Resonance Suppression in Multi-Degree-of-Freedom Rotating Flexible Structures Using Order-Tuned Absorbers

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ABSTRACT

This paper considers the dynamic response and order-tuning of vibration absorbers fitted to a rotating flexible structure under traveling wave (TW) engine order excitation. Of specific interest is the extension of previous results on the so-called no-resonance zone, that is, a region in linear tuning parameter space in which the coupled structure/absorber system does not experience resonance over all rotation speeds. The no-resonance feature was shown to exist for cyclically-coupled systems with one structural and one absorber degree-of-freedom (DOF) per sector. This work uses a higher-fidelity structural model to investigate the effects of higher modes on the cyclically-coupled system. It is shown that the no-resonance zone is replaced by a resonance-suppression zone in which one structural mode is suppressed, but higher-order resonances still exist with the addition of the absorbers. The results are general, in the sense that one vibration mode can be eliminated using a set of identically-tuned absorbers on a rotating structure with arbitrarily many DOFs per sector.

1 INTRODUCTION

Many rotating flexible structures consist of an array of interconnected constituent parts whose geometry and structural properties are rotationally periodic (i.e., cyclically symmetric). In a bladed disk assembly, for example, the fundamental substructure is one blade plus the corresponding segment of the disk, which is collectively referred to as a sector. During steady operation these systems rotate at a constant speed and are subjected to TW dynamic loading, or engine order (e.o.) excitation, which is characterized by excitation frequencies that are proportional to the mean rotational speed of the rotor. Such excitations result in component vibrations that can lead to high cycle fatigue failure, noise, reduced performance, and other undesirable effects. Order-tuned vibration absorbers exploit the centrifugal field from rotation of the primary system and are thus ideally-suited to address component vibration in rotating flexible structures. They essentially consist of masses that ride along designer-specified paths relative to the primary system. When properly tuned, these absorbers effectively counteract fluctuating loads applied to the primary system over all rotation speeds [1].

The dynamic performance, characteristics and features of
order-tuned absorbers are well-understood in typical situations, and there are numerous examples of their implementation. They have been investigated by Shaw et al. [2–6] and others for torsional vibration reduction in rotating systems. Applications include light aircraft engines [7], helicopter rotors [8], diesel camshafts [9], and advanced technology automotive engines [10].

In previous works Olson et al. [11–15] investigated the performance of order-tuned vibration absorbers applied to lumped-parameter models of a cyclically-coupled bladed disk assembly under engine order excitation. In these models, the sector dynamics are captured by two DOFs: one for the blade and one for the absorber. A key finding from the linearized coupled system is the existence of the so-called “no-resonance zone” in linear tuning parameter space. It was shown that there exists a range of absorber tuning values, close to the ideal tuning that exactly matches the excitation order, for which there are no resonances over the entire range of rotation speeds. This result was first reported by Shaw et al. [16] for a model consisting of a single, isolated sector (i.e., one blade and one absorber attached to a rotating disk). Olson et al. showed that the no-resonance zone persists in the (coupled) multi-sector linear system [11–13] and that it qualitatively persists when weak nonlinearities are taken into account via the absorber paths [12, 14, 15]. Existence of the no-resonance zone allows for the possibility of robustly-tuned absorbers that can function effectively in the presence of model and parameter uncertainties.

This paper investigates if and how the no-resonance zone persists when higher-fidelity models are used to describe the bladed disk system. It will suffice to employ a 3-DOF sector model that consists of two DOFs for each blade/disk and one for the attendant absorber. The no-resonance zone is shown to persist in a restricted sense: one structural mode can be eliminated, but higher mode resonances still exist at some rotor speed for any absorber tuning. These results generically hold for models with many blade/disk DOFs because of the nature of dynamic coupling between the absorber and blade dynamics, which results in characteristic eigenvalue veering as the rotor speed is varied [17, 18]. Suppression of multiple resonances may be achieved, for example, by implementing multiple sets of absorbers. This is left for future work.

