Why a magnetized quantum wire can act as an active laser medium

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The fundamental issues associated with the magnetoplasmon excitations are investigated in a quantum wire characterized by a confining harmonic potential and subjected to a perpendicular magnetic field. Essentially, we embark on the device aspects of the intersubband collective (magnetoroton) excitation which observes a negative group velocity between the maxon and the roton. The computation of the gain coefficient suggests an interesting and important application: the electronic device based on such magnetoroton modes can act as an active laser medium.

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The current progress in nanofabrication technology and the ability to tailor potentials and interactions is stimulated by the world-wide quest to develop exotic high-speed, low-power devices that are small enough, sharp enough, and uniform enough to behave the way theory says they should. Ever increasing excitement behind the research interest in the systems of reduced dimensionality can now reasonably be attributed to the discovery of the quantum Hall effects. These are the man-made semiconductor quantum structures generally known as quantum wells, quantum wires, and quantum dots in which the charge carriers exposed to external probes such as electric and/or magnetic fields can exhibit unprecedented quantal effects that strongly modify their behavior [1]. In the present work, we are concerned with the theoretical investigation of quantum wires or (more realistically) a quasi-one dimensional electron gas (Q1DEG) originally proposed by Sakaki in 1980 [see, e.g., Ref. 2].

The proposal of semiconductor quantum wire structures was motivated by the suggestion [2] that 1D k-space restriction would severely reduce the impurity scattering, thereby substantially enhancing the low-temperature electron mobilities. Consequently, the technological promise that emerges is the route to the minimum are called rotons.

The motivation behind this communication is grounded crucially in the device aspects emerging from the fact that the intersubband collective (magnetoroton) excitation which changes the sign of its group velocity twice before merging with the respective single-particle continuum. By definition, a roton is an elementary excitation whose dispersion relation shows a linear increase from the origin, but exhibits first a maximum, and then a minimum in energy as the momentum increases. Excitations with momenta in the linear region are called phasons; those with momenta near the maximum are called maxons; and those with momenta close to the minimum are called rotons.

In Q1DEG, the magnetoroton mode was predicted within the framework of Hartree-Fock approximation in 1992 [4] and it was soon verified in the resonant Raman scattering experiments [5]. A rigorous theoretical finding of the magnetoroton (MR) modes in the realistic quantum wires had, however, been elusive until quite recently [6-9]. After a brief theoretical background, we recall the relevant aspects of the MR mode from our recent studies on Q1DEG [7-9], compute the gain coefficient as an evidence, and systematically argue and substantiate that the electronic device based on such MR modes can act as an active laser medium.

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the magnetic length, the effective magnetic length, and the renormalized effective mass. Here the hybrid frequency $\tilde{\omega} = \sqrt{\omega^2 + \omega_0^2}$. The effective magnetic length $l$ refers to the typical width of the wave function and reduces to the magnetic length $l_0$ if the confining potential is zero (i.e., if $\omega_0 = 0$). In the limit of a strong magnetic field, the renormalized mass $m_r$ becomes infinite and the system undergoes a cross-over to the 2DES and hence the Landau degeneracy is recovered.

For the illustrative numerical examples, we have been focusing on the narrow channels of the narrow-gap In$_{1-x}$Ga$_x$As system. The material parameters used are: effective mass $m^* = 0.042m_0$, the background dielectric constant $\varepsilon_b = 13.9$, 1D charge density $n_{1D} = 1.0 \times 10^6$ cm$^{-2}$, confinement energy $h\omega_0 = 2.0$ meV, and the effective confinement width of the harmonic potential well, estimated from the extent of the Hermite function, $w_{eff} = 40.19$ nm. Notice that the Fermi energy $E_F$ varies in the case where the charge density ($n_{1D}$), the magnetic field ($B$), or the confining potential ($h\omega_0$) is varied. Thus we set out on the extensive investigation associated with the magnetoplasmon excitations in a Q1DEG subjected to a perpendicular magnetic field $B$ at $T=0$ K [7-9].

