On Exact Supersymmetry in DLCQ

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Abstract

In recent years a supersymmetric form of discrete light-cone quantization (hereafter ‘SDLCQ’) has emerged as a very powerful tool for solving supersymmetric field theories. In this scheme, one calculates the light-cone supercharge with respect to a discretized light-cone Fock basis, instead of working with the light-cone Hamiltonian. This procedure has the advantage of preserving supersymmetry even in the discretized theory, and eliminates the need for explicit renormalizations in 1 + 1 dimensions. In order to compare the usual DLCQ prescription with the supersymmetric prescription, we consider two dimensional SU($N$) Yang-Mills theory coupled to a massive adjoint Majorana fermion, which is known to be supersymmetric at a particular value of the fermion mass. After studying how singular-valued amplitudes and intermediate zero momentum modes are regularized in both schemes, we are able to establish a precise connection between conventional DLCQ and its supersymmetric extension, SDLCQ. In particular, we derive the explicit form of the (irrelevant) interaction that renders the DLCQ formulation of the theory exactly supersymmetric for any light-cone compactification. We check our analytical results via a numerical procedure, and discuss the relevance of this interaction when supersymmetry is explicitly broken.
1 Introduction

The properties of supersymmetric gauge theories are of particular interest nowadays because of intriguing connections that are believed to exist between large $N$ super Yang-Mills theories and string/M theory [1, 2, 3, 4]. A string/M theoretic interpretation of a class of super Yang-Mills theories at finite $N$ was also provided by Susskind [5], and it was in this context that DLCQ emerged as an interesting conceptual tool in the non-perturbative formulation of M theory.

Remarkably, DLCQ turns out to be very useful in practical bound state calculations [8], and this fact has been readily exploited in the context of many studies of two dimensional field theories (see [9] for a review). In more recent times, a DLCQ prescription preserving exact supersymmetry for any discretization of the total light-cone momentum (originally proposed in [10]) has been employed in a study of a large class of two dimensional supersymmetric matrix models ([11, 12, 13, 14, 15]).

Despite these developments, a detailed knowledge of DLCQ in the context of supersymmetric theories is still lacking, although steps in that direction have been taken [6, 7]. In the present paper, we delve into this issue further by investigating the relation between conventional DLCQ, and the supersymmetric form of DLCQ (hereafter ‘SDLCQ’) proposed in [10]. For this purpose, we employ both DLCQ prescriptions in a study of an SU($N$) gauge theory coupled to a single Majorana adjoint fermion, which is known to be supersymmetric for a particular value of the fermion mass $m_1$ [19, 20].

The content of this paper may be summarized as follows; in Section 2, we establish a precise connection between conventional DLCQ and SDLCQ for an SU($N$) gauge theory coupled to an adjoint Majorana fermion. It turns out that the relationship between the two prescriptions hinges on the regularization of a singular amplitude governing a two-body $\to$ two-body interaction with zero momentum exchange. The difference in the two prescriptions may be encoded as an operator, and we present its explicit form. In Section 3, we test our analytical results with a numerical analysis, and study the behavior of this operator at the supersymmetric point, and when supersymmetry is broken explicitly. Different boundary conditions for the fermions are also considered. We conclude in Section 4 with a summary of our results, and speculate on the enhanced significance of DLCQ when supersymmetry is present.

A numerical study of this model using conventional DLCQ was initially carried out in [17]. A subsequent study of the model at the supersymmetric point can be found in [18].
2 Zero Modes and Supersymmetric Regularization.

