Particle and/or wave features in neutron interferometry

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Abstract. Neutron interferometry provides a powerful tool to investigate particle and wave features in quantum physics. Single particle interference phenomena can be observed with neutrons and the entanglement of degrees of freedom, i.e., contextuality can be verified and used in further experiments. Entanglement of two photons, or atoms, is analogous to a double slit diffraction of a single photon, neutron or atom. Neutrons are proper tools for testing quantum mechanics because they are massive, they couple to electromagnetic fields due to their magnetic moment, they are subject to all basic interactions, and they are sensitive to topological effects, as well. The $4\pi$-symmetry of spinor wave functions, the spin-superposition law and many topological phenomena can be made visible, thus showing interesting intrinsic features of quantum physics. Related experiments will be discussed. Deterministic and stochastic partial absorption experiments can be described by Bell-type inequalities. Neutron interferometry experiments based on post-selection methods renewed the discussion about quantum non-locality and the quantum measuring process. It has been shown that interference phenomena can be revived even when the overall interference pattern has lost its contrast. This indicates a persisting coupling in phase space even in cases of spatially separated Schrödinger cat-like situations. These states are extremely fragile and sensitive against any kind of fluctuations and other decoherence processes. More complete quantum experiments also show that a complete retrieval of quantum states behind an interaction volume becomes impossible in principle, but where and when a collapse of the wave-field occurs depends on the level of experiment.

1. Basic Relations
Neutrons are usually considered as particles but throughout this article they are considered as waves underlying the duality features of quantum physics. Experiments reported in this article have been performed with monochromatic low energy neutrons from a research reactor and with neutron interferometers based on wave-front and amplitude division [1, 2, 3, 4]. The most frequently used neutron interferometer is the perfect crystal interferometer first tested in 1974 at our 250 kW TRIGA reactor in Vienna. The wide beam separation of several centimeters and the relatively high intensity make it useful for fundamental-, nuclear- and solid-state physics [5] (Fig. 1). This kind of neutron interferometry is based on the undisturbed arrangement of atoms in a monolithic perfect silicon crystal [6, 2]. An incident beam is split coherently at the first crystal plate, reflected at the middle plate and coherently superposed at the third plate. From general symmetry considerations follows immediately that the wave functions in both beam paths, which compose the beam in the forward direction behind the interferometer, are equal ($\psi_0^I = \psi_0^H$), because they are transmitted-reflected-reflected (TRR) and reflected-reflected-transmitted (RRT), respectively. The theoretical treatment of the diffraction process from the perfect crystal is described by the dynamical diffraction theory [7, 8]. To preserve the interference
properties over the length of the interferometer, the lattice planes have to be parallel within one lattice constant and the dimensions of the monolithic system have to be accurate on a scale comparable to the so-called Pendellösung length ($\sim 50 \mu m$). The whole interferometer crystal has to be placed on a stable goniometer table under conditions avoiding temperature gradients and vibrations. A phase shift between the two coherent beams can be produced by nuclear, magnetic or gravitational interactions. In the first case, the phase shift for non-absorbing and weakly absorbing materials is most easily calculated using the index of refraction \[ n = \frac{k_{\text{in}}}{k_{0}} = 1 - \frac{\lambda_{0}^{2} N}{2\pi} \sqrt{b_{c}^{2} - \frac{\sigma_{T}^{2}}{2\lambda}} + i\frac{\sigma_{T} N\lambda}{2\pi} \simeq 1 - \frac{\lambda_{0}^{2} N b_{c}}{2\pi}, \] where $b_{c}$ is the coherent scattering length, $\sigma_{T}$ the attenuation cross section, and $N$ is the particle density of the phase shifting material. The different $k$-vector inside the phase shifter of thickness $D$ causes a spatial shift of the wave packet which depends on the orientation of the sample surface $\hat{s}$ and which is related to the scalar phase shift $\chi$ by \[ \psi \rightarrow \psi_{0} e^{i\Delta \cdot \vec{k}} = \psi_{0} e^{i(k_{\text{in}} - k_{0})D} = \psi_{0} e^{-iN b_{c} \lambda D} = \psi_{0} e^{i\chi}, \] where $\chi$ can be written as a path integral of the canonical momentum $k_{c}$ along the beam paths $\chi = \oint k_{c} d\vec{s}$ \[ [11]. \] Therefore, the intensity behind the interferometer becomes \[ I_{0} \propto |\psi_{0} + \psi_{10}|^{2} \propto (1 + \cos \chi). \] (3)

The intensity of the beam in the deviated direction $I_{H}$ follows from particle conservation ($I_{0} + I_{H}$ = const.). Thus, the intensities behind the interferometer vary as a function of the thickness $D$ of the phase shifter, the particle density $N$ and the neutron wavelength $\lambda$.

