Vortices In Density Wave Systems
Subject To Transverse Electric Fields

Akakii Melikidze
Physics Department, Princeton University, Princeton, NJ 08544
(February 11, 1998)

In this paper we predict many interesting new properties of vortices in highly anisotropic density wave systems subject to strong transverse electric fields. We mainly concentrate on ground state properties. Besides electric field-induced vortices we consider also thermally activated vortices. A new type of temperature-driven transition between two different phases of density waves in strong fields is predicted and several new properties of those phases are reported.

I. INTRODUCTION

It is now well realized that the dynamics of topological defects plays an important role in low dimensional electronic systems. In a particular case of density wave (DW) ground states (for a review see [1]) it was found that topological defects are responsible for such interesting phenomena as generation of narrow-band noise [2], non-linear current-voltage characteristics in strong fields (the phenomenon associated with the so-called “phase slip-page”) [3,4]. More recently, quantum phase slip through creation of vortices was proposed [5] to explain low-temperature properties of spin-density wave compound (TM)TsF₂PF₆ [6,7].

This paper was originally initiated by the prediction [8] that DW systems should possess a novel phase in strong electric fields applied transverse to the direction of highest conductivity. This novel phase was called "mixed state" in analogy with the mixed state of type-II superconductors, and was characterized by the presence of vortices. We have undertaken an extensive study of the predicted transition. The possibility of transverse electric field-induced transition to metallic state (type-I-like) was also investigated and a temperature-driven transition between type-I and type-II regions of the phase diagram was predicted. We report these findings here as well as some interesting anticipated properties of the above-mentioned phases such as, e.g., non-exponential screening of electric fields by DW systems in "mixed state".

II. GINSBURG-LANDAU THEORY

We adopt macroscopic mean-field description of the dynamics of DW systems and make use of (time-independent) Ginsburg-Landau free energy [9] which is a functional of complex order parameter \( \Delta(\mathbf{r}) = |\Delta(\mathbf{r})| \exp(i\phi) \), absolute value of which is DW gap in the electronic spectrum. For the case of highly anisotropic system with an open Fermi surface: \( \mathbf{k} = (\pm k_F(1 + \gamma \cos(bk_y)), k_y, k_z) \) (here \( x \) is the direction of highest conductivity - usually the direction along conducting chains, \( y \) is the direction of strongest Fermi surface wrapping, \( \gamma \) is the anisotropy parameter, \( b \) is lattice constant along \( y \); we neglect dispersion along \( z \); we neglect it) this functional is [9]:

\[
F_{inhom} = \int d\mathbf{r} \frac{K}{2|\Delta_0|^2} \left[ \partial_x|\Delta|^2 + \gamma^2|\partial_y\Delta|^2 \right] \tag{1}
\]

We would like to emphasize the fact that the two terms in \( F_{inhom} \) are of totally different origin. As is well known [9], in the system described above a density modulation appears with the wavevector: \( \mathbf{q} = \mathbf{q}_0 = (2k_F, \pi/b, 0) \); \( \rho_{spin} \propto \Delta \cos(\mathbf{q}\cdot\mathbf{r}) \). Spatial variation of \( \Delta \) gives rise to the effective shift in \( \mathbf{q} \) by \( \delta q = -\gamma \sqrt{\Delta} \). Thus the gap is developed at the wave-vector which is slightly off its optimal value \( \mathbf{q}_0 \). This costs extra energy: \( \delta E \propto |\delta \mathbf{q}| \) - it is linear with \( \delta \mathbf{q} \) which is quite costly for small deviations. Instead the system rearranges its electron density in such a way that locally \( 2k_F \) coincides with the new DW wave-vector. There is no more linear contribution to the energy, however another contribution arises - the energy of the non-homogenous electron density distribution. But the fact is that this density (per chain) is:

\[
\rho = \frac{1}{\pi}(-i\partial_x) \log \Delta \tag{2}
\]

and the extra energy is: \( \delta E \propto \rho^2 \propto (\delta q_x)^2 \) - it is quadratic in \( \delta q_x \) and therefore wins for small distortions of DW gap. This phenomenon is usually described as follows: "particle density follows the variations of the gap". At the same time the gradient in \( y \)-direction does not lead to any change in the local charge density; its contribution to the free energy is of purely elastic character.

