Coulomb interaction for Lévy sources

Bálint Kurgyis\textsuperscript{1}, Dániel Kincses\textsuperscript{1}, Márton I. Nagy\textsuperscript{1}, and Máté Csanád\textsuperscript{1}

\textsuperscript{1}Eötvös Loránd University, Hungary, H-1117 Budapest, Pázmány P. s. 1/A, e-mail: csanad@elte.hu

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Abstract

During the study of Bose-Einstein correlations in heavy ion collisions, one has to take into account the final state interactions, amongst them the Coulomb interaction playing a prominent role for charged particles. In some cases, measurements have shown that the correlation function can be best described by Lévy sources, and three-dimensional measurements have indicated the possibility of deviation from spherical symmetry. Therefore, one would like to study the Coulomb interaction for non-spherical Lévy sources. We resort to numerical methods which are most commonly used to consider the Coulomb interaction for such measurements. Here, we utilize the Metropolis-Hastings algorithm. The symmetric Lévy distribution that describes the source can be characterized by three Lévy scale parameters and the Lévy exponent. We investigate the roles of these parameters in the correlation function. We show the results for the Bose-Einstein correlation functions for ellipsoidal Lévy sources with Coulomb interaction. We also compare our results with previous ways to treat the Coulomb interaction in the presence of Lévy sources.

1 Introduction

The investigation of Bose-Einstein-correlations or HBT measurements offers a way to get information about heavy-ion collisions on a femtosopic scale. These analyses can yield information about the space-time geometry of the collision, particle production mechanisms and could even indicate critical phenomena \cite{1, 2, 3, 4}.

For the study of Bose-Einstein correlation functions, one usually makes an assumption for the source function. Recent measurements indicate that there are cases when there is a long-range component to the source and the most suitable choice is a Lévy-type source \cite{4, 5, 6}. Additionally, the available statistics can be more often utilized and the measurement simplified by spherical symmetry i.e. when the correlation function is measured as a function of only one momentum variable, the length of the momentum difference. However, three-dimensional measurements can yield further information about the collision, thus it is desirable to perform these also whenever possible \cite{3, 7}.

These measurements are often carried out with pairs of identical charged particles, e.g. pions or kaons. As such, one has to take into account the Coulomb repulsion between the outgoing particles. This final state interaction is handled by a Coulomb correction in experimental analyses \cite{8, 9}. At the moment, the Coulomb correction is at hand for spherically symmetric Lévy
HBT measurements [10]. Our goal is twofold: first, to investigate the Coulomb correction for three-dimensional Lévy sources and determine a sound method to use in experimental works. Second, we are looking at the implications of using different coordinate frames for the measurements and calculations, namely longitudinally comoving system (LCMS) and pair center of mass system (PCMS).

1.1 Two-particle correlation functions

The $n$-particle correlation functions are defined as

$$C_n = \frac{N_n(k_1, \cdots k_n)}{\prod_{i=1}^{n} N_1(k_i)},$$

where $N_n$ is the $n$-particle invariant momentum distribution which we can write up using the $n$-particle wave-function $\psi_n(x_1, \cdots x_n, k_1, \cdots k_n)$ and the source function $S(x, k)$ as

$$N_n(k_1, \cdots k_n) = \int |\psi_n(x_1, \cdots x_n, k_1, \cdots k_n)|^2 \prod_{i=1}^{n} S(x_i, k_i) dx_i.$$

Instead of the single-particle source function it is useful to introduce the pair-distribution $D(\rho, K)$, which is the auto-convolution of the source function in the first variable and with $k = K$. If we assume that the particles are of similar momentum ($k_1 \approx k_2$) the two-particle correlation function then can be expressed with properly normalized wave-function and source as

$$C_2(q, K) = \int |\psi_2(q, K, \rho, R)|^2 D(\rho, K) d\rho dR,$$

where we introduced relative ($q, \rho$) and average ($K, R$) quantities.

