Using 385 fb$^{-1}$ of $e^+e^-$ collisions, we study the amplitudes of the singly Cabibbo-suppressed decay $D^0 \to K^-K^+\pi^0$. We measure the strong phase difference between the $D^0$ and $D^0$ decays to $K^*(892)^+K^-$ to be $-35.5 \pm 1.9^\circ$ (stat) $\pm 2.2^\circ$ (syst), and their amplitude ratio to be $0.599 \pm 0.013$ (stat) $\pm 0.011$ (syst). We observe contributions from the $K\pi$ and $K^-K^+$ scalar and vector amplitudes, and analyze their angular moments. We find no evidence for charged $\kappa$, nor for higher spin states. We also perform a partial-wave analysis of the $K^-K^+$ system in a limited mass range.

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The amplitudes describing $D$ meson weak decays into three-body final states are dominated by intermediate resonances that lead to highly nonuniform intensity distributions in the available phase space. Analyses of these distributions have led to new insights into the role of the light-meson systems produced. The $K^-\pi^+$ systems produced in $\bar{p}p$ collisions at and around 10.58 GeV center-of-mass energy can be an input for extracting the contributions in the available phase space. Analyses of these contributions have led to new insights into the role of the light-meson systems produced in $\bar{p}p$ collisions at and around 10.58 GeV center-of-mass energy. The complex quantum mechanical amplitude $f$ is a coherent sum of all relevant quasi-two-body $D^0 \to r \to AB/C$ isobar model resonances, $f = \sum \alpha R e^{i\delta r} A_i(s)$. Here $s = m_{AB}^2$, and $A_i$ is the resonance amplitude. We obtain coefficients $\alpha$ and $\delta F$ from a likelihood fit. The probability density function for signal events is $|f|^2$. We model incoherent background empirically using events from the lower sideband of the $m_{D^0}$ distribution.

For $D^0$ decays to spin-1 ($P$-wave) and spin-2 states, we use the Breit-Wigner amplitude,

$$A_{BW}(s) = M_L(s, p) \frac{1}{\sqrt{p^2 s - m_0^2 - i m_0 \Gamma(s)}},$$

$$\Gamma(s) = \Gamma_0 \left( \frac{p}{p_0} \right)^{2L+1} \frac{F_L(p)}{F_L(p_0)},$$

where $m_0$ ($\Gamma_0$) is the resonance mass (width), $L$ is the angular momentum quantum number, $p$ is the momentum of either daughter in the resonance rest frame, and $p_0$ is the value of $p$ when $s = m_0^2$. The function $F_L$ is the Blatt-Weisskopf barrier factor:

$$F_L = 1, F_1 = 1/\sqrt{1 + R^2}, \text{ and } F_2 = 1/\sqrt[4]{9 + 3R^2 + R^4}.$$

where we take the meson radial parameter $R$ to be 1.5 GeV$^{-1}$. We define the spin part of the amplitude, $M_L$, as: $M_0 = M_{D^0}^2, M_1 = -2 p_{A'G} p_{C}, \text{ and } M_2 = \frac{4}{3} [3\sqrt{3} p_{A'G} p_{C}^2 - p_{A}^2 p_{C}^2] M_{D^0}^2$, where $M_{D^0}$ is the nominal $D^0$ mass, and $p_{A}$ is the 3-momentum of particle $i$ in the resonance rest frame.

For $D^0$ decays to $K^{\pm}\pi^0$ $S$-wave states, we consider three amplitude models. One model uses the LASS amplitude for $K^-\pi^+ \to K^-\pi^+$ elastic scattering:

$$A_{K^\mp\pi^+}(s) = \sqrt{s} \sin \delta(s) e^{i\phi(s)},$$

$$\delta(s) = \cot^{-1} \left( \frac{1}{p^4 - b} \right) + \cot^{-1} \left( \frac{M_0^2 - s}{M_0^2 - \frac{M_0^2}{s} \sqrt{2} \frac{p_0}{p}} \right),$$

where $M_0$ ($\Gamma_0$) refers to the $K^0(1430)$ mass (width), $a = 1.95 \pm 0.09$ GeV$^{-1}c$, and $b = 1.76 \pm 0.36$ GeV$^{-1}c$. The unitary nature of Eq. 6 provides a good description of the amplitude up to 1.45 GeV$^{-1}$ (i.e., $K^{\eta'}$ threshold).

