Sequently Optimal Pricing under Informational Robustness

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Abstract. A seller sells an object over time but is uncertain how the buyer learns their willingness-to-pay. We consider informational robustness under limited commitment, where the seller offers a price each period to maximize continuation profit against worst-case information arrival. Our formulation maintains dynamic consistency by considering the worst case sequentially. Under general conditions, we characterize an essentially unique equilibrium where the buyer does not delay to learn more later. Furthermore, we identify a condition that ensures the equilibrium price path is “reinforcing,” so even dynamically inconsistent information arrival would not lower the seller’s payoff below the equilibrium level.

Keywords. Limited commitment, informational robustness, reinforcing solution.
1. INTRODUCTION

We consider a seller aiming to sell one unit of an indivisible good to a buyer. Our focus is on situations where the buyer need not initially know her willingness-to-pay, but can learn about it according to a general information arrival process while considering purchase. We propose a new approach, motivated by the active literature on informational robustness, to analyze these settings when the seller selects prices each period to maximize payoffs from that time on.

A buyer may not know their objective willingness-to-pay for a product for many reasons. Consider a new parent negotiating to buy a house from a previous homeowner who has already vacated. Neighborhood characteristics (e.g., annual average NOx level, risk associated with natural disasters, school quality, etc.) may not be immediately apparent. A first-time buyer may even be learning how characteristics of their potential new home should translate into true willingness-to-pay.

Our starting point is the simple observation that a multitude of possibilities for buyer learning may abound in situations like these. The buyer may be perfectly informed initially, or possibly not. The buyer may or may not be consulting family or different online sources. The buyer could be learning about when these updates will arrive. Perhaps an “unexpected” offer influences the sources the buyers considers. The varied possibilities suggest that any particular structure on buyer learning may fail to reflect the actual problem of interest. And the potential for rich learning possibilities is a fundamental feature of many settings beyond housing markets.

Our interest is in applications where the seller cannot commit to a strategy in advance, but instead will seek to optimize their continuation payoff after an offer is rejected. While this possibility is recognized as economically significant, relatively little has been said regarding the combination of limited commitment and unrestricted learning. In principle, the possibility of learning could fundamentally alter behavior in this class of models. This claim follows from traditional intuition that the extent to which delay selects low willingness-to-pay buyers shapes

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1Such negotiations often lack mechanisms to avoid price revisions. In fact, the sale of land is the leading example in Coase (1972). Han and Strange (2015) discuss recent evidence on post-match price revisions in housing markets.

2The variation in, and influence of, information availability regarding idiosyncratic value-relevant characteristics is particularly well-documented empirically in housing markets. Fairweather et al. (2023) conduct an experiment showing that the provision of information regarding flood risk has a significant impact on buyer strategies. Ainsworth et al. (2023) show households often lack information about local school quality; among various observation, they find variation in accuracy of these beliefs predicts child achievement. Bergman et al. (2020) similarly show experimentally that providing information on schools influences household location decisions for families.

3As we discuss Section 1.4, existing precedents impose particular structure on learning, generally ruling out many of the possible technologies outlined. As Pavan (2017) states: “The literature on limited commitment has made important progress in recent years.... However, this literature assumes information is static, thus abstracting from the questions at the heart of the dynamic mechanism design literature. I expect interesting new developments to come out from combining the two literatures.” We view our contribution in the spirit of this agenda.
equilibrium pricing (see Section 1.4 for discussion). While incentives to delay given a price path are pinned down by the expected surplus from purchase absent learning, the same conclusion need not hold if learning is possible. For some price paths, the prospect of future learning may induce buyers to delay *irrespective* of perceived willingness-to-pay.

In fact, without restrictions on information arrival, some learning processes can sustain constant price paths and equilibrium multiplicity, a dramatic departure from the outcomes in these situations without learning (see Section 6.2). We view this result as a proof-of-concept that some structure is necessary in order to derive interpretable heuristics behind equilibrium pricing, or properties of equilibrium outcomes. But our motivation is that frequently such restrictions lack economic justification. A different perspective is necessary to provide a useful baseline.

To make progress, we focus on situations where the seller, in turn, is completely ignorant about how the buyer learns. While even an inexperienced seller might form a refined prior over the buyer’s true willingness-to-pay using market data (e.g., publicly available transaction prices), he may not know anything about her information sources. A seller unfamiliar with up-to-date resources for home buyers may lack the confidence needed to form a prior over the set of learning processes, and thus may not anticipate a particular one. Assuming confidence necessary to form a prior over buyer learning may be unnatural, particularly as information is typically intangible. It is not always clear how a seller might form a prior over such an object after every contingency. This benchmark reflects our economic motivation of studying situations where sellers do not consider any information arrival process as a priori impossible.

This motivation of studying the implications of uncertainty over buyer information is shared with other work in the active literature on informational robustness [Bergemann et al. 2017; Du 2018; Brooks and Du 2021 2023; Deb and Roesler 2023]. The typical formalization of “good performance given complete ignorance” associates this goal with achieving a favorable worst-case guarantee across the set of possibilities considered. This formulation is also frequently utilized when uncertainty is not related to information per se [Bergemann and Morris 2005; Carroll 2015 2017; Lopomo et al. 2020; He and Li 2022; Che and Zhong 2022]. Yet despite enormous interest in this approach, there is no consensus on how to even *formalize* worst-case optimality in a dynamic setting, when one relaxes commitment. As [Carroll 2019] notes, “trying to write dynamic models with non-Bayesian decision makers leads to well-known problems of dynamic inconsistency, except in special cases (e.g., Epstein and Schneider 2007). This may be one reason why there has been relatively little work to date on robust mechanism design in dynamic settings.” To see the relevance of this comment for us, consider the following stylized scenario: Suppose that, when deciding on the second-period price, the seller is concerned about the buyer perfectly learning

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4One reason for the popularity of the *informationally* robust approach in particular is due to the influence of the *Wilson Critique*, that strong epistemic assumptions made by mechanism design severely limits its applicability.
her value at time 10, encouraging a “wait-and-see” strategy. But once time 10 arrives, complete learning might not be the worst-case, as a buyer with value slightly above the price could be kept ignorant and dissuaded from purchase. With the caveat that these conjectures are hypothetical, we see tension between the worst-case objective at time 2 and this objective at time 10.

This gap in the literature is unfortunate, as it implies dynamic models are more tied to assumptions of correctly specified Bayesianism than static ones, even though this assumption is especially imposing under dynamics. If anything, it seems plausible that in some cases the economic circumstances preventing a seller from forming a precise prior (e.g., limited experience with selling one’s house) might also inhibit commitment. Thus, we hope our work provides a concrete starting point for an agenda described in [Bergemann and Valimaki (2019)], writing that dynamic mechanism design has so far involved “… Bayesian solutions and relied on a shared and common prior of all participating players. Yet, this clearly is a strong assumption and a natural question would be to what extent weaker informational assumptions, and corresponding solution concepts, could provide new insights into the format of dynamic mechanisms.” While our approach described below will also involve strong assumptions, we believe that providing some alternative articulating the conceptual issues concretely is the natural first step toward fulfilling the assigned task.

1.1. Our Approach

We start with a protocol which allows (1) the seller to post prices over time and (2) the buyer to learn about willingness-to-pay. In every period, the seller offers to sell the object to the buyer at a price chosen in that period, and the buyer can either accept or reject the offer after receiving their information; the interaction either continues indefinitely, or lasts until some terminal date. This environment reduces to a classic one under the assumption that the buyer knows willingness-to-pay; otherwise, a traditional Bayesian approach would posit the seller forms a prior over the buyer’s information arrival process (a potentially infinite-dimensional object).

We instead introduce a specification of the robust objective for this environment whereby the seller considers a dynamically consistent worst case (that is, the worst case the seller anticipates for tomorrow will still be the worst case when tomorrow arrives). Specifically, we posit the seller chooses prices imagining that at every time, the buyer’s information structure will minimize the seller’s profit from that period on. We call such an information arrival process sequentially worst case. Starting with the case where dynamic consistency is imposed strikes us as sensible toward extending the robust approach to settings without commitment, given the conceptual issues that emerge without it. That said, dynamic consistency in maxmin models has been well-studied in the decision theory literature, and whether it is per se desirable is the topic of much debate.

5For instance, [Al-Najjar and Weinstein (2009)] present a number of apparently behavioral anomalies which emerge
While engaging in this debate further is beyond the scope of this paper, we emphasize that our work is not a single-agent problem, unlike notable past work applying such formulations (Malladi 2023; Auster et al. 2022). A defining feature of robust mechanism design is the presence of an agent; in our setting, the buyer is allowed to have any behavior consistent with rationality.

We will describe information structures as resulting from the choices of an “adversarial Nature.” This phrasing is a common expositional device; in our context it facilitates our explanation of how to think about sequentially worst-case information. In our formulation, at every period, Nature chooses an information structure for that period to minimize (discounted) continuation profit. The worst-case information arrival process thus emerges as an equilibrium object. Positing that Nature’s choices arise from equilibrium pins down the seller’s conjectures of the information arrival process, on-and-off-path—it amounts to requiring (1) in every period the conjecture minimizes seller profit, and (2) conjectures over time are consistent with one another.

1.2. Our Results

Our main observations in the paper are the following twin results:

1. We characterize equilibrium outcomes under conditions that roughly speaking ensure the interaction ends in finite time. The seller’s payoff is unique. In each period, Nature’s information choice has the property that buyers who do not purchase would be indifferent between purchasing or not if no further information were to arrive in the future. Thus, in equilibrium, learning does not induce additional delay.

2. We provide a permissive condition such that among all learning possibilities, sequentially worst-case information arrival minimizes seller profit given the equilibrium pricing strategy. Thus, dynamic consistency need not be at odds with full worst-case optimality.

Formally, our first main result shows that the worst-case choices of Nature minimize the probability of sale within each period given the equilibrium price path. We refer to these Nature strategies as fully-myopic. We emphasize that our model assumes Nature seeks to minimize the seller’s total discounted profit. In principle, Nature could promise more information in the future in order to deter the buyer from buying in the present. However, we show that this does not occur in equilibrium. The reduction to fully-myopic information arrival allows us to characterize under dynamic maxmin models without dynamic consistency; however, Siniscalchi (2009) argues that several of these may be natural. The interested reader can also see discussion in Epstein and Schneider (2003) on why dynamic consistency may be per se desirable. Note that dynamic inconsistencies may arise in settings studying ambiguity in sequential games, even outside of worst-case; Battigalli et al. (2019a,b) discuss these issues under versions of smooth-ambiguity preferences, a general class which can approximate maxmin in a limiting case.
equilibrium, recovering traditional intuition about the form of equilibrium price-paths. In particular, we show that it implies the seller does not randomize in equilibrium except possibly in the initial period; from this observation, payoff uniqueness immediately follows.

Here is the broad intuition for why fully-myopic information arrival is sequentially worst case. In the final period, nature’s information choice must have the property that any buyer who is not purchasing must be indifferent between purchasing or not. Otherwise another information structure can be found that lowers the probability of purchase and thus seller profit. Because of this indifference property, the buyer’s expected payoff in the final period is the same as if no information were provided. Now, anticipating this, the buyer in the second-to-last period makes purchase decisions as if there would be no future information. This buyer behavior in turn reduces Nature’s problem in the penultimate period to a static problem in which it seeks to minimize the probability that the buyer buys in that period. Thus Nature’s optimal strategy in the second-to-last period is also fully myopic, so on and so forth.

Our second main result addresses the (admittedly informal) question of whether the equilibrium outcome qualifies as a “true worst case.” We do not see a definitive consensus of what formal criterion this title should correspond to. But it strikes us as a fundamental issue to the agenda of Bergemann and Valimaki (2019). The reason is that to achieve dynamic consistency, we assume information is “reoptimized” in every period, just as the seller reoptimizes prices. The natural question is if and when this step requires relaxing the motivation for the worst-case objective.

We interpret the “true worst case” criteria as asking the following: Does the equilibrium information arrival process minimize the seller’s ex-ante expected profit, given their pricing strategy, across all possible information processes, including those that are not sequentially worst-case? Our second main result shows that the above equilibrium outcome is a true worst case under a simple assumption on the buyer’s value distribution. Roughly speaking this condition requires that not too much mass be toward the top of the support of the prior willingness-to-pay distribution. Under this assumption, whenever the seller optimizes against sequentially worst-case information arrival, no other information arrival process can further lower seller expected profit.

We call a pricing strategy with this property a reinforcing solution; even if the seller is misspecified about how Nature selects the buyer’s information process, this misspecification cannot possibly hurt. If the seller believed that Nature did not have commitment (as under sequentially worst-case information arrival), then his profit would not be lower against any arbitrary information process. This result is quite subtle, and is driven by the fact that the seller does not react to the information arrival process as we consider richer information arrival possibilities.

This result is reminiscent of Mailath and Samuelson (2020), albeit in a very different context, which describes conditions under which agents with misspecified models behave similarly to those with correctly specified ones.
1.3. Our Message

Our work achieves twin goals. The first is to provide a clear baseline articulating how the possibility of learning interacts with limited commitment to prices. Despite several subtleties we elucidate, we recover traditional intuition and heuristics regarding how the seller sets prices, lowering them over time to sell to the residual (lower-value) buyers. We hope this benchmark will be useful in understanding outcomes under limited commitment absent restrictive structure on learning—especially since (a) Bayesian approaches have not provided a clear benchmark for this, and (b) insofar as there could be a Bayesian benchmark, it would most likely resemble “anything goes.”

The framework, however, involves enriching the traditional informationally robust approach. This literature (e.g., [Du (2018); Brooks and Du (2023)], among many others) often points to difficulties with determining practically relevant learning as economic motivation. Our suspicion is that limited commitment has been ignored not due to economic relevance, but instead theoretical conceptual challenges. Our second goal is to provide a template to concretely interrogate these. Our result that sequentially worst-case pricing does no worse against arbitrary information arrival suggests dynamic consistency issues may be less prohibitive than originally thought.

1.4. Relevant Literature

The literature on robust mechanism design was initially motivated by the goal of relaxing strong common knowledge assumptions implicit in Bayesian mechanism design (Bergemann and Morris, 2005). While early work in this line of research assumed agents knew their preferences, subsequent work relaxed this assumption and allowed designers to also face uncertainty about what the agents know about their own preferences. As far as we are aware of, there have been relatively few papers that study robust mechanism design in dynamic settings, and none of them relax commitment. Our suspicion is that complexities related to dynamic consistency assumptions as per Carroll (2019) may be responsible for the lack of work on this topic.

Other papers have accommodated other conceptual issues that arise under the robust approach. Bolte and Carroll (2020) study the problem of a principal who can choose investment in the course of interacting with an agent, and show that this provides a foundation for linear contracts, echoing an earlier result of Carroll (2015). Ocampo Diaz and Marku (2019) also extend Carroll (2015), but they consider the case of competing principals in a common agency game. Both of these papers address a similar conceptual issue, namely how the strategic choices of the designer should

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7 A recent paper that relaxes commitment in a similar way to our paper is Ravid et al. (2020). They consider the problem of buyer-optimal information (in a one-period model) when the choice of information is unobservable to the seller. A difference is that they assume information is costly. However, relaxing commitment to the information structure as in Ravid et al. (2020) is similar to relaxing Nature’s commitment to the information process as we do.
interact with the maxmin objective. Libgober and Mu (2021) studied robust dynamic pricing with commitment, thus avoiding the issue of consistency and the complexities associated with characterizing equilibria. In all of these papers (and most others in this literature), the worst-case is only considered once. Fundamentally, limited commitment requires considering the worst-case multiple times. Our contribution is to study properties of the solution under our formalization.

A less related literature considers mechanism design where agents (instead of the designers) have non-Bayesian preferences, including the maxmin case (Bose and Renou 2014; Wolitzky 2016; Di Tillio et al. 2017). However, typical motivation in this literature relates to how the designer should exploit this. Some papers in this literature explicitly consider exploitation of dynamic inconsistency (Bose et al. 2006; Bose and Daripa, 2009).

If the buyer knew realized willingness-to-pay, our model would reduce to textbook durable goods monopoly without commitment, e.g., Fudenberg et al. (1985). But while we use some technical results from this literature, our agenda is almost entirely orthogonal. We address dynamic consistency issues due to maxmin, and the $\delta \to 1$ limit is not our focus, unlike most papers on this topic. A notable exception is Fuchs and Skrypacz (2013), which provides a characterization of non-trivial pricing dynamics given “frequent offers” and a finite time to trade, depending on how the offer frequency and time horizon approach their limits jointly.

It is worth noting that past work has shown that changing preferences may qualify the conclusions of the literature on the Coase conjecture (for instance, Ortner (2017, 2023); Acharya and Ortner (2017)), and information arrival can be interpreted as a preference change. Relatedly, Lomys (2018), Duraj (2020) and Laiho and Salmi (2020) study how Coasian dynamics are influenced by the presence of buyer learning, albeit under restrictive assumptions on either the type distribution or the learning process. These departures are driven by aforementioned interaction between learning and selection, and the importance of the direction of selection for Coasian dynamics is explored in Tirole (2016) and Ali et al. (2023). In contrast to these, one interpretation of our main result is that Coasian dynamics are exactly restored under the dynamically-consistent informationally robust objective. In Appendix B we discuss alternative dynamic worst-case formulations under which the prospect of learning introduces additional forces, implying this conclusion no longer holds.

2. MODEL

We start with the basic primitives of the environment. Then, we move onto the particular interaction between the buyer and seller, describing how information arrival works and defining strategies and beliefs. Section 2.5 introduces our worst-case notion. To avoid distraction, our discussion of model assumptions is deferred to Section 6.4 and a full-scale discussion of alternative worst-case notions to Appendix B.
2.1. Underlying Environment

A seller (he) of a durable good (e.g., a house) interacts with a single buyer (she) in discrete time until some terminal date \( T \), where \( T \leq \infty \); we will handle the case of \( T = \infty \) and \( T < \infty \) separately. The buyer can purchase the good at any time \( t = 1, \ldots, T \). The buyer has unit demand for the seller’s product, and obtains utility \( v \) from purchasing, where \( v \) is drawn from a continuous distribution \( F \) with density \( f \), which the buyer and seller commonly know. We take \( v \) to be drawn once, at time 0, and fixed throughout the game. We assume that the support of \( F \) is an interval \([v, \bar{v}]\). Payoffs by both buyer and seller are discounted according to a discount factor \( \delta \in (0, 1) \).

However, neither the buyer nor the seller know the realization of \( v \) itself. Instead, the buyer will learn about \( v \) over time according to an information arrival process. We define an information structure to be a pair \((S, I)\), where each \( s \in S \) denotes a possible signal and \( I : [v, \bar{v}] \rightarrow \Delta(S) \) determines the distribution over signals for every \( v \in [v, \bar{v}] \). We assume throughout the paper that the signals drawn under an information structure are observed only by the buyer, and not by the seller. We will allow the buyer to obtain signals according to different information structures over time, in a history dependent way. For now, observe that the buyer can form her posterior expectation following \((I_1, \ldots, I_t)\) and \((s_1, \ldots, s_t)\) as:

\[
E[v \mid I_1, s_1, \ldots, I_t, s_t],
\]

via Bayesian updating (and with no other information).

2.2. Actions

Each period \( t \) begins with the seller choosing a price \( p_t \in \mathbb{R}_+ \). While the seller has the ability to randomize according to a distribution \( \gamma_t \in \Delta(\mathbb{R}_+) \), we assume that the buyer observes \( p_t \) prior to deciding whether or not to purchase. As alluded to above, the buyer also observes a signal drawn according to an information structure in every period before deciding whether to purchase. We specify how this information structure is determined below.

