Magnetic Field Distribution Due To Domain Walls
In Unconventional Superconductors

N. A. Logoboy
Racah Institute of Physics, The Hebrew University of Jerusalem, Jerusalem 91904, Israel
(Dated: February 2, 2008)

Steady-state properties of 180° Bloch domain wall (DW) in superconducting ferromagnet (SCFM) are studied. The distribution of magnetic field above and below the surface of the SCFM due to the permanent magnetization supercurrent flowing in the DW plane is calculated by solving Maxwell equations supplemented by London equation. It is shown that part of the magnetic flux of the two neighboring domains closures in the nearest vicinity of the surface of the sample giving rise to declination of the line of the force from being parallel to the DW plane. As a result, the value of the normal component of magnetic field at the surface of the sample reaches only half of the value of the bulk magnetic flux. At the distances of the order of value of London penetration depth the magnetic field decreases as inverse power law due to the long-range character of dipole-dipole interaction. The last two circumstances are important for comparison the calculated magnetic field with the data obtained by the methods on measurement of normal component of magnetic field, e.g. Hall probe technique, aimed to confirm the existence of magnetic order parameter in unconventional superconductor.

PACS numbers: 74.25.Ha, 74.90.+n, 75.60.-d

I. INTRODUCTION

Unconventional superconductors are a subject of intensive experimental and theoretical studies during past decade [1]-[8]. Coexistence of superconductivity and magnetism results in a number of unusual phenomena, which have both fundamental and application interests. Unconventional superconductors, such as Sr$_2$RuO$_4$ [2], ZrZn$_2$ [4] and UGe$_2$ [5], which are characterized by the state with broken time-reversal symmetry (TRSB), possess the magnetic structure related to non-zero orbital magnetic moment due to spin-triplet $p$-wave state of multi-component SC order parameter. The properties of such superconductors are not trivial. In particular, the macroscopic magnetization is not well-defined quality, and the equations of motion cannot be expressed in terms of local magnetization [7].

Starting from the pioneering work [10], [11] on investigation of non-uniform states of superconducting order parameter (see, also review [12]), the planar defects such as domain walls (DW) and surfaces have been attracted recently a lot of interest [13], [14], [15], [16] due to new experiment possibilities, e.g. scanning superconducting quantum interference device (SQUID) and Hall probe technique, on investigation of magnetic field distribution resulting from spontaneously generated superconducting currents which can serve as a prove of TRSB origin of superconducting order parameter in unconventional chiral superconductors.

Although, the magnetostatic fields are screened by superconducting current, metastable domain walls (DWs), as topologically stable planar defects, may exist even in the Meissner state [4]. Thus, the domain structure of SCFM cannot be ignored. The anomalies in the local magnetization loop for Sr$_2$RuO$_4$ near $B = 0$ are considered as a strong indication of the presence of chiral domains [15]. The strong coordinate dependence of magnetization of such a 2D magnetic defect creates the intrinsic magnetic field which interacts with superconducting (SC) current. For unconventional superconductors the discontinuity of magnetic induction at the DW can be interpreted as an effective magnetization, and the contribution of DW current to the energy density can be transformed to an effective Zeeman term providing the possibility of excitation of orbital magnetization waves by incident electromagnetic field [7].

In present publication we solve the Maxwell equations for distribution of magnetic field inside the sample and above it due to the presence of planar defect, such as a DW, show rapid decrease of the magnetic field in the vicinity of sample surface, discuss the long-range origin of this field resulting in existence of the tails at the distance $\sim \lambda$ (London penetration depth) and compare our results with recent experimental data on measurements of the magnetic field in Sr$_2$RuO$_4$ by SQUID [13] and Hall probe technique [19]. It is shown that for infinitely thin DW the stray fields above the sample depend only the magnetic field jump at the DW, but not on the details of DW structure.

