Quantum annealing of an infinite-range transverse-field Ising model

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Abstract. The adiabatic quantum evolution of the Lipkin-Meshkov-Glick (LMG) model across its quantum critical point is studied. The dynamics is realized by linearly switching the transverse field from an initial large value towards zero. We concentrate our attention on the residual energy after the quench in order to estimate the level of diabaticity of the evolution. We discuss a Landau-Zener approximation of the finite size LMG model, that is successful in reproducing the behavior of the residual energy as function of the transition rate in most of the regimes considered. The system proposed is a paradigm of infinite-range interaction or high-dimensional models.

1. Introduction

Understanding the adiabatic quantum dynamics of many-body systems is central to many areas of physics and information science. In quantum annealing (QA), see Ref.[1] for a review, alias adiabatic quantum computation [2], the ultimate goal is to find the ground state of a complex system by adiabatically transforming the underlying Hamiltonian. Indeed any quantum algorithm can be efficiently implemented through the adiabatic evolution of a system from an initial exactly known state towards a unknown final one in which the answer to the specific computational task is encoded [3]. Once the time scale on which the Hamiltonian is varied is large compared to the typical inverse spectral gap of the system, the quantum adiabatic theorem [4] ensures that once the system was prepared in the ground state of the initial Hamiltonian, the evolved state will be the ground state of the final one. The bottleneck to the speed at which the algorithm is performed is thus given by those places where the instantaneous Hamiltonian has a spectrum where the gap closes in the thermodynamic limit, i.e. as the number of qubits increases. If the minimum gap closes faster than a power of the number of qubits then the corresponding computational task is intractable.

The closing of a gap between the ground state and the first excited level in the thermodynamic limit is a distinct feature of second order quantum phase transition. It is responsible for the critical slowing down [5], and the evolution becomes necessarily not adiabatic. The problem of adiabatic dynamics close to a critical point, and the consequent defects formation, has originally born in the study of phase transitions in the early universe [6, 7]. The recent extension to the
quantum case [8, 9] has stimulated an intense theoretical activity [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27].

In the search for a deeper understanding of the loss of adiabaticity on crossing a quantum critical point an important role is played by exactly solvable models. The problem under consideration involves both non-equilibrium and many-body physics and the help of a tractable exactly solvable system is of interest in itself and of great help in testing approximate approaches. Most of the work done so far in this direction have concentrated on one-dimensional quantum systems with short range interaction. In this paper we would like address a complementary limit, i.e. a model with infinite coordination (in the thermodynamic limit), but still amenable of an exact solution: the Lipkin-Meschkov-Glick model (LMG). First introduced by Lipkin, Meschkov and Glick [28] in the context of nuclear physics, it was then adopted by the condensed matter community as paradigm of an infinitely coordinated solvable system [29]. The result of a sudden quench in this model was recently discussed in [30]. Here we present results in the opposite limit in which the system is dragged adiabatically through the critical point. As it will be shown in the following, although the phase transition is of mean field nature, the dynamics leads to non-trivial results.

The paper is organized as follows: In Sec. 2 we introduce the Lipkin-Meschkov-Glick model and briefly review its properties which are important for the purposes of this work. In the same section we also discuss how we solve numerically the dynamics, Sec. 2.1, and the observables used to quantify the departure from the adiabatic ground state, Sec. 2.2. In this work we use the residual energy (the excess energy as compared to the adiabatic limit), the incomplete magnetization (the deficit magnetization as compared to the adiabatic limit), Sec. 3. In the final section we also discuss how we solve numerically the dynamics, Sec. 2.1, and the observables used to quantify the departure from the adiabatic ground state.

2. The Model

The properties of the LMG model have been thoroughly scrutinized in the literature (see, e.g. [31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43] and references therein). Below, we briefly recall a few results that are relevant to the present paper. The LMG Hamiltonian describes a system of spins (1/2 in this work) interacting through an infinite-range exchange coupling and immersed in a transverse field. Assuming that the field is directed along the z-direction, the Hamiltonian can be written as

$$H = -\frac{1}{N S} \sum_{i<j} (S_i^x S_j^x + \gamma S_i^y S_j^y) - \Gamma \sum_i S_i^z,$$

where $N$ is the number of the spins in the system, $S_i$ are the Pauli operators, $\gamma$ is the XY-anisotropy parameter and $\Gamma$ is the transverse field. By introducing the total spin operator $\mathbf{S} = \sum_i S_i$, the Hamiltonian can be rewritten, apart from a additive constant, as $H = -\frac{1}{N} [S_x^2 + \gamma S_y^2] - \Gamma S_z$. The Hamiltonian hence commutes with $S^2$ and does not couple states having a different parity of the number of spins pointing in the magnetic field direction: $[H, S^2] = 0$ and $[H, \prod_i S_i^z] = 0$. In the XY-isotropic case $\gamma = 1$ also the z-component $S_z$ is conserved, $[H, S_z] = 0$.

