Nuclear equation of state from observations of short gamma-ray burst remnants

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The favored progenitor model for short γ-ray bursts (SGRBs) is the merger of two neutron stars that triggers an explosion with a burst of collimated γ-rays. Following the initial prompt emission, some SGRBs exhibit a plateau phase in their X-ray light curves that indicates additional energy injection from a central engine, believed to be a rapidly rotating, highly magnetized neutron star. The collapse of this “protomagnetar” to a black hole is likely to be responsible for a steep decay in X-ray flux observed at the end of the plateau. In this paper, we show that these observations can be used to effectively constrain the equation of state of dense matter. In particular, we show that the known distribution of masses in binary neutron star systems, together with fits to the X-ray light curves, provides constraints that exclude the softest and stiffest plausible equations of state. We further illustrate how a future gravitational wave observation with Advanced LIGO/Virgo can place tight constraints on the equation of state, by adding into the picture a measurement of the chirp mass of the SGRB progenitor.

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Recent observations of long and short γ-ray bursts (SGRBs) show plateau phases in the X-ray light curves that last hundreds of seconds \[ t \approx 10^2 \text{ s} \] and provide evidence for ongoing energy injection through a central engine \[ t \approx 10^2 \text{ s} \]. The main candidate for the central engine in SGRBs is a rapidly rotating, highly magnetized neutron star (NS) \[ t \approx 10^2 \text{ s} \] that forms following the coalescence of two NSs \[ t \approx 10^2 \text{ s} \]. Recent analytic fits to X-ray light curves support this “protomagnetar” interpretation of a central engine for both long \[ t \approx 10^2 \text{ s} \] and short GRBs \[ t \approx 10^2 \text{ s} \]. Excitingly, some objects exhibit an abrupt cutoff in the X-ray flux \( \sim 100 \text{ s} \) after the initial trigger \[ t \approx 10^2 \text{ s} \]. This has been interpreted as the metastable protomagnetar collapsing to form a black hole.

From a theoretical perspective, the coalescence of binary NSs can follow a number of evolutionary paths. If the merger remnant is sufficiently massive, it immediately collapses to a black hole, or forms a dynamically unstable hypermassive NS that is supported by strong differential rotation and thermal pressure \[ t \approx 10^2 \text{ s} \]. Magnetic braking terminates differential rotation on the Alfvén timescale \[ t \approx 10^2 \text{ s} \] implying that the object collapses in \~{} 10–100 ms. If the merger remnant is less massive it forms a supramassive, metastable protomagnetar in which centrifugal forces support the remnant \( t \approx 10^2 \text{ s} \) until the centrifugal force is insufficient to support the mass, at which point it collapses to a black hole. The recent discovery of \~{} \( 2 \, M_\odot \) NSs \[ t \approx 10^2 \text{ s} \] in which centrifugal forces from uniform rotation support a higher mass than the nonrotating Tolman-Oppenheimer-Volkoff (TOV) maximum mass \[ t \approx 10^2 \text{ s} \]. Such a supramassive star spins down until the centrifugal force is insufficient to support the mass, at which point it collapses to a black hole. The recent discovery of \~{} \( 2 \, M_\odot \) NSs \[ t \approx 10^2 \text{ s} \] demonstrates that the equation of state (EOS) permits massive enough NSs for supramassive stars to be created from the merger of two NSs \[ t \approx 10^2 \text{ s} \]. Finally, a merger remnant that is less massive than the TOV maximum mass will survive as a stable NS.

In this paper, we focus on the possibility that protomagnetars drive the plateau phases of SGRB X-ray light curves. The loss of rotational energy from the NS powers the emission, and a simple spin-down model can be fit to the light curve to obtain the initial spin period, \( p_0 \), and surface dipolar magnetic field, \( B_p \), of the protomagnetar \[ t \approx 10^2 \text{ s} \]. When an abrupt decay in X-ray luminosity is also observed, this is interpreted as the star having spun down to the point at which centrifugal forces can no longer support its mass against gravity \[ t \approx 10^2 \text{ s} \]. The time between the initial prompt emission and the decay, \( t_{\text{col}} \), is hence interpreted as the collapse time of the protomagnetar. Given \( p_0 \) and \( B_p \), the time it takes the NS to collapse will depend only on its initial mass and the EOS. We thus have almost all of the ingredients needed to determine the EOS, with the exception that the initial mass of the NS is not known. In the following, we show how one can constrain the EOS using these observations and the observed distribution of NS masses in binary NS systems \[ t \approx 10^2 \text{ s} \]. We also show how the EOS constraints will improve given a gravitational wave (GW) measurement of the binary inspiral (i.e., prior to coalescence) with Advanced LIGO and Virgo.