1.2 Lévy sources

For the source function we assume that we have a symmetric Lévy distribution [11]:

$$S(r, K) = \mathcal{L}^{(4D)}(r^\mu, \alpha(K), R_{\sigma\nu}^2(K)) = \int \frac{d^4 q}{(2\pi)^4} e^{iq_\mu r^\mu} e^{-\frac{1}{2} |q^\sigma R_{\sigma\nu} q^\nu|^\alpha/2},$$

where $\alpha$ is the Lévy-exponent and $R_{\sigma\nu}^2$ is a 2-index symmetric tensor containing the squares of the Lévy-scale parameters; these parameters carry the momentum dependence of the source. The auto-convolution of such a Lévy-distribution is itself a Lévy-distribution but with scale parameters $R_{\sigma\nu}^2 = 2^{2/\alpha} R_{\sigma\nu}^2$. By choosing a coordinate frame and making some assumptions we constrain the form of the $R_{\sigma\nu}^2$ matrix. Most measurements are done in the LCMS system [3,4], thus we will use the assumption that our source can be described by a spatially three-dimensional symmetric Lévy (only diagonal terms) and that the freeze-out is simultaneous for particles with the same average momentum (no temporal part). Therefore, the $R_{\sigma\nu}^2$ tensor has the following form:

$$R_{\sigma\nu}^2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & R_{\text{out}}^2 & 0 & 0 \\ 0 & 0 & R_{\text{side}}^2 & 0 \\ 0 & 0 & 0 & R_{\text{long}}^2 \end{pmatrix},$$

(5)
where we used the out, side, and long to indicate that we are using Bertsch-Pratt coordinates \cite{12,13}. We can then simplify the four-dimensional Lévy distribution as a product of a Dirac delta function and a three-dimensional symmetric Lévy distribution:

\[
\mathcal{L}^{(4D)} = \delta(t^L) \mathcal{L}^{(3D)}(r^L, \alpha, R_{\text{out}}, R_{\text{side}}, R_{\text{long}}),
\]

\[
\mathcal{L}^{(3D)}(r^L, \alpha, R_{\text{out}}, R_{\text{side}}, R_{\text{long}}) = \int \frac{d^3q}{(2\pi)^3} e^{-i\vec{q}\vec{r}^L} e^{-|q_{\text{out}}^2 R_{\text{out}}^2 + q_{\text{side}}^2 R_{\text{side}}^2 + q_{\text{long}}^2 R_{\text{long}}^2|^\alpha/2},
\]

where the \( L \) superscript indicates that these coordinates are in LCMS. Without final state interactions we can easily get the form of the two-particle correlation function in the LCMS with the above mentioned source \cite{11}:

\[
C_2^{(0)}(\vec{q}, \alpha, R_{\text{out}}, R_{\text{side}}, R_{\text{long}}) = 1 + e^{-|q_{\text{out}}^2 R_{\text{out}}^2 + q_{\text{side}}^2 R_{\text{side}}^2 + q_{\text{long}}^2 R_{\text{long}}^2|^{\alpha/2}}.
\]

2 Material and method

2.1 Coulomb interaction

To take into account the Coulomb interaction one has to use the Coulomb interacting two particle wave function. We get that as the solution of the two-particle Schrödinger equation with repulsive Coulomb force. It can be solved in the PCMS \cite{8,14}, and the fully symmetric wave function for identical bosons is

\[
\psi(\vec{R}^P, \vec{r}^P, \vec{K}^P, \vec{k}^P) = \frac{\mathcal{N}}{\sqrt{2}} e^{-i2\vec{k}\vec{R}} \left[ e^{i\vec{k}\vec{r}} F(-i\eta, 1, i(kr - \vec{k}\vec{r})) + e^{-i\vec{k}\vec{r}} F(-i\eta, 1, i(kr + \vec{k}\vec{r})) \right],
\]

where we used \( \vec{k} = \vec{q}/2, k = |\vec{k}| \) and the following notations:

\[
\eta = \frac{mc^2\alpha}{2\hbar ck}, \text{ where } \alpha \text{ is the fine-structure constant},
\]

\[
\mathcal{N} = e^{-\frac{\eta^2}{4}} \Gamma(1 + i\eta),
\]

