i-process yields from multi-cycle evolution of rapidly-accreting white dwarfs for a range of metallicities

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ABSTRACT

Rapidly accreting white dwarfs (RAWDs) are characterized by stable H burning intermittent with strong He-shell flashes on their surfaces. We have modelled the multi-cycle evolution of these objects, within the metallicity range $-2.6 < [\text{Fe/H}] < 0$. We have calculated the nucleosynthesis yields of these models, and we confirm the activation of the intermediate neutron-capture process ($i$ process), as the dominant process for the production of heavy elements beyond Fe. The $i$ process occurs when convection driven by the He-shell flash ingests protons from the accreted H-rich surface layer, which results in maximum neutron densities $N_{n,\text{max}} \approx 10^{13} - 10^{15} \text{ cm}^{-3}$. The H-ingestion rate and the convective boundary mixing parameter $f_{\text{top}}$ adopted in the one-dimensional nucleosynthesis and stellar evolution models are constrained through 3D hydrodynamic simulations. We confirm our previous result that the high-metallicity RAWDs have a low mass retention efficiency ($\eta < \sim 10\%$), therefore it is highly unlikely that their masses will reach the Chandrasekhar limit. A new result is that RAWDs with $[\text{Fe/H}] < \sim -2.6$ have $\eta > \sim 20\%$, therefore they may eventually explode as SNeIa. This result and the good fits of the $i$-process yields from the metal-poor RAWDs to the observed chemical composition of the CEMP-r/s stars suggest that some of the present-day CEMP-r/s stars could be former distant members of triple systems, orbiting close binary systems with RAWDs that may have later exploded as SNeIa.

Key words: binaries: close – stars: abundances – stars: evolution – stars: interiors – supernovae: general

1 INTRODUCTION

The type Ia supernovae (SNeIa) are thermonuclear explosions of carbon-oxygen (CO) white dwarfs (WDs) (e.g. Hillebrandt & Niemeyer 2000; Hillebrandt et al. 2013; Churazov et al. 2014; Livio & Mazzali 2018). In the single degenerate (SD) channel of SNIa progenitors, that was originally proposed by Schatzman (1963) and Whelan & Iben (1973), it is assumed that the CO WD is a primary star of a close binary system with a main-sequence, subgiant or red-giant-branch companion. The secondary star fills its Roche lobe and donates matter from an H-rich envelope to the WD, and, as a result, the WD will explode when its growing mass $M_{\text{WD}}$ approaches the Chandrasekhar limit $M_{\text{Ch}} \approx 1.38M_\odot$. Initially, the primary star of such a binary system was an intermediate-mass star with $M \approx 2.5 - 7M_\odot$, the upper boundary of this mass interval depending on the amount of convective boundary mixing and C-burning rate (Chen et al. 2014). It left a core — the CO WD — after having lost the rest of its mass during a common-envelope event, when it arrived at the asymptotic-giant branch (AGB) and filled its Roche lobe. Given that the CO cores of AGB stars

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can grow in mass only up to $\sim 1M_\odot$ (Chen et al. 2014), the SD channel can work only if the accreted H-rich matter is first processed into He and then into C and O, while being efficiently retained on the WD.

Figure 9 of Nomoto (1982) summarizes the results of the previous investigations of H accretion onto CO WDs at different rates $M_{\text{acc}}$ (an update of this figure can be found in Nomoto et al. 2007). It shows that stable burning of accreted H occurs in a very narrow interval of $M_{\text{acc}}$ around a value of $\sim 10^{-7}M_\odot\text{yr}^{-1}$ that linearly increases with $M_{\text{WD}}$. At the lower rates, H is processed into He via thermal flashes that become stronger when $M_{\text{acc}}$ decreases, eventually leading to thermonuclear runaways typical for the classical novae. At the higher rates, the non-processed H accumulates in an expanding envelope, so that such a rapidly accreting WD would be resembling a red giant.

However, an important aspect of the evolution of rapidly accreting white dwarfs is the fact that even if accreted H is burning stably the He shell will ignite in a thermonuclear runaway when enough He has been accumulated as H-shell burning ash, just as in a thermal-pulse AGB star (Cassisi et al. 1998). A few consecutive He-shell flashes at the end of H accretion were computed by Idan et al. (2013). Following Nomoto (1982), various outcomes of pure He accretion onto CO WDs at different values of $M_{\text{acc}}$ have been studied in a number of papers (e.g. Piersanti et al. 2014; Wang et al. 2015). The data obtained in such simulations are used in models of binary population synthesis to estimate a theoretical SNIa rate for the SD channel, simply assuming that the He accretion rate matches that of H in the regime of stable H burning (e.g. Han & Podsiadlowski 2004).

Recently, Denissenkov et al. (2017a, hereafter Paper I) have presented the results of the first stellar evolution computations in which rapid accretion and stable burning of H on CO WDs were repeatedly interrupted by strong He-shell flashes that were followed by mass loss caused either by the super-Eddington luminosity wind or by the common-envelope event resulting from the expansion of the WD envelope overflowing its Roche lobe, after which the H accretion resumed. These simulations show a low efficiency of He retention ($\eta_{\text{hL}} \lesssim 10\%$) and, because all the processes that accompany the rapid H accretion were taken into account, they provide estimates of the low retention efficiency of the total accreted mass ($\eta \approx \eta_{\text{hL}}$) by the rapidly-accreting WDs (RAWDs)$^1$. In one of the models even a negative value of $\eta_{\text{hL}}$ was found meaning that $M_{\text{WD}}$ was decreasing with time. Given that the binary population synthesis models predict an order of magnitude lower SNIIa rates for the SD channel when optimistically assuming that $\eta_{\text{hL}} = 100\%$, this result of Paper I makes the SD channel highly unlikely. Fortunately, there are a number of alternative channels of SNIIa progenitors, various pros and cons of which are discussed by Wang (2018) and Livio & Mazzali (2018).

Paper I has proposed a new role for the former SD channel, namely, instead of growing in mass towards $M_{\text{Ch}}$ and exploding as SNeIa, the RAWDs could be a stellar site of the intermediate (i) process of neutron captures by heavy isotopes (Cowan & Rose 1977). This idea is based on the similarity of physics that one encounters in the RAWD models and in the model of the post-AGB star Sakurai’s object (V4334 Sagittarii) in which the elemental abundance signature of the i-process nucleosynthesis was observed by Asplund et al. (1999) and interpreted by Herwig et al. (2011). Indeed, in both cases the He-shell burning drives convection that ingests protons from an H-rich envelope. This triggers the reactions $^{12}\text{C} (p,\gamma)^{13}\text{N}$, $^{13}\text{N} (e^+\nu)^{13}\text{C}$ and $^{13}\text{C} (\alpha, n)^{16}\text{O}$, the first one taking place close to the top and the third one near the bottom of the He convective zone, and $^{14}\text{N}$ decaying into $^{13}\text{C}$ while being transported downwards.

