Abstract: Forecasting process may become cumbersome under erratic demand structures where no conventional methodology seems to provide plausible outcomes. The company investigated in this paper is subject to this condition and needs to forecast the workload in order to plan the indispensable process enhancement activities. Two parameters to be forecasted are the total production volume and the ratio of in-house production ratio with respect to the total production. The required forecasts for relatively regular structured in-house production to total production ratio are computed via static forecasting. The more important erratic structured total production volume forecasts are computed using Fast Fourier Transform algorithm. Total production is further observed during the next year to check the reliability of the forecasts. The resulting respective Mean Absolute Percentage Errors of approximately 11% (in-house ratio) and 14 - 22% (total production) are considered acceptable under the erratic demand structure.

Keywords: Forecasting, erratic demand, discrete Fourier transform, fast Fourier transform.
Öz: Düzensiz talep altında öngörüde bulunma oldukça zahmetli bir süreç olup, klasik öngörü yöntemlerinden verim almak oldukça zordur. Bu çalışmada incelenen firmanın üretim yapısı da düzensiz seyretmektedir ve firma vazgeçilmez süreç geliştirmeye faaliyetlerini planlayabilmek için iş yükü öngörüsüne ihtiyaç duymaktadır. Öngörülecek iki parametre toplam üretim hacmi ve fason üretim miktarının toplam üretim oranını şeklinde belirlemiştir. Göreceli daha düzenli bir yapı sergileyen fason üretim oranı öngörüleri durağan öngörü yöntemyle gerçekleştirmiştir. Daha kritik olan düzensiz toplam üretim miktar ise Hızlı Fourier Dönüşüm algoritmasıyla hesaplanmıştır. Toplam üretim parametresi ayrıca tahmin güvenilirliği ölçümü amacıyla bir yıl boyunca gözlenmiştir. Görülen Ortalama Mutlak Hata Yüzdeleri %11 (fason üretim oranı) ve %14-%22 (toplam üretim miktar) olarak hesaplanmış ve düzensiz talep göz önüne alınmadığında kabul edilebilir olarak değerlendirilmiştir.

Anahtar Sözcükler: Öngörü, düzensiz talep, kesikli fourier dönüsümü, hızlı fourier dönüsümü.
INTRODUCTION

Management science has been providing answers to many planning problems faced in production discipline including those about inventory management, lot sizing and scheduling, workforce and aggregate planning. Most of the proposed approaches either start with the assumption of deterministic demand or that the researcher actually has an idea of the underlying probability distribution of a possible stochastic demand. Unless the demand is deterministic, it must be forecasted. Even in the absence of these assumptions, it is always possible to utilize simple approaches like demand levelling where an excess demand of one period may be transferred to the next in order to smooth out the general demand structure.

Obviously, well behaving data sets containing dominant explainable components are easier to forecast, whereas data sets which display random or erratic behaviour are harder to be represented by a functional form. As noted by Treharne and Sox (2002) and Shang (2012), it is very rare to deal with a real life demand data which does not show nonstationary behaviour. It is common practice to resort to regression or time series based analyses when they are applicable. Problems start to emerge under the absence of a set of explaining variables or when certain stationarity assumptions such as mean, variance and autocorrelation structure are violated. Using conventional methods, it is even harder to obtain a plausible forecast with an added fact of randomly fluctuating data or an erratic behaviour. Whereas the first option is to bind the problem with assumptions instead of dealing with the seemingly untameable data (Shang, 2012), depending on the case tackling the data as it is an inevitable task. In such cases, the remedy would be any methodology with sufficiently acceptable outcomes.

This study examines such a case. The company in question is a branch of one of the main actors of defence industry settled in Turkey. They produce their own products along with in-house products as a subcontractor. Process enhancement activities are considered indispensable whereas they must be planned for periods with relatively lower workload for both internal and external products. For this purpose, the company decided to benefit from demand forecasting. By identifying the periods which they were likely to face low production volumes as well as a low in-house production ratio, they would have more admissible conditions for the purpose.

