QUIET-SUN MAGNETIC FIELD MEASUREMENTS BASED ON LINES WITH HYPERFINE STRUCTURE

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ABSTRACT

The Zeeman pattern of Mn i lines is sensitive to hyperfine structure (HFS), and because of this, they respond to hectogauss magnetic field strengths differently than the lines commonly used in solar magnetometry. This peculiarity has been employed to measure magnetic field strengths in quiet-Sun regions, assuming the magnetic field to be constant over a resolution element. This assumption is clearly insufficient, biasing the measurements. The diagnostic potential of Mn i lines can be fully exploited only after one understands the sense and magnitude of such bias. We present the first syntheses of Mn i lines in realistic quiet-Sun model atmospheres. The Mn i lines weaken with increasing field strength. In particular, kilogauss magnetic concentrations produce Mn i $\lambda 5538$ circular polarization signals (Stokes $V$) that can be up to 2 orders of magnitude smaller than what the weak magnetic field approximation predicts. The polarization emerging from an atmosphere having weak and strong fields is biased toward the weak fields, and HFS features characteristic of weak fields show up even when the magnetic flux and energy are dominated by kilogauss fields. For the HFS feature of Mn i $\lambda 5538$ to disappear, the filling factor of kilogauss fields has to be larger than the filling factor of subkilo gauss fields. Since the Mn i lines are usually weak, Stokes $V$ depends on magnetic field inclination according to the simple cosine law. Atmospheres with unresolved velocities produce very asymmetric line profiles, which cannot be reproduced by simple one-component model atmospheres. Using the HFS constants available in the literature, we reproduce the observed line profiles of nine lines with varied HFS patterns.

Subject headings: line: profiles — Sun: magnetic fields — Sun: photosphere

1. INTRODUCTION

When polarimetric sensitivity and angular resolution exceed the required thresholds, magnetic signals show up almost everywhere on the solar photosphere. The signals in supergranulation cell interiors are particularly weak; however, most of the solar surface is in the form of cell interiors, and therefore, these weak signals probably set the total unsigned magnetic flux and magnetic energy existing in the photosphere at any given time (see, e.g., Unno 1959; Stenflo 1982; Yi et al. 1993; Sánchez Almeida 1998, 2004; Schrijver & Title 2003). The importance of these ubiquitous quiet-Sun magnetic fields depends, to a large extent, on the magnetic field strengths that characterize them. For example, the magnetic field and the magnetic energy scale as powers of the field strength, and the connectivity between photosphere and corona is probably provided by the magnetic concentrations with the largest field strengths (e.g., Domínguez Cerdeña et al. 2006a). Unfortunately, measuring quiet-Sun magnetic field strengths is not a trivial task. All measurements are based on the residual polarization left when a magnetic field of complex topology is observed with finite angular resolution (e.g., Emonet & Cattaneo 2001; Sánchez Almeida et al. 2003; Trujillo Bueno et al. 2004). Measuring implies assuming many properties of the unresolved complex magnetic field, and in doing so, the measurements become model dependent and prone to bias. Depending on the technique used for measuring, the real quiet Sun exhibits weak decagauss fields (e.g., Stenflo 1982; Faurobert-Scholl 1993; Bommier et al. 2005), intermediate hectogauss fields (e.g., Lin & Rimmele 1999; Khomenko et al. 2003; López Ariste et al. 2006), or strong kilogauss fields (e.g., Sánchez Almeida & Lites 2000; Lites 2002; Domínguez Cerdeña et al. 2003). Such discrepancies can be naturally understood if the true quiet Sun contains a continuous distribution of field strengths going all the way from 0 to 2 kG. Different techniques are biased differently, and therefore they tend to pick out a particular part of the distribution. This scenario is very much consistent with realistic numerical simulations of magnetoconvection (Cattaneo 1999; Stein & Nordlund 2006; Vögler et al. 2005; Vögler & Schüssler 2007). In order to provide a comprehensive observational description of the quiet-Sun magnetic field strengths, one has to combine different methods carefully chosen to have complementary biases. Then the full distribution can be assembled, taking the biases into account (Domínguez Cerdeña et al. 2006a).

In an effort to complement the existing magnetic field strength diagnostic techniques, López Ariste et al. (2002) proposed the use of spectral lines whose Zeeman patterns are sensitive to hyperfine structure (HFS). The formalism to deal with the HFS of spectral lines in magnetic atmospheres was developed more than 30 years ago (Landi Degl’Innocenti 1975). According to this formalism, the polarization of the HFS lines varies with magnetic field strength very differently from that of the lines commonly used in solar magnetometry. This unusual behavior was invoked by López Ariste et al. (2002) when proposing the use of HFS Mn i lines as a diagnostic tool for magnetic field strengths. López Ariste et al. (2006, 2007) and Asensio Ramos et al. (2007) have applied the idea to measure the magnetic field strengths in quiet-Sun regions. Since the number of observables is limited, they minimize the statistical error of the measurement by minimizing the number of free parameters to be tuned. Milne–Eddington (ME) atmospheres are used to synthesize the polarization of the lines. The magnetic field is assumed to be constant, and therefore, the measurements provide some kind of weighted average of the true field strengths existing in the resolution elements. As we pointed out above, the topology of the quiet-Sun magnetic field is complex, with a distribution of field strengths and polarities coexisting in a typical resolution element. Thus, the ill-defined average provided

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by the Mn i lines is expected to be biased toward a particular range of field strengths, as happens with the rest of the quiet-Sun measurements (e.g., the near-IR Fe i lines overlook kilogauss magnetic concentrations [Sánchez Almeida & Lites 2000; Socas-Navarro & Sánchez Almeida 2003]; the traditional visible Fe i lines exaggerate the contribution of kilogauss fields [Bellot Rubio & Collados 2003; Martínez González et al. 2006]; and the Hanle scattering polarization signals are not sensitive to hecto- or kilogauss fields [Fauré Robert et al. 2001; Sánchez Almeida 2005]).

