Supplemental Material for:
High-Q trampoline resonators from strained crystalline InGaP for integrated free-space optomechanics

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(Dated: May 18, 2023)

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S1. FABRICATION AND INGaP MATERIAL PROPERTIES

A. Fabrication

Fig. S1 shows an SEM image of a fabricated and suspended InGaP trampoline. One can clearly see that the etched GaAs substrate surface under the trampoline is elevated and uneven. The unevenness will contribute to scattering loss of the optical cavity mode that forms between the suspended PhC trampoline and the substrate surface. The reason for this unevenness comes from the wet etch process. The PhC pattern determines the height profile of the "imprinted" surface onto the substrate. Basically, the wet etch process is faster on the large open areas outside of the trampoline pad than through the small diameter PhC holes. As a result, the material below the trampoline is initially etched slower than the material outside of the pad. At the end of the wet etch process, one obtains an elevated height profile mimicking the shape of the trampoline on the substrate surface.

Fig. S1. Tilted SEM image of a PhC trampoline resonator. The etched GaAs substrate under the trampoline is clearly seen to exhibit an elevated and uneven surface with a height profile mimicking the PhC trampoline.

B. Critical thickness

Fig. S2 shows the critical thickness of In$_{1-x}$Ga$_x$P on GaAs using the People and Bean model [1]. We observe that the thickness of In$_{0.43}$Ga$_{0.57}$P from our work is well below the critical thickness criterion for the corresponding gallium content and hence the device layer does not relax to its native lattice constant and will therefore be tensile strained.

C. Anisotropic elasticity

The general linear relation between the components of the stress tensor $\sigma_{ij}$ and strain tensor $\epsilon_{kl}$ is given by the fourth order stiffness tensor $C_{ijkl}$ as

$$\sigma_{ij} = C_{ijkl}\epsilon_{kl}. \quad (S1)$$

As InGaP has a zincblende crystal structure with a cubic symmetry, one can relate stress and strain with a...
Gallium content (x) in In_{1-x}Ga_xP represents the content that we use in this work (x = 0.5658).

The horizontal line represents the thickness of InGaP in this work, (73 nm). The gray area represents the range of gallium content estimated via X-ray diffraction and the vertical line represents the content that we use in this work (x = 0.5658).

Reduced number of independent components by

\[
\begin{pmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{zz} \\
\sigma_{yz} \\
\sigma_{xz} \\
\sigma_{xy}
\end{pmatrix} =
\begin{pmatrix}
c_{11} & c_{12} & c_{12} & 0 & 0 & 0 \\
c_{12} & c_{11} & c_{12} & 0 & 0 & 0 \\
c_{12} & c_{12} & c_{11} & 0 & 0 & 0 \\
0 & 0 & 0 & c_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & c_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & c_{44}
\end{pmatrix}
\begin{pmatrix}
\epsilon_{xx} \\
\epsilon_{yy} \\
\epsilon_{zz} \\
\epsilon_{yz} \\
\epsilon_{xz} \\
\epsilon_{xy}
\end{pmatrix},
\tag{S2}
\]

where x, y, z denote the crystal directions [1 0 0], [0 1 0], and [0 0 1], respectively. The elastic constants of In_{1-x}Ga_xP, c_{11}(x), c_{12}(x), c_{44}(x) [2], are listed in Tab. S1. The simplified matrix for C from Eq. S2 can be expressed in terms of material constants \( \nu_{ij} = -\frac{\epsilon_{ij}}{\epsilon_{ii}}, E_i = \sigma_{ii}/\epsilon_{ii}, G_{ij} = \sigma_{ij}/\epsilon_{ij} \) [3] as

