A consistent thermodynamic treatment for quark mass density-dependent model

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Abstract

The ambiguities and inconsistencies in previous thermodynamic treatments for the quark mass density-dependent model are addressed. A new treatment is suggested to obtain the self-consistent results. A new independent variable of effective mass is introduced to make the traditional thermodynamic calculation with partial derivative still practicable. The contribution from physical vacuum has been discussed. We find that the properties of strange quark matter given by quark mass density-dependent model are nearly the same as those obtained by MIT bag model after considering the contribution of the physical vacuum.

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I. INTRODUCTION

It is generally accepted that the effective masses of particles will change with density due to medium effects. Many theoretical considerations, including the finite temperature quantum hadron dynamics (QHD) model\cite{1}, Brown-Rho scaling\cite{2}, finite temperature quark-meson coupling (QMC) model\cite{3}, and etc., had been suggested to investigate the effective masses of mesons and nucleons. Besides theoretical study, many experimental results which predict the changes of particle masses with density have been shown. In particular, the experiments of TAGX collaboration had shown directly that when the density of the nucleon medium equals to $0.7n_0$ where $n_0$ is the saturation density, the effective mass of neutral $\rho$-meson reduces to 610 MeV\cite{4}. Both theoretical and experimental results confirm that the medium effects are important for studying the nuclear or quark systems.

To illustrate the medium effects more transparently in theory, instead of first principle calculation, many authors introduced different hypothesis to represent the medium contributions, for example, supposed the density-dependent vacuum energy $B(\rho)$ to modify QMC model\cite{5, 6}, suggested the density-dependent NN$\rho$ coupling to address liquid-gas phase transition\cite{7}, and etc.. Employing these hypothesis, many physical properties of nuclear matter, quark matter, nucleon system and hyperon system had been discussed. In particular, to simplify the calculations, many ideal quasiparticle models in which the effective mass depends on the density and/or the temperature had been suggested for studying the quark gluon plasma \cite{8, 9}, gluon plasma \cite{10} and strange quark matter \cite{11}.

The quark mass density-dependent (QMDD) model \cite{11} is one of such candidates. It was first suggested by Fowler, Raha and Weiner. In this model, the masses of $u$, $d$ quarks and $s$ quark are given by

$$m_q^* = \frac{B}{3\rho_B}, \quad (q = u, \pi, d, \bar{d}), \quad (1)$$

$$m_s^* = m_{s0} + \frac{B}{3\rho_B}, \quad (2)$$

where $B$ is a constant and $m_{s0}$ is the current mass of the strange quark; and $\rho_B$, the baryon density, is defined as

$$\rho_B = \frac{1}{3}(\Delta \rho_u + \Delta \rho_d + \Delta \rho_s), \quad (3)$$

where

$$\Delta \rho_i = \rho_i - \rho_T = \frac{g_i}{(2\pi)^3} \int d^3 k \left( \frac{1}{\exp[\beta(\epsilon_i(k) - \mu_i)] + 1} - \frac{1}{\exp[\beta(\epsilon_i(k) + \mu_i)] + 1} \right), \quad (4)$$
where $g_i$ is the degeneracy, $i (i = u, s, d)$ and $\bar{7}$ correspond to the quarks and antiquarks, respectively. The ansätze in Eqs.(1) and (2) corresponds to a quark confinement mechanism because if quark goes to infinity, the volume of the system tends to infinity and the baryon number density goes to zero, then $m_q^*$ approaches to infinity according to Eqs.(1) and (2). The infinite mass prevents the quark from going to infinity.

Employing QMDD model and considering weak processes

$$u + d \leftrightarrow u + s, \quad s \rightarrow u + e^- + \bar{\nu}_e, \quad d \rightarrow u + e^- + \bar{\nu}_e, \quad u + e^- \rightarrow d + \nu_e,$$

and the condition of charge neutrality

$$2\Delta \rho_u = \Delta \rho_d + \Delta \rho_s + 3\Delta \rho_e,$$

many physical properties of strange quark matter [12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25] and strange quark star [26, 27] have been studied, and the results are in good agreement with those given by MIT bag model [28].

