Multiple universes, cosmic coincidences, and other dark matters

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Abstract. Even when completely and consistently formulated, a fundamental theory of physics and cosmological boundary conditions may not give unambiguous and unique predictions for the universe we observe; indeed inflation, string/M theory, and quantum cosmology all arguably suggest that we can observe only one member of an ensemble with diverse properties. How, then, can such theories be tested? It has been variously asserted that in a future measurement we should observe the a priori most probable set of predicted properties (the ‘bottom-up’ approach), or the most probable set compatible with all current observations (the ‘top-down’ approach), or the most probable set consistent with the existence of observers (the ‘anthropic’ approach). These inhabit a spectrum of levels of conditionalization and can lead to qualitatively different predictions. For example, in a context in which the densities of various species of dark matter vary among members of an ensemble of otherwise similar regions, from the top-down or anthropic viewpoints—but not the bottom-up—it would be natural for us to observe multiple types of dark matter with similar contributions to the observed dark matter density. In the anthropic approach it is also possible in principle to strengthen this argument and the limit the number of likely dark matter sub-components. In both cases the argument may be extendible to dark energy or primordial density perturbations. This implies that the anthropic approach to cosmology, introduced in part to explain ‘coincidences’ between unrelated constituents of our universe, predicts that more, as-yet-unobserved coincidences should come to light.

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1. Introduction

Our surrounding physical universe is very well described by general relativity and the (quantum mechanical) standard model of particle physics, combined with initial conditions specifying a hot big-bang cosmology dominated by dark matter and dark energy. But this description is fundamentally incomplete: explanations for dark matter and dark energy lie outside of the standard model, general relativity is not a quantum theory, and general relativity and the standard model cannot address the big-bang singularity or initial conditions.

The current most highly regarded candidate theories of more fundamental physics and of big-bang initial conditions appear to be, respectively, string/M theory and inflation. Although it is not clear how (or whether) these theories may be combined into some consistent quantum cosmology, it is interesting—and somewhat disquieting—that string/M theory, inflation, and current formulations of quantum cosmology all share an important feature: it is far from clear that any of the three components (even less their potential combination) makes a single, unique set of predictions regarding our observable world.

In current formulations of string/M theory, low-energy physics—such the cosmological constant $\Lambda$, the particle physics coupling constants, and the cosmological parameters—is governed by dynamical degrees of freedom called \textit{moduli} that are in turn governed by a low-energy effective potential which is itself determined by a set of field fluxes. Metastable minima of this potential constitute vacua, one of which would determine the physics of our world. There appears to be no unique vacuum (there is a continuous space of different supersymmetric $\Lambda = 0$ vacua), nor any good reason to suppose that there is just one (or even a small number of) non-supersymmetric $\Lambda > 0$ vacua such as could describe our universe (see, e.g. [1]–[5] but also [6]).
In generic models for inflation, the observable universe is but one of many thermalized regions spawned within an eternally inflating background [7]–[9]. Due to the exponential expansion, each region may be described by a Friedmann cosmology with some governing parameters, but fields governing these parameters (including the M-theory moduli) may be globally inhomogeneous so that the parameters vary from one region to another. Thus such inflation can lead to an ensemble of ‘sub-universes’ in which both particle physics coupling constants and cosmological parameters such as the amplitude of primordial fluctuations [9], the cosmological constant [10,11] or the density of dark matter [12] may vary.

In formulations of quantum cosmology such as the ‘no boundary proposal’ using Euclidean quantum gravity, a prescription is given for computing the amplitude of a given spatial section and field configuration. These are then pieced together to give amplitudes for cosmological histories. In the ‘many worlds’ interpretation of quantum mechanics [13], all of these histories are equally real and coexist in superposition. Cosmological parameters such as the curvature scale [14,15] or others [16] can then take a random value in each world, drawn from a fairly well-defined probability distribution.

If we are fortunate, these ways of understanding of quantum gravity, inflation, and quantum cosmology will all turn out to be in some essential way incorrect or inapplicable to a true ‘fundamental theory’ of cosmology that will instead give unique predictions. But if not, and at least in the meantime, we must face the rather thorny issue of how to extract meaningful predictions from a theory that predicts a ‘multiverse’, by which we mean an ensemble of physically realized systems with different properties, only one of which may possibly be observed by any given observer. This is the subject of the present paper.

Three approaches to this problem have been widely adopted: the ‘bottom-up’, ‘top-down’, and ‘anthropic’ approaches. The meaning of these terms will become clear in section 2. This paper argues for three basic points. First, that in a cosmological context with no unique predictions, these different approaches are of more than academic or philosophical interest: they correspond to different specific questions being implicitly asked, and can lead to genuinely different answers to the more general question of ‘what will we observe?’ Second, that the top-down and bottom-up approaches should be seen as two ends of a spectrum, with anthropic arguments in between. Third, that when applied to the specific subject of dark matter, what we expect to see in future observations depends qualitatively on which method of making predictions is adopted, and that in the top-down or anthropic approaches a rather counterintuitive general prediction emerges.

The plan of the paper is as follows. In section 2 we discuss the three basic ways of making predictions, and how they may be viewed in a unified way. In section 3 we make generic predictions regarding dark matter in three different ways, in the context where there are many possible types of dark matter with densities that vary from one member to another of an ensemble of regions that otherwise resemble our observable universe. The extension of the arguments to other cosmological quantities is discussed in section 4, and the general issue of ‘cosmic coincidences’ is analysed in section 5. We draw general conclusions in section 6.

