Numerical investigation for optimization of spark-ignition internal combustion engine knock by Taguchi-Grey method

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Abstract
This article carried out a numerical investigation of knock, using the Taguchi method and the grey relational analysis method to determine the importance and the contribution rate of multiple parameters on the peak pressure in the cylinder and the knock tendency under heavy load conditions. Four parameters, namely, compression ratio, spark timing, EGR rate, and inlet temperature, were set at four levels. The simulation was designed using a design of experiment method based on Taguchi’s \textsuperscript{L}_{16} orthogonal array. The simulation results of knock tendency and peak in-cylinder pressure were analyzed by the Taguchi-Grey method. According to the analysis results of the Taguchi-Grey method, the optimal level, the importance rank, and the contribution rate of factors on the knock tendency, peak in-cylinder pressure, and equivalent response were determined. The results demonstrate that the contribution rate of compression ratio, spark timing, EGR rate, and inlet temperature to the knock tendency is 45.9\%, 22.98\%, 19.46\%, and 11.66\%, respectively. The compression ratio, spark timing, EGR rate, and inlet temperature contribution to the peak in-cylinder pressure is 40.56\%, 31.03\%, 24.94\% and 3.47\%, respectively. The optimal conditions for the minimum knock tendency and the maximum peak in-cylinder pressure are obtained at \textsuperscript{CR}_1\textsuperscript{EGR}_4\textsuperscript{IT}_1\textsuperscript{ST}_1 and \textsuperscript{CR}_4\textsuperscript{EGR}_1\textsuperscript{IT}_1\textsuperscript{ST}_4, respectively.

Keywords
Knock tendency, peak in-cylinder pressure, 3D engine modeling, spark-ignition engine

Introduction
Due to the energy crisis and the ever-increasing stringent automobile emission regulations, the traditional internal combustion engine industry confronts more severe challenges. Energy saving and emission reduction have become major trends in the
development of engines. In improving the thermal combustion efficiency and power performance of internal combustion engines, increasing the compression ratio is one of the most beneficial ways to improve internal combustion engines’ thermal efficiency and power performance. However, the increase in compression ratio will also raise the risk of knock in spark-ignition engines. The engine knock phenomenon is caused by the collision between the normal flame front and the flame front produced by the unburned end gas, which causes the pressure in the cylinder to fluctuate and the internal combustion engine body to resonate. Recently, engine knock has become a significant limitation that restricts the improvement of the economy and power of spark-ignited internal combustion engines. Therefore, many scholars have been attracted to solving spark-ignition engine knock under high load conditions. It has been shown that several operating parameters influence the knocking tendency of an internal combustion engine. With the development of CFD (Computational Fluid Dynamics) technology, more and more scholars use simulation methods to investigate the knock of spark-ignition engines. Li et al. developed a SAGE model with chemical reaction kinetics to explore the influence of EGR (Exhaust Gas Recirculation) on a high compression ratio engine knock. It was found that EGR can reduce the knock intensity and delay the moment when a knock occurs. Zou et al. propose a two-zone combustion model to explore the influence of inlet temperature and compression ratio on knock. The results show that changes in inlet temperature and compression ratio will promote the occurrence of knock. If the inlet temperature is controlled below 336 K and the compression ratio is less than 15, it will not cause the knock. In one study conducted by Chuahy et al., it was shown that spark timing and compression ratio affect knock, and the degree of ignition time’s influence on knock is not as significant as the compression ratio. However, the contribution rate of the two parameters to the specific impact of knock is not given. Based on a predictive knock model, Kumano, Kengo investigated the effect of spark timing and EGR on knock tendency. The results show that the spark timing at the critical knock point can be advanced by 1 CAD (Crankshaft Angle Degree) per 1% of the EGR used. According to Zhen et al., spark timing retarding and EGR technology has outstanding performance in suppressing the knock of high compression ratio internal combustion engines. However, just delaying the spark timing caused a drastic decrease in the in-cylinder pressure, so it is necessary to use EGR technology to suppress knock.

Based on the existing literature, previously published research mainly focused on exploring the independent influence of each factor of compression ratio, inlet temperature, spark timing, and EGR rate on knock. However, few studies highlight the contribution rate and importance ranking of compression ratio, inlet temperature, spark timing, and the EGR rate to knock tendency, which brings difficulties to the control and suppression of spark ignition engine knock. Therefore, this article aims to determine the order of multiple parameters to the influence of knock tendency and peak in-cylinder pressure. After studying the ranking of the impact of multiple parameters on the knock tendency and the peak pressure in the cylinder, it becomes more meaningful to further explore the contribution rate of each parameter to the knock and peak pressure in the cylinder. The quantification of the effect of multiple parameters on knock will also provide a pavement for the synergistic adjustment of multiple parameters to control knock.
The Taguchi method is a mathematical-statistical method proposed by Japanese Genichi Taguchi, which is employed to investigate the impact of multiple design parameters on a single target result. The least orthogonal array experiments can be used to obtain an ideal multi-parameters response analysis effect. This method was often used in product design and development experiment analysis in the past, which could efficiently find the best combination of parameters and determine the influence and contribution rate of each parameter on the response through the signal-to-noise ratio. Although 3D simulation research is more convenient than bench experiments, it is still time-consuming work. Thus, the Taguchi method has gradually been applied to internal combustion engine numerical simulation analysis to solve the time consumption problem. It performs well for exploring multiple parameters for single-objective optimization. However, the Taguchi method can’t accurately explore the influence of multiple parameters on two or more responses simultaneously. With the purpose of investigating the impact of multiple factors on two or more responses, some methods should be adopted to convert two or more responses into an equivalent response. Therefore, the grey relational analysis approach is introduced to analyze the influence of multiple parameters on multiple responses based on the Taguchi method.

This paper aims to fill the gap in quantifying the contribution rate of multi-parameters to knock, and quantify the contribution rates of compression ratio, spark timing, EGR rate, and inlet temperature to the knock tendency and the peak in-cylinder pressure. First, a CFD model is developed and validated to predict the combustion and detonation of a 3-cylinder internal combustion engine. Next, this paper will combine the Taguchi method and gray relational analysis to design an L16 orthogonal test table and use CFD technology to explore the effect of compression ratio, spark timing, EGR rate, and inlet temperature on the knock tendency and the maximum in-cylinder pressure of the engine under a high load operating condition. Finally, each factor’s contribution rates and importance ranking to the knock tendency and the peak in-cylinder pressure are quantified and ranked, respectively.

