Study of kinetic Alfven wave (KAW) in plasma – sheet-boundary-layer

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Abstract. The effect of parallel electric field with general loss-cone distribution function on the dispersion relation and damping rate/growth rate of the kinetic Alfven wave (KAW) is evaluated by kinetic approach. The generation of KAW by the combined effect of parallel electric field and loss-cone distribution indices \( J \) at a particular range of \( k_{\perp}\rho_i \) \( (k_{\perp}\rho_i < 1 \text{ and } k_{\perp}\rho_i > 1) \) is noticed, where \( k_{\perp} \) is perpendicular wave number and \( \rho_i \) is the ion-gyro radius. Thus the propagation of KAW and loss of the Poynting flux from plasma sheet boundary layer (PSBL) to the ionosphere can be explained on the basis of present investigation. It is found that the present study also shows that the loss-cone distribution index is an important parameter to study KAW in the PSBL.

1. Introduction

In magnetohydrodynamics, there are three linear wave modes: Fast, Slow and Alfven or (intermediate) modes. These waves have frequencies which are low compared to the ion cyclotron frequencies in the plasma, and their wavelengths are much longer than the ion gyro-radius, \( \rho_i \) or the ion inertial lengths, \( V_A/\omega_{ci} \), where \( V_A \) is the Alfven speed and \( \omega_{ci} \) is the ion cyclotron frequency. The Alfven mode has probably attracted the much attention in space physics [Hollweg, 1999]. It is weakly damped and can propagate long distance; its Poynting flux is along the ambient magnetic field, \( B_0 \). The Kinetic Alfven wave is obtained when the MHD Alfven wave develops a large wave number, \( k_{\perp} \), transverse to \( B_0 \). In contrast to the MHD Alfven mode, which is linearly non compressive, the kinetic Alfven wave becomes linearly compressive. The Kinetic Alfven wave is the “low frequency electromagnetic wave” which propagates obliquely to the magnetic fields while the magnetohydrodynamics Alfven wave is often called a “high-frequency” wave [Hollweg, 1999]. Hasegawa [1976] first suggested that small-scale kinetic Alfven waves possesses parallel electric fields and could be an efficient mechanism for accelerating particles on plasma sheet field lines. Shukla et al. [2008] stated that the Polar and Cluster satellites have observed that large amplitude KAWs are observed throughout the plasma sheet, particularly at the Plasma sheet boundary layer (PSBL) during substorms [Wygant et al., 2000, 2002; Keiling et al., 2002; Angelopoulos, 2002].

The plasma sheet is particularly associated with the central plane of the magnetotail where the magnetic field reverses. The pressure of the particles in the sheet balances the magnetic pressure in the tail lobes. The outer parts of plasma sheet are generally called as plasma sheet boundary layer. Plasma sheet particles are energized within the magnetotail and they are important in auroral activity and the behavior of the high latitude ionosphere. The plasma sheet boundary layer lies between the hot dense plasma sheet and the tenuous lobe plasma of earth’s magnetotail. Most of the magnet tail plasma is concentrated around the tail midplane in an about 10 \( R_E \) thick plasma sheet. The plasma sheet boundary layer (PSBL), which is located between the auroral acceleration region and the distant magnetotail, is of great importance in auroral substorms physics as it is regarded as a region of strong energy transfer between the distant tail and the auroral ionosphere [Eastman et al., 1984; Tiwari et al., 2008]. Wygant et al. [2000] have observed the first evidence for small scale kinetic Alfven waves in the plasma sheet boundary layer (PSBL), which may play an important role in the local heating of the
plasma sheet and also may produce earthward, magnetic field-aligned electron beams and transversely heated ions.

By two Polar case studies the electric fields in the PSBL at 4-6Re were associated with Alfvén waves, which carried large and sufficient Poynting flux toward the ionosphere to power magnetically conjugate auroral emissions [Wygant et al. 2002]. Keiling et al. [2000] have shown that large Alfvénic Poynting flux (i.e. the energy flowing per unit area, per unit time transported by the fields is called the Poynting Flux, \( S = (E \times B)/\mu_0 \)) often occurs in the PSBL during the expansion phase of substorms. These recent Polar observations have clearly demonstrated the connection between auroral phenomena and energy transfer processes by large Alfvén waves in the PSBL. Schriver et al., [2003] concluded that the Poynting Flux/kinetic Alfvén waves: occurs during high and moderately active times at the highest latitude edge of the auroral zone, these waves lead to strong earthward electron accelerations and precipitation.

