In this talk we discuss the next-to-leading-order electroweak (EW) corrections to W-boson + jet hadroproduction and compare the full result to a simple approximation assuming factorization of EW and QCD corrections for the charged-current Drell–Yan process. The W-boson resonance is treated consistently using the complex-mass scheme, and all off-shell effects are taken into account. The corresponding next-to-leading-order QCD corrections have also been recalculated. All the results are implemented in a flexible Monte Carlo code. Selected numerical results for this Standard Model benchmark process are presented for the LHC. The comparison of our result to an approximation based on the EW corrections to W-boson production without additional jets is a step towards a better understanding of the interplay between QCD and EW effects for W-boson production in general.
1. Introduction

The production of electroweak (EW) W and Z bosons with subsequent leptonic decays is one of the cleanest and most frequent Standard Model (SM) processes at the Tevatron and the LHC. The charged-current Drell–Yan process allows for a precision measurement of the W-boson mass and width, can deliver important constraints in the fit of the parton distribution functions, may serve as a luminosity monitor at the LHC, and offers the possibility to search for new charged W' gauge bosons. For more details we refer the reader for example to Ref. [2] and references therein.

At hadron colliders, the EW gauge bosons are (almost) always produced together with additional QCD radiation. The production cross section of W bosons in association with a hard, visible jet,

\[ pp/\bar{p}\bar{p} \rightarrow W + \text{jet} \rightarrow l\nu_l + \text{jet} + X, \]

is still large. The jet recoil can lead to strongly boosted W bosons, i.e. to events with high-\(p_T\) charged leptons and/or neutrinos. Hence, W+jet(s) production is not only a SM candle process, it is also an important background for a large class of new physics searches based on missing transverse momentum. Moreover, the process offers the possibility for precision tests concerning jet dynamics in QCD.

To match the prospects and importance of this process class, an excellent theoretical accuracy has already been achieved for the prediction of inclusive W-boson production including NNLO calculations, resummation, parton-shower matching, NLO EW corrections, and leading higher-order corrections. The production of W bosons in association with jets is now known in NLO QCD up to 3 jets [3]. An extensive list of references can be found in Ref. [1].

So far, the EW corrections in the SM have been assessed for W+1jet production in an on-shell approximation where the W boson is treated as a stable external particle [4]. For W bosons at large transverse momentum, i.e. at large centre-of-mass energy, this is a good approximation since the EW corrections are dominated by large universal Sudakov logarithms.

In this work, we summarize a calculation of the NLO EW corrections for the physical final state in W-boson hadroproduction, i.e. \( pp/\bar{p}\bar{p} \rightarrow l\nu_l + \text{jet} + X \), described in full detail in Ref. [1]. In contrast to the on-shell approximation, all off-shell effects due to the finite width of the W boson are included. Moreover, we can incorporate the experimental event selection based on the charged-lepton momentum and the missing transverse momentum of the neutrino in our fully flexible Monte Carlo code which is able to calculate binned distributions for all physically relevant W+1jet observables. Our calculation, introduced in Section 2, is completely generic in the sense that it can predict observables which are dominated by W bosons close to their mass shell as well as observables for which the exchanged W boson is far off-shell. Moreover, we have recalculated the NLO QCD corrections at \(\mathcal{O}(\alpha_s^2\alpha^2)\), supporting a phase-space dependent choice for the factorization and renormalization scales. Selected results are discussed in Section 3.

The calculation of the EW corrections to W production in association with a hard jet is also a step towards a better understanding of the interplay between QCD and EW corrections for W production in general. More specifically, our calculation allows to test the approximation which assumes factorization for EW and QCD corrections in W production in a simple but well controlled setup: Calculating the EW corrections to single-W production and taking into the account
the emission of the additional jet in a subsequent step is compared to our calculation, which constitutes a part of the full NNLO mixed EW/QCD corrections for single-W production, in Section 4. The understanding of the interplay between QCD and EW effects—including a full treatment of off-shell W bosons—is mandatory to match the envisaged experimental accuracy for the W-mass measurement at the Tevatron and the LHC.

