Nonlinear Charged Black holes in anti-de Sitter Quasi-topological Gravity

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Abstract

In this paper, we present the static charged solutions of quartic quasitopological gravity in the presence of a nonlinear electromagnetic field. Two branches of these solutions present black holes with one or two horizons or a naked singularity depending on the charge and mass of the black hole. The entropy of the charged black holes of fourth order quasitopological gravity through the use of Wald formula is computed and the mass, temperature and the charge of these black holes are found as well. We show that black holes with spherical, flat and hyperbolical horizon in quasitopological gravity are stable for any allowed quasitopological parameters.

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One main reason for investigating the black holes of higher dimensions with a negative cosmological constant is finding the correspondence between the gravitating fields in an anti-de Sitter (AdS) space-time and conformal field theory living on the boundary of the AdS space-time \[1\]. The Ads/CFT correspondence has become a key concept in the calculation of action and thermodynamical quantities for every reference space-time \[2-5\]. This presumption has been taken into account for asymptotically de Sitter space-times \[6, 7\]. Most applications of the (A)dS/CFT correspondence is for infinite boundaries, but sometimes it is used for finite boundaries in order to obtain the conserved and thermodynamic quantities \[8\]. One of the key issues, in recent years, has been has been the thermodynamics of black holes in Ads space-time. If the AdS/CFT correspondence be used, then, by studying the thermodynamics of Ads black holes, we can certainly gain deep insights in the characteristics and phase structures of strong ’t Hooft coupling CFTs by studying thermodynamics of AdS black holes. As regards the topological characteristics, the asymptotically AdS black holes are different from the asymptotically flat or dS black holes. One of the main differences is that the black hole horizon topology should be a round sphere S2 in four dimensional asymptotically flat or dS spaces \[9\]. It should be noted that, however, the constant curvature of black holes horizon should be zero or negative in asymptotically AdS spaces. Many researchers have investigated the thermodynamical properties of asymptotically anti-de Sitter space-times with nonspherical horizon \[4, 10\]. In Einstein gravity \[11\], the black holes of Gauss-Bonnet gravity with hyperbolic horizon is found stable \[12\]. It seems that the Lovelock terms may have no effect on the stability of topological black holes. In Ref. \[13\] the authors have shown that an asymptotically flat uncharged black hole of third order Lovelock gravity may have two horizons, which does not happen in lower order Lovelock gravity. Also in Ref. \[14\] the authors investigate the effects of third order Lovelock term on the stability of a spherical black hole of third order Lovelock gravity.

Some special features of higher curvature gravities have been mentioned in Ref. \[15\]. The asymptotically Ads black hole solutions and their thermodynamic properties have also been discussed in quartic quasitopological gravity in Ref. \[11\]. It should be noticed that the equations of motion are only second order in derivatives (for spherical symmetry) in quartic quasitopological and the quartic topological action \[15\] yields nontrivial second-order field
equations in all space-time dimensionalities except 8 (beginning with the five-dimensional). In Ref. [16] the cubic quasitopological is introduced. They investigate the holographic properties of the cubic Lagrangian as well, furthermore, they prove that the generic perturbations around an AdS background for such theory fulfill a second order equation. In Ref. [17] a Lagrangian out of a linear combination of cubic invariants have been constructed which generically gives fourth order field equations. But such field equations reduce to second order when evaluated on spherically, hyperbolically or planar symmetric spacetimes. They have also introduced a Lagrangian containing quartic terms in the curvature with similar properties to Ref. [15] for dimensions greater that seven. Due of special features of quartic quasi topological gravity in this paper, we find the asymptotically Ads charged black hole solutions and study the stability of these kinds of black holes. It is shown that black holes of quasitopological gravity with arbitrary curvature horizon are thermodynamically stable.

The outline of this paper is as follows: We introduce the action of quartic quasitopological gravity in $(n+1)$ dimensions in the presence of nonlinear electromagnetic field in Sec. (II). In section (III), we obtain the general solutions of quasitopological gravity. Section (IV) is devoted to calculate the entropy, temprature and mass of charged black holes in quasitopological gravity. We also investigate the stability of the charged black holes with curved horizons in Sec. (V), and finally, we finish our paper with some concluding remarks.