2. The spectrum of conditionalization

Imagine that we have a physical theory and set of cosmological boundary conditions (or ‘wavefunction of the universe’), \( \mathcal{T} \), that predicts an ensemble of physically realized systems, each of which is approximately homogeneous in some coordinates and can be
Multiple universes and other dark matters characterized by a set of parameters that may vary from one system to another. Denote each such system a ‘universe’ and the ensemble a ‘multiverse’. Given that we can observe only one of these universes, what conclusions can we draw regarding the correctness of $T$, and how? This is the problem apparently confronting—at varying levels—eternal inflation, quantum cosmology in the many-worlds interpretation, and string/M theory.

Because the alternative is to give up at the outset, let us assume that we have some quantitative way of comparing these universes so that we may specify the joint probability distribution $P(\alpha_1, \ldots, \alpha_N)$ for observables $\alpha_i (i = 1 \ldots N)$. To see what this might involve, consider the combination of the string/M theory landscape with eternal inflation. Let $\alpha_i$ be low-energy particle physics or cosmological parameters that may be compared to observations. To realize predictions of these $\alpha_i$ we might use the following successively definable quantities:

(i) The relative number $N_{\text{vacua}}(\alpha_i, \Delta\alpha_i)$ of vacua with low-energy constants in the range $[\alpha_i, \alpha_i + \Delta\alpha_i]$ for each $i$.

(ii) The relative number $N_{\text{real}}$ of these vacua that actually come into being in the cosmological context; this may differ exponentially from $N_{\text{vacua}}$ because of exponentially suppressed tunnelling (or transition) rates between vacua.

(iii) The relative number $N_V$ of physical Planck volumes on the post-inflation reheating surface in all FRW cosmologies for which the range of low-energy constants holds. This can easily differ exponentially from $N_{\text{vacua}}$ due to different periods of inflation (and may also involve ambiguous comparisons between infinite volumes [19, 20, 7, 21, 22]).

(iv) The relative number $N_B$ of baryons that exist in these volumes. These may be quite different from $N_V$ depending on the physics of baryogenesis.

None of these quantities promises to be easy to calculate, and each is harder than the previous one. But without calculating at least one, the theory makes no predictions whatsoever. For present purposes we will call $P(\alpha_i)$ a priori probabilities which are calculated using one of $N_{\text{vacua}}, N_{\text{real}}, N_V$ or $N_B$. For example, we may use $N_B$ to calculate $P_B(\alpha)$, the probability per unit $\alpha$ that a randomly chosen baryon would inhabit a universe in which $\alpha$ takes the given value. (For instance, suppose that only one observable $\alpha$ varies among the universes, that each universe has a finite baryon number $B$ depending only on $\alpha$, and that each universe has a value of $\alpha$ drawn at random from some cumulative probability distribution $p(\alpha)$; then $P_B(\alpha) \propto B(\alpha)dp(\alpha)/d\alpha$ would be the probability per unit $\alpha$ that a randomly chosen baryon would inhabit a universe in which $\alpha$ takes the given value.)

With $P(\alpha_i)$ in hand, we wish to connect our observation of the $\alpha_i$ to $T$; but how? The first approach one might consider, which may be termed the ‘bottom-up’ approach, is to use $P(\alpha_i)$ as directly as possible and assert, for example, that for each $k$ we should observe $\alpha_k$ to be near the peak of the probability distribution obtained by marginalizing

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4 For this reason we will not consider here truly disjoint multiverses with no unifying physics $T$; see [17, 18] for a discussion of such multiverses and the rather daunting difficulties associated with them.

5 The idea of this nomenclature is that $T$, being fundamental, provides the foundation for a logical structure—of which the succession of quantities $N_{\text{vacua}}, N_{\text{real}}, N_V$, and $N_B$ would be part—with our particular low-energy, local observations at the top.
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$P(\alpha_i)$ over $i \neq k$. If not, then $T$ is ruled out at some confidence level depending upon the shape of $P(\alpha_i)$ and the observed value of $\alpha_k$. This seems straightforward, but hides an unavoidable choice that was made between weighting by baryons (using $P_B$) rather by volume (using $P_V$), or giving each universe equal weight (i.e., using $P\text{vacua}$ or $P\text{real}$). Implicitly we are asking: ‘What sort of universe should I live in, given that I am a randomly chosen baryon?’, rather than ‘What should I observe given that I am a randomly chosen volume element?’ or ‘Given that I am a randomly chosen universe, what type of universe should I be?’ It is rather unclear which, if any, of these questions bears upon an observation that we make. Indeed, the only completely unambiguous situation would seem to be that in which $\alpha$ is completely independent of any other physical property and so is simply a random variable that attains, in each universe, a value drawn from the distribution $P(\alpha)$. But none of the cosmological parameters are of this nature.

A second, quite different approach—which may be called ‘top-down’—asks the question differently: ‘Given everything thus far measured, along with $T$, what should I measure for $\alpha_k$?’ In making a prediction, one thus conditions on all available data (including $\alpha_i$ for $i \neq k$), implicitly discarding from consideration all regions with properties different than those observed in our region. The danger of this approach is that it leads to the acceptance of proposals for $T$ for which the properties of our observed universe are incredibly rare. Outside of cosmology, this may not be worrisome: in order to test a theory we may construct a very special experimental arrangement and not worry that is ‘improbable’. But in cosmology there is no wider context, and the acceptance would go directly against normal scientific methodology: if a theory predicts value $a_1$ for some observable $A$ with 99.99999% probability, and result $a_2$ with 0.00001%, one would be reluctant to accept the theory if a single trial were performed and result $a_2$ were obtained; why should this change if $A$ has been measured previously and one is now measuring observable $B$? For the top-down approach to make sense, it seems, one must implicitly posit that there is some reason to neglect the discarded regions, so that the conditionalization is justified.

