Partitioning of diluted anyons reveals their braiding statistics

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Correlations of partitioned particles carry essential information about their quantumness. Partitioning full beams of charged particles leads to current fluctuations, with their autocorrelation (namely, shot noise) revealing the particles’ charge. This is not the case when a highly diluted beam is partitioned. Bosons or fermions will exhibit particle antibunching (owing to their sparsity and discreteness). However, when diluted anyons, such as quasiparticles in fractional quantum Hall states, are partitioned in a narrow constriction, their autocorrelation reveals an essential aspect of their quantum exchange statistics: their braiding phase. Here we describe detailed measurements of weakly partitioned, highly diluted, one-dimension-like edge modes of the one-third filling fractional quantum Hall state. The measured autocorrelation agrees with our theory of braiding anyons in the time domain (instead of braiding in space); with a braiding phase of $2\theta = 2\pi/3$, without any fitting parameters. Our work offers a relatively straightforward and simple method to observe the braiding statistics of exotic anyonic states, such as non-abelian states, without resorting to complex interference experiments.

Fractional quantum Hall (FQH) systems host exotic quasiparticles (QPs), named anyons, that carry fractional charges and obey fractional statistics. An adiabatic braiding of abelian anyons leads to an added fractional statistical phase $2\theta$, whereas for non-abelian anyons, the original state transforms into another degenerate state. The charge of the QPs can be determined by partitioning a full beam of QPs, leading to excess shot noise (autocorrelation of charge fluctuations). Here we demonstrate that partitioning a dilute anyon beam reveals the braiding phase of the QPs in the autocorrelation’s Fano factor.

The traditional strategy to observe the statistics of QPs of FQH states involves interference in a Fabry–Pérot interferometer or a Mach–Zehnder interferometer, where edge modes circulate localized QPs in the insulating bulk. Another recent approach exploited a configuration of three quantum point contacts (QPCs) where two highly dilute beams, partitioned by two side QPCs, ‘collided’ at a central QPC (a typical Hong–Ou–Mandel configuration). Measured for the anyonic one-third filling FQH state, the cross-correlation of the back-scattered QPs beams was interpreted as a partly anyonic bunching at the central QPC.

A different origin of the three-QPC outcome is based on time-domain braiding between the two impinging dilute anyon beams and the thermally (or vacuum) excited particle–hole anyon pairs at the central QPC. To test this scenario, we focused on a two-QPC geometry where one QPC dilutes an anyon beam, further partitioned by a second QPC, leading to excess shot noise (autocorrelation). Testing under different conditions, such as beam dilution, the second QPC’s transmission and beam travel distance, we found an anomalous autocorrelation Fano factor ($F_{\text{dilute}}$) that agrees with our theory of time-domain braiding at the second (partitioning) QPC (without any fitting parameters).

Notably, although the theoretical description of the time-domain anyon braiding in a QPC is based on the chiral Luttinger liquid (CLL) theory (or the equivalent conformal field theory), the saddle potential in the QPCs is far from the ideal barrier in the CLL theory. To overcome this difficulty, we developed a theoretical description that hybridizes the CLL theory and a phenomenological theory in the spirit of the successful ubiquitous approach of charge determination via autocorrelation measurements.

Shot noise of full beam

Our experimental set-up is shown in Fig. 1a (Supplementary Note I). The source (S) is biased by voltage $V_S$, injecting a full QP beam with current $I_S = GV_S$, flowing chirally along Edge1, with conductance $G = e^2/h$ at filling factor $v = 1/3$, where $e$ is the electron charge and $h$ is the Planck constant. The full beam is highly diluted by QPC1, with a reflection probability $R_{\text{QPC1}}$ and thus current $I_{\text{QPC1}} = I_S R_{\text{QPC1}}$. The dilute beam flows chirally along Edge2, impinging at QPC2 (being 2 $\mu$m away), where it is further partitioned. The scattered current fluctuations are measured after being amplified by amplifiers A and B, with the spectral densities $S_A, S_B$ and $S_{\text{AB}}$ measured. The charge of the diluted QPs $e^*$ was determined from the autocorrelation shot noise of QPC2.