\[
C =
\begin{pmatrix}
\frac{1-\nu_{xx}\nu_{yy}}{E_x} & \frac{\nu_{xx}+\nu_{yy}+\nu_{xy}}{E_x} & \frac{\nu_{xx}+\nu_{yy}+\nu_{xz}}{E_x} & 0 & 0 & 0 \\
\frac{\nu_{xx}+\nu_{yy}+\nu_{xy}}{E_x} & \frac{1-\nu_{xx}\nu_{yy}}{E_x} & \frac{\nu_{xx}+\nu_{yy}+\nu_{yz}}{E_x} & 0 & 0 & 0 \\
\frac{\nu_{xx}+\nu_{yy}+\nu_{xz}}{E_x} & \frac{\nu_{xx}+\nu_{yy}+\nu_{yz}}{E_x} & \frac{1-\nu_{xx}\nu_{yy}}{E_x} & 0 & 0 & 0 \\
0 & 0 & 0 & G_{yx} & 0 & 0 \\
0 & 0 & 0 & 0 & G_{xx} & 0 \\
0 & 0 & 0 & 0 & 0 & G_{xy}
\end{pmatrix}
\tag{S3}
\]

\[
\Delta = \frac{1-\nu_{xx}\nu_{yy}-\nu_{yz}\nu_{yx}-\nu_{xz}\nu_{zx}-2\nu_{xy}\nu_{yx}+\nu_{xy}}{E_x E_y E_z}
\]

The inverse of the stiffness matrix is the compliance matrix and relates strain to stress, \( \epsilon_{ij} = S_{ijkl}\sigma_{kl} \), via

\[
S = \begin{pmatrix}
\frac{1}{E_x} & -\frac{\nu_{xx}}{E_y} & -\frac{\nu_{xx}}{E_z} & 0 & 0 & 0 \\
-\nu_{xx} & \frac{1}{E_y} & -\frac{1}{E_z} & 0 & 0 & 0 \\
-\frac{\nu_{xx}}{E_x} & -\frac{1}{E_y} & \frac{1}{E_z} & 0 & 0 & 0 \\
0 & 0 & 0 & G_{yz} & 0 & 0 \\
0 & 0 & 0 & 0 & G_{xx} & 0 \\
0 & 0 & 0 & 0 & 0 & G_{xy}
\end{pmatrix}
\tag{S4}
\]

Thus, one can easily obtain Young’s modulus in the x direction as \( E_x = 1/s_{11} \). By rotating the matrices around the [0 0 1]-direction [4], Young’s modulus in the (0 1 0) plane can be obtained as [5]

\[
E(x, \theta) =
8 \left[ c_{11}(x) - c_{12}(x) \right] \left[ c_{11}(x) + 2c_{12}(x) \right] c_{44}(x)
\frac{c_{11}(x) - 2c_{12}(x) [c_{12}(x) - 2c_{44}(x)] + c_{11}(x) [c_{12}(x) + 6c_{44}(x)] + \left[ c_{11}(x) + 2c_{12}(x) \right] [c_{11}(x) - c_{12}(x) - 2c_{44}(x)] \cos (4\theta)}{c_{11}(x) - 2c_{12}(x) [c_{12}(x) - 2c_{44}(x)] + c_{11}(x) [c_{12}(x) + 6c_{44}(x)] + \left[ c_{11}(x) + 2c_{12}(x) \right] [c_{11}(x) - c_{12}(x) - 2c_{44}(x)] \cos (4\theta)}.
\tag{S5}
\]

For a gallium content of x = 0.5658, E(x, \theta) is plotted in Fig. S3(a).

\[
\begin{align*}
E(x, \theta) &=
8 \left[ c_{11}(x) - c_{12}(x) \right] \left[ c_{11}(x) + 2c_{12}(x) \right] c_{44}(x)
\frac{c_{11}(x) - 2c_{12}(x) [c_{12}(x) - 2c_{44}(x)] + c_{11}(x) [c_{12}(x) + 6c_{44}(x)] + \left[ c_{11}(x) + 2c_{12}(x) \right] [c_{11}(x) - c_{12}(x) - 2c_{44}(x)] \cos (4\theta)}{c_{11}(x) - 2c_{12}(x) [c_{12}(x) - 2c_{44}(x)] + c_{11}(x) [c_{12}(x) + 6c_{44}(x)] + \left[ c_{11}(x) + 2c_{12}(x) \right] [c_{11}(x) - c_{12}(x) - 2c_{44}(x)] \cos (4\theta)}.
\end{align*}
\]

As we are interested in the effect of the stress component along the string, we consider Poisson’s ratio \( \nu = \nu_{xy} \). This value can be obtained in the (0 0 1) plane from the transformed compliance matrix as \( \nu = -s_{21}/s_{11} \). The resulting Poisson’s ratio is shown in Fig. S3(b).

