Positronium Decays with Dark $Z$ and Fermionic Dark Matter

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We investigate the invisible decay of positronium to probe the fermionic light dark matter mediated by the dark $Z$ boson. Too tiny is the invisible decay rate of positronium through weak interaction in the standard model to be detected in the experiment. We show that it can be enhanced to be observed in the future if the dark matter is lighter than the electron in the dark $Z$ model. We also compute the relic abundance of such light dark matter and discuss the Big Bang Nucleosynthesis constraint with an alternative thermal history scenario.

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I. INTRODUCTION

Positronium (Ps) is a leptonic atom which consists of an electron-positron bound state and the lightest bound state in the standard model (SM). The wavefunction of the Ps in the leading order is obtained by solving Schrödinger equation of the hydrogen atom with the reduced mass equal to half of the electron mass. The lowest states of Ps are the spin singlet ($^1S_0$) and the spin triplet states ($^3S_1$), called para-positronium (p-Ps) and ortho-positronium (o-Ps), respectively. The energy spectra and lifetimes of Ps states can be calculated in QED with high accuracy since theoretical study of Ps is free from hadronic uncertainty. Combined with precise measurements, the study of Ps allows us to test our understanding of bound state structure of QED [1,2].

The dominant decay channel of the p-Ps is two-photon decay with the lifetime $\tau = 798.6060(2)^{-1} \mu s$ [3], while that of the o-Ps three-photon decay with the lifetime $\tau = (7.0380 - 7.0417)^{-1} \mu s$ [4]. The triplet state o-Ps may decay into neutrino pairs via weak interaction to leave invisible final state in the SM. However the SM invisible decay rate of the o-Ps is extremely small such that the branching ratios $\sim 6.2 \times 10^{-18}$ (for $\nu_e$) and $\sim 9.5 \times 10^{-21}$ (for $\nu_{\mu,\tau}$) [5]. The p-Ps may also decay into neutrino pairs via weak interaction, but its decay rates are much smaller than those of the o-Ps because the weak decay rates into neutrinos for the p-Ps are proportional to the squares of the neutrino masses. Thus the Ps invisible decays can be a good testing laboratory of the new physics beyond the SM in both o-Ps and p-Ps decays. Actually sizable invisible decays of o-Ps are predicted in many new physics models, e.g. millicharged particles, paraphotons etc. [6–8].

Experimental searches for invisible decays of Ps have been performed but observed no signals so far. The most stringent upper bound on the o-Ps invisible decay branching ratio is set to be [9]

$$\text{Br}(o-Ps \rightarrow \text{invisible}) < 4.2 \times 10^{-7}$$

at 90% C.L.. Recently Vigo et al. [10] also set the model independent upper limit, $\text{Br}(P_{s} \rightarrow \text{invisible}) < 1.7 \times 10^{-5}$ at 90 % C.L. in an alternative experiment. Note that decays of o-Ps are more manageable in experiments due to its longer lifetime.

One of the most important challenges in particle physics at present is resolving nature and origin of nonbaryonic dark matter (DM). Many theoretical models have predicted the Weakly Interacting Massive Particles (WIMP) of mass of the electroweak scale as DM candidates. The WIMPs are assumed to be produced by the thermal freeze-out mechanism in the early Universe and can be fermions or scalars or vector bosons depending on the model. As no signals of the WIMP have been observed in the high energy colliders and direct detection experiments, however, much interest is devoted to the possibility of light dark matter (LDM) in the keV–MeV mass range. [6].

In this paper, we consider a fermionic LDM model which is suggested in Ref [11] where the hidden sector is mediated by an additional SU(2) scalar doublet. When we consider singlet fermions as DM candidates, usually singlet scalars are introduced together as a mediator between hidden sector and the SM and an additional mass scale also introduced by the vacuum expectation value (VEV) of the singlet scalar [12–16]. However, we take the scalar doublet as a mediator, neither singlet scalar nor new mass scale is required. Instead the U(1)$_X$ symmetry is required for the fermionic DM candidate to couple to the mediator scalar doublet. Thus the hidden sector in this model is QED-like, which consists of a SM singlet fermion and a hidden U(1)$_X$ gauge field. Since the mediator scalar is the SU(2) doublet and also carries the hidden U(1)$_X$ charge, the U(1)$_X$ symmetry is broken by the electroweak symmetry breaking (EWSB) and the corresponding gauge boson gets its own mass by the electroweak VEV. The new massive gauge boson is mixed with the Z boson and is called the dark Z boson. This model satisfies strong electroweak constraints from the low energy experiments and high energy collider phenomenology on neutral current (NC) interactions.