The bias presented by the HFS Mn i lines is so far unknown, and the existing and forthcoming measurements based on these lines will be fully appreciated only when the sources of systematic effects are properly acknowledged and quantified. In order to explore the magnitude and the sense of the expected effects, we have undertaken the synthesis of Mn i lines in a number of realistic quiet-Sun atmospheres with complex magnetic field distributions. The main trends are presented here.

This paper is organized as follows: Section 2 describes the software developed to carry out the syntheses. First, a ME code is needed to compare our syntheses with the results in the literature. Then, a plane-parallel one-dimensional code allows us to explore the influence of realistic thermodynamic conditions on the polarization of lines with HFS. Finally, a microstructured magnetic atmosphere (MISMA) code provides additional realism to the modeling, since it includes coupling between magnetic field strengths and thermodynamic conditions, magnetic fields varying along and across the line of sight (LOS), mixed polarities in the resolution element, etc. All these codes are based on the original FORTRAN routine written by Landi Degl’Innocenti (1978). The analysis is focused on the line most often used in observations, namely, Mn i 5538. We discuss its intensity and circular polarization, since the linear polarization signals are very weak and they remain undetected so far. Single-component and multi-component syntheses of this line are described in §§ 3 and 4, respectively. An exploratory attempt to consider unresolved mixed polarities and velocity fields is carried out in § 5. The polarization of other Mn i lines is also considered, in § 6. The implications of these syntheses are discussed in § 7.

2. DESCRIPTION OF THE SYNTHESIS PROCEDURES

The synthesis of Stokes profiles has been carried out using three different approaches to solve the radiative transfer equation for polarized light,

\[ \frac{dI}{ds} = -K(I - S), \]

where \( I = (I, Q, U, V) \) is the column vector containing the Stokes parameters, \( S = (B_T, 0, 0, 0)^T \) is the source vector with \( B_T \) the LTE source function, \( K \) is the \( 4 \times 4 \) absorption matrix, and finally, the variable of integration \( s \) corresponds to the length along the LOS. (For details on matrix elements and the rest of the equation, see Landi Degl’Innocenti 1976.) The three codes differ in the model atmosphere used to describe the photosphere. The first one solves equation (1) under the ME hypothesis. The source function is assumed to vary linearly with the continuum optical depth \( \tau \). Then the radiative transfer equation admits an analytic solution involving the two coefficients of the source function \( (B_T = B_0 + B_1 \tau) \) and the seven coefficients of the absorption matrix, which are assumed to be constant (see eq. [14] of Landi Degl’Innocenti 1975).

The second code solves equation (1) for one-dimensional atmospheres. Realistic temperatures, pressures, and magnetic field vectors are considered, but they only vary along the LOS, so that a single ray suffices to synthesize the spectrum. The third code solves a different version of equation (1), which is obtained by a spatial averaging under the MISMA hypothesis (Sánchez Almeida et al. 1996). It assumes the solar atmosphere to have inhomogeneities smaller than the photon mean free path, whose details are washed out by the radiative transfer process. Only their average properties matter. The new equation to be solved is

\[ \frac{dI}{ds} = -\langle K \rangle (I - S'), \]

where \( S' = \langle K \rangle^{-1}(K.S) \) is a mean source vector. The angle brackets in equation (2) denote an average over a volume whose dimension along the LOS is on the order of the photon mean free path. As a result of the averaging, \( \langle K \rangle \) and \( S' \) are slowly varying functions of \( s \), with this slow variation accounting for the large-scale structure in the atmosphere. Pursuing the idea that optically thin details are irrelevant for the final spectrum, Sánchez Almeida (1997) puts forward a type of model MISMA made of distinct components with different temperatures, pressures, and magnetic field vectors. The coexistence of different components imposes a number of physical constraints to be satisfied; in particular, the lateral pressure balance causes the magnetic field to be coupled with the thermodynamics (see the original reference for an exhaustive description). These model MISMAs, with their constraints, provide the degree of realism required to carry out our exploratory work. Note that the model MISMAs reproduce all kinds of Stokes profiles observed in the quiet Sun, including those with extreme asymmetries (Sánchez Almeida & Lites 2000; Socas-Navarro & Sánchez Almeida 2002; Domínguez Cerdeña et al. 2006b).

Equation (1) is formally identical to equation (2); they are integrated using a predictor-corrector method with the variables equally spaced in a fixed grid of atmospheric heights. As mentioned in § 1, the linear polarization signals \( Q \) and \( U \) are very weak, and we do not analyze them here despite the fact that they are synthesized together with \( I \) and \( V \).

