THE DIFFICULTY IN USING ORPHAN AFTERGLOWS TO MEASURE GAMMA-RAY BURST BEAMING

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ABSTRACT

If gamma-ray burst (GRB) emission is strongly collimated, then GRBs occur throughout the universe at a rate much higher than is detected. Since the emission from the optical afterglow is thought to be more isotropic than the gamma-ray emission, it has been hypothesized that a search for orphan afterglows (those without the triggering GRB) would allow strong constraints to be placed on the degree of GRB collimation. In this paper we establish that, within the context of leading models of GRB jet evolution, measurement of the GRB beaming angle $\theta_{\text{jet}}$ using optical orphan searches is extremely difficult, perhaps impossible in practice. We show that this is because, in the leading model of GRB jets, the effective afterglow beaming angle scales with the jet angle, $\Omega_{\text{opt}} \propto \Omega_{\text{jet}}$ for small angles, and so the ratio of detected orphan afterglows to GRBs is independent of the jet opening angle. Thus, the number of expected afterglow detections is the same for moderate jet angles ($\sim 20^\circ$) as for arbitrarily small jet angles ($\lesssim 0.1^\circ$). For nearly isotropic GRB geometry, or for radio afterglow searches in which the jet has become nonrelativistic, the ratio of afterglows to GRBs may give information on collimation. However, using a simple model we estimate the expected number of orphan detections in current supernova surveys and find this number to be less than 1 for all jet opening angles. Even for future supernova surveys, the small detection rate and lack of dependence on collimation angle appear to ruin the prospects of determining GRB beaming by this method. Radio searches may provide the best hope to find the missing orphans.

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1. INTRODUCTION

Gamma-ray bursts (GRBs) are observed at a rate of about one per day and are in some cases accompanied by optical afterglows that are useful in determining properties of the burst such as its redshift. Current models that attempt to explain the enormous apparent energy release, the multiband spectra, and the temporal behavior of afterglow light curves typically invoke a highly collimated relativistic jet, which is beamed toward the Earth. The light curve from a highly collimated jet is expected initially to decay as a low power of $t$ (e.g., flux $\sim t^{-1.1}$) and then break to a steeper slope (e.g., flux $\sim t^{-2.4}$) at late times. A number of GRB afterglows show evidence for just this behavior (Sari, Piran, & Halpern 1999). This has led Frail et al. (2001) to conclude that the GRB emission is collimated into opening angles between $3^\circ$ and $25^\circ$ with $\sim 4^\circ$ as an average.

One important feature of the jet model of GRBs is that the vast majority of GRBs are beamed in directions away from the Earth and therefore not observed in gamma-rays. However, the afterglow emission, occurring at much later times and at longer wavelengths, should be beamed into a larger fraction of the sky because of the decay of the jet Lorentz factor with time. There should therefore be afterglows that are observable when the associated GRB is not (Rhoads 1997; Perna & Loeb 1998; Mészáros, Rees, & Wijers 1999; Rhoads 2001a). As first suggested by Rhoads (1997), the ratio of the "orphan" afterglow rate to the GRB rate might allow measurement of the GRB collimation angle $\theta_{\text{jet}}$ via the equation $\frac{N_{\text{opt}}}{N_{\text{jet}}} = \frac{N_{\text{grb}}}{N_{\text{opt}}} \Omega_{\text{opt}}/\Omega_{\text{jet}}$, where $\Omega_{\text{opt}}$ is the solid angle subtended by the jet, $\Omega_{\text{jet}}$ is the angle into which the afterglow is beamed (these angles are illustrated in Fig. 1), and $N_{\text{opt}}$ is the efficiency corrected event rate for optical afterglows. Thus, by counting the rate at which orphan afterglows are detected, we could measure the average GRB jet angle and independently check other methods (Frail et al. 2001) of measuring GRB collimation.

Rhoads' method has already been applied several times to extant searches for Type Ia supernovae (SNe) and other optical transients. Rhoads argued that the lack of afterglow detections by the Supernova Cosmology Project and the High-z Supernova Search roughly limited $\Omega_{\text{opt}}/\Omega_{\text{jet}} \lesssim 100$, on the basis of a lack of detections over 2 yr of exposure. The continued nondetection of orphans since that time has been used by several groups to set stronger collimation limits. For example, Rees (1999) found $\Omega_{\text{opt}}/\Omega_{\text{jet}} \lesssim 20$, and Rhoads (2001b) quotes a transient search by Schaefer et al. to claim $\Omega_{\text{opt}}/\Omega_{\text{jet}} \lesssim 100$. If optical afterglow emission is only mildly beamed (e.g., $\Omega_{\text{opt}} \approx 1$), then this is marginally inconsistent with the collimation factors ($\sim 500$) claimed by Frail et al. (2001) and could render untenable most solar mass progenitor models, given the inefficiency of converting rest mass into gamma-ray emission (Kumar 1999).