The third approach, the ‘anthropic’, attempts to remedy the shortcomings of the others by supplying the appropriate weighting quantity lacked by the bottom-up approach, and at the same time providing the reason required by the top-down approach for discarding regions unlike our own. The anthropic approach innocuously enough attempts to derive the probability distribution of observed parameter values, and thus implicitly asks: ‘Given that I am a typical observer, what value of $\alpha$ should I expect to measure?’ (The key assumption, that we should expect to observe the same value as a typical observer, been called the ‘principle of mediocrity’ [33].) The anthropic approach is, therefore, the bottom-up approach weighted by ‘observers’, and at the same time an attempt to justify the top-down approach by asserting that regions very different from our own have few or no observers in them. The clear problem of principle in the anthropic approach is in how to define ‘observer.’ One might ask, for example, ‘Given that I am a living being, what value of the cosmological constant $\Lambda$ should I observe?’ and obtain indeed a prediction for $P(\Lambda)$, but the concomitant prediction that one is most likely an insect or bacterium. Or it might be asked: ‘Given that I am in a galaxy, what is $P(\Lambda)$?’ But the results would then depend on the galaxy mass chosen, and may be wrong if galaxies are either not required, or not all that is required, for the existence of observers.

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6 See [32,18] for some discussion of the great technical problems.
The bottom-up and top-down approaches form the ends of a spectrum of conditionalization, with the anthropic approach (or any other approach that conditions on some but not all available observations) in between.\(^7\)

The bottom-up approach has the maximal power to rule out proposals for \(T\), but may rule out the correct one. The top-down approach is sure to allow the correct theory, but may additionally allow many other erroneous ones. The desirable compromise (which seems to be the implicit hope of proponents of the anthropic principle) would appear to be to condition on as little as possible, while still making entirely accurate predictions of all data not conditioned on. But it is unclear how we can know when minimal necessary conditionalization has been reached. This is the heart of the conundrum.

One might hope that the distinctions between top-down, bottom-up, etc would be academic when it comes to making real predictions about the real universe. There is, however, no reason to believe that this is the case. Clearly different conditionalizations will lead to different probabilistic predictions from the same candidate \(T\). But just as clearly, this will lead to different inferences of the types of \(T\)s capable of matching observations—a \(T\) that would reproduce our observations only by astonishing luck or coincidence (and hence which we would be apt to discard) under one conditionalization might seem very natural under another. Thus what may ‘naturally’ be observed in future observations may differ, in a rather general way, upon which type of reasoning (i.e., what conditionalization level) is used.

In the next section, we more explicitly argue that this is the case, using an extended and explicit example of predictions for dark matter.

3. Application to dark matter

3.1. The context

Consider, as discussed above, a proposal for \(T\) that predicts an ensemble of ‘universes’, each of which may be accurately modelled as a big-bang described by \(N\) cosmological parameters with values \(\alpha_k (k = 1 \ldots N)\). Now suppose, contrary to convention, that there are \(N_{\text{DM}} > 1\) different independent substances that act as collisionless, nonbaryonic dark matter, all of which exist in the low-energy phenomenology of \(T\). Many such particles exist in the literature, for example neutralinos and other supersymmetric particles, axions, sterile neutrinos, Kaluza-Klein particles, cryptons, light scalar particles, ‘little Higgs’ particles, Q-balls, monopoles, WIMPzillas, LIMPs, CHAMPS, D-matter, Brane-world dark matter, mirror matter, quark nuggets, primordial black holes, etc. Not all dark matter candidates in the literature can even in principle coexist—for example there can be only one lightest supersymmetric particle in each universe. But let us assume that the phenomenology of \(T\) is rich enough that some number \(N_{\text{DM}}\) of them are present in our full ensemble, with the density of each species \(i\) in each universe characterized by a dark-matter-to-baryon ratio \(\eta_i \equiv \Omega_{\text{DM},i}/\Omega_b\), which may be zero. Examples of ways in which densities for a number of dark matter candidates may become probabilistic are given in [26].

\(^7\) We note that Hartle [24] has outlined the problem in quantum cosmology in somewhat similar terms, and Bostrum’s notion of a ‘reference class’ [25] is also similar.
For tractability, let us assume that each is governed by an independent normalizable cumulative probability distribution \(p_i(\eta_i)\) that describes the probability that a randomly chosen baryon resides in a universe with the given \(\eta_i\). Of course in reality the probabilities may well not be independent, nor independent of the \(\alpha_k\); but the present purpose is to explore differences in predictions for a given \(p_i[\eta_i]\), and independent probabilities are the simplest case. For convenience we will generally work with the differential distribution in \(\log \eta_i\), \(P_i \equiv dp(\eta_i)/d\ln \eta_i = \eta_i dp(\eta_i)/d\eta_i\). Let the remaining \(N - N_{DM}\) parameters \(\alpha_k\) be described by a similarly defined probability distribution \(p(\alpha_1, \ldots, \alpha_{N-DM})\). These probability distributions may also describe parameters taking discrete values by composing \(p_i(\eta_i)\) and \(p(\alpha_1, \ldots, \alpha_{N-DM})\) of \(\delta\)-functions, but we will assume that the distributions are smooth, or that the allowed discrete values of \(\eta_i\) are closely enough spaced to be described by a smooth distribution.

3.2. Argument using bottom-up/minimal conditionalization

Let us now ask, using the bottom-up approach, what we expect to find in future experiments bearing upon the nature of the dark matter. First, we must suppose that our \(T\) proposal predicts that the probability distribution \(P(\alpha_1, \ldots, \alpha_{N-DM})\) for already measured parameters \(\alpha_k\) is such that those values are reasonably probable, or even uniquely and correctly predicted—otherwise we should already have discarded \(T\). (We are thus assuming, here and in subsequent bottom-up reasoning, that the parameters we observe are not highly unlikely given \(T\).)

