Orientifolds of Warped Throats from Toric Calabi-Yau Singularities

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1605.01732 by A.R. & A. Uranga
Introduction

Consider Type IIB String Theory (ST) on a Calabi-Yau (CY) threefold

\[ ds^2 = e^{2A(r)} \eta_{\mu\nu} \, dx^\mu \, dx^\nu + e^{-2A(r)} \left( dr^2 + r^2 d\Omega^2_{X_5} \right) \]

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Warped throats are local (non-compact), cone-like and warped geometries.

\[ ds^2 = e^{2A(r)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{-2A(r)} \left( dr^2 + r^2 d\Omega_{X_5}^2 \right) \]

- Interesting because warping can be used to create hierarchies

Dasgupta, Rajesh, Sethi ; H. Verlinde ; Giddings, Kachru, Polchinski

- Nice feature: we can study geometry using probe D3-branes.

**Holography**

- Orientifolds of throats recently used for several applications: nilpotent goldstino in String Theory

Garcia-Etxebarria, Kallosh, Quevedo, Uranga, Valandro, Wrase... , de Sitter with Dynamical SUSY Breaking

A.R. , Uranga
Introduction

How to achieve this warping in 10d supergravity solutions? Let us use the prototypical example: the Klebanov, Strassler throat.

- It is based on the deformed conifold:

\[ xy - uv = L \quad x, y, u, v \in \mathbb{C} \]

This is a cone over \( X_5 \sim S^2 \times S^3 \).
- For \( L = 0 \) it is singular at \( x = y = u = v = 0 \). When \( L \neq 0 \), the \( S^3 \) of the base remains finite at the tip of the cone.
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- In order to keep the \( S^3 \) at finite size use fluxes (ISD fluxes for SUSY)

\[ \int_{S^3} F_3 \sim M \quad ; \quad \int_{S^2 \times \mathbb{R}^+} H_3 \sim K(r) \quad ; \quad N(r) \sim \int_{X_5} F_5 \sim M^2 \log(r/\varepsilon) \]

\[ e^{-4A(r)} \sim \frac{\alpha'^2 g_s^2 N(r)}{r^4} \quad L \sim \exp \left(-\frac{2\pi K}{M g_s} \right) \]

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- Warping comes from fluxes on \( S^3 \).

**General:** Conical manifold with finite size 3-cycle (from fluxes) \( \Rightarrow \) Warping
Introduction

The question we addressed

How to construct an orientifold of a warped throat?
or
Which warped throats accept orientifold involutions?
or
Which conical singularities accept both orientifold involutions and complex deformations (growing a finite size 3-cycle)?
Toric Calabi-Yau Singularities

We will focus on this particular type of conical singularities. They are defined as hypersurfaces of a toric manifold via a GLSM.

The conifold example see e.g. Aganagic, Vafa

\[
\begin{array}{c|cccc}
U(1)_s & x_1 & x_2 & x_3 & x_4 \\
\hline
1 & 1 & -1 & -1
\end{array}
\]

\[|x_1|^2 + |x_2|^2 - |x_3|^2 - |x_4|^2 = s\]

Gauge invariant observables are

\[x = x_1 x_3, \quad y = x_2 x_4, \quad u = x_1 x_4, \quad v = x_2 x_3; \quad xy - uv = 0\]
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Another way of seeing it. The conifold as a \(\mathbb{C}^* \times \mathbb{C}^*\) fibration over \(\mathbb{C}\):

\[xy = z; \quad uv = z; \quad (x, y, u, v) \rightarrow \left(x e^{i\theta_a}, ye^{-i\theta_a}, ue^{i\theta_b}, ve^{-i\theta_b}\right)\]
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Fixed points under these $U(1)$'s

\begin{align*}
(x_1, x_2, x_3, x_4) \\
\downarrow \\
(x_1 e^{i(\theta_a+\theta_b)}, x_2 e^{-i\theta_b}, x_3 e^{i\theta_b}, x_4 e^{-i\theta_a}) \\
\downarrow \\
(x_1 e^{i(\theta_a+\theta_b)/2}, x_2 e^{-i(\theta_a+\theta_b)/2}, x_3 e^{i(\theta_a-\theta_b)/2}, x_4 e^{-i(\theta_a-\theta_b)/2})
\end{align*}
For $s = 0$ we can perform the deformation

$$xy - uv = L \quad \Rightarrow \quad z = xy \quad ; \quad z = uv + L$$

and the fixed points under the U(1) actions $\theta_a$ & $\theta_b$ no longer touch.
Toric Calabi-Yau Singularities

For \( s = 0 \) we can perform the deformation

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This diagram describing the fixed points of the U(1)'s is the web diagram

Moreover, it describes the geometry & its possible complex deformations
Toric Calabi-Yau Singularities