In the thermodynamical limit the LMG model undergoes a second order quantum phase transition at $\Gamma_c = 1$ characterized by mean-field critical exponents [31]. The magnetization in the $x$-direction (or in the $XY$-plane, for $\gamma = 1$) vanishes when $\Gamma \to 1^-$ as $m = (1 - \Gamma^2)^{1/2}$, for all values of the anisotropy parameter $\gamma$. For $\Gamma > \Gamma_c = 1$ and for any $\gamma$, the ground state is non-degenerate; for $\Gamma < \Gamma_c = 1$ the ground state is doubly degenerate in the thermodynamical limit for any $\gamma \neq 1$, signaling the breaking of the $Z_2$ symmetry. The gap vanishes at the transition as $\Delta = [(\Gamma - 1)(\Gamma - \gamma)]^{1/2}$ for $\Gamma \geq \Gamma_c = 1$. 

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For any finite $N$ both the magnetization and the gap are modified (as any other physical observable). The finite size scaling behavior is available in literature in all the relevant regimes (see, e.g., [31, 38]). The deviation from the thermodynamic limit for the gap $\delta \Delta_N = \Delta_N - \Delta$, for instance, scales, at the critical point $\Gamma = \Gamma_c = 1$, as $\delta \Delta_N \sim N^{-1/3}$, for $\gamma \neq 1$ and as $\delta \Delta_N \sim N^{-1}$ for $\gamma = 1$. The scaling behavior of the gap is important in order to distinguish the various dynamical regimes in the adiabatic annealing. We should stress at this point that the equilibrium gap is not necessarily the one responsible for the loss of adiabaticity. As we will see in the following section, due to the parity conservation the relevant gap for the dynamics is different from the equilibrium one, but the corresponding scaling behavior is identical.

2.1. Adiabatic dynamics
The adiabatic dynamics is implemented by changing the external transverse field from an initial value $\Gamma \gg 1$ at $t_{in} = -\infty$, where the ground state of $H(t_{in})$ is completely dominated by the transverse field term with all the spins aligned along the $+\hat{z}$ direction, to $\Gamma = 0$, where the ground state is ordered in the $XY$ plane. The annealing time is characterized by a time scale $\tau$. More specifically, we reduce the magnetic field linearly in time, as often in this type of problems,

$$\Gamma(t) = -t/\tau \quad \text{for} \quad t \in (-t_{in}, 0]$$

with $t_{in} \gg \tau$.

The problem we want to discuss is further simplified by the following observation. The initial state, the ground state of $H(t_{in})$, belongs to the sector of maximum spin $S = N/2$. Since $S$ is a constant of motion, it is sufficient to restrict our attention to this subspace only. From now on we assume $S = N/2$ (for simplicity we consider $N$ even). In the basis $|N/2,S^z\rangle$ ($S^z = -N/2,...,N/2$), the Schrödinger evolution of the state

$$|\psi(t)\rangle = \sum_{j=1}^{N/2+1} u_{2j-1}(t) |N/2, -N/2 - 2 + 2j\rangle,$$

amounts to solving the following set of coupled equations

$$i\frac{du_{2j-1}}{dt} = \sum_k A_{j,k} u_{2k-1}(t).$$

The odd amplitudes $|N/2, -N/2 - 1 + 2j\rangle$ do not couple because of parity conservation. In Eq. (4) $A$ is a $(N/2 + 1) \times (N/2 + 1)$ symmetric matrix whose non-zero entries are given by

