Achieving Better Than 1 Minute Accuracy in the Heliocentric and Barycentric Julian Dates

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ABSTRACT. As the quality and quantity of astrophysical data continue to improve, the precision with which certain astrophysical events can be timed becomes limited not by the data themselves, but by the manner, standard, and uniformity with which time itself is referenced. While some areas of astronomy (most notably pulsar studies) have required absolute time stamps with precisions of considerably better than 1 minute for many decades, recently new areas have crossed into this regime. In particular, in the exoplanet community, we have found that the (typically unspecified) time standards adopted by various groups can differ by as much as a minute. Left uncorrected, this ambiguity may be mistaken for transit timing variations and bias eccentricity measurements. We argue that, since the commonly-used Julian Date, as well as its heliocentric and barycentric counterparts, can be specified in several time standards, it is imperative that their time standards always be reported when accuracies of 1 minute are required. We summarize the rationale behind our recommendation to quote the site arrival time, in addition to using BJD$_{\text{TDB}}$, the Barycentric Julian Date in the Barycentric Dynamical Time standard for any astrophysical event. The BJD$_{\text{TDB}}$ is the most practical absolute time stamp for extraterrestrial phenomena, and is ultimately limited by the properties of the target system. We compile a general summary of factors that must be considered in order to achieve timing precisions ranging from 15 minutes to 1 $\mu$s. Finally, we provide software tools that, in principal, allow one to calculate BJD$_{\text{TDB}}$ to a precision of 1 $\mu$s for any target from anywhere on Earth or from any spacecraft.

1. INTRODUCTION

Precise timing of astrophysical events is one of the fundamental tools of astronomy, and is an important component of essentially every area of study. There are two basic sources of uncertainty in timing: the astrophysical data characterizing the event, and the time stamp with which the event is referenced. Unfortunately, since the accuracy of the time stamp is something that is often taken for granted, the improvements in data are sometimes not accompanied (or are not uniformly accompanied) by the requisite improvements in accuracy of the time stamp used. This situation can lead to confusion, or even spurious inferences.

Timing plays a particularly important role in the study of exoplanets. Indeed, many of the ways in which exoplanets are discovered involve the detection of transient or time-variable phenomena, including the radial velocity, transit, microlensing, and astrometry techniques. Furthermore, in some cases much can be learned about planetary systems from the precise timing of these phenomena. As examples, the measurement of terrestrial parallax in microlensing allows one to infer the mass of the primary lens and so the planetary companion (e.g., Gould et al. 2009), and one can constrain the eccentricity of transiting planets by comparing the times of primary transits and secondary eclipses (e.g., Deming et al. 2005; Charbonneau et al. 2005; Knutson et al. 2007). Possibly the most promising application of timing in exoplanets, however, comes from transit timing variations (TTVs). With an exquisitely periodic phenomena like transiting planets, we will be able to measure many effects using the departures from strict periodicity, such as the gravitational perturbations from additional planets (Miralda-Escudé 2002; Holman & Murray 2005; Agol et al. 2005), trojans (Ford & Gaudi 2006; Ford & Holman 2007), and moons (Kipping 2009), stellar quadrupoles (Miralda-Escudé 2002), tidal deformations, general relativistic precession (Jordán & Bakos 2008; Pál & Kocsis 2008; Heyl & Gladman 2007), orbital decay (e.g., Sasselov 2003), and proper motion (Rafikov 2009).

Because of their great potential, TTVs have become the focus of many groups. The typical data-limited transit timing precisions of most observations are around 1 minute, with the best transit time precision yet achieved of a few seconds (Pont et al. 2007). However, as discussed above, accuracies of transit times are limited not only by the data themselves, but also by the time stamp used. In order to make these difficult measurements useful, it is critical that a time stamp be used that is considerably more accurate than the uncertainty due to the data themselves. Furthermore, since thorough characterization of TTVs will require the use of all available data spanning many years from several groups, this time stamp must be stable in the long term, and all groups must clearly convey how it was calculated.
Unfortunately, we have discovered that, in the exoplanet community, the Julian Date (JD) and its geocentric (GJD), heliocentric (HJD), and barycentric (BJD) counterparts are currently being quoted in several different, and often unspecified, time standards. In addition, the site arrival time and its time standard is not quoted. This general lack of homogeneity and specificity leaves quoted time stamps ambiguous at the 1 minute level. More alarmingly, the most commonly-used time standard, the Coordinated Universal Time (UTC), is discontinuous and drifts with the addition of each leap second roughly each year.

The pulsar community has solved the problem of precise timing well beyond the level that is currently necessary for exoplanet studies, and we can benefit from the techniques they have developed over the past 40 years. In particular, their current state of the art program (TEMPO2) models pulsar arrival times to ±1 ns precision (Hobbs et al. 2006; Edwards et al. 2006). This program is highly specialized and generally cannot be applied outside of pulsar timing observations, but many of the effects they consider are relevant to optical observers in the exoplanet community.

In this article, we summarize the effects one must consider in order to achieve timing accuracy of 1 µs—well beyond the accuracy that will likely be required by the exoplanet community for the foreseeable future. Section 2 provides the background required to understand each of the effects that could change the arrival time of a photon. They are listed in order of decreasing magnitude, so latter subsections can be ignored for low-precision measurements. Section 3 discusses the practical limitations to achieving high-precision timing. We begin with the effects which may cause errors that are comparable to or exceed the BJD correction. These should be read and understood by everyone. We continue with remaining effects, in order of decreasing magnitude, which can be ignored for low-precision (>30 ms) measurements. We conclude § 3 by listing additional effects, the errors due to which are negligible (<1 µs).

We begin § 4 by detailing the procedure one must follow in order to calculate the BJD_{TDB}, which is designed to be a useful reference for those already familiar with the concepts of precision timing. In the latter part of this section, we describe our particular IDL and web-based implementation of this procedure. Lastly, in the Appendix, we discuss some of our specific findings about the time stamps currently in use and how these are calculated throughout the exoplanet community.

