Additional Information from Astrometric Gravitational Microlensing Observations

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Abstract. Astrometric observations of microlensing events were originally proposed to determine the lens proper motion with which the physical parameters of lenses can be better constrained. In this proceeding, we demonstrate that besides this original usage astrometric microlensing observations can be additionally used in obtaining various important information about lenses. First, we demonstrate that the lens brightness can be determined with astrometric observations, enabling one to know whether the event is caused by a bright star or a dark lens. Second, we show that with additional information from astrometric observations one can resolve the ambiguity of the photometric binary lens fit and thus uniquely determine the binary lens parameters. Finally, we propose two astrometric methods that can resolve the degeneracy in the photometric lens parallax determination. Since one can measure both the proper motion and the parallax by these methods, the lens parameters of individual events can be uniquely determined.

1. Introduction

When a source is microlensed, it is split into two images. The flux sum of the individual images is greater than that of the unlensed source, and thus the source becomes brighter during the event. The sizes and brightnesses of the individual images change as the lens-source separation changes due to their transverse motion. Therefore, microlensing events can be detected either by photometrically monitoring the source brightness changes or by directly imaging the two separated images. However, with the current instrument direct imaging of the separate images is impossible due to the low precision of the instrument. As a result, current microlensing observations have been and are being carried out only by using the photometric method (Aubourg et al. 1993; Alcock et al. 1993; Udalski et al. 1993; Alard & Guibert 1997).

However, if an event is astrometrically observed by using the planned high precision interferometers from space-based platform, e.g. the Space Interferometry Mission (SIM), and ground-based interferometers soon available on 8-10 m class telescope, e.g. the Keck and the Very Large Telescope, one can measure the shift of the source star image centroid caused by microlensing. The astrometric centroid shift vector as measured with respect to the position of the unlensed
source is related to the lens parameters by

\[
\delta \theta_{c,0} = \frac{\theta_E}{u^2 + 2} \left[ \left( \frac{t - t_0}{t_E} \right) \hat{x} + \beta \hat{y} \right],
\]

(1)

where \( \theta_E \) is the angular Einstein ring radius, \( t_E \) is the time required for the source to cross \( \theta_E \) (Einstein time scale), \( t_0 \) is the time of the closest lens-source approach (and thus the time of maximum amplification), and \( \beta \) is the separation at this moment (i.e. impact parameter). The notation \( \hat{x} \) and \( \hat{y} \) represent the unit vectors with their directions that are parallel and normal to the lens-source proper motion. If one defines \( x = \delta \theta_{c,x} \) and \( y = \delta \theta_{c,y} - \beta \theta_E/2(\beta^2 + 2) \), equation (1) becomes

\[
x^2 + \frac{y^2}{q^2} = a^2,
\]

(2)

where

\[
a = \frac{\theta_E}{2(\beta^2 + 2)^{1/2}},
\]

(3)

and

\[
q = \frac{\beta}{(\beta^2 + 2)^{1/2}}.
\]

(4)

Therefore, during the event the image centroid traces out an elliptical trajectory (hereafter astrometric ellipse) with a semi-major axis \( a \) and an axis ratio \( q \). In Figure 1, we present astrometric ellipses for several example microlensing events with various lens-source impact parameters.

The greatest importance of astrometric microlensing observation is that one can determine \( \theta_E \) from the observed astrometric ellipse (Høg, Novikov & Polarev 1995; Walker 1995; Paczyński 1998; Boden, Shao, & Van Buren 1998). This is because the size (i.e. semi-major axis) of the astrometric ellipse is directly proportional to \( \theta_E \) [see equation (3)]. Once \( \theta_E \) is determined, the lens proper motion...
Figure 2. Left part: Astrometric behavior of bright lens events. The ellipses represents the trajectories of image centroid shift for events caused by bright lenses with various lens/source flux ratio $\ell_L/\ell_S$. The straight arrows represent the position vectors of the image centroid at different times during events. The curved arrow represents the direction of centroid motion with the progress of time. All example events have the same impact parameter of $\beta = 0.5$. Right part: Difference between the impact parameters determined from the centroid shift trajectory, $\beta$, and the angular speed curve, $\beta_0$, as a function of lens-source brightness difference.

is determined by $\mu = \theta_E/t_E$ with the independently determined $t_E$ from the light curve. While the photometrically determine $t_E$ depends on the three physical lens parameters of the lens mass ($M$), location ($D_{ol}$), and the transverse motion ($v$), the astrometrically determined $\mu$ depends only on the two parameters of $M$ and $D_{ol}$. Therefore, by measuring $\mu$ one can significantly better constrain the nature of lens matter. However, we note that to completely resolve the lens parameter degeneracy, it is still required to additionally determine the lens parallax (see more details in § 4).