We focus on the observations presented in Rowlinson et al. \[ t \approx 10^2 \text{ s} \], in which X-ray plateaus were observed following initial SGRB triggers using \textit{Swift}. The light curves fit the prediction of a protomagnetar that is being spun down through dipole electromagnetic radiation \[ t \approx 10^2 \text{ s} \] (as noted in Rowlinson et al. \[ t \approx 10^2 \text{ s} \]), this is consistent with the late-time residual spin-down phase being driven by a relativistic magnetar wind \[ t \approx 10^2 \text{ s} \], allowing the authors to obtain \( p_0 \) and \( B_p \) from the model.

Rowlinson et al. \[ t \approx 10^2 \text{ s} \] present data for a number of ob-
Table I: The SGRB sample containing central engines used in this article, with all data and fits from Ref. 5. *z*, *p₀*, *B_p* and *t_col* are, respectively, the redshift, initial spin period, surface dipolar magnetic field, and collapse time. The bottom four SGRBs do not collapse within 10⁴−10⁵ s.

| GRB       | *z*   | *p₀* (ms) | *B_p* (10¹⁵ G) | *t_col* (s) |
|-----------|-------|-----------|----------------|-------------|
| 060801    | 1.13  | 1.95 ± 0.15 | 11.24 ± 1.93 | 326         |
| 070724A   | 0.46  | 1.80 ± 1.04 | 28.72 ± 1.29  | 90          |
| 080905A   | 0.122 | 9.80 ± 0.78 | 39.26 ± 10.24 | 274         |
| 101219A   | 0.718 | 0.95 ± 0.05 | 2.81 ± 0.47   | 138         |
| 051221A   | 0.55  | 7.79 ± 0.31 | 1.80 ± 0.14   | -           |
| 070809    | 0.219 | 5.54 ± 0.48 | 2.06 ± 0.48   | -           |
| 090426a   | 2.6   | 1.89 ± 0.07 | 4.88 ± 0.88   | -           |
| 090510    | 0.9   | 1.86 ± 0.04 | 5.06 ± 0.27   | -           |

*Duration (T90=1.2s) suggests GRB090426 is a SGRB, however its host and prompt characteristics remain ambiguous [e.g. 37, 38].

Objects with accurate redshift measurements. As this is required to determine the rest-frame light curve, and hence *p₀* and *B_p*, we omit any SGRBs for which the redshift is not known. We are left with four SGRBs that collapse and four that are long-term stable, which are presented in Table I.

The values of *B_p* and *p₀* are derived assuming electromagnetic dipolar spin-down, with perfect efficiency in the conversion between rotational energy and electromagnetic radiation. We discuss the possibility of a lower efficiency below. Note that a mass of 1.4 *M_☉* and radius of 10 km were also assumed, although the dependence on these parameters is weak 4, 5. The standard spin-down formula is 39

\[ p(t) = p₀ \left(1 + \frac{4\pi^2 B_p R^6}{3c^2 I p₀ t}\right)^{1/2}, \]  

(1)

where *R* and *I* are the radius and moment of inertia, respectively, of the NS. This spin-down law is implicitly used in the fits to the X-ray light curves [4]; a deviation from dipole spin-down would result in a different power-law exponent (see also [11]). Moreover, it has recently been shown that randomly distributed magnetic fields lead to similar spin-down luminosities than ordered magnetic fields [42].

For a given EOS, one can write the maximum gravitational mass, *M_max*, as a function of the star’s rotational kinetic energy [39, 41], and hence *p*. For slow rotation

\[ M_{\text{max}} = M_{\text{TOV}} \left(1 + \alpha p^2\right), \]  

(2)

where in Newtonian gravity \( \beta = -2 \) and \( \alpha \) is a function of the star’s mass, radius, and moment of inertia. We evaluate equation 2 in relativistic gravity by creating equilibrium sequences of \( M_{\text{max}}(p) \) using the general relativistic hydrostatic equilibrium code RNS [42]. That is, for various values of the spin period we calculate equilibrium sequences and find the local maximum in the \( M\sim p_c \) curve (where \( p_c \) is the central energy density) that indicates the maximum mass. We then calculate a functional fit to these equilibrium sequences to get \( \alpha \) and \( \beta \) for each EOS.