While we focus on the effects of timing on the optical/infrared exoplanet community, timing precision of order 1 minute is necessary for many other areas, such as the study of rapidly rotating white dwarfs (Euchner et al. 2006). This article should be equally applicable in such cases.

2. THEORETICAL TIMING PRECISION

The biggest source of confusion comes from the fact that time standards and reference frames are independent from one another, even though there are many overlapping concepts between the two. We will use the following terminology: “reference frame” will refer to the geometric location from which one could measure time—different reference frames differ by the light-travel time between them; “time standard” will refer to the way a particular clock ticks and its arbitrary zero point, as defined by international standards; and “time stamp” is the combination of the two, and determines the timing accuracy of the event.

The BJD_{TDB}, the time stamp we advocate, can be calculated using the equation:

\[ \text{BJD}_{\text{TDB}} = \text{JD}_{\text{UTC}} + \Delta_{R\odot} + \Delta_C + \Delta_{S\odot} + \Delta_{E\odot}, \]

where \text{JD}_{\text{UTC}} is the Julian Date in Coordinated Universal Time; \Delta_{R\odot} is the Rømer Delay, discussed in § 2.1; \Delta_C is the clock correction discussed in § 2.2; \Delta_{S\odot} is the Shapiro delay discussed in § 2.3; and \Delta_{E\odot} is the Einstein delay, discussed in § 2.4.

The order of these terms is such that they are of decreasing magnitude, so one need only keep the terms up to the precision required. The timing precision required by current exoplanet studies (~1 s) requires only the terms up to and including \Delta_C.

Because future solar system ephemerides may enable more precise calculations of the arrival time at the barycenter, or in order to allow others to check that the original conversion was done accurately enough for their purpose, the site arrival time (e.g., the JD_{\text{UTC}}) should always be quoted in addition to the BJD_{TDB}.

2.1. Reference Frames: The Rømer Delay

Due to the finite speed of light, as the Earth travels in its orbit, light from an astrophysical object may arrive early or be delayed by as much as 8.3 minutes from the intrinsic time of the extraterrestrial event. This is called the Rømer delay, \Delta_{R\odot}, in honor of Ole Rømer’s demonstration that the speed of light is finite. Since most observers cannot observe during daylight, a bias is introduced and in practice the delay (as distinct from the early arrival time) is only as much as 7 minutes, for a peak-to-peak variation of 15 minutes. Figure 1 shows an example of this effect for a maximally-affected object on the ecliptic. In order to show the observational bias, our example assumes the object is at 0° right ascension and 0° declination. This curve shifts in phase with ecliptic longitude and in amplitude with ecliptic latitude. We also place our observer at the Earth’s equator, but note that the asymmetry will be larger at different latitudes.

The solution to this problem is to calculate the time when a photon would have arrived at an inertial reference frame. This time delay is the dot product of the unit vector from the observer to the object, \hat{n}, and the vector from the origin of the new reference frame to the observer, \vec{r}:

\[ \Delta_{R\odot} = \vec{r} \cdot \hat{n} / c, \]
where \( c \) is the speed of light and \( \tilde{n} \) can be written in terms of its right ascension (\( \alpha \)) and declination (\( \delta \)),

\[
\tilde{n} = \begin{pmatrix}
\cos(\delta) \cos(\alpha) \\
\cos(\delta) \sin(\alpha) \\
\sin(\delta)
\end{pmatrix}.
\quad (3)
\]

This equation is general as long as \( \alpha \), \( \delta \), and \( \tilde{r} \) are in the same coordinate system (e.g., Earth mean equator J2000) and the object located at (\( \alpha \), \( \delta \)) is infinitely far away. Other forms of this equation in the literature assume that we have the angular coordinates of the new origin or that the Earth and the new origin are in the same plane (e.g., Binnendijk 1960; Henden & Kaitchuck 1982; Hirshfeld & Sinnott 1997), but we explain in § 4 why this form is most practical for calculating the delay.

The HJD, which uses the Sun as the origin of the new reference frame, is only accurate to 8 s because of the acceleration of the Sun due primarily to Jupiter and Saturn (Fig. 2). It was popular when people first began considering this effect because it is relatively simple to calculate from tables without a computer (Landolt & Blondeau 1972), and remains popular because self-contained algorithms exist to approximate it without any external tables (e.g., Duffett-Smith 1989). However, because the HJD is not useful when accuracies of better than 8 s are needed, most of the algorithms in use today use approximations that are only precise at the 1 s level, and it becomes impossible to back out the original JD from the HJD unless we know the exact algorithm used.

Because of these problems, the HJD was formally deprecated by International Astronomical Union (IAU) Resolution A4 in 1991, in favor of the BJD, a time referenced to the solar system barycenter (SSB).

The analogous correction to the Römer delay in our solar system can also be significant in the target system. We refer to this as \( \Delta_R \). For example, for transiting planets with \( a \approx 0.06 \text{ AU} \), \( \Delta_R \) can be as large as 30 s. In general, the position of the planet during primary transit has become the unspoken standard reference frame for transiting planets, while the host star’s photosphere is the unspoken standard for radial velocity (RV) planets.

In theory, the timing would be much more stable in the target’s barycentric reference frame, but the accuracy with which we can convert to this frame depends on the measurements of the system. Since different observers may use different values as measurements improve, quoting the JD in the frame of the target’s barycenter may obfuscate the long-term reliability of timing. Therefore, we argue it is better to quote Julian Date in the SSB reference frame, and correct for \( \Delta_R \) only when comparing observations at different phases in the planet’s orbit.

This \( \Delta_R \) correction is not necessary for TTVs of the primary transit, since the planet is always in the same phase. Nevertheless, we should explicitly state the object’s reference frame to avoid any potential ambiguity, particularly when comparing any combination of primary transits, secondary transits, RVs, and another primary transit of a different planet in the same system, when it may not be obvious which origin is being used.

