DC current through a superconducting two-barrier system.

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We analyze the influence of the structure within a SNS junction on the multiple Andreev resonances in the subgap I-V characteristics. Coherent interference processes and incoherent propagation in the normal region are considered. The detailed geometry of the normal region where the voltage drops in superconducting contacts can lead to observable effects in the conductance at low voltages.

\textbf{I. INTRODUCTION}

The physics of SNS junctions of microscopic size have attracted a great deal of attention recently \cite{1,2}. A variety of experiments can be understood by modelling the constriction by a number of one dimensional superconducting channels in parallel \cite{3}. Each channel is described in terms of its transmission coefficient. This model assumes that, at finite voltages, the potential drop occurs in a normal region close to the barrier. The size of this region is implicitly fixed when specifying the boundary conditions for the quasiparticle wavefunctions in the two superconducting leads. The matching conditions used so far are equivalent to the assumption that the width of the normal region is much smaller than other relevant length scales in the system, such as the coherence length, $\xi_0$. If this is the case, Anderson’s theorem can be used to justify the lack of variation of the superconducting properties of the leads in the proximity of the constriction \cite{4}.

Corrections to the preceding picture are expected when the potential drop, and the effective barrier region are of sizes comparable to the scales which determine the superconducting properties. We can expect that, near the constriction of size $L$, the mean free path of the superconductor, $l$, will not exceed $L$. The effective coherence length is given by $\xi = \sqrt{\xi_0 l} \sim \sqrt{\xi_0 L}$. According to Anderson’s theorem, the relative corrections to the superconducting properties due to imperfections of size $d$ are of order $O(L/\xi)$. Thus, the small parameter which justifies the existence of an abrupt barrier at the junction is $\propto \sqrt{L/\xi_0}$. For contact regions $L \approx 10\text{nm}$ and larger, these effects need not be negligible for materials such as Al, and should be more relevant for Pb or Nb junctions.

In this work, we analyze the leading corrections to the abrupt barrier limit by assuming that the barrier region has an internal structure. We allow the transmission of the central region to depend on energy. The simplest such situation is to consider that the junction is made up of two barriers, which define the central region of the contact, and that that transport is ballistic in between the barriers. Recent experiments suggest that such a geometry can be manufactured with existing technologies \cite{3,4}.

The study of microscopic models of Andreev reflections was started in 1982 by Blonder et al. \cite{10}, who analyzed the current which flows through a normal metal interface. They considered the case in which scattering takes place only at the interface. The Landauer’s approach is generalized to account for Andreev reflection. An electron incident from the normal part reaches the interface and its probability to cross it or been reflected is calculated. Electrodes are in equilibrium. Thus the probability of each departing process and the occupation of the states is controlled by the Fermi function. Adding all possible processes current is calculated.

In that reference \cite{10}, it was shown that at large voltages the IV curves corresponding to a NS interface are linear with a slope equal to the normal state conductance. However they are displaced with respect to the normal state IV curve by an amount called excess current. At low voltages $V \leq \Delta$ the shape of the IV curve is strongly dependent on the transmission of the interface. At large values of the transmission, the current at low voltages is finite \cite{11}.

Later Octavio et al. \cite{12}, proposed a model to analyze transport in superconducting constrictions. They modeled the system as a superconducting-normal-superconducting (SNS) system and analyzed the transport through two SN systems connected in series. Normal scattering was modeled with $\delta$ barriers at the NS interfaces.

Their model is semiclassical. All possible scattering processes are added, weighted by its probability. Probabilities and not quantum mechanical amplitudes are considered. The interference between scattering processes is neglected. With this model, it was possible to explain qualitatively the appearance of subharmonic gap structure in the IV characteristics of superconducting weak links. The computation of IV curves in superconducting constructions was analyzed by Arnold \cite{14,15}, within the formalism of Green’s functions. In the last years, several authors have improved

\textbf{REFERENCES}

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4. INTRODUCTION

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these calculations, have calculated the fully coherent quantum mechanical I-V curves using different approaches and have generalized the calculations to the case in which transport takes place by resonant tunneling.

