Nilpotent Symmetries in Super-Group Field Cosmology

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In this paper we study the gauge invariance of the third quantized super-group field cosmology which is a model for multiverse. Further, we propose both the infinitesimal (usual) as well as the finite superfield-dependent BRST symmetry transformations which leave the effective theory invariant. The effects of finite superfield-dependent BRST transformations on the path integral (so-called void functional in the case of third quantization) are implemented. Within the finite superfield-dependent BRST formulation, the finite superfield-dependent BRST transformations with specific parameter switch the void functional from one gauge to another. We establish this result for the most general gauge with the help of explicit calculations which holds for all possible sets of gauge choices at both the classical and the quantum levels.

I. INTRODUCTION

The biggest challenge for all the schemes that quantize gravity (in particular the background-independent schemes), beside obtaining a solid description of the fundamental degrees of freedom for quantum spacetime, is to produce the testable predictions [1, 2]. In the background-independent approaches to quantize gravity, the loop quantum gravity is a powerful candidate quantizing gravity in mathematically rigorous and non-perturbative way [3, 4]. In this approach the Hamiltonian constraints are reformulated in terms of Ashtekar-Barbero variables, the densitized triad and the Ashtekar connection [5–10]. These basic variables, after the consideration of a holonomies of the connection and the fluxes of the triads, are promoted to the basic quantum operators. In the loop quantum gravity, the complete quantum dynamics of spin network states are obtained by putting the quantum states into the larger framework of group field theories. The group field theories are basically the field theories on group manifolds (or their Lie algebras) which provide a background-independent third quantized formalism for gravity in any dimension and signature [11, 12]. In the group field theories, both the geometry and the topology are dynamical and thus described in purely algebraic and combinatorial terms. The Feynman diagrams of such theories have an interpretation of the spacetimes and therefore the quantum amplitudes for these diagrams can be interpreted as algebraic realization of a path integral description of gravity [13, 14].

The topology of loop quantum gravity is fixed and hence its dynamics can be studied by second quantization. However, the topology changing processes can not be analyzed by second quantization approach. Therefore, to analyze the theories, which are described by topology changing processes, the “third quantization” is mandatory [15–18]. Incidentally, the third quantization of loop quantum gravity leads to the group field theory [19–22]. The basic idea behind the third quantization formalism is to treat the theory of multiverse as a quantum field theory on superspace [23]. However, the Wheeler-De Witt (minisuperspace) approximations of such group field theory are known as the group field cosmology [24–31].

On the other hand, the supersymmetry has been proved as a prominent candidate for the dark matter [32]. Supersymmetry is also important in the study of many phenomenological models beyond the standard model [33–37]. Recently, a supersymmetric generalization for group field cosmology has been made which is known as super-group field cosmology [38]. The super-group field cosmology is a gauge symmetric model of the multiverse and hence the theory contains some spurious degrees of freedom. To quantize the theory correctly one should remove these spurious degrees of freedom by putting some
well-defined constraint, called as the gauge-fixing, in the theory. To achieve this goal at quantum level
a gauge-fixing term is added in the classical action of the theory. This gauge-fixing term reflects an
introduction of Faddeev-Popov ghosts in the void functional (the vacuum functional of third quantization)
which helps in defining the physical Hilbert state of the effective theory. The supersymmetric BRST
transformation and the unitarity of super-group field cosmology has been studied recently [39]. Further,
the Slavnov-Taylor identity and renormalizability of the theory has been investigated [53].

The generalization of usual BRST transformations, so-called the finite field-dependent BRST (FF-
BRST) transformation, has been studied in great details in the context of gauge theories [41]. The
FFBRST transformations have found many important applications in the second quantized gauge field
theories [41 50]. Therefore, it is worthwhile to study the FFBRST formulation in the case of third
quantized super-group field cosmology. With this motivation we feel that this is a glaring omission.

