FITTING THE SUPERNOVA TYPE Ia DATA
WITH THE CHAPLYGIN GAS

J. C. Fabris1, S.V.B. Gonçalves2 and P.E. de Souza3

Departamento de Física, Universidade Federal do Espírito Santo, CEP29060-900, Vitória, Espírito Santo, Brazil

Abstract

The supernova type Ia observational data are fitted using a model with cold dark matter and the Chaplygin gas. The Chaplygin gas, which is characterized by a negative pressure varying with the inverse of density, represents in this model the dark energy responsible for the acceleration of the universe. The fitting depends essentially on four parameters: the Hubble constant, the velocity of sound of the Chaplygin gas and the fraction density of the Chaplygin gas and the cold dark matter. The best fitting model is obtained with \( H_0 = 65\, \text{km/Mpc.s} \), \( c_s^2 \sim 0.92c \) and \( \Omega_c = 1, \Omega_m = 0 \), that is, a universe completely dominated by the Chaplygin gas. This reinforces the possibility that the Chaplygin gas may unify dark matter and dark energy, as it has already been claimed in the literature.

PACS number(s): 98.80.Bp, 98.65.Dx, 98.80.Es

The combined data from the measurements of the spectrum of anisotropies of the cosmic microwave background radiation [1] and from the observations of high redshift supernova type Ia [2, 3] indicate that the matter content of the universe today may be very probably described by cold dark matter and dark energy, in a proportion such that \( \Omega_{dm} \sim 0.3 \) and \( \Omega_{de} \sim 0.7 \). The distinction between cold dark matter and dark energy lies on the fact that both manifest themselves through their gravitational effects only and, at the same time, on the fact that a fraction of this dark matter agglomerates at small scales (cold dark matter) while the other fraction seems to be a smooth component (dark energy). The dark energy must exhibit negative pressure, since it would be the responsible for the present acceleration of the universe, as induced by the supernova type Ia observations, while the cold matter must have zero (or almost zero) pressure, in order that it can gravitationally collapse at small scales.

The nature of these mysterious matter components of the universe is the object of many debates. The cold dark matter may be, for example, axions which result from the symmetry breaking process of Grand Unified Theories in the very early universe. But, since the Grand Unified Theories, and their supersymmetrical versions, remain a theoretical proposal, the nature of cold dark matter is an open issue.

A cosmological constant is, in principle, the most natural candidate to describe the dark energy. It contributes with a homogeneous, constant energy density, its fluctuation being strictly zero. However, if the origin of the cosmological constant is the vacuum energy, there is

---

1e-mail: fabris@cce.ufes.br
2e-mail: sergio@cce.ufes.br
3e-mail: patricia.ilus@bol.com.br
a discrepancy of about 120 orders of magnitude between its theoretical value and the observed value of dark energy [1]. This situation can be ameliorate, but not solved, if supersymmetry is taken into account. Another candidate to represent dark energy is quintessence, which considers a self-interacting scalar field, which interpolates a radiative era and a vacuum dominated era [4, 6, 7]. But the quintessence program suffers from fine tuning of microphysical paremeters.

Recently, an alternative to both the cosmological constant and to quintessence to describe dark energy has been proposed: the Chaplygin gas [8, 9, 10, 11]. The Chaplygin gas is characterized by the equation of state

\[ p = -\frac{A}{\rho}, \]  

where \( A \) is a constant. Hence, the pressure is negative while the sound velocity is positive, avoiding instability problems at small scales [12]. The Chaplygin gas has been firstly been conceived in studies of adiabatic fluids [13], but recently it has been identified in interesting connection with string theories [15]. In fact, considering a \( d \)-brane in a \( d + 2 \) dimensional space-time, the introduction of light-cone variables in the resulting Nambu-Goto action leads to the action of a newtonian fluid with the equation of state (1). The symmetries of this "newtonian" fluid have the same dimension as the Poincaré group, revealing that the relativistic symmetries are somehow hidden in the equation of state (1).

Considering a relativistic fluid with the equation of state (1), the equations for the energy-momentum conservation relations leads, in the case of a homogeneous and isotropic universe, to the following relation between the fluid density and the scale factor \( a \):

\[ \rho = \sqrt{A + \frac{B}{a^6}}, \]  

where \( B \) is an integration constant. This relation shows that initially the Chaplygin gas behaves as a pressureless fluid, acquiring later a behaviour similar to a cosmological constant. So, it interpolates a non-accelerated phase of expansion to an accelerate one, in a way close to that of the quintessence program.

