Quantum Zeno Dynamics and Inhibition of Geometric phase

Arun Kumar Pati and Suresh V. Lawande

Theoretical Physics Division, 5th Floor, Central Complex,
Bhabha Atomic Research Centre, Mumbai - 400 085, INDIA.

(August 3, 2018)

Quantum Zeno dynamics refers to the unitary evolution of a quantum system interrupted by a sequence of measurements. We investigate the effect of sequence of measurements on the geometric phase under quantum Zeno dynamics and show that it can inhibit the development of the geometric phase under a large number of measurement pulses. We show that the path dependent memory of a system can be erased by a sequence of measurements, a result that can be tested by neutron and photon interference experiments.

PACS NO: 03.65.Bz
email:apati@apsara.barc.ernet.in

Quantum theory is confronted with two types of basic dynamical evolutions, namely, one is the unitary, reversible evolution leading to deterministic solutions and other is non-unitary, irreversible evolution leading to probabilistic predictions. The former is referred to as Schrödinger evolution and the latter corresponds to von Neumann’s collapse mechanism in the standard ‘Copenhagen interpretation’ of quantum theory. Although these processes cannot be reconciled they can be allowed to compete and when they do, some interesting effects occur. Unitary evolution creates a linear superposition of all possible eigenstates and measurement leads to the collapse of the linear superposition to a particular eigenstate corresponding to the eigenvalue of the observable being measured. On the other hand, if the unitary evolution is interrupted successively at short time intervals by measurements (we call such a dynamics as quantum Zeno dynamics (QZD)) the quantum system continues to remain in its initial state. This inhibition of quantum transition under frequent observations in rapid succession is called quantum Zeno effect (QZE). Misra and Sudarshan [14] were the first to predict the QZE for an unstable quantum system resulting in the inhibition of decay. There were various proposals [2,3] to test the prediction of QZE. Following a recent proposal of Cook [4], Itano et al [5] have verified the inhibition of a quantum transition in Be\(^+\) ions which are laser-cooled and trapped in a Penning trap. Such a system provides a clean environment to test fundamental quantum predictions.

In quantum theory it is known that if we prepare a system in the state \(\Psi_i\) and a measurement is performed to see if it is in a state \(\Psi_f\) at a later time, then the probability of transition from \(\Psi_i\) to \(\Psi_f\) is given by the modulus square of the transition amplitude. The transition amplitude between two non-orthogonal states is given by \(\langle \Psi_i | \Psi_f \rangle = \langle \Psi_i | \Psi_f \rangle |\exp(i\Phi_{if})|\), which has a modulus and a phase. Both of them have physical meaning. However, the issue of phase is always very subtle in the quantum world and in recent years our understanding of various phases (dynamical phases, geometric phases, topological phases etc.) has been advanced considerably. Under QZD the transition probability is affected giving rise to QZE. It is pertinent here to ask an intriguing question: what happens to the phase under QZD? Specifically, we will be concerned with the evolution of the geometric phase under QZD. There are some deep reasons for this. First, in quantum theory the physical quantity of interest should not
depend on a particular choice of gauge. The transition probability and geometric phase are in fact gauge invariant under $U(1)$ action of the state vectors. Second, both these quantities are ray space objects and hence they are on the same footing. It is thus natural to study the behaviour of the geometric phase and not all other phases under QZD.

The phase $\Phi_{i,f}$ is quite general. If the final state has been obtained according to unitary and reversible dynamics, then the transition amplitude is $<\Psi_i|\Psi_f> = <\Psi_i|U(t_f-t_i)|\Psi_i>$. The unitary evolution in general introduces non-cyclic dynamical phase and geometric phase \cite{15}. On the other hand if the state of the system follows non-unitary evolution due to sequence of measurements of non-commuting observables (hence the corresponding projectors also do not commute) then the system acquires a pure geometric phase. This type of geometric phase was first encountered by Pancharatnam \cite{6} but later discussed in the quantum measurement context by Samuel and Bhandari \cite{7}. The important point here is that if the successive measurements are carried out along shortest geodesics the $|\Psi_i>$ is in phase with $|a>$ and $|a>$ is in phase with $|b>$ but $|b>$ need not be in phase with $|\Psi_i>$. This non-transitivity of phase preserving relation is often responsible for giving rise to geometric phase \cite{8}.

