Baryon Electric Dipole Moments from Strong CP Violation

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Ottnad, Kubis, Meißner, Guo, PLB687(2010)42; Guo, Meißner, JHEP07(2012)097
...it may be that the next exciting thing to come along will be the discovery of a neutron or atomic or electron electric dipole moment. These electric dipole moments ... seem to me to offer one of the most exciting possibilities for progress in particle physics.

— Steven Weinberg (1992)
Outline

1 Introduction

2 Baryon EDMs in chiral perturbation theory

3 Finite volume corrections

4 Summary
Electric dipole moment (EDM)

- EDM measures the polarity of a charged system, \( \vec{d} = \sum_i q_i \vec{r}_i \)
- For a hadron or any elementary particle at rest, \( \vec{d} = d \frac{\vec{S}}{|\vec{S}|} \)
- Hamiltonian for a dipole interacting with an electric field

\[
\mathcal{H}_{edm} = -\vec{d} \cdot \vec{E} = -d \frac{\vec{S} \cdot \vec{E}}{|\vec{S}|}
\]

- P and T(CP) violation
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**Introduction**

**Neutron and proton EDMs**

- In the Standard Model, no EDM at the first order of weak interaction

\[ |d_n(\text{SM})| \sim 10^{-31} e\text{cm} \]

- Sensitive to physics beyond the Standard Model: some new physics models’ predictions are orders-of-magnitude larger

- Current experimental upper limit

\[ |d_p| < 7.9 \times 10^{-25} e\text{cm} \]

  calculated from the limit on the EDM of the $^{199}$Hg atom Griffith et al (2009)

\[ |d_n| < 2.9 \times 10^{-26} e\text{cm} \]

  ultracold neutron experiment Baker et al (2006)

\[ |d_\Lambda| < 1.5 \times 10^{-16} e\text{cm} \]

  $3 \times 10^6 \Lambda \rightarrow p\pi^-$ decays Pondrom et al (1981)
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Introduction

History of the neutron EDM measurements

![Graph showing the history of neutron EDM measurements]

- ORNL, Harvard
- MIT, BNL
- LNPI
- Sussex, RAL, ILL

Supersymmetry Predictions

Standard model Predictions

Year of Publication
Introduction
Current and near future experiments

- Neutron EDM
  - nEDM Collaboration @ PSI:
    - Ultracold neutron experiment
    - Goal for 2012&2013 data: $5 \times 10^{-27} \text{e cm} (95\% \text{ CL})$
    - n2EDM (2015 – ?) with a sensitivity goal of $5 \times 10^{-28} \text{e cm}$
    - CryoEDM @ ILL with a sensitivity goal of $10^{-28} \text{e cm}$
    - TRIUMF neutron EDM (2018 – ?): $1 \times 10^{-28} \text{e cm}$
    - R. Brock et al, arXiv:1205.2671[hep-ex];
      - http://nedm.web.psi.ch/EDM-world-wide/

- EDMs of charged particles (proton, deuteron and other light nuclei)
  - Storage ring experiment
    - Goal: $10^{-29} \text{e cm}$
    - Storage ring EDM Collaboration @ BNL (?), Fermilab (?)
    - JEDI Collaboration @ Jülich (?)

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- Supersymmetry Predictions
- Standardmodel Predictions [Electro-weak]

Year of Publication

| Year | 1950 | 1960 | 1970 | 1980 | 1990 | 2000 | 2010 |
|------|------|------|------|------|------|------|------|
| Value | $10^{-32}$ | $10^{-31}$ | $10^{-30}$ | $10^{-29}$ | $10^{-28}$ | $10^{-27}$ | $10^{-26}$ | $10^{-25}$ | $10^{-24}$ | $10^{-23}$ | $10^{-22}$ | $10^{-21}$ | $10^{-20}$ | $10^{-19}$ |

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Introduction
Theta-term in QCD & Strong CP problem

