PROBABILISTIC MASSIVE MIMO CHANNEL ESTIMATION
WITH BUILT-IN PARAMETER ESTIMATION

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ABSTRACT
In order to reduce hardware complexity and power consumption, massive multiple-input multiple-output (MIMO) system uses low-resolution analog-to-digital converters (ADCs) to acquire quantized measurements \( y \). This poses new challenges to the channel estimation problem, and the sparse prior on the channel coefficients \( x \) in the angle domain is used to compensate for the information lost during quantization. By interpreting the sparse prior from a probabilistic perspective, we can assume \( x \) follows some sparse prior distribution and recover it using approximate message passing (AMP). However, the distribution parameters are unknown in practice and need to be estimated. Due to the increased computational complexity in the quantization noise model, previous works either use an approximated noise model or manually tune the noise distribution parameters. In this paper we treat both the signal and parameters as random variables and recover them jointly within the AMP framework. This leads to a much simpler parameter estimation method and allows us to work with the true quantization noise model. Experimental results show that the proposed approach achieves state-of-the-art performance under various noise levels and does not require parameter tuning, making it a practical and carefree approach for channel estimation.

Index Terms— Channel estimation, low-resolution ADCs, massive MIMO, approximate message passing, parameter estimation

1. INTRODUCTION
Massive multiple-input multiple-output (MIMO) technology has shown great potential in the next generation wireless systems such as 5G cellular network [1] [6]. It deploys large numbers of antennas (in the order of hundreds or more) at the base station to serve multiple users simultaneously within the same channel. This not only produces high throughput but also boosts the array gain so that the noise and interference can be greatly reduced. However, the large antenna array also increases the complexity of hardware design and the power consumption drastically. For example, the power consumed by analog-to-digital converters (ADCs) in the hardware grows exponentially with the number of quantization bits [7]. Currently it is either too expensive or impractical to use high-resolution ADCs in base stations and portable devices [8]. As a result, there has been a growing interest in low-resolution ADCs that output 1 ~ 3 bits in recent years. In particular, 1-bit ADC does not require automatic gain control or linear amplifier, and is much preferred in wideband millimeter wave (mmWave) communication systems that require high sampling frequency [9].

In order to achieve reliable data transmission and maximize channel capacity in the communication system, we need to estimate its channel state information so that we can compensate distortion effects such as scattering, fading, and power decay. The nonlinear (quantized) measurements from low-resolution ADCs pose new challenges to channel estimation. Dithering has been used in [10] [11] to alleviate the nonlinearity of the quantization noise, however it becomes less effective for MIMO channel where signals from multiple antennas are linearly combined before quantization. On the other hand, the “channel sparsity” of the MIMO system can be exploited to improve the channel estimation performance from quantized measurements. In the massive MIMO channel, the channel energy is mostly concentrated in small regions in the multipath angular spread, and the channel matrix is thus approximately sparse in the angle domain [12] [13]. In the mmWave MIMO channel, the channel matrix is also approximately sparse in both the angle domain and delay domain [14]. Compressive sensing techniques then enable us to incorporate this sparse prior information into the channel estimation problem [15] [19].

Various sparse signal recovery methods can be used to estimate the channel matrix from quantized measurements, and they generally differ in how the sparse prior is enforced. For example, methods like iterative hard thresholding (IHT) [20] [21], OMP [22] [23] and CoSaMP [24] impose a constraint on the sparsity of the signal \( x \), i.e. \( \| x \|_0 \leq K \), where \( K \) is the number of nonzero entries in \( x \). Whereas some other methods promote sparse solutions through regularization functions like the \( l_1 \)-norm \( \| x \|_1 \), [25] [26], the \( l_p \)-norm \( \| x \|_p \), [27] [29], and generalized entropy functions [30].

