Recent MEG Results and Predictive SO(10) Models

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Abstract

Recent MEG results of a search for the lepton flavor violating (LFV) muon decay, $\mu \rightarrow e \gamma$, show 3 events as the best value for the number of signals in the maximally likelihood fit. Although this result is still far from the evidence/discovery in statistical point of view, it might be a sign of a certain new physics beyond the Standard Model. As has been well-known, supersymmetric (SUSY) models can generate the $\mu \rightarrow e \gamma$ decay rate within the search reach of the MEG experiment. A certain class of SUSY grand unified theory (GUT) models such as the minimal SUSY SO(10) model (we call this class of models “predictive SO(10) models”) can unambiguously determine fermion Yukawa coupling matrices, in particular, the neutrino Dirac Yukawa matrix. Based on the universal boundary conditions for soft SUSY breaking parameters at the GUT scale, we calculate the rate of the $\mu \rightarrow e \gamma$ process by using the completely determined Dirac Yukawa matrix in two examples of predictive SO(10) models. If we interpret the 3 events in MEG experiment as a positive signal and combine it with other experimental constraints such as the relic density of the neutralino dark matter and recent results on muon $g - 2$, we can pin down a parameter set of the universal boundary conditions. Then, we propose benchmark sparticle mass spectra for each predictive SO(10) model, which will be tested at the Large Hadronic Collider.

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The search for new physics beyond the Standard Model (SM) has been performed in a variety of energy scales. In addition to the direct searches for new physics at the Large Hadronic Collider (LHC), being the collider experiment with the highest energy at present, a search for the LFV processes at low energies is also very important in discovering new physics. This is because the LFV processes are highly suppressed in the SM, and any positive signal once observed can be an evidence of new physics.

Recently MEG collaboration [1] has reported new results of a search for the $\mu \rightarrow e\gamma$ decay and a maximally likelihood analysis sets an upper limit at 90% C.L. on the branching ratio, $BR(\mu \rightarrow e\gamma) < 1.5 \times 10^{-11}$, which is at the same level of the current smallest limit set by the MEGA experiment [2], $BR(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11}$. Very interestingly, the results of the MEG experiment also show 3 events as the best value for the number of signals in the maximally likelihood fit, which corresponds to $BR(\mu \rightarrow e\gamma) = 3 \times 10^{-12}$

for the center value. Although this result is still far from the evidence/discovery in statistical point of view, it might be a sign of a certain new physics beyond the Standard Model.

Non-zero masses and flavor mixings of neutrinos observed through the neutrino oscillation phenomena imply that the lepton flavor in each generation is not individually conserved, and therefore, LFV processes among the charged-leptons exist. However, in simply extended models of the SM so as to incorporate massive neutrinos, the rate of the LFV processes are strongly suppressed and far out of the reach of the experimental detection. This is because of the GIM mechanism which leads to a suppression factor of the ratio between the neutrino mass scale and the electroweak scale. It has been known [3, 4] that SUSY models can generate the experimentally detectable rate of the LFV processes through the LFV sources in soft SUSY breaking terms.

The minimal supersymmetric Standard Model (MSSM) is one of the most promising candidate for new physics, providing a solution to the gauge hierarchy problem of the SM. In addition, the successful gauge coupling unification at $M_{GUT} \simeq 2 \times 10^{16}$ GeV supported by the low energy data of the SM gauge couplings strongly suggests the paradigm of SUSY GUT. Thus, the SUSY GUT models with unified gauge groups are well-motivated theories at high energies. Among them, models based on the gauge group SO(10) are probably the most compelling ones in terms of neutrino physics, because quarks and leptons in each generation are unified into a single $16$ representation along with a right-handed neutrino, and the smallness of the neutrino masses can be naturally explained through the seesaw mechanism [5]. Furthermore, a class of SO(10) models such as the minimal SUSY SO(10) model [6] has a very interesting feature. Because of the complete unification of quarks and leptons into a single $16$ representation and introduction of the minimal set of Higgs multiplets, fermion Yukawa matrices are highly constrained, and it is very non-trivial to fit all the current data of fermion masses and mixing angles including the neutrino sector. It has been shown [7] that the minimal SO(10) model can simultaneously reproduce all the
observed quark-lepton mass matrix data involving the neutrino oscillation data. It is very interesting that after the data fitting, no free parameters are left and hence, all the fermion Yukawa matrices, in particular, the neutrino Dirac Yukawa matrix, are unambiguously determined.

