Joint effects of entanglement and symmetrization:
physical properties and exclusion

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Abstract

Entanglement and symmetrization lead to non-separable states that can modify physical properties. Using the example of atomic absorption we compare both types of effects when they are relevant at once. The presence of multi-particle superpositions largely alters the absorption rates of identical atoms, even inhibiting the dependence on overlapping for fermions. We also identify a set of non-standard excluded states related to multi-fermion superposition that naturally emerge in this context. We propose an arrangement based on the dissociation of molecules to test these ideas.

Keywords: Entanglement; Symmetrization; Excluded states; Atomic absorption

1 Introduction

There are two types of non-separability in quantum theory, these associated with entangled and symmetrized states. Both types have been extensively studied. However, the interplay between them is yet an open subject. There is a vivid debate about their relationship, mainly centered in two points: the definition of entanglement in systems of identical particles, and the possibility of extracting useful entanglement from symmetrized states.

With respect to the first point, it is well-known that there is no general agreement on the definition of adequate measures of entanglement for identical particles, in spite of the fact that several proposals have been presented in the literature. These proposals are based on so different ideas as the use of analogues of the Schmidt decomposition [1], the determination of properties that can be ascribed to the system [2, 3], the exploration of the mathematical structure of the observables [4], and the introduction of states without labels for identical particles [5]. As a consequence of this lack of consensus, it is even discussed if some states of indistinguishable particles are entangled or not.
In the same vein there is some controversy about the second point. One can ask if the symmetrization of identical particles alone can be used as a resource in physical tasks. Several authors have proposed schemes that convert the pure symmetrization into practical entanglement, answering in the positive this question [6, 7, 8]. These results suggest that entanglement and symmetrization are two manifestations of a more general phenomenon, non-separability. However, it would be desirable a deeper and more quantitative understanding of the relation between them when both forms of non-factorizability are simultaneously present.

We present in this paper an analysis in this line, which is based on a simple example, atomic absorption. The associated physical property, the light absorption rate, is dependent on the non-separability of atomic systems. This dependence has been studied in [9] for symmetrized states and in [10] for entangled ones. They are only two examples of the general dependence of light-matter interactions on non-factorizability [11, 12, 13]. Using this dependence we can compare situations where symmetrization acts alone with others where simultaneously we have multi-particle superpositions associated with entanglement. The absorption rates of identical atoms are drastically modified, both qualitatively and quantitatively, by the multi-particle superpositions. They can lead to large changes of the intensity of the rates, to reverse the analytical form of the curves or to the inhibition of the dependence on the overlap degree for fermions.

The joint effects of entanglement and symmetrization are by no means restricted to modifications of some physical properties. We show the existence of a set of non-standard excluded states. They do not obey the standard Pauli’s exclusion principle [14]. Instead, they are related to the presence of multi-fermion superpositions. In [15] it was suggested, from a more formal perspective, that in the case of entangled systems of identical fermions there can be excluded states beyond the scope of Pauli’s exclusion principle. Our example shows that these hypothetical states could be present in realistic systems.

It is possible, in principle, to test the above ideas. We propose an experimental arrangement based on light absorption by atoms prepared in the photodissociation of molecules. Photodissociation of molecules composed of identical atoms has been experimentally used to study the role of entanglement in spontaneous emission [16, 17]. Later, it was demonstrated that the processes of disentanglement [18] and symmetrization must be taken into account to give a complete description of the experiments [19]. That type of experiment can be easily adapted to test the simultaneous effects of symmetrization and multi-particle superposition.
2 The system

First of all we describe the system considered in this paper. It consists of pairs of identical atoms, either bosons or fermions, prepared in the initial state

$$|\Phi> = N_0 [a(|\psi>_1 |\phi>_2 \pm |\phi>_1 |\psi>_2) +$$

$$b(|\varphi>_1 |\chi>_2 \pm |\chi>_1 |\varphi>_2)]|g>_1 |g>_2$$

(1)

where $a$ and $b$ are two complex coefficients. The center of mass (CM) one-particle states of the atoms are $\psi$, $\phi$, $\varphi$ and $\chi$. They are normalized, $<\psi|\psi> = 1, \ldots$. On the other hand, in general, they are not orthogonal: $<\psi|\phi> \neq 0, \ldots$. The internal or electronic states of the atoms are $g$ and $e$ that refer to the ground and excited states. In the double sign $\pm$, the upper one holds for bosons and the lower one for fermions. The normalization coefficient is denoted by $N_0$ and given by

$$N_0 = (2|a|^2 + |b|^2) \pm 2|a|^2 |<\psi|\phi> |^2 \pm 2|b|^2 |<\psi|\phi> |^2 +$$

$$4Re(a^*b <\psi|\varphi > <\phi|\chi>) \pm 4Re(a^*b <\psi|\chi > <\phi|\varphi>)^{1/2}$$

