Cosmological Evolution of Quintessence and Phantom with a New Type of Interaction in Dark Sector

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ABSTRACT

In the present work, motivated by the work of Cai and Su [Phys. Rev. D 81, 103514 (2010)], we propose a new type of interaction in dark sector, which can change its sign when our universe changes from deceleration to acceleration. We consider the cosmological evolution of quintessence and phantom with this type of interaction, and find that there are some scaling attractors which can help to alleviate the cosmological coincidence problem. Our results also show that this new type of interaction can bring new features to cosmology.

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I. INTRODUCTION

In dark energy cosmology (see e.g. [1] for reviews), the cosmological coincidence problem is one of the well-known conundrums, which asks: why are we living in an epoch in which the densities of dark energy and matter are comparable? To alleviate this coincidence problem, it is natural to consider the possible interaction between dark energy and dark matter in the literature (see e.g. [2,11,17]). In fact, since the nature of both dark energy and dark matter is still unknown, there is no physical argument to exclude the possible interaction between them. On the contrary, some observational evidences of the interaction in dark sector have been found recently. For instance, Bertolami et al. [12] showed that the Abell Cluster A586 exhibits evidence of the interaction between dark energy and dark matter, and they argued that this interaction might imply a violation of the equivalence principle. On the other hand, Abdalla et al. [13] found the signature of interaction between dark energy and dark matter by using optical, X-ray and weak lensing data from the relaxed galaxy clusters. So, it is reasonable to consider the interaction between dark energy and dark matter in cosmology.

In the literature, it is usual to assume that dark energy and dark matter interact through a coupling term $Q$, according to

$$\dot{\rho}_m + 3H \rho_m = Q,$$

$$\dot{\rho}_{de} + 3H (\rho_{de} + p_{de}) = -Q,$$  

where $\rho_m$ and $\rho_{de}$ are the densities of dark matter and dark energy (we assume that the baryon component can be ignored); $p_{de}$ is the pressure of dark energy; $H \equiv \dot{a}/a$ is the Hubble parameter; $a$ is the scale factor; a dot denotes the derivative with respect to cosmic time $t$. Notice that Eqs. (1) and (2) preserve the total energy conservation equation $\dot{\rho}_{tot} + 3H (\rho_{tot} + p_{tot}) = 0$, where $\rho_{tot} = \rho_m + \rho_{de}$. Since there is no natural guidance from fundamental physics on the interaction $Q$, one can only discuss it to a phenomenological level. The familiar interactions extensively considered in the literature (see e.g. [2,11]) include $Q = 3\alpha H \rho_m$, $Q = 3\beta H \rho_{tot}$, and $Q = 3\eta H \rho_{de}$.

Recently, Cai and Su [14] investigated the interaction in a way independent of specific interaction forms by using the latest observational data. They divided the whole range of redshift $z$ into a few bins and set the interaction term $\delta (z) = Q/(3H)$ to be a constant in each redshift bin. From the latest observational data, they found that $\delta (z)$ is likely to cross the non-interacting line ($\delta = 0$), namely, the sign of interaction $Q$ changed in the approximate redshift range of $0.45 \lesssim z \lesssim 0.9$. In fact, this result raises a remarkable problem. Indeed, most interactions extensively considered in the literature, such as $Q = 3\alpha H \rho_m$, $Q = 3\beta H \rho_{tot}$ and $Q = 3\eta H \rho_{de}$, are always positive or negative and hence cannot give the possibility to change their signs. As noted by the authors of [14], some new interaction forms should be proposed to address this problem.

In the present work, we are interested to propose such a type of interaction and consider its implications to cosmology. The authors of [14] found that the sign of interaction $Q$ changed in the approximate redshift range of $0.45 \lesssim z \lesssim 0.9$. We note that this redshift range is coincident with the one of our universe changing from deceleration to acceleration [4]. So, a simple idea naturally comes to our mind. If the interaction $Q$ is proportional to the deceleration parameter

$$q \equiv -\frac{\ddot{a}}{aH^2} = 1 - \frac{\dot{H}}{H^2} \equiv s - 1,$$  

the sign of $Q$ can change when our universe changes from deceleration ($q > 0$) to acceleration ($q < 0$). Noting that the deceleration parameter $q$ is dimensionless, from Eqs. (1) and (2), $Q \propto q\dot{\rho}$ and $Q \propto qH\rho$ are both viable from the dimensional point of view. To be general, we consider the linear combination of these two, namely

