Generation of strong magnetic fields via the small-scale dynamo during the formation of the first stars

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Here we summarize our recent results of high-resolution computer simulations on the turbulent amplification of weak magnetic seed fields showing that such fields will be exponentially amplified also during the gravitational collapse reminiscent to the situation during primordial star formation. The exponential magnetic field amplification is driven by the turbulent small-scale dynamo that can be only observed in computer simulations if the turbulent motions in the central core are sufficiently resolved. We find that the Jeans length, which determines the central core region, has to be resolved by at least 30 grid cells to capture the dynamo activity. We conclude from our studies that strong magnetic fields will be unavoidably created already during the formation of the first stars in the Universe, potentially influencing their evolution and mass distribution.

1 Introduction

Magnetic fields are ubiquitous in the local Universe\textsuperscript{10} and there is growing evidence of their presence also at high redshift\textsuperscript{11,12}. The seeds for these fields could be a relic from the early Universe, possibly arising due to inflation or some other phase transition process\textsuperscript{13}. Alternatively, they could be generated by the Biermann battery\textsuperscript{14} or the Weibel instability\textsuperscript{15}. Regardless of their physical origin, most models predict weak field strengths and/or have large uncertainties\textsuperscript{16,17}. Consequently, magnetic fields are thought to be irrelevant for the formation of the first stars and galaxies\textsuperscript{18}. Here we show that strong and dynamically important magnetic fields will be generated from weak initial magnetic seed fields in the presence of the small-scale turbulent dynamo during the collapse of primordial halos\textsuperscript{13,14}.
The properties of the small-scale dynamo have been explored both in computer simulations of driven turbulence without self-gravity and in analytic models\textsuperscript{15, 16, 17, 18} as well as in the context of magnetic fields in galaxy clusters\textsuperscript{19}. Analytic estimates show that the small-scale dynamo could be important already during the formation of the first stars and galaxies\textsuperscript{20, 21, 22}.

Field amplification via the small-scale dynamo requires both turbulent gas motions and high magnetic Reynolds numbers. The presence of such turbulence is suggested by cosmological hydrodynamical simulations of first star formation\textsuperscript{23, 24, 25} where it also plays an important role in regulating the transport of angular momentum. High magnetic Reynolds numbers are expected in primordial gas as it follows closely the conditions of ideal magnetohydrodynamics (MHD).

2 Numerical method and initial conditions

In this study, we focus on the gravitational collapse and magnetic field amplification of the inner parts of a contracting primordial halo. The initial conditions for our computer simulation were motivated from larger-scale cosmological models\textsuperscript{23, 24, 25}. Thus, we set up a super-critical Bonnor-Ebert (BE) sphere with a core density of $\rho_{\text{BE}} \simeq 4.68 \times 10^{-20}$ g cm$^{-3}$ ($n_{\text{BE}} = 10^4$ cm$^{-3}$) and a small amount of rotation, $E_{\text{rot}}/E_g = 0.04$. We use a random initial velocity field with transonic velocity dispersion, and a weak random magnetic field with $B_{\text{rms}} \sim 1$ nG. Both the turbulent energy and magnetic field spectra were initialized with the same power law dependence, $\propto k^{-2}$, with most power on large scales. Consistent with previous studies that follow the thermodynamics during the collapse, we adopt an effective equation of state with $\gamma = d \log T/d \log \rho + 1 = 1.1$ for all densities relevant to this study\textsuperscript{26, 27}. For this setup, we solve the equations of ideal magnetohydrodynamics (MHD) including self-gravity with an adaptive-mesh refinement technique\textsuperscript{28}, which guarantees that the critical length scale of gravitational collapse is always resolved with the same number of cells regardless of density. This scale, the local Jeans length

$$\lambda_J = \left( \frac{\pi c_s^2}{G \rho} \right)^{1/2},$$

where $c_s$ and $G$ are sound speed and gravitational constant, is set by the competition between gravity and thermal pressure. This is also the turbulent injection scale below which the small-scale dynamo is active. We note that the efficiency of the dynamo process depends on the Reynolds number and is thus related to how well the turbulent motions are resolved\textsuperscript{29}. Higher resolution results in larger field amplification. To demonstrate this effect, we perform five computer simulations where we resolve $\lambda_J$ by either 8, 16, 32, 64 or 128 cells. Our results indicate that a minimum resolution of 30 cells per Jeans length is required to see an exponential growth of the magnetic field.

3 Results

3.1 Physical properties

We find that the dynamical evolution of the system is characterized by two distinct phases. First, as the initial turbulent velocity field decays the system exhibits weak oscillatory
behavior and contracts only slowly. Soon, however, the run-away collapse sets in. Figure 1 shows a snapshot of the central region of the collapsing core in our highest-resolution simulation at a time when the central density has increased by a factor of $\sim 10^5$. The magnetic field strength has grown by a factor of $10^6$, reaching peak values of about 1 mG. The left image shows density and velocity structure, and the right image shows magnetic field strength and morphology.

To understand the behavior of the system more quantitatively, we need to account for its dynamical contraction. First, we note that the physical time scale becomes progressively shorter during collapse. We therefore define a new time coordinate $\tau$, $\tau = \int dt/t_{ff}(t)$, based on the local free-fall time, $t_{ff}(t) = \sqrt{3\pi/(32G\rho_m(t))}$, where $\rho_m(t)$ is the mean density of the contracting central region. We can also define a critical volume for gravitational collapse, the so-called Jeans volume, $V_J = 4\pi(\lambda_J/2)^3/3$, with $\lambda_J$ given by equation (1). We obtain all dynamical quantities of interest as averages within the central Jeans volume. This approach ensures that we always average over the relevant volume for collapse and field amplification.

