Investigating the heavy quarkonium radiative transitions with the effective Lagrangian method

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Abstract

In this work, we study the radiative decay of heavy quarkonium states by using the effective Lagrangian approach. Firstly, we construct the spin-breaking terms in the effective Lagrangian for the np ↔ mS transitions and determine some of the coupling constants by fitting the experimental data. Our results show that in χcJ, ψ(2S), Υ(2S), and Υ(3S) radiative decays, the spin-breaking effect is so small that can be ignored. Secondly, we investigate the radiative decay widths of the c¯c(1D) states and find the if ψ(3770) is a pure 3D1 state its radiative decay into χcJ + γ roughly preserve the heavy-quark spin symmetry, while if it is a S−D mixing state with mixing angel 12◦ the heavy quark-spin symmetry in its radiative decay and in the radiative decay of ψ(3686) will be largely violated. In the end, we show that combining the radiative decay and the light hadron decay of P-wave χbJ(1, 2P) can provide another way to extract the information of the color-octet matrix element in the context of non-relativistic QCD (NRQCD) effective theory, and our result is consistent with potential NRQCD hypothesis.

Keywords: radiative decay, spin-breaking, color-octet matrix elements

1. Introduction

The heavy quarkonium states that are constituted by heavy-quark (Q) and anti-quark (Q̄) pair provide an ideal laboratory to study the dynamics of strong interaction from both perturbative and non-perturbative aspects. In recent years, thanks to the large amount data accumulated in electron-positron colliders and hadronic colliders, more accurate and new properties of them have been obtained. Especially, many new resonances were discovered, which gave rise to a great renewed theoretical interest in studying their spectra and decays (for recent reviews see Ref.[1] and references therein).

For the states below open flavor threshold (D ¯D for charmonium and BB for bottomonium), they have relative narrow width because they can not decay through the Okubo-Zweig-Iizuka allowed decay mechanism. Their radiative decay width could reach hundred kev level, therefore, contributes a considerable branching ratio. On the experimental side, the radiative transitions among heavy quarkonia also play an important role in searching for the new states. Theoretically, the heavy quarkonium states are approximate nonrelativistic systems. Their annihilation decays are sensitive to the wave functions of QQ at small distance, on the contrary the radiative decay can help us to probe the behavior of wave functions at long-distance. Besides the intrinsic scale ΛQCD, heavy quarkonia are characterized by a hierarchy of three energy scales, mQ the heavy quark mass, mQvQ and mQvQ2 the typical momentum and energy of the heavy quark, where vQ ≪ 1 is velocity of the heavy quark in the rest frame of the heavy meson. The nonrelativistic effective field theories, nonrelativistic QCD (NRQCD) [2, 3, 4], and potential NRQCD (pNRQCD) [5, 6, 7] are suitable tools to separate the physics in different energy scales. Recently, the magnetic dipole (M1) transition as well as the radiative decay of X(3872) was studied within the framework of pNRQCD [8]. Besides from the model-independent perspective, the radiative transitions among heavy quarkonia have already been extensively studied within potential model approach (here we refer Ref.[9] as a comprehensive review).

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In this letter, we will not only calculate the radiative decay widths, but will also do some further analysis by taking into account the spin-breaking effect or the $S - D$ mixing effect in the radiative decays. We will also combine the radiative decay with the light hadron (LH) decay of the $\chi_J^0(nP)$ states to extract the information of the color-octet (CO) matrix elements in NRQCD\cite{4} and pNRQCD\cite{10,11}. In this work, we plan to employ the effective Lagrangian approach, which can exploit the heavy quark spin symmetry order by order through the expansion of $1/m_Q$. The rest of this letter is organized as follows. A brief description of the effective Lagrangian approach will be given in Section 2, and then is used to study the $S \leftrightarrow P$ transitions in $c\bar{c}$ and $b\bar{b}$ systems by taking into account spin-breaking effect. In section 3, we will study the $\psi(3D_1) \rightarrow \chi_{cJ}^0 + \gamma$ transitions. In section 4, we will relate the transitions of $\chi_J^0(nP) \rightarrow \Upsilon(nS) + \gamma$ to the LH decay of $\chi_J^0(nP)$ states to determine the ratios of the CO matrix elements $m_Q^n\mathcal{H}(nP)$ to the corresponding color-singlet (CS) matrix elements $\mathcal{H}(nP)$, where $\mathcal{H}(nP) = \langle \psi(nP)\bar{Q}_1(\gamma P)_J|\chi_J^0(nP)\rangle$, and $\mathcal{H}(nP) = \langle \chi_J^0(nP)|\bar{Q}_1(\gamma P)_J|\psi(nP)\rangle$ \cite{4}. A short summary and conclusion will be presented in the last section.