In this paper we generalize previous calculations to the case in which there are two barriers in a one-dimensional superconducting constriction, SISIS system. Another interesting and related system has been recently studied by Ingerman et al. In that work the current through a SNINS with finite length normal regions, is analyzed. They find that in these systems the subgap current is enhanced in comparison to that of superconducting constrictions. The effect is most pronounced in low transparency junctions.

We will consider two cases: in Section II the case in which quantum-coherence is maintained between the barriers is analyzed. The details of the calculation are given in Appendix A. In Section III we study the case in which both barriers are added in series, without considering quantum interference between the scattering processes at both barriers. We end with the main conclusions.

II. QUANTUM-COHERENT PROPAGATION BETWEEN THE BARRIERS

In this section, we generalize the previous calculations to the case in which there are two barriers, separated by a distance $L$, assuming that phase-coherence in maintained between the barriers, see Fig. 1. Except for the existence of these two barriers, the superconductor is perfectly clean.

FIG. 1. Model-system considered to analyze transport in a superconducting constriction when two barriers are present.

Each superconductor is described by Bogoliubov de Gennes (BdG) equations and is assumed to be in equilibrium at its own chemical potential. Boundary conditions on the wave function and its derivative must be given at the interface. If there is a finite potential drop $V$ between two superconductors, the chemical potential of both superconductors is not equal, $\epsilon V = \mu_2 - \mu_1$. Here $i = 1, 2$ label the left and right electrodes. A common reference level must be chosen. When both superconductors are referred to the same chemical potential the order parameter acquires a time-dependent phase. Thus the time-dependent BdG equations must be solved. The time dependence of the relative phase $\delta \phi = \phi_2 - \phi_1$ is given by the Josephson relation

$$\frac{d\delta \phi}{dt} = \frac{2eV}{\hbar}$$

(1)

To visualize the processes which contribute to the current and perform the calculations, it is easier to work in a scattering formalism scheme, in which each barrier is inside a normal region (see Fig. 1), as developed in [9]. This normal region, of length $d \ll \xi$, with $\xi$ the coherence length of the superconductor, can be introduced without losing generality as the condition $d \ll \xi$ makes the superconducting properties of the constriction irrelevant. The superconductor acts as a source of quasiparticles. These quasiparticles incide at the barrier and can be transmitted or reflected. Moreover, Andreev reflection processes can occur at the normal-superconducting (NS) interfaces. Assuming
that $\Delta \ll \mu_i$, the Andreev approximation can be used. For simplicity, in the following, we assume that all the superconducting regions are ideal BCS ones and that the order parameter in all regions is equal.

Andreev reflection at the NS interface is given by the amplitude $a(\epsilon)$ of suffering an Andreev reflection process. If the superconductor is of the BCS type \[13\]

$$a(\epsilon) = \begin{cases} 1 & \epsilon > \Delta, \\ \frac{\epsilon - s \text{sgn}(\epsilon)(\epsilon^2 - \Delta^2)^{1/2}}{\epsilon} & |\epsilon| < \Delta \end{cases}$$

In the absence of a magnetic field, which breaks time-reversal invariance, each barrier is characterized by a scattering matrix.

$$\begin{pmatrix} r_i & t_i \\ t_i & -r_i^* t_n \end{pmatrix}$$

In the neck, between the barriers, electrons propagate without suffering scattering events, but acquire a phase $e^{ikL}$, where $k$ is the electronic momentum.

As a result of all multiple Andreev reflection processes (MAR) and their interference with the normal scattering ones, the wave function in the normal region, in zone I, II (a and b) and III can be written as \[19\]

$$\Psi^e_I = \sum_{m,n} \left[ (a_{2n}^m A_n^m + J_0 \delta_{m0} \delta_{n0}) e^{ikx} + B_n^m e^{-ikx} \right] e^{i(\epsilon + 2meV_1 + 2meV_2)t}$$

$$\Psi^h_I = \sum_{m,n} \left[ A_n^m e^{ikx} + a_{2n}^m B_n^m e^{-ikx} \right] e^{-i(\epsilon + 2meV_1 + 2meV_2)t}$$