**Modeling approach and model validation**

**Numerical mesh setup**

The 3D CFD numerical calculations were performed with the commercial CFD package AVL FIRE. After verification of grid independence, the computational mesh used for numerical calculation from IVC (Intake Valve Closing, 365CA) to EVO (Exhaust Valve Opening, 856CA) of the cycle is shown in Figure 1. The total number of grids in Figure 1 is 0.7 million, the maximum grid size is 1 mm, and the minimum grid size is 0.01 mm. In addition, the intake valves, the combustion chamber, and the piston had been refined. Based on the original internal combustion engine model with a compression ratio of 10.5, without changing the intake port model, by changing the height of the piston, grid models with compression ratios of 12, 13, and 14 were generated, respectively. Based on these four grid models with varying compression ratios, the impact of the compression ratio, the EGR rate, the inlet temperature, and the spark timing on the knock tendency and maximum pressure are studied.
Turbulent modeling and combustion modeling. The $z$-$\epsilon$-$f$ model proposed by Hanjalic K is used to simulate the turbulent flow in the cylinder. This four-equation model has high calculation accuracy and good stability, which can better describe the in-cylinder turbulent movement under the premise of good adaptability to mesh uniformity. Researchers widely use the ECFM (Extended Coherent Flame Model) model to simulate and analyze the combustion of spark-ignition internal combustion engines. In the ECFM model, the combustion in the cylinder is divided into two areas by the flame surface. One is the combustion area, and the other is the unburned area. The flame density is determined by equation (1).

$$\frac{\partial \sum}{\partial t} + \frac{\partial u_i \sum}{\partial x_i} = \frac{\partial}{\partial x_i} \left( \frac{\mu}{Sc} + \frac{\mu_t}{Sc_t} \right) \cdot \frac{\partial \sum / \rho}{\partial x_i} + (P_1 + P_2 + P_3) \cdot \sum - D + P_k \tag{1}$$

where $\sum$ is the flame surface density, $\mu$ is the laminar viscosity, $\mu_t$ is the turbulent viscosity, $Sc$ and $Sc_t$ represent laminar and turbulent Schmidt numbers, respectively. $P_1 = \alpha K_t$: Flame surface due to turbulent stretch, $P_2 = (2/3)(\partial u_i / \partial x_i)$: mean flow stretch production term, $P_3 = (2/3)S_f(1-c)\sum$: thermal expansion and curvature production term, $P_k$: spark plug ignition production term, $D = \beta \sum / (\sum^2 / 1 - c)$: the destruction, where $\beta$ is a constant, $c$ is the mean progress variable, and $S_f$ is the mean laminar flame speed.

Knock modeling. Currently, many models are used to predict engine knock, and each model weighs the contradiction between model simulation accuracy and simulation calculation complexity. Based on the ECFM combustion model, the AnB knock model is employed in this article. The AnB knock model is based on auto-ignition delay and uses two chemical reaction processes to simulate the knock in the cylinder.

In the first step, the AnB knock model calculates the growth of a hypothetical knock “precursor”. When the “precursor” quality exceeds the set unburned fuel quality threshold, the chemical oxidation reaction of knock begins. In this second stage, the fuel...
consumption rate can be determined by equation (2).

\[
\frac{dY_{Fu}}{dt} = Y_{Fu} \times 10^4 e^{-3500/T_{gb}} \tag{2}
\]

where \(Y_{Fu}\) means the mass fraction of fuel, and \(T_{gb}\) is the local temperature of the burned mixture.

The AnB knock model determined the crank angle at the start of the auto-ignition delay by equation (3).

\[
\theta = A \left( \frac{RON}{100} \right)^{3.4017} P^{-n} T_{Fr}^{B} \tag{3}
\]

where \(RON\) is the octane number of the fuel, \(P\) and \(T_{Fr}\) respectively represent the pressure and temperature of the unburned mixture, \(A\), \(n\), and \(B\) are the adjustable parameters in the knock model.

During the auto-ignition delay period, the change of the chemical reaction kinetics in the cylinder is far from non-linear. The concentration of the “precursor” showed an approximately exponential change over time. The concentration change of the “precursor” can be described by equation (4).

\[
\frac{dY_{P}}{dt} = Y_{TFu} F(\theta) \tag{4}
\]

with

\[
F(\theta) = \sqrt{(a\theta)^2 + 4(1-a\theta)(Y_{P} / Y_{TFu})} / \theta \tag{5}
\]

where \(\alpha\) is equal to 1 \(s^{-1}\), \(Y_{TFu}\) is the mass fraction of the fuel tracer. When the mass fraction of the precursor locally exceeds the tracer mass fraction, the chemical reaction of knock will be triggered. Therefore, when there is no combustion, the fuel mass fraction equals the tracer mass fraction.\(^{25}\) After auto-ignition, the mass fraction of the “precursor” can be determined by equation (6).

\[
\frac{dY_{P}}{dt} = \beta Y_{TFu} e^{-3500/T_{gb}} \left( \frac{\rho}{\rho_{gf}} \times \frac{Y_{Fu}}{Y_{TFu}} \right) \tag{6}
\]

where \(\beta = 1000 s^{-1}\), \(\rho\) and \(\rho_{gf}\) are the burnt mixture’s density and fresh gas’s density, respectively.

The parameters of the AnB knock model are presented in Table 1. The Knock Reaction Rate is a dimensionless quantity defining the knock tendency.\(^{26,27}\) The larger the value, the higher the probability of knock.

**Boundary conditions and initial conditions.** The operation condition of the simulation is set at 3600 rpm, with 90% whole throttle opening. The model’s initial conditions are demonstrated in Table 2. Table 3 illustrates the temperature boundary condition setting of the simulation calculation. The thermodynamic boundary conditions and initial flow
Table 1. Anb knock model parameters.

| Parameter | Unit  | Value   |
|-----------|-------|---------|
| A         | K     | 0.01932 |
| n         | bar   | 1.7     |
| B         | m²/s³ | 3430    |

Table 2. Boundary conditions of the simulation model.

| Boundary     | Type    | Value (K) |
|--------------|---------|-----------|
| Intake port  | Wall    | 363       |
| Intake valves| Movement| 363       |
| Piston       | Movement| 450       |
| Chamber      | Wall    | 500       |
| Liner        | Wall    | 350       |

Table 3. Initial conditions of the simulation model.

| Type                               | Intake port | Chamber   |
|------------------------------------|-------------|-----------|
| Temperature (K)                    | 322         | 837.75    |
| Pressure (bar)                     | 1.12        | 1.09      |
| Turbulent energy (m²/s²)           | 5           | 5         |
| Turbulence scale (m)               | 0.001       | 0.001     |
| Turbulence dissipation rate (m²/s³)| 1837.12     | 1837.12   |

conditions are determined by the actual operating conditions of the test bench and the GT-power one-dimensional simulation calculation.

