A piecewise negative stiffness mechanism and its application in dynamic vibration absorber

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Abstract

A new type of piecewise negative stiffness (NS) mechanism is designed and the relationship between the force and displacement is studied. At first, the prototype of the piecewise NS mechanism is established, and the stiffness characteristic of this mechanism is analyzed. Then, the piecewise NS mechanism is applied to dynamic vibration absorber (DVA) system to establish a dynamic model with the piecewise linearity. The differential motion equations are derived according to Newton’s law of mechanics. The approximate analytical solution and the amplitude frequency curve of the system with the piecewise NS are obtained by means of the averaging method. The correctness of the analytical solution is proved by comparing with the numerical solution. In the end, the comparisons with two other traditional DVAs show that the system in this paper has better vibration reduction effect under the condition of harmonic excitation and random excitation.

KEYWORDS

amplitude-frequency curve, dynamic vibration absorber, piecewise negative stiffness, vibration control

1 | INTRODUCTION

The general definition of stiffness $K$ is the rate of deformation caused by the load to which the elastic element is subjected.\textsuperscript{1,2} Elastic elements of positive stiffness are very common in engineering practices, such as most steel springs and elastic mechanisms. The negative stiffness (NS) elastic elements are rare. Most NS elastic elements need a specific mechanism or elastic element to realize under certain conditions.\textsuperscript{3} The direction of force presented by the NS element or mechanism is the same as that of the displacement, and it has a lot of advantages, such as high bearing capacity, small deformation, good control performance and low natural frequency. Therefore, the research on realizing methods and application of NS mechanism has been more and more in recent years. In 1989, Alabuzhev\textsuperscript{4} published the first monograph about NS system, which comprehensively introduced the realization forms and theoretical analysis of NS mechanism associated with the application in vibration control. A NS mechanism based on compression linkage was designed by Platus\textsuperscript{5} and applied in an ultra-low frequency vibration isolation mechanism. In the Ref.\textsuperscript{6}, a NS device based on a compressed rod and spring was designed, and a quasi-zero stiffness isolator was studied in parallel with a positive stiffness spring. Sarlis et al.\textsuperscript{7} connected a rotating rod with springs to design a NS device which was applied to resistant seismic system of earthquake, and its vibration reduction characteristics were simulated and analyzed. The quasi-zero stiffness isolators usually consisted of an elastic element with positive stiffness and a NS corrector which were widely used in vibration control.\textsuperscript{8-10}

Except to achieving a mechanical NS structure, Carrella et al.\textsuperscript{11} used three attractive magnets to form the NS properties and
experimentally verified the feasibility of this device. Hoque et al.\(^{12}\) proposed a zero power control of magnetic spring with NS characteristics which formed a vibration isolator in series with a positive stiffness. This positive stiffness and NS isolator in series had a good effect on resisting to the direct disturbance on the load of the isolator. Braun et al.\(^{13}\) presented a compliant actuator that employed a passive NS mechanism in conjunction with an effectively tunable positive stiffness mechanism. Alabузhev et al.\(^{8}\) introduced a stiffness-correcting device with NS characteristics based on a linear spring so that the isolator had quasi-zero stiffness characteristics within a certain displacement and summarized the characteristic calculation of quasi-zero stiffness isolator and nonlinear vibration isolator. Shen et al.\(^{14,15}\) designed two new types of dynamic vibration absorbers (DVAs) with NS characteristics, and found that NS DVAs had better control performance than traditional DVAs. Aiming at the different beam structures, Hossein et al.\(^{16-18}\) designed the vibration control strategies and analyzed the complex dynamical behaviors.

