related computing tasks, such as \textit{Link prediction} (predicting \(t\) given \(h\) and \(r\) or \(h\) given \(r\) and \(t\)) and \textit{Triplet classification} (to discriminate whether a triplet \((h, r, t)\) is correct or wrong). Two latest studies \[12\],
used subsets of WordNet (WN) and Freebase (FB) data to evaluate their models and reported the performance on the two tasks respectively.

In order to conduct solid experiments, we compare our model (LMNNE) with many related studies including state-of-the-art and baseline approaches involving in the two tasks, i.e. Link prediction and Triplet classification. All the datasets, the source codes and the learnt embeddings for entities and relations can be downloaded from [http://1drv.ms/1pUPwzP](http://1drv.ms/1pUPwzP).

A. Link prediction

One of the benefits on knowledge embedding is that we can apply simple vector calculations to many reasoning tasks. For example, Link prediction is a valuable task that contributes to completing the knowledge graph. Specifically, It aims at predicting the missing entity or the relation given the other two elements in a mutilated triplet.

With the help of knowledge embeddings, if we would like to tell whether the entity \( h \) has the relation \( r \) with the entity \( t \), we just need to calculate the distance between \( h + r \) and \( t \). The closer they are, the more possibility the triplet \((h, r, t)\) exists.

1) Datasets: Bordes et al. [17], [12] released two benchmark datasets which were extracted from WordNet (WN18) and Freebase (FB15K). Table 1 shows the statistics of these two datasets. The scale of FB15K dataset is larger than WN18 with much more relations but fewer entities.

| TABLE I. STATISTICS OF THE DATASETS USED FOR LINK PREDICTION TASK. |
|---------------------------------|---------|--------|
| DATASET                        | WN18   | FB15K  |
| #ENTITIES)                     | 40,943 | 14,951 |
| #RELATIONS)                    | 18     | 1,345  |
| #TRAINING EX.)                 | 141,442| 483,142|
| #VALIDATING EX.)               | 5,000  | 50,000 |
| #TESTING EX.)                  | 5,000  | 59,071 |

2) Evaluation Protocol: For each testing triplet, all the other entities that appear in the training set take turns to replace the head entity. Then we get a bunch of candidate triplets associated with the testing triplet. The dissimilarity of each candidate triplet is firstly computed by the scoring functions, then sorted in ascending order, and finally the rank of the ground truth triplet is recorded. This whole procedure runs in the same way for replacing the tail entity, so that we can gain the mean results. We use two metrics, i.e. Mean Rank and Mean Hit@10 (the proportion of ground truth triplets that rank in Top-10), to measure the performance. However, the results measured by those metrics are relatively inaccurate, as the procedure above tends to generate the false negative triplets. In other words, some of the candidate triplets rank rather higher than the ground truth triplet just because they also appear in the training set. We thus filter out those triplets to report more reasonable results.

3) Experimental Results: We compare our model LMNNE with the state-of-the-art TransE and other models mentioned in [17] and [18] evaluated on the WN18 and FB15K. We tune the parameters of each previous model based on the validation set, and select the combination of parameters which leads to the best performance. The results are almost the same as [12]. For LMNNE, we tried several combinations of parameters: \( d = \{20, 50, 100\} \), \( \gamma = \{0.1, 1.0, 2.0, 10.0\} \), \( \alpha = \{0.01, 0.02, 0.05, 0.1, 0.5, 1.0\} \), \( \beta = \{0.01, 0.02, 0.05, 0.1, 0.5, 1.0\} \) and \( \mu = \{0.2, 0.4, 0.5, 0.6, 0.8\} \), and finally chose \( d = 20, \gamma = 2.0 \), \( \alpha = \beta = 0.02 \) and \( \mu = 0.6 \) for WN18 dataset; \( d = 50, \gamma = 1.0 \), \( \alpha = \beta = 0.02 \) and \( \mu = 0.6 \) for FB15K dataset. Moreover, we adopted different distance metrics, such as \( L_1 \) norm, \( L_2 \) norm and inner product, for the scoring function. Experiments show that \( L_2 \) norm and \( L_1 \) norm are the best choices to measure the distances in \( L_{pull} \) and \( L_{push} \) for both of the two datasets, respectively. Table 2 demonstrates that LMNNE outperforms all the prior arts, including the baseline model Unstructured [18], RESCAL [19], SE [13], SME (LINEAR) [18], SME (BILINEAR) [18], LFM [16] and the state-of-the-art TransE [17], [12], measured by Mean Rank and Mean Hit@10.