$$Q = q(\alpha \dot{\rho} + 3\beta H \rho),$$  

where $\alpha$ and $\beta$ are both dimensionless constants. It is not surprising to find $\dot{\rho}$ in the interaction $Q$. We note that in the literature (see e.g. [10]) the derivatives of energy density have already been allowed to appear in the general forms of $Q$. However, the key point of our new interaction is the deceleration parameter $q$ in $Q$, which makes our proposal different from the previous works. This new feature gives the
possibility that interaction $Q$ can change its sign, and hence brings some interesting results to cosmology. In this work, we would like to consider three interactions of this type, namely

$$Q = q(\alpha \dot{\rho}_m + 3\beta H \rho_m),$$  
(5)

$$Q = q(\alpha \dot{\rho}_{\text{tot}} + 3\beta H \rho_{\text{tot}}),$$  
(6)

$$Q = q(\alpha \dot{\rho}_{\text{de}} + 3\beta H \rho_{\text{de}}).$$  
(7)

In the present work, we consider the cosmological evolution of quintessence and phantom with the above type of interaction. In Sec. II we present the dynamical system of interacting quintessence and phantom. In Secs. III—V we discuss the cases with $Q$ given in Eqs. (5)—(7), respectively. In Sec. VI a brief conclusion is drawn.

II. DYNAMICAL SYSTEM OF INTERACTING QUINTESSENCE AND PHANTOM

In this work, we consider a flat Friedmann-Robertson-Walker (FRW) universe. The Friedmann and Raychaudhuri equations are given by

$$H^2 = \frac{\kappa^2}{3} \rho_{\text{tot}} = \frac{\kappa^2}{3}(\rho_{\text{de}} + \rho_m),$$  
(8)

$$\dot{H} = -\frac{\kappa^2}{2}(\rho_{\text{tot}} + p_{\text{tot}}) = -\frac{\kappa^2}{2}(\rho_m + \rho_{\text{de}} + p_{\text{de}}),$$  
(9)

where $\kappa^2 \equiv 8\pi G$. The role of dark energy is played by quintessence or phantom, namely

$$\rho_{\text{de}} = \rho_\phi = \frac{1}{2} \epsilon \dot{\phi}^2 + V(\phi),$$  
(10)

$$p_{\text{de}} = p_\phi = \frac{1}{2} \epsilon \dot{\phi}^2 - V(\phi),$$  
(11)

in which $\epsilon = +1$ (quintessence) or $\epsilon = -1$ (phantom); $V(\phi)$ is the potential. In this work, we consider the exponential potential

$$V(\phi) = V_0 e^{-\lambda \kappa \phi},$$  
(12)

where $\lambda$ is a dimensionless constant. Without loss of generality, we choose $\lambda$ to be positive, since we can make it positive through field redefinition $\phi \rightarrow -\phi$ if $\lambda$ is negative.

We consider the cosmological evolution of interacting quintessence (phantom) by using the method of dynamical system [15]. Following [2–4, 6], we introduce the following dimensionless variables

$$x \equiv \frac{\kappa \dot{\phi}}{\sqrt{6H}}, \quad y \equiv \frac{\kappa \sqrt{V}}{\sqrt{3H}}, \quad z \equiv \frac{\kappa \sqrt{\rho_m}}{\sqrt{3H}}.$$  
(13)

With the help of Eqs. (8)—(11), the evolution equations (1) and (2) can then be rewritten as a dynamical system, namely

$$x' = (s - 3)x + \frac{1}{\epsilon} \left( \sqrt{\frac{3}{2}} \lambda y^2 - Q_1 \right),$$  
(14)

$$y' = sy - \sqrt{\frac{3}{2}} \lambda xy,$$  
(15)

$$z' = \left( s - \frac{3}{2} \right) z + Q_2,$$  
(16)

where

$$Q_1 \equiv \frac{\kappa Q}{\sqrt{6H^2}}, \quad Q_2 \equiv \frac{z Q}{2H \rho_m},$$  
(17)
a prime denotes derivative with respect to the so-called e-folding time $N \equiv \ln a$, and

$$s \equiv -\frac{\dot{H}}{H^2} = 3ex^2 + \frac{3}{2}z^2.$$  \hspace{1cm} (18)

The Friedmann constraint equation (8) becomes

$$ex^2 + y^2 + z^2 = 1.$$  \hspace{1cm} (19)

The fractional energy densities of dark energy and dark matter are given by

$$\Omega_{de} = \Omega_\phi \equiv \frac{\kappa^2 \rho_\phi}{3H^2} = ex^2 + y^2,$$

$$\Omega_m \equiv \frac{\kappa^2 \rho_m}{3H^2} = z^2.$$  \hspace{1cm} (20)

Once the interaction $Q$ is given, we can obtain the critical points $(\bar{x}, \bar{y}, \bar{z})$ of the autonomous system Eqs. (14)—(16) by imposing the conditions $\bar{x}' = \bar{y}' = \bar{z}' = 0$. Of course, they are subject to the Friedmann constraint Eq. (19), namely,

$$\bar{x}^2 + \bar{y}^2 + \bar{z}^2 = 1.$$ Note that these critical points must satisfy the conditions $\bar{y} \geq 0$ and $\bar{z} \geq 0$ by definition (13), and the requirement of $\bar{x}$, $\bar{y}$, $\bar{z}$ all being real. Then, we can discuss the existence and stability of these critical points. An attractor is one of the stable critical points of the autonomous system.