We consider the 1 + 1 dimensional SU($N$) gauge theory coupled to an adjoint Majorana fermion. The light-cone quantization of this model in the light-cone gauge and large $N$ limit has been dealt with explicitly before [17, 18], and we choose to adopt the same notation here\textsuperscript{2}. The expressions for the light-cone momentum $P^+$ and light-cone Hamiltonian $P^-$ for this model are

\begin{align}
P^+ &= \int dx^- \text{tr}(i\psi \partial_- \psi), \\
P^- &= \int dx^- \text{tr}(\frac{-i}{2}m^2 \psi \frac{1}{\partial_-} \psi - \frac{g^2}{2} \frac{1}{\partial_-} J^+ - \frac{1}{\partial_-} J^+).
\end{align}

Here $J^+_{ij} = 2\psi_{ik} \psi_{kj}$ is the longitudinal current. It is well known that at a special value of fermionic mass (namely $m^2 = g^2N/\pi$) this system is supersymmetric [19]. This special value of the fermion mass will be denoted by $m_{\text{SUSY}}$. At this supersymmetric point, the supercharge is given by

\begin{equation}
Q^- = 2^{1/4} \int dx^- \text{tr}(2\psi \psi \frac{1}{\partial_-} \psi)
\end{equation}

which satisfies the supersymmetry relation $\{Q^-, Q^-\} = 2\sqrt{2}P^-$. This may be checked explicitly by using the anticommutator at equal $x^+$:

\begin{equation}
\{\psi_{ij}(x^-), \psi_{kl}(y^-)\} = \frac{1}{2} \delta_{il} \delta_{jk} \delta(x^- - y^-).
\end{equation}

In the DLCQ formulation, the theory is regularized by a light-like compactification, and either periodic or antiperiodic boundary conditions may be imposed for fermions. If $P^+$ denotes the total light-cone momentum, light-like compactification is equivalent to restricting the light-cone momentum of partons to be non-negative integer multiples of $P^+/K$, where $K$ is some positive integer that is sent to infinity in the decompactified limit\textsuperscript{3}. Anti-periodic boundary conditions will in general explicitly break the supersymmetry in the discretized theory, although supersymmetry will be restored in the decompactification limit $K \to \infty$ [18]. If we wish to maintain supersymmetry at any finite $K$, we must at least impose periodic boundary conditions for the fermions. This, however, leads to the notorious "zero-mode problem"\textsuperscript{4}. From a numerical perspective, omitting

\textsuperscript{2} For a treatment of this model at finite $N$, the reader is referred to [16].

\textsuperscript{3} $K$ is sometimes called the ‘harmonic resolution’, or just ‘resolution’.

\textsuperscript{4} For anti-periodic boundary conditions, the light-cone momentum of partons is restricted to odd integer multiples of $P^+/K$, and so there are no zero-momentum modes in such a formulation.
zero-momentum modes in our analysis is absolutely necessary, since it guarantees a finite Fock basis for each finite resolution $K$. The mass spectrum of the continuum theory may then be extracted after an appropriate extrapolation of masses obtained from diagonalizing a sequence of finite mass matrices for $M^2 = 2P^+P^-$. But are we really justified in omitting the zero-momentum modes? To date, the general consensus is that omitting zero momentum modes in a two dimensional interacting field theory does not affect the spectrum of the decompactified theory, where $K \to \infty$. Actually, the numerical results of Section 3 are consistent with this viewpoint.

However, the goal of this work is to understand the structure of a supersymmetric theory at finite resolution. As we will see shortly, understanding why the DLCQ and SDLCQ prescriptions differ involves studying certain intermediate zero-momentum processes. But first, we need to be more precise about the form of the light-cone operators of the theory. If we expand the fermion field $\psi_{ij}$ in terms of its Fourier components, we may express the uncompactified light-cone supercharge and Hamiltonian in a momentum space representation involving fermion creation and annihilation operators: ([19, 17, 18]):

\[
Q^- = i2^{-1/4} \frac{g}{\sqrt{\pi}} \int_0^\infty dk_1dk_2dk_3 \delta(k_1 + k_2 - k_3) \left( \frac{1}{k_1} + \frac{1}{k_2} - \frac{1}{k_3} \right) \times \\
\times \left( b^\dagger_{ik}(k_1)b^\dagger_{kj}(k_2)b_{ij}(k_3) + b^\dagger_{ij}(k_3)b_{ik}(k_1)b_{kj}(k_2) \right),
\]

