Chargino mass determination
at a muon collider

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Abstract

We analyze the prospects at a muon collider for measuring chargino masses in the $\mu^+\mu^- \rightarrow \tilde{\chi}^+\tilde{\chi}^-$ process in the threshold region. We find that for the lighter chargino of a mass $100 - 200$ GeV, a measurement better than $50 - 300$ MeV should be possible with $50$ fb$^{-1}$ integrated luminosity. The accuracy obtained here is better than with other techniques or at other facilities. The muon sneutrino mass, which enters through the $\tilde{\nu}_\mu$ exchange diagram, can also be simultaneously measured to a few GeV if it is not too heavy.
I. INTRODUCTION

Particle masses can be measured to high precision through threshold production cross sections at lepton colliders. This has been demonstrated at LEP II in $W$ pair production at $\sqrt{s} = 161$ GeV, just above $2M_W$. We have recently shown that future high-luminosity $e^+e^-$ and $\mu^+\mu^-$ colliders can measure the $W$ boson, top quark and Higgs boson masses at high precisions in the processes $\ell^+\ell^- \rightarrow W^+W^-, t\bar{t}, ZH$ \cite{1, 2}. Initial state radiation from muons is reduced compared to electrons, and muon colliders have negligible beamstrahlung which increasingly becomes a problem at linear electron colliders as the energy increases. Muon colliders thus could be very useful in precision measurements of particle masses, widths, and couplings \cite{3–7}.

In this paper we study the achievable accuracy in measuring the mass of the lighter chargino in the minimal supersymmetric standard model (MSSM) via the cross section for

$$\mu^+\mu^- \rightarrow \chi^+\chi^-$$ (1)

near the threshold. The measurement of the chargino mass via the threshold cross section has been considered previously for $e^+e^-$ machines in Ref. \cite{8, 9}. The narrower energy spread and the negligible beamstrahlung at a muon collider offer a distinct advantage over most electron-positron designs. We assume in this paper that the muon collider has a relatively modest (rms) beam energy spread of $R = 0.1\%$. We consider a measurement with high integrated luminosity (50 fb$^{-1}$), carefully taking into account beam smearing effects and optimization of cuts to eliminate the background in the threshold region.

A precision measurement of the chargino mass is a highly desirable goal to test patterns of supersymmetry breaking. For example the relationship between the lightest neutralino and the lighter chargino masses can be used to test the existence of a universal soft SUSY-breaking parameter. Renormalization Group Evolution (RGE) from the grand unification scale leads to the approximate prediction $m_{\tilde{\chi}^\pm} \simeq m_{\tilde{\chi}^0_2} \simeq 2m_{\tilde{\chi}^0_1}$ \cite{10}. The predictions for chargino pair production have recently been investigated beyond the tree-level \cite{11}. A pre-
cision measurement of the cross section can test radiative corrections coming from heavy squarks, since the corrections depend on \( \log(M_{\tilde{Q}}/m_{\tilde{\ell}}) \).

The cross section of the chargino pair production depends not only on \( m_{\tilde{\chi}^\pm} \) but also on the mass of the muon sneutrino \( (m_{\tilde{\nu}}) \) which enters through a \( t \)-channel diagram. As we show in Sec. II, a simultaneous measurement of both \( m_{\tilde{\chi}^\pm} \) and \( m_{\tilde{\nu}} \) is possible. In Sec. III, we compare our results with that achievable at an \( e^+e^- \) linear collider and with the kinematical endpoint technique. We also comment on the benefits with polarized muon beams in studying the chargino mass and properties.

**II. ACHIEVABLE ACCURACY IN \( m_{\tilde{\chi}^\pm} \)**

If the lighter chargino is gaugino-dominated as expected [12,13], then changing the parameters of the chargino mass matrix essentially changes the mass but does not significantly change its couplings. The chargino mass matrix is

\[
M_C = \begin{pmatrix}
M_2 & \sqrt{2} M_W \sin \beta \\
\sqrt{2} M_W \cos \beta & -\mu
\end{pmatrix},
\]

and in supergravity models the diagonal terms are expected to be larger than the off-diagonal ones. As a typical illustration, we choose the representative MSSM parameters

\[
M_2 = 120 \text{ GeV}, \quad \mu = 400 \text{ GeV}, \quad \tan \beta = 4,
\]

where \( M_2 \) is the gaugino mass parameter, \( \mu \) the Higgs mixing and \( \tan \beta = v_2/v_1 \) the ratio of the vevs of the two Higgs doublets in the MSSM. The choice of Eq. (3) is motivated by the “gaugino point” of Ref. [14], so that the lighter chargino is gaugino-like \( (M_2 < |\mu|) \). This choice corresponds to \( m_{\tilde{\chi}^\pm} = 123 \text{ GeV} \).