In most of the theoretical work carried out so far in the analysis of kinetic Alfvén waves (KAW) the velocity distribution function have been assumed either an ideal Maxwellian or bi Maxwellian ignoring the steepness of the loss cone feature. Plasma particles in the PSBL which are imbedded with curved and converging field lines have very high anisotropies in their transverse and parallel velocity components [Keiling et al., 2005; Wygant et al., 2002, Angelopoulos et al., 2002] and therefore, considerably depart from Maxwellian distribution and have the loss-cone distribution of particles. A loss-cone distribution is a distribution that has a deficiency of particles for small values of \( v_\perp \), or small values of pitch angle \( \alpha \) defined by \( \alpha = \tan^{-1}(v_\perp/v_\parallel) \). It is usually the case that for a loss-cone distribution there is a region of velocity \((v_\perp, v_\parallel)\) space such that \( \frac{\partial f}{\partial v_\perp} = 0 \). Loss cone distribution typically arise when a plasma is confined in a magnetic trap (or ‘bottel’) such as those occurring in planetary magnetosphere, the Solar (or a stellar) atmosphere and in laboratory plasma containment machines [Summer and Throne, 1995]. Thus kinetic Alfvén waves generated in highly anisotropic plasma sheet boundary layer and propagating towards auroral ionosphere may be suitably discussed with general distribution function. The importance of using the loss-cone distribution to study various instabilities has been previously predicted by various authors [Shukla et al., 2007, 2008; Duan et al., 2005; Ahirwar et al., 2006].

The purpose of this paper is to investigate the effect of static parallel electric field on the kinetic Alfvén wave in the plasma sheet boundary layer using the general loss-cone distribution function. The parallel electric field considered in the present paper is external in nature [Sterlitsov and Marklund, 2006; Morikawa et al., 2007]. In the recent past, the method of particle aspect analysis was used [Tiwari et al., 2008; Ahirwar et al. 2006; Varma et al., 2007] to analyze different instabilities in the presence of parallel electric field. Shukla et al.[ 2007 & 2008] have previously studied the KAW in the presence of the general loss-cone distribution and ion-electron beams using the kinetic approach.

2. Basic Assumption

To determine the dispersion relation and damping rate, we use the general distribution function of the form [Gomberoff and Cupermann, 1981; Summers and Throne, 1995; Wang et al., 1998; Varma and Tiwari, 1993]

\[
F_j = n_0 F_j(\bar{V}_j) F_j(v_j)
\]

where \( n_0 \) is the zeroth-order density.

\[
F_j(\bar{V}_j) = \frac{v_{j!}}{\pi v_{j!}^3} \exp \left( -\frac{\bar{V}_j^2}{v_{j!}^2} \right)
\]

(1a)
\begin{align}
F_i(v_i) &= \frac{1}{\pi^{2/3} v_{T_{ie}}^{2}} \exp \left[ -\frac{v_i^2}{v_{T_{ie}}^2} \right] \\
T_{ie} &= T_{ie} \left[ 1 + \frac{i\epsilon E_{||}}{k_i (k_i T_{ie})} \right]
\end{align}

(1b)  
(1c)  

J is an integer known as the loss cone index, $v_{T_{ie}}^2 = \frac{2 T_{ie}}{m}$, and $v_{T_{i\perp}}^2 = (J + 1)^2 \frac{2 T_{i \perp}}{m}$ are the squares of parallel and perpendicular velocities. Here, $m$ is the mass and $T_{i \perp}$ and $T_{i \parallel}$ are the temperatures of charged particles perpendicular and parallel to the electric field $B_\parallel$. The expression for $T_{ie}$ is originally derived by Pines and Schrieffer [1961] adopting the rigorous treatment of kinetic approach for collective behavior of solid state plasma.

3. Dispersion relation

To evaluate the general dispersion relation we have used the determinant

$$
\begin{vmatrix}
\frac{c^2 k_\perp}{\omega^2} - \frac{c^2 k_\parallel}{\omega^2} & \frac{\varepsilon_\parallel}{\omega} - \frac{c^2 k_\perp^2}{\omega^2} \\
\frac{c^2 k_\parallel}{\omega^2} & \varepsilon_\parallel - \frac{c^2 k_\perp^2}{\omega^2}
\end{vmatrix} = 0
$$

(2)

