Speeding up antidynamical Casimir effect with nonstationary qutrits

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The antidynamical Casimir effect (ADCE) is a term coined to designate the coherent annihilation of excitations due to resonant external perturbation of system parameters, allowing for extraction of quantum work from nonvacuum states of some field. Originally proposed for a two-level atom (qubit) coupled to a single cavity mode in the context of nonstationary quantum Rabi model, it suffered from very low transition rate and correspondingly narrow resonance linewidth. In this paper we show analytically and numerically that the ADCE rate can be increased by at least one order of magnitude by replacing the qubit by an artificial three-level atom (qutrit) in a properly chosen configuration. For the cavity thermal state we demonstrate that the dynamics of the average photon number and atomic excitation is completely different from the qubit’s case, while the behavior of the total number of excitations is qualitatively similar yet significantly faster.

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I. INTRODUCTION

The broad term dynamical Casimir effect (DCE) refers to the generation of excitations of some field (Electromagnetic, in the majority of cases) due to time-dependent boundary conditions, such as changes in the geometry or material properties of the system [11-14] (see [15,16] for reviews; see also [7,9] for the related problem of a particle in a wall with moving boundaries). In the so called cavity DCE one considers nonadiabatic (periodic or not) modulation of the cavity natural frequency by an external agent, investigating the accumulation of intracavity photons or the photon emission outside the cavity [11,10,12]. The additional interaction of the cavity field with a stationary ‘detector’ during the modulation (harmonic oscillator, few-level atom or a set of two-level atoms in the simplest examples) may dramatically alter the photon generation dynamics, for instance, altering the field statistics, shifting the resonance frequency and inhibiting the photon growth [13,19] (see [20] for a short review). Moreover, the degree of excitation of the detector varies according to the regime of parameters, and entanglement can be created between the cavity field and the detector, or between the set of atoms coupled to the field [21-25].

Over the past ten years a new path has attracted attention of the community working on nonstationary phenomena in cavity Quantum Electrodynamics (QED). Instead of changing the cavity frequency, different studies suggested the parametric modulation of the ‘detector’ instead, promoting it from a passive to an active agent responsible for both the generation and detection of photons [26-55]. Beside eliminating the inconvenience of time-dependent Fock states of the field associated to time-varying cavity frequency [10], this scheme makes full use of the counter-rotating terms in the light–matter interaction Hamiltonian and does not require the inclusion of additional parametric down-conversion terms in the formalism [26,31,34,35]. Moreover, it benefits from recent advances in the coherent control and readout of microscopic few-level quantum devices developed in the realm of the circuit QED for applications in Quantum Information Processing (see [36] for a recent review).

The area of circuit QED investigates the interaction of artificial superconducting atoms, formed by a sophisticated array of Josephson Junctions, and the Electromagnetic field confined in increasingly complex microwave resonators, ranging from waveguide resonators or 3D cavities [37,41]. The advances in engineering allowed for implementation of multi-level atoms, with controllable transition frequencies and coupling strengths, that can interact with multiple cavities and other atoms controlled independently [32,38,42-48]. Moreover, circuit QED allows for unprecedented atom–field coupling strength, in what became known as ultrastrong and deep strong coupling regimes [49,52]. In the context of DCE, thequisite control over the parameters of the Hamiltonian allows for multi-tone multi-parameter modulations [26,53,55], while quantum optimal control strategies can be used to enhance the desired effects [56].

Photon generation is not the only phenomenon induced by parametric modulations in circuit QED. It was shown recently that the counter-rotating terms can also be employed to annihilate excitations of the Electromagnetic field from nonvacuum initial states, in what became known as antidynamical Casimir effect (ADCE) [34]. This effect was predicted in the context of the quantum Rabi model, which describes the interaction of the cavity field with a two-level atom [57,59], and consists in

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the coherent annihilation of three photons accompanied by the excitation of the far-detuned atom \(^{60, 61}\) (four photons could be annihilated by employing a two-tone modulation \(^{54}\)). Thus an amount of energy \(\lesssim\) photons could be annihilated by employing a two-tone by the excitation of the far-detuned atom \(^{60, 61}\) (four the coherent annihilation of three photons accompanied
the lowest energy state. In Sec. V we evaluate analyt-

dynamics in the dressed-states basis. In Sec. III we dis-
malism to obtain approximate expressions for the system
our problem and derive the general mathematical for-
additional technical issues.
This paper is organized as follows. In Sec. II we define
and the general mathematical formalism to obtain approximate expressions for the system
dynamics in the dressed-states basis. In Sec. III we discuss
three specific configurations of the qutrit for which
the overall behavior is most easily inferred: the double-
divergent regimes. In Sec. IV we identify the regimes of parameters and the transitions
for which excitations can be annihilated from the cavity thermal state, assuming that the atom was initially in the lowest energy state. In Sec. V we evaluate analytically the transition rates associated to ADCE between different dressed states and compare our predictions to the exact numerical solution of the Schrödinger equation, demonstrating that the ADCE rate can undergo almost 50-fold increase compared to the qubit’s case while the amount of annihilated excitations is roughly the same. Our conclusions are summarized in Sec. VI.

