Parallel interaction-free measurement using spatial adiabatic passage

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Abstract. Interaction-free measurement (IFM) is a surprising consequence of quantum interference, where the presence of objects can be sensed without any disturbance of the object being measured. Here, we show an extension of IFM using techniques from spatial adiabatic passage, specifically multiple recipient adiabatic passage. Due to subtle properties of the adiabatic passage, it is possible to image an object without interaction between the imaging photons and the sample. The technique can be used on multiple objects in parallel and is entirely deterministic in the adiabatic limit. Unlike more conventional IFM schemes, this adiabatic process is driven by the symmetry of the system, and not by more usual interference effects. As such it provides an interesting alternative quantum protocol that may be applicable to photonic implementations of spatial adiabatic passage. We also show that this scheme can be used to implement a collision-free quantum routing protocol.

S Online supplementary data available from \url{stacks.iop.org/NJP/13/125002/mmedia}

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1. Introduction

One of the counter-intuitive effects of quantum mechanics is that of interaction-free measurement (IFM). Classically, one can only obtain information about the location of an object by interacting particles with it: for example, by absorption or deflection of the particles. In every case there is an inevitable interaction between the sensing particles and the object being sensed. However, Dicke [1] and later Elitzur and Vaidman [2] showed that it is possible to perform an IFM, i.e. a measurement where the sensing particles have never interacted with the object and yet provide definitive information about its existence. To illustrate IFMs, we summarize the canonical IFM: the ‘quantum bomb’ problem.

Assume that we are searching for a bomb. However, this bomb is so sensitive that if a single photon touches it, it will explode: an undesirable outcome. How can we design an optical sensing system that detects the presence or absence of the bomb without setting it off? Elitzur and Vaidman showed that this task can be achieved non-deterministically. They considered a Mach–Zehnder interferometer, balanced so that when both arms are unobstructed, photons always exit via the bright port. However, if a bomb is placed in one of the arms, then there is a 50% chance of detecting the photon at the dark port and hence the detection at the dark port proves the presence of a bomb without the photon having interacted with the bomb.

The quantum bomb protocol is non-deterministic: the bomb is triggered 50% of the time, and the detection of a photon at the bright port does not provide information about the presence or absence of the bomb. Nonetheless, this is an important protocol that performs a task not possible classically. The success probability of this protocol can be asymptotically increased to unity [3]. It is also possible to perform imaging and this was used to demonstrate high-resolution images [4]. Variants of IFM lead to the possibility of counter-factual quantum computation, where the result of a quantum algorithm is determined without the qubits actually performing the algorithm [5].

All of the above protocols have at their heart interferometry. As such they are sensitive to issues such as alignment and/or timing. We are considering an alternative approach to IFM based on adiabatic passage. As such, instead of relying on interferometry to carry out the measurement, we use the symmetry of the problem, in particular the composition of the null space of the solution to the adiabatic network, as will be described below. This gives a fundamentally different approach to the task of IFM, which gives rise quite naturally to a parallel search with robustness and deterministic sensing. Returning to the quantum bomb
Figure 1. Multi-waveguide concept diagram for a four-leaf MRAP device, such as can be realized using direct-write lithography. Waveguides are represented as circular tubes through the device. In this device, the adiabatic protocol coherently sends photons from the input node, $|0_e\rangle$, to an equally weighted superposition $(1/2)(|11_e\rangle + |1\bar{1}_e\rangle + |\bar{1}1_e\rangle + |\bar{1}\bar{1}_e\rangle)$, as indicated.

analogy, we can think of our scheme as allowing the imaging of a quantum minefield, with multiple quantum bombs distributed at unknown spatial locations. The adiabatic network can best be achieved using an integrated multi-waveguide approach, leveraging the advances in direct-write lithography [6] for quantum photonics [7] in the spirit of the waveguide coherent tunnelling adiabatic passage (CTAP) [8–12]. A concept diagram of our envisaged device is shown in figure 1.

Techniques for adiabatic passage are used to evolve quantum states so that the system remains in an instantaneous eigenstate, but the Hamiltonian is varied as a function of time. The requirement for adiabaticity implies that the Hamiltonian must be changed sufficiently slowly so that population is unable to leak between the eigenstates. Such techniques are well known, especially in atomic and molecular systems [13]. Advances in the construction of quantum systems have allowed new perspectives in adiabatic passage, in particular the opportunity to move particles adiabatically through space. To explain this we first discuss CTAP as a precursor adiabatic passage protocol, before showing how this can be extended to multiple recipients.

