The Semiclassical Approach to Small \( x \) Physics\(^{\circ}\)

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The semiclassical approach to small-\( x \) physics is briefly reviewed, where the main emphasis is on the determination of the gluon distribution at NLO.

1 Introduction

The semiclassical approach to deep inelastic scattering (DIS) at small \( x \) exploits the target rest frame point of view\(^{\dagger}\). In this frame the virtual photon interacts via its partonic fluctuation with the target. The target itself is considered as localized soft color field. In the small \( x \) limit the partons are fast, and therefore their interaction with the target can be treated in an eikonal approximation\(^{\ddagger}\).

This picture of DIS allows a combined description of both inclusive and diffractive events. In particular in the case of diffractive scattering, like diffractive dissociation and the diffractive production of high-\( p_{\perp} \) jets, several interesting results can be obtained, even without explicit numerical calculations\(^{\S}\).

Here we focus on the most recent development in the semiclassical approach, namely the determination of the gluon density at next-to-leading order (NLO)\(^{\|}\), which can serve as input in the evolution equation. The gluon density is expressed in terms of a (non-perturbative) Wilson loop and can be evaluated in any model of the target color field.

2 The Gluon Distribution

To extract the gluon density it is convenient to use a ‘scalar photon’ (denoted by \( \chi \)) coupled directly to the gluon field\(^{\|\|}\). The gluon density is then derived by matching the semiclassical and the parton model approach. To leading order this means that we have to equate the cross section for the transition \( \chi \rightarrow g \) in an external field with the cross section of the process \( \chi g \rightarrow g \) as given in

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\(^{\dagger}\)For a comprehensive review of these topics see Ref.\(^{\|}\) and references therein.
the parton model. The result reads
\[xg^{(0)}(x, \mu^2) = \frac{1}{12\pi^2\alpha_s} \int d^2x_\perp \left| \frac{\partial}{\partial y_\perp} W^A_{\perp}(y_\perp) \right|^2_{y_\perp=0},\] (1)

where \(W^A\) indicates a Wilson loop in the adjoint representation describing the eikonalised interaction of a gluon in the external color field of the target. The gluon distribution \(xg^{(0)}(x, \mu^2)\) is a constant, and measures the averaged local field strength of the target.

At NLO, we write the gluon density as
\[xg(x, \mu^2) = xg^{(0)}(x, \mu^2) + xg^{(1)}(x, \mu^2),\] (2)
with \(xg^{(1)}(x, \mu^2)\) denoting the (scheme dependent) NLO correction. To extract this correction, the cross section for the transition \(\chi \to gg\) in an external field has to be equated with the parton model cross section of the process \(\chi g \to gg\).

Without providing any details of the calculation, we quote here only the final result of the distribution in the \(\overline{\text{MS}}\) scheme at NLO,
\[xg^{(1)}(x, \mu^2) = \frac{1}{\pi^3} \left( \ln \frac{1}{x} \right) \int_{r^2(\mu)}^{\infty} \frac{dy_\perp^2}{y_\perp} \left\{ - \int d^2x_\perp \text{tr} W^A_{\perp}(y_\perp) \right\}.\] (3)

The scheme dependence enters through the short-distance cutoff \(r^2(\mu)\). The NLO gluon density shows a \(\ln(1/x)\) enhancement at small \(x\), and is sensitive to the large-distance structure of the target.

Using the model of a large hadron to describe the color field of the target, allows a comparison of our result with the one of Mueller. We find agreement for both the integrated distribution in (3) and the unintegrated one not shown here. However, in Refs. 5, 6, where the main focus is on parton saturation, the scale dependence has not been discussed. Therefore, we provide for the first time a quantitative relation between the short-distance cutoff in Eq. (3) and the scale of the gluon distribution, which can only be achieved by matching the semiclassical approach with a treatment in the parton model.

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