REVERSED CIRCULAR DICHROISM OF
ISOTROPIC CHIRAL MEDIUMS WITH NEGATIVE REAL
PERMEABILITY AND PERMITTIVITY

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\textbf{ABSTRACT:} Negative real parts of the permittivity and permeability lead an
isotropic chiral medium to exhibit circular dichroism that is reversed with respect
to that exhibited by an identical medium but the real parts of whose permittivity
and permeability are positive.

\textbf{Key words:} chiral medium; circular dichroism; negative real permeability; negative
real permittivity

\section{Introduction}

The Drude–Born–Fedorov constitutive relations of a homogeneous, isotropic chiral
medium (ICM) are stated as

\begin{equation}
\begin{aligned}
D(r, \omega) &= \epsilon(\omega) [E(r, \omega) + \beta(\omega) \nabla \times E(r, \omega)] \\
B(r, \omega) &= \mu(\omega) [H(r, \omega) + \beta(\omega) \nabla \times H(r, \omega)]
\end{aligned}
\end{equation}

The permittivity of the ICM is denoted by $\epsilon(\omega)$, and its permeability by $\mu(\omega)$,
whilst $\beta(\omega)$ is the chirality parameter. An $\exp(-i\omega t)$ time–dependence is implicit
throughout and the dependences on the angular frequency $\omega$ are understood from here onwards.

When a linearly polarized plane wave is normally incident on an ICM slab of infinite lateral dimensions, the transmitted plane wave is elliptically polarized with its vibration ellipse rotated with respect to direction of the incident electric field phasor [2, 3]. These two effects are quantified via the optical rotation $\delta$ and the circular dichroism $\psi$, expressed on a per–unit–thickness basis as

$$\delta = \text{Re} \left[ \gamma_1 - \gamma_2 \right] / 2,$$

$$\psi = \text{Im} \left[ \gamma_1 - \gamma_2 \right] / 2,$$

where the wavenumbers

$$\gamma_1 = \frac{\omega \sqrt{\epsilon \mu}}{1 - \omega \sqrt{\epsilon \mu} \beta},$$

$$\gamma_2 = \frac{\omega \sqrt{\epsilon \mu}}{1 + \omega \sqrt{\epsilon \mu} \beta};$$

hence,

$$\delta + i\psi = \frac{\omega^2 \epsilon \mu}{1 - \omega^2 \epsilon \mu \beta^2} \beta.$$

A related quantity of interest is the impedance

$$\eta = \sqrt{\frac{\mu}{\epsilon}},$$

which appears in the decomposition of the electric and the magnetic field phasors in terms of Beltrami fields [4].

Typically, $\beta$ is a small parameter that appears to manifest chiefly via $\delta$ and $\psi$. The influence of $\beta$ is immediately obvious from (3) and (4): a change in its sign results in the interchange of the values of the two wavenumbers, which thus reverses the signs of both the optical rotation and the circular dichroism [1]. Pairs of mediums differing only in $\beta$ and/or in $\eta$ have been contrasted earlier in this journal [5, 6].
The objective of the present communication is to present the observable consequences of a very recent development on isotropic chiral mediums: Shelby et al. have constructed and tested a dielectric/magnetic composite material which appears to possess permeability and permittivity with negative real parts in a particular frequency range \[7\]. Although their material is not exactly as advertised, future possibilities mandate the examination of observable repercussions on ICMs.

2 Analysis

Let us consider electromagnetic field phasors in an ICM at a frequency that does not lie in an absorption band \[4\]. Accordingly,

\[
\begin{align*}
\epsilon &= a \left( \pm 1 + i \tan \delta_{\epsilon} \right) \epsilon_0 \\
\mu &= b \left( \pm 1 + i \tan \delta_{\mu} \right) \mu_0
\end{align*}
\]

where \(a > 0, b > 0, \tan \delta_{\epsilon} \geq 0 \) and \(\tan \delta_{\mu} \geq 0\) are real-valued, and we impose the condition of low loss (i.e., \(\text{Im} \left[ \gamma_{1,2} \right] \leq \text{Re} \left[ \gamma_{1,2} \right] \)). The upper signs in the foregoing equations hold for the normal case, the lower for the case suggested by the experimental results of Shelby et al. \[7\]. The permeability and the permittivity of free space are denoted, respectively, by \(\mu_0\) and \(\epsilon_0\).

