The holographic Ricci dark energy and its possible doomsdays.

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Abstract It is well known that the holographic Ricci dark energy can induce some future doomsdays in the evolution of the universe. Here we analyse the possible avoidance of those doomsdays by invoking a modification to general relativity on the form of curvature effects.

1 Introduction

A possible approach to explain the current acceleration of the universe is based on the holographic dark energy [1, 2]. Such a phenomenological model is based on the idea that the energy density of a given system is bounded by a magnitude proportional to the inverse square of a length characterising the system [3, 4]. When this principle is applied to the universe as a whole, we obtain the holographic dark energy [1, 2]. It turns out that there are many different ways of characterising the size of the universe and one of them is related to the inverse of the Ricci curvature of the universe. When the size of the universe is characterised in such a way, we end up with the holographic Ricci dark energy (RDE) model [5], whose energy density...
\[ \rho_H = 3 \beta M_P^2 \left( \frac{1}{2} \frac{dH^2}{dx} + 2H^2 \right), \]  
\[ (1) \]

where \( M_P \) is the Planck mass, \( x = - \ln(z + 1) = \ln(a) \), \( z \) is the redshift and \( \beta \) is a dimensionless parameter that measures the strength of the holographic component.

A spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) universe filled with this kind of matter accelerates and therefore the RDE can play the role of dark energy on the Universe. It turns out that the asymptotic behaviour of the Universe depends crucially on the value acquired by \( \beta \): (i) if \( \beta \leq 1/2 \) the universe is asymptotically de Sitter, otherwise (ii) the universe faces a big rip singularity [6] in its future evolution.

Our main purpose in this paper is to see if we can appease the big rip appearing in some cases on the RDE by invoking some infra-red and ultra-violet curvature corrections. This two corrections can be quite important to remove the big rip singularity which takes place on the future and at high energy. The curvature corrections will be modeled within a 5-dimensional brane-world model with an induced gravity (IG) term on the brane and a Gauss-Bonnet term in the bulk [7].

### 2 The RDE model with curvature corrections

We consider a DGP brane-world model, where the bulk action contains a GB curvature term. The bulk corresponds to two symmetric pieces of a 5-dimensional (5d) Minkowski space-time. The brane is spatially flat and its action contains an IG term. We assume that the brane is filled with matter and RDE. Then, the modified Friedmann equation reads [7]:

\[ H^2 = \frac{1}{3M_P^2} \rho + \frac{\epsilon}{r_c} \left( 1 + \frac{8 \alpha}{3} H^2 \right) H, \]

\[ (2) \]

where \( H \) is the brane Hubble parameter, \( \rho = \rho_m + \rho_H \) is the total cosmic fluid energy density of the brane which can be described through a cold dark matter component (CDM) with energy density \( \rho_m \) and an holographic Ricci dark energy component with energy density \( \rho_H \). The parameters \( r_c \) and \( \alpha \) correspond to the cross over scale and the GB parameter, respectively, both of them being positive. The parameter \( \epsilon \) in Eq. \( (2) \) can take two values: \( \epsilon = 1 \), corresponding to the self-accelerating branch in the absence of any kind of dark energy [7]; and \( \epsilon = -1 \), corresponding to the normal branch which requires a dark energy component to accelerate at late-time (see for example [8, 9, 10]). For simplicity, we will keep the terminology: (i) self-accelerating branch when \( \epsilon = 1 \) and (ii) normal branch when \( \epsilon = -1 \).

The modified Friedmann equation \( (2) \) can be rewritten as

\[ \frac{dE}{dx} = - \frac{\Omega_m e^{-3x} + (2\beta - 1) E^2 + 2\epsilon \sqrt{4r_c(1 + \Omega_\alpha E^2)} E}{\beta E}, \]

\[ (3) \]
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Fig. 1 Plot of the parameters $\beta_+$ and $\beta_{\lim}$, defined in Eqs. (6) and (7), respectively, versus the parameter $\Omega_\alpha$. We have used the values $q_0 \sim -0.7$ and $\Omega_m \sim 0.27$. The parameter $\beta_{\lim}$ defines the border line between the normal branch ($\beta_{\lim} < \beta$) and the self-accelerating branch ($\beta < \beta_{\lim}$).

where $E(z) = H/H_0$ and

$$
\Omega_m = \frac{\rho_{m0}}{3M_P^2H_0^2}, \quad \Omega_\rho = \frac{1}{4\pi^2H_0^2}, \quad \text{and} \quad \Omega_\alpha = \frac{8}{3}\alpha H_0^2
$$

(4)

are the usual convenient dimensionless parameters and the subscripts 0 denotes the present value (we will follow the same notation as in [10, 8, 11]). By evaluating the modified Friedmann equation at present and imposing that the brane is currently accelerating, we obtain a constraint on the parameter $\beta$ which depends on the chosen brane

$$
\begin{cases}
\beta < \beta_{\lim} \text{ for } \varepsilon = +1, \\
\beta > \beta_{\lim} \text{ for } \varepsilon = -1,
\end{cases}
$$

(5)

where

$$
\beta_{\lim} = \frac{1 - \Omega_m}{1 - q_0}.
$$

(6)

An estimation of $\beta_{\lim}$ can be obtained as follows: the brane would behave roughly (to be consistent with the present observations) as the $\Lambda$CDM leading to $\beta_{\lim} \sim 0.43$.

Even though the modified Friedmann equation (3) cannot be solved analytically, we can obtain the future asymptotic behaviour of the brane which reads: (i) If $\beta < \beta_{\lim}$ or $\beta_- \leq \beta$, the brane is asymptotically de Sitter. (ii) If $\beta_{\lim} < \beta < \beta_-$, the brane faces a big freeze singularity in its future [12], where (see also Fig. 1)

$$
\beta_{\pm} = \frac{1 + \Omega_\alpha \pm 2\sqrt{\Omega_\rho(1 - \Omega_m)}}{2\left[1 + \Omega_\alpha \pm \sqrt{4\Omega_\rho(1 - q_0)}\right]}.
$$

(7)

We have completed and confirmed those results by solving numerically the cosmological evolution of the brane. We refer the reader to [11] for more details. Our
analysis shows that even though the infra-red and ultra-violet effect can appease the big rip appearing on the RDE model, it cannot remove them completely. We would like as well to point out that when the GB term is switched off a little rip event [13] can show up which is much milder that a big rip or a big freeze. The little rip has been previously found on brane-world model [13].

3 Conclusions

We present an HRD energy brane-world model of the Dvali-Gabadadze-Porrati scenario with a GB term in the bulk. The reason for invoking curvature corrections, for example through a brane-world scenario, is to try to smooth the doomdays present on a standard 4-dimensional HRD energy model. It turns out that the model presented here can only partially remove those doomdays.

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