Localization of the Standard Model via Higgs mechanism and a finite electroweak monopole from non-compact five dimensions

Masato Arai\textsuperscript{1}, Filip Blaschke\textsuperscript{2,3}, Minoru Eto\textsuperscript{4} and Norisuke Sakai\textsuperscript{5}

\textsuperscript{1}Faculty of Science, Yamagata University, Kojirakawa-machi 1-4-12, Yamagata, Yamagata 990-8560, Japan
\textsuperscript{2}Faculty of Philosophy and Science, Silesian University in Opava, Bezručovo nám. 1150/13, 746 01 Opava, Czech Republic
\textsuperscript{3}Institute of Experimental and Applied Physics, Czech Technical University in Prague, Horská 3a/22, 128 00 Praha 2, Czech Republic
\textsuperscript{4}Department of Physics, Yamagata University, Kojirakawa-machi 1-4-12, Yamagata, Yamagata 990-8560, Japan
\textsuperscript{5}Department of Physics, and Research and Education Center for Natural Sciences, Keio University, 4-1-1 Hiyoshi, Yokohama, Kanagawa 223-8521, Japan and iTHEMS, RIKEN, 2-1 Hirasawa, Wako, Saitama 351-0198, Japan

We propose a minimal and self-contained model in non-compact flat five dimensions which localizes the Standard Model (SM) on a domain wall. Localization of gauge fields is achieved by the condensation of Higgs field via a Higgs dependent gauge kinetic term in five-dimensional Lagrangian. The domain wall connecting vacua with unbroken gauge symmetry drives the Higgs condensation which provides both electroweak symmetry breaking and gauge field localization at the same time. Our model predicts higher-dimensional interactions $|H|^{2n}(F_{\mu\nu})^2$ in the low-energy effective theory. This leads to two expectations: The one is a new tree-level contribution to $H \rightarrow \gamma\gamma$ ($H \rightarrow gg$) decay whose signature is testable in future LHC experiment. The other is a finite electroweak monopole which may be accessible to the MoEDAL experiment. Interactions of translational Nambu-Goldstone boson is shown to satisfy a low-energy theorem.
I. INTRODUCTION

The hypothesis that our four-dimensional world is embedded in higher-dimensional spacetime has been a hot topic in high energy physics for decades. Indeed, many mysteries of the Standard Model (SM) can be explained in this way. In particular, the discovery of D-branes in superstring theories [1] has intensified the research of the brane-world scenarios more than anything else. Then the seminal works [2-5] provided the basic templates for further studies.

The biggest advantage of models in extra dimensions is to utilize geometry of the extra dimensions. A conventional setup, common among the extra-dimensional models, is that extra dimensions are prepared as a compact manifold/orbifold. Namely, our four-dimensional spacetime is treated differently compared with extra dimensions.

In order to make things more natural, we can harness the topology of extra dimensions in addition to the geometry. The idea is quite simple and dates back to early 80’s [6], namely that the seed of dynamical creation of branes in extra-dimensions is a spontaneous symmetry breaking giving rise to a topologically stable soliton/defect on which our four-dimensional world is localized. The topology ensures not only stability of the brane but also the presence of chiral matters localized on the brane [6, 7]. In addition, graviton can be trapped [8-13]. Thus, the topological solitons provide a natural framework bridging gap between extra dimensions and four dimensions.

In contrast, localizing massless gauge bosons, especially non-Abelian gauge bosons, is quite difficult. There were many works so far [14-37]. However, each of these has some advantages/disadvantages and there seems to be only little universal understanding. Then, a new mechanism utilizing a field dependent gauge kinetic term (field dependent permeability)

\[-\beta \phi_i^2 F_{MN}F^{MN}, \quad (M, N = 0, 1, 2, 3, 4), \tag{1.1}\]

came out in Ref. [38] where \(\phi_i\) are scalar fields. This is a semi-classical realization of the confining phase [2, 39, 41] rather than Higgs phase outside the solitons. The authors have continuously studied brane-world models with topological solitons by using (1.1) [45, 51]. Let us highlight several results: We investigated the geometric Higgs mechanism which is the conventional Higgs mechanism driven by the positions of multiple domain walls in an extra dimension in Ref. [49]. Then we proposed a model in which the brane world on five domain walls naturally gives \(SU(5)\) Grand Unified Theory in Ref. [50]. Furthermore, we have clarified how to derive a low-energy effective theory on the solitons in the models with a
non-trivial gauge kinetic term (I.1) by extending the $R_\xi$ gauge in any spacetime dimensions $D$ [51]. Another group also recently studied the SM in a similar model with $\beta^2$ taken as a given background in $D = 5$ [52, 53]. They have also discussed phenomenology involving Nambu-Goldstone (NG) bosons for broken translation.

In this paper, we propose a minimal and self-contained model in non-compact flat five dimensions which localizes the SM on a domain wall. A striking difference from the previous works [45–51] is that we do not need extra scalar fields $\phi_i$ which were introduced only for localizing gauge fields via Eq. (I.1). Instead, we put the SM Higgs in that role. As a consequence, localization of massless/massive gauge fields and the electroweak symmetry breaking have the same origin. In other words, the Higgs field is an active player in five dimensions with a new role as a localizing agent of gauge fields on the domain wall, in addition to the conventional roles giving masses to gauge bosons and fermions. Since our model does not need extra scalar fields $\phi_i$, it is not only very economical in terms of field content but also we are free from a possible concern that $\phi_i$ would give an undesirable impact on the low-energy physics. We also study the translational NG boson $Y(x^\mu)$. Due to a low-energy theorem, it should have a derivative coupling with all other particles including Kaluza-Klein (KK) particles. We find a new vertex $\bar{\psi}^{(KK)}(y)\gamma^\mu \partial_\mu Y \psi^{(SM)}$ which provides a new diagram for the production of KK quarks $\psi^{(SM)} + \psi^{(SM)} \to \psi^{(KK)} + \psi^{(KK)}$ in the LHC experiment. This should be a dominant production process compared to the usual gluon fusion, and can easily violate experimental bounds. To avoid this, we will set a fundamental five-dimensional energy scale sufficiently large, providing all the KK modes supermassive. However, surprisingly, the Higgs dependent gauge kinetic term (I.1) can naturally leave masses of localized lightest particles to be of order the SM energy scale. Thus, all KK particles and the NG boson have no impact on the low-energy physics. Nevertheless, as a consequence of Eq. (I.1), regardless of the extra particles, our model still has a new experimental signature in $H \to \gamma\gamma$ ($H \to gg$) decay channel at tree level which is testable in future LHC experiment. Furthermore, we point out that the localization via Eq. (I.1) yields higher dimensional interactions $|H|^{2n}(F_{\mu\nu})^2$ in the low-energy effective theory and it provides a natural reason to have a finite electroweak monopole solution. Its mass has been previously estimated [69, 70] as $\lesssim 5.5$ TeV, so that it could be pair-produced at the LHC and accessible to the MoEDAL experiment [71, 72]. Thus, our model can pay the price for an electroweak monopole.

The paper is organized as follows. In Sec. II we explain all the essential ingredients in a simple toy model of Abelian-Higgs-scalar model in $D = 5$. We explain how a domain wall
drives condensation of the Higgs field and at the same time localizes massless/massive gauge
bosons and also chiral fermions. Phenomenological viability, the translational zero mode,
and relevance of the $H \rightarrow \gamma \gamma$ decay channel are addressed in Sec. III. We present a realistic
model localizing the SM in Sec. IV and discuss the finite electroweak monopole in Sec. V.
Our results are summarized and discussed in Sec. VI. Appendix A is devoted to define mode
expansion on the stable background. Mode expansion and effective potential on unstable
background is given in Appendix B. Some formulae for KK fermion pair production by NG
boson exchange are given in Appendix C.

II. LOCALIZATION VIA HIGGS MECHANISM

In order to illustrate a novel role of the Higgs mechanism besides the conventional roles of
giving masses to gauge fields and chiral fermions in a gauge invariant manner, let us consider
a simple Abelian-Higgs-scalar model in $D = 5$ flat spacetime as a toy model. The following
arguments are quite universal so that it is straightforward to apply them to non-Abelian
gauge theories, such as the SM which we discuss in Sec. IV and also to models with $D \geq 5$
[61].

A simple Abelian-Higgs-scalar model in $D = 5$ reads[1]

$$\mathcal{L} = -\beta(\mathcal{H})^2 \mathcal{F}_{MN}^2 + |D_M \mathcal{H}|^2 + (\partial_M \mathcal{T})^2 - V
+i\bar{\Psi}\Gamma_M D^M \Psi + \bar{\Psi}\Gamma_M \partial^M \Psi + \left(\eta\mathcal{T}\bar{\Psi}\Psi - \bar{\eta}\mathcal{T}\bar{\Psi}\Psi + \chi\mathcal{H}\bar{\Psi}\Psi + \text{h.c.}\right),$$

$$V = \Omega^2 |\mathcal{H}|^2 + \lambda^2 \left(|\mathcal{H}|^2 + \mathcal{T}^2 - v^2\right)^2,$$  \hspace{1cm} (II.1)

with $\mathcal{F}_{MN} = \partial_M \mathcal{A}_N - \partial_N \mathcal{A}_M$. Here $\mathcal{T}$ is a real scalar field, and $\mathcal{H}$ is the Higgs field which
interacts with $\mathcal{A}_M$ not only via the covariant derivative $D_M \mathcal{H} = \partial_M \mathcal{H} + iq_M \mathcal{A}_M \mathcal{H}$, but also
through non-minimal gauge kinetic term with the field-dependent function $\beta^2$ defined by

$$\beta(\mathcal{H})^2 = \frac{|\mathcal{H}|^2}{4\mu^2}. \hspace{1cm} (II.3)$$

The covariant derivative of the charged fermion field is defined by $D_M \Psi = \partial_M \Psi + iq_M \mathcal{A}_M \Psi$.
$\bar{\Psi}$ is a neutral fermion. The bosonic part of the model has $Z_2$ symmetry $\mathcal{T} \rightarrow -\mathcal{T}$. Mass
dimensions of the fields and parameters are summarized as $[\mathcal{H}] = [\mathcal{T}] = \frac{3}{2}$, $[\mathcal{A}_M] = 1$,
$[\Psi] = [\bar{\Psi}] = 2$, $[\mu] = [\Omega] = [\chi^{-2}] = [\eta^{-2}] = [\bar{\eta}^{-2}] = [\mathcal{H}] = [v^2] = 1$, and $[\beta] = \frac{1}{2}$.

