Entanglement, intractability and no-signaling

R Srikanth
Poornaprajna Institute of Scientific Research, Sadashivnagar, Bangalore-560080, India
Raman Research Institute, Sadashivnagar, Bangalore-560080, India
E-mail: srik@ppisr.res.in

Received 21 September 2009
Accepted for publication 9 April 2010
Published 11 May 2010
Online at stacks.iop.org/PhysScr/81/065002

Abstract
We consider the problem of deriving the no-signaling condition from the assumption that, as seen from a complexity theoretic perspective, the universe is not an exponential place. A fact that disallows such a derivation is the existence of polynomial superluminal gates, hypothetical primitive operations that enable superluminal signaling but not the efficient solution of intractable problems. It therefore follows, if this assumption is a basic principle of physics, either that it must be supplemented with additional assumptions to prohibit such gates or, improbably, that no-signaling is not a universal condition. Yet a gate of this kind is possibly implicit, although not recognized as such, in a decade-old quantum optical experiment involving position–momentum entangled photons. Here, we describe a feasible modified version of the experiment that appears to explicitly demonstrate the action of this gate. Some obvious counter-claims are shown to be invalid. We believe that the unexpected possibility of polynomial superluminal operations arises because some practically measured quantum optical quantities are not describable as standard quantum mechanical observables.

PACS numbers: 03.67.–a, 03.65.Ud, 03.65.Ta, 03.30.+p

1. Introduction
In a multipartite quantum system, any completely positive (CP) map applied locally to one part does not affect the reduced density operator of the remaining part. This fundamental no-go result, called the ‘no-signaling theorem’, implies that quantum entanglement [1] does not enable nonlocal (‘superluminal’) signaling [2] under standard operations and is thus consistent with relativity, in spite of the counterintuitive, stronger-than-classical correlations [3] that entanglement enables. For simple systems, no-signaling follows from non-contextuality, the property that the probability assigned to projector \( \Pi_1 \), given by the Born rule, \( \text{Tr}(\rho \Pi_1) \), where \( \rho \) is the density operator, does not depend on how the orthonormal basis set is completed [4, 5]. No-signaling has also been treated as a basic postulate to derive quantum theory [6].

It is of interest to consider the question of whether/how computation theory, in particular intractability and uncomputability, matters to the foundations of (quantum) physics. Such a study, if successful, could potentially allow us to reduce the laws of physics to mathematical theorems about algorithms and thus shed new light on certain conceptual issues. For example, it could explain why stronger-than-quantum correlations that are compatible with no-signaling [7] are disallowed in quantum mechanics (QM). One strand of thought leading to the present work, earlier considered by us in [8], was the proposition that the measurement problem is a consequence of basic algorithmic limitations imposed on the computational power that can be supported by physical laws. In the present work, we would like to see whether no-signaling can also be explained in a similar way, starting from computation theoretic assumptions.

The central problem in computer science is the conjecture that two computational complexity classes, \( \mathbf{P} \) and \( \mathbf{NP} \), are distinct in the standard Turing model of computation. \( \mathbf{P} \) is the class of decision problems solvable in polynomial time by a (deterministic) TM. \( \mathbf{NP} \) is the class of decision problems whose solution(s) can be verified in polynomial time by a deterministic TM. \( \mathbf{NP} \) is the class of counting problems associated with the decision problems in \( \mathbf{NP} \). The word ‘complete’ following a class denotes a problem \( \mathcal{X} \) within the class, which is maximally hard in the sense that any other problem in the class can be solved in poly-time.
using an oracle giving the solutions of \( X \) in a single clock cycle. For example, determining whether a Boolean formula is satisfied is \( \text{NP} \)-complete, and counting the number of Boolean satisfactions is \( \#\text{P} \)-complete. The word ‘hard’ following a class denotes a problem not necessarily in the class, but to which all problems in the class reduce in poly-time.

\( \text{P} \) is often taken to be the class of computational problems that are ‘efficiently solvable’ (i.e. solvable in polynomial time) or ‘tractable’, although there are potentially larger classes that are considered tractable such as \( \text{RP} \) and \( \text{BQP} \), the latter being the class of decision problems efficiently solvable by a quantum computer (see footnote 1).

\( \text{NP} \)-complete and potentially harder problems, which are not known to be efficiently solvable, are considered intractable in the Turing model. If \( \text{P} \neq \text{NP} \) and the universe is a polynomial—rather than an exponential—place, physical laws cannot be harnessed to efficiently solve intractable problems, and \( \text{NP} \)-complete problems will be intractable in the physical world.

That classical physics supports various implementations of the Turing machine is well known. More generally, we expect that computational models supported by a physical theory will be limited by that theory. Witten [10] identified expectation values in a topological quantum field theory (QFT) with values of the Jones polynomial that are \#\( \text{P} \)-hard. There is evidence that a physical system with a non-Abelian topological term in its Lagrangian may have observables that are \( \text{NP} \)-hard or even \#\( \text{P} \)-hard [11].

Other recent related works that have studied the computational power of variants of standard physical theories from a complexity or computability perspective are, respectively, [8, 12–15] and [8, 12]. Aaronson [14] noted that \( \text{NP} \)-complete problems do not seem to be tractable using resources of the physical universe and suggested that this might embody a fundamental principle, christened the \( \text{NP} \)-hardness assumption (also cf [16]). Gruska [17] studied how insights from quantum information theory could be used to constrain physical laws. We will informally refer to the proposition that the universe is a polynomial place in the computational sense (to be strengthened below) as well as in the communication sense by the expression ‘the world is not hard enough’ (WNHE).

Recently, Bennett et al [18] posed the question whether nonlinear quantum evolution can be considered as providing any help in solving otherwise hard problems, on the grounds that under nonlinear evolution, the output of such a computer on a mixture of inputs is not a convex combination of its output on the pure components of the mixture. We circumvent this problem here by adopting the standpoint of information realism, the position that physical states are ultimately information states registered in some way sub-physically but objectively by Nature. At this stage, we will not worry about the details except to note an implication for the present situation, which is that from the perspective of ‘Nature’s eye’, there are no mixed states. Therefore, in describing nonlinear physical laws or specifying the working of non-standard computers based on such laws, it suffices for our purpose to specify their action on (all relevant) pure state inputs.

In [8], we pointed out that the assumption of WNHE (and further that of \( \text{P} \neq \text{NP} \)) can potentially give a unified explanation of (a) the observed ‘insularity-in-theory-space’ of QM, namely that QM is exactly unitary and linear, and requires measurements to conform to the \(|\psi|^{2} \) Born rule [13, 19]; (b) the classicality of the macroscopic world; and (c) the lack of quantum physical mechanisms for non-signaling superquantum correlations [7].

In (a), the basic idea is that departure from one or more of these standard features of QM seems to invest quantum computers with super-Turing power to solve hard problems efficiently, thus making the universe an exponential place, contrary to assumption. The possibility (b) arises for the following reason. It is proposed that the WNHE assumption holds not only in the sense that hard problems (in the standard Turing model) are not efficiently solvable in the physical world, but in the stronger sense that any physical computation can be simulated on a probabilistic TM with at most a polynomial slowdown in the number of steps (the strong Church–Turing thesis). Therefore, the evolution of any quantum system computing a decision problem could asymptotically be simulated in polynomial time in the size of the problem and thus lies in \( \text{BPP} \), the class of problems that can be efficiently solved by a probabilistic TM.

Assuming that \( \text{BPP} \neq \text{BQP} \), this suggests that although at small scales standard QM remains valid with characteristic BQP-like behavior, at sufficiently large scales classical (‘BQP-like’) behavior should emerge. and that therefore there must be a definite scale—sometimes called the Heisenberg cut—where the superposition principle breaks down [20], so that asymptotically, quantum states are not exponentially long vectors. In [8], we speculate that this scale is related to a discretization of Hilbert space. This approach provides a possible computation theoretic resolution to the quantum measurement problem. In (c), the idea is that in a polynomial universe, we expect that phenomena in which a polynomial amount of physical bits can simulate exponentially large (classical) correlations, thereby making communication complexity trivial, would be forbidden.

In the present work, we are interested in studying whether the no-signaling theorem follows from the WNHE assumption. The paper is divided into two parts: part I, concerned with the computer scientific aspects, giving a complexity theoretic motivation for the work; part II, concerned with the quantum optical implementation of a test suggested by part I.

In part I, firstly, some results concerning non-standard operations that violate no-signaling and help efficiently solve

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1 In complexity theory, \( \text{RP} \) is the class of decision problems for which there exists a probabilistic TM (a deterministic TM with access to genuine randomness) such that: it runs in polynomial time in the input size. If the answer is ‘no’, it returns ‘no’. If the answer is ‘yes’, it returns ‘yes’ with a probability of at least 0.5 (else it returns ‘no’). \( \text{BQP} \) is the class of decision problems solvable by a quantum TM [9] in polynomial time, with an error probability of at most 1/3 (or, equivalently, any other fixed fraction smaller than 1/2) independently of input size.

2 That is, ‘the universe is not hard enough to not be simulable using polynomial resources’. The expression is non-technically related to the statement ‘The world is not enough’ (‘orbsis non sufficit’), the family motto of, as well as a motion picture featuring, a well-known Anglo-Scottish secret agent!