In the present paper we first review the basic ideas of super-group field cosmology. Then, we discuss the
importance of different gauge-fixing conditions and different sets of infinitesimal BRST transformation for
such model of super-group field cosmology. The Jacobian of functional measure under such infinitesimal
BRST transformations has found constant. Further we generalize the third quantized BRST transforma-
ion by making the transformation parameter finite and superfield-dependent because the theory resides
on the superspace. After-that we define the supersource free void functional for third quantized cos-

The paper is organized as follows. In section II, we discuss the third quantized supersymmetric group
cosmology. The different gauge conditions and BRST symmetries are studied in the section III. The
generalization of usual BRST symmetry of super-group field cosmology is discussed in section IV. Fur-
thermore, in section V, we relate the different gauge-fixing conditions of the theory under the effect of
finite superfield-dependent BRST transformations. The last section is reserved for results and conclusions.

II. SUPERSYMMETRIZATION OF GROUP FIELD COSMOLOGY

In this section we provide a description within loop quantum cosmology of the spatially at, homogeneous
and isotropic universe with a massless scalar field as matter. The four-dimensional metric described in
terms of three metric $g_{ab}$ is then defined by

$$ds^2 = -N^2(t)dt^2 + a^2(t)g_{ab}dx^adx^b,$$

where $N(t)$ and $a(t)$ are lapse function and scale factor respectively. Here Latin indices $a$ and $b$ denote the
spatial indices. In loop quantum gravity the phase space is described by a $SU(2)$ gauge connection, the
Ashtekar-Barbero connection $A^A_a$ and its canonically conjugate momentum and the densitized triad $E^a_A$
which plays the role of an “electric field”. The capital alphabets $A, B, ..., $ are $SU(2)$ indices and label new
degrees of freedom introduced when passing to the triad formulation. To define these objects, first one
introduces the co-triad $e^a_A$ defined as $g_{ab} = e^a_Ae^B_B\delta_{AB}$, where $\delta_{AB}$ stands for the Kronecker delta in the
three dimensions. Then we define the triad, $e^a_A$, as its inverse $e^a_Ae^B_B = \delta^a_B\delta^B_A$. Now, the Ashtekar-Barbero
connection reads $A^A_a = \Gamma^A_a + \gamma K^A_a$, where $\gamma$ is the Barbero-Immirzi parameter and $K^A_a$ is the extrinsic
curvature in triadic form, and $\Gamma^A_a$ is the spin connection compatible with the densitized triad. The
curvature of connection $\Lambda^A_a$ in the loop quantum cosmology is expressed through the holonomy around
a loop such that the area of a loop cannot be smaller than a fixed minimum area because the smallest
eigenvalue of the area operator in loop quantum gravity is nonzero. Now, one defines the eigenstates of
the volume operator (finite cell) $\mathcal{V}$ with a basis, $|\nu\rangle$, as follows: $\mathcal{V}|\nu\rangle = 2\pi\gamma G|\nu\rangle$, where gravitational
conguration variable $\nu = \pm a^2\nu_0/2\pi\gamma G$ has the dimensions of length. The Hamiltonian constraint in the
Plank units for a homogeneous isotropic universe is defined as [38]

$$-B(\nu)(E^2 - \partial^2_{\nu})\Phi(\nu, \phi) = K^2\Phi(\nu, \phi) = 0,$$
where $\Phi(\nu, \phi)$ is a wavefunction on configuration space and $E^2$ is a difference operator defined as

$$-E^2\Phi(\nu, \phi) = \frac{C^+(\nu)}{B(\nu)}\Phi(\nu + \nu_0, \phi) + \frac{C^0(\nu)}{B(\nu)}\Phi(\nu, \phi) + \frac{C^-(\nu)}{B(\nu)}\Phi(\nu - \nu_0, \phi), \quad (3)$$

and $K^2 = -B(\nu)[E^2 - \partial_0^2]$. Here $\nu_0$ is an elementary length unit, usually defined by the square root of the area gap and the functions $B(\nu), C^+(\nu), C^0(\nu)$ and $C^-(\nu)$ that depend on the choice of the lapse function and on the details of quantization scheme. For example, in an improved dynamic scheme, these functions for particular choice of lapse function, i.e. $N = 1$, have the following form [57]:

$$B(\nu) = \frac{3\sqrt{2}}{8\sqrt{3\gamma G}} \left| \nu + \nu_0 \right|^3 - \nu - \nu_0 \right|^3,$$

$$C^+(\nu) = \frac{1}{12\gamma\sqrt{2\sqrt{3}}} \left| \nu + \nu_0 \right|^2 - \nu - \frac{3\nu_0}{4},$$

$$C^0(\nu) = -C^+(\nu) - C^-(\nu - \nu_0),$$

$$C^-(\nu) = C^+(\nu - \nu_0). \quad (4)$$

Now, the classical actions for the bosonic group field cosmology is defined by [29, 38]

$$S_{base} = \sum_\nu \int d\phi \ L_{base} = \sum_\nu \int d\phi \ \Phi(\nu, \phi)K^2\Phi(\nu, \phi). \quad (5)$$