The remainder of this study is organised as follows. Section one provides a literature review along with information on data and methodology. Section two consists of the theoretical framework, analyses and findings, followed by a conclusion.
1. LITERATURE REVIEW AND METHODOLOGY

This paper aims to provide consistent forecasts of total amount of monthly production and of a reliable ratio of in-house production over total production for a leading defence industry company from Turkey. Based on these objectives two research problems are constructed:

- Analysis and forecast of the in-house production ratio
- Analysis and forecast of the total production

The past data of interest was obtained from the company. These included the total and contracted amounts of in-house production for the convenient January 2011–December 2015 period. The problems were tackled with a time dependent manner since the total production volume did not present any structural shape and showed no evidence of a possible causal model approach that could have been attempted to carry out. For the first problem, the time frame was narrowed down to January 2014–December 2015 period. The time frame for the second problem was February 2012–December 2015.

First problem was rather easily solved via static forecasting which is basically a time dependent regression model utilising a seasonal effect component. The method is similar to those presented by Ferbar Tratar et al. (2016) and Ferbar Tratar and Strmčnik (2016) except that the parameters are not updated with each newly observed demand.

Second problem could not be forecasted using conventional forecasting approaches due to its random and erratic behaviour, so FFT (Fast Fourier Transform) (Van Loan, 1992), which provided the most acceptable outcome, was employed. The second research problem, estimating the total production volumes which present erratic behaviour, is the main target of the study. Some possible reasons for the erratic behaviour are considered to be exchange rate volatility and changes in economic conditions (Song, Zipkin, 1993).

Even though FTs (Fourier Transforms) are widely utilized for forecasting purposes, this is not the case in operations management. In Table 1, we present a summary of studies which involve Fourier transform for forecasting purposes. Most of these cases deal with data sets with obvious trend and/or seasonal structures (ie, González-Romera et.al., 2008; Beiraghi, Ranjbar, 2011, Dong et.al., 2011; Ichinose et. al., 2012, Zong-chang, 2013) and use FT along with other methods as noted. Nevertheless, FTs have not been very popular in industrial demand forecasting literature where they might be used as a solution to forecast erratic behaviour which is common to these type of data sets.
Table 1. A Summary of Forecasting Literature Using FTs

| Problem focus          | Methodology                  | Reference                  |
|------------------------|------------------------------|----------------------------|
| Electricity demand     | FT and regression            | Yukseltan et al., 2017     |
| Electricity demand     | FT and neural networks       | González-Romera et al., 2008|
| Electricity demand     | FT and ARIMA                 | Beiraghi, Ranjbar, 2011    |
| Stock market volatility| FT and time series analysis  | Malliavin, Mancino, 2002   |
| Stock market volatility| FT and non-harmonic analysis| Ichinose et al., 2012      |
| Real time urban water  | FT and support vector regression | Brentan et al., 2017     |
| demand                 |                              |                            |
| Wind power             | FT and discrete wavelet transform | Bifaraf et al., 2015 |
| Traffic flow           | FT and AR and neighbourhood regression | Dong et al., 2011 |
| Air temperature movement | FT                           | Zong-chang, 2013           |
| Monthly reservoir inflows | FT                           | Yu et al., 2018            |
| Fashion                | FT                           | Fumi et al., 2013          |

Forecasting smooth demand is relatively easy. There are many studies on lumpy and intermittent demand. Researchers also focused on forecasting erratic demand but the volume of these studies are considered less than expected whereas erratic demand is as common as the other types. The methodologies vary, and FT is not one of the most popular methods. Zhang et al. (2010) utilized support vector machines. Amjady et al. (2010) practised a two level forecast approach where the lower level consists of neural networks and evolutionary algorithms and the upper level optimises the performance via an enhanced differential evolution algorithm. Onyeocha et al. (2014) studied the erratic demand’s underlying statistical distributions via discrete event simulations. Molina et al. (2016) compared artificial neural networks and ARIMA performances under erratic demand in medicine industry. We propose using FFT directly for forecasting erratic demand.

