Fracture analysis of center crack functionally graded material plate under tensile biaxial loading by extended finite element method

Achchhe Lal 1*, B.M Sutaria2 and Kundan Mishra3

1 Assistant Professor, Department of Mechanical Engineering SVNIT, Surat, India.
2 Associate Professor, Department of Mechanical Engineering SVNIT, Surat, India.
3 Research Scholars, Department of Mechanical Engineering SVNIT, Surat., India.
*Email: lalachchhe@yahoo.co.in

Abstract: The fracture analysis of a centrally cracked functionally graded material (FGM) plate under biaxial stresses using extended finite element method (XFEM) is studied. In this present study the effect of crack length and crack angle on mixed mode stress intensity factor (MSIF) of center cracked FGM plate under biaxial stress is presented. The mathematical formulation is carried out in MATLAB by XFEM using interaction integral and partition of unity method. The results obtained by this present approach are compared with the result from the literature. This study is carried out to find MMSIF of center cracked FGM plate to understand the behavior of crack under biaxial state of stress.

Keywords: FGM plate, extended finite element method, MMSIF.

1. Introduction

Functionally graded material (FGM) is one of the emerging advanced composite where the material properties vary in desired direction. This concept was first utilized by Japan while working on a project where heat resisting material was required. Now FGM materials are being used for many engineering applications where high thermal resistance and high resistance to crack propagation are required.

In spite of sophisticated and advanced manufacturing technology, components contain cracks, which become very dangerous as components may fail under small loadings. So, researchers are giving more effort in analyzing the behavior of cracked plate under different loading conditions. In FGM material properties can be tailored in such a way that crack propagation can be minimized. In this direction Kim and Paulino [1] presented interaction Integral (M-integral) method to calculate MMSIF of orthotropic FGM plate with straight or curved crack under loading. The proposed work is accurately compared with available analytical and numerical solutions, Bayesteh and Mohammadi [2] Studied fracture mechanics by XFEM, where singular stress field is reproduced by using orthotropic crack tip enrichments. In the present work it is verified that orthotropic XFEM needs lesser DOFs as compared to conventional FEM. Lal and Palekar [3] presented the stochastic MMSIF of cracked composite plate with random material properties. The proposed work is done by using XFEM and the present model is seen to be very accurate for uncertainty analysis. Mohammadi [4] presented fracture analysis by XFEM, where MMSIF of cracked FGM plate by using interaction integral and partition of unity method are discussed. Ebrahimi et al [5] investigated and modeled crack in orthotropic composite plate under uniaxial and bi-axial load by XFEM where re-meshing is not required while progress of crack. In the present study it is observed that crack tip domain size has not any effect on SIF when it reaches one third of crack length. Lal et al [6] investigated stochastic fracture analysis cracked composite beam under different loadings. The sensitivity of individual parameter on MMSIF
is also investigated and here it is also confirmed that SIF is get reduced when orthotropic angle is above 40°. So it is essential to keep orthotropic angle more than 40° for safety point of view.

Meek and Ainsworth [7] presented finite element analysis with lower and upper bond theory of center cracked plate under biaxial loading. In this present study a perfect limit load can be evaluated for a various biaxial stress ratio. Lim et al. [8] presented fracture analysis of cracked orthotropic plate under biaxial loading. In the present study it is confirmed that only singular term is not sufficient to predict the crack extension. Goldstein and Shifrin [9] solved first mode crack deviation of orthotropic plane under biaxial loading where, a crack is considered as a thin elliptical hole. In this present work condition for crack stability and crack deviation is obtained and the present results are compared with the results of the conventional model of the crack, where crack is considered as ideal cut. Khatri and Lal [10] presented the stochastic fracture analysis of isotropic plate with hole and emanating crack under biaxial loadings by XFEM. In this present study different stochastic methods are used to see the seniority of different random variables in fracture analysis.

In the present paper MMSIF of center cracked FGM plate under bi-axial tensile load is evaluated for different fracture parameter like crack length and crack angle. XFEM based fracture analysis is utilized in the present study and the result obtained from present approach is also validated with the results from literature.

2. Problem formulation

In conventional FEM it is necessary to update mesh while solving fracture problem. This difficulty can be avoided in XFEM where updation of mesh is not required. The displacement vector in XFEM is written as.

\[ u'(\mathbf{x}) = \sum_{i=1}^{n_c} N_i(x) \mathbf{u}_i + \sum_{i=1}^{n_t} N_i(x) \mathbf{t}_i + \sum_{i=1}^{n_c} \Phi_\alpha(x) \mathbf{b}_1 + \sum_{i=1}^{n_c} \Phi_\beta(x) \mathbf{b}_2 \]

(1)

Where \( \mathbf{u}_i, \mathbf{a}_i, \mathbf{b}_1^\alpha \) and \( \mathbf{b}_2^\alpha \) are the conventional degrees of freedom (dof), dof for enrichment function, crack tip and crack face asymptotic functions. A body with crack of area \( \Omega \) and its outer boundary \( \Gamma \) is shown in Figure 1. The body is having volume / body loads \( b \) and the surface traction at the boundary \( \Gamma' \). The Boundary surface is \( \Gamma = \Gamma_u + \Gamma_t + \Gamma_c \), where \( u, t \) and \( c \) are displacement, traction and discontinuity.