$$\Psi_{II-a}^I = \sum_{m,n} \left[ J_n^m e^{ikx} + G_n^m e^{-ikx} \right] e^{i\epsilon + (2n+1)eV_1 + 2meV_2}$$

$$\Psi_{II-a}^I = \sum_{m,n} \left[ J_n^m e^{ikx} + H_n^m e^{-ikx} \right] e^{-i\epsilon + (2n-1)eV_1 + 2meV_2}$$

$$\Psi_{II-b}^I = \sum_{m,n} \left[ j_n^m e^{ikx} + G_n^m e^{-ikx} \right] e^{i\epsilon + (2n+1)eV_1 + 2meV_2}$$

$$\Psi_{II-b}^I = \sum_{m,n} \left[ j_n^m e^{ikx} + H_n^m e^{-ikx} \right] e^{-i\epsilon + (2n-1)eV_1 + 2meV_2}$$

$$\Psi_{III}^I = \sum_{m,n} \left[ E_n^m e^{ikx} + a_{2n+1}^m E_n^m e^{-ikx} \right] e^{i\epsilon + (2n+1)eV_1 + (2m+1)eV_2}$$

$$\Psi_{III}^I = \sum_{m,n} \left[ a_{2n-1}^m E_{n-1}^m e^{ikx} + F_{n-1}^m e^{-ikx} \right] e^{-i\epsilon + (2n-1)eV_1 + 2meV_2}$$

where $V_1$ and $V_2$ are the voltage drops at the two barriers and $a_n^m(\epsilon) = a(\epsilon + meV_1 + neV_2)$ with $a(\epsilon)$ the Andreev reflection amplitude. Region II-a refers to the superconducting part situated just after the left barrier, while region II-b refers to that one just before the right barrier.

The current is a time-dependent quantity, which oscillates with all the harmonics of the Josephson frequency $\omega_J = 2eV/h$.

$$I(t) = \sum_n I_n e^{i\omega_j}$$

The time-dependence of the current arises from the time-dependence of the superconducting phase induced by the voltage drop. Here we will be only interested in the dc current ($I_0$ component) which is the one experimentally measured.

The scattering matrices relates electron and hole coefficients at regions I, II-a, b and III. Scattering matrices of electrons and holes are related by $S_b(\epsilon) = S^*_{el}(\epsilon)$. As explained in appendix A the matching conditions lead to a set of matrix equations between the coefficients in the wave functions, which can be recursively solved.

There it is shown that if the barriers are a distance $L$ far apart, the problem can be mapped into another one with a momentum-dependent single barrier $T_{eff}$ given by

$$T_{eff}(k) = \frac{T_1 T_2}{1 + R_1 R_2 - 2\sqrt{R_1 R_2 \cos(\phi + 2kL)}}$$
Here \( \phi \) is defined as

\[
r'_{12} = \sqrt{R_1 R_2} e^{i \phi}
\]  

(7)

where \( r'_{1} = -r^*_1 t_1 / t^*_1 \) is the amplitude of probability of reflexion for those electrons which incide from the right onto the first barrier. This effective transmission would be the same in the case of a normal central region and/or normal left and right electrodes. The superconducting nature of the electrodes enters in the calculation of the wave functions’ coefficients.

Due to the dependence on the momentum, the transmission depends on energy and the equations of previous section must be generalized to the case in which transmission and reflection are energy-dependent. A similar situation was found in \[28\]. We assume that Andreev approximation still holds, what implies that the chemical potential is much larger than any other energy scale in the problem. Assuming a linear dispersion relation (6) can be written as

\[
T(\epsilon) = \frac{T_1 T_2}{1 + R_1 R_2 - 2 \sqrt{R_1 R_2} \cos (\phi + 2f \frac{\epsilon}{\Delta})}
\]

(8)

with \( f = \frac{\Delta}{\hbar v_F} \).

The procedure to calculate the coefficients \( A_n \) and \( B_n \) and the current is detailed in appendix A. Only those coefficients \( A_n = A^n_{nm} \delta_{nm} \) and \( B_n = B^n_{nm} \delta_{nm} \) are non-zero.