II. MATHEMATICAL FORMALISM

We consider a three-level artificial atom (qutrit) interacting with a single cavity mode of constant frequency \(\omega_0\), as described by the Hamiltonian (we set \(\hbar = 1\))

\[
\hat{H} = \omega_0 \hat{n} + \sum_{k=0}^{2} E_k \hat{a}^\dagger \hat{a} \hat{\sigma}_{k,k} + \sum_{k=0}^{1} G_k (\hat{a}^\dagger \hat{a}) (\hat{\sigma}_{k+1,k} + \hat{\sigma}_{k,k+1}).
\]

(1)

\(\hat{a}\) \((\hat{a}^\dagger)\) is the cavity annihilation (creation) operator and \(\hat{n} = \hat{a}^\dagger \hat{a}\) is the photon number operator. The atomic
eigenenergies are \(E_0 = 0, E_1\) and \(E_2\), with the corre-

represent the externally prescribed modulation, where
the collective index \(l\) denotes \(\{E; k = 1, 2\}\) or \(\{G; k = 0, 1\}\). Constants \(0 \leq w_l^{(j)} \leq 1\) and \(\phi_l^{(j)}\) are the weight and the phase corresponding to the harmonic modulation of \(l\) with frequency \(\eta_l^{(j)}\), and the index \(j\) runs over all the imposed frequencies (in this paper at most 2-tone modulations will be examined). We normalize the weights so that \(\sum_j w_l^{(j)} = 1\) for any set \(l\), so that \(\varepsilon_l\) characterizes completely the modulation strength (in our examples we shall set \(w_l^{(j)} = 1\) and \(\phi_l^{(j)} = 0\) unless stated otherwise).

To obtain a closed analytical description we first rewrite the Hamiltonian as \(\hat{H} = \hat{H}_0 + \hat{H}_c\), where

\[
\hat{H}_0 = \omega_0 \hat{n} + \sum_{k=0}^{2} \left[ E_{0,k} \hat{a}^\dagger \hat{a} \hat{\sigma}_{k,k} + G_{0,k} (\hat{a}^\dagger \hat{a} \hat{\sigma}_{k+1,k} + \hat{\sigma}_{k,k+1}) \right]
\]

(3)
is the bare Hamiltonian in the absence of modulation and counter-rotating terms (to shorten the formulas we defined formally \(G_{0,2} = \varepsilon_{G,2} = 0\)). For the realistic weak coupling regime \((G_{0,0}, G_{0,1} \ll \omega_0)\) we expand the wavefunction corresponding to the total Hamiltonian \(\hat{H}\) as

\[
|\psi(t)\rangle = \sum_{n=0}^{\infty} \sum_{S} e^{-it\lambda_{n,S}} A_{n,S}(t) |\varphi_{n,S}\rangle,
\]

(4)

where \(\lambda_{n,S}\) and \(|\varphi_{n,S}\rangle\) are the \(n\)-excitations eigenvalues and eigenstates (dressed states) of the Hamiltonian \(\hat{H}_0\) and the index \(S\) labels different states with a fixed number of excitations \(n\), which is the quantum number associated to the operator \(\hat{N} = \hat{n} + |1\rangle\langle 1| + 2|2\rangle\langle 2|\). As shown in Sec III the range of values of \(S\) depends on \(n\), and we denote such degeneracy with \(g(n)\). Moreover, the number of excitations in the subspace having the atom in its ground \((|0, n\rangle)\).