Where $k_\parallel$ and $k_\perp$ refer to wave vector components along and across the external magnetic field respectively. Hence the general dispersion relation of the KAW is obtained as

$$
\omega \left( \frac{1 + \alpha^2}{k_i v_{T_{ie}}} \sqrt{\frac{\pi}{2}} R^{-\gamma/2} \sin \left( \frac{3}{2} \theta \right) \right) + \omega^2 - \omega \left( \frac{k_i v_{A}^2}{\delta_{0j}} - \frac{\lambda_j}{\delta_{0j} - D_{ij}^j(\lambda_j)} \right) \left( \frac{1 + \alpha^2}{k_i v_{T_{ie}}} \sqrt{\frac{\pi}{2}} R^{-\gamma/2} \sin \left( \frac{3}{2} \theta \right) \right) - k_i v_{A}^2 \left( \delta_{0j} - D_{ij}^j(\lambda_j) \right) \frac{\lambda_j}{\delta_{0j} - D_{ij}^j(\lambda_j)} \delta \frac{m_p}{m_i} \frac{k_i v_{ex}^2}{\omega^2_p} \left( 1 + \alpha^2 \right) = 0
$$

(3)

where

$$
D_{ij}^j(\lambda_j) = \int_0^1 \left[ 1 - \frac{J v_{T_{i\perp}}^2}{v_{T_{i\perp}}^2} \right] d\jmath \frac{v_{T_{i\perp}}^2}{v_{T_{i\perp}}^2} \frac{k_i v_{\perp}}{\omega_{\jmath}} \frac{v_{T_{i\perp}}^2}{v_{T_{i\perp}}^2} \exp \left( \frac{-v_i^2}{v_{T_{i\perp}}^2} \right)
$$

$$
C_{ij}^j(\lambda_j) = \int_0^{2\pi} \int_0^{2\pi} \left[ \frac{k_i v_{\perp}}{\omega_{\jmath}} \right] F_i \left( \jmath \right) \, d\theta \, d\phi
$$

We have assumed the following identities

$$
\int_0^\infty \left[ -s^2 \right] J_{s}^i(\jmath x) \exp(-x^2) = \exp \left( \frac{-s^2}{2} \right) I_{s} \left( \jmath \right)
$$

$$
\int_0^\infty \left[ -s^2 \right] x J_{s}^i(\jmath x) \exp(-x^2) = \exp \left( \frac{-s^2}{2} \right) \left[ \frac{1}{2} \right] s^2 \left( I_{s} \left( \jmath \right) - I_{s} \left( \jmath \right) + I_{s} \left( \jmath \right) \right)
$$

3
and plasma dispersion function \( Z(\zeta) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} x \exp\left(-\frac{x^2}{2}\right) dx \). \( Z(\zeta) \approx 1 \) for ions \( \zeta = \frac{\omega - n e a}{k_i v_{thi}} \) and \( v_{thi} \) is the ion thermal velocity, \( \lambda_c = \frac{1}{2}(J+1)k_i^2\rho_i = \frac{1}{2}(J+1)\rho_i \). Thus in the frequency range \( \omega << \omega_c \) and we define

\[
\varepsilon_z = \frac{\omega^2}{\omega_{ce}^2} \sum_{n=\infty}^{\infty} D_n^2(\lambda_c) = \frac{\omega^2}{\omega_{ce}^2} \left( \delta_{0j} - D_n(\lambda_c) \right)
\]

Where, \( \delta_{0j} = 1 \) for \( J = 0 \), and \( \delta_{0j} = 0 \) if \( J \neq 0 \)

Also the use is made of the identities [Goswami and Buti, 1975; Hirose, 1976]

\[
\sum_{n=\infty}^{\infty} C_n^2(\lambda_c) = 1 \text{ for all } J's
\]

The summation over \( n \) is performed as discussed by Lysak and Lotko [1996]. Similarly,

\[
\varepsilon_z = C_0^2(\lambda_c) \frac{\omega_{ce}^2}{\omega} \zeta^2 Z(\zeta) Z'(\zeta)
\]

\[
Z'(\zeta) = -2 \left( 1 + i \frac{\omega}{2 k_i v_{thc}^2} \right) \zeta^2 = \frac{\omega}{2 k_i v_{thc}^2} \sqrt{2} \omega_{ce}
\]

\[
\varepsilon_z = C_0^2(\lambda_c) \frac{1}{\delta} \left( \frac{m_i}{m_e} \frac{\omega_{pi}^2}{k_i^2 v_{thc}^2} \right) \left( 1 + i \frac{\omega}{2 k_i v_{thc}^2} \right)
\]

Here, \( v_{thc} \) is an electron thermal velocity, and \( \delta = \frac{n_e}{n_{ce0}} > 1 \) is a measure of the negative charge density which signifies the presence of dust grains [Salimullah and Rosenberg, 1999]. However in the present investigation we have assumed \( \delta = 1 \) to avoid dust grain effects for magnetospheric plasma.