Model validation

The internal combustion engine to verify the model is a three-cylinder four-stroke water-cooled naturally aspirated gasoline engine with a displacement of 1 L. The main structural parameters of the engine are illustrated in Table 4. The experimental test diagram for the original engine with a compression ratio of 10.5 is shown in Figure 2.

To validate the combustion model, the comparison of the measured pressure in the bench test and the simulation in-cylinder pressure of 3600 rpm under a no-knock condition is analyzed, as shown in Figure 3. The comparison in Figure 3 shows that the simulation results agree well with the experimental pressure history. The in-cylinder peak error between the experimental data and the simulation results is within 5%. The combustion simulation model can accurately describe the changes in internal combustion engine
pressure. The experimental validation of the knock model often brings irreversible harm to the engines. In addition, many scholars have verified and used the AnB model, which has been proven to have good knock simulation performance.6,24–28

Table 4. Engine specifications and conditions.

| Engine type          | DAM10E                      |
|----------------------|-----------------------------|
| Number of cylinders  | 3, inline                   |
| Displacement (L)     | 1.0                         |
| Bore (mm)            | 74                          |
| Stroke (mm)          | 77.4                        |
| Speed (rpm)          | 3600                        |
| Compression ratio (–) | 10.5                       |
| Top dead center (CAD)| 720                         |
| Fuel injection time (CAD) | 600                      |
| Cooling water temperature (K) | 358 ± 2                   |
| Fuel/air mixture equivalence ratio (–) | 1.0 ± 0.01          |
| Maximum torque at 5200 rpm (N*m) | 92.3                  |
| Maximum power at 6000 rpm (kW)  | 55                       |

Figure 2. Diagram of the research engine testbed. 1-air cleaner, 2-air flow meter, 3-surge tank, 4-throttle, 5-fuel injectors, 6-fuel flow meter, 7-fuel tank, 8-crank angle encoder, 9-engine, 10-ignition system, 11-in-cylinder pressure sensor, 12-dynamometer, 13-EGR cooler, 14-three-way catalytic, 15-electronic control unit, 16-EGR valve, 17-PC control, 18-combustion analyzer.
The Taguchi method is a mathematical-statistical approach proposed by Japanese Genichi Taguchi in 1995, which uses experimental design and analysis of experimental data to evaluate the importance of multiple design parameters to the target result. The best experimental results can be obtained with the least number of experiments. Using the Taguchi method orthogonal design table for experimental parameter design can effectively reduce the number of experiments. The sum of the number of experiments with four factors and four levels requires 256 times. However, only 16 times the orthogonal test is needed after using Taguchi orthogonal array design method. The Taguchi method adopts the signal-to-noise(S/N) ratio to obtain the order of influence of each factor on the results. Finally, each parameter’s importance and contribution rate to the response can be determined by the difference between the parameters’ maximum and minimum mean S/N ratio. The best combination of design parameters can also be obtained.

Parameters selection and levels selection

This paper mainly studies the influence of spark timing, EGR rate, inlet temperature, and compression ratio on an internal combustion engine’s knock tendency and peak in-cylinder pressure. The spark timing should not be set too small. If it is too small, it will cause the combustion to deviate from the ideal cycle and reduce the combustion heat efficiency. Thus, the spark timing is set to 7 BTDC (original engine spark timing), 9 BTDC, 11 BTDC, and 13 BTDC at four levels. The EGR rate is set to four levels: 0%, 4%, 8% and 12%. Because when the EGR rate exceeds 15%, it will affect

Figure 3. Pressure comparison between experiment and simulation without knock combustion.
the stability of internal combustion engine operation.\textsuperscript{33} The inlet temperature is controlled from 313 K to 383 K.\textsuperscript{34} Therefore, the inlet temperature in this study is set to four levels: 330 K, 340 K, 350 K, and 360 K. For exploring the impact of different compression ratios on the knock tendency and the highest in-cylinder pressure of the spark-ignition engine, the combustion chambers with compression ratios of 12, 13, and 14 were generated based on the original internal combustion engine with a compression ratio of 10.5. The parameters levels and values of the engine are displayed in Table 5. The L\textsubscript{16} orthogonal array for simulation is shown in Table 6.

\textbf{Signal-to-noise ratio}

The Taguchi method uses a loss function to express the difference between the simulation results and the ideal value. Then, the loss function is then transformed into Signal-to-Noise (S/N) ratio.\textsuperscript{35} The Taguchi method has three kinds of loss functions: the S/N ratio has a characteristic of “smaller is better”, the S/N ratio has a characteristic of “larger is better”, and the S/N ratio has a characteristic of “nominal-the-better”.\textsuperscript{36}

The S/N ratio has the characteristic of “smaller is better”:

$$\frac{S}{N} = -10 \log \left[ \frac{1}{n} \left( \sum_{i=1}^{n} \frac{1}{y_i^2} \right) \right]$$

The S/N ratio has the characteristic of “larger is better”:

$$\frac{S}{N} = -10 \log \left[ \frac{1}{n} \left( \sum_{i=1}^{n} y_i^2 \right) \right]$$

The S/N ratio has the characteristic of “nominal-the-better”:

$$\frac{S}{N} = 10 \log \left[ \frac{\bar{y}}{S^2} \right]$$

where $y_i$ represents the response of the $i$th experiment within the orthogonal array, $n$ is the total number of the experiments, $S^2$ and $\bar{y}$ are the variance and average of the observations $y_i$, respectively.

Since the study’s main objective is to obtain a higher peak in-cylinder pressure with a minor knock tendency, the S/N ratio with a “smaller is better” characteristic (equation 7) will convert the knock tendency response result into S/N. The S/N ratio with a “larger is better” characteristic (equation 8) will be chosen for the peak in-cylinder pressure.

\begin{table}[h]
\centering
\caption{Parameters levels and their values.}
\begin{tabular}{llllll}
\hline
Engine parameters & Symbol & Unit & Level 1 & Level 2 & Level 3 & Level 4 \\
\hline
Compression ratio & CR & -- & 10.5 & 12 & 13 & 14 \\
EGR rate & EGR & % & 0 & 4 & 8 & 12 \\
Inlet temperature & IT & K & 330 & 340 & 350 & 360 \\
Spark timing & ST & CAD & 7 & 9 & 11 & 13 \\
\hline
\end{tabular}
\end{table}
Grey relational analysis

The S/N ratio of the Taguchi method is often adopted to quantify the effect of multiple parameters on one sole response characteristic. However, the Taguchi method has poor adaptability when applied to optimizing multiple objectives with different signal-to-noise ratios. Therefore, to investigate the influence of multiple factors on multiple responses, the gray relation analysis method can be introduced to convert multiple responses into an equivalent response by considering the gray relation between multiple responses. There are three main steps in the grey correlation analysis method.

