Non-Minimal $B-L$ Inflation with Observable Gravity Waves

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We consider non-minimal $\lambda\phi^4$ inflation in a gauged non-supersymmetric $U(1)_{B-L}$ model containing the gravitational coupling $\xi R \Phi^4$, where $R$ denotes the Ricci scalar and the standard model singlet inflaton field $\Phi$ spontaneously breaks the $U(1)_{B-L}$ symmetry. Including radiative corrections, the predictions $0.956 \lesssim n_s \lesssim 0.984$ and $0.007 \lesssim r \lesssim 0.1$ for the scalar spectral index and tensor to scalar ratio $r$ lie within the current WMAP 1-$\sigma$ bounds. If the $B-L$ symmetry breaking scale is of order a TeV or so, one of the three right handed neutrinos is a plausible cold dark matter candidate. Bounds on the dimensionless parameters $\lambda$, $\xi$ and the gauge coupling $g_{B-L}$ are obtained.

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A non-minimal gravitational coupling of inflaton has been known to play an important role in models of chaotic inflation [1]. Recently, this idea has received a fair amount of attention [2]–[15] arising from the possibility of taking the Standard Model (SM) Higgs boson as an inflaton. In the simplest scenarios of this kind, the SM Higgs doublet $H$ has a relatively strong non-minimal gravitational interaction $\xi R H\phi^4$, where $R$ is the Ricci scalar and $\phi$ a dimensionless coupling whose magnitude is estimated to be of order $10^4$ based on WMAP data [16]. This SM Higgs-based inflationary scenario is currently mired in some controversy stemming from the arguments put forward in [17] that for $\xi \gg 1$, the energy scale $\lambda^{1/4}m_F/\sqrt{\xi}$ during non-minimal SM inflation exceeds the effective ultraviolet cut-off scale $\Lambda = m_F/\xi$. Here $\lambda$ of order unity denotes the SM Higgs quartic coupling and $m_F \simeq 2.43 \times 10^{18}$ GeV represents the reduced Planck mass. This point has been further elaborated in Refs. [18]–[19]. However, it has recently been argued [20] that if the Higgs field is perturbed around its non-zero classical background, the effective cut-off can become larger than the energy scale of inflation. (See [21] for other possible solutions to the unitarity problem.) As we will see below, the above problem has a negligible impact on our conclusions in this paper. Even though the inflation in our case carries a $U(1)_{B-L}$ charge, a satisfactory scenario imposes relatively mild but nonetheless important constraints on the $U(1)_{B-L}$ gauge coupling.

In this paper we implement non-minimal $\phi^4$ inflation by supplementing the standard model with a gauged $U(1)_{B-L}$ symmetry [22]. (For inflation with local supersymmetric $U(1)_{B-L}$, see [23]–[24] and references therein. For global $U(1)_{B-L}$ inflation see [20].) The well-known advantages of a spontaneously broken gauge $U(1)_{B-L}$ symmetry include seesaw physics [22] to explain neutrino oscillations, and baryogenesis via leptogenesis [25]–[26] arising from the right handed neutrinos that are present to cancel the gauge anomalies. In the inflation model that we consider the symmetry breaking scale of $U(1)_{B-L}$ is arbitrary as long as the lower bound from LEP experiments $\gtrsim 3$ TeV [30], is satisfied. One interesting possibility is to break it at the TeV scale [31], and it has been shown [32]–[33] that the minimal $U(1)_{B-L}$ model with additional classical conformal invariance naturally predicts the symmetry breaking scale to be at TeV. This means that the new particles, the $Z'$ gauge boson, the $B-L$ Higgs boson $\Phi$ and the RH neutrinos $N^i$ have TeV scale masses, and they can be observed at Large Hadron Collider (LHC) [34]–[35]. Furthermore, with TeV scale RH neutrinos we can explain the origin of the baryon asymmetry through resonant leptogenesis [36]–[37].

