Power Efficient Scheduling under Delay Constraints over Multi-user Wireless Channels

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Abstract—In this paper, we consider the problem of power efficient uplink scheduling in a Time Division Multiple Access (TDMA) system over a fading wireless channel. The objective is to minimize the power expenditure of each user subject to satisfying individual user delay. We make the practical assumption that the system statistics are unknown, i.e., the probability distributions of the user arrivals and channel states are unknown. The problem has the structure of a Constrained Markov Decision Problem (CMDP). Determining an optimal policy under the CMDP faces the problems of state space explosion and unknown system statistics. To tackle the problem of state space explosion, we suggest determining the transmission rate of a particular user in each slot based on its channel condition and buffer occupancy only. The rate allocation algorithm for a particular user is a learning algorithm that learns about the buffer occupancy and channel states of that user during system execution and thus addresses the issue of unknown system statistics. Once the rate of each user is determined, the proposed algorithm schedules the user with the best rate. Our simulations within an IEEE 802.16 system demonstrate that the algorithm is indeed able to satisfy the user specified delay constraints. We compare the performance of our algorithm with the well known M-LWDF algorithm. Moreover, we demonstrate that the power expended by the users under our algorithm is quite low.

Index Terms—Multi-user Fading Channel, Markov Decision Process, Energy Efficient Scheduling

I. INTRODUCTION

Broadband wireless networks like IEEE 802.16 [1] and 3G cellular [2] are expected to provide Quality of Service (QoS) for emerging multimedia applications. One of the challenges in providing QoS is the time varying nature of the wireless channel due to multipath fading [3]. Moreover, for portable and hand-held devices, energy efficiency is also an important consideration.

In a multi-user wireless system, recent studies [4], [5] suggest that since the wireless channel fades independently across different users, this diversity can be exploited by opportunistically scheduling the user with the best channel gain. This leads to significant performance gain in terms of total system throughput. Such scheduling algorithms that exploit the characteristics of the physical channel to satisfy some network level QoS performance metrics are referred to as cross layer scheduling algorithms [6]. Power required to transmit reliably at a certain rate under better channel conditions is much less than that required under poorer channel conditions at the same rate [3]. This suggests that in order to save power, one should transmit at higher rates under better channel conditions, this leads to queuing delays. Moreover, since transmission power is an increasing and strictly convex function of the transmission rate [3], power efficiency can also be achieved by transmitting the data at lower rates, albeit at the cost of increased queuing delay thus leading to a power-delay tradeoff.

In this paper, we consider a single cell multi-user wireless uplink system with Time Division Multiple Access (TDMA). For such a system, we consider the problem of determining the user to be scheduled in each time slot so that the average transmission power expended by each user is minimized subject to a constraint on the average queuing delay experienced by each user. Moreover, we assume a peak power constraint, i.e., in each slot the transmission power of a user is less than or equal to a certain maximum value. This scenario may correspond to a base station scheduling users on an uplink in an IEEE 802.16 system to satisfy delay constraint of each user.

There is a copious literature on cross layer scheduling algorithms. See [7] for a succinct review. The scheduling problem is typically formulated as an optimization problem with an objective of efficiently allocating resources such as time, frequency bands, power, codes etc. to the users under physical layer (wireless channel) and/or network layer QoS constraints. Various QoS constraints have been considered in the literature like system throughput, minimum rate, maximum delay, delay bound, queue stability and fairness. A scheduling policy is an allocation rule that allocates these resources based on parameters like channel conditions of the users, their queue lengths etc. In this paper, we concentrate on efficiently allocating power and rates to users based on their channel condition and queue length. The power allocation policy is considered feasible if it satisfies certain average or peak power constraints. On the other hand, the rate allocation policy is considered feasible if the physical layer can deliver the data reliably to the users at a given rate. The set of all feasible rate tuples is called the feasible capacity region [7].

A scheduling policy is considered stable if the expected queue lengths are bounded under the policy. Many scheduling policies proposed in the literature have considered stability as a QoS criterion. In [4], the authors determine the throughput capacity region of a multi-access system, i.e., the set of all feasible rates with average power constraints. In [8], the authors have shown that the throughput capacity region is same as the multi-access stability region (i.e., the set of all arrival vectors for which there exists some rate and power allocation policies that keep the system stable.). A scheduler is
termed throughput-optimal if it can maintain the stability of the system as long as the arrival rate is within the stability region. Throughput optimal scheduling policies have been explored in [4, 9]. Longest Connected Queue (LCQ) [10], Exponential (EXP) [11], Longest Weighted Queue Highest Possible Rate (LWQHPR) [12] and Modified Longest Weighted Delay First (M-LWDF) [13] are other well known throughput optimal scheduling policies. In [14], the authors define the notion of delay limited capacity of a multi-access system, i.e., the maximum rate achievable such that the delay is independent of the fading characteristics.

While throughput-optimal scheduling policies maintain the stability of the queueing system, they do not necessarily guarantee small queue lengths and consequently lower delays. Delay-optimal scheduling deals with optimal rate and power allocation such that the average queue length and hence average delay are minimized for arrival rates within the stability region under average and peak power constraints. Due to the nature of the constraints, there is no loss of optimality in choosing the rate and power allocation policies separately [7]. Hence to simplify the problem, one can choose any stationary power allocation policy that satisfies the peak and average power constraints. The delay optimal policy therefore deals with optimal rate allocation for minimizing delays under a given power allocation policy. It has been shown that the Longest Queue Highest Possible Rate (LQHPR) policy [15] (besides being throughput optimal) is also delay optimal for any symmetric power control under symmetric fading provided that the packet arrival process is Poisson and packet length is exponentially distributed.

Apart from throughput and delay optimal policies, opportunistic scheduling with various fairness constraints have been explored in [16], [17].

In this paper, our focus is on rate allocation with a constraint on peak power as well as average queueing delay which acts as the QoS metric. This problem for a single user wireless channel without the peak power constraints has been explored in the pioneering work of [18]. The problem with many generalizations on arrival and channel gain processes have been considered in subsequent papers [19], [20], [21], [22], [23], [24], [25]. In most of these papers, the scheduling policy has been formulated as a control policy within the Markov Decision Process (MDP) framework. However, only structural results of the optimal policy are available under various assumptions and that too for a single user scenario only. There is very little work for extending the vast body of literature on delay constrained power efficient scheduling to multi-user scenario. Recently, in [26], the author has extended the asymptotic analysis of Berry-Gallger [18] for exploiting the power-delay tradeoff in multi-user system. The objective is to minimize the total power on the downlink subject to user queue stability constraints. The author using the concept of Lyapunov Drift Steering has also given an algorithm that comes within a logarithmic factor of achieving the Berry-Gallger power-delay bound. However, on the downlink, the base station typically transmits with a constant maximum power sufficient to reach the farthest user and hence power minimization in not a major concern. Moreover, minimizing the sum power can lead to unfairness, i.e., users with better average channel conditions might get a far higher share of the bandwidth than the users who have relatively poor average channel conditions. On the other hand, for the uplink, the problem is to minimize the power of each user subject to individual delay constraint, which has not been addressed in the literature so far.

