An Incentive Compatible Mechanism for Market Coupling

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Abstract

The term market “coupling” refers to mechanisms used to price and allocate the available transmission capacity between interconnected electricity markets. In this paper, we propose a market coupling mechanism that is guaranteed to provide the necessary incentives for identifying the efficient allocation of interconnection capacity. Each individual area (or market) operator participates in this coupling mechanism by iteratively submitting bids for trading energy across interties and the prices for interconnection capacity is adjusted as a function of excess demand. The mechanism is scalable as the informational demands placed on each market operator at each iteration are limited. We show that the mechanism’s outcome converges to the optimal intertie flows when market operators are truthful. We devise incentive transfers based on the individual area contribution to total operational cost savings that guarantee truthful participation is an approximate Nash equilibrium, i.e., no individual market operator has an incentive to manipulate bids on energy import/export over interties if all other market operators are truthful. We identify a sufficient condition on a uniform participation fee ensuring the mechanism incurs no deficit. The proposed decentralized mechanism is implemented on the three-area IEEE Reliability Test System where the simulation results showcase the efficiency of proposed model.

1 Introduction

The interconnection of individual electricity markets by means of tielines provides increased efficiency by enabling access to cheaper generation and flexibility in the form of reserves. If the information on generation costs, demand and technical characteristics of all markets is available in a centralized location, a joint economic dispatch problem for the interconnected power grid as a whole could be solved in order to identify the efficient allocation of interconnection capacity. However, such economic and technical information cannot be aggregated in a central location due to legal or regulatory restrictions and/or privacy concerns. In practice, the control and operation of interconnected electricity markets (e.g. Western Europe, Northeast US) is conducted in a decentralized manner by independent agencies (e.g. system operators ISOs in the US, power exchanges PXs and transmission system operators TSOs in Europe). The coordinated operation of interconnected electricity markets (i.e. pricing and allocation of available interconnection transmission capacity) is generally referred to as market coupling.

The Price Coupling of Regions (PCR) project is perhaps the largest example of a market coupling mechanism. In such mechanism, on a day-ahead basis market participants submit their orders to their respective power exchange (PX). These orders are collected and submitted to a market-clearing algorithm

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(Euphemia) which identifies the prices and the trades that should be executed so as to maximize the gains from trade generated by the executed orders while ensuring the available transfer capacity is not exceeded [1].

In the United States, a different approach referred to as coordinated transaction scheduling (CTS) has been approved by FERC and implemented in NYISO, PJM, and MISO. In this method, proxy buses at the interties are used as trading points for enabling bids from external market participants. Thus offers to sell (buy) from external market participants are represented by injections (withdrawals) at such proxy buses.

Nonetheless, there are important concerns regarding both the PCR and CTS designs. First, participants may exercise market power by reporting information (e.g. orders to buy or sell) that is not consistent with the overall goal of maximizing social surplus. That is, the schemes are not guaranteed to be incentive compatible. Thus, for example, with distorted information as input, it does not follow that a sophisticated market-clearing algorithm such as Euphemia results in the efficient allocation and pricing of available interconnection capacity. A second concern pertains to market architecture. The PCR design implies individual area operators play a largely passive role as conveyors of bidding information. This goes against the widely accepted notion that with increasing intermittency (due to increased renewable capacity) individual area operators are likely to have superior information regarding acceptable trade-offs in cost vs security for the operation of their respective areas. A uniform protocol such as PCR limits the discretion of area operators in implementing heterogeneous approaches for market design and operation. CTS is limited to scheduling interchanges between two neighboring areas. This clearly limits efficiency gains as areas may need to interact with more than one neighboring area simultaneously in order to identify maximum gains from trade. In addition, the fictitious nature of proxy buses may imply that the actual power flow may differ from the CTS schedule. Thus, it is of interest to identify alternative market coupling designs ensuring efficiency with incentive compatibility guarantees while giving a more active role to individual area operators.

In this paper we present a market coupling mechanism that meets these goals. In the proposed market coupling design, individual area operators participate on behalf of agents (generators, retailers) active in their respective area. Specifically, area operators provide bids for energy import/export in the form of quantities and prices at incident tielines. In addition, each area operator exchanges voltage phase angles of boundary buses with adjacent area operators. The market coupling design is iterative: a capacity price for each intertie is updated at every round (or iteration) to reflect excess demand and then area operators are allowed to revisit their bids by recomputing their respective economic dispatch problem (modeled as a local DC-OPF problem). We assume that internal transactions associated with intertie trades are handled by each area operator. In order to focus our attention on incentive compatibility issues arising from pricing and allocation rules of intertie capacity and not from internal market design, we assume that area operators identify optimal internal economic dispatch at every iteration.

In the first result of this paper (Theorem 1) we show that if area operators report their bids in a manner that truthfully reflects their local economic dispatch, the sequence of outcomes of the proposed market coupling mechanism converges to the optimal allocation and pricing of intertie capacity, i.e. the limit outcome corresponds to the solution of the joint (centralized) DC-OPF for all areas.

However, the market coupling mechanism may not be compatible with the incentives of individual area operators. Failure to bid truthfully (reporting the results of internal economic dispatch) may lead to inefficient allocation and/or pricing of intertie trades and capacity. In the second result (Theorem 2) we show truthful participation in iterative bids an \(\epsilon\)-Nash equilibrium within the class of convex bidding strategies, i.e. no individual area operator benefits by more than \(\epsilon > 0\) by deviating from truthfully reporting the

1 https://www.nyiso.com/documents/20142/3036853/CTS+ISONE.pdf
2 https://www.pjm.com/-/media/committees-groups/stakeholder-meetings/pjm-nyiso/20180402/20180402-item-02b-coordinated-transaction-scheduling-metrics.ashx
3 https://www.misoenergy.org/stakeholder-engagement/issue-tracking/miso-pjm-coordinated-transaction-scheduling-cts/
results of their internal economic dispatch. The results rely on incentive transfers with a rationale that can be roughly described as follows: at every iteration, the reported information by area operators (i.e. desired intertie flows and locational marginal prices at boundary nodes) is used to estimate the change in value they perceive at every iteration. Thus, at each iteration any given area receives a transfer which is an estimate of the difference in value to all other participating areas with and without the area in question joining the coupling. Hence, the incentive transfers align the incentives of each individual area with that of minimizing the total cost (net of value of intertie trades) and the gross benefit that each area receives from coupling is its own marginal contribution the coupling. This is similar to the Clarke pivot rule in Vickrey-Clarke-Groves (VCG) mechanism [2], [6] and [8]. However, unlike the VCG mechanism (which requires agents to report high dimensional data [5] and a centralized solution of social surplus maximization), in the proposed market coupling reported information is elicited in an iterative pointwise manner and only a finite number of queries is required to approximate in a decentralized fashion the solution of the optimal allocation and pricing of intertie capacity. To ensure the mechanism does not incur a deficit, each area is assessed a participation fee which must be bounded below by the value of the lowest individual surplus across areas. We identify a sufficient condition for ensuring no deficit which corresponds to the case in which the information rents, i.e. the costs incurred to ensure incentive compatibility do not exceed the system-wide benefits obtained from coupling.

Related Literature: The iterative mechanism is related to decentralized approaches for solving the joint interconnected economic dispatch problem in a distributed fashion via primal decomposition methods [6], [7], [8], [9] or dual decomposition methods [10], [11], [12]. However, these methods do not address incentive compatibility concerns, i.e. area operators submitting truthful information is implicitly assumed.

Our paper is related to game theoretic approaches for allocating transmission investment costs in planning (see, e.g. [13], [14]). Although our focus is the short-run coupling of interconnected markets, we use game theoretic concepts for allocating savings that result from coupling. For example, in [14] the authors propose a planning model for interconnection capacity assuming complete information on generation costs. To ensure the coupling (seen as a coalition of markets) is stable, intertie investment costs are allocated using the Shapley value [15]. The proposed mechanism in this paper also ensures the coupling is stable (in the sense that it is each area’s best interest to participate). However, our work (which only deals with short-run operation of existing intertie capacity) does not rely on assuming complete information on generation costs.

The market coupling problem is also related to the literature in distributed optimization algorithms for the resource allocation problem [11, 16–23]. The motivation for a distributed solution to the economic dispatch problem, e.g., in a microgrid, is to increase scalability, robustness to failure and cyber-security of the operations. In contrast, in this paper we focus on multiple markets connected via interties wherein each market operates independently to solve its own DC-OPF. Finally, our paper is related to literature on improvements to existing market coupling designs [24], [25] that provide improvements and extensions to these designs.

Organization: The structure of this paper is as follows. In section 2 we introduce the centralized DC-OPF program with multiple areas. Section 2.2 describes a primal decomposition approach for decentralized solution. In section 2.3 we introduce the market coupling mechanism describing how information flows between areas as well as incentive transfers and participation fees. In section 3, we analyze the proposed mechanism. We start showing the convergence of capacity prices in Lemma 2. This result is then used to prove that if area operators report their bids in a manner that truthfully reflects their local economic dispatch, the sequence of outcomes of the proposed market coupling converges to the optimal allocation and pricing of intertie capacity (Theorem 1). In section 4, we analyze the incentive transfers to show that the coupling mechanism is incentive compatible. In addition, we show that each area is better-off by participating in the coupling mechanism. Finally, we identify sufficient conditions for ensuring the market
coupling implementation does not incur a deficit. In section 5, we show these results continue to hold when only reported information is used during the implementation of the mechanism. In section 6, we provide a numerical illustration with a three-zone IEEE Reliability Test System. We end the paper with conclusions.

2 Tieline Capacity Allocation and Pricing

Consider a power system composed of \( A > 1 \) operational areas, as schematically shown in Fig. 1, where the areas are corresponded to the associated electricity markets governed by separate ISOs. Each area \( a \in \mathcal{A} = \{1, 2, \ldots, A\} \) is represented by a directed graph \((\mathcal{N}_a, \mathcal{F}_a)\) where \( \mathcal{N}_a = \{1, 2, \ldots, N_a\} \) and \( \mathcal{F}_a = \{(i, j)|i, j \in \mathcal{N}_a, j \equiv j(i)\} \) respectively represent the set of nodes (buses) and edges (transmission lines). For area \( a \in \mathcal{A} \), the bus voltage angles and nodal loads form respectively the vectors \( \boldsymbol{\theta}_a = (\theta_{a,1}, \theta_{a,2}, \ldots, \theta_{a,N_a})^T \) and \( \boldsymbol{D}_a = (D_{a,1}, D_{a,2}, \ldots, D_{a,N_a})^T \). The \( N_a \times N_a \) admittance matrix of each area is denoted by \( \mathbf{B}_a \). The set of \( G_a \) generating units at each area is represented by \( \mathcal{G}_a = \{1, 2, \ldots, G_a\} \), the power generation of units form the vector \( \mathbf{P}_a = (P_{a,1}, P_{a,2}, \ldots, P_{a,G_a})^T \), and the \( N_a \times G_a \) incidence matrix \( \mathbf{M}_a \) maps the generating units to buses. The tielines entering/leaving area \( a \) are defined by set \( \mathcal{T}_a = \{(i, j)|i \in \mathcal{N}_a, j \in \mathcal{N}_{a'}, a' \in \mathcal{A} \setminus a\} \), the associated tieline power flows form the vector \( \mathbf{T}_a = (T_{a,(i,j)}), (i, j) \in \mathcal{T}_a \), and the incidence matrix mapping tieline power flows to buses is shown by \( \mathbf{R}_a \).

![Figure 1: Generic multi-area power transmission network.](image)

2.1 Centralized DC-OPF

In describing the operation of the proposed market coupling mechanism, it will be useful to refer to the centralized DC-OPF problem where a single system operator optimally operates the entire power system.

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4In the rest of this paper we use the terms “areas”, “markets”, and “ISOs” interchangeably.
with complete information regarding all the areas’ generators supply costs. This is a hypothetical exercise as in practice there is no single market operator and information on generators costs is not available to any single party. In the formulation below, the areas are separated and the transmission line power flows are distinguished from tieline power flows. This arrangement would facilitate formulating the decentralized DC-OPF, which is the backbone of proposed tieline capacity allocation.