In this paper, we consider such a class of SUSY SO(10) models (predictive SO(10) models). We assume a suitable Higgs sector which breaks SO(10) to the MSSM gauge group at the GUT scale with generating masses for the right-handed neutrinos. Below the GUT scale, the low energy effective theory of the model is described as the MSSM with the right-handed neutrino chiral multiplets. In this effective theory, the superpotential in the leptonic sector is given by

$$ W = Y^\nu_{ij} (\nu^c_R)_i \ell_j H_u + Y^e_{ij} (e^c_R)_i \ell_j H_d + \frac{1}{2} M_{R_{ij}} (\nu^c_R)_i (\nu^c_R)_j + \mu H_d H_u, $$

(2)

where the indices $i, j$ run over three generations, $H_u$ and $H_d$ denote the up-type and down-type MSSM Higgs doublets, respectively, and $M_{R_{ij}}$ is the heavy right-handed Majorana neutrino mass matrix. We work in the basis where the charged-lepton Yukawa matrix $Y_e$ and the mass matrix $M_{R_{ij}}$ are real-positive and diagonal matrices: $Y^e_{ij} = Y^e_{i} \delta_{ij}$ and $M_{R_{ij}} = \text{diag}(M_{R_1}, M_{R_2}, M_{R_3})$. Thus the off-diagonal components of the neutrino Dirac Yukawa coupling matrix $Y_\nu$ break the conservation of the lepton flavor in each generation. Note that once we fix a predictive SO(10) model, all components in the neutrino Dirac Yukawa matrix $Y_\nu$ and the mass spectrum of the right-handed neutrinos are completely determined. The soft SUSY breaking terms in the leptonic sector is described as

$$ -\mathcal{L}_{\text{soft}} = \tilde{\ell}_i^\dagger \left( m^2_{\tilde{\ell}} \right)_{ij} \tilde{\ell}_j + \tilde{\nu}_{Ri}^\dagger \left( m^2_{\tilde{\nu}} \right)_{ij} \tilde{\nu}_{Rj} + \tilde{e}_{Ri}^\dagger \left( m^2_{\tilde{e}} \right)_{ij} \tilde{e}_{Rj} + m^2_{H_u} H_u^\dagger H_u + m^2_{H_d} H_d^\dagger H_d + \left( B \mu H_d H_u + \frac{1}{2} B_\nu M_{R_{ij}} \tilde{\nu}_{Ri}^\dagger \tilde{\nu}_{Rj} + h.c. \right) + \left( A^\nu_{ij} \tilde{\nu}_{Ri}^\dagger \tilde{\nu}_{Rj} + A^e_{ij} \tilde{e}_{Ri}^\dagger \tilde{e}_{Rj} + h.c. \right) + \left( \frac{1}{2} M_1 \tilde{B}^\dagger \tilde{B} + \frac{1}{2} M_2 \tilde{W}^a \tilde{W}^a + \frac{1}{2} M_3 \tilde{G}^a \tilde{G}^a + h.c. \right). $$

(3)

Current experimental results severely constrain soft SUSY breaking parameters to be almost flavor blind and real. In our analysis, we adopt the most commonly considered scenario, the constraint MSSM (CMSSM), and impose the universal boundary conditions for soft SUSY breaking parameters at the GUT scale:

$$ \left( m^2_{\tilde{\ell}} \right)_{ij} = \left( m^2_{\tilde{\nu}} \right)_{ij} = \left( m^2_{\tilde{e}} \right)_{ij} = m^2_0 \delta_{ij}, $$

$$ m^2_{H_u} = m^2_{H_d} = m^2_0, $$

$$ A^\nu_{ij} = A_0 Y^\nu_{ij}, \quad A^e_{ij} = A_0 Y^e_{ij}, $$

$$ M_1 = M_2 = M_3 = M_{1/2}, $$

(4)

and extrapolate the soft SUSY breaking parameters to the electroweak scale according to their renormalization group equations (RGEs). Although the boundary conditions
are flavor-blind, the LFV terms in the soft SUSY breaking parameters such as the off-diagonal components of \((m_{\tilde{\ell}}^2)_{ij}\) and \(A_{ij}^\ell\) are induced by the LFV interactions, namely, the neutrino Dirac Yukawa couplings. For example, the LFV effect most directly emerges in the left-handed slepton mass matrix through the RGEs such as