Therefore, it is quite clear that unitary processes introduce dynamical as well as geometric phases and projections (measurements) introduce only geometric phase. What is the situation when both the evolutions are present? In this letter we investigate the behaviour of geometric phase when unitary evolution is interrupted by successive measurements of the von Neumann type. We predict that the geometric phase of the quantum system is also inhibited like the transition probability itself under quantum Zeno dynamics. We may refer to this effect as quantum Zeno phase effect (QZPE). Incidentally, this issue was examined by the present authors \cite{9} earlier based on a specific model of continuous quantum measurements \cite{10,11}. The dynamical equation of \cite{9,11} was non-unitary thereby invoking the irreversibility of the measurement process but in a sense the description was beyond von Neumann’s (orthodox) quantum mechanics. Another limitation was that one measures observables which commute with the Hamiltonian of the system. The present proof does not assume this restricted class of observables. Also our earlier prediction was difficult to implement in laboratory whereas the present results can be tested by means of suitable interference experiments with neutrons or photons.

The geometric phase that we are concerned with here is quite general, going beyond the one that was first discovered by Berry \cite{12} in his pioneering work on quantum adiabatic theorem. The Berry phase basically attributes a ‘memory’ to the quantum system because it depends purely on the geometry of the evolution path thereby remembering its history. This concept was further generalised to no-adiabatic, cyclic evolutions by Aharonov and Anandan \cite{13}. Further, it was generalised to non-adiabatic, non-cyclic evolutions by Samuel and Bhandari \cite{7}. A kinematic and group theoretic approach to geometric phase was provided by Mukunda and Simon \cite{14}. More recently, it is shown by one of the present authors \cite{15} that the geometric phase also arises for non-adiabatic, non-cyclic, non-unitary and non-Schrödinger evolutions of quantum systems. By considering sequence of incomplete measurements Anandan and Pines \cite{17} have obtained geometric phase. Bhandari \cite{18} has argued that in Einstein’s gedanken experiment for which path information in a double-slit interference experiment the geometric phase plays the role of a random phase which washes out the interference pattern. Recently, Aharonov et al \cite{19} have shown that the back reaction due to measurement process can induce a gauge potential leading to an observable geometric phase change. In view of all these studies it is quite amazing that a counter-intuitive effect results when a sequence of measurements performed
on a quantum system leads to an inhibition of geometric phase in the limit of continuous measurement.

Let us consider a quantum system whose state vector $|\Psi(t)\rangle \in \mathcal{H}$ the Hilbert space of the system with finite or infinite dimension. The dynamical evolution is effected by a Hamiltonian $H$. The state vector of the system evolves according to Schrödinger equation (in the absence of measurement process)

$$i\hbar \frac{d}{dt}|\Psi(t)\rangle = H(t)|\Psi(t)\rangle \quad (1)$$

Consider an arbitrary evolution (in general a non-cyclic) of a quantum system for a time $T$. It is shown by one of the present author \cite{15,16} that the dynamical phase and geometric phase are given respectively, by

$$\Phi_d = \frac{1}{\hbar} \int_0^T \langle \Psi(t)|H(t)|\Psi(t)\rangle \, dt,$$

$$\Phi_g = i \int_0^T \langle \chi(t)|d\chi(t)\rangle, \quad (2)$$

where $|\chi(t)\rangle$ is the reference-state of the system introduced in \cite{15,16}, given by $|\chi(t)\rangle = \frac{\langle \Psi(t)|\Psi(0)\rangle}{\langle \Psi(t)|\Psi(0)\rangle} |\Psi(t)\rangle$. Here, $i \langle \chi(t)|d\chi(t)\rangle$ is basically a connection-form whose line integral along open path in the projective Hilbert space of the quantum system gives the desired geometric phase. This geometric phase is gauge invariant and does not depend on the detailed dynamics of the system. There could be an infinite number of open-paths in the Hilbert space but for all of them the geometric phase defined above is the same for a given projection of the open-path in the projective Hilbert space of the quantum system. This phase is in general, non-additive in nature and assigns a memory to the quantum system.