To sum up, in the centralized DC-OPF formulation the total operational cost of power system in is minimized subject to the operation constraints:

\[
\min_{\mathbf{P}_{a,g}} \sum_{a \in \mathcal{A}} \sum_{g \in \mathcal{G}_a} C_{a,g}(P_{a,g}) ,
\]

\[
\mathbf{B}_a \mathbf{\theta}_a + \mathbf{R}_a \mathbf{T}_a = \mathbf{M}_a \mathbf{P}_a - \mathbf{D}_a , \quad \mathbf{\alpha}_a , \quad \forall a \in \mathcal{A},
\]

\[
\mathbf{P}_a \leq \mathbf{P}_a \leq \mathbf{P}_a , \quad (\nu_{a,g}, \lambda_{a,g}) \quad \forall a \in \mathcal{A},
\]

\[
-F_{a(i,j)} \leq \frac{\theta_{a,i} - \theta_{a,j}}{x_{a(i,j)}} \leq F_{a(i,j)} , \quad (\kappa_{a(i,j)}, \eta_{a(i,j)}) , \quad \forall a \in \mathcal{A}, \forall (i, j) \in \mathcal{F}_a ,
\]

\[
T_{a(i,j)} = \frac{\theta_{a,i} - \theta_{a,j}}{x_{a(i,j)}} , \quad (\xi_{a(i,j)}) , \quad \forall a \in \mathcal{A}, \forall (i, j) \in \mathcal{T}_a ,
\]

\[
-T_{a(i,j)} \leq T_{a(i,j)} \leq T_{a(i,j)} , \quad (\bar{\kappa}_{a(i,j)}, \bar{\eta}_{a(i,j)}) , \quad \forall a \in \mathcal{A}, \forall (i, j) \in \mathcal{T}_a ,
\]

\[
\theta_{1,1} = 0 ,
\]

where \(C_{a,g}(P_{a,g})\) denotes the strictly convex cost functions of generating units, the vectors \(\mathbf{P}_a\) and \(\mathbf{P}_a\) respectively represent the minimum and maximum generation limits of units, and \(\mathcal{F}_a\) and \(\bar{\mathcal{F}}_a\) respectively represent the power flow limits of transmission lines and tielines. The nodal power balance is secured through \(\mathcal{G}_a\), the generation of units are confined to their limits in \(\mathcal{T}_a\), transmission line power flows are maintained within thermal limits in \(\mathcal{F}_a\) where \(x_{a(i,j)}\) represents the reactance of each line, and the tieline power flows are calculated and constrained to their thermal limits in \(\mathcal{T}_a\) and \(\bar{\mathcal{F}}_a\) where \(\bar{x}_{a(i,j)}\) represents the tieline reactances. Further, the voltage phase angle of the slack bus, which is assumed to be the bus 1 of area 1, is set to zero in \(\mathcal{T}_a\). Note that \(\bar{T}_{a(i,j)} = \bar{T}_{a(i,j)}\) and \(\bar{x}_{a(i,j)} = \bar{x}_{a(i,j)}\).

The first order conditions with respect to \(\theta_{a,i}, \theta_{a,j}\), and \(T_{a(i,j)}\) for tieline \((i, j) \in \mathcal{T}_a\) in the centralized DC-OPF problem \((1)-(7)\) are:

\[
\sum_{j \mid (i,j) \in \mathcal{F}_a} \frac{1}{x_{a(i,j)}} (\alpha_{a,i}^* - \alpha_{a,j}^* + \eta_{a(i,j)}^* - \kappa_{a(i,j)}^*) - \frac{1}{\bar{x}_{a(i,j)}} \xi_{a(i,j)}^* = 0
\]

\[
\sum_{j \mid (i,j) \in \mathcal{F}_a'} \frac{1}{x_{a(i,j)}} (\alpha_{a',i}^* - \alpha_{a',j}^* + \eta_{a'(i,j)}^* - \kappa_{a'(i,j)}^*) + \frac{1}{\bar{x}_{a(i,j)}} \xi_{a(i,j)}^* = 0
\]

\[
\alpha_{a,i}^* - \alpha_{a',j}^* + \tilde{\eta}_{a(i,j)}^* - \tilde{\kappa}_{a(i,j)}^* + \xi_{a(i,j)}^* = 0
\]

The centralized model presented in this section leads to an efficient tieline capacity allocation, yet it requires the areas to reveal their private information (network configuration and cost functions of units) and leads to solving an optimization problem of high dimensionality. However, the decentralized DC-OPF formulation and the counterpart iterative capacity allocation method presented in the following sections reduce the size of optimization problems and preserve the information privacy of areas.
2.2 Decentralized DC-OPF

In this section, we introduce a decentralized approach for solving the DC-OPF problem. Given locational marginal prices \((\alpha_{a',j})\) and angles \((\theta_{a',j})\) of adjacent areas and the capacity price \(\mu_{(i,j)}\), the problem for area \(a \in A\) is given by:

\[
\min \sum_{g \in \mathcal{G}_a} C_{a,g}(P_{a,g}) - \sum_{(i,j) \in T_a} \alpha_{a',j} T_{a,(i,j)} + \sum_{(i,j) \in T_a} \frac{\mu_{(i,j)}}{2} \left( |T_{a,(i,j)}| - \bar{T}_{a,(i,j)} \right)
\]

\[
B_a \theta_a + R_a T_a = M_a P_a - D_a, \quad (\alpha_a),
\]

\[
P_a \leq \bar{P}_a, \quad (\nu_{a,g}, \lambda_{a,g}),
\]

\[-F_{a,(i,j)} \leq \frac{\theta_{a,i} - \theta_{a,j}}{x_{a,(i,j)}} \leq F_{a,(i,j)}, \forall (i,j) \in \mathcal{T}_a, \quad (\kappa_{a,(i,j)}, \eta_{a,(i,j)})\]

\[T_{a,(i,j)} = \frac{\theta_{a,i} - \theta_{a,j}}{x_{a,(i,j)}}, \quad \forall (i,j) \in \mathcal{T}_a, \quad (\xi_{a,(i,j)})\]

\[
\theta_{a,1} = 0, \text{ if } a = 1,
\]

Note the objective function in \[(11)\] not only includes area \(a\)'s total generation cost but also accounts for the capacity costs of incident tielines as well as the cost/revenue of the energy import/export from/to neighbouring areas. It is assumed that the interconnected areas split the capacity cost over tielines, meaning that the two incident areas of a tieline are both charged for the excess capacity usage at the rate of \(\frac{\mu_{(i,j)}}{2}\). This is done with no loss of generality: other conventions (e.g. exporter is assigned capacity cost, importer assigned capacity cost) can also be used without altering the results in this paper.

The following lemma makes precise in which way decentralization is made possible in this formulation.

**Lemma 1.** If the locational marginal prices, angles of adjacent areas and the capacity price of interties correspond to the optimal solution of centralized OPF (i.e. \(\alpha_{a',j}^*, \theta_{a',j}^*\) and \(\mu_{(i,j)}^*\) \(\text{sign}(T_{a,(i,j)}^*) = \bar{\eta}_{a,(i,j)}^* - \bar{\kappa}_{a,(i,j)}^*\)), then the solution to the decentralized OPF problem for area \(a \in A\) corresponds to the solution of the centralized OPF.

**Proof.** Consider the first order conditions with respect to \(\theta_{a,i}\) and \(T_{a,(i,j)}\) in the decentralized DC-OPF problem \[(11)-(16)\] for area \(a \in A\):

\[
\sum_{j|(i,j) \in \mathcal{T}_a} \frac{1}{x_{a,(i,j)}} (\hat{\alpha}_{a,i} - \bar{\alpha}_{a,j} + \bar{\eta}_{a,(i,j)} - \bar{\kappa}_{a,(i,j)}) - \frac{1}{x_{a,(i,j)}} \hat{\xi}_{a,(i,j)} = 0
\]

\[
\hat{\alpha}_{a,i} + \hat{\xi}_{a,(i,j)} = \alpha_{a',j}^* - \frac{\mu_{(i,j)}^*}{2} \text{sign}(\hat{T}_{a,(i,j)})
\]

where \(\hat{T}_{a,(i,j)}\) is the optimal transfer flow for area \(a\) and \(\text{sign}(x) = 1\) if \(x \geq 0\) and \(\text{sign}(x) = -1\) if \(x < 0\).

We check now that the centralized DC-OPF solution is a solution to each decentralized DC-OPF problem, namely that conditions \[(17)-(18)\] are satisfied with \(\hat{\alpha}_{a,i} = \alpha_{a',i}^*, \hat{\xi}_{a,(i,j)} = \xi_{a,(i,j)}^*\) and

\[
\hat{T}_{a,(i,j)} = \begin{cases} 
T_{a,(i,j)}^* & T_{a,(i,j)}^* \geq 0 \\
-T_{a,(i,j)}^* & T_{a,(i,j)}^* < 0 
\end{cases}
\]
Thus, it can be seen that equation (8) is equivalent to (17). Similarly, equation (18) corresponds to equation (10) above. The remaining optimality conditions related to internal operation of area \( a \) match those of the centralized OPF problem.

**Remark 1.** The decentralization approach in [6] has each area internalizing the tieline capacity constraint. However, adjacent areas have no means of ensuring they agree on the marginal value of intertie capacity. Hence, one cannot conclude that the first order conditions for the decentralized problems match those of the centralized DC-OPF. The decentralization approach in [6] only works if interties have no capacity constraints (so that marginal value of capacity \( \mu_{(i,j)} \) is null) or if an additional consensus constraint of the form

\[
\bar{\eta}_{a,(i,j)} - \bar{\eta}_{a',(i,j)} + \bar{\eta}_{a',(i,j)} - \bar{\eta}_{a,(i,j)} = 0
\]

is introduced where \( \bar{\eta}_{a,(i,j)} \) (respectively, \( \bar{\eta}_{a',(i,j)} \)) is the Lagrange multiplier associated with upper bounds on intertie flow from \( a \) to \( a' \) (respectively, from \( a' \) to \( a \)) and \( \bar{\eta}_{a,(i,j)} \) (respectively, \( \bar{\eta}_{a',(i,j)} \)) is the multiplier associated with lower bound on intertie flow.

### 2.3 Iterative Mechanism for Tieline Capacity Allocation and Pricing

In this section we elaborate the proposed capacity allocation and pricing model. In the proposed model, the individual ISOs associated with areas are self-interested agents seeking to maximize the gains from trade for their respective area. Each ISO has private information of their internal generation resources resulting from the operation of an internal market within each area.

The basic steps of the mechanism are as follows—see Algorithm 1 for details. At each iteration \( k > 0 \) of the mechanism,

1. (Information exchange) Each area reports the terms of trade for each interconnection in the form of intertie flows \( (\bar{T}^{k-1}_{a,(i,j)}) \), locational marginal prices \( (\bar{\alpha}^{k-1}_{a,(i,j)}) \) and angles of adjacent areas \( (\bar{\theta}^{k-1}_{a',(i,j)}) \), where \( \alpha^0_{a',j} = 0 \).

2. (Updating Intertie Flows) Given a capacity price \( \mu^{k-1}_{i,j} \), each area \( a \in \mathcal{A} \) solves (11) - (16) and reports the desired flows \( \hat{T}^k_a \), corresponding angles at interties \( \hat{\theta}^k_a \) and locational marginal prices \( \hat{\alpha}^k_a \) to the agent in charge of executing the mechanism which updates the flows, angles and locational marginal prices along the interties according to:

\[
\bar{T}_a^k = (1 - \rho_k)\bar{T}_a^{k-1} + \rho_k \hat{T}_a^k \quad (19)
\]

\[
\bar{\theta}_a^k = (1 - \rho_k)\bar{\theta}_a^{k-1} + \rho_k \hat{\theta}_a^k \quad (20)
\]

\[
\bar{\alpha}_a^k = (1 - \rho_k)\bar{\alpha}_a^{k-1} + \rho_k \hat{\alpha}_a^k \quad (21)
\]

with \( \rho_k \to 0^+ \), \( \sum_k \rho_k = +\infty \) and \( \sum_k \rho_k^2 < \infty \).

3. (Updating Intertie Capacity Prices)

\[
\mu^k_{i,j} = \max\{\mu^{k-1}_{i,j} + \beta\left(\frac{|\hat{T}^k_{a,(i,j)}| + |\hat{T}^k_{a',(i,j)}|}{2} - \bar{T}_{a,(i,j)}\right), 0\} \quad (22)
\]

where \( \beta \in (0, 1) \) and \( \mu^0_{i,j} = 0 \).
4. A monetary transfer $\Delta \pi^k_a$ is allocated to each area $a \in A$. Upon stopping after $T > 0$ iterations a coupling participation fee $R > 0$ is assessed to each area $a \in A$.