Step 1:
Typically, the analyzed responses often have many differences due to their dimensions and units, bringing difficulties to the multi-response conversion in the grey relational analysis methods. Thus, the grey relational analysis method first normalizes the response results $Y_{ij}$ as $X_{ij}$ ($0 < X_{ij} < 1$) to eliminate the above difficulties. There are three different formulas used to normalize the corresponding results.

The normalization formula of “the bigger, the better”:

$$X_{ij} = \frac{Y_{ij} - \min (Y_{ij})}{\max (Y_{ij}) - \min (Y_{ij})}$$

The normalization formula of “the smaller, the better”:

$$X_{ij} = 1 - \frac{|Y_{ij} - Y_{target}|}{\max (Y_{ij}) - Y_{target}}$$

| Case. no | CR  | EGR | IT  | ST |
|----------|-----|-----|-----|----|
| 1        | 10.5| 0   | 330 | 7  |
| 2        | 10.5| 4   | 340 | 9  |
| 3        | 10.5| 8   | 350 | 11 |
| 4        | 10.5| 12  | 360 | 13 |
| 5        | 12  | 0   | 340 | 11 |
| 6        | 12  | 4   | 330 | 13 |
| 7        | 12  | 8   | 360 | 11 |
| 8        | 12  | 12  | 350 | 9  |
| 9        | 13  | 0   | 350 | 13 |
| 10       | 13  | 4   | 360 | 11 |
| 11       | 13  | 8   | 330 | 9  |
| 12       | 13  | 12  | 340 | 7  |
| 13       | 14  | 0   | 360 | 9  |
| 14       | 14  | 4   | 350 | 7  |
| 15       | 14  | 8   | 340 | 13 |
| 16       | 14  | 12  | 330 | 11 |
The normalized formula of “the closer the target value, the better”:

$$X_{ij} = 1 - \frac{|Y_{ij} - Y_{target}|}{\max(Y_{ij}) - Y_{target}}$$  \hspace{1cm} (12)

where $X_{ij}$ is the normalized response result of the $j$th response of the $i$th simulation, $Y_{ij}$ is the $j$th response result of the $i$th simulation, and $Y_{target}$ represents the desired target value.

The peak in-cylinder pressure simulation results are normalized by equation (10); meanwhile, the knock tendency is normalized according to equation (11).

**Step 2:**

The gray correlation coefficient is calculated using equations (13)–(16) to express the connection between the ideal and normalized simulation results. Equations (14)–(16) are adopted to calculate the deviation between the reference and comparability sequences.\textsuperscript{31}

$$n_{ij} = \frac{\Delta_{min} + \rho \Delta_{max}}{\Delta_i(j) \rho \Delta_{max}} \hspace{1cm} (13)$$

$$\Delta_{min} = \min_{i,j} |x^0_j - x_{ij}| \hspace{1cm} (14)$$

$$\Delta_{max} = \max_{i,j} |x^0_j - x_{ij}| \hspace{1cm} (15)$$

$$\Delta_i(j) = |x^0_j - x_{ij}| \hspace{1cm} (16)$$

where $n_{ij}$ represents the gray correlation coefficient of the $j$th response in the $i$th simulation, $x^0_j$ means the normalized ideal value of the $j$th response result (the value in the reference sequence is 1). $x_{ij}$ represents the normalized value of the $j$th response result of the $i$th trial (the value of the comparison sequence). The value of $\rho$ is 0.5.\textsuperscript{40}

In this study, the deviation $\Delta_i(j)$ between the $j$th normalized response results (comparability sequence) of the $i$th simulation and the ideal normalized results (reference sequence) of the $j$th response is calculated by formula 16, where $i$ is from 1 to 16, and $j$ is from 1 to 2.

**Step 3:**

To convert the two responses ($\eta_{11}$ and $\eta_{12}$) into an equivalent response, equation (17) is used to calculate the gray correlation gradient (GCG) of the equivalent response.

$$\eta_i = \frac{1}{\sum_{j=1}^{m} w_j} \sum_{j=1}^{m} w_j n_{ij} \hspace{1cm} (17)$$

where $m$ represents the number of responses, $\eta_i$ means the grey relational grade of the $i$th equivalent response, $w_j$ denotes the normalized weight value of the $j$th response, $w_1 = 0.5$, and $w_2 = 0.5$.

The results of the normalized response, grey relational coefficients, and grey relational grade of the equivalent response calculated based on peak in-cylinder pressure and knock tendency are displayed in Table A1 (in the Appendix).
Results and discussions

According to the simulation sequence in Table 6, the results of the knock tendency and the in-cylinder pressure for each simulation number are illustrated in Figures 4 and 5, respectively. For obtaining the influence order and contribution rate of four parameters on the knock tendency and the peak in-cylinder pressure, the simulation results will be converted into an S/N ratio for analysis purposes. The S/N ratio for the knock tendency and the peak in-cylinder pressure is tabulated in Table 7.

Knock tendency response

Based on Table 7, the S/N ratio of each level under each parameter for the knock tendency can be calculated. For example, to calculate the four levels of signal-to-noise ratio $CR_{SN1}$ (case sequence 1–4), $CR_{SN2}$ (case sequence 4–8), $CR_{SN3}$ (case sequence 9–12), and $CR_{SN4}$ (case sequence 13–16) under the compression ratio of the parameters affecting the knock

Figure 4. The results of 16 sets of knock tendency simulation based on Table 6.
tendency:

\[
CR_{SN1} = ((-16.0993) + (-32.5580) + (-23.0781) + (-23.5795)) / 4 = -23.8287
\]

\[
CR_{SN2} = ((-38.1042) + (-37.2550) + (-30.0450) + (-31.1979)) / 4 = -34.1505
\]

\[
CR_{SN3} = ((-39.9565) + (-37.4042) + (-35.6966) + (-29.8479)) / 4 = -35.7263
\]

\[
CR_{SN4} = ((-38.9884) + (-36.0897) + (-38.1191) + (-35.9574)) / 4 = -37.2886
\]

The different levels of other influencing parameters for knock tendency can also be calculated, like equation (18). Table 8 presents the S/N ratio for the knock tendency at different levels under different parameters. According to Table 7, the main influence diagrams for the knock tendency (Figure 6) S/N ratio can be obtained at different levels under different parameters. No matter which S/N conversion formula is used to convert the response to S/N, the larger the S/N ratio of each level under each parameter in Figure 6, the greater the level is the best in the parameter. The difference in Table 8 between the biggest value and smallest value of the S/N ratio at different levels for each parameter indicates the importance of the parameter’s impact on the knock tendency. The greater the difference, the higher the importance of the parameter. The difference between maximum and minimum is finally converted into the contribution rate of each parameter to the knock effect, as displayed in Figure 8.

