PUMPING CURRENT IN A QUANTUM DOT BY AN OSCILLATING MAGNETIC FIELD

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We investigate spin and charge current through a quantum dot pumped by a time-varying magnetic field. Using the density matrix method, quantum rate equations for the electronic occupation numbers in the quantum dot are obtained and solved in the stationary state limit for a wide set of setup parameters. Both charge and spin current are expressed explicitly in terms of several relevant parameters and analyzed in detail. The results suggest a way of optimizing experimental setup parameters to obtain a maximal spin current without the charge current flow.

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Spintronics is an emerging active research field, which is based on the effective control of electron spin in addition to its charge degree of freedom. One common operational principle for spin-based devices is how to generate a spin current. It is known that a spin current is usually accompanied by a charge current, which generates heat. And this may pose a severe problem in microelectronics as the devices become smaller and smaller, since heat destabilizes the operation of the devices. Therefore proposals for the spintronics devices without creation of charge current flow would be greatly desirable.

Some spin-based devices have been already proposed. However, one of the most effective way to create a spin current is using semiconductor quantum dots. Martin et al. proposed a scheme for electrical detection of electron spin resonance(ESR) of an electron trap. This idea intrigues subsequent studies on how to achieve a desirable spin current through a quantum dot or multiple quantum dots either in the strong Coulomb regime or in the Kondo regime. The basic idea is first Zeeman splitting the dot level by a perpendicular constant magnetic field and then pumping electron from the low-lying spin-up state to the high-lying spin-down state by an oscillating magnetic field. We notice that special parameters are chosen to facilitate the studies. It is the purpose of this work to relax the parameter constraints to gain a general physical picture about spin pump effect in a quantum dot by a time-varying magnetic field.

The quantum dot spin pump consists of a quantum dot coupled to two electronic leads by tunnel barriers. No voltage bias is applied to the device to allow for a direct current through the quantum dot. To realize a spin pump with such a system, one apply a perpendicular constant magnetic field $B_0 \hat{z}$ and a lateral time-varying field $B_{rf}(\cos(\omega_{rf}t)\hat{x} + \sin(\omega_{rf}t)\hat{y})$ to the quantum dot. Due to Zeeman effect in the presence of a constant magnetic field $B_0 \hat{z}$, the dot level will be split into two spin-dependent levels: $\varepsilon_d \rightarrow \varepsilon_\sigma = \varepsilon_d - \sigma E_Z = g\mu_B B_0/2$, where $g$, $\mu_B$ are the effective $g$ factor and Bohr magneton of the quantum dot. The oscillating magnetic field serves a machine to pump low-lying spin-up electrons to the high-lying spin-down level, which is described by a Hamiltonian $H_{rf}$ given below. We consider the case that the Coulomb interaction between electrons inside the dot is strong enough to prohibit double occupation of the dot. A spin-up electron tunnels into the quantum dot from either the left or right lead, and tunnels out of it after being pumped to the high-lying state. This process persists repeatedly and a steady charge and spin current is generated.
The Hamiltonian of this device is

$$H = \sum_\sigma \varepsilon_\sigma d_\sigma^\dagger d_\sigma + \sum_{k, \sigma} \varepsilon_{\alpha k} c_{\alpha k}^\dagger c_{\alpha k} + \sum_{k, \sigma} t_{\alpha k} (c_{\alpha k}^\dagger d_\sigma + d_\sigma^\dagger c_{\alpha k}) + H_{rf}(t),$$

(1)

where \(d_\sigma^\dagger (d_\sigma)\) creates (annihilates) an electron with spin \(\sigma = \uparrow, \downarrow\) in the quantum dot at the level \(\varepsilon_\sigma\). \(c_{\alpha k}^\dagger (c_{\alpha k})\) creates (annihilates) a spin-\(\sigma\) electron in the lead \(\alpha = l, r\). The third term in Eq. (1) describes tunneling between the dot and the leads, while the last term \(H_{rf}(t)\) denotes the pumping mechanism and can be written as \(H_{rf}(t) = \Omega_R (d_\sigma^\dagger d_\sigma e^{-i\omega_{rf} t} + d_\sigma^\dagger d_\sigma e^{i\omega_{rf} t})/2\), where \(\Omega_R = g \mu_B B_{rf}/2(\hbar\text{ is set to be unity throughout this work})\) is the Rabi frequency.

A gate voltage must be applied to the quantum dot and so adjusted that the chemical potential of the leads lies between the spin-up and spin-down levels of the dot. Initially the system lies in its ground state \(|G\rangle\) with electrons filling up to the chemical potential in the leads and without electron inside the dot. When tunneling is turned on, the wave function of the whole system can be written in the following form

$$|\Psi(t)\rangle = \{b_0(t) + \sum_{k} b_{\alpha k}(t) c_{\alpha k}^\dagger, b_{\alpha k}(t)(d_\sigma^\dagger c_{\alpha k})\} + \sum_{k, k'} b_{\alpha k}(t) c_{\beta k'}^\dagger c_{\alpha k} + \sum_{k, k', k''} b_{\alpha k}(t) c_{\beta k'}^\dagger c_{\gamma k''} c_{\beta k''}^\dagger + \cdots \} |G\rangle,$$

where \(b(t)\)'s denote the probability amplitudes for finding the system in the corresponding states at time \(t\), and initially all the \(b(0)\)'s except \(b_0(0)\) are zero.