The buyer then decides whether to purchase at price \( p_t \). Let \( d_t \) denote the buyer’s decision, where \( d_t = 1 \) denotes the event that she buys and \( d_t = 0 \) denotes the event she does not. If the buyer purchases or \( t = T \), the game is over. Otherwise, the game proceeds to the next period.

2.3. Defining Histories

We now define histories and information sets. Our main solution concept requires optimal actions at three different kinds of information sets—those for (1) Nature (it), (2) the seller and (3) the buyer.

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8Assuming \( v > 0 \) holds is known as the “gap case,” whereas the “no-gap case” refers to when \( v = 0 \).
Since the buyer only decides whether to purchase, and since the game ends when she does, in defining histories we will assume that the buyer has not yet purchased, thus omitting this decision from our definition of histories. Define $h^1_1 = \emptyset$, $h^1_2 = (p_1)$, and for each $t \geq 2$, we define:

$$h^1_t = (p_1, I_1, s_1, \ldots, p_{t-1}, I_{t-1}, s_{t-1}),$$

$$h^2_t = (p_1, I_1, s_1, \ldots, p_{t-1}, I_{t-1}, s_{t-1}, p_t).$$

We are interested in a protocol where within each period, first the seller moves, then Nature moves, and finally the buyer moves. In our main model, we assume that when seller moves at time $t$, he only observes $p_1, \ldots, p_{t-1}$, that is, the past prices charged, and not the buyer’s information. When Nature moves at time $t$, it sees all of $h^2_t$. When the buyer moves at time $t$, she sees all of $h^1_{t+1}$. We this in mind, let $H^t_B, H^t_S, H^t_N$ denote the set of possible information sets for the buyer, seller, and Nature, respectively, at time $t$. Let $H$ denote the set of all histories thus defined.

We have not yet discussed how information arrival is determined. Still, it is worth pausing and noting that so far the framework is fairly standard. For instance, suppose $I_1$ were perfectly informative. Then the above reduces to textbook bargaining with one-sided private information. The only innovation in the above is that the buyer can learn about $v$ over time.

### 2.4. Defining Strategies and Beliefs

To complete the model, we must specify how the seller and buyer’s actions are chosen. Part of our contribution is in formulating a robust objective for the seller in this environment, which as discussed in the introduction, has proved elusive. We do not know of any consensus approach.

We start with strategies. A buyer strategy is a function $\sigma : \cup_t H^t_B \rightarrow \Delta(\{0, 1\})$; for each buyer information set (which, for the buyer—as well as Nature—is a singleton history), $\sigma$ specifies a probability distribution over (a) the event 0, corresponding to “not buying” and (b) the event 1, corresponding to “buying.” A pricing strategy is a function $\gamma : \cup_t H^t_S \rightarrow \Delta(\mathbb{R}^+)$; that is, for each seller information set, $\gamma$ specifies a distribution over prices. A price path is a sequence $(p_1, \ldots, p_t, \ldots)$. We denote $p^t = (p_1, \ldots, p_t)$. An information arrival process is a function $IS : \cup_t H^t_N \rightarrow \Delta((\mathcal{P}(X) \setminus \emptyset) \times \{I : [v, \overline{v}] \rightarrow \Delta(X)\})$, where the first coordinate of any element in the image is a set $S \subset X$, and the second coordinate is restricted to be a function from $[v, \overline{v}]$ to $\Delta(S) \subset \Delta(X)$. For technical reasons, we will take $X$ to be finite, and omit the formal details necessary to allow for $X$ with infinite cardinality.

We now turn to beliefs. Let $T = \{1, \ldots, T, \infty\}$ denote the set of possible dates at which the

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9Strictly speaking this rules out fully informative information structures; however, no conceptual issues arise when simply adding one additional information structure into the set of possible choices, so this is not essential. Note that the worst-case information structures we identify do involve a finite number of signal realizations.
buyer could purchase the good, where \( T = \infty \) corresponds to the event that the buyer does not purchase. Define a pair of histories \( h^s_t \) and \( h^B_t \) to be non-contradictory if they coincide with one another whenever possible (e.g., contain the same pricing strategies, information structures, etc., at every time up to and including \( t \)). Given a strategy triple, \( \sigma, \gamma, IS \), as per the above, we define:

\[
P^*_\sigma,\gamma,IS : H \rightarrow \Delta([v, \tau] \times T \times H),
\]

\( P^*_\sigma,\gamma,IS \) to be the induced probability distribution over possible buyer values, terminal nodes, and future histories, given that players are at history \( h \in H \) and the strategies \( \sigma, \gamma, IS \) are being used. Note that, for every \( h \in H \), \( P^*_\sigma,\gamma,IS(h) \) is concentrated on histories non-contradictory with \( h \). Knowing the distribution over possible buyer values and purchase times suffices to define the expected payoffs of each of the players—i.e., a probability distribution over future outcomes, induced by these strategies. A belief system for player \( j \in \{ S, N, B \} \) is a function:

\[
\mu_j : \bigcup_t H^j_t \rightarrow \Delta([v, \tau] \times H),
\]

where, at every information set, \( \mu_j \) is supported on \( h \in H \) within each player’s information set. Let \( \mu = (\mu_S, \mu_N, \mu_B) \). A belief system “satisfies Bayes rule where possible” if, for \( t < s \) and any \( i \), \( h^i_t \) non-contradictory with \( h^i_s \), \( \mu(h^i_s) \) can be derived from \( \mu(h^i_t) \) via Bayes rule. Note since Nature and the buyer observe the complete history, their belief systems are only about the private type \( v \). The seller can not observe the information structure chosen and the signal realization, so his belief system is not only about the private type but also the history.

We take belief systems to satisfy “no signalling what you don’t know” ([Fudenberg and Tirole 1991]): Specifically, we restrict \( \mu \) so that beliefs over \( v \) do not depend on the price charged, and also that (1) defines the buyer’s posterior expectation of \( v \), even if the seller (or Nature, for that matter) were to deviate. This assumption ensures that when the seller deviates, this deviation does not lead the buyer to updating beliefs about his own value.\(^{10}\) It also requires that the buyer maintains Bayesian updating even if Nature were to deviate as well.

### 2.5. Equilibrium Assumptions

We can now specify our solution concept. Fix an arbitrary triple \((\sigma, \gamma, IS)\) and consider the corresponding \( \mu, P^*_\sigma,\gamma,IS \) induced by it. Let \( h^S_t, h^N_t, h^B_t \) denote representative decision nodes for the seller, Nature, and buyer, respectively, at time \( t \). We say that the buyer’s strategy is sequentially rational given \( \mu \) if, for all \( t \) and \( h^B_t, \sigma(h^B_t) > 0 \) implies:

\(^{10}\)Otherwise, one could construct equilibria whereby a deviation is deterred by the buyer adopting a belief that \( v = 0 \) with probability 1, even if this is outside of the support of \( F \).
\[ \mathbb{E}_{v \sim F}[v - p_t \mid h_B^t] \geq \mathbb{E}_{\mu_B, P^*_S, \gamma, IS} \left[ \max_{\tau : t < \tau \leq T} \delta^\tau \mathbb{E}_{v \sim F}[v - p_\tau \mid h_B^\tau] \mid h_B^t \right]. \]

and \( \sigma(h_B^t) < 1 \) implies this inequality is flipped. Note that the buyer can determine the left hand side of this inequality simply by observing \( h_B^t \); the right hand side requires the buyer taking an expectation over future (strategic) variables, which explains the \( \mu(h_B^t) \) subscript.

If the buyer purchases at some time \( s \) at a price of \( p_s \), then from the perspective of time \( t < s \) the seller obtains payoff \( \delta^{s-t} p_s \). Given \( \gamma \), let \( \gamma_t \) denote the corresponding seller strategy at time \( t \); similarly define \( IS_t \). The seller’s pricing strategy is sequentially rational given \( \mu \) if, for all \( t \) and \( h_S^t \), the seller chooses \( \gamma_t(h_S^t) \) to maximize the expectation of:

\[
p_t \mathbb{P}_{\mu_S, P^*_S, \gamma, IS}[d_t = 1 \mid h_S^t, \hat{\gamma}, p_t] + \\
\mathbb{E}_{\mu_S, P^*_S, \gamma, IS} \left[ \sum_{k=t}^{T} \delta^{k-t+1} p_{k+1} \mathbb{P}_{\mu_S, P^*_S, \gamma, IS}[d_{k+1} = 1 \mid h_S^{k+1}, \gamma_{k+1}, p_{k+1}] \mid h_S^t, \hat{\gamma}, p_t \right],
\]

over \( \hat{\gamma} \); recall that \( d_t \in \{0, 1\} \) denotes the buyer decision at time \( t \).

At this point, we again note that so far, all that matters for determining the seller’s optimal choice at time \( t \) is the distribution over future histories given the current history, for \( s > t \). To close the model, a Bayesian approach would require us to specify a distribution over the information arrival process as a function of \( h_S^t \) and \( h_B^t \), subject to restrictions associated with update.

Instead, we impose the following requirement, the substance of our approach: We say an information arrival process is sequentially worst-case given \( \mu \) if, for all \( t, S_t(h_N^t), I_t(h_N^t) \) minimizes:

\[
p_t \mathbb{P}_{\mu_N, P^*_N, \gamma, IS}[d_t = 1 \mid h_N^t, \hat{S}, \hat{I}] + \\
\mathbb{E}_{\mu_N, P^*_N, \gamma, IS} \left[ \sum_{k=t}^{T} \delta^{k-t+1} p_{k+1} \mathbb{P}[d_{k+1} = 1 \mid h_N^{k+1}, S_{k+1}, I_{k+1}] \mid h_N^t, \hat{S}, \hat{I} \right],
\]

taken over \( \hat{S}, \hat{I} \).

**Definition 1.** Let \( \sigma, \gamma, IS \) denote strategies for the buyer, seller, and Nature (respectively) and let \( \mu \) be a belief system induced by them satisfying the assumptions in Section 2.4. This quadruple forms an equilibrium if and only if:

- \( \sigma \) is sequentially rational for the buyer,
- \( \gamma \) is sequentially rational for the seller,
- \( IS \) is sequentially worst-case.

11
We say that the quadruple forms an equilibrium without Nature rationality if the first two bullets hold (whereas the third need not).

Solving for equilibrium outcomes is the primary focus of this paper. Note that there will typically be a plethora of equilibria without Nature rationality, since IS can be specified arbitrarily.

3. THE $T = 2$ CASE WITH UNIFORM VALUES

We first describe equilibrium in the special case where $F = U[0, 2]$ and there are only two periods to sell. We articulate how the sequentially worst-case objective influences the selection of buyers, while also illustrating how possible dynamic consistency issues might arise. Sections 3.1 and 3.2 discuss the take-aways from this special case for our two main messages. Missing formal details are provided in the proof of Theorem 1 in the Appendix.

Given any discount factor $\delta$, the seller’s equilibrium prices are:

$$p^*_1 = \frac{(2 - \delta)^2}{8 - 6\delta}, \quad p^*_2 = \frac{(2 - \delta)}{8 - 6\delta}.$$ 

In period $t \in \{1, 2\}$, Nature informs the buyer whether or not $v > y^*_t$, where:

$$y^*_1 = \frac{4 - 2\delta}{4 - 3\delta}, \quad y^*_2 = \frac{4 - 2\delta}{8 - 6\delta}.$$ 

It can be verified that given the equilibrium prices and information arrival process, the buyers with value $v > y^*_1$ optimally purchase in period 1. The buyers with value $v \leq y^*_1$ is indifferent between purchasing in period 1 or waiting till period 2, and they optimally purchase in period 2 (breaking ties against the seller) if $y^*_1 \geq v > y^*_2$. Finally, the buyers with value $v \leq y^*_2$ is indifferent between purchasing in period 2 or never, and they optimally do not purchase. Thus the seller’s profit $\pi$ can be calculated as (recalling $F(y) = \frac{y}{2}$ for the uniform distribution):

$$\pi = p^*_1 (1 - F(y^*_1)) + \delta p^*_2 (F(y^*_1) - F(y^*_2)) = \frac{(2 - \delta)^2}{4(4 - 3\delta)}.$$

We now walk through the key steps to derive this equilibrium:

**Step One: Nature’s Second Period Choice is Threshold Information** Given any second-period price $p_2$, Nature faces an information design problem a la Kamenica and Gentzkow (2011), since its goal is to choose an information structure to convince the buyer (acting as a receiver) not to purchase. If $p_2 \geq \mathbb{E}[v | s_1]$, then the buyer will not purchase even if Nature provides no information, yielding payoff of 0; so suppose $s_1$ is such that this inequality is not satisfied, and further that $p_2$ is above the minimum of the support of $F(v | s_1)$ (as otherwise Nature cannot
provide *any* information to dissuade the buyer from purchasing). Note that since we assume that the buyer’s expected willingness-to-pay is $\mathbb{E}[v \mid s_1, s_2]$, even if Nature deviates (as per no-signalling-what-you-don’t-know), posterior expectations do not depend on whether Nature’s choices are in equilibrium or not.

Since the receiver has a binary action choice and the set of states forms a continuum, the solution follows from a result due to Kolotilin (2015): Nature informs the buyer whether or not $v > y_{s_1}$, where $y_{s_1}$ satisfies:

$$p_2 = \int_{y_{s_1}} v f(v \mid s_1) dv$$

(4)

For this threshold, Nature’s “less-preferred action” (purchasing) is taken when the state is above the threshold, while the “more-preferred action” (not purchasing) is taken when the state is below the threshold. The condition that pins down the threshold is that the buyer is indifferent whenever taking the more preferred action. Intuitively, if the buyer is recommended to not purchase, then he must believe that his expected value following such a signal is less than or equal to the price. With this threshold information structure, making the buyer purchase with even lower probability requires increasing her expectation about $v$ when recommended to not purchase. But doing so would make the buyer prefer purchasing, so this cannot be done.\(^{11}\)

**Step Two: Find the Second-Period Price** The result in Step One describes Nature’s second-period choice following any $p_2$. Assume, for this computation, that Nature’s first-period information structure choice also involves a threshold, above which the buyer purchases in the first period and below which the buyer delays. In this case, (4) implies that Nature informs the buyer whether his value is above or below $y_2 = 2p_2$, which satisfies the indifference property for $F = U[0, 2]$. If we let $y_1(p_1)$ denote Nature’s equilibrium first-period threshold choice, then since the buyer buys whenever $v > 2p_2$, the seller will choose $p_2$ to maximize $p_2 \left(1 - \frac{2p_2}{y_1(p_1)}\right)$, so that

$$p_2(p_1) = \frac{y_1(p_1)}{4}.$$ 

**Step Three: Determine the First-Period Indifference Point for the Buyer** In equilibrium, a function $p_2(p_1)$ will specify the second-period price given any on-or-off-path choice for $p_1$. Before considering Nature’s problem following an arbitrary $p_1$, we note the following: Since the buyer is indifferent between purchasing and not whenever she does not purchase by Step One, her expected payoff would be unchanged even if she were to *always purchase in the final period*. We conclude

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\(^{11}\)One additional technical detail relative to “standard information design” is that our analysis requires assumptions on off-path buyer behavior; however, Nature can provide strict incentives to the buyer while only increasing seller profit an arbitrarily small amount. Thus, we can again assume the buyer breaks ties against the seller, even off-path.
that a buyer indifferent between purchase and delay at time 1 is also indifferent between purchase at time 1 and purchase at time 2. Given a signal $s_1$, the buyer will be indifferent whenever:

$$E_{v \sim U[0,2]}[v | s_1] - p_1 = \delta \left( E_{v \sim U[0,2]}[v | s_1] - p_2(p_1) \right) \Rightarrow E_{v \sim U[0,2]}[v | s_1] = \frac{p_1 - \delta p_2(p_1)}{1 - \delta}.$$ 

This expression characterizes buyer indifference, even off-path: In Step One, the solution for Nature’s problem did not depend on whether the realized price $p_2$ was an equilibrium choice of the seller. Thus, this relationship must characterize buyer indifference even if the seller deviates from the equilibrium conjecture of $p_1$. As per no-signalling-what-you-don’t-know (in Step One), $E_{v \sim U[0,2]}[v | s_1]$ must be the buyer’s belief after observing $s_1$, whether or not $I_1$ is on-path.

**Step Four, The Most Important Step: Find the First-Period Information Structure** Now consider Nature’s problem in the first period. One potential strategy is the myopic one, which simply tries to minimize the probability the buyer purchases in *that* period (i.e., time 1). Doing so makes the problem identical to the single-period problem, except that now the buyer is indifferent between his actions (purchasing and not) whenever their expected value is $w(p_1)$. Repeating the argument from Step One, the myopic policy thus involves Nature informing the buyer whether or not $v$ is above or below $2w(p_1)$—i.e., $y_1(p_1) = 2w(p_1)$.

The crucial observation is that Nature cannot hurt the seller any more than under this strategy, precisely because information cannot be used to induce additional delay. Specifically, Nature’s last-period strategy necessarily makes the buyer indifferent between purchasing and not, given the price $p_2(p_1)$. As a result, the buyer will be indifferent between purchasing and delaying *if and only if* their expected willingness-to-pay is $w(p_1)$—even if Nature deviates, nothing will influence this possibility, given their last-period choice. Nature’s problem is identical to that in Step One, except with a different value at which the buyer is indifferent between purchasing and not.

**Step Five: Put Everything Together to Find the First-Period Optimal Price** Since we have shown that $p_2 = y_1(p_1)/4$ in Step Two, and $y_1(p_1) = 2w(p_1)$ in Step Four, we have $p_2(p_1) = w(p_1)/2$. Plugging this into the definition of $w(p_1)$ in Step 3, we find that:

$$w(p_1) = \frac{2p_1}{2 - \delta}.$$ 

The seller’s profit, given a price of $p_1$, is:

$$p_1 \left( 1 - \frac{y_1(p_1)}{2} \right) + \delta p_2(p_1) \left( \frac{y_1(p_1)}{2} - \frac{y_2(p_1, p_2(p_1))}{2} \right).$$

Since $y_1(p_1) = 2w(p_1)$ and $y_2(p_1, p_2(p_1)) = 2p_2(p_1) = w(p_1)$, we substitute and rewrite this as:
\[ p_1 \left( 1 - \frac{2p_1}{2 - \delta} \right) + \delta \frac{p_1}{2 - \delta} \left( \frac{2p_1}{2 - \delta} - \frac{p_1}{2 - \delta} \right). \]

Maximizing this expression with respect to \( p_1 \) yields the optimal choice \( p_1^* = \frac{(2-\delta)^2}{8-6\delta} \), as stated before. From this, we can readily verify that the solutions for \( y_1^*, y_2^* \) and \( p_2^* \) are also as claimed.

3.1. Lesson One: Irrelevance of Information

The first take-away from the above calculation is that the prospect of future information does not influence Nature’s problem. When it is the last period, Step One above yields a function \( y(p) \) which maps prices to thresholds that Nature uses. Threshold information structure remains optimal in the first period if Nature were myopic—i.e., if it simply minimizes the probability that the buyer purchases at that time. It is clear that the same function \( y \) would induce the worst-case outcome, so long as its argument “\( p \)” is adjusted to take into account the buyer’s continuation payoff when choosing not to buy in the current period.

The possibility of learning more about \( v \) in the second period does not induce the buyer to delay with higher probability, and therefore justifies Nature’s myopic choice in the first period. Specifically, the value at which the buyer is indifferent between purchase and delay depends only on the conjectured price path, and not the possibility of information arrival. This key observation, as discussed in Step Four, holds due to Nature’s equilibrium behavior in the second period. In contrast, suppose for exogenous reasons the buyer were to learn \( v \) perfectly in the second period (for example), then the first-period worst-case threshold would need to be adjusted. Indeed, a buyer with \( \mathbb{E}[v \mid s_1] \leq w(p_1) \) would then strictly prefer to delay, since the fact that she can avoid purchasing in the second period when \( v < p_2 \) delivers additional surplus. Thus, Nature could hurt the seller by raising the first-period’s threshold to induce additional delay. This point is further discussed in the next section.