\[
\mu \frac{d}{d\mu} \left( m_{\tilde{\ell}}^2 \right)_{ij} = \left. \mu \frac{d}{d\mu} \left( m_{\tilde{\ell}}^2 \right)_{ij} \right|_{\text{MSSM}} + \frac{1}{16\pi^2} \left( m_{\tilde{\ell}}^2 Y_\nu^d Y_\nu + Y_\nu^d Y_\nu m_{\tilde{\ell}}^2 + 2Y_\nu^d m_{\tilde{\ell}}^2 Y_\nu + 2m_{\tilde{H}_u}^2 Y_\nu Y_\nu + 2A_{ij}^\ell A_\nu \right)_{ij},
\]

where the first term in the right hand side denotes the normal MSSM term with no LFV. In the leading-logarithmic approximation, the off-diagonal components \((i \neq j)\) of the left-handed slepton mass matrix are estimated as

\[
(\Delta m_{\tilde{\ell}}^2)_{ij} \sim -\frac{3m_0^2 + A_0^2}{8\pi^2} \left( Y_\nu^d L Y_\nu \right)_{ij},
\]

where the distinct thresholds of the right-handed Majorana neutrinos are taken into account by the matrix \(L = \log[M_{\text{GUT}}/M_R] \delta_{ij}\).

The effective Lagrangian relevant for the LFV processes \((\ell_i \to \ell_j \gamma)\) is described as

\[
\mathcal{L}_{\text{eff}} = -\frac{e^2}{2} m_{\ell_i} \bar{\ell}_j \sigma_{\mu\nu} F^{\mu\nu} \left( A_{ij}^L P_L + A_{ij}^R P_R \right) \ell_i,
\]

where \(P_{R,L} = (1 \pm \gamma_5)/2\) is the chirality projection operator, and \(A_{L,R}\) are the photon-penguin couplings of 1-loop diagrams in which chargino-sneutrino and neutralino-charged slepton are running. The explicit formulas of \(A_{L,R}\) etc. used in our analysis are summarized in [8]. The rate of the LFV decay of charged-leptons is given by

\[
\Gamma(\ell_i \to \ell_j \gamma) = \frac{e^2}{16\pi} m_{\ell_i}^5 \left( |A_{ij}^L|^2 + |A_{ij}^R|^2 \right).
\]

The following approximation formula [8] is useful to understand the order of magnitude of the LFV decay rate:

\[
\Gamma(\ell_i \to \ell_j \gamma) \sim \frac{e^2}{16\pi} m_{\ell_i}^5 \times \left( \frac{\alpha_2}{4\pi} \right)^2 \frac{\left( \Delta m_{\tilde{\ell}}^2 \right)_{ij}^2}{M_S^2} \tan^2 \beta,
\]

where \(M_S\) is the average slepton mass at the electroweak scale, and \(\left( \Delta m_{\tilde{\ell}}^2 \right)_{ij}\) is the slepton mass estimated in Eq. (6). From these formulas, once we obtain the information of the neutrino Dirac Yukawa coupling matrix and the right-handed neutrino masses, we can predict the LFV decay rate, for a given set of universal boundary conditions and \(\tan \beta\).

Recent various cosmological observations, in particular, the Wilkinson Microwave Anisotropy Probe (WMAP) satellite [9], have established the ΛCDM cosmological model.
with a great accuracy. The relic abundance of the cold dark matter (CDM) in 2σ range has been measured as

$$\Omega_{CDM} h^2 = 0.1131 \pm 0.0034.$$  \hspace{1cm} (10)$$

As is well-known, in SUSY models with the R-parity conservation, a neutralino, if it is the lightest sparticle (LSP), is a promising candidate for the CDM in the present universe. When we apply the WMAP result to the relic density of the neutralino LSP, the parameter space of the CMSSM is dramatically reduced into the narrow stripe due to the great accuracy of the WMAP data [10]. We take into account this cosmological constraint in our analysis of the LFV processes below. For $\tan \beta = 45$, $\mu > 0$ and $A_0 = 0$, for example, we can find the approximate relation between $m_0$ and $M_{1/2}$ such as

$$\left( \frac{m_0}{1 \text{ GeV}} \right) = 125.3 + 0.329 \left( \frac{M_{1/2}}{1 \text{ GeV}} \right) + 5 \times 10^{-5} \left( \frac{M_{1/2}}{1 \text{ GeV}} \right)^2,$$  \hspace{1cm} (11)$$

along which the observed relic abundance, $\Omega_{CDM} h^2 = 0.113$, is realized. In our analysis, we have employed the SOFTSUSY 3.1.4 package [11] to solve the MSSM RGEs and produce mass spectrum. Then, the relic abundance of the neutralino LSP is calculated by using the micrOMEGAs 2.4 [12] with the output of SOFTSUSY in the SLHA format [13].

