Analysis of the real estate market in Las Vegas: Bubble, seasonal patterns, and prediction of the CSW indexes

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Abstract

We analyze 27 house price indexes of Las Vegas from Jun. 1983 to Mar. 2005, corresponding to 27 different zip codes. These analyses confirm the existence of a real-estate bubble, defined as a price acceleration faster than exponential, which is found however to be confined to a rather limited time interval in the recent past from approximately 2003 to mid-2004 and has progressively transformed into a more normal growth rate comparable to pre-bubble levels in 2005. There has been no bubble till 2002 except for a medium-sized surge in 1990. In addition, we have identified a strong yearly periodicity which provides a good potential for fine-tuned prediction from month to month. A monthly monitoring using a model that we have developed could confirm, by testing the intra-year structure, if indeed the market has returned to “normal” or if more turbulence is expected ahead. We predict the evolution of the indexes one year ahead, which is validated with new data up to Sep. 2006. The present analysis demonstrates the existence of very significant variations at the local scale, in the sense that the bubble in Las Vegas seems to have preceded the more global USA bubble and has ended approximately two years earlier (mid-2004 for Las Vegas compared with mid-2006 for the whole of the USA).

Key words: Econophysics, Real estate market, Periodicity, Power law, Prediction

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1 Introduction

Zhou and Sornette (2003) analyzed the deflated quarterly average sales prices $p(t)$ from December 1992 to December 2002 of new houses sold in all the states in the USA and by regions (northeast, midwest, south and west) and found that, while there was undoubtedly a strong growth rate, there was no evidence of a bubble in the latest six years (as qualified by a super-exponential growth). Then, Zhou and Sornette (2006) analyzed the quarterly average sale prices of new houses sold in the USA as a whole, in the northeast, midwest, south, and west of the USA, in each of the 50 states and the District of Columbia of the USA up to the first quarter of 2005, to determine whether they have grown faster-than-exponential (which is taken as the diagnostic of a bubble). Zhou and Sornette (2006) found that 22 states (mostly Northeast and West) exhibit clear-cut signatures of a fast growing bubble. From the analysis of the S&P 500 Home Index, they concluded that the turning point of the bubble would probably occur around mid-2006. The specific statement found at the bottom of page 306 of Ref. [Zhou and Sornette (2006)] is: “We observe a good stability of the predicted $t_c \approx$ mid-2006 for the two LPPL models (2) and (3). The spread of $t_c$ is larger for the second-order LPPL fits but brackets mid-2006. As mentioned before, the power-law fits are not reliable. We conclude that the turning point of the bubble will probably occur around mid-2006.” It should be stressed that these studies departed from most other reports by analysts and consulting firms on real estate prices in that Zhou and Sornette (2003, 2006) did not characterize the housing market as overpriced in 2003. It is only in 2004-2005 that they confirmed that the signatures of an unsustainable bubble path has been revealed.

Let us briefly analyze how this prediction has fared. The upper panel of Figure 1 shows the quarterly house price indexes (HPIs) in the 21 states and in the District of Columbia (DC) from 1994 to the fourth quarter of 2006 released by the OFHEO. It is evident that the growth in most of these 22 HPIs has slowed down or even stopped during the year of 2006. When we look at the S&P Case-Shiller Home Indexes of the 20 major US cities, as illustrated in the lower panel of Figure 1, we observe that the majority of the S&P/CSIs had a maximum denoted by a solid dot in the middle of 2006, validating the prediction of Zhou and Sornette (2006). Specifically, the times of the maxima are respectively 2006/06/01, 2006/09/01, 2005/11/01, 2006/05/01, 2006/08/01, 2006/05/01, 2006/12/01, 2006/07/01, 2006/08/01, 2006/09/01, 2005/09/01, 2005/12/01, 2006/09/01, 2006/09/01, 2006/08/01, 2006/06/01, 2006/07/01, 2006/09/01, 2006/08/01, 2006/12/01, 2006/06/01, and 2006/07/01 for the 20 cities shown in the legend of the lower panel. The only two cities with a max-

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imum occurring later towards the end of 2006 (2006/12/01) are Miami and Seattle. However their growth rates decreased remarkably in 2006 as shown in the figure. Furthermore, the S&P/CS Home Price Composite-10 reached its historical high 226.29 on 2006/06/01 and the Composite-20 culminated to 206.53 on 2006/07/01, again confirming remarkably well the validity of the forecast of Zhou and Sornette (2006).

![Graph showing OFHEO HPIs and S&P/CSIs from 1994 to 2007]  

Fig. 1. Evaluation of the prediction of Zhou and Sornette (2006) that “the turning point of the bubble will probably occur around mid-2006” using the OFHEO HPI data (upper panel) and the S&P CSI data (lower panel).

In this note, we provide a more regional study of the diagnostic of bubbles and the prediction of their demise. Specifically, we analyze the Case-Shiller-Weiss (CSW) Zip Code Indexes of 27 different Las Vegas regions calculated with a monthly rate from June-1983 to March-2005. The CSW Indexes are based on the so-called repeat sales methods which directly measure house price appreciations. The key to these data is that they are observations of multiple transactions on the same property, repeated over many properties and then pooled in an index. Prices from different time periods are combined to create “matched pairs,” providing a direct measure of price changes for a given property over a known period of time. Bailey et al. (1963) proposed the basic repeat sales method over four decades ago, but only after the work by Case and Shiller (1987, 1989, 1990) did the idea receive significant attention.
in the housing research community.