According to Figures 6 and 8, to obtain the slightest knock tendency, the best parameters for knock tendency are selected as the compression ratio at level 1 \((CR = 10.5)\), the EGR rate at level 4 \((EGR = 12\%)\), inlet temperature at level 1\(\,(IT = 330\, K)\) and the

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**Figure 5.** The results of in-cylinder pressure simulation based on Table 6.
Table 7. Simulation plan of L16 for the knock tendency and the peak in-cylinder pressure with their S/N ratio.

| Case no. | Compression ratio (-) | EGR rate (%) | Inlet temperature (K) | Spark timing (CA) | Knock tendency (-) KT | Peak in-cylinder pressure (MPa) P_max | S/N |
|----------|-----------------------|--------------|-----------------------|------------------|-----------------------|---------------------------------------|-----|
| 1        | 10.5                  | 0            | 330                   | 7                | 6.382                 | 56.1824                               | −16.0993 34.9920 |
| 2        | 10.5                  | 4            | 340                   | 9                | 42.452               | 60.9610                               | −32.5580 35.7010 |
| 3        | 10.5                  | 8            | 350                   | 11               | 14.253               | 59.7079                               | −23.0781 35.5206 |
| 4        | 10.5                  | 12           | 360                   | 13               | 15.100               | 58.6805                               | −23.5795 35.3699 |
| 5        | 12                    | 0            | 340                   | 11               | 80.391               | 95.5783                               | −38.1042 39.6072 |
| 6        | 12                    | 4            | 330                   | 13               | 72.904               | 97.5154                               | −37.2550 39.7815 |
| 7        | 12                    | 8            | 360                   | 7                | 31.787               | 54.3621                               | −30.0450 34.7059 |
| 8        | 12                    | 12           | 350                   | 9                | 36.299               | 57.3859                               | −31.1979 35.1761 |
| 9        | 13                    | 0            | 350                   | 13               | 99.500               | 119.2320                              | −39.9565 41.5279 |
| 10       | 13                    | 4            | 360                   | 11               | 74.167               | 103.3680                              | −37.4042 40.2877 |
| 11       | 13                    | 8            | 330                   | 9                | 60.930               | 81.5219                               | −35.6966 38.2255 |
| 12       | 13                    | 12           | 340                   | 7                | 31.074               | 54.4108                               | −29.8479 34.7137 |
| 13       | 14                    | 0            | 360                   | 9                | 89.006               | 113.3000                              | −38.9884 41.0846 |
| 14       | 14                    | 4            | 350                   | 7                | 63.751               | 87.4364                               | −36.0897 38.8338 |
| 15       | 14                    | 8            | 340                   | 13               | 80.529               | 122.1800                              | −38.1191 41.7400 |
| 16       | 14                    | 12           | 330                   | 11               | 62.787               | 97.2322                               | −35.9574 39.7562 |
spark timing at level 1 ($ST = 7\text{BTDC}$). According to Table 8 and Figure 8, it is observed that for the knock tendency, the contribution rates of compression ratio, EGR rate, spark timing, and inlet temperature are 45.9%, 19.46%, 22.98%, and 11.66%, respectively. The compression ratio has the most significant influence on knock tendency. The effect of spark timing and EGR rate on knock tendency is similar in importance and lower than the compression ratio. The change of inlet temperature has the slightest impact on knock tendency compared with the other three parameters. Therefore, the order of the most influential factors of knock tendency is compression ratio, spark timing, EGR rate, and inlet temperature. The knock is caused by the spontaneous combustion of the end mixture, which is inseparable from the temperature and pressure of the mixture. According to Figure 6, the minimum knock tendency is obtained at the smallest value of the compression ratio. A smaller compression ratio allows the temperature and pressure of the end mixture to change less drastically than a larger compression ratio, which also reduces the knock tendency. The reason for the reduction in knock tendency with EGR technology is that EGR increases the specific heat capacity of the gas in the

| Parameters | CR       | EGR       | IT        | ST       |
|------------|----------|-----------|-----------|----------|
| Level 1    | -23.8287 | -33.2871  | -31.2521  | -28.0205 |
| Level 2    | -34.1505 | -35.8267  | -34.6573  | -34.6102 |
| Level 3    | -35.7263 | -31.7347  | -32.5806  | -33.6360 |
| Level 4    | -37.2886 | -30.1457  | -32.5043  | -34.7275 |
| Max-Min    | 13.3999  | 5.6810    | 3.4052    | 6.7070   |
| Rank       | 1        | 3         | 4         | 2        |

Table 8. Response table for S/N ratios (KT).

Figure 6. Main effect plots for S/N ratios (response: KT).
cylinder and thus reduces the temperature and pressure in the cylinder. The higher the EGR rate, the more pronounced the knock suppression. When the inlet temperature increases, it causes the mixture temperature in the cylinder to rise after the compression stroke, which increases the propensity for detonation. The earlier the spark timing, the more the mixture burns during the compression stroke, which increases the temperature and pressure to enhance the knock tendency. Therefore, the retardation of spark timing is beneficial to suppress knock.