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1 The bosonic part is a simple extension of the well-studied model [62] in which the Higgs field $\mathcal{H}$ is
replaced by a real scalar field.
The five-dimensional Gamma matrix $\Gamma^M$ is related to four-dimensional one as $\Gamma^\mu = \gamma^\mu$ and $\Gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = i\gamma^5$.

There are two discrete vacua $T = \pm v$ with $H = 0$. The vacua break the $Z_2$ symmetry but preserve $U(1)$ gauge symmetry which is necessary to localize the massless $U(1)$ gauge field on a domain wall \[38\,45\,51\]. Therefore, the Higgs mechanism does not take place in the vacua.

However, spontaneous breaking of the $Z_2$ symmetry gives rise to a topologically stable domain wall, connecting these two discrete vacua. Depending on the values of the parameters, the following stable domain wall solutions are obtained

$$T_0' = v \tanh \lambda vy, \quad H_0 = 0, \quad (\lambda v \leq \Omega),$$

$$T_0 = v \tanh \Omega y, \quad H_0 = \bar{v} \text{sech} \Omega y, \quad (\lambda v > \Omega),$$

with $\bar{v} = \sqrt{v^2 - \Omega^2/\lambda^2}$ and $y = x^4$. We are not interested in the former solution (II.4) since the $U(1)$ is unbroken everywhere and the gauge field is not dynamical due to $\beta^2 = 0$. On the other hand, as we will show below, the latter solution (II.5) localizes the $U(1)$ gauge field by $\beta^2 \propto \text{sech}^2 \Omega y$. When the Higgs is neutral ($q_H = 0$), the lightest mode of the localized gauge field is precisely massless \[49\,51\] whereas, as we will see, it becomes massive when the Higgs is charged ($q_H \neq 0$).

To understand the mechanism for the localized massless gauge field to become massive, let us compute the low-energy effective potential for the effective Higgs field in four dimensions in the parameter region

$$0 < \epsilon^2 \ll 1, \quad \epsilon^2 \equiv \frac{\lambda^2\bar{v}^2}{\Omega^2} = \frac{\lambda^2v^2 - \Omega^2}{\Omega^2}.$$  

From the linearized field equation around the background of the domain wall solution (II.5), we find that there is a mass gap of order $\Omega$, and two discrete modes much lighter than the mass gap. The lowest mode is exactly massless Nambu-Goldstone (NG) boson corresponding to spontaneously broken translation symmetry along the $y$ direction. Its interactions with all other effective fields are generally suppressed by inverse powers of large mass scale, whose characteristics will be discussed in Sec. III B and Sec. IV. Disregarding the NG boson, we retain only one light boson, whose wave function is well-approximated by the same functional form as the background solution $H_0(y)$ in (II.5). When $\lambda\bar{v} = 0$, this wave function gives the zero mode exactly, corresponding to the condensation mode at the critical point $\lambda v = \Omega$, where the $H$ field begins to condense. After $H$ condenses, this mode becomes slightly
massive above the critical point (II.6) with the mass of order $\lambda\bar{v}$, whose wave function receives small corrections suppressed by powers of $\epsilon$ (including an admixture of fluctuations of $\mathcal{T}$). Combining the background solution and the fluctuation, we introduce the following effective field $H(x)$ (a quasi-moduli) corresponding to the Higgs field in the low-energy effective field theory

$$\mathcal{H}(x, y) = \sqrt{\frac{\Omega}{2}} H(x) \text{sech} \Omega y.$$  (II.7)

Inserting this Ansatz into the Lagrangian and integrating over $y$, we obtain effective action as

$$\mathcal{L}_{\text{Higgs}}(H) = |D_\mu H|^2 - V_H, \quad V_H = \lambda_2^2 |H|^2 + \lambda_4^2 |H|^4,$$  (II.8)

$$\lambda_2^2 = -\frac{4\lambda^2 \bar{v}^2}{3}, \quad \lambda_4^2 = \frac{2\lambda^2 \Omega}{3},$$  (II.9)

where the effective gauge field in the covariant derivative $D_\mu$ is more precisely defined below, see Eq. (II.17). The possible corrections suppressed by powers of $\epsilon^2$ can be systematically computed as described in Appendix A. This is just a conventional Higgs Lagrangian which catches all the essential features. First, note that the sign of the quadratic term is determined by $\bar{v}^2 = v^2 - \Omega^2/\lambda^2$. When $\bar{v}^2 = 0$ ($v\lambda = \Omega$), the Higgs is massless corresponding to the condensation zero mode in (II.7). When $\bar{v}^2 < 0$ ($v\lambda < \Omega$), the vacuum expectation value (VEV) is $\langle H \rangle = 0$. Thus, we reproduce the solution (II.4). On the other hand, when $\bar{v}^2 > 0$ ($v\lambda > \Omega$), we have non zero VEV for the effective Higgs field $H(x)$

$$\langle H \rangle = \sqrt{\frac{2}{\Omega}} \bar{v} \equiv v_h \sqrt{2},$$  (II.10)

which correctly gives the solution (II.5). Note that the VEV $v_h$ can also be obtained directly from the five-dimensional field $H$ as

$$\frac{v_h^2}{2} = \int_{-\infty}^{\infty} dy \mathcal{H}^2_0 = \frac{2\bar{v}^2}{\Omega}.$$  (II.11)

The mass of physical Higgs boson can be read from Eq. (II.8) as

$$m_h^2 = \frac{8}{3} \lambda^2 \bar{v}^2 = \frac{8}{3} \Omega^2 \epsilon^2,$$  (II.12)
FIG. 1. The black lines show the Schrödinger potentials $V_S = \beta''/\beta$ for $\beta^2$ given in Eq. (II.3) (a), in Eq. (III.17) (b), and in Eq. (V.2) (c). The potential of (c) is multiplied by 0.1 for clarity. The horizontal axis is $\Omega y$. The red curves show the corresponding zero mode wave functions.

which is of order $\epsilon^2$ as we expected. Thus, the $y$-dependent Higgs condensation $\mathcal{H}_0(y)$ of Eq. (II.5) in $D = 5$ which is driven by the domain wall $T_0(y)$ connecting two unbroken vacua gives indeed the Higgs mechanism through Eq. (II.8). To complete the picture, we next calculate the mass of gauge bosons. We will assume $v\lambda > \Omega$ in the rest of paper, so that the solution (II.5) always applies.

To figure out the spectrum of the gauge field, first of all, we use canonical normalization $A_M = 2\beta A_M$. The linearized equation of motion for $A_\mu$ in the generalized $R_\xi$ gauge [51, 61] is

$$\left\{ \eta^{\mu\nu} \Box - \left( 1 - \frac{1}{\xi} \right) \partial^\mu \partial^\nu + \eta^{\mu\nu} \left( -\partial^2_y + \frac{(\partial_y^2 \beta)}{\beta} + 2q_H^2 \mu^2 \right) \right\} A_\nu = 0.$$  \hspace{1cm} (II.13)

Thus, the Kaluza-Klein (KK) spectrum is identical to eigenvalues of 1D quantum mechanical problem with the Schrödinger potential $V_S = (\partial_y^2 \beta)/\beta + 2q_H^2 \mu^2$. Fig. (a) shows the corresponding Schrödinger potential. The eigenvalues $m_n^2$ and eigenfunctions $\phi_n(y)$ can be easily obtained [51]. There is a unique bound state

$$\phi_0(y) = \frac{\sqrt{2} v}{v_h \sech \Omega y}, \quad m_0^2 = 2q_H^2 \mu^2.$$  \hspace{1cm} (II.14)

No other bound states exist and a continuum of scattering modes parametrized by the momentum $k$ corresponds to the eigenvalues $m_k^2 = k^2 + \Omega^2 + q_H^2 \mu^2$. Thus, the mass gap between the unique bound state $\phi_0$ and the higher KK modes is of order $\Omega$ (under the assumption $\Omega \gg \mu$) which is the inverse width of the domain wall. In terms of the original
field $A_\mu$, the lightest massive gauge boson $A_\mu^{(0)}(x)$ is given by

$$A_\mu = \frac{A_\mu}{2\beta} = \frac{\mu\phi_0}{\mathcal{H}_0} A_\mu^{(0)}(x) + \cdots = \frac{\sqrt{2}\mu}{\nu_h} A_\mu^{(0)}(x) + \cdots,$$

(II.15)

where the ellipses stand for the heavy continuum modes. The mass of the lightest massive
gauge boson is

$$m_A = m_0 = \sqrt{2} q_H \mu.$$

(II.16)

One can show that the fifth gauge field $A_y$ has no physical degrees of freedom [51].