The sparse prior can also be interpreted from a probabilistic perspective. In this case the signal \( x \) is assumed to follow some sparse prior distributions \( p(x|\lambda) \) like Laplace distribution and Bernoulli-Gaussian mixture distribution, where \( \lambda \) contains the distribution parameters. Let \( y \) denote the quantized measurements. The posterior distribution \( p(x|y) \) can be computed via approximate message passing (AMP) [31] [33], and the minimum mean squared error (MMSE) estimation of the signal is then \( \hat{x} = E[x|y] \). AMP has been used for channel estimation in massive MIMO and mmWave MIMO channels due to its computational efficiency and superior performance [34] [38]. In practice the distribution parameters are unknown and need to be estimated. Previous works used the expectation maximization (EM) to find the parameters that maximize the likelihood [39]. However, the involved computation becomes increasingly difficult in the case of complicated probability models such as the quantization noise model of low-resolution ADCs. As a result, the simple additive white Gaussian noise (AWGN) model was adopted in [34] [37] [40] [41] to approximate the quantization noise, which leads to suboptimal performance. The approach in [36] used the actual quantization noise model, but the noise distribution parameter needs to be manually tuned for different types of channels, which greatly
limits its applicability in real applications.

By treating the distribution parameters as random variables, we proposed an extension to the AMP framework in [42] where the posteriors of the signal and parameters can be computed and used to recover them jointly. We later presented a more computationally efficient approach to perform parameter estimation in [43]. We showed that our proposed approach has a wider applicability and could directly work with the complicated quantization noise model. In this paper we apply the proposed approach to solving the channel estimation problem in broadband mmWave MIMO channel. Compared to previous AMP-based channel estimation methods that either use an approximated noise model or manually tune the noise distribution parameters, the propose approach estimates the channel matrix based on the true quantization noise model and comes with “built-in” parameter estimation.

This paper proceeds as follows. In Section 2, we introduce the sparse channel estimation problem in the angle domain and the probability models of the signal and noise. In Section 3, we discuss the AMP framework with built-in parameter estimation, and use it to compute the MMSE estimation of the channel coefficients. In Section 4, we conduct experiments to compare the proposed approach with the other-state-of-the-art methods, followed by a conclusion in Section 5.

2. PROBLEM FORMULATION

When the channel matrix $H$ represents the baseband channel impulse response from the transmitter to the receiver in the “antenna domain”, it is typically dense. If $H$ is transformed into the “angle domain” [12], the transformed channel coefficients in $X$ become approximately sparse. We next briefly introduce the different domain representations of the channel in a MIMO system according to the derivations outlined in [36].

Suppose there are $N_t$ antennas at the transmitter and $N_r$ antennas at the receiver, and the delay spread is limited to $L$ intervals. The quantized measurements $y[k] \in \mathbb{C}^{N_r}$ at time $k$ are

$$y[k] = Q \left( \sum_{l=0}^{L-1} H[l] s[k-l] + w[k] \right),$$

where $H[l] \in \mathbb{C}^{N_t \times N_r}$ is the channel matrix at the $l$-th lag, $s[k] \in \mathbb{C}^{N_t}$ is the transmitted symbol, $w[k]$ is the noise, and the quantization operator $Q$ quantizes the real and imaginary parts of a complex number respectively. The channel matrix $H[l]$ in the angle domain can be expressed in terms of the angle domain representation $X[l] \in \mathbb{C}^{N_r \times N_t}$:

$$H[l] = B_{N_r} X[l] B_{N_t}^t,$$

where $B_{N_r} \in \mathbb{C}^{N_r \times N_t}$ and $B_{N_t} \in \mathbb{C}^{N_t \times N_r}$ are the steering matrices for the receiver and transmitter arrays respectively, and they can be constructed from unitary discrete Fourier transform (DFT) matrices. Rearranging the quantized measurements $y[k]$ from all the transmission blocks as in [36], we can simplify the notations of the quantized measurement model as follows

$$y = Q(Ax + w),$$

where $y \in \mathbb{C}^M$ are the quantized measurements, $A \in \mathbb{C}^{M \times N}$ is the measurement matrix, $x \in \mathbb{C}^N$ are the (approximately) sparse channel coefficients in the angle domain, and $w \in \mathbb{C}^N$ are the pre-quantization noises.