II. CHARGED ACTION OF QUARTIC QUASI-TOPOLOGICAL GRAVITY IN $(n+1)$ DIMENSIONS

The action of 4-th order quasitopological gravity in $(n+1)$ dimensions in the presence of the nonlinear electromagnetic field can be formulated as follows:

$$I_G = \frac{1}{16\pi} \int d^{n+1}x \sqrt{-g}[-2\Lambda + \mathcal{L}_1 + \mu_2 \mathcal{L}_2 + \mu_3 \mathcal{X}_3 + \mu_4 \mathcal{X}_4 + L(F)],$$

where $\Lambda = -n(n-1)/2l^2$ is cosmological constant, $\mathcal{L}_1 = R$ is the Einstein-Hilbert Lagrangian, $\mathcal{L}_2 = R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2$ is the Gauss-Bonnet Lagrangian, $\mathcal{X}_3$ is the
The curvature-cubed Lagrangian \[ \mathcal{X}_3 = R_{ab} R_{cd} R^{ef} R_{ef} R_{ab}^{\phantom{ab} cd} R_{cd} R^{ef} f R_{ef} R_{ab}^{\phantom{ab} cd} R_{cd} R^{ef} \frac{1}{(2n-1)(n-3)} \left( \frac{3(3n-5)}{8} R_{abcd} R_{abcd} R^{abcd} R^{abcd} \right) \]

\[ \mathcal{X}_4 = c_1 R_{abcd} R_{efgh} R^{efgh} R_{efgh} + c_2 R_{abcd} R_{efgh} R^{efgh} R_{efgh} + c_3 R_{abcd} R_{efgh} R^{efgh} R_{efgh} + c_4 \left( R_{abcd} R_{abcd} \right)^2 \]

\[ + c_5 R_{ab} R_{cd} R_{efgh} R^{efgh} R_{efgh} + c_6 R_{abcd} R_{efgh} R^{efgh} R_{efgh} + c_7 R_{abcd} R_{efgh} R^{efgh} R_{efgh} + c_8 R_{abcd} R_{efgh} R^{efgh} R_{efgh} + c_9 R_{abcd} R_{efgh} R^{efgh} R_{efgh} \]

\[ + c_{10} R_{abcd} R_{efgh} R^{efgh} R_{efgh} \]

\[ + c_{12} R_{abcd} R_{efgh} R^{efgh} R_{efgh} \]

\[ + c_{13} R_{abcd} R_{efgh} R^{efgh} R_{efgh} \]

\[ + c_{14} R_{abcd} R_{efgh} R^{efgh} R_{efgh} \]

where

\[
\begin{align*}
    c_1 &= -\left( n - 1 \right) \left( n^7 - 3 n^6 - 29 n^5 + 170 n^4 - 349 n^3 + 348 n^2 - 180 n + 36 \right), \\
    c_2 &= -4 \left( n - 3 \right) \left( 2 n^6 - 20 n^5 + 65 n^4 - 81 n^3 + 13 n^2 + 45 n - 18 \right), \\
    c_3 &= -64 \left( n - 1 \right) \left( 3 n^2 - 8 n + 3 \right) \left( n^2 - 3 n + 3 \right), \\
    c_4 &= -\left( n^8 - 6 n^7 + 12 n^6 - 22 n^5 + 114 n^4 - 345 n^3 + 468 n^2 - 270 n + 54 \right), \\
    c_5 &= 16 \left( n - 1 \right) \left( 10 n^4 - 51 n^3 + 93 n^2 - 72 n + 18 \right), \\
    c_6 &= -32 \left( n - 1 \right) \left( n^2 - 3 n + 3 \right) \left( n^2 - 8 n + 3 \right), \\
    c_7 &= 64 \left( n - 2 \right) \left( n - 1 \right)^2 \left( 4 n^3 - 18 n^2 + 27 n - 9 \right), \\
    c_8 &= -96 \left( n - 1 \right) \left( n - 2 \right) \left( 2 n^4 - 7 n^3 + 4 n^2 + 6 n - 3 \right), \\
    c_9 &= 16 \left( n - 1 \right)^3 \left( 2 n^4 - 26 n^3 + 93 n^2 - 117 n + 36 \right), \\
    c_{10} &= n^5 - 31 n^4 + 168 n^3 - 360 n^2 + 330 n - 90, \\
    c_{11} &= 2 \left( 6 n^6 - 67 n^5 + 311 n^4 - 742 n^3 + 936 n^2 - 576 n + 126 \right), \\
    c_{12} &= 8 \left( 7 n^5 - 47 n^4 + 121 n^3 - 141 n^2 + 63 n - 9 \right), \\
    c_{13} &= 16 n \left( n - 1 \right) \left( n - 2 \right) \left( 3 n^2 - 8 n + 3 \right), \\
    c_{14} &= 8 \left( n - 1 \right) \left( n^7 - 4 n^6 - 15 n^5 + 122 n^4 - 287 n^3 + 297 n^2 - 126 n + 18 \right),
\end{align*}
\]