One important feature missing in the above TeV scale $U(1)_{B-L}$ model is non-baryonic dark matter (DM). To circumvent this, following Ref. [40], we introduce an unbroken $Z_2$ parity under which one of the three RH neutrinos is taken to be odd, while all other fields are even. In this case the $Z_2$-odd RH neutrino is absolutely stable and a viable DM candidate. Note that the two remaining RH neutrinos are sufficient to reconcile theory with the observed neutrino oscillation data. The model also predicts that one of the three observed neutrinos is essentially massless. Thus, without introducing any additional dynamical degrees of freedom, the DM particle can be incorporated in the minimal gauged $U(1)_{B-L}$ model.

In this paper we consider non-minimal $\lambda\phi^4$ inflation by taking $\phi (\equiv \sqrt{2} \Re(\Phi))$, to be the inflaton field which is charged under $B-L$. We take into account quantum corrections to the inflationary potential arising from the inflaton interactions with the $U(1)_{B-L}$ gauge field. We find that the tensor to scalar ratio $r \gtrsim 0.007$ and the scalar spectral index $n_s \gtrsim 0.956$. More generally, in this non-minimal $\lambda\phi^4$ inflation model, the predictions $0.956 \lesssim n_s \lesssim 0.984$ and $0.007 \lesssim r \lesssim 0.1$ lie within the WMAP 1-$\sigma$ bounds for $10^{-12} \lesssim \lambda \lesssim 0.3$ and $10^{-3} \lesssim \xi \lesssim 10^4$. Recall that the corresponding tree level predictions for minimal ($\xi = 0$) $\lambda\phi^4$ chaotic inflation, namely $n_s \simeq 0.95$ and $r \simeq 0.26$, lie outside the WMAP 2-$\sigma$ bounds.

Our inflation model is based on the gauge group...
SU(3)_c × SU(2)_L × U(1)_Y × U(1)_{B-L} and the particle content is listed in Table 1 [33]. The SM singlet scalar (Φ) breaks the U(1)_{B-L} gauge symmetry down to Z_2\{B-L\} by its vacuum expectation value (vev), and at the same time generates the right-handed neutrino masses. The Lagrangian terms relevant for the seesaw mechanism are given by

\[ \mathcal{L} = -Y_{\ell \nu}^2 \overline{N^i}H^\dagger \ell_L^i - \frac{1}{2} Y_{\nu \nu}^2 \Phi N^i \nu N^i + \text{h.c.}, \]

where the first term yields the Dirac neutrino mass after electroweak symmetry breaking, while the right-handed neutrino Majorana mass term is generated by the second term associated with the B − L gauge symmetry breaking. Without loss of generality, we use the basis which diagonalizes the second term and makes \( Y_{\nu}^i \) real and positive.

Consider the following tree level action in the Jordan frame:

\[ S_{J}^{tree} = \int d^4x \sqrt{-g} \left[ -\left( \frac{m_p^2}{2} + \xi_H H^\dagger H + \xi \Phi^\dagger \Phi \right) \mathcal{R} + (D_\mu H)^\dagger \mathcal{G}^{\mu\nu} (D_\nu H) - \lambda_H \left( H^\dagger H - \frac{v^2}{2} \right)^2 + (D_\mu \Phi)^\dagger \mathcal{G}^{\mu\nu} (D_\nu \Phi) - \lambda \left( \Phi^\dagger \Phi - \frac{v_{B-L}^2}{2} \right)^2 - \lambda' \left( \Phi^\dagger \Phi \right) (H^\dagger H) \right], \]

where \( v \) and \( v_{B-L} \) are the vevs of the Higgs fields \( H \) and \( \Phi \) respectively. To simplify the discussion, we assume that \( \lambda' \) is sufficiently small so it can be ignored, and also \( \xi_H \ll \xi \).