For the chargino pair production under discussion, the sneutrino contribution in the \( t \)-channel interferes destructively with the \( s \)-channel graphs. Therefore one can envision a measurement of the cross section that essentially depends on just two parameters, \( m_{\tilde{\chi}^\pm} \) and \( m_{\tilde{\nu}} \). Figure 1 illustrates the total cross sections versus the center-of-mass energy near
threshold for various values of sneutrino mass, with other parameters as in Eq. (3). The rapid rise of the cross section near threshold is due to the $S$-wave pair production of spin-1/2 particles with small decay widths. The cross section is typically of order 1 pb. Thus a large signal sample of order $5 \times 10^4$ chargino events would be obtained with the assumed collider luminosity.

A simultaneous measurement of the chargino and sneutrino masses requires a sampling of the cross section at least two points. As in other threshold measurements, the statistical precision on the chargino mass is maximized at CM energy $\sqrt{s}$ just above $2m_{\tilde{\chi}^{\pm}}$. However as is evident from Fig. 1, a change in the cross section at $\sqrt{s} = 2m_{\tilde{\chi}^{\pm}} + 1$ GeV can also be due to a variation in the sneutrino mass, so a second measurement of the cross section must be made at a higher $\sqrt{s}$ where the dependence of the cross section on the chargino mass and the slepton mass is different. It turns out to be advantageous for the chargino mass measurement to choose this higher energy measurement at a $\sqrt{s}$ point where the chargino cross section is not flat. The precision that can be obtained in the chargino mass depends substantially on the chargino mass itself since the cross-section decreases with increasing $m_{\tilde{\chi}^{\pm}}$. The heavier the chargino is, the less accurate the measurement for a given luminosity.

The chargino decay mode is $\tilde{\chi}^{\pm} \rightarrow \tilde{\chi}^{0} f \bar{f}'$, resulting in large missing energy due to $\tilde{\chi}^{0}$ in the final state, which is stable in the MSSM and thus escapes the detector. If $m_{\tilde{\chi}^{\pm}} - m_{\tilde{\chi}^{0}} > M_W$ then real $W$ contributions (two-body decay) dominate and the $\chi^+\chi^-$ final state is comprised of 49% purely hadronic events, 42% mixed hadronic-leptonic events, and 9% purely leptonic events (these ratios are determined by the $W$ branching fractions). To effectively suppress the backgrounds, we concentrate on the pure hadronic channel. The width of the chargino, typically less than a few MeV, has a negligible impact on the threshold cross section even for the two-body decay case, provided that the lighter chargino is gaugino-dominated. Based on the cross sections given in Fig. 1 and including the decay branching ratios and signal efficiencies, the signal rate at $\sqrt{s} = 2m_{\tilde{\chi}^{\pm}} + 1$ GeV would be about 20 fb for most values of the sneutrino mass. With 50 fb$^{-1}$ integrated luminosity, the cross section could be measured to a statistical accuracy of about 3%. Thus an understanding of the
background to at least this level is necessary.

There are several backgrounds to the chargino pair signal, by far the largest being $\mu^+\mu^- \rightarrow W^+W^-$. The backgrounds have been studied in Refs. [15,16], and signal efficiencies were obtained for the various final states when the center-of-mass energy is $\sqrt{s} = 500$ GeV. The dominant $W^+W^-$ background can be effectively eliminated by angular cuts because the $W$'s are produced in the very-forward direction. However, if the energy is reduced for running in the chargino threshold region, then the effectiveness of the angular cuts would be reduced since the background events become more spherical. Therefore we reinvestigate the acceptance criteria near the threshold.

