Can planets be pushed into a disc inner cavity by a resonant chain?

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ABSTRACT

Context. The orbital distribution of exoplanets indicates an accumulation of compact Super-Earth sized planetary systems close to their host stars. Assuming an inward disc-driven migration scenario for their formation, these planets could have been stopped and eventually parked at an inner edge of the disc, or be pushed through the inner disc cavity by a resonant chain. This topic has not been properly and extensively studied.

Aims. Using numerical simulations we investigate how much the inner planets in a resonant chain can be pushed into the disc inner cavity by outer planets.

Methods. We perform hydrodynamical and N-body simulations of planetary systems embedded in their nascent disc. The inner edge of the disc is represented in two different ways, resembling either a dead zone (DZ) inner edge or a disc inner boundary (IB), where the main difference lies in the steepness of the surface density profile. The innermost planet has always a mass of 10 M\textsubscript{Earth}, with equal or higher mass additional outer planets.

Results. A steeper profile is able to stop a chain of planets more efficiently than a shallower profile. The final configurations in our DZ models are usually tighter than in their IB counterparts, and therefore more prone to instability. We derive analytical expressions for the stopping conditions based on power equilibrium, and show that the final eccentricities result from torque equilibrium. For planets in thinner discs, we found, for the first time, clear signs for overstable librations in the hydrodynamical simulations, leading to very compact systems. We also found that the popular N-body simulations may overestimate the number of planets in the disc inner cavity.

Key words. Hydrodynamics - Methods: numerical-Planetary systems - Protoplanetary disks - Planet-disk interactions

1. Introduction

The notable number of short period Super-Earths is one of the puzzles introduced by the exoplanet discovery surveys (e.g. Winn & Fabrycky 2015; Dressing & Charbonneau 2015). To answer the question of how these planets could accumulate at the sub-au proximity to their host stars, several scenarios have been proposed, such as: a combination of planet trapping at the magnetospheric truncation radius of the disc and the star-planet tidal interaction (Lee & Chiang 2017), planet-planet interaction aided by the eccentricity damping of the gas disc (Chatterjee et al. 2008; Ogihara & Ida 2009), planet trapping at the disc inner edge (Benítez-Llambay et al. 2011; Miranda & Lai 2018), and pushing of the inner planets in a resonant chain by the outer planets in the system (Coleman & Nelson 2016; Carrera et al. 2019). The common core idea in all of these scenarios is the existence of a disc inner edge, where the planets can be trapped. This trapping can either lead to the formation of a resonant chain that might become unstable or push the inner planets into the disc inner cavity, or simply stall the migration a single planet.

Soon after the discovery of the first exoplanet, 51 Peg b, it was suggested that this short-period planet has been stopped by the inner edge of its natal disc (Lin et al. 1996). By the means of two-dimensional (2D) hydrodynamical simulations, Masset et al. (2006) showed that the migration of a low-mass planet can be halted at a steep surface density transition thanks to the change in the Lindblad torque and enhancement of the positive co-rotation torque. In order to sustain a transition in the surface density, they increased the viscosity inside the inner disc in their models. This viscosity transition can itself modifies the co-rotation torque. Without assuming a transition in the viscosity, Romanova et al. (2019) supported the trapping of low-mass planet at the disc inner edge by three-dimensional (3D) hydrodynamical simulations. In their study, the disc inner edge is modelled as a region with transition in density and temperature such that the pressure equilibrium is sustained. Therefore, the trapping of a single low-mass planet at the inner edge of the disc seems to be a robust phenomenon. However, whether an inner edge could also stop the migration of multiple planets is still questionable.

Trapping of multi-planet systems at the disc inner edge is often observed in N-body simulations (e.g Izidoro et al. 2017; Raymond et al. 2018). In these simulations, the migration and eccentricity damping of the planets are modelled using the formulae that are obtained from the results of hydrodynamical simulations for a single planet in a power-law disc. It has not been investigated if those formulae is applicable close to the inner edge of a disc, where a sharp gradient in the surface density exists. On the other hand, the trapping of multi planets in studies that performed hydrodynamical simulations has not been vigorously investigated. Papaloizou & Szuszkiewicz (2005) and Cui et al. (2019) inspected the convergent migration of low-mass two-planet systems in a disc with a surface density profile that resembles the disc inner edge. The surface density in the inner part of their disc rises linearly from the inner boundary up to a
given distance and then becomes constant. In the outer part it becomes a decreasing power-law. There is no report of pushing the inner planet into the inner region of the disc except in one figure of Papaloizou & Szuszkiewicz (2005), where the planets become very close to the inner boundary. Therefore, the trapping of multi planets using hydrodynamical simulation needs to be studied more vigorously.

Trapping of multi-planet systems at the disc inner edge can differ from a single planet in two aspects. Firstly, trapped multi planets are usually in resonance, that results into the excitation of their eccentricities (e.g. Papaloizou & Szuszkiewicz 2005). Eccentricity damping of a single planet and multi-planet system have been investigated in power-law discs (Cresswell & Nelson 2006, 2008;Bitsch & Kley 2010), while it is not obvious if a trapped eccentric planet at the disc inner edge behaves similarly. Ogihara et al. (2010) used analytical calculations along with orbital integration and found that an inner eccentric planet in a resonant chain, that is close to the inner edge of a disc, can halt the migration of the whole chain. This type of trapping, named eccentricity trap, arises from different eccentricity damping rate of the inner planet at the apocentre and pricentre. Although, they did not consider the change of the torque on the inner planet corresponding to the surface density jump, their study hints on different behaviour of the eccentric planets at the disc edge. Secondly, multi planets in a resonant chain migrate as one entity. Whether trapping the inner planet can stop the whole chain or whether a massive enough system would overcome and break the trap need to be probed properly.

The outcome of N-body simulations of multi planets at the disc inner edge highly depends on what equation of motion is used or how the inner edge is modelled (Brasser et al. 2018; Ida et al. 2020). One advantage of hydrodynamical simulations is that the migration of a planetary system is directly calculated through the disc-planet and planet-planet gravitational interactions. Therefore, the main complexity would refer to the modelling of the inner edge.

The detailed modelling of the inner edge of a disc, where the material is transferred to the central star, can be very complicated (Romanova et al. 2002; Long et al. 2005; Romanova & Lovelace 2006; Zanni & Ferreira 2009). However, we can simply define it as where the gas leaves the disc and settles to the star. Near to the inner edge of the disc can be other traps as well such as a dead-zone edge. Flock et al. (2017) studied the inner region of protoplanetary discs around solar-like stars using radiative magneto hydrodynamical simulations and show that the silicate sublimation at ∼ 0.08 au leads to the formation of a density jump at ∼ 0.5 au. This location is the inner edge of the dead-zone, where the turbulence dramatically changes due to the change of the ionization degree. Therefore, there can be two radii close to the inner edge, where planets can be trapped: (1) the dead-zone edge, and (2) the inner boundary. Both of them can trap planets if the disc surface density and temperature profiles provide a strong enough positive torque on the planet. However, the planets have to overpass the traps to approach the cavity between the disc and the star.

In this work, we use 2D hydrodynamical simulations to study the trapping of a planetary system at the inner edge of a disc and also a dead-zone. We examine resonant systems, mostly composed of Super-Earths, and monitor their migration to find out whether the inner planet is able to stop the chain or will be pushed into the cavity. For this purpose we construct two types of traps that differ in their steepness and long term evolution. In Sec. 2, we describe how these two types of traps are modelled, which code is used, and how the disc and planets are set up. We performed simulations with two, three, and more planets with different mass combinations, and also for a lower disc viscosity and smaller aspect ratio. The results are listed in Sec. 3. After that, in Sec. 4, we check if the results depend on the width of our dead-zone edge model. A power and torque analysis is presented in Sec. 5, where we show the halting of a resonant system happens when the total power of the system vanishes. In some of our models, we observe overstability, that is discussed in Sec. 7. Finally, our findings are summarised in Sec. 9.

2. Modelling the inner edge: setup and numerics

As described in the introduction it is expected that a protoplanetary disc disrupts near the stellar surface and has an inner edge, where the disc surface density drops significantly. Another location that is also close to the inner edge and has transition in surface density is the dead-zone edge. Both of them can trap planets if the disc provides a strong enough co-rotation torque. However, the planets have to overpass the trap to get closer to the star. Using 2D hydrodynamical simulations, we study the migration of planets and their trapping at these two types of density transition. We analyse the final configuration with respect to their resonances and monitor if the inner planet can be pushed into the inner cavity.

Bringing the planets into a resonance, allowing the inner planet to reach the trap, and properly modelling the migration of planets at the disc inner edge is cumbersome and time-consuming. Even if a planetary system is initialised close to a resonance commensurability, it might need thousands of orbits to reach a steady final state. On the other hand, properly resolving the co-rotation torque for a low or moderate mass planet needs at least six cells in a half horseshoe width of the planet (Paardekooper et al. 2011). Having such high resolution at the distances very close to the star would decrease the time-step so dramatically that the simulations become infeasible. To evade this issue, we shift the density transition to ∼ 1 code length unit. This way, we can properly test if a resonant migration can push the inner planet out of the trap and into the cavity.

2.1. The edge models

We construct two types of traps: a dead-zone inner edge (DZ) and a disc inner boundary (IB). To ease understanding of the results, we stick to this colour code for the rest of the manuscript when describing these two cases. These planetary traps are constructed in the following way:

**Dead-zone inner edge:** We model a dead-zone inner edge by decreasing the viscosity over a distance of the order of disc scale height, using the the method of Masset et al. (2006). In this model the inner part has a higher viscosity while the outer part has a lower value. As a consequence the inflow velocity is larger inside the cavity and smaller outside, that gives rise to a density maximum just outside of the viscosity transition region. The enforced viscosity determines the shape of the density jump. Because the viscosity profile is fixed, it leads to a surface density profile that does not evolve much by time. This fixed surface density profile is the advantage of this model. However, as it turned out, the strong viscosity variation around the location of the planetary trap and consequent vortex formation complicates the saturation of the co-rotation torque.

**Disc inner boundary:** We assume that the region between the disc edge and the star is emptied by some mechanism. Hence, when the gas reaches to the inner edge, it is taken out from the
simulations such that an extremely low surface density is created at the inner boundary. The inner disc evolves correspondingly and adopts its profile to this inner boundary condition. A similar method has been used by Romanova et al. (2019) to study trapping of a planet in 3D hydro-simulations. The drawback of this model is that this boundary condition causes the surface density profile, which determines both the Lindblad and co-rotation torques, slowly evolves in time. However, the smooth viscosity can be considered as its advantage.

