The Surface polaritons with negative group velocity at the vacuum-resonant dielectric interface

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Abstract. Surface polaritons most clearly manifest their properties when the processes of energy dissipation are neglected. In this case, they are collective excitations that propagate exclusively along the interface, and the energy flux of surface polaritons deep into the medium becomes zero. A more realistic model of a surface polariton assumes that dissipation processes are always present. In this case, the surface polariton partially penetrates deep into the medium due to the appearance of the real part of the propagation constant in this direction. It should be emphasized that in the region of existence of a surface polariton, the dielectric constant of the medium has strong frequency dependence; therefore, by virtue of the Kramers-Kronig relations, the dielectric constant must have an imaginary part, which is precisely responsible for the dissipation processes. In this paper we show that when surface polaritons are considered at the vacuum-resonance insulator boundary and strict allowance for dissipation, frequency regions arise in which surface polaritons have a negative group velocity and the dielectric acquires the properties of a metamaterial.

1. Introduction
The study of surface polaritons and surface plasmons has long been a separate field in which scientists, researchers and engineers of many laboratories and universities are successfully working. Surface polaritons (SP) are collective excitations arising at the boundary of two media, one of which is resonant and the dielectric constant of which can take negative values [1]. In [2], the authors consider the surface plasmon resonance spectroscopy as a sensitive tool for analyzing the optical characteristics of thin films and for monitoring chemical and electrochemical reactions. In [3] the authors have studied the dispersion characteristics of the main surface plasmon polariton modes supported by strip and plate waves of ZnO (zinc oxide). In [4] the authors found the spectral region where dispersion waves, phase and group velocities of surface waves are most sensitive to changes in the chemical potential of graphene layers. In [5] the authors have studied the relationship between propagating surface plasmon polaritons on silver films and excitons in CdS quantum dots. In [6] gold is considered as a plasmon material for the excitation of a surface plasmon to determine the refractive index. In [7] the authors have analyzed the optical properties of biological objects by the method of plasmon resonance spectroscopy.

On the other hand, in the last decade, active research has been carried out in the search for artificially created media - metamaterials with unique optical properties [8]. Film nanostructures with negative group velocity of plasmon-polaritons were investigated in [9]. In our joint works with Yu.M. Aleksandrov [10, 11], the cases of the appearance of surface plasmons with negative group velocity are considered.

However, it should be emphasized that all the characteristic properties of SP are mainly manifested in the neglect of dissipative processes. But with a significant frequency dependence of the medium
parameters under the conditions of surface polaritons excitation, such an account is fundamentally necessary due to the Kramers-Kronig relations. In this case, the SP partially acquires the properties of a body wave and gradually decays as it propagates along the interface.

1.1. Formulation of the problem
Statement of the problem to be solved in this work: to show that with strict allowance for dissipation at the boundary of a resonant dielectric, the existence of a surface polariton with a negative group velocity is possible. Find possible frequency regions in which the group velocity of the surface polariton acquires negative values. In addition, to analyze the length of the SP travel and the parameters of the propagation of the SP from the frequency.

1.2. Theoretical consideration
To solve the problem, we will consider the case of a dielectric with one resonant frequency, taking into account dissipation, bordering on vacuum. The dielectric constant of the material is described by the one-resonance Lorentzian model

\[ \varepsilon(\omega) = \varepsilon_{\infty} \frac{\Omega_L^2 - \omega^2 - i\omega \Gamma}{\Omega_T^2 - \omega^2 - i\omega \Gamma} \]  

(1)

where \( \Omega_L, \Omega_T \) are the frequencies of the longitudinal and transverse phonons, respectively, \( \varepsilon_{\infty} \) is the dielectric constant at \( \omega \to \infty \), \( \Gamma \) is the dissipation parameter, \( \omega \) is the frequency of the SP. Consider the case of p-polarized radiation at the interface between the two media. The dispersion equation for the SP in this case has the following form:

\[ k_p^2 = k_0^2 \frac{\varepsilon(\omega)\varepsilon_1}{\varepsilon(\omega) + \varepsilon_1} \]  

(2)

where \( k_p \) is the magnitude of the wave vector of SP, \( k_0 = \omega/c, \varepsilon_1 \) is the dielectric constant of vacuum, equal to 1, \( c \) is the speed of light in vacuum. Having entered the designation, using formulas (1) - (3), the dispersion equation for PP can be represented in the following form:

\[ n_p^2 = \frac{\varepsilon_1 \varepsilon_{\infty}}{\varepsilon_1 + \varepsilon_{\infty}} \frac{\Omega_L^2 - \omega^2 - i\omega \Gamma}{\Omega_T^2 - \omega^2 - i\omega \Gamma} \]  

(3)

here \( \Omega_3 \) is the frequency of the SP, which is given by the formula

\[ \Omega_3 = \Omega_T \sqrt{\frac{\varepsilon_1 + \varepsilon_0}{\varepsilon_1 + \varepsilon_{\infty}}} \]  

(4)

From formula (4) it can be seen that the quantity proportional to the parameter of the propagation of the SP is complex

\[ n_p = n + i\kappa \]  

(5)

This means that the SP will decay as it propagates along the interface. After a detailed analysis, you can get the following exact expressions for the real and imaginary parts of the parameter \( n_p \)

\[ n = \sqrt{\frac{\beta}{2}} R_0, \quad \kappa = \sqrt{\frac{\beta}{2}} \frac{\text{Im} F_0}{R_0} \]  

(6)
\[ \beta = \frac{\varepsilon_i \varepsilon_\infty}{\varepsilon_i + \varepsilon_\infty}, \quad F_0 = \frac{\Omega_1^2 - \omega^2 - i\omega \Gamma}{\Omega_2^2 - \omega^2 - i\omega \Gamma} \]  

(7)

The complex value \( F_0 \) is represented in the form of real and imaginary parts in the usual way

\[ F_0 = F_0' + iF_0'' \]

(8)

\( R_0 \) is calculated by the formula

\[ R_0 = \sqrt{R_0'^2 + R_0''^2} \]

(9)

The group velocity is calculated using the formula

\[ v_g = \frac{\partial \omega}{\partial k_p} = \frac{c}{n + u \frac{\partial n}{\partial u}} \]

(10)

Here \( u = \omega / \Omega_f \) is the relative frequency of the SP.

