Bending of light: A classical analysis

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In this work we applied Soldner’s classical approach for bending of light in the context of Newtonian corpuscular theory. We show that there is a good evidences to exist a massive photon in new scenario of gravity due to Verlinde[17].

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I. INTRODUCTION

Johann Georg von Soldner is now mostly remembered for having concluded due to the Newton’s corpuscle theory of the light that light would be diverted by heavenly bodies[1]. Soldner’s work on the effect of gravity on light became considered less relevant during the Nineteenth Century, as “corpuscular” theories and calculations based on them were increasingly considered to have been discredited in favor of wave theories of light. Other present work that became unpopular and largely forgotten for similar reasons included Henry Cavendish’s light-bending calculations[2], John Michell’s 1783 study of gravitational horizons and the spectral shifting of light by gravity[3], and even Isaac Newton’s study in “Principia” of the gravitational bending of the paths of “corpuscles”, and his description of light bending in “Opticks”[4]. Einstein speculated in 1911[5] whether the relation $m = E/c^2$ for the inert mass of radiation energy may be inserted in the gravitational field to describe the deflection of light rays from remote stars by the sun. In 1901 the German astronomer J. Soldner had made a similar calculation in which he described the light as a Newtonian particle with the velocity $c$. Calculate the deflection angle of a photon grazing the border of the sun with the assumption that the photon passes the sun with the velocity $c$ on a straight line. Let the component of the gravitational force perpendicular to the path of flight $F \cos(\theta)$ integrated over the entire flight orbit provide the transverse momentum component. Albert Einstein calculated and published a numerically similar value for the amount of gravitational light-bending in light skimming the Sun in 1911, leading Philipp Lenard to accuse Einstein of plagiarizing Soldner’s result.

Einstein included the cosmological constant as a term in his field equations for general relativity because he was dissatisfied that otherwise his equations did not allow, apparently, for a static universe: gravity would cause a universe which was initially at dynamic equilibrium to contract. To counteract this possibility, Einstein added the cosmological constant. Ironically, the cosmological constant is still of interest, as observations made in the late 1990s of distance-redshift relations indicate that the expansion of the universe is accelerating. When combined with measurements of the cosmic microwave background radiation these implied a value of a result which has been supported and refined by more recent measurements. There are other possible causes of an accelerating universe, such as quintessence, but the cosmological constant is in most respects the most economical solution. Thus, the current standard model of cosmology, the Lambda-CDM model, includes the cosmological constant, which is measured to be on the order of $10^{-57} \text{m}^{-2}$, or $10^{-47} \text{GeV}^4$, or $10^{-29} \text{g/cm}^3$, or about $10^{-120}$ in reduced Planck units Setting $\Lambda$ to this small value ensures that the acceleration of the universe is a fairly recent phenomenon giving rise to the cosmic coincidence conundrum according to which we live during a special epoch when the density in matter and $\Lambda$ are almost equal. Latter Wolfgang Rindler and et al.[6, 7] show that when the Schwarzschild de Sitter geometry is taken into account, $\Lambda$ does indeed contribute to the bending. Other describe this effect from different points of view[8-12].

In this work following the most old corpuscle assumption about light and by supposing that the interaction between massive photons and Dark energy is classical we derived a classical value for $\Lambda$ to light bending correction in the context of Newtonian gravity. Therefor we mentioned here that although the massive photon gauge fields (which is frequently used in theory) has not detected until now, but these hypothesis particles can be construct a lensing effect for cosmological constant.

II. ABOUT THE EXISTENCE OF A HYPOTHETICAL MASSIVE PHOTON

The addition of mass to the photon would introduce a frequency-dependent dispersion relationship, and also modify Maxwells equations in an ultimately testable fashion. Experimental considerations have constrained $m_\gamma < 10^{-49} - 10^{-54} \text{ kg}[13]$. A massive graviton would possess a similar dispersion relation[14], which would manifest itself as signal arrival-time delays (or even inversions) in gravity wave detectors[15]. Such a property could also be used to model long-range deviations from general relativity, and hence provide an explanation for galaxy rotation curves and late-time cosmolog-

