REVISITING STUDIES OF THE STATISTICAL PROPERTY OF A STRONG GRAVITATIONAL LENS SYSTEM AND MODEL-INDEPENDENT CONSTRAINT ON THE CURVATURE OF THE UNIVERSE

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ABSTRACT

In this paper, we use a recently compiled data set, which comprises 118 galactic-scale strong gravitational lensing (SGL) systems to constrain the statistical property of the SGL system as well as the curvature of the universe without assuming any fiducial cosmological model. Based on the singular isothermal ellipsoid (SIE) model of the SGL system, we obtain that the constrained curvature parameter $\Omega_k$ is close to zero from the SGL data, which is consistent with the latest result of Planck measurement. More interestingly, we find that the parameter $f$ in the SIE model is strongly correlated with the curvature $\Omega_k$. Neglecting this correlation in the analysis will significantly overestimate the constraining power of SGL data on the curvature. Furthermore, the obtained constraint on $f$ is different from previous results: $f = 1.105 \pm 0.030$ (68% confidence level [C.L.]), which means that the standard singular isothermal sphere (SIS) model ($f = 1$) is disfavored by the current SGL data at more than a 3$\sigma$ C.L. We also divide all of the SGL data into two parts according to the centric stellar velocity dispersion $\sigma$ and find that the larger the value of $\sigma$, the more favored the standard SIS model is. Finally, we extend the SIE model by assuming the power-law density profiles for the total mass density, $\rho = \rho_0 (r/r_0)^{-\nu}$, and luminosity density, $\nu = \nu_0 (r/r_0)^{\delta}$, and obtain the constraints on the power-law indices: $\alpha = 1.95 \pm 0.04$ and $\delta = 2.40 \pm 0.13$ at a 68% C.L. When assuming the power-law index $\alpha = \delta = \gamma$, this scenario is totally disfavored by the current SGL data, $\chi^2_{\min,\gamma} - \chi^2_{\min,SIE} \approx 53$.

Key words: cosmological parameters – cosmology: theory – gravitational lensing: strong

1. INTRODUCTION

The curvature of the universe, which is often parameterized by $\Omega_k$, is a fundamental parameter in cosmology. It determines whether our universe is open ($\Omega_k > 0$), flat ($\Omega_k = 0$), or closed ($\Omega_k < 0$). Currently, most cosmological observations favor that $\Omega_k$ is very close to zero, such as the latest constraint from the Planck measurement, $|\Omega_k| < 0.005$ (Planck Collaboration et al. 2015). However, these constraints on $\Omega_k$ are obtained by using a model-dependent method. Therefore, constraints on curvature using the model-independent method are still very attractive in the literature (Bernstein 2006; Clarkson et al. 2007; Oguri et al. 2012; Li et al. 2014; Räsänen et al. 2015; Cai et al. 2016; Yu & Wang 2016). For instance, in Bernstein (2006), based on the basic distance sum rule, they used weak lensing to test the curvature in a model-independent way. Recently, this method was also used in Räsänen et al. (2015) with the strong gravitational lensing (SGL) systems to test the curvature of the universe and the Friedmann–Lemaître–Robertson–Walker (FLRW) metric. Räsänen et al. (2015) collected 38 SGL systems and used them to constrain $\Omega_k$, finding that the SGL sample gives a consistent constraint with the flat universe, although the statistical error is very large, $-1.22 < \Omega_k < 0.63$ at a 95% confidence level (C.L.).

As an important prediction of general relativity (GR), SGL has become a powerful tool to test cosmology, astrophysics (the structure, formation, and evolution of galaxies and galaxy clusters), and the gravity theories. The first discovery of the SGL system Q0957+561 (Walsh et al. 1979) hints at the possibility of using the galactic lensing systems in the study of cosmological parameters and the galaxy structure. In a specific strong-lensing system, the background source (high-redshift quasar, supernova, or galaxy) will reveal itself as multiple images, due to the gravitational field of the intervening lens (usually an elliptical galaxy) between the observer and the source. Meanwhile, as the image separation in the system depends on angular diameter distances to the lens and to the source, the observation of SGL can provide the information of $d_{ls}/d_s$, where $d_{ls}$ is the angular diameter distance between the source and lens and $d_s$ is that between the source and observer. Following this direction, many recent works have provided successful applications of different SGL samples in the investigation of the structure and evolution of galaxies (Zhu & Wu 1997; Treu et al. 2006; Cao et al. 2016), the post-Newtonian parameter describing the deviations from GR (Schwab et al. 2010), the dynamical properties of dark energy (Zhu 2000; Chae et al. 2004; Cao et al. 2015; Li et al. 2016), and the curvature of our universe (Bernstein 2006; Räsänen et al. 2015).

