**Abstract**

The study of $B \to \tau^+ \tau^- (X)$ decays can provide us with a better understanding of the third generation, and can be a useful probe of physics beyond the standard model. We present a model independent analysis of these decays. We classify new physics that can largely enhance the decay rates and we discuss the constraints implied by other processes. Experimentally, flavor changing neutral current $B$ decays into final state $\tau$'s are still unconstrained. Searches for $B$ decays with large missing energy at LEP provide the first limits. We estimate that existing data already imply bounds on the $B_d \to \tau^+ \tau^-$, $B_s \to \tau^+ \tau^-$, and $B \to X \tau^+ \tau^-$ decay rates at the few percent level. Although these bounds are over four orders of magnitude above the standard model predictions, they provide the first constraints on some leptoquarks, and on some R-parity violating couplings.
I. INTRODUCTION

The standard model (SM) of the strong and electroweak interactions provides an accurate description of the low energy properties of the known elementary particles. However, experimental results involving fermions of the third generation are far less precise than for the first two generations. For example, little is known about decays involving more than one third generation fermions. From the theoretical point of view, a better knowledge of the physics of the third generation could help us understand the hierarchy of the fermion masses and mixing angles. As it is the case in some models of new physics \[1\], the third generation might even be essentially different from the first two.

The LEP experiments have provided us with several new results on third generation fermions, and in particular on the $b$ quark. One type of measurement for which the LEP environment has advantages over symmetric $B$ factories (like CLEO) or hadron colliders (like CDF) is the study of $B$ decay modes that produce a large amount of missing energy due to neutrinos in the final state. The main background to such analyses is the tail of the semileptonic decay distribution, so decay modes yielding a harder missing energy spectrum can be effectively measured or constrained. The excess of events with large missing energy measured at LEP was interpreted as the signature of the decay $B \rightarrow X_c \tau \bar{\nu}_\tau$ followed by $\tau \rightarrow \nu X$ \[2\] \[4\], yielding

$$\text{BR}(B \rightarrow X_c \tau \bar{\nu}) = 2.68 \pm 0.34\%.$$ \hspace{1cm} (1.1)

This is in good agreement with the SM prediction $\text{BR}(B \rightarrow X_c \tau \bar{\nu}) = 2.30 \pm 0.25\%$ \[3\], and constrains certain new physics contributions \[3\]. The ALEPH Collaboration also searched for events with very large missing energy, $E_{\text{miss}} > 35 \text{ GeV}$ \[4\]. The absence of excess events over the background yielded the 90% confidence level upper limit on the exclusive leptonic decay $B \rightarrow \tau \bar{\nu}$ \[2\]

$$\text{BR}(B \rightarrow \tau \bar{\nu}) < 1.8 \times 10^{-3}.$$ \hspace{1cm} (1.2)

In a recent paper \[6\] we showed that the same data also imply a bound on the flavor
changing $B \to X \nu \bar{\nu}$ decay rate, and we discussed the resulting constraints on several possible sources of new physics. Based of the full LEP–I data sample, the ALEPH Collaboration has recently announced a preliminary 90% confidence level limit on this decay mode \[8\]

$$\text{BR}(B \to X \nu \bar{\nu}) < 7.7 \times 10^{-4}.$$  \(1.3\)

This limit is only one order of magnitude above the SM prediction, and provides important constraints on several new physics scenarios \[7\].

Besides these decay modes, the exclusive $B \to \tau^+ \tau^-$ and inclusive $B \to X \tau^+ \tau^-$ decays are also associated with sizable missing energy due to the neutrinos from the $\tau$ decays. Since these processes are presently unconstrained, it is interesting to see whether any useful limit can be established by analyzing the LEP data on large missing energy events. In the SM, $B$ decays into a pair of charged leptons are highly suppressed. However, certain kinds of new physics can enhance these decay rates up to several orders of magnitude above the SM predictions.

In section II we study the $B_{d,s} \to \tau^+ \tau^-$ decays from a theoretical point of view. Since we are mainly interested in possible new physics contributions, we present a model independent analysis. In section III we estimate the limits on the $B \to \tau^+ \tau^- (X)$ branching ratios that could be established from the LEP data. In section IV we discuss the constraints that the limits on these decays imply on some new physics models. Section V contains a summary and our conclusions.