Neutron optics is a part of quantum optics and many phenomena can be described properly in that terminology where the coherence function plays an important role \[ [12,13] \]

\[ \Gamma(\Delta) = \langle \psi(0) \psi(\Delta) \rangle, \] which is the autocorrelation function of the wave function. Using a wave packet description for the wave functions (amplitude $a(\vec{k})$)

\[ \psi(\vec{x}) \propto \int a(\vec{k}) e^{i\vec{k} \cdot \vec{x}} d\vec{k}, \] one obtains

\[ I_{0}(\Delta) \propto |\psi_{0}^{1} + \psi_{10}^{1}|^{2} \propto 1 + |\Gamma(\Delta)| \cos \chi_{0} = 1 + |\Gamma(\Delta)\cos(\Delta_{0} \cdot \vec{k}_{0})|, \] (6)
where $\Delta$ and $\chi_0$ denote the phase shifts at the mean momentum $\vec{k}_0$. This gives:

$$ \left| \Gamma(\Delta) \right| \propto \left| \int g(\vec{k}) e^{i \vec{k} \cdot \vec{\Delta}} d^3 \vec{k} \right|. $$

(7)

$\Gamma(\Delta)$ depends on the phase shift and therefore each interference fringe is slightly different from any other and this shows that each interference fringe has a distinct identity. The absolute value of the coherence function can be obtained from the fringe visibility $\left| \Gamma(\Delta) \right| = \left( I_{\text{Max}} - I_{\text{Min}} \right) / \left( I_{\text{Max}} + I_{\text{Min}} \right)$ or as the Fourier transform of the momentum distribution $g(\vec{k}) = \left| a(\vec{k}) \right|^2$.

The mean square distance related to $\left| \Gamma(\Delta) \right|$ defines the coherence length $\Delta_c$ which is for Gaussian distribution functions directly related to the minimum uncertainty relation ($\Delta_c \delta k_i = \frac{1}{2}$). Similar relations can be obtained for time-dependent phenomena where the spectral distribution $g(\omega)$ and the temporal coherence function come into play.

Any experimental device deviates from the idealized situations: the perfect crystal can have slight deviations from its perfectness, and its dimensions may vary slightly; the phase shifter contributes to such deviations by variations in its thickness and due to its inhomogeneities; and even the neutron beam itself contributes to a deviation from the idealized situation because of its momentum spread $\delta k$. Therefore, the experimental interference patterns have to be described by a generalized relation

$$ I \propto A + B \left| \Gamma(\Delta) \right| \cos(\chi + \Phi). $$

(8)

where $A$, $B$ and $\Phi$ are characteristic parameters of a certain set-up. It should be mentioned, however, that the idealized behavior described by Eq. (3) can be approached by a well balanced set-up (Fig. 1). Phase shifts can be applied in the longitudinal, transverse and vertical directions and the related coherence properties can be measured [14]. In the transverse direction the phase shift becomes wavelength independent ($\chi_T = -2d_{hkl}Nb_kD_0$; $d_{hkl}$ reflecting lattice plane distance), which implies a much larger coherence length in that direction.

All the results of interferometric measurements obtained up to now can be explained well in terms of the wave picture of quantum mechanics and the complementarity principle of standard quantum mechanics. Nevertheless, one should bear in mind that neutrons also carry well defined particle properties, which have to be transferred through the interferometer. These properties are summarized in Table 1 together with a formulation in the wave picture. Both particle and wave properties are well established and, therefore, neutrons seem to be a proper tool for testing quantum mechanics with massive particles, where the wave-particle dualism becomes very obvious.

