Time-domain solution of Cole-Cole model with induced polarization method

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Abstract: The spectral parameters of the Cole-Cole model can be used to distinguish the IP anomalies of different geological bodies. However, the frequency domain IP requires a lot of observation data and its efficiency in field work is low. In view of this reality which is very common to use the data of the time-domain IP. It is necessary to study the law of the time-domain solution of Cole-Cole model with induced polarization method. First of all, this paper transforms the complex resistivity in the frequency domain into the time-varying resistivity in the time domain through the inverse Laplace transform, then by changing different Cole-Cole parameters in the time domain so that we can observe the changing law of time-varying resistivity time-spectrum characteristics caused by different parameters.

1. Introduction

When studying the frequency spectrum characteristics and time-spectrum characteristics of rocks and ores, some researchers have adopted quantitative methods to evaluate the changes in the complex resistivity time-spectrum characteristics of rocks and ores. Taking the research of Pelton\textsuperscript{[1]} et al as an example, they proposed and verified a new Cole-Cole model. Through a large number of measurements of the complex resistivity of rocks, ore outcrops and specimens, they verified the feasibility of the proposed new Cole-Cole model. And later many researchers also have conducted research on this and from their research data\textsuperscript{[1-4]} we can find that different rocks and ores have different Cole-Cole model parameters. These differences can be used to distinguish the distribution characteristics of underground geological bodies. The researchers also derive the expression of the Cole-Cole model of time-varying resistivity through the inverse Laplace based on the expression of the Cole-Cole model of complex resistivity in the frequency domain. In reality, time-domain IP data is very common and easy to obtain. Therefore, this paper studies the time-varying resistivity of the time-domain induced polarization Cole-Cole model, by assigning values to the parameters of the Cole-Cole model, the time-varying resistivity is obtained and its changing law is observed.

2. Frequency Domain Cole-Cole Model

After the substitution operation of the formula, the Cole-Cole expression of the complex resistivity of Pelton's new Cole-Cole model in the frequency domain is obtained\textsuperscript{[1,5]}:

\[
\rho(\omega) = \rho_0[1 - m[1 - (j\omega \tau)^c]^{-1}] \tag{1}
\]

In the formula:

\[m = 1/[1 + (R_2/R_1)] \quad \tau = (R_1/m)^{1/c} \cdot X\]

In this Cole-Cole expression of complex resistivity, \(\rho_0, m, \tau, c\) separately represent the...
resistivity at zero frequency, polarizability, time constant, and frequency correlation coefficient, which is called Cole-Cole parameters. The unit of \( \rho_0 \) is \( \Omega \), the unit of \( \tau \) is \( s \), \( m \) and \( c \) are dimensionless. As shown above the formula of complex resistivity in the frequency domain is the same as the formula used by K. S. Cole and R. H. Cole[6] to describe dielectric polarization in the early 1940s, so it is called the Cole-Cole model. The four parameters included in the Cole-Cole model can be used to describe the abnormal conditions of the spectral induced polarization.

3. Time domain Cole-Cole expression

Using the inverse Laplace transform, the frequency domain expression of the Cole-Cole impedance can be transformed into the time domain. When the step current is supplied, using this way can obtain using the Cole-Cole parameter represented the electric field response and power-off electric field decay response.

According to the inverse Laplace transform, the time domain impedance can be obtained as[7]:

\[
A(T) = R \left\{ 1 - mL^{-1} \{ (s \tau)^c \cdot \{ s[1 + (s \tau)^c] \}^{-1} \} \right\}
\]  

\( s = j \omega R = Z(0) \) is the impedance after a long time power supply in the time domain, \( L^{-1} \) is the symbol representing the inverse Laplace transform. Therefore, the potential difference function in the time domain is

\[
\Delta U(T) = IA(T) = \Delta U \left\{ 1 - mL^{-1} \{ (s \tau)^c \cdot \{ s[1 + (s \tau)^c] \}^{-1} \} \right\}
\]  

In the formula: \( \Delta U \) is the potential difference after long-term power supply, \( s[1 + (s \tau)^c]^{-1} = s^{-1} \cdot [1 + (s \tau)^{-c}]^{-1} \)

When \( T \) is small, \( (s \tau)^c \) is large, so \( (s \tau)^{-c} \) is small, the above formula can be extended to a power series:

\[
(s \tau)^c \cdot \{ s[1 + (s \tau)^c] \}^{-1} = \sum_{n=0}^{\infty} (-1)^n \tau^{-nc} \cdot s^{-(1+nc)}
\]  

Substituting equation (5) into equation (3) to get

\[
\Delta U(T) = \Delta U \left[ 1 - m \sum_{n=0}^{\infty} (-1)^n \tau^{-nc} L^{-1} \{ s^{-(1+nc)} \} \right]
\]  

According to the inverse Laplace transform formula, formula (6) can be transformed into

\[
\Delta U(T) = \Delta U \left[ 1 - m \sum_{n=0}^{\infty} (-1)^n \tau^{-nc} \Gamma(1 + nc) \right] \cdot (T \cdot \tau^{-1})^{nc}
\]  

In the formula \( \Gamma(1 + nc) \) is the gamma function.