We assume that the as-grown strain along 0\(^\circ\) and 90\(^\circ\) is the same, i.e., \( \epsilon_{xx} = \epsilon_{yy} = \epsilon \). Then, the as-grown stress of In_{1-x}Ga_xP for x = 0.5658 is \( \sigma_{\text{grown}} = \sigma_{xx} = E_x\epsilon/(1-\nu) \). We obtain 472 MPa for the 0\(^\circ\) direction.

The released stress in the string resonators can be obtained by taking into account Poisson’s ratio, \( \sigma_{\text{grown}} (1-\nu) \), resulting in 460 MPa and 319 MPa for a string oriented along 0\(^\circ\) and 45\(^\circ\), respectively, see Fig. S4.
D. Yield strength

We fabricated nanostring resonators that are attached to the substrate with a tapered clamp of varying ratio $r = w_{\text{clamp}}/w_{\text{string}}$, where $w_{\text{clamp}}$ is the width of the string at the clamping point and $w_{\text{string}}$ is the width at the center of the string, see Fig. S5(a). The width at the clamping point determines the strain redistribution along the string resonator. The strain in the clamping region can be obtained from the strain-width relation as $\epsilon_{\text{clamp}} = \epsilon_{r=1} \cdot w_{\text{string}}/w_{\text{clamp}} = \epsilon_{r=1}/r$ [6]. As stress is proportional to strain, the stress at the clamping region is enhanced for $r < 1$ [Fig. S5(b)] and an increase above the yield stress of the material results in the fracturing of the beam. The corresponding yield stress is (for details see Ref. [6])

$$\sigma_{\text{yield}} = \sigma_{r=1}/r_{\text{yield}}. \quad (S6)$$

We fabricated 15 arrays of 40 µm-long and 2 µm-wide string resonators with $r$ varying from 0.001 to 1 along the three aforementioned crystal directions. We observe that strings oriented along $0^\circ$ [Fig. S5(c)], $45^\circ$, and $90^\circ$ break for $r_{\text{yield}}$ of $(0.062 \pm 0.005)$, $(0.061 \pm 0.007)$, and $(0.067 \pm 0.005)$, respectively. We determined $\sigma_{r=1}$ from a measurement of the fundamental mode frequency of the $r = 1$ resonator. We obtain yield stresses $\sigma_{\text{yield}}$ using Eq. S6 of $(5.5 \pm 0.8)$ GPa, $(3.3 \pm 0.5)$ GPa, and $(3.7 \pm 0.5)$ GPa along $0^\circ$, $45^\circ$, and $90^\circ$, respectively.

To verify the analytic model for determining the yield strength, we perform independent FEM simulations of the corresponding tapered geometries [Fig. S5(b)] (parameters see Appendix [7]). Using $r_{\text{yield}}$, FEM simulations predict yield stresses of $\sigma_{\text{yield}}(0^\circ, 90^\circ) \approx 4.14$ GPa, $\sigma_{\text{yield}}(45^\circ) \approx 3.2$ GPa and yield strain of $\approx 0.029$, in reasonable agreement with the experimental values.

E. Optical properties

We measured the refractive index and absorption coefficient of the 73 nm-thick In$_{0.43}$Ga$_{0.57}$P layer using an ellipsometer. The results are shown in Fig. S6 and resemble the values found in the literature for the wavelength range of interest in our work [8].

![Fig. S6](image)

**Fig. S6.** Refractive index $n$ and extinction coefficient $k$ for the 73 nm-thick InGaP film determined from ellipsometer measurements. The refractive index $n$ is in agreement with Ref. [8].

S2. MECHANICAL DAMPING

The total mechanical quality factor of a mechanical resonator is given by [9, 10]

$$Q^{-1} = Q_{\text{int}}^{-1} + Q_{\text{ext}}^{-1} = \sum_i (Q_{\text{int}}^i)^{-1} + \sum_j (Q_{\text{ext}}^j)^{-1}, \quad (S7)$$

where $Q_{\text{int}}^i$ originates, amongst others, from surface loss, thermoelastic damping, or loss processes related to ma-
terial defects, and $Q_{\text{ext}}^i$ from, amongst others, gas damping or clamping loss. For an in-depth discussion, see Refs. [9, 10]. In the following, we analyze mechanical loss processes relevant to this work.