$$S_{\text{QPC1}} = 2e^*I_{\text{QPC1}}(1 - R_{\text{QPC1}}) \left[ \coth \left( \frac{e^*V_S}{2k_B T} \right) - \frac{2k_B T}{e^*V_S} \right].$$

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which was determined by $S_{\text{QPC1}} = S_0 + S_A + 2S_{\text{AB}}$, with the electron temperature $T$ and the Boltzmann constant $k_B$ (Fig. 1b and Methods). The data agree well with equation (1) with $e^* = e/3$ (a similar measurement was performed with QPC2 (Supplementary Note III)).

We now elaborate on the phenomenological hybridization of the non-interacting expression in equation (1) and the interacting theory of the CLL. In the limit of very large $V_s/T$ and very small $R_{\text{QPC}}$, equation (1) agrees with the prediction of the CLL theory. In the CLL theory, the current and shot noise are expressed as $I_{\text{QPC}} = e^* (W_{1\rightarrow2} - W_{2\rightarrow1})$ and $S_{\text{QPC2}} = 2e^2 W_{1\rightarrow2}/(k_B T)$. When a full (undiluted) biased beam obeys $e^* V_s > k_B T$, the rate $W_{1\rightarrow2}$ is exponentially suppressed compared with $W_{2\rightarrow1}$. Resulting in $S_{\text{QPC2}} = 2e^2 V_s /k_B T$. The phenomenological binomial factor $(1 - R_{\text{QPC}})$ in equation (1) relates to charge fluctuation of non-interacting particles in the QPC. The temperature-dependent term emanates from the detailed balance principle.

**Time-domain braiding by diluted QPC**

We extend equation (1) to the two-QPC configuration. When a diluted beam is partitioned by QPC2, the spectral density $S_{\text{QPC2}}$ of the excess autocorrelation of current fluctuations in QPC2 can be expressed as

$$S_{\text{QPC2}} = S_{\text{dilute}} \times 2e I_{\text{QPC1}} R_{\text{QPC2}} (1 - R_{\text{QPC2}}) \left[ \coth \left( \frac{e^* V_s}{2 k_B T} \right) - \frac{2 k_B T}{e^* V_s} \right].$$

with $S_{\text{dilute}}$ being dependent on the diluting $R_{\text{QPC2}}$ of the beam (Supplementary Note III), and $R_{\text{QPC2}}$ is the reflection probability of QPC2. This expression has the same structure as equation (1), with the replacement of $I_0$ with $I_{\text{QPC1}}$ and $R_{\text{QPC1}}$ with $R_{\text{QPC2}}$. In the limit of large $V_s$ and small $R_{\text{QPC2}}$, it becomes $S_{\text{QPC2}} = 2e^2 I_{\text{QPC1}} R_{\text{QPC2}}$ with the current $I_{\text{QPC1}} = |I_{\text{QPC2}}| R_{\text{QPC1}}$. It is noted that for free fermions $S_{\text{dilute}} = 1$.

The Fano factor $S_{\text{dilute}}$ distinguishes between different partitioning processes. We consider the limits of large $V_s$ and small $R_{\text{QPC2}}$, where $I_{\text{QPC2}} = e^* (W_{1\rightarrow2} - W_{2\rightarrow1})$, with spectral density $S_{\text{QPC2}} = 2e^2 (W_{1\rightarrow2} + W_{2\rightarrow1})$, and $S_{\text{dilute}} = (W_{1\rightarrow2} + W_{2\rightarrow1})/(W_{1\rightarrow2} - W_{2\rightarrow1})$. Among possible partitioning processes, we first consider the trivial partitioning where an anyon in the dilute beam directly tunnels at QPC2 from Edge2 to Edge3 (Fig. 2a). This ubiquitous partitioning manifests particle antibunching $e^*$ + $e^*$, regardless of whether the particle is a boson, a fermion, or an anyon. Here $S_{\text{dilute}} = 1$ as the rate $W_{1\rightarrow2}$ at high enough voltage $(e^* V_s \gg k_B T)$, in a similar fashion to the partitioning of a full beam.

