1/4 BPS AdS$_3$/CFT$_2$

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We discuss new solutions in massive Type IIA supergravity with AdS$_3$×S$^2$ factors, preserving $\mathcal{N} = (0, 4)$ SUSY. We propose a duality with a precise family of quivers that flow to $\mathcal{N} = (0, 4)$ fixed points at low energies. These quivers consist on two families of linear quivers coupled by matter fields. Physical observables such as the central charges provide stringent checks of the proposed duality. A formal mapping is presented connecting our backgrounds with those dual to six dimensional $\mathcal{N} = (1, 0)$ CFTs, suggesting the existence of a flow across dimensions between the CFTs.

I. INTRODUCTION

An important by-product of the Maldacena conjecture [1], has been a thorough study of supersymmetric and conformal field theories (SCFTs) in various dimensions. In particular, the last two decades have witnessed a large effort in the classification of Type II or M-theory backgrounds with AdS$_{d+1}$ factors, see for example [2],[3]. The solutions are conjectured to be dual to CFTs in $d$ dimensions with different amounts of SUSY, that can then be studied holographically.

Major progress has been achieved when the CFT preserves half of the maximum number of allowed supersymmetries. For the case of $\mathcal{N} = 2$ CFTs in four dimensions, the field theories studied in [4] have holographic duals discussed in [5] and further elaborated (among other works) in [6]-[11]. The case of five-dimensional CFTs was analysed from the field theoretical and holographic viewpoints in [12]-[18], among many other interesting works. An infinite family of six-dimensional $\mathcal{N} = (1, 0)$ CFTs was discussed from both the field theoretical and holographic points of view in [19]-[26]. For three-dimensional $\mathcal{N} = 4$ CFTs, the field theories presented in [27] were discussed holographically in [28]-[31], among other works.

The case of two-dimensional CFTs and their AdS duals is particularly attractive, due to the interest that CFTs in two dimensions and AdS$_3$ solutions present in other areas of theoretical physics. This applies in particular to the microscopic study of black holes, where major progress has been achieved in [32]-[37]. This motivated various attempts at finding classifications of AdS$_3$ backgrounds and studying their dual CFTs [38]-[56]. Small $\mathcal{N} = (0, 4)$ AdS$_3$ solutions remains however largely unexplored, with known cases following mostly from orbifolding, string dualities or F-theory constructions. Two-dimensional SCFTs with $\mathcal{N} = (0, 4)$ supersymmetry constructed in the literature [57]-[61] awaited as well their holographic description. An important recent development has been the complete classification of AdS$_3$ solutions in massive IIA with $\mathcal{N} = (0, 4)$ (and SU(2) structure) achieved in [56]. In this letter we add a new entry to the dictionary between CFTs and string backgrounds with an AdS-factor by proposing explicit CFTs dual to these solutions. We define our CFTs as the IR fixed points of $\mathcal{N} = (0, 4)$ UV finite QFTs. These QFTs are described by quivers, consisting of two long rows of gauge groups connected by hypermultiplets and Fermi multiplets. We show that the new background solutions to massive IIA supergravity constructed in [56] contain the needed isometries to be dual to our CFTs. We give an example (elaborated in [62] where additional examples can be found) that shows agreement between the field theory and holographic calculations of the central charge. Finally, we provide a formal mapping to the AdS$_3$ solutions constructed in [20] that suggests the existence of a flow across dimensions [63] between the dual CFTs.

II. THE GEOMETRY

The backgrounds of massive IIA supergravity constructed in [56] were proposed to be dual to $\mathcal{N} = (0, 4)$ CFTs in two dimensions. These solutions have SL(2)×SU(2) isometries and eight (four Poincare plus four conformal) supercharges. In this paper we will consider a particular case of the geometries in [56], those referred as class I therein. In string frame, the background reads,

$$ds^2 = g_1 \left( ds^2(\text{AdS}_3) + g_2 ds^2(S^2) \right) + g_3 ds^2(CY_2) + \frac{d\rho^2}{g_4},$$
$$e^{-\Phi} = g_4, \quad B_2 = g_5 \text{vol}(S^2), \quad \hat{F}_0 = g_6, \quad \hat{F}_2 = g_7 \text{vol}(S^2),$$
$$\hat{F}_4 = g_8 d\rho \wedge \text{vol(AdS$_3$)} + g_9 \text{vol(CY}_2).$$

