Chaotic resonance in Izhikevich neural network motifs under electromagnetic induction

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Abstract Chaotic resonance (CR) is the response of a nonlinear system to weak signals enhanced by internal or external chaotic activity (such as the signal derived from Lorenz system). The triple-neuron feed-forward loop (FFL) Izhikevich neural network motifs with eight types are constructed as the nonlinear systems in this paper, and the effects of EMI on CR phenomenon in FFL neuronal network motifs are studied. It is found that both the single Izhikevich neural model under electromagnetic induction (EMI) and its network motifs exhibit CR phenomenon depending on the chaotic current intensity. There exists an optimal chaotic current intensity ensuring the best detection of weak signal in single Izhikevich neuron or its network motifs via CR. The EMI can enhance the ability of neuron to detect weak signals. For T1-FFL and T2-FFL motifs, the adjustment of EMI parameters makes T2-FFL show a more obvious CR phenomenon than that for T1-FFL motifs, which is different from the impact of system parameters (e.g., the weak signal frequency, the coupling strength, and the time delay) on CR. Another interesting phenomenon is that the variation of CR with time delay exhibits quasi-periodic characteristics. Our results showed that CR effect is a robust phenomenon which is observed in both single Izhikevich neuron and network motifs, which might help one understand how to improve the ability of weak signal detection and propagation in neuronal system.

Keywords Izhikevich neuron model · Chaotic resonance · Electromagnetic induction · Fourier coefficient · Lorenz system · Network motifs

1 Introduction

Resonance is a term which is used with very high frequency in physics [1]. It refers to the situation that a system vibrates with a greater amplitude than other frequencies and wavelengths at a specific frequency and wavelength [2]. A small periodic vibration can produce a large vibration at this resonance frequency and resonance wavelength [3]. In general, a system (whether mechanical, thermal or electronic) has multiple resonance frequencies, and it is easy to vibrate at these frequencies but difficult to vibrate at other frequencies [4]. The concept of “stochastic resonance” (SR) was first proposed by Benzi et al. [5] and used to explain the Quaternary glacier problem. After that, it was widely used to describe the phenomenon that the presence of internal noise or external noise in nonlinear system can increase the response of the system output [6]. In the process of signal analysis, noise is often regarded as a nuisance,
because the presence of noise reduces the signal-to-noise ratio and affects the extraction of useful information [7, 8]. However, in some specific nonlinear systems, the presence of noise can enhance the detection ability of weak signals [9, 10].

It is well known that the occurrence of resonance must meet three conditions at the same time, namely the existence of nonlinear system, noise and weak signal [11]. It is found that the role of noise can be replaced by high-frequency (HF) signal with the deepening of research, so that in this case the resonance phenomenon of nonlinear system can also be observed [12, 13]. This resonance phenomenon is called vibrational resonance (VR), from which one can observe the system’s response to weak low-frequency (LF) signal becomes maximal by an appropriate choice of vibration amplitude for the HF signal [14, 15]. As far as we know, vibration resonance refers to the mechanism that one can make use of the optimal amplitude of HF driving to enhance the response of excitable system to LF sub-threshold signals; therefore the utilization of double-frequency signal to explore signal detection and transmission has been widely studied [16, 17]. This double-frequency signal exists in many different fields, including brain dynamics, telecommunications, neural networks and so on [18, 19].

The brain’s cognitive functions (such as working memory, selective attention and sensory coding) are realized through irregular neuronal discharges [20]. For neuroscience, exploring, revealing and understanding brain connections is one of the most important problems [21]. Different interconnected states are considered to be the basis of different cognitive functions of the human brain [22]. Meanwhile, some neurological diseases, such as Parkinson’s disease, have been proved to be related to brain connectivity disorders [23]. For nervous system or neural network, it contains a large number of neurons in different discharge states, and these neurons receive and produce different neural signals in the process of interaction with each other [24]. In this way, the signal transmission of the nervous system becomes complex and chaotic, resulting in some neurons in chaotic discharge state [25]. For nervous system, chaotic signal comes from two parts, one of which is the chaotic signal generated by external chaotic activities, and the other is the chaotic signal generated by internal chaotic activities [26]. Recent studies have shown that the response to weak signals can be amplified by chaotic activity rather than noise or HF driving, and this phenomenon is defined as chaotic resonance (CR) [27–30].

On the one hand, it was found that CR effect is a robust phenomenon, which can be observed in both single neuron and neural networks [29]. In 2019, Baysal et al. [30] studied the influence of external chaotic signals on weak signal detection performance of Hodgkin–Huxley (HH) neurons through numerical simulation. After that, the effects of autapse on CR phenomenon in single HH neuron and small-world (SW) neuronal networks were investigated in 2020 [31]. Tokuda et al. [32] investigated the role of CR in cerebellar learning, they pointed out that CR widens the range of noise intensity within which efficient learning can be realized, and they suggested that the spiking activity induced by chaos can be more economical than that induced by noise from an energetic viewpoint. CR was found by Nobukawa and Nishimura [33] to arise with appropriate coupling strength for weak signal and have the frequency response characteristic as with general resonance phenomena. Chew et al. [29] considered the resonant effects of chaotic fluctuations on a strongly damped particle in a bistable potential driven by weak sinusoidal perturbation. Ishimura et al. [34] studied CR in forced Chua’s oscillators. A large number of studies further show the universality and importance of CR.