All neutron interferometric experiments pertain to the case of self-interference, where during a certain time interval, only one neutron is inside the interferometer, if at all. Usually, at that time the next neutron has not yet been born and is still contained in the uranium nuclei of the reactor fuel. Although there is no interaction between different neutrons, they have a certain common history within predetermined limits which are defined, e.g., by the neutron moderation process, by their movement along the neutron guide tubes, by the monochromator crystal and by the special interferometer set-up. Therefore, any interferometer pattern contains single particle and ensemble properties together.

2. CLASSIC NEUTRON INTERFERENCE EXPERIMENTS

Here only short comments on these experiments are given since at least some of them provide the basis of the more recent investigation described in the following chapters. More details can be found in a related book [5].
PARTICLE PROPERTIES

\begin{align*}
\text{\( m = 1.674928(1) \cdot 10^{-27} \text{kg} \)} & \quad \text{CONNECTION} \quad \text{\( \lambda_c = \frac{h}{mc} = 1.319695(20) \cdot 10^{-15} \text{m} \)} \\
\text{\( s = \frac{1}{2} \hbar \)} & \quad \text{de Broglie} \quad \text{\( \lambda_B = \frac{h}{mv} \)} \\
\text{\( \mu = -9.6491783(18) \cdot 10^{-27} \text{J/T} \)} & \quad \text{for thermal neutrons:} \quad \lambda_B = \frac{h}{mv} = 1.8 \cdot 10^{-10} \text{m} \\
\text{\( \tau = 887(2) \text{s} \)} & \quad \text{Schrödinger} \quad \text{\( \lambda_B = \frac{h}{mv} = 1.8 \cdot 10^{-10} \text{m} \)} \\
\text{\( R = 0.7 \text{fm} \)} & \quad \text{boundary conditions} \quad \text{\( \Delta_p = v \cdot \Delta t \simeq 10^{-2} \text{m} \)} \\
\text{\( \alpha = 12.0(2.5) \cdot 10^{-4} \text{fm}^3 \)} & \quad \text{\( \Delta_t = v \cdot t = 1.942(5) \cdot 10^6 \text{ m} \)} \\
\text{\( \text{u - d - d - quark structure} \)} & \quad 0 \leq \chi \leq 2 \pi(4 \pi) \\
\end{align*}

\begin{tabular}{ll}
\text{m mass, } s \text{ spin, } \mu \text{ magnetic moment,} & \text{\( \lambda_c \) Compton wavelength,} \\
\text{\( \tau \beta \)-decay lifetime, } R \text{ (magnetic)} & \text{\( \lambda_B \) de Broglie wavelength,} \\
\text{confinement radius,} & \text{\( \Delta_c \) coherence length,} \\
\text{\( \alpha \) electric polarizability; all other} & \text{\( \Delta_p \) packet length,} \\
\text{measured quantities like electric} & \text{\( \delta k \) momentum width,} \\
\text{charge, magnetic monopole, and} & \text{\( \Delta t \) chopper opening time,} \\
\text{magnetic dipole moment are} & \text{\( v \) group velocity,} \\
\text{compatible with zero.} & \text{\( \chi \) phase.} \\
\end{tabular}

| Table 1. Properties of neutrons. |

2.1. 4\pi-Spinor symmetry

This is probably one of the most intensively discussed interference experiments done with matter waves. Based on elementary principles of quantum mechanics, the propagation of a wave function can be described by a unitary transformation, given by the relevant Hamiltonian. For magnetic interaction, \( H_m = -\vec{\mu} \vec{B} \), the propagation of the two-component \textit{spinor wave function}, which describes the neutron as a fermion, can be represented as follows:

\[
\psi(t) = e^{iHt/h} \psi(0) = e^{-i\vec{\sigma} \vec{B} t/h} \psi(0) = e^{-i\vec{\sigma} \vec{B} t/h} \psi(0) = \psi(\alpha),
\]

where \( \alpha \) means the \textit{Larmor precession angle}

\[
|\alpha| = \frac{2\mu}{\hbar} \int Bdt = \frac{2\mu}{\hbar v} \int Bds. \tag{10}
\]

When inserting the Pauli spin operators, one can easily show that \( \psi(\alpha) \) has a \( 4\pi \)-\textit{symmetry}, and not the \( 2\pi \)-symmetry which we are used to with respect to expectation values and within the scope of classical physics,

\[
\psi(2\pi) = -\psi(0) \\
\psi(4\pi) = \psi(0). \tag{11}
\]