$$A_{j,j+1} = -(1/4N)(1 - \gamma)a_{-N/2-2+2j}a_{-N/2+2j-1}$$

and $A_{j,j} = (1/4N)(1 + \gamma)[a_{-N/2-3+2j}^2 + a_{-N/2-2+2j}^2 - \Gamma(-N/2) - 2 + 2j] + (1/4)(1 + \gamma)$, in terms of the usual angular momentum raising operator matrix elements $a_j = [(N/2)(N/2 + 1) - j(j + 1)]^{1/2}$. Special values have the boundary terms of $A$, given by $A_{1,1} = -(1/4N)(1 + \gamma)a_{-N/2}^2 - \Gamma(N/2) + (1/4)(1 + \gamma)$ and $A_{N/2+1,N/2+1} = (1/4N)(1 + \gamma)a_{N/2-1}^2 - \Gamma(N/2) + (1/4)(1 + \gamma)$. The equations (4) were integrated via a standard fourth-order Runge-Kutta integrator with initial conditions given by the amplitudes of the ground state of $H(t = t_{in})$.

2.2. Residual energy and incomplete magnetization
A natural way of quantifying the degree of adiabaticity of the evolution is to measure the residual energy, defined as

$$E_{res} = E_{fin} - E_{gs},$$

where $E_{gs}$ is the ground state energy of $H(t_{fin})$, and $E_{fin} = \langle \psi(t_{fin})|H(t_{fin})|\psi(t_{fin})\rangle$ is the average energy of the final time-evolved state $|\psi(t_{fin})\rangle$. Obviously $E_{fin}$, and hence $E_{res}$, depends on the
annealing time $\tau$; the slower the evolution, the closer the final energy to $E_{gs}$, hence the small the residual energy.

An alternative way of quantifying the degree of adiabaticity of the evolution is in terms of the incomplete magnetization in the final state, defined by

$$m_{inc} = m_{gs} - m(t)$$  \hfill (6)

where $m_{gs}$ is the static magnetization of the ground state at $\Gamma = 0$ and $m_{gs}$ is the average magnetization of the final evolved state. Following Botet et al [29], the magnetization $m$ has been defined through

$$m^2 = \frac{4}{N^2} \langle \psi | \Sigma^2_x + \delta_{\gamma,1} \Sigma^2_y | \psi \rangle,$$  \hfill (7)

where the expectation value can be taken either the ground state, for $m_{gs}$, or in the evolved state, for $m(t)$. As discussed in Ref. [29], the previous definition differs from that of the spontaneous magnetization; however, it is more amenable for finite size systems and it reduces to the spontaneous magnetization in the thermodynamic limit. Since we are dealing with a model where the coupling has an infinite range, the incomplete magnetization is an appropriate way of characterizing the loss of adiabaticity. In this case a correlation length characterizing the typical distance between defects, along the lines followed for short range models, cannot be introduced.

In the Ising limit, $\gamma = 0$, at $\Gamma(t = 0) = 0$, the residual energy and the incomplete magnetization are related, as they both depend only on the average value $\langle \psi(t = 0) | \Sigma^2_x | \psi(t = 0) \rangle$: The residual energy per site can be expressed as $E_{res}/N = -(1/N^2) \langle \psi(t = 0) | \Sigma^2_x | \psi(t = 0) \rangle + 1/4$, and the incomplete magnetization is given by $m_{inc} = 1 - (2/N) \sqrt{\langle \psi(t = 0) | \Sigma^2_x | \psi(t = 0) \rangle}$.  \vspace{0.5cm}

3. Results

The results presented below were obtained by integrating numerically Eq. (4). We verified that, as for the initial time of the evolution, it is enough to consider $t_{in} = -5\tau$ for faster sweeps ($1 < \tau < 500$) and $t_{in} = -2\tau$ for slower ones ($\tau > 500$). We checked (data not reported) that our results do not depend on the precise value of $t_{in}$, as long as it is not too small as compared to the previous values. We considered systems up to $N = 1024$ spins and annealing times up to $\tau \sim 10^3 - 10^4$.

*Residual energy and incomplete magnetization -*

In Fig. 1 and we show the behaviour of the residual energy per site and of the incomplete magnetization versus $\tau$, for two values of $\gamma$. The behaviour is qualitatively independent from the value of the anistropy parameter $\gamma$, as long as $\gamma \neq 1$, where the system acquires XY-symmetry. Hereafter, only the case $\gamma = 0$ will be discussed. Fig. 2, shows more in detail the results for the residual energy in an extended $\tau$-range, for $\gamma = 0$. Inspection of Fig. 2 reveals three different regimes. For fast quenches the dynamics involves almost all the instantaneous levels of the Hamiltonian. The residual energy per site is close to its maximum and shows very little dependence on the size of the system. For larger values of $\tau$, a second intermediate region appears in which a power-like decay emerges, with $E_{res} \sim \tau^{-3/2}$. Finally, by further slowing the quench rate, a third large-$\tau$ regime characterized by a different power-law, $E_{res} \sim \tau^{-2}$, emerges. We briefly discuss the emergence of the last two regimes by means of a Landau-Zener approach adapted to the present problem.