In this paper we shall be interested in the response of DW system to external electric fields. We take account of the effect of electric field by introduction of the following term into the free energy functional [9]:

\[
F_{el} = \int d\mathbf{r} \left[ J \epsilon \rho \mathbf{\varphi} - \frac{1}{8\pi} \left( \epsilon |\nabla \mathbf{\varphi}|^2 + \lambda_{\varphi}^{-2} \mathbf{\varphi}^2 \right) + \varphi \rho^{ext} \right] \tag{3}
\]

Here the first term describes coupling of electric field to the charge induced by DW distortion, the term in parenthesis represents energy associated with electric field itself and with the coupling of electric field to free charge
carriers excited over the DW-gap. The last term represents the interaction of electric field with external charges \[\lambda_{TF}\]. Strictly speaking Eqn.(3) is the action of the system taken with minus sign. However we shall use the term "free energy functional" to denote a functional of which an extremum determines the ground state. We also emphasize that electric field couples only to x-component of the gradient of the gap - only this component induces electric charge!

Important point here is that our action is essentially different from that of ref. [8] in that it contains wave vector-dependent dielectric constant \(\epsilon = \epsilon(k)\). It appears because the ground state of DW, in complete analogy with the low-T ground state of semiconductors, is highly polarizable due to excitations of virtual electron-hole pairs. This should not be mixed neither with the screening due to thermally excited quasiparticles, nor should it be mixed with the screening due to spatial distortion of the order parameter (see below). We stress that this dielectric function is generic for a gapped electronic system. It can be calculated using simple anisotropic two-band semiconductor model [9]:

\[
\epsilon(k) \sim \frac{\lambda_{TF}^{-2}}{k^2 + \xi(k)^{-2}}
\]

Here \(\lambda_{TF}\) is Tomas-Fermi screening length in the metallic state, \(\xi(k)\) is anisotropic coherence length in DW state. (The value \(\epsilon(0)\) is usually huge: for instance \(\epsilon(0) \sim 10^5\) for (TMTSF)$_2$PF$_6$). Now we turn to the analysis of the response of DW systems to external electric fields.

### III. WEAK FIELDS

In the absence of external fields the phase of the order parameter \(\Delta = |\Delta| \exp(i\theta)\) is the so-called degeneracy parameter: the free energy is globally \(U(1)\) invariant - it does not change if we make a global substitution \(\theta \rightarrow \theta + \text{const}\). Consequently there is a Goldstone (the so-called phason) gapless mode associated with this degeneracy. At the same time the orthogonal amplitudon mode, in which \(|\Delta|\) is varied, has a gap in the spectrum at \(k = 0\). Therefore we expect that in the limit of weak fields the response of the system is given by the excitation of phason mode only. Thus we set \(|\Delta|\) to be constant. After Fourier transform:

\[
F_{inhom} + F_{el} = \int \left[ \frac{K k_x^2}{2} |\theta_k| + J e \dot{\varphi}_k i k_x \theta_k \right] - \frac{1}{8\pi} \left( e k_x^2 + \lambda_q^{-2} \right) |\varphi_k|^2 + \varphi_k \varphi^* \frac{v_F k_y}{k_z^2}
\]

\[
= \int \left[ \frac{K k_x^2}{2} |\theta_k| - i 2 e k_x \varphi_k / v_F k_y^2 \right]^2 - \frac{1}{8\pi} \left( e k_x^2 + A_k^{-2} \right) |\varphi_k|^2 + \varphi_k \varphi^* \frac{v_F k_y^2}{k_x^2}
\]

the minimization with respect to \(\theta_k\) becomes trivial. Then the remaining effective action for the electric field leads to an anisotropic screening length \(\lambda\). Screening length in y-direction is \(\lambda_y = \sqrt{\epsilon(\lambda_q^{-1}) \lambda_q}\) and is due to interaction of electric field with quasiparticles excited over the gap. On the other hand screening in x-direction is now determined by both quasiparticles and phasons leading to screening length \(\lambda_x = \lambda_{TF}\) [12]. This is again a manifestation of the fact that only the phason mode along x-direction couples to electric field [13].

### IV. STRONG FIELDS

As we have just seen, electric fields in the x-direction (along the chains) are screened at the short lengths \(\sim \lambda_{TF}\) at all temperatures. In contrast to that, \(E_y\) component of the field can penetrate far enough into the bulk since its screening is determined by thermally excited quasiparticles the concentration of which is exponentially decreased at low temperatures. However in strong electric fields the situation changes: \(\Delta\) can no longer be considered constant and it may be energetically favourable for the system to expel electric field from the sample by spatial variation of the magnitude of the gap [4]. Firstly, it’s obvious that in the limit \(E_y \rightarrow \infty\) the state of the system is metallic. A simple argument for this is that in competition between condensate energy \(-n(\epsilon_F) \Delta^2 / 2\) (here \(n(\epsilon_F) = N_0 / \pi v_F\) is density of states at Fermi level, \(N_0\) is density of chains in the plane perpendicular to the chains) and electrostatic energy \(E D / 8\pi\) the latter always wins in the limit of strong fields: the gap vanishes giving rise to screening. Thus in the region of strong fields there is a thin layer near the edge of the sample in which the state is metallic. Electric field in this layer is screened at the distance \(\sim \lambda_{TF}\). However, taking into account that actually the continuous model used here is not applicable at such small distances, one should substitute \(\lambda_{TF}\) for interchain distance. This corresponds to a metallic layer with the thickness of as small as a single interchain distance. However, in the bulk the electric field is absent and DW state is restored.

Thus we establish that there must exist a critical value of \(E_y\) for which a transition occurs from DW to some other phase. As the field is increased from small values two possibilities may occur: \(\Delta\) can jump to a smooth other phase. As the field is increased from small values two possibilities may occur: \(\Delta\) can jump to a smooth configuration (Type-I) or a configuration with topological defects [8] (Type-II). The latter means that the ground state is characterized by the presence of vortices - centers the in-plane circulation of the phase of the order parameter around which is non-zero. These two possibilities correspond to the two types of superconductors in magnetic field. An ordinary isotropic superconductor can be distinguished between these two types by the
evaluation of Ginzburg-Landau parameter \( \kappa = \lambda / \xi \). Values \( \kappa < 1/\sqrt{2} \) and \( \kappa > 1/\sqrt{2} \) correspond to type-I and type-II respectively. In our case this simple argument is not applicable since DW materials are usually highly anisotropic. Indeed, usually along \( x \)-direction one has \( \lambda_x \ll \xi_x \) whereas along \( y \)-direction either of \( \lambda_y \ll \xi_y \) and \( \lambda_y \gg \xi_y \) can hold depending on temperature: \( \lambda_y \) exponentially increases with decreasing temperature (see below for a more detailed discussion). Instead we shall use a more physical argument to decide between Type-I and Type-II. Namely, we shall evaluate the critical field for these two cases. Then we can argue that the system will make a transition to the state for which the critical field is lower.

**A. Type I**

The critical field for this case can be obtained by equating condensate and electrostatic free energy densities:

\[
\frac{ED}{4\pi} = \frac{1}{2} n(\epsilon_F)\Delta_0^2
\]
\[
D'_c = \frac{\Delta_0}{2e\lambda_{TF}} \sim \frac{d_0}{\xi_x \lambda_{TF}}
\]

Here we have introduced \( d_0 = v_F/2e \) - a parameter with the dimensionality of electric dipole moment (see below). N.B. We use the value of the dielectric constant \( \epsilon \sim 1 \) because the metallic breakdown will occur in a thin layer near the boundary with the thickness \( \sim \lambda_{TF} \ll \xi_y \) - polarization will not develop at such small scales!

**B. Type II - Vortices**

First of all let us clarify what is a vortex in a DW system. DW state is a ground state with broken \( U(1) \) global invariance. This means that the order parameter is a complex number: \( \Delta = |\Delta|\exp(i\theta) \). The order parameter, as a function of coordinates, should be univalued. This implies that, as we travel along a closed contour, the phase of the order parameter can get an increment \( 2n\pi \) where \( n \) is integer called winding number. The corresponding texture of the order parameter is called \( n \)-times quantized vortex. Let us now consider a singly-quantized vortex located at \( \vec{r} = 0 \):

\[
\Delta(\vec{r}) = |\Delta(0)|\exp(i\phi)
\]

where \( \phi \) is is azimuthal angle in the \( x-y \) plane. The absolute value of the gap is assumed to be constant everywhere except regions of the size \( \xi \) (correlation length) near the vortex centers. This assumption constitutes the so-called London approximation. From Eqn.(7) we can calculate induced charge density (per chain) using Eqn.(8):

\[
e\rho = \frac{e}{\pi} \sin \phi \frac{\phi}{r}
\]

We see that it falls off as \( 1/r \) - it is highly non-local! However the screening makes the fall-off exponential. To see how this happens we make the following substitution in Eqn.(8) in order to take vortex degree of freedom into account:

\[
(\vec{\nabla}\theta)_k \rightarrow ik\hat{\theta}^{\text{phason}} + (i\hat{k} \times \hat{z}/k^2)n_k
\]