Studying the Las Vegas database is particularly suitable since Las Vegas belongs to a state which was identified by Zhou and Sornette (2006) as one of the 22 states with a fast-growing bubble in 2005. With access to 27 different CSW Zip Code Indexes of Las Vegas, we are able to obtain more reliable and fine-grained measures, which both confirm and extend the previous analyses of Zhou and Sornette (2003, 2006). The next section recalls the conceptual background underlying our empirical approach. Then, section 3 analyzes the regional CSW indexes for Las Vegas, showing that there is a regime shift separated by a bubble around year 2004. Section 4 identifies and then analyzes the yearly periodicity and intra-year pattern detected in the growth rate of the regional CSW indexes. Section 5 offers a preliminary forecast based on the periodicity analyses in Sec. 4. Section 6 concludes.

2 Conceptual background of our empirical analysis

2.1 Humans as social animals and herding

Humans are perhaps the most social mammals and they shape their environment to their personal and social needs. This statement is based on a growing body of research at the frontier between new disciplines called neuroeconomics, evolutionary psychology, cognitive science, and behavioral finance. This body of evidence emphasizes the very human nature of humans with its biases and limitations, opposed to the previously prevailing view of rational economic agents optimizing their decisions based on unlimited access to information and to computation resources.

Here, we focus on an empirical question (the existence and detection of real-estate bubbles) which, we hypothesize, is a footprint of perhaps the most robust trait of humans and the most visible imprint in our social affairs: imitation and herding. Imitation has been documented in psychology and in neuro-sciences as one of the most evolved cognitive process, requiring a developed cortex and sophisticated processing abilities. In short, we learn our basics and how to adapt mostly by imitation all along our life. It seems that imitation has evolved as an evolutionary advantageous trait, and may even have promoted the development of our anomalously large brain (compared with other mammals). It is actually “rational” to imitate when lacking sufficient time, energy and information to take a decision based only on private information and processing, that is..., most of the time. Imitation, in obvious or subtle forms, is a pervasive activity of humans. In the modern business, economic and financial worlds, the tendency for humans to imitate leads in
its strongest form to herding and to crowd effects.

Based on a theory of cooperative herding and imitation, we have shown that imitation leads to positive feedbacks, that is, an action leads to consequences which themselves reinforce the action and so on, leading to virtuous or vicious circles. We have formalized these ideas in a general mathematical theory which has led to observable signature of herding, in the form of so-called log-periodic power law acceleration of prices. A power law acceleration of prices reflects the positive feedback mechanism. When present, log-periodicity takes into account the competition between positive feedback (self-fulfilling sentiment), negative feedbacks (contrariant behavior and fundamental/value analysis) and inertia (everything takes time to adjust). Sornette (2003) presented a general introduction, a synthesis and examples of applications.

2.2 Definition and mechanism for bubbles

The term “bubble” is widely used but rarely clearly defined. Following Case and Shiller (2003), the term “bubble” refers to a situation in which excessive public expectations of future price increases cause prices to be temporarily elevated. During a housing price bubble, homebuyers think that a home that they would normally consider too expensive for them is now an acceptable purchase because they will be compensated by significant further price increases. They will not need to save as much as they otherwise might, because they expect the increased value of their home to do the saving for them. First-time homebuyers may also worry during a housing bubble that if they do not buy now, they will not be able to afford a home later. Furthermore, the expectation of large price increases may have a strong impact on demand if people think that home prices are very unlikely to fall, and certainly not likely to fall for long, so that there is little perceived risk associated with an investment in a home.

What is the origin of bubbles? In a nutshell, speculative bubbles are caused by “precipitating factors” that change public opinion about markets or that have an immediate impact on demand, and by “amplification mechanisms” that take the form of price-to-price feedback, as stressed by Shiller (2000). A number of fundamental factors can influence price movements in housing markets. On the demand side, demographics, income growth, employment growth, changes in financing mechanisms or interest rates, as well as changes in location characteristics such as accessibility, schools, or crime, to name a few, have been shown to have effects. On the supply side, attention has been paid to construction costs, the age of the housing stock, and the industrial organization of the housing market. The elasticity of supply has been shown to be a key factor in the cyclical behavior of home prices. The cyclical process that we observed in the 1980s in those cities experiencing boom-and-bust cycles
was caused by the general economic expansion, best proxied by employment
gains, which drove demand up. In the short run, those increases in demand
encountered an inelastic supply of housing and developable land, inventories
of for-sale properties shrunk, and vacancy declined. As a consequence, prices
accelerated. This provided an amplification mechanism as it led buyers to
anticipate further gains, and the bubble was born. Once prices overshoot or
supply catches up, inventories begin to rise, time on the market increases,
vacancy rises, and price increases slow down, eventually encountering down-
ward stickiness. The predominant story about home prices is always the prices
themselves (see Shiller, 2000; Sornette, 2003); the feedback from initial price
increases to further price increases is a mechanism that amplifies the effects of
the precipitating factors. If prices are going up rapidly, there is much word-of-
mouth communication, a hallmark of a bubble. The word of mouth can spread
optimistic stories and thus help cause an overreaction to other stories, such as
stories about employment. The amplification can also work on the downside
as well. Price decreases will generate publicity for negative stories about the
city, but downward stickiness is encountered initially.

2.3 Was there a bubble? Status of the argument based on the ratio of cost of
owning versus cost of renting

In recent years, there has been increasing debates on whether there was a real
estate bubble or not in the United States of America. Case and Shiller (2003),
Shiller (2006) and Smith and Smith (2006) argued that the house prices over
the period 2000-2005 were not abnormal as they reflected only the convergence
of the prices to their fundamentals from below. In contrast, Zhou and Sornette
(2006) and Roehner (2006) have suggested that there was a bubble, which be-
came identifiable only after 2003, that is, after the work of Zhou and Sornette
(2003).