For RV measurements, which are taken at many different phases, the effect is much smaller and can generally be ignored because the star’s orbit around the barycenter is small. For a typical hot Jupiter (i.e., a Jupiter mass planet in a 3 day orbit around a solar mass star), the maximum time difference in the RV signal (for an edge-on orbit) is 20 ms, which would change the measured RV by \( \sim 50 \mu \text{m s}^{-1} \). While planets farther
out will cause a larger timing offset, the difference in the measured RV is even smaller.

2.2. Time Standards: The Clock Correction

To be clear, the JD can be specified in many time standards (Seidelmann 1992), and while the IAU has made no explicit statement regarding the allowed time standards of the GJD, HJD, or BJD, their meaning in any given time standard is unambiguous. Unfortunately, they have been specified in many standards, usually implicitly.

However, the particular time standard used affects how useful the time stamp is as an absolute reference. We must be careful not to directly compare BJDs or HJDs in different time standards, as each has different offsets, periodic terms, and/or rates, which can introduce systematic errors of over 1 minute. For this reason, it is critical that any stated BJD or HJD also specify the time standard used when 1 minute accuracies are important, and the uncertainty of a time that is quoted without a standard should be assumed to be at least 1 minute.

First, it may be useful to summarize the relevant standards of time:

Universal Time, UT1—Defined by the mean solar day, and so drifts forward and backward with the speeding and slowing of the Earth’s rotation. Generally, it slows due to the tidal braking of the Moon, though changes in the Earth’s moment of inertia and complex tidal interactions make its exact behavior unpredictable. It is rarely used directly in astronomy as a time reference, but we mention it for context.

International Atomic Time, TAI—Based on an average of atomic clocks all corrected to run at the rate at 0 K, with 1 s equal to “9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom,” as defined by Resolution 1 of the thirteenth meeting of the Conférence Générale des Poids et Mesures (CGPM) in 1967. This definition is based on the duration of the Ephermeris Time second, which was previously defined as 1/31,556,925.9747 of the tropical year for 1900 January 0 at 12 hours Ephermeris Time by Resolution 9 of the eleventh CGPM in 1960. TAI is the fundamental basis for many other time standards, and is the default time standard of the Sloan Digital Sky Survey.

Coordinated Universal Time, UTC—Runs at the same rate as TAI, except that it is not allowed to differ from UT1 by more than 0.9 s. Every 6 months, at the end of 31 December and 30 June, the International Earth Rotation and Reference Systems Service (IERS) may elect to add (or subtract) a leap second to UTC in order to keep it within 0.9 s of UT1. UTC is therefore discontinuous and drifts relative to TAI with the addition of each leap second, which occur roughly once per year. As of 2009 January, the current number of leap seconds, \( N \), is 34. The full table of leap seconds is available online and is typically updated several months in advance of when an additional leap second is to be added. UTC is the current international standard for broadcasting time. As a result, when a modern, network-connected computer’s clock is synchronized to a network time protocol (NTP) server, it will be in UTC. Thus, this is the system of time most familiar to astronomers and nonastronomers alike (modulo time zones and daylight savings time).

Universal Time, UT—An imprecise term, and could mean UT1, UTC, or any of several other variations. In general, such imprecise language should be avoided, as the potential ambiguity is up to 1 s. In the context of a time stamp, it is likely UTC, but some people may intentionally use UT to imply 1 s accuracy. While explicitness is preferred (i.e., UTC ± 1 s), any time stamp quoted in UT should be assumed to be uncertain at the 1 s level unless the time standard has been independently verified.

Terrestrial Time, TT(TAI)—A simple offset from TAI of 32.184 s released in real time from atomic clocks and never altered. This offset is to maintain continuity between it and its predecessor, the Ephemeris Time (ET).

Terrestrial Time, TT(BIPM)—A more precise version of TT (TAI). The International Bureau of Weights and Measures (BIPM) reanalyzes TT(TAI) and computes a more precise scale to be used for the most demanding timing applications. The current difference between TT(TAI) and TT(BIPM) is \( \sim 30 \mu s \), and must be interpolated from a table maintained by the BIPM and published online with a 1 month delay.

Terrestrial Time (TT)—Sometimes called Terrestrial Dynamical Time (TDT), can refer to either TT(TAI) or TT(BIPM). From this point on, we will not make the distinction, but when accuracies of better than 30 \( \mu s \) are required, TT(BIPM) must be used.

Barycentric Dynamical Time, TDB—Corrects TT for the Einstein delay to the geocenter, \( \Delta E \), which is the delay due to time dilation and gravitational redshift from the motions of the Sun and other bodies in the solar system. The conversion from TT to TDB cannot be written analytically, but is usually expressed as a high-order series approximation (Irwin & Fukushima 1999). The difference is predominately a periodic correction with a peak-to-peak amplitude of 3.4 ms and a period of 1 yr. TDB was slightly modified by IAU Resolution B3 in 2006, converging on the same definition as the JPL ephemeris time, \( T_{\text{eph}} \), also called coordinate time (CT) in the JPL ephemeres of solar system objects.

Barycentric Coordinate Time, TCB—Physically and mathematically equivalent to the TDB as defined in 2006 (Standish 1998), and differs only by an offset and rate of about 0.5 s yr\(^{-1} \) due primarily to time dilation in the Sun’s gravitational potential. TDB and TCB were roughly equal at 1977 January 1.0 TAI, and now differ by about 16 s.

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\(^1\) At http://www.iers.org.

\(^2\) At ftp://maia.usno.navy.mil/ser7/tai-utc.dat.

\(^3\) At ftp://tai.bipm.org/TFG/TT%28BIPM%29.
Caution must be always be exercised, however, as these definitions are subject to change at any point, though usually with a few year’s notice.

Assuming the time is measured according to current definition of UTC, then the clock correction, from UTC to TDB, can be written as the sum of the corrections from UTC to TAI, TAI to TT, and TT to TDB:

\[ \Delta_C = N + 32.184s + (TDB - TT). \]  

Of course, if one wishes to express the BJD in another time standard (or start with something other than UTC), the clock correction would change accordingly. However, not every time standard is well suited to precise, astronomical time stamps, and the use of any time standard other than TDB should be viewed simply as an adequate approximation to TDB.