Here we have introduced \( \hat{k}^{\text{phason}} \) and \( n_k \) is a complex number: \( \Delta = \Delta e^{i\phi} \). With this simplification the potential is given by:

\[
F_{inhom} + F_{el} = \int d^2k \left[ \frac{K\gamma^2}{2k^2} |n_k - \frac{ie\lambda_{av}}{K}\phi_k|^2 - \frac{1}{8\pi}(ek^2 + \lambda_{av}^{-2})|\phi_k|^2 + \phi_k\epsilon_{ext} \right]
\]

Electric potential produced by vortices is obtained by the minimization of Eqn.(8) with respect to \( \phi_k \) and is given by:

\[
\phi_k = \frac{4\pi i e\gamma^2 J}{ek^2 + \Lambda_k^2} n_k
\]

**C. Single Vortex**

Now \( n(r) = 2\pi\delta(r) \) and the electrostatic potential produced by a single vortex is given by the Fourier transform of Eqn.(8). It is hard to obtain an analytical expression in the general case, but one can get an idea of what this potential looks like from the consideration of the purely isotropic case: \( \gamma = 1, \Lambda_k = \lambda = \text{const} \). With this simplification the potential is given by:

\[
\phi(\vec{r}) = -2e\gamma^2 J \sin \theta f(r)
\]

\[
f(r) = \int \frac{dk}{ek^2 + \Lambda^2} J_1(kr)
\]

\[
= \begin{cases} \frac{1}{\lambda^2} \log \frac{e^{1/2}\Lambda}{r} & \text{for } r \ll e^{1/2}\Lambda, \\ \frac{\Lambda}{r} & \text{for } r \gg e^{1/2}\Lambda. \end{cases}
\]

Here \( J_1 \) is first Bessel function. The effect of huge actual anisotropy in \( \Lambda \) can be taken into account qualitatively in the following way. First of all we may notice that in the large- \( r \) region \( f(r) \) is still angle-independent: it is obtained by \( f(r) \approx \Lambda_{av}/r \) where \( \Lambda_{av} \) is some "average" screening length. Important point here is that the total charge density in the large- \( r \) region falls off exponentially: one can check that by taking laplacian of \( \phi(\vec{r}) \). But the above mentioned extreme anisotropy comes to play in the small- \( r \) region - the core of a vortex is highly anisotropic and hard to analyze.

Some of the vortex properties, however, allow exact description. From Eqn.(10) it can be inferred that the
total charge of a vortex is zero, but there is a non-zero electric dipole moment directed along $y$:

$$d_y = 2\pi \varepsilon J \lambda^{2}_{qp} = d_0 \frac{\epsilon f_s}{(1 - f_s)} \quad (12)$$

Here $d_0 = v_F/2e$. $d_y$ is exponentially increased as $T \to 0$. It should be noted that this expression was derived for a vortex in the bulk; it is expected that for vortices near boundary (the case which is relevant for vortices produced by external fields - see below) the induced dipole moment is reduced.

D. Critical Field

Now we estimate the critical field at which an appearance of a single vortex becomes energetically favourable. In order to do that we substitute Eqn.(11) (with $n_k = 2\pi$) back into Eqn.(10):

$$F_{eff} = \int d^2k \frac{K\gamma^2}{2k^2} \left( \frac{\epsilon k^2 + \lambda_{TF}^2}{\epsilon k^2 + \Lambda_k^2} \right) - d_y E^y_{\text{ext}} \quad (13)$$

$$= \frac{\pi K}{2} \frac{\gamma \lambda_{qp}}{\lambda_{TF}} \log \frac{W}{\epsilon^{1/2} \lambda_{qp}} - d_y E^y_{\text{ext}} \quad (14)$$

Here $W$ is the size of the system in the $y$-direction. From Eqn.(14) and Eqn.(12) we obtain the critical field:

$$D^I_c = \frac{\gamma d_0}{4\lambda_{qp} \lambda_{TF}} \log \frac{W}{\epsilon^{1/2} \lambda_{qp}} \quad (15)$$

