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QCD in hadronic B decays *

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The perturbative QCD approach is based on \( k_T \) factorization, including the Sudakov form factors so that to avoid the endpoint singularity. In this approach, we calculate the charmless B decays like \( B \to \pi\pi \) decays etc. to produce the right number of branching ratios and also CP asymmetry parameters. For final states with at least one charmed meson, like \( B \to D\pi \) decays, our results also agree with the experiments.

Keywords: perturbative QCD; hadronic B decays; CP violation.

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1. Introduction

The hadronic B decays are important for the CKM matrix elements measurements and CP violation detection. Understanding nonleptonic B meson decays is crucial for testing the standard model, and also for uncovering the signal of new physics. The simplest case is two-body hadronic B meson decays, for which Bauer, Stech and Wirbel (BSW) proposed the naive factorization assumption (FA) in their pioneering work. Considerable progress, including generalized FA and QCD-improved FA (QCDF) has been made since this proposal. On the other hand, technique to analyze hard exclusive hadronic scattering was developed by Brodsky and Lepage, based on collinear factorization theorem in perturbative QCD (PQCD). A modified framework based on \( k_T \) factorization theorem was then given, and extended to exclusive B meson decays. The infrared finiteness and gauge invariance of \( k_T \) factorization theorem was shown explicitly. Using this so-called PQCD approach, we have investigated dynamics of nonleptonic B meson decays.

Although the predictions of branching ratios agree well with experiments in most cases, there are still some theoretical points unclear in FA and QCDF. First, it relies strongly on the form factors, which cannot be calculated by FA itself. Secondly, the generalized FA shows that the non-factorizable contributions are important in a group of channels, which can not be done reliably in FA and QCDF. The reason

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of this large non-factorizable contribution needs more theoretical studies. Thirdly, the strong phase, which is important for the CP violation prediction, is quite sensitive to various approaches. The mechanism of this strong phase is quite different for different method, and give quite different results. The recent experimental results can make a test for the validity of these approaches.

2. Formalism of PQCD Approach

In this section, we will introduce the idea of PQCD approach. The three scale PQCD factorization theorem has been developed for non-leptonic heavy meson decays \cite{9}, based on the formalism by Brodsky and Lepage \cite{5}, and Botts and Sterman \cite{6}. In the non-leptonic two body $B$ decays, the $B$ meson is heavy, sitting at rest. It decays into two light mesons with large momenta. Therefore the light mesons are moving very fast in the rest frame of $B$ meson. In this case, the short distance hard dynamic dominates the decay amplitude. The reasons can be ordered as: first, because there are not many resonance near the energy region of $B$ mass, so it is reasonable to assume that final state interaction is not important in two-body $B$ decays. Second, With the final light mesons moving very fast, there must be a hard gluon to kick the light spectator quark (with small momentum) in the $B$ meson to form a fast moving light meson. So the dominant diagram in this theoretical picture is that one hard gluon from the spectator quark connecting with the other quarks in the four quark operator of the weak interaction. Unlike the usual factorization approach, the hard part of the PQCD approach consists of six quarks rather than four. We thus call it six-quark operators or six-quark effective theory. There are also soft (soft and collinear) gluon exchanges between quarks. Summing over those leading soft contributions gives a Sudakov form factor, which suppresses the soft contribution to be dominant. Therefore, it makes the PQCD reliable in calculating the non-leptonic decays. With the Sudakov resummation, we can include the leading double logarithms for all loop diagrams, in association with the soft contribution.

There are three different scales in the $B$ meson non-leptonic decay. The QCD corrections to the four quark operators are usually summed by the renormalization group equation \cite{15}. This has already been done to the leading logarithm and next-to-leading order for years. Since the $b$ quark decay scale $m_b$ is much smaller than the electroweak scale $m_W$, the QCD corrections are non-negligible. The third scale $1/b$ involved in the $B$ meson exclusive decays is usually called the factorization scale, with $b$ the conjugate variable of parton transverse momenta. The dynamics below $1/b$ scale is regarded as being completely non-perturbative, and can be parameterized into meson wave functions. The meson wave functions are not calculable in PQCD. But they are universal, channel independent. We can determine them from experiments, and it is constrained by QCD sum rules and Lattice QCD calculations. Above the scale $1/b$, the physics is channel dependent. We can use perturbation theory to calculate channel by channel.