Our goal is to recover the channel coefficients $x$ given $y$ and $A$. We use AMP to compute the posterior distribution $p(x|y)$ and the MMSE estimation $\hat{x}$

$$\hat{x} = \int x \cdot p(x|y) \, dx.$$

2. Sparse signal model: Here we assume the entries of the sparse signal $x$ are i.i.d. and follow the Bernoulli and Gaussian mixture distribution

$$p(x_n) = (1 - \kappa) \cdot \delta(x_n) + \kappa \cdot \sum_{i=1}^D \xi_i \cdot \mathcal{CN}(x_n | \mu_i, \nu_{x_i}),$$

where $\delta(x_n)$ is the Dirac delta function, $\kappa$ is the probability that $x_n$ takes a non-zero value, $\xi_i$ is the Gaussian mixture weight, $\mu_i$ and $\nu_{x_i}$ are the mean and variance of the $i$-th complex Gaussian component, $D$ is the number of complex Gaussian mixture components. We assume the pseudo-covariance (relation) of $x_n$ is 0. The probability density function $\mathcal{CN}(x_n | \mu_i, \nu_{x_i})$ is then

$$\mathcal{CN}(x_n | \mu_i, \nu_{x_i}) = \frac{1}{\pi \cdot \nu_{x_i}} \exp \left( \frac{-|x_n - \mu_i|^2}{\nu_{x_i}} \right).$$

2. Quantization noise model: Generally we assume the pre-quantization noise $w$ are circularly symmetric complex Gaussian with variance $\tau_w$. The probability density function of the noise $w_m \in \mathbb{C}$ is

$$p(w_m) = \frac{1}{\pi \cdot \tau_w} \exp \left( -\frac{|w_m|^2}{\tau_w} \right).$$

Let $z = Ax$ denote the noiseless measurement, $K$ denote the number of bits of the quantized measurement $y_m$, and $\{b_k | k = 1, \cdots, 2^K\}$ denote the quantization symbol. As shown in Fig. 1, the quantizer $Q$ is applied on the real and imaginary parts of $z_m + w_m$ respectively.

- If the real coefficient $\text{Re}(z_m + w_m) \in [a_{k-1}, a_k)$,

  $$\text{Re}(y_m) = Q(\text{Re}(z_m + w_m)) = b_k,$$

  where $a_{k-1}$ and $a_k$ are the lower and upper thresholds corresponding to the symbol $b_k$.

- If the imaginary coefficient $\text{Im}(z_m + w_m) \in [a_{l-1}, a_l)$,

  $$\text{Im}(y_m) = Q(\text{Im}(z_m + w_m)) = b_l,$$

  where $a_{l-1}$ and $a_l$ are the lower and upper thresholds corresponding to the symbol $b_l$. 

Fig. 1. The quantizer $Q$ is applied on the real and imaginary parts of a complex measurement respectively.
3. AMP WITH BUILT-IN PARAMETER ESTIMATION

According to the distributions in \(5\) and \(7\), we have the following two sets of parameters in the signal prior and the noise prior respectively.

\[
\lambda = \{ \kappa, \xi_i, \mu_i, \nu_{x_i} \mid i = 1, \cdots, D \} \quad (10) \\
\theta = \{ \tau_w \}. \quad (11)
\]

The computation involved in parameter estimation by previous AMP approaches becomes too difficult for the quantization noise model. As a result, they either use an approximated AWGN model or manually tune the noise distribution parameters.

As illustrated in Fig. 2 in this paper we choose the AMP framework introduced in \[42\] where the distribution parameters are treated as unknown random variables. The posteriors of the signal \(x\) and the parameters \(\{ \lambda, \theta \}\) can then be jointly computed. Compared to previous parameter estimation approaches that maximize the likelihood \[39\] or the Beth free entropy \[44, 45\], our proposed approach maximizes the posteriors of the parameters and could handle complicated distributions with ease \[43\].