Imagine a situation where we have prepared our quantum system initially in one of the eigenstates of some observable $O$, which we are interested in measuring. The Hermitian operator $O$ has an eigenvalue spectrum $\{O_n\}$ and a complete set of eigenfunctions $\{|\Psi_n\rangle\}$. The spectrum is assumed to be discrete and non-degenerate. The observable $O$ need not commute with the Hamiltonian of the system that drives the system. Now, we allow the system to evolve under the Hamiltonian $H$ during the time interval $[0,T]$ such that one performs a series of measurements at times $\tau, 2\tau, \ldots , (N-1)\tau, N\tau = T$. During the interval $[0,\tau]$ which is very short the system evolves unitarily. The sequence of measurements that are carried out are idealised to be discrete and instantaneous. We are interested in knowing how the geometric phase is affected due to $N$ number of measurements on the quantum system. The fact that the geometric phase is non-additive in nature implies that the sum of the geometric phases acquired by the system from time $0$ to $\tau$, $\tau$ to $2\tau$, $\ldots$, $(N-1)\tau$ to $N\tau$ is not the same as the geometric phase acquired by the system from $0$ to $N\tau = T$, unlike the dynamical phase which is additive.

Here we proceed to give a simple derivation of the result where the dependence of the number of measurement on the geometric phase can be seen explicitly. Actually, starting from the initial state $|\Psi_n\rangle$ the state at time $\tau$ is given by

$$|\Psi(\tau)\rangle = U(\tau)|\Psi_n\rangle = e^{-iH\tau/\hbar}|\Psi_n\rangle. \quad (3)$$

After performing a von Neumann measurement at time $\tau$ (denoted by a projection operator $P_n = |\Psi_n\rangle \langle \Psi_n|$) to know whether the eigenvalue of the observable is still $O_n$, the state is given by $|\Psi_{am1}(\tau)\rangle = |\Psi_n\rangle \langle \Psi_n|U(\tau)|\Psi_n\rangle$, where $|\Psi_{am1}(\tau)\rangle$ is the state of the system just after first measurement at time $\tau$. Similarly, the state of the system
number of measurements tend to infinity. Then the geometric phase behaves as \( \Phi \propto -\tan^{-1}\left(\frac{\frac{T}{N\hbar}}{\frac{T^2}{2N^2\hbar^2}}\right) \). If we proceed with \( N \) number of measurement steps, the state of the system just after \( N \)th measurement is given by

\[
|\Psi_{amN}(\tau)\rangle = |\Psi_n(U(\tau)|\Psi_n> \ldots <\Psi_n(U(\tau)|\Psi_n>|\Psi_n>.
\]

This is the final state of the system at time \( T \) under quantum Zeno dynamics. Therefore, the total phase of the system can be obtained by taking the argument of the inner product of the initial and final state, in the sense of Pancharatnam. Thus, the total phase is given by

\[
\Phi_g(N)(T) = \arg\left[<\Psi_n|U(\tau)|\Psi_n> \ldots <\Psi_n|U(\tau)|\Psi_n>\right].
\]

On the other hand, the dynamical phase (which is an additive quantity) of the system under quantum Zeno dynamics is given by

\[
\Phi_d(N)(T) = -<\Psi_n|H|\Psi_n> \frac{T}{\hbar},
\]

which does not depend on the number of measurements performed on the quantum system undergoing unitary evolution. Therefore, the dynamical phase is insensitive to the quantum Zeno dynamics.

The geometric phase \( \Phi_g \) is given by

\[
\Phi_g(N)(T) = \arg\left[<\Psi_n|U(\tau)|\Psi_n> \ldots <\Psi_n|U(\tau)|\Psi_n>\right] + <\Psi_n|H|\Psi_n> \frac{T}{\hbar}.
\]

The above expression clearly shows the dependence of the geometric phase on the number of measurements (i.e., the number of times the system’s wavefunction has been collapsed) that has been performed on the quantum system.