In [27], the author has extended the analysis for single user case to the multi-user case, albeit with only two users which can be applicable for the uplink also. Beyond two users, the problem becomes unwieldy to gain any useful insight, primarily due to large state space. For the two user case, the author has given an elegant near optimal policy where each user’s rate allocation is determined by the joint channel states across users and the user’s own queue state. Thus each user’s queue evolution process behaves as if it were controlled by a single user policy. However, computation of user’s transmission power still takes into account the joint channel and queue state processes.

Even for the single user case [19], [20], [21], [22], [23], [24], [25], practical implementation of optimal policy is far from simple. This is because a knowledge of the probability distributions of the arrival and channel gain process is required for computing the optimal policy. This knowledge is not available in practice. While, we have addressed this limitation by formulating an on-line algorithm within stochastic approximation framework in [28], this algorithm still deals with the single user scenario. This algorithm does not assume any explicit knowledge of the probability distributions of the channel gain and arrival processes. In this paper, we consider a multi-user wireless system. The state of the system is defined as the minimum information required by the scheduler for making scheduling decisions. For the multi-user scenario considered in this paper the state space is considerably large as compared to that for the single user scenario. We illustrate this with a simple example. Let us assume that the channel condition of a user can be represented using 8 states. This is a practical assumption and has been justified in [29]. Let us assume that each user has a buffer in which at most 50 packets can be stored. For a single user system, the channel state and buffer occupancy of the user forms the state of the system in any time slot. The number of states is 8 × 50 = 400. Now consider a multi-user system with 4 users. In this case, the state of the system consists of the channel state and buffer occupancy of each user. The state space consists of 504 × 44 = 2.56 × 1050 states. Furthermore, the number of states increases exponentially with the users. Hence determining the optimal policy by estimating the dynamic programming value function would take prohibitively long time. Hence in this paper, we propose a alternate approach.

In our approach, each user’s queue evolution behaves as if it were controlled by a single user policy. Depending on each user’s channel state and queue size, the algorithm allocates a certain rate to each user in a slot using the single user algorithm outlined in this paper. The algorithm then schedules the user with the highest rate in a slot. From the structural properties of optimal policy for a single user scenario, it is well known that the optimal policy is increasing in queue
length and channel gain [27]. Thus more number of packets are transmitted when the queue length is greater or the channel gain is higher. Hence a user transmitting at a high rate has either very good channel condition, or large queue length. Scheduling such a user, therefore, either leads to its power savings or aids in satisfying the delay constraint.

The scheduling algorithms proposed in the literature like EXP scheduler [11], LQHP scheduler [15], M-LWDF [13] scheduler require the queue length information for determining the scheduling decision. In the downlink scenario, this information is readily available to the scheduler residing at the base station. However, in the uplink scenario, this information needs to be communicated by the users to the scheduler. Communicating the queue length information poses a significant overhead. In our approach, each user determines the rate at which it would transmit if it were scheduled in a slot. All the users inform these rates to the base station. The base station then schedules the user with the highest rate. Thus by communicating the rates directly, we avoid the queue length communication overhead.

The IEEE 802.16 system is an emerging system for broadband wireless access and is expected to provide QoS to the users. Through our simulations in an IEEE 802.16 system, we demonstrate that the algorithm is indeed able to satisfy the delay constraints of the users. Moreover, we demonstrate that the power expenditure of a user is commensurate with its delay constraints of the users. Moreover, we demonstrate that the power expenditure of a user is commensurate with its delay requirements, the average arrival rate and average channel conditions. The higher the delay, lower the average arrival rate and better the average channel conditions, the lower is the power expenditure.

The contributions of this paper are summarized as follows:

1) We formulate the problem of minimizing the average power expended by each user subject to a constraint on individual user delay as a constrained optimization problem. To the best of our knowledge, this problem has not been studied in a multi-user uplink scenario.

2) We propose an online algorithm that does not require the knowledge of the probability distributions of the channel states and the arrivals of the users.

3) The computational complexity of our approach increases only linearly with the number of users.

4) The communication overhead of our approach is low and hence the algorithm is suitable for practical implementation. The algorithm satisfies the delay constraints of the users. We demonstrate the power efficiency our algorithm through comparison with the M-LWDF algorithm within an IEEE 802.16 system simulation.

The rest of the paper is organized as follows. In Section II, we present the system model. We formulate the problem as an optimization problem in Section III, where we show that the problem has the structure of a Constrained Markov Decision Problem (CMDP). We discuss the issues like large state space and unknown system model in determining an optimal solution using the traditional CMDP solution techniques. In Section IV, we consider and extension to the traditional single user scenario based on transmitter induced errors. In Section V, we propose an online algorithm that is based on the extension to the single user scenario detailed in Section IV. We also discuss the implementation issues.

We present the simulation setup and discuss results in Section VI. Finally, we conclude in Section VII.

II. SYSTEM MODEL

As illustrated in Figure 1, we consider uplink transmissions in a TDMA system with $N$ users, i.e., time is divided into slots of equal duration and only one user is allowed to transmit in a slot. We assume that the slot duration is normalized to unity. The base station is a centralized entity that schedules the users in every slot. We assume a fading wireless channel where the channel gain is assumed to remain constant for the duration of the slot and to change in an independent and identically distributed (i.i.d.) manner across slots. This model is called the block fading model [18]. We assume that the fading across users is also i.i.d. Under these assumptions, if a user $i$ transmits a signal $y_n^i$ in slot $n$, then the received signal $Y_n^i$ can be expressed as,

$$Y_n^i = H_n^i b_n^i + G_n,$$

where $H_n^i$ denotes the complex channel gain due to fading and $G_n$ denotes the complex additive white Gaussian noise with zero mean and variance $N_0$. Let $X_n^i = |H_n^i|^2$ be the channel state for user $i$ in slot $n$. Practically $H_n^i$ is a continuous random variable and hence so is $X_n^i$. However, in this paper we assume that $X_n^i$ takes only finite and discrete values from a set $\mathcal{X}$. This assumption has been justified in [18], [19]. In this paper, we assume that the distribution of $H_n^i$ and hence that of $X_n^i$ is unknown.