Piecewise linear systems are special nonlinear systems which depend on state transition.\(^{19}\) Shen et al.\(^{20-22}\) analytically studied some semiactive control oscillators with piecewise nonlinearities, and presented the optimization method for system parameters based on the obtained approximate algebra equations. Hu et al.\(^{24}\) studied a semiactive DVA with piecewise linear stiffness, and the vibration frequency of the absorber was controlled by the clearance of the elastic element. Some work on the single-degree-of-freedom semiactive vibration isolation system had been studied so that it provided an effective method for design, analysis, transformation, and control for this kind of semiactive control nonlinear system. Lei et al.\(^{25}\) analyzed properties of single degree-of-freedom system of piecewise linear vibration control, and the amplitude response equation of the system was obtained.

A piecewise NS mechanism is designed in this paper, and it is applied to the DVA system to reduce the vibration response of the primary system in Section 2. In Section 3 the differential motion equations are derived according to Newton’s law of mechanics, and the approximately analytical solution of the system is obtained by means of the averaging method. The amplitude-frequency curve of DVA system with the piecewise NS characteristics is obtained, and the correctness of the analytical solution is certified by comparing with the numerical solution in Section 4. Under the condition of harmonic excitation and random excitation, the better control performances of the new DVA system in this paper are verified in the following parts.

## 2 | THE MODEL OF THE PIECEWISE NS MECHANISM

The prototype of the piecewise NS mechanism based on rollers-moving cam device is shown in Figure 1. When a weak disturbance is applied on the moving cam, the vertical stiffness of the model is zero at static equilibrium position and nearby. When the disturbance increases, the rollers will move on the arc surface of the moving cam.
The expression of the relationship between the force and displacement for the realization of NS is given by

$$K = \frac{dF}{dx} = 2k_0 \left[-1 + \frac{(R + r - d)(R + r)^2}{\sqrt{(R + r)^2 - (x - \Delta)^2}}\right].$$ \hspace{1cm} (4)

From Equation (4), it can be found that the designed mechanism shows NS characteristic within a certain range of vertical displacement. When the vertical displacement exceeds a certain interval, the stiffness changes from negative to positive one.

Supposing \( \hat{k} = \frac{K}{2k_0} \), one could obtain the dimensionless stiffness \( \hat{k} \) as follows:

$$\hat{k} = -1 + \frac{(1 - \varepsilon)}{\sqrt{1 - \varepsilon^2}}.$$ \hspace{1cm} (5)

When \( x \leq -\Delta \), the analysis procedure is similar to the above analysis, and the dimensionless stiffness \( \hat{k} \) is obtained as follows:

$$\hat{k} = -1 + \frac{(1 - \varepsilon)}{\sqrt{1 - \varepsilon^2}}.$$ \hspace{1cm} (6)

where

$$\hat{k} = \frac{x + \Delta}{R + r}$$

Thus, the stiffness expression of the piecewise NS mechanism could be summarized as

$$\hat{k} = \begin{cases} 1 + \frac{(1 - \varepsilon)}{\sqrt{1 - \varepsilon^2}} & \text{if } x \geq \Delta \leq -\Delta, \\ 0 & \text{if } -\Delta \leq x \leq \Delta, \\ -1 + \frac{(1 - \varepsilon)}{\sqrt{1 - \varepsilon^2}} & \text{if } x \leq -\Delta \leq \Delta \end{cases}$$ \hspace{1cm} (7)

From Equation (7), it can be seen that the stiffness value of the piecewise stiffness mechanism is primarily related with the parameter \( \varepsilon \) and \( x \). Figure 3 presents the effect of the two parameters on the stiffness characteristics of the mechanism. Under the condition of \( |x| \geq \Delta \), the stiffness of the piecewise stiffness mechanism is negative within a certain vertical displacement range when \( \varepsilon < 1 \). Furthermore, the stiffness of the piecewise stiffness mechanism is always negative when \( \varepsilon \geq 1 \). With the increase of disturbance, the absolute value of the NS will be changed abruptly.

### 3 APPROXIMATE SOLUTION FOR DVA WITH PIECEWISE NS CHARACTERISTICS

A piecewise NS mechanism is designed by choosing appropriate mechanism parameters in Table 1, and the relationship between force and displacement is shown in Figure 4. The piecewise NS mechanism is applied to the classical Voigt DVA system to establish a dynamic model with the piecewise NS mechanism shown in Figure 5, where \( m_1, m_2, k_1, \) and \( k_2 \) are the masses and linear stiffness coefficients of the primary system and DVA, respectively. \( x_1 \) and \( x_2 \) are the displacements of the primary system and DVA. \( F_0 \) and \( \omega \) are the amplitude and frequency of the force excitation. \( c \) is the damping coefficient of DVA. The ground spring \( k \) is the stiffness of the piecewise NS mechanism.

Based on the Newton's second law, the motion equation could be established

$$\begin{align*}
m_1\ddot{x}_1 + k_1x_1 + k_0(x_1 - x_2) + c(\dot{x}_1 - \dot{x}_2) &= F_0 \cos \omega t \\

m_2\ddot{x}_2 + k_0(x_2 - x_1) + c(\dot{x}_2 - \dot{x}_1) + F(x_2) &= 0
\end{align*}$$ \hspace{1cm} (8)

Using the following parametric transformation

$$\begin{align*}
\omega_1 &= \sqrt{\frac{k_1}{m_1}}, \quad \omega_2 = \sqrt{\frac{k_2}{m_2}}, \quad \tau = \omega t, \quad r_1 = \frac{\omega_1}{\omega_2}, \quad r_2 = \frac{\omega_2}{\omega_1}, \\
q &= \frac{m_2}{m_1}, \quad \xi_2 = \frac{c}{2m_2\omega_2}, \quad f = \frac{F_0}{k_0}, \quad a = \frac{k}{k_0}, \quad \lambda = \frac{\omega}{\omega_1}, \\
A &= \frac{x_2}{m_1k_0}, \quad \delta = \arccos\frac{\Delta}{a_2}
\end{align*}$$

Equation (8) becomes

$$\begin{align*}
r_1^2\frac{d^2x_1}{dt^2} + px_1 + \frac{\dot{q}}{\dot{p}}(x_1 - x_2) + 2\xi_2\dot{q}\frac{dx_1}{dt} - \frac{dx_2}{dt} &= fp \cos \tau \\
r_2^2\frac{d^2x_2}{dt^2} + x_2 - x_1 + 2\xi_2\frac{dx_2}{dt} - \frac{dx_1}{dt} + F(x_2) &= 0
\end{align*}$$ \hspace{1cm} (10)
All the perturbation methods are almost equivalent when they are used to find the first-order approximate solution of nonlinear oscillator. Here we select the averaging method because it is simple to realize for the piecewise linear system. When the averaging method is applied to obtain the approximately analytical solution of Equation (10), the solution has the form as follows:

\[ x_1 = \alpha_1(t) \cos \varphi_1(t) \]

\[ \frac{dx_1}{dt} = -\alpha_1(t) \sin \varphi_1(t) \]  

\[ \text{(11a)} \]
By differentiating Equations (11a) and (11b) with respect to $\tau$, the following results could be obtained:

\[
\frac{d^2x_1}{d\tau^2} = \frac{d^2x_2}{d\tau^2} = \frac{d^2x_3}{d\tau^2} = \frac{d^2x_4}{d\tau^2} = \frac{d^2\phi_1}{d\tau^2} = \frac{d^2\phi_2}{d\tau^2} = \frac{d^2\phi_3}{d\tau^2} = \frac{d^2\phi_4}{d\tau^2} = 0
\]

\[
\frac{dx_1}{d\tau} = \frac{dx_2}{d\tau} = \frac{dx_3}{d\tau} = \frac{dx_4}{d\tau} = \frac{d\phi_1}{d\tau} = \frac{d\phi_2}{d\tau} = \frac{d\phi_3}{d\tau} = \frac{d\phi_4}{d\tau} = 0
\]

The four unknown variables could be obtained from Equation (14)
PARAMETERS OPTIMIZATION AND NUMERICAL SIMULATION

To illustrate the precision of the approximate solution, the system simulation parameters shown in Table 2 are chosen. According to the actual engineering and stability characteristics of the piecewise NS system, the parameter ranges of the piecewise NS system are summarized in Table 3. Through optimization theory, the optimum parameters are found and shown in Table 4.