On the other hand, as the basic structural unit of neural network, neuron performs its function in a network receiving contacts from about $\times 10^4$ presynaptic neurons [35]. Such a dense connectivity profile for a single neuron may lead to a huge and complex neuronal topology, which may be difficult to understand the potential mechanism of neural function and disease [36]. A large number of experimental data from neuroanatomical research have found that neural networks include some recurrent microcircuit topologies, known as network motifs, which are used as feature building modules of complex networks [37, 38]. Jiao et al. [39] pointed out that network motifs play an important role in network classification and network attribute analysis, and they proposed a method for analyzing the effective connectivity of functional magnetic resonance imaging data by using network motifs. Milo et al. [40] investigated motifs in networks from biochemistry, neurobiology, ecology...
and engineering. Alon et al. [41] reviewed network motifs and their functions, and focused on using experiments to study neural motifs. Therefore, it is generally believed that a clear explanation of the dynamic and functional characteristics of these network motifs can be used as the first step to understand large networks [42, 43].

Although there are many studies on CR in single neuron or SW neural network, the reports on CR in neural network motifs are still few [44, 45]. In particular, there is basically no research on CR of Izhikevich neural network motifs under the action of EMI [46]. Thus it is very important to investigate CR phenomenon of network motifs and the influence of EMI on CR, and it has guiding significance for the detection performance and transmission ability of neural signals in chaotic environment [47]. Therefore, some interesting questions now arise: Is there exists CR phenomenon in Izhikevich neuronal systems? How does chaotic signal affect the membrane potential of Izhikevich neuronal model under EMI? What are the effects of both chaotic signal and EMI on triple-neuron feed-forward loop (FFL) neural network motifs?

In this paper, to address above issues, the triple-neuron FFL Izhikevich neuronal network motifs are constructed, and the effects of EMI on CR in single Izhikevich neural model and Izhikevich network motifs with eight types are investigated, respectively. Firstly, the Izhikevich neural model in the presence of chaotic current is modeled by introducing the chaotic current into the Izhikevich neural model. Secondly, the EMI effect is taken into consideration according to Faraday’s law of electromagnetic induction and the Izhikevich neuron model under the effects of EMI is constructed. Thirdly, the triple-neuron network motifs are constructed by using a FFL network composed of three Izhikevich neurons. Finally, the CR in single Izhikevich neural model and the triple-neuron network motifs are discussed by calculating the response of the neural system to the chaotic signal (Fourier coefficient) during 500 periods, as well as the effects of EMI on CR phenomenon in FFL neuronal network motifs. The results show that CR can also be observed in the triple-neuron network motifs, and the EMI can enhance the ability of neuron to detect weak signals. Meanwhile, the variation of CR with time delay exhibits quasi-periodic characteristics. Look at the overall situation, CR has been observed before, but CR in triple-neuron network motifs has not been reported. More importantly, there is no literature to study the effect of EMI on CR of triple-neuron network motifs. Our results might provide some theoretical support for construction of large-scale neural networks and detection or transmission of neural information.

2 Model description

2.1 Izhikevich neural model in the presence of chaotic current

Izhikevich neuronal model is a neuronal discharge model proposed by Eugene M. Izhikevich [48–50] in 2004, which combines the advantages of Integrate-and-Fire (IF) neuronal model and HH neuronal model. It is not only close to the discharge characteristics of real biological neurons, but also convenient for large-scale simulation [51]. From the work of Kafraj et al. [52], we know that Izhikevich and Gerald M. Edelman had successfully simulated the thalamocortical system of mammals by using Izhikevich neural model. Considering that the firing neurons of this model have the characteristics of associative memory, and it is closer to real biological neurons, so it has become the focus of scientific research [53].

When considering the existence of chaotic signal and weak signal, Izhikevich neuronal model is a two-dimensional system of ordinary differential equations, which is governed by the following two equations [54]:

\[
\begin{align*}
\frac{dv}{dt} &= 0.04v^2 + 5v + 140 - u + I_{syn} + I_{ext}, \\
\frac{du}{dt} &= a(bv - u).
\end{align*}
\]

(1)

with the auxiliary after-spike resetting

\[
\text{if, } v \geq 30 \text{ mV then, } \begin{cases} v = c, \\ u = u + d. \end{cases}
\]

(2)

where \(v\) and \(u\) are dimensionless variables, and \(v\) represents the membrane potential of neuron, \(u\) denotes the membrane recovery variable. Once if the peak of membrane potential reaches its peak (+ 30 mV), the membrane potential and recovery variable are reset according to Eq. (2) [55].
Here, $a, b, c, d$ are four dimensionless parameters which are used to determine the neuron type [56]. On the basis of previous original literature, parameter $a$ is used to describe the time scale of recovery variable $u$, the parameter $b$ describes the sensitivity of recovery variable $u$ to subthreshold fluctuations of membrane potential $v$ [57].

Regular spiking (RS) neurons ($a = 0.02, b = 0.2, c = -65, d = 8$) are the most typical neurons in cortex, and they are used to model the excitatory neurons [58]. Fast spiking (FS) neurons ($a = 0.1, b = 0.2, c = -65, d = 2$) can fire periodic trains of action potentials with extremely high frequency practically without any adaptation (slowing down), and they are utilized to model inhibitory neurons [59].

