The Supersymmetric Stueckelberg Mass and Overcoming the Fayet-Iliopoulos Mechanism for Breaking Supersymmetry

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1 Abstract

Gauge invariant generation of mass for a supersymmetric \( U(1) \) vector field through use of a chiral Stueckelberg superfield is considered. When a Fayet-Iliopoulos \( D \) term is also present, no breaking of supersymmetry ever occurs so long as the Stueckelberg mass is not zero. A moduli space in which gauge symmetry is spontaneously broken arises in this case.

2 Introduction

The Stueckelberg mechanism for generating a mass for a \( U(1) \) vector field is well understood [1]; it also has a supersymmetric generalization in which the Stueckelberg field is a chiral superfield [2]. In this case, both the photon and photino field develop a degenerate mass.

The breaking of a supersymmetry in a supersymmetric \( U(1) \) gauge theory can be accomplished through the presence of a so-called Fayet-Iliopoulos \( D \) term [3].

In this paper, we consider what happens when a \( U(1) \) supersymmetric gauge theory is supplemented by both a Stueckelberg chiral superfield and a Fayet-Iliopoulos \( D \) terms. It is demonstrated that if the Stueckelberg mass is non-zero, then supersymmetry remains unbroken for any value of the Fayet-Iliopoulos parameter \( \xi \), and that gauge symmetry is broken in a moduli space characterized by the vacuum expectation value of the scalar matter field.

3 A \( U(1) \) Supersymmetric Model

We begin with a real \( U(1) \) superfield \( V = V^* \), the associated field strength \( W_\alpha = \overline{\partial}^2 D_\alpha V \), and a chiral matter superfield \( \Phi \). (The conventions are those of [4].) The Lagrangian

\[
\mathcal{L}_{CL} = \frac{1}{32} (W^\alpha W_\alpha)_F + \left( \Phi^\dagger e^{2gV} \Phi \right)_D
\]

possesses the gauge invariance

\[
V \rightarrow V' = V + i \left( \Lambda - \Lambda^\dagger \right)
\] (2a)
where $\Lambda$ is a chiral (gauge) superfield. One can supplement $\mathcal{L}_{CL}$ of eq. (1) by the Fayet-Iliopoulos term [3]

\[ \mathcal{L}_{FI} = \xi V \big|_D \]  \hspace{1cm} (3)

without breaking gauge symmetry. If $\xi g < 0$, then spontaneous breaking of gauge symmetry occurs, leaving supersymmetry unbroken, while if $\xi g > 0$ supersymmetry is broken and gauge symmetry is unbroken.

One can also introduce a chiral superfield $S$ that acts as a Stueckelberg field [2]. It is possible then to have a gauge invariant mass term

\[ \mathcal{L}_M = m^2 \left[ V + \frac{i}{m} \left( S - S^\dagger \right) \right]^2 \bigg|_D \]  \hspace{1cm} (4)

provided

\[ S \rightarrow S' = S - m\Lambda \]  \hspace{1cm} (5)

when the transformation of eq. (2) occurs.

In the Wess-Zumino gauge in which $V$ becomes

\[ V = \theta \sigma^\mu \overline{\sigma} \nu \phi + i \theta \theta \phi \sigma^\mu \overline{\phi} - \frac{i}{\sqrt{2}} \theta \theta \partial_\mu \psi \theta \sigma^\mu \overline{\psi} + \frac{1}{4} \theta \theta \theta \theta \overline{\theta} \sigma^\mu \overline{\theta} \partial_\mu \overline{\sigma} \]  \hspace{1cm} (6)

and the matter field $\Phi$ can be expanded as

\[ \Phi = \phi + \sqrt{2} \theta \phi + \theta \theta F + i \partial_\mu \phi \theta \sigma^\mu \overline{\phi} - \frac{i}{\sqrt{2}} \theta \theta \partial_\mu \psi \theta \sigma^\mu \overline{\psi} - \frac{1}{4} \theta \theta \theta \theta \overline{\theta} \sigma^\mu \overline{\theta} \partial_\mu \overline{\theta} \]  \hspace{1cm} (7a)

\[ \Phi^\dagger = \phi^\dagger + \sqrt{2} \overline{\theta} \overline{\phi} + \overline{\theta} \overline{\theta} F^\dagger - i \partial_\mu \phi^\dagger \theta \sigma^\mu \overline{\phi} + \frac{i}{\sqrt{2}} \overline{\theta} \overline{\theta} \theta \sigma^\mu \partial_\mu \overline{\theta} \psi - \frac{1}{4} \theta \theta \phi^\dagger \theta \theta \overline{\theta} \theta \overline{\theta} \]  \hspace{1cm} (7b)