Following the approach detailed in \(^{31, 31}\) we propose a change of variables that maps each group of \(g(m)\) variables \(A_{m,T}\) into another set \(b_{m,T}\), so that \(A_{m,T} = \sum_{T} a_{T} b_{m,T}\). In particular, we consider the following
transformation:

\[ A_{m,T} = \frac{1}{\sqrt{2^j}} \sum_{S(m) \neq T} e^{i\Phi_{m,T}(t)} \left\{ e^{-i\nu_{m,T}b_{m,T}(t)} - \frac{1}{2} \sum_{S(m) \neq T} e^{-i\nu_{m,S}b_{m,S}(t)} \sum_{k=0}^{2} \sum_{L=E,G} \gamma_{L,k,T,S} e^{-i\phi_{L,k}} \right\} \]

\[ \Phi_{m,T}(t) = \sum_{j} \sum_{k=0}^{2} \frac{\gamma_{L,k,m,T,S}}{\eta_{j}} \left[ \cos(\eta_{j} t) + \phi_{j,L,k} - \cos \phi_{j,L,k} \right] \]

where we divided the sum in two parts: \( \sum_{j}^f \) runs over ‘fast’ frequencies \( \eta_{j} \sim \lambda_{m+2,S} - \lambda_{m,T} \) and \( \sum_{j}^s \) runs over the ‘slow’ ones \( \eta_{j} \sim |\lambda_{m,S} - \lambda_{m,T}| \). The small frequency shift \( \nu_{m,T} \) will be given in Eq. 13 and we introduced constant coefficients \( (k = 0, 1, 2) \)

\[ \gamma_{E,k,j}^{m,T,S} = \varepsilon_{E,k} u_{E,k}^{(j)} \langle \varphi_{m,T} | \hat{\sigma}_{k,k} | \varphi_{m,S} \rangle \]

\[ \gamma_{G,k,j}^{m,T,S} = \varepsilon_{G,k} w_{G,k}^{(j)} \langle \varphi_{m,T} | (\hat{a}_{\sigma_{k+1,k}}^{+} + \hat{a}_{\sigma_{k,k+1}}) | \varphi_{m,S} \rangle \].

After substituting \( \Lambda_{m,T} \) into the Schrödinger equation and systematically eliminating the rapidly oscillating terms via Rotating Wave Approximation (RWA) 31, to the first order in \( \varepsilon_{E,k} \) and \( \varepsilon_{G,k} \) we obtain the approximate differential equation for the effective probability amplitude

\[ \dot{b}_{m,T} = \sum_{S(m) \neq T} \zeta_{m,T,S} e^{it(\lambda_{m,T} - \lambda_{m,S})} b_{m,S} \]

\[ + \sum_{j} \sum_{S(m) \neq T} \Xi_{m,T,S}^{(j)} e^{it\nu_{m,T}S} \delta_{m,T} - \eta_{j}) b_{m,S} \]

\[ + \sum_{j} \left[ \sum_{S(m+2)} \Theta_{m+2,T,S}^{(j)} e^{it(\lambda_{m+2,S} - \lambda_{m,T} - \eta_{j})} b_{m+2,S} \right] \]

\[ - \sum_{S(m-2)} \Theta_{m-2,T,S}^{(j)} e^{it(\lambda_{m-2,S} - \lambda_{m,T} - \eta_{j})} b_{m-2,S} \right] \]

The time-independent transition rates between the dressed states are

\[ \zeta_{m,T,S} = \frac{1}{i} \sum_{k,l=0} G_{0,k} G_{0,l} \left\{ \sum_{R(m-2)} \frac{\Lambda_{k,m+2,T,R} \Lambda_{l,m+2,S,R}}{\lambda_{m+2,R} - \lambda_{m,S}} \right\} \]

\[ \Theta_{m+2,T,S}^{(j)} = \sum_{k=0}^{1} \frac{G_{0,k}}{2} \left[ -\frac{\delta_{G,k}}{\lambda_{m+2,R} - \lambda_{m+2,S} + \eta_{j}} \right] \]

\[ \Theta_{m-2,T,S}^{(j)*} e^{it(\lambda_{m-2,R} - \lambda_{m,T} - \eta_{j})} \]

\[ \lambda_{m,T} \neq \lambda_{m,R} \]

\[ \lambda_{m,T} \neq \lambda_{m,R} + \eta_{j} \]

Here \( \varpi_{m,T} \equiv sign(\lambda_{m,T} - \lambda_{m,S}) \) and we introduced the complex modulation depth \( \varepsilon_{E,k} \equiv \varepsilon_{G,k} \exp(i\phi_{j}) \).

Moreover, we defined the corrected eigenfrequencies

\[ \tilde{\lambda}_{m,T} \equiv \lambda_{m,T} + \nu_{m,T} + \Delta \nu, \]

where the correction due to counter-rotating terms reads

\[ \nu_{m,T} = \left[ \sum_{S(m-2)} \left( \sum_{k=0}^{1} G_{0,k} \Lambda_{k,m+2,S,T} \right)^{2} \right] \left[ \frac{1}{\lambda_{m+2,R} - \lambda_{m,T}} \right] \]

and \( \Delta \nu \) denotes the neglected contributions smaller than \( \nu_{m,T} \) and the terms of the order \( \theta_{m,T}^{2} / \omega_{0} \), \( \theta_{m,T}^{2} / \omega_{0} \).