Most readily available programs that calculate the time stamp assume that the user has already applied the UTC-to-TT part of this clock correction, which is often not true. We feel this assumption has contributed to the widespread confusion regarding time stamps.

As this last point is our primary motivation for writing this article, we elaborate here on the effects of time standards on the reliability of time stamps. For the sake of simplicity, we only discuss the effects of time standards on the BJD. Each of these effects also applies to the HJD, though the improvement in accuracy of the time stamp is negligible compared to the accuracy of the HJD reference frame for all but the UTC time standard.

The least preferred, though most commonly used, time standard for the BJD is UTC (BJD_{UTC}), and is equivalent to ignoring \( \Delta_C \) altogether. Because UTC is discontinuous and drifts with the addition of each leap second, comparing two BJD_{UTC} time stamps could result in spurious differences if any leap seconds have been introduced between observations. Therefore, 1 s timing accuracies cannot be achieved using the BJD_{UTC} over a span that straddles the addition of one or more leap seconds (roughly 1 yr). Figure 3 shows the difference between the BJD_{UTC} time stamp with the uniform BJD_{TDB} time stamp (described below) from 1961 January 1, when UTC was defined (though its definition has evolved over the years), to 2010 December 31, the furthest future date for which the value of UTC can be accurately predicted at this writing.

BJD in TT (BJD_{TT}), which is equivalent to ignoring the (TDB-TT) term in equation (4), corrects for the discontinuity and drift introduced by leap seconds and is appropriate for timing accurate to 3.4 ms.

BJD in TDB (BJD_{TDB}) is usually the best time stamp to use in practice, as it further corrects the BJD_{TT} for all known effects on the motions, and therefore rates, of our atomic clocks. While BJD_{TDB} is not perfect, any more accurate time stamp is unique for each target.

\[ \Delta_{S\odot} = \frac{2GM_{\odot}}{c^3} \log(1 - \cos \theta), \]  

This can be as large as 0.1 ms for observations at the limb of the Sun, but for objects more than 30° from the Sun, the correction is less than 20 \( \mu \)s.

There is also the analogous correction, \( \Delta_S \), for the target system. Similar to \( \Delta_{S\odot} \), \( \Delta_S \) depends on the measurements of the target, which may be refined over time. Therefore, the time

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Footnote: Ignoring the TT(BIPM) correction.
should generally be quoted without $\Delta_S$, but include it when comparing times where this could be significant.

2.4. Einstein Delay

As we discussed in § 2.2, relativity dictates that the motion of the observer influences the rate at which the observed clock ticks. The use of TDB corrects for an observer moving with the geocenter, but in reality we observe from the surface of the Earth or from a satellite, for which there is an additional term to $\Delta_{E\odot}$:

$$\Delta_{E\odot} = \frac{\vec{r}_{\odot} \cdot \vec{v}_{\odot}}{c^2}. \quad (6)$$

Here, $\vec{r}_{\odot}$ is the location of the observer with respect to the geocenter, and $\vec{v}_{\odot}$ is the velocity of the geocenter. Again, there is an analogous correction, $\Delta_{E_t}$, for the target system, which should be ignored when quoting the time but included when comparing times if necessary.

3. PRACTICAL CONSIDERATIONS

Of course, the accuracy of the output time stamp is only as good as our assumptions and the inputs needed to specify the time standard and reference frame. Here we discuss their effects on the accuracy of the time stamps. The first four subsections, through § 3.4, have no reasonable upper bound. In each case, the accuracy of the inputs must be evaluated depending on the accuracy of time stamp required. The later three subsections through § 3.7 are organized in decreasing magnitude, all of which can be ignored for accuracies no better than $21.3$ ms. Finally, § 3.8 discusses the effects of less than $1\mu s$ that we have ignored.

3.1. Coordinates

An error of $1'$ in the position of the target amounts to a timing error of as much as $0.28$ s in the BJD (Fig. 4). Such error would be common if the coordinates of the field center are used instead of the specific object’s coordinates. In particular, if doing a survey, one may wish to assign the same BJD to all objects in a given frame. However, with a $10^\circ$ offset, which is possible with some wide-field transit surveys, the error can be as large as $200$ s for objects at the edge of the field. An error of $0.25''$ will yield $1$ ms timing offsets, and $0.25$ mas accuracy is necessary for $1\mu s$ timing.

3.2. Computer Clock

The accuracy of a typical computer clock depends on its intrinsic stability, the computer workload, its operating system, and the reliability of the network connection. Older computers with a parallel port CCD interface may produce unreliable timing because the clock may slow or stop completely during CCD readout. Without any special effort, a modern Windows machine with a network connection is accurate to $\sim2$ seconds\(^5\), and with third-party software like Dimension 4,\(^6\) we have found it to be stable to $0.1$ s. An NTP-synchronized Linux machine is typically accurate to $\sim50$ ms.

Of course, the stability of the clock only sets a lower limit on the absolute accuracy of the time recorded in the FITS image header. NTP synchronization attempts to measure and compensate for network latency, but the accuracy of time stamps also depends on the particular software package taking the image and the hardware it uses, which is difficult to calibrate. Unless independently verified, the time recorded in image headers should not be trusted to better than $0.25$ s. However, various solutions exist for higher precision timing, such as GPS-triggered shutters.

In particular, it is worth emphasizing the $1$ s error in the Hubble Space Telescope (HST) clock and potential $6.5$ s error in the Kepler clock, described in more detail in the Appendix, both of which have already achieved data-limited transit timing precisions of that order (e.g., Pont et al. 2007; Kipping & Bakos 2010).