Now let us consider two example models in the class of predictive SO(10) models. The first example is the minimal SUSY SO(10) model analyzed in [7], in which only one $10$ and one $126$ Higgs multiplets have Yukawa couplings with $16$ matter multiplets such as

$$W_Y = Y^{ij}_{10} 16_i H_{10} 16_j + Y^{ij}_{126} 16_i H_{126} 16_j,$$  \hspace{1cm} (12)$$

where $16_i$ is the matter multiplet of the $i$-th generation, $H_{10}$ and $H_{126}$ are the Higgs multiplet of $10$ and $126$ representations under SO(10), respectively. By virtue of the gauge symmetry, the Yukawa couplings, $Y_{10}$ and $Y_{126}$, are, in general, complex symmetric $3 \times 3$ matrices.

We assume some appropriate Higgs multiplets, whose vacuum expectation values (VEVs) correctly break the SO(10) gauge symmetry into the Standard Model one at the GUT scale, $M_{GUT} \simeq 2 \times 10^{16}$ GeV. Note that $H_{10}$ and $H_{126}$, respectively, includes one pair of up-type and down-type Higgs doublets in the same representation as the pair in the MSSM. Suppose that the Higgs doublets in the MSSM are realized as one light linear combination of the two pairs of Higgs doublets and the other combination is as heavy as the GUT scale, realizing the MSSM as the low energy effective theory. The Higgs doublet in the representation $(\overline{10}, 1, 3)$ under the Pati-Salam subgroup, $G_{422} = \text{SU}(4)_c \times \text{SU}(2)_L \times \text{SU}(2)_R$, is embedded in the $126$ representation, whose nonzero VEV breaks the Pati-Salam subgroup to the SM one and at the same time, generates the masses of the right-handed neutrinos. Providing suitable VEVs for the Higgs multiplets, the quark and
lepton mass matrices are characterized by only two basic mass matrices, \( M_{10} \) and \( M_{126} \), and three complex coefficients, \( c_{10} \), \( c_{126} \) and \( c_R \), at the GUT scale:

\[
\begin{align*}
M_u &= c_{10}M_{10} + c_{126}M_{126} \\
M_d &= M_{10} + M_{126} \\
M_D &= c_{10}M_{10} - 3 c_{126}M_{126} \\
M_e &= M_{10} - 3 M_{126} \\
M_R &= c_R M_{126},
\end{align*}
\]

where \( M_u, M_d, M_D, M_e, \) and \( M_R \) denote the up-type quark, down-type quark, neutrino Dirac, charged-lepton, and right-handed neutrino Majorana mass matrices, respectively. Except for \( c_R \), which is used to determine the overall neutrino mass scale, this system has fourteen free parameters in total \[14\], and the strong predictability to the fermion mass matrices.

In Ref. \[7\], thirteen electroweak data of six quark masses, three mixing angles and one phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix, and three charged-lepton masses are extrapolated to the GUT scale according to the RGEs with a given \( \tan \beta \), and are used as inputs at the GUT scale. Solving the GUT mass matrix relation among quarks and charged-leptons obtained by Eq. (13), we can describe the neutrino Dirac mass matrix \( M_D \) and \( M_{126} \) as functions of only one free parameter \( \sigma \), the phase of the combination \((3_{10} + c_{126})/(−c_{10} + c_{126})\). It has been shown \[7\] that the appropriate value of \( \sigma \) and \( c_R \) can reproduce the light neutrino mass matrix consistent with the observed neutrino oscillation data (as of 2002). For \( \tan \beta = 45 \) and \( \sigma = 3.198 \) fixed, the right-handed Majorana neutrino mass eigenvalues are found to be (in GeV) \[7\]

\[
M_{R_1} = 1.64 \times 10^{11}, \quad M_{R_2} = 2.50 \times 10^{12}, \quad M_{R_3} = 8.22 \times 10^{12},
\]

where \( c_R \) is fixed so that \( \Delta m_{32}^2 = 2 \times 10^{-3} \text{eV}^2 \). In the basis where both of the charged-lepton and right-handed Majorana neutrino mass matrices are diagonal with real and positive eigenvalues, the neutrino Dirac Yukawa coupling matrix at the GUT scale is unambiguously determined and explicitly given by

\[
Y_\nu = \begin{pmatrix}
-0.000135 - 0.00273i & 0.00113 + 0.0136i & 0.0339 + 0.0580i \\
0.00759 + 0.0119i & -0.0270 - 0.00419i & -0.272 - 0.175i \\
-0.0280 + 0.00397i & 0.0635 - 0.0119i & 0.491 - 0.526i
\end{pmatrix}.
\]