3.2. Lesson Two: Implication of Dynamic Consistency

Does the fact that information must be sequentially worst-case limit Nature’s ability to hurt the seller? Let us suppose for the moment that Nature were not subject to sequential rationality constraints. Consider the following information arrival process:

- In the first period, the seller charges some \( p_1^* \) on-path; following any \( p_1 \), the buyer learns whether \( v > \tilde{v}(p_1) \) and buys if and only if it is. We leave \( \tilde{v}(p_1) \) as to-be-specified for now.

- In the second period, the seller charges price \( \frac{\tilde{v}(p_1)}{8} \), the buyer receives no additional information, and the buyer purchases.
• If the seller deviates in the second period to a price $\hat{p}_2 \neq \frac{\tilde{v}(p_1)}{8}$, the buyer learns whether or not $v > 2\hat{p}_2$.

For an appropriate choice of $\tilde{v}(p_1)$, this information arrival process delivers lower expected profit than the sequentially worst-case one we identified. We now explain why. Denote the described information arrival process by $I^*_1, I^*_2$.

Since all remaining buyers in the second period have $v \leq \tilde{v}(p_1)$, the above construction ensures that the seller has no (strictly) profitable second-period deviation following any first period price. Indeed, under $I^*_2$, on-path the seller obtains profit $\frac{\tilde{v}(p_1)}{8} \cdot \frac{\tilde{v}(p_1)}{2}$, where $\frac{\tilde{v}(p_1)}{8}$ is the price and $\frac{\tilde{v}(p_1)}{2}$ is the probability that $v \leq \tilde{v}(p_1)$. But as shown by Step Two above, the best alternative price $p_2$ for the seller is $p_2 = \frac{\tilde{v}(p_1)}{4}$, which delivers the same profit level $\frac{\tilde{v}(p_1)}{4} \cdot \frac{\tilde{v}(p_1)}{4}$ where $\frac{\tilde{v}(p_1)}{4}$ is both the price and the probability that $v \leq \frac{\tilde{v}(p_1)}{2}$, the threshold that Nature would use.

Let us hypothetically consider $\tilde{v}(p_1) = 2w(p_1)$, which by Step Four is Nature’s first-period threshold in the sequentially worst-case information arrival process. Then by the previous paragraph, the seller’s profit would be the same under $I^*_1, I^*_2$ as under the sequentially worst-case information arrival process. However, with first-period threshold $\tilde{v}(p_1) = 2w(p_1)$, the buyer would strictly prefer to delay because the second period price is lower. By contrast, indifference holds for sequentially worst-case information arrival. Explicitly, compare the buyer’s second-period expected payoff under $I^*_2$ when $\tilde{v}(p_1) = 2w(p_1)$ versus under sequentially worst-case information:

$$\int_0^{2w(p_1)} \left( v - \frac{2w(p_1)}{8} \right) dv > \int_{w(p_1)}^{2w(p_1)} \left( v - \frac{w(p_1)}{2} \right) dv .$$

(5)

The left hand side of this inequality is the buyer’s expected second-period payoff under $I^*_2$; the right hand side is the buyer’s equilibrium expected payoff against sequentially worst-case information. Intuitively, the buyer does better under $I^*_2$ since the efficient outcome arises—since the buyer always buys—but the seller obtains the same profit as under sequentially worst-case information arrival. Since there is an efficiency gain but the seller does no better, the buyer must do better.

Therefore, by continuity, the buyer would still prefer delay when told $v \leq \tilde{v}(p_1)$ if we set $\tilde{v}(p_1)$ slightly larger than $2w(p_1)$. In words, a higher first-period threshold is possible if the second-period information structure is $I^*_2$ rather than the sequentially worst-case one. A higher threshold means more delay, and more delay makes the seller worse off due to discounting (optimal price in the second period is always lower than the first period).

Actually, we will see that this example is the worst case for the seller when nature can commit to arbitrary information arrival processes after the seller posts a price in the first period. When $\tilde{v}(p_1)$ is chosen optimally to minimize seller profit (in the sense that nature can commit to an
Figure 1: Comparison of seller profit between (1) sequentially worst-case information and (2) profit from the information arrival processes in Section 3.2, taking $T = 2$ and $v \sim U[0, 2]$ case.

arbitrary information arrival process after the seller chooses the price in the first period, not before), it turns out that when $\delta \geq \frac{4}{5}$, no sale occurs in the first period and the seller’s profit is $\frac{4}{5}$. When $\delta < \frac{4}{5}$, the seller’s worst-case profit is $\frac{(4-3\delta)^2}{64(1-\delta)}$ (see the Appendix for details).

Figure 1 plots, as a function of $\delta$, the profit the seller obtains against sequentially worst-case information (blue curve) to the profit the seller obtains in the equilibrium under this different information process (orange curve). We see that the orange curve is uniformly lower, except when $\delta = 0$ or $\delta = 1$, in which case the seller’s problem is essentially static (with only the first period mattering in the former case, and all sale happening in the second period in the latter case).

Of course, $I^*_1, I^*_2$ is not sequentially worst-case since $I^*_2$ does not minimize the seller’s second-period profit given the on-path price. Following $p_2 = \frac{v(p_1)}{4}$, rather than providing no information, Nature could instead tell the buyer whether $v \leq \frac{v(p_1)}{4} - \varepsilon$ for some small $\varepsilon$. A buyer learning this would strictly prefer to not purchase, hurting the seller.

4. EQUILIBRIUM FOR THE GENERAL CASE

4.1. Arbitrary Finite Horizon

Our analysis in the previous section suggests future information cannot be used to induce additional buyer delay, when time-consistency is imposed. We now articulate this concept more formally.

**Definition 2.** Let $(p_t)_{t=1}^T$ be some arbitrary deterministic weakly decreasing price path with possible randomization at $t = 1$, where

$$w_t(p_t) := \frac{p_t - \delta p_{t+1}}{1 - \delta}.$$
is weakly decreasing in $t$ (taking $p_{T+1} = p_T$). The **fully-myopic information arrival outcome** involves Nature informing the buyer, at every time $t$, if $v > y_t(p_t)$, where $y_t(p_t)$ solves:

$$w_t(p_t) = \mathbb{E}_{v \sim F}[v \mid v \leq y_t(p_t)],$$

(6)

and where, whenever this expectation is induced, the buyer does not purchase.

As discussed in Section 3, the threshold $y_t$ solves the (static) Bayesian Persuasion problem of minimizing the probability the buyer buys at time $t$, given the continuation price path $p_t, \ldots$ and assuming a prior that $v \sim F$. The name “fully-myopic” reflects the idea that the seller’s future profit is not relevant for Nature. Our first main result is the following:

**Theorem 1.** When $T < \infty$, an equilibrium exists, and the seller’s equilibrium payoff is unique. Any sequentially worst-case strategy of Nature implements the same buyer behavior as the fully-myopic information arrival outcome.

The proof of this Theorem actually presents a stronger characterization of equilibrium outcomes—namely, that the seller’s pricing strategy is deterministic and decreasing after the initial period. On the other hand, it stops short of ensuring uniqueness of the equilibrium price path; doing so generally requires the seller to have a single-valued optimization at every time, a common issue in similar finite-horizon problems (e.g., Fuchs and Skrypacz (2013)). A sufficient condition for this is that the derivative of the single-period profit with respect to price is decreasing; this property holds for, e.g., the uniform distribution, and otherwise appears natural. However, given the seller’s indifference in the initial period whenever randomizing, we still obtain uniqueness of the seller’s payoff without any added assumptions.

For our purposes, of greater interest than the price path itself is the form of worst-case information arrival. Theorem 1 shows that the conclusion of Section 3.1 holds for general distributions and time horizons (without, e.g., restricting the seller to deterministic pricing). The intuition essentially matches that articulated in Section 3.3. But a challenge in our exercise is that characterizing outcomes **requires solving for equilibrium strategies in a (three-player) game.** Uniqueness is non-trivial precisely for this reason. On top of requiring us to consider both on-and-off-path strategies as emphasized in Section 3, Theorem 1 rules out a non-fully-myopic choice of Nature sustained using some conjectured future choice of the buyer (and visa versa). To handle this, our proof shows buyer behavior is uniquely determined even off-path, as Nature can perturb information away from fully-myopic by an arbitrarily small amount to induce strict preferences.

We mention two technical issues in our proof beyond those discussed in the $T = 2$ case above. First, the informal construction solved for the second-period information structure **assuming** the first-period information structure were partitional. This property need not hold in equilibrium, as
other information structures can induce buyer indifference following a recommendation to not buy. More significant, however, is that our proof technique in Theorem 1 requires us to consider arbitrary (past) information arrival, as we cannot rule them out a priori. Showing that even in these cases the intuition from Section 3.1 are maintained is significantly more involved.

Second, we also cannot rule out seller randomization. The issue is that backward induction prevents us from imposing assumptions on the buyer’s posterior value distribution at any given time that would be needed to rule out some “early” randomizations. In particular, the posterior value distribution depends endogenously on Nature’s choices. When inducting, we allow for the seller to have randomized early expecting an exotic information structure to arise later. Thus, we first consider the information design problem fixing the seller’s strategy, doing so without assuming it is deterministic. The proof outlines a more general definition of fully-myopic information arrival which accommodates randomization. A consequence of our Theorem, however, is that in any equilibrium, any seller randomization can only occur in the initial period.

4.2. The Gap Case with $T = \infty$

We now turn to the case infinite horizon case. The obvious complication this introduces is that backward induction no longer applies, but we can recover the key intuitions when $v > 0$.

Readers familiar with the bargaining literature may associate the assumption that $v > 0$ with the conclusion that the market clears in finite time—that is, the existence of some period, say $\hat{T}$, by which the buyer has purchased with probability 1. This conclusion, however, requires certain assumptions on the distribution of the buyer’s willingness-to-pay, e.g., Lipschitz continuity around the minimum of the support of the willingness-to-pay distribution (see Gul et al. (1986)). This assumption can easily be violated for the posterior willingness-to-pay distribution conditional on delay, for certain choices of Nature. The question is whether such strategies could be supported in equilibrium—but if the worst-case against such prices paths is not fully myopic, then the buyer can “punish” Nature were it to deviate.

The following Section presents results which show this possibility can not emerge for broad classes of willingness-to-pay distributions. Here, we show that we can alternatively recover uniqueness under an intuitive condition on the buyer’s equilibrium strategy:

**Definition 3.** Let $F_t$ denote a candidate buyer’s posterior belief over $v$ in period $t$. We say an equilibrium $\sigma, \gamma, IS, \mu$ is a **monotone equilibrium** if and only if whenever $F_1 \text{ FOSD } F_2$ for a fixed $t$, $0 \in \sigma_t(F_1, p_t)$ implies $\sigma_t(F_2, p_t) = 0$.

In the special case where $F_1$ and $F_2$ are point-masses on $v$, then monotonicity reduces to the skimming property—i.e., the property is, that the remaining buyers are those with lower willingness-to-pay.
to-pay. Monotonicity requires that if buyer’s perceived distribution of willingness-to-pay becomes uniformly less favorable, eagerness to delay cannot increase.

If the buyer anticipates no further information at period $t$, then her buying decision will only depend on her current posterior, $p_t$, and $E[p_{t+1}]$. Thus, we immediately have the following:

**Corollary 1.** An equilibrium with fully-myopic information arrival outcome is monotone.

Due to the corollary, we can show the existence of monotone equilibria by showing the existence of fully-myopic equilibria. Using the same assumption in [Gul et al. (1986)](gul1986), we recover uniqueness:

**Proposition 1.** Suppose $T = \infty$, $\nu > 0$, and that $F^{-1}$ is Lipschitz-continuous at 0. A monotone equilibrium exists, and the seller’s (monotone) equilibrium payoff is unique. Any sequentially worst-case strategy of Nature implements the same buyer behavior as the fully-myopic information arrival outcome. Furthermore, given any price path which either (i) is on-path or (ii) follows a single deviation of the seller, there exists some finite $\hat{T} < \infty$ such that the market clears by time $\hat{T}$.

The proof of this result shows that monotonicity restores the equilibrium to be one featuring finite-time market clearing. As long as this property holds, sequentially worst-case information will imply that information cannot be used to induce delay. To determine the equilibrium price path, we take Nature to implement the fully-myopic outcome, on-and-off path, as it implies that the following representation of the seller’s value function at time $t$:

$$V(y_{t-1}(p_{t-1})) = \max_{p_t} p_t(F(y_{t-1}(p_{t-1})) - F(y_t(p_t))) + \delta V(y_t(p_t)),$$

with $y_0 = \nu$ and $y_t(\cdot)$ given by (6). Several corollaries regarding the structure of equilibrium—for instance, its stationarity, uniqueness, and absence of on-path randomization after the initial period—follow as immediate corollaries from the existence of this representation by applying results from, for instance, [Fudenberg et al. (1985)](fudenberg1985) and [Gul et al. (1986)](gul1986).

### 4.3. Discussion of the Solution

Theorem 1 and Proposition 1 provide a sharp characterization of sequentially worst-case information arrival. The key economic take-away is that under sequentially worst-case pricing, the

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12Note that this value-function representation is only guaranteed to hold on-path. It is thus a result, rather than an assumption, that this characterizes price-setting behavior. The proof of Proposition 1 (as well as Theorem 1) does not restrict the possible choices of Nature, and hence does not assume a value function of this form.

13We call a candidate equilibrium profile stationary if the seller’s choice at time $t$ depends only on the seller’s belief over the buyer’s posterior belief, $\phi \in \Delta(\Delta([\nu, \pi]))$, and if the buyer’s acceptance decision depends only on $\phi$, $s^t$ and $p_t$. This definition matches the usual one when the buyer knows $\nu$ under an added restriction to beliefs which are truncations of the priors. However, it is not clear how to define “truncations” given arbitrary beliefs over $\nu$. 

prospect of learning does not induce extra delay. Indeed, the myopic policy determines a particular mapping from thresholds to buyer values, say \( y^* \), satisfying \( x = \mathbb{E}_{v \sim F}[v \mid v \leq y^*(x)] \). Given this mapping, the seller’s discounted expected profit from time \( t \) on (taking \( w_0 = \bar{v} \)) is:

\[
\sum_{s=t}^{T} \delta^{s-t} p_s \frac{F(y^*(w_{s-1})) - F(y^*(w_s))}{F(y^*(w_1))}.
\]

By contrast, when the buyer knows \( v \) perfectly, the only difference is that the threshold determining purchase in period \( s \) is just \( w_s \) instead of \( y^*(w_s) \). Since information plays no role in inducing delay, this benchmark reflects outcomes that are indistinguishable from the known-values case (solved in closed-form by, e.g., Fuchs and Skrypacz (2013), under some added assumptions on the value distributions). All differences are captured by \( y^*(x) \), which can be computed from primitives.

5. ALLOWING ARBITRARY INFORMATION ARRIVAL

Theorem \( \Pi \) provides a sharp characterization of sequentially worst-case information arrival—it involves descending partitional thresholds which make the buyer indifferent between purchasing and not were Nature to provide no further information. Here, we consider worst-case information arrival without imposing sequential rationality on Nature. The insight from Section 3.2 (Lesson Two) imply that dropping this restriction can support equilibria with worse seller profit:

**Proposition 2.** The information arrival process minimizing seller payoff, given any price \( p_1 > v \), over all equilibria without Nature rationality, induce second-period outcomes where (a) the buyer receives no information, and (b) the seller chooses price equal to continuation profit. Seller profit is lower than any equilibrium with Nature rationality where the market does not clear at time 2.

The proof of the proposition shows that information arrival processes qualitatively similar to the one described in Example from Section 3.2 do indeed form the worst-case (from the perspective of time 1) without Nature rationality. At time 2, the described information arrival process is no longer worst-case, as information can be provided to dissuade purchase—so, without imposing Nature rationality, time 1 worst-case information will generally differ from the time 2 worst-case.

Note that if the market did clear at time 2 in the sequentially worst-case benchmark, then Nature’s only nontrivial choice would be at time 1; in this case, allowing richer information arrival processes cannot lower the seller’s profit given the price path. However, whenever this condition is violated, sequentially worst-case information arrival leaves additional scope to transfer surplus to the buyer in order to induce additional delay. Our characterization is therefore sharp.

Should this discussion lead us to conclude that the sequentially worst-case benchmark is not a “true worst-case?” Consider the following criterion aimed at addressing this question:
Definition 4. An equilibrium pricing strategy is a **reinforcing solution** if the seller’s equilibrium payoff is the worst-case profit using the same pricing strategy against an arbitrary dynamic information arrival processes.

We view this condition as natural even beyond the scope of our problem; however, we restrict our definition of this concept to our model to maintain focus. We interpret this criterion as reflecting whether a given price path qualifies as a true worst-case. Constructing a pricing strategy while imposing dynamic consistency requires that the seller ignore any information arrival process that is not itself worst-case in the future. Worrying about such information arrival processes may lead the seller to depart from the strategy specified in Theorem 1. However, for a given strategy, it may or may not be that the seller would do worse if Nature could deviate arbitrarily. If not, the price path is a reinforcing solution—and hence, qualifies as a true worst case.

The following condition turns out to be sufficient for Theorem 1’s price paths to be reinforcing:

Definition 5. Let \( y(w) \) satisfy \( w = \mathbb{E}_{v \sim F}[v \mid v \leq y(w)] \). We say that a distribution \( F \) satisfies **threshold-ratio monotonicity** if \( \frac{v}{y(v)} \) is weakly decreasing in \( v \).

Recall that the myopic strategy of Nature involves informing the buyer whether or not their value is above or below \( y(w_t(p_t)) \), where \( w_t(p_t) = \frac{p_t - \delta p_{t+1}}{1 - \delta} \). So, the condition states that the threshold Nature uses to induce an expectation \( w_t \) does not increase that much more slowly than \( w_t \). Intuitively, the definition rules out cases where too much mass is located at the top of the distribution. In this case, a small increase in the threshold used to induce the buyer to delay leads to a larger change in the expectation of \( \mathbb{E}[v \mid v \leq y] \).

Our second main result of the paper is the following:

**Theorem 2.** Suppose the value distribution satisfies threshold-ratio monotonicity. Then an equilibrium pricing strategy against fully-myopic information arrival (in Theorem 1 and Proposition 1) is a reinforcing solution—that is, if the seller uses the outlined strategy, no information arrival process leads to lower expected seller payoff than the fully-myopic one.

The Theorem explicitly solves for Nature’s information structure under the assumption of threshold-ratio monotonicity, and shows that this involves the same information structure choice as in Theorem 1. The key difference between this exercise and that of finding sequentially worst-case information stems from the conjectures the buyer may make regarding information arrival. Nature’s sequential rationality constraints matter as the seller realizes the buyer will not expect information that is not itself sequentially worst-case; in Theorem 2, we fix the price path, but in principle let Nature provide more information to the buyer to induce added delay. We mention that the proof of Theorem 2 uses the fact that the equilibrium price-path is deterministic (after the initial period) but otherwise allows for general seller strategies.
A useful preliminary observation is that given a price path, worst-case information is parti-
tional. But even with this observation, however, substantial work remains since the worst-case
thresholds could differ from those that would emerge from myopic minimization. Note that
myopic minimization requires that whenever the buyer delays, she is indifferent between delay
and purchase. Put differently, threshold information structures could still induce the buyer to strictly
prefer to delay purchase. Nature’s optimization still involves a non-trivial choice of a
threshold for each time period, subject to the buyer’s obedience conditions.