Having Eq. (II.15) at hand, we are now able to read the effective gauge coupling constant. By plugging Eqs. (II.7) and (II.15) into Eq. (II.1) and integrating it over $y$, we have the
kinetic term for $H$ as

$$\int_{-\infty}^{\infty} dy \ |D_\mu \mathcal{H}|^2 = \left| \left( \partial_\mu + i q_H \frac{\sqrt{2}\mu}{\nu_h} A_\mu^{(0)} \right) H \right|^2 + \cdots = |D_\mu H|^2 + \cdots,$$

(II.17)

where the ellipses stand for the massive modes. Thus, the effective four-dimensional gauge
coupling reads

$$e = \frac{\sqrt{2}\mu}{\nu_h}.$$  

(II.18)

Combining Eq. (II.18) with Eq. (II.16), and Eq. (II.10) with Eq. (II.12), we see that what
happens here is perfectly consistent with the ordinary Higgs mechanism

$$m_A = q_H \epsilon \nu_h, \quad m_h = \lambda_4 \nu_h.$$

(II.19)

Finally, let us investigate the domain wall fermions $\Psi$ and $\bar{\Psi}$ [6, 7]. In the region (II.6)
with a phenomenological condition explained later, our parameters should satisfy the following inequality

$$\frac{\eta \nu}{\Omega} \simeq \frac{\eta \nu}{\Omega} \simeq 1 \gg \frac{\chi \bar{\nu}}{\Omega}.$$  

(II.20)

Then we can treat the Yukawa term $\mathcal{H} \bar{\Psi} \Psi$ in Eq. (II.1) as a perturbation. In order to study
the unperturbed Dirac equation, we decompose five-dimensional fermions as

$$\Psi = \sum_n \left( f_L^{(n)}(y) \psi_L^{(n)}(x) + f_R^{(n)}(y) \psi_R^{(n)}(x) \right),$$

(II.21)
where \( \psi^{(n)}_L \) and \( \psi^{(n)}_R \) are left-handed \( (\gamma^5 \psi_L = -\psi_L) \) and right-handed \( (\gamma^5 \psi_R = \psi_R) \) spinors in four-dimensions

\[
\begin{align*}
    i\partial_n \psi^{(n)}_L &= M_n \psi^{(n)}_R, \\
    i\partial_n \psi^{(n)}_R &= M_n \psi^{(n)}_L,
\end{align*}
\] (II.22)

and the mode functions \( f^{(n)}_L \) and \( f^{(n)}_R \) satisfies

\[
\begin{align*}
    Q f^{(n)}_L + M_n f^{(n)}_R &= 0, \\
    Q^\dagger f^{(n)}_R + M_n f^{(n)}_L &= 0,
\end{align*}
\] (II.23)

with \( Q = \partial_y + \eta T_0 \), and \( Q^\dagger = -\partial_y + \eta T_0 \). Assuming the five-dimensional Yukawa coupling to satisfy \( \eta > 0 \), we find a unique zero mode

\[
\begin{align*}
    f^{(0)}_L (y) &= N_{L,0} (\cosh \Omega y)^{-\eta v} , \\
    f^{(0)}_R (y) &= 0, \\
    M_0 &= 0,
\end{align*}
\] (II.24)

where \( N_{L,0} \) is a normalization constant. The number of excited bound KK states corresponds to \( n = \lfloor \eta v \Omega \rfloor \) (\( \lfloor \rfloor \) is the floor function). For example, the first excited bound state exists when \( \frac{\eta v}{\Omega} \geq 1 \) and its wave function and mass are given by

\[
\begin{align*}
    f^{(1)}_L &= N_{L,1} \sinh \Omega y (\cosh \Omega y)^{-\eta v} , \\
    f^{(1)}_R &= N_{R,1} Q f^{(1)}_L, \\
    M^2_1 &= \left( 2 \frac{\eta v}{\Omega} - 1 \right) \Omega^2.
\end{align*}
\] (II.25)

The mass gap between the zero mode and the KK modes is again of order \( \Omega \) for the parameter region given in Eq. (II.20). The analysis for \( \bar{\Psi} \) can be done similarly by replacing \( \eta \) with \( \tilde{\eta} \) and by exchanging \( L \) and \( R \).

The interaction between the lightest massive gauge boson \( A^{(0)}_\mu \) and the fermionic zero mode \( \psi^{(0)}_L \) is obtained as

\[
\begin{align*}
    \int_{-\infty}^{\infty} dy \, i\bar{\Psi} \Gamma^\mu D_\mu \Psi &= i \bar{\psi}^{(0)}_L \gamma^\mu \\
    &\quad \left( \partial_\mu + i q f \sqrt{2} \frac{A^{(0)}_\mu}{\nu} \right) \psi^{(0)}_L + \cdots ,
\end{align*}
\] (II.26)

where the ellipses stand for the massive modes. Notice the gauge coupling is the same as in Eq. (II.18). We have to emphasize that the effective gauge coupling \( e \) is the same for any localized fields. The universality is ensured by the fact that the wave function of the lightest mode of \( A_\mu \) is always constant.

We can also easily derive an effective Yukawa coupling as follows,

\[
\begin{align*}
    \int_{-\infty}^{\infty} dy \, \chi \bar{\Psi} \bar{\psi}^{(0)}_L \psi^{(0)}_R &= \chi \bar{\psi}^{(0)}_L \bar{\psi}^{(0)}_R , \\
    b &\equiv \frac{\eta v}{\Omega}, \\
    \tilde{b} &\equiv \frac{\tilde{\eta} v}{\Omega},
\end{align*}
\] (II.27)
with a dimensionless constant \( \tau(b, \tilde{b}) = \frac{\Gamma\left(\frac{1+b+\tilde{b}}{2}\right)}{\Gamma\left(\frac{2+b+\tilde{b}}{2}\right)} \sqrt{\frac{\Gamma(b+\frac{1}{2})\Gamma(\tilde{b}+\frac{1}{2})}{\Gamma(b)\Gamma(\tilde{b})}} \), where \( \Gamma(x) \) is the gamma function. Thus the Yukawa coupling in the four dimensions reads

\[
\chi_4 = \frac{\tau(b, \tilde{b})\chi \tilde{v}}{v_h} \simeq \chi \sqrt{\Omega},
\]

where we assume that \( \tau(b, \tilde{b}) \) is of order one because of \( b \simeq \tilde{b} \simeq 1 \).

Before closing this section, let us comment on the Higgs field. The Higgs condensation occurs at the five-dimensional level leading to the localization of the massless/massive gauge bosons in our model. A new feature of our Higgs mechanism is that the order parameter \( \mathcal{H} \) induced by domain wall is position-dependent. As a consequence, effective Higgs field is localized and only the massive physical Higgs boson \( h \) remains in the low-energy physics. In contrast, if one uses other neutral scalar fields \( \phi_i \) to localize the gauge fields \([45–51]\), one has to prepare another trick to localize the Higgs fields too. For example, in recent papers \([52, 53]\), the kinetic term of the Higgs field is not minimal but multiplied by a function \( \beta^2(\phi) \). In such models, the Higgs field (massive Higgs boson and massless NG boson) is localized on the domain wall and the Higgs condensation occurs in the low-energy effective theory. Namely, the Higgs field plays no active roles at the five-dimensional level.

### III. PHENOMENOLOGICAL IMPLICATIONS

#### A. Mass scales

In order to have a phenomenologically viable model, we need to explain observed mass \( m_A \) of a gauge boson, vacuum expectation value \( v_h \) of four-dimensional Higgs field, and mass \( m_h \) of physical Higgs boson. These observables are necessary and sufficient to fix the parameters of gauge-Higgs sector of the SM (gauge coupling \( e \), and quadratic and quartic couplings of Higgs scalars). We can regard all these masses to be of order \( 10^2 \text{ GeV} \), taking the four-dimensional gauge coupling\(^2\) \( e \) and Higgs quartic coupling \( \lambda_4 \) to be roughly of order unity\(^3\). On the other hand, we have four parameters, \( \Omega, v, \lambda, \mu, \) in the bosonic part of the five-dimensional Lagrangian (II.1). It is convenient to take \( \Omega \) as the fundamental mass scale of the high energy microscopic theory. Three other parameters can be put into two mass scales \( \mu, \lambda \tilde{v} \), and one dimensionless combination \( \lambda^2 = 2\lambda^2\Omega/3 \) in Eq. (II.9), where

\(^2\) Here we have just one gauge coupling, because of our simplification of \( U(1) \) instead of \( SU(2) \times U(1) \) gauge group.

\(^3\) Actually they are somewhat less than unity experimentally, in conformity with the perturbativity of SM.
\( \bar{v} = \sqrt{v^2 - (\Omega/\lambda)^2} \). From Eqs. (II.10), (II.12), and (II.16), masses of the low-energy effective theory are given in terms of parameters of the five-dimensional theory as

\[
m_A = \sqrt{2q_H \mu}, \quad v_h = \frac{2}{\sqrt{\Omega}} \bar{v}, \quad m_h = \sqrt{\frac{8}{3}} \lambda \bar{v}.
\]

(III.1)



Fitting these masses to experimentally observed values, we still have one mass scale \( \Omega \) completely free. Therefore we can choose the energy scale \( \Omega \) of the five-dimensional theory as large as we wish, leaving phenomenologically viable model at low-energies.

For instance, if we choose the ratio of the high energy scale and SM scale to be parametrized as

\[
\epsilon^2 = \frac{\lambda^2 \bar{v}^2}{\Omega^2} \sim 10^{-2a} \ll 1,
\]

(III.2)

we find the scale of parameters in the model as

\[
\lambda \bar{v} \sim 10^2 \text{ GeV} \ll \lambda v \sim 10^{2+a} \text{ GeV},
\]

(III.3)

implying \( \lambda \sim 10^{-1-a/2} \text{ GeV}^{-1/2}, v \sim 10^{3+3a/2} \text{ GeV}^{3/2}, \) and \( \bar{v} \sim 10^{3+a/2} \text{ GeV}^{3/2} \). This large mass gap allows us to use the low-energy effective field theory retaining only light fields with the mass of order \( \lambda \bar{v} \) or less. In order to achieve this hierarchy, we need a fine-tuning of parameters \( \lambda \bar{v} \ll \Omega \), as in Eq. (III.3).

For the fermionic sector, we require Eqs. (II.20) and (II.28). Therefore, we have \( \eta \sim \bar{\eta} \sim 10^{-1-a/2} \text{ GeV}^{-1/2} \). In order to obtain appropriate values of the four-dimensional Yukawa couplings, for example, for the top Yukawa coupling to be of order one, we need the five-dimensional Yukawa coupling as

\[
\chi \simeq \frac{\chi_{4,\text{top}}}{\sqrt{\Omega}} \sim 10^{-1-a/2} \text{ GeV}^{-1/2}.
\]

(III.4)

Thus, the five-dimensional Yukawa couplings \( \eta, \bar{\eta} \) and \( \chi \) are naturally set to be the same order. Note also that this justifies Eq. (II.20). To understand the hierarchy of lighter fermion masses, we can use the usual mechanism of splitting of position of localized fermions as explained briefly in Sec. IV.

In summary, for having the SM at the low-energy, all the dimension full parameters in the five-dimensional Lagrangian are set to be of the same order as

\[
\Omega \sim \lambda^{-2} \sim v^2 \sim \eta^{-2} \sim \bar{\eta}^{-2} \sim \chi^{-2} \sim 10^{2+a} \text{ GeV}.
\]

(III.5)
We need a fine-tuning for two small parameters of mass dimension: \( \lambda \bar{v}, \mu \sim 10^2 \) GeV. Estimate of the lower bound for the parameter \( \Omega \sim 10^{2+a} \) GeV will be discussed in Sec. IV using constraints from the LHC data.