In the numerical investigation, we choose the fourth-order Runge-Kutta method to integrate the original differential equation and obtain the steady-state response by disposing the transient one. The total setting time is 1000 s, and the maximum values of the stationary response in the last 200 s are taken as the amplitudes. In the analytical researching procedure, the steady-state response could be obtained by solving the abovementioned system of nonlinear algebra equations. The approximate solution result is shown in Figure 6 for DVA with piecewise NS characteristics. From Figure 6 it could be found that the approximately analytical solution has satisfactory precision, and agrees well with the numerical solution. In the two resonance ranges and the zone nearby zero-frequency, there are some errors between the approximately analytical solution and the numerical result, which are small enough and may have little effect on the analysis of DVA with the piecewise NS characteristics. In fact, the analytical solving procedure is primarily for main resonance, so that the errors are acceptable.

To illustrate the optimal parameters of the system in Table 4, Figures 7–9 show normalized amplitude-frequency curves under different NS ratios, piecewise clearances, and damping ratios. From the comparison of amplitude-frequency curves, it can be clearly seen that the system has a good vibration control effect under the optimal parameters.

COMPARISON WITH OTHER DYNAMIC VIBRATION ABSORBERS

5.1 Comparison of the responses under harmonic excitation

To certify the vibration reduction effect of DVA with piecewise NS characteristics, the model is compared with two classical DVAs, that is, the Voigt model by Ormondroyd and Den Hartog26 as well as grounded model by Ren.33 When the same mass ratio is selected as $\mu = 0.1$, one can obtain all the amplitude frequency curves shown in Figure 10. From this figure, it can be easily concluded that the DVA with the piecewise NS characteristics in this paper can not only significantly suppress the amplitude of the primary system in resonance region, but also extend the efficient frequency range of vibration absorption under the same mass ratio. That means it has distinct advantages compared to the existing models when subjected to harmonic excitation.

### Table 2 Main system simulation parameters

| Main system          | Values  |
|----------------------|---------|
| Structure mass $m_1$/kg | 1       |
| DVA mass $m_2$/kg   | 0.1     |
| Structure stiffness $k_1$/[N m⁻¹] | 100     |
| Amplitude of the force excitation $F$/N | 1000    |

Abbreviation: DVA, dynamic vibration absorber.

### Table 3 Parameter ranges of DVA

| Parameters of DVA       | Range        |
|-------------------------|--------------|
| DVA stiffness $k_2$/[N m⁻¹] | $10 \leq k_2 \leq 100$ |
| Negative stiffness ratio $\alpha$ | $-0.8 \leq \alpha \leq -0.1$ |
| Damping ratio of DVA $\xi_2$ | $0.01 \leq \xi_2 \leq 1$ |
| Piecewise interval $\Delta$/m | $0.01 \leq \Delta \leq 0.5$ |

Abbreviation: DVA, dynamic vibration absorber.

### Table 4 Optimal parameters of DVA

| Optimal parameters of DVA | Values  |
|---------------------------|---------|
| DVA stiffness $k_2$/[N m⁻¹] | 19.6    |
| Negative stiffness ratio $\alpha$ | -0.71   |
| Damping ratio of DVA $\xi_2$ | 0.36    |
| Piecewise clearance $\Delta$/m | 0.1     |

Abbreviation: DVA, dynamic vibration absorber.

4 | PARAMETERS OPTIMIZATION AND NUMERICAL SIMULATION

To illustrate the precision of the approximate solution, the system simulation parameters shown in Table 2 are chosen. According to the actual engineering and stability characteristics of the piecewise NS system, the parameter ranges of the piecewise NS system are summarized in Table 3. Through optimization theory, the optimum parameters are found and shown in Table 4.