However, the trivial partitioning process of a highly diluted anyonic beam with a high source voltage $V_s$ leads to only a subdominant contribution to the observables. Instead, a more dominant process, which involves anyon braiding, takes place $2\delta$. In this process, which we call time-domain braiding, the anyon that tunnels at QPC2 is not an arriving anyon of the dilute beam but a thermally (or virtually) excited anyon. The excited anyon tunnels between Edge2 and Edge3 (for example, from Edge2 to Edge3) at time $t_1$, leaving a hole behind (on Edge2). This anyon tunnels back at time $t_2$ and is ‘pair annihilated’ with the hole as long as $t_2 - t_1 \leq h/k_B T$, where $h$ is the reduced Planck constant. These probabilistic events of the particle–hole excitation and recombination form a loop in the time domain. The time-domain loop of the thermal anyon in QPC2 braids with the anyons in the diluted beam that arrive at QPC2 during the time interval $t_2 - t_1$ (Fig. 2b), thus gaining a braiding phase (see below). The time-domain braiding dominates over the trivial partitioning as, according to the CLL theory, anyon tunnelling at a QPC becomes suppressed at higher energy. Within QPC2, anyon tunnelling for a thermal particle–hole pair excitation (with energy approximately $k_B T$) happens much more frequently than the tunnelling of an arriving diluted anyon with energy approximately $e^* V_s \approx k_B T$, and required for the trivial partition.

Being fundamental in our experiment, we stress the time-domain braiding process again. The thermal particle–hole excitation happens at QPC2 between Edge2 and Edge3 either before (at $t_1$) or after (at $t_2$) the arrival of the diluted anyons at QPC2. These two subprocesses differ by an exchange phase, as the spatial order of the anyons (the thermal particle–hole and the arriving dilute anyons) on Edge2 differs between the subprocesses (Supplementary Fig. 9 and Supplementary Video 1). The interference between the subprocesses forms the time-domain loop of the thermal anyons that braids the diluted anyons. This braiding process leads to a modified Fano factor $S_{\text{dilute}}$ (Methods and Supplementary Note III)

$$S_{\text{dilute}} = \cot \delta \cot \left( \frac{\pi}{2} - \theta \right) (2\delta - 1) = 3.27,$$
when \( R_{QPC} \approx 1 \). Here, \( \delta \) is the scaling dimension of anyon tunnelling at QPC2, and \( 2\theta = (0, 2\pi) \) is the braiding angle. The value \( F_{\text{dilute}} = 3.27 \) is obtained with the ideal \( \nu = 1/3 \) state, with the corresponding \( \delta = 1/3 \) and \( \theta = \pi/3 \).

As measuring the excess autocorrelation of a highly diluted beam is challenging, we developed a phenomenological theory for a moderately diluted beam. Going beyond the CLL theory, the critical step is the identification of the average braiding phase in the time-domain braiding process

\[
(\epsilon^{2i\theta})_{\text{binomial}} = \sum_{k=0}^{n} P_k \epsilon^{2i\theta} = (1 - R_{QPCQ} + R_{QPCQ} \epsilon^{2i\theta})^n, \tag{4}
\]

where \( k \) denotes the number of anyons in the dilute beam which arrive at QPC2 in the time interval \( t_\tau - t_\tau \). The phase term \( \epsilon^{2i\theta} \) corresponds to the braiding phase of a thermally excited anyon with each of the arriving anyons. The probability \( P_k \) of the \( k \) anyon event is naturally assumed to follow the binomial distribution \( P_k = \binom{n}{k} (R_{QPCQ})^k (1 - R_{QPCQ})^{n-k} \), that is, the probability for \( k \) anyons being reflected by QPC1 with reflection probability \( R_{QPC} \). The maximum value of \( k \) is \( n = I(\tau_\tau - \tau_\tau)/e\). The average braiding phase is implemented in the calculation of \( F_{\text{dilute}} \) using the ideal CLL parameters (as above) and integrating over the time difference \( t_\tau - t_\tau \). As the beam is less diluted (that is, fuller), the trivial partitioning process is also considered in the above expression, although its contribution is small (Methods and Supplementary Note III). It is noted that the average braiding phase is \( (\epsilon^{2i\theta})_{\text{binomial}} = 1 \) for fermions \( (\theta = \pi) \) and for bosons \( (\theta = 0) \).

