Article

Negative exponential behavior of image mutual information for pseudo-thermal light ghost imaging: observation, modeling, and verification

Junhui Li a, Bin Luo b, Dongyue Yang c, Longfei Yin c, Guohua Wu c, Hong Guo a,⇑

a State Key Laboratory of Advanced Optical Communication Systems and Networks, School of Electronics Engineering and Computer Science, and Center for Quantum Information Technology, Peking University, Beijing 100871, China
b State Key Laboratory of Information Photonics and Optical Communications, Beijing University of Posts and Telecommunications, Beijing 100876, China
c School of Electronic Engineering, Beijing University of Posts and Telecommunications, Beijing 100876, China

A R T I C L E   I N F O

Article history:
Received 25 March 2017
Received in revised form 8 April 2017
Accepted 10 April 2017
Available online 18 April 2017

Keywords:
Ghost imaging
Image mutual information
Information theory
Image quality assessment

A B S T R A C T

When using the image mutual information to assess the quality of reconstructed image in pseudo-thermal light ghost imaging, a negative exponential behavior with respect to the measurement number is observed. Based on information theory and a few simple and verifiable assumptions, semi-quantitative model of image mutual information under varying measurement numbers is established. It is the Gaussian characteristics of the bucket detector output probability distribution that leads to this negative exponential behavior. Designed experiments verify the model.

© 2017 Science China Press. Published by Elsevier B.V. and Science China Press. All rights reserved.

1. Introduction

Image quality assessment is known to be difficult so far [1]. Besides traditional image quality measures based on error estimation, assessments of different types are introduced, first by the signal processing community (cf. [2] for a review). Among others, the mutual information (MI), representing the amount of information shared by two random variables in information theory [3], was introduced to account the similarity between images [4,5], and has been successfully applied in different circumstances to assess image quality [6,7].

Being different from the usual “single snapshot” imaging process, ghost imaging (GI) is built on a large number of consecutive measurements on two quantities: light intensity registered by a “bucket” detector with no spatial resolution, and a spatial profile that never reaches the object—either an “idler” reference light field [8], a modulation pattern [9], or the calculated diffraction profile of that field [10]. As a consecutive process, modeling the performance under varying measurement numbers is of great significance to GI. One would naturally expect the image quality to improve with increasing measurement number n, and converge when n → ∞. Unfortunately, previous studies of image quality focus on the influence of either the noise level [11–13] or relative spatial/temporal scale [14], and no quantitative analysis concerning measurement number has been published according to our knowledge, except for a few qualitative observations [9,15] and an untight lower bound [16].

GI has been applied to various scenarios, ranging from entangled photon pairs [8], pseudo-thermal light [17], thermal light from hollow-cathode lamp [18], sunlight [19], all the way up to X-ray [20,21]. Although different types of GI image quality assessments have been studied and compared, e.g., mean square error (MSE) [9], signal-to-noise ratio (SNR) [14], and contrast-to-noise ratio (CNR) [22], as Ref. [23] pointed out, these are all “posterior” assessments depending on specified image content, i.e., one has to get full knowledge of the “ideal” pattern beforehand, thus are inappropriate for design and optimization of a general imaging system. Ref. [23] pioneers a promising subject of prior GI image quality assessment by introducing MI between a random object and its GI image. However, they simply assumed that MI grew linearly with the measurement number n, which had not been examined, and appears to be not true according to our experiment.

In this contribution, we use image mutual information (IMI) between the object O and the reconstructed image Y to assess image quality of pseudo-thermal light GI. Semi-quantitative fitting shows that IMI I(O; Y) is a negative exponential function of the measurement number n. An information-theory-based model explains this behavior. All the assumptions are validated. Designed

⇑Corresponding author.
E-mail address: hongguo@pku.edu.cn (H. Guo).
http://dx.doi.org/10.1016/j.scib.2017.04.008
2095-9273/© 2017 Science China Press. Published by Elsevier B.V. and Science China Press. All rights reserved.
further experiments demonstrate highly agreement with the predictions of the model.

