String Inspired Quintom Model with Non-minimally Coupled Modified Gravity

J. Sadeghi $^{a*}$, M. R. Setare $^{b,c}$ $^\dagger$, A. Banijamali $^{a}$ $^\ddagger$

$^a$ Sciences Faculty, Department of Physics, Mazandaran University, P.O. Box 47415-416, Babolsar, Iran
$^b$ Department of Science, Payame Noor University, Bijar, Iran
$^c$ Research Institute for Astronomy and Astrophysics of Maragha, P. O. Box 55134-441, Maragha, Iran.

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Abstract

In this paper we consider a quintom model of dark energy with non-minimal coupling between scalar field and modified gravity which is known $f(R)$ gravity. The Lagrangian for scalar field has been inspired by tachyonic Lagrangian in string theory. Then we obtain the equation of state (EoS), and the condition required for the model parameters when $\omega$ crosses over $-1$. This model shows that for having $\omega$ across over -1, one doesn’t need to add some higher derivative operator in the tachyonic part of action (the way that usually used to obtain crossing of the phantom divide line for EoS parameter).

Keywords: Quintom model; Tachyon; Non-minimal coupling; Modified gravity.
1 Introduction

Nowadays it is strongly believed that the universe is experiencing an accelerated expansion, and this is supported by many cosmological observations, such as SNe Ia [1], WMAP [2], SDSS [3] and X-ray [4]. There are two ways to explain the current accelerated expansion of the universe. The first one is to introduce some unknown matter, which is called dark energy in the framework of general relativity. Although the nature and origin of dark energy could perhaps understood by a fundamental underlying theory unknown up to now, physicists can still propose some paradigms to describe it. The most obvious theoretical candidate for dark energy is the cosmological constant [5, 6, 7] which has the equation of state $\omega = -1$. However, it leads to the two known difficulties [8], namely the “fine-tuning” problem (why is the current vacuum energy density so small), and the “cosmic coincidence” one (why are the densities of vacuum energy and dark matter nearly equal today since they scale very differently during the expansion history). In the other side the analysis of the properties of dark energy from recent observations mildly favor models with $\omega$ crossing -1 (phantom divide line) in the near past [9]. In the framework of general relativity, the crossing of the phantom divide line has been realized in the literatures in different approaches, such as scalar tensor theories with non-minimal coupling between scalar field and curvature [10], two scalar field models [11] string-inspired models [15] and so on. In this framework the general belief is that the crossing of the phantom divide is not admissible in simple minimally coupled models and its explanation requires models with non-minimal coupling between scalar field and gravity [16]. As it was indicated in the literature [14], the consideration of the combination of quintessence [12] and phantom [13] in a unified model, leads to the fulfillment of the aforementioned transition through the $w = -1$ divide. This model, dubbed quintom, can produce a better fit to the observational data.

The second way to account for the current accelerated expansion of the universe is to modify the gravitational theory and in the simplest case replace $R$ with $f(R)$ in the action which is well known as $f(R)$ gravity. Here $f(R)$ is an arbitrary function of scalar curvature (for recent reviews see [17, 18]). Although there are some works with related subjects on crossing of the phantom divide line in the framework of modified gravity [17, 19, 20], but Ref. [21] was the first paper that has investigated a modified gravity model realizing $\omega$ across -1. The authors of Ref. [21] have shown an explicit model of modified gravity in which a crossing of the phantom divide can occur and relation between scalar field theories with property of $\omega$ crossing -1 and the corresponding modified gravity theories have been investigated.

In the present paper we would like to explore the consequence of possibility of a crossing of the phantom divide line in modified gravity non-minimally coupled with scalar field. In this model, the tachyon field in the world volume theory of the open string stretched between a D-brane and an anti-D-brane or a non-BPS D-brane plays the role of scalar field [22, 23]. Although crossing of the phantom divide line can be realized by using tachyonic matter, but it has been shown that one needs to add a higher derivative operator in the action and the extra term plays a important role for having $\omega$ across over -1. We will show that if we consider non-minimal coupling between modified gravity and tachyon matter the
modification of tachyon action is not necessary and crossing of the phantom divide line can occur. An outline of this paper is as follows. In section 2 we introduce action for tachyon non-minimally coupled to modified gravity. In order to discuss the equation of state we derive the corresponding energy density and pressure for this model. By solving this equation we obtain the conditions required for the $\omega$ across -1. Section 3 is devoted to discussion of our results.