Now we introduce the reduced density matrix \(\rho_{ij}\) spanned in the Fock space of the quantum dot: \(|0\rangle \rightarrow \text{ empty state, } |\uparrow\rangle \rightarrow \text{ spin-up state is occupied, } |\downarrow\rangle \rightarrow \text{ spin-down state is occupied.} The diagonal elements of the density matrix \(\rho_{ii}\) give the probabilities of finding the dot being either empty or occupied by a spin-\(\sigma\) electron, while the off-diagonal elements describe coherent superposition of the spin-up and spin-down states. The density matrix \(\rho_{ij}\) can be obtained by tracing out the degrees of freedom of the leads in the full density matrix \(\rho_{\alpha}\) \(=\sum_{n_{\downarrow}, n_{\uparrow}} \rho_{ij}^{(n_{\downarrow}, n_{\uparrow})}, \) where \(\rho_{ij}^{(n_{\downarrow}, n_{\uparrow})}\) represent the probabilities of finding the dot in the state \(ij\) with \(n_{\uparrow}\) and \(n_{\downarrow}\) spin-up electrons tunneling out of the left and right leads, and \(n_{\downarrow}\) and \(n_{\uparrow}\) spin-down electrons tunneling into the left and right leads. We find \(\rho_{00} = |b_0(t)|^2 + \sum_{k, k'} |b_{\alpha k}(t)|^2 + \cdots, \rho_{\alpha\beta} = \sum_{k} |b_{\alpha k}(t)|^2 + \sum_{k, k', k''} |b_{\alpha k}(t) b_{\alpha k'}(t) b_{\alpha k''}(t) + \cdots|^2.\) Solving the schrödinger equation \(i\partial_t |\Psi(t)\rangle = H |\Psi(t)\rangle\) results in an infinite set of coupled linear differential equations for \(b(t)\)'s, which can be finally transformed into an infinite set of algebraic equations after the Laplace transform \(b(E) = \int_0^\infty db \tilde{b}(t) e^{iEt}.\) Performing inverse Laplace transform and summing up the relevant terms, we obtain the following quantum rate equations for the density matrix \(\rho_{ij}\)

$$\dot{\rho}_{00} = -(\Gamma_{\uparrow\uparrow} + \Gamma_{\uparrow\downarrow}) \rho_{00} + (\Gamma_{\downarrow\uparrow} + \Gamma_{\downarrow\downarrow}) \rho_{\downarrow\downarrow},$$

(2)

$$\dot{\rho}_{\uparrow\uparrow} = (\Gamma_{\uparrow\uparrow} + \Gamma_{\uparrow\downarrow}) \rho_{00} + \frac{\Omega_R}{2} (e^{i\omega_{rf} t} \sigma_{\uparrow\downarrow} - e^{-i\omega_{rf} t} \rho_{\downarrow\uparrow}),$$

(3)

$$\dot{\rho}_{\downarrow\downarrow} = -(\Gamma_{\downarrow\uparrow} + \Gamma_{\downarrow\downarrow}) \rho_{00} - \frac{\Omega_R}{2} (e^{i\omega_{rf} t} \rho_{\downarrow\uparrow} - e^{-i\omega_{rf} t} \rho_{\uparrow\downarrow}),$$

(4)

$$\dot{\rho}_{\uparrow\downarrow} = (iE_Z - \frac{\Gamma_{\downarrow\uparrow} + \Gamma_{\downarrow\downarrow}}{2}) \rho_{\uparrow\downarrow} + \frac{\Omega_R}{2} e^{i\omega_{rf} t} (\rho_{\downarrow\uparrow} - \rho_{\uparrow\downarrow}),$$

(5)

$$\dot{\rho}_{\downarrow\uparrow} = (iE_Z - \frac{\Gamma_{\uparrow\uparrow} + \Gamma_{\downarrow\uparrow}}{2}) \rho_{\downarrow\uparrow} + \frac{\Omega_R}{2} e^{-i\omega_{rf} t} (\rho_{\uparrow\downarrow} - \rho_{\downarrow\uparrow}),$$

(6)

where \(\Gamma_{\alpha\beta} = 2\pi \sum_k |t_{\alpha k}|^2 \delta(\omega - \varepsilon_{\alpha k})\) are the line-width functions characterizing the coupling strength between the dot and the leads. We have introduced phenomenologically an additional relaxation term \(1/T_{\perp}\) to describe the transverse spin relaxation process.

