Branching ratios, forward-backward asymmetries and angular distributions of $B \to K_2^* l^+ l^-$ in the standard model and two new physics scenarios

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We analyze the $B \to K_2^*\to K^* l^+ l^-$ (with $l = e,\mu,\tau$) decay in the standard model and two new physics scenarios: vector-like quark model and family non-universal $Z'$ model. We derive its differential angular distributions, using the recently calculated form factors in the perturbative QCD approach. Branching ratios, polarizations, forward-backward asymmetries and transversity amplitudes are predicted, from which we find a promising prospective to observe this channel in the future experiment. We update the constraints on effective Wilson coefficients and/or free parameters in these two new physics scenarios by making use of the $B \to K^* l^+ l^-$ and $b \to sl^+ l^-$ experimental data. Their impact on $B \to K_2^* l^+ l^-$ is subsequently explored and in particular the zero-crossing point for the forward-backward asymmetry in these new physics scenarios can sizably deviate from the standard model. In addition we also generalize the analysis to a similar mode $B_s \to f_2'(1525)(\to K^+ K^-) l^+ l^-$. 

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I. INTRODUCTION

Discoveries of new degrees of freedom at TeV energy scale, with contributions to our understanding of the origin of the electroweak symmetry breaking, can proceed in two different ways. One is a direct search of the Higgs boson, the last piece to complete the standard model (SM), and particles beyond the SM, to establish new physics (NP) theories. The other effort, already ongoing, is to investigate processes in which SM is tested with higher experimental and theoretical precision. Among the latter category, rare $B$ decays are among ideal probes. Besides constraints on the Cabibbo-Kobayashi-Maskawa (CKM) matrix including apexs and angles of the unitary triangle, which have been contributed by semileptonic $b \to u/c$ and nonleptonic $B$ decays respectively, the electroweak interaction structure can also be probed by, for instance, the $b \to s\gamma$ and $b \to sl^+ l^-$ modes which are induced by loop effects in the SM and therefore sensitive to the NP interactions.

Unlike $b \to s\gamma$ and $B \to K^*\gamma$ that has only limited physical observables, $b \to sl^+ l^-$ especially $B \to K^* l^+ l^-$, with a number of observables accessible, provides a wealth of information of weak interactions, ranging from the forward-backward asymmetries (FBAs), isospin symmetries, polarizations to a full angular analysis. The last barrier to access this mode, the low statistic with a branching faction of the order $10^{-6}$, is being cleared by the $B$ factories and the hadron collider$^{[4]}$. The ongoing LHCb experiment can accumulate 6200 events per nominal running year of $2 fb^{-1}$ with $\sqrt{s} = 14$ TeV $^{[5]}$, which allows to probe the short-distance physics at an unprecedented level. For instance the sensitivity to zero-crossing point of FBAs can be reduced to $0.1 GeV^2$ and might be further improved as $0.01 GeV^2$ after the upgrade $^{[6]}$. This provides a good sensitivity to discriminate between the SM and different models of new physics. There are also a lot of opportunities on the Super B factory $^{[7,12]}$. Because of these virtues, theoretical research interests in this mode have exploded and the precision is highly improved, see Refs. $^{[8,12]}$ for an incomplete list.

Toward the direction to elucidate the electroweak interaction, $B \to K^* l^+ l^-$ and its SU(3)-related mode $B_s \to s l^+ l^-$ are not unique. In this work, we shall point out that $B \to K_2^*(1430) l^+ l^-$ and the $B_s$-counterpart $B_s \to f_2'(1525) l^+ l^-$, which so far have not been investigated in detail$^{[23,24]}$, are also useful in several aspects. Due to the similarities

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‡ Hereafter will use $K_2^*$ and $f_2'$ to abbreviate $K_2^*(1430)$ and $f_2'(1525)$. 

between $K^*$ and $K^*_2$, all experiment techniques for $B \to K^* l^+ l^-$ are adjustable to $B \to K^*_2 l^+ l^-$. The main decay product of $K^*_2$ is a pair of charged kaon and pion which are easily detected on the LHCb. Moreover as we will show in the following, based on either a direct computation in the perturbative QCD approach \cite{27} or the implication of experimental data on $B \to K^*_2 \gamma$ process, the branching ratio (BR) of $B \to K^*_2 l^+ l^-$ is found sizable. Therefore thousands of signal events can be accumulated on the LHCb per nominal running year.

As a consequence of the unitarity of quark mixing matrix, tree level flavor-changing neutral-current (FCNC) is forbidden in the SM. When higher order corrections are taken into account, $b \to s l^+ l^-$ arises from photonic penguin, Z penguin and W-box diagram. The large mass scale of virtual states leads to tiny Wilson coefficients in $b$ quark decays and thus $b \to s l^+ l^-$ would be sensitive to the potential NP effects. In certain NP scenarios, new effective operators out of the SM scope can emerge, but in a class of other scenarios, only Wilson coefficients for effective operators are modified. Among the latter category, vector-like quark model (VQM) \cite{28–36} and family non-universal $Z'$ model \cite{37–42} are simplest and therefore of theoretical interest. In this work we shall also elaborate the impacts of these models on $B \to K^*_2 l^+ l^-$. The rest of the paper is organized as follows. In Sec. II we collect the necessary hadronic inputs, namely form factors. Sec. II contains the analytic formulas for differential decay distributions and integrated quantities. In Sec. IV we give a brief overview of two NP models whose effects we will study. Sec. V is our phenomenological analysis: the predictions in the SM; update of the constraints on the VQM and $Z'$ model parameters; the NP effect on the physical quantities. We conclude in the last section. In the appendix, we give the effective Hamiltonian in the SM and the helicity amplitude method.

II. $B \to K^*_2$ FORM FACTORS

$B \to K^*_2 l^+ l^-$ decay amplitudes contain two separate parts. Short-distance physics, in which contributions at the weak scale $\mu_W$ is calculated by perturbation theory and the evolution between $m_W$ and $b$ quark mass scale $m_b$ is organized by the renormalization group. These degrees of freedom are incorporated into Wilson coefficients and the obtained effective Hamiltonian responsible for $b \to s l^+ l^-$ in the appendix A. The low-energy effect characterizes the long-distance physics and will be parameterized by hadronic matrix elements of effective operators, which are usually reduced to heavy-to-light form factors in semileptonic $B$ decays.

The spin-2 polarization tensor, which satisfies $\epsilon_{\mu\nu} P_2^\nu = 0$ with $P_2$ being the momentum, is symmetric and traceless. It can be constructed via the spin-1 polarization vector $\epsilon$:

$$
\epsilon_{\mu\nu}(\pm 2) = \epsilon_{\mu}(\pm) \epsilon_{\nu}(\pm), \quad \epsilon_{\mu\nu}(\pm 1) = \frac{1}{\sqrt{2}}[\epsilon_{\mu}(\pm)\epsilon_{\nu}(0) + \epsilon_{\nu}(\pm)\epsilon_{\mu}(0)],
$$

$$
\epsilon_{\mu\nu}(0) = \frac{1}{\sqrt{6}}[\epsilon_{\mu}(+)\epsilon_{\nu}(-) + \epsilon_{\nu}(+)\epsilon_{\mu}(-)] + \sqrt{\frac{2}{3}}\epsilon_{\mu}(0)\epsilon_{\nu}(0).
$$

In the case of the tensor meson moving on the $z$ axis, the explicit structures of $\epsilon$ in the ordinary coordinate frame are chosen as

$$
\epsilon_{\mu}(0) = \frac{1}{m_{K^*_2}}(|\vec{p}_{K^*_2}|, 0, 0, E_{K^*_2}), \quad \epsilon_{\mu}(\pm) = \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0),
$$

where $E_{K^*_2}$ and $|\vec{p}_{K^*_2}|$ is the energy and the momentum magnitude of $K^*_2$ in $B$ meson rest frame, respectively. In the following calculation, it is convenient to introduce a new polarization vector $\epsilon_T$

$$
\epsilon_{T\mu}(h) = \frac{1}{m_B}\epsilon_{\mu\nu}(h)P_B^\nu,
$$

which satisfies

$$
\epsilon_{T\mu}(\pm 2) = 0, \quad \epsilon_{T\mu}(\pm 1) = \frac{1}{m_B}\frac{1}{\sqrt{2}}\epsilon_{\mu}(0) \cdot P_B\epsilon_{\mu}(\pm), \quad \epsilon_{T\mu}(0) = \frac{1}{m_B}\sqrt{\frac{2}{3}}\epsilon_{\mu}(0) \cdot P_B\epsilon_{\mu}(0).
$$

The contraction is evaluated as $\epsilon(0) \cdot P_B/m_B = |\vec{p}_{K^*_2}|/m_{K^*_2}$ and thus we can see that the new vector $\epsilon_T$ plays a similar role to the ordinary polarization vector $\epsilon$, regardless of the dimensionless constants $\frac{1}{\sqrt{2}}|\vec{p}_{K^*_2}|/m_{K^*_2}$ or $\sqrt{\frac{2}{3}}|\vec{p}_{K^*_2}|/m_{K^*_2}$.
The parametrization of $B \to K^*_2$ form factors is analogous to the $B \to K^*$ ones \cite{25,27,43}.