These facts, which were not previously regarded as verifiable, can be elucidated very easily with neutron interferometry by observing the intensity modulations, while one of the coherent beams
Figure 2. Results of the neutron interferometric $4\pi$ experiment [20].

passes through a magnetic field,

$$I_0 = |\psi_0(0) + \psi_0(\alpha)|^2 \propto \left(1 + \cos \frac{\alpha}{2}\right).$$ (12)

The above relations are valid for polarized as well as for unpolarized neutrons, which points to the inherent symmetry properties of fermions. From Eqs. (13) and (14) one recognizes that only for $\alpha = 4\pi$ the original state is reproduced. This was verified, nearly simultaneously, in measurements by Rauch et al. [15] and by Werner et al. [16]. Afterwards, this effect was also proven through several other methods and for a series of other fermion systems.

A distinction between dynamical and topological phases will be discussed in Chapter 4.

2.2. Spin-superposition

Spin superposition is a frequently used principle of quantum mechanics. Its curiosity value has been stressed by Wigner [17]. The wave function of both coherent beams is originally polarized in $|z\rangle$-direction. One beam is then inverted to a polarization in $|-z\rangle$-direction, whereas the other remains unchanged. Both beams are then superimposed. This spin flip can be produced, for example, by Larmor precession around a magnetic field perpendicular to $z$-direction. The result of superposition of these two beams can be obtained by applying the rotation operator to the spin-flipped beam for a rotation of $180^\circ$ in $y$-direction (Equ. (11)). If we also allow for a nuclear phase shift, one gets

$$\psi(\chi, \pi) = e^{i\chi} e^{-i\sigma_y \pi/2} |+z\rangle = -i\sigma_y e^{i\chi} |+z\rangle = e^{i\chi} |-z\rangle.$$ (13)

The total wave function $\psi = |+z\rangle + e^{i\chi} |-z\rangle$ leads to the following intensity and polarization of the out-going beam

$$I = \text{const.}, \quad \vec{p} = \frac{\psi \ast \vec{\sigma} \psi}{\psi \ast \psi} = \begin{pmatrix} \cos \chi \\ \sin \chi \\ 0 \end{pmatrix}. $$ (14)

Thus, the intensity does not show any dependence on the phase shift but the polarization shows a marked $\chi$ dependence where the polarization vector lies in the $x,y$-plane, and is perpendicular to the polarizations of the two superimposed, coherent beams. The results of a related experiment are shown in Fig. 4. This implies that a pure quantum state in $|z\rangle$-direction, e.g. for $c = 0$, has
been transformed into a quantum state in \( |x>\)-direction, and, in the sense of self-interference, which definitively applies here, it seems that each neutron has information about the physical situation in both of the widely separated coherent beams. The experiment by Summhammer et al. \[18\] has fully confirmed this process. Intensity modulations appear only when the interference analysis is done in the \( x,y\)-plane.

The experiment mentioned above has been repeated with a Rabi resonance flipper, where an energy exchange of \( h\nu_1 = 2mB_0 \) occurs which causes a Larmor rotation of the polarization vector behind the interferometer \[19\].

In connection with these results, the obvious question arises whether the measurement of the energy transfer makes a determination of the beam path possible. One can, however, show that this is impossible, because interference vanishes in the presence of a measurable energy shift (i.e., larger than the energy width of the beam), and because the measurement of the energy change of the flip-field is impossible due to the photon number-phase uncertainty relationship (\( D\phi DN > 1 \)).

2.3. Stochastic versus deterministic beam path detection

A certain beam attenuation can be achieved either by a semi-transparent material or by a proper chopper or slit system. The transmission probability in the first case is defined by the attenuation cross section \( \sigma a \) of the phase shifting material \( [a = I/I_0 = \exp(-\sigma a ND)] \). The change of the wave function is obtained directly from the complex index of refraction (Eqn. (1)):

\[
\psi \rightarrow \psi_0 e^{(n-1)kD} = \psi_0 e^{i\chi} e^{-\sigma a ND/2} = \psi_0 e^{i\chi} \sqrt{2} \psi,
\]

Therefore, the beam modulation behind the interferometer is obtained in the following form

\[
I_0 \propto |\psi_0^1 + \psi_0^2|^2 \propto [(1 - a) + 2\sqrt{2} \cos \chi].
\]