It is worthwhile to analyse the fermionic distribution of universes also which might lead to the correct value of the cosmological constant. Since the significant value of cosmological constant is not obtained by considering only bosonic distributions of the universes in the multi-universe scenario [58]. Keeping this point in mind, the free action corresponding to the fermionic group field cosmology is constructed as [38]

$$S_{fermi} = \sum_\nu \int d\phi \ \Psi^i(\nu, \phi)K^i_\nu\Psi^i(\nu, \phi), \quad (6)$$

where $\Psi_1(\nu, \phi) = (\Psi_1(\nu, \phi), \Psi_2(\nu, \phi))$ is a fermionic spinor field and $K_{ij} = (\gamma^\mu)_{ij}K_{\mu}$ is an operator. The spinor indices are raised and lowered by the second-rank antisymmetric tensors $C^{ij}$ and $C_{ij}$ respectively. These tensors satisfy following condition $C_{ij}C^{ij} = \delta^{ij}$. The above bosonic and the fermionic actions describe the bosonic and the fermionic universes in the multiverse and hence, it is worthwhile to construct a supersymmetric gauge invariant model describing multiverse. For this purpose we define two complex scalar super-group fields $\Omega(\nu, \phi, \theta)$ and $\Omega^l(\nu, \phi, \theta)$ and a spinor super-group field $\Gamma_a(\nu, \phi, \theta)$, which are suitably contracted with generators of a Lie algebra, $[T_A, T_B] = if_{AB}^C T_C$, as

$$\Omega(\nu, \phi, \theta) = \Omega^A(\nu, \phi, \theta)T_A,$$

$$\Omega^l(\nu, \phi, \theta) = \Omega^l_A(\nu, \phi, \theta)T_A,$$

$$\Gamma_a(\nu, \phi, \theta) = \Gamma^a_A(\nu, \phi, \theta)T_A. \quad (7)$$

The extra variable $\theta$ is Grassmannian in nature which defines the extra direction in superspace. The super-covariant derivatives of these superfields are defined by [38]

$$\nabla_a \Omega^A(\theta) = D_a\Omega^A(\theta) - if_{ABC}^A \Gamma^C_B(\theta)\Omega_B(\theta),$$

$$\nabla_a \Omega^A(\theta) = D_a\Omega^A(\theta) + if_{ABC}^A \Gamma^C_B(\theta)\Omega_B(\theta),$$

where super-derivative $D_a = \partial_a + K_a^\theta \theta_b$ and superspace variables $(\nu, \phi, \theta) := \theta$. We define the field-strength for a matrix valued spinor field ($\Gamma^A_a$) as follows $\omega^A_a(\theta) = \nabla^a \nabla_a \Gamma^A_a(\theta)$.

Now, we are able to construct the classical action for the super-group field cosmology as [38]

$$S_0 = \sum_\nu \int d\phi \ \left[ D^2 \Omega^A_a(\theta)\nabla^2 \Omega^A(\theta) + \omega^A_a(\theta)\omega^A_a(\theta) \right] $$
where \( '| \) refers \( \theta_a = 0 \) at the end of calculation. This classical action remains invariant under following gauge transformations

\[
\delta \Omega^A(\varphi) = if_{CB}^A \Omega^C(\varphi) \Omega^B(\varphi), \\
\delta \Omega^{A\dagger}(\varphi) = -if_{C\varphi}^A \Omega^{C\dagger}(\varphi) \Lambda^B(\varphi), \\
\delta \Gamma^a_a(\varphi) = \nabla_a \Lambda^A(\varphi).
\]

where the bosonic transformation parameter \( \Lambda^A \) is infinitesimal in nature.