Each user possesses a finite buffer of $B$ bits. Bits arrive into the user buffer and are queued until they are transmitted. The arrival process for each user is assumed to be i.i.d. across slots. Let $A_n^i$ denote the number of bits arriving into the user $i$ buffer in slot $n$. We assume that the random variable $A_n^i$ takes values from a finite and discrete set $\mathcal{A} \triangleq \{0, \ldots, A\}$. Like $X_n^i$, we assume that the distribution of $A_n^i$ is unknown. Let $Q_n^i$ denote the queue length or buffer occupancy of user $i$ in slot $n$. Let $U_n^i$ denote the number of bits transmitted by user $i$ in slot $n$. We assume that $U_n^i$ takes values from the set $\mathcal{U} \triangleq \{0, \ldots, B\}$. Let $I_n^i$ be an indicator variable that is set to 1 if user $i$ is scheduled in slot $n$ and is set to 0 otherwise. Let $I_n$ be the vector $[I_n^1, \ldots, I_n^N]$. Note that since only one user can transmit in a slot, only one element of $I_n$ is equal to 1 and the rest are 0. Let $T$ be the set of all possible $N$ dimensional vectors with one element equal to 1 and the rest being 0. Let $K_n^i$ denote the number of bits that the user $i$ transmits in a slot.
if it is scheduled. Then \( U_n^i \) can be represented as \( U_n^i = I_n^i K_n^i \).
Moreover, since a user can at most transmit all the bits in its buffer in a slot, \( K_n^i \leq Q_n^i \). Since we assume that the slot length is normalized to unity, \( U_n^i \) is the rate at which user \( i \) transmits in slot \( n \). Let \( U_n \) be the vector \([U_n^1, \ldots, U_n^N]\), \( U_n \in \mathcal{U}^N \).

The buffer evolution equation for user \( i \) can be expressed as,
\[
Q_{n+1}^i = \max \{ Q_n^i - U_n^i, 0 \} + A_{n+1}^i.
\]
The buffer size \( B \) is large as compared to the arrival rate, thus we can neglect the buffer overflow in the buffer evolution equation.

From [18], the power required for error-free or reliable communication at a rate \( K_n^i = u \) bits/sec when \( X_n^i = x \) is given by,
\[
P(x, u) = \frac{W N_u}{x} (2\pi - 1).
\]
where \( W \) is the bandwidth in Hz. Note that for a given \( x \), the transmission power \( P(x, u) \) is an increasing and strictly convex function of \( u \). Let \( \hat{P} \) denote the peak power constraint. Let \( K_{n}^i \) be the maximum rate at which user \( i \) can transmit in a slot \( n \) when the channel condition is \( X_n^i = x \) while satisfying the peak power constraint (i.e., \( P(K_n^i, X_n^i) \leq \hat{P} \)). Then the set of feasible rates for user \( i \) in slot \( n \), \( F_n^i \triangleq \{0, \ldots, \min(K_n^i, Q_n^i)\} \).

We assume that the users specify their QoS requirements in terms of the average packet delay requirements. These delay requirements of the users are known a priori to the scheduler. By Little’s law [30], the average delay \( \bar{D} \) is related to the average queue length \( \bar{Q} \) as,
\[
\bar{Q} = \bar{a} \bar{D},
\]
where \( \bar{a} \) is the average arrival rate. In the rest of the paper, we treat average delay as synonymous with average queue length and ignore the proportionality constant \( \bar{a} \).

### III. Problem Formulation

In this section, we formulate the problem as a constrained optimization problem within the Constrained Markov Decision Process (CMDP) framework.

#### A. Formulation as a Constrained Optimization Problem

The user devices have limited battery power, hence it is essential to design transmission policies that conserve battery power. The power-delay tradeoff can be exploited to save power at the expense of extra delay. Moreover, multi-user diversity can be exploited to schedule a user with better channel state. Such a user requires less power while transmitting at a certain rate as compared to when it has a poorer channel state. However, this also incurs additional delay. The objective is to design a joint rate allocation and scheduling scheme that minimizes the power expenditure of each user subject to the satisfaction of the individual delay constraints. The average power consumed by a user \( i \) over a long period of time can be expressed as,
\[
\bar{P}^i = \limsup_{M \to \infty} \frac{1}{M} \mathbb{E} \sum_{n=1}^{M} P(X_n^i, u_n^i K_n^i).
\]
The average queue length of a user \( i \) over a long period of time can be expressed as,
\[
\bar{Q}^i = \limsup_{M \to \infty} \frac{1}{M} \mathbb{E} \sum_{n=1}^{M} Q_n^i.
\]
Each user \( i \) wants its average queue length to remain below a certain value, say, \( \bar{\delta}^i \). Hence the problem becomes,
\[
\text{Minimize } \bar{P}^i \text{ subject to } \bar{Q}^i \leq \bar{\delta}^i, \ i = 1, \ldots, N.
\]

**Remark 1:** Dependence between problems: Note that the \( N \) problems formulated in (7) are not independent. This is because in a TDMA system, only one user can be scheduled in a slot. Consequently, the scheduling decision in a slot impacts the buffer occupancy of all the users in the future slots.

#### B. Notion of Optimal Solution

The problem in (7) is a multi-objective optimization problem with \( N \) objectives and \( N \) constraints. There can be multiple average power vectors that can be considered as optimal. Hence it is necessary to precisely define the properties of an optimal solution sought by us. We seek Pareto optimal solutions [31]. Let vector \([\bar{P}^1_{\psi}, \ldots, \bar{P}^N_{\psi}]\) be the power expenditure vector under the rate allocation policy \( \psi \). We say that the rate allocation policy \( \psi \) is Pareto optimal if and only if there exists no rate allocation policy \( \zeta \) with the corresponding power expenditure vector \([\bar{P}^1_{\zeta}, \ldots, \bar{P}^N_{\zeta}]\) having the following properties,
\[
\forall i \in \{1, \ldots, N\} P^i_{\psi} \leq P^i_{\zeta} \land \exists i \in \{1, \ldots, N\} |P^i_{\psi} < P^i_{\zeta}.
\]
The Pareto optimal solution is generally not unique and the set of Pareto optimal solutions is called the set of non-dominated solutions. The weighted sum approach is a common approach for solving a multi-objective optimization problem [31]. In this approach, one aggregates the \( N \) objective functions into a single objective function. The resultant problem has a single objective function and \( N \) constraints and can be expressed as,
\[
\text{Minimize } \bar{P} = \gamma^1 \bar{P}^1 + \ldots + \gamma^N \bar{P}^N,
\]
subject to,
\[
\bar{Q}^i \leq \bar{\delta}^i, \ i = 1, \ldots, N.
\]
where \( \gamma \triangleq [\gamma^1, \ldots, \gamma^N] \) is the weight vector. It is generally assumed that \( \gamma^i \in [0, 1], \forall i, \sum_{i=1}^{N} \gamma^i = 1 \) implying that \( \bar{P} \) is a convex combination of the individual powers. In general, the non-dominated set (i.e., the set of all Pareto optimal policies) may be a non-convex set. By varying the weight vector in the weighted sum approach we can determine the Pareto optimal policies within a convex subset of the non-dominated set. However, choosing the weight vector in order to obtain a particular solution is not straightforward. In the next section, we formulate the problem in (9) within the CMDP framework.
C. The CMDP Framework