All the codes are based on the FORTRAN procedure HYPER by Landi Degl’Innocenti (1978), which computes the Zeeman pattern of any line with hyperfine structure. The pattern depends on the externally imposed magnetic field together with a set of atomic parameters, namely, the HFS constants, the quantum numbers of the two levels involved in the transition, the relative isotopic abundance, and the isotope shifts. When the Zeeman splitting is comparable to the HFS splitting, the Stokes profiles are strongly disrupted, showing typical HFS features. The pattern depends on the so-called HFS constants \( A \) and \( B \), which account for the two first terms of the Hamiltonian describing the interaction between the electrons in an atomic level and the nuclear magnetic moment—\( A \) corresponds to the magnetic dipole coupling, whereas \( B \) describes the electric quadrupole coupling. The constants \( A \) and \( B \) can be determined empirically or theoretically. The values of \( A \) adopted for our Mn i syntheses are listed in Table 1. The table includes the appropriate references, which indicate that \( B \) is negligibly small in all cases.

2.1. Testing the Codes

We carry out different types of tests using the line Mn i 5538 as target. Intensity profiles are compared with the unpolarized Fourier transform spectrometer (FTS) solar atlas of Neckel (1999). The syntheses are also compared with the (ME) Stokes profiles existing in the literature. The codes are cross-checked one with respect to another, and with existing synthesis codes that neglect the HFS. All these efforts are summarized below.
The first series of tests is based on nonmagnetic atmospheres. We start by recovering the ME Stokes $I$ profile from Figure 3 of López Ariste et al. (2002) with use of the appropriate ME parameters (A. López Ariste 2007, private communication). In addition, Figure 1 shows a reasonable ME fit to the FTS solar spectrum using a Doppler width $\Delta \lambda_D = 0.029$ mA, a damping constant $a = 0.3$, an absorption coefficient $\eta = 2.7$, a source function given by $B_0 = 1.0$ and $B_1 = 1.1$, and the atomic parameters in Table 1. Figure 1 also includes a fit to the observed profile using the one-dimensional code. The line is synthesized in the quiet-Sun model atmosphere of Maltby et al. (1986) using the atomic parameters in Table 1. The fit was carried out by trial and error, varying the velocity along the LOS, the macroturbulence ($v_{\text{mac}} = 1.5$ km s$^{-1}$), and the Mn abundance (log [Mn] = 5.31). The MISMA code was also tested against the observed Stokes $I$, assuming the thermodynamic parameters of the model atmosphere to be given by the model quiet Sun in § 4.1 of Sánchez Almeida (1997). As in the case of one-dimensional atmospheres, the atomic parameters are those listed in Table 1, and we vary the velocity along the LOS, the macroturbulence ($v_{\text{mac}} = 1.5$ km s$^{-1}$), and the Mn abundance (log [Mn] = 5.31). One additional consistency test was carried out with the one-dimensional code. We compared the synthesis obtained neglecting the HFS (i.e., imposing $A = B = 0$) with the intensity from the one-dimensional code from which we inherited the absorption coefficient routines (§ 5 of Sánchez Almeida 1992). The profiles of the two syntheses are identical within the numerical precision.

The second series of tests refers to magnetic atmospheres. Employing the atomic parameters and the atmospheres described above, we include a constant magnetic field vector. We use three different magnetic field strengths, $B = 100, 300, \text{and } 900$ G, with the same inclination, $\theta = 60^\circ$, and azimuth, $\phi = 0^\circ$. The ME syntheses reproduce the Stokes profiles in Figure 4 of López Ariste et al. (2002). Stokes $V$ profiles synthesized with the three codes are shown in Figure 2. The agreement between the syntheses supports their internal consistency—if the codes are tuned to produce similar Stokes $I$ profiles (Fig. 1), then for the same field strength they also produce similar Stokes $V$ (Fig. 2).

The Stokes $V$ profiles represented in Figure 2 correspond to (and are consistent with) the two behaviors of Mn i $\lambda 5538$ described by López Ariste et al. (2002). Figure 2a shows the kind of profile for $B \lesssim 700$ G, with a characteristic reversal not far from the line core (hereafter the "HFS hump"). Figure 2b corresponds to values $B \gtrsim 700$ G, which do not generate HFS humps at the core. The presence or absence of such a reversal is used by López Ariste and coworkers to distinguish between subkilogauss fields (present) and kilogauss fields (absent).

The results in § 6 can be regarded as a third independent test. The same nonmagnetic quiet-Sun model MISMA that reproduces the observed intensity of Mn i $\lambda 5538$ also accounts for eight additional Mn i lines with very different HFS patterns. Such agreement demonstrates the realism and self-consistency of the syntheses.

### 3. SYNTHESSES OF Mn i $\lambda 5538$

### IN ONE-DIMENSIONAL ATMOSPHERES

The weak magnetic field approximation is routinely used in solar magnetometry to, for example, calibrate magnetograms. It is also valid for lines with HFS (Landi Degl’Innocenti 1975; Landi Degl’Innocenti & Landolfi 2004), and so it was used by...
López Ariste et al. (2006) to estimate the relative contribution of weak and strong fields to the observed quiet-Sun signals. When the approximation holds, the Stokes $V$ profile is proportional to the longitudinal component of the magnetic field, $B \cos \theta$:

$$V = -c \frac{dI}{d\lambda} B \cos \theta,$$

where the constant $c$ depends on the particular spectral line and where the derivative of the intensity profile, $dI/d\lambda$, gives the variation with wavelength of Stokes $V$ (Landi Degl’Innocenti 1975; Landi Degl’Innocenti & Landolfi 2004). We tested the approximation with the quiet-Sun model atmospheres described in § 2.1, which are realistic in the sense that they reproduce the observed intensity profile when the magnetic field strength is close to zero (Fig. 1). The syntheses assume the magnetic field to be constant along the LOS, with the inclination $\theta$ set to 60°. Figure 3 shows the variation of the maximum Stokes $V$ signal as a function of magnetic field strength. The weak-field approximation breaks down for fairly low field strengths, $B \gtrsim 400$ G (compare the dashed and dotted lines). Moreover, the deviations are very important for kilogauss magnetic field strengths. When the field strength is 1.5 kG, the weak-field approximation yields a Stokes $V$ signal twice as large as the synthetic signals in the one-dimensional model atmospheres (Fig. 3).