II. BASIC EQUATIONS

Let us consider the 180° DW of Bloch type in a semi-infinite sample of superconducting ferromagnet occupying $y \leq 0$. We assume, that the surface of a crystal is parallel to the $x-z$ plane at $y = 0$, and the domain wall, being parallel to the $y-z$ plane, separates two domains with magnetization $M(x)$ along the $+y$ or $-y$ direction provided that the quality parameter $\alpha = H_K/4\pi M_0 > 1$, e.g. the field of magnetic crystallographic anisotropy of the easy-axis ($y$-axis) type ($H_K$) is high enough in or-
is the magnetization, and $M_0$ parameter, $\Phi$ where $\phi$ is the phase of the superconducting order parameter, $\Phi_0 = hc/2e$ is the magnetic flux quantum, $M$ is the magnetization, and $M_\perp$ is the magnetization component perpendicular to the easy axis $\hat{z}$, which is the normal to the sample surface. The vector potential $A$ determines the magnetic induction $B = \text{curl} \ A$. The first and second terms in Eq. (1) describe the magnetic crystallographic and exchange energies correspondingly, the third term is kinetic energy of superconducting current, and the last two terms relate to magnetostatic energy. The length $\Delta$ characterizes the stiffness of the spin system and is also the DW width. The magnetization can be expressed in polar coordinates, $M = M_0(\sin \phi \sin \theta, \cos \theta, \cos \phi \sin \theta)$ with $M_0 = g\mu_B s/\alpha^3$ the saturation magnetization and $\alpha$ the lattice constant.

A. Ground state of $180^\circ$ Bloch domain wall

The ground state of the SCFM is defined by:

$$\frac{\delta F}{\delta q_i} = 0,$$

where $q_i = \theta, \phi, A$. Minimization of the free energy density Eq. (1) with respect to azimuthal, $\phi$, polar, $\theta$, angles and vector potential $A$ leads to the system of non-linear differential equations describing the ground state of SCFM with DW of Bloch type. In case of $\delta = \Delta/\lambda < 1$

![Figure 1](image1.png)

**FIG. 1:** Shown are the $y$-component ($a$) and $z$-component ($b$) of magnetic induction for Bloch DW at constant value of the domain wall width $\Delta$ and variable London penetration depth $\lambda$. The indexes 1, 2 and 3 correspond to $\delta = \Delta/\lambda = 0.1, 0.3$ and 0.6 respectively. The dashed line shows the $y$-component ($a$) and $z$-component ($b$) of magnetization which are the limiting values for corresponding components of magnetic induction at $\lambda \rightarrow \infty$, e.g. in neglecting of screening by superconducting current.

![Figure 2](image2.png)

**FIG. 2:** Shown are the $y$-component ($a$) and $z$-component ($b$) of magnetic field for Bloch DW at constant value of the domain wall width $\Delta$ and variable London penetration depth $\lambda$. The indexes 1, 2 and 3 correspond to $\delta = \Delta/\lambda = 0.1, 0.3$ and 0.6 consequently.
FIG. 3: Shown are the $y$-components of magnetic induction (a) and magnetic field (b) for Bloch DW at constant value of the London penetration depth $\lambda$ and variable DW width $\Delta$. The indexes 1, 2 and 3 correspond to the solution of these equations are as follows: 

$$
\begin{align*}
\theta_0 &= 2 \tan^{-1} e^{x/\lambda}, \quad \phi_0 = 0, \\
b_y^{(0)} &= \cos \theta_0 e^{x/\lambda} \cos \phi_0, \quad b_z^{(0)} = \sin \theta_0 e^{x/\lambda} \cos \phi_0, 
\end{align*}
$$

(3)

where we used the reduced values for the components of magnetic induction, $b_y^{(0)} = B_y^{(0)}/4\pi M_0$ and $b_z^{(0)} = B_z^{(0)}/4\pi M_0$. The condition $\delta < 1$ allows us to neglect the influence of Meissner current on the structure of the DW, which is defined entirely by the magnetic anisotropy and exchange energy. The screening action of the current results in decreasing of magnetic induction at the distance of $\sim \lambda$ from the center of the domain wall $x = 0$. The graphical representation of the solutions (3) are shown in Fig. 1 for different values of London penetration depth $\lambda$ at constant DW width $\Delta$. The distribution of magnetic field $\mathbf{H} = \mathbf{B} - 4\pi \mathbf{M}$, created by Meissner current, are shown in Fig. 2. It follows from Fig. 1 and Fig. 2 that the screening effect of superconducting current decreases with increasing of London penetration depth $\lambda$.