To study the stability of the critical points of Eqs. (14)—(16), we substitute the linear perturbations $x \to \bar{x} + \delta x$, $y \to \bar{y} + \delta y$ and $z \to \bar{z} + \delta z$ about the critical point $(\bar{x}, \bar{y}, \bar{z})$ into Eqs. (14)—(16) and linearize them. Because of the Friedmann constraint Eq. (19), there are only two independent evolution equations, namely

$$\delta x' = (\bar{s} - 3)\delta x + \bar{x}\delta s + \frac{1}{\epsilon} \left( \sqrt{6}\lambda \bar{y}\delta y - \delta Q_1 \right),$$  \hspace{1cm} (21)

$$\delta y' = \bar{y}\delta s + \bar{s}\delta y - \sqrt{\frac{3}{2}} \lambda (\bar{x}\delta y + \bar{y}\delta x),$$  \hspace{1cm} (22)

where

$$\bar{s} = \frac{3}{2} (ex^2 - \bar{y}^2 + 1), \quad \delta s = 3 (\epsilon \bar{x}\delta x - \bar{y}\delta y),$$  \hspace{1cm} (23)

and $\delta Q_1$ is the linear perturbation coming from $Q_1$. The two eigenvalues of the coefficient matrix of the above equations determine the stability of the critical point.

In the following sections, we will study the dynamics of quintessence (phantom) with the interaction $Q$ given in Eqs. (5)—(7), respectively.

**III. THE CASE OF $Q = q(\alpha \dot{\rho}_m + 3\beta H \rho_m)$**

Firstly, we consider the case of $Q = q(\alpha \dot{\rho}_m + 3\beta H \rho_m)$ given in Eq. (5). Substituting it into Eq. (1), one can find that

$$\dot{\rho}_m = \frac{\beta q - 1}{1 - \alpha q} \cdot 3H \rho_m.$$  \hspace{1cm} (24)

Then, substituting into Eq. (5), we can finally obtain

$$Q = \frac{\beta - \alpha}{1 - \alpha q} \cdot 3qH \rho_m.$$  \hspace{1cm} (25)

At first glance, this interaction form is very similar to the familiar $Q = 3\eta H \rho_m$ in which $\eta$ is a constant. However, the deceleration parameter $q$ in Eq. (25) makes difference. Note that $q = -1 - \dot{H}/H^2$ is a variable function of time, which changes its sign when the universe changes from deceleration to acceleration.
Critical Point \((\bar{\varphi}, \bar{\gamma}, \bar{z})\)

| Label | Critical Point \((\bar{\varphi}, \bar{\gamma}, \bar{z})\) |
|-------|--------------------------------------------------|
| M.1p  | \(+1/\sqrt{\varphi}, 0, 0\)                      |
| M.1m  | \(-1/\sqrt{\varphi}, 0, 0\)                      |
| M.2p  | \(\sqrt{(r_1 - r_2 - 1)/\varphi}, \sqrt{2 - r_1 + r_2}\) |
| M.2m  | \(-\sqrt{(r_1 - r_2 - 1)/\varphi}, \sqrt{2 - r_1 + r_2}\) |
| M.3p  | \(+\sqrt{(r_1 + r_2 - 1)/\varphi}, \sqrt{2 - r_1 - r_2}\) |
| M.3m  | \(-\sqrt{(r_1 + r_2 - 1)/\varphi}, \sqrt{2 - r_1 - r_2}\) |
| M.4   | \(\lambda/(\sqrt{\varphi}), \sqrt{1 - \lambda^2/(6\varphi)}, 0\) |
| M.5   | \(\sqrt{3/2}(r_1 - r_2)/\lambda, \sqrt{1 - r_1 + r_2 + 3\epsilon(r_1 - r_2)^2/(2\lambda^2)}, \sqrt{r_1 - r_2 - 3\epsilon(r_1 - r_2)^2/\lambda^2}\) |
| M.6   | \(\sqrt{3/2}(r_1 + r_2)/\lambda, \sqrt{1 - r_1 - r_2 + 3\epsilon(r_1 + r_2)^2/(2\lambda^2)}, \sqrt{r_1 + r_2 - 3\epsilon(r_1 + r_2)^2/\lambda^2}\) |

TABLE I: Critical points for the case of \(Q = q(\alpha\rho_m + 3\beta H\rho_m)\).

