The Wave Making Resistance Evaluation of The Fishing Ship With Skeg Using Tent Function Method

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Abstract. The numerical computation method to explore the wave-making resistance for a case of the small ship with skeg in calm water’s surface condition is described. The problem is solved using the theory of Thin ship since it suitable for a case of slenderness of ship’s hull. The distribution of source along the hull’s centre plane is written in the Green function which is a form of Havelock source in ideal flow beneath free surfaces. In numerical computation, the approach of Tent function is proposed to discretize the form of the ship’s hull developed from the data offset for solving the Michell integral problem. Employing the numerical methods developed, the wave resistance prediction for the case of the Wigley hull and the traditional fishing ship ‘Madura’ in Indonesia are conducted within several of Froude numbers. Numerical results show an effect of the hull’s skeg on the ship's resistance due to wave after body significantly.

1. Introduction

Froude's hypothesis says that the component of resistance in a ship consists of friction resistance and residuary resistance [1]. Friction resistance is a liquid that experiences shear forces on the body surface in the tangential direction. The amount of friction resistance that occurs on a ship hull is influenced by the body’s wetted surface, the number of Reynolds, and the shape factor of hull. Residuary resistance involves all possible resistances other than friction resistance, and to obtain it by reducing total resistance with friction resistance. The wave resistance is understood as the energy transfer of a water wave and places itself as a force that opposes the forward ship. It is the main part of residuary resistance and can be evaluated through the potential flow theory.

The first equation for the wave resistance problem was theoretically introduced by Michell [2] by considering the ship as a thin plate that moves on the surface of the ideal fluid. The Michell Integral is basically solved as a double Fourier transformation of the velocity potential. Gadd [3] and Dawson [4] initiated the Rankine source method which was followed by researchers to deal with the Michell’s integral solution. In this method, the singularity should be distributed on both surfaces of the water and the ship. In contrast to the Rankine source method, the Havelock source is only distributed on the surface of the ship’s body, and automatically fulfils the conditions of a water surface as well as a far-field [5]. Havelock's moving source is as the Green’s function of wave making which contains some complicated singularities and the function itself is very oscillating.

In numerical computation, Hsiung [6] introduced the tent function to solve the numerical integration problem of the ship’s hull in the context of solving wave resistance integral. With this method, the hull geometry is easily explained by a linear form function into the data offset of hull which is applied to any number hull of ship. This approach was successfully used to solve the wave resistance problem of the single, double, and triple hull of ships [7] and the underwater body [8]. This...
paper solves the wave resistance of the fishing vessel without transom equipped with skegs by adopting the tent function approach in solving Michell's integrals.

2. Formulation of Problems
The system of xyz co-ordinate for the body-fixed problem is set up as expressed in figure 1.

Figure 1. System of cartesian co-ordinate

Where B is a breadth, Lwl is a length of water line, T is a draft, c is a steady forward velocity, S is ship’s surface, and $S_f$ is water surface.

2.1. Boundary Conditions
Fluid is assumed ideal which is an inviscid, incompressible, and irrotational, fulfils the Laplace’s equation, as shown in equation (1).

$$\nabla^2 \varphi(x, y, z) = 0 \quad (1)$$

For the problem solution, the boundary conditions are set up as follow.

a. Kinematics boundary on water surface;
The Sf is described as $z = F(x, y)$. The normal velocity of fluid at the surface of calm water and the normal velocity of the body surface itself must be equal $\varphi_n = V \cdot n$ that is the substance derived from $z = F(x, y) = 0$ or vanishes on the surface of water, see equation (2).

$$\varphi_z + F_z (c - \varphi_x) - \varphi_y F_y = 0 \quad \text{on} \quad z = F(x, y) \quad (2)$$

b. Dynamics boundary on water surface;
Assumed a pressure at the water free surface equal to the ambient pressure, and a steady flow, see equation (3)

$$F(x, y) - \frac{c \varphi_x}{g} + \frac{1}{2g} (\varphi_x^2 + \varphi_y^2 + \varphi_z^2) = 0 \quad \text{on} \quad z = F(x, y) \quad (3)$$

c. Kinematic condition on the ship’s hull;
Given the ship’s surface $S$ is described by $y = \pm F(x, z)$ the normal potential velocity of the fluid and the hull must be equal hence,

$$\varphi_y \pm F_y (c \mp \varphi_x) - \varphi_z F_z = 0 \quad \text{on} \quad y = \mp F(x, z) \quad (4)$$

d. Radiated wave condition
In order to ensure further waves propagating behind the hull ship that we specify, [7]
\[ \varphi(x, y, z) = O\left(\sqrt{x^2 + y^2}\right) \quad \text{as} \quad x^2 + y^2 \to \infty, \quad x < 0 \]
\[ = O(1) \quad \text{as} \quad x^2 + y^2 \to \infty, \quad x > 0 \]

or that there is no disturbance far ahead,

\[ \lim_{x \to \infty} \left( \varphi_x^2 + \varphi_y^2 + \varphi_z^2 \right) R = 0 \quad \text{where} \quad R = \sqrt{x^2 + y^2} \]

Solutions that meet the boundary conditions can be superimposed to obtain the final solution that refers to the method of Green’s function. The Havelock moving source as Green’s function must meet the requirements of the boundary of 1st order-free surface and the governing Laplace equation.

