Orbital maneuvers for asteroids using genetic algorithm

Guilherme M Neves¹, Denilson P S dos Santos ², Rita C Domingos³, Jorge K S Formiga⁴

¹,²,³ UNESP - Campus de São João da Boa Vista - SP, Brazil
⁴ UNESP - Campus de São José dos Campos - SP, Brazil
E-mail: guilherme.neves@unesp.br; denilson.santos@unesp.br; rita.domingos@unesp.br; jorge.formiga@unesp.br

Abstract. Near Earth objects (NEOs) are comets and asteroids that orbit the vicinity of the Earth. The scientific interest in comets and asteroids is due in large part to their status as remnants remaining relatively unchanged from the process of forming the solar system some 4.6 billion years ago, and some asteroids are made of precious metals such as platinum. Genetic algorithms are a particular class of evolutionary algorithms that use techniques inspired by evolutionary biology such as heredity, mutation, natural selection, and recombination. They can also be defined as global optimization algorithms and model a solution to a specific problem. This method provides a way to find solutions to problems that would be unlikely to be analytically feasible. The present work aims to use this method to optimize the consumption of fuel in Rendezvous maneuvers in interplanetary missions in a context where one wishes to send a probe to an asteroid to study it.

1. Introduction
The study of orbital maneuvers that optimize fuel expenditure is very important to decrease the cost of interplanetary missions. Currently, ways of sending spacecraft to some asteroids [1, 2] are being studied in order to study their chemical composition or to change their orbit due to the risk of colliding with Earth. In this context, the present work aims to study impulsive orbital maneuvers between the Earth and some asteroids, using a genetic algorithm to optimize the fuel consumption to perform the maneuver [3, 4]. Rendezvous bi-impulsive maneuvers [5] were considered to arrive at asteroids such as Apophis [6].

The purpose of the work is the application of a genetic algorithm in the optimization of orbital maneuvers [7–9]. The implementation of such a numerical method starts with a population of individuals, which are compared to each other by the function to be optimized, so that the chromosomes that represent a better solution are more likely to reproduce than those representing a worse solution. The definition of a better or worse solution is usually related to the current population. The algorithm still predicts the occurrence of mutations, which allows the creation of chromosomes that were not present in the population. And as generations go by, the algorithm converges to the best value.

In addition, the literature is extensive with respect to impulsive maneuvers, transfer orbits were studied in a context of a restricted three-body problem [10, 11] and in a context of non-keplerian trajectories [12].
The study of optimization of trajectories for asteroids is an evolving field of study due to its importance, approximately 1000 asteroids are currently known to have orbits that approach significantly the trajectory of the Earth in space, so constituting a potential threat to the planet. These asteroids are usually designated by the initials NEA (Near Earth Asteroid). The asteroids studied in this work were 99942 Apophis, 101955 Bennu and 162173 Ryugu. The goal in investigating their orbits and calculating maneuvers to intercept them would be the interest that the scientific community has in studying their chemical compositions for the purpose of understanding the formation of the solar system and how life originated, in addition there is a chance of a impact of these bodies with the Earth. So, this work aims the optimization of the cost of the interplanetary missions to asteroids.

2. The problem description
Let \( m \) be a particle of negligible mass that moves around a body of mass \( M \). Considering the formulation of the two-body problem, the particle trajectory is a conic, being an ellipse, a parable or a hyperbola depending on the energy associated with the particle. Given position and velocity vectors of a particle, its orbit can be determined. Can be determined its Keplerian elements, which are \( a \) the major semi-axis of the ellipse, \( e \) eccentricity, \( i \) inclination with respect to the plane of the ecliptic, \( \omega \) the argument of the periapsis, \( \Omega \) argument of the ascending node and \( M \) mean anomaly.

2.1. Lambert’s Problem
The applicability of this problem is extensive in astrodynamics, it can be used in lunar and interplanetary transfers, in rendezvous maneuvers [13–15]. Also known as the Gauss problem, Lambert’s problem is used to determine the trajectory of an object in orbit between two position vectors in space given the flight time and the direction of motion (Figure 1a).

