Extension of warm inflation to noncanonical scalar fields

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We extend the warm inflationary scenario to the case of the noncanonical scalar fields. The equation of motion and the other basic equations of this new scenario are obtained. The Hubble damped term is enhanced in noncanonical inflation. A linear stability analysis is performed to give the proper slow-roll conditions in warm noncanonical inflation. We study the density fluctuations in the new picture and obtain an approximate analytic expression of the power spectrum. The energy scale at the horizon crossing is depressed by both noncanonical effect and thermal effect, and so is the tensor-to-scalar ratio. Besides the synergy, the noncanonical effect and the thermal effect are competing in the case of the warm noncanonical inflation.

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I. INTRODUCTION

Inflation is a quasiexponential expansion (strictly speaking, the inflation is an accelerated expansion and often taken to be a regime of quasiexponential expansion in the majority of models considered in the literatures) in the very early Universe \[L\] which can give successful explanation to the problems such as the horizon and the flatness. As a necessary supplement to the standard cosmological model, the inflation can also produce seeds to give rise to the large scale structure and to the observed little anisotropy of cosmological microwave background (CMB) \[L \] through vacuum fluctuations. Besides the standard inflation, there is also another type of inflation called warm inflation which is proposed by Berera and Fang \[L \]. Radiation is produced constantly through the interaction \[L \int\] between the inflaton field and other subdominated boson or fermion fields during warm inflation so there is no reheating phase. The Universe can smoothly go into the big-bang phase. And the density fluctuations originate mainly from the thermal fluctuations \[L \] rather than vacuum fluctuation. Many problems suffered in standard inflation such as eta problem \[\delta\] and the overlarge amplitude of the inflaton \[\eta\] can be cured in warm inflation. With an additional thermal damped term \[\Gamma\phi\] added to the evolution equation of the inflaton, the slow-roll conditions are much more easily satisfied \[\eta\].

Usually, the inflation can be realized using the canonical scalar field which has the Lagrangian density \[L(X, \phi)\], where \(X = \frac{1}{2} g_{\mu\nu} \partial_\mu \phi \partial_\nu \phi\) and \(V_0\) is the potential of inflaton. But the noncanonical fields have many novel features as the inflaton when the Universe accelerates, such as that the equations of motion remain second order and that the slow-roll conditions become more easily satisfied compared to canonical inflationary theory \[\eta\]. The tensor-to-scala ratio can drop considerably in the most plausible noncanonical models \[\eta\] or increase in some phenomenological models \[\eta\]. Much work has been done about noncanonical standard inflation \[\eta\] and noncanonical fields are the more universal case with a general Lagrangian density satisfied some conditions \[\eta\]. However, warm inflation as a kind of new and realizable inflationary scenario, has always dealt with canonical fields except in \[\eta\] where a warm Dirac-Born-Infeld (DBI) inflationary model was proposed. In this paper we try to extend warm inflation to a general noncanonical scalar field and thus the inflation can have a greater and broader scope. Through the new picture, we can find whether its predictions can be fitted to the observation and what attractive and new features can be obtained.

The paper is organized as follows: In Sec. II we introduce a new noncanonical warm inflationary scenario and get the basic equations of the new picture. Then in Sec. III we propose slow-roll inflation in the new picture and make a fully linear stability analysis to obtain the conditions that guarantee the slow-roll approximation is valid. The scalar and tensor perturbations in the new scenario are performed in Sec. IV. Finally, we draw the conclusions in Sec. V.