In this paper, however, we argue that current SN searches place no constraints upon GRB collimation. One reason for this is that present surveys are too shallow, narrow, and infrequent to detect significant off-axis emission. Using a simplified model of GRB afterglow emission and survey detection efficiency, we show that present SN surveys are generically expected to detect (far) less than one orphan afterglow. We further show that a deeper reason for the impotence of present surveys is the fact that, within the context of the leading GRB afterglow model (e.g., Sari et al. 1999; Rhoads 1999), the effective afterglow beaming angle\textsuperscript{1}

\textsuperscript{1} Note that as the jet Lorentz factor $\gamma$ decays with time, the beaming angle at that time, $\gamma^{-1}$, steadily increases. By "effective" beaming angle we mean the solid angle subtended by the jet emission at the time when the flux falls below the detection threshold.
scales with the GRB jet angle, \( \theta_{\text{jet}} \). Because of this scaling, the ratio \( \Omega_{\text{opt}}/\Omega_{\text{jet}} \) is unchanged even as the GRB jet angle varies from moderate angles (\( \theta_{\text{jet}} \approx 20^\circ \)) to arbitrarily small angles (\( \theta_{\text{jet}} \approx 0.1^\circ \)), and so the ratio of detected orphan afterglows to GRBs cannot be used to measure \( \theta_{\text{jet}} \).

We derive this scaling below using a simplified model of GRB jet emission; however, we can understand this result in more general terms. From relativistic kinematics, we know that the jet Lorentz factor \( \gamma \), beaming angle \( \theta \), and observer time \( t \) are related by simple power laws, and as mentioned above the observed flux \( f \) is a simple power law in time. Thus, the beaming angle is a (scale-free) power law in \( \gamma \), \( t \), and \( f \). In the limit \( \gamma \gg 1 \) the only angular scale in the problem becomes \( \theta_{\text{jet}} \), so we expect the effective beaming angle to scale as the jet angle \( \theta_{\text{opt}} \propto \theta_{\text{jet}} \). This should break down in the nonrelativistic limit \( \gamma \sim 1 \).

This scaling of \( \theta_{\text{opt}} \) with \( \theta_{\text{jet}} \) severely hampers the ability of optical orphan searches to measure or constrain GRB collimation. In fact, we argue in this paper that measurement of the GRB jet angle may be impossible using either present or future optical SN surveys. The plan of the paper is as follows. In § 2 we describe our simple model of emission from a GRB afterglow. In § 3 we estimate the afterglow detection rate using this simple model. In § 4 we discuss the results of our calculations, and in § 5 we give our conclusions, along with some discussion and caveats.

### 2. A SIMPLE MODEL OF OPTICAL AFTERGLOW EMISSION

GRB-alerted afterglows ("on-axis afterglows") are now well studied. In most cases, their time dependence can be characterized as a broken power law, \( f \sim t^{-x} \), with a power-law break from \( x \approx 1.1 \) to 2.4 occurring at late times when the relativistic beaming angle \( \gamma^{-1} \) exceeds the jet opening angle \( \theta_{\text{jet}} \). Here \( \gamma \) is the bulk Lorentz factor of the relativistic jet. This break occurs at a time (Sari et al. 1999)

\[
    t_{\text{break}} \approx 6.2(E_{\text{b}}/n_1)^{1/3}(\theta_{\text{jet}}/0.1)^{8/3} \text{ hr} ,
\]

where \( E_{\text{b}} \) is the inferred energy of the ejecta assuming isotropic expansion, in units of \( 10^{52} \text{ ergs} \), and \( n_1 \) is the density of the surrounding interstellar medium in \( \text{cm}^{-3} \). GRB optical emission has a wide range in apparent brightness, ranging from 9th magnitude for the prompt optical emission associated with GRB 990123, down to the limiting magnitude \( R \sim 23–24 \) of follow-up searches for afterglows. Fynbo et al. (2001) have argued that most (~70%) afterglows are even fainter than this and have therefore eluded detection.

Observers of orphan afterglows, on the other hand, would see off-axis light curves, which differ from on-axis light curves. The behavior of off-axis emission is governed by relativistic effects, as well as the lateral spreading and internal structure of the jet. We will first consider the effects of relativistic kinematics and then consider lateral spreading and jet internal structure. Since the internal structure and time evolution of GRB jets is quite uncertain, we will consider throughout two limiting cases, which we hope will bracket reality.