To illustrate the different behaviours of these two traps, we display the evolution of the surface density and the specific torque on an imaginary 10 Earth-mass planet in Fig. 1. Here and in subsequent plots, the left panels (green frames) refer to the Dead-zone inner edge (DZ) and the right panels (purple frames) to the Disc inner boundary (IB) model. The top row shows that the DZ model yield a very steep surface density transition that becomes stationary with time. In the bottom row, 2D maps of the specific torque, $\Gamma$, acting on a fictitious 10 Earth-mass planet at every location in the disc at a specific time during the disc’s evolution are displayed. The torque is calculated using the formulae in Paardekooper et al. (2010a, 2011). The positive density slope at the inner disc’s edge produces a region where the torque acting on the planet becomes positive. The black contours indicate where the torque is zero. The actual planetary trap is at the outer edge of the positive torque region. At the trap, the planet feels a negative torque if it is shifted outwards and a positive torque if slightly displaced inwards. In the DZ model, there is a sharp transition from the negative to positive torque at the trap, whereas in the IB model, the gradient of the torque is very shallow around the zero-torque location. Said differently, if the planet is pushed slightly inwards, it would feel a very strong positive torque in the DZ model and is sent back to the zero-torque location, while in the IB model, the planet would feel a little positive torque which can be easily overcome by the push of an outer planet. Therefore, based on the torque map of our disc models, we anticipate that a resonant chain of planets would be more successful in push a planet out of the planetary trap in the IB than the DZ model. In the next section, we will explain in more detail the numerical setups of these two models.

2.2. Setup and numerics

We used a modified version of the 2D hydrodynamical code FARGO\(^1\). This code simulates a locally isothermal disc in the cylindrical coordinate. We used a fixed coordinate frame centred on the star and took into account the indirect terms both on the planets and the disc.

The aspect ratio of the disc is $h = h_0 (r/r_0)^{0.5}$ with $r_0 = 1$ being our length unit. A value of $h_0 = 0.05$ is the reference model, but all parameter sequences include models with the smaller value of 0.03. Our time unit, named orbit, defined as one Keplerian orbital time at $r_0$. To be initially in viscous equilibrium, we chose $\Sigma = \Sigma_0 (r/r_0)^{-1.5} [M_*/c_s^2]$ with $\Sigma_0 = 2 \times 10^{-4}$ except for few simulations which have double of this value. $\Sigma_0 = 2 \times 10^{-4}$ corresponds to $\sim 1800$ g/cm\(^2\) for a Solar-mass star $M_* = 1M_\odot$ and $r_0 = 1$ au. We note that this is the basic profile in the disc that is modified the edge at the inner part of the disc for each disc model as we explain later. Given this setup, the disc inner edge in our models will be located near 1 au, while in real discs it is approximately 10 times smaller. Our choice was done for numerical reasons, because a smaller radius would require many more time steps due to the shorter dynamical timescale, not allowing an extensive parameter study. As we deal with locally isothermal discs the results should be scalable to smaller radii, using the value $h_0 = 0.03$ (Flock et al. 2017).

For the disc viscosity $\nu$, we use the $\alpha$-viscosity model $\nu = \alpha c_s H$, where $c_s = H/\Omega$ is the sound speed with $\Omega$ being the Keplerian angular velocity, and $H = hr$ is the disc scale height. In most of the simulations $\alpha = 5 \times 10^{-4}$ except in few models, in which the effect of a ten times smaller viscosity is examined.

In the DZ model, the viscosity becomes a hundred times larger inside the active zone. The transition happens over a distance of $A = 1.2H(r_i)$ at $r_i = 1$ using the following function (same method as Masset et al. 2006)

$$\alpha = 5 \times 10^{-4} \times \begin{cases} 1 & r < r_1 \\ 100 \alpha_{\text{ref}} & r_1 \leq r \leq r_2 \\ 1 & r > r_2 \end{cases}$$

with $r_1 = r_i - \lambda/2$ and $r_2 = r_i + \lambda/2$. For the IB model, $\alpha$ is constant through the whole disk.

In the DZ models, the disc is meshed into $N_r = 929$ logarithmic segments in the radial, and $N_\phi = 1024$ equal grids in the azimuthal direction. The inner boundary is outflow and located at $r_{in} = 0.3$. We set our outer boundary far enough at $r_{out} = 20$ to avoid any influence of the outer boundary on the migration of outer planets. All quantities are damped to the initial value within $r = 18$ to 20, using the method in de Val-Borro et al. (2006).

In the IB models, the outer boundary is identical to the DZ model. The inner boundary is located at $r_{in} = 0.8$ in order to have the planetary trap around $r = 1$. We changed the radial resolution to $N_r = 712$ while keeping the azimuthal grid the same as the DZ model to have similar resolution in both models. Following the explanation in Sec. 2.1, we successively decrease

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\(^{1}\) http://fargo.in2p3.fr/-Legacy-archive-
the surface density\(^2\) within the interval \(r \in [0.8 - 0.9]\) until it reaches a floor value of \(\Sigma_{\text{floor}} = 10^{-11}\). The radial velocity in this region is outflow. This slow decreasing of surface density prevents producing an instability by guaranteeing that the surface density has enough time to be adopted to the new condition. We would like to point out that any other choice of inner boundary, either location or damping width, could produce a different profile and consequently dissimilar incommensurability than what we present here.

The inner planet is planted at \(r = 1.2\) in both models. Other planets are located just outside of 2:1 commensurability of their inner neighbours. The potentials are gradually applied on the disc during the first 50 orbits. This time is long enough for the bump to be established in both models. In some of the simulations, instead of embedding all of the planets from beginning, we add the next planet after the inner pairs get into a resonance. In these models, the tapering time only applies on the first planet. The smoothing length for the potential of the planets is \(\epsilon = 0.6\) \(M \times H(r)\) (Müller et al. 2012). For calculating the torque on the planets, we only used the perturbed surface density to avoid any false resonances (Ataiee & Kley 2020). The mass of the inner planet is always \(M_1 = 10 M_\oplus\) where \(M_\oplus = 3 \times 10^{-6} M_\star\) is the mass of Earth for a Solar-like star. The rest of the planets are more massive than or as massive as the inner one. The maximum number of the planets used in our models is 6 (See Table 1 for the details).

3. Results

In this section we summarize the main results of our study. To analyse the impact of an inner edge on the final configuration of embedded planets, we ran an extensive suite of simulation using various disc parameter and planet masses. The set of models is summarized in Table 1, and we refer to the quoted model identifiers in describing their outcome. The simulations are continued until all planet pairs settle into resonances or the system becomes unstable. As we found that the result of each simulation is individual, we present and explain them one by one in the following. To make the whole text more readable we display additional results on the individual models in appendix B.

3.1. A single planet

In the first step, we checked trapping of a single planet in our disc models. The top panels of Fig. 2 show the migration of a 10 M\(\oplus\) (blue) and a Jupiter-mass planet (orange) in a dead-zone inner edge (left) and a disc inner boundary (right) model (1p10 and 1pJup in Tab. 1). As pointed out, the colour of axes specifies the type of disc model. In the bottom rows, the surface density profiles are presented at \(t = 50\) orbits (in black), when the planet’s potential is fully applied on the disc, and \(t = 10^4\) orbits.

In the DZ model (with steep density slope), the low-mass planet is trapped at the zero torque location \(r \sim 1.14\) and its semi-major axis does not evolve, as the torque map in Fig. 1 also suggests. The small oscillations in the semi-major axis is due to the presence of some vortices outside of the edge. These vortices exert some torques on the planet as they pass by and produce small wiggles. In contrast, the massive planet continues its inward migration as it opens a planetary gap and deforms the density profile which completely suppressing the positive corotation torque.

\(^2\) The surface density in each timestep is multiplied by a factor of 0.999

Fig. 2. Evolution of a single planet in a disc with a dead-zone inner edge (left) and at the disc inner boundary (right). Top: Migration path, \(a(t)\). In each panel, every line belongs to a separate simulations with only one planet. Bottom: The surface density profiles at \(t = 50\) (black), when the planet is applied on the disc, and \(t = 10^4\) orbit. The colour code is the same as for the top row and the final location of the planets are marked by circles. The low mass planet is trapped the DZ model and slowly migrates outwards in the IB model. In contrast, the massive planet (Jupiter-mass) continues its migration to the inner disc thanks to the deformation of the bump by its gap.

Fig. 3. Semi-major axis evolution of the inner 10 M\(\oplus\) planet for all models with 2 planets, top for DZ models and bottom for the IB ones. The single planet model (1p10) is added for comparison.

In the IB model, the low mass planet moves slowly outward as the trap regions widens and the zero torque location moves slowly outwards. The massive planet in this model behaves similar to the DZ case and continues its inward migration by reshaping the surface density profile.

These results imply that massive gap-opening planets could possibly push and shepherd small planets that are trapped at the edge toward the star. In the next section, we add a second planet with various masses to these models and also examine the effect of lower viscosity and smaller aspect ratio.
models, the outer planet in most cases is able to reverse models. In the model (green frame) and the bottom panel (purple frame) indicates that the inner planet remains almost in unstable shows, in the simulation, after the outer planet catches the outwards IB simulation, the outer planet migrates inwards but with a slower rate than the one of outer planet is not strong enough to overcome the positive torque on the inner planet. On the contrary, in the IB model, after the planets get into the 3:2 resonance, the outer planet pushes the inner planet inwards and reverses its slow outward migration. 2p20: Figure 5 shows semi-major axis, eccentricity, and orbital period ratio for 2p20 models. In the DZ simulation, the outer planet migrates inwards and gets into the 4:3 resonance with the trapped inner planet. Because the outer planet is twice as massive as the 2p10 model, the inner planet is pushed slightly into the transition zone (top panel of Fig. 3). After forming the final resonance, the migration of the planets stops. On the contrary, in the IB simulation, after the outer planet catches the outwards migrating inner planet in the 3:2 resonance, they continue their migration inwards but with a slower rate than the one of outer planet before the resonance.

2p10: In these models, the masses of both planets are 10 M⊕. As Fig. 4 shows, in the DZ model, the planets cross several resonances and finally settle in the 5:4. The blue line in the top panel of Fig. 3 indicates that the inner planet remains almost in its initial trapping location. In this case, the push by the outer planet is not strong enough to overcome the positive torque on the inner planet. On the contrary, in the IB model, after the planets get into the 3:2 resonance, the outer planet pushes the inner planet inwards and reverses its slow outward migration.