\[
\langle K^*_2(P_2, \epsilon)|\bar{s}\gamma^\mu b|B(P_B)\rangle = -\frac{2V(q^2)}{m_B + m_{K^*_2}} e^{\nu\rho\sigma} \epsilon_{\nu T}^* P_{B\rho} P_{2\sigma},
\]

\[
\langle K^*_2(P_2, \epsilon)|\bar{s}\gamma^\mu \gamma_5 b|B(P_B)\rangle = 2i m_{K^*_2} A_0(q^2) \frac{\epsilon_T \cdot q}{q^2} + i (m_B + m_{K^*_2}) A_1(q^2) \left[ \frac{\epsilon_T \cdot q}{q^2} \right] - i A_2(q^2) \frac{\epsilon_T \cdot q}{m_B + m_{K^*_2}} \frac{p_\mu - m_B^2 - m_{K^*_2}^2}{q^2} q^\mu,
\]

\[
\langle K^*_2(P_2, \epsilon)|\bar{s}\sigma^{\mu\nu} q_{b}|B(P_B)\rangle = -2i T_1(q^2) \frac{\epsilon^{\nu\rho\sigma} \epsilon_{T\rho}^* P_{B\sigma} P_{2\sigma}}{m_B + m_{K^*_2}},
\]

\[
\langle K^*_2(P_2, \epsilon)|\bar{s}\sigma^{\mu\nu} \gamma_5 q_{b}|B(P_B)\rangle = T_2(q^2) \left[ (m_B^2 - m_{K^*_2}^2) \epsilon_T \cdot P - \epsilon_T \cdot q P^\mu \right] + T_3(q^2) \left[ \frac{\epsilon_T \cdot q}{q^2} \right] \left[ q^\mu - \frac{q^2}{m_B^2 - m_{K^*_2}^2} P^\mu \right],
\]

where $q = P_B - P_2, P = P_B + P_2$. We also have the relation $2m_{K^*_2} A_0(0) = (m_B + m_{K^*_2}) A_1(0) - (m_B - m_{K^*_2}) A_2(0)$ in order to smear the pole at $q^2 = 0$.

Using the newly-studied light-cone distribution amplitudes \cite{44}, we have computed $B \to K^*_2$ form factors \cite{27} in the perturbative QCD approach (PQCD) \cite{43}. At the leading power, our predictions are found to obey the nontrivial relations derived from the large energy symmetry. This consistence may imply that the PQCD results for the form factors are reliable and therefore suitable for the study of the semileptonic B decays. The recent computation in light-cone QCD sum rules \cite{43} is also consistent with ours. Results in the light-cone sum rules in conjunction with $B$-meson wave functions \cite{46}, however, are too large and thus not favored by the $B \to K^*_2 \gamma$ data. In our work the $B \to K^*_2$ form factors are $q^2$-distributed as \cite{27}

\[
F(q^2) = \frac{F(0)}{(1 - q^2/m_B^2)(1 - a(q^2/m_B^2) + b(q^2/m_B^2)^2)},
\]

where $F$ denotes a generic form factor among $A_0, A_1, V, T_{1-3}$. Neglecting higher power corrections, $A_2$ is related to $A_0$ and $A_1$ by

\[
A_2(q^2) = \frac{m_B + m_{K^*_2}}{m_B - q^2} \left[ (m_B + m_{K^*_2}) A_1(q^2) - 2m_{K^*_2} A_0(q^2) \right].
\]

Numerical results for the $B \to K^*_2$ and $B_s \to f'_2(1525)$ form factors at maximally recoil point and the two fitted parameters $a, b$ are collected in table \cite{4}. The two kinds of errors are from: decay constants of $B$ meson and shape parameter $\omega_b$; AQCQD, the scales $t_5$ and the threshold resummation parameter $\epsilon$ \cite{27}.

### III. Differential Decay Distributions and Spin Amplitudes

In this section, we will discuss the kinematics of the quasi four-body decay $B \to K^*_2(\to K\pi)l^+l^-$, define angular observables and collect the explicit formulas of helicity amplitudes and/or transversity amplitudes.

#### A. Differential decay distribution

At the quark level, the decay amplitude for $b \to sl^+l^-$ is expressed as

\[
\mathcal{M}(b \to sl^+l^-) = \frac{G_F \alpha_{em}}{\sqrt{2}} \pi \bar{V}_b V^*_l \times \left( \frac{C_9 + C_{10}}{4} [\bar{s}b]_{V-A} [\bar{l}l]_{V+A} + \frac{C_9 - C_{10}}{4} [\bar{s}b]_{V-A} [\bar{l}l]_{V-A} + C_{7L} m_b \bar{s}i\sigma_{\mu\nu}(1 + \gamma_5) b \frac{q^\mu}{q^2} \times [\bar{l}l]^\nu + C_{7R} m_b \bar{s}i\sigma_{\mu\nu}(1 - \gamma_5) b \frac{q^\mu}{q^2} \times [\bar{l}l]^\nu \right),
\]

where $C_{7L} = C_7$ and $C_{7R} = \frac{m_b}{m_s} C_{7L}$ in the SM. Sandwiching Eq. \cite{28} between the initial and final states and replacing the spinor product $[\bar{s}b]$ by hadronic matrix elements, one obtains the decay amplitude for hadronic $B$ process. For the process under scrutiny in this work, the decay observed in experiment is actually $B \to K^*_2(\to K\pi)l^+l^-$ which is
TABLE I: $B \rightarrow K^*_2$ and $B_s \rightarrow f'_2(1525)$ form factors in the PQCD approach. $F(0)$ denotes results at $q^2 = 0$ point while $a, b$ are the parameters in the parametrization shown in Eq. (6). The two kinds of errors are from: decay constants of $B$ meson and shape parameter $\omega_b$; $\Lambda_{QCD}$, factorization scales $\mu_s$ and the threshold resummation parameter $c$.

| $F$ | $F(0)$ | $a$ | $b$ |
|-----|--------|-----|-----|
| $V_{BK^*_2}^T$ | $0.21\pm 0.04 \pm 0.05$ | $1.73\pm 0.02 \pm 0.03$ | $0.66\pm 0.04 \pm 0.05$ |
| $A_0^{BK^*_2}$ | $0.18\pm 0.04 \pm 0.04$ | $1.70\pm 0.04 \pm 0.05$ | $0.64\pm 0.04 \pm 0.04$ |
| $A_1^{BK^*_2}$ | $0.13\pm 0.03 \pm 0.03$ | $0.78\pm 0.01 \pm 0.03$ | $-0.11\pm 0.02 \pm 0.04$ |
| $A_2^{BK^*_2}$ | $0.08\pm 0.02 \pm 0.02$ | $-0.02\pm 0.01$ | $-0.03\pm 0.02$ |
| $T_1^{BK^*_2}$ | $0.17\pm 0.04 \pm 0.04$ | $1.73\pm 0.04 \pm 0.03$ | $0.69\pm 0.04 \pm 0.05$ |
| $T_2^{BK^*_2}$ | $0.17\pm 0.03 \pm 0.03$ | $0.79\pm 0.04 \pm 0.02$ | $-0.06\pm 0.03 \pm 0.00$ |
| $T_3^{BK^*_2}$ | $0.14\pm 0.03 \pm 0.03$ | $1.61\pm 0.03 \pm 0.03$ | $0.52\pm 0.05 \pm 0.01$ |
| $V_{B_s f'_2}^T$ | $0.20\pm 0.04 \pm 0.05$ | $1.75\pm 0.02 \pm 0.03$ | $0.69\pm 0.05 \pm 0.08$ |
| $A_0^{B_s f'_2}$ | $0.16\pm 0.03 \pm 0.03$ | $1.69\pm 0.04 \pm 0.03$ | $0.64\pm 0.03 \pm 0.02$ |
| $A_1^{B_s f'_2}$ | $0.12\pm 0.02 \pm 0.03$ | $0.80\pm 0.02 \pm 0.03$ | $-0.11\pm 0.03 \pm 0.00$ |
| $A_2^{B_s f'_2}$ | $0.09\pm 0.02 \pm 0.02$ | $-0.02\pm 0.01$ | $-0.01\pm 0.01$ |
| $T_1^{B_s f'_2}$ | $0.16\pm 0.03 \pm 0.04$ | $1.75\pm 0.01 \pm 0.05$ | $0.71\pm 0.03 \pm 0.06$ |
| $T_2^{B_s f'_2}$ | $0.16\pm 0.03 \pm 0.03$ | $0.82\pm 0.03 \pm 0.03$ | $-0.08\pm 0.03 \pm 0.08$ |
| $T_3^{B_s f'_2}$ | $0.13\pm 0.03 \pm 0.03$ | $1.64\pm 0.02 \pm 0.06$ | $0.57\pm 0.04 \pm 0.05$ |

FIG. 1: Kinematics variables in the $\overline{B} \rightarrow K^*_2 (\rightarrow K^- \pi^+) l^- \bar{l}^-$ process. The moving direction of $K^*_2$ in $B$ rest frame is chosen as the $z$ axis. The polar angle $\theta_K$ ($\theta_l$) is defined as the angle between the flight direction of $K^-$ ($\mu^-$) and the $z$ axis in the $K^*_2$ (lepton pair) rest frame. The convention also applies to $B_s \rightarrow f'_2(\rightarrow K^+ K^-) l^- \bar{l}^-$ transition.

a quasi four-body decay. The convention on the kinematics is illustrated in Fig. 1. The moving direction of $K^*_2$ in $B$ meson rest frame is chosen as $z$ axis. The polar angle $\theta_K$ ($\theta_l$) is defined as the angle between the flight direction of $K^-$ ($\mu^-$) and the $z$ axis in $K^*_2$ (lepton pair) rest frame. $\phi$ is the angle defined by decay planes of $K^*_2$ and the lepton pair.