Now the interesting situation arises, when one takes the large \( N \) limit, i.e., in the limit of continuous measurement, a continuous measurement is understood as the limit of a sequence of discrete, instantaneous measurements when the number of measurements tends to infinity. Then the geometric phase behaves as

\[
\lim_{N \to \infty} N \tan^{-1}\left[\frac{-<\Psi_n|H|\Psi_n> \frac{T}{N\hbar}}{1-<\Psi_n|H^2|\Psi_n> \frac{T^2}{2N^2\hbar^2}}\right] + <\Psi_n|H|\Psi_n> \frac{T}{\hbar} \to 0.
\]

where we have replaced \( \tan^{-1} x = x \) for small \( x \). This shows that continuous measurement in quantum Zeno dynamics setting can completely inhibit the development of the geometric phase. Since the geometric phase attributes a ‘memory’ to the quantum system the above result shows that under quantum Zeno dynamics the ‘memory’ of a system can be erased. This is the main result of this letter. It is immaterial which observable (commuting or non-commuting) of the system is being monitored repeatedly. The prediction of quantum Zeno phase effect requires only unitary Schrödinger evolution and the projection postulate. (Recently, we \cite{20} have shown that the QZE in fact occurs for a wide class of
systems, obeying non-linear, non-unitary equations and we hope that the QZPE can also be predicted for non-linear systems).

The above idea can be illustrated with the neutron spin undergoing “free evolution” and measurement of its spin in its initial state. Let us consider a source which sends a spin-up neutron that passes through several identical magnetic field regions. The magnetic field could have components along all the three directions $x, y$ and $z$. When the spin of neutron passes through the magnetic field, it undergoes precession. During precession the spin state can acquire a geometric phase in addition to the usual dynamical phase arising from instantaneous rotation. The interaction of the spin with the magnetic field $\mathbf{B}$ can be described by a Hamiltonian $H = \mu \sigma \cdot \mathbf{B}$, where $\mu$ is the magnetic moment, $\sigma$ is the Pauli spin matrices. If the initial state is prepared in the state $|\Psi(0)\rangle = |\uparrow\rangle$, then the state after a time $t$ is given by

$$|\Psi(t)\rangle = e^{-i\mu \sigma \cdot \mathbf{B} \bar{\hbar} t} |\uparrow\rangle = a(t)|\uparrow\rangle + b(t)|\downarrow\rangle.$$  \hfill (9)

Here, $a(t) = (\cos \frac{\omega t}{2} - i n_z \sin \frac{\omega t}{2})$, $b(t) = (n_y - in_x) \sin \frac{\omega t}{2}$, $\omega = \frac{2\mu B}{\hbar}$ and $n = (n_x, n_y, n_z)$ is the unit vector in the direction of the magnetic field. If the evolution is not interrupted by the measurement process the non-cyclic geometric phase is given by

$$\Phi_g(t) = -\tan^{-1}(n_z \tan \frac{\omega t}{2}) + n_z \frac{\omega t}{2}.$$  \hfill (10)

which can be interpreted as half the solid angle subtended by the closed curve obtained by joining the end points of the open curve with a shortest geodesic. The open curve can be parametrised by the azimuthal angle $\phi = \frac{\omega t}{2}$ and polar angle $\theta = \cos^{-1}(n_z)$, on a sphere which is the projective Hilbert space for the neutron.

Let us now investigate the situation when the neutron spin is monitored $N$ number of times during its evolution over a time period $T$ and undergoing precession in several magnetic field regions each with a length $l$ (here $l$ is very small). Let there be a device to select and detect the spin component of the neutron as in the proposed experiment to test the QZE by Nakazato et al [21–23]. When netron passes through a magnetic field region for a small time $\tau = l/v$, with $v$ being the speed of neutron in the field region, it undergoes rotation. The state of the neutron after passing through sequence of rotation and projection is $|\Psi_{amN}(N\tau)\rangle = a(\tau)^N |\uparrow\rangle$. Now the geometric phase shift relative to the initial state during a quantum Zeno dynamics process is given by

$$\Phi_g(N) = -N \tan^{-1}(n_z \tan \frac{\omega T}{2N}) + n_z \frac{\omega T}{2}.$$  \hfill (11)

In the limit of large number of measurements the geometric phase for neutron spin goes to zero, demonstrating the quantum Zeno phase effect, in conformity with our prediction.