Although the density-dependent quark masses Eqs.(1) and (2) can mimic the quark confinement mechanism, when we discuss the thermodynamic behaviors of the system with such quarks, many difficulties will emerge. The dispersion relation for quark

$$\epsilon(k, \rho) = [k^2 + m^*(\rho)^2]^{1/2}$$

will make many usual thermodynamic relations with partial derivative no longer satisfied. As is well known in thermodynamics, a proper choice of independent variables will have a suitable characteristic thermodynamical function, from which all the thermodynamic quantities can be obtained by partial derivatives without integration. For example, with variables temperature $T$, volume $V$ and chemical potential $\mu$, the characteristic function is the thermodynamical potential $\Omega = \Omega(T, V, \mu)$. From the differential relation for reversible process

$$d\Omega = -SdT - pdV - N d\mu,$$

we have

$$S = -\left(\frac{\partial \Omega}{\partial T}\right)_{V, \mu}, \quad p = -\left(\frac{\partial \Omega}{\partial V}\right)_{T, \mu}, \quad N = -\left(\frac{\partial \Omega}{\partial \mu}\right)_{T, V},$$

where $S$, $p$ and $N$ are entropy, pressure and average particle number respectively. Other thermodynamic quantities such as internal energy $U$, Helmholtz free energy $F$, enthalpy $H$,
Gibbs function $G$, etc., can be calculated by the combination of the quantities we obtained, based on their definitions or relations.

But for the QMDD model, $\Omega$ is not only a function of $T, V, \mu$, but also depends explicitly on the quark density $\rho$ because of Eq.(7), $\Omega = \Omega(T, V, \mu, m^*(\rho))$. How to tackle the thermodynamics self-consistently is still a problem. There have been many wrangles in present references [12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25]. The difficulty comes from the first and second laws of reversible process thermodynamics expressed by Eq.(8) and the partial derivatives by Eq.(9). Obviously, some extra terms involving the derivatives of $m^*$ will emerge when partial derivatives are calculated following Eq.(9). Unfortunately, these extra terms for different treatments in different references contradict each other. For example, for ideal quark gas system of quasiparticle with effective quark mass $m^* = m^*(\rho)$, the pressure and energy density $\varepsilon$ were given by

$$p = -\tilde{\Omega} \equiv -\frac{\Omega}{V},$$

$$\varepsilon \equiv \frac{U}{V} = \tilde{\Omega} + \sum_i \mu_i \rho_i - T \frac{\partial \tilde{\Omega}}{\partial T},$$

in Ref.[12, 13, 14]; or given by

$$p = -\left(\frac{\partial (\tilde{\Omega}/\rho)}{\partial (1/\rho)}\right)_{T,\{\mu_i\}} = -\tilde{\Omega} + \rho \left(\frac{\partial \tilde{\Omega}}{\partial \rho}\right)_{T,\{\mu_i\}} + \sum_i \mu_i \rho_i - T \left(\frac{\partial \tilde{\Omega}}{\partial T}\right)_{\{\mu_i,\rho\}}.$$  \hspace{1cm} (12)

$$\varepsilon = \tilde{\Omega} - \rho \left(\frac{\partial \tilde{\Omega}}{\partial \rho}\right)_{T,\{\mu_i\}} + \sum_i \mu_i \rho_i - T \left(\frac{\partial \tilde{\Omega}}{\partial T}\right)_{\{\mu_i,\rho\}}.$$  \hspace{1cm} (13)

in Ref.[15, 16]; and given by

$$p = -\tilde{\Omega} + \rho \left(\frac{\partial \tilde{\Omega}}{\partial \rho}\right)_{T,\{\mu_i\} \rho},$$

$$\varepsilon = \tilde{\Omega} + \sum_i \mu_i \rho_i - T \left(\frac{\partial \tilde{\Omega}}{\partial T}\right)_{\{\mu_i,\rho\}},$$

in Ref.[17].

For the quark mass density and temperature dependent (QMDTD) model [18, 19, 20, 21, 22, 23, 24] with quark mass

$$m_q = \frac{B(T)}{3\rho_B}, \quad (q = u, \bar{u}, d, \bar{d}),$$

$$m_{s,\bar{s}} = m_{s0} + \frac{B(T)}{3\rho_B},$$

(16)
where
\[ B(T) = B_0 \left[ 1 - \left( \frac{T}{T_c} \right)^2 \right], \] (18)
p and \( \varepsilon \) read
\[ p = -\tilde{\Omega} - V \frac{\partial \tilde{\Omega}}{\partial V} + \rho \sum_i \frac{\partial \tilde{\Omega}}{\partial m_i} \frac{\partial m_i}{\partial \rho}, \] (19)
\[ \varepsilon = \tilde{\Omega} - \sum_i \mu_i \frac{\partial \tilde{\Omega}}{\partial \mu_i} - T \frac{\partial \tilde{\Omega}}{\partial T} - T \sum_i \frac{\partial \tilde{\Omega}}{\partial m_i} \frac{\partial m_i}{\partial T}, \] (20)
in Ref.[18], respectively. The ambiguity arises from the variable \( \rho \) in \( m^* \), because it is not one of the characteristic variables of thermodynamic potential.