Using the technique of helicity amplitudes described in the appendix B, we obtain the partial decay width

$$
\frac{d^4\Gamma}{dq^2d\cos\theta_Kd\cos\theta_l d\phi} = \frac{3}{8}|M_B|^2,
$$

(9)
with the mass correction factor $\beta_i = \sqrt{1 - 4m_i^2/q^2}$. The function $|\mathcal{M}_B|^2$ is decomposed into 11 terms

$$
|\mathcal{M}_B|^2 = \left[ I_1^2 C^2 + 2I_1 S^2 + (I_2 C^2 + 2I_2 S^2) \cos(2\theta_i) + 2I_3 S^2 \sin^2 \theta_i \phi + 2\sqrt{2}I_4 CS \sin(2\theta_i) \cos \phi \\
+ 2\sqrt{2}I_5 CS \sin \theta_i \cos \phi + 2I_6 S^2 \cos \theta_i + 2\sqrt{2}I_7 CS \sin(\theta_i) \sin \phi \\
+ 2\sqrt{2}I_8 CS \sin(2\theta_i) \sin \phi + 2I_9 S^2 \sin^2 \theta_i \sin(2\phi) \right],
$$

with the angular coefficients

$$
I_1^2 = (|A_{L0}|^2 + |A_{R0}|^2) + \frac{3 m_f^2}{q^2} \text{Re}[A_{L0} A_{R0}^*] + \frac{4 m_f^2}{q^2} |A_i|^2,
$$

$$
I_1^2 = \frac{3}{4} |A_{L\perp}|^2 + |A_{L||}|^2 + |A_{R||}|^2 + |A_R|^2 \left( 1 - \frac{4 m_f^2}{3 q^2} \right) + \frac{4 m_f^2}{q^2} \text{Re}[A_{L\perp} A_{R\perp}^* + A_{L||} A_{R||}^*],
$$

$$
I_2^2 = -\beta_i^2 (|A_{L0}|^2 + |A_{R0}|^2),
$$

$$
I_2^2 = \frac{1}{4} \beta_i^2 (|A_{L\perp}|^2 + |A_{L||}|^2 + |A_{R||}|^2)^2,
$$

$$
I_3^2 = \frac{1}{2} \beta_i^2 (|A_{L\perp}|^2 - |A_{L||}|^2 + |A_{R||}|^2)^2,
$$

$$
I_4^2 = \frac{1}{\sqrt{2}} \beta_i^2 \text{Re}[A_{L0} A_{L\perp}^*] + \text{Re}[A_{R0} A_{R\perp}^*],
$$

$$
I_5^2 = \sqrt{2} \beta_i^2 (\text{Re}[A_{L0} A_{L\perp}^*] - \text{Re}[A_{R0} A_{R\perp}^*]),
$$

$$
I_6^2 = 2 \beta_i^2 (\text{Re}[A_{L||} A_{L\perp}^*] - \text{Re}[A_{R||} A_{R\perp}^*]),
$$

$$
I_7^2 = \sqrt{2} \beta_i^2 \text{Im}[A_{L0} A_{L\perp}^*] - \text{Im}[A_{R0} A_{R\perp}^*],
$$

$$
I_8^2 = \sqrt{2} \beta_i^2 \text{Im}[A_{L||} A_{L\perp}^*] + \text{Im}[A_{R||} A_{R\perp}^*].
$$

$C = C(K^*_3)$ and $S = S(K^*_3)$ for $B \to K^*_3 l^+ l^-$. Without higher order QCD corrections, $I_7$ is zero and $I_8, I_9$ are tiny in the SM and the reason is that only $C_9$ has an imaginary part. In this sense these coefficients can be chosen as an ideal window to probe new physics signals.

The amplitudes $A_i$ are generated from the hadronic $B \to K^*_3 V$ amplitudes $\mathcal{H}_i$ through

$$
A_i = \sqrt{\frac{\sqrt{3} m_f^2 \beta_i}{2 \pi m_K^2 q^2}} B(K^*_3 \to K^0 \pi) \mathcal{H}_i
$$

$$
A_{L0} = N_{K^*_3} \frac{\sqrt{\lambda}}{\sqrt{6} m_B m_{K^*_3}^2} \frac{1}{2 m_{K^*_3}} \sqrt{q^2} \left[ (C_9 - C_{10})(m_B^2 - m_{K^*_3}^2 - q^2)(m_B + m_{K^*_3}) A_1 - \frac{\lambda}{m_B + m_{K^*_3}} A_2 \right] + 2 m_b (C_{7L} - C_{7R})(m_B^2 + 3 m_{K^*_3}^2 - q^2) T_2 - \frac{\lambda}{m_B + m_{K^*_3}} T_3),
$$

$$
A_{L\perp} = N_{K^*_3} \frac{\sqrt{\lambda}}{\sqrt{8} m_B m_{K^*_3}^2} \left[ (C_9 - C_{10})(m_B + m_{K^*_3}) A_1 \mp \frac{\sqrt{\lambda}}{m_B + m_{K^*_3}} V \right] - \frac{2 m_b (C_{7L} + C_{7R})}{q^2}(\pm \sqrt{\lambda} T_1) + \frac{2 m_b (C_{7L} - C_{7R})}{q^2}(m_B^2 - m_{K^*_3}^2) T_2),
$$

$$
A_{L||} = N_{K^*_3} \frac{\sqrt{\lambda}}{\sqrt{6} m_B m_{K^*_3}^2} (C_9 - C_{10}) \frac{\sqrt{\lambda}}{\sqrt{q^2}} A_0,
$$

with $N_{K^*_3} = \sqrt{\frac{C_7^2 q^2}{3 \cdot 2^{6} m_B m_{K^*_3}^2}} |V_{tb} V_{ts}^{*}| l^{1/2} \left( 1 - \frac{4 m_f^2}{q^2} \right)^{1/2} B(K^*_3 \to \bar{K} \pi) l^{1/2}$. For convenience, we have introduced transversity amplitudes as

$$
A_{L\perp / ||} = \frac{1}{\sqrt{2}} (A_{L+} \mp A_{L-}),
$$

$$
A_{L\perp} = -\sqrt{2} \frac{\sqrt{\lambda}}{\sqrt{8} m_B m_{K^*_3}^2} N_{K^*_3} \left[ (C_9 - C_{10}) \frac{\sqrt{\lambda} V}{m_B + m_{K^*_3}} + \frac{2 m_b (C_{7L} + C_{7R})}{q^2} \sqrt{\lambda} T_1 \right],
$$

$$
A_{L||} = \sqrt{2} \frac{\sqrt{\lambda}}{\sqrt{8} m_B m_{K^*_3}^2} N_{K^*_3} \left[ (C_9 - C_{10})(m_B + m_{K^*_3}) A_1 + \frac{2 m_b (C_{7L} - C_{7R})}{q^2}(m_B^2 - m_{K^*_3}^2) T_2 \right].
$$
and the right-handed decay amplitudes are similar

\[ A_{Rt} = A_{Lt}|_{C_{10} \rightarrow -C_{10}}. \]  

(14)

The combination of the timelike decay amplitude is used in the differential distribution

\[ A_t = A_{Rt} - A_{Lt} = 2N_{K_2}^2 \frac{\sqrt{\lambda}}{\sqrt{6m_Bm_{K_2}^*}} C_{10} \frac{\sqrt{\lambda}}{q^2} A_0. \]  

(15)

**B. Dilepton spectrum distribution**

Integrating out the angles \( \theta_l, \theta_K \) and \( \phi \), we obtain the dilepton mass spectrum

\[ \frac{d\Gamma}{dq^2} = \frac{1}{4} (3I_1^\pm + 6I_1^0 - 2I_2^0), \]  

(16)

and its expression in the massless limit

\[ \frac{d\Gamma_i}{dq^2} = (|A_L|^2 + |A_Rt|^2), \]  

(17)

with \( i = 0, \pm 1 \) or \( i = 0, \perp, || \). After some manipulations in the appendix, the correspondence of the above equations and Eq. (20) with results in Ref. [25] can be shown.