**B. Translational zero mode**

Here we study interactions of the translational Nambu-Goldstone (NB) mode, and their impact on low-energy phenomenology. Symmetry principle gives low-energy theorems, dictating that the NG bosons interact with corresponding symmetry currents as derivative interactions (no interaction at the vanishing momentum of NG bosons). Hence their interactions are generally suppressed by powers of large mass scale. In order to understand the interactions of the NG bosons, it is most convenient to consider the moduli approximation where the moduli are promoted to fields in the low-energy effective Lagrangian. Let us consider a general theory with a number of fields \( \phi^i(x, y) \) admitting a solution (soliton) of field equation, which we take as a background. When the theory is translationally invariant, the position \( Y \) of the soliton is a moduli. It is contained in the solution as \( \phi^i(x, y - Y) \). In the moduli approximation, we promote the moduli parameter \( Y \) to a field \( Y(x) \) slowly varying in the world volume of the soliton. We call this moduli field \( Y(x) \) as NG field. By introducing the NG boson decay constant \( f_Y \) to adjust the mass dimension of the NG field to the canonical value \( [Y(x)] = 1 \), we obtain

\[
\mathcal{L}_{\text{NG}} = \int dy \mathcal{L} \left( \phi \left( x, y - \frac{1}{f_Y} Y(x) \right) \right). \tag{III.6}
\]

The precise value of the decay constant \( f_Y \) is determined by requiring the kinetic term of NG boson to be canonical as illustrated in the subsequent explicit calculation. By integrating over \( y \), we can obtain the effective interaction of the NG field. One should note that the constant part \( Y \) of NG field \( Y(x) \) is nothing but the position of the wall, which can be absorbed into the integration variable \( y \) by a shift \( y \rightarrow y - Y \) because of the translational invariance. Hence the constant \( Y \) disappears from the effective action after \( y \)-integration is done. This fact guarantees that \( Y(x) \) must appear in the low-energy effective theory always with derivatives, i.e. \( \partial_{\mu} Y(x) \). Let us examine how this fact fixes the interactions of NG particle in the effective Lagrangian to produce the low-energy theorem. Derivative \( \partial_{\mu} \) can

---

4 In our concrete model, we have fields such as \( A_M, T, H, \Psi \) and \( \tilde{\Psi} \).

5 This definition is, in general, a nonlinear field redefinition of the effective field that arises in the mode analysis of fluctuation fields, such as in Appendix.
only come from the derivative term in the original action $\mathcal{L}$, giving terms linear in the NG particle $Y(x)$ as

$$\mathcal{L}_{\text{NG}} = - \int dy \frac{\partial \mathcal{L}}{\partial \phi^i} \frac{\partial \mu Y}{\partial \phi^i \partial y} f_Y + \cdots = - \frac{1}{f_Y} \partial_\mu Y(x) \left[ \int dy T^{\mu y} \right] + \cdots, \quad (III.7)$$

where the energy-momentum tensor $T^{MN}$ of matter in five dimensions is given by

$$T^{MN} = \frac{\partial \mathcal{L}}{\partial \partial_M \phi^i} \partial_N \phi^i - \eta^{MN} \mathcal{L}. \quad (III.8)$$

This is the low-energy theorem of the NG particle for spontaneously broken translation. Thus we find that there are no nonderivative interactions that remain at the vanishing momentum of NG bosons, including KK particles. For instance, the possible decay amplitude of a KK fermion into an ordinary fermion and a NG boson should vanish at zero momentum of NG boson and will be suppressed by inverse powers of large mass scale such as $\Omega$. In this way, we can compute the effective action of NG field in powers of derivative $\partial_\mu$. Usually we retain up to second order in derivatives, but higher derivative corrections can be obtained systematically with some labor [76].

Let us compute the effective Lagrangian of NG field $Y(x)$ more explicitly by using the moduli approximation in our model as

$$T = v \tanh \left( \Omega y - \frac{1}{f_Y} Y(x) \right), \quad \mathcal{H} = \sqrt{\frac{\Omega}{2}} H(x) \text{sech} \left( \Omega y - \frac{1}{f_Y} Y(x) \right). \quad (III.9)$$

The wall position moduli in wave functions of fermions must also be promoted to NG field $Y(x)$, i.e.

$$f^{(n)}_{L,R}(y) \rightarrow f^{(n)}_{L,R} \left( y - \frac{1}{f_Y} Y(x) \right), \quad (III.10)$$

although we only retain the zero mode given in Eq. (II.24) in order to obtain low-energy effective Lagrangian for light particles. Plugging these Ansatz into the four-dimensional kinetic terms of $T$, $\mathcal{H}$ and $\Psi$ and integrating over $y$, we obtain the effective Lagrangian containing the NG field. Requirement of canonical normalization of the NG field $Y(x)$ fixes the decay constant $f_Y$ as

$$f_Y = \frac{2\sqrt{2} v}{\sqrt{3} \Omega}, \quad (III.11)$$
We finally obtain the effective Lagrangian for low-energy particles as:

\[ \mathcal{L}_{NG} = \frac{1}{2} \partial_{\mu} Y \partial^{\mu} Y \left( 1 + \frac{\Omega}{2v^2} |H|^2 \right). \]  

(III.12)

A few features can be noted. First of all, the NG bosons have only derivative interactions, as required by the above general consideration. Secondly, the derivative interaction produces higher-dimensional operators coupled to NG bosons. The required mass parameter in the coefficient of the interaction term is given by the high energy scale as \( \Omega/(2v^2) \sim 1/\Omega^2 \). Therefore the interaction is suppressed by a factor of \((\text{momentum})/\Omega\). Thirdly, the interaction linear in the NG particle in Eq.(III.7) happens to be absent in this model. This is a result of a selection rule in our model. The Lagrangian (II.1) and the background solution (II.5) allows us to assign generalized parity under the reflection symmetries \( y \rightarrow -y \), as a conserved quantum number to all modes including KK modes. Since NG boson has odd parity, whereas all other low-energy particles including fermion have even parity, we end up in the quadratic interaction for the NG boson \( Y(x) \), as given in Eq. (III.12). The parity quantum number under \( y \rightarrow -y \) may not be conserved in more general models, and can have nonvanishing interaction linear in \( \partial_{\mu} Y(x) \) given in Eq. (III.7).

Only when we take into account the heavy KK modes [53], we have interactions linear in \( \partial_{\mu} Y \). For example, including the lightest KK fermion given in Eq. (II.25) (\( b = \frac{v}{\Omega} > 1 \) in order to have a discrete state) we obtain a vertex

\[ \int_{-\infty}^{\infty} dy \ i\bar{\Psi} M^D M \Psi \supset i\alpha \sqrt{\frac{\Omega}{v}} \partial_{\mu} Y \left( \bar{\psi}_{L}^{(1)} \gamma^{\mu} \psi_{L}^{(0)} - \bar{\psi}_{L}^{(0)} \gamma^{\mu} \psi_{L}^{(1)} \right), \]  

(III.13)

where \( \alpha \) is a dimensionless constant of order one defined by \( \alpha = \frac{\sqrt{3}}{4} \frac{b}{\sqrt{b-1}} \frac{B(b+\frac{1}{2},b-\frac{1}{2})}{B(b+1,b-1)} \), where \( B(x,y) \) is the beta function. The above interaction gives the decay process \( \psi_{L}^{(1)} \rightarrow Y \psi_{L}^{(0)} \). \( \bar{\Psi} \) yields similar interactions between \( Y \) and \( \psi_{R}^{(0)} \). Although the NG boson amplitudes are generally suppressed by the ratio \( p_{\mu}/\Omega \) with the large mass scale \( \Omega \), it can give a significant decay rate in the case of two-body decay like here. Moreover, this type of vertex provides a new diagram for the production of KK quarks \( \psi_{L,R}^{(1)} \) out of quarks \( \psi_{L,R}^{(0)} \) in the colliding nucleons via the NG boson exchange

\[ \psi_{i}^{(0)} + \psi_{j}^{(0)} \rightarrow \psi_{i}^{(1)} + \psi_{j}^{(1)}, \quad (i,j = L, R) \]  

(III.14)

\(^6\) Note that a non-derivative coupling \( Y \psi_{L}^{(0)} \psi_{R}^{(1)} \) from \( T \bar{\Psi} \Psi \) was recently studied in Ref. [53]. However, the symmetry principle of NG boson for translation does not allow coupling without the derivative \( \partial_{\mu} \).

\(^7\) Note that \( \alpha \rightarrow 0 \) as \( b \rightarrow 1 \).
in the LHC experiment. This should be the dominant production mechanism because of large momentum fraction of quarks as given by their distribution function inside nucleons. The production process (III.14) tells us the lower bound of the KK quark masses. We will estimate it in Sec. IV where the Standard Model is embedded in our framework.