Although we will use the data in Eqs. (14) and (15) for our numerical analysis below, the neutrino oscillation parameters given by these equations are, unfortunately, more than 3\( \sigma \) away from the current neutrino oscillation data \[13\]. Some extension of the minimal SO(10) model is necessary to improve the data fitting for the neutrino oscillation parameters \[16\]. However, the resultant neutrino Dirac Yukawa couplings and the right-handed neutrino mass spectrum are not so much changed by this improvement, and we
believe that our result for the LFV process in this paper gives, at least, the correct order of magnitude for the prediction of a slightly extended minimal SUSY SO(10) model.

As another example of predictive SO(10) models, we consider a simple SUSY SO(10) GUT in five dimensions (5D) proposed in [17], which can ameliorate several theoretical problems in the minimal (renormalizable) SUSY SO(10) model (see [17] for detailed discussions on the theoretical problems of the minimal SUSY SO(10) model). This model is defined in five dimensions with the fifth dimension compactified on the $S^1/(Z_2 \times Z_2')$ orbifold [18]. The SO(10) gauge symmetry in five dimensions is broken by the orbifold boundary conditions to the Pati-Salam (PS) symmetry $SU(4)_c \times SU(2)_L \times SU(2)_R$. All matter and Higgs multiplets are arranged to reside only on a brane (PS brane) at one orbifold fixed point where the PS gauge symmetry is manifest, so that low energy effective four dimensional description of this model is nothing but the PS model with a special set of matter and Higgs multiplets. The PS symmetry is broken at the normal GUT scale, $M_{GUT} \simeq 2 \times 10^{16}$ GeV, in usual four dimensional manner, while the full gauge coupling unification is realized after incorporating threshold corrections of Kaluza-Klein modes in the bulk gauge multiplets. Phenomenology of sparticles [19] and applications of the model to cosmology such as inflation [20] and baryogenesis via leptogenesis [21] have been investigated.

In this model with a simple set of Higgs multiplets, the structure of fermion Yukawa couplings are almost the same as the one in the minimal SUSY SO(10) model. Assuming appropriate VEVs for Higgs multiplets, fermion mass matrices are obtained as (see [17] for the detailed definition of the mass matrices and the coefficients)

$$
M_u = c_{10}M_{1.2.2} + c_{15}M_{15.2.2}, \\
M_d = M_{1.2.2} + M_{15.2.2}, \\
M_D = c_{10}M_{1.2.2} - 3c_{15}M_{15.2.2}, \\
M_e = M_{1.2.2} - 3M_{15.2.2}, \\
M_R = c_RM_{10.1.3},
$$

(16)

which are characterized by three fundamental mass matrices $M_{1.2.2}$, $M_{15.2.2}$ and $M_{10.1.3}$ and complex coefficients $c_{10}$, $c_{15}$ and $c_R$. Comparing these expressions with Eq. (13), we can see that the combination of two mass matrices of $M_{1.2.2}$ and $M_{15.2.2}$ among $M_u, M_d, M_D$, and $M_e$ is the same as that of $M_{10}$ and $M_{126}$ in the minimal SUSY SO(10) model and, therefore, the procedure for fitting the realistic Dirac fermion mass matrices is the same as in the minimal SO(10) model. On the other hand, $M_R$ is fully independent on the above four Dirac fermion mass matrices, whereas in the minimal SUSY SO(10) model it is described by $M_{126}$ and not independent. This fact enables us to improve the data fitting of the neutrino oscillation parameters.