2. Effective Lagrangian For Radiative Transitions

The heavy charmonium states can be classified according to the spectroscopic notation $n^S L J$, where $n = 1, 2, \ldots$ is the radial quantum number, $S = 0, 1$ is the total spin of the heavy quark pair, $L = 0, 1, 2 \ldots$ (or $S, P, D \ldots$) is the orbital angular momentum, and $J$ is the total angular momentum. They have parity $P = (-1)^{L+S}$ and charge conjugation $C = (-1)^{S+1}$. As mentioned in the introduction, NRQCD and pNRQCD are a good starting point to approximate symmetry of the heavy quarkonium states, that is inherited in the subsequent effective theories. Hence, it is most convenient to introduce hadronic spin-symmetry multiplets, in an analogous way as it was initially done in Heavy Quark Effective Theory (HQET)\cite{12}.

For heavy quarkonium states, this formalism was developed in Ref.\cite{13}. The states have the same radial number $n$ and the same orbital momentum $L$ can also be expressed by means of a single multiplet: $J^{\mu_1...\mu_L}$\cite{13},

$$J^{\mu_1...\mu_L} = \frac{1 + \gamma}{2}(H_{L+1}^{\mu_1...\mu_L})^q + \frac{1}{\sqrt{L+1}} \sum_{i=1}^{L} e^{\mu_0\beta\gamma_i} v_\beta H_L^{\mu_1...\mu_L}$$

$$+ \frac{1}{L} \sum_{i=1}^{L} (\gamma_i - \gamma_i H_{L-1}^{\mu_1...\mu_L}$$

$$- \frac{2}{L} \sum_{i=1}^{L} (\gamma_i - \gamma_i H_{L-1}^{\mu_1...\mu_L}$$

$$+ K_L^{\mu_1...\mu_L} \gamma_0 \frac{1 - \gamma}{2}$$

(1)

where $v_i$ is the four-velocity associated to the multiplet $J^{\mu_1...\mu_L}$ (not to be mistaken by $v_Q$, the typical velocity of the heavy quark in the heavy quarkonium rest frame), $K_L^{\mu_1...\mu_L}$ represents the spin-singlet effective field, and $H_{L-1}^{\mu_1...\mu_L}$ and $H_L^{\mu_1...\mu_L}$ represent the three spin-triplet effective fields with $J = L - 1, L$, and $L + 1$ respectively. The four tensors are all completely symmetric and traceless and satisfy the transpose condition

$$v_\mu K_L^{\mu_1...\mu_L} = 0 \quad v_\mu H_L^{\mu_1...\mu_L} = 0$$

(2)

$i = 1, \ldots, L, \ j = 1, \ldots, J$. The properties of $H$ and $K$ under parity, charge conjugation and heavy quark spin transformations can be easily obtained by assuming that the corresponding transformation rules of the multiplet $J^{\mu_1...\mu_L}$ follow as:

$$J^{\mu_1...\mu_L} \xrightarrow{P} \gamma_0 J^{\mu_1...\mu_L} \gamma_0, \ v_\mu \xrightarrow{P} v_\mu$$

(3a)

$$J^{\mu_1...\mu_L} \xrightarrow{C} (-1)^{L+1} C[J^{\mu_1...\mu_L}]^T C,$$

(3b)

$$J^{\mu_1...\mu_L} \xrightarrow{S} S J^{\mu_1...\mu_L} S^T,$$

(3c)
where $C$ is the charge conjugation matrix ($C = iγ^2γ^0$ in the Dirac representation), and $S \in SU(2)Q$ and $S' \in SU(2)\bar{Q}$ correspond to the heavy quark and heavy antiquark spin symmetry groups ($[S, \bar{S}] = [S', \bar{S}] = 0$).

Since we are going to consider the $S$, $P$, and $D$ wave states radiative decay, it will be helpful to give the explicit expressions of the $S$-, $P$-, and $D$-wave multiplets that follow from Eq. (1). For the $L = S$ case, we have

$$ J = \frac{1 + \hat{γ}}{2}(H^μ_1γ_μ - K_0γ^5)\frac{1 - \hat{γ}}{2}, $$

for the $L = P$ case,

$$ J^μ = \frac{1 + \hat{γ}}{2}H^μ_2γ_μ + \frac{1}{\sqrt{2}}e^{μνβγ}γ_νy_βH^γ_1y_μ + \frac{1}{\sqrt{3}}(γ^μ - γ^ν)H_0 + K^μ_1γ^5\frac{1 - \hat{γ}}{2}, $$

and for $L = D$ case,

$$ J^{μν} = \frac{1 + \hat{γ}}{2}(H^μ_3γ_μ + \frac{1}{\sqrt{2}}\epsilon^{μνβγ}γ_νy_βH^γ_2y_μ + \epsilon^{νμβγ}γ_βy_γH^β_3y_μ)
+ \sqrