Quark core formation in spinning-down pulsars.

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Abstract

Pulsars spin-down due to magnetic torque reducing its radius and increasing the central energy density. Some pulsar which are born with central densities close to the critical value of quark deconfinement may undergo a phase transition and structural re-arrangement. This process may excite oscillation modes and emit gravitational waves. We determine the rate of quark core formation in neutron stars using a realistic population synthesis code.

1 Introduction

Black holes (BHs) and neutron stars (NSs) are certainly two major potential sources of gravitational waves (GWs). Unlike BHs, whose gravitational waveforms are specified essentially by their masses and angular momenta, the characteristics of the gravitational emission from NSs depend on the properties of the nuclear matter.

Different mechanisms related to NSs susceptible to produce large amounts of GWs have been investigated in the past years (see de Freitas Pacheco 2001 for a recent review). In particular, the mini gravitational collapse induced by a phase transition in the core[1]. If quark deconfinement occurs in the central region of the NS, the core will have a softer equation of state, inducing the system to search for a new equilibrium configuration, which will be more compact and having a larger binding energy. The energy difference will partially cover the cost of the phase transition and will be partially used to excite mechanical modes, which will be damped either by heat dissipation or gravitational wave emission. Radial oscillation modes are more likely to be excited after the mini-collapse if the star has a slow rotation. In this case, most of the mechanical energy will be dissipated in the form of heat and radiated away. If the NS has an important rotation when the conditions for deconfinement are attained, then non-radial oscillations and radial modes coupled to rotation[2] may lead to an important GW emission, whose energy amounts to about $10^{52-53} \text{erg}$.

For a given equation of state for the hadronic and for the quark matter, the deconfinement occurs when the Gibbs conditions are satisfied, e.g., equality
between pressure and chemical potential of both phases. If the baryonic mass of the configuration is high enough, the pressure and the energy density in the central regions attain values required for a phase transition to occur. However, if the star is in rapid rotation, the central pressure and energy density are below the critical values and the NS has an internal structure constituted essentially by hadrons. If, as expected, the NS has a magnetic field, the rotation velocity will decrease due to the canonical magnetic dipole braking mechanism. Thus, after a certain time, the phase transition conditions are reached and the star develops a quark core.

If one assumes that the above scenario is correct, then a natural question appears. What is the expected frequency of these mini-collapse events? In a recent essay [3] based on very simple arguments and supposing that Soft Gamma Repeaters or cosmological Gamma-Ray Bursts are a consequence of NSs which have undergone a quark-hadron phase transition, estimated a frequency of \( \sim 10^{-5} \) yr\(^{-1}\) per galaxy for these events. In the present work, a more detailed estimate of the occurrence of these events is given. Rotating NS models have been computed for the equation of state derived by MVP02 in order to estimated, for a given baryonic mass, what is the critical rotation velocity below which the phase transition is possible. Then, using numerical simulations as in [4], the NS flux (in the \( P-\dot{P} \) plane) crossing the critical region where the transition occurs was estimated and, as a consequence, the event frequency.

Using the RNS rotation code[5], we are able to compute the evolution track of neutron stars with constant baryonic masses. Stars with baryonic masses lower than 1.05 \( M_\odot \), which represent 10% of the whole population using a gaussian distribution centered in the 1.4 \( M_\odot \), do never form quark cores due to its low central pressure. In opposite, 70% of the stars are already born with quark cores due o its high central pressure. The figure 1 describes the stars with masses between these two values and the formation of a quark core in a precise rotation frequency.

## 2 The phase transition frequency

As we have seen in the previous section, only NSs born in the mass interval 1.05 < \( M < 1.26 \) will develop a deconfined core. For a NS of a given mass within that range, if its rotation velocity is not zero, the central density will be below the critical value required for the phase transition to occur. During its evolution the rotation frequency decreases due to magnetic torques and the central density will eventually reaches the transition point. The timescale for the occurrence of the transition depends on the initial rotation period and magnetic field. The equation governing the number of NS in the rotation period \( P \) space is

\[
\frac{\partial N(P,t)}{\partial t} + \frac{\partial (\dot{P}N(P,t))}{\partial P} = S(P,t)
\] (1)
Figure 1: Evolutionary tracks for neutron stars of different masses in the diagram $\epsilon_0 - \Omega$. Rotation periods evolve according to the magnetic dipole braking mechanism. In the upper shadowed region stars are already born with a quark core whereas in the lower shadowed region, stars will never develop a quark core. The region between the two horizontal lines corresponds to the energy density jump during the phase transition.

where $N(P, t)$ is the number of $NS$s at the instant $t$ with period $P$ in the interval $P - (P + dP)$, $S(P, t)$ is the source function and $\dot{P} \equiv dP/dt = f(P, H)$ describes the deceleration mechanism. Formally, in terms of the Green’s function, the solution of this equation can be written

$$N(P, t) = \int_{-\infty}^{t} dt_0 \int dP_0 S(P_0, t) G(P - P_0, t - t_0)$$

which depends on the initial distribution of periods ($P_0$) and magnetic fields defining the evolution rate $dP/dt$. It worth mentioning that, as consequence of this relation, the period $P$ is unambiguously connected with the initial period $P_0$ at $t_0$.