(1)

The functions $g_i$ are defined in terms of three functions, $u(\rho), h_4(\rho), h_8(\rho)$, according to,

$$g_1 = \frac{u}{\sqrt{h_4 h_8}}, \quad g_2 = \frac{h_8 h_4}{4 h_4 h_8 + (u')^2}, \quad g_3 = \sqrt{\frac{h_4}{h_8}},$$
$$g_4 = \frac{h_8^2}{2 h_4^2} \sqrt{4 h_8 h_4 + (u')^2}, \quad g_6 = h_8', \quad g_9 = -\partial_\rho h_4.$$
which also imply the continuity of the functions \( \hat{h}_4, h_8 \) across intervals. The first derivatives present discontinuities at \( \rho = 2k\pi \) where D8 and D4 sources are located.

**Page charges:**

The Page charges are important observable quantities characterising a supergravity solution. They are quantised, hence the quantisation of some of the constants in eqs. (2)-(3) is implied. The Page charge of Dp-branes is given by the integral of the magnetic part of the Page \( \hat{F}_{A-p} \) form, according to 

\[
Q_{Dp} = \frac{(2\pi)^{p-7}}{2\pi} g_8 \alpha' (7-p)/2 Q_{Dp} = \int_{\Sigma_p} \hat{F}_{A-p}.
\]

In what follows, we set \( \alpha' = g_8 = 1 \). In the interval \( [2k\pi, 2(k+1)\pi] \), we find,

\[
Q_{D8} = 2\pi F_0 = \nu_k, \quad Q_{D6} = \frac{1}{2\pi} \int_{S^2} \hat{F}_2 = \mu_k.
\]

\[
Q_{D4} = \frac{1}{8\pi^2} \int_{CY_2} \hat{F}_4 = Y \frac{\text{Vol}(CY_2)}{16\pi^2} \beta_k, \quad Q_{D2} = \frac{1}{32\pi^5} \int_{CY_2 \times S^2} \hat{F}_6 = Y \frac{\text{Vol}(CY_2)}{16\pi^4} \alpha_k.
\]

We have used that the magnetic part \( \hat{F}_{6, mag} = \hat{f}_6 \) is

\[
\hat{f}_6 = \frac{1}{2} (h_4 - h_4'\rho - 2k\pi) \text{vol}(S^2) \wedge \text{vol}(CY_2).
\]

We count one NS-five brane every time we cross the value \( \rho = 2k\pi \) (for \( k = 1, \ldots, P \)). The total number of NS-five branes is

\[
Q_{NS} = \frac{1}{2\pi} \int_{\Sigma S^2} H_3 = (P + 1).
\]

The study of the Bianchi identities (see [62] for the details), shows that dissolved in flux, we have “colour” D2 and D6 branes. We also find that D4 and D8 branes play the role of “flavour”, appearing explicitly as delta-function corrections of the Bianchi identities. For the interval \( [2\pi(k-1), 2\pi k] \), we calculate

\[
N_{D_6}^{k-1,k} = \nu_{k-1} - \nu_k, \quad N_{D_4}^{k-1,k} = \beta_{k-1} - \beta_k, \quad N_{D_8}^{k-1,k} = \mu_k = \sum_{i=0}^{k-1} \nu_i, \quad N_{D_4}^{k-1,k} = \alpha_k = \sum_{i=0}^{k-1} \beta_i.
\]

We then have a Hanany-Witten brane set-up [64], that in the interval \( [2\pi(k-1), 2\pi k] \) (bounded by NS-five branes), has \( N_{D_6}^{k-1,k}, N_{D_4}^{k-1,k} \) colour branes and \( N_{D_8}^{k-1,k}, N_{D_4}^{k-1,k} \) (see table I and figure 1).

| \( D2 \) | \( D4 \) | \( D6 \) | \( D8 \) |
|---|---|---|---|
| \( x \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \times \time
III. THE FIELD THEORY