2. Methods and observation

2.1. Experiment setup

A conventional GI setup is implemented as in Fig. 1. Output of a 532 nm laser passes through a rotating ground glass (R.G.G., Edmund 100 mm diameter 220 grit ground glass diffuser), turning it into pseudo-thermal light [24], whose intensity fluctuates randomly both in the space and time domain. This pseudo-thermal light is then split into two arms by the beam splitter (BS). The signal arm penetrates a transmissive object mask, followed by a focus lens, to be registered as a whole into a temporal intensity sequence $B(t)$ by a bucket detector which has no spatial resolution. The spatial profile of the reference arm, $R(x; t)$, which never reaches the object, is recorded by a commercial CMOS camera (Thorlabs DCC3240C) synchronically with the bucket detector. The second order fluctuation correlation (2nd FC) [25] between corresponding $B(t)$ and $R(x; t)$ yields the reconstructed image $Y(x)$,

$$Y(x) \propto \frac{\langle [R(x; t) - \langle R(x; t) \rangle_t] \times [B(t) - \langle B(t) \rangle_t] \rangle_t}{\langle R(x; t) \rangle_t \langle B(t) \rangle_t},$$

where $\langle \cdot \rangle_t$ denotes average over all the measurements.

2.2. Image mutual information

**Mutual information** In information theory, MI between two random variables $A$ and $B$ is defined as

$$I(A; B) = H(A) - H(A|B),$$

where $H(A) = -\sum_a p_A(a) \log_2 p_A(a)$ is the Shannon entropy of $A$ with probability distribution function (PDF) $p_A(a)$, denoting the amount of information one reveals when gets full knowledge of $p_A(a)$, and $H(A|B)$ is the conditional entropy of $A$ given $B$, representing the amount of the remain unknown information of $A$ even when the probability distribution of $B$ is totally determined,

$$H(A|B) = -\sum_a \sum_b p_{A|B}(a|b) \log_2 p_{A|B}(a|b),$$

where $p_{A|B}(a|b)$ is the joint probability of $A = a$ and $B = b$, and $p_{A|B}(a|b)$ is the conditional probability of $A = a$ given $B = b$. Eq. (2) shows that $I(A; B)$ denotes the amount of information shared by two partite $A$ and $B$, thus can be a measure of how similar the two variables are, since identical variables have the largest MI, while totally independent ones have the smallest.

**Image mutual information** When MI is applied, image $A(x)$ of $N$ pixels is treated as a one-dimensional random variable $A$ of length $N$. MI between two images $A(x)$ and $B(x)$ is defined as MI between two random variables $A$ and $B$, which denotes the similarity between the two images. If $A(x)$ and $B(x)$ are set to be the object and image of an imaging system, respectively, MI can assess the image quality, since the goal of imaging is to accomplish a duplicate as similar to the object as possible. In fact, by maximizing MI, imaging distortion and relative displacement can be corrected profoundly—known as the image registration technique (cf. [6] for a review). Here we want to note that, in order to reduce the influence made by image distortion or relative displacement on the image quality assessment, the image and object should be aligned at first when uses MI to assess image quality. What is more, unlike other assessments, e.g., mean square error (MSE), MI is indifferent to any change of prior knowledge within the area of interest (AOI), i.e., specified object spatial profile $Y(x)$. This unique property, on one hand, emphasizes the importance of image alignment, and suggests the potential to develop a content-free image quality assessment on the other, which is a general measure of image quality regardless of what pattern it has for a specified image.

2.3. Observation of negative exponential behavior

Reconstructed image $Y(x)$ is recorded against different measurement numbers. The AOI contains $120 \times 120$ pixels. To ensure alignment, the image that is the most over-sampled ($n = 50,000$). $Y_n(x)$, serves as an almost-identical approximation of the object $O(x)$, assuming that after so many measurements, the image has been a stable, nearly perfect duplicate to the object. MI between $O(x)$ and $Y(x)$, $I(O; Y)$, is calculated under varying $n$ to assess the image quality—the higher $I(O; Y)$ is, the better quality image one gets. For our system, the image quantization bit length when calculates MI is set to be 9, according to the first part of the Electronic Appendix (Online). The result is shown in Fig. 2. Curve fitting with both linear and nonlinear regression shows that the negative exponential function fits the experiment result best, i.e.,

$$I(O; Y) = C_1 - C_2 \exp \left( -\frac{n}{C_3} \right),$$

where fitting parameter $C_1$ denotes the upper limit of $I(O; Y)$ when $n \rightarrow \infty$, and parameter $C_2$ represents the converge speed, i.e., $C_2$ measurements are required to reduce the uncertainty between image and object to the $1/e$ of its initial value. The larger $C_3$ is, the more measurements one needs to achieve the same level of image quality.

