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**Straight-line drawings of 1-planar graphs.** (English) Zbl 07738814
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Summary: A graph is 1-planar if it can be drawn in the plane such that each edge is crossed at most once. However, there are 1-planar graphs that do not admit a straight-line 1-planar drawing. We show that every 1-planar graph has a straight-line drawing with a two-coloring of the edges such that edges of the same color do not cross. Thus 1-planar graphs have geometric thickness two. In addition, the drawing is nearly 1-planar, that is, it is 1-planar if all fan-crossed edges are removed. An edge is fan-crossed if it is crossed by edges with a common vertex if it is crossed more than twice. The drawing algorithm uses high precision arithmetic with numbers with $O(n \log n)$ digits and computes the straight-line drawing from a 1-planar drawing in linear time on a real RAM.

**MSC:**

05C62 Graph representations (geometric and intersection representations, etc.)
05C10 Planar graphs; geometric and topological aspects of graph theory
05C85 Graph algorithms (graph-theoretic aspects)

**Keywords:**

graph drawing; straight-line drawings; shift method; 1-planar graphs; geometric thickness

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**References:**

[1] Ackerman, E.; On a note on 1-planar graphs, Discrete Appl. Math., 175, 104-108 (2014) - Zbl 1298.05081
[2] Ackerman, E.; Fox, J.; Pach, J.; Suk, A., On grids in topological graphs, Comput. Geom., 47, 7, 710-723 (2014) - Zbl 1292.05087
[3] Aggarwal, A.; Klawe, M. M.; Shor, P. W., Multilayer grid embeddings for VLSI, Algorithmica, 6, 1, 129-151 (1991) - Zbl 0703.68044
[4] Alam, M. J.; Brandenburg, F. J.; Kobourov, S. G., Straight-line drawings of 3-connected 1-planar graphs, (Wismath, S.; Wolff, A., Proc. GD 2013. Proc. GD 2013, LNCS, vol. 8242 (2013), Springer), 83-94 - Zbl 1406.68054
[5] Alekseev, V.; Gončakov, V., The thickness of arbitrary complete graphs, Math. Sb., 30, 2, 187-202 (1976) - Zbl 0381.05034
[6] Bárány, I.; Rote, G., Strictly convex drawings of planar graphs, Doc. Math., 11, 369-391 (2006) - Zbl 1108.05065
[7] Bekos, M. A.; Bruckdorfer, T.; Kaufmann, M., The book thickness of 1-planar graphs is constant, Algorithmica, 79, 2, 444-465 (2017) - Zbl 1372.05049
[8] Bekos, M. A.; Kaufmann, M.; Raftopoulou, C. N., On optimal 2- and 3-planar graphs, (Aronov, B.; Katz, M. J., SoCG 2017. SoCG 2017, LIPIcs, vol. 77 (2017), Schloss Dagstuhl - Leibniz-Zentrum für Informatik), 16:1-16:16 - Zbl 1435.05057
[9] Bekos, M. A.; Kaufmann, M.; Klute, F.; Pupyrev, S.; Raftopoulou, C. N.; Ueckerdt, T., Four pages are indeed necessary for planar graphs, J. Comput. Geom., 11, 1, 332-353 (2020) - Zbl 1486.05055
[10] Bodendiek, R.; Schumacher, H.; Wagner, K., Bemerkungen zu einem Sechsfarbenproblem von G. Ringel, Abh. Math. Semin. Univ. Hamb., 53, 41-52 (1983) - Zbl 0495.05020
[11] Bodendiek, R.; Schumacher, H.; Wagner, K., Über 1-optimale Graphen, Math. Nachr., 117, 323-339 (1984) - Zbl 0558.05017
[12] Bouchet, C.; Felsner, S.; Mosbah, M., Convex drawings of 3-connected plane graphs, Algorithmica, 47, 4, 399-420 (2007) - Zbl 1118.68100
[13] Brandenburg, F. J., 1-visibility representation of 1-planar graphs, J. Graph Algorithms Appl., 18, 3, 421-438 (2014) - Zbl 1301.05238
[14] Brandenburg, F. J., A first order logic definition of beyond-planar graphs, J. Graph Algorithms Appl., 22, 1, 51-66 (2018) - Zbl 1377.05028
[15] Brandenburg, F. J., T-shape visibility representations of 1-planar graphs, Comput. Geom., 69, 16-30 (2018) - Zbl 1381.05048
[16] Brandenburg, F. J., On fan-crossing and fan-crossing free graphs, Inf. Process. Lett., 138, 67-71 (2018) - Zbl 1458.68139
[17] Brandenburg, F. J., Characterizing and recognizing 4-map graphs, Algorithmica, 81, 5, 1818-1843 (2019) - Zbl 1423.05030
[18] Brandenburg, F. J., On fan-crossing graphs, Theor. Comput. Sci., 841, 39-49 (2020) - Zbl 1461.68145
[19] Brandenburg, F. J., Fan-crossing free graphs and their relationship to other classes of beyond-planar graphs, Theor. Comput.


