Renormalization Group Flows from Five-Dimensional Supergravity †

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1. Introduction

Perhaps the most extensively tested and studied version of the AdS/CFT correspondence is that of $\mathcal{N}=4$ supersymmetric Yang-Mills theory in $(3+1)$ dimensions and the compactification of $IIB$ supergravity on $AdS^5 \times S^5$ [1]. The relationship between supergravity fields and Yang-Mills operators can most easily be mapped out from the representation structure of the extended superconformal algebra. Taken at face value one might be tempted to conclude that the AdS/CFT correspondence itself may be little more than a convenient way to encode this representation theory, but this view is too facile, and perhaps one of the remarkable lessons from the last year has been just how deeply this correspondence works, even far from the conformal regime.

One of the purposes of this talk is to describe one of the more non-trivial tests of the correspondence [2, 3], a test in which one moves far from the $\mathcal{N}=4$ superconformal regime of the Yang-Mills theory: I will discuss how supergravity correctly describes an $\mathcal{N}=1$ supersymmetric renormalization group flow, and how this flow can be checked rather well against known field theory results. A by-product of this study of renormalization group flows is the discovery of a general $c$-theorem for any such flow described via supergravity. The $c$-function does everything that one wants of such a function: it is monotonic decreasing on all flows to the infra-red, and is only stationary at conformal fixed points, and at such fixed points it is equal to the central charge of the fixed-point theory. On a more philosophical level, given that the renormalization group

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flow is being modelled by a five-dimensional cosmological solution, it is also satisfying that the central charge is related to the inverse of the cosmological entropy density.

Using supergravity to describe renormalization group flows was first discussed in \cite{5, 6}, but these flows were non-supersymmetric and they went to unstable supergravity vacua. The lack of supersymmetry makes it hard to independently verify the results via field theory, and the lack of stability suggests that the IR field theory limit may in fact be pathological (non-unitary). The solution and flow discussed here has neither of these problems, and indeed the supergravity computations match perfectly with independent field theory results.

A secondary purpose of this talk will be to revisit some ancient issues in supergravity, most particularly that of consistent truncation, and discuss the relevance of these issues to the phase structure and flows in Yang-Mills theory. The final part of my talk will focus on how supergravity distinguishes between perturbations of the Yang-Mills action and states in the Hilbert space. As an illustration of this I will discuss the supergravity description of part of the Coulomb branch of the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory \cite{4}.

2. Gauged $\mathcal{N} = 8$ Supergravity in Five Dimensions

2.1. Ancient Issues: Embedding and Consistent Truncation

Maximal supergravities in any dimension can usually be obtained by trivial dimensional reduction of the eleven dimensional supergravity on a torus. Such supergravities are “ungauged” in that all the vector fields have abelian gauge symmetries, and these gauge fields are not minimally coupled to the fermions. Gauging refers to converting part of the global symmetry into a local one by introducing the appropriate couplings by hand, with a coupling constant, $g$, and then constructing the supersymmetric model by the “Noether procedure.” If the gauge group is non-abelian then one can show very generally that any ground state that leaves the symmetries unbroken is necessarily anti-de Sitter space, with a cosmological constant proportional to $g^2$. There are sometimes several options as to the choice of gauge group, but here I will only consider the gauged theories with maximal supersymmetry and the maximal $SO(M)$ gauge symmetry. Such theories are related to sphere compactification of IIB and eleven dimensional supergravity.

It is important to stress that gauged supergravity theories are consistent, classical field theories in their own right. Their structure is generically very non-linear, particularly in the scalar sector. However, it is also important to remember that the complete structure of these theories is completely determined by: (i) Supersymmetry, gauge and global symmetries, and (ii) The perturbative field content. Given this, and the assumption of only second-derivative actions, the Noether procedure is essentially unambiguous.
It is a separate issue as to how gauged supergravities may be related to compactifications of higher dimensional supergravities, and indeed, one of the key issues is whether they are actually embedded in the higher dimensional theory. Since the lower dimensional theory is a truncation of the higher dimensional theory down to some subset of “lowest modes” there is the obvious question as to whether the truncation is consistent with the equations of motion of the higher dimensional field theory, or whether the lower dimensional field theory is merely some effective low energy truncation. In this context, “embedding” is the strongest option: it means that the equations of motion of the higher dimensional field theory are exactly satisfied if one satisfies the equations of motion of the lower dimensional theory. In particular, a solution to the lower dimensional theory automatically yields a solution to the higher dimensional theory.