CTAP is an all-spatial variant of the well-known STIRAP (STImulated Raman Adiabatic Passage) technique [14]. The simplest form of CTAP requires a single particle that can be placed in a coherent superposition of three spatially defined quantum states; particle motion between the two outermost sites is effected by adiabatic variation of the tunnel matrix elements (TMEs) between neighbouring sites. For clarity, we label the sites as $|1\rangle$, $|2\rangle$ and $|3\rangle$, with TMEs between $|1\rangle$ and $|2\rangle$ defined as $A$ and those between $|2\rangle$ and $|3\rangle$ defined as $B$. CTAP uses the counter-intuitive pulse sequence, where the TME between the sites where the particle is not present is initially high, whereas the TME connected to the site with the particle is initially zero. So, for the CTAP of a particle from $|1\rangle$ to $|3\rangle$, initially $A = 0$ and $B$ should be at a maximum. The ratio between the TMEs is varied smoothly as a function of time so that at the end of the protocol the TMEs are reversed, i.e. $B = 0$ and $A$ at a maximum. The counter-intuitive pulse sequence affords considerable robustness to the transport and has the surprising property that the particle is never found at the central site. In addition to the waveguide studies mentioned earlier, CTAP has been studied theoretically for electrons around dopants in semiconductors [15–18], quantum dots [19, 20], atoms [21–24], Bose–Einstein condensates [25, 26] and superconductors [27].

Techniques based on spatial adiabatic passage have a considerable advantage over their more familiar quantum optical counterparts. With atomic and molecular systems, the Hilbert
space of the system is defined by the physics of the system under investigation. In contrast, advances in nano-fabrication give the intriguing ability to engineer a desired Hilbert space by technologies such as lithography or the application of spatially varying optical fields. This ultimately provides new flexibility and the potential to realize novel quantum devices. CTAP can be extended using linear schemes that extend the number of sites over which transport can occur. The quantum optical versions of these extensions are the alternating [28] and straddling [29] STIRAP schemes, and these have been investigated in the context of CTAP. The straddling CTAP scheme was first considered in [15] and later demonstrated with photons [11], and the alternating scheme has also been investigated [19, 30]. The alternating scheme is particularly interesting for interferometric schemes and non-trivial loop topologies because of the transient population, which allows for sensing, and in this context a scheme for an adiabatic electrostatic Aharanov–Bohm interferometer has been proposed [31].

This paper is organized as follows. We first discuss the extension of the multiple recipient adiabatic passage (MRAP) protocol to account for large-scale quantum networks, and discuss some of the properties of the null space of the resulting Hamiltonian. We then show an application of this protocol to the task of parallel IFM in the quantum minefield problem. Following this, we explore some practical limitations in terms of ensuring the adiabaticity of the protocol. Finally, we show how this protocol can be converted into a collision-free quantum routing technique.

2. Multiple recipient adiabatic passage (MRAP) and quantum tree networks

The possibility of engineering Hilbert spaces by spatial location of quantum sites provides many opportunities for the realization of novel devices. Here, we focus on the ability to engineer branched quantum networks. One proposal for a branched adiabatic passage scheme is MRAP [32]. This was investigated as a form of quantum fanout, useful for the direct synthesis of operator measurements [33].

The dynamics of the MRAP tree are governed by the system Hamiltonian and in particular the adjacency matrix. Although the actual geometry is arbitrary, it is helpful to keep a definite geometry in mind, and this is depicted schematically in figure 2. We assume that the energy of the particle at any site is the same and hence the Hamiltonian is solely defined by the adjacency matrix. To label the tree, first note that each node other than the zeroth node either has two connections or three connections, corresponding to whether there is an odd number of links to the initial node or an even number, respectively. We choose to label those sites with an odd number of links as o and those with an even number as e. We next number all of the e sites by the path taken to reach them using a balanced ternary notation according to figure 2, where a link to the left of the previous site is denoted by adding the digit 1 to the right-hand side (least significant digit) of the number, and connections to the right by the digit 1. The o sites are numerically labelled with the same number as the site to their immediate left. The depth of the tree is understood to increase with increasing distance from the initial state, \(|0_e\rangle\), i.e. up in figure 2, and is the length of the string describing the node in ternary digits. The tree depth is denoted by \(d\).