In this context, it is instructive to comment on the study by Himmelberg et al.
(2005), from the Federal Reserve Bank of New York, as it reflects the never
ending debate between tenants of the fundamental valuation explanation and
those invoking speculative bubbles. We are resolutely part of the second group.
Himmelberg et al. (2005) constructed measures of the annual cost of single-
family housing for 46 metropolitan areas in the United States over the last 25
years and compared them with local rents and incomes as a way of judging the
level of housing prices. In a nutshell, they claimed in 2005 that conventional
metrics like the growth rate of house prices, the price-to-rent ratio, and the
price-to-income ratio can be misleading and lead to incorrect conclusions on
the existence of the real-estate bubble. Their measure showed that, during
the 1980s, houses looked most overvalued in many of the same cities that
subsequently experienced the largest house price declines. But they found that
from the trough of 1995 to 2004, the cost of owning rose somewhat relative to the cost of renting, but not, in most cities, to levels that made houses look overvalued.

The rosy conclusion of Himmelberg et al. (2005), that 2004-2005 prices were justifiable and that there was no risk of deflation as no bubble was present, is based on a particularly curious comparison between cost of owning and cost of renting, as noticed by Jorion (2005), in a letter to the Wall Street Journal. Indeed, they candidly revealed however that their “cost of owning” calculations imply an “expected appreciation on the property” coefficient. The value for this factor is no doubt derived from figures for appreciation as currently observed on the housing market, meaning they regarded the current appreciation level as a reasonable assumption for what would indeed happen next – which is precisely what our analyses and that of others question. In other words, the authors had unwittingly hard-wired into their model the assertion that there was no housing bubble; little wonder then that this is also what they felt authorized to conclude. The circularity of their reasoning is particularly obvious in an illustration they gave for San Francisco where for more than 60 years the price-to-rent ratio has exceeded the national average, which, so they claimed, “does not necessarily make owning there more expensive than renting.” The reason why is that “high financing costs are offset by above-average expected capital gains.” Translated, this means that as long as there is a bubble, prices will go up and investing in a house remain a profitable operation. This trivial statement is hollow; the real question is whether the trend that is observed now remains sustainable.

In addition to this criticism put forward by Jorion (2005), there are other reasons to doubt the validity of the conclusion of Himmelberg et al. (2005). In the own words of Himmelberg et al. (2005), “the ratio of the cost of owning to the cost of renting is especially sensitive to the real long-term interest rates.” They are right in their rosy conclusion... as long as the long-term interest rates remain exceptionally low. It is particularly surprising that their estimation of the ratio of the cost of owning to the cost of renting was based on the most recent rates over the preceding year of their analysis (2004), while the price of a house is a long-term investment: what will be the long-term rates in 10, 20, 30, or 50 years? Another problem is that their analysis was “mono-dimensional”: they proposed that everything depends only on the ratio of the cost of owning to the cost of renting. But they missed the interest rates as an independent variable. As a consequence, it is not reasonable to compare the 1980s and the present time, as the long-term interest rates had nothing in common. Another problem with their analysis is that they assumed “equilibrium,” while people are sensitive to the history-dependent path followed by the prices. In other words, people are sensitive to the way prices reach a certain level, if there is an acceleration that can self-fuel itself for a while, while Himmelberg et al. (2005) discussed only the mono-dimensional level of the price, and not how it
We think that this general error made by “equilibrium” economists constitutes a fundamental flaw which fails to capture the real nature of the organization of human societies and their decision process. In the sequel, we actually focus our attention on signatures of price trajectories that highlight the importance of history dependence for prediction.

This discussion is reminiscent of the proposition by Mauboussin and Hiler (1999), offered close to the peak of the Internet and new technology bubble that culminated in 2000, that better business models, the network effect, first-to-scale advantages, and real options effect could account rationally for the high prices of dot.com and other New Economy companies. These interesting views expounded in early 1999 were in synchrony with the bull market of 1999 and preceding years. They participated in the general optimistic view and added to the strength of the herd. Later, after the collapse of the bubble, these explanations seem less attractive. This did not escape the then U.S. Federal Reserve chairman Alan Greenspan (1997), who said: “Is it possible that there is something fundamentally new about this current period that would warrant such complacency? Yes, it is possible. Markets may have become more efficient, competition is more global, and information technology has doubtless enhanced the stability of business operations. But, regrettably, history is strewn with visions of such new eras that, in the end, have proven to be a mirage. In short, history counsels caution.”

3 Regime shift in the CSW Zip Code Indexes of Las Vegas

3.1 Description of the data

We now turn to the analysis of the CSW indexes of 27 different Las Vegas zip regions obtained with a monthly rate. The 27 monthly CSW data sets start from June-1983 and end in March-2005. Figure 2 shows the price trajectories of all the 27 CSW indexes. Visual inspection shows (i) a very similar behavior of all the different zip codes and (ii) a sudden increase of the indexes since Mid-2003. Let us now analyze this data quantitatively.

3.2 Power law fits

The simplest mathematical equation capturing the positive feedback effect and herding is the power law formula (see Broekstra et al., 2005, for a simple introduction in a similar context)

$$I(t) = A + B|t_c - t|^m ,$$  (1)
where $B < 0$ and $0 < m < 1$ or $B > 0$ and $m < 1$. Others cases do not qualify as a power law acceleration. For $B < 0$ and $0 < m < 1$ or $B > 0$ and $m < 0$, the trajectory of $I(t)$ described by (1) expresses the existence of an accelerating bubble, which is faster than exponential. This is taken as one hallmark of the existence of a bubble.