- Nothing forbids a $\theta$-term in the QCD Lagrangian

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} G^{a}_{\mu\nu} G^{a,\mu\nu} + \bar{q} (i \gamma - M_q) q + \theta \frac{g^2}{32\pi^2} G^{a}_{\mu\nu} \tilde{G}^{a,\mu\nu}$$

Here, $\tilde{G}^{a,\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\alpha\beta} G^{a}_{\alpha\beta}$

- The contribution of the $\theta$-term to the neutron EDM

$$d_n \sim 10^{-16} \theta e\text{ cm}$$

Crewther et al (1979)

- Strong CP problem: $\theta \lesssim 10^{-10}$, why is it so small?
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- Generally, $M_q$ is complex and non-diagonal in the Standard Model. We will encounter a $U_A(1)$ transformation $\exp(i\alpha) \equiv \exp[i(\varphi_R - \varphi_L)/2]$

$$M_q \rightarrow e^{-i\varphi_L}M_qe^{i\varphi_R}, \quad q_L \rightarrow e^{-i\varphi_L}q_L, \quad q_R \rightarrow e^{-i\varphi_R}q_R$$

$$\Rightarrow \arg(\det M_q) \rightarrow \arg(\det M_q) + 2N_f\alpha$$

- For massless quarks

  - Classically, $\partial_\mu J_5^\mu = 0$
  
  - Quantum $U_A(1)$ anomaly: $\partial_\mu J_5^\mu = \frac{N_f g^2}{16\pi^2} G_{\mu\nu}^a \tilde{G}^{a,\mu\nu}$

$$\Rightarrow \theta \rightarrow \theta - 2N_f\alpha$$

- One cannot solve the strong CP problem by simply imposing CP on QCD

The measurable quantity is not $\theta$ but $\theta_0 = \theta + \arg(\det M_q)$
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Lattice calculations

Calulations from several groups.
For a review, see
Shintani, talk at ConfinmentX (2012)

From lattice to reality
- Finite lattice spacing: $a \to 0$
- Finite volume: $V \to \infty$
- Quark masses: normally $M_{\pi}^{\text{lattice}} > M_{\pi}^{\text{physical}}$, chiral extrapolation

Our purpose: to calculate baryon EDMs in U(3) CHPT — chiral extrapolation and finite volume corrections
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Baryon EDMs in CHPT
Baryon electromagnetic form factors

\[ \langle B(p')| J_{\text{em}}^\nu | B(p) \rangle = \bar{u}(p') \left[ \gamma^\nu F_1(q^2) - \frac{iF_2(q^2)}{2m_B} \sigma^{\mu\nu} q_\mu \right. \\
+ i \left( \gamma^\nu q^2 \gamma_5 - 2m_B q^\nu \gamma_5 \right) F_A(q^2) - \frac{F_3(q^2)}{2m_B} \sigma^{\mu\nu} q_\mu \gamma_5 \left. \right] u(p) \]

- \( F_{1,2}(q^2) \): Dirac and Pauli form factors, preserve P and CP
- \( F_A(q^2) \): anapole form factor, P-violating
- \( F_3(q^2) \): electric dipole form factor (EDFF), P- and CP-violating

The EDM is related to \( F_3(0) \)

\[ d_B = \frac{F_{3,B}(0)}{2m_B} \]
Chiral perturbation theory (CHPT) is the low-energy effective theory of QCD (Weinberg (1979), Gasser & Leutwyler, (1984,1985))

- CHPT has the same symmetries as QCD
- Chiral symmetry is broken both spontaneously and explicitly
  - a double-expansion in both small momentum and light quark masses,
    \[ \frac{p}{\Lambda_\chi} \sim \frac{M_\pi}{\Lambda_\chi} \ll 1 \]
  - Framework for extrapolation from unphysical to physical quark masses
- Construct the most general effective Lagrangian up to a certain order
  - more and more parameters: low-energy constants (LECs)
- Nonrenormalizable, but can be renormalized to a given order

Calculations of \( d_{n(p)} \) in CHPT:
- Crewther, Di Vecchia, Veneziano, Witten (1979); Pich, de Rafael (1991); Borasoy (2000); Narison (2008), Hockings, van Kolck (2005); Mereghetti et al (2011); …
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Framework for extrapolation from unphysical to physical quark masses

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Framework for extrapolation from unphysical to physical quark masses.