II. NONCANONICAL WARM INFLATIONARY SCENARIO

In warm inflationary case, the Universe is a multicomponent system; thus, the total matter action can be given as:

\[
S = \int d^4 x \sqrt{-g} [L(X, \phi) + L_R + L_{\text{int}}]
\]

where the Lagrangian density of the noncanonical field is \(L_{\text{non-can}} = L(X, \phi)\), which can be an arbitrary function of the inflaton field \(\phi\) and the kinetic term \(X\), and for brevity we use \(L\) to stand for \(L(X, \phi)\), \(L_R\) denotes the Lagrangian of the radiation fields and \(L_{\text{int}}\) denotes the interaction term between inflaton and other fields. In order to have a uniform normalization of the field, we will make the Lagrangian density in a form that can reduce...
to canonical case (i.e., $L = X - V_0$) in small $X$ limit. The noncanonical Lagrangian density should satisfy the conditions: $L_X \geq 0$ and $L_{XX} \geq 0$ (where a subscript $X$ here denotes a derivative while the subscripts in $L_R$ and $L_{int}$ are just labels) to obey the null energy condition and the physical propagation of perturbations \[22, 23\]. Through these two conditions and normalization of the field, we can obtain $L_X \geq 1$. The equation of motion can be obtained by taking the variation of the action:

$$\frac{\partial (L(X, \phi) + L_{int})}{\partial \phi} - \sqrt{-g} \partial_{\mu} \left[ \sqrt{-g} \frac{\partial L(X, \phi)}{\partial (\partial_{\mu} \phi)} \right] = 0. \quad (2)$$

In the spatially flat Friedmann-Robertson-Walker Universe, the mean inflaton field is homogeneous, i.e. $\phi = \phi(t)$; hence the equation of motion reduces to

$$\left[ \frac{\partial L(X, \phi)}{\partial X} + 2X \frac{\partial^2 L(X, \phi)}{\partial X^2} \right] \dot{\phi} + \left[ 3H \frac{\partial L(X, \phi)}{\partial X} + \dot{\phi} \frac{\partial^2 L(X, \phi)}{\partial X \partial \phi} \right] \dot{\phi} - \frac{\partial (L(X, \phi) + L_{int})}{\partial \phi} = 0, \quad (3)$$

where $X = \frac{1}{2} \dot{\phi}^2$. Through the energy-momentum tensor of $\phi : T^{\mu\nu} = (\partial L/\partial X) (\partial^\mu \phi \partial^\nu \phi) - g^{\mu\nu} L$, we can get the energy density and pressure of the field: $\rho(\phi, X) = 2X (\partial L/\partial X) - L$, $p(\phi, X) = L$. An important parameter of the noncanonical field is the sound speed which can describe the traveling speed of scalar perturbations: $c_s^2 = p_X(\phi, X)/\rho_X(\phi, X) = (1 + 2X L_{XX}/L_X)^{-1}$, where the subscript $X$ denotes a derivative.

Now we consider a special case that the Lagrangian density can be written in a separable form for the kinetic term and the potential term, i.e. $L = K(X) - V_0(\phi)$, where $K$ is the noncanonical kinetic term that is weakly dependent or independent on $\phi$ \[23\], so we assume $K$ is only the function of $X$. In this case we have $L_X = 0$ and $K_X = L_X$. The general Lagrangian mainly contains two kinds: a series-form Lagrangian and a closed-form Lagrangian \[23\]. The second form can be reduced to a canonical or DBI inflation in the specific gauge $L_X = c_s^{-2} \tilde{L}_X$ \[23\]. The interaction term $L_{int}$ in Eq. (1) is only the function of zero order of the inflaton and other fields but not of the derivative of the fields. The most successful explanation of the interaction between the inflaton and other fields is the supersymmetric two-stage mechanism \[24, 27\]. We use $\Gamma \dot{\phi}$ to describe the dissipation effect of $\phi$ to all other fields \[8, 10, 11\], which is a thermal damping term. The other terms that do not contain $\phi$ in the $\partial L_{int}/\partial \phi$ of Eq. (3) and the term $\partial L(X, \phi)/\partial \phi$ are assumed as the effective potential $V_{eff}$, which is the thermal correction potential and is the function of inflaton and temperature. The detailed introduction of the temperature $T$ in warm inflation can be found in \[8, 23\] etc. The temperature appearing in the effective potential is that of the radiation bath and does not fall to zero thanks to the dissipations of the inflaton to the bath provided that the dependence of temperature in the dissipative coefficient satisfies the condition Eq. (33) obtained in Sec. \[11\]. Under these assumptions the equation of motion can be finally gotten:

$$L_X c_s^{-2} \ddot{\phi} + (3H L_X + \Gamma) \dot{\phi} + V_{eff, \phi}(\phi, T) = 0. \quad (4)$$