Notice also that this formula expresses the existence of a singularity at time $t_c$, which should be interpreted as a change of regime (the mathematical singularity does not exist in reality and is rounded off by so-called finite-size effects and the appearance of a large susceptibility to other mechanisms). This critical time $t_c$ must be interpreted as the end of the bubble and the time where the regime is transiting to another state through a crash or simply a plateau or a slowly moving correction.

We have fitted each of the 27 individual CSW indexes using the pure power law model (1). The data used for fitting is from Dec-1995 to Jun-2005. We do not show the results as the signature of a power law growth is not evident, essentially because the acceleration is only over a rather short period of time from approximately 2002 to 2004. As a consequence, power law fits give unreliable critical time $t_c$ too much in the future (like 2008 and beyond). We have thus redone the fits of the 27 CSW indexes over a shorter time interval from Aug-2001 to Jun-2005. A typical example is shown in Fig. 3. All other 26 CSW are very similar, with some variations of the parameters, but the message is the same: while there is a clear faster-than-exponential acceleration over most of the time interval, the price trajectory has clearly transitioned into another regime in the latter part of the time interval considered here. The transition occurred smoothly from mid-2004 to mid-2005 (the end of the time period analyzed here).

It is important to recognize that the power law regime is expected only rela-
Fig. 3. Typical evolution of a CSW index from Aug-2001 to Jun-2005 and its fit by 
a power law, showing both the faster-than-exponential growth up to mid-2004 and 
the smooth transition to a much slower growth at later times. The root-mean-square 
χ of the residuals of the fit as well at $t_c$ and $m$ are given inside the figure.

tively close to the critical time $t_c$, while other behaviors are expected far from 
$t_c$. The simplest model is to consider that, far from $t_c$, the price follows an 
exponential growth with an approximately constant growth rate $\mu$:

$$I(t) = a + be^{\mu t}.$$  \hspace{1cm} (2)

A fuller description is thus to consider that formula (2) holds from the begin-
ing of the time series up to a cross-over time $t^*$, beyond which expression (1) 
takes over. Any given price trajectory should thus be fitted by (2) from some 
initial time $t_{start}$ to time $t^*$ and then by (1) from $t^*$ to the end of the time 
series. Technically, $t^*$ is known from the parameters $a, b, \mu, A, B, t_c, m$ by the 
condition of continuity of $I(t)$ at $t = t^*$, that is, both formulas give the same 
value at $t = t^*$. We can further determine one of the parameters $a, b$ or $\mu$ by 
imposing a condition of differentiability at $t^*$, that is, the first time-derivative 
of $I(t)$ is continuous at $t^*$. This approach is known in numerical analysis as 
“asymptotic matching” (see Bender and Orszag, 1978).

A simplified description of such a cross-over between a standard exponential 
growth and the power law super-exponential acceleration is obtained by using 
a more compact formulation

$$I(t) = A + B \tanh\left[\left(\frac{t_c - t}{\tau}\right)^m\right],$$ \hspace{1cm} (3)

where tanh denotes the hyperbolic tangent function. This expression derives 
from a study of the transition from the non-critical to critical regime in rup-
ture processes (of which bubbles and their terminal singularity belong to) 
conducted by Sornette and Andersen (1998). This expression has the virtue 
of providing automatically a smooth transition between the exponential behavior (2) and the pure power law (1), since $\tanh\left[\left(\frac{t_c - t}{\tau}\right)\right] \approx \left(\frac{t_c - t}{\tau}\right)$ for
$t_c - t < \tau$ and $\tanh[(t_c - t)/\tau] \approx 1 - 2e^{2(t-t_c)/\tau} \text{ for } t_c - t > \tau$. In this later case $t_c - t > \tau$, expression (3) becomes of the form (2) with $m = 1$ and

\begin{align}
a &= A + B , \\
b &= -2Be^{-2t_c/\tau} , \\
\mu &= 1/\tau .
\end{align}

In contrast, for $t_c - t < \tau$, expression (3) becomes of the form (1) with the correspondence $B/\tau^m \rightarrow B$. Expression (3) has only five free parameters, in contrast with the model involving the cross-over from (2) to (1) which has 7 free parameters ($a, b, \mu, A, B, t_c, m$) while $t^*$ is determined by the asymptotic matching). The pure power law formula (1) has 4 parameters while the exponential law (2) has just 3 parameters. The problem with expression (3) is that it does not recover a pure exponential growth even for $t_c - t > \tau$, when $m \neq 1$. Thus, expression (3) is limited in fully describing a possible cross-over from a standard mild exponential growth and an super-exponential power law acceleration. Our tests (not shown) find that a fit with model (3) retrieve the pure power law model (1) with the same critical time $t_c$ and exponent $m$ and the same root-mean-square residual r.m.s. (the fit adjusts the parameter $\tau$ to a very large value, ensuring that the fit is always in the regime $t_c - t \ll \tau$ so that the hyperbolic tangential model reduces to the pure power law model). Thus, contrary to our initial hopes, this approach does not provide any additional insight.