### Table 1. The list of our main simulations. The information about the DZ and IB models are given by their corresponding colours. The data in black are the same in both trap models.

| # planets | Name          | h₀     | Σ₀     | α     | rp     | M₀ [M⊕] | Result     |
|-----------|---------------|--------|--------|-------|--------|---------|------------|
| 1         | 1p10          | 0.05   | 2 × 10⁻⁴ | 5 × 10⁻⁴ | 1.2     | 10      | –          |
|           | 1p1up         | 0.05   | 2 × 10⁻⁴ | 5 × 10⁻⁴ | 1.2     | 10      | –          |
|           | 2p10          | 0.05   | 2 × 10⁻⁴ | 5 × 10⁻⁴ | 1.2     | 1.95    | 10, 10    | 6:5, 3:2   |
|           | 2p20          | 0.05   | 2 × 10⁻⁴ | 5 × 10⁻⁴ | 1.2     | 1.95    | 10, 20    | 4:3, 3:2   |
|           | 2p100         | 0.05   | 2 × 10⁻⁴ | 5 × 10⁻⁴ | 1.2     | 1.95    | 10, 100   | 29:20°, 2:1 |
|           | 2p1up         | 0.05   | 2 × 10⁻⁴ | 5 × 10⁻⁴ | 1.2     | 1.95    | 10, 333   | 2:1, 2:1   |
|           | 2p20LA        | 0.05   | 2 × 10⁻⁴ | 5 × 10⁻⁵ | 1.2     | 1.95    | 10, 20    | unstable, 3:2 |
|           | 2p20LH        | 0.03   | 2 × 10⁻⁴ | 5 × 10⁻⁴ | 1.2     | 1.95    | 10, 20    | 4:3, 5:4   |
|           | 2p20HS        | 0.05   | 4 × 10⁻⁴ | 5 × 10⁻⁴ | 1.2     | 1.95    | 10, 20    | 5:4, 3:2   |
| 3         | 3p1020        | 0.05   | 2 × 10⁻⁴ | 5 × 10⁻⁴ | 1.2     | 1.95, 3.2 | 10, 10, 20 | unstable, 3:2 |
|           | 3p2020        | 0.05   | 2 × 10⁻⁴ | 5 × 10⁻⁴ | 1.2     | 1.95, 3.2 | 10, 10, 20 | unstable, 3:2 |
|           | 3p10100       | 0.05   | 2 × 10⁻⁴ | 5 × 10⁻⁴ | 1.2     | 1.95, 3.2 | 10, 10, 100 | unstable, 3:2 |
|           | 3p20100       | 0.05   | 2 × 10⁻⁴ | 5 × 10⁻⁴ | 1.2     | 1.95, 3.2 | 10, 10, 200 | unstable, 3:2 |
|           | 3p2020LA      | 0.05   | 2 × 10⁻⁴ | 5 × 10⁻⁵ | 1.2     | 1.95, 3.2 | 10, 10, 20 | unstable, 3:2 |
|           | 3p2020LH      | 0.03   | 2 × 10⁻⁴ | 5 × 10⁻⁴ | 1.2     | 1.95, 3.2 | 10, 10, 20 | 4:3, 5:4, 3:2 |
|           | 3p2020HS      | 0.05   | 4 × 10⁻⁴ | 5 × 10⁻⁴ | 1.2     | 1.95, 3.2 | 10, 10, 20 | unstable, 3:2 |
| >3        | onebyone      | 0.05   | 2 × 10⁻⁴ | 5 × 10⁻⁴ | 1.2     | 1.9, 2.43, 2.98 | 10, 20, 20 | 2:1, 2:1, 2:1, 2:1 |
|           | onebyoneLH    | 0.03   | 2 × 10⁻⁴ | 5 × 10⁻⁴ | 1.2     | 1.9, 2.43, 2.98 | 10, 20, 20 | 2:1, 2:1, 2:1 |

3.2. A planet pair

In this section, we add a second planet just off the 2:1 commensurability and monitor the evolution of the system. Mass of the inner planet in all cases is 10 M⊕ but the outer one has different masses. We denote the inner planet by an index ‘i’ and the outer by ‘o’. Taking the case with Mᵢ = 20 M⊕ (model 2p20) as the reference model, we additionally examined the effect of lower viscosity, smaller aspect ratio, and higher surface density for this model.

Figure 3 embodies the migration of inner planet for all models with two planets. The top panel contains the results for the DZ model (green frame) and the bottom panel (purple frame) demonstrate those of the IB model. The figure clearly shows that the second planet undoubtedly has an affect on the migration of the inner planet specially if it is more massive than the inner one. Moreover, comparing the top and the bottom panels implies that it is easier to push the inner planet to the inner disc in the IB models than the DZ models. In the DZ models, the inner planet insists on staying at the trapping location while in the IB models, the outer planet in most cases is able to reverse the migration of the inner planet and pushes it inward after they get into resonance. In the following, we describe the results of the models individually in more details.
Fig. 4. Evolution of semi-major axis, eccentricity, and orbital period ratio for 2p10 models. The inner planet is given by the blue line and the outer by the orange one. In the bottom panels resonances are marked by horizontal lines.

Fig. 5. Similar to Fig. 4 but for the 2p20 model.

e-0.000 0.000 0.025 0.050
0 0.25 0.50 0.75 1.0 1e4
0 0.25 0.50 0.75 1.0 1e4
T /T ×

Fig. 6. The three top rows are the same as Fig. 4 but for the 2p100 model. The forth row shows the final surface density with the locations of planets indicated by markers, whose colours correspond to the top panels.

els opens a partial gap, not as strong as the one of the Jupiter mass planet in 1pJup to destroy the bump, but can still affect it. The left panels in this figure, that contain the results for the DZ model, show that the outer planet cages the inner planet between the edges of the dead-zone and its gap. After temporarily settling into the 3:2 MMR, they rearrange and settle near a 29:20 commensurability. We checked the resonant arguments but found no definitive pattern. In the IB model (right panels), the planets get into the 2:1 resonance and keep this configuration until the end of the simulation. Both planets continue their inward migration and the inner planet, which becomes very eccentric with the eccentricity of ∼ 0.4, is pushed to the inner part of the disc.

2pJup: In these models the outer planet has one Jupiter mass and opens a gap similar to the 1pJup model. For both edge models, the inner planet traps into the 2:1 resonance with the outer one (see Fig. B.1 in the appendix B). The massive planet pushes the inner planet into the inner part of the disc regardless of which inner disc model is used. The low-mass planet in both models becomes very eccentric. We stopped the simulations because for such large eccentricities ∼ 0.3, the inner planet passes the area that is close to the inner boundary of our computational domain. The overall behaviour of the planets in both of these models are very similar and it seems that the type of the trap has only little influence on the evolution. Therefore, we will not examine more models with a Jupiter-mass planet.

2p20LA: These models have identical setups as the reference 2p20 models except that α is lowered to 5×10⁻⁵. The lower viscosity can ease the partial gap opening and consequently reduce both of the Lindblad and co-rotation torques. In addition, it can change the saturation of the co-rotation torque (Masset 2001; Kley & Nelson 2012). In the bottom panels of Fig. 7 the surface density profiles at the end of the simulations are displayed. We can distinguish a shallow gap around the outer planet with the mass of 20 M ⊕. Comparing the two upper rows of Fig. 7 with those of Fig. 5 shows that the planets in the DZ model managed to get closer and settle into the 6:5 resonance. However, they finally closely interact and the inner planet is sent to an orbit outside of the bump, and the order of the planet is reversed. A comparison with the top panel in Fig. 3 shows that the inner planet here is initially trapped more inside than the planet in the 2p20 and 1p10 models with higher viscosity. This is consistent with the idea that the lower viscosity reduces the positive co-rotation torque due to the saturation effects.

In the IB model, similar to its 2p20 counterpart, the planets pass the 2:1 resonance (around ∼ 2000 orbits when their eccentricities peak), get into the 3:2 resonance, and after that the inner planet is pushed to the inner disc. A closer look at the semi-
The planets are initially in 5:4 resonance, but for 2p20LA models which have a reduced viscosity.

In the 2p20 model, the inner planet initially is migrating inwards which indicates that the co-rotation torque is more saturated. After the outer planet passes the 2:1 resonance, the inner planet changes its migration direction, moves outwards, and halts until the outer planets catch it in the 3:2 resonance. Thereafter, its migration is governed by the outer planet.

2p20LH: These models have a smaller aspect ratio than 2p20, here $h = 0.03$. A smaller aspect ratio can change the torque in two ways. Firstly, the torque from the disc on the planet is proportional to $h^{-2}$ (Baruteau et al. 2014, and references therein), and therefore the torque is larger for smaller $h$. Secondly, the planets are liable to open a partial gap and the smaller surface density around the planet’s orbit decreases the torque. The brown lines in Fig. 3 shows that the semi-major axes of the inner planets for the 2p20LH models are smaller than those of the reference model 2p20.

The left panels of Fig. 8 show that the planets in the DZ model finally settle into the 4:3 resonance and remain there. In the IB model, whose results are displayed in the right panels of Fig. 8, the inner planet migrates inwards until outer planet captures it in the 5:4 resonance at around $t \approx 7500$ orbit. As long as they are in this configuration, they migrate very slowly outward until they leave the resonance. A similar behaviour happens later, when they get into the 4:3 resonance. Finally, they come out of the 4:3 and eventually stay in the 5:4 resonance while they continue their inward migration. Their semi-major axis and particularly their eccentricity evolutions evidence overstable behaviour in the 3:2 and 4:3 resonances. We will discuss about overstable models later in Sec. 7.

2p20HS: The surface density in these models is twice that of the 2p20 models. Hence, the planets feel larger torques compared to 2p20. The migrations and eccentricities are almost identical to the 2p20 model, except the evolution is faster. It can be seen in the lower panel of Fig. 3 that in the IB model the behaviour of the inner planet is very similar in the 2p20 and 2p20HS models but the resonance capture happens earlier in 2p20HS. However, the results are slightly different for the DZ model. The planets are initially get into 5:4 resonance, then are pushed into and stay in the deeper 6:4 resonance for about 10000 orbits. After that, they return back to 5:4 because of the planet-planet gravitational interaction (see Fig. B.2 in the appendix B). Despite changing of the resonance configuration, the semi-major axis of the inner planet does not vary and remains the same as in the 2p20 model after the first resonance (upper panel of Fig. 3).

### 3.3. Three planets

In this section, we will present the results of the simulations with three planets. Mass of the outer planet is either 20 or 100 $M_\oplus$, the middle 10 or 20 $M_\oplus$, and the inner planet is always 10 $M_\oplus$, see Tab. 1 for details. The outer planet in these models is added just outside of 2:1 commensurability with the middle planet. Figure 8 shows the migration of the inner planet for all models with three planets. In the top panel, which shows the results for the DZ models, the inner planet in many simulations undergoes an instability and is expelled. However, none of the inner planets is ejected in the IB model (the bottom panel of Fig. 9). In the following, we describe the results of each model individually.