and \( L(F) \) is the Lagrangian of power Maxwell invariant theory \[15\]:

\[ L(F) = (-F)^s \]
where \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \) is the electromagnetic tensor field and \( A_\mu \) is the vector potential. We should mention that in the limit \( s = 1 \), \( L(F) \) reduces to the standard Maxwell Lagrangian.

### III. \((n + 1)\)-DIMENSIONAL SOLUTIONS IN QUASI-TOPOLOGICAL GRAVITY

The purpose of this section is to introduce the charged black hole solutions of quasitopological gravity in the presence of nonlinear Maxwell field. The action per unit volume can be written as,

\[
I_G = \frac{(n-1)}{16\pi l^2} \int dt d\rho [N(\rho) \left[ \rho^n (1 + \psi + \hat{\mu}_2 \psi^2 + \hat{\mu}_3 \psi^3 + \hat{\mu}_4 \psi^4) \right]'] + \frac{2s l^2 \rho^{(n-1)}}{(n-1)N(\rho)^{2s-1}}], \tag{5}
\]

where \( \psi = l^2 \rho^{-2}(k - f) \) and the dimensionless parameters \( \hat{\mu}_2, \hat{\mu}_3 \) and \( \hat{\mu}_4 \) are defined as:

\[
\hat{\mu}_2 \equiv \frac{(n-2)(n-3)}{l^2} \mu_2, \quad \hat{\mu}_3 \equiv \frac{(n-2)(n-5)(3n^2 - 9n + 4)}{8(2n-1)l^4} \mu_3,
\]

\[
\hat{\mu}_4 \equiv \frac{n(n-1)(n-2)^2(n-3)(n-7)(n^5-15n^4+72n^3-156n^2+150n-42)}{l^6} \mu_4.
\]

In order to obtain the action (5), the static spherically symmetric metric and the vector potential as follows have been used

\[
ds^2 = -N(\rho)^2 f(\rho) dt^2 + \frac{d\rho^2}{f(\rho)} + \rho^2 d\Omega^2, \tag{6}
\]

\[A_\mu = h(\rho) \delta^0_\mu \tag{7}\]

where

\[
d\Omega^2 = \begin{cases} 
  d\theta_1^2 + \sum_{i=2}^{n-1} \prod_{j=1}^{i-1} \sin^2 \theta_j d\theta_i^2 & k = 1 \\
  d\theta_1^2 + \sinh^2 \theta_1 d\theta_2^2 + \sum_{i=3}^{n-1} \prod_{j=2}^{i-1} \sin^2 \theta_j d\theta_i^2 & k = -1 \\
  \sum_{i=1}^{n-1} d\phi_i^2 & k = 0
\end{cases}
\]

represents the line element of an \((n-1)\)-dimensional hypersurface with constant curvature \((n-1)(n-2)k\) and volume \( V_{n-1} \). Varying the action (5) with respect to \( f(\rho) \), we obtain \( N(\rho) = 1 \). Therefore, the variation with respect to \( h(\rho) \) gives

\[
(2s-1)\rho h'' + (n-1)h' = 0, \tag{8}
\]
The solution of this differential equation is

\[ h(\rho) = \begin{cases} 
q \ln(\rho), & s = n/2 \\
-q\rho^{-(n-2s)/(2s-1)}, & 1/2 < s < n/2
\end{cases} \tag{9} \]

here \( q \) is an integration constant which is related to the charge parameter. Since the potential should be finite at infinity for \( s \neq n/2 \), the interval \( 1/2 < s < n/2 \) is chosen. One can find the electric charge according to the Gauss theorem as:

\[ Q = \frac{1}{4\pi} \int_{r \rightarrow \infty} F_{\mu\nu} \sqrt{-g} d^{n-1}x = \frac{2^s s(n-2s)^{2s-1} V_{n-1} q^{2s-1}}{8\pi (2s-1)^{2s-1}} \tag{10} \]

varying the action \( \mathcal{S} \) with respect to \( N(\rho) \) yields

\[ \psi^4 + \frac{\hat{\mu}_3}{\hat{\mu}_4} \psi^3 + \frac{\hat{\mu}_2}{\hat{\mu}_4} \psi^2 + \frac{1}{\hat{\mu}_4} \psi + \frac{1}{\hat{\mu}_4} \kappa = 0, \tag{11} \]

where

\[ \kappa = 1 - \frac{m}{\rho^n} + \frac{2s l^2 q^2(n - 2s)^{2s-1}}{(n - 1)(2s - 1)(2s - 2) \rho^{2s(n-1)/(2s-1)}}, \tag{12} \]

and \( m \) is an integration constant which can be evaluated as the geometrical mass of black hole solutions in terms of the horizon radius

\[ m = \left( 1 + k \frac{l^2}{\rho_+^2} + \frac{\hat{\mu}_2}{\hat{\mu}_4} \frac{l^4}{\rho_+^4} + \frac{\hat{\mu}_3}{\rho_+^6} + \frac{\hat{\mu}_4}{\rho_+^8} \right) \frac{2^s l^2 q^2(n - 2s)^{2s-1} (\rho_+)^{2s(1-n)/(2s-1)}}{(n - 1)(2s - 1)(2s - 2)} \rho_+^n \tag{13} \]

In order to find the black hole solutions, we choose two solutions of \( f(\rho) \) as

\[ f_1(\rho) = k + \frac{\rho^2}{l^2} \left( \frac{\hat{\mu}_3}{4\hat{\mu}_4} + \frac{1}{2} R - \frac{1}{2} E \right), \tag{14} \]

\[ f_2(\rho) = k + \frac{\rho^2}{l^2} \left( \frac{\hat{\mu}_3}{4\hat{\mu}_4} - \frac{1}{2} R + \frac{1}{2} K \right). \tag{15} \]

where

\[ R = \left( \frac{\hat{\mu}_3^2}{4\hat{\mu}_4^2} - \frac{\hat{\mu}_2}{\hat{\mu}_4} y_1 \right)^{1/2}, \tag{16} \]

\[ E = \left( \frac{3\hat{\mu}_3^2}{4\hat{\mu}_4^2} - \frac{2\hat{\mu}_2}{\hat{\mu}_4} - R^2 - \frac{1}{4R} \left[ 4\hat{\mu}_2 \hat{\mu}_3 - 8 \frac{\hat{\mu}_3^3}{\hat{\mu}_4^3} \right] \right)^{1/2}, \tag{17} \]

\[ K = \left( \frac{3\hat{\mu}_3^2}{4\hat{\mu}_4^2} - \frac{2\hat{\mu}_2}{\hat{\mu}_4} - R^2 + \frac{1}{4R} \left[ 4\hat{\mu}_2 \hat{\mu}_3 - 8 \frac{\hat{\mu}_3^3}{\hat{\mu}_4^3} \right] \right)^{1/2}, \tag{18} \]

\[ \Delta = \frac{H^3}{27} + \frac{D^2}{4}, \quad H = \frac{3\hat{\mu}_3 - \hat{\mu}_2}{3\hat{\mu}_4^2} - \frac{4\kappa}{\hat{\mu}_4}, \quad K = \frac{H^2}{4}, \quad \hat{\mu}_i = \hat{\mu}_i(\rho), \quad 0 \leq s \leq 2, \quad \rho \leq \rho_+, \quad \rho_+ < \rho_\infty. \tag{19} \]

\[ D = \frac{2\hat{\mu}_3}{27 \hat{\mu}_4^2} - \frac{1}{3} \left( \frac{\hat{\mu}_3}{\hat{\mu}_4} + 8 \frac{\kappa}{\hat{\mu}_4} \right) \frac{\hat{\mu}_2}{\hat{\mu}_4} + \frac{\hat{\mu}_3^2}{\hat{\mu}_4^2} + \frac{1}{\hat{\mu}_4^2}. \tag{20} \]
and \( y_1 \) is the real root of following equation:

\[
y^3 - \frac{\mu_2 y^2}{\mu_4} + \left( \frac{\mu_3}{\mu_4^2} - \frac{4 \kappa}{\mu_4} \right) y - \frac{4 \mu_2 \kappa}{\mu_4^2} - \frac{1}{\mu_4^2} = 0 \tag{21}
\]