Now, each \(\eta_i\) might in principle take any order of magnitude; for definiteness, let us say that each \(\log \eta_i\) has a range of \(M \gg 1\).\(^8\) It would then require fine-tuning for any given universe two dark matter components would have \(\eta\) of the same order. This is because each component will have a \(P_i\) that is either large only across a narrow range of \(\log \eta\) (in which case it is unlikely for that range to overlap with the \(\eta\) of the second component), or will have a \(P_i\) that is large over a very wide range of \(\log \eta\) (in which case it is unlikely that in the given universe the \(\eta\) chosen from this wide distribution will agree with the second component). Very roughly, we might estimate the degree of fine-tuning by assuming that the \(P_i\) peak at uniformly random values of \(\log \eta\), and thus that the probability \(P\) of the \(j\) highest-density components having the same order of magnitude would just be given by the binomial distribution:

\[
P = p^{(j-1)}(1-p)^{N_{DM}-j} \frac{(N_{DM} - 1)!}{(N_{DM} - j)!(j-1)!},
\]

where \(p = 1/M\), and the average number \(\bar{j}\) of components coincident with the dominant one will simply be \(\bar{j} = (N_{DM} - 1)/M\). Both numbers will be very small unless \(N_{DM} \gtrsim M\), i.e., \(N_{DM}\) is extremely large.

By this argument, in the absence of any genuine information on the \(P_i\), the only likely occurrence from the bottom-up approach is to have the dark matter dominated by one species, with all other possibly existent species giving negligible contributions. If the dominant species is predicted to have \(\eta \approx \eta_{obs} \approx 5\) (the observed value) then the

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\(^8\) Of course, a specific \(T\) will only allow a specific range for each \(\log \eta_i\), but this is taken into account by the \(P_i\) which would be nonzero only over this range.
A proposal may be accepted; otherwise it must be discarded and the argument repeated with some new proposal for $\mathcal{T}$.

The preceding elaborate argument has thus arrived at the standard conclusion that given several dark matter types with independent physics (i.e., in the absence of any physical reason why the $P_i$ would peak narrowly near the same values), we would be surprised if they all contributed similarly to $\Omega_{\text{DM}}$, for just the same reason it is generally considered surprising that the energy densities of dark matter, baryons, neutrinos and dark energy are all presently comparable.

Note that this is not, of course, a deductive prediction that could be obtained from any given $\mathcal{T}$. Rather, it is an inference drawn by comparing many imagined prospective $\mathcal{T}$-candidates to a set of observations. This is something we do routinely: for example, in ‘deriving’ cosmological parameters from astronomical observations we implicitly have in mind an imaginary ensemble of universes which we can compare in turn with our observations to select out the set that is in reasonable accord with those observations. In doing that analysis, bottom-up reasoning weighted by universe is generally used, which seems sensible given that the ensemble is imaginary; but the weighting could be done differently (i.e., different ‘priors’ could be chosen, or alternatively a different measure could be placed over the space of theories) and different inferences would be so obtained. In general we know of no reasonable way to define such a measure over ‘theory space’ and we made a simple assumption in calculating equation (1); however, we note that this is also implicitly done in any discussion of whether a given theory is fine tuned. The present argument is of just this character.

### 3.3. Argument using top-down/maximal conditionalization

Let us now perform the analysis from the top-down point of view. We again assume that there is an ensemble of regions with $N_{\text{DM}}$ species of dark matter of densities $\eta_i$ present in each. Now, however, we further assume that we inhabit one member of a sub-ensemble of universes in which the $\alpha_i$ all take values compatible with current observations; in particular we assume that whatever values the individual $\eta_i$ take, the total dark matter density $\eta \equiv \sum_i \eta_i$ satisfies $\eta = \eta_{\text{obs}}$, since this is essentially all that is known about the dark matter$^9$. This sub-ensemble may constitute a tiny part of the full ensemble, but in the top-down approach we refuse to be troubled by this.

While we know $\eta$, we do not know the individual $\eta_i$, so we can predict their most probable values using our candidate $\mathcal{T}$. This requires that we maximize the total probability

$$P_{\text{tot}} \propto \prod_i P_i(\log \eta_i),$$

subject to $\eta = \eta_{\text{obs}}$. Each $P_i$ may favour (i.e., have substantial probability at) ‘low’ values $\eta_i \ll \eta_{\text{obs}}$, ‘high values’ $\eta_i \gg \eta_{\text{obs}}$, the observed value $\eta_i \sim \eta_{\text{obs}}$, or some combination of these. Now pick some component $i$. We have argued above that it is a priori unlikely that only $\eta_i \sim \eta_{\text{obs}}$ will be likely. Then, if low values are favoured (and whether or not

$^9$ Implicitly, we are assuming that all of the components genuinely are cold, weakly interacting, particles that could not have been detected by current experiments. Stronger constraints could be imposed on, for example, warm or hot dark matter.
high or observed values are), then they are probably attained, since there is no conflict with the conditionalization $\eta \simeq \eta_{\text{obs}}$; therefore each such component would be expected to have an unobservably small density in our universe.

This leaves us with $N_{\text{big}}$ components with probable values near or far above $\eta_{\text{obs}}$. Let us then examine the set of $\mathcal{P}_i$ for these near $\eta_i \sim \eta_{\text{obs}}$, where they may have small (but nonzero\footnote{If for any of the components favouring high $\eta_i$, no universes have $\log\eta_i < \log\eta_{\text{obs}}$, then of course $T$ is ruled out; this would be wonderful as it would be a definite prediction of $T$.}) probability.