Inspired by these tests, we could propose the following modified model

\begin{equation}
I(t) = a + be^{\mu t}(t_c - t)^m .
\end{equation}

It has 5 adjustable parameters, like model (3), but it seems more flexible to describe the looked-for cross-over: for large $t_c - t$, the power law term $(t_c - t)^m$ changes slowly, especially for $0 < m < 1$ as is expected here; for small $t_c - t$, the power law term changes a lot while the exponential term is basically constant. But, this model is correct for a critical point only if $m < 0$ so that $b > 0$; otherwise, if $0 < m < 1$, $b < 0$ and for $t_c - t$ large, the exponential term which dominate does not describe a growth but an exponentially accelerating decay. For $0 < m < 1$, we thus need a different formulation. We propose

\begin{equation}
I(t) = a + be^{\mu t} + c(t_c - t)^m .
\end{equation}

We have fitted this formula to the data over the four periods 1983 - Oct. 2004, 1991 - Oct. 2004, 1983 - Mar. 2005, 1991 - Mar. 2005 and, while the fits are reasonable, the critical time $t_c$ is found to overshoot to 2007-2008, which is a typical signature that the model is not predictive.

In conclusion of this first preliminary study, the presence of a bubble (faster-than-exponential growth) is confirmed but the determination of the end of
this phase is for the moment unreliable.

3.3 Dependence of the growth rate on the index value

The monthly growth rate $g(t)$ of a given CSW index at time $t$ is defined by

$$g(t) = \ln[p(t)/p(t - 1)],$$

where $p(t)$ is the price of that CSW index at time $t$. Figure 4 shows the evolution of the growth rates of the 27 CSW indexes from June-1983 to March-2005. While there are some variations, all 27 CSW indexes follow practically the same pattern. We clearly observe a large peak of growth over the period 2003-2005. Notice that this recent peak is much larger and coherent than the previous one ending in 1991, which was followed by a price stabilization and even a price drop in certain cases. This figure stresses that the acceleration in growth rate is a very localized event which occurred essentially in 2003-2004 and the subsequent growth rate has leveled off to pre-bubble times. We can conclude that there has been no bubble from 1990 to 2002, approximately, then a short-lived bubble until mid-2004 followed by a smoothed transition back to normal.

Fig. 4. Evolution of the growth rates of the 27 regional CSW indexes from June-1983 to March-2005.

Fig. 5 plots the price growth rate $g(t)$ versus the price $p(t)$ itself for the 27 CSW indexes. A linear regression of the data points on Fig. 5, shown as the red straight line, gives a correlation coefficient of 0.494. If we perform linear regression for each index, then we find an average correlation coefficient $0.503 \pm 0.036$, confirming the robustness of this estimation of the correlation between growth rate and price level. The obtained relation between $g$ and $p$ obtained from this correlation analysis is captured by the following mathe-
matical regression
\[ g = 0.00922 \times \frac{p}{100} - 0.00747 . \] (10)

In words, if \( p \) is large, then \( g \) is large on average, which confirms the concept of a positive feedback of price on its further growth. The continuous time limit of \( g(t) \) defined by (9) is
\[ g(t) = \frac{d \ln p}{dt} = \frac{1}{p} \frac{dp}{dt} . \] (11)

This last equation together with (10), that we write as \( g(t) = \alpha p - \beta \) (with \( \alpha = 0.00922/100 \) and \( \beta = 0.00747 \)), implies the following ordinary differential equation
\[ \frac{dp}{dt} = \alpha p^2 - \beta p , \] (12)

which indeed gives a power law acceleration \( p(t) \sim 1/(t_c - t) \) asymptotically close to the critical time \( t_c \). Note that this critical time is determined by the initial conditions, and is called in mathematics a movable singularity. We conclude from this first analysis that the rough linear growth of the growth rate confirms the existence of a bubble growing faster than exponential according to an approximate power law. But of course, the exponent of this power law is poorly constrained, in particular from the fact that the growth rate \( g(t) \) exhibits significant variability and furthermore nonlinearity, as can be seen in Fig. 5.

Fig. 5. Dependence on the data price \( p \) for all CSW indexes of its growth rate \( g \). The overall correlation coefficient is 0.494. The red line is the linear fit of the data points.

It is useful to refine this analysis by separating the whole time interval into three distinct intervals. The corresponding plot of the growth rate \( g \) as a function of price is shown in Fig. 5 with different symbols: period 1 is Jul. 1983 to Sept. 2003, period 2 is from Oct. 2003 to Sept. 2004, and period 3 is from Oct. 2004 to Mar. 2005. An anomaly can be clearly outlined, associated
with the red dots which correspond to the anomalous peak in the growth rate in the period from Oct. 2003 to Sept. 2004. Notice also that the most recent time interval from Oct. 2004 to Mar. 2005 shows practically the same behavior as the first period before 2003. In other words, when removing the data in red for the period from Oct. 2003 to Sept. 2004, the growth rate \( g(t) \) is practically independent of \( p \), which qualifies the normal regime. We can thus conclude that this so-called “phase-portrait” of the growth rate versus price has identified clearly an anomalous time interval associated with extremely fast accelerating prices followed by a more recent period where the price growth has resumed a more normal regime.

4 Yearly periodicity and intra-year structure

4.1 Yearly periodicity from superposed year analysis and spectral analysis

In Fig. 4, the time dependence of the monthly growth rate exhibits a clear seasonality (or periodicity), which appear visually to be predominantly a yearly phenomenon. This visual observation is made quantitative by performing a spectral Fourier analysis. The power spectrum of a typical CSW index is shown in Fig. 6 (all CSW indexes show the same power spectrum). Since the unit of time used here is one year, the frequency \( f \) is in unit of \( 1/\text{year} \). A periodic behavior with period one year should translate into a peak at \( f = 1 \) plus all its harmonics \( f = 2, 3, 4, \cdots \), which is indeed observed in Fig. 6. Note also that the spectrum has large peaks at \( f = 4 \) and \( f = 8 \) among the harmonics of \( f = 1 \), which indicates a weak periodicity with period of one quarter. This is consistent with Fig. 7 where four oscillations in the averaged monthly growth rates can be observed.