In this work, we will constrain the parameters associated with the Chaplygin gas using the supernova type Ia data. Specifically, we will consider a model where the dynamics of the universe is driven by pressureless matter and by the Chaplygin gas. The luminosity distance for this configuration is evaluated, from which a relation between the magnitude and the redshift \( z \) is established. The observational data are then considered, and they are fitted using four free parameters: the density fraction, with respect to the critical density \( \rho_c \), today of the pressureless matter and of the Chaplygin gas, \( \Omega_{m0} \) and \( \Omega_{c0} \) respectively, the sound velocity of the Chaplygin gas today \( \bar{A} \), in terms of the velocity of light, and the Hubble parameter \( H_0 \). The sound velocity of the Chaplygin gas today is given by

\[ \bar{A} = \frac{A}{\rho_{c0}^2}. \]  

It will be verified that the best fitting is obtained when \( \bar{A} \sim 0.92, \Omega_{m0} = 0, \Omega_{c0} = 1 \) and \( H_0 \sim 65 \text{ km/Mpc.s} \). This result becomes quite interesting if we take into account some recent considerations about a unification of cold dark matter and dark energy in Chaplygin gas models [11].
The equation governing the evolution of our model is
\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \left\{ \frac{\rho_m 0}{a^3} + \sqrt{A + \frac{B}{a^6}} \right\} .
\] (4)

It can be rewritten as
\[
\left( \frac{\dot{a}}{a} \right)^2 = H_0 \left\{ \Omega_m 0 \frac{a^3}{a^3} + \Omega_c 0 \sqrt{\bar{A} + \frac{1 - A}{a^6}} \right\}
\] (5)

where \( H_0 \) is the Hubble parameter today and the scale factor was normalized to unity today, \( a_0 = 1 \). We will consider only a flat spatial curvature section, which seems to be favoured by observations [1].

The luminosity distance is obtained by a standard procedure [7], using the equation for the light trajectory in the above specified background, and its definition,
\[
D_L = \left( \frac{1}{4\pi r} \right)^{1/2}
\] (6)

where \( L \) is the absolute luminosity of the source, and \( l \) is the luminosity measured by the observer. This expression can be rewritten as
\[
D_L = (1 + z) r ,
\] (7)

\( r \) being the co-moving distance of the source. Taking into account the definitions of absolute and apparent magnitudes in terms of the luminosity \( L \) and \( l \), \( M \) and \( m \), respectively, we finally obtain the relation
\[
m - M = 5 \log \left\{ (1 + z) \int_0^z \frac{dz'}{[\Omega_m 0 (1 + z')^3 + \Omega_c 0 \sqrt{\bar{A} + (1 - A)(1 + z')^6}]^{1/2} \right\} .
\] (8)

We proceed by fitting the supernova data using the model described above. Essentially, we compute the quantity
\[
\mu_0 = 5 \log \left( \frac{D_L}{Mpc} \right) + 25 ,
\] (9)

and compare the same quantity as obtained from observations. The quality of the fitting is characterized by the parameter
\[
\chi^2 = \sum_i \frac{[\mu_{0,i} - \mu_{0,i}^t]^2}{\sigma_{\mu_0,i}^2 + \sigma_{mz,i}^2} .
\] (10)

In this expression, \( \mu_{0,i} \) is the measured value, \( \mu_{0,i}^t \) is the value calculated through the model described above, \( \sigma_{\mu_0,i}^2 \) is the measurement error, \( \sigma_{mz,i}^2 \) is the dispersion in the distance modulus due to the dispersion in galaxy redshift caused by peculiar velocities. This quantity will be taken as
\[
\sigma_{mz} = \frac{\partial \log D_L}{\partial z} \sigma_z ,
\] (11)

where, following [3, 15], \( \sigma_z = 200 \text{ km/s} \). We evaluate, in fact, \( \chi^2 \), the estimated errors for degree of freedom.
Table 1: The SN Ia data