2. The Calculation

In this section we highlight specific aspects of the calculation which are particularly important for the presented corrections and which are not part of the standard framework for NLO calculations. For an extensive discussion of the calculational setup we refer the reader to Ref. [1].

The potentially resonant W bosons require a proper inclusion of the finite gauge-boson width in the propagators. We use the complex-mass scheme [5]. In this approach the W-boson mass (as well as the Z-boson mass) is consistently considered as a complex quantity,

\[ \mu_W^2 = M_W^2 - i M_W \Gamma_W , \]  

(2.1)

defined as the location of the propagator pole in the complex plane, where \( M_W \) is the conventional real mass and \( \Gamma_W \) denotes the W-boson width. This leads to complex couplings and, in particular, a complex weak mixing angle. The underlying (real) Lagrangian does not change since the introduced width is compensated by adding a corresponding complex counterterm. The scheme fully respects all relations that follow from gauge invariance.

The experimental event definition for final-state muons usually selects so-called “bare” muons which are measured without any special treatment of collinear bremsstrahlung photons. Technically, the two collinear particles are not recombined into a single pseudo-particle and the observable is not collinear safe. Therefore, the KLN theorem does not apply and the corresponding EW corrections include terms which are enhanced by logarithms of the (small) muon mass. The enhanced corrections are phenomenologically relevant and cannot be calculated by the standard subtraction methods which assume collinear safety. Accordingly, we use an extended dipole subtraction method [6] which has been specifically designed to deal with non-collinear-safe observables. The logarithms are extracted analytically and we can still work with matrix elements in the massless muon approximation.

To form collinear-safe quantities, QCD partons and also photons have to be recombined into a single jet if they are sufficiently collinear. However, the recombination induces a problem if the bremsstrahlung photon and a gluon are accidentally collinear. In this case, soft gluons can still pass the jet selection due to the recombination procedure. Hence, a soft-gluon divergence is induced that would be canceled by the virtual QCD corrections to W + photon production. To avoid the singularity, one has to distinguish W + photon and W + jet production by means of a more precise event definition employing a cut on the maximal energy or transverse-momentum fraction of a photon inside a given jet. However, this procedure spoils the collinear safety of the event definition in partonic processes with final-state quarks. Using again the subtraction formalism [8] to extract the problematic collinear terms, the appearance of an unphysical quark-mass logarithm in the final result signals the necessity to include non-perturbative physics to properly describe the emission of a photon by a quark. The relevant collinear physics can be factorized from the underlying hard
process and can be cast into a process-independent quark-to-photon fragmentation function \cite{7}, which has been measured at LEP in photon+jet events \cite{8}. We employ this fragmentation function to achieve both, a realistic event selection and a theoretically consistent result.

To reach the accuracy of $\mathcal{O}(\alpha_s \alpha^3)$ throughout the calculation we have also included the photon-induced partonic processes and the respective NLO QCD corrections. Also non-trivial interference terms between EW and QCD diagrams within the real corrections have been included at this order. However, these contributions are phenomenologically irrelevant and will not be discussed in this talk.

### 3. Results

We define W+1 jet events by requiring a jet and a charged lepton with transverse momentum $p_T > 25 \text{GeV}$ as well as missing transverse momentum larger than 25 GeV. The jet and the lepton have to be central with a rapidity smaller then 2.5 in absolute value. The details of the event selection as well as the numerical input values for the calculation can be found in Ref. \cite{1}. All results are presented for the LHC running at 14 TeV.

For the inclusive cross section, we find negative percent-level EW corrections. When we focus on events in the tails of the transverse-momentum distributions of the charged-lepton $p_{T,l}$ or the jet $p_{T,jet}$ (or the transverse-mass distribution of the final-state leptons $M_{T,l\nu}$) we observe the well-known universal Sudakov enhancement of EW corrections in the high-energy regime. For example, at $p_T = 1 \text{TeV}$ for the leading jet, the EW corrections rise to $-25\%$. In the Sudakov regime, where the on-shell result is a good approximation, the transverse-momentum distribution for the leading jet agrees at the percent level with the previous on-shell results \cite{4}.