This paper involves from an attempt to clear above ambiguity and suggests a method to calculate the thermodynamic quantities from partial derivatives self-consistently. In next section, we will address the traditional thermodynamic treatment with partial derivative for QMDD model transparently and emphasize that the difficulty can not be overcome by usual method. In fact, the thermodynamic inconsistency for an ideal quasiparticle system with effective mass \( m^*(\rho, T) \) to describe the quark gluon plasma or gluon plasma had been pointed out by many authors previously [8, 9, 10]. To avoid this difficulty, in Sec.III, we argue that we can calculate the thermodynamic quantities from equilibrium state and show that if we choose the quasiparticle effective mass \( m^* \) as a new independent degree of freedom, we can calculate the thermodynamic quantities along reversible process by usual partial derivative self-consistently. In Sec.IV, we will apply this new treatment to study the QMDD model. We show that we can get rid of previous difficulties with partial derivative and take clear, reasonable and self-consistent results in the fresh, which coincide with the results in equilibrium state. We will consider the vacuum correction on the pressure and compare our result with that given by MIT bag model in Sec.V. We will show that even though our results are in agreement with that given by Ref.[15, 16], but the mechanism and the physical reasons are complete different. In Ref.[15, 16], the negative pressure comes from the partial derivative terms of \( m^* \). But in our treatment, it comes from the physical vacuum. Our mechanism is just the same as that of MIT bag model. The last section includes a summary and discussion.
II. INCONSISTENCY OF TRADITIONAL THERMODYNAMIC TREATMENTS WITH PARTIAL DERIVATIVE

We first repeat the traditional treatment based on Eqs.(1) and (2). It has been established in Ref.[15, 16, 17] systematically. In Ref.[17], they derived

\[ p = - \left( \frac{\partial \Omega}{\partial V} \right)_{T,\mu} = - \left( \frac{\partial (V \tilde{\Omega})}{\partial V} \right)_{T,\mu} = - \left( \frac{\partial (\tilde{\Omega} / \rho)}{\partial (1 / \rho)} \right)_{T,\mu} = - \tilde{\Omega} + \rho \left( \frac{\partial \tilde{\Omega}}{\partial \rho} \right)_{T,\mu}. \]  

(21)

Without losing generality, from here on we will display the formulae with only one component for simplicity and clarity. Eq.(21) is different from that of the normal expression \( p = -\Omega / V = -\tilde{\Omega} \) by the last additional term due to the density-dependence of effective mass. But Eq.(21) seems not so solid if we pay attention to the invariables. Noting that the chemical potential \( \mu \) is determined by the particle density constraint \( \rho = \bar{N} / V \), and the partial derivative of Eq.(21) is with respect to \( V \) but fixed \( T \) and \( \mu \), or equivalently fixed \( T \) and \( \rho \), in this process \( \bar{N} \) must be changed. It is not a constant and so the third equal mark in Eq.(21) can not hold. The result in Eq.(21) is error.

To avoid this ambiguity, in Ref.[18], in stead of the thermodynamic potential \( \Omega \), the authors introduced the Helmholtz free energy \( F \) to change the characteristic variable \( \mu \) to the average particle number \( \bar{N} \)

\[ dF = -SdT - pdV + \mu d\bar{N}. \]  

(22)

By using \( F = \Omega + \mu \bar{N} \), they got

\[ p = - \left( \frac{\partial F}{\partial V} \right)_{T,\bar{N}} = - \left( \frac{\partial \Omega}{\partial V} \right)_{T,\bar{N}} - \bar{N} \frac{\partial \mu}{\partial V}, \]  

(23)

\[ S = - \left( \frac{\partial F}{\partial T} \right)_{V,\bar{N}} = - \left( \frac{\partial \Omega}{\partial T} \right)_{V,\bar{N}} - \bar{N} \frac{\partial \mu}{\partial V}. \]  

(24)

After some calculations, they gave

\[ p = - \frac{\partial \Omega}{\partial V} - \left( \bar{N} + \frac{\partial \Omega}{\partial \mu} \right) \frac{\partial \mu}{\partial V} + \rho \frac{\partial \Omega}{\partial \rho} \frac{\partial m^*}{\partial \rho}, \]  

(25)

\[ S = - \frac{\partial \Omega}{\partial T} - \left( \bar{N} + \frac{\partial \Omega}{\partial \mu} \right) \frac{\partial \mu}{\partial T} - \frac{\partial \Omega}{\partial m^*} \frac{\partial m^*}{\partial T}. \]  

(26)

For the internal energy, they required

\[ \bar{N} = - \left( \frac{\partial \Omega}{\partial \mu} \right)_{V,m^*,T}, \]  

(27)
and found
\[ U = \Omega + \mu N + TS = \Omega + \mu N - T \frac{\partial \Omega}{\partial T} - T \frac{\partial \Omega}{\partial m^*} \frac{\partial m^*}{\partial T}. \] (28)