On the other hand, the transmission probability of a chopper wheel or another shutter system is given by the open to closed ratio, \( t = t_{open}/(t_{open} + t_{closed}) \), and one obtains after straightforward calculations

\[
I \propto [(1 - a) |\psi_0|^2 + a |\psi_0^1 + \psi_0^2|^2] \propto [(1 - a) + 2a \cos \chi].
\]

i.e., the contrast of the interference pattern is proportional to \( \sqrt{2} \), in the first case, and proportional to \( a \) in the second case, although the same number of neutrons are absorbed in both cases. The absorption represents a measuring process in both cases, i.e., a beam path detection, because compound nuclei are produced with an excitation energy of several MeV, which are usually de-excited by capture gamma rays. The measured contrast verifies the “stochastic” and “deterministic” predictions (Eqs. (16) and (17)) \[20, 21\]. The different contrast becomes especially obvious for low transmission probabilities. The discrepancy diverges for \( a \) but it has been shown that in this regime the variations of the transmission due to variations of the thickness or of the density of the absorber plate have to be taken into account which shifts the points below the \( \sqrt{2} \)-“stochastic” curve \[22\]. The region between the linear and the square root behavior can be reached by very narrow chopper slits or by narrow transmission lattices, where one starts to lose information about which individual slit the neutron went through. This is exactly the region which shows the transition between a deterministic and a stochastic situation, i.e., between a particle-like and a wave-like behavior.

The stochastic limit corresponds to the quantum limit when one does not know anymore through which individual slit the neutron went. Which situation is given depends on how the slit widths \( l \) compare to the coherence lengths in the related direction. In case that the slit widths become comparable to the coherence lengths, the wave functions behind the slits show distinct diffraction peaks which correspond to new quantum states (\( n \neq 0 \)). The creation of the new quantum states means that those labeled neutrons carry information about the chosen beam path and, therefore, do not contribute to the interference amplitude \[23\] (Fig. 3). A related
Figure 3. Lattice absorber in the interferometer approaching the classical limit when the slits are oriented horizontally and the quantum limit when they are oriented vertically [30].

Experiment has been carried out by rotating an absorption lattice around the beam axis where one changes from $l \ll \Delta_x$ (vertical slits) to $l \gg \Delta_y$ (horizontal slits). Thus, the attenuation factor $a$ has to be generalized including not only nuclear absorption and scattering processes but also lattice diffraction effects if they remove neutrons from the original phase space. The partial absorption and coherence experiments are closely connected to the quantum duality principle which states that the observation of an interference pattern and the acquisition of which-way information are mutually exclusive. Various inequalities have been formulated to describe this mutual exclusion principle [24, 25, 26]. The most concise formulation reads as

$$V^2 + P^2 \leq 1 \quad (18)$$

where $V$ denotes the fringe visibility (Eq. 6) and $P$ is the predictability of the path through the interferometer, which is a quantitative measure of the a priori which-way knowledge.

3. Post-selection Experiments

Various post-selection measurements in neutron interferometry have shown that interference features can be restored by proper filtering methods even in cases when the overall beam does not exhibit any interference fringes due to spatial phase shifts larger than the coherence lengths of the interfering beams [27, 28]. Post-selection procedures can be applied to various parameters of an experiment:

(i) spatial post-selection
(ii) momentum post-selection
(iii) counting statistic post-selection
(iv) phase post-selection
(v) topology post-selection

In each case more information about the quantum system can be extracted than without post-selection. Fig. 4 shows some of them schematically. Here we discuss momentum post-selection and phase phase-echo experiments and for other methods we refer the reader to the literature [29, 30].

3.1. Post-selection of momentum states

The experimental arrangement with an indication of the wave packets at different parts of the interference experiment is shown in Fig. 4. An additional monochromatization is applied behind
the interferometer by means of Bragg diffraction from single crystals or by time-of-flight systems. The momentum-dependent intensity for Gaussian momentum distributions reads as:

\[ I_0(k) = \exp\left[-\frac{(k - k_0)^2}{2\delta k^2}\right] \left\{ 1 + \cos \left( \chi_0 \frac{k_0}{k} \right) \right\} \] (19)