Throughout the derivation of the formula (9) we have assumed the constraints

\[ |\lambda_{m,T} - \lambda_{m,S}|, |\Gamma_{m,L,j,T,S}|, \left| \frac{G_{0,k} \Lambda_{m+2,S,T}}{\lambda_{m+2,R} - \lambda_{m,T}} \right| G_{0,1} \ll \omega_{0} \]

Under these approximations we have \( |A_{m,T}| \approx |b_{m,T}| \), so from Eq. [9] one can easily infer the evolution of populations of the dressed states. Besides, the generalization of our method for \( N \)–level atoms and second-order effects is straightforward 31.

It is worth noting that the occurrence of ADCE is essentially governed by the transition rates \( \Theta_{m,T,S}^{(j)} \) that couple states belonging to subspaces with different numbers of excitations. Of course the whole dynamics is determined also by the transitions occurring inside each subspace, but the annihilation of (two) excitations is possible only in the presence of non negligible \( \Theta \)-terms.
III. ANALYTICAL REGIMES

We shall confine ourselves to three different regimes of parameters when the dressed states have simple analytical expressions. With the aid of these formulas we shall be able to evaluate analytically the coefficients \( \Theta^{(j)}_{m+2,T,S} \) in the section IV.

The ground state of \( \hat{H}_0 \) is \( |\varphi_0\rangle = |0, 0\rangle \) and the corresponding eigenenergy is \( \lambda_0 = 0 \). In this paper we denote \( |k, n\rangle \equiv |k\rangle_{\text{atom}} \otimes |n\rangle_{\text{field}} \), where \( k \) stands for the atomic level and \( n \) stands for the Fock state. Moreover, we define the bare atomic transition frequencies as

\[
\begin{align*}
\Omega_{01} &= E_{0,1} - E_{0,0} \equiv \omega_0 - \Delta_1, \\
\Omega_{12} &= E_{0,2} - E_{0,1} = \omega_0 - \Delta_2,
\end{align*}
\]

where \( \Delta_1 \) and \( \Delta_2 \) are the bare detunings.

A. Two-level atom (2L)

We include this case \( (G_{0,1} = 0) \) to compare the advantages and disadvantages of using qutrits instead of qubits. The exact expressions for \( m \geq 1 \) read

\[
\lambda_{m,\pm} = \omega_m - \frac{\Delta_1}{2} \pm D\frac{\beta_m}{2}
\]

(15)

\[
|\varphi_{m,\pm}\rangle = \frac{1}{\sqrt{\beta_m}} \left[ \sqrt{\beta_{m,\pm}}|0, m\rangle \pm D\sqrt{\beta_{m,\pm}}|1, m-1\rangle \right],
\]

(16)

where \( \beta_m = \sqrt{\Delta_1^2 + 4G_{0,0}^2m} \) and we introduced the detuning symbol \( D = \pm 1 \) for \( \Delta_1 \geq 0 \) and \( D = -1 \) for \( \Delta_1 < 0 \).

For the qutrits we can use Eqs. (15) – (16) for the subspace containing a single excitation, \( m = 1 \); the dressed states with \( m \geq 2 \) excitations are presented below.

B. Double-resonant regime (RR)

When both \( G_{0,0} \) and \( G_{0,1} \) are nonzero, first we consider the special case when \( \Delta_2 = -\Delta_1 \), so that we have the double-resonance \( \Omega_{02} = E_{0,2} - E_{0,0} = 2\omega_0 \). The exact formulas read (for \( m \geq 2 \))

\[
\lambda_{m,0} = m\omega_0, \quad \lambda_{m,\pm} = m\omega_0 \pm D\varrho_{m,\pm}
\]

(17)

\[
|\varphi_{m,0}\rangle = N^{-1}_{m,0} \left[ -G_{0,1}\sqrt{m-1}|0, m\rangle + \sqrt{m}G_{0,0}|2, m-2\rangle \right],
\]

\[
|\varphi_{m,\pm}\rangle = N^{-1}_{m,\pm} \left[ \sqrt{m}G_{0,0}|0, m\rangle \pm D\varrho_{m,\pm}|1, m-1\rangle + \sqrt{m-1}G_{0,1}|2, m-2\rangle \right],
\]

where we defined

\[
\varrho_m = \sqrt{\Delta_1^2/4 + mG_{0,0}^2 + (m-1)G_{0,1}^2}.
\]