Construct the most general effective Lagrangian up to a certain order.

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Baryon EDMs in CHPT

SU(3) CHPT at $\mathcal{O}(p^2)$

Leading order Lagrangian for SU($N_f$) mesonic CHPT

$$\mathcal{L}^{(2)}_{SU(3)} = \frac{F^2}{4} \text{Tr} \left[ \nabla_{\mu} U^\dagger \nabla^{\mu} U \right] + \frac{F^2}{4} \text{Tr} \left[ \chi U^\dagger + U \chi^\dagger \right]$$

Here $\nabla_{\mu} U = \partial_{\mu} U + i \ell_{\mu} U - i U r_{\mu}$, $\chi = 2B_0(s + ip)$

External sources: $\ell_{\mu}, r_{\mu}, s, p$

Quark masses can be included by $s = M_q = \text{diag}(m_u, m_d, m_s)$

The Goldstone bosons are included in $U = \exp \left( i \frac{\sqrt{2}}{F} \phi \right)$

$$\phi = \begin{pmatrix}
\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta_8 & \pi^+ & K^+ \\
\pi^- & -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta_8 & K^0 \\
K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}} \eta_8
\end{pmatrix}$$
Baryon EDMs in CHPT
U(3) CHPT at $\mathcal{O}(p^2)$

- When the $\theta$-term is absent, $\mathcal{L}_{\text{QCD}}$ has a $U_L(3) \times U_R(3)$ symmetry. Treating $\theta$ as an external field,

$$\theta(x) \xrightarrow{\text{U(1)}} \theta(x) - N_f(\varphi_R - \varphi_L)$$

$\theta(x) = \theta_0$ gives QCD.

- Nine Goldstone bosons: $(\pi, K, \eta_8, \eta_0)$ are collected in $\tilde{U}(x)$

$$\tilde{U}(x) \xrightarrow{\text{U(1)}} e^{i\varphi_R} \tilde{U}(x)e^{-i\varphi_L} \Rightarrow \ln \det \tilde{U}(x) \rightarrow \ln \det \tilde{U}(x) + iN_f(\varphi_R - \varphi_L)$$

$$\bar{\theta}(x) = \theta(x) - i \ln \det \tilde{U}(x) \text{ is chirally invariant}$$

- The most general Lagrangian up to $\mathcal{O}(p^2)$ Gasser, Leutwyler (1985)

$$\mathcal{L}^{(2)}_{\text{U(3)}} = -V_0 + V_1 \text{Tr}[\nabla_\mu \tilde{U}^\dagger \nabla^\mu \tilde{U}] + V_2 \text{Tr}[\chi^\dagger \tilde{U} + \chi \tilde{U}^\dagger] + iV_3 \text{Tr}[\chi^\dagger \tilde{U} - \chi \tilde{U}^\dagger]$$
$$+ V_4 \text{Tr}[\tilde{U} \nabla_\mu \tilde{U}^\dagger] \text{Tr}[\tilde{U}^\dagger \nabla^\mu \tilde{U}] + V_5 \text{Tr}[\nabla_\mu \theta \nabla^\mu \theta]$$

$V_0, \ldots, 5$ are functions of $\bar{\theta}(x)$
Baryon EDMs in CHPT

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$V_0,...,5$ are functions of $\bar{\theta}(x)$
Baryon EDMs in CHPT

U(3) CHPT at $O(p^2)$

- $\tilde{U} = \sqrt{U_0} U \sqrt{U_0}$ where $U_0$ describes the vacuum and

$$U = \exp \left( i \sqrt{\frac{2}{3}} \frac{\eta_0}{F_0} + i \sqrt{\frac{2}{F}} \phi \right)$$