If \( f = 0 \) (barriers are at the same point) the IV curves are the ones corresponding to a superconducting constriction with a non-energy dependent transmission of value

\[
T^{f=0} = \frac{T_1 T_2}{1 + R_1 R_2 - 2 \sqrt{R_1 R_2} \cos \phi}
\]

(9)

and no new features appear. New I-V curves appear when \( f \neq 0 \). Then, the transmission depends on energy and oscillates between the values \( T_{\text{min}} \) and \( T_{\text{max}} \) given by

\[
T_{\text{max}} = \frac{T_1 T_2}{(1 - \sqrt{R_1 R_2})^2}
\]

(10)

\[
T_{\text{min}} = \frac{T_1 T_2}{(1 + \sqrt{R_1 R_2})^2}
\]

(11)

If one of the barriers is equal to unity, the current is controlled by the other barrier and we again recover the single constriction case. The effect of a finite value of \( f \) will be more important for larger difference between \( T_{\text{max}} \) and \( T_{\text{min}} \). Note that, in particular, if both barriers are equal, equal to \( T \), \( T_{\text{max}} = 1.0 \) and \( T_{\text{min}} = T^2/(2 - T)^2 \), which decreases with decreasing \( T \). Some examples are shown if Figs. 2 to 5.

![FIG. 2. IV curve for the case of coherent transport in a superconducting constriction with two barriers with parameters \( T_1 = 0.9 \), \( T_2 = 0.1 \) \( \phi = 0 \) and several values of \( f \)](image-url)
FIG. 3. IV curve for the case of coherent transport in a superconducting constriction with two barriers with parameters $T_1 = T_2 = 0.9 \phi = 0$ and several values of $f$

FIG. 4. IV curve for the case of coherent transport in a superconducting constriction with two equal barriers for the case $f = 0.1$, $\phi = 0$ and several values of the transmission.

In Fig. 3 curves corresponding to $T_1 = 0.9$, $T_2 = 0.1$ and $\phi = 0.0$ are shown. The maximum and minimum value corresponding to these transmissions are $T_{\text{max}} = 0.184$ and $T_{\text{min}} = 0.05$. These values are relatively similar and a very strong effect of a finite $f$ is not expected. In fact, as $f$ increases the conductance is slightly reduced. The fact that the conductance is reduced and not increased is because we have started from the maximum value at $f = 0$.

Another example is shown in Fig. 4, in which case curves for $T_1 = T_2 = 0.9$, $\phi = 0.0$ and several values of $f$ are plotted. Maximum and minimum values are $T_{\text{max}} = 1$ and $T_{\text{min}} = 0.67$, which are again not too different. The main effect of a finite value of $f$ is the suppression of the current at zero voltage. This effect is also observed in Fig. 4, where several curves in the case in which both barriers are equal, $f = 0.1$ and $\phi = 0$ are shown. In all the cases the transmission for $f = 0$ would be equal to unity. The deviation of this curve is more pronounced for small transmission of the barrier. For all transmissions current at zero voltage is completely suppressed. The reason of this suppression is that even at $V = 0$, all the energies contribute to the current and the transmission is not equal to unity at all energies.
FIG. 5. IV curve for the case of coherent transport in a superconducting constriction with two barriers with parameters $T_1 = T_2 = 0.1 \phi = 0$ and several values of $f$.

The effect of a finite value of $f$ is specially strong in figure 5 as $T_{\text{min}} = 0.002$. The maximum value is 500 times the minimum one. The effect is very strong even for a very small values of $f$, as in the case $f = 0.02$.

III. INCOHERENT PROPAGATION BETWEEN THE BARRIERS

The calculation and features of the IV curves in the case in which propagation between the barriers is not coherent are completely different to those of previous section. From current conservation $I_1 = I_2 = I$, where $I_i$ is the current which cross junction $i$. In the normal state, in the absence of coherence $I_i = g_i V_i$ determines the voltage drop in each contact, $V_i = g_i/(g_1 + g_2)V$ and, as a result the total current is obtained adding the resistances in series.