3.3. Flux-weighted Mean Time of Exposure

When calculating the time of exposure of an image, we typically use the time at midexposure. However, the precise time of exposure that should be used is the flux-weighted mean time of exposure. The magnitude of this error depends on the intrinsic stability of source, the stability of the atmosphere, and the exposure time. In the diabolical case of a cloud completely

\(^{5}\) At http://support.microsoft.com/kb/939322.  
\(^{6}\) At http://www.thinkman.com/dimension4.
covering the object during one-half of the exposure, the error could be as large as half the exposure time. For a typical hot Jupiter that dims by 1% over the course of a 15 minute ingress, the error introduced into the time stamp during ingress or egress by using the mid-exposure time is 0.25% of the exposure time—150 ms for a 1 minute exposure. Near the peak of some high-magnification microlensing events (i.e., Gould et al. 2009), the flux may double in as little as 6 minutes. During such instances, using the mid-exposure time will result in a time stamp error of as much as 1/4 of the exposure time—15 s for a 1 minute exposure.

However, this Taylor expansion is divergent when \( d < |\vec{r}| \). It should never be used for objects inside 1 AU and may still be inadequate for other objects inside the solar system. It has a maximum error of 1 day for the Moon, 20 s for the Main Asteroid Belt, and 40 ms for the Kuiper Belt. In these cases, using the exact formula (eq. [7]) may be easiest.

Therefore, we recommend that for precise calculations of any Solar System body, equation (7) should be used. For better than 1 ms timing of any object outside the solar system, equation (8) should be used.

### 3.4. Plane-Wave Approximation

Equation (2) assumes the object is infinitely far away, and therefore the incoming wavefronts are plane waves. In reality, the wavefronts are spherical, which introduces a distance-dependent, systematic error. The maximum error introduced by the plane-wave approximation is 1000 s for the Moon,\(^7\) 100 s for the Main Asteroid Belt, 5 s for the Kuiper Belt, 1 ms at the distance of Proxima Centauri, and 150 ns at the distance to the Galactic center.

The fully precise equation, assuming spherical wavefronts, is

\[
\Delta R_0 = \frac{|\vec{r} + \vec{n}d| - d}{c}, \tag{7}
\]

where \( d \) is the distance from the observer to the target. In such instances, the distances must be derived from precise ephemerides for both the target and observer.

Although this form is generally applicable, it is not generally practical because at large distances, double precision floating point arithmetic cannot reliably recover the small difference between \(|\vec{r} + \vec{n}d|\) (the distance from the barycenter to the target) and \(d\).

To solve this problem, the pulsar community (e.g., Lorimer 2008\(^8\)) uses the two-term Taylor expansion of equation (7) about \(1/d = 0\):

\[
\Delta R_0 \approx \frac{\vec{r} \cdot \vec{n} + \vec{r} \cdot \vec{n} - (\vec{r} \cdot \vec{n})^2}{2cd}. \tag{8}
\]

One may recognize the first term as plane wave approximation (eq. [2]). In practice, the accuracy of this two-term approximation exceeds the accuracy of the “exact” calculation using double precision at a distance of 10,000 AU (0.05 pc). At the distance of Proxima Centauri, the accuracy of this approximation is at worst 1 ns.

\(^7\)The maximum error for the Moon is not because of the departure from the plane wave, but because the plane-wave formalism can place the Moon on the wrong side of the SSB.

\(^8\)Note: the sign of the second term in Lorimer 2008, eq. 9, should be − rather than +.
3.8. Negligible (<1 μs) Effects

The Shapiro delay occurs for other bodies as well, but observations at the limb of Jupiter only delay light by 200 ns. Typical modern, commercial GPS units use the World Geodetic System (WGS84), which is referenced to the International Terrestrial Reference System (ITRS) with an error of about 15 m, which amounts to a 50 ns error in the time stamp.

The index of refraction of the atmosphere is not exactly 1 and changes with its composition, temperature, and pressure, which changes the speed of light. However, the largest reasonable deviation due to this effect is only tens of ns.

The pulsar community must specify a frequency-dependent dispersion measure delay. At radio wavelengths (21 cm), the delay can be as much as 1 s, but the dispersion delay contributes at less than 1 μs shortward of ∼300 μm.

4. CALCULATING THE BJD\(_{TDB}\)

The most practical way to precisely calculate the BJD\(_{TDB}\) time stamp is using JPL’s DE405 ephemeris.\(^\text{10}\) It contains the position of thousands of bodies in the solar system, including the Sun, planets, spacecraft, moons, asteroids, and comets. It is oriented to the International Celestial Reference Frame (ICRF), which is consistent with the FK5 system at J2000.0 within the 50 mas error of FK5, and has its origin at the SSB with its axes fixed with respect to extragalactic objects (Arias et al. 1995). Therefore, it is recommended to use the 3-space Cartesian coordinates retrieved from the JPL DE405 ephemeris directly with the J2000 object coordinates in equation (2).

The following is an outline of the steps required to properly calculate the BJD\(_{TDB}\).

1. Calculate the midexposure time in JD\(_{UTC}\). Most FITS image headers give DATE-OBS in UTC at the beginning of exposure. If high precision is required (<1 s), read the caveats about clock precision in § 3.2 carefully; depending on the sky conditions or variability of the object, one may need to account for the flux-weighted mean time of exposure.

2. Convert the midexposure time to JD\(_{TDB}\) by applying Δ\(_C\) (eq. [4]). For times accurate to 3.4 ms, one can use the simpler JD\(_{TT}\) and calculate BJD\(_{TT}\) (using the positions of planets delayed by the TT-TDB offset). The difference between BJD\(_{TT}\) and BJD\(_{TDB}\) is no more than ∼200 ns, which is well below the precision of the BJD\(_{TT}\). If better than 30 μs precision is required, the TT(BIPM)-TT(TAI) offset must be applied.