We use the GAMP formulation \[46\] to compute the messages passed between the variable nodes and the factor nodes in Fig. 2. Taking the messages between the factor node \(\Phi_m\) and the variable node \(x_n\) for example, we use the following notations for the messages:

- \(\Delta_{\Phi_m \rightarrow x_n}\) denotes the message from \(\Phi_m\) to \(x_n\),
- \(\Delta_{x_n \rightarrow \Phi_m}\) denotes the message from \(x_n\) to \(\Phi_m\).

Both \(\Delta_{\Phi_m \rightarrow x_n}\) and \(\Delta_{x_n \rightarrow \Phi_m}\) can be viewed as functions of \(x_n\), and they are expressed in the “log” domain. The posteriors of the signal \(x\) and the distribution parameters \(\lambda, \theta\) can be obtained via belief propagation, aka sum-product message passing:

\[
p(x_n | y) \propto \exp \left( \sum_m \Delta^{(t+1)}_{\Phi_m \rightarrow x_n} + \sum_n \Delta^{(t+1)}_{x_n \rightarrow \Phi_m} \right) \quad (12a)
\]

Detailed derivations of the messages in \(12\) can be found in our earlier work \[43\]. Note that the messages are derived for the real case in \[43\], the extension to the complex case for channel estimation should be pretty straightforward. The distribution parameters \(\lambda, \theta\) can be estimated by maximizing the posteriors in \(12\). \(\lambda\) and \(\theta\) are the vector parameters.

\[
\hat{\lambda}^{(t+1)} = \arg \max_{\lambda} \sum_n \Delta^{(t+1)}_{\Omega_n \rightarrow \lambda} \\
\hat{\theta}^{(t+1)} = \arg \max_{\theta} \sum_m \Delta^{(t+1)}_{\phi_m \rightarrow \theta} . \quad (13a)
\]

We use the computationally efficient approach in \[43\] to find the maximizing distribution parameters. It combines EM with the second-order method, and is several orders of magnitude faster than vanilla gradient descent. The AMP with built-in parameter estimation approach is summarized by Algorithm 1.

**Algorithm 1 AMP with built-in parameter estimation** \[43\]

1. Initialize \(\lambda^{(0)}, \theta^{(0)}\).
2. for \(e = \{0, 1, \cdots, E\}\) do
3. Perform belief propagation via GAMP.
4. Find the parameters that maximize the posteriors

\[
\hat{\lambda}^{(e+1)} = \arg \max_{\lambda} \sum_n \Delta^{(e+1)}_{\Omega_n \rightarrow \lambda_l} \\
\hat{\theta}^{(e+1)} = \arg \max_{\theta} \sum_m \Delta^{(e+1)}_{\Phi_m \rightarrow \theta_k} . \quad (15)
\]
5. if convergence is reached then
6. \(\lambda = \hat{\lambda}^{(e+1)}, \theta = \hat{\theta}^{(e+1)}\);
7. break;
8. end if
9. end for
10. return Output \(\hat{x}, \hat{\lambda}, \hat{\theta}\).

\[
p(\lambda | y) \propto \exp \left( \sum_n \Delta^{(t+1)}_{\Omega_n \rightarrow \lambda} \right) \quad (12b) \\
p(\theta | y) \propto \exp \left( \sum_m \Delta^{(t+1)}_{\phi_m \rightarrow \theta} \right) . \quad (12c)
\]

4. EXPERIMENTAL RESULTS

We next compare the proposed AMP with built-in parameter estimation approach (AMP-PE) with the other state-of-the-art approaches such as the iterative hard thresholding (IHT) \[20\], basis pursuit (BP) \[49\] and the baseline least square approach (LS). In particular, IHT requires the knowledge of the sparsity of the signal, BP requires the knowledge of the noise level. These parameters need to be manually tuned in practice. Although LS does not require parameter tuning, it usually does not have the state-of-the-art performance and thus serves as a baseline. We shall also include the results from the oracle AMP approach (AMP-oracle) where the distribution parameters are assumed to be known, which corresponds to the best performance that could be achieved by AMP-based approach.
Fig. 3. Comparison of the true channel $\mathbf{x}$ and the recovered channel $\hat{\mathbf{x}}$ from 1-bit measurements using the proposed AMP-PE approach when $\text{SNR} = 10$ dB. The NMSE of the recovered channel $\hat{\mathbf{x}}$ is $-10.98$ dB.