4. Growth / Damping Rate

We assumed \( \omega \to \omega_r + i \gamma_L \) with \( \gamma_L < \omega_r \), to obtain the growth/damping rate

\[
\gamma_L = -\frac{\text{Im} D(\omega, k)}{\text{Re} D(\omega, k)}
\]

thus obtained an expression for the collisionless growth/damping rate of the kinetic Alfven wave

\[
\gamma_L = \left[ \frac{\omega \cdot (1 + a^2)^{3/2}}{(1 + a^2)^{1/2}} \frac{\pi}{2} \cos\left(\frac{3}{2} \theta\right) - a_1 k_{v_{thc}} \right]
\]

\[
\left[ (1 + a^2)^{1/2} \frac{\pi}{2} \sin\left(\frac{3}{2} \theta\right) + (1 + a^2)^{1/2} \frac{1}{C_0^2(\lambda_c)} \frac{m_i}{m_e} \delta \cdot \frac{k_i^2 v_{thc}^2}{2} \omega_{ce}^2 c_i^2 k_i^2 \right]
\]

(4)
We can also easily show that for \( J=0, 1, 2 \ldots \), general dispersion relation and growth/damping rate can be obtained using recurrence relations. We have considered both the smaller and larger wavelengths limit for ions. In the case when \( J=0 \) and \( E_\parallel = 0 \), we recover the growth/damping rate and dispersion relation as derived by Sallimullah and Rosenberg [1999].

5. Result and discussion

In the numerical calculation of the growth/damping rate and dispersion relation we have used the following parameters for the plasma sheet boundary layer [Wygant et al., 2000, 2002]

\[
B_0 = 400\text{nT}, \ n_0 = 0.4 \text{ cm}^{-3}, \ T_{\parallel e} = 2\text{keV}, \ T_{\perp i} = 4 \text{ keV}, \ E_\parallel = 0-100 \text{ mV/m}, \ k_\parallel = 1.0 \times 10^{-10} \text{cm}^{-1}
\]

Eq. (3) & (4) has been evaluated numerically using the Newton-Raphson method to solve the dispersion relation for wave frequency \( \omega (s^{-1}) \) and growth/damping rate \( (\gamma_L) \).

In the present analysis we have considered weak electric field only which does not affect the motion of ions and modified the electron thermal velocity parallel to the ambient magnetic field. This weak electric field produces the drift velocity of electrons much smaller than the phase velocity of kinetic Alfvén wave. However, the moderate and strong parallel electric fields lead the ion beam and electron current which must be accounted altogether as discussed by Voitenko [1998] and may be considered as further investigations. In our calculations \( k_\parallel = 1.0 \times 10^{-10} \text{cm}^{-1} \) and for such a long parallel wavelengths the condition \( \frac{k_\parallel T_e}{eE_{Dr}} < 1 \) is always satisfied for the small Dreicer fields \( E_{Dr} \).

Thus, the electron currents can be ignored.

Fig.1(a,b,c) shows the variation of wave frequency \( \omega (s^{-1}) \) with perpendicular wave number \( (k_\perp \rho_i) \) for both \( b_i<1 \) and \( b_i>1 \) at different values of downward (positive) and upward (negative) parallel electric field \( (E_\parallel) \) and for distribution index \( J=0 \). For electric field \( E_\parallel = 0 \) frequency \( \omega (s^{-1}) \) is increasing with wave number \( (k_\perp \rho_i) \) and is pertaining to the observed values [Wygant et al., 2000]. It is also noticed that the higher electric fields can reduce the wave frequency. For the upward (negative) electric fields the wave frequency \( \omega \) is negative representing the negative phase velocity. Thus, we can predict that the parallel electric field may result the reflection of kinetic Alfvén wave resulting upward/down current pattern driven by KAW usually reported in auroral region at substorm times.

Fig.2(a,b,c) shows the variation of growth/damping rate of the KAW for different values of parallel electric field for bi-Maxwellian distribution of particles i.e. \( J=0 \). The wave is highly damped in case of no parallel electric field as the usual characteristic of kinetic Alfvén wave. The growth of wave starts when parallel electric field exits as seen fig. 2a. It is observed that the growth rate decreases with \( k_\perp \rho_i \). The damping in the wave may be due to the transfer of energy from the wave to the particles through the wave-particle interaction. The effect of positive (downward) electric field is to reduce the growth rate. At higher values of \( k_\perp \rho_i \) the wave may even cease to exist as predicted by Cranmer and Van Ballegooijen [2003].