The paper is organized as follows. A higher-fidelity model of a rotating bladed disk assembly under engine order excitation is developed in Section 2, where each blade is fitted with an order-tuned vibration absorber. Section 3 introduces a tuning strategy in which the absorber tuning order is set relative to the order of the TW excitation. Features of the forced response are discussed in Section 4, which contains the main results of this work. The paper closes in Section 5 with a discussion and suggestions for future work.

2 SYSTEM MODEL

A lumped-parameter model of the rotating bladed disk assembly is shown in Fig. 1a. It consists of a cyclic array of \( N \) identical, identically-coupled sectors (Fig. 1b), each with one blade and one absorber plus a corresponding segment of the disk. The disk component of each sector is composed of a rigid part of radius \( H \), which rotates at a constant speed \( \Omega \) about a fixed axis through \( O \). This component is referred to as the rotor. A second flexible portion vibrates about a point \( Q \) at the periphery of the rigid rotor, which is modeled by a pendulum of length \( d \) and mass \( M_d \) and is referred to as the disk. The blade dynamics are captured by a second pendulum of length \( L \) and mass \( M_b \) that is attached to the disk pendulum at the vertex \( A \). There are \( N \) such double pendulum systems, which are uniformly distributed.
along the circumferential of the rotor. The sector model shown in Fig. 1b could also represent a 2-DOF blade model attached to a rigid disk/rotor. The key feature is multiple DOFs per sector, in this case two, which is sufficient to capture the effects of order-tuned absorbers when there are arbitrarily many disk/blade DOFs per sector.

The flexural stiffness associated with the $i^{th}$ blade and disk is modeled using linear torsional springs of stiffness $K_b$ and $K_d$, respectively. Adjacent blades (resp. disk portions) are coupled via translational springs with stiffness $k_b$ (resp. $k_d$) at a distance $b$ (resp. $h$) relative to their attachment points to the disk (resp. rotor). It is assumed that the springs are unstressed when the disks and blades are in a purely radial configuration, that is, when $\phi_i = \theta_i = 0$ for each $i \in \mathcal{N} = \{1, 2, \ldots, N\}$. Each stiffness element is paired with a corresponding linear viscous damper (not shown in Fig. 1a) to capture the effects of dissipation. The disk and inter-blade coupling is modeled by linear torsional and translational dampers with constants $C_d$ and $c_d$ (resp. $C_b$ and $c_b$). It is convenient to model the absorber damper with an effective torsional damper with constant $C_a$ that acts at point $B$ (attachment point of absorber pendulum).

The blades are fitted with identical vibration absorbers, which generally consist of particle masses $M_a$ that ride on user-specified paths. Each absorber path is described by a vector of length $R_i(S_i) = R_i (-S_i)$ relative to the outermost blade pendulum, where $S_i$ is the path arc length relative to its vertex $V$. Polchi [19] derived the governing EOM for the system shown in Fig. 1a fitted with the general-path absorbers using the path formulation described by Olson et al. [12, 14, 15]. However, for the small-amplitude linearized model to be developed, the absorbers can be regarded as simple pendulums with length $r$ and mass $M_a$ that are attached a distance $aL$ along the outermost blade pendulum relative to point A. Their linearized dynamics are described by the relative angular coordinates $\psi_i = S_i / r$. Thus, each sector consists of a double-pendulum disk/blade model and a simple-pendulum (circular path) absorber attached to the outermost double-pendulum link.

The $i^{th}$ disk/blade double pendulums are harmonically forced in the transverse sense by e.o. excitation of order $n$ such that the cyclic system is circumferentially-forced by a TW. The e.o. excitation is modeled by [12, 20]

$$F_i(t) = F_0 e^{j \phi_i} e^{j n \Delta \Omega},$$

where $\phi_i = 2 \pi S_i (i-1)$ is the inter-sector phase angle, $F_0$ is the excitation strength, and $j = \sqrt{-1}$. The e.o. is restricted such that $n \in \mathcal{N}$, which includes all practically relevant situations [12].