### II. THEORETICAL ANALYSIS

The most general effective four-fermion interaction involving a $b$ quark, a $q = d$ or $s$ quark, and a pair of $\tau^+ \tau^-$ leptons can be written in the form

$$L_{\text{eff}}^{q b} = G_F \sum_a C_a^q (\bar{q} \Gamma a b) (\bar{\tau} \Gamma a \tau) + G_F \sum_a C_a^{q'} (\bar{q} \Gamma a b) (\bar{\tau} \Gamma a' \tau),$$  \(2.1\)

where $\Gamma a = \{I, \gamma_5, \gamma^\mu, \gamma^\mu \gamma_5, \sigma^{\mu \nu}\}$, $\Gamma a' = \Gamma a \gamma_5$ and $a = \{S, P, V, A, T\}$. In (2.1) we have factored out the Fermi constant $G_F$ so that the coefficients $C_a^q$ and $C_a^{q'}$ are dimensionless.
While the $B \to X_q \tau^+ \tau^-$ decay depends on all ten coefficients $C^q_a$ and $C^q_a'$, fewer operators contribute to the leptonic $B_q \to \tau^+ \tau^-$ decay.

Let us consider the matrix element $\langle 0 | \bar{q} \Gamma b | B \rangle$. It can only depend on the four-momentum of the $B$ meson, $p_B^\mu$, and therefore it must vanish for $\Gamma_T = \sigma_{\mu\nu}$ which is antisymmetric in the Lorentz indices. The matrix elements of the parity-even operators ($\Gamma_S = I$ and $\Gamma_V = \gamma^\mu$) also vanish due to the pseudoscalar nature of the $B$ meson. Finally, for on-shell leptons, the contribution of the axial-vector operator $\langle 0 | \bar{q} \gamma^\mu \gamma_5 b | B \rangle \propto p_B^\mu = p_{\tau^+}^\mu + p_{\tau^-}^\mu$ also vanishes when contracted with the leptonic vector current $\bar{\tau} \gamma^\mu \tau$. Hence, leptonic $B$ decays are induced only by the following three operators

$$C^q_P (\bar{q} \gamma_5 b) (\bar{\tau} \gamma_5 \tau), \quad C^q_P' (\bar{q} \gamma_5 b) (\bar{\tau} \tau), \quad C^q_A (\bar{q} \gamma^\mu \gamma_5 b) (\bar{\tau} \gamma^\mu \gamma_5 \tau).$$

(2.2)

Using the PCAC relations

$$\langle 0 | \bar{q} \gamma^\mu \gamma_5 b | B \rangle = -i f_B p_B^\mu,$$

$$\langle 0 | \bar{q} \gamma_5 b | B \rangle = i f_B \frac{m^2_B}{m^2_B + m^2_q} \simeq i f_B m_B,$$

(2.3)

the most general amplitude for the $B_q \to \tau^+ \tau^-$ decay reads

$$\mathcal{A}_q = i f_B m_B G_F \left[ \left( C^q_P + \frac{2m_\tau}{m_B} C^q_A \right) (\bar{\tau} \gamma_5 \tau) + C^q_P' (\bar{\tau} \tau) \right],$$

(2.4)

where, for simplicity, we omitted the subscripts $q = d, s$ for the $B$ mass and decay constant. All three operators in (2.2) appear in the SM. Thus, the general result for the total decay rate can be read off from Ref. [3]

$$\Gamma(B_q \to \tau^+ \tau^-) = \frac{G^2_F f_B^2 m^2_B}{8\pi} \sqrt{1 - \frac{4m^2_\tau}{m^2_B}} \left[ \left| C^q_P + \frac{2m_\tau}{m_B} C^q_A \right|^2 + \left( 1 - \frac{4m^2_\tau}{m^2_B} \right) \left| C^q_P' \right|^2 \right].$$

(2.5)

In the SM, $C^q_P'$ and $C^q_P$ gets contributions from penguin diagrams with physical and unphysical neutral scalar exchange, and are suppressed as $\sim (m_b/m_W)^2$. Then, the dominant contribution to the decay rate comes from

*We thank David London for bringing this point to our attention.
\[
(C_A^q)^{SM} = |V^*_{tb} V_{ts}| \frac{\alpha_{em}(M_W)}{\sqrt{8 \pi} \sin^2 \theta_W} \sqrt{V^{*}_{tb} V_{tb}^*} \left( m_t^2 \right) \frac{1}{m_W^2}, \tag{2.6}
\]

where, at leading order, \( Y(x) = (x/8) \left[ (4 - x)/(1 - x) + 3x \ln x/(1 - x)^2 \right] \) \[^10\]. Including the small next-to-leading order correction, the SM result for the branching ratio is \[^11\]

\[
\text{BR}^{SM}(B_s \to \tau^+ \tau^-) = 8.9 \times 10^{-7} \left[ \frac{f_{B_s}}{230 \text{ MeV}} \right]^2 \left[ \frac{m_t(m_t)}{170 \text{ GeV}} \right]^{3.12} \left( \frac{|V_{ts}|}{0.040} \right)^2 \left( \frac{\tau_{B_s}}{1.6 \text{ ps}} \right). \tag{2.7}
\]

Compared to this result, the \( B_d \to \tau^+ \tau^- \) decay rate has an additional suppression of about \( |V_{td}/V_{ts}|^2 \sim 10^{-2} \).

