Research on the peak prediction and intelligent computing of reversal error for the tilt feed system installed on the precision machine tools

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Abstract. To effectively predict the peak of reversal error of tilt feed system and reduce reversal error caused by friction and gravity components, a peak prediction method of reversal error for tilt feed system on the precision NC machine tool is proposed. According to the load, tilt angle, motion trajectory, maximum static friction torque and relevant dynamic characteristic information, the peak prediction formula of the reversal error for the tilt feed system is established by mathematical derivation based on the kinematics, dynamics and torque balance during the process of reversal. Thus, the peak of reversal error for the tilt feed system can be obtained. The experimental results show that this method can achieve a good prediction effect, and can predict the peak of reversal error before the machining. It provides a theoretical basis for the reversal error suppression.

Keywords: reversal error; feed system; motion error; peak prediction; machine tools

1. Introduction
The tilt feed system is often used in CNC machine tools[1]-[2]. Compared with horizontal feed system, it has the advantages of compact structural design and high structural stiffness[3]-[5]. However, its movement will be adversely affected by external gravity components[6]. Reversal error is the motion error of a feed system during the reversal[7], [8]. Different from the reversal error of a horizontal feed system, the reversal error of a tilt feed system is determined by reverse backlash, geometric error caused by assembly and manufacturing, external gravity component and nonlinear friction force[9]. However, the full closed-loop tilt feed system of precision NC machine tools, the backlash and the geometric error caused by assembly manufacturing are relatively small. Thus, these motion errors can be ignored. Due to the friction between the dynamic and static contact surfaces of the feed system changes nonlinearly during the reversal, the traditional servo motion controller established by PID control structure can not effectively suppress the friction force, resulting in friction error[10]-[12]. The friction error adversely affects contour accuracy[13], [14]. The tilt feed system will also be affected by the external gravity component during the process of reversal. These combined activities will lead to the reversal error, which seriously affects the motion accuracy[15].

To overcome the external gravity component for the tilt feed system, a gravity component compensation method for the NC machine tools is often used to counteract the negative impact of the
initial external gravity component. However, there are often large changes of the load (such as adding or replacing the cutter and power head) during the machining process, which lead to the surplus of non counteract gravity component[16]. It and the friction force cause large spike of reversal error and seriously affects the motion accuracy of tilt feed system[17], [18]. Therefore, before the actual machining, there is an urgent demand that can accurately predict the peak of reversal error. It is very important to research the reversal error, reduce machining error and ensure machining quality [21], [22].

A peak prediction method of reversal error for the tilt feed system is proposed in this paper and the peak prediction formula is established. The experimental results show that the peak prediction method of reversal error for the tilt feed system on the precision NC machine tool is verified and effective.

2. Peak prediction of reversal error

The initial load mass and tilt angle of the feed system on the NC machine tools can be obtained by the NC machine tool product manual. The initial gravity component torque of the tilt feed system can be calculated. It can be expressed as:

$$T_{g0} = M_g g \sin \theta r_g$$

(1)

Where g and rg are gravitational acceleration and transmission ratio, respectively. Input the initial gravity component torque to the corresponding parameters of the gravity compensation (GCP) on the NC system, and set the GCP to take effect to counteract the gravity component torque. The torque of the servo motor can be expressed as:

$$T_m = T_{ce} + T_{g0}$$

(2)

Where is Tce the torque generated by the servo controller. The servo motor torque Tm is generated by the torque control variable u, which can be expressed as:

$$T_m = u(t) K_t$$

(3)

Where Kt is the torque constant and it can be obtained from servo driver manual. The torque control variable u can be expressed as:

$$u = u_{ce} + u_{g0}$$

(4)

Where uce is the torque control variable of the servo controller. u0g is the torque variable which counteracts the gravity component torque, and it can be expressed as:

$$u_{ce} = \frac{T_{ce}}{K_t}, u_{g0} = \frac{T_{g0}}{K_t}$$

(5)

The trajectory command x, motion trajectory speed Vr and motion trajectory acceleration Ar of the tilt feed system can be obtained by G code. The servo control parameters of the tilt feed system can be obtained by the operation setting interface as follows: position loop ratio gain Kpp, speed loop proportional gain Kvp, speed loop integral gain Kv, speed feedforward coefficient KvF, acceleration
feedforward coefficient $K_{AF}$, speed loop sampling period $T$. The control system of tilt feed system is shown in Figure 1.