As a simple model, consider a radiating plasma moving relativistically along the jet axis, with bulk Lorentz factor \( \gamma \) relative to us, and radiating isotropically in its own frame. The measured power, as a function of angle \( \theta \) with respect to the jet axis in our frame, is (Rybicki & Lightman 1979)

\[
    \frac{dP}{d\Omega} = \frac{F}{\gamma^4(1 - \beta \cos \theta)^3} ,
\]

where \( F \) is the (isotropic) power per unit solid angle in the plasma frame, and \( \beta = (1 - \gamma^{-2})^{1/2} \) as usual. Let us assume that the Lorentz factor \( \gamma \) and rest-frame flux \( F \) evolve with time to give an observed on-axis (\( \theta = 0 \)) light curve \( f(t) \) matching the light curves observed for optical GRB afterglows, i.e., \( f(t) \sim t^{-x} \). For early times before the break \( \alpha \approx 1.1 \), while at late times after the break \( \alpha \approx 2.4 \). Since \( \gamma \sim t^{-\alpha} \), with \( \mu = \frac{\alpha}{2} \) before the break and \( \mu = \frac{\alpha}{2} \) after the break (Sari et al. 1999), this means \( f \propto \gamma^{\mu} \). This then requires \( F \propto \gamma^{\mu} (dP/d\Omega(\theta = 0))^{-1} \). With our assumption of rest-frame isotropy, we obtain the flux seen by off-axis
observers as a function of angle $\theta$ and Lorentz factor $\gamma$,  
\[ f(t) = f_{\text{break}} \left( \frac{t}{t_{\text{break}}} \right)^{-\alpha} \left( \frac{1 - \beta}{1 - \beta \cos \theta} \right)^m. \]  
(3)

Here $f_{\text{break}}$ is the flux seen by an on-axis observer at the break time $t_{\text{break}}$ and we have generalized the exponent 4 to the parameter $m$ for later use. This function with $m = 4$ is plotted as a function of $t$ for several values of $\theta$ in Figure 2. Several features follow from equation (3) and Figure 2. First, the light curve seen at angle $\theta$ peaks at a time $t_{\text{peak}}$, which can be estimated by setting $\gamma \approx \theta^{-1}$. Next, the flux rises to the peak roughly as $t^{2m-2}$ and decays after peak as $t^{-\alpha}$. Also, the flux is nearly independent of angle for $\theta \ll \gamma^{-1}$; that is, the various light curves all match together at late times when the viewing angle becomes small relative to the beaming angle. Finally, when $\gamma < \theta^{-1}$, an off-axis viewer at $\theta > \theta_{\text{jet}}$ should see comparable flux to an on-axis observer inside $\theta_{\text{jet}}$, and the off-axis afterglow light curve should decay with roughly the same slope as the on-axis afterglow.

Note that off-axis ($\theta > \theta_{\text{jet}}$) afterglows do not peak until after the on-axis power-law break, which typically occurs hours to days after the GRB (Frail et al. 2001). This implies that orphan afterglows, even at peak brightness, should be far fainter than their GRB-triggered siblings and thus should be much more difficult to detect. We can roughly estimate how much fainter as follows. After the break, the afterglow flux decays as $t^{-2.4}$, while the Lorentz factor decays as $\gamma \propto t^{-1.2}$ (Sari et al. 1999), so the flux scales as $\gamma^{0.8}$. For a reasonable estimate of the jet angle, we would like to detect orphans at large viewing angles, say $\theta \approx 0.2$–0.3. This means detecting emission at times when the Lorentz factor has decayed to $\gamma \approx 3$–4. Taking $\gamma \approx (4')^{-1} \approx 15$ at the break, we see that the flux is 7 or 8 magnitudes fainter by the time $\gamma \approx 3$–4 than at the break.

This immediately shows why current SN searches are not expected to detect orphan afterglows. Typical limiting magnitudes for such searches are near $R \approx 23$, which Fynbo et al. (2001) claim is too shallow to detect most on-axis afterglows, let alone far fainter off-axis orphan afterglows. This implies that detectable orphan afterglows are mostly contained in a cone of opening angle not much larger than $\theta_{\text{jet}}$, so we do not expect many more orphan afterglows to appear in SN surveys than on-axis afterglows. The same reasoning applies for non-SN survey searches for orphans. For example, the ROTSE III experiment$^2$ with a 4 deg$^2$ field of view and sensitivity down to 19th magnitude would likely detect only on-axis afterglows. In addition, we show in the next section that the relatively infrequent monitoring characteristic of current SN surveys (e.g., $t_{\text{mon}} = 21$ days) causes most bright orphans to be missed.

