Maximal \( CP \) Violation Hypothesis and Phase Convention of the CKM Matrix

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Abstract

The maximal \( CP \) violation hypothesis depends on the phase convention of the Cabibbo-Kobayashi-Maskawa matrix. A phase convention which leads to successful prediction under the maximal \( CP \) violation hypothesis is searched, and thereby, possible structures of the quark mass matrices are speculated.

PACS numbers: 11.30.Er, 12.15.Hh and 12.15.Ff

1 Introduction

Recent remarkable progress of the experimental \( B \) physics [1] has made possible to know the magnitude of the \( CP \) violation in the quark sector. We are interested what logic can give the observed magnitude of the \( CP \) violation. For this subject, for example, we know an attractive hypothesis, the so-called “maximal \( CP \) violation” hypothesis [2]. However, the conventional “maximal \( CP \) violation” hypothesis cannot give the observed magnitude of the \( CP \) violation, as we discuss later.

We are also interested that, which quark mass matrix element, the \( CP \) violation originates in (in other words, which of quark mass matrix elements is accompanied by a \( CP \) violating phase). However, it is usually taken that this question is meaningless, because we know that the observable quantities are invariant under the rephasing of the Cabibbo-Kobayashi-Maskawa (CKM) [3][4] matrix. For example, we cannot physically distinguish the standard CKM matrix phase convention [5]

\[
V_{SD} = R_1(\theta_{23})P_3(\delta_{13})R_2(\theta_{13})P_3^T(\delta_{13})R_3(\theta_{12})
\]

from the original CKM matrix phase convention by Kobayashi and Maskawa (KM) [4]

\[
V_{KM} = R_1^T(\theta_2)P_3(\delta_{KM} + \pi)R_3(\theta_1)R_1(\theta_3)
\]
\[
\begin{pmatrix}
  c_1 & -s_1c_3 & -s_1s_3 \\
  s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta_{KM}} & c_1c_2s_3 + s_2c_3e^{i\delta_{KM}} \\
  s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta_{KM}} & c_1s_2s_3 - c_2c_3e^{i\delta_{KM}}
\end{pmatrix},
\]

where

\[
R_1(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{pmatrix}, \quad R_2(\theta) = \begin{pmatrix} c & 0 & s \\ 0 & 1 & 0 \\ -s & 0 & c \end{pmatrix}, \quad R_1(\theta) = \begin{pmatrix} c & s & 0 \\ -s & c & 0 \\ 0 & 0 & 1 \end{pmatrix},
\]

\[
P_3(\delta) = \text{diag}(1, 1, e^{i\delta}),
\]

\(s = \sin \theta\) and \(c = \cos \theta\).

Although there are many different versions of the maximal \(CP\) violation hypothesis, the conventional one demands that the nature takes a value of the \(CP\) violating phase so that the rephasing invariant quantity \(J\) takes its maximal value. In the standard CKM matrix phase convention, the quantity \(J\) is given by

\[
J = c_{13}s_{13}c_{12}s_{12}c_{23}s_{23}\sin \delta_{13},
\]

i.e.

\[
J = \frac{|V_{11}||V_{12}||V_{33}||V_{23}||V_{13}|}{1 - |V_{13}|^2} \sin \delta_{13}.
\]

The maximal \(CP\) violation hypothesis demands \(\sin \delta_{13} = 1\), so that we obtain

\[
J \simeq |V_{us}||V_{cd}||V_{ub}|,
\]

where we have used the observed fact \(1 \gg |V_{us}|^2 \gg |V_{cd}|^2 \gg |V_{ub}|^2\). The choice \(\delta_{13} = \pi/2\) also predicts

\[
|V_{td}| = \sqrt{(s_{23}s_{12})^2 + (c_{23}c_{12}s_{13})^2} = 0.00976 \pm 0.00016,
\]

\[
\alpha = 68.5^\circ \pm 3.2^\circ \simeq \sin^{-1}(|V_{us}||V_{cb}|/|V_{td}|),
\]