We avoid this issue by identifying a particular adjustment of the partition thresholds in order
to lower discounted profit, whenever some threshold does not induce exact indifference following
the recommendation to not buy. While lowering the threshold induces more sale in that period, we
adjust Nature’s previous period’s threshold so that the buyer’s obedience condition is maintained.
The Appendix verifies that this leads to a loss of profit under threshold-ratio monotonicity.

While the threshold-ratio monotonicity condition appears restrictive, we note that it will
always hold in some neighborhood of the lower bound of the value distribution:

**Proposition 3.** For any differentiable distribution $v \sim f$ in the gap case, there exists some $y^* > v$
such that the distribution of $v$ conditional on being less than $y^*$ satisfies threshold-ratio monotonicity.

We also present the following sufficient condition for threshold-ratio monotonicity, which depends
only on the underlying distribution $F$:

**Proposition 4.** For differentiable $f$, threshold-ratio monotonicity holds if $\frac{w f(w)}{F(w)}$ is decreasing in $w$.

We close this Section by noting that if threshold-ratio monotonicity holds, the restriction to
monotone equilibria in Proposition 1 can be significantly relaxed.

**Definition 6.** We say an equilibrium $\sigma, \gamma, IS, \mu$ is a deterministic equilibrium if and only if the
pricing strategy is deterministic except perhaps at the first period.

Note the actual price path starting at the second period may depend on the realization of the first
period price. Equilibria in the known-values case under the Lipschitz condition all satisfy this
restriction (see Gul et al. (1986)).

**Corollary 2.** A fully myopic equilibrium is a deterministic equilibrium.

**Corollary 3.** Suppose $T = \infty$, $\underline{v} > 0$, and that $F^{-1}$ is Lipschitz-continuous at 0. If $F$ satisfies
threshold-ratio monotonicity, then (the unique) fully myopic equilibrium is the unique deterministic
equilibrium.
6. CONCLUSION

6.1. Interpretations of Reinforcing Solutions

One way of interpreting the reinforcing solution condition is that it reflects the impact of the seller’s misspecification about Nature’s commitment power. Suppose the seller believed information to be sequentially worst case, even though it could in principle actually be unrestricted. Given the incorrect conjecture, we can ask how much (expected discounted) profit the seller is guaranteed when information can be arbitrary. If the price path does not perform any worse, even with arbitrary commitment on the part of Nature, then the solution is reinforcing.

Reinforcing solutions are also appealing as resolving an “optimism-pessimism” exercise:

- A seller chooses a model of how buyers learn about their values, doing so in an optimistic way in order to maximize their own profits.

- After doing this, however, the seller becomes pessimistic and reconsiders, as he worries that perhaps his conjecture was wrong and lacks confidence to say otherwise. The seller abandons a model if some information arrival process could deliver lower expected profit.

While an optimistic seller may want to assume information arrival will deliver high profits, he might reconsider given his lack of understanding. But even a highly pessimistic seller may wonder why this pessimism is inevitable. If a price path is a reinforcing solution, a seller may be willing to adopt it, and could rest assured their profit guarantee would not change even if they were wrong—no matter how pessimistic.

6.2. Outcomes under Equilibrium without Nature Rationality

Part of the motivation for our robust approach is the observation that classical Bayesian approaches have thus far failed to allow for general classes of information arrival possibilities. Here, we argue that insofar as this could be allowed, the benchmark results would likely resemble “anything goes” more than a sharp characterization. Specifically, we observe that allowing for general unrestricted information arrival without imposing some restriction (such as our sequentially worst-case requirement) may yield dramatic departures from the known-values equilibrium.\(^\text{14}\)

We note that with worst-case information arrival, constant prices cannot possibly emerge as an}

\(^{14}\)This exercise is in the spirit of robust predictions; Liu (2022) performs such an exercise when the seller may obtain extra information about the buyer’s value, showing that a rich set of payoffs may emerge in the frequent-offer limit.
equilibrium outcome, as the seller is always tempted to undercut himself to sell to the remaining market. However, information arrival can influence the composition of buyers remaining in the market, thus influencing this temptation. The following result shows that indeed this possibility enables constant-price path equilibria to emerge without Nature rationality:

Proposition 5. Fix $F, \delta$. Suppose the equilibrium outcome when the buyer knows $v$ does not involve purchase at time 1 with probability 1. Then there exists an monotone equilibrium without Nature rationality such that:

- The seller uses a constant price path.
- The seller’s expected payoff is $v^*$, given any $v^*$ less than $E_F[v]$ and larger than the equilibrium payoff identified in Proposition 1.
- The market does not clear in any finite time (i.e., there is no $\hat{T}$ such that the buyer buys before time $\hat{T}$ with probability 1 on-path).

A monotone equilibrium without Nature rationality satisfying the first two points exists when $T < \infty$ as long as $v^*$ is larger than the equilibrium payoff identified in Theorem 1 in this equilibrium the market does not clear before time $T$.

This result highlights (i) the possible equilibrium multiplicity for a fixed information arrival process, and (ii) the lack of a finite time horizon by which the market clears, neither of which holds in the known-values gap case. Proposition 5 shows that if arbitrary information arrival is possible, then severe departures from the known-values predictions can emerge obtained.

The information arrival used to prove Proposition 5 is surprisingly simple—it involves a deviation by the seller triggering a release of information to the buyer, and no information being provided if the seller sticks to the constant price path. This enables an equilibrium where the buyer randomizes, with a probability that induces the seller to stick to the constant price path. Indeed, triggering the release of information can lead to quite bad outcomes for the seller. The key property of this information arrival process is that information is used to shape equilibrium outcomes. The outcome, however, is not fully-myopic. Indeed, if Nature sought to minimize the

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15For a proof sketch of this claim with an infinite horizon even without monotonicity, we note that in those cases, worst-case information arrival involves a single signal provided at time 1 (as this outcome coincides with the seller-commitment outcome, studied in Libgober and Mu (2021)) with no delayed sales. However, if the seller anticipated such an outcome, he would have a profitable deviation to lower the price in the second-period—unless this initial price were $v$, in which case offering a higher price would be a profitable deviation whenever the equilibrium in Proposition 1 did not involve such a price.

16The combination of a constant price path is also distinct, although constant prices would arise in the degenerate case where the market clears at time 1.
seller’s profit within a given period, then given a conjecture of a constant price path of $E_{v \sim F}[v]$, it could inform the buyer whether $v < v - \varepsilon$, inducing profit $\approx 0$.

The take away from this discussion is to underscore that our robust approach has an appealing property, in that we recover much of the usual intuition from the well-studied literature on known-values bargaining. From this perspective, our previous result—that sequentially worst-case information arrival induces an outcome exactly mimics the known-values gap case—is even more striking. It is worth emphasizing that a conclusion as sharp as what we derive need not hold under other formulations of the robust objective (as we discuss in Appendix B). For instance, we discuss one formulation where the seller may never attempt to sell at all, intuitively since the buyer faces very strong incentives to delay whenever any sale is attempted.

6.3. Discussion of the No-Gap Case

While our main analysis focused on either the finite-horizon or gap-cases, many of our main insights apply to the no-gap case as well:

**Proposition 6.** Suppose $T = \infty$, $\underline{v} = 0$. An equilibrium exists where Nature implements fully-myopic information arrival outcome.

Fully-myopic equilibria maximize seller profit across all equilibria given the pricing strategies, intuitively since Nature has the ability to induce the myopic thresholds and no conjecture of the buyer could induce a violation of obedience (i.e., imply that the buyer would no longer wish to follow the action recommendation). However, given that it is typically impossible to impose finite-time market clearing when $\underline{v} = 0$, monotonicity does not suffice to rule out non-fully-myopic information arrival. But the threshold-ratio monotonicity condition on $F$ does still imply that this outcome forms the same class of equilibria with possible multiplicity:

**Proposition 7.** Suppose $T = \infty$, $\underline{v} = 0$. Under threshold-ratio monotonicity, any deterministic equilibrium is a fully-myopic equilibrium.

The proof of these results follow the exact same steps as Corollary 3 and are hence omitted. This proposition highlights the relevance of fully-myopic information arrival even in the no-gap case. Note that in the no-gap case, we no longer obtain unique equilibrium price paths, just as in the known-values case—identical constructions for these apply once nature’s strategy is restricted to being fully myopic.

6.4. Discussion of Model Assumptions

We discuss some of our model’s substantive assumptions. Note that, though Nature maximizes the negative of the seller’s payoff (so that the sum of the seller’s payoff and Nature’s payoff is 0),
strictly speaking this is not a zero-sum game as there are three players (i.e., the buyer as well).

Note that the assumption that \( \text{(I)} \) holds is a restriction on how the buyer updates about \( v \) following a deviation of Nature. The imposition of "no-signalling-what-you-don’t-know" requires that the buyer does not react to a deviation by the seller as suggesting \( v = v \). This avoids, for instance, commitment power emerging due to deviations triggering pessimistic beliefs from the buyer regarding their value, which could not emerge on-path in any equilibrium.

We provide an in-depth exploration of alternative formulations in Appendix B. The explicit use of “Nature" as a player is primarily an expositional device to explain why one might expect dynamic-consistency of the information structure to be maintained. In equilibrium, actions are required to maximize payoffs, given that future actions are determined by the equilibrium profile (and in turn, these actions must satisfy the same requirements). While ours is an incomplete information game (complicating backwards induction), this reasoning drives key intuitions.

Our introduction contrasted our use of a maxmin objective from others in past work. Epstein and Schneider (2007) propose a “rectangularity” condition on the set of priors which characterizes when the maxmin decision rule is dynamically consistent. Aside from the presence of a buyer, our seller also considers the worst-case not over only a set of priors, but a set of information arrival processes. That said, we acknowledge that our exercise is in spirit similar to their proposal. We simply find it more direct, in our setting, to impose dynamic consistency by relaxing assumptions on “Nature’s commitment power,” rather than on the set of information arrival processes.

On this note, we have in mind situations where the buyer can consult any source that comes to mind costlessly. The restrictions on learning reflect the possibilities the buyer may consider or have access to at any time, rather than the effort involved with obtaining them. We share this assumption on how information is generated with most of the informational robustness literature, e.g., Du (2018); Brooks and Du (2021, 2023); Deb and Roesler (2023), among others.

One general issue for robust objectives relates to the timing of the worst-case. We have in mind situations where buyers may have time to respond to a seller’s offer, or where a new offers are made very shortly after rejection (so that enough time would exists for the buyer to obtain additional information while considering the seller’s offer). While the seller can randomize, information in a given period can depend on the seller’s realized actions (i.e., the price). For instance, after seeing a price revision, the home buyer may realize it is less likely their family will move again and hence start looking into school quality. Ke and Zhang (2020) provide decision theoretic foundations for this assumption; various other work has considered cases where price-dependence may emerge, with or without a worst-case objective (Xu and Yang (2022); Liu et al. (2023); Libgober and Mu (2021); Ichihashi and Smolin (2023)), or equivalently, where Nature’s moves can condition on the maximizer’s realized choices (Carroll (2015); Malladi (2023); Guo and Shmaya (2023); Chen (2023); see also the discussion of optimal reserve prices in Bergemann et al. (2017)).
For us, allowing the worst-case to condition on realized seller choices is uniquely more compelling for three reasons. First, allowing information to depend on the randomization but not the realization of this randomization itself evokes commitment, since it requires a seller who observes the outcome of the randomization to not reconsider. Our interest is in cases where no commitment ability is present. In fact, the possible inability to commit to a randomization is traditionally used as motivation for restricting to deterministic mechanisms more generally (See Laffont and Martimort [2002, p. 67] for a discussion of this point). Second, the assumption that information is independent of realized seller choices implies information cannot not react to a seller deviation. But a seller may very well be deterred from deviating by the prospect of directly influencing the informational environment (e.g., attracting attention from unexpected actions), so allowing for at least limited price dependence seems natural. Since we do not see natural restrictions, we take this to be complete. Third, given that our goal is to study a seller completely unsure of how their actions influence buyer learning, it seems natural to start with the case where Nature has as much power as possible. More to the point, these issues strike us as orthogonal to issues related to dynamic consistency; we view it as sensible to start with the simplest setting.

6.5. Final Remarks

In this paper, we propose a new approach to modelling a seller (re)optimizing a dynamic worst-case objective. By treating the adversarial Nature as another player in the dynamic game, we obtain a dynamically consistent worst-case objective, and sharp characterizations of equilibrium outcomes. A seller of a house should not worry about the possibility of delay caused by, for instance, the buyer wanting to wait for the chance to hear advice from a family member. The sequentially worst-case information arrival process simply minimizes the seller’s payoff period-by-period, given the conjectures of how prices will evolve. Even if there are other possible learning processes, in many cases this is immaterial, as no prospective future information would hurt the seller.

Durable goods pricing is a natural first place to study informational robustness while relaxing commitment, as the buyer’s decision simply involves deciding when to purchase. However, our analysis uncovers potentially underappreciated conceptual issues that arise when relaxing commitment under a robust approach more broadly. Toward further clarifying this, Appendix B discusses alternative specifications of the robust objective, concluding that ours leads to the most intuitive and tractable solution for informationally robust sales. We do not claim this conclusion holds in all problems. We simply argue that in our application, the potential for dynamic inconsistency may be less severe than previously thought.

For instance, a seller who considers the worst-case over all information arrival processes but does not realize this might change over time might never attempt to sell at all, even if only moderately patient. See Appendix B.1.
We hope that our work encourages the agenda aimed at filling the gap outlined by Bergemann and Valimaki (2019), by suggesting a way of addressing the dynamic consistency issues involved in worst-case optimization, as outlined in Carroll (2019). On this note, we hope we have provided a template that can be used to extend the reach of the robust approach to dynamic interactions beyond the dynamic pricing. The price path against a myopic information arrival outcome in Theorem 1 illustrates that equilibrium can be fairly tractable if the seller is only concerned about dynamically-consistent information processes. While one may argue that focusing on this restricted worst-case is at odds with the robust objective, we formally showed that this criticism often has little bite in our setting. By introducing the notion of a reinforcing solution, we hope other researchers will be able to derive tractable solutions to other dynamic models, and plausibly argue that such solutions do not compromise the initial motivation for adopting a robust approach.
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A. PROOFS

A.1. Characterizing Fully Myopic Information Arrival as Sequentially Worst-Case

Proof of Theorem 1. We first argue that it is without loss in terms of game outcomes to assume that Nature only provides binary recommendations to the agent.

Lemma 1. Given any history $h^t$ and strategies $IS, \sigma, \gamma$ and their corresponding belief systems, there exist strategies $\tilde{IS}, \tilde{\sigma}, \tilde{\gamma}$ and corresponding belief systems where $\tilde{IS}$ only provides two signals in every period and the outcome under the new profile is equivalent to that of the original profile. In particular, if the original profile forms an equilibrium, so does the new profile.

Proof of Lemma 1. Let $(I_t, S_t) = IS(h^t, r_t)$ denote a realization of an information structure chosen by Nature at period $t$, where $r_t$ is the randomization device. Let $\sigma_t(h^t, I_t, s_t)[1]$ denote the probability the buyer buys after signal $s_t$ at period $t$. We first argue that $\sigma_t$ can be taken to be deterministic without loss of generality. The idea is Nature can do all the randomization for the buyer by providing richer signals. Consider any signal $s_t \in S_t$ such the buyer randomizes following $s^b_t$. Let $(\tilde{I}_t, \tilde{S}_t)$ be a new information structure such that $s^b_t$ is realized with probability $\sigma(h^t, I_t, s_t)[1]$ and $s^n_t$ with complementary probability, conditional on $s_t$ is realized, and let $\tilde{\sigma}_t$ otherwise coincide with $\sigma_t$ except the buyer always buys following $s^b_t$ and never buys following $s^n_t$.

Consider strategy $\tilde{IS}$ where Nature chooses $(\tilde{I}_t, \tilde{S}_t)$ instead of $(I_t, S_t)$, but otherwise all actions are the same as in $IS$; suppose the buyer’s strategy $\tilde{\sigma}$ at any information set other than following signals $s^b_t$ or $s^n_t$ to be as the same as in $\sigma$. It is clear that the game outcome under the new profile is the same as (equivalent to) that of the original profile. Note we can perform this operation for any history $h^t$ and induct on $t$, so it is without loss of generality to assume $\sigma$ is deterministic.

Thus taking $\sigma$ to be deterministic at every information set, for any information structure of Nature $I_t = IS(h^t, r_t)$, we can find a replacement $\tilde{I}_t : [\underline{v}, \overline{v}] \rightarrow \{s_t, \overline{s}_t\}$, with the buyer buying following $s_t$ and not following $s^n_t$—i.e., with Nature pooling all signals where the buyer takes the same action.

We define a new strategy $\tilde{IS}$ of Nature such that $\tilde{IS}(h^t, r_t) = (\tilde{I}_t, \{s_t, \overline{s}_t\})$, and

$$\tilde{IS}(h^t, \tilde{I}_t, \tilde{s}_t, p_{t+1}, r_{t+1}) = ((I_t, S_t); IS(h^t, I_t, s_t, p_{t+1}, r_{t+1})).$$

In other words, Nature gives the minimum information for the buyer to make the same decision at period $t$, and gives the original information which it is supposed to give the buyer at period $t$ together with the information which it is supposed to give the buyer at period $t + 1$ at period $t + 1$. Define the buyer’s new strategy accordingly. It is clear that the game outcome under the new profile is the same as (equivalent to) that of the original profile. It follows that at arbitrary
history $h'_2$, we can assume Nature provides only two signals. Note that we can induct on $t$ and this induction does not rely on the game’s horizon being finite.

Fix a history $h'_1$, let $v_{h'_1}$ denote the infimum of the support of $F_{h'_1}$, the remaining buyer’s posterior value distribution given a particular history $h'_1$ given the buyer has not bought. Note because at every $t$ a certain mass of buyers may leave, and we will do all the calculations in terms of unconditional probability, so here for $F_{h'_1}(\cdot)$ we mean a generalized cumulative distribution function such that $F_{h'_1}(+\infty)$ may not equal 1, but other properties of a cumulative distribution function will still hold. We now define threshold information structures given a particular history. Define $y_{h'_1}(x)$ as the solution to:

$$\inf\{y \geq v_{h'_1} \text{ such that } x \leq \mathbb{E}[v \mid h'_1, v \leq y]\}. \tag{8}$$

Note that since the prior $F$ is continuous, at any period besides possibly one at which the buyer is expected to have purchased with probability 1, we can assume that the distribution $F_{h'_1}$ is continuous everywhere. This holds due to the fact that we take the set of possible signal realizations to be finite, although applying Lemma[1] we note that the same holds more generally under the restriction to information structures which provide binary recommendations. But since $F_{h'_1}$ is continuous everywhere, it follows that the inequality in (8) will either hold with equality, or be equal to $v_{h'_1}$. Furthermore, $y_{h'_1}(\cdot)$ is continuous in $x$ since $F$ is continuous.

Note the seller cannot observe the information structure or signal realization. So the seller cannot detect any deviation except the market should have been cleared by the end of period $\bar{T}$ at some $\bar{T} < T$ on the equilibrium path but the game proceeds. In this case, the seller will know that some other players have deviated, and we need to define the belief system and strategies of the seller. Note by [Fudenberg et al., 1985], if the seller charges $v$, then the market has to be cleared in any equilibrium. Thus under any seller’s equilibrium belief this event has probability 0, and we can let the seller charge the price $v$ again.