Substituting Eq. (25) into Eq. (17), we find that the corresponding \(Q_1\) and \(Q_2\) are given by

\[
Q_1 = \frac{3}{2} \cdot \frac{(\beta - \alpha)q}{1 - \alpha q} \cdot \frac{\varphi^2}{x}, \quad Q_2 = \frac{3x}{2} \cdot \frac{(\beta - \alpha)q}{1 - \alpha q}. \tag{26}
\]

Notice that \(q = s - 1\) and \(s\) is given in Eq. (18). Then, substituting them into the autonomous system Eqs. (14)–(16), we can find the critical points and present them in Table I. Note that \(r_1\) and \(r_2\) are given by

\[
r_1 \equiv \frac{2 + 2\alpha + 3\beta}{6\alpha}, \quad r_2 \equiv \frac{\sqrt{4\varphi^2 + (2 + 3\beta)^2 - 4\alpha(4 + 3\beta)}}{6\alpha}. \tag{27}
\]

If \(\alpha = 0\), only the first two critical points (M.1p) and (M.1m) in Table I can exist, which are trivial solutions in fact. If \(\alpha \neq 0\), for convenience, we can regard \(r_1\) and \(r_2\) as the model-parameters, in place of \(\alpha\) and \(\beta\). By reversing Eq. (27), we can express \(\alpha\) and \(\beta\) as functions of \(r_1\) and \(r_2\), namely

\[
\alpha = \frac{2}{4 - 12r_1 + 9r_1^2 - 9r_2^2}, \quad \beta = -\frac{2(2 - 6r_1 + 3r_1^2 - 3r_2^2)}{4 - 12r_1 + 9r_1^2 - 9r_2^2}. \tag{28}
\]

Now, we discuss the existence of the critical points in Table I. Obviously, Points (M.1p) and (M.1m) can exist only for \(\epsilon = +1\) (namely quintessence). They are both quintessence-dominated solutions, because the corresponding \(\Omega_m = \varphi^2 = 0\). For Points (M.2p) and (M.2m), if \(\epsilon = -1\) (namely phantom), we should have \(r_1 - r_2 \leq 1\) by requiring \(x\) to be real. However, in this case \(\Omega_m = \varphi^2 = 1\) which is physically meaningless. Therefore, Points (M.2p) and (M.2m) can exist only for \(\epsilon = +1\) (namely quintessence) and \(1 \leq r_1 - r_2 \leq 2\). Similarly, Points (M.3p) and (M.3m) can exist only for \(\epsilon = +1\) (namely quintessence) and \(1 \leq r_1 + r_2 \leq 2\). For Point (M.4), if \(\epsilon = +1\) (namely quintessence), it can exist under condition \(\lambda^2 \leq 6\); on the other hand, if \(\epsilon = -1\) (namely phantom), it can exist for any \(\lambda\). In fact, it is a dark-energy-dominated solution, because the corresponding \(\Omega_m = \varphi^2 = 0\). Point (M.5) exists under condition \(1 - r_1 + r_2 + 3\epsilon(r_1 - r_2)^2/(2\lambda^2) \geq 0\) and \(1 \geq r_1 - r_2 - 3\epsilon(r_1 - r_2)^2/\lambda^2 \geq 0\). Point (M.6) exists under condition \(1 - r_1 - r_2 + 3\epsilon(r_1 + r_2)^2/(2\lambda^2) \geq 0\) and \(1 \geq r_1 + r_2 - 3\epsilon(r_1 + r_2)^2/\lambda^2 \geq 0\). Obviously, Points (M.2p), (M.2m), (M.3p), (M.3m), (M.5) and (M.6) are all scaling solutions, because the corresponding \(\Omega_m = \varphi^2 \geq 0\).

To study the stability of these critical points, by linearizing \(Q_1\), we obtain

\[
\delta Q_1 = \frac{3}{2x} \cdot \frac{\beta - \alpha}{1 - \alpha q} \cdot \left\{ (1 - \epsilon x^2 - y^2) \frac{\delta q}{1 - \alpha q} - q \left[ 2\epsilon x \delta x + 2y \delta y + (1 - \epsilon x^2 - y^2) \frac{\delta x}{x} \right] \right\}, \tag{29}
\]