Embedding, or consistent truncation is trivial for toroidal compactifications where one takes all fields to be independent of the toroidal coordinates, but it is highly non-trivial for the sphere compactifications that lead to maximal gauged supergravities. Embedding of gauged supergravity is almost completely proven for the $S^7$ compactification and $S^4$ compactification of eleven dimensional supergravity \[7\]. It has not been proven for gauged $N = 8$ supergravity in five dimensions and the $S^5$ compactification of the IIB theory. However there is a significant body of evidence to support this correspondence, and the similarities with the other sphere compactifications make the case very convincing†. I will therefore assume that the five-dimensional gauged supergravity is indeed embedded in the IIB theory, and proceed.

2.2. The Field/Operator Correspondence

The field content of gauged $N=8$ supergravity is a graviton, eight spin-$3/2$ fields, twelve tensor gauge fields, fifteen vector gauge fields, forty-eight spin-$1/2$ fields, and forty two scalars. The ungauged theory has a global $E_6(6)$ symmetry and a composite local $USp(8)$ symmetry, with the scalars parametrized in terms of the coset $E_6(6)/USp(8)$. In the gauged supergravity theory \[9\] the global $E_{6(6)}$ is replaced by a local $SO(6)$ symmetry and a global $SL(2, \mathbb{R})$. The latter can be thought of as the $SL(2, \mathbb{R})$ symmetry inherited from the IIB theory, with the dilaton and axion corresponding to the coset $\frac{SL(2, \mathbb{R})}{U(1)}$. At the quantum level this $SL(2, \mathbb{R})$ must therefore be broken to $SL(2, \mathbb{Z})$.

Even though the $E_{6(6)}$ symmetry is broken in the gauged theory it is still convenient to think of the scalar fields as living on $\frac{E_{6(6)}}{USp(8)}$, but now the scalars also have a potential $V(\phi)$ proportional to $g^2$. This potential is invariant under $SO(6) \times SL(2, \mathbb{R})$.

The AdS/CFT correspondence implies \[10, 11\] that this $N = 8$ gauged supergravity multiplet must correspond to the $N = 4$ Yang-Mills energy-momentum tensor supermultiplet, with the local $SO(6)$ symmetry dual to the $R$-symmetry.\[†\]

† Consistent truncation has been recently proven for a subsector of the theory \[8\].
The scalar fields of the supergravity lie in the following $SO(6)$ representations:
\[ 1 \oplus 1 \oplus 10 \oplus 10' \oplus 20', \]
with the two singlets being the ten-dimensional dilaton and axion. From this one can deduce that these supergravity scalars correspond, respectively, to the gauge coupling, the $\theta$-angle, the fermion bilinear operators and the scalar bilinear operator of the Yang-Mills theory. The latter have the form:
\[ \text{Tr} \left( \lambda^i \lambda^j \right), \text{Tr} \left( \bar{\lambda}^i \bar{\lambda}^j \right), \text{Tr} \left( X^A X^B \right) - \frac{1}{6} \delta^{AB} \text{Tr} \left( X^C X^C \right). \] (1)

### 2.3. Phases and operator product algebras

When one combines the embedding of gauged supergravity with the AdS/CFT correspondence one is led to some interesting statements about the Yang-Mills theory. Consistent truncation implies that the operator product of fields in the energy-momentum supermultiplet must be closed at large $N$, a fact that has been demonstrated in [12]. Conversely, the fact that this operator product algebra does not close at finite $N$ implies that gauged $\mathcal{N} = 8$ supergravity is an incomplete quantum theory. The presence and exact knowledge of the supergravity potential (and other highly non-linear structure in supergravity) implies a completely determined, highly non-trivial operator algebra. Furthermore, critical points of the potential give rise to anti-de Sitter vacua in supergravity, which will, under appropriate conditions, imply non-trivial conformal fixed points for the Yang-Mills theory. The supergravity scalars whose VEV’s lead to the new critical point tell one exactly what relevant operators in the Yang-Mills theory would drive a flow to this fixed point in the infra-red. Moreover, constructing the supergravity kink solutions that interpolate between these AdS ground states should be equivalent to constructing explicitly the renormalization group flow [5, 6, 3]. Thus, in a very real sense, the supergravity potential should map out the phase diagram of the $\mathcal{N}=4$ Yang-Mills theory under mass perturbations.

