Possible effects of hybrid gravity on stellar kinematics in elliptical galaxies

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Abstract. We use the Fundamental Plane of Elliptical Galaxies to constrain the so-called Hybrid Gravity, a modified theory of gravity where General Relativity is improved by further degrees of freedom of metric-affine Palatini formalism of \( f(R) \) gravity. Because the Fundamental Plane is connected to the global properties of elliptical galaxies, it is possible to obtain observational constraints on the parameters of Hybrid Gravity in the weak field limit. We analyze also the velocity distribution of elliptical galaxies comparing our theoretical results in the case of Hybrid Gravity with astronomical data for elliptical galaxies. In this way, we are able to constrain the Hybrid Gravity parameters \( m_0 \) and \( \phi_0 \). We show that the Fundamental Plane, i.e., \( v_c/\sigma \) relations, can be used as a standard tool to probe different theories of gravity in the weak field limit. We conclude that Hybrid Gravity is able to explain elliptical galaxies with different stellar kinematics without the dark matter hypothesis.

1 Introduction

The Fundamental Plane (FP) is connected to the global properties of elliptical galaxies. It is represented by: the central projected velocity dispersion \( \sigma_0 \), the effective radius \( r_e \), and the mean effective surface brightness (within \( r_e \)) \( I_e \) \([1,2]\). Elliptical galaxies are usually confined in a narrow logarithmic plane of their configuration space which is referred to as the FP \([1,2]\). It is possible to show that any of the three parameters may be estimated from the other two, and together they describe a plane that falls within a more general three-dimensional phase-space. In other words, the FP is considered a bi-variate manifold in a parametric space. The FP is defined and discussed in detail in several papers, see, e.g., \([3–11]\) and references therein. This important empirical relation is given by the following equation \([5]\):

\[
\log(r_e) = a \log(\sigma_0) + b \log(I_e) + c, \tag{1}
\]

with \( a \) and \( b \) being the FP coefficients which, in general, are fixed by observations. This relation gives us the possibility to obtain observational constraints on the structure, formation, and evolution of early-type galaxies. Reversing the argument, the FP can be adopted to fix parameters of a given theory of gravity, once they are constrained by observations.

In this paper, we shall adopt Eq. (1) to constrain parameters of the so-called Hybrid Gravity, which is a recently proposed theory of gravity \([12]\) where further degrees of freedom coming from extensions of General Relativity \([13]\) are represented in the metric-affine formalism to cure shortcomings of \( f(R) \) gravity in both metric and Palatini formalism.

Specifically, in order to describe the velocity of stellar populations, one can define rotational velocity of a group of stars \( v_c \). Also, one can define a dispersion \( \sigma \) which represents the characteristic random velocity of stars. The obtained ratio \( v_c/\sigma \) is a relation which characterizes the kinematics of galaxies. In case of spiral galaxies (kinematically cold systems) \( v_c/\sigma \gg 1 \), while elliptical galaxies (kinematically hot systems) are characterized by \( 0 < v_c/\sigma < 1 \). It is the main characteristic which differentiates spiral from elliptical galaxies.

Here, we study Extended Theories of Gravity using astronomical observations for FP. In this way, we want to give constrains to parameters of Hybrid Gravity. Extended Gravity is presented in several review papers like \([13–17]\). Some experimental limits related
to Extended Theories of Gravity are reported in [18–21] and references therein.

The content of this paper is as follows. In Sect. 2, we present basics of Hybrid Gravity. In Sect. 3, we recover the FP of elliptical galaxies in the framework of this theory. In Sect. 4, we find constraints on Hybrid Gravity parameters by FP and we study the \( v_c/\sigma \) relation in the \( (m_\phi, \phi_0) \) parameter space. Section 5 is devoted to conclusions.

2 A summary of hybrid gravity

Let us present here the basic formalism for Hybrid Metric-Palatini Gravity within the equivalent scalar-tensor representation (we refer the reader to [12,22–25] for more details). The action for Hybrid Gravity is given by

\[
S = \int d^4x \sqrt{-g} \left[ R + f(R) + 2\kappa^2 L_m \right].
\]

where \( \kappa^2 \equiv 8\pi G \) is the Einstein–Hilbert term defined with the Levi–Civita connection, \( R \equiv g^{\mu\nu}R_{\mu\nu} \) is the Palatini curvature with the connection \( \hat{\Gamma}_{\mu\nu} \) independent of the metric \( g_{\mu\nu} \). As discussed in [12], it is possible to recast such an action as

\[
S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} [R + \phi R - V(\phi)] + S_m,
\]

where the scalar field \( \phi \) is derived from \( f'(R) \), the first derivative in \( R \) of \( f(R) \).