The equations of motion (EOM) are formulated using Lagrange’s method, linearized for small disk/blade and absorber motions, and made dimensionless according to the non-dimensional parameters defined in Table 1. The matrix-vector form of the sector EOM is given by

$$M \ddot{z}_i + C \dot{z}_i + K z_i + C_c (-z_{i-1} + 2z_i - z_{i+1}) + K_c (-z_{i-1} + 2z_i - z_{i+1}) = f e^{j \phi_i} e^{j n \Delta \Omega},$$

where the elements of the $3 \times 3$ sector mass, damping, and stiffness matrices $(M, C, K)$ are defined in Table 2, the vector $z_i = (\phi_i, \dot{\phi}_i, \psi_i)^T$ captures the sector dynamics, and $\Delta \Omega = d(\cdot) / d\tau$ denotes differentiation with respect to dimensionless time. Inter-sector coupling is captured by the matrices

$$C_c = \begin{bmatrix} \xi_{ac} & \xi_{bc} & 0 \\ \xi_{bc} & \xi_{cc} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad K_c = \begin{bmatrix} v_a^2 & v_b^2 & 0 \\ v_b^2 & v_c^2 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

Table 1. Selected list of dimensionless parameters.

| Parameters | Description |
|------------|-------------|
| $\delta$   | $H / d$     | Radius of the rigid rotor disk |
| $\rho$     | $L / d$     | Length of blade pendulum |
| $\alpha$   |             | Distance between points $A$ and $B$ |
| $\gamma$   | $r / d$     | Length of absorber pendulum |
| $\mu_a$    | $M_a / M_d$ | Absorber mass |
| $\mu_b$    | $M_b / M_d$ | Blade mass |
| $f_0$      | $F_0 (L + d) / K_d$ | Strength of e.o. excitation ($\phi_i$-dynamics) |
| $f_\theta$ | $F_0 L / K_d$ | Strength of e.o. excitation ($\theta_i$-dynamics) |
| $\lambda$  | $\sqrt{K_d}$ | Square of the blade torsional stiffness |
| $\nu_a$    | $\sqrt{2\pi \rho a d^2} k_b$ | Stiffness coupling between sectors |
| $\nu_b$    | $\sqrt{2\pi \rho b d^2} k_d$ | Stiffness coupling between sectors |
| $\nu_c$    | $\sqrt{2\pi \rho c d^2} k_b$ | Stiffness coupling between sectors |
| $\xi_{ac}$ | $\frac{1}{2} \sqrt{K_b} \frac{a \rho}{M_a d^2}$ | Damping coupling between sectors |
| $\xi_{bc}$ | $\frac{1}{2} \sqrt{K_b} \frac{b \rho}{M_b d^2}$ | Damping coupling between sectors |
| $\xi_{cc}$ | $\frac{1}{2} \sqrt{K_b} \frac{c \rho}{M_c d^2}$ | Damping coupling between sectors |
| $\xi_{ad}$ | $\frac{1}{2} \sqrt{K_b} \frac{a \rho}{M_a d^2}$ | Absorber torsional damping constant |
| $\xi_{bd}$ | $\frac{1}{2} \sqrt{K_b} \frac{b \rho}{M_b d^2}$ | Blade torsional damping constant |
| $\xi_{cd}$ | $\frac{1}{2} \sqrt{K_b} \frac{c \rho}{M_c d^2}$ | Disk torsional damping constant |
| $\sigma$   | $\Omega / \omega_0$ | Angular speed of the rotor |

Throughout the remainder of this work it is understood that variables and equations with subscripts $i$ are defined for each $i \in \mathcal{N}$.
where the undamped natural frequencies are given by

\[ s = \frac{\sqrt{\alpha + \delta + 1}}{\gamma}, \]

and the sector forcing vector is given by \( \mathbf{f} = (f_\theta, f_\phi, 0)^T \), where the dimensionless entries are defined in Table 1. The 3N linearized EOM defined by Eq. (2) serve as the basis for the analysis that follows.