Gravitational compression during the collapse of a primordial gas cloud can at most lead to an amplification of the magnetic field strength by a factor of $\sim \rho^{2/3}$ in the limit of perfect flux freezing (i.e., ideal MHD). A stronger increase implies the presence of an additional amplification mechanism. Starting from an initial field strength of $\sim 1\mu G$, our simulations show a total magnetic field amplification by six orders of magnitude, leading to a field strength of about $\sim 1$ mG for the case where we resolve the local Jeans length by 128 cells. This is illustrated in Fig. 2a. Fig. 2b shows that the obtained field amplification is indeed stronger than what is expected for purely adiabatic compression, which demonstrates that the small-scale turbulent dynamo provides significant additional field
Figure 2. Evolution of the dynamical quantities as a function of $\tau = \int \frac{dt}{\tau}$, defined in equation (2) for five runs with different number of cells to resolve the local Jeans length. Panel (a) - the rms magnetic field strength $B_{\text{rms}}$, amplified to 1 mG from an initial field strength of 1 nG, (b) - the evolution of $B_{\text{rms}}/\rho_{\text{m}}^{2/3}$, showing the turbulent dynamo amplification by dividing out the maximum possible amplification due to pure compression of field lines, (c) - the evolution of the mean density $\rho_{\text{m}}$ and (d) - the rms velocity $v_{\text{rms}}$. The onset of runaway collapse commences at about $\tau \sim 6$. [Figure taken from 13]

amplification over compression. A closer comparison of Fig. 2a and 2b shows that the amplification of the field by compression and by the turbulent dynamo are roughly comparable. The time evolution of the central density $\rho_{\text{m}}$ is depicted in Fig. 2c, while Fig. 2d shows the corresponding rms velocity. The presence of turbulence delays the collapse until $\tau \sim 6$. During this time, the mean density shows some oscillations while the rms velocity decreases as the turbulence decays. A comparison between Fig. 2b and Fig. 2d indicates that dynamo amplification takes place throughout the entire duration of the simulation, that is during the initial phase of turbulent decay, i.e., for $\tau \lesssim 6$, as well as during the run-away collapse phase ($\tau \gtrsim 6$). We note that the time constant for the field amplification is roughly the same in both regimes.
3.2 Implications for numerical resolution

We also point out, that sufficient numerical resolution is a crucial issue when studying the small-scale turbulent dynamo. We see that a minimum of 30 cells per Jeans length is required to unambiguously identify the exponential growth of the magnetic field.\cite{14,18}

Seeing the dynamo process is computationally extremely demanding, and even our \(128^3\) high-resolution run is by no means numerically converged. This reflects the fact that no numerical scheme to date is able to reach the enormous Reynolds numbers, which determines the dynamo growth rate, of star-forming, turbulent gas. Therefore, this simulations show no signs of saturation, thus we simply stop the calculation when the numerical cost becomes prohibitively high. In our subsequent study\cite{30} we investigate the saturation behavior when back-reactions of the Lorentz force become important. Here we find typical values of \(E_{\text{mag}}/E_{\text{kin}}\) of 0.2–0.4 (see Fig.\,3) in agreement with calculations of non-selfgravitating MHD turbulence\cite{31,18}. The physical dissipation scales are much smaller than the Jeans length and thus the growth rates obtained in our simulations are lower limits on the physical growth rates. In Fig.\,4 we visualize various quantities of our \(128^3\) resolution run to give an impression of the complex structure within the central core.
Figure 4.  (a) Spherical slice of the gas density inside the Jeans volume at $\tau = 12$ for our run with 128 cells per Jeans length. (b) Velocity streamlines on a linear color scale ranging from dark blue (0 km s$^{-1}$) to light gray (5 km s$^{-1}$). (c) Magnetic field lines, showing a highly tangled and twisted magnetic field structure typical of the small-scale dynamo; yellow: 0.5 mG, red: 1 mG. (d) Four randomly chosen, individual field lines. The green one, in particular, is extremely tangled close to the center of the Jeans volume. (e) Contours of the vorticity modulus, showing elongated, filamentary structures typically seen in subsonic turbulence. (f) Spherical slice of the divergence of the velocity field, white: compression, red: expansion. [Figure taken from 14]
4 Conclusions

Taken all together, our results strongly indicate that dynamically important magnetic fields are generated during the formation of the first stars. This has important consequences for our understanding of how the first stars form and how they influence subsequent cosmic evolution. We know from modeling galactic star-forming clouds that the presence of magnetic fields can reduce the level of fragmentation, and by doing so strongly influences the stellar mass spectrum. Furthermore, we also know that dynamos in accretion disks produce jets and outflows, which remove a significant fraction of the mass and angular momentum. Again, this influences the stellar mass spectrum. There are first attempts to study this process in the context of first star formation, but more sophisticated initial conditions and more appropriate magnetic field geometries need to be considered. Once the first stars have formed, they are likely to produce a copious amount of ionizing photons, which drive huge HII regions, bubbles of ionized gas, expanding into the low-density gas between the halos. These dynamics could be substantially different if magnetic outflows drive a cavity-wave into the surrounding gas. The magnetic field may further affect fluid instabilities near the ionization front.

The mechanisms discussed here are likely to work not only during the formation of the first stars, but in all types of gravitationally bound, turbulent objects. Highly magnetized gas is thus expected already in the first galaxies, which subsequently needs to become more coherent to form the typical field structures that are observed in the present-day Universe.

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