Testable \((g - 2)_{\mu}\) contribution due to a light stabilized radion in the Randall-Sundrum model

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Abstract
In this paper we calculate the \((g - 2)_{\mu}\) contribution due to a light stabilized radion using the radion couplings both to the kinetic energy and the mass term of the muon. We find that the \((g - 2)_{\mu}\) contribution due to radion diverges logarithmically with the cut off. We then show that the bound from precision EW data on radion phenomenology allows a sizable shift in the radion mediated muon anomaly that could be detected or tested with the present precision and certainly with the future precision for measuring muon anomaly.

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Recently there has been a lot of interest in studying the phenomenology of models of large \([1]\) and small \([2]\) extra dimensions. Various phenomenological data have been used to put bounds on the unknown parameters of both large and small extra dimensions. In particular the precision electroweak (EW) data \([3]\) and the \((g-2)\) value of the muon \([4]\) have been used to constrain these parameters. In this paper we calculate the \((g-2)\) due to a stabilized radion \([5]\) in the Randall-Sundrum model. Using the radion couplings both to the kinetic energy and mass term of the muon we find that the radion contribution to the muon anomaly diverges logarithmically with the cut off. For a light radion with a mass of few tens of GeV and a radion vev of around a TeV, we obtain a muon anomaly of the order of a few times \(10^{-9}\). The values of \(m_\phi\) and \(<\phi>\) used by us in arriving at our numerical results are chosen so as to be consistent with the bounds implied by precision EW data on radion phenomenology.

The Feynman diagrams that give rise to the radion contribution to the muon anomaly are shown in Fig 1.

The Feynman rules that are necessary for evaluating these diagrams can be found in Ref \([6]\). We find that

\[
I_a = -\frac{e}{<\phi>^2} \int \frac{d^4l}{(2\pi)^4} \frac{\frac{3}{2}(l + p + 2q) - 4m_\mu}{[(l + q)^2 - m_\mu^2]([l - m_\mu^2][l - p]^2 - m_\phi^2]} \left[\frac{3}{2}(l + p + q) - 4m_\mu\right] (1)
\]

and

\[
I_b = \frac{3e}{<\phi>^2} \int \frac{d^4l}{(2\pi)^4} \frac{\frac{3}{2}(l + p + q) - 4m_\mu}{[(l - m_\mu^2)][(l - p - q)^2 - m_\phi^2]} (2)
\]

The expressions for the loop integrals \(I_a\) and \(I_b\) given above arise from Figs 1a and 1b respectively.
\[ e^\mu(q) \] is the photon polarization vector. We have chosen the incoming muon momentum \( p \) and the photon momentum \( q \) to express \( I_a \) and \( I_b \). It can be shown that Fig 1c and Fig 1d do not give rise to a term proportional to \( q^\nu \sigma_{\mu\nu} \). These two diagrams therefore do not contribute to the anomalous magnetic moment of the muon and can be omitted from further discussion. The contributions of Figs 1a and 1b to the anomalous magnetic moment of the muon can be shown to be given by
\[ I_a = -\frac{9ie}{2} m_\mu \epsilon^\mu(q) q^\nu \sigma_{\mu\nu} \int xdx dy \int \frac{d^4l}{(2\pi)^4} \frac{l^2}{D_a} + \ldots \]  \hspace{1cm} (3)

and

\[ I_b = \frac{3ie}{2} m_\mu \epsilon^\mu(q) q^\nu \sigma_{\mu\nu} \int (3 - 5x) dx \frac{1}{D_b} + \ldots \]  \hspace{1cm} (4)

where

\[ D_a = l^2 - (1 - x)^2 q^2 + 2xy(1 - x)p.q + -x^2 y^2 p^2 + (1 - x)^2 q^2 \]
\[ + p^2 xy - m_\mu^2 (1 - xy) - m_\phi^2 xy \]  \hspace{1cm} (5)

and

\[ D_b = (l - x(p + q))^2 + x(1 - x)(p + q)^2 - x m_\phi^2 - (1 - x) m_\mu^2 \]  \hspace{1cm} (6)

In the above we have dropped terms proportional to \(m_\mu^3 q^\nu \sigma_{\mu\nu}\) from \(I_a\) and \(I_b\). The contributions arising from these terms will be suppressed compared to those that are proportional \(m_\mu\) for \(m_\phi^2 > m_\mu^2\). Further in this paper we shall be interested only in the static values of the muon magnetic moment. In this static or low energy approximation we can set \(p^2 = p'^2 = p.q = 0\) in the denominator after the magnetic moment term proportional to \(q^\nu \sigma_{\mu\nu}\) has been extracted out from the numerator.