In this section we discuss the two-dimensional CFTs dual to the backgrounds given by eqs.(1)-(3). They are defined as the strongly coupled IR fixed points of QFTs that in the (weakly coupled) UV are constructed from the “building block” depicted in figure 2. We have an SU($N$) gauge group with the matter content of a two-dimensional $\mathcal{N} = (4,4)$ vector multiplet in the adjoint of SU($N$). This gauge group is joined with other (gauged or global) symmetry groups SU($\hat{P}$), SU($R$) and SU($Q$). The connection with the SU($\hat{P}$) symmetry group is mediated by $\mathcal{N} = (4,4)$ hypermultiplets running over the black solid line. The connection with the SU($R$) symmetry group is via $\mathcal{N} = (0,4)$ hypermultiplets that propagate over the grey lines. Finally, over the dashed line run $\mathcal{N} = (0,2)$ Fermi multiplets. Notice that a similar (but not the same) field content was used in [59].

The cancellation of gauge anomalies constrains the ranks of the different symmetry groups. Using the contribution to the gauge anomaly coming from each multiplet, see [67], we find that for the SU($N$) gauge group the cancellation of the anomaly imposes,

$$2R = Q. \quad (10)$$

Our quiver gauge theories are then obtained by “assembling” the building blocks of figure 2 such that there is anomaly cancellation. In turn, the central charge of the IR CFT is calculated by associating it with the correlation function of U(1)- R-symmetry currents (computed in the UV-description above). At the conformal point, the central charge is related to the two point, U(1)$_c$ current correlation function, such that (see for example [68]),

$$c = 6(n_{\text{hyp}} - n_{\text{vec}}). \quad (11)$$

The central charge is then obtained by counting the number of $\mathcal{N} = (0,4)$ hypermultiplets and substracting the number of $\mathcal{N} = (0,4)$ vector multiplets in the UV description.

The proposed duality:

Our proposal relates the backgrounds in eqs.(1)-(3) with quiver field theories obtained by assembling building blocks such as the one depicted in figure 2. For generic functions $h_4, h_8$ this results in the quiver shown in figure 3, associated to the Hanany-Witten set-up in figure 4. The reader can check that the cancellation of gauge anomalies implies for a generic SU($\alpha_k$) colour group, in the interval $[2\pi(k - 1), 2\pi k]$

$$F_{k-1} + \mu_{k+1} + \mu_{k-1} = 2\mu_k \rightarrow F_{k-1} = \nu_{k-1} - \nu_k, \quad (12)$$

which, according to (7), is precisely the number of flavour D8 branes in the $[2\pi(k - 1), 2\pi k]$ interval of the brane set-up. Things work analogously if D6 are replaced by D2 (or $\mu_k \leftrightarrow \alpha_k$) and D8 by D4 ($\nu_k \leftrightarrow \beta_k$), and we work with a generic lower-row gauge group SU($\mu_k$).

We can calculate the central charge of the quiver by counting the numbers of $\mathcal{N} = (0,4)$ hypermultiplets and vector multiplets. We find,

$$c = \frac{6}{\pi} \sum_{j=1}^{P} (\alpha_j \mu_j - \alpha_j^2 - \mu_j^2 + 2) + \sum_{j=1}^{P-1} (\alpha_j \alpha_{j+1} + \mu_j \mu_{j+1}). \quad (13)$$

In [62] we present various examples in which this expression agrees with the holographic central charge computed according to (9). This should hold in the limit in which the number of nodes $P$ and the ranks of each gauge group $\alpha_i, \mu_i$ are large, which is when the supergravity backgrounds are trustable. We present one such example below. Notice that both the global symmetries and isometries (space-time, SUSY and flavour), as well as the ranks of the gauged (colour) groups do match in both descriptions, the latter being given by the numbers $\alpha_k, \mu_k$ in (4).