Moreover, we divide FB15K into different categories (i.e. ONE-TO-ONE, ONE-TO-MANY, MANY-TO-ONE and MANY-TO-MANY) based on the mapping properties of facts. According to TransE [12], we set 1.5 as the threshold to discriminate ONE and MANY. For example, given a pair \((h, r)\), if the average number of tails appearing in the dataset is upon 1.5, we can categorize the triplets involving in this relation \( r \) into the ONE-TO-MANY class. We evaluate the performance of Filter Hit@10 metric on each category. Table 3 shows that LMNNE performs best on most categories. The result proves that the proposed approach can not only maintain the characteristic of modeling the ONE-TO-ONE triplets, but also better handle the facts with multi-mapping properties.

B. Triplet classification

Triplet classification is another task proposed by Socher et al. [14] which focuses on searching a relation-specific distance threshold \( \sigma_r \) to tell whether a triplet \((h, r, t)\) is plausible.

1) Datasets: Similar to Bordes et al. [17], [12], Socher et al. [14] also constructed two standard datasets i.e. WN11 and FB13, sampled from WordNet and Freebase. However, both of the datasets contain much fewer relations. Therefore, we build another dataset following the principle proposed by Socher et al. [14] based on FB15K which owns much more relations. The head or the tail entity can be randomly replaced with another one to produce a negative triplet, but in order to build much tough validation and testing datasets, the principle emphasizes that the picked entity should once appear at the same position. For example, (Pablo Picaso, 3) The datasets can be downloaded from [https://www.hds.utc.fr/everest/doku.php?id=en:transe](https://www.hds.utc.fr/everest/doku.php?id=en:transe).

4) All the codes for the related models can be downloaded from [https://github.com/glorotxa/SME](https://github.com/glorotxa/SME).

5) To conduct fair comparisons, we re-implemented TransE based on the SGD algorithm (not mini-batch SGD) and fed the same random embeddings for initializing both LMNNE and TransE. That’s the reason why our experimental results of TransE are slightly different from the original paper [17], [12].
TABLE II. LINK PREDICTION RESULTS. WE COMPARED OUR PROPOSED LMNNE WITH THE STATE-OF-THE-ART METHOD (TransE) AND OTHER PRIOR ARTS.

| DATASET | WN11 | FB15K |
|---------|------|-------|
| METRIC  |      |       |
|         | MEAN RANK | MEAN HIT@10 | MEAN RANK | MEAN HIT@10 |
|         | Raw | Filter | Raw | Filter | Raw | Filter | Raw | Filter |
| Unstructured RESCAL | 315 | 304 | 35.3% | 38.2% | 1,074 | 979 | 4.5% | 6.3% |
| SE | 1,180 | 1,163 | 37.2% | 52.8% | 828 | 68.3 | 28.4% | 44.1% |
| SME (LINEAR) | 1,011 | 985 | 68.5% | 80.5% | 273 | 162 | 28.8% | 39.8% |
| SME (BILINEAR) | 545 | 533 | 65.1% | 74.1% | 274 | 154 | 30.7% | 40.8% |
| LFHM | 526 | 509 | 54.7% | 61.3% | 284 | 158 | 31.3% | 41.3% |
| TransE | 469 | 456 | 71.4% | 81.6% | 283 | 164 | 26.0% | 33.1% |
| LMNNE | 294.4 | 283.2 | 70.4% | 80.2% | 243.3 | 139.9 | 36.7% | 44.3% |

TABLE III. RESULTS OF MAP HITS@10 (IN %) ON FB15K CATEGORIZED BY DIFFERENT DISTRIBUTION PROPERTIES OF FACTS (M. STANDS FOR MANY).

| TASK | Predicting head | Predicting tail |
|------|----------------|----------------|
| REL. Mapping | 1-TO-1 | 1-TO-M. | M.-TO-1 | M.-TO-M. | 1-TO-1 | 1-TO-M. | M.-TO-1 | M.-TO-M. |
| Unstructured | 34.5% | 1.4% | 6.1% | 6.6% | 34.3% | 4.2% | 1.9% | 6.6% |
| SE | 35.6% | 1.4% | 17.2% | 37.5% | 34.9% | 14.6% | 68.3% | 41.3% |
| SME (LINEAR) | 35.1% | 1.4% | 19.0% | 40.3% | 32.7% | 14.9% | 61.6% | 43.3% |
| SME (BILINEAR) | 30.9% | 1.4% | 19.9% | 38.6% | 28.2% | 13.1% | 76.0% | 41.8% |
| TransE | 43.7% | 1.4% | 65.7% | 18.2% | 47.2% | 43.7% | 19.7% | 66.7% | 50.0% |
| LMNNE | 59.2% | 1.4% | 77.8% | 17.5% | 55.5% | 58.6% | 20.0% | 80.9% | 51.2% |

TABLE IV. STATISTICS OF THE DATASETS USED FOR TRIPLET CLASSIFICATION TASK.

| DATASET | WN11 | FB13 | FB15K |
|---------|------|------|-------|
| #ENTITIES | 38,096 | 75,043 | 14,931 |
| #RELATIONS | 11 | 13 | 1,345 |
| #TRAINING EX. | 112,581 | 316,232 | 483,142 |
| #VALIDATING EX. | 5,218 | 11,816 | 100,000 |
| #TESTING EX. | 21,088 | 47,466 | 118,142 |