In Eq. 6, the first term is a nonresonant contribution defined by a scattering length $a$ and an effective range $b$, and the second term represents the $K^0(1430)$ resonance. The phase space factor $\sqrt{s}/p$ converts the scattering amplitude to the invariant amplitude. Our second model uses the E-791 results for the $K^-\pi^+$ $S$-wave amplitude.
from an energy-independent partial-wave analysis in the decay $D^+ \rightarrow K^− \pi^+ \pi^+$ [14]. The third model uses a coherent sum of a uniform nonresonant term, and Breit-Wigner terms for the $\kappa(800)$ and $K^0_a(1430)$ resonances. In Fig. 2 we compare the $K\pi$ S-wave amplitude from the E-791 analysis [14] to the LASS amplitude of Eqs. [5-6]. For easy comparison, we have normalized the LASS amplitude in Fig. 2a approximately to the E-791 measurements with $\sqrt{s} > 1.15$ GeV/$c^2$, and have reduced the LASS phase, $\delta(s)$, in Fig. 2b by 80°. We then observe good agreements in the mass dependence of amplitude and phase for $\sqrt{s} > 1.15$ GeV/$c^2$. As the mass decreases from 1.15 GeV/$c^2$, the E-791 amplitude increases while the LASS amplitude decreases, with the ratio finally reaching ~1.7 at threshold. At the same time, their phase difference increases to ~40° at threshold. This behavior might be due to the form factor describing $D^0$ decay to a $K\pi$ S-wave system and a bachelor $K$. Since no centrifugal barrier is involved, such an effect should be more significant for S-wave than for higher spin waves because of the larger overlap between the initial and final state wave functions. However, the inverse momentum of the $K\pi$ system in the $D^0$ rest frame increases from 0.27 Fermi at $K\pi$ threshold to 0.48 Fermi at 1.15 GeV/$c^2$, therefore any form factor effect would decrease with increasing $K\pi$ mass. If the effect is essentially gone by 1.15 GeV/$c^2$, similar mass dependence of amplitude and phase in $D^0$ decay and $K\pi$ scattering would be observable at higher mass values, in agreement with Fig. 2. In the present analysis, we make an attempt to distinguish between the two rather different $K\pi$ S-wave mass dependences in the region below ~1.15 GeV/$c^2$. In each case, we also allow the fit to determine the strength and phase of these amplitudes relative to the $K^*(892)^+$ reference.

We describe the $D^0$ decay to a $K^- K^+ S$-wave state by a coupled-channel Breit-Wigner amplitude for the $f_0(980)$ and $a_0(980)$ resonances, with their respective couplings to $\pi\pi$, $KK$ and $\eta\pi$, $KK$ final states [14],

$$A_{f_0[a_0]}(s) = \frac{M_{f_0[a_0]}^2}{M_{f_0[a_0]}^2 - s - i(g_1^2 \rho_{\pi\pi[\pi\pi]} + g_2^2 \rho_{KK})}.$$  (7)

Here $\rho$ represents Lorentz invariant phase space, $2p/\sqrt{s}$.
the data well. Both yield almost identical behavior in

invariant mass (Fig. 1I-II) and angular distribution (Fig. 1II). We use LASS amplitude to describe the $K\pi$ $S$-wave amplitudes in both the isobar models (I and II). We summarize the results of the best fits (Model I: $\chi^2/\nu = 702.08/714$, probability 61.9%; Model II: $\chi^2/\nu = 718.89/717$, probability 47.3%) in Table 1. We also list the fit fraction for each resonant process $r$, defined as $f_r = \int |a_r A_r|^2 d\tau / \int |f_{D^0}|^2 d\tau$, where $d\tau = dm_{K^-\pi^0}^2 dm_{K^+\pi^0}^2$ in Table 1. Due to interference among the contributing amplitudes, the $f_r$ do not sum to one in general. We find that the $K\pi$ $S$-wave is not in phase with the $P$-wave at threshold as it was in the LASS scattering data. For Model I (II), the $S$-wave phase relative to the $K^*(892)^+$ is $\sim 180^\circ$ ($150^\circ$) for the positive charge and $135^\circ$ ($110^\circ$) for the negative charge.