We can compute the maximum of the total probability $\mathcal{P}_{\text{tot}}$ subject to the condition of fixed $\eta = \eta_{\text{obs}}$ using a Lagrange multiplier:

$$\max\left\{ \ln \mathcal{P}_{\text{tot}} - \lambda \sum_i \eta_i \right\}$$

gives

$$\max \sum_i [\ln \mathcal{P}_i - \lambda \eta_i]$$

and the solution

$$d \ln \mathcal{P}_i / d \eta_i = \lambda. \quad (3)$$

In other words, the optimal values $\eta_i$ have the intuitive property that increasing any of them by a small amount increases $\ln \mathcal{P}_{\text{tot}}$ by the same factor $\lambda$.

For a more explicit solution we will assume that each is locally a power law, i.e.,

$$\mathcal{P}_i \propto \eta_i^{\beta_i}. \quad (4)$$

Since we are considering a narrow range of $\eta \sim \eta_{\text{obs}}$, this should be a good approximation unless there is a preferred scale in the probability distribution of order $\eta_{\text{obs}}$, which is a priori unlikely (i.e., would require fine-tuning). We also assume $\beta_i > 0$ as we are only considering the $N_{\text{big}}$ components with probabilities peaked at high $\eta$.

The maximization of $\mathcal{P}_{\text{tot}}$ subject to $\eta = \eta_{\text{obs}}$ yields, using equation (3),

$$\eta_i = \left( \frac{\beta_i}{\beta} \right) \eta_{\text{obs}}, \quad \beta \equiv \sum_j \beta_j, \quad (5)$$

thus the contributions to $\eta$ of the $N_{\text{big}}$ components will be different by orders of magnitude only if the power laws governing the probability distributions at $\eta_i \sim \eta_{\text{obs}}$ are. This cannot be ruled out (although this may be possible in the anthropic approach as discussed below), but it may be argued for some specific cases that this is unlikely; for example, in discussing the cosmological constant in a similar context, Weinberg \cite{28} (see also \cite{27,10,29,26}) argues that because the range considered (here $\eta \sim \eta_{\text{obs}}$) is so small compared to the characteristic scale governing $P(\eta_i)$, the latter should locally be nearly flat (i.e., $\beta = 1$ here).

Three conclusions have been drawn using the top-down approach, summarized as follows. First, it is very likely that we live in a very improbable ensemble member, because at least one dark matter component $i$ is likely to have $\mathcal{P}_i$ peaked at $\eta_i \gg \eta_{\text{obs}}$. But we have decreed that we shall accept this in light of observed facts. Second, as in the bottom-up case, a number of components should contribute $\eta \ll \eta_{\text{obs}}$, and we shall
probably not detect these. Third, several components should contribute roughly equally to $\eta^{\text{obs}}$, unless one of the components has a probability distribution that is both peaked at $\eta \gg \eta^{\text{obs}}$ and is quickly varying at $\eta \sim \eta^{\text{obs}}$. We shall call this the ‘principle of equal representation’ (PER): when a conditionalization is placed on a sum $\alpha \equiv \sum \alpha_i$ of parameters with (relatively slowly) rising independent probability distributions $P(\alpha_i)$, the most probable combination subject to the conditionalization is that all $\alpha_i$ are of similar order.

Thus the generic prediction of the top-down case is directly contrary to the result obtained in the bottom-up case; whereas in the latter we would expect one dominant dark matter form, here it is quite reasonable to expect several (even many) forms to have comparable contributions. However, we cannot eliminate the possibility of an extremely high $\beta_i$ value that would allow one component to be dominant, nor have we explained why we live in a very improbable region.

### 3.4. Anthropic argument/partial conditionalization

The final approach we may take is that of partial conditionalization, which includes the anthropic approach. Starting with the same ensemble of universes each with $N_{\text{DM}}$ dark matter components, we calculate probabilities as in the bottom-up approach, but with an additional weighting (or conditionalization) factor $W(\alpha_k)$ applied to the baryon-weighted (or volume-weighted) probabilities. To reproduce the top-down approach, for example, we may set $W(\alpha_k) = 0$ if $\alpha_k$ is incompatible with our observations for any $k$, and $W = 1$ otherwise. Then if the probabilities are re-normalized, this is equivalent to limiting the analysis to a sub-ensemble of universes compatible with our current observations. But $W$ can be much less restrictive. For example, $W(\alpha_k)$ could count the number of some object $X$ (say a large spiral galaxy) per baryon in a universe with parameters $\alpha_k$.

The (re-normalized) $W$-weighted probabilities can be interpreted in two different ways. First, they could be said to describe what we expect to measure for $\alpha_k$ given only that we observe an $X$. One might then view this ‘partial conditionalization’ approach simply as relaxing the top-down assumptions, in order to gain more predictive power using less assumptions [30, 14, 23]; but why keep some conditions, and not all, or none?

Alternatively, one might with a slight philosophical shift take the anthropic approach of attempting to ask: ‘Given that I am a randomly chosen observer, what should I observe?’ Then the $X$-object should be an observer, and the $W$-weighted probabilities would describe the probability that a randomly chosen observer inhabits a universe with parameters $\alpha_k$. This clears up some of the ambiguity of the partial conditionalization approach in that there is a justification conditionalizing on observers. However, since there is no obvious definition of what ‘observer’ actually means, one is forced to adopt some proxy—stars, galaxies, or universes ‘pretty much’ like ours—and hence the ambiguity remains.

A third possibility, often employed in the literature in making anthropic predictions, is to focus on a single parameter, say $\Lambda$, fix all other cosmological parameters to the observed values, then weight $P(\Lambda)$ by some proxy for observers, for example by a $W(\Lambda)$ that is the number of galaxies per baryon (or unit volume) in a universe of the given $\Lambda$. This approach is generally taken for purposes of tractability, but is really justified only if $\Lambda$ alone varies across the ensemble; the predictions made by varying only one parameter
will generally not be the same as if several are varied (see section 5 and [31, 32, 34, 35]),
and if several parameters vary, it seems hard to justify treating only a subset of them
anthropically.