(2)

This state corresponds to the (anti)symmetrization of the unnormalized entangled state $|\Psi> = [a|\psi>_1 |\phi>_2 + b|\varphi>_1 |\chi>_2]|g>_1 |g>_2$. When the state describing two identical particles is the (anti)symmetrization of a superposition of two-particle states we have symmetrization and entanglement effects at once. In effect, using for instance the approach in [2], we have that the (anti)symmetrization of $\Psi$, given by $|\Phi>_{12} = |\Psi>_{12} \pm |\Psi>_{21}$, is in general entangled when the two coefficients $a$ and $b$ are not null. On the other hand, when the CM-states are not orthogonal we have overlap between the atoms and consequently exchange effects.

After the preparation, the two atoms interact with light. The light beam contains the absorption frequency of the atoms. The intensity of the beam is low, we assume that the probability of more than one absorption is negligible. This way we simplify the calculations, avoiding two-photon absorption or other non-linear processes.

Next, we discuss a viable realization of this arrangement. We are looking for a scheme where the symmetrization and entanglement effects are simultaneously relevant. In addition, we must be able to compare the single absorption probability for various overlap degrees between the identical atoms. Our proposal is based on the photodissociation of molecules. This process has already been used in two experiments [16, 17] to study entanglement in spontaneous emission. A detailed analysis of the experiments shows that symmetrization and entanglement (plus disentanglement [18]) are simultaneously necessary to explain the results [19]. Thus, this scheme fulfills our first demand.
Figure 1: Schematic representation of the arrangement. LS denotes a light source, BS the beam splitter, T and S the transmitted and reflected light rays, M the molecule composed of two identical atoms, and LB the light beam interacting with the atoms released in the photodissociation.

In these experiments identical atoms in excited states were prepared by molecular photodissociation. In our case, we want to have pairs of identical atoms, but in their ground states. In our proposal a source produces a light beam tuned to the photodissociation frequency of the molecule. The beam interacts with a beam splitter leading to the superposition \(|\gamma\rangle \rightarrow a|\gamma_T\rangle + b|\gamma_R\rangle\), provided that the transmission and reflection coefficients are \(a\) and \(b\). Using glasses or other optical elements the beams are directed towards the molecule. We can choose different incidence angles for both beams (although arriving simultaneously). This way, we can have different movement directions and overlap degrees for the atoms. Because of the superposition of the incident photon the two atoms after the dissociation will be in a two-atom superposition of the type described by Eq. (1). In any photodissociation process the atoms show strong quantum momentum correlations, reminiscent of the classical law of momentum conservation.

Next, we make interact the atoms with another light beam tuned to the excitation frequency of the atoms. If the delay between the dissociation and the light-matter interaction is short enough the overlap between the atoms will not be negligible. Varying the delay we can study the variation of the absorption rate with the overlap degree. The number of absorptions can be measured detecting the subsequent spontaneous emissions. The scheme demands an extreme
3 Absorption probabilities

In this section we evaluate the absorption rates of the atoms after the interaction with the light. The absorption event can happen in four different ways: to be absorbed by an atom in the CM state $\psi$, in the $\phi$, or in the $\chi$. After the absorption the CM-state changes from $\psi$, $\phi$, $\chi$ to $\psi^*$, $\phi^*$, $\chi^*$ because of the recoil. As the final state must be symmetrized we have four different (unnormalized) states representing the absorption:

\[ |I > = |\psi >_1 |e >_1 |\phi >_2 |g >_2 \pm |\phi >_1 |g >_1 |\psi >_2 |e >_2 \] (3)

\[ |II > = |\psi >_1 |g >_1 |\phi^* >_2 |e >_2 \pm |\phi^* >_1 |e >_1 |\psi >_2 |g >_2 \] (4)

\[ |III > = |\phi^* >_1 |e >_1 |\chi >_2 |g >_2 \pm |\chi >_1 |g >_1 |\phi >_2 |e >_2 \] (5)

\[ |IV > = |\phi >_1 |g >_1 |\chi^* >_2 |e >_2 \pm |\chi^* >_1 |e >_1 |\phi >_2 |g >_2 \] (6)