![Graph](image)

Fig. 1: The cross section for $\mu^+\mu^- \rightarrow \tilde{\chi}^+\tilde{\chi}^-$ in the threshold region for various sneutrino masses, with the parameters in Eq. (3). The sneutrino mass dependence arises from a $t$-channel contribution which interferes destructively with the $s$-channel diagrams. The muon collider is assumed to have a beam energy spread of $R = 0.1\%$, and initial state radiation is included.

Based on the characteristic kinematics of the signal, we impose the following cuts to remove the backgrounds, mainly from $W^+W^- \rightarrow 4$ jets:
• A cut on missing mass, roughly $2M_{\tilde{\chi}^0} < M(\text{miss}) < 2M_{\tilde{\chi}^0} + 20 \text{ GeV}$.

• Require $\cos(\theta_{W-\text{miss}}) > -0.8$ where $\cos(\theta_{W-\text{miss}})$ is the minimum cosine of the angle between the reconstructed faster $W$ and the missing momentum.

• Require the reconstructed $W$’s be in the central region: $|\cos(\theta_W)| < 0.7$.

These cuts greatly reduce the $WW$ background to a negligible level. The overall signal efficiency with these cuts is about 10% for the fully hadronic decays.

![Graph](image)

**Fig. 2:** The $1\sigma$ precision obtainable in the chargino mass taking $m_\chi = 300$ and 500 GeV assuming 50 fb$^{-1}$ integrated luminosity. The precision on $m_{\tilde{\chi}^\pm}$ is better for larger sneutrino mass (see Fig. 1).

Figure 2 shows the expected precision of $m_{\tilde{\chi}^\pm}$ from fully hadronic decays with 50 fb$^{-1}$ integrated luminosity and a sneutrino mass of 300 and 500 GeV. For a lighter sneutrino, for which the destructive interference between the $s$-channel and $t$-channel graphs is more severe, the precision of $m_{\tilde{\chi}^\pm}$ is less. In the range of $m_{\tilde{\chi}^\pm} = 100 - 200 \text{ GeV}$, a measurement better than $50 - 300 \text{ MeV}$ is possible, much below the 1% level. The precision decreases with increasing chargino mass since the production cross section decreases.
Fig. 3: The $\Delta \chi^2 = 1$ contours in the chargino mass - sneutrino mass plane, taking the parameters in Eq. (3) and $m_{\tilde{\nu}} = 300$ and 500 GeV. The curves assume 25 fb$^{-1}$ of integrated luminosity is devoted to $\sqrt{s} = 2m_{\tilde{\chi}^\pm} + 1$ GeV, and 25 fb$^{-1}$ is applied at $\sqrt{s} = 2m_{\tilde{\chi}^\pm} + 20$ GeV.

The result of a fit to the chargino event rate is shown in Fig. 3, taking the parameters in Eq. (3) and assuming an integrated luminosity of 50 fb$^{-1}$. The cross section is measured just above the threshold $\sqrt{s} = 2m_{\tilde{\chi}^\pm} + 1$ GeV, and at a point well above the threshold, $\sqrt{s} = 2m_{\tilde{\chi}^\pm} + 20$ GeV (with 25 fb$^{-1}$ at each measurement). The chargino mass determination is better for higher sneutrino mass. The cross section is more sensitive to $m_{\tilde{\nu}}$ when it is lighter, resulting in a better measurement of the sneutrino mass. The sneutrino mass can be measured to about 6 GeV accuracy for $m_{\tilde{\nu}} = 300$ GeV and to about 20 GeV accuracy for $m_{\tilde{\nu}} = 500$ GeV. This provides an indirect method of measuring the sneutrino mass\cite{14}, which would be especially valuable when the threshold for sneutrino pair production is not open.
Comparing our results with similar studies for e+e− colliders, we find that the beam energy spread can cause a significant reduction in the precision of the threshold measurement. The most recent TESLA design envisions an electron beam energy spread of $R = 0.2\%$ \cite{17} while the Next Linear e+e− Collider (NLC) design anticipates a beam energy spread of $R = 1.0\%$. The NLC will be able to achieve precisions which are from 15% to 90% worse (for $m_{\tilde{\chi}^\pm}$ from 100 GeV to 200 GeV) than for the muon collider considered here, while the TESLA design should achieve precisions less than 10% worse than the muon collider. A high energy e+e− collider in a Very Large Hadron Collider (VLHC) tunnel would have a beam spread of $\sigma_E = 0.26$ GeV \cite{18} and would obtain results with a precision comparable to those considered here.