---

![Fig. 1. (Color online) Experiment setup. R.G.G. = rotating ground glass. BS = beam splitter.](image-url)
3. Modeling

3.1. GI equivalent model

In order to explain Eq. (4), an equivalent model of GI concerning only the data processing part is established, as shown in Fig. 3. Light field in the signal arm penetrates the object to be registered by the bucket detector. Under a classical approximation, this light field is identical to its counterpart in the reference arm, up to a 

\[ \text{B} \]

light before the beam splitter in Fig. 1. Therefore, in each measurement, \( O(x) \) is “encoded” by the current reference light field \( R(x; t) \) into the bucket detector output \( B(t) \). It is the \( n \) realizations of \( R(x; t) \) and \( B(t) \), i.e., \( r_1, r_2, \ldots, r_n \) and \( b_1, b_2, \ldots, b_n \), respectively, that lead to the reconstructed image \( Y(x) \) through second order fluctuation correlation of \( R(x; t) \) and \( B(t) \), i.e., \( Y = Y(\{b_i\}, \{r_n\}) \). Therefore, any quantity concerning \( O(x) \) and \( Y(x) \) should be fully determined by random variables \( O, \{b_i\} \) and \( \{r_n\} \).

3.2. Assumptions

Based on the GI equivalent model, derivation of Eq. (4) is shown in the second part of the Electronic Appendix (Online). Here we want to summary and double check all the involving assumptions.

**Fixed object** This assumes that object \( O(x) \) is fixed, so that \( H(O) \) is a constant. This is true since we use a static object, and the most over-sampling image \( Y_{\text{fg}}(x) \) is a good-enough approximation of \( O(x) \).

**Independent imaging system** This assumes that when calculating IMI, the imaging system’s optical transfer function (OTF, cf. Ref. [27] §6.3.1) is independent of specified intensity values of neither the object nor the image. With the fixed object assumption in mind, and as revealed in Section 3.1, a natural corollary would be

\[ \sum_{a} \sum_{y} p_{a|y}(o, y) = p_{O}(O) = p_{B}(\{b_i\}, \{r_n\}). \]  

(5)

**Independent reference light field** This assumes that reference light field \( R(x; t) \) is independent of the bucket detector output \( B(t) \). Considering the random nature of the reference light field, both in spatial and time domain, caused by the pseudo-thermal property, and the fixed object assumption, conditional probability

\[ p_{\text{ref}}(t|b) \propto p_{b}(b) \]

for all \( b \) and \( r \). This is due to the ergodicity of the pseudo-thermal light, since all possible spatial profiles has the equal probability to appear, then the number of corresponding spatial realizations given the bucket detector output against a fixed object should be proportional to the probability of that specified bucket value. We calculate the normalized mutual information (NMI, or “symmetric uncertainty” in Ref. [28]) between the time sequences of bucket detector output \( \{b_i\} \) and reference light field \( \{r_n\} \) (as for the value of \( r_n \), both the summation over the whole AOI and the intensity in a fixed pixel are applied) to verify this independence

\[ I_{\text{mut}}(B; R) = \frac{2I(B; R)}{H(B) + H(R)}. \]

(7)

where the denominator is the entropy summation of \( \{b_i\} \) and \( \{r_n\} \). Two totally correlated random variables have NMI of unity, while totally independent ones get zero. For varying measurement number \( n \), \| \text{I}_{\text{mut}}(B; R) \| \leq 10^{-14} \), indicating the independence of \( \{r_n\} \) on \( \{b_i\} \).
Gaussian PDF bucket detector output This assumes the output of bucket detector has a Gaussian probability distribution function,

\[ p_{\beta}(b_i) \propto \exp \left[ -\frac{(b_i - \bar{b})^2}{2\sigma^2} \right], \quad i = 1, 2, \ldots, n. \]  