Figure 2 summarizes the key update steps within an area and the information exchange between two coupled areas. In updating the intertie flows and angles, and locational marginal prices, each area uses an inertial update in (19)-(21) by weighting the current optimal solution, i.e., $\hat{T}^k_a, \hat{\theta}^k_a, \hat{\alpha}^k_a$, with the previous step, rather than using the current optimal solution to the decentralized DC-OPF. From the updated capacity price in (22), both sending and receiving areas use the same capacity price in their DC-OPF problem at each iteration $k$. Moreover, the capacity price is updated with a constant step size ($\beta$), while the inertial updates assume a diminishing step size. The convergence analysis is based on the exploitation of this fast update of the capacity prices versus the slow update of intertie flows and angles, and locational marginal prices. The update mechanism is complemented by an incentive mechanism (a money transfer and participation fee) in Step 4, which makes sure that areas are truthful in their reporting. We detail the incentive scheme in Section 4 after discussing the convergence of the updates (Steps 1-3) under truthful participation of areas.

\textbf{Algorithm 1: Iterative Tieline Capacity Allocation}

| Require: Initialize $x^0_a = (T^0_a, \theta^0_a, \alpha^0_a)$, and $\mu^0_{(i,j)}$ for all areas and tielines. |
| Require: Maximum number of iterations $T + 1$. Set $k = 1$ |
| while $k \leq T + 1$ do |
| • Run DC-OPF problems of all areas and obtain $\hat{x}^k_a = (\hat{T}^k_a, \hat{\theta}^k_a, \hat{\alpha}^k_a)$. |
| • Calculate $x^k_a = (1 - \rho_k)x^{k-1}_a + \rho_k\hat{x}^k_a$ for all areas. |
| • Calculate the capacity prices $\mu^k_{(i,j)} = \max\{\rho^{k-1}_a + \beta(\frac{|T^k_a(i,j)| + |T^k_a'(i,j)|}{2} - \bar{T}_{a,(i,j)}), 0\}$ for all tielines. |
| • Exchange $x^k_a$ between neighboring areas. |
| • Set $k = k + 1$. |
| end |

3 Convergence Analysis

We make the following standing assumption on the feasibility of each area in the absence of tielines.

\textbf{Assumption (Area feasibility):} There exists a feasible solution to the decentralized DC-OPF problem (11)-(16) for area $a \in A$ when $T_{a,(i,j)} = 0$ for all $(i, j) \in T_a$.

The above assumption states that each market can meet the demand in its own area without relying on the intertie power flows. A manifestation of this assumption on coupled markets is the existence of a maximum intertie capacity price $\bar{\mu}_{(i,j)} > 0$ for $(i, j) \in T_a$ such that for $\mu_{(i,j)} \geq \bar{\mu}_{(i,j)}$ the optimal solution to (11) - (16) will have no flow along the intertie, i.e. $T_{a,(i,j)} = 0$—see Proposition 2 in the Appendix.

In the first part of the convergence analysis, we show that the capacity multipliers converge.
Lemma 2. For every intertie \((i, j)\) it holds that \(\mu^k_{(i,j)} \to \mu^\infty_{(i,j)} \geq 0\) and

\[
\mu^k_{(i,j)}(\frac{|T^k_{a,(i,j)}| + |T^k_{a',(i,j)}|}{2} - \bar{T}_{a,(i,j)}) \to 0
\]  

(23)

Proof. Suppose the sequence \(\{\mu^k_{(i,j)} : k > 0\}\) does not converge. So either \((i)\), \(\mu^k_{(i,j)} \to +\infty\) or \((ii)\), it oscillates with \(\lim \inf \mu^k_{(i,j)} < \lim \sup \mu^k_{(i,j)}\). Case \((i)\) is easily discarded since from the assumption on maximum intertie capacity price:

\[
\mu^k_{(i,j)} \to +\infty \Rightarrow |T^k_{a,(i,j)}| + |T^k_{a',(i,j)}| \to 0
\]

which is a contradiction. Now let us consider case \((ii)\). Capacity price oscillation implies that for some \(\epsilon > 0\)

\[
\lim \sup_k \left\{ \frac{|T^k_{a,(i,j)}| + |T^k_{a',(i,j)}|}{2} - \bar{T}_{a,(i,j)} \right\} \geq \epsilon > 0 \geq \lim \inf_k \left\{ \frac{|T^k_{a,(i,j)}| + |T^k_{a',(i,j)}|}{2} - \bar{T}_{a,(i,j)} \right\}
\]

Thus the sequence \(\left\{ \frac{|T^k_{a,(i,j)}| + |T^k_{a',(i,j)}|}{2} - \bar{T}_{a,(i,j)} \right\}\) exhibits infinitely many upcrossings of \(\frac{\epsilon}{4}\) and \(\frac{3\epsilon}{4}\). Let \(k > 0\) denote an index for an iteration in which an upcrossing of \(\frac{3\epsilon}{4}\) takes place, i.e.:

\[
\frac{|T^k_{a,(i,j)}| + |T^k_{a',(i,j)}|}{2} - \bar{T}_{a,(i,j)} > \frac{3\epsilon}{4}, \quad \text{and} \quad \frac{|T^{k-1}_{a,(i,j)}| + |T^{k-1}_{a',(i,j)}|}{2} - \bar{T}_{a,(i,j)} < \frac{\epsilon}{4}.
\]
Since \( T_{a,(i,j)}^{k+1} - T_{a,(i,j)}^k = \rho_k (\hat{T}_a^k - T_a^k) \) it follows that
\[
\sum_{\ell=1}^{\tau(k)} (T_{a,(i,j)}^{k+\ell} - T_{a,(i,j)}^{k+\ell-1}) = \sum_{\ell=1}^{\tau(k)} \rho_{k+\ell} (\hat{T}_a^{k+\ell} - T_a^{k+\ell}) \leq 2 T \sum_{\ell=1}^{\tau(k)} \rho_{k+\ell}
\]
This allows us to obtain a lower bound on number of iterations \( \tau(k) > 0 \) needed to upcross \( \frac{3\epsilon}{4} \) from \( \frac{\epsilon}{4} \):
\[
2T \sum_{\ell=1}^{\tau(k)} \rho_{k+\ell} > \frac{3\epsilon}{4} - \frac{\epsilon}{4} = \frac{\epsilon}{2}
\]
Since \( \tau(k) \rho_k > \sum_{\ell=1}^{\tau(k)} \rho_{k+\ell} \), it follows that \( \tau(k) > \frac{2T}{\rho_k} \to +\infty \) since \( \rho_k \to 0^+ \). We also have \( \mu_{(i,j)}^{k+\ell} \geq \mu_{(i,j)}^k + \beta \tau \frac{\epsilon}{4} \) for all \( 0 < \tau \leq \tau(k) \). Since \( \sum_{\ell=1}^{\tau(k)} \rho_{k+\ell} \to 0^+ \), it follows that there exists \( \tau > 0 \) such that \( \mu_{(i,j)}^{k+\ell} > \hat{\mu}_{(i,j)} \) and \( \hat{T}_a^{k+\ell} = 0 \) for all \( \tau < \ell \leq \tau(k) \). Hence, \( |T_{a,(i,j)}^{k+\ell}| = |T_{a,(i,j)}^{k+\tau}| \) for all \( \tau < \ell \leq \tau(k) \) which is a contradiction to upcrossing \( \frac{3\epsilon}{4} \).

The non-negativity of \( \mu_{(i,j)}^\infty \) follows by the max operation in (22).

The above result implies that the asymptotic capacity price \( \mu_{(i,j)}^\infty \) is dual feasible and satisfies complementary slackness condition of the centralized DC-OPF problem as per (23).

### 3.1 Convex Reporting Strategies

We now show that if all areas participate **truthfully**, that is, they report accurate information to the agent implementing the mechanism, then the mechanism converges to the optimal solution of centralized DC-OPF problem.

Before defining **truthful participation** we consider the possibility of strategic behavior by participating areas in the form of reporting strategies that are consistent with some choice of strictly convex generation cost functions but not necessarily the true cost function. Specifically, in the course of the execution of the trading mechanism, we consider the possibility that area \( a \) reports values of the form \( \hat{x}_{a}^{D,k} = (\hat{T}_a^{D,k}, \hat{\theta}_a^{D,k}, \hat{\alpha}_a^{D,k}) \) that corresponds to the optimal solution of the problem:

\[
\min \sum_{g \in \mathcal{G}_a} C_{a,g}^D (P_{a,g}) + \sum_{(i,j) \in \mathcal{T}_a} \left[ \frac{1}{2} |T_{a,(i,j)} - \alpha_{a,i,j} - T_{a,(i,j)} + \theta_{a,i,j}^k| \right] + \frac{1}{2} \alpha_{a,i,j}^k |T_{a,(i,j)} - \alpha_{a,i,j} - T_{a,(i,j)} + \theta_{a,i,j}^k|,
\]

s.t. (12), (13), (14), (16)

\[
T_{a,(i,j)} = \frac{\theta_{a,i,j} - \theta_{a,i,j}^{k-1}}{x_{a,(i,j)}}, \quad \forall (i,j) \in \mathcal{T}_a,
\]

where \( C_{a,g}^D(\cdot) \) are strictly convex differentiable functions for each \( g \in \mathcal{G}_a \). We shall refer to this form of reporting as **convex**.

**Definition 1** (Convex reporting strategy) Area \( a \in \mathcal{A} \) follows a convex reporting strategy if and only for every iteration \( k > 0 \), such area reports the values \( \hat{x}_{a}^{D,k} = (\hat{T}_a^{D,k}, \hat{\theta}_a^{D,k}, \hat{\alpha}_a^{D,k}) \) corresponding to the solution to (24) with \( C_{a,g}^D(\cdot), g \in \mathcal{G}_a \) strictly convex.
We consider two cases of the capacity price: \( \mu \). We begin by showing that the decentralized OPF problem.

In the second part of the proof we show that the iterative mechanism satisfies the KKT conditions of the decentralized DC-OPF solution that is feasible for all areas satisfies all the conditions of the centralized DC-OPF problem.

By definition of the update rule, it holds that

\[
x_{a}^{k+1} - x_{a}^{k} = (1 - \rho_{k+1})x_{a}^{k} + \rho_{k+1}(\hat{x}_{a}^{k+1} - x_{a}^{k+1}) - \rho_{k}\hat{x}_{a}^{k}
\]

\[
= (1 - \rho_{k})(x_{a}^{k} - x_{a}^{k-1}) + \rho_{k}(\hat{x}_{a}^{k+1} - \hat{x}_{a}^{k}) + (\rho_{k+1} - \rho_{k})(\hat{x}_{a}^{k+1} - x_{a}^{k})
\]

Since the optimal solution and Lagrange multipliers for each area \( a \in A \) are Lipschitz continuous (see e.g. [26]), it follows that

\[
\|x_{a}^{k+1} - \hat{x}_{a}^{k}\| \leq \sum_{a' \in A(a)} L_{a} \|x_{a'}^{k+1} - x_{a'}^{k}\| + L_{a} \|\mu_{a}^{k} - \mu_{a}^{k-1}\| = O(\rho_{k}) + L_{a} \|\mu_{a}^{k} - \mu_{a}^{k-1}\|
\]

for some \( L_{a} > 0 \). Thus for finite \( T < \infty \) it follows that

\[
\|x_{a}^{k+T} - x_{a}^{k}\| \leq \sum_{\ell=1}^{T} \rho_{k+\ell} \|x_{a}^{k+\ell} - x_{a}^{k+\ell-1}\| + \rho_{k+\ell} \|\hat{x}_{a}^{k+\ell+1} - \hat{x}_{a}^{k+\ell}\| + |\rho_{k+\ell+1} - \rho_{k+\ell}| \|\hat{x}_{a,i}^{k+\ell+1} - x_{a,i}^{k+\ell}\|
\]

\[
\leq \sum_{\ell=1}^{T} \rho_{k+\ell}O(\rho_{k+\ell}) + \sum_{\ell=1}^{T} \sum_{(i,j) \in T_{a}} \rho_{k+\ell}(\mu_{(i,j)}^{k+\ell} - \mu_{(i,j)}^{k+\ell-1}) + O(1) \sum_{\ell=1}^{T} |\rho_{k+\ell+1} - \rho_{k+\ell}|
\]

Thus the sequence \( \{x_{a}^{k} : k > 0\} \) is Cauchy and converges to \( x_{a}^{\infty} \). Since \( x_{a}^{k} \rightarrow x_{a}^{\infty} \) for all adjacent areas \( a' \in A \), by continuity of optimal solutions and Lagrange multipliers it follows that the sequence \( \{\hat{x}_{a}^{k} : k > 0\} \) has a limit. By construction, \( x_{a}^{k} \) is a weighted average of \( \{\hat{x}_{a}^{k} : k > 0\} \), hence \( \|x_{a}^{k} - x_{a}^{\infty}\| \rightarrow 0 \) and \( \hat{x}_{a}^{k} \rightarrow x_{a}^{\infty} \).