We now turn to the induction in Nature’s problem. We use $w_t = w_t(h'_t+1)$ to denote the buyer cutoff type indifferent between buying and delaying (which is decided by strategies in the future). We also define two correspondences

$$M_t(F_{h'_t}) = \arg\max_{p_t} V^t(F_{h'_t}(\cdot))$$

and

$$m_{h'_t}(b) = \arg\max_{p_{t+1}} V^{t+1}(F_{h'_t}^b(\cdot))$$

where $F_{h'_t}^b(\cdot)$ is the lower part of $F_{h'_t}(\cdot)$ truncated at $b$, and $V^t(\cdot)$ is the seller’s expected payoff at
period $t$\textsuperscript{18}

**The Base Case: Nature’s Last-Period Problem** Consider $t = T$. If market has already been cleared, then any strategy is optimal for Nature. In particular, Nature can provide the trivial threshold information structure which corresponds to no information. Now suppose there is a remaining posterior distribution of buyers $F_{h_T^1}$, if the realized $p_T \leq y_{h_T^1}$, then the buyer must buy with probability 1 in equilibrium, irrespective of Nature’s choices. In this case, the claim that Nature uses a myopic threshold such that the buyer’s payoff is the same as if no information were provided is immediate. So, suppose $p_T > y_{h_T^1}$. Following the argument from Section 3 Step One, we note that the worst-case information structure takes a threshold form in this period, with threshold $y_{h_T^1}(p_T)$. Since this information structure involves either (1) the buyer being indifferent between purchasing and not when recommended to not purchase, or (2) Nature’s choices having no influence on buyer behavior, we have that the buyer’s expected payoff in this case is the same as if no information were provided, as claimed. Thus, using sequential rationality of Nature in the final period, given an arbitrary history $h_{T-1} = (h_T^1, p_T)$, the final period information structure will be a threshold-form, and hence the myopic policy.

For later reference, we also note that Nature can ensure profit arbitrarily close to this worst-case profit while ensuring that the buyer has strict incentives to purchase, for any $p_T > y_{h_T^1}$, by informing the buyer if $v \leq y_{h_T^1}(p_T) - \varepsilon$ for $\varepsilon$ small. This observation will be useful later.

**The Base Case for the Seller’s Strategy** Now consider history $p_1, \ldots , p_{T-1}$. If according to $IS$ and $\sigma$ (Nature and the buyer play according to equilibrium strategies with possible seller deviation) the market should have been cleared but the game proceeds, then the seller charges $v$ again. If the market should not have been cleared and is indeed not cleared, then the only rational belief is that Nature and the buyer have played according to $IS$ and $\sigma$, with remaining posterior distribution of buyers $F_{h_T^1}$ (note the seller’s belief of $h_T^1$ may not be the actual $h_T^1$, but we slightly abuse the notation here). By the optimal following strategy of Nature in this case, for any $p_T$ in the support of the seller’s optimal strategy $\gamma_T$, we have:

$$p_T \in M_T(F_{h_T^1}) = \arg\max_{p_T} V_T(F_{h_T^1}(\cdot)) = \arg\max_{p_T} p_T(F_{h_T^1}(+\infty) - F_{h_T^1}(y_{h_T^1}(p_T))).$$

Since $F_{h_T^1}(\cdot)$ and $y_{h_T^1}(\cdot)$ are continuous, and if we equip the space of $F_{h_T^1}(\cdot)$ with weak topology (the topology induced by Lévy metric), then by Berge’s Maximum Theorem, $V_T(F_{h_T^1}) = \max p_T(F_{h_T^1}(+\infty) - F_{h_T^1}(y_{h_T^1}(p_T)))$ is continuous in $F_{h_T^1}$. And $M_T(F_{h_T^1})$ is non-empty and com-

\textsuperscript{18}Note since we have not proved any uniqueness result, these value functions $V(\cdot)$ may not be uniquely defined. Thus when we say $V(\cdot)$ in the proof, we are referring to a particular equilibrium.
pact. Furthermore, for any $p_T$ in the support of the seller’s optimal strategy $\gamma_T$, we must have $w_{T-1} \geq p_T$. This is because on the equilibrium path the remaining buyers must have posterior distribution expectation less or equal to $w_{T-1}$. If $p_T \geq w_{T-1}$, Nature can provide no information to the buyer, and the buyer will not buy and the seller will have a profit 0. This also immediately gives us

$$w_{T-1} - p_{T-1} = \mathbb{E}_{\gamma_T}[\delta(w_{T-1} - p_T)]$$

because at $t = T - 1$ there is no ex-ante information value for the buyers to delay, and thus $w_{T-1}$ defined in this way is the unique cutoff anticipating the future play. This also further gives $p_{T-1} \geq \mathbb{E}_{\gamma_T}[p_T]$.

**The Inductive Step for Nature** Now let’s consider $t = k < T$ and again the market has not been cleared. Now suppose there is a remaining posterior distribution of buyers $F_{h_{k+1}}$, by the same argument if the realized $p_k \leq w_k$, then the buyer must buy with probability 1 in equilibrium irrespective of Nature’s choices. Suppose this is not the case, by the inductive hypothesis, we assume that Nature’s future equilibrium strategy uses the myopic policy thresholds corresponding to $w_{k+s}$ for all $s = 1, 2, \ldots$, noting that these (by induction) give the buyer the same expected payoff as-if Nature provided no information. Thus we know the unique cutoff is defined by

$$w_k - p_k = \mathbb{E}_{\gamma_k+1}[\delta(w_k - p_k+1)].$$

We also assume that Nature has a continuation strategy which delivers profit arbitrarily close to the fully-myopic information arrival outcome profit, while providing the buyer strict incentives to not purchase whenever recommended not to do so, as we saw held in the base-case as well.

Next let’s consider Nature’s equilibrium information structure $(I_k, \{s_k, \bar{s}_k\}) = IS(h_k^1, p_k, r_k)$. If $\mathbb{E}[v \mid h_k^1, I_k, s_k] > w_k$, then the buyer must strictly prefer to buy at $t = k$; with the reverse inequality the buyer would strictly prefer to delay. In particular, this observation holds due to the observation that in equilibrium, Nature’s future equilibrium information structure choices do not influence the expected payoff the buyer. If $\bar{s}_k$ occurs with positive probability, then $s_k$ must lead to expected value exactly $w_k$. To see this, suppose not. Consider a new choice of information structure $(\tilde{I}_k, \{\tilde{s}_k, \tilde{\bar{s}}_k\})$ which following a realization of signal $\bar{s}_k$ (recalling that this signal involves buyers purchasing), changes the signal realization to be $s_k$ with probability $\varepsilon$. For $\varepsilon$ sufficiently small, we still have $\mathbb{E}[v \mid h_k^1, \tilde{I}_k, \tilde{s}_k] < w_k$. In the period $k + 1$, Nature then separate $I_k^{-1}(s_k)$ and $I_k^{-1}(\bar{s}_k) \setminus I_{k+1}^{-1}(\bar{s}_k)$ in period $k + 1$ and reveal fully-myopic information arrival policy for each population later on. If so, the population $I_k^{-1}(s_k)$ will act exactly the same as under the original Nature’s strategy, and therefore the revenue collected from them is the same (the seller’s future strategy is fixed because he cannot observe the information structure chosen). But given the
replacement, the population \( I_k^{-1}(s_k) \setminus I_{k-1}(s_k) \) no longer purchase in period \( k \).

Given the continuation play—which, even if the buyer only delays when strictly profitable to do so, can be made arbitrarily close to what is achieved under the fully-myopic information arrival policy—the profit can be no larger than if these buyers were to buy with probability 1 in the subsequent period. In this case, the seller’s realized profit on \( I_k^{-1}(s_k) \setminus I_{k-1}(s_k) \) is at most \( \delta \cdot E_{\gamma_{k+1}}[p_{k+1}] \). Since any price \( p_t \) that is charged with positive probability must have \( p_k \geq E_{\gamma_{k+1}}[p_{k+1}] > \delta E_{\gamma_{k+1}}[p_{k+1}] \), we have that this deviation would be profitable for Nature.

We can now show that the seller’s profit is minimized when Nature reveals the myopic policy threshold with threshold \( y_{h_{1}^{k}}(w_k) \) (and the buyer breaks indifference against the seller). Note that if \( w_k \geq E[v | h_{1}^{k}] \), in this case Nature optimally only provides a single signal \( \bar{s}_k \), corresponding to no information.

If instead \( w_k < E[v | h_{1}^{k}] \), then \( \bar{s}_k \) must occur with positive probability. Thus, \( \bar{s}_k \) leads to expected value \( E[v | h_{1}^{k}, I_k, \bar{s}_k] \) exactly \( w_k \). We claim that \( \bar{s}_k \) must correspond to all the buyer types below the myopic policy threshold with threshold \( y_{h_{1}^{k}}(w_k) \). Suppose this is not the case; then we can find positive measure of \( v' \) in \( I_{k-1}(s_k) \) and \( v'' \) in \( I_{k-1}(\bar{s}_k) \) such that \( v' > v'' \). If Nature were to “swap” \( v' \) and \( v'' \) with small probability, then the expected value following the modified \( \bar{s}_k \) would still exceed \( w_k \), leading to the same buyer action. Moreover, the entire posterior distribution following the modified \( \bar{s}_k \) is shifted down in the FOSD sense, so profit is weakly decreased (because pricing strategy is fixed since deviation cannot be detected by the seller). Now since the expected value following the modified \( \bar{s}_k \) is strictly less than \( w_k \), there is room for further reducing the profit using the scheme as described above. Hence the desired contradiction. Lastly, we note that as before, Nature can lower the threshold by an arbitrarily small amount to provide strict incentives to the buyer.

From here, we know \( w_k \) is defined by

\[
w_k - p_k \in \delta[w_k - \tilde{m}_{h_{1}^{k}}(y_{h_{1}^{k}}(w_k))],
\]

where \( \tilde{m}_{h_{1}^{k}}(\cdot) \) is the convexification of \( m_{h_{1}^{k}}(\cdot) \). The above is equivalent to

\[
p_k \in (1 - \delta)w_k + \delta \tilde{m}_{h_{1}^{k}}(y_{h_{1}^{k}}(w_k)).
\]

First note by induction hypothesis we already know \( m_{h_{1}^{k}}(b) \) is non-empty and compact, and

\[
m_{h_{1}^{k}}(b) = \arg \max p_{k+1}(F_{h_{1}^{k}}(b) - F_{h_{1}^{k}}(y_{h_{1}^{k+1}}(w_{k+1}(p_{k+1})))) + \delta V^{k+2}(F_{h_{1}^{k}}(\cdot))
\]
It is easy to see

\[ p_{k+1}(F_{h_1}^k(b) - F_{h_1}^k(y_{h_1}^{k+1}(w_{k+1}(p_{k+1})))) + \delta V^{k+2}(F_{h_1}^{y_{h_1}^{k+1}}(\cdot)) \]

satisfies single crossing property in \((p_{k+1}; b)\) because of the \(p_{k+1}(F_{h_1}^k(b))\) term, and thus by the monotonicity theorem of Milgrom and Shannon [1994], we know \(m_{h_1}^k(b)\) is non-decreasing in strong set order w.r.t to \(p_{k+1}\). Furthermore, we claim \(m_{h_1}^k(b)\) and \(m_{h_1}^k(b')\) can intersect at most one point for \(b \neq b'\). This is because if \(p_{k+1}\) and \(p'_{k+1}\) are two common maximizers, then

\[
p_{k+1}(F_{h_1}^k(b) - F_{h_1}^k(y_{h_1}^{k+1}(w_{k+1}(p_{k+1})))) + \delta V^{k+2}(F_{h_1}^{y_{h_1}^{k+1}}(\cdot)) = p'_{k+1}(F_{h_1}^k(b) - F_{h_1}^k(y_{h_1}^{k+1}(w_{k+1}(p'_{k+1})))) + \delta V^{k+2}(F_{h_1}^{y_{h_1}^{k+1}}(\cdot))
\]

and

\[
p_{k+1}(F_{h_2}^k(b') - F_{h_2}^k(y_{h_1}^{k+1}(w_{k+1}(p_{k+1})))) + \delta V^{k+2}(F_{h_1}^{y_{h_1}^{k+1}}(\cdot)) = p'_{k+1}(F_{h_2}^k(b') - F_{h_2}^k(y_{h_1}^{k+1}(w_{k+1}(p'_{k+1})))) + \delta V^{k+2}(F_{h_1}^{y_{h_1}^{k+1}}(\cdot))
\]

Then subtracting them we have

\[
(p'_{k+1} - p_{k+1})(F_{h_1}^k(b') - F_{h_2}^k(b)) = 0
\]

which is impossible. This result together with non-decreasing in strong set order we know what for any \(b' > b\), we must have

\[
\min m_{h_1}^k(b') \geq \max m_{h_1}^k(b),
\]

which means \(\tilde{m}_{h_1}^k(b)\) is a single compact interval (including a singleton) and each \(\tilde{m}_{h_1}^k(b)\) is disjoint with others. This together with the fact that \(y_{h_1}^k(w_k)\) is continuous and strictly increasing in \(w_k\) means there is a unique function \(w_k(p_k)\) satisfying

\[
p_k \in (1 - \delta)w_k + \delta \tilde{m}_{h_1}^k(y_{h_1}^k(w_k)).
\]

and this \(w_k(p_k)\) is continuous and non-decreasing in \(p_k\). This means that after the buyer and Nature observes \(p_k\), the buyer’s cutoff is uniquely pinned down by this fixed point argument induced by rationality of the buyer and Nature.

This further eliminates all the randomization in period \(k + 1\), because to induce a cutoff \(w_k = c\), the seller can always charges \(p_k = \max w_k^{-1}(c)\). Compared with other \(p \in w_k^{-1}(c)\), this will have
no affect on future play and today’s buying population but will maximize today’s profit. From another perspective, let
\[ p_{k+1}^{\text{max}} = \max\{p \mid p \in \text{supp}\gamma_{k+1}\}. \]
At \( t = k \), we have \( w_k = (p_k - E_{\gamma_{k+1}}[p_{k+1}])/(1 - \delta) \). Now we consider price \( \hat{p}_k \) defined by
\[ (\hat{p}_k - p_{k+1}^{\text{max}})/(1 - \delta) = w_k, \]
then \( y_{h_t}^{k+1}(w_k) \) is the same, but since \( \hat{p}_k > p_k \), the seller’s profit at \( t = k \) increases. Hence, we have improved upon the stochastic strategy with a deterministic one.

Thus, we can change
\[ w_k - p_k = E_{\gamma_{k+1}}[\delta(w_k - p_{k+1})] \]
to
\[ w_k - p_k = \delta(w_k - p_{k+1}). \]
This is similar to the result in Fudenberg et al. (1985) and Gul et al. (1986).

The Inductive Step For the Seller Now we consider the seller’s pricing strategy at period \( t = k \). Now consider history \( p_1, \ldots, p_{k-1} \). If according to \( IS \) and \( \sigma \) (Nature and the buyer play according to equilibrium strategies with possible seller deviation) the market should have been cleared but the game proceeds, then the seller charges \( \nu \) again. If the market should not have been cleared and is indeed not cleared, then the only rational belief is that Nature and the buyer have played according to \( IS \) and \( \sigma \), with remaining posterior distribution of buyers \( F_{h_t}^k \) (note the seller’s belief of \( h_t \) may not be the actual \( h_t \), but we slightly abuse the notation here). By the optimal strategy of Nature in the future, for any \( p_k \) in the support of the seller’s optimal strategy \( \gamma_k \), we have:

\[ p_k \in M_k(F_{h_t}^k) = \arg\max_{p_k} [p_k(F_{h_t}^k(\infty) - F_{h_t}^k(y_{h_t}^k(w_k(p_k)))) + \delta V^{k+1}(F_{h_t}^{k+1}(\cdot))], \]
and
\[ V^k(F_{h_t}^k) = \max p_k(F_{h_t}^k(\infty) - F_{h_t}^k(y_{h_t}^k(w_k(p_k)))) + \delta V^{k+1}(F_{h_t}^{k+1}(\cdot)). \]
Note \( F_{h_t}^{k+1}(\cdot) \) is continuous in \( y \), and \( w_k(p_k) \) by induction hypothesis is continuous w.r.t \( p_k \), and \( y_{h_t}^k(w_k) \) is continuous w.r.t \( w_k \), and by induction hypothesis \( V^{k+1}(\cdot) \) is continuous, we know
\[ p_k(F_{h_t}^k(\infty) - F_{h_t}^k(y_{h_t}^k(w_k(p_k)))) + \delta V^{k+1}(F_{h_t}^{k+1}(\cdot)) \]
is jointly continuous in \( p_k \) and \( F_{h_t}^k \). Apply Berge’s Maximum Theorem again, we immediately
have
\[ V^k(F_{h_1^k}) = \max p_k(F_{h_1^k}(+\infty) - F_{h_1^k}(h_{k_1^k}(w_k(p_k)))) + \delta V^{k+1}(F_{h_1^{k+1}}(\cdot)) \]
is continuous w.r.t to \( F_{h_1^k} \), and \( M_k(F_{h_1^k}) \) is non-empty and compact. Furthermore, for any \( p_k \) in the support of the seller’s optimal strategy \( \gamma_k \), we must have \( w_{k-1} \geq p_k \). This is because on the equilibrium path the remaining buyers must have posterior distribution expectation less or equal to \( w_{k-1} \). If \( p_k \geq w_{k-1} \), Nature can provide no information to the buyer, and the buyer will not buy at \( t = k \), in which case mimicking the strategy from time \( k + 1 \) onward, but starting at time \( k \), meaning that the seller’s profit in this case is the same as it would be starting at time \( k + 1 \), divided by \( \delta \), and will yield a gain in seller profit. This means \( p_k \geq w_{k-1} \) cannot be optimal to the seller. Now, given that the time \( k \) information structure will make the buyer indifferent between purchasing and not whenever not purchasing, we have that the buyer payoff is exactly the same as if they were to always purchase at time \( k \) following delay. Therefore, we must have:
\[ w_{k-1} - p_{k-1} = \delta \cdot \mathbb{E}_{\gamma_k}[w_{k-1} - p_k], \]
where \( w_{k-1} \) defined in this way is the unique cutoff anticipating the future play. This also further gives \( p_{k-1} \geq \mathbb{E}_{\gamma_k}[p_k] \).

The last thing we need to verify is that buyer has no profitable deviation. Note if the buyer from waiting to buying then there is nothing to consider since the game basically ends. On the other hand, if the buyer deviates from buying to waiting, since the future price will not change, and her current information is already enough for her to make her optimal decision, this also cannot be profitable.

We now complete the proof by specifying the strategy profile.

- The strategies of the buyer, the seller, and Nature are all deterministic, except possible seller randomization at \( t = 1 \).

- As mentioned, Nature uses a myopic threshold information structure, i.e., informing the buyer whether \( v < y_{h_t^t}(w_t(p_t)) \), in every period, note this \( h_t^t \) can be on or off path, which means even after a deviation from the seller or the buyer.

- Given that we assume without loss\(^{[19]} \) that Nature uses a threshold equilibrium, the seller chooses a price to solve:

\(^{[19]} \)As discussed in the main text, this property need not hold in every equilibrium, although for the purposes of describing seller profit this detail is not significant.
\[
\max_{p_t} \sum_{s=t}^{T} \delta_{s-t} p_s(p_t) \frac{F(y(w_{s-1}(p_t))) - F(y(w_s(p_t)))}{F(y(w_{t-1}))}.
\]

- The buyer buys following any signal where \( v > y_t \) and not otherwise, and reacts to any deviation by assuming Nature uses a threshold strategy.

\[\square\]

**Proof of Proposition** We first prove that in any monotone equilibrium, Nature must use threshold information structure in every period. Suppose for an arbitrary equilibrium, we define a random variable \( \lambda_t \), which is the probability that the buyer is recommended to purchase in period \( t \) on path, \( (\lambda_t \text{ is adapted to the realized price path } p^t) \). We further define \( \lambda_\infty \) to be the probability that the buyer is never recommended to purchase, and \( y_\infty \) to be the expected value conditional on this event. We then construct a threshold process with price-dependent thresholds \( \infty = v_0 \geq v_1 \geq v_2 \geq \cdots \geq v_\infty = 0 \), such that \( v_t \) depends (only) on realized price path \( p^t \), and \( P[v_t < v \leq v_{t-1} | p^t] = \lambda_t \) for every \( t \) and every price history \( p^t \). At each period \( t \), Nature recommends the buyer to buy if \( v > v_t \) and recommends to wait if \( v \leq v_t \).