Fig.3 (a, b, c) shows the variation of wave frequency for different values of parallel electric field in the presence of the loss-cone distribution index \( (J=1) \). It is observed that in the presence of the loss-cone distribution the frequency of the wave increases as compared to that for bi-Maxwellian distribution \( J=0 \) (see fig. 1) suggesting that as the Alfvén wave propagates from the PSBL to the ionosphere it becomes kinetic. The effect of higher electric fields is to reduce the wave frequency, as observed previously for \( J=0 \). For \( J=1 \) also the upward (negative) electric field may gives rise to the reflected waves as is evident from the negative frequency at higher \( k_\perp \rho_i \). The negative electric fields become effective at lower \( k_\perp \rho_i \) for \( J=1 \) as compared to \( J=0 \) to produce the reflection of the wave.

Fig. 4(a,b,c) shows the variation of growth/damping rate of KAW for different values of parallel electric field in the presence of the loss-cone distribution \( (J=1) \). It is observed that the effect of loss-cone distribution is to enhance the growth rate of the KAW as compared to that for \( J=0 \) (fig.2). Here it is noticed that both the upward and downward electric fields can generate kinetic Alfvén waves.
depending upon \(k_{\perp}\rho_i\) in the presence of steep loss-cone distribution functions. Hence, the loss-cone distribution acts as a source of free energy and enhances the growth rate of the wave through the wave particle interaction in the presence of parallel electric fields.

Fig. 5 (a, b, c) shows the variation of wave frequency for different values of parallel electric field and for higher index of loss-cone distribution \((J=2)\). Negative frequency of the wave obtained for a larger range of \(k_{\perp}\rho_i\) show that at higher distribution index \((J=2)\) the wave is more readily reflected even for lower parallel electric field. This is in accordance with the fact that at the poles, which have higher distribution index, the wave is readily reflected back. In this figure the effectiveness of parallel electric fields is noticed with the increasing value of loss-cone index. The negative electric fields can support the reflected waves depending upon \(k_{\perp}\rho_i\) in the presence of steep loss-cone distribution function.

Fig. 6(a, b,c) shows the variation of growth/damping rate for different values of parallel electric field at higher index of loss-cone distribution \((J=2)\). It is observed that as compared to \(J=1\) the growth rate of the wave increases for \(J=2\). The damping in the wave also increases for \(J=2\). It is seen that the combined effect of general loss-cone distribution and parallel electric field is operative move effectively towards lower \(k_{\perp}\rho_i\) to control the direction of propagation and the growth/damping rate of the wave. The kinetic Alfven wave may be generated by parallel electric fields in the presence of general loss-cone distribution function.

6. Conclusions

(1). The results of the present study have shown that in the presence of the loss-cone distribution the parallel electric fields support the incident and reflected waves there by standing wave pattern may result. Hence, the loss-cone distribution is an important factor to study KAWs in the PSBL as the distribution index alters the propagating properties of KAW.

(2). The kinetic Alfven waves may be excited in the plasma sheet boundary layer by the combined effect of parallel electric fields and steep loss-cone distribution of particles carrying sufficient amount of Poynting flux that would have propagated toward the ionosphere leading to the acceleration of charged particles.

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Fig. 1 Variation of wave frequency ω(s⁻¹) with perpendicular wave number (k⊥ρ) for different values of parallel electric field (E||) and for distribution index J=0.
Fig. 2 Variation of growth/damping rate \( \gamma \) (s\(^{-1}\)) with perpendicular wave number \( k_\perp \rho_i \) for different values of parallel electric field \( E_\parallel \) and for distribution index \( J=0 \).
Fig. 3 Variation of wave frequency $\omega$ (s$^{-1}$) with perpendicular wave number ($k_{\perp}\rho_i$) for different values of parallel electric field ($E_{||}$) and for distribution index $J=1$. 
Fig. 4 Variation of growth/damping rate $\gamma_L (s^{-1})$ with perpendicular wave number ($k_\perp \rho_i$) for different values of parallel electric field ($E_\parallel$) and for distribution index $J=1$. 
Fig. 5 Variation of wave frequency $\omega (s^{-1})$ with perpendicular wave number ($k_{\perp \rho_i}$) for different values of parallel electric field ($E_{||}$) and for distribution index $J=2$. 
**Fig. 6** Variation of growth/damping rate $\gamma (s^{-1})$ with perpendicular wave number ($k_{\parallel} \rho_i$) for different values of parallel electric field ($E_||$) and for distribution index $J=2$. 