In the second part of the proof we show that the iterative mechanism satisfies the KKT conditions of the centralized OPF problem.

We begin by showing that \( \{x_{a}^{\infty}\}_{a \in A} \) is a feasible solution of the centralized OPF problem. Note that a decentralized DC-OPF solution that is feasible for all areas satisfies all the conditions of the centralized DC-OPF except \( \mu_{g} \). The convergence of the sequence \( x^{k} \) to \( x^{\infty} \) implies that \( T_{a,(i,j)}^{\infty} = -T_{a',(i,j)}^{\infty} \) for \( (a, a') \in T \).

We consider two cases of the capacity price: \( \mu_{(i,j)} = 0 \) and \( \mu_{(i,j)} > 0 \) to argue that inter-tie capacity limits are obeyed. If \( \mu_{(i,j)} = 0 \), then it must be that \( T_{a,(i,j)}^{\infty} \leq T_{a,(i,j)} \) because of the capacity price updates in \([22]\). If \( \mu_{(i,j)} > 0 \), then \( T_{a,(i,j)}^{\infty} = T_{a,(i,j)} \) by the condition in \([23]\). Thus, the inter-tie capacity limits are satisfied by \( x_{a}^{\infty} \). By Lemma \([2]\) we have that the dual feasibility and the complementary slackness conditions hold.
We continue by showing that the first order stationarity conditions of the centralized problem are satisfied by $T_a^\infty$ and $\theta_a^\infty$. In the limit as $k \to \infty$, let $L_A(x_A, \bar{x}_B^\infty, \alpha_A^\infty, \mu^\infty)$ and $L_B(x_B, \bar{x}_A^\infty, \alpha_B^\infty, \mu^\infty)$ denote the Lagrangian of the decentralized DC-OPF problem for areas $A$ and $B$, respectively. The terms $\bar{x}_B^\infty, \alpha_B^\infty$, and $\mu^\infty$ in $L_A(\cdot)$ represent the terms in the objective $\|P\|_2$ and tie-flow constraint (15) of Area $A$ coming from the decoupling of constraints in the centralized DC-OPF. We first note that $\partial L_A(\cdot) = \partial P^A$ and $\partial L_B(\cdot) = \partial P_B^A$, $\forall(i, j) \in F_a$ where $L(\cdot)$ is the Lagrangian of the centralized DC-OPF.

Next we consider the first order stationarity conditions with respect to $\theta_A, i, \theta_B, j$ and $T_{A,(i,j)}$ for tie-line $(i,j) \in T_A$ in the decentralized DC-OPF problem and show their equivalence to the corresponding first order conditions of the centralized DC-OPF in (8)-(10). The first order conditions with respect to $T_{a,(i,j)}$ for areas $a \in \{A, B\}$, i.e., $\frac{\partial L_A(x_A, \bar{x}_B^\infty, \alpha_A^\infty, \mu^\infty)}{\partial T_{A,(i,j)}}$ and $\frac{\partial L_B(x_B, \bar{x}_A^\infty, \alpha_B^\infty, \mu^\infty)}{\partial T_{B,(i,j)}}$, are respectively as follows

$$\begin{align*}
\hat{\alpha}_{A,i} + \tilde{\xi}_{A,(i,j)} &= \alpha_{B,j}^\infty - \frac{\mu_{(i,j)}^\infty}{2} \text{sign}(\hat{T}_{A,(i,j)}), \\
\hat{\alpha}_{B,j} + \tilde{\xi}_{B,(i,j)} &= \alpha_{A,i}^\infty - \frac{\mu_{(i,j)}^\infty}{2} \text{sign}(\hat{T}_{B,(i,j)}).
\end{align*}$$

From Lemma [1], we know that if $\alpha_{B,j}^\infty = \alpha_{A,j}^\ast$ and $\mu_{(i,j)}^\infty \text{sign}(\hat{T}_{A,(i,j)}) = \bar{\eta}_{(i,j)}^\ast - \bar{\kappa}_{(i,j)}^\ast$, then the centralized solution $(\hat{\alpha}_{A,i} = \alpha_{A,i}^\ast$ and $\tilde{\xi}_{A,(i,j)} = \xi_{A,(i,j)}^\ast)$ satisfies the stationarity condition in (25) for the decentralized DC-OPF for area $A$. The same argument applies to the condition in (26) for area $B$. Consider the optimal dual variables $(\bar{\eta}_{A,(i,j)}^\ast$ and $\bar{\kappa}_{A,(i,j)}^\ast)$ associated with the intertie capacity constraints (6). The intertie capacity price $(\frac{\mu_{(i,j)}^\infty}{2} \text{sign}(\hat{T}_{A,(i,j)}))$ satisfies the identical complementary slackness conditions as $\bar{\eta}_{A,(i,j)}^\ast - \bar{\kappa}_{A,(i,j)}^\ast$. Same applies to the optimal dual variables of area $B$ $(\bar{\eta}_{B,(i,j)}^\ast$ and $\bar{\kappa}_{B,(i,j)}^\ast$). Thus, we have $\frac{\partial L_A(x_A, x_B^\infty, \alpha_A^\infty, \mu^\infty)}{\partial T_{A,(i,j)}} = 0$ if $\alpha_A^\infty = \alpha_A^\ast$ and $\frac{\partial L_B(x_A^\infty, x_B, \alpha_B^\ast, \mu^\infty)}{\partial T_{B,(i,j)}} = 0$ if $\alpha_B^\infty = \alpha_B^\ast$. For these two conditions to hold at the same time, it must be that $x_A^\infty = x_A^\ast$ and $x_B^\infty = x_B^\ast$.

The main steps in the proof of Theorem [1] involves showing convergence of the capacity prices (Lemma [2]) and the intertie flows, angles, and locational marginal prices ($x_a^\infty$) to some finite value. In showing their convergence, we rely on the updates of $x_a^\infty$ with diminishing step size in (19)-(21) and Lipschitz continuity of solutions to the decentralized DC-OPF problem. In the second part of the proof, we show that these convergence points have to correspond to the optimal solution of the centralized DC-OPF via the equivalence of KKT conditions between the decentralized and centralized DC-OPF problems.

The main assumption in Theorem [1] is the truthful participation of the areas in the update process, which is not guaranteed when the areas are selfish and strategic. We lift this assumption by designing a monetary incentive mechanism in the following section.

### 4 An Incentive Compatible Design for Market Coupling

We identify incentive transfers based on *marginal contribution to the coupling* of each area to ensure that engaging in strategic manipulation, e.g., reporting information that is inconsistent with solving each DC-OPF problem with respect to true generation costs is not advantageous.

The marginal contribution of an area $a \in A$ refers to the cost savings of other areas $a' \in A \setminus a$ as a result of area $a$ participating in the coupling. Let $\hat{P}_{-a}$ denote the solution of the DCOPF problem without area
a’s participation (i.e., area a’s initial power flow and exchanges with adjacent areas remains fixed). With \( \bar{C}_a := \sum_{g \in \mathcal{G}_a} C'_{a,g}(\bar{P}_{a,g}) \), the cost savings induced by area a’s participation in the coupling can be expressed as 
\[
M_a := \bar{C}_a - C^* - a
\]
where \( \bar{C}_a = \sum_{a' \in \mathcal{A}, a} \bar{C}_{a'} \) and \( C^* = \sum_{a' \in \mathcal{A}, a} C'_{a'} \).

In what follows, we will describe an iterative design with incentive transfers that upon stopping (approximately) gives each area its marginal contribution to the coupling. Thus, upon stopping, area a is expected to have a net cost reduction equal to:
\[
C_{a,0} - C^* + M_a = C_{a,0} - C^* + \bar{C}_a - C^* - a = C_{a,0} + \bar{C}_a - C^* > 0
\]
where \( C_{a,0} \) is the cost of area a under the initial power flow. The inequality follows from the fact that area a’s initial dispatch \( P_{a,0} \) and the resulting coupled dispatch \( \bar{P}_a \) is a feasible power dispatch for the interconnected power system with all areas. To ensure incentive transfers and cost savings are balanced, we introduce a minimum participation fee \( R > 0 \) that is assessed to each market (area) participating in the proposed design for market coupling.

### 4.1 Marginal Contribution to a Coupling

We will use \( \bar{z}^k = (\bar{T}^k, \bar{\theta}^k, \bar{\alpha}^k) \) to denote the sequence of the flows between areas, angles and nodal prices for relevant nodes in adjacent areas at iteration \( k \) generated by the updates in Algorithm 1 without area a’s participation in the coupling. The cost of area a’ associated with the updates \( \bar{z}^k \) is denoted using \( \bar{C}_{a',k} := \sum_{g \in \mathcal{G}_{a'}} C'_{a',g}(\bar{P}^k_{a',g}) \). Similarly we use \( C_{a',k} \) to denote the cost of area a’ associated with the sequence \( \bar{z}^k \) when area a is participating in the coupling. The incremental change in cost of area a’ from iteration \( k \) to iteration \( k + 1 \) with and without area a’s participation are respectively given as 
\[
\Delta \bar{C}_{a',k} := \bar{C}_{a',k+1} - \bar{C}_{a',k}
\]
and 
\[
\Delta C_{a',k} := C_{a',k+1} - C_{a',k}.
\]
The difference between the incremental changes in costs \( \Delta \bar{C}_{a',k} - \Delta C_{a',k} \) represents the incremental cost saving for area a’ \( \in \mathcal{A} \setminus \{a\} \) caused by area a’s participation in the iterative mechanism. Formally, we define the marginal contribution of area a at step \( k \) as the change in cost to all areas \( \mathcal{A} \setminus \{a\} \) relative to the change in cost when area a is not part of the coupling,
\[
M_{a,k} := \sum_{a' \in \mathcal{A} \setminus \{a\}} \Delta C_{a',k} - \Delta \bar{C}_{a',k}.
\]

Let \( V_{a,k} \) denote the optimal value of area a’s DC-OPF problem (11)-(16) at iteration \( k \),
\[
V_{a,k} = C_{a,k} - r_{a,k},
\]
where we define
\[
r_{a,k} := \sum_{(i,j) \in T_a} \alpha_{a',j}^{k-1} \bar{T}_{a,(i,j)} - \sum_{(i,j) \in T_a} \frac{\mu_{a,(i,j)}^{k-1}}{2} \left( T^k_{a,(i,j)} - T_{a,(i,j)} \right).
\]

Using the definitions above, we define the incentive transfer for area a as follows:
\[
\Delta \pi_{a,k} = M_{a,k} + r_{a,k+1} - r_{a,k}
\]
The incentive payments return area a’s marginal contribution to area a, and offsets the terms associated with tieline flow prices in the value function.
Remark 2. The incentive transfers defined (30) can not be readily computed since the change in costs $\Delta C_{a,k}$ is privately known by area $a$’s operator. However, as we shall show in section 5, a fairly accurate estimate of $\Delta C_{a,k}$ can be obtained using only reported information (i.e. desired transfer flows, angles and locational marginal prices) for each area $a \in A$.

Remark 3. The computation of $\Delta \tilde{C}_{a,k}$ for each $a \in A$ requires a total of $|A| + 1$ parallel executions of Algorithm 1: one for the complete coupling of all areas and $|A|$ for different coupling excluding each one of the areas. Figure 3 shows all possible subsets of coupling with one area excluded from the coupling of three areas $A = \{A,B,C\}$. For instance, Areas B and C need to exchange information and bids according to Algorithm 1 as if area A is not part of the coupling—see blue lines in the figure. This will generate sequence of nodal prices, angles and intertie flows, denoted by $\tilde{x}^k = \{(\tilde{T}_{a,k}, \tilde{\theta}_{a,k}, \tilde{\alpha}_{a,k})\}_{a \in A}$, different from the sequence $\hat{x}^k = \{(\hat{T}_{a,k}, \hat{\theta}_{a,k}, \hat{\alpha}_{a,k})\}_{a \in A}$ generated with all areas included in the coupling. In practice, the number of markets in the coupling would be small for these hypothetical updates to be computationally demanding.

Figure 3: Marginal contribution to a coupling. The dashed and solid lines respectively represent the data exchange and coupling territory. The black lines refer to the case where all areas participate in market coupling while the blue, red, and green lines are respectively attributed to the cases where either area A, B, or C is excluded from coupling.