$I_{\text{ext}}$ is the weak signal that applied to Izhikevich neuron, with the form of $I_{\text{ext}} = A \sin(\omega t)$ [60]. In which $A$ is the amplitude of weak signal and $\omega$ is frequency of weak signal [61]. Neurons interact with the external environment, so they may receive some nerve signals from the surrounding environment, and the total synaptic input received from the environmental neurons can be introduced into system by $I_{\text{syn}} = I_0$.

$I_{\text{chaos}}$ [62]. The total synaptic current $I_{\text{syn}}$ consists of two parts, one of which is a constant direct current (DC) stimulation signal, or called slow current; the other part is chaotic current stimulation signal, which can also be called fast current [63].

As can be seen from Fig. 1, the size of $I_0$ controls the frequency of neuronal spike discharge [64]. Therefore, in order to avoid the influence of constant DC signal on chaotic resonance, we define $I_0 = 0$ if there is no special description [27]. Chaotic information generated by environmental neurons is represented by chaotic current $I_{\text{chaos}}$, and its specific form is given by

$$I_{\text{chaos}} = \varepsilon x$$

where $\varepsilon$ represent chaotic current intensity, and $x$ describes the external chaotic signal derived from Lorenz system [65]. Lorenz system is the first dissipative system with chaotic motion found in numerical experiments, and it is given by following equations [66]:

$$\frac{dx}{dt} = \sigma(y - x),$$
$$\frac{dy}{dt} = \rho x - y - xz,$$
$$\frac{dz}{dt} = xy - \beta z.$$  \hspace{1cm} (4)

All chaotic signals discussed here are generated by above equations, and the chaotic system parameters are chosen as $\sigma = 10, \rho = 28, \beta = 8/3$.

In order to explore the effects of chaotic current intensity $\varepsilon$ and amplitude of weak signal on CR, the evolution of inter-spike interval (ISI) of the regular spiking neuron with respect to the chaotic current intensity and the amplitude of weak signal are plotted in Fig. 2. ISI denotes the inter-spike interval in membrane potential series, from which one can know the complexity of neuronal discharge.

It can be seen clearly from Fig. 2a that the Izhikevich neuron undergoes a variety of discharge

![Fig. 1](image_url) Evolution of membrane potential versus time of Izhikevich neuron to external current $I_0$: a, e regular spiking (RS) neuron and b, d fast spiking (FS) neuron
patterns as the chaotic current is injected into the system. Firstly, the Izhikevich neuron approximately presents a multi-cycle firing state when the chaotic current intensity is small (for $e < 1.0$), but it is still an irregular discharge state on the whole. The reason for the occurrence of multiple ISIs may come from the complexity of chaotic signal and nonlinear system, which leads to the irregular discharge of neuron. For the larger chaotic current intensity, the firing frequency of neuron increases, resulting in the gradual decrease of ISI. This is the same as common sense, that is, the stronger the external input current, the faster the neuronal discharge rhythm. Therefore, the role of chaotic current is dominant when the chaotic current intensity is large.

Figure 2b shows the bifurcation evolution of neuronal membrane potential when the amplitude of weak signal changes. It is revealed that the amplitude of weak signal is directly related to the degree of chaotic firing state. When the external weak signal amplitude is large (for $A > 2.0$), the external signal plays a dominant role in the neuronal system, and the neuronal discharge state is relatively regular. On the other hand, the firing state of neuron appears very disordered when the amplitude $A$ is small, which indicates that the chaotic signal is not suppressed. Therefore, the overall change of neuronal firing activity is that it changes from a chaotic firing state to a rhythmic firing.

2.2 Izhikevich neuron model under the effects of EMI

There are a large number of flowing charged ions inside and outside the cell membrane [61, 67]. Therefore, EMI effect can be introduced into Izhikevich neuronal model according to Faraday’s law of electromagnetic induction [68]. Thus, the dynamical equations for the improved Izhikevich neural model are described by as follows [69]:

$$\frac{dv}{dt} = 0.04v^2 + 5v + 140 - u - k_1(x + 3\beta \phi^2)v + I_{\text{syn}} + I_{\text{ext}},$$

$$\frac{du}{dt} = a(bv - u),$$

$$\frac{d\phi}{dt} = k_2v - k_3\phi. $$

in which the newly introduced variable $\phi$ describes the magnetic flux across membrane, and the term $\rho(\phi) = x + 3\beta \phi^2$ is memory conductance of a magnetic flux-controlled memristor [70]. $x$ and $\beta$ are fixed parameters. $k_1$ and $k_2$ are parameters that describe the interaction between membrane potential and magnetic flux [71]. The term $k_3\phi$ describes the leakage of magnet flux [72]. The total synaptic current $I_{\text{syn}}$ and external current $I_{\text{ext}}$ are the same as that in Eq. (1) [73].