3 More formally, \( \text{BPP} \) is the class of decision problems solvable by a probabilistic TM in polynomial time, with an error probability of at most 1/3 (or, equivalently, any other fixed fraction smaller than 1/2) independently of input size.
intractable problems are surveyed, in sections 2.1 and 2.2, respectively. Then, in section 2.3, we introduce the concept of a polynomial superluminal gate, a hypothetical primitive operation that is prohibited by the assumption of no-signaling, but allowed if instead we only assume that intractable problems should not be efficiently solvable by physical computers. We examine the relation between the above two classes of non-standard gates. We also describe a constant gate on a single qubit or qutrit, possibly the simplest instance of a polynomial superluminal operation.

In part II, firstly, we present a quantum optical realization of the constant gate and its application to an experiment involving entangled light generated by parametric downconversion in a nonlinear crystal in section 3.1. Physicists who could not care less about computational complexity aspects could skip directly to this section. They may be warned that the intervening sections of part I will involve mangling QM in ways that may seem awkward, and whose consistency is, unfortunately, not obvious! On the other hand, computer scientists unfamiliar with quantum optics may skip section 3.1, which is essentially covered in section 3.2, which discusses quantitative and conceptual issues surrounding the physical realization of the constant gate. Finally, we conclude with section 4 by surveying some implications of a possible positive outcome of the proposed experiment and discussing how such an unexpected physical effect may fit in with the mathematical structure of known physics. We present a slightly abridged version of discussions in this work in [21].

2. Part I: computation theoretic motivation

2.1. Superluminal gates

Even minor variants of QM are known to lead to superluminal signaling. An example is a variant incorporating nonlinear evolution. Given the entangled state $|\psi\rangle$, unless the nonlinearity is confined to immediately before a measurement, the distance between the states being greater for larger $m$ (cf section 2.2), leading to a superluminal signal from Alice to Bob.

More generally, we may allow non-unitary and irreversible evolution but still conform to no-signaling, provided the corresponding set of operator(s) is complete, i.e., constitutes a partition of unity. Suppose that Alice and Bob share the state $\rho_{AB}$, and Alice evolves her part of $\rho_{AB}$ locally through the linear operation given by the set $P$ of (Kraus) operator elements $\{E_j \equiv e_j \otimes I_B, j = 1, 2, 3, \ldots\}$ [9], where $I_B$ is the identity operator in Bob’s subspace. Bob’s reduced density operator $\rho_B'$ conditioned on her performing the operation and after normalization is

$$\rho_B' = N^{-1} Tr_A \left[ \sum_j E_j \rho_{AB} E_j^\dagger \right] = N^{-1} Tr_A \left[ \sum_j E_j^\dagger E_j \rho_{AB} \right],$$

where $N$ is the normalization factor. We satisfy the no-signaling condition $\rho_B' = \rho_B$ only if $\rho_{AB}$ is unentangled or $P$ satisfies the completeness relation

$$\sum_j e_j^\dagger e_j = I_A,$$

which guarantees that the operation preserves norm $N$. Here $I_A$ is the identity operator in Alice’s subspace. If the norm is not preserved, renormalization is required, making the evolution effectively nonlinear. If system A is subjected to unitary evolution or non-unitary evolution due to noise, or to standard projective measurements or more general measurements described by positive operator valued measures, the corresponding map satisfies equation (3) and $\rho_B' = \rho_B$. For terminological brevity, we call a (non-standard) gate like $G$, or a non-complete operation $P$ that enables superluminal signaling, a ‘superluminal gate’, and denote the set of all superluminal gates by $‘C’$. For the purpose of this work, $‘C’$ is restricted to qubit or qutrit gates. Non-unitary superquatum cloning and deleting, introduced in [28], which lead to superluminal signaling, are other examples of non-complete operations.

Even at the single-particle level, if the measurement is non-complete, there is a superluminal signaling due to breakdown in non-contextuality coming from the renormalization. As a simple illustration, suppose that we are given two observers Alice and Bob sharing a delocalized qubit, $\cos(\theta/2)\ket{0} + \sin(\theta/2)\ket{1}$, with eigenstate $\ket{1}$ localized near Alice and $\ket{0}$ near Bob. With an $m$-fold application of $G$ (which can be thought of as an application of imaginary phase on Alice’s side, leading to selective augmentation of amplitude) on this state, Alice produces the

$$G = \begin{pmatrix} 1 & 0 \\ 0 & 1 + \epsilon \end{pmatrix},$$

where $\epsilon > 0$ is a real number. The resultant state $\sum_x |\alpha_x| \ket{x}$ must be normalized by dividing it by the normalization factor $\sqrt{\sum_x |\alpha_x|^2}$ immediately before a measurement, making measurements nonlinear. Given the entangled state $(1/\sqrt{2})(|01\rangle + |10\rangle)$ that Alice and Bob share, to transmit a superluminal signal, Alice applies either $G^m$ (where $m \geq 1$ is an integer) or the identity operation $I$ to her qubit. Bob’s particle is left, respectively, in the state $\rho_B^{(1)} \simeq 1/2(|0\rangle \langle 0| + (1 + \epsilon)2|1\rangle \langle 1|) + \rho_B^{(0)} = 1/2(|0\rangle \langle 0| + |1\rangle \langle 1|)$, which can in principle be distinguished, the distance between the states being greater for larger $m$ (cf section 2.2), leading to a superluminal signal from Alice to Bob.
2.1.2. Nonlinear evolution. As a simple illustration of a superluminal gate arising from nonlinear evolution, we consider the action of the nonlinear two-qubit ‘OR’ gate $R$, whose action in a preferred (say, computational) basis is given by

$$
\begin{align*}
|00\rangle & \mapsto |11\rangle \\
|01\rangle & \mapsto |10\rangle \\
|11\rangle & \mapsto |01\rangle,
\end{align*}
$$

(4)

If the two qubits are entangled with other qubits, then the gate is assumed to act in each subspace labeled by states of the other qubits in the computational basis. Alice and Bob share the entangled state $|\Psi\rangle = 2^{-1/2}(|00\rangle - |11\rangle)$. To transmit a bit superluminally Alice measures her qubit in the computational basis or the diagonal basis $|\pm\rangle \equiv 2^{-1/2}(|0\rangle \pm |1\rangle)$, leaving Bob’s qubit’s density operator in a computational basis ensemble or a diagonal basis ensemble, which are equivalent in standard QM. However, with the nonlinear operation $R$, the two ensembles can be distinguished. Bob prepares an ancillary qubit in the state $|\psi\rangle$, and applies a CNOT on it, with his system qubit as the control. On the resulting state he performs the nonlinear gate $R$ and measures the ancilla. The computational (resp., diagonal) basis ensemble yields the value 1 with probability $\frac{1}{2}$ (resp., 1). By a repetition of the procedure a fixed number $m$ of times, a superluminal signal is transmitted from Alice to Bob with exponentially small uncertainty in $m$. Analogous to equation (4), one can define a ‘nonlinear AND’, which, again, similarly leads to a non-local signaling.

2.1.3. Departure from the Born $|\psi|^2$ probability rule. Gleason’s theorem shows that the Born probability rule that identifies $|\psi|^2$ as a probability measure, and more generally, the trace rule, is the only probability prescription consistent in three or larger dimensions with the requirement of non-contextuality [4]. Suppose we retain unitary evolution, which preserves the two-norm, but assume that the probability of a measurement on the state $\sum_j \alpha_j |j\rangle$ is of the form $|\alpha_j|^p/\sum_k |\alpha_k|^p$ for outcome $j$, and $p$ is any non-negative real number. The renormalization will make the measurement contextual, giving rise to a superluminal signal. One might consider more general evolution that preserves a $p$-norm, but there are no linear operators that do so except permutation matrices [13].

For example, let Alice and Bob share the two-qubit entangled state $\cos \theta |00\rangle + \sin \theta |11\rangle$ ($0 < \theta < \pi/2$). The probability of Alice measuring her particle in the computational basis and finding $|0\rangle$ (resp., $|1\rangle$) must be the same as that for a joint measurement in this basis to yield $|00\rangle$ (resp., $|11\rangle$). Therefore Bob’s reduced density operator is given by the state $\rho^{(1)} = (\cos^p \theta |0\rangle \langle 0| + \sin^p \theta |1\rangle \langle 1|)/\sqrt{2^{p-1}(\cos^p \theta + \sin^p \theta)}$. On the other hand, if Alice employs an ancillary, third qubit prepared in the state $|0\rangle$ and applies a Hadamard on it conditioned on her qubit being in the state $|0\rangle$, she produces the state $\cos^p \theta |000\rangle + \sin^p \theta |001\rangle + \sin \theta |110\rangle$. The probability that Alice obtains outcomes 00, 01, or 10 must be that for a joint measurement to yield 000, 001, or 110. Along similar lines as in the above case, we find that she leaves Bob’s qubit in the state

$$
\rho^{(2)} = \frac{2^{(1-p/2)} \cos^p \theta |0\rangle \langle 0| + \sin^p \theta |1\rangle \langle 1|}{2^{(1-p/2)} \cos^p \theta + \sin^p \theta}.
$$

(5)

Since $\rho^{(1)}$ and $\rho^{(2)}$ are probabilistically distinguishable, with sufficiently many shared copies Alice can signal Bob one bit superluminally, unless $p = 2$.

