The temporal evolution of quantal Joule heating of 2D electrons in GaAs quantum well placed in quantizing magnetic fields is studied using a difference frequency method. The method is based on measurements of the electron conductivity oscillating at the beat frequency \( f = f_1 - f_2 \) between two microwaves applied to 2D system at frequencies \( f_1 \) and \( f_2 \). The method provides direct access to the dynamical characteristics of the heating and yields the inelastic scattering time \( \tau_n \) of 2D electrons. The obtained \( \tau_n \) is strongly temperature dependent, varying from 0.13 ns at 5.5K to 1 ns at 2.4K in magnetic field \( B=0.333T \). When temperature \( T \) exceeds the Landau level separation the relaxation rate \( 1/\tau_n \) is proportional to \( T^2 \), indicating the electron-electron interaction as the dominant mechanism limiting the quantal heating. At lower temperatures the rate tends to be proportional to \( T^4 \), indicating considerable contribution from electron-phonon scattering.

An alternative experimental method accessing the inelastic relaxation processes is certainly needed to settle this important issue.

In this Letter we present an experimental method providing direct access to the temporal evolution of the electron transport affected by quantal heating. The method yields the inelastic relaxation time \( \tau_n \). At high temperatures the time is found to be in a good agreement with the inelastic time obtained in \textit{dc} experiment\textsuperscript{3}. At low temperatures a disagreement between these two times is observed. Presented below results significantly support the existing theory\textsuperscript{4}.

The dynamics of quantal heating was studied in a two-dimensional system of highly mobile electrons in Corbino geometry. The Corbino disc with inner radius \( r_1 =0.9 \) mm and outer radius \( r_2 =1 \) mm was fabricated from a selectively doped single GaAs quantum well sandwiched between AlAs/GaAs superlattice barriers. The width of the well was 13 nm. The structure was grown by molecular beam epitaxy on a (100) GaAs substrate. AuGe eutectic was used to provide electric contacts to the 2D electron gas. The 2D electron system with electron density \( n=8\cdot10^{15} \text{ m}^{-2} \) and mobility \( \mu =112 \text{ m}^2/\text{Vs} \) at \( T=4.8K \) was studied at different temperatures from 2.4K to 6K in magnetic fields up to 1T.

Fig\textsuperscript{4} shows the experimental setup. Two microwave sources supply the radiation to the sample through a semi-rigid coax at two different frequencies \( (f_1, f_2) \). The interference between these sources forms microwave radiation with amplitude modulation at the difference (beat) frequency \( f = f_1 - f_2 \). The modulated microwave induces oscillations of Joule heating and, thus, the sample conductance \( \delta G_f \) at the frequency \( f \). Application of a \textit{dc} voltage \( V_{dc} \) to the structure produces current oscillations \( \delta I_f = \delta G_f V_{dc} \), which propagate back to a microwave analyzer through the same coax. The analyzer detects the
current oscillations at frequency \( f \) (\( f \)-signal). In addition the setup contains a bias-tee providing measurements in the \( dc \) domain. These measurements are essential for a frequency calibration of the microwave setup.

In frequency experiments frequency \( f_1 = 8 \) GHz was fixed while frequency \( f_2 \) was scanned from 5.5 to 7.999 GHz. To take into account variations of the microwave power \( P_2 \) delivered to the sample in the course of the frequency scan, a \( dc \) measurement of the resistance variation induced by the same applied microwave power \( P_2 \) is done. At a small applied power the induced resistance variation is proportional to \( P_2 \), thereby providing the power calibration. A similar calibration is done for the receiver channel at frequency \( f \) and is based on the reciprocal property of the microwave setup.

Figure 2 presents the magnetic field dependence of the resistance of the sample, \( R \), with neither \( dc \) bias nor microwaves applied (thin solid line). As expected in the Corbino geometry the resistance shows the classical parabolic increase with the magnetic field \( B \). The thick solid line presents the nonlinear response of the sample (\( f \)-signal) measured at difference frequency \( f = 1 \) MHz. The nonlinear response is very weak at small magnetic fields \( B < 0.1 \) T. At these fields the Landau level separation \( \hbar \omega_c \) is much smaller than the levels width \( \Gamma \) and both the quantization of the electron spectrum and quantal heating are absent\(^2\). Above 0.1 Tesla the Landau quantization occurs and quantal heating starts to grow, reaching maximums at about 0.3 and 0.45 T. At higher magnetic fields the \( f \)-signal decreases due to a decrease of the cyclotron radius of electron orbits leading to significant reduction of the spatial and, thus, the spectral separation \( \Gamma_{\text{sp}} \). At \( T = 4.8 \) K and \( B > 0.5 \) T, Shubnikov de Haas (SdH) oscillations are visible in both the resistance and the \( f \)-signal. The frequency dependence of the \( f \)-signal was studied at magnetic field \( B = 0.333 \) T corresponding to a maximum of the sample conductivity at low temperatures (not shown).