3p1020: These simulations have the same setup as 2p10 except that a third planet with the mass of 20 $M_\oplus$ has been added.
model, the outer planet pushes the system into a tighter configuration, which is more prone to instability.

In the IB model, the outer planet gets into 2:1 resonance with the middle one but it has no effect on the migration of the inner planet (right panels of Fig. 10). As the bottom panel of Fig. 9 shows, the semi-major axis of the inner planet follows that of the single planet until it gets into 3:2 resonance with the middle planet. Afterwards, it is pushed into the inner disc with a faster migration rate than its 2p10 counterpart.

3p2020: The trapping positions of the inner planets in these models are very similar to their two-planet counterparts and the outer planet does not play a notable role. In the DZ model, although the outer planet does not push the most inner one, it can destabilise the system after it enters into 2:1 resonance with the middle planet (Fig. B.3). After the two outer planets spend some time in this resonance, the inner and middle planets switch their orbits. In the IB model the middle planet enters 3:2 resonance with the inner one at around 8000 orbits, and it causes the eccentricity of the inner planet increases, exactly as in the 2p20 model, see Fig. 5. Later, the outer planet catches up and a 3:2:1 resonant chain forms with period ratios 3:2 and 2:1 (from inside out). Upon the capture, the eccentricities of the two inner planets increase further. Otherwise no change in the migration rate of the inner planet happens.

3p10100: These models are complementary to the 2p10 and 3p1020 models in understanding the effect of the third planet. Comparing the DZ simulations in the three mentioned models, shown in the left panels of Fig. 4, 10, and 11, indicates that the two inner planets form tight configurations in all three models and the role of the outer planet is mostly destabilising the system. The more massive the outer planet is, the earlier the system becomes unstable. Differently, in the IB simulations, the third planet has a notable impact. When the third planet is more mas-

With the aid of the black lines in Fig. 3 and 9, that show the semi-major axis of a 10M⊕ planet in the single-planet models, we can compare the effect of adding the third planet to the evolution of the inner planet. Comparing the blue lines in the upper panels of the mentioned figures indicates that, in the DZ model, the presence of the third planet hardly affects the migration of the inner planet. When the third planet catches the other two in a 6:5:3 resonant chain (with period ratios 6:5 and 5:3 from inside out) at ~ 350000 orbits (see left panels of Fig. 10), the inner planet is slightly pushed inwards. Later, the outer planet comes out of the chain and continues its inward migration until the two inner planets closely interact and switch their orbits. Therefore, in 

Fig. 9. Semimajor axis evolution of the innermost planet. Similar to Fig. 3 but for models with three planets. The inner planet has always 10M⊕ and the labels give the masses of the other two in units of M⊕, see Tab. 1.

Fig. 10. Top and middle panels display the semi-major axis and eccentricity for 3p1020 models. The bottom panels show the orbital period ratio for each consecutive pair of planets, where the colour corresponds to the outer planet. For example, the green curve demonstrates the orbital period ratio of the third planet to the second planet when counting from inside out.

Fig. 11. Similar to Fig. 10 but for 3p10100 model.
sive, the reversal of the inner planet’s migration happens earlier, and the inward migration is faster after the reversal. In all of these IB models, the two inner planets are in 3:2 resonance and when a third planet exists, the planets form a 3:2:1 resonant chain, which enhances the eccentricity of the lower mass planets.

The last row of Fig. 11 shows the azimuthally averaged surface density of the 3p10100 models at given times. In both cases, the outer planet opens a gap. In the DZ model, the two inner planets are sandwiched between the dead-zone edge and the partial gap of the outer planet. In contrast to the DZ model that become unstable, the planets in the IB model migrate inwards while keeping their resonant chain.

3p20100: Evolution of these models are very similar to 3p2020 but faster due to the faster migration of the more massive outer planet. Similar to 3p2020, the DZ model becomes unstable after about 10000 orbits. The planets in the IB model get into a 3:2:1 chain. The only difference is that the eccentricities of the planets in this configuration are larger than those of 3p2020, due to the larger mass of the outer planet. Looking at the migration of the inner planet for this model and 3p10100 in the lower panel of Fig. 9 indicates that the behaviour of the inner planet after reversal is determined completely by the third planet which is more massive than the second one. (The eccentricity and orbital period ratio for these models are brought in Fig B.4.)

3p2020LA: These models with a smaller $a$ behave very similar to their high viscosity counterparts (as shown in Fig. B.5). The DZ model becomes unstable when the outer planet gets close to the inner two, which are in 4:5 resonance. The planets in the IB model, similar to the previous IB models with three planets, continue their inward migration as they stay in the 3:2:1 resonant chain.

3p2020LH: These models, with a smaller aspect ratio, show different behaviours than other three-planet models. The DZ model here is the only one among other DZ models with three planets that does not go unstable after being in a 4:3:2 chain for more than 5000 orbits. Because the inner planet barely moved from its trapping location (upper panel of Fig. 9), the migration of whole system halts as they form the resonant chain. One can see this in the left panels of Fig. 12. In the IB model, as the lower panel of Fig. 9 shows, the inner planet passes various phases of inward and outward migration depending on in which resonance it is with the outer planet. Even after the final configuration forms, it does not migrate inwards. The right panels of Fig. 12 demonstrate the evolution of this system. The two inner planets get into various resonances, namely 3:2, 4:3, and 9:7, but these resonances are broken by the overstability. They finally dwell in 5:4 resonance. The outer planet, however, remains in the 3:2 resonance with the middle planet after they enter into it.

3p2020HS: The outcome of these models with higher surface density and 3p2020 models are alike, except that the ones here have faster evolution. Therefore, we omit showing the full results. We can see in Fig. 9 that the inner planets in these models behave very similar to their low surface density counterparts but their inward movements have started earlier. As before, the IB model ends up in the 3:2:1 chain and the DZ model becomes finally unstable.

3.4. More than three planets

In this section, we examine the effect of additional planets in the systems and its effect on the innermost planet. We take the 2p20 as the reference model, add more planets with the mass of 20 $M_{\oplus}$, and observe the behaviour of the innermost planet. We start with the inner 10 $M_{\oplus}$ planet and allow it to find its trapping location. Then the second planet with the mass of 20 $M_{\oplus}$ is placed just out of 2:1 commensurability. We continue the simulation until the planets get trapped into a resonance. The next planets are added one by one in a similar way when the inner pair form a resonance. We continue until the system becomes unstable or we find that adding more planets has no effect on the location of the inner planet. In the following the results of these simulations for discs with $h = 0.05$ and $h = 0.03$ for both inner edge models are presented.

Onebyone: The evolution of the system for the DZ model is shown in Fig. 13. On the top left, the semi-major axes of the planets are plotted. This panel, besides the migration of the planets, demonstrate where and when the planets are added into the disc. As the top right panel shows, the eccentricities are about 0.07 unless when the planets closely interact. By the help of the bottom panels we can check how the location of the inner planet is affected by the outer planets. The second planet (orange
five $20M_\oplus$ planets to the system in the DZ model. All of them stay in either 3:2 or 4:3 resonances but the inner planet is not pushed inwards. The only effect of the outer planets on the first planet is producing some jitters in its semi-major axis by enhancing its eccentricity.

The IB model, on the other hand, as Fig. 16 demonstrates, becomes unstable after the forth planet comes into resonance by the third one and forms a 5:4:3:2 resonant chain. From the top right panel, we can infer that the two inner planets undergo over-stability. The inner planet shows absolutely no monotonic behaviour. Depending on which resonance it forms with the other planets, it can move inwards or outwards.

4. Transition width and vortices

For our choice of the viscosity transition in the DZ models, some vortices are created and persist during the simulations. They form because of the narrow transition width and a small discontinuity in $\alpha$ at the edges of the transition zone. In this section, we examine different widths and profiles in order to inspect whether the vortices and transition width can affect the migration and final settling of the planets. For the models 2p20 and 3p2020, we ran additional simulations using different viscosity profiles, either by changing $\lambda$ in Eq. (1), or by smoothing the transition.

The top panel of Fig. 17 shows four azimuthally averaged surface density profiles for models with viscosity profile given by relation (1) using $\lambda = 0.06, 0.3, 1.0$, and one which has the same transition width as $\lambda = 0.3$ but using a smoother function at the edges of the transition zone. The bottom panel of Fig. 17 presents the orbital period ratio for these simulations. In all of them, the planets end up in 3:2 resonance except the one with the narrowest transition width $\lambda = 0.06$ (our reference model), for which the outer planet could get closer to the inner planet. The corresponding two-dimensional surface densities are displayed in Fig. 18 at the time just before the planets get into resonance. In the three simulations with our standard profile, there is at least one vortex visible, while the one with the smoother transition shows none. In all of these simulations, regardless of the existence of a vortex/vortices, the inner planet is trapped at the edge of the transition zone. In the model with the steepest transition, $\lambda = 0.06$, there are many small vortices at the transition edge. Although they can hardly be distinguished in Fig. 18, their spirals are clearly visible (Paardekooper et al. 2010b). Both the vortices and their spirals can interact with the...
planets and produce small perturbation in their orbits. However, because these interactions are random, their torques/powers on the planets do not influence the overall migration. For the models with the wider transitions \( \lambda = 0.3 \) and \( \lambda = 1 \), only a single vortex is established and the inner planet is trapped azimuthally between the two ends of the vortex (Ataiee et al. 2014). In all of these models, after the planets get into resonance and their eccentricities are excited, the vortex/vortices fade.

In the model with the smooth transition (labelled with \( \lambda = 0.3 \) smooth), as shown in Figs. 17 and 18, the surface density profile is almost identical to the model with \( \lambda = 0.3 \) but no vortex is created during the evolution. In both of these models (smooth and non-smooth) the planets get trapped into 3:2 resonance. However, the inner planet in the model with smooth transition is stopped slightly inwards than the one with the standard profile. These additional tests show that the existence of vortices near the edge does not drastically alter the results, but they create some noisiness in the simulations.

The planets in 3p2020 simulations with \( \lambda = 0.3 \) and 1.0 are trapped into 3:2 resonance and do not becomes unstable during the simulation time. In spite of the model with \( \lambda = 0.06 \), where the system goes into 5:4 resonance and becomes unstable, the inner planet in the simulation with \( \lambda = 0.3 \) is trapped at the transition edge, while in the widest model (\( \lambda = 1 \)) the outer planet pair managed to push the inner planet out of the trapping point (Fig. B.6). Therefore, it appears that, similar to the IB models, the inner planet can be pushed into the inner disc if the transition zone in the DZ models are extremely wide.