But in their formulae, the invariable quantities for partial derivative have not been written down explicitly. To show the errors of their calculation, we write down these fixed quantities in the following by using the standard mathematical chain rule of partial derivative with composition function:

\[
p = -\left( \frac{\partial F}{\partial V} \right)_{T,N,\mu,m^*} - \left( \frac{\partial \Omega}{\partial \mu} \right)_{T,N,m^*,V} \left( \frac{\partial \mu}{\partial V} \right)_{T,N} - \left( \frac{\partial \Omega}{\partial \mu} \right)_{T,N,m^*,V} \left( \frac{\partial m^*}{\partial V} \right)_{T,N} - \left( \frac{\partial \Omega}{\partial m^*} \right)_{T,N,\mu,V} \left( \frac{\partial \mu}{\partial \rho} \right)_{T,N} \tag{29}
\]

\[
S = -\left( \frac{\partial \Omega}{\partial T} \right)_{V,N,\mu,m^*} - \left[ \frac{\partial \Omega}{\partial \mu} \right]_{V,N,m^*,T} \left( \frac{\partial \mu}{\partial T} \right)_{V,N} - \left( \frac{\partial \Omega}{\partial m^*} \right)_{V,N,\mu,T} \left( \frac{\partial m^*}{\partial T} \right)_{V,N}. \tag{30}
\]

Eqs.(29) and (30) are just Eqs.(25) and (26) with the unchanging variable explicitly written out. Note that many partial derivatives are with four fixed variables. The term \( \left( \frac{\partial \Omega}{\partial m^*} \right)_{V,N,\mu,T} \) must vanish because \( m^* = m^*(T, \rho) \) depends on \( \rho = N/V \) to fix \( V \) and \( T \), so essentially to fix \( m^* \). It means all derivatives involving \( m^* \) in Eqs.(29) and (30) should be zero, therefore the extra terms \( \left( \frac{\partial \Omega}{\partial m^*} \right)_{T,N,\mu,V} \left( \frac{\partial \mu}{\partial \rho} \right)_{T,N} \) and \( \left( \frac{\partial \Omega}{\partial m^*} \right)_{V,N,\mu,T} \left( \frac{\partial m^*}{\partial T} \right)_{V,N} \) can not survive. In fact, according to the definition of Gibbs function \( G = \mu N = U - TS + pV \), we can directly find \( \Omega = -pV \).

The extra terms involving derivative of \( m^* \) will surely destroy this relation.

The inconsistency does not only appear in the expression of \( p \) and \( S \), but also in \( \bar{N} \). If we calculate the particle number \( \bar{N} \) from Eqs.(8) and (9), we obtain

\[
\bar{N} = \left( \frac{\partial \Omega(T,V,\mu,m^*)}{\partial \mu} \right)_{T,V,m^*} - \left( \frac{\partial \Omega(T,V,\mu,m^*)}{\partial m^*} \right)_{T,V,\mu} \left( \frac{\partial \mu}{\partial \rho} \right)_{T,V} \tag{31}
\]

where the summation is over all the quantum states with degeneracy \( g_i \). Since \( \rho = \bar{N}/V \) is usually taken as a constraint to decide the value of \( \mu \), \( \left( \frac{\partial \mu(T,\rho)}{\partial \rho} \right)_{T,V} \) does not vanish if \( m^* \) depends on \( \rho \) explicitly, so the extra term in Eq.(31) modifies the particle number of the system. For an ideal quasiparticle system, the number of quasiparticle \( \bar{N} = \sum_i g_i n_i \), which is just the first term of Eq.(31), then we come to a conclusion that the result of Eq.(31) is
incorrect. Besides, this result in Eq.(31) also conflicts with the requirement in Eq.(27), then the inconsistency of the traditional treatment with partial derivative is transparent.

In fact, the inconsistency of usual thermodynamic treatment for ideal quasiparticle system has been shown by many authors [8, 9, 10, 25]. This section is not a new result. Our aim is to demonstrate the inconsistency for thermodynamic treatment with partial derivative along a reversible process explicitly.

III. A THERMODYNAMICALLY CONSISTENT TREATMENT

In Sec.II, we have shown the inconsistency of traditional thermodynamic treatment with partial derivative along reversible process. In this section, we will suggest a consistent thermodynamic treatment for quasiparticle system with density and/or temperature dependent particle mass $m^*(\rho, T)$. We want to point out that our method is universal and QMDD model is just a special example for using this method. Our method can also be applied to discuss the thermodynamic behavior of the quark mass density and temperature dependent (QMDTD) model [18, 19, 20, 21, 22, 23, 24] or other ideal quasiparticle system.