Figure 1: An arbitrary body with center crack with traction \( \mathbf{T} \) and displacement \( \mathbf{u} \)

\[
\text{dofs} = \text{size} (n_c) + \text{size} (n_t) + \text{size} (n) \quad \text{Where,} \quad n = n_n + n_c
\]

(2)

Where, \( n_n \), \( n_c \) and \( n_t \) represent the nodes for FEM mesh, crack face and crack tip respectively. \( H(x) \) is the Heaviside function for the enrichment and the crack tip asymptotic functions are denoted by \( \Phi_\alpha \) and \( \Phi_\beta \).
\[ \Phi_1^\partial = \sqrt{r} \sin \left( \frac{\theta}{2} \right), \quad \Phi_2^\partial = \sqrt{r} \cos \left( \frac{\theta}{2} \right) \]  
(3)

\[ \Phi_1^\partial = \sqrt{r} \sin \theta \cos \left( \frac{\alpha}{2} \right), \quad \Phi_2^\partial = \sqrt{r} \sin \theta \cos \left( \frac{\beta}{2} \right) \]  
(4)

The J-integral in the form of SIF is represented as,

\[ J = \frac{K_1^2 + K_0^2}{E_{\text{eff}}} \]  
Where, \( E_{\text{eff}} \) = \( \frac{E}{1-\nu^2} \) for plane stress \( \frac{E}{1-\nu^2} \) for plane strain  
(5)

The J integral \[10\] for the cracked body is given in equation (6). The two states of J is represented by equation (7).

\[ J = \int \left( W \delta_{ij} - \sigma_{ij} \frac{\partial u_i}{\partial x_j} \right) \rho \, d\Gamma \]  
(6)

\[ J^{(i)} = \int \left[ \frac{1}{2} \left( \sigma_{ij}^{(i)} \right) \left( \delta_{ij}^{(i)} \right) - \left( \sigma_{ij}^{(i)} \right) \frac{\partial u_i}{\partial x_j} \right] \rho \, d\Gamma \]  
(7)

On further solving, we get,

\[ J^{(i)} = J^{(2)} + \frac{2}{E_{\text{eff}}} (K_1^{(i)} K_{1}^{(i)} + K_0^{(i)} K_{0}^{(i)}) \]  
(8)

Now, comparing the Equations. (5) and (8) we get the interaction integral ,

\[ J^{(i,j)} = \frac{2}{E_{\text{eff}}} (K_1^{(i)} K_{1}^{(j)} + K_0^{(i)} K_{0}^{(j)}) \]  
(9)

The SIF in XFEM for two states are evaluated by putting \( K_1^{(2)} = 1 \) and \( K_0^{(2)} = 0 \) and \( K_1^{(2)} = 0 \) and \( K_0^{(2)} = 1 \) in equation (9), we get,

\[ K_1^{(0)} = \frac{M^{(1,\text{Mode I})} E_{\text{eff}}}{2} \quad K_0^{(0)} = \frac{M^{(1,\text{Mode II})} E_{\text{eff}}}{2} \]  
(10)

The following normalized mean MMSIF for first mode SIF and second mode SIF under tensile loading are represented as

\[ K_1 = \frac{K_1^{(0)}}{\sigma \sqrt{\pi a}} \quad K_1^{(0)} = \frac{K_1^{(0)}}{\sigma \sqrt{\pi a}} \]  
(11)

### 3. Result and Discussion

In present study a FGM plate with dimension \((W = L = 1)\) under biaxial tensile loading is studied. In Figure 2 center crack FGM plate under Bi-axial loading, X-FEM meshing and enrichment for crack tip and face are shown. The material properties for FGM plate are \( E_1 = 10^4, E_2 = 10^3, \nu = 0.3 \). Equation 12 represents the change of material properties along width.

\[ E_1(x) = E_1 e^{\beta x}, \quad E_2(x) = E_2 e^{\beta x} \quad G_{12} = \frac{0.5 E}{1+\nu} \]  
(12)
Figure 2: Plate under biaxial tensile stress (a) geometry and loading condition (b) X-FEM meshing of the plate with centre inclined crack (c) enrichment for crack tip and crack face by X-FEM.

In Figure 3 validation study of FGM plate with center crack of crack length (a=0.1) under uniform uniaxial loading is presented, where MMSIF $K_I$ is calculated for gradation factor ($\beta$). It is observed that result of present model is very close to the result from the literature.