A supramassive protomagnetar collapses when the star’s period becomes large enough that \( M_p = M_{\text{max}}(p) \), where \( M_p \) is the mass of the protomagnetar. The collapse time, *t_col*, is found by substituting 11 into 2 with \( t = t_{\text{col}} \) and \( M_{\text{max}} = M_p \). Solving for *t_col* gives

\[
t_{\text{col}} = \frac{3c^3 I}{4\pi^2 B_p^2 p_0} \left[\left(\frac{M_p - M_{\text{TOV}}}{\alpha M_{\text{TOV}}}\right)^{2/\beta} - p_0^2\right].
\]  

(3)

Equation 3 gives the time for a supramassive protomagnetar to collapse to a black hole given observed parameters (*p₀*, *B_p*, *M_p*) and parameters related to the EOS (\( M_{\text{TOV}} \), *R*, and *I*). Note that Eq. 3 does not account for several effects, such as how *I* and *B_p* change with time as the star spins down or how the presence of matter outside the star affects the spin-down torque, a point we discuss below.

The observations in Ref. 5 give *B_p*, *p₀* and *t_col*, implying that we require *M_p* in Eq. 3 to constrain the EOS. We obtain *M_p* statistically from the observed masses of NSs in binary NS systems 33, 52, where the most up-to-date measurements give \( M = 1.32^{+0.11}_{-0.09} \) *M_☉*, with the errors being the 68% posterior predictive intervals 52. Numerical simulations of binary NS mergers and observations of SGRBs indicate that \( \lesssim 0.01 \) *M_☉* of material is ejected during the merger [e.g. 24, 42 and references therein]. Modulo this lost mass, which we ignore in the following, it is the rest mass of a system that is conserved through the merger. An approximate conversion between gravitational and rest masses is \( M_{\text{rest}} = M + 0.075 M^2 \) 44, which leads to a gravitational mass for the protomagnetar following an SGRB merger of \( M_p = 2.46^{+0.13}_{-0.15} \) *M_☉*.

In Fig. 11 we plot the collapse time, *t_col*, as a function of the protomagnetar mass, *M_p*, for each of the SGRBs listed in Table I. We utilize five EOSs that are consistent with current observations and have a range of maximum masses: SLy 45 \( (M_{\text{TOV}} = 2.05 \) *M_☉*, \( R = 9.97 \) km; black curve), APR 46 \( (2.20 \) *M_☉*, \( 10.00 \) km; orange), GM1 47 \( (2.37 \) *M_☉*, \( 12.05 \) km; red), AB-N 48 \( (2.67 \) *M_☉*, \( 12.90 \) km; green) and AB-L 48 \( (2.71 \) *M_☉*, \( 13.70 \) km; blue).
Consider GRB 060801 in Fig. 1 with $B_p$ and $p_0$ given in Table I. EOS GM1 (red curves) requires $M_p \approx 2.38 \, M_\odot$ for it to collapse 326 s following the initial burst. On the other hand, EOS SLy (black curves) requires $M_p \approx 2.06 \, M_\odot$, which falls well outside the 2σ posterior mass distribution. The quoted errors for $B_p$ and $p_0$ have little effect on this result. Similarly, GRB 101219A requires $M_p \approx 3.00 \, M_\odot$ for AB-L and $M_p \approx 2.82 \, M_\odot$ for AB-N, which both lie at the extreme high-mass end of the distribution. In this sense, all of the GRBs plotted in the two left-hand columns of Fig. 1 favor the intermediate EOSs. It is worth noting that the EOSs we plot are a representative sample that covers a wide range of maximum masses; many more EOSs fit into the intermediate regime that would be satisfied by the constraints we are placing herein. For an up-to-date review of plausible EOSs see Ref. [32].

It is worth paying special attention to GRB 080905A. Rowlinson et al. [4] found relatively large $p_0$, implying slow spin-down from electromagnetic torques. In the 274 s before GRB 080905A collapses, the protomagnetar has spun down from $p_0 = 9.8$ ms to between $p = 10.2$ ms and $p = 10.9$ ms depending on the EOS. For any EOS, this requires a fine tuning in the protomagnetar mass. There are many interpretations for this fine tuning. Fan et al. [49] proposed that this is evidence that the protomagnetar was predominantly spun down through GW losses as opposed to electromagnetic torques. This is possible, although we note that the ellipticity of the star needs to be $\sim 10^{-2} - 10^{-3}$, which requires an average internal toroidal field of almost $10^{17}$ G for a star with $M \gtrsim 2.5 \, M_\odot$ [50]. On the other hand, the isotropic efficiency of turning rotational energy into electromagnetic energy is assumed to be 100%. Reducing the assumed efficiency or beam opening angle also leads to a reduction of the initial spin period. Other possibilities include a chance alignment that led to a false host-galaxy identification, or ongoing accretion or propelling that is affecting the pulsar spin-down [51]. It is clear that these are crucial issues that have to be dealt with in a more systematic study if our method is to be used to obtain a strong, quantitative constraint on the EOS.