It turns out that this model favors rather light dark Z boson and fermionic DM. If the DM candidate is lighter than the electron, the o-Ps can annihilate into the DM pair through the dark Z boson and the final state is invisible in this model. The dark Z boson being light, predictions of invisible decay rates of o-Ps into the DM pair can be much enhanced compared to the weak invisible decay rate in the SM, which is a clear signal of the new physics.

On the other hand, LDM with the mass less than MeV suffers from tension with cosmological observables when it is in thermal equilibrium with the bath of the SM particles in the early Universe [17–19]. It is because the temperature where the BBN started is affected by extra relativistic degrees of freedom and the predictions of the abundance of light elements would be altered. Recently Berlin and Blinov reported that sub-MeV LDM is allowed when the equilibrium of the light state with the SM is later than the neutrino decoupling [20,21]. We take this scenario to accept our fermionic LDM candidate lighter than the electron here.

In this paper we investigate the exotic decays of Ps including invisible decays and single photon decays when the DM candidate is lighter than the electron in the singlet fermionic DM model with hidden U(1)$_X$ gauge group and an additional scalar doublet mediator. The outline of this paper is as follows: We briefly describe the model with electroweak constraints in section 2. In section 3, we present the predictions of the positronium decays in this model. The dark matter phenomenology is elaborated in relation to the positronium decays in section 4. We finally conclude in section 5.
II. DARK Z PHENOMENOLOGY

The hidden sector of the model consists of a SM gauge singlet Dirac fermion $\psi_X$ as a DM candidate and a gauge field for a new U(1)$_X$ gauge symmetry. We assume no kinetic mixing between the hidden U(1)$_X$ and the SM U(1)$_Y$ and the gauge charge of $\psi_X$ to be (1, 1, 0, X) based on SU(3)$_c \times$ SU(2)$_L \times$ U(1)$_Y \times$ U(1)$_X$ gauge group. The SM fields do not carry the U(1)$_X$ gauge charge and do not couple to the hidden sector fermion directly.

We introduce an additional SU(2) scalar doublet $H_1$ as a mediator field between the hidden sector and the SM sector and the content of Higgs fields is the 2 Higgs Doublet Model (2HDM). There are three free parameters, U(1)$_X$ gauge coupling $g_X$ and the U(1)$_X$ charges of $H_1$ and $\psi_X$, but they just appear in the form of $g_X X$. Thus we have freedom to fix only one of three parameters. We take the U(1)$_X$ charge of $H_1$ to be 1/2 for convenience. Then we let the charge assignments of $H_1$ and the SM-like Higgs doublet $H_2$ be (1, 2, 1, 2) for $H_1$, and (1, 2, 1, 0) for $H_2$, respectively. Due to the U(1)$_X$ charge, $H_1$ does not couple to the SM fermions and the $H_2$ couplings to the SM fermions are same as the SM Yukawa interactions as in the Higgs sector in the 2HDM of type I. The Higgs sector lagrangian is given by

$$\mathcal{L}_H = (D^\mu H_1)\partial_\mu H_1 + (D^\mu H_2)\partial_\mu H_2 - V(H_1, H_2) + \mathcal{L}_Y(H_2),$$

where $V(H_1, H_2)$ is the Higgs potential, $\mathcal{L}_Y$ the Yukawa couplings, and the covariant derivative defined by

$$D^\mu = \partial^\mu + igW^{\mu a}T^a + igB^\mu Y + ig_X A'^\mu_X X$$

with $T^a$ ($a = 1, 2, 3$) being the SU(2) generators. Here $X$ is the hidden U(1)$_X$ charge operator and $A'^\mu_X$ the corresponding gauge field. The Higgs potential is given by