The exact mechanism of how this convective flow operates in different cases is still under investigation. Sakurai’s object is believed to be a very-late thermal pulse, i.e. the He-shell flash occurred when the H-burning shell of the star had already turned off and the star had evolved around the knee in the HRD. Stellar evolution models predict an immediate split of the He-shell flash convection zone which would prevent any mixing of protons and the burning products of p capture on $^{12}\text{C}$ into the hottest bottom region of the pulse-driven convection zone. Herwig et al. (2014) have shown that in three-dimensional hydrodynamic simulations the H ingestion triggers a Global Oscillation of Shell H-ingestion (GOSH) which drastically rearranges the structure of the He-shell flash convection zone. So far it has not been possible to follow the long-term evolution of this event past the first GOSH. The one-dimensional, spherically symmetric nucleosynthesis simulations of Herwig et al. (2011) adopted the approach that mixing between the upper layer in which protons and $^{14}\text{C}$ react and the bottom layer in the pulse-driven convection zone, where neutrons can be released via the $^{14}\text{C} (\alpha, n)^{17}\text{O}$ reaction, continues until the observed abundance pattern has been reproduced. This delayed-split scenario is not yet fully supported by the initial three-dimensional hydrodynamic simulation results by Herwig et al. (2014). The initial GOSH in those simulations happens at a time when the amount of protons consumed is still insufficient to explain the neutron exposure required to explain observations, and the evolution past the initial GOSH in three dimensions remains unclear.

With an ongoing supply of protons, most of which are transformed into neutrons in the above reactions, the neutron number density at the bottom of the He zone can reach a value of $N_n \sim 10^{15}$ cm$^{-3}$ intermediate between the values characteristic of the $s$ ($N_n \lesssim 10^{11}$ cm$^{-3}$) and $r$ ($N_n \gtrsim 10^{20}$ cm$^{-3}$) process. Very-late thermal pulse objects, such as the post-AGB star Sakurai’s object, will only experience one H-ingestion event. Their nucleosynthesis production is very unique, but the impact on a galactic chemical evolution scale is negligible. RAWDs, on the other hand, continuously accrete from a close binary companion, and can potentially experience dozens of He-shell flashes followed by mass-loss episodes before the concomitant changes in the binary system parameters will terminate the rapid H accretion. Along with their low mass retention efficiency, this makes the RAWDs a potentially important galactic source of heavy elements, with distinct elemental and isotopic abundance signatures different from those produced in the $s$ and $r$ process. For example, $i$ process from RAWDs can make a sig-

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$^1$ We define an RAWD as a WD that accretes H rapidly enough for its stable burning to be maintained on the WD surface. This definition is slightly different from the one used by Lepo & van Kerkwijk (2013) whose RAWDs can have the higher accretion rates and lose mass via the optically thick wind.
nificant contribution to the first n-capture peak of the solar system abundance distribution, as demonstrated by combining RAWD i-process yields with stellar population synthesis and galactic chemical evolution models (Côté et al. 2018).

In Paper I, we have considered the rapid ($M_{\text{acc}} = 1-2 \times 10^{-7} M_\odot \text{yr}^{-1}$) accretion of only solar-composition matter onto CO WDs with the masses $0.65 M_\odot$, $0.73 M_\odot$ and $1 M_\odot$. The present work extends the set of our RAWD models to sub-solar metallicities, while keeping their masses close to $M_{\text{WD}} \approx 0.73 M_\odot$. The main goals of this paper are to describe the methods that we use to simulate the multi-cycle evolution of RAWDs (Section 2.1) and the i-process nucleosynthesis in their He-flash convection zones during H ingestion (Section 2.2), and to present the results of our new computations of the RAWD evolution and i-process yields (Section 4) that have been used in Côté et al. (2018). Section 3 describes the 3D hydrodynamic simulations that support our 1D estimates of H-ingestion rates for the RAWD models. Section 5 concludes the paper.

2 METHODS FOR ONE-DIMENSIONAL STELLAR EVOLUTION AND NUCLEOSYNTHESIS SIMULATIONS

2.1 The RAWD evolution

Our RAWD models are computed with the revision 7624 of the MESA stellar evolution code (Paxton et al. 2011, 2013). We use the reaction rates from the JINA Reaclib database (Cyburt et al. 2010) and the MESA default equation of state. The nuclear network includes 31 species, from neutron to $^{28}$Si, that can participate in 60 reactions of the pp chains, four CNO cycles, NeNa and MgAl cycles, as well as He and C burning. The initial mixtures of elements and isotopes are prepared using the solar-system chemical composition of Asplund et al. (2009) that is scaled to specified values of $[\text{Fe}/\text{H}]$. We assume that for $[\text{Fe}/\text{H}] \lesssim -0.7$ the initial mixtures are α-element enhanced with $[\alpha/\text{Fe}] = +0.4$. The appropriate Type 1 and Type 2 (with enhanced C and O abundances) OPAL and low-temperature molecular opacities have been prepared for such mixtures (Denisenkov et al. 2017b) and used in our computations.

The CO WD models are made with the inlists from the MESA test suite example make_co.wd. With these inlists the MESA code first computes the evolution of an intermediate-mass star from the pre-main sequence to the completion of the first He-shell thermal pulse on the AGB (blue curves in Figure 1, initial masses given in each panel), then the Blöcker AGB wind parameter (Blöcker 1995) is increased from 0.1 to 5 to mimic the enhanced mass loss caused by a common-envelope interaction in a close binary system in which the AGB star overflows its Roche lobe. As a result, the model star leaves the AGB and evolves towards and down the WD cooling track (green curves in the same Figure).

Our computations include convective boundary mixing (CBM) adopting the exponentially decaying diffusion model

\[ D(r) = D_0 \left( \frac{-2r - r_0}{H_P} \right), \]

where $D_0$ is a value of the convective diffusion coefficient provided by the mixing-length theory (MLT) and $H_P$ is the pressure scale height, both evaluated at $r = r_0$ in the vicinity of the respective convective boundary. For the boundaries of the H and He convective cores we have adopted the value of $f = 0.014$ that is close to the one constrained by the position of the terminal-age main sequence in a large number of stellar clusters (Herwig 2000). For the top and bottom boundaries of the He-flash convective zone we use the values of $f_{\text{top}} = 0.1$ and $f_{\text{bot}} = 0.008$ that are equal or close to those obtained in the multi-dimensional hydrodynamic simulations of the He-shell flash convection by Herwig et al. (2007). We provide further support from new hydrodynamic simulations for our choice of $f_{\text{top}}$ in §3. The value for $f_{\text{bot}}$ is consistent with a number of abundance observables of s-process elements and H-deficient stars (Herwig 2005; Werner & Herwig 2006; Battino et al. 2016).

