Research of the Lifting Wavelet Arithmetic and Applied in Rotary Mechanic Fault Diagnosis

S Q Zhang, N He, J T Lv, X H Xu and X Y Zang

Electrical Engineering Institute, Yanshan University, Qinhuangdao, China, 066004

E-mail: zhshq-yd@163.com

Abstract. The lifting wavelet transform is completely based on the space-time area instead of relying on Fourier transform, so it can construct wavelet in non-shift area to achieve the separation of signal in different frequency bands. In this paper, the lifting scheme of wavelet and its multiphase expression are analysed, and applied to fault diagnosis of gears, rolling bearings and rotor rubbings. Simulation was accomplished based on the analysis of fault vibration signals. The results indicate that the lifting scheme of wavelet is effective to pick up fault characteristics.

1. Introduction

The vibration signal and its frequency components generated by large rotary machine are tightly related with the type, the position and the causation of faults, which is important for fault diagnosis. Because the running velocity of rotary machine is instable and the burthen changes and so on, the vibration signal would be non-stationary [1]. The wavelet transform is widely used in the area of fault diagnosis, because its perfect localization in both time and frequency domain and it is fit for the process of non-stationary signals [2].

In the middle of 1990s, Sweldens [3-5] brought forward the lifting wavelet transform. It is based on the theory of biorthogonal wavelet and perfect-reconstruction filter bank. It improves the performance of wavelet and its dualization by lifting and dual lifting keeping the biorthogonal characteristic. Comparing with traditional wavelet, the lifting scheme does not need additional memory and is easy to realize in chips; its arithmetic is simple for parallel disposal, so its calculation is faster; it can be transformed in integers. Hence, its application is wide [6-7].

2. Research of the wavelet lifting arithmetic

2.1. The principle of lifting arithmetic

Generally, lifting scheme contains 3 steps to decompose signal, that is, Split, Predict and Update, which is shown as figure 1.

The original signal is $s[n]$. It is transformed into approached signal in low frequency $c[n]$ and detail signal $d[n]$. 
Figure 1. The decomposition of signal in lifting scheme.

(1) Split: In this step, the original signal $s[n]$ is split into two subsets which do not overlap with each other: $s_e[n]$ (even sequence) and $s_o[n]$ (odd sequence), that is

$$
\begin{align*}
    s_e[n] &= s[2n] \\
    s_o[n] &= s[2n+1]
\end{align*}
$$

(2) Predict: If the original signal is locally coherent, the subsets $s_e[n]$ and $s_o[n]$ are also coherent, so one subset can be predicted by another. Commonly we use even sequence to predict odd sequence,

$$
d[n] = s_o[n] - P(s_e[n])
$$

Where $P$ is the predict operator and reflects the degree of correlation of data. $P(s_e[n])$ implies that the value of $d[n]$ can be predicted by the value of $s_e[n]$.

(3) Update: $c[n]$ in figure 1 is the approach signal which has been decomposed. One of the important features is that its average value should be equal to the average value of original signal $s[n]$. So we can use detail subset $d[n]$ to update the signal $s_e[n]$, expressed by $c[n]$:

$$
c[n] = s_e[n] + U(d[n])
$$

Where the operator $U$ denotes some combination of $d[n]$. If we continue decomposing the approach signal $c[n]$ for two levels above, we can get a multilevel decomposed signal.

Figure 2. The reconstruction of signal in lifting scheme.

Figure 2 shows the reconstruction of lifting scheme, which can be expressed as following:

$$
\begin{align*}
    s_e[n] &= c[n] - U(d[n]) \\
    s_o[n] &= d[n] + P(s_e[n]) \\
    s[n] &= \text{Merge}(s_e[n], s_o[n])
\end{align*}
$$

2.2. The polyphase expression of wavelet lifting

The traditional biorthogonal wavelet transform may find their implementation by filter bank in figure3. $h_1$ and $g_1$ are decomposition filters, $h$ and $g$ are reconstruction filters.

Figure 3. The structure of filter bank.

Then the decomposition of wavelet can be written as:
Where \( P(z) \) is polyphase matrix, 
\[
P(z) = \begin{bmatrix} h_{e}(z) & g_{e}(z) \\ h_{o}(z) & g_{o}(z) \end{bmatrix} \tag{6}
\]

The decomposition of the polyphase matrix can be implemented by Euclidean arithmetic of Laurent polynomial [8]:
\[
P(z) = \prod_{i=1}^{m} \begin{bmatrix} 1 & 1 \ 0 & 1 \end{bmatrix} \begin{bmatrix} k & 0 \ 0 & k \end{bmatrix}
\]
\[
\tilde{P}(z^{-1})^T = \prod_{i=1}^{m} \begin{bmatrix} 1 & 1 \ 0 & 1 \end{bmatrix} \begin{bmatrix} -s_{i}(z) & 1/k \ 0 & 0 \end{bmatrix}
\tag{8}
\]

Therefore, given \( P(z) \), we can do the lifting scheme as above steps to get \( s(z) \) and \( t(z) \).

3. Simulation of the arithmetic

3.1. Fault analysis and Simulation of gear

Gear is one of the most important elements in machine equipments. The frequency of faults in gear contains two parts, one is the carrier wave signal of meshing frequency and its harmonic wave; the other is the modulation signal which contains varieties of amplitude and phase in low frequency.