The two right-hand columns of Fig. 1 are those SGRBs that are not observed to collapse. Their relatively high initial spin periods and low surface magnetic field strengths imply that they do not spin down significantly in $\sim 100 - 1000$ s. Therefore, as with GRB 080905A, each EOS curve is almost a vertical line. If $M_p \lesssim M_{\text{TOV}}$, these objects are stable magnetars, and will never collapse from loss of centrifugal support. On the other hand, they may still have $M_p \gtrsim M_{\text{TOV}}$, in which case they are unstable with $t_{\text{col}} \gg 10^5$ s. If the latter is true, these GRBs could be candidate “blitzars” [52] that are a proposed physical mechanism behind fast radio bursts (FRBs) [53, 54]. In principal, if the blitzer model is correct one could utilize the method described herein to also constrain the EOS using FRBs, although a method for determining $t_{\text{col}}$ would be required. A method for testing the blitzer model, in particular the connection between FRBs and GRBs, has recently been described in Ref. [55].
The apparent bias in the protomagnetar mass posterior distribution from Advanced LIGO-only measurements (dotted black curve of Fig. 2) is caused by the bias in the estimation of \( \eta \) for some GW waveform templates \([57]\). Individual templates can, however, be used to estimate \( \eta \) with percent-level precision, which, when combined with measurements of \( M_p \), would render the binary NS mass distributions irrelevant in constraining protomagnetar masses.

How often does one expect a coincident GW and electromagnetic detection of an SGRB with an X-ray plateau? Using a conservative beaming angle of \( 8^\circ \) and references therein) and the \textit{Swift} sample of SGRBs corrected for dominant selection biases \([50]\), we obtain an intrinsic rate of \( 820 \text{ Gpc}^{-3} \text{s}^{-1} \). With a binary NS horizon distance for coincident Advanced LIGO and Virgo detections \([56, 60]\) and assuming 50\% of all SGRBs have X-ray plateaus \([5]\), we get a rate of 0.2 coincident electromagnetic and GW detections per year. The Space-based multi-band astronomical Variable Object Monitor (SVOM) has a decrease in sensitivity of a factor \( \sim 2 \) compared to \textit{Swift}, but the higher triggering energy band may be more optimal for the detection of spectrally harder SGRBs. Assuming that these two effects cancel, the increased sky coverage of SVOM over \textit{Swift} implies \( \sim 0.4 \) coincident events per year. Finally, ISS-Lobster, a proposed all-sky X-ray imaging telescope, has been estimated to see about two coincident SGRBs per year \([61]\), corresponding to about one per year with X-ray plateaus.

In this paper we have shown how one can constrain the nuclear EOS from observations of SGRBs that exhibit X-ray plateaus. We have outlined how current understanding of the mass distribution in NS binaries can already be used to place constraints on the EOS of dense matter and how a future coincident detection of a GW and X-ray signal from a binary NS merger could place significantly stronger constraints. This is an exciting prospect, and the rates we have estimated for coincident detection suggest that it is a very real possibility.

Given this encouraging starting point, it is crucial for future work to build on the method presented here and address in a more systematic way the caveats we have mentioned above (e.g., more detailed torque modeling, more accurate fits to light curves). This has the potential to allow for strong and truly quantitative constraints to be placed on the EOS of dense matter.

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[57] J. Aasi et al., Phys. Rev. D 88, 062001 (2013).
[58] I. Bartos, P. Brady, and S. Márka, Classical and Quantum Gravity 30, 123001 (2013), 1212.2289.
[59] D. M. Coward, E. J. Howell, T. Piran, G. Stratta, M. Branchesi, O. Bromberg, B. Gendre, R. R. Burman, and D. Guetta, Mon. Not. R. Astron. Soc. 425, 2668 (2012).
[60] LIGO Scientific Collaboration, Virgo Collaboration, J. Aasi, et al. (2013), arXiv:1304.0670.
[61] J. Camp, S. Barthelmy, L. Blackburn, K. G. Carpenter, N. Gehrels, J. Kanner, F. E. Marshall, J. L. Racusin, and T. Sakamoto, Experimental Astronomy (2013).