$$V(H_1, H_2) = \mu_1^2 H_1^\dagger H_1 + \mu_2^2 H_2^\dagger H_2 + \lambda_1 (H_1^\dagger H_1)^2 + \lambda_2 (H_2^\dagger H_2)^2$$

$$+ \lambda_3 (H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_4 (H_1^\dagger H_2)(H_2^\dagger H_1),$$

where $\mu_1^2, \mu_2^2, \lambda_1, \lambda_2, \lambda_3, \lambda_4$ are dimension 2 couplings for quadratic terms while $\lambda_{1,2,3,4}$ dimensionless quartic couplings. Note that the soft $Z_2$ symmetry breaking terms of the $H_1^\dagger H_2$ quadratic term and the quartic term $(H_1^\dagger H_2)^2$ with $\lambda_5$ coupling are forbidden by the U(1)$_X$ gauge symmetry.

The physical gauge bosons after the EWSB, photon, $Z$ boson and the extra $Z$ boson ($Z'$) are defined by

$$A_X = c_X Z' + s_X Z,$$
$$W_3 = -s_X c_W Z' + c_X c_W Z + s_W A,$$
$$B = s_X s_W Z' - c_X s_W Z + c_W A,$$

where $s_W = \sin \theta_W = g' / \sqrt{g'^2 + g'^2}$, $c_W = \cos \theta_W$ is the Weinberg angle, and $s_X = \sin \theta_X$, $c_X = \cos \theta_X$ the $Z - Z'$ mixing defined by

$$\tan 2\theta_X = \frac{-2g_X g' s_W \cos^2 \beta}{g'^2 - g_X^2 s_W^2 \cos^2 \beta}.$$  

The VEVs of two Higgs doublets, $\langle H_i \rangle = (0, v_i / \sqrt{2})^T$ with $i = 1, 2$, define tan $\beta = v_2 / v_1$. We have the neutral gauge boson masses

$$m_{Z, Z'}^2 = \frac{1}{8} \left( g_X^2 v_1^2 + (g^2 + g'^2) v^2 \pm \sqrt{ (g_X^2 v_1^2 - (g^2 + g'^2) v^2)^2 + 4 g_X^2 (g^2 + g'^2) v_1^2 } \right),$$

where $v^2 = v_1^2 + v_2^2$. Note that only two mixing angles are required to diagonalize the neutral gauge boson mass matrix in this model.

The NC interactions of $Z$ and $Z'$ bosons are given by

$$\mathcal{L}_{NC} = (c_X Z'^\dagger + s_X Z'^{2}) \left( g_Y \tilde{f} \gamma_{\mu} f + g_A \tilde{f} \gamma_{\mu} \gamma^5 f \right),$$

where $g_Y$ and $g_A$ are the SM $Z$ couplings to the fermions. Since the $Z'$ interactions are same as the SM $Z$ interactions except for the overall suppression by $s_X$, we call it the dark $Z$ boson.

The $Z$ boson mass is shifted in this model such that

$$m_Z^2 = m_W^2 c_W^2 c_X^2 - m_Z^2 s_X^2 c_X^2.$$
which leads to the shift of the $\rho$ parameter

$$\frac{1}{\rho} = \frac{m_Z^2 c_W^2}{m_W^2} = \frac{1}{c_X^2} - \frac{m_\pi^2 s_X^2}{m_W^2} \approx 1 + s_X^2 \left( 1 - \frac{m_Z^2 c_W^2}{m_W^2} \right) \equiv 1 + \Delta \rho_{Z'}$$

in the leading order of $s_X^2$. Moreover there exist also new scalar contributions to the $\Delta \rho$ in this model, given by

$$\Delta \rho^{(1)}_{NS} = \frac{\alpha}{16 \pi m_W^2 s_W^2} \left( m_{\mu}^2 - \frac{m_H^2 m_\mu^2}{m_{Z'}^2 - m_{\mu}^2} \log \left( \frac{m_H^2}{m_{\mu}^2} \right) \right)$$