- Vacuum alignment:

$$U_0 = \text{diag} \left( e^{-i\phi_u}, e^{-i\phi_d}, e^{-i\phi_s} \right)$$

is determined by minimizing the potential energy density

$$V_0 - V_2 \text{Tr} \left[ \chi^\dagger U_0 + \chi U_0^\dagger \right] - iV_3 \text{Tr} \left[ \chi^\dagger U_0 - \chi U_0^\dagger \right]$$

For $\theta_0 = 0$, the vacuum solution is trivial $U_0 = 1$.

- Expanding $V_i$’s around $\tilde{\theta}_0 = \theta_0 - i \ln \det U_0$, parameters determined by normalization of the kinetic terms, $\eta - \eta'$ mixing, ...

Herrera-Siklódy et al, (1998)
Baryon EDMs in CHPT
U(3) baryon CHPT up to NLO

Baryon octet

\[
\begin{pmatrix}
\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda & \Sigma^+ & p \\
\Sigma^- & -\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda & n \\
\Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}} \Lambda
\end{pmatrix}
\]

- Up to NLO, about 10 parameters in the Lagrangian

\[
i \text{Tr} [\bar{B} \gamma^\mu [D_\mu, B]] - \bar{m} \text{Tr}[\bar{B}B] - \frac{D}{2} \text{Tr} [\bar{B} \gamma^\mu \gamma_5 \{u_\mu, B\}] - \frac{F}{2} \text{Tr} [\bar{B} \gamma^\mu \gamma_5 [u_\mu, B]] + \frac{w_0}{2} \text{Tr} [\bar{B} \gamma^\mu \gamma_5 B] \text{Tr}[u_\mu] + b_D \text{Tr} [\bar{B} \{\tilde{\chi} +, B\}] + b_F \text{Tr} [\bar{B} [\tilde{\chi} +, B]]
\]

\[
+ b_0 \text{Tr}[\bar{B}B] \text{Tr}[\tilde{\chi} +] + i \left( w'_{13} \bar{\theta}_0 + w_{13} \frac{\sqrt{6}}{F_0} \eta_0 \right) \text{Tr} [\bar{B} \sigma^{\mu\nu} \gamma_5 \{F_{\mu\nu}^+, B\}]
\]

\[
+ 4 \mathcal{A} w'_{10} \frac{\sqrt{6}}{F_0} \eta_0 \text{Tr}[\bar{B}B] + i \left( w'_{14} \bar{\theta}_0 + w_{14} \frac{\sqrt{6}}{F_0} \eta_0 \right) \text{Tr} [\bar{B} \sigma^{\mu\nu} \gamma_5 [F_{\mu\nu}^+, B]]
\]

- Only two combinations up to NLO in total 8 baryon EDMs:
  \[ [w_{13}, w'_{13}(\mu)] \text{ and } [w_{14}, w'_{14}(\mu), w_0, w'_{10}, b_0] \]
Loops are regularized using the method of **infrared regularization**: Lorentz covariant, well-defined power counting

Divergences are absorbed into the renormalization of $w'_{13}$ and $w'_{14}$

Becher, Leutwyler (1999)
Baryon EDMs in CHPT

$|d_n| \neq |d_p|$
Baryon EDMs in CHPT

Pion mass dependence of $d_{n(p)}$

Lattice data: Shintani et al (2008); Shintani, talk given at Confinement X (2012)

Grey bands: LO loop results

Two parameter combinations can be fixed to the two data points at $M_\pi = 530$ MeV
Grey bands: LO loop results