A. Clamping loss

Clamping loss is determined by the geometry of the clamping of the mechanical resonator to the substrate. Clamping results in the transfer of acoustic energy from the mechanical resonator to its environment. In the following, we will estimate the clamping loss for string-type resonators and compare it to the one of a cantilever and a fully clamped square membrane which would give the lower and upper bounds for clamping loss, respectively. Further, we will examine the clamping loss of a fully clamped membrane to obtain an upper limit to the clamping loss of a trampoline resonator, which we also independently simulated with FEM.

For string resonators in this work, we can write the total mechanical quality factor as

$$Q_{\text{total}}^{-1} = \frac{1}{D_n \cdot Q_{\text{int}}} + \frac{1}{Q_{\text{clamp,s}}}.$$  \hspace{1cm} (S8)

Similar to Ref. [11], we assume $Q_{\text{clamp,s}} = \eta L/h$, where $\eta$ is a free parameter. For the string resonators in our work and using Eq. S8, we obtain $Q_{\text{clamp,s}} \sim 10^{12}$.

We can compare the clamping loss of the doubly-clamped string resonators to the one of singly-clamped cantilevers, where the latter will give an upper bound on $Q_{\text{clamp}}$ for string resonators. The clamping loss of a cantilever of thickness $h$, width $w$ and length $L$ is given by [10]

$$Q_{\text{clamp,c}}^{-1} = 0.31 \frac{w^4}{L} \left( \frac{h}{L} \right)^4,$$  \hspace{1cm} (S9)

for the case where the thickness of the substrate is comparable to the wavelength of the acoustic wave. The fundamental mode of a 155$\mu$m-long and 2$\mu$m-wide cantilever has $Q_{\text{clamp,c}} = 5 \cdot 10^{15}$, which is larger than $Q_{\text{clamp,s}}$ of a string with the same dimensions.

Similarly, we obtain the lower bound on $Q_{\text{clamp,s}}$ by examining a worst-case scenario of a fully clamped membrane. The clamping loss-limited $Q$ factor of a tensile-stressed square membrane clamped to a thick substrate is given by [10, 12]

$$Q_{\text{clamp,m}} \approx \frac{3}{2} \sqrt{\frac{\rho_{\text{InGaP}}}{\rho_{\text{GaAs}}}} \left( \frac{E_{\text{GaAs}}}{\sigma} \right)^{3/2} \frac{n^2 m^2}{(n^2 + m^2)^{3/2}} \frac{L}{h},$$  \hspace{1cm} (S10)

where $h$ is the thickness of the membrane, $L$ the length of one of its sides, $\rho_{\text{InGaP}}$ and $\rho_{\text{GaAs}}$ are the densities of the membrane and the substrate, respectively, $E_{\text{GaAs}}$ is Young’s modulus of the GaAs substrate, $n$ and $m$ are the eigenmode numbers of the mechanical mode. For a 155$\times$155$\mu$m$^2$ square membrane with a uniform tensile stress of 415 MPa, $Q_{\text{clamp,m}}$ for the fundamental mode is about $7 \cdot 10^6$. As we observe $Q$ factors of string resonators $\leq 3 \cdot 10^6$, we conclude that they are not limited by clamping loss.

Now, we estimate a lower bound on $Q_{\text{clamp}}$ for the trampoline resonators by using Eq. S10. For a 1130$\times$1130$\mu$m$^2$ square membrane (corresponding to a trampoline with tether length of 750$\mu$m and central pad size of 100$\mu$m) with uniform tensile stress of 415 MPa, $Q_{\text{clamp,m}}$ for the fundamental mode is about $5 \cdot 10^7$. This value yields a lower limit for the clamping loss-related $Q$ factor of trampoline resonators. We also performed FEM simulations to estimate $Q_{\text{clamp,m}}$ for the trampoline resonators using perfectly matched layers [13]. We find that $Q_{\text{clamp}} > 10^9$ [14]. Therefore, we consider that the micromechanical resonators of this work are not clamping-loss limited.

B. Gas damping

Gas damping is another extrinsic mechanical damping process [9]. For a mechanical resonator, such as a string- or membrane-type resonator, oscillating at frequency $\Omega_m$ at low pressures $P$, i.e., in the ballistic regime, where the mean free path of the gas molecules is larger than the dimensions of the mechanical resonator, $Q_{\text{gas}}$ is given as [15]

$$Q_{\text{gas}}^{-1} = 4 \sqrt{\frac{2}{\pi}} \frac{1}{\rho h \Omega_m} P \sqrt{\frac{M}{RT}},$$  \hspace{1cm} (S11)

where $M$ is the molar mass of the gas molecules, $R$ is the molar gas constant, and $T$ is the temperature of the gas.

Fig. S7 shows pressure-dependent measurements for InGaP string resonators of length 155$\mu$m and width 2$\mu$m oriented along 0°. For pressures $P > 10^{-2}$ mbar, the $Q$ factor of the strings is limited by gas damping, while for pressures $P < 10^{-2}$ mbar, $Q$ is constant, indicating another limiting damping mechanism.

C. Thermoelastic damping

Thermoelastic damping (TED) is related to the irreversible conversion of mechanical energy into heat [9]. During the oscillation of the mechanical resonator, one of its sides is compressed, while the opposite side is stretched. This deformation produces a temperature gradient and, thus, an irreversible transport of heat, in case the thermal expansion coefficient $\alpha$ is non-zero.

For a thin beam of length $L$, thickness $h$, and width $w$ at temperature $T$ the quality factor limited by TED, $Q_{\text{TED}}$, is given by [16]

$$Q_{\text{TED, L-R}}^{-1} = \frac{\alpha^2 ET}{C_p} \left( \frac{6}{\xi^2} - \frac{6}{\xi^3} \sinh \xi + \sin \xi \right),$$  \hspace{1cm} (S12)
where $\xi = \frac{h}{\sqrt{\frac{\omega_0}{\pi}}} \cdot \frac{1}{1 + \frac{F_{cr}}{F}}$ is a dimensionless variable, with $\omega_0$ the isothermal eigenfrequency, $\chi = \kappa / \rho C_p$ the thermal diffusivity, $C_p$ the heat capacity per unit volume at constant pressure, and $\kappa$ the thermal conductivity. At room temperature, the linear thermal expansion coefficient is $\alpha \approx 4.28 \times 10^{-6}$ 1/K, based on the experimental values for InP and GaP [17]. For this isotropic model, we set Young’s modulus to $E = 85$ GPa. Based on Eq. S12, we obtain the straight line in Fig. S8 for $Q_{TED, L-R}$. We independently performed FEM simulations of beams with the same dimensions and material parameters and obtain very similar results (see Fig. S8).

As our InGaP resonators are strained we need to modify the thermal relaxation time and the oscillation frequency in the model above [18]. Following the derivation from Ref. [19] the pre-stress is included via an additional frictional term $\frac{F_{cr}}{F}$, which leads to $Q_{TED}$ for strained beams

$$Q_{TED}^- = \frac{1}{1 + \frac{F_{cr}}{F}} \cdot Q_{TED, L-R}(\xi), \quad (S13)$$

where $F_{cr} = \pi EI/L^2$, $a = 0.97$ is a factor that depends on the boundary condition [20], $I$ is the moment of inertia, and $\xi$ gets modified by inserting $\omega_0 = \frac{\pi}{\sqrt{E I / \rho \pi \kappa \sqrt{\pi^2 + \frac{F L^2}{EI}}}$. For a stress of $\sigma_0 = 300$ MPa, $Q_{TED}$ of a strained beam is shown as the dashed line in Fig. S8 and considerably larger than $Q_{TED, L-R}$. We also simulated such stressed beams in FEM and obtain a good agreement with the analytical estimate Eq. S13 (see Fig. S8).

Estimating TED of the trampoline patterned with a PhC should take into account the periodic perforation of the material, which leads to an effective medium with a weaker elasticity matrix [21] and periodic stress redistribution. The periodic strain variation results in a temperature gradient around the PhC hole edges and, therefore, in an increase of TED loss [22].