at one-loop level [22]. Here $m_H$ is the SM Higgs mass and $m_{\pm}$ is the charged Higgs mass. Then $\Delta \rho$ predicted in this model is $\Delta \rho_{New} = \Delta \rho_{Z'} + \Delta \rho_{NS}^{(1)}$. The present limit on $\Delta \rho$ reads from the measurements $\alpha(m_Z)^{-1} = 127.955 \pm 0.010$ and Peskin-Takeuchi $T$ value $T = 0.07 \pm 0.12$ [23] as

$$-0.00039 < \Delta \rho < 0.001485.$$  

When $m_{\pm} \geq 120$ GeV, $\Delta \rho_{NS}$ exceeds 0.001485 and no points of $(m_{Z'}$, $-s_X)$ are allowed. Consequently 120 GeV is an upper limit on $m_{\pm}$ in this model. Actually $\Delta \rho$ is insensitive to $m_{\pm}$ below 120 GeV.

The precise measurement of the atomic parity violation (APV) also provides a strong constraint on the exotic NC interactions. The dark $Z$ exchange also contributes to the shift of the weak charge

$$Q_W = Q_{SM}^{W} \left( 1 + \frac{m_Z^2}{m_{Z'}^2} s_X^2 \right)$$

in the leading order of $s_X$. The SM prediction of the Cs atom is $Q_{SM}^{W} = -73.16 \pm 0.05$ [24, 25], and the present experimental value is $Q_{exp}^{W} = -73.16 \pm 0.35$ [26], which yields the bound

$$\frac{m_Z^2}{m_{Z'}^2} s_X^2 \leq 0.006$$

at 90 % C.L.. The $\Delta \rho$ and APV constraints are depicted in Fig. 1 by the red region and red line, respectively.

We assume that the hidden sector lagrangian is QED-like,

$$\mathcal{L}_{hs} = -\frac{1}{4} F_{X \mu \nu} F_{X \mu \nu} + \bar{\psi}_X i \gamma^\mu D_\mu \psi_X - m_X \bar{\psi}_X \psi_X,$$

where

$$D^\mu = \partial^\mu + ig_X A_\mu^X.$$  

After the EWSB, the $U(1)_X$ is broken and $\psi_X$ is connected to the SM through $Z$ and dark $Z$ bosons, given in Eq. (5). Note that the fermion mass $m_X$ is a free parameter. We will set $m_X < m_e$ in the next section.

### III. POSITRONIUM DECAYS

In this model Ps can annihilate into the DM fermion pair through the dark $Z$ boson when the DM fermion is lighter than the electron, $m_X < m_e$. We obtain the dark $Z$ contribution to the invisible decays as

$$\Gamma(\alpha\text{-Ps} \to Z' \to \bar{\psi}_X \psi_X) = \frac{1}{12 \pi m_e^3} s_X^2 c_X^2 d_X^2 (g_X X)^2 \left[ \left( 1 - \frac{m_Z^2}{4 m_e^2} \right)^2 + \frac{m_Z^2 \Gamma_{Z'}^2}{16 m_e^4} \right]^{-2} \times \sqrt{1 - \frac{m_Z^2}{2 m_e^2}} \frac{m_X^2}{2 m_e^2} |\psi(0)|^2,$$

where the square of the Ps wavefunction at the origin is $|\psi(0)|^2 = m_0^2 \alpha^3 / 8 \pi$. The dark $Z$ couplings to the SM particles are suppressed by the mixing angle $s_X$ while not suppressed to the DM fermions. Thus the decay width of the dark $Z$ boson is dominated by $Z' \to \bar{\psi}_X \psi_X$, where $\Gamma_{Z'} = \Gamma(Z' \to \bar{\psi}_X \psi_X) \sim 10^{-2}$ MeV, when $m_{Z'} > 2m_X$.