Let \( P_1 \) and \( P_2 \) are points belonging to the orbit of the mass particle \( m \), at these points \( \vec{r}_1 \) and \( \vec{r}_2 \) are the position vectors \( \vec{v}_1 \) and \( \vec{v}_2 \) the mass velocity vectors \( m \) at the respective points (Figure 1a). Given \( f \) and \( g \) Lagrange coefficients [15] we can write (Equation 1),

\[
\begin{align*}
\vec{r}_2 &= f \vec{r}_1 + g \vec{v}_1 \\
\vec{v}_2 &= f \vec{r}_1 + g \vec{v}_1
\end{align*}
\]

(1)

\[
\begin{align*}
\vec{v}_1 &= \vec{r}_2 - f \vec{r}_1 \\
\vec{v}_2 &= \vec{r}_2 - \vec{r}_1
\end{align*}
\]

(2)

2.2. Genetic Algorithm - GA
The process starts with the creation of a random population of 1000 individuals that take into account a Julian date and a flight time obtained randomly within domains established a priori in the simulation. The domain of the Julian date is calculated given the domain among the Gregorian dates: 2018-12-05 and 2100-01-01. The flight time domain was 2 weeks to 3 years, however this domain was discretized in sub-domains of 1 week (partition) and in each sub-domain an optimum total impulse was calculated and thus a total impulse curve was obtained by time of flight (\( FT \)), finally, the algorithm uses as domain of flight time a domain that contains the global minimum and thus obtains the optimal maneuver (value optimized by the GA) for the asteroid in question.

The julian date and flight time of each individual are transformed into binary vectors that would be the chromosomes of each individual, after this procedure the algorithm allows the individuals crossover and generate new individuals, during this process there is a chance of a mutation occurring in the individuals. After that there is a competition between them according to the fitness function that one wants to optimize, so that the ones that have better results in the fitness
Figure 1. Figure a illustrates the Lambert problem that was used in the proposed method and figure b shows the algorithm flowchart implemented.

function survive and thus a new generation is formed, the algorithm realizes 50000 generations. The algorithm crossover rate is 65% and the mutation rate is 0.15% when the flight time domain is 0 to 1 month and 0 to 6 months and is 2% when the domain for flight time is 0 to 12 months (Figure 1b).

**Fitness function** Each individual carries with it the value of a Julian date and a flight time which are the inputs of the fitness function that will return the total impulse of the maneuver as follows:

(i) Let \( n \) be the mean motion, \( \lambda_0 \) the average length, \( \tau \) the time and \( \varpi \) the longitude of the periapsis of a particle orbiting a central body, the mean anomaly of the Earth is calculated by the equation 3, where \( t_i \) is a data of departure in julian date.

\[
M_e = n_e t_i - \lambda_{0e} - \varpi_e
\]  

The mean anomaly of the asteroid, is calculated according to the equation 4, where \( t_f \) is a date of arrival in julian date.

\[
M_a = n_a (t_f - \tau_a)
\]  

\( t_f = t_i + FT \), where \( FT \) is the flight time in years.

(ii) For both the Earth and the asteroid the eccentric anomaly \( E \) is calculated by the Kepler equation (5)

\[
M = E - e \sin E
\]

Now the Earth’s and asteroids true anomaly \( \nu \) is calculated by the equation 5.

\[
tg \left( \frac{\nu}{2} \right) = \sqrt{\frac{1 + e}{1 - e}} \cdot tg \left( \frac{E}{2} \right)
\]
(iii) The position vectors for the start and end of the transfer are then calculated on the basis of the calculated true anomalies and the orbiting elements of the Earth and the asteroid respectively.

(iv) Assuming a pro-maneuver the transfer orbit the position vectors and the flight time are calculated using the Lambert’s problem and then the impulses ($\Delta v_1$ and $\Delta V_2$) are calculated. Finally the function returns the total impulse of the maneuver ($\Delta v_{\text{total}} = \Delta V_1 + \Delta V_2$).