In any case, once a conditionalization (or weighting) factor is chosen, a calculation can
be done to predict the various dark matter densities by computing the total probability
that an $X$ resides in a universe with defining parameters $\alpha_k$ and dark matter ratios $\eta_i$:

$$P_{\text{tot}} = P(\alpha_k) \times \prod_i P_i \times W(\alpha_k, \eta),$$

(6)

where $W$ is the number of $X$'s per baryon in a universe with parameters $\alpha_k$ and total dark
matter density $\eta$.$^{11}$

Now there are three possibilities. Either (1) $P_{\text{tot}}$ allows reasonable probability of the
observed values of $\alpha_k$ and $\eta$, and $W(\eta)$ is falling near $\eta \sim \eta_{\text{obs}}$, or (2) the same, but $W(\eta)$
is rising at $\eta \sim \eta_{\text{obs}}$, or (3) $P_{\text{tot}}$ has little probability near the observed values.

In cases (1) and (2), we have a good candidate for an anthropic explanation of
the cosmological parameters $\alpha_k$ and $\eta$ if we can make a strong argument that our $X$-
object is a good proxy for an observer. But what do we predict for the individual dark
matter densities $\eta_i$? In case (1), the situation is quite similar to that in the top-down
approach, and the same arguments lead to the conclusion that it is natural for several
dark components of comparable density to contribute to $\eta$, with several other components
existing at undetectable levels. Thus the PER applies to the anthropic standpoint, if there
is a high-$\eta$ cutoff in the hospitality factor $W(\eta)$. In case (2), however, the PER will not
hold, because $P(\eta_i)$ must be falling for all $i$ (or else $P_{\text{tot}}$ could not fall off on both sides of
$\eta_{\text{obs}}$, and we would instead be discussing case (3)). In this case $P_{\text{tot}}$ will be greatest when
only one dark matter component is required to take an improbable value $\eta_i \sim \eta_{\text{obs}}$.

Similarly, additional inferences can in principle be made using case (3) that $P_{\text{tot}}$ does not
peak near the observed parameter values. In this case we are led to conclude that the fundamental theory and its $P$ are incorrect. Given a form of $W$, we can in this way
constrain the form of $P$. To see this more explicitly, suppose that we have a candidate
$T$ with $N_{\text{DM}}$ dark matter components for which $P_i \propto \eta_i^{\beta_i}$ at $\eta \sim \eta_{\text{obs}}$, and for which all
other parameters have fixed, unique values. Then let us maximize

$$P_{\text{tot}}(\eta_i) \propto \prod_i \eta_i^{\beta_i} W(\eta).$$

(7)

Because the conditionalization factor $W$ depends only on the total dark matter density
$\eta$, the maximum of $P_{\text{tot}}$ will occur where each $\eta_i = \beta_i/\beta$ (where $\beta \equiv \sum_i \beta_i$), as in the
top-down approach. But furthermore, the peak will occur at a value $\eta_{\text{max}}$ that depends
upon $\beta$.

For example, let us model $W$ (for demonstrational purposes only) by either a power-
law falloff (with index $\gamma$) or Gaussian decline for $\eta > \eta_0$, i.e.,

$$W(\eta) \propto \frac{1}{1 + (\eta/\eta_0)^\gamma} \quad \text{or} \quad W(\eta) \propto e^{-\eta^2/2\eta_0^2}.$$  

(8)

If $P(\eta_i) \propto \eta_i^{\beta_i}$, then it is readily shown that

$$P(\eta) \propto \eta^{\beta},$$

(9)

$^{11}$ We are assuming here that only the total dark matter density is important in forming $X$-objects.

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and the maximum of $P_{\text{tot}}$ occurs at

$$\eta_{\text{max}} = \eta_0 \left( \frac{\beta}{\gamma - \beta} \right)^{-1/\gamma} \quad \text{or} \quad \eta_{\text{max}} = \beta^{1/2} \eta_0,$$

respectively.

Knowledge of $\eta_0$ and $\gamma$ would then allow us to limit the maximum allowed $\beta_i$ for our candidate $\mathcal{T}$. If $W$ has a power-law cutoff, we must have $\gamma > \beta$, or else the probability would be dominated by $\eta \gg \eta_0$ and we would discard the $\mathcal{T}$ leading to that $\beta$. The Gaussian falloff in the conditionalization factor will defeat any power-law indices $\beta_i$, but if $\beta \approx 0$, then the probability would be peaked at $\eta \gg \eta_{\text{obs}}$ unless $\eta_0 \ll \eta_{\text{obs}}$. For example, if $\beta = 2$, 4 or 8, then 95% of the probability in $P(\beta)W(\beta)$ lies at $\eta > \eta_0 \times 0.31$, 1.04 and 1.8, respectively. Thus for our observations to be compatible with $\beta = 8$ at 95% confidence, we would have to (uncomfortably) assert that $\eta_0 < \eta_{\text{obs}}/1.8$, i.e., that we are well into the exponentially cutoff in $W$, and, for example, that a universe with only slightly more dark matter would have far fewer observers.

An upper limit on allowed $\beta$ can be used in two ways. First, we can derive an upper limit to the number components that should, by the previous arguments, have comparable $\eta_i$, since these components must all have $\beta_i \geq 1$. No such limit exists in the top-down approach. Second, a limit on $\beta$ can rule out one component $i$ having a $\beta_i$ orders of magnitude greater than the rest, strengthening (as compared to the top-down approach) the argument that components with rising $W$ should be comparable in density—though as noted above the anthropic argument for multiple components holds only if $W(\eta)$ is decreasing near $\eta_{\text{obs}}$.

