The Swap Matching Problem Revisited

Pritom Ahmed¹, Costas S. Iliopoulos², A.S.M. Sohidull Islam¹, and M. Sohel Rahman¹

¹ A/EDA Group, Department of Computer Science, BUET, Dhaka, 
{sritom.11,sohansayed}@gmail.com, msrahman@cse.buet.ac.bd
http://teacher.buet.ac.bd/msrahman
² Algorithm Design Group, Department of Computer Science, King's College London, University of London
csi@dcs.kcl.ac.uk
http://www.dcs.kcl.ac.uk/staff/csi

Abstract. In this paper, we revisit the much studied problem of Pattern Matching with Swaps (Swap Matching problem, for short). We first present a graph-theoretic model, which opens a new and so far unexplored avenue to solve the problem. Then, using the model, we devise two efficient algorithms to solve the swap matching problem. The resulting algorithms are adaptations of the classic shift-and algorithm. For patterns having length similar to the word-size of the target machine, both the algorithms run in linear time considering a fixed alphabet.

Key words: Algorithms; Strings; Swap Matching; Graphs.

1 Introduction

The classical pattern matching problem is to find all the occurrences of a given pattern \( P \) of length \( m \) in a text \( T \) of length \( n \), both being sequences of characters drawn from a finite character set \( \Sigma \). This problem is interesting as a fundamental computer science problem and is a basic need of many practical applications such as text retrieval, music information retrieval, computational biology, data mining, network security, among many others. In this paper, we revisit the Pattern Matching with Swaps problem (the Swap Matching problem, for short), which is a well-studied variant of the classic pattern matching problem. In this problem, the pattern \( P \) is said to swap match the text \( T \) at a given location \( i \), if adjacent pattern characters can be swapped, if necessary, so as to make the pattern identical to the substring of the text ending (or equivalently, starting) at location \( i \). All the swaps are constrained to be disjoint, i.e., each character is involved in at most one swap.

Amir et al. [1] obtained the first non-trivial results for this problem. They showed how to solve the problem in time \( O(nm^{1/3} \log m \log \sigma) \), where \( \sigma = \min(|\Sigma|, m) \). Amir et al. [3] also studied certain special cases for which \( O(n \log^2 m) \) time solution can be obtained. However, these cases are rather restrictive. Later,
Amir et al. [2] solved the Swap Matching problem in time $O(n \log m \log \sigma)$. We remark that all the above solutions to swap matching depend on Fast Fourier Transformation (FFT) technique. Recently, Cantone and Faro [9] presented a dynamic programming approach to solve the swap matching problem which runs in linear time for finite character set $\Sigma$, when patterns are compatible with the word size of the target machine. Notably the work of [9] avoids the use of FFT technique. Cantone, Faro and Campanelli presented another approach in [10] to solve the Swap matching problem. Though the algorithm of [10] runs in $O(nm)$ time for patterns compatible with the word size of the target machine, in practice it achieves quite good result. In fact as it turns out, the algorithm of [10] outperforms the algorithm of [9] most of the time. Notably, approximate swapped matching [4] and swap matching in weighted sequences [7] have also been studied in the literature.

1.1 Our Contribution

The contribution of this paper is as follows. We first present a graph-theoretic approach to model the swap matching problem. Using this model, we devise two efficient algorithms to solve the swap matching problem. The resulting algorithms are adaptation of the classic shift-and algorithm [6] and runs in linear time if the pattern size is similar to the size of word in the target machine, assuming a fixed alphabet size. Notably, some preliminary results of this paper were presented in [8]. In [8], an algorithm running in $O(m/w \log m)$ time was presented, where $w$ is the machine word size. For short patterns, i.e., pattern size similar to machine word size, this runtime becomes $O(n \log m)$. Hence the result in this paper clearly improves the results of [8] and matches the result of [9]. Finally, we present experimental results to compare the non-FFT algorithms of [9, 10] and our work.

1.2 RoadMap

The rest of the paper is organized as follows. In Section 2, we present some preliminary notations and definitions. In Section 3, we present our model to solve the swap matching problem. In Section 4, we present two different algorithms to solve the swap matching problem. Section 5, presents the experimental results. Finally, we briefly conclude in Section 6.

2 Preliminaries

A string is a sequence of zero or more symbols from an alphabet, $\Sigma$. A string $X$ of length $n$ is denoted by $X[1..n] = X_1X_2 \ldots X_n$, where $X_i \in \Sigma$ for $1 \leq i \leq n$. The length of $X$ is denoted by $|X| = n$. A string $w$ is called a factor of $X$ if $X = uvw$ for $u, v \in \Sigma^*$; in this case, the string $w$ occurs at position $|u| + 1$ in $X$. The factor $w$ is denoted by $X[|u|+1..|u|+|w|]$. A $k$-factor is a factor of length $k$. A prefix (or suffix) of $X$ is a factor $X[x..y]$ such that $x = 1$ ($y = n$),
1 \leq y \leq n \ (1 \leq x \leq n). \ We \ define \ the \ i-th \ prefix \ to \ be \ the \ prefix \ ending \ at \ position \ i, \ i.e., \ X[1..i], \ 1 \leq i \leq n. \ On \ the \ other \ hand, \ the \ i-th \ suffix \ is \ the \ suffix \ starting \ at \ position \ i, \ i.e., \ X[i..n], \ 1 \leq i \leq n.

**Definition 1.** A swap permutation for \( X \) is a permutation \( \pi : \{1, \ldots, n\} \to \{1, \ldots, n\} \) such that:

1. if \( \pi(i) = j \) then \( \pi(j) = i \) (characters are swapped).
2. for all \( i, \pi(i) \in \{ i - 1, i, i + 1 \} \) (only adjacent characters are swapped).
3. if \( \pi(i) \neq i \) then \( X_{\pi(i)} \neq X_i \) (identical characters are not swapped).

For a given string \( X \) and a swap permutation \( \pi \) for \( X \), we use \( \pi(X) \) to denote the swapped version of \( X \), where \( \pi(X) = X_{\pi(1)}X_{\pi(2)} \ldots X_{\pi(n)} \).

**Definition 2.** Given a text \( T = T_1 T_2 \ldots T_n \) and a pattern \( P = P_1 P_2 \ldots P_m \), \( P \) is said to swap match at location \( i \) of \( T \) if there exists a swapped version \( P' \) of \( P \) that matches \( T \) at location\(^3 \) \( i \), i.e. \( P'_j = T_{i-m+j} \) for \( j \in \{1..m\} \).

**Problem “SM” (Pattern Matching with Swaps).** Given a text \( T = T_1 T_2 \ldots T_n \) and a pattern \( P = P_1 P_2 \ldots P_m \), we want to find each location \( i \in \{1..n\} \) such that \( P \) swap matches with \( T \) at location \( i \).

**Definition 3.** A string \( X \) is said to be degenerate, if it is built over the potential \(|\Sigma| - 1\) non-empty sets of letters belonging to \( \Sigma \).