The metric function \( f(\rho) \) for the uncharged solution \((q = 0)\) is real in the whole range \(0 \leq \rho < \infty\). But for the charged solution, the spacetime should be restricted to the region \( \rho \geq r_0 \), we introduce a new radial coordinate \( r \) as

\[
r = \sqrt{\rho^2 - r_0^2} \Rightarrow d\rho^2 = \frac{r^2}{r^2 + r_0^2} dr^2 \tag{22}
\]

where \( r_0 \) is the largest real root of \( \Delta_0 = \Delta(\kappa = \kappa_0), R_0 = R(\kappa = \kappa_0), E_0 = E(\kappa = \kappa_0) \) and \( K_0 = K(\kappa = \kappa_0), \) and \( \kappa_0 \) is

\[
\kappa_0 = 1 - \frac{m l^2}{r_0^n} + \frac{2 l^2 (n - 2)}{(n - 1) r_0^{2(n - 1)}} \tag{23}
\]

So the metric of an \((n + 1)\)-dimensional static spherically symmetric spacetime changes to:

\[
ds^2 = - f(r) dt^2 + \frac{r^2 dr^2}{f(r)(r^2 + r_0^2)} + (r^2 + r_0^2) d\Omega^2 \tag{24}
\]

and the functions \( h(r) \) and \( \kappa \) changes to

\[
h(r) = - q (r^2 + r_0^2)^{(2s - n)/(4s - 2)} \tag{25}
\]

\[
\kappa = 1 - \frac{m}{(r^2 + r_0^2)^{n/2}} + \frac{2^s l^2 q^2 (n - 2s)^{2s - 1}}{(n - 1)(2s - 1)(2s - 2)(r^2 + r_0^2)^{s(n - 1)/(2s - 1)}} \tag{26}
\]

**IV. THE THERMODYNAMICS QUANTITIES OF THE BLACK HOLES**

In order to use Wald’s formula \cite{20}, one can evaluate the entropy per unit volume \( V_{n-1} \) \cite{15}:

\[
S = \frac{\eta_{+}^{-1}}{4} \left( 1 + 2 k \bar{\mu}_2 \frac{(n - 1) l^2}{(n - 3) \eta_+^2} + 3 k^2 \bar{\mu}_3 \frac{(n - 1) l^4}{(n - 5) \eta_+^4} + 4 k^3 \bar{\mu}_4 \frac{(n - 1) l^6}{(n - 7) \eta_+^6} \right) \tag{27}
\]

where \( \eta_+ = \sqrt{r_+^2 + r_0^2} \). This entropy reduces to the area law of entropy for \( \bar{\mu}_2 = \bar{\mu}_3 = \bar{\mu}_4 = 0 \) (Einstein gravity). Furthermore the temperature of the event horizon by the standard method of analytic continuation of the metric can be calculated as

\[
T = \frac{f'(r_+)}{4 \pi} \sqrt{1 + \frac{r_0^2}{r_+^2}}
\]

\[
= \frac{n \bar{\mu}_0 \eta_+^8}{4 \pi} \left[ (n - 2) k l^2 \eta_+^6 + (n - 4) k^2 \bar{\mu}_2 l^4 \eta_+^4 + (n - 6) k^3 \bar{\mu}_3 l^6 \eta_+^2 + (n - 8) k^2 \bar{\mu}_4 l^8 \right]
\]

\[
\frac{(\eta_+^6 + 2 k \bar{\mu}_2 l^2 \eta_+^4 + 3 k^2 \bar{\mu}_3 l^4 \eta_+^2 + 4 k^3 \bar{\mu}_4 l^6)}{4 \pi l^2 \eta_+}
\]

\[
\frac{q^2 2^s (n - 2s)^{2s} (\eta_+^{-1} (n - 1) - 2s - 1) \eta_+}{4 \pi l^2 (4 \bar{\mu}_4 k \eta_+ - 6 l^6 + 3 \eta_+^{-4} \bar{\mu}_3 k^2 l^4 + 2 \eta_+^{-2} \bar{\mu}_2 k l^2 + 1) (n - 1)(2s - 1)^{2s - 1} \eta_+} \tag{28}
\]
The electric potential \( \Phi \), which is measured at infinity with respect to the horizon, is introduced by \[21, 22\]