In [2] the critical points of supergravity potential with at least $SU(2)$ invariance were classified. The results are summarized in Table 1. From the work of [13] one can deduce that the central charge at the new infra-red fixed point is related to the central charge at the UV fixed point according to:
\[ c_{IR} = \left( \frac{\Lambda_{IR}}{\Lambda_{UV}} \right)^{-3/2} c_{UV}, \] (2)
where $\Lambda$ denotes the cosmological constant of the corresponding five-dimensional supergravity solution.

To be a stable solution of the supergravity theory the solution must either be supersymmetric [14] or the small oscillations must satisfy the Breitenlohner-Freedman (BF) condition [15]. If there are supergravity modes that fail the BF-condition then the corresponding Yang-Mills operators seem to have complex conformal dimension, violating unitarity, and so corresponding phases appear to be pathological. The stability
analysis for the $SO(5)$ point was done in [3], and the instability of the other non-
supersymmetric points was established by Pilch [10].

It thus seems that only the last critical point represents a viable, non-trivial phase of large $N$ Yang-Mills theory. It is, of course natural to ask whether this should also represent a phase at finite $N$, and the AdS/CFT correspondence implies that this question is equivalent to asking whether the supergravity solution is a good vacuum for the IIB string. The general dogma in this area states that this is true for $N = 4$ supersymmetric supergravity solutions, and suggests that there should be a string vacuum “nearby” for $N = 2$ supersymmetric supergravity solutions. Thus finding a corresponding Yang-Mills phase represents a very non-trivial, non-linear test of the AdS/CFT correspondence (and of the general dogma about supersymmetric solutions).

| Unbroken Gauge Symmetry | Cosmological Constant | Unbroken Supersymmetry | CentralCharge $c_{IR}/c_{UV}$ | Stable? |
|-------------------------|-----------------------|------------------------|-------------------------------|---------|
| $SO(6)$                 | $-\frac{3}{14}g^2$   | $N = 8$                | $1$                           | Yes     |
| $SO(5)$                 | $-\frac{3}{2}g^2$    | $N = 0$                | $\frac{2\sqrt{2}}{3}$        | No      |
| $SU(3)$                 | $-\frac{27}{32}g^2$  | $N = 0$                | $\frac{16\sqrt{2}}{27}$      | No      |
| $SU(2) \times U(1) \times U(1)$ | $-\frac{3}{8}(\frac{1}{4})^{1/3}g^2$ | $N = 0$ | $\frac{1}{5}$ | No |
| $SU(2) \times U(1)$    | $-\frac{21}{32}g^2$  | $N = 2$                | $\frac{27}{32}$              | Yes     |

Table 1: The $SU(2)$ invariant critical points of the $N=8$ supergravity potential.

From the supergravity scalar vevs at this non-trivial, supersymmetric critical point one finds that in the $N = 4$ Yang-Mills theory one is turning on a mass for one of the four fermions, and two of the six scalars. Indeed, writing everything in terms of $N = 1$ superfields, one sees that this new phase arises from giving a mass to exactly one of the three adjoint hypermultiplets. Based upon this, several authors [17, 3] have identified this infra-red phase as precisely one of those discovered by Leigh and Strassler in 1995 [18]. The global symmetries, supersymmetry, the UV end of the flow, the central charge and the IR anomalous dimensions have all been checked, and the match is perfect. Of course, once one has established the existence of such a phase, a number of the properties, like anomalous dimensions, follow as a consequence of the $N = 1$ superconformal symmetry. However, the remarkable thing is that supergravity knows about the existence of the fixed point, and that it requires the extremely non-linear structure of the potential to see it. Conversely, had supergravity not found this fixed point, it would have cast significant doubt on the ability of AdS/CFT to go beyond the most basic, second order perturbative details.

