Correlation Analysis between Gross Parameters and Performance Parameters of the Light Aircraft

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Abstract. Based on the elementary concepts of aircraft design, statistical analysis are made between the gross parameters and performance parameters of modern two-seater and four-seater light aircraft. The relations among these parameters are found. 12 experiential expressions are obtained and some parameters’ value scopes are determined. These expressions and data are proved to be useful for the conceptual design of two-seater and four-seater light aircraft.

1. Introduction

In conceptual design stage, aircraft design is largely dependent on previous design experience, so each aircraft design and research institutions typically has analyzed a large amount of statistical data, and has got a series of empirical formulas for overall parameters and performance parameters. Through these empirical formulas, the aircraft designers can select, analyze and evaluate the overall parameters and performance parameters of the scheme.

A large number of studies have been conducted on the light aircraft [1-3]. As the new technique, new materials, new technology are developed and applied, a lot of the new type light aircraft have been developed in recent years. Now the relationship between overall and performance parameters reflecting the current technical conditions has not been set up. Based on these considerations above, the paper collects and sorts the related parameters of light aircraft of the past and the recent at home and abroad, and the correlation of the overall and performance parameters of light aircraft is analyzed.

2. The source and classification of the overall and performance parameters

At present, there are many kinds of light aircraft. Due to the difference of mission requirements and design characteristics, there are different statistical rules between the overall and performance parameters of different types of light aircraft. Light aircraft are mainly used in tourism, pilot training and other aspects, and most of them are two-seater and four-seater in the market. Therefore, this paper mainly studies two-seater and four-seater light aircraft. All the data cited in this paper are from the literature [4-6], with a total of 106 models, of which 64 are two-seater light aircraft and 42 are four-seater light aircraft.

The general and performance parameters of the aircraft are many, and the weight characteristic of the aircraft, the wing loading and the power to weight ratio are the three key parameters of the conceptual design phase. The wing area and the aspect ratio are important parameters affecting the aerodynamic characteristics of the aircraft. Endurance performance and flight speed are the main parameters...
reflecting the performance of light aircraft\cite{7-8}. The following equation shows the weight characteristics of the aircraft:

$$W_{TO} = W_E + W_{PL} + W_F$$

(1)

In the formula, $W_{TO}$ is the take-off weight, kg; $W_E$ is the empty weight, kg; $W_{PL}$ is the payload, kg; $W_F$ is the fuel weight, kg.

Take-off weight is composed of empty weight, payload and fuel weight. Among them, the empty weight including structure weight, power system weight, avionics system weight and control system weight. The deformation of equation (1) can be obtained:

$$W_{TO} = W_{PL} / \left(1 - \frac{W_E}{W_{TO}} - \frac{W_F}{W_{TO}}\right)$$

(2)

where, $W_E/W_{TO}$ is the coefficient of the empty weight, and $W_F/W_{TO}$ is the coefficient of fuel weight. Generally, $W_{PL}$ is the data specified in the requirements of the aircraft design. Therefore, if the relationship between $W_E/W_{TO}$ and $W_F/W_{TO}$ is known, or the relationship between $W_E/W_{TO}$ and $W_F/W_{TO}$ and $W_{TO}$ is known, then the take-off weight can be obtained.

If we find out the mathematical relation between the wing loading and the power to weight ratio and the take-off weight, then the wing area and engine power can be obtained. The aircraft performance mainly depends on the overall parameters of the aircraft. After the overall parameters are determined, the aircraft performance will be roughly determined.

Based on the above considerations, the overall parameters and performance parameters studied in this paper are shown in Table 1 and Table 2.

| Table 1. Overall parameters. |
|-----------------------------|
| Parameter                  | Symbol | Unit  |
| Empty weight               | $W_E$  | kg    |
| Wing area                  | $S$    | m$^2$ |
| Aspect ratio               | $A$    | /     |
| Engine power               | $P$    | kW    |
| Take-off wing loading      | $W_{PL}$ | kg/m$^2$ |
| Take-off power to Weight ratio | $W_{TO}$ | kW/kg |

| Table 2. Performance parameters. |
|-----------------------------|
| Parameter                  | Symbol | Unit  |
| Take-off weight            | $W_{TO}$ | kg  |
| Fuel weight                | $W_F$  | kg   |
| Maximum level speed        | $V_{MAX}$ | m/s |
| Stall speed                | $V_S$  | m/s  |

