On the UPMSat-2 Attitude Determination and Control Subsystem’s magnetometers integration

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Abstract. In the present work, a method for magnetometer calibration through least squares fitting is presented. This method has been applied over the magnetometer’s data set obtained during the integration tests of the Attitude Determination and Control Subsystem (ADCS) of UPMSat-2. The UPMSat-2 mission is a 50-kg satellite designed and manufactured by the Technical University of Madrid (Universidad Politécnica de Madrid), and finally launched in September 2020. The satellite has three fluxgate magnetometers (one of them experimental) whose calibration is critical to obtain correct measurements to be used by the ADCS. Among several mathematical methods suitable to obtain the calibration parameters, an ordinary least squares fitting algorithm is selected as a first step of the calibration process. The surface estimated is an ellipsoid, surface represented by the magnetometer’s measures of the Earth magnetic field in a point of the space. The calibration elements of the magnetometers are related to the coefficients of the estimated ellipsoid.

1. Introduction

The UPMSat-2\(^1\) (see Table 1 and Figure 1) is a 50 kg microsatellite designed, developed, tested and integrated at the IDR/UPM Institute of Universidad Politécnica de Madrid, for educational and technology demonstration purposes. The satellite was launched on September 3rd, 2020. More information on this mission can be found on [1]–[3].

A purely magnetic Attitude Determination and Control Subsystem (ADCS) is used by the UPMSat-2, as it is both simple and reliable. The design of this subsystem includes three ZARM (ZARM Technik AG) magnetorquers, and two SSBV (SSBV Space & Ground Systems) fluxgate magnetometers. The second magnetometer is carried on board to increase the reliability of the subsystem. In addition, a third fluxgate magnetometer by Bartington, declared on board as payload, is available for the ADCS during the mission. The control law is based on a modification of the “B-dot” strategy, modification that allows the satellite angular rotation rate to be controlled, and also to set the rotation axis of the satellite perpendicular to the satellite orbit’s plane (see [4] for more information). The satellite orbits in a sun-synchronous noon/midnight orbit in which the sun radiation will be almost perpendicular to the rotation axis (Z-axis of the UPMSat-2). This kind of orbit allows a better thermal

\(^1\) The UPMSat-2 was preceded by a former one, the UPMSat-1, developed by the same team and launched in 1995.
control and a high orbit-average energy production (solar panels are located at +X, −X, +Y and −Y axes of the UPMSat-2, see Figure 1).

![Figure 1. CAD drawing of the UPMSat-2 satellite.](image)

**Table 1. Outline of the UPMSat-2 Mission**

| Category                  | Details                                                                 |
|---------------------------|-------------------------------------------------------------------------|
| Mission Life              | 2 years                                                                |
| Orbit                     | Sun-synchronous:                                                       |
|                           | - 10:30                                                                |
|                           | - Altitude: 500 km                                                     |
| Dimensions                | 0.5 m × 0.5 m × 0.6 m                                                  |
| Attitude Control          | Magnetic:                                                              |
|                           | - SSBV24 magnetometers                                                 |
|                           | - ZARM Technik AG25 magnetorquers                                      |
|                           | - Control law designed by IDR/UPM                                      |
| Thermal Control           | Passive                                                                |
| Power                     | Based on solar photovoltaic panels and batteries:                     |
|                           | - 5 body-mounted solar panels Selex Galileo                           |
|                           | - Li-ion battery designed by SAFT26                                    |
|                           | - Direct Energy Transfer (DET)                                          |
| On board electronic box (E-BOX) | Based on FPGA (designed by Tecnobit S.L.27 and programmed by STRAST/UPM28).|
|                           | Includes:                                                              |
|                           | - On-board computer                                                    |
|                           | - Data handling                                                        |
|                           | - Power supply control                                                 |
|                           | - Power supply distribution                                            |
| Communications            | Link at 436 Mhz frequency                                              |
1.1. Aim of the present work
In the present work a linear least square fitting algorithm is evaluated as the method to estimate the ellipsoid surface that the magnetometer data describe, in order to calibrate the sensor. This action is part of the ADCS sensors integration and testing campaign.