3. Renormalization Group Flows
3.1. The $\mathcal{N}=1$ flow

Consider now the supergravity scalars $\varphi_1, \varphi_2$, that are dual to the mass parameters of a fermion and the two scalar fields that constitute an $\mathcal{N}=1$ Yang-Mills hypermultiplet. On this subsector one can write the supergravity potential in the canonical form:

$$V = \frac{g^2}{8} \sum_{j=1}^{3} |\partial W/\partial \varphi_j|^2 - \frac{g^2}{3} |W|^2,$$

where

$$W = \frac{1}{4\rho^2} \left[ \cosh(2\varphi_1) \left( \rho^6 - 2 \right) - (3\rho^6 + 2) \right],$$

and $\rho \equiv e^{1/\sqrt{6} \varphi^2}$. The scalar kinetic term has the usual form: $\frac{1}{2} \sum_j g^{\mu\nu}(\partial_\mu \varphi_j)(\partial_\nu \varphi_j)$.

The $\mathcal{N}=8$ supersymmetric critical point is at $\varphi_j \equiv 0$, and there is a $\mathbb{Z}_2$ equivalent pair of $\mathcal{N}=2$ supersymmetric critical points at:

$$\varphi_1 = \pm \frac{1}{2} \log(3), \quad \varphi_2 = \frac{1}{\sqrt{6}} \log(2).$$

To construct the kink corresponding to the renormalization group flow one requires the five-metric to take the form:

$$ds^2 = e^{2A(r)} \eta_{\mu\nu} dx^\mu dx^\nu - dr^2.$$

(I will adopt the conventions of [3] throughout.) As $r \to \infty$ the scalars, $\varphi_j$, must vanish and one must have $A(r) \sim \text{const.}$ so that the background is asymptotic to the $\mathcal{N}=8$ supersymmetric AdS background at infinity. At the other end of the flow ($r \to -\infty$), the scalars must approach either of the critical values (5), and once again $A(r) \sim \text{const.}$ as one approaches the new conformal fixed point.

Since the flow is $\mathcal{N}=1$ supersymmetric one should be able to find a five-dimensional kink that preserves this amount of supersymmetry. The equations of motion for such a flow can be obtained by finding solutions to the vanishing of the supersymmetry variations of the supergravity fermions: $\delta \psi^a_\mu = 0, \delta \chi_{abc} = 0$. From this one can show that there is a supersymmetric flow if and only if:

$$\frac{d\varphi_j}{dr} = \frac{g}{2} \frac{\partial W}{\partial \varphi_j}, \quad \text{and} \quad A'(r) = -\frac{g}{3} W.$$  

These equations mean that the flow is determined by the steepest descent of the superpotential, and that the cosmology ($A(r)$) is determined directly from this steepest descent.

Figure 1 shows the contour maps of the potential $V$, and the superpotential, $W$. These functions have a $\mathbb{Z}_2$ symmetry under $\varphi_1 \to -\varphi_1$, and thus there are two equivalent copies of each non-trivial critical point. The contour map of $V$ shows five critical points corresponding to the first, third and fifth entries in Table 1. Critical points of $W$ yield
supersymmetric vacua, and Figure 1 also shows these as well as a steepest descent path. Note that the steepest descent path has a parabolic form near the central critical point:

\[ \varphi_1 \sim a_0 e^{-r}, \quad \varphi_2 \sim \sqrt{\frac{8}{3}} a_0^2 r e^{-2r} + a_1 e^{-2r}, \quad \text{as } r \to \infty. \]

(8)

This is, of course, precisely what one needs for the Yang-Mills scalar masses to be the square of the Yang-Mills fermion mass along the supersymmetric flow.

3.2. The \(c\)-Function

If one wishes to describe a general renormalization group flow in terms of holography then one considers a five-metric of the form (6) with some five-dimensional matter on the brane. It is relatively elementary to show \cite{3} that if the energy-momentum tensor of this matter satisfies the null energy condition then the following function is a \(c\)-function for the flow:

\[ C(r) = \frac{C_0}{(A'(r))^3}, \]

(9)

where \(C_0\) is a suitably chosen constant.

