The Analyzing of the Dynamic Characteristic and the Design of the Torque Measuring System in Extra-High Rotation Speed

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Abstract. Torque measuring in the extra-high rotation speed has not been resolved ideally yet, due to the restriction of the rough conditions. The dynamic characteristics of the developed digital plane grating angular-torquer measuring system are analyzed in this paper. The paper mainly calculates and designs the parameters of elastic shafts with different cross-section. Via the compare of dynamic characteristics and compactness of the elastic shafts with different cross-section, the optimized design is proposed at the end of the paper. The achievements of this paper can supply references to the study and design, maintaining and improvement of relative instruments.

1. Introduction
In the servocontrol systems of the aircrafts, devices working on extra-high speed become more and more, the speed is averagely around 10⁵r/min. Scholars are paying more attention to the measurement of the torque in extra-high rotation speed. The quiver is inevitable during the measurement, for the exceeding high speed, so dynamic characteristic of the system deserves attention. The torque measuring system can mainly work in a static state or low speed, previously. It is hard to assess the dynamic characteristic of torque measurement, because no normative equipment has been produced. Only the dynamic characteristic of electric circuit could be analyzed and measured.

The assignment has developed a digital grating torque measuring system to measure the torque of extra-high rotation speed shaft. The measuring system converts the torque to rotation angle via an elastic shaft, transforms the angle information into light pulse with plane gratings and acquires the torque as the form of digital information. The dynamic characteristic of the system can be considered from two aspects, mechanism and electric circuit. Under the existing conditions, analyzing of mechanic aspect looks more difficult but important. The paper analyzes and studies the dynamic characteristic of the mechanisms around the elastic shaft, providing essential references to designing and studying of the system.

2. Time course and dynamic response characteristic
There are two types of torque, static torque and dynamic torque. Static one dose not change or changes very slowly and fluctuates a little along with time, while, dynamic torque changes rapidly and fluctuates a lot. The static and dynamic here is relative, it is independent of whether the transmission shaft rotating or not. The shaft transmitting static torque is not always stock-still, similarly, the shaft transmitting dynamic torque is not always rotating.
Although the transmitting shaft of the subject is of high speed, the output torque should be pulse torque when the system is working steadily. The mechanic transmission system studied is a system which contains infinite particles quivering and being retorted. Through analyzing the quiver torque via Fourier Transform ignoring high-order harmonic, parameters of low-order can be gained of a certain precision range. When the system is working steadily, if the fluctuation margin exceeds 3~5% of the average, the measured value has transformed to quiver torque from pulse torque.

The system works in the air, if the damp of lube in the bearing and the damp of the air are considered, the system can be looked on as a two-order system made up of spring, mass and damp. The mathematic model can be described as:

\[ J \frac{d^2 \varphi(t)}{dt^2} + C \frac{d\varphi(t)}{dt} + K_n \varphi(t) = b_0 T(t) \]  

In the formula, \( \varphi \) is the torsion angle; \( T \) is the output torque of the system. \( J \) is the moment of inertia, \( K_n \) (the torque corresponded to one unit angle) is the torsion stiffness of the elastic shaft. \( C \) is the damp coefficient; \( b_0 \) is a constant relative to the parameters of the devices. The four parameters can be cut short to three: \( K \) is Static Sensitivity, \( \omega_n \) is Natural Angular Frequency, \( \zeta \) is Damp Ratio.

\[ K = \frac{b_0}{K_n} \]  \hspace{1cm} (2)

\[ \omega_n = \sqrt{\frac{K_n}{J}} \]  \hspace{1cm} (3)

\[ \zeta = \frac{C}{2\sqrt{JK_n}} \]  \hspace{1cm} (4)

So the formula (1) is equivalent to

\[ \left( \frac{D^2}{\omega_n^2} + \frac{2\zeta D}{\omega_n} + 1 \right) \varphi(t) = KT(t) \]  

Transfer function of the system

\[ G(s) = \frac{K}{s^2\omega_n^{-2} + 2\zeta s\omega_n^{-1} + 1} \]

Frequency response characteristic of the second-order measurement system can be deduced, when \( s \) is replaced by \( j\omega \),

\[ G(j\omega) = \frac{K}{(j\omega\omega_n^{-1})^2 + (2\zeta j\omega\omega_n^{-1}) + 1} \]  \hspace{1cm} (6)

Amplitude frequency characteristic curve and phase-frequency characteristic curve being described by formula (6) is shown as figure 1.

![Figure 1. Frequency response of the measurement system.](image-url)
According to the analyzing above, conclusions can be drawn as follows:

- Natural frequency $\omega_n$ should be increased to measuring torque signals with relatively more frequencies of pulse or quiver. As it is shown in Figure.1, the increasing of the natural frequency will expand the linear part of the amplitude frequency characteristic curve.

- According to the two characteristic curves of Figure.1, $\zeta$ should be chosen in the range 0.6~0.7, then the linear part of the amplitude frequency characteristic curve is the widest, and the phase-frequency characteristic curve is relatively more close to a line. The output $\varphi(t)$ still can recur the measurand $T(t)$ correctly, though the response lags in phase.

The choosing of dynamic parameters in particular depends on the practical specification and the measurand. The paper mainly analyzes and discusses about the increasing of free frequency $\omega_n$, to offer the specific ideas for structural design.

3. The design of dynamic parameters of the measurement system

3.1. The compendium of the structure rationale of the measurement system

The elastic shaft adopted and the relative frames are shown in Figure 2.

![Figure 2. Principle of the torque sensor.](image)

The torque sensor is made up of elastic shaft 1 and signal generator 2 and 3. Two plane gratings and two optical fiber detectors opposite to the gratings constitute the signal generators. The gratings are columnar; the reticles along the generatrix of columnar gratings are made adopting photo-etch method. There will be phase difference between two signals received by the optical fiber detectors, which is caused by the torsion angle $\varphi$ of the elastic shaft, when the shaft begins to offer output power. The phase difference can be gained via photoelectric converter measuring circuit.