When the CO WD model cools down to $\log_{10} L/L_\odot = -2$, we initiate a slow accretion of H-rich matter on it with $M_{\text{acc}} \approx 10^{-8} M_\odot \text{yr}^{-1}$ using the MESA mass-change control parameter in a new inlist with the same as before input physics. A higher value of $M_{\text{acc}}$ at this stage would result in MESA iterations having not converged. We assume that the accreted matter has the initial chemical composition of the binary. Because the accretion rate is now lower than the one required for stable H burning, our model exhibits mild H-shell flashes, each of them being followed by the expansion of its envelope. To stop the model from becoming a red giant, we enforce a mass loss by implementing the MESA super-Eddington wind prescription with an artificially reduced value of $L_{\text{Edd}}$, so that the model returns to the accretion phase and continues to make nova-like loops on the Hertzsprung-Russel diagram (a grey curve in Figure 1 that is shown only for model A). During the stable H burning at a higher accretion rate a RAWD remains near the high-$T_{\text{eff}}$ and high-$L$ “knee” of a nova loop (e.g. Wolf et al. 2013), therefore we switch the accretion rate to a higher value in a model that is located near the knee. We adjust a value for $M_{\text{acc}}$ that would guarantee stable H burning and allow relatively large time steps between consecutive evolutionary models.

The RAWDs spend most of their time stably burning the accreted H at the knees of nova loops, during which they should be seen as super-soft X-ray sources, unless being obscured by the ejected circum-binary matter (van den Heuvel et al. 1992; Lepo & van Kerckwijk 2013; Woods & Gilfanov 2016). Just as in thermal-pulse AGB stars, when a critical mass of He is accreted from the H-burning shell, a He-shell flash occurs causing an expansion of the accreted envelope (Figures 2a and 2f). Whereas the mass loss via the radiation-driven super-Eddington wind becomes less efficient at sub-solar metallicities, a fast and efficient mass loss can still be assumed for a star in a close binary system when it expands and overflows its Roche lobe. Therefore, a slightly modified MESA scheme for the Roche-lobe wind has been implemented in the present work to model the mass loss by our RAWD models during their expansion driven by the He-shell flashes. While the MESA code approximates the Roche-lobe mass-loss

\[^{2} \frac{A}{B} = \log_{10}(N(A)/N(B)) - \log_{10}(N_\odot(A)/N_\odot(B)), \]

where $N(A)$ and $N(B)$ are the abundances (number densities or mass fractions) of the nuclides A and B.
The evolutionary tracks of the progenitors of our RAWD models (blue and green) and the tracks of the multi-cycle RAWD evolution (orange). The grey curve in model A panel shows the nova-like evolutionary loops that are used to reach the starting point, at the high-\(T_{\text{eff}}\) and high-\(L\) knee of the nova loop, of the RAWD evolution modelling. Text in the panels indicates the models and the serial numbers of their He-shell thermal pulses (TP) that are chosen for the post-processing simulations of the \(i\)-process nucleosynthesis, the black curves showing their tracks. The other parameters of these models are listed in Table 1.

2.2 The RAWD \(i\)-process nucleosynthesis

The \(i\) process in an RAWD commences when the top of the He convective zone reaches the bottom of its H-rich surface layer, soon after the He-shell flash peak luminosity. At this moment, the He-shell convection begins to ingest protons (the left dotted line in Figure 2d). It should be noted that our \textsc{Mesa} computations find such H ingestion in all of our RAWD models even when \(f_{\text{op}} = 0\) (Paper I), i.e. even when convective boundary mixing at the top boundary of the He convective zone is not included (although the H ingestion rate \(\dot{M}_H\) does positively correlate with \(f_{\text{op}}\)).

The \(i\) process in our RAWD models is simulated in post-processing computations similar to those carried out...
to model the $\i$-process nucleosynthesis in Sakurai's object (Herwig et al. 2011; Denissenkov et al. 2018). These computations use the NuGrid multi-zone post-processing nucleosynthesis parallel code mppnp (Pignatari et al. 2016) customized for the H-ingestion He-shell problem. The input data for this code include a static or time-dependent structure of the He convective zone (this work uses the first option), i.e. the radius $r$, temperature $T$, density $\rho$, and convective diffusion coefficient $D_{\text{conv}}$ at each point of its mass mesh, the chemical compositions of the He zone and of the ingested matter, the mass ingestion rate $\dot{M}_{\text{ing}}$ and its duration $t_{\text{ing}}$.

Whereas the ingested matter has the initial chemical composition of the binary, the composition of the He convective zone at the beginning of H ingestion is obtained by processing the initial mixture through complete H burning followed by its processing via partial He burning, until the increasing C abundance matches its value from the corresponding MESA RAWD model, for which we use the NuGrid single-zone code ppn (cf. Denissenkov et al. 2018).

Our $\i$-process nucleosynthesis simulations include $\sim 1000$ isotopes and $\sim 15000$ reactions. The reaction rates for these simulations are taken from the same list of references as in Denissenkov et al. (2018). We adopt an equally spaced $\sim 100$-zone mass grid for the He shell region by interpolating the stellar structure variables to the new mesh. At each time step $\Delta t$, we add $X_k \dot{M}_{\text{ing}} \Delta t$ mass of the $k$th isotope from the envelope to the top $\Delta M = 1-4 \times 10^{-4} M_\odot$ of the He shell that occupies $\sim 10$ mass zones, as described in Appendix A.

### 2.3 Ingestion rates

The one-dimensional nucleosynthesis simulations require an ingestion rate $\dot{M}_{\text{ing}}$ and duration $t_{\text{ing}}$ as input. For $t_{\text{ing}}$, we adopt the values estimated from our MESA RAWD models and check that it does not exceed its upper limit constrained by the condition $t_{\text{ing}} < M_{\text{env}}/\dot{M}_{\text{ing}}$. $\dot{M}_{\text{ing}}$ is determined from a combination of constraints from three-dimensional simulations (see §3) and the MESA RAWD stellar evolution simulations.

Given the modelling choices described in the previous section the 1D stellar evolution simulations predict the in-
gestion of H-rich envelope material into the He-shell flash coordinate zone as it expands outward in Lagrangian coordinate. The protons ingested into the convection zone lead to a H-burning luminosity $L_H$ that reflects the mass ingestion rate through the relation $M_H = X_{\text{surf}} M_{\text{ing}}$, where $X_{\text{surf}}$ is the H mass fraction at the RAWD surface, and

$$M_H \approx \frac{L_H}{\epsilon_H},$$

where $\epsilon_H$ is the energy released per one gram of burned H.

Because only the first two reactions of the CNO cycle are fast enough to occur in the He-shell convective zone during a convective overturn time, we assume that $\epsilon_H = 0.667 \epsilon_{\text{CNO}}$, where $\epsilon_{\text{CNO}} \approx 6.3 \times 10^{18}$ erg g$^{-1}$ is the energy released per one gram of H transformed into He in the full CNO cycle, and the factor 0.667 is the fraction of this energy produced per one gram of consumed H in the reactions $^{12}$C(p,$\gamma$)$^{13}$N and $^{13}$N(e+$\nu$)$^{13}$C.

The resulting ingestion rate evolution is shown as an example for model C in Figure 2c, and its adopted time-average values used in the different simulations are summarized in Table 1.

3 INGESTION RATES AND CONVECTIVE BOUNDARY MIXING PARAMETERS FROM 3D HYDRODYNAMIC SIMULATIONS

The ingestion of material from the stable layer into the convection zone is the result of complex mixing processes at the convective boundary that may involve global, large-scale flow modes revealed in full 4\pi three-dimensional hydrodynamic simulations (Woodward et al. 2015). As mentioned in §2.1 we adopt in one-dimensional simulations the exponentially decaying CBM model with an efficiency parameter $f_{\text{top}}$ to describe this mixing.