![Figure 1. Servo control system diagram](image)

The tilt feed system completes the movement of multiple speeds range from 0 to 2mm/s, and the speed interval is 0.2mm/s. The servo motor torque control variable $u$ is sampled during the movement process. The data in acceleration and deceleration sections are removed. The average $u_{a}$ of torque control variable $u$ in constant speed section. Moreover, the servo motor average torque $T_{m,a}$ is obtained. Based on the principle of torque balance, the friction torque $T_{f}$ of positive and negative motion directions can be expressed as:

$$T_f = T_{m,a} \cdot g \cdot T_{g0} \Rightarrow T_f = u_{a} \cdot K_{f} \cdot T_{g0}$$  \hspace{1cm} (6)

Finally, the corresponding relationship between the friction torque $T_{f}$ and the speed under each constant speed condition can be obtained. Through the variation trend of the friction torque $T_{f}$ under the moving speed of 0.2mm/s and 0.4mm/s, the maximum static friction torque $T_{fs}$ in the positive and negative moving directions at zero speed can be obtained. If a tool or power head needs to be added during the processing of the NC machine tool, resulting in the load mass of the NC machine tool inclined feed system becoming, the gravity component torque can be expressed as:

$$T_{g1} = M_{i} \cdot g \cdot \sin \theta \cdot r_{g}$$  \hspace{1cm} (7)

The torque control variable $u$ of servo motor is sampled during the reversal. The servo motor torque $T_{m}$ can be obtained. Combined with static friction torque $T_{fs}$ in the positive and negative moving directions and based on the principle of torque balance, the following formula is satisfied:

$$T_{m} = T_{g1} + T_{fs} \Rightarrow T_{ce} + T_{g0} = T_{g1} + T_{fs} \Rightarrow T_{ce} = T_{g1} + T_{g0} + T_{fs} \Rightarrow T_{ce} = \Delta T_{g} + T_{fs}$$  \hspace{1cm} (8)

Where $\Delta T_{g}$ is margin of unmatched gravity component after the gravity compensation of CNC system takes effect. According to the kinematics of the reversing process and assuming that the worktable of the tilt feed system reaches the reversing position at the $t_{0}$ moment, starts sliding at the $t_{s}$ moment and reaches the peak of reversal error at the $t_{e}$ moment, the peak $e_{p}$ of reversal error for the tilt feed system can be expressed as:
\[ e_p = e_r (t_e) = \int_0^t (v_r - v) \, dt + \int_{t_e}^t (v_r - v) \, dt + \int_{t_e}^t (v_r - v) \, dt = \]

\[ x_e (t_e) - x(t_e) + \int_{t_e}^t (v_r - v) \, dt + \Delta e + D_o \]

(9)

Where \( v \) is the actual speed of the worktable; \( \Delta e \) is the error caused by measurement. \( D_o \) is the elastic constraint. The value of \( \Delta e \) and \( D_o \) are both small and can be ignored. From \( t_0 \) moment to \( t_s \) moment, it can be considered that the feedback position \( x \) basically remains unchanged, and its value is approximately \( x(t_0), x(t) \approx x(t_0), t \in [t_0, t_s] \). Thus, the peak of reversal error can be expressed as:

\[ e_p = x_r (t_s) - x(t_0) + \int_{t_e}^t (v_r - v) \, dt + \Delta e + D_o \approx x_r (t_s) - x(t_0) + \int_{t_e}^t (v_r - v) \, dt \]

(10)

And, the equation (10), the peak of reversal error can be further expressed as:

\[ e_r (t) \approx x_r (t) - x_r (t_0), t \in [t_0, t_s] \]

(11)

The duration from the sliding time \( t_s \) moment to the peak time \( t_e \) moment is relatively small, so it can be considered that the velocity change curvature is changes linearly. At the \( t_s \) moment, the worktable speed of the tilt feed system changes from zero. At the \( t_s \) moment, \( v_r = v \), the reversal error reaches the maximum peak, so as to form a triangle with bottom \( v_r (t_s) \) and height \( (t_s - t_0) \). The triangle area is the reversal error during the \( (t_s - t_0) \) period. Therefore,

\[ \int_{t_0}^{t_s} (v_r - v) \, dt \approx \frac{1}{2} v_r (t_s) (t_s - t_0) \]

(12)

Substituting equation (12) into equation (10):

\[ e_p \approx x_r (t_s) - x_r (t_0) + \int_{t_0}^{t_s} (v_r - v) \, dt \approx x_r (t_s) - x_r (t_0) + \frac{1}{2} v_r (t_s) (t_s - t_0) \]