3.2. The design of several elastic shafts with different cross-section

According to the practical specification, the maximum output torque is 7Nm; the chosen material of the shaft is Be-bronze with high-point shearing stress $\tau_0=5000\text{kgf/cm}^2$ and modulus of elasticity in shear $G=5\times10^5\text{kgf/cm}^2$. The design of elastic shaft is classified into 3 instances.

- Solid column torsion shaft

$$\varphi = \frac{32TL}{\pi Gd^4} = \frac{2\pi L}{Gd} \quad (7)$$

$$\tau = \frac{16T}{\pi d^3} \quad (8)$$

Then,

$$d = 1.73\sqrt[3]{\frac{T}{\tau}} \quad (9)$$
\[ L = 0.1 \frac{Gd^4}{T} \varphi \]  \hspace{1cm} (10)

In formula (9), \( [\tau] \) is permissible stress. Considering the extra-high speed working condition, \( [\tau]=0.1 \), \( \tau_0=5000\text{kgf/cm}^2 \). The diameter \( d \) of the shaft equals to 0.9cm, via calling \( T \) and \( [\tau] \) of formula (9) by values \( T=7\text{Nm} \) and \( [\tau]=0.1 \), \( \tau_0=5000\text{kgf/cm}^2 \), deducing from the formula (10), supposing the maximum torsion angle \( \varphi=1^\circ \).

- The design of solid shaft with rectangular cross-section

\[
\varphi = \frac{TL}{GK_2 \frac{ab}{3} \frac{\tau L}{GK_2}} = K_2 \frac{\tau L}{GK_2} \hspace{1cm} (11)
\]

\[
\tau = \frac{T}{K_3 \frac{ab}{3}} \hspace{1cm} (12)
\]

In the formula (11) and (12), \( a \) is the height of the cross-section, and \( b \) is the width, \( L \) is the effective length of the torsion shaft, \( K_2 \) and \( K_3 \) is the coefficient relative to \( a/b \), they can be obtained looking-up the table [4].

Supposing \( a/b=3 \), then \( K_2=0.27/cm \), \( K_3=0.27/cm \). Other parameters have no difference to the round cross-section shaft. It is calculated according to formula (12) that \( b=1.12\text{cm} \), \( a=0.37\text{cm} \) and \( L=17.5\text{cm} \), according to formula (11).

- Solid torsion shaft with square cross-section

According to \( a/b=1 \), it can be looked up from the table that \( K_2=0.13/cm \), \( K_3=0.22/cm \), \( a=b=0.9\text{cm} \) and \( L=10.3\text{cm} \), according to formula (11) and (12).

3.3. Analyzing of dynamic characteristic torsion shaft with different cross-section

The free frequency \( \omega_n \) mentioned forehead is the one of the most important parameters of the dynamic characteristic of the system. Free frequency \( \omega_n \) of the shafts with different cross-section will be introduced following:

\[ \omega_n = \sqrt{\frac{K_n}{J}} \]  \hspace{1cm} (13)

As the calculating above showing, the shafts with different cross-section share the same torsion rigidity \( K_n=401.1\text{Nm/rad} \), according to criterion to sensitivity of the torsion shaft, namely, torsion angle \( \varphi=1^\circ \) for \( T=7\text{Nm} \).

- Torsion shaft with round cross-section

\[ J = \frac{1}{2} MR^2 \]  \hspace{1cm} (14)

\[ M = \rho \pi R^2 L \]  \hspace{1cm} (15)

\( M \) is the mass of the torsion shaft; \( R \) is the cross-section radius of the torsion shaft; \( \rho \) is material density. Calling formula (14), (15) by values of \( R, L \) and \( \rho=8.23\text{g/cm}^3 \), \( J \) can be obtained. Then, it can be acquired from formula (13) that: \( \omega_n =3.076\times10^4/s \)

- Torsion shaft with rectangle cross-section

\[ J = \frac{M}{12} (a^2 + b^2) \]  \hspace{1cm} (16)

Calling formula (12) by the relative values, it can be drawn that: \( \omega_n =2.409\times10^4/s \)

- Torsion shaft with square cross-section

For \( a=b \), it can be obtained from formula (16) that: \( \omega_n =2.080\times10^4/s \)

4. Conclusion

Compare of the torsion shafts with different cross-section is shown in table1. Torsion shaft with round cross-section excels any other shaft listing in table 1, either in compactness or in dynamic characteristic. So the design adopts torsion shaft with round cross-section.
### Table 1. Characteristic of torsion shafts with different cross-section.

| Cross-section         | Compactness   | Dynamic Characteristic | Applicability               |
|-----------------------|---------------|------------------------|-----------------------------|
| Round cross-section   | Good (L=8.0cm) | Good ($\omega_n=3\times10^4$/s) | Dynamic, high rotation speed |
| Square cross-section  | Worse (L=10.3cm) | Worse ($\omega_n=2.4\times10^4$/s) | Static, low rotation speed   |
| Rectangle cross-shaft | Worst (L=17.5cm) | Worst ($\omega_n=2\times10^4$/s) | Facile to stick strain foil  |

Only the effective length of the torsion shaft has been considered in the analyzing above, namely the part generating distortion. The size of non-distortion part, such as the axial size of plane gratings and driving wheel, should be tried to deduce to ensure the better dynamic characteristic of the system. In the design of relative structure, the radial size of the plane gratings should be minimized to assure the free frequency larger. Actually, there is no plane grating developed in this task. While, the grating reticles are etched directly on the torsion shaft.

### References

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