Optimal Design of Sound Insulation Frequency Band of Local Resonance Membrane Material

Jian Zhang¹, Qizheng Zhou¹*, Deshi Wang¹ and Shengyao Gao²

¹Naval University of Engineering, Wuhan 430033, China; ²No. 92578 Troops of PLA, Beijing 100161, China.

*Corresponding author email: zqizheng@126.com

Abstract. In order to broaden the sound insulation frequency band of local resonance membrane materials, established an acoustic-vibration coupling model of multilayer membrane materials, based on the derived analytical solutions of the coupling equations. Simulate the sound insulation performance of the designed single-layer membrane materials with four different mass structures. Then combine different membrane materials to analyse the changes in the sound insulation frequency band. Calculations proves that: (1) The four single-layer membrane structures have a maximum TL peak of 40.08dB and the frequency of the smallest TL peak is at 204.56Hz in the frequency range of 100-1000Hz.(2) Combination of two same structures will significantly increase the sound insulation at the sound insulation peak;(3) The combination of single-layer membrane with different structures can broaden the sound insulation frequency band and enhance the sound insulation performance.

Keywords: Sound insulation, local resonance, combine different membrane materials.

1. Introduction

Sound insulation materials have a wide range of applications in aerospace, naval, environmental noise and other fields. However, affected by the law of mass action, the sound absorption effect of traditional sound insulation materials ranges the low frequency is not ideal. The emergence of local resonance membrane materials provides new design ideas for the development of low-frequency sound insulation. The local resonance membrane material is composed of additional masses on the membrane and adhered to the rigid frame. The membrane and the mass are similar to the spring and vibrator structure in the "vibration absorber", which increases the absorption of elastic waves by the structure, thereby increasing low-frequency sound insulation. The transmission loss of this type of material is significantly increased relative to the results calculated by the law of mass action.

The membrane-mass structure constitutes a local resonant unit, which is arranged periodically will show extraordinary characteristics that are completely different from its composition. Current research shows that band gaps are formed when elastic waves propagate in periodic elastic composite materials. The research of Yang found that a small mass is placed in the centre of the membrane. Between the two eigenmodes of the membrane material, there is a frequency point that makes the in-plane average of vibration displacement is zero, resulting in very small far-field acoustic radiation and realizing the total reflection of sound waves. However, the structure with a mass attached to the centre of the membrane still has a limited sound insulation frequency band. In order to optimize the sound insulation performance of the local resonance membrane material and broaden its sound insulation frequency band, we designed four membrane structures by changing the shape, numbers and relative position of the masses. The basic units are respectively a symmetric semi-circular mass, asymmetric semi-circular mass,
symmetric circular mass, and asymmetric circular mass attached to a rectangular membrane. In the following description, these four structures are represented by A, A, B, and B respectively. Combine different single-layer membrane structures into multi-layer membrane-mass structures and study the changes in sound insulation performance.

This paper starting from the forced vibration equation of a single-layer membrane, the analytical solution of the vibration-acoustic coupling equation of the membrane-mass structure is derived, and expand to multilayer membrane structure, which provide a theoretical basis for finite element calculation. On this basis, the finite element analysis method is used to calculate the sound insulation performance of four single-layer membrane structures, and different single-layer membrane structures are combined into different multi-layer membrane structures to realize the broadening of the sound insulation frequency band.

2. Method

2.1 Acoustic-vibration Coupling Model and Formulation

The acoustic-vibration coupling model of the double-layer local resonance membrane material is established, and the analytical solution of the model is derived based on the acoustic reflection theory and the modal superposition method. Figure 1(a) is a schematic diagram of the model. The two-layer membrane divides the air domain into three parts, the distance between the two membranes is \( d \), the \( z \) axis is orthogonal to the membrane, and the coordinate origin is \( o \). Figure 1(b) is a schematic diagram of the structure of a single-layer membrane. The single-layer membrane is composed of a rectangular membrane with the length of \( a \) and width of \( b \) and a mass attached to the membrane.