2.3 Triple-neuron feed-forward loop (FFL) network motifs

The Izhikevich neural model discussed here is used to build the triple-neuron FFL neuronal network motifs, and the connection patterns is given in Fig. 3 [74]. Figure 3 shows us how to construct the triple-neuron FFL network motifs, it can be seen clearly that the interaction between the 3 neurons is that neuron 1 drives neuron 2, and then both jointly drive neuron 3 [75]. Different neurons are connected by chemical synapse coupling, and the specific expression of chemical synapse is given in Eq. (7).

The dynamics of the studied motifs is governed by following equations [76]:

Fig. 2 Bifurcation diagram associated with $a$ the chaotic current intensity and $b$ the amplitude of weak signal. ISI denotes the inter-spike interval in membrane potential series. The parameters are selected as $k_1 = 0.001, k_2 = 0.01, k_3 = 0.2, \omega = 0.05$ and $a A = 2.0, b e = 0.6$. 
where \( i = 1, 2, 3 \) index the neurons. \( v_i \) is the membrane potential of the neuron \( i \), \( u_i \) represents the membrane recovery of neuron \( i \), \( \phi_i \) denotes the magnetic flux across membrane of neuron \( i \) [77]. The weak signal is input from neuron 1 (which is considered as the input port of the network motifs), the chaotic current is applied to all three neurons, and the output signal is detected on neuron 3 (which is considered as the output port of the network motifs) [78].

\( I_i^{\text{syn}} \) denotes the total synaptic current and it is the linear sum of all incoming chemical synaptic current onto neuron \( i \) from neuron \( j \), which has the form [21]:

\[
I_i^{\text{syn}} = \sum_j I_{ij}^{\text{syn}}
\]  

(7)

and \( I_{ij}^{\text{syn}} \) is given as follows:

\[
I_{ij}^{\text{syn}}(t) = g_{ij} r_j [E_x - v_i(t)]
\]  

(8)

where \( g_{ij} \) describes the coupling strength of the synapse from neuron \( j \) to neuron \( i \) [79]. For simplicity, it is assumed that all connections have the same coupling strength [47]. \( E_x \) is defined as the reversal potential, which can be used to determine the type of synapse [80]. The synapse between neurons is expressed in two different forms, i.e., \( E_x = 0 \) mV for excitatory synapse and \( E_x = -80 \) mV for inhibitory synapse [81]. What needs to be explained here is how to define the type of synapse. As far as we know, excitatory or inhibitory synapses are determined by the types of pre-synapse neuron according to Ref. [74]. For excitatory neuron, its role is to transmit excitatory neurotransmitters to postsynaptic neurons and cause postsynaptic neurons to respond or discharge or rest duly [82]. On the other hand, for inhibitory neuron, its role is to transmit inhibitory neurotransmitters to postsynaptic neurons and cause postsynaptic neurons to respond accordingly [83].

In Eq. (8), \( r_j \) represents the synapse variable, which is the fraction of post-synaptically bound neurotransmitter obeying the first-order kinetics, and it has the form as follows [74]:

\[
\frac{dr_j}{dt} = \frac{1 - r_j}{1 + e^{-(v_j - \tau)}} - \frac{r_j}{10},
\]  

(9)

where \( \tau \) represents the length of time delay.

To quantitatively determine the correlation between the weak LF periodic signal and the output activity of Izhikevich neuron and its neural network motifs in the presence of chaotic signal, the Fourier coefficient \( Q \) during \( M = 500 \) periods of weak signal is calculated as follows [84, 85]:

\[
Q_{\sin} = \frac{\omega}{2\pi M} \int_{T_0}^{T_0 + \frac{2\pi M}{\omega}} 2 v(t) \sin(\omega t) dt,
\]

\[
Q_{\cos} = \frac{\omega}{2\pi M} \int_{T_0}^{T_0 + \frac{2\pi M}{\omega}} 2 v(t) \cos(\omega t) dt,
\]  

(10)

\[
Q = \sqrt{Q_{\sin}^2 + Q_{\cos}^2}.
\]

where \( \omega \) is the frequency of weak LF signal, \( T_0 = 10 \) is defined as the initial integration time [86]. The maximum of Fourier coefficient \( Q \) shows the best phase synchronization between weak input signal and output-neuron firing, which indicates the higher correlation between the weak periodic signal and the output activity of Izhikevich neuron system [87].

According to the existing research conclusions, the value of \( Q \) detected at the output neuron 3 is too small and has no research significance if input neuron 1 is an inhibitory neuron [74]. Therefore, only T1-FFL and T2-FFL motifs are discussed in detail in this paper.
3 Results and discussions

In order to fully understand whether there is CR phenomenon in Izhikevich neuronal model and explore the influence of EMI on CR in Izhikevich neural network motifs, the Euler algorithm is used to calculate the Fourier coefficient $Q$, and the time step is chosen as 0.01 [20]. Unless otherwise specified, other parameters are set as follows, $\alpha = 0.4$, $\beta = 0.02$, $k_1 = 0.01$, $k_2 = 0.01$, $k_3 = 0.2$, $\tau = 10$, $A = 0.2$, $\omega = 0.05$ [88] Table 1.

By calculating the Fourier coefficient $Q$, the influence of external chaotic activity obtained by Lorentz system on the weak signal detection performance of single Izhikevich neuron is systematically analyzed [89]. Since the improved model discussed here is closer to the real situation, there are two cases worth studying: one is to consider the role of EMI, and the other is without considering the effects of EMI [90].