\[
P^- = \frac{m^2}{2} \int_0^\infty dk b^\dagger_{ij}(k)b_{ij}(k) + \frac{g^2N}{\pi} \int_0^\infty dk \int_0^k \frac{k^2}{(p-k)^2} b^\dagger_{ij}(k)b_{ij}(k) + \\
+ \frac{g^2}{2\pi} \int_0^\infty dk_1dk_2dk_3dk_4 \left[ \delta(k_1 + k_2 - k_3 - k_4) A(k) b^\dagger_{kj}(k_3)b^\dagger_{ji}(k_4)b_{ki}(k_1)b_{li}(k_2) + \\
+ \delta(k_1 + k_2 + k_3 - k_4) B(k) (b^\dagger_{kj}(k_3)b_{ki}(k_1)b_{li}(k_2)b_{ji}(k_4) - b^\dagger_{kj}(k_3)b^\dagger_{ji}(k_4)b_{li}(k_3)b_{ki}(k_1)) \right]
\]

with

\[
A(k) = \frac{1}{(k_4 - k_2)^2} - \frac{1}{(k_1 + k_2)^2}, \\
B(k) = \frac{1}{(k_3 + k_2)^2} - \frac{1}{(k_1 + k_2)^2}.
\]

As we mentioned earlier, the continuum theory is supersymmetric for a special value of fermion mass. We will therefore consider only the case $m = m_{\text{SUSY}}$. In the DLCQ

\[\text{In the continuum theory, the (arbitrarily small) region surrounding the zero mode } k^+ = 0 \text{ becomes important, rather than the single mode } k^+ = 0, \text{ which has measure zero.}\]
formulation, one simply restricts integration of the light-cone momenta \( k_i \) in expression (1.4) for \( P^- \) above to be positive integer multiples of \( P^+/K \). i.e. one simply drops the zero-momentum mode. The DLCQ mass spectrum is then obtained by diagonalizing the mass operator \( M^2 = 2P^+P^- \). Similarly, in the SDLCQ formulation, the light-cone momenta \( k_i \) in expression (1.5) for \( Q^- \) are restricted to positive integer multiples of \( P^+/K \). One then simply defines \( P^- \) to be the square of the supercharge: 
\[
2\sqrt{2}P^- = \{Q^-,Q^-\}.
\]
The mass operator \( M^2 = 2P^+P^- \) is then easily constructed and diagonalized to obtain the SDLCQ spectrum.

In general, the following observations are made; at finite resolution, the DLCQ spectrum of a supersymmetric theory is not supersymmetric. However, supersymmetry is restored after extrapolating to the continuum limit \( K \to \infty \) (see [18], for example). In contrast, for any finite resolution, the SDLCQ spectrum is supersymmetric. The DLCQ and SDLCQ spectra agree only in the decompactified limit \( K \to \infty \).

Not surprisingly, the difference in the DLCQ and SDLCQ prescriptions at finite resolution may be understood as a zero-mode contribution. What is surprising is that we can encode the effect of these zero-mode contributions into a simple well defined operator. The main result of this paper will be to state the precise operator form of this contribution at finite \( K \).

In order to motivate our argument, note that the anticommutator for the supercharge \( Q^- \) in the continuum theory involves products of terms of the form \( b\dag(k)b\dag(0)b(k) \) and \( b\dag(p)b(0)b(p) \), and these provide contributions to \( P^- \) that may be expressed in terms of non-zero momentum modes. The problem is exacerbated by the fact that the coefficients of these terms behave singularly. To examine this more closely, we consider the discretized theory where the light-cone momenta \( k_i \) in the expression for \( Q^- \) [eqn(1.5)] are restricted to positive integer multiples of \( P^+/K \). We also include the effects of zero-momentum modes by introducing an ‘\( \epsilon \) regulated zero mode’, which are modes with momentum \( k_i = \epsilon \), where \( \epsilon \) is much less than \( P^+/K \), and is sent to zero at the end of the calculation.