### 2.2. Green Function Solution

The Havelock moving source or the Green’s function, \( G \), for the problem of 1st order boundary value can be examined in three parts as shown in equation (7).

\[ \phi(x, y, z) = \Phi(x, y, z, x_1, y_1, z_1) + \Phi(x, y, z, x_1, y_1, z_1) + \Phi(x, y, z, x_1, y_1, z_1) \]

Where the integral from 0 to \( \infty \) is a principal value integral of Cauchy and \( k_0 = g/c^2 \), \((x, y, z)\) is field coordinate, \((x_1, y_1, z_1)\) is body coordinate, and \( \theta, m \) is dummy variables for integration. \( G_i \) represents the singularity system on the body and also its image above the free surface level. \( G_j \) can be regarded as the far-field influence on the solution while \( G_k \) is the local disturbance. All three comprise the solution subject to the boundary set up within the linearized free surface. The potential velocity is then obtained with formula as present in equation (8).

\[ \varphi(x, y, z) = \int_{\Sigma_0} \sigma(x, y, z) G(x, y, z, x_1, y_1, z_1) dS \]

Where \( \sigma \) is the strength function of the source/sink distribution. Since the terms of the potential velocity in equation (8) is known, the resistance formula can be calculated, however, the strength of the source is still unknown. The center plane distribution has long since been determined using the ‘thin-ship’ assumption that on the center plane follow Peng [7].

\[ \varphi_y(x, \pm 0, z) = \lim_{y \to \pm 0} \varphi_y(x, y, z) = \pm 2\pi \sigma(x, z) \]
2.3. Wave Resistance Formulation
The continuous distribution of sources and sinks moving in under deep surface as presented in equation (10) and (11) which is follow to Michell [2], represent the resistance equation due to generated wave after body.

\[ R = 16\pi \rho k_o^2 \int_0^{\pi/2} \left[ P(\theta)^2 + Q(\theta)^2 \right] \sec^3 \theta \, d\theta \]  
\[ \left\{ \begin{array}{l}
  P(\theta) \\
  Q(\theta)
\end{array} \right\} = \int_S \sigma \exp(k_o z \sec^2 \theta) \sin(\cos(k_o x \sec \theta + k_o(y \tan \theta \sec \theta)) \, dS \]  

The previously derived potential is used, and the source strength \( \sigma = -c f_x(x, z)/2\pi \) can be assumed to be the center of the ship's hull.

3. Numerical Procedure
The tent function is applied to handle the solution of numerical integration, see figure 2. The hull function \( f(x, y) \) is explained as the hull slope function with concern to the x-direction under the boundary condition, which can be described as equation (12) and (13) follows:

\[ f_x(x, z) = \sum_{ij} f_x^{ij} \frac{-c}{2\pi} (x, z) \]  
\[ f_x^{ij}(x, z) = \left[ \frac{1}{x_{j+1} - x_j} \right] \left[ \frac{z - z_{j+1}}{z_j - z_{j+1}} \right] y_{i,j} + \left[ \frac{1}{x_{i+1} - x_i} \right] \left[ \frac{z - z_{i+1}}{z_i - z_{i+1}} \right] y_{i+1,j} \]  
\[ + \left[ \frac{1}{x_{i+1} - x_i} \right] \left[ \frac{z - z_{j+1}}{z_j - z_{j+1}} \right] y_{i+1,j+1} + \left[ \frac{1}{x_{i+1} - x_i} \right] \left[ \frac{z - z_{j+1}}{z_j - z_{j+1}} \right] y_{i+1,j+1} \]  

Figure 2. Unit Tent Function
This $P$ and $Q$ can be expanded in more detail as,

$$\begin{align*}
   \left\{ \begin{array}{l}
   P(\theta) \\
   Q(\theta)
   \end{array} \right\} &= -\frac{c}{2\pi} \int_{S} f_{s}(y,z) \exp\left(k_{o}z\sec^{2}\theta\right) \left[ \sin \right] \left( k_{o}x\sec\theta \right) ds \\
   &= -\frac{c}{2\pi} \sum_{j} \int_{X} \sin \left( k_{o}x\sec\theta \right) dx
\end{align*}$$

(14)

$$\begin{align*}
   \left\{ \begin{array}{l}
   A_{ij} \int_{Z_{j}}^{Z_{j+i}} \left( z-z_{j+i} \right) \exp\left(k_{o}z\sec^{2}\theta\right) dz + B_{ij} \int_{Z_{j}}^{Z_{j+i}} \left( z-z_{j} \right) \exp\left(k_{o}z\sec^{2}\theta\right) dz \\
   \end{array} \right\}
\end{align*}$$