(18)

\[
\varrho_{m,\pm} = \varrho_m \pm |\Delta_1|/2, \quad \varrho_{m,0} = \sqrt{mG_{0,0}^2 + (m-1)G_{0,1}^2}
\]

\[
N_{m,0} = \varrho_{m,0}, \quad N_{m,\pm} = \varrho_{m,\pm}/\varrho_{m,0}.
\]

For example, if \( G_{0,1} \sim G_{0,0} \) and \( |\Delta_1| \gg G_{0,0}\sqrt{m} \) for all relevant values of \( m \) we have approximately \( |\varphi_{m,\pm}\rangle \sim |1, m-1\rangle, |\varphi_{m,0}\rangle \sim (|0, m\rangle + |2, m-2\rangle)/\sqrt{2} \), while for \( |\Delta_1| \ll G_{0,0}, G_{0,1} \) (near the atom–field resonance) we get

\[
|\varphi_{m,\pm}\rangle \sim (|0, m\rangle \pm \sqrt{2}|1, m-1\rangle + |2, m-2\rangle)/2.
\]

C. Dispersive regime (DR)

Now we assume that both the atomic transition frequencies are far-detuned from the cavity frequency

\[
|\Delta_1|, |\Delta_2|, |\Delta_1 + \Delta_2| \gg G_{0,0}\sqrt{m}, G_{0,1}\sqrt{m-1}.
\]

(19)

From the perturbation theory we obtain to the 4th order in \( G_{0,0}/\Delta_1 \) and \( G_{0,1}/\Delta_2 \)

\[
\lambda_{m,0} = m\omega_0 + \delta_1 m \left[ 1 + \frac{G_{0,1}^2(m-1)}{\Delta_1(\Delta_1 + \Delta_2)} - \frac{G_{0,0}^2m}{\Delta_1^2} \right]
\]

\[
|\varphi_{m,0}\rangle = N^{-1}_{m,0} \left[ |0, m\rangle + \frac{r_{m,0}G_{0,0}\sqrt{m}}{\Delta_1}|1, m-1\rangle + \frac{r_{m,0}G_{0,0}\sqrt{m}}{\Delta_1}|2, m-2\rangle \right],
\]

\[
|\varphi_{m,1}\rangle = N^{-1}_{m,1} \left[ |1, m-1\rangle - \frac{\rho_{m,1}G_{0,0}\sqrt{m}}{\Delta_1}|0, m\rangle + \frac{r_{m,1}G_{0,1}\sqrt{m-1}}{\Delta_2}|2, m-2\rangle \right],
\]

\[
|\varphi_{m,2}\rangle = N^{-1}_{m,2} \left[ |2, m-2\rangle - \frac{\rho_{m,2}G_{0,1}\sqrt{m-1}}{\Delta_2}|1, m-1\rangle + \frac{r_{m,2}G_{0,0}\sqrt{m}}{\Delta_2}|0, m\rangle \right],
\]

where we defined the dispersive shifts \( \delta_1 \equiv G_{0,1}^2/\Delta_1 \) and \( \delta_2 \equiv G_{0,1}^2/\Delta_2 \). We adopted an intuitive notation in which the second index in \( |\varphi_{m,s}\rangle \) represents the most probable atomic state in a given dressed state (for example, in the expansion of \( |\varphi_{m,0}\rangle \) the bare state \(|0, m\rangle \) appears with the highest weight). The parameters \( \rho_{m,s}, r_{m,s} \) and \( N_{m,s} \) are equal to 1 to the first order in \( G_{0,0}/\Delta_1, G_{0,1}/\Delta_2 \) and are summarized in [22].
D. Mixed regime (MR)

In the mixed regime we assume $\Delta_2 = 0$ and

$$|\Delta_1| \gg G_{0,0}\sqrt{m}, G_{0,1}\sqrt{n-1},$$

(19)
i.e., the atomic transition $|1\rangle \to |2\rangle$ is resonant with the cavity mode, while the transition $|0\rangle \to |1\rangle$ is far-detuned. To the second order in $G_{0,0}/\Delta_1$ we obtain

$$\lambda_{m,0} = m\omega_0 + \Delta_1 G_{0,0,m}^{2} \frac{\Delta_1 G_{0,0}^{2}}{\Delta_1^2 - G_{0,1}^{2}(m-1)}$$

$$|\varphi_{m,0}\rangle = N_{m,0}^{-1} \left\{ G_{0,1}\sqrt{m} - i \rho_{m,0} |2,m-2\rangle + \rho_{m,0} |\Delta_1,1,m-1\rangle + |0,m\rangle \right\}$$

$$\lambda_{m,\pm D} = m\omega_0 - D \left( |\Delta_1| + G_{0,1}\sqrt{m-1} \right)$$

$$\pm D (1 + r_{m,\pm,m-1} + |\rho_{m,\pm,m-1}\rangle \langle 0,m\rangle)$$

where we defined

$$\rho_{m,\pm} = \frac{G_{0,0}\sqrt{m}}{G_{0,1}\sqrt{m-1} + |\Delta_1|}$$

$$r_{m,\pm} = \frac{1}{4} \frac{G_{0,0}^2 m}{G_{0,1}\sqrt{m-1}(G_{0,1}\sqrt{m-1} + |\Delta_1|)}$$

$$N_{m,0} = \sqrt{1 + \rho_{m,0}^2 \left( \Delta_1^2 + (m-1) G_{0,1}^2 \right)}$$

$$N_{m,\pm} = \sqrt{2 + 2 r_{m,\pm,m-1} + \rho_{m,\pm,m-1}}$$

IV. ADCE

Our goal is to study the coherent annihilation of system excitations from the initial separable state $\hat{\rho}_0 = |0\rangle\langle 0| \otimes \hat{\rho}_{th}$, where $\hat{\rho}_{th} = \sum_{m=0}^{\infty} \rho_{m} |m\rangle \langle m|$ is the cavity thermal state with $\rho_{m} = \bar{n}^{m}/(\bar{n} + 1)^{m+1}$. Here $\bar{n} = (e^{\omega_0} - 1)^{-1}$ is the average initial photon number, $\beta^{-1} = k_B T$, $T$ is the absolute temperature and $k_B$ is the Boltzmann’s constant. From Eq. (10) it is clear that such process can be implemented via transition of the form $|\varphi_{m,T}\rangle \to |\varphi_{m-2,S}\rangle$ when the modulation frequency is $\eta^{(res)} = \lambda_{m,T} - \lambda_{m-2,S}$. So first we must determine the dressed states for which the initial population of the state $|\varphi_{m,T}\rangle$, denoted as $P_{m,T}$, is larger than $P_{m-2,S}$.

We assume a small integer $m$ (for the sake of illustration we choose $m = 4$, although the overall behavior is similar for other values of $m$) and set the realistic parameters $G_{0,0} = 6 \times 10^{-3} \omega_0$ and $\bar{n} = 1.5$. We verified numerically that when $G_{0,1}$ is of the same order of $G_{0,0}$ the exact value of $G_{0,1}$ does not affect qualitatively the results, so in this paper we set $G_{0,1} = 1.2 G_{0,0}$. See [62] for an illustration of the quantitative differences in the results when $G_{0,1} = G_{0,0}$ or $G_{0,1} = 0.8 G_{0,0}$.

Figure 1: (color online) Difference of initial populations $P(m,T,S) = P_{m,T} - P_{m-2,S}$ for $m = 4$ and different regimes as function of the absolute value of the detuning $\Delta_1$. Regimes: 2-level atom (2L), double-resonant regime (r), dispersive regime (d) and mixed regime (m). Only the states for which $P(m,T,S) > 0$ are plotted and the values $(T,S)$ are indicated alongside the curves. (Here $G_{0,1} = 1.2 G_{0,0}$.)

In Fig. 1 we plot the initial population difference $P(m,T,S) = P_{m,T} - P_{m-2,S}$ as function of $|\Delta_1|$ for $m = 4$. Only positive values of $P(4,T,S)$ are plotted and the values $(T,S)$ are indicated next to the curves, where the index stands for 2-level (2L), double-resonant (r), dispersive (d) and mixed (m) regimes. In the dispersive and mixed regimes we assume $|\Delta_1|/G_{0,0} > 4$ in order to satisfy the approximations (18) and (19). Besides, throughout this paper we set $\Delta_2 = 6 G_{0,0} \text{sign}(\Delta_1)$ in the dispersive regime so that $|\Delta_1 + \Delta_2|$ never approaches zero, as required by the inequality (18). One can see that large detuning $|\Delta_1|$ favors the implementation of ADCE; the transitions $(1,2)_d$ and $(D,\pm D)_m$ are not particularly useful since the population differences are always small and are inversely proportional to the detuning. As already known, for a qubit the ADCE relies on the transition $(D,\pm D)_2L$. From Fig. 1 we discover that for a qutrit we have the following candidates for the realization of ADCE: $(D,\pm D)$, and $(0,\pm D)$, in the double-resonant regime; $(0,1)_d$ and $(0,2)_d$ in the dispersive regime; $(0, D)_m$ and $(0, -D)_m$ in the mixed regime.