In order to test the dependence on $\cos \theta$ predicted by equation (3), we also carried out syntheses with various magnetic field inclinations. This dependence turns out to be closely followed by the synthetic signals, even for strong kilogauss magnetic field strengths. This behavior was to be expected, since Mn i λ5538 is a weak line satisfying the weakly polarizing medium approximation (Sánchez...
In this approximation, Stokes $V$ scales with its specific absorption coefficient, which is proportional to $\cos \theta$ independently of whether the spectral line has HFS.

The quiet Sun contains plasmas with all magnetic field strengths from 0 to 2 kG ($B = 100$ G). According to a mechanism originally put forward by Spruit (1976) and now well established, one expects a strong correlation between the magnetic field strength and the temperature of the observable photospheric layers. Plasmas having kilogauss fields must be strongly evacuated to stay in mechanical balance within the photosphere. Consequently, strongly magnetized plasmas are transparent, showing light coming from deep and therefore hot (sub-)photospheric layers. In order to explore the dependence of the Mn $I \lambda 5538$ Stokes $V$ signals on the thermodynamics, we compare synthetic profiles formed under different thermodynamic conditions but at the same magnetic field strength. The line is synthesized in a quiet-Sun model atmosphere (Maltby et al. 1986), a network model atmosphere (Solanki 1986; Solanki & Steenbock 1988), and a plage model atmosphere (Solanki 1986; Solanki & Steenbock 1988), with the temperature increasing from quiet Sun to network. The results are shown in Figure 4.

First note that the continuum intensity of Stokes $I$ increases from quiet Sun (the coolest model) to network (the hottest). In anticorrelation with continuum intensity, the Stokes $V$ signals decrease with increasing temperature (Fig. 4b). This behavior is not attributable to the HFS but is due to the ionization balance of Mn in the photosphere. Mn $I$ is the minor species, so that a small modification of the ionization equilibrium does not alter the Mn $II$ abundance, but it drastically changes the number of Mn $I$ atoms available for absorption. Increasing the temperature increases the ionization, and the Mn $I$ lines weaken. Figure 5 shows the relative abundance of Mn $I$ as a function of height in the atmosphere for a constant density representative of the layers where the lines are formed (say, 100 km above a continuum optical depth of 1). According to Figure 5, if the presence of a strong magnetic field decreases the formation height of a spectral line by 100 km, then the Mn $I$ is depleted by more than an order of magnitude (compare the values at 100 and 0 km). Note that this behavior is common to all Mn $I$ lines, and it is also typical of other minor species such as Fe $I$ (e.g., Fig. 5, dotted line). In short, the Mn $I$ lines are expected to weaken with increasing magnetic field strength.

The coupling between magnetic field strength and temperature is fully accounted for in the MISMA syntheses, which partly explains the dimming of the kilogauss Stokes $V$ signals shown in Figure 3 (solid line). Another part of the dimming is due to non-magnetic plasma obscuring the magnetic plasma, an effect specific to MISMAs and elaborated by Sánchez Almeida (2000).

4. MULTICOMPONENT SYNTHESIS OF Mn $I \lambda 5538$

The syntheses carried out so far assumed very simple magnetic configurations. The magnetic field is either constant or, in the case of model MISMAs, one magnetic component varies along
the LOS. Because of the complex topology of quiet-Sun magnetic fields, our limited angular resolution, and the spatial averages carried out to obtain appropriate signals, those simplifications are insufficient to represent observed Stokes profiles. Real polarization signals are formed in plasmas having a range of magnetic properties; therefore, we improve the realism of the synthesis by assuming multicomponent model atmospheres. Specifically, we assume

\[ I = \int_0^{B_{\text{max}}} P(B) I_B \, dB, \quad (4) \]

\[ V = \int_0^{B_{\text{max}}} (\cos \theta) P(B) V_B \, dB. \quad (5) \]

The quantities \( I_B \) and \( V_B \) represent the Stokes \( I \) and \( V \) profiles produced by an atmosphere with a single longitudinal magnetic field of strength \( B \), whereas \( P(B) \) stands for the fraction of a resolution element occupied by such atmosphere. \( P(B) \) is usually referred to as the magnetic field strength probability density function (PDF). It is normalized to 1 and can be envisaged as the filling factor corresponding to each \( B \). The maximum field strength existing in the quiet Sun sets \( B_{\text{max}} \) to some 2 kG. As we show in the Appendix, equations (4) and (5) hold even for an arbitrary distribution of magnetic field directions, provided the average inclination factor \( \langle \cos \theta \rangle \) is computed properly. We would like to make clear, however, that the PDF approach for representing the complications of quiet-Sun magnetic fields also has limitations. Equations (4)
and (5) imply a one-to-one relationship between the magnetic field and the thermodynamic structure of the atmosphere. Although a coupling between magnetic field and thermodynamics is to be expected, the real relationship should have a significant scatter (see, e.g., Vogler 2003). Magnetic field variations along the LOS are restricted, and the overlapping along the LOS of structures with different magnetic fields is ignored. Our PDF approach is only a first approximation to the problem.