| SN Ia | z     | $\mu_0$($\sigma_{\mu_0}$) | SN Ia | z     | $\mu_0$($\sigma_{\mu_0}$) |
|-------|-------|---------------------------|-------|-------|---------------------------|
| 1992bo| 0.018 | 34.72(0.16)               | 1992br| 0.087 | 38.21(0.19)               |
| 1992bc| 0.020 | 34.87(0.11)               | 1992bs| 0.064 | 37.61(0.14)               |
| 1992aq| 0.111 | 38.41(0.15)               | 1993O | 0.052 | 37.03(0.12)               |
| 1992ae| 0.75  | 37.80(0.17)               | 1993ag| 0.050 | 36.80(0.17)               |
| 1992P | 0.026 | 35.76(0.13)               | 1996E | 0.43  | 42.03(0.22)               |
| 1990af| 0.050 | 36.53(0.15)               | 1996H | 0.62  | 43.01(0.15)               |
| 1992ag| 0.026 | 35.37(0.23)               | 1996I | 0.57  | 42.83(0.21)               |
| 1992al| 0.014 | 33.92(0.11)               | 1996J | 0.30  | 40.99(0.25)               |
| 1992bg| 0.035 | 36.26(0.21)               | 1996K | 0.38  | 42.21(0.18)               |
| 1992bh| 0.045 | 36.91(0.17)               | 1996U | 0.43  | 42.34(0.17)               |
| 1992bl| 0.043 | 36.26(0.15)               | 1997cl| 0.44  | 42.26(0.16)               |
| 1992bp| 0.080 | 37.75(0.13)               | 1997cj| 0.50  | 42.70(0.16)               |
| 1997ck| 0.97  | 44.30(0.19)               | 1995K | 0.48  | 42.49(0.17)               |
Table 2: Value of $\chi^2$ for the case where $H_0 = 65 \text{ km/Mpc.s}$

| $\Omega_m/H_0$ | $A$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1   |
|----------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 0.0/1.0        | 4.689 | 4.226 | 3.761 | 3.297 | 2.838 | 2.391 | 1.974 | 1.619 | 1.412 | 1.749 |
| 0.1/0.9        | 4.734 | 4.316 | 3.893 | 3.467 | 3.041 | 2.619 | 2.209 | 1.828 | 1.518 | 1.445 |
| 0.2/0.8        | 4.780 | 4.406 | 4.026 | 3.642 | 3.253 | 2.861 | 2.470 | 2.087 | 1.729 | 1.458 |
| 0.3/0.7        | 4.825 | 4.497 | 4.162 | 3.820 | 3.471 | 3.116 | 2.753 | 2.386 | 2.017 | 1.660 |
| 0.4/0.6        | 4.871 | 4.588 | 4.298 | 4.001 | 3.696 | 3.381 | 3.056 | 2.718 | 2.364 | 1.986 |
| 0.5/0.5        | 4.917 | 4.680 | 4.437 | 4.186 | 3.926 | 3.657 | 3.375 | 3.077 | 2.758 | 2.399 |
| 0.6/0.4        | 4.963 | 4.773 | 4.576 | 4.373 | 4.162 | 3.941 | 3.708 | 3.459 | 3.188 | 2.874 |
| 0.7/0.3        | 5.010 | 4.866 | 4.717 | 4.563 | 4.402 | 4.233 | 4.054 | 3.860 | 3.647 | 3.397 |
| 0.8/0.2        | 5.055 | 4.959 | 4.860 | 4.756 | 4.647 | 4.532 | 4.410 | 4.277 | 4.129 | 3.955 |
| 0.9/0.1        | 5.102 | 5.054 | 5.003 | 4.951 | 4.896 | 4.837 | 4.775 | 4.707 | 4.631 | 4.541 |
| 1.0/0.0        | 5.148 | 5.148 | 5.148 | 5.148 | 5.148 | 5.148 | 5.148 | 5.148 | 5.148 | 5.148 |
Table 3: Value of $\chi^2$ for the case where $H_0 = 75 \text{ km/Mpc.s}$

