Design of Control and Switching Frequency Optimization of DC/DC Power Converter for Super-capacitor

The article discusses design of mathematical model buck and boost converter and the analysis of the optimal frequency choice of the PWM signal for IGBT. The converter is dedicated for charging and discharging of hybrid car drive super-capacitor energy storage. The main role of the converter is to optimize energy content in super-capacitor storage used to acceleration and deceleration during driving period. Mathematical model of the system in MATLAB/Simulink and converter prototype with Freescale Digital Signal Processor 56F8257 were designed. Comparing of simulation results and results measured on the prototype are discussed on the end of the article.

Key words: Optimal switching frequency, DC/DC Converter, Super-capacitor, LQ Controller

1 INTRODUCTION

Increasing of living standards increased demands for people transport. Gas emission and fuel consumption are actual problems for global environment quality on the Earth. Gas emission of car internal combustion engines brings many ecological problems in big cities specially. Electric vehicles avoid this problem partially. They do not produce gas emissions on places where they are driving but they displace it to places where the energy for batteries produced. The quality of electric energy production technology in power stations is much more better. Electric energy can be stored in batteries, but the disadvantages are the limited battery capacity and inconvenient capacity/mass ratio. The compromise between vehicles with combustion engine and electric vehicles (EV) are hybrid electric vehicles (HEV). HEV combine the combustion engine and electric motor for motion [1].

The experimental stand of hybrid drive has been created at Czech Technical University on Department of Electric Drives and Traction in Prague. This stand simulates combined hybrid drive with electric power splitter. The diagram of this stand is shown in Fig. 1. The parts of this drive are two identical induction motors. The first motor simulates a combustion engine (ICE). The traction load (BRAKE) is simulated by the second motor.

The electric power splitter (EPS) is a synchronous machine, which has two rotating parts - two rotors. On the first rotor are mounted permanent magnets. This rotor is driven by ICE. The second rotor is provided with three-phase winding, which is connected by brushes to the input of AC/DC converter (AFE). The output DC voltage that supplies the DC bus is controlled by PWM in range...
0 - 500 V (typically 200 V). The DC bus is connected to the DC/DC converter for charging/discharging the super-capacitor and to the DC/AC frequency converter which supplies the traction motor (TM). More information about experimental stand of hybrid drive are presented in [2] and [3].

2 MATHEMATICAL MODEL OF BUCK/BOOST CONVERTER

The DC/DC converter for super-capacitor is used for charging and discharging. It is needed to transfer energy from high voltage side to the low voltage side and vice versa. Therefore converter must be bi-directional.

![Diagram of bi-directional DC/DC converter](image)

The converter is able to transfer the energy from higher voltage level $V_{PRI}$ to lower voltage level $V_{SEC}$ and the other way round. The diagram is shown in Fig. 2. Other bi-directional converters are presented in [4]. To transfer the energy from higher voltage level to lower voltage level, the DC/DC converter is operating in buck mode and to transfer from lower voltage level to higher voltage level, the DC/DC converter is operating in boost mode.

2.1 Buck Mode

Buck mode is the operational mode of DC/DC converter during which the energy is being stored from the higher voltage level source to super/capacitor with lower voltage. The diagram is described in Fig. 3.

![Diagram of DC/DC converter operating in buck mode](image)

The transistor Q1 has two states: $u = 1$, when transistor is switched ON and $u = 0$, when transistor is switched OFF. For ideal assumptions:

- ideal switch $R_{ON} = 0$
- ideal capacitor $R_{SC} = 0$
- ideal inductor $R_{L} = 0$

the buck converter can be described using ordinary differentiation equations as follows:

\[
SC \frac{dv_{SC}}{dt} = i_{L} - \frac{v_{SC}}{R},
\]

\[
v_{SEC} = v_{SC} + R_{SC} \frac{dv_{SC}}{dt},
\]

\[
L \frac{di_{L}}{dt} = u \cdot v_{C} - v_{SEC} - R_{L}i_{L},
\]

\[
C \frac{dv_{C}}{dt} = \frac{v_{PRI} - v_{C}}{R_{s}} - i_{L}.
\]

With respect non-zero resistance of inductor $R_{L} \neq 0$ and nonzero Equivalent Series Resistor (ESR) of the super-capacitor $R_{SC} \neq 0$, the equations 1 can be described as follows:

\[
SC \frac{dv_{SC}}{dt} = i_{L} - \frac{v_{SEC}}{R},
\]

\[
v_{SEC} = v_{SC} + R_{SC} \frac{dv_{SC}}{dt},
\]

\[
L \frac{di_{L}}{dt} = u \cdot v_{C} - v_{SEC} - R_{L}i_{L},
\]

\[
C \frac{dv_{C}}{dt} = \frac{v_{PRI} - v_{C}}{R_{s}} - i_{L}.
\]