In this static approximation we get

\[ I_a \approx \frac{9em_\mu \epsilon^\mu(q)}{64\pi^2} \frac{q^\nu \sigma_{\mu\nu}}{m_\phi^2} \ln \frac{\Lambda^2}{m_\phi^2} + \ldots \]  \hspace{1cm} (7)

and
\[ I_b \approx -3e\mu e^\mu(q) \frac{q^\nu \sigma_{\mu\nu}(\ln \frac{\Lambda^2}{m_\phi^2} + \frac{5}{2}) + \ldots}{64\pi^2 < \phi >^2} \]  

(8)

where \( \Lambda \) is an ultraviolet momentum cut off. In the Randall-Sundrum model the cut off \( \Lambda \) can be identified with the mass of the lightest Kaluza-Klein mode of the graviton in the several TeV range. To arrive at the above result we have assumed that \( m_\phi^2 \gg m_\mu^2 \). The above contributions to the muon magnetic moment can be put in the form of an effective Lagrangian

\[ L_{\text{eff}} \approx \frac{3e\mu}{32\pi^2 < \phi >^2} \partial^\nu A^\mu \bar{\psi} \sigma_{\mu\nu} \psi (\ln \frac{\Lambda^2}{m_\phi^2} - \frac{5}{4}) \]  

(9)

The radion contribution to the muon magnetic moment is therefore given by

\[ a_{\mu}^r \approx \frac{3m_\mu^2}{16\pi^2 < \phi >^2} (\ln \frac{\Lambda^2}{m_\phi^2} - \frac{5}{4}) \]  

(10)

The UV cut off \( \Lambda \) for low energy radion phenomenology can be estimated by using naive dimension analysis (NDA) [7]. The NDA estimate stipulates that \( \Lambda = 4\pi < \phi > \), since \( \frac{1}{< \phi >} \) acts as the expansion parameter for non-renormalizable radion couplings to muon. In general however the cut off \( \Lambda \) can be related to \( < \phi > \) via \( \Lambda = k < \phi > \) where \( k \) lies between 1 and 4\( \pi \). In the numerical results presented in this paper we shall take \( k \) to be equal to the geometric mean of 1 and 4\( \pi \). We shall also ensure that the values of \( m_\phi \) and \( < \phi > \) used to estimate \( a_{\mu}^r \) satisfies the precision EW constraints.
The oblique EW parameter $T$ has been used to put bounds on $m_\phi$ and $<\phi>$ \[ 8 \]. These bounds can be represented in terms of an allowed region and forbidden region in the $m_\phi - <\phi>$ plane [see Fig 2].

![Graph](https://via.placeholder.com/150)

Figure 2: $\rho$ parameter constraints on radion vev $<\phi>$ and radion mass $m_\phi$. The allowed region lies above the curve.

For a light radion with a mass of 10 Gev, the precision EW constraint forces the radion mediated muon anomaly to be less than or equal to $2.7 \times 10^{-9}$. On the other hand for a heavy radion with a mass of 500 Gev, the $T$ parameter constraint on radion phenomenology allows a muon anomaly of $1.5 \times 10^{-9}$ as shown in Fig. 3.
Figure 3: Plot of muon anomaly $a_\mu$ against the radion mass $m_\phi$ (TeV). The horizontal line corresponds to the ultimate precision of the experiment.

We would like to note that the log divergence of our result arises from the radion coupling to the kinetic energy term of the muon which gives rise to a stronger divergence to the loop integral. Previous estimates of radion mediated muon anomaly used the radion coupling only to the mass term of the muon. In fact their radion coupling to fermion is similar to that of the Higgs boson. Therefore they do not get the log divergence. Actually they get a subdominant contribution proportional to $m_\mu^4$ which is correct for the Higgs boson but not for the radion. It can be shown that the radion couplings to the muon reduces to the mass term of the muon only if both the muon lines are on shell. However in calculating the loop diagrams shown in Fig 1
one certainly cannot assume that muon lines at each vertex are on shell and hence their result is not trustable.

The Muon \((g - 2)_{\mu}\) collaboration has reported a new improved measurement of positive muon anomaly \[1\]

\[a_{\mu}(\text{expt}) = (11659202 \pm 14 \pm 6) \times 10^{-10}\]

The muon anomaly expected in the SM according to the latest calculations is given by

\[a_{\mu}(SM) = (11659176.96 \pm 6.4) \times 10^{-10}\]

This shows a discrepancy from the experimental value given by \(\delta a_{\mu} = (26 \pm 16) \times 10^{-10}\). The ultimate goal of the Collaboration is to reduce the error to \(4 \times 10^{-10}\). In this paper we have shown that the T parameter constraint on \(m_{\phi}\) and \(\langle \phi \rangle\) gives rise to a radion mediated muon anomaly which is of the same order as the present precision \((1.5 \times 10^{-9})\) for measuring the muon anomaly. However with the ultimate precision of the experiment the level of muon anomaly presented in this paper can certainly be detected or tested.

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