For simplicity, we write $V_{eff}$ as $V$ hereinafter, and the subscript $\phi$ denotes a derivative. We can see that the Hubble damping term is $L_X$ times larger than that in canonical inflation.

The total energy density of the multicomponent Universe is

$$\rho = 2XK_X - K(X) + V(\phi, T) + Ts, \quad (5)$$

where $s$ is entropy density. Through the thermodynamics relation $U = F + TS$, we can get the free energy density of the warm inflationary Universe:

$$f = 2XK_X - K(X) + V(\phi, T). \quad (6)$$

Through the definition of entropy in thermodynamics, we can get the expression for $s$:

$$s = -\partial f/\partial T = -V_T(\phi, T). \quad (7)$$

So, the total pressure of the Universe is

$$p = K(X) - V(\phi, T). \quad (8)$$

Combining the total energy conservation equation $\dot{\rho} + 3H(\rho + p) = 0$ and Eq. (4) we can get the entropy production equation:

$$T \dot{s} + 3HTs = \Gamma \ddot{\phi}^2. \quad (9)$$

If the thermal corrections to the potential are little enough (i.e. $b \ll 1$, which can be obtained in slow-roll valid regime; see next section), the radiation energy can be written as $\rho_r = 3T^4s/4$ and Eq. (9) is equivalent to the radiation energy density producing equation:

$$\dot{\rho}_r + 4H \rho_r = \Gamma \dot{\phi}^2. \quad (10)$$

To get a successful inflation that has enough number of e-folds, we should make

$$\epsilon_H = -\frac{\dot{H}}{H^2} = \frac{3}{2} \frac{2X K_X + Ts}{2X K_X - K + V + Ts} \ll 1, \quad (11)$$

which means

$$Ts \ll V, \quad XK_X \sim K \ll V \quad (12)$$

i.e. the noncanonical warm inflation should be potential dominated. The number of e-folds is

$$N = \int H dt = \int \frac{H}{\dot{\phi}} d\phi \approx - \frac{1}{M_p^2} \int_{\phi_*}^{\phi_{end}} \frac{V(L_X + r)}{V_\phi} d\phi, \quad (13)$$

where $r = \Gamma/3H$ is the parameter that describes the damping strength of warm inflation.
III. STABILITY ANALYSIS

In order to make a systematic stability analysis, we define some potential slow-roll parameters which are different from but have relations with the Hubble slow-roll parameters \[ \epsilon = \frac{M_p^2}{2} \left( \frac{V_{,\phi}}{V} \right)^2, \quad \eta = \frac{M_p^2}{2} \frac{V_{,\phi\phi}}{V}, \quad \beta = \frac{M_p^2}{2} \frac{V_{,\phi}}{V} \left( \frac{\dot{V}_{,\phi}}{V} \right) \]
(14)
and two parameters about the temperature dependence:
\[ b = \frac{TV_{,\phi}}{V_{,\phi}}, \quad c = \frac{T\Gamma_{,T}}{\Gamma}. \]
(15)

We define \( u = \dot{\phi} \), and Eqs. (4) and (9) can be rewritten as
\[ \dot{u} = -\mathcal{L}_X u^2 c_s^2 \left[ (3H\mathcal{L}_X + \Gamma)u + V_{,\phi}(\phi, T) \right], \]
(16)
and
\[ \dot{s} = -3Hs + \frac{\Gamma u^2}{T}. \]
(17)

The Friedmann equation is \( H^2 = \frac{\rho}{3M_p^2} \).