Let $X_n = [X_{n,1}, \ldots, X_{n,N}]$ and $Q_n = [Q_{n,1}, \ldots, Q_{n,N}]$. The state of the system at time $n$ can be described by the tuple, $S_n = [Q_n, X_n]$, comprising of the queue length and the channel state of each of the $N$ users. Note that the system state space $S = Q^N \times X^N$ is discrete and finite. Let $\{S_n\}$ denote the state process. In each slot, the scheduler sets the rate vector $U_n = [U_{n,1}, \ldots, U_{n,N}]$, where $U_{n,i} = I_{n,K_i}$. $U_n$ takes values from the finite action space $U^N$. $\{U_n\}$ denotes the control process. This problem has the structure of a CMDP with finite state and action spaces. Since we are considering average power expended and average delay suffered, it is an average cost CMDP. The scheduler objective is to determine an optimal rate allocation policy, i.e., a mapping from past history of states and actions to a rate allocation vector $U_n$ for every slot $n$. For a CMDP with finite state and action spaces, it is well known that an optimal stationary randomized policy exists [32], i.e., the rate allocation policy is a mapping from the current system state to a probability distribution on the set of feasible rate vectors in slot $n$. However, the traditional computational approaches based on Linear Programming [32] cannot be used to determine the optimal policy because of the following reasons:

1) **Large state space:** The state system space is large even for very few users. We have already illustrated this with an example in Section I. Moreover, the state space grows exponentially with number of users, hence the computational complexity of the traditional approaches also grows exponentially with number of users.

2) **Unknown user/system statistics:** The probability distributions of $X^N$ and $A^N$ are unknown. The traditional approaches rely on the knowledge of these distributions for determining the optimal policy.

We now extend the approach suggested in [28] to determine an optimal policy for the problem in (9).

Let user $i$ be scheduled in slot $n$. Then the state of the system immediately after user $i$ transmits the data can be represented as $S_n = [Q_n, X_n] = [(q_{n,1}^i, \ldots, \max(0, q_{n,1}^i - u_{n,1}^i), \ldots, q_{n,N}^i, x_{n,1}^i, \ldots, x_{n,N}^i)$. Let $A_n = [a_{n,1}, \ldots, a_{n,N}]$ denote the arrival vector in slot $n$. The state of the system at the beginning of slot $n+1$ can be represented as $S_{n+1} = [Q_{n+1}, X_{n+1}] = [(q_{n+1,1}^i + a_{n+1,1}^i, \ldots, \max(0, q_{n+1,1}^i - u_{n+1,1}^i + a_{n+1,1}^i), \ldots, q_{n+1,N}^i + a_{n+1,N}^i, x_{n+1,1}^i + a_{n+1,1}^i, \ldots, x_{n+1,N}^i)]$. The queue transition equation in the vector form can be written as

$$Q_{n+1} = \max(0, Q_n + A_{n+1} - U_n).$$

Let $\Delta = [\delta^1, \ldots, \delta^N]$ denote the delay constraint vector. The problem in (9) can be converted into an unconstrained problem using the Lagrangian approach. The unconstrained problem can be expressed as

$$\text{Minimize } \tilde{P} + \sum_{i=1}^{N} \lambda^i(Q^i - \tilde{\delta}^i).$$

The immediate cost $b(\cdot, \cdot, \cdot, \cdot)$ incurred in scheduling a user $i$ in state $S_n$ when the LM vector is $\lambda$ and rate vector $U_n$ (user $i$ is scheduled in slot $n$) can be expressed as,

$$b(\lambda, S_n, U_n) = P(x_n^i, u_n^i) + \sum_{i=1}^{N} \lambda^i(Q_n^i - \delta^i)$$

Let $\tilde{S}^0 = [q^0, x^0]$ denote a fixed state. Let $\tilde{V}(\cdot)$ denote the dynamic programming value function based on the state $\tilde{S}$ reached immediately after taking the scheduling decision but before the arrivals. Let $\tilde{F}_n = [\tilde{F}_{n,1}, \ldots, \tilde{F}_{n,N}]$ be the set of feasible rate vectors in slot $n$. Let $\{f_n\}$ and $\{e_n\}$ be two sequences that have the following properties,

$$f_n \to 0, \quad e_n \to 0, \quad \sum_n f_n^2 < \infty, \quad \sum_n (e_n)^2 < \infty, \quad \sum_n f_n = \infty, \quad \sum_n e_n = \infty, \quad \sum_n (f_n^2 + e_n^2) < \infty, \quad \lim_{n \to \infty} e_n/f_n \to 0.$$  

The significance of these properties is explained later. We now present an optimal online primal-dual algorithm for solving the constrained problem in (9):

$$U_{n+1} = \arg\min_{U_n \in \tilde{F}_n} \left\{ (1 - f_n)\tilde{V}(\tilde{S}_n) + f_n \times \right. \left\{ b(\lambda, (Q_n + A_{n+1}, X_{n+1}), V) \right. \left. + \tilde{V}_n((Q_n + A_{n+1} - V, X_{n+1})) \right\} \right. \left. - \tilde{V}_n(\tilde{S}_n^0) \right\}.$$  

The algorithm in (16) and (17) is an online version of the well known Relative Value Iteration Algorithm (RVIA) [33]. It iteratively determines the optimal value function and hence the optimal policy one state at a time for a fixed value of the LM vector $\lambda$. To determine the optimal LM vector we augment the above algorithm with a dual LM iteration:

$$\lambda_{n+1} = \Lambda[\lambda_n + e_n(Q_n - \Delta)],$$

where $\Lambda$ is a projection operator for ensuring that the LMs are non-negative and finite. The properties of the update sequences in (14) ensure that the sequences $\{f_n\}$ and $\{e_n\}$ converge to 0 sufficiently fast to eliminate the noise effects when the iterates are close to their optimal values $\tilde{V}(\cdot, \cdot)$ and $\lambda^*$, while those in (13) ensure that they do not approach 0 too rapidly to avoid convergence of the algorithm to non-optimal values. Furthermore, (15) ensures that the update rates of primal iterations, i.e., the value function iterations and the dual iterations, i.e., the LM iterations are different. Since $e_n$ approaches 0 much faster than $f_n$, the update rate of the value function iterations is much higher than the update rate of the LM iterations. This ensures that even though both the primal and duals are updated simultaneously, both converge to their
optimal values [34], (16), (17) and (18) constitute the optimal algorithm. The proof of optimality is exactly similar to that in [28].

However, compared to the single user case, the state space here is too large for the algorithm to converge in reasonable number of iterations. We therefore motivate an alternate approach. We incorporate the possibility of transmitter induced errors in the single user scenario. We then motivate the multi-user solution by making use of this extension to the single user scenario.

