Numerical modeling of interaction of the aircraft engine with concrete protective structures

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Abstract. The paper presents numerical modeling results considering interaction of Boeing 747 aircraft engine with nuclear power station protective shell. Protective shell has been given as a reinforced concrete structure with complex scheme of reinforcement. The engine has been simulated by cylinder projectile made from titanium alloy. The interaction velocity has comprised 180 m/s. The simulation is three-dimensional solved by finite element method using the author’s own software package EFES. Fracture and fragmentation of materials have been considered in calculations. Program software has been assessed to be used in calculation of multiple-contact objectives.

1. Introduction
Recent authors’ works describe results of simulation of Boeing 747 interaction with protective shell [1]; in that case contact interaction of airplane body with the shell has been replaced by impulse. Such approach is justified when initial stage of interaction is described, taking that concrete structure contacts quite thin aluminium body (from 3 to 8 mm thick). In this case one does not need to calculate contact boundaries thus simplifying significantly the objective and enabling obtaining the results in short terms. When aircraft engine impacts concrete structure it is crucial to consider contact interaction, deformation and fracture of both protective shell material and engine material. Reinforced concrete structures used for nuclear power stations protection are generally designed with quite complex scheme of reinforcement. Modeling of projectiles penetration into such structures can be held by means of two approaches depending on the reinforcement design, size of reinforcing elements and the assigned objectives. The first approach suggests clear separation of reinforcing elements and therefore one needs to calculate multiple contact boundaries thus complicating the given objective solution. The second approach is based on separation of locations in reinforced concrete structure which are adjacent to areas of reinforcement placement where material is described by mixture model, the properties of the material in such areas are determined on the basis of the volume fractions of the mixture components—steel and concrete. In this case there is only one contact boundary which is between the projectile and the barrier, that allows to reduce the computation time. The first approach can find application when it is necessary to obtain information on the influence of boundaries on shock-wave processes, to evaluate the adhesion properties of contact surfaces. However when assessment of integral parameters is required, second approach is appropriate to be applied. The given work contains calculations using both approaches.
2. Mathematical model

The set of equations describing non-stationary adiabatic motions of compressible medium in general coordinates \((i = 1, 2, 3)\), includes the following equations [2]:

- **continuity equation**
  \[
  \frac{\partial \rho}{\partial t} + \rho \nabla_i v^i = 0,
  \]
  (1)

- **motion equation**
  \[
  \rho a^k = \nabla_i \sigma^i k + F^k,
  \]
  (2)

  where
  \[
  a^k = \frac{\partial v^k}{\partial t} + v^i \nabla_i v^k,
  \]

- **energy equation**
  \[
  \frac{dE}{dt} = \frac{1}{\rho} \sigma^{ij} e_{ij},
  \]
  (3)

where \(F^k\) is components of mass force vector; \(\Gamma^i_{jk}\)—Christoffel symbols; \(\sigma^{ij}\)—contravariant components of symmetric stress tensor; \(E\)—specific internal energy; \(e_{ij}\)—components of symmetric strain velocity tensor; \(\rho\)—density of medium; \(\bar{\nu}\)—velocity vector; \(a^k\)—components of acceleration vector;

\[
\varepsilon_{ij} = \frac{1}{2} \left( \nabla_i v_j + \nabla_j v_i \right).
\]

(4)

Behavior of materials under study both metal and concrete has been described by elasto-plastic model.

Stress tensor is presented as a sum of deviatoric \(S^{ki}\) and spherical part \(P\):

\[
\sigma^{ij} = -Pg^{ij} + S^{ij},
\]

(5)

where \(g^{ij}\)—metric tensor. Pressure inside the materials has been calculated using Mie–Gruneisen equation as a function of specific internal energy \(E\) and density \(\rho\):

\[
P = \sum_{n=1}^{3} K_n \left( \frac{V}{V_0} - 1 \right)^n \left[ 1 - K_0 \left( \frac{V}{V_0} - 1 \right)/2 \right] + K_0 \rho E,
\]

(6)

where \(K_0, K_1, K_2, K_3\)—material constants, \(V_0\)—initial specific volume, \(V\)—current specific volume.