The four PDFs in Figure 6 were used for synthesis. They correspond to one of the semiempirical quiet-Sun PDFs obtained by Domínguez Cerdeña et al. (2006a; labeled “DC”), plus the PDFs of the three magnetoconvection numerical simulations by Vogler (2003), representing magnetic fluxes of 10, 50, and 200 G (“V10,” “V50,” and “V200,” respectively).

When ME atmospheres, one-dimensional atmospheres, and model MISMAs are combined according to these PDFs, one obtains the Stokes $V$ profiles shown in Figure 7. In this case the magnetic field inclination is constant and set to $60^\circ$, that is, $\langle \cos \theta \rangle = \frac{1}{2}$.

The corresponding Stokes $I$ profiles for the ME and the one-dimensional cases are practically identical to those in Figure 1. The MISMA synthesis has a continuum intensity 5% lower than that of the quiet Sun, since the thermodynamic structure of the model MISMA used for synthesis is cooler than typical quiet-Sun model atmospheres (see Fig. 11 of Sánchez Almeida & Lites 2000).

**Fig. 8.—** Relative contribution to the average Stokes $V$ profile of a multicomponent atmosphere characterized by the PDF labeled DC in Fig. 6. The integral of the function represented in the plot is proportional to the Stokes $V$ signal at a particular wavelength. The three curves correspond to any wavelength under the weak-field approximation (solid line), the wavelength where the average Stokes $V$ signal is maximum (dashed line), and the wavelengths where the HFS hump shows up (dotted line). The units of the ordinates are relative, since the curves have been scaled to the solid line when $B \to 0$.

**Fig. 9.—** Least-squares fits of Stokes $V$ profiles synthesized in multicomponent atmospheres (solid lines) using single-component ME atmospheres (triangles): (a) synthesis in ME atmospheres; (b) synthesis in model MISMAs. The values for $\alpha_g$ and $B$ retrieved from the fits are included in the figures. The two synthetic profiles correspond to the same PDF (“DC”). The profiles are normalized to the quiet-Sun continuum intensity.
Note that the Stokes $V$ profiles in Figure 7 always show HFS humps. This result warns us against simplistic interpretations of the presence of HFS features as the unequivocal signature of an atmosphere dominated by hectogauss magnetic fields. Despite the fact that most of the atmospheric volume has very low field strengths, the magnetic flux and the magnetic energy of some of the PDFs used for synthesis are dominated by the tail of kilogauss fields. The extreme case corresponds to V200 in Figure 6, where 60% of the magnetic flux and 87% of the energy is in field strengths higher than 700 G, which is the field strength at which the HFS hump disappears according to the ME syntheses. Even in this case, the Stokes $V$ profiles present clear HFS humps (see Fig. 7, triple-dot–dashed lines). In order to understand this behavior one has to realize that the various magnetic field strengths existing in the atmosphere do not contribute to the observable signal in proportion to their magnetic flux, as would be expected if the weak magnetic field approximation were satisfied (eq. [3]).

The strong kilogauss fields are always underrepresented (§ 3), and this bias is particularly severe at line center, where the HFS hump shows up. The effect is shown in Figure 8, which contains the integrand of equation (5) for one of the PDFs in Figure 6 ("DC").

The integrand is given for three cases: (1) assuming that the weak-field approximation holds (solid line), (2) at the wavelength where the average Stokes $V$ is maximum (dashed line), and (3) at the line core (dotted line). The individual spectra have been synthesized in model MISMAs, but the qualitative behavior is common to all other model atmospheres. According to Figure 8, the contribution of the kilogauss fields to the HFS hump of the average Stokes $V$ signal is some 10 times smaller than that predicted by the weak-field approximation. The relative contribution of the various field strengths to the hump is worked out as part of § 4.1.

4.1. On the Diagnostic Provided by the HFS Hump

The observations of Mn I $\lambda$5538 analyzed in the literature employ single-component ME atmospheres (see § 1). The observed Stokes $V$ profiles are fitted with synthetic profiles to obtain mean magnetic field strengths. In order to explore the diagnostic content of this inversion technique when applied to complex magnetic atmospheres, we developed a simple least-squares minimization procedure to fit single-component ME profiles $V_B$ to the synthetic multicomponent $V$.

The procedure fits

$$V = \alpha_B V_B,$$

where the factor $\alpha_B$ and the field strength $B$ are the only two unknowns. Figure 9 shows the single-component ME fit of the multicomponent Stokes $V$ produced by the DC PDF. In the case that the syntheses are based on ME atmospheres (Fig. 9a), the match is excellent. If the synthesis is based on model MISMAs, the fit worsens a bit but the agreement is still fair (Fig. 9b). In both cases the field strength resulting from the ME inversion is about 550 G. We have unsuccessfully tried to link this average magnetic field strength with a particular property of the underlying PDF (Fig. 6, solid line). It is neither the unsigned flux ($\sim 150$ G) nor the most probable field strength ($\sim 13$ G). Figure 8 (solid line) shows $B\langle P(B) \rangle$, that is, the unsigned flux per unit of magnetic field strength. It peaks at 50 and 1700 G, which, again, do not seem to bear any obvious relationship with the average ME magnetic field strength ($\sim 550$ G).