In general, new physics that can induce large contributions to the coefficients in \((2.3)\) is also likely to enhance the rates for other rare processes. The existing experimental limits already imply that in several models, the \( B_q \to \tau^+ \tau^- \) decays can only occur at rates far below the sensitivity achievable at LEP. Therefore, it is useful to classify the contributions which are already tightly constrained by other measurements, and the ones which are still unconstrained (or only weakly constrained). In doing so, we will avoid referring to any specific model, but we will use \( SU(2) \) gauge invariance to relate operators contributing to the \( B_q \to \tau^+ \tau^- \) decays to operators which induce other transitions. In the presence of new physics, operators which are not manifestly \( SU(2) \) invariant can also appear \[^12\], suppressed by inverse powers of some large mass scale. They will not be considered here, since it is unlikely that through such contributions the \( B_q \to \tau^+ \tau^- \) decay rates could get the few order of magnitude enhancement required in order to be observable at LEP.

\( SU(2) \) invariants can be built out of four SM fermions by combining four singlets, four doublets, or two singlets and two doublets. While for \( 1^4 \) and \( 1^2 \times 2^2 \) there is only one \( SU(2) \) invariant, two different \( SU(2) \) invariants arise from \( 2^4 \). Taking into account all inequivalent permutations of the \( b, q, \) and \( \tau \) fields, and recalling that matrix elements of tensor operators vanish, the following operators can contribute to the \( B_q \to \tau^+ \tau^- \) decay:

\[
(\mathcal{O}_0^{MN})^q = 4 G_F (g_0^{MN})^q (\bar{q}_M \gamma^\mu b_M) (\bar{\tau}_N \gamma_\mu \tau_N),
\]

\[
(\mathcal{O}_0^{LL})^q = 4 G_F (g_0^{LL})^q (\bar{\tau}_L \gamma^\mu b_L) (\bar{q}_L \gamma_\mu \tau_L),
\]
\((\mathcal{O}_{1/2}^{MN})^q = 4G_F (g_{1/2}^{MN})^q (\bar{q}_M b_N) (\tau_N \tau_M), \quad (M \neq N)\)

\((\mathcal{O}_1^{LL})^q = 4G_F (g_1^{LL})^q (\bar{q}_L \gamma^\mu b_L) (\bar{\tau}_L \gamma_\mu \tau_L). \quad (2.8)\)

In these equations the subscripts 0, 1/2, and 1 denote the isospin of the field bilinears, and
\(M, N = L, R\) label the fields’ chirality. In terms of \((g_I^{MN})^q\), the \(C_q^a\) coefficients in \((2.3)\), which are directly constrained by the experimental data, read

\[
C_p^q = -(g_{1/2}^{LR})^q - (g_{1/2}^{RL})^q, \quad C_{p'}^q = (g_{1/2}^{LR})^q - (g_{1/2}^{RL})^q,
\]

\[
C_A^q = (g_0^{LL})^q + (g_0^{RR})^q - (g_0^{LR})^q - (g_0^{RL})^q + (g_1^{HL})^q + (g_1^{LL})^q. \quad (2.9)\)

Some of the heavy states which generate the effective operators in \((2.8)\) appear in nontrivial \(SU(2)\) multiplets. Assuming that the mass splittings between different members of the same multiplet are not too large, \(SU(2)\) rotations leave the overall coefficients in \((2.8)\) invariant to a good approximation. This allows us to obtain model independent relations between contributions to different transitions of some of the operators in \((2.8)\). The operators whose contributions to \(B \to \tau^+ \tau^-\) can be constrained in this way are listed in Table I. Operators corresponding to the effective couplings in the first column are related through \(SU(2)\) rotations to operators which induce the decays \(B \to X_q \nu \bar{\nu}, B \to \tau \bar{\nu}, \) and \(B \to X_q \tau \bar{\nu},\) as given in the second column. These decay modes provide the strongest constraints on various new physics contributions to \(B \to \tau^+ \tau^-\). The bounds on \(BR(B \to X_q \nu \bar{\nu})\) \((1.3)\) and on \(BR(B \to \tau \bar{\nu})\) \((1.2)\) imply that the contributions to \(B \to \tau^+ \tau^-\) proportional to the coefficients in the first two lines of Table I are much below the present experimental sensitivity.