2. Calculation results

We have carried out numerous calculations of the group velocity for various values of the parameters of the resonant dielectric. In these calculations, the fulfillment of the equality following from the boundary conditions was controlled:

\[ \frac{K_1}{\varepsilon_1} + \frac{K_2}{\varepsilon_2} = 0 \]

(11)

here

\[ K_j = \sqrt{k_p^2 - k_j^2 \varepsilon_j}, \quad j = 1, 2, \quad \varepsilon_j = \varepsilon(\omega) \]

(12)

This condition was fulfilled with a high accuracy of the order \( 10^{-15} \), the calculations were carried out using the compiled program in the MATLAB environment. For the parameters of the resonant dielectric, the following parameters were selected: \( \Omega_\perp = 1060 \text{ cm}^{-1}, \ \Omega_L = 1063 \text{ cm}^{-1}, \ \varepsilon_\infty = 1.2 \). To clarify the role of dissipation, the calculation was carried out for several values of the dissipation parameter; here the results are presented for two values of \( \Gamma \): \( \Gamma = 5 \text{ cm}^{-1} \) and \( \Gamma = 10 \text{ cm}^{-1} \). In the graphs below, the physical quantities for these parameter values are labeled with subscripts 05 and 10; respectively figure 1 shows the dependence of the real parts of the parameter \( n_p \) for various parameters \( \Gamma \).
**Figure 1.** Frequency dependence of the real parts of the complex parameter $n_p$ for two values of the damping parameter $\Gamma$: $n05 - \Gamma = 5 \text{ cm}^{-1}$, $n10 - \Gamma = 10 \text{ cm}^{-1}$.

**Figure 2.** Frequency dependence of the imaginary parts $\kappa$ of the complex parameter $n_p$ for two values of the damping parameter $\Gamma$: $k05 - \Gamma = 5 \text{ cm}^{-1}$, $k10 - \Gamma = 10 \text{ cm}^{-1}$.

As seen from figure 2, in this frequency range the imaginary part $\kappa$ of the complex parameter $n_p$ for a larger value of the attenuation parameter takes smaller values. We obtained this result by rigorously
calculating the real and imaginary parts of the complex parameter of propagation of the surface polariton. As seen from Figure 2, in contrast to ordinary polaritons, both parts – real and imaginary – of the parameter $n_p$ are of a comparatively close order in magnitude. It should be emphasized here that we are talking about the frequency range $\Omega_p < \omega < \Omega_s$, where, according to the classical theory of surface polaritons [1], when attenuation is neglected, surface polaritons arise.

Figure 3 shows the frequency dependence of the path length of the PP for the same values of the attenuation parameter. It can be seen that, as expected after studying the behavior of the imaginary parts of $\kappa$, the path length for the case $\Gamma = 10 \text{ cm}^{-1}$ is greater than for the smaller value $\Gamma = 5 \text{ cm}^{-1}$.

Figure 4 shows the dependence of the group velocity on the frequency in the region of SP excitation. We see that with these parameters in this entire range, the group velocity has negative values. The phase velocity in the same range is positive. We emphasize that we used the usual definition of the group velocity, known from the theory of waves - formula (10). Thus, the dielectric in this case acquires the properties of a metamaterial. It should be noted that we obtained this result for positive values of the refractive index. This is in full agreement with the approach to metamaterials described in [12], where the difference in the signs of the phase and group velocities is the main feature of metamaterials.

![Figure 3](image.png)

**Figure 3.** Frequency dependence of the path length in microns for two values of the attenuation parameter $\Gamma$: $L_05 - \Gamma = 5 \text{ cm}^{-1}$, $L_{10} - \Gamma = 10 \text{ cm}^{-1}$.
Figure 4. Dependence of the group velocity of the surface polariton normalized to the speed of light on the frequency for two values of the damping parameter $\Gamma$: $\text{G}05\text{vgroup} /c - \Gamma = 5 \text{ cm}^{-1}$, $\text{G}10\text{vgroup} /c - \Gamma = 10 \text{ cm}^{-1}$.

3. Conclusion
In this work, we carried out a thorough analysis of the behavior of surface polaritons excited at the vacuum-resonant dielectric interface. It is shown that in the entire range of the existence of the SP with an accurate allowance for the dissipation, several important features of the propagation of the SP were revealed. First, the imaginary part of the propagation parameter for a larger value of the dissipation parameter $\Gamma$ turned out to be less than for a smaller value $\Gamma$. Second, the path length of the SP also turned out to be greater in the first case than in the second. Third, it was shown that in the entire range of excitation of the SP, the group velocity of the surface polariton turned out to be negative, which means that such a dielectric acquires the properties of a metamaterial, which is characterized by a difference in the signs of the phase and group velocities of the surface wave propagation. This behavior of the SP can be explained by the fact that in the region of its excitation, strict allowance for dissipation leads to comparatively close values of the real and imaginary parts of the complex parameter of propagation of the SP.

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