Furthermore, precise spectroscopic and astrometric observations, obtained with well-defined selection criteria, may help us to study the statistical properties of lensing galaxies. Compared with late-type and unknown-type galaxies, early-type galaxies (or elliptical galaxies), which contain most of the stellar mass in the universe, provide most of the lensing galaxies in the available galactic SGL systems. Therefore, the singular isothermal ellipsoid (SIE) model, which has an elliptical projected mass distribution (Ratnatunga et al. 1999), and the singular isothermal sphere (SIS) model, which has a spherical symmetrical mass distribution, are two useful assumptions and good first-order approximations in statistical gravitational lensing studies. For an SIE lens, the Einstein radius of an
SGL system can be calculated by the theoretical expression
\[ \theta_E = 4\pi f^2 \sigma_c^2 d_h / c^2, \]
where \( \theta_E \) is the Einstein radius, \( \sigma_c \) is the central velocity dispersion of the lensing galaxy, \( c \) is the speed of light, and \( f \) is a phenomenological coefficient that includes several systematic errors caused by the difference between the observed stellar velocity dispersion and that of the SIS model, the assumption of the SIS model in calculating the theoretical Einstein radius, and the softened isothermal sphere potentials (Ofek et al. 2003; Cao et al. 2012). For the standard SIS model, the coefficient parameter reduces to \( f = 1 \). In order to take the uncertainty of \( f \) into account, a prevalent procedure in the literature is to directly include a prior on \( f \) with a 20% uncertainty in the analysis. This method was first introduced by Kochanek et al. (2000) and Ofek et al. (2003) and extensively applied in the recent works by Räsänen et al. (2015) and Holanda et al. (2016). However, the statistical quality of parameter \( f \) is still not well understood.

Recently, based on a gravitational lens data set including 70 galactic systems from the Sloan Lens ACS Survey (SLACS) and the Lens Structure and Dynamics (LSD) survey, Cao et al. (2012) treated \( f \) as a free parameter to fit the matter energy density \( \Omega_m \) and the equation of state of dark energy \( w \). In the framework of a flat universe, their results showed that on the statistical level the 68% C.L. constraint is \( f^2 = 1.01 \pm 0.02 \). However, in a more recent work by Räsänen et al. (2015), the authors found that the parameter \( f \) might be correlated with the cosmic curvature \( \Omega_k \), although they did not fully take the uncertainty of \( f \) into account properly. Given the availability of a new sample of 118 lenses (Cao et al. 2015) observed by SLACS, the BOSS emission-line lens survey (BELLS), the LSD survey, and the Strong Lensing Legacy Survey (SL2S), the purpose of this work is to reconsider the studies on the curvature from this latest SGL system and fully consider the effect of the uncertainty of the parameter \( f \) in the analysis. The structure of this paper is organized as follows. In Section 2 we introduce the method and the SGL data used in this work. Then we show our numerical results in Section 3. Finally, some discussion and a summary are given in Section 4.

2. METHOD AND DATA

2.1. Distance Sum Rule Method

The basic distance sum rule is a simple geometric rule. Imagining that there are three points A, B, and C on a straight line and B lies between A and C, the distances among them are \( d_{AB}, d_{AC}, \) and \( d_{BC} \) respectively. Then, obviously, there is a relation that \( d_{AC} = d_{AB} + d_{BC} \). However, if this line is not straight, the relation will become invalid. For example, assuming that the line is a part of arc, then we have \( d_{AC} < d_{AB} + d_{BC} \). This rule is the same in the universe. Considering an SGL system in the universe, the distances between the observer and the lens galaxy and the sources are \( d_l \) and \( d_s \) respectively, while the distance between the lens galaxy and the source is \( d_h \). Therefore, we have the equation \( d_s = d_l + d_h \) if our universe is flat (\( \Omega_k = 0 \)). Otherwise, there is \( d_s > d_l + d_h \) or \( d_s < d_l + d_h \) for \( \Omega_k > 0 \) or \( \Omega_k < 0 \), respectively (see Figure 1 of Bernstein 2006 for an illustration).