with \( \Gamma(z) \) being the gamma function and \( F(a, b, z) \) is the confluent hypergeometric function. To evaluate the two-particle correlation function we need the norm squared of the wave function, with that the \( \vec{R} \) and \( \vec{K} \) dependence is lost:

\[
|\psi(\vec{r}^P, \vec{K}^P)|^2 = \frac{2\pi\eta}{e^{2\pi\eta} - 1} \left[ |F(-i\eta, 1, i(kr + \vec{k}\vec{r}))|^2 + e^{2i\vec{k}\vec{r}} F(-i\eta, 1, i(kr - \vec{k}\vec{r})) F(i\eta, 1, -i(kr + \vec{k}\vec{r})) \right].
\]

To get the two-particle correlation function one has to evaluate a \( d^4r \) integral over the whole space-time, this could be performed in any coordinate frame. We have several options to explore:

1. Let us assume, that the \( R_{\sigma\nu}^2 \), thus the source is the same in PCMS and LCMS, this is an approximation of \( \vec{K} \approx 0 \). However this is a rather strong approximation and one of the goals of HBT measurements is to explore the average momentum (or transverse mass) dependence of the parameters that describe the source.
2. There are two objects, one in PCMS (the wave function) and the other in LCMS (the source function). We could try to transform the wave function from PCMS to LCMS and then use the simple form of source function and get the result in LCMS coordinates. However, the two-particle wave function of eq. 12 is not a relativistic expression, thus we refrain from trying to come up with the right transformation of this object.

3. The third option is to evaluate the integral in PCMS as the two-particle Coulomb wave function is only known in PCMS. This means that the Lévy-source has to be transformed from LCMS to PCMS.

Below we proceed with the third option listed above. We introduce some further notations: the mass of the particles \( m \) (e.g. pion mass), the average transverse momentum in LCMS \( K_T \), the transverse mass \( m_T = \sqrt{m^2 + K_T^2} \) and the \( \beta_T = K_T / m_T \) factor. The Lorentz-boost from LCMS to PCMS is then

\[
\Lambda^\nu_{\mu} = \frac{1}{m} \begin{pmatrix}
m_T & -K_T & 0 & 0 \\
-K_T & m_T & 0 & 0 \\
0 & 0 & m & 0 \\
0 & 0 & 0 & m
\end{pmatrix},
\]

(13)

The Lévy distribution then transforms as a scalar from LCMS to PCMS, which means that we have to evaluate eq. 6 at the coordinates \( r' = \Lambda^{-1} r \), where the transformation is the following:

\[
\begin{pmatrix}
t^L_L \\ \rho^L_L
\end{pmatrix} = \frac{1}{m} \begin{pmatrix}
m_T t^P + K_T r^P_{\text{out}} \\
K_T t^P + m_T r^P_{\text{out}} \\
r^P_{\text{side}} \\
r^P_{\text{long}}
\end{pmatrix},
\]

(14)

The temporal integral then can be easily evaluated and then we are left with the expression (where \( 2\vec{k} = \vec{q} \))

\[
C_2^{(C)} (\vec{q}) = \int d^3r |\psi(\vec{k}, \vec{r})|^2 L^{(3D)} \left( \sqrt{1 - \beta_T^2 r_{\text{out}}, r_{\text{side}}, r_{\text{long}}, \alpha, R_{\text{out}}, R_{\text{side}}, R_{\text{long}}} \right),
\]

(15)

where we dropped the \( P \) superscripts for simplicity but every momentum and spatial coordinate is in PCMS. We can further work on the expression for the three-dimensional Lévy-distribution and obtain the following relationship:

\[
L^{(3D)} \left( \sqrt{1 - \beta_T^2 r_{\text{out}}, r_{\text{side}}, r_{\text{long}}, \alpha, R_{\text{out}}, R_{\text{side}}, R_{\text{long}}} \right) \sim L^{(3D)} \left( \vec{r}, \alpha, R_{\text{out}}/\sqrt{1 - \beta_T^2}, R_{\text{side}}, R_{\text{long}} \right).
\]

(16)

Then the integral we would like to calculate is

\[
C_2^{(C)} (\vec{q}, \alpha, R_1, R_2, R_3) = \int d^3r |\psi(\vec{k}, \vec{r})|^2 L^{(3D)} (\vec{r}, \alpha, R_1, R_2, R_3),
\]

(17)

where \( R_1 = R_{\text{out}}/\sqrt{1 - \beta_T^2}, R_2 = R_{\text{side}}, R_3 = R_{\text{long}} \). This expression can be evaluated numerically, we utilize the Metropolis-Hastings algorithm.
2.2 Numerical simulations

The Metropolis-Hastings algorithm can be used to evaluate integrals of the form

\[ I = \int_{\Omega} f(x) \cdot g(x) \, dx, \]  

where \( f(x) \) can be thought of as a probability distribution and \( g(x) \) is the function of interest [15, 16]. In our case the three-dimensional symmetric Lévy distribution is the probability distribution and the function of interest is from eq. 12:

\[ f(x) \, dx := \mathcal{L}^{(3D)}(\vec{r}, \alpha, R_1, R_2, R_3) \, d^3r, \]  
\[ g(x) := |\psi(\vec{k}, \vec{r})|^2. \]

We can utilize two transformations. First, with the reflection relations of the confluent hypergeometric functions the second term in eq. 12:

\[ e^{2i\vec{k}\vec{r}}F(-i\eta, 1, i(kr - \vec{k}\vec{r}))F(i\eta, 1, -i(kr + \vec{k}\vec{r})) = F(1 + i\eta, 1, -i(kr - \vec{k}\vec{r}))F(1 - i\eta, 1, -i(kr + \vec{k}\vec{r})). \]  

Additionally, we can transform the 3D symmetric Lévy distribution:

\[ \mathcal{L}^{(3D)}(\vec{r}, \alpha, R_1, R_2, R_3) = \mathcal{L}^{(1D)}(s(\vec{r}), \alpha, 1) \, R_1 R_2 R_3, \]  
\[ s(\vec{r}) = \sqrt{\frac{r_{\text{out}}^2}{R_1^2} + \frac{r_{\text{side}}^2}{R_2^2} + \frac{r_{\text{long}}^2}{R_3^2}}. \]

The integral was performed using spherical coordinates on the domain \( \Omega = [0, r_{\text{max}}] \times [0, 2\pi] \times [0, \pi] \), with an \( r_{\text{max}} \) chosen so that the integral of the Lévy distribution \( I = \int \mathcal{L} \) to be a maximum of 1% less than 1 \( (I \geq 0.99) \).

3 Results

First we are going to look at the comparison of our three-dimensional calculations and other available, spherically symmetric calculations for Lévy sources. Then, we are going to investigate the implications of the fact that most measurements are in LCMS, and the source is assumed to be spherical there for one-dimensional analyses, but the integral of eq. 17 is in PCMS.

3.1 Three-dimensional measurements

The three-dimensional calculation is rather time-consuming and its numerical precision could be also problematic for implementing it for experimental analyses. Our approach here was, that we fixed a set of parameters \( (\alpha, R_1, R_2, R_3) \) and evaluated the integral at \( 100^3 \) points in momentum space. This gives us a fine enough resolution in momentum space for purposes of comparison. First let us compare the two-particle correlation functions in PCMS. On fig. 1 we can see the Bose-Einstein correlation functions with Coulomb interaction (Full BEC) and without any final state interaction (Free BEC) from our 3D calculation, from 1D calculation with quadratic and arithmetic average scale parameters and the angle averaged values of the 3D calculation. In the spherical case, on the left hand side plot, everything is the same as we would
Correlation functions in PCMS

\[
q_{\text{long}} = q_{\text{side}} = q_{\text{out}} \quad [\text{MeV/c}] \quad \text{(for 3D: } q_0 = 20, 40, 60, 80, 100 \ldots \text{mean R)}
\]

1D Free BEC with quadratic mean R
1D Full BEC with arithmetic mean R
1D Free BEC with arithmetic mean R

Figure 1: On the left hand side the two-particle correlation functions are shown in a spherical case for the three dimensional calculation in comparison with one dimensional calculations in presence of Coulomb interaction in without final state interactions. On the right hand side a non-spherical three dimensional calculation is shown alongside with one dimensional calculations with quadratic and arithmetic mean scale parameters.

expect; but on the right hand side plot, when we have a non-spherical source for the 3D calculation we can see that there is large difference between the correlation functions, both in the Coulomb interacting and in the free case. However, we are interested in the question whether we could use the 1D calculation for the purposes of Coulomb correction only, viz. the ratio of the full and free BEC functions \( K = C_2^{(\text{C})}/C_2^{(0)} \). We can see the comparison of Coulomb corrections on fig. [2] with two sets of parameters, both non-spherical. The Full BEC functions are here the Coulomb corrected three-dimensional correlation functions (FullBEC = \( K \cdot C_2^{(0)} \)). The one dimensional Coulomb corrections are evaluated at \(|\vec{q}|\) in PCMS, thus at \( q_{\text{inv}} \) and at an average \( R \) for \( R_1, R_2 \) and \( R_3 \). Although the correlation functions were quite different, we can see that the Coulomb corrections are very much the same. Now, we would just like to point out the fact that one-dimensional and three-dimensional Coulomb corrections are very similar, therefore in an experimental analysis it is sufficient to use a one-dimensional Coulomb correction, with the right parameter values. The error caused by the spherical Coulomb correction could be estimated, but it is not in the scope of this paper to give a quantitative limit on this uncertainty. The application of the Coulomb correction in three-dimensional analyses is quite straightforward: if the measurement is in LCMS and we have momenta \( q^L = (q_{\text{out}}^L, q_{\text{side}}^L, q_{\text{long}}^L) \) and \( \dot{\text{L}} \)évy scale parameters \( R_{\text{out}}, R_{\text{side}}, R_{\text{long}} \) for particles with an average transverse momentum of \( K_T \), which gives us \( \beta_T \). Then using the assumption that the Coulomb correction transforms as a scalar we evaluate the Coulomb correction (which was calculated in PCMS) at momenta \( q^P = (\sqrt{1 - \beta_T^2} q_{\text{out}}^L, q_{\text{side}}^L, q_{\text{long}}^L) \) and scale parameters \( R_1 = R_{\text{out}}/\sqrt{1 - \beta_T^2}, R_2 = R_{\text{side}} \) and \( R_3 = R_{\text{long}} \). Accordingly, we use \( q_{\text{inv}} = \sqrt{(1 - \beta_T^2) q_{\text{out}}^L + q_{\text{side}}^L + q_{\text{long}}^L} \) and some average of \( R_1, R_2 \) and \( R_3 \) if we use a 1D Coulomb correction, for example the quadratic average:

\[
R_{\text{PCMS}} = \sqrt{\frac{R_{\text{out}}^2}{1 - \beta_T^2} + R_{\text{side}}^2 + R_{\text{long}}^2} \quad (24)
\]
Therefore the Coulomb-correction that can be applied in a three-dimensional measurement is the following:

\[
K_{3D} = \frac{C_{2,1D}^{(C)}(q_{\text{inv}}, R_{\text{PCMS}}, \alpha)}{1 + \exp \left( -|q_{\text{inv}} R_{\text{PCMS}}|^{\alpha} \right)},
\]  

(25)

where \(C_{2,1D}^{(C)}\) is the result from the integral of Eq. 17 in a spherical case with radius of \(R_{\text{PCMS}}\) according to Eq. 24, and at momentum \(q_{\text{inv}}\) which can be calculated for every point in a three-dimensional measurement in LCMS.