Membrane I is excited by a plane acoustic wave with a frequency \( \omega \) in the \(-z\) direction, and the incident plane acoustic wave can be expressed as

\[
p_i(z,t) = A_i e^{i(\omega t - kz)}
\]

where \( k = \omega / c_a \), \( c_a \) is the sound speed of air. The incident pressure field and transmission pressure field from the membrane are written as

\[
p_2(z,t) = A_2(x,y) e^{i(\omega t + kz)}
\]

\[
p_3(z,t) = A_3(x,y) e^{i(\omega t - kz)}
\]

The coefficients \( A_i \), \( A_2 \), \( A_3 \) represent the sound pressure amplitude of the incident pressure field, reflected pressure field and transmission pressure field, respectively.
Under the assumptions that the bending stiffness of the membrane ignored and the mass does not prevent the bending of the membrane segment, the motion differential equations of the double membrane-type acoustic metamaterials are given by

\[
\rho_1 \left( \frac{\partial^2 w_1}{\partial t^2} \right) + \sum_{i=1}^{0} \rho h(x,y,x_0,y_0,\Delta l_1,\Delta l_2) \left( \frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial y^2} \right) = p_1 + p_3 - p_2 \quad \text{for membrane I} \quad (4)
\]

\[
\rho_2 \left( \frac{\partial^2 w_2}{\partial t^2} \right) + \sum_{i=1}^{0} \rho h(x,y,x_0,y_0,\Delta l_1,\Delta l_2) \left( \frac{\partial^2 w_2}{\partial x^2} + \frac{\partial^2 w_2}{\partial y^2} \right) = p_4 + p_6 - p_5 \quad \text{for membrane II} \quad (5)
\]

where, \( p_3 = p_1|_{x=0}, p_4 = p_2|_{x=a}, p_5 \), and \( T_i \) are the density and tension of membrane I, \( \rho, \rho, \) and \( T_2 \) are the density and tension of membrane II. \( h(x,y,x_0,y_0,\Delta l_1,\Delta l_2) \) is the superposition of four Heaviside step functions.

\[
h(x,y,x_0,y_0,\Delta l_1,\Delta l_2) = H(x-x_0,y-y_0) - H(x-x_0,y-y_0-\Delta l_1) - H(x-x_0-\Delta l_1,y-y_0) + H(x-x_0-\Delta l_1,y-y_0-\Delta l_2)
\]

(6)

Where, \( H(x-x_0,y-y_0) = \begin{cases} 0 & x < x_0 \text{ or } y < y_0 \\ 1 & x \geq x_0 \text{ and } y \geq y_0 \end{cases} \).

For the fixed boundary of membrane, \( w_1(x,y,t) \) and \( w_2(x,y,t) \) can be written as a combination of mode functions and time generalized function, generalized coordinate \( e^{i\omega t} \) based on the mode superposition theory. That is

\[
w_1(x,y,t) = \sum_{m=1}^{M} \sum_{n=1}^{N} W_{1mn} \sin k_m x \sin k_n y e^{i\omega t}
\]

(7)

\[
w_2(x,y,t) = \sum_{m=1}^{M} \sum_{n=1}^{N} W_{2mn} \sin k_m x \sin k_n y e^{i\omega t}
\]

(8)

where, \( k_m = m\pi / a, k_n = n\pi / b, W_{1mn} \) and \( W_{2mn} \) are the modal coefficients.

Taking membrane I as an example, the coupling surface should satisfy the boundary condition between the membrane and the acoustic medium, that is

\[
p_1 - p_3 = \rho_s c_s \frac{\partial w_1}{\partial t} \quad z = 0^-
\]

(9)

\[
p_2 = \rho_s c_s \frac{\partial w_1}{\partial t} \quad z = 0^+
\]

(10)

\[
p_1 + p_3 - p_2 = 2p_1 - 2\rho_s c_s \frac{\partial w_1}{\partial t} \quad z = 0^-
\]

(11)