IV. SINGLE USER SCENARIO IN PRESENCE OF TRANSMITTER ERRORS

We describe the scenario in brief. Consider a point to point transmission system over a fading wireless channel. Time is divided into slots of unit duration. We consider the block fading model as described in Section II. The scheduler is unaware of the probability distribution of the arrivals and the channel state at the beginning of slot. The objective is to minimize the average transmission power subject to average packet delay constraints. In [28], we have determined an online algorithm that determines the optimal transmission rate in each slot so as to minimize the average power expenditure subject to average packet delay constraints. We consider the following extension to the above problem: suppose that after the online algorithm has determined the rate $U_n \in F_n$, with a certain unknown random probability $\theta_n \in [0,1]$, the transmitter is unable to proceed with the transmission. We assume that the probability distribution of $\theta_n$ is not known. Under this assumption, the queue evolution equation can be expressed as,

$$Q_{n+1} = Q_n + A_{n+1} - I_n U_n,$$

where $I_n$ is an indicator variable that is set to 1 if the transmitter is successful in transmitting the packets and is set to 0 otherwise. We now formulate the rate allocation problem for this scenario. The long term power expenditure can be expressed as,

$$\bar{P}_e = \limsup_{M \to \infty} \frac{1}{M} E \sum_{n=1}^{M} P(X_n, I_n U_n)$$

The average queue length over a long period of time can be expressed as,

$$\bar{Q}_e = \limsup_{M \to \infty} \frac{1}{M} E \sum_{n=1}^{M} Q_n,$$

Hence the rate allocation problem can be stated as,

Minimize $\bar{P}_e$ subject to $\bar{Q}_e \leq \delta.$  \hspace{1cm} (22)

Note that the problem in (22) has the structure of a CMDP with a state space for the single user case and average cost criterion. The objective is to determine an optimal policy $\mu^*$ such that the power expended under this policy is minimum possible while satisfying the delay constraint.

A. The Primal Dual Approach

The constrained problem in (22) can be converted into an unconstrained problem using the Lagrangian approach [32]. Let $\lambda \geq 0$ be a real number called as the Lagrange Multiplier (LM). Let $B$ be the set $\{0,1\}$. $c : \mathcal{R}^+ \times \mathcal{Q} \times \mathcal{X} \times \mathcal{B} \times \mathcal{U} \to \mathcal{R}$ be defined as the following,

$$c(\lambda, Q_n, X_n, I_n, U_n) \triangleq P(X_n, I_n U_n) + \lambda(Q_n - \delta),$$

where $U_n$ is determined using the rate allocation policy $\mu : \mathcal{Q} \times \mathcal{X} \to \mathcal{U}$. The unconstrained problem is to minimize,

$$L(\mu, \lambda) = \limsup_{M \to \infty} \frac{1}{M} \sum_{n=1}^{M} c(\lambda, Q_n, X_n, I_n, \mu(Q_n, X_n)).$$

(24)

$L(\cdot, \cdot)$ is called the Lagrangian. Our objective is to determine the optimal rate allocation policy $\mu^*$ and optimal LM $\lambda^*$ such that the following saddle point optimality condition is satisfied,

$$L(\mu^*, \lambda^*) \leq L(\mu, \lambda^*) \leq L(\mu^*, \lambda). \hspace{1cm} (25)$$

For a fixed LM $\lambda$, the problem is an unconstrained Markov Decision Problem (MDP) with finite state and action spaces with the average cost criterion. The following dynamic programming equation [35] gives a necessary condition for optimality of a solution,

$$V(q, x) = \min_{r \in F} \left[ c(\lambda, q, x, I, r) - \beta + \sum_{a',x'} p((q, x), r, (q + a' - r, x')) \times V(q + a' - r, x') \right], a' \in A, x' \in X,$$

(26)

where $V(\cdot, \cdot)$ is the value function, $\beta \in \mathcal{R}$ is the unique optimal power expenditure. Let $(q^0, x^0) \in \mathcal{Q} \times \mathcal{X}$ be a fixed state. If we impose $V(q^0, x^0) = 0$, then $V(\cdot, \cdot)$ is unique [35]. $p(s, r, s')$ is the probability of reaching a state $s'$ upon taking an action $r$ in state $s$. The traditional approaches for computing the optimal policy for an unconstrained average cost MDP such as the Relative Value Iteration Algorithm [35] require the knowledge of $p(\cdot, \cdot, \cdot)$ which in this case is dependent on the probability distributions of the arrivals and channel states which is not known. Note that determining the optimal value function as defined in (26) is not sufficient because the unconstrained solution for a particular $\lambda$ does not ensure that the constraints would be satisfied. To ensure constraint satisfaction, the optimal LM needs to be determined.

B. The Online Rate Allocation Algorithm

We now present the rate allocation algorithm. Let the user state at the beginning of slot $n$ be $(Q_n, X_n) = (q, x)$. Suppose that $u$ bits are transmitted in slot $n$. The following primal-dual algorithm can be used to compute the rate $U_{n+1} = r_{n+1}$ at
which the transmitter should transmit in slot $n+1$,

$$r_{n+1} = \arg \min_{v \in \mathcal{F}_{n+1}} \left\{ (1 - f_n) \bar{V}_n(\bar{q}, \bar{x}) + f_n \times \right.$$

$$\left. \left\{ c(\lambda_n, \bar{q} + A_{n+1}, X_{n+1}, 1, v) + \bar{V}_n(\bar{q} + A_{n+1} - v, X_{n+1}) - \bar{V}_n(\bar{q}, \bar{x}) \right\} \right\},$$

(27)

$$\bar{V}_{n+1}(\bar{q}, \bar{x}) = (1 - f_n) \bar{V}_n(\bar{q}, \bar{x}) + f_n \times$$

$$\left\{ c(\lambda_n, \bar{q} + A_{n+1}, X_{n+1}, I_{n+1}, r_{n+1}) + \bar{V}_n(\bar{q} + A_{n+1} - I_{n+1}r_{n+1}, X_{n+1}) - \bar{V}_n(\bar{q}, \bar{x}) \right\},$$

(28)

$$\lambda_{n+1} = \lambda(\lambda_n + e_n (Q_n - \delta)).$$

(29)

These equations are explained below:

1) (27), (28) and (29) constitute the rate allocation algorithm. It consists of two phases: rate determination phase and update phase. (27) constitutes the rate determination phase of the algorithm, i.e., it is used to determine the rate at which a user transmits in a slot if the transmission is successful. (28) is the primal iteration to determine the optimal value function and thereby the optimal policy, while (29) is the coupled dual iteration for determining the optimal LM. They constitute the update phase of the algorithm.

2) If in a state $(Q_n, X_n) = (q, x)$, the transmitter decides to transmit $u \leq q$ bits, then $\bar{q} \triangleq q - u$, and $\bar{x} \triangleq x$.

3) (28) determines the optimal value function based on this new virtual state $(\bar{q}, \bar{x})$. Note that the value function for this new state is related to the usual value function as $\bar{V}_n(\bar{q}, \bar{x}) = E^{A,X}[\bar{V}_n(Q_n, X_n)]$.

4) The rate determination phase (27) determines the rate assuming that the transmitter would be successful in transmitting in slot $n+1$ ($I_{n+1}$ is assumed to be equal to 1). However in (28), updating the value function requires the knowledge of whether the transmission is successful or not. This because the immediate cost function $c(\cdot, \cdot, \cdot, \cdot, \cdot)$ depends on $I_{n+1}$, i.e., whether the transmission is successful or not. Thus the update phase updates the value function and LM in each slot based on the success of the transmission.