The data of a three axes fluxgate magnetometer is distributed, for a fixed point, in an ellipsoid that represents the distorted modulus of the Earth Magnetic Field in that point, modulus that theoretically would be measured as an spherical data distribution around the measurement point. The distortion of the modulus measured by the sensor is due to environmental and sensor noise, which convert the theoretical spherical distribution into an ellipsoidal distribution.

The least squares problem consists on the fitting of a parametrized mathematical model to a set of points, while minimizing an objective function. In the case of a surface, as the ellipsoid, the cost function is quadratic in the parameters; therefore, this function can be minimized with respect to the parameters in one step by solving a linear matrix equation. Due to the problem’s linear nature, a linear least squares model is proposed, the ordinary least squares algorithm. In order to avoid the trivial zero solution, a scale normalization is used. The ordinary least squares method has one of the simples scale normalizations, imposing the norm of the array of estimated parameters to be one.

The estimated parameters from the least squares method are related to the magnetometer calibration elements as it is explained in section 2. Therefore, once the ellipsoid coefficients are estimated, the calibration parameters can be calculated. The data set used for the described process has been collected during the satellite integration tests, with the satellite subsystems in their nominal mode. As a reference for the Earth magnetic field modulus value, a high precision magnetometer has been used. Considering the theoretical Earth magnetic field modulus, the error introduced in the magnetic field modulus calculation by the estimated magnetometer’s calibration parameters is evaluated as part of the ADCS performance.

This work is organized as follows:
- In Section 2 an introduction to Earth magnetic field and fluxgate magnetometers performance in introduced.
- In Section 3 the linear least squares method theoretical base is presented.
- In Section 4 the results from the UPMSat-2 magnetometers calibration are evaluated.
- Finally, in Section 5 the conclusions from this work are shown.

2. Magnetometer behaviour modelling
2.1. The Earth Magnetic Field Model
The Earth magnetic field can be represented by analytical models. There are many analytical models of the Earth’s magnetic field which considered the Main Field (Earth Magnetic field modelled as a magnet centred in the centre of the Earth, which represents the 90% of the total Earth magnetic field), as well as other contributions (solar flux in high altitudes, contribution of Earth crust), and the evolution of the two aforementioned factors with time. The most well-known analytical models are the World Magnetic Model and the Geomagnetic Reference Field, IGRF. The main difference between the two models is that the WMM is a predictive model while IGRF model is a historic model. In the UPMSat-2 project, the IGRF model is the one selected in the satellite simulations.

The IGRF model is valid for periods of five years, and the Earth magnetic field potential is expressed as:

$$\psi (r, \theta, \lambda, t) = R \sum_{l=1}^{L} \sum_{m=-l}^{l} \left( \frac{R}{r} \right)^{l+1} \left[ g_l^m \cos(m\lambda) + h_l^m \sin(m\lambda) \right] P_l^m (\cos \theta),$$

where $L$ is the maximum degree of the expansion, $\lambda$ is the East longitude, $\theta$ is the colatitude, $R$ is the Earth’s radius, $g_l^m$ and $h_l^m$ are Gauss coefficients, which are functions of time, and $P_l^m$ are Schmidt
normalized associated Legendre functions, of degree \( l \) and order \( m \). Once the magnetic scalar potential is defined, the geomagnetic field \( H \) can be easily derived:

\[
H = - \nabla \psi .
\]  

(2)

The IGRF model is the one developed in the UPMSat-2 orbit simulator in order to test the performance of the ADCS. Once the calibration parameters are calculated, they are introduced in this SW simulator, in order to verify the robustness of the subsystem facing the error introduced in the sensor measurements by the estimated calibration elements.

2.2. The Fluxgate Magnetometers

The three axes fluxgate magnetometers measure the magnetic field vector, \( B_m \), expressing its components in sensor body axes, in volts units. Due to environmental and sensor errors introduced in sensor measurement, the vector \( B_m \) does not represent directly the measured geomagnetic field, but the following transfer function is required to obtain the Earth magnetic field vector value:

\[
H_m = C^{-1}(B_m - b) .
\]  

(3)

The errors introduced in the magnetometer measurements include biases, scale factors and misalignments, classified in two main groups: environmental errors and instrumental errors. Regarding the environmental errors they are divided into hard iron error (permanent magnets, magnetic hysteresis), presented as a constant bias \( b^{hi} \), and soft iron error (interaction with ferromagnetic materials), presented as a change of the measured magnetic field intensity and direction represented by a rotation matrix, \( C^{si} \).