In the present work, a different approach was adopted. We have assumed that the initial distribution of rotation periods and magnetic fields are independent on the NS mass, which obeys a Gaussian distribution as we have already mentioned. Under these conditions, the number of evolving $NS$s that cross the interval $P - (P + dP)$ is given by the current $J$ (Phinney & Blandford 1981)

$$J = \frac{1}{\delta P} \sum \frac{dP}{dt}$$

The current $J$ was calculated using the population synthesis code developed
by Regimbau & de Freitas Pacheco(2001), up-graded to take into account new pulsars discovered at high frequencies by the Parkes Multibeam Survey and more recent models for the natal kick distribution (7). Pulsars are generated at a constant rate using a Monte Carlo procedure and their evolution in the Galaxy is followed during hundreds of million years. An interactive method permits modifications of the initial distributions until observed distributions like those of the period, period derivative, magnetic field, distances are well reproduced when selection effects are taken into account (for details, the reader is referred to [4]). The derived parameters defining the properties of pulsars at birth are given in table 4. The estimated number of “active” pulsars in the Galaxy is about 250,000, their birth rate is about one every 90 years and their mean lifetime is about 22 million years.

Table 1: Parameters of the initial period $P_0$ and magnetic braking timescale $\ln \tau_0$ distribution, assumed to be Gaussian

| mean          | dispersion |
|---------------|------------|
| $P_0$ (ms) = 240 ± 20 | $\sigma_{P_0} = 80 \pm 20$ |
| $\ln \tau_0$ (s) = 11 ± 0.5 | $\sigma_{\ln \tau_0} = 3.6 \pm 0.2$ |

In figure 2 is shown the calculated current $J$ within bins of $P - (P + \delta P)$. These bins are equivalent to mass bins $\delta M$ in the interval $M_{\text{min}} - M_{\text{sup}}$ and the periods correspond to critical values for which the transition occurs, which depends on the NS mass. The sum of the current in each bin, weighted by the number of pulsars in that mass interval and averaged over the pulsar lifetime gives a phase transition frequency of $1.2 \times 10^{-5}$ events per year.

3 Gravitational Waves

Once we have determined the maximum distance probed (see [1]) by gravitational wave detector and the rate of events in our galaxy, we assume that extragalactic pulsars are formed with the same characteristics of those found in our galaxy. We have extrapolated our results to extragalactic calculations using the data found in ([9, 8]) from where we extracted the luminosity of close galaxy and clusters. We have used the luminosity as an indication of the formation of stars and consequently of the formation of NS. The luminosity of the enclosed members was divided by the Milky Way luminosity to derive the total number of detectable events. From ([9, 8]) we can extract, for example, the Local Group of Galaxies, where our Galaxy is the most important, followed by M31.
Figure 2: We show the neutron stars flow by second for each period and the flow of hybrid star formation in the filled bars.

and M33. We can also find at larger distances, objects like NGC672 (D~6 Mpc and L~ 6.7 × 10^{10}L_{\odot}). Using this results we estimate the maximum detection rate of only one event each 800 years for the present Virgo planned sensitivity, which is low rate of events. This value get better by a factor of 2 if we are able to detect event up to 10 Mpc (Adv. Ligo planned sensitivity) and reach one event each 100 years if the detectors are able to see this kind of events in the Virgo cluster.

4 Conclusions

Neutron stars evolution has been described. They were generated with periods and masses that reproduces the characteristics of the detected pulsars. Than, we have performed the spin-down due to magnetic torque, which increases the central density of the stars. Some of these pulsars, borned with central densities close to the deconfinement density, may undergo a phase transition and suffer a micro-collapse. The rate such events were determined as well as the possible rate of GW detection that come from them.

Most of the neutron stars cores are formed during the supernovae event. The stars which form a quark core after the supernovae explosion have their evolution determined by the initial mass and period. Our model has predicted a rate of 10^{-5} events \cdot year^{-1} \cdot galaxy^{-1}. As these events can be detected by the gravitational wave detectors for distances close to 7 Mpc (13 Mpc) by the Virgo (Adv. Ligo) detectors, we can, in first approximation, extrapolate the results obtained from our galaxy to a greater distance which encloses more events and
estimate a rate of detection of about one each 800 years for the Virgo detector. The planned advanced Ligo detector shall be able to see such events close to the Virgo cluster and possibly detect one event each 100 years. This rate of events can still get better if one takes into account the stars that undergo a phase transition due to mass accretion, however we do not expect a great change in the actual value since there are only few low-mass stars that could undergo the phase transition.

References

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