| $\Omega_{m0}/\Omega_{c0}$ | $A$   | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1   |
|---------------------------|-------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 0.0/1.0                   |       |     |     |     |     |     |     |     |     |     | 12.525 |
| 0.1/0.9                   |       |     |     |     |     |     |     |     |     |     | 12.608 |
| 0.2/0.8                   |       |     |     |     |     |     |     |     |     |     | 12.690 |
| 0.3/0.7                   |       |     |     |     |     |     |     |     |     |     | 12.772 |
| 0.4/0.6                   |       |     |     |     |     |     |     |     |     |     | 12.855 |
| 0.5/0.5                   |       |     |     |     |     |     |     |     |     |     | 12.937 |
| 0.6/0.4                   |       |     |     |     |     |     |     |     |     |     | 13.020 |
| 0.7/0.3                   |       |     |     |     |     |     |     |     |     |     | 13.102 |
| 0.8/0.2                   |       |     |     |     |     |     |     |     |     |     | 13.184 |
| 0.9/0.1                   |       |     |     |     |     |     |     |     |     |     | 13.267 |
| 1.0/0.0                   |       |     |     |     |     |     |     |     |     |     | 13.349 |
In order to compute $\chi^2$, we use data from [2, 18]. The relevant data are listed in table 1. We compute $\chi^2$ varying $H_0$, $\Omega_{m0}$, $\Omega_{c0}$ and $\tilde{A}$. As an example, in tables 2 and 3 the values for $\chi^2$ are listed for the cases where $H_0 = 65 \text{ km/Mpc.s}$ and $H_0 = 75 \text{ km/Mpc.s}$. In general, the best results are obtained, in each case when $\Omega_{m0} = 0$, $\Omega_{c0} = 1$ and $\tilde{A} = 1$. These cases represent a pure cosmological constant model. However, the best fitting is in fact obtained when $\Omega_{m0} = 0$, $\Omega_{c0} = 1$, $H_0 = 65 \text{ km/Mpc.s}$ and $\tilde{A} \sim 0.92$. This case represents a universe containing just the Chaplygin gas, which exhibits a behaviour close to a cosmological constant. In this case, the universe begins to accelerate at $z \sim 0.7$. In figure 1 the fitting for this case is exhibited.

The fact that the best fitting is achieved by a model with the Chaplygin gas as the only component of the matter content of the universe may be seen as a negative feature of the results discussed above. However, some comments must be made about this point. First, we have neglected the baryon content, which must contribute with a factor $\Omega \sim 0.2 \, h^{-1}$, where $h = H_0/(100 \text{ km/Mpc.s})$, as deduced from the primordial nucleosynthesis and from the spectrum of the anisotropy of the cosmic microwave background radiation. But, the introducing of the baryon component does not change substantially the results above. Second, there is evidence that the Chaplygin gas may unify the cold dark matter and dark energy scenarios [11], in the sense that it can behave as cold dark matter at small scales and as dark energy at large scales. Hence, our results may be an indication that such a unification of dark matter and dark energy through the Chaplygin gas must be taken more seriously. In order to confirm this, the analysis of the anisotropy of cosmic microwave background radiation in this scenario may be performed. We hope to present this result in the future.

**Acknowledgements:** We thank Kirill Bronniko and Flávio Gimenes Alvarenga for careful reading of the manuscript. This work has receiveid partial financial supporting from CNPq (Brazil).
References

[1] C.H. Lineweaver, *Cosmological parameters*, astro-ph/0112381.

[2] A.G. Riess et al, Astron.J. 116, 1009(1998);

[3] S. Perlmutter et al, Astrophys. J. 517, 565(1998);

[4] S.M. Carroll, Liv. Rev. Rel. 4, 1(2001);

[5] R.R. Caldwell, R. Dave and P. Steinhardt, Phys. Rev. Lett. 80, 1582(1998);

[6] V. Sahni, *The cosmological constant problem and quintessence*, astro-ph/0202076;

[7] Ph. Brax and J. Martin, Phys. Lett. B468, 40(1999);

[8] A. Kamenshchik, U. Moschella and V. Pasquier, Phys. Lett. B511, 265(2001);

[9] J.C. Fabris, S.V.B. Gonçalves and P.E. de Souza, *Density perturbations in a universe dominated by the Chaplygin gas*, gr-qc/0103083, to appear in *General Relativity and Gravitation*;

[10] N. Bilic, G.B. Tupper and R.D. Viollier, Phys. Lett. B535, 17(2002);

[11] M.C. Bento, O. Bertolami and A.A. Sen, *Generalized Chaplygin Gas, accelerated expansion and dark energy-matter unification*, gr-qc/0202064;

[12] J.C. Fabris and J. Martin, Phys. Rev. D55, 5205(1997);

[13] S. Chaplygin, Sci. Mem. Moscow Univ. Math. Phys. 21, 1(1904);

[14] M. Bordemann and J. Hoppe, Phys. Lett. B317, 315(1993);

[15] N. Ogawa, Phys. Rev. D62, 085023(2000);

[16] R. Jackiw, *A particle field theorist’s lectures on supersymmetric, non-abelian fluid mechanics and d-branes*, physics/0010042.

[17] P. Coles and F. Lucchin, *Cosmology*, Wiley, New York(1995);

[18] Y. Wang, Astrophys. J. 536, 531(2000).