5) $(q^0, x^0)$ is any pre-designated state. On the RHS in (28), the value function corresponding to this state is subtracted in order to keep the iterates bounded.

6) The LM iteration in (29) ensures that the specified delay constraint is satisfied.

7) The sequences $f_n$ and $e_n$ have properties specified in (13), (14) and (15). The reasons for imposing these properties have been explained in Section III-C.

C. Proof of Convergence

**Theorem 1:** For the rate determination algorithm (27), (28) and (29), the iterates $(V_n, \lambda_n) \to (V, \lambda^*)$.

**Proof:** The proof of convergence is exactly similar to that in [28]. The probability of transmission failure in each slot serves as an extra noise term. The algorithm being a stochastic approximation based online algorithm, averages out this extra noise term and determines the optimal policy and the optimal LM.

V. AN ONLINE PRIMAL DUAL ALGORITHM FOR THE MULTI-USER PROBLEM

In this section, we propose a suboptimal approach to solve the problem in (7). To avoid the state space explosion, in the proposed algorithm, we determine the rate $R^*_n$ in $\mathcal{F}^*_n$ for a user $i$ in a slot $n$, if it is scheduled, based on its state $S^*_n = [Q^*_n, X^*_n]$ alone instead of the entire system state $S_n = [Q_n, X_n]$. Note that $S^*_n \in \mathcal{Q} \times \mathcal{X}$. The rate $R^*_n$ is determined using a rate allocation policy $\rho^i$, i.e., a mapping from the history of states and rate allocations for user $i$ to its transmission rate. Once the rate $R^*_n$ for each user $i$ is determined, the next task is to determine a user to be scheduled in that slot. The user selection policy $\kappa$ is a mapping, $\kappa : \mathcal{F}^*_n \times \ldots \times \mathcal{F}^{n}_n \to \mathcal{I}$.

A. Rate Allocation Algorithm for a User

The rate allocation algorithm for each user behaves as if it were controlled by a single user policy in the presence of transmitter errors as explained in Section IV. Each user $i$ determines the rate $R_{n+1}^i$ at which it would transmit in slot $n+1$ if it were to be scheduled in slot $n+1$ and informs this rate to the base station. The base station uses the user selection algorithm to schedule a user. The users who are not scheduled in a slot update their value function assuming transmitter errors, while the user who is scheduled updates its value function assuming successful transmission.

**Remark 2:** In the case of the single user scenario with transmitter errors, the probability with which a transmission is unsuccessful is independent of the scheduler action, i.e., the transmission rate determined by the online algorithm. In the multi-user scenario, this independence does not hold. This makes the problem a multi-agent learning problem [36], [37] where each agent (user) attempts to learn the optimal strategy and the actions taken by an agent (a user) influences the actions taken by the other agents (users).

B. User Selection Algorithm

The user selection algorithm is simple: select the user with the largest $R^*_n$, i.e., select the user with the best rate. The intuition behind this is the following. The rate allocation algorithm of a user $i$ would direct it to transmit at a high rate $R^*_n$ under two circumstances: either the channel condition for that user is very good, in which case, transmission at high rate saves power, or the delay constraints of that user are not being satisfied. Thus selecting a user with a high rate results in either power savings or the user delay constraint being satisfied.

C. Algorithm Details and Implementation

The rate allocation algorithm is implemented on the user devices while the user selection algorithm is implemented at the base station, as illustrated in Figure 2. From (27) note that the rate determination phase requires $X^*_n$, i.e., the knowledge of the channel state at the base station. The communication
overhead incurred by the base station in informing a user the channel state perceived by it depends on the number of states used to represent the channel. We represent the channel using 8 states. Hence the base station needs 3 bits per slot in order to inform a user the channel state perceived by it. The users inform the base station the rate at which they would transmit if they were to be scheduled. We allocate 3 bits for conveying this information, i.e., the system can employ 8 rates. The user selection algorithm then determines the user to be scheduled and all the users are informed about this decision. The rate allocation algorithm at each user then enters the update phase where the value function and the LM for each user are appropriately updated using (28) and (29). The algorithm thus continues in each slot $n$. The rate allocation algorithm that is executed at each user device is illustrated in Algorithm 1 where, steps 4-8 represent the rate determination phase, while steps 10-14 represent the update phase. The user selection algorithm executed at the base station is detailed in Algorithm 2.

D. Discussion

Here we discuss certain aspects of the online algorithm:

1) **Computational complexity:** The computational complexity of the rate allocation algorithm executed at a user device is independent of the number of users in the system. This is because the rate allocation algorithm for any user $i$ is directly dependent on the user $i$ state $S^i$ only and is independent of the states of the other users. The user selection algorithm has to determine the maximum of $N$ numbers and hence is linear in $N$. Thus the computational complexity of the user selection algorithm grows only linearly with the number of users.

2) **An auctioning interpretation:** The solution can be interpreted as an auction, where the user selection algorithm auctions each time slot. The users bid in the form of their transmission rates to the user selection algorithm, which allocates the time slot to the user bidding the highest rate. The rate bid by a user is dependent on its channel state and queue length constraint violation (i.e., the difference between the current queue length and the queue length constraint). If the channel state is quite good and queue constraint violation is large, the user bids a high rate. This is because transmitting at a high rate when the channel state is good saves power, while doing it when the queue length constraint violation is large aids in satisfying the delays. Note that the users do not bid unnecessarily high rates because that might result in higher power consumption. For a user, not winning an auction in a certain slot, implies that other users either have better channel conditions or higher queue length constraint violation or both. If a

---

**Algorithm 1:** The Rate Allocation Algorithm at the User $i$ Device

```
1: Initialize the value function matrix $\hat{V}^i(q, x) \leftarrow 0 \ \forall q \in Q, x \in X$
2: Initialize LM $\lambda^i \leftarrow 0$
3: Initialize slot counter $n \leftarrow 1$
4: Initialize queue length $q^i \leftarrow 0$
5: Initialize channel states $x^i \leftarrow 0$, $x^i' \leftarrow 0$
6: Reference state $s^{i,0} = (0, x^1)$, where $x^1 \in X$
7: while TRUE do
8:    while Base station has not informed the channel state $x^i'$ do
9:       wait
10:   end while
11:   Determine the number of arrivals $A_{n+1}^i = a^i$ in the current slot
12:   Determine the queue length in the current slot $Q_n^i = q^i$
13:   Use the rate determination phase of the rate allocation algorithm, i.e., (27) to determine the rate $r^i$, for transmission
14:   Determine the power $P(x^i', r^i)$ required to transmit $r^i$ bits using (3)
15:   Inform the base station of the rate $r^i$
16:   while Base station has not scheduled a user do
17:      wait
18:   end while
19:   if user $i$ is scheduled then
20:      $u^i \leftarrow r^i$
21:   else
22:      $u^i \leftarrow 0$
23:   end if
24:   Update the component $(q^i, x^i)$ of the value function matrix $\hat{V}^i$ using (28). Rest of the components of the matrix remain unchanged
25:   Update the LM $\lambda^i$ using (29) ($Q_n^i = q^i$)
26:   $q^i \leftarrow q^i + a^i - u^i$
27:   $x^i \leftarrow x^i'$
28:   $n \leftarrow n + 1$
29: end while
```
user does not win the auction for a certain number of slots successively, its queue length grows thus forcing it to bid a higher rate. Motivated by this interpretation, we refer to the scheduling scheme proposed in this paper as the Auctioning Algorithm (AA).