In order to ensure that the signal input to system meets the research requirements, the parameters of Lorentz system are adjusted to make it in a chaotic oscillation state. In this way, the single Izhikevich neuron model and its neural network motifs are exposed to chaotic signal and weak periodic signal. In order to clearly know the demonstration form of chaotic signal with time, the time series of chaotic signal generated by Lorentz system in and the phase diagram of chaotic Lorenz system are plotted in Fig. 4.

As shown in Fig. 4, the time series of chaotic signal $x$ derived from Lorenz system and $x - z$ phase plane diagram of chaotic Lorenz system are presented, it is clearly that the signal $x$ generated by Lorentz system is indeed in a chaotic oscillation state. Chaotic signals injected into the system can come from different chaotic systems. There is no special reason why choose Lorentz system to generate chaotic signals in this paper, and it is also feasible to choose other forms of chaotic signals.

3.1 Chaotic resonance in single Izhikevich neuron

As it is shown in Fig. 5, the response of single Izhikevich neuron to weak signal, which is measured by Fourier coefficient $Q$, is plotted as a function of $\varepsilon$ for different frequencies of weak signal. In order to compare the influence of EMI on Fourier coefficient $Q$, Fig. 5a shows the results considering EMI, while Fig. 5b shows the results without considering EMI. As can be seen from Fig. 5, a maximum similar to resonance peak appears when $Q$ changes with the chaotic current intensity $\varepsilon$, which indicates the generation of CR. For both cases, there is an optimal chaotic current intensity to maximize the $Q$ value whether the EMI is considered or not, it means the best detection of weak signal.

Further analysis shows that the $Q$ value is larger in the case of EMI is considered than that in the case of EMI is not considered if the same system parameters are selected. The $Q$ curves exhibit a more sensitive dependence on weak signal frequency $\omega$ when EMI is taken into account, as shown in Fig. 5a. Moreover, the CR phenomenon tends to disappear when the frequency of weak signal is large, and the maximum of $Q$ can hardly be distinguished, which may indicate a phase transition in the neural system. Meanwhile, the

| Table 1 | Eight possible and frequently discussed FFL motifs types |
|---------|--------------------------------------------------------|
| Type of motifs | Neuron 1 | Neuron 2 | Neuron 3 |
| T1-FFL | E | E | E |
| T2-FFL | E | I | E |
| T3-FFL | E | E | I |
| T4-FFL | E | I | I |
| T5-FFL | I | E | E |
| T6-FFL | I | I | E |
| T7-FFL | I | E | I |
| T8-FFL | I | I | I |
larger the value of $\omega$, the larger the $\varepsilon$ value corresponding to the maximum value of $Q$. The most intuitive phenomenon one can see is that the position of the peak gradually moves to right. However, the difference is that the change of $Q$ curve with $\varepsilon$ always presents a more perfect bell-shaped-like relationship if EMI is not considered.

In order to fully understand how weak signal detection performance of Izhikevich neuron is modulated by the chaotic current intensity $\varepsilon$ and frequency of weak signal $\omega$, the dependence of $Q$ on $\varepsilon$ and $\omega$ is calculated in a relatively wide range of the chaotic current intensity $\varepsilon \in [0–3.5]$ and the frequency of weak signal $\omega \in [0.001–1$ ms$^{-1}]$. It is seen that Fig. 6a and b shows similar change rules on the whole, i.e., a red area in which the weak signal can be best detected is observed. However, they also show differences in details. The range of maximum $Q$ value in Fig. 6a is wider than that in Fig. 6b, and the maximum value of $Q$ is larger. A deeper analysis of the red region shows that the introduction of EMI makes Izhikevich neuron still have a certain response to weak signals when $\varepsilon$ is small. These phenomena are completely consistent with the analysis results in Fig. 5, which shows that EMI has a certain impact on the detection ability of weak signal in Izhikevich neuron.

In order to provide a clearer perspective and explain the obtained results in more detail, the membrane potential of Izhikevich neuron and weak signal are presented as a function of time for three different chaotic current intensities. Among them, three chaotic current intensities of different sizes are defined as small value, intermediate value and large value, and their corresponding values are chosen as 0.1, 0.6 and 1.0, respectively. For comparison, both FS and RS neuron cases are given in Figs. 7 and 8.

As illustrated in Figs. 7 and 8, the Izhikevich neuron is mostly in its quiescent state for small values of $\varepsilon$ ($\varepsilon = 0.1$), and even one firing peak cannot be seen.
As we all know, neurons mainly rely on discharge spikes to transmit signals, so the Izhikevich neuron cannot transmit any weak signals effectively for the given conditions here, that is, the input weak signal cannot be detected at the output either. For the medium chaotic current intensity ($\varepsilon = 0.6$), one can see that the membrane potential at the output is strongly correlated with the input weak signal, and neuron fires a spike in each positive half cycle of weak signal resulting in a high encoding performance for the neuron. In this case, the weak signal received by neurons can be well detected. Further increasing the chaotic current intensity to $\varepsilon = 1.0$, the time of neuron firing spike is no longer strongly correlated with the positive half cycle of weak signal. In this situation, the neuronal discharge activity does not match the period of weak signal, thus the disordered firing pattern is observed. Particularly, neuronal discharges can also be observed in the negative half cycle of weak signal, which indicates the lack of ability to detect weak signal. On the other hand, above results also show that large chaotic current intensity makes Izhikevich neuron in spiking state in the absence of weak signal. Another important conclusion is that FS neurons have faster firing rate than RS neurons. Therefore, FS neurons can fire more spikes under the optimal chaotic current intensity.
indicating that they can carry and transmit more neural information.