4. Equal representation in other cosmological quantities

The arguments given concerning dark matter that lead to the principle of equal representation (PER) could also be applied to other cosmological parameters such as the neutrino density parameter $\Omega_\nu$, the amplitude $Q$ of density inhomogeneities, and the cosmological constant $\Lambda$. In all cases we could place a constraint on the total value (either from observation or from anthropic considerations), so we might apply the PER to a situation in which there were multiple possible contributions to $\Omega_\nu$, $Q$, or $\Lambda$. The argument is, however, somewhat different for each parameter.

In the case of $Q$, inflation tends to quash rival pre-inflationary perturbation sources; thus a second significant component would have to be imprinted later, and would likely be either non-Gaussian, non-adiabatic, or non-scale-invariant (as we presently know no post-inflation perturbation source without at least one of these properties). The present data suggest none of these.

As for $\Lambda$, the argument is complicated by the fact that $\Lambda$ can be either positive or negative, so that contributions of large magnitude may cancel. Also, to be meaningfully distinguishable the contributors must be dynamically different, so the arguments would have to be applied to multiple parameters describing the details of the dark energy components.

12 Of course, if $0 \leq \beta_i \leq 1$, there could be more components; but if the power law extends to $\eta \ll \eta_{\text{obs}}$ then the probability lies mostly there, so there is no reason to expect $\eta \sim \eta_{\text{obs}}$. 
Massive neutrinos provide an interesting context in which to make detailed calculations because it is known that there are three neutrino species with nonzero mass, and because the cosmological effects of massive neutrinos are relatively well understood.

Neutrinos with masses in the eV range slow the growth of density perturbations on small scales, thereby suppressing galaxy formation. This effect has been used \cite{36,34} to derive an anthropic prediction of \( \Omega_\nu \) (or equivalently the sum \( \sum_i m_i \) of the neutrino masses \( m_i \)) under the assumptions that (1) either only \( m_\nu \), or only \( m_\nu \) and \( \Lambda \) vary across the ensemble, (2) the number of observers is proportional to the collapse fraction of dark matter into bound halos, and (3) the prior distribution of either \( m_\nu \) or of the individual neutrino species’ masses are flat:

\[
P(m_\nu) \propto m_\nu \quad \text{or} \quad P(m_i) \propto m_i.
\]

The suppression of galaxies provides a near-exponential cutoff in \( W(m_\nu) \) when combined with \( P(m_\nu) \) gives 95% of the probability in the range 0.13–5 eV. (When variation in \( \Lambda \) is included and marginalized over, this shifts to 0.035–5 eV.)

What would we expect for the neutrino masses using our different methods of reasoning? The two measured neutrino mass splittings are \( \delta m_{23}^2 = 2.5 \times 10^{-3} \) eV\(^2\) and \( \delta m_{12}^2 = 8 \times 10^{-5} \) eV\(^2\) \cite{37,38}, and cosmological observations bound \( m_\nu \lesssim 0.42 \) eV \cite{39}. From the bottom-up approach, we would require a \( T \) with high probability to have all three masses \( m_i \lesssim 0.13 \) eV, and with one family of mass > 0.007 eV and another of mass > 0.05 eV. This would be surprising if the families had independent physics (as in the posited dark matter species discussed above) but of course they probably do not, and we would simply expect a successful \( T \) to predict an overall neutrino mass scale \( \sim 0.01–0.05 \) eV, with splittings of the same order.

In the top-down approach, the predictions would be similar, except that it would be possible for the overall neutrino mass scale to be highly improbable.

The anthropic approach is potentially more interesting. Under the assumption that all three neutrino species were governed by independent \( P(m_i) \propto m_i \), the anthropic prediction (as derived by \cite{36} under the above assumptions) would be \( m_\nu \sim 3 \) eV. The probability distribution for either of the two mass splittings can be similarly computed, and is centered about zero with a width of \( \sim 1 \) eV, so we would expect all three neutrino masses to be \( \sim \) eV in a nice exhibit of the PER. (For the same \( P(m_i) \propto m_i \), bottom-up reasoning would predict very large \( m_\nu \) and large mass splittings.) However, as \( m_\nu \sim 3 \) eV violates the cosmological bound and \( \sim 1 \) eV splittings far exceed the observed ones, we must rule out any \( T \) with \( P(m_i) \propto m_i \). Instead, we must assume \( P(m_\nu) \propto m_\nu \), in which case the predictions of \cite{34} are compatible at 95% confidence with the observed bounds. In this case the mass splittings would presumably follow from the neutrino physics, rather than the PER.

5. Variations in multiple parameters, and cosmic coincidences

One of the surprising aspects of the standard cosmological model is that so many energetic constituents of the present-day universe have such similar densities, i.e., \( \Omega_A/\Omega_{DM} \approx 2 \), \( \Omega_{DM}/\Omega_b \approx 5 \), and \( \Omega_b/\Omega_\nu \lesssim 30 \), even though many or all of these densities are thought to arise from essentially independent physics.

Anthropic (or other partial-conditionalization) reasoning affords an opportunity to explain these ‘coincidences’ because even if \( P(\alpha_i) \) factorizes into \( P(\alpha_1) \ldots P(\alpha_N) \), the
conditionalization factor $W(\alpha_i)$ almost certainly will not, and therefore the product $PW$ used in making predictions from the theory $T$ will have correlations between the $\alpha_i$. Part of the purpose of the present paper is to point out that if the true explanation of the coincidences in these parameters is that their values are bound together by the necessity of observers (which provide a particular $W$), then we may expect new, as-yet-unobserved coincidences. These might consist of similar contributions to currently unresolvable components such as dark matter, or, in principle, of new coincidental aspects of the universe. The ‘preposterous’ universe [40] may get even worse.