| Constrained operators | Decay mode |
|-----------------------|------------|
| \((g_0^{LL})^q, \quad (g_1^{LL})^q\) | \(B \to X_q \nu \bar{\nu}\) |
| \((g_{1/2}^{LR})^d, \quad \bar{g}_{0}^{LR})^d, \quad (g_1^{LL})^d\) | \(B \to \tau \bar{\nu}\) |
| \((g_{1/2}^{LR})^q, \quad \bar{g}_0^{LL})^q, \quad (g_1^{LL})^q\) | \(B \to X_q \tau \bar{\nu}\) |

**TABLE I.** Effective couplings of the operators that contribute to \(B_q \to \tau^+ \tau^-\), which are constrained by the limits on the decays listed in the second column.
The coefficients \((g_{1/2}^{LR})^s\) and \((g_0^{LL})^s\) in the third line are only weakly constrained by the data on \(\text{BR}(B \to X_c \tau \bar{\nu})\) \([\text{I.I}].\) This is because the SM contribution to this decay is large, and its possible interference with the new physics cannot be neglected. Since the sign of the interference is not known, \([\text{I.I}].\) does not yield definite limits on the model independent parameters. Finally, the contributions proportional to \((g_0^{MR})^q\) \((M = L, R)\) and to \((g_{1/2}^{RL})^q\) are presently unconstrained, as they are not related to any existing experimental limit.

\[\text{III. EXPERIMENTAL STATUS}\]

To date, no dedicated experimental search for \(B \to \tau^+ \tau^- (X)\) decays has been carried out. In this section we discuss the possibilities of searching for these decays in current experiments.

The CLEO Collaboration has established limits on \(B_d\) decays into any pair of charged leptons \([\text{I3}].\), including different final state flavors, except for the \(B_d \to \tau^+ \tau^-\) decay mode. The reason is that in all but this case the final state contains a muon or electron with a well-defined energy that can be easily searched for. The CDF Collaboration has recently established strong limits on \(\text{BR}(B_{d,s} \to \mu^+ \mu^-)\) \([\text{I4}].\). These limits follow from the absence of muon pairs with invariant mass matching \(m_{B_d}\) or \(m_{B_s}\). Because of these selection criteria, the CDF analysis does not constrain \(B \to \tau^+ \tau^- (X)\) followed by \(\tau \to \mu\).

The stringent UA1 bound \(\text{BR}(B \to X \mu^+ \mu^-) < 5 \times 10^{-5}\) \([\text{I5}].\) has been used to constrain the product of branching ratios \(\text{BR}(B \to X \tau^+ \tau^-) \times [\text{BR}(\tau \to \mu \nu \bar{\nu})]^2\), and thus to infer an indirect limit on \(\text{BR}(B \to X \tau^+ \tau^-)\) \([\text{I10}].\). However, the UA1 Collaboration searched for muons pairs with large invariant mass, \(3.9 \text{ GeV} < m_{\mu\mu} < 4.4 \text{ GeV}\). Muons from \(\tau\) decays would not have passed this cut, so the limit inferred in \([\text{I10}].\) does not hold.

We conclude that the existing data still allow for \(B \to \tau^+ \tau^- (X)\) branching ratios up to \(\mathcal{O}(10\%)\). Therefore, in searching for \(B\) decays with a \(\tau^+ \tau^-\) pair in the final state, measurements at LEP can be competitive with other searches, and may even yield the best bounds until asymmetric \(B\) factories will start operating. Unlike at \(B\) factories running on
the \( \Upsilon(4S) \), both \( B_d \) and \( B_s \) meson decays can be studied at LEP. Since the LEP \( b \) hadron sample contains about 40\% \( B_d \) and 12\% \( B_s \) mesons \(^{[17]}\), the limits on \( B_s \) decays are weaker than the limits on the corresponding \( B_d \) decays by about a factor of 3.3.