The spatial phase shift-dependent intensity is given by Eq. (6). The formula show that the overall interference fringes disappear for spatial phase shifts much larger than the coherence lengths \[ \Delta_i \geq \Delta_i^c = 1/(2\delta k_i) \]. The surprising feature is that \( I_0(k) \) becomes oscillatory for large phase shifts where the interference fringes disappear (27). This indicates that interference in phase space has to be considered (31). The amplitude function of the packets arising from beam paths I and II determines the spatial shape of the packets behind the interferometer

\[ I_0(x) = |\psi(x) + \psi(x + \Delta)|^2 \] (20)

which separates for large phase shifts (\( \Delta \gg \Delta^c \)), into two peaks. The related state can be interpreted as a superposition state of two macroscopically distinguishable states, that is a stationary Schrödinger cat-like state (31, 32) - here, for the first time, for massive particles. These states - separated in ordinary space and oscillating in momentum space - seem to be notoriously fragile and sensitive to dephasing and decoherence effects (32, 33, 34, 35, 36, 37).
Measurements of the wavelength (momentum) spectrum were made with an additional silicon crystal placed behind the interferometer with a rather narrow mosaic spread (high resolution) which reflects in the parallel position a rather narrow band of neutrons only \((\delta k' / k_0 \approx 0.0003)\) causing a restored visibility even at large phase shifts \((28, \text{Fig. 5})\). This feature shows that an interference pattern can be revived even behind the interferometer by means of a proper post-selection procedure. In this case the overall beam does not show interference fringes anymore and the wave packets originating from the two different beam paths do not overlap. The momentum distribution has been measured by scanning the analyzer crystal through the Bragg-position. These results clearly demonstrate that the predicted spectral modulation (Eq. (19)) appears when the interference fringes of the overall beam disappear. The modulation is somehow smeared out due to averaging processes across the beam due to various imperfections, unavoidably existing in any experimental arrangement. The contrast of the empty interferometer was 60\%.

It should be mentioned that momentum post-selection in typical Bell experiments (EPR-experiments) with entangled photons may also provide a less mystic view about these experiments since more information can be extracted when a momentum post-selection is added \((27)\).

General conclusions about wave function properties should only be drawn if all accessible information about it is included. Thus the completeness of a quantum experiment has to be seen in a new light. The non-locality phenomenon of quantum mechanics can be understood as the far reaching action of the plane wave components of the wave function as well.
3.2. Contrast retrieval by phase-echo
A large phase shift ($\Delta > \Delta^c$) can be applied in one arm of the interferometer, which can be compensated by a negative phase shift acting in the same arm or by the same phase shift applied to the second beam path [38]. Because the phase shift is additive, the coherence function depends on the net phase shift only. Thus, the interference pattern can be restored as it is shown in form of an experimental example in Fig. 6. The phase-echo method can also be applied behind the interferometer loop when multi-plate interferometers are used [30]. In this case, the situation becomes even more similar to the situation discussed in the previous section. The experimental results completely confirmed that behavior. Phase echo is a similar technique to spin echo [3], which is routinely used in neutron spectroscopy and which represents an interference experiment as well.

Nevertheless it should be mentioned that a complete retrieval seems to be impossible due to theoretical and practical limitations [39, 40].

4. Topological Effects
Topological and geometrical effects appear in the solution of the Schrödinger equation due to special geometric forms of the interaction [41, 42, 43]. Thus they are part of quantum mechanics but they are easily overlooked by pure intensity experiments. It also shows that a wave-function often carries more information than those extracted in a standard experiment. A typical example is the spin superposition experiment discussed in Chap. 2.3, where the result also depends around which axis the spin has been rotated into the opposite direction. In this respect the action of a Hamiltonian can be separated into a part related to its strength (dynamical) and its geometry, which results from the sum of state changes along the excursion in phase space

$$\phi = - \frac{1}{\hbar} \int_0^T <\psi(t)|H|\psi(t)> \ dt + i \int_0^T \phi(t) \frac{d}{dt} |\phi(t)> \ dt = \alpha + \phi_g,$$

with: $|\phi(t)> = e^{i\phi} |\psi(t)>$.

Wagh et al. [44] did recently a related experiment and showed clearly the existence of the topological phase. In a similar sense the scalar and the vector Aharonov-Bohm effects of neutrons have been verified by neutron interferometric methods [45, 46]. In the case of an adiabatic excursion the geometrical phase becomes half the solid angle of the excursion seen on the Bloch
This has been verified recently with a high accuracy with ultra-cold neutrons guided by slowly varying magnetic fields $\phi_g = -0.51(1)\Omega$ \[54\].