3. Numerical Results
The main goal of the present paper is to find optimal trajectories for a spacecraft to reach the asteroids Apophis (2004 MN4), Bennu (1999 RQ36) and Ryugu (1999 JU3) which were part of the group of asteroids that have orbits near the Earth (Table 1). These necessity about send an spacecraft to an asteroid is putting to the study about it chemical composition, or a possible crash on Earth. Besides that, exist the possibility of make a mining of precious metals in these celestial bodies.

| Table 1. Orbital Elements and characteristics of asteroids. |
|----------------------------------------------------------|
| Apophis (2004 MN4) | Bennu (1999 RQ36) | Ryugu (1999 JU3) |
| $\tau$ (TDB) | 2454733.5 | 2455625.5 | 2455907.5 |
| 2008-Sep-24.00 | 2011-Jan-01.00 | 2011-Dec-12.00 |
| a (ua) | 0.9224 | 1.1264 | 1.1896 |
| e | 0.1912 | 0.2037 | 0.1902 |
| i (degree) | 3.3314 | 6.0349 | 5.8837 |
| $\omega$ (degree) | 126.4019 | 66.2231 | 211.4258 |
| $\Omega$ (degree) | 204.4460 | 2.0609 | 251.6197 |
| Estimated diameter (km) | 0.325 | 0.492 | 0.98 |

3.1. Apophis
The asteroid Apophis has been accurately analyzed by several researchers because of the near-pass and high probability of impact with Earth [6]. The Figure 2 shows the curve of the $\Delta V_{\text{total}}$ per flight time of the spacecraft. We can notes that from a given moment the more we increase the flight time the greater the total impulse of the maneuver, this is due to the fact that the bigger the flight time ($FT$), the more the transfer orbit approaches a hyperbolic orbit. Figures 2 - 3 and Table 2 show the maneuver for the global minimum, using the proposed algorithm.
Figure 2. Minimum total impulse $\Delta V$ vs. flight time.

Figure 3. Optimal maneuver to asteroid Apophis using to GA optimization algorithm.

3.2. Bennu
The asteroid Bennu has about 500 m of equatorial diameter and close approach to Earth every 6 yrs. This is a rare asteroid (primitive and carbon-rich), which is expected to have organic compounds and water-bearing minerals like clays [16]. The Figures 4 - 5 and Table 2 expose the behavior of total impulse ($\Delta V_{total}$) curve by time and then the optimal maneuver (value optimized by the GA).
3.3. Ryugu

Ryugu is an Apollo-type asteroid, a group of asteroids with orbits close to Earth (Near-Earth asteroids). In simulations the total impulse first due to the flight time and then the maneuver with the least impulse. The Figures 6 - 7 and Table 2 expose the behavior of total impulse ($\Delta V_{\text{total}}$) curve by time and then the optimal maneuver.
Figure 6. Minimum total impulse vs. flight time.

(a) Transfer maneuver leaving the Earth (12/23/2068) and arriving at the asteroid Ryugu (06/106/2069).

(b) Convergence of the genetic algorithm with 50000 generations

Figure 7. Optimal maneuver to asteroid Ryugu using to GA optimization algorithm.

Table 2. Simulation results for asteroids using the GA (genetic algorithm) with 50000 generations, crossover rate 65% and the mutation rate 0.15%.

|                  | Apophis (2004 MN4) | Bennu (1999 RQ36) | Ryugu (1999 JU3) |
|------------------|--------------------|--------------------|------------------|
| Departure        | 14-Jun-2027        | 04-Dec-2042        | 23-Dec-2068      |
| Arrival          | 16-Apr-2028        | 29-Mar-2043        | 10-Jun-2069      |
| Flight time (days)| 277.1751           | 139.9075           | 169.2592         |
| $\Delta V_1$ (km/s) | 1.4467             | 1.8849             | 2.6909           |
| $\Delta V_2$ (km/s) | 2.9210             | 3.2123             | 1.6159           |
| $\Delta V_{total}$ (km/s) | 4.3678             | 4.3450             | 4.3068           |
4. Final Remarks
Evolutionary optimization was used to solve the Lambert Problem associated with those transfers and searches for the best trajectories. From the analysis of the obtained results, the genetic algorithm implemented here showed that this technique can obtain results for the proposal to optimize maneuvers using stochastic algorithms, it was verified that the total impulse to perform a bi-impulsive maneuver between Earth and an asteroid is intrinsically connected with the total time in which the maneuver is to be performed, i.e., the time in which the spacecraft take to get to the asteroid.