Through the seesaw mechanism [5], the light neutrino mass matrix is given by

$$
m_\nu = Y_\nu^T M_R^{-1}Y_\nu v_u^2 = U_{MNS}D_\nu U_{MNS}^T
$$

(17)
in the basis where the mass matrix of charged lepton is diagonal with real and positive eigenvalues. Here $v_u$ is the VEV of the up-type Higgs doublet in the MSSM, $D_\nu$ is the diagonal mass matrix of light neutrinos, and $U_{MNS}$ is neutrino mixing matrix. This is equivalent to the expression of the right-handed neutrino mass matrix as

$$M_R = v_u^2 \left( Y_\nu U_{TBM}^* D_\nu^{-1} U_{TBM}^T Y_\nu^T \right). \quad (18)$$

Once the information of the Dirac Yukawa coupling, the mass spectrum of the light neutrinos, and the neutrino mixing matrix is obtained, we can fix the right-handed neutrino mass matrix. In order to determine $M_R$, we follow the manner in [21].

We consider the normal hierarchical case for the light neutrino mass spectrum, for simplicity, and describe $D_\nu$ in terms of the lightest mass eigenvalue $m_1$ and the mass squared differences:

$$D_\nu = \text{diag} \left( m_1, \sqrt{\Delta m_{12}^2 + m_1^2}, \sqrt{\Delta m_{13}^2 + m_1^2} \right). \quad (19)$$

Here we adopted the neutrino oscillation data [15]:

$$\Delta m_{12}^2 = 7.59 \times 10^{-5} \text{ eV}^2, \quad \Delta m_{13}^2 = 2.43 \times 10^{-3} \text{ eV}^2 \quad (20)$$

In addition, we assume the neutrino mixing matrix of the so-called tri-bimaximal form [22]

$$U_{TBM} = \begin{pmatrix}
\sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\
-\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\
-\sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}}
\end{pmatrix}, \quad (21)$$

which is in very good agreement with the current best fit values of the neutrino oscillation data [15]. As has been discussed above, the data fit for the realistic charged fermion mass matrices is the same as in the minimal SO(10) model, and we here use the numerical value of $Y_\nu$ in Eq. (15) at the GUT scale for $\tan \beta = 45$, in the basis where the charged lepton mass matrix is diagonal. Substituting Eqs. (15), (19), (20), and (21) into Eq. (18), we obtain the right-handed neutrino mass matrix as a function of only $m_1$. Since it has been shown [21] that the simple 5D SO(10) model can reproduce the observed baryon asymmetry of the present universe for $m_1 \simeq 1.8 \times 10^{-3} \text{ eV}$ through non-thermal leptogenesis, we take this value as a reference value for $m_1$. In this way, $M_R$ is now completely determined, but not yet diagonalized. Changing the basis to diagonalize $M_R$, we find the neutrino Dirac Yukawa matrix,

$$Y_\nu = \begin{pmatrix}
-0.00119 + 0.0000268i & -0.00108 - 0.000485i & -0.000392 + 0.000421i \\
0.00135 + 0.00167i & -0.0253 + 0.00154i & 0.0237 + 0.000851i \\
-0.0265 - 0.0173i & 0.0609 + 0.0275i & 0.790 - 0.0436i
\end{pmatrix}, \quad (22)$$
and the diagonal right-handed neutrino mass matrix with eigenvalues (in GeV)

\[ M_{R_1} = 1.03 \times 10^{10}, \quad M_{R_2} = 7.55 \times 10^{11}, \quad M_{R_3} = 3.22 \times 10^{15}. \] (23)

Now we are ready to analyze the $\mu \rightarrow e\gamma$ decay rate by using the completely determined neutrino Dirac Yukawa coupling matrix and the right-handed neutrino mass eigenvalues \[^{1}\] Eqs. (14) and (15) for the minimal SO(10) model while Eqs. (22) and (23) for the simple 5D SO(10) model. For $\tan\beta = 45$, $A_0 = 0$ and $\mu > 0$, the branching ratio of $\mu \rightarrow e\gamma$ for the minimal SO(10) model is shown in Fig. 1 as a function of the universal gaugino mass at the GUT scale, along the relation of Eq. (11) to satisfy the observed relic density for the neutralino dark matter. The short dashed line corresponds to the branching ratio of Eq. (11) obtained from the MEG results if the three events are considered as a positive signal, while the long-dashed line to the upper limit of the branching ratio set by the MEGA experiments. The universal gaugino mass $M_{1/2} \simeq 790$ GeV (which corresponds to $m_0 \simeq 415$ GeV) reproduces the MEG results.

Fig. 2 depicts the same result as in Fig. 1, but for the simple 5D SO(10) model. In this case, the universal gaugino mass reproducing $\text{BR}(\mu \rightarrow e\gamma) = 3 \times 10^{-12}$ is relatively light, $M_{1/2} \simeq 420$ GeV (which corresponds to $m_0 \simeq 272$ GeV). This is because the components in the neutrino Dirac Yukawa matrix relevant to the LFV between 1st and 2nd generations are smaller than those in the minimal SO(10) model and a lighter sparticle mass spectrum is necessary in order to achieve the same branching ratio (see Eq. (9)).