The relevant one-loop renormalization group improved effective action can be written as [41]

\[ S_J = \int d^4x \sqrt{-g} \left[ -\left( \frac{m_p^2}{2} + \xi G(t)^2 \phi^2 \right) \mathcal{R} + \frac{1}{2} G(t)^2 (\partial \phi)^2 - \frac{1}{4} \lambda(t) G(t)^4 \phi^4 \right], \]

where \( t = \ln(\phi/\mu) \) and \( G(t) = \exp(- \int_0^t dt' \gamma(t'))/(1 + \gamma(t')) \), with

\[ \gamma(t) = \frac{1}{(4\pi)^2} \left( \frac{1}{2} \sum_i (Y_{\nu}^i(t))^2 - 12 g_{B-L}^2 \right) \]

being the anomalous dimension of the inflaton field, \( g_{B-L} \) denotes the \( U(1)_{B-L} \) gauge coupling and \( \mu \) the renormalization scale. In the presence of the nonminimal gravitational coupling, the one loop renormalization group equations (RGEs) of \( \lambda, g_{B-L}, \xi \) and \( Y_{\nu}^i \) are given by [32, 33]

\[ (4\pi)^2 \frac{d\lambda}{dt} = (2 + 18 s^2) \lambda^2 - 48 \lambda g_{B-L}^2 + 96 g_{B-L}^4 \]

\[ + 2\lambda \sum_i (Y_{\nu}^i)^2 - \sum_i (Y_{\nu}^i)^4, \]  

\[ (4\pi)^2 \frac{dg_{B-L}}{dt} = \left( \frac{32 + 4 s^2}{3} \right) g_{B-L}^3, \]  

\[ (4\pi)^2 \frac{d\xi}{dt} = ((1 + s^2) \lambda - 2 \gamma), \]  

\[ (4\pi)^2 \frac{dY_{\nu}^i}{dt} = (Y_{\nu}^i)^3 - 6 g_{B-L}^2 Y_{\nu}^i + \frac{1}{2} Y_{\nu}^i \sum_j (Y_{\nu}^j)^2, \]

where the \( s \) factor is defined as

\[ s(\phi) = \frac{(1 + \xi \phi^2)}{1 + (6\xi + 1) \frac{\phi^2}{m_p^2}}. \]

In the Einstein frame with a canonical gravity sector, the kinetic energy of \( \phi \) can be made canonical with respect to a new field \( \sigma = \sigma(\phi) \),

\[ \left( \frac{d\sigma}{d\phi} \right)^2 = \frac{G(t)^2 \Omega(t) + 3 m_p^2 (\partial_\phi \Omega(t))^2}{\Omega(t)^2}/2, \]

where,

\[ \Omega(t) = 1 + \xi G(t)^2 \phi^2/m_p^2. \]

The action in the Einstein frame is then given by

\[ S_E = \int d^4x \sqrt{-g_E} \left[ -\frac{1}{2} m_p^2 \mathcal{R}_E + \frac{1}{2} (\partial E \sigma)^2 - V_E(\sigma) \right], \]

with

\[ V_E(\phi) = \frac{1}{4} \lambda(t) G(t)^4 \phi^4 \left( 1 + \frac{\xi \phi^2}{m_p^2} \right)^2. \]

In our numerical work, we employ above potential with the RGEs given in Eqs. (5-8). However, for a qualitative discussion it is reasonable to use the following leading-log approximation of the above potential:

\[ V_E(\phi) \approx \left( \frac{\phi}{\mu} \right)^{1 + \frac{96 g_{B-L}^2}{16 \pi^2} \ln \left( \frac{\phi}{\mu} \right)} \left( 1 + \frac{\phi^2}{m_p^2} \right)^{2}, \]
where we have assumed \( \gamma \approx 0 \), \( \frac{dV'_{B}}{dt} \approx 0 \), \( \frac{dg_{B-L}}{dt} \approx 0 \), \( \frac{d\xi}{dt} \approx 0 \), \( \phi_{B-L}^2 \ll (\lambda, (Y_{i}^{2})^{2}) \), and \( d\lambda/\lambda \approx 96\phi_{B-L}^4/(4\pi) \) with \( \lambda_{0} \equiv \lambda(t = 0) \). We have checked that for a broad range of parameters the above expression can be regarded as a valid approximation for the potential given in Eq. \([13]\). In our numerical calculations we fix the renormalization scale \( \mu = 1 \) TeV.