20. Brandenburg, F. J.; Eppstein, D.; Gleißner, A.; Goodrich, M. T.; Hanauer, K.; Reislhuber, J., On the density of maximal 1-planar graphs, (van Kreveld, M.; Speckmann, B., Proc. GD 2012. Proc. GD 2012, LNCS, vol. 7704 (2013), Springer), 327-338

21. Chen, Z.; Grigni, M.; Papadimitriou, C. H., Recognizing hole-free 4-map graphs in cubic time, Algorithmica, 45, 2, 227-262 (2006) · Zbl 1095.68076

22. Cheong, O.; Har-Peled, S.; Kim, H.; Kim, H., On the number of edges of fan-crossing free graphs, Algorithmica, 73, 4, 673-695 (2015) · Zbl 1330.05048

23. Chiba, N.; Yamanouchi, T.; Nishizeki, T., Linear time algorithms for convex drawings of planar graphs, (Progress in Graph Theory (1984), Academic Press), 153-173

24. Chiba, N.; Onoguchi, K.; Nishizeki, T., Drawing plane graphs nicely, Acta Inform., 22, 2, 187-201 (1985) · Zbl 0545.68057

25. Chrobak, M.; Kant, G., Convex grid drawings of 3-connected planar graphs, Int. J. Comput. Geom. Appl., 7, 3, 211-223 (1997) · Zbl 0875.68452

26. Chrobak, M.; Payne, T., A linear-time algorithm for drawing a planar graph on a grid, Inf. Process. Lett., 54, 241-246 (1995) · Zbl 0851.68086

27. de Fraysseix, H.; Pach, J.; Pollack, R., How to draw a planar graph on a grid, Combinatorica, 10, 41-51 (1990) · Zbl 0728.05016

28. Di Battista, G.; Eades, P.; Tamassia, R.; Tollis, I. G., Graph Drawing: Algorithms for the Visualization of Graphs (1999), Prentice Hall · Zbl 1057.68653

29. Didimo, W., Density of straight-line 1-planar graph drawings, Inf. Process. Lett., 113, 7, 236-240 (2013) · Zbl 1259.05107

30. Dillencourt, M. B.; Eppstein, D.; Hirschberg, D. D., Geometric thickness of complete graphs, J. Graph Algorithms Appl., 4, 3, 5-15 (2000) · Zbl 0955.05028

31. Eppstein, D., Separating thickness from geometric thickness, (Kobourov, S. G.; Goodrich, M. T., Proc. GD 2002. Proc. GD 2002, LNCS, vol. 2528 (2002), Springer), 150-161 · Zbl 1037.68582

32. Fáry, I., On straight line representation of planar graphs, Acta Sci. Math. Szeged, 11, 229-233 (1948) · Zbl 0032.52501

33. Gabow, H. N.; Westermann, H. H., Forests, frames, and games: algorithms for matroid sums and applications, Algorithmica, 7, 465-497 (1992) · Zbl 0771.05026

34. Harary, F., Research problem, Bull. Am. Math. Soc., 67, 542 (1961)

35. Harvey, D.; van der Hoeven, J., Integer multiplication in O(n logn), Ann. Math., 193, 2, 563-617 (2021) · Zbl 1480.11162