2.2. Exponential gates

As superluminal quantum gates like $G$ or $R$ are internally consistent, one can consider why no such operation occurs in Nature, whether a fundamental principle prevents their physical realization. One candidate principle is of course no-signaling itself. Alternatively, since we would like to derive it, linearity of QM may be taken as an axiom. Since all the above non-standard operations involve an overall nonlinear evolution, the assumption of strict quantum mechanical linearity can indeed rule out such non-standard gates. Yet it must be admitted that, from a purely physics viewpoint, assuming that QM is linear affords no greater insight than assuming it to be a non-signaling theory. We would like to suggest that the absence of such operations may have a complexity theoretic basis.

Both superluminal gates and hypothetical gates that allow efficient solving of intractable problems involve some sort of communication across superposition branches. In particular, the superluminal gates of section 2.1 can be turned into the latter type of gates, as discussed below.

2.2.1. Non-complete quantum gates. It can be easily seen that the gate $G$ in equation (1) can be used to solve NP-complete problems efficiently. Consider solving Boolean satisfiability (SAT), which is NP-complete: given an efficiently computable black box function $f : \{0, 1\}^n \rightarrow \{0, 1\}$, to determine if there exists $x$ such that $f(x) = 1$. With the use of an oracle that computes $f(x)$, we prepare the $(n+1)$-qubit entangled state

$$
|\Psi_{\text{eq}}\rangle = 2^{-n/2} \sum_{x \in {0, 1}^n} |x\rangle |f(x)\rangle
$$

and then apply $G^m$ to the second, one-qubit register, where $m$ is a sufficiently large integer, before measuring the register. In particular, suppose that at most one solution exists. The unnormalized ‘probability mass’ of obtaining outcome 1 becomes 1 (and the normalized probability about 1/2) when $m = n/2 \log(1+e)$ if there is a solution or remains 0 if no solution exists. Repeating the experiment a fixed number of times and applying the Chernoff bound, we find that to solve SAT, we only require $m \in O(n)$. For terminological
brevity, we will call an ‘exponential gate’ such a non-standard gate that enables the efficient computation of \( \text{NP} \)-complete problems and denote by \( E \) the set of all exponential gates, restricted in the present work to qubits and qutrit gates.

2.2.2. Nonlinear quantum gates. The nonlinear operation \( R \) in equation (4) can be used to efficiently simulate non-determinism. We prepare the state \( |\psi\rangle \) in equation (6), where the first \( n \) qubits are called the ‘index’ qubits and the last one the ‘flag’ qubit. There are \( 2^{n-1} \) 4-dim subspaces, consisting of the first index qubit and the flag qubit, labeled by the index qubits \( 2, \ldots, n \). On each such subspace, the first index qubit and flag qubit are in one of the states \( |00\rangle + |11\rangle \), \( |01\rangle + |10\rangle \), \( |00\rangle + |10\rangle \). The operation equation (4) is applied \( n \) times, pairing each index qubit sequentially with the flag. The number of terms with 1 on the flag bit doubles with each operation so that after \( n \) operations, it becomes disentangled and can then be read off to obtain the answer [15]. A slight modification of this algorithm solves \#P-complete problems efficiently, by replacing the flag qubit with \( \log n \) qubits and the 1-bit nonlinear OR operation with the corresponding nonlinear counting. The final readout is then the number of solutions to \( f(x) = 1 \) [15]. Applying the nonlinear OR and AND alternately to the state \( |\psi\rangle \) in equation (6) allows one to efficiently solve the quantified Boolean formula problem, which is \( \text{PSPACE} \)-complete.

2.2.3. Non-Gleasonian gates. By employing polynomially many ancillas in the method of (2.1.3) in the previous subsection, one can show that non-Gleasonian quantum computers (for which \( p \neq 2 \)) can solve \( \text{PP} \)-complete problems efficiently. Defining \( \text{BQP}_p \) as similar to \( \text{BQP} \), except that the probability of measuring a basis state \( |x\rangle \) equals \( \alpha_x^p / \sum_y \alpha_y^p \) (so that \( \text{BQP}_2 = \text{BQP} \)), it can be shown that \( \text{PP} \subseteq \text{BQP}_p \) for all constants \( p \neq 2 \) and that, in particular, \( \text{PP} \) exactly characterizes the power of a quantum computer with even-valued \( p \) (except \( p = 2 \)) [13].

In view of the connection between the two classes of gates, we now propose, as we earlier did in [8], that the reason for the absence in Nature of the superluminal gates of section 2.1 is WNHE: in a universe that is a polynomial place, exponential gates like \( G \) and \( R \) are ruled out. In the next section, we will consider in further detail the viability of the WNHE assumption as an explanation for no-signaling.

2.3. Polynomial superluminal gates

Even though WNHE excludes the type of superluminal gates considered above, for the exclusion to be general, it would have to be shown that every superluminal gate is exponential, i.e. \( C^c \subseteq E \). It turns out that this cannot be done, because one can construct hypothetical polynomial superluminal gates, which are superluminal operations that are not exponential. In fact, it is probably true that \( E \subset C^c \). To see this, let us consider solving the \( \text{NP} \)-complete problem associated with equation (6) via Grover search [29], which is optimal for \( \text{QM} \) [30] but offers only a quadratic speed-up, thus leaving the complexity of the problem exponential in \( n \), at least in the black box setting. The optimality proof relies on showing that, given the problem of distinguishing an empty oracle \( \langle y, A(x) = 0 \rangle \) and a non-empty oracle containing a single random unknown string \( y \) of known length \( n \) (i.e. \( A(y) = 1 \), but \( \forall y \neq y^*\), \( A(x) = 0 \)), subject to the constraint that its overall evolution be unitary and linear (so that in a computation with a non-empty oracle, all computation paths querying empty locations evolve exactly as they would for an empty oracle), the speed-up over a classical search is at best quadratic.

Any degree of amplitude amplification of the marked state above the quadratic level would then require empty superposition branches being ‘made aware’ of the presence of a non-empty branch, i.e. a non-linearity of some sort. Let us suppose that Bob can perform a trace-preserving nonlinear transformation \( \rho_j \rightarrow \tilde{\rho}_j \) of the above kind on an unknown ensemble of separable states. Further, let Alice and Bob share an entangled state, by which Alice is able to prepare, employing two different positive operator-valued measures (POVMs), two different but equivalent ensembles of Bob. Then, depending on Alice’s choice, his reduced density matrix evolves as \( \rho_B = \sum_j p_j \rho_j \rightarrow \sum_j p_j \tilde{\rho}_j \equiv \rho'' \) or \( \rho_B = \sum_j p_k \rho_k \rightarrow \sum_j p_k \tilde{\rho}_k \equiv \rho'' \), where \( (\rho_j, p_j) \) and \( (\rho_k, p_k) \) are distinct, equivalent ensembles [31]. The assumption of linearity is sufficient to ensure that \( \rho'' = \rho'' \). This is not guaranteed in the presence of nonlinearity, leading to a potential superluminal signal. In a nonlinearity of the above kind, the result would depend on whether the particular ensemble remotely prepared by Alice has states that include \( |\psi\rangle \) in the superposition. This would lead to a scenario similar to that encountered with nonlinear gate \( R \) in section 2.1.

Possibly the simplest examples of polynomial superluminal gates are the non-invertible constant gates, which map any state in an input Hilbert space to a fixed state in the output Hilbert space and have the form \( |\xi\rangle \otimes \sum_j |j\rangle \), for some fixed \( \xi \). Examples in matrix notation are:

\[
Q = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad Q' = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{7}
\]

acting in Hilbert space \( \mathcal{H}_2 \equiv \text{span}(|0\rangle, |1\rangle) \) and \( \mathcal{H}_3 \equiv \text{span}(|0\rangle, |1\rangle, |2\rangle) \), respectively. They have the effect of mapping any input state in \( \mathcal{H}_2 \) to a fixed (apart from a normalization factor) state \( |\xi\rangle \), in this case \( |\xi\rangle \) being \( |0\rangle \). In equation (7), we do not in general require the input and output bases to be the same, nor indeed that the input and output Hilbert subspaces be the same (for example, as with the distinct incoming and outgoing modes of a scattering problem).

Both \( Q \) and \( Q' \) are non-complete, inasmuch as \( Q^I Q \neq I \) and \( (Q')^I Q' \neq I \), and represent superluminal gates. For example, by applying or not applying \( Q \) to her register in the state \( (1/\sqrt{2})(|01\rangle + |10\rangle) \) shared with Bob, Alice can remotely prepare his state to be the pure state \( (1/\sqrt{2})(|0\rangle + |1\rangle) \) or leave it as a maximal mixture, respectively, corresponding to a superluminal signal of about 0.3 bits (determined by the Holevo bound). Similarly, by choosing to apply, or not, \( Q' \),
on her half of the state \((1/\sqrt{2})(|11⟩ + |22⟩)\) shared with Bob, Alice can superluminally signal him.