In the weak gravitational field limit, it is possible to demonstrate that Newtonian potential for Hybrid Gravity is given as [25]:

\[
\Phi(r) = -\frac{G}{1 + \phi_0} \left[ 1 - \frac{\phi_0}{3} e^{-m_\phi r} \right] \frac{M(r)}{r},
\]

where the main parameters of hybrid gravity are \( m_\phi \) and \( \phi_0 \), derived from the self-interaction potential \( V(\phi) \).

For modeling the stellar kinematics in the elliptical galaxies, we assumed that the mass distribution within them may be described by the singular isothermal sphere (SIS) model: \( M(r) = 2\sigma_{SIS}^2 G^{-1} r \), like in our previous paper [26].

3 Hybrid gravity fundamental plane

Hybrid gravity parameters can be constrained by the FP.

Let us label the Newtonian potential with \( \Phi_N(r) \) and circular velocity in this potential with \( v_N(r) \). Then, \( \Phi_N(r) = -\frac{GM(r)}{r} \) and \( v_N^2(r) = r \cdot \Phi_N(r) \). In the case of hybrid modified potential, supposing the spherically distributed mass in elliptical galaxies, for circular velocity we have \( v_c^2(r) = r \cdot \Phi'(r) \). Here, we start from the

\[
\Phi(r) = -\frac{GM(r)}{1 + \phi_0} \left[ 1 - \frac{\phi_0}{3} e^{-m_\phi r} \right] \frac{GM(r)}{r},
\]

\[
\Phi'(r) = -\frac{1}{1 + \phi_0} \left( \frac{GM(r)}{r} \right)' + \frac{\phi_0}{3(1 + \phi_0)} \left( \frac{GM(r)}{r} \right)' e^{-m_\phi r} + \frac{\phi_0}{3(1 + \phi_0)} \left( \frac{GM(r)}{r} \right) (e^{-m_\phi r})',
\]

\[
v_c^2(r) = \frac{1}{1 + \phi_0} \Phi_N'(r) - \frac{\phi_0}{3(1 + \phi_0)} \Phi_N'(r)e^{-m_\phi r} + \frac{\phi_0}{3(1 + \phi_0)} \Phi_N'(r)(-m_\phi e^{-m_\phi r});
\]

\[
r\Phi'(r) = \frac{1}{1 + \phi_0} r\Phi_N'(r) - \frac{\phi_0}{3(1 + \phi_0)} r\Phi_N'(r)e^{-m_\phi r} + \frac{\phi_0 m_\phi r}{3(1 + \phi_0)} r\Phi_N'(r)e^{-m_\phi r}.
\]

\[
v_c^2(r) = \frac{1}{1 + \phi_0} \left[ 1 - \frac{\phi_0}{3} (m_\phi r + 1) e^{-m_\phi r} \right].
\]

Having in mind that the Newtonian circular velocity at the effective radius, i.e., for \( r = r_e \), is \( v_N(r_e) = \sigma_0 \), where \( \sigma_0 \) is the observed velocity dispersion see Table 1 in Ref. [27] in relation to our considered sample of elliptical galaxies, the circular velocity at the \( r_e \) can be written in the following form:

\[
v_c^2(r_e) = \frac{\sigma_0^2}{1 + \phi_0} \left[ 1 - \frac{\phi_0}{3} (m_\phi r_e + 1) e^{-m_\phi r_e} \right].
\]

Furthermore, let us introduce the following variable:

\[ w = m_\phi r_e. \]

Then, one can obtain the following \( v_c/\sigma \) relation at \( r_e \) for the ellipticals in Hybrid Gravity:

\[
\frac{v_c}{\sigma} = \frac{v_c(r_e)}{\sigma_0} = \sqrt{\frac{1}{1 + \phi_0} \left[ 1 - \frac{\phi_0}{3} (w + 1) e^{-w} \right]}.
\]

4 Results and discussion

Let us study now the \( v_c/\sigma \) relation in the \( (m_\phi, \phi_0) \) parameter space of Hybrid Gravity. We constrain the parameters using a sample of elliptical galaxies given in [27]. Also, we study velocity dispersion \( \sigma_{\text{theor}} \) as a function of the effective radius \( r_e \). In order to check how
different kinematical properties influence the Hybrid Gravity parameters, we constrain the gravitational parameters with the following values of $v_c/\sigma$ relation: 1, 0.5 and 0.3. One should have in mind that, in the observed sample, $\sigma_0$ is equal to the Newtonian circular velocity. The method that we are using is described in detail in references [25, 26, 28–30] and references therein.