Both the mass ingestion rate and the convective boundary parameter $f_{\text{top}}$ can be determined from hydrodynamic simulations, as demonstrated by Jones et al. (2017). For this purpose we have performed 3D hydrodynamic simulations of the He-shell flash convection zone and H ingestion in an RAWD using the PPMSSTAR code (Woodward et al. 2015). The radial stratification of simulations is based on model A from Paper I. We consider the point in time 7.44 hr after the beginning of the second He-shell flash, when the He luminosity has dropped to $4.10 \times 10^{10} L_\odot$ from its maximum value of $7.4 \times 10^{10} L_\odot$, and H ingestion into the He shell has just started.

The initial stratification of the 3D simulations follows the same approach as in Woodward et al. (2015) and approximates the 1D model with three polytropes: a lower stable layer (radial range 6 Mm < $r$ < 7.4 Mm), a convection zone (7.4 Mm < $r$ < 33.5 Mm), and an upper stable layer (33.5 Mm < $r$ < 50.0 Mm). We neglect radiation pressure, which contributes less than 25% to the total pressure in the 1D model, and we use the equation of state for a monatomic ideal gas. To obtain a similar overall stratification with the slightly different difference of state, we use a small mean molecular weight $\mu_1 = 0.3$ for the fluid $F_1$ initially filling the upper stable layer. The rest of the simulation domain contains fluid $F_2$ with $\mu_2 = 1.4$. The two fluids are allowed to react with each other via the $^{12}$C(p,$\gamma$)$^{13}$N reaction, assuming that $F_1$ contains 88.6% of protons and $F_2$ contains 20.4% of $^{12}$C by number. The subsequent decay of $^{13}$N is not considered. Convection in the He shell is driven by volume heating applied between the radii 7.9 Mm and 8.9 Mm.

The 3D simulations are done in 4\pi geometry on a Cartesian grid. We measure the luminosity dependence of the mass ingestion rate using runs E8, E13, and E15 (see Table 2), which cover a range of 1.4 dex in the driving luminosity $L_{\text{He}}$ at the grid resolution of 768$^3$, and one high-resolution run E10 (1536$^3$) with the same driving luminosity as its 768$^3$ equivalent E8.

Hydrogen ingestion starts as soon as the first upwelling plumes reach the upper convective boundary. After a few convective overturns a balance between hydrogen ingestion and burning is reached and the convective-reactive flow becomes quasi-stationary. We do not find a GOSH in these H-ingestion simulations which differs from the results found in the case of Sakurai’s object (Herwig et al. 2014). This corresponds to the result from 1D stellar evolution reported in Paper I and also found in most simulations here that the H-ingestion does not cause a split of the convection zone, with the exception of model A where a split happens in very late phases (see §4).

Figure 3 shows that the ingestion process is dominated by large scales. The average hydrogen luminosity $L_{\text{He}}$ reaches 2–3% of the driving luminosity $L_{\text{He}}$ (Table 2). Ingestion events localised in time and space can have a much stronger influence on the flow than these small values suggest, but they are not strong enough to launch a global ingestion instability such as the GOSH phenomenon observed by Herwig et al. (2014) in their 3D simulations of Sakurai’s object.

The amount of mass $M_{\text{ing}}(t)$ ingested into the convection zone by a time $t$ is the sum of mass $M_b(t)$ present in the convection zone at this time and mass $M_b(t)$ burnt in the convection zone by this time. The radius $r_{\text{ab}}$ of the upper boundary of the convection zone increases in time as a result of both mass ingestion and thermal expansion. We define $r_{\text{ab}}$ to be the radius at which the radial gradient of the rms horizontal velocity $v_h(r)$ reaches a local maximum and we integrate the density of the ingested fluid up to the radius $r_{\text{top}} = r_{\text{ab}} - H_{\text{c,ab}}$, where the velocity scale height $H_{\text{c,ab}} = (\partial v_h/\partial r)^{-1}$ is evaluated at $r_{\text{ab}}$. The subtraction of $H_{\text{c,ab}}$ mitigates issues related to the large contrast in the concentration of the ingested fluid between the boundary region and the bulk of the convection zone (for details, see Jones et al. 2017). To find the burnt mass $M_b(t)$, we compute the mass burning rate from spherically-averaged profiles of temperature, density, and fractional volume of ingested fluid at regularly-spaced points in time; $M_b(t)$ is then obtained by time integration. The resulting time dependence of $M_b$, $M_b$, and $M_{\text{ing}}$ in run E10 is shown in Fig. 4. We obtain the ingestion rate $M_{\text{ing}}$ by fitting a straight line to $M_{\text{ing}}(t)$.

Figure 5 shows that the ingestion rate scales in proportion to the driving luminosity $L_{\text{He}}$, in agreement with the results of Jones et al. (2017) on the O-shell convection in massive stars. This is likely to be caused by the fact that the ingestion rate is limited by the amount of work needed to be done to overcome the buoyancy of the ingested material, as argued by Spruit (2015). The ingestion rate in run E8 (768$^3$) is only 26% lower than in run E10 (1536$^3$) of the same luminosity, which provides an idea of the resolution dependence of these entrainment rate results.
The ingestion rates measured in 1D RAWD models B–F are close to the scaling relation established by the 3D hydrodynamic simulations, which may not have been expected given the numerous differences between the 1D and 3D models. For the RAWD models in Figure 5, we have used the He-shell luminosity $L_{\text{He}}^{\text{ing}}$ at the moment when the top of the He convective zone reaches the bottom of the H-rich envelope and, as a result of this, the H-burning luminosity quickly increases. These luminosities are somewhat lower than their corresponding peak He luminosities $L_{\text{He}}^{\text{max}}$ (Table 1 and panel d in Figure 2). In any case, the ingestion rates adopted for our nucleosynthesis simulations are consistent with the results obtained with the 3D hydrodynamic simulations in the sense that they are close to the scaling relation between the driving luminosity of the convection and the ingestion rate established by three 768-grid hydrodynamic simulations.

We can also obtain information on the convective boundary mixing efficiency at the top of the He-shell flash convection zone. As in Jones et al. (2017) we determine the $f$ parameter from the evolution of the spherically averaged radial profiles of the abundance of the H-rich fluid that is entrained into the He-shell flash convection zone (Figure 6). This is done by solving the inverted Lagrangian diffusion equation which gives the diffusion coefficient profile that would have been needed to advance from an abundance profile at a time $t$ to a profile at a time $t + \Delta t$ by means of a
1D diffusion process. The time difference $\Delta t$ is usually taken to be one or a few convective overturning time scales. The details of this procedure have been improved somewhat over the approach in Jones et al. (2017) and will be described elsewhere in detail. For the high-resolution E10 simulation we determine $f_{\text{top}} = 0.434$.