The absence of \( B \) decays associated with very large missing energy, which yielded the limit on \( \text{BR}(B \rightarrow \tau \bar{\nu}) \) \(^{[12]}\), also constrains \( \text{BR}(B \rightarrow \tau^+ \tau^-) \). However, compared to the decay \( B \rightarrow \tau \bar{\nu} \), the \( B \rightarrow \tau^+ \tau^- \) mode yields a much softer missing energy spectrum, since in this case both neutrinos come from secondary decays. In addition, to reject background from semileptonic \( b \) and \( c \) decays, events with charged leptons in the final state are rejected. This weakens the limit by an additional factor of about 65\%, corresponding to the hadronic \( \tau \) branching fraction. Still, for sufficiently large \( B \rightarrow \tau^+ \tau^- \) branching ratios some events would have been seen in the large \( E_{\text{miss}} \) region studied by ALEPH \(^{[2]}\). To obtain the bound implied by the absence of such events, we need to evaluate the probability of \( B \rightarrow \tau^+ \tau^- \) decay events to pass the \( E_{\text{miss}} > 35 \text{ GeV} \) cut, relative to that of \( B \rightarrow \tau \bar{\nu} \) decays. We estimated this probability with a Monte Carlo simulation similar to that in \(^{[7]}\), except that now we used the hadronic invariant mass spectrum in \( \tau \) decays as measured by CLEO.\(^{\dagger}\) We also made some simplifying approximations which are not always conservative, and could be avoided in a dedicated experimental analysis. For example, we neglected the effects of the correlation between the direction of the missing momenta from the \( \tau^+ \) and \( \tau^- \) decays. Nevertheless, we think that our results give a reasonable estimate of the limits that can be established by a dedicated experimental analysis of the LEP data. We obtain the following bounds:

\[
\text{BR}(B_d \rightarrow \tau^+ \tau^-) < 1.5\% ,
\]

\[
\text{BR}(B_s \rightarrow \tau^+ \tau^-) < 5.0\% . \quad (3.1)
\]

It is interesting to mention that the first of these limits is probably close to the bound that CLEO may be able to obtain using the fully reconstructed \( B \) decay sample \(^{[18]}\). As we shall

\(^{\dagger}\)We thank Alan Weinstein for providing us with this spectrum.
discuss in the next section, in spite of being over four orders of magnitude above the SM predictions, the limits (3.1) yield the first constraints on some new physics parameters.

Neutrinos from the $B \to X \tau^+ \tau^-$ decay yield a missing energy spectrum which is too soft to produce any signal in the $E_{\text{miss}} > 35$ GeV region. However, for large enough branching ratios, events from $B \to X \tau^+ \tau^-$ would enhance the signal in the missing energy region used to measure the $B \to X_c \tau \bar{\nu}$ decay. Taking into account that also for $B \to X \tau^+ \tau^-$ selecting hadronic $\tau$ decays weakens the limit by a factor of about 65%, we estimate that a bound

$$\text{BR}(B \to X \tau^+ \tau^-) < \mathcal{O}(5\%)$$

(3.2)
is within the reach of LEP sensitivity.

Before concluding this section, we mention that the radiative decay $B \to \gamma \nu \bar{\nu}$ is also associated with large missing energy. The corresponding branching ratios in the SM have been recently estimated to be of order $10^{-9}$ for $B_d$ and $10^{-8}$ for $B_s$ [19]. We can obtain limits on these decays by assuming a missing energy spectrum for the $B \to \gamma \nu \bar{\nu}$ decay, similar to that in $B \to X \nu \bar{\nu}$ in the limit of zero invariant mass for the final state hadron system. Taking into account that only neutral $B$ mesons can decay into $\gamma \nu \bar{\nu}$, while all $b$ hadrons can decay via the $b \to s \nu \bar{\nu}$ transition, we estimate the bounds

$$\text{BR}(B_d \to \gamma \nu \bar{\nu}) < 1 \times 10^{-3},$$
$$\text{BR}(B_s \to \gamma \nu \bar{\nu}) < 3 \times 10^{-3}.$$  (3.3)

Our estimates indicate that the rates for $B \to \tau^+ \tau^- (X)$ decays can be constrained by the missing energy method only at the few percent level. By refining the details of the analysis (for example, by optimizing the $E_{\text{miss}}$ cuts), dedicated experimental searches could probably establish better limits. However, regardless of possible such improvements, it is unlikely that the missing energy method could yield much stronger bounds. Therefore, it is appropriate to discuss whether similar (or better) limits can be obtained by different analyses. A back-of-an-envelope estimate shows that a limit competitive with our results could be established at LEP by looking for two charged leptons from $\tau$ decays, coming from
a secondary vertex in the hemisphere opposite to a tagged $b$. It seems to us that also the inclusive lepton spectrum in $B_d$ decays measured by CLEO [20] cannot yield more severe constraints than those found above. Finally, the decay mode $B \to \gamma \nu \bar{\nu}$ could be effectively searched for, by looking for the decay photon in the hemisphere opposite to a tagged $b$. Such a search may yield significantly better limits than our estimates (3.3). To what extent some of these analyses could improve the constraints derived above can only be decided on the basis of more detailed experimental studies.