Off-diagonal and non-adiabatic geometrical phases have been predicted as well \[43, 48\]. Detailed proposals and related experiments have been done \[49, 50\]. In a Poincaré representation diagonal phases are given by the solid angle opened up by the excursion line $|\psi_i\rangle$ to $|\psi_f\rangle$ and their geodesics to the pole, whereas off-diagonal phases are given by two excursion lines and their connection line in form of geodesics. In a related experiment non-adiabatic and non-cyclic phases have been verified with a double loop interferometer where two phase shifters (PS) and an absorber (A) permit quite peculiar state excursions as shown in Fig 7 \[51\].

It should be mentioned that just geometric phases show a high robustness against fluctuation and dissipative effects as predicted by DeChiara and Palma \[59\]. This has been verified experimentally by Filipp \textit{et al.} \[47\], which may have remarkable consequences for quantum communication systems.

5. Quantum contextuality

A. Einstein, B. Podolsky and N. Rosen \[53\] argued that quantum mechanics may not be complete since non-local correlations between spatially separated systems are predicted, which stimulated the discussion about “hidden” variables and a more “realistic” theory. J. Bell \[54\] formulated inequalities which can decide between the quantum mechanical and the “realistic” view \[55, 56\]. Related experiments with entangled photons verified the non-local view of quantum mechanics \[57, 58, 59, 60\]. Entanglement does not only exist between two particles (photons), but also between different degrees of freedom of a single system (neutron). This yields to the concept of “contextuality”, which states that independent measurements of independent observables are correlated. In our case the beam path through the interferometer and the spin states are taken as independent observables. In this case a Bell-like inequality can be formulated, which can be measured from the counting rates $N$ at different values of the phase shift $\chi$ and the spin rotation angle $\alpha$ \[50\].

\[-2 \leq S \leq 2,\]

\[S = E(\alpha_1, \chi_1) + E(\alpha_1, \chi_2) - E(\alpha_2, \chi_1) + E(\alpha_2, \chi_2),\]
Figure 8. Sketch of the experimental setup for the contextuality experiment. The phase \( \chi \) and the polarization rotation \( a \) could be varied independently \[50\].

\[
\begin{align*}
E(\alpha, \chi) &= \frac{N(\alpha, \chi) + N(\alpha + \pi, \chi + \pi) - N(\alpha, \chi + \pi) - N(\alpha + \pi, \chi)}{N(\alpha, \chi) + N(\alpha + \pi, \chi + \pi) + N(\alpha, \chi + \pi) + N(\alpha + \pi, \chi)}.
\end{align*}
\]

(23)

The maximal violation of this inequality due to quantum mechanics happens for the following parameters: \( \alpha_1 = 0, \alpha_2 = \pi/2, \chi_1 = \pi/4 \) and \( \chi_2 = -\pi/4 \) and amounts to \( S = 2\sqrt{2} = 2.82 \).

The measurement scheme is shown in Fig. 8. The entangled neutron state has been produced by rotating the neutron spin in beam path I into the \( |\text{y}\rangle \) and in beam path II into the \( |\text{y}\rangle \) direction respectively. The precise determination of the related counting rates at the parameter values given above yielded a value for \( S \) of \[50\]:

\[
S = 2.051 \pm 0.019,
\]

which is by a 3s-limit above 2, verifying for the first time the contextuality principle of quantum mechanics. The maximal violation of \( S = 2.82 \) has not been achieved because the contrast of the interference pattern and the neutron polarization were below unity. In this kind of measurements these quantities play a similar role than the finite efficiency of the photon detectors in entangled photon experiments. In a subsequent and improved experiment a violation up to \( S = 2.291 \pm 0.008 \) has been measured \[61\].