4.2 Information Rents from Coupling

Our first result identifies a sufficient condition on the participation fee $R > 0$ to ensure all areas stand to obtain (approximately) a surplus (net benefit) from participation. This surplus should be interpreted as an information rent. It is the benefit an area derives from the fact that its cost information is private so that incentive transfers are needed to elicit truthful participation.

Proposition 1. Consider the mechanism defined by the incentive payments $\Delta \pi_{a,k}$ in (30) and assume the participation fee satisfies the following condition:

$$R \leq \min_{a \in A} \{C_{a,0} + \tilde{C}_a - C^*\}$$

(31)
Truthful participation in the mechanism yields approximately the following change in optimal value, i.e.,

$$V_{a,T+1} - V_{a,0} + \sum_{k=0}^{T} \Delta \pi_{a,k+1} + R \leq \delta_T,$$

(32)

where $T$ denotes the iteration at which the algorithm stops, and $\delta_T \geq 0$. Further, $\delta_T \to 0$ as $T \to \infty$.

Proof. At the $k$-th iteration, the change in optimal value (inclusive of incentive transfer) for area $a$ is

$$V_{a,k+1} - V_{a,k} + \Delta \pi_{a,k} = C_{a,k+1} - C_{a,k} + M_{a,k},$$

where the equality follows by the cancellation of the term $r_{a,k+1} - r_{a,k}$ when we substitute in (28) for $V_{a,k}$ and (30) for $\Delta \pi_{a,k}$. When we expand the definition of the marginal contribution in (27), we obtain

$$V_{a,k+1} - V_{a,k} + \Delta \pi_{a,k} = C_{a,k+1} - C_{a,k} + (C_{a,k+1} - C_{a,k}) - (\tilde{C}_{a,k+1} - \tilde{C}_{a,k})$$

(34)

where we recall the following notation: $C_{a,k} = \sum_{g \in G_a} C_g(P_{a,g}^k)$, $C_{a,k} = \sum_{a' \in A \setminus \{a\}} C_{a',k}$, and $\Delta C_{a',k} = C_{a',k+1} - C_{a',k}$, and use $M_{a,k} = (C_{a,k+1} - C_{a,k}) - (\tilde{C}_{a,k+1} - \tilde{C}_{a,k})$. Upon stopping after $T$-iterations, the total change in the optimal value for area $a$ with the incentive transfers is given by

$$\sum_{k=0}^{T} [V_{a,k+1} - V_{a,k} + \Delta \pi_{a,k}] = \sum_{k=0}^{T} (C_{a,k+1} - C_{a,k}) + \sum_{k=0}^{T} [(C_{a,k+1} - C_{a,k}) - (\tilde{C}_{a,k+1} - \tilde{C}_{a,k})]$$

(35)

$$= C_{a,T+1} - C_{a,0} + C_{a,T+1} - \tilde{C}_{a,T+1}$$

(36)

$$= C_{T+1} - (C_{a,0} + \tilde{C}_{a,T+1}).$$

(37)

The first equality follows when we substitute in (34) for the corresponding terms inside the first summation. The second equality follows from telescoping sum of terms. The last equality follows by the definition $C_k = C_{a,k} + C_{a,k}$ for $k = T + 1$.

Next, we consider the total benefits to area $a$ by including the change in optimal value, incentive transfers and the participation fee:

$$\sum_{k=0}^{T} [V_{a,k+1} - V_{a,k} + \Delta \pi_{a,k}] + R \leq (C^* - C_{a,0} - \tilde{C}_{a}) + \min_{a \in A} \{C_{a,0} + \tilde{C}_{a} - C^*\} + \delta_T \leq \delta_T$$

(38)

where we define $\delta_T := C_{T+1} - C^* + \tilde{C}_{a} - \tilde{C}_{a,T+1}$. We obtain the first inequality in (38) by adding and subtracting $C^*$ and $\tilde{C}_{a}$ to (37), and using the lower bound for $R$ in (31). The second inequality follows from the observations that $(C^* - C_a, 0 - \tilde{C}_a) < 0$ and $|C^* - C_{a,0} - \tilde{C}_a| \geq \min_{a \in A} |(C^* - C_{a,0} - \tilde{C}_a)|$ for all $a \in A$.

By Theorem 1, the outcome sequence generated, $\tilde{x}^k = (\tilde{T}^k, \tilde{\theta}^k, \tilde{\alpha}^k)$, converges to the optimal solution of the centralized DC-OPF without area $a$ participating in the coupling, i.e., the optimal solution when the tieline flows that involve area $a$ are set to zero, i.e., $\tilde{x}^k \to \tilde{x}$, and, $C_{a',k} \to \tilde{C}_{a'}$. Thus, $\delta_T \to 0$ as $T \to \infty$. 

\qed
In Proposition 1, we define individual rationality using the net gain at the end of the iterative mechanism. While at each iteration some areas may lose money, no area will lose more than $\delta_T$ amount by time $T$ with $\delta_T$ shrinking to zero. The proof uses the fact that the convergence result in Theorem 1 also applies in the case that area $a$ is excluded from the coupling. In particular, we have $\bar{V}_a^{T'} \rightarrow \bar{V}_a^\epsilon$. This implies that the total costs of all areas except area $a$ at time $T$, $C_{-a,T} = \sum_{a' \in A \setminus \{a\}} \bar{C}_{a',T}$, converges to $\bar{C}_{-a}$ which is the optimal cost to the centralized DC-OPF problem when area $a$ is excluded from the coupling.

### 4.3 Incentive Compatibility

Our next result again relies on the convergence of the iterative mechanism to show that truthful participation strategy is an $\epsilon$-Nash equilibrium strategy.

**Theorem 2.** Given incentive transfers as in (30), truthful participation in the mechanism is $\epsilon(T)$-Nash equilibrium in convex reporting strategies where $\epsilon(T) \rightarrow 0$ as $T \rightarrow \infty$.

**Proof.** Consider area $a$ reporting information consistent with strictly convex differentiable cost functions $C_{a,g}^\epsilon \neq C_{a,g}$ yielding mechanism output denoted by $\hat{\bar{x}}^k = (\hat{T}^k, \hat{\theta}^k, \hat{\alpha}^k)$. Under such untruthful reporting, let us denote by $C_{a,k}^\epsilon$ the true total cost for area $a$ after $k$ iterations and $r_{a,k}$ the intertie flow payments, $C_{a,k}^\epsilon := \sum_{g \in g_a} C_{a,g}(\hat{P}_{a,g}^k)$, and $r_{a,k}^\epsilon := \sum_{(i,j) \in T} \hat{V}_{a',j}^k \hat{a}_{(i,j)}^k - \sum_{(i,j) \in T} \hat{F}_{a(i,j)}^k \left( \hat{T}_{a(i,j)}^k - \hat{T}_{a(i,j)}^k \right)$. We also define $C_{-a,k}^\epsilon$ as the true total cost of all areas except area $a$ at iteration $k$ when area $a$ is being untruthful. Upon stopping after $T$-iterations the total change in cost inclusive of incentive transfers for untruthful reporting is

$$\sum_{k=0}^{T} [V_{a,k+1} - V_{a,k} + \Delta p_{a,k}^\epsilon] = \sum_{k=0}^{T} (C_{a,k+1}^\epsilon - C_{a,k}^\epsilon) + \sum_{k=0}^{T} [(C_{-a,k+1}^\epsilon - C_{-a,k}^\epsilon) - (\bar{C}_{-a,k+1} - \bar{C}_{-a,k})] \tag{39}$$

$$= C_{a,T+1}^\epsilon - C_{a,0}^\epsilon + C_{-a,T+1}^\epsilon - \bar{C}_{-a,T+1} \tag{40}$$

$$= C_{T+1}^\epsilon - (C_{a,0}^\epsilon + \bar{C}_{-a,T+1}) \tag{41}.$$}

(40) is derived using telescoping sum and the fact that initial costs for truthful and untruthful reporting is equal, that is, $C_{a,0}^\epsilon = C_{a,0}$ since the initial power flow for each area is fixed. (41) follows by $C_{T+1}^\epsilon = C_{a,T+1}^\epsilon + C_{-a,T+1}^\epsilon$. Thus, upon stopping after $T$-iterations the total change in cost inclusive of incentive transfers and participation fee for area $a$ can be written as the following when we substitute in (41) for total change in cost:

$$\sum_{k=0}^{T} [V_{a,k+1} - V_{a,k} + \Delta p_{a,k}^\epsilon] + R = (C_{T+1}^\epsilon - (C_{a,0}^\epsilon + \bar{C}_{-a})) + R + \delta_{a,T}^F \tag{42}$$

where $\delta_{a,T}^F = (C_{T+1}^\epsilon - C_{-a}^\epsilon + \bar{C}_{-a} - \bar{C}_{-a,T+1})$. We compare the total changes in cost of area $a$ when it is
untruthful \((\ref{eq:untruthful})\) with when it is truthful \((\ref{eq:truthful})\),
\[
\sum_{k=0}^{T} [V_{a,k+1}^{F} - V_{a,k}^{F} + \Delta \pi_{a,k}^{F}] + R - \left(\sum_{k=0}^{T} [V_{a,k+1} - V_{a,k} + \Delta \pi_{a,k}] + R\right) = (C^{F} - C^{*}) + (\delta_{a,T}^{F} - \delta_{a,T}) \geq \inf_{C_{a,g}^{F}} (\delta_{a,T}^{F} - \delta_{a,T}). \tag{44}
\]

Note that we get \((\ref{eq:untruthful})\) by substituting in \((\ref{eq:truthful})\) and \((\ref{eq:untruthful})\) for the truthful and untruthful costs respectively, and canceling out the participation fees. The inequality follows from optimality of truthful solution, i.e., \(C^{*} \leq C^{F}\). By convergence of the mechanism’s output to optimal values, it follows that \(\inf_{C_{a,g}^{F}} (\delta_{a,T}^{F} - \delta_{a,T}) \to 0\) as \(T \to \infty\). Defining \(\epsilon(T) := \inf_{C_{a,g}^{F}} (\delta_{a,T}^{F} - \delta_{a,T})\), the result follows.

\section{Budget Balance}

Finally we show the mechanism obtains a surplus associated with the value of trade at the tielines under a sufficient condition on the participation fee \(R > 0\).

We define the mechanism’s net budget after \(T\) iterations as the sum of all incentive payments \((\ref{eq:incentive})\) and participation fees,
\[
B(T) := |A| R + \sum_{a \in A} \sum_{k=0}^{T} \Delta \pi_{a,k+1}. \tag{45}
\]

\textbf{Theorem 3.} \textit{If \(R \geq \min_{a \in A} \{\tilde{C}_{-a} + C_{a,0} - C^{*}\} \) and}
\[
C_{0} - C^{*} \geq \sum_{a \in A} \left[ (\tilde{C}_{-a} + C_{a,0} - C^{*}) - \min_{a} \{\tilde{C}_{-a} + C_{a,0} - C^{*}\} \right], \tag{46}
\]

then the net budget is bounded below by the sum of all trades,
\[
\lim_{T \to \infty} B(T) \geq \sum_{(i,j) \in T} (\alpha_{a,i,j}^{*} - \alpha_{a,i}^{*}) T_{a,(i,j)}, \tag{47}
\]

where \(T := \bigcup_{a \in A} T_{a}\) denotes the set of all tielines.