2 Non-minimally coupled modified gravity with tachyon field

We consider the following action for non-minimally coupled $f(R)$ gravity and Born-Infeld type action for tachyon field,

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_P^2}{2} f(R)h(\phi) - V(\phi)\sqrt{1 + \alpha'\nabla_\mu \phi \nabla^\mu \phi} \right],$$

(1)

where $h(\phi)$ is a function of the tachyon $\phi$ and corresponds to the non-minimal coupling factor. Here $V(\phi)$ is the tachyon potential which is bounded and reaching its minimum asymptotically. $M_P = \frac{1}{\sqrt{8\pi G}}$ is reduced Planck mass.

The equation of motion of the scalar field is as follows;

$$\alpha'\nabla_\mu \left( \frac{V(\phi)\nabla^\mu \phi}{u} \right) - V_\phi(\phi)u + \frac{M_P^2}{2} f(R)h_\phi(\phi) = 0,$$

(2)

where

$$V_\phi(\phi) = \frac{dV(\phi)}{d\phi}, \quad h_\phi(\phi) = \frac{dh(\phi)}{d\phi}, \quad u = \sqrt{1 + \alpha'\nabla_\mu \phi \nabla^\mu \phi}.$$ 

By using the definition of the energy momentum tensor,

$$\delta g_{\mu\nu}S = -\int d^4x \sqrt{-g} T^{\mu\nu} \delta g_{\mu\nu},$$

(3)

one can obtain the result as;

$$T_{\mu\nu} = g_{\mu\nu} \left( \frac{M_P^2}{2} f(R)h(\phi) - V(\phi)u \right)$$

$$- M_P^2 \left[ f'(R)h(\phi) R_{\mu\nu} + (g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu) f'(R)h(\phi) \right] + \frac{\alpha' V(\phi)\nabla_\mu \phi \nabla^\mu \phi}{u},$$

(4)

where $f'(R) = \frac{df(R)}{dR}$.

For a flat Friedman- Robertson- Walker (FRW) spacetime with the metric

$$ds^2 = -dt^2 + a^2(t)(dr^2 + r^2d\Omega^2)$$

(5)
and a homogenous scalar field $\phi$, the equation of motion can be written by the following equation,

$$
\ddot{\phi} + 3H\dot{\phi} = -\frac{\alpha'\dot{\phi}^2\ddot{\phi}}{1 - \alpha'\dot{\phi}^2} - \frac{V_\phi(\dot{\phi})\dot{\phi}^2}{V(\phi)} - \frac{M_P^2}{2\alpha'} \frac{f(R)h_\phi(\phi)}{V(\phi)}(1 - \alpha'\dot{\phi}^2)^{\frac{3}{2}},
$$

(6)

while energy density, pressure and Friedman equation are

$$
\rho = \frac{V(\phi)}{\sqrt{1 - \alpha'\dot{\phi}^2}} - \frac{M_P^2}{2} \left[ f(R)h(\phi) + 6H\frac{\partial}{\partial t}(f'(R)h(\phi)) - 6f'(R)h(\phi)(\dot{H} + H^2) \right],
$$

(7)

$$
p = -V(\phi)\sqrt{1 - \alpha'\dot{\phi}^2} + \frac{M_P^2}{2} \left[ f(R)h(\phi) + 4H\frac{\partial}{\partial t}(f'(R)h(\phi)) + 2\frac{\partial^2}{\partial t^2}(f'(R)h(\phi)) - 2f'(R)h(\phi)(\dot{H} + 3H^2) \right],
$$

(8)

$$
H^2 = \frac{1}{3M_P^2} \frac{V(\phi)}{\sqrt{1 - \alpha'\dot{\phi}^2}} - \frac{1}{6} \left[ f(R)h(\phi) + 6H\frac{\partial}{\partial t}(f'(R)h(\phi)) - 6f'(R)h(\phi)(\dot{H} + H^2) \right].
$$

(9)

Where we have used the following components of $R_{\mu\nu}$ in FRW spacetime,

$$
R_{00} = -3(\dot{H} + H^2), \quad R_{0i} = 0, \quad R_{ij} = (\dot{H} + 3H^2)g_{ij}.
$$

(10)

$H = \frac{\dot{a}}{a}$ is the Hubble parameter and $a$ is the scale factor.