### 3 ABSORBER TUNING

The linear absorber tuning order follows from the special case where the disks and blades are locked in their zero positions relative to the spinning rotor [11, 12]. This leads to a system of identical, dynamically isolated and unforced single-DOF absorbers that oscillate freely under the influence of centrifugal effects. Their dynamics follow from Eq. (2) by setting \( \psi = \psi'' = \psi' = 0 \) and \( \theta = \theta'' = \theta' = 0 \). Then

\[ \mu_\gamma \psi'' \xi + \xi_\sigma \psi' + \mu_\gamma (\alpha + \delta + 1) \sigma^2 \psi = 0, \]

where the undamped natural frequencies are given by

\[ \omega_{33} = \frac{\omega_{33}}{\omega_0} = \sqrt{\frac{\alpha + \delta + 1}{\gamma}}. \]

Physically, each absorber acts like a centrifugally-driven pendulum of length \( \gamma \) whose pivot point is located a distance \( \alpha + \delta + 1 \) from the rotor axis \( O \). The base accelerates towards the center or rotation at a constant rate \( (\alpha + \delta + 1)\sigma^2 \), which is the effective gravity of the pendulum. This results in a centrifugal restoring force that scales with \( \sigma^2 \) and an undamped natural frequency that scales with \( \sigma \) according to the constant

\[ \tilde{n} = \sqrt{\frac{\alpha + \delta + 1}{\gamma}}, \]

which is defined to be the absorber tuning order. Because the isolated absorber eigenfrequencies \( \tilde{\omega}_{33} \) and the e.o. excitation frequency scale directly with the rotor speed, the absorbers are tuned to a given order of the excitation as opposed to a particular frequency [21].

Absorber tuning refers to a choice of absorber geometric parameters, which are selected to attenuate the blade responses over a range of operating speeds, particularly near resonance. This is done by choosing a value for \( \tilde{n} \) and then selecting the dimensionless parameters \( \alpha, \delta, \) and \( \gamma \) to satisfy Eq. (6). It is shown that there exists an ideal absorber tuning that results in suppression of one of the blade resonances, where the attendant absorber amplitudes depend primarily on their mass and placement along the blade lengths and the level of the applied loads. Ideal tuning is defined by

\[ \tilde{n} = n, \]

in which case the isolated absorber natural frequency \( \tilde{\omega}_{33} \) identically matches that of the excitation. Exact tuning is effective for all rotation speeds, but it is susceptible to uncertainties of the model or absorber parameters due to in-service wear, environmental effects, or the effects of tolerances. To account for such effects, and to allow for intentionally detuned designs, we let

\[ \tilde{n} = n(1 + \beta), \]

where \( \beta \) is a detuning parameter. Ideal linear tuning corresponds to \( \beta = 0 \) while undertuning (resp. overtuning) corresponds to \( \beta < 0 \) (resp. \( \beta > 0 \)). One of the main goals of absorber design is to select \( \beta \) to achieve robust vibration reduction of the primary system.

### 4 FEATURES OF THE FORCED RESPONSE

This section reports the main results of the paper. The linearized EOM are analyzed to determine the effects of absorber tuning on the steady-state TW response to TW e.o. excitation.
The special case for which the absorbers are locked at their vertices is analyzed in Section 4.1. These results, which are typical for a cyclic system with two DOF per sector, are used as a baseline against which the absorber performance is assessed. The full model, in which all three sector DOF are free to move, is considered in Section 4.2. Of particular interest are the effects of varying the absorber tuning and whether or not the no-resonance zone reported by Olson et al. [11–13] exists for this higher fidelity model.

The forced response can be obtained by assembling the sector models defined by Eq. (2) into a single matrix EOM and proceeding in the usual way [22]. For large N, however, it is desirable to simplify the analysis by exploiting the system (cyclic) symmetry, as it is done by Olson et al. [11–13] and others [23]. If \( P \) is the number of DOF in each sector \( (P = 3 \) for the model shown in Fig. 1), the \( PN \times PN \) system mass, damping, and stiffness matrices are block circulant with \( P \times P \) blocks. The \( PN \)-DOF model can be block decoupled to a set of \( N, P \)-DOF modal systems via a unitary transformation involving the complex Fourier matrix [24–26]. The \( PN \) natural frequencies are preserved under the transformation and can be obtained in sets of \( P \) from the \( N \) decoupled modal EOM. The forced response follows accordingly, and allows for the full system response to be determined by considering a set of uncoupled three DOF models.