### 5. Power and torque analysis of 2p20 models

The results of two-planet models show that, after the resonance formation, the inner planet in the DZ simulations is slightly pushed inwards, where the torque is expected to be positive. After that the migration of the whole system halts. In the IB models, in spite of the initial outward migration of the inner planet, the resonant system pushes the inner planet back to the inner disc and the whole system migrates inwards with a different migration rate than those of the individual planets. In this section we analyse the torque and power for 2p20 model (displayed in Fig. 5), and show that the halting position of the whole system is at that location where the total power vanishes.

The energy \( E \), and angular momentum \( L \), of a planet is given in terms of its semi-major axis and eccentricity as

\[
E = -\frac{GM_\star M_p}{2a} \quad \text{and} \quad L = M_p \sqrt{GM_\star a(1-e^2)}.
\]

As the energy depends only on the semi-major axis of the planet, its migration rate is determined by the disc power, \( P_{\text{disc}} \), acting
on it. The power gives the total energy change of a planet
\[ E = P_{\text{disc}}. \] (3)

On the other hand, the disc torque, \( \Gamma_{\text{disc}} \) determines the rate of angular momentum change of a planet
\[ L = \Gamma_{\text{disc}}, \] (4)

which is a combination of eccentricity and semi-major axis change (see e.g. Cresswell et al. 2007). For a single planet on a circular orbit, torque and power are equivalent. In this case, it is expected that the migration of planet halts where the torque vanishes. However, for a planet on an eccentric orbit, the halting location will be given by vanishing power. In order to analyse the halting process of a planet pair, we monitored the disc torque and power continuously during the evolution of the system. Figure 19 shows the torque (top) and power (bottom) acting on the planets as a function of time for a single planet model (1p10) on the left and the reference two-planet model (2p20) on the right. In these plots, the contribution of the indirect term has been taken into account in the calculation of the torque and power, as our coordinate system is centred on the star. Hence, the torque/power is the sum of the disc contribution, denoted by subscript ‘disc’, and that of the indirect term, labelled by ‘ind’. The disc torque and the disc power are calculated as
\[ \Gamma_{\text{disc}} = \sum_{i,j=0}^{N_p,N_p} r_p \times F_{\text{pi}j}, \quad P_{\text{disc}} = \sum_{i,j=0}^{N_p,N_p} r_p \cdot F_{\text{pi}j}, \] (5)

where \( r_p \) is the location of the planet and \( F_{\text{pi}j} \) is the gravitational force between the planet and the mass in the \( i \)-th cell of the disc. Since the calculations have been performed in the star-centred non-inertial frame, there is an additional torque/power component due to the frame acceleration \( a_{\ast} \), which is the acceleration on the star exerted by the disc. The indirect torque/power vanishes when the disc is completely symmetric and there is only one planet in the system. Otherwise, this term must be taken into account. The indirect torque \( \Gamma_{\text{ind}} \) and indirect power \( P_{\text{ind}} \) are calculated using the following equations
\[ \Gamma_{\text{ind}} = M_p r_p \times a_{\ast}, \quad P_{\text{ind}} = M_p r_p \cdot a_{\ast}. \] (6)

The left column of Fig. 19 shows that the torque and power are zero for the single planet of 1p10 DZ model. On the right, similar quantities are presented for the 2p20 DZ model with two planets (left column in Fig. 5). When the planets are trapped in their final halting positions, at \( t \sim 10^4 \) orbits, the torque and power acting on the inner planet are positive and those of the outer planet are negative, while their sum cancels out. If we consider each planet individually, we would expect the inner planet to move outwards because of the positive power, and oppositely for the outer planet. To maintain an equilibrium, the two powers need to be equal but have opposite sign. This results in zero net migration for the system as a whole. While, the migration of the system is governed by the total power on the system\(^1\), the vanishing total torque indicates that the eccentricities in the system are also not changing anymore.

\(^1\) The planet-planet interaction can be neglected since these quantities, which are very oscillatory, average out when the system has reached equilibrium.

\(^2\) If the calculation is carried out in the centre of mass frame, the sum of the power on all objects including the star should vanishes to maintain the equilibrium.

To examine our claim in more detail, we performed additional N-body simulations using analogous parameters to the 2p20 DZ model. To calculate an equilibrium situation, we consider a system of two planets, where the inner planet is migrating outwards and the outer planet inwards. Upon capture in 4:3 resonance (as observed in the hydrodynamical simulation), they should come to a halt and reach an equilibrium. When the migration of the planetary system stops, its energy and angular momentum stays constant, on average. This implies that the total power should vanish, \( P_i + P_o = 0 \). Here, we neglect the interaction energy between the two planets as it averages out after they reach their parking position at the inner edge of the disc. Using this equilibrium condition and the relation
\[ P = |E| \frac{\dot{a}}{a} \equiv \frac{|E|}{\tau_a}, \] (7)

which follows from eq. (3), we obtain the ratio of the migration rate of the inner to the outer planet as
\[ \frac{\tau_{a_i}}{\tau_{a_o}} = \frac{a_i M_o}{a_o M_i} = -\left(\frac{T_o}{T_i}\right)^{-2/3} \frac{M_o}{M_i}. \] (8)

Because our system is in a 4:3 resonance, \( T_i/T_o = 4/3 \), and the mass of outer planet is twice of the inner, we get \( |\tau_{a_i}/\tau_{a_o}| \approx 1.65 \). Substituting the power and the final semi-major axis of the inner planet from Figs. 19 and 5 into eq. (7), we obtain \( \tau_{a_i} \approx 1.6 \times 10^5 \). The migration of the outer planet is simply 1.65 times slower than the inner planet and in opposite direction, meaning \( \tau_{a_o} \approx -2.7 \times 10^7 \). We input these timescales in an N-body code (Kley et al. 2005) that solves the equation of motion of the planets by the method in Lee & Peale (2002). To assure that the system settles into the desired resonance, we place the planets just outside
of the 4:3 resonance. Besides the migration timescales, eccentricity damping timescales $\tau_e$ and $\tau_o$ are also needed. For simplicity, we assume they are equal and find $\tau_e = \tau_o = -4 \times 10^3$ to give a good match between the N-body and hydrodynamical results. From eqs. (4) and (2) we have

$$\frac{\Gamma}{\mathcal{L}} = \frac{1}{2} \frac{\dot{a}}{a} - \frac{e^2}{1-e^2} \dot{e} \equiv \frac{1}{2} \frac{1}{\tau_e} - \frac{e^2}{1-e^2} \tau_e. \quad (9)$$

Using the obtained values for the damping timescales, the corresponding torques are $\Gamma_i \approx 1.18 \times 10^{-10}$ and $\Gamma_o \approx -1.19 \times 10^{-10}$. These values are indicated in the upper right panel of Fig. 19 with dashed lines, and agree well with the averaged torque on the planets obtained from the hydrodynamical simulation.

Figure. 20 demonstrates the results of the N-body compared to the final part of the hydrodynamical simulation. The fact that in the N-body simulation, the final resonance and locations match with the hydrodynamical results implies that the proper ratio of migration timescales has been used, in agreement with eq. (8). For a different ratio, the planets would not come to an equilibrium but would migrate jointly, either outwards or inwards. One needs to keep in mind that the power and torque need to be calculated by including the initial torque term when working in an accelerated reference frame.

From the equilibrium of the torque, $\Gamma_i + \Gamma_o = 0$, we can derive another important relation for the eccentricity damping timescales at the final parking position of the planet pair. Using eq. (2) for angular momentum and energy, eq. (8) from power equilibrium, and assuming that $e^2 \ll 1$ (which is well fulfilled in our cases), we find

$$\frac{1 - 2e^2 \tau_o}{1 - 2e^2 \tau_e} = \frac{T_o}{T_i}. \quad (10)$$

This implies that the final eccentricities in equilibrium are determined by the ratio of migration over eccentricity timescale $\tau_o/\tau_e$. One should remember that $\tau_o$ is positive and $\tau_e$ negative. The eccentricity damping timescales are always negative and much shorter than the migration timescales. Using the specified timescales and the equilibrium eccentricities of the N-body simulations, $e_i = 0.05$ and $e_o = 0.027$, one finds that the relation (10) is excellently fulfilled.

As another example, we show the power for the 2p20 IB model in Fig. 21. This system migrates inwards after the resonance forms, albeit with a slower rate than that of the outer planet before the resonance. The total power on this system is negative, indicating inward migration, but its magnitude is smaller than that of the outer planet. Without considering the indirect term, the total power on the system would be about zero, not consistent with the inward migration of the system. This second example demonstrates again the necessity to include the indirect term, and shows that it is the total power on the system that determines the migration of the resonant chain.

6. Comparing N-body to Hydrodynamical simulations

In this section we compare the outcome of selected hydrodynamical models to customized N-body simulations, using the method described in appendix A. Adopting the density and temperature distributions from the hydrodynamical models the N-body simulations calculate the migration of the planets. As in our hydrodynamical simulations, we used $\epsilon = 0.6$ for the smoothing length of the gravitational potential of the planet (see Sect. 2). The temperature is constant that translates to $\beta = 0$. For calculating $\alpha_\Sigma$, we took the surface density profile from the hydrodynamical simulation at two specific times. The viscosity transition is also taken into account as in eq. (1).

In Fig. 23, we display the results of two sets of N-body simulations using the initial conditions of 2p20 and 3p2020 DZ models, where the mass of inner planet is $10 M_{\oplus}$ and the outer planet(s) $20 M_{\oplus}$. Each set contains two identical setups and differ only in their surface density profile, which is chosen at $t = 50$ orbit (unperturbed) and $t = 13000$ orbit (evolved). We chose two snapshots to take a possible change of the surface density in the hydrodynamical simulations into account. The left column in this figure contains the migration of the planets, and on the right the used surface density profiles are plotted. The thicker sections in the right panels mark the region where the torque is positive. The upper/lower rows show the results for the model with two/three planets. In all of these models, the inner planets in each model is pushed by the outer planets into the positive torque region regardless of which surface density profile is used. The only difference between the outcome of these simulations is the slower migration of the planets in the model with evolved surface density profile. This slower migration is because of the flattened bump that is created by the planets in the hydrodynamical simulations.