This value is larger than the one appropriate for our 1D simulations because the E10 hydrodynamic simulation has been performed at a driving luminosity that is 141 times higher than, for example, the He-burning luminosity in the stellar evolution run C ($\log L_{\text{He}} = 7.9$). In order to scale the $f$ value obtained in our higher-luminosity hydro simulation to the actual lower luminosity of the stellar evolution RAWD model we use the scaling relationship

$$f_{\text{top}} \propto L_{\text{drive}}^{1/3}.$$  \hspace{1cm} (4)

between the driving luminosity of a shell convection and the convective boundary mixing parameter $f_{\text{top}}$ at the top of the convection zone. This relationship has been derived from a series of new 1536-grid 3D hydrodynamic simulations of O-shell convection as in Jones et al. (2017), but with an improved version of the PPMSTAR code (Figure 7). These simulations, the $f$ determination and the resulting scaling law will be described in detail elsewhere. However, this relationship can be motivated within a simplistic picture in which the CBM $f$ parameter is a measure of how much convective plumes can deform the convective boundary and penetrate into it before being decelerated by their negative buoyancy upon entering the stably stratified regions. It would be the momentum of the convective plume that determines the level of penetration and boundary deformation. While the density perturbation of the plume would vary little with driving luminosity the convective velocity scales with one third power of the driving luminosity $L_{\text{drive}}$.

$$f_{\text{top}} \propto L_{\text{drive}}^{1/3}.$$  \hspace{1cm} (4)

$$f_{\text{top}} \propto L_{\text{drive}}^{1/3}.$$  \hspace{1cm} (4)

Applying relation 4 to scale $f_{\text{top}} = 0.434$ from the E10 driving luminosity of $\log L_{\text{He}} = 9.95$ to the driving luminosity of the stellar evolution model C in the middle of H ingestion ($\log L_{\text{He}} = 7.9$) we obtain $f_{\text{top}} = 0.09$. This provides strong support for the value $f_{\text{top}} = 0.10$ that we have adopted in the RAWD stellar evolution simulations which are all at a similar He-burning luminosity at the time of ingestion.

**Figure 5.** Ingestion rates as functions of He-burning luminosity that is driving convection according to the RAWD stellar evolution models and 3D hydrodynamic simulations. The black line is the $L_{\text{He}}$-scaling relation for the mass ingestion rate from the 3D hydrodynamic simulations that used the RAWD model A from Paper I for the initial setup. The filled black circles are the results of the individual $(768)^3$ hydro simulations and the filled black square is the result of the $(1536)^3$ hydro run (Table 2). The star symbols with adjacent horizontal dashed line segments are the average mass ingestion rates estimated for our 1D RAWD models as described in Section 4.2, while the arc curves of the same colors show smooth fits to the actual variations of $\dot{M}_{\text{ing}}$ with $L_{\text{He}}$ in these models. The star symbols are located at $L_{\text{He}} = L_{\text{He}}^{1/3}$ that corresponds to the beginning of H ingestions in 1D models (Table 1).

**Figure 6.** Determination of the $f_{\text{top}} = f_{\text{CBM}}$ mixing parameter from the high-grid-resolution 3D hydrodynamic simulation E10.

**Figure 7.** Scaling relation of the convective boundary mixing parameter $f_{\text{top}}$ versus the driving luminosity based on a series of 1536-grid simulations of O-shell convection with the same setup as in Jones et al. (2017).
According to these models the SD channel of SNIa progenitors (the last row in Table 1) compared to models E and F. Accordingly, the solar-metallicity RAWD models was computed the evolution of seven RAWD models with the metallicity and WD masses listed in Table 1 along with other model parameters. The evolutionary tracks of six of these models are shown in Figure 1, where their initial masses are also indicated. Except the solar-metallicity model A, we have simulated the RAWD evolution for many cycles, typically more than five (orange curves in Figure 1). The multi-cycle evolution of the solar-metallicity RAWD models was discussed in Paper I. Black curve segments in Figure 1 highlight the relatively short-lasting evolutionary phase of the He-shell thermal pulse (TP) whose serial number is specified for each model and that has been chosen for the \( i \)-process post-processing nucleosynthesis computations in Section 4.2.

All of the RAWD models have nearly the same initial central temperature with \( \log_{10} T_\odot \approx 7.2 \) and use the same WD Roche-lobe radius \( R_{\text{RL,WD}} = 2R_\odot \) in Equation 2 that corresponds to the orbital period \( P \approx 1.2 \text{ days for a secondary mass of } \sim 2M_\odot \).

The new computations of the RAWD multi-cycle evolution confirm the conclusion about their low mass retention efficiency (the last column of Table 1 and Figure 8) made in Paper I, at least for \([\text{Fe/H}] \gtrsim -2\). A new result is that \( \eta \) increases when \([\text{Fe/H}] \) decreases below \(-2\) to fractions of \( \approx 20\) to \(30\)% in models E, F and G. The 0.75\(M_\odot \) RAWD model with \([\text{Fe/H}] = -2.6\) (model G) has the highest value of \( \eta = 29\% \) and a lower He peak luminosity of \( \log_{10} L_{\text{He,peak}} = 8.5 \) (the last row in Table 1) compared to models E and F. According to these models the SD channel of SNIa progenitors still works at very low metallicities.

The revealed trend of \( \eta \) with \([\text{Fe/H}]\) is probably caused by the lower opacity of the metal-poor accreted matter. It allows the accumulated He layer to cool down, and therefore be compressed by the gravity more efficiently. As a result, the He flash starts at a lower mass of the He shell and achieves a lower peak luminosity, which is reflected in the different amplitudes of the RAWD mass changes in the models with \([\text{Fe/H}] < -1.55\) (E and F) compared to the models with \([\text{Fe/H}] \geq -1.55\) (B, C and D) in Figure 8, and in their different values of \( \log_{10} L_{\text{He,peak}}/L_\odot \) (Table 1).

Besides, during the expansion of the RAWD following the He-shell flash its envelope cools down faster, because of the lower opacity, in the low-metallicity models, and the RAWD returns to the accretion phase after having lost a smaller fraction of the accreted matter.

As for the RAWD models with \([\text{Fe/H}] \geq -1.55\), they stubbornly want to expand to red-giant dimensions and it is only because of our implementation of the Roche-lobe mass loss that they can expand only up to \( R \approx R_{\text{RL,WD}} \) (Figure 2a) and retain this radius until a significant fraction of the accreted matter is gone with the wind. To test if this behaviour is affected by our choice of the MESA mass-loss algorithm, we have switched to the super-Eddington wind prescription in model C, enforcing it to work only when \( R \approx R_{\text{RL,WD}} \). With this modification, we have reproduced the results obtained for model C with the Roche-lobe mass-loss prescription.

We have also addressed the frequently raised concern that a much larger number of He-shell flashes than we have simulated in our RAWD models could lead to significant changes in the RAWD multi-cycle evolution. To test this, we have extended the computed number of the He-shell flashes in model C up to 42 from the initial 10. The results of this long-run simulation are plotted in Figure 9. Its top panel demonstrates that the WD central temperature has increased only by 6%. Our analysis of the RAWD mass variations presented in the middle panel shows that the mass retention efficiency has varied between 5.8% and 8.8% in this run, still remaining below 10%, as it was in the initial 10 cycles. Finally, the bottom panel reveals that the He peak luminosity has not changed at all.