The simulations showed that there are values for the time (FT) in which the total impulse is minimized and total impulse becomes proportional to the time of flight. Minimum values were found for the problem variables and the procedure converged to optimized values.

Simulations involving the Bennu asteroid, no close dates were found, since the best moment of the encounter between the spacecraft and the asteroid, found as a solution of the problem in several simulations, converges to this specific date. The results for the Apophis asteroid converged to a great flight time around 270 days and for the asteroid Ryugu the algorithm converged satisfactorily and with a maneuver time of about 170 days.

In the future, we intend to use multi-impulsive maneuvers to study the behavior of the total impulse of the maneuver in relation to the flight time of the mission, and we will extend this study to maneuvers involving low thrust.

5. Acknowledgments
The authors wish to express their appreciation for the support provided by Grants #2018/18811-9, #2016/15675-1, #2017/04643-4 from São Paulo Research Foundation (FAPESP) and Institute of Science and Technology, UNESP - São Paulo State University. The contribution of researcher Antonio Fernando Bertachini de Almeida Prado of the National Institute for Space Research, INPE, Brazil, in the discussion and revision of this paper is gratefully acknowledged.

References
[1] Colombo C, Vasile M and Radice G 2009 Optimal low-thrust trajectories to asteroids through an algorithm based on differential dynamic programming. Celestial Mechanics and Dynamical Astronomy v.105. doi: 10.1007/s10569-009-9224-3
[2] Bulirsch R, Callies R 1992 Optimal trajectories for a multiple rendezvous mission to asteroids. Acta Astronautica v.26. doi: https://doi.org/10.1016/0094-5765(92)90149-D
[3] Cacciatore F and Toglia C 2008 Optimization of orbital trajectories using genetic algorithms. J Aerosp Eng Sci Appl 1(1):58–69
[4] Santos, D P S and Formiga, J K S 2015 Application of a genetic algorithm in orbital maneuvers Comp. Appl. Math. 34: 437. https://doi.org/10.1007/s40314-014-0151-x
[5] Prussing J E 1970 Optimal two- and three-impulse fixed-time rendezvous in the vicinity of a circular orbit. AIAA J 8(7):1221–1228
[6] Santos, D P S and Prado, A F B A 2012 Optimal low-thrust trajectories to reach the asteroid apophis WSEAS Mechanical Engineering Series, v.7 p. 241-251
[7] Santos D P S, Prado A F B A, Colasurdo G 2012 Four-impulsive rendezvous maneuvers for spacecrafts in circular orbits using genetic algorithms. Math Probl Eng (493507). doi: 10.1155/2012/493507
[8] Rosa S M, Casalino L 2006 Genetic algorithm and indirect method coupling for low-thrust trajectory optimization. AIAA No. 064468
[9] Santos, D P S and Prado, A F B d A Minimum fuel multi-impulsive orbital maneuvers using genetic algorithms. Advances in the Astronautical Sciences, Univelt, v. 145, p. 1137–1150, 2012.
[10] Prado A F B A, Broucke R A 1995 Transfer orbits in restricted problem. J Guid Control Dyn 18(3):593–598
[11] Prado, A. F. B. A. 2001 Trajetórias espaciais e manobras assistidas por gravidade. São José dos Campos: INPE 169
[12] Hinckley D W and Jr, H D L. Evolutionary Approach to Lambert’s Problem for Non-Keplerian Spacecraft Trajectories. Aerospace. 2017; 4(3):47.
[13] Fernandes, S S and Zanardi, M C F P S 2018 *Fundamentos de astronautica e suas aplicações*. São Bernardo do Campo - SP: Ed. UFABC. v. 1.

[14] Marec J P 1979 *Optimal space trajectories*. Elsevier, New York

[15] Curtis, H. D. 2013 *Orbital mechanics for engineering students*, Elsevier Aerospace Engineering Series

[16] NASA Jet Propulsion Laboratory. www.nasa.gov/press-release/nasas-osiris-rex-spacecraft-arrives-at-asteroid-bennu. (accessed in 04/04/2019)