In order to understand the relationship between the HFS hump and the magnetic fields existing in the multicomponent atmosphere, we adopt a strategy that avoids ME fits. The presence or absence of a hump is certainly associated with the balance between weak fields and strong fields, but it is unclear how the hump is related to the fraction of atmosphere filled by weak and strong fields. One can quantify the presence of the hump using the derivative of the Stokes $V$ profile at a wavelength $\lambda_c$ slightly blueward of the HFS maximum (i.e., centered in linear growth that ends up in the hump; see Fig 2a, where $\lambda_c \approx 0$). Then the sign of

$$\left. \frac{dV}{d\lambda} \right|_{\lambda_c}$$

indicates the presence ($>0$) or absence ($<0$) of a hump. According to equation (5),

$$\left. \frac{dV}{d\lambda} \right|_{\lambda_c} = \int_{0}^{B_{\text{max}}} (\cos \theta) \langle P(B) \rangle \frac{dV_B}{d\lambda} \mid_{\lambda_c} dB,$$

where $\theta$ indicates the angle between the DC magnetic field at $\lambda_c$ and the DC magnetic field at $\lambda_c'$. This criterion assumes positive magnetic polarity, implying a positive Stokes $V$ blue lobe. The same rule applies to negative polarity if one reverses the sense of the inequalities.

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5 Following Sánchez Almeida (2004) and Domínguez Cerdeña et al. (2006a), the unsigned magnetic flux and the magnetic energy are computed as the two first moments of the PDF.
so the slope of the mean Stokes $V$ is just a weighted mean of the slopes that characterize each field strength. Consequently, $dV_B/d\lambda_c$ yields the contribution of each field strength to the sign of the mean profile, or in other words, it yields the contribution of each field strength to the presence or absence of a hump in the average profile.

We have applied equation (8) to understand the HFS humps of the synthetic profiles in Figure 7. Figure 10a shows the Stokes $V$ derivative at line core ($\lambda_c = 0$) for the ME, the one-dimensional, and the MISMA model atmospheres. The magnetic field strength at which this derivative is zero indicates when $V_B$ changes from having a hump to lacking one. As the figure shows, this transition field strength depends on the model atmosphere. The positive and negative lobes of the curves in Figure 10a have similar amplitudes, which indicates that for the average Stokes profile to lack a hump, the filling factor of the kilogauss fields in the atmosphere has to be similar to the filling factor in weak fields. The requirement is not fulfilled by the PDFs in Figure 6, whose filling factors are dominated by weak fields. Such superiority explains why all the profiles in Figure 7 show HFS humps.

Equation (8) also explains why the magnetic flux in the form of kilogauss structures has to exceed by far the flux in weak fields for the HFS hump to disappear. The HFS hump goes away when weak fields and strong fields have similar filling factors, but then the kilogauss fields dominate the flux, since the (unsigned) magnetic flux scales as the filling factor times the field strength. For example, consider a resolution element filled half-and-half with structures of 100 G and 1 kG. The HFS hump is not present, but the magnetic flux in kilogauss fields is 10 times the weak-field flux.

Among the three curves in Figure 10a, the one corresponding to MISMAAs reflects the solar conditions best, since it includes the dimming of the line with increasing field strength. This curve has positive and negative lobes of approximately the same area, and therefore, the presence or absence of HFS humps in the Mn i $\lambda 5538$ Stokes $V$ profiles is controlled by the fraction of photospheric atmosphere with field strength smaller or larger than $\sim 1$ kG, respectively (i.e., the zero-crossing point of the curve).

Interpreting real observations with this rule of thumb requires disregarding the atmospheric volume with very weak magnetic fields, which produce no polarization and therefore do not contribute to the observed signal (see Fig. 10b).

5. MIXED POLARITIES AND UNRESOLVED VELOCITIES

We expect the quiet-Sun resolution elements to contain mixed polarities (Sánchez Almeida & Lites 2000; Socas-Navarro & Sánchez Almeida 2002), which leads to a reduction of the Stokes $V$ signal. In the parlance of the multicomponent approximation (eq. [3]), having unresolved mixed polarities implies

$$\langle \cos \theta \rangle \ll \langle |\cos \theta| \rangle,$$

Fig. 11.— Examples of Stokes $V$ profiles synthesized in semiempirical model MISMAAs having unresolved mixed polarities correlated with velocity. They correspond to different classes from Sánchez Almeida & Lites (2000): (a) class 6, (b) class 9, and (c) class 7. In (d) it is shown an offspring of the class 7 model atmosphere in which the two polarities have the same sign and the velocity difference has been increased. The profiles are normalized to the quiet-Sun continuum intensity.
since positive and negative $\cos \theta$ tend to cancel when carrying out the averages. If the positive polarity ($\cos \theta > 0$) and the negative polarity ($\cos \theta < 0$) have the same distribution of inclinations but they fill different fractions of a resolution element, then

$$\langle \cos \theta \rangle = \langle |\cos \theta| \rangle (2f - 1),$$

(10)

with $f$ the fraction filled by the magnetic fields of positive polarity. If $f$ does not depend on the magnetic field strength, the effect of mixed polarities is only a global scaling of the Stokes $V$ profile, with no effect on Stokes $I$ (eqs. [4], [5], and [10]). However, the above view of the effect produced by unresolved mixed polarities is too simplistic. It implicitly neglects the existence of unresolved velocities in the resolution element coupled with changes of polarity. When the sense of the velocity and the polarity are correlated, the Stokes $V$ profiles of opposite polarities are Doppler shifted. They do not perfectly cancel, leaving observable residuals. Such coupling has been observed in Fe lines, and it produces very asymmetric Stokes $V$ profiles (Sánchez Almeida & Lites 2000). In order to illustrate the effect, Figure 11 shows Stokes $V$ profiles of Mn $\text{i}\lambda5538$ synthesized in empirical model MISMA with mixed polarities and different velocities (Sánchez Almeida & Lites 2000).