To calculate the IV curves in the superconducting state we proceed in the same way. $I(V) = I_1(T_1, V_1) = I_2(T_2, V_2)$. $I_i(T_i, V_i)$ is computed according to the calculations for a constriction with a single barrier of transmission $T_i$ [19]. Then the voltage in each junction is numerically determined from the current conservation equation. Note that SGS will not appear at values $V = 2\Delta/n$, but at values $V_i = 2\Delta/n$.

FIG. 6. IV curves corresponding to transport in superconducting constrictions when there are two barriers in the structure and coherence is lost between the barriers. The figure shows several curves for the case in which both barriers are equal.

Examples of IV curves determined by this method are shown in Figs. 6 and 7. In Fig. 6 several curves, corresponding to the case in which both barriers are equal, are plotted. Subharmonic gap singularities appear at $V = 4e\Delta/n$, as the
voltage in each barrier is equal to $V/2$. The curves are equivalent to the IV curve corresponding to the transmission of the barrier, but for a doubled voltage. Fig. 7 show curves for the case in which both barriers are different. The position at which SGS appears depends on the value of both barriers.

![IV curves of a superconducting constriction in presence of two different barriers, when there is no phase-coherence between the barriers.](image)

**FIG. 7.** IV curves of a superconducting constriction in presence of two different barriers, when there is no phase-coherence between the barriers.

**IV. CONCLUSIONS**

We have studied the influence of the structure of the normal region where the voltage drops in superconducting contacts. We present detailed expressions to analyze the interference effects which arise due to the Andreev reflections at different positions within the contact.

The observed I-V characteristics at low voltages can differ from the usually considered single abrupt barrier case if the size of the contact, $L$, is comparable to the effective coherence length, $\sqrt{\xi_0 L}$, where $\xi_0$ is the coherence length in the clean limit. The influence of inelastic scattering in the normal region, leading to loss of coherence, has also been investigated.

In particular, we have derived the equations for the case in which the transmission through the constriction depends on energy. We have analyzed in detail the case in which the dependence in energy arises from the existence of two barriers in the contact, in between which the phase is maintained. The I-V curves differ from the ones obtained with only one non energy-dependent barrier with any transmission. The effect is strongest when both barriers have similar, and low, transmission coefficients. In this case, the energy-dependent transmission oscillates between two values very different in magnitude. Even when at zero energy the transmission is equal to unity, we find that current at zero voltage is suppressed.

In the case in which propagation between both barriers is incoherent, current is derived from the current conservation requirement. The main feature of the I-V curves is the appearance of subharmonic structure at voltages different from $2\Delta/n$.

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**VI. APPENDIX A**

The coefficients in the wave functions are related by the scattering matrices giving a set of equations which allows us to obtain them. For the electronic part these equations are
where

\[
\begin{bmatrix}
B_n^m \\
J_n^m \\
G_n^m \\
E_n^m
\end{bmatrix} = \begin{bmatrix}
r_1 & t_1 & r^*_1 & t^*_1 \\
t_1 & r^*_1 & t^*_1 & r^*_1 \\
0 & e^{ikL} & 0 & e^{ikL} \\
r_2 & t_2 & r^*_1 & t^*_1
\end{bmatrix} \begin{bmatrix}
a_{2n}^m A_n^m + J \delta_{n0} \\
a_{2n+1}^m A_n^m \\
a_{2n}^m \delta_{n0} \\
a_{2n+1}^m \delta_{n0}
\end{bmatrix}
\]

(12)

and similar equations for the holes. From it the coefficients in region I and III are related by

\[
\begin{bmatrix}
B_n^m \\
F_n^m
\end{bmatrix} = S_{el} \begin{bmatrix}
a_{2n}^m A_n^m + J \delta_{n0} \\
a_{2n+1}^m A_n^m \\
a_{2n}^m \delta_{n0} \\
a_{2n+1}^m \delta_{n0}
\end{bmatrix}
\]

(15)

with

\[
S_{el} = \frac{1}{t_1^* + t_1 r_1^* r_2} \begin{bmatrix}
t_1^* r_1 e^{-ikL} + t_1 r_2 e^{ikL} & T_1 t_2 \\
T_1 t_2 & -(t_1 r_1^* e^{ikL} + t_1^* r_2^* e^{-ikL})
\end{bmatrix}
\]

