Imprint of cosmic neutrino decoupling in the spectrum of inflationary gravitational waves

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Abstract. In this paper we study the effect of cosmic neutrino decoupling on the spectrum of cosmological gravitational waves (GWs). Before decoupling, namely at temperatures $T \gg 1\,\text{MeV}$, neutrinos constitute a perfect fluid and do not hinder GW propagation, while well after decoupling, for $T \ll 1\,\text{MeV}$, they free-stream generating an effective viscosity that damps cosmological GWs by a constant amount. In the intermediate regime, corresponding to the time of neutrino decoupling, the damping is frequency-dependent. GWs entering the horizon during neutrino decoupling have a frequency $f \sim 1\,\text{nHz}$; in particular, we show how neutrino decoupling induces a step in the spectrum of cosmological GWs just below 1 nHz, a frequency region that will be probed by Pulsar Timing Arrays (PTAs). We briefly discuss the conditions for a detection of this feature and conclude that it is unlikely to be observed by PTAs.

Introduction. The presence in the Universe today of a stochastic background of gravitational waves (GWs) is a quite general prediction of several early cosmology scenarios. In fact, the production of GWs is the outcome of many processes that could have occurred in the early phases of the cosmological evolution, like the amplification of vacuum fluctuations in inflationary [1] and pre-big-bang cosmology scenarios [2], phase transitions [3], and finally the oscillation of cosmic strings loops [4]. In most of these cases, the predicted spectrum of GWs extends over a very large range of frequencies.

The detection of GWs produced in the early Universe would be a major breakthrough in cosmology and high energy physics. This is because GWs decouple from the cosmological plasma at very early times, when the temperature of the Universe is of the order of the Planck energy. In this way, relic GWs provide us a “snapshot” of the Universe at the time of their production, in a similar way as the cosmic microwave background (CMB) radiation images the Universe at the time of recombination. The extremely low frequency region ($f \lesssim 10^{-15}\,\text{Hz}$) in the spectrum of primordial GWs can be probed through the anisotropies of the CMB. In particular, GWs leave a distinct imprint in the so-called magnetic or B-modes of its polarization field [5, 6].

The frequency region between $\sim 10\,\text{Hz}$ and few kHz is probed by operating large scale interferometric GW detectors like LIGO [7] and VIRGO [8], that, although designed with the aim to detect astrophysical signals, can possibly also detect signals of cosmological origin [9]. The LISA space interferometer [10], that will hopefully operate in the 2020s, will probe the frequency region between $10^{-4}$ and 1 Hz. Finally, pulsar observations can be used to obtain...
information on the stochastic GW background, through the technique known as pulsar timing. The so-called “Pulsar Timing Arrays” will probe the region from $\sim 1$ to $\sim 100$ nHz, with a maximum sensitivity between $\sim 3$ and $\sim 10$ nHz [11].

In order to compare the theoretical predictions with the expected instrument sensitivities, one needs to evolve the GWs from the time of their production to the present. This is often done by assuming that GWs propagate freely across the Universe. In this case, the only effect is that the amplitude of the wave decreases due to the expansion of the Universe. However, GWs are sourced by the anisotropic stress part of the energy-momentum tensor of matter, so that the above assumption is well-motivated only when anisotropic stress can be neglected. It is already known that the anisotropic stress of free streaming neutrinos acts as an effective viscosity, absorbing GWs in the low frequency region, thus resulting in a damping of the B-modes of CMB [12, 13, 14, 15, 16]. In Ref. [18], the evolution of cosmological GWs in the presence of neutrino free-streaming was studied up to second order in perturbation theory. Moreover, features in the spectrum are also induced by entropy generating events (like the $e^+e^-$ annihilation) occurring during the cosmological evolution [17]. The authors of Ref. [17] also consider the effect of neutrino free streaming but assume an instantaneous neutrino decoupling.