We shall quantify these broad statements in §3 with a calculation of the detection rates expected in surveys, using this simple model of off-axis afterglow emission. Before proceeding, we pause to note that we have, so far, neglected the effects of finite jet width and jet spreading. Both of these should lead to greater off-axis emission than indicated by equation (3). To obtain an upper limit to the effects of the uncertain jet evolution (and thus bracket the range of possible fluxes seen by off-axis observers), we consider the following simple picture. The effect of finite jet width on the off-axis light curve is to point some of the emitting material more closely to the observer’s line of sight and to point other emitting material farther from the observer’s line of sight. A proper calculation of this effect would involve specifying the emission profile in the jet and integrating over the jet face (Woods & Loeb 1999), but in the spirit of setting upper bounds on the detection rate, we can get a sense of the effect by simpler means. Consider the emission from the part of the jet face most closely pointing to a fiducial off-axis observer at $\theta$. This material is moving at an angle $\theta - \theta_{\text{jet}}$ from the observer’s line of sight, so we can approximate the flux from this part of the jet face by replacing $\theta$ with $\theta - \theta_{\text{jet}}$ in equation (3). We immediately see that the asymptotic scalings remain unchanged (i.e., $t^{2m-2}$ and $t^{-\alpha}$) but that the peak time changes to $\gamma^{-1} \approx \theta - \theta_{\text{jet}}$. This makes sense because we roughly expect the flux to beam into an angle $\sim \theta_{\text{jet}} + \gamma^{-1}$. This of course overestimates the total flux and the true visible angle, because emission from the rest of the jet face is less blueshifted and therefore less bright. Also note that the emission from other regions of the jet face should peak at later times so that we expect a broader light-curve peak for off-axis viewers than on-axis viewers. This broadening of the peak is seen in numerical simulations of relativistic jet hydrodynamics and emission (Granot et al. 2001).

An upper limit on the effects of lateral spreading of the jet on the off-axis light curve may be obtained by noting that if in the jet comoving rest frame the emitting material is expanding laterally at speed $v$, then the apparent angular size of the jet face becomes $\theta_{\text{jet}} = \theta_{\text{jet,0}} + (v/c)\gamma^{-1}$. Then our

\[ f(t) = f_{\text{break}} \left( \frac{t}{t_{\text{break}}} \right)^{-\alpha} \left( \frac{1 - \beta}{1 - \beta \cos \theta} \right)^m. \]  

Fig. 2.—Apparent magnitude of a GRB afterglow as a function of time for various viewing angles as given by eq. (3). From top to bottom, the curves are for viewing angles of $\theta = 0.01, 0.05, 0.1, 0.2,$ and 0.5 radians, respectively. The GRB is taken to be at a redshift of 1 and to have absolute magnitude $M = -25$ at the break time of 1 day in the GRB rest frame (corresponding to observed break time of $t_{\text{break}} = 2$ days). The jet opening angle is $\theta_{\text{jet}} = 0.07$ radians, and it is assumed that the flux is concentrated at the jet center and lateral spreading of the jet is not included. Note that the flux peaks near the time when $\gamma \approx \theta^{-1}$. Before the peak, the flux rises as roughly $t^{2m-2}$, and after the peak it decays as $t^{-\alpha}$.

$^2$ See http://www.umich.edu/~rotse.
previous discussion applies, merely replacing the constant jet angle with this spreading jet size. To bracket possible jet morphologies and evolution, for a lower bound on off-axis emission we use equation (3), which has all emission coming from the jet center and no lateral spreading, and for an upper bound on off-axis emission we use equation (3) with $\theta$ replaced by $\theta - \theta_{\text{jet}} - \gamma^{-1}$, which has all emission coming from the nearest jet edge and lateral jet spreading at the speed of light.

3. EVENT DETECTION RATE

Observing programs that aim to measure GRB collimation using orphans will measure the rate of orphan afterglow detection and compare this to the rate of GRB-triggered (on-axis) afterglow occurrence. Optical searches for orphan afterglows naturally piggyback on top of Type Ia SN searches, which repeatedly monitor fixed regions of the sky and use differential photometry to detect transient optical sources. Ideally, one would like to detect every afterglow occurring in the observed region, but some afterglows will be missed either because they are too faint or because the monitoring is not frequent enough to catch them. Note, however, that if one can accurately estimate the efficiency at which afterglows are found, then orphan searches may be useful even if many afterglows are missed.