where \(\bar{q} = \bar{s} - 1\) and \(\delta q = \delta s\), while \(\bar{s}\) and \(\delta s\) are given in Eq. (23). Substituting this \(\delta Q_1\) into Eqs. (21) and (22), the two eigenvalues of the coefficient matrix of Eqs. (21) and (22) determine the stability of the
critical point. In Table II we present the eigenvalues for the first 7 critical points in Table I. For Point (M.1p), noting that its existence requires $\epsilon = +1$ (namely quintessence), it can be stable under condition $(4 - 4r_1 + r_1^2 - r_2^2)/(4r_1 + 3r_1^2 - 3r_2^2)/0 > 0$ and $\lambda = \sqrt{6}$. For Point (M.1m), noting that its existence requires $\epsilon = +1$ (namely quintessence), it is unstable because the second eigenvalue is positive (nb. $\lambda$ is positive). For Point (M.2p), noting that its existence requires $\epsilon = +1$ (namely quintessence) and $1 \leq r_1 - r_2 \leq 2$, it can be stable under condition $r_2/(2 - 5r_1 + 3r_1^2 + 2 - 3r_2^2) / 0 > 0$ and $r_1 - r_2 \leq \lambda \sqrt{2}[r_1 - r_2 - 1]/(3\epsilon)$. For Point (M.2m), noting that its existence requires $\epsilon = +1$ (namely quintessence) and $1 \leq r_1 - r_2 \leq 2$, it is unstable because the second eigenvalue is positive (nb. $\lambda$ is positive). For Point (M.3p), noting that its existence requires $\epsilon = +1$ (namely quintessence) and $1 \leq r_1 - r_2 \leq 2$, it can be stable under condition $r_2/(2 - 5r_1 + 3r_1^2 + 2 - 3r_2^2) / 0 > 0$ and $r_1 + r_2 \leq \lambda \sqrt{2}[r_1 + r_2 - 1]/(3\epsilon)$. For Point (M.3m), noting that its existence requires $\epsilon = +1$ (namely quintessence) and $1 \leq r_1 + r_2 \leq 2$, it is unstable because the second eigenvalue is positive (nb. $\lambda$ is positive). For Point (M.4), noting that its existence requires $1 - \lambda^2/(6\epsilon) \geq 0$, it can be stable under condition $-9r_1^2/(2 - 4r_1 + 3r_1^2 - r_2^2 + 3r_2^2) / 0 \geq 0$ and $r_1 - r_2 \leq \lambda \sqrt{2}[r_1 - r_2 - 1]/(3\epsilon)$. Finally, the eigenvalues of Points (M.5) and (M.6) are considerably involved, and hence we do not present them here. We find that they can exist and are stable in proper parameter-space.

In summary, for the case with interaction $Q = q(\alpha\dot{\rho}_{tot} + 3\beta H\rho_m)$, we find that there are two dark-energy-dominated attractors (M.1p) and (M.4), and four scaling attractors (M.2p), (M.3p), (M.5) and (M.6). These scaling attractors can help to alleviate the cosmological coincidence problem. In e.g. [28], it has been found that there is no scaling solution in the interacting phantom model with the familiar interaction $Q = 3qH\rho_m$ in which $\eta$ is a constant. This fact shows that our new interaction $Q = q(\alpha\dot{\rho}_{tot} + 3\beta H\rho_m)$, with $\beta = \alpha$,

\[\frac{3}{2} - \alpha s\] 3qH\rho_m (nb. Eq. (25)) can bring new results to cosmology.

IV. THE CASE OF $Q = q(\alpha\dot{\rho}_{tot} + 3\beta H\rho_m)$

Here, we consider the case of $Q = q(\alpha\dot{\rho}_{tot} + 3\beta H\rho_{tot})$ given in Eq. (6). From Eq. (5), it is easy to find $\rho_{tot} = 3H^2/\kappa^2$. Substituting into Eq. (5), we can finally obtain

\[Q = \frac{6qH^3}{\kappa^2} \left( \frac{3}{2} - \alpha s \right) . \]

Substituting into Eq. (17), we find that the corresponding $Q_1$ and $Q_2$ are given by

\[Q_1 = \frac{q}{z} \left( \frac{3}{2} - \alpha s \right) , \quad Q_2 = \frac{q}{z} \left( \frac{3}{2} - \alpha s \right) . \]

Notice that $q = s - 1$ and $s$ is given in Eq. (18). Then, substituting them into the autonomous system Eqs. (13) – (16), we can find that there are 5 critical points and present the first 4 points in Table II All
the 4 points in Table III are scaling solutions because $\Omega_m = \bar{z}^2 \geq 0$. The last Point (T.3) is considerably involved and hence we do not present it here, except to mention that it is also a scaling solution. Note that $r_3$ and $r_4$ are given by

$$r_3 \equiv \frac{2 - 4\alpha + 3\beta}{4 + 6\alpha}, \quad r_4 \equiv \frac{\sqrt{4 - 24\alpha + 4\alpha^2 + 20\beta - 12\alpha\beta + 9\beta^2}}{4 + 6\alpha}.$$ (32)