In the same transformation method, the total field potential difference in the time domain can be expressed as:

\[
\Delta U(T) = \Delta U \left[ 1 - m \sum_{n=0}^{\infty} (-1)^n \tau^{-nc} \Gamma(1 + nc) \right] \cdot (T \cdot \tau^{-1})^{nc}
\]  

\( 0 < T \cdot \tau^{-1} \leq 2\pi \)

\[
\Delta U(T) = \Delta U \left[ 1 - m \sum_{n=1}^{\infty} (-1)^{n+1} \tau^{-nc} \Gamma(1 - nc) \right] \cdot (T \cdot \tau^{-1})^{nc}
\]  

\( T \cdot \tau^{-1} > 2\pi \)

In the same way, the expression for the attenuation of the secondary potential difference in the time domain similar to Pelton, Guptaarma, and Liangkui Fu[1,8-10] can be obtained as:

\[
\Delta U_2(t) = \Delta U_2(0) \cdot \sum_{n=0}^{\infty} (-1)^n \tau^{-nc} \Gamma(1 + nc) \cdot (T \cdot \tau^{-1})^{nc}
\]  

\( 0 < t \cdot \tau^{-1} \leq 2\pi \)

\[
\Delta U_2(t) = \Delta U_2(0) \sum_{n=1}^{\infty} (-1)^{n+1} \tau^{-nc} \Gamma(1 - nc) \cdot (T \cdot \tau^{-1})^{nc}
\]  

\( t \cdot \tau^{-1} > 2\pi \)

In the same way, the expression of the time-varying resistivity corresponding to the total field potential difference is:

\[
\rho(T) = \rho(\infty) \left[ 1 - m \sum_{n=0}^{\infty} (-1)^n \tau^{-nc} \Gamma(1 + nc) \right] \cdot (T \cdot \tau^{-1})^{nc}
\]  

\( 0 < T \cdot \tau^{-1} \leq 2\pi \)

\[
\rho(T) = \rho(\infty) \left[ 1 - m \sum_{n=1}^{\infty} (-1)^{n+1} \tau^{-nc} \Gamma(1 - nc) \right] \cdot (T \cdot \tau^{-1})^{nc}
\]  

\( T \cdot \tau^{-1} > 2\pi \)

3. Time domain solution

According to the time domain resistivity expressions (2.11) and (2.12) to program and assign values to
the Cole-Cole parameters, then use computer software to make the time-spectrum response figure of the
time-varying resistivity under different parameters.

At first, for different frequency correlation coefficient \(c\) (\(c=0.15, c=0.25, c=0.35, c=0.45, c=0.55\)),
when \(m=0.5, \tau = 1s\), the time-varying resistivity response diagram at different times is shown in
figure 1.

![Different frequency correlation coefficients](image1)

**Figure 1** Different frequency correlation coefficients, the time-varying resistivity changes with time

Then, for different time constant values \(\tau\) (\(\tau = 0.001s, \tau = 0.01s, \tau = 0.1s, \tau = 1s, \tau = 10s, \tau = 100s\)),
when \(m = 0.5, c = 0.25\), the time-varying resistivity response diagram at different times is shown in
figure 2.

![Different time constants](image2)

**Figure 2** Different time constants, the time-varying resistivity changes with time

Finally, for different polarizability \(m\) (\(m = 0.1, m = 0.2, m = 0.3, m = 0.4, m = 0.5\)), when \(\tau =1, c=0.25\),
the time-varying resistivity response diagram at different times is shown in figure 3.
4. Conclusion

It can be seen from figure 1 that when the time constant and the polarizability are fixed values, in the coordinate axis where the time axis is a logarithmic scale and the time-varying resistivity is arithmetic scale. The time-varying resistivity time spectrum curve is centrally symmetric and the time value corresponding to this centrally symmetric point is the value of the time constant. At the point far from the center symmetric point, as the value of the frequency correlation coefficient increases, the slope of the time-varying resistivity curve with time changes gradually decreases. While near the center symmetric point, as the value of the frequency correlation coefficient increases, the slope of the time-varying resistivity curve with time changes gradually increases.

It can be seen from figure 2 that when the polarizability and frequency correlation coefficient are constant, in the coordinate axis where the time axis is a logarithmic scale and the time-varying resistivity is arithmetic scale. The change trend of the time-varying resistivity vs. time curve corresponding to different time constants is consistent and the time-varying resistivity curve corresponding to different time constants can be obtained by shifting left and right.

It can be seen from figure 3 that when the time constant and frequency correlation coefficient are constant, in the coordinate axis where the time axis is a logarithmic scale and the time-varying resistivity is arithmetic scale. The time-varying resistivity spectrum has the following characteristics: 1. Constantly positive, its value increases monotonously with time; 2. When power is supplied for a long time, the time-varying resistivity tends to \( \rho(\infty) \), when power is supplied for a short time, the time-varying resistivity tends to \( \rho(\infty)(1 - m) \).

Understanding the changing law of time-spectrum characteristics of time-varying resistivity caused by different Cole-Cole parameters is very important for dealing with the induced polarization effect of different underground geological bodies.

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