The TME connecting states follows the alternating convention, which can also be considered as an \(A - B\) chain. The strength of a connection between states in the order e–o is \(A\), and between o–e is \(B\), as shown in figure 2. The magnitudes of all of the \(A\) TMEs are the same, as are all the \(B\). The values of the couplings are varied according to the
Figure 2. Schematic diagram showing the tree suitable for imaging in a four-leaf device. Nodes are labelled according to their position in the tree. Subscripts 'o' and 'e' represent odd and even numbers of edges, whereas the $i$ and $j$ nodes denote the imaging nodes. The quantum minefield is realized by the quantum bombs at sites that break the connection between the $i$ and $j$ sites, represented by cubes. In this case there are two quantum bombs, between $|1\bar{1}_i\rangle$ and $|1\bar{1}_j\rangle$ and between $|\bar{1}_1\bar{1}_i\rangle$ and $|\bar{1}_1\bar{1}_j\rangle$. The strength of interaction between two waveguides is labelled as 'A' and 'B', and the depth of the tree is labelled as $d$, which is the number of ternary digits used to label the nodes at that depth.

counter-intuitive pulse sequence (also employed in CTAP). This pulse sequence initially has coupling $B$ on, rather than the intuitive way of operating which would raise $A$ first. For simplicity we choose a sinusoidal variation of TME with time, $t$, i.e.

$$A(t) = \sin^2(\pi t/2T), \quad B(t) = \cos^2(\pi t/2T),$$

where $T$ is the total time of the protocol and the maximum values of the TMEs are normalized to unity.

The general form of our MRAP Hamiltonian is

$$H = A \sum_k \begin{pmatrix} a_{k,o}^\dagger a_{k,e} \\ a_{k,e}^\dagger a_{k,o} \end{pmatrix} + B \sum_k \begin{pmatrix} a_{3k+1,e} a_{3k,o} + a_{3k-1,e} a_{3k,o} \\ a_{3k,e}^\dagger a_{3k,o} + a_{3k-1,e}^\dagger a_{3k,o} \end{pmatrix} + A \sum_k (a_{k,i}^\dagger a_{k,e}) + B \sum_k (a_{k,j}^\dagger a_{k,i}) + h.c.,$$

where $a$ ($a^\dagger$) is the usual annihilation (creation) operator, and $k$ sums over all sites in the tree. The particle distribution via the tree is achieved by the $e$ and $o$ sites, as per the first two sums in the Hamiltonian, while the second sums effect the CTAP-like imaging plane.

The simplest form of MRAP is a four-site system, as shown in the lowest four-tree nodes of figure 2, i.e. $|0_o\rangle$, $|0_e\rangle$, $|1_o\rangle$ and $|1_e\rangle$. The Hamiltonian required to understand this is equivalent to that discussed in the context of geometric phase gates in the tripod atom [34, 35]. Our scheme takes this structure as our starting point and extends to realize an MRAP tree, similar in spirit to NAND trees [36]. The essence of the imaging scheme is that objects in the imaging plane
break the symmetry of the tree and remove sections of the Hamiltonian from the null space. The adiabatic passage can only explore the null space and hence pathways that have been removed cannot have population, thereby realizing IFMs.

Although we discuss the MRAP protocol in an arbitrary, decoherence-free setting, it is clear that decoherence will be deleterious to the protocol. The ideal method of realizing this scheme would therefore be in an optical setting using waveguides such as those demonstrated in [10, 11]. In such schemes, each site is replaced by a waveguide, and the TME between sites controlled by varying the proximity of the waveguides. Varying proximity changes the evanescent tails of the modes and hence their overlap. In this way the temporal variation of the adiabatic passage is translated into a spatial evolution, but the essential characteristics of the evolution are unaltered between representations.

The Hamiltonian of a two-leaf MRAP structure with base node $|k_e\rangle$ is

$$H^{(2)}_{k_e} = Aa_{k,e}^\dagger a_{k,e} + B(a_{3k+1,e}^\dagger + a_{3k-1,e}^\dagger) a_{k,o} + \text{h.c.}$$

The null space of this Hamiltonian is spanned by [33]

$$|D^{(2)}_1\rangle = \frac{B|k_e\rangle - A(3k + 1)e\rangle}{\sqrt{A^2 + B^2}}, \quad |D^{(2)}_2\rangle = \frac{B|k_e\rangle - A(3k - 1)e\rangle}{\sqrt{A^2 + B^2}}.$$  

This spanning ensures that under adiabatic evolution, a particle initially in $|0_e\rangle$ will be transported to a coherent superposition $-((|1_e\rangle + |3_e\rangle))/\sqrt{2}$, without any population even transiently being present in site $|0_o\rangle$. The evolution into an equal superposition of the null states can also be understood by considering the reverse evolution, i.e. evolution from the superposition $-(|1_e\rangle + |3_e\rangle))/\sqrt{2}$ to $|0_e\rangle$. It can be easily shown that no other superposition state can evolve back to the base node via the reversed MRAP [33, 34, 37], and hence evolving from base node to the leaves in our (highly symmetric) case must lead to the equally weighted superposition, as is also observed in our direct numerical simulations shown below.