2. ANALYSIS AND DISCUSSIONS

2.1. Forecasting the in-House Production Ratio

At the start of the analysis, total production amount and in-house production ratio were both investigated. It was noticed at this point that the in-house production ratio exhibited a characteristic pattern, that of a typical run including a level, a trend and seasonality at the same time. Figure 1 presents the proportioned data. Along with the ratio run, linear regression line and a border showing the periodicity for the seasonal
analysis is also added. The periodicity, 12, was observed as a monthly change through a whole year. The seasonal similarity of the two years can readily be seen around the dashed line. On all the graphs, months are presented as numbers starting from 1 denoting February, 2012.

**Figure 1. The Proportion of In-house Production over Total Production**

To forecast the in-house production ratio $CPF_p$, we propose an additive static forecast model;

$$CPF_p = L + T \ast p + SF_p + u_p$$  \hspace{1cm} (1)

where $p$ denotes the period, $L$ is the basic level, $T$ is the trend, $SF_p$ is the seasonal factor of period $p$ and $u_p$ is the random error term.

After smoothing the data with respect to periodicity using moving averages to obtain deseasonalised proportions, a linear regression was carried out where $p$ is the explaining variable. The resulting equation where $DS_p$ denotes the deseasonalised data was found to be;

$$DS_p = 0.02097 + 0.25427 \ast p$$

The differences of real proportions and the proportions estimated via regression are found and averaged period wise (with lag 12) to obtain the seasonal factors to be added. The seasonal factor array for the 12 month period is;
Finally the seasonal factors are added to the regression outcomes to get the final forecasts. The resulting errors have a standard deviation of 0.06 and a MAPE of 0.11 which is considered acceptable. Forecasts are presented in Figure 2, along with the original data and the regression line.

**Figure 2. Static Forecast of In-house Production Ratio**

The seasonal structure is promising in terms of better future planning of process enhancement activities. It may be expected that the in-house production will consume most of the production capacity in July and October; therefore process enhancement activities should take place during August and September where the in-house production ratios are relatively lower. Because of the linear structure, it may be argued that the in-house production ratio may, at some point, exceed 100% which would then imply a possible need for a capacity expansion. Furthermore, the linearity assumption is not always realistic and recalculations are required as new data accumulates.

### 2.2. Forecasting the Total Production

The only factor affecting the process enhancement activities is not the in-house production ratio; the company is also interested in forecasting the total production volume. The total production volume has a slight upward trend, yet it does not fit into any conventional category for forecasting due to its erratic nature. Owing to the lack of any causal variables to explain the high volatility and the fact that data does not respond well to other regression, smoothing or time based methodologies, the effort was made to obtain a deterministic mathematical procedure. A data set should not include a noise

\[SF_p = \{0.174 - 0.052 - 0.175 - 0.122 \times 0.054, - 0.188 \times 0.227, - 0.008 - 0.079 \times 0.254 - 0.032 - 0.041\}\]
term if it needs to be described mathematically. In this sense, first we examined the data to verify an obvious lack of random change.

2.2.1. Cross-Period Correlation Analysis

It is common to propose that unless two variables are correlated, at least one of them is regarded as noise and the two cannot be bound using a mathematical model. With the aid of this fact, 28 subsets of the 47-period data set were constructed in a rolling horizon approach (i.e. the first set consisted of periods 1 to 20, the second set consisted of periods 2 to 21 and so on). Pearson correlations of these 28 subsets were calculated. Figure 3 shows the matrix correlation heat map and Figure 4 shows the 3D correlation change graph both of which were analysed in Matlab. P values for significance of the calculated correlations are given in Figure 5.

Figure 3. Correlation Heat Map Graph of Period Subsets
In Figure 5, the red cells indicate marginal significance at a 95% confidence level and thus reveal strong correlations (greater than 0.50). The yellow cells indicate marginal significance at a 90% confidence level and are therefore moderately strong correlations (0.50 to 0.30). The remaining green cells indicate low or no correlations. Even though the green cells are higher in frequency, there is sufficient
evidence of correlation. Therefore we proceed to the FFT in order to model the total production volumes.