Now we are in position to evaluate the ADCE rate in different regimes according to Eq. (10). For the trans-
The main finding of the paper is the observation that in the double-resonant regime the ADCE rate is at least one order of magnitude larger than for the qubit, and the difference increases for larger $|\Delta_1|$, as can be seen from Fig. 2a. Besides, in this regime the population differences $P(m, D, -D)$ and $P(m, 0, -D)$ also increase proportionally to $|\Delta_1|$, achieving sufficiently large values for $|\Delta_1| \sim 8G_{0,0}$ (see Fig. 1). Thus, it seems that one could speed up ACDE by at least one order of magnitude using three-level atoms in the double-resonant configuration instead of qubits, provided the detuning $|\Delta_1|$ is large enough.

In real circuit QED setups it might be tricky to modulate only one parameter at a time, while keeping the other parameters constant. So in figure 2b, we consider the simultaneous modulation of $E_1$ and $E_2$ (with the same modulation frequency $\eta^{(\text{res})} = \lambda_{m, T} - \lambda_{m-2, S}$) assuming parameters $\varepsilon_{E,1} = 5 \times 10^{-2}\Omega_0$, $\varepsilon_{E,2} = 5 \times 10^{-2}\Omega_1$, $\phi_{E,1} = 0$ and $\phi_{E,2} = \pi$. Conveniently the ADCE transition rates increase even more when compared to an isolated modulation of either $E_1$ or $E_2$.

In Fig. 2c we illustrate in details the transition rates and the population differences for different values of $G_{0,1}$ and isolated modulations of $E_2$, $G_0$ and $G_1$. It is found that the modulation of $G_0$ does not speed up significantly the transition rate in comparison to a qubit, whereas the modulation of $E_2$ or $G_1$ does increase the transition rate in the double-resonant regime by at least one order of magnitude. We also verified that under the simultaneous modulation of all the parameters ($E_1$, $E_2$, $G_0$ and $G_1$) the total transition rate is still substantially higher than for a qubit, provided the phases are properly adjusted. Hence, the simultaneous modulation of several parameters is not an issue from the experimental point of view, provided one can manage the phases $\phi^{(j)}_t$ corresponding to different modulation components.

V. NUMERICAL VERIFICATION

Now we proceed to the numerical verification of the phenomenon predicted in the previous section, namely, the enhancement of the ADCE rate in the double-resonant regime. We solved numerically the Schrödinger equation for the Hamiltonian (1) using the initial local thermal state $\hat{\rho}_0 = |0\rangle \langle 0| \otimes \hat{\rho}_{th}$ and parameters $m = 4$, $G_{0,0} = 6 \times 10^{-2}\omega_0$, $G_{0,1} = 1.2G_{0,0}$, $\bar{n} = 1.5$ and $\Delta_1 = -\Delta_2 = -8G_{0,0}$. One downside of using the double-resonant regime for qutrits is clear from Fig. 1 both the populations differences $(0, -D)_y$ and $(D, -D)_y$, involved in the ADCE, are roughly twice smaller than the population difference $(D, -D)_{2L}$ for the qubit. Hence, considering the connection between ADCE and quantum
thermodynamic processes recently analyzed in Ref. [43], we can say that the work extraction would be half smaller if one used qutrits instead of qubits. This nuisance can be readily bypassed by employing 2-tone modulation with frequencies \( \eta^{(1)} = \lambda_{m,0} - \lambda_{m-2,-D} \) and \( \eta^{(2)} = \lambda_{m,D} - \lambda_{m-2,-D} \) that drives simultaneously the transitions \( |\varphi_{m,0}\rangle \rightarrow |\varphi_{m-2,-D}\rangle \) and \( |\varphi_{m,D}\rangle \rightarrow |\varphi_{m-2,-D}\rangle \).

In figure 3b, we illustrate the dynamics of the average photon number \( n_{ph} = \langle \hat{n} \rangle \), the average number of atomic excitations \( n_{at} = (\sum_{k=1}^{2} k \hat{a}_{k,k}) \) and the total average number of excitations \( n_{tot} = n_{ph} + n_{at} \) for a qubit (setting momentarily \( G_1 = 0 \)) with modulation depth \( \varepsilon_{E,1} = 5 \times 10^{-2} \Omega_{01} \). We observe the sinusoidal oscillation of \( n_{ph}, n_{at} \) and \( n_{tot} \) with typical period \( \tau \approx 4 \times 10^3 G_{01}^{-1} \). The coherent annihilation of excitations does take place, but since the initial population of the state \( |\varphi_{4,D}\rangle \) was \( P_{4,D} \approx 5 \times 10^{-2} \), the average number of annihilated excitations is \( \sim 2 P_{4,D} \approx 0.1 \), in agreement with the numerical data.