The argument follows closely the one given in [8]. The probability of exciting the system into the first excited state, obtained from the Landau-Zener formula

$$P_{LZ} \simeq e^{-\alpha \Delta^2 \tau},$$  \hfill (8)
with $\alpha = \pi/4$, gives a lower bound to the true transition probability, as it ignore the transitions to all the other excited levels. A crucial question is, now, how the relevant critical gap $\Delta$ scales in the large-$N$ limit. The characteristic time scale for breaking of adiabaticity is however not given by the equilibrium smallest gap. As noticed in the previous Section, the dynamics is restricted to the subspace with fixed total spin $S = N/2$ and can involve only states with the same parity of $S_z$ [44]. Hence, the first gap relevant for the dynamics, that we call dynamical gap, is the energy difference between the ground state and the second excited state, the smallest gap being forbidden by parity conservation of the total spin along the $z$-axis. As shown in Fig. 3, the dynamical gap exhibits the same critical behavior as the first gap [31]: both close polynomially in the thermodynamical limit with the same dynamical exponent $z = 1/3$, $\Delta_c \sim N^{-z}$. Using this scaling of the critical point gap with the number of spins, $\Delta \sim N^{-1/3}$, it is possible to
Figure 4. (Color online) Comparison between the excitation probabilities as function of $\Gamma$ of the LMG model with $N = 32$ and of its effective LZ approximation for different values of $\tau$.

determine maximum system size for a defect-free quench once the probability for this to occur is fixed to the value $\tilde{P}_\text{ex}$. This gives:

$$\frac{1}{N_{\text{free}}} \sim \left( \frac{\ln \tilde{P}_\text{ex}}{\alpha} \right)^{3/2} \frac{1}{\tau^{3/2}}.$$  \hspace{1cm} (9)

One can consider $1/N_{\text{free}}$ as an estimate of the fraction of the flipped spins after the quench. The residual energy per site in the LMG model can then be evaluated to be

$$\frac{E_{\text{res}}}{N} \sim \frac{1}{N^2 N_{\text{free}}} N \sim \text{const.} \frac{1}{\tau^{3/2}}.$$  \hspace{1cm} (10)

This simple estimate is in good agreement with the numerical data in the intermediate regime of Fig. 2.

**Effective two-level approximation** - As already mentioned there is, for slower quenches, a further crossover to a different power-law, $E_{\text{res}}/N \sim \tau^{-2}$. This $\tau^{-2}$ regime is usually described as the general deviation from adiabaticity deriving by the adiabatic theorem for very slow evolutions [45]. Can one explain also this behavior by using a Landau-Zener argument? To this end, it is important to refine this comparison and to understand to which extent the dynamics of a many-body system described by the LMG model can be described by two (many-body) levels. In general, in a many-body system there will be a number of avoided crossings and multiple LZ transitions, including interference between them. Only when a single avoided crossing is dominant and well separated from the others a two-level approximation is appropriate. A detailed analysis of this issue is summarized in Fig. 4, where we discuss the case of $N = 32$ as an example. Our analysis starts by extracting, through a fit, the best dynamical minimum gap. From here we compare the results of the full LMG model with those obtained using the fitted gap and the simple LZ theory. As shown in Fig. 4, the excitation probability in the LMG model for fast enough quenches coincides with that of the effective LZ problem. It appears that this approximation is good also in the estimate of the asymptotic value of the probability for $10 < \tau < 100$. Deviations come predominantly from the more enhanced oscillations of the post crossing region in the LMG model. For larges $\tau$’s, the asymptotic value obtained from the effective two-level system is wrong. This can be understood because of the presence of more
than one crossing, which is obviously neglected in the two-level approximation. As found by Vitanov [46], it is possible to define the duration of a single LZ as the time required by the probability for jumping from zero to its asymptotic value, linearly and with the slope calculated at the crossing point. Using $\Gamma$ as time-scale one can write:

$$\Gamma_{\text{jump}} \sim \frac{P(\infty)}{P'(\Gamma_{\text{cross}})}.$$  

(11)

This time turns out to be exponentially divergent with $\tau$ for large $\tau$ [46]. This means that for slow quenches, consecutive LZ transitions are not independent.