Each state leads to an absorption probability amplitude. These amplitudes are indistinguishable because they are compatible with the same absorption process and, consequently, they must be added to get the full absorption probability. To add the amplitudes is equivalent to consider a final state representing the superposition of these states and to evaluate the transition rates with respect to that final state, which reads

\[ |\Phi_f > = N_f (a(|I > +|II >) + b(|III > +|IV >)) \] (7)

Note that we must include the coefficients $a$ and $b$ in this expression. If, instead, we would have taken the sum of the four states with equal coefficients we would have erroneous results. For instance, in the limit of no initial multi-particle superposition ($b = 0$), the states $III$ and $IV$ would contribute to the matrix element representing the transition from the initial to the final state, a non-sense behavior. The normalization coefficient is

\[ N_f^{-2} = 4(|a|^2 + |b|^2) + 4Re(a^*b < \psi^*|\phi^* > < \phi|\chi >) + \\
4Re(ab^* < \chi^*|\phi^* > < \phi|\psi >) \pm 4|a|^2 Re(< \psi^*|\phi^* > < \phi|\psi >) \pm \\
4Re(ab < \psi^*|\chi^* > < \phi|\phi >) \pm 4Re(ab^* < \phi^*|\phi^* > < \chi|\psi >) \pm \\
4|b|^2 Re(< \phi^*|\chi^* > < \chi|\phi >) \] (8)

Next, we evaluate the matrix element $M =< 0_{EM}| < \Phi_f |\hat{U}|\Phi > |1_{EM} >$ representing the probability amplitude for absorption. In this expression $\hat{U}$ is the evolution operator of the complete system, and $1_{EM}$ and $0_{EM}$ represent the states of the electromagnetic field of the light with 1 and 0 photons (remember
that we have assumed a low intensity beam with only one possible absorption). Because the two atoms do not interact the evolution operator can be expressed in the factored form $\hat{U} = \hat{U}_1 \otimes \hat{U}_2$. The atom-light interaction is represented by the usual electric-dipole Hamiltonian. Moreover, at the first order of perturbation theory the matrix element can be expressed in the form $< 0_{EM} | \Phi | \hat{H}_1 \otimes \hat{H}_2 | \Phi > | 1_{EM} >$, with $\hat{H}_i$ the one-atom $(i = 1, 2)$ interaction electric-dipole Hamiltonian. It is proportional to $D \mathbf{E}$, with $D$ the total electric-dipole moment of atom $i$ (as the atoms are identical both momenta are equal) and $\mathbf{E}$ the electromagnetic field of the light.

Let us explicitly carry out the evaluation of one of the multiple matrix elements included in $\mathcal{M}$, for instance,

\[ N_f N_0 a < 0_{EM} | 1 < \psi^* | 1 < e | 2 < \phi | 2 < g | \hat{H}_1 \otimes \hat{H}_2 | \psi >_1 | g >_1 | \phi >_2 | g >_2 | 1_{EM} > = N_f N_0 a D < \psi^* | \psi > \]  \hspace{1cm} (9)

with $D = D_0 < e | D | g > . < 0_{EM} | \mathbf{E} | 1_{EM} >$, where the coefficient $D_0$ contains all the constant terms appearing in the evaluation and whose explicit form is not relevant for our discussion.

The rest of matrix elements can be calculated in the same way, and the full matrix element becomes

\[ \mathcal{M} / 2 N_0 N_f D = |a|^2 (|\psi^* | \psi > + |\phi^* | \phi >) + |b|^2 (|\varphi^* | \varphi > + |\chi^* | \chi >) + a^* b < \psi^* | \varphi > < \phi | \chi > + a^* b < \phi^* | \chi > < \psi | \varphi > + a b^* < \varphi^* | \psi > < \chi | \phi > + a b^* < \chi | \phi > < \varphi | \psi > + b^* < \phi^* | \chi > < \psi | \varphi > + |a|^2 < \psi^* | \psi > < \varphi | \phi > + |b|^2 < \chi^* | \chi > < \varphi | \psi > (10) \]

With this expression we can study the dependence of the absorption rate, $|\mathcal{M}|^2$, on symmetrization and entanglement.

