We study the decoherence process associated with the scattering of stochastic gravitational waves. We discuss the case of macroscopic systems, such as the planetary motion of the Moon around the Earth, for which gravitational scattering is found to dominate decoherence though it has a negligible influence on damping. This contrast is due to the very high effective temperature of the background of gravitational waves in our galactic environment.

1. Decoherence and the micro/macro transition

Decoherence is a general phenomenon which occurs in principle for any physical system coupled to its environment. The fluctuations associated with this coupling tend to wash out quantum coherences - i.e. superpositions of different quantum states - on a time scale which becomes extremely short for systems with a large degree of classicality - i.e. for superpositions of quantum states with sufficiently different classical properties.

Decoherence thus plays an important role in the transition between microscopic and macroscopic physics. For large macroscopic masses, say the Moon orbiting around the Earth, decoherence is expected to be so efficient that the classical de-
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scription of the motion is sufficient. Precisely, the decoherence time scale is so short that the observation of any quantum coherence effect is impossible. For microscopic masses in contrast, decoherence is expected to be so inefficient that we are left with the ordinary quantum description of the system. In other words, if we consider for example an electron orbiting around a proton, the decoherence time scale is so long that decoherence can be forgotten.

A lot of theoretical papers have been devoted to decoherence but only a few experiments have shown evidence for the phenomenon and this can be understood from the simple arguments sketched in the previous paragraph. In order to observe decoherence, one must deal with ‘mesoscopic’ systems for which the decoherence time is neither too long nor too short. The transition from quantum to classical behaviour is then characterized through the variation of this time with some parameter measuring the degree of classicality of the system.

These experimental challenges have been met with a few specific systems such as microwave photons stored in a high-Q cavity or an ion in a trap. In these model systems where the fluctuations are particularly well mastered, the quantum/classical transition has been shown to fit the predictions of decoherence theory.

It must be emphasized that the details of the quantum/classical transition depend on the coupling mechanisms between the system under consideration and its environment as well as on the spectral properties of the fluctuations. Furthermore, a given system may be coupled to various environments which contribute differently to decoherence. It is only after having studied these points that one may obtain a reliable estimation of the decoherence time scale and, hence, a precise description of the position of the frontier between quantum and classical behaviours.

For motions in the solar system for instance, decoherence is often attributed to the scattering of the electromagnetic fluctuations associated with solar radiation or cosmic microwave background. In fact, as discussed here, the decoherence of planetary motions is not dominated by electromagnetic processes but rather by the scattering of stochastic gravitational waves present in our galactic environment.

2. Decoherence and gravity

The idea that the quantum/classical transition might be related to fundamental fluctuations of space-time has often been proposed. It can be presented in quite simple words relying essentially on dimensional arguments.

Fundamental fluctuations of space-time, associated for example with quantum gravity models, are expected to become important on length scales of the order of the Planck length, i.e. the length built on the constants \( \hbar, c \) and \( G \)

\[
\ell_P = \sqrt{\frac{\hbar G}{c^3}} \sim 10^{-35} \text{ m}
\]  

(1)

This Planck length is extremely small when compared not only to ordinary macroscopic scales but also to the smallest microscopic scales experimentally explored to date. The same argument holds for the Planck time \( t_P = \frac{\ell_P}{c} \).
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Meanwhile, the Planck scale for mass lies on the borderland between microscopic and macroscopic masses

\[ m_P = \sqrt{\frac{\hbar c}{G}} \sim 22 \mu g \]  

(2)

Hence, microscopic and macroscopic values of mass \( m \) may be delineated by comparing the associated Compton length \( \ell_C \) to the Planck length \( \ell_P \)

\[ m < m_P \iff \ell_C = \frac{\hbar}{mc} > \ell_P \]
\[ m > m_P \iff \ell_C = \frac{\hbar}{mc} < \ell_P \]  

(3)

It is quite tempting to consider that this coincidence is not just accidental but that it might be a consequence of the existence of fundamental gravitational fluctuations. The idea was already present in the Feynman lectures on gravitation and it was more thoroughly developed and popularized by a number of authors, for example.

However, the simple dimensional arguments given above are not by themselves sufficient to reach a precise conclusion. As already stated, the description of the quantum/classical transition for a given system must depend on the details of the coupling of this system to its environment as well as on the noise spectrum characterizing the amplitude of fluctuations at the frequencies of interest for the motion of the system.