C. \( h \to \gamma \gamma \)

As explained above, our model provides a domain wall inside which all the SM particles are localized. All the KK modes are separated by the mass gap \( \Omega \sim 10^{2+\alpha} \) GeV. Furthermore, for the minimal \( \beta^2 \) as given in Eq. (II.3), there are no additional localized KK modes of the gauge fields [51]. At first sight, one might wonder if the low-energy theory would be distinguishable from the conventional SM if \( \alpha \) is sufficiently large. However, a significant difference between these two theories is an additional interaction between the Higgs boson and the gauge bosons due to the field dependent gauge kinetic term. For illustration, suppose that \( A_M \) is the electromagnetic gauge field and the Higgs boson is neutral with \( q_H = 0 \). Nevertheless, the field-dependent gauge kinetic term yields an interaction between the photon and the neutral Higgs boson. This mechanism is valid also for the physical Higgs boson in our model. To see this, let us consider the fluctuation of physical Higgs boson \( h(x) \) by perturbing \( H \) in Eq. (II.7) about \( H = v_h \)

\[
\mathcal{H} = \bar{v} \left( 1 + \frac{\sqrt{2} h(x)}{v_h} \right) \text{sech} \, \Omega y.
\] (III.15)

Then the first term of Eq. (II.1) yields

\[
- \int_{-\infty}^{\infty} dy \, |\beta|^2 (F_{MN})^2 = -\frac{1}{4} \left( 1 + 2 \frac{\sqrt{2} h}{v_h} + \frac{2 h^2}{v_h^2} \right) (F_{\mu\nu}^{(0)})^2.
\] (III.16)

Thus, there is a new tree-level amplitude for \( h \to \gamma \gamma \). In the SM, the Higgs boson decays into two photons mediated by top or \( W \) bosons at one-loop level. The operator of interest is \( c \frac{h}{v_h} (F_{\mu\nu}^{(0)})^2 \), whose coefficient is bounded by the LHC measurement as \( c \sim 10^{-3} \) [65, 66]. However, our simplest model has \( c = \frac{1}{2} \), so is strongly excluded experimentally.
D. Generalized models

To have a phenomenologically acceptable $h \to \gamma\gamma$ decay amplitude, we can modify the field dependent gauge kinetic term, for example, as

$$\beta^2(\mathcal{H}) = \frac{1}{2\mu^2} \left( |\mathcal{H}|^2 - \frac{3}{4} \frac{|\mathcal{H}|^4}{\bar{v}^2} \right). \quad (III.17)$$

The background configuration of the Higgs field $\mathcal{H} = \mathcal{H}_0(y)$ remains the same as in Eq. (II.5) since the $\beta^2 F_{MN}^2$ term does not contribute to the background solution. The reason for selecting this specific modification will be explained below soon. Before that, however, let us mention that the modification comes with a price. The linearized equation of motion in the generalized $R_\xi$ gauge for the gauge field with a generic $\beta$ reads $[51, 61]$

$$\left\{ \eta^{\mu\nu} \Box - \left( 1 - \frac{1}{\xi} \right) \partial^\mu \partial^\nu + \eta^{\mu\nu} \left( -\partial_y^2 + \frac{(\partial_y^2 \beta)}{\beta} + q_H^2 \frac{\mathcal{H}_0^2}{2\beta^2} \right) \right\} A_\nu = 0. \quad (III.18)$$

Then, determining the physical spectrum corresponds to solving the eigenvalue problem

$$\left( -\partial_y^2 + \frac{(\partial_y^2 \beta)}{\beta} + q_H^2 \frac{\mathcal{H}_0^2}{2\beta^2} \right) \phi_n = m_n^2 \phi_n. \quad (III.19)$$

If $\beta^2$ is quadratic in $\mathcal{H}$ as was the case in Eq. (II.3), the third term on the left-hand side is constant. Therefore, the problem is of the same complexity as if $q_H = 0$. On the other hand, when $\beta^2$ is not purely quadratic, the eigenvalue problem is essentially different from that of $-\partial_y^2 + \frac{(\partial_y^2 \beta)}{\beta}$. Fig. 1 (b) shows the corresponding Schrödinger potential. In case of $\beta^2$ as in Eq. (III.17) the Schrödinger equation in terms of the dimensionless coordinate $z = \Omega y$ is given by

$$\left( -\partial_z^2 + \frac{\partial^2 \beta_0}{\beta_0} + q_H^2 \frac{\mu^2}{\Omega^2} \left( 1 - \frac{3}{4} \frac{\mathcal{H}_0^2}{\bar{v}^2} \right) \right) \phi_n = \frac{m_n^2}{\Omega^2} \phi_n. \quad (III.20)$$

Note that this is independent of $\bar{v}$ because of $\mathcal{H}_0 = \bar{v} \sech z$. Although we cannot solve this exactly, we can still solve this problem perturbatively for $\Omega \gg \mu$ by treating the third term on the left-hand side as a small correction. The lowest eigenfunction and eigenvalue are approximately given by

$$\phi_0 = \frac{\mu \sqrt{2\Omega}}{\bar{v} \beta_0}, \quad m_0^2 \simeq 2q_H^2 \mu^2 \int dy \phi_0^2 \left( 1 - \frac{3}{4} \frac{\sech^2 y}{\Omega y} \right)^{-1} = 2q_H^2 \mu^2. \quad (III.21)$$
This is just the same as Eq. (II.14), and, therefore, the mass of the lightest massive gauge boson is of order $\mu$, which justifies our assumption $\Omega \gg \mu$. Since the situation is almost the same as in the simplest model, we have $v^2_h/2 = \int dy H^2_0 = 2\bar{v}^2/\Omega$, and the effective gauge coupling is $e \sim \mu/v_h \sim 1$. Thus the modified model defined by Eq. (III.17) provides the SM at low energies in the same manner as the simplest model does.

Now, let us turn to the problem of $h \rightarrow \gamma\gamma$. So we set $q_H = 0$ and Eq. (III.21) becomes exact wave function of the massless photon. As before, we put $\mathcal{H}$ given in Eq. (III.15) into the gauge kinetic term $-\beta^2 \mathcal{F}^2_{MN}$ with $\beta^2$ given in Eq. (III.17). Then, we find

$$-\int_{-\infty}^{\infty} dy \beta^2 (\mathcal{F}_{\mu\nu})^2 = \left[ -\frac{1}{4} + \frac{2h^2}{v^2_h} + \mathcal{O}\left(\frac{h^3}{v^3_h}\right) \right] (F_{\mu\nu}^{(0)})^2. \quad (III.22)$$

As we see, the term $h(F_{\mu\nu}^{(0)})^2$ does not exist. Therefore, the modified model is compatible with the bound given by the current experimental measurement of $h \rightarrow \gamma\gamma$.

If the factor in front of the quartic term of Eq. (III.17) deviates slightly from $\frac{3}{4}$, the term $h(F_{\mu\nu}^{(0)})^2$ comes back with a tiny factor. We can compare the contribution of this tree-level term to $h \rightarrow \gamma\gamma$ with those mediated by top/W-boson loop in the SM. If a sizable discrepancy is found in the future experiments in $h \rightarrow \gamma\gamma$ channel compared with the SM prediction, it can be a signature of our model.

Of course, the modification in Eq. (III.17) is just an example. There are other modifications which forbid $h \rightarrow \gamma\gamma$ process at the tree-level. For instance, in addition to $h\gamma\gamma$, one can eliminate other higher-dimensional interactions such as $hh\gamma\gamma$ vertex by appropriately choosing $\beta^2$.

The above consideration holds for another similar process of $h \rightarrow gg$ (two gluons). An experimental signature should be the decay of physical Higgs particle to hadronic jets. Moreover, it will affect the production rate of physical Higgs particles from hadron collisions.

Recently, it was proposed that another interesting signature from the localized heavy KK modes of gauge bosons and fermions [52, 53], although the presence and/or the number of localized KK modes is more dependent on details of models. Our model has the same signatures too but they are subdominant in our model since they are 1-loop effects of the supermassive KK modes.
IV. THE STANDARD MODEL

Let us briefly describe how our mechanism works in the SM. The minimal five-dimensional Lagrangian is

\[
\mathcal{L} = -\beta (\mathcal{H})^2 \left[ (G_{MN}^2 + (W_{MN}^i)^2 + B_{MN}^2) + |D_M \mathcal{H}|^2 + (\partial_M T)^2 - V \right]
+ iUT^M D_M U + iQT^M D_M Q + \eta_R (T - m) \bar{U}U - \eta_L T\bar{Q}Q + \chi \bar{Q}HU + h.c.,
\]

\[
V = \Omega^2 |\mathcal{H}|^2 + \lambda^2 (T^2 + |\mathcal{H}|^2 - v^2)^2,
\]

where \(G_{MN}, W_{MN}, \) and \(B_{MN}\) are the field strengths of \(SU(3)_C, SU(2)_W\) and \(U(1)_Y\) gauge fields, respectively. More explicitly, they are given by \(W_{MN} = \partial_M W_N - \partial_N W_M + iq [W_M, W_N]\), and so on. The Higgs field \(\mathcal{H}\) is an \(SU(2)_W\) doublet with the covariant derivative \(D_M \mathcal{H} = (\partial_M + \frac{i}{2} q W_M + \frac{i}{2} q' B_M) \mathcal{H}\), with \(q\) and \(q'\) being five-dimensional gauge couplings for \(SU(2)_W\) and \(U(1)_Y\) relative to that of \(SU(3)_C\). We will assume \(\tan \theta_w = q'/q\) to reproduce the SM in the low-energy. The fermions \(Q\) and \(U\) are doublet and singlet of \(SU(2)_W\), respectively. Flavor indices for \(U, Q\) and the couplings are implicit.

As before, there are two discrete vacua \(T = \pm v\) and \(\mathcal{H} = 0\). The background domain wall solution in the parameter region \(\lambda v > \Omega\) is given by

\[
T_0 = v \tanh \Omega y, \quad \mathcal{H}_0 = \begin{pmatrix} 0 \\ \bar{v} \sech \Omega y \end{pmatrix}.
\]

The Higgs doublet \(H(x)\) in the four-dimensional effective theory is found in \(\mathcal{H}\) as is done in Eq. (II.7). The Higgs potential is identical to that in Eq. (II.8). One can show that the upper component and the imaginary part of the lower component are localized NG bosons and are absorbed by the \(W\) and \(Z\) bosons. Indeed, the spectrum of \(W_{\mu}^\pm = 2\beta W_{\mu}\) and \(Z_{\mu} = 2\beta Z_{\mu}\) are determined by the 1D Schrödinger problems

\[
-\partial_y^2 + \frac{(\partial_y^2 \beta)}{\beta} + \frac{q^2 \mathcal{H}_0^2}{4 \, 2\beta^2}, \quad -\partial_y^2 + \frac{(\partial_y^2 \beta)}{\beta} + \frac{q^2}{4 \cos^2 \theta_w} \frac{\mathcal{H}_0^2}{2\beta^2}.
\]

The details of the derivation will be given elsewhere [61]. On the other hand, the photon \(A_{\mu} = 2\beta A_{\mu}\) and gluon \(G_{\mu} = 2\beta G_{\mu}\) are determined by \(-\partial_y^2 + \frac{(\partial_y^2 \beta)}{\beta}\). Therefore, the lightest modes \(\phi_0 \propto \beta\) of photon and gluon are exactly massless. The results so far are independent of \(\beta^2\). To be concrete, let us choose the simplest function \(\beta^2 = |\mathcal{H}|^2/4\mu^2\). Then the effective
SU(2)W gauge couplings and the electric charge are given by

\[
g = \frac{\sqrt{2}q\mu}{v_h}, \quad g' = \frac{\sqrt{2}q'\mu}{v_h}, \quad e = \frac{qq'}{\sqrt{q^2 + q'^2}} \frac{\sqrt{2}\mu}{v_h} = \frac{gg'}{\sqrt{g^2 + g'^2}},
\]

(IV.5)

where \(v_h\) is given in Eq. (II.10). Masses of \(W\) and \(Z\) are easily read from Eq. (IV.4) as

\[
m^2_W = \frac{q^2\mu^2}{2} = \frac{g^2v_h^2}{4}, \quad m^2_Z = \frac{q^2\mu^2}{2\cos^2\theta_w} = \frac{g^2v_h^2}{4\cos^2\theta_w}.
\]

(IV.6)

For the fermions, we assume \(\eta_L > 0\) and \(\eta_R > 0\). Then the left-handed fermion from \(Q\) is localized at the zero of \(\mathcal{T}\), while the right-handed fermion from \(U\) is localized at the zero of \(\mathcal{T} - m\). The Yukawa term \(\chi\bar{Q}HU\) is responsible for giving non-zero masses to the localized chiral fermions, which is necessarily exponentially small for \(m \neq 0\) since the left- and right-handed fermions are split in space. By distinguishing parameters such as \(m\) for different generations as was done in many models with extra dimensions [73, 74], the hierarchical Yukawa coupling can be naturally explained in our model.

