Accretion and Ejection—The GR/MHD View

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Abstract.

The inward flow of matter through accretion disks is driven by MHD turbulence. Global general relativistic MHD simulations shed quantitative light on this process, revealing a number of aspects of accretion previously unrecognized. Among them are strong stresses in the marginally stable and plunging regions near the black hole and electromagnetically-dominated conical relativistic jets that can form spontaneously from the accretion flow. The energy release associated with both of these effects can significantly augment the classical energy-release estimates based on purely hydrodynamic models.

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1. OLD POINTS OF VIEW AND NEW

Speaking in Sicily, home of many distinguished relics of the Classical world, it would only be appropriate to begin this talk with a reference to the classical view of accretion disks around black holes and the outflows they drive. In this view, created in three seminal papers published thirty-plus years ago, matter drifts slowly inward through accretion disks due to the action of a mysterious “viscosity” while dissipating an amount of energy per unit mass that is a function solely of the black hole’s spin. More quantitatively, the “viscous” stress is assumed, on the basis of dimensional analysis, to be \( \alpha p \), where \( \alpha \) is some unknown number less than or comparable to unity, and \( p \) is the internal pressure of the accretion flow [17]. The reason why the heat liberated is supposed to be a function of black hole spin alone is that all forces are imagined to end at the innermost stable circular orbit (the ISCO), so that the radiative efficiency can be identified with the binding energy per unit mass of that orbit [15]. Meanwhile, relativistic jets are driven outward along the rotation axes of spinning black holes, even in the absence of ongoing accretion, as frame-dragging creates an effective electric field parallel to the large-scale magnetic field threading the hole’s event horizon [6]. The luminosity of these jets is controlled entirely by the strength of the field on the horizon and the spin of the black hole.

In recent years, this classical view has been substantially revised. The stress that transports angular momentum outward so that matter can move inward is now thought to be the result of fluctuating, but correlated magnetic fields: orbital shear forces \( \langle B_r B_\phi \rangle < 0 \) whenever the orbital frequency gradient \( d\Omega/dr < 0 \). Because this stress has nothing to do with the sort of kinetic processes that the name “viscosity” connotes, unlike a conventional fluid viscosity, it does not necessarily dissipate any energy where the stress takes place. That the field is strong enough to bring matter inward at a reasonable rate is due to continual stirring by a robust underlying MHD instability, the magneto-rotational instability [3, 4]. Because this instability has a very rapid linear growth rate (comparable
to the orbital frequency) and its conditions for growth are very modest (good enough electrical conductivity to justify the MHD approximation, a seed magnetic field with energy density no more than the total pressure), it should be present throughout any disk surrounding a black hole, as well as in the great majority of disks around other, less extreme, gravitating objects. Thus, electromagnetic stress is the primary vehicle for angular momentum transport in accretion disks.

A corollary of attributing disk stresses to magnetic fields is that the classical boundary condition at the ISCO is likely inappropriate. Its heuristic basis rested on purely hydrodynamic reasoning: as matter accelerates inward through the ISCO, passing from a domain of stable orbits to one where they are unstable, its density falls and its pressure drops. It would be hard to envision circumstances in which the small amount of matter resident inside the ISCO could exert much force upon the far heavier disk outside. However, as first recognized long ago [16] and recently recalled to mind [11, 8], magnetic fields severely undercut this argument—their effectiveness at exerting stresses has nothing to do with the inertia or pressure of the matter. Consequently, there is no particular reason for their stresses to cease at the ISCO. In fact, if one estimates the magnetic stress at the ISCO by supposing that the field is strong enough to drive accretion at the requisite rate in the main disk body and is then advected inward by flux-freezing, the magnetic stress there is likely to be quite sizable [11, 8].

A second corollary of this picture is that magnetic fields arrive at the event horizon because they are brought there by the accretion flow. The field strength at the horizon, the basic parameter of the Blandford-Znajek [6] mechanism, should then be determined by the history and current state of accretion.