**Peak in-cylinder pressure response**

Table 7 shows the main influence diagram of the peak in-cylinder pressure signal-to-noise ratio at different levels under different parameters. After a calculation similar to equation (18), the S/N ratio of each level under each parameter for the peak in-cylinder pressure is presented in Table 9. The contribution rate of each parameter to peak the in-cylinder pressure is also displayed in Figure 8. For the maximum pressure in the cylinder, the compression ratio at level 4 \((CR = 14)\), EGR rate at level 1 \((EGR = 0\%)\), inlet temperature at level 1 \((IT = 330 \text{ K})\), and spark timing at level 4 \((ST = 13\text{ BTDC})\) are selected to get the biggest peak in-cylinder pressure. According to Table 9 and Figure 8, it can be identified that for the maximum in-cylinder pressure, the contribution rates of compression ratio, EGR rate, spark timing, and inlet temperature are 40.56%, 24.94%, 3.47%, and 31.03%, respectively. The change of compression ratio has a significant influence on the knock tendency. The change of spark timing has a degree of effect on knock tendency. The spark timing has a more significant influence than the EGR rate. The change of inlet temperature has no significant influence on knock tendency compared with the other three parameters. Thus, the factors that have the greatest influence on the peak in-cylinder pressure are

![Main Effect Plots for S/N Ratios](image_url)

**Figure 7.** Main effect plots for S/N ratios (response: \(P_{\text{max}}\)).
compression ratio, spark timing, EGR rate, and inlet temperature. By looking at Figure 7, the maximum in-cylinder pressure is obtained at the maximum compression ratio. This is because the compression ratio becomes bigger, and the temperature and pressure of the mixture in the cylinder become larger at the end of the compression stroke, thus making the maximum in-cylinder pressure of the internal combustion engine larger. As the EGR increases the specific heat capacity of the cylinder and slows down the flame propagation in the cylinder, the maximum pressure in the cylinder decreases as the EGR rate increases. The maximum in-cylinder pressure is obtained at the lowest level of the EGR rate. An earlier spark timing allows an earlier flame formation in the cylinder. As the flame spreads and the mixture burns in the cylinder, the pressure in the cylinder increases rapidly. Therefore, the maximum in-cylinder pressure is obtained at the maximum spark timing angle.

**Equivalent response**

The gray relational analysis method converts the two responses of knock tendency and peak in-cylinder pressure into an equivalent response to analyzing multiple parameters’

![Figure 8. The contribution rate of each parameter to KT, Pmax, and equivalent response.](image)

**Table 9.** Response table for S/N ratios (Pmax).

| Parameters | CR   | EGR  | IT   | ST   |
|------------|------|------|------|------|
| Level 1    | 35.3959 | 39.3029 | 38.1888 | 35.8114 |
| Level 2    | 37.3177 | 38.6510 | 37.9405 | 37.5468 |
| Level 3    | 38.6887 | 37.5480 | 37.7646 | 38.7929 |
| Level 4    | 40.3537 | 36.2540 | 37.8620 | 39.6048 |
| Max-Min    | 4.9578  | 3.0489  | 0.4242 | 3.7934  |
| Rank       | 1      | 3      | 4    | 2     |

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effects on the two responses. For applying the grey relational analysis method to analyze the effect of multiple parameters on the knock tendency and the maximum pressure in the cylinder, it is necessary to normalize the S/N of the knock tendency and the peak in-cylinder pressure in Table 7. By using equations (10), (11), (13), and (17), the results of the normalized response, grey relational coefficients, grey relational grade, and ranks are listed in Table A1. The fluctuation of the grey relational grade is presented in Figure 9. The gray relational grade determines the equivalent response characteristics, indicating that a large gray relational grade represents a good characteristic of the equivalent response. The parameters of the 15th simulation can obtain the best equivalent response characteristics in these 16 simulations. Based on the data in Table A1, using the calculation methods in Tables 8 and 9, the average grey relational grade of each level under each factor can be obtained, as listed in Table 10. Figure 10 displays the main influence of different levels under each factor on the gray relational grade of equivalent response characteristics. The contribution rate of each parameter to the equivalent response is also listed in Figure 8.

For the equivalent response, according to Figures 9 and 10, and Table 10, the larger the grey relational grade value of each level under each parameter, it means that the level is the best in the parameter. To obtain the best equivalent response, the compression ratio at level 1 \((CR = 10.5)\), the EGR rate at level 1 \((EGR = 0\%)\), the inlet temperature at level 4 \((IT = 360\, K)\), and the spark timing at level 4 \((ST = 13\, BTDC)\) should be selected.

**Figure 9.** Grey relational grade for the equivalent response.

**Table 10.** Mean of grey relational grade (equivalent response).

| Parameters | CR   | EGR  | IT   | ST   |
|------------|------|------|------|------|
| Level 1    | 0.5821 | 0.5865 | 0.5342 | 0.5311 |
| Level 2    | 0.4838 | 0.4879 | 0.5099 | 0.4926 |
| Level 3    | 0.5257 | 0.5611 | 0.5444 | 0.5291 |
| Level 4    | 0.5636 | 0.5198 | 0.5467 | 0.6025 |
| Max-Min    | 0.0984 | 0.0986 | 0.0168 | 0.1100 |
| Rank       | 3     | 2    | 4    | 1    |
According to Figure 8 and Table 10, it is found that for the equivalent response, the contribution rate of the compression ratio is 30.39%, the contribution rate of the EGR rate is 30.45%, the contribution rate of the inlet temperature is 5.19%, and the contribution rate of the spark timing is 33.97%. The spark timing has the most significant influence on the equivalent response compared with the other three parameters. The EGR rate takes a slightly more significant influence on the equivalent response than the compression ratio. The slightest effect on the equivalent response is the inlet temperature. The most influential factors on the equivalent response are spark timing, EGR rate, compression ratio, and inlet temperature. With a combination of in-cylinder burst tendency and maximum in-cylinder pressure, the best equivalent response is obtained at the smallest compression ratio. This is because larger compression ratios have a severe knock tendency. However, the use of larger EGR rates results in significant pressure losses. A smaller EGR and a smaller compression ratio allow the internal combustion engine to use an earlier spark timing to increase the maximum in-cylinder pressure without causing severe knock.

Conclusion

This paper proposes for the first time to quantify the effect of multiple engine parameters on the knock and the peak in-cylinder pressure using the Taguchi-gray method. Firstly, an accurate engine 3D mesh model was created and calibrated based on the bench experiment data. Next, the L16 orthogonal simulation with four parameters and four levels was designed by Taguchi’s method. Then, the simulation results of the burst tendency and the maximum pressure in the cylinder were analyzed using Taguchi’s gray method. Finally, the following conclusions can be obtained.

1. Exploring the influence of multiple parameters on the knock tendency separately, the contribution rate of compression ratio, spark timing, EGR rate, and inlet temperature.
temperature to the knock tendency is 45.9%, 22.98%, 19.46%, and 11.66%, respectively. Therefore, the factors significantly impacting the knock tendency are compression ratio, spark timing, EGR rate, and inlet temperature. The parameter selection: \( CR = 10.5, \ EGR = 12\% , \ IT = 330\ K, \) and \( ST = 7\text{BTDC} \) can obtain the minimum internal combustion engine knock tendency.

