Inferences of $H_0$ in presence of a non-standard recombination

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Measurements of the Hubble parameter from the distance ladder are in tension with indirect measurements based on the cosmic microwave background (CMB) data and the inverse distance ladder measurements at 3-4 $\sigma$ level. We consider phenomenological modification to the timing and width of the recombination process and show that they can significantly affect this tension. This possibility is appealing, because such modification affects both the distance to the last scattering surface and the calibration of the baryon acoustic oscillations (BAO) ruler. Moreover, because only a very small fraction of the most energetic photons keep the early universe in the plasma state, it is possible that such modification could occur without affecting the energy density budget of the universe or being incompatible with the very tight limits on the departure from the black-body spectrum of CMB. In particular, we find that under this simplified model, with a conservative subset of Planck data alone, $H_0 = 73.44_{-6.77}^{+5.06}$ km s$^{-1}$ Mpc$^{-1}$ and in combination with BAO data $H_0 = 68.86_{-1.35}^{+1.51}$ km s$^{-1}$ Mpc$^{-1}$, decreasing the tension to $\sim 2\sigma$ level. However, when combined with Planck lensing reconstruction and high-$\ell$ polarization data, the tension climbs back to $\sim 2.7\sigma$, despite the uncertainty on non-ladder $H_0$ measurement more than doubling.

I. INTRODUCTION

The tension between the local measurement of the Hubble parameter $H_0$ based on distance ladder and indirectly derived values of Hubble parameter from higher redshift datasets is a particularly interesting tension, because both measurements seem to have passed many internal consistency checks (see e.g. Refs. [1–4] for reanalysis of the local distance ladder and Refs. [5–7] for Planck consistency) and would naturally point to a physics beyond the standard $\Lambda$CDM model [8].

The latest distance ladder measurements [9] observed the bright Cepheids in both optical and near-infrared to mitigate saturation as well as to reduce pixel-to-pixel calibration errors, and the distances of the Cepheids are measured by the Gaia DR2 parallaxes and HST photometry. This gives $H_0 = 73.52 \pm 1.62$ km s$^{-1}$ Mpc$^{-1}$ (we shall refer to this as Riess measurement), which is in tension with the Planck 2018 cosmic microwave background (CMB) data that give $H_0 = 67.36 \pm 0.54$ km s$^{-1}$ Mpc$^{-1}$ from all Planck measurements combined [10], under the assumption of $\Lambda$CDM model. More importantly, this tension is robust to a few obvious resolutions. First, as shown in several papers [11–16], it is robust to change of the CMB data: replacing Planck with a combination of WMAP and finer resolution experiments gives the same results. Second, it is robust to very-low redshift modifications to expansion history (caused, for example, by a rapidly evolving equation of state of the dark energy): supernovae Ia data can be used to translate the baryon acoustic oscillations (BAO) determination of Hubble parameter at $z \sim 0.5$ into a $z = 0$ measurement under rather general assumption about smoothness of the evolution of dark energy component, a method also known as an inverse distance ladder. Results from this method are competitive and consistent with $\Lambda$CDM determination using Planck alone. Third, it is also robust to removing CMB data altogether: calibrating BAO as a standard ruler assuming standard pre-recombination physics, but using just the big-bang nucleosynthesis (BBN) measurement of the baryon density. This does not relieve this tension very significantly either [17].

Therefore, any solution to this problem must involve two ingredients: i) it must modify the early universe calibration of the BAO ruler and ii) it must do so while maintaining the excruciatingly precise measurements of the CMB power spectrum. One possible solution would be a change in the radiation content of the early universe and/or adding massive sterile neutrinos. This improves the fit somehow, but does not resolve the tension, as the $N_{\text{eff}}$ required is inconsistent with the damping tail measurements of the CMB [18].

II. MODIFICATION TO RECOMBINATION

Here we propose an alternative solution: modifying the recombination history of the universe. Changing recombination would change the sound horizon scale of photon-baryon plasma. While the observed angle of the sound horizon at the photon decoupling, $\theta_s$, remains the same, different sound horizon requires a different angular diameter distance to the last scattering surface. Hence, the expansion history as well as the Hubble parameter will be influenced. In addition, using BAO as a standard ruler requires the determination of the comoving sound horizon at the end of the baryonic-drag epoch, $r_{\text{dr}}$, from CMB, so modifying the recombination would also affect the BAO constraint on the Hubble parameter.