Inserting the second equation from (2) into the first from (2) leads to:

\[
SC \frac{dv_{SC}}{dt} = i_{L} - \frac{v_{SEC}}{R} - R_{SC} \frac{dv_{SC}}{dt},
\]

\[
SC(1 + \frac{R_{SC}}{R}) \frac{dv_{SC}}{dt} = i_{L} - \frac{v_{SEC}}{R}.
\]

Hence,

\[
u_{SEC} = u_{SC} \frac{R}{R + R_{SC}} + i_{L} \frac{R R_{SC}}{R + R_{SC}}.
\]
The mathematical model has been created using equations (5) in MATLAB/Simulink. The diagram of this model is shown in Fig. 4. The integrator of $i_L$ has limited output in range (0; 1). In real buck converter is negative current limited by diode (see diode D2 in Fig. 3).

2.2 Boost Mode

Boost mode is the operational mode of DC/DC converter during which the energy is being restored to the higher voltage level source from super-capacitor with lower voltage level. The diagram is described in Fig. 5.

With respect non-zero resistance of inductor $R_L$ and non-zero equivalent series resistor (ESR) of the super-capacitor $R_{SC}$, the equations (6) can be described as follows:

$$\frac{dv_{SEC}}{dt} = \frac{v_{SEC}}{R} + \frac{R_{SC}}{R} \frac{dv_{SEC}}{dt},$$

Inserting the second equation from (7) into the first from (7) leads to:

$$\frac{dv_{SEC}}{dt} = \frac{v_{SEC}}{R} + \frac{R_{SC}}{R} \frac{dv_{SEC}}{dt},$$

Hence,

$$v_{SEC} = \frac{v_{SC} R_{SC}}{R + R_{SC}} - \frac{i_L R_{SC} R}{R + R_{SC}}.$$
schema of this model is shown in Fig. 6. The integrator of $i_L$ has limited output in range $(0; 1)$. In real buck converter is negative current to limit by diode (see diode D1 in Fig. 5). The integrator of $v_{SC}$ has limited output in range $(0; 1)$ and it is necessary to set initial condition, because for boost converter is super-capacitor voltage source.

![Diagram of boost converter in MATLAB/Simulink](image)

**Fig. 6. Diagram of boost converter in MATLAB/Simulink**

3 SWITCHING LOSSES OF IGBT

The instantaneous power dissipation $p(t)$ had been determined by multiplication of instantaneous current $i_d$ and voltage $u_d$ values. The integral of $p(t)$ reflects the total IGBT losses over whole period.

$$W_{LOSS}(t) = \frac{1}{T} \int_0^T u_d(t) \cdot i_d(t) dt. \quad (11)$$

The typical waveform for the IGBT transistor during turn ON is shown in the Fig. 7. There are shown the waveforms for current through IGBT and voltage between collector and emitter. The total power is calculated by multiplication of current and voltage (as shown in the equation (11)). It is clear from Fig. 7 and from equation (11) that the switching losses increasing proportionally to the frequency.

4 SWITCHING FREQUENCY ANALYSIS

4.1 Buck Mode

By switching the transistor Q1, we are able to regulate the mean value of voltage on secondary side. The mean value of voltage depends linearly on the time when transistor is switched on (12).

$$V_{SEC} = V_{PRI} \cdot \text{duty} = \frac{V_{PRI} \cdot t_{ON}}{t_{ON} + t_{OFF}}. \quad (12)$$

4.2 Boost Mode

For switching frequency optimization in boost mode an analysis is needed, if optimal energy amount is stored in the inductor before the transistor is switched off. The circuit, when the transistor Q2 is switched on, can be described by following equations (we are setting the resistor $R = 0$):

$$v_{SC}(t) = \frac{\partial i_L(t)}{\partial t} \cdot L, \quad (13)$$

$$-i_L(t) = \frac{\partial v_{SC}(t)}{\partial t} \cdot SC. \quad (14)$$

And setting the conditions at the time $t=0$:

$$v_{SC}(0) = V_0, \quad i_L(0) = 0. \quad (15)$$

The result of this equations are the equations for current through inductor $i_L$ and the voltage on super-capacitor $v_{SC}$.

$$i_L(t) = \frac{\sqrt{SC} \cdot V_0 \cdot \sin \left( \frac{t}{\sqrt{SC \cdot L}} \right)}{\sqrt{L}}, \quad (16)$$
\[ v_{SC}(t) = V_0 \cdot \cos \left( \frac{t}{\sqrt{SC \cdot L}} \right). \]  \hspace{1cm} (17)

From equations (16) and (17) we are able to calculate the power which describes how energy is being transferred into inductor.