\textit{Proof.} The mechanism’s net revenue comprised of incentive transfers \((\Delta \pi_{a,k})\) and participation fees \(|A| R\) over all regions can be written as by substituting \((\ref{eq:incentive})\) for \(\Delta \pi_{a,k}\) :
\[
B(T) = |A| R + \sum_{a \in A} \sum_{k=0}^{T} \Delta C_{a,k} - \Delta \tilde{C}_{a,k} + r_{a,k+1} - r_{a,k}. \tag{48}
\]

Telescoping sum over \(k\) for the second term on the right hand side yields
\[
B(T) = |A| R + \sum_{a \in A} \left[ C_{a,T+1} - C_{a,0} - (\tilde{C}_{a,T+1} - \tilde{C}_{a,0}) + r_{a,T+1} - r_{a,0} \right] \tag{49}
\]
\[
= |A| R + \sum_{a \in A} \left[ C_{T+1} - \tilde{C}_{a,T+1} - C_{a,0} + r_{a,T+1} - r_{a,0} \right] + \sum_{a \in A} \left( C_{a,0} - C_{a,T+1} \right) \tag{50}
\]

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where the second equality follows by adding and subtracting $\sum_{a \in A} C_{a,T+1}$ and $\sum_{a \in A} C_{a,0}$, and by using $C_{T+1} = C_{a,T+1} + C_{-a,T+1}$ and $C_{a,0} = C_{a,0}$. We get the following lower bound by using the minimum value of the participation fee

$$B(T) \geq |A| \min_a \{ \tilde{C}_{-a} + C_{a,0} - C^* \} + \sum_{a \in A} [C_{T+1} - \tilde{C}_{-a,T+1} - C_{-a,0} + r_{a,T+1} - r_{a,0}] + \sum_{a \in A} (C_{a,0} - C_{a,T+1}).$$

(51)

We reorganize the terms to get

$$B(T) \geq \sum_{a \in A} [C_{T+1} - (\tilde{C}_{-a,T+1} + C_{a,0}) + \min_a \{ \tilde{C}_{-a} + C_{a,0} - C^* \}] + \sum_{a \in A} (r_{a,T+1} - r_{a,0}) + C_0 - C_{T+1}$$

(52)

$$= \sum_{a \in A} [C^* - (\tilde{C}_{-a} + C_{a,0}) + \min_a \{ C_{a,0} + \tilde{C}_{-a} - C^* \}] + \sum_{a \in A} (r_{a,T+1} - r_{a,0}) + C_0 - C^* + \delta_T$$

(53)

where $\delta_T := \sum_{a \in A} [C_{T+1} - C^* - (\tilde{C}_{-a,T+1} - \tilde{C}_{-a})] + C^* - C_{T+1}$. We get (53) by adding and subtracting $\sum_{a \in A} C^*$, $\sum_{a \in A} \tilde{C}_{-a}$, and $C^*$. Now we use (46) to get

$$B(T) \geq \sum_{a \in A} [r_{a,T+1} - r_{a,0}] + \delta_T. $$

(54)

Setting initial locational marginal prices and capacity prices associated with the tielines to zero, we have $r_{a,0} = 0$. As before, we have $\delta_T \to 0$ as $T \to \infty$ by Theorem 1. Thus we get

$$\lim_{T \to \infty} B(T) \geq \sum_{a \in A} \sum_{(i,j) \in T_a} \alpha^*_{a,i,j} T^*_{a,(i,j)} - \sum_{(i,j) \in T_a} \frac{\mu^*_{(i,j)}}{2} (|T^*_{a,(i,j)}| - \tilde{T}_{a,(i,j)})$$

(55)

by taking the limit of $r_{a,T+1}$ in (20) as $T \to \infty$. Note that the capacity price associated terms above are zero via complementary slackness (23). The relation in (55) follows by noting that $T_{a,(i,j)} = -T^*_{a',(j,i)}$ and letting $T$ be the set of all tielines.

Remark (Efficiency vs Information Rents): Condition (46) can be interpreted as a requirement on the efficiency of coupling all areas in $A$. The right hand side is the total sum of individual surpluses or sum total of information rents (i.e. the cost entailed by coupling all areas in a truthful manner). The left hand side is the total system benefit from coupling $C_0 - C^*$. Therefore, if the inequality does not hold then the total information rent (i.e. costs of coupling all areas in $A$) exceeds the benefits from coupling.

Remark (Participation vs Incentive Compatibility): Proposition 1 and Theorem 3 characterize the trade-offs between a lower participation fee (which ensures all areas benefit from coupling) versus a higher participation fee (which ensures there is no deficit involved when coupling all areas). Only when the participation fee is set to equal the minimum individual net cost reduction across all areas, i.e. $R = \min_{a \in A} \{ \tilde{C}_{-a} + C_{a,0} - C^* \}$ the mechanism can offer non-negative net cost reductions for all areas and incur no deficit.
4.5 A Toy Example

Suppose the abstract test system in Fig. 4, incorporating three areas A, B, and C, with the generation cost functions, power generation limits, and loads specified below/beside each area, and power flow capacity and reactances specified above and below the tielines. We show the operation costs of areas in Table 1 for five coupling scenarios, ranging from full coupling to independent operation of areas, and furnish the optimal coupling decisions in Table 2.

$$C_A(P_A) = 0.05P_A^2 + 15P_A + 40$$
$$0 \leq P_A \leq 600MW$$
$$D_A = 150MW$$

$$C_B(P_B) = 0.07P_B^2 + 20P_B + 50$$
$$0 \leq P_B \leq 600MW$$
$$D_B = 150MW$$

$$C_C(P_C) = 0.1P_C^2 + 30P_C + 60$$
$$0 \leq P_C \leq 600MW$$
$$D_C = 550MW$$

Figure 4: Three-bus test system

Table 1: Operation costs of areas w.r.t coupling scenarios for the three-bus test system (the symbols “+/−” between areas indicate whether the associated areas are coupled/decoupled)

| Coupling Scenarios | (A+B+C) | A,(B+C) | (A+C),B | (A+B),C | A,B,C |
|--------------------|---------|---------|---------|---------|-------|
| Operation Cost of A| 15,568  | 3,415   | 11,415  | 4,895   | 3,415 |
| Operation Cost of B| 8,454   | 15,625  | 4,625   | 2,893   | 4,625 |
| Operation Cost of C| 9,436   | 22,810  | 22,810  | 46,810  | 46,810 |
| Total Operation Cost| 33,457  | 41,850  | 38,850  | 54,598  | 54,850 |

4.5.1 Sensitivity Analysis w.r.t Participation Fee

Given the data in Table 1, we report the marginal contributions of areas to coupling and their total cost reductions in Table 3 for the exact estimation of the participation fee (i.e., $R = \min_{a \in A} \{C_a - C_{a,0} - C^*\}$), as well as its overestimation and underestimation. The exact value of the participation fee equals to $5,393, as shown in the first row of Table 3 while the total cost reduction of each area amounts to the sum of
Table 2: Optimal coupling decisions of the three-bus test system: locational marginal prices, tieline power flows, and capacity prices

| Locational Marginal Prices ($/MWh) | Power Flows (MW) | Capacity Prices ($/MWh) |
|-------------------------------------|-------------------|-------------------------|
| \(\alpha_{A,i} \) | \(\alpha_{B,j} \) | \(\alpha_{C,k} \) | \(T_{A,(i,j)} \) | \(T_{A,(i,k)} \) | \(T_{B,(j,k)} \) | \(\mu_{(i,j)} \) | \(\mu_{(i,k)} \) | \(\mu_{(j,k)} \) |
| 57.7 | 52.5 | 68.2 | 118.1 | 159.0 | 200.0 | 0 | 0 | 21.0 |

participation fee, marginal contribution to coupling, and the internal cost saving of that area. Since none of the areas incur any loss, the exact participation fee corroborates the individual rationality of the proposed mechanism. The congestion rent in Table 2 represents the net monetary value of all tieline transactions, given by the term \(\sum_{(i,j) \in T}(\alpha_{a,j}^* - \alpha_{a,i}^*)T_{a,(i,j)} \) in the right hand side of (47), which we calculate using the data in Table 2. In addition, the mechanism’s surplus indicates the net budget of the mechanism (defined as \(B(T) \) in (45)) minus the congestion rent. Note that we subtract the congestion rent from the net budget as it has a different claimant.

Now, imagine the market maker overestimates or underestimates the participation fee by $2,500, as we show in the second and third rows of Table 3, respectively. In the former case, area B incurs a loss of $2,500 as a result of participating in market coupling, and probably opts out, while in the latter case all areas benefit from participating in the market coupling, yet the mechanism leads to a deficit of $4,855. It is worth highlighting that, in the cases of exact estimation and overestimation of participation fees both conditions of the Theorem 3 hold, thus, the mechanism’s surplus \((B(T) \text{ minus congestion rent})\) is non-negative as anticipated by Theorem 3. However, in the case of underestimated participation fee, the first condition of Theorem 3 is violated leading to a negative surplus and violation of budget balance.

Table 3: Summary of areas cost reduction w.r.t changes in participation fee

| Marginal Contributions to Coupling ($) | Total Cost Reduction ($) | Congestion Rent ($) | Mechanism’s Surplus ($) |
|---------------------------------------|-------------------------|---------------------|-------------------------|
| A | B | C | A | B | C | A | B | C |
| 5,393 | -20,545 | -9,222 | 16,234 | 3,000 | 0 | 15,748 | 4,193 | 2,645 |
| 7,893 | -20,545 | -9,222 | 16,234 | 500 | -2,500 | 13,248 | 4,193 | 10,145 |
| 2,893 | -20,545 | -9,222 | 16,234 | 5,500 | 2,500 | 18,248 | 4,193 | -4,855 |

4.5.2 Untruthful Reporting

Here we aim to investigate how the untruthful reporting of area A impacts its total cost reduction. To this end we consider two cases: Case 1 where area A uses a cost function greater than its truthful cost function by a factor of 1.1, i.e., \(C_A^D(P_A) = 1.1C_A(P_A)\); Case 2 where area A uses a cost function lower than its truthful cost function by a factor of 0.9, i.e., \(C_A^D(P_A) = 0.9C_A(P_A)\). We present the cost reduction analysis of the two cases in Table 4 for the participation fee equal to $5,393. As shown in Table 4, and compared to the first row of Table 3 any deviation from truthful reporting leads to a lower cost reduction for Area A, thus, area A is better off reporting truthfully. In addition, untruthful reporting of area A may cause other areas undergo loss and opt out, such as area B in Case 1, or may lead to reducing the mechanism’s surplus compared to truthful reporting, as in Case 2.
Table 4: Summary of areas cost reduction for untruthful reporting of area A: Cases 1 and 2 respectively refer to overbidding and underbidding.

| Participation Fee ($) | Marginal Contributions to Coupling ($) | Total Cost Reduction ($) | Congestion Rent ($) | Mechanism’s Surplus ($) |
|-----------------------|----------------------------------------|--------------------------|---------------------|-------------------------|
|                       | A | B | C | A | B | C | A | B | C |
| Case 1                | 5,393 | -18,795 | -9,958 | 16,555 | 2,919 | -330 | 14,742 | 3,982 | 3,981 |
| Case 2                | 5,393 | -22,408 | -8,609 | 15,837 | 2,899 | 511 | 16,884 | 4,428 | 998 |

5 A Practical Implementation

In proving theorems 2 and 3 we assume the marginal contributions to coupling were readily available to the market maker. In practice, these marginal contributions have to be estimated based only on the reported information by areas. In the following, we show how estimates of the marginal contribution to coupling that only rely on information shared during the distributed iterative mechanism in Algorithm 1, i.e., locational marginal and capacity prices, and intertie flows. We show that the results in Theorem 2 and 3 carry over with an additional step-size dependent error term for approximate incentive transfer and participation fees.

We use the first order term Taylor expansion of the optimal value to define the incentive payments at each step,

\[ V_{a,k+1} = V_{a,k} + \Delta V_{a,k} + \mathcal{O}(\beta^2) \] (56)

where we can state the change in optimal value for area \( a \) by

\[ \Delta V_{a,k} = - \sum_{(i,j) \in T_a} \left[ \frac{\hat{T}^k_{a,(i,j)}}{2} (\alpha_{a,i}^k - \alpha_{a,j}^{k-1}) - \frac{1}{2} \frac{\hat{T}^k_{a,(i,j)}}{\hat{x}_{a,(i,j)}} (\mu_{(i,j)}^k - \mu_{(i,j)}^{k-1}) - \frac{\hat{x}_{a,(i,j)}}{2} (\theta_{a,i}^k - \theta_{a,j}^{k-1}) \right]. \] (57)

See Proposition 3 in the Appendix for the derivation of (57). Using (28), the change in cost for area \( a' \in A \) can be written as:

\[ \Delta C_{a',k} = V_{a,k+1} - V_{a,k} + r_{a,k+1} - r_{a,k} \] (58)

\[ = \Delta V_{a,k} + r_{a,k+1} - r_{a,k} + \mathcal{O}(\beta^2). \] (59)

The second equality follows by using (56) in (58). We have the identical expression for the scenario with area \( a \) excluded from the coupling, i.e., \( \Delta C_{a',k} = \Delta V_{a,k} + r_{a,k+1} - r_{a,k} + \mathcal{O}(\beta^2) \). We define the estimated changes in cost for area \( a' \) by getting rid of the \( \mathcal{O}(\beta^2) \) terms in (59), i.e.,

\[ \hat{\Delta} C_{a',k} = \Delta V_{a',k} + r_{a',k+1} - r_{a',k}. \] (60)

We note that all of the terms of \( \hat{\Delta} C_{a',k} \) in (60) can be computed using publicly available information, i.e., locational marginal prices, and intertie flows and angles, \( x^k_a = (T^k_a, \theta^k_a, \alpha^k_a) \), and intertie capacity prices \( \mu_{(i,j)}^k \). In particular, \( r_{a',k} \) in (29) can be computed using locational marginal and capacity prices, and intertie flows shared among areas. Moreover, \( \hat{\Delta} V_{a',k} \) in (57) can be computed using the neighboring areas’ local information \( x^k_a \), and the following first order condition,

\[ \hat{\alpha}_{a,(i,j)}^k = \alpha_{a,i}^{k-1} - \frac{\hat{T}^k_{a,(i,j)}}{2} \text{sign}(\hat{T}^k_{a,(i,j)}) - \alpha_{a,j}^k. \] (61)
The above condition implies $\hat{\xi}^k_{a,(i,j)}$ is a function of locally available information. Using the definitions above, we define the approximate incentive transfer for area $a$ as follows:

$$\hat{\Delta} \pi_{a,k} = \sum_{a' \in A \backslash \{a\}} \hat{\Delta} C_{a',k} - \hat{\Delta} C_{a',k} + r_{a,k+1} - r_{a,k}. \quad (62)$$

Next, we provide the corresponding individual rationality, incentive compatibility and budget balance results given approximate incentive payments (62)—see Appendix 8.2 for proofs.