Figure 3 presents the dependence of the \( f \)-signal and differential resistance on the \( dc \) bias at different frequencies \( f \) as labeled. At small \( dc \) biases \( V_{dc} \) the \( f \)-signal is proportional to \( V_{dc} \) while the differential resistance \( r_{xx} \sim V_{dc}^2 \). These data agree with the relation \( j = \sigma_0 E + \alpha E^2 \) between the current density \( j \) and the electric field \( E \), which is expected at small magnitude \( E \). Here \( \sigma_0 \) is the ohmic conductivity (linear response). In this perturbative regime the nonlinear current density \( j_\omega \) at angular frequency \( \omega = 2\pi f \) (the \( f \)-signal) should be proportional to applied \( dc \) (\( E_0 \)) and microwave \( (E_1, E_2) \) electric fields: \( j_\omega = \alpha E_0 E_1 E_2 \). The observed microwave power dependence (not shown) of the \( f \)-signal is in complete agreement with the expected behavior at small microwave power. At higher \( dc \) biases the \( f \)-signal demonstrate an additional interesting features, which were, however, beyond the scope of the present work.

The frequency dependence of the nonlinear response can be understood from an analysis of the spectral diffusion equation for the electron distribution function \( f(\epsilon) \)\(^3\)

\[
\frac{\partial f(\epsilon)}{\partial t} + E^2 \frac{\sigma_D}{\nu(\epsilon)} \partial_\epsilon \left[ \nu^2(\epsilon) \partial_\epsilon f(\epsilon) \right] = \frac{f(\epsilon) - f_T(\epsilon)}{\tau_{\text{in}}(\epsilon)} 
\]

(1)

Here, \( \sigma_D(B) \) is the Drude conductivity in a magnetic field \( B \), \( \nu = \nu/\nu_0 \) is ratio of the density of electron states (DOS) \( \nu(\epsilon) \) to the DOS at zero magnetic field \( \nu_0 \) and \( f_T \) represents the Fermi-Dirac distribution at a temperature \( T \). Below we consider the case of a low difference frequency \( \omega = \omega_1 - \omega_2 \ll \omega_1, \omega_2 \) corresponding to the experiments. At small electric field
versely proportional to frequency. In this regime, the frequency dependence expected from Eq. (3): 

\[ \frac{\sigma}{\sigma_D} = \frac{1}{\omega + 1/\tau_{in}} \cdot \Sigma \] 

Here \( \sigma_L \) is the conductivity at energy \( \epsilon \) and \( \Sigma = -\sigma_D \int \sigma_L \partial_t (\Theta^2 \partial_t f_T) / \nu \, dt \). Eq. (3) indicates that at high difference frequency \( \omega \gg 1/\tau_{in} \) the f-signal is inversely proportional to frequency. In this regime, microwave radiation is "on" for a short time \( \Delta t \sim 1/\omega \), which is not enough to considerably change the electron distribution.

Figure 4 presents the frequency dependence of the f-signal at different temperatures as labeled. The observed f-signal is nearly frequency independent at low frequencies and is inversely proportional to the frequency in the high frequency limit. The solid lines represent the frequency dependence expected from Eq. (3): \( j_\omega = A/|1 + \omega \tau_{in}| \) with amplitude \( A \) and time \( \tau_{in} \) as fitting parameters. The figure indicates a good agreement between the data and the frequency dependence described by Eq. (3). The insert to the figure presents the temperature dependence of the inelastic scattering time \( \tau_{in} \) obtained from the fit. The high temperature behavior of the inelastic time is consistent with \( T^2 \) decrease indicating the dominant contribution of electron-electron interactions to the inelastic electron relaxation. At low temperatures a deviation from the \( T^2 \) behavior is found, indicating a suppression of the \( e-e \) contribution. The suppression is expected at low temperatures, when \( kT < \hbar \omega_e \), where \( \omega_e \) is the cyclotron frequency.

It is important to compare the obtained inelastic time \( \tau_{in} \) with the time \( \tau_{in}^{dc} \) obtained from the nonlinear response in the dc domain. Fig. 5(a) presents the dependence of the normalized conductivity of the sample \( \sigma/\sigma_D \) on the applied electric field \( E \approx V_{dc}/(r_2 - r_1) \). In the Corbino geometry the Drude conductivity \( \sigma_D \) is obtained from a comparison of the magneto-conductivity of the sample \( \sigma(B) \) with the expected classical behavior: \( \sigma_D(B) \sim 1/(1 + (\omega_e \tau_{tr})^2) \), where \( \tau_{tr} \) is the transport scattering time. Solid lines present experimental dependences obtained at different temperatures. Filled circles present results of numerical simulations of the nonlinear response based on the spectral diffusion equation (1). The simulation uses a Gaussian approximation for the electron density of states. Comparison between the experiments and numerical simulations yields the inelastic relaxation time \( \tau_{in}^{dc} \).

Fig. 5(b) shows temperature dependencies of the time \( \tau_{in}^{dc} \) and the inelastic time obtained from the dynamics of the nonlinear response (f-signal). At high temperatures both times are close to each other. At lower temperatures there is a considerable difference between the two times suggesting that important parameters responsible for the nonlinear response are not well understood or remain unknown. The dc-domain inelastic time \( \tau_{in}^{dc}(T) \) follows \( 1/T^3 \) decrease with the temperature \( T \), while the
time $\tau_{in}$, obtained from $f$-signal, is mostly proportional to $1/T^2$ with a tendency to $1/T^3$ at low temperatures.

Previous $dc$ domain investigations have shown that in the regime when only few Landau levels provide the electron transport ($kT < \hbar \omega_c$) the electron-electron scattering is ineffective for the relaxation of the overheated electrons since the scattering conserves the total electron energy. In this regime the experiments show the $1/T^3$ temperature dependence of the time $\tau_{in}^{dc}(T)$ attributed to contributions of the electron-phonon interaction to the electron energy relaxation. At small magnetic fields ($kT \gg \hbar \omega_c$) the inelastic time $\tau_{in}^{ec}(T)$ was found to follow $1/T^3$ dependence attributed to electron-electron scattering in accord with the theory. The transition between two regimes is poorly understood. The presented results have been obtained in the transitional regime ($kT \sim \hbar \omega_c$). The discrepancy between $\tau_{in}(T)$ obtained in the high frequency and the $dc$ domains suggests the complexity of actual inelastic processes in this 2D system.

Furthermore, another mechanism of nonlinear transport, which is comparable with the effects of quantal heating, has been recently identified in the regime of SdH oscillations. This mechanism was related to $dc$ bias induced electron spatial redistribution which may have very different (most likely very slow) relaxation and, thus, may not have been captured by the $f$-signal in the studied frequency range.

Shown in Fig. 5(b), the discrepancy between the two times may also be related to assumptions and approximations used to extract the inelastic scattering rate in the $dc$ domain. In particular the expression for conductivity used in previous investigations $\sigma(\nu) = \sigma_D \nu^2$ may not be quite adequate for separated Landau levels. In contrast to the $dc$ domain result, the frequency dependence of the $f$-signal is a direct measurement and is not subject to assumptions and/or approximations. Thus the dynamical measurements provide more accurate and reliable information on the inelastic relaxation of 2D electrons placed in quantizing magnetic fields.

In summary the dynamics of the strongly nonlinear response of 2D highly mobile electrons placed in quantizing magnetic fields is observed. The obtained results indicate the dominant contribution of quantal heating to the response and significantly support the existing theoretical predictions. The presented method provides the direct measurement of the inelastic relaxation in 2D electron systems.

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[1] I. A. Dmitriev, M. G. Vavilov, I. L. Aleiner, A. D. Mirlin, and D. G. Polyakov, Phys. Rev. B 71, 115316 (2005).
[2] J.Q. Zhang, S. Vitkalov, and A.A. Bykov, Phys. Rev. B 80, 045310 (2009).
[3] N. Romero Kalmanovitz, A. A. Bykov, Sergey Vitkalov, and A. I. Toropov, Phys. Rev. B 78, 085306 (2008).
[4] A.A. Bykov, Jing-Qiao Zhang, Sergey Vitkalov, A.K. Kalaquin, A.K. Bakarov, Phys. Rev. Lett. 99, 116801 (2007).
[5] A. A. Bykov, Sean Byrnes, Scott Dietrich, Sergey Vitkalov, I. V. Marchishin, and D. V. Dmitriev, Phys. Rev. B 87, 081409 (2012).
[6] I. A. Dmitriev, A. D. Mirlin, D. G. Polyakov, M. A. Zudov, Rev. Mod. Phys. 84, 1709 (2012).
[7] see ref. [5] in paper.
[8] In the Corbino geometry the electric field $E(r) \sim \ln(r)$ varies with the radius $r$. For Corbino disc with a narrow conducting channel $r_2 - r_1 \ll r_1, r_2$ the field variation is small and the electric field is approximated by an average value $E \approx V_{dc}/(r_2 - r_1)$.
[9] At $B=0.333T$ the $f$-signal demonstrates both the high amplitude (see Fig. 2) and significant temperature variations of the frequency response within the available frequency range from 1 to 2500 MHz.
[10] Scott Dietrich, Sean Byrnes, Sergey Vitkalov, D. V. Dmitriev and A. A. Bykov Phys. Rev. B 85, 155307 (2012).
[11] Scott Dietrich, Sean Byrnes, Sergey Vitkalov, A. V. Goran and A. A. Bykov J. Appl. Phys. 113, 053709 (2013).
[12] In a magnetic field corresponding to a maximum of the conductivity ($B=0.333T$) and at small $dc$ bias the redistribution mechanism and the quantal heating work together enhancing the total nonlinearity and narrowing.
the conductivity drop shown in Fig. 5(a). It can make the extracted $\tau_{\text{dc}}$ to be longer.