This way, the SM particles are correctly localized on the domain wall in our framework.

Before closing, we evaluate the lower bound of KK quark mass by using the KK quark production process in Eq. (III.14) via Nambu-Goldstone boson exchange. If we take the initial quarks of different flavor for simplicity, we have only single Feynman diagram depicted in Fig. 2(a). In the process (III.14) followed by \(\psi^{(1)}_{L,R} \rightarrow \chi Y \psi^{(0)}_{L,R}\), the final state contains two SM fermion jets and a missing energy of the NG boson \(Y\), whose signature is similar to

![Feynman diagrams](image)

FIG. 2. Feynman diagrams for the processes (a) \(ud \rightarrow u^{(1)}d^{(1)}\) and (b) \(ud \rightarrow \tilde{u}\tilde{d}\).
squark pair production, where a squark decays into the partner SM quark and a gluino or neutralino in the simplified supersymmetric models \[54\, 56\). In most of kinematical regions, a dominant processes for squark pair production is given by Feynman diagram depicted in Fig. 2(b). Since both processes involve the same valence quark distribution functions, we can compare these cross-sections directly to obtain an order of magnitude estimate of the lower bound for KK quark mass using the analysis for squark mass bound. As shown in Appendix C, the differential cross section \(\frac{d\sigma}{dt}\) of (III.14) producing a pair of the first KK fermion with mass \(M_1\) is given by summing contributons from initial state of different chiralities (LL, RR, LR, RL) as

\[
\frac{d\sigma}{dt}(ud \to u^{(1)}d^{(1)}) = \frac{\alpha^4}{576\pi^2} \frac{\Omega^2 (1 - \beta_{M_1} \cos \theta)^2 (1 - \beta_{M_1}^2)}{v^4 (\beta_{M_1}^2 + 1 - 2\beta_{M_1} \cos \theta)^2},
\]

where \(\beta_{M_1} = \sqrt{1 - \frac{M_1^2}{E^2}}\), \(E\) is the center of mass energy of incoming particles, and \(\theta\) is the scattering angle. We ignore masses of the SM quarks and all the parameters are taken to be common for the different quarks just for simplicity. We can assume \(v \approx \frac{\Omega^3}{2}\) and \(M_1 \approx \Omega\) for simplicity.

The squarks production \(ud \to \tilde{u}\tilde{d}\) cross section \[57\, 59\) is

\[
\frac{d\sigma}{dt}(ud \to \tilde{u}\tilde{d}) = \frac{g_s^4}{288\pi} \frac{1 + \beta_{m_{\tilde{q}}}^2 \cos^2 \theta + (m_{\tilde{g}} - m_{\tilde{q}})^2/E^2}{(2E^2(1 - \beta_{m_{\tilde{q}}} \cos \theta) + m_{\tilde{g}}^2 - m_{\tilde{q}}^2)^2},
\]

with \(\beta_{m_{\tilde{q}}} = \sqrt{1 - \frac{m_{\tilde{q}}^2}{E^2}}\). The SU(3)_C gauge coupling and gluino mass are denoted as \(g_s\) and \(m_{\tilde{g}}\), and a common mass \(m_{\tilde{q}}\) is assumed for squarks of different flavors and chiralities.

To obtain the bound for the production of heavy particles, we can expect that the cross-section near threshold (\(\beta = 0\)) is a good guide for the order of magnitude estimate. Both differential cross-sections become constants without angular dependence at the threshold, and their ratio is given as

\[
\frac{\frac{d\sigma}{dt}(ud \to u^{(1)}d^{(1)})|_{E=M_1}}{\frac{d\sigma}{dt}(ud \to \tilde{u}\tilde{d})|_{E=m_{\tilde{q}}}} = \frac{1}{2\pi} \frac{\alpha^4 m_{\tilde{g}}^2 m_{\tilde{q}}^2}{g_s^4} \frac{\Omega^4}{\left(1 + \frac{m_{\tilde{q}}^2}{m_{\tilde{g}}^2}\right)^2}.
\]

The simplified analysis for squark production gives \(m_{\tilde{q}} > 1.5\) TeV, assuming \(m_{\tilde{q}} = m_{\tilde{g}}\) \[60\). The identical bound for the KK fermion mass \(M_1 \sim \Omega > 1.5\) TeV is obtained for \(2\alpha^4/(\pi g_s^4) \approx 1\). Since \(\Omega = 10^{2+a}\) GeV, we have the lower bound for \(a\) as \(a \gtrsim 1\). If the coupling \(\alpha\) of KK fermion is larger than \(g_s\), we obtain larger lower bound for its mass. To determine how much larger requires a more detailed analysis of data.
V. FINITE ELECTROWEAK MONOPOLES

The SM has a point magnetic monopole which is the so-called Cho-Maison (CM) monopole \[67\]. It is different from either a Dirac monopole or a Nambu electroweak monopole \[68\]. Unfortunately, its mass diverges due to a singularity at the center of the monopole. Cho, Kim and Yoon (CKY) \[69\] have proposed a modification of the SM in four dimensions which includes the field dependent gauge kinetic term as \[\mathcal{L} \in -\frac{\epsilon |H|/v_h}{4} (B_{\mu\nu})^2.\]

In order to have the conventional SM at the electroweak vacuum \(|H| = v_h\), the normalization should be fixed as \(\epsilon (|H| \to v_h) = 1\). It was found that this modification makes the CM monopole regular if \(\epsilon \sim |H|^n\) with \(n > 4 + 2\sqrt{3} \approx 7.46\) as \(|H| \to 0\). However, it has recently been pointed out by Ellis, Mavromatos and You (EMY) \[70\] that the original CKY model is incompatible with LHC measurements of Higgs boson \(H \to \gamma\gamma\). They have proposed generalizations of the CKY model which are compatible with LHC measurements. Their conclusion is that the monopole mass is \(\lesssim 5.5\) TeV so that it could be pair-produced at the LHC and accessible to the MoEDAL experiment \[71, 72\].

Neither CKY nor EMY does discuss the underlying rationale for their modifications to the SM. In contrast, our five-dimensional model has a clear motivation for the field dependent gauge kinetic term, which is the domain wall induced Higgs mechanism. For example, one of the EMY’s proposals is \[70\]

\[
\epsilon_1 = 5 \left( \frac{H}{v_h} \right)^8 - 4 \left( \frac{H}{v_h} \right)^{10}. \tag{V.1}
\]

This can be derived from our model with

\[
\beta^2 = \frac{|\mathcal{H}|^2}{\mu^2} \left( 10 \frac{|\mathcal{H}|^6}{\bar{v}^6} - 9 \frac{|\mathcal{H}|^8}{\bar{v}^8} \right). \tag{V.2}
\]

The background solution is still \(\mathcal{H}_0 = \bar{v} \text{sech } \Omega y\). Fig. 1 (c) shows the corresponding Schrödinger potential. Then the wave function of the massive \(U(1)_Y\) gauge field reads

\[\phi_0 \approx \sqrt{\frac{35\Omega}{6\pi^2}} \left( 10 \mathcal{H}_0^8 - 9 \frac{\mathcal{H}_0^{10}}{\bar{v}^2} \right).\]

As before, we identify the four-dimensional Higgs field \(H(x)\) as \(\mathcal{H} = \bar{v} \frac{H(x)}{v_h} \text{sech } \Omega y\) with \(v_h = \sqrt{\frac{2}{\Omega}} \bar{v}\). We find the EMY’s model from the five dimensions via the domain wall and the Higgs mechanism as

\[- \int_{-\infty}^{\infty} dy \beta^2 (\mathcal{B}_{\mu\nu})^2 = - \int_{-\infty}^{\infty} dy \beta^2 \frac{\phi_0^2}{4\beta_0^2} (B_{\mu\nu}^{(0)})^2 = -\frac{\epsilon_1}{4} (B_{\mu\nu}^{(0)})^2, \tag{V.3}\]

where we ignored contributions from the massive KK modes.
Note that the $\beta^2$ modifies not only the gauge kinetic term of $U(1)_Y$ but also that of the $SU(2)_W$. An electroweak monopole in such theory also has a finite mass \cite{77}.

CKY have claimed that discovery of an electroweak monopole is a real final test for the SM \cite{69}. For us, it is not only the topological test of the SM but also would give constraints for restricting the $\beta^2$ factor of the five-dimensional theory.

\section{Conclusions and Discussion}

We proposed a minimal model in flat non-compact five dimensions which realizes the SM on a domain wall. In our approach, the key ingredients for achieving this result are the following: (i) the spacetime is five-dimensional, (ii) there is an extra scalar field $T$ which is responsible for the domain wall, (iii) there is a field-dependent gauge kinetic term as a function of the absolute square of the Higgs field.