**Proposition 1’**. Consider the mechanism defined by the approximate incentive payments $\hat{\Delta} \pi_{a,k}$ in (62) and assume the participation fee satisfies the following condition:

$$R \leq \min_{a \in A} \{C_a + \tilde{C}_a - C^*\}. \quad (63)$$

Truthful participation in the mechanism yields a cost reduction that is bounded by $\epsilon_1(\beta, T)$, i.e.,

$$V_{a,T+1} - V_{a,0} + \sum_{k=0}^{T} \hat{\Delta} \pi_{a,k} + R \leq \epsilon_1(\beta, T)$$

where $T$ denotes the iteration at which the algorithm stops. Further, $\epsilon_1(\beta, T) \to 0$ as $\beta \to 0^+, T \to \infty$.

**Theorem 2’**. Given approximate incentive transfers as in (62), truthful participation in the mechanism is $\epsilon_2(\beta, T)$-Nash equilibrium in convex reporting strategies where $\epsilon_2(\beta, T) \to 0$ as $\beta \to 0^+, T \to \infty$.

**Theorem 3’**. We define the mechanism’s approximate net budget after $T$ iterations as the sum of all approximate incentive payments (62), and participation fees, $\hat{B}(T) := |A|R + \sum_{a \in A} \sum_{k=0}^{T} \hat{\Delta} \pi_{a,k}$. If $R \geq \min_{a \in A} \{\tilde{C}_a + C_a, 0 - C^*\}$ and

$$C_0 - C^* \geq \sum_{a \in A} [(\tilde{C}_a + C_a, 0 - C^*) - \min_{a} \{\tilde{C}_a + C_a, 0 - C^*\}], \quad (64)$$

the approximate net budget is bounded below by the sum of all trades,

$$\lim_{T \to \infty} \hat{B}(T) \geq \sum_{(i,j) \in T} (\alpha_{a',j}^* - \alpha_{a,i}^*)T_{a,(i,j)}, \quad (65)$$

where $T := \bigcup_{a \in A} T_a$ denotes the set of all tielines.

The results above show that the market maker can approximate the marginal contribution of each area using locally shared information during the iterative updates. The accuracy of the approximation is a function of the step-size $\beta$ and iteration steps $T$. It reduces when step-size is smaller and iteration time $T$ is longer.

### 6 Numerical Testbed

Here we provide the numerical results for the three-zone IEEE Reliability Test System (IEEE-RTS) with 96 generating units, 73 buses, 115 transmission lines, and 5 tielines. The topology of the test system under study, the operation limitations of generating units and lines/tielines, as well as the nodal load data are available in [27]. We make the following changes to the original network in order to simulate the occasion where transmission lines/tielines are congested:
• The capacity of transmission line located in area 1 which connects the buses (16, 17) is reduced from 500MW to 200MW, e.g., $F_{1,(16,17)}=200\text{MW}$.

• The capacity of transmission line located in area 2 which connects the buses (3, 24) is reduced from 400MW to 150MW, e.g., $F_{2,(3,24)}=150\text{MW}$.

• The capacity of tieline 4 is reduced from 500MW to 100MW.

For quick accessibility, we also provide the tieline data in Table 5, where the set of areas is defined as $A = \{A, B, C\}$, in accordance with [27].

| Tieline Number | Sending End (Area-Bus) | Receiving End (Area-Bus) | Reactance (pu) | Capacity (MW) |
|----------------|------------------------|--------------------------|----------------|--------------|
| TL1            | A-7                    | B-3                      | 0.16           | 175          |
| TL2            | A-13                   | B-15                     | 0.08           | 500          |
| TL3            | A-23                   | B-17                     | 0.07           | 500          |
| TL4            | C-25                   | A-21                     | 0.10           | 100          |
| TL5            | C-18                   | B-23                     | 0.10           | 500          |

We carry out the simulation for a single hour corresponding to the peak load of the three-zone IEEE-RTS, and assume all generating units are online. In addition to the simulation results of the proposed incentive-compatible approach, we also furnish the results associated with locational marginal price remuneration scheme, where the areas are merely paid/charged for the exported/imported energy over tielines at the locational marginal prices of boundary buses. This remuneration scheme is in accordance with the definition of optimal value in (28), and serves as a benchmark case. The cost functions are quadratic.

6.1 Numerical Results of Iterative Capacity Allocation

Here we present the numerical results of implementing the proposed iterative capacity allocation method on the three-area IEEE-RTS. The constant step-size associated with the capacity price updates is $\beta = 0.3$, the diminishing step-size associated with intertie power flow updates as well as the voltage phase angles and locational marginal price updates are deemed as $\rho_k = \frac{1}{1 + \log(k)}$.

The simulation results indicate that the values obtained from the iterative capacity allocation converge to that of the centralized DC-OPF model, confirming the efficiency of the proposed method. Figures 5-(a) and (b) respectively represent the intertie power flows and the associated capacity prices for each iteration, converging after 175 iterations. The initial values of intertie power flows are all zero, meaning that the initial state corresponds to the independent operation of areas. In addition, the capacity prices in Fig. 5-(b) are all initialized at $50$ per MWh, which is slightly greater than the highest incremental cost rate of the most expensive generator. The power flow of tieline number 4 converges to its maximum limit, 100MW, and bears a capacity price of $15.6$ per MWh, while the power flows of other tielines remain within their operating limits and all the associated capacity prices converge to zero. We present the locational marginal prices at tieline incident buses, or indeed the import/export quotes of the areas, in Fig. 6 for all iterations. Further, we provide the optimal intertie power flows, capacity prices, and the locational marginal prices at incident buses in Table 6 as they are essential to calculating the congestion rent.
Figure 5: (a) Tieline power flows, $T^k_{a,(i,j)}$ (MW) (b) Tieline capacity prices, $\mu^k_{a,(i,j)}$ ($/MW$)

Table 6: Optimal coupling decisions for the three-area IEEE-RTS: Tieline power flows, capacity prices, and locational marginal prices at incident buses

| Tieline Number | Power Flow (MW) | Capacity Price ($/MWh$) | Sending End LMP ($/MWh$) | Receiving End LMP ($/MWh$) |
|----------------|----------------|--------------------------|---------------------------|-----------------------------|
| TL1            | 13.1           | 0                        | 31.1                      | 47.8                        |
| TL2            | -136.5         | 0                        | 12.7                      | 7.3                         |
| TL3            | -34.3          | 0                        | 13.0                      | 10.2                        |
| TL4            | -100.0         | 15.6                     | 15.9                      | 7.5                         |
| TL5            | -29.3          | 0                        | 16.3                      | 16.8                        |

6.2 Incentive Payment Analysis

Here we first provide the results of truthful reporting and next consider the untruthful behaviour of areas.

6.2.1 Truthful Reporting

We provide the operation costs of areas in Table 7 for the full coupling and independent operation of areas, as well as coupling with one area left out. As expected, any coupling leads to a reduction in total operation cost of the system compared to independent operation of areas, and the highest system saving, $3,866, is realized for the case of full coupling.

Given the information in Table 7, the best estimate of the participation fee equals to $1,311, as shown in Table 8. Area B receives the highest reward due to marginal contribution to coupling and the area A comes next; however, the marginal contribution of area C to coupling is a positive number meaning that it is charged rather than being rewarded (Table 8). The sum of participation fee, marginal contribution to coupling, and the internal cost saving of each area amounts to its total cost reduction. The proposed incentive payment approach results in non-negative cost reductions for all areas, where areas A and C respectively benefit the most and least. Since the participation fee meets the first condition of Theorem 3 and the second condition
Table 7: Operation costs of areas w.r.t coupling scenarios for the three-area IEEE-RTS (the symbols “+”/“,” between areas indicate whether the associated areas are coupled/decoupled)

| Coupling Scenarios  | (A+B+C) | A,(B+C) | (A+C),B | (A+B),C | A,B,C |
|----------------------|---------|---------|---------|---------|-------|
| Operation Cost of A  | 29,822  | 31,843  | 32,757  | 28,986  | 31,843|
| Operation Cost of B  | 34,062  | 35,295  | 33,366  | 33,667  | 33,366|
| Operation Cost of C  | 25,995  | 26,162  | 26,534  | 28,537  | 28,537|
| Total Operation Cost | 89,879  | 93,300  | 92,657  | 91,190  | 93,745|

also holds true (sum of area cost reductions is less than the system cost saving, i.e., $3,576 < $3,866), the mechanism bear a non-negative surplus as suggested by Theorem 3.

6.2.2 Untruthful Reporting

In this part we consider three untruthful reporting cases where at each case only one area attempts to use untruthful generation cost functions in its internal DC-OPF model. The devised untruthful cost functions are greater than truthful cost functions by a factor of 1.1, meaning that, the areas A, B, and C respectively use the cost functions $C_D^{A,g}(P_{A,g}) = 1.1C_A(g,P_{A,g}), g \in G_A$, $C_D^{B,g}(P_{B,g}) = 1.1C_B(g,P_{B,g}), g \in G_B$, and $C_D^{C,g}(P_{C,g}) = 1.1C_C(g,P_{C,g}), g \in G_C$. We have provided the total cost reductions of areas in Tables 9 and 10 respectively for the proposed incentive-compatible approach and the locational marginal price remuneration scheme. These tables contain the results for both truthful and untruthful reporting cases.

None of the areas manage to improve their cost reduction through untruthful reporting, and all the attempts lead to lower cost reductions compared to the truthful counterpart (Table 9). Thus, there is no incentive for the areas to deviate from truthful reporting. However, areas B and C acquire a greater cost reduction through reporting untruthfully under the locational marginal price remuneration scheme (Table 10). The locational marginal price remuneration scheme is prone to untruthful behaviour of areas, whereas the proposed approach is not.
Table 8: Summary of areas cost reduction for truthful reporting

| Participation Fee ($) | Marginal Contributions to Coupling ($) | Total Cost Reduction ($) | Congestion Rent ($) | Mechanism’s Net Budget ($) |
|-----------------------|---------------------------------------|--------------------------|---------------------|---------------------------|
| A B C A B C           |                                       |                          |                     |                           |
| 1311 -1,400 -3,474    | 1,230                                 | 2110 1466 0             | 1,872 290           |                           |

Table 9: Cost reductions of areas for truthful and untruthful reporting under proposed incentive payment approach

| Total Cost Reduction ($) |
|--------------------------|
| A B C                    |
| Truthful Reporting       |
| 2,110 1,466 0           |
| A Reports Untruthfully   |
| 2,074 1,796 101         |
| B Reports Untruthfully   |
| 2,272 1,458 47          |
| C Reports Untruthfully   |
| 2,089 1,525 −5          |

7 Conclusions

We have introduced and analyzed a market design for scheduling and pricing power flows between interconnected electricity markets. Each area operator participates by iteratively submitting bids for trading energy across interties and the prices for interconnection capacity are adjusted as a function of excess demand. The proposed design allows individual area operators to retain local control and can be easily implemented after existing short-run markets (e.g. day ahead, hour ahead) have cleared. Incentive transfers (updated at each iteration) remunerate each area with its marginal contribution (i.e. cost savings) to all other participating areas. Thus, the incentives for each area are aligned with the identification of power flows across areas that maximize total economic surplus. Finally, we identify a sufficient condition on a uniform participation fee ensuring the mechanism incurs no deficit.