\[ p(t) = v_{SC}(t) \cdot i_L(t) = \sqrt{C} \cdot V_0^2 \cdot \cos \left( \frac{t}{\sqrt{SC \cdot L}} \right) \cdot \sin \left( \frac{t}{\sqrt{SC \cdot L}} \right). \]  \hspace{1cm} (18)

The next step is to enumerate the function of mean value of power with dependence on time during the transistor is switched on \( t_{ON} \).

\[ P_{MEAN}(t_{ON}) = \int_0^{t_{ON}} \frac{p(t)}{t_{ON}} dt, \]  \hspace{1cm} (19)

\[ P_{MEAN}(t_{ON}) = \frac{SC \cdot V_0^2 \cdot \sin^2 \left( \frac{t_{ON}}{\sqrt{SC \cdot L}} \right)}{2 \cdot t_{ON}}. \]  \hspace{1cm} (20)

The result is plotted in the Fig. 8 (\( L = 4mH, SC = 100F, V_0 = 50V \)).

![Fig. 8. Graph of power mean value as function \( t_{ON} \)](image)

It is clear from the figure, that the maximal mean power depends linearly on the maximal time \( t_{ON} \). This is valid only for small \( t_{ON} \), because the equation (20) is not linear. For example: for the time \( t_{ON} = 0.005s \) the mean power is about \( P_{MEAN}(t_{ON}) = 2.5kW \) and the switching frequency should be 100 Hz with the respect \( t_{ON} = t_{OFF} \) for controller operating range. For this frequency we have made a graph of maximal mean value of power depending on voltage on super-capacitor (Fig. 9). From this graph we have decided that the working voltage on super-capacitor will be 25 - 50 V with the respect the range of \( P_{MEAN} \) which is from 550 to 2500 W.

Moreover the switching frequency has influence on the selection of the other passive components of the converter’s power part as discussed in [4].

5 DESIGN OF CONTROL FOR DC/DC CONVERTER

Possible control strategies are discussed in [5], [6] and [7]. This article deals with the design of LQR (Linear Quadratic Regulator) controller for DC/DC converter and with the switching frequency optimization. Designed controller will optimize the switching frequency with these criteria:

- Minimal switching losses
- Maximal transferred electrical power
- Minimal current ripple

The maximal current ripple is 20% for buck converter. The maximal difference between input and super-capacitor voltage is 180V, the inductor has the inductance \( L = 4mH \). The minimal time \( t_{ON} \) is about 50\( \mu s \), if the \( t_{ON} = t_{OFF} \), the switching frequency is about 10kHz for all transferred electrical power with respect to minimal switching losses. The time \( t_{ON} \) for boost converter depends on the transferred electrical power and it is calculated using (20). In accordance with the Minimal current ripple criterium we need to calculate the time \( t_{ON} \) for every using of boost mode. The switching frequency for this mode is from 50Hz to 500Hz.

5.1 LQR Controller

The mathematical model is linear time-invariant system of 3rd order and the state-variable equations are given by:

\[ \dot{x} = Ax + Bu. \]  \hspace{1cm} (21)

where \( x \) is 3 \( \times \) 1 state vector and \( u \) is 1 \( \times \) 1 input vector. Matrix \( A \) is the system matrix, \( B \) is the input matrix, \( C \) is the output matrix and \( D \) is the direct feed matrix.

The states variables are:

- \( v_{SC} \): voltage on super-capacitor
We consider this performance index:

\[
J(u) = \int_0^\infty (x^T Q x + u^T R u + 2 x^T R u) dt.
\]

(22)

where \(Q\) and \(R\) is given \(3 \times 3\) and \(1 \times 1\) real symmetry weight matrix, respectively \(Q\) is semi-positive definite and \(R\) positive definite. When \((A, B)\) is controllable and \((A, C)\) is observable, the optimal state-feedback law is \[u = -K x\] such that \(J\) is minimized \[8\]. A simple feedback control diagram is shown in the Fig. 10 where \(v(t)\) is the new external control input.

![Fig. 10. Output feedback](image)

The design procedure for finding the state-feedback \(K\) is:

1. Select parameters of \(Q\) and \(R\) matrices
2. Solve the algebraic Riccati equation for \(P\):
   \[
   A^T P + PA + Q - PBR^{-1} B^T P = 0
   \]
3. Find the state-feedback \(K = R^{-1} B^T P\)
4. Verify the design. If not suitable, go to first step and design different matrices \(Q\) and \(R\).

The possible algorithm, how to find the \(Q\) and \(R\) matrices is described for example in \[8\].