it follows that

\[ \mathcal{L}_{CL} + \mathcal{L}_{FI} = -\frac{1}{4} V_{\mu \nu} V^{\mu \nu} + i \lambda \sigma^\mu \partial_\mu \lambda - \frac{1}{4} V_{\mu \nu} \ast V^{\mu \nu} + \frac{1}{2} D^2 \]  

\[ + \left( D_\mu \phi \right)^\dagger (D^\mu \phi) + i \psi \sigma^\mu D_\mu \overline{\psi} \]  \hspace{1cm} (8)

\[ + F^\dagger F + i \sqrt{2} g \left( \phi^\dagger \psi \lambda - \phi \overline{\psi} \lambda \right) \]
\[ D_\mu = \partial_\mu + igV_\mu \text{ and } V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu. \]

If we now parameterize the Stueckelberg field as
\[
S = \left( \frac{A - iB}{2} \right) + \sqrt{2} \theta \chi + \theta \theta F_S + i\partial_\mu \left( \frac{A - iB}{2} \right) \theta \sigma^\mu \overline{\theta} + \frac{i}{\sqrt{2}} \theta \partial_\mu \chi \sigma^\mu \overline{\theta} - \frac{1}{4} \partial^2 \left( \frac{A - iB}{2} \right) \theta \theta \overline{\theta} \theta \tag{9}
\]
then in the Wess Zumino gauge, \( \mathcal{L}_M \) in (4) becomes
\[
\mathcal{L}_M = \frac{m^2}{2} \left( V_\mu - \frac{m}{2} \partial_\mu A \right)^2 - \frac{1}{2} B \partial^2 B - i (\chi \sigma^\mu \partial_\mu \overline{\chi} - \partial_\mu \chi \sigma^\mu \overline{\chi}) + 2F_S^\dagger F_S + mBD - \sqrt{2}m \left( \overline{\chi} \chi + \chi \lambda \right) + 2F_S^\dagger F_S + mBD - \sqrt{2}m \left( \overline{\chi} \chi + \chi \lambda \right), \tag{10}
\]
The residual gauge invariance in (8) and (10)
\[
V_\mu \rightarrow V_\mu + \partial_\mu \Lambda \tag{11a}
\]
\[
A \rightarrow A + m\Lambda \tag{11b}
\]
\[
\phi \rightarrow e^{-ig\Lambda} \phi \tag{11c}
\]
\[
\psi \rightarrow e^{-ig\Lambda} \psi \tag{11d}
\]
can be broken by a so-called “U gauge”,
\[
A = 0 \tag{12}
\]
or by an “R gauge” with a gauge fixing Lagrangian
\[
\mathcal{L}_{GF} = -\frac{1}{2\alpha} \left[ \partial \cdot V + \alpha mA \right]^2. \tag{13}
\]
In the former case, \( A \) is completely eliminated and \( V^\mu \) is a massive vector with the longitudinal polarization present while in the latter case, \( A \) just decouples from \( V \) and the longitudinal contribution of \( V_\mu \) vanishes as \( \alpha \) goes to zero. If one were to neglect the axial anomaly that occurs, the renormalizability of the model is apparent in the \( R \) gauge of eq. (13).

The potential from (8) and (10) is given by
\[
V = - \left\{ \frac{1}{2} D^2 + F^\dagger F + 2F_S^\dagger F_S + g\phi^\dagger \phi D + \xi D + mBD \right\}. \tag{14}
\]
Using the equation of motion for $D$, $F$ and $F_S$ this becomes

$$V = \frac{1}{2} \left( g\phi^\dagger \phi + \xi + mB \right)^2.$$  \hspace{1cm} (15)

It is evident that provided $m \neq 0$, the minimum of $V$ is zero irrespective of the values of $g$ and $\xi$. If $\phi$ has a vacuum expectation value of $\phi_0$, then the vacuum expectation value of $B$ is

$$B_0 = -\frac{\xi + g\phi_0^\dagger \phi_0}{m},$$  \hspace{1cm} (16)

in order to minimize $V$. Since this minimum occurs at $V = 0$, supersymmetry is unbroken. There is thus a “moduli space” for the scalar fields $\phi$, $\phi^\dagger$ and $B$ in which supersymmetry is unbroken; if $m \neq 0$ then supersymmetry is in fact never broken. Only if $m = 0$ can the Fayet-Iliopoulos mechanism for breaking of supersymmetry be operative.