\[
\beta = 21.5^\circ \pm 3.2^\circ \simeq \sin^{-1}(|V_{ub}|/|V_{td}|),
\]

\[
\gamma = 89.96^\circ \pm 0.00^\circ \simeq \sin^{-1}(1),
\]

where angles \(\alpha, \beta\) and \(\gamma\) are defined by

\[
\alpha = \text{Arg} \left[ -\frac{V_{31}V_{33}^*}{V_{11}V_{13}} \right] = \sin^{-1} \left[ \frac{|V_{12}||V_{22}|}{|V_{31}|(1 - |V_{13}|^2)} \sin \delta_{13} \right],
\]

\[
\beta = \text{Arg} \left[ -\frac{V_{21}V_{23}^*}{V_{31}V_{33}} \right] = \sin^{-1} \left[ \frac{|V_{11}||V_{12}||V_{13}|}{|V_{21}||V_{31}|(1 - |V_{13}|^2)} \sin \delta_{13} \right],
\]
\[
\gamma = \text{Arg} \left[ -\frac{V_{11}V_{13}^*}{V_{21}V_{23}^*} \right] = \sin^{-1} \left[ \frac{|V_{12}||V_{33}|}{|V_{21}|(1 - |V_{13}|^2)} \sin \delta_{13} \right], \tag{1.14}
\]

and we have used the observed values [7]

\[
|V_{us}| = 0.2200 \pm 0.0026, \\
|V_{cb}| = 0.0413 \pm 0.0015, \\
|V_{ub}| = 0.00367 \pm 0.00047. \tag{1.15}
\]

The world average value of \(\beta\) [7] which has been obtained from \(B_d\) decays is

\[
\sin 2\beta = 0.736 \pm 0.049 \quad \left( \beta = 23.7^\circ \pm 2.2^\circ \right), \tag{1.16}
\]

so that the prediction (1.10) is in good agreement with the observed value. However, on the other hand, the best fit for the CKM parameters [7] gives

\[
\gamma = 60^\circ \pm 14^\circ, \quad \beta = 23.4^\circ \pm 2^\circ, \tag{1.17}
\]

so that the prediction of \(\gamma\), (1.11), is entirely in disagreement with the experiments. Therefore, the maximal \(CP\) violation hypothesis must be ruled out.

However, note that this maximal \(CP\) violation hypothesis depends on the phase convention of the CKM matrix. If we use the original KM phase convention, the rephasing invariant quantity \(J\) is given by

\[
J = c_1 s_1^2 c_2 s_2 c_3 s_3 \sin \delta_{KM}, \tag{1.18}
\]

i.e.

\[
J = \frac{|V_{11}||V_{12}||V_{13}||V_{21}||V_{31}|}{1 - |V_{11}|^2} \sin \delta_{SD}, \tag{1.19}
\]

and the requirement \(\delta_{KM} = \pi/2\) predicts

\[
J \simeq |V_{ub}||V_{td}|, \tag{1.20}
\]

\[
|V_{ub}| = s_1 s_2 \simeq |V_{us}||V_{cb}| \sqrt{1 - \xi^2}, \tag{1.21}
\]

where

\[
\xi = \frac{|V_{ub}|}{|V_{us}||V_{cb}|}. \tag{1.22}
\]

(The relations between \(V_{SD}\) and \(V_{KM}\) can, for instance, be found in Ref. [8].) From the observed values (1.15), we obtain the numerical results

\[
|V_{td}| = 0.0084 \pm 0.0005, \tag{1.23}
\]

\[
\alpha = 89.96^\circ \pm 0.00^\circ, \tag{1.24}
\]

\[
\beta = 23.2^\circ \pm 3.8^\circ. \tag{1.25}
\]
\( \gamma = 66.8^\circ + 3.8^\circ - 3.5^\circ \). \hfill (1.26)

These results are in good agreement with the observed values (1.16) and (1.17).