(16)

and

\[
\begin{bmatrix}
A_n^m \\
F_{n-1}^m
\end{bmatrix} = S_h \begin{bmatrix}
a_{2n}^m B_{n-1}^m \\
a_{2n-1}^m B_{n-1}^m
\end{bmatrix}
\]

(17)

with \( S_h = S_{el}^* \). From the existence of a unique source term, \( J \delta_{n0} \), it can be shown that \( A_n^m \) and \( B_n^m \) will vanish except for \( n = m \). Thus, the superindex can be dropped and the problem is equivalent to one in which there is a single barrier with transmission given by (11). Due to the dependence on the momentum, the transmission depends on energy.

In the following, the transmission is characterized by a scattering matrix

\[
\begin{pmatrix}
r(\epsilon) \\
t(\epsilon)
\end{pmatrix} = \begin{pmatrix}
r(\epsilon) & t(\epsilon) \\
t(\epsilon) & r^{-1}(\epsilon) t(\epsilon)
\end{pmatrix}
\]

(18)

As usual, transmission and reflection coefficients satisfy

\[ R(\epsilon) + T(\epsilon) = 1 \]

(19)

where \( R(\epsilon) = |r(\epsilon)|^2 \) and \( T(\epsilon) = |t(\epsilon)|^2 \). In the two-barrier case, the dependence in energy comes from the dependence in momentum. We assume that momentum does not change when the particle cross the barrier, thus the reflexion and transmission coefficient depends on the energy of the originally incident particle. However, in the following we include both the possibility that transmission and reflexion coefficients depend, as in our case, only on the energy of the original quasiparticle, and then we define

\[
r_n(\epsilon) = r(\epsilon)
\]

(20)

and the case in which both depend on the energy of the quasiparticle, \( \epsilon + 2n\epsilon \), which is crossing the barrier and has emerged after \( 2n \) Andreev reflections. Then

\[
r_n(\epsilon) = r(\epsilon + 2n\epsilon)
\]

(22)

\[
t_n(\epsilon) = t(\epsilon + 2n\epsilon)
\]

(23)

\( R_n(\epsilon) \) and \( T_n(\epsilon) \) are analogously defined.

As before we can find the relations between the coefficients of the wavefunctions. After some algebra the equations which relate the \( A_n \) and \( B_n \) coefficients are

\[
t_n^*(\epsilon) A_{n+1} - t_{-(n+1)}^*(\epsilon) a_{2n+1} a_{2n} A_n =
\]

\[
t_n^*(\epsilon) r_{n+1}^*(\epsilon) a_{2n+2} B_{n+1} - t_{-(n+1)}^*(\epsilon) a_{2n+1} r_{n+1}^*(\epsilon) t_n^* B_n + t_{-(n+1)}^*(\epsilon) a_{2n+1} J \delta_{n0}
\]

(24)
In the case which is simpler than (25) and reduces to the expression obtained in [19], but including explicitly the energy dependence.

We define

\[ S_B = \text{probability (1-0)} \]

and

\[
T_n(\epsilon)T_{-(n+1)}(-\epsilon)a_{2n+2}a_{2n+1}L_nB_{n+1} - \{a_{2n}^2L_{n+1}(\epsilon)[t_{n-1}(\epsilon)t_{n+1}^*(-\epsilon)a_{2n-1}^* - r_{n-1}(\epsilon)r_{n+1}(\epsilon)t_{n-1}(\epsilon)t_{n+1}(\epsilon) - r_{n-1}(\epsilon)t_{n+1}(\epsilon)t_{n-1}(\epsilon)] + L_n(\epsilon)[t_{-(n+1)}(-\epsilon)t_{n}^*(-\epsilon) - r_{-(n+1)}(-\epsilon)r_{n}^*(\epsilon)t_{n}^*(\epsilon)a_{2n+1}^2]\}
\]

with

\[ L_n(\epsilon) = -r_n(-\epsilon)t_{n-1}^*(-\epsilon)t_{n-1}(\epsilon)a_{2n-1}^2 - r_{n-1}(\epsilon)t_{n-1}^*(\epsilon) \]