2. NUMERICAL SIMULATIONS

Analytic tools for studying turbulence are very limited; large-scale numerical simulation is the only technique we have for obtaining quantitative results. In the past few years both the capabilities of the hardware and the quality of the computer codes available for this effort have improved dramatically, leading to rapid improvements in accretion disk studies. Relatively recently, it was possible to study disks only in the shearing-box approximation, in which an annular segment is modelled by straightening it out into a rectangular geometry, while retaining a plane-parallel shear, a tidal potential and a Coriolis force to approximate the dynamics of orbital motion. Given the nature of this approximation, it made sense to perform these simulations in the Newtonian limit.

Today’s codes, on the other hand, can treat global disks, initially with Newtonian gravity and now in full general relativity [9, 7, 2, 14]. Under the assumption of ideal MHD (modulo the inevitable dissipation that occurs on the gridscale), they can follow fully three-dimensional dynamics for very long timescales; durations $\sim 10^4 M$ (for black hole mass $M$, setting $G = c = 1$). These codes do an excellent job of conserving angular momentum, whether associated with the matter or electromagnetic fields, but their fidelity to Nature is much more limited in regard to thermodynamics and radiation. The De Villiers-Hawley [7] and Anninos et al. [2] codes evolve an internal energy equation and treat the gas equation of state as adiabatic with $\gamma = 5/3$, adding entropy only in association with shocks through the use of an artificial bulk viscosity. They therefore
capture neither the bulk of the heat production that is a concomitant of accretion nor the photon radiation that cools the gas and produces the light we see. In a sense, the first omission compensates for the second. The Gammie-McKinney-Tóth [9] and Mizuno et al. [14] codes evolve a total energy equation. By construction, losses in kinetic and magnetic energy are transferred into heat, thereby modeling the turbulent dissipation process, but like the others, these codes also wholly ignore radiation.

Most of the global disk simulations done to date by various groups assume either an initial large scale vertical net field threading the disk (most of these have been Newtonian simulations), or an initial disk that has no net flux, i.e., all field lines closing within the disk. Simulations with strong net flux generally have little difficulty producing jets and outflows, but we have no way of knowing whether such a strong field passing through the central region of the accretion flow is realistic. The zero net flux initial condition is the simplest in terms of guessed parameters, as it means the field is simply zero all around the outer boundary. This is the approach we have adopted for our simulations.

To avoid the numerical difficulties arising from subsonic flow across the boundary, all our simulations place their entire mass supply within the computational volume. This fact has the consequence that angular momentum removed from the inner accretion flow is given to the mass in the outer disk, which must then move outward. In other words, only the part of the flow inside a certain radius (variously 12–25\(M_\odot\), depending on the simulation) actually participates in a genuine accretion flow. The outer boundary is generally located much farther out (\(r_{\text{out}} \simeq 100M_\odot\)).

### 3. PROPERTIES OF THE ACCRETION FLOW

A number of results of qualitative importance have been derived from these studies. Most notably, the order-of-magnitude estimates predicting that magnetic stresses at the ISCO should be important have been strongly vindicated [12]. As Figure [1] demonstrates vividly, whereas the classical Novikov-Thorne picture predicts that the stress ends abruptly at the ISCO, the actual MHD stresses continue quite smoothly inward toward the black hole. In the case of the non-rotating black hole, there can be no outward stress at the event horizon because there is no source of angular momentum to supply the outward flow of angular momentum that constitutes the stress. When the black hole rotates, however, the black hole itself is an ample reservoir of angular momentum, and there is no reason why the stress must end somewhere outside the event horizon. Although this process might appear to violate causality, there is no such violation in reality: This process can equally well be thought of as an inward flow of negative angular momentum facilitated by the rotating black hole’s ergosphere, where both matter and electromagnetic fields can have negative energy-at-infinity.