5.2 Design of LQR Controller

The design of LQR controller is based on the mathematical model. For the complete mathematical model we need these parameters:

- \(SC = 100F\)
- \(C = 100mF\)
- \(L = 4mH\)
- \(R = 1000\Omega\)

The state-space matrices for the buck converter (where \(u=1\) are:

\[
A = \begin{pmatrix}
-9.9995 \cdot 10^{-6} & 0.01 & 0 \\
-249.9875 & -19.9994 & 250 \\
0 & 0 & -10000
\end{pmatrix},
\]

\[
B = \begin{pmatrix}
0 \\
0 \\
10000
\end{pmatrix},
\]

\[
C = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix},
\]

\[
D = \begin{pmatrix}
0 \\
0 \\
0
\end{pmatrix}.
\]

The state-space system based on these matrices is fully controllable (the controllability matrix has full rank). We need for the design of LQR controller the matrices \(Q\) and \(R\). We want to use the designed controller for the inductor current control. The state \(i_L\) is in the second row in matrix \(A\). The first and third row of matrix \(Q\) are zeros. The maximal current of the transfer function depends on the value of second row. The physical limit of DC/DC converter is 50A - for this value the matrices are:

\[
Q = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0.2 & 0 \\
0 & 0 & 0
\end{pmatrix},
\]

\[
R = (0.5).
\]

The main design of LQR controller is in MATLAB. The state-feedback \(K\) is solved using lqr function.

\[
K = \begin{pmatrix}
-0.0144 & 0.5564 & 0.0138
\end{pmatrix}.
\]

The design for the boost converter is similar. The output from controller is connected on PWM (Pulse-Width Modulation) generator input, which generates pulses for IGBT switching transistor in accordance with the frequency switching optimization using (20). The reference signals are voltage on super-capacitor and charging/discharging current. The inductor has an inductance 5 mH, super-capacitor has a capacity 100 F and maximal voltage 56 V. Those parts are used for building prototype of DC/DC converter and comparing of simulated and measured signals.
The MATLAB/Simulink model is shown in Fig. 11. The model has three main parts. First is a part with mathematical models of buck and boost converter. Second part is a PWM generator. The PWM generator transfers reference signal to PWM signal for IGBT transistors. Third part is controller, where are included both controllers. The output from this block is signal for PWM generator.

6 PROTOTYPE OF DC/DC CONVERTER FOR HEV

The prototype of DC/DC converter was developed for validation of simulation results. The control algorithm is programmed in Digital Signal Processor from Freescale using a processor board of Freescale TOWER System. Freescale TOWER System is modular development platform, where it is possible to combine different processor platforms and different peripheral modules. The TOWER System used for control of DC/DC converter is shown in the Fig. 12. The processor board (blue board) has included processor 56F8257 type and it was designed by Freescale. The peripheral board was designed especially for this converter. The board has included 2 CANNON connectors: 1st for PWM signals and 2nd for measuring of current and voltage.

7 EXPERIMENTAL RESULTS

It was created the model in MATLAB/Simulink and the same algorithm was implemented to the Freescale processor 56F8257. The converter prototype was created for validation of simulated data. For comparing simulation and prototype the next experiment was designed. The initial condition is the voltage 35 V on super-capacitor and 200 V on DC bus. In time 1 s it is started the discharging of super-capacitor by the power 0.1 kW. The super-capacitor reference voltage is 20 V. In time 320 s is started the charging by power 0.15 kW with voltage reference 25 V. In time 450 s is started again charging by power 0.15 kW with voltage reference 38 V. The results are shown in Fig. 13. From 1 s to 300 s super-capacitor is discharged by power 0.1 kW. The source in model is simulated like ideal part with zero internal resistance. The discharging is stopped after the voltage on super-capacitor is 20 V. The next is buck mode with overcurrent protection. The controller limits the current, if the voltage is lower than minimal value 120 V. In time 420 s the source voltage was lower than minimal value, charging was stopped and the charged was started in time 465 s, where the voltage was greater than minimal value.

The result from simulation and result obtain from measurement are in accordance therefore we can say the controller was designed correctly and the buck/boost converter with the super-capacitor can be implemented to the prototype model of hybrid drive.

8 CONCLUSION

The first part of article discusses design of mathematical model of buck and boost converter. The model includes internal resistances of inductor and super-capacitor and the capacitor on source primary side. The second part of article discusses choosing of optimal frequency for PWM signal. In third part the presented model was designed in MATLAB/Simulink and it was designed the control. For validation of simulated data, the prototype of DC/DC converter was created. Designed control algorithm corresponds with measured data from DC/DC converter prototype.

ACKNOWLEDGMENT

This research has been realized using the support of Technological Agency, Czech Republic, programme Centres of Competence, project TE01020020 Josef Božek Competence Centre for Automotive Industry. This support is gratefully acknowledged.
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Received: 2013-08-23
Accepted: 2015-08-20