The constant gate is linear and presumes no re-normalization following its non-complete action. The probability of occurrence of a constant gate \(C\) when it is applied to a state \(|ψ⟩\) is simply given by \(∥C|ψ⟩∥^2\), as per the standard prescription. One consequence is that it could not be used to violate no-signaling without the use of entanglement. As an illustration: in \(H_3\), let the states \(|0⟩\) and \(|1⟩\) be localized near Alice and \(|2⟩\) near Bob. Applying \(Q\) on the state \(|ψ⟩ = a|0⟩ + b|1⟩ + c|2⟩\), Alice obtains the (unnormalized) state \(Q^{|ψ⟩} = (a + b)|0⟩ + c|2⟩\). If renormalization were allowed, Alice could contextually (i.e. non-locally) influence Bob’s probability to find \(|2⟩\) to be \(\frac{1}{2}|a + b|^2 + |c|^2\) or \(|c|^2\), depending on whether she applies \(Q\) or not. The linearity of the constant gate requires the interpretation that following her application of \(Q\), Alice can detect the particle with probability \(|a + b|^2\), while for Bob, the probability remains \(|c|^2\). Although non-complete operations do not necessarily conserve probability, still, we will find below and later that in situations of interest they can exactly or effectively conserve probability.

On the other hand, none among \(Q\), and \(Q'\) and a general constant gate is an exponential gate: each of them simply used to violate no-signaling without the use of entanglement. As an illustration: in equation (5) can be extended to a more general class of polynomial superluminal operations acting on qubits, qudits and higher dimensional qudits, such as

\[
Q_2(φ) = \begin{pmatrix}
1 & e^{iφ} \\
0 & 0
\end{pmatrix}, \quad Q_3(φ_1, φ_2) = \begin{pmatrix}
1 & e^{iφ_1} & e^{iφ_2} \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}, \text{ etc.}
\]

By definition, \(Q = Q_2(0)\) and \(Q' = Q_3(0, 0)\). To see that \(Q_1(φ)\) is a polynomial operation, it suffices to show that it can be simulated using only a polynomial amount of standard quantum mechanical resources, which we do in the following theorem. Let \(\text{BQP}_c\) denote the complexity class of problems that can be efficiently solved on a standard quantum computer that can access a constant gate. Then:

**Theorem 1.** \(\text{BQP}_c = \text{BQP}\).

**Proof.** It is clear that any problem in \(\text{BQP}\) can be efficiently solved using resources of \(\text{BQP}_c\), by simply not using the constant gates. Now let us consider the simulation the other way. Given an arbitrary \((n + 1)\)-qubit state \(|ψ⟩ = |α⟩|0⟩ + |β⟩|1⟩\), where the \(n\)-qubit states \(|α⟩\) and \(|β⟩\) are neither necessarily mutually orthogonal nor normalized and with \(∥|α⟩∥^2 + ∥|β⟩∥^2 = 1\), the action of \(Q_2(φ)\) on the last qubit is to produce \(Q_2|ψ⟩ = (|α⟩ + e^{iφ}|β⟩)|0⟩ ≡ |ψ⟩|0⟩\), which is interpreted as \(N ≡ (|ψ⟩|φ⟩ = 1 + 2[\cos(φ)|R(⟨α|β⟩)| − \sin(φ)|S(⟨α|β⟩)|]\) copies of the normalized state \(|ψ⟩|φ⟩ ≡ |ψ⟩/\sqrt{N}\) and a copy of \(|0⟩\). If \(|α⟩\) and \(|β⟩\) are mutually orthogonal and thus the reduced density operator for the last qubit is diagonal in the computational basis, then \(N = 1\), and no such special interpretation is needed.

To simulate the production of \(|ψ⟩\) with standard quantum resources, one first applies a phase gate \(\begin{pmatrix}1 & 0 \0 & e^{iφ}\end{pmatrix}\) followed by a Hadamard on the last qubit, to obtain the state \((1/\sqrt{2})(|ψ⟩|0⟩ + |ψ⟩|1⟩)\), where \(|ψ⟩ = |α⟩ − e^{iφ}|β⟩\). Measurement on the last qubit in the computational basis yields \(|0⟩\) and hence \(|ψ⟩|1⟩\) in the first register, with probability \(∥|ψ⟩|1⟩/2^2 = N/2\), which is to say that the simulation of \(Q_2\) succeeds with probability \(1/2\), irrespective of \(n\). Similar arguments hold for \(Q_3\), etc. Therefore, the class of problems efficiently solvable with standard quantum computation augmented by the non-standard family of constant gates is in \(\text{BQP}\).

□

It is worth noting that the constant gate is quite different from the following two operations that appear to be similar but are, in fact, quite distinct. The first operation is a standard quantum mechanical CP map, polynomial and not superluminal; the second is exponential and consequently superluminal.

(i) To begin with, a constant gate is not a quantum deletor [32], in which a qubit is subjected to a complete operation, specifically, a contractive CP map that prepares it asymptotically in a fixed state \(|0⟩\). The action of a quantum deletor is given by an amplitude damping channel [9], which has an operator sum representation, respectively

\[
ρ_2 → \sum_j E_j ρ_2 E_j^{†}, \quad ρ_3 → \sum_j E_j′ ρ_3 E_j^{†},
\]

in the qubit case or when extended to the qudit case, with the Kraus operators given by equation (10a) or (10b), respectively

\[
E_1 ≡ \begin{pmatrix}1 & 0 \\
0 & 0
\end{pmatrix}, \quad E_2 ≡ \begin{pmatrix}0 & 1 \\
0 & 0
\end{pmatrix},
\]

\[
E_1′ ≡ \begin{pmatrix}1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}, \quad E_2′ ≡ \begin{pmatrix}0 & 1 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}, \quad E_3′ ≡ \begin{pmatrix}0 & 0 & 1 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}.
\]

Unlike in the case of \(Q\), \(Q'\) or \(Q''\), there is no actual destruction of quantum information, but its transfer through dissipative decoherence into correlations with a zero-temperature environment. The reduced density operator of Bob’s entangled system remains unaffected by Alice’s application of this operation on her system. The deleting action, although nonlinear at the state vector level, nevertheless acts linearly on the density operator.

(ii) Next we note that the constant gate is quite different from the ‘post-selection’ operation, which is a deterministic rank-1 projection [13]. Verbally, if the constant gate corresponds to the operation ‘for all input states \(|j⟩\) in the computational basis, set the output state to \(|ξ⟩\), independently of \(j\), except for a global phase’, where \(|ξ⟩\) is some fixed state, then post-selection corresponds to the action ‘for all input states \(|j⟩\), if \(j ≠ ξ\), then discard branch \(|j⟩\)’. Post-selective equivalents of \(Q\) and \(Q'\) are

\[
Q_{ps} = \begin{pmatrix}1 & 0 \\
0 & 0
\end{pmatrix}, \quad Q_{ps}' = \begin{pmatrix}1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}.
\]
followed by renormalization. In particular, whereas the action of $Q$ on the first of two particles in the state $(1/\sqrt{2})(|00\rangle + |11\rangle)$ leaves the second particle in the state $(1/\sqrt{2})(|0\rangle + |1\rangle)$, that of $Q_{PS}$ leaves the second particle in the state $|0\rangle$. It is straightforward to see that post-selection is an exponential operation: performing it on the second qubit of $|\Psi_{nc}\rangle$ in equation (6), and post-selecting on 1, we obtain the solution to SAT in one time step.

The seemingly immediate conclusion due to the fact $C^< \not\subseteq E$ is that the WNHE assumption is not strong enough to derive no-signaling, and would have to be supplemented with additional assumption(s), possibly purely physically motivated ones, prohibiting the physical realization of polynomial superluminal gates.

An alternative, highly unconventional reading of the situation is that WNHE is a fundamental principle of the physical world, while the no-signaling condition is in fact not universal, so that some polynomial superluminal gates may actually be physically realizable. Quite surprisingly, we may be able to offer some support for this viewpoint. We believe that constant gates of the above type can be quantum optically realized when a photon detection is made at a path singularity, defined as a point in space where two or more incoming paths converge and terminate. In graph theoretic parlance, a path singularity is a terminal node in a directed graph, of degree greater than 1.

We describe in section 3.1 an experiment that possibly physically realizes $Q$. In principle, a detector placed at the focus of a convex lens realizes such a path singularity. This is because the geometry of the ray optics associated with the lens requires rays parallel to the lens axis to converge to the focus after refraction, while the destructive nature of photon detection implies the termination of the path. Although conceptually and experimentally simple, the high degree of mode filtering or spatial resolution that the experiment requires will be the main challenge in implementing it. Indeed, we believe that this is the reason why such gates have remained undiscovered so far.

Our argument here has implicitly assumed that $P \neq NP$. If it turns out that $P = NP$, then even the obviously non-physical operations such as $G$ or $R$ would be polynomial gates, and the WNHE assumption would not be able to exclude them. Nevertheless, the question of existence and testability of certain superluminal gates, which is the main result of this work, would still remain valid and of interest. If polynomial superluminal gates are indeed found to exist (and given that other superluminal gates do not seem to exist anyway), this would give us greater confidence that $P \neq NP$ (or, to be safe, that even Nature does not ‘know’ that $P \neq NP$) and that the assumption of WNHE is indeed a valid and fruitful one.