IV. NEW PHYSICS

In this section we study the constraints on new physics implied by the limits on $B \to \tau^+ \tau^-$ ($X$) decays. By comparing (2.5) with the limits on BR($B \to \tau^+ \tau^-$) given in (3.1) we obtain the following constraint on the coefficients $C^q_{P}$, $C^q_{P}'$, and $C^q_{A}$:

$$\left| C^q_P + \frac{2}{3} C^q_A \right|^2 + \frac{5}{9} \left| C^q_{P}' \right|^2 \lesssim 2.0 \times 10^{-4} \left( \frac{190 \text{ MeV}}{f_B} \right)^2 \left( \frac{1.5 \text{ ps}}{\tau_B} \right) \left[ \frac{\text{BR}(B_q \to \tau^+ \tau^-)}{1.0 \times 10^{-2}} \right]$$

$$\simeq \begin{cases} 
3.0 \times 10^{-4}; & \text{for } q = d, \\
1.0 \times 10^{-3}; & \text{for } q = s. 
\end{cases} \quad (4.1)$$

In the next two subsections, we give two examples of models in which these bounds yield the first constraints on some parameters: models with light leptoquarks, and SUSY without R-parity. First we express in terms of the model parameters the effective couplings of the operators $(g^{MN}_I)^q$ in (2.8), which arise from integrating out the new heavy states. Then, by inserting the expressions (2.9) for the relevant $C^q_a$ coefficients into (4.1), we derive constraints on various couplings.

A. Leptoquarks

Leptoquarks (LQ) carry both baryon and lepton number, and hence couple directly leptons to quarks. A comprehensive analysis of the experimental constraints on the LQ couplings has been given in [16], and is summarized in Table 15 of this reference.
discussed above, the limit on BR($B \rightarrow X \mu^+ \mu^-$) \cite{15} does not constrain BR($B \rightarrow X \tau^+ \tau^-$). Therefore, some of the limits on LQ Yukawa couplings in \cite{16} involving the third generation do not apply.

Several different types of LQ are possible, and most of them can induce the $B \rightarrow \tau^+ \tau^-(X)$ decays. However, in many cases LQ also mediate the decays $B \rightarrow X \nu \bar{\nu}$ and $B \rightarrow \tau \bar{\nu}$, which are tightly constrained. Therefore, we will concentrate only on those cases where transitions involving neutrinos are not induced. This can happen either because of the particular electric charge of the LQ (for example $|Q| = 4/3$), or when the LQ couplings to the left-handed lepton doublets vanish. We adopt here the notations of \cite{16}. Scalar and vector LQ are denoted as $S$ and $V$, while $SU(2)$ singlets, doublets, and triplets are respectively labeled with a lower index 0, 1/2, and 1. The types of LQ relevant for $B \rightarrow \tau^+ \tau^-(X)$ decays, and for which no strong constraints exist from other processes, are

$$\tilde{S}_0, \quad S_{1/2}, \quad V_0^\mu, \quad V_{1/2}^\mu.$$ \hspace{1cm} (4.2)

The relevant scalar and vector terms in the interaction Lagrangian can be found in \cite{16}. Schematically, they read

$$\mathcal{L}_{LQ} = \lambda_{ij}^{LQ} \ell_i q_j \phi_{LQ},$$ \hspace{1cm} (4.3)

where $\ell_i$ and $q_j$ denote respectively a lepton and quark fields, $\phi_{LQ}$ represents one of the LQ in (4.2), and $i$ and $j$ are generation indices. In deriving our constraints, we assume that all the $\lambda_{ij}^{LQ}$ couplings are real, that only one type of LQ is present at a time (for other possibilities see \cite{21}), and we neglect the rotation from the interaction to the mass basis \cite{22}. Integrating out the LQ fields and Fiertz transforming, we obtain the expressions for the unconstrained coefficients of the relevant effective four-fermion operators given in (2.8)

$$\begin{align*}
(g_{0}^{RR})^q &= \frac{(\lambda_{S}^{R})_{3q} (\lambda_{S}^{R})_{33}}{8 G_F \, m_{S_0}^2}, \quad \frac{(\lambda_{V}^{R})_{3q} (\lambda_{V}^{R})_{33}}{4 G_F \, m_{V_0}^2}; \\
(g_{0}^{LR})^q &= \frac{(\lambda_{S}^{SL})_{3q} (\lambda_{S}^{SL})_{33}}{8 G_F \, m_{S_{1/2}}^2}, \quad \frac{(\lambda_{V}^{L})_{3q} (\lambda_{V}^{L})_{33}}{4 G_F \, m_{V_{1/2}}^2};
\end{align*}$$

