Vacuum energy, holography and a
quantum portrait of the visible
Universe

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Abstract

Describing the presently observable Universe as a self-sustained condensate of gravitons of size $H_0^{-1}$, with large occupation number $N$, we argue that the most probable value for the quantum vacuum energy is of the order of the critical energy density, as observed.
1 Computing vacuum energy in the context of gravity

Vacuum energy, which we define loosely as the energy of the fundamental state, is a measurable quantity only in the context of gravity. Indeed, in a non-gravitational context, only differences of energy, such as forces, can be measured (a well-known example is the Casimir effect). On the other hand, the presence of the energy-momentum tensor on the right-hand side of Einstein’s equations shows that any form of energy impacts on the geometry of spacetime. In particular, the Friedmann equation which can be deduced expresses the fact that any form of energy participates to the expansion of the Universe. Hence energy, in particular the energy of the vacuum, can be measured absolutely.

We know from observation that space is flat; hence the energy density of the vacuum $\rho_{\text{vac}}$ is smaller than the critical density:

$$\rho_{\text{vac}} < \rho_c \equiv \frac{3H_0^2}{8\pi G_N}. \tag{1}$$

This should be compared with a naive guess estimate: in the context of a quantum theory of gravity, one expects on dimensional grounds that $\rho_{\text{vac}}$ scales as $m_P/l_P^3$, where $m_P$ is the (reduced) Planck mass scale of quantum gravity i.e. $m_P \equiv \sqrt{\hbar c/(8\pi G_N)}$ and $l_P \equiv h/(m_P c)$ is the Planck length (from now on, we set $k = c = 1$). In other words,

$$\rho_{\text{vac}} \sim \frac{m_P}{l_P^3} = \frac{1}{h(8\pi G_N)^2}. \tag{2}$$

This however does not take into account the specificity of gravity. Let us consider a spherical region of radius $R$. It cannot contain more mass-energy than a black hole of same size i.e. of Schwarzschild radius $R_S = R$. Hence, using $R_S = 2G_NM$ ($M$ mass of black hole), the energy $E$ in this spherical region satisfies the relation $E < M = R/(2G_N)$ and, disregarding constants of order one,

$$\rho_{\text{vac}} = E/(4\pi R^3/3) < \frac{3}{8\pi G_N R^2}. \tag{3}$$

If we extend this to the whole observable Universe of radius $H_0^{-1}$, this gives

$$\rho_{\text{vac}} < \frac{3H_0^2}{8\pi G_N}. \tag{4}$$
Let us note the surprising similarity with the observational constraint \( \text{(1)} \).

The rationale behind the limit \( \text{(3)} \) is the fact that, from a gravitational point of view, each Planck cell of size \( l_P = \hbar/m_P \) within a volume \( R^3 \) cannot host a maximal energy \( m_P \): this would lead to a total mass \( R^3/l_P^3 \times m_P \) or \( M_{BH} (R/l_P)^2 \), where \( M_{BH} \sim R m_P^2/\hbar \) is the mass of a black hole with size \( R \), and thus to gravitational collapse whenever we consider a volume element larger than an elementary scale (i.e. \( R > l_P \)). Hence gravitational collapse prevents the ultraviolet cut-off of the quantum theory to reach its maximal value \( m_P \) in a large fraction of elementary shells. At the level of the whole observable universe, this provides a connection between the microscopic ultraviolet scale \( m_P \) and the cosmological infrared scale \( H_0^{-1} \) which is expressed as the bound \( \text{(3)} \).

Obviously, these ideas are reminiscent of the ones associated with holography and entropy bounds \([1, 2]\) (see \([3]\) and references therein), as applied to cosmology \([4, 5, 6, 7, 8, 9]\). Indeed, the largest number of degrees of freedom that we can pack in a region of size \( R \) is the one corresponding to a black hole of size \( R \), i.e. \( N < (R/l_P)^2 \) since the degrees of freedom of a black hole lie on the horizon surface. Since for each individual cell the Poissonian fluctuation in energy is \( \Delta \epsilon \sim m_P \), the energy for the overall fluctuations is \( \Delta E^2 = N m_P^2 \), which corresponds to an energy density \( \rho_{\text{vac}} = \frac{\Delta E}{R^3} = \frac{\sqrt{N} m_P}{R^3} < \frac{m_P^2}{\hbar R^2} \), again consistent with \( \text{(3)} \).

Since the upper value corresponds to the maximal entropy, one expects on statistical grounds (see next section) that

\[
\rho_{\text{vac}} = \frac{m_P^2}{\hbar R^2} ,
\]

which would yield when applied to the whole observable Universe \( R = H_0^{-1} \) the well known relation \([10, 11]\)

\[
\rho_{\text{vac}} = \frac{\hbar}{l_P^2 H_0^{-2}} .
\]

2 A quantum portrait of the visible Universe

The preceding ideas regarding vacuum energy are following the same lines as the discussion of gravity theory as a classicalized theory. We will see that
there are indeed some strong similarities between some of the concepts developed within classicalized gravity and a possible description of our observable universe.