In this section, we estimate the number of afterglows that an observing program would detect and the properties (such as viewing angle from the center of the jet) of those detected afterglows. We can estimate these quantities using the simple model of off-axis emission described in §2. First, we give a simple analytic calculation of the detection rate, appropriate in the limit of frequent or continuous monitoring, which captures the effects of jet behavior. To determine the effects of infrequent monitoring, we then perform a Monte Carlo simulation of a simplified observing program.

3.1. Analytic Estimate

Consider a perfect orphan afterglow search capable of detecting any orphan afterglow above limiting flux $f_{\text{lim}}$. We would like to determine which viewing angles $\theta$ see apparent fluxes above threshold, $f > f_{\text{lim}}$, as a function of time $t$. Using equation (3) and $\gamma = \theta_{\text{jet}}(t/t_{\text{break}})^{\alpha}$ and taking $\gamma \gtrsim$ few, we see that $f > f_{\text{lim}}$ for viewing angles $\theta < \theta_{\text{max}}$, where $\theta_{\text{max}}$ is given by

$$1 - \cos \theta_{\text{max}} \approx \frac{1}{2} \theta_{\text{max}}^2 = \frac{1}{2} \theta_{\text{jet}}^2 \left( \frac{t}{t_{\text{break}}} \right)^{2\mu} \left[ \left( \frac{f_{\text{lim}}}{f_{\text{break}}} \right)^{1/m} - \left( \frac{t}{t_{\text{break}}} \right)^{-\alpha/m} - 1 \right].$$

(4)

Recall that before the break, $\mu = \frac{3}{2}$ and $\alpha \approx 1.1$, while after $t_{\text{break}}$ we have $\mu = \frac{1}{2}$ and $\alpha \approx 2.4$. To compute the effective afterglow beaming angle, we maximize equation (4) with respect to time. Let us call $t_{\text{max}}$ the time at which the visible angle is maximized. Depending upon whether or not $t_{\text{max}}$ coincides with the break time, the $\theta_{\text{max}}$ expression becomes

$$\theta_{\text{max},0} = \theta_{\text{jet}} + \left( 1 + \frac{\mu}{c} \right) \theta_{\text{max}},$$

(6)

where $\theta_{\text{max},0}$ is defined as in equation (5). We emphasize that this is an upper limit to the visible angle and that a more careful treatment would give a $\theta_{\text{max}}$ somewhere in between that given by equations (5) and (6). Our previous expression for $\theta_{\text{max},0}$ should underestimate off-axis emission, and so these two cases should bracket all possible jet emission.

In order to proceed, we next need to define the GRB properties. There are large uncertainties here, and we expect these to introduce substantial uncertainties in our estimates of the event rates. However, we argue that this will not affect our main conclusions.

First we follow Wijers et al. (1998) and Woods & Loeb (1998) in hypothesizing that the comoving rate at which GRBs occur is proportional to the star formation rate, $R_{\text{SFR}}$. For simplicity we approximate the star formation rate, as measured by Steidel et al. (1999), as

$$\log_{10} R_{\text{SFR}}(z) = \left\{ \begin{array}{ll} Cz & z < 1 \\ C & 1 < z < 10 \end{array} \right.$$

(7)

with an arbitrary cutoff for $z > 10$. For the $\Omega_M = 1$, $H_0 = 50$ km s$^{-1}$ Mpc Einstein-de Sitter universe that Steidel et al. use, $C \approx 1$ appears adequate, and correcting this to the flat $\Omega_M = 0.3$, $\Omega_\Lambda = 0.7$, $H_0 = 70$ km s$^{-1}$ Mpc cosmology that we adopt gives instead $C \approx 0.75$. We absorb the overall normalization of $R_{\text{SFR}}$ into our total event rate, which we normalize so that the overall rate of GRBs is $\sim 666$ yr$^{-1}$ over the full sky (Paciesas et al. 1999).3

Next we must make an assumption concerning the luminosity function of GRB afterglows. For simplicity we will assume that afterglows are standard candles with absolute magnitude $M = -25$ at the time of the light-curve break. This number is chosen so that $\sim 70\%$ of well-followed GRBs do not show afterglows because the afterglows are too faint at the time of observation, which we assume to be near the break time. Of course, it is known that the dozen or so afterglows with measured redshifts are widely distributed in absolute magnitude, but $M = -25$ is a typical value. Since the effective beaming angle depends strongly on the apparent luminosity function (see eq. [5]), uncertainty here will lead to substantial uncertainty in the event rate, as we discuss below.