### III. THE INFINITESIMAL BRST SYMMETRIES FOR DIFFERENT GAUGE CONDITIONS

In this section, we will construct the infinitesimal nilpotent BRST symmetries for the theory. For this purpose we need to fix a gauge before quantizing the theory as the theory is gauge invariant and therefore possesses some redundant degrees of freedom. The general gauge-fixing condition for the theory is given by

\[
F^A[\Gamma^A_a(\varphi)] = 0. \tag{11}
\]

This can be incorporated at a quantum level by adding an appropriate gauge-fixing term to the classical action. The linearized gauge-fixing term with the help of Nakanish-Lautrup auxiliary superfield \( B^A(\nu, \phi, \theta) \) is given by

\[
S_{gf} = \sum_{\nu} \int d\phi \left[ D^2 \left\{ B^A(\varphi) F^A[\Gamma^A_a(\varphi)] \right\} \right]. \tag{12}
\]

Since the gauge-fixing condition produces the Faddeev-Popov ghost term in the effective theory. Therefore, in this case, the ghost term induced by (12) is

\[
S_{gh} = \sum_{\nu} \int d\phi \left[ D^2 \left\{ \bar{c}^A(\varphi) s_b F^A[\Gamma^A_a(\varphi)] \right\} \right], \tag{13}
\]

where \( c^A(\varphi) \) and \( \bar{c}^A(\varphi) \) are the ghost and anti-ghost superfields respectively, however, \( s_b \) denotes the Slavnov variation. Now, the total effective action for super-group field cosmology having general gauge choice is written by

\[
S_T = S_0 + S_{gh} + S_{gf}. \tag{14}
\]

However, for specific choice (say covariant gauge choice) \( F^A = D^a \Gamma^A_a(\varphi) = 0 \), the above action \( S_T \) reduces to [39]:

\[
S_T = S_0 + \sum_{\nu} \int d\phi \left[ D^2 \left\{ B^A(\varphi) D^a \Gamma^A_a(\varphi) + \bar{c}^A(\varphi) D^a \nabla_a c^A(\varphi) \right\} \right], \tag{15}
\]

which remains invariant under following third quantized infinitesimal BRST transformations [39]

\[
\delta_b \Omega^A(\varphi) = if_{CB}^A \bar{c}^C(\varphi) \Omega^B(\varphi) \delta \lambda, \\
\delta_b \Omega^{A\dagger}(\varphi) = -if_{C\varphi}^A \bar{c}^{C\dagger}(\varphi) \Lambda^B(\varphi) \delta \lambda, \\
\delta_b c^A(\varphi) = f_{BC}^A(\varphi) \Omega^B(\varphi) \delta \lambda, \\
\delta_b \Gamma^a_a(\varphi) = \nabla_a c^A(\varphi) \delta \lambda, \\
\delta_b \bar{c}^A(\varphi) = B^A(\varphi) \delta \lambda, \\
\delta_b B^A(\varphi) = 0. \tag{16}
\]
where $\delta \lambda$ is an infinitesimal, anticommuting and global parameter. These transformations are nilpotent in nature, i.e., $\delta \lambda^2 = 0$. Utilizing the above BRST transformation we are able to write the sum of gauge-fixing and ghost parts of the action given in (12) and (13) in terms of BRST variation of gauge-fixed fermion as follows

$$S_{gf} + S_{gh} = \sum_\nu \int d\phi \ s_\nu \left[ D^2 \{ \bar{c}_A(\varrho) F^A[\Gamma_\alpha(\varrho)] \} \right].$$

Furthermore, to analyse the theory in massless Curci-Ferrari gauge (which is a non-linear gauge) we make the parameter, $\delta \lambda$ anticommuting and global. These observations give us a freedom to generalize the BRST transformation characterized by infinitesimal parameter $\delta \lambda$ is given by

$$\delta_b \Omega^A(\varrho) = if_{ABC}c^C(\varrho)\Omega^B(\varrho) \delta \lambda,$$

$$\delta_b \Omega^{A\dagger}(\varrho) = -if_{ABC}^\ast \Omega^C(\varrho)c^B(\varrho) \delta \lambda,$$

$$\delta_b c^A(\varrho) = f_{BC}^A c^C(\varrho)c^B(\varrho) \delta \lambda,$$

$$\delta_b \Gamma_\alpha^A(\varrho) = \nabla_a c^A(\varrho) \delta \lambda,$$

$$\delta_b \bar{c}^A(\varrho) = B^A(\varrho) \delta \lambda - \frac{1}{2} f_{BC}^A \bar{c}^B(\varrho)c^C(\varrho) \delta \lambda,$$

$$\delta_b B^A(\varrho) = -\frac{1}{2} f_{BC}^A B^B(\varrho)c^C(\varrho) \delta \lambda$$

$$- \frac{1}{8} f_{BC}^A f_{GHB}^C c^B(\varrho)c^G(\varrho)\bar{c}^H(\varrho) \delta \lambda,$$

which is also nilpotent in nature, i.e., $\delta_b^2 = 0$.