Specifically, \(C(r)\) is monotonic decreasing along renormalization group flows to the infra-red. It is only stationary at conformal fixed points of the flow, and by proper choice of \(C_0\) the function \(C(r)\) is equal to the central charge of the conformal theory at any
fixed point. For the flows of $\mathcal{N} = 4$ Yang-Mills considered earlier, one sets $C_0 = g^3 N^2 / 32$, which gives $C = a = c = N^2 / 4$ at the $\mathcal{N} = 4$ supersymmetric point. The same $c$-function and a restricted version of the $c$-theorem was also obtained in [3].

Interestingly enough, for the supersymmetric flows one can use (7) to obtain:

$$C(r) = -\frac{27}{g^3} \frac{C_0}{W^3} ,$$

(10)

where $W$ is the superpotential. This form of the $c$-function is much more in the spirit of Zamolodchikov’s $(1 + 1)$-dimensional $c$-function in that it is explicitly written as a function of the couplings, whereas (9) only has an implicit dependence on the couplings. It is also interesting to note that the renormalization group flows (7) also represent steepest descents of the $c$-function (10).

4. Flows to Hades and the Coulomb Branch

In the previous section I considered a very specific flow that follows a ridge line of the superpotential (4) to the non-trivial fixed point. However, any solution to (7) represents a supersymmetric solution to the five-dimensional supergravity theory: indeed Figure 2 shows a family of supersymmetric flows superimposed on the contours of $W$. It is evident that all the flows (other than the ridge-line flow) go to infinite values of the scalar fields, and thus to infinite values of $W$ (called Hades by one of my illustrious collaborators). Since these flows are part of the supergravity theory, and have some interpretation in terms of perturbations of the gauge theory at its UV fixed point, it would seem that all these flows should also have some gauge theoretic interpretation. However instead of tackling this problem directly, I will consider a larger, but simpler family of flows to Hades; a family that includes the flows along the horizontal axis in Figure 2.

Consider all the supergravity scalars in the $20'$ of $SO(6)$. These are naturally parametrized by a matrix $S \in SL(6, \mathbb{R})/SO(6)$, and on this subsector the potential and superpotential have the form:

$$V = -\frac{g^2}{32} \left[ (\text{Tr}(M))^2 - 2\text{Tr}(M^2) \right] , \quad W = -\frac{1}{4} \text{Tr}(M) ,$$

(11)

where $M \equiv S S^T$. Using the $SO(6)$ symmetry one can take $M$ to have the form $M = \text{diag}(e^{2\beta_1}, e^{2\beta_2}, e^{2\beta_3}, e^{2\beta_4}, e^{2\beta_5}, e^{2\beta_6})$, with $\sum_a \beta_a = 0$. If $\varphi_j$ are orthonormal linear combinations of the $\beta_a$, then once again one obtains supersymmetric solutions to the supergravity theory if one satisfies (11). However, these solutions preserve $\mathcal{N} = 4$ supersymmetry on the brane. One can also show that while the flows near the UV fixed point are generically complicated, far from the UV fixed point the flows settle down to one of five classes:
Fig. 2: The contour map of the superpotential (4) showing families of steepest descent paths from the central critical point. Each such path represents an $\mathcal{N} = 1$ supersymmetric solution.

- Two $SO(5)$ invariant flows defined by $\vec{\beta} = \pm(1,1,1,1,-5)\mu$
- Two $SO(4) \times SO(2)$ invariant flows defined by $\vec{\beta} = \pm(1,1,1,-2,-2)\mu$
- An $SO(3) \times SO(3)$ invariant flow defined by $\vec{\beta} = (1,1,1,-1,-1,-1)\mu$

where $\mu$ is a parameter. The corresponding five-metrics are regular for the $SO(5)$ and $SO(4) \times SO(2)$ invariant flows with $\mu > 0$ and the “+” choice. The regularity, and physical interpretation of the corresponding ten-dimensional metrics will be discussed in the talk by Steve Gubser.