In relation to the instrumental errors, they are composed of a constant bias, \( b^{so} \), a scale factor, represented by a diagonal matrix \( S \), and a misalignment error, represented by a symmetric matrix \( E \).

Considering all the aforementioned errors, the measurements model of a fluxgate magnetometer is described by the following expression:

\[
B_m = SE(C^{si}H_m + b^{hi}) + b^{so} + v ,
\]  

(4)

where \( v \) stands for a noise vector. The terms of the above expression can be grouped in order to reach a simpler equation:

\[
B_m = CH_m + b .
\]  

(5)

The purpose of the sensors is to provide the value of the Earth magnetic field vector, \( H_m \). Therefore, the expression (3) from the beginning of the section (and reproduced below):

\[
H_m = C^{-1}(B_m - b) .
\]  

(6)

can be easily derived. Commonly, \( C^{-1} \) is called the calibration matrix, whereas, as said, \( b \) is the offset vector. Considering that the modulus of the Earth magnetic field has to be equal to the local magnetic field modulus:

\[
\|B_m\|^2 = H_0^2 .
\]  

(7)

Combining equations (6) and (7) it is possible to obtain:

\[
(B_m - b^*)^T C^{**-T} C^{**-1} (B_m - b^*) - H_0^2 = 0 ,
\]  

(8)
where $C^{*\rightarrow T} = C^{*\rightarrow -T}$.

Finally, the expression of the ellipsoid which coefficients wanted to be calculated, appears in the following expression:

$$B_m^T Q B_m^T + u^T B_m + d = 0,$$

where $Q = C^{*\rightarrow T} C^{*\rightarrow -1}$, $u = -2Qb^*$ and $d = b^T Qb^* - H^2_0$. The expression (9) is a quadratic expression in the variables $x, y, z$ (the measurements components gather in $B_m$), that represents an ellipsoid. The ellipsoid, expressed in $x, y, z$, looks as follows:

$$\theta_0 x^2 + \theta_1 2xy + \theta_2 y^2 + \theta_3 2xz + \theta_4 2yz + \theta_5 z^2 + \theta_6 2x + \theta_7 2y + \theta_8 2z + \theta_9 = 0.$$  

In the equation (10) the coefficients $\theta_0$ to $\theta_9$ are the variables that are going to be estimated with the parametric estimation algorithms, and they are related to the calibration elements as:

$$\begin{align*}
\theta_0 &= q_{00}, \\
\theta_1 &= q_{01}, \\
\theta_2 &= q_{11}, \\
\theta_3 &= q_{02}, \\
\theta_4 &= q_{02}, \\
\theta_5 &= q_{22}, \\
\theta_6 &= -(q_{01} b_1^1 + q_{00} b_0^0 + q_{02} b_2^2), \\
\theta_7 &= -(q_{01} b_1^1 + q_{11} b_1^2 + q_{12} b_2^2), \\
\theta_8 &= -(q_{02} b_0^0 + q_{12} b_1^2 + q_{22} b_2^2), \\
\theta_9 &= 2q_{01} b_0^0 b_1^1 + 2q_{02} b_0^0 b_2^2 + 2q_{12} b_1^2 b_2^2 + q_{00} b_0^0 + q_{11} b_1^2 + q_{22} b_2^2 - H^2_0,
\end{align*}$$

where:

$$Q = \begin{bmatrix} 
q_{00} & q_{01} & q_{02} \\
q_{01} & q_{11} & q_{12} \\
q_{02} & q_{12} & q_{22}\end{bmatrix} = C^{*\rightarrow T} C^{*\rightarrow -1}$$

In the next section the ordinary least squares method, used to estimate the ellipsoid described by the magnetometers’ measurement data set, is briefly described.

3. Ordinary Linearized Least Squares Method

Parametric fitting consists on finding the parameters of a model that fit a given data set. In the problem at hands, the parameters to find are the coefficients of an ellipsoid, surface that the data set is supposed to describe.