To study the quantum effects for third quantize super-group field cosmology we first define the source free void functional as follows:

$$\langle 0 | 0 \rangle = Z[0] = \int \mathcal{D}Me^{iS_T(\varrho)},$$

where $\mathcal{D}M \equiv \mathcal{D}\Omega \mathcal{D}\Omega^\dagger \mathcal{D}\Gamma_\alpha \mathcal{D}B \mathcal{D}c \mathcal{D}\bar{c}$ is the path integral measure. This path integral measure remains invariant under the infinitesimal BRST transformation given in (16) because the Jacobian of path integral measure, $\mathcal{D}M$, for such BRST transformations comes unit.

### IV. Finite Superfield-Dependent BRST Symmetry for Super-Group Field Cosmology

In this section, we construct the finite superfield-dependent BRST transformations for the third quantized super-group field cosmology. The properties of the usual BRST transformation do not depend on whether the parameter $\delta \lambda$ is (i) finite or infinitesimal, (ii) superfield-dependent or not, as long as it is anticommuting and global. These observations give us a freedom to generalize the BRST transformation by making the parameter, $\delta \lambda$ finite and superfield-dependent without affecting its other properties.

In order to do that we first make all the generic superfields $\Phi_i(\varrho, \kappa) = \Phi_i^A(\varrho, \kappa)T_A$, where $\Phi_i^A = (\Omega^A, \Omega^{1A}, \Gamma_\alpha^A, B^A, c^A, \bar{c}^A)$, to depend on a continuous parameter, $\kappa : 0 \leq \kappa \leq 1$, in such a manner that
Φ_i(ρ, 0) are the initial superfields and Φ_i(ρ, 1) are the transformed superfields. We also define a functional
Θ[Φ_i(ρ)] with odd Grassmann parity. Now, we make the infinitesimal parameter in the BRST transforma-
tion superfield dependent and hence the infinitesimal superfield-dependent BRST transformation takes
the following form [41]:
\[ \frac{d}{d\kappa} Φ_i(ρ, \kappa) = s_b Φ_i(ρ, \kappa) \epsilon[Φ_i(ρ, \kappa)], \] (21)
where \( \epsilon[Φ_i(ρ, \kappa)] \) is an infinitesimal superfield-dependent parameter and \( s_b Φ_i \) is the BRST variation of
superfields without parameter known as the Slavnov variation. By integrating these equations from \( \kappa = 0 \)
to \( \kappa = 1 \), it is shown that the \( Φ_i(ρ, 1) \) are related to \( Φ_i(ρ, 0) \) by the finite superfield-dependent BRST
transformation as follows
\[ Φ^f(ρ, 1) = Φ_i(ρ, 0) + s_b Φ_i(ρ, 0) Θ[Φ_i(ρ)], \] (22)
where
\[ Θ[Φ_i(ρ)] = \int_0^1 d\kappa' \epsilon[Φ_i(ρ, \kappa')]. \] (23)
Consequently, the finite superfield-dependent version of BRST transformation [10] for the super-group
field cosmology in linear gauge is demonstrated by
\[ f \Omega^A(ρ) = if^{c}_{[AB]}(c^B(ρ)) Ω^A(ρ) Θ[Φ_i], \]
\[ f \Omega^A(ρ) = -if^{A}_{[BC]}(c^C(ρ)) Ω^B(ρ) Θ[Φ_i], \]
\[ f c^A(ρ) = f^{c}_{ABC}(c^B(ρ)) c^C(ρ) Θ[Φ_i], \]
\[ f Γ^A(ρ) = \nabla_a c^A(ρ) Θ[Φ_i], \]
\[ f \bar{c}^A(ρ) = B^A(ρ) Θ[Φ_i], \]
\[ f B^A(ρ) = 0, \] (24)
The above finite BRST symmetry transformation is a symmetry of the effective action [15] only but
not of the corresponding functional measure defined in the Eq. (20) because the Jacobian for path integral
measure in the expression of void functional does not appear as a constant factor due to superfield-
dependent nature of transformation parameter. Under such transformation the Jacobian changes as
\[ \mathcal{D}M = J[Φ_i(ρ, \kappa)] \mathcal{D}M(ρ, \kappa), \] where Jacobian depends on superfields. It has been shown that this non-
trivial Jacobian can be replaced within the functional integral as
\[ J[Φ_i(ρ, \kappa)] \to e^{iS_1[Φ_i(ρ, \kappa)]}, \] (25)
where \( S_1[Φ_i(ρ, \kappa)] \) is some local functional (here local stands for superfield dependent). The Jacobian
\( J[Φ_i] \) can be incorporated in the functional integral without changing the physical theory if and only if
[41]
\[ \frac{1}{J[Φ_i(ρ, \kappa)]} \frac{dJ[Φ_i(ρ, \kappa)]}{d\kappa} - \frac{i}{J[Φ_i(ρ, \kappa)]} \frac{dS_1[Φ_i(ρ, \kappa)]}{d\kappa} = 0. \] (26)
Following the formulation given in Ref. [41], we calculate the infinitesimal change in Jacobian with the help of the following expression,
\[ \frac{1}{J[Φ_i(ρ, \kappa)]} \frac{dJ[Φ_i(ρ, \kappa)]}{d\kappa} = - \sum \int d\phi \left[ -s_b \Omega^A(ρ, \kappa) \frac{\delta Φ_i(ρ, \kappa)}{\delta Ω^A(ρ, \kappa)} + s_b \Gamma^A(ρ, \kappa) \frac{\delta Φ_i(ρ, \kappa)}{\delta Γ^A(ρ, \kappa)} - s_b c^A(ρ, \kappa) \frac{\delta Φ_i(ρ, \kappa)}{\delta c^A(ρ, \kappa)} + s_b B^A(ρ, \kappa) \frac{\delta Φ_i(ρ, \kappa)}{\delta B^A(ρ, \kappa)} \right], \] (27)
Therefore, the Jacobian under finite superfield-dependent BRST transformation extends the effective action $S_T(\varrho)$ within the void functional by a terms $S_1(\varrho)$ as follows:

$$Z[0] \equiv \int \mathcal{D}\varrho e^{iS_T(\varrho)} \longrightarrow Z'[0] \equiv \int \mathcal{D}\varrho e^{i(S_T(\varrho) + S_1(\varrho))}, \quad (28)$$

where $S_T + S_1$ is an extended effective action. In the next section, we will elaborate this in more detailed way.

V. EFFECTS OF FINITE SYMMETRY ON THE THIRD QUANTIZED PATH INTEGRAL

In this section, we discuss the effect of finite superfield-dependent BRST transformation on functional measure given in Eq. (20) for a particular choice of finite superfield-dependent parameter. In this regard we establish a connection between two different but arbitrary gauge choices of super-group field cosmology. For this purpose, we first identify an appropriate superfield-dependent parameter $\Theta[\Phi_i]$ involved in the Eq. (24). In this case $\Theta[\Phi_i]$ is obtainable from the following infinitesimal superfield-dependent parameter

$$\epsilon[\Phi_i(\varrho, \kappa)] = -i \sum_\phi \int d\phi \ D^2 \left[ \bar{c}_A(\varrho, \kappa) F^A_1[\Gamma^A(\varrho, \kappa)] - c_A(\varrho, \kappa) F^A_2[\Gamma^A(\varrho, \kappa)] \right]_1, \quad (29)$$

using the relation (23). Here $F^A_1[\Gamma^A(\varrho, \kappa)]$ and $F^A_2[\Gamma^A(\varrho, \kappa)]$ are two arbitrary gauge-fixing conditions for the theory of super-group field cosmology.