If $4 + 6\alpha = 0$, all the 4 critical points in Table III cannot exist. If $4 + 6\alpha \neq 0$, for convenience, we can regard $r_3$ and $r_4$ as the model-parameters, in place of $\alpha$ and $\beta$. By reversing Eq. (32), we can express $\alpha$ and $\beta$ as functions of $r_3$ and $r_4$, namely

$$\alpha = \frac{-2(-1 + 2r_3 + 3r_3^2 - 3r_4^2)}{1 + 6r_3 + 9r_3^2 - 9r_4^2}, \quad \beta = \frac{-2(-1 + 2r_3 + 7r_3^2 - 7r_4^2)}{1 + 6r_3 + 9r_3^2 - 9r_4^2}.$$ (33)

Here, we briefly discuss the existence of the critical points. For Points (T.1p) and (T.1m), if $\epsilon = -1$ (namely phantom), we should have $r_3 + r_4 \leq 0$ by requiring $\bar{x}$ to be real. However, in this case $\Omega_m = \bar{z}^2 \geq 1$ which is physically meaningless. Therefore, Points (T.1p) and (T.1m) can exist only for $\epsilon = +1$ (namely quintessence) and $1 \geq r_3 + r_4 \geq 0$. Similarly, Points (T.2p) and (T.2m) can exist only for $\epsilon = +1$ (namely quintessence) and $1 \geq r_3 - r_4 \geq 0$. Point (T.3) can exist in proper parameter-space [16].

To study the stability of these critical points, by linearizing $Q_1$, we obtain

$$\delta Q_1 = \frac{1}{\bar{x}} \left[ \left( \frac{3}{2} \beta - \alpha \bar{s} \right) \left( \delta \bar{q} - \frac{\delta \bar{s}}{\bar{x}} \right) - \alpha \bar{q} \delta \bar{s} \right],$$ (34)

where $\bar{q} = \bar{s} - 1$ and $\delta q = \delta s$, while $\bar{s}$ and $\delta s$ are given in Eq. (23). Substituting this $\delta Q_1$ into Eqs. (21) and (22), the two eigenvalues of the coefficient matrix of Eqs. (21) and (22) determine the stability of the critical point. In Table IV, we present the eigenvalues for the first 4 critical points in Table III. For Point (T.1p), noting that its existence requires $\epsilon = +1$ (namely quintessence) and $1 \geq r_3 + r_4 \geq 0$, it can be stable under condition $-6(r_3 - r_4)(r_3 + r_4)^3 - 12r_3(r_3 + r_4)\lambda^2 + 18\lambda^4 \leq 0$ and $\frac{2}{\bar{x}} + \frac{3\lambda^2}{2(r_3 + r_4)} - \lambda \sqrt{\frac{3\lambda^2}{2(r_3 + r_4)}} \leq 0$. For Point (T.1m), noting that its existence requires $\epsilon = +1$ (namely quintessence) and $1 \geq r_3 + r_4 \geq 0$, it is unstable because the second eigenvalue is positive (nb. $\lambda$ is positive). For Point (T.2p), noting that its existence requires $\epsilon = +1$ (namely quintessence) and $1 \geq r_3 - r_4 \geq 0$, it can be stable under condition $-6(r_3 + r_4)(r_3 - r_4)^3 - 12r_3(r_3 - r_4)\lambda^2 + 18\lambda^4 \leq 0$ and $\frac{2}{\bar{x}} + \frac{3\lambda^2}{2(r_3 - r_4)} - \lambda \sqrt{\frac{3\lambda^2}{2(r_3 - r_4)}} \leq 0$. For Point (T.2m), noting that its existence requires $\epsilon = +1$ (namely quintessence) and $1 \geq r_3 - r_4 \geq 0$, it is unstable because the second eigenvalue is positive (nb. $\lambda$ is positive). Finally, the eigenvalues of Point (T.3) are considerably involved, and hence we do not present them here. We find that it can exist and is stable in proper parameter-space [16].

So, in the case of $Q = q(\alpha \dot{\rho}_{tot} + 3\beta H \rho_{tot})$, there are 3 scaling attractors (T.1p), (T.2p) and (T.3). These scaling attractors can help to alleviate the cosmological coincidence problem. Of course, these scaling solutions are also different from the ones in the interacting quintessence or phantom model with the usual interaction $Q = 3\eta H \rho_{tot}$ in which $\eta$ is a constant. Our new interaction $Q = q(\alpha \dot{\rho}_{tot} + 3\beta H \rho_{tot})$ brings new results.
Eqs. (14)—(16), we can find that there are 8 critical points and present the first 4 points in Table V. All $r$ and $(D.6)$ are considerably involved and hence we do not present them here, except to mention that they $\alpha = \frac{3H}{1 + aq} \cdot (\rho_{de} + p_{de} + \beta q_{de})$.

