The $D_s$, $D^+$, $B_s$ and $B^+$ decay constants from $2 + 1$ flavor lattice QCD

Fermilab Lattice and MILC collaborations
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The $D$ and $B$ decay constants

- Among simplest Weak matrix elements to compute.
- Test of lattice technology: *e.g.*, heavy quark mass sensitivity, HQ formalisms, chiral extrapolations.
- The $D$ and $B$ decay constants provide tests of the CKM picture *e.g.*, $B^0 \rightarrow \mu^+\mu^-$ in SM and $|V_{ub}|$ from $B^+ \rightarrow \tau^+\nu$.
- This talk: Decay constants from three-flavor MILC asqtad lattices using asqtad light and clover (Fermilab interpretation) heavy valence quarks. PRELIMINARY.
- $D$ decay constants with HISQ charm on MILC four-flavor HISQ lattices see talks by:
  
  - C. Bernard Wed 11:00 session: 6C
  - D. Toussaint Thu 17:50 session: 8C

We have published results from our previous asqtad study...
Conclusions from our previous study

PRD.85.114506, arXiv:1112.3051

Contributions in quadrature to percent error

\[
\begin{align*}
\text{D-meson system} & \\
f_{D^+} &= 218.9 \pm 11.3 \text{ MeV} \\
f_{D_s} &= 260.1 \pm 10.8 \text{ MeV} \\
f_{D_s}/f_{D^+} &= 1.188 \pm 0.025 \\
\end{align*}
\]

\[
\begin{align*}
\text{B-meson system} & \\
f_{B^+} &= 196.9 \pm 8.9 \text{ MeV} \\
f_{B_s} &= 242.0 \pm 9.5 \text{ MeV} \\
f_{B_s}/f_{B^+} &= 1.229 \pm 0.026 \\
\end{align*}
\]

Our current study addresses many of these sources of error...
Conclusions from our previous study

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Contributions in quadrature to percent error

Improvements in current study:

- No u0 adjustment.
- HQ discretization, LQ discr., chiral extrap.

and statistical errors reduced by including finer $a \approx 0.058$ and 0.043 fm lattices and higher 2-pt statistics.

- Nearer to physical $m_l/m_h = 1/20$ helps in chiral extrap.
Conclusions from our previous study

PRD.85.114506, arXiv:1112.3051

Contributions in quadrature to percent error

... and more improvements:

- Better kappa tuning to reduce HQ mass errors.
- Better runs to compute \( Z_{V_{QQ}^4} \) and \( Z_{V_{qq}^4} \).
- New 2-pt fit technology to control 2-pt fit errors.
MILC asqtad $N_f = 2 + 1$

Current study includes

| id | a [fm] | beta  | $m_l/m_h$ | $a m_h$ | $m_h/m_s$ | $r_1/a$ | $N_{config}$ | $N_{tsrc}$ |
|----|--------|-------|-----------|---------|-----------|---------|--------------|------------|
| A  | 0.043  | 7.81  | 0.2       | 0.014   | 1.079     | 7.208   | 801          | 4          |
| B  | 0.059  | 7.46  | 0.1       | 0.018   | 1.019     | 5.307   | 827          | 4          |
| C  | 0.058  | 7.465 | 0.139     | 0.018   | 1.024     | 5.330   | 801          | 4          |
| D  | 0.058  | 7.47  | 0.2       | 0.018   | 1.028     | 5.353   | 673          | 8          |
| E  | 0.058  | 7.48  | 0.4       | 0.018   | 1.037     | 5.399   | 593          | 4          |
| F  | 0.083  | 7.075 | 0.05      | 0.031   | 1.255     | 3.738   | 791          | 4          |
| G  | 0.083  | 7.08  | 0.1       | 0.031   | 1.256     | 3.755   | 1015         | 4          |
| H  | 0.083  | 7.085 | 0.15      | 0.031   | 1.262     | 3.772   | 984          | 4          |
| I  | 0.082  | 7.09  | 0.2       | 0.031   | 1.267     | 3.789   | 1931         | 4          |
| J  | 0.081  | 7.11  | 0.4       | 0.031   | 1.290     | 3.858   | 1996         | 4          |
| K  | 0.11   | 6.76  | 0.1       | 0.05    | 1.489     | 2.739   | 2099         | 4          |
| L  | 0.11   | 6.76  | 0.14      | 0.05    | 1.489     | 2.739   | 2110         | 4          |
| M  | 0.11   | 6.76  | 0.2       | 0.05    | 1.489     | 2.739   | 2259         | 4          |
| N  | 0.11   | 6.79  | 0.4       | 0.05    | 1.534     | 2.821   | 2052         | 4          |
| O  | 0.14   | 6.572 | 0.2       | 0.0484  | 1.156     | 2.222   | 631          | 24         |