Recently geometric phase has been measured experimentaly by Wagh et al [24,25] for neutron spin undergoing precession. Here, we sketch briefly a neutron interference experiment to observe this new effect (QZPE). Let there be an incident neutron beam which is polarised along up direction and the beam is split coherently into two parts. In one arm (say 1) of the interferometer let there be several regions of magnetic field and $N$ number of detection devices. In the other arm (say 2) of the interferometer let there be a magnetic field present over a distance $L$ such that
L = vT. The magnetic field is applied only along z-direction (with a magnitude equal to $B \cos \theta$) just to compensate the dynamical phase shift in the interference pattern. The state of the neutron passing through the arm 1 of the interferometer is $|\psi_1\rangle = K |\uparrow\rangle = a(\tau)^N |\uparrow\rangle$, where $K$ denotes the sequence of rotation and projection operation. The state in the arm 2 is $|\psi_2\rangle = \exp(-i\mu B \cos \theta \sigma_z T/\hbar) |\uparrow\rangle$. When the two neutron beams are recombined the intensity of the beam is given by

$$I(N) \propto |||\psi_1\rangle + |\psi_2\rangle||^2 \propto \left[1 + |a(\tau)|^{2N} + 2|a(\tau)|^N \cos(\Phi_g(N))\right].$$

This shows the interference pattern is not simply of the type $1 + \cos \phi$. The modification in the first term is due to the non-unitary operation in the arm 1. The interference contrast and the phase shift of geometric origin depends on the number of measurements that have been performed in the arm 1 of the interferometer. This is an interesting observation and by changing the number of measurements we can have a different interference pattern. In the limit of large number of measurements $|a(\tau)|^{2N} = \exp(-(n_x^2 + n_y^2)\omega^2 T^2/4N) \rightarrow 1$ and the geometric phase shift tends to zero, thus giving us a full maximum in the interference pattern.

In conclusion we have predicted a new effect (QZPE), which says that under frequent measurements the geometric phase of any quantum system can be inhibited. This in turn implies that repeated measurements can erase the ‘memory’ of a quantum system. We have illustrated the idea for a neutron spin under going successive rotations and projections and suggested a method to observe this effect in neutron interference experiments. This can provide a new way of controlling phase shift by means of measurements which could be of interest in many branches of physics.

[1] B. Misra and E. C. G. Sudarshan, J. Math. Phys. 18, 756 (1977).
[2] H. Dehmelt, Proc. Natl. Acad. Sci. U.S.A. 83, 2291 (1986); ibid 3074.
[3] G. J. Milburn, J. Opt. Soc. Am. B 5, 1317 (1988).
[4] R. J. Cook, Phys. Scr. T 21, 49 (1988).
[5] W. H. Itano et al. Phys. Rev. A 41, 2295 (1990).
[6] S. Pancharatnam, Proc. Indian Acad. Science A 44, 247 (1956).
[7] J. Samuel and R. Bhandari, Phys. Rev. Lett. 60, 2339 (1988).
[8] J. Anandan, Nature, 360, 307 (1996).
[9] A. K. Pati and S. V. Lawande, Phys. Lett. A 223, 233 (1996).
[10] A. A. Kulaga, Phys. Lett. A 202, 7 (1995).
[11] U. Tambini, C. Presilla and R. Onofrio, Phy. Rev. A 51, 967 (1995); C. Presilla, R. Onofrio and U. Tambini, Ann. Phys. 248, 95 (1996).
[12] M. V. Berry, Proc. R. Soc. A 392, 45 (1984).
[13] Y. Aharonov and J. Anandan, Phys. Rev. Lett. 58, 1593 (1987).
[14] N. Mukunda and R. Simon, Ann. Phys. 228, 20 (1993).
[15] A. K. Pati, Phys. Rev. A. 52, 2576 (1995).
[16] A. K. Pati, J. Phys. A. 28, 2087 (1995).
[17] J. Anandan and A. Pines, Phys. Lett. A 45, 141, 335 (1989).
[18] R. Bhandari, Phys. Rev. Lett. 69, 3720 (1992).
[19] Y. Aharonov, T. Kaufherr, S. Popescu, B. Reznik, Phys. Rev. Lett. 80, 2023 (1998).
[20] A. K. Pati and S. V. Lawande, Phys. Rev. A 58, .. (1998).
[21] S. Pascazio and M. Namiki, Phys. Rev. A 50, 4582 (1994).
[22] H. Nakazato, M. Namiki, S. Pascazio and H. Rauch, Phys. Lett. A 199, 27 (1995).
[23] Z. Hardil, H. Nakazato, M. Namiki and S. Pascazio, Phys. Lett. A 239, 333 (1998).
[24] A. G. Wagh et al, Phys. Rev. Lett. 78, 755 (1997).
[25] A. G. Wagh and V. C. Rakhecha, Phys. Lett. A, 197, 107 (1995); ibid 112 (1995).