In order to study this issue, we analyzed the effect of adding a second crossing to a simple LZ system; the three-level Hamiltonian so obtained is given by:

$$H = \begin{pmatrix} -\Delta_1 \Gamma & \Omega_1 & 0 \\ \Omega_1 & \Delta_1 \Gamma & \Omega_2 \\ 0 & \Omega_2 & \Delta_3 \Gamma + a_3 \end{pmatrix}.$$  

(12)

We focused the attention on two situations: in the first, the slope of the second crossing ($\Delta_3$) is kept constant and its position ($a_3$) is varied; in the second, the position is constant and the absolute value of the slope is increased. In both cases, we chose the parameters in such a way the third level crosses only the highest of the first two (i.e., it doesn’t alter the ground state energy), see Figs 5 and 6 for the spectrum ($\Omega_1 = \Omega_2 = 0.2, \Delta_1 = 1, \Gamma_i = -5, \Gamma_f = 2$). The behavior of the total excitation probability as a function of the quench time $\tau$ is summarized in Figs 7 and 8. The presence of a power-law regime $\sim \tau^{-2}$ for extremely slow dynamics is a clear consequence of the finite duration of the evolution. In the original works by Landau and by Zener, the final time is supposed to be $t_f = \infty$; here the evolution is stopped at $\Gamma_f = t_f/\tau = 2$. An accurate analysis of the finite-time Landau-Zener model (FTLZ) has been done in Ref. [46], where it is shown that the transition probability reads, in this case [47]:

$$P_{\text{ex}}(\tau) \sim P_{\text{LZ}}(\tau) + (1 - 2P_{\text{LZ}}(\tau)) \frac{1}{16\Omega_1^4 + \left(1 + \frac{\Delta_1^2}{\Omega_1^2}\Gamma_f^2\right) \frac{\Delta_3^2}{\Delta_1^2}}.$$  

(13)
where all parameters refer to first crossing of the 3-level system and $P_{LZ}(\tau) = e^{-\pi \Omega^2 \tau / \Delta_1}$. As it can be immediately seen from the previous equation, by sending the final time to infinity the usual LZ probability is recovered. From Figs 7 and 8 it emerges that for moderate absolute value of the slope of the third diabatic level and for far enough crossings, the total excitation probability for the 3-level system coincides with the 2-level case. Instead considering the adiabatic limit (large $\tau$ region), both position and slope can influence the process, modifying the effective duration of the first FTLZ. In a first crude approximation, we can guess that the transposition of the effect in the LMG model is simply to stop the probability from relaxing towards the asymptotic value when the system has reached the second avoided crossing. In Fig. 9 we compared the excitation probabilities of LMG systems of different sizes with their single-LZ approximations. The probabilities for the effective models are evaluated for three different final
time: \( \Gamma_f = 0, \Gamma_1, \Gamma_2 \), where the last two are the positions, respectively, of the minimum gap between the ground state and the second excited level, and the minimum gap between the first and the second excited levels. As it can be seen, the agreement is quite good and one can reproduce in this way also the regime with the \( \tau^{-2} \) behavior.

