Phase diagram of the D3/D5 system in a magnetic field and a BKT transition

Nick Evans,* Astrid Gebauer,† Keun-Young Kim,‡ and Maria Magou§
School of Physics and Astronomy,
University of Southampton,
Southampton, SO17 1BJ, UK

We study the full temperature and chemical potential dependence of the D3/D5 2+1 dimensional theory in the presence of a magnetic field. The theory displays separate transitions associated with chiral symmetry breaking and melting of the bound states. We display the phase diagram which has areas with first and second order transitions meeting at two critical points similar to that of the D3/D7 system. In addition there is the recently reported BKT transition at zero temperature leading to distinct structure at low temperatures.

I. INTRODUCTION

There has been recent interest in holographic descriptions of the phase structure of gauge theories in the presence of magnetic fields [1–8]. The D3/D7 holographic system describes a confining 3+1d gauge theory with quarks [9]. The magnetic field induces chiral symmetry breaking. The symmetry breaking and quark confinement are lost at high temperature and density. Between is a rich structure of phase transitions of both first and second order meeting at critical points. These transitions have been explored in [4] and the summary phase diagram is displayed in Fig 1a. Here the theory is interesting as a loose analogue for QCD which is also a confining and chiral symmetry breaking gauge theory but where we can not as yet compute the precise phase diagram.

Interest has also turned to the D3/D5 system [10] that describes fundamental representation matter fields on a 2+1d defect within a 3+1d gauge theory. This system may have some lessons for condensed matter systems. In [7] an analysis of the D3/D5 system at finite density (d) and at zero temperature (T) revealed that the chiral symmetry breaking transition with increasing magnetic field (B) is not second order but similar to a Berezinskii-Kosterlitz-Thouless (BKT) transition [11] (see also the holographic example in [8, 12]). That is order parameters across the transition grow as \( \exp(-a/\sqrt{\nu_c - \nu}) \) where \( a \) is a constant and \( \nu_c \) the critical value for the transition. For small \( T \) the authors of [7] showed the BKT transition returns to a second order nature. This difference from the D3/D7 case is surprising so it seems worth fleshing out the entire phase diagram for the theory to see if other surprises are present. In this letter we can not as yet compute the precise phase diagram.

II. THE HOLOGRAPHIC DESCRIPTION

The N=4 super Yang-Mills gauge theory at finite temperature has a holographic description in terms of an AdS5 black hole geometry (with N D3 branes at its core) [13]. The geometry can be written as

\[
ds^2 = \frac{w^2}{R^2} (-g_t dt^2 + g_x d\vec{x}^2 + g_y dy^2) + \frac{R^2}{w^2} (d\rho^2 + \rho^2 d\Omega^2_2 + dL^2 + L^2 d\Omega^2_2),
\]

where \( \vec{x} \) is two dimensional, \( y \) will be the D3 coordinate not shared by our D5, we have split the transverse six plane into two three planes each with a radial coordinate \( \rho, L \) and a two sphere, \( R^4 = 4\pi g_s N\alpha'^2 \) and

\[
g_t := \frac{(w^4 - w^4_H)^2}{2w^4(w^4 + w^4_H)}, \quad g_x := \frac{w^4 + w^4_H}{2w^4}.
\]

The temperature of the theory is given by the position of the horizon, \( w_H = \pi R^2 T \).

We include our 2+1d defect with fundamental matter fields by placing a probe D5 brane in the D3 geometry. The probe limit corresponds to the quenched limit of the gauge theory. The D5 probe can be described by its DBI action

\[S_{DBI} = -T_{D5} \int d^6 \xi \sqrt{-\det (P[G]_{ab} + 2\pi \alpha' F_{ab})},\]

where \( P[G]_{ab} \) is the pullback of the metric and \( F_{ab} \) is the gauge field living on the D5 world volume. We will use \( F_{ab} \) to introduce a constant magnetic field (eg \( F_{12} = -F_{21} = B \)) [1] and a chemical potential associated with baryon number \( A_t(\rho) \neq 0 \) [14, 15]. We embed the D5 brane in the \( t, \vec{x}, \rho \) and \( \Omega_2 \) directions of the metric but to allow all possible embeddings must include a profile \( L(\rho) \).