On the other hand, if $m_{Z'} < 2m_X$, then only the neutrino channels are allowed and $\Gamma_{Z'} = 3 \Gamma(Z' \to \nu \bar{\nu}) = 3 \Gamma(Z \to \nu \bar{\nu}) (s_X / c_X)^2 (m_{Z'} / m_Z) \leq 10^{-13}$ MeV at most. Therefore we can neglect $\Gamma_{Z'}$ to calculate the invisible decay rate except for around the resonance region.
FIG. 1. Exclusion regions by the positronium invisible decays in the plane of the model parameter \((m_{Z'}, |\sin \theta_X|)\). The magenta region is excluded by the current limit \(\text{Br}(o-Ps \to \text{invisible}) < 4.2 \times 10^{-7}\), the green region by the future limit \(< 10^{-9}\), and the blue region by the future limit \(< 10^{-11}\). Electroweak constraints of \(\Delta \rho\) is the red region (overlapped by the Ps exclusion regions) and the APV constraints above the red line.

The branching ratio of the o-Ps invisible decay is

\[
\text{Br}(o-Ps \to \text{invisible}) = \frac{\Gamma(o-Ps \to \text{invisible})}{\Gamma_0 + \Gamma(o-Ps \to \text{invisible})},
\]

where the SM decay rate \(\Gamma_0\) is dominated by the three photon decay given by

\[
\Gamma(o-Ps \to \gamma\gamma\gamma) = \frac{2(\pi^2 - 9)m_\gamma\alpha^6}{9\pi} \left(1 - 10.28661 \frac{\alpha}{\pi} + \mathcal{O}(\alpha^2)\right) \approx 7.0382 \mu s^{-1}.
\]

Figure 1 depicts the exclusive region by the invisible positronium decays with the present data of Eq. (1) and the future experimental reaches \(\text{Br}(o-Ps \to \text{invisible}) < 10^{-9}\) and \(< 10^{-11}\)\[27\]. This model has two more free parameters on the DM sector, the hidden \(U(1)_X\) charge and the mass of \(\psi_X\). We take \((g_X X)^2 = 2\pi\) and \(m_X = m_e / 2\) in Fig. 1 as benchmark values which are chosen to probe the parameter region below the APV bound and to accommodate the DM phenomenology as will be discussed in the next section.

The para-positronium dominantly decays into two photons. In this model, p-Ps can decay into a photon and a dark \(Z\) boson, then we will observe the single photon decay process. The decay rate into \(\gamma Z'\) is

\[
\Gamma(p-Ps \to \gamma Z') = \frac{8\alpha}{3m_{Z'}^2} s_X g_A^2 \left(1 - \frac{m_{Z'}^2}{4m_e^2}\right) \frac{1 + m_{Z'}^2}{4m_e^2} |\psi(0)|^2.
\]

Meanwhile o-Ps can also decay into the \(\gamma Z'\) final state due to the axial coupling of the dark \(Z\) such as

\[
\Gamma(o-Ps \to \gamma Z') = \frac{16\alpha}{3m_{Z'}^2} s_X g_A^2 \left(1 - \frac{m_{Z'}^2}{4m_e^2}\right) \frac{1 + m_{Z'}^2}{4m_e^2} |\psi(0)|^2.
\]

We estimate the branching ratios \(\text{Br}(p-Ps \to \gamma Z') < 10^{-12}\) and \(\text{Br}(o-Ps \to \gamma Z') < 10^{-13}\) with the allowed values of \((m_{Z'}, |\sin \theta_X|)\) given in Fig. 1 which are smaller than the future experimental reaches considered in Fig. 1. Neither the experimental limits for the single photon decay of p-Ps nor those of o-Ps have not been reported yet.

IV. DARK MATTER PHENOMENOLOGY

We calculate the relic abundance \(\Omega_{CDM} h^2\) in the thermal freeze-out scenario using the micrOMEGAs\[28\] with the allowed values of parameters \((m_{Z'}, |\sin \theta_X|)\) given in the previous section and show that the model prediction can
FIG. 2. Behaviors of relic density for a few benchmark points, as functions of the interaction strength between DM pairs and $Z'$ boson.

accommodate the present measurements with high precision

$$\Omega_{\text{CDM}}h^2 = 0.1186 \pm 0.0020$$  \hspace{1cm} (22)

from the anisotropy of the cosmic microwave background (CMB) and of the spatial distribution of galaxies. Since we are interested in the parameter set with which the dark sector has a sizable effect on the positronium physics, we consider a light DM (LDM) scenario that constrains the DM mass smaller than the electron mass. In this case, the model is barely constrained by both the direct [29–32] and indirect [33, 34] detection experiments as discussed in Ref. [11].