3.2 Chaotic resonance in Izhikevich neural network motifs

In early research, most of the researchers and scholars focuses on neuronal model and its numerical analysis. In the later research, complex networks in biological systems also aroused great interest of researchers. As far as the current situation is concerned, the complex networks studied mainly include feed-forward networks, small-world networks, scale-free networks and so on.

An unavoidable problem is that more and more studies show that some significant recurring nontrivial patterns of interconnections, termed as “network motifs”, are the basic units of various complex neural networks. Since network motifs are considered to be basic building blocks of various neuronal networks, the function, neural information transmission or coding of network motifs have aroused widespread interest. In the following section, we will focus on the CR of Izhikevich neural network motifs under the effect of EMI.

As seen in Fig. 9, the response of Izhikevich neural network motifs to weak signal is plotted as a function of chaotic current intensity \( \varepsilon \) for different weak signal frequency \( \omega \) under the effects of EMI. Considering the fact that meaningful results can be seen only when the input is excitatory neuron, the focus here is on T1-FFL and T2-FFL motif. It is obvious that the response curves of the two FFL network motifs to weak signals exhibit bell-shaped dependence on the chaotic current intensity, and the shape of this bell is more perfect, which indicates that the FFL coupled by three neurons plays a positive role in inducing CR.

Obtained results provide a profound enlightenment, that is, the triple-neuron FFL neuronal network motifs improve the signal detection and transmission ability in nervous system. So here comes the question: what about the information coding ability of the feedforward network composed of FFL? This question may be our further work to consider in the future, and the conclusion of this paper may provide inspiration for the answer to this question.

Numerical results in Fig. 9 reveal a fact that there is an optimal chaotic current intensity (equaling approximately \( \varepsilon = 0.6 \) for T1-FFL motif and \( \varepsilon = 0.4 \) for T2-FFL motif) which can maximize \( Q \) if the frequency of weak signal is fixed. On the other hand, there is also an optimal weak signal frequency (equaling approximately \( \omega = 0.05 \)) to maximize \( Q \). Comparing these two types of FFL network motifs, it can be found that if the three neurons constituting the network motifs are all excitatory neurons (i.e. T1-FFL motif), the signal detection ability is better. The \( Q \) values of T2-FFL motif containing inhibitory neurons are smaller than T1-FFL network motif, indicating that inhibitory neurons inhibit the induction of CR.

To fully understand the impact of weak signal frequency and chaotic current intensity for FFL network motifs, the contour plot of the Fourier coefficient \( Q \) in parametric space (\( \varepsilon-\omega \)) for Izhikevich neural network motifs under the effects of EMI are presented in Fig. 10. The conclusion in Fig. 10 verifies that the detection performance of T1-FFL motif for weak signals is stronger than T2-FFL motif, and the range of parameters that maximize \( Q \) can also be found out intuitively.

For T1-FFL motif, the dependence of \( Q \) on chaotic current intensity and weak signal frequency exhibits

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Fig. 9 Response of Izhikevich neural network motifs to weak signal (\( Q \)) are plotted as a function of chaotic current intensity \( \varepsilon \) for different weak signal frequency \( \omega \) under the effects of EMI at \( A = 2.0, \tau = 10, g = 0.9, k_1 = 0.01, k_2 = 0.02, k_3 = 0.2 \). a T1-FFL; b T2-FFL.

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an obvious maximum, and a smaller extreme value (yellow area) appears near the maximum, which is defined as sub-harmonic resonance. For T2-FFL motif, most of the parameter areas are not able to guarantee the effective detection and transmission of weak signals. It is unearthed that the connection between inhibitory neurons and inhibitory synapses play an important role in the neural signals transmission. In addition, Fig. 10b also reveals an interesting phenomenon that no sub-harmonic resonance is observed in the contour plot of the Fourier coefficient \( Q \) in parametric space \((\varepsilon, \omega)\) for T2-FFL motifs.

To further investigate the effects of weak signal frequency and coupling strength on the detect performance of Izhikevich neural network motifs, the response of Izhikevich neural network motifs to weak signal is plotted as a function of weak signal frequencies \( \omega \) for different coupling strength \( g \) under the effects of EMI, as presented in Fig. 11. Obviously, the curve of \( Q \) with respect to \( \omega \) exhibits two maxima, of which the maximum on the left has the best signal detection ability, and the maximum on the right is called sub-harmonic resonance. But on the whole, there are two maxima in the \( Q \) curve, which can also be called chaotic double resonance. According to the definition of vibrational bi-resonance, this phenomenon can also be called chaotic bi-resonance. Another meaningful phenomenon is that the larger the coupling strength, the larger the corresponding \( Q \) value. This is consistent with the response of neural system to external stimulus. The larger the coupling strength represents the larger the signal input, and the corresponding response is also larger.