In the FLRW metric, the dimensionless comoving angular diameter distance between the lens galaxy, \( z_l \), and the source, \( z_s \), in a certain direction can be expressed as
\[ d(z_l, z_s) = (1 + z_s)H_0D_A(z_l, z_s), \]
where
\[ d(z_l, z_s) = \frac{1}{\sqrt{|\Omega_k|}} \sinh \left[ \sqrt{|\Omega_k|} \int_{z_l}^{z_s} \frac{dz'}{E(z')} \right]. \]

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the SL2S data for a total of 31 lenses were taken from Sonnenfeld et al. (2013a, 2013b).

The Einstein radius $\theta_E$ is defined to be the radius at which the mean surface mass density $\Sigma$ equals the critical density $\Sigma_c$ of the lensing configuration. However, for the lens galaxies one needs to assume a specific lens model to obtain the measurements of Einstein radii from observed strong-lens systems in various lensing surveys. For all of the lenses from LSD, SLACS, BELLs, and SL2S used in this paper, the resulting Einstein radii were obtained on the basis of an SIE lens-mass model. Compared with the SIS counterpart, SIE includes a two-dimensional (2D) potential of similar concentric and aligned elliptical isodensity contours, with minor-to-major axis ratio $q_{\text{SIE}}$ (Kassiola & Kovner 1993; Kormann et al. 1994; Keeton & Kochanek 1998). We remark here that, although the resulting Einstein radii are slightly model dependent and SIE might not be accurate enough for some cosmological applications (Saha 2000; Rusin et al. 2002), the assumption of an SIE model does not significantly bias the determination of the value of Einstein radii used in our analysis (see Sonnenfeld et al. 2013a, 2013b, for details).

In these SGL data, they provided 118 SGL systems with detailed information about the redshift of lens and source galaxies $z_l$ and $z_s$, the Einstein radius $\theta_E$, and the central stellar velocity dispersion $\sigma_*$ of the lens galaxies. These 118 SGL systems spread in the redshift range of $0.075 \leq z \leq 1.004$, and the source galaxies spread in the redshift range of $0.196 \leq z \leq 3.595$. In practice, we subtract those SGL systems with $z_s > 1.4$, because the maximal redshift of the supernova data set we are using to determine the cosmological distance is about 1.4 (see details in the next subsection). Finally, there are 83 SGL systems (20 samples from BELLs, 57 samples from SLACS, and 6 samples from SL2S) left in our sample.

### 2.2. Determination of the Distances

In order to constrain the curvature of the universe using Equation (3), we still need to have the distance information of $d_l$ and $d_s$, besides the distance ratio $d_0/d_s$. To avoid the model dependence, here we do not use the standard $\Lambda$CDM model to calculate the dimensionless comoving angular diameter distance information. Instead, we use the current observation of the Type Ia supernova (SN Ia) to determine the distance of the lens galaxy $d_l$ and that of the source galaxy $d_s$. Suzuki et al. (2012) provided the SN Ia Union 2.1 compilation of 580 data samples from the Hubble Space Telescope Supernova Cosmology Project. The data are usually presented as tabulated distance moduli with errors. In this catalog, the redshift spans $0 < z < 1.414$, and about 95% of the samples are in the low-redshift region $z < 1$. The authors also provided the covariance matrix of data with and without systematic errors. In order to be conservative, we include systematic errors in our calculations.

Each SN sample in the Union 2.1 compilation gives the redshift $z$ and the luminosity distance $d_L$. Using the relation between comoving angular diameter distance and luminosity distance, we can obtain that

$$d(z) = \frac{H_0 d_L(z)}{c(1 + z)}.$$  

(5)

It should be pointed out that the parameters of the SN sample in the Union 2.1 compilation are fitted with the parameters of the cosmological model simultaneously. Therefore, the distance information of SN data is dependent on the input cosmological model. However, this effect caused by the assumed cosmological model can be omitted compared with the current larger uncertainties in modeling the SGL systems (Räsänen et al. 2015). Therefore, we simply use the distances and the uncertainties of the Union 2.1 compilation reported in Suzuki et al. (2012).