For each model, we have pinned down the soft SUSY breaking mass parameters in the CMSSM, in the light of the MEG results. The set of the CMSSM parameters proposes a good benchmark point for the SUSY search at the LHC, and we present (sparticle) mass spectra for each model in Table 1, along with other observables which can be compared with the current experimental bounds. In both results, the lower bound on the Higgs boson mass $m_h \geq 114.4$ GeV \[^{2}\] is satisfied. Other phenomenological constraints we consider here are

\begin{align*}
2.85 \times 10^{-4} \leq \text{BR}(b \rightarrow s\gamma) & \leq 4.24 \times 10^{-4} \quad (2\sigma) \quad \text{(24)} \\
\text{BR}(B_s \rightarrow \mu^+\mu^-) & < 5.8 \times 10^{-8} \quad \text{(26)}
\end{align*}

The results for the minimal SO(10) model satisfy these constraints, while $\text{BR}(b \rightarrow s\gamma)$ for the simple 5D SO(10) model is marginal (about $3.4\sigma$ away from the center value).

The muon anomalous magnetic dipole moment (muon aMDM) has been measured in a great precision as \[^{27}\]

\[ a_\mu^{\text{exp}} = 11659208.0(6.3) \times 10^{-10}, \] (26)

where the number in parentheses shows $1\sigma$ uncertainty. On the other hand, the SM predictions were calculated \[^{28}\] (see also \[^{29,30,31,32,33}\]),

\[ a_\mu^{\text{SM}} = 11659193.2(5.2) \times 10^{-10}, \]

\[^{1}\] See \[^{23}\] for previous analysis on the minimal SO(10) model with more general parameter sets.
\[ a_{\mu}^{\text{SM}}[e^+e^-] = 11659177.7(5.1) \times 10^{-10}, \] (27)

by using data in the hadronic \( \tau \) decay and \( e^+e^- \) annihilation to hadron, respectively, in calculating the hadronic contributions to the muon aMDM, \( a_{\mu}^{\text{SM}}[\tau] \) and \( a_{\mu}^{\text{SM}}[e^+e^-] \). The deviations of the SM predictions from the experimental result are given by

\[
\Delta a_{\mu}[\tau] \equiv a_{\mu}^{\exp} - a_{\mu}^{\text{SM}}[\tau] = 14.8(8.2) \times 10^{-10},
\]

\[
\Delta a_{\mu}[e^+e^-] \equiv a_{\mu}^{\exp} - a_{\mu}^{\text{SM}}[e^+e^-] = 30.3(8.1) \times 10^{-10},
\] (28)

which correspond to 1.8\( \sigma \) and 3.7\( \sigma \) deviations, respectively. These deviations may be from the sparticle contributions. Table 1 also include the sparticle contributions to \( \Delta a_{\mu} \). We can see that the result for the minimal 5D SO(10) model favors the deviation obtained by using the data of the hadronic \( \tau \) decay, while the deviation from the \( e^+e^- \) data is favored by the result for the simple 5D SO(10) model.

With \( M_{1/2} \) (and \( m_0 \)) determined in each SO(10) model, we can also predict the LFV decay rates of tau lepton in the same analysis as for \( \mu \to e\gamma \). For the minimal SO(10) model, we find

\[
\text{BR}(\tau \to \mu\gamma) \simeq 5.74 \times 10^{-10} \quad \text{and} \quad \text{BR}(\tau \to e\gamma) \simeq 1.32 \times 10^{-10},
\] (29)

while for the simple 5D SO(10) model

\[
\text{BR}(\tau \to \mu\gamma) \simeq 5.53 \times 10^{-10} \quad \text{and} \quad \text{BR}(\tau \to e\gamma) \simeq 1.34 \times 10^{-10}.
\] (30)

These predicted values are well below the current experimental upper bounds \([34]\), \text{BR}(\tau \to \mu\gamma) \leq 4.4 \times 10^{-8} \quad \text{and} \quad \text{BR}(\tau \to e\gamma) \leq 3.3 \times 10^{-8}. \) Note that improvements by one or two orders of magnitude are expected at future super B-factories \([35]\) and the branching ratio of order \( 10^{-10} \) can be tested in the near future.