4 Graphical representation

In this section we represent the relative single absorption rate for pairs of bosonic and fermionic atoms. It is defined as the absorption rate normalized with respect to that of the same atoms in a product state, $R = |\mathcal{M}|^2 / |\mathcal{M}_{pro}|^2$. The absorption rate in a product state can be easily evaluated. If the initial state is $|e >_1 | g >_1 | \mu >_2 | g >_2$ the final one after absorption is $(|e^* >_1 | e >_1 | g >_2 + |\mu^* >_1 | g >_1 | \mu >_2) / \sqrt{2}$. The matrix element reads $\mathcal{M}_{pro} = D (|e^* | e > + |\mu^* | \mu >) / \sqrt{2}$. We model the CM scalar products with one recoil as $\langle \xi^* | \Upsilon > = \alpha_0 < \xi | \Upsilon >$ with $\alpha_0$ a constant coefficient. As the recoil is in general a small effect we take $\alpha_0 = 0.9$. Moreover, we assume that it is independent of the CM state. With this parametrization we have $\mathcal{M}_{pro} = \sqrt{2} \alpha_0 D$. 

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In addition to the parametrization for scalar products with a recoil we must also specify these with two recoils. We take \( <\xi^*|\Upsilon^*>=\alpha^2 <\xi|\Upsilon> \), where the coefficient has the form \( \alpha = \alpha_0 + (1 - \alpha_0)c \). With this choice we guarantee that if initially \( |\xi> = |\Upsilon> \), at the end we will have \( <\xi^*|\Upsilon^*> = 1 \). If this condition would not be preserved we would obtain erroneous results in the limit \( |\xi> \rightarrow |\Upsilon> \). In the particular case that \( \xi \) and \( \Upsilon \) do not vary, \( <\xi|\Upsilon> \) is constant, we cannot take that limit and we must use \( \alpha = \alpha_0 + (1 - \alpha_0) <\xi|\Upsilon> \). We have extensively checked that this choice leads to the correct results in the limits of complete overlapping, absence of multi-particle superposition, ... Any other election gives errors in some of these limiting situations.

![Figure 2: Relative single absorption rate R versus initial overlap c, both in arbitrary units, for the choices (i) left and (ii) right. The red and blue curves correspond respectively to bosons and fermions. The continuous, dashed and dotted lines represent the cases \( a = 1, 0.8 \) and \( 1/\sqrt{2} \), with \( |a|^2 + |b|^2 = 1 \). In order to see more clearly the case with only symmetrization we include on the left top corner of (i) the curves for this choice alone.](image)

We represent the relative rate for four different choices of the CM overlaps. For each choice we consider three different values of \( a \) and \( b \), that is, of the relative weight of the two-atom states in the superposition. One of the three cases is \( b = 0 \) \( (a = 1) \), which corresponds to symmetrization without superposition. This way we can compare the effects generated by symmetrization alone with those present when symmetrization and superposition are simultaneously taken into account. The four choices are: (i) \( <\psi|\phi>=c \), with \( c \) real and varying in the interval \([0,1]\), \( <\psi|\phi>=c \), \( <\varphi|\chi>=0.9 \), \( <\psi|\varphi>=<\psi|\chi> \).
\[ <\phi|\varphi> = <\phi|\psi><\psi|\varphi> \quad \text{and} \quad <\phi|\chi> = <\phi|\varphi><\varphi|\chi>. \] The last three relations are equal for the rest of the cases. 

(ii) \[ <\psi|\phi> = 0.8, <\psi|\varphi> = c, <\varphi|\chi> = 0.9. \]

(iii) \[ <\psi|\phi> = c, <\psi|\varphi> = 0.9, <\varphi|\chi> = c. \]

(iv) \[ <\psi|\phi> = 0.8, <\psi|\varphi> = 0.9, <\varphi|\chi> = c. \]

We represent choices (i) and (ii) in Fig. 2 and (iii) and (iv) in Fig. 3. When only symmetrization is taken into account \((b = 0)\) we have different behaviors for bosons and fermions in (i) and (iii) (in (ii) and (iv) the overlap between \(\psi\) and \(\phi\) is constant and there are no variations). For null overlap there are no differences between them because the exchange effects disappear. When the overlap is not null the situation changes. The relative rate for fermions steadily decreases, progressively reaching larger deviations with respect to the values associated with product states. In contrast, the probability for bosons increases for small overlaps reaching its maximum at intermediate values of \(c\) and decreasing later. For \(c = 1\) the initial bosonic state is \(|\psi_1\rangle|\psi_2\rangle\), a product one with no effects of symmetrization. With the exception of \(c = 0\) and \(c = 1\) the absorption rates of bosonic states are larger than those of product ones.