The full-fledged MR mode was observed in the similar physical conditions in the magnetized quantum wires made up of narrow-gap In$_{1-x}$Ga$_x$As systems in a rather different context [6]. Subsequently, we extensively studied the dependence of its propagation characteristics on the charge-density, confinement potential, and magnetic field [7]. Since the existence of this MR mode is exclusively attributed to the applied (perpendicular) magnetic field, we also scrutinized its group velocity for numerous values of the magnetic field in order to judge when and where this otherwise regular intersubband mode starts taking up the magnetoroton character. It was found that there is a minimum (threshold) value of the magnetic field ($B_{th}$) below which this MR does not exist and a regular intersubband magnetoplasmon survives. The $B_{th}$ for the present system, is defined as $B_{th} \approx 1.0$ T. There it was also established that both maxon and roton are the higher density of excitation states.

Later, we also embarked on investigating the inverse dielectric functions (IDF) for this system under the similar physical conditions [8]. The motivation there was not solely to reaffirm the fact that the poles of the IDF and the zeros of the dielectric function (DF) yield exactly identical excitation spectrum, but also to pinpoint the advantage of the former over the latter. For instance, the imaginary (real) part of the IDF sets to furnish a significant measure of the longitudinal (Hall) resistance in the system. Moreover, the quantity $\text{Im} \left[\frac{1}{\varepsilon(q,\omega)}\right]$ also implicitly provides the reasonable estimates of the inelastic electron (or Raman) scattering cross-section $S(q)$ for a given system. For the details of the formalism of the IDF’s for the quasi-n dimensional systems, a reader is referred to Ref. 11.

The main concern here is the MR mode between maxon and roton where it exhibits negative group velocity (NGV). We believe that the occurrence of NGV is quite unusual and hence must have some dramatic consequences. It turns out that the NGV matters – in the striking phenomena such as techy-on-like (superluminal) behavior [10], anomalous dispersion in the gain medium, a state with inverted population (likely) characterized by negative temperature, a medium having the ability of amplifying a small optical signal and hence serving as an active laser medium, ...etc. [7]. We focus on the latter and choose to compute the gain coefficient $\alpha(\omega)$ in order to materialize the notion of a quantum wire acting as an active laser medium.

FIG. 1: (Color online) The gain coefficient $\alpha(\omega)$ as a function of the excitation energy $\hbar\omega$ for several values of the damping factor and for the given values of the magnetic field $B$, the charge density $n_{1D}$, and the confinement energy $h\omega_0$. The parameters are as given inside the picture. The inset on the top-left shows the model quantum wire investigated here. The inset on the top-right illustrates the magnetoplasmon excitation spectrum in a two-subband model within the full RPA [8]. The crux of the matter here is the intersubband magnetoplasmon (or magnetoroton) mode which observes one maximum [the maxon] and one minimum [the roton] (after starting at $q = 0$ and $\hbar\omega = 10.65$ meV) before merging with the respective single-particle continuum.

In the classical electrodynamics, it makes sense to express $\alpha(\omega)$ in terms of $\text{Im} \left[\chi(\omega)\right]$, with $\chi$ being the susceptibility. But the story takes a different turn when it comes to the quantum systems, as is the case here. Despite the fact that it is the $\chi$ that, generally, gives rise to the (mathematically) complex nature of the DF or the IDF, we must recognize that $\chi$ contains only the single-particle response, whereas
the IDF can provide both single-particle and collective responses. Since our concern here is the MR which happens to be the intersubband collective (magnetoplasmon) excitation, we sought to search \( \alpha(\omega) \) in terms of Im \( \chi(\omega) \) rather than Im \( \chi_{inter}(q, \omega) \).

Figure 1 illustrates the computation of the gain coefficient \( \alpha(\omega) \) as a function of the excitation energy \( h\omega \) for various values of the damping factor \( \gamma \). The gain coefficient in the context of the laser amplification is defined as \( \alpha(\omega) = (\omega / 2c) \) Im \( \chi_{inter}(q, \omega) \); \( c \) being the speed of light in vacuum. The symbol Im refers to the imaginary part of a nonlinear, dynamic, inverse dielectric function (considered only for the relevant intersubband [or magnetoroton] excitations). The gain coefficient that persists due to the electronic transitions shows a maximum at \( h\omega \sim 14.26 \text{ meV} \) for the damping factor \( \gamma = 0.010 \). It is not just by chance that the peak position occurs at the expected values of \( (q, \omega) \) in the excitation spectrum. As it is intuitively expected, the peak turns towards the lower energy with increasing damping factor. It is reasonable that an amplifier device such as a laser gain medium cannot maintain a fixed gain for arbitrarily high input powers, because this would require adding arbitrary amounts of power to the amplified signal. Therefore, the gain must be reduced for high input powers; this phenomenon is called gain saturation. In the case of a laser gain medium, it is widely known that the gain does not instantly adjust to the level according to the optical input power, because the gain medium stores some amount of energy, and the stored energy determines the gain.