At constant $\lambda$ in limit case when the DW width $\Delta \to 0$, the jump of the tangential component of magnetization $M_y$ at the plane of the geometric domain boundary $y = z$ defines the current sheet responsible for the jump of tangential component of magnetic induction $B_y$ [8]. The distributions of tangential components of magnetic induction $B_y$ and magnetic field $H_y$ of Meissner current at different values of the DW width $\Delta$ and constant London penetration depth $\lambda$ are shown in Fig. 3. The results of our calculation based on Eq. (3) show the decrease of magnetic induction splitting with increase of DW width $\Delta$ (see, Fig. 3), which confirms the effect of smoothing due to finite DW width [20].

It follows from Maxwell equation $\nabla \times \mathbf{H} = \mathbf{j}$ that there exists a two-component screening current $\mathbf{j}(x) = (0, j_y(x), j_z(x))$, which is the result of 2-dimensional structure of the DW Eq. (3). 

III. RESULTS AND DISCUSSION

To calculate the distribution of magnetic field near the surface of SCFM we neglect the domain wall $\Delta$, assuming that $\Delta \ll \lambda$. This assumption does not affect the results qualitatively, but essentially simplifies the problem. In the end we shall discuss the effects of finite DW width $\Delta \neq 0$.

A. Infinitely thin DW ($\Delta \to 0$)

The difference between a normal FM and a SCFM is important at distances larger than $\Delta$: while in normal FMs the magnetic field $\mathbf{H} = \mathbf{B} - 4\pi \mathbf{M}$ vanishes and the magnetic induction $\mathbf{B} = 4\pi \mathbf{M}$ is constant inside domains, in SCFMs the magnetic induction $\mathbf{B}$ is confined in the Meissner layers of width $\lambda$ [8]:

$$
B_y = \pm \frac{1}{2} \delta B_y e^{\pm x/\lambda},
$$

(4)

where the upper and the lower signs correspond to $x < 0$ and $x > 0$ respectively, and $\delta B_y$ is the magnetic induction splitting. Thus the Meissner currents $j_z = -(c/4\pi) \partial_x B_y$ screen out the main bulk of domains from the magnetic induction. This screened magnetic induction Eq. (4) influences at the distribution of magnetic field above the sample surface and, in principle, can be detected by Hall probes. To find the distribution of magnetic field above the surface of the sample due to magnetic induction Eq. (4) the standard procedure of solving Maxwell equation in magnetic media ($y < 0$) and vacuum ($y > 0$) with appropriate boundary conditions at the surface located at $y = 0$, e. g. the continuity of normal component of magnetic induction and tangential component of magnetic field, is used (see, e. g. [18]). The results of calculation can be represented in explicit form:

$$
\begin{align*}
\delta B_y &= \frac{2}{\pi} \int_0^{+\infty} dk \frac{k \exp (-ky)}{k(k+k)} \cos (kx), \\
\delta B_y &= \frac{2}{\pi} \int_0^{+\infty} dk \frac{k \exp (-ky)}{k(k+k)} \sin (kx)
\end{align*}
$$

(5)
for the components of the reduced magnetic field $h_i = 2H_i/δB_i$ in vacuum, and

$$
b_x = -\frac{2}{\pi} \int_0^{+\infty} dk \frac{k \exp(\tilde{ky})}{k(k + \tilde{k})} \cos(kx),$$

$$
b_y = \frac{2}{\pi} \int_0^{+\infty} dk \frac{k[k + k(1 - \exp(\tilde{ky}))]}{k^2(k + \tilde{k})} \sin(kx) \tag{6}$$

for the components of the reduced magnetic induction $b_i = 2B_i/δB_i$ in the sample. In Eqs. (5), (6) we used the notation $\tilde{k} = (k^2 + \lambda^2)^{1/2}$. The expression for $h_y$ (last equation in (5)) in slightly different form was calculated in [19].