There are several methods for using the SN data to calibrate the distance, such as the Padé approximation of order $(3, 2)$ (Liu & Wei 2015; Lin et al. 2016) and the linear or cubic interpolation method (Liang et al. 2008; Wang et al. 2016). In this work, we use a simple third-order polynomial function with constraint conditions that $d(0) = 0$ and $d'(0) = 1$ to fit the distance information of SNe. It can be expressed as

$$d(z) = a_1z + a_2z^2 + a_3z^3,$$

(6)

where $a_i$ are two free parameters that can be constrained from the SN Union 2.1 compilation. Consequently, we can get the distance information of $d_l$ and $d_s$ from the SN observational data. Going from the third-order case to the fourth-order case does not improve the goodness of fit, which indicates that the third-order polynomial function is sufficient.

### 3. RESULT

In our analysis, we perform a global fitting using the COSMOMC package (Lewis & Bridle 2002), a Markov chain Monte Carlo code. In the program, we make several modifications that allow us to use the SGL data to constrain the curvature of the universe. In the calculations, we have some free parameters that should be constrained from the SGL and SN data sets simultaneously:

$$P = (a_1, a_2, \Omega_k, \text{SGL}_i),$$

(7)

where $\text{SGL}_i$ denotes the free parameter of the specific model of the SGL system. Furthermore, in the FLRW framework, if the curvature $\Omega_k < 0$, the space will be a hypersphere and the comoving angular diameter distance will have an upper limit $d(z) \leq 1/\sqrt{-\Omega_k}$. The current observation of cosmic microwave background radiation gives an angular diameter distance at $z = 1090$ of about $D_k(1090) = 12.8 \pm 0.07$ Mpc (Vollanthen et al. 2010; Audren et al. 2013; Audren 2014). In the meantime we also have the direct probe on the current Hubble constant $H_0$ obtained from the reanalysis of Riess et al. (2011) Cepheid data made by Efstathiou (2014) by using a revised geometric maser distance to NGC 4258 from Humphreys et al. (2013): $H_0 = 72.5 \pm 2.5$ km s$^{-1}$ Mpc$^{-1}$. Therefore, we add a prior about the lower limit of the curvature:

$$\Omega_k \geq -\frac{c}{D_k(1090)\left(1 + 1090\right)H_0} \simeq -0.1.$$  

(8)

### 3.1. SIE Model

We start from the SIE model of the SGL system. In this model, we only need one free parameter $f$ to describe the SGL system using Equation (4).

First, we try to reproduce the numerical results of model Ia in Räsänen et al. (2015), which were obtained from a small sample of the SGL system. Following their steps, we take $f = 1$.
and assign an error of 2% on $\theta_E$ and a minimum error of 5% on $\sigma_\epsilon$. In Figure 1, the black solid line in the one-dimensional (1D) distribution plot of $\Omega_\kappa$ is the result we obtain. The 95% C.L. upper limit of the curvature is $\Omega_\kappa < 0.84$, which is similar to the result in the previous work (Räsänen et al. 2015).

Then, we take the uncertainty of $f$ into account in the analysis. Different from the method in Räsänen et al. (2015), in which they assigned an extra Gaussian error of 20% on $d_0/d_s$ from $f^2$, we treat the parameter $f$ as a free parameter and obtain the constraints on both $f$ and the curvature $\Omega_\kappa$ from the SGL data simultaneously. The obtained 1D and 2D constraints on $\Omega_\kappa$ and $f$ are plotted in Figure 1. We find that the constraint on the curvature is quite different from the result presented in Räsänen et al. (2015). The 95% upper limit of the curvature is

$$\Omega_\kappa < 7.02,$$

which is much weaker than that obtained in the case with $f = 1$. The reason is that the curvature is strongly correlated with the parameter $f$ of the SGL system, as shown in the bottom left panel of Figure 1. And the SGL data do not favor the standard SIS model at more than 2σ C.L.:

$$f = 1.079 \pm 0.034 \ (68\% \ C.L.).$$  \hfill (10)

If we forcibly set parameter $f = 1$, the obtained constraint on $\Omega_\kappa$ will be obviously biased. The constraining power of the SGL data is significantly overestimated. Furthermore, we also notice that our result is much weaker than that obtained for model Ib in Räsänen et al. (2015), which included an extra Gaussian error of 20% on $d_0/d_s$ from $f^2$. The reason is that including an extra Gaussian error from $f^2$ does not fully take the effect of $f$ into account. The strong correlation we find here is totally neglected in the analysis. Since this strong degeneracy between $f$ and $\Omega_\kappa$ is broken in their analysis, the constraint on $\Omega_\kappa$ becomes significantly tight. Furthermore, the current SGL data can already give a very good constraint on $f$ (see Equation (10)); the uncertainty of $f$ is much smaller than 20%.