Let's first consider such a deviation of Nature to use such a threshold information arrival process. Because the seller can only observe the price path, he cannot detect this deviation, and the pricing strategy will not change. And because for every price path \( p^t \), the buyer, if recommended to wait, will face a new posterior which is inferior than the original posterior of the same price path \( p^t \) in the FOSD sense, by monotonicity, the buyers will wait when they are recommended to wait.

Now let’s show this is a profitable deviation of Nature. First note when Nature deviates in this way, for any price history \( p^t \) and any period \( t \), at least the same amount mass of buyers will be deferred as in the equilibrium, so the total social surplus can only decreased. For the buyers who are recommended to buy, they can always have the option to buy or wait, so they cannot be worse off. Thus, the seller’s utility must decrease, and thus Nature’s utility must increase. And generically, this increase should be strict, because at any period \( t \) the remaining posterior should be inferior in the FOSD sense. Thus, we have shown in generically every equilibrium Nature should use threshold information arrival process.

**Step Two** Now we show that if Nature uses threshold information arrival process, then the market will be cleared in finite time. Using the notation before, for any \( b \), let \( F^b \) denote the cdf of
the lower part of the prior $F$ truncated at $b$. We want to show that there exists $b^*$ such that if the seller faces a posterior $F^{b^*}$, he will choose to set a price $v$ to clear the market.

Now suppose the seller faces posterior $F^b$ at $h_t$. Then if the seller charges $p_t$, one feasible strategy for Nature is to disclose all information to the buyer, basically let the buyer know its type. Then we have

$$V(F^b) \leq p_t(F(b) - F(w_t)) + \delta F(w_t)w_t \leq w_t(F(b) - F(w_t)) + \delta F(w_t)w_t$$

because $p_t \leq w_t$. On the other hand, we have $V(b) \geq F(b)v_t$. Note here the value function is defined on a particular equilibrium because there may exist multiple equilibria. Because $F^{-1}$ is Lipschitz-continuous at 0, there exists $q^*$ such that

$$F^{-1}(q) - v \leq Lq$$

for $q^* \geq q \geq 0$. This implies

$$v - v \leq LF(v)$$

for $F^{-1}(q^*) \geq v \geq v_t$.

Let $b \leq F^{-1}(q^*)$, combining them together, we have

$$0 \geq F(b)v - w_t(F(b) - F(w_t)) - \delta F(w_t)w_t$$

$$\geq (1 - \delta)F(w_t)v - LF(w_t)(F(b) - F(w_t)) - \delta F(w_t)LF(w_t)$$

$$\geq F(w_t)((1 - \delta)v - F(b)L + F(w_t)L - \delta LF(w_t))$$

$$\geq F(w_t)((1 - \delta)v - F(b)L - \delta F(b)L)$$

Note the left hand side is positive when $b$ is sufficiently small, which means $w_t = v$, which implies $p_t = v_t$.

Thus, we conclude that there exists $b^*$ such that if the seller faces a posterior $F^{b^*}$, he will choose to set a price $v$ to clear the market.

Now suppose Nature uses a threshold information arrival process on path with thresholds

$$\infty = v_0 \geq v_1 \geq v_2 \geq \cdots \geq v_\infty = 0,$$

By Nature’s sequential rationality, in each $t$ the buyers will wait if they are recommended to do so. Let $y_t$ denote the upped bound of the support of the remaining posterior. we claim that $y_t \leq b^*$ will happen in finite time and the market will be cleared in finite time. Suppose not, because $y_t$
is non-increasing on-path, if the market doesn’t clear in finite time, then the sequence $y_t$ must converge to a $y^* \geq b^*$. Suppose we are at $y < y^* + \epsilon$, because starting at $y$, \{\{y_t\}\} will never drop below $y^*$, we must have

$$V(F^y) \leq \bar{v}(F(y) - F(y^*)),$$

and this can be made arbitrarily small as we decrease $\epsilon$ since $F$ is continuous. On the other hand, we must have

$$V(F^y) \geq \underline{v}F(y),$$

which gives us a contradiction.

We can also give a bound $\bar{T}$ on the time horizon of market clearing. Starting at any $y > b^*$, and any mass $\epsilon > 0$, there exists a finite number $k$ such that $\epsilon$ mass of buyers will leave after $k$ periods. For the sake of contradiction, if this is not the case, we will have

$$V(F^y) \leq \epsilon \bar{v} + \delta^k \bar{v}.$$

It is easy to see that the bound can be made arbitrarily small if we take $\epsilon \to 0$ and $k \to \infty$. But we also have

$$V(F^y) \geq F(y)\underline{v} \geq F(b^*)\underline{v}.$$

Thus, the market will be cleared within $\bar{T} = \lceil k(1 - F(b^*)) \epsilon + 1 \rceil$.

**Step Three** Now because we know the market will be cleared in finite time, we can pin down the equilibrium by backward induction on $t$ and $T$ at the same time as in Theorem 1 and we know this backward induction will end at $T = \bar{T}$. Suppose market will clear the market in $T^* \leq \bar{T}$, then this equilibrium should have the same features as that of a $T^*$ period game, including the uniqueness of equilibrium price path except the first period. And if the seller ever deviates, by sequential rationality, Nature should still use the fully-myopic information structure, and the following game will still end in finite time.

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**A.2. Richer Nature Commitment**

**Proof of Proposition 2** For an arbitrary information arrival process, suppose the market has not been cleared by period 2. Denote the seller’s expected surplus (unscaled by the remaining mass) starting from period 2 by $\Pi_2$, then the seller’s expected surplus is

$$mp_1 + \delta(1 - m)\Pi_2.$$

where $m$ denotes the mass of buyer buying in the first period.

Now suppose nature commits to
1. If the seller charges $\Pi_2$, nature provides no information

2. If the seller charges any price else, nature will do the same as the original strategy.

Then it is easy to see the seller is indifferent between following this original strategy and charging $\Pi_2$, both yielding a profit of $(1 - m)\Pi_2$ given all buyers in the second period will buy.

Now we prove that the buyers who wait to the second period still prefer to wait to the second period under this new information arrival process. This is because by clearing the market by the second period, the total surplus starting at second period will be maximized because additional delay can only decrease total surplus. Because the seller will have the same expected surplus, the buyer’s expected surplus can only increase, and the buyer will become more eager to wait to the second period if she originally would like to do so.

Of course, it is possible that in the first period the buyers who originally want to buy in the first period will delay to the second period under the new information arrival process, but this can only hurt the seller because by delaying the total surplus can only decrease but the buyer surplus cannot decrease because she is always optimizing.

Thus for any information arrival process by nature, we can find another one where the market is cleared by the second period, and the seller is worse off.

It is easy to show further that the first period information structure must be a threshold form, as in our other proof.

\textit{Details for Example in Section 3.2} We can use the result form Proposition 2. First, recall that $\pi^*(\tilde{v}) = \frac{\tilde{v}}{8}$. Since $\mathbb{E}[v \mid v < \tilde{v}] = \frac{\tilde{v}}{2}$, if the buyer learns that $v < \tilde{v}$ in period 1 and does not buy, then the buyer obtains $\frac{3\tilde{v}}{8}$ in the second period. Therefore, the value of $\tilde{v}$ such that the buyer is indifferent between buying at time 1 and delaying purchase to time 2 satisfies:

\[
\frac{\tilde{v}}{2} - \frac{3\tilde{v}}{8} = \frac{8p_1}{4 - 3\delta}.
\]

Suppose that Nature, in the first period, tells the buyer whether her value is above or below $\frac{8p_1}{4 - 3\delta}$. Given this information structure (as well as understanding that the seller will follow the equilibrium strategy), the buyer will delay if told her value is below the threshold and not if it is above the threshold. Let us assume for the moment that this solution involves purchase in each period with positive probability, handling the case where this does not occur separately. Since the probability the buyer’s value is above the first period threshold is $1 - \frac{4p_1}{4 - 3\delta}$ (since $v \sim U[0, 2]$), the seller’s profit can be written:

\[
p_1 \left( 1 - \frac{4p_1}{4 - 3\delta} \right) + \delta \frac{4p_1}{4 - 3\delta} \frac{p_1}{4 - 3\delta} \Rightarrow 1 - \frac{8p_1}{4 - 3\delta} + \frac{8p_1\delta}{(4 - 3\delta)^2} = 0 \Rightarrow p_1 = \frac{(4 - 3\delta)^2}{32(1 - \delta)}.
\]
Profit at this price is:

\[
\frac{(4 - 3\delta)^2}{32(1 - \delta)} \left( 1 - \frac{4(4 - 3\delta)}{32(1 - \delta)} \right) + \delta \frac{4(4 - 3\delta)^2}{(32(1 - \delta))^2} = \frac{(4 - 3\delta)^2(32(1 - \delta) - 4(4 - 3\delta) + 4\delta)}{(32(1 - \delta))^2} = \frac{(4 - 3\delta)^2}{64(1 - \delta)}
\]

Unlike with the previous case, however, we need to check that this solution does indeed involve sale at both periods. Given \(p_1\), we have \(\tilde{v} = 2\) if:

\[
1 - \frac{(4 - 3\delta)^2}{32(1 - \delta)} = \frac{\delta}{4} \Rightarrow \delta = \frac{4}{5}.
\]

So, if \(\delta < \frac{4}{5}\), this scheme involves profit exactly as above. If \(\delta \geq \frac{4}{5}\), all buyers delay to the second period and no sale occurs in the first period, meaning the total profit is \(\delta/4\).

**Proof of Theorem 2** We fix an arbitrary deterministic price path \(p_1, p_2, \ldots\). By Proposition 3 in Libgober and Mu (2021), the worst-case information structure against an arbitrary price path is a threshold process (but in general, not a fully-myopic one). It follows that Nature’s choice of information structure is determined by thresholds \(y_1 \geq y_2 \geq \cdots\), with the buyer purchasing at the first time \(t\) satisfying \(v > y_t\).

Consider any threshold-information arrival process with \(y_1 \geq y_2 \geq \cdots\) (i.e., where the buyer is told at time \(t\) whether \(v \geq y_t\)). Note the incentive compatible constraint at time \(t\) is

\[
\int_v^{y_t} (v - p_t) f(v) dv \geq \sum_{s=t+1}^{T} \delta^{s-1} \left( \int_{y_s}^{y_{s-1}} (v - p_s) f(v) dv \right),
\]

and the profit of the seller is

\[
\sum_{s=1}^{T} \delta^{s-1} \left( \int_{y_{s-1}}^{y_s} p_s f(v) dv \right).
\]

We show the following claim in order to prove the Theorem: at every time \(t\), either

1. \(y_t = y_{t-1}\).

2. the buyer must be indifferent between purchasing and continuing whenever informed that \(v \leq y_t\), namely

\[
\int_v^{y_t} (v - p_t) f(v) dv = \sum_{s=t+1}^{T} \delta^{s-t} \left( \int_{y_s}^{y_{s-1}} (v - p_s) f(v) dv \right).
\]
First let $p_0 = +\infty$ and $t_k$ be the $k$th price on the price path such that $\delta^{t_k-t_{k-1}} p_{t_k} \leq p_{t_{k-1}}$ (note $p_1 = p_1$).

First we prove that for any $k$, it is optimal to set

$$y_{t_k} = y_{t_k+1} = \cdots = y_{t_{k+1}}$$

No matter what $y_{t_k}, y_{t_k+1}, \ldots, y_{t_{k+1}}$ we have from the start, by setting $y_{t_k} = y_{t_k+1} = \cdots = y_{t_{k+1}}$ ($y_{t_{k+1}}$ is fixed), we claim that it is still incentive compatible for the buyer to follow the recommendation at $t_k, t_{k+1}, \ldots, t_{k+1} - 1$. Note because by definition

$$p_{t_k} \leq \delta p_{t_{k+1}} \leq \cdots \leq \delta^{t_{k+1}-t_k} p_{t_{k+1}}$$

we have

$$(v - p_{t_k}) > \delta (v - p_{t_{k+1}}) > \cdots > \delta^{t_{k+1}-t_k} (v - p_{t_{k+1}}).$$

Thus the buyers originally recommended to buy during these periods will have regret not to buy at $t_k$. Now by pooling the types between and for each $k$ will reduce the profit. Of course, because we increase the ‘value’ of waiting, buyers who originally prefer to buy may prefer to wait. However, since buying actions can only happen at $t_1, t_2, \ldots$, and by definition

$$p_{t_1} \geq \delta^{t_2-t_1} p_{t_2} \geq \cdots,$$

so deferring buying can always reduce the profit.

Thus, we’ve reduced the problem to optimize $y_{t_1}, y_{t_2}, \ldots$ subject to

$$\int_v^{y_{t_k}} (v - p_{t}) f(v) dv \geq \sum_{s=k+1} \delta^{t_{s}-t_k} \left( \int_{y_{t_{s}}}^{y_{t_{s-1}}} (v - p_{t}) f(v) dv \right).$$

(12)

We first prove the constraint is binding at $k = 1$ (note $t_1 = 1$). This is immediate because by increasing $y_{t_1}$ until the constraint at $t_1 = 1$ is binding, profit will reduce because $p_{t_1} \geq \delta^{t_2-t_1} p_{t_2}$.

Now we use induction. Suppose we have shown that $y_{t_k}$ is set so that either

1. $y_{t_k} = y_{t_{k-1}}$
2. the buyer is indifferent at time \( t_k \) between purchasing and continuing when given the recommendation to not purchase (i.e., when \( v < y_t \)).

We will now show that the same should be true for \( y_{t+1} \). First without loss of generality, we can assume the second case holds for \( t_k \) namely

\[
\int_{v}^{y_{t_k}} (v - p_{t_k}) f(v) dv = \sum_{s=k+1} \delta^{t_{s}-t_k} \left( \int_{y_{t_s}}^{y_{t_{s-1}}} (v - p_{t_s}) f(v) dv \right). \tag{13}
\]

This is because if this doesn’t hold, then we must have \( y_{t_k} = y_{t_{k-1}} \), and we can go backwards one by one to find the first \( j < k \) such that

\[
\int_{v}^{y_{t_j}} (v - p_{t_j}) f(v) dv = \sum_{s=j+1} \delta^{t_{s}-t_j} \left( \int_{y_{t_s}}^{y_{t_{s-1}}} (v - p_{t_s}) f(v) dv \right). \tag{14}
\]

because we know this holds at least at \( t_1 = 1 \).

We will identify a perturbation of \( y_{t_k} \) and \( y_{t_{k+1}} \) that lowers the seller’s profit if neither of the two hypothesis conditions hold. We note that showing that this perturbation decreases seller profit will make use of threshold-ratio monotonicity.

The particular perturbation we use is the following: For any \( y_{t_{k+1}} \) such that the buyer strictly prefers to continue and \( y_{t_{k+1}} < y_{t_k} \), we increases \( y_{t_{k+1}} \) while lowering \( y_{t_k} \) so that (13) is maintained. Note by doing this, because we keep the constraint at \( t_k \) binding, all the incentive compatible constraints before \( t_k \) still hold. To sign the change in seller’s profit from such a perturbation, we simply implicitly differentiate (10) using (13).

Following this strategy, we write \( y_{t_k}(y_{t_{k+1}}) \) to be the threshold \( y_{t_k} \) that satisfies (13) given Nature’s choice \( y_{t_{k+1}} \). We will determine \( y'_{t_k}(y_{t_{k+1}}) \) by differentiating (13) with respect to \( y_{t+1} \), holding all thresholds other than \( y_t \) and \( y_{t+1} \) fixed. The derivative of the right hand side of (13) with respect to \( y_{t_{k+1}} \), holding fixed \( y_{t_k} \) and \( y_{t_s} \) for \( s > k + 1 \) is:

\[
\delta^{t_{k+1}-t_k} (- (y_{k+1} - p_{t_{k+1}}) + \delta^{t_{k+2}-t_{k+1}} (y_{k+1} - p_{t_{k+2}})) f(y_{k+1}).
\]

Let \( (1 - \delta^{t_{k+2}-t_{k+1}}) \bar{y}_{t+1} = p_{t_{k+1}} - \delta^{t_{k+2}-t_{k+1}} p_{t_{k+2}} \) and rewrite this derivative as:

\[
\delta^{t_{k+1}-t_k} (1 - \delta^{t_{k+2}-t_{k+1}})(\bar{y}_{t+1} - y_{k+1}) f(y_{k+1}). \tag{15}
\]

We now differentiate the indifference condition with respect to \( y_{t_k} \) (instead of \( y_{t_{k+1}} \)), after the term on the right hand side of (13) involving \( y_{t_k} \) is added to the left hand side:
We first claim $y_{t_k} > \bar{v}_{t_k}$. This is because by induction hypothesis, we know at time $t_k$ the incentive constraint is binding. If $y_{t_k} \leq \bar{v}_{t_k}$, then the buyer receives the signal $v < y_{t_k}$ will know that strictly prefer to wait at least to $t_{k+1}$, which is a contradiction.

We are now ready to differentiate (10). Under the particular perturbation listed, since only $y_{t_k}$ and $y_{t_{k+1}}$ adjust, we have it suffices to differentiate:

$$p_{t_k} \left(1 - F(y_{t_k}(y_{t_{k+1}}))\right) + \delta^{t_{k+1}-t_k}p_{t_{k+1}}(F(y_{t_k}(y_{t_{k+1}})) - F(y_{t_{k+1}})) + \delta^{t_{k+2}-t_{k+1}}p_{t_{k+2}}F(y_{t_{k+1}}),$$

as all other terms are constant. Differentiating yields:

$$-p_{t_k} f(y_{t_k}(y_{t_{k+1}}))y'_{t_k}(y_{t_{k+1}}) + \delta^{t_{k+1}-t_k}p_{t_{k+1}}(f(y_{t_k}(y_{t_{k+1}}))y'_{t_k}(y_{t_{k+1}}) - f(y_{t_{k+1}})) + \delta^{t_{k+2}-t_{k+1}}p_{t_{k+2}}f(y_{t_{k+1+1}}).$$

Now, multiply through by $(y_{t_k} - \bar{v}_{t_k})$ (which we recall is positive), and use (17) to eliminate the right hand side wherever it appears in the derivative of profit with respect to $y_{t_{k+1}}$; doing this and factoring out terms, we have that the derivative of profit with respect to $y_{t_{k+1}}$ is proportional to:

$$(-p_{t_k} + \delta^{t_{k+1}-t_k}p_{t_{k+1}})\frac{\delta^{t_{k+1}-t_k}(1 - \delta^{t_{k+2}-t_{k+1}})}{1 - \delta^{t_{k+1}-t_k}}(\bar{v}_{t_{k+1}} - y_{t_{k+1}}) - \delta^{t_{k+1}-t_k}p_{t_{k+1}}(y_{t_k} - \bar{v}_{t_k}) + \delta^{t_{k+2}-t_{k+1}}p_{t_{k+2}}(y_{t_k} - \bar{v}_{t_k}).$$

Dividing by $\delta^{t_{k+1}-t_k}(1 - \delta^{t_{k+2}-t_{k+1}})$ and substituting in for $\bar{v}_{t_k}$ and $\bar{v}_{t_{k+1}}$, we have the change in profit from increasing the $y_{t_{k+1}}$ threshold is proportional to:

$$\bar{v}_{t_k}(y_{t_{k+1}} - \bar{v}_{t_{k+1}}) + \bar{v}_{t_{k+1}}(\bar{v}_{t_k} - y_{t_k}) = \bar{v}_{t_k}y_{t_{k+1}} - \bar{v}_{t_{k+1}}y_{t_k}. \tag{18}$$

First, if $\bar{v}_{t_{k+1}} \geq \bar{v}_{t_k}$, then because $y_{t_k} \geq y_{t_{k+1}}$, this expression is always weakly negative, and we
should always set $y_{t+1}$ as large as possible to decrease profit. Thus, we either have $y_{t+1} = y_t$ or the incentive compatible constraint at $t_{k+1}$ binding.