2. Exploring the influence of multiple parameters on the average maximum pressure in the cylinder separately, The contribution rate of compression ratio, spark timing, EGR rate, and inlet temperature to the peak in-cylinder pressure is 40.56%, 31.03%, 24.94, and 3.47%, respectively. Thus, the factors that have the most significant influence on the peak in-cylinder pressure are compression ratio, spark timing, EGR rate, and inlet temperature. Parameter selection: \( CR = 14, \ EGR = 0\% , \ IT = 330\ K \) and \( ST = 13\text{BTDC} \) can obtain the maximum in-cylinder pressure.

3. Although the four parameters are ranked in the same order for effect on the knock tendency and the maximum in-cylinder pressure responses, the optimal level of the parameters to obtain the minimum knock tendency and the maximum in-cylinder pressure is different. The contribution rate of each parameter to the knock tendency and the maximum in-cylinder pressure is also different. Compared to the other three parameters, the compression ratio has a dominant influence on the knock tendency and peak in-cylinder pressure responses.

4. Exploring the influence of multiple parameters on the equivalent response, The contribution rate of compression ratio, spark timing, EGR rate, and inlet temperature to the propensity to detonation is 30.39%, 33.97%, 30.45%, and 5.19%, respectively. The most influential factors for equivalent response are spark timing, EGR rate, compression ratio, and inlet temperature. Parameter selection: \( CR = 10.5, \ EGR = 0\% , \ IT = 360\ K \) and \( ST = 13\text{BTDC} \) can obtain the best equivalent response.

The contribution rates of the four parameters to knock are different based on the results. Using the difference in parameter contribution rate provides a new idea for coordinated adjustment of parameters for knock suppression optimization. Because Taguchi’s method uses a limited number of simulations to explore the optimal parameters of knock, it is impossible to find the optimal parameter combination in the overall situation. Therefore, future research uses optimization algorithms to analyze the optimal parameter combination that affects knock globally.

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Author contributions

Yalin Kou designed the study and defined intellectual content, literature search, data acquisition, data analysis, and manuscript preparation. Ying Gao provided funding assistance for the study.
Yurang Wang and Yuelin You revised the manuscript. Yuelin You carried out grammar modification and manuscript editing.

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**Availability of data and material**

The authors confirm that the data supporting the findings of this study are available within the article.

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**References**

1. Chang C and Wei M. Effect of key parameters on knock suppression in a two-stroke spark ignition engine with aviation kerosene fuel. *Proc Inst Mech Eng, Part A* 2019; 233: 1047–1055.
2. Nakama K, Kusaka J and Daisho Y. Study of knock control in small gasoline engines by multidimensional simulation. *SAE Trans* 2006; 115: 999–1013.
3. Kawahara N and Tomita E. Visualization of auto-ignition and pressure wave during knocking in a hydrogen spark-ignition engine. *Int J Hydrogen Energy* 2009; 34: 3156–3163.
4. Nates RJ and Yates ADB. Knock damage mechanisms in spark-ignition engines. *SAE Trans* 1994; 103: 1970–1980.
5. Zhen X, Wang Y, Xu S, et al. The engine knock analysis—an overview. *Appl Energy* 2012; 92: 628–636.
6. Wang Z, Liu H and Reitz RD. Knocking combustion in spark-ignition engines. *Prog Energy Combust Sci* 2017; 61: 78–112.
7. Corrigan D and Fontanesi S. Knock: a century of research. *SAE Int J Engines* 2022; 15: 57–127.
8. Li Y, Wang P, Wang S, et al. Quantitative investigation of the effects of CR, EGR and spark timing strategies on performance, combustion and NOx emissions characteristics of a heavy-duty natural gas engine fueled with 99% methane content. *Fuel* 2019; 255: 115803.
9. Li X, Zhen X, Wang Y, et al. The knock study of high compression ratio SI engine fueled with methanol in combination with different EGR rates. *Fuel* 2019; 257: 116098.
10. Zou F, Zeng H, Wang H, et al. Implementation and parameter analysis of the knock phenomenon of a marine dual-fuel engine based on a two-zone combustion model. *Processes* 2021; 9: 602.
11. Chuahy DF, Splitter D, Boronat V, et al. Enabling high compression ratio in boosted spark ignition engines: thermodynamic trajectory and fuel chemistry effects on knock. *Combust Flame* 2020; 222: 446–459.
12. Kumano K and Yamaoka S. Analysis of knocking suppression effect of cooled EGR in turbo-charged gasoline engine. *SAE Tech Pap* 2014: 2014-01-1217. https://doi.org/10.4271/2014-01-1217.

13. Zhen X, Wang Y, Xu S, et al. Study of knock in a high compression ratio spark-ignition methanol engine by multi-dimensional simulation. *Energy* 2013; 50: 150–159.

14. Karna SK and Sahai R. An overview on Taguchi method. *Int J Eng Math Sci* 2012; 1: 1–7.

15. Antony J and Antony FJ. Teaching the Taguchi method to industrial engineers. *Work Study* 2013; 14: 141–149.

16. Xiao S, et al. Application of CFD, Taguchi method, and ANOVA technique to optimize combustion and emissions in a light duty diesel engine. *Math Probl Eng* 2014; 2014: 1–9.

17. Aoki H, Hayakawa K and Suda N. Numerical analysis on effect of surface asperity of piston skirt on lubrication performance. *Procedia Manuf* 2018; 15: 496–503.

18. Sanjeevannavar MB, Banapurmath NR, Soudagar MEM, et al. Performance indicators for the optimal BTE of biodiesels with additives through engine testing by the Taguchi approach. *Chemosphere* 2022; 288: 132450.

19. Julong D. Introduction to grey system theory. *J Grey Syst-UK* 1989; 1: 1–24.

20. Hanjalić K, Popovac M and Hadžiabdić M. A robust near-wall elliptic-relaxation eddy-viscosity turbulence model for CFD. *Int J Heat Fluid Flow* 2004; 25: 1047–1051.

21. Colin O, Benkenida A and Angelberger C. 3D modeling of mixing, ignition and combustion phenomena in highly stratified gasoline engines. *Oil Gas Sci Technol* 2003; 58: 47–62.

22. Mittal G, Subhash M and Gwalwansi M. Effect of initial turbulence on combustion with ECFM-3Z model in a CI engine. *Mater Today: Proc* 2021; 46: 11007–11010.

23. Duclos JM, Bruneaux G and Baritaud TA. 3D modelling of combustion and pollutants in a 4-valve SI engine; effect of fuel and residuals distribution and spark location. *SAE Trans* 1996; 105: 2048–2062.