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ical acceleration. LIGO/VIRGO and LISA-scale gravitational wave searches from e.g. massive compact binary coalescence could potentially constrain the gravi-

tons Compton wavelength to be $\lambda_g = 10^{12} - 10^{16}$ km, respectively, yielding an upper mass limit of about $m_g = 10^{-58} - 10^{-62}$ kg [14]. Other theoretical and model-
dependent considerations provide similar estimates in the range $m_g 10^{-55} - 10^{-69}$ kg Recently as was shown by Mureika and Mann[16] there is a bounding range repre-
sents the smallest non-zero mass for any particle quanta in the entropic gravity framework due to Verlinde en-
tropic scenario for gravity[17] and thermodynamic description of the Einstein General relativity of Padmanab-
han[18].As was stated by Mann if the information bits transfer between test particle near the screen and holo-
graphic screen (the equipotential surfaces for example Black hole horizon as was postulated by Susskind[19]) in entropic gravity respects both the uncertainty principle and causality.and in more details when we insert the GUP effects[20] and also the backreaction portion[21] to the usual Unruh[22]-Bekenstein[23] Hawking[24] temperature, a lower limit on the number of quantum bits in and mass of the universe may be derived. Furthermore, these limits indicate particles that travel at the speed of light the photon and/or graviton have a non-zero mass $m \geq 10^{-68}$kg.

## III. SOLDNER’S MODEL

In this section we present original form of classical deflec-
tion of light near a heavily body first calculated by Von Soldner in (1901). We must take some notifications about main physics which is hidden behind this simple calculations. First assumption is that a massive pho-
ton is exist which according to Newtonian theory can be attrac-
ted by any other massive objects by a simple in-
verse square force. This assumption means that we treat photon as a classical object that interact with mass clas-
ically. That is there is no quantization in this model. Another assumption is that the gravitational field needs no correction from photon’s mass. In next section we generalize Soldner method to the cosmological constant case and derive a smaller order result as Rindler et al derived.

### A. deflection of light in the constant gravitational field

The deflection causes that an observer supposes the pos-
tion of the star to be along the extension of the straight line. Thus the direction of the star seems to be displaced . We assume that photon grazing the border of the sun with the assumption that the photon passes the sun with the velocity $c$ on a straight line. Let the component of the gravitational force perpendicular to the path of flight $F \cos(\theta)$ integrated over the entire flight orbit provide the transverse momentum component. The transverse momentum component $\Delta p = \int F \cos(\theta) dt$ is calculated between the limits $\pm \infty$ whereby the origin of the path $x, 0$ is put into the point of contact. The momentum is $p = E/c$ and $dt = dx/c$. Explicitly we have:

$$\Delta p = \frac{2GMm}{Rc}$$

We thus obtain for the deflection angle $\alpha \approx \tan(\alpha) = \Delta p/p$.

$$\alpha = \frac{2GM}{Rc^2}$$

Insertion of the numerical values $M = 1.99 \times 10^{30} kg$ , $R = 6.96 \times 10^{8} m$ , $G = 6.67 \times 10^{-11} m^3/(kgs^2)$ , $c = 2.998 \times 10^8 m/s$ yields a deflection angle of $\alpha = 0.875''$ , a result that at first is believed as quantitatively correct in special conditions. Surprisingly, in the general theory of relativity the calculation of the deflection of a light ray in the Schwarzschild field yields the same value except for a factor of 2, thus $\alpha = 4GM/Rc^2 = 1.75''$. Experimental investigations between 1919 and 1954 yielded values between 1.5'' and about 3'' (Finlay-Freundlich, 1955; von Kluber, 1960). These measurements on the average seem to yield 2.2'', which would be too large by 25 percent. In 1952 van Biesbroeck found in a precision experiment the value 1.7'' $\pm 0.1''$. More measurements from 1970 (Hill, 1971; Sramek, 1971) at Mullard Radio Astronomy Observatory of Cambridge University and at the National Radio Observatory (USA) essentially confirm the value obtained by van Biesbroeck, which agrees well with the theoretical prediction. The most accurate mea-
surements of the deflection of radio waves grazing the sun using state-of-the-art long-baseline interferometry yield a confirmation of the general relativistic prediction for the deflection of 0.9998 $\pm 0.0008$. It should be noted that the most sophisticated optical observations during solar eclipses can give no better confirmation than 0.95 $\pm 0.11$ of the Einstein prediction.