*Electronic address: evans@soton.ac.uk
†Electronic address: ag806@soton.ac.uk
‡Electronic address: k.kim@soton.ac.uk
§Electronic address: mm21g08@soton.ac.uk
at constant $y, \Omega_2$. The full DBI action we will consider is then

$$S = \int d^6 \xi \mathcal{L}(\rho) = \left( \int_{S^2} \epsilon_2 \int dt dx \right) \int d\rho \mathcal{L}(\rho), \quad (4)$$

where $\epsilon_2$ is a volume element on the 2-sphere and

$$\mathcal{L} := -N_f T_{D5} \frac{\rho^2}{2\sqrt{2}} \left( 1 - \frac{w_H^4}{w^4} \right) \times \sqrt{(1 + (\partial_\rho L)^2 - \frac{2w^4(w^4 + w_H^4)}{(w^4 - w_H^4)^2}(2\pi c'(\partial_\rho A_t)^2)} \times \sqrt{\left( 1 + \frac{w^4}{w_H^4} + \frac{4R^4}{w^4 + w_H^4} B^2 \right)}. \quad (5)$$

Since the action is independent of $A_t$, there is a conserved quantity $d := \frac{\delta S}{\delta F_{\mu t}}$ and we can use the Legendre transformed action

$$\tilde{S} = S - \int d^6 \xi \frac{\delta S}{\delta F_{\mu t}} \int d^6 \xi \frac{\delta S}{\delta F_{\mu t}} = \left( \int_{S^2} \epsilon_2 \int dt dx \right) \int d\rho \tilde{\mathcal{L}}(\rho), \quad (6)$$

where

$$\tilde{\mathcal{L}} := -N_f T_{D5} \frac{(w^4 - w_H^4)}{2\sqrt{2}w^4} \sqrt{K(1 + (\partial_\rho L)^2)} \quad (7)$$

$$K := \left( \frac{w^4 + w_H^4}{w^4} \right) \rho^4 + \frac{4R^4 B^2}{w^4 + w_H^4} \rho^4 + \frac{4w^4}{(w^4 + w_H^4)(N_f T_{D5} 2\pi c')^2} \rho^2. \quad (8)$$

To simplify the analysis we note that we can use the magnetic field value as the intrinsic scale of conformal symmetry breaking in the theory - that is we can rescale all quantities in (7) by $B$ to give

$$\tilde{\mathcal{L}} = -N_f T_{D3} (R\sqrt{B})^3 \frac{\tilde{w}^4 - \tilde{w}_H^4}{\tilde{w}^4} \sqrt{\tilde{K}(1 + \tilde{L}^2)}, \quad (9)$$

$$\tilde{K} = \left( \frac{\tilde{w}^4 + \tilde{w}_H^4}{\tilde{w}^4} \right) \tilde{\rho}^4 + \frac{1}{\tilde{w}^4 + \tilde{w}_H^4} \tilde{\rho}^4 + \frac{\tilde{w}^4}{(\tilde{w}^4 + \tilde{w}_H^4)^2} \tilde{\rho}^2. \quad (10)$$

where the dimensionless variables are defined as

$$\tilde{\mathcal{L}} = \left( \frac{w}{R\sqrt{B}}, \frac{L}{R\sqrt{2B}}, \frac{\rho}{R\sqrt{2B}}, \frac{d}{(R\sqrt{B})^2 N_f T_{D5} 2\pi c'} \right). \quad (11)$$

In all cases the embeddings become flat at large $\rho$ taking the form

$$\tilde{L}(\tilde{\rho}) \sim \tilde{m} + \frac{\tilde{c}}{\tilde{\rho}}, \quad (12)$$

In the absence of temperature, magnetic field and density the regular embeddings are simply $L(\tilde{\rho}) = \tilde{m}$, which is the minimum length of a D3-D5 string, allowing us to identify it with the quark mass as shown. $\tilde{c}$ should then be identified with the quark condensate.

We will classify the D5 brane embeddings by their small $\tilde{\rho}$ behavior. If the D5 brane touches the black hole horizon, we call it a black hole embedding, otherwise, we call it a Minkowski embedding. We have used Mathematica to solve the equations of motion for the D5 embeddings resulting from (9). Typically in what follows, we numerically shoot out from the black hole horizon (for black hole embeddings) or the $\tilde{\rho} = 0$ axis (for Minkowski embeddings) with Neumann boundary condition for a given $\tilde{d}$. Then by fitting the embedding function with (12) at large $\tilde{\rho}$ we can read off $\tilde{m}$ and $\tilde{c}$.