Then the anticommutator of \( Q^- \) gives contributions of the following form,
\[
\left\{ \left( \frac{1}{\epsilon} + \frac{\epsilon}{k(k+\epsilon)} \right)b\dag(k)b\dag(\epsilon)b(k+\epsilon), \left( \frac{1}{\epsilon} + \frac{\epsilon}{p(p+\epsilon)} \right)b\dag(p+\epsilon)b(\epsilon)b(p) \right\} = \left( \frac{1}{\epsilon^2} + \frac{1}{p(p+\epsilon)} + \frac{1}{k(k+\epsilon)} + \frac{\epsilon^2}{pk(p+\epsilon)(k+\epsilon)} \right), \tag{1.8}
\]
where any terms involving an \( \epsilon \) regularized zero mode on the right-hand-side are dropped.\footnote{In the DLCQ formulation of \( P^- \), we omit zero modes.}

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We have suppressed all matrix indices in this expression. In the limit $\epsilon \to 0$ the last term on the right-hand-side in the brackets vanishes, while the first term is the pure momentum–independent divergence that was identified in an earlier study of this model [18], and is canceled if we adopt a principal value prescription for singular amplitudes in the definition of $P^-$. The second term however, is clearly a finite contribution to $P^-$, although it arises from the $\epsilon$ regulated zero modes in $Q^-$, which are not present in the SDLCQ prescription for defining $Q^-$. Consequently, in order to ensure the supersymmetry relation $\{Q^-, Q^-\} = 2\sqrt{2}P^-$ in the discretized formulation, we must include an $\epsilon$ regularization of the zero modes in the definition for $Q^-$, and then apply a principal value prescription in the presence of any singular processes to eliminate $1/\epsilon$ divergences.

Stated slightly differently, we may decompose the supercharge into a part without zero modes $Q^-_{SDLCQ}$ (i.e. $k_i = nP^+/K, n = 1, 2, \ldots$), and terms with $\epsilon$ regularized zero modes, $Q^-$. The anti-commutator $\{Q_{SDLCQ}, Q^-\}$ contains only terms with $\epsilon$ regulated zero-modes. Since $Q^- = Q^-_{SDLCQ} + Q^-_{\epsilon}$ one finds

$$\{Q_{SDLCQ}, Q^-_{SDLCQ}\} = 2\sqrt{2}P^-_{SDLCQ} = 2\sqrt{2}P^-_{DLCQ} - \{Q^-_{\epsilon}, Q^-_{\epsilon}\}_{PV}, \quad (1.9)$$

after dropping any $\epsilon$ regulated zero-mode terms in the calculated expression for $\{Q^-, Q^-\}$. Note that the first equality above is just the definition for the light-cone Hamiltonian $P^-$ in the SDLCQ prescription. The $PV$ abbreviation on the right hand side indicates a principal value regularization prescription, which is tantamount to dropping all $1/\epsilon$ terms as $\epsilon \to 0$. The procedure is well known in the context of the present model [18]. It is clear that our definition for $P^-_{SDLCQ}$ gives rise to the supersymmetry relation $[Q^-_{SDLCQ}, P^-_{SDLCQ}] = 0$, which yields a supersymmetric spectrum for any finite resolution $K$. Moreover, we know that $P^-_{SDLCQ}$ and $P^-_{DLCQ}$ yield the same spectrum\(^7\) in the continuum limit $K \to \infty$, so it remains to calculate the difference at finite resolution $K$. We will write this difference in terms of their respective mass operators: $M^2 = 2P^+P^-$. A straightforward calculation of the anticommutator on the right-hand-side of (1.9) leads to the result:

$$M^2_{SDLCQ} - M^2_{DLCQ} = M^2_\Delta = -\frac{g^2NK}{\pi} \sum_n \frac{1}{n^2} B^\dagger_{ij}(n)B_{ij}(n) - \frac{g^2NK}{\pi} \sum_{mn} \left( \frac{1}{m^2} + \frac{1}{n^2} \right) \frac{1}{N} B^\dagger_{kj}(m)B^\dagger_{ji}(n)B_{kl}(m)B_{ti}(n). \quad (1.10)$$