(8)

where \( \bar{b} \) and \( \sigma \) are the expectation and standard derivation of \( \{b_i\} \), respectively. Though this can be a natural result for being the summation of a large number of independent random variables, considering the central limit theorem in statistics, the strict derivation for a general case is complex, since specific object pattern is involved. The experiment result is shown in Fig. 4. Similar result has been reported in Ref. [29]. Note that this assumption is sufficient but not necessary for Eq. (4), as we will see later in Section 4.

Bucket detector output iid. This assumes that the bucket detector output values are independent and identical distributed (iid.), so that \( p_{\beta}(\{b_i\}) = \prod_{i=1}^{n} p_{\beta}(b_i) \). This is guaranteed by the pseudo-thermal nature of the reference light field. Since \( R(x,t) \) is random in both the space and time domain [24], and \( b_i = b_i(O,t_i) \) for any \( i \in [1,n] \), as mentioned in Section 3.1, the iid. property of \( \{b_i\} \) is natural. Note that this is different from the linear MI assumption made by Ref. [23]. Every single measurement can be iid., but IMI concerns the ultimate performance accumulating \( n \) measurements.

Slow-varying image entropy This assumes that the image entropy

\[ H(Y) \equiv \text{const}. \]  

(9)

against varying measurement number \( n \), comparing with the IMI behavior. This is true due to the fact that the increasing knowledge of object revealed by the image with growing number of measurement is mainly caused by the image being more and more similar to the object, rather than simply being more and more complicated. The experiment results are shown in Fig. 4b,c: the image entropy did not change much even though the probability distribution function did.

Large measurement number This assumes \( n \gg 1 \), which is fulfilled in our experiment. This also indicates that error of Eq. (4) would be large when \( n \) is small, as Fig. 2 shows.

Image alignment This assumes alignment of image to the object. Instead of the real object pattern, we use the most oversampled image of the same consecutive measurement sequence as the approximation, which ensures perfect alignment as long as the imaging system is stable. Image registration is also conducted. This assumption is not necessary to derive Eq. (4), but is vital for IMI to be a good image quality assessment as mentioned in Section 2.2.

3.3. Further verification

Experiments are designed to verify the above model from different perspectives.

Post-selection on bucket fluctuation Derivation in the second part of the Electronic Appendix (Online) uses property

\[ \lim_{n \to \infty} \frac{\sum_{i=1}^{n} (b_i - \bar{b})^2}{n} = \sigma^2, \]  

(10)

which is true for all the \( \{b_i\} \) as a whole. If one picks up the measurements whose bucket detector output \( b_i \) satisfying \( (b_i - \bar{b})^2 \geq m_1 \sigma^2 \), i.e., those with larger fluctuations away from the mean value \( \bar{b} \). The variance of selected \( b_i \) satisfies

\[ \sigma_i^2 = \frac{\sum_{i=1}^{n_i} (b_i - \bar{b})^2}{n_i} = m_2 \sigma^2, \]  

(11)

in which \( m_2 > 1 \). Back to Eq. (4), one has \( C_i = C_i/m_2 \), which indicates that if the average fluctuation of selected measurements increases, \( n \) required measurement number for IMI to converge, i.e., to get an image of same quality, decreases with the equal proportion. In other words, one can achieve the same image quality with fewer measurements after post-selection on the fluctuation amplitude of bucket detector output. Similar phenomena has been reported in Ref. [29], and here we present a quantitative relationship this post-selection process should obey

\[ \sigma_i^2 \propto \frac{1}{C_i}. \]  

(12)

At the same time, according to the second part of the Electronic Appendix (Online) and the fixed object approximation, no matter how \( m_i \) varies,

\[ C_i = H(Y) = \text{const}. \]  

(13)

Experiment results are shown in Fig. 5a. The good agreement with Eqs. (12) and (13) further validates our model.