4.1 Response with the Absorbers Locked

If the absorbers are locked relative to the rotating blades, then \( \psi_i = \psi_i' = \psi_i'' = 0 \) and each sector model has two DOFs. Figure 2a shows the \( 2N \) natural frequency loci in a Campbell diagram and the corresponding disk/blade amplitude frequency response (\( |\psi_i|, |\theta_i| \)) for a representative model with \( N = 10 \) sectors, order \( n = 3 \) excitation, \( \alpha = 1.4, \delta = 1.117, \rho = 1.67, \lambda = 0.1, \gamma = (\alpha + \delta + 1)/\delta^2, \mu_0 = 0.01, \mu_0' = 0.1, \nu_0 = 0.5, \nu_0' = \nu_0'' = 0.1, f_0 = 0.0016, \) and \( f_0' = 0.001 \). There are two groups of \( N \) natural frequencies whose spread depends primarily on the strength of the inter-sector coupling. For nonzero coupling they generally appear in repeated pairs, except for mode \( p = 1 \) and mode \( p = (N + 2)/2 \) if \( N \) is even. For zero coupling they collapse onto a single pair of values [11, 12]. The bottom-most group corresponds to in-phase motions of the two DOF in each sector and the top-most curves represent out-of-phase motions of the disk/blade DOFs. The natural frequencies increase slightly with increasing rotor speed due to centrifugal stiffening.

It is well-known that e.o. excitation excites only one mode in each eigenfrequency group due to special modal orthogonality properties of cyclic systems [11, 12]. The resonant rotor speeds can be identified in Fig. 2a by the intersections of the natural frequency loci corresponding to mode \( p = n + 1 \) with the e.o. line defined by \( \sigma \alpha \).\(^2\) The corresponding resonances are shown in the frequency response diagrams of Fig. 2b. Of course, mis-

\(^2\)The excited mode is \( p = n \mod N + 1 \) when considering \( n \in \mathbb{Z}_+ \) [12].

4.2 Response with the Absorbers Free

When the absorbers are free to move there are three natural frequencies corresponding to each index \( p \in \mathbb{N}_c \). The three groups of \( N \) frequency loci \( \tilde{\omega}_{p}\) are plotted in Fig. 3 for increasing inter-sector coupling and increasing absorber mass while holding all other parameters fixed. Each natural frequency corresponds to a system mode, which, together with fixed inter-sector phase relationships, form a forward TW, backward TW, or standing wave response among the sectors, depending on the value of \( \bar{n} \) relative to \( N \) [11, 12, 23]. The natural frequencies are tightly-spaced for weak inter-sector coupling (Fig. 3a) and they spread out as the coupling is increased (Fig. 3c). Each group exhibits the eigenvalue veering phenomenon, which results from small dynamic coupling between the disks and blades and between the blades and absorbers [17, 18, 29]. The ratio of absorber mass to the disk and blade masses is essential for the curve veering phenomenon. When the mass ratio is very small (Fig. 3d), the natural frequency curves are close to each other in the veering regions. (In Figs. 3d-f only one frequency locus \( \tilde{\omega}_{p}\) is shown in each group, which corresponds to zero inter-sector coupling.) A larger difference between the groups of frequency loci can be obtained by increasing the mass ratio (Fig. 3f), although in most applications this mass ratio is small.
Possible resonances can be identified by the intersections of $\bar{\omega}_i^{(p)}(\sigma)$ with the e.o. line $n\sigma$. However, there is a system resonance in the perfectly symmetric case only when

$$n\sigma = \bar{\omega}^{(n+1)}_{i,2,3}(\sigma)$$

is satisfied because only mode $n+1$ is excited in the steady-state [11, 12]. The principle objectives of this work are to determine if any resonances persist for near-ideal absorber tuning (i.e., for $\beta$ close to zero); whether or not any new resonances are introduced for multi-DOF disk/blade models; and, if so, whether the absorbers can be tuned to avoid these resonances over a range of rotor speeds.