Then, substituting into Eq. (17), we find that the corresponding $Q$ is given in Eq. (18). Then, substituting them into the autonomous system

$$Q = \frac{3Hq}{1 + aq} \cdot \left[ (\beta - 2 - \alpha) \epsilon \dot{\phi}^2 + \beta V \right].$$

Substituting into Eq. (17), we find that the corresponding $Q_1$ and $Q_2$ are given by

$$Q_1 = \frac{3q}{2(1 + aq)} \cdot \left[ (\beta - 2 - \alpha) \epsilon \dot{x}^2 + \beta y^2 \right],$$

$$Q_2 = \frac{3q}{2(1 + aq)} \cdot \left[ (\beta - 2 - \alpha) \epsilon \dot{y}^2 + \beta x^2 \right].$$

Notice that $q = s - 1$ and $s$ is given in Eq. (18). Then, substituting them into the autonomous system Eqs. (13)—(16), we can find that there are 8 critical points and present the first 4 points in Table V. All the 4 points in Table V are scaling solutions because $\Omega = \epsilon^2 \geq 0$. The last 4 Points (D.3), (D.4) (D.5) and (D.6) are considerably involved and hence we do not present them here, except to mention that they are also scaling solutions. Note that $r_5$ and $r_6$ are given by

$$r_5 \equiv \frac{2 + 4\alpha - 3\beta}{6\alpha}, \quad r_6 \equiv \sqrt{\frac{4\alpha^2 + 4\alpha(10 - 3\beta) + (2 - 3\beta)^2}{6\alpha}}.$$

If $\alpha = 0$, all the 4 critical points in Table V cannot exist. If $\alpha \neq 0$, for convenience, we can regard $r_5$ and $r_6$ as the model-parameters, in place of $\alpha$ and $\beta$. By reversing Eq. (39), we can express $\alpha$ and $\beta$ as functions of $r_5$ and $r_6$, namely

$$\alpha = \frac{8}{9r_6^2 - 9r_5^2 + 6r_5 - 1}, \quad \beta = \frac{2(3r_6^2 - 3r_5^2 - 6r_5 + 5)}{9r_6^2 - 9r_5^2 + 6r_5 - 1}.$$

Here, we briefly discuss the existence of the critical points. For Points (D.1p) and (D.1m), if $\epsilon = -1$ (namely phantom), we should have $r_6 \leq r_5$ by requiring $\dot{x}$ to be real. However, in this case $\Omega = \epsilon^2 \geq 1$ which is physically meaningless. Therefore, Points (D.1p) and (D.1m) can exist only for $\epsilon = +1$ (namely quintessence) and $r_6 \geq r_5$. Similarly, Points (D.2p) and (D.2m) can exist only for $\epsilon = +1$ (namely quintessence) and $r_5 + r_6 \leq 0$. Points (D.3), (D.4), (D.5) and (D.6) can exist in proper parameter-space.

| Point | Eigenvalues |
|-------|-------------|
| T.1p  | $-\lambda \pm \frac{4a^2 - 12a^2 + 18\lambda}{2(1 + aq)}$, $\gamma + \frac{a^2}{2(1 + aq)} - \lambda \sqrt{\frac{2(1 + aq)}{2(1 + aq)}}$ |
| T.1m  | $-\lambda \pm \frac{4a^2 - 12a^2 + 18\lambda}{2(1 + aq)}$, $\gamma + \frac{a^2}{2(1 + aq)} + \lambda \sqrt{\frac{2(1 + aq)}{2(1 + aq)}}$ |
| T.2p  | $-\lambda \pm \frac{4a^2 - 12a^2 + 18\lambda}{2(1 + aq)}$, $\gamma + \frac{a^2}{2(1 + aq)} - \lambda \sqrt{\frac{2(1 + aq)}{2(1 + aq)}}$ |
| T.2m  | $-\lambda \pm \frac{4a^2 - 12a^2 + 18\lambda}{2(1 + aq)}$, $\gamma + \frac{a^2}{2(1 + aq)} + \lambda \sqrt{\frac{2(1 + aq)}{2(1 + aq)}}$ |