(13)

Set \( (t_s - t_0) = \lambda (t_s - t_0) \), where \( \lambda \) is coefficient of response time and its value is determined by dynamic performance and the trajectory command \( x_r \). The value of response time coefficient \( \lambda \) is generally set as 0.2. The transition time \( T_b \) is the interval between the reversal moment \( t_0 \) and the sliding moment \( t_s, T_b = (t_s - t_0) \). Therefore, the equation (13) can be written as:

\[ e_p \approx x_r (t_s) - x_r (t_0) + \frac{1}{2} v_r (t_s) (t_s - t_0) \approx x_r (t_s) - x_r (t_0) + \frac{1}{2} v_r (t_s) T_b = \]

\[ x_r (t_0 + T_b) - x_r (t_0) + 0.5 \lambda v_r (t_0 + T_b) T_b \]

(14)

Where the peak prediction formula of the reversal error for the tilt feed system is established. From the equation (14), the key to predict the peak of reversal error is to calculate the transition time \( T_b \). The control system of the tilt feed system is a discrete control system. Let the worktable reach the reversing position at the \( iTh \) moment, that is, \( t_0 = iT \). The worktable starts sliding at the \( t_i \) moment, \( t_i = (i + N)T, T_b = N \cdot T, \) where \( N \) is the number of iterations. Under the condition of worktable sliding, combined with equations (8) and (11), considering the influence of discrete equation step, the relevant equations of torque and error are approximately equal. The equation for solving the transition time \( T_b \) for the tilt feed system is established, as shown in the following formula:
\[
\begin{aligned}
&\left\{ \begin{array}{l}
T_c((i+N)T) \approx \Delta T_g + T_b \\
e((i+N)T) \approx x((i+N)T) - x(iT)
\end{array} \right. \\
&\text{Solving the transition time } T_b \text{ is to calculate the number of iterations } N. \text{ When the worktable reaches the reserving position at the } iT \text{ moment, before the } (i+N)T \text{ moment, the reversal error can be expressed as:}
\end{aligned}
\]
\[
e((i+N)T) \approx x((i+N)T) - x(iT)
\]

Where the initial number of iterations \( N \) is 1, that is, \( N = 1 \). At the moment \((i+N)T\), the speed loop speed command \( v_c \), actual motion speed \( v \) and speed loop error term \( e \) can be expressed as:
\[
\begin{aligned}
v_c((i+N)T) &= e((i+N)T)\|K_{pp} \\
v((i+N)T) &= 0 \\
e((i+N)T) &= v_f((i+N)T) + v_c((i+N)T) - v((i+N)T)
\end{aligned}
\]

Where \( v_f \) is the velocity feedforward output term, \( v_f = K_{pf} \cdot v_c \). At the \((i+N)T\) moment, the proportional gain term \( v_{pe} \) and the integral term \( v_{ie} \) of speed control loop can be expressed as:
\[
\begin{aligned}
v_{pe}((i+N)T) &= K_{pp} \cdot e((i+N)T) \\
v_{ie}((i+N)T) &= K_{in} \cdot T \cdot e((i+N)T) + v_{ie}((i+N-1)T)
\end{aligned}
\]

Where at the \( iT \) moment, \( v_{ie}(iT) = 0 \). At the \((i+N)T\) moment. The torque control variable of the servo controller \( u_{ce} \) can be expressed as:
\[
\begin{aligned}
u_{ce}((i+N)T) &= v_{pe}((i+N)T) + v_{ie}((i+N)T) + a_f((i+N)T)
\end{aligned}
\]

Where \( a_f \) is the acceleration feedforward output item, \( a_f = K_{af} \cdot a_c \). At the \((i+N)T\) moment. The torque generated by the servo controller \( T_{ce} \) can be expressed as:
\[
\begin{aligned}
T_{ce}((i+N)T) &= u_{ce}((i+N)T)\|K_t
\end{aligned}
\]

The number of iterations \( N \) is updated by \( N = N + 1 \). Repeat the process (16) to (20) until the approximate equation (20) is satisfied and the iterative algorithm ends. The number of iterations \( N \) can be obtained and it is the solution of transition time equation (15). Then the transition time \( T_b \) is obtained, \( T_b = N \cdot T \). Combined with the trajectory command \( x_c \), reversal moment \( T_b \), worktable command speed \( v_c \) and coefficient \( \lambda \) of response time, it is substituted into the peak prediction formula of the reversal error for the tilt feed system. Then the peak \( e_f \) of reversal error for the tilt feed system can be obtained.