The components of magnetic field in vacuum Eqs. (5) can be calculated by introducing the complex potential $\psi = \psi(w)$ which is analytical function of complex variable $w = y - ix$. Thus, the complex magnetic field

$$
h(w) = h_y(x, y) + ih_x(x, y) \tag{7}$$

is derived from $\psi(w)$ by $h(w) = -\partial_w \psi(w)$. The complex potential $\psi(w)$ can be expressed through special functions

$$
\psi(w) = -\frac{1}{\pi w} - i\partial_w[H_0(w) - N_0(w)], \tag{8}
$$

where $H_0(w)$ and $N_0(w)$ are zero-order Struve and Neumann functions correspondingly (see, e. g. [21]). For $|w| \geq \lambda$, the difference $H_0(w) - N_0(w) \approx 2/\pi w$, therefore, the main contribution to the potential $\psi(w)$ is due to the first term in Eq. (5) which allows to calculate the asymptotic distribution of the component of magnetic field above the sample ($y \geq \lambda$)

$$
h_x = \frac{2}{\pi} \lambda^2 \frac{x^2 - y^2}{(x^2 + y^2)^2}, \quad h_y = \frac{4}{\pi} \lambda^2 \frac{xy}{(x^2 + y^2)^2}. \tag{9}
$$

In particular, it follows from Eq. (9), that at $|w| > 1$ the value of magnetic field decreases as inverse power law, e. g. $|h| \sim |w|^{-2}$, due to long-range magnetostatic interaction.

The results of numerical calculations of normal to the sample surface component of magnetic field in vacuum above the surface of the sample $h_y$, Eq. (9) and in the sample $b_y$ Eq. (6) are represented in Fig. 4 which shows that at the sample surface $y = 0$, magnetic induction splitting equals half of it bulk value $\delta B_y$ and are characterized by the rapid decrease with the distance above the DW. Thus, at the distance of $y = 0.1 \lambda$ above the DW the magnetic field decays till the third of the bulk value $\delta B_y$. At $y \sim \lambda$, the magnetic field splitting reaches about 0.05 of it bulk value and decays with $x$ as inverse power law Eq. (9) due to the long-range origin of dipole-dipole interaction.

In a limit of infinitely thin DW, $\Delta \to 0$, the stray fields in vacuum above the superconducting ferromagnet depend only on the jump of magnetic induction at DW plane, but not on the details of DW structure. In this case, the tangential to the sample surface component of magnetic field is non-analytic function of $y$ and is characterized by the discontinuous jump at the DW position, $y = 0$. In next subsection we take into account the effects of small ($\Delta << \lambda$), but finite DW width $\Delta \neq 0$.

### B. Finite DW width ($\Delta \neq 0$)

To consider the effects related to the finite DW width, $\Delta \neq 0$, we assume the linear distribution of magnetic induction in the DW deep in the bulk, e.g. $B_y \approx (1/2)\delta B_y(x/\Delta)$ at $|x| \leq \Delta$, which is not affected by superconducting currents, inasmuch as $\Delta << \lambda$. This assumption significantly simplifies the calculation and allows to express the results in analytic form. It can be shown that for finite DW width the kernels in Eqs. (5) and (6) are modified by the multiplier $\sin(k\Delta)/k\Delta$. It results in slight suppression of $y$-component of magnetic field and restores the analyticity of $x$-component of magnetic field. The discontinuous jump of tangential component of magnetic field at the position of DW is replaced by the value $h_y^{max} = (1/\pi)\ln(\gamma\delta)$, where $\gamma \approx 1.78107$ is Euler-Mascheroni constant.

Recent experiments on imaging of magnetic field distribution above the surface of the sample of Sr$_2$RuO$_4$ by scanning SQUID and Hall probe microscopy has revealed no evidence for existence of DWs in this unconventional superconductor [19]. This negative result can
be understood in the framework of the developed theory. If the DWs exist, the maximum value of magnetic induction due to permanent current flowing in the plane of the DW does not exceed the lower critical field, e. g. $\delta B_y/2 \sim H_{c1} \approx 30G$ above which the domain structure is unstable due to formation of Abrikosov vortices. Thus, at the distance of $y = \lambda \approx 190\text{ nm}$, the normal component of magnetic field due to the presence of DW is of order of 3G which definitely improbable by the used methods.