If now $f$ and $b$ are the quantum fluctuations of $\phi$ and $B$ respectively about the background field, so that

$$\phi_0 = \phi_0^\dagger = h,$$  \hspace{1cm} (17a)

$$\phi = h + f, \quad \phi^\dagger = h + f^\dagger,$$  \hspace{1cm} (17b)

$$B = -\left( \frac{\xi + gh^2}{m} \right) + b,$$  \hspace{1cm} (17c)

then we find from (8), (10), (13) and (15) that in the Wess-Zumino $R$ gauge the component field form of our Lagrangian will have a term bilinear in $f$ and $V^\mu$. To eliminate this cross term, we modify the gauge fixing of eq. (13) in a way suggested by ’t Hooft [5,6] so that

$$\mathcal{L}_{GF} = -\frac{1}{2\alpha} \left[ \partial \cdot V + \alpha (mA + 2ihf) \right] \left[ \partial \cdot V + \alpha (mA - 2hf^\dagger) \right].$$  \hspace{1cm} (18)

Together, (8), (10), (15) and (18) leave us with the total Lagrangian

$$\mathcal{L} = -\frac{1}{2} \left[ (\partial_\mu V_\nu) (\partial^\mu V^\nu) - \left( 1 - \frac{1}{\alpha} \right) (\partial \cdot V)^2 + \frac{1}{2} \left( m^2 + 2g^2h^2 \right) V_\mu V^\mu \right. $$

$$+ \left. (D_\mu F)^\dagger (D^\mu f) + \frac{1}{2} (\partial_\mu A)^2 - \frac{1}{2} b \partial^2 b \right) $$

$$-\frac{1}{2} \alpha (mA + 2ihf) \left[ mA - 2hf^\dagger \right].$$  \hspace{1cm} (19)
\[-\frac{1}{2} \left[ gh (f + f^\dagger) + mb \right]^2 - g f^\dagger f \left[ gh (f + f^\dagger) + mb \right] \]
\[-\frac{1}{2} \left( gf^\dagger f \right)^2 + i \lambda \sigma \cdot \partial \lambda - i \left( \chi \sigma \cdot \partial \chi - \partial \chi \cdot \sigma \chi \right) \]
\[+ i \bar{\psi} \sigma \cdot D^\dagger \psi - \sqrt{2} m \left( \bar{\chi} \chi + \chi \lambda \right) \]
\[+ i \sqrt{2} gh \left( \psi \lambda - \bar{\psi} \lambda \right) + i \sqrt{2} g \left( f^\dagger \psi \lambda - f \bar{\psi} \lambda \right) \]

once the auxiliary fields have been eliminated. For all values of the vacuum expectation value \( h \), this model does not have spontaneously broken supersymmetry provided \( m \neq 0 \). The vector field has a mass \( (m^2 + 2g^2 h^2)^{\frac{1}{2}} \).

4 Discussion

We have considered a \( U(1) \) gauge model in which a real vector superfield \( V \) has been coupled to a chiral matter superfield \( \Phi \). This has been supplemented by a Fayet-Iliopoulos term and, through the use of an additional chiral superfield \( S \), a Stueckelberg mass term. It is found that the model always has a supersymmetric ground state, and that there is a moduli space for the vacuum expectation value \( h \) of the scalar component of \( \Phi \) much as there is in non-Abelian \( N = 2 \) supersymmetric models. If \( h \neq 0 \), gauge symmetry is spontaneously broken.

One could also consider what happens when more then one chiral matter superfield couples to the vector superfield as in the Wess Zumino model of QED [7] with a Fayet-Iliopoulos \( D \)-term [3]. We have found that if a Stueckelberg mass term is also present, it is again not possible to have spontaneous breaking of supersymmetry provided the Stueckelberg mass is non-zero. However, in this case, if there is a gauge invariant mass term for the matter fields, the vacuum state is unique and this vacuum state, the expectation value of the scalar matter fields is zero.

There are several avenues of investigation that suggest themselves. One might consider coupling directly the Stueckelberg superfield \( S \) to the matter superfield \( \Phi \) in a gauge invariant fashion. This has been done in [8]; the results indicate the possibility of devising a model in which supersymmetry is broken and the vector field remains massless. It is also tempting
to consider a non-Abelian generalization of the model considered in the preceding section. Possibly the divergences normally encountered in conventional non-Abelian Stueckelberg models [9] are mitigated by supersymmetry.

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