**Example 1.** Suppose we are considering DNA alphabet, i.e., \( \Sigma = \Sigma_{DNA} = \{A, C, T, G\} \). Then we have 15 non-empty sets of letters belonging to \( \Sigma_{DNA} \). In what follows, the set containing \( A \) and \( T \) will be denoted by \([AT]\) and the singleton \([C]\) will be simply denoted by \( C \) for ease of reading. The set containing all the letters, namely \([ACTG]\), is known as the don’t care character in the literature.

**Definition 4.** Given two degenerate strings \( X \) and \( Y \) each of length \( n \), we say \( X[i] \) matches \( Y[j], 1 \leq i, j \leq n \) if, and only if, \( X[i] \cap Y[j] \neq \emptyset \).

**Example 2.** Suppose we have degenerate strings \( X = AC[CTG][TG][AC]C \) and \( Y = TC[AT][AT]TTC \). Here, \( X[3] \) matches \( Y[3] \) because \( X[3] = [CTG] \cap Y[3] = [AT] = T \neq \emptyset \).

### 3 The Graph-Theoretic Model for Swap Matching

In this section, we propose the graph-theoretic model to solve the swap matching problem. In this model, both the text and the pattern are viewed as two separate graphs. We start with the following definitions.

\(^3\) Note that, we are using the end position of the match to identify it.
Definition 5. The $\mathcal{T}$-graph is defined in the following way:

Given a text $T = T_1 \ldots T_n$ of Problem SM, a $\mathcal{T}$-graph, denoted by $T^G = (V^T, E^T)$, is a directed graph with $n$ vertices and $n - 1$ edges such that $V^T = \{1, 2, \ldots, n\}$ and $E^T = \{(i, i + 1) \mid 1 \leq i < n\}$. For each $i \in V^T$, we define label($i$) = $T_i$ and for each edge $e \equiv (i, j) \in E^T$, we define label($e$) $\equiv$ label($\{(i, j)\}) = (T_i, T_j)$.

Note that, the labels in the above definition may not be unique. Also, we normally use the labels of the vertices and the edges to refer to them.

$$a \rightarrow c \rightarrow a \rightarrow c \rightarrow b \rightarrow a \rightarrow c \rightarrow b \rightarrow a \rightarrow c \rightarrow b \rightarrow a$$

Fig. 1. The corresponding $\mathcal{T}$-graph of Example 3

Example 3. Suppose, $T = acacbacbacaba$. Then the corresponding $T$-graph is shown in Figure 1.

Definition 6. The $\mathcal{P}$-graph is defined in the following way:

Given a text $P = P_1 \ldots P_m$ of Problem SM, a $\mathcal{P}$-graph, denoted by $P^G = (V^P, E^P)$, is a directed graph with $3m - 2$ vertices and at most $5m - 9$ edges. The vertex set $V^P$ can be partitioned into three disjoint vertex sets, namely, $V^P_{(-1)}, V^P_0, V^P_{(+1)}$ such that $|V^P_{(+1)}| = |V^P_{(-1)}| = m - 1$ and $|V^P_0| = m$. The partition is defined in a $3 \times m$ matrix $M[3, m]$ as follows. For the sake of notational symmetry we use $M[-1], M[0]$ and $M[+1]$ to denote respectively the rows $M[1], M[2]$ and $M[3]$ of the matrix $M$.

1. $V^P_{(-1)} = \{M[-1, 2], M[-1, 3], \ldots, M[-1, m]\}$
2. $V^P_0 = \{M[0, 1], M[0, 2], \ldots, M[0, m]\}$
3. $V^P_{(+1)} = \{M[+1, 1], M[+1, 2], \ldots, M[+1, m - 1]\}$

The labels of the vertices are derived from $P$ as follows:

1. For each vertex $M[-1, i] \in V^P_{(-1)}, 1 < i \leq m, \text{label}(M[-1, i]) = P_{i-1}$
2. For each vertex $M[0, i] \in V^P_0, 1 \leq i \leq m, \text{label}(M[0, i]) = P_i$
3. For each vertex $M[+1, i] \in V^P_{(+1)}, 1 \leq i \leq m - 1, \text{label}(M[+1, i]) = P_{i+1}$

The edge set $E^P$ is defined as the union of the sets $E^P_{(-1)}, E^P_0$ and $E^P_{(+1)}$ as follows:

1. $E^P_{(-1)} = \{(M[-1, i], M[0, i + 1]), (M[-1, i], M[+1, i + 1]) \mid 2 \leq i \leq m - 2\}$
2. $E^P_0 = \{(M[0, i], M[0, i + 1]) \mid 1 \leq i \leq m - 1\} \cup \{(M[0, i], M[+1, i+1]) \mid 1 \leq i \leq m - 2\}$
3. $E^P_{(+1)} = \{(M[+1, i], M[-1, i + 1]) \mid 1 \leq i \leq m - 1\}$
Fig. 2. $\mathcal{P}$-graph of the Pattern $P = acbab$

The labels of the edges are derived from using the labels of the vertices in the obvious way.

Example 4. Suppose, $P = acbab$. Then the corresponding $\mathcal{P}$-graph $P^G$ is shown in Figure 2.

Definition 7. Given a $\mathcal{P}$-graph $P^G$, a path $Q = u_1 \rightarrow u_\ell = u_1 u_2 \ldots u_\ell$ is a sequence of consecutive directed edges $((u_1, u_2), (u_2, u_3), \ldots, (u_{\ell-1}, u_\ell))$ in $P^G$ starting at node $u_1$ and ending at node $u_\ell$. The length of the path $Q$, denoted by $\text{len}(Q)$, is the number of edges on the path and hence is $\ell - 1$ in this case. It is easy to note that the length of a longest path in $P^G$ is $m - 1$.

Definition 8. Given a $\mathcal{P}$-graph $P^G$ and a $\mathcal{T}$-graph $T^G$, we say that $P^G$ matches $T^G$ at position $i \in [1..n]$ if, and only if, there exists a path $Q = u_1 u_2 \ldots u_m$ in $P^G$ having $u_1 \in \{M[0,1], M[+1,1]\}$ and $u_m \in \{M[-1,m], M[0,m]\}$ such that for $j \in [1..m]$ we have $\text{label}(u_j) = T_{i-m+j}$.

This completes the definition of the graph theoretic model. The following Lemma presents the idea to solve the swap matching problem using the presented model.

Lemma 1. Given a pattern $P$ of length $m$ and a text $T$ of length $n$, suppose $P^G$ and $T^G$ are the $\mathcal{P}$-graph and $\mathcal{T}$-graph of $P$ and $T$, respectively. Then, $P$ swap matches $T$ at location $i \in [1..n]$ of $T$ if and only if $P^G$ matches $T^G$ at position $i \in [1..n]$ of $T^G$.