VI. SIMULATION SETUP AND RESULTS

We demonstrate the performance of our algorithm under the IEEE 802.16 [1] framework through our simulations in a discrete event simulator. Specifically, we intend to demonstrate the following:

1) The algorithm satisfies the delay constraints of all the users.
2) The algorithm is efficient in terms of the power consumed for each of the users. Moreover, power consumed is commensurate with the delay requirements, average arrival rates and the channel states of the users.
3) The sum power consumed under our algorithm is marginally more than that consumed under the optimal algorithm in Section III-C.
4) Average power expended by the users under our algorithm, is much less than that expended under the M-LWDF scheme [13].

A. M-LWDF Algorithm Details

The M-LWDF scheduler [13] attempts to minimize the user delay. It also considers the probability with which a user’s queue length is allowed to exceed a certain target queue length. We assume that this probability is the same for all the users and hence adapt the M-LWDF scheme for our scenario by ignoring it in the present simulations. Specifically, the adapted M-LWDF scheme schedules a user \( i \) in each slot such that,

\[
i = \arg \max_j \tau_n^j \times U_n^j,
\]

where \( \tau_n^j \) is the delay experienced by the head of the line packet for user \( j \). M-LWDF scheme transmits at a constant maximum power in each time slot. We first determine the average delays experienced by the users under the M-LWDF scheme for various transmission powers. We consider the values of these delays to be the delay constraints for our algorithm. We determine the average delays experienced by the users under our algorithm and also the power expended by the users under our algorithm for the same maximum transmission power in each slot as in the M-LWDF scheme. We compare the average power expended by the users under our algorithm, with that expended under the M-LWDF scheme. We perform the simulations within the framework of an IEEE 802.16 system. Next, we provide some details regarding the IEEE 802.16 system.

B. The IEEE 802.16 System

The IEEE 802.16 standard specifies two modes for sharing the wireless medium: point-to-multipoint (PMP) and mesh. In this paper, we concentrate on the PMP mode where a centralized base station (BS) serves multiple subscriber stations (SSs). We consider the uplink (UL) transmissions. IEEE 802.16 medium access control (MAC) specifies four different scheduling services in order to meet the QoS requirements of various applications. These are: unsolicited grant service (UGS) (for real-time applications with strict delay requirements), real-time polling service (rtPS) (for real-time applications with less stringent delay requirements), non-real-time polling service (nrtPS) and best effort (BE) (for applications that do not have any delay requirements). However, unlike BE connection, nrtPS connection is reserved a minimum amount of bandwidth. We consider the residential scenario as in [38]. It consists of a BS providing Internet access to the subscribers. Although the standard does not specify any QoS class for providing average delays, we argue that the nrtPS must be extended to cater to the average delay requirements of the users. The unicast polling service of nrtPS can be extended to inform a user the channel state perceived by the base station as well as to determine the rate at which a user would transmit if it were to be scheduled. The scheduling algorithm can thus be implemented as a part of nrtPS.

The system can be operated in either time division duplex (TDD) or frequency division duplex (FDD) mode. We assume the FDD mode of operation where all SSs have full-duplex capability. We consider a single carrier system (WirelessMAN-SC) with a frame duration of 1 msec and bandwidth \( W \) of 10 MHz. We assume that the users transmit at a rate such that data is delivered reliably to the base station. Hence we do not consider retransmissions and Automatic Repeat Request (ARQ). The SSs employ the following modulations: 64 Quadrature Amplitude Modulation (QAM), 16 QAM, Quadrature Phase Shift Keying (QPSK), QPSK with 1/2 rate convolutional code which provide us with 4 rates of transmission.

C. Simulation Setup and Results

Internet traffic is modeled as a web traffic source [38], [39]. Packet sizes are drawn from a truncated Pareto distribution (shape factor 1.2, mode = 2000 bits, cutoff threshold = 10000 bits) which provides us with an average packet size of 3860 bits. In each time frame, we generate the arrivals for all the users using Poisson distribution\(^2\). Arrivals are generated in

\(^2\)AA does not rely on the Poisson arrival process of the users, we simulate using the Poisson process only for the purpose of illustration.

\[\text{Algorithm 2: The User Selection Algorithm at the Base Station}
\]

1: while TRUE do
2:   for \( i \in 1, \ldots, N \) do
3:     Estimate the channel state \( X_{n+1}^i = x^i \) in the current slot for user \( i \)
4:   end for
5:   while Rate of each user is not known do
6:     wait
7:   end while
8:   Inform \( x^i \) to user \( i \)
9: end while
10: Schedule user \( k \) in the current slot
11: end while
an i.i.d. manner across frames. We fragment the packets into fragments of size 2000 bits each. Fragments of size less than 2000 bits are padded with extra bits to make them of size 2000 bits. Since all fragments are of equal size, we determine the transmission rate for users in terms of number of fragments per frame instead of bits per frame. We simulate a Rayleigh fading channel\(^3\) for each user. For a Rayleigh model, channel state\(^{X_i}\) is an exponentially distributed random variable with probability density function given by \(f_{X_i}(x) = \frac{1}{\alpha^i} e^{-\frac{x}{\alpha^i}}\), where \(\alpha^i\) is the mean of \(X^i\). We know from (3) that the power required for transmitting \(u\) fragments of size \(\ell\) bits when the channel state is \(x\) is given by, \(P(x, u) = \frac{N_0 W}{\alpha^i} (2^{u/\ell} - 1)\), where \(N_0\) is the power spectral density of the additive white Gaussian noise and \(W\) is the received signal bandwidth. We assume that the product \(W N_0\) is normalized to 1. We measure the sum of queuing and transmission delays of the packets and ignore the propagation delays. In all the scenarios described below, a single simulation run consists of running the algorithm for 100000 frames and the results are obtained after averaging over 20 simulation runs.

### Scenario 1: Comparison with the Optimal Algorithm
This scenario demonstrates that the sum power expended by AA is very close to that expended by the optimal algorithm (OA) suggested in Section III-C. For this scenario, we assume \(N = 2\), i.e., two users. The channel state can be either bad (\(\alpha^1 = 0.1422 (-8.47 \text{ dB})\)) or good (\(\alpha^2 = 2.0796 (3.18 \text{ dB})\)). We assume a buffer of size 10 packets (\(B = 10\)) at each user. In each frame the arrivals are generated using the Poisson distribution with mean 0.05 packets/msec (0.184 Mbits/sec/user). Packet lengths are Pareto distributed with parameters as discussed previously in this section. In each frame, we generate a Rayleigh random variable with mean 0.9817 (–0.08 dB). If the value taken by the random variable is greater than 2.0796, the channel state is assumed to be good, else channel state is assumed to be bad. The peak transmission power in any slot is fixed at 3 Watts. We compare the sum power for the two users for the two schemes in Table I. It can be seen that both the schemes satisfy the delay constraints. The power required by AA is marginally more than that required for the OA.