We consider a general phenomenological model that modifies the timing and width of the recombination, and we apply the changes to CLASS [19]. Specifically, when CLASS computes the free electron fraction $\chi_e$ during recombination, we first numerically locate the redshift $z_{\text{rec}}^{0.5}$ such that $\chi_e(z_{\text{rec}}^{0.5}) = 0.5$. The value $z_{\text{rec}}^{0.5}$ is cosmology
dependent. We then introduce a mapping redshift
\[ \tilde{z} = z_{0.5}^{\text{rec}} + (z - z_{0.5}^{\text{rec}} - \delta z_{0.5}^{\text{rec}})/(1 + \Delta z_{0.5}^{\text{rec}}), \]
where \( \delta z_{0.5}^{\text{rec}} \) and \( \Delta z_{0.5}^{\text{rec}} \) parametrize respectively the shift in recombination timing and the change in recombination width. Finally, we compute the new free electron fraction \( \chi_e(\tilde{z}) = \chi_e(z) \) outside this range. By construction, for \( \delta z_{0.5}^{\text{rec}} = \Delta z_{0.5}^{\text{rec}} = 0 \), we have \( \tilde{z} = z \) and the standard recombination is recovered. In our convention, positive \( \delta z_{0.5}^{\text{rec}} \) corresponds to a later recombination timing, and positive \( \Delta z_{0.5}^{\text{rec}} \) corresponds to a narrower recombination width. The actual value of \( r_{\text{drag}} \) would depend on the other parameters. We vary the values for the redshift range for modifying the recombination history, and find that the parameter constraints using CMB and BAO data are insensitive to the choice, so we stick with the reported values. Figure 1 illustrates the free electron fraction for various recombination parameters for the Planck 2015 fiducial cosmology [20]. A similar exercise has been performed in Ref. [21], but without the shift parameter. Authors find that CMB data alone gives a tight constraint on the width of recombination process. Another related work has also been done in Ref. [16] to measure the sound horizon using the distance ladder calibration.

![FIG. 1. The free electron fraction for various values of recombination parameters \( \delta z_{0.5}^{\text{rec}} \) and \( \Delta z_{0.5}^{\text{rec}} \), where \( \delta z_{0.5}^{\text{rec}} \) shifts the recombination timing and \( \Delta z_{0.5}^{\text{rec}} \) changes the recombination width (see the main text for detailed description of our phenomenological model). In our convention, in terms of redshift, positive \( \delta z_{0.5}^{\text{rec}} \) corresponds to a later recombination timing, and positive \( \Delta z_{0.5}^{\text{rec}} \) corresponds to a narrower recombination width.](image)

### III. RESULTS

To study the impact of the modification to the recombination on the parameter constraints, we perform Markov chain Monte Carlo (MCMC) using MontePython [22, 23] with our modified version of CLASS. We apply the Gelman-Rubin convergence criteria [24] and confirm that the chains achieve \( R - 1 \leq 0.03 \) for all parameters of interest. We adopt a flat \( \Lambda \)CDM cosmology with the minimum neutrino mass, i.e. \( m_\nu = 0.06 \) eV and \( N_{\text{eff}} = 3.046 \), as well as the primordial Helium fraction consistent with BBN.

We use two choices of CMB datasets based on 2015 Planck data release [25]: Planck uses the conservative choice of likelihood consisting of Planck low-1 TTTEEE and Planck high-1 TT: Full Planck is more aggressive by replacing the high-\( \ell \) part with Planck high-1 TTTEEE and adding the lensing reconstruction power spectrum [26]. In other words, the Planck dataset refers to mainly the temperature power spectrum aided by the large-scale polarization to break certain degeneracies (most importantly with the optical depth). The Full Planck on the other hand uses the complete information available, including weak lensing reconstruction which sets the amplitude of matter fluctuations.

In addition to the CMB data, we consider the BAO measurement from the completed SDSS-III Baryon Oscillation Spectroscopic Survey [27], which we refer to as BAO. Specifically, the measurement is done with the anisotropic galaxy clustering with the reconstruction technique, and is the largest galaxy redshift survey sample to date hence offers the most stringent constraint. Since using BAO as a standard ruler requires the knowledge of \( r_{\text{drag}} \), modifying the recombination affects \( r_{\text{drag}} \) and will impact the parameter constraint from BAO.

Finally, we use the BBN prior from on the value of physical baryon density parameter \( \Omega_b h^2 \). We choose to measurements from Ref. [17, 28], giving a simple prior 100\( \Omega_b h^2 \) = 2.200 \( \pm 0.034 \). In the past, the BBN has been used to allow the inverse distance-ladder argument to be applied to the BAO data without CMB prior. However, in the modified recombination model, this will not work, since one would need to apply priors on not only \( \Omega_b h^2 \), but also to \( \delta z_{0.5}^{\text{rec}} \) and \( \Delta z_{0.5}^{\text{rec}} \) parameters. Instead, since these parameters are degenerate with \( \Omega_b h^2 \) in the case of CMB alone, we will apply this prior to CMB data, where it will have a regularizing function of bringing the model back closer to \( \Lambda \)CDM.

We have calculated MCMC chains for four possible combinations of the two Planck datasets with or without the BAO data and for the standard \( \Lambda \)CDM model and the modified recombination model. The modified recombination model has two more parameters than the standard \( \Lambda \)CDM model (\( \delta z_{0.5}^{\text{rec}} \) and \( \Delta z_{0.5}^{\text{rec}} \)).

We apply the BBN prior by importance sampling the chains without the BAO data. We show the full set of results in Table I.