The mass of the chargino can also be measured by finding the endpoint in the spectrum (or by fitting to the full spectrum) of the chargino decay products \cite{1,13,19–21}. The endpoint is determined strictly by the kinematics of the decay $\tilde{\chi}^\pm \rightarrow \tilde{\chi}^0 f\bar{f}'$, so it is sensitive to both the chargino and neutralino masses. However the expected precision of the endpoint method with 50 fb−1 of integrated luminosity is greater than 1.5%.

A further advantage of the threshold measurement is that the chargino mass measurement is largely independent of how it subsequently decays. Distributions in the final state observables, say e.g. $E_{jj}$ from the decay $\tilde{\chi}^\pm \rightarrow \tilde{\chi}^0 jj$ \cite{15}, depend on the neutralino mass. The cross section for chargino pair production, on the other hand, is independent of the final state decays, and only the branching fractions and detector efficiencies for the various final states impact this measurement (if $m_{\tilde{\chi}^\pm} - m_{\tilde{\chi}^0} > M_W$ the branching fractions of chargino decay is given essentially in terms of the $W$ branching fractions).

We have assumed here that the chargino is lighter than the muon sneutrino, as is normally the case in mSUGRA models \cite{12,13}. If that is not so, the chargino has a new decay mode: $\tilde{\chi}^\pm \rightarrow \ell^\pm {\bar{\nu}}$. The signal efficiency of the cuts against background would need to be reconsidered if this mode is kinematically allowed.
It is expected that the both beams of a muon collider can be partially polarized, although with some loss of luminosity for high polarization [7]. Polarization could prove a useful tool for studying the chargino pair production. When the chargino is gaugino-dominated, it couples to the left-handed $\mu^-$ because it is then dominantly the partner to the $W$. Should the chargino be Higgsino-dominated, one would want to employ right-handed $\mu^-$ polarization since the $W^+W^-$ background would then be largely reduced. In addition, the $t$-channel sneutrino exchange contribution can be turned off by operating with a right-handed polarized $\mu^-$ beam.

For the gaugino-dominated chargino considered here, both the signal and background are approximately proportional to $(1 - P)^2$ where $P \equiv P_{\mu^-} = -P_{\mu^+}$ is the polarization of the two muon beams ($P = -1$ for a pure left-handed $\mu^-$ or a right-handed $\mu^+$). The background ($W^+$ pairs) and the $t$-channel sneutrino signal contribution couple to the left-handed $\mu^-$ (and right-handed $\mu^+$) beam. In the limit of $SU(2) \times U(1)$ symmetry, the $U(1)$ gauge boson couples only to the Higgsino component of the lighter chargino [14,22]. So the $s$-channel graph also couples predominantly to the left-handed $\mu^-$ when the lighter chargino is gaugino-like as considered here. Thus for 100% polarized $\mu^+$ and $\mu^-$ beams the mass determination would improve by a factor of two assuming the same integrated luminosity.

We have assumed in this study that the overall normalization of the chargino cross section is theoretically known, apart from the contribution from the $t$-channel diagram from a sneutrino of unknown mass. One can relax this assumption and allow the cross section normalization to be another free parameter. Then at least three measurements for the cross section would be required to extract the two masses ($m_{\tilde{\chi}^\pm}, m_{\tilde{\nu}}$) and the cross section normalization. This would test the theoretical prediction for radiative corrections from which the mass scale of squarks might be inferred [11]. On the other hand, if the sneutrino is discovered independently and its mass reasonably well measured, one could carry out the two-point measurement, as presented this paper, to determine $m_{\tilde{\chi}^\pm}$ and the cross section normalization.
IV. CONCLUSIONS

A measurement of the lighter chargino mass to better than $50 - 300$ MeV is possible for $m_{\tilde{\chi}^{\pm}} = 100 - 200$ GeV by measuring the pair production cross section near threshold at a muon collider with $50$ fb$^{-1}$ luminosity. This is superior to other techniques, such as the kinematical end-point method, or at other colliders. Only modest beam energy resolution ($R \sim 0.1\%$) is needed for the threshold measurements. The muon sneutrino mass can also be simultaneously measured to a few GeV if it is not too heavy.

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