The ellipsoid model can be represented by the expression:

$$f(\theta, r) = \theta^T e(r),$$

where $\theta$ is the coefficients vector, the set of parameters to calculate:

$$\theta = [\theta_0, \theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7, \theta_8, \theta_9]^T,$$

and $e(r)$ is the measurement vector, the input data to our model, being $x$, $y$, and $z$ the Cartesian coordinates of the data points:
Finally, the implicit expression of the ellipsoid (10) is obtained by combining equations (12), (13), and (14).

The coefficients calculated by the fitting algorithms have some uncertainty due to data noise, normally modelled by a Gaussian probability distribution. If this uncertainty is acceptable for the problem that is being solved, then the fitting process is done. However, if the uncertainty of the estimated coefficients is not acceptable, it shall be reduced by collecting more data or by reducing the measurement error and collecting new data. In the case of study, the fitting will be considered acceptable if the error introduced by the calibration parameters calculated do not affect the ADCS subsystem performance (this error threshold is described in [7]).

Focusing on the ordinary least squares method as the fitting method, this estimator is a linear model that minimizes the sum of squared residuals, which are the differences between the observed value (the value of a point from the data set $S$) and the fitted value provided by the model. The cost function to minimize represents, in the case of a curve or a surface, the minimization of the algebraic distance. In the case of the ellipsoid, this cost function is as follows:

$$
\gamma_S^2(\theta) = \frac{1}{n} \sum_{i=0}^{n} \| f(\theta, p_i) \|^2 = \frac{1}{n} \sum_{i=0}^{n} (\epsilon(p_i) \theta)^T \epsilon(p_i) \theta = \theta^T \left( \frac{1}{n} \sum_{i=0}^{n} \epsilon(p_i) \epsilon(p_i)^T \right) \theta = \theta^T M_S \theta,
$$

where $\gamma_S^2(\theta)$ is called the mean square error and $n$ represents the total number of points in the data set $S$. The matrix $M_S$ is defined as:

$$M_S = \frac{1}{n} \sum_{i=0}^{n} \epsilon(p_i) \epsilon(p_i)^T.
$$

If equation (15) is derived by the coefficients vector, $\theta$, to obtain the value of $\theta$ that minimizes the cost function, the trivial solution is obtained, $\theta = 0$. In order to avoid the trivial solution, a scale normalization is imposed in $\theta$. In the case of the ordinary least squares, the normalization condition is:

$$\theta^T \theta = 1.
$$

Considering the aforementioned condition, the minimization of (15) will lead to an eigenvector problem [8]:

$$M_S \theta = \lambda \theta.
$$

The problem represented by (18) is a linear matrix equation which solution is the eigenvector of $M_S$ correspondent to the eigenvalue with the smallest absolute value.

4. **UPMSat-2 Magnetometer’s Calibration**

The input data used for the estimation problem is the magnetometers data collected during the UPMSat-2 integration tests, which consists on bunches of 409 measurements space distributed. The mathematical code developed for the estimation problem and the calibration parameters calculation has been developed in python, using Numpy and Scipy libraries.

Related to the UPMSat-2 integration tests, in the case of ADCS subsystem the goal is to rule out the existence of errors in the hardware and software of data acquisition and discard the uncertainty in the wiring of the hardness. In addition, the mounting axes of the ADCS magnetometers do not coincide with the satellite’s body axes; therefore the change of axes between aforementioned references has to be validated. In order to fulfil the defined goals, the following tests have been done:
• Initial tests over the magnetometers and magnetorquers in order to rule out faults in the wiring and in the acquisition of instrument’s data.
• Static tests with the aim of ensuring that the attitude control correctly operates the magnetorquers.
• Dynamic tests looking to ensure that the derivative of the magnetic field is correctly interpreted by the attitude control.
• Magnetometers data acquisition in different satellite positions with the presence of a high precision magnetometer as reference.

Thanks to the action described in the last point above, a bunch of measures, along with the reference magnetic field value, have been collected as to fulfilled the nominal and payload magnetometers calibration.