![Power spectrum](image)

Fig. 6. Spectrum analysis to confirm the strong periodicity in \( g(t) \).
Note that the power spectrum itself is periodic with a period of 12, which is the sampling frequency, equal to the double of the Nyquist frequency. There are also many peaks in the low-frequency region (larger than one-year time scale) close to $f = 0$, which are associated with the time scales of the global trends produced by the big peaks in $g(t)$ around year 2004 as well as around 1990.

To further explore this seasonal variability of the price growth rates, we calculate the averages of the growth rates for given months, where the average is performed over all years. Consider for instance the month of January: we look up the growth rate for all the data over all years for the month of January and take the average. We do the same for each successive month. The result is shown in Figure 7 for two time periods, which gives the average growth rate $\langle g \rangle$ for different months of the year. The red dash line and circles give the resultant $\langle g \rangle$ for all the data and the black dash line and triangles give the standard deviation $\sigma_g$ for all data (which is a measure of the variability from year to year and from zip code to zip code around the average). The difference between the two time periods is precisely the time interval from June 2003 to March 2005: this period is responsible for a significant increase of the average growth rate (compare the red dashed line (filled circles) with the red continuous line (open circles)) and an even larger increase of the variability (compare the black continuous line (filled triangles) with the dashed black line (open triangles)), again confirming the evidence of an anomalous behavior in that period. In 2005, it appears that the growth rate relaxed back to the normal level (according to the historical record).

Fig. 7. Monthly average growth rate (circles) and its standard deviation (triangles) as a function of the month within the year. Dash: results obtained over all 27 indexes over the period from Jun. 1983 to Mar. 2005; Solid: results obtained over all 27 indexes over the period from Jun. 1983 to May 2003.
4.2 Yearly periodicity and intra-year structure with a scale and translation modulated model

Inspired by these results, we propose the following quantitative model. Consider a time $t$ in units of month. We write $t = 12T + m$, where $T$ is the year and $m$ is the month within that year and thus goes from 1 (January) to 12 (December). For instance, $t = 26$ corresponds to $T = 2$ and $m = 2$ (February), while $t = 38$ corresponds to $T = 3$ and again the same month $m = 2$ (February) within the year. We propose to model the intra-year structure of the growth rate $g(t)$ together with possible yearly variations by the following expression

$$g(t = 12T + m) = f(T)h(m) + j(T).$$  \hspace{1cm} (13)

In words, the growth rate has an intra-year structure $h(m)$ modulated from year to year in amplitude by $f(T)$ up to a possible overall translation $j(T)$ which can also vary from year to year. We can expect $f(T)$ and $j(T)$ to be approximately constant for most years, except around 1990 and 2004 for which we should see an anomaly in either or both of them, since these two periods had bubbles. Note that this model (13) gives an exact yearly periodicity if $f(T)$ and $j(T)$ are constant. A non-constant $f(T)$ describes an amplitude modulation of the yearly periodicity. In particular, we expect a strong peak around $T = 2004$. With this model, we can focus on predicting $f(T)$ and $j(T)$ only, because we have removed the complex intra-year structure.

We have thus fitted the model (13) to three subsets of the whole available time series for the growth rate $g(t)$ and also to the whole set taken globally, in order to test for the robustness of the model. For this, we use the cost function

$$
\sum_{T=1}^{T_{\text{max}}} \sum_{m=1}^{12} \left[ g(t = 12T + m) - f(T)h(m) - j(T) \right]^2  
$$  \hspace{1cm} (14)

which is minimized with respect to the 12 unknown variables $h(1), ..., h(12)$ and the $2 \times T_{\text{max}}$ variables $[f(1), j(1)], ..., f(T_{\text{max}}), j(T_{\text{max}})$. There are $12T_{\text{max}}$ terms in the sum and $12 + 2 \times T_{\text{max}}$ unknown variables. This shows that the system is well-constrained as soon as $T_{\text{max}} \geq 2$. For instance for $T_{\text{max}} = 20$, we have 52 unknown variables to fit and 240 terms in the sum to constrain the fit.

Figure 8 illustrates the result of the fit of model (13) to the growth rate over the whole time interval from 1985 to 2005. As expected, we can observe a clear peak in the amplitude $f(T)$ corresponding to the year 2004, while there is not appreciable peak around 1990. This means that the recent bubble appears significantly stronger than any other episodes in the last 20 years and dwarfs them. The anomalous nature of the recent bubble is reinforced by the existence of a peak in $j(T)$ for the same year 2004, showing that both the amplitude and
The translation components of the growth rates has been completely anomalous in 2004. The middle graph of the top panel of figure 8 shows the intra-year pattern captured by the model, which is in remarkable agreement with the pattern shown in figure 7: one can observe a peak in March, May, August and December, the largest peak being in May. The bottom panel of figure 8 shows visually how well (or badly) the model fits the actual data. The quality of the fit is excellent, except in 2004-2005. In other words, we clearly identify a very anomalous or exceptional behavior in 2004-2005, again providing a confirmation that something exceptional or anomalous has occurred during that period.