### 2.2.2. Fast Fourier Transform

Fourier Transform (FT) (Bracewell, 1965) is a method to map and transform a function in time domain to frequency domain. Discrete Fourier Transform (DFT) is a special case of Fourier Transform where the function consists of equally spaced samples. Computing an N-point DFT for N samples of a FT means applying a computation of N points on a unit circle on z plane N equally spaced samples ($w_k = 2\pi k/N$). FFT’s (Van Loan, 1992) are efficient algorithms used for computing N-point DFTs which provide less computational complexity.

DFT computation for a series $\{x_k\}_{k=1}^{N}$ is as follows;

$$c_k = \sum_{j=0}^{N-1} x_j w_N^{jk}, k = 0, \ldots, N - 1$$  \hspace{1cm} (2)

where $w_N = e^{-j(2\pi/N)}$.

The idea is to convert the total production data from time domain to frequency domain using FFT and to analyse the dominant frequencies. For this purpose, the previously constructed 28 samples of 20 elements were converted to frequency domain via FFT. As long as the resulting distribution has a dominant frequency in most of the samples, it may be put forward that the data set can be modelled by a mathematical function. A dominant sinusoidal frequency observed in the main function must also be dominant in other functions which derive from the main function.

For the generic function (3), suppose $A_1 \sin(B_1 * t - C_1)$ function is as shown on Figure 6 and $A_2 \sin(B_2 * t - C_2)$ function is as shown on Figure 7. Then function f which is a summation of these two terms will be as Figure 8.
It can clearly be observed from Figure 8 that the dominant sub function is $A_1 \times \sin(B_1 \times t - C_1)$ in which case the dominant frequency is $f = 1/B_1$. In our case, the samples were transformed via FFT using Matlab and the outcome is shown in Figure 9.
It can readily be seen that the dominant frequency is the first one for all the sets, followed by third and the last. This means that the original data set can be explained by a mathematical model.

2.2.3. Mathematical Forecast

The aim is to model the total production volume mathematically, therefore we decided on a sinusoidal function of the generic type $Y = A \sin (B \cdot t - C)$ which is similar to the fluctuations of the original data. The terms A, B and C represent amplitude, period and phase respectively. Emanating from the variability observed in the original production volume data, it is straightforward to conclude that all these three parameters vary dynamically. Hence, the main function is a summation of sub functions of the sinusoidal form. We represent this as:

$$Y = \sum a_j \sin (b_j \cdot t + c_j)$$ (4)
The Matlab solution for the parameters for $j = \{1, \ldots, 8\}$ is presented in Table 2 and the resulting function can be observed in Figure 10.

### Table 2. Main Function Coefficients

| $a_1$ | 8777.0 | $b_1$ | 0.062 | $c_1$ | 0.123 |
|-------|--------|-------|-------|-------|-------|
| $a_2$ | 7615.0 | $b_2$ | 0.069 | $c_2$ | 3.111 |
| $a_3$ | 434.3  | $b_3$ | 0.501 | $c_3$ | 2.538 |
| $a_4$ | 400.7  | $b_4$ | 0.682 | $c_4$ | 2.793 |
| $a_5$ | 261.2  | $b_5$ | 0.931 | $c_5$ | -1.929 |
| $a_6$ | 276.7  | $b_6$ | 1.588 | $c_6$ | 0.187 |
| $a_7$ | 292.9  | $b_7$ | 2.701 | $c_7$ | 1.748 |
| $a_8$ | 196.4  | $b_8$ | 1.232 | $c_8$ | 3.119 |

### Figure 10. Total Production Volume Function over Time

The MAPE for the resulting in-sample forecast was found as 24.11% which is somewhat large yet acceptable considering the erratic behaviour of the present data. It was considered as insightful in order to carry out the appropriate process enhancement activity planning. Under the expectation that the dominant sub functions will keep on effecting the change in production volume, forecasts were made for the upcoming year. These are given in Table 3 and visualised along with past data and in-sample forecasts in Figure 11.
Table 3. Total Production Forecasts for 2016

| Period | Forecasted Production | Period | Forecasted Production |
|--------|-----------------------|--------|-----------------------|
| Jan 16 | 1355                  | Jul 16 | 841                   |
| Feb 16 | 1942                  | Aug 16 | 689                   |
| Mar 16 | 1278                  | Sep 16 | 1229                  |
| Apr 16 | 1883                  | Oct 16 | 430                   |
| May 16 | 1471                  | Nov 16 | 735                   |
| Jun 16 | 1432                  | Dec 16 | 618                   |