We have also considered the possible contributions from other resonant states such as: $K_2^+(1430)$, $f_2(1270)$, $f_0(1370)$, and $f_0(1510)$. We find that none of them is needed to describe the Dalitz plot, they all provide small

FIG. 2: (Color online) LASS (solid line, blue) and E-791 (dots with error bars) $K\pi$ $S$-wave amplitudes (a), in arbitrary units, and phase (b). The double headed arrow (red) indicates the mass range available in the decay $D^0 \rightarrow K^-K^+\pi^0$.

TABLE I: The results obtained from the $D^0 \rightarrow K^-K^+\pi^0$ Dalitz plot fit. We define amplitude coefficients, $a_r$ and $\phi_r$, relative to those of the $K^*(892)^+$, The errors are statistical and systematic, respectively, We show the $a_0(980)$ contribution, when it is included in place of the $f_0(980)$, in square brackets. We denote the $K\pi$ $S$-wave states here by $K^\pm\pi^0(S)$. We use LASS amplitude to describe the $K\pi$ $S$-wave states in both the isobar models (I and II).

![Table](Image)

FIG. 3: (Color online) The phase-space-corrected $K^-K^+\pi^0$ Dalitz plot fit. We define amplitude coefficients, $a_r$ and $\phi_r$, relative to those of the $K^*(892)^+$, The errors are statistical and systematic, respectively, We show the $a_0(980)$ contribution, when it is included in place of the $f_0(980)$, in square brackets. We denote the $K\pi$ $S$-wave states here by $K^\pm\pi^0(S)$. We use LASS amplitude to describe the $K\pi$ $S$-wave states in both the isobar models (I and II).

![Table](Image)
contributions and lead to smaller $\chi^2$ probabilities.

Angular distributions provide a more detailed information on specific features of the amplitudes used in the description of the Dalitz plot. We define the helicity angle $\theta_H$ for the decay $D^0 \rightarrow (r \rightarrow AB)C$ as the angle between the momentum of $A$ in the $AB$ rest frame and the momentum of $AB$ in the $D^0$ rest frame. The moments of $\cos \theta_H$, defined as the efficiency-corrected and background-subtracted invariant mass distributions of events weighted by spherical harmonic functions, $Y_l^0(\cos \theta_H) = \sqrt{\frac{2l+1}{4\pi}} P_l(\cos \theta_H)$, where the $P_l$ are Legendre polynomials of order $l$, are shown in Fig. 4 for the $K^+\pi^0$ and $K^-K^+$ channels, for $l = 0-7$. The $K^0\pi^0$ moments are similar to those for $K^+\pi^0$.

The mass dependent $K^-K^+$ $S$- and $P$-wave complex amplitudes can also be obtained directly from our data in a model-independent way in a limited mass range around $1 \text{ GeV}/c^2$. In a region of the Dalitz plot where $S$- and $P$-waves in a single channel dominate, their amplitudes are given by the following Legendre polynomial moments,

$$P_0 = \frac{|S|^2 + |P|^2}{2}, \quad P_1 = \sqrt{2}|S||P| \cos \theta_{SP}, \quad P_2 = \sqrt{\frac{2}{5}} |P|^2,$$

using $\frac{1}{2} P_1 P_m d(\cos \theta_H) = \delta_{lm}$. Here $|S|$ and $|P|$ are, respectively, the magnitudes of the $S$- and $P$-wave amplitudes, and $\theta_{SP} = \theta_S - \theta_P$ is the relative phase between them. We use these relations to evaluate $|S|$ and $|P|$, shown in Fig. 8 for the $K^-K^+$ channel in the mass range $m_{K^-K^+} < 1.15 \text{ GeV}/c^2$. The measured values of $|S|$ agree well with those obtained in the analysis of the decay $D^0 \rightarrow K^-K^+K^0$ [18]. They also agree well with either the $f_0(980)$ or the $a_0(980)$ lineshape. The measured values of $|P|$ are consistent with a Breit-Wigner lineshape for $\phi(1020)$. Results for $\cos \theta_{SP}$ and $\theta_{SP}$

![Image](https://via.placeholder.com/150)

FIG. 4: (Color online) The mass dependence of the spherical harmonic moments of $\cos \theta_H$ after efficiency corrections and background subtraction: $K^+\pi^0$ (columns I, II) and $K^-K^+$ (columns III, IV). The circles with error bars are data points and the curves (red) are derived from the fit functions (see text). For the sake of visibility, we do not show error bars on the curves.
and their amplitude ratio, $\text{Cosine of relative phase}$