Figure 3: MMSIF $K_I$ for center cracked FGM plate subjected to uniaxial tensile loading

In Table 1 MMSIF $K_I$ and $K_{II}$ of the center crack isotropic plate under uniform uni-axial loading is calculated and this result is compared with the result of FGM plate of same dimension and loading. It is observed that MMSIF $K_I$ and $K_{II}$ in FGM plate is less as compared to Isotropic plate. So, from this study it is concluded that FGM provides better fracture resistance than Isotropic material. The maximum reduction of $K_I$ and $K_{II}$ is observed as 4.41% and 15.38% respectively.

| $\alpha$ | $K_I$ | Isotropic Plate | FGM Plate |
|---------|-------|-----------------|-----------|
| 0.1     | 1.202 | 1.188           |           |
|         | $K_{II}$ | 0.003           | 0.002     |
| 0.15    | $K_I$   | 1.284           | 1.283     |
In Figure 4 MMSIF $K_I$ of center crack FGM plate for $a = 0.1$ and $\alpha = 0$ with respect to $\beta$ under uni-axial and bi-axial loadings is presented. In this study it is observed that the value of MMSIF $K_I$ is get reduced in bi-axial (B=1) loading as compared to uni-axial loading of same magnitude. This is due to loading in lateral direction which reduces the effect of loading in Y direction. The maximum reduction in MMSIF $K_I$ is 8.21 % for $\beta = 0$, where as the minimum reduction is 2.24 % for $\beta = 1$. This study is further extended to the effect of bi-axial (B=2) loading and it is observed that for center cracked FGM plate the value of MMSIF $K_I$ is get almost double as compared to the bi-axial (B=1) loading.

| $\beta$ | $K_I$ | $K_I$ |
|--------|------|------|
| 0.2    | 0.009| 0.008|
| 0.25   | 1.360| 1.340|
| 0.25   | 0.022| 0.019|
| 0.25   | 1.495| 1.429|
| 0.25   | 0.039| 0.033|

Figure 4: MMSIF $K_I$ for center cracked FGM plate subjected to uniaxial tensile loading for different values of $\beta$.

In Figure 5 MMSIF $K_I$ of center cracked FGM plate for $\beta = 0.25$ and $\alpha = 0$ with respect to $a$ under uni-axial and bi-axial loadings is presented. In this study it is observed that the value of MMSIF $K_I$ is get reduced in bi-axial (B=1) loading as compared to uni-axial loading of same magnitude. This is due to loading in lateral direction which reduces the effect of loading in Y direction. The maximum reduction in MMSIF $K_I$ is 84.76 % for $a = 0.3$, where as the minimum reduction is 6.77 % for $a = 0.1$. This study is further extended to the effect of bi-axial (B=2) loading and it is observed that for center crack FGM plate the value of MMSIF $K_I$ is get increased with maximum 7.13 times for $a = 0.3$ and minimum 2.07 times for $a = 0.1$.
In Figure 6 MMSIF $K_I$ and $K_{II}$ for center crack FGM plate where $\beta = 0.25$, $a = 0.2$ are calculated for different values of crack angle ($\alpha$) under uni-axial and bi-axial ($B=1$ and $B=2$). In this study it is observed that for uni-axial loading as crack angle increases from $0^0$ to $50^0$ MMSIF $K_I$ decreases by 59.19 % whereas $K_{II}$ increases by 31.85 times. This is due to shearing component of uni-axial tensile load. In center cracked FGM plate under bi-axial stress it is observed that as crack angle increases from $0^0$ to $50^0$ MMSIF $K_I$ increases whereas maximum value of $K_{II}$ is observed at $40^0$ crack angle. It is also observed that as crack angle increases from $0^0$ to $50^0$ the MMSIF $K_I$ is more and $K_{II}$ is less in bi-axial ($B=2$) loading as compared to bi-axial ($B=1$) for all values of crack angle.

Figure 6: MMSIF $K_I$ and $K_{II}$ for center cracked FGM plate under uni-axial and Bi-axial loadings for different values of $\alpha$

4. Conclusions

In this present work MMSIF of center cracked FGM plate under bi-axial loading is presented. The mathematical formulation is done by XFEM using $J$–integral in MATLAB. The conclusions of this present work are summarized as

- MMSIF $K_I$ and $K_{II}$ in FGM plate is less as compared to Isotropic plate. This is due to better fracture resistance in FGM material than Isotropic material. The maximum
reduction of $K_I$ and $K_{II}$ is observed as 4.41 % and 15.38 % respectively when crack length varies from 0.1 to 0.25.

- It is observed that the value of MMSIF $K_I$ is get reduced in Bi-axial (B=1) loading as compared to uni-axial loading of same magnitude. This is due to loading in lateral direction which reduces the effect of loading in Y direction. It is also observed that MMSIF $K_I$ is get almost double for bi-axial (B=1) loading as compared to the bi-axial (B=1) loading when gradation factor varies from 0.1 to 1.

- It is observed that the value of MMSIF $K_I$ is get reduced in Bi-axial (B=1) loading as compared to uni-axial loading of same magnitude as crack length ($a$) varies from 0.1 to 0.3. This is due to loading in lateral direction which reduces the effect of loading in Y direction.

- In this study it is observed that for uni-axial loading as crack angle increases from $0^0$ to $50^0$ MMSIF $K_I$ decreases by 59.19 % whereas $K_{II}$ increases by 31.85 times .

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