Given the sequence of instances manifesting in the system, it makes sense to reason that the applied magnetic field drives the system to the metastable (or non-equilibrium) state \([12]\) that gives rise to the population inversion so that the gain rather than absorption occurs at the frequencies of interest. This is attested by the existing negative group velocity (NGV) associated with the anomalous dispersion in the frequency range [between maxon and roton].

It is evident from Fig. 1 that the lower the damping (or ohmic or scattering loss), the higher the gain in the medium. The bandwidth of the laser amplifier is seen to becoming narrower with increasing gain. The concept of bandwidth in the laser amplification is different from that in the band structure. Conventionally, the bandwidth of an amplifier is defined as the full distance between the frequency (or energy) points at which the amplifier gain has dropped to half the peak value. Another important issue is the nature of the electronic transitions: an amplifying (absorbing) transition implies a positive (negative) values of Im \( \chi_{inter}(q, \omega) \) and hence of \( \alpha(\omega) \). Of course, one can always associate a suitable \( \pm \) sign with \( \alpha(\omega) \) in order to give it the proper meaning for either amplifying or absorbing medium.

One may argue that the sign specifying gain or loss should result from the calculus. As a matter of fact, this is what we should expect if we are not sure of the characteristics of the medium. But if we know (as is the case here) that NGV implies gain rather than loss in a certain frequency range [between maxon and roton], then we do have the freedom to choose the sign of the damping term. Similar remarks as made on the sign convention here can be seen in the biblical textbooks on lasers.

These characteristics open the possibility of designing MR-based electron device capable of amplifying a small optical signal of definite wavelength. Figure 1 then clearly provides a platform for realizing the potential of a magnetized quantum wire to act as an active laser medium. The situation is analogous to the (quasi-two dimensional) superlattices where the crystal can exhibit a negative resistance: it can refrain from consuming energy like a resistor and instead feed energy into an oscillating circuit.

This paper reports on the device aspects of the magnetoroton excitations in the magnetized quantum wires in a two-subband model within the framework of Bohm-Pines’ full RPA. We have computed and discussed the gain coefficient versus the excitation energy of the magnetoroton. This fundamental investigation suggests an interesting, important, and significant application: the electronic device based on such magnetorotons can act as an active laser medium. We believe that all the parameters involved in the process [such as the charge density, magnetic field, confinement po-
tential, propagation vector, etc.] that lead us to infer this proposal are in the reach of the current technology. It is premature to predict whether the tantalizing concept of magnetized quantum wire as an active laser medium will emerge as an exciting idea to be engaged in by the researchers. But certainly no other system of reduced dimensions has spoiled scientists and engineers with as many appealing features to pursue.

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[12] The metastable state is an important concept, from both a fundamental and a practical point of view, in the condensed matter physics. It is, by definition, a state that may exist even though it is much less stable than its ultimate equilibrium state. Irradiation of the system with the light of suitable wavelength allows its electrons to jump to an excited state. When the incoming radiation is removed, the excited electron goes back to its original (lower) state. However, when an electron goes to a metastable state, it remains there for a relatively longer duration. This process leads to accumulation of electrons in the metastable state, since the rate of addition of electrons to the metastable state is higher than the rate of their de-excitation. This leads to the phenomenon called population inversion, which forms the basis of lasing action of lasers. There are different ways to represent the metastable state for a given system. We choose to compute the life-time of magnetoroton in the $\omega-\mathbf{q}$ space until it propagates as a bonafide intersubband excitation [i.e., before it merges with the respective single-particle continuum] (see Fig. 2). Just as expected, the roton stands as an unstable transition state between the metastable state near the maxon and the ultimate equilibrium state at lower wavelengths. The picture speaks clearly: the stronger the magnetic field, the shorter the life-time, and hence more vulnerable the metastable state. The damping can be interpreted as the inverse of the life-time of the excited quantum state. This implies that the smaller magnetic fields favor the optimum case for the higher gain coefficients.