![Figure 3: The same as Fig. 2 for (iii) left and (iv) right. The continuous, dashed and dotted lines correspond respectively to \(a = 1, b = 0; a = 0.8, b = 0.2\) and \(a = b = 0.5\) for (iii) and the same values of \(a\) with \(|a|^2 + |b|^2 = 1\) for (iv).](image-url)

When multi-particle superposition is simultaneously taken into account the general picture notoriously changes. First of all, note that in various cases the relative rates of bosons and fermions for \(c = 0\) are not equal. This is so because in these cases there is overlap between some states of the superposition and the
exchange effects are present even in this limit. For fermions in (ii) the relative rate does not depend on the overlap, that is, the antisymmetrization effects have been canceled. Multi-particle superposition can inhibit the antisymmetrization effects. This situation corresponds to fixed overlap between the two states of each term of the multi-particle superposition. In contrast, this inhibition does not happen for bosons. Note that, although constant, the values of the fermionic rates are very different depending on the values of \( a \) and \( b \).

We have also observed (not represented here) the behavior that can be considered complementary of the above inhibition. In the case (iii) with \( |a|^2 + |b|^2 = 1 \), for any choice of \( a \) the fermion curves coincide with that of \( a = 1 \). There are antisymmetrization effects, but the superposition ones have been canceled. For both bosons and fermions the relative rates can be larger or smaller than one. Another notorious difference between bosons and fermions is that, in the presence of entanglement, in the first case the relative rates always increase with the overlap. In contrast, in the second case can increase (i), decrease (iii and iv) or be constant (ii).

5 Excluded states

In addition to the examples presented in the previous section we have analyzed many other situations. One specially interesting occurs for the set of initial states \( |\phi> = c|\psi> + d|\zeta> \), \( |\varphi> = e|\psi> + f|\zeta> \) and \( |\chi> = g|\psi> + h|\zeta> \), where \( \zeta \) is a state orthogonal to \( \psi \), \( <\zeta|\psi>=0 \). All the coefficients obey the normalization conditions \( |c|^2 + |d|^2 = 1 \), \( |e|^2 + |f|^2 = 1 \) and \( |g|^2 + |h|^2 = 1 \). Note that we do not include the internal states of the atoms because they are irrelevant here.

We represent for fermions the particular case \( e = f = 1/\sqrt{2} \) and \( |\chi> = c|\varphi> + d|\varphi^+> \), with \( \varphi^+ \) orthogonal to \( \varphi \). If we take \( |\varphi^+> = (|\psi> - |\zeta>) / \sqrt{2} \), the coefficients are \( g = (c + d)/\sqrt{2} \) and \( h = (c - d)/\sqrt{2} \). The results are presented in Fig. 4. For superpositions with equal weights we observe a non-regular behavior. This behavior can be explained analytically invoking the unnormalized form of Eq. (1) for our set of states:

\[
|\tilde{\Phi}> = (ad + b(\bar{c} - fg))(|\psi> + |\zeta> - |\zeta> + |\psi>) \tag{11}
\]

Replacing the values used in the graphical representation we see that for \( a = b \), \( |\tilde{\Phi}> = 0 \). For the normalized state we have \( |\tilde{\Phi}> / ||\tilde{\Phi}>|| = 0/0 \), an undefined form that is typical from excluded states.

This is the reason for the observed non-regular behavior. When we introduce this state in the graphical representation program, it deals with an undefined expression of the type \( 0/0 \). However, because of numerical round-off errors can give an output (the pieces of line that are represented), which anyway are null. In contrast, for other values the programm gets an undefined answer and cannot represent a value.

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This is not the only state with this form. From Eq. (11) we can see that any state whose coefficients obey the relation \( ad + beh - bfg = 0 \) has the same undefined form. As a particular case, we have that when there is not superposition, \( b = 0 \), the above condition reads \( d = 0 \), or equivalently, \( \phi = \psi \). For non-entangled states this is the usual Pauli’s condition for exclusion. We have found a set of excluded entangled states. The exclusion condition in this case is different from the usual one and does not require the one-particle states to be equal. The Pauli-type condition is only recovered for non-entangled states.