TABLE V. THE ACCURACY OF TRIPLET CLASSIFICATION COMPARED WITH THE STATE-OF-THE-ART METHOD (TransE) AND OTHER PRIOR ARTS.

| DATASET | WN11 | FB13 | FB15K |
|---------|------|------|-------|
| DISTANCE Mode | 53.0% | 75.2% | - |
| Hadamard Model | 70.0% | 63.7% | - |
| Single Layer Model | 69.9% | 85.3% | - |
| Bilinear Model | 73.8% | 84.1% | - |
| NTN | 70.4% | 87.1% | 66.7% |
| TransE | 77.5% | 67.5% | 85.8% |
| LMNNE | 78.6% | 74.8% | 86.8% |

VI. QUALITATIVE ANALYSIS

In addition to the quantitative experiments of evaluating the performance on Link prediction and Triplet classification with several benchmark datasets, we analytically compare the parameter complexity among the approaches that we have mentioned as well. Table 6 lists the theoretical costs on representing triplets \((h, r, t)\) based on the scoring functions of nearly all the classical models. Except for Unstructured, TransE and LMNNE, the other approaches regard the relation \(r\) as a transportation matrix serving for the entities \(h\) and \(t\). These models need more resources on storing and computing embeddings. Unstructured costs least, but does not contain any information on relations. LMNNE and TransE embed both entities and relations into the low-dimensional vector spaces from different aspects of observations: LMNNE regards a relation as the inner connections of word embedding calculations, and TransE considers it as a kind of translation from one entity to another. Despite the varies angles of modeling, both of them are relatively efficient models for knowledge representation.

VII. CONCLUSION AND FUTURE WORK

Knowledge embedding is an alternative way of representing knowledge besides displaying in triplets, i.e. \((h, r, t)\). Its essence is to learn a distributed representation for each entity and relation, to make the knowledge computable, e.g. \(h + r \approx t\).

To achieve higher quality of embeddings, we propose LMNNE, a both effective and efficient model on learning...
TABLE VI. THE SCORING FUNCTION AND PARAMETER COMPLEXITY ANALYSIS FOR ALL THE MODELS MENTIONED IN THE EXPERIMENTS. FOR ALL THE MODELS, WE ASSUME THAT THERE ARE A TOTAL OF \( n_e \) ENTITIES, \( n_r \) RELATIONS (IN MOST CASES, \( n_e \gg n_r \)), AND EACH ENTITY IS EMBEDDED INTO A \( d \)-DIMENSIONAL VECTOR SPACE, I.E. \( \mathbf{h}, \mathbf{t} \in \mathbb{R}^d \). WE ALSO ASSUME THAT THERE ARE \( s \) SLICES IN A TENSOR FOR THE NEURAL-NETWORK RELATED MODELS, I.E. SINGLE LAYER MODEL AND NEURAL TENSOR NETWORK.

| Model                       | Scoring Function                                                                 | Parameter Complexity |
|-----------------------------|----------------------------------------------------------------------------------|----------------------|
| Unstructured                | \[ ||\mathbf{h} - \mathbf{t}|| \]                                               | \( n_e d \)          |
| Distance Model (SE)         | \[ ||W_{kr}\mathbf{h} - W_{kr}\mathbf{t}|| \]                                | \( n_e d + 2n_r d^2 \) |
| Single Layer Model          | \( u_k^r \tanh(W_{kh}\mathbf{h} + W_{kt}\mathbf{t} + b_k) \)                  | \( n_e d + 2n_r (sd + s) \) |
| Bilinear Model              | \( \mathbf{h}^T W_{rt}; W_r \in \mathbb{R}^{d \times d} \)                     | \( n_e d + n_r d^2 \) |
| Neural Tensor Network (NTN) | \( u_k^r \tanh(h_k^r W_{rh}\mathbf{h} + W_{rt}\mathbf{t} + b_r) \)              | \( n_e d + n_r (sd^2 + 2sd + 2s) \) |
| TransE and LMNNE            | \( ||\mathbf{h} + \mathbf{r} - \mathbf{t}|| \)                               | \( n_e d + n_r d \) |

a low-dimensional vector representation for each entity and relation in Knowledge Bases. Some canonical tasks, such as Link prediction and Triplet classification, which were ever based on hand-made logic rules, can be truly facilitated by means of the simple vector calculation. The results of extensive experiments on several benchmark datasets and complexity analysis show that our model can achieve higher performance without sacrificing efficiency.

In the future, we look forward to paralleling the algorithm which can encode a whole KB with billion of facts, such as Freebase and YAGO. Another direction is that we can apply this new way of Knowledge Representation on reinforcing some other related studies, such as Relation Extraction from free texts and Open Question Answering.

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