To discuss the predictions of this model it is useful to first recall the basic results of the slow roll assumption. The inflationary slow-roll parameters are given by

\[
\epsilon(\phi) = \frac{1}{2}m_{P}^{2}\left(\frac{V'^{2}}{V_{E}^{2}}\right),
\]

\[
\eta(\phi) = m_{P}^{2}\left[\frac{V''^{2}}{V_{E}^{2} (\sigma'^{2})} - \frac{V'^{2} \sigma'^{2}}{V_{E}^{2} (\sigma'^{2})^{3}}\right],
\]

\[
\zeta(\phi) = m_{P}^{2}\left(\frac{V'}{V_{E}^{2} (\sigma')}\right)\left(\frac{V''^{2}}{V_{E}^{2} (\sigma')} - 3 \frac{V'^{2} \sigma'^{2}}{V_{E}^{2} (\sigma')^{3}}\right) + 3 \frac{V'^{2} \sigma'^{2}}{V_{E}^{2} (\sigma')^{3}} - \frac{V'^{2}}{V_{E}^{2} (\sigma')^{3}},
\]

where a prime denotes a derivative with respect to \( \phi \).

The slow-roll approximation is valid as long as the conditions \( \epsilon \ll 1 \), \( |\eta| \ll 1 \), and \( \zeta \ll 1 \) hold. In this case the scalar spectral index \( n_{s} \), the tensor-to-scalar ratio \( r \), and the running of the spectral index \( \frac{d n_{s}}{d \ln k} \) are approximately given by

\[
n_{s} \approx 1 - 6 \epsilon + 2 \eta,
\]

\[
r \approx 16 \epsilon.
\]

\[
\frac{d n_{s}}{d \ln k} \approx 16 \epsilon - 24 \epsilon^{2} - 2 \zeta.
\]

The number of e-folds after the comoving scale \( l \) has crossed the horizon is given by

\[
N_{l} = \frac{1}{\sqrt{2}m_{P}} \int_{\phi_{c}}^{\phi_{l}} \frac{d \phi}{\sqrt{\epsilon(\phi)}} \left( \frac{d \sigma}{d \phi} \right),
\]

where \( \phi_{l} \) is the field value at the comoving scale \( l \), and \( \phi_{c} \) denotes the value of \( \phi \) at the end of inflation, defined by \( \text{max}(\epsilon(\phi_{c}), |\eta(\phi_{c})|, |\zeta(\phi_{c})|) = 1 \).

The amplitude of the curvature perturbation \( \Delta_{K} \) is given by

\[
\Delta_{K} = \frac{V_{E}}{24 \pi^{2} m_{P}^{2} \epsilon(\phi)} \bigg|_{k_{0}},
\]

where \( \Delta_{K}^{2} = (2.43 \pm 0.11) \times 10^{-9} \) is the WMAP7 normalization at \( k_{0} = 0.002 \) Mpc\(^{-1} \) \([16]\). Note that for added precision, we include in our calculations the first order corrections \([12]\) in the slow-roll expansion for the quantities \( n_{s} \), \( r \), \( \frac{d n_{s}}{d \ln k} \), and \( \Delta_{K} \).