3. Experiment verification

Experimental verification is carried out on a three-axis precision servo worktable. High-resolution scales were adopted on the platform to realize full closed-loop motion control. Its vertical Z-axis (tilt angle \( \theta = 90^\circ \)) is a typical tilt feed system, and the main parameters are as following: \( K_{pp} = 98.5 \text{s}^{-1} \), \( K_{pp} = 0.112 \text{V} \cdot \text{s/mm} \), \( K_v = 19.3 \text{V} \cdot \text{s/mm} \), \( K_{at} = 0.0013 \text{V} \cdot \text{s}^2/\text{mm} \), \( K_{vt} = 1 \text{V} \cdot \text{s}^2/\text{mm} \), \( K_{e} = 2.6875 \text{ N-m/V} \);
\( r_s = 2.5465 \text{ mm}/\text{rad}, T_s = 1 \text{ ms}, M_0 = 80 \text{ kg} \). Set the GCP to take effect to offset the gravity component of the Z-axis worktable. The servo motor torque control variable \( \mu \) is sampled during reversal at the multiple constant speed sections. The maximum static friction torque \( T_{fs} \) are \( T_{fs} = 1.0841 \text{ N·m} \) in the positive direction and \( T_{fs} = -1.2548 \text{ N·m} \) in the negative direction respectively. Adding a mass block 10\text{ kg} to the Z-axis worktable, the load mass of the Z-axis worktable becomes 90\text{ kg}. With amplitude \( R = 25 \text{ mm} \) and angular velocity \( \omega = 0.33 \text{ rad/s} \), a sinusoidal trajectory command \( x_r \) of Z-axis is shown in Figure 2, and there are two reversal processes of B and C.

![Figure 2. Sinusoidal trajectory command \( x_r \) of Z-axis](image)

The peak prediction of reversal error at C area is taken as an example to illustrate. It can be obtained from the trajectory command \( x_r \) that the reversal position was reached at the moment \( t_0, t_0 = 14.137 \text{ s} \). The transition time \( T_b \) was calculated by the above solution process, \( T_b = 0.086 \text{ s} \). Then, the value of transition time \( T_b \) was substituted into the peak prediction formula of the reversal error. The peak \( e_p \) of reversal error can be obtained as follows:

\[
e_p \approx x_r(t_0 + T_b) - x_r(t_0) + 0.5 \mu v(t_0 + T_b) T_b
\]

\[
= R \sin(\omega(t_0 + T_b)) - R \sin(\omega t_0) + 0.5 \mu R \rho \cos(\omega(t_0 + T_b)) T_b
\]

\[
= 25 \sin(0.33(14.137 + 0.086)) - 25 \sin(0.33(14.137)) + 0.5 \times 0.25 \times 0.33 \cos(0.33(14.137 + 0.086)) \times 0.086 \approx 0.0123 \text{ mm} = 12.3 \mu \text{ m}
\]

The reversal error at the C area during Z-axis movement is shown in Figure 3. The actual peak value of the reversal error at the C area is 11.2 \( \mu \text{ m} \). The predicted peak value of reversal error is 12.3 \( \mu \text{ m} \). The prediction deviation is 12.3-11.2 = 1.1 \( \mu \text{ m} \). The prediction deviation is caused by the integral approximation in the calculation process, and the value of response time coefficient \( \lambda \) is difficult to be accurate. However, the predicted deviation is smaller than the actual peak of reversal error, so the predicted peak of reversal error can truly reflect the actual counterpart, which can be used to effectively predict the peak of reversal error for tilt feed system.
4. Conclusions
In this paper, a peak prediction method of reversal error for tilt feed system installed on the precision NC machine tool is proposed. Experiments show that this method can predict the peak of reversal process. The prediction deviation is mainly caused by integral calculation in the solution process of transition time $T_b$, difficulty in accurately determining the value of response time coefficient $\lambda$, and deviation of control system model, etc. The predicted peak deviation of reversal error is only 1.1um, while the actual peak of reversal error at the C area is 11.2 $\mu$m which is far smaller than the actual peak of reversal error. It is illustrated that the predicted peak of reversal error can truly reflect the actual counterpart, and the peak prediction method of reversal error proposed is verified. Applying this method, the peak of reversal error under different working conditions can be predicted before machining, so as to lay a theoretical foundation to take actions such as regulating motion speed of the machining process in advance.

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