**C. Polarization distribution**

The longitudinal polarization distribution for \( \bar{B} \rightarrow K_{2}^* l^+ l^- \) is defined as

\[ \frac{df_L}{dq^2} = \frac{d\Gamma_0}{dq^2} = \frac{3I_1^\pm - 3I_2^0}{3I_1^\pm + 6I_1^0 - 2I_2^0}, \]  

(18)

in which \( \frac{df_r}{dq^2} \) can be reduced into \( I_1^\pm \) in the case of \( m_l = 0 \) since \( I_1^\pm = -I_2^0 \). The integrated polarization fraction is given as

\[ f_L \equiv \frac{\Gamma_0}{\Gamma} = \int \frac{dq^2}{dq^2} \frac{df_L}{dq^2}. \]  

(19)

**D. Forward-backward asymmetry**

The differential forward-backward asymmetry of \( \bar{B} \rightarrow K_{2}^* l^+ l^- \) is defined by

\[ \frac{dA_{FB}}{dq^2} = \int_0^1 - \int_{-1}^0 d\cos \theta_l \frac{d\Gamma}{dq^2} d\cos \theta_l = \frac{3}{4} I_6, \]  

(20)

while the normalized differential FBA is given by

\[ \frac{dA_{FB}}{dq^2} = \frac{dA_{FB}}{dq^2} = \frac{3I_6}{3I_1^\pm + 6I_1^0 - 2I_2^0}. \]  

(21)

In the massless limit, we have

\[ \frac{dA_{FB}}{dq^2} = \frac{\lambda M_{F_2}^2}{8m_B^2m_{K_2}^*} |V_{tb}V_{ts}^*|^2 \text{Re} \left[ C_9C_{10}A_1V + C_{10}(C_{7L} + C_{7R}) \frac{m_b(m_B + m_{K_2}^*)}{q^2} A_1T_1 \right. \]

\[ \left. +C_{10}(C_{7L} - C_{7R}) \frac{m_b(m_B - m_{K_2}^*)}{q^2} T_2V \right]. \]  

(22)

In the SM where \( C_{7R} \) is small, the zero-crossing point \( s_0 \) of FBAs is determined by the equation

\[ C_9 A_1(s_0)V(s_0) + C_{7L} \frac{m_b(m_B + m_{K_2}^*)}{s_0} A_1(s_0)T_1(s_0) + C_{7L} \frac{m_b(m_B - m_{K_2}^*)}{s_0} T_2(s_0)V(s_0) = 0. \]  

(23)
E. Spin amplitudes and transverse asymmetries

Using the above helicity/spin amplitudes, it is also possible to construct several useful quantities which are ratios of different amplitudes. The following ones, widely studied in the $B \to K^*$ case, are stable against the uncertainties from hadronic form factors

\[
A_T^{(1)} = \frac{\Gamma_- - \Gamma_+}{\Gamma_- + \Gamma_+} = \frac{-2\text{Re}(A_{||}A_{||}^*)}{|A_{||}|^2 + |A_{\perp}|^2},
\]

\[
A_T^{(2)} = \frac{|A_{\perp}|^2 - |A_{||}|^2}{|A_{||}|^2 + |A_{\perp}|^2},
\]

\[
A_T^{(3)} = \frac{|A_{L0}A_{L0}^* + A_{R0}A_{R0}^*|}{\sqrt{|A_{L0}|^2|A_{R0}|^2}},
\]

\[
A_T^{(4)} = \frac{|A_{L0}A_{L0}^* - A_{R0}A_{R0}^*|}{|A_{L0}A_{L0}^*| + |A_{R0}A_{R0}^*|},
\]

with the notation

\[
A_iA_j^* = A_{Li}A_{Lj}^* + A_{Ri}A_{Rj}^*.
\]

Due to the hierarchy in the SM $\Gamma_- \gg \Gamma_+$, $A_T^{(3)}$ is close to 1 and therefore its deviation from 1 is more useful to reflect the size of the NP effects.

IV. TWO NP MODELS

The $b \to s l^+l^-$ has a small branching fraction since the SM is lack of tree level FCNC. It is not necessarily the same in extensions. In this section we will briefly give an overview of two NP models, which allow tree-level FCNC. Both of these two models, vector-like quark model and family non-universal $Z'$ model, do not introduce new type operators but instead modify the Wilson coefficients $C_9, C_{10}$. To achieve this goal, they introduce an $SU(2)$ singlet down-type quark or a new gauge boson $Z'$.

A. Vector-like quark model: $Z$-mediated FCNCs

In the vector-like quark model, the new $SU(2)_L$ singlet down quarks $D_L$ and $D_R$ modify the Yukawa interaction sector

\[
\mathcal{L}_Y = \bar{Q}_L Y_D H d_R + h_D \bar{Q}_L H D_R + m_D \bar{D}_L D_R + h.c.,
\]

where the flavor indices have been suppressed. $Q_L$ ($H$) is the $SU(2)$ quark (Higgs) doublet, $Y_D$ and $h_D$ are the Yukawa couplings and $m_D$ is the mass of exotic quark before electroweak symmetry breaking. When the Higgs field acquires the vacuum expectation value (VEV), the mass matrix of down type quark becomes

\[
m_d = \begin{pmatrix}
y_{ij}^D & h_i^D \\
- & - \\
0 & m_D
\end{pmatrix},
\]

which can be diagonalized by two unitary matrices

\[
m_d^{\text{dia}} = V_D^L m_d V_D^{R\dagger}.
\]

The SM coupling of $Z$-boson to fermions is flavor blind, and the flavor in the process with exchange of $Z$-boson is conserved at tree level. Unlikely although the right-handed sector in the VQM is the same as the SM, the new
left-handed quark is $SU(2)_L$ singlet, which carries the same hypercharge as right-handed particles. Therefore the gauge interactions of left-handed down-type quarks with $Z$-boson are given by

$$
\mathcal{L}_Z = \tilde{Q}_L \frac{g}{\cos\theta_W} (I_3 - \sin^2\theta_W Q) \tilde{Z} Q_L + \bar{D}_L \frac{g}{\cos\theta_W} (-\sin^2\theta_W Q) \bar{Z} D_L,
$$

where $g$ is the coupling constant of $SU(2)_L$, $\theta_W$ is the Weinberg’s angle, $P_{R(L)} = (1 \pm \gamma_5)/2$. $I_3$ and $Q$ are operators for the third component of the weak isospin and the electric charge, respectively.

Since the ratio $\xi_D$ of the coupling constants deviates from unity: $\xi_D = -\sin^2\theta_W Q_D / (I_3^L - \sin^2\theta_W Q_F)$, tree level FCNC can be induced after the diagonalization of the down-type quarks. For instance, the interaction for $b \to s l^+ l^-$ in the VQM is given by

$$
\mathcal{L}_{b \to s} = \frac{g\lambda_{sb}}{\cos\theta_W} \bar{s} \gamma^\mu P_L b Z_\mu + h.c.,
$$

where $\lambda_{sb}$ is introduced as the new free parameter:

$$
\lambda_{sb} = (\xi_D - 1) (V_D^L)^* s D (V_D^L)_{sD} \equiv |\lambda_{sb}| \exp(i\theta_s).
$$

Using Eq. (30), the effective Hamiltonian for $b \to s l^+ l^-$ mediated by $Z$-boson is found by

$$
\mathcal{H}_{b \to s l^+ l^-}^Z = \frac{2G_F}{\sqrt{2}} \lambda_{sb} c_L^e (\bar{s} b)_{V-A} \left[ c_L^e (\bar{\ell} \ell)_{V-A} + c_R^e (\bar{\ell} \ell)_{V+A} \right].
$$

The Wilson coefficients $C_{9,10}$ are modified accordingly

$$
C_{9}^{VLQ} = C_{9}^{SM} - 4\pi \alpha_{em} \lambda_{sb} c_L^e (c_L^e + c_R^e) / V_{tb}^2 V_{tb}, \quad C_{10}^{VLQ} = C_{10}^{SM} + 4\pi \alpha_{em} \lambda_{sb} c_L^e (c_L^e - c_R^e) / V_{tb}^2 V_{tb}.
$$

Making use of the experimental data of $b \to s l^+ l^-$, our previous work [47] has placed a constraint on the new coupling constant

$$
|\lambda_{sb}| < 1 \times 10^{-3},
$$

but its phase $\theta_s$ is less constrained. In the following, we shall see that the constraint can be improved by taking into account the experimental data of the exclusive process $B \to K^* l^+ l^-$. 

**B. Family non-universal $Z'$ model**

The SM can be extended by including an additional $U(1)'$ symmetry, and the currents can be defined as following in a proper gauge basis

$$
J_{Z'}^\mu = g' \sum_i \bar{\psi}_i \gamma^\mu \left[ \epsilon_i^{\psi_L} P_L + \epsilon_i^{\psi_R} P_R \right] \psi_i,
$$

where $i$ is the family index and $\psi$ labels the fermions (up- or down-type quarks, or charged or neutral leptons). According to some string construction or GUT models such as $E_6$, it is possible to have family non-universal $Z'$ couplings, namely, even though $\epsilon_i^{L,R}$ are diagonal the gauge couplings are not family universal. After rotating to the physical basis, FCNCs generally appear at tree level in both LH and RH sectors. Explicitly,

$$
B^{\psi_L} = V^{\psi_L}_{\psi_L} \epsilon^{\psi_L} V^{\psi_L}_{\psi_L}^+, \quad B^{\psi_R} = V^{\psi_R}_{\psi_R} \epsilon^{\psi_R} V^{\psi_R}_{\psi_R}^+.
$$

Moreover, these couplings may contain CP-violating phases beyond that of the SM.