In the case \( t_n(\epsilon) = t(-\epsilon) \), equation (25) can be simplified to

\[
T(\epsilon)a_{2n+1}a_{2n+2}B_{n+1} + T(\epsilon)a_{2n-1}a_{2n}(1 - a_{2n-1}^2)B_{n-1} - [a_{2n}^2(1 - a_{2n+1}^2)(a_{2n-1}^2 - R(\epsilon)) + (1 - R(\epsilon)a_{2n+1}^2)(1 - a_{2n-1}^2)]B_n = -r(1 - a_{2n-1}^2)(1 - a_{2n+1}^2)J_0\delta_{n0}
\]

which is simpler than (23) and reduces to the expression obtained in [19], but including explicitly the energy dependence.

Including the contributions from quasiparticles incident on the constriction from both superconductors (electrons from the left superconductor - with momentum \( k \) and probability \( f(\epsilon) \) and holes from the right one - with momentum \( -k \) and probability \( 1 - f(\epsilon) \)) the dc-current, as calculated in zone I, is given by

\[
I_0 = -\frac{e}{\pi h} \int_{-\mu}^\mu d\epsilon \text{sgn}(\epsilon)\epsilon [J^2 + |Ja_0A_0^* + Ja_0A_0 + \sum_n (1 + |a_{2n}|^2)(|A_n|^2 - |B_n|^2)\mid_{\epsilon \to \infty}]
\]

This expression was defined in [19] and takes into account the contribution of all scattering processes of quasiparticles with both spins. One must solve (25) or (27) and obtain the \( B_n \) coefficients and, using this solution, determine \( A_n \) from (24)

It only rests now to discuss how equation (25) or (27) can be solved. Let us write these equations in the form (29)

\[
V_nB_{n+1} + U_nB_n + H_nB_{n-1} = F_n\delta_{n0}
\]

where \( V_n, U_n, H_n \) and \( F_n \) depend on energy and are given by the coefficients of the corresponding equation (25) or (27). Consider \( n \neq 0 \). Then (25)

\[
V_n \frac{B_{n+1}}{B_n} + U_n + H_n \frac{B_{n-1}}{B_n} = 0
\]

We define

\[
S_{n>0} = \frac{B_n}{B_{n-1}}
\]

\[
S_{n<0} = \frac{B_n}{B_{n+1}}
\]

In terms of these factors \( S_n \)

\[
B_{n>0} = \prod_n S_{n>0} \cdots S_1B_0
\]

\[
B_{n<0} = \prod_n S_{n<0} \cdots S_{-1}B_0
\]

If \( n > 0 \), substituting the definition of \( S_n \) in (30)

\[
V_nS_{n+1} + U_n + H_n \frac{S_{n-1}}{S_n} = 0
\]

and from it
\[ S_{n>0} = -\frac{H_n}{U_n + V_n S_{n+1}} \] (36)
equivalently
\[ S_{n>0} = -\frac{H_n}{U_n - \frac{V_n H_{n+1}}{U_{n+1} - \frac{V_n H_{n+2}}{U_{n+2} - \frac{V_n H_{n+3}}{U_{n+3} - \cdots}}}} \] (37)

Analogously, for \( n < 0 \)
\[ \frac{V_n}{S_n} + U_n + H_n S_{n-1} = 0 \] (38)
and \( S_{n<0} \) is given from
\[ S_{n<0} = -\frac{V_n}{U_n + H_n S_{n-1}} \] (39)
which in term of recurrent fractions is written
\[ S_{n<0} = -\frac{V_n}{U_n - \frac{H_n V_{n-1}}{U_{n-1} - \frac{H_n V_{n-2}}{U_{n-2} - \frac{H_n V_{n-3}}{U_{n-3} - \cdots}}}} \] (40)
Thus, from (37) and (40), the terms \( S_n \) can be easily evaluated. To calculate \( B_0 \), we write (15) for \( n = 0 \) as
\[ [V_0 S_1 + U_0 + H_0 S_{-1}] B_0 = F_0 \] (41)
Thus
\[ B_0 = \frac{F_0}{V_0 S_1 + U_0 + H_0 S_{-1}} \] (42)
and the rest of coefficients are determined from (33) and (34).