The same set-up as shown in Fig. 8 has been used to perform experiments related to the Kochen-Specker theorem \[62\] and the Mermin inequalities \[63\], where even stronger violations of classical hidden variable theories can be verified. For neutron matter-waves a related proposal came from Basu et al. \[49\]. In this experiment the beam paths could be closed alternatively by means of an absorber sheet \[64\]. The measurement of the product observable \( (\sigma_x^s\sigma_y^p) \cdot (\sigma_y^s\sigma_z^p) \) was done by measuring \( (\sigma_z^s\sigma_y^p) \) and using \textit{a priori} the non-contextuality relation. The measurable quantity is defined by a sum of product observables

\[
C' = \hat{I} - \sigma_x^s\sigma_x^p - \sigma_y^s\sigma_y^p - (\sigma_x^s\sigma_y^p) \cdot (\sigma_y^s\sigma_z^p).
\]

(24)
In any experiment expectation values only can be measured. For non-contextual models the last term can be separated:

\[ < (\sigma_x^s \sigma_y^p) > < (\sigma_y^s \sigma_y^p) > = < \sigma_x^s > < \sigma_y^p > < \sigma_y^s > < \sigma_x^p > \]  

which gives

\[ C_{nc} = \pm 2, \]  

whereas quantum mechanics predicts

\[ C_{qm} = 4. \]

The measured value was

\[ C_{exp} = 3.138 \pm 0.0115, \]

which is well above the non-contextuality (classical) limit of 2 and provides an all-versus-nothing-type contradiction. It is also a Peres-Mermin proof of quantum-mechanics against non-contextual hidden variable theories.

A debate in literature [65, 66] criticized the \textit{a priori} use of the non-contextuality relation \((\sigma_x^s \sigma_y^p) \cdot (\sigma_y^s \sigma_x^p) = (\sigma_x^s \sigma_x^p)\) and in this connection the use of an absorber to measure this quantity. In a follow-up proposal [67] and subsequent experiment [68] the previous result (Eq. 28) has been verified and an even stronger violation has been observed. In this case a quantum erasure has been used instead of an absorber and, therefore, all quantities required for Eq. 24 could be measured within the same context.

6. Discussion

It has been shown that more information about a quantum system can be extracted when more accessible parameters are measured, i.e., when post-selection methods are applied. It becomes obvious that a system may remain coupled in phase space even when it becomes separated in any other parameter space. Thus, interference properties can be shifted from one parameter space to another one and back again. Related bands of plane wave components which compose the wave packets may be considered as a responsible factor for the understanding of the coupling and non-locality phenomena in quantum mechanics. It looks like these plane wave components of the wave packets, i.e., narrow bands, interact over much larger distances than the sizes of the overall packets. This interaction guides neutrons of certain momentum bands to the 0- or H-beam, respectively. These phenomena throw a new light on the discussion on Schrödinger-cat-like situations in quantum mechanics [53, 55]. It may be considered as a contribution to speakable and unspeakable aspects of quantum mechanics [56]. Spatially separated packets remain entangled (correlated) in phase space and non-locality appears as a result of this entanglement. Since entanglement exists not only between objects but also between different degrees of freedom, Bell-type experiments can also be done in single particle experiments [56]. In this respect, contextuality experiments with neutrons (Chap. 5) may be of special value since they show that the experimental outcomes when measuring commuting observables (spin and beam path) are intrinsically correlated and quantum contextuality may be considered as an important feature of quantum physics. This also shows that quantum systems contain stronger correlations than classical ones and contextuality may cause an additional loophole for the deviation of Bell inequalities [69].

The summaries drawn for the different experimental situations discussed in this article are followed by statements that the retrieval of the interference properties by several post-selection procedures became increasingly more difficult the wider the separation in any parameter space of the quantum system happened before. This is caused by fluctuations, which are unavoidable due to residual quantum fluctuations inherent to any physical system.
Unavoidable fluctuations (even zero-point fluctuations) cause an irreversibility effect which becomes more influential for widely separated Schrödinger-cat like states. All these effects can be described by an increasing entropy inherently associated with any kind of interaction. This also supports the idea that irreversibility is a fundamental property of nature and reversibility an approximation only, as stated by several authors [70, 71, 72, 73].

All the results of the neutron interferometric experiments are well described by the formalism of quantum mechanics. According to the complementarity principle of the Copenhagen interpretation, the wave picture has to be used to describe the observed phenomena. The question of how the well-defined particle properties of the neutron are transferred through the interferometer is not a meaningful one within this interpretation, but from the physical point of view it should be an allowed one.

More complete quantum experiments show that a complete retrieval of all wave components behind an interaction the quantum system experienced becomes impossible, in principle. It also shows commuting variables are still correlated in the sense of quantum contextuality.

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