Thus, the results from the maximal CP violation hypothesis depend on the phase convention. (Note that we have applied the maximal CP violation hypothesis to the CKM phase convention \( V_{KM} \), (1.2), under the rotation parameters fixed. If we apply the hypothesis to \( V_{KM} \) under \( |V_{us}|, |V_{cb}| \) and \( |V_{ub}| \) fixed, the results are same as in the standard phase convention.) Such phase-convention dependence, in spite of the rephasing invariance of the CKM matrix, is due to that we tacitly assume that only the phase parameter \( \delta_{13} (\delta_{KM}) \) is free and it is independent of the rotation parameters \( s_{ij} (s_i) \).

In the present paper, we systematically investigate whether there is other phase convention which gives successful predictions or not, and we will find an interesting phase convention which speculates successful relations for quark masses \( m_q \) and the CKM matrix elements \( |V_{ij}| \).

2 Phase conventions and the expressions of \( J \)

Let us give the CKM matrix \( V \) as

\[
V = V(i, k) \equiv R_i^T P_j R_j R_k \quad (i \neq j \neq k),
\hfill (2.1)
\]

where \( R_i \) (\( i = 1, 2, 3 \)) are defined by Eqs. (1.3), and \( P_i \) are given by \( P_1 = \text{diag}(e^{i\delta}, 1, 1) \)

\[
P_2 = \text{diag}(1, e^{i\delta}, 1), \quad \text{and} \quad P_3 = \text{diag}(1, 1, e^{i\delta}),
\]

we can show that the magnitudes of the CKM matrix elements, \( |V_{i1}|, |V_{i2}|, |V_{i3}|, |V_{ik}|, |V_{2k}| \) and \( |V_{3k}| \), do not depend on the phase parameter \( \delta \), and the rephasing invariant quantity \( J \) is given by

\[
J = \frac{|V_{i1}| |V_{i2}| |V_{i3}| |V_{ik}| |V_{2k}| |V_{3k}|}{(1 - |V_{ik}|^2)|V_{ik}|} \sin \delta.
\hfill (2.2)
\]

Note that the expression (2.2) is only dependent on \( i \) and \( k \), and it is independent of \( j \). Therefore, we have nine cases of \( V(i, k) \). (This has been pointed out by Fritzsch and Xing [9].) The expressions \( V(1,3) \) and \( V(1,1) \) correspond to the standard and original KM phase conventions, respectively.

For the observed fact \( 1 \gg |V_{us}|^2 \simeq |V_{cd}|^2 \gg |V_{cb}|^2 \simeq |V_{ts}|^2 \gg |V_{ub}|^2 \), the results (2.2) are approximately given as follows:

\[
J \simeq |V_{us}| |V_{cb}| |V_{ub}| \sin \delta, \hfill (2.3)
\]

for \( V(1,2), \ V(1,3), \ V(2,1) \) and \( V(2,3) \)

\[
J \simeq |V_{ub}| |V_{td}| \sin \delta, \hfill (2.4)
\]

for \( V(1,1) \) and \( V(3,3) \)

\[
J \simeq |V_{cb}|^2 \sin \delta, \hfill (2.5)
\]
for $V(2,2)$, and

$$J \simeq |V_{us}| |V_{cb}| |V_{td}| \sin \delta,$$

(2.6)

for $V(3,1)$ and $V(3,2)$. The cases which can give reasonable predictions for unitary triangle under the maximal $CP$ violation hypothesis are only the cases $V(1,1)$ and $V(3,3)$.