FIG. 5: (Color online) Results of the partial-wave analysis of the $K^-K^+$ system using Eq. [S] described in the text. (a) Cosine of relative phase $\theta_{BP} = \theta_B - \theta_P$, (b) two solutions for $\theta_{SP}$, (c) $P$-wave phase taken from Eqs. [B3] for the $\phi(1020)$ meson, and (d) $S$-wave phase derived from the upper solution in (b). Solid bullets are data points, and open circles (blue) and open triangles (red) correspond, respectively, to isobar models I and II. The number of simulated events used for the two models is 10 times larger than data. Errors for quantities from the isobar models arise from Monte Carlo statistical limitations, and differ from errors derived from Eq. [S].

parameters are changed by one standard deviation ($\sigma$). Similarly, we estimate the experimental uncertainty from the variation in results when either the signal efficiency parameters are varied by $1\sigma$, or the background shape is taken from simulation instead of the data sideband, or the ratio of particle-identification rates in data and simulation is varied by $1\sigma$. Model and experimental systematics contribute almost equally to the total uncertainty. As a consistency check, we analyze disjoint data samples, in bins of reconstructed $D^0$ mass and laboratory momentum, and find consistent results.

Neglecting $CP$ violation, the strong phase difference, $\delta_D$, between the $\overline{D}^0$ and $D^0$ decays to $K^+(892)^+K^-$ state and their amplitude ratio, $r_D$, are given by

$$r_D e^{i\delta_D} = \frac{A_{D^0\rightarrow K^-K^+\pi^0}}{A_{\overline{D}^0\rightarrow K^+K^-\pi^0}} e^{i(\delta_{K^+K^-\pi^0} - \delta_{K^-K^+\pi^0})}.$$  

Combining the results of models I and II, we find $\delta_D = -35.5^\circ \pm 1.9^\circ$ (stat) $\pm 2.2^\circ$ (syst) and $r_D = 0.599 \pm 0.013$ (stat) $\pm 0.011$ (syst). These results are consistent with the previous measurements [13], $\delta_D = -28^\circ \pm 8^\circ$ (stat) $\pm 11^\circ$ (syst) and $r_D = 0.52 \pm 0.05$ (stat) $\pm 0.04$ (syst).

In conclusion, we have studied the amplitude structure of the decay $D^0 \rightarrow K^-K^+\pi^0$, and measured $\delta_D$ and $r_D$. We find that two isobar models give excellent descriptions of the data. Both models include significant contributions from $K^*(892)$, and each indicates that $D^0 \rightarrow K^{*+}K^-$ dominates over $D^0 \rightarrow K^{*-}K^+$. This suggests that, in tree-level diagrams, the form factor for $D^0$ coupling to $K^{*-}$ is suppressed compared to the corresponding $K^-$ coupling. While the measured fit fraction for $D^0 \rightarrow K^{*+}K^-$ agrees well with a phenomenological prediction [20] based on a large SU(3) symmetry breaking, the corresponding results for $D^0 \rightarrow K^{*-}K^+$ and the color-suppressed $D^0 \rightarrow \phi\pi^0$ decays differ significantly from the predicted values. It appears from Table I that the $K^+\pi^0$ $S$-wave amplitude can absorb any $K^*(1410)$ and $f_2(1515)$ if those are not in the model. The other components are quite well established, independent of the model. The $K\pi$ $S$-wave amplitude is consistent with that from the LASS analysis, throughout the available mass range. We cannot, however, completely exclude the behavior at masses below $\sim 1.15$ GeV/$c^2$ observed in the decay $D^+ \rightarrow K^-\pi^+\pi^+$ [3, 14]. The $K^-K^+ S$-wave amplitude, parametrized as either $f_0(980)$ or $a_0(980)^0$, is required in both isobar models. No higher mass $f_0$ states are found to contribute significantly. In a limited mass range, from threshold up to 1.02 GeV/$c^2$, we measure this amplitude using a model-independent partial-wave analysis. Agreement with similar measurements from $D^0 \rightarrow K^-K^+K^0$ decay [18], and with the isobar models considered here, is excellent.

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