Noticing that at a fixed instant of reversible process, the system is at an equilibrium state. Denote the temperature and density of this system as $T_0$ and $\rho_0$, respectively, then the effective mass of the quasiparticle becomes constant $m^*(T_0, \rho_0) = m_0$. The system reduces to a usual ideal gas system with constant mass $m_0$ quasiparticles. For this equilibrium state, the corresponding thermodynamic quantities can be directly obtained. For example, for the case of one component Fermi system:

\begin{align*}
\bar{N} &\equiv \rho V = \sum_i g_i n_i = \sum_i \frac{g_i}{e^{\beta(\sqrt{m_0^2 + k_i^2} - \mu)} + 1}, \\
\Omega &\equiv -\sum_i g_i kT \ln(1 + e^{-\beta(\sqrt{m_0^2 + k_i^2} - \mu)}), \\
U &\equiv \sum_i g_i n_i \epsilon_i = \sum_i \frac{g_i \sqrt{m_0^2 + k_i^2}}{e^{\beta(\sqrt{m_0^2 + k_i^2} - \mu)} + 1}, \\
G &\equiv \bar{N} \mu = \sum_i g_i n_i \mu = \sum_i \frac{g_i \mu}{e^{\beta(\sqrt{m_0^2 + k_i^2} - \mu)} + 1}, \\
S &\equiv \frac{U - \Omega - G}{T}, \\
p &\equiv -\frac{\Omega}{V},
\end{align*}

where $n_i$ is the particle number of the $i$th state and $g_i$ is the corresponding degeneracy.
It can be clearly seen that these formulae are the same as those of the standard ideal gas. In fact, for example, if we derive the energy from the very beginning of the statistical definition in grand canonical distribution, we have

\[
U = \mathcal{E} = \frac{1}{\Xi} \sum_N E_N e^{-\beta (E_N - \mu N)}
\]

\[
= \frac{1}{\Xi} \sum_N \left[ \left( -\frac{\partial}{\partial \beta} \right)_{\mu, m^*, V} + \mu N \right] e^{-\beta (E_N - \mu N)}
\]

\[
= -\left( \frac{\partial \ln \Xi}{\partial \beta} \right)_{\mu, m^*, V} + \mu N,
\]

(38)

where the invariables of \(\mu\) is clearly seen, while \(m^*\) and \(V\) are fixed since \(E_N\) is a function of \(m^*\) and \(V\) for each \(N\). Take the quasiparticle ideal gas thermodynamic potential, Eq.(33), which means the grand partition function satisfies

\[
\ln \Xi = \sum_i g_i \ln(1 + e^{-\beta (\sqrt{m^*^2 + k_i^2} - \mu)}),
\]

(39)

it is easy to find

\[
U = -\left( \frac{\partial \ln \Xi}{\partial \beta} \right)_{\mu, m^*, V} + \mu N
\]

\[
= \sum_i g_i n_i \left( \sqrt{m^*^2 + k_i^2} - \mu \right) + \mu N
\]

\[
= \sum_i g_i n_i \epsilon(m^*, k_i),
\]

(40)

where \(n_i = \frac{1}{e^{\beta (\sqrt{m^*^2 + k_i^2} - \mu)} + 1}\) is the average particle number of the \(i\)th state. This result consists with the interaction-free quasiparticle picture of the QMDD model.

We see, from Eqs.(32)-(37), the contribution of medium effect is included in the effective value of mass, and appears in the exponential of the Fermi distribution. A remarkable property of these formulae is that the extra terms related to the partial derivative of \(m^*\) do not appear. This is reasonable because these thermodynamic quantities are functions of equilibrium state, they do not depend on the change of the quasiparticle mass, since at this state the quasiparticle mass \(m^*(T, \rho)\) becomes constant \(m_0(T_0, \rho_0)\).

To compare above treatment with those given in Sec.II, as an example, we calculate the entropy of the ideal quasiparticle system. Denote the entropy calculated by equilibrium state as \(S_{sta}\). From Eq.(34)-(37), we have:

\[
S_{sta} = \frac{U - G - \Omega}{T}
\]
\[
\sum_i g_i \left[ \frac{n_i \sqrt{m_0^2 + k_i^2} - n_i \mu}{T} + k \ln(1 + e^{-\beta(\sqrt{m_0^2 + k_i^2} - \mu)}) \right] = k \sum_i g_i \left[ n_i \beta(\sqrt{m_0^2 + k_i^2} - \mu) + \ln \left( \frac{e^{\beta(\sqrt{m_0^2 + k_i^2} - \mu)}}{e^{\beta(\sqrt{m_0^2 + k_i^2} - \mu) + 1}} \right) \right]
\]

\[
= -k \sum_i g_i [n_i \ln(n_i) + (1 - n_i) \ln(1 - n_i)] \quad \text{(41)}
\]