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Text follows the mathematical content:

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[1] E. Scheer, P. Joyez, D. Esteve, C. Urbina and M. H. Devoret, Phys. Rev. Lett. 78, 3535 (1997).
[2] E. Scheer, N. Agrait, A. Cuevas, A. Levy-Yeyati, B. Ludolph, A. Martin-Rodero, G. Rubio, J. M. van Ruitenbergen and C. Urbina, Nature 394, 154 (1998).
[3] H. Suderow, E. Bascones, W. Belzig, F. Guinea and S. Vieira, Europhys. Lett. 50, 749 (2000).
[4] P. W. Anderson, J. Phys. Chem. Solids 11, 26 (1959).
[5] H. Takayanagi, T. Akazaki, and J. Nitta, Phys. Rev. B 51, 1374 (1995).
[6] A. Chrestin, T. Matsuyama, and U. Merkt, Phys. Rev B 55, 8457 (1997).
[7] J. Kutchinsky, R. Taboryski, T. Clausen, C. B. Sorensen, A. Kristensen, P. E. Lindelof, J. Bindslev Hansen, C. Schelde Jacobsen, and J. L. Skov, Phys. Rev. Lett. 78, 931 (1997).
[8] G. Bastian, E. O. Gobel, A. B. Zorin, H. Schulze, J. Niemeyer, T. Weimann, M. R. Bennett and K. E. Singer, Phys. Rev. Lett. 81, 1686 (1998).
[9] P. Dubos, H. Courtois, O. Buisson, B. Pannetier, preprint [cond-mat/0107194].
[10] G.E. Blonder, M. Tinkham and T.M. Klapwijk, Phys. Rev. B 25, 4515 (1982).
[11] G.E. Blonder and M. Tinkham, Phys. Rev. B 27, 112 (1983)
[12] M. Octavio, M. Tinkham, G.E. Blonder and T.M. Klapwijk, Phys. Rev. B 27, 6739 (1983)
[13] T.M. Klapwijk, G.E. Blonder and M. Tinkham, Physica B&C 109, 1657 (1982)
[14] G.B. Arnold, J. Low Temp. Phys. 59, 143 (1985)
[15] G.B. Arnold, J. Low Temp. Phys. 68, 1 (1987)
[16] K. Flensberg, J. Bindslev Hansen and M. Octavio, Phys. Rev. B 38, 8707 (1988).
[17] E.N. Bratus, V.S. Shumeiko and G. Wendin, Phys. Rev. Lett. 74, 2110 (1995).
[18] S. Chaudhuri and P. Bagwell, Phys. Rev. B 51, 16936 (1995).
[19] D. Averin and A. Bardas, Phys. Rev. Lett. 75, 1831 (1995).
[20] J.C. Cuevas, A. Martín-Rodero and A. Levy Yeyati, Phys. Rev. B 54, 7366 (1996).
[21] M. Hurd, S. Datta and P. Bagwell, Phys. Rev. B 54, 6557 (1996).
[22] A. Bardas and D.V. Averin, Phys. Rev. Lett. 79, 3482 (1997).
[23] A.V. Zaitsev and D.V. Averin, Phys. Rev. Lett. 78, 4821 (1997).
[24] A. Martín Rodero, A. Levy Yeyati and J.C. Cuevas, Superlatt. and Microst. 25, 925 (1999).
[25] A. Levy Yeyati, A. Martín Rodero and F. Flores, Phys. Rev. B 56, 1 (1997).
[26] G. Johansson, E.N. Bratus, V.S. Shumeiko and G. Wendin, Phys. Rev. B, 60, 1382 (1999).
[27] E. Bascones and F. Guinea, cond-mat 9809352.
[28] A. Ingerman, G. Johansson, V.S. Shumeiko and G. Wendin, cond-mat 0101283.
[29] E. N. Bratus and V.S. Shumeiko, Phys. Solid State 21, 1509 (1979)