3. Part II: quantum optical test

3.1. An experiment with entangled pairs of photons

Our proposed implementation of $Q'$, based on the use of entanglement, is broadly related to the type of quantum optical experiments encountered in [33] and closely related to an experiment performed in Innsbruck that elegantly illustrates wave–particle duality by means of entangled light [34, 35]. In the Innsbruck experiment, pairs of position–momentum entangled photons are produced by means of type-I spontaneous parametric downconversion (SPDC) at a nonlinear source, such as a BBO crystal. The two outgoing conical beams from the nonlinear source are presented ‘unfolded’ in figure 1. One of each pair, called the ‘signal photon’, is received by Alice, while the other, called the ‘idler’, is received and analyzed by Bob. Alice’s photon is registered by a detector behind a lens.

Bob’s photon is detected after it enters a double-slit assembly. If Alice’s detector, which is located behind the
Figure 2. The modified Innsbruck experiment (not to scale): the same configuration as in figure 1, except that Bob’s photon (the idler), before entering the double-slit assembly, traverses a direction filter that permits only (nearly) horizontal modes to pass through, absorbing the other modes at the filter walls. The direction filter acts as a state filter that ensures that Bob receives only the pure state consisting of the horizontal modes. Thus if Alice makes no measurement or makes a detection at \( f \), Bob’s corresponding photon builds an interference pattern of modes 2 and 5 in the singles counts. On the other hand, if Alice positions her detector in the imaging plane, she knows the path the idler takes through the slit assembly. Thus, no interference pattern is found on Bob’s screen even in the coincidence counts.

3.1.1. Quantum optical description of the Innsbruck experiment. Here the state of the SPDC field of figure 2 is modeled by a 6-mode vector:

\[
|\Psi\rangle = \left(1 + \frac{\epsilon}{\sqrt{6}} \sum_{j=1}^{6} a_j^\dagger b_j^\dagger\right) |\text{vac}\rangle,
\]

where \( |\text{vac}\rangle \) is the vacuum state, \( a_j^\dagger \) (resp., \( b_j^\dagger \)) are the creation operators for Alice’s (resp., Bob’s) light field on mode \( j \), as per the mode numbering scheme in figure 2. The quantity \( \epsilon (\ll 1) \) depends on the pump field strength and the crystal nonlinearity. The coincidence counting rate for corresponding measurements by Alice and Bob is proportional to the square of the second-order correlation function and given by

\[
R_\alpha(z) \propto \langle \Psi | E_\alpha^{(-)} E_\alpha^{(-)} E_\alpha^{(+)} E_\alpha^{(+)} |\Psi\rangle = ||E_\alpha^{(+)} E_\alpha^{(+)}|\Psi\rangle||^2 = ||E_\alpha^{(\pm)} E_\alpha^{(\pm)}|\Psi\rangle||^2 \quad (\alpha = f, f'', l, m, \ldots),
\]

where \( E_\alpha^{(\pm)} \) represents the positive frequency part of the electric field at a point on Alice’s focal or imaging plane, and \( E_\alpha^{(\pm)} \) that of the electric field at an arbitrary point \( z \) on Bob’s screen.

We have

\[
E_\alpha^{(\pm)} = e^{ik_\alpha r_0} \left(e^{ik_\alpha r_2} \hat{b}_2 + e^{ik_\alpha r_3} \hat{b}_3\right) + e^{ik_\alpha r_2} \left(e^{ik_\alpha r_3} \hat{b}_1 + e^{ik_\alpha r_2} \hat{b}_4\right),
\]

where \( k \) is the wavenumber, \( r_0 \) the distance from the EPR source to the upper/lower slit on Bob’s double-slit diaphragm (the length of the segment \( \overline{y} \) or \( \overline{z} \)); \( r_2 \) (resp., \( r_3 \)) is the distance from the lower (resp., upper) slit to \( z \). The other two terms in equation (14), pertaining to the other two pairs of modes, are obtained analogously. We study two cases, corresponding to Alice making a remote position or remote momentum measurement on the idler photons.

Case 1. Alice remotely measures the position (path) of the idler. Suppose that Alice positions her detector at the imaging
plane and detects a photon at $l$ or $m$. The corresponding field at her detector is

$$ E_m^{(x)} = e^{ikr_m} (\hat{a}_1 + \hat{a}_2 + \hat{a}_3), \quad E_l^{(x)} = e^{ikr_l} (\hat{a}_4 + \hat{a}_5 + \hat{a}_6), \quad (15) $$

where $s_m$ (resp., $s_l$) is the path length along any ray path from the source point $p$ (resp., $q$) through the lens up to image point $m$ (resp., $l$). By Fermat’s principle, all paths connecting a given pair of source and image points are equal. Setting $\alpha = l, m$ in equation (13) and substituting equations (12), (14) and (15) in equation (13), we find the coincidence counting rate for detections by Alice and Bob to be

$$ R_m(z) \propto e^2[|e^{ikr_m} + e^{ikr_1}|^2, $$

$$ R_l(z) \propto e^2[|e^{ikr_l} + e^{ikr_1}|^2, $$

which is essentially a single-slit diffraction pattern formed behind, respectively, the upper and lower slits. The intensity pattern Bob finds on his screen in the singles count, obtained by averaging $R_m(z)$ over $\alpha = l, m$, is thus not a double-slit interference pattern, but an incoherent mixture of the two single-slit patterns. A similar lack of interference pattern is obtained by Bob if Alice makes no measurement.

**Case 2. Alice remotely measures the momentum (direction of the idler.** Alice positions her detector on the focal plane of the lens. If she detects a photon at $f, f'$ or $f''$, the field at her detector is, respectively,

$$ E_f^{(x)} = e^{ikr_f} \hat{a}_2 + e^{ikr'_f} \hat{a}_3 = e^{ikr_f} (\hat{a}_2 + \hat{a}_3), \quad (17a) $$

$$ E_f^{(x)} = e^{ikr_f} \hat{a}_1 + e^{ikr'_f} \hat{a}_4 = e^{ikr_f} (\hat{a}_1 + \hat{a}_4), \quad (17b) $$

$$ E_f^{(x)} = e^{ikr_f} \hat{a}_5 + e^{ikr'_f} \hat{a}_6 = e^{ikr_f} (\hat{a}_5 + \hat{a}_6), \quad (17c) $$

where $r_{2f}$ (resp., $r_{3f}$) is the distance from $p$ (resp., $q$) along the path 2 (resp., 5) through the lens up to point $f$. The distances along the two paths being identical, $r_{2f} = r_{3f} \equiv r_f$. The distances $r_{1f}, r_{4f}, r_{3f}$ and $r_{5f}$ are defined analogously. Substituting equations (12), (14) and (17) into equation (13), we find that the coincidence counting rate is given by

$$ R_f(z) \propto e^2[1 + \cos(k \cdot [r_2 - r_3])], \quad (18a) $$

$$ R_f(z) \propto e^2[1 + \cos(k \cdot [r_1 - r_4 + \omega_{14})], \quad (18b) $$

$$ R_f(z) \propto e^2[1 + \cos(k \cdot [r_1 - r_3 + \omega_{36})], \quad (18c) $$

where $\omega_{14} \equiv k(r_{4f} - r_{1f})$ and $\omega_{36} \equiv k(r_{6f} - r_{3f})$ are fixed for a given point on the focal plane. Each equation in equation (18) represents a conventional Young’s double-slit pattern. Conditioned on Alice detecting photons at $f$, Bob finds the pattern $R_f(z)$, and similarly for points $f'$ and $f''$. In his singles count, Bob perceives no interference, because he is left with a statistical mixture of the patterns (18a), (18b), (18c), etc., corresponding to all the points on Alice’s focal plane illuminated by the signal beam.

### 3.1.2. The proposed experiment.

The experiment proposed here, presented earlier by us in [36], is derived from the Innsbruck experiment and is therefore called ‘the modified Innsbruck experiment’. It was claimed to manifest superluminal signaling, although it was not clear what the exact origin of the signaling was, and in particular, which assumption that goes to proving the no-signaling theorem was being given up. The modified Innsbruck experiment is revisited here in order to clarify this issue in detail in the light of the discussions of the previous sections. This will help crystallize what is, and what is not, responsible for the claimed signaling effect. In [37], we studied a version of nonlocal communication inspired by the original Einstein–Podolsky–Rosen thought experiment [1]. Recently, similar experiments, also based on the Innsbruck experiment, have been independently proposed in [38, 39].

Firstly, we present a qualitative overview of the modified Innsbruck experiment. The only material difference between the original Innsbruck experiment and the modified version we propose here is that the latter contains a ‘direction filter’, consisting of two convex lenses of the same focal length $G$, separated by distance $2G$. Their shared focal plane is covered by an opaque screen, with a small aperture $o$ of diameter $\delta$ at their shared focus. We want $\delta$ to be small enough so that only almost horizontal modes are permitted by the filter to fall on the double-slit diaphragm. The angular spread (about the horizontal) of the modes that fall on the aperture is given by $\Delta \theta = \delta/G$; we require that $(\delta/G) \sigma \ll \lambda$, where $\sigma$ is the slit separation, to guarantee that only modes that are horizontal or almost horizontal are selected to pass through the direction filter, to produce a Young’s double-slit interference pattern on the screen plane. On the other hand, we do not want the aperture to be so small that it produces significant diffraction; thus $\delta \gg \lambda$. Putting these conditions together, we must have

$$ 1 \ll \frac{\delta}{\lambda} \ll \frac{G}{\sigma}. \quad (19) $$

The ability to satisfy this condition, while preferable, is not crucial. If it is not satisfied strictly, the predicted signal is weaker but not entirely suppressed. The point is clarified further down.