In the two-planet simulations, the inner planet is first trapped at the zero torque location and then pushed further in by the incoming (more massive) outer planet. The migration of the outer planet is halted when it reaches the trap at outer edge of the viscosity transition. By then the inner planet has been pushed into the negative torque region again and migrates slowly inwards. However, in the three-planet cases, the two inner planets are first trapped in a resonant configuration while the outer planet is approaching. Later, at $t \approx 50000$, the outer planet catches the inner pair in resonance and pushes them out of the positive torque region into the inner disc. The migration of the inner planets can only be halted if they are still in the positive torque region at the moment the outer planet reaches its final trapping location. In all of our N-body cases, we saw that the outermost planet was able to push the inner planets inward beyond the trap region into the negative torque region, such that they continue a very slow inward migration. In strong contrast, in the corresponding hydrodynamical simulations, the inner planet was able to stop the inward migration of the planetary system, even though an unstable evolution occurred in the three planet case.

Our results hint that the strong positive torques experienced by the planets at the trap in the hydrodynamical cases are not
Fig. 22. Panel (a)–(c): Evolution of the eccentricity and resonant angle for the 2p20LH IB model when the planets are engaged in 3:2, 4:3, and 5:4 resonances, respectively. Panel (d) shows the evolution of orbital period ratio. The colour-coding corresponds to the right panel, and indicates the time evolution of the noted resonances in panels (a)–(c). Explicitly said, the blue colour in each panel (a)–(c) indicates the time before the system gets into the resonance, which is noted on top of each panel and also in panel (d), and the red colour represents the time after the system is in resonance. The exact time can be extracted from comparing the colour in panel (d) with the ones in panel (a)–(c).

Fig. 23. The right panels show the surface density in the horizontal axis and distance from the star on the vertical. The surface density profiles at \( t = 50 \) and \( t = 13,000 \) orbit are denoted by blue and orange. The thick sections represent where the torque on a 10 \( M_\odot \) circular planet is positive. The left panels show the results of N-body simulations for the initial conditions as 2p20 (top) and 3p2020 (bottom) DZ models. The colours correspond to the surface density profiles shown on the right panels. One can see that the push by the outer planet in each pair can shove the inner planet out of the trapping point.

Fig. 24. Torque from the hydrodynamical and N-body simulations versus the semi-major axes of the planet for the inner (top) and outer planet (bottom) in 2p20 DZ model. The grey lines denote the torque calculated from the formulae used in the N-body code as a function of distance from the star for a circular (solid) and an eccentric (dashed) planet. The values of the used eccentricities equal those of the hydrodynamical simulation after equilibrium. Comparing the grey lines and the outputs of the simulations show that these formulae are capable of correctly calculating the torque only for the outer planet before it approaches transition zone.

Fully reflected by the N-body simulations. This is depicted in Fig. 24, which shows the torque from the hydrodynamical and N-body simulations (using the unperturbed surface density profile) against the semi-major axes of the planets. The grey solid and dashed lines are torques calculated using the formulae in the N-body code for a circular and an eccentric planet. For the outer planet (lower panel), the torque agrees excellently between the hydrodynamical and N-body models down to \( a \sim 1.5 \). The discrepancy increases when approaching the transition edge. 

Specialy, in the hydrodynamical model, the torque on the inner planet (upper panel of Fig. 24) is positive and relatively large compared to the value given by formulae. It is why it can compensate the push by the outer planet and consequently halts at \( r \sim 1.4 \). In the N-body simulation, the torque on the inner planet is mostly negative. Even when the torque is positive inside the transition zone, it is much smaller than that of the hydrodynamical simulation. This indicates that the actual dynamical values for torque and eccentricity damping (\( r_1, r_2 \) inside and close to a transition zone differ from those calculated from the formulae which are estimated from relatively smooth disc surface density profiles. We would like to mention that the N-body torque would be even smaller if the evolved surface density profile was used in Fig. 24 because both the smaller surface density and also flatter profile would result in smaller torques.

For illustrative purposes and to connect to the next example, we show the inverse of the migration timescales\(^5\) for the hy-

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\(^5\) Because the migration timescale is extremely large for the trapped planets, showing its inverse is more proper for comparison.
The migration timescale of the inner planet in the hydrodynamical simulation is much smaller than its N-body counterpart. For the outer planet, the migration timescale is very similar in both simulations until $a \sim 1.5$. As the planet approaches the transition zone, the migration timescale in the hydrodynamical model deviates from that of the N-body.

To verify our suggestion that the difference between hydrodynamical and N-body simulations is hinged to a very small positive torque near and inside of the trap region in the N-body runs, we performed a set of additional N-body simulations in which the migration timescale is given by an analytical model. For each planet, we choose a negative constant reference migration time, $\tau_{\text{ref}}$, except inside a transition zone where it becomes positive in order to mimic the planet trap. Mathematically speaking, the migration timescale is given by $\tau_a = \tau_{\text{ref}}/\tau_{\text{trans}}$ with $\tau_{\text{trans}}$ defined as

$$f_{\text{trans}} = 1 + a \tanh \left( \frac{r - r_{\text{ref}}}{\Delta} - 1 \right) + b \left( \frac{r - (1 + c\Delta)r_{\text{ref}}}{\Delta} \right) \exp \left( -\frac{(r - r_{\text{ref}})^2}{\Delta^2} \right),$$

where $r_{\text{ref}}$ is the location of the transition, $\Delta$ determines the width, and $a$, $b$, $c$ control the shape of the transition zone: $a$ the difference between inner and outer inward migration, $b$ strength of the trap, and $c$ the bump strength. The analytical form of this function is motivated by the results from our hydrodynamical modelling. Different sets of these parameters corresponds to different shape for the disc inner edge. For example, the third term adjusts the torque if a surface density bump exists before the zero-torque location. Attention should be paid that finding a reverse process which gives the exact shape of the disc inner edge is not trivial because of the torque calculation complexities. In fact, it is very hard, if not impossible, to find the shape and the disc properties only by having the torque on the planet.  

This function is defined such that it gives a faster migration in the outer than the inner disc plus a region where the migration becomes very slow and outwards. Far from $r_{\text{ref}}$, where $f_{\text{trans}} = 1$, the migration of the planet is simply inwards with the timescale of $\tau_{\text{ref}}$. As the planet approaches $r_{\text{ref}}$, $f_{\text{trans}}$ becomes initially larger than unity. This is where the migration of the planet becomes faster due to the existence of a surface density bump before the disc edge. Afterwards, $f_{\text{trans}}$ decreases until it becomes zero and brings the migration of the planet to a halt. If by some means, for example the push of another planet, the planet moves to where $r < r_{\text{ref}}$, a negative $f_{\text{trans}}$ causes an outward migration similar to a disc inner edge. If the planet is pushed even more inwards, such than the planet is out of the disc inner edge, we expect the migration of the planet becomes again inwards but very slow due to the low surface density. This is what $f_{\text{trans}}$ does for $r < r_{\text{ref}}$. We would like to insist that this factor does not produce the torque from the asymmetric features such as vortices, which produce usually a stochastic torque.

We examined two sets of models, one with a deep and one with a shallow drop in $f_{\text{trans}}$, resembling the hydrodynamical and N-body simulations, respectively. In each set, one has a bump outside of the drop and one does not. The latter models were performed in order to check the role of the bump, which speeds up the inward migration just outside of the transition zone, as seen in the hydrodynamical models. The upper panels of Fig. 26 show $f_{\text{trans}}$ for the deep (left) and shallow (right) drop models. The used parameters in these profiles are listed in table 2.

We used the same masses and initial locations as 2p20 and considered $\tau_a = -10^4$ and $\tau_{\text{ref}} = -2 \times 10^4$. The eccentricity timescales are fixed to $\tau_e = \tau_\nu = -3 \times 10^3$. These numbers are similar to those of Sec. 5. The result of the simulations are shown in the bottom panels of Fig. 26 with the corresponding colours to each of the transition profiles.

In all of the models, the planets get into 3:2 resonance after about 2800 orbits. In the deep models (left panel), the migration of the planets is halted where the total power acting on both planets vanishes. This location is near $f_{\text{trans}} = 0$ that is denoted by a vertical dashed line. There is no significant difference between the models with and without bump. In the shallow model

![Fig. 25. Inverse of migration timescale for the hydrodynamical and N-body simulation of 2p20 DZ model.](image)

![Fig. 26. The results of four N-body simulations in which a migration timescale transition zone, as depicted with the same colour on the top panels, is used. The vertical dashed line corresponds to $f_{\text{trans}} = 0$, and marks the transition from positive to negative migration.](image)

| Parameter | DB | D | SB | S |
|-----------|----|---|----|---|
| $\Delta$  | 0.12 | 0.12 | 0.12 | 0.12 |
| $r_{\text{ref}}$ | 1.13 | 1.05 | 1.13 | 1.09 |
| $a$       | 0.45 | 0.45 | 0.45 | 0.45 |
| $b$       | 20  | 8  | 3  | 1.4 |
| $c$       | 0.2 | 1.1 | 0.2 | 1.1 |

Table 2. The list of parameters for $f_{\text{trans}}$ profiles shown in the top row of Fig. 26. The letters refer to, D: deep trap, S: shallow trap, B: bump.

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Equation 11 is a general profile that can be used for any type of planetary trap only by proper adjustment of the parameters, either the trap is a dead-zone or disc inner edge. However, we note that in the IB mode, the parameters would be time-dependent due to the change of the surface density profile by time.
with a bump, the inner planet continues its migration to the transition zone and is not able to stop the migration of the outer planet. Finally, the inner planet leaves the transition region while the outer planet stays at the trapping location and therefore, the resonance breaks. However, in the model without bump, the migration of the inner planet halts when the outer planet reaches the trapping point and they remain in 3:2 resonance. This different behaviour is due to the fact that, in spite of the S model, the inner planet has passed the transition zone before the outer planet approaches the trapping point in the SB model.

To summarise, based on the results in this section, we found that the N-body simulations favour resonant pushing, unlike hydrodynamical simulations. This can be due to the fact that the formulae, which are used in the N-body calculations, do not give proper values where the surface density deviates from the simple power-law profiles. This point requires more detailed analyses in a separate study.

7. Overstability

Among the results of our simulations in Sec. 3, the IB models with smaller aspect ratio, $h_0 = 0.03$, show overstable behaviour (see runs 2p20LH, 3p2020LH, and OnebonyeLH, as displayed in Figs. 8, 12 and 16). Overstability for planets at the inner edge of a protoplanetary disc was observed in N-body simulations by Xiang-Gruess & Papaloizou (2015), but has not been reported yet convincingly in hydrodynamical simulations.

In an overstable system, the planets enter successively various first-order resonances that are broken after a short time. In the transient resonances, the eccentricities oscillate around an equilibrium value with increasing amplitudes until the resonance breaks and the planets enter an even tighter resonance. Overstability occurs when the eccentricity damping is not efficient enough. As a result, the eccentricities of the planets, that are excited due to their mutual gravitational interaction, gradually grow while the libration amplitude increases (Goldreich & Schlichting 2014). This growth continues until the resonance is broken eventually.