Acknowledgments

The stimulated inspiring discussions with Prof. E. B. Sonin are highly appreciated. This work has been supported by the grant of the Israel Academy of Sciences and Humanities.

[1] I. Felner, U. Asaf, Y. Levi, and O. Milo, Phys. Rev. B 55, 3374 (1997).
[2] M. R. Eskildsen, K. Harada, P. L. Gammel, A. B. Abrahamson, N. H. Andersen, G. Ernst, A. P. Ramirez, D. J. Bishop, K. Mortensen, D. G. Naugle, K. D. D. Rathnayaka, and P. C. Canfield, Nature (London) 393, 242 (1998).
[3] G. M. Luke, Y. Fudamoto, K. M. Kojima, M. I. Larkin, J. Merrin, B. Nachumi, Y. J. Uemura, Y. Maeno, Z. Q. Mao, Y. Mori, H. Nakamura and M. Sigrist, Nature (London) 394, 558 (1998).
[4] C. Pfleiderer, M. Uhlarz, S. M. Hayden, R. Vollmer, Hv. Löhneyesen, N. R. Bernhoeft, and G. G. Lonzarich, Nature (London) 412, 58 (2001).
[5] S. S. Saxena, P. Agarwal, K. Ahilan, F. M. Grosche, R. K. W. Haselwimmer, M. J. Steiner, E. Pugh, I. R. Walker, S. R. Julian, P. Monthoux, G. G. Lonzarich, A. Huxley, I. Sheikin, D. Braithwaite, and J. Flouquet, Nature (London) 406, 587 (2000).
[6] E. B. Sonin and I. Felner, Phys. Rev. B 57, R14000 (1998).
[7] V. Braude and E. B. Sonin, Phys. Rev. B 74, 064501 (2006).
[8] E. B. Sonin, Phys. Rev. B 66, 100504 (2002).
[9] V. Braude and E. B. Sonin, Phys. Rev. Lett. B 93, 117001 (2003).
[10] G. E. Volovik and L. P. Gor’kov, Sov. Phys. JETP 61, 843 (1985).
[11] M. Sigrist, T. M. Rice, and K. Ueda, Phys. Rev. Lett. 63, 1727 (1989).
[12] M. Sigrist and K. Ueda, Rev. Mod. Phys. 63, 239 (1991).
[13] P. G. Björnsson, Y. Maeno, M. E. Huber, and K. A. Moller, Phys. Rev. B 72, 012504 (2005).
[14] P. G. Kealey, T. M. Riseman, E. M. Forgan, L. M. Galvin, A. P. Mackenzie, S. L. Lee, D. M. Paul, R. Cubitt, D. F. Agterberg, R. Heeb, Z. Q. Mao, and Y. Maeno, Phys. Rev. Lett. 84 6094 (2000).
[15] T. Tamegai, K. Yamazaki, M. Tokunaga, Z. Mao, and Y. Maeno, Physica C Volumes 388-389 499 (2003).
[16] V. O. Dolocan, C. Veauvy, F. Servant, P. Lejay, K. Haselbach, Y. Liu, and D. Mailly, Phys. Rev. Lett. 95, 097004 (2005).
[17] D. Vollhardt and P. Wölfle, The Superfluid Phases of Helium 3 (Taylror and Francis, London, 1990).
[18] S. Chikazumi, Physics of Ferromagnetism (Clarendon Press, Oxford, England 1997).
[19] J. R. Kirtley, C. Kallin, C. W. Hicks, E.-A. Kim, Y. Liu, K. A. Moler, Y. Maeno, and K. D. Nelson, Phys. Rev. B 76, 014526 (2007).
[20] H. Bluhm, [arXiv:0705.4118v1 [cond-mat.sup-con]],
[21] M. Abramowitz and I. A. Stegun, Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables (Dover, New York, 1972).