To get an estimate of $Q_{TED}$ for a PhC trampoline resonator, we performed FEM simulations (using $r_{PhC} = 552$ nm, $a_{PhC} = 1323$ nm) and obtained $Q_{TED} \approx 10^{10}$ for the fundamental mode at a frequency of $\Omega_m/2\pi = 163$ kHz, which is higher than the $Q$ we observed in the experiment.

### D. Dilution factor in FEM

$Q_{int}$ can be generally written as the ratio of the stored versus the lost energy over one cycle of oscillation [10]:

$$Q_{int} = 2 \pi \frac{W_{elongation} + W_{bending}}{\Delta W_{elongation} + \Delta W_{bending}}, \quad (S14)$$

where $W_{elongation}$ and $W_{bending}$ are the potential energy stored in the elongation and bending of the resonator, respectively. Similarly, $\Delta W_{elongation}$ and $\Delta W_{bending}$ are the corresponding loss processes.

The quality factor can be increased by diluting the intrinsic material friction. This can be achieved by utilizing materials with high intrinsic tensile stress, which introduces $W_{tensile}$ as an additional contribution to the potential energy. The increased quality factor $Q_D$ is then given by [6, 23]

$$Q_D = D \cdot Q_{int}, \quad (S15)$$

where $D$ is the dilution factor that depends on the pre-stain of the material, resonator geometry, and the displacement mode profile. The dilution factor is in general given by [23]

$$D = 1 + \frac{W_{tensile}}{W_{elongation} + W_{bending}}. \quad (S16)$$

We see that by increasing the elastic energy $W_{tensile}$, the mechanical quality factor of the resonator can be increased to a considerable extent, with record values of $Q > 10^{10}$ in highly tensile-strained crystalline silicon [24].
For thin two-dimensional membranes (i.e., membranes with a large length-to-thickness ratio) with the out-of-plane displacement \( u_z = w(x,y) \), the dilution factor can be calculated using FEM. In the case of \( D_Q \gg 1 \), the first term in Eq. S16 can be neglected. Then, the dissipation dilution factor can be calculated by integrating over the entire membrane area as [25]

\[
D_Q = \frac{\rho h \Omega_m^2}{D_p} \frac{\iint \left\{ \left( \frac{\partial^2 w(x,y)}{\partial x^2} + \frac{\partial^2 w(x,y)}{\partial y^2} \right)^2 + 2(1-\nu) \left[ \frac{\partial^2 w(x,y)}{\partial x \partial y} - \frac{\partial^2 w(x,y)}{\partial x^2} \frac{\partial^2 w(x,y)}{\partial y^2} \right] \right\} dx dy}{\iint w(x,y)^2 dx dy} \tag{S17}
\]

where \( D_p = Eh^3/[12(1-\nu^2)] \) is the flexural rigidity of the material and \( \nu \) is Poisson’s ratio. The numerator yields the kinetic energy of the membrane and the denominator yields the energy stored in the membrane bending [26].

S3. EXPERIMENTAL PARAMETERS

Tab. S1 summarizes the material and device parameters of this work.

| Parameter                     | Symbol | Value |
|-------------------------------|--------|-------|
| **Material properties**       |        |       |
| Ga content                    | \( x \) | 0.5658 |
| Density (kg/m³)               | \( \rho \) | \((4.81 - 0.67x) \cdot 10^n\) |
| Thickness (nm)                | \( h, d_{PSC} \) | 73 |
| Stress (MPa)                  | \( \sigma(0^\circ) \) | 467.7 ± 7.1 |
|                               | \( \sigma(45^\circ) \) | 313.3 ± 5.4 |
|                               | \( \sigma(90^\circ) \) | 374.9 ± 16.4 |
| Elastic constants (Pa)        | \( c_{11} \) | \((10.11 + 3.94x) \cdot 10^m\) at 300 K |
|                               | \( c_{12} \) | \((5.61 + 0.59x) \cdot 10^{10}\) |
|                               | \( c_{44} \) | \((4.56 + 2.47x) \cdot 10^{10}\) |
| Lattice constant [Å]          | \( a_{11-2 GaAs} \) | \(GaAs\) | 5.65325 |
| In\(_{1-x}\)-Ga\(_x\)P       | \( a_{GaAs} \) | \( (5.8687 - 0.4182x) \) |
| Thermal diffusivity (W/K)     | \( \kappa \) | 66 |
| Heat capacity (J/kgK)         | \( C_p \) | 380 |
| Linear thermal expansion      | \( \alpha \) | \(4.28 \cdot 10^{-6}\) |
| coefficient at RT (1/K)       |        |       |