This is a familiar formula for equilibrium state whose physical meaning is transparent. While we denote the entropy calculated by the partial derivative of \( \Omega \) following Eqs.(8) and (9) as \( S_{\text{der}} \), we have

\[
S_{\text{der}} = -\left( \frac{\partial \Omega(T, V, \mu, m^*(T, \rho))}{\partial T} \right)_{V, \mu, m^*} \quad \text{and for the equilibrium state with } m^*(T_0, \rho_0) = m_0,
\]

\[
= k \sum_i g_i \ln \left( 1 + e^{\mu - \sqrt{m_0^2 + k_i^2}/kT} \right) + kT \sum_i g_i \frac{e^{\mu - \sqrt{m_0^2 + k_i^2}/kT}}{1 + e^{\mu - \sqrt{m_0^2 + k_i^2}/kT}}
\]

\[
= -k \sum_i g_i [n_i \ln(n_i) + (1 - n_i) \ln(1 - n_i)] = S_{\text{sta}}. \quad \text{(43)}
\]

The first term of Eq.(42) is just the result given by equilibrium state. The difference between these two treatments is significant. They can not be accorded together. Noticing that the contribution of medium effect at equilibrium state is included within the value of \( m^* \) in the distribution, and the entropy describing disorder of quasiparticles in a system dose not depend on the intrinsic quantity such as effective mass of quasiparticle, the correctness of \( S_{\text{sta}} \) is obvious. The thermodynamic consistency of the new treatment is transparent because it is based on an equilibrium state and \( m_0(T_0, \rho_0) \) is constant.

If one hopes to extend this treatment to a reversible process, the accordance between the results from calculation along a reversible process and results obtained at a fixed equilibrium state must hold. So in order to get a self-consistent calculation between equilibrium state and
reversible process in thermodynamics, we are compelled by above discussions to introduce an intrinsic degree of freedom $m^*$ for quasiparticle in thermodynamic system to describe medium effect. We should rewrite Eq.(8) as:

$$d\Omega = -SdT - pdV - \overline{N}d\mu + Xdm^*, \quad (44)$$

then Eq.(9) becomes

$$S = -\left( \frac{\partial \Omega}{\partial T} \right)_{V,\mu,m^*}, \quad p = -\left( \frac{\partial \Omega}{\partial V} \right)_{T,\mu,m^*}, \quad \overline{N} = -\left( \frac{\partial \Omega}{\partial \mu} \right)_{T,V,m^*}, \quad X = \left( \frac{\partial \Omega}{\partial m^*} \right)_{T,V,\mu}, \quad (45)$$

where $X$ is an extensive quantity corresponding to the intensive variable $m^*$. In Eq.(44), the intrinsic degree of freedom $m^*$ for quasiparticle has been added as an independent variable in thermodynamic system. The quantities $S$, $p$ and $\overline{N}$ shown in Eq.(45) are in agreement with the results obtained by the formulae of equilibrium state in Eqs.(32)-(37), because we have fixed $m^*$ as an unchanged parameter in the partial derivative calculations. For example, the pressure given by Eq.(9) is

$$p = -\left( \frac{\partial \Omega(T,V,\mu,m^*(\rho_B))}{\partial V} \right)_{T,\mu} = -\left( \frac{\partial \Omega(T,V,\mu,m^*)}{\partial V} \right)_{T,V,\mu} \left( \frac{\partial m^*(\rho_B)}{\partial V} \right)_{T,\mu}, \quad (46)$$

while given by Eq.(45) as

$$p = -\left( \frac{\partial \Omega(T,V,\mu,m^*)}{\partial V} \right)_{T,\mu,m^*} = -\frac{\Omega}{V}. \quad (47)$$

In Eq.(46), the first term is just the result of equilibrium state, but the second term, according to the treatment in Sec.II, will lead to inconsistency in thermodynamics.

Usually the thermodynamic parameters such as $S$, $p$, $T$... depend on the whole system. They are independent with the intrinsic property of the particle or subsystem, no matter the subsystem is a simple point particle or a quasiparticle with inner structure and different intrinsic properties. Ordinary thermodynamic variables depend on the collection of the subsystem only. Similarly, the mass is an intrinsic quantity of a particle, it does not affect on collective thermodynamic properties of the whole system. When we consider the
medium effect or the confinement mechanism, and summarize this effect into the effective mass $m^*(T, \rho)$ under quasiparticle approximation, the dynamic interaction can be concentrated on the effective mass $m^*$ of quasiparticle by using the finite temperature quantum field calculation \cite{29} or directly by confinement anst"aże. But the macro thermodynamic variables cannot describe these micro dynamic interactions. We must choose new variables to represent these dynamic interactions or the medium effect. Obviously, the effective mass $m^*$ appears as the suitable independent variable. Furthermore, we want to emphasize that the application of this new independent variable $m^*$ to rewrite Eqs.(8) and (9) is limited to quasiparticle approximation only. In quasiparticle approximation, the interaction between particles, the medium effect or the confinement mechanism are summarized in the effective mass $m^*$. Then $m^*$ becomes an independent variable to represent all those physical effects, its dependence on density and temperature is due to equilibrium condition of the whole system. Introducing $m^*$ in quasiparticle physical picture to represent the medium effect and taking it as a variable is a twin in thermodynamics of quasiparticle system. If we employ finite temperature quantum field theory to calculate the thermodynamic potential beyond quasiparticle approximation \cite{30,31,32}, we can use Eq.(2) to calculate the thermodynamic quantities and investigate the interaction between particles by different orders of Feynman diagrams. Then we will not need to introduce the quasiparticle and use the corresponding Eqs.(44) and (45).