The Hamilton’s equations from (6) are $\partial_\rho d = \frac{\delta \tilde{\mathcal{L}}}{\delta \tilde{A}_t}$ and $\partial_\rho A_t = -\frac{\delta \tilde{\mathcal{L}}}{\delta \tilde{d}}$. The first simply means that $\tilde{d}$ is the con-
served quantity. The second reads as

\[
\partial_\tilde{\rho} \tilde{A}_t = d \frac{\tilde{w}_4 - \tilde{w}_H^4}{\tilde{w}^4 + \tilde{w}_H^4} \sqrt{1 + (\tilde{L})^2} K,
\]

(13)

where \( \tilde{A}_t := \frac{\sqrt{2\pi\rho^4 A_0}}{K^{\frac{3}{2}}} \).

There is a trivial solution of (13) with \( \tilde{d} = 0 \) and constant \( A_t \) [16]. The embeddings are then the same as those at zero chemical potential. For a finite \( \tilde{d} \), \( A_t \) is singular at \( \tilde{\rho} = 0 \) and requires a source. In other words the electric displacement must end on a charge source. The source is the end point of strings stretching between the D5 brane and the black hole horizon. The string tension pulls the D5 branes to the horizon resulting in black hole embeddings [14]. For such an embedding the chemical potential(\( \tilde{\mu} \)) is defined as

\[
\tilde{\mu} := \lim_{\tilde{\rho} \to \infty} \tilde{A}_t(\tilde{\rho}) = \int_{\tilde{\rho}_H}^{\infty} d\tilde{\rho} \frac{\tilde{w}_4 - \tilde{w}_H^4}{\tilde{w}^4 + \tilde{w}_H^4} \sqrt{1 + (\tilde{L})^2} K, \]

(14)

where we fixed \( \tilde{A}_t(\tilde{\rho}_H) = 0 \) for a well defined \( A_t \) at the black hole horizon.

The generic analysis below with massless quarks and \( B, T \) and \( \mu \) all switched on involve four types of solution of the Euler Lagrange equations. All of these approach the \( \tilde{\rho} \) axis at large \( \rho \) to give a zero quark mass. Firstly, there are Minowski embeddings that avoid the black hole so have a non-zero condensate \( \tilde{c} \) - these solutions have \( \tilde{d} = 0 \) so \( \tilde{A}_t = \mu \). Secondly, there can be generic black hole solutions with both of \( \tilde{c} \) and \( \tilde{d} \) non zero. Finally there are solutions that lie entirely along the \( \tilde{\rho} \) axis so that \( \tilde{c} = 0 \) but with \( \tilde{d} \) either zero or non zero. In fact the flat embeddings with \( \tilde{d} = 0 \) are always the energetically least preferred but the other three all play a part in the phase diagram of the theory.

To compare these solutions we compute the relevant thermodynamic potentials. The Euclideanized on shell bulk action can be interpreted as the thermodynamic potential of the boundary field theory. The Grand potential (\( \Omega \)) is associated with the action (5) while the Helmholtz free energy (\( \overline{\Omega} \)) is associated with the Legendre transformed action (6):

\[
\overline{F}(\tilde{w}_H, \tilde{d}) := -\frac{\tilde{S}}{N_f T D_5 (R V B)^3 \text{Vol}} = \int_{\tilde{\rho}_H}^{\infty} d\tilde{\rho} \frac{\tilde{w}_4 - \tilde{w}_H^4}{\tilde{w}^4 + \tilde{w}_H^4} \sqrt{1 + (\tilde{L})^2} \sqrt{K},
\]

(15)

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\overline{\Omega}(\tilde{w}_H, \tilde{\mu}) := -\frac{\tilde{S}}{N_f T D_5 (R V B)^3 \text{Vol}} = \int_{\tilde{\rho}_H}^{\infty} d\tilde{\rho} \frac{\tilde{w}_4 - \tilde{w}_H^4}{\tilde{w}^4 + \tilde{w}_H^4} \sqrt{1 + (\tilde{L})^2} \sqrt{K(\tilde{d} = 0)}
\]

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where \( \text{Vol} \) denote the trivial 5-dimensional volume integral except \( \tilde{\rho} \) space, so the thermodynamic potentials defined above are densities, strictly speaking. Since \( K \sim \tilde{\rho}^4 \), both integrals diverge as \( \tilde{\rho}^2 \) at infinity and need to be renormalized.

FIG. 2: A plot of the condensate vs the quark mass to show the first order phase transition at zero chemical potential induced by temperature. The solid line corresponds to the black hole embedding and the dotted line to a Minkowski embedding. From bottom to top the curves correspond to temperatures \( \tilde{w}_H = 0.25, 0.3435, 0.45 \).