In the numerical investigation, we choose the fourth-order Runge-Kutta method to integrate the original differential equation and obtain the steady-state response by disposing the transient one. The total setting time is 1000 s, and the maximum values of the stationary response in the last 200 s are taken as the amplitudes. In the analytical researching procedure, the steady-state response could be obtained by solving the abovementioned system of nonlinear algebra equations. The approximate solution result is shown in Figure 6 for DVA with piecewise NS characteristics. From Figure 6 it could be found that the approximately analytical solution has satisfactory precision, and agrees well with the numerical solution. In the two resonance ranges and the zone nearby zero-frequency, there are some errors between the approximately analytical solution and the numerical result, which are small enough and may have little effect on the analysis of DVA with the piecewise NS characteristics. In fact, the analytical solving procedure is primarily for main resonance, so that the errors are acceptable.

To illustrate the optimal parameters of the system in Table 4, Figures 7–9 show normalized amplitude-frequency curves under different NS ratios, piecewise clearances, and damping ratios. From the comparison of amplitude-frequency curves, it can be clearly seen that the system has a good vibration control effect under the optimal parameters.
5.2 | Comparison of responses under random excitation

The primary system is usually subjected to random excitation in practical engineering, so that it is very important and meaningful to investigate the statistical system response under random excitation. At first, the parameters of the primary system are still selected as Table 2 and the piecewise clearance $\Delta = 0.005\text{m}$ is used to ensure the NS device works normally. To approach realistic situation, 50 s random force excitation is constructed. This excitation is simply composed of 5000 normalized random numbers with zero mean value and variance as 9. Here the parameters of excitation can be selected almost arbitrary, and it will not affect the qualitative results between the different DVAs. Based on some numerical methods, one could obtain the time history of the random force excitation shown in Figure 11.

Based on the numerical integration method, the response of the primary system without DVA and with different DVAs can be obtained. The time histories for the displacements of these primary systems can be drawn in Figures 12–15 based on the computation results. Moreover, the response variance of the primary system is...
often associated with the system energy after vibration control, so that one can get all the response variances and decrease ratios summarized in Table 5 to intuitively compare the performance of vibration reduction.

From these figures and Table 5, it could be easily found that the DVA with piecewise NS characteristics could reduce not only the peak value of the displacement response remarkably, but also the statistical response in whole-frequency range excellently. That means the DVA in this paper has more advantages even if the primary system is subjected to random excitation, which verifies the feasibility of this model in practical engineering.

6 | CONCLUSIONS

A new type of piecewise NS mechanism is designed and the relationship between the force and displacement is obtained. The presented piecewise NS mechanism is applied to the DVA system to reduce the vibration response of the primary system. The approximately analytical solution of the primary system is obtained by means of the averaging method, and the correctness of the analytical solution is certified by comparing with the numerical solution. Moreover, it is proved that the model of DVA with the piecewise NS mechanism in this paper can not only significantly suppress the amplitude of the primary system in resonance region, but also extend the efficient frequency range of vibration absorption under the condition of harmonic excitation. Furthermore, the DVA system with piecewise NS characteristics has

| Model of DVA                     | Variances | Decrease ratios (%) |
|----------------------------------|-----------|---------------------|
| Without DVA                     | 0.0403    |                     |
| DVA by Ormondroyd and Den Hartog| 0.0029    | 92.80               |
| DVA by Ren                      | 0.0025    | 93.79               |
| Our result                      | 0.0017    | 95.78               |

Abbreviation: DVA, dynamic vibration absorber.
better vibration reduction effect than other DVAs through comparison under random excitation. The better control performance of this system to harmonic and random excitation verifies its feasibility in practical engineering. The analytical procedure and results are meaningful to the vibration control of similar engineering structures.

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CONFLICT OF INTEREST

The authors declare that there are no conflict of interest.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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