**Experimental results**

We measured the excess spectral density \( S_\nu \) of the excess autocorrelation for two partitioning cases: injection of a full beam and injection of a dilute beam. We first performed these measurements in the integer regime (the outer edge mode of filling factor \( \nu = 3 \)). The Fano factors in both cases agree with trivial partitioning \( F_{\text{dilute}} = 1 \), with the expected electronic charge \( e^* = e \) (Supplementary Note II). Similar measurements were performed at filling \( \nu = 1/3 \). Injecting a full beam led to \( S_\nu \) agreeing with equation (I) with charge \( e^* = e/3 \) (Supplementary Note II). Injecting a dilute beam, with \( R_{QPCQ} = 0.11 \approx 1 \), the experimental values of \( F_{\text{dilute}} \) were found close to \( F_{\text{dilute}} = 3.27 \) (equations (3) and (4), and Fig. 3); ruling out the trivial process \( F_{\text{dilute}} = 1 \) and substantiating the time-domain braiding process. Here we utilized that \( S_\nu \) coincides with \( S_{QPCQ} \) at large voltages (Supplementary Note IV).

In Fig. 4, the spectral density \( S_\nu \) of the autocorrelation was measured with varying dilutions, \( R_{QPCQ} \), and different partitioning, \( R_{QPCQ} \). With less dilution (‘fuller’ beam), the time-domain braiding process gives rise to smaller \( F_{\text{dilute}} \) and the trivial partitioning contribution to \( F_{\text{dilute}} \) is higher, albeit still small. Notice the excellent agreement between the experimental data and the phenomenological theory over a wide range of \( \nu V / T, R_{QPCQ} \) and \( R_{QPCQ} \) without fitting parameters. The deviation of the data from the theory at large \( \nu V \), complicated with less dilution (larger \( R_{QPCQ} \)), is probably due to the variation of the QPC reflection with the source voltage \( V_\nu \) (not taken into account in the theory).

**Promise of time-domain braiding**

It might be worthwhile to compare our two-QPC configuration with a recent work based on a three-QPC setup25. In the latter work, the measured cross-correlation (of partitioned dilute 1/3-filling beams) agreed with quantum calculations8,19, and was attributed to ‘anyon bunching by collision’ following a classical lattice model25. The collision is a different process from the time-domain braiding, providing only a subdominant contribution to the cross-correlation (similarly to trivial partitioning)9. In the collision process, two dilute anyons, injected from two side QPCs, simultaneously arrive at the central QPC and the presence of one anyon alters the tunnelling of the other one (at the central QPC) owing to anyonic bunching. Consequently, we tested our theory by performing a three-QPC experiment and found the results to agree well with our phenomenological approach (at a relatively large \( R_{QPCQ} \)), supporting the underlying physics of the time-domain anyon braiding (Supplementary Note VI). Therefore, we believe that the previous three-QPC experimental results35 should be regarded as time-domain braiding rather than anyon bunching. We note that two recent experiments also support the time-domain braiding process26,27.