Inflation is often associated with slow-roll approximation, which consists of neglecting the highest order terms in Eqs. (14) and (15). The slow-roll approximation implies that the energy is potential dominated, the evolution of inflaton is slow and the production of radiation is quasistatic.

We use \( u_0, \phi_0 \) and \( s_0 \) to denote the slow-roll solutions that satisfy slow-roll equations below:
\[ (3H\mathcal{L}_X + \Gamma)u_0 + V_{,\phi}(\phi, T) = 0, \]
(18)
\[ 3H_0 T_0 s_0 - \Gamma u_0^2 = 0. \]
(19)

The variables \( u, \phi \) and \( s \) can be expanded around the slow-roll solutions: \( u = u_0 + \delta u, \phi = \phi_0 + \delta \phi, s = s_0 + \delta s \). The perturbation terms \( \delta u, \delta s, \) and \( \delta \phi \) are much smaller than the background ones \( u_0, s_0 \) and \( \phi_0 \). The stability is done around the slow-roll solutions, for we should obtain the conditions to guarantee they can really act as formal attractor solutions for the dynamical system.

Using the new variable, \( X = \frac{1}{2} u^2 \), then \( \delta X = u \delta u \), and \( \delta \mathcal{L}_X = \mathcal{L}_X u \delta u \). Varying the Friedmann equation we obtain \( 2H_0 \delta H = \frac{1}{M_p^2} \left[ \mathcal{L}_X c_s^2 u_0 \delta u + V_{,\phi} \delta \phi + T_0 \delta s \right] \).

Through the thermal relation \( s = -V_{,T} \), we have \( \delta s = -V_{,T} \delta T - V_{\phi T} \delta \phi \). Then we can get the variations of \( V, \Gamma \) etc. by using the definition of the slow roll parameters.

Taking the variation of Eqs. (16) (17), we can get
\[ \begin{pmatrix} \delta \phi \\ \delta u \\ \delta s \end{pmatrix} = E \cdot \begin{pmatrix} \delta \phi \\ \delta u \\ \delta s \end{pmatrix} - F. \]
(20)

The matrices \( E \) and \( F \) can be expressed as
\[ E = \begin{pmatrix} 0 & 1 & 0 \\ A & \lambda_1 & B \\ C & D & \lambda_2 \end{pmatrix}, \quad F = \begin{pmatrix} 0 \\ u_0 \\ s_0 \end{pmatrix}. \]
(21)

The matrix elements of \( E \) can be calculated out:
\[ A = \frac{3H_0^2 u_0}{T_0} \left[ \frac{c_s}{(\mathcal{L}_X + r)^2} \begin{pmatrix} \mathcal{L}_X - \eta \mathcal{L}_X - \beta \mathcal{L}_X + r \mathcal{L}_X \end{pmatrix} b + \mathcal{L}_X (1 - c) b \right], \]
(22)
\[ B = \frac{H_0 T_0}{u_0} c_s \begin{pmatrix} -\frac{\epsilon}{(\mathcal{L}_X + r)^2} \left( \frac{c_{s^2}}{\mathcal{L}_X + r} b + \mathcal{L}_X + r \right) \end{pmatrix}, \]
(23)
\[ C = \frac{3H_0^2 u_0}{T_0} \left[ \frac{r}{\mathcal{L}_X + r} - \frac{r}{\mathcal{L}_X + r} \beta + (\mathcal{L}_X + r)(1 - c) b \right], \]
(24)
\[ D = \frac{H_0 u_0}{T_0} \left[ 6r - \frac{r}{\mathcal{L}_X c_s^2}(\mathcal{L}_X + r)^2 \right], \]
(25)
\[ \lambda_1 = -3H_0 \left( 1 + \frac{r c_s}{c_{s^2}} \right) - H_0 c_{s^2} \mathcal{L}_X (\mathcal{L}_X + r)^2, \]
(26)
\[ \lambda_2 = -H_0 (4 - c) - H_0 \frac{r c_s}{c_{s^2}}. \]
(27)