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III. CHIRAL SYMMETRY RESTORATION BY TEMPERATURE

The chiral symmetry restoration transition by temperature is first order [6] (a transition related to the thermal transition for non-zero mass at B=0 [17]). The transition on the gravity side is between a Minkowski embedding that avoids the black hole to an embedding that lies along the \( \tilde{\rho} \) axis ending on the black hole. The string tension pulls the D5 branes to the horizon resulting in black hole embeddings [14]. For such an embedding the chemical potential(\( \tilde{\mu} \)) is defined as

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\]

(16)
IV. CHIRAL SYMMETRY RESTORATION BY DENSITY

At zero temperature we find two phase transitions with increasing chemical potential. At low chemical potentials the preferred embedding is a Minkowski embedding with $A_L = \mu$ so there is no quark density. There is then a transition to a black hole embedding with non-zero quark density, $\tilde{d}$. This transition, whilst appearing first order in terms of the brane embeddings, displays second order behaviour in all field theory quantities such as the condensate or density (which grows smoothly from zero). The transition also corresponds to the onset of bound state melting since the black hole embedding has quasi-normal modes rather than stable fluctuations.

The chiral symmetry transition induced by density at zero temperature is distinct and also a continuous transition. It has been shown to be of the BKT type for this D3/D7 case [7] as opposed to a mean-field type second order transition as seen in the D3/D7 case [4, 5].

The chiral symmetric phase corresponds to the trivial embedding, $L = 0$. Chiral symmetry breaking is signaled by the instability of small fluctuation around the $L = 0$ embedding. The Free energy (15) with (9) at zero $T$ reads

$$\tilde{F} \sim \sqrt{1 + \tilde{L}^2} \frac{1}{\tilde{\rho}^2} \sqrt{\tilde{\rho}^4 + \frac{\tilde{d}^4}{\tilde{\rho}^4} + \tilde{d}^2},$$ \hspace{1cm} (17)$$

which can be expanded up to the quadratic order in $\tilde{L}$ as

$$\tilde{F} \sim \frac{1}{2} \sqrt{1 + \tilde{\rho}^4 + \tilde{d}^2} \tilde{L}^2 + \frac{\tilde{L}^2}{\tilde{\rho}^2 \sqrt{1 + \tilde{\rho}^4 + \tilde{d}^2}}$$ \hspace{1cm} (18)$$

At $\tilde{\rho} \gg 1$, $\frac{\tilde{d}}{\tilde{\rho}}$ behaves as a scalar with $m^2 = -2$ in AdS$_4$, while at small $\tilde{\rho} \ll 1$ and $\tilde{\rho} \ll \tilde{d}$ it behaves as a scalar with $m^2 = -\frac{1}{1 + \tilde{d}^2}$ in AdS$_2$. The Breitenlohner-Freedman (BF) bound of AdS$_2$ is $-\frac{1}{4}$, so below $\tilde{d}_c = \sqrt{7}$ the BF bound is violated and the embedding $\tilde{L} = 0$ is unstable [7]. This critical density corresponds to the critical chemical potential $\tilde{\mu} \sim 2.9$ as can be computed from (14). In [7] it was shown that the condensate scales near this transition as

$$- \tilde{c} \sim - e^{-T} \sqrt{\frac{1}{\tilde{L}^2} + \tilde{d}^2},$$ \hspace{1cm} (19)$$

which corresponds to BKT scaling [11]. This transition is an example of the analysis in [12] where it was shown that if a scalar mass in a holographic model could be tuned through the BF bound a BKT transition would be seen at the critical point.

V. PHASE DIAGRAM IN $\mu$-T PLANE

To compute the full phase diagram we work on a series of constant $T$ slices. We have found the four relevant embeddings discussed above and found those that minimize the relevant thermodynamic potential. For more details of the method and relevant analysis we refer to [4], where we studied D3/D7 system using the same methods. Fig 3 shows some example plots of the dependence of the condensate on the density on fixed $T$ slices. It shows that the Minkowski embedding with $\tilde{d} = 0$ is preferred at low $\tilde{\mu}$, a black hole embedding with growing $\tilde{d}$ at intermediate $\tilde{\mu}$, before finally a transition to a flat embedding occurs at high chemical potential.

Qualitatively the phase diagram, shown in Fig 1, is almost the same as the D3/D7 case - the two second order transitions at zero temperature converge at two critical points to form the first order transition identified at zero density. The only difference is induced by the chiral phase transition at zero $T$. Comparing to the D3/D7 case we see there is a long tail near zero $T$, the end point of which corresponds to the BKT transition. However even infinitesimal temperature turns it into mean-field type second order transition[7, 8]. In Fig 3bc we plot the condensate against $\mu$ at a very low temperature ($\tilde{w}_H = 10^{-5}$) to show the second order nature.

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