In our model, all spatial dimensions are treated on the same footing at the beginning. The effective compactification of the fifth dimension happens as a result of the domain wall formation breaking the $Z_2$ symmetry spontaneously. The presence of domain wall automatically localizes chiral fermions \cite{6,7}. The key feature of our model is that the Higgs dependent gauge kinetic term drives the localization of SM gauge bosons and the electroweak symmetry breakdown \textit{at the same time}. The condensation of the SM Higgs field inside the wall for $\Omega < \lambda v$ can be understood as follows. As we let the parameter $\Omega$ decrease across $\lambda v$, we find a massless mode emerges at the critical point $\Omega = \lambda v$, which becomes tachyonic below the critical point and condenses until a new stable configuration is formed. It is interesting to observe that this thought-process is analogous to a second-order phase transition if we regard the parameter $\Omega$ as temperature.

Contrary to the conventional wisdom in domain wall model-building, where the formation of the domain wall happens separately from the Higgs condensation to break electroweak symmetry, we succeeded in our model to combine both mechanisms and keep the Higgs field active even in five dimensions. In other words, our model is very economical in terms of field content. Naively, one may expect that this means that domain-wall mass scale must coincide with the SM scale, but surprisingly, that does not have to be so. As we have argued in Sec. \[11\] all light modes are separated from all KK modes by the mass scale $\Omega$, which is of order $10^{2+a}$ GeV, where $a$ can be large at the cost of only mild fine-tuning. We found a natural bound $a \gtrsim 1$ in Sec. \[14\] The reason why this separation of scales happens naturally is that we are near the critical point of the domain-wall induced Higgs condensation. In
short, our model can be viewed as an enrichment of the conventional domain wall model-building toolbox by a new instrument, which is the domain-wall induced condensation where the Higgs field plays a role of a position-dependent order parameter.

In addition to the conceptual advantages listed above, we investigated a new interaction $h\gamma\gamma$ (and $hgg$) coming from Eq. (I.1). This should be bounded by the LHC measurement [65, 66], therefore it gives a constraint to $\beta^2$. However, a small deviation from exactly vanishing amplitude $h\gamma\gamma$ from tree-level coupling is allowed, which can be a testable signature in the future experiment at the LHC. This possibility of the tree-level coupling of $h\gamma\gamma$ is a new signature of our model of domain-wall-induced Higgs condensation and gauge field localization. This feature is in contrast to similar models of gauge field localization without the active participation of Higgs field in the localization mechanism [52, 53]. For instance, these models generally give only loop-effects of KK particles, instead of the tree-level $h\gamma\gamma$ coupling. Therefore we can have a testable signature of $h\gamma\gamma$ even if there are no low-lying KK particles, unlike these models. Furthermore, our five-dimensional model explains higher dimensional interaction as Eq. (V.1) that allows the existence of a finite electroweak monopole, whereas previous studies have failed to provide the origin of such higher-dimensional operators [69, 70]. The monopole mass was estimated [69, 70] as $\lesssim 5.5$ TeV, so that it can be pair-produced at the LHC and accessible to the MoEDAL experiment [71, 72]. If an electroweak monopole will be found, it provides an indirect evidence for the extra dimensions and the domain wall. Our domain wall model can account for the hierarchical Yukawa coupling in the SM from position difference of localized wave functions of matters as was done in many models with extra dimensions [73, 74].

If we introduce the other scalar fields $\phi_i$ to localize the gauge field and the Higgs field via $\beta(\phi_i)$ as in Eq. (I.1), they would give an impact on the low-energy physics like $\phi_i \rightarrow hh$, $\phi_i \rightarrow \gamma\gamma$, and $\phi_i \rightarrow gg$. Therefore, we have to be very cautious for including the extra scalar fields $\phi_i$. Our model is free from this kind of concern, which is one of the important progress achieved in this work.

Although we did not explain it in detail, the absence of additional light scalar boson from $A_y$ is one of the important properties of our model [51, 53]. Moreover, the fact that the localization of gauge fields via Eq. (I.1) automatically ensures the universality of gauge charges is also important.

In summary, the particle contents appearing in the low-energy effective theory on the domain wall are identical to those in the SM. All the KK modes can be sufficiently separated from the SM particles as long as we set $\Omega \sim 10^{2+\alpha}$ GeV be sufficiently large. Nevertheless,
our model is distinguishable from the SM by the new tree-level decay $h \to \gamma\gamma \ (h \to gg)$ and a finite electroweak monopole. A possible concern in our model is the additional massless particle $Y(x)$ which is inevitable because it is the NG mode for spontaneously broken translational symmetry. However, thanks to the low-energy theorems, all the interactions including $Y(x)$ must appear with derivatives $\partial_\mu Y(x)$. Consequently, they are suppressed by the large mass scale $\Omega$ and have practically no impact on phenomena at energies much lower than the large mass scale $\Omega$. The KK quark pair production via NG particle exchange gives a lower bound for $\Omega$ which is larger than 1.5 TeV. Larger $\Omega$ requires severer fine-tuning, but is safer phenomenologically, whereas smaller $\Omega$ requires less fine-tuning and can be disproved more easily by experimental data.

Let us discuss possible effects of radiative corrections in our low-energy effective theory. The particle content of effective theory below the mass scale $\Omega$ is identical to SM except the NG boson $Y(x)$ for translation. The higher dimensional operators of NG boson interactions are suppressed by powers of the large mass $\Omega$. Hence they do not contribute for phenomena at energies much below the scale $\Omega$, in the spirit of effective Lagrangian approach. The only possible exception is the Higgs coupling of gauge fields expressed by higher dimensional operators with the small mass scale $\mu$ in the gauge kinetic function. This coupling of Higgs boson and gauge fields such as in Eq. (III.22) is given by Higgs vacuum expectation value $\langle h \rangle$. We need to assume that the higher dimensional coupling of Higgs boson and gauge fields are fine-tuned to that value when the Higgs vacuum expectation value is fine-tuned to a value much smaller than $\Omega$. With this assumption, we expect that the radiative corrections to quantities such as physical Higgs boson mass should be essentially the same as nonsupersymmetric SM. For instance we need to implement supersymmetry if we wish to make the fine tuning less severe in our model.

Models with warped spacetime \cite{4, 5} exhibit features similar to our model, except that the usual assumption of delta-function-like brane in models with warped spacetime is replaced by a smooth localized energy density (fat brane) in our model. Previously we have studied BPS domain-wall solutions embedded into four- and five-dimensional supergravity \cite{11, 13}. These solutions are quite similar to BPS domain wall solutions in our present model. From these examples, we expect that our model can be coupled to gravity giving a fat brane embedded into warped spacetime. The resulting model should give physics in warped spacetime with finite wall width. We expect that phenomenology of our model will not be affected too much as long as we consider phenomena at energies below the gravitational (Planck) scale.

Finally, our model offers an interesting problem for the study of the cosmological evolution
of the universe. Let us restrict ourselves in the region of temperature around the scale $\lambda \bar{v} \sim 10^2$ GeV, where the analysis using effective potential is applicable. As we calculate explicitly in Appendix A and B, we find that the effective potential computed on the stable background with $\langle H \rangle = v_h/\sqrt{2}$ is slightly different from that computed on the unstable background with vanishing Higgs $\langle H \rangle = 0$. More explicitly, only the quadratic term has different coefficient $\lambda^2_2$: it changes from $-4(\lambda \bar{v})^2/3$ at $\langle H \rangle = v_h/\sqrt{2}$ to $-(\lambda \bar{v})^2$. We can understand this phenomenon as follows. The definition of effective Higgs field depends on the background solution on which we expand the quantum fluctuation. The off-shell extrapolation of the effective potential computed on a particular background is different from that computed on a different background. Consequently, even though the extrapolated effective potential can give the position of another neighboring stationary point correctly, the curvature (mass squared) around it need not reproduce the value of mass squared computed on that point, since the background is different. This feature is in contrast to ordinary local field theory, and perhaps can be interpreted as a composite nature of fluctuation fields on solitons. The coefficient $\lambda^2_2$ is directly related to the transition temperature of phase transition during the cosmological evolution. At zero temperature, our effective potential (II.9) calculated on the background of $\langle H \rangle = v_h/\sqrt{2}$ is valid, since we assume $\Omega < \lambda v$. As we heat up the universe starting from this situation, finite temperature effects come in to raise the effective potential for nonzero values of Higgs field. Eventually around a certain temperature of order $\lambda \bar{v}$, we will find a phase transition to the phase without Higgs condensation, namely $SU(2) \times U(1)$ gauge symmetry restoration. To estimate this transition temperature, we need to study the change of effective potential during this process. As we noted, the coefficient $\lambda^2_2$ is likely to change gradually from $-4(\lambda \bar{v})^2/3$ to $-(\lambda \bar{v})^2$. Therefore we need to take account of the change of $\lambda^2_2$ besides the finite temperature effects. This is an interesting new challenge to determine the transition temperature in this kind of models. We leave this issue for a future study.

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Appendix A: Mode equations on the stable BPS solution

Here we define mode expansions for Higgs and other fields in order to compute low-energy effective action in four dimensions. We need to choose a solution of field equations as a background on which we expand fluctuation fields. Since we are interested in the parameter region (II.6), we should choose the stable BPS solution in Eq.(II.5). With this background, we define fluctuation fields \( \delta T \) and \( \delta H_R, \delta H_I \) as

\[
T = v \tanh \Omega y + \frac{\delta T}{\sqrt{2}}, \quad H = \frac{\bar{v}}{\cosh \Omega y} + \frac{\delta H_R + i \delta H_I}{\sqrt{2}}.
\]

(A.1)

The quadratic part of the bosonic Lagrangian is given by means of Hamiltonians \( K_{TR}, K_I \)

\[
\mathcal{L}^{(2)} = \mathcal{L}_{TR}^{(2)} + \mathcal{L}_I^{(2)},
\]

(A.2)

\[
\mathcal{L}_{TR}^{(2)} = \frac{1}{2} \Phi^T [-\partial_\mu \partial^\mu - K_{T,R}] \Phi, \quad \Phi^T = (\delta T, \delta H_R),
\]

\[
K_{TR} = -\partial_y^2 1_2 + \left( \begin{array}{cc} 2\lambda^2 (\mathcal{H}_0^2 + 3\mathcal{T}_0^2 - v^2) & 4\lambda^2 \mathcal{T}_0 \mathcal{H}_0 \\ 4\lambda^2 \mathcal{T}_0 \mathcal{H}_0 & \Omega^2 + 2\lambda^2 (3\mathcal{H}_0^2 + \mathcal{T}_0^2 - v^2) \end{array} \right),
\]

(A.3)

\[
\mathcal{L}_I^{(2)} = \frac{1}{2} \delta \mathcal{H}_I [-\partial_\mu \partial^\mu - K_I] \delta \mathcal{H}_I,
\]

\[
K_I = -\partial_y^2 + \Omega^2 + 2\lambda^2 (\mathcal{H}_0^2 + \mathcal{T}_0^2 - v^2).
\]

(A.4)

Once we obtain eigenfunctions of these Hamiltonians, we can obtain mode expansions of the 5D fields into KK towers of effective fields, such as

\[
\Phi_i(x,y) = \sum_{n=0}^{\infty} \phi_n(x) u_i^{(n)}(y), \quad i = T, R.
\]

(A.5)
where the \( n \)-th eigenstate generally has components in both 5D fields \( \delta T \) and \( \delta H_R \), since they have coupled Hamiltonian \( K_{TR} \). The label of eigenstates \( n \) contains also continuum states.