Proof. The proof basically follows easily from the definition of the $\mathcal{P}$-graph. At each column of the matrix $M$, we have all the characters as nodes considering the possible swaps as explained below. Each node in row $(-1)$ and $(+1)$ represents a swapped situation. Now consider column $i$ of $M$ corresponding to $P^G$. According to definition, we have $M[-1,i] = P_{i-1}$ and $M[+1,i-1] = P_i$. These two nodes represents the swap of $P_i$ and $P_{i-1}$. Now, if this swap takes place, then in the
resulting pattern, \(P_{i-1}\) must be followed by \(P_i\). To ensure that, in \(P^G\), the only edge starting at \(M[1, i-1]\), goes to \(M[-1, i]\). On the other hand, from \(M[-1, i]\) we can either go to \(M[0, i+1]\) or to \(M[1, i+1]\): the former is when there is no swap for the next pair and the later is when there is another swap for the next pair. Recall that, according to the definition, the swaps are disjoint. Finally, the nodes in row 0 represents the normal (non-swapped) situation. As a result, from each \(M[0, i]\) we have an edge to \(M[0, i+1]\) and an edge to \(M[1, i+1]\): the former is when there is no swap for the next pair as well and the later is when there is a swap for the next pair. So it is easy to see that all the paths of length \(m - 1\) in \(P^G\) represents all combinations considering all possible swaps in \(P\). Hence the result follows. □

Since the number of the possible paths of length \(m - 1\) in \(P^G\) is exponential in \(m\), we exploit the above model in a different way and apply a modified version of the classic shift-and [5] algorithm to solve the swap matching problem.

4 Our Algorithms for Swap Matching

In this section, we use the model proposed in Section 3 to devise two novel efficient algorithms for the swap matching problem. Both of the algorithms are modified versions of the classic shift-and algorithm for pattern matching. We start with a brief review of the shift-and algorithm below. In Sections 4.2 and 4.4 we present the modifications needed to adapt it to solve the swap matching problem.

4.1 Shift-And Algorithm

The shift-and algorithm uses the bitwise techniques and is very efficient if the size of the pattern is no greater than the word size of the target processor. The following description of the shift-and algorithm is taken from [6] after slight adaptation to accommodate our notations.

Let \(R\) be a bit array of size \(m\). Vector \(R_j\) is the value of the array \(R\) after text character \(T_j\) has been processed. It contains information about all matches of prefixes of \(P\) that end at position \(j\) in the text. So, for \(1 \leq i \leq m\) we have:

\[
R_j[i] = \begin{cases} 
1 & \text{if } P[1..i] = T[j - i + 1..j], \\
0 & \text{Otherwise.}
\end{cases} 
\]  

(1)

The vector \(R_{j+1}\) can be computed after \(R_j\) as follows. For each \(R_j[i] = 0\):

\[
R_{j+1}[i + 1] = \begin{cases} 
1 & \text{if } P_{i+1} = T_{j+1}, \\
0 & \text{Otherwise.}
\end{cases} 
\]  

(2)

and
$$R_{j+1}[0] = \begin{cases} 1 & \text{if } P_0 = T_{j+1}, \\ 0 & \text{Otherwise.} \end{cases}$$ (3)

If \(R_{j+1}[m] = 1\) then a complete match can be reported. The transition from \(R_j\) to \(R_{j+1}\) can be computed very fast as follows. For each \(c \in \Sigma\) let \(D_c\) be a bit array of size \(m\) such that for \(1 \leq i \leq m\), \(D_c[i] = 1\) if and only if \(P_i = c\). The array \(D_c\) denotes the positions of the character \(c\) in the pattern \(P\). Each \(D_c\) for all \(c \in \Sigma\) can be preprocessed before the pattern search. Then the computation of \(R_{j+1}\) reduces to two simple operations, namely, shift and and as follows:

\[R_{j+1} = \text{SHIFT}(R_j) \land D_{T_{j+1}}\]

### 4.2 The First Algorithm: SMALGO-I

In this section, we present a modification of the shift-and algorithm to solve the swap matching problem using the graph model presented in Section 3. In what follows the resulting algorithm shall be referenced to as SMALGO-I. The idea of SMALGO-I is described below.

First of all, the shift-and algorithm can be extended easily for the degenerate patterns [5]. In our swap matching model the pattern can be thought of having a set of letters at each position as follows: \(\tilde{P} = [M[0,1]M[+1,1] [M[-1,2]M[0,2]M[+1,2]] \ldots [M[-1,m-1]M[0,m-1]M[+1,m-1]] [M[-1,m]M[0,m]]\). Note that we have used \(\tilde{P}\) instead of \(P\) above because, in our case, the sets of characters in the consecutive positions in the pattern \(P\) don’t have the same relation as in a usual degenerate pattern. In particular, in our case, a match at position \(i+1\) of \(P\) will depend on the previous match of position \(i\) as the following example shows.

**Example 5.** Suppose, \(P = acbab\) and \(T = bchaaabca\). The \(P\)-graph of \(P\) is shown in Figure 2. So, in line of above discussion, we can say that \(\tilde{P} = [ac][acb][cha][ba][ab]\).

Now, as can be easily seen, if we consider degenerate match, then \(\tilde{P}\) matches \(T\) at Positions 2 and 6. However, \(P\) swap matches \(T\) only at Position 6; not at Position 2. To elaborate, note that at Position 2, the match is due to ‘c’. So, according to the graph \(\tilde{P}^G\) the next match has to be an ‘a’ and hence at Position 2 we can’t have a swap match.

For the sake of convenience, in the discussion that follows, we refer to both \(\tilde{P}\) and the pattern \(P\) as though they were equivalent; but it will be clear from the context what we really mean. Suppose we have a match up to position \(i < m\) of \(\tilde{P}\) in \(T[j - i + 1..j]\). Now we have to check whether there is a match between \(T_{j+1}\) and \(P_{i+1}\). For simple degenerate match, we only need to check whether \(T_{j+1} \in P_{i+1}\) or not. However, as Example 5 shows, for our case we need to do more than that.

In what follows, we present a novel technique to adapt the shift-and algorithm to tackle the above situation. Suppose that \(T_j = c = M[\ell,i]\). Now, from the \(P\)-graph we know which of the \(M[k,i+1], k \in [-1,0,+1]\) will follow \(M[\ell,i]\) and...
which of the $M[k, i + 2], k \in [-1, 0, +1]$ will follow $M[q, i + 1]$. So, for example, even if $M[q, i + 1] = T[j + 1]$ we can’t continue if there is no edge from $M[\ell, i]$ to $M[q, i + 1]$ or from $M[q, i + 1]$ to $M[r, i + 2]$ in the $P$-graph.

To tackle this, we define a new notion. Consider 3-member vertex sets $\{u_0, u_1, u_2\}$ and $\{x_0, x_1, x_2\}$ of $P$-graph such that there exist edges $(u_i, u_{i+1})$ and $(x_i, x_{i+1})$, for all $i$ where $0 \leq i < 2$. Then the edge $(u_0, u_1)$ and $(x_0, x_1)$ are considered to be same if and only if, $label(u_i) = label(x_i)$ for all $i$ where $0 \leq i < 2$.