3. Retrieve the JPL ephemeris of the observing station for the times spanning the observing window. JPL’s HORIZONS system\(^\text{11}\) is designed for this. To return inputs for use with equation (2) and J2000 target coordinates: select “Vector Table”, the SSB as the coordinate origin, and “Earth mean equator and equinox of reference epoch” for the reference plane. This will return the Cartesian coordinates of the observing station with respect to the SSB in the J2000 Earth mean equator reference frame at steps as small as 1 minute in CT, which is the same as TDB.

4. If the observing station is on Earth and better than 20 ms timing is required, another ephemeris must be generated (from

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\(^{10}\) At ftp://ssd.jpl.nasa.gov/pub/eph/export/DE405/de405iom.ps.

\(^{11}\) At http://ssd.jpl.nasa.gov/?horizons.
HORIZONS) for the observer’s position with respect to the non-rotating geocenter, and added to the geocentric positions. We note that the precise conversion from latitude, longitude, and elevation to the Cartesian coordinates with respect to the non-rotating geocenter is not trivial, and requires tables of measured precession and nutation of the Earth.

5. Interpolate the positions of the observing station to each midexposure time JD

6. Input the interpolated X, Y, and Z positions of the observing station, and the target’s J2000 Earth mean equator coordinates into equation (2). Depending on the distance to the target and the precision required, equation (7) (and the target’s ephemeris) or equation (8) may be required. One must be careful to use sufficiently accurate target coordinates.

7. If greater than 0.1 ms precision is required, apply the Shapiro correction (eq. [5]). An ephemeris of the Sun is required, which can be generated from HORIZONS.

8. If greater than 1 µs precision is required, apply the additional Einstein correction for the observing station’s position with respect to the geocenter. The geocentric velocity is required, which can also be given by HORIZONS.

4.1. Our Code

Our IDL code implementing this procedure is available online. It requires the JD

TDB

as inputs. We outline its procedure here. More explicit details, as well as the calling procedure and dependencies, are commented inside the code.

We compute ΔC using Craig Markwardt’s TAI.UTC program to read the leap second table, and his TDB2TDT program to compute the TT-TDB correction, which uses a 791-term Fairhead and Bretagnon analytical approximation to the full numerical integration, with an error of 23 ns (Fairhead & Bretagnon 1990).

Our code will automatically update its leap second table the first time it runs after every January 1 or July 1, but this requires a periodic Internet connection and the use of the wget program. It will terminate on failure to update, but this protection can be bypassed for those that elect to (or have to) update their table by hand.

By default, we ignore the ~30 µs TT(BIPM)-TT(TAI) correction, which would require a constant Internet connection, would not apply to data acquired in the previous month, and is likely negligible for most applications. However, our code can optionally correct for it if an up-to-date file is supplied.

To read and interpolate the ephemeris from JPL, we use Craig Markwardt’s routines JPLEPHREAD and JPLEPHINT to the Earth, Sun, and other planets. If the observing station is space-borne, the smaller ephemeris used with those programs does not include satellites, so we use an Expect script to automate a telnet session to the HORIZONS system and automatically retrieve the ephemeris, which we quadratically interpolate to the desired times using IDL’s INTERPOL. The accuracy of this interpolation depends on how quickly the object’s position is changing and the step size of the ephemeris. HORIZONS can only return ~90,000 data points per query, so the smallest step size (1 minute) limits the calculation to a range of 60 days. For the geocenter, a 100 minute step size is sufficient for 60 ns accuracy, but, for example, a 2 minute step size is required for 1 µs accuracy for the HST (though it is still limited by its clock accuracy). We have found that a 10 minute step size is adequate for 1 ms timing for most objects and allows a range of nearly 2 yr.

If the observer is on the Earth, and the coordinates (latitude, longitude, and elevation) are given, we correct for the additional delay. If no observer-specific information is given, we assume the observer is at the geocenter, and the result will be biased by ~10 ms (Fig. 5).

If the target’s ephemeris can be returned by HORIZONS and its unique name is given, we use our Expect script to generate its ephemeris too, and calculate the exact ΔH⊙ (eq. [7]). If not, and instead the distance is given, we use the two-term approximation to the spherical wave solution (eq. [8]). Otherwise, we use the plane wave approximation (eq. [2]).

Lastly, we include the Shapiro correction and the additional Einstein correction due to the position of the observer with respect to the geocenter, either from the surface of the Earth (if given the coordinates), or the spacecraft.

In the geocentric case, our code agrees with BARYCEN to 200 ns (peak to peak), and the authors of BARYCEN report that their code agrees with FXBARY to 1 µs. The ephemeris we generate for a location on the surface of the Earth agrees with HORIZONS to 20 nano-its, and the geocentric BJDs we calculate from HORIZONS ephemeris agree with the BJDs we calculate using Craig Markwardt’s routines within 10 ns.

The near-exact agreements between these methods is not surprising, and do not necessarily indicate that they are accurate to better than 1 µs. Our code was inspired by BARYCEN and both rely on Craig Markwardt’s routines (the difference comes from the fact that we index the JPL ephemeris with JD

TDB

instead of JD

TT

), and all methods use JPL’s DE405 ephemeris.

The primary advantage of our code is that it includes the JD

UTC

to JD

TT

correction (but can optionally ignore it). The choice of starting with JD

UTC

is a departure from what is typically done with such time stamp calculators, but we feel this is a far more robust starting point. The current confusion has shown that many assume JD

UTC

as the starting point, which is likely due to a lack of explicitness in the programs and/or unfamiliarity with various time standards. Our hope is that people are unlikely

12 At http://astrouils.astronomy.ohio-state.edu/time.
13 At http://cow.physics.wisc.edu/~craigm/idl/ephem.html.
14 At http://astro.uni-tuebingen.de/software/idl/aitlib/astro/barycen.pro.
15 At http://heasarc.nasa.gov/lheasoft/ftools/fhelp/xbary.txt.
to make the opposite mistake (assume the input should be JD_{TT} instead of JD_{UTC}) since our code is very explicit and calculating the JD_{UTC} is almost always a trivial calculation from the DATE-OBS FITS header keyword.

Additionally, our code can easily correct for the observer’s position on the Earth or from a spacecraft, and can include the spherical wave correction.