Now we use the latest observation with 83 SGL samples to perform the global analysis. In Figure 2 we show the 1D and 2D constraints on $\Omega_\kappa$ and $f$ from the SGL data (blue dashed lines). Apparently, these new SGL data provide much stronger constraining power on $\Omega_\kappa$ and $f$ than the above small SGL sample used in Räsänen et al. (2015), since the strong degeneracy between $\Omega_\kappa$ and $f$ is partly broken. Consequently, the constraints on $\Omega_\kappa$ and $f$ are much tighter than above:

$$\Omega_\kappa < 0.16 \ (95\% \ C.L.),$$

$$f = 1.083 \pm 0.011 \ (68\% \ C.L.).$$  \hfill (11)

We also check the minimal $\chi^2$ value of the best-fit model and find that $\chi^2_{\text{min}} = \chi^2_{\text{SN}} + \chi^2_{\text{SGL}} = 545 + 243 = 788$. The $\chi^2$ of SN data looks normal, $\chi^2_{\text{SN}/\text{dof}} = 545/580 = 0.94$, while the SGL data give a too large $\chi^2$ value, $\chi^2_{\text{SGL}/\text{dof}} = 243/83 = 2.93$. This might imply that there are some unknown uncertainties on the SGL samples.

In order to take this unknown effect into account in the analysis, we use D’Agostini’s likelihood (D’Agostini 2005):

$$L_D(\Omega_\kappa, f, \sigma_{\text{int}}) \propto \prod_i \frac{1}{\sqrt{\sigma_{\text{int}}^2 + [\Delta(\sigma_{\text{int}})]^2}} \times \exp\left(-\frac{\left[\sigma_{\text{obs}}(\text{th}) - \sigma_{\text{obs}}(\text{obs})\right]^2}{2(\sigma_{\text{int}}^2 + [\Delta(\sigma_{\text{int}})]^2)}\right).$$  \hfill (12)

where $\sigma_{\text{int}}$ is the intrinsic scatter, which represents any other unknown uncertainties except for the observational statistical ones, $\sigma_{\text{th}}$ and $\sigma_{\text{obs}}$ are the theoretical prediction and observation of the central stellar velocity dispersion, respectively, and $\Delta(\sigma_{\text{int}})$ is the observed statistical error bar of SGL samples. By maximizing D’Agostini’s likelihood, or, equivalently, by minimizing the $\chi^2$, we could obtain the best-fit intrinsic scatter $\sigma_{\text{int}}$ and its uncertainty. Then, we put a top-hat prior on $\sigma_{\text{int}}$ and use the equation

$$\chi^2 = \sum_{i=1}^{83} \frac{[\sigma_{\text{obs}}(\text{th}) - \sigma_{\text{obs}}(\text{obs})]^2}{(\sigma_{\text{int}}^2 + [\Delta(\sigma_{\text{int}})]^2)},$$  \hfill (13)

to perform the whole calculations.

In Figure 2 we also show the 1D and 2D constraints on $\Omega_\kappa$ and $f$ from the SGL data with the intrinsic scatter $\sigma_{\text{int}}$ included (red solid lines). Apparently, because of the presence of the intrinsic scatter, the constraints on $\Omega_\kappa$ and $f$ become weaker, namely,

$$\Omega_\kappa < 0.60 \ (95\% \ C.L.),$$

$$f = 1.105 \pm 0.030 \ (68\% \ C.L.),$$

$$\sigma_{\text{int}} = 31.8 \pm 4.2 \ (68\% \ C.L.).$$  \hfill (14)

When comparing with the CMB and baryon acoustic oscillation measurements, the constraint on the curvature from the SGL data is very weak. However, this is a model-independent constraint based only on geometrical optics; thus, it could be the useful complement to model-specific analyses. On the other hand, similar to the above analysis, the standard SIS model with $f = 1$ is strongly disfavored by the SGL data at more than 3σ C.L. More importantly, the minimal $\chi^2_{\text{SGL}}$ now is about 85, and we consequently obtain the reduced value $\chi^2_{\text{SGL}/\text{dof}} = 1.03$. Therefore, in the following calculations, we always include the intrinsic scatter $\sigma_{\text{int}}$ to represent any other unknown uncertainties of the SGL data.