Figure 11. Total Past Production and Forecasts for 2016

CONCLUSION

It is evident that modelling demand mathematically is a rather farfetched assumption unless the demand has an obvious reason to follow it. Causal models satisfy this assumption by definition, and function based models can only hope for a good future fit. There is no other way to verify this expectation other than observing the future data. We took this path, expecting actual data to match our forecasts, implying that sinusoidal functions stand for possible causal variables which were unknown.
Following actual production data was observed during year 2016 to analyze the efficiency of the forecasts. We present the values in Table 4 and the graph of actual and forecasted values for the in-house production amount which was the main concern of the study in Figure 12.

Table 4. Actual and Forecasted Production Volumes for 2016

| Period | Forecasted Production | Actual Production | Absolute Error | Absolute Percentage Error |
|--------|-----------------------|-------------------|----------------|--------------------------|
| Jan 16 | 1355                  | 1528              | 173            | 0.113                    |
| Feb 16 | 1942                  | 2106              | 164            | 0.078                    |
| Mar 16 | 1278                  | 1659              | 381            | 0.229                    |
| Apr 16 | 1883                  | 1728              | 155            | 0.089                    |
| May 16 | 1471                  | 1344              | 127            | 0.094                    |
| Jun 16 | 1432                  | 1110              | 322            | 0.290                    |
| Jul 16 | 841                   | 690               | 151            | 0.218                    |
| Aug 16 | 689                   | 728               | 39             | 0.053                    |
| Sep 16 | 1229                  | 1086              | 143            | 0.131                    |
| Oct 16 | 430                   | 1040              | 610            | 0.586                    |
| Nov 16 | 735                   | 1162              | 427            | 0.367                    |
| Dec 16 | 618                   | 1149              | 531            | 0.462                    |

Figure 12. Actual and Forecasted Production Volume for 2016

It can be observed from Figure 12 that DFT can actually estimate the changes even for such a volatile data structure. Forecast errors increase towards the end of the year, which can be accepted as a common phenomena based on previous studies applying different methods. This was the case for Zhang et al. (2010), Amjady et al.
KOYUNCU, TURFAN | Hızlı Fourier Dönüşümü Algoritması ile Düzensiz Talep Altında İş Yükü...

(2010), Ichinose et al. (2012) and Kochak and Sharma (2015) who all used different methods for forecasting erratic patterns.

Apart from the last three months of 2016, the forecasts have a tendency to predict the actual values with a MAPE of 14.4 % which rises to a 22.6 % including the off-target values during the last quarter of 2016. Another possible advantage is that, even in the long run companies may benefit from the fact that FFT almost always is able to predict the direction change even when the discrepancy between the forecast and the actual value is relatively high. The correlations of two series are 0.885 for the first nine months and 0.782 for the 12-month period. We believe these outcomes are useful, and convincing in terms of MAPEs they reveal, for a data set which reveals no inner structural consistency and does not allow for any conventional estimation approaches. It is not surprising to obtain MAPEs well above 40 % (Fumi et al., 2013) which may still be deemed useful depending on the case.

The company benefited from the forecasts, planning their process enhancement activities during the June – August period where the production volume was expected to follow a local minimum and that was actually the case. Even though, later on October production turned out to be even lower than that of August’s, a two month minimum would have been preferred since those activities took more than a month to complete. FFT performed well in defining the low demand periods without the aid of additional methodologies, yet improvements are deemed necessary if the forecasted demand is used directly as in an inventory planning problem. Researching a possible relation between FFT coefficients and cause variable coefficients of a multivariate regression, when they are visible and accessible, could justify the direct use of already well performing FFT for forecasting erratic demand.

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