Current \(I_{\alpha\sigma}\) is calculated as the evolution rate of electron number tunneling into or out of the lead \(\alpha: I_{\alpha\sigma} = e \dot{N}_{\alpha\sigma}(t) = \sum_{n_{\alpha\sigma}} n_{\alpha\sigma} \rho_{n_{\alpha\sigma}}^{(n_{\downarrow}, n_{\uparrow})}.\) In the stationary state limit, \(\dot{\rho}_{ij} = 0,\) one finds the following expressions for the charge current \(I^c_{\alpha} = I_{\alpha\sigma} + I_{\alpha\sigma}^c\) and the spin current \(I^s_{\alpha} = I_{\alpha\sigma} - I_{\alpha\sigma}^c\) flowing into the left or right lead

$$I^c_{\alpha} = I^c_{\tau} = e \Omega_R (\Gamma_{\downarrow\tau} - \Gamma_{\uparrow\tau}) \Theta(E_Z, \Omega_R, \omega_{rf}, T_{\perp}, \Gamma_{\alpha\sigma}),$$

(7)
\[ I^r_\ell = -e\Omega^2_R(2\Gamma_{\ell\uparrow}\Gamma_{\ell\downarrow} + \Gamma_{r\uparrow}\Gamma_{r\downarrow} + \Gamma_{l\uparrow}\Gamma_{r\uparrow})\Theta(E_Z, \Omega_R, \omega_{rf}, T_{\perp}, \Gamma_{\alpha\sigma}), \]

\[ I^r_r = e\Omega^2_R(2\Gamma_{r\uparrow}\Gamma_{r\downarrow} + \Gamma_{r\uparrow}\Gamma_{l\downarrow} + \Gamma_{r\downarrow}\Gamma_{r\uparrow})\Theta(E_Z, \Omega_R, \omega_{rf}, T_{\perp}, \Gamma_{\alpha\sigma}), \]

where the resonance function is

\[ \Theta(E_Z, \Omega_R, \omega_{rf}, T_{\perp}, \Gamma_{\alpha\sigma}) = \Upsilon(T_{\perp}, \Gamma_{\alpha\downarrow})/\Lambda(E_Z, \Omega_R, \omega_{rf}, T_{\perp}, \Gamma_{\alpha\sigma}), \]

\[ \Upsilon(T_{\perp}, \Gamma_{\alpha\downarrow}) = \Gamma_{l\downarrow} + \Gamma_{r\downarrow} + \frac{2}{T_{\perp}}, \]

\[ \Lambda(E_Z, \Omega_R, \omega_{rf}, T_{\perp}, \Gamma_{\alpha\sigma}) = 4(E_Z - \omega_{rf})^2(\Gamma_{l\uparrow} + \Gamma_{r\uparrow})(\Gamma_{l\downarrow} + \Gamma_{r\downarrow}) + \Omega^2_R(2\Gamma_{l\uparrow} + 2\Gamma_{r\uparrow} + \Gamma_{l\downarrow} + \Gamma_{r\downarrow})\Upsilon + (\Gamma_{l\uparrow} + \Gamma_{r\uparrow})(\Gamma_{l\downarrow} + \Gamma_{r\downarrow})\Upsilon^2. \]

We notice that Dong et al.\textsuperscript{7} considered a particular case with all line-width functions being the same, and drew the conclusions of zero charge current and same values for the spin current in the two leads. We see from Eq. (7) that the pumped charge current in the left and right leads has the same magnitude and direction no matter what the parameter values. It is a result of current conservation. Charge current disappears when the line-width functions satisfy the relation \( \Gamma_{l\uparrow}/\Gamma_{l\downarrow} = \Gamma_{r\uparrow}/\Gamma_{r\downarrow} \). While the condition to have the same magnitude for the spin current in the left and right leads is \( \Gamma_{l\uparrow}\Gamma_{l\downarrow} = \Gamma_{r\uparrow}\Gamma_{r\downarrow} \). In the spin-independent tunneling case \( \Gamma_{\alpha\uparrow} = \Gamma_{\alpha\downarrow} \), charge current is always zero, and a maximal spin current can be expected when the time-varying field is resonantly coupled to the dot, i.e., \( \omega_{rf} = E_Z \), and the dot is coupled to the leads in an extremely asymmetric way. The ratio \( |I^r_\ell/I^r_r| \) between the magnitudes of spin current in the left and right leads is directly proportional to the coupling asymmetry factor \( \Gamma_{l}/\Gamma_{r} \) in the spin-dependent tunneling case.

In summary, we have derived explicit expressions for the charge and spin current in terms of setup parameters of a quantum dot pump device, and discussed a possible way of optimizing the relevant parameters to achieve a maximal spin current without charge current flow in the leads. The influences of finite coulomb interaction and spin-flip process on the charge and spin current in such a device in the nonequilibrium situation are expected to be more interesting and the study is underway.

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