Also, given an edge $(u_0, u_1) \equiv (M[i_1, j_1], M[i_2, j_2])$, we say that edge $(u_0, u_1)$ ‘belongs to’ column $j_2$, i.e., where the edge ends; and we say $col((u_0, u_1)) \equiv col((M[i_1, j_1], M[i_2, j_2])) = j_2$. Now we traverse all the edges and construct a set of sets $S = \{S_1 \ldots S_\ell\}$ such that each $S_i, 1 \leq i \leq \ell$ contains the edges that are ‘same’. The set $S_i$ is named by $\{u_0, u_1, u_2\}$ and we may refer to $S_i$ using its name. Now, we construct $P$-masks $P_{S_j}, 1 \leq j \leq \ell$ such that $P_{S_i}[g] = 1$ if and only if, there is a set of three vertices $\{u_0, u_1, u_2\}$ such that there exists an edge between each $(u_i, u_{i+1})$ where $0 \leq i \leq 1$ and $\{u_0, u_1, u_2\} \in S_j$ having $col((u_0, u_1)) = g$. With the $P$-masks at our hand, we compute $R_{j+1}$ as follows:

$$R_{j+1} = RSHIFT(R_j) \text{ AND } DT_{j+1} \text{ AND } LSHIFT(DT_{j+2}) \text{ AND } P(T_j, T_{j+1}, T_{j+2})$$  \hspace{1cm} (4)

Here, RSHIFT indicates right shift, LSHIFT indicates left shift and AND is the usual bitwise AND operation. Note that, to locate the appropriate $P$-mask, we again need to perform a look up in the database constructed during the construction of the $P$-masks. Since a particular $P$-mask involves a set of 3 (consecutive) vertices, we need a 3D array to ensure constant time reference to it. Note that in Equation 4, we have referred to the $P$-mask using the 3 vertices of the corresponding vertex set. Example 6 presents a complete execution of our algorithm.

**Example 6.** Suppose, $P = acbab$ and $T = acbbababab$. The $D$-masks of $P$ are shown in Table 1. The $P$-masks of $P$ are shown in Table 2. Table 3 shows the detail computation of $R$. Explanation of the terms used in the Table 3 are as follows:

- $R$ Right Shift Operation on the previous column
- $D_x$ $D$-Mask value for character ‘x’
- $L D_x$ Left Shift Operation on $D_x$
- $P(x, y, z)$ $P$-Mask value of the set $(x, y, z)$
- $R_j$ Value of $R$ after $T_j$ has processed ( ‘1’ in m-1 th row of $R_j$ column indicates that a match has been found ending at the corresponding column.)

The preprocessing is formally presented in Algorithm 1. The main algorithm is presented in Algorithm 2.

### 4.3 Analysis of SMALGO-I

The running times of the different phases of SMALGO-I is listed in Table 4. In the algorithm, we first initialize all the entries of $P$-masks which requires
Algorithm 1 Computation of Preprocessing $[P$-masks$]$ for SMALGO-I

Require: Pattern $p$

1: $x \leftarrow \text{pattern}_\text{length}$
2: for $i = 0$ to $x - 2$ do
3: \hspace{1em} $p_{\text{mask}}[p[i]] [p[i + 1]] [p[i + 2]] \leftarrow p_{\text{mask}}[p[i]] [p[i + 1]] [p[i + 2]](1 << (x - i - 2))$
4: \hspace{1em} $p_{\text{mask}}[p[i]] [p[i + 2]] [p[i + 1]] \leftarrow p_{\text{mask}}[p[i]] [p[i + 2]] [p[i + 1]](1 << (x - i - 2))$
5: end for
6: for $i = 0$ to $x - 3$ do
7: \hspace{1em} $p_{\text{mask}}[p[i]] [p[i + 1]] [p[i + 3]] \leftarrow p_{\text{mask}}[p[i]] [p[i + 1]] [p[i + 3]](1 << (x - i - 2))$
8: end for
9: for $i = 1$ to $x - 1$ do
10: \hspace{1em} $p_{\text{mask}}[p[i]] [p[i - 1]] [p[i + 1]] \leftarrow p_{\text{mask}}[p[i]] [p[i - 1]] [p[i + 1]](1 << (x - i - 1))$
11: end for
12: for $i = 1$ to $x - 2$ do
13: \hspace{1em} $p_{\text{mask}}[p[i]] [p[i - 1]] [p[i + 2]] \leftarrow p_{\text{mask}}[p[i]] [p[i - 1]] [p[i + 2]](1 << (x - i - 1))$
14: end for
15: for $i = 0$ to $x - 3$ do
16: \hspace{1em} $p_{\text{mask}}[p[i]] [p[i + 3]] [p[i + 2]] \leftarrow p_{\text{mask}}[p[i]] [p[i + 3]] [p[i + 2]](1 << (x - i - 3))$
17: end for
18: for $i = 0$ to $x - 4$ do
19: \hspace{1em} $p_{\text{mask}}[p[i]] [p[i + 2]] [p[i + 4]] \leftarrow p_{\text{mask}}[p[i]] [p[i + 2]] [p[i + 4]](1 << (x - i - 3))$
20: end for
21: for $i = 0$ to $x - 3$ do
22: \hspace{1em} $p_{\text{mask}}[p[i]] [p[i + 2]] [p[i + 3]] \leftarrow p_{\text{mask}}[p[i]] [p[i + 2]] [p[i + 3]](1 << (x - i - 3))$
23: end for
24: return $P$-mask $p_{\text{mask}}[]$ for Pattern $p$
Here, $P(x,x,x)$ indicates the edges that are not present in the $P$-graph.

Table 3. Detailed Calculation for text in Example 6 for SMALGO-I
Algorithm 2 SMALGO-I

1: \( R_0 \leftarrow 2^{\text{pattern length}} - 1 \)
2: \( R_0 \leftarrow R_0 \& D_T \)
3: \( R_1 \leftarrow R_0 >> 1 \)
4: \( x \leftarrow 2 \)
5: \( i := 0 \)
6: for \( j = 0 \) to \((n - 3)\) do
7: find \( D_j \) for \text{Text}[j]
8: \( R_j \leftarrow R_j \& \text{pmask}(T_j, T_j+1, T_j+2) \& D_{T_{j+1}} \& (D_{T_{j+2}} << 1) \)
9: if \( (R_j \& x) = x \) then
10: Match found ending at position \((j - 1)\)
11: end if
12: \( R_{j+1} \leftarrow R_j >> 1 \)
13: end for

| Phase                  | Running Time          |
|------------------------|-----------------------|
| Computation of \(D\)-masks | \(O(m/w(m + |\Sigma|))\) |
| Computation of \(P\)-masks  | \(O(m/w(m + |\Sigma|^3))\) |
| Running time of Algorithm 2 | \(O(m/w \ n)\) |

Table 4. Running times of the different phases of SMALGO-I

\(O(m/w|\Sigma|^3)\) time. Then, we start traversing the edges and corresponding \(P\)-masks in a name database (3-D array). Finding and updating the \(P\)-mask of corresponding edges can be done in constant time. As we have \(O(m)\) edges, the total time needed for the computation of \(P\)-masks is \(O(m/w(m + |\Sigma|^3))\).

The computation of \(D\)-masks takes \(O(m/w(m + |\Sigma|))\) time [5] when pattern is not degenerate. However, in our case, we need to assume that our pattern has a set of letters in each position. In this case, we require \(O(m/w(m' + |\Sigma|))\) time where \(m'\) is the sum of the cardinality of the sets at each position [5].

In general degenerate strings, \(m'\) can be \(m|\Sigma|\) in the worst case. However, in our case, \(m' = |V^P| = O(m)\), where \(V^P\) is the vertex set of the \(P\)-graph. So, computation of the \(D\)-mask requires \(O(m/w(m + |\Sigma|))\) time in the worst case.