24. Lafossas FA, Castagne M, Dumas JP, et al. Development and validation of a knock model in spark ignition engines using a CFD code. *SAE Trans* 2002; 111: 1252–1263.

25. Galloni E, Fontana G and Staccone S. Numerical and experimental characterization of knock occurrence in a turbo-charged spark-ignition engine. *Energy Convers Manage* 2014; 85: 417–424.

26. Jamrozik A and Tutak W. Application of numerical modeling to optimize the thermal cycle of the internal combustion engine. *IMACS* 2012; 11: 43–52.

27. Tutak W. Possibility to reduce knock combustion by EGR in the SI test engine. *Journal of KONES* 2011; 18: 485–492.

28. Li J, Zhou L, Zhao Z, et al. Research on knocking characteristics of kerosene spark-ignition engine for unmanned aerial vehicle (UAV) by numerical simulation. *Therm Sci Eng Prog* 2019; 9: 1–10.

29. Balki MK, Sayin C and Sarıkaya M. Optimization of the operating parameters based on Taguchi method in an SI engine used pure gasoline, ethanol and methanol. *Fuel* 2016; 180: 630–637.

30. Ganapathy T, Murugesan K and Gakckhar RP. Performance optimization of Jatropha biodiesel engine model using Taguchi approach. *Appl Energy* 2009; 86: 2476–2486.

31. Kumar BR, Saravanan S, Sethuramasamyraja B, et al. Screening oxygenates for favorable NOx/smoke trade-off in a DI diesel engine using multi-response optimization. *Fuel* 2017; 199: 670–683.

32. Sayin C. The impact of varying spark timing at different octane numbers on the performance and emission characteristics in a gasoline engine. *Fuel* 2012; 97: 856–861.

33. Cha J, Kwon J, Cho Y, et al. The effect of exhaust gas recirculation (EGR) on combustion stability, engine performance and exhaust emissions in a gasoline engine. *KSME Int J* 2001; 15: 1442–1450.

34. Cinar C, Uyumaz A, Solmaz H, et al. Effects of intake air temperature on combustion, performance and emission characteristics of a HCCI engine fueled with the blends of 20% n-heptane and 80% isooctane fuels. *Fuel Process Technol* 2015; 130: 275–281.
35. Kuar AS, Acherjee B, Ganguly D, et al. Optimization of Nd: YAG laser parameters for micro-drilling of alumina with multi-quality characteristics via grey–Taguchi method. *Mater Manuf Processes* 2012; 27: 329–336.

36. Yuvarajan D, Ravikumar J and Babu MD. Simultaneous optimization of smoke and NOx emissions in a stationary diesel engine fuelled with diesel–oxygenate blends using the grey relational analysis in the Taguchi method. *Anal Methods* 2016; 8: 6222–6230.

37. Mehat NM, Zakaria NS and Kamaruddin S. Investigating the effects of blending ratio and injection parameters on the tensile properties of glass fiber-filled nylon 66 composite gear. *Appl Mech Mater* 2014; 548: 43–47.

38. Kuo Y, Yang T and Huang GW. The use of grey relational analysis in solving multiple attribute decision-making problems. *Comput Ind Eng* 2008; 55: 80–93.

39. Bose PK, Deb M, Banerjee R, et al. Multi objective optimization of performance parameters of a single cylinder diesel engine running with hydrogen using a Taguchi-fuzzy based approach. *Energy* 2013; 63: 375–386.

40. Kuram E and Ozcelik B. Multi-objective optimization using Taguchi based grey relational analysis for micro-milling of Al 7075 material with ball nose end mill. *Measurement* 2013; 46: 1849–1864.

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Table A1. Normalized response, grey relational coefficients, and grey relational grade for the equivalent response.

| Case no | Reference sequence | KT | $P_{\text{max}}$ | $\Delta_i$ | $\Delta_j$ | $\eta_i$ | $\eta_j$ | Rank |
|---------|--------------------|----|-----------------|-------------|-------------|---------|---------|------|
| 1       | 1                  | 1.0000 | 0.0268 | 0.0000 | 0.9732 | 1.0000 | 1.0000 | 0.6697 | 2    |
| 2       | 2                  | 0.6126 | 0.0973 | 0.3874 | 0.9027 | 0.5635 | 0.5635 | 0.4600 | 15   |
| 3       | 3                  | 0.9155 | 0.0788 | 0.0845 | 0.9212 | 0.8554 | 0.8554 | 0.6036 | 4    |
| 4       | 4                  | 0.9064 | 0.0637 | 0.0936 | 0.9363 | 0.8423 | 0.8423 | 0.5952 | 5    |
| 5       | 5                  | 0.2052 | 0.6077 | 0.7948 | 0.3923 | 0.3862 | 0.3862 | 0.4733 | 13   |
| 6       | 6                  | 0.2856 | 0.6363 | 0.71443 | 0.3637 | 0.4117 | 0.4117 | 0.4953 | 9    |
| 7       | 7                  | 0.7272 | 0.0000 | 0.2728 | 1.0000 | 0.6470 | 0.6470 | 0.4902 | 11   |
| 8       | 8                  | 0.6787 | 0.0446 | 0.3213 | 0.9554 | 0.6088 | 0.6088 | 0.4762 | 12   |
| 9       | 9                  | 0.0000 | 0.9565 | 1.0000 | 0.0435 | 0.3333 | 0.3333 | 0.6267 | 3    |
| 10      | 10                 | 0.2721 | 0.7226 | 0.7279 | 0.2774 | 0.4072 | 0.4072 | 0.5252 | 7    |
| 11      | 11                 | 0.4142 | 0.4005 | 0.5858 | 0.5995 | 0.4605 | 0.4605 | 0.4576 | 16   |
| 12      | 12                 | 0.7348 | 0.0000 | 0.2652 | 0.9993 | 0.6535 | 0.6535 | 0.4935 | 10   |
| 13      | 13                 | 0.1126 | 0.8691 | 0.8873 | 0.1309 | 0.3604 | 0.3604 | 0.5764 | 6    |
| 14      | 14                 | 0.3839 | 0.4877 | 0.6161 | 0.5123 | 0.4480 | 0.4480 | 0.4710 | 14   |
| 15      | 15                 | 0.2037 | 1.0000 | 0.7963 | 0.0000 | 0.3857 | 0.3857 | 0.6929 | 1    |
| 16      | 16                 | 0.3943 | 0.6321 | 0.6057 | 0.3679 | 0.4522 | 0.4522 | 0.5142 | 8    |