Besides the hard gluon exchange with the spectator quark, the soft gluon ex-
changes between quark lines give out the double logarithms $\ln^2(Pb)$ from the overlap of collinear and soft divergence, $P$ being the dominant light-cone component of a meson momentum. The resummation of these double logarithms leads to a Sudakov form factor $\exp[-s(P, b)]$, which suppresses the long distance contributions in the large $b$ region, and vanishes as $b > 1/\Lambda_{QCD}$. So this term includes the double infrared logarithms. The corresponding Sudakov factor can be derived in PQCD as a function of the transverse separation $b$ and of the momentum fraction $x$ carried by the spectator quark [8]. The Sudakov factor suppresses the large $b$ region, where the quark and antiquark are separated by a large transverse distance and the color shielding is not so effective. It also suppresses the $x \sim 1$ region, where a quark carries all of the meson momentum, and intends to emit real gluons in hard scattering. The Sudakov factors from $k_T$ resummation [10] for the $B$ and $D^{(*)}$ mesons are only associated with the light spectator quarks, since the double logarithms arise from the overlap of the soft and mass (collinear) divergences. It is shown in ref. [10] that $e^{-s}$ falls off quickly in the large $b$, or long-distance, region, giving so-called Sudakov suppression. This Sudakov factor practically makes PQCD approach applicable. For the detailed derivation of the Sudakov form factors, see ref. [8, 17].

With all the large logarithms resummed, the remaining finite contributions are absorbed into a perturbative $b$ quark decay subamplitude $H(t)$. Therefore the three scale factorization formula is given by the typical expression,

$$C(t) \times H(t) \times \Phi(x) \times \exp \left[-s(P, b) - 2 \int_{1/b}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})) \right],$$

(1)

where $C(t)$ are the corresponding Wilson coefficients, $\Phi(x)$ are the meson wave functions and the variable $t$ denotes the largest mass scale of hard process $H$, that is, six-quark effective theory. The quark anomalous dimension $\gamma_q = -\alpha_s/\pi$ describes the evolution from scale $t$ to $1/b$. Since logarithm corrections have been summed by renormalization group equations, the above factorization formula does not depend on the renormalization scale $\mu$ explicitly.

As shown above, in the PQCD approach, we keep the $k_T$ dependence of the wave function. In fact, the approximation of neglecting the transverse momentum can only be done at the non-endpoint region, since $k_T \ll k^+$ is qualified at that region. At the endpoint, $k^+ \to 0$, $k_T$ is not small any longer, neglecting $k_T$ is a very bad approximation. By, keeping the $k_T$ dependence, there is no endpoint divergence as occurred in the QCD factorization approach, while the numerical result does not change at other region. Furthermore, the Sudakov form factors suppress the endpoint region of the wave functions. Recently another type of resummation has been observed. The loop correction to the weak decay vertex produces the double logarithms $\alpha_s \ln^2 x_2$ [18]. Using the wave functions from light-cone sum rules, at the endpoint region, these large logarithms are important, they must be resummed. The threshold resummation for the jet function results in Sudakov suppression, which decreases the contribution of endpoint region of wave functions. Therefore,
the main contributions to the decay amplitude in PQCD approach comes not from the endpoint region. The perturbative QCD is applied safely.

The main input parameters in PQCD are the meson wave functions. It is not a surprise that the final results are sensitive to the meson wave functions. Fortunately, there are many channels involve the same meson, and the meson wave functions should be process independent. In all the calculations of PQCD approach, we follow the rule, and we find that they can explain most of the measured branching ratios of B decays. For example: $B \to \pi\pi$ decays, $B \to \pi\rho$, $B \to \pi\omega$ decays, $B \to K\pi$ decays, $B \to KK$ decays, the form factor calculations of $B \to \pi$, $B \to \rho$, $B \to K\eta$ ($\eta'$), $B \to K\phi$ decays etc.

