Reconstructing the Equation of State of Tachyon

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Abstract: Recent progress in theoretical physics suggests that the dark energy in the universe might be resulted from the rolling tachyon field of string theory. Measurements to SNe Ia can be helpful to reconstruct the equation of state of the rolling tachyon which is a possible candidate of dark energy. We present a numerical analysis for the evolution of the equation of state of the rolling tachyon and derive the reconstruction equations for the equation of state as well as the potential.

I. Introduction

In the evolution model of the universe, the underlying dynamics is mostly described as a single scalar field rolling in certain potential. This scenario is preferred because of its simplicity and catches the most attention since its proposition. Furthermore, many seemingly more complicated models can be rewritten in such a framework. An important feature of the spectra of density perturbation and gravitational perturbation, which has been widely studied in inflation theory, is that they can be linked by a consistency equation. Given a particular set of observations of certain accuracy, one may attempt the bold task of reconstructing the potential of the scalar field from the observations. The study on the reconstruction equation was firstly introduced when investigating the inflation universe, in which it was usually referred to as "perturbation reconstruction". In this scheme, one has found the relationship between observations of microwave anisotropies and that of the large-scale structure, and tried to connecting them with the potential of the scalar field that drives the inflation. In this approach, the consistency equation and scalar potential are determined as an expansion about a given point (regarded either as a single scale in the spectra or as a single point on the potential), allowing the reconstruction of a region of the potential about the point. The second round of study on reconstruction equations appears in the investigation of quintessence, in which the connection between the red-shift of the supernovae and the potential of the quintessence is established.

In this paper, following the ingenuity of the above reconstruction, we consider the reconstruction of the equation of state as well as the potential of the rolling tachyon, which has been proposed recently to play the role of dilaton or quintessence. The potential of the rolling tachyon could be derived from string theory and its dynamics has been widely studied. On the other hand, one can also reconstruct the effective potential of the tachyon field by the observations. It should be noted that the CMB anisotropic spectra mainly probe the universe at redshift \( z \approx 1000 \) when the ratio of dark energy to matter is less than \( 10^{-6} \). While the Supernovae observations reflect that the universe at recent epoch when the dark energy is beginning to dominate the universe. Therefore, if we assume that the rolling tachyon is the candidate for dark energy, we’d better reconstruct the potential of the tachyon field by the red-shifts data of SNe Ia.

It is worth noting that the effective potential of the tachyon field should be derived from string theory by considering the corresponding process. While it can also be reconstructed from the observations. It would be more appropriate to consider the reconstruction equation as a criteria with which one can judge whether the effective potential from string theory can fit the SNe Ia red-shift data. The outline of this paper is as follows: In section II, we exhibit the reconstruction of the potential of tachyon field. Section III contains a brief numerical analysis for the dynamical evolution of tachyon and the conclusions are discussed in section IV.

II. Reconstruct the Potential of Tachyon field

The effective Lagrangian density of tachyon of string theory in a flat Robertson-Walker background is as following:

\[
L = -V(T)\sqrt{1+g_{\mu\nu}\partial_{\mu}T\partial_{\nu}T}
\] (1)

where

\[
ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2)
\] (2)

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is the flat Robertson-Walker metric and \( V(T) \) is the potential resulted from string theory. When we consider the existence of non-relativistic matter and the tachyon field \( T \), the Einstein equations for the evolution of the background metric, \( G_{\mu\nu} = \kappa T_{\mu\nu} \) can be written as:

\[
H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{\kappa}{3} (\rho_T + \rho_M) \tag{3}
\]

and

\[
\frac{\ddot{a}}{a} = -\frac{\kappa}{3} \left( \frac{1}{2} \dot{\rho}_T + \frac{3}{2} p_T + \frac{1}{2} \rho_M \right) \tag{4}
\]