The aim of this paper is to show that reliable conclusions can be drawn at least for some well defined problems. We will study the decoherence of planetary motions, such as the motion of the Moon around the Earth, associated with the scattering of the stochastic background of gravitational waves present in our celestial environment. We will estimate the decoherence rate and show that it is dominated by this contribution of gravitational fluctuations.

3. Gravitational backgrounds

We now explain how we describe the fundamental fluctuations of space-time and their effect on the motion of matter although we don’t have a complete theory of quantum gravity at our disposal.

The basic idea is that the frequency range of interest, which depends on the specific system of interest, lies in any case far below Planck frequency. For the motion of the Moon for instance, this frequency range lies from the orbital frequency, roughly speaking \( 1 \mu \text{Hz} \), to frequencies larger by a few orders of magnitude. At these frequencies, general relativity is known to be an accurate description of gravitational phenomena, and this statement is essentially independent of the modifications of the theory which will have to take place when a complete theory of quantum gravity will be available.
Stated in different terms, general relativity is certainly not the final word but it can be used quite safely as an effective theory of gravitation at the frequencies involved in planetary motions. We may then conclude that the fundamental space-time fluctuations we have to consider are simply the gravitational waves which are predicted by the linearized version of Einstein theory of gravity. Precisely, the fundamental fluctuations in our gravitational environment are the stochastic backgrounds of gravitational waves which are thoroughly studied in relation with the ongoing experimental development of gravitational wave detectors.

The effect of gravitational perturbations may in principle be described in a manifestly gauge-invariant manner. As soon as this is checked out, a specific gauge can be chosen. Here the calculations are made simpler by choosing the transverse traceless (TT) gauge with metric perturbations differing from zero only for purely spatial components \( h_{ij} \) (i,j=1,2,3 stand for the spatial indices whereas 0 will represent the temporal index).

Then the gravitational waves are conveniently described through a mode decomposition

\[
h_{ij} (x) = \int \frac{d^4k}{(2\pi)^4} h_{ij} [k] e^{-ik_\mu x^\mu} \quad h_{00} = h_{0i} = 0
\]  

Any Fourier component is a sum over the two circular polarizations \( h^\pm \)

\[
h_{ij} [k] = \left( \frac{\varepsilon_{i1}^+ \varepsilon_{j1}^+}{\sqrt{2}} \right)^* h^+ [k] + \left( \frac{\varepsilon_{i1}^- \varepsilon_{j1}^-}{\sqrt{2}} \right)^* h^- [k]
\]  

with the gravitational polarization tensors obtained as products of the polarization vectors \( \varepsilon_i^\pm \) well-known from electromagnetic theory. The gravitational waves correspond to wavevectors \( k \) lying on the light cone (\( k^2 = 0 \)) and they are transverse with respect to this wavevector (\( k_i h_{ij} = 0 \)).

We will consider for simplicity the case of stationary, unpolarized and isotropic backgrounds. Such backgrounds correspond to correlation functions which are completely characterized by the number \( n_{gr} \) of gravitons per mode.

\[
\langle h^+ [k] h^+ [k'] \rangle = \langle h^- [k] h^- [k'] \rangle = (2\pi)^4 \delta(k + k') \delta(k^2) 32\pi^2 G c^3 h_{n_{gr}}
\]

We now rewrite the noise spectrum characterizing gravitational fluctuations by performing a few formal transformations. First, backgrounds are usually written in terms of one metric component, say \( h \equiv h_{12} \), at a fixed spatial point as a function

\( ^a \)Taking into account quantum fluctuations of gravity, and not only classical gravitational waves, one should replace \( n_{gr} \) by \( \left( \frac{1}{2} + n_{gr} \right) \) with the term \( \frac{1}{2} \) representing vacuum fluctuations. These vacuum fluctuations have been shown to lead to ultimate fluctuations of geodetic distances of the order of Planck length. Here we ignore this subtlety because the gravitational backgrounds discussed below correspond to the classical limit where the number of gravitons per mode is extremely large \( n_{gr} \gg 1 \).
of time. The fluctuations of $h$ are described by a noise spectrum $C_{hh}$ obtained by summing over the directions of momentum for a given value of frequency

$$\langle h(t) h(0) \rangle = \int \frac{d\omega}{2\pi} C_{hh} [\omega] e^{-i\omega t}$$

(7)

$C_{hh}$ is the spectral density of strain fluctuations considered in most papers on gravitational wave detectors. We introduce an effective noise temperature $T_{gr}$ for measuring the noise energy per mode and measure this temperature as a frequency $\Theta_{gr}$

$$k_B T_{gr} \equiv \hbar \omega n_{gr} \equiv \frac{\hbar}{\pi} \Theta_{gr}$$

(8)

with $k_B$ the Boltzmann constant. Then the spectral density $C_{hh}$ is simply the product of the frequency $\Theta_{gr}$ by the square of the Planck time $t_P$

$$C_{hh} [\omega] = \frac{16 G}{5c^5} k_B T_{gr} = \frac{16}{5\pi} \Theta_{gr} t_P^2$$

(9)

$C_{hh}$ has the dimension of the inverse of a frequency.