Varying AOI Ref. [14] reported that for a thermal light GI, the ultimate imaging signal-to-noise ratio (SNR) after many measurements should be a rational function of the size of AOI, approximately inversely proportional to the square root of AOI scale parameter \( I \). We vary the AOI by choose the \( 1 \times 1 \) pixels in the middle of the recorded reference light field only in each measurement, and calculate corresponding IMI against measurement number \( n \)—the same as in Section 2.3. Experiment shows that Eq. (4) holds for

![Fig. 4.](image-url) (Color online) Verification of assumptions. (a) Gaussian PDF bucket detector output. Asterisks are experiment results, and solid line is the Gaussian fitting. Adjust \( R^2 = 0.98041 \). (b) Image entropy vs. \( n \). \( H(Y) \) changes little for different measurement numbers. (c) Image PDF vs. \( n \). Change of PDF is obvious only when \( n \) is small.
all the different A0Is. The results are shown in Fig. 5b. Fitting parameter $C_1$, which stands for the ultimate IMI when $n \to \infty$, fits a power-law function of the A0I length $l$ (equivalent to the “resolution parameter $R$” in Ref. [14]) well,

$$C_1 \propto l^a; \quad (14)$$

where $a = -0.49252 \pm 0.01991$. This result agrees with Ref. [14] nicely, suggesting that IMI has very similar behavior with a usual image quality assessment—imaging SNR. What is more, according to our model, the converge parameter $C_3$ should be irrelevant to the size of A0I—which is sort of counter-intuitive—indicating that the number of measurement required before the image quality converges and the image gets stable is the same for both large and small images. However, experiment verifies this conjecture, thus further validates our model. Note that bucket detector remains the same when changing A0I due to our setup. More precise experiment should be implemented in the future, which can change the sensitive area of bucket detector accordingly.

**Contribution of reference field** Our model suggests that the reference light field has no direct contribution to the negative exponential behavior of the IMI vs. $n$ relationship. To verify this, a comparative experiment is designed that bucket detector output is replaced by summation of reference light field intensities within A0I in each measurement. We calculate IMI between the object and images reconstructed this way, $Y_{rf}(x)$, against measurement number $n$, as shown in Fig. 5c. No explicit change of IMI under varying $n$ has been found, which supports our argument. Furthermore, we define “differential” IMI as

$$I_{diff}(O;Y) = I(O;Y) - I(O;Y_{rf}); \quad (15)$$

which is the difference between the ordinary IMI and the IMI accomplished by replacing bucket detector output by reference field intensity summation. We expect $I_{diff}(O;Y)$ to represent the part of IMI revealed “solely” by the bucket detector, and the name “differential” comes from the differential entropy in information theory [3]. $I_{diff}(O;Y)$ also satisfies Eq. (4), dominating the negative exponential behavior with one more advantage over $I(O;Y)$—it has a zero initial value when $n \to 0$, making it closer to practical applications since one gets no knowledge of the object before conducting any measurements. This makes $I_{diff}(O;Y)$ another promising image quality assessment.

**4. Discussion and conclusion**

**Comparison with correspondence GI** Results of bucket fluctuation post-selection in Section 3.3 are quite similar to those in Fig. 5b. Fitting parameter $C_1$, which stands for the ultimate IMI when $n \to \infty$, fits a power-law function of the A0I length $l$ (equivalent to the “resolution parameter $R$” in Ref. [14]) well,
reported in the correspondence GI [29]. For example, using only the measurements with large fluctuations can reduce the number of measurement involved in the reconstructed image calculation, and the larger fluctuations are, the fewer measurements are needed. Rather than a coincident, we believe this is due to the same dynamics behind, even though the image calculation formula is different. In both systems, the bucket detector output has a Gaussian PDF, which decreases as the fluctuation increases. In our case, this means measurements with larger fluctuations from the mean bucket detector output value are rarer, and in the language of information theory, one can reveal more information when these measurements appear. We think this explanation also applies to their case. Furthermore, Fig. 5d shows the relationship between converge parameter $C_7$, which denotes the number of measurement required in the bucket fluctuation post-selection calculation, and the “effective” ratio—how many measurements in all that being complemented can fulfill the varying fluctuation requirements. Fitting result suggests a linear relationship. Since the total number of measurement stays the same, one can see that reduction of required measurement number is merely due to that fewer measurements can meet the harsher fluctuation requirement—which makes the satisfying measurements rarer, and cannot reduce the total number of measurement to be conducted at the first place. Therefore, correspondence GI can only reduce the number of post-selection measurement, meanwhile still needs the same number of measurement to be complemented.