In Tables 2-4 it can be seen, for each UPMSat-2 magnetometer, the results obtained, consisting in set of data collected during the integration tests and the estimated ellipsoid by the ordinary least squares algorithm, along with the calibration parameters obtained from the ellipsoids coefficients using equation system expressed in equations (11).

The mean error has been calculated over all the sensor measurements resulting earth magnetic field module. For magnetometers SSBV07 (Table 2) and SSBV08 (Table 3), the error returned by the calculated calibration parameters are less than 2%, whereas for the Bartington (Table 4) magnetometer we obtain an error below 5%. Observing the graph shown in Figure 7 (Table 4), it seems that during the integration test the sensor faced an increase of noise level over the average process noise level during the data acquisition. The same phenomenon can be appreciated in the Bartington ellipsoid figure, Figure 6, where clearly a group of measurements do not follow the ellipsoidal patron of the rest of measurements.

As demonstrated in [7], the ADCS control is robust enough as to absorb the error presented by the two nominal magnetometers in the ADCS function, as well as the error presented by Bartington magnetometer, in the case this sensor would be used as ADCS nominal sensor.
Table 2. Calibration results for UPMSat-2 nominal ADCS magnetometer SSBV07.

\[
\begin{bmatrix}
-23801.08 & -21844.01 & -18084.80 \\
-21844.01 & 25864.60 & -1358.73 \\
-18084.80 & -1358.73 & 31071.48
\end{bmatrix}
\]

\[
b = [2.51\ 2.57\ 2.49]^T
\]

Figure 2. SSBV07 ellipsoidal coefficients estimation result

Figure 3. SSBV08 Earth magnetic field measurement error

Mean Magnetic Field Modulus Error = 1.54 %
### Table 3. Calibration results for UPMSat-2 nominal ADCS magnetometer SSBV08.

#### Magnetometer SSBV08 Calibration

![SSBV08 calibration result](image)

**Figure 4.** SSBV08 ellipsoidal coefficients estimation result

\[
C^{-1} = \begin{bmatrix} -27618.09 & -6566.56 & 24110.82 \\ -6566.56 & 33696.66 & 2201.74 \\ 24110.82 & 2201.74 & 22175.37 \end{bmatrix}
\]

\[b = [2.62 \quad 2.57 \quad 2.49]^T\]

![SSBV08 Earth magnetic field measurement error](image)

**Figure 5.** SSBV08 Earth magnetic field measurement error

Mean Magnetic Field Modulus Error = 1.55 %
Table 4. Calibration results for UPMSat-2 payload magnetometer Bartington.

Magnetometer Bartington Calibration

| Calibration Results | Value  |
|---------------------|--------|
| $C^{-1}$            |        |
|                     | 10095.34 | 335.02 | 77.54 |
|                     | 335.02  | 10118.54 | 216.36 |
|                     | 77.54   | 216.36   | 10365.80 |
| $b$                 | $[-0.02, -0.16, -0.18]^{T}$ |

Mean Magnetic Field Modulus Error = 4.12 %
5. Conclusions

A first step of the UPMSat-2 magnetometer’s calibration is introduced in the present work. The three axes fluxgate magnetometers on board of the UPMSat-2 satellite measures the modulus Earth magnetic field in a point in the form of an ellipsoid instead of a sphere, due to the presence environmental and sensor noise. The coefficients of the described ellipsoid are related with the parameters that define the transference function, function that relates the magnetometer’s electrical measurements with the value of the Earth magnetic field. In order to estimate the ellipsoid’s coefficients, an ordinary least squares algorithm is selected, as a first approach model to the fitting problem.

The magnetometer’s measurements set acquired during the integration tests is the data used in the estimation process. Once the coefficients of the ellipsoid for each of the UPMSat-2 magnetometers are estimated, the calibration parameters are obtained. The error introduced by the whole process does not affect the ADCS performance, as it is demonstrated by the in-orbit satellite simulator.

Regarding future studies, other linear least squares options are going to be analysed for different environmental conditions. In the case of the magnetometer’s work environment, there has to be considered noisy data distributions and scarce data distributions as the input to in orbit magnetometer calibration. Therefore, tests related to the performance of different fitting methods facing these kinds of environments are going to be included in works to come. This investigation process is focused on future in orbit calibration of the UPMSat-2 magnetometers.

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