If Alice makes no measurement, the idler remains entangled with the signal photon, which renders incoherent the beams coming through the upper and lower slits on Bob’s side, so that he will find no interference pattern on his screen. Similarly, if she detects her photon in the imaging plane, she localizes Bob’s photon at his slit plane, and so, again, no interference pattern is seen. So far, the proposed experiment has the same effect as the Innsbruck experiment.

On the other hand, if Alice scans the focal plane and makes a detection, she remotely measures Bob’s corresponding photon’s momentum and erases its path information, thereby (non-selectively) leaving it as a mixture of plane waves incident on the direction filter. However, only the fraction that makes up the pure state comprising the horizontal modes passes through the filter. Diffracting through the double-slit diaphragm, it produces a Young’s double-slit interference pattern on Bob’s screen. Those plane waves coincident with Alice’s detecting her photon away from focus $f$ are filtered out and do not reach Bob’s double-slit assembly. It follows that an interference pattern will emerge in Bob’s singles counts, coinciding with Alice’s detection
We single out, in the following section, the key assumption that

\[ E_{z}^{(r)} = e^{ik_{z}r_{D}} \left( e^{i\theta_{1}}b_{2} + e^{i\theta_{2}}b_{3} \right), \quad (20) \]  

where \( r_{D} \) now represents the distance from the EPR source to the upper/lower slit on Bob’s double-slit diaphragm (the length of the segment \( R \psi_{\text{opt}} \) or \( R \psi_{\text{th}} \)); \( r_{2} \) (resp., \( r_{3} \)) is the distance from the upper (resp., lower) slit to \( z \). Detection of a signal photon at or near \( f \) is the only possible event on the focal plane such that Bob detects the twin photon at all. Focal plane detections sufficiently distant from \( f \) will project the idler into non-horizontal modes that will be filtered out before reaching Bob’s double-slit assembly. Therefore, the interference pattern equation (18a) is in fact the only one seen in Bob’s singles counts. We denote by \( R^{(z)}(z) \), this pattern, which Bob obtains conditioned on Alice measuring in the focal plane. By contrast, in the Innsbruck experiment Bob in his singles counts sees a statistical mixture of the patterns (18a), (18b), (18c), etc, corresponding to all points on Alice’s focal plane illuminated by the signal beam.

When Alice measures in the imaging plane, as in the Innsbruck experiment, Bob finds no interference pattern in his singles counts. Setting \( \alpha = l, m \) in equation (13) and substituting equations (12), (20) and (15) into equation (13), we find the coincidence counting rate for detections by Alice and Bob to be

\[ R_{\alpha}(z) \propto \epsilon^{2}, \quad (\alpha = l, m), \quad (21) \]

which is a uniform pattern (apart from an envelope due to single-slit diffraction, which we ignore for the sake of simplicity). It follows that Bob’s observed pattern in the singles counts conditioned on Alice measuring in the imaging plane, \( R^{(z)}(z) \), is also the same, i.e. \( R^{(z)}(z) \propto \epsilon^{2} \).

Our main result is the difference between the patterns \( R^{(l)}(z) \) and \( R^{(z)}(z) \), which implies that Alice can signal Bob one bit of information across the spacelike interval connecting their measurement events, by choosing to measure her photon in the focal plane or not to measure. In practice, Bob would need to include additional detectors to sample or scan the \( z \)-plane fast enough. This procedure can potentially form the basis for a superluminal quantum telegraph, bringing into sharp focus the tension between quantum non-locality and special relativity.

Considering the far-reaching implications of a positive result to the experiment, we may pause to consider the following: whether our analysis so far can be correct and—under the possibility (however limited) that it is—how such a signal may ever arise, in view of the no-signaling theorem. It may be easy to dismiss a proof of putative superluminal communication as ‘not even wrong’, yet it is less easy to spot where the purported proof fails and to provide a mechanism for thwarting the signaling. For one, the prediction of the non-local signaling is based on a model that departs only slightly from our quantum optical model of section 3.1.1, which explains the original Innsbruck experiment quite well. There have been various attempts at proving that quantum non-locality somehow contravenes special relativity. The author has read some of their accounts, and it was not difficult to spot a hidden erroneous assumption that led to the alleged conflict with relativity. Armed with this lesson, the present claim will be different in the following three ways:

- **We discuss in the following section various possible objections to our claim and demonstrate why each of them fails.** Perhaps they do not cover some erroneous but subtle assumption, but even so, our present exercise could still be instructive in yielding new theoretical insights. For example, a proposal for superluminal communication based on light amplification was eventually understood to fail because it violates the no-cloning theorem, a principle that had not been discovered at the time the proposal was made (cf [41]).

- **We single out, in the following section, the key assumption responsible for the superluminality (that Alice’s momentum measurement implements a polynomial superluminal gate).** This singling out of the non-standard element at play makes it easier for the reader to judge whether the proposal is wrong, not even wrong, or—as we believe is the case—worth testing experimentally.

- **We have furnished computation and information theoretic grounds for why superluminal gates could be possible,** according to which no-signaling could be a nearly-universal-but-not-quite side effect of the computation theoretic properties of physical reality; elsewhere [42], we show how the relativity principle could be a consequence of conservation of information.

In the last section, we clarify how non-complete measurements, if experimentally validated, could possibly fit in with known physics. There we will argue that they arise owing to the potential fact that practically measurable quantities resulting from QFT are not described by Hermitian operators, at variance with a key axiom of orthodox quantum theory [40].

### 3.2. The question of existence and origin of the signaling

In the section, we will consider a number of possible objections to our main result and demonstrate quantitatively why each of them fails.

#### 3.2.1. Spreading at the direction filter

It can be shown that the effect of spreading at the direction filter only lowers—but does not eliminate—the distinguishability between the two kinds of pattern that Bob receives. For illustration, suppose that \( \delta = 10\lambda \), and as a result, nearly only horizontal modes \( r_{2} \) and \( r_{3} \) are selected, but the diffractive spreading at the filter is strong, assumed to be given by \( \frac{e^{i\delta}}{\sin\theta} \) in the space spanned by modes 2 and 5, where \( \theta \) is determined by the
geometry of the filter. In place of equations (14) and (16), we now have

\[ E_{e}^+(\alpha) = e^{i\alpha} \left( e^{i\alpha} (\cos \theta \hat{b}_2 + \sin \theta \hat{b}_5) + e^{i\alpha} (\cos \theta \hat{b}_5 - \sin \theta \hat{b}_2) \right), \]

\[ R'_\alpha(z) \propto \epsilon^2 [1 \pm \sin(2\theta) \cos(k \cdot \hat{r}_2 - r_3)] \]

(with \( \pm \) according as \( \alpha = l, m \)). (22)

The pattern found by Bob in his singles counts is \( R'_\alpha(z) \propto \epsilon^2 [1 \pm \cos(2\theta) \cos(k \cdot \hat{r}_2 - r_3)] \). (23)

Except in the case \( \theta = \pi/4 \), which is highly unlikely and in any case can be precluded by altering \( \delta \) or \( G \), the two patterns are in principle distinguishable.

3.2.2. Alice’s focal plane measurement implements a constant gate in the subspace of interest. The state (12) is now represented in a simple way as the unnormalized state \( |\Psi(1)\rangle = \sum_b |j, j\rangle \), where for simplicity the vacuum state, which does not contribute to the entanglement-related effects, is omitted, and it is assumed that each mode contains at most one pair of entangled photons (i.e. no higher excitations of the light field). Further, because of the direction filter, it suffices to restrict our attention to the state

\[ |\psi(2)\rangle \propto \frac{1}{\sqrt{2}} (|2, 2\rangle + |5, 5\rangle), \]

(24)

the projection of \( |\Psi(1)\rangle \) onto \( \mathcal{H}_2 \otimes \mathcal{H}_2 \), where \( \mathcal{H}_2 \) is the subspace spanned by \( \{2\}, \{5\} \). Under these assumptions, Alice’s position measurement in this subspace, represented by the operators \( \hat{a}_2 \) and \( \hat{a}_5 \), can be written as the Kraus operators

\[ \hat{a}_2 \equiv |0\rangle\langle 2| \quad \text{and} \quad \hat{a}_5 \equiv |0\rangle\langle 5|. \]

Within \( \mathcal{H}_2 \) these operators form a complete set since \( \hat{a}_2^\dagger \hat{a}_2 + \hat{a}_5^\dagger \hat{a}_5 = |2\rangle\langle 2| + |5\rangle\langle 5| = I_2 \). Thus, Alice’s measurement on \( |\Psi(2)\rangle \) in the position basis does not non-locally affect Bob’s reduced density operator, which is proportional to \( I_2/2 \).