The results are summarized in Fig. 4, which shows the eigenfrequency loci in Campbell diagrams and the corresponding disk, blade, and absorber amplitude frequency responses ($|\varphi_i|, |\theta_i|, |\psi_i|$) for a model with $N = 10$ sectors, order $n = 3$ excitation, various absorber tuning values that range from undertuning ($\beta < 0$) to overtuning ($\beta > 0$), and for the same parameter values used in Fig. 2. Inter-sector coupling is assumed to be quite strong to more clearly show the effects of detuning on the excited modes; this does not affect the approach nor the conclusions. In many situations the eigenfrequency loci $\bar{\omega}_i^{(p)}$ cross the e.o. line twice. For absorber overtuning (Fig. 4a) or sufficiently large undertuning (Fig. 4b) there are two system resonances, which are indicated by circles in the Campbell diagrams. Insets 1 and 2 show close-up views of the veering regions in Fig. 4a, including the resonances corresponding to $\bar{\omega}^{(n+1)}_{i,2,3}$. Figure 4c shows that the fundamental resonance disappears for ideal tuning ($\beta = 0$), but the resonance corresponding to $\bar{\omega}^{(n+1)}_{i,2,3}$ persists. In this case, there is only one system resonance over the full range of possible rotor speeds.

One of the main findings of Olson et al. is the existence of a no-resonance zone in the 2-DOF sector model dynamics, that is, a range of absorber tuning values which results in complete resonance suppression over the full range of possible rotor speeds.
Figure 4. Campbell diagrams and the corresponding amplitude frequency response.
5 CONCLUSIONS AND DIRECTIONS FOR FUTURE WORK

The performance of centrifugally-driven, order-tuned vibration absorbers has been investigated using a higher-fidelity lumped-parameter model of a bladed disk assembly under e.o. excitation. The model consists of a cyclic array of $N$ identical, identically-coupled sectors, each with three DOFs that capture the effects of the disk, blades, and absorbers. The principle finding of this work is the existence of an absorber undertuning region in which one resonance is robustly avoided. The range is bounded from above by ideal tuning and from below by a critical linear undertuning, which forms a resonance-suppression zone. Thus, while the absolute no-resonance zone reported by Olson et al. [11–13] does not extend completely to higher-fidelity sector models, the absorbers do provide a range of speeds over which one resonance can be suppressed.

It is important to note that adding more DOFs per sector does not qualitatively alter this picture. That is, groups of higher-order modes will still experience resonance, albeit at higher rotor speeds. This is due to the structure of the natural frequency curves shown in Fig. 4, which veer only once and then flatten out (asymptotically approach the absorbers-locked frequencies [11, 12]) as the rotor speed is increased. Therefore, the e.o. line generically crosses all but one natural frequency group. Adding more DOFs per sector simply adds more such curves at higher frequencies, thereby introducing more resonances at higher rotor speeds. The main conclusion is that an identical set of order-tuned absorbers can suppress only one resonance.

There remains much work to be done on this class of problems, which can be divided into four general categories. First, very few systematic experiments have been performed. The work by Duffy and co-workers on self-tuned impact dampers makes use of an impacting mode of motion for order-tuned absorbers, and demonstrates that they are effective in attenuating vibrations in a rotating plate [30, 31].

The second group of problems address nonlinear effects. Shaw and Pierre [16] have investigated a single-sector model with an order-tuned absorber that undergoes impacts when it reaches a certain amplitude. Olson and Shaw [12, 14, 15] analyzed the response of a two-DOF-per-sector model in which absorber path nonlinearity is taken into account, and they make some recommendations about nonlinear absorber tuning. Specifically, the absorber path, which is generalized to be non-circular, should be slightly softening for large amplitudes. Polchi [19] has some preliminary results along the same lines for the three-DOF-per-sector model considered here, which show that the linear resonance structure essentially persists for weakly nonlinear absorber motions. However, a detailed analysis with recommendations for path design remains to be completed.

The third category deals with systems of multiple absorbers, for which there are many possible studies. For example, is it necessary to have an absorber in every sector? The answer to
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