In order to schedule observations, even \pm 10 minute precision is generally good enough, and one can approximate BJD_{TDB} \approx JD_{UTC}: for more demanding observing schedules, we provide software to iteratively calculate the reverse correction.

Along with the IDL source code, we provide a web-based interface to our codes,\(^\text{16}\) though not every feature is enabled. Specifically, it is limited to 1 ms precision, can only do one target at a time, only does the plane wave approximation, and is limited to 10,000 JDs at a time. Those with applications for which these features are too limited should download our source code and run it locally.

5. CONCLUSION

Timing of transient events is a powerful tool for characterizing many astronomical phenomena. In the field of exoplanets in particular, the search for variations in the times of primary transits and secondary eclipses, or transit timing variations, is one of the most promising new techniques for studying planetary systems.

The accuracy with which transit times, and indeed any transient phenomenon can be measured is limited not only by the data themselves, but by the time stamp to which the transit time is referenced. As the quality of transit timing data crosses the threshold of 1 minute precision, the precise time standard and reference frame in which event times are quoted becomes important. Achieving uniform and accurate time stamps with accuracies of better than 1 minute that can reliably be compared to one another requires extraordinary care in both our techniques and our terminology. We have found that the time standards adopted by various groups that measure transit times can differ by as much as a minute, and are typically left unspecified. As these ambiguities can be significant compared to the timing precisions that are quoted, they may therefore lead to spurious detections of transit timing variations or biased eccentricity measurements.

Here we have summarized the effects one must consider in order to achieve timing precision of 1 \(\mu s\). We argue that the BJD_{TDB} is nearly the ideal time stamp, being as reliable as any time stamp can be without being unique to each target system. On the other hand, BJD_{UTC} and the HJD in any form should be avoided whenever possible. Most importantly, we emphasize that the time standard should always be explicitly stated. Any time stamp that is quoted without a time standard should be assumed to be uncertain to at least 1 minute. Unless the time standards used in programs or algorithms have been independently confirmed, one should avoid using ones that do not precisely specify the input and output time standard.

In addition, the arrival time at the observing site along with its time standard (e.g., JD_{UTC}) should also be specified. This will remove any ambiguity in the time stamp, allow others to apply improved corrections should more precise ephemerides become available in the future, and allow others to check that original conversion was done accurately enough for their purpose.

Finally, we have written an IDL program for general use that facilitates the use of BJD_{TDB} to an accuracy of 1 \(\mu s\), provided that the inputs are sufficiently precise, and we provide a web-based interface to its most useful features.

We would like to thank Craig Markwardt for his fundamental routines that make ours possible, the help desks at the various space telescopes and at IERS for answering our questions, the anonymous referee, Steve Allen, Richard Pogge, Joseph Harrington, Roberto Assef, Andrew Becker, Mercedes Lopez-Morales, Christopher Campo, Drake Deming, Ryan Hardy, Heather Knutson, Eric Agol, and Joshua Winn for useful discussions, and Wayne Landsman and Jörn Wilms for managing the IDL astronomy libraries.

APPENDIX

A1. Software

Ground-based observers have typically used one of the following methods to calculate the BJD. However, most FITS image headers give the DATE-OBS and TIME-OBS keywords in UTC. We have found that most people, when starting with JD_{UTC}, end up quoting HJD_{UTC} or BJD_{UTC}.

A1.1. HORIZONS

JPL’s HORIZONS ephemeris calculator, which is used by many to calculate the BJDs from space telescopes, and can be used to calculate ground-based BJDs, returns the time in JD_{CT} = T_{eph} = JD_{TDB} when the ephemeris type is “Vector

\(^{16}\) At http://astroutils.astronomy.ohio-state.edu/time/utc2bjd.html.
Table.” Any conversion that uses a HORIZONS ephemeris in $J_{\text{TDB}}$ but indexes it with $J_{\text{UTC}}$, as had been done by several people we spoke with, will calculate $BJD_{\text{UTC}}$, which can be offset from the true $BJD_{\text{UTC}}$ by up to $13.4$ ms (as shown in Fig. 6), and offset from the uniform $BJD_{\text{TDB}}$ by more than $1$ minute (as shown in Fig. 3).

A1.2. IRAF

IRAF’s $\text{setjd}$ calculates the HJD, but calls for UT, which is likely to be interpreted as $J_{\text{UTC}}$. In this case, it would calculate the drifting quantity $HJD_{\text{UTC}}$. If TT were used instead, it would calculate the $HJD_{\text{TT}}$, accurate to $\sim 8$ s.

A1.3. IDL

The IDL routines HELIOJ (for HJD), from the IDL Astronomy Library, curated by Wayne Landsman, and BARYCENT (for BJD), from the Institut für Astronomie und Astrophysik IDL Library, maintained by Jörn Wilms, both call for the GJD, which, we remind the reader, can be specified in any time standard. Often, this is interpreted as $J_{\text{UTC}}$, in which case they would calculate $HJD_{\text{UTC}}$ or $HJD_{\text{UTC}}$, respectively. If TT were used, they would calculate the $HJD_{\text{TT}}$, accurate to $\sim 8$ s, or $BJD_{\text{TDB}}$, accurate to the geocentric correction (21.3 ms).

A1.4. AXBARY

NASA’s High Energy Astrophysics Science Archive Research Center (HEASARC) created the tools $\text{AXBARY}$ and later the improved version $\text{FAXBARY}$, both of which call $\text{AXBARY}$ to calculate the BJD. Their documentation is precise and correct, but quite long, and may be difficult for the uninitiated to follow. Therefore, it would not be surprising for users of $\text{AXBARY}$ to input either UTC or TT, in which case they could generate either the $BJD_{\text{UTC}}$ to the accuracy of the leap seconds or the $BJD_{\text{TDB}}$ to the accuracy of the geocentric correction (21.3 ms).