Comparing this expression with its superconducting analog one may call $d_0$ an "electric dipole moment quantum". The dependence of the critical field on temperature is given through: $\lambda^{2}_{qp} = \lambda^{2}_{TF} \sqrt{2\pi \Delta_0/T} \exp(-\Delta_0/T)$. So the critical field for vortex mixed state can be significantly lowered by increasing the screening length in the $y$-direction as $T \to 0$. As we do so, an interesting situation can occur: $D^I_c$ can be made smaller than $D_c^{\ast}$ signalling a temperature-driven transition between Type-I and Type-II-like ground states! A more intuitive explanation of this transition is obvious: one can, in principle, evaluate $\kappa_x = \lambda_x/\xi_x$ and $\kappa_y = \lambda_y/\xi_y$. Usually one has $\kappa_x \ll 1$ while $\kappa_y$ is exponentially temperature dependent. One can argue then that the type of the ground state is determined by the geometric mean of the two kappas: $\kappa = \sqrt{\kappa_x \kappa_y}$. This "mean" Ginzburg-Landau parameter $\kappa$ is also temperature-dependent. This raises a possibility of Type-I/Type-II temperature-driven transition as some critical value $\kappa_c \sim 1$ is passed by $\kappa$ in a temperature sweep. To our knowledge this is the first time that such type of transition is predicted.

E. Screening Properties

From the physical point of view the reason why the appearance of vortices becomes favourable at high fields is that vortices can screen external electric fields. In order to show this we first introduce a "dense limit approximation" in which the average distance between vortices is assumed to be much less than the characteristic length of the external electric field variation. Then one can take the vorticity $n(\vec{r})$ to be a continuous function rather than the sum of delta-functions. The minimization of the effective energy Eqn.(3) with respect to $n_k$ becomes trivial: $n_k = \delta^{\ast} E^y_k$ and leaves us with the effective free energy which describes the screening with the screening length $\lambda = \lambda_{TF}$. In fact this contradicts to the above made "dense limit approximation", nevertheless the conclusion about screening remains valid if we assume that the screening length is rather equal to the average inter-vortex distance: $\lambda = n^{-1/2}$. But now the screening length becomes coordinate dependent leading to a non-exponential screening. Indeed, Poisson equation $\varphi'' - \lambda^2 \varphi = 0$ in this case reads $\varphi'' + d_0^{-1} \varphi \varphi' = 0$. Its solution is:

$$\varphi(y) = \frac{2d_0}{y + d_0 \varphi(0) - 1} \quad (16)$$

At distances $\sim \lambda_{qp}$, however, the exponential screening due to quasiparticles comes to play. So in the bulk the DW state is restored. Thus the electric field-induced transition to a vortex state is a surface effect. Therefore it is unlikely to be detected in any transport measurements. On the other hand the best way to observe such a transition would be to measure directly the screening length. It would be expected that the screening length be $\sim \lambda_{qp}$ for field below critical whereas for fields above critical it collapses to some small value. Such experiments are currently underway.

V. THERMALLY EXCITED VORTEXES

An alternative to the production of vortices in DW systems by applying transverse electric field at low temperature is their thermal activation in the temperature region near $T_c$. We have already calculated the energy required to produce a singly-quantized vortex in the absence of external electric field (Eqn.(4)) - this energy is given by:

$$E_a = \frac{\pi K}{2} \frac{\gamma \lambda_{qp}}{\lambda_{TF}} \log \frac{W}{\epsilon^{1/2} \lambda_{qp}} \quad (17)$$

At low temperature this energy is big, but as we get closer to the transition point $E_a$ decreases rapidly because of decreasing rigidity $K$ (it is proportional to the condensate density). Moreover, $K$ is renormalized to smaller values in the vicinity of critical point by the fluctuations of the order parameter. The description of the dynamics of thermally excited vortices in external transverse electric fields is beyond the scope of this article and will be reported elsewhere; however a simple picture can be outlined here. The key point is that there is a gas of
thermally activated vortices at some finite temperature. We shall restrict ourselves to only two kinds of vortices, namely those with winding numbers $n_i = \pm 1$ and dipole momenta $d_{iy} = \pm |d_{iy}|$ respectively. It is assumed that the excitation of vortices with higher winding numbers is suppressed by their bigger energies. Then applying transverse electric field one would be able to polarize vortex gas. The dielectric constant of the vortex gas in the case of weak fields is:

$$\epsilon = \frac{4\pi nd_{iy}^2}{T}$$

Here $n$ is the density of vortices. $n$ also enters the expression for the correlation length in the gas of vortex dipoles. Thus one would be able to extract $n$ from experiments concerned with the response of DW materials to transverse electric fields. Such experiments are currently underway [16].