Figure 1 shows $v_c/\sigma$ relation for elliptical galaxies in Hybrid Gravity representation in the $(m_\phi \cdot r_e, \phi_0)$ parameter space. The values of $v_c/\sigma$ are calculated numerically and represented by different color shades. Some chosen values for $v_c/\sigma$ are designated by lines: 0.2, 0.3, 0.35, 0.4, 0.45 and 0.5, respectively. For a specific value of the $v_c/\sigma$ relation, these lines represent the values of the parameters for which we expect good agreement between theoretical predictions and observations. For example, if we choose the value $v_c/\sigma = 0.3$, in the area along line designated with 0.3 and in nearby region, the agreement between theory and observations is very good. If the parameters $(m_\phi \cdot r_e, \phi_0)$ are more scattered with respect to the designated lines, then this agreement will be worse. We can see that, if we increase the value of the parameter $m_\phi \cdot r_e$ in region from 0 to 6, the values for parameter $\phi_0$ change drastically. It means that, in this region, for fixed values of parameter $\phi_0$, values of $v_c/\sigma$ relation is very sensitive on parameter $m_\phi \cdot r_e$. For values $m_\phi \cdot r_e \gtrsim 6$, the lines become almost horizontal for all studied values of parameter $\phi_0$. It means that, in this region, for a fixed value of parameter $\phi_0$, values of the $v_c/\sigma$ relation are not influenced by parameter $m_\phi \cdot r_e$.

Figure 2 shows the $v_c/\sigma$ relation for elliptical galaxies in the Hybrid Gravity like Fig. 1, but for different values of parameter $\phi_0$ and $v_c/\sigma$ relation: the values of $v_c/\sigma$ relation: 0.5, 0.6, 0.7, 0.8, 0.9 and 0.95.
are designated by lines for 0.5, 0.6, 0.7, 0.8, 0.9 and 0.95, respectively. We can notice the same tendency like in Fig. 1, i.e., that if we change the value of parameter $m_\phi \cdot r_e$, in region from 0 to 3, the corresponding values for parameter $\phi_0$ change drastically. The values of $v_e/\sigma$ relation are more influenced by increasing of $m_\phi \cdot r_e$ for smaller values of $v_e/\sigma$. For values $m_\phi \cdot r_e \gtrsim 3$, studied lines become almost horizontal for all six values of $v_e/\sigma$ ratio. The main difference from Fig. 1 is that in this case saturation of $v_e/\sigma$ relation happens for $m_\phi \cdot r_e \gtrsim 3$, since that the corresponding curve becomes constant. This means that for higher values of $v_e/\sigma$ ratio its saturation could be expected for smaller values of $m_\phi \cdot r_e$ product.

Figure 3 shows theoretical values for the velocity dispersion $\sigma^{\text{theor}}$, which is assumed to be equal to $v_e(r_e)$ as a function of the effective radius $r_e$ for elliptical galaxies. The values for $\sigma^{\text{theor}}$ are presented for 4 different products of $m_\phi \cdot r_e$: 2, 4, 6 and 8. In the same figure, it is presented the Newtonian velocity dispersion at the effective radius $\sigma_0$ [27]. Theoretical values of velocity dispersion $\sigma^{\text{theor}}$ are calculated for the three different values of Hybrid Gravity parameter $\phi_0$: 0.01, 0.5 and 5.0. We can conclude that in all 4 cases for the $m_\phi \cdot r_e$ product, the agreement between theoretical results and astronomical observation is very poor in case of $\phi_0 = 0.05$. In case of $\phi_0 = 0.5$ and $\phi_0 = 5.0$ agreement is excellent. Also in case $\phi_0 = 8.0$, and for the value of $m_\phi \cdot r_e = 8$, agreement is satisfactory. For the values of $m_\phi \cdot r_e = 4$ and 6, the agreement is also satisfactory. If we look position of line designated by 0.3 in the $(m_\phi \cdot r_e, \phi_0)$ parameter space (see Fig. 1), we can conclude that these results are expected.