The Lagrangian of $Z'$'s couplings is given as

$$
\mathcal{L}_{Z'}_{FCNC} = -g'(B_{sV}^L \bar{s} L \gamma_\mu b_L + B_{sV}^R \bar{s} R \gamma_\mu b_R) Z'^\mu + h.c..
$$

It contributes to the $b \to s l^+ l^-$ decay at tree level with the effective Hamiltonian

$$
\mathcal{H}_{Z'}_{eff} = \frac{8G_F}{\sqrt{2}} (\rho_{sV}^L \bar{s} L \gamma_\mu b_L + \rho_{sV}^R \bar{s} R \gamma_\mu b_R) (\rho_{lV}^L \bar{\ell} L \gamma_\mu \ell_L + \rho_{lV}^R \bar{\ell} R \gamma_\mu \ell_R),
$$

(37)
where
\[ \rho'_{ff'}^{L,R} = \frac{g'M_{Z'}B_{ff'}^{L,R}}{gM_{Z'}} \]  
(38)

with the coupling \( g \) associated with the \( SU(2)_L \) group in the SM. In this paper we shall not take the renormalization group running effects due to these new contributions into consideration because they are expected to be small. For the couplings are all unknown, one can see from Eq. (38) that there are many free parameters here. For the purpose of illustration and to avoid too many free parameters, we put the constraint that the FCNC couplings of the \( Z' \) and quarks only occur in the left-handed sector. Therefore, \( \rho'_{sb} = 0 \), and the effects of the \( Z' \) FCNC currents simply modify the Wilson coefficients \( C_9 \) and \( C_{10} \) in Eq. (39). We denote these two modified Wilson coefficients by \( C'_9 \) and \( C'_{10} \), respectively. More explicitly,
\[ C'_9 = C_9 - \frac{4\pi}{\alpha_{em}} \frac{\rho'^L_{bb}(\rho'^L_{ll} + \rho'^R_{ll})}{V_{tb}V_{ts}^*}, \quad C'_{10} = C_{10} + \frac{4\pi}{\alpha_{em}} \frac{\rho'^L_{sb}(\rho'^L_{ll} - \rho'^R_{ll})}{V_{tb}V_{ts}^*}. \]  
(39)

Compared with the Wilson coefficients in the vector-like quark model in Eq. (32), we can see that the \( Z' \) contributions in Eq. (39) have similar forms and the correspondence lies in the coupling constants
\[ \lambda_{sb} c_{L,R} \rightarrow \rho'^L_{bb}, \quad c'_{L,R} \rightarrow \rho'^{L,R}_{ll}. \]  
(40)

However the number of free parameters is increased from 2 to 4 since \( c'_{L,R} \) in the VQM is the same as the SM.

V. PHENOMENOLOGICAL ANALYSIS

In this section, we will present our theoretical results in the SM, give an update of the constraints in the above two NP models and investigate their effects on \( B \to K_2^* \mu^+ \mu^- \) and \( B_s \to f_2^* \mu^+ \mu^- \). For convenience, branching ratios of \( K_2^* \) and \( f_2^* \) decays into \( K\pi \) and \( K\bar{K} \) will not be taken into account in the numerical analysis.

A. SM predictions

With the \( B \to K_2^* \) form factors computed in the PQCD approach [27], the BR, zero-crossing point of FBAs and polarization fractions are predicted as
\[ B(B \to K_2^* \mu^+ \mu^-) = (2.5^{+1.6}_{-1.1}) \times 10^{-7}, \]
\[ f_L(B \to K_2^* \mu^+ \mu^-) = (66.6 \pm 0.4)\%, \]
\[ s_0(B \to K_2^* \mu^+ \mu^-) = (3.49 \pm 0.04)\text{GeV}^2, \]
\[ B(B \to K_2^* \tau^+ \tau^-) = (9.6^{+6.2}_{-4.5}) \times 10^{-10}, \]
\[ f_L(B \to K_2^* \tau^+ \tau^-) = (57.2 \pm 0.7)\%. \]  
(41)

The errors are from the form factors, namely, from the \( B \) meson wave functions and the PQCD systematic parameters. Most of the uncertainties from form factors will cancel in the polarization fractions and the zero-crossing point \( s_0 \). Similarly results for \( B_s \to f_2 l^+ l^- \) are given as
\[ B(B_s \to f_2 l^+ l^-) = (1.8^{+1.1}_{-0.7}) \times 10^{-7}, \]
\[ f_L(B_s \to f_2 l^+ l^-) = (63.2 \pm 0.7)\%, \]
\[ s_0(B_s \to f_2 l^+ l^-) = (3.53 \pm 0.03)\text{GeV}^2, \]
\[ B(B_s \to f_2^* \tau^+ \tau^-) = (5.8^{+3.7}_{-2.1}) \times 10^{-10}, \]
\[ f_L(B_s \to f_2^* \tau^+ \tau^-) = (53.9 \pm 0.4)\%. \]  
(42)

We also show the \( q^2 \)-dependence of their differential branching ratios (in units of \( 10^{-7} \)) in Fig. 2.

Charm-loop effects, due to the large Wilson coefficient and the large CKM matrix element, might introduce important effects. In a very recent work [20], the authors have adopted QCD sum rules to investigate both factorizable
diagrams and nonfactorizable diagrams. Their results up to the region \( q^2 = m_{J/\psi}^2 \) are parameterized in the following form,

\[
\Delta C_9^{(i)B \rightarrow K^*}(q^2) = \frac{r_1^{(i)}(1 - \frac{q^2}{Q^2}) + \Delta C_9^{(i)B \rightarrow K^*}(q^2)q^2}{1 + r_2^{(i)}q^2/m_{J/\psi}^2},
\]

where the three results correspond to different Lorentz structures: \( i = 1, 2, 3 \) for terms containing \( V, A_1 \) and \( A_2 \) respectively. The numerical results are quoted as follows

\[
\begin{align*}
\Delta C_9^{(1)}(q^2) &= 0.72^{+0.57}_{-0.37}, \quad r_1^{(1)} = 0.10, \quad r_2^{(1)} = 1.13, \\
\Delta C_9^{(2)}(q^2) &= 0.76^{+0.70}_{-0.41}, \quad r_1^{(2)} = 0.09, \quad r_2^{(2)} = 1.12, \\
\Delta C_9^{(3)}(q^2) &= 1.11^{+1.14}_{-0.70}, \quad r_1^{(3)} = 0.06, \quad r_2^{(3)} = 1.05.
\end{align*}
\]

It should be pointed out that not all charm-loop effects in \( B \rightarrow K_2^*l^+l^- \) are the same as the ones in \( B \rightarrow K^*l^+l^- \). Among various diagrams the factorizable contributions, which can be simply incorporated into \( C_9 \) given in Eq. (A3), are the same. The nonfactorizable ones are more subtle. In particular the light-cone sum rules (LCSR) with \( B \)-meson distribution amplitudes are adopted in Ref. [20], in which intermediate states like \( K^* \) are picked up as the ground state. The generalization is not straightforward to the case of \( K_2^* \) since in this approach states below \( K_2^* \) may contribute in a substantial manner. However in another viewpoint, i.e. the conventional LCSR, they may be related. In our previous work we have shown that the light-cone distribution amplitudes of \( K_2^* \) is similar with \( K^* \) in the dominant region of the PQCD approach. If it were also the same in the conventional LCSR, one may expect that the charm-loop effects in the processes under scrutiny have similar behaviors with the ones in \( B \rightarrow K^*l^+l^- \). Therefore as the first step to proceed, we will use their results to estimate the sensitivity in our following analysis and to be conservative, we use

\[
\Delta C_9^{(i)B \rightarrow K^*}(q^2) = (1 \pm 1)\Delta C_9^{(i)B \rightarrow K^*}(q^2)
\]

in the region of \( 1 \text{GeV}^2 < q^2 < 6 \text{GeV}^2 \). The central values for \( q^2 \)-dependent parameters will be used for simplicity and in this procedure, the factorizable corrections to \( C_9 \) given in Eq. (A3) should be set to 0 to avoid double counting.

FIG. 2: Differential branching ratios of \( B \rightarrow K_2^*l^+l^- \) (upper) and \( B_s \rightarrow f_2^0l^+l^- \) (lower) (in units of \( 10^{-7} \)): the left panel for \( l = \mu \) and the right panel for \( l = \tau \).
With the above strategy, our theoretical predictions are changed to
\[ f_L(B \to K^*_2 \mu^+ \mu^-) = (66.6^{+1.4}_{-0.7})\%, \]
\[ s_0(B \to K^*_2 \mu^+ \mu^-) = (3.49^{+0.19}_{-0.39}) \text{GeV}^2, \]
\[ f_L(B_s \to f_2^\prime \mu^+ \mu^-) = (63.5^{+1.5}_{-0.9})\%, \]
\[ s_0(B_s \to f_2^\prime \mu^+ \mu^-) = (3.55^{+0.19}_{-0.39}) \text{GeV}^2. \]  
(46)

The uncertainties in the zero-crossing point of FBAs are enlarged to 0.4 GeV$^2$. We also show the $q^2$-dependence of the differential polarization in Fig. 3 and the normalized forward-backward asymmetries in Fig. 4.

As a parallel way, the BR of $B \to K^*_2 l^+ l^-$ can also be estimated by making use of the data of radiative $B \to K^*(K^*_2)^\gamma$ decays  
\[ B(\bar{B}^0 \to K^*_2 \gamma) = (12.4 \pm 2.4) \times 10^{-6}, \]
\[ B(B^0 \to K^* \gamma) = (43.3 \pm 1.5) \times 10^{-6}. \]  
(47)

The ratio of the above BRs $R \equiv \frac{B(K^*_2 \gamma)}{B(K^* \gamma)} = 0.29 \pm 0.06$ and the measured data of $B \to K^* l^+ l^-$ shown in Tab II give the implication
\[ B_{\text{exp}}(B^0 \to K^*_2^0 l^+ l^-) = (3.1 \pm 0.7) \times 10^{-7}. \]  
(48)

which are remarkably consistent with our theoretical predictions within uncertainties.