In order to understand the interesting constraint on the parameter $f$ better, we first use the SGL systems in each survey...
to constrain $f$ separately. As we said before, in this SGL subsample, there are 20 samples from BELLS, 57 samples from SLACS, and 6 samples from SL2S. Therefore, we use 20 samples from BELLS and 57 samples from SLACS to constrain $f$ and obtain the 68% C.L. constraints, respectively:

$$f_{\text{BELLS}} = 2.06 \pm 0.37,$$

$$f_{\text{SLACS}} = 1.07 \pm 0.03,$$  \hspace{1cm} (15)

which imply that the standard SIS model ($f=1$) is still ruled out at more than $2\sigma$ C.L. in each survey. Interestingly, we find that the median value of $f$ from BELLS is far away from unity. However, due to the lack of an SGL sample, the error bar of $f$ from the BELLS data is very large. Meanwhile, the constraint on $f$ from SLACS is more similar to that from all data (Equation (14)). The constraining power on $f$ mainly comes from the samples of SLACS.

Then, we divide these SGL data into two parts according to the centric stellar velocity dispersion $\sigma_c$ of the SGL system: $\sigma_c < 240$ km s$^{-1}$ ($n = 41$ lenses) and $\sigma_c > 240$ km s$^{-1}$ ($n = 42$ lenses). In Figure 3 we show the 1D and 2D marginalized distributions with $1\sigma$ and $2\sigma$ contours for the parameters $\Omega_k$ and $f$ constrained from the two subsamples, which imply that the standard SIS model is still ruled out at more than $1\sigma$ C.L. and $2\sigma$ C.L., respectively. For comparison, we also show the constraints from all 83 SGL samples (black solid lines).

The standard SIS model is consistent with this large velocity dispersion subsample within 95% C.L. The larger value of velocity dispersion $\sigma_c$ the subsample has, the more favored the standard SIS model with $f = 1$ is.

Finally, we also separate these SGL data into two parts according to the redshift of the lens galaxy: $z_l < 0.24$ ($n = 39$ lenses) and $z_l > 0.24$ ($n = 44$ lenses). Different from the above analysis, here we find that the constraints on $f$ are consistent with each other at about $1\sigma$ C.L., namely,

$$f_{z_l < 0.24} = 1.10 \pm 0.04 \ (68\% \ C.L.),$$

$$f_{z_l > 0.24} = 1.17 \pm 0.06 \ (68\% \ C.L.).$$  \hspace{1cm} (18)

The standard SIS model ($f=1$) is still ruled out at more than $2\sigma$ C.L. in each subsample.

### 3.2. Extended SIE Model

Besides the standard SIE model with one free parameter $f$, in our analysis we also consider the more complex SGL model. As we know, the measurement of central velocity dispersion $\sigma_c$ can provide a model-dependent dynamical estimate of the mass, based on the assumption of the power-law mass density profile $\rho(r)$ and luminosity density of stars $\nu(r)$:

$$\rho(r) = \rho_0 \left(\frac{r}{r_0}\right)^{-\alpha},$$

$$\nu(r) = \nu_0 \left(\frac{r}{r_0}\right)^{-\delta},$$  \hspace{1cm} (19, 20)

where $r$ is the spherical radial coordinate from the lens center. Therefore, besides the parameters $a_1, a_2, \Omega_k$, and $\sigma_{\text{int}}$, we have two more free parameters, $\alpha$ and $\delta$. Following Cao et al. (2016), we can obtain the expression of the observed velocity dispersion:

$$\sigma_c^2 = \left(\frac{c^2}{4} \frac{d\ln \theta_E}{d\ln \xi}\right)^2 \frac{2}{\sqrt{\pi} (\xi - 2\beta)} \left(\frac{\theta_{\text{ap}}}{\theta_E}\right)^{2-\alpha} \times \left[\frac{\lambda(\xi) - \beta \lambda(\xi + 2)}{\lambda(\alpha) \lambda(\delta)}\right] \frac{\Gamma(3 - \xi/2)}{\Gamma(3 - \delta/2)},$$