Using Eqs. \([14]\), \([29]\) above we can obtain various predictions of the radiatively corrected non-minimal \( \phi^{4} \) model of inflation. Once we fix the parameters \( \xi \) and \( g_{B-L} \), and the number of e-foldings \( N_{0} \), we can predict \( n_{s} \), \( r \), and \( \frac{d n_{s}}{d \ln k} \). The tree level \( (g_{B-L} = 0) \) predictions for minimal \( \phi^{4} \) inflation are readily obtained as:

\[
n_{s} = 1 - \frac{24}{\phi_{0}^{2}} = 1 - \frac{3}{N_{0}},
\]

\[
r = \frac{128}{\phi_{0}^{2}} = \frac{16}{N_{0}},
\]

\[
\frac{d n_{s}}{d \ln k} = \frac{192}{\phi_{0}^{4}} = -\frac{3}{N_{0}}.
\]

For \( N_{0} = 60 \) (\( N_{0} = 50 \)), we find \( n_{s} \approx 0.95 \) (\( n_{s} \approx 0.94 \)), \( r \approx 0.26 \) (\( r \approx 0.31 \)) and \( \frac{d n_{s}}{d \ln k} \approx -8 \times 10^{-3} \) (\( \frac{d n_{s}}{d \ln k} \approx -10^{-3} \)). As expected, the predictions of tree level minimal \( \phi^{4} \) inflation lie outside the 2-\( \sigma \) WMAP bounds \([16]\).

However, the situation is improved once the radiative corrections are included \([43]\). The impact of these radiative corrections on the tree level predictions of various inflationary models have been studied in Refs. \([44, 45]\). Furthermore, the nonminimal gravitational coupling also plays an important role in making the tree level predictions consistent with the WMAP data. Indeed, the radiative corrections smear out the tree level predictions of nonminimal inflationary models \([12]\). A similar behavior is observed in our situation. The approximate potential in Eq. \([14]\) effectively behaves as a nonminimal \( \lambda_{\phi} \phi^{4}/4 \) potential with a running coupling constant \( \lambda_{\phi} \approx \lambda_{0} + \frac{96 g_{B-L}^{2}}{4 \pi^{2}} \ln \left( \frac{\phi_{0}}{\mu} \right) \).

In the limit \( \xi \ll 1 \), assuming \( \lambda_{\phi} \) to be approximately constant, the scalar spectral index, the tensor to scalar ratio and the running of the spectral index for the radiatively corrected non-minimal \( \phi^{4} \) inflation are given by \([12]\)

\[
n_{s} \approx 1 - \frac{3(1 + 16 \xi N_{0}/3)}{N_{0}(1 + 8 \xi N_{0})},
\]

\[
r \approx \frac{16}{N_{0}(1 + 8 \xi N_{0})}.
\]
sharp transitions in the predictions of a tiny correction to its tree level prediction. Note the vicinity of \( \xi_n \) and Eqs. (27) and (28).

The expression for the inflationary potential given in Eq. (14) we obtain the following results for \( \varphi = 0 \).

\[
\frac{dn_s}{d \ln k} \simeq \frac{2}{N_0^2} \left( 1 - \frac{2}{N_0} \right)
\]

\[
\frac{3 \left( 1 + 4 \left( 8 \xi N_0 / 3 - 5 \right) \left( 8 \xi N_0 \right)^2 - 2 \left( 8 \xi N_0 \right)^3 \right)}{N_0^2 \left( 1 + 8 \xi N_0 \right)^4},
\]  

(29)

These results exhibit a reduction in the value of \( r \) and an increase in the value of \( n_s \) compared to their minimally coupled tree level predictions (Eqs. (24-26)), as can be seen in Figs. 16. In our analysis, we set \( \lambda_0 = 0 \) limit for simplicity. From the WMAP 1-\( \sigma \) bounds \( r \sim 0.1 \) and \( n_s \sim 0.96 \), we obtain a lower bound of \( \xi \gtrsim 3 \times 10^{-3} \) with \( N_0 = 60 \) e-foldings [2]. The value of \( \frac{dn_s}{d \ln k} \) receives a tiny correction to its tree level prediction. Note the sharp transitions in the predictions of \( n_s \) and \( r \) in the vicinity of \( \xi \approx 10^{-3} \). This can be understood from the expression for the inflationary potential given in Eq. (14) and Eqs. (27) and (28).