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\[ (g_0^{LL}q) = \frac{(\lambda_L^{V_0})_{3q} (\lambda_L^{V_0})_{33}}{4G_F m_{V_0}^2}; \]
\[ (g_{1/2}^{LR})^q = \frac{(\lambda_L^{V_0})_{3q} (\lambda_L^{V_0})_{33}}{2G_F m_{V_0}^2}; \]
\[ (g_{1/2}^{RL})^q = \frac{(\lambda_R^{V_0})_{3q} (\lambda_L^{V_0})_{33}}{2G_F m_{V_0}^2}. \]

(4.4)

For the different products of LQ couplings \((\lambda_{LM}^{LQ})_{32}, (\lambda_{LN}^{LQ})_{33}\) involving the second and third generation fermions, from \((2.9)\) and \((4.1)\) we obtain the following limits:

\[ \lambda_R^{S_0} \lambda_R^{S_0}, \lambda_R^{S_{1/2}} \lambda_R^{S_{1/2}} < 4.4 \times 10^{-2} \left( \frac{m_{LQ}}{100 \text{ GeV}} \right)^2; \]
\[ \lambda_R^{V_0} \lambda_R^{V_0}, \lambda_L^{V_0} \lambda_L^{V_0}, \lambda_R^{V_{1/2}} \lambda_R^{V_{1/2}} < 2.2 \times 10^{-2} \left( \frac{m_{LQ}}{100 \text{ GeV}} \right)^2; \]
\[ \lambda_L^{V_0} \lambda_R^{V_0}, \lambda_R^{V_0} \lambda_L^{V_0} < 5.9 \times 10^{-3} \left( \frac{m_{LQ}}{100 \text{ GeV}} \right)^2, \]

(4.5)

where the indices \((32)\) and \((33)\) respectively for the first and second coupling in the products are understood. The bounds on the analogous products \((\lambda_{LM}^{LQ})_{31}, (\lambda_{LN}^{LQ})_{33}\) involving a first generation \((q = d)\) quark are about a factor of 1.8 stronger. From Table I we see that for \(q = d\) the products of the LQ couplings in \((4.4)\) corresponding to \((g_0^{LL})^q\) and \((g_{1/2}^{LR})^q\) are already tightly constrained by the limit on \(B \to \tau \bar{\nu}\) decays, while for \(q = s\) they are weakly constrained by the upper bound on \(B \to X_c \tau \bar{\nu}\). For all the other combinations, the bounds obtained from \(B \to \tau^+ \tau^- (X)\) are the strongest.

**B. SUSY without R-parity**

In SUSY models, it is usually assumed that R-parity is a good symmetry. However, this is not necessarily the case, and phenomenologically viable models have been constructed where R-parity is not imposed as an exact symmetry \([23]\). In the absence of R-parity, additional baryon and lepton number violating terms are allowed in the superpotential. Some of these terms can induce a large enhancement for certain rare \(B\) decay modes. Denoting by \(L^i_L, Q^i_L, \ell^i_R, \) and \(d^i_R\) respectively the chiral superfields of the \(i\)’th generation containing the left-handed lepton and quark doublets, the right-handed lepton, and the down-type quark singlet, the R-parity violating terms relevant for \(B\) decays read
\[ W_R = \lambda_{ijk} L^i_L L^j_L L^k_R + \lambda'_{ijk} L^i_L Q^j_L d^k_R. \]  

(4.6)

The terms in (4.6) give rise to two types of diagrams that can mediate \( B \to \tau^+ \tau^- (X) \). Exchanging a left- or right-handed squark gives rise to effective operators proportional to \( \lambda' \lambda' \). Since squark exchange induces also the decay \( B \to X \nu \bar{\nu} \), these operators are already tightly constrained, and hence irrelevant for the present discussion. The exchange of left-handed sleptons generates operators proportional to \( \lambda' \lambda \). These operators do not induce \( B \to X \nu \bar{\nu} \), but they can still contribute to \( B \to \tau \bar{\nu} \) and to \( B \to X_c \tau \bar{\nu} \). The corresponding effective couplings are:

\[
(g_{1/2}^{LR})^q = \frac{\lambda'_{k3q} \lambda_{k33}}{4G_F m_{\tilde{L}_k}^2}, \quad (g_{1/2}^{RL})^q = \frac{\lambda'_{k3q} \lambda_{k33}}{4G_F m_{\tilde{L}_k}^2},
\]