Substituting formula (1) and formula (11) into formula (4) to sort out

\[
\rho_1 \left( \frac{\partial^2 w_1}{\partial t^2} \right) + \sum_{i=1}^{0} \rho h(x,y,x_0,y_0,\Delta l_1,\Delta l_2) \left( \frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial y^2} \right) + 2\rho_s c_s \frac{\partial w_1}{\partial t} - TV^2 w_1 = 2A e^{i\omega t}
\]

(12)

In the same way, the differential equation of motion of membrane II can be sorted into equation (13)

\[
\rho_2 \left( \frac{\partial^2 w_2}{\partial t^2} \right) + \sum_{i=1}^{0} \rho h(x,y,x_0,y_0,\Delta l_1,\Delta l_2) \left( \frac{\partial^2 w_2}{\partial x^2} + \frac{\partial^2 w_2}{\partial y^2} \right) + 2\rho_s c_s \frac{\partial w_2}{\partial t} - TV^2 w_2 = 2A e^{i\omega t}
\]

(13)

Integrate the above formula, we can get relation between \( w_1 \) and \( w_2 \).
\[ w_2 = \frac{2j\omega \rho \sigma c_m w_i}{-\omega^2 \rho_j - \omega^2 \sum_{i=1}^{l} \rho_i h(x_i, y_i, x_0, y_0, l_x, l_y) + 2j\omega \rho \sigma c_m - T \left(k_n^2 + k_n^0\right)} \] (14)

The transmission sound pressure coefficient of the membrane structure is the ratio of the transmission sound pressure to the incident sound pressure. The transmission sound pressure can be obtained by averaging the surface vibration speed of the membrane structure. The transmission sound pressure coefficient of the single-layer membrane structure and the double-layer membrane structure can be expressed for

\[ t_{p_1} = \frac{p_2}{A e^{j\omega \tau}} = \frac{j\rho c_{ma} \omega w_i}{A_{ab}} \] (15)

\[ t_{p_2} = \frac{p_1}{A e^{j\omega \tau}} = \frac{j\rho c_{ma} \omega w_2}{A_{ab}} \] (16)

Then the transmission loss expression of the membrane structure can be obtained

\[ TL_1 = 20\log_{10} \left(1/t_{p_1}\right) \] (17)

\[ TL_2 = 20\log_{10} \left(1/t_{p_2}\right) \] (18)

Using the same idea, the calculation formula of transmission loss can be extended to multi-layer membrane structures.

2.2 Membrane Structural Design and Analysis

The analysis of the acoustic and vibration characteristics theory provides a theoretical basis for the finite element simulation of the local resonance type membrane material, and then the finite element analysis method is used to calculate the frequency regime of 100-1000Hz. In our calculation, the length, width and thickness of the rectangular membrane are 30mm, 15mm, 2mm, respectively. The material is silicone rubber, and the Young's modulus, Poisson's ratio and density are \(1.9 \times 10^6\)Pa, 0.48 and 980kg/m³, respectively. While Young's modulus, Poisson's ratio and density for the masses are \(205 \times 10^9\), 0.28, 980kg/m³, the material of the mass is structural steel. The radius of the semi-circular mass and the circular mass are both 5mm, the thickness are both 1mm, the horizontal distance between the centre of the masses is 14mm, the symmetrical mass is placed on the central axis of the membrane, the asymmetric mass is offset from the centre axis by 0.75mm and -0.75mm, respectively. Acoustic pressure of 1 atm, and speed of sound in air of \(c=340\)m/s were used. Analyse the TL peak and sound insulation frequency band in the calculation result. Here, the frequency band greater than 15dB is defined as the effective sound insulation frequency band.
Figure 2. The TL of four single-layer membrane structures in the 100-1000Hz frequency regime, the detail values at the TL peak are given in Table 1. (A-structure represents the additional symmetrical semi-circular mass structure on the membrane; A- structure represents the additional symmetrical semi-circular mass structure on the membrane; B- structure represents the additional symmetrical circular mass structure on the membrane; B- structure represents the additional asymmetric circular mass structure on the membrane.)