But the possibility of explaining cosmic coincidences comes at a price: degeneracies in $W$ imply that the anthropic prediction for one parameter will likely change when additional parameters are allowed to vary; and there is every reason to believe that degeneracies exist in $W$ [32,31,35]. Thus an anthropic explanation of the observed $\alpha_i$ ($i = 1 \ldots N$) really requires the computation of both $P$ and $W$ over the $N$-dimensional parameter space; and until all $N$ are considered, there is no reason to hope that adding an additional parameter will not spoil the correct prediction. In the context of the spectrum of conditionalization, one is working from the top-down toward a fully anthropic approach; and as the conditionalization factor is loosened, more and more theories can be ruled out; each ‘successful’ anthropic prediction is only provisional.

6. Summary and conclusions

While our observable universe is well described by a simple big-bang FRW cosmological model, attempts to understand that model at a more fundamental level through inflation, quantum cosmology, and string/M theory have raised the spectre of multiple quasi-FRW regions (‘universes’) with different properties. We must then ask: ‘Given a fundamental theory of physics and cosmology $T$, how can we extract from it predictions for the single universe we can observe?’ It has been variously asserted that, in a future measurement, we should observe the most probable set of predicted properties (the ‘bottom-up’ approach), or the most probable set compatible with all current observations (the ‘top-down’ approach), or the most probable set consistent with the existence of observers (the ‘anthropic’ approach). These correspond to three different implicit questions of the form: ‘Given that $X$, what should I observe in future measurement $Y$’, with $X$ being a conditionalization that is minimal in the bottom-up approach, and maximal in the top-down approach. Given a theory that predicts a multiverse, these questions will have different answers, and hence different implications on what we will observe in a future measurement. This paper has been a rough exploration of these different questions and their differing implications.

There are both problems of principle and great technical challenges in mathematically describing the ensemble of universes and defining a measure so that an $a$ priori probability distribution $P(\alpha_k)$ can be defined for the parameters $\alpha_k$ describing low-energy physics and cosmology. Does each possible universe, or each realized universe, or each volume element, or each baryon receive equal weight? Making this choice constitutes a definition of the bottom-up probabilities, and the ambiguity in this choice may be termed the ‘measure problem’ [41].

To investigate the top-down and anthropic approaches, we have optimistically assumed that these $a$ priori probabilities can in principle be defined and computed, and have made simple assumptions about the form of $P(\alpha_k)$ in order to draw some exemplary
and qualitative conclusions, focusing on the particular issue of dark matter, in the context of an imagined ensemble in which there are $N_{DM}$ physically independent species of dark matter, with different densities $\eta_i$ ($i = 1 \ldots N_{DM}$) in each ensemble member. Among these conclusions are:

- Using bottom-up reasoning, we would generically expect to see one dominant form of dark matter, because it would be unlikely for any of the other independent dark components to have a comparable density by chance.

- Using top-down reasoning, we can fix the total dark matter density $\eta \equiv \sum_i \eta_i$ but not the individual contributions. But these can be predicted by assuming that we live in a rather typical universe consistent with the observed $\eta$. If $P(\eta_i)$ varies smoothly near $\eta_i \sim \eta$ for each $i$, the overall probability will be maximized for all $\eta_i \sim \eta/N_{DM}$. The prediction of many dark matter components of similar density is directly contrary to the bottom-up prediction, and is a demonstration of what we call the principle of equal representation.

- Using anthropic reasoning, we will obtain the same predictions for the dark matter components as in top-down reasoning, if the conditionalization factor (which essentially constitutes a choice of definition of ‘observer’) forces the prediction for $\eta$, as well as the other cosmological parameters, to accord with the observed values (and in particular provides a cutoff at $\eta \gtrsim \eta_{\text{obs}}$). Moreover, for some types of ‘observers’, the conditionalization factor $W(\alpha_k)$ can be actually estimated, and used to constrain $P(\alpha_k)$ by eliminating theories for which $P(\alpha_k)W(\alpha_k)$ does not peak near the observed values of $\alpha_k$. This strengthens the argument for the coincidence between components, but suggests that the number of coincident dark components cannot be large.

None of the three methods (or those elsewhere on the spectrum) is obviously ‘correct’, and each has serious issues. The bottom-up approach suffers from an ambiguity in the measure, and may rule out the correct $T$. The top-down approach may allow an incorrect theory, by simply asserting that we inhabit an exceedingly unlikely member of the ensemble, while providing no reason why this is the case. And the anthropic method, while providing a measure and a reason for excluding many ensemble members, requires the definition of an observer—which is itself quite ambiguous.

Perhaps we may hope that nature may give us a clue as to which sort of reasoning we should employ, for a key point of this paper is that if the anthropic effects are the explanation of the parameter values—and coincidences between them—that we see, then it ought to predict that new coincidences will be observed in future observations. If in the next several decades dark matter is resolved into several equally important components, dark energy is found to be, say, five independent substances, and several other ‘cosmic coincidences’ are observed, even the most die-hard sceptic might accede that the anthropic approach may have some validity. On the other hand, if we are essentially finished in defining the basic cosmological constituents, then anthropic reasoning might be able to explain some of these, but (beyond its arguably successful prediction of a small but nonzero cosmological constant) would have missed its chance to predict anything not yet observed.

Probably the best we can hope to do is, starting with a $T$, work our way down the conditionalization spectrum, knowing that any successful predictions are provisional as the theory could be ruled out each successive stage of weaker conditionalization. But we must keep in mind that if there is a correct $T$, using it to make successful predictions may
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really require some conditionalization, and we will never be able to be completely sure
that there is not a different \( T \) that could require less. When dealing with multiverses, the
dream of a final theory may be just that.

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