Velocities and other magnetic quantities are also expected to be coupled. For example, strong kilogauss fields appear in intergranular lanes with downflows, whereas weak fields prefer granules with upflows (see, e.g., Cattaneo 1999; Socas-Navarro et al. 2004). This combination of Doppler shifts and magnetic field strengths renders profiles like the two-component average shown in Figure 12b (solid line). It corresponds to the sum of the profiles in Figure 12a, which are shifted by some 3.5 km s$^{-1}$. The HFS hump now appears elevated from the zero level and the Stokes $V$ blue wing is broadened, with a conspicuous double absorption structure. These two features are common among the observed Stokes $V$ profiles of Mn $\text{i}\lambda5538$ (e.g., Figs. 8 and 9 of López Ariste et al. 2006). The same features appear in the MISMA synthesis of Figure 11d.

In short, mixed polarities and velocity gradients are properties of real quiet-Sun magnetic fields that affect the observed polarization in a significant way. They are not easy to deal with but must be taken into account for a realistic interpretation of the observations.

6. SYNTHESIS OF OTHER Mn $\text{i}$ LINES

The purpose of synthesizing Mn $\text{i}$ lines other than Mn $\text{i}\lambda5538$ is threefold. First, it confirms that our LTE syntheses reproduce all kinds of observed HFS patterns. Second, it offers a chance to
explore whether other Mn i lines show a sensitivity to magnetic fields complementary to that of Mn i λ5538. Finally, it proves that the uncertainty in the HFS constants does not limit the use of Mn i lines for magnetometry.

All the syntheses in this section employ the nonmagnetic quiet-Sun MISMA that reproduces the Mn i λ5538 profile in Figure 1. Varying the macroturbulence, and in some cases also tuning the oscillator strength, we managed to fit the observed Stokes I profiles of all the lines we tried. A total of eight Mn i lines reproduced by the code are shown in Figures 13 and 14.

The dependence of the various lines on magnetic field strengths has been studied by synthesizing them in the magnetic quiet-Sun model MISMA used in § 4, increasing the field strength from ~100 to ~2000 G. All the lines share a common behavior—they weaken with increasing field strength, and the HFS features tend to disappear. This behavior is also shared by the near-IR line Mn i λ5516, recently pointed out by Asensio Ramos et al. (2007) (Fig. 14, bottom right). On top of this general trend, and depending on the actual HFS constants, the Stokes V profiles of different lines present specific features. The line Mn i λ5516 behaves like Mn i λ5538, with a single HFS reversal at the line core. It is thus possible to relate the presence or absence of the hump to the distribution of field strengths in the atmosphere, exactly as the analysis carried out in § 4.1. The patterns of Mn i λ5420 and λ8741 are more complicated, and we have not been able to agree on a particular feature of the Stokes V profiles that can be treated as in § 4.1.

7. DISCUSSION AND CONCLUSIONS

The Zeeman pattern of the Mn i lines depends on hyperfine structure, which confers upon them a sensitivity to hectogauss magnetic field strengths different from that of the lines traditionally used in solar magnetometry. This peculiarity has been used to measure magnetic field strengths in quiet-Sun regions (see § 1). The methods applied so far assume the magnetic field strength to be constant in the resolution element, an approximation driven by feasibility rather than based on physical or observational arguments. Actually, it is not a good approximation, since the
magnetic fields of the quiet Sun are expected to vary on very small spatial scales, with field strengths spanning from 0 to 2 kG. Under these extreme conditions, all diagnostic techniques employed in quiet-Sun magnetometry are strongly biased toward a particular range of field strengths, and the Mn\textsuperscript{i} signals are not expected to be the exception. Consequently, the diagnostic content of the Mn\textsuperscript{i} lines cannot be fully exploited unless their biases are properly understood. This task was undertaken in this paper by exploring the response of Mn\textsuperscript{i} lines in a number of realistic quiet-Sun scenarios.

Three complementary LTE synthesis codes have been written and tested (ME, one-dimensional, and MISMA; § 2). They provide a number of relevant results, the first being the ability to reproduce all observed Mn\textsuperscript{i} HFS patterns. We reproduce the observed unpolarized line profile of nine assorted lines, corresponding to all HFS sensitivities (§ 6). This study is focused on the line most often used in magnetometry, Mn\textsuperscript{i} \lambda 5538; however, its behavior should be representative of the other lines. According to the weak magnetic field approximation, the Stokes V signals scale with the longitudinal component of the magnetic field—that is, the magnetic field strength times the cosine of the magnetic field inclination with respect to the LOS. We verify that the scaling with cosine holds very tightly. The scaling with the magnetic field strength, however, breaks down quickly (when $B \geq 400$ G for Mn\textsuperscript{i} \lambda 5538). Even for ME atmospheres, the weak-field approximation predicts a Stokes V signal twice as large as the synthetic one for $B \approx 1.5$ kG. When the expected coupling between the thermo-dynamic conditions and the magnetic field strengths is taken into account, the dimming of the kilogauss Stokes V signals can be as large as 2 orders of magnitude (see Figs. 3 and 8). The dimming of the polarization signals formed in kilogauss magnetic concentrations affects all Mn\textsuperscript{i} lines (§ 3), and it has significant observational implications.