To create the Hamiltonian for the entire MRAP tree, the Hamiltonian $H^{(2)}$ is recursively added to the creation operators for the leaf nodes by the Hamiltonian (with appropriate relabelling). So, for example, the four-leaf MRAP tree with base node $|0_e\rangle$ is

$$H^{(4)} = H^{(2)}_{0,e} + H^{(2)}_{1,e} + H^{(2)}_{1,e},$$

$$= Aa_{0,0}^\dagger a_{0,0} + B(a_{1,0}^\dagger + a_{1,0}^\dagger) a_{0,0} + Aa_{1,0}^\dagger a_{1,0} + B(a_{1,1}^\dagger + a_{1,1}^\dagger) a_{1,0} + Aa_{1,1}^\dagger a_{1,1} + B(a_{1,1}^\dagger + a_{1,1}^\dagger) a_{1,0} + \text{h.c.}$$

The null space is spanned by the vectors

$$|D^{(4)}_{11}\rangle = \frac{B^2|0_e\rangle - AB|1_e\rangle + A^2|11_e\rangle}{\sqrt{A^4 + A^2B^2 + B^4}},$$

$$|D^{(4)}_{11}\rangle = \frac{B^2|0_e\rangle - AB|1_e\rangle + A^2|11_e\rangle}{\sqrt{A^4 + A^2B^2 + B^4}},$$

$$|D^{(4)}_{11}\rangle = \frac{B^2|0_e\rangle - AB|1_e\rangle + A^2|11_e\rangle}{\sqrt{A^4 + A^2B^2 + B^4}},$$

$$|D^{(4)}_{11}\rangle = \frac{B^2|0_e\rangle - AB|1_e\rangle + A^2|11_e\rangle}{\sqrt{A^4 + A^2B^2 + B^4}}.$$
where the subscript on the null vectors denotes the numerical value of the final (leaf) state. Any state which is a superposition of the null states is also in the null space. So, in particular, if we initialize our system in the state \(|0_e\rangle\), then it will evolve into an equally weighted superposition of all of the null states, and hence the adiabatic evolution is described by the state

\[
\frac{2B^2|0_e\rangle - AB(|1_e\rangle + |\bar{1}_e\rangle) + \frac{A^2}{2}(|11_e\rangle + |1\bar{1}_e\rangle + |\bar{1}1_e\rangle + |\bar{1}\bar{1}_e\rangle)}{\sqrt{A^4 + 2A^2B^2 + 4B^4}}.
\] (11)

Our notation allows us to inductively explore both the total tree structure and the vectors spanning the null space. So, for example, consider adding another MRAP-type branch to an even node connected to site \(|0_e\rangle\) via a null state \(|D(j)\rangle_i\). The effect of adding these new links is to replace the state \(|D(j)\rangle_i\) with two new states, which are

\[
|D_{3+i+1}\rangle = B|D_j\rangle - A^{i+1}|3i + 1\rangle,
\] (12)

\[
|D_{3+i-1}\rangle = B|D_j\rangle - A^{i+1}|3i - 1\rangle,
\] (13)

where the subscript on \(D\) is represented in decimal form for simplicity. The proof that these unnormalized states constitute a valid null space is a straightforward application of the Hamiltonian \(H\),

\[
H_{2j} |D_{3+i+1}\rangle = H_{2j} |BD_j\rangle - A^{i+1}H_{2j}|3i + 1\rangle
\] (14)

\[
= 0 + A (A^j) B|3i\rangle - A^{i+1}B|3i\rangle
\] (15)

\[= 0.
\] (16)

Given that a single node is a valid null state, it follows by induction that each such state \(|D_{3+i+1}\rangle\) and \(|D_{3+i-1}\rangle\) is also a valid null state.

Although the tree structures are shown as if all leaf nodes are the same distance from the origin, in fact there is no requirement for this and the essential features of the transport protocol are unaffected by leaf nodes of varying distance to the origin.