### 4 RESULTS

#### 4.1 The RAWD multi-cycle evolution

Using the methods described in Section 2.1, we have computed the evolution of seven RAWD models with the metallicities and WD masses listed in Table 1 along with other model parameters. The evolutionary tracks of six of these models are shown in Figure 1, where their initial masses are also indicated. Except the solar-metallicity model A, we have simulated the RAWD evolution for many cycles, typically more than five (orange curves in Figure 1). The multi-cycle evolution of the solar-metallicity RAWD models was discussed in Paper I. Black curve segments in Figure 1 highlight the relatively short-lasting evolutionary phase of the He-shell thermal pulse (TP) whose serial number is specified for each model and that has been chosen for the \( i \)-process post-processing nucleosynthesis computations in Section 4.2.

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#### 4.2 The RAWD \( i \)-process nucleosynthesis yields

Before presenting the yields, we will describe how we choose the input data for the RAWD \( i \)-process nucleosynthesis computations. For this purpose, we use model C (Figure 2), as an example. We begin with selecting a representative RAWD He-shell flash that does not stands out, therefore this cannot be the first He-shell flash that usually is stronger than the others. The only exception to this rule in the present paper is the solar-metallicity model A. We find it to be much more difficult to compute multiple cycles with standard He burning interrupted by strong He-shell flashes at a high metallicity, probably because of the high opacity of the envelope matter. Because the multi-cycle evolution of the solar-metallicity RAWD models has already been discussed in Paper I, and the \( i \)-process yields calculated for the first He-shell flash in the solar-metallicity RAWD model in this work are similar to those presented in Paper I, we have followed only one He-shell flash in the new model A. As for model C, we have chosen the 6th flash (panels a, b and c in Figure 2). We have used its corresponding H-burning luminosity (panel d) to estimate the H-ingestion rate with Equation 3 (the purple curve in panel e). The dashed line in panel e is our estimate of an average \( \dot{M}_H \) value for this model that was used to calculate the parameter \( \dot{M}_{\text{ing}} = (\dot{M}_H/X_{\text{surf}}) \) listed in Table 1. The vertical dotted lines in panel e constrain \( \dot{M}_{\text{ing}} \). The left line is chosen close to the beginning of H ingestion. The position of

\[ \frac{M_{\text{RAWD}}}{M_\odot} \]

\[ \text{time (10^4 yr)} \]

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Figure 8. The changes of the total mass of our RAWD models caused by the accretion and Roche-lobe mass loss.

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the right line is less certain. It marks the beginning of the fast decline of $M_H$. The H-ingestion time in the RAWD models is usually less than one month ($t_{\text{ing}} < 0.08$ yr), except the first phase of H-ingestion in model A. Like in the solar-metallicity model A from Paper I, the updated model A has two phases of H-ingestion, the longer-lasting slow-ingestion phase with $M_{\text{ing}} = 2.2 \times 10^{-12} M_\odot$ s$^{-1}$ and $t_{\text{ing}} = 0.17$ yr followed by the shorter fast-ingestion phase with $M_{\text{ing}} = 3.5 \times 10^{-11} M_\odot$ s$^{-1}$ and $t_{\text{ing}} = 0.024$ yr, both included in our nucleosynthesis computations.

It usually takes about hundred time steps for the MESA code to evolve a RAWD model through the entire H-ingestion phase, meaning that the time interval between two consecutive models on this evolutionary phase is hundreds of minutes, corresponding to tens of convective turn-over times of the He-shell flash convection. This is too long for the $i$-process nucleosynthesis simulations that usually require a time step of the order of minutes (Herwig et al. 2011). We cannot reduce the MESA time steps, because the mixing-length theory (MLT) adopted in MESA to describe convection is formulated in terms of time and spatial averages. Time steps smaller than about ten times the convective turnover time would violate this assumption. Therefore, we simply take the temperature, density, radius and MLT convective diffusion coefficient profiles from a MESA model in the middle of the H-ingestion phase, when the accreted envelope is already expanding (e.g., the dashed $T$ profile in Figure 2f), and use them to set up our post-processing nucleosynthesis simulations. Note, that Herwig et al. (2011) were able to reproduce the surface abundances of heavy elements measured in Sakurai’s object by Asplund et al. (1999) using a 1D model similar to this one.

The post-processing simulations of the $i$-process nucleosynthesis were carried out using the methods described in

Figure 9. Central temperature, total RAWD mass and H- and He-burning luminosity as a function of time for the long-term multi-cycle evolution of model C.

Figure 10. The evolution of the maximum neutron number density in the He convective zones of our RAWD models. In model A, the jump in the evolution of $N_{n,\text{max}}$ at the end is caused by the switching to the second phase of H-ingestion that is shorter but faster than the previous phase (see text).

Figure 11. The elemental $i$-process yields (solar-scaled mass fractions) from our RAWD models. For comparison, the abundances of the first peak n-capture elements measured in Sakurai’s object by Asplund et al. (1999) are shown as filled black circles with errorbars. They were interpreted by Herwig et al. (2011) as results of the $i$-process nucleosynthesis in the convective He shell during its very late thermal pulse in a model of Sakurai’s object with a half-solar metallicity.
The nucleosynthesis simulations provide the abundance distributions in the He convective zone and the surface abundances at the top of the He shell for the selected (≈1000) isotopes for $0 \leq t \leq t_{*nug}$. At the end, we allow the surface abundances to decay for 1 Gyr. Figure 10 shows the evolution of the maximum neutron number density in the He shell for our RAWD models. The peak value of the $N_{n, \text{max}}(t)$ curves increases with a decrease of $[\text{Fe/H}]$ because the total mass fraction of the isotopes that capture neutrons decreases with the metallicity, while the production of neutrons, that is controlled by the $^{13}\text{C}$ abundance in the He shell and the total amount of ingested H, remains approximately the same. In other words, the neutron source is primary, while the $i$-process seeds are secondary. The steep increase of $N_{n, \text{max}}$ at the end of its evolution in model A marks the beginning of the short fast-ingestion phase that appears to be common for solar-metallicity RAWDs (Paper I). It demonstrates that $N_{n, \text{max}}$ increases with $M_{\text{ing}}$ for models with the same metallicity.

Figure 11 shows the final surface elemental abundances divided by the solar abundances from Asplund et al. (2009) of the RAWD models. As in the $s$ process in AGB stars the global heavy-element distribution shifts to higher mass elements at lower metallicity (Clayton 1968; Busso et al. 2001). This is due to the primary nature of the $^{13}\text{C}$(α,n) neutron source, for both the $^{13}\text{C}$ pocket and the $i$ process following H ingestion events, and the secondary Fe seeds. Therefore, generally speaking, we expect to see local elemental $i$-process signatures in higher-mass second- and third-peak species at lower metallicity, whereas the $i$-process signature may be most prominently detected in lower-mass, first-peak elements at higher, solar-like metallicities. Accordingly, it had been proposed in Paper I that the solar-metallicity RAWDs could be contributors of first-peak elements to the solar system abundance distribution. Côté et al. (2018) have used our RAWD $i$-process yields in a framework that included Galactic chemical evolution and binary-star population synthesis models to confirm this hypothesis.