Correct to the first order in both \(\tan \delta_{\epsilon}\) and \(\tan \delta_{\mu}\), we obtain

\[
\gamma_1 \approx \frac{k_0 m}{1 - k_0 m \beta} \left[ 1 \pm \frac{i}{1 - k_0 m \beta} \frac{\tan \delta_{\mu} + \tan \delta_{\epsilon}}{2} \right]
\]

and

\[
\gamma_2 \approx \frac{k_0 m}{1 + k_0 m \beta} \left[ 1 \pm \frac{i}{1 + k_0 m \beta} \frac{\tan \delta_{\mu} + \tan \delta_{\epsilon}}{2} \right]
\]

from (3) and (6), where \(k_0 = \omega \sqrt{(\epsilon_0 \mu_0)}\) is the free-space wavenumber and \(m = \)
Therefore, the right side of (4) can be approximated as follows:

\[ \delta + i\psi \approx \frac{k_0^2 m^2}{1 - k_0^2 m^2 \beta^2} \left[ 1 \pm i \frac{\tan \delta_\mu + \tan \delta_\epsilon}{1 - k_0^2 m^2 \beta^2} \right] \beta. \]  

(9)

In the same way,

\[ \eta \approx \sqrt{\left( \frac{b}{a} \right)} \left[ 1 \pm i \frac{\tan \delta_\mu - \tan \delta_\epsilon}{2} \right] \eta_0, \]  

(10)

where \( \eta_0 = \sqrt{\left( \mu_0/\epsilon_0 \right)} \) is the intrinsic impedance of free space.

3 Conclusions

Equation (9) shows that the optical rotation remains unaffected if the real parts of both the permittivity and the permeability change signs, but the circular dichroism reverses in sign. This conclusion holds provided \( \text{Im}[\beta] \) can be ignored, i.e., at frequencies away from a circular dichroism band \([4]\). Otherwise, optical rotation is also affected. Equation (10) shows that only the imaginary part of \( \eta \) changes sign, if the signs of the real parts of both the permittivity and the permeability are altered.

Suppose two ICMs labeled \( p \) and \( q \) possess identical response properties at a certain frequency — except that \( 0 < \text{Re}[\epsilon_p] = -\text{Re}[\epsilon_q] \) and \( 0 < \text{Re}[\mu_p] = -\text{Re}[\mu_q] \).

Correct to the zeroth order in both \( \tan \delta_\epsilon \) and \( \tan \delta_\mu \), the two ICMs are isoimpedant as well as isorefractive; yet, they could be distinguished from each other by employing obliquely incident plane waves. Of course, (9) and (10) show that the two ICMs are neither isoimpedant nor isorefractive, correct to the first order in \( \tan \delta_\epsilon \) and \( \tan \delta_\mu \).
References

[1] A. Lakhtakia, Beltrami fields in chiral media, World Scientific, Singapore, 1994.

[2] D. Pye, Polarised light in science and nature, Institute of Physics, Bristol, UK, 2001.

[3] O.N. Singh and A. Lakhtakia (eds), Electromagnetic fields in unconventional materials and structures, Wiley, New York, 2000.

[4] E. Charney, The molecular basis of optical activity, Wiley, New York, 1979.

[5] A. Lakhtakia, Isorefactive chiral media, Microwave Opt Technol Lett 19 (1998), 350–352.

[6] A. Lakhtakia, Cross-refractive chiral media and constitutive contrasts, Microwave Opt Technol Lett 20 (1999), 337–339.

[7] R.A. Shelby, D.R. Smith and S. Schultz, Experimental verification of a negative index of refraction, Science 292 (2001), 77–79.