**Optical properties**

| Refractive index | \( n \) | 3.15 |
| Refractive index | \( L_{gap} \) | 14.8 |

**Mechanical properties**

| Effective mass (ng) | \( m_{eff} \) | 9.3 |

TABLE S1. Parameters used in FEM and RCWA simulations.

S4. APPLICATIONS

We have demonstrated InGaP trampoline mechanical resonators with a quality factor of \( 10^7 \) at room temperature with a resonance frequency of 38 kHz, resulting in a \( Q \cdot f_m \) product of \( 7 \cdot 10^{11} \) Hz.

In the following, we discuss future device improvements and mention applications, where improved InGaP trampoline resonators are particularly fitting. We focus our discussion on InGaP mechanical resonators that offer a central area amenable to patterning with a photonic crystal (PhC). We deliberately choose the use of a suspended PhC as engineering the reflectance of the mechanical resonator via the PhC enables efficient transduction of mechanical displacement to out-of-plane light fields, as was recently demonstrated [27]. Furthermore, the (Al,Ga)As combined with InGaP material platform provides the opportunity to realize integrated free-space optomechanical microcavities [28]. Another interesting avenue would be to pattern phononic-shielded string-like resonators in InGaP [29, 30], which we will not discuss in the following.

a. Improvements of mechanical performance

Figures of merit in nanomechanics are the mechanical frequency \( f_m \), the mechanical \( Q \) factor, and the effective mass \( m_{eff} \) of the resonator. An increase in \( Q \) and \( f_m \) can be achieved by increasing the Ga content of InGaP to obtain a tensile stress of up to 1 GPa. This is a factor of two higher than the stress in the current samples and would translate into a factor of about \( \sqrt{2} \) enhancement of \( f_m \). In general, the tensile strain of the InGaP layer allows employing more advanced strain engineering techniques than we have currently used. This comprises optimized trampoline designs (with \( Q \) factors up to \( 10^8 \) at \( f_m \sim 150 \) kHz in SiN [31]), hierarchical clamping (\( Q \) factor larger than \( 10^8 \) at \( f_m \sim 100 \) kHz in SiN [29]), or phononic-shield membranes (\( Q \)-factor values larger than \( 10^8 \) at \( f_m \sim 700 \) kHz in SiN [32]). Furthermore, decreasing the thickness of the InGaP layer by a factor of two to about 35 nm, would lead to a further increase of \( Q \), with a concomitant decrease of \( f_m \).

Overall, we can expect to reach \( Q \) factors of InGaP trampoline-like resonators larger than \( 10^8 \) at frequencies of between 50 to 100 kHz with an effective mass between 1 to 10 ng. Furthermore, we expect that operating InGaP resonators at low temperatures will result in a further enhancement of \( Q \), as observed in other crystalline materials such as diamond [33] or silicon [24, 33], with an increase of up to a factor of five.
b. Force sensing For our current InGaP trampolines, we estimate a thermal noise limited force sensitivity of $F_{\text{th}} \approx 50 \text{aN}/\sqrt{\text{Hz}}$ (for an effective mass of 10 ng). Improved InGaP trampoline resonators with an effective mass of 1 ng at a frequency of 50 kHz and a $Q$ of $10^8$ would yield a thermal noise limited force sensitivity at room temperature of $7 \text{aN}/\sqrt{\text{Hz}}$, which would be better than the one achieved with SiN trampolines ($19.5 \text{aN}/\sqrt{\text{Hz}}$ [31]) or phononic-shield membranes ($37 \text{aN}/\sqrt{\text{Hz}}$ [32, 34]) and similar to other mechanical resonator geometries designed for force sensing [35–37]. When operating the devices at lower temperatures, e.g., at 4 K, we estimate 0.37 aN/√Hz when also assuming a $Q$ increase by a factor of 5 upon cooling. This value is close to the sensitivity required to detect the magnetic moment of a single proton [38].