An example for solar-like metallicity $i$-process abundances are those of Sakurai’s object which indeed show large enhancements in the first peak $n$-capture elements (around $Z = 40$) as shown in Fig. 11. The details of the abundance patterns of Sakurai’s object are however better reproduced by the very-late thermal pulse post-AGB star models of Herwig et al. (2011) which feature peak neutron densities and H ingestion rates that are both more than two orders of magnitude higher compared to the RAWD models.

### 4.3 A possible relation of the metal-poor RAWDs to the CEMP-r/s stars

An example for $i$-process abundance patterns in the second-peak elements expected at lower metallicity may be found in the subclass of carbon-enhanced metal-poor (CEMP) stars with abundance patterns that appear to be enhanced with both $r$- and $s$-process elements (e.g. Beers & Christlieb 2005; Masseron et al. 2010; Lugaro et al. 2012; Bisterzo et al. 2012). Dardelet et al. (2014) and Hampel et al. (2016) have done one-zone nucleosynthesis simulations of $i$-process conditions to demonstrate that the abundances of heavy elements observed in these CEMP-r/s stars can be reproduced by an $n$-capture process with neutron densities $N_{n} = 10^{12} - 10^{15} \text{cm}^{-3}$.

As an example, Figure 12 shows the best $\chi^2$ fit of the $i$-process elemental yields from our RAWD model G to the surface chemical composition of the CEMP-r/s star CS31062-050. The only exception to the otherwise excellent agreement is the discrepant Ba abundance that requires a further investigation. For other CEMP-r/s stars a good agreement with the RAWD $i$-process yields can be found as well. The low-metallicity RAWD models are at this point the first and only complete models in which nucleosynthesis calculation directly post-processing complete stellar evolution models can reproduce the complete abundance patterns observed in CEMP-r/s stars. We therefore propose that CEMP-r/s stars that have been well reproduced with $i$-process models should be referred to as CEMP-i stars.

Our findings suggest a new scenario for the formation of CEMP-r/s, or in this case CEMP-i stars, that takes into account our finding that the RAWDs with $[\text{Fe/H}] \lesssim -2$ may reach the Chandrasekhar mass and explode as SNe Ia. It is based on the fact that the mass retention efficiency of our RAWD models significantly increases when $[\text{Fe/H}]$ decreases below $-2$ (the last column of Table 1), and it is supported by our calculations of the evolution of binary-star parameters, results of which are presented in Figures 13 and 14. For these calculations, we have used the isotropic re-emission model of mass transfer in which a fraction $\beta$ of matter accreted by the primary star (RAWD) is lost from the binary system. In our case, $\beta = 1 - \eta$, where $\eta$ is the mass retention efficiency. This model and its equations are described by Postnov & Yungelson (2014) in their Section 3.3.3. Our Figures 13 and 14 show the evolution of the semi-major axis $a$, the mass ratio $q = M_1/M_2$, and the RAWD mass $M_1$ for two sets of the binary initial parameters. Figure 13 corresponds to the case of $\eta = 10\%$ that includes the RAWD models with $[\text{Fe/H}] \gtrsim -2$, while Figure 14 demonstrates a possible evolution of the binary-system parameters for RAWDs with $[\text{Fe/H}] \lesssim -2$. In both cases, we have stopped the calculations at $q = 2$.

In the second case, $M_1$ reaches the Chandrasekhar limit and the RAWD ends its life as a SNIa. If this is true, then some of the present-day CEMP-i stars could be former tertiary members of triple systems in which they had been orbiting a close binary system with a RAWD. A series of dozens He-shell flashes on the RAWD, each being followed by the RAWD expansion and mass loss, could enrich the tertiary star with the products of $i$-process nucleosynthesis. Finally, when the RAWD exploded as SNIa, the tertiary star, that became a CEMP-i star by that moment, would leave the triple system. Those CEMP-i stars would not be binaries anymore. If the RAWD does not explode as a SN Ia the CEMP-i star would be possibly in a wider orbit around compact binary and would show a long binary period superimposed with the very short period of the compact RAWD binary. Thus, CEMP-i stars can be both single stars and binaries in this scenario.

This scenario can potentially explain the observational...
bias against finding CEMP-r/s stars in globular clusters that may be caused by the destruction of wide triple systems through close star encounters in dense cores of globular clusters. Our scenario differs from the triple-system scenario for the formation of CEMP-r/s stars discussed by Abate et al. (2016), in which a primary massive star was assumed to produce the r-process elemental abundances during its SN explosion and the presence of a secondary AGB star was required to make s-process elements.

Figure 12. Abundances of heavy elements observed in the CEMP-r/s star CS31062-050 (Aoki et al. 2002; Johnson & Bolte 2004) and the best-fit abundance distribution from the time evolution of the RAWD model G diluted with 99.58% of the initial abundances.

Figure 13. The evolution of the binary-system parameters, the semi-major axis a, the mass ratio q = M1/M2, and the RAWD mass M1, for the isotropic re-emission model of mass transfer in which \( \beta = 1 - \eta \), where \( \eta \) is the RAWD mass retention efficiency. This set of the initial binary parameters corresponds to the RAWD models with [Fe/H] \( \gtrsim -2 \) that have \( \eta \lesssim 10\% \). The calculations stop when \( q = 2 \).

Figure 14. Same as in Figure 13, but for the RAWD models with [Fe/H] \( \lesssim -2 \) that have \( \eta \geq 20\% \). In this case that assumes \( \eta = 30\% \), the RAWD mass may reach the Chandrasekhar limit.

5 CONCLUSIONS

We have used the MESA stellar evolution code (Paxton et al. 2011, 2013) to compute the models of CO white dwarfs (WDs) rapidly accreting H-rich matter, assuming this matter is donated by normal (main-sequence, sub-giant or red-giant-branch) components of the WDs in close binary systems. Such stellar configurations can result from common envelope events, after the primary AGB star components fill their Roche lobes, loose almost entire envelopes above the WD cores in unstable mass transfers, and the binary systems become tighter via the transformation of their orbital energies into the kinetic energy of the ejecta. When the secondary components of these post-common-envelope systems expand and fill their own Roche lobes by or after the end of the main-sequence evolution, they begin to donate H-rich matter to their, by this time cooled-down, WD primary components. We assume that the mass accretion rate is rapid enough, \( \dot{M}_{\text{acc}} \sim 10^{-7} \ M_{\odot} \ yr^{-1} \), for the accreted H to be stably burning on the WD surface, resulting in the accumulation of a He shell. When its mass reaches a critical value, the He shell will experience a thermal flash in which some fraction of He will be transformed into C. If the sequence of the stable H burning intermittent with the He-shell flashes is not accompanied by a significant mass loss by the WD, then its mass may eventually reach the Chandrasekhar limit, \( M_{\text{Ch}} \approx 1.38 M_{\odot} \), and the CO WD will explode as a SNIa (Figure 14). This is the classical single degenerate (SD) channel of SNIa progenitors (Schatzman 1963; Whelan & Iben 1973).