So the whole preprocessing takes \(O(m/w(m + |\Sigma|^3 + m + |\Sigma|))\) time. Assuming constant alphabet \(\Sigma\) and pattern of size compatible with machine word length the preprocessing time becomes \(O(m)\).

With the \(P\)-masks and \(D\)-masks at our hand, for our problem, we simply need to compute \(R_j\) using Equation 4. So, in total the construction of \(R\) values require \(O(m/w \ n)\) which is \(O(n)\) if \(m \sim w\). Therefore, in total the running time for SMALGO-I, is linear assuming a constant alphabet and a pattern size similar to the word size of the target machine.
4.4 The Second Algorithm: SMALGO-II

In this section, we present another algorithm which is more space efficient. Instead of a 3-D array we need only 2-D arrays here. In order to understand the new algorithm, we need the following definitions.

Definition 9. A level change indicates a change of row in the Matrix M having one of the following cases:

- A Upward Change, i.e., going from a position \((+1, j)\) to \((-1, j + 1)\);
- A Downward Change, i.e., going from a position \((i, j)\) to \((+1, j + 1)\) where \(i = 0\) or \(i = -1\);
- A End of Swap/Middle-ward Change, i.e., going from a position \((-1, j)\) to \((0, j + 1)\).

In this approach, we only need to know which of the \(M[k, i + 1]_{k \in [-1, 0, +1]}\) will follow \(M[\ell, i]\) in the \(P\)-graph. Thus we have to generate \(P\)-masks in the following way. Here we change the notion of two edges being 'same' as follows.

Two edges \((u, v), (x, y)\) of the \(P\)-graph are said to be 'same' if \(label(u) = label(x)\) and \(label(v) = label(y)\), i.e., if the two edges have the same labels. Also, given an edge \((u, v) = (M[i_1, j_1], M[i_2, j_2])\), we say that edge \((u, v)\) 'belongs to' column \(j_2\), i.e., where the edge ends; and we say \(col((u, v)) = col((M[i_1, j_1], M[i_2, j_2])) = j_2\). Now we traverse all the edges and construct a set of sets \(S = \{S_1 \ldots S_\ell\}\) such that each \(S_i, 1 \leq i \leq \ell\) contains the edges that are 'same'. The set \(S_i\) is named by the (same) label of the edges it contains and we may refer to \(S_i\) using its name.

Now, we construct \(P\)-masks \(P_{S_i}, 1 \leq i \leq \ell\) such that \(P_{S_i}[k] = 1\) if, and only if, there is an edge \((u, v) \in S_i\) having \(col((u, v)) = k\). Clearly here, \(\ell = O(m)\).

Note that, to locate the appropriate \(P\)-mask, we again need to perform a look up in the database constructed during the construction of the \(P\)-masks. To maintain all \(P\)-masks we are keeping a 2D array indexed by the consecutive vertices of an edge.

|   | \(P_{(a,a)}\) | \(P_{(a,b)}\) | \(P_{(a,c)}\) | \(P_{(b,a)}\) | \(P_{(b,b)}\) | \(P_{(b,c)}\) | \(P_{(c,a)}\) | \(P_{(c,b)}\) | \(P_{(c,x)}\) |
|---|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| 1 | 1            | 1            | 1            | 1            | 1            | 1            | 1            | 1            | 1            |
| 2 | 0            | 1            | 1            | 0            | 0            | 1            | 0            | 0            | 0            |
| 3 | 1            | 1            | 0            | 0            | 1            | 1            | 1            | 0            | 0            |
| 4 | 0            | 1            | 0            | 1            | 0            | 1            | 1            | 0            | 0            |
| 5 | 0            | 1            | 0            | 1            | 1            | 0            | 0            | 0            | 0            |

* Here, \(P_{(x,x)}\) indicates the edges that are not present in the \(P\)-graph.

Table 5. \(P\)-masks for pattern \(acbab\)

However, the \(P\)-masks as defined above, and \(D\)-masks defined before are not sufficient to solve the SM problem as shown below with an example. Please note that the definition of \(P\)-mask in SMALGO-II (i.e., the current algorithm) is different than that of SMALGO-I (i.e., the algorithm presented in section 4.2).
Example 7. In Table 5, at the column named ‘P(b,b)’, the value is 10011 for pattern acbab. The ‘1’ in the rightmost bit indicates that either one or both edge \((M[-1,4], M[0,5]), (M[+1,4], M[0,5])\) exists, as shown in Figure 3. We cannot find out which one actually exists because our \(P\)-mask values are only dependent on the column positions (i.e., the edge starts at Column 4 and ends at Column 5) irrespective of row positions \((-1,0,+1)\). So the algorithm will accept acbbb as a swapped version of the pattern acbab which is clearly a false match.

To solve the problem we need to be able to tell which level change has occurred, Upward Change or Middleward Change. So, we introduce three new masks called up-mask, down-mask and middle-mask as discussed below.

1. We construct up-masks, \(up_{S_j}, 1 \leq j \leq \ell\) such that \(up_{S_j}[g] = 1\) if and only if \(edge(u_0, u_1) \equiv (M[+1,g-1], M[-1,g])\) exists with \(col((u_0, u_1)) = g\).

2. We construct down-masks, \(down_{S_j}, 1 \leq j \leq \ell\) such that \(down_{S_j}[g] = 1\) if and only if either \(edge(u_0, u_1) \equiv (M[-1,g-1], M[+1,g])\) or \(edge(u_0, u_1) \equiv (M[0,g-1], M[+1,g])\) exists with \(col((u_0, u_1)) = g\).

3. We construct middle-masks, \(middle_{S_j}, 1 \leq j \leq \ell\) such that \(middle_{S_j}[g] = 1\) if and only if either \(edge(u_0, u_1) \equiv (M[0,g-1], M[0,g])\) or \(edge(u_0, u_1) \equiv (M[-1,g-1], M[0,g])\) exists with \(col((u_0, u_1)) = g\).

The motivation and usefulness of the 3 masks defined above will be clear from the following discussion. It is easy to see that, to get a match, after a level change at a particular position \((i_1, j)\), another level change must occur at the next position, i.e., at position \((i_2, j+1)\) in the Matrix \(M\); otherwise there can be no match. So we do the following based on the structure of the \(P\)-graph.

1. If a Downward change has occurred then we have to check whether an Upward Change occurs at the next position. We can do that by saving the previous down-mask \(down(T_{j+1}, T_j)\) and matching that value with the current up-mask \(up(T_j, T_{j+1})\) and \(R_j\). Otherwise there can be no match.

2. If an Upward Change has occurred then we have to check whether Downward change or a Middle-ward change occurs at the next position. We can do that by saving the previous up-mask \(up(T_{j-1}, T_j)\) and matching that value with
current down-mask \( (\text{down}(T_j, T_{j+1})) \), middle-mask \( (\text{middle}(T_j, T_{j+1})) \) and \( R_j \).
Otherwise there can be no match.

3. This process continues repeatedly until either an End of Swap occurs or an end of pattern is encountered. To check whether an end of swap occurs we have to keep previous up-mask \( (\text{up}(T_{j-1}, T_j)) \) and match that value with current middle-mask \( (\text{middle}(T_j, T_{j+1})) \) and \( R_j \).