It is worth stressing that $T_{gr}$ is an effective noise temperature, that is an equivalent manner for representing the noise spectrum $C_{hh}$, but certainly not a real temperature. In particular, we have not supposed that $T_{gr}$ is independent of frequency. And, in any case, the value obtained below for this temperature is much higher than the thermodynamical temperature associated with any known phenomenon. This reveals that the motion of matter is so weakly coupled to the gravitational fluctuations that it remains always far from the thermodynamical equilibrium. In other words, the characteristic time which could be associated to the potential thermalization would be extremely long.

We now use the information available from the studies devoted to the detectability of gravitational background by interferometers. As already noticed, the orbital frequency of the Moon is close to 1\(\mu\)Hz and the frequency range of interest for our problem is roughly 1-100\(\mu\)Hz. In this range, the gravitational background is dominated by the confusion noise due to gravitational waves emitted by unresolved binary systems in our galaxy and its vicinity. Figure 1 of reference shows that this 'binary confusion background' corresponds to a nearly flat function $C_{hh}$ in the frequency range of interest

$$10^{-6} \text{Hz} < \frac{\omega}{2\pi} < 10^{-4} \text{Hz} \quad C_{hh} \sim 10^{-34} \text{Hz}^{-1}$$

(10)

After the conversion already discussed, this corresponds to an equivalent noise temperature nearly constant on the frequency range but with an extremely large value $T_{gr} \sim 10^{41}$ K. Equivalently, this corresponds to $\Theta_{gr} \sim 10^{52}$ Hz or to a graviton

$^5$This temperature is even larger than Planck temperature $\frac{\hbar}{k_B t_P} \sim 10^{32}$ K, which emphasizes its unconventional character from the point of view of thermodynamics.
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number per mode \( n_{\text{gr}} \sim 10^{57} \) at \( \omega \sim 1 \mu \text{Hz} \). These numbers clearly correspond to the high temperature limit \( k_B T_{\text{gr}} \gg \hbar \omega \) where fluctuations may be described as classical.

It is worth noticing that these fluctuations are indeed evaluated as the classical gravitational waves emitted by binary systems in the galaxy or its vicinity. They may be treated as stochastic variables because of the large number of unresolved and independent sources. As a consequence of the central limit theorem, they may even be considered to obey a gaussian statistics, a property which will be used later on.

The estimations discussed here correspond to the confusion background of gravitational waves emitted by binary systems in our galaxy or its vicinity. They thus rely on the laws of physics and astrophysics as they are known in our local celestial environment. There also exist predictions for gravitational backgrounds associated with a variety of cosmic processes. These predictions depend on the parameters used in the cosmic models and they have a more speculative character than local astrophysical predictions. The associated temperatures vary quite rapidly with frequency and they are usually thought to be dominated by the confusion binary background in the \( \mu \text{Hz} \) frequency range. Should they surpass the binary confusion background, the latter would have to be considered as a minimum noise level in our gravitational environment and most conclusions to be drawn in the following would be essentially preserved.

4. Tidal perturbation of planetary motions

As the next step in our derivation, we now discuss the effect of gravitational fluctuations on the motion of matter. As far as the non relativistic limit of macroscopic motion with velocities much smaller than the velocity of light is concerned, this effect is essentially described by the standard theory of gravitational wave emission and gravitational wave detection by mechanical detectors.

In the non relativistic limit, the gravitational perturbation on a planetary system may be represented as a tidal acceleration acting on each mass

\[
x_i'' (t) = -R_{i0j0} (t) x_j (t)
\]

The tidal tensor \( R_{i0j0} \) is built up from components of the Riemann curvature tensor and the prime denotes a time derivative. In the transverse traceless (TT) gauge, the tidal tensor is the second order derivative of the metric perturbation \( h_{ij} \) evaluated

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6At this point, we may address an interesting objection which states that, since these fluctuations are classical, they can be monitored and their effect corrected. This remark is valid as a matter of principle but irrelevant for the problem considered in this paper. Indeed, macroscopic bodies such as the Moon follow passively their geodesics and have therefore their motion affected by gravitational fluctuations. Only actively driven bodies, such as dragfree satellites, have the ability to correct their trajectory from the effect of external forces. And this ability may apply to the effect of gravitational waves only if the satellite can simultaneously monitor these waves!
at the center of mass of the planetary system

\[ R_{00} (t) = -\frac{1}{2} h''_{00} (t) \]  

(12)

The interaction is equivalently described by a perturbation coupling the quadrupole momentum of the system to the tidal tensor.