Fig. 4. Comparison of the proposed AMP-PE approach with the least square (LS), iterative hard thresholding (IHT), basis pursuit (BP) and the oracle AMP (AMP-oracle) across different noise levels. The quantized measurements are from 1-bit, 2-bit and 3-bit ADCs. The proposed AMP-PE automatically estimates the distribution parameters, whereas the parameters of QIHT, BP and AMP-oracle need to be manually tuned or pre-specified.
Following the experimental setting in [36], we estimate a broadband mmWave massive MIMO channel that has \( N_{cl} = 4 \) clusters (each has 10 paths) and a delay spread of at most \( L = 16 \) symbols. There are \( N_t = 64 \) antennas at the transmitter and \( N_r = 64 \) antennas at the receiver. The length of the channel coefficient vector \( \hat{x} \) is then \( N = N_t N_r L \). A random QPSK block transmission [50] of length \( N_y = 2048 \) is used for training, producing \( M = N_r N_p \) quantized measurements in \( y \) with an oversampling rate \( \frac{M}{N} = 2 \).

The AMP approach used in [36] assumes the noise variance \( \theta = \{ \nu_w \} \) is already known, it only estimates the signal prior parameters \( \lambda = \{ \kappa, \xi, \mu, \nu_w \} \). Our proposed AMP-PE automatically estimates all the distribution parameters, and enjoys a wider applicability in practice where \( \nu_w \) is unknown. Let \( r = Ax + w \) denote the “pre-quantization” measurements. The signal-to-noise ratio (SNR) of \( r \) varies between 0 and 40 dB in the experiments, ranging from low-SNR to high-SNR regimes. Normalized mean squared error (NMSE) of the recovered signal is computed and used to evaluate the performances of all the approaches.

\[
\text{NMSE}(\hat{x}) = \mathbb{E} \left[ \frac{\| \hat{x} - x \|_2^2}{\| x \|_2^2} \right].
\]  

Fig. 3 shows a comparison of the true channel \( x \) and the recovered channel \( \hat{x} \) from 1-bit measurements using the proposed AMP-PE approach when SNR = 10 dB. We are able to achieve a decent recovery with NMSE = −10.98 dB from 1-bit measurements. The performance can be further improved with more measurements or higher-resolution ADCs.

The average NMSEs from 100 random trials using different approaches are shown in Fig. 4. The proposed AMP-PE automatically estimates the distribution parameters by maximizing their posteriors. It enjoys a probabilistic perspective where the signal and parameters can be jointly recovered.

Although in some cases IHT performs better than AMP-PE, it requires the knowledge of the signal sparsity or the noise level, which are unknown in real applications. The proposed AMP-PE approach offers a more practical and carefree alternative where the signal and parameters can be jointly recovered.

5. CONCLUSION

Using the sparse prior of the massive MIMO channel coefficients \( x \) in the angle domain, we perform channel estimation from a probabilistic perspective where the signal \( x \) and the quantized measurements \( y \) follow some distributions with unknown parameters. We propose to jointly recover the signal and parameters within the extended AMP framework where they are both treated as random variables. Due to the nonlinearity of the quantization noise, previous AMP-based channel estimation methods either use an approximated noise model or manually tune the noise distribution parameters. Our proposed approach works with the true quantization noise model and estimates the parameters by maximizing their posteriors. It enjoys a wider applicability in practice compared to methods that require parameter tuning for different types of signals and noise levels.

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