On the other hand, if Alice measures momentum, her measurement is represented by the field operator \( E_f^{(+)} \) in equation (17). We have in the above notation

\[ E_f^{(+)} \propto \hat{a}_2 + \hat{a}_5 \equiv |0\rangle\langle 2| + |5\rangle\langle 5|. \]

(25)

This is just the polynomial superluminal gate \( Q \) in equation (10), with the output basis given by \( \{|0\rangle, |0\rangle\} \), where \( |0\rangle \) is any basis element orthogonal to the vacuum state.

We note that the operator \( E_f^{(+)} \propto \hat{a}_2 - \hat{a}_5 \equiv |0\rangle\langle 2| - |5\rangle\langle 5| \), which would complete \( E_f^{(+)} \) in that \( E_f^{(-)} E_f^{(+)} + E_f^{(-)} E_f^{(-)} = I \) in the space span (\( |2\rangle, |5\rangle \)). However, \( E_f^{(+)} \) is necessarily non-physical in the given geometry since modes 2 and 5 meet only at \( f \), where the electric field operator is indeed \( \propto \hat{a}_2 - \hat{a}_5 \).

We further note that, in spite of the non-completeness of \( E_f^{(+)} \), the structure of \( |\psi(2)\rangle \) in equation (24) guarantees that \( E_f^{(+)} |\psi(2)\rangle \) is by default normalized and hence poses no problem with respect to probability conservation.

By contrast, Bob’s measurement is complete (which rules out a Bob-to-Alice superluminal signaling). Each element of Bob’s screen \( z \)-basis is a possible outcome, described by the annihilation operator approximately of the form \( \hat{E}_f^{(-)} \propto \hat{a}_2 + e^{-i2\theta} \hat{a}_5 \), where \( \gamma = \gamma(k, z) \) is the phase difference between paths 2 and 5 from the slits to a point \( z \) on Bob’s screen. This represents a POVM of the form \( \hat{E}_f^{(+)} \hat{E}_f^{(-)} = |2\rangle\langle 2| + e^{-i2\theta} |5\rangle\langle 5| \). Even though \( \hat{E}_f^{(+)} \) has the same form as Alice’s operator \( \hat{E}_f^{(+)} \) (as a Kraus operator describing the absorption of two interfering modes at a point \( z \)), yet, when integrated over his whole ‘position basis’, Bob’s measurement is seen to form a complete set, because, as can be shown, \( \int_{z} \hat{E}_f^{(+)} \hat{E}_f^{(-)} dz = |2\rangle\langle 2| + |5\rangle\langle 5| \). In the case of Alice’s momentum measurement, because the detection happens at a path singularity, a similar elimination of cross-terms via integration is not possible, whence the non-completeness. It is indeed somewhat intriguing how geometry plays a fundamental role in determining the completeness status of a measurement. This has to do with the fact that the direct detection of a photon is practically a determination of position distribution. For example, even in remotely measuring the idler’s momentum, Alice measures her photon’s position at the focal plane. We will return again to this issue in the final section.

3.2.3. Role of the direction filter. A simple model of the action of the perfect direction filter is

\[ D = \sum_{j=2.5}^{|j|} |j\rangle\langle j| + \sum_{j=2.5}^{|-j|} |j\rangle\langle -j| \]

(26)

acting locally on the second register of the state of \( |\Psi(1)\rangle \). Here \( |j\rangle \) can be thought of as a state orthogonal to all \( |j\rangle \)’s and other \( |-j\rangle \)’s, which removes the photon from the experiment, for example, by reflecting it out or by absorption at the filter. It suffices for our purpose to note that \( D \) can be described as a local, standard (linear, unitary and hence complete) operation. Since the structure of QM guarantees that such an operation cannot lead to nonlocal signaling, the conclusion is that the superluminal signal, if it exists, must remain even if the direction filter is absent.

We will employ the notation \( |j + k + m\rangle \equiv (1/\sqrt{3})(|j\rangle + |k\rangle + |m\rangle) \). To see that the non-local signaling is implicit in the state modified by Alice’s actions even without the application of the filter, we note the following: if Alice measures ‘momentum’ on the state \( |\Psi(1)\rangle \) and detects a signal photon at \( f \), she projects the corresponding idler into the state \( |2 + 5\rangle \). Similarly, her detection of a photon at \( f'' \) projects the idler into the state \( |3 + 6\rangle \), and her detection at \( f' \) projects the idler into the state \( |1 + 4\rangle \). Therefore, in the absence of the direction filter, Alice’s remote measurement of the idler’s momentum leaves the idler in a (assumed uniform for simplicity) mixture given by

\[ \rho_P \propto |2 + 5\rangle\langle 2 + 5| + |1 + 4\rangle\langle 1 + 4| + |3 + 6\rangle\langle 3 + 6|. \]

(27)

Her momentum measurement is non-complete, since the summation over the corresponding projectors (rhs of
\( \rho_Q \propto (1 + 2 + 3)(1 + 2 + 3) + (4 + 5 + 6)(4 + 5 + 6) \). (28)

Here again, her position measurement is non-complete, reflected by the fact that the summation over the corresponding projectors (rhs of equation (28)) is not \( \mathbb{I}_6 \). Since \( \rho_P \neq \rho_Q \), we are led to conclude that the violation of no-signaling is already implicit in the Innsbruck experiment. Yet, since Bob measures in the \( z \)-basis rather than the ‘mode’ basis, in the absence of a direction filter (as is the case in the Innsbruck experiment), Bob’s screen will not register any signal, for the following reason. In the case of Alice’s focal plane measurement, the integrated diffraction-interference pattern corresponding to different outcomes will wash out any observable interference pattern. On the other hand, in the case of Alice’s imaging plane measurement, each of Bob’s detections comes from the photon’s incoherent passage through one or the other slit, and hence—again—no interference pattern is produced on his screen. Thus, measurement at Bob’s screen plane \( z \) without a direction filter will render \( \rho_P \) effectively indistinguishable from \( \rho_Q \). The role played by the direction filter is to prevent modal averaging in the case of Alice’s momentum measurement, by selecting one set of modes. The filter does not create, but only exposes, a superluminal effect that otherwise remains hidden.

3.2.4. Complementarity of single- and two-particle correlations. It is well known that path information (or particle nature) and interference (or wave nature) are mutually exclusive or complementary. In the two-photon case, this takes the form of mutual incompatibility of single- and two-particle interference [43, 44], because entanglement can be used to monitor path information of the twin particle and is thus equivalent to ‘particle nature’. One may thus consider single- and two-particle correlations as being related by a kind of complementarity relation that parallels wave- and particle-nature complementarity. A brief exposition of this idea is given in the following paragraph.

For a particle in a double-slit experiment, we restrict our attention to the Hilbert space \( \mathcal{H} \), spanned by the states \( |0\rangle \) and \( |1\rangle \) corresponding to the upper and lower slits of a double-slit experiment. Given the density operator \( \rho \), we define coherence \( C \) by \( C = 2|\rho_{01}| = 2|\rho_{01}| \), a measure of cross-terms in the computational basis not vanishing. The particle is initially assumed to be in the state \( |\psi_0\rangle \), and a 'monitor', initially in the state \( |0\rangle \), interacting with each other by means of an interaction \( U \), parameterized by variable \( \theta \) that determines the entangling strength of \( U \). It is convenient to choose \( U = e^{i \theta \mathbb{I}} \otimes I + i \sin \theta \mathbb{C} \), where \( \mathbb{C} \) is the operation \( I \otimes |0\rangle \langle 0| + X \otimes |1\rangle \langle 1| \), where \( X \) is the Pauli \( X \) operator. Under the action of \( U \), the system particle goes to the state \( \rho = U \rho U^\dagger = \frac{1}{2} + \frac{1}{2} (|\psi_0\rangle \langle \psi_0| + |\psi_1\rangle \langle \psi_1|) \), where \( U \) indicates taking trace over the monitor. Applying the above formula for coherence to \( \rho \), we calculate that coherence \( C = \cos \theta \). Let \( \lambda_{\pm} \) denote the eigenvalues of \( \rho \). Quantifying the degree of entanglement by concurrence [50], we have \( E = \frac{\lambda_+ - \lambda_-}{\sqrt{2}} \sin \theta \). We thus obtain a trade-off between coherence and entanglement given by \( C^2 + E^2 = 1 \), a manifestation of the complementarity between single-particle and two-particle interference.

In the context of the proposed experiment, this could raise the following purported objection to our proposed signaling scheme: as the experiment happens in the near-field regime, where two-particle correlations are strong, one would expect that Bob should not find an interference pattern in his singles counts. Yet, contrary to this expectation, equation (18) implies that such an interference pattern does appear. The reason is that in the focal plane measurement, Alice is able to erase her path information in the subspace \( \mathcal{H}_2 \), and thus, by virtue of the associated non-completeness, she does so in only one way, namely via the non-complete operation \( E_0^{(s)} \) associated with her measurement. If her measurement were complete, she would erase path information in more than one way, and the corresponding conditional single-particle interference patterns would mutually cancel each other in the singles count. This is clarified in the following section.