We demonstrate generic behaviors of $\Omega_{\text{CDM}}h^2$ for some benchmark points near the resonance region of Ps decay in Fig. 2. We find that the relic abundance is very sensitive to the value of $g_X X$ in the resonance region around $m_{Z'} = 1$ MeV. In Fig. 3, we show the branching ratios of the o-Ps invisible decay for points that accommodate the present relic density with $3\sigma$ range as a function of the $Z'$ mass in unit of MeV. For this analysis, we scan the DM mass less than 0.5 MeV and the $Z'$ mass in the range 0.01 − 4 MeV to investigate possible correlations between the DM and positronium physics. As shown in Fig. 3, the branching ratio of o-Ps invisible decay can be enhanced significantly in the resonance region around $m_{Z'} \simeq 1$ MeV. Though relic density depends on various model parameters, it is largely determined by the interaction strength between DM pair and $Z'$ boson, $g_X X$, generically. Perturbativity bound on the size of the coupling between the DM pair and $Z'$ is also imposed.

We also compute the branching ratios of o-Ps and p-Ps decays into a photon and a $Z'$ boson, that will be observed as single photon decays, with parameter sets allowed by the electroweak constraints and DM relic density and show results in Fig. 4. The predicted branching ratios of o-Ps and p-Ps are at most $O(10^{-12})$ and $O(10^{-13})$ as estimated in the previous section, which are rather far from the reach of precision for the search of the invisible decays of positronium in near future.

A few comments are in order. Generically, dark matter mass below 1 MeV is disfavored by BBN and CMB data through modifying the effective number of neutrino species when employing conventional thermal freeze-out scenarios [18, 35, 36]. Recently the authors of Ref. [20, 21] suggested an alternative cosmological scenarios that can alleviate the problem for sub-MeV DM. It is dubbed as ‘delayed equilibration scenario’ in which sub-MeV DM thermalizes with the SM sector below the neutrino-photon decoupling temperature. In this scenario, the SM bath is cooled down by the equilibration and is heated again by the freeze-out of DM. In the dark $Z$ model, a DM pair can convert to an electron-positron pair through $Z'$ to be constrained from the supernova. However after comprehensive analysis about the supernova constraints on the dark photon portal models the authors in Ref. [37] found that such constraints can be evaded for a dark sector with dark fine structure constant $\alpha_D \gtrsim 10^{-7}$. We find that the dark $Z$ model, in which DM coupling to $Z'$ is taken to be large enough to accommodate the present relic density as in Fig. 2, can avoid the supernova constraints. We conclude that the parameter region studied in this paper is still valid.
FIG. 3. Branching ratios of the o-Ps invisible decay for points that accommodate the observed relic density of DM within $3\sigma$ range as a function of $Z'$ mass in unit of MeV. In the plot, two distinct kinds of points are overlapped. One is the case of $m_{Z'} < 2m_X$, where the decay width of $Z'$ is relatively suppressed compared with the opposite case, $m_{Z'} > 2m_X$. Green horizontal line corresponds to the present limit on the branching ratio of the o-Ps invisible decay, $4.2 \times 10^{-7}$.

FIG. 4. Branching ratios of o-Ps and p-Ps decay into a photon and a $Z'$ boson for points that accommodate the observed relic density of DM within $3\sigma$ range as a function of the $Z'$ mass in unit of MeV. They appear as the conversion of positronium to single photon.

V. CONCLUDING REMARKS

We study the invisible decay of o-Ps into fermionic DM pair through the dark $Z$ boson when the fermionic DM particle is lighter than the electron. Still the predictions of the Ps invisible decay rates in the dark $Z$ model are less than the present experimental reach but much enhanced compared to the SM predictions through the weak interaction. The fermionic LDM scenario with the light mediator can also satisfy the relic abundance and is not constrained by the present direct detection experiment and indirect observations. We discuss that the LDM model discussed in this work can be accommodated in the recent delayed equilibration scenario. In conclusion the Ps invisible decay provides attractive phenomenology of the dark $Z$ model with fermionic DM which is independent of collider and dark matter phenomenology.
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