Cf. our previous study:

- Two finer lattice spacings: 0.043 and 0.058 fm (id=A-E).
- Sea quarks nearer to physical: $m_l/m_h = 1/20$ (F).
- Better statistics: around $3.6 \times$ more $N_{config} \cdot N_{tsrc}$. 
Charm and Bottom 2-pt functions

On each ensemble, for $H = \text{charm, bottom}$ and a range of $m_q$ compute six 2-pt functions

\[
C^{(j,k)}(t) = \langle O_{H_q}^{(j)\dagger}(t) O_{H_q}^{(k)}(0) \rangle
\]

\[
C^{(k)}_{A_4}(t) = \langle A_{4\dagger}(t) O_{H_q}^{(k)}(0) \rangle
\]

with smearings $j, k \in \{\text{point, smeared}\}$. The quantity $\phi_{H_q} = f_{H_q} \sqrt{M_{H_q}}$ is found from the overlap

\[
\frac{\langle 0 \mid A_\mu \mid H_q(p) \rangle}{\sqrt{m_{H_q}}} = (p_\mu / m_{H_q}) \phi_{H_q}
\]

$O(a)$-improved $A_4$ is matched to continuum by factor

\[
\sqrt{Z_{V_4}^{QQ} Z_{V_4}^{qq} \rho_{A_4}^{Qq}}
\]

with nonperturbative $Z_{V_4}$ and small one-loop $\rho_{A_4}$ correction.
2-pt fitting details

- Fit four (five) 2-pt functions.
- New two stage fit process:
  1. Set empirical Bayesian priors for ground-state from $1+1$ state fits at large time.
  2. Use (broadened) priors in fit over a wide range of smaller non-overlapping times.
- Good isolation of ground-state using $4+4$ or $5+5$ states
- Bootstrap fits clearly show (expected) correlations.
Clover $c$- and $b$-quark kappa tuning

Checks: $D_s$ HFS

Want $m_2(\kappa) = m(D_s)$ or $m(B_s)$

$$E(\vec{p})^2 = m_1^2 + \left(\frac{m_1}{m_2}\right)^2 \vec{p}^2 + O(p^4)$$

- High-statistics 2-pt tuning runs on $m_l = 0.2m_h$ ensembles.
- $E(\vec{p})$ vs $\vec{p}$ fits include priors for $O(p^4)$ effects.
- Tunings corrected for sea-quark effects.
- Predict tuned kappa for all ensembles.

Expt. $\Delta M$ shown at zero

... and $B_s$ HFS
Chiral fits

- Correct simulation $\phi = f\sqrt{M}$ for any kappa mistuning.
- NLO expression for $\phi$ from partially-quenched staggered chiral perturbation theory [Aubin and Bernard, arXiv:hep-lat/0510088].
- Add NNLO analytic (quadratic in quark mass) terms.
- Model both light- and heavy-quark discretization effects in the fits.
- Distance scale $r_1$, quark masses $m_s$, $m_d$ and $m_u$ and $O(a^2)$ LECs from MILC light meson fits.
Extrapolations for $D^+$ and $D_s$

**PRELIMINARY** analysis and data are still blinded.
$B$ system fit

PRELIMINARY

| $a$          | $m_l/m_h$ |
|--------------|-----------|
| 0.043 fm     | 0.2       |
| 0.058 fm     | 0.2       |
| 0.059 fm     | 0.2       |
| 0.083 fm     | 0.1       |
| 0.082 fm     | 0.2       |
| 0.081 fm     | 0.4       |
| 0.11 fm      | 0.4       |
| 0.11 fm      | 0.1       |
| 0.11 fm      | 0.2       |
| 0.14 fm      | 0.2       |
| 0.14 fm      | 0.4       |
| 0.059 fm     | 0.1       |
| 0.083 fm     | 0.15      |
| 0.058 fm     | 0.14      |
| 0.083 fm     | 0.2       |
| 0.11 fm      | 0.2       |
| 0.11 fm      | 0.14      |
| 0.14 fm      | 0.2       |

$r_1^2m_\pi^2$ vs $r_1^2/r_\phi$
Extrapolations for $B^+$ and $B_s$

**PRELIMINARY** analysis and data are still blinded.
Summary

Current study (predicted)

- Error budget is work in progress.
- Current projections shown on left.
- Anticipate reductions in systematic errors compared to our previous study.
- Underway: final cross-checks of this analysis and full error budget for this three-flavor asqtad study.
- Next: clover bottom quark calculations on MILC four-flavor HISQ lattices – including e.g., $f_{B^+}/f_{D^+}$ with HISQ charm.

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