IV. QMDD MODEL

To illustrate the basic difference between our treatment based on Eqs.(44) and (45) and those of traditional treatment based on Eqs.(8) and (9), we employ QMDD model as an example to calculate the thermodynamic property of strange quark matter. We fix the parameters in Eqs.(1) and (2) $B = 170$ MeVfm$^{-3}$, $m_{s0} = 150$ MeV, as in Ref.[21]. The temperature is set at $T = 0$, as that of Ref.[12,15,17], for convenience of comparison. Our results are shown in Fig.1 and Fig.2. In Fig.1, we draw the curves of energy per baryon $\varepsilon/\rho_B$ vs. baryon number density $\rho_B$, where the solid curve refers to our treatment and the dashed curve refers to treatment in Ref.[15]. We see that the saturation point of the solid curve is $\varepsilon/\rho_B = 906.3$ MeV at $\rho_B = 0.433$ fm$^{-3}$, which locates in a reasonable range. The saturation point of the dashed curve is $\varepsilon/\rho_B = 1042.8$ MeV at $\rho_B = 0.692$ fm$^{-3}$. It is higher
than our result because it has an additional term $\rho_B \left( \frac{\partial \tilde{\Omega}}{\partial \rho_B} \right)_{T,\{\mu_i\}}$.

The equation of state for different treatments are shown in Fig.2, where the solid curve refers to our treatment, and the dotted curve, dashed curve refers to that given by Ref.[15] and Ref.[17], respectively. We see from Fig.2 that there are two remarkable differences between the solid curve and others. Firstly, the pressure is always positive in our treatment. This is of course reasonable because our Hamiltonian $H = \sum_{k,s} \varepsilon_{ks} \hat{a}_{ks}^+ \hat{a}_{ks}$ is an ideal gas Hamiltonian, it cannot give negative pressure in principle. But for the dashed curve and dotted curve, the pressure becomes negative in the small energy density regions due to the incorrect modification term

$$\rho_B \left( \frac{\partial \tilde{\Omega}}{\partial \rho_B} \right)_{T,\{\mu_i\}} = -\sum_i \frac{g_i}{48\pi^2} \frac{4m_i^* B}{\rho_B} \left[ \mu_i \sqrt{\mu^2_i - m_i^2} - m_i^2 \ln \left( \frac{\mu_i + \sqrt{\mu_i^2 - m_i^2}}{m_i^*} \right) \right].$$

Secondly, although the tendencies of all the curves are similar in large energy density region, in small energy density region, their behavior are very different. In particular, the end points at $\rho_B \to 0$ for these three curves are different. They are $\{\varepsilon = 170\text{MeV fm}^{-3}, p = 0\}$ for solid curve, $\{\varepsilon = 340\text{MeV fm}^{-3}, p = -170\text{MeV fm}^{-3}\}$ for dashed curve and $\{\varepsilon = 170\text{MeV fm}^{-3}, p = -170\text{MeV fm}^{-3}\}$ for dotted curve, respectively.

Can we get negative pressure pressure in QMDD model, as that of the MIT bag model? We will answer this question in next section.

V. THE CONTRIBUTION OF PHYSICAL VACUUM

To answer this question, let us recall that in the MIT bag model the negative pressure comes from the physical vacuum. If we extend the quasiparticle Hamiltonian to

$$H = \sum_{k,s} \varepsilon_{ks} \hat{a}_{ks}^+ \hat{a}_{ks} + H_0,$$

where $H_0$ is the system energy in the absence of quasiparticle excitations. Noting that in a quasiparticle system, the physical vacuum energy $H_0$ is possibly a function of $T$ and $\rho$ since $\varepsilon_{ks}$ depends on $T$ and $\rho$ via effective mass $m^*$. Corresponding to $H_0$, the additional term of thermodynamic potential is $\Omega_0$, and the pressure becomes

$$p' = -\frac{\Omega + \Omega_0}{V} = p - B_0,$$
the energy density becomes $\varepsilon + B_0$, but the entropy density $s = (\varepsilon + p - \sum_i \mu_i \rho_i)/T$ is unchanged.