We consider the simple case of a circular planetary orbit in the plane \( x_1 x_2 \). The two masses \( m_a \) and \( m_b \) are also described by the reduced mass \( m = \frac{m_a m_b}{m_a + m_b} \) and the total mass \( M = m_a + m_b \). The radius \( \rho \), that is the constant distance between the two masses, and the orbital frequency \( \Omega \) are related by the third Kepler law

\[ \rho^3 \Omega^2 = GM \]  

(13)

In the following we will also use as characteristic parameters the tangential velocity \( v = \rho \Omega \) and the normal acceleration \( a = \rho \Omega^2 = \frac{v^2}{\rho} \).

In this simple configuration, we characterize the gravitational perturbation through a relative force \( F \) projected along the mean motion

\[ F (t) = \frac{m \rho}{2 \sqrt{2}} (h''_{00} (t) e^{2i\Omega t} + c.c.) \quad \rho = \frac{h_{12}}{\sqrt{2}} + \frac{h_{22} - h_{11}}{2i\sqrt{2}} \]  

(14)

\( F \) is written in terms of the circular polarization \( h \) which fits the circular motion of the planetary system in the plane \( x_1 x_2 \) of the orbit. Precisely, the force \( F \) is driven by gravitational waves through a frequency transposition due to the evolution of the quadrupole momentum at the frequency \( 2\Omega \).

The effect of this force on motion is essentially a momentum diffusion. As a matter of fact, if we define the momentum perturbation \( p_t \) integrated over an interaction time \( t \), we obtain a result typical of Brownian motion with the variance of \( p_t \) proportional to the elapsed time \( t_p \)

\[ p_t = \int_0^t ds \ F (s) \quad \Delta p_t^2 \equiv \langle p_t^2 \rangle = 2D_{gr} t \]  

(15)

The momentum diffusion coefficient \( D_{gr} \) is obtained as

\[ 2D_{gr} = 4m^2 a^2 C_{hh} [2\Omega] \]  

(16)

with \( a \) the acceleration and \( C_{hh} \) the gravitational noise at the frequency \( 2\Omega \) of evolution of the quadrupole.

The diffusion coefficient may be written under the form of an Einstein fluctuation-dissipation relation, i.e. as the product of the effective temperature \( T_{gr} \) by a damping rate associated with the emission of gravitational waves by the planetary system

\[ D_{gr} = m \Gamma_{gr} k_B T_{gr} \quad \Gamma_{gr} = \frac{32Gma^2}{5c^3} \]  

(17)

We have used the assumption of an unpolarized and isotropic background, which seems reasonable for a background of extragalactic origin but not necessarily for a background of galactic origin. The generalization of this result to a non isotropic background would however not modify strongly the orders of magnitude discussed in the following.
This formula connects the Einstein fluctuation-dissipation relation on Brownian motion \(^21\) and the Einstein quadrupole formula for gravitational wave emission \(^22\). The latter does not depend on the effective temperature. For the Moon, the associated damping is so small, \(\sim 10^{-34}\text{s}^{-1}\), that it does not affect the classical motion. It is only for strongly bound binary systems that gravitational damping has a noticeable effect \(^24\). In contrast, the damping rate \(\Gamma_{\text{gr}}\) vanishes at the limit of a null acceleration, that is also at the limit of an inertial motion.

5. Decoherence of planetary motions

We come now to the final step of our evaluation of decoherence for planetary motions. To this aim, we consider two neighbouring motions on the circular orbit of the Moon around the Earth. More precisely, we consider two motions characterized by the same spatial geometry but slightly different values of the epoch - i.e. the time of passage at a given space point. For simplicity, the difference is measured by the spatial distance \(\Delta x\) between the two motions which is constant on a circular orbit.

As the gravitational wave fluctuations depend on time, these two motions undergo different perturbations. This effect is conveniently described by the differential perturbation \(\delta S_t\) on the action integral after an interaction time \(t\). In a first order perturbation theory, \(\delta S_t\) is simply evaluated as

\[
\delta S_t = \int_0^t ds F(s) \Delta x = p_t \Delta x
\]

where \(p_t\) is the momentum perturbation integrated over \(t\) for a single motion.