\(^7\)This fact is expected from physical grounds, but one can also check this numerically.
We also write down the expression for $M_{DLCQ}^2$ in the theory with periodic fermions:

$$M_{DLCQ}^2 = \frac{g^2 N K}{\pi} \sum_n B_{ij}^\dagger(n) B_{ij}(n) \left( \frac{x}{n} + \sum_{m=1}^{n-1} \frac{2}{(n-m)^2} \right) +$$

$$+ \frac{g^2 K}{\pi} \sum_{n_i} \left\{ \delta_{n_1+n_2,n_3+n_4} \left[ \frac{1}{(n_2-n_4)^2} - \frac{1}{(n_1+n_2)^2} \right] B_{kj}^\dagger(n_3) B_{ji}^\dagger(n_4) B_{kl}(n_1) B_{li}(n_2) \right. +$$

$$\left. + \frac{1}{(n_2+n_3)^2} - \frac{1}{(n_1+n_2)^2} \right\} \right\}. \quad(1.11)$$

In this expression the variable $x = \frac{m x^2}{g N}$ is a dimensionless mass parameter, and for the supersymmetric point we have $x = 1$. The sums are performed over positive integers, $0 < n_i < K$, and we employ a principal value prescription in sums labeled as $\sum'$, which implies that terms of the form $1/(k-k)^2$ are dropped. In the SDLCQ procedure we calculate $Q^-$ which is non-singular and requires no principal value prescription.

The term $M_{\Delta}^2$ appears to be non-trivial due to the presence of $B^\dagger B^\dagger B B$ terms on the right hand side of (1.10). However, the action of this term on any SU($N$) Fock state turns out to be equivalent to the first term, although with opposite sign, and twice the magnitude. Thus the action of the right hand side of (1.10) is equivalent to the single quadratic operator:

$$M_{\Delta}^2 = \frac{g^2 N K}{\pi} \sum_n \frac{1}{n^2} B_{ij}^\dagger(n) B_{ij}(n). \quad(1.12)$$

Fortunately, we are able to test this analytical result by performing direct numerical simulations of this model using both prescriptions, and comparing the differences observed with the above prediction. Interestingly, although this result was derived for large $N$, agreement turns out to be perfect for both finite and large $N$, which was verified using the finite $N$ DLCQ algorithms developed in [16]. We discuss this further in the next section.

3 Numerical Results

First, let us review the numerical results for this model which were discussed in [18]. The authors imposed anti-periodic boundary conditions for the fermion (which guarantees the absence of a zero-momentum mode) and showed that at the supersymmetric point, the

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8 Results for finite $N$ can be found in ref [16].
extrapolated \((K = \infty)\) mass squared \(M^2\) for the lightest fermion and boson bound states are equal, and approximately 25.9 in units \(g^2N/\pi\). The convergence was relatively slow, and the mass squared obtained at the highest resolution \((K = 25)\) was still 15\% from the final extrapolated value. It was also stated that periodic boundary conditions reproduce the same result, although convergence is much slower. Our numerical calculations are consistent with these observations.

We then repeated the calculation in SDLCQ. We find of course exact fermion-boson mass degeneracies at every resolution, and by calculating masses up to resolution \(K = 10\), we find an extrapolated mass squared \(M^2 = 26.4\), in units \(g^2N/\pi\). The mass squared calculated at the highest resolution \((K = 10)\) was only 7\% from the extrapolated value. Moreover, the convergence of \(M^2\) appeared to be very close to a linear function of \(1/K\), and so the extrapolated value can be determined to high precision.

We then repeated the calculation using the usual DLCQ prescription for \(P^-\), but with the additional term \((1.12)\) that was calculated in the previous section. As predicted, this ‘modified DLCQ’, and the SDLCQ results just mentioned are in perfect agreement.