Fig. 8. Upper panels: three graphs showing the three functions $f(T)$, $h(m)$ and $j(T)$ fitted on the growth rate over the whole time interval from 1985 to 2005. Lower panel: Comparison between the growth rate data (empty blue circles) and the model (red line).

Figure 9 is the same as figure 8 for the period from 1985 to 1990. One can clearly here observe a peak in the scaling amplitude $f(T)$ at $T = 1988$ and in the translation term $j(T)$ at $T = 1986$, suggesting that the first bubble of the 1985-2005 period occurred over a relatively large time period 1985-1990, with two successive contributions. The intra-year structure $h(m)$ has also its peaks...
on March, May, August and December, but this intra-year structure is weaker than for other sub-periods. The lower panel of figure 9 shows that the model captures very well the overall trend as well as the intra-year structure. The main discrepancies are in the amplitude of the large peaks and valleys, which are not fully predicted.

Fig. 9. Same as figure 8 for the period from 1985 to 1990.

Figure 10 is the same as figure 8 for the period from 1991 to 2000. One can clearly here observe a peak in the scaling amplitude \( f(T) \) at \( T = 1995 \) and in the translation term \( j(T) \) at \( T = 1994 \). This thus identifies a small bubble in the mid-1990s. The intra-year structure \( h(m) \) has also its peaks on March, May, August and December, with very large amplitudes. The lower panel of figure 10 shows a truly excellent fit.

Fig. 10. Same as figure 8 for the period from 1991 to 2000.

Figure 11 is the same as figure 8 for the period from 2001 to 2005. One can clearly here observe a peak in the scaling amplitude \( f(T) \) at \( T = 2004 \) and in the translation term \( j(T) \) also at \( T = 2004 \). This thus clearly identifies the bubble as peaking in 2004. The intra-year structure \( h(m) \) has also its peaks on March, May, August and December, with very large amplitudes and very good agreement with the other three figures. The lower panel of figure 11 shows an excellent fit up to the early 2003 and then a rather large discrepancy starting early 2003 all the way to the last data point approaching mid-2005. In particular, note that the intra-year structure is washed out by the anomalous growth rate culminating in mid-2004. Symmetrically, the intra-year structure
is also absent in the fast decay of the growth rate back to normal. We do not have enough data to ascertain if the growth rate has resumed its normal intra-year pattern. We believe that this is a very important diagnostic to characterize abnormal behavior and this could be a very useful variable to monitor on a monthly basis.

The four figures validate model (13): in particular, they show the very robust intra-year structure with peaks in March, May, August and December.

One possible contribution to this quarterly periodicity comes from the construction of the CSI: the monthly indexes use a three-month moving average algorithm. Home sales pairs are accumulated in rolling three-month periods, on which the repeat sales methodology is applied. The index point for each reporting month is based on sales pairs found for that month and the preceding two months. For example, the December 2005 index point is based on repeat sales data for October, November and December of 2005. This averaging methodology is used to offset delays that can occur in the flow of sales price data from county deed recorders and to keep sample sizes large enough to create meaningful price change averages. A three month rolling window construction corresponds in general to a convolution of the bare price with a kernel which possesses a three month periodicity (or size). The Fourier transform of the convolution is the product of Fourier transforms. Thus the spectrum of the signal should contain the peaks of the Fourier spectrum of the kernel, which by construction contains a peak at three months. However, our synthetic tests (not shown) suggest that this effect is by far too small to explain the strong amplitude of the observed quarterly periodicity. It would be important to understanding why such intra-year structure develops: is it the result of a natural intra-day organization of buyers’ behaviors associated with taxes/ income constraints or a problem of reporting or perhaps the effect of other calendar regularities? Or is it the result of patterns coming from the supply part of the equation, namely home-builders, developers, and perhaps in the time modulation of the rates of allocated permits? Answering these questions is important to determine how much emphasis one should give to these results. But if indeed the intra-day structure is a genuine non-artificial
phenomenon, we believe that it offers a remarkable opportunity for monitoring in real time the normal versus abnormal evolution of the market and also for developing forecasts on a month time horizon.

4.3 Intra-year pattern from signs of growth rate increments

The existence of a strong and robust intra-year structure in the price growth rate can be further demonstrated by studying the sign of $g(t + 1) - g(t)$. A positive (negative) sign mean that the growth rate tends to increase (decrease) from one month to the next.

Based on the seasonality of the growth rate, we are able to answer the following question: given the current growth rate $g(t)$, will the growth rate increase or decrease at time $t + 1$? This amounts to asking what is the sign of $g(t + 1) - g(t)$? Technically, we construct the (unconditional) number of times the sign of the increment $g(t + 1) - g(t)$ is positive or negative irrespective of what is $g(t)$. From Fig. 4, we obtain a sequence of signs: $-+-+-+-++$. For each month, we calculate the percentage of positive and negative signs, respectively. The second and the third rows of Table 4 gives the percentage of positive and negative signs for each month. The third and fourth rows gives the signs and the associated percentages.