One additional concern is the $k$-correction. At high frequencies, the afterglow spectrum is expected to behave as $F_\nu \sim \nu^{-\alpha/2}$ (Sari et al. 1999), so as a function of redshift $z$, $f_{\text{break}}(z) = f_{\text{break}}(0) \times (1 + z)^{-\alpha/2}$.

Finally, we assume that all GRBs have the same jet opening angle $\theta_{\text{jet}}$, the same power-law indices $\alpha$ and $\mu$ specified in §2, and the same rest-frame break time $t_{\text{break}} \approx 1$ day. Note that here by rest frame we mean as viewed by an observer at the same redshift so that there is a $(1 + z)$ correction. We do not mean the plasma rest-frame time, for

3 Note the number $666$ yr$^{-1}$ includes both long and short GRBs, while afterglows have been detected only for long bursts. We conservatively assume here, however, that all bursts give rise to afterglows; this will soon be testable with experiments such as HETE-2 or Swift.
curves assume emission from the near edge of the jet (eq. [6]) with degree as a function of jet angle in radians for our simple approximation, which there is an additional $\gamma^2$ correction relative to observer time. Clearly there is a wide spread in the observed $t_{\text{break}}$ values and power-law indices $x$ for previous GRBs (Frail et al. 2001), and this will lead to uncertainties on the order of a few in our final numerical results.

We can now compute the expected rate of orphan afterglow detections. Let us write the effective beaming (solid) angle as $\Omega(z) = 4\pi[1 - \cos \theta_{\text{max}}(z)]$. With our assumption of standard candle afterglows, we need only specify the redshift distribution to obtain the afterglow luminosity function. Assuming as above that the GRB rate is proportional to the star formation rate, we may write the observed afterglow redshift distribution as

$$\frac{dN}{dz} = A \frac{dV_{\text{co}}}{dz} R_{\text{SFR}}(z) \frac{\Omega(z)}{1 + z},$$

where $dV_{\text{co}}/dz$ is the comoving volume element, $z$ is the GRB redshift, and $A$ is the normalization constant. The $k$-correction is incorporated as above, and we set $A$ as above by assuming that all on-axis GRBs are bright enough to be detected; that is, we set $\theta_{\text{max}} = \theta_{\text{jet}}$ in equation (8) and integrate over redshift giving $N_{\text{GRB}} = A \Delta \Omega_{\text{jet}} \int_0^z dz (dV_{\text{co}}/dz) R_{\text{SFR}}/(1 + z)$ 666 yr$^{-1}$ over the full sky.

The calculation described above elucidates the overall behavior but neglects effects that may affect numerical predictions for the overall detection rate. For example, we have extrapolated the scaling formulas for $\gamma$ to the nonrelativistic regime where the scalings break down; fortunately, we will show that the regions of low $\gamma$ make a small contribution to the integral for all but the deepest surveys. We have also assumed continuous monitoring, which is certainly not the case in current SN searches. In order to take into account the effect of imperfect monitoring and to check the above analytic calculation, we ran a Monte Carlo simulation of a simplified observing program.

3.2. Monte Carlo Simulation

We define an observing program by its limiting magnitude $m_{\text{lim}}$ (and corresponding limiting flux $f_{\text{lim}}$), by the time between exposures of a given field $t_{\text{mon}}$, and by the total survey exposure (number of deg$^2$ yr). In an actual survey additional parameters and various experimental efficiencies would also be taken into account but these are not crucial for our considerations here. The observing program simulation is run for a given value of $\theta_{\text{jet}}$ by distributing GRBs randomly throughout the universe, proportionally to the product of comoving volume and star formation rate, and at a random viewing angle to the observer. The light curve is started at a random time and observed every $t_{\text{mon}}$ days. The afterglow is considered to be detected if any of the observations find a flux above the limiting flux of the observing program. The total event rate and the distribution of detected events as a function of redshift, observing angle, etc., is thus easily found.

4. RESULTS

Now all the elements are in place to calculate the detection rate and average beaming factor for a given survey. Let us first consider a Type Ia SN search (Schmidt et al. 1998; Perlmutter et al. 1999), with limiting magnitude of $R = 23$ and monitoring repeat baseline $t_{\text{mon}} \approx 21$ days. Adopting an opening angle of $\theta_{\text{jet}} = 4^\circ$ (Frail et al. 2001), our Monte Carlo calculation gives an expected rate of $\sim 0.0008$ deg$^{-2}$ yr$^{-1}$ if the flux is concentrated at the jet center and there is no lateral spreading, and $\sim 0.014$ deg$^{-2}$ yr$^{-1}$ if lateral spreading is maximal and the flux is concentrated at the jet edge.

