Parallel efficiency of monolithic and fixed-strain solution strategies for poroelasticity problems

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Motivation

- Thermo-hydro-mechanical processes near radioactive waste repositories
- Coupled multi-physical problems
- Efficient parallel solvers are a must
- Poroelasticity is a simple first step
- Scalability of different concepts can be tested

GeRa (Geomigration of Radionuclides) – subsurface simulator

gera.ibrae.ac.ru
Poroelasticity

Flow in porous media

Elastic deformation of the media
Governing equations: Biot model

- Groundwater flow: mass conservation + Darcy’s law + volume change

\[ s_{stor} \frac{\partial h}{\partial t} - \nabla \cdot (K \nabla h) + \alpha \nabla \cdot \frac{\partial u}{\partial t} = Q \]

- Elasticity: mechanical equilibrium + Hooke’s law + water pressure

\[ \nabla \cdot \left( C \frac{(\nabla u) + (\nabla u)^T}{2} - \alpha PI \right) = f \]

- Primary variables are water head \( h \) and solid displacement \( u \)
Numerical solution: challenges

Unstructured grids:
- Layered domains
- "Flat" cells
- Cells can be general polyhedra

- Strong heterogeneity
- Anisotropy
The finite volume method (FVM):

- Locally conservative
- Can handle wide class of cell shapes
- Easy to implement and is widely used
- Flux approximation is the key issue
Recently introduced virtual element method (VEM):

- Works on arbitrary cells
- Is algorithmically similar to conventional FEM
- Grows in popularity, gains theory
- Is used in multiphysics with FVM!
Temporal discretization

Fully implicit (backward Euler) scheme:

- Conventional for subsurface modeling
- Unconditionally stable
- Produces a linear system
Structure of the coupled system

\[
\begin{bmatrix}
A_f & A_{fm} \\
A_{mf} & A_m
\end{bmatrix}
\begin{bmatrix}
h \\
u
\end{bmatrix}
=
\begin{bmatrix}
b_f \\
b_m
\end{bmatrix}
\]

FVM for flow

VEM for the coupling terms

VEM for mechanics
## Solution strategies

### Monolithic

- Solving the full system
  - Unconditionally stable
  - Large matrix
  - Complicated matrix pattern

### Coupling

**Fixed-strain**

- Sequential flow and mechanics substeps
  - Can use tailored solvers
  - Less memory-consuming
  - Conditionally stable
  - Adds an iterative loop on each time step
The INMOST (www.inmost.org) numerical platform written in C++ provides:

- Unstructured mesh handling
- Automatic differentiation tools for systems assembly
- Linear solvers
- MPI parallelization:
  - Mesh partitioning
  - Parallel linear solvers
The idea:

- Fixed-strain strategy solves smaller systems with simpler structure
- A general-purpose black-box linear solver with no tuning can work better
- INMOST solver Inner_MPTILUC was used
Problem A: faulted reservoir

A 3-layer domain with fault

- 1,700,000 unknowns
- 4 time steps, 127 years
- 8-100 cores
Problem A: results

Fixed-strain scales better

- Assembly takes larger fraction of time
- Assembly naturally scales better
Problem B: real-life domain

A 9-layer domain, 11 media

- Injection in 8\textsuperscript{th} layer
  - 5 460 000 unknowns
  - 2 time steps, 6 years
  - 40-600 cores
Problem B: results

Both scale *superlinearly*!

Monolithic even scales better
Problem B: why superlinear?

The reason is superior scaling of MPTILUC preconditioner, default drop tolerance makes it closer to full $LU$-decomposition.
Problem B: why sublinear assembly scaling?

- Non ideal mesh partitioning
- Assembly takes larger fraction of time in fixed-strain strategy
- It’s the reason why fixed-strain scales worse
Conclusions

- Efficient solvers are required for multyphics
- Monolithic and splitting strategies are considered for poroelasticity problems discretized on unstructured meshes
- Strategies were tested in parallel with no tuning of linear solver or mesh partitioner
- No clear answer on which scales better
- Side note: INMOST linear solvers can handle coupled systems
Thank you for your attention!