When the large energy symmetry is exploited, the seven $B \to K^*_2$ form factors can be reduced into two independent ones $\zeta_\perp$ and $\zeta_\parallel$. Based on these nontrivial relations, Ref. [25] has used the experimental data of $B \to K^*_2 \gamma$ to extract $\zeta_\perp$. With the assumption of a similar size for $\zeta_\parallel$, the authors also estimated the branching ratio and forward-backward
asymmetries of $B \to K^*_2 l^+ l^-$. Explicitly they have employed
\[ \zeta_\perp = 0.27 \pm 0.03^{+0.00}_{-0.01}, \quad 0.8 \zeta_\perp < \zeta_\parallel < 1.2 \zeta_\perp, \]
which are comparable with our results \[27\]
\[ \zeta_\perp = (0.29 \pm 0.09), \quad \zeta_\parallel = (0.26 \pm 0.10). \]  
As a consequence, the predicted results of BR, forward-backward asymmetries and polarizations are compatible with each other.

Our results for angular coefficients, $\tilde{I}_i = I_i/\exp_{\text{err}}$, are depicted in Fig. 5 for $B \to K^*_2 \mu^+ \mu^-$ and Fig. 4 for $B_s \to f'_2 \mu^+ \mu^-$. Since the predictions for $\tilde{I}_7, \tilde{I}_8, \tilde{I}_9$ in the SM are typically smaller than 0.03, we shall not show them. The corresponding transversity asymmetries are shown in Fig. 7 and Fig. 8 respectively. One particular feature is that most of these results are stable against the large uncertainties from the form factors.

For the experimental purpose, it is valuable to estimate the minimum size of the averaged value of an angular distribution coefficient so that it can be measured in experiment. To establish any generic asymmetry with the averaged value $\langle A \rangle$ of a particular decay at $n\sigma$ level, events of the number $N = n^2/\langle (A) \rangle^2$ should be accumulated. For instance on the LHCb there are 6200 events for the $B \to K^* l^+ l^-$ process per nominal running year \[4\]. Incorporating all differences between $K^*_2$ and $K^*$, we may expect roughly 1000 events of $B \to K^*_2 (\to K^\pi) l^+ l^-$. Therefore if one wants to observe an asymmetry at $n\sigma$ level, its averaged value should be larger than $\langle A \rangle_{\text{min}} = \sqrt{n^2/1000} \approx 0.03n$.

Before closing this subsection, it is necessary to point out that the above estimation might be too optimistic. In the first few running years of LHCb, the central energy in the $pp$ collision may not reach 14 TeV and its luminosity will be below $2\,fb^{-1}$. Thus in the first stage not enough data are available for a precise determination of some angular coefficients. Nevertheless this will not affect our analysis of branching fractions and many angular coefficients.

B. Constraint on NP parameters and the NP effects on $B \to K^*_2 \mu^+ \mu^-$

In this subsection we will first update constraints of free parameters in the above two NP models, and particularly we use the experimental data of $b \to s l^+ l^-$ and $B \to K^* l^+ l^-$. Decay width of the inclusive process $b \to s l^+ l^-$ is given as \[48\]
\[ \frac{d\Gamma(b \to s l^+ l^-)}{ds} = \frac{\Gamma(b \to c\bar{\nu}_\tau)}{\exp_{\text{err}}(\alpha_m^2/4\pi^2)(1-\hat{s})^2C_{\text{leading}}(0,0)} \times \left[ 1 + 2\hat{s} \frac{(C_9 - 1) + 4C_7^2}{\hat{s}} \right] \left| C_7 \right|^2 + 12C_7\text{Re}C_9, \]
\[ f(z) = 1 - 8\hat{s}^2 + 8\hat{s}^6 - \hat{s}^8 - 24\hat{s}^4 \ln \hat{s}, \]
\[ k(z) = 1 - \frac{2\alpha_s^{\text{QCD}}}{3\pi} \left[ \frac{\hat{s}^2 - \frac{31}{4}}{(1-\hat{s})^2 + \frac{3}{4}} \right], \]  
where $\hat{s} = q^2/m_b^2$, and $\Gamma(b \to c\bar{\nu}_\tau)$ is used to cancel the uncertainties from the CKM matrix elements and the factor $m_b^2$. For $B \to K^* l^+ l^-$, the FBAs, polarizations, and BR have been measured in different kinematic bins \[2\]. The other relevant experimental data collected in Tab. III are from Refs. \[49,50\].

We will adopt a least-$\chi^2$ fitting method to constrain the free parameters, in which the $\chi^2$ is defined by
\[ \chi^2 = \frac{(B_i^{\text{the}} - B_i^{\text{exp}})^2}{(B_i^{\text{err}})^2}, \]  
where $B_i$ denotes one generic quantity among the physical observables. The $B_i^{\text{the}}, B_i^{\text{exp}}$ and $B_i^{\text{err}}$ denotes the theoretical prediction, the central value and $1-\sigma$ error of experimental data, respectively. The total $\chi^2$ is obtained by adding the individual ones. It is necessary to point out that although the errors in experiment may correlate, for instance the measurement of $B$, $f_L$ and $A_{FB}$ proceed at the same time in the fitting of angular distributions \[2\], we have not taken into account their correlation in our theoretical results.

As shown in the previous section, these two NP models have the similarity that only $C_{9,10}$ are modified. One difference lies in the coupling with the leptons, the newly introduced down-type quark in VQM will not modify the
lepton sector and the coupling with leptons is SM-like; on the contrary, one new gauge boson is added in the $Z'$ model and its coupling with leptons are completely unknown.

Embedded in the VQM, the two parameters, real part and imaginary part of $\lambda_{sb}$, are found as

$$\text{Re}\lambda_{sb} = (0.07 \pm 0.04) \times 10^{-3}, \quad \text{Im}\lambda_{sb} = (0.09 \pm 0.23) \times 10^{-3},$$ (53)
from which we obtain $|\lambda_{ab}| < 0.3 \times 10^{-3}$ but the phase is less constrained again. The corresponding constraint on Wilson coefficients are

$$|\Delta C_9| = |C_9 - C_9^{SM}| < 0.2, \quad |\Delta C_{10}| = |C_{10} - C_{10}^{SM}| < 2.8.$$  

(54)

Our result of $\chi^2/d.o.f.$ in the fitting method is $49.3/(23 - 2)$. 

FIG. 6: Similar as Fig. 5 but for $B_s \to f_2 \mu^+ \mu^-$
FIG. 7: Spin amplitudes and transversity asymmetries of $B \to K^*_2 \mu^+ \mu^-$

FIG. 8: Similar with Fig. 7 but for $B_s \to f_2 \mu^+ \mu^-$
TABLE II: Experimental data used in the least $\chi^2$-fitting method

| $b \to d\bar{\nu}$ [50] | $b \to s^{\pm}l^\mp$ [49] | $B^0 \to K^*l^+l^-$ [49] |
|-------------------------|-------------------|------------------|
| $(10.58 \pm 0.15) \times 10^{-2}$ | $(3.66^{+0.76}_{-0.77}) \times 10^{-6}$ | $(1.09^{+0.12}_{-0.11}) \times 10^{-6}$ |
| $q^2$(GeV$^2$) | $B(10^{-7})$ | $F_L$ | $-A_{FB}$ $^a$ |
| [0, 2] | 1.46 ± 0.41 | 0.29 ± 0.21 | 0.47 ± 0.32 |
| [2, 4.3] | 0.86 ± 0.32 | 0.71 ± 0.25 | 0.11 ± 0.37 |
| [4.3, 8.68] | 1.37 ± 0.61 | 0.64 ± 0.25 | 0.45 ± 0.26 |
| [10.09, 12.86] | 2.24 ± 0.48 | 0.17 ± 0.17 | 0.43 ± 0.20 |
| [14.18, 16] | 1.05 ± 0.30 | −0.15 ± 0.28 | 0.70 ± 0.24 |
| > 16 | 2.04 ± 0.31 | 0.12 ± 0.15 | 0.66 ± 0.16 |
| [1, 6] | 1.49 ± 0.47 | 0.67 ± 0.24 | 0.26 ± 0.31 |

$^a$The different convention on $\theta_i$ introduces a minus sign to the forward-backward asymmetry.

Turning to family nonuniversal $Z'$ model in which the coupling between $Z'$ and a lepton pair is unknown, the two Wilson coefficients, $C_9$ and $C_{10}$, can be chosen as independent parameters. Assuming $\Delta C_9$ and $\Delta C_{10}$ as real, we find

$$\Delta C_9 = 0.88 \pm 0.75, \quad \Delta C_{10} = 0.01 \pm 0.69,$$

with $\chi^2/d.o.f. = 48.4/(23 - 2)$. Removal of the above assumption leads to

$$\Delta C_9 = -0.81 \pm 1.22 + (3.05 \pm 0.92)i, \quad \Delta C_{10} = 1.00 \pm 1.28 + (-3.16 \pm 0.94)i$$

with $\chi^2/d.o.f. = 45.6/(23 - 4)$. If the $\mu$-lepton mass is neglected, the imaginary part of $C_{10}$ will not appear in the expressions for the differential decay widths and the polarizations. Moreover, for the forward-backward asymmetry as shown in Eq. (22), the imaginary part of $C_{10}$ contributes in the combination Re[$C_9 C_{10}$], thus the inclusion of Im[$C_{10}$] will have little effect on the $\chi^2$.