In this work, the evolution of such rapidly-accreting WDs (RAWDs) has for the first time been followed through multiple strong He-shell flashes. Each He-shell flash leads to the expansion of the WD envelope that overflows the WD Roche lobe. We assume that the matter leaving the WD Roche lobe is lost from the binary system because of its interaction with the secondary component, like in the common-envelope event. The important new result found in our simulations is that the WD envelope remains inflated at the WD Roche-lobe radius until its significant fraction is lost, after which it shrinks, and the mass accretion onto the
WD resumes. As a result, we have obtained relatively low mass retention efficiencies, $\eta \lesssim 10\%$, for our RAWD models, at least for the metallicities $[\text{Fe/H}] \lesssim -2$ (Table 1). It is unlikely that the masses of such RAWDs will ever approach the Chandrasekhar limit. Therefore, the SD channel should not work in these cases. As for the RAWD models with $[\text{Fe/H}] \lesssim -2$, they are found to have $\eta \gtrsim 20\%$ and, therefore, we cannot exclude that their masses will ultimately reach the Chandrasekhar limit.

The ignition of He at the bottom of the He shell triggers convection. When the top of the He-flash convective zone reaches the bottom of the H-rich envelope, convective boundary mixing (convective shear mixing and convective overshooting) starts to ingest protons into the He shell. There, they are quickly captured by the abundant $^{12}\text{C}$ nuclei producing the unstable $^{13}\text{N}$ that has the lifetime of $\sim 10$ minutes and decays into $^{13}\text{C}$ while being transported by convection downwards. When $^{13}\text{C}$ nuclei arrive at the bottom of the He shell they capture $\alpha$ particles releasing neutrons. This convective-reactive process transforms almost every proton ingested at the top of the He shell into a neutron at its bottom, therefore at sufficiently high mass ingestion rates, $M_{\text{ing}} \sim 10^{-12} - 10^{-11} M_\odot$ s$^{-1}$, the neutron density can be as high as $N_{n,\text{max}} \sim 10^{12} - 10^{14}$ cm$^{-3}$. These values are intermediate between those characteristic of the s and r processes, therefore the ensuing n-capture process is called the i process (Cowan & Rose 1977).

The important input parameters — the ingestion rate and convective boundary mixing efficiency — have been determined and constrained through a series of three-dimensional hydrodynamic simulations of the RAWD He-shell flash convection. The estimates of the mass ingestion rate obtained from 1D RAWD models are consistent with the 3D hydrodynamic simulations. The convective boundary mixing efficiency parameter adopted in our stellar evolution simulations at the top of the He-shell flash convection boundary are in agreement with the luminosity scaling law for the CBM $f_{\text{top}}$ parameter presented here.

It is interesting that the i-process nucleosynthesis yields predicted by our metal-poor ($[\text{Fe/H}] \lesssim -2$) RAWD models almost perfectly fit the abundances of heavy elements measured in some CEMP-$r/s$ stars (Figure 12). These are the CEMP-i stars. Given that the same RAWD models have the higher mass retention efficiencies and can potentially become SNeIa, we propose that CEMP-i stars used to be distant systems with RAWDs. When the RAWD exploded as a SNIa, the tertiary star, polluted by the products of i-process nucleosynthesis that had taken place on the RAWD, got loose from the system and is now seen as a single star. If the RAWD has not exploded yet, the CEMP-i star can still be a member of a triple system, and would then show signs of binarity.

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The change of the average mass fraction of the $k$th isotope in the ingestion zone is therefore

$$\Delta \langle X_i^k \rangle = \frac{\Delta M_{\text{ing}}^k}{\Delta M} = \frac{X_i^k(M_{\text{max}}) - X_i^k(M_{\text{min}})}{\Delta M} \Delta M_{\text{ing}} + \frac{1}{2} \left( X_i^k(M_{\text{max}}) - X_i^k(M_{\text{min}}) \right) \left( \frac{\Delta M_{\text{ing}}}{\Delta M} \right)^2.$$

This change can be applied either as a step increase of the previous linear distribution

$$\widetilde{X}_i^k(M) = X_i^k(M) + \Delta \langle X_i^k \rangle,$$

or as a ramp increase

$$\widetilde{X}_i^k(M) = X_i^k(M) + \frac{2(M - M_{\text{min}})}{\Delta M} \Delta \langle X_i^k \rangle$$

for $M_{\text{min}} \leq M \leq M_{\text{max}}$. In the present work, we have chosen the latter option.

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APPENDIX A: THE IMPLEMENTATION OF MASS INGESTION IN RAWD MODELS

Let us denote $X_i^k$ and $X_i^k$ the mass fractions of the $k$th isotope, respectively, inside and outside the convective He shell, in the vicinity of its outer boundary. We assume that the envelope matter ingested into the He shell gets immediately distributed within an ingestion zone with the mass $\Delta M < M_{\text{top}} - M_{\text{bot}}$, where $M_{\text{bot}}$ and $M_{\text{top}}$ are the mass coordinates of the bottom and the top of the He shell. The input parameters here are $X_i^k$, $\Delta M$, and the mass ingestion rate $\dot{M}_{\text{ing}}$. Because of a small size of the ingestion zone, we assume that all the isotopes are linearly distributed in it, i.e.

$$X_i^k(M) = X_i^k(M_{\text{max}}) - \frac{(M_{\text{max}} - M)}{\Delta M} [X_i^k(M_{\text{max}}) - X_i^k(M_{\text{min}})],$$

where $M_{\text{min}} \leq M \leq M_{\text{max}}$, $M_{\text{max}} = M_{\text{top}}$, and $M_{\text{min}} = M_{\text{bot}} - \Delta M$. The total mass of the $k$th isotope in this distribution is

$$\Delta M_i^k = \int_{M_{\text{min}}}^{M_{\text{max}}} X_i^k(M) dM = \frac{1}{2} [X_i^k(M_{\text{max}}) + X_i^k(M_{\text{min}})] \Delta M.$$

After $\Delta M_{\text{ing}} = \dot{M}_{\text{ing}} \Delta t$ of the envelope matter is ingested, the mass of the $k$th isotope in the ingestion zone becomes

$$\Delta M_i^k = \Delta M_i^k + X_i^k \Delta M_{\text{ing}} - \delta M_i^k,$$

where

$$\delta M_i^k = \int_{M_{\text{max}} - \Delta M_{\text{ing}}}^{M_{\text{max}}} X_i^k(M) dM = X_i^k(M_{\text{max}}) \Delta M_{\text{ing}} - \frac{1}{2} \frac{\Delta M_{\text{ing}}}{\Delta M} [X_i^k(M_{\text{min}}) - X_i^k(M_{\text{max}})]$$

is the mass of the $k$th isotope in the mass $\Delta M_{\text{ing}}$ that replaces the ingested mass outside the He shell (we assume that the mixing between the He shell and the envelope does not change the chemical composition, $X_i^s$, of the latter.)