In summary, recent MEG results of a search for the \( \mu \to e\gamma \) decay might show a sign of a certain new physics beyond the Standard Model. The primary candidate for new physics which can give the rate of the LFV decay accessible to the MEG experiment is supersymmetric models. A certain class of SUSY SO(10) GUT models (predictive SO(10) models) can unambiguously determine fermion Yukawa coupling matrices, in particular, the neutrino Dirac Yukawa matrix and the mass spectrum of the right-handed neutrinos. As an example of such models, we have considered the minimal SO(10) model and a simple 5D SO(10) model. The effective theory of the SO(10) models below the GUT scale is described as the MSSM with the right-handed neutrinos. In these models, even if the universal boundary conditions for the soft SUSY breaking parameters at the GUT scale, LFV terms are induced in the sparticle sector through the RGE effects with the neutrino Dirac Yukawa matrix, which give rise to a sizable rate of the \( \mu \to e\gamma \) decay. Since the neutrino Dirac Yukawa matrix and the mass spectrum of the right-handed neutrinos are completely determined, the LFV rate can be predicted once the set of input parameters
in the CMSSM is given. Requiring to reproduce $\text{BR}(\mu \rightarrow e\gamma) = 3 \times 10^{-12}$ given by the MEG results and combining this requirement with the cosmological constraint on the observed relic abundance of neutralino dark matter, we have pinned down the parameter set in the CMSSM and then, have proposed benchmark points for each SO(10) model. We have checked other phenomenological constraints for each benchmark point. Finally, our strategy in this paper is applicable also to other SUSY (GUT) models if they can completely determine the neutrino Dirac Yukawa matrix and the right-handed neutrino mass spectrum. Depending on models, different benchmark points can be pinned down, which will be tested at the LHC.

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Figure 1: The branching ratio BR(\(\mu \to e\gamma\)) for the minimal SO(10) model as a function of \(M_{1/2}\) (GeV) along the cosmological constraint of Eq. (11). The short dashed line corresponds to the MEG result, BR(\(\mu \to e\gamma\)) = \(3 \times 10^{-12}\) when three evens are considered as a positive signal, while the long dashed line to the upper limit on BR(\(\mu \to e\gamma\)) = \(1.2 \times 10^{-11}\) given by the MEGA experiment.

Figure 2: The same figure as Fig. 1 but for the simple 5D SO(10) model.
Table 1: Mass spectra (in GeV) and phenomenological constraints for the two SUSY SO(10) models with the universal boundary conditions in the CMSSM

|                  | minimal SO(10) model | simple 5D SO(10) model |
|------------------|----------------------|------------------------|
| $m_0$            | 415                  | 272                    |
| $M_{1/2}$        | 790                  | 420                    |
| $A_0$            | 0                    | 0                      |
| $\tan \beta$    | 45                   | 45                     |
| $h_0$            | 119                  | 115                    |
| $H_0$            | 786                  | 449                    |
| $A_0$            | 787                  | 449                    |
| $H^\pm$          | 791                  | 457                    |
| $\tilde{g}$      | 1756                 | 981                    |
| $\tilde{\chi}_0^{0,1,2,3,4}$ | 333,631,928,938  | 171,324,535,548            |
| $\tilde{\chi}_1^{\pm}$           | 631,938               | 324,548               |
| $d, \tilde{s}_{R,L}$   | 1576,1645            | 898,934               |
| $\tilde{u}, \tilde{c}_{R,L}$  | 1582,1643            | 901,931               |
| $\tilde{b}_{1,2}$   | 1409,1473            | 784,849               |
| $\tilde{t}_{1,2}$   | 1266,1475            | 698,864               |
| $\nu_{e,\mu,\tau}$ | 667,667,619          | 386,386,355           |
| $\tilde{c}, \tilde{\mu}_{R,L}$ | 511,672             | 317,395               |
| $\tilde{\tau}_{1,2}$ | 342,642              | 186,392               |
| $\text{BR}(b \to s\gamma)$ | $3.27 \times 10^{-4}$ | $2.36 \times 10^{-4}$ |
| $\text{BR}(B_s \to \mu^+\mu^-)$ | $1.04 \times 10^{-8}$ | $4.95 \times 10^{-9}$ |
| $\Delta a_{\mu}$    | $12.0 \times 10^{-10}$ | $37.7 \times 10^{-10}$ |
| $\Omega h^2$      |                      | 0.113                  |