For all results in this talk we employ a variable scale choice (var) which reflects the kinematics of the process and has been chosen to stabilize the QCD corrections (see Ref. \cite{1}). Concerning the QCD corrections, we only briefly note that a veto against a second hard QCD jet has to be used to carefully define the W+1 jet observable, in particular for the $p_{T,jet}$ distribution. Otherwise,
the differential cross section is completely dominated by QCD dijet production, where a quark jet radiates a W boson, i.e. by a completely different process which is not related to a generic NLO contribution.

In contrast to the integrated cross sections, the transverse-mass distribution is quite sensitive to the specific treatment of final-state photons, in particular close to the Jacobian peak of the distribution at $M_{T,l\nu} \sim M_W$, where the correction for bare muons, $\delta_{\text{EW}}^{\mu+\nu_{\mu}\text{var}}$, reaches almost $-10\%$ (see left panel of Figure 1). As expected, the corrections for bare muons are larger than the corrections with lepton–photon recombination, $\delta_{\text{EW}}^{\text{rec, var}}$, since photons, being radiated collinearly to the charged lepton, carry away transverse momentum. The region around the Jacobian peak, $M_{T,l\nu} \sim M_W$, is of particular interest for the precise determination of the W-boson mass.

The EW corrections for $M_{T,l\nu} \sim M_W$ near the Jacobian peak resemble the corrections for the inclusive W-boson sample for which no additional jet is required (see, e.g., Figure 2 in Ref. [9]). The fact that an additional jet due to QCD initial-state radiation does not have a large effect on the EW corrections indicates that EW and QCD effects approximately factorize for the transverse-mass distribution close to $M_W$. For the transverse momentum of the charged lepton, $p_{T,l}$ (see right panel of Figure 1), the EW corrections are quite different from the single-W results (Figure 1 in Ref. [9]) and we discuss the question of factorization for this observable in detail in the next section.

4. Testing Factorization of QCD and EW Corrections in W Production

In this section, we compare the EW corrections to $W + 1\text{jet}$ with a simple approximation based on the EW corrections for $W$ production without any additional jet activity. This comparison can shed some light on the important question how the available EW and QCD corrections can be combined to obtain the most accurate predictions for the charged-current Drell-Yan process while a full calculation for the mixed $\mathcal{O}(\alpha\alpha_s)$ corrections is missing. We test the assumption that the EW and QCD corrections factorize, motivated by the fact that QCD does not couple to the leptonic final state and that the EW corrections are dominated by collinear final-state radiation from the charged lepton. In general, in this approximation a given observable can be first calculated including the EW corrections for $W$ production but ignoring all QCD effects. Then all the relevant known QCD corrections can be applied to this result, e.g. fixed-order and/or resummed corrections and/or parton-shower evolution of the final state. For a recent discussion combining several tools and estimating the theoretical error of different approximations see Ref. [10].

Here we follow this prescription for leptonic observables in events where QCD radiation produces an additional jet. For the EW corrections to the underlying single-$W$ production, we use the results from Ref. [9] tuned to our $W + 1\text{jet}$ setup employing the complex-mass scheme. To describe the QCD radiation resulting in a jet our approach is very modest: we simply use the tree-level $W + 1\text{jet}$ matrix elements to describe the first QCD emission. Hence, we do not seek for most accurate predictions. But on the other hand, we can test the assumed factorization because we can compare to the complete EW corrections to $W + 1\text{jet}$ production which include all possible cross-talk between QCD emission and EW effects at the level of $\mathcal{O}(\alpha\alpha_s)$ corrections to the Drell–Yan process for this specific contribution.