So, taking an exposure of 10 deg$^2$ yr, we predict between 0.008 and 0.14 detections from previous Type Ia SN surveys. We note that in the limit of very small $t_{\text{mon}}$ the Monte Carlo results are close to the analytic results, which finds 5–10 times more events depending on jet morphology.

Now the important point to note is that these numbers, besides being small, are fairly independent of $\theta_{\text{jet}}$. The reason for this should be clear. The only dependence of equation (8) on $\theta_{\text{jet}}$ is the $\Omega(z)$ factor, which was the effective beaming angle as a function of redshift. Since $\theta_{\text{max}} \propto \theta_{\text{jet}}$, $\Omega(z) \propto \Omega_{\text{jet}}$. But since the total GRB rate scales like $\Omega_{\text{jet}}^{-1}$ to hold fixed the number of observed GRBs, this means that the expected number of detected orphans is also held fixed. So the nondetection of orphan afterglows is not only consistent with opening angles of $4^\circ$, it is consistent with almost any opening angle.

Next we consider a more powerful future observing program with limiting magnitude $R_{\text{lim}} \approx 27$ and frequent enough monitoring that our analytic estimates are accurate. Here the number of predicted afterglow detections can rise above unity if many tens of square degrees are monitored. The number of expected events is shown in Figure 3 as a function of the opening angle $\theta_{\text{jet}}$. For opening angles ranging from moderate ($\sim 20^\circ$) to arbitrarily small ($< 0.01^\circ$) there is essentially no change in the predicted number of detections. Note, however, that for large jet angles ($> 30^\circ$) there can be a significant decrement in the predicted number of detected orphans, in part because the relativistic scalings we have employed break down, and in part because the emission begins to subtend the full sky. However, it is clear that for the angles being considered in the leading jet models, the ratio of GRB detection to afterglow detection will give no information on the GRB collimation.

5. DISCUSSION AND CONCLUSIONS

We have shown that within the framework of the leading model for the late-time GRB jet evolution, current optical surveys cannot constrain GRB beaming. Part of the
problem is that current surveys are too infrequent, narrow, and shallow.

A more basic problem is that the effective afterglow beaming angle scales as the jet angle for small (<20°) jet angles, and so the ratio of $\Omega_{\text{on}}/\Omega_{\text{jet}}$ cannot be used to measure $\theta_{\text{jet}}$. This conclusion depends on the simple scaling laws of flux and Lorentz factor that apply during the relativistic phase of the expansion. In order to see emission from large viewing angles, one must wait until the jet has become only moderately relativistic and therefore dim. In Figure 4 we plot the distribution of viewing angles for several different limiting magnitudes. Only the $R = 30$ light curve has any sensitivity to large viewing angles, and it is still dominated by small $\theta$. Finding afterglows this faint or fainter will be difficult owing to, for example, host galaxy domination.

Absent the capability to detect emission from viewing angles near unity, measuring the GRB jet angle using orphans is all but impossible. This is shown in Figure 3, in which we plot the number of expected afterglow detections per year per square degree as a function of jet opening angle, for a limiting magnitude of $R = 27$. As mentioned above, there is essentially no variation in the predicted number of detections as the jet solid angle $\Omega_{\text{jet}}$ is varied by orders of magnitude until nearly isotropic jets ($\theta_{\text{jet}} \gtrsim 30°$) are reached. Now, once the GRB afterglow luminosity function is well measured, as should be possible with future missions such as Swift or HETE-2, and once GRB jet dynamics are better understood, then a prediction can be made for the expected number of afterglows any given survey can detect. A significant downward departure from this expected rate would be hard to reconcile with small jet angles, within the context of accepted models. A statistically significant detection of such a deficit would require a very deep survey expected to detect hundreds of orphans over a large region of sky.

To establish our conclusion of the difficulty of using optical afterglow searches to constrain small beaming angles, we have presented two sets of detection rates in an attempt to bracket the effects of uncertain jet morphology and evolution. We caution that our upper bound is probably not realistic because it assumes that the jet emits all of its flux as near as possible to the direction of the observer. In addition, for both the upper and lower bounds, we have neglected the frequency redshift of photons received by off-axis observers relative to on-axis observers. This should be taken into account for observers interested in the narrow frequency range within the optical band. For a narrow pencil beam moving along the $\theta = 0$ axis, the relative redshift of photons received by an observer at $\theta_{\text{obs}}$ is $v(\theta_{\text{obs}})/v(\theta = 0) = (1 - \beta \cos \theta_{\text{obs}})/(1 - \beta)$. If the afterglow spectrum behaves as $F_v \propto v^{-\alpha/2}$ (Sari et al. 1999), then the redshift effect drives the exponent in equation (3), which we previously found to be $m = 4$, to $m = (4 + \alpha/2)$. Replacing $m = 4$ in our equations with $m = 4 + \alpha/2$ slightly diminishes the expected number of detections by $\lesssim 30\%$. Were an orphan afterglow search to be performed, these corrections, as well as a detailed study of the jet evolution and emission, would be required to calculate a more accurate estimate of the expected rate.