The explicit expression of $V(1,1)$ has already been given by Eq. (1.2). The explicit expression of $V(3,3)$ is given by

$$V(3,3) = R_T^T(\theta_{12}^u) P_3(\delta) R_1(\theta_{23}) R_3(\theta_{12}^d)$$

$$= \begin{pmatrix}
  e^{i\delta} c_{12}^u c_{12}^d + c_{23} s_{12}^u s_{12}^d & e^{i\delta} c_{12}^u s_{12}^d - c_{23} s_{12}^u c_{12}^d & -s_{23} s_{12}^u \\
  e^{i\delta} s_{12}^u c_{12}^d - c_{23} c_{12}^u s_{12}^d & e^{i\delta} s_{12}^u s_{12}^d + c_{23} c_{12}^u c_{12}^d & s_{23} c_{12}^u \\
  -s_{23} s_{12}^u & -s_{23} c_{12}^u & c_{23}
\end{pmatrix},$$

(2.7)

which has been proposed by Fritzsch and Xing [10]. For the expression (2.7), we obtain the expression of $J$

$$J = c_{23} s_{12}^u c_{12}^u s_{12}^u c_{12}^d s_{12}^d \sin \delta \frac{|V_{13}| |V_{23}| |V_{33}| |V_{32}| |V_{31}|}{1 - |V_{33}|^2} \sin \delta,$$

(2.8)

and the relations

$$\frac{s_{12}^u}{c_{12}^u} = \frac{|V_{ub}|}{|V_{cb}|},$$

(2.9)

$$\frac{s_{12}^d}{c_{12}^d} = \frac{|V_{td}|}{|V_{ts}|},$$

(2.10)

$$s_{23} = \sqrt{|V_{cb}|^2 + |V_{ub}|^2}.$$

(2.11)

Under the maximal $CP$ violation hypothesis, since the matrix element $|V_{us}|$ is given

$$|V_{us}| = \sqrt{(c_{12}^u s_{12}^d)^2 + (c_{3} s_{12}^u c_{12}^d)^2},$$

(2.12)

the value of $s_{12}^d$ can be fixed by the observed values of $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}|$. It is approximately given by

$$s_{12}^d \simeq |V_{us}| \sqrt{1 - \xi^2},$$

(2.13)

where $\xi$ is defined by Eq. (1.22). When we use the observed values of $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}|$, (1.15), the numerical predictions without approximation are as follows:

$$J = (3.01^{0.22}_{-0.22} + 0.10) \times 10^{-5},$$

(2.14)

$$|V_{td}| = 0.00842 \pm 0.00052,$$

(2.15)
\[ \alpha = 88.95^\circ \pm 0.14^\circ, \]  
\[ \beta = 23.2^\circ \pm 3.8^\circ \pm 3.5^\circ, \]  
\[ \gamma = 67.8^\circ \pm 2.7^\circ \pm 4.4^\circ. \]

These numerical results are approximately same as those in the original KM phase convention, but are slightly different from the results \((1.8)–(1.11)\).

3 Speculation on the quark mass matrix form

In the maximal \(CP\) violation hypothesis, we have, so far, assumed that the rotation parameters are fixed and only free parameter is the \(CP\) violation phase \(\delta\). This suggests the following situation. The phase factors in the quark mass matrices \(M_f\) \((f = u, d)\) are factorized by the phase matrices \(P_f\) as

\[ M_f = P_f^\dagger \tilde{M}_f P_f, \quad (3.1) \]

where \(P_f\) are phase matrices and \(\tilde{M}_f\) are real matrices. The real matrices \(\tilde{M}_f\) are diagonalized by rotation (orthogonal) matrices \(R_f\) as

\[ R_f^\dagger \tilde{M}_f R_f = D_f \equiv \text{diag}(m_{f1}, m_{f2}, m_{f3}), \quad (3.2) \]

(for simplicity, we have assumed that \(M_f\) are Hermitian or symmetric matrix, i.e. \(P_fR = P_fL\) or \(P_fR = P_f^\dagger L\) respectively), so that the CKM matrix \(V\) is given by

\[ V = R_u^T P R_d, \quad (3.3) \]

where \(P = P_{uL} P_{dL}\). The quark masses \(m_{fi}\) are only determined by \(\tilde{M}_f\). In other words, the rotation parameters are given only in terms of the quark mass ratios, and independent of the \(CP\) violating phases. In such a scenario, the maximal \(CP\) violation hypothesis means that the \(CP\) violation parameter \(\delta\) takes its maximum value without changing the quark mass values.