c. Quantum optomechanics at room temperature Improved InGaP trampoline resonators with a frequency of 100 kHz and a $Q$ of $10^8$ would yield a $Q$-factor of $10^7$, which places such devices in the regime of quantum optomechanics at room temperature. As the trampoline can be patterned with a high-reflectivity PhC, it can be directly used as a mirror in an optical cavity, which enables feedback cooling of a trampoline mechanical mode to the ground state [39, 40].

d. Optomechanical microcavity We have recently demonstrated an optomechanical microcavity based on an (Al,Ga)As heterostructure [28], which accesses the regime of ultra-strong optomechanical coupling, with $\omega_m \sim 0.3g_0$. However, these experiments were limited by the large optical loss rate of hundreds of GHz, placing the system deep in the bad cavity regime, and the low mechanical $Q \sim 10^4$ of the suspended GaAs-based mechanical resonator.

Using InGaP trampolines for such an integrated optomechanical microcavity would allow reaching $Q$ factors that are four orders of magnitude larger compared to GaAs-based mechanical resonators. Furthermore, the strained InGaP layer enables patterning of a PhC on a trampoline with a sufficiently large central pad of 250 μm×250 μm, such that diffraction loss and collimation or finite size effects [41] of the PhC when illuminated with a 50 μm waist are estimated to be negligible. A reflectivity above 0.9998 has been recently demonstrated with a similarly sized suspended PhC membrane at telecommunication wavelength [27], yielding a Finesse $F > 3 \cdot 10^4$.

An envisioned InGaP-based optomechanical microcavity with a length $L_c \sim \lambda/2$ at $\lambda_0 = 1550 \text{nm}$, would yield an optical loss rate $\kappa = \pi c/(2L_c F) < 2\pi \cdot 3 \text{GHz}$. Thanks to the short length of $\lambda_0/2$ of the microcavity a single-photon coupling strength $g_0 \sim 2\pi \cdot 0.9 \text{MHz}$ is within reach. This optomechanical microcavity would be placed in the ultra-strong coupling ($g_0/\omega_m \sim 9$ with $\omega_m = 2\pi \cdot 100 \text{kHz}$) and bad cavity regime ($g_0/\kappa \sim 3 \cdot 10^{-4}$). With a $Q$ of $10^8$ ($5 \cdot 10^8$) at room temperature (4K), a single-photon cooperativity $C_0 = (g_0)^2/\kappa\gamma n_{th}$ of $4 \cdot 10^{-3}$ (1.6) can be achieved, with the phonon occupation $n_{th} = k_B T/\hbar \omega_m$. Such a system will allow studying novel optomechanical effects in the ultra-strong coupling regime [42] and realization of mechanical squeezing as recently proposed in Ref. [43].

An integrated optomechanical microcavity may also be able to access the single-photon strong coupling regime, which is so far elusive in nano- or micromechanics. One approach relies on multi-element optomechanics [44], which aims at increasing $g_0$ by virtue of collective mechanical effects between highly reflective mechanical elements placed within a cavity. We have taken steps in this direction in the (Al,Ga)As-based system by realizing two-element mechanics on a chip [28]. Using InGaP instead of GaAs-based mechanics would thereby allow reaching higher $Q$ mechanical resonators. Further, a key challenge with this approach is to match the mechanical frequency of the resonators within their linewidth. As InGaP is a piezoelectric material, one can tune the mechanical frequency in-situ via application of a static voltage, which is a desired feature to realize the stringent requirements of Ref. [44].

Another approach to reach the single-photon strong coupling regime relies on minimizing the optical loss rate $\kappa$ via the use of photonic bound states in the continuum [45]. A system composed of two InGaP-based suspended PhC mirrors at a well-defined spacing, e.g., defined via heterostructure growth, can realize such a photonic bound state in the continuum, where it has to be seen in the experiment how small $\kappa$ can be realized given fabrication imperfections of the PhC and the material’s absorption.

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