In our calculations we have assumed particular values for the parameters describing GRB afterglows. These parameters are the power-law exponents $\alpha$ and $\mu$, the absolute magnitude at break, and the break time $t_{\text{break}}$. Uncertainties in these parameters will introduce uncertainties in the overall rates. For example, changing $\alpha$ from 2.4 to 1.8 increases the detection rate by a factor of $\sim 3$. Similarly, changing the absolute magnitude at break from $M = -25$ to $-29$ (an increase in luminosity by a factor of 40) increases the overall detection rate by a factor of 4–5, as expected from equation (5). Changing $t_{\text{break}}$ does not affect our calculations for continuous monitoring but can change the detection efficiency depending upon the monitoring time. In the limit of small efficiency the rate should scale as $t_{\text{break}}^{-1}$. Most importantly, the constancy of the calculated detection rate with respect to the jet opening angle remains valid for $\theta_{\text{jet}} \lesssim 20°$.

One caveat to our estimate for the orphan afterglow detection rate and to our constraint on the observed ratio of orphans to GRBs is the possibility of processes that lead to weakly beamed optical emission while remaining consistent with the good agreement between the expected and observed power-law decay and break of the late-time afterglow. For example, the GRB central engine might emit,

![Graph](https://via.placeholder.com/150)

**Fig. 4.**—Plot of $\frac{dN}{d\theta}$, in units of events per year per square degree, as a function of viewing angle $\theta$, for an assumed jet angle $\theta_{\text{jet}} = 4°$ and for our analytic approximation. The different curves within each panel show the effect of varying the limiting magnitude. From top to bottom, they are $m_{\text{lim}} = 30$, 27, 24, and 21, respectively. The different panels correspond to different assumptions of the effects of jet width and lateral spreading. In the first panel, off-axis emission is estimated using eq. [5], that is, it is assumed to come only from the jet center. In the second panel, off-axis emission is estimated using eq. [6] with $v = c$, corresponding to emission from the nearest jet edge and maximal spreading. Note that $\frac{dN}{d\theta}$ is independent of limiting magnitude for $\theta < \theta_{\text{jet}}$ when we include the finite jet width, since on-axis afterglows in our model can be arbitrarily bright at early times, and we assume continuous monitoring in our analytic approximation. [See the electronic edition of the Journal for a color version of these figures.]
coincident with an ultrarelativistic jet, a moderately relativistic spherical shell that leads to isotropic optical emission. This might occur in central engine models where rotation leads to small baryon contamination and ultrarelativistic flow along the rotation axis but also to “dirtier,” moderately relativistic flow away from the rotation axis (e.g., in the collapsar model; MacFadyen & Woosley 1999). Such an isotropic component, if bright enough to be detected off-axis but faint enough to avoid on-axis detection, would clearly confound our analysis and allow direct measurement of GRB collimation. On the other hand, it seems unlikely that we have underestimated the off-axis optical emission from the jet itself, because this estimate derives principally from simple relativistic kinematics. Similarly, the upper limit to the degree of angular spreading in the jet comes from the maximal assumption that the lateral expansion velocity is \(c\) in the jet comoving rest frame. Recent numerical hydrodynamics simulations (Granot et al. 2001) of the relativistic jet dynamics appear to indicate that the afterglow emission is indeed more tightly beamed in the forward direction than we have assumed, suggesting that our calculations neglecting spreading are more realistic than our calculations including lateral spreading.

Finally, we consider afterglow emission at other frequencies. The analysis presented above clearly applies to X-ray emission, so it appears that searches for orphan X-ray afterglows (Grindlay 1999; Greiner et al. 2000) will similarly be unable to constrain GRB collimation. Radio afterglows are not subject to the analysis presented here. They do not decay as quickly as their higher frequency counterparts and remain visible until the afterglow emission is effectively isotropic. Perna & Loeb (1998) have proposed that a full-sky survey sensitive to 0.1 mJy could legitimately test the jet model of GRB emission. It appears that this may be the best way to implement Rhoads’ suggestion of using burstless afterglows to measure the GRB jet angle.

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