In our algorithm, each of the previous checkings have to be done while we process each character. The algorithm is formally presented in Algorithm 4. The preprocessing of the algorithm is presented in Algorithm 3. In the algorithm, we are using a 2-D array for \( P \)-masks, up-masks, down-masks and middle-masks.

Example 8 shows a complete execution of our algorithm.

Example 8. Suppose, \( P = acbab \) and \( T = acbbabcabab \). The \( D \)-masks of \( P \) are shown in Table 1. The \( P \)-masks of \( P \) are shown in Table 5. The new mask values are shown in Table 6 and Table 7 shows the detail computation of \( R \) bit array. Explanation of the terms used in Table 7 are as follows:

| SH | D-Mask value for character ‘x’ |
|----|--------------------------------|
| \( D_x \) | \( P \)-Mask value for the set \( (x,y) \) |
| \( P(x,y) \) | \( R_j \) Value of \( R \) after \( T_j \) has been proceed ( ‘1’ in \( m \)-th row of \( R_j \) column indicates that a match has been found ) |

| up-mask | middle-mask | down-mask |
|---------|-------------|-----------|
| (a,a)   | 00000       | 00000     | 00100     |
| (a,b)   | 00010       | 00101     | 01000     |
| (a,c)   | 00000       | 01000     | 10000     |
| (b,a)   | 00001       | 00011     | 00000     |
| (b,b)   | 00000       | 00001     | 00010     |
| (b,c)   | 00100       | 00000     | 10000     |
| (c,a)   | 01000       | 00010     | 00100     |
| (c,b)   | 00000       | 00100     | 00010     |

Table 6. Masks for Algorithm 2 of the pattern in Example 8

4.5 Analysis of SMALGO-II

The running times of the different phases of SMALGO-II is listed in Table 8. In SMALGO-II, we first initialize all the entries of \( P \)-masks which requires \( O(m/w|\Sigma|^2) \) time. Then, we start traversing the edges and corresponding \( P \)-masks in a name database (2-D array). Finding and updating the \( P \)-mask of corresponding edges can be done in constant time. As we have \( O(m) \) edges, the
Here, $P_{(x,x)}$ indicates the edges that are not present in the $P$-graph.

Table 7. Detailed Calculation for text in Example 8
**Algorithm 3** Algorithm for Computation of all the Masks [Preprocessing] for SMALGO-II

**Require:** Pattern p

1: \( x \leftarrow \text{pattern}_k \)
2: for \( i = 0 \) to \( x - 2 \) do
3: \( \text{pmask}[p[i]][p[i + 1]] \leftarrow \text{pmask}[p[i]][p[i + 1]](1 << (x - i - 2)) \)
4: \( \text{middle}[p[i]][p[i + 1]] \leftarrow \text{middle}[p[i]][p[i + 1]](1 << (x - i - 2)) \)
5: end for
6: for \( i = 0 \) to \( x - 3 \) do
7: \( \text{pmask}[p[i]][p[i + 2]] \leftarrow \text{pmask}[p[i]][p[i + 2]](1 << (x - i - 2))(1 << (x - i - 3)) \)
8: \( \text{down}[p[i]][p[i + 2]] \leftarrow \text{down}[p[i]][p[i + 2]](1 << (x - i - 2)) \)
9: \( \text{middle}[p[i]][p[i + 2]] \leftarrow \text{middle}[p[i]][p[i + 2]](1 << (x - i - 3)) \)
10: end for
11: for \( i = 1 \) to \( x - 1 \) do
12: \( \text{pmask}[p[i]][p[i - 1]] \leftarrow \text{pmask}[p[i]][p[i - 1]](1 << (x - i - 1)) \)
13: \( \text{up}[p[i]][p[i - 1]] \leftarrow \text{up}[p[i]][p[i - 1]](1 << (x - i - 1)) \)
14: end for
15: for \( i = 1 \) to \( x - 4 \) do
16: \( \text{pmask}[p[i]][p[i + 3]] \leftarrow \text{pmask}[p[i]][p[i + 3]](1 << (x - i - 3)) \)
17: \( \text{down}[p[i]][p[i + 3]] \leftarrow \text{up}[p[i]][p[i + 3]](1 << (x - i - 3)) \)
18: end for
19: for \( i = 1 \) to \( x - 1 \) do
20: \( d[p[i]] \leftarrow d[p[i]]((1 << (x - i - 1))(1 << (x - i - 1))((1 << (x - i - 2))) \)
21: end for
22: return P-masks pmask[], D-masks d[], up-masks up[], down-masks down[] and middle-masks middle[] for Pattern p

| Phase                          | Running Time          |
|-------------------------------|-----------------------|
| Computation of D-masks        | \( O(m/w(m + \Sigma^*)) \) |
| Computation of P-masks        | \( O(m/w(m + \Sigma)) \) |
| Computation of Up-masks       | \( O(m/w(m + \Sigma)) \) |
| Computation of Down-masks     | \( O(m/w(m + \Sigma^*)) \) |
| Computation of Middle-masks   | \( O(m/w(m + \Sigma^*)) \) |
| Running time of Algorithm 4   | \( O(m/w n) \)         |

**Table 8.** Running times of the different phases of of SMALGO-II
Algorithm 4 SMALGO-II

Require: Text T, up-mask up, down-mask down, middle-mask middle, P-mask pmask, D-mask D for given pattern p

1: \( R_0 \leftarrow 2^{\text{pattern length}} - 1 \)
2: checkup \( \leftarrow \) checkdown \( \leftarrow 0 \)
3: \( R_0 \leftarrow R_0 \& D_{T_0} \)
4: \( R_1 \leftarrow R_0 >> 1 \)
5: \( x \leftarrow 1 \)
6: \textbf{for} \( j = 0 \) to \( (n - 2) \) \textbf{do}
7: \( R_j \leftarrow R_j \& \text{pmask} (T_j, T_{j+1}) \& D_{T_{j+1}} \)
8: \( \text{temp} \leftarrow \text{prevcheckup} >> 1 \)
9: \( \text{checkup} \leftarrow \text{checkup} \mid \text{up} (T_j, T_{j+1}) \)
10: \( \text{checkup} \leftarrow \text{checkup} \& \sim \text{down} (T_j, T_{j+1}) \& \sim \text{middle} (T_j, T_{j+1}) \)
11: \( \text{prevcheckup} \leftarrow \text{checkup} \)
12: \( R_j \leftarrow (\text{temp} \& \text{checkup}) \& R_j \)
13: \( \text{temp} \leftarrow \text{prevcheckdown} >> 1 \)
14: \( \text{checkdown} \leftarrow \text{checkdown} \mid \text{down} (T_j, T_{j+1}) \)
15: \( \text{checkdown} \leftarrow \text{checkdown} \& \sim \text{up} (T_j, T_{j+1}) \)
16: \( \text{prevcheckdown} \leftarrow \text{checkdown} \)
17: \( R_j \leftarrow (\text{temp} \& \text{checkdown}) \& R_j \)
18: \textbf{if} \( (R_j \& x) = x \) \textbf{then}
19: \hspace{1em} Match found ending at position \( (j - 1) \)
20: \textbf{end if}
21: \( R_{j+1} \leftarrow R_j >> 1 \)
22: \( \text{checkup} \leftarrow \text{checkup} >> 1 \)
23: \( \text{checkdown} \leftarrow \text{checkdown} >> 1 \)
24: \textbf{end for}
total time needed for computation of \(P\)-mask is \(O(m/w(m + |\Sigma|^2))\). Similarly, the computation of up-masks, down-masks and middle-masks can be done in \(O(m/w(m + |\Sigma|^2))\) time as well. The computation of \(D\)-mask takes \(O(m/w(m + |\Sigma|))\) time. So the whole preprocessing takes \(O(m/w(4(m + |\Sigma|^2) + m + |\Sigma|)))\) time. Assuming constant alphabet \(\Sigma\) and pattern of size compatible with machine word length the preprocessing time becomes \(O(m)\).