In figure 3b we consider the qutrit under 2-tone mod-

ulation of \( E_1 \) with the previous amplitude \( \varepsilon_{E,1} = 5 \times 10^{-2} \Omega_{01} \), weights \( w_{E,1}^{(1)} = 10/17, w_{E,1}^{(2)} = 7/17 \) and phases \( \phi_{E,1}^{(1)} = 0, \phi_{E,1}^{(2)} = \pi \) (the weights were adjusted to equalize the two transition rates). We see that the total number of excitation exhibits the same qualitative behavior as for the qubit, but the transition rate undergoes a 30-fold enhancement. The behavior of \( n_{ph} \) and \( n_{at} \) differs drastically from the one observed for the 2-level atom partly due to the oscillations between the bare states \( |0,k\rangle \leftrightarrow |2,k-2\rangle \) for \( k \geq 2 \), and partly due to the oscillations between the dressed states \( |\varphi_{k,D}\rangle \leftrightarrow |\varphi_{k,0}\rangle \), as will be discussed shortly. In figure 3c we consider the simultaneous two-tone modulation of \( E_1 \) and \( E_2 \) with parameters \( \varepsilon_{E,1} = 5 \times 10^{-2} \Omega_{01}, \varepsilon_{E,2} = 9 \times 10^{-2} \Omega_{12}, w_{E,1}^{(1)} = w_{E,1}^{(2)} = 10/17, w_{E,1}^{(2)} = 7/17 \) and phases \( \phi_{E,1}^{(1)} = \phi_{E,1}^{(2)} = 0, \phi_{E,2}^{(1)} = \phi_{E,2}^{(2)} = \pi \). We see that the ADCE rate suffers an additional 50% enhancement compared to the sole modulation of \( E_1 \), while the average number of total annihilated excitations is roughly the same as in the previous cases.

Finally, in Fig. 4 we plot the probabilities of finding the system in the dressed states \( P(m,S) = |\text{Tr}(\hat{\rho}(t) |\varphi_{m,S}\rangle \langle \varphi_{m,S}|) | \) as function of time for the 2-tone double-modulation discussed in Fig. 3. As predicted by Eq. 9 there is a simultaneous periodic transfer of populations from the states \( |\varphi_{4,D}\rangle \) and \( |\varphi_{4,0}\rangle \) to the state \( |\varphi_{2,-D}\rangle \), which correspond to the coherent annih-
lation of two system excitations. Other states $|\varphi_{k\neq 2,-D}\rangle$ are not affected by the modulation, as illustrated for the state $|\varphi_{3,-D}\rangle$ which undergoes just minor fluctuations due to off-resonant couplings neglected under RWA. Moreover, one also observes periodic oscillations between the dressed states $|\varphi_{k,D}\rangle \leftrightarrow |\varphi_{k,0}\rangle$ for $k \geq 2$. This occurs because for large $|\Delta_1|$ we have $\lambda_{k,0} \approx \lambda_{k,D}$, as seen from Eq. (17), hence the first term on the RHS of Eq. (9) becomes nearly resonant and couples these states with the strength $|\Delta\rangle[k,D,0]$ [this behavior is due solely to the counter-rotating terms in Eq. (1) and is independent of modulation].

VI. CONCLUSIONS

In conclusion, we showed that the resonant external modulation of a three-level artificial atom is highly advantageous for the implementation of the antidynamical Casimir effect (ADCE) in comparison to a two-level atom, since the transition rate can suffer almost 50-fold increase while the total amount of annihilated excitations is roughly the same. The strongest enhancement takes place in the double-resonant regime (when $\Delta_1 = -\Delta_2$, so that $\Omega_{12} = 2\omega_0$) and for large detuning $|\Delta_1|$, though weaker enhancement may occur also in other regimes. Beside speeding up the ADCE, the use of qutrits also loosens the requirements for accurate tuning of the modulation frequency, and reproduces the characteristic ADCE behavior of a qubit when all the atomic transitions are largely detuned from the cavity field (and $\Omega_{12} \neq 2\omega_0$). However, for the optimum annihilation of excitations from a thermal state the usage of qutrits also brings some inconveniences, such as two-tone driving and the necessity of controlling the phase difference between different components of the modulation. Nevertheless, our results indicate that the substantial gain in the transition rate compensates for the additional complexity in the external control, favoring the experimental implementation of ADCE.

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