(4.7)

where \( k = 1, 2 \) due to the antisymmetry in the first two indices of the \( \lambda \) couplings. Neglecting possible cancelations between the contributions from \( \tilde{L}_1 \) and \( \tilde{L}_2 \) exchange, and assuming that one of the two couplings \( \lambda'_{k3q} \) and \( \lambda'_{q3} \) is dominant, from (2.9) and (4.1) we obtain

\[
\lambda'_{23k} \lambda_{k33}, \lambda'_{32k} \lambda_{k33} < 1.2 \times 10^{-2} \left( \frac{m_{\tilde{L}_k}}{100 \text{ GeV}} \right)^2.
\]

(4.8)

The bounds on the analogous combinations involving the couplings of the \( d \) quark are about a factor of 1.8 stronger. From Table I we see that more stringent bounds already exist on the combination involving \( \lambda'_{k13} \), while for \( \lambda'_{k23} \) additional weak constraints can be derived from \( B \to X_c \tau \bar{\nu} \). However, for the products involving the \( \lambda'_{k3q} \) couplings, these are the strongest limits.

**V. SUMMARY AND CONCLUSIONS**

To date no experiment has searched for \( B \to \tau^+ \tau^- (X) \) decays, thus no experimental bounds exist on the corresponding branching ratios. \( B \) decays into final states involving \( \tau \) leptons can be searched for by looking for the missing energy associated with the neutrinos from \( \tau \) decay. Such searches have been carried out at LEP to measure the \( B \to X_c \tau \bar{\nu} \) decay rate [2, 4] and to set bounds on the \( B \to \tau \bar{\nu} \) and \( B \to X_s \nu \bar{\nu} \) decays [2, 8]. In this paper
we pointed out that similar analyses can also set bounds on $B \rightarrow \tau^+ \tau^- (X)$ decays. We estimated that limits at the few percent level [see eqs. (3.1) and (3.2)] are within the reach of the LEP sensitivity.

The SM predictions for the $B \rightarrow \tau^+ \tau^- (X)$ branching ratios are of order $10^{-6}$ or below. Therefore, the current experimental sensitivity only allows us to derive constraints on physics beyond the SM. To identify what kind of new physics could yield large enhancements, we performed a model independent analysis of these decays. While the decay $B \rightarrow X \tau^+ \tau^-$ can depend on all the ten possible Dirac structures (2.1), $B_{d,s} \rightarrow \tau^+ \tau^-$ can be induced only by the three operators in (2.2). Thus, for studying new physics, these decay modes are complementary to one another. Much could be learned from the individual rates, and also from their ratio.

Operators which can induce the $B \rightarrow \tau^+ \tau^- (X)$ decays can be related to operators contributing to other processes through $SU(2)$ rotations. In particular, when the dominant operators involve left-handed lepton doublets, $B \rightarrow \tau^+ \tau^- (X)$ can be related to $B \rightarrow X \nu \bar{\nu}$ and to $B \rightarrow \tau \bar{\nu}$. In these cases the limits on these processes [8,4] provide strong constraints. Therefore, only operators involving right-handed $\tau$’s can induce $B \rightarrow \tau^+ \tau^- (X)$ at the level of current experimental sensitivity. This restricts the types of new physics models that can be constrained through these decays. We gave two examples of such models: light lepto-quarks, and SUSY without R-parity. In both cases, the rather weak limits that we have estimated already yield the strongest constraints on some of the model parameters.

In the future, $B$ factories will provide significantly larger samples of $B$ decays. Hopefully, the $B \rightarrow \tau^+ \tau^- (X)$ decays will be observed, even if their rates are as small as predicted by the SM. Measurements of various $B$ decay rates into final states involving $\tau$’s (or $\nu_\tau$’s) as well as of other observables (like the $\tau$ polarization [24]) would provide very valuable information. Even if the experimental difficulty of these measurements will only allow establishing tighter limits on the decay rates, this will still yield strong constraints on possible physics beyond the SM, and might provide us with a better understanding of the third generation.
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