With all the masks at our hand, for our problem, we simply need to compute \(R_j\) by some simple calculation. Each step of the calculation, including locating the appropriate masks, needs constant amount of time. So, in total the construction of \(R\) values require \(O(m/w n)\) which is \(O(n)\) when \(w \sim m\).

Therefore, in total the running time for SMALGO-II, is linear assuming a constant alphabet and a pattern size similar to the word size of the target machine.

5 Experimental Results

We have conducted extensive experiments to compare the actual running time of the existing (non FFT) swap matching algorithms in the literature [9, 10] with ours. In this section, we present our findings based on the experiments conducted. The following acronyms are used in the presented results to identify different algorithms.

- CS CROSS-SAMPLING algorithm of [9]
- BPCS BP-CROSS-SAMPLING algorithm of [9]
- BCS BACKWARD-CROSS-SAMPLING algorithm of [10]
- BPBCS BP-BACKWARD-CROSS-SAMPLING algorithm of [10]
- ALG-I SMALGO-I of this paper
- ALG-II SMALGO-II of this paper

We have chosen to exclude the naive algorithm and all algorithms in the literature based on FFT techniques from our experiments, because, the overhead of such algorithms is quite high resulting in a bad performance. All algorithms have been implemented in Microsoft Visual C++ in Release Mode on a PC with Intel Pentium D processor of 2.8 GHz having a memory of 2GB.

5.1 Datasets

All algorithms have been tested on random texts, on a Genome sequence, on a Protein sequence and on a natural language text buffer with patterns of length, \(m = 4, 8, 12, 16, 20, 24, 28, 32\). In the Tables below running times have been expressed in the hundredth of a second and best results are highlighted.

In the case of random texts we have adopted a similar strategy of [9, 10]. In particular, the algorithm has been tested on six \(Rand\Sigma\) problem sets (for \(|\Sigma| = 4, 8, 16, 32, 64\) and 128). Each \(Rand\Sigma\) problem consists in searching a set of 100 random patterns for any given length value in a 4MB long random text over
a common alphabet of size $|\Sigma|$. In order to make the test more effective, in our experiments half of the patterns are randomly chosen and rests are picked from the text randomly so that they surely appear in the text at least once.

We also follow a strategy similar to that of [9, 10] for the tests on real world problems. We have been performed tests on a Genome sequence, on a protein sequence and on a natural text buffer. The genome sequence we used for the tests is a sequence of 4,638,690 base pairs of *Escherichia coli* taken from the file *E.coli* of the large Canterbury Corpus [11]. The tests on the Protein sequence have been performed using a 2.4MB file containing a protein sequence from the Human Genome with 22 different characters. The experiments on the natural language text buffer have been done on the file *world192.txt* (The CIA World Fact Book) of the Large Canterbury Corpus [11]. This file contains 2,473,400 characters drawn from an alphabet of 93 different characters.

5.2 Running Times of Random Problems

The running times for different algorithms for this experiment are reported in Tables 9 - 14. From the results, we see that, in general, ALG-I (SMALGO-I) runs faster than BPBCS for smaller patterns and small alphabet size whereas BPBCS performs better when pattern and alphabet size are relatively large.

| m  | 4     | 8     | 12    | 16    | 20    | 24    | 28    | 32    |
|----|-------|-------|-------|-------|-------|-------|-------|-------|
| CS | 61.247| 61.137| 61.252| 61.043| 61.468| 63.472| 68.489| 66.090|
| BCS| 33.366| 22.865| 18.523| 16.580| 15.289| 14.585| 13.628| 13.087|
| BPCS| 1.914 | 1.867 | 1.849 | 1.864 | 1.861 | 1.908 | 1.859 | 1.860 |
| BPBCS| 3.552 | 2.001 | 1.451 | 1.124 | 0.968 | 0.835 | 0.737 | 0.672 |
| ALG-II| 4.062 | 4.081 | 4.090 | 4.093 | 4.124 | 4.082 | 4.085 | 4.095 |
| ALG-I| 0.631 | 0.626 | 0.631 | 0.631 | 0.636 | 0.636 | 0.640 | 0.629 |

Table 9. Running time for Rand4 problems

| m  | 4     | 8     | 12    | 16    | 20    | 24    | 28    | 32    |
|----|-------|-------|-------|-------|-------|-------|-------|-------|
| CS | 52.447| 52.407| 52.356| 52.357| 52.457| 52.424| 52.443| 52.424|
| BCS| 23.139| 16.532| 12.539| 10.810| 9.701 | 9.111 | 8.502 | 7.996 |
| BPCS| 1.861 | 1.858 | 1.857 | 1.864 | 1.896 | 1.905 | 2.001 | 1.853 |
| BPBCS| 2.156 | 1.319 | 0.944 | 0.743 | 0.621 | 0.533 | 0.476 | 0.421 |
| ALG-II| 4.061 | 4.066 | 4.051 | 4.062 | 4.060 | 4.059 | 4.064 | 4.063 |
| ALG-I| 0.674 | 0.684 | 0.671 | 0.675 | 0.654 | 0.676 | 0.633 | 0.633 |

Table 10. Running time for Rand8 problems
### Table 11. Running time for Rand16 problems

| m     | 4      | 8      | 12     | 16     | 20     | 24     | 28     | 32     |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|
| CS    | 51.632 | 52.622 | 53.139 | 53.450 | 51.829 | 55.243 | 53.136 | 52.847 |
| BCS   | 18.718 | 13.350 | 11.130 | 9.084  | 7.711  | 7.045  | 6.578  | 6.103  |
| BPCS  | 1.864  | 1.870  | 1.857  | 1.844  | 1.862  | 1.851  | 1.856  | 1.864  |
| BPBCS | 1.340  | 0.917  | 0.700  | 0.555  | 0.459  | 0.393  | 0.349  | 0.314  |
| ALG-II| 4.063  | 4.062  | 4.066  | 4.063  | 4.075  | 4.083  | 4.062  | 4.074  |
| ALG-I | **0.636** | **0.634** | **0.662** | 0.631  | 0.635  | 0.640  | 0.632  | 0.643  |