Two parameter combinations can be fixed to the two data points at $M_\pi = 530$ MeV
Predictions at physical $M_\pi$ in units of $10^{-16} \theta_0 e cm$

\[
d_n = -2.9 \pm 0.9, \quad d_p = 1.1 \pm 1.1
\]
\[
d_\Lambda = -2.5 \pm 0.4, \quad d_{\Sigma^+} = -0.7 \pm 1.1
\]
\[
d_{\Sigma^0} = 0.7 \pm 0.4, \quad d_{\Sigma^-} = 2.2 \pm 0.5
\]
\[
d_{\Xi^0} = -3.4 \pm 0.9, \quad d_{\Xi^-} = 0.6 \pm 0.5
\]

Upper limit of $\theta_0$:

\[
|\theta_0| \lesssim 1.5 \times 10^{-10}
\]
Predictions at physical $M_\pi$ in units of $10^{-16} \theta_0 \, e \, cm$

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\begin{align*}
    d_n &= -2.9 \pm 0.9, \\
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    d_{\Xi^0} &= -3.4 \pm 0.9, \\
    d_p &= 1.1 \pm 1.1, \\
    d_{\Sigma^+} &= -0.7 \pm 1.1, \\
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\end{align*}
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$$|\theta_0| \lesssim 1.5 \times 10^{-10}$$
Finite volume corrections

Introduction

- Lattice calculations are performed in a finite volume
  \( \Rightarrow \) momentum is quantized

\[ \vec{q} = \vec{n} \frac{2\pi}{L} \]

- The continuum Lagrangian can still be used
  Loop integrals become summations

\[
i \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_1^2)[(k + q)^2 - m_2^2]} \rightarrow \frac{i}{L^3} \sum_{\vec{n}} \int \frac{dk^0}{2\pi} \frac{1}{(k^2 - m_1^2)[(k + q)^2 - m_2^2]} \]

- Finite volume corrections:

\[ \delta_L[\mathcal{D}] = \mathcal{D}(L) - \mathcal{D}(\infty) \]
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\[
i \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_1^2)[(k + q)^2 - m_2^2]} \to i \sum \int \frac{dk^0}{L^3} \frac{1}{2\pi} \frac{1}{(k^2 - m_1^2)[(k + q)^2 - m_2^2]}\]

- Finite volume corrections:

\[ \delta_L[\mathcal{D}] \equiv \mathcal{D}(L) - \mathcal{D}(\infty) \]
Finite volume corrections

Results for $d_n$

Finite volume corrections to $d_n$ at LO

\[ \delta_L[d_n^{\text{LO}}] = \frac{M^2}{4\pi^2 F^2} e \theta_0 \sum_{\vec{n} \neq 0} \left[ (D + F) (b_D + b_F) K_0 (LM_{\pi} |\vec{n}|) - (D - F) (b_D - b_F) K_0 (LM_K |\vec{n}|) \right] \]

$K_0(z)$: modified Bessel function of the second kind

\[
\Rightarrow \text{NLO results}
\]

Upper lines: NLO
Lower lines: LO
Finite volume corrections to $d_n$ at LO

O’Connell, Savage (2006); Guo, Meißner (2012)

$$\delta_L [d_n^{LO}] = \frac{M_\pi^2 e \theta_0}{4 \pi^2 F_\pi^2} \sum_{\vec{n} \neq 0} \left[ (D + F) (b_D + b_F) K_0 (LM_\pi |\vec{n}|) - (D - F) (b_D - b_F) K_0 (LM_K |\vec{n}|) \right]$$

$K_0(z)$: modified Bessel function of the second kind

$\Leftarrow$ NLO results

Upper lines: NLO
Lower lines: LO

F. K. Guo (Uni. Bonn)
Summary

- Neutron and proton EDMs: *P* and *CP* violating, sensitive to new physics, under intense experimental investigations
- Many lattice calculations: at unphysical pion masses and in a box
- CHPT: chiral extrapolation and finite volume corrections

\[ d_n = (-2.9 \pm 0.9) \times 10^{-16} \theta_0 \text{ e cm}, \quad d_p = (1.1 \pm 1.1) \times 10^{-16} \theta_0 \text{ e cm} \]

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Coupled-channel effects in radiative charmonium transitions

Time: 10:00am, next Tuesday