Figure 27 shows the overstable behaviour in more details for the 2p20LH IB model, in which the system becomes overstable during the 3:2 and 4:3 resonances (see also Fig. 8). Panels (a), (b), and (c) demonstrate how the eccentricity $e$ and the resonant angle $\phi$ of the inner planet are evolving during the 3:2, 4:3, and 5:4 resonances. The distance of every point in these panels from the centre (0, 0) represents the eccentricity, and its angle with the horizontal axis is a measure of resonant angle. The colour scheme indicates the time evolution as it is shown in panel (d), where the time evolution of the orbital period ratio is plotted. The resonant angle for a first-order $j + 1 : j$ resonance is defined as $\phi_* = (j + 1)h_0 - jd_L - \pi\alpha$, where $\lambda$ and $\pi\alpha$ denote the mean longitude and longitude of the pericentre, respectively. If $\phi_*$, which measures the angle between the conjunction and the pericentre of the inner planet, librates around a constant value, the system is in a resonance. Panel (a) implies that as the planets settle into the 3:2 resonance, the eccentricity of the inner planet grows to $-0.04$ and the resonant angle librates around $3\pi$. The amplitude of the libration, as well as the eccentricity, increases until it circulates and the system leaves the resonance. The situation for the 4:3 resonance in panel (b) is very similar, except that the equilibrium eccentricity and resonant angle are smaller. In contrast, the final resonance 5:4 seems to be stable around $e \sim 0.025$ and $\phi_* = 0$.

The stability of first-order mean motion resonances between planets assuming constant eccentricity damping and migration rates was studied by Goldreich & Schlichting (2014). They showed that such resonance configurations are stable if $\tau_e/\tau_a$ is small enough. They also present a condition between the equilibrium eccentricity $e_{eq}$ and mass of the planets. They suggest that if $e_{eq} \lesssim (M_0/M_*)^{1/3}$, the system remains stable. For the inner planet in our 2p20LH IB model, $(M_0/M_*)^{1/3} \sim 0.03$. Therefore, their stability criterion is only satisfied in our case for the 5:4 resonance. They also found that $e_{eq} \sim (\tau_e/\tau_a)^{1/2} - h$ that predicts $e_{eq} \sim 0.03$ which roughly agrees with our results. However, the study by Xu et al. (2018), that uses a migration model based on planet-disc interaction studies, suggests that the equilibrium eccentricity of the resonant planets are larger than those given by the studies that used constant migration and eccentricity damping rates. Our results seem to match better with the work of Goldreich & Schlichting (2014).

It has been shown that $\tau_e/\tau_a$ is of the order of $h^2$ (e.g. Tanaka & Ward 2004). Therefore, we expect that overstability is more probable to occur in thicker discs while we observe it for our thinner models. This may be explained by noting that the planets in the models with $h_0 = 0.03$ open a partial gap around their orbits. Although this partial emptiness around the orbit of the planets slow down their migration, it also decreases the efficiency of the eccentricity damping. Therefore, it seems that in our simulations the net effect of the gap opening is more in the favour of larger $\tau_e/\tau_a$. Figure 27 shows $\tau_e/\tau_a$ scaled by $h^2$ for the model 2p20LH and 2p20 for when the planets are in a resonance. We can see that, in 2020LH model, $\tau_e/\tau_a$ is larger at the overstable resonances than that of the stable resonance. Interestingly, the scaled ratio has similar values for the stable resonance in both 2p20 and 2p20LH models. However, this speculation needs to be justified by more rigorous studies.

8. Stability of the systems and comparing with observation

Formation of planetary systems is a long journey from when they form in a protoplanetary disc until the disc dispersal. Thus, it is uncertain if those of our systems that survive from instability during the simulation time could remain stable afterwards. Because continuing all of our simulations would be computationally very costly, we only examined whether DZ and IB 2p20 and 2p20LH models retain their configuration during the disc dis-
persal. We include the X-ray photo-evaporation model of Owen et al. (2012) with the X-ray luminosity of $L_x = 10^{32}$ erg/s \(^8\) and continue the mentioned simulations for over thousands of orbits until the disc is dispersed around the planets. The outcome of these tests show that none of them become unstable (Fig. 28 and B.7). Similar outcomes can be seen in the results of the N-body simulations which include photo-evaporation (e.g. Migaszewski 2015, 2016).

To inspect the stability of other models, we used the criteria presented by Chambers et al. (1996) and Gladman (1993), which suggest that a low-mass planetary system would remain stable if $\Delta/R_H > 2 \sqrt{3}$ for two-planet systems, and always be unstable if $\Delta/R_H < 10$ for more than two planets. Here, $\Delta$ is the initial difference of the semi-major axes of the planets, and $R_H$ is their mutual Hill radius. For all of our two-planet models, this condition is well satisfied. Among the multi-planet models, it is only IB 3p1020 that has $\Delta/R_H$ above 10. Therefore, it is the only two-planet system that would survive after a long-term evolution.

We also compared these stable systems with observed exoplanets \(^9\) and found two systems, Kepler-804 and K2-189, that have configurations close to our results. Kepler-804 has two planets revolving around a Solar-type star (Morton et al. 2016) with the size of the outer planet about twice of the inner one (that is almost an Earth-size planet) and the outer to inner period ratio of $\sim 14.3/9.6 \approx 1.5$. Using the mass-radius relation in Wolfgang et al. (2016) for sub-Neptune-sized planets, the mass and period ratios of this system resemble our DZ 2p20 model (although the masses are smaller than ours). The other system, K2-189, has also two planets with the radii of about 2.48 and 1.51 times of the Earth that are revolving around a star with the mass of 0.93$M_\oplus$ Mayo et al. (2018). The period ratio of the planets is about 1.3 that is near 4:3 commensurability. Using the relation in Wolfgang et al. (2016) gives a mass ratio of the outer to inner $\sim 1.9$. These mass and orbital period ratios are similar to our IB 2p20 model. Therefore, both of our tested inner edge models could be used for the explaining the formation of these observed close-in exo-planetary systems.

\(^8\) This high value of $L_x$ is chosen to speed up the simulations.

\(^9\) http://www.openexoplanetcatalogue.com/

9. Summary and conclusion

Using hydrodynamical simulations, we investigated whether a trapped planet at a dead-zone inner edge (DZ) or a disc inner boundary (IB) can be pushed into the inner cavity towards the central star by a resonant chain. We examined systems with two and three planets where the planets were placed in the disc simultaneously, and systems with more than three planets in which the planets were added one by one after the previous planets formed a resonant chain. The DZ model has a very narrow and steep transition in the surface density, and the IB model has a much shallower profile. The mass of the inner planet was always 10$M_\oplus$ while the outer planets had either the same mass or were more massive. We summarize and discuss our findings in the following.

The steepness of the inner disc edge, i.e. the gradient of the surface density plays a key role in the dynamical evolution of our systems. The DZ model, where the sign of the power (and torque) abruptly changes at the edge of the transition zone, in which the planet experiences positive power, acts as a dam that can stop an incoming resonant chain of planets. The inner planet cannot pass this dam unless an instability in the system hurls it to the inner disc. On the other hand, in the IB model, which has a smoother transition and a less steep surface density profile, a resonant chain can push the inner planet to the inner cavity. However, even in this case it requires multiple, and/or more massive planets to push the inner planet over the trap. Test calculations using a broader transition zone in the DZ setups produce wider resonant systems which are less prone to instability. These tests indicate that if the viscosity transition zone is very wide, the inner planet might be pushed out of its trapping point, similar to the IB models.

In the two-planet cases, the final configurations in the DZ models are usually tighter than in their IB counterparts. A typical outcome for a 10 and 20$M_\oplus$ combination is the 4:3 resonance. Most of the three-planet systems in the DZ models became unstable and a scattering event between the two inner planets occurred, while in the IB models, the planets were trapped in a 3:2:1 resonant chain. In the models where we added the planets one by one, increasing the number of planets has no effect on the innermost planet and only raises the chance of instability.

Planets in the models with $h_0 = 0.03$ behave differently than those with $h_0 = 0.05$. In the cooler discs, the planets open partial gaps around their orbits. The gap opening changes both the migration rate and eccentricity damping such that overstable oscillations of the planets’ eccentricities occur (Goldreich & Schlichting 2014) and the planets reach successively tighter resonances, jumping for example from 3:2 to 4:3 to 5:4. Our results are the first hydrodynamical simulations that clearly show the overstable behaviour in resonant planetary systems, see Fig. 22. For computational reasons we focussed our study on transitions located at 1 au with typical aspect ratio of $h = 0.05$. However, the inner edge of discs lies about 10 times closer to the star where the disc is much thinner $h = 0.02$ to 0.03, and we expect that under these conditions overstability may occur frequently. This raises and highlights the need for more detailed studies in this topic.

In addition to the aspect ratio, we varied the disc mass (surface density, $\Sigma_0$) and viscosity ($\nu$). Using a higher surface density for the disc, only speeds up the evolution while having no specific effect on the final results. This is in agreement with early studies of resonant capture by Lee & Peale (2002) who showed that it is the ratio of migration over eccentricity damping that determines the outcome of capture, and not the absolute timescale.
Although the planets in the less viscous disc were able to slightly modify the surface density profile of the disc through partial gap opening, the outcome of the models are not significantly different than those with higher viscosity. In the DZ simulations the steep positive surface density profile gives rise to the creation of vortices, that can potentially interfere in the migration of the planets. Our results show that the presence or number of vortices at the edge of a dead-zone does not influence the evolution of the system greatly, besides a small enhancement of the susceptibility to unstable evolutions.

The eccentricity of the planets remain small as long as no instability happens or they are not pushed into the inner disc, where the surface density is very low. This indicates that the eccentricity damping timescales in our models are still much shorter than the migration timescales. The comparison of our hydrodynamical and N-body simulations show indeed a ratio of $\tau_e/\tau_4 \approx 40$ for the inner 10 $M_\oplus$ planet.

We analysed in detail the conditions for an equilibrium parking of a system of two planets at the inner edge. In agreement with previous results for migrating planets (Cresswell et al. 2007), we show that it is determined by vanishing total power, where all objects have to be taken into account. This could be formulated as a condition for the ratio of migration timescales, see eq. (8). The additional requirement of vanishing total torque yields a condition for the equilibrium eccentricities, see eq. (10). This last equation implies particularly that resonant planetary systems created by trapping must have a non-zero eccentricity. The validity of these relations was confirmed in our hydrodynamical as well as N-body simulations.