Figure 4: The same as in Fig. 2 for fermions in the states discussed in this section. The continuous, dashed and dotted lines correspond to \( a = 0.64, 0.67 \) and \( 1/\sqrt{2} \), with \( |a|^2 + |b|^2 = 1 \).

The possibility of excluded entangled states beyond the scope of Pauli’s principle [14] has been previously considered in more theoretical grounds [15]. The fundamental idea in that proposal is that in entangled systems you can define the multi-fermion state but not the one-fermion ones and, consequently, the exclusion conditions must be based on different considerations. The results of this paper show that there are non-standard excluded entangled states in realistic systems.

6 Conclusions

The interplay between symmetrization and entanglement is a subtle and difficult subject. Topics like the definition of entanglement measures in systems of identical particles or the conversion of the identity correlations into useful entanglement have deserved a lot of attention, but there are other problems, like the physical modifications associated with their joint effects, that remain almost unexplored.
We have shown that there is not an universal behavior of the absorption rates when symmetrization and multi-particle superposition act at once. Depending on the parameters of the problem, the coefficient $a$ and $b$ in the superposition and the choice of overlaps between the spatial CM states, we have a big variety of absorption rate patterns. The presence of two-particle superpositions drastically changes the analytical form of the symmetrized absorption rates of identical particles. The observed effects cannot be considered as the addition of those associated with the contributions of symmetrization and superposition. One could associate the exchange terms (those with the sign $\pm$) with the effects of symmetrization and the rest with superposition. However, the normalization coefficients contain exchange- and non-exchange-type contributions and the above separation is not possible. We must also remark the notorious differences between the behavior of bosons and fermions in all the cases.

We have also found excluded entangled states beyond the scope of Pauli’s principle. From the experimental point of view, these states could be tested noting that fixing the values of the overlaps and approaching the limit $a = b$ we would see a progressive decreasing of the absorption rates towards a null value (see Fig. 4). The arrangement discussed in the second section seems to be well suited to implement the experimental scheme.

References

[1] J. Schliemann, J. I. Cirac, M. Kus, M. Lewenstein, D. Loss, Phys. Rev. A 64 (2001) 022303.
[2] G. C. Ghirardi, L. Reusch, T. Weber, J. Stat. Phys. 108 (2002) 49-122.
[3] M. C. Tichy, F. Mintert, A. Buchleitner, J. Phys. B 44 (2011) 192001.
[4] F. Benatti, R. Floreanini, K. Titimbo, Open Syst. Inf. Dyn. 21 (2014) 1440003.
[5] R. Lo Franco, G. Compagno, Sci. Rep. 6 (2016) 20603.
[6] D. Cavalcanti, L. M. Malard, F. M. Matinaga, M. O. T. Cunha, M. F. Santos, Phys. Rev. B 76 (2007) 113304.
[7] N. Killoran, M. Cramer, M. B. Plenio, Phys. Rev. Lett. 112 (2014) 150501.
[8] R. Lo Franco, G. Compagno, Phys. Rev. Lett. 120 (2018) 240403.
[9] P. Sancho, Ann. Phys. 336 (2013) 482-488.
[10] P. Sancho, Eur. J. Phys. 40 (2019) 015404.
[11] Z. Ficek, R. Tanaš, Phys. Rep. 372 (2002) 369-443.
[12] H. You, S. M. Hendrickson, J. D. Franson, Phys. Rev. A 78 (2008) 053803.
[13] J. P. Dowling, Contemp. Phys. 49 (2008) 125-143.
[14] W. Pauli, Z. Physik 31 (1925) 765-783.
[15] P. Sancho, arXiv:1806.02631 [quant-ph] (2018).
[16] T. Tanabe, T. Odagiri, M. Nakano, Y. Kumagai, I. H. Suzuki, M. Kitajima, N. Kouchi, Phys. Rev. A 82 (2010) 040101(R).
[17] X. Urbain, A. Dochain, C. Lauzin, B. Fabret, J. Phys. Conf. Ser. 635 (2015) 112085.
[18] T. Yu, J. H. Eberly, Science 323 (2009) 598-601.
[19] P. Sancho, Phys. Rev. A 95 (2017) 032116.