36. Hong, S.; Nagamochi, H., Re-embedding a 1-plane graph for a straight-line drawing in linear time, Theor. Comput. Sci., 892, 132-154 (2021) · Zbl 1514.68219

37. Hong, S.-H.; Eades, P.; Liotta, G.; Poon, S.-H., Fáry’s theorem for 1-planar graphs, (Gudmundsson, J.; Mestre, J.; Viglas, T., COCOON 2012. COCOON 2012, LNCS, vol. 7434 (2012), Springer), 335-346 · Zbl 1364.68308

38. Hutchinson, J. P.; Shermer, T.; Vince, A., On representations of some thickness-two graphs, Comput. Geom., 13, 161-171 (1999) · Zbl 0953.68116

39. Kainen, P., Thickness and coarseness of graphs, Abh. Math. Semin. Univ. Hamburg, 39, 88-95 (1973) · Zbl 0264.05108

40. Kant, G., Drawing planar graphs using the canonical ordering, Algorithmica, 16, 4-32 (1996) · Zbl 0851.68086

41. Kaufmann, M.; Ueckerdt, T., The density of fan-planar graphs, J. Comput. Geom. 7, 465-497 (2016) · Zbl 0875.68452

42. Kobourov, S. G.; Liotta, G.; Montecchiani, F., An annotated bibliography on 1-planarity, Comput. Sci. Rev., 25, 45-67 (2017) · Zbl 1398.68402

43. Kainen, P.; Grigni, M.; Papadimitriou, C. H., Recognizing hole-free 4-map graphs in cubic time, Algorithmica, 45, 2, 227-262 (2006) · Zbl 1095.68076

44. Mutzel, P.; Odenthal, T.; Scharbrodt, M., The thickness of graphs: a survey, Graphs Comb., 14, 1, 59-73 (1998) · Zbl 0896.05020

45. Nash-Williams, C. S. J., A., Edge-disjoint spanning trees of finite graphs, J. Lond. Math. Soc., 36, 1, 445-450 (1961) · Zbl 0102.38805

46. Nash-Williams, C. S. J., A., Edge-disjoint spanning trees of finite graphs, J. Lond. Math. Soc., 36, 1, 445-450 (1961) · Zbl 0102.38805

47. Preparata, F., Shamos, M., Computational Geometry: An Introduction (1985), Springer · Zbl 0759.68037

48. Schnyder, W., Embedding planar graphs on the grid, (ACM-SIAM Symposium on Discrete Algorithms, SODA 1990 (1990), SIAM), 138-147 · Zbl 0786.05029

49. Stein, S., Convex maps, Proc. Am. Math. Soc., 2, 464-466 (1951) · Zbl 0042.42004

50. Steinhardt, E., Rademacher, H., Vorlesungen über die Theorie der Polyeder (1934), Julius Springer: Julius Springer Berlin · Zbl 0042.42004

51. Thomassen, C., Rectilinear drawings of graphs, J. Graph Theory, 12, 3, 335-341 (1988) · Zbl 0649.05051

52. Tutte, H., How to draw a graph, Proc. Lond. Math. Soc., 13, 743-768 (1963) · Zbl 0115.40805

53. Tutte, H., The thickness of a graph, Indag. Math., 25, 567-577 (1963) · Zbl 0123.17002

54. Tutte, W. T., Convex representations of graphs, Proc. Lond. Math. Soc., 10, 302-320 (1960) · Zbl 0094.36301

55. Wagner, K., Bemerkungen zum Vierfarbenproblem, Jahresber. Dtsch. Math.-Ver., 46, 26-32 (1936) · Zbl 0695.05020

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[56] Yannakakis, M., Embedding planar graphs in four pages, J. Comput. Syst. Sci., 31, 1, 36-67 (1989) · Zbl 0673.05022
[57] Yannakakis, M., Planar graphs that need four pages, J. Comb. Theory, Ser. B, 145, 241-263 (2020) · Zbl 1448.05055

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