We can use web diagrams to build more complicated warped throats
Franco, Hanany, Uranga
Toric Calabi-Yau Singularities

These geometries have a nice description in terms of dual gauge theory. We can understand it by performing T-duality (conifold example): Uranga

\[
xy - uv = 0 \quad ; \quad x \to xe^{i\theta}, \quad y \to ye^{-i\theta}
\]

Fiber degeneration $\Rightarrow$ NS5-brane \quad Bershadsky, Vafa, Sadov

\[
\begin{array}{|c|c|c|c|c|c|c|c|c|c|}
\hline
 & x_0 & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 \\
\hline
D4 & x & x & x & x & - & - & x & - & - & - \\
NS5_1 (x = 0) & x & x & x & x & x & x & - & - & - & - \\
NS5_2 (y = 0) & x & x & x & x & - & - & - & - & x & x \\
\hline
\end{array}
\]

We can read the gauge theory from this picture:

$\mathcal{N} = 1 \quad SU(N) \times SU(N)$ with $A_1, A_2 \text{ in } (\square, \overline{\square})$ & $B_1, B_2 \text{ in } (\overline{\square}, \square)$
Toric Calabi-Yau Singularities

Another T-duality along the U(1) parametrized by $\theta_b$ takes to this brane configuration see e.g. Garcia-Etxebarria, Heidenreich

- We get two extra NS5-branes: the configuration is brane-tiling

![Dimer Diagram](image)

- **NS5**
  - $\text{Inst}_2$ $\text{D}_5^1$ $\text{Inst}_2$
  - $\text{D}_5^2$ $\text{Inst}_1$ $\text{D}_5^2$
  - $\text{Inst}_2$ $\text{D}_5^1$ $\text{Inst}_2$

Dimer Diagrams: Bipartite graphs on $T^2$ describing the gauge theory [Feng, Franco, Hanany, He, Kenyon, Kennaway, Vegh, Wetch...].
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From here, we see a correspondence for toric CY singularities

Gauge Theory $\Longleftrightarrow$ Dimer diagram $\Longleftrightarrow$ Web diagram $\Longleftrightarrow$ Geometry

We can construct the dimer/web diagram if we know the other one using the "fast forward/inverse" algorithms Hanany, Vegh

External leg on web diagram with $(p,q)$ $\Longleftrightarrow$ NS5-brane on dimer with $(p,q)$

Complex deformations of geometry $=$ Removal of lines from web diagram $=$ Removal of NS5-branes from brane tiling $=$ Gauge group(s) confine
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The deformation on the dimer (more in detail)

A gauge group condensates when $N_c \geq N_f$ (square in dimer).

$n_1 = n_2 = n_4 = M$ , $n_3 = 2M$ ; $N_c = 2M$ , $N_f = 2M$

The superpotential is:

$$W_0 = X_{12}X_{21}X_{14}X_{41} + X_{34}X_{43}X_{32}X_{23} - X_{32}X_{21}X_{12}X_{23} - X_{34}X_{41}X_{14}X_{43}$$
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Confinement:

\[
\det \begin{bmatrix}
M_{22} & M_{24} \\
M_{42} & M_{44}
\end{bmatrix} = 
\begin{bmatrix}
X_{23}X_{32} & X_{23}X_{34} \\
X_{43}X_{32} & X_{43}X_{34}
\end{bmatrix} = \varepsilon^{4M}
\]

\[ W_{0+np} = X_{12}X_{21}X_{14}X_{41} + M_{42}M_{24} - M_{22}X_{21}X_{12} - M_{44}X_{41}X_{14} + \lambda (M_{22}M_{44} - M_{42}M_{24}) \]

\[ \Rightarrow W_{\text{eff}} = X_{12}X_{21}X_{14}X_{41} - X_{21}X_{12}X_{41}X_{14} \]
Toric Calabi-Yau Singularities

- Orientifolds of these geometries are interesting e.g. to get $\text{SO}(N)/\text{Sp}(N)$ gauge groups and matter in (anti)symmetric representations.
- Two possible effects on dimer: fixed line/points

Their effect on the gauge theory/dimer studied by Franco, Hanany, Krefl, Park, Uranga, Vegh
Summary of background

We can encode GLSM/geometry in terms of web diagram. Deformations of geometry = removal of subweb. Dimer diagram encodes dual gauge theory. Deformation in gauge theory = confinement. Dimer/web diagrams related via NS5-branes/external legs. Orientifolds leave fixed lines/points on dimer.
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Orientifolds of deformed toric CY singularities

**Question:** which CY singularities are compatible with orientifolds?
Orientifolds of deformed toric CY singularities

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We study first the orientifold line case \( \Omega = (-1)^{F_L} \sigma R \)

1) \( \rightarrow \) 2) because \( \sigma : B_2 \rightarrow -B_2 \), so NS5 \( \rightarrow '-' \text{NS5} \)

2) \( \rightarrow \) 3) because \( R : w \rightarrow \bar{w} \)

So, \( \Omega : (p,q) \rightarrow (-p,q) \)
Orientifolds of deformed toric CY singularities

Another possibility with orientifold lines

![Diagram showing toric CY singularity with orientifold lines](image)

Criterion for toric CY singularities accepting orientifold lines:
A toric CY singularity can have orientifold lines on its dimer if its web diagram has a $\mathbb{Z}_2$ symmetry that leaves a line invariant. Moreover, if the fixed line on the web diagram is horizontal/vertical, the orientifold line will invert one coordinate on the dimer; whereas for diagonal fixed lines on the web diagram, it will exchange its two coordinates.
Orientifolds of deformed toric CY singularities

**Question:** which singus are compatible with orientifold lines and deformations?
Orientifolds of deformed toric CY singularities

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- Warped throat = complex deformation = remove external legs
- Orientifold lines possible if web diagram has $\mathbb{Z}_2$ symm. with fixed line.
Orientifolds of deformed toric CY singularities

**Question:** which singus are compatible with orientifold lines and deformations?

- Warped throat = complex deformation = remove external legs
- Orientifold lines possible if web diagram has $\mathbb{Z}_2$ symm. with fixed line.
- **Combine:** The deformation is compatible with orientifold lines if the removed external legs have the same $\mathbb{Z}_2$ symm. as the web diagram.

![Diagram](image_url)
Orientifolds of deformed toric CY singularities

**Question:** which CY singularities are compatible with orientifold points?

Now the orientifold action on NS5-branes is $\Omega: (p,q) \rightarrow (p,q)$.

- 4 fixed points & $n_z$ NS5-branes mapped to themselves. $4 \geq n_z$
- Define $n_z^{\text{min}}$ as amount of odd subsets of NS5-branes/external legs
- For the example above $n_z^{\text{min}} = 2$. Can have $n_z = 2$ & $n_z = 4$
Orientifolds of deformed toric CY singularities

Criterion for toric CY singularities accepting orientifold points:
A toric CY singularity can have orientifold points if $n_{Z}^{\text{min}}$ is $n_{Z}^{\text{min}} \leq 4$.

Different possibilities exist for different $n_{Z}^{\text{min}}$ ($n_{Z}^{\text{min}}$ odd)

- $n_{Z}^{\text{min}} \neq 1$
- $n_{Z}^{\text{min}} = 3$ only accepts $n_{Z} = 3$
Orientifolds of deformed toric CY singularities

Different possibilities exist for different $n_{Z}^{\text{min}}$ ($n_{Z}^{\text{min}}$ even)

- $n_{Z}^{\text{min}} = 0$ accepts $n_{Z} = 0$ and $n_{Z} = 4$
- $n_{Z}^{\text{min}} = 2$ accepts $n_{Z} = 2$ and $n_{Z} = 4$ (we saw an example)
- $n_{Z}^{\text{min}} = 4$ only accepts $n_{Z} = 4$
- $n_{Z}^{\text{min}} \geq 4$ does not allow for orientifold points
Orientifolds of deformed toric CY singularities

**Question:** which singularities are compatible with orientifold points & deformations?

\[
\begin{align*}
\text{i)} & \quad \text{Deformation allows removing NS5-branes mapped to themselves.} \\
\text{ii)} & \quad \text{Only NS5-brane pairs can be removed on deformations.} \\
\text{iii)} & \quad \text{Orientifold points accept no deformation.}
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**Strategy:** use gauge dual. (Recall: confining groups = rectangles for \( N_c \geq N_f \)).

- In i) orientifold point ‘touches’ confining groups (field in (anti)symm.)
  - Doesn’t cause a deformation

- So case i) is not a possibility.
Orientifolds of deformed toric CY singularities

In case ii) orientifold points do not touch confining groups
Orientifolds of deformed toric CY singularities

In case ii) orientifold points do not touch confining groups

Criterion: Deformations of dimers with orientifold points require subtracting pairs of NS5-branes that are ampped to each other by the orientifold.
Summary

- Toric CY singularities are useful to easily build warped throats.
  - Web diagrams allow to easily “cook” interesting geometries.
  - Gauge dual description in terms of dimer diagrams is a useful tool.
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- The effect of orientifolds on the throat gets reflected on dimer.
- Orientifold lines:
  - Identify NS5-branes with “different” winding numbers.
  - Criterion: $\mathbb{Z}_2$ symmetry with fixed line on web-diagram. Deformation possible if compatible with $\mathbb{Z}_2$ symmetry.
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