\[
\Phi = A_\mu \xi^\mu \bigg|_{r \to \infty} - A_\mu \xi^\mu \bigg|_{r=r_+}
\] (29)

where \( \xi^\mu \) is the null generator of the horizon, \( A_\mu \) is the vector potential. Therefore the following can be calculated

\[
\Phi = q \left( \frac{r_+^2 + r_0^2}{(n-2s)/2(2s-1)} \right)
\] (30)

The ADM mass of black hole can be obtained by using the behavior of the metric at large \( r \). One can show that the mass of the black hole per unit volume, \( V_{n-1} \), is

\[
M = \frac{(n-1)}{16\pi} m
\] (31)

meanwhile the first law of thermodynamics, \( dM = TdS + \Phi dQ \), is satisfied. This is due to the fact that the intensive quantities

\[
T = \left( \frac{\partial M}{\partial S} \right)_Q, \quad \Phi = \left( \frac{\partial M}{\partial Q} \right)_S
\] (32)

are in agreement with those given in Eqs. (28) and (30).

Note that all the thermodynamic quantities obtained in this section reduce to those of Einstein gravity given in \[1\] for \( \hat{\mu}_2 = \hat{\mu}_3 = \hat{\mu}_4 = 0 \). Figures 1 and Figures 2 show the metric function \( f_1(r) \) for different values of charge parameters with \( k = -1 \) and \( k = +1 \) respectively. The charge of the extreme black hole may be obtained by using Eq. (13) and computing \( q_{\text{ext}} = q(r_{\text{ext}}) \). Then, our solution presents a black hole with inner and outer horizons provided \( q > q_{\text{ext}} \), an extreme black hole if \( q = q_{\text{ext}} \), and a naked singularity for \( q < q_{\text{ext}} \). Figure 3 shows the metric function \( f_2(r) \) for different values of charge parameters with \( k = -1 \). Therefore, the charge of the extreme black hole may be obtained by using Eq. (13) and evaluating \( q_{\text{ext}} = q(r_{\text{ext}}) \).

V. STABILITY IN THE CANONICAL ENSEMBLE

The local stability analysis in any ensemble can be modified by finding the determinant of the Hessian matrix \( |\partial^2 S/\partial X_i \partial X_j| \), where \( X_i \)s are the thermodynamic variables of the system \[21\]. The number of the thermodynamic variables depends on the ensemble which is used. Since the charge is a fixed parameter in the canonical ensemble, therefore, the
positivity of the thermal capacity $C_Q$ is a sufficient factor to have the local stability. The thermal capacity $C_Q$ at constant charge is

$$C_Q = T \frac{\partial S}{\partial T} \quad (33)$$

as a result, the slope of $logS$ versus $logT$ shows the thermodynamic stability of the black holes. For $k = 0$ the temperature and the entropy do not depend on quasitopological coefficients. Figures 4 and 5 show that the black holes with flat horizons are stable. We will discuss the stability of hyperbolic and spherical black holes in the rest of this paper.

It should be mentioned that the action of quartic quasitopological gravity contributes to the equations of motion in any dimensions except 8. Figure 3 shows the metric function $f_2(r)$ for different values of charge parameters with $k = -1$. In the case of $k = -1$, it can be seen that there are two values for the charge of the extreme black hole $q_{1\text{ext}} = q(r_{1\text{ext}})$ and $q_{2\text{ext}} = q(r_{2\text{ext}})$. For black holes with $q < q_{2\text{ext}}$, the charge of the extreme black hole is $q_{1\text{ext}}$, while for black holes with $q > q_{2\text{ext}}$, the charge of the extreme black hole is $q_{2\text{ext}}$. Where $r_{1\text{ext}}$ and $r_{2\text{ext}}$ are real root(s) for the equation $T = 0$.

In the case of $k = -1$ the critical charge $q_{\text{crit}}$, can be calculated numerically, if we choose $q < q_{\text{crit}}$ the function $f_2(r)$ to become imaginary. For hyperbolic black holes, the $logS$ versus $logT$ is shown in Fig. 4 in which the slope of the $logS$ versus $logT$ is always positive, and therefore these black holes are thermodynamically stable.