Here we briefly summarize our results concerning a possible signature of neutrino decoupling on the spectrum of cosmological GWs, as reported in Refs. [25, 26]. During decoupling, the effective viscosity of neutrinos is increasing from zero (its value at high temperatures, when the neutrinos are tightly coupled to the cosmological plasma) to a finite value. Neutrino decoupling happens when the temperature of the Universe is $T \simeq 1$ MeV, or redshift $z \simeq 10^{10}$; GWs entering the horizon at that time have a frequency $f \simeq 1$ nHz. Since the viscosity results in a damping of the wave, we expect to have a frequency-dependent absorption of GWs in this frequency range. A spectral feature in the nHz range would be potentially interesting for PTAs. PTAs combine the fact that pulsars are very stable clocks with the fact that the time of arrival of the pulse also depends on the GW background between the pulsar and the Earth [11]. The use of a pulsar array allows to correlate the electromagnetic signals from different pulsars, thus eliminating variations in the arrival times that do not depend on the GW background. The dominant contribution to the GW background in the PTAs sensitivity range is that of supermassive black hole binaries following galaxy mergers [19, 20, 21]. This has an astrophysical origin and would obviously not be affected by the decoupling of cosmological neutrinos. The dominant cosmological contribution to the GW signal in the nHz range is expected to be that of cosmic strings, that are topological defects left after symmetry-breaking phase transitions occurring in the early Universe. Other, possibly important, contributions in this frequency range are those of cosmic superstrings (i.e., topological defects arising in string-theory inspired inflationary models) and of inflation-generated GWs (through the usual amplification of quantum zero-point fluctuations). For our purposes, one important difference between the cosmic (super)strings-generated GWs on one side, and the inflationary GWs on the other, is that the former are created by causal processes at horizon and subhorizon scales, while the latter are generated by the stretching of quantum fluctuations to super-horizon scales. In the following we will concentrate on GWs that were outside the horizon at some time before neutrino decoupling, so that our results rigorously applies to inflationary GWs but not necessarily to string-generated GWs. It should be noted that the inflationary signal is, in the nHz range, the weakest of the cosmological signals mentioned above, and is probably beyond the reach of the ongoing PTA projects. It could be however within the reach of future instruments [11, 22] like the Square Kilometer Array (SKA) [23].

**Basic statements.** We consider a GW, described by a metric perturbation $h_{ij}$, propagating on the background of a flat Friedmann Universe. In synchronous gauge, the spatial components of
the perturbed metric are written as \(^1\)

\[ g_{ij} = a^2(\eta) [\delta_{ij} + h_{ij}] \]  

while the other components are left unperturbed: \(g_{00} = -1\) and \(g_{0i} = 0\). We will consider only the transverse traceless part of \(h_{ij}\). With this restriction, \(h_{ij}\) only has two degrees of freedom, corresponding to the two polarizations of the GW. Here \(a\) is the cosmological scale factor and \(\eta\) is the conformal time, related to the synchronous time \(t\) by \(dt = a(\eta)d\eta\). With this choice, a GW \(h_{ij}\) with wave number \(k\) evolves according to \(^2\)

\[ \ddot{h}_{ij} + 2\mathcal{H}\dot{h}_{ij} + k^2h_{ij} = 16\pi G\Pi_{ij}, \]  

where dots denote derivatives with respect to \(\eta\), \(\mathcal{H}\) is the conformal Hubble constant \(\mathcal{H} = \dot{a}/a\), and \(\Pi_{ij} = T_{ij} - \delta_{ij}T/k^2/3\) is the anisotropic stress, i.e. the traceless part of the three dimensional energy-momentum tensor \(T^i_j\) of the cosmological fluid, representing dissipative effects. We assume that the anisotropic stress is relevant only for one component of the cosmological fluid (the neutrinos), providing a fraction \(f_\nu\) of the total background density \(\bar{\rho}\), while for the other components (e.g., the photons) it can be safely neglected. The anisotropic stress can be computed by solving the Boltzmann equation for the neutrino phase space density \(f(x^i, P_i, \eta)\), from which the energy-momentum tensor is obtained by integrating over momenta. Since the Boltzmann equation itself contains the metric \(h_{ij}\), it forms a coupled system together with the Einstein equation (2). In the following, we will restrict our attention to waves entering the horizon well before the time of matter-radiation equality, corresponding to a redshift \(z \simeq 10^4\). This corresponds to waves with a present frequency \(\nu > 10^{-15}\) Hz.

**Coupled evolution of GWs and neutrinos.** The thermal evolution of neutrinos can be divided into two distinct regimes. In the early Universe, when the temperature was sufficiently high, neutrinos were kept in thermal equilibrium with the cosmological plasma by frequent reactions like \(\nu\bar{\nu} \leftrightarrow e^+e^-\), \(\nu e \leftrightarrow \nu e\), etc. As the Universe expanded and cooled down, the interaction rate \(\Gamma_\nu\) eventually became smaller than the expansion rate \(H\) and the neutrinos decoupled from the cosmological plasma. From this point on, neutrinos can be considered as freely streaming through the Universe. The temperature \(T_{\text{dec}}\) that separates the collisional from the non-collisional regime, defined as the temperature when the interaction rate is equal to the expansion rate, i.e. \(\Gamma_\nu(T_{\text{dec}}) \simeq H(T_{\text{dec}})\), is found to be \(T_{\text{dec}} \simeq 1\) MeV, corresponding to \(z \simeq 10^{10}\). A wave that enters the horizon at that time has a frequency \(f \sim 1\) nHz. Higher frequency waves enter the horizon before neutrino decoupling, and vice versa. Then, the following three regimes for the evolution of a GW can be correspondingly identified:

- **When** \(T \gg 1\) MeV, the neutrinos are tightly coupled to the cosmological plasma, that behaves like a single, perfect fluid. The anisotropic stress of neutrinos is negligible and the GW evolves according to the homogeneous version of Eq. (2). This regime is relevant for waves with \(f \gg 1\) nHz.
- **When** \(T \simeq 1\) MeV, the neutrinos are decoupling from the plasma. They cannot be considered as a perfect fluid because the mean free path of particles is getting large, so that the anisotropic stress cannot be neglected. However, the collisions have still to be taken into account. The GW evolution has to be calculated from the full Einstein-Boltzmann system, taking into account neutrino interactions. This regime is relevant for waves with \(f \simeq 1\) nHz.

\(^1\) We use the \((-+++)\) signature for the metric. We shall also use, all throughout the paper, natural units in which \(c = \hbar = k_B = 1\).

\(^2\) The spatial dependence of all quantities has been Fourier transformed.
• When $T \ll 1$ MeV the neutrinos are collisionless and behave as free particles. The anisotropic stress cannot be neglected: the GW evolution has to be calculated from the full Einstein-Boltzmann system, but neutrino interactions can be neglected. This regime is relevant for waves with $f \ll 1$ nHz.

In the first, high-temperature regime, the metric perturbation evolves as $h_{ij} = \tilde{h}_{ij} \equiv \lambda_{ij} \sin(k \eta)/k \eta$ (assuming the initial velocity vanishes), where $\lambda_{ij}$ is the initial value of the perturbation. In other words, the GW oscillates with a time-dependent amplitude $\propto 1/k \eta$. This can be seen as the energy loss of the wave due to the expansion of the Universe. The third, low-temperature regime was first studied in Ref. [14], where it was realized that the anisotropic stress of neutrinos would lead to a partial absorption of the GW. In this limit (waves entering the horizon well after neutrino decoupling) the absorption is frequency independent, and only depends on the neutrino fraction $f_{\nu}$; in particular, for the standard value $f_{\nu} = 0.4052$ (corresponding to three neutrino families in thermal equilibrium with a present temperature $T_\nu = 1.9$K) the intensity of the wave is reduced to $\sim 75\%$ of its value in the absence of stress [25]. In the second, intermediate regime the effect of neutrino interactions should be taken into account. We have modeled neutrino collisions by means of a suitable collision term in the Boltzmann equation, and integrated the Boltzmann-Einstein system for different values of the GW frequency between $10^{-12}$ Hz and $10^{-8.5}$ Hz $\simeq 3$ nHz, as described in Refs. [25, 26]. We have assumed the standard value for $f_{\nu}$. We have defined a frequency-dependent damping factor $D_f \equiv \sqrt{2((k \eta h_{ij})^2)}$ where the brackets $\langle \ldots \rangle$ denote the average over many periods, taken when the wave is well inside the horizon ($k \eta \gg 1$). The definition is chosen so that when $h_{ij} = \sin(k \eta)/k \eta$ then $D_f = 1$. The damping factor basically quantifies how much the amplitude of a GW produced in the early Universe is reduced due to the anisotropic stress of neutrinos, with respect to propagation in a perfect fluid. We express our results in terms of the GW density parameter $\Omega_{GW}(f) = (d \rho_{GW}/d \ln f)/\rho_c$, where $\rho_{GW}$ is the energy density of GWs and $\rho_c$ is the critical density of the Universe. The quantity $\Omega_{GW}$ is used by theorists to quantify the intensity of the cosmological GW background predicted in a given scenario, and experimental sensitivities are also often quoted in terms of it. The present intensity is usually computed simply by rescaling the GW amplitude at the source by a factor $1/k \eta$ to take into account the redshift due to the expansion. This should be corrected to take into account also the anisotropic stress of neutrinos; since $\rho_{GW} \propto h_{ij}^2$, the correction factor is $D_f^2$. Calling $\Omega_{GW}(f)$ the present intensity in absence of stress, we have that the observed spectrum $\Omega_{GW}(f) = D_f^2 \Omega_{GW}(f)$. The damping factor introduces an additional dependence on frequency that was not present at the source. In the left panel of Fig. 1 we show $D_f^2$ as a function of frequency.