Regarding the Shakura-Sunyaev \(\alpha\) parameter as merely a measure of the stress in pressure units, we find that it rises rapidly with diminishing radius, beginning well outside the ISCO and continuing through the plunging region (Fig. [2]). In the disk body, \(\alpha \simeq 0.02–0.1\). As one moves closer to the ISCO, the pressure diminishes (in relative terms) even while the magnetic stress increases. The result is that the nominal \(\alpha\) is 5–10 times larger near the ISCO and another factor of 5 or more larger still deep inside the plunging region.
Throughout the disk, wherever there are stresses transporting angular momentum radially, there is also work done by one ring on its neighbor. The energy flow into a ring due to the imbalance between the work it does and the work done on it is not necessarily cancelled by the net change in its orbital (and electromagnetic energy) content due to accretion. When Novikov and Thorne first calculated the radial profile of dissipation on the basis of energy and angular momentum conservation alone, it was exactly this mismatch that lay at the basis of their work. Stress at the ISCO therefore alters the total dissipation possible in the disk outside that radius \([1]\).

Recently, we have attempted to estimate the total radiative efficiency, including heat liberated inside the ISCO (and therefore beyond the reach of the Agol-Krolik \([1]\) formalism) and also allowing for losses due to photons following orbits that bring them to the black hole rather than infinity \([5]\). Unfortunately, as mentioned earlier, we do not actually compute the dissipation rate directly. Instead, we must seek some way of
estimating the dissipation rate based on local physical quantities. Although we cannot prove that a diagnostic based on guesswork is correct, we can eliminate those that must be wrong: in the disk body, where the inflow time is long compared to the thermal time, energy conservation in the context of a time-steady disk demands a definite dissipation profile. With no stress at the ISCO, this is the profile predicted by Novikov and Thorne; with stress, this form is generalized, as worked out by Agol and Krolik. Any candidate prescription for estimating the dissipation must therefore match the profile required by energy conservation in the time-steady portion of the disk. After trying a number of possibilities, all heuristically linked to energy production one way or another, we found that the best agreement with the requirements of energy conservation in a time-steady disk was achieved with the assumption that there is a uniform resistivity $\eta$, so that the rate of heat generation is $\eta J_\mu J_\mu$, where $J_\mu$ is the four-current density. To compute the luminosity observed at infinity, we then assumed that photons are emitted isotropically in the fluid frame at a rate proportional to that dissipation function and traced their trajectories until they reach infinity or are swallowed by the hole.

As it turns out, most of the dissipation predicted by this rule takes place outside the ISCO. In addition, many of the photons radiated from within the ISCO are on orbits that ultimately lead to capture by the black hole. Consequently, the bulk of the additional radiative efficiency seen by distant observers has its origin near and just outside the ISCO.

Table 1 shows the numbers derived from a single snapshot from each of four simulations. The entries in the second column are the efficiencies predicted by the Novikov-Thorne model, but after allowance for photons captured by the black hole. They are therefore smaller than the standard quoted values, by a very small amount for $a/M = 0$, where the ISCO is relatively far from the horizon, by a more substantial amount (about a 10% fractional reduction) for the case of $a/M = 0.998$. Ranges rather than single values are shown because some photon orbits intersect the equatorial plane before heading off to infinity. When they do, they may interact with the matter in the accretion flow. Scattering or absorption followed by reradiation puts the photons onto a new orbit that may or may not go to infinity. Which end-result obtains depends very much on the details. To allow for this very model-dependent uncertainty, we quote ranges in which the lower number is the result of assuming that none of the energy from photons on plane-crossing orbits reaches infinity while the upper number is the result of the opposite assumption.

Comparing the Novikov-Thorne numbers with those based on this estimate from the simulations indicates that interesting augmentations in radiative efficiency may be

### Table 1. Estimated radiative efficiency from accretion, in rest-mass units.

| $a/M$ | $\varepsilon_{NT}$ | $\varepsilon_{MHD}$ |
|-------|--------------------|---------------------|
| 0.0   | 0.055–0.056        | 0.067–0.07          |
| 0.5   | 0.077–0.079        | 0.13–0.14           |
| 0.9   | 0.137–0.145        | 0.16–0.18           |
| 0.998 | 0.250–0.290        | 0.29–0.41           |
achieved. The greatest fractional increase is for $a/M \simeq 0.5$ because increasing spin results in a trade-off: more stresses at small radii, but also larger probabilities of photon capture.