In this proceeding, we demonstrate that besides this original usage astrometric microlensing observations can be additionally used in obtaining various important information about lenses. First, we show that the lens brightness can be determined with astrometric observations, enabling one to know whether the event is caused by a bright star or a dark lens (§ 2). Second, we demonstrate that additional astrometric microlensing observations allow one to uniquely determine the binary lens parameters (§ 3). Finally, we propose two astrometric methods that can uniquely determine the lens parallax, with which one can completely break the lens parameter degeneracy along with the measured proper motion (§ 4).

2. Lens Brightness Determination

If an event is caused by a bright lens (i.e. star), the centroid shift trajectory is distorted by the brightness of the lens. The lens brightness affects the centroid shift trajectory in two ways. First, the lens makes the image centroid further
shifted toward the lens. Second, the bright lens makes the reference of centroid shift measurements changed from the position of the unlensed source to the one between the source and the lens. By considering these two effects of the bright lens, the resulting centroid shift vector is computed by

$$\delta \theta_c = \frac{1 + f_L + f_L([u^2 + 2] - u(u^2 + 4)^{1/2})}{(1 + f_L)(1 + f_L u(u^2 + 4)^{1/2}/(u^2 + 2))} \delta \theta_{c,0},$$

where $f_L = \ell_L/\ell_S$ is the flux ratio between the lens and the source star.

In the left part of Figure 2, we present the trajectories of astrometric centroid shifts for events caused by bright lenses with various brightnesses. From the figure, one finds that the trajectories are also ellipses like those of dark lens events. As seen from the view of identifying bright lenses from the distorted trajectories, this is a bad news because one cannot identify whether the event is caused by a bright lens or not just from the shape of the trajectory (Jeong, Han, & Park 1999). One also finds that as the lens becomes brighter, the observed astrometric ellipse becomes rounder and smaller (measured by $a$).

Fortunately, identification of bright lenses is possible by measuring the angular speed ($\omega$) of the image centroid motion around the unlensed source position (Han & Jeong 1999). In the left part of Figure 2, we present the position vectors (arrows with straight lines) of the image centroid at different times during events for both the dark and bright lens events. From the figure, one finds that the position vector at a given moment directs towards the same direction for both the dark and bright lens events, implying that $\omega$ is the same regardless of the lens brightness. The angular speed does not depend on the lens brightness because lens always lies on the line connecting the two images, and thus additional shift caused by the bright lens occurs along this line. As a result, although the amount of shift changes due to lens brightness, the direction of the shift does not change. Since the angular speed is related to the lensing parameters of $(\beta, t_E, t_0)$ by

$$\omega(t) = \frac{\beta t_E}{(t - t_0)^2 + \beta^2 t_E^2},$$

these parameters can be determined from the observed angular speed curve. Note that these parameters are the same regardless of the lens brightness because the angular speed curve is not affected by the lens brightness. By contrast, the impact parameter determined from the shape of the observed centroid shift trajectory [see equation (4)] differs from the true value because the shape of the astrometric ellipse for a bright lens event differs from that of a dark lens event.

Then, if an event is caused by a bright lens, the impact parameter determined from the observed centroid shift trajectory, $\beta$, will differ from that determined from the angular speed curve, $\beta_0$. Therefore, by comparing $\beta$ and $\beta_0$, one can identify the bright lens and measure its flux. In the right part of Figure 2, we present $\delta \beta = \beta - \beta_0$ as a function of lens-source brightness difference in magnitudes.

3. Resolving Binary-Lens Parameter Degeneracy

If an event is caused by a binary lens, the resulting light curve deviates from that of a single lens event. Detecting binary lens events is important because one
can determine important binary lens parameters such as the mass ratio ($q$) and separation ($\ell$). These parameters are determined by fitting model light curves to the observed one.

For many cases of binary lens events, however, it is difficult to uniquely determine the solutions of the binary lens parameters with the photometrically constructed light curves alone. In Figure 3, we illustrate this ambiguity of the photometric binary lens fit. In the left part of the figure, we present the observed light curve of the binary lens event OGLE-7 (dots with error bars, Udalski et al. 1994) and several example model light curves (solid curves) obtained from the fit to the observed light curve by Dominik (1999). In the right part of the figure, we also present the binary lens system geometries for the individual solutions responsible for the model light curves. The binary lens parameters ($\ell$, $q$, and $t_E$) for each model are marked in the corresponding panel. From the figure, one finds that despite the dramatic differences in the binary lens parameters between different solutions, the resulting light curves fit the observed light curve very well, implying that unique determination of lens parameters is difficult by using the photometrically measured light curve alone.