Suppose that the principle of minimum work of true stresses on the increments of plastic deformations is true for the medium, than the connection of component of strain velocity tensor and stress deviator is as follows:

\[
2G \left( g^{im} g^{jk} e_{mk} \frac{1}{3} g^{mn} e_{mk} g^{ij} \right) = \frac{DS^{ij}}{Dt} + \lambda S^{ij} \quad (\lambda \geq 0);
\]

(7)

in this case time derivatives of stress tensor are accepted by Jaumann definition:

\[
\frac{DS^{ij}}{Dt} = \frac{dS^{ij}}{dt} - g^{im} \omega_{mk} S^{kj} - g^{im} \omega_{mk} S^{ik},
\]

where \(\omega_{ij} = \frac{1}{2} \left( \nabla_i v_j - \nabla_j v_i \right)\), \(G\)—shear modulus.
Consider that material behaves in elastic manner ($\lambda=0$), in case when von Mises criterion is followed:

$$S^{ij}S_{ij} \leq \frac{2}{3}\sigma_d^2,$$

and it behaves in plastic manner ($\lambda > 0$), when the criterion is not followed. Here $\sigma_d$—dynamic tensile yield stress that can in the general case be the function of deformations velocity, pressure and temperature. The dependency of yield stress on the pressure has been considered for concrete [3]:

$$\sigma_d = 7.7 + \frac{11.398P}{13.9 + 0.82P},$$

in case the condition (8) is violated, we apply the procedure of correction of stresses considering the material plasticity for calculation of the component of stress deviator. Components of $S^{ij}$ are multiplied by normalizing factor, that equals to description of medium behavior in plastic zone as proved by equations of Prandtl–Reuss.

Limiting value of plastic strain intensity is accepted as a local criterion of shear fracture in metals:

$$e_u = \frac{\sqrt{2}}{3}\sqrt{3T_2 - T_1^2},$$

where $T_1$, $T_2$—first and second invariants of strain tensors.

To describe concrete fracture we use Hoffman criterion [4]. The criterion considers the differences in ultimate tensile and compressive strength:

$$C_1(\sigma_{22} - \sigma_{33})^2 + C_2(\sigma_{33} - \sigma_{11})^2 + C_3(\sigma_{11} - \sigma_{22})^2 + C_4\sigma_{11} + C_5\sigma_{22} + C_6\sigma_{33} + C_7\sigma_{12}^2 + C_8\sigma_{23}^2 + C_9\sigma_{31}^2 \leq 1,$$

where $C_i$ is defined from the following formulas:

- $C_1 = [(Y_tY_c)^{-1} + (Z_tZ_c)^{-1} - (X_tX_c)^{-1}] / 2$,
- $C_2 = [(X_tX_c)^{-1} + (Z_tZ_c)^{-1} - (Y_tY_c)^{-1}] / 2$,
- $C_3 = [(X_tX_c)^{-1} + (Y_tY_c)^{-1} - (Z_tZ_c)^{-1}] / 2$,
- $C_4 = (X_t^{-1} - X_c^{-1})$,
- $C_5 = (Y_t^{-1} - Y_c^{-1})$,
- $C_6 = (Z_t^{-1} - Z_c^{-1})$,
- $C_7 = S_{yz}^{-2}$,
- $C_8 = S_{xx}^{-2}$,
- $C_9 = S_{xy}^{-2}$,

where $X_t$, $X_c$, $Y_t$, $Y_c$, $Z_t$, $Z_c$—ultimate strength along axes $X$, $Y$, $Z$ under tension and compression respectively, while $S_{xy}$, $S_{yz}$, $S_{xx}$—ultimate shear strength along the corresponding axes. In case of isotropic material $X_t = Y_t = Z_t = R_t$, $X_c = Y_c = Z_c = R_c$, $S_{xy} = S_{yz} = S_{xx} = R_s$.

It is suggested that concrete fracture in conditions of intensive dynamic loads occurs the following way [1]: -in case strength criterion (11) is violated under conditions of compression ($e_{kk} \leq 0$), than further material behavior is described by hydrodynamic model; -in case criterion (11) is violated under conditions of tension ($e_{kk} > 0$), than material is considered to be completely destructed and components of stress tensor are supposed to be equal to zero. Experiments have shown that dynamic loads lead to increasing strength properties of concrete [5].
Figure 1. Ultimate strength of concrete as a function of deformation velocity: 1—under tension; 2—under compression.