An example:

Let us discuss an example that illustrates the duality proposed above. We consider the quiver with two rows of linearly increasing colour groups depicted in Figure
5. For an intermediate gauge node $\text{SU}(k\nu)$ we have $Q = 2k\beta, R = k\beta$. This implies that (10) is satisfied and any generic intermediate gauge group is not anomalous. If we refer to the last gauge group in the upper-row $\text{SU}(P\nu)$ we have that $Q = (P + 1)\beta + (P - 1)\beta = 2P\beta$ and $R = P\beta$. As a consequence (10) is satisfied and the gauged group $\text{SU}(P\nu)$ is also not anomalous. The same occurs for the lower-row gauge groups.

The counting of $(0, 4)$ hypermultiplets and vector multiplets gives

$$n_{vec} = \sum_{j=1}^{P} (j^2(\nu^2 + \beta^2) - 2),$$

$$n_{hyp} = \sum_{j=1}^{P-1} j(j + 1)(\nu^2 + \beta^2) + \sum_{j=1}^{P} j^3\nu\beta,$$

from which the central charge of the IR CFT can be computed,

$$c = 6\nu\beta\left(\frac{P^3}{3} + \frac{P^2}{2} + \frac{P}{6} - 3(\nu^2 + \beta^2)(P^2 + P) + 12P\right) \sim 2\nu\beta P^3.$$  (15)

In turn, the holographic description of the system is in terms of the functions,

$$h_8(\rho) = \begin{cases} \frac{2P\rho}{2\pi} & 0 \leq \rho \leq 2P\pi \\ \frac{\pi}{2} & 2\pi P \leq \rho \leq 2P(\pi + 1) \end{cases},$$

$$\hat{h}_4(\rho) = \begin{cases} \frac{\pi}{2} & 0 \leq \rho \leq 2P\pi \\ \frac{\pi}{\beta} & 2\pi P \leq \rho \leq 2\beta P(\pi + 1) \end{cases}. $$ (16) (17)

Using (9) and a convenient choice for the constant $\Upsilon$, gives rise to the holographic central charge,

$$c_{hot} = 2\nu\beta P^3(1 + \frac{1}{P}) \sim 2\nu\beta P^3.$$  (18)

We thus find perfect agreement between the field theory and holographic calculations. In [62] other examples of dual holographic pairs are discussed that provide stringent support to our proposed duality.

IV. MAPPING TO $\text{AdS}_7$ BACKGROUNDS

A sub-class of the solutions discussed in section II can be related to the $\text{AdS}_7$ solutions to massive IIA constructed in [20]. As opposed to the mappings in [19], this mapping is not one-to-one, due to the presence of additional D2-D4 branes in the $\text{AdS}_7$ solutions, whose backreaction introduces 4-form and 6-form fluxes, and reduces the supersymmetries by a half. Using this map it is possible to give an interpretation to the 2d CFTs dual to the $\text{AdS}_7$ solutions as associated to D2-D4 defects in the D6-NS5-D8 brane set-ups dual to the $\text{AdS}_7$ solutions in [20], wrapped on the CY$_2$. Thus, the word defect is here used to indicate the presence of extra branes in Hanany-Witten brane set-ups that would otherwise arise from compactifying higher dimensional branes.

The explicit map, discussed in detail in [69], reads,

$$\rho \leftrightarrow 2\pi z, \ u \leftrightarrow \alpha, \ h_8 \leftrightarrow -\frac{\bar{\alpha}}{81\pi^2}, \ \hat{h}_4 \leftrightarrow \frac{81}{\bar{\alpha}},$$

$$ds^2(\text{AdS}_3) + \frac{4}{\bar{\alpha}} ds^2(\text{CY}_2) \leftrightarrow 4 ds^2(\text{AdS}_7).$$  (19)

This transforms the original backgrounds in (1) into the $\text{AdS}_7$ backgrounds constructed in [20]

$$\frac{ds^2}{\pi\sqrt{2}} = 8\sqrt{-\frac{\alpha}{\bar{\alpha}}} ds^2(\text{AdS}_7) + \sqrt{-\frac{\bar{\alpha}}{\alpha}} dz^2 + \frac{\alpha^{3/2}/(\bar{\alpha})^{1/2}}{\Delta} ds^2(S^2),$$

$$e^{2\Phi} = 2^{5/2} \pi^5 8 \left(\frac{-\alpha/\bar{\alpha}}{\Delta^2 - 2\alpha\bar{\alpha}}\right), \ F_0 = \frac{\bar{\alpha}}{16\pi^3},$$