Applicable range One may think that the above negative exponential behavior of image quality versus measurement number relationship is limited to situations with Gaussian PDF or take mutual information as the image quality assessment. Some of our recent contributions [30–32] suggest the other way. Our model applies to a quite general scenario thanks to the fundamentality of information theory, including general imaging process [33–37] and transformational optics [38].

Significance Our work is important to the GI community in several ways. First, we provide a semi-quantitative model of the image quality versus measurement number relationship, enabling accurate prediction of the expected image quality after certain number of measurement, and the necessary measurement number to meet any given image quality requirement. Second, explicit connections between GI parameters and concepts in information theory may lead to a new perspective of interdisciplinary research, which we hope could benefit the comparative new and less developed GI study in return. Last but not least, the similar behavior of image mutual information with usual image quality assessments, e.g., mean square error, contrast-to-noise ratio, imaging signal-to-noise ratio, meanwhile the unique insensitivity to specified prior knowledge, i.e., specified object pattern, suggests that IMI (and differential IMI) is a promising candidate for content-free image quality assessment.

In conclusion, we report the observation, modeling, and verification of the negative exponential behavior when use image mutual information to assess the image quality of pseudo-thermal light ghost imaging, with respect to the measurement number. Rooted in the fundamental information theory, the model applies to a much more general scenario.

Conflict of interest

The authors declare that they have no conflict of interest.

Acknowledgments

We want to thank Prof. Shensheng Han for helpful discussions. This work was supported by the National Natural Science Foundation of China (61631014, 61401036, 61471051 and 61531003), the National Science Fund for Distinguished Young Scholars of China (61225003), the China Postdoctoral Science Foundation (2015M580008), and the Youth Research and Innovation Program of BUPT (2015RC12).

Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.scib.2017.04.008.