Table 1. The detail values at the TL peak.

|               | First TL peak (Hz) | TL (dB) | Second TL peak (Hz) | TL (dB) | Average TL (dB) |
|---------------|--------------------|---------|---------------------|---------|-----------------|
| A             | 294.18             | 33.67   | 405.74              | 20.64   | 10.79           |
| A             | 296.34             | 32.76   | 409.81              | 18.94   | 10.65           |
| B             | 236.73             | 19.92   | 518.51              | 40.08   | 26.62           |
| B             | 204.56             | 32.01   | 261.63              | 17.03   | 17.56           |

The sound transmission loss curves of A-structure and A-structure structures are basically the same, and the sound insulation frequency band of the two structures is 240Hz-480Hz; The average TL and TL peak of the B-structure are the largest among the four structures, reaching to 26.62dB and 40.08dB respectively, and the effective sonic band gap is 200Hz-1000Hz, it is the widest of them; the first sound insulation peak of the B-structure is 204.56Hz, and the effective sonic band gap is 180Hz-280Hz, which is the structure with the best low-frequency sound insulation performance among the four structures. The four structures all have good sound insulation performance for different frequency bands. The four structures are stacked in different ways to expand the sound insulation frequency band of the local resonance film material, these four structures are stacked in different ways to expand the sound insulation frequency band of the local resonance membrane material.

According to the sound insulation performance of the above four single-layer structures, two double-layer membrane structures are built (two A-structures are combined; A and B-structure are combined), and two three-layer membrane structures are built (A, B and A-structure are combined, and A, B and B-structure are combined). Through calculation, the result shown in Figure 5 and Figure 6.
As shown in Figure 3(a), compared with a single layer, the overall sound insulation performance has been improved. The changes in the two TL peaks are more obvious. The TL of the two peaks has increased by about 7dB, but the sound insulation frequency band has not changed. In figure 3(b), it can be clearly seen that the first TL peak of the double-layer structure is similar to the TL peak of the A-structure, and the second TL peak is similar to the TL peak of the B-type structure. The sound insulation frequency bands of the two single-layer structures are superimposed on the double-layer structure, which broadens the sound insulation frequency band to 180Hz-1000Hz, and the average TL between 100-1000Hz is 27.54Hz, the sound insulation performance is significantly improved relative to the single layer.

As shown in Figure 4(a), The first and second TL peak of the three-layer membrane structure reached 41.22dB and 43.25dB, respectively, the average TL reached 28.46dB, and the effective sonic band gap is 190-1000Hz. In figure 4(b), The peak and average TL of this structure reached 55.56dB and 34.82Hz, and the sound insulation frequency band is 175Hz-1000Hz. Compared with the double-layer structure, the three-layer structure can achieve wider frequency band sound insulation.

From the above analysis, it is found that the sound insulation performance of the multilayer membrane structure is not a simple superposition of the multiple single-layer membrane structures. The frequency point at the TL peak of each single-layer structure has a greater impact to the sound insulation performance of the multilayer membrane structure. Taking the double-layer structure as an example, it can be seen that the increase in the TL peak is the most obvious. In the double-layer membrane structure
AA, the increase in the first and second TL peaks reached 20% and 34%; In the double-layer structure membrane AB, the TL peaks of structure A and structure B are superimposed on the double-layer structure AB. In the three-layer structure, a situation similar to the double-layer structure can also be seen. The sound insulation frequency band of different single-layer structures will be superimposed in multiple layers, and the TL will increase.

3. Conclusion
In summary, we can get the following conclusions: Changing the shape and position of the mass in the local resonance membrane material can isolate sound waves in different frequency bands; Membrane materials with the same structure form a multilayer structure, which will greatly enhance the sound insulation performance in a certain frequency band; Combining single-layer materials with different structures can broaden the sonic band gap. Through the design of single-layer structure and different combinations of single-layer structure, The local resonance type membrane material can enhance the sound insulation performance in the low frequency broadband range.

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