Now, suppose $\tau_{t+1} < \tau_t$, threshold-ratio monotonicity means that the expression

$$\tau_t \overline{\gamma}_{t+1} - \tau_{t+1} \overline{\gamma}_t$$

is weakly negative where $\overline{\gamma}_{t+1}$ satisfies $E[v \mid v \leq \overline{\gamma}_{t+1}] = \tau_{t+1}$ and $\overline{\gamma}_t$ satisfies $E[v \mid v \leq \overline{\gamma}_t] = \overline{\tau}_t$; indeed threshold ratio monotonicity requires that $\frac{v}{\overline{v}(v)}$ is decreasing in $v$; since $\overline{\tau}_t > \overline{\tau}_{t+1}$ we must have $\frac{\overline{\tau}_{t+1}}{\overline{\tau}_t} \geq \frac{\overline{\tau}_{t+1}}{\overline{\tau}_t}$. Now let’s consider the trajectory of $y_t(y_{t+1})$ using (17). Note we already know $y_{t_k} > \tau_{t_k}$, thus $y'_{t_k}(y_{t+1}) > 0$ when $y_{t+1} < \tau_{t_k}$ and $y'_{t_k}(y_{t+1}) < 0$ when $y_{t+1} > \tau_{t_k+1}$. Let $y^*_{t+1}$ be the largest $y_{t+1}$ possible without violating incentive compatible constraint at $t_{k+1}$ (which is exactly when the constraint binds), it is easy to see the maximum of

$$\tau_t y_{t+1} - \tau_{t+1} y_t$$

will either be achieved at $y_{t+1} = 0$ or $y_{t+1} = y'_{t+1}$ by the sign of $y'_{t_k}(y_{t+1})$. When $y_{t+1} = 0$, the expression is clearly negative. When $y_{t+1} = y^*_{t+1}$, first we immediately have $y_t(y^*_{t+1}) = \overline{\tau}_t$ because if the incentive constraint binds at both at $t_k$ and $t_{k+1}$ then $y_t$ will just be pinned down as this. Now we claim we must have $y^*_{t+1} \leq \overline{\gamma}_{t+1}$. This is because otherwise the information at $t_{k+1}$ will carry value for the buyer receiving $v < y_{t_k}$ at time $t_k$ and they will strictly prefer to delay, violating the inductive hypothesis. As a result, we have

$$\tau_t y^*_{t+1} - \tau_{t+1} y_t(y^*_{t+1}) \leq \tau_t \overline{\gamma}_{t+1} - \tau_{t+1} \overline{\gamma}_t \leq 0,$$

which means the expression is non-negative globally. Thus, an improvement is found by increasing $y_{t+1}$ as much as possible until either $y_{t+1} = y_t$ or the incentive constraint at $t_{k+1}$ binds.

We have shown that for a fixed deterministic pricing path, if $F$ satisfies threshold-ratio monotonicity, then even if Nature has commitment (w.r.t to the buyer not the seller) and sets arbitrary information structures each period and the buyer moves after understanding the information structure every period, at each period $t$ we will always either have $y_t = y_{t-1}$ or the incentive constraint binding.

Now we consider a fully-myopic equilibrium. By seller’s sequential rationality, at each period setting $y_t = y_{t-1}$ will result market clearing; such a strategy is clearly not optimal for Nature. Hence, Nature’s optimal information structure must have the incentive compatibility constraint binding at each period. Thus, we can conclude that the fully myopic equilibrium is an reinforcing solution. □
Proof of Corollary 3. We first show the following Lemma, of independent interest which plays a role in the proof of this Corollary.

**Lemma 2.** For any given deterministic price path \((p_t)_{t=1}^T\), Nature can implement the optimal thresholds information structures described in Theorem 2 sequentially.

**Proof.** Note in each period \(t_k\), since the incentive constraint is binding, by the construction, this means 
\[ \mathbb{E}[v | v \leq y_{t_k}] = \bar{v}_{t_k} \]
and the buyer is indifferent between buying today and waiting to \(t_{k+1}\) even if she conjectures that no information will be provided in the future. On the other hand, for any other \(t\), we must have \(y_t = y_{t_k}\) for some \(t_k\) before, because we know at \(t_k\) the buyer is indifferent, so as time goes by because no information is given between \(t_k\) and \(t_{k+1}\) the buyer can only become more eager to wait (strictly prefer).

\[ \square \]

Now note for any given deterministic price path generated by equilibrium strategy, since the seller cannot detect any deviation, Nature can always deviate to implement the information arrival process sequentially as shown in the previous lemma. In fact, it is always optimal for Nature to do so due to threshold-ratio monotonicity. Thus, in any deterministic equilibrium Nature will use such a strategy. Under this condition, solving the equilibrium requires determining the optimal seller price path under seller’s sequential rationality. The rest of the argument follows the same steps as the previous proof. \[ \square \]

A.3. Distributional Assumptions Yielding Threshold-Ratio Monotonicity

**Proof of Proposition 3.** We consider the derivative of \(\frac{v}{y(v)}\):
\[ \frac{d}{dv} \frac{v}{y(v)} \propto y(v) - v \frac{d}{dv} y(v). \]
Consider the function \(L(y) = \mathbb{E}[v \mid v \leq y]\), noting that \(y(v)\) is the threshold that induces expectation \(v\) and is hence the inverse function of \(L\). We can thus use the inverse function theorem to differentiate \(y = L^{-1}\) as follows:
\[ \frac{d}{dv} y(v) \bigg|_{y=\tilde{v}} = \frac{1}{L'(y)}, \]
where \(y\) is the threshold that leads to \(\mathbb{E}[v \mid v \leq y] = \bar{v}\). As will become important later, we note that \(\lim_{\tilde{v} \to \mu} L^{-1}(\tilde{v}) = v\).

Since \(L(y) = \frac{\int_{y}^{\mu} w f(w) dw}{F(y)}\), we can differentiate the function \(L(y)\) as follows:
\[ L'(y) = \frac{f(y) \left( yF(y) - \left( \int_{\hat{v}}^{y} w f(w) dw \right) \right)}{F(y)^2}. \]

We note that this function shares the same differentiability properties as \( F \) whenever \( y > \hat{v} \). In order to prove the proposition, we study the limit of this expression as \( y \to \hat{v} \). Notice that in the limit as \( y \to \hat{v} \), both the numerator and the denominator approach 0. By L’Hopital’s rule, however, to evaluate this limit, we can differentiate the numerator and the denominator twice to obtain:

\[
\lim_{y \to \hat{v}} L'(y) = \lim_{y \to \hat{v}} \frac{(f(y))^2 + 2F(y)f'(y) + (yF(y) - \int_{\hat{v}}^{y} w f(w) dw) f''(y)}{2(f(y))^2 + F(y)f'(y)}.
\]

However, since \( F(\hat{v}) = 0 \), we have that this limit reduces very simply to \( \frac{1}{2} \).

Returning to the original limit, and recalling that \( \lim_{\hat{v} \to \hat{v}} y(\hat{v}) = \tilde{v} \), we therefore put this together to obtain the following:

\[
\lim_{\hat{v} \to \hat{v}} \frac{d}{dv} \frac{v}{y(\hat{v})} \bigg|_{v=\hat{v}} = \frac{v - \hat{v}}{\frac{1}{2}} = -\hat{v} < 0.
\]

Using the differentiability properties of the distribution, we therefore have that threshold-ratio monotonicity condition is satisfied in some neighborhood of \( \hat{v} \), as desired. \( \square \)

**Proof of Proposition 4** Our goal is to show that \( \frac{w}{y(w)} \) is decreasing in \( v \). Let \( w = y(v) \), then \( v = \mathbb{E}[x \mid x \leq w] = \frac{\int_{x \leq w} x f(x) dx}{F(w)} \) so that

\[
\frac{v}{w} = \frac{\int_{x \leq w} x f(x) dx}{w F(w)}.
\]

The derivative of the RHS with respect to \( w \) is

\[
\frac{\partial(v/w)}{\partial w} = \frac{w f(w) \cdot w F(w) - (w f(w) + F(w)) \cdot (\int_{x \leq w} x f(x) dx)}{w^2 F(w)^2}.
\]

Rearranging, this derivative is non-positive if and only if

\[
\int_{x \leq w} x f(x) dx \geq \frac{w^2 f(w) F(w)}{w f(w) + F(w)}.
\]

The above inequality holds at \( w = \hat{v} \), so a sufficient condition for it to hold at every \( w \) is that
the derivatives of two sides are ordered. That is, we want
\[ w f(w) \geq \left( \frac{w^2 f(w) F(w)}{w f(w) + F(w)} \right)'. \]

We can compute the derivative of \( \frac{w^2 f(w) F(w)}{w f(w) + F(w)} \) to be
\[
(\frac{w f(w) + F(w))}{(w f(w) + F(w))^2} \cdot (2 f(w) F(w) + w^2 f'(w) F(w) + w^2 f(w)^2) - w^2 f(w) F(w) \cdot (2 f(w) + w f'(w))
\]
which simplifies to
\[
\frac{w^3 f(w)^3 + w^2 f(w)^2 F(w) + w^2 f'(w) F(w)^2 + 2 w f(w) F(w)^2}{(w f(w) + F(w))^2}.
\]

This expression is smaller than \( w f(w) \) if and only if
\[
w f(w)(w f(w) + F(w))^2 \geq w^3 f(w)^3 + w^2 f(w)^2 F(w) + w^2 f'(w) F(w)^2 + 2 w f(w) F(w)^2.
\]

After some more algebra, the desired inequality becomes
\[
w^2 f(w)^2 F(w) \geq w^2 f'(w) F(w)^2 + w f(w) F(w)^2.
\]

Dividing both sides by \( w F(w) \), this is equivalent to
\[
w f(w)^2 \geq w f'(w) F(w) + f(w) F(w).
\]

We can further divide both sides by \( F(w)^2 \) to arrive at
\[
w \frac{f(w)^2}{F(w)^2} \geq w \frac{f'(w)}{F(w)} + \frac{f(w)}{F(w)}.
\]

Let \( h(w) = \frac{f(w)}{F(w)} \) with \( h'(w) = \frac{f'(w)}{F(w)} - \frac{f(w)^2}{F(w)^2} \). The above inequality then becomes
\[
wh'(w) + h(w) \leq 0.
\]

Note that \( w h'(w) + h(w) \) is the derivative of \( w h(w) \), so this requires \( w h(w) \) to be decreasing in \( w \). \(\square\)
A.4. Using Learning to Sustain Constant Price Paths and Equilibrium Multiplicity

Proof of Proposition 5. We consider two cases for this proof; we first consider the case where $T = \infty$, and subsequently describe the modified argument for $T < \infty$. In both cases we consider the following equilibrium without Nature rationality:

- On-path, the seller chooses a price equal to the buyer’s expected value, and no information is provided.

- Meanwhile, the buyer randomizes purchase with probability to be specified—in particular, to be specified so that the seller has incentives to follow the equilibrium strategy.

- If the seller deviates, the equilibrium reverts to the worst-case outcome outlined in Theorem 1/Proposition 1


We now prove this profile forms an equilibrium without Nature rationality. It is immediate that following a deviation of the seller, buyer’s strategy forms an equilibrium, by Theorem 1/Proposition 1. The same holds on-path, since the buyer’s purchasing decision does not depend on their value, the on-path distribution of $v$ conditional on not having purchased at time $t$ is simply $F$. Thus, the buyer is indifferent between purchasing and delaying on-path, since both deliver payoff 0, making them willing to randomize. The seller’s continuation strategy following a deviation also forms an equilibrium, by construction. Note that, since we assume the buyer (strictly) randomizes, it is not possible for them to deviate, since all actions occur with positive probability on-path.

Thus all that is left to show is that the seller does not prefer to deviate on-path, for appropriately chosen randomization probabilities. Letting $\pi^*$ denote the profit achieved in the equilibrium from Proposition 1/Theorem 1 the seller obtains at most $\pi^*$ following a deviation; in particular since the buyer’s posterior distribution on-path is always $F$, and since the horizon is infinite, this property holds at every time. Suppose we seek an equilibrium where the seller’s continuation value is $\tilde{v}$ at every point in time, for $\tilde{v} > \pi^*$. In this case, set the buyer’s purchase probability to be $\rho$ in every period, where $\rho$ satisfies:

$$
\tilde{v} = \rho \mathbb{E}_{F}[v] + (1 - \rho) \delta \pi^* \Rightarrow \rho = \frac{\pi^* (1 - \delta)}{\mathbb{E}_{F}[v] - \delta \pi^*},
$$

where $\rho \in (0, 1)$ whenever $\tilde{v} \in (\pi^*, \mathbb{E}_{v \sim F}[v])$

Thus, by charging $\mathbb{E}_{F}[v]$, the seller obtains a higher payoff than what they could obtain from deviating. We thus verify the conditions are satisfied in the proposition: First, the seller uses a constant price path. Second, the profit obtains is the arbitrary $\tilde{v} \in (\pi^*, \mathbb{E}_{v \sim F}[v])$. And lastly, the market does not clear by any finite time; since $\rho$ is constant, the probability the buyer has
not bought at or before time $K$ is $(1 - \rho)^K > 0$.

It is straightforward to see this equilibrium is monotone, as there is a degenerate distribution on-path, and off-path we have buyers with more favorable distributions are more eager to delay.

For the $T < \infty$ case, we define $v_T = \mathbb{E}_F[v]$. Given any $v_{t+1}$ defined with $t < T$, we consider $v_t$ and $\rho_t$ satisfying:

$$v_t = \rho_t \mathbb{E}_F[v] + (1 - \rho_t)\delta v_{t+1}.$$ 

Letting $\pi^*_t$ denote the $T = t$ equilibrium payoff identified in Theorem 1 for this part of the proof, we note that this value is non-increasing in $T$. Thus, the equilibrium in this case can take an identical form above, as long as $v_1 = v^*$ and $\rho_1, v_2, \rho_2, \ldots, v_{t-1}, \rho_{t-1}$, given that $v_T = \mathbb{E}_F[v]$, are such that $v_t > \pi^*_t$ for all $t$. In this case, the seller obtains higher payoff under the constant price path than from deviating, and the buyer is indifferent between purchasing at any time and hence is willing to follow the mixed strategy.

**B. DYNAMICALLY INCONSISTENT INFORMATIONALLY ROBUST OBJECTIVES**

As we hope the analysis in this paper will be useful generally even beyond pricing applications, it may be instructive to discuss precisely which alternative assumptions we could have made instead. We hope this detour delivers some deeper appreciation for our main benchmark, while also clarifying the challenges which may emerge in future work. We articulate alternative benchmarks, and describe why these are less compelling in the informationally robust dynamic durable goods setting. Clearly this conclusion may not be true in other applications, so it is worth mentioning what some alternative approaches could be.

Fully articulating each benchmark formally would take us too far afield; instead, we use examples or simplifications to clarify why each one would have influenced the analysis, thus providing intuition for what the impact of our modelling choices were. Throughout this section, we focus again exclusively on the gap case, otherwise we fully maintain the basic structure of the game we analyze; therefore, Sections 2.1, 2.2 and 2.4 should be understood as applying in their entirety. Instead, we will consider alternative solution notions different from Definition 1.

In all three cases we discuss, we assume the seller chooses prices at each time assuming that Nature commits to arbitrary information arrival processes after the seller chooses his price in

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20In the case of a finite horizon, the proof is identical except in the last period, we assume the buyer purchases with probability $v/\mathbb{E}_F[v]$; here, we note that the seller’s minmax continuation payoff following a deviation is time dependent, although no matter what the time horizon is it is always strictly bounded away from $\mathbb{E}_F[v]$ (indeed, it is always lower than the seller’s static monopoly profit, which is lower than $\mathbb{E}_F[v]$). Accommodating this is straightforward and thus omitted.
each given period. They differ on the following two dimensions:

1. Whether the seller chooses prices anticipating the worst-case will change over time.

Section B.1 considers the naive case, where the seller does not take these changes into account, and thus assumes the \( t \) price is would be part of an equilibrium outcome given the worst-case information arrival process at time \( t \), for all \( t > 1 \); Sections B.2 and B.3 consider sophisticated sellers, who realize the time \( t \) price will be optimal against a different information arrival process than the worst-case at time \( t \).

2. Whether the worst-case information arrival process at time \( t > 1 \) restricts to the information structures Nature would have chosen in equilibrium at time \( \bar{t} < t \).

Section B.3 considers the case where Nature can re-optimize over past information structures; in Sections B.1 and B.2 it cannot.

B.1. Naiveté over Future Actions

In Section 2, we showed that there generally exists an information arrival process and equilibrium under which the seller’s profit is lower than in the main model. Therefore, considering the worst case over the “set of all possible information arrival processes and equilibria” requires the seller to no longer choose a maxmin optimal price at time 2.

An alternative would be to insist that the seller does consider the worst-case over all information arrival processes, but does not realize that this worst-case will change over time, and correspondingly, does not realize that his future choices will be different. This modifies both (a) the “Bayesian updating whenever possible” requirement, and (b) the requirement that information is sequentially-worst case.

Specifically, suppose at every time \( t \), for \( t = 1, 2, \ldots \) the seller chooses \( p_t \) to maximize profit against all possible information arrival processes following price \( p_t \), and sequential equilibria (i.e., seller pricing strategies and buyer purchasing strategies) under any such particular information arrival processes. In addition, we assume that at time \( t > 1 \), the seller restricts Nature’s choices at time \( \bar{t} < t \) to be those that Nature would have chosen given the conjectured equilibrium at time \( \bar{t} \). In other words, the seller chooses \( p_t \) assuming nature commits to the information structure following this price.

Under this assumption, the seller displays naivitée, in the sense that he simply expects himself to take certain actions in the future, and considers a worst-case with respect to those actions, failing to realize that such actions would not be worst-case in the future. The fact that this may
not be a sensible model for a “sufficiently introspective” seller is immediate, since calculating the optimal action in the future would reveal that these are not maxmin optimal.

To better illustrate the ideas of this setting, we describe the $T = 2$ example with uniform $[0, 2]$; in particular, the worst-case information structure is the one described in Section 3.2:

- In the first period, the seller charges some $p_1^*$ on-path; following any $p_1$, the buyer learns whether $v > \tilde{v}(p_1)$ and buys if and only if it is.
- In the second period, the seller charges price $\tilde{v}(p_1)/8$, the buyer receives no additional information, and the buyer purchases.
- If the seller deviates in the second period to a price $\hat{p}_2 \neq \tilde{v}(p_1)/8$, the buyer learns whether or not $v > 2\hat{p}_2$.

The optimal price in the first period given $\tilde{v}(p_1)$ is chosen to minimize the seller’s profit is $p_1 = (4 - 3\delta)^2/32(1 - \delta)$. Thus, in the first period, the on-path outcome is exactly as the example in section 3.2, where nature can commit to an arbitrary information arrival process after the seller charges the price.

However, in the second period, providing no information to the buyer is not worst-case. Thus, instead of charging $\tilde{v}(p_1)/8$ as in section 3.2, the seller will charge $\tilde{v}(p_1)/4$. Once the second period starts, the seller updates his conjecture (realizing nature will not implement the commitment solution in the first period), and thus the seller will charge the optimal price in this subgame, which is exactly $\tilde{v}(p_1)/4$.