However, the binary lens parameter degeneracy can be lifted if events are additionally observed astrometrically (Han, Chun, & Chang 1999). To demonstrate this, we compute the expected astrometric centroid shifts of the binary lens events resulting from the lens parameter solutions responsible for the model light curves in Figure 3, and the resulting trajectories are presented in Figure 4. From the figure, one finds that the trajectories are dramatically different each other. Therefore, with the additional information provided by the astrometric
microlensing observations, one can completely resolve the ambiguity of the photometric binary lens fit and thus uniquely determine the binary lens parameters.

4. Resolving Parallax Degeneracy

Although the astrometrically determined $\mu$ better constrains the lens parameters than the photometrically determined $t_E$ does, $\mu$ still results from the combination of the lens mass and location, and thus the lens parameter degeneracy is not completely resolved. To completely resolve the lens parameter degeneracy, it is required to determine the transverse velocity projected on the source plane ($\tilde{v}$, hereafter simply projected speed). Determination of $\tilde{v}$ is possible by measuring the lens parallax $\Delta u$ from photometric observations of the source light variations from two different locations, one from ground and the other from a helio-centric satellite (Gould 1994, 1995). Once both $\mu$ and $\tilde{v}$ are determined, the individual lens parameters are determined by

$$M = \left(\frac{c^2}{4G}\right) t_E^2 \tilde{v} \mu,$$

$$D_{ol} = \frac{D_{os}}{\mu D_{os}/\tilde{v} + 1},$$

$$v = \frac{1}{[\tilde{v}^{-1} + (\mu D_{os})^{-1}]-1}.$$

However, the elegant idea of lens parallax measurements proposed to resolve the lens parameter degeneracy suffers from its own degeneracy. The parallax degeneracy is illustrated in Figure 5. In the upper panel, we present two light curves of an event observed from the Earth and the satellite. Presented in the
Figure 5. Degeneracy in the determination of parallax and astrometric resolution of the degeneracy. The two curves in the upper panel represent the light curves of an event observed from ground and the satellite. The two possible lens system geometries that can produce the light curves in the upper panel are presented in the middle panels. The dotted circle represents the Einstein ring. The vector $\Delta u$ connecting the two points ($S_S$ for the source seen from the satellite and $S_E$ seen from the Earth) on the individual trajectories (straight lines) represent the displacements of the source positions (i.e. parallaxes) observed at a given time. In the lower panels, we present two sets of astrometric ellipses as seen from the Earth and the satellite corresponding to the source trajectories in the middle panels.

middle panels are the two possible lens system geometries that can produce the light curves in the upper panel. From the figure, one finds that depending on whether the source trajectories as seen from the Earth and the satellite are on the same or opposite sides with respect to the lens, there can be two possible values of $\Delta u$.

Astrometric microlensing observations are useful in resolving the degeneracy in parallax determination (Han & Kim 2000). The first method is provided by simultaneous astrometric observations from the ground and the satellite instead of photometric observations. Note that the SIM will have a heliocentric orbit, and thus can be used for this purpose. In the lower panels of Figure 5, we present two sets of the astrometric ellipses as seen from the Earth and the satellite that are expected from the corresponding two sets of source trajectories in the middle panels. One finds that these two sets of astrometric ellipses have opposite orientations, and thus can be easily distinguished from one another.
The parallax degeneracy can also be resolved if the event is astrometrically observed on one site and photometrically observed on a different site, instead of simultaneous astrometric observations from the ground and the satellite. This is possible because astrometric observations allow one to determine the lens-source proper motion $\mu$. Then with the known Earth-satellite separation vector, which is parallel to $\Delta u$, one can uniquely determine the angle between $\mu$ and $\Delta u$, allowing one to select the right solution of $u$.

5. Summary

In this proceeding, we demonstrate various additional usages of astrometric microlensing observations besides the original usage of the lens proper motion determination. These are summarized as follows.

1. By astrometrically observing a microlensing event caused by a bright lens, one can identify the bright lens and measure its flux.

2. With additional information from astrometric observations one can resolve the ambiguity of the photometric binary lens fit and thus uniquely determine the binary lens parameters.

3. With application of the two proposed astrometric methods, the degeneracy in the photometric lens parallax determination can be resolved, allowing one to completely break the lens parameter degeneracy along with simultaneously determined lens proper motion.

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