![Graph](image)

FIG. 1: $f_1(r)$ vs. $r$ for $k = -1, l = 1, n = 4, \hat{\mu}_0 = 1, \hat{\mu}_2 = .1, \hat{\mu}_3 = .2, \hat{\mu}_4 = .01, m = .5$ and $q > q_{\text{ext}}, q = q_{\text{ext}}$ and $q < q_{\text{ext}}$ from up to down, respectively.

The solution for $k = 1$ presents a black hole with inner and outer horizons provided $q > q_{\text{ext}}$, an extreme black hole if $q = q_{\text{ext}}$, and a naked singularity for $q < q_{\text{ext}}$. 
FIG. 2: $f_1(r)$ vs. $r$ for $k = 1$, $l = 1$, $n = 4$, $\hat{\mu}_0 = 1$, $\hat{\mu}_2 = -.01$, $\hat{\mu}_3 = .2$, $\hat{\mu}_4 = .001$, $m = 1.5$ and $q > q_{\text{ext}}$, $q = q_{\text{ext}}$ and $q < q_{\text{ext}}$ from up to down, respectively.

FIG. 3: $f_2(r)$ vs. $r$ for $k = -1$, $l = 1$, $n = 4$, $\hat{\mu}_0 = 1$, $\hat{\mu}_2 = .2$, $\hat{\mu}_3 = -.1$, $\hat{\mu}_4 = -.06$, $m = .3$ and $q > q_{1\text{ext}}$, $q = q_{1\text{ext}}$, $q_{2\text{ext}} < q < q_{1\text{ext}}$, $q = q_{2\text{ext}}$ and $q < q_{2\text{ext}}$ from up to down, respectively.

It is known that the negative slope in a temperature-entropy plot produces a situation in which the black hole will be thermodynamically unstable. Considering AdS black holes, we learn from Fig. 4 that no phase transition takes place for spherical black holes because the slope is positive in all aspect. The most important result of this paper is to clarify this fact that quartic quasitopological term has no effect on the stability of the black holes (Fig. 5).

VI. CONCLUDING REMARKS

We calculated the entropy and the temperature of quasitopological black holes and found that the entropy of fourth gravity reduces to the area law of the entropy for $\hat{\mu}_2 = \hat{\mu}_3 = \hat{\mu}_4 = 0$. Using Gauss theorem, we also calculate the charge of the black hole. In addition, it is shown
FIG. 4: $\log S$ vs. $\log T$ for $n = 4$, $q = .1$, $l = 1$, $\hat{\mu}_2 = .2$, $\hat{\mu}_3 = .1$, $\hat{\mu}_4 = .007$, $k = 1$(dashed line), $k = 0$(solid line) and $k = -1$(bold line).

FIG. 5: $\log S$ vs. $\log T$ for $n = 4$, $q = .1$, $l = 1$, $\hat{\mu}_2 = .2$, $\hat{\mu}_3 = .1$, $\hat{\mu}_4 = .06$, Quartic-quasi (dashed line), Cubic-quasi (dotted line), Gauss-Bonnet(solid line) and Einsteinian (bold line).

that the first law of thermodynamics is satisfied. For the case $k = 1$ and $k = -1$, we obtained the solution that presents a black hole with inner and outer horizons provided $q > q_{\text{ext}}$, an extreme black hole if $q = q_{\text{ext}}$, and a naked singularity for $q < q_{\text{ext}}$. At the same time, for quasitopological black holes with hyperbolic horizon is obtained two values for the charge of the extreme black hole $q_{1\text{ext}} = q(r_{1\text{ext}})$ and $q_{2\text{ext}} = q(r_{2\text{ext}})$. For black holes with $q < q_{2\text{ext}}$, the charge of the extreme black hole is $q_{1\text{ext}}$, while for black holes with $q > q_{2\text{ext}}$, the charge of the extreme black hole is $q_{2\text{ext}}$. In Einstein and Gauss-Bonnet gravities, the topological black holes with hyperbolic horizon are stable [11, 12]. The Lovelock terms, for black holes with flat horizons, do not change the stability phase structure [23]. In this paper, we can show that the black holes in quartic quasitopological gravity are thermodynamically stable.
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