For instance, the table says that the “probability” of the sign of $g(t = \text{Feb}) - g(t = \text{Jan})$ being “-” is about 92.1%. If we know $g(t = \text{Jan})$, we can say that it is very probable that the growth rate of February will be less than this January value. Thus, this table has predictive power in the sense that the probabilities to predict the signs are much higher than the value of 75% obtained under the null hypothesis that $g(t)$ is a white noise process (see Sornette and Andersen, 2000). This table is another way to rephrase and expand on our preceding analysis on the yearly periodicity by identifying a very strong and robust intra-year structure.

| Mon | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| +%  | 7.91| 17.2| 88.0| 5.64| 97.7| 8.47| 8.47| 91.4| 6.57| 8.92| 84.2| 82.2|
| -%  | 92.1| 82.8| 12.0| 94.4| 2.29| 91.5| 91.5| 8.59| 93.4| 91.1| 15.8| 17.8|
| sign| -   | -   | +   | -   | +   | -   | -   | +   | -   | -   | +   | +   |
| %   | 92.1| 82.8| 88.0| 94.4| 97.7| 91.5| 91.5| 91.4| 93.4| 91.1| 84.2| 82.2|

Table 1
Analysis of the signs of $g(t + 1) - g(t)$. The second and the third rows gives the percentage of positive and negative signs for each month. The third and fourth rows give the sign for each month that dominates and the associated percentages.
Since our initial analysis performed in the summer of 2005 which used data up to March 2005, new data for the 27 CSW indexes has become available which covers the interval from Apr. 2005 to Sept. 2006. It is very interesting to check if the sign of the growth variations obtained in Table 1 using the data until March 2005 still applies to the new data. The realized signs of the newly available months are calculated and the sequence of signs is the following: - (Apr. 2005, 27 CSW indexes out of 27), + (May. 2005, 27 out of 27), - (Jun. 2005, 27 out of 27), - (Jul. 2005, 27 out of 27), + (Aug. 2005, 27 out of 27), - (Sep. 2005, 27 out of 27), - (Oct. 2005, 27 out of 27), + (Nov. 2005, 27 out of 27), + (Dec. 2005, 21 out of 27), - (Jan. 2006, 27 out of 27), - (Feb. 2006, 27 out of 27), + (Mar. 2006, 27 out of 27), - (Apr. 2006, 27 out of 27), + (May. 2006, 27 out of 27), - (Jun. 2006, 27 out of 27), - (Jul. 2006, 27 out of 27), + (Aug. 2006, 27 out of 27), and - (Sep. 2006, 27 out of 27). Thus, Table 1 predicts exactly the signs of the growth rate variations of all 27 CSW indexes for all months except for Dec. 2005 for which there are 6 errors: Table 1 predicts that the growth rate variation from Dec. 2005 to Jan. 2006 should be +, which is correct for 21 CSW indexes out of 27, corresponding to a success ratio of 77% (close to the white noise case). This score is slightly lower than the previously estimated probability of 82.2% for the month of December, which is the lowest among all months. Overall, the success rate is remarkably high, adding further evidence that the Las Vegas property market has returned to a more normal phase (no bubble from April 2005 to Sept. 2006).

5 Predicting the monthly growth rate

Conditional of the evidence that the anomalous faster than exponential growth has ended, let us attempt to predict the future evolution of the CSW indexes based only on the strong seasonality of the growth rate. Figure 12 presents the predictions one year ahead for the 27 regional CSW indexes. Two different prediction schemes are used. The RED lines are based on the average growth rate obtained from all 27 indexes, while the MAGENTA lines are based on the average growth rate obtained from the individual index under investigation. There is not discernable difference.

A similar prediction of the Clark County (Las Vegas MSA) indexes (NVC003Q and NVC003C) has also been made using the average growth rates obtained from all 27 regional indexes. Since these two indexes are only available from July-2000 to March-2005, we do not have enough data to calculate the average growth rates using the indexes themselves. The results are shown in Fig. 13.
Fig. 12. Predicting regional CSW indexes one year ahead. Red lines: Prediction using average growth rate obtained from all 27 indexes; Magenta lines: Prediction using average growth rate obtained from the individual index under investigation. The two kinds of prediction are almost undistinguishable.

Fig. 13. Predicting Clark County (Las Vegas MSA) indexes (NVC003Q and NVC003C) one year ahead.

6 Conclusion

We have analyzed 27 house price indexes of Las Vegas from Jun. 1983 to Mar. 2005, corresponding to 27 different zip codes. These analyses confirm the existence of a real-estate bubble, defined as a price acceleration faster than exponential. This bubble is found however to be confined to a rather limited time interval in the recent past from approximately 2003 to mid-2004 and has progressively transformed into a more normal growth rate in 2005. The data up to mid-2005 suggests that the current growth rate has now come back to pre-bubble levels. We conclude that there has been no bubble from 1990 to 2002 except for a medium-sized surge in 1995, then a short-lived but very strong bubble until mid-2004 which has been followed by a smoothed
transition back to what appears to be normal. It thus seems that, while the strength of the real-estate bubble has been very strong over the period 2003-2004, the price appreciation rate has returned basically to normal.

In addition, we have identified a strong yearly periodicity which provides a good potential for fine-tuned prediction from month to month. As the intra-year structure is likely a genuine non-artificial phenomenon, it offers a remarkable opportunity for monitoring in real time the normal versus abnormal evolution of the market and also for developing forecasts on a monthly time horizon. In particular, a monthly monitoring using a model that we have developed here could confirm, by testing the intra-year structure, if indeed the market has returned to “normal” or if more turbulence is expected ahead. In addition, it would provide a real-time observatory of upsurges and other anomalous behavior at the monthly scale. This requires additional technical developments and tests beyond this report.

Compared with previous analysis of Zhou and Sornette (2003, 2006) at the scale of states and whole regions (northeast, midwest, south and west), the present analysis demonstrates the existence of very significant variations at the local scale, in the sense that the bubble in Las Vegas seems to have preceded the more global USA bubble and has ended approximately two years earlier (mid 2004 for Las Vegas compared with mid-2006 for the whole of the USA).

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