Using the expression (23), the change in Jacobian for the above $\epsilon[\Phi_i(\varrho, \kappa)]$ is calculated by

$$\frac{1}{\mathcal{J}} \frac{d\mathcal{J}}{dk} = -i \sum_\phi \int d\phi \ D^2 \left[ -B_A(\varrho, \kappa) \{F^A_1[\Gamma^A(\varrho, \kappa)] - F^A_2[\Gamma^A(\varrho, \kappa)] \} \right. \left. + \left\{ s_b F^A_1[\Gamma^A(\varrho, \kappa)] - s_b F^A_2[\Gamma^A(\varrho, \kappa)] \right\} c_A(\varrho, \kappa) \right]_1,$$

$$= \sum_\phi \int d\phi \ D^2 \left[ -B_A(\varrho, \kappa) \{F^A_1[\Gamma^A(\varrho, \kappa)] - F^A_2[\Gamma^A(\varrho, \kappa)] \} \right. \left. - c_A(\varrho, \kappa) \left\{ s_b F^A_1[\Gamma^A(\varrho, \kappa)] - s_b F^A_2[\Gamma^A(\varrho, \kappa)] \right\} \right]_1 \quad (30)$$

The Jacobian $\mathcal{J}$ can be written as $e^{iS_i[\Phi_i(\varrho, \kappa)]}$ when the condition (26) is satisfied. We make the following ansatz for the functional $S_1$ in this case:

$$S_1[\Phi_i(\varrho, \kappa)] = \sum_\phi \int d\phi \ \left[ D^2 \{ \xi_1(\kappa) B_A(\varrho, \kappa) F^A_1[\Gamma^A(\varrho, \kappa)] \right. \left. + \xi_2(\kappa) B_A(\varrho, \kappa) F^A_2[\Gamma^A(\varrho, \kappa)] \right. \left. + \xi_3(\kappa) \bar{c}_A(\varrho, \kappa) s_b F^A_1[\Gamma^A(\varrho, \kappa)] \right. \left. + \xi_4(\kappa) c_A(\varrho, \kappa) s_b F^A_2[\Gamma^A(\varrho, \kappa)] \right]_1, \quad (31)$$

where $\xi_1(\kappa), \xi_2(\kappa), \xi_3(\kappa)$ and $\xi_4(\kappa)$ are arbitrary $\kappa$-dependent constants which satisfy following boundary conditions

$$\xi_1(\kappa = 0) = \xi_2(\kappa = 0) = \xi_3(\kappa = 0) = \xi_4(\kappa = 0) = 0. \quad (32)$$

Further, equations (29), (31) and (26) yields

$$\sum_\phi \int d\phi \ \left[ D^2 \{ (\xi'_1 + 1) B_A(\varrho, \kappa) F^A_1[\Gamma^A(\varrho, \kappa)] \right.$$
+ (ξ'_2 - 1)B_A(ϕ, κ)F^A_2[Γ^A(ϕ, κ)] \\
+ (ξ'_3 + 1)\bar{c}_A(ϕ, κ)s_1F^A_1[Γ^A(ϕ, κ)] \\
+ (ξ'_4 - 1)\bar{c}_A(ϕ, κ)s_1F^A_2[Γ^A(ϕ, κ)] \\
+ (ξ_1 - ξ_3)B_A(ϕ, κ)s_1F^A_1[Γ^A(ϕ, κ)][c[Φ_i(ϕ, κ)] \\
+ (ξ_2 - ξ_4)B_A(ϕ, κ)s_1F^A_2[Γ^A(ϕ, κ)][c[Φ_i(ϕ, κ)]])_1 = 0, \tag{33}

where prime denotes the differentiation w.r.t. κ. Now, equating the coefficients of L.H.S. and R.H.S. of the above equation, we get the following differential equations

\[ ξ'_1 + 1 = 0, \quad ξ'_2 - 1 = 0, \quad ξ'_3 + 1 = 0, \quad ξ'_4 - 1 = 0, \]

which satisfy the condition, \( ξ_1 - ξ_3 = ξ_2 - ξ_4 = 0 \). The solutions of the differential equations given in (34), satisfying boundary conditions mentioned in Eq. (32), are

\[ ξ_1(κ) = -κ, \quad ξ_2(κ) = κ, \quad ξ_3(κ) = -κ, \quad ξ_4(κ) = κ. \tag{35} \]