4. OUTFLOWS

One of the most interesting results from recent disk simulations has been the discovery that Poynting-flux jets can form spontaneously within a cone aligned with the black hole’s rotation axis. When accreting matter approaches the black hole’s event horizon, there is an effective resistivity due to general relativistic kinematics that lets matter cross field lines and enter the black hole, even while the field lines remain just outside. One way to understand this process is to recall that gravity immediately outside the event horizon is so powerful that it can overcome the Lorentz force binding electric charges to their Larmor orbits, forcing them to deviate away from their usual guiding center motion along a field line.

Field lines thus shorn of their inertia find it easy to float upward. In fact, if there were no magnetic field at high latitudes around the event horizon, the $\nabla B^2$ force would quickly cause them to spread into that region. Once there, they can expand upward and outward. Indeed, we find that in simulations with non-rotating black holes a small amount of accretion can suffice to create a magnetic field in a cone around the rotation axis that is almost exactly radial and stretches to the outer boundary of the simulation.

Very little matter penetrates into this cone around the rotation axis for the simple reason that there is a strong centrifugal barrier. Even matter with the small amount of angular momentum permitting an orbit at the ISCO has too much angular momentum to approach the axis above the black hole. Numerical difficulties prevent us from quantifying just how small the matter density is, but we can say with confidence that it must be very small. As a result, this region is strongly magnetically-dominated and highly relativistic, moving outward with a Lorentz factor that we can only bound from below at $\simeq 5–10$. In other words, we have found that, in the right circumstances, relativistic jets can spontaneously form as a result of accretion.

When the black hole rotates, the frame-dragging at small radius pulls the field into rotation (see Fig. 3) and creates a transverse component. Moving transverse magnetic field, of course, carries Poynting flux. In its fundamental character, this mechanism resembles the classical Blandford-Znajek mechanism. However, this version differs in that, rather than operating independent of accretion, the very existence of the field is due to accretion, and its strength is regulated by it. In addition, we are unrestricted in black hole spin rate (the original model calculation involved a perturbation expansion in $a/M$) and compute the field shape self-consistently.

The magnitude of the electromagnetic luminosity associated with this jet can be sizable. Phrased in terms of efficiency in rest-mass units, when the black hole spins rapidly the jet power can be competitive with the radiative luminosity expected from the accretion disk (Table 2 taken from [10]; see also [13]).

Not all magnetic geometries are this efficient, however. The numbers in Table 2 were computed on the basis of an initial magnetic field shape that is “dipolar” in the sense that the field lines wrap concentrically around the pressure maximum of the original matter
distribution. When the initial field is, instead, “quadrupolar” in the sense that the field is divided into two sectors, with one set of field loops centered on a ring offset above the equatorial plane, the other positioned symmetrically below, reconnection in the inner accretion flow is so efficient that the Poynting flux in the jet is greatly reduced.

5. SUMMARY

With the progress of the past fifteen years, there have been substantial advances in our understanding of the physics of accretion disks. The origin of the stress that plays such a fundamental role in driving accretion is no longer unknown—it is MHD turbulence, driven by a powerful and robust instability.

Although the equations governing such a system are strongly nonlinear, they are based on very well-understood physics: nothing more than classical electromagnetism and relativistic gravity. Because we can build upon such a solid foundation, it makes sense to undertake the arduous task of constructing large-scale simulations. These are now so feasible that there are several competing groups performing them.
Much remains to be explored in this rich territory, but already several important results have been established:

- Contrary to expectations based on hydrodynamic intuition, MHD stresses continue to be strong all the way through the ISCO and (if the black hole rotates) in to the event horizon. Precise estimates of the additional efficiency due to these stresses are not yet available, but we can begin to make estimates.

- When the magnetic field has the right geometry, relativistic electromagnetically-dominated jets can form along the black hole rotation axis. Driven by the rotation of the black hole, these jets can carry Poynting flux to infinity at a rate that may increase the power output per unit rest mass accreted by factors of order unity relative to that radiated directly from the accretion disk proper.

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