8 Appendix

8.1 Preliminary results

Proposition 2. If there exists a feasible solution to the decentralized DC-OPF problem (11)-(16) for area a ∈ A when T_{a,(i,j)} = 0 for all (i, j) ∈ T_a, then for each intertie (i, j), there exists a critical price \( \mu_{(i,j)} > 0 \) such that for \( \mu_{(i,j)} \geq \mu^*_{(i,j)} \), the optimal solution to (11) - (16) will have no flow along the intertie, i.e. \( T_{a,(i,j)} = 0 \).

Proof. We prove by contradiction. Suppose that there exists an optimal solution such that for a tieline \( (i^*, j^*) \in T_a \), we have \( T^*_{a,(i^*, j^*)} = \epsilon \neq 0 \) for any capacity price \( \mu_{(i^*, j^*)} > 0 \). Denote the power generation and tieline flows of this optimal solution with \( P^*_{a,g} \) and \( T^*_{a,(i,j)} \), respectively. In this case, the optimal objective value in (11) is given by

\[
\sum_{g \in \mathcal{G}_a} C_{a,g}(P^*_{a,g}) - \epsilon \alpha_{a^*,j^*} + \frac{\mu_{(i^*, j^*)}}{2} (|\epsilon| - \bar{T}_{a,(i^*, j^*)}) - \sum_{(i,j) \in T_a \setminus (i^*, j^*)} \frac{\mu_{(i,j)}}{2} \bar{T}_{a,(i,j)}. \tag{66}
\]
Table 10: Cost reductions of areas for truthful and untruthful reporting under locational marginal price remuneration scheme

|                        | Total Cost Reduction ($) | A     | B     | C     |
|------------------------|--------------------------|-------|-------|-------|
| Truthful Reporting     |                          | 2,886 | 1,556 | 1,296 |
| A Reports Untruthfully |                          | 2,711 | 1,669 | 1,206 |
| B Reports Untruthfully |                          | 2,883 | 1,712 | 1,223 |
| C Reports Untruthfully |                          | 3,015 | 1,571 | 1,298 |

Consider a feasible solution with \( T_{a,(i,j)} = 0 \) for all \((i,j) \in T_a \). We denote the power generation of the feasible solution with zero tieline flows by \( P_{a,g}^0 \). Given the feasible solution, the objective value is given by

\[
\sum_{g \in G_a} C_{a,g}(P_{a,g}^0) - \sum_{(i,j) \in T_a} \mu_{a,(i,j)} T_{a,(i,j)}.
\]

Considering the difference between the objective value in (66) with that of the feasible solution when \( T_{a,(i^*,j^*)} = 0 \), we have

\[
\sum_{g \in G_a} C_{a,g}(P_{a,g}^{\epsilon}) - \epsilon \alpha_{a',j^*} + \frac{\mu_{(i^*,j^*)}}{2} |\epsilon| - \sum_{g \in G_a} C_{a,g}(P_{a,g}^0) < 0
\]

which must be negative by the assumption that \( (P_{a,g}^{\epsilon}, T_{a,(i,j)}) \) is optimal. Observe now that there exists a capacity price value \( \mu_{(i^*,j^*)} > 0 \) such that the difference in objective values (67) becomes positive for any \( \epsilon \neq 0 \). Thus, for each intertie \((i,j)\), there exists a price \( \mu_{(i,j)} > 0 \) such that for \( \mu_{(i,j)} \geq \mu_{(i,j)}^c \) the optimal solution will have no flow along the intertie, i.e. \( T_{a,(i,j)} = 0 \).

**Proposition 3.** A first order approximation to the change in optimal value for area \( a \) is given by:

\[
\Delta V_{a,k} = - \sum_{(i,j) \in T_a} \left[ \hat{T}_{a,(i,j)} (\alpha_{a',j}^k - \alpha_{a',j}^{k-1}) - \frac{1}{2} \hat{T}_{a,(i,j)} |\epsilon| - \frac{\mu_{(i,j)}}{2} \right] - \sum_{g \in G_a} C_{a,g}(P_{a,g}^0) < 0
\]

**Proof.** Change in optimal value is given by

\[
\Delta V_{a,k} \triangleq \sum_{(i,j) \in T_a} \left[ \frac{\partial V_{a,k}}{\partial \alpha_{a',j}} (\alpha_{a',j}^k - \alpha_{a',j}^{k-1}) + \frac{\partial V_{a,k}}{\partial \mu_{(i,j)}} (\mu_{(i,j)}^k - \mu_{(i,j)}^{k-1}) + \frac{\partial V_{a,k}}{\partial \theta_{a',j}} (\theta_{a',j}^k - \theta_{a',j}^{k-1}) \right].
\]

By the envelope theorem, we have

\[
\frac{\partial V_{a,k}}{\partial \alpha_{a',j}} = -\hat{T}_{a,(i,j)} \quad \frac{\partial V_{a,k}}{\partial \mu_{(i,j)}} = \frac{1}{2} \hat{T}_{a,(i,j)} \quad \frac{\partial V_{a,k}}{\partial \theta_{a',j}} = \frac{\hat{\xi}_{a,(i,j)}}{x_{a,(i,j)}}.
\]

**8.2 Proofs**

**Proof of Proposition 1.**

\[
\square
\]
At the $k$-th iteration, the change in optimal value inclusive of incentive transfer for area $a$ is

$$V_{a,k+1} - V_{a,k} + \hat{\Delta} \pi_{a,k} = C_{a,k+1} - C_{a,k} + \sum_{a' \in A \setminus \{a\}} [\hat{\Delta} C_{a',k} - \hat{\Delta} \tilde{C}_{a',k}], \quad (68)$$

where the equality follows by the cancellation of the term $r_{a,k+1} - r_{a,k}$ when we substitute in (28) for $V_{a,k}$ and (62) for $\hat{\Delta} \pi_{a,k}$.

$$V_{a,k+1} - V_{a,k} + \hat{\Delta} \pi_{a,k} = C_{a,k+1} - C_{a,k} + (C_{a,k+1} - C_{a,k}) - (\tilde{C}_{a,k+1} - \tilde{C}_{a,k}) + 2 |A - 1| O(\beta^2)$$

where

$$C_{a,k} = \sum_{g \in G_a} C_g(\hat{T}^k_{a,g}) \quad C_{a,k} = \sum_{a' \in A \setminus \{a\}} C_{a',k}.$$

Upon stopping after $T$-iterations the total change in cost inclusive of incentive transfers and participation fee for area $a$ is

$$\sum_{k=0}^{T} |V_{a,k+1} - V_{a,k} + \hat{\Delta} \pi_{a,k}| + R \leq (C^* - (C_{a,0} + \tilde{C}_{-a})) + \min \{C_{a,0} + \tilde{C}_{-a} - C^*\} + \epsilon_1(\beta, T) \quad (69)$$

$$\leq \epsilon_1(\beta, T). \quad (70)$$

where the inequality follows optimality of coupling all areas and

$$\epsilon_1(\beta, T) = C_{T+1} - C^* + \tilde{C}_{-a} - \tilde{C}_{-a,T+1} + 2T |A - 1| O(\beta^2)$$

By Theorem 1, $\epsilon_1(\beta, T) \to 0$ as $\beta \to 0^+$ and $T \to \infty$.

**Proof of Theorem 2.**

Consider area $a$ reporting information consistent with strictly convex differentiable cost functions $C^{F}_{a,g}(\cdot) \neq C_{a,g}(\cdot)$ yielding mechanism output denoted by

$$\hat{x}^{F,k} = (\hat{T}^{F,k}, \hat{\theta}^{F,k}, \hat{\alpha}^{F,k})$$

Upon stopping after $T$-iterations the total change in cost inclusive of incentive transfers and participation fee for area $a$ is

$$\sum_{k=0}^{T} |V_{a,k+1}^{F} - V_{a,k}^{F} + \hat{\Delta} \pi_{a,k}^{F}| + R \leq (C^F - (C_{a,0}^{F} + \tilde{C}_{-a}^{F})) + R + \delta_{T,\beta}^{F} \quad (71)$$

where

$$\delta_{T,\beta}^{F} = C_{T+1}^{F} - C^F + \tilde{C}_{-a}^{F} - \tilde{C}_{-a,T+1}^{F} + 2T |A - 1| O(\beta^2) \quad (72)$$

Similarly, we can write the total change in cost from truthful reporting:

$$\sum_{k=0}^{T} |V_{a,k+1} - V_{a,k} + \hat{\Delta} \pi_{a,k}| + R \leq (C^* - (C_{a,0} + \tilde{C}_{-a})) + R + \delta_{T,\beta} \quad (73)$$

where

$$\delta_{T,\beta} = C_{T+1} - C^* + \tilde{C}_{-a} - \tilde{C}_{-a,T+1} + 2T |A - 1| O(\beta^2). \quad (74)$$
Here again, by convergence of the mechanism’s output it follows that
\[ \inf \]
where the inequality follows from optimality of truthful solution and \( \epsilon_2(\beta, T) \geq 0 \) as \( \beta \to 0^+ \) and \( T \to \infty \).

**Proof of Theorem 3.**
The mechanism’s approximate net revenue comprised of approximate incentive transfers \( \Delta \pi_{a,k} \) and participation fees \( |A| \cdot R \) over all regions can be written as by substituting 62 for \( \Delta \pi_{a,k} \):
\[
\hat{B}(T) = |A| \cdot R + \sum_{a \in A} \sum_{k=0}^{T} \Delta \hat{C}_{a,k} - \hat{C}_{a,k} + r_{a,k+1} - r_{a,k}.
\]  
(76)

Telescoping sum over \( k \) for the second term on the right hand side yields
\[
\hat{B}(T) = |A| \cdot R + \sum_{a \in A} [C_{a,T+1} - C_{a,0} - (\hat{C}_{a,T+1} - \hat{C}_{a,0}) + r_{a,T+1} - r_{a,0}] + 2 |A - 1| \cdot \mathcal{O}(\beta^2)
\]  
(77)

where the second equality follows by adding and subtracting \( \sum_{a \in A} C_{a,T+1} + \sum_{a \in A} C_{a,0} \) and by using \( C_{T+1} = C_{a,T+1} + C_{a,0} \) and \( \hat{C}_{a,0} = C_{a,0} \). We get the following lower bound by using the minimum value of the participation fee
\[
\hat{B}(T) \geq |A| \cdot \min_a [\hat{C}_{a} + C_{a,0} - C^*] + \sum_{a \in A} [C_{T+1} - \hat{C}_{a,T+1} - C_{a,0} + r_{a,T+1} - r_{a,0}] + 2 |A - 1| \cdot \mathcal{O}(\beta^2).
\]  
(79)

We reorganize the terms to get
\[
\hat{B}(T) \geq \sum_{a \in A} [C_{T+1} - (\hat{C}_{a,T+1} + C_{a,0}) + \min_a (\hat{C}_{a} + C_{a,0} - C^*)] + \sum_{a \in A} (r_{a,T+1} - r_{a,0}) + C_0 - C^* + \delta_T
\]  
(83)

29
where \[ \delta_{T,\beta} := \sum_{a \in A} [C_{T+1} - C^* - (\tilde{C}_{-a,T+1} - \tilde{C}_{-a})] + C^* - C_{T+1} + 2|A - 1|O(\beta^2). \] (84)

We get (83) by adding and subtracting \( \sum_{a \in A} C^* \), \( \sum_{a \in A} \tilde{C}_{-a,0} \), and \( C^* \). Now we use (64) to get

\[ \hat{B}(T) \geq \sum_{a \in A} [r_{a,T+1} - r_{a,0}] + \delta_{T,\beta}. \] (85)

Setting initial locational marginal prices and capacity prices associated with the tielines to zero, we have \( r_{a,0} = 0 \). We have \( \lim_{T \to \infty} \delta_T = 2|A - 1|O(\beta^2) \). Thus we get

\[ \lim_{T \to \infty} \hat{B}(T) \geq \sum_{a \in A} \sum_{(i,j) \in T_a} \alpha_{a,i,j}^* T_{a,(i,j)}^* - \sum_{(i,j) \in T_a} \frac{\mu_{r_{a,i,j}}}{2} \left( |T_{a,(i,j)}^*| - \bar{T}_{a,(i,j)} \right) + 2|A - 1|O(\beta^2) \] (86)

by taking the limit of \( r_{a,T+1} \) in (29) as \( T \to \infty \). Note that the capacity price associated terms above are zero via complementary slackness (23). The relation in (47) follows by noting that \( T_{a,(i,j)}^* = -T_{a,(j,i)}^* \) and letting \( T \) be the set of all tielines. The result follows if we let the step size \( \beta \to 0^+ \).

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