Since the \( \delta H_I \) will be absorbed by the gauge boson by the Higgs mechanism, we will consider only the coupled linearized field equation for \( \delta T \) and \( \delta H_R \). Since the coupled equation is difficult to solve exactly, we solve it starting from the \( \lambda \bar{v} = 0 \) case as a perturbation series in powers of the small parameter \( \epsilon^2 = (\lambda \bar{v}/\Omega)^2 \).

At \( \lambda \bar{v} = 0 \), the Hamiltonian \( K_{TR} \) becomes diagonal and the \( T \) and \( H_R \) linearized field equations decouple

\[
K_T = -\partial_y^2 + 4\Omega^2 - \frac{6\Omega^2}{\cosh^2 \Omega y}, \\
K_R = -\partial_y^2 + \Omega^2 - \frac{2\Omega^2}{\cosh^2 \Omega y}.
\]

(A.6) (A.7)

Eigenvalues of the Hamiltonian give mass squared \( m^2 \) of the corresponding effective fields.

In the parameter region (II.6), we find two discrete bound states for \( \delta T \), and a continuum of states with the threshold at \( (m_T^{(2)})^2 = (2\Omega)^2 \)

\[
u_T^{(0)}(y) = \frac{\sqrt{3\Omega}}{2} \frac{1}{\cosh^2 \Omega y}, \quad (m_T^{(0)})^2 = 0,
\]

(A.8)

\[
u_T^{(1)}(y) = \sqrt{\frac{3\Omega \tanh \Omega y}{2 \cosh \Omega y}}, \quad (m_T^{(1)})^2 = 3(\Omega)^2.
\]

(A.9)

We recognize that the massless mode is precisely the Nambu-Goldstone boson for spontaneously broken translation. For the fluctuation \( \delta H_R \), we find that there is only one discrete bound state below the threshold at \( \Omega^2 \)

\[
u_R^{(0)}(y) = \sqrt{\frac{\Omega}{2}} \frac{1}{\cosh \Omega y}, \quad (m_R^{(0)})^2 = 0.
\]

(A.10)

This is the massless particle at the critical point where condensation of \( H_R \) starts. It is not an accident that the functional form of this mode function is identical to the condensation of \( H_R \) in Eq. (II.5). This mode will become massive physical Higgs particle when we switch on the perturbation \( (\lambda \bar{v})^2 > 0 \).

We can now systematically compute the perturbative corrections in powers of small parameter \( \epsilon \). The lowest order correction to the eigenvalue can be obtained by taking the expectation value of the perturbation Hamiltonian in terms of the lowest order wave function. Therefore we obtain the mass eigenvalue of the physical Higgs particle up to the leading
order

\[(m_h)^2 = \int dy u_R^{(0)}(y) [(K_{TR})_{22} - K_R] u_R^{(0)}(y) = \frac{8}{3} (\lambda \bar{v})^2. \quad \text{(A.11)}\]

This result agrees with the result of the analysis using the effective potential (II.8). In fact, we can reproduce the effective potential by evaluating the cubic and quartic terms in fluctuation field \(\delta \mathcal{H}_R\). With the perturbation theory, we can compute corrections to the Higgs mass to any desired order of \(\epsilon\).

\[V_H = -\frac{4(\lambda \bar{v})^2}{3} |H|^2 + \frac{\lambda^2 \Omega^2}{3} |H|^4, \quad \text{(A.12)}\]

in agreement with Eq. (II.8).

**Appendix B: Mode equations on the unstable BPS solution**

We can choose another BPS solution (II.4) as background, which becomes stable in the parameter region \(\Omega < \lambda v\). We define a small fluctuation around this background as

\[\mathcal{T} = v \tanh \lambda vy + \delta \mathcal{T}'/\sqrt{2}, \quad \mathcal{H} = (\delta \mathcal{H}_R^I + i \delta \mathcal{H}_I^R)/\sqrt{2}. \quad \text{(B.1)}\]

The linearized field equation, in this case, is decoupled with the Hamiltonian \(K_T', K_R', K_I\) as

\[K_T' = -\partial_y^2 + 4(\lambda v)^2 - \frac{6(\lambda v)^2}{\cosh^2 \lambda vy}, \quad \text{(B.2)}\]

\[K_R' = K_I' = -\partial_y^2 + \Omega^2 - \frac{2(\lambda v)^2}{\cosh^2 \lambda vy}. \quad \text{(B.3)}\]

We find exact mode functions in this case. We find two discrete bound states for \(\delta \mathcal{T}'\) and a continuum of states with the threshold at \((m_{T'}^{(2)})^2 = (2\lambda v)^2\)

\[u_T^{(0)}(y) = \frac{\sqrt{3\lambda v}}{2} \frac{1}{\cosh^2 \lambda vy}, \quad (m_{T'}^{(0)})^2 = 0, \quad \text{(B.4)}\]

\[u_T^{(1)}(y) = \frac{\sqrt{3\lambda v} \tanh \Omega y}{2 \cosh \lambda vy}, \quad (m_{T'}^{(1)})^2 = 3(\lambda v)^2. \quad \text{(B.5)}\]
The massless mode gives an exact NG boson mode function in this case. For the fluctuation \( \delta \mathcal{H}'_R \), we find that there is only one discrete bound state below the threshold at \( \Omega^2 \)

\[
    u'^{(0)}_R(y) = \frac{\sqrt{\lambda v}}{2} \frac{1}{\cosh \lambda vy}, \quad (m'^{(0)}_R)^2 = -(\lambda v)^2.
\]

This is precisely the tachyonic mode at the unstable background solution. We note that the value of (negative) mass squared is different from the corresponding value \(-4(\lambda \bar{v})^2/3\) of the off-shell extension to \( H = 0 \) of the effective potential computed on the stable BPS solution in Eq. (II.9). This is due to the fact that a different background solution gives a different spectrum of fluctuations, even though they are qualitatively similar.

Once the exact mode function is obtained, on the background of the unstable solution, we only need to insert the following Ansatz into the 5D Lagrangian and integrate over \( y \), in order to obtain the effective potential of the effective Higgs field \( H'(x) \).

\[
    \mathcal{T} = v \tanh \lambda vy, \quad \mathcal{H} = H'(x) \frac{\sqrt{\lambda v}}{2} \frac{1}{\cosh \lambda vy}.
\]

After integrating over \( y \), we obtain the effective action as

\[
    \mathcal{L}_{\text{Higgs}}(H') = |D_\mu H'|^2 - V_{H'}, \quad V_{H'} = -(\lambda \bar{v})^2 |H'|^2 + \frac{\lambda^2 \Omega}{3} |H'|^4.
\]

The quadratic term agrees with the mass squared eigenvalue of the mode equation of fluctuations. It is interesting to observe that the coefficient of the quadratic term is different from that computed on the stable BPS solution as background, although the quartic term is identical.

Appendix C: Cross section for KK fermion pair production by NG boson exchange

Here we calculate the differential cross section \([IV.7]\). First we consider the process \( u_L d_L \rightarrow u^{(1)}_L d^{(1)}_L \), whose Feynman diagram is shown in Fig. 2(a). The amplitude is given in terms of spinor wave functions \( u_{uL} \) and \( u_{dL} \) of incoming SM fermions, and \( u_{u^{(1)}L} \) and \( u_{d^{(1)}L} \) of outgoing KK quarks as

\[
    i\mathcal{M} = \frac{\alpha^2 \Omega}{v^2} \frac{i}{l} (\bar{u}_{u^{(1)}L}(k_1)i(\not{\phi}_1 - \not{k}_1)u_{uL}(p_1))(\bar{u}_{d^{(1)}L}(k_2)i(\not{\phi}_1 - \not{k}_1)u_{dL}(p_2)),
\]

\[\text{(C.1)}\]
with \( t = (p_1 - k_1)^2 \). We approximate SM quarks to be massless, and assume the same vertex couplings \( \alpha \) for \( uu^{(1)}Y \) and \( dd^{(1)}Y \) for simplicity, although they can be different since fermion wave functions for \( u, u^{(1)} \) and \( d, d^{(1)} \) are in general different. The squared amplitude is

\[
|\mathcal{M}|^2 = \frac{4 \alpha^4 \Omega^2}{v^4 t^2} \left\{ 2(p_1 \cdot (p_1 - k_1))(k_1 \cdot (p_1 - k_1)) - (p_1 \cdot k_1)t \right\} \\
\times \left\{ 2(p_2 \cdot (p_1 - k_1))(k_2 \cdot (p_1 - k_1)) - (p_2 \cdot k_2)t \right\} \\
= \frac{4 \alpha^4 \Omega^2}{v^4} E^4 \left( 1 - \beta \cos \theta \right)^2 \left( 1 - \beta^2 \right),
\]

(C.2)

which leads to the differential cross section

\[
\frac{d\sigma}{dt}(u_Ld_L \rightarrow u^{(1)}_Ld^{(1)}_L) = \frac{\alpha^4 \Omega^2}{576 \pi^2 s} \frac{E^4 \left( 1 - \beta_{M_1} \cos \theta \right)^2 \left( 1 - \beta_{M_1}^2 \right)}{\left( \beta_{M_1}^2 + 1 - 2 \beta_{M_1} \cos \theta \right)^2},
\]

(C.3)

with \( s = 4E^2 \). Other combinations of initial quark chiralities \( RR, LR, RL \) are found to give identical differential cross sections. Hence we find \( (IV.7) \).

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