The slow-roll solution can be an attractor for warm inflationary dynamic system only when the eigenvalues of the matrix \( E \) are negative or possibly positive but of order \( \mathcal{O}(\frac{\epsilon}{\mathcal{L}_X + r}) \) (i.e. we would have slow growth) and the "forcing term" \( F \) is small enough, i.e. \( \left| \frac{\delta s}{u_0} \right|, \left| \frac{\delta u}{s_0} \right| \ll 1 \). Now we study the forcing term \( F \) first. Taking the derivative of the slow roll equations (13) (14), we get
\[ \frac{\dot{u}_0}{H_0 u_0} = \frac{c_s^2}{\Delta} \left[ \frac{1}{\mathcal{L}_X + r} \left( 4 - 3 \frac{c_{s^2}}{c_s^2} + \frac{4r}{\mathcal{L}_X} \right) \epsilon + \frac{1}{\mathcal{L}_X + r} \left( \frac{c - 4}{\mathcal{L}_X} \frac{\epsilon}{\mathcal{L}_X + r} \right) \right], \]
(28)
\[ \frac{\dot{s}_0}{H_0 s_0} = \frac{c_s^2}{\Delta} \left[ \frac{1}{\mathcal{L}_X + r} \left( 6 \frac{3}{c_s^2} + \frac{3r}{\mathcal{L}_X} \right) \epsilon - \frac{6}{r} \frac{\eta}{\mathcal{L}_X} \right] \]
\[ + \frac{1}{\mathcal{L}_X + r} \left( \frac{9r}{\mathcal{L}_X} + \frac{3r}{c_s^2} \right) \beta + 6 \frac{\mathcal{L}_X + r}{\mathcal{L}_X r} \left[ 3r(3c - 1) \frac{\epsilon}{\mathcal{L}_X} - \frac{3(1 - c)}{c_s^2} b \right], \]
(29)
where $\Delta \approx (4 - c) + (c + 4) \frac{c^2}{\xi^2}$. The Hubble parameter should also be slowly varying, i.e. $\frac{\dot{H}}{H^2} \approx -\frac{1}{3}$. Then we can get the sufficient conditions to satisfy the above requirements:

$$\epsilon \ll \frac{\mathcal{L}_X + r}{c_s^2}, \quad \beta \ll \frac{\mathcal{L}_X + r}{c_s^2}, \quad \eta \ll \frac{\mathcal{L}_X}{c_s^2}, \quad b \ll \text{min}\left\{\frac{\mathcal{L}_X}{c_s^2}, \frac{\mathcal{L}_X + r}{c_s^2}\right\}$$

where $c_s^2$ is not far less than unity, and when $c_s^2 \ll 1$,

$$\epsilon \ll \frac{\mathcal{L}_X + r}{9}, \quad \beta \ll \frac{\mathcal{L}_X + r}{9}, \quad \eta \ll \frac{\mathcal{L}_X}{9(\mathcal{L}_X + r)}.$$  \hfill (31)

We can reach the conclusion that the slow-roll conditions in our new case are much broader than the canonical warm inflation, let alone standard inflation. The good features are guaranteed by the two large overdamped terms: the larger Hubble damped term and the thermal damped term in Eq. (9). Thus the potential can have a much broader choice and many new models can be embedded into the cosmological inflation. This is the synergy of the two kind effect. And noncanonical effect and thermal effect also has competitive effect. If thermal dissipation dominates over Hubble damping effect, i.e. $r >> \mathcal{L}_X$, the case approximates to the canonical warm inflationary one. In the opposite case $r < \mathcal{L}_X$, thermal effect is weak but still different from cold noncanonical inflation, and the reason we will see later in this paper.