Since the SDLCQ prescription (or DLCQ with an additional term \((1.12)\)) yields the same spectrum in the decompactified limit \(K \to \infty\) as the standard DLCQ prescription for \(P^-\) (i.e. DLCQ without additional term), we conclude that the operator \((1.12)\) is irrelevant in the continuum limit \(K \to \infty\), since it gives a zero contribution. Its only role is to guarantee supersymmetry in the compactified theory.

Interestingly, this situation changes when we choose to break the supersymmetry explicitly by choosing a fermion mass \(m \neq m_{SUSY}\) (or adding a mass term to the definition of \(P^-\) in the SDLCQ prescription). Namely, for a non-supersymmetric value of the fermion mass, the DLCQ spectrum with and without the additional term \((1.12)\) appear to differ even in the decompactified limit \(K \to \infty\). Interestingly, small perturbations in fermion mass away from the supersymmetric point \(m = m_{SUSY}\) were considered by Boorstein and Kutasov \([20]\), and an explicit formula was derived that measured the splittings in the fermion and boson masses in terms of the supersymmetry-breaking mass. This result may be checked explicitly in both the DLCQ and SDLCQ prescriptions. Surprisingly, the mass splittings observed in both prescriptions agree extremely well with the Boorstein-Kutasov analysis, although the absolute values for the masses in both schemes differ away from the supersymmetric point.

It is tempting to suggest that the discrepancy in continuum masses away from the supersymmetric point reflects extremely slow convergence, and would disappear if we
were able to probe larger enough values for $K$. Our numerical analyses so far cannot confirm such a view at present. A more dramatic interpretation is to propose that there is a possible phase transition at the supersymmetric point $[21]$, and we might be comparing two different phases in this theory.

4 Discussion

We have considered the connection between supersymmetric discrete light-cone quantization (SDLCQ) and the standard prescription for DLCQ in numerical calculations. SDLCQ preserves supersymmetry even in the discretized theory, and requires no explicit renormalization of interactions. In DLCQ, one works with the light-cone Hamiltonian $P^+$, which involves amplitudes that must be regulated (typically by the principal value prescription) in addition to the usual regularization imposed by neglecting $k^+ = 0$ modes. In contrast to the SDLCQ scheme, DLCQ does not preserve supersymmetry at finite resolutions, although supersymmetry is restored in the decompactified limit $K \to \infty$.

In a simple example of a two dimensional SU($N$) gauge theory coupled to an adjoint Majorana fermion, we determined an operator that connects DLCQ (regularized using principal value) to SDLCQ. We were able to check our results numerically, and obtained exact agreement for finite and large $N$. The derivation of this operator involved a careful treatment of the zero modes that are normally omitted in SDLCQ, and the principal value regularization prescription in DLCQ. In particular, we demonstrated that certain zero mode contributions may be succinctly encoded as an operator involving non-zero momentum modes.

Our numerical results indicated that the operator (1.12) which restores supersymmetry when added to the usual DLCQ Hamiltonian is irrelevant in the decompactified limit, since both DLCQ and SDLCQ schemes agree in this limit. However, for a non-supersymmetric choice of fermion mass, this operator appears to become relevant even in the continuum limit. In particular, the extrapolated masses at $K = \infty$ using both schemes differ, although the mass splittings between bosons and fermions in each scheme were in agreement with the analysis of ref[21].

One suggestion is that the model exhibits very bad convergence behavior, although a detailed analysis conducted so far reveals very little evidence for this. It is tempting to speculate that the zero-mode degrees of freedom that are irrelevant at the supersym-
metric point become relevant when supersymmetry is broken. Clearly, this transition reflects a dynamical property of the theory, since one can only check the relevance or irrelevance of an operator by explicitly solving for the bound states in the theory for different fermion mass, as we have done here. It would be interesting to connect this observation with a proposal that the theory undergoes a phase transition at the supersymmetric point. Whether we are observing different phases in a theory would have very interesting ramifications in understanding the nature of supersymmetry – and perhaps supersymmetry breaking – in a non-perturbative setting.

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