With regard to IFM, the inductive method for generating the null states is particularly useful for understanding the properties of the network as a whole, but we can also use this method essentially in reverse to understand the effect of removing couplings from the network. Observe that for every null state, the population in all of the o nodes is zero. Trivially, if the TME connecting to a site with zero population with lower depth is also zero, then the population in any subsequent sites must be zero. In other words, a break in the chain immediately after an o site ensures that there cannot be any population in the subsequent sites: their contributions are removed from the null space. This insight immediately allows us to base an IFM protocol based on MRAP networks. An inductive proof for the properties of the null space of the MRAP tree, and the populations in the network, can be found in the appendix.

3. MRAP for imaging the quantum minefield

Here, we turn to the task of how to use our protocol on a parallel IFM of a quantum minefield. This is a situation where there are a number of sites \(n\) and a number of quantum bombs \(m < n\), where \(m\) may be unknown. Our task is to determine the locations of the bombs without setting them off. This task may be accomplished by the network shown in figure 2, where the minefield
is located between the sites labelled \( i \) and \( j \). Because breaks in the coupling for the tree remove states from contributing to the null space and because the population in the site immediately before the broken chain is zero, the adiabaticity of the transport protocol forces every photon to avoid the broken link. This satisfies all of the requirements of an IFM. To more explicitly show how MRAP can be used for imaging objects, figure 2 shows a typical configuration where this can be achieved. An excitation, such as a photon, starting from the root of the tree, \(|0_o\rangle\), is transported via adiabatic passage to the leaves of the tree. An object, placed just before the leaf nodes, blocks the TME on the final o–e link (or not) revealing its presence or absence. This happens despite the fact that the photon is never found in a location adjacent to the bomb.

The observation that the null space of the MRAP tree is spanned by all of the null vectors that connect the base to the leaves immediately suggests a parallel IFM scheme. By altering the symmetry of a pathway, we can remove it from the null space, \emph{without affecting the other null vectors}. Consider the tree depicted in figure 2. In this arrangement, the MRAP tree is terminated by three-site CTAP pathways, which preserve the overall symmetry of the null space. However, by removing the TME between the final two sites, the symmetry of the pathway is destroyed, and the vector connecting the leaf to the base of the tree is removed from the null space. We term the line between the final two sites as the imaging plane, and the object in this case is assumed to occlude some subset of the paths. A particle adiabatically evolving through the network will be unable to explore the occluded pathways, and neither will it be able to occupy the site directly before the object. Hence, the particle performs an IFM of the image plane in parallel, with the fidelity set by the adiabaticity of the overall protocol.

In the arrangement shown in figure 2, the population of all odd states (e.g. \(|0_o\rangle\) and \(|1_o\rangle\)), in addition to the states \(|1_i\rangle\) and \(|1_j\rangle\), is (in the adiabatic limit) always zero. This follows from the fact that neighbours of leaf nodes always have zero amplitude. Therefore, any excitation in the system has no probability of actually interacting with the object being imaged (or even undergoing adiabatic passage down a branch of the tree in which an object is located). In this example, the excitation undergoes adiabatic passage from state \(|0\rangle\) to states \(|1_j\rangle\) and \(|1_j\rangle\), where it can be safely detected. Similarly, for any combination of objects obscuring the imaging nodes, the excitation proceeds directly to the imaging nodes, which are not occluded and never undergoes adiabatic passage down the branches of the tree that lead to an object.

Returning to the initial quantum bomb analogy, we can now imagine the parallel version—a quantum minefield placed in the imaging plane. The sensing particles will adiabatically explore the minefield, and will be unable to interact with sites with bombs (as they are removed from the null space). Measurements of the particles at the leaf nodes will definitively mark sites without bombs, and eventually the pattern of the bombs will be discovered to arbitrarily high accuracy, without the loss of any particles.

The evolution for each of the inequivalent imaging configurations for a four-leaf tree is shown in figure 3, with accompanying animation depicted in figure 4. In particular, figure 3(c) shows the results of evolution for the case depicted in figure 2, i.e. with two obscured imaging sites (\(|1_j\rangle\) and \(|1_j\rangle\)). If we write the state as \(|\psi\rangle\), then the evolution is represented by \(|\langle i|\psi\rangle|^2\) for all sites \(|i\rangle\). In this case adiabatic passage takes the excitation from the initial state \(|0_o\rangle\), to the superposition \(|1_j\rangle + |1_j\rangle\)/\(\sqrt{2}\). In the adiabatic limit, all of the \(o\) and \(j\) sites are unpopulated, as are \(|1_e\rangle\), \(|1_j\rangle\), \(|1_j\rangle\) and \(|1_j\rangle\), as expected by the IFM. Transient population is observed in intermediate states, as expected for the alternating protocol. The first maximum corresponds to the superposition (at time \((t/T) \approx 0.49\)) \(|1_o\rangle + |1_e\rangle\), while the second maximum (at time \((t/T) \approx 0.55\)) to the superposition \(|1_e\rangle + |1_e\rangle\). The evolutions for the other configurations are