One can express energy density and pressure in terms of derivatives of $f(R)$ and $h(\phi)$ with respect to their arguments and time derivatives of $R$ as follows,

$$
\rho = \frac{V(\phi)}{\sqrt{1 - \alpha'\dot{\phi}^2}} - \frac{M_P^2}{2} \left[ f(R)h(\phi) - 6f'(R)h(\phi)(\dot{H} + H^2) + 6H\dot{R}f''(R)h(\phi) + 6\dot{\phi}f'(R)h_\phi(\phi) \right],
$$

(11)

$$
p = -V(\phi)\sqrt{1 - \alpha'\dot{\phi}^2} + \frac{M_P^2}{2} \left[ f(R)h(\phi) - 2f'(R)h(\phi)(\dot{H} + 3H^2) + 4H\dot{R}f''(R)h(\phi) 
+ 4H\dot{\phi}f'(R)h_\phi(\phi) + 2\dot{R}^2f''(R)h(\phi) + 2\dot{R}f''(R)h(\phi) + 4\dot{\phi}\dot{R}f''(R)h_\phi(\phi) 
+ 2\dot{\phi}f'(R)h_\phi(\phi) + 2\dot{\phi}^2f'(R)h_\phi(\phi) \right],
$$

(12)

We now study the cosmological evolution of equation of state for the present model. The equation of state is $p = \omega \rho$. To explore the possibility of the $\omega$ across -1, we have to check
\( \frac{d}{dt}(\rho + p) \neq 0 \) when \( \omega \to -1 \).

From equations (11) and (12) one can obtain the following expression,

\[
\rho + p = \frac{\alpha'V(\phi)\dot{\phi}^2}{\sqrt{1 - \alpha'\dot{\phi}^2}} + \frac{M_p^2}{2} \left[ 4\dot{H}f'(R)h(\phi) - 2H\ddot{R}f''(R)h(\phi) + 2\ddot{R}^2 f''(R)h(\phi) + 2\ddot{R}f''(R)h(\phi) \right. \\
\left. + 4\phi\ddot{R}f''(R)h_{\phi}(\phi) + 2\phi\dot{f}'(R)h_{\phi}(\phi) - 2H\phi\dot{f}'(R)h_{\phi}(\phi) + 2\phi f'(R)h_{\phi}(\phi) \right].
\]

(13)

Since \( \rho + p = (1 + \omega)\rho \), if we assume \( \dot{\phi} = 0 \) when \( \omega \to -1 \), the following condition take place,

\[
4\dot{H}\ddot{f}'(R)h(\phi) - 2H^2\dddot{f}'(R)h(\phi) = -2H \left( \dddot{R}f''(R)h(\phi) + \dddot{f}'(R)h(\phi) + \phi f'(R)h_{\phi}(\phi) \right).
\]

(14)

By using above condition as well as \( \dot{\phi} = 0 \), when \( \omega \) crosses -1, one can obtain,

\[
\frac{d}{dt}(\rho + p) \sim h(\phi) \left[ \dddot{f}'(R)(\ddot{H} - H^2) + 2f'(R)(H\dddot{H} + \dddot{H}) + \dddot{f}'(R) + 3\dddot{R}f''(R) + \dddot{f}'(R) \right] \\
+ h_{\phi}(\phi) \left[ \dddot{f}'(R) + 3\dddot{R}\phi f''(R) \right].
\]

(15)

One can see from (15) that, even if \( \dddot{\phi} = 0 \) and \( \dddot{\phi} = 0 \), crossing -1 can be happen. This result is in contrast with the result of Ref.[24], where the authors have added a term \( \phi\Box\phi \) in the square root part of action (1) and concluded that for having crossing over -1 in case of \( \dot{\phi} = 0 \), one needs \( \dddot{\phi} \neq 0 \) and \( \frac{d}{dt}\Box\phi \neq 0 \) which means \( \dddot{\phi} \neq 0 \) when \( \omega \) crosses -1. Also Ref.[25] considered a dimension-6 operator \( \Box\phi\Box\phi \) in the Lagrangian of phantom field to propose a model which admits \( \omega \) across over -1. In this note we haven’t added a higher derivative operator in the Lagrangian but we considered non-minimal coupling between matter and modified gravity. So, it seems that we don’t need to add some terms in square root part of action (1).
3 Conclusion

In this paper, we have considered a crossing of the phantom divide in modified gravity non-minimally coupled with tachyon matter. As a result we have shown that instead of modification in tachyonic square root action, the non-minimally coupled \( f(R) \) gravity can play an important role to realize EoS across over -1. We assumed when crossing over -1 occur, \( \dot{\phi} = 0 \) and concluded that even if \( \ddot{\phi} = 0 \) and \( \phi = 0 \), our model can admit crossing of the phantom divide line. In this model we have shown that the modification of tachyon Lagrangian with higher derivative operator in Ref [24] is not require for crossing over \(-1\) for equation of state. It will be interesting to examine this model for the special potential \( V(\phi) \), \( h(\phi) \) and \( f(R) \) in Refs. [26, 27].

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