Technically, the comparison is realized as follows: We first calculate a tree-level $W + 1\text{jet}$ event. Then we reweight this event according to the corresponding EW corrected prediction for
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Figure 2: EW corrections for the $p_T,\ell$ distribution obtained from the full calculation and the factorization approximation by reweighting the single-W result, as explained in the text. The left plot shows the corrections for events with $20\text{ GeV} < p_T,\text{jet} < 25\text{ GeV}$, the right plot for events with $3\text{ GeV} < p_T,\text{jet} < 5\text{ GeV}$.

the underlying W production. The reweighting factor is obtained by boosting the event into the W-boson rest frame and looking up the EW correction for single-W production in the histogram for the leptonic observable under consideration, e.g. $p_T,\ell$ for the results discussed in the following.

Here, we focus on the transverse momentum $p_T,\ell$ of the charged lepton where the direct sensitivity of the observable to the jet recoil clearly obscures or may even spoil the factorization approximation. Indeed, the approximation fails for events including hard jets which are present in our default setup. The more complicated kinematical situation cannot be captured by the simple reweighting procedure advertised above. However, this is not the kinematical region where the combination of EW and QCD effects is most needed for the W-mass measurement, for which events with small QCD recoil are selected.

In Figure 2 (left), we show the full EW corrections and the result from the reweighting approximation for a restricted class of events with $20\text{ GeV} < p_T,\text{jet} < 25\text{ GeV}$ for the transverse momentum of the jet. Around $p_T,\ell \sim 55\text{ GeV}$, where the EW corrections show a dip due to the remnant of the Jacobian peak of the cross section in this region, the factorization approximation works quite well. However, for smaller $p_T,\ell$ the approximation underestimates the full result by an amount which is as big as the correction itself. In this region, final-state configurations of decaying on-shell W bosons often fail to pass the missing $p_T$ cut for the given $p_T,\text{jet}$ and $p_T,\ell$. In the full calculation, events with real photon emission populate the region suppressed at tree level and reduce the negative EW corrections. This, of course, is an effect the reweighting procedure cannot account for. The harder the jets in the events the more such kinematical effects related to cuts are relevant for the total EW corrections, and it is not surprising that the factorization approximation fails for the inclusive $p_T,\ell$ distribution, where different regions of the distribution are dominated by events with different $p_T,\text{jet}$.

On the other hand, for events with little QCD activity, corresponding to low $p_T,\text{jet}$ in our simple approach, the factorization approximation can be expected to work. The tree-level approximation
in QCD for the $W+1\text{jet}$ cross section, of course, breaks down at low $p_{T,\text{jet}}$. However, the test of factorization may still be performed since only the EW corrections are relevant, not the cross section itself. As expected, for events with $3\text{GeV} < p_{T,\text{jet}} < 5\text{GeV}$, the approximation is almost exact, as shown in Figure 2 (right). The region subject to kinematical complications at the edge of the distribution is very small.

Similar considerations apply for the $M_{T,l\nu}$ distribution. However, since $M_{T,l\nu}$ is not sensitive to initial-state radiation, the region $M_{T,l\nu} \sim M_W$ is not strongly affected by the discussed kinematical effects. Therefore, the factorization close to the Jacobian peak is visible already by directly comparing the single-$W$ and the $W+1\text{jet}$ results for the EW corrections. Using the proposed approximation allows to reproduce the full $W+1\text{jet}$ result even more closely.

5. Conclusion

We have extended the theoretical effort for the precise prediction for $W$-boson production at the Tevatron and the LHC by an important step: We have presented the first calculation of the full electroweak NLO corrections for $W$-boson hadroproduction in association with a hard jet where all off-shell effects are taken into account in the leptonic $W$-boson decay, i.e. we have studied final states with a jet, a charged lepton, and missing transverse momentum at NLO in the EW coupling constant within the SM. All results are implemented in a flexible Monte Carlo code which can model the experimental event definition at the NLO parton level. Comparing our calculation with a simple approximation indicates that EW corrections to $W$ production approximately factorize from the underlying QCD dynamics for certain observables in limited kinematical regions.

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