Combding the above results, we can see that the NP contributions in both cases satisfy

$$|\Delta C_9| < 3, \quad |\Delta C_{10}| < 3.$$

To illustrate, we choose $\Delta C_9 = 3e^{i\pi/4,i\pi/4}$ and $\Delta C_{10} = 3e^{i\pi/4,i\pi/4}$ as the reference points and give the plots of branching ratios, FBAs and the polarizations in Fig. 9. The black (solid) line denotes the SM result, while the dashed (blue) and thick (red) lines correspond to the modification of $C_9$. The dot-dashed (green) and dotted lines are obtained by modifying $C_{10}$. From the figure for $A_{FB}$, we can see that the zero-crossing point $s_0$ can be sizably changed, which can be tested on the future collider or can be further constrained.

One last process to explore is $B_s \to \mu^+\mu^-$, of which the branching fraction is

$$\mathcal{B}(B_s \to \mu^+\mu^-) = \tau_{B_s} \frac{G_F^2 \alpha_s^2}{16\pi^3} |V_{ts}^* V_{tb}|^2 m_{B_s} f_{B_s}^2 m_{\mu}^2 |C_{10}|^2 \left( 1 - \frac{4m_{\mu}^2}{m_{B_s}^2} \right)^{1/2}.$$

Using the same inputs as those in our computation of $B \to K^*_2 l^+ l^-$, we have

$$\mathcal{B}(B_s \to \mu^+\mu^-) = 3.50 \times 10^{-9} \left( \frac{f_{B_s}}{230\text{MeV}} \right)^2 \left( \frac{|C_{10}|}{4.67} \right)^2.$$

Even if $C_{10}$ is enhanced by a factor of 2, the above result is still consistent with the recent measurement $[51]$

$$\mathcal{B}(B_s \to \mu^+\mu^-) < 5.1 \times 10^{-8}.$$

VI. SUMMARY

In this work we have explored $B \to K^*_2(\to K\pi) l^+ l^-$ (with $l = e, \mu, \tau$) decays and a similar mode $B_s \to f_2^*(1525)(\to K^+ K^-) l^+ l^-$ in the standard model and two new physics scenarios: vector-like quark model and family non-universal
FIG. 9: The impacts of the NP contributions on differential branching ratios (in unit of $10^{-7}$), polarization fractions and normalized forward-backward asymmetry of $B \to K^*_2l^+l^-$. 

$Z'$ model. Besides branching ratios, forward-backward asymmetries and transversity amplitudes, we have also derived the differential angular distributions of this decay chain. The sizable production rates lead to a promising prospective to observe this channel on the future experiment.

Using the experimental data of the inclusive $b \to sl^+l^-$ and $B \to K^*l^+l^-$, we have updated the constraints on effective Wilson coefficients and/or free parameters in these two new physics scenarios. In the VQM, we find that the constraint on the coupling constant is improved by a factor of 3 compared with our previous work. Their impact on $B \to K^*_2l^+l^-$ is elaborated and in particular the zero-crossing point for the forward-backward asymmetry in these NP scenarios can sizably deviate from the SM. These results will be tested on the future hadron collider.

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The effective Hamiltonian governing $b \rightarrow s\ell^+\ell^-$ is given by

$$H_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu),$$

where $V_{tb} = 0.999176$ and $V_{ts} = -0.03972$ are the CKM matrix elements and $C_i(\mu)$ are Wilson coefficients for the effective operators $O_i$. In this paper, we will adopt the Wilson coefficients up to the leading logarithmic accuracy \cite{48}, and their values in SM are listed in Tab. III. Since the NP scenarios considered in the present paper would not introduce any new operator, the SM operators will form a complete basis for our analysis

\begin{table}[h]
\centering
\begin{tabular}{cccccccc}
\hline
$C_1$ & $C_2$ & $C_3$ & $C_4$ & $C_5$ & $C_6$ & $C_7^{\text{eff}}$ & $C_9$ & $C_{10}$ \\
\hline
1.107 & -0.248 & -0.011 & -0.026 & -0.007 & -0.031 & -0.313 & 4.344 & -4.669 \\
\hline
\end{tabular}
\end{table}

**Appendix A: Effective Hamiltonian**

The left-handed and right-handed operators are (\bar{\psi}_b \gamma_\mu b_s \bar{\psi}_s \gamma_\mu b_s). In this paper, we will adopt the Wilson coefficients up to the leading logarithmic accuracy \cite{48}, and their values in SM are listed in Tab. III. Since the NP scenarios considered in the present paper would not introduce any new operator, the SM operators will form a complete basis for our analysis

$$O_1 = (\bar{s}_\alpha c_\beta)_{V-A}(\bar{c}_\beta b_\alpha)_{V-A}, \quad O_2 = (\bar{s}_\alpha c_\beta)_{V-A}(\bar{c}_\beta b_\alpha)_{V-A},$$

$$O_3 = (\bar{s}_\alpha c_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V-A}, \quad O_4 = (\bar{s}_\alpha c_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V-A},$$

$$O_5 = (\bar{s}_\alpha c_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V-A}, \quad O_6 = (\bar{s}_\alpha c_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V-A},$$

$$O_7 = \frac{e m_b}{8 \pi^2} \bar{s}_\alpha \mu^\nu (1 + \gamma_5) \bar{b}_F \mu \nu + \frac{e m_b}{8 \pi^2} \bar{s}_\alpha \mu^\nu (1 - \gamma_5) \bar{b}_F \mu \nu,$$

$$O_9 = \frac{\alpha_{\text{em}}}{2 \pi} (\bar{f}_{\gamma} \mu l) (\bar{s} \gamma^\mu (1 - \gamma_5) b), \quad O_{10} = \frac{\alpha_{\text{em}}}{2 \pi} (\bar{f}_{\gamma} \mu l) (\bar{s} \gamma^\mu (1 - \gamma_5) b).$$

The left-handed and right-handed operators are (\bar{q}_i q_2)_{V-A}(\bar{q}_j q_4)_{V+A} \equiv (\bar{q}_i \gamma^\mu (1 - \gamma_5) q_2)_{V-A} (\bar{q}_j \gamma^\mu (1 + \gamma_5) q_4)_{V+A}$. $m_b = 4.8$ GeV and $m_s = 0.95$ GeV are $b$ and $s$ quark masses in the MS scheme and $\alpha_{\text{em}} = 1/137$ is fine structure constant. The double Cabibbo suppressed terms, proportional to $V_{ub} V_{us}^*$, have been neglected.

At the one-loop level accuracy, the matrix element of $b \rightarrow s\ell^+\ell^-$ transition receives loop contributions from $O_1 - O_6$. Since the factorizable loop terms can be incorporated into the Wilson coefficients $C_7$ and $C_9$, it is convenient to define combinations $C_7^{\text{eff}}$ and $C_9^{\text{eff}}$

$$C_7^{\text{eff}} = C_7 - C_5/3 - C_6,$$

$$C_9^{\text{eff}}(q^2) = C_9(\mu) + h(\bar{m}_c, \bar{s}) C_0 - \frac{1}{2} h(\bar{0}, \bar{s}) (4 C_3 + 4 C_4 + 3 C_5 + C_6),$$

with $\bar{s} = q^2/m_b^2$, $C_0 = C_1 + 3 C_2 + 3 C_3 + C_4 + 3 C_5 + C_6$, and $\bar{m}_c = m_c/m_b$. The auxiliary functions used above are

$$h(z, \bar{s}) = -\frac{8}{9} \ln \frac{m_b}{\mu} - \frac{8}{9} \ln z + \frac{8}{27} + \frac{4}{9} - \frac{2}{9} (2 + x) |1 - x|^{1/2} \left\{ \ln \frac{\sqrt{1+z} + 1}{\sqrt{1+z} - 1} - i \pi \right\} \begin{cases} 2 \arctan \frac{1}{\sqrt{x-1}} & \text{for} \ x \equiv \frac{4 x^2}{z} < 1 \\ 1 \end{cases},$$

$$h(0, \bar{s}) = -\frac{8}{9} \ln \frac{m_b}{\mu} - \frac{4}{9} \ln \bar{s} + \frac{8}{27} + \frac{4}{9} - i \pi.$$

In the following, we shall also drop the superscripts for $C_9^{\text{eff}}$ and $C_7^{\text{eff}}$ for convenience.