The phenomena of resonance, such as stochastic resonance, vibrational resonance and chaotic resonance, in dynamical systems with time delay feedback have been widely studied in various fields. Especially in neuronal system, it was believed that time delay can induce multi-resonance and enhance neural synchronization. It was recently reported that different information transmission delays could induce stochastic resonance appear intermittently.

Generally speaking, due to the transmission of matter, energy and information, the time delay arises...
in dynamical system. In view of this, then the effect of time delay in synaptic variables on response of the system to weak signals is discussed. In order to investigate the effect of time delay feedback on dynamics of the system, the response of Izhikevich neural network motifs to weak signal are plotted as a function of time delay $\tau$ for different coupling strength $g$ under the effects of EMI, as shown in Fig. 12.

It can be observed from Fig. 12 that there exist some bright strip-like regions of high values of $Q$, where CR can be realized. Where resonance occurs, neurons fire and information can be transmitted. From an overall perspective, one can confirm that a periodic resonance behavior with the period approximately equal to 28 ms for the system is found, which can be defined as quasi-periodic chaotic resonance (QPCR). Indeed, with the increasing of information transmission time delay, T1-FFL motif exhibits intermittent appearance of CR.

The response of T2-FFL motifs to weak signal is generally the same with T1-FFL motifs, but it also shows different aspects. On the whole, the contour plot of the Fourier coefficient $Q$ versus parameters ($g$, $t$) still shows the characteristics of QPCR, along with roughly the same period. For small coupling strength, the response of both T1-FFL and T2-FFL motifs to weak signal is low regardless of the delay time. However, when the coupling strength $g$ is relatively large (for $g > 1.5$), the $Q$ value of some parameter regions of T2-FFL is larger than T1-TTL, as shown in the red region in Fig. 12b. The above conclusions verify again that time delay can not only promote and improve the synchronization of neurons, but also induce multiple resonance leading to many interesting phenomena. Further, these analyses tell us that the time delay mainly originate from the limited speed of action potential propagation on neuronal axons and the time delay of dendritic and synaptic processes are inevitable in intermediate neuronal communication and inherent in the nervous system, so it is worth discussing.

The coupling strength determines the strength of the connection between neurons, so it is also of practical significance to investigate the influence of coupling strength on system response. As depicted in Fig. 13, the dependence of Fourier coefficient $Q$ on chaotic current intensity $\varepsilon$ is presented. With the increase of chaotic current intensity $\varepsilon$, the $Q$-$\varepsilon$ curve shows a perfect bell-shape relationship, which indicates the occurrence of CR. The maximum in $Q$ curve represents the best chaotic current intensity that optimize the weak signal detection performance of neural network motifs. Moreover, it can be seen that the larger the coupling strength $g$, the higher the peak height of the $Q$ curve. However, large coupling strength may lead to the too fast discharge of neurons that make up the neural network motifs; hence too large coupling strength is not discussed here.

On the other hand, the $Q$ value of T2-FFL motif is much smaller than that of T1-FFL motif, indicating that the addition of inhibitory neuron to neural network motif has a great impact on signal detection performance of the neural network motifs. When the signal is transmitted directly from neuron 1 to neuron 3, the information transmission efficiency will not decrease. However, if weak signals are transmitted from neuron 1 to neuron 2 and then to neuron 3, the propagation efficiency decreases because neuron 2 is an inhibitory neuron. Above results can provide some theoretical support for the construction of neural network, so that one can clearly know how to build neural network better.

![Fig. 12](image-url) Contour plot of the Fourier coefficient $Q$ in parametric space ($g$, $\tau$) for Izhikevich neural network motifs under the effects of EMI at $A = 2.0$, $\omega = 0.2$, $\varepsilon = 0.2$, $g = 0.5$, $k_1 = 0.01$, $k_2 = 0.02$, $k_3 = 0.2$. a T1-FFL; b T2-FFL.
3.3 Effects of electromagnetic induction on chaotic resonance in Izhikevich neural network motifs

As a new physical quantity introduced into neural model, EMI has attracted the attention of a large number of researchers. Considering the widespread existence of EMI, it is of practical significance to discuss the influence of EMI on CR of neural network motifs. Results shown in Fig. 14 clarify the findings that the $Q$ curves of T1-FFL and T2-FFL neural network motifs exhibit resonance-like behaviors dependence on $k_1$, and they have the same change trend with the increasing of $k_1$, that is, a smaller maximum appears first, and then a larger maximum appears. The first maximum of $Q$ for T1-FFL motif occurs when $k_1 = 0.06$ ($k_1 = 0.10$ for T2-FFL motif), while the second maximum occurs when $k_1 = 0.24$ ($k_1 = 0.26$ for T2-FFL motif).

For T1-FFL motif, the $Q$ value drops to a lower level when the value of $k_1$ is larger than 0.54, which means that too large $k_1$ value is not conducive to the detection of weak signals. However, for T2-FFL motif, the $Q$ value quickly decreases to a lower level when the value of $k_1$ is larger than 0.31, and continues to decrease approximately equal to 0 with the increase of $k_1$. If the value of $Q$ is equal to 0, it means that the detection performance of neural network motifs for weak signals is significantly diminished, or even tends to disappear. Above results show that there is an optimal EMI, which can make the signal detection ability of neural network motifs best. Furthermore, too strong EMI may destroy the information coding function of neural network motifs, resulting in the lack of signal detection ability.