### Table 12. Running time for Rand32 problems

| m     | 4      | 8      | 12     | 16     | 20     | 24     | 28     | 32     |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|
| CS    | 52.644 | 54.104 | 51.123 | 50.795 | 50.045 | 54.296 | 53.458 |
| BCS   | 16.628 | 11.472 | 8.682  | 7.618  | 6.684  | 5.966  | 5.474  | 5.318  |
| BPCS  | 1.862  | 1.863  | 1.866  | 1.858  | 1.862  | 1.851  | 1.910  | 1.856  |
| BPBCS | 0.950  | 0.637  | **0.532** | **0.453** | **0.394** | **0.363** | **0.307** | **0.274** |
| ALG-II| 4.100  | 4.094  | 4.104  | 4.093  | 4.102  | 4.108  | 4.099  | 4.101  |
| ALG-I | **0.636** | **0.633** | 0.630  | 0.632  | 0.629  | 0.631  | 0.631  | 0.629  |

### Table 13. Running time for Rand64 problems

| m     | 4      | 8      | 12     | 16     | 20     | 24     | 28     | 32     |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|
| CS    | 49.482 | 47.775 | 50.448 | 49.668 | 52.965 | 51.000 | 52.663 | 53.103 |
| BCS   | 14.892 | 9.850  | 7.615  | 6.481  | 6.692  | 5.484  | 5.242  | 5.066  |
| BPCS  | 1.846  | 1.855  | 1.864  | 1.919  | 1.923  | 1.851  | 1.917  | 1.863  |
| BPBCS | 0.739  | **0.475** | **0.370** | **0.334** | **0.294** | **0.282** | **0.271** | **0.233** |
| ALG-II| 4.334  | 4.341  | 4.342  | 4.336  | 4.347  | 4.355  | 4.390  | 4.341  |
| ALG-I | **0.635** | 0.635  | 0.686  | 0.626  | 0.637  | 0.636  | 0.632  | 0.630  |

### Table 14. Running time for Rand128 problems

| m     | 4      | 8      | 12     | 16     | 20     | 24     | 28     | 32     |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|
| CS    | 49.939 | 48.608 | 50.570 | 52.593 | 50.797 | 50.075 | 50.075 | 49.481 | 49.640 |
| BCS   | 14.411 | 9.541  | 7.513  | 6.546  | 6.769  | 5.242  | 4.743  | 4.573  |
| BPCS  | 1.855  | 1.866  | 1.849  | 1.851  | 1.855  | 1.858  | 1.861  | 1.864  |
| BPBCS | 0.688  | **0.430** | **0.325** | **0.288** | **0.274** | **0.229** | **0.216** | **0.204** |
| ALG-II| 4.429  | 4.431  | 4.426  | 4.477  | 4.408  | 4.032  | 4.422  | 4.422  |
| ALG-I | **0.651** | 0.661  | 0.661  | 0.640  | 0.653  | 0.638  | 0.633  | 0.644  |
| m   | 4   | 8   | 12  | 16   | 20   | 24   | 28   | 32   |
|-----|-----|-----|-----|------|------|------|------|------|
| CS  | 74.771 | 74.729 | 74.890 | 74.564 | 73.251 | 77.605 | 71.991 | 73.472 |
| BCS | 78.499 | 52.828 | 43.492 | 38.730 | 35.401 | 33.078 | 31.815 | 30.234 |
| BPCS| 2.224 | 2.157 | 2.152 | 2.146 | 2.152 | 2.159 | 2.164 |      |
| BPBCS| 3.977 | 2.228 | 1.619 | 1.283 | 1.084 | 0.930 | 0.830 | 0.740 |
| ALG-II | 4.732 | 4.734 | 4.739 | 4.744 | 4.745 | 4.730 | 4.709 | 4.711 |
| ALG-I | 0.600 | 0.619 | 0.639 | 0.596 | 0.611 | 0.606 | 0.582 | 0.583 |

Table 15. Running time for a genome sequence ($\Sigma = 4$)

| m   | 4   | 8   | 12  | 16   | 20   | 24   | 28   | 32   |
|-----|-----|-----|-----|------|------|------|------|------|
| CS  | 55.764 | 54.316 | 55.594 | 51.737 | 50.797 | 50.491 | 50.154 | 50.889 |
| BCS | 33.825 | 24.180 | 21.250 | 17.179 | 14.344 | 13.464 | 12.506 | 12.075 |
| BPCS| 1.852 | 1.863 | 1.866 | 1.875 | 1.859 | 1.859 | 2.027 |      |
| BPBCS| 1.107 | 0.771 | 0.617 | 0.504 | 0.428 | 0.367 | 0.314 | 0.282 |
| ALG-II | 4.064 | 4.086 | 4.098 | 4.062 | 4.068 | 4.056 | 4.076 | 4.075 |
| ALG-I | 0.613 | 0.606 | 0.599 | 0.611 | 0.618 | 0.602 | 0.613 | 0.622 |

Table 16. Running time for a protein sequence ($\Sigma = 22$)

| m   | 4   | 8   | 12  | 16   | 20   | 24   | 28   | 32   |
|-----|-----|-----|-----|------|------|------|------|------|
| CS  | 30.205 | 33.177 | 30.626 | 33.344 | 28.872 | 31.382 | 30.744 | 29.362 |
| BCS | 29.917 | 26.244 | 25.232 | 28.203 | 30.407 | 26.468 | 30.966 | 26.778 |
| BPCS| 1.178 | 1.166 | 1.181 | 1.154 | 1.146 | 1.150 | 1.146 | 1.132 |
| BPBCS| 0.793 | 0.703 | 1.657 | 0.723 | 0.768 | 0.691 | 0.738 | 0.694 |
| ALG-II | 2.508 | 2.510 | 2.506 | 2.507 | 2.546 | 2.501 | 2.510 | 2.505 |
| ALG-I | 0.632 | 0.620 | 0.625 | 0.616 | 0.609 | 0.634 | 0.618 | 0.608 |

Table 17. Running time for a natural language text buffer ($\Sigma = 93$)
5.3 Running Times for Real World Problems

The running time of different algorithms in these different experiments are reported in Tables 15 - 17. From the experiments, we see that ALG-I runs faster for all pattern lengths in genome sequence and natural language text buffer. However in protein sequence, ALG-I performs best for smaller patterns whereas BPBCS performs better for larger patterns.

6 Conclusion

In this paper, we have revisited the Swap Matching problem, a well-studied variant of the classic pattern matching problem. In particular, we have presented a graph theoretic model to solve the swap matching problem and devised two novel algorithms based on this model. The resulting algorithms are adaptations of the classic shift-and algorithm [6] and runs in linear time for finite alphabet if the pattern-length is similar to the word-size in the target machine. Note that our algorithms like the work of [9, 10] does not use FFT techniques. Though both algorithms are based on the same classic shift-and algorithm, they are different in their technique/approach. Moreover, the techniques used in our algorithms are quite simple and easy to implement as well as understand.

We believe that our graph theoretic model could be used to devise more efficient algorithms and a similar approach can be taken to model similar other variants of the classic pattern matching problem. Furthermore, it would be interesting to ‘swap’ the definitions of $T$-graph and $P$-graph and investigate whether efficient pattern matching techniques for Directed Acyclic Graphs can be employed to devise efficient off-line and online algorithms for swap matching.

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