Here we demonstrate a relatively simple experimental configuration that identifies the statistical phase of abelian anyons in the FQH states. Our findings are also substantial considering the long-time disagreements between experiments (conductance and shot noise) and the chiral Luttinger theory28. For example, the theoretical voltage dependence of reflection probability in a QPC, \( R_{QPC} \approx V^{2\nu-2.2} \), has not been confirmed experimentally (Supplementary Note II). As such, it is worth examining the robustness of our Fano factor, \( F_{\text{dilute}} \) with respect to a
The dependence of the autocorrelation (amplifier B) on beam dilution ($R_{\text{QPC}}$) and on $R_{\text{QPC}}$ in Fig. 4 | The dependence of the autocorrelation (amplifier B) on beam dilution ($R_{\text{QPC}}$). From more to less dilution via $R_{\text{QPC}}$: a $R_{\text{QPC}} = 0.117$, b $R_{\text{QPC}} = 0.192$, c $R_{\text{QPC}} = 0.297$, and d $R_{\text{QPC}} = 0.358$; excess autocorrelation (shot noise, blue dots) in the two-QPC configuration for different values of beam dilution. The yellow dashed lines are the theoretical predictions according to the phenomenological theory of equation (2). The black dashed lines are for the trivial process. The black dotted lines are for the predictions according to the phenomenological theory of equation (2). The different values of beam dilution. The yellow dashed lines are the theoretical autocorrelation (shot noise, blue dots) in the two-QPC configuration for dilution (smaller $R_{\text{QPC}}$) minimizes the contribution of the trivial partitioning to the data, allowing the Fano factor of the autocorrelation to reach $F_{\text{dilute}} = 3.27$. See also Supplementary Fig. 5.

Fig. 4 | The dependence of the autocorrelation (amplifier B) on beam dilution ($R_{\text{QPC}}$) and on $R_{\text{QPC}}$. From more to less dilution via $R_{\text{QPC}}$: a $R_{\text{QPC}} = 0.117$, b $R_{\text{QPC}} = 0.192$, c $R_{\text{QPC}} = 0.297$, and d $R_{\text{QPC}} = 0.358$; excess autocorrelation (shot noise, blue dots) in the two-QPC configuration for different values of beam dilution. The yellow dashed lines are the theoretical predictions according to the phenomenological theory of equation (2). The black dashed lines are for the trivial process. The black dotted lines are for the predictions according to the phenomenological theory of equation (2). The different values of beam dilution. The yellow dashed lines are the theoretical autocorrelation (shot noise, blue dots) in the two-QPC configuration for dilution (smaller $R_{\text{QPC}}$) minimizes the contribution of the trivial partitioning to the data, allowing the Fano factor of the autocorrelation to reach $F_{\text{dilute}} = 3.27$. See also Supplementary Fig. 5.

Fig. 5 | Two-QPC configuration with an inter-QPC distance of 20 μm. a, Scanning electron microscope image of the experimental set-up. The gates are marked in yellow. The 2 μm QPC separation structure is shown (for comparison) in the white-bordered inset. The blue dots are the measured excess autocorrelation with dilution of $R_{\text{QPC}} = 0.261$ and $R_{\text{QPC}} = 0.176$. The measurement results agree with the trivial model (that is, integer filling factor) in equation (1) with $R_{\text{QPC}} = R_{\text{QPC}}$, suggesting energy loss and dephasing due to the long propagation distance. The black dotted line is the ideal anyonic behaviour with Fano factor $F_{\text{dilute}} = 3.27$.