3. Analysis and processing of statistical data

3.1. Statistical analysis of some parameters

Because different aircraft have different design requirements, for a particular parameter, the variation range of statistical data is generally large, which makes the inaccurate statistical relationship between the parameters. For this reason, the statistical values of all parameters are treated as logarithm, and the processed values are relatively concentrated, so the mathematical relations between parameters can be established. Before establishing the mathematical relationship between the overall and performance parameters, this paper first gives the value range of some parameters determined according to the
statistical results, which can provide useful data and reference for the same type of light aircraft in the conceptual design stage, as shown in table 3.

| Parameter                        | Two-seater aircraft | Four-seater aircraft |
|----------------------------------|---------------------|----------------------|
|                                  | Average             | Standard deviation   | Average     | Standard deviation   |
| Take-off wing load (kg/m²)       | 58.075              | 23.143               | 94.258      | 25.566               |
| Take-off power to weight ratio (kW/kg) | 67.028             | 11.031               | 178.101     | 57.985               |
| Engine power (kW)                | 62.266              | 14.422               | 134.692     | 34.982               |
| Wing area (m²)                   | 10.947              | 1.855                | 13.824      | 2.812                |
| Aspect                           | 9.21                | 0.93                 | 8.279       | 1.785                |
| Maximum level speed (km/h)       | 219.514             | 41.215               | 295.628     | 44.825               |
| Stall speed (km/h)               | 73.846              | 16.144               | 98.029      | 26.095               |

3.2. Empty weight and take-off weight

The magnitude of the take-off weight determines the value of the empty weight. Generally, the larger take-off weight will cause the larger empty weight. Statistics of empty weight and take-off weight of 64 two-seater light aircraft are shown in figure 1. The fitting equation is as follows:

\[ \frac{W_E}{W_{TO}} = 0.04W_{TO}^{0.410} \]  

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\[ y = 1.410x - 3.208 \]
\[ R^2 = 0.924 \]

Figure 1. Empty weight and take-off weight of two-seater light aircraft.

Statistics of empty weight and take-off weight of 29 four-seater light aircraft are shown in figure 2. The fitting equation is as follows:

\[ \frac{W_E}{W_{TO}} = 0.056W_{TO}^{0.334} \]  

\[ W_E/W_{TO} = 0.056W_{TO}^{0.334} \]
It can be seen from Figure 1 and Figure 2 that there is a good linear relationship between the empty weight and the take-off weight, and the empty weight increases with the increase of the take-off weight. Equation (3) and equation (4) are the relation between the empty weight coefficient and the take-off weight. These formulas can provide a reference basis for the estimation of the take-off weight, and it can be seen that the empty weight coefficient increases with the increase of the take-off weight.

3.3. Fuel weight and take-off weight

The aircraft’s fuel weight includes the mission fuel weight and the dead fuel weight, and the mission fuel weight depends on the flight mission, the aerodynamic characteristics of the aircraft and the fuel consumption characteristics of the engine. Statistical data of fuel weight and takeoff weight of 43 two-seater light aircraft are shown in figure 3, and the fitting equation is as follows:

\[ \frac{W_F}{W_{TO}} = 0.00013W_{TO}^{1.035} \]  \hspace{1cm} (5)

Statistical data of fuel weight and takeoff weight of 21 four-seater light aircraft are shown in figure 4, and the fitting equation is as follows:
Figure 4. Fuel weight and take-off weight of four-seater light aircraft.

It can be seen from figure 3 and figure 4 that the fuel weight of most aircraft increases with the increase of the take-off weight, while a few aircraft do not show this feature due to mission requirements. Equation (5) and equation (6) are the relation between the fuel weight coefficient and the take-off weight. These formulas can provide a reference basis for the estimation of the take-off weight of light aircraft, and it can be seen that the fuel weight coefficient increases with the increase of the take-off weight.

3.4. Engine power and take-off weight

At the conceptual design stage, suitable engines are usually selected based on the required power of the aircraft. The engine power is higher, and the more fuel it takes to carry out the entire mission, thus the take-off weight of the aircraft will increase to complete the design mission. The statistical data of engine power and takeoff weight of 40 two-seater light aircraft are shown in figure 5. The fitting equation is as follows:

$$P/W_{TO} = 0.00118W_{TO}^{0.742}$$

Figure 5. Engine power and take-off weight of two-seater light aircraft.
The statistical data of engine power and takeoff weight of 24 four-seater light aircraft are shown in figure 6. The fitting equation is as follows:

\[ \frac{P}{W_{TO}} = 0.0429W_{TO}^{0.146} \]  

Equation (7) and equation (8) are the relation between the power to weight ratio and the take-off weight. If the take-off weight is known, the required engine power can be estimated.