VI. CONCLUSIONS

We have performed an extensive analysis of vortices in highly anisotropic quasi-1D DW systems. This work builds a more detailed and correct picture of the structure and dynamics of vortices as compared to [3] and also predicts several unique properties of such vortices never encountered before with the case of their superconducting counterparts. Among those are the possibility of a temperature-controlled phase transition between Type-I and Type-II-like phases; the non-exponential screening of strong electric fields by vortices at comparably large scales as opposed to the incorrect conclusion about short range exponential screening drawn in [3]; temperature controlled ”flux quantum” of a vortex - its dipole moment. At the end we would like to point out that the exploration of the dynamics of topological defects in DW systems is a relatively new and actively developing area of condensed matter physics with many surprises and, possibly, some connections to other branches of physics (see e.g. a recent paper [17] and references therein).

AKNOWLEDGEMENTS

We would like to thank S. Sondhi and O. Motrunich for stimulating discussions and M. V. Feigelman for pointing out some of the relevant bibliography. We are also grateful to I. Nemenman and O. Motrunich for careful reading of this article and making numerous useful comments. We would like to especially acknowledge numerous insightful comments on this work by D. Huse and P. M. Chaikin.

[1] G. Gruner, ”Density Waves in Solids”, Addison-Wesley, 1994
[2] N. P. Ong, K. Maki, Phys. Rev. B32 (1985) 6582
[3] S. Ramakrishna, M. P. Maher, V. Ambegaokar, U. Eckern, Phys. Rev. Lett. 68 (1992) 2066
[4] J-M. Duan, Phys. Rev. B48 (1993) 4860
[5] K. Maki, Phys. Lett. A202 (1995) 313
[6] G. Mihaly, Y. Kim, G. Gruner, Phys. Rev. Lett. 67 (1991) 2713; Y. Kim, R. Gaal, B. Alavi, G. Gruner, Phys. Rev. B50 (1994) 13867
[7] F. Nad', P. Monceau, K. Bechgaard, Sol. St. Comm. 95 (1995) 655
[8] M. Hayashi, H. Yoshioka, PRL 77 (1996) 3403
[9] The coefficients in the equation are given as follows [10]: $K = N_1 v_F f_s / 2\pi$, $N_1$ is density of chains in the plane perpendicular to chains, $f_s(T) \approx 7\xi(3)\Delta^2 / 4\pi^2 T_c^2$ for $T \approx T_c$ and $f_s(T) \approx 1 - \sqrt{2\pi\Delta_0} / T \exp (\Delta_0 / T)$ for $T \approx T_c$ is the dimensionless condensate density $\rho_{c}/\rho_{0}$, $\Delta_0 = 1.7T_c$ is the gap at $T = 0$; $\gamma^2 = \xi_x^2 / \xi_y^2$ is the anisotropy parameter, $\xi_x$ and $\xi_y$ are correlation lengths in $x$ and $y$ directions respectively.
[10] H. Fukuyama, P. A. Lee, Phys. Rev. B17 (1978) 535; S. Brazovskii, I. E. Dzyaloshinskii, Sov. Phys. JETP 44 (1976) 1233
[11] Here $J = N_1 f_s / \pi$; $\lambda_{\parallel}^{-2} = (1 - f_s)\lambda_{TF}^{-2}$ is inverse squared screening length due to quasiparticles excited over the gap, $\lambda_{TF}^{-2} = 8N_1 e^2 / v_F$ is inverse squared Tomas-Fermi screening length in the metallic state. See main text for the discussion of $\epsilon$.
[12] This conclusion is true for the case of weak pinning which is a common situation in usual DW materials; see M.V.Feigelman in ”Charge Density Waves in Solids”, edited by L.P.Gorkov and G.Gruner (1989)
[13] The polarization contribution to the screening in the x-direction is omitted because $\epsilon(\lambda_{TF}^{-1}) \approx 1$.
[14] The variation of the phase of the order parameter cannot screen electric field in y-direction.
[15] This is true for field induced vortices. However, there are expected to be the so-called commensuration vortices in the bulk which can, in dirty systems, contribute to conductivity due to the phenomenon of spectral flow [17].
[16] A.Melikidze, Experimental Project, Princeton Univ.
[17] M.Hayashi, ”Spectral Flow and the Dynamics of Dislocations in Charge Density Waves”, cond-mat/9801094
[18] This is the energy of a vortex per unit length. For a vortex in a single plane one should multiply this expression by interplane distance.