References

[1] Eskicioglu AM, Fisher PS. Image quality measures and their performance. IEEE Trans Commun 1995;43:2595–65.
[2] Wang Z, Bovik AC. Mean squared error: love it or leave it? A new look at signal fidelity measures. IEEE Signal Process Mag 2009;26:98–117.
[3] Cover TM, Thomas JA. Elements of information theory. New York: Wiley-Interscience; 1991.
[4] Miles J. $R^2$, adjusted $R^2$. Wiley StatsRef: Statistics Reference Online.
[5] Cover TM, Thomas JA. Elements of information theory. New York: Wiley-Interscience; 1991.
[6] Yu H, Lu R, Han S, et al. Fourier-transform ghost imaging with hard X rays. Phys Rev A 2009;80:033811.
[7] Chen H, Peng T, Shih YH. 100% correlation of chaotic thermal light. Phys Rev A 2010;82:031808.
[8] Ashok A, Baheti PK, Neifeld MA. Compressive imaging system design using task-specific image sensing. IEEE Trans Image Process 2013;22:137–54.
[9] Pluim JPW, Maintz JBA, Viergever MA. Mutual-information-based registration of medical images: a survey. IEEE Trans Med Imaging 2003;22:986–1004.
[10] Guo B, Gunn SR, Damper RI, et al. Band selection for hyperspectral image classification using mutual information. IEEE Geosci Remote Sens Lett 2006;3:522–6.
[11] Pittman TB, Shih YH, Strekalov DV, et al. Optical imaging by means of two-photon quantum entanglement. Phys Rev A 1995;52:R3429.
[12] Duarte MF, Davenport MA, Takbar D, et al. Single-pixel imaging via compressive sampling. IEEE Signal Process Mag 2008;25:83–91.
[13] Shapiro JH. Computational ghost imaging. Phys Rev A 2008;78:061802.
[14] Jain A, Moulin P, Miller MI, et al. Information-theoretic bounds on target recognition performance based on degraded image data. IEEE Trans Pattern Anal Mach Intell 2002:24:1153–66.
[15] Neifeld MA, Ashok A, Baheti PK. Task-specific information for imaging system analysis. J Opt Soc Am A 2007;24:825–41.
[16] Ashok A, Baheti PK, Neifeld MA. Compressive imaging system design using task-specific information. Appl Opt 2008;47:4457–71.
[17] Brida G, Chekhova MV, Fornaro GA, et al. Systematic analysis of signal-to-noise ratio in bipartite ghost imaging with classical and quantum light. Phys Rev A 2011;83:063807.
[18] Wang W, Wang YP, Li J, et al. Iterative ghost imaging. Opt Lett 2014;39:5150–3.
[19] Sarvorham S, Baron D, Baraniuk RG. Measurements vs. bits: Compressed sensing meets information theory. In Allerton Conference on Communication, Control and Computing; 2006.
[20] Valencia A, Scarcelli G, D’Angelo M, Shih YH. Two-photon imaging with thermal light. Phys Rev Lett 2005;94:063601.
[21] Zhang D, Zhai YH, Wu LA, et al. Correlated two-photon imaging with true thermal light. Opt Lett 2005;30:2354–6.
[22] Liu X, Chen XH, Yao X, et al. Lensless ghost imaging with sunlight. Opt Lett 2014;39:2314–7.
[23] Yu H, Lu R, Han S, et al. Fourier-transform ghost imaging with hard X rays. Phys Rev Lett 2016;117:113902.
[24] Pelliccia D, Rack A, Scheel M, et al. Experimental X-ray ghost imaging. Phys Rev Lett 2016;117:113902.
[25] Chen KWC, O’Sullivan MN, Boyd RW. Optimization of thermal ghost imaging: high-order correlations vs. background subtraction. Opt Express 2010;18:5562–73.
[26] Li JH, Chen M, Gong W, et al. Mutual information of ghost imaging systems. Acta Opt Sin 2013;33:1211003.
[27] Martienssen W, Spiller E. Coherence and fluctuations in light beams. Am J Phys 1964;32:919–26.
[28] Chen H, Peng T, Shih YH. 100% correlation of chaotic thermal light. Phys Rev A 2013;88:023808.
[29] Miles J. $R^2$, adjusted $R^2$. Wiley StatsRef: Statistics Reference Online; 2014.
[30] Goodman JW. Introduction to Fourier optics. 2nd ed. McGraw-Hill; 1996.
[31] Witten IH, Frank E, Hall MA, et al. Data mining: practical machine learning tools and techniques. Amsterdam: Morgan Kaufmann; 2016.
[32] Luo RH, Huang BQ, Zheng WM, et al. Nonlocal imaging by conditional averaging of random reference measurements. Chin Phys Lett 2012;29:074216.
[33] Li JH, Luo B, Yang DY, et al. Modeling the behavior of signal-to-noise ratio for repeated snapshot imaging. Arxiv: 1603.00371.
[34] Li JH, Luo B, Yang DY, et al. Source coding model for repeated snapshot imaging. Arxiv: 1604.02515.
[35] Li JH, Yang DY, Luo B, et al. Binary sampling ghost imaging: add random noise to fight quantization caused image quality decline. Arxiv: 1702.08687.
[33] Zhong M, Xu P, Lu L, et al. Resolution of ghost imaging with entangled photons for different types of momentum correlation. Sci China-Phys Mech Astron 2016;59:670311.

[34] Cao H. von Neumann measurement-related matrices. Sci China-Phys Mech Astron 2016;60:020332.

[35] Sobieranski AC, Inci F, Tekin HC, et al. Portable lensless wide-field microscopy imaging platform based on digital inline holography and multi-frame pixel super-resolution. Light Sci Appl 2015;4:e346.

[36] Cui TJ, Liu S, Li LL. Information entropy of coding metasurface. Light-Sci Appl 2016;5:e16172.

[37] Yang Z, Magana-Loaiza OS, Mirhosseini M, et al. Digital spiral object identification using random light. Light Sci Appl 2017;6:e17013.

[38] Zhao R, Luo Y, Pendry JB. Transformation optics applied to van der Waals interactions. Sci Bull 2016;61:59–67.