3.5. Wing area and take-off weight

The take-off weight determines the maximum lift, which affects the wing area. Wing load is the weight of aircraft divided by wing area, which has a great influence on the determination of take-off weight. If the wing loading decreases, the wing area will be increased, which can improve the aircraft performance, however, additional drag and empty weight which are generated by the increase of wing area will result in the increase of take-off weight for the completion of the mission. The statistical data of wing area and take-off weight of 28 two-seater light aircraft are shown in figure 7. The fitting equation is as follows:

\[ \frac{W_{TO}}{S} = 438.781W_{TO}^{-0.362} \]  

Equation (7) and equation (9) are the relation between the wing area and the take-off weight. If the take-off weight is known, the required wing area can be estimated.

Figure 6. Engine power and take-off weight of four-seater light aircraft.

Figure 7. Wing area and take-off weight of two-seater light aircraft.
The statistical data of wing area and take-off weight of 32 four-seater light aircraft are shown in figure 8. The fitting equation is as follows:

$$\frac{W_{TO}}{S} = 17.584W_{TO}^{0.222} \quad (10)$$

Equation (9) and equation (10) are the relation between the wing load and the take-off weight. If the take-off weight is known, the wing area can be estimated.

3.6. Stall speed and take-off wing loading

The aircraft stall speed is the main factor which affects the flight safety. It is directly determined by the aircraft wing loading and maximum lift coefficient. The statistical data of the stall speed and take-off wing loading of 26 two-seater light aircraft are shown in figure 9. The fitting equation is as follows:

$$V_S = 2.4303\left(\frac{W}{S}\right)_{TO}^{0.529} \quad (11)$$

Equation (9) and equation (11) are the relation between the wing load and the stall speed. If the wing loading is known, the stall speed can be estimated.
The statistical data of stall speed and take-off wing loading of 35 four-seater light aircraft are shown in figure 10. The fitting equation is as follows:

\[ V_s = 0.3379 (W/S)^{0.965} \]  

Equation (11) and equation (12) are the relation between the stall speed and the take-off wing loading. If the take-off wing loading is known, the stall speed can be estimated.

3.7. Maximum level speed and aspect
In general, the drag of the high aspect ratio wing is less than the low aspect ratio wing. Therefore, the aspect ratio directly affects the maximum lift-to-drag ratio of aircraft. Statistics of maximum level speed and aspect ratio of 31 light aircraft with two seats are shown in figure 11. The fitting equation is as follows:

\[ V_{\text{MAX}} = 0.0746A^{3.533} \]  

Equation (11) and equation (12) are the relation between the stall speed and the take-off wing loading. If the take-off wing loading is known, the stall speed can be estimated.

\[ V_s = 0.3379 (W/S)^{0.965} \]  

Figure 10. Stall speed and take-off wing loading of four-seater light aircraft.

Figure 11. Aspect ratio and maximum level speed of two-seater light aircraft.
The statistical data of aspect ratio and maximum level speed of 42 four-seater light aircraft are shown in figure 12. The fitting equation is as follows:

\[ V_{\text{MAX}} = 19.6288A^{0.678} \]  

(14)

**Figure 12.** Aspect ratio and maximum level speed of four-seater light aircraft.

Equation (13) and equation (14) are the relation between the aspect ratio and the maximum level speed. If the aspect ratio is known, the maximum level speed can be estimated.

### 4. Analysis of example

Taking P2010 four-seater aircraft as an example, the statistical formula of four-seater light aircraft is used to calculate and compare with the real data, as shown in table 4. The results show that the satisfactory results can be obtained quickly by applying these formulas in the conceptual design stage.

**Table 4.** Comparison of estimated value and actual value of the P2010 four-seater aircraft parameters.

| Parameter                  | Unit   | Estimate value | Actual value |
|----------------------------|--------|----------------|--------------|
| Empty weight               | kg     | 681.937        | 710.000      |
| Fuel weight                | kg     | 148.676        | 151.200      |
| Take-off wing loading      | kg/m²  | 84.222         | 79.452       |
| Take-off power to weight ratio | kW/kg | 0.120          | 0.116        |
| Stall speed                | km/h   | 87.726         | 88.000       |
| Maximum level speed        | km/h   | 278.290        | 280.000      |

### 5. Conclusion

Based on the statistical analysis of the overall and performance parameters of 106 kinds of light aircraft, this paper obtained 6 empirical formulas for two-seater aircraft and 6 empirical formulas for four-seater aircraft. The calculation example proved that the parameter estimation and selection in the conceptual design stage of light aircraft had certain guiding significance and practical value.
Because exhaust emissions of oil aircraft have an impact on air quality and climate, and with the constant improvement of the battery energy density, now the light electric aircraft begins to grow [9-10]. The empirical formula in this paper is given according to the statistical data of oil aircraft, but some empirical formula can also be applied to light electric aircraft conceptual design phase.

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