where $\beta$ is an anisotropy parameter to characterize the anisotropic distribution of three-dimensional velocity dispersion and has a Gaussian distribution $\beta = 0.18 \pm 0.13$, based on the well-studied sample of nearby elliptical galaxies from Gerhard et al. (2001); $\theta_{\text{ap}}$ is the spectrometer aperture radius; $\xi$ has the notation $\xi = \alpha - \delta - 2$; and $\lambda(\alpha) = \Gamma\left(\frac{\alpha - 1}{2}\right) / \Gamma\left(\frac{\alpha}{2}\right)$ denotes the ratio of Euler’s gamma functions. Finally, we have the $\chi^2$ equation in the calculations

$$\chi^2 = \sum_{i=1}^{83} \left(\frac{\sigma_{c,i} - \sigma_{c,i}(\text{obs})}{\Delta \sigma_{c,i}}\right)^2.$$  \hspace{1cm} (21)

In Figure 4 we show the 1D and 2D marginalized distributions with $1\sigma$ and $2\sigma$ contours for the parameters $\Omega_k$ and the power-law indices $\alpha$ and $\delta$ constrained from the SGL data. The constraint on the curvature is identical to that obtained in the standard SIE model: $\Omega_k < 0.60$ at 95% C.L. The power-law indices $\alpha$ and $\delta$ can also be well constrained.
namely, the 68% C.L. limits are
\[ \alpha = 1.97 \pm 0.04, \]
\[ \delta = 2.40 \pm 0.13, \]
in which the constraint of \( \alpha \) is consistent with the SIS model at 95% C.L., while the \( \delta \) constraint is ruled out for the SIS model at more than 5\( \sigma \) C.L. Note that the slope \( \delta \) can be directly measured from the observations, and the average mean value is \( \delta = 2.39 \) with 1\( \sigma \) error bar 0.05 (Schwab et al. 2010), which is consistent with the result we obtain here. In the calculation, the minimal \( \chi^2 \) we obtain is about 84 and \( \chi^2_{\text{SGL}}/\text{dof} = 1.03 \), which is quite similar to those in the SIE model with the free parameter \( f \).

We remark here that the above fitting results were obtained without considering the seeing effect. Considering the effects of aperture (\( \theta_{ap} \)) with atmospheric blurring (\( \sigma_{\text{atm}} \)) and luminosity-weighted averaging (see Schwab et al. 2010; Cao et al. 2016), for details), the constraints become \( \alpha = 2.05 \pm 0.05 \) and \( \delta = 2.61 \pm 0.15 \) at 68% C.L., as shown in the top right panel of Figure 4, which is consistent with previous works (Koopmans et al. 2009; Sonnenfeld et al. 2013b; Oguri et al. 2014; Cao et al. 2016).

Furthermore, we can also derive the parameter \( f' \) by using
\[ f' = \left\{ \frac{2}{\sqrt{\pi} (\xi - 2\beta)} \left[ \frac{\lambda(\xi) - \beta \lambda(\xi + 2)}{\lambda(\alpha) \lambda(\delta)} \right] \right\}^{-1/2}, \]
and obtain the constraint from the SGL data:
\[ f' = 1.108 \pm 0.030 \] (68% C.L.).

Note that this term \( f' \) is different from the parameter \( f \) in the standard SIE model by a term \( (\theta_{ap}/\theta_T)^{2-\alpha} \). However, this neglected term is very close to unity when \( \alpha \approx 2 \). Therefore, the posterior distribution of \( f' \) in this model and that of \( f \) in the SIE model are almost identical (see the bottom right panel of Figure 4).

If we assume that the radial profile of the luminosity density \( \nu(r) \) follows that of the total mass density \( \rho(r) \), namely, \( \alpha = \delta = \gamma \), we can obtain the constraint
\[ \gamma = 2.04 \pm 0.02 \] (68% C.L.),
which is consistent with the standard SIS model at the 2\( \sigma \) level.

The 68% C.L. limit on the derived term is \( f' = 1.00 \pm 0.01 \), which is quite different from those results above. The reason is that when setting \( \alpha = \delta = \gamma \), the term \( f' \) in Equation (25) is limited to being not far away from unity. As we know, the SGL data do not favor the model with \( f = 1 \). Therefore, when the term \( f' \) is close to unity by force, the minimal \( \chi^2_{\text{SGL}} \) we obtain is much larger than those in the above models, \( \chi^2_{\text{min},\gamma} - \chi^2_{\text{min},\text{SIE}} \approx 53 \). This assumption with \( \alpha = \delta = \gamma \) has been ruled out by the current SGL data at very high significance.