Online content

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1. Hanbury Brown, R. & Twiss, R. Q. A test of a new type of stellar interferometer on Sirius. Nature 178, 1046 (1956).
2. de-Picciotto, R. et al. Direct observation of a fractional charge. Nature 389, 162 (1997).
3. Saminadayar, L. et al. Observation of the e/3 fractionally charged Laughlin quasiparticle. Phys. Rev. Lett. 79, 2526 (1997).
4. Kimble, H. J., Dagenais, M. & Mandel, L. Photon antibunching in resonance fluorescence. Phys. Rev. Lett. 39, 699 (1977).
5. Henry, M. et al. The fermionic Hanbury Brown and Twiss experiment. Science 284, 296 (1999).
6. Oliver, W. D. et al. Hanbury Brown and Twiss-type experiment with electrons. Science 284, 299 (1999).
7. Lee, B., Han, C. & Sim, H.-S. Negative excess shot noise by anyon braiding. Phys. Rev. Lett. 123, 016803 (2019).
8. Nayak, C., Simon, S. H., Stern, A., Freedman, M. & Das Sarma, S. Non-abelian anyons and topological quantum computation. Rev. Mod. Phys. 80, 1083 (2008).
9. Lee, J.-Y. M. & Sim, H.-S. Negative excess shot noise by anyon braiding. Phys. Rev. Lett. 123, 016803 (2019).
10. Leinaas, J. M. & Myrheim, J. On the theory of identical particles. Nuovo Cimento B 37, 1 (1977).
11. Arovas, D., Schrieffer, J. R. & Wilczek, F. Fractional statistics and the quantum Hall effect. Phys. Rev. Lett. 53, 722–723 (1984).
12. Nakamura, J. et al. Direct observation of anyonic braiding statistics. Nat. Phys. 16, 931 (2020).
13. de C. Chamon, C., Fried, D. E., Kivelson, S. A., Sondhi, S. L. & Wen, X. G. Two point-contact interferometer for quantum Hall systems. Phys. Rev. B 55, 2331–2343 (1997).
14. Kundu, H.K., Biswas, S., Ofek, N. et al. Anyonic interference and braiding phase in a Mach-Zehnder interferometer. Nat. Phys. 19, 515–521 (2023).
15. Bartolomei, H. et al. Fractional statistics in anyon collisions. Science 368, 6487 (2020).
16. Hong, C. K., Ou, Z. Y. & Mandel, L. Measurement of subpicosecond time intervals between two photons by interference. Phys. Rev. Lett. 59, 2044 (1987).
17. Liu, R. et al. Quantum interference in electron collision. Nature 391, 263 (1998).
18. Bocquillon, E. et al. Coherence and indistinguishability of single electrons emitted by independent sources. Science 339, 1054 (2013).
19. Rosenow, B., Levkovskiy, I. P. & Halperin, B. I. Current correlations from a mesoscopic anyon collider. Phys. Rev. Lett. 116, 156802 (2016).
20. Chung, Y.C. et al. Anomalous chiral Luttinger liquid behavior of diluted fractionally charged quasiparticles. Phys. Rev. B 67, 201004(R) (2003).
21. Heiblum, M. Fractional Charge Determination via Quantum Shot Noise Measurements, in Perspectives of Mesoscopic Physics: Dedicated to Joseph Imry’s 70th Birthday (eds Ahnony, A. & Enn-In-Whileman, O.) 115–136 (World Scientific, 2010).
22. Feldman, D. E. & Heiblum, M. Why a noninteracting model works for shot noise in fractional charge experiments. Phys. Rev. B 95, 115308 (2017).
23. Trauzettel, B., Roche, P., Glattli, D. C. & Salleur, H. Effect of interactions on the noise of chiral Luttinger liquid systems. Phys. Rev. B 70, 233301 (2004).
24. Rosenow, B. & Halperin, B. I. Nonuniversal behavior of scattering between fractional quantum Hall edges. Phys. Rev. Lett. 88, 096404 (2002).
25. Blanter, Y. M. & Buttiker, M. Shot noise in mesoscopic conductors. Phys. Rep. 336, 1–116 (2000).
26. Glidic, P. et al. Cross-correlation investigation of anyon statistics in the $\nu = 1/3$ and 2/5 fractional quantum Hall states. Phys. Rev. X 13, 011030 (2023).
27. Ruelle, M. et al. Comparing fractional quantum Hall Laughlin and Jain topological orders with the anyon collider. Phys. Rev. X 13, 011031 (2023).
28. Glattli, D. C. Tunneling Experiments in the Fractional Quantum Hall Effect Regime in The Quantum Hall Effect Progress in Mathematical Physics Vol 45 (eds Doucet, B. et al.) 163–197 (Birkhauser, 2005).