For a spatially homogenous tachyon field \( T \), we have the equation of motion

\[
\ddot{T} + 3H \dot{T}(1 - \dot{T}^2) + \frac{V'(T)}{V(T)} (1 - \dot{T}^2) = 0 \tag{5}
\]

where the overdot represents the differentiation with respect to \( t \) and the prime denotes the differentiation with respect to \( T \). Eq. (5) is also equivalent to the entropy conservation equation. The constant \( \kappa = 8\pi G \) where \( G \) is Newtonian gravitation constant. The density \( \rho_T \) and the pressure \( p_T \) are defined as following:

\[
\rho_T = \frac{V(T)}{\sqrt{1 - \dot{T}^2}} \tag{6}
\]

\[
p_T = -V(T)\sqrt{1 - \dot{T}^2} \tag{7}
\]

the equation of state is

\[
w_T = \frac{p_T}{\rho_T} = \dot{T}^2 - 1 \tag{8}
\]

It is clear that when tachyon field is dominant and if the tachyon field can accelerate the expansion of the universe, there must be \( \dot{T}^2 < \frac{2}{3} \) and \(-1 < w_T < -\frac{1}{3}\).

Now, we will correlate the potential with the observable red-shift of SNe Ia. To do so, following the earlier study in this field\(^2\), we introduce the quantity

\[
r(z) = \int_{t(z)}^{t_0} \frac{du}{a(u)} = \int_0^z \frac{dx}{H(x)} \tag{9}
\]

which is the Robertson-Walker coordinate distance to an object at red-shift \( z \). Also, we denote

\[
\rho_M = \Omega_M \rho_{\text{crit}} = \frac{3\Omega_M H_0^2 (1 + z)^3}{\kappa} \tag{10}
\]

where \( H_0 \) is the present Hubble constant, \( \Omega_M \) is the fraction of non-relativistic matter to the critical density and \( \rho_{\text{crit}} \) is the critical energy density of the universe.

We then readily have

\[
\left( \frac{\dot{a}}{a} \right)^2 = H(z)^2 = \frac{1}{(dr/dz)^2} \tag{11}
\]

\[
\frac{\ddot{a}}{a} = \frac{1}{(dr/dz)^2} + (1 + z) \frac{d^2r/dz^2}{(dr/dz)^3} \tag{12}
\]

\[
\frac{dz}{dt} = -(1 + z) H(z) = -(1 + z) \frac{dr}{dz} \tag{13}
\]
Through Eq.(3) to Eq.(13), one can express the potential of the rolling tachyon in term of the red-shift \( z \), \( dr/dz \) and \( d^2r/dz^2 \) as:

\[
V^2(T(z)) = \frac{3}{\kappa (dr/dz)^2} - \rho_M \tag{14}
\]

\[
\times \left[ \frac{3}{\kappa (dr/dz)^2} + \frac{2(1 + z)(d^2r/dz^2)}{(dr/dz)^3} \right]
\]

\[
\left( \frac{dT}{dz} \right)^2 = \frac{(dr/dz)^2}{(1 + z)^2} - \frac{\kappa^2 (dr/dz)^6 V(T(z))^2}{(1 + z^2)[3 - \kappa \rho_M (dr/dz)^2]^2} \tag{15}
\]

\[
w_T = -\frac{\kappa^2 (dr/dz)^4 V(T(z))^2}{[3 - \kappa \rho_M (dr/dz)^2]^2} \tag{16}
\]

It is clear that if we know the coordinate distance as a function of the red-shift \( z \), which can be attained by fitting the observation data, we can reconstruct the quantities such as potential and the equation-of-state by the above reconstruction equations. Also, it must be pointed out that there are sign ambiguities in Eq.(14) and Eq.(15), which suggests that the reconstruction equations can not completely determine the potential as well as the variation of the field. However, the equation of state \( w_T \) can be determined uniquely by the reconstruction equation (16). Especially, from Eq.(16), one can find that \( w_T \) is always negative no matter what is the form of the \( r(z) \) and \( V(T(z)) \). This is compatible with the property of dark matter which possesses a negative pressure.