3.2.5. Polarization and ‘interferometric quantum computing’. \( Q \)-like gates describe the situation where two converging modes at the path singularity have the same polarization. The quantum optics formalism implies that if the polarizations of the two incoming modes are not parallel when interfering, then the polarization states add vectorially (that is, superpose), with amplitudes being added componentwise along each polarization/dimension, and the resulting probability being the squared magnitude of this vector sum. One can define a corresponding more general constant gate (a tensor sum of constant gates over the internal dimensions), and a correspondingly potentially larger \( BQP \). It can be shown that theorem 1 still holds. Here we will content ourselves to illustrate it by a simple example.

Suppose that we have this ‘interferometric quantum computer’: a 2\(^n\)-level atom, whose spin part is prepared initially in the state \( |a\rangle \equiv (2^{\lambda_2})|1\rangle + |2\rangle + \cdots + |2^{\lambda_n} - 1\rangle + |2^n\rangle \). The spatial part of the atom’s ‘matter wave’ is now split into two subwaves by an appropriate beamsplitter and then refocused onto a path singularity. On the second subwave, before the two subwaves reach the region of spatial overlap, an oracle operation is applied that in a single step inverts the sign of all the kets, except the ‘marked’ state \( |2^n\rangle \), yielding \( |b\rangle \equiv (2^{\lambda_2})|2^{\lambda_2} - 1\rangle - |2^{\lambda_2} - 2\rangle - \cdots - |2^n - 1\rangle + |2^n\rangle \). According to the above prescription, the output at the path singularity should be \( |a\rangle + |b\rangle \equiv (2^{\lambda_2})|2^{\lambda_2}/2^n\rangle \), i.e. a particle is detected with exponentially low probability \( \| |a\rangle + |b\rangle \|^2 = 4 \times 2^{-n} \).
and detection leaves the particle in the state $|2^n\rangle$. The oracle together with detection at the path singularity is equivalent to the non-complete operation $\bigoplus_{j=1}^{2^n} Q_2^{(j)}(\pi) \oplus Q_2^{(2^n)}(0)$.

If the marked state is designated to be a possible solution to a SAT problem, the measurement would have to be repeated an exponentially large number of times or performed once on an exponentially large number of atoms, to detect a possible ‘yes’ outcome. Either way, the physical situation is compatible with the WNHE assumption, but not with no-signaling. (We observe that augmenting the detection with a renormalization following vector addition would in fact implement the post-selection gate.)

Finally, let us clarify the sense in which non-complete operations like $Q$ of potential physical interest may effectively conform to probability conservation. In the modified Innsbruck experiment, Alice’s application of $Q$ conforms exactly to probability conservation, because the state $|\psi(2)\rangle$ in equation (24) has a Schmidt form, with $Q$ defined in Bob’s Schmidt basis. However, this is not the general situation. In such cases, one seems to find that the spreading of the wavefunction produces a pattern of bright and dark interferometric fringes at and around the path singularity such that, even though locally there is an excess or deficit over the average probability density, there is overall probability conservation across the fringes. This is somewhat comparable to the situation with Bob’s POVM $\hat{E}^{(1)} \hat{E}^{(3)}$, which, even though locally a $Q$-like operation, yields identity when integrated over $\epsilon$. This conservation mechanism is not applicable to the modified Innsbruck experiment, which is performed in the near-field limit, where spreading is minimal and two-particle correlations are strong. However, as noted above, probability conservation is inherently exact for the situation in the experiment, and the mechanism need not be invoked.

As an illustration of the mechanism, let the angle at which the two interfering beams of the ‘interferometric quantum computer’ converge towards a spatial overlapped region be $\theta$. The fringes are given by a stationary pattern with spatial frequency $k' = k \sin \theta \approx k d = k(S/d)$, where $S$ is the spatial separation between two optical elements (say, mirrors) that are, respectively, reflecting the beams along the two interferometric arms towards $q$, and $d$ is the distance from the central point between these mirrors to the center of region $q$. The width of each fringe is about $2\pi/k' = \lambda(d/S)$. Now the initial beam width must be of the order of several wavelengths, and the diffractive spread rate of each beam at least $\lambda / S$, so that beam width $> \lambda d / S$. Thus, the spreading of (quantum) waves guarantees that there will always be compensatory fringes, and hence overall conservation of probability, even though locally the dark and bright bands contain less than or more than the average probability density.

Applied to the above atom interferometer example, the state vector at the interference screen will have the form $\kappa(\theta) (|a\rangle + e^{i\theta} |b\rangle)$ with $\theta$ running from $-\infty$ to $+\infty$, where $\kappa(\theta)$ is a narrow Gaussian-like function centered at $\theta = 0$. When $\theta = 0$, $2\pi, 4\pi, \ldots$, one obtains dark fringes with the ‘solution’ $|2^n\rangle$ at exponentially low intensity, as noted above. When $\theta = \pi, 3\pi, 5\pi, \ldots$, one obtains bright fringes of near maximum intensity, diminished by only an exponentially small amount, corresponding, again, to the ‘solution’. Thus, the interference pattern is a band of bright and dark fringes at spatial frequency $k'$ with the bright ones very slightly dimmer than if $|a\rangle$ and $|b\rangle$ had the same polarization, and the dark ones very slightly brighter.

### 4. Discussions and conclusions

Considering the far-reaching implications of a positive result to our proposed experiment, even though we have ruled out in section 3.2 all the (as far as we know) obvious objections, we have to remain open to the possibility that there may be a subtle error, possibly a hidden unwarranted assumption, somewhere in our analysis. In the surprising event that the proposed experiment yields a positive outcome, no-signaling would no longer be a universal condition, and the issue of ‘speed of quantum information’ [45] would assume practical significance. It would also bolster the case for believing that the WNHE assumption is a basic principle of quantum physics and that considerations of intractability, and by extension uncomputability, can serve as an informal guide to basic physics.

Physical space would be regarded as a type of information, and physical dynamics a kind of computation, with physical separation being not a genuine obstacle to rapid communication in the way it would be when seen from the perspective of causality in conventional physics. On the other hand, the barrier between polynomial-time and hard problems would be real, and the physical existence of superluminal signals would thus not be as surprising as that of exponential gates. Interestingly, polynomial superluminal operations exist even in classical computation theory. The random access machine (RAM) model [46], a standard model in computer science wherein memory access takes exactly one time step irrespective of the physical location of the memory element, illustrates this idea. RAMs are known to be polynomially equivalent to Turing machines.

Even granting that the noncomplete gate $Q'$ turns out to be physically valid and realizable, this brings us to another important issue: how would non-completeness fit in with the known mathematical structure of the quantum properties of particles and fields? We assume that the answer has to do with the nature of and relationship between observables in QM, on the one hand, and those in quantum optics, and more generally, in QFT, on the other hand.

It is frequently claimed that QFT is just the standard rules of first quantization applied to classical fields, but this position can be criticized [40, 47, 48]. For example, the relativistic effects of the integer-spin QFT imply that the wavefunctions describing a fixed number of particles do not admit the usual probabilistic interpretation [48]. Again, fermionic fields do not really have a classical counterpart and do not represent quantum observables [40].

In practice, measurable properties resulting from a QFT are properties of particles—of photons in quantum optics. Particulate properties such as number, described by the number operator constructed from fields, or the momentum operator, which allows the reproduction of single-particle QM in momentum space, do not present a problem. The problem is the position variable, which is considered to be a parameter, and not a Hermitian operator, both in QFT and...
single-particle relativistic QM, and yet relevant experiments measure particle positions. The experiment described in this work involves measurement of the positions of photons, such as, for example, Alice’s detection of photons at points on the imaging or focal plane, or Bob’s detection at points on the z-plane, respectively. There seems to be no way to derive from QFT the experimentally confirmed Born rule that the nonrelativistic wavefunction $\psi(x, t)$ determines quantum probabilities $|\psi(x, t)|^2$ of particle positions. In most practical situations, this is really not a problem. The probabilities in the above experiment were computed according to standard quantum optical rules to determine the correlation functions at various orders [49], which serve as an effective wavefunction of the photon, as seen for example from equations (13). In QFT, particle physics phenomenologists have developed intuitive rules to predict distributions of particle positions from scattering amplitudes in momentum space.

Nevertheless, there is a problem in principle, and this leads us to ask whether QFT is a genuine quantum theory [40]. If we accept that properties like position are valid observables in QM, the answer seems to be ‘no’. We see this again in the fact that the effective ‘momentum’ and ‘position’ observables that arise in the above experiment are not seen to be Hermitian operators of standard QM (see footnote 6).

Since nonrelativistic QM and QFT are presumably not two independent theories describing entirely different objects, but do describe the same particles in many situations, the relationship between observables in the two theories needs to be better understood. Perhaps some quantum mechanical observables are a coarse-graining of QFT ones, having wide but not universal validity. For example, Alice’s detection of a photon at a point in the focal plane was quantum mechanically understood to project the state of Bob’s photon into a one-dimensional subspace corresponding to a momentum eigenstate. Quantum optically, however, this ‘eigenstate’ is described as a superposition of a number of parallel, in-phase modes originating from different downconversion events in the nonlinear crystal, producing a coherent plane wave propagating in a particular direction.

Acknowledgments

I am grateful to Professor S Rangwala and Mr K Ravi for their insightful elucidation of experimental issues and to Professors J Sarfatti, J Cramer and A Shiekh for helpful discussions.

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