Now we turn to determine $\Omega_0$ in QMDD model. By means of the vacuum correction to guarantee the thermodynamic consistency of quasiparticle system, it was first suggested by Gorenstein and Yang [10], they gave a set of constraints. While in Ref.[25], $\Omega_0$ is chosen to satisfy

$$\left(\frac{\partial (\tilde{\Omega} + \tilde{\Omega}_0)}{\partial \rho_B}\right)_{\left\{\mu_i\right\}} = 0,$$  \hspace{1cm} (51)

where $\tilde{\Omega}_0 = \Omega_0/V$ is virtually $B_0$ in Eq.(50). At zero temperature, Eq.(51) reduces to

$$d\tilde{\Omega}_0(\rho_B) = -\left(\frac{\partial \tilde{\Omega}}{\partial m^*}\right)_{T=0,V,\mu} \frac{dm^*}{d\rho_B} d\rho_B,$$  \hspace{1cm} (52)

because $m^*$ depends on $\rho_B$ only, $\tilde{\Omega}_0$ can be obtained by integration. Based on Eqs.(1), (2) and (33), we obtain

$$d\tilde{\Omega}_0(\rho_B) = -\left(\frac{\partial \tilde{\Omega}}{\partial m^*}\right)_{T=0,V,\mu} \frac{dm^*}{d\rho_B} d\rho_B,$$  \hspace{1cm} (53)

therefore

$$\tilde{\Omega}_0(\rho_B) = \int_{\rho_0}^{\rho_B} \sum_i \frac{g_i B m_i^*}{12\pi^2 \rho_B^2} \mu_i \sqrt{\mu_i^2 - m_i^*} - m_i^* \ln \left(\frac{\mu_i + \sqrt{\mu_i^2 - m_i^*}}{m_i^*}\right) d\rho_B + \tilde{\Omega}_0(\rho_0).$$ \hspace{1cm} (54)

Chosen $\tilde{\Omega}_0(\infty) = B_\infty$, as that of Ref.[25], we get

$$\tilde{\Omega}_0(\rho_B) = -\int_{\rho_B}^{\infty} \sum_i \frac{g_i B m_i^*}{12\pi^2 \rho_B^2} \mu_i \sqrt{\mu_i^2 - m_i^*} - m_i^* \ln \left(\frac{\mu_i + \sqrt{\mu_i^2 - m_i^*}}{m_i^*}\right) d\rho_B + B_\infty.$$ \hspace{1cm} (55)

Eqs.(53)-(55) are just Eqs.(7),(8) and (14) in Ref.[25], then the stability and the negative pressure have been shown there. The negative pressure and the stable point dose exist, we do not recalculate the numerical results here again.

Of course, the constraint Eq.(51) is one of the possible ways to get $\tilde{\Omega}_0$ only. The aim of this choice is to demonstrate that we can get negative pressure and a stable strange quark matter. We must emphasize here again that the negative pressure as well as the stability of the strange quark matter come from physical vacuum. It is completely different from that of Ref.[15, 16, 17], in which the negative pressure comes from the extra partial
derivative term \(\rho_B \frac{\partial \Omega}{\partial \rho_B}\). As has been shown in Sec.II, this term cannot exist since it destroy the thermodynamic consistency. Our mechanism is the same as that of MIT bag model. The conclusion [15] that the properties of strange quark matter in QMDD model are nearly the same as those obtained in the MIT bag model is still valid, but their argument was incorrect. The correct argument is shown by Sec.III-V.

VI. SUMMARY AND DISCUSSION

In summary, we have shown the shortcomings of the previous treatments for QMDD model, which are based on the partial derivative of thermodynamical functions along reversible process. A new method is suggested. For the reversible process, we have introduced a new intrinsic degree of freedom \(m^*\) for the system of quasiparticle. We have proved that the thermodynamic quantities calculated by the partial derivatives concerning this independent variable are in agreement with those obtained by the equilibrium state. The difficulties and controversies in previous references are removed. We also find that the properties of strange quark matter in QMDD model are nearly the same as those obtained in MIT bag model [23], if we take the vacuum contribution into account. Our method is applicable if the quasiparticle with effective mass is introduce to represent the medium effect, since such quasiparticle models are quite commonly used in nuclear matter, quark matter and quark-gluon plasma, we hope our discussion might help the theoretical study in these fields.

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FIG. 1: Energy per baryon as a function of the baryon density $\rho_B$ at $T = 0$ for different treatments of the QMDD model. The solid curve represents our treatment and the dashed line refers to the treatment in Ref. [15].
FIG. 2: Pressure $P$ as a function of the energy density $\varepsilon = U/V$ for different treatments of the QMDD model. The solid curve, dotted curve and dashed curve represent our treatment, the treatment in Ref. [15] and the treatment in Ref. [17], respectively. The tendencies of these curves are similar at large energy density region, but at small energy density region, different treatments have quite different behaviors. In our treatment, the pressure never goes to negative.