For example, the choices of the standard and original KM phase conventions suggest the quark mass matrix structures

\[ \tilde{M}_u = R_2(\theta_{13}^u)R_1(\theta_{23})D_u R_1^T(\theta_{23})R_2^T(\theta_{13}^u), \]
\[ \tilde{M}_d = R_2(\theta_{13}^d)R_3(\theta_{12})D_d R_3^T(\theta_{12})R_2^T(\theta_{13}^d), \quad (3.4) \]

with \(\theta_{13} = \theta_{13}^d - \theta_{13}^u\) and

\[ \tilde{M}_u = R_3(\theta_1^u)R_1(\theta_2)D_u R_1^T(\theta_2)R_3^T(\theta_1^u), \]
\[ \tilde{M}_d = R_3(\theta_1^d)R_1(\theta_3)D_d R_1^T(\theta_3)R_3^T(\theta_1^d), \quad (3.5) \]

with \(\theta_1 = \theta_1^d - \theta_1^u\), respectively. The success of the maximal \(CP\) violation hypothesis, \((1.23)–(1.26)\), suggest that the mass matrix structure \((3.5)\) is preferable to the structure \((3.4)\). However,
another candidate of $V$ which gives the magnitude of $J$, (2.4), also gives successful results (2.14)–(2.18). The case $V(3,3)$ suggests the following quark mass matrix structure:

$$\tilde{M}_u = R_1(\theta_{23}^u)R_3(\theta_{12}^u)D_uR_3^T(\theta_{12}^u)R_1^T(\theta_{23}^u),$$
$$\tilde{M}_d = R_1(\theta_{23}^d)R_3(\theta_{12}^d)D_dR_3^T(\theta_{12}^d)R_1^T(\theta_{23}^d),$$

(3.6)

with $\delta = \delta_d - \delta_u$ and $\theta_{23} = \theta_{23}^d - \theta_{23}^u$. The mass matrix structure (3.6) is explicitly given by the form

$$\tilde{M}_f = \begin{pmatrix}
m_{f1}c_{12}^f + m_{f2}s_{12}^f & (m_{f2} - m_{f1})c_{12}^f s_{12}^f c_{23}^f & -(m_{f2} - m_{f1})c_{12}^f s_{12}^f s_{23}^f \\
(m_{f2} - m_{f1})c_{12}^f s_{12}^f c_{23}^f & (m_{f1}s_{12}^f + m_{f2}c_{12}^f)c_{23}^f + m_{f3}s_{23}^f & (m_{f3} - m_{f2}c_{12}^f - m_{f1}s_{12}^f)c_{23}^f s_{23}^f \\
-(m_{f2} - m_{f1})c_{12}^f s_{12}^f s_{23}^f & (m_{f3} - m_{f2}c_{12}^f - m_{f1}s_{12}^f)c_{23}^f s_{23}^f & (m_{f1}s_{12}^f + m_{f2}c_{12}^f)s_{23}^f + m_{f3}c_{23}^f
\end{pmatrix}.$$

(3.7)

In the mass matrix (3.7), the ansatz $\tilde{M}_{11}^d = 0$ leads to the well–known relation \[11\]

$$|V_{us}| \simeq s_{12}^d \simeq \sqrt{\frac{m_d}{m_s}} \simeq 0.22.$$

(3.8)