Moreover the dependency of ultimate tensile and compressive strength differs. Connection of static and dynamic ultimate strength is expressed by means of dynamic response factor:

\[ K_d = \frac{R_d}{R_s}, \]

where \( R_d \)—dynamic strength, \( R_s \)—static strength.

Based on the experimental data [5] approximation dependences for dynamic response factor has been obtained for concrete under tension \( K_{dt} \) and compression \( K_{dc} \):

\[ K_{dt} = 0.00158333e^5 + 0.0252855e^4 + 0.15255e^3 \]
\[ + 0.047898e^2 + 1.01959e + 2.3603, \] 
\[ K_{dc} = 0.000832308e^5 + 0.0110547e^4 + 0.0447734e^3 \]
\[ + 0.0475887e^2 + 0.0184316e + 1.20895. \] 

\( (12) \)

\( (13) \)

Corresponding curves are given in figure 1, where curve 1 describes dependency of dynamic response factor under tension on deformations velocity, curve 2 describes dependency under compression.

Ultimate shear strength of concrete is defined from the values of ultimate compressive and tensile strength [3]: \( R_s = 0.55\sqrt{R_cR_t} \)

Important aspect while numerical modeling of impact interaction is selecting the algorithm of contact boundaries calculation. Generally, the existing program software use algorithms “element–node” and “node–node” to define the possible penetration of one body into another. The given work suggests the algorithm of “element–element” type [6], proved oneself to be appropriate while solving the three dimensional objectives and enabling to use possibilities of parallel computing to a maximum extent.
3. Numerical results

In order to evaluate the adequacy of mathematical model and computational algorithm numerical studies of the engine model interaction with mesh reinforcement used in protective shell have been held. Figure 2 show at sequential time points computation configuration of the process of regular interaction between deformable titanium projectile simulating the aircraft engine (4 m long and 0.9 m in diameter) with reinforcement mesh made from steel.

Initial projectile velocity is 180 m/s. Projectile weight is 4299 kg. Reinforcement mesh is given as a welded structure from steel bars of different diameter (20, 36 and 40 mm), located in orthogonally related directions. Longitudinal reinforcement contains four layers of double mesh; diameter of reinforcement from the side of projectile is 36 mm, and from the back side is 40 mm. Back side contains steel base 50 mm thick. Total thickness of reinforcement mesh is 1.5 m.

The given results illustrate well the deformation process and fracture of reinforcement and accuracy of contact interaction description. Figure 3 shows transverse cross-section of projectile and barrier. Projectile deformation in this case is insignificant and is observed only in the body, projectile penetrates the barrier and almost do not decelerate—its center of mass velocity decreases by 1% only during the whole interaction process.

Clear separation of reinforcement elements and calculation of contact boundaries requires larger resources and one not always needs such calculations. To assess protective properties of reinforced concrete shell, to describe the general structural response on impact, it is quite enough to apply mixture model where in some locations the effective properties of mixture are
defined from volume ratio of the components. In this case we define the locations along the reinforcing elements and inside those locations effective mixture properties are defined. The mentioned approach includes calculations of titanium projectile interaction (velocity—180 m/s) with reinforced concrete barrier 1.5 m thick.

Figure 4 presents configuration of projectile and barrier for this case. One can observe active concrete fracture in front of projectile in the area between the reinforcement locations. The fracture area increases with time in transverse directions as well. Fracture of barrier material occurs due to shear and tensile stresses.

4. Conclusions
Resulting from the conducted studies it can be concluded as follows:

- The suggested algorithm for calculation of contact boundaries enables solving multiple contact objectives of deformable solid bodies interaction considering their fracture and fragmentation.
- The suggested model of concrete behavior with metal reinforcement describes appropriately the stress-strain state, the processes of fracture and crack formation.
- The implemented algorithm and calculation methodology enables to study behavior of separate elements and whole structures in three dimensional dynamic setting.
- Based on the developed methodology it is possible to conduct multiple-parameters numerical experiments to select optimal structural solutions.
Acknowledgments
The work has been conducted with the grant of the President of the Russian Federation No. MK-413.2017.1 and the grant of the Ministry of Education and Science of the Russian Federation (project No. 9.6814.2017/BP).

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