The slow-roll condition for $b$ implies thermal correction to the inflaton potential should be small as in canonical warm inflation [13, 14]. Thus the total energy density can have a nearly separable form $\rho \approx \rho(\phi, X) + \rho_r$.

Now we study the matrix $E$ to give an additional slow-roll condition. Through the slow-roll conditions we have got, we obtain that

$$\det(\lambda - E) = \left| \begin{array}{ccc} \lambda & -1 & 0 \\ -A & \lambda - \lambda_1 & -B \\ -C & -D & \lambda - \lambda_2 \end{array} \right| = (\lambda - \lambda_2) [\lambda(\lambda - \lambda_1) - A] - BD\lambda - BC$$

$$= 0 \quad \hfill (32)$$

has a very small eigenvalue $\lambda \approx \frac{BC - AL}{A - \lambda_2 - BD}\ll \lambda_1, \lambda_2$. The other two eigenvalues satisfy $\lambda^2 - (\lambda_1 + \lambda_2)\lambda + \lambda_1\lambda_2 - BD = 0$. The two eigenvalues are both negative when $\lambda_1, \lambda_2 \ll 0$ and $\lambda_1\lambda_2 - BD > 0$. Finally we get

$$|c| < 4. \quad \hfill (33)$$

The radiation energy density is dominated during the slow-roll inflationary epoch: $\rho_r = \frac{3c^2}{8\pi} \approx 1$ which is consistent with the requirement that the inflaton is potential dominated.

IV. COSMOLOGICAL PERTURBATIONS

Now we develop the theory of cosmological perturbations in the warm noncanonical inflationary theory.
defined as $\frac{\epsilon \phi^2 + m^2}{(3H^2 + 1)H} = 1$. The mass term is negligible compared to other terms in slow-roll inflation. Then we can work out

$$k_F = \sqrt{\frac{3H^2(\mathcal{L}_X + r)}{\mathcal{L}_X}}. \quad (38)$$

Based on the field perturbation relation $\delta \phi^2 = \frac{k_F \phi}{2\pi^2}$, we can finally get the scalar power spectrum in warm non-canonical inflationary model:

$$P_R = \frac{H^3 r}{2\pi^2 V^2} \frac{3(\mathcal{L}_X + r)}{\mathcal{L}_X} = \frac{9H^5 r (\mathcal{L}_X + r)^{\frac{1}{2}}}{2\pi^2 V^2} \sqrt{\frac{3}{\mathcal{L}_X}}. \quad (39)$$

CMB observations provide a good normalization of the scalar power spectrum $P_R \approx 10^{-9}$ on large scales, so we can see from the $(\mathcal{L}_X + r)^{5/2}$ in the numerator that the energy scale when horizon crossing can be depressed by both the noncanonical effect and thermal effect, which is good news to the assumption that the Universe inflation can be described well by effective field theory. The spectral index

$$n_s = 1 = \frac{d\ln P_R}{d\ln k}$$

is given by

$$n_s - 1 = \alpha_1 \frac{c^2}{\mathcal{L}_X + r} + \alpha_2 \frac{c^2}{\mathcal{L}_X + r} + \alpha_3 \frac{c^2}{\mathcal{L}_X + r} + \alpha_4 \frac{c^2}{\mathcal{L}_X + r} + \alpha_5 \frac{c^2}{\mathcal{L}_X + r}, \quad (40)$$

where the expressions for $\frac{\dot{L}}{L}$, $\frac{\dot{c}}{c(L)}$ and $\frac{\dot{r}}{r(L)}$ are used. The parameters $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ and $\alpha_5$ are given by:

$$\alpha_1 = -3c^2 + \frac{rc^2}{2(\mathcal{L}_X + r)} + \frac{1}{\Delta} \left\{ 6 + \frac{3r}{c^2} + \frac{3r}{\mathcal{L}_X} \left[ 1 + \frac{cr}{2(\mathcal{L}_X + r)} \right] - 3 \left( 4 - \frac{cr}{\mathcal{L}_X} \right) \left[ 2 + \frac{r(c_s^2 - 1)}{2(\mathcal{L}_X + r)} \right] \right\}, \quad (41)$$