Let us consider our observable Universe. It consists approximately of 30% of matter and 70% of dark energy. We will make the hypothesis that dark energy is vacuum energy and will neglect for the time being the subdominant matter component: we attempt to describe first a Universe with only vacuum energy.

This Universe is obviously a classical object, the most classical of all in some sense but, because fundamental forces are described by quantum physics, it should have as well a quantum description: a graviton bound-state with a very high occupation number $N \gg 1$.

Let us consider first an individual quantum state of graviton with energy $\epsilon \sim \hbar k \sim \hbar H_0$ (since $k$ is typically the inverse of the Hubble horizon length $H_0^{-1}$). Then the total energy $E$ in the observable universe is simply $N\epsilon$ where $N$ is the total occupation number. Now we have seen in the preceding section that $E < H_0^{-1}/(2G_N)$, hence

$$N = \frac{E}{\epsilon} < \frac{1}{G_N \hbar H_0^2} = \frac{1}{l_P^2 H_0^2},$$

where we used $l_P^2 = \hbar G_N$. The limit (8) represents the highest possible value for $N$; in other words, maximal classicality of the observable universe is reached for

$$N = (l_P H_0)^{-2}.$$

This is reminiscent of the quantum N-portrait of a black hole as described by Dvali and Gomez [12, 13, 14]. This is obviously not surprising since the black hole represents the most classical object within a region of a given size. We will pursue the analogy and describe some of the properties of the universe seen as a Bose-Einstein condensate of $N = (l_P H_0)^{-2}$ soft gravitons of wavelength $\lambda \sim H_0^{-1}$ which are weakly interacting (their dimensionless coupling is $\hbar G_N/\lambda^2 \sim (l_P H_0)^2$).

First, as emphasized in [12], the condensate is self-sustained only if its size $H_0^{-1}$ does not overcome too much its Schwarzschild radius $R_S$ otherwise not only the gravitons are extremely weakly coupling to one another but also the interaction of one graviton with the collective gravitational energy is negligible. This suggests that our own observable universe saturates the bound (8) and thus satisfies $N = (l_P H_0)^{-2}$, i.e. it is maximally classical.
Second, just as in the case of a black hole \cite{12}, there is a ‘thermal spectrum of temperature
\[ T = \frac{\hbar}{\sqrt{N l_P}} = \hbar H_0. \] (10)
This should be compared to the famous result according to which an observer in de Sitter space (with Hubble parameter $H$) feels as if he is in a thermal bath of temperature $T = \hbar H/(2\pi)$. The result (10) is thus consistent with the fact that, if the vacuum energy density is dominant, we are in a de Sitter phase.

Just as a black hole evaporates, one expects that the whole observable universe will decay after a time:
\[ t_{\text{dec}} = N^{3/2} l_P = l_P^{-2} H_0^{-3} = N H_0^{-1}, \] (11)
which is thus much larger than the present age of the Universe\footnote{This is to be contrasted with the result obtained with dark energy models where the Universe collapses within a time of the order of $H_0^{-1}$ \cite{17}.}

Finally, one may define the entropy of the visible Universe as \cite{12}
\[ S = N \] (12)
Following a Boltzmann distribution, we expect that the probability for $N$ to have a value in the interval between $N$ and $N + dN$ will be given by:
\[ w(N) dN = \text{cst} \ e^S = \text{cst} \ e^N. \] (13)
The probability is maximum for the largest possible value of $N$ compatible with the limit \cite{8}, that is for $N = (l_P H_0)^{-2}$ which corresponds to
\[ \rho_{\text{vac}} = \frac{N \epsilon}{(H_0^{-1})^3} = \frac{\hbar}{l_P^2 H_0^{-2}}. \] (14)
as in \cite{7}.

We note that, by writing
\[ e^S = e^N = e^{l_P^{-2} H_0^{-2}} = e^{\hbar/(\rho_{\text{vac}} l_P^3)} = e^{1/(\Lambda l_P^2)}, \] (15)
where we introduced the cosmological constant $\Lambda = 8\pi G_N \rho_{\text{vac}}$, we recover the distribution $w(\Lambda) d\Lambda = \text{cst} \ e^{1/(\Lambda l_P^2)}$ proposed by Horava and Minic \cite{15}. However, while these authors inferred from such a distribution that a vanishing cosmological constant $\Lambda$ has maximal probability, we draw a different conclusion: taking $l_P$ as given by the theory and $H_0$ as imposed by observation, we infer that the maximal probability corresponds to maximal $N$ and thus to $\rho_{\text{vac}}$ given by (14).
3 The cosmological evolution of our Universe

The cosmological scenario that emerges from the preceding considerations is both familiar and very different from what is usually described. We assume that the Universe emerges from the quantum epoch (characterized by a length scale $l_P$) in a quantum state which has many classical realizations i.e. which can be projected onto many different classical states. Projection occurs through the measurement process, that identifies a (visible) Universe of size $H_0^{-1}$. In the absence of matter, this Universe is a classical condensate of weakly interacting gravitons. Its stability imposes the condition (4) on the vacuum energy density. Moreover, the upper limit $\rho_{\text{vac}} = \hbar/(l_P H_0^{-1})^2$ corresponds to maximal probability and maximal classicality.

It should be stressed that the relation $\rho_{\text{vac}} = \hbar/(l_P H_0^{-1})^2$ is only valid at the time of measurement i.e. at the time where the state of the Universe is projected onto the classical state. The classical Universe then deploys itself in time (backward and forward) in the standard way. In particular, in the case where matter and radiation are negligible, the Universe falls in de Sitter expansion as we have discussed above (with associated radiation at temperature $T \sim \hbar H_0$). The classical description has obviously a limited range of validity, namely $kT \ll m_P$.

A useful analogy is provided by the simple double slit interference experiment with electrons. If one is interested in the time of flight of the electrons from the source to the screen, one may follow step by step the motion of the electron (and thus identify which slit it went through): this time of flight is a classical quantity and can thus be measured classically. On the other hand, one will be losing the interference pattern on the screen. If one wants to recover the interference pattern, one should avoid tracing the electron through its evolution. Similarly, if one wants to understand the amount of vacuum energy (a quantum observable), one should not trace the Universe through its evolution. Instead, one may compute probabilities for measuring a given value, once we observe the Universe (that is, now). Other aspects of cosmology which are purely classical (from the point of view of gravity) may reliably be computed by following the evolution of the Universe.

This should be contrasted with cosmologies proposed along similar lines which assumed the relation $\rho_{\text{vac}} \sim \hbar/(l_P H^{-1})^2$ throughout the evolution of the Universe. It is easily seen that they cannot describe dark energy \[16\].

\[2\] Indeed, if $\rho_{\text{vac}} = \alpha H^2/(8\pi G_N)$, with $\alpha$ an unknown constant, then the Friedmann
One may now add quantum fields to describe matter, radiation, inflaton into our description of the Universe\(^3\). By doing this, one may wonder whether past phase transitions or inflationary epochs may lead to a change in the vacuum energy. But again, in the framework presented, the vacuum energy has the value it has because the observed Universe has the size it has \((H_0^{-1})\). An inflation scenario is still needed in order to explain the flatness of the Universe but the true (present) ground state of the inflaton field must correspond to \(\rho_{\text{vac}} \sim h/(l_P H_0^{-1})^2\) in order to comply with the observation that the Universe is large (of size \(H_0^{-1}\)).

To conclude, we propose to identify the presently observable universe to the same Bose-Einstein condensate of gravitons that describes a black hole. This seems at first to contradict our view of a black hole as a very dense object, but one should remember that the density of a black hole decreases as the inverse square of its radius. Indeed, the density of the presently observable universe (of radius \(H_0^{-1}\)) has the right order of magnitude. Moreover, it appears plausible that the Universe, when we observe it, is a self-sustainable condensate of gravitons with a classical behaviour. This is exactly what is a black hole, only at a different length scale.

This allows us to understand the order of magnitude of the vacuum energy density, in agreement with observation. The value obtained is such because the observed universe is large. This provides a new twist to the question “Why does vacuum energy become dominant now?” and correspondingly a different solution to this problem.

We focused in this paper on the main component of the Universe i.e. dark energy (which, in our case, is vacuum energy). This departs from the standard attitude which, for historical reasons, considers dark energy as an “extra” component. To us, it appears that one should first explain the dark universe before addressing the question of luminous matter, which appears to be a detail (though an important one) in the present Universe. When one equation \(H^2 = (8\pi G_N/3)(\rho_{\text{vac}} + \rho)\) may be rewritten as \(H^2 = 8\pi G_N\rho/(3 - \alpha)\), which amounts to a mere rescaling of Newton’s constant.

\(^3\)One could imagine applying the preceding ideas not just to vacuum energy but to the total energy density \(\rho_T\). In this case, one would reach the conclusion that \(\rho_T = 3H_0^2/(8\pi G_N)\) i.e. that the Universe is spatially flat. But again, our argument only applies to the quantum observable that is the vacuum energy. There remains the possibility of bosonic dark matter participating to the energy budget (just as bosons may account for the baryonic component in a black hole\(^{15}\)).
tries to add matter to the proposed scheme, new and interesting possibilities arise, in particular for dark matter.

Let us conclude by stressing again the main difference with respect to the standard cosmological scenarios. Measurement (in the quantum mechanical sense) realizes the Universe as we know it, among the many possible classical realizations. The type of graviton condensate that forms our classical space-time allows to identify continuous time and space, and to describe classically the Universe and its evolution (backward and forward in time). Obviously, the description is valid only for $kT \ll m$. In a certain sense there is no big bang but a multiplicity of different potential classical universes.

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