### 3.2 Spherical (one-dimensional) HBT measurements

Below we investigate the implications of our calculations for one-dimensional HBT measurements. When we perform a one-dimensional measurement in LCMS we assume that the source here is spherical, thus \(R = R_{\text{out}} = R_{\text{side}} = R_{\text{long}}\) and we have a single momentum variable \(q_{\text{LCMS}} = \sqrt{q_{\text{out}}^2 + q_{\text{side}}^2 + q_{\text{long}}^2}\), but the Coulomb correction is calculated in PCMS with \(R_1, R_2, R_3\). This means, that a spherical source in LCMS would imply a non-spherical \((R_1 = R/\sqrt{1 - \beta_T^2}, R_2 = R_3 = R)\) source in PCMS and the need for a three-dimensional Coulomb-correction. However, we have seen above that the non-spherical Coulomb-correction can be well approximated with a spherical Coulomb-correction if we use the right average \(R\), viz. instead of \(R_{\text{LCMS}} = R\) we have to use

\[
R_{\text{PCMS}} = \sqrt{\frac{1 - \frac{2}{3} \beta_T^2}{1 - \beta_T^2} R},
\]  

(26)

if we use a quadratic average \(R\). Another problem stems from the fact that we can not reconstruct \(q_{\text{inv}}\) from \(q_{\text{LCMS}}\). An obvious solution would be to measure all momentum variables instead of just the length of the momentum difference, but then the advantages of the 1D measurement over the 3D would be lost. We can try to overcome this obstacle in some other ways, one solid
Figure 3: The Coulomb corrections and the Coulomb corrected three-dimensional two-particle correlation function is shown in LCMS, when the source is spherical in LCMS but not for the calculation. On the left hand side we take the three-dimensional Coulomb correction along a diagonal line and on the right hand side along the $q_{\text{out}}$ axis.

approximation could be the following: we measure an $A(q_{\text{LCMS}}, q_{\text{inv}})$ distribution of particle pairs and then we use this to obtain a weighted Coulomb-correction:

$$K_{\text{weighted}}(q_{\text{LCMS}}) = \frac{\int A(q_{\text{LCMS}}, q_{\text{inv}})K(q_{\text{inv}})dq_{\text{inv}}}{\int A(q_{\text{LCMS}}, q_{\text{inv}})dq_{\text{inv}}}.$$  

(27)

On fig. 3 we can see the Coulomb correction and the corrected three-dimensional two-particle correlation functions for $K_T = 0.8$ GeV/c in LCMS. The parameters are chosen so, that in the LCMS we have an approximately spherically symmetric source ($R_{\text{out}} = 2.06$ fm, $R_{\text{side}} = R_{\text{long}} = 2$ fm). We can see that there is a clear difference between the two one-dimensional corrections, one with an LCMS average $R$, the other with an average in accordance with eq. 26. In the low-$q$ region there is some difference between the angle averaged, the one-dimensional and the three dimensional Coulomb-corrections, also the numerical precision of the three-dimensional calculation make it difficult to decide between the options. However, we can clearly see that from $q > 20$ MeV/c the angle averaged and the three-dimensional Coulomb correction are in good agreement with the one-dimensional Coulomb-correction with the average $R$ of eq. 26 and there is consistent difference from the other one. The fact that the angle averaged case is most similar to the one dimensional with the transformed average $R$ of eq. 26 indicates that it is best to use the latter for one-dimensional measurements. On the left hand side, the three-dimensional correlation function is taken at a diagonal line in LCMS ($q_{\text{out}} = q_{\text{side}} = q_{\text{long}}$) and on the right hand side along the out axis. For the one-dimensional Coulomb correction we did not rely on a weighted average, as we could calculate $q_{\text{inv}}$. Let us now list the possible approaches to deal with the Coulomb interaction in one-dimensional measurements that are carried out in LCMS. We only list the options that make use of a one-dimensional calculation for the integral of eq. 17 in these cases the factor of ref. 10 can be used. A more simplistic solution would be to use the Gamow-factor, where the source size is neglected. The most sophisticated approach would be to use the angle averaged Coulomb correction from a three-dimensional calculation, but this would be an overly intricate solution. The possibilities for
making use of a one-dimensional Coulomb integral calculation are the following, in increasing sophistication:

1. Simply use $C_2^{(C)}(q_{LCMS}, R_{LCMS})$, which means that we formally substitute $q_{LCMS} = q_{inv}$ and $R_{PCMS} = R_{LCMS}$.