In the large \( \xi \) limit, again assuming \( \lambda_0 \) to be constant, we obtain the following results for \( n_s \), \( r \) and \( \frac{dn_s}{d \ln k} \):

\[
n_s \simeq 1 - \frac{2}{N_0},
\]

\[
r \simeq \frac{12}{N_0^2},
\]

(30)

(31)

We obtain \( 0.007 \lesssim r \lesssim 0.1 \) and \( 0.956 \lesssim n_s \lesssim 0.984 \) consistent with the WMAP 1-\( \sigma \) bounds. The running of the spectral index \( \frac{dn_s}{d \ln k} \) varies from \( -6 \times 10^{-3} \) to \( -8 \times 10^{-3} \).

Note the second sharp transitions in the predictions of \( n_s \) and \( r \) around \( \xi \approx 10^4 \). Actually, in this limit the approximation of the potential given in Eq. (14) does not hold as the value of the gauge coupling \( g_{B-L} \) becomes large and we can no longer ignore its running.

Finally in Figs. 16 we display the relation among the parameters \( g_{B-L}(\phi_0) \), \( g_{B-L}\text{(TeV)} \), \( \lambda(\phi_0) \) and \( \xi(\phi_0) \). Within 1-\( \sigma \) bounds of WMAP data, these parameters take values in the range \( 3 \times 10^{-3} \lesssim g_{B-L}(\phi_0) \lesssim 0.46 \), \( 3 \times 10^{-4} \lesssim g_{B-L}\text{(TeV)} \lesssim 0.32 \), \( 10^{-12} \lesssim \lambda(\phi_0) \lesssim 0.3 \) and \( 10^{-3} \lesssim \xi(\phi_0) \lesssim 10^4 \). However, if we require that \( \lambda^{1/4} \lesssim \Lambda \equiv m_P / \xi \), then more stringent upper bounds, \( g_{B-L}(\phi_0) \lesssim 0.043 \), \( g_{B-L}\text{(TeV)} \lesssim 0.043 \), \( \lambda(\phi_0) \lesssim 7 \times 10^{-5} \) and \( \xi(\phi_0) \lesssim 300 \) are obtained. Although, there is some uncertainty in the calculations of cut-off (i.e., \( \Lambda \) is ar-
predicted by some (Ref. 20) to be larger than $m_P/\xi$ during inflation, interesting $B - L$ related LHC physics still looks viable even with the more stringent bound on $g_{B-L} (\lesssim 0.043)$.

To summarize, we have considered non-minimal $\lambda \phi^4$ chaotic inflation in a minimal gauged $U(1)_{B-L}$ extension of SM. Among the very well-known attractive features of this model are the natural presence of three RH neutrinos, seesaw mechanism to understand non-zero neutrino masses, and explanation of the baryon asymmetry via leptogenesis. With an extra $Z_2$ symmetry one of the three RH neutrino can be a viable dark matter candidate. To realize inflation we utilize the SM gauge singlet inflaton $\Phi$ which is charged under $B - L$. In addition to the non-minimal gravitational coupling, we have also included the effect of inflaton-gauge coupling $g_{B-L}$. For $10^{-12} \lesssim \lambda(\phi_0) \lesssim 0.3$, $10^{-3} \lesssim \xi(\phi_0) \lesssim 10^4$ and $3 \times 10^{-3} \lesssim g_{B-L}(\phi_0) \lesssim 0.46$ we obtain the inflationary predictions $0.956 \lesssim n_s \lesssim 0.984$ and $0.007 \lesssim r \lesssim 0.1$ that are consistent with the WMAP 1-σ bounds and will be tested by the Planck satellite.

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