3. Numerical Analysis for the Dynamical Evolution of Tachyon

The evolution of the tachyon is determined by its potential, which may depend on the underlying (bosonic or supersymmetric) string field theory. In this section, we analyze the evolution of the equation of state \( w_T \) of the rolling tachyon in the potential \cite{24, 25}

\[
V(T) = V_0 (1 + \frac{T}{T_0}) \exp(-\frac{T}{T_0}) \tag{17}
\]

Here, we consider the case that tachyon field is dominant over the non-relativistic matter. In an earlier paper, we have shown, with the aid of phase-plane analysis, that there is no stable critical point in the evolution of the tachyon field \cite{24}. The critical point is a saddle point, which implies the evolution of the tachyon field is very sensitive to its initial condition. Rescale the tachyon field and the time variable by setting \( T = xT_0 \) and \( t = sT_0 \). Introducing a new dimensionless variable \( y = \frac{dT}{ds} \), one can reduce the equation of motion of the tachyon field in the potential Eq.(17) to two first order differential equations as following:

\[
\frac{dx}{ds} = y \tag{18}
\]

\[
\frac{dy}{ds} = \frac{x(1 - y^2)}{1 + x} - \beta y(1 - y^2)^{\frac{3}{4}}(1 + x)^{\frac{3}{2}} \exp(-\frac{x}{2}) \tag{19}
\]

where \( \beta = \sqrt{3V_0 \kappa}T_0 \) is a dimensionless parameter. Now, it is straightforward to carry out numerical analysis on the above equations and the following are the numerical results (we choose \( \beta \) as 0.9, 1.0 and 1.1 respectively).

From Fig.1, one can find that the tachyon field increases steadily with respect to \( s \). From Fig.2, one knows that the variation of the field will approach an asymptotic value. The Fig.3 shows that the equation of state \( w_T \) is -1 at the beginning and then increases to 0 as time evolves. If one consider the tachyon as dark energy that accelerates the expansion of current universe, then this acceleration will eventually stop and then the universe will recover its decelerating expansion.

IV. Discussion and Conclusion

In this paper, we derive the relation between the equation of state of rolling tachyon of string theory and the observable coordinate distance in term of red-shift \( z \) of SNe Ia. the prerequisite of such a relation is that we consider
FIG. 1: The plot of $x$ against $s$, curve a, b and c correspond to the cases of $\beta = 0.9, 1.0, 1.1$ respectively.

FIG. 2: The plot of $y$ against $s$, curve a, b and c correspond to the cases of $\beta = 0.9, 1.0, 1.1$ respectively.
FIG. 3: The plot of equation of state $w$ against $s$, curve a, b and c correspond to the cases of $\beta = 0.9, 1.0, 1.1$ respectively.

the tachyon as the dark energy that dominates the universe and accelerates its expansion at recent epoch. In some earlier works$^3$, such a possibility has been pointed out and the theoretical predication and the reconstructed results have been compared with the simulated SNe Ia data, which is an important and interesting topic.

The importance of the reconstruction equations lies in that it may serve as the intersection between theory and observations. We analyzed numerically the dynamical evolution of the equation of state of tachyon in the potential derived from string theory, and found that it will increase steadily with time, which implies that the expansion of the universe will eventually slow down and become decelerating. This could be compared with the observed SNe Ia data through the reconstruction equation in the former part of this paper. It is worth noting that the evolution of the equation of state in this paper is under the initial condition that the tachyon is rolling down from the top of the potential with 0 initial speed. The initial condition could be choose otherwise so that the equation of state evolves toward -1, that is, the tachyon rolls up to the top of potential although this is not very natural. In some other effective potentials, one may find different evolution of the $w_T$, which can be compared with the reconstructed evolution of $w_T$.

On the other hand, the tachyon here we considered is resulted from string theory, while there are also other possibilities. For example, the tachyon inflation has also been considered for phenomenological potentials that are not derived from string theory$^6$. Such models are related to phenomenological "k-inflation"$^2$. Clearly, this work can also be further generalized to these kinds of phenomenological theories.

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