2. Take into account the fact that $q_{inv} \neq q_{LCMS}$ but neglect the same for the scale parameters, and use the weighting method of eq. (27). However, not for the Coulomb correction but for the correlation function instead. Thus use $C_{2,\text{weighted}}(q_{LCMS}, R_{LCMS})$ for the fitting:

$$C_{2,\text{weighted}}(q_{LCMS}, R_{LCMS}) = \frac{\int A(q_{LCMS}, q_{inv})C_2(q_{inv}, R_{LCMS})dq_{inv}}{\int A(q_{LCMS}, q_{inv})dq_{inv}}. \quad (28)$$

3. The same approach as above, use $R_{LCMS}$ for the Coulomb correction, and use a weighted average but for the Coulomb correction this time. This approach is more sensible if we consider fig. 1, where we saw that the correlation functions can look rather different even if on fig. 2 the Coulomb corrections look very much the same. Now we use $K_{\text{weighted}}(q_{LCMS}, R_{PCMS}) \cdot C_2^{(0)}(q_{LCMS}, R_{LCMS})$ for fitting.

4. One improvement to the above mentioned methods is to take into account the transformation of scale parameters, so use the average of eq. (26). The simpler version is the same as no. 3 above, when we weigh the correlation function and use $C_{2,\text{weighted}}(q_{LCMS}, R_{PCMS})$ for fitting. Here however, we lose the explicit form of $C_2^{(0)}$ in LCMS which is known.

5. The most sophisticated option would be to use $R_{PCMS}$ only for the Coulomb correction, and use the weighting of eq. (27). The function used for fitting is now $K_{\text{weighted}}(q_{LCMS}, R_{PCMS}) \cdot C_2^{(0)}(q_{LCMS}, R_{LCMS})$.

6. Finally, an approach that is easier to implement than the previous ones making use of a distribution $A(q_{LCMS}, q_{inv})$, is to make an approximation for the $q_{LCMS}$-$q_{inv}$ relationship that is appropriate for the Coulomb correction. One could be motivated by the left hand side plot of fig. 3 as the one-dimensional Coulomb correction with $R_{PCMS}$ and the angle averaged three-dimensional calculation are in a relatively good agreement. The relationship $q_{inv} = \sqrt{1 - \beta^2 T}/3q_{LCMS}$ could be used, as it would hold for the diagonal line $q_{out} = q_{side} = q_{long}$ line. Therefore the function we could use for fitting would be $K(\sqrt{1 - \beta^2 T}/3q_{LCMS}, R_{PCMS}) \cdot C_2^{(0)}(q_{LCMS}, R_{LCMS})$.

Additionally, either the a distribution of particle pairs from same events (usually denoted with $A$) or some background distribution, that has no quantum-statistical effects ($B$) could be used for weighting $C_2$ and $K$ [4]. Here, one could argue in the favor of the latter, however it is not expected to make a significant difference. The soundest approach for one dimensional analyses is no. 5.

### 4 Conclusions

We have investigated the Coulomb interaction for HBT measurements in presence of Lévy sources. Our results can be applied to three-dimensional and one-dimensional measurements alike. The results hold for Gaussian or Cauchy sources as well because these are special cases.
of the Lévy source ($\alpha = 2$ for Gaussian and $\alpha = 1$ for Cauchy). We have learned that a one-dimensional Coulomb correction can be reasonably well applied for three-dimensional measurements if we use the the average scale parameter of eq. 24 and use $q_{\text{inv}}$ as the momentum variable for the Coulomb correction. For one dimensional measurements in LCMS we saw that we should use the average scale parameter of eq. 26 and we should also evaluate the Coulomb correction at $q_{\text{inv}}$ as we calculated this in PCMS, which in practice can be estimated with a weighted Coulomb correction according to the option no. 5 in the previous section. The above detailed treatment of Coulomb interaction in heavy-ion collisions could be readily applied to experimental measurements.

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