### B. Cosmological constant case

Now we add cosmological constant term to previous formalism that is responsible for acceleration. We know that the gravitational potential of a mass $M$ in the presence of a cosmological constant $\Lambda$ according to the New-
tonian gravity is ( in units $G = c = 1$):

$$\phi(R) = -\frac{M}{R} + \frac{1}{6}\Lambda R^2$$

The force exerted to a photon with hypothesis mass $m$ is:

$$\vec{F} = -m\nabla \phi(R)$$

Now we suppose that photon interact with cosmological constant with the same mass $M$ , i.e; we assume that
vacuum energy can interacts with massive photon commonly. So this force is:

\[ |\vec{F}| = -\frac{mM}{R^2} + \frac{1}{3}m\Lambda R \]

The transverse momentum component \( \Delta p = \int |\vec{F}|\cos(\theta)dt \) can be calculated to obtain:

\[ \Delta p_{\Lambda \neq 0} = mb( -\frac{M}{R^3} + \frac{1}{3}\Lambda \int dx) \]

In which

\[ R = \sqrt{x^2 + b^2} \]

Now we have a problem to choose integration limits. Classically if we assume that \( x \) varies in the range \(-\infty \leq x \leq \infty \) then the first integral leads to same result for case \( \Lambda = 0 \) that is Soldner’s deflection. But the second integral diverges. We use regularization scheme that is used frequently in QFT. We calculated the integral diverges. We use regularization scheme that is Soldner deflection . But the second cutoff for \( r \) i.e; \( 0 \leq \infty \) leads to the same result for \( \int dx \) and expanding in series form.

\[ C \int dx = Lim_{\alpha \to 0}(\int_0^\infty e^{-\alpha x}dx - \sum_{n=0}^\infty e^{-\alpha n}) = -\frac{1}{2} \]

Where \( C \) is denoted for Casimir value of this integral which is defined as:

\[ C \int f(x)dx = Lim_{\alpha \to 0}(\int_0^\infty f(x)e^{-\alpha x}dx - \sum_{n=0}^\infty f(n)e^{-\alpha n}) \]

Substituting this value in (4) we finally similar to (2) we have:

\[ \alpha_{\Lambda \neq 0} = \alpha_{\Lambda = 0} - \frac{1}{3}b^2\Lambda \]

Or equivalently in terms of de-Sitter radius we have:

\[ \alpha_{\Lambda \neq 0} = \alpha_{\Lambda = 0} - \left(\frac{b}{a}\right)^2 \]

Another method is using the De-Sitter radius as a UV cutoff for \( r \) i.e; \( 0 \leq r \leq a \) where \( a = \sqrt{\frac{2}{\Lambda}} \). This limits \( x \) to the \(-\sqrt{a^2 - b^2} \leq x \leq \sqrt{a^2 - b^2} \). If we substitute this boundaries in relation for \( \alpha \) and expanding in series form in terms of cosmological constant we obtain:

\[ 2\frac{M}{b} - \frac{2}{3}b\sqrt{3}\Lambda - \frac{1}{3}bMA + \frac{1}{9}b^3\sqrt{3}\Lambda^{3/2} + O(\Lambda^2) \]

except to the first term which correspond to Soldner deflection the next terms are cosmological constant correction. So in this case :

\[ \alpha_{\Lambda \neq 0} = \alpha_{\Lambda = 0} - \frac{2}{3}b\sqrt{3}\Lambda - O(\Lambda) \]

This result can be written in terms of de-Sitter radius as:

\[ \alpha_{\Lambda \neq 0} = \alpha_{\Lambda = 0} - \frac{2b}{3a} \]

Comparing (6) and (7) it seems that the cutoff or regularization schemas is inserted a smaller correction by a factor of 2 in comparing with different integration interval.

IV. PHYSICAL MEANING OF \( \Lambda \) REGULARIZED CORRECTION

In this section we want to describe the meaning of regularization which is done in (4). Obviously we must determine the deviation of light ray with respect to the straight line. But we know that according to GR there is no straight line in a spherically symmetric spacetime. Thus if we want to correct our classical results (6) or (7) there is ambiguity in definition of deviation. Mathematically we must determine an iterative solution for orbit equation in the presence of a cosmological constant term. It seems that this solution is begins with a straight line solution which belongs to only a flat space time. Now we take another suitable definition of deviation (Bending) and next relate it to the regularization schemas which is used in (4). In the absence of \( \Lambda \) term which is responsible for repulsive force we know that only mass can deviated light. We take this angle by a superscript \( \Lambda = 0 \) in (6). The correction which is added to it and has negative sign is in fact the subtraction of the cosmological case (i.e; a mass which we assume is located on origin and also a homogenous constant energy vacuum is surrendered it) from a single mass. The difference between energy of these two different configurations is completely physical and is hidden behind the regularization schemas (4). In this case the meaning of a Casimir’s like regularization is very bright. In fact Casimir regularization means that we subtracted cosmological constant plus mass portion from a unique mass or in the language of GR we eliminated the contribution of a mass and cosmological constant from pure mass.

V. SUMMARY

In this work first we review Soldner’s classical method for calculation light bending. Then we generalized it to a cosmological constant term to obtain a new correction that is different in one order from other authors results.

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