We emphasize that nonfactorizable and annihilation diagrams are indeed subleading in the PQCD formalism as $M_B \to \infty$. This can be easily observed from the hard functions in appendices of ref. 12, 13. When $M_B$ increases, the $B$ meson wave function enhances contributions to factorizable diagrams. However, annihilation amplitudes, being independent of $B$ meson wave function, are relatively insensitive to the variation of $M_B$. Hence, factorizable contributions become dominant and annihilation contributions are subleading in the $M_B \to \infty$ limit. Although the non-factorizable and annihilation diagrams are subleading for the branching ratio in color enhanced decays, they provide the main source of strong phase, by inner quark or gluon on mass shell. This mechanism of strong phase is negligible in the PQCD approach, since it is at next-to-leading order $O(\alpha_s)$ corrections. In fact, the factorizable annihilation diagrams are Chirally enhanced. They are not negligible in PQCD approach.

In the PQCD framework based on $k_T$ factorization theorem, an amplitude is expressed as the convolution of hard $b$ quark decay kernels with meson wave functions in both the longitudinal momentum fractions and the transverse momenta of
partons. In the $B \rightarrow D\pi$ like decays with at least one heavy meson in the final states, our PQCD formulas are derived up to leading-order in $\alpha_s$, to leading power in $m_D/m_B$ and in $\Lambda/m_D$, and to leading double-logarithm resummations.

### 3. Numerical Results and Discussion

The PQCD predictions for each term of the $B \rightarrow D\pi$ decay amplitudes \[23\] are exhibited in Table 1. The theoretical uncertainty comes only from the variation of the shape parameter for the $D$ meson exhibited in Table 1. The theoretical uncertainty comes only from the variation of the $D$ meson distribution amplitude, 0.6 < $C_D$ < 1.0. It is expected that the color-allowed factorizable amplitude $f_\pi \xi_{\text{ext}}$ dominates, and that the color-suppressed factorizable contribution $f_D \xi_{\text{int}}$ is smaller due to the Wilson coefficient $C_1 + C_2/N_c \sim 0$. The color-allowed non-factorizable amplitude $\mathcal{M}_{\text{ext}}$ is negligible: since the pion distribution amplitude is symmetric under the exchange of $x_3$ and $1-x_3$, the contributions from the two diagrams Figs. 3(c) and 3(d) cancel each other in the dominant region with small $x_2$. It is also down by the small Wilson coefficient $C_1/N_c$. For the color-suppressed non-factorizable contribution $\mathcal{M}_{\text{int}}$, the above cancellation does not exist in the dominant region with small $x_3$, because the $D$ meson distribution amplitude $\phi_D(x_3)$ is not symmetric. Furthermore, $\mathcal{M}_{\text{int}}$, proportional to $C_2/N_c \sim 0.3$, is not down by the Wilson coefficient. It is indeed comparable to the color-allowed factorizable amplitude $f_\pi \xi_{\text{ext}}$, and produces a large strong phase. Both the factorizable and non-factorizable annihilation contributions are vanishingly small.

The predicted branching ratios in Table 1 are in agreement with the averaged experimental data \[24\]. We extract the parameters $a_1$ and $a_2$ from our calculations of amplitudes. That is, our $a_1$ and $a_2$ do not only contain the non-factorizable amplitudes as in generalized FA, but the small annihilation amplitudes, which was first discussed in \[25\]. We obtain the ratio $|a_2/a_1| \sim 0.43$ with 10% uncertainty and the phase of $a_2$ relative to $a_1$ about $\text{Arg}(a_2/a_1) \sim -42^\circ$. Note that the experimental data do not fix the sign of the relative phases. The PQCD calculation indicates that $\text{Arg}(a_2/a_1)$ should be located in the fourth quadrant. It is evident that the short-distance strong phase from the color-suppressed nonfactorizable amplitude is already sufficient to account for the isospin triangle formed by the $B \rightarrow D\pi$ modes. Hence, it is more reasonable to claim that the data just imply a large strong phase, but do not tell what mechanism generates this phase \[26\]. From the viewpoint of PQCD, this strong phase is of short distance, and produced from the non-pinched

### Table 1. Predicted $B \rightarrow D\pi$ decay amplitudes in units of $10^{-2}$ GeV, and branching ratios in units of $10^{-3}$.