4. DISCUSSION AND SUMMARY

With the growing importance of the SGL system in astronomy research, investigating the statistical properties of SGL systems becomes more and more important. In this paper, we use the latest SGL data observed by four surveys, SLACS, BELLS, LSD, and SL2S, to study the curvature of the universe model independently, based on the basic distance sum rule method, as well as the statistical properties of SGL systems in detail. Here we summarize our main conclusions in more detail:

1. Based on the standard SIS model with one free parameter \( f \), we reproduce the analysis of Räsänen et al. (2015) on the constraint of the curvature from a small sample of SGL data. When we fix the parameter \( f = 1 \), we obtain a similar constraint on the curvature to that in the previous work. However, when we set \( f \) as a free parameter, due to the strong correlation between \( \Omega_k \) and \( f \), the constraint on the curvature is significantly enlarged, namely, the 95% C.L. upper limit \( \Omega_k < 7.02 \). In the meantime, we also obtain the limit on the parameter \( f : f = 1.079 \pm 0.034 \) (68% C.L.), which implies that the standard SIS model with \( f = 1 \) has been ruled out at more than 2\( \sigma \) C.L. This result is different from that obtained in Räsänen et al. (2015), because in their analysis they did not fully take the uncertainty of \( f \) into account, but only assigned a Gaussian error of 20\% from \( f^2 \). This will produce a bias on the determination of \( \Omega_k \) and overestimate the constraining power of the SGL data on the curvature.

2. We perform the global analysis on the curvature and the parameter \( f \) by using the latest SGL data and find that these new SGL data can significantly improve the constraint on the curvature by a factor of 10, namely, \( \Omega_k < 0.60 \) (95% C.L.), when we introduce the intrinsic scatter, \( \sigma_{\text{int}} \), which represents any other unknown uncertainties, into the analysis. On the other hand, the constraint on the parameter \( f \) is also improved: \( f = 1.105 \pm 0.030 \) (68% C.L.), which tells us that the SGL data do not favor the SIS model at more than 3\( \sigma \) C.L. We also use the SGL samples in each survey to constrain \( f \) separately and find that the most constraining power comes from SLACS. Due to the lack of samples in BELLS, the limit on \( f \) is very weak.
3. In order to understand this result on f better, we divide the whole SGL sample into two parts according the centric stellar velocity dispersion $\sigma_c$ of the SGL system: $\sigma_c < 240 \, \text{km s}^{-1}$ and $\sigma_c > 240 \, \text{km s}^{-1}$. Due to the smaller SGL data in two subsamples, the upper limit constraints on the curvature are weaker. More interestingly, the standard SIS model is consistent with the constraint of f from the large velocity dispersion subsample within 95\% C.L.: $f = 1.03 \pm 0.02$ (2\sigma C.L.). The larger value of velocity dispersion $\sigma_c$ the subsample has, the more favored the standard SIS model with f = 1 is. We also divide the sample into two parts according to the redshift of the lens galaxy, $z_l < 0.24$ and $z_l > 0.24$, and find that the constraints on f are consistent with each other at about 1\sigma C.L. and different from unity at more than 2\sigma C.L.

4. Besides the standard SIE model with one free parameter f, in our analysis we also consider the more complex SGL model by assuming the power-law mass density profile $\rho(r) = \rho_0 (r/r_0)^{-\alpha}$ and luminosity density of stars $\nu(r) = \nu_0 (r/r_0)^{-\delta}$. Using all of the SGL data, we obtain the constraints on the power-law indices, $\alpha = 1.95 \pm 0.04$ and $\delta = 2.40 \pm 0.13$ at 68\% C.L., which are consistent with the direct measurement from the observations on $\delta$. Comparing with the SIE model, we also obtain the constraint on the derived parameter $f'$, which is almost identical to the constraint on f in the SIE model.

5. We also assume that the radial profile of the luminosity density $\nu(r)$ follows that of the total mass density $\rho(r)$, namely, $\alpha = \delta = \gamma$. This model strongly suggests that $f'$ is very close to unity, which is disfavored by the SGL data. Therefore, the minimal $\chi^2_{\text{SGL}}$ we obtain is much larger than those in the above models, $\chi^2_{\text{min, SGL}} - \chi^2_{\text{min, SIE}} \approx 53$. This assumption with $\alpha = \delta = \gamma$ has been ruled out by the current SGL data.

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