One point is that the buyer is indifferent between $p_1$ and $\tilde{v}/8$ given $\tilde{v}$. Thus if the buyer knew the game would proceed to the second period, $p_2$ will be $\tilde{v}/4$, then she would buy in the first period no matter what her recommendation. However, our interest is in describing how the seller determines prices, for a given conjecture of rational buyer behavior, rather than predicting what the buyer would “actually” do; thus, this non-consistent buyer behavior is not our concern.

**Example 1.** Take $T = \infty$ and $v \sim U[0, 2]$. The fully-myopic information arrival outcome with $v \sim U[0, 2]$ coincides with the known-value case with $v \sim U[0, 1]$. The Coasian equilibrium with $v \sim U[0, 1]$ is solved in Gul et al. (1986) and Stokey (1981). In the known-values case with $v \sim U[0, 1]$, the seller’s profit when $\tilde{v}$ is the highest buyer value remaining is given by:

$$\pi^*(\tilde{v}) = \frac{1}{2} \left( 1 - \frac{1}{\delta} + \frac{1}{\delta} \sqrt{1 - \delta} \right) \tilde{v}^2$$

One can verify that $\lim_{\delta \to 1} \pi^*(1) = 0$, as predicted by the Coase conjecture.

21 Note that the known-values case has a unique outcome, for a fixed $\delta$, when $v \sim U[\epsilon, 1]$, which converges to the Coasian outcome as $\epsilon \to 0$. For our purposes, the same point would remain by considering a sufficiently small $\epsilon$. 

58
We now construct a stationary naivete equilibrium. In this equilibrium,

1. At $t = 1$, the seller charges $p_1$, threshold $\tilde{v}$ is revealed.

2. At $t = 2$, the seller charges $p_2$, and all buyers will buy, nature gives no information.

Using that $\tilde{v}$ must make the buyer indifferent between purchasing and not when learning $v < \tilde{v}$, we have:

$$\frac{\tilde{v}}{2} - p_1 = \delta(\frac{\tilde{v}}{2} - p_2).$$

The implied profit is:

$$p_1 \left( 1 - \frac{1}{\tilde{v}} \right) + \delta \frac{1}{\tilde{v}} p_2$$

By uniformity, we note that price is linear in $\bar{v}$. Thus, we suppose the seller’s (optimal) price is $p_1 = k_1 \bar{v}$ and $p_2 = k_2 \tilde{v}$. By the indifference condition we have

$$\tilde{v} = \frac{k_1}{2 - \delta/2 + \delta k_2} \bar{v}.$$ 

And profit:

$$\bar{v} \left[k_1 \left( 1 - \frac{k_1}{1/2 - \delta/2 + \delta k_2} \right) + \delta k_2 \left( \frac{k_1}{1/2 - \delta/2 + \delta k_2} \right)^2 \right]$$

In second period, since we must have the seller charges a price equal to available surplus, we have

$$k_2 \frac{\bar{v}}{\tilde{v}} = \bar{v} \left[k_1 \left( 1 - \frac{k_1}{1/2 - \delta/2 + \delta k_2} \right) + \delta k_2 \left( \frac{k_1}{1/2 - \delta/2 + \delta k_2} \right)^2 \right]$$

Solving yields:

$$k_2 \left( 1/2 - \delta/2 + \delta k_2 \right) = \left( 1/2 - \delta/2 + \delta k_2 \right)^2 - k_1 \left( 1/2 - \delta/2 + \delta k_2 \right)^2 + \delta k_1 k_2$$

So we have:

$$k_1 = \frac{1}{2} \left( 1 - 2k_2 - \delta + 4k_2 \delta - 4k_2^2 \delta \right)$$

Now suppose in the first period, the seller charges $k \bar{v}$; in that case, we must have

$$\frac{\tilde{v}}{2} - k \bar{v} = \delta \left( \frac{\tilde{v}}{2} - k_2 \tilde{v} \right)$$

Solving yields $\tilde{v} = \frac{k}{1/2 - \delta/2 + \delta k_2} \bar{v}$. Thus, the profit is

$$k \bar{v} \left( 1 - \frac{k}{1/2 - \delta/2 + \delta k_2} \frac{1}{\bar{v}} \right) + \delta \frac{k}{1/2 - \delta/2 + \delta k_2} \frac{1}{\tilde{v}} \frac{1}{\bar{v}} k k_2$$
Note that this implicitly uses the one-shot deviation principle to determine how \( p_2 \) changes as \( p_1 \) changes. Taking the first order w.r.t to \( k \), we have

\[
k^* = \frac{(1/2 - \delta/2 + \delta k_2)^2}{1 - \delta}
\]

Then because we must have \( k^* = k_1 \) in equilibrium, we use

\[
\frac{(1/2 - \delta/2 + \delta k_2)^2}{1 - \delta} = \frac{1}{2}(1 - 2k_2 - \delta + 4k_2\delta - 4k_2^2\delta)
\]

to pin down \( k_2 \). Putting this together, we have \( k_2 = \frac{1 - \delta}{4 - 2\delta} \). We find \( k_1 \) as \( \frac{1 - \delta}{(2 - \delta)^2} \).

Thus the seller first period price is \( \frac{1 - \delta}{(2 - \delta)^2} \bar{v} \), and \( \bar{v} = \frac{1}{(2 - \delta)^2} \tilde{v} \). Note that indeed when \( \delta = 0 \), this reduces to one period static case, as expected.

This example is similar to Section 3.2 which highlighted that for high discount factors the seller might not attempt to sell in the first period. We mention that this phenomenon can also emerge in Bayesian models with a finite horizon (e.g., [Fershtman and Seidmann (1993)]).

Here, when \( \delta \geq 2 - \sqrt{4} \approx 0.4126 \), we again have the seller does not attempt to sell in the first period. But a major difference is that now the horizon is infinite. As a result, the seller’s problem at time 2 looks identical to the time 1, whenever sale occurs with probability 0 at time 1.

This observation shows that the seller would never induce a sale in this alternative, for this specification with a sufficiently high \( \delta \)—and for that matter, \( \delta \) does not have to be particularly close to one for this to occur. After waiting one period, the seller would “reset” the worst-case. This property is unusual, and highlights how in principle the use of the maximin objective can dramatically change the pricing strategies a seller might adopt. We are not aware of other environments where the seller does not even try to sell in equilibrium. On the other hand, this result also provides a reason why our dynamically consistent benchmark may be more useful as a benchmark compared to the fully-pessimal-and-naive case. It seems hard to imagine that a seller, capable of computing discounted payoffs, would not further anticipate not even trying to sell under this objective.

**B.2. Sophistication**

While the previous section shows that the worst-case information structure for the seller at \( t = 1 \) will generally induce an equilibrium where the seller does not optimize against the worst-case at time \( t = 2 \), one might instead insist on maintaining that the seller maximizes against the worst-case information arrival process, but acknowledges that this may change over time. Such a seller is dynamically inconsistent, but aware of this.
To be precise, this alternative induces the following assumption regarding the objectives of each of the players is as follows:

- At time 1, the seller chooses \( p_1 \) anticipating the equilibrium strategies \( p_2(p_1) \) (note because the seller cannot observe other stuff so \( p_2 \) is only a function of \( p_1 \)) that he would make at time 2; in particular, \( p_1 \) is chosen to maximize profit, against the worst-case information arrival process given \( p_1 \) and \( p_2(p_1) \). Denote this information arrival process by \( I_1, I_2, I_1, \).

- Nature then provides \( I_1 \) from the previous step to the buyer.

- At time 1, the buyer decides whether to purchase or not as a function of \( I_1, I_2, I_1, \).

- At time 2, the seller maximizes profit assuming the worst case information structure at time 2, holding fixed \( I_1 \). Denote this information structure \( I_2, I_2, \). This determines the equilibrium strategy \( p_2(p_1) \).

- At time 2, the buyer decides whether to purchase depending on \( p_2 \) and the information provided by \( I_1, I_2, I_2, \), breaking indifference against the seller.

This model is substantially more complicated than the benchmark model, because it requires us to solve for an information arrival process at every time the seller acts. Rather than solving a single information design problem, as in our benchmark model, this version requires us to solve as many information design problems as time periods, and for the seller to optimize over all of these.

We make two comments on this alternative. First, this alternative benchmark provides a new way of interpreting Theorem 2: Under threshold-ratio monotonicity, the price path chosen by a sophisticated maxmin seller will coincide with the price path from the main model. The reason is simple: The full worst-case information structure in the second bullet point always coincides with the no-commitment worst case. Under threshold-ratio monotonicity, a dynamically consistent and correct seller is also “sophisticated and fully-worst-case.”

In general, however, the sophisticated benchmark differs from the one in this model. We present an example of this in Section B.2.1—one featuring discrete values—where the worst-case information structure is not the one necessary to induce the outcome described in Theorem 1.

We are not able to say much more than this. Solving for the equilibrium price paths for this alternative, even in simple examples, is beyond the scope of our existing techniques we are aware 22The assumption of discrete values not change the analysis relative to the continuous value distribution; we discuss why the continuous distributions which approximate discrete ones will typically violated threshold-ratio monotonicity.
of, and thus for now we leave it as an open problem\footnote{For instance, the approach of Auster et al. (2022), who derive an HJB representation for a sophisticated maxmin decision maker, does not work in our setting, at least not immediately, since it is not clear which state variable one could use. The natural choice (and the choice in Auster et al. (2022)) would be the set the seller has uncertainty over at time $t$; but the set of possible Nature choices from time $t$ on does not pin down the seller’s payoff, since past information structures will influence which buyers have already purchased or remain in the market, and thus matter for the seller’s continuation value. Note that in Auster et al. (2022), Nature’s choice at time $t$ is the initial prior, making their setting closer to Section B.3 than Section B.2.} While we expect the resulting price paths to be qualitatively similar, for our purposes the key point is the following: the resulting equilibrium can be interpreted as displaying non-Coasian forces, since both our model and this alternative induce identical single-period problems, but different dynamic solutions.

### B.2.1. Example of Sophisticated Maxmin Differing from Sequentially Worst-Case

Consider the discrete distribution where $v = 1$ with probability $1/2$ and $v = 0$ with complementary probability. Note that the concavification arguments from Kamenica and Gentzkow (2011) immediately imply that the worst-case makes the buyer indifferent between purchasing and not whenever recommended to purchase, and therefore in the static problem we have that given a price of $p$, the information structure recommends purchase with probability $r$ when $v = 1$, where $r$ satisfies:

$$p = \frac{(1 - r)q}{(1 - r)q + 1 - q} \Rightarrow r = \frac{q - p}{q(1 - p)},$$

where $q$ is the prior that $v = 1$. When $q = 1/2$, we have the profit given worst-case information following price $p$ is $p^{1-2p}$, yielding optimal static price of $\frac{1}{2}(2 - \sqrt{2})$ and optimal static profit of $\approx .1718$. While our model assumed a continuous value distribution, this was not essential to deliver Theorem 1 in the two-period case; in the second period, Nature will induce expectation $p_2$, which induces no additional option value, and in the first period, Nature will induce expectation $w(p_1)$, the indifferent value for a consumer following price $p_1$. Note that, given $w(p_1)$, the second period price will maximize:

$$p_2 \left( \frac{w(p_1) - p_2}{w(p_1)(1 - p_2)} \right),$$

since $w$ is also the probability that $v = 1$ in the second period. Maximizing this over $p_2$, we see that $p_2 = 1 - \sqrt{1 - w}$. Using this, we can solve for $w(p_1)$, using the identity that $w(p_1) - p_1 = \delta(w(p_1) - p_2)$. Given a solution for $w(p_1)$, and assuming it is interior, we therefore have $p_1$ is
chosen to maximize:
\[
\frac{1}{2} \cdot p_1 \left( \frac{1/2 - w(p_1)}{(1/2)(1 - w(p_1))} \right) + \left( \frac{1}{2} + \frac{1}{2} \left( 1 - \frac{1/2 - w(p_1)}{(1/2)(1 - w(p_1))} \right) \right) \cdot w(p_1) \delta p_2 \left( \frac{w(p_1) - p_2(p_1)}{w(p_1)(1 - p_2(p_1))} \right).
\]

This expression can be maximized numerically; doing so for \( \delta = 2/3 \) yields the following solution:

\[
p_1 \approx 0.2609, \quad w(p_1) \approx 0.3700, \quad p_2 \approx 0.2072, \quad \text{Seller Payoff} \approx 0.0763.
\]

For this price path, it is straightforward to show that the resulting solution is not reinforcing, and thus that the sophisticated fully-maxmin seller would use a different pricing strategy than we outlined. Suppose to that the seller charged prices \( p_1 \) and \( p_2 \) as above, and suppose Nature used an information structure which perfectly revealed the value to the buyer in the second period. In this case, the buyer would find it optimal to delay as a result, since when \( \delta = 2/3 \):

\[
(1/2) - 0.2609 < (1/2)(2/3)(1 - 0.2072).
\]

On the other hand, the seller’s payoff under this alternative—where the buyer buys at time 2 whenever \( v = 1 \)—is \((2/3)(1/2)p_2 \approx 0.0691 < 0.0763\). Thus, the fully worst-case information structure is not the one identified.

While threshold-ratio monotonicity is only defined for continuous distributions, we note that it will be violated for continuous distributions which approximate this discrete distribution—for instance, taking \( n \) even and sufficiently large and considering \( f(v) = (v - 1/2)^n (1 + n)2^n \). Intuitively, for moderate values of \( v \)—say, in the range \([1/4, 1/3]\)—for \( n \) very large, the threshold \( y^*(v) \) will be very close to 1 for all values in this range. As a result, over this range, \( y(v) \) will increase only slightly as \( v \) increases, even for large changes of \( v \). Hence the ratio \( \frac{v}{y(v)} \) will increase as well.

### B.3. Worse Past Information

We have assumed that the seller posits all past actions of Nature as “sunk.” Since the seller knows Nature has already moved, the seller who chooses a price at time \( t \) does not consider the worst-case information structure at time \( s < t \)—that is, this information structure is assumed to be known. However, if the seller at time \( t \) considered the worst-case over all information arrival processes, these could very well include past information as well.

Specifically, assume the following, and for simplicity\(^{24}\) take \( T = 2 \).

\(^{24}\)While there is no conceptual difficulty in considering the general time horizon case, doing so formally requires
At time 1, the timing protocol is exactly as in the main model.

At time 2, the seller chooses a price to maximize the profit guarantee, taken over all $I_1, I_2$, and conditional on the buyer not having purchased at time 1.

To obtain a coherent statement while avoiding conceptual difficulties, we treat the buyer as a completely passive player and do not consider their incentives, taking $\hat{p}_2(p_1)$ as a primitive—a more complete model would require an assumption about how this is set.

For this model, Section 8.3.1 presents a result showing that in the two-period model and under the restrictive assumption that outcomes are such that sale occurs in both periods with positive probability, the worst-case "past information structure" from the perspective of time 2 involves time-1 information having been the most favorable to the seller. This involves the buyer being informed whether $v$ is above or below a threshold, but where the buyer is indifferent between buying and not whenever buying (as opposed to whenever delaying).

The reason this result holds is, intuitively, because Nature can condition on the fact that the buyer has not bought when choosing a "past information structure." This is still restricted, since for this to be coherent the information structure must be such that the buyer would have been willing to purchase given the conjecture. But choosing past information in this way means that the buyers who remain have the lowest possible values.

We do not present a full characterization of equilibrium for this benchmark for two reasons. First, to do this formally requires specifying how the seller resolves his time inconsistency, as well as how the seller believes the buyer resolves his time inconsistency. At time 1, the problem appears to the seller exactly as in the model described in Section 2, but at time 2 the problem seems very different; thus we have (at least) two possible candidates for $\hat{p}_2(p_1)$, and so without an assumption on (the seller’s belief of) buyer equilibrium behavior, we cannot specify which first-period indifference threshold is relevant.

Second, characterizing the full equilibrium requires finding primitive conditions which ensure sale occurs in both periods with positive probability, in order to avoid making assumptions on endogenous objects. Without this assumption, the seller could form a time 2 conjecture that would imply the buyer should have bought at time 1 with probability 1. If this were possible, the seller would then believe himself at a probability 0 event whenever the game continues to time 2. We wish to avoid taking a stand on how the seller disciplines beliefs here.

Still, this discussion clarifies the Nature of the dynamic inconsistency issues that arise when the seller allows the worst-case to extend to past information. The result suspiciously suggests the seller always believes the past information was chosen favorably despite future information being unfavorable. We leave our analysis of this alternative to this observation.

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spelling out more technical details regarding the definition of equilibrium.
B.3.1. Proof of the above claim

We now present a formal statement of the result alluded to in the previous section:

**Proposition 8.** Suppose \( T = 2 \), and suppose that at time 2, the seller seeks to maximize the profit guarantee over the worst-case choices of \( \tilde{I}_1 : [v, \overline{v}] \to \Delta(S_1) \) and \( I_2 : [v, \overline{v}] \times S_1 \to \Delta(S_2) \). Suppose that, at time 2, the seller conjectures that the buyer anticipated a second period price of \( \hat{p}_2(p_1) \). Let \( v^* = \frac{p_1 - \hat{p}_2(p_1)}{1 - \delta} \), and suppose that \( v^* > \mathbb{E}_F[v] \). Then given a price of \( p_1 \), the worst-case \( \tilde{I}_1 \) (for the seller at time 2) involves the buyer learning whether \( v > y^* \), and \( y^* \) is either equal to \( v \) or characterized by:

\[
\mathbb{E}[v \mid v > y^*] = \frac{p_1 - \delta \hat{p}_2(p_1)}{1 - \delta}.
\]

**Proof of Proposition 8** To prove the proposition, we solve a nested information design problem. The argument from Theorem 1 shows that the second-period information structure is characterized by a (possibly signal dependent) threshold such that the buyer purchases if and only if \( v \) is above this threshold. Furthermore, this threshold makes the buyer indifferent between purchase and not.

Consider the time 1 problem. Given an arbitrary information structure from Nature, we can without loss assume all signals are collapsed to action recommendations, via a revelation argument. Thus it suffices to show that the time-1 recommendation to not buy involves a threshold which is as low as possible.

The argument then follows from two claims:

**Claim 1:** For general distributions \( H_t \), where \( H_t \) is FOSD decreasing in \( t \), the seller’s optimal profit \( \max_p p(1 - H_t(p)) \) is decreasing in \( t \). This argument is standard and thus omitted.

**Claim 2:** The FOSD minimal distribution \( F_s \) that can be induced in Nature’s problem for the time 1 information structure is partitional. Note that, given a quantile \( q \) and signals \( s, s' \) in a binary information structure, we must have:

\[
\mathbb{P}[v \leq F(x)] = \mathbb{P}[v \leq F(x) \mid s] \mathbb{P}[s] + \mathbb{P}[v \leq F(x) \mid s'] \mathbb{P}[s']
\]

Since by assumption \( \mathbb{E}_{v \sim F}[v] < \frac{p_1 - \delta \hat{p}_2}{1 - \delta} \), the information structure which provides no information to the buyer does not solve the constraint that the buyer must be willing to delay purchase if recommended to not buy. The above equation, however, implies immediately that the maximum value for \( F_s(x) \), given \( x \), is \( F(x) \). But recall that \( H_2 \) FOSD dominates \( H_1 \) if \( H_1(x) \geq H_2(x) \) for all \( x \). Thus, the only candidate for FOSD minimizing distributions are such that \( F_s(x) = F(x) \) for all \( x < x^* \) for some \( x^* \), since any other distributions inducing a given expectation FOSD dominate some distribution in this class (i.e., simply choose \( x^* \) so that the mean is the same).
Given the previous argument reduces the candidate information structures to being a threshold, it suffices to find the optimal threshold. Indeed, the FOSD minimal one within this class is as low as possible, and therefore induces indifference when the information recommends that the buyer purchase. The result follows.