Should we associate a quantum phase \(\Phi\) to a motion of the Moon, the two neighbouring motions would suffer a differential dephasing characterized by the exponential

\[
e^{i\delta \Phi_t} = \exp \frac{i\delta S_t}{\bar{\hbar}}
\]

Since \(\delta \Phi_t\) is linearly driven by gravitational waves behaving as classical fluctuations, we can treat it as a gaussian classical stochastic variable and this allows us to get the mean value of the exponentiated dephasing as

\[
\langle e^{i\delta \Phi_t} \rangle = \exp \left( -\frac{\Delta \Phi_t^2}{2} \right)
\]

\[
\Delta \Phi_t^2 \equiv \langle \delta \Phi_t^2 \rangle = \frac{\Delta S_t^2}{\bar{\hbar}^2}
\]

Using the Einstein relation for momentum diffusion, we obtain the final characterization of decoherence between two neighbouring trajectories

\[
\langle e^{i\delta \Phi_t} \rangle = \exp \left( -\Lambda_{\text{gr}} \Delta x^2 t \right)
\]

\[
\Lambda_{\text{gr}} = \frac{D_{\text{gr}}}{\bar{\hbar}^2} = \frac{32Gm^2a^2}{5c^5\bar{\hbar}^2}k_B T_{\text{gr}}
\]
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As expected from general discussions, decoherence is described by a factor decreasing exponentially with time $t$, the inverse $\Lambda_{\text{gr}} \Delta x^2$ of the decoherence time becoming larger when the distance $\Delta x$ increases. For the motion of the Moon, the coefficient $\Lambda_{\text{gr}}$ is so large $\sim 10^{75} \text{s}^{-1} \text{m}^{-2}$ that an extremely short decoherence time is obtained even for ultrasmall distances $\Delta x$. To fix ideas, this time lies in the $10 \mu\text{s}$ range for $\Delta x$ of the order of the Planck length.

6. Discussion

This result confirms the idea that decoherence is so efficient for large macroscopic masses that their motion can be treated as classical. But it also leads to more specific conclusions.

First, we want to come back to the simple scaling arguments presented in the introduction. To this aim, we rewrite the decoherence factor

$$\langle e^{i\delta \Phi} \rangle = \exp \left( \frac{32}{5} \left( \frac{m v^2}{m_P c^2} \right)^2 \left( \frac{\Delta x}{\rho} \right)^2 \Theta_{\text{gr}} t \right)$$

(22)

The ratio $\frac{m^2}{m_P^2}$ which appears in this expression is clearly reminiscent of the scaling arguments showing the role of the Planck mass $m_P$ as a natural reference on the borderland between microscopic and macroscopic masses. However, the presence of other terms in the formula implies that these arguments are not sufficient for obtaining quantitative estimations. The correct result depends on the velocity $c$, on the geometrical factor $\frac{\Delta x}{\rho}$, and on the frequency $\Theta_{\text{gr}}$ which measures the amplitude of gravitational fluctuations at the frequency of interest for the motion under study.

Then, the gravitational contribution to decoherence studied in this paper is found to overshadow the other contributions which dominate the damping and are for this reason usually studied. The damping of the main motion of Moon, revealed by the secular variation of lunar rotation, is due to the interaction of Earth and Moon tides and the corresponding damping rate is $\sim 10^{16}$ larger than the gravitation contribution $\Gamma_{\text{gr}}$. The effect of radiation pressure of solar photons is $\sim 10^{10}$ larger than $\Gamma_{\text{gr}}$ and even the scattering of the cosmic microwave background would dominate $\Gamma_{\text{gr}}$ by a factor larger than 100 for the case of Moon. At the same time however, the effective gravitational temperature has such a large value that the gravitational decoherence coefficient $\Lambda_{\text{gr}}$ is much larger than the coefficients associated with tide interactions and electromagnetic scattering processes.

This entails that the ultimate fluctuations of the motion of Moon, and the associated decoherence mechanisms, are determined by the classical gravitation theory which also explains its mean motion. In other words, the environment to be considered when dealing with macroscopic motion consists in the gravitational waves of the confusion binary background. This background is naturally defined in the reference frame of the galaxy if it is dominated by galactic contributions or in a

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*In fact, the decoherence factor scales as the square of the kinetic energy on one hand and on the elapsed time on another hand.*
reference frame built on a larger region of the universe if extragalactic contributions have to be taken into account. In any case, the gravitational background allows to define a reference frame built on our celestial environment which has an effect on local phenomena.

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