This table contains the main results and we will start by highlighting some of the main conclusions from this data. Figure 2 displays the triangle plot for Planck data alone. We see that the basic picture holds: by relaxing the physical connection between recombination timing the constraints on the basic \( \Lambda \)CDM parameters relax.
TABLE I. 68% marginalized constraints for a subset of relevant cosmological parameters. $H_0^{\text{Riess}} = 73.52\pm 1.62$ km s$^{-1}$ Mpc$^{-1}$ is the mean value of distance ladder measurements from Ref. [9] and $\sigma_{\text{tot}}$ refers to errors from both measurement added in quadrature. $S_8$ is defined as $S_8 = \sigma_8 (\Omega_m/0.3)^{0.5}$ [29].

The relaxation is not complete, because a recombination occurring at a different redshift would result in a radically different power spectrum. Nevertheless, the parameter constraints inflate prominently. In particular, we see that the timing of recombination significantly affects the implied constraint on $r_{\text{drag}}$, the ruler size for BAO. Most importantly, the error on $H_0$ increases by a factor of 7 with a central value that is delightfully close to the Riess measurement. The main question is whether this survives the addition of other datasets.

In figure 3 we see that adding other data to Planck dataset helps bring some of the parameters back closer to their fiducial values. We find that BBN constraint helps in this case, but the errors on $H_0$ remain large. The BAO data, however, pull $H_0$ back to the fiducial value, albeit with a 1$\sigma$ shift in the central value and a factor of $\sim 2$ increase in error bars. The tension with the Riess measurement is relaxed by about one standard deviation and remains above 2$\sigma$.
The results on $H_0$ constraints are summarized in figure 4. The additional recombination parameters $\delta z_{0.5}^{\text{rec}}$ and $\Delta z^{\text{rec}}$ significantly enlarges the constraint compared to the standard recombination, making the $H_0$ measurement from the early universe more consistent with the distance ladder measurement. In particular, for the Planck data combinations the $H_0$ central values are shifted to a higher values, closer to the Riess measurement. However, adding Planck lensing reconstruction and high-$\ell$ polarization data pulls $H_0$ back to the values of standard recombination. Nevertheless, the increase of uncertainty still relieves the tension with the distance ladder measurement.

\section{Discussion}

In this paper we have shown that there exist a phenomenological expansion of the $\Lambda$CDM model that is capable of significantly decreasing the tension between distance ladder Hubble parameter measurements and indirect determinations relying on early universe physics. The main feature of this model is that it simultaneously affects the standard CMB parameter fitting, while also changing the BAO ruler calibration.

When considering the Planck data alone, the new parameters bring degeneracies that relieve the tension completely. However, the additional data, such as BAO data and the Full Planck data result in tension reappearing, but is now at $2 - 3\sigma$ level rather than $3 - 4\sigma$ level.

For completeness, we also calculate the effect on other parameters, such as the matter density and the lensing amplitude parameter $S_8$ and find that they are not significantly affected.

Importance sampling our most constrained parameter combination of Full Planck+BAO with the $H_0$ prior from the Riess measurement, we find the marginalized values of $\delta z_{0.5}^{\text{rec}} = -17.07^{+8.97}_{-9.91}$ and $\Delta z^{\text{rec}} = -0.0183^{+0.0100}_{-0.0111}$. This indicates that a higher $H_0$ values favors an earlier and wider recombination in terms of redshift, and the exact value of $r_{\text{drag}}$ will depend on the other parameters. The same qualitative trend is seen for using Planck dataset alone, but the constraints on the modified recombination parameters are statistically consistent with zero.

In this exploratory paper we do not propose a physical model that could implement this idea. Nevertheless, we make some relevant remarks. An important point is that due to the large over-abundance of photons compared to baryons, with the photo-to-baryon ratio of $n_\gamma/n_b \sim 6 \times 10^{-10}$, only a very small fraction of photons deep in the exponential suppression of the distribution participate in keeping the universe ionized up to recombination. These photons are today redshifted to frequencies $\nu \gtrsim 1.5 \text{ THz}$ and are therefore not constrained by the precise FIRAS measurement of CMB black-body spectrum $[30, 31]$. Second, while these photons an order of magnitude more energetic compared to typical CMB photons, they contribute negligibly to the energy density of radiation fluid simply because they are so few of them (by construction, their total number is suppressed by the baryon fraction). Therefore, it is entirely possible to have $O(1)$ modifications to the distribution of these photons, without disturbing the energetics of the universe at all. If the modification is the high-energy tail of the CMB spectrum, we have shown that this solutions requires fewer photons in the tail in order to recombine at a higher redshift. Future CMB spectral distortion observations can help determine the recombination physics more accurately, further testing the validity of resolving the $H_0$ tension by a non-standard recombination.

Finally, we conclude by saying that we did not obsess over the details of model comparison, because it is premature. We reiterate that this model is completely phenomenological and that a more concrete, physically based model could provide better fits for the union of data. Presumably, any model grounded in actual physics will have a somewhat more complicated properties that are not well described by a simple shift and stretch parameterization. Nonetheless, we have demonstrated that should the data continue to show an ever increasing tension in the two determinations of $H_0$, that modified recombination offers a possible reconciliation mechanism.
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