$$\alpha_2 = \frac{1}{\Delta} \left\{ -6 - \frac{3cr}{\mathcal{L}_X + r} + (12 - 3c) \left[ 2 + \frac{r}{2(\mathcal{L}_X + r)} (c_s^2 - 1) \right] \right\}, \quad (42)$$

$$\alpha_3 = \frac{1}{\Delta} \left\{ \left( \frac{9r}{\mathcal{L}_X} + \frac{3}{c^2} \right) \left[ 1 + \frac{cr}{2(\mathcal{L}_X + r)} \right] - 12 \frac{r}{\mathcal{L}_X} \left[ 2 - \frac{r}{2(\mathcal{L}_X + r)} (c_s^2 - 1) \right] - \frac{4r}{\mathcal{L}_X} \right\}, \quad (43)$$

$$\alpha_4 = \frac{1}{\Delta} \left\{ \frac{3(2rc - 6r - 3)}{\mathcal{L}_X} - \frac{cr}{2(\mathcal{L}_X + r)} \left[ \frac{3r(3c - 1)}{\mathcal{L}_X} - \frac{3(c - 1)}{c_s^2} \right] \right\}$$

$$- \frac{1}{\Delta} \left\{ \frac{2 + \frac{r}{2(\mathcal{L}_X + r)} (c_s^2 - 1)}{2(\mathcal{L}_X + r)} \right\} - \frac{1}{\Delta} \left\{ \frac{min\{\mathcal{L}_X, r\}}{\mathcal{L}_X} \right\}$$

$$- \frac{1}{\Delta} \left\{ \frac{r}{2(\mathcal{L}_X + r)} (c_s^2 - 1) \right\} - \frac{1}{\Delta} \left\{ \frac{min\{\mathcal{L}_X, r\}}{\mathcal{L}_X} \right\}, \quad (44)$$

$$\alpha_5 = \frac{3}{\Delta} \left\{ 2 \left[ 1 + \frac{cr}{2(\mathcal{L}_X + r)} \right] - (c - 4) \left[ 2 + \frac{r(c_s^2 - 1)}{2(\mathcal{L}_X + r)} \right] \right\}. \quad (45)$$

The five parameters above are all of order unity, so we can find that $n_s - 1$ is of order $O \left( \frac{c_s^2}{\mathcal{L}_X + r} \right) \ll 1$, where $\epsilon$ refer to the slow-roll parameters in general. We obtained a nearly scale-invariant power spectrum that is consistent with observations. The running of the spectral index $\alpha_s = \frac{\dot{n_s}}{n_s}$ is calculated to find that it is of order $\left( \frac{c_s^2}{\mathcal{L}_X + r} \right)^2 \ll (n_s - 1)$, which coincides with observations qualitatively. And we can study some concrete models in the new theory numerically and fix the physical quantities by comparing with new observations given by PLANCK satellite in the future.

The tensor perturbations do not couple to the thermal background, and so gravitational waves are only generated by the quantum fluctuations as in standard inflation.
The spectral index of tensor perturbation is $n_T = -\frac{\epsilon}{1 + \epsilon}$, and the tensor-to-scalar ratio is
\[
R = \frac{P_T}{P_R} = \frac{H}{T} \frac{2c\sqrt{L_X}}{\sqrt{3(L_X + r)^{3/2}}}.
\] (48)