New Journal of Physics 13 (2011) 125002 (http://www.njp.org/)
Figure 3. Evolution through the imaging protocol for each inequivalent pattern of obscurants for the tree shown in figure 2. The traces show populations at each occupied node as a function of time through the MRAP protocol. In each case the population is initialized in the state $|0_e\rangle$ and the thumbnail to the left shows the imaging configuration using the same notation as figure 2. (a) No obscurants. Here the population evolves smoothly from $|0_e\rangle$ to an equally weighted superposition of $|11_j\rangle$, $|1\bar{1}_j\rangle$, $|\bar{1}1_j\rangle$ and $|\bar{1}\bar{1}_j\rangle$, via the intermediate states as shown. (b) One obscurant. The population evolves adiabatically as shown. The total final population fraction in the two right-hand nodes ($|\bar{1}1_j\rangle$ and $|\bar{1}\bar{1}_j\rangle$) is $\sim 28\%$ each, which exceeds the population fraction in the unobscured left-hand imaging node, $|1\bar{1}_k\rangle$, which is $\sim 44\%$. Panels (c) and (d) show the inequivalent imaging configurations for two obscurants. In each case an equally weighted superposition of the unobscured nodes is observed; however, the transient populations in the intermediate states are perforce different. (e) Three obscurating nodes lead to simple adiabatic passage to the unobscured node. (f) With all nodes obscured there is no null space, and the population undergoes non-adiabatic oscillations that explore every state connected to $|0_e\rangle$. 

New Journal of Physics 13 (2011) 125002 (http://www.njp.org/)
Figure 4. The evolution of population and variations in coupling through the MRAP protocol, with the shading of the sites corresponding to the populations seen in figure 3(c). The TMEs are also depicted with the transparency of the lines corresponding to the strength of the transitions, with bold lines corresponding to maximum TME, and no line corresponding to the case of no coupling. An animation showing this figure is provided in the supplementary data, available at stacks.iop.org/NJP/13/125002/mmedia.

also shown and follow similar patterns, with the exception of figure 3(f) where all of the nodes are obscured and hence there is no null state. In this case the population exhibits oscillations that are essentially unresolved on the scale of the figure. This highlights that for a practical imaging solution, one would always design a pathway that is never obscured to use as a reference.

To ensure adiabatic evolution, a critical consideration is the energy gap between the ground ($E = 0$) state and the first excited state. A typical energy level diagram for a two-level tree, with no object obscuring the imaging plane, is shown in figure 5.

Initially, when $A = 0$, $B = 1$ there are five distinct, but degenerate energy levels. These are at $E = 0$, which has been discussed in detail in this paper, at $E = \pm 1$, which are contributions from symmetric and anti-symmetric superpositions of the imaging plane, and at $E = \pm \sqrt{2}$ corresponding to symmetric and anti-symmetric superpositions across different levels of the tree.

As evolution proceeds, the minimum energy gap between the $E = 0$ (i.e. null) state and the excited state narrows. According to the adiabaticity criteria, for any two eigenstates $|\psi\rangle$ and $|\phi\rangle$, adiabatic evolution is ensured for a fixed number of nodes when

$$\frac{|\langle\psi| \hat{H} |\phi\rangle|}{(E_{|\psi\rangle} - E_{|\phi\rangle})^2} \ll 1.$$  

(17)
Figure 5. Energy level diagram for a two-level tree, with no object obscuring the imaging plane. The minimum energy gap occurs approximately halfway through the protocol, in accordance with expectations from other adiabatic passage protocols (CTAP, STIRAP and MRAP). Robustness of the protocol is guaranteed provided the rate of evolution is slow compared with this minimum energy gap.

Figure 6. Minimum energy gap between the $E = 0$ state, and its complement for different sized trees, for the case that $A_{\text{max}} = B_{\text{max}} = 1$. Note the monotonic decrease in the energy gap with increasing tree depth, which implies that evolution through more complicated trees must proceed commensurately more slowly than for shorter depth trees.