From the Yang-Mills perspective, the scalars in the $20'$ of $SO(6)$ are dual to the last set of operators in (1). However these supergravity scalars cannot correspond to turning on masses since the flows described above all preserve the full $\mathcal{N} = 4$ supersymmetry. These supergravity scalars must therefore represent moduli of the $\mathcal{N} = 4$ theory, and indeed these flows must be along the Coulomb branch. One can see this fact directly by constructing the corresponding ten-dimensional metrics, and looking at the distribution of branes that make up the sources (see [4], and the talk by S. Gubser). More generally, this situation raises the issue of when a supergravity scalar represents a mass term or a field theory modulus. The resolution of this was given in [11, 19], and there is a direct characterization that can be expressed entirely from the five dimensional perspective.

Recall that a general (non-supersymmetric) flow in the supergravity scalar sector is determined by a second order differential equation:

$$\frac{d^2 \phi_j}{dr^2} + 4A'(r) \frac{d \phi_j}{dr} = \frac{\partial V}{\partial \phi_j}. \quad (12)$$

Near the UV critical point this equation has a general solution with asymptotic
behaviour:

$$\varphi_j \sim a_j \, e^{-2x} + b_j \, e^{-2x}.$$  \hspace{1cm} (13)

The first of these modes is non-normalizable on the $AdS_5$, and so couples non-trivially to operators on the boundary, and thus represents a “mass insertion” on the brane. The second mode represents a normalizable wave-function on $AdS_5$, and thus must represent a state in the Hilbert space, which here means a modulus of the Coulomb branch of the $\mathcal{N} = 4$ theory. Thus a general flow can involve a mixture of mass terms and vevs. The supersymmetric flows are determined by first order equations (7), and they select a unique solution, and one can easily check that the flows considered in this section involve only normalizable modes ($a_j = 0$ in (13)), whereas the $\mathcal{N} = 1$ flow discussed earlier has asymptotics (8), which manifestly involves non-normalizable modes.

Thus even the apparently pathological flows to Hades in the five-dimensional supergravity have a sensible physical interpretation on the brane. Moreover the five-dimensional supergravity also captures the subtleties of operators and states in the AdS/CFT correspondence.

5. Final Comments

As outlined in the introduction, the primary purpose here was to describe some extremely non-trivial (and successful) tests of the AdS/CFT correspondence, and its generalization to renormalization group flows. I also wished to highlight the utility of five-dimensional gauged supergravities as a tool for examining the non-linear structure of field theories on the brane, and to show how some ancient issues of gauged supergravity theories have new relevance to holographic field theories. I would like to conclude in a similar spirit by raising some related, but as yet unresolved issues.

First, in the work that led up to \cite{4, 4} we also found a broad family of $\mathcal{N} = 1$ and $\mathcal{N} = 2$ flows to Hades. We are still looking at some of the details, but these flows certainly represent combinations of massive and Coulomb branch flows.

It is an empirical fact that supersymmetric flows appear to be governed by steepest descent of a superpotential, $W$, and hence of the $c$-function. It is not clear how far this can be generalized. Specifically, we have found a $c$-function for general flows, but it is not written directly as a function of the couplings, and it is not known whether a general RG flow is necessarily a steepest descent of this $c$-function. However, the recent results of \cite{20} suggest that some non-supersymmetric generalizations are indeed possible.

Returning to ancient supergravity issues: there are non-supersymmetric critical points, and all the known ones are unstable. Is this true in general, i.e. does stability also imply supersymmetry? Additionally, what is the significance of the unstable critical points for the field theory on the brane? Are they merely some large $N$ pathology?
It would obviously be interesting to determine all critical points of the potential, or find some way of controlling the problem. For black hole solutions based on Calabi-Yau compactifications one finds the “flow” to the horizon induces a potential on the Calabi-Yau moduli space, and that that potential selects certain natural arithmetic points as attractors for the flow \[21\]. It would be interesting if there were some arithmetic principle in $E_{6(6)}$ for classifying the critical points of the supergravity potential.

One last issue: it would be very interesting to find supersymmetric flows similar to those discussed here, but in which the dilaton runs. Most particularly it would be nice to do this with the recently discovered $\mathcal{N}=1$ flows of \[22\], and thereby obtain a flow to pure $\mathcal{N}=1$ gauge theory with a running coupling.

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