We can see that the tensor perturbation can be much weaker thanks to both the noncanonical effect and thermal effect if both the effects are strong, which is another synergy of both effects. Considering the slow-roll condition $\epsilon < \frac{L_X + r}{c^2}$, we can get the upper bound of the tensor-to-scalar ratio $R < \frac{H}{T} \frac{2c\sqrt{L_X}}{\sqrt{3(L_X + r)^{3/2}}}$. As BICEP2 suggests recently, the tensor-to-scalar ratio is significant at a level $R = 0.2 \pm 0.05$ [32], the upper bound of $R$ in our case should be large enough. We can obtain that a significant $R$ prefers a weaker ($r \ll 1$) noncanonical warm inflationary scenario with a big sound speed ($c_s$ is order of unity). The insignificant non-Gaussianity suggested by PLANCK [33] also prefers a big sound speed of the noncanonical inflaton [17, 22]. The amount of expansion is $\Delta N \simeq 4$ while the scales corresponding to $2 \leq l \leq 100$ are leaving the horizon, the corresponding variation of field is $\frac{\Delta \phi}{\phi} = \frac{\Delta N}{N} \simeq 5.2\left(\frac{T}{H}\right)^{1/2}(1 + \frac{L_X}{r})^{1/4}R^2/3$. The field variation can be smaller than Planck scale opposite to standard inflation $\left|\frac{\Delta \phi}{\phi}\right| \sim 0.5\left(\frac{H}{T}\right)^{1/2}$ [30] in strong regime of warm inflation, which can cure the large amplitude of inflaton in standard inflation. The consistency equation becomes $R = -\frac{H}{T} \frac{L_X^{1/2}(1 + r)}{\sqrt{3(L_X + r)^{3/2}}}n_T$, which is not a fixed relation as in standard inflation ($R = -6.2n_T$) [30] anymore.

The radiation energy density and the universal temperature has the Stefan-Boltzmann relationship $\rho_r = \pi^2 g_s T^4/30$, and using the slow-roll equations we obtained
\[
\frac{T}{H} = \left(\frac{r}{g_s P_R}\right)^{1/3} \left(\frac{45}{4\pi^2}\right)^{1/3} \left[3 \left(1 + \frac{r}{L_X}\right)\right]^{1/6}
\] (49)

from the scalar power spectrum. The ratio $T/H$ is smaller than that of warm canonical inflation [13, 15] for the variable $L_X$ in the denominator in the last factor. We can see that a larger $r/L_X$ can enhance the ratio $T/H$; thus, the thermal effect is more obvious and the case is opposite when we have a smaller $r/L_X$, which is the competitive effect of the noncanonical effect and thermal effect. The criterion for the happening of warm inflation $T > H$ can be easily and sufficiently satisfied by $r > g_s P_R$ by analyzing Eq. (49). Considering that $g_s$ is of order $O(10^2)$ and $P_R$ is of order $O(10^{-3})$, we can find very small amounts of dissipation that can result in warm inflation. So the warm inflation can describe the very early Universe more realizably and even in weak dissipative regime $r \ll L_X$, and the thermal fluctuation amplitude dominates over its quantum counterpart, which is the consequence of Eqs. (39) and (49).

V. CONCLUSIONS

We summarize with a few remarks. We develop a theory of warm noncanonical inflationary scenario and generalize the scope of the inflation. Through the action of the warm Universe system, we get the equation of motion for the inflaton and other basic equations of the new scenario. The Hubble damping term is enhanced by an important physical quantity $L_X$ in noncanonical field. The stability analysis is made to give out broader slow-roll conditions thanks to the thermal and noncanonical effect. We obtain a new form but still nearly scale-invariant scalar power spectrum and we find the energy scale during horizon crossing can be depressed by the synergy of the two effects. The tensor-to-scalar ratio can be significant in weak noncanonical warm inflation with a big sound speed and insignificant in the opposite case. Warm noncanonical inflation in strong regime is also a kind of scenario to cure the eta problem and large amplitude of the inflaton. We will focus on some concrete models of the new theory to give more precise comparison with the observations in the future. And the detailed issue of non-Gaussianity in the new scenario also deserves more cognization and research.

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