For this reason, the energy gap is of crucial importance for maintaining the adiabaticity of the operation. Fortunately, for any reasonable level of tree, it is possible to determine the size of this gap and show that it does not grow too small. Figure 6 shows a numerical calculation of the size of the minimum energy gap for up to a depth of eight, or 256 imaging nodes. Note that to rigorously determine the adiabaticity requires a full treatment including the degeneracy of the system, which we do not provide here [38].

*New Journal of Physics* **13** (2011) 125002 (http://www.njp.org/)
This example may be extended to much larger trees, of much larger depth, in a straightforward way. Additional nodes may be added in a tree-like structure to provide many more leaf nodes, and therefore many more locations for interaction-free imaging.

If objects are placed in the imaging plane, then it is possible (after several iterations of the MRAP procedure) to determine the position of these objects to arbitrary accuracy. It should be noted that if objects can be placed at any level of the tree, it is only possible to determine which branches of the tree have been obscured by the objects, but not necessarily to determine their exact positions. For example, in our scheme one cannot distinguish between two bombs placed immediately before sites $|11_i⟩$ and $|1\bar{1}_i⟩$ and one bomb placed immediately before site $|1_o⟩$.

4. Collision-free routing

In this section, we point out that the MRAP scheme as discussed offers another surprising result arising from the fact that a local change in the Hamiltonian produces a global alteration of the null states. This feature gives rise to the possibility of collision-free routing, which we describe below.

Imagine a distribution network where particles (possibly containing information, or halves of Bell pairs) need to be distributed from a starting node, to multiple recipients. Examples of protocols that could take advantage of such a network are quantum cryptographic networks where a base station wants to share random numbers for use as quantum keys with many users perhaps using a modification of the BB84 [39] or E91 [40] protocols. So, for concreteness, let us assume that we are employing the MRAP tree to distribute qubits, and that there is no coupling between the qubit degree of freedom and the spatial degree of freedom. Returning to figure 2, imagine that Alice is at site $|0_e⟩$ and wants to distribute qubits to Bobs at the leaf nodes of the MRAP tree. Alice does not care which qubit goes to which Bob, but she does need to know the time stamp of arrival of the qubits (so that appropriate correlations can be made during public basis comparison). Let us further assume that the Bobs need a certain amount of time to process the arrival of a particle (e.g. detector dead time), or that for other reasons (for example, they have a full buffer of qubits) they do not wish to receive qubits. However, they do not wish to communicate with Alice or the other Bobs when they want to receive particles.

In a classical setting with conventional routing, Alice would need to decide to which Bob she would send a particle. To avoid the dead time problem, there are many solutions which involve some scheduling of the distribution of qubits; however, it seems that any such system will either enforce an effective clock rate, and possibly entail the ‘collision’ of qubits in the network, for example if two qubits are sent to the same recipient (Bob) during the dead time of the detection network. Remarkably, the MRAP approach provides an elegant solution to this problem.

For the IFM protocol discussed above, the breaking of a connection led to an alteration of the symmetry of the null space, thereby removing certain states from the null space. Adiabaticity of evolution then prevented particles from exploring the removed states. However, another way to remove states from the null space is simply to apply a small energy shift to the state. For example, if one of the Bobs, say at site $|11_e⟩$ in figure 2, shifts the energy of their site, they will immediately remove their site from the null space, without affecting the rest of the null space, meaning that a local change in the Hamiltonian affects the entire null space. However, this change is not observable by the other Bobs or Alice. The net result is that to have a probability of receiving particles, the Bobs should ensure that their receiving site has zero energy. But if
at any time they wish to cease receiving qubits, then they shift their energy. Each Bob will receive particles randomly, collecting a fraction of the distributed particles sent by Alice equal to the rate of particles transmitted divided by the number of Bobs receiving particles. Finally, Alice need not know who is on the network; she only needs to continue transmitting at the rate available to her.

This surprising routing solution is interesting and would seem to be ideally applied in an optical setting, again taking advantage of the structures similar to the CTAP waveguide networks.

5. Conclusion

We have explored the extension of the MRAP protocol to explore adiabatic passage through tree-like structures. This MRAP tree affords a new method of realizing IFM. The original quantum bomb problem was non-deterministic and could only sense a single bomb. Our approach allows for deterministic parallel IFM arising from the adiabaticity of the protocol, and the perturbation to the symmetry of the null space introduced by the objects being imaged. Due to the adiabaticity of the protocol, this approach should be robust with respect to perturbations in the distribution network.

We have also shown that with a minor variation to the MRAP protocol, it can be used to effect a novel distribution network, where collision-free routing can be achieved. In particular where the source distributes particles, but does not need to specify any of the routing or scheduling. We are not aware of any classical protocol that can achieve these outcomes.