TABLE IV: The corresponding eigenvalues for the first 4 critical points in Table III.
To study the stability of these critical points, by linearizing $Q_1$, we obtain
\[
\delta Q_1 = \frac{3}{2(1 + \alpha \bar{q})} \left\{ \bar{q} \left[ (\beta - 2\alpha) \epsilon \delta x - \frac{\beta y^2}{\bar{x}} \delta x + \frac{2\beta y}{\bar{x}} \delta y \right] + \left[ (\beta - 2\alpha) \epsilon \bar{x} + \frac{\beta y^2}{\bar{x}} \right] \right\},
\] (41)
where $\bar{q} = \bar{s} - 1$ and $\delta q = \delta s$, while $\bar{s}$ and $\delta s$ are given in Eq. (23). Substituting this $\delta Q_1$ into Eqs. (21) and (22), the two eigenvalues of the coefficient matrix of Eqs. (21) and (22) determine the stability of the critical point. In Table VI we present the eigenvalues for the first 4 critical points in Table V. For Point (D.1p), noting that its existence requires $\epsilon = +1$ (namely quintessence) and $r_6 \geq r_5$, it can be stable under condition $r_6 / [(3r_5 - 3r_6 - 1)(1 + r_5 + r_6)] \geq 0$ and $1 - r_5 + r_6 - \lambda \sqrt{2(r_5 - r_6)/(3\epsilon)} \leq 0$. For Point (D.1m), noting that its existence requires $\epsilon = +1$ (namely quintessence) and $r_6 \geq r_5$, it is unstable because the second eigenvalue is positive (nb. $\lambda$ is positive). For Point (D.2p), noting that its existence requires $\epsilon = +1$ (namely quintessence) and $r_5 + r_6 \leq 0$, it can be stable under condition $r_6 / [(1 - 3r_5 - 3r_6)(1 + r_5 - r_6)] \geq 0$ and $1 - r_5 - r_6 - \lambda \sqrt{-2(r_5 + r_6)/(3\epsilon)} \leq 0$. For Point (D.2m), noting that its existence requires $\epsilon = +1$ (namely quintessence) and $r_5 + r_6 \leq 0$, it is unstable because the second eigenvalue is positive (nb. $\lambda$ is positive). Finally, the eigenvalues of Points (D.3), (D.4), (D.5) and (D.6) are considerably involved, and hence we do not present them here. We find that they can exist and are stable in proper parameter-space.

So, in the case of $Q = q(\alpha \dot{\rho}_{de} + 3\beta H \rho_{de})$, there are 6 scaling attractors (D.1p), (D.2p), (D.3), (D.4), (D.5) and (D.6). These scaling attractors can help to alleviate the cosmological coincidence problem. Of course, these scaling solutions are also different from the ones in the interacting quintessence or phantom model with the usual interaction $Q = 3\eta H \rho_{de}$ in which $\eta$ is a constant. Our new interaction $Q = q(\alpha \dot{\rho}_{de} + 3\beta H \rho_{de})$ brings new results.

VI. CONCLUSION

In the present work, motivated by the recent work of Cai and Su [14], we proposed a new type of interaction in dark sector, which can change its sign when our universe changes from deceleration to acceleration. We considered the cosmological evolution of quintessence and phantom with this type of interaction. We found that there are some scaling attractors which can help to alleviate the cosmological coincidence problem. Our results also showed that this new type of interaction can bring new features to cosmology.

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TABLE VI: The corresponding eigenvalues for the first 4 critical points in Table V.

| Point | Eigenvalues |
|-------|-------------|
| D.1p  | \(\frac{24r_2(r_5-r_6)}{(5r_5-4r_6-1)(1+r_5+r_6)}\), (3/2) \(1 - r_5 + r_6 - \lambda \sqrt{2(r_6-r_5)/(3\epsilon)}\) |
| D.1m  | \(\frac{24r_2(r_5-r_6)}{(5r_5-4r_6-1)(1+r_5+r_6)}\), (3/2) \(1 - r_5 + r_6 + \lambda \sqrt{2(r_6-r_5)/(3\epsilon)}\) |
| D.2p  | \(\frac{24r_2(r_5+r_6)}{(1-r_5+3r_6)(1+r_5+r_6)}\), (3/2) \(1 - r_5 - r_6 - \lambda \sqrt{2(r_5+r_6)/(3\epsilon)}\) |
| D.2m  | \(\frac{24r_2(r_5+r_6)}{(1-3r_5+3r_6)(1+r_5+r_6)}\), (3/2) \(1 - r_5 - r_6 + \lambda \sqrt{2(r_5+r_6)/(3\epsilon)}\) |

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[16] In fact, the particular parameter-space for the existence and/or stability of this critical point is considerably
involved and verbose. Since our main aim is to point out the fact that it can exist and is stable, the corre-
sponding parameter-space is not necessary to be presented here. One can work it out comfortably with the
help of computer, especially with the help of Mathematica.

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