The discrete vibration signals of gear can be written as:
\[
x_{0}(n) = \sum A_{i} \cos(2\pi f_{1} n / f_{s} + \phi_{i}) + \sum A_{j} \cos(j \cdot 2\pi f_{2} n / f_{s} + \phi_{j}) + x_{1}(n) \tag{9}
\]

Where, \( f_{1} \) is the rotary frequency of axes, \( f_{2} \) is meshing frequency of gear, \( f_{s} \) is sampling frequency, \( x_{1} \) is noise.

When local damages occur, the spot corrosion in gear’s surface generates break vibrating pulse signal whose frequency is equal to that of the axes. So the vibration signals of gear with local damages in surface can be expressed as:
\[
x(n) = x_{0}(n) + (\sum A'_{i} \cos(i \cdot 2\pi f_{1} n / f_{s} + \phi'_{i})) \cdot \sum R_{N}(n - m \cdot k \cdot l) \tag{10}
\]

Where, \( R_{N}(n) \) is a rectangular sequence with \( N \) in length, \( m \) is initial point of damages, \( l \) is periodicity of mutations.

Now we assume the meshing frequency of gear is 300Hz, the rotary frequency of axes is 10Hz, the frequency of fault signal is 10Hz. Figure 4 shows the results of decomposition and reconstruction with wavelet lifting scheme. The sampling frequency is 1000Hz, with 300 dates in length.

![Figure 4. The reconstruction of coefficients in gear with the lifting wavelet.](image)

As shown in the figure 4, there are meshing frequency of gear in the frequency band d3 (125Hz—250Hz) and the second frequency multiplication and the third frequency multiplication in the band d2 (250—500Hz), there are also pulse signals with same interval. There are about 3 peaks within 0.3s, so the periodicity of peak value is 0.1s. Its frequency is 10Hz, being close to the fault frequency. So it can
be said that lifting scheme is effective to pick up fault characteristic frequency. Besides, the initial point of fault signal \( m \) and the interval \( l \) can be read clearly in the figure.

3.2. Fault analysis and Simulation of rolling bearing

When local damages come out, the parts of rolling bearing will strike the impaired loci periodically during running and the impulse force drives bearing block and bracing structures, so a series of damped oscillation generated. Its frequency is characteristic frequency of faults, which is determined by rotational speed and physical dimension and position of impaired loci of rolling bearings. So it is possible to find fault and fix it by checking the characteristic frequency.

The vibration signals of rolling bearings with local damages can be expressed as:

\[
x(n) = x_0(n) + \left( \sum A_j \cos(2\pi f_j n + \phi_j) \right) \times R_N (n - m - k \cdot l) + A^* \cos(f_0 n + \phi_0) \times R_N (n - m - k \cdot l)
\]

Here, the second item is impulse signal, the third item is cyclical sympathetic vibration signal, \( m \) is initial point of damages, \( l \) is interval of impulse signal. Now the frequency of fault signal is 50Hz, and the sampling frequency is 2500Hz, data is 500 in length. Figure 5 shows the results of decomposition and reconstruction of rolling bearing’s fault signal with wavelet lifting scheme.

![Figure 5](image.png)

**Figure 5.** The reconstruction of coefficients of rolling bearing’s fault signal with the lifting wavelet.

Apparently, there are pulse signals with same interval in the frequency band of detail signal \( d_1 \) (1250Hz—2500Hz), that is to say, there are about 10 pulses within 0.2s, so the frequency is about 50Hz. The initial point of fault signal \( m \) and the interval \( l \) can be gained.

3.3. Fault analysis and Simulation of rotor rubbing

Rubbing is an ordinary fault in engineering. The impulse vibrating signal by rubbing is usually short-lived, with high frequency signals increasing clearly, but peak value of rotating frequency and its low-level harmonics changes little. Based on the characteristics above, the unit rubbing vibrating signal is a response to impulse of rubbing between static and dynamic part, the rubbing signal is a typical impulse response.

When a rubbing fault occurs, there will be mutations in the wave. So considered the mutations and noise, the signal can be expressed as:

\[
x(n\Delta t) = \sum_{i=1}^{N} A_i \cos(\omega_i n\Delta t + \phi_i) + x_i(n\Delta t) + x_s(n\Delta t)
\]

Where, the first item is rotor’s frequency and its harmonics; the second is fault signal; the third is noise in system. If rotor’s frequency is 50Hz, fault signal’s frequency is 30Hz; sampling frequency is 3000Hz, the sum of sampling data is 600, the results of decomposition and reconstruction with wavelet lifting scheme are shown in figure 6.

![Figure 6](image.png)

From the figure, it is obvious that there are high frequency signals in detail signal \( d_1 \), which are corresponding to original signals. The information about time and intensity of rubbing fault has been indicated clearly in the detailed graph \( c_d1 \). So we can find the periodicity of mutations in the figure is 0.033s, frequency is 30.3Hz, which is close to the fault frequency. Of course, we should improve wavelet’s performance to meet the requirement of practical application in real application.
4. Conclusion
In this paper, the lifting scheme of wavelet and its multiphase expression are analyzed. Then it is applied in fault diagnosis of gear, rolling bearing and rotor rubbing. Simulations accomplished at the same time indicates that the wavelet lifting arithmetic is effective in early diagnosis of mechanical faults and has been proved to be a very powerful tool for real time fault diagnosis in mechanical equipments.

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