If the resolution elements contains both weak and strong fields, then the kilogauss fields tend to be underrepresented in the average profile. We have modeled the bias assuming a multi-component atmosphere, where the synthetic signals are weighted means of the Stokes profiles corresponding to each single field strength. The weight is given by the fraction of the atmosphere filled by each field strength, that is, by the magnetic field strength probability density function (eqs. [4] and [5]). According to our modeling, even when the (unsigned) magnetic flux and the magnetic energy are dominated by kilogauss magnetic fields, the Stokes V profile of Mn\textsuperscript{i} \lambda 5538 can show an HFS reversal at line core characteristic of hectogauss fields (Fig. 7). A purely morphological inspection of the Stokes profiles does not suffice to infer which is the dominant magnetic field strength in the resolution element. For the HFS hump of Mn\textsuperscript{i} \lambda 5538 to disappear, the kilogauss filling factor has to be larger than the subkilogauss filling factor, and consequently, when the HFS hump disappears the magnetic flux and magnetic energy of the atmosphere are completely dominated by kilogauss fields (§ 4.1).

Detecting Stokes V profiles with HFS features indicates the presence of hectogauss fields in the resolution element. However,
this observation alone does not tell one whether the hectogauss field strengths dominate. There seem to be two extreme alternatives to exploit the diagnostic potential of these lines. The first is to improve the spatial resolution of the observations to a point where the quiet-Sun magnetic structures can be regarded as spatially resolved. Unfortunately, this possibility does not seem feasible at present. Realistic simulations of magnetoconvection indicate that quiet-Sun magnetic fields are uniform only at the diffusive length scales (e.g., Cattaneo 1999; Vögler & Schüssler 2007), which are on the order of a few kilometers in the photosphere (e.g., Schüssler 1986). These scales are very far from the angular resolution of current measurements and even much smaller than the length scale for the radiative transfer average along the LOS (Sánchez Almeida et al. 1996). We prefer the alternative possibility, namely, developing inversion techniques in which complex magnetic atmospheres are included in the diagnostics. Using appropriate constraints, the number of free parameters required to describe such atmospheres can be maintained within reasonable limits (e.g., the model MISMAs in Sánchez Almeida 1997). Dealing with unresolved velocities also favors detailed inversion codes (§ 5). We are presently working on these improvements needed to develop the diagnostic technique pioneered by López Ariste and coworkers.

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APPENDIX

MULTICOMPONENT ATMOSPHERE WITH MAGNETIC FIELD VECTORS VARYING IN DIRECTION

The equations used for the multicomponent syntheses (eqs. [4]–[5]) hold even when the magnetic field vector presents a distribution of magnetic field inclinations θ and azimuths φ. In the general case, the PDF depends on B, θ, and φ, \(P(B, \theta, \phi)\), so that the average Stokes \(I\) and Stokes \(V\) profiles are

\[
I = \int_0^{2\pi} \int_0^{\pi} \int_0^{B_{\text{max}}} P(B, \theta, \phi) S_I(B, \theta, \phi) dB d\theta d\phi, \tag{A1}
\]

\[
V = \int_0^{2\pi} \int_0^{\pi} \int_0^{B_{\text{max}}} P(B, \theta, \phi) S_V(B, \theta, \phi) dB d\theta d\phi, \tag{A2}
\]

with \(P(B, \theta, \phi)\) normalized to 1. The quantities \(S_I(B, \theta, \phi)\) and \(S_V(B, \theta, \phi)\) denote the Stokes \(I\) and \(V\) profiles corresponding to each magnetic field vector. As the tests carried out in § 3 show, and as is to be expected from the weakly polarizing medium approximation (Sánchez Almeida & Trujillo Bueno 1999), the intensity of the weak Mn i lines is independent of inclination and azimuth, so that \(S_I(B, \theta, \phi) \approx I_B\). Similarly, Stokes \(V\) is independent of \(\phi\) and depends on \(\theta\) through the factor \(\cos \theta\); \(S_V(B, \theta, \phi) = V_B \cos \theta\). Here \(V_B\) represents the Stokes \(V\) profile for a longitudinal magnetic field (\(\cos \theta = 1\)). Then, from equations (A1) and (A2) one retrieves equations (4) and (5) in the main text,

\[
I = \int_0^{B_{\text{max}}} P(B) I_B dB, \quad V = \int_0^{B_{\text{max}}} (\cos \theta) P(B) V_B dB, \tag{A3}
\]

with

\[
P(B) = \int_0^{2\pi} \int_0^{\pi} P(B, \theta, \phi) d\theta d\phi, \quad (\cos \theta) = \frac{1}{P(B)} \int_0^{2\pi} \int_0^{\pi} \cos \theta P(B, \theta, \phi) d\theta d\phi. \tag{A4}
\]

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