On the hadron level resonant states, such as vector charmonia generated from the $b \rightarrow c\bar{c}s$, may annihilate into a lepton pair. Therefore they will also contribute in a long distance manner \cite{53 55}. But these contributions can be subtracted with a kinematic cutoff in experiment. Moreover our following analysis of differential distributions will be mainly dedicated to the region of $1 \text{GeV}^2 < q^2 < 6 \text{GeV}^2$, also excluding contributions from the charmonia.
Appendix B: Helicity amplitudes

Within a graphic picture $B \rightarrow K_2^* (\rightarrow K\pi) l^+ l^-$ proceeds via three steps: $B$ meson first decays into an off-shell strange meson plus a pair of leptons; the $K_2^*$ meson propagates followed by its strong decay into $K\pi$. To evaluate the decay width of multibody decays, we shall adopt the helicity amplitude which mainly uses

$$g_{\mu \nu} = - \sum_{\lambda} \epsilon_{\mu}(\lambda) \epsilon^{\nu*}(\lambda) + \frac{g_{\mu \nu}}{q^2}, \quad (B1)$$

$\epsilon$ is the polarization vector with the momentum $q$ and $\lambda$ denotes the three kinds of polarizations. The last term can be formally identified as a timelike polarization $\epsilon_{\mu}(t) = \frac{q_{\mu}}{\sqrt{q^2}}$, and thus the metric tensor $g_{\mu \nu}$ can be then understood as summations of the four polarizations. For the purpose of illustration we will first evaluate the decay amplitude of $B \rightarrow K_2^* l^+ l^-$. In the SM, the lepton pair in the final state is produced via an off-shell photon, a $Z$ boson or some hadronic vector mesons. These states may have different couplings but they share many commonalities: the Lorentz structure for the vertex of the lepton pair is either $V - A$ or $V + A$ or some combination of them. Therefore the decay amplitudes of $\bar{B} \rightarrow K_2^* l^+ l^-$ can be rewritten as

$$A(\bar{B} \rightarrow K_2^* l^+ l^-) = \mathcal{L}^\mu(L) \mathcal{H}_\nu(L) + \mathcal{L}^\mu(R) \mathcal{H}_\nu(R), \quad (B2)$$

in which $\mathcal{L}_\mu(L), \mathcal{L}_\mu(R)$ are the lepton pair spinor products:

$$\mathcal{L}_\mu(L) = \bar{l} \gamma_\mu (1 - \gamma_5) l, \quad \mathcal{L}_\mu(R) = \bar{l} \gamma_\mu (1 + \gamma_5) l, \quad (B3)$$

while $\mathcal{H}$ incorporates the remaining $B \rightarrow K_2^*$ part. In the case of massless leptons, left-handed and right-handed sectors decouple, which will greatly simplify the analysis. The identity in Eq. (B1) results in a factorization of decay amplitudes

$$A(\bar{B} \rightarrow K_2^* l^+ l^-) = \mathcal{L}_\mu(L) \mathcal{H}_\nu(L) g^{\mu \nu} + \mathcal{L}_\mu(R) \mathcal{H}_\nu(R) g^{\mu \nu}$$

$$= - \sum_{\lambda} \mathcal{L}_{L\lambda} \mathcal{H}_{L\lambda} - \sum_{\lambda} \mathcal{L}_{R\lambda} \mathcal{H}_{R\lambda}, \quad (B4)$$

where $q^{\mu}$ is the momentum of the lepton pair and $\mathcal{L}_{L\lambda} = \mathcal{L}^\mu(L) \epsilon^{\nu*}_{\lambda}(\lambda)$ and $\mathcal{L}_{R\lambda} = \mathcal{L}^\mu(R) \epsilon^{\nu*}_{\lambda}(\lambda)$ denote Lorentz invariant amplitudes for the lepton part. It is also similar for the Lorentz invariant hadronic amplitudes: $\mathcal{H}_{L\lambda} = \mathcal{H}^\mu(L) \epsilon^{\nu*}_{\lambda}(\lambda)$ and $\mathcal{H}_{R\lambda} = \mathcal{H}^\mu(R) \epsilon^{\nu*}_{\lambda}(\lambda)$. The timelike polarization gives vanishing contributions in the case of $m_l = 0$ for $l = e, \mu$; using equation of motion, this term is proportional to the lepton mass.

An advantage of the helicity amplitudes is that both hadronic amplitudes and leptonic amplitudes are Lorentz invariant. Such a good property allows to choose different frames in the evaluation. For instance leptonic amplitudes are evaluated in the lepton pair central mass frame, while hadronic $B$ decay amplitudes are directly obtained in the $B$ rest frame. Since $K_2^*$ and $K^*$ have several important similarities, $B \rightarrow K_2^* l^+ l^-$ differential decay widths can be simply obtained from the ones of $B \rightarrow K^* l^+ l^-$ in a comparative manner.

- Longitudinal and transverse $B$ decay amplitudes are obtained by multiplying the factor $\sqrt{X} \sqrt{6m_B m_{K_2^*}}$ and $\sqrt{X} \sqrt{6m_B m_{K_2^*}}$ respectively. The function $\lambda$ is the magnitude of the $K_2^*$ momentum in $B$ meson rest frame:

$$\lambda \equiv \lambda(m_B^2, m_{K_2^*}^2, q^2) = 2m_B |p_{K_2^*}|, \quad \text{and} \quad \lambda(a^2, b^2, c^2) = (a^2 - b^2 - c^2)^2 - 4b^2c^2. \quad \text{This replacement is an output of the fact that the polarization vector } \epsilon \text{ is replaced by } \epsilon_T \text{ in the form factor definitions. Explicitly, these}$$
hadronic amplitudes are
\[ H_{L0} = N \frac{\sqrt{\lambda}}{\sqrt{8m_Bm_{K^*_2}}} \frac{1}{2m_{K^*_2}\sqrt{q^2}} \left( (C_9 - C_{10})((m_B^2 - m_{K^*_2}^2 - q^2)(m_B + m_{K^*_2})A_1 - \frac{\lambda}{m_B + m_{K^*_2}}A_2) + 2m_b(C_7L - C_7R)((m_B^2 + 3m_{K^*_2}^2 - q^2)T_2 - \frac{\lambda}{m_B + m_{K^*_2}}T_3) \right), \]
\[ H_{L\pm} = N \frac{\sqrt{\lambda}}{\sqrt{6m_Bm_{K^*_2}}} \left( (C_9 - C_{10})((m_B + m_{K^*_2})A_1 \mp \frac{\sqrt{\lambda}}{m_B + m_{K^*_2}}V) - \frac{2m_b(C_7L + C_7R)}{q^2}(\pm\sqrt{\lambda}T_1) + \frac{2m_b(C_7L - C_7R)}{q^2}(m_B^2 - m_{K^*_2}^2)T_2 \right), \]
\[ H_{Li} = N \frac{\sqrt{\lambda}}{\sqrt{8m_Bm_T}}(C_9 - C_{10})\frac{\sqrt{\lambda}}{\sqrt{q^2}}A_0, \]
\[ H_{Ri} = H_{Li}|_{C_{10} \rightarrow -C_{10}} \]
with \( N = -i \frac{G_F}{\sqrt{2}} \frac{\alpha_{em}}{\pi} V_{tb} V_{ts}^* \).

- In the propagation of the intermediate strange meson, the width effect of \( K_2^* \) could be more important since \( \Gamma_{K_2^*} \approx 100\text{MeV} \gg \Gamma_{K^*} \approx 50\text{MeV} \). Nevertheless, since the \( K_2^* \) width is only larger than that of \( K^* \) by a factor of 2, the narrow-width approximation, which has been well used in the case of \( K^* \), might also work for \( K_2^* \). In this sense, there is no difference except that the \( B(K^* \rightarrow K\pi) \) is replaced by \( B(K_2^* \rightarrow K\pi) \).

- Incorporation of the \( K_2^* \rightarrow K\pi \) decay gives the complete results for differential decay distribution of \( B \rightarrow K_2^*(\rightarrow K\pi)l^+l^- \). Angular distributions of \( K_2^* \) and \( K^* \) strong decays are described by spherical harmonic functions: \( Y^2_1(\theta, \phi) \) for \( K^* \) and \( Y^2_3(\theta, \phi) \) for \( K_2^* \). In particular we find the relations
  \[ \sqrt{\frac{3}{4\pi}}\cos(\theta_K) \equiv C(K^*) \rightarrow \sqrt{\frac{5}{16\pi}}(3\cos^2\theta_K - 1) \equiv C(K_2^*) \],
  \[ \sqrt{\frac{3}{8\pi}}\sin(\theta_K) \equiv S(K^*) \rightarrow \sqrt{\frac{15}{32\pi}}\sin(2\theta_K) \equiv S(K_2^*). \] (B6)

Our formulas for branching fractions and forward-backward asymmetries can be shown compatible with the ones in Ref. [25] through the following relations
\[ A_{L0} = N_{K^*_2}\alpha_L m_B^3 \frac{1}{2m_{K^*_2}\sqrt{q^2}} \left( - (1 - \hat{m}_{K^*_2}^2 - q^2)F + \hat{\lambda}G + (1 - \hat{m}_{K^*_2}^2 - q^2)B - \hat{\lambda}C \right), \]
\[ A_{L\pm} = -\sqrt{2\lambda}N_{K^*_2}\beta_T \frac{1}{2m_B}(A - E), \]
\[ A_{Li} = \sqrt{2\lambda}N_{K^*_2}\beta_T \frac{1}{2m_B}(B - F), \]
\[ A_t = N_{K^*_2}\alpha_L \sqrt{\frac{\lambda}{m_{K^*_2}}} \sqrt{q^2} [F - (1 - \hat{m}_{K^*_2}^2)G - \hat{q}^2H], \]
where the coefficients \( A, B, E, F, G, H \) are defined in Eqs. (49,50,53, 54) in Ref. [25] but the coefficient \( C \) in Eq. (51) contains a typo and should be read as
\[ C = \frac{1}{1 - \hat{m}_{K^*_2}^2} \left[ (1 - \hat{m}_{K^*_2}^2)e_{\gamma}(s)A_{K^*_2}(s) + 2\hat{m}_b e_{\tau}(T_{3\tau}K^*_2(s) + \frac{1 - \hat{m}_{K^*_2}^2}{s}T_{2\tau}K^*_2(s)) \right]. \] (B11)

The dimensionless constants are given as \( \hat{\lambda} = \lambda/m_B^4, \hat{m}_{K^*_2} = m_{K^*_2}/m_B, \hat{m}_b = m_b/m_B \) and \( \hat{q}^2 = q^2/m_B^2 \). \( \alpha_L = \sqrt{2/3} \) and \( \beta_T = 1/\sqrt{2} \).

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