Now, plugging these values back in Eq. (31), the expression of \( S_1 \) precisely becomes

\[ S_1[Φ_i(ϕ, κ)] = \sum_φ \int dφ \left[ D^2\{-κB_A(ϕ, κ)F^A_1[Γ^A(ϕ, κ)] + κB_A(ϕ, κ)F^A_2[Γ^A(ϕ, κ)] - κ\bar{c}_A(ϕ, κ)s_1F^A_1[Γ^A(ϕ, κ)] + κ\bar{c}_A(ϕ, κ)s_1F^A_2[Γ^A(ϕ, κ)]\} \right], \tag{36} \]

which vanishes at κ = 0. However, at κ = 1 (under finite superfield-dependent BRST transformation) the void functional \( Z[0] \) defined with gauge condition \( F^A_1[Γ^A(ϕ)] = 0 \) transforms as follows:

\[ Z[0] \left( = \int \mathcal{D}Me^{iS_T} \right) \rightarrow \frac{finit BRST}{Z'[0]} \left( = \int J(= e^{iS_{1}|κ=1})\mathcal{D}Me^{iS_T} \right) \rightarrow \int \mathcal{D}Me^{i(S_T + S_{1}|κ=1)} \tag{37} \]

where \( S_T \) refers to the total effective action for the gauge condition \( F^A_1[Γ^A(ϕ)] = 0 \) as

\[ S_T = S_0 + \sum_φ \int dφ \left[ D^2\{B_A(ϕ)F^A_1[Γ^A_0(ϕ)] + \bar{c}_A(ϕ)s_1F^A_1[Γ^A_0(ϕ)]\} \right], \tag{38} \]

and hence \( S_T + S_{1}|κ=1 \) becomes

\[ S_T + S_{1}|κ=1 = S_0 + \sum_φ \int dφ \left[ D^2\{B_A(ϕ)F^A_2[Γ^A_0(ϕ)] + \bar{c}_A(ϕ)s_1F^A_2[Γ^A_0(ϕ)]\} \right], \tag{39} \]

which is nothing but the complete effective actions corresponding to another set of gauge choice \( F^A_2[Γ^A_0(ϕ)] = 0 \). Consequently, \( Z'[0] \) refers to the void functional of third quantized super-group field cosmology defined for different gauge-fixing condition \( F^A_2[Γ^A_0(ϕ)] = 0 \). More concretely, the finite superfield-dependent BRST transformation with parameter \( c[Φ_i(ϕ, κ)] = i \sum_φ \int dφ \left[ f^B_C\bar{c}_A\bar{c}^B\bar{c}^C \right] \) will connect the effective actions of super-group field cosmology in linear and non-linear gauges. Hence, remarkably, we conclude that the finite super-field dependent transformations with specific transformation parameter map two different gauges of super-group field cosmology which will certainly help in explaining the different pole structures of propagators of the theory.
VI. CONCLUSION

In this paper we have discussed the supersymmetric group field cosmology which is a model for homogeneous and isotropic multiverse. In the multiverse scenario, the gauge and the matter fields describe the different universes. Furthermore, we have constructed the third quantized infinitesimal BRST transformations of the super-group field cosmology defined for the most general gauge-fixing condition. The finite superfield-dependent BRST symmetries, characterized by an arbitrary superfield-dependent parameter, have been demonstrated by integrating the infinitesimal BRST transformation of the super-group field cosmology. Within the formalism, we have defined the void functional (so-called the generating functional in the case of second quantized field theories) for this third quantized super-group field cosmology. The effects of infinitesimal and finite superfield-dependent of BRST transformations on the functional measure of void functional have also been reported. Within the analyses, we have found that the infinitesimal BRST transformation leaves both the effective action and the functional measure invariant. However, the novelty of finite superfield-dependent version of BRST symmetry is that it leaves only effective action of the theory symmetric but not the functional measure. Remarkably, we have found that with appropriate choices of transformation parameter this finite superfield-dependent BRST transformation switches the void functional from one gauge-fixing condition to another gauge-fixing condition. We have established this connection for an arbitrary set of gauges. To be more specific, the connection of linear and non-linear gauges of the super-group field cosmology has also been discussed. These results hold at both classical and quantum levels. Since the different gauge choices correspond to the different propagators and therefore our formulation will be helpful in connecting different propagators of super-group field cosmology also. Further generalizations of nilpotent BRST symmetry are the subject of future investigations which might have some interesting implications.

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