A1.5. Online Tools

Currently, common Google results turn up various applets, spreadsheets, programs, or algorithms to calculate the HJD that explicitly call for $J_{\text{UTC}}$ or $J_{\text{UT}}$ as an input. Unless explicitly mentioned otherwise, it is usually safe to assume the time standard used as input will be the time standard used throughout their calculation. Thus, these algorithms and applets will very likely calculate $HJD_{\text{UTC}}$. However, they are perfectly capable of calculating $HJD_{\text{TT}}$ if given $J_{\text{TT}}$ as an input.

A2. Space Telescopes

A2.1. EPOXI

EPOXI has the mid-exposure time $BJD_{\text{TDB}}$ in the header for the intended pointing under the FITS header keyword KPKSSBJT. This can be used directly, as long as one is careful about the intended target, so it would be very surprising if a BJD from EPOXI was not $BJD_{\text{TDB}}$.

We recalculated the $BJD_{\text{TDB}}$ using the HORIZONS ephemeris as described in § 4 and an example FITS header given to us by the EPOXI help desk. In their example FITS header, they pointed at the Moon, which is not infinitely far away, so we must use equation (7) for $\Delta_{R\odot}$. With this method, we agree with the KPKSSBJT header value to $\sim 1$ ms, the limit of the precision of the keyword.

We also redid the $BJD_{\text{TDB}}$ calculation of HAT-P-4b as described in the Report on the Calibration of EPOXI spacecraft timing and reduction to Barycentric Julian Date of 2009 August by Hewagama et al. We find agreement with the quoted KPKSSBJT FITS header keyword to 47 ms. While this is much better than the 0.41 s difference calculated by Hewagama et al. (a difference they attribute to “cumulative rounding limits” in their method), we believe our method to be far more precise. However, we could not obtain access to the original headers and were unable to determine the source of the discrepancy. Given the very good agreement with the calculation of the Moon above, our best guess is that the target coordinates used by the EPOXI pipeline differed from the published values for HAT-P-4b. The 47 ms difference could be explained by a $\sim 27^\circ$ discrepancy in R.A., a $\sim 45^\circ$ discrepancy in declination, or some combination thereof.

A2.2. Chandra X-Ray Observatory

Chandra stores their DATE-OBS keyword in TT. Their more precise TSTART and TSTOP keywords are expressed in seconds after 1998 January 1, 00:00:00 TT. This departs from what is typically done, which may lead to confusion, but it makes the conversion to a uniform time stamp much more straightforward and less likely to drift by the leap seconds. They provide extensive directions online to calculate the $BJD_{\text{TDB}}$ using $\text{AXBARY}$, so it is likely that anyone using Chandra who quotes a BJD is using $BJD_{\text{TDB}}$.

A2.3. Hubble Space Telescope

The FITS headers of $\text{HST}$ data state that their DATE-OBS and TIME-OBS keywords are UT. We contacted the $\text{HST}$ help desk for clarification, since UT is ambiguous. The $\text{HST}$ help desk response stated that their clock reports UTC accurate to $\sim 10$ ms, but “due to variabilities and quantization in the particular science instruments’ operations, the actual time light begins falling on the detector is not known to better than about $\sim 1$ second, $\pm 50\%$ (rough estimate).” It is possible for the $\text{HST}$ engineering

17 At http://idlastro.gsfc.nasa.gov/ftp/pro/astro/helio_jd.pro.
team to calibrate these variations, but they have limited resources and have no plans to do so. It is thought that this error is some combination of random and systematic errors, but the precise breakdown is unknown.

This potential ∼1 s systematic error may have important implications for the reliability of the transit times quoted with HST observations, most importantly, the 3 s error of the transit time of HD189733b (Pont et al. 2007).

HST does not calculate the HJD or BJD at any point, leaving the calculation up to each individual observer. Our experience with ground-based observers suggests that most people will end up quoting an \( \text{HJD}_\text{UTC} \) or \( \text{BJD}_\text{UTC} \).

### A2.4. Kepler

The Kepler Data Release Notes 2 describe how to calculate the BJD from UTC, but do not include the correction to TDB. They mention the HORIZONS ephemeris, but neglect to mention that its output time is in CT, not UTC; thus it appears they calculate \( \text{BJD}_\text{UTC} \), though we were unable to confirm this. In addition, the time stamp uncertainty may be much larger than typical, so it is worth quoting from the Kepler Data Release Notes 5 (released 2010 June 4): The advice of the DAWG (Data Analysis Working Group) is not to consider as scientifically significant relative timing variations less than the read time (0.5 s) or absolute timing accuracy better than one frame time (6.5 s) until such time as the stability and accuracy of time stamps can be documented to near the theoretical limit.

### A2.5. Spitzer Space Telescope

The Spitzer pipeline calculates the HJD\(_\text{UTC} \) for the intended pointing (presumably the target) at the end of the exposure, subtracts the full exposure time, and records the result in the header as HJD. Depending on the exposure time, this will produce roughly a 10 ms effect similar to that shown in Figure 6, and depending on how close the intended pointing was to the object of interest, may produce a ∼0.1 s effect similar to Figure 4. However, this effect is negligible compared to both the ∼8 s accuracy of the HJD (Fig. 2) and the number of leap seconds that may have elapsed between observations (Fig. 3).

One typically quotes the HJD at the midexposure time. Since Spitzer quotes the HJD\(_\text{UTC} \) at the beginning of the exposure, using the unmodified Spitzer HJDs would produce a systematic offset of half the exposure time, though experienced observers correct for this.

Also contributing to this confusion, the FITS header keyword UTC\(_\text{S_OBS} \) is incorrectly documented. While the documentation states that it is seconds after J2000 ET, it is actually seconds after January 1st, 2000 12:00 UTC + \( N – 32 \) Therefore, trusting the documentation as is will unwittingly lead to a difference of \( N + 32.184 \) s.

Most of the people we have asked opted to calculate their own BJD using the HORIZONS ephemeris. However, they have typically quoted the BJD\(_\text{UTC} \).

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