For the rest of the scenarios, we discretize the channel into eight equal probability bins, with the boundaries specified by \(\{(-\infty, -8.47 \text{ dB}), [-8.47 \text{ dB}, -5.41 \text{ dB}), [-5.41 \text{ dB}, -3.28 \text{ dB}), [-3.28 \text{ dB}, -1.59 \text{ dB}), [-1.59 \text{ dB}, -0.08 \text{ dB}), [-0.08 \text{ dB}, 1.42 \text{ dB}), [1.42 \text{ dB}, 3.18 \text{ dB}), [3.18 \text{ dB}, \infty)\}\}. For each bin, we associate a channel state and the state space \(X = \{\text{bins}\}\). We determine the channel state based on the bin that contains \(X^i\) as explained above. We perform multiple experiments. In the symmetric scenario, in successive experiments, the delay constraints of all the users are fixed at 25, 50, 75, 100, 125, 150, 175 msecs respectively. We measure the average delay experienced and the average power expended.

### Table I: Comparison between Optimal Algorithm (OA) and the Auctioning Algorithm (AA)

| Delay Constraint | Achieved Delay for OA | Achieved Delay for AA | Power for OA | Power for AA |
|------------------|-----------------------|-----------------------|--------------|--------------|
| 3 msec           | 3.09119               | 3.07970               | 0.26359      | 0.26522      |
| 5 msec           | 3.59760               | 3.40740               | 0.24756      | 0.26097      |

### Table II: Comparison between M-LWDF and the AA

| Power Constr. | Achieved Delay - M-LWDF | Achieved Delay - AA | Average Power - M-LWDF | Average Power - AA |
|---------------|-------------------------|---------------------|------------------------|-------------------|
| 1.5           | 28.71332                | 28.12900            | 0.07499                | 0.04206           |
| 2             | 28.18142                | 28.01677            | 0.09999                | 0.08175           |
| 2.5           | 22.57460                | 22.03732            | 0.12499                | 0.05530           |
| 3             | 22.12825                | 18.27730            | 0.14999                | 0.10907           |
| 3.5           | 21.95025                | 16.36487            | 0.17499                | 0.07026           |
| 4             | 20.09445                | 16.39282            | 0.19999                | 0.07073           |
| 4.5           | 20.09445                | 14.74080            | 0.22397                | 0.07074           |

\(^3\)AA does not rely on the Rayleigh channel, we simulate using a Rayleigh channel only for the purpose of illustration.
by each user in each experiment. These quantities for a user selected at random are plotted in Figure 3. In the asymmetric case, the delay constraint of the users in Group 1 are fixed at 100 msec in each experiment, while the delay constraints of the users in Group 2 are fixed at 25, 50, 75, 100, 125, 150, 175 msec in successive experiments. Average delay suffered by a user selected at random from Group 1 and Group 2 and power consumed by them are plotted in Figure 4. It can be observed from Figures 3(a) and 4(a) that the delay constraints are satisfied in both the cases. Moreover, from Figures 3(b) and 4(b) it can be observed that power expended is a convex decreasing function of the delay constraint imposed by the user. Larger delay constraints imply that much lesser power is required to satisfy the constraint.

**Scenario 4:** In this scenario, we demonstrate that the AA satisfies the user specified delay constraints for various channel conditions. We consider two cases: symmetric and asymmetric. The delay constraints of all the users are kept constant at 100 msec. For the symmetric case, we fix $\alpha_i$ as $-13$ dB, $-8.47$ dB, $-5.41$ dB, $-3.28$ dB, $-1.59$ dB, $-0.08$ dB, $1.42$ dB, $V_i$ in successive experiments. Rest of the parameters are the same as in Scenario 3. We measure the average delay suffered by each of the users and the average power consumed by each of them. These quantities are plotted in Figure 5. In the asymmetric case, we maintain the average channel state for users in Group 1 constant for all the experiments, i.e., $\alpha_i = -0.08$ dB, $i \in 1, \ldots, 10$. For the users in Group 2, i.e., $\alpha_i$ for $i \in 11, \ldots, 20$, the average channel state is fixed at $\alpha_i = -13$ dB, $-8.47$ dB, $-5.41$ dB, $-3.28$ dB, $-1.59$ dB, $-0.08$ dB, $1.42$ dB, in successive experiments. Average delay suffered by a user in Group 1 and in Group 2 and power consumed by them are plotted in Figure 6. It can be observed from Figures 5(a) and 6(a) that the delay constraints are satisfied even for extremely poor channel conditions. Moreover, from Figures 5(b) and 6(b) it can be observed that the scheme is able to satisfy the delay constraints above a certain average channel state\(^4\). Better channel conditions imply that much lesser power is required to satisfy the delay constraints.

**Scenario 5:** In this scenario we demonstrate the range of arrival rates for which the AA satisfies the user specified delay constraint of 100 msec. We consider two cases - symmetric and asymmetric. In the symmetric case, the arrival rates of all the users are fixed at 0.2702 to 0.5018 Mbits/sec (0.05 to 0.12 packets/msec) in successive experiments. Rest of the parameters are same as in Scenario 3. We measure the average delay suffered and average power expended by each user. These quantities for a user chosen at random are plotted in Figure 7. In the asymmetric case, the arrival rate of the users in Group 1 is fixed at 0.386 Mbits/sec (0.15 packets/msec) for all the experiments, while the arrival rates of the users in Group 2 are increased from 0.1351 to 0.2509 Mbits/sec (0.07 to 0.13 packets/msec) in 8 steps in successive experiments. Rest of the parameters are same as in Scenario 3. Average delay suffered by a user from Group 1 and Group 2 (each selected at random) and power consumed by them are plotted in Figure 8. It can be observed from Figures 7(a) and 8(a) that the delay constraints are satisfied in both the cases. From Figures 7(b) and 8(b) it can be seen that power expended is an increasing function of the average arrival rates for the same delay constraint. Higher the arrival rate, higher is the power expended.

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Fig. 3. Variation of achieved delay and power consumed for various delay constraints - symmetric case

Fig. 4. Variation of achieved delay and power consumed for various delay constraints - asymmetric case

Fig. 5. Variation of achieved delay and power consumed for various channel conditions - symmetric case
Fig. 6. Variation of achieved delay and power consumed for various channel conditions - asymmetric case

Fig. 7. Variation of achieved delay and power consumed for various arrival rates - symmetric case

Fig. 8. Variation of achieved delay and power consumed for various arrival rates - asymmetric case