One issue with the scheme is that the MRAP protocol requires spatial coherence of the particle exploring the network. For this reason, a photonic implementation would seem to be the only practical system to realize this network, although it is clear from the quantum mechanics that, in principle, any quantum system could exploit these effects provided that the spatial decoherence rate is sufficiently long.

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Appendix

In this appendix, we provide two inductive proofs. Firstly, we show that the amplitude of the null state at any odd node is zero. Secondly, we show that the amplitude of a null state at nodes leading to only obscured leaf nodes is also zero.

We will first show that the amplitude of this state at each odd site, \( |s_o\rangle \), is zero. The argument we will give is an inductive argument and proceeds from the base of the tree to the leaves. Applying the Hamiltonian \( H \) to the base of the tree, \( |0_e\rangle \), implies that for any null state \( |D\rangle \),

\[
\langle D | H | 0_e \rangle = A \langle D | 0_e \rangle,
\]

(A.1)
and therefore, provided $A \neq 0$,
\[ \langle D|0_o \rangle = 0. \]  
(A.2)

Therefore, any null state $|D\rangle$ has no support of $|0_o\rangle$.

Induction now proceeds in terms of the depth of the tree: assuming that the amplitude is zero at each odd site of depth $d$ implies that the amplitude is also zero for each odd site of depth $d + 1$. Consider $s_o$, a site of depth $d$, and $t_o$, a site descended from $s_o$ of depth $d + 1$. This is shown in figure A.1. Applying $H$ at the site $t_o$ yields
\[ \langle D|H|t_o \rangle = A\langle D|t_o \rangle + B\langle D|s_o \rangle, \]  
(A.3)

and therefore
\[ A\langle D|t_o \rangle = 0, \]  
(A.4)

where we have used the inductive assumption that $\langle D|s_o \rangle = 0$ and the fact that $H|D\rangle = 0$.

Therefore, provided $A \neq 0$, there is also no support on $t_o$. A similar argument applies to all odd sites of depth $d + 1$. Therefore, since there is no support on the site $0_o$ of depth 1 and the lack of support on the odd nodes of depth $d$ implies that there is no support on nodes of depth $d + 1$, there is no support on any of the odd nodes in the tree.

The same argument shows that there is no support of imaging nodes $s_i$ in the null space (since imaging nodes take the place of the final odd node). Therefore in the null space the excitation is never directly adjacent to the object being imaged.

The result can be made stronger: an excitation will not even proceed into a branch of a tree (via either odd or even nodes) where all the leaf nodes of that branch are obscured by the object. We now show that in an obscured branch (i.e. a branch leading to only obscured nodes) not only the odd nodes, but also the even nodes have no support. This is an inductive argument (based on the depth) which proceeds from the leaf nodes towards the base of the tree. In the base case: consider an object obscuring the leaf node $t_i$ in figure A.2. In this case,
\[ A\langle D|t_i \rangle = \langle D|H|t_i \rangle, \]  
(A.5)
\[ A\langle D|t_o \rangle = 0, \]  
(A.6)
Figure A.2. A completely obscured branch, as shown here, has no support on any node which leads to only obscured leaf nodes.

where \( t_i \) is the last node before the object, and \( t_e \) is its adjacent even node. Therefore every last even node before the object has no support. This is true for every node in a branch leading to obscured nodes (such as \( \bar{t}_e \) in the diagram). We now show that no support in a branch for even nodes of depth \( d \) implies that there is no support on nodes of depth \( d - 1 \) in that branch. Consider three nodes: \( t_o \), and its two children, \( t_e \) and \( \bar{t}_e \) at depth \( d \) and a node \( s_e \) adjacent to it at depth \( d - 1 \). Using the inductive assumption we assume that the amplitude at both \( t_e \) and \( \bar{t}_e \) is zero. In this case,

\[
B \langle D | t_e \rangle + B \langle D | \bar{t}_e \rangle + A \langle D | s_e \rangle = \langle D | H | t_o \rangle
\]

and therefore

\[
A \langle D | s_e \rangle = 0.
\]

Therefore the fact there is no support at depth \( d \) implies there is also no support at depth \( d - 1 \). Therefore, by induction, there is no support on any even node which leads to only obscured leaves of the tree.

Neither even nor odd nodes of an obscured branch of the tree have any support. In the adiabatic limit, an excitation will not propagate into any part of an obscured branch provided the null space exists.

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New Journal of Physics 13 (2011) 125002 (http://www.njp.org/)
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