The term $k_2\nu$ in Eq. (5) represents the influence of membrane on magnetic flow, and $k_3\phi$ describes the leakage of magnet flux. Then the response of Izhikevich neural network motifs to weak signal are plotted as a function of $k_2$ and $k_3$ under the effects of EMI, as displayed in Figs. 15–16. It is clearly seen that the weak signal detection performance of both T1-FFL and T2-FFL neural network motifs display a resonance-like dependence...
For the two different FFL neural network motifs discussed here, the $Q$ value of T2-FFL motif is larger than that of T1-FFL motif under the same parameter conditions. Thus, one can conclude that changing the parameters of EMI or external current input (including chaotic current and weak signal current) has different effects on detection performance of the two FFL neural network motifs. Through the comparison between Sects. 3.2 and 3.3, it can be found that the detection performance of T1-FFL motif for weak signal is better than T2-FFL motif when the parameters of external current are changed. However, the detection performance of T2-FFL motif for weak signal is in turn stronger than T1-FFL motif when the parameters of EMI are changed.

As can be seen from Fig. 15, the $Q$ curve shows a resonance-like dependence on $k_2$, with the maximum value of $Q$ for T1-FFL motif appears at $k_2 = 0.04$ and the maximum value of $Q$ for T2-FFL motif appears at $k_2 = 0.07$. However, it is coincidentally found that the $k_3$ value which maximize $Q$ are obtained at $k_3 = 0.03$ for both T1-FFL and T2-FFL neural network motifs, as shown in Fig. 16.

In order to fully understand the influence of EMI on CR, the contour plot of $Q$ in $k_1$-$k_2$ parameters plane under the effects of EMI for both T1-FFL and T2-FFL neural network motifs are depicted in Fig. 17. Since the $k_2$ value corresponding to the maximum value of $Q$ is small, and large $k_2$ value has an adverse impact on signal detection performance of neural network motifs. Therefore, only the influence of small $k_1$ and $k_2$ values ($k_1$ and $k_2 \in [0,0.1]$) on $Q$ is considered. It is found that a series of striped ribbons are observed, and the $Q$ value of T2-FFL motif is obviously larger than that of T1-FFL motif.

4 Conclusions

An improved Izhikevich neuronal model in the presence of EMI is proposed, and the triple-neuron FFL neural network motifs are constructed in this paper. The CR phenomenon of Izhikevich neural model under the effects of EMI is discussed by calculating the response amplitude of chaotic signal to weak signal input the system, and the detection performance of FFL neural network motifs composed
of excitatory and inhibitory Izhikevich neurons for weak signals is studied. Meanwhile, how EMI affects the CR of single Izhikevich neuron and its neural network motifs is also explored in detail.

Obtain results show that there exists an optimal chaotic current intensity, which can make the single Izhikevich neuron have the best detection performance for weak signals. Further results uncover the fact that the introduction of EMI enhances the ability of single Izhikevich neuron to detect weak signals. It can be seen from the time series of membrane potential of RS and FS neurons that the most coherent firings with the weak signal occur at the intermediate level of chaotic current intensity, this is consistent with the phenomenon that the Fourier coefficient exhibits bell-shaped dependence on the chaotic current intensity. When the CR for T1-FFL and T2-FFL neural network motifs are checked under the effects of EMI, it is found that the detection performance of T2-FFL motifs to weak signals is worse than T1-FFL motifs due to the existence of inhibitory neurons. Furthermore, the effect of time delay in synaptic variables on response of the neural network motifs to weak signals is discussed, and the quasi-periodic CR is observed with the increasing of information transmission time delay. In addition, the influence of EMI on CR has also been studied in detail, and one can conclude that the introduction of EMI enhances the weak signal detection performance of T2-FFL neural network motifs regardless of the presence of inhibitory neurons. The triple-neuron network motifs is an extension of the coupling of two neurons, and previous studies have shown that, a novel electrical model of the coupling medium constructed by the coupling of two neurons examined effects of the model parameters on the synchronization of those neurons [91]. What attracts us is that, a neuronal coupling medium model that accords with the neuron’s intrinsic electrical properties was obtained by using the knowledge of electrical circuits and biological-experimental evidence. Take a deeper look, a synchronization scheme of coupling ring communication topology under external electrical stimulation and subject to directional coupling medium and disturbance is proposed for the first time by using the ring network constructed by four FHN neurons [92]. This also brings inspiration to our work, that is, the number of neurons in the neural network motifs can be further expanded to 4 or even more, and the coupling between neurons also has a great impact on the dynamic properties of the network motifs. These works will also be a direction of our later research.

Results in this paper have revealed that CR effect is a robust phenomenon which can be observed both in single Izhikevich neuron and network motifs, and these conclusions can help people understand how to improve the ability of weak signal detection and propagation in chaotic neuronal system.

Acknowledgements This project is supported by National Natural Science Foundation of China under Grants Nos 12175080 and 11775091.

Data availability All data generated or analyzed during this study are included in this published article.

Declarations

Conflict of interest The authors declare that they have no potential conflict of interest.

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