On the other hand, in the mass matrix structure (3.5), there is no simple relation such as (3.8). Therefore, the mass matrix structure (3.6) [i.e. (3.7)] [and also the phase convention (2.7) ] is more attractive to us compared with the alternative one (1.2) (the original KM phase convention). Furthermore, in the mass matrix (3.7), if we assume $\tilde{M}_{11}^u = 0$ analogous to $\tilde{M}_{11}^d = 0$, we obtain

$$\frac{s_{12}^u}{c_{12}^u} \simeq \sqrt{\frac{m_u}{m_c}} = 0.059,$$

(3.9)

where quark mass values \[12\] at $\mu = m_Z$ have been used. Compared with the experimental value of $|V_{ub}|/|V_{cb}|$

$$\frac{s_{12}^u}{c_{12}^u} = \frac{|V_{ub}|}{|V_{cb}|} = 0.089^{-0.015}_{+0.014},$$

(3.10)

the prediction (3.9) is slightly small. However, this discrepancy should not be taken seriously, because the present speculation on the quark mass matrices is made only for main framework of the mass matrices. The purpose of the present paper is to investigate a possible phase convention form which can give successful predictions for the shape of the unitary triangle under the maximal $CP$ violation hypothesis, and not to find a phenomenologically successful quark mass matrix form, we do not go into the phenomenology of the mass matrix form (3.7) any more.

4 Conclusion
The predictions from the maximal $CP$ violation hypothesis depend on the phase conventions of the CKM matrix $V$. We have systematically investigated whether the hypothesis can give successful predictions for the magnitude of the rephasing invariant quantity $J$ and the shape of the unitary triangle or not. In conclusion, we have found that, of the nine possible phase conventions $V(i, k) = R_i^T P_j R_j R_k$, only two, $V(1, 1)$ (the original KM phase convention) and $V(3, 3)$ (the Fritzsh–Xing phase convention), can yield successful predictions. Furthermore, we have speculated possible quark matrix forms which are suggested from the expressions $V(i, k)$. Since a texture-zero requirement $M^d_{11} = 0$ in the mass matrix form (3.7) can lead to the well–known relation $|V_{us}| \simeq m_d/m_s$, the new phase convention $V(3, 3)$ is very attractive to us rather than the original KM phase convention $V(1, 1)$. (Of course, for experimental data analysis, the standard phase convention $V(1, 3)$ [i.e. (1.1)] is the most useful expression. Only for discussing the relations between the CKM matrix and the quark mass matrix forms $M_f$, the expression $V(3, 3)$ [i.e. (2.7)] will be useful.)

Of course, we cannot ruled out a possibility that the maximal $CP$ violation hypothesis is not true. Then, from the view point of a simple texture-zero ansatz, the phase convention $V(2, 3)$ is also attractive to us, because the case suggests the quark mass matrix structure $\tilde{M}_u = R^u_1 R^u_2 D_u R^u_1 R^u_2$ and $\tilde{M}_d = R^d_1 R^d_3 D_d R^d_1 R^d_3$. The texture-zero requirements $\tilde{M}^u_{11} = 0$ and $\tilde{M}^d_{11} = 0$ predicts $|V_{ub}| \simeq \sqrt{m_u/m_t} = 0.0036$ and $|V_{us}| \simeq \sqrt{m_d/m_s} = 0.22$, respectively. Those predictions are in good agreement with the observed values (1.15).

If we apply the mass matrix structure (3.7) to the lepton sector, we obtain

$$|U_{e3}| \simeq \frac{1}{\sqrt{2}} \sqrt{\frac{m_e}{m_\mu}} = 0.049,$$

(4.1)

for the $V(3, 3)$ model, while

$$|U_{e3}| \simeq \frac{1}{\sqrt{2}} \sqrt{\frac{m_e}{m_\tau}} = 0.012,$$

(4.2)

for the $V(2, 3)$ model, where we have taken $s_{23} = c_{23} = 1/\sqrt{2}$ from the observed fact $\sin^2 2\theta_{atm} \simeq 1$. If a near future experiment confirms the relation (4.1), the $V(3, 3)$ model which is suggested from the maximal $CP$ violation hypothesis will become promising.

Acknowledgment

This work was supported by the Grant-in-Aid for Scientific Research, the Ministry of Education, Science and Culture, Japan (Grant Number 15540283).
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