Figure 4 represents the same as in Fig. 3, but for $\sigma^{\text{theor}}(r_e) = 0.5 \times v_e(r_e)$ and the following three values of $\phi_0$: 0.01, 2.0 and 3.0. We can conclude that, in all 4 cases, of the $m_\phi \cdot r_e$ product, the agreement between theoretical results and astronomical observation is very poor in case of $\phi_0 = 0.01$. In case of $\phi_0 = 2.0$ and 3.0 agreement is better in case $\phi_0 = 2.0$, agreement is excellent. Also in case $\phi_0 = 3.0$ agreement is better in case $\phi_0 = 0.01$. If we look at Fig. 2, we can conclude that for higher values of $v_e/\sigma$ ratio its saturation could be expected for smaller values of $m_\phi \cdot r_e$ product.

Figure 5 represents the same as in Figs. 3 and 4, but for $\sigma^{\text{theor}}(r_e) = 0.5 \times v_e(r_e)$ and the following three values of $\phi_0$: 0.01, 4.0 and 8.0. We can see that, in all 4 cases of the $m_\phi \cdot r_e$ product, the agreement between theoretical results and observations is very poor in case of $\phi_0 = 0.01$. In case of $\phi_0 = 4.0$ and for the value of $m_\phi \cdot r_e = 2$, agreement is excellent. Also in case $\phi_0 = 8.0$, and for the value of $m_\phi \cdot r_e = 8$, agreement is satisfactory. For the values of $m_\phi \cdot r_e = 4$ and 6, the agreement is also satisfactory. If we look position of line designated by 0.3 in the $(m_\phi \cdot r_e, \phi_0)$ parameter space (see Fig. 1), we can conclude that these results are expected.

For all the three studied values of $\sigma^{\text{theor}}$, a good agreement between theoretical calculations and astronomical observations is achieved for different values of Hybrid Gravity parameter $\phi_0$. We can conclude that the parameter $\phi_0$ is very sensitive on the value of $v_e/\sigma$ relation.
Fig. 4 The same as in Fig. 3, but for $\sigma^{\text{theor}}(r_e) = 0.5 \times v_c(r_e)$ and the following three values of $\phi_0$: 0.01, 2.0 and 3.0.

Fig. 5 The same as in Figs. 3 and 4, but for $\sigma^{\text{theor}}(r_e) = 0.3 \times v_c(r_e)$ and the following three values of $\phi_0$: 0.01, 4.0 and 8.0.

5 Conclusions

We studied the $v_c/\sigma$ relation in the $(m_\phi \cdot r_e, \phi_0)$ parameter space of Hybrid Gravity. We constrain its parameters using a sample of elliptical galaxies given in [27].

According to the above results, we can conclude that the Hybrid Gravity is able to explain elliptical galaxies with different stellar kinematics described by $v_c/\sigma$ relations shown in Figs. 1 and 2, without introducing the dark matter hypothesis. For theoretical values $v_c/\sigma = 1$, we determine parameters $\phi_0$ and $m_\phi$ of Hybrid Gravity, since the same condition was assumed at the observed sample for the Newtonian gravity [27]. In order to get good agreement between numerical calculation and observations, we obtain that the value parameter $\phi_0$ has to be close to $\lesssim 0.01$ for all values...
of the product $m_{\phi} \cdot r_e$ in the studied range. In order to check how different kinematical properties influence the parameters, we also tested what happens if a given system of elliptical galaxies is kinematically hotter. We decrease the value of $v_c/\sigma$ to 0.5 and to 0.3, and the best agreement was obtained for larger value of parameter $\phi_0$. Also, we can notice that, in these cases (kinematically hotter systems), the influence of $m_{\phi} \cdot r_e$ product should be taken into account. From Figs. 3, 4 and 5, we can notice that the Hybrid Gravity parameter $\phi_0$ is very sensitive to the ratio $v_c/\sigma$. Therefore, the $v_c/\sigma$ relation and the FP can be used as standard tools to probe the Hybrid Gravity parameters in the weak gravitational field limit, as well as to constrain these parameters. Clearly, the same procedure can be adopted to test other theories of gravity.

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Author contributions

All coauthors participated in calculation and discussion of obtained results. The authors contributed equally to this work.

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