The fact that the collisions introduce a frequency dependence on the amplitude of the GW background (in addition to any dependence that could be already present at the source) is a more interesting case with respect to a frequency independent suppression, since the latter could not be disentangled from our ignorance on the amplitude of the original spectrum (that usually depends on the free parameters of the theory and thus cannot be determined a priori without any additional experimental input). On the other hand, a frequency-dependent effect can be disentangled by using this same dependence. To better quantify this, let us suppose that the GW spectrum at the source, has a featureless power law behaviour $\propto f^\alpha$. After correcting for the expansion, the shape is still the same. Then the spectrum, once the effect of anisotropic stress has been taken into account, will be $\Omega_{GW}(f) \propto D_f^2 f^\alpha$. The logarithmic slope of the spectrum in a given point is given by $\alpha' = d \log \Omega_{GW}/d \log f$, so that, defining the deviation $\Delta \alpha = \alpha' - \alpha$, we have:

$$\Delta \alpha = \frac{d \log \Omega_{GW}}{d \log f} - \alpha = \frac{d \log D_f^2}{d \log f}, \quad (3)$$
In the right panel of Fig. 1, we show $\Delta \alpha$ as a function of frequency. We find that it has a maximum value of $\Delta \alpha = 0.15$ at $f \simeq 0.1 \text{ nHz}$.

**Observational prospects** The nHz region of the spectrum of GWs can be probed by PTAs, combining the fact that pulsars are very stable clocks with the fact that the time of arrival of the pulse also depends on the GW background between the pulsar and the Earth [11]. Correlating the electromagnetic signals from different pulsars could, in principle, enable a positive detection of the GW background in the galaxy. PTAs are sensitive to GWs with frequency between roughly 1 and 100 nHz, with a maximum sensitivity in the region between 3 and 10 nHz. The lower bound comes from the fact that standard pulsar timing techniques absorb any low-frequency signal, and so the time span of the data (currently $\sim 30$ years) gives a lower bound on the observable frequencies. The principal source of GWs in the region of maximum sensitivity are believed to be coalescing supermassive binary black-hole systems in the centre of merging galaxies. However, a signal of cosmological origin could also be present in that same frequency range, for example GWs produced during the inflationary era or from the oscillation of cosmic string loops.

Can the effect of anisotropic stress leave an imprint that could be, in the future, detected using PTAs? This could be possible, at some conditions. First of all, obviously, a signal of cosmological origin has to be present in the nHz range and it has to be strong enough to be detectable by PTAs. As noted in the introduction, the oscillation of cosmic (super)strings loops could give a detectable signal in the nHz range. However, GWs generated in this way are produced at scales smaller or equal than the horizon, so that the analysis made here does not rigorously apply. Although it is sensible to expect that GWs with wavelengths comparable to the horizon would behave in a similar way to the super-horizon GWs considered here, we cannot assess the effect on the overall spectrum and we defer this generalization to a future work. This leaves the GWs produced during inflation by the amplification of quantum vacuum fluctuations, that are too weak to be detected by ongoing PTA projects but could be within the reach of future experiments like SKA. In any case, an independent observation of the signal at larger frequencies, like those probed by interferometers, would also be useful to normalize the spectrum at the source. Secondly, the cosmological signal should be clearly separated from the astrophysical signal, like that produced by black hole binaries. The third point is the more
problematic. We have shown that the effects of the damping are more evident at frequencies between 0.1 and 1 nHz. As explained above, the lower limit to the detectable frequency is given by the time span of the observations, currently 30 years. In 70 years from now, with a total of 100 years of observations, the lower limit will be $f = 1/(100 \text{years}) \simeq 0.3 \text{nHz}$, where the intensity of the wave is reduced by only 5% and the deviation from a featureless power law is $\Delta \alpha = 0.05$. The maximum change of the slope is at $f \simeq 0.1 \text{nHz}$, corresponding to 300 years of observations. It should also be noted that, even if we were willing to wait such a long time, it is not certain that it would actually increase the sensitivity, because of intrinsic low-frequency instabilities in the timing data. In conclusion, we think that the possibility to detect this effect are quite small, and that they will depend crucially on the existence of a sizeable inflationary-like GW background at the frequencies of interest and on the capability of estimating the logarithmic slope of the spectrum in the region just below 1 nHz.

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