As hydrodynamical simulations are very time-consuming, we performed additional customised N-body simulations in order to test the often-used approach of taking analytical formulæ for the planet’s torque and eccentricity damping. In the appendix, we demonstrated that the direct usage of the disc parameters, namely density and temperature profiles, from the hydrodynamical simulations as inputs for the forces in the N-body formulæ does result in different evolutions of the planetary systems. In particular, the inner planet in the N-body simulations was not able to stop the resonant chain. This indicates that the positive torque (power) contribution from the inner edge of the disc is underestimated by the analytical formulæ.

The main conclusion of our study is that it is difficult for the planets to be pushed into the cavity created by a dead-zone inner edge through resonant migration due to its steep density slope. However, for a smoother planetary trap, such as a disc inner boundary a resonant chain can help pushing the planets inwards. Therefore, if we consider a dead-zone inner edge that does not coincide the the inner boundary of the disc but lies further out, the formed planets inside the dead-zone would have difficulties to get into the inner disc, unless a the resonant chain becomes unstable. Nonetheless, if they manage to pass the dam of the viscosity transition, they can easily move towards the disc inner edge by the help of resonant migration.

In order to compare our results with the observed exoplanetary system, we also examined the stability of our models and found that the two-planetary systems are more probable to remain stable. Among our stable models, we only found two observed systems, Kepler-804 and K2-189, that are similar to the outcome of our 2p20 models. Although, we can not make a conclusion based on these two observed systems, we suggest that the more packed systems are more likely to form in a disc with a steeper inner edge.

We would like to note that our results are obtained using locally isothermal disc. The study by Faure & Nelson (2016), who investigated the planet trapping at a dead-zone edge in a non-isothermal disc using 2D and 3D magnetohydrodynamical simulations, hints that the inner edge of a dead-zone can be a mass-dependent barrier for the migration of vortices, which formed inside the dead-zone, and also planets.

As mentioned before, for computational reasons, we placed the inner boundary at a distance of 1 au using typical values for the disc surface density, aspect ratio, and viscosity at that location. The models with smaller aspect ratio or higher surface density may resemble more the conditions closer to the star. In particular, the thinner disc models ($h_0 = 0.03$) give an insight towards how the resonant pushing can be different in the inner disc in the later stages of the disc evolution. Hence, we might expect that overstable librations with subsequent tightening of the resonant chains are a frequent outcome and may help in producing very compact resonant systems such as Kepler-223 (Mills et al. 2016).

This study has been carried out using a background disc profile which dictates divergent migration in the absence of a planetary trap. If one used a different disc setup that governs the converging migration, instability would be more probable.

One of the questions that remain open in this study is that what happens to the resonant planets once they are pushed into the inner cavity. In some IB models, where the inner planet is pushed all the way to the region with very low surface density, its eccentricity increased to large values. Whether the system will remain stable afterwards is a question we could not answer using our hydrodynamical simulations.

The tidal interaction between the star and the planets can be an important phenomenon for the planets located very close to the star. In this work, as the first hydrodynamical study which tries to simulate multi-planets at the disc inner edge, we ignored this effect for simplicity. How much the evolution can be changed by this effect is a question for future works.

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Appendix A: N-body simulations - methods

N-body simulations of migrating planets are a common alternative for the lengthy and computationally expensive hydrodynamical simulations. In studies where the long term evolutions of many planetary systems are desired, using hydrodynamical simulation is not only expensive but can even be unfeasible. Therefore, using N-body simulations augmented with a suitable migration treatment is a good option (e.g. Papaloizou & Larwood 2000; Ogihara & Ida 2009; Coleman & Nelson 2014; Izidoro et al. 2017). The basic ingredients for performing such a simulation is an N-body code, an expression for the disc torque acting on the planet, a relation for calculating the eccentricity damping timescale, and an equation of motion that modifies the acceleration of the planets accordingly.

In order to examine how well the results of such simulations agree with their hydrodynamical counterparts, we employed the REBOUND code\(^{10}\) (Rein & Liu 2012) with the WHFAST integrator (Rein & Tamayo 2015) with a timestep of \(dt = 10^{-3}\) orbits, and implemented the formulae of Paardekooper et al. (2010a, 2011) into it for calculating the torques on the planets. The torque has two components: Lindblad, \(\Gamma_{\ell}\), and corotation, \(\Gamma_c\). Both of these are altered if the planet is eccentric. The corresponding correction factors for the Lindblad and corotation torques, \(\Delta_{\ell}\) and \(\Delta_c\), respectively, are taken from Cresswell & Nelson (2008) and Fendyke & Nelson (2014). The formulae we use are identical to Coleman & Nelson (2014) and Izidoro et al. (2017) except that: (a) our simulations are 2D and therefore the inclination is set to zero, (b) we applied the additional correction factors for the smoothing length of the planet as in Paardekooper et al. (2010a) for consistency purpose, and (c) because the disc is locally isothermal, we consider the thermal diffusivity \(\chi\) as infinite and set the adiabatic index to \(\gamma = 1\). For completeness and reference, we summarize the relevant formulae briefly. The total torque is given by

$$\Gamma = \Delta_{\ell} \Gamma_{\ell} + \Delta_c \Gamma_c, \quad \text{(A.1)}$$

where \(\Gamma_c\) is the sum of the two components, barotropic and entropy-related co-rotation torque, and each of them has a linear torque and a horseshoe drag (see Paardekooper et al. 2011). Therefore, the co-rotation torque reads

$$\Gamma_c = \Gamma_{c, \text{baro}} + \Gamma_{c, \text{ent}}, \quad \text{(A.2)}$$

$$\Gamma_{c, \text{baro}} = \Gamma_{c, \text{hs, baro}} F(p_r) G(p_r) + \Gamma_{c, \text{lin, baro}} (1 - K(p_r)), \quad \text{(A.3)}$$

$$\Gamma_{c, \text{ent}} = \Gamma_{c, \text{lin, ent}} \sqrt{(1 - K(p_r))}, \quad \text{(A.4)}$$

The functions \(F(p_r), G(p_r),\) and \(K(p_r)\) regulate the saturation of the co-rotation torque due to the disc viscosity, where \(p_r\) is defined as \(2/3 \sqrt{\sqrt{\Omega^2 \Sigma^2}}/2\pi r \), with \(x_h\) being the half-horseshoe width of the planet. For brevity, we avoid bringing the definition of the functions and refer the readers to Paardekooper et al. (2011) or Izidoro et al. (2017). Please note that the entropy-related horseshoe drag vanishes due to the locally isothermal assumption. The Lindblad torque and the components of the co-rotation torque are as following

$$\frac{\Gamma_{L}}{\Gamma_0} = (-2.5 - 1.7\beta + 0.1\alpha^2) \left(\frac{0.4}{\epsilon}\right)^{0.71}, \quad \text{(A.5)}$$

$$\frac{\Gamma_{c, \text{hs, baro}}}{\Gamma_0} = 1.1(1.5 - \alpha^2) \left(\frac{0.4}{\epsilon}\right), \quad \text{(A.6)}$$

$$\frac{\Gamma_{c, \text{lin, baro}}}{\Gamma_0} = 0.7(1.5 - \alpha^2) \left(\frac{0.4}{\epsilon}\right)^{1.26}, \quad \text{(A.7)}$$

$$\frac{\Gamma_{c, \text{lin, ent}}}{\Gamma_0} = 2.2\beta \left(\frac{0.4}{\epsilon}\right)^{0.71} - 1.4\beta \left(\frac{0.4}{\epsilon}\right)^{1.26}, \quad \text{(A.8)}$$

where \(\Gamma_0\) is the torque normalization at the planet’s semi-major axis, \(a\), which is given by

$$\Gamma_0 = \left(\frac{q}{h}\right)^3 \Sigma(a) a^4 \Omega^2. \quad \text{(A.9)}$$

The radial slopes of the density and temperature stratifications are denoted by

$$\alpha^\Sigma = -\frac{\partial \log \Sigma}{\partial \log r} \quad \text{and} \quad \beta = -\frac{\partial \log \nu}{\partial \log r}, \quad \text{(A.10)}$$

with \(\nu\) being the temperature. The eccentricity correction factors for the torque are calculated using

$$\Delta_{\ell} = \frac{1 - (\frac{q}{h})^2}{1 + (\frac{q}{h})^2 + (\frac{q}{h})^6}, \quad \text{(A.11)}$$

$$\Delta_c = \exp\left(\frac{e}{0.5h + 0.01}\right). \quad \text{(A.12)}$$

The calculated torque along with the eccentricity and semi-major axis of the planets give the timescale of angular momentum change

$$\tau_a = L/\Gamma. \quad \text{(A.13)}$$

The second ingredient, the eccentricity damping timescale \(\tau_e\), is obtained from eq. (11) in Cresswell & Nelson (2008) and reads

$$\tau_e = 1.282 \left[1 - 0.14\left(\frac{e}{h}\right)^2 + 0.06\left(\frac{e}{h}\right)^3\right] \frac{h^2 a^3 \Omega}{\Gamma_0}. \quad \text{(A.14)}$$

And finally, we use accelerations given by Papaloizou & Larwood (2000) as

$$a = \frac{v}{\tau_L} - \frac{2(r \cdot v)}{r^2 \tau_e} r. \quad \text{(A.15)}$$

The connection of the actual migration timescale, \(\tau_a\), and the angular momentum change, \(\tau_L\), is given by

$$\frac{1}{\tau_a} = \frac{2}{\tau_L} + \frac{\epsilon^2}{1 - \epsilon^2} \tau_e^{-1}. \quad \text{(A.16)}$$

see eq. (9). Only for a circular orbit, the migration timescale equals half of the angular momentum loss timescale.

Appendix B: Supplementary figures

In order to keep the main text more concise we moved supplementary figures to this appendix. Refer to the main text and Table 1 for details on the specific model setup.

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\(^{10}\) The N-body code REBOUND is freely available at http://github.com/hannorein/rebound.
Fig. B.1. Similar to Fig. 4 but for 2pJup models.

Fig. B.2. Orbital period ratio for the 2p20HS DZ model, whose surface density is twice that of 2p20.

Fig. B.3. Similar to Fig. 10 but for 3p2020 model.

Fig. B.4. Eccentricity and orbital period ratio of for 3p20100 models.

Fig. B.5. Evolution of eccentricity and orbital period ratio for 3p2020LA which has a smaller viscosity.

Fig. B.6. The two top panels are similar to Fig. 17 but for 3p2020 model. In the middle panel, the lines without/with marker correspond to the results for the inner/outer planet pair. The bottom panel shows the migration of the most inner planet in these three simulations.
Fig. B.7. Similar to Fig. 28 but for 2p20LH models.