$$B_2 = \pi \left(-z + k + \frac{\alpha}{\Delta}\right) \text{vol}(S^2),$$

$$\bar{F}_2 = \frac{1}{16\pi^2} \left(\bar{\alpha} - \bar{\alpha}(z - k)\right) \text{vol}(S^2), \ \Delta = \alpha^2 - 2\alpha\bar{\alpha},$$

where the function $\alpha(z)$ satisfies the equation $\bar{\alpha} = -16\pi^3 F_0$. As analysed in [20], $\alpha(z)$ encodes the information about the 6d (1,0) dual CFTs, which are realised in D6-NS5-D8 Hanany-Witten set-ups. Using the mapping defined by (19) it is possible to obtain an $\text{AdS}_7$ solution in the class of [20] from an $\text{AdS}_3$ solution. However, starting from an $\text{AdS}_7$ solution it does not allow one to compute the $u$ and $h_8$ functions needed to fully determine the $\text{AdS}_3$ solution.

In the paper [69], we show that the D6-NS5-D8 sector of the $\text{AdS}_3$ solution is simply obtained by compactifying on the CY$_2$ the D6-NS5-D8 branes that underlie the $\text{AdS}_7$ solution, while it is necessary to add the D2-D4 sector, encoded by the functions $u$ and $h_4$ (see [69] for the details) to achieve conformality.

The holographic central charge of the 6d CFTs dual to the $\text{AdS}_7$ solutions was computed in [23],

$$c_{\text{AdS}_7} = \frac{1}{G_N} \int \frac{2^4}{3^8} \frac{dz}{z} (-\alpha\bar{\alpha}).$$  (21)

In turn, the holographic central charge of the 2d CFTs is given in (9). Using the mapping given by (19) this becomes,

$$c_{\text{AdS}_3} \leftrightarrow 3 \frac{\text{Vol}(\text{CY}_2)}{2\pi G_N} \int dz (-\alpha\bar{\alpha}) = \frac{3^9}{2^7} \frac{\text{Vol}(\text{CY}_2)}{3} c_{\text{AdS}_7}.$$

FIG. 5. Quiver consisting of two rows of linearly increasing colour groups, terminated with the addition of flavour groups.
This kind of relations are ubiquitous when calculating the holographic central charges for "flows across dimensions". See for example [63]. The result strongly suggests that we can obtain our CFTs by compactifying on a CY$_2$ the D6-NS5-D8 system underlying the 6d (1,0) CFT. The conformality in the lower dimensional theory however requires the presence of “defect” D2 and D4 branes, represented by the fluxes $F_4, F_2$ in (1). Flows of this type were studied in [70],[71], but those do not reach an AdS$_3$ fixed point. It would be interesting to find the explicit RG flows that deform the six-dimensional $\mathcal{N} = (1, 0)$ CFT to reach a two-dimensional $\mathcal{N} = (0, 4)$ conformal fixed point in the IR. Aside from this, the mapping in eqs.(19) can be used to find a “completion” of the background obtained through non-Abelian T-duality on AdS$_3 \times S^3 \times$ CY$_2$. The details are discussed in [69].

V. CONCLUSIONS

This letter presents a new entry in the mapping between CFTs and AdS-supergravity backgrounds, for the case of two-dimensional small $\mathcal{N} = (0, 4)$ CFTs and backgrounds with AdS$_3 \times S^3$ factors. We have constructed new solutions of the type AdS$_3 \times S^3 \times$ CY$_2$, belonging to class I in the classification in [56], with compact CY$_2$. The defining functions are piecewise continuous. We commented on their regime of validity and matched the background isometries and the global symmetries (both space-time and flavour) of the CFTs. We computed Page charges for flows that deform the six-dimensional $\mathcal{N} = (1, 0)$ CFT to reach a two-dimensional $\mathcal{N} = (0, 4)$ conformal fixed point in the IR. Aside from this, the mapping in eqs.(19) can be used to find a “completion” of the background obtained through non-Abelian T-duality on AdS$_3 \times S^3 \times$ CY$_2$. The details are discussed in [69].

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