| Quantities        | $C_D = 0.6$ | $C_D = 0.8$ | $C_D = 1.0$ | Data           |
|-------------------|-------------|-------------|-------------|----------------|
| $\mathcal{M}_1$   | 6.39 - 0.35i| 6.88 - 0.38i| 7.35 - 0.40i|                |
| $\mathcal{M}_2$   | -1.53 + 1.48i| -1.49 + 1.48i| -1.45 + 1.45i|                |
| $\mathcal{M}_3$   | 8.56 - 2.45i| 8.99 - 2.47i| 9.40 - 2.46i|                |
| $B(B^0 \rightarrow D^+\pi^-)$ | 2.37 | 2.74 | 3.13 | 3.0 ± 0.4 |
| $B(B^0 \rightarrow D^0\pi^0)$  | 0.26 | 0.25 | 0.24 | 0.29 ± 0.05 |
| $B(B^- \rightarrow D^0\pi^-)$    | 4.96 | 5.43 | 5.91 | 5.3 ± 0.5 |

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Table 2. Direct CP asymmetries calculated in FA, QCDF and PQCD for $B \to \pi\pi$ and $B \to K\pi$ decays together with the experimental results at percentage.

| Quantities | FA          | QCDF | PQCD       | Data       |
|------------|-------------|------|------------|------------|
| $B^0 \to \pi^+\pi^-$ | $-5 \pm 3$ | $-6 \pm 12$ | $+30 \pm 20$ | $+37 \pm 11$ |
| $B^0 \to \pi^+K^-$  | $+10 \pm 3$ | $+5 \pm 9$  | $-17 \pm 5$  | $-10.9 \pm 1.9$ |
| $B^+ \to K^0\pi^+$ | $+1.7 \pm 0.1$ | $+1 \pm 1$  | $-1.0 \pm 0.5$ | $-2.0 \pm 3.4$ |
| $B^+ \to K^+\pi^0$ | $+8 \pm 2$  | $+7 \pm 9$  | $-13 \pm 4$  | $+4 \pm 4$   |

singularity of the hard kernel. Certainly, under the current experimental and theoretical uncertainties, there is still room for long-distance phases from final-state interaction. Other decays with one $D$ meson in final states, like $B \to D_s^{(*)}K^{(*)}$, $B \to D^{(*)}\eta$, and $B \to D_s\phi$ etc. also agree with experiments.

As discussed in the previous section, the strong phase generated from PQCD approach is quite different from the FA and QCDF approaches. The direct CP asymmetry is proportional to the sine of the strong phase difference of two amplitudes. Therefore the direct CP asymmetry will be different if strong phase is different. The predicted CP asymmetry by the three methods are shown in table. It is easy to see that the FA and QCDF results are quite close to each other, since the mechanism of strong phase is the same for them. Recently the two B factories measure at least two channels with non-zero direct CP asymmetry: $B^0 \to \pi^+\pi^-$ and $B^0 \to \pi^+K^-$ which are shown in table. It is easy to see that our PQCD results of direct CP asymmetry agree with the experiments. Although FA and QCDF are not yet ruled out by experiments, but the experiments at least tell us that the dominant strong phase should come from the mechanism of PQCD not QCDF. Charm quark loop mechanism, which gives the central value of strong phase in QCDF, is argued next-to leading order in PQCD. This argument is now proved by B factories experiments.

4. Summary

In the PQCD approach, the form factors are calculable, which are dominant by short distance contribution. By including the $k_T$ dependence and Sudakov suppression, there is no endpoint divergence. In the PQCD formalism non-factorizable amplitudes are of the same order as factorizable ones in powers of $1/M_B$, which are both $O(1/(M_B\Lambda_{QCD}))$. The smaller magnitude of nonfactorizable amplitudes in color enhanced decays are due to the cancellation of the two non-factorizable diagrams. From the viewpoint of power counting, they are of the same order. In case of $B \to D\pi$ decays, the cancellation is absent. The power counting changes, so that we can also calculate the non-factorizable contribution dominant processes. In this case, we give the right branching ratios for $B \to D^0\pi^0$ decay.

The strong phase comes mainly from the annihilation and non-factorizable diagrams in PQCD approach, which is quite different from the FA and QCDF approaches. The experimentally measured direct CP asymmetry implies that PQCD
gives at least the dominant strong phase than other approaches. This will be further
tested by experiments.

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