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Methods

Theory of the Fano factor

In the CLL theory and the equivalent conformal field theory, the time-domain braiding process is described by a non-equilibrium correlator $C_{\text{corr}}(t_1, t_2)$ of the anyon tunnelling operator at QPC2 in the presence of a dilute anyon beam impinging at QPC2. It is expressed as

$$C_{\text{corr}}(t_1, t_2) = \langle e^{i\theta_{t_1} - i\theta_{t_2}} \rangle,$$

where $\langle \cdot \rangle$ represents the ensemble average over the tunnelling currents and noise at QPC2 in the cases of fermions or bosons, but only with anyons.

In the dilute limit of $R_{\text{QPC}} \ll 1$, the multiplicative factor $1 - R_{\text{QPC}} + R_{\text{QPC}} e^{2i\theta_{t_1} - i\theta_{t_2}}$ is reduced to the factor $e^{2i\theta_{t_1} - i\theta_{t_2}}$ found in a previous work. Employing $C_{\text{corr}}(t_1, t_2)$ as an integral over $t_1 - t_2$, it is straightforward to compute the rates of anyon tunnelling (back and forth) at QPC2 in the time-domain braiding process. At zero temperature and $R_{\text{QPC}} \ll 1$, we get

$$W_{\text{braid}}^{(2)} = 2\theta e^{-i\theta_{t_1} - i\theta_{t_2}},$$

with the full expressions given in Supplementary Note III. In contrast to the trivial process where $W_{\text{braid}}^{(2)}$ is exponentially suppressed compared with $W_{\text{braid}}^{(2)}$ and $W_{\text{braid}}^{(2)}$, the non-negligible in the time-domain braiding. The appearance of the combination $e^{i\theta_{t_1} - i\theta_{t_2}}$ in equation (6) implies that the rates $W_{\text{braid}}^{(2)}$ and $W_{\text{braid}}^{(2)}$ vanish, and thus do not contribute to the tunnelling currents and noise at QPC2 in the cases of fermions ($\theta = \pm \pi$) or bosons ($\theta = 0$). Hence the time-domain braiding does not exist with fermions or bosons, but only with anyons.

When the time-domain braiding process dominates over other processes, the Fano factor is written as

$$S_{\text{Fano}} = 1 - R_{\text{QPC}} + 2R_{\text{QPC}} e^{2i\theta_{t_1} - i\theta_{t_2}},$$

and with the experimental measured $R_{\text{QPC}}$ as input of the calculation. We note that $W_{\text{braid}}^{(2)}$ and $W_{\text{braid}}^{(2)}$ are not negligible but much smaller than $W_{\text{braid}}^{(2)}$ and $W_{\text{braid}}^{(2)}$ for the values of $R_{\text{QPC}}$ studied in our experiments.

Obtaining $S_{\text{QPC}}$ in a two-QPC configuration

While performing the two-QPC measurements, the noise generated by QPC1 ($S_{\text{QPC1}}$) is not directly accessible (owing to the locations of the amplifiers). However, current conservation can be used to relate $S_{\text{QPC1}}$ to the correlations measured in the experiment. By current conservation in QPC2

$$I_{\text{QPC2}} = I_{\text{QPC1}} + I_{\text{QPC1}},$$

where $I_{\text{QPC1}}$ is the dilute current generated by QPC1 and $I_{\text{QPC2}}$ is the output current of QPC2 that reaches amplifier A/B (Fig. 1a). The same relation also holds for the averages

$$\langle I_{\text{QPC1}} \rangle = \langle I_{\text{QPC2}} \rangle + \langle I_{\text{QPC1}} \rangle,$$

subtracting these two equations and taking the square we arrive at a relation between the current correlations

$$S_{\text{QPC1}} = S_A + S_B + 2S_{\text{any}},$$

which allows us to obtain $S_{\text{QPC1}}$ by summing the autocorrelations ($S_A$ and $S_B$) and the cross-correlation ($S_{\text{any}}$) measured in the experiment.

Data availability

Source data are provided with this paper. All other data related to this paper are available from the corresponding authors upon reasonable request.