Let the following equation which represents each separate integral be given, and hence

$$\begin{align*}
   \left\{ \begin{array}{l}
   P(\theta) \\
   Q(\theta)
   \end{array} \right\} &= -\frac{c}{2\pi} \sum_{j} \left\{ \begin{array}{l}
   E_{j} \\
   F_{j}
   \end{array} \right\} \left( A_{ij}C_{j} + B_{ij}D_{j} \right) 
\end{align*}$$

(15)

where

$$A_{ij} = \left[ \frac{1}{x_{i} - x_{i+1}} \right] Y_{i,j} + \left[ \frac{1}{z_{j} - z_{j+1}} \right] Y_{i+1,j}$$

(16)

$$\begin{align*}
   B_{ij} &= \left[ \frac{1}{x_{i} - x_{i+1}} \right] Y_{i,j+1} + \left[ \frac{1}{z_{j+1} - z_{j}} \right] Y_{i+1,j+1} 
\end{align*}$$

(17)

$$C_{j} = \int_{Z_{j}}^{Z_{j+i}} \left( z-z_{j+i} \right) \exp\left(k_{o}z\sec\theta\right) dz$$

(18)

$$D_{j} = \int_{Z_{j}}^{Z_{j+i}} \left( z-z_{j} \right) \exp\left(k_{o}z\sec^{2}\theta\right) dz$$

(19)

$$E_{j} = \int_{X_{j}}^{X_{j+i}} \cos\left(k_{o}x\sec\theta\right) dx$$

(20)

$$F_{j} = \int_{X_{j}}^{X_{j+i}} \sin\left(k_{o}x\sec\theta\right) dx$$

(21)
4. Numerical Results
To confirm the acceptable thin ship modified results, the first step is required a common ship’s hull of the Wigley which satisfies this requirement. The offsets data are a quadratic function of the station and waterline, being zero at bow, stern and baseline, symmetric around the mid ship station where the maximum beam occurs at the design of waterline as indicated in equation (22).

\[ y(x, z) = \frac{B}{2} \left[ 1 - \left( \frac{x}{L} \right)^2 \right] \left[ 1 - \left( \frac{z}{T} \right)^2 \right] \]  

(22)

The lines plan of the hull is developed and presented in figure 3. Implement the computer program developed follow the present algorithm to evaluate the wave resistance after the Wigley’s body, and the computational results as seen at figure 4. Figure 4 describes the comparison between the results calculated by this method on the coefficient value of the wave resistance, and the experimental results refer to [9] and the computational results in [10] for the case of Wigley's parabolic hull; and it shows a very close agreement with each other.

![Figure 3. Wigley parabolic hull](image)

After validation of the developed numerical program, the evaluation of wave-making resistance for the traditional fishing ship of Madura Island in Indonesia is computed by present method. Two cases of the hulls with and without skeg as presented in figure 5 are conducted. In practical, the skeg is used with purpose to improve ship’s construction more strength longitudinally. The skeg is constructed at fore, bottom, and after of hull continuously. For the case of hull without skeg, the main dimensional of ship are Length of water line (Lwl) 9.63 m, Breadth (B) 2.89 m, and Draft (T) 0.5. And for the hull with skeg, the length of water line is differently about 10.65 m, and the same value for others dimensional.

![Figure 4. Result comparison of the wave-making resistance for the Wigley’s hull](image)
4.1. Style and spacing

4.2. 4.3. 4.4. 4.5. 4.6.

Figure 5. Lines plan of ‘Madura’ Fishing vessel

The computational results of the wave making resistance coefficients ($C_w$) for both cases of the fishing ship with and without skeg are presented in figure 6. In general, the coefficient value of wave making resistance inclines for increasing value of Froude number. Except for certain values of the Froude number, the wave resistance coefficient decreases about three times after experiencing the peak of the curve, see figure 6. It shows the effect of additional skeg can reduce the magnitude of wave making resistance significantly, even if the area of wetted surface on the hull with skeg about 22.1 m$^2$ that is upper value than the hull without skeg about 18.4 m$^2$. The reduction of wave making resistance for hull with skeg is shown at Froude number of 0.3 and after 0.45 that its are respectively around 34% and 19%.

Figure 7 shows the wave contour that is generated by the Madura fishing ship with skeg for Froude number about 0.4.

Figure 6. Wave-making resistance coefficient of the ‘Madura’ ship with and without skeg
Figure 7. Contour of wave surface generated by the Madura fishing ship for Fr = 0.4

5. Conclusions
Numerical investigations are performed to explore the waves resistance of the fishing ship without and with skegs. Using the linear theory of wave equation and the assumption of thin ship, the computational program is fully developed for the solution. The numerical integration technique is performed to calculate resistance of the initiated wave after body and give the similar results with the thin ship theory. The Wigley model is used to validate the numerical technique of Tent function method in evaluating the coefficient of wave making resistance. The computational results show similar with the early works for all Froude number considered. The verification of the computer program is conducted for two cases of the Madura fishing ship such as the ship’s hull with and without skeg. The results also show the additional skeg can reduce the resistance of wave making on the body especially for a high number of Froude significantly.

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