4. Conclusions
In this paper we have studied the adiabatic quantum dynamics of an infinite-range transverse field Ising system, the LMG model, across its quantum critical point. We focused our attention on the residual energy after the quench analyzing its behavior as a function of the annealing time, in order to evaluate the extent of diabaticity of the evolution. The dynamics is restricted to a subspace of definite total spin and parity of its projection along the \( z \)-axis, due to the symmetries of the Hamiltonian. Results appeared to be qualitatively independent of the value of the \( XY \)-anisotropy parameter \( \gamma \), except for the fully isotropic \( XX \) case at \( \gamma = 1 \), where the further conservation of \( S_z \) plays an important role. By starting the evolution in the ground state for very large values of the transverse field \( \Gamma \), and then reducing \( \Gamma(t) \) linearly to zero, three regimes in the residual energy are identifiable: the first one, corresponding to fast quenches, is strongly not adiabatic, involves transitions from the ground state towards many excited states, and is characterized by a residual energy near its saturation value. In the intermediate regime, the lowest critical dynamically accessible gap starts dominating the evolution, inducing a residual energy per site that decays in a power-like manner, like \( \tau^{-3/2} \). The third large-\( \tau \) region, where the residual energy decays like \( \tau^{-2} \), usually described as the general deviation from adiabaticity deriving from the adiabatic theorem for very slow evolutions [45], can be understood in terms of a finite-time Landau-Zener sweep. It is important to realize that in a generic model with a critical point, the final universal \( \tau^{-2} \) regime could be, in practice, of academic interest, as it could set in at very large values of the annealing time \( \tau \), much larger than the typical critical-gap related Landau-Zener behaviour. It is therefore important that the critical-gap \( \Delta \) scales to zero slow enough, as \( N \to \infty \), in such a way that the intermediate \( \tau \) regime shows a fast power-law behaviour. One dimensional Ising models with disorder, where the critical gap scales to zero exponentially in \( N \), are particularly detrimental to quantum annealing (and to classical annealing as well) [21], and the final \( \tau^{-2} \) regime is completely out of reach.

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References
[1] Santoro G E and Tosatti E 2006 J. Phys. A: Math. Gen. 39 R393–R431
[2] Farhi E, Goldstone J, Gutmann S, Lapan J, Lundgren A and Preda D 2001 Science 292 472
[3] Aharonov D, van Dam W, Kempe J, Landau Z, Lloyd S and Regev O 2004 Proceedings of the 45th Annual IEEE Symposium of Foundations of Computer Science (FOCS’04) (IEEE Computer Society, Washington DC, USA) p 42
[4] Messiah A 1962 Quantum mechanics vol 2 (Amsterdam: North-Holland)
[5] Sachdev S 1999 Quantum Phase Transition (Cambridge University Press)
[6] Kibble T W B 1980 Phys. Rep. 67 183
[7] Zurek W H 1996 Phys. Rep. 276 177
[8] Zurek W H, Dorner U and Zoller P 2005 Phys. Rev. Lett. 95 105701
[9] Polkovnikov A 2005 Phys. Rev. B 72 161201(R)
[10] Damski B 2005 Phys. Rev. Lett. 95 035701
[11] Dziarmaga J 2005 Phys. Rev. Lett. 95 245701
[12] Cherng R W and Levitov L 2006 Phys. Rev. A 73 043614
[13] Dziarmaga J 2006 Phys. Rev. B 74 064416
[14] Schutzhold R, Uhlmann M, Xu Y and Fischer U R 2006 Phys. Rev. Lett. 97 200601
[15] Cinco L, Dziarmaga J, Rams M M and Zurek W H 2007 Phys. Rev. A 75 052321
[16] Cucchietti F M, Damski B, Dziarmaga J and Zurek W H 2007 Phys. Rev. A 75 023603
[17] Fubini A, Falci G and Osterloh A 2007 New J. Phys. 9 134
[18] Polkovnikov A and Gritsev V 2007 Breakdown of the adiabatic limit in low dimensional gapless systems to be published in Nat. Phys. (Preprint arXiv:0706.0212)
[19] Damski B and Zurek W H 2007 Phys. Rev. Lett. 99 130402
[20] Lamacraft A 2007 Phys. Rev. Lett. 98 160401
[21] Caneva T, Fazio R and Santoro G E 2007 Phys. Rev. B 76 144427
[22] Sengupta K, Sen D and Mondal S 2008 Phys. Rev. Lett. 100 077204
[23] F Pellegrini S Montangero G S and Fazio R 2008 Phys. Rev. B 77 140404
[24] Mukherjee V, Divakaran U, Dutta A and Sen D 2008 Quenching along a gapless line: a new exponent for defect density (Preprint arXiv:0805.3328)
[25] Divakaran U, Dutta A and Sen D 2008 Phys. Rev. Lett. 99 130402
[26] Lipkin H J, Meshkov N and Glick A J 1965 Nucl. Phys. 62 188
[27] Castaños O, López-Peña R, Hirsch J and López-Moreno E 2006 Phys. Rev. B 74 104118
[28] Unanyan R and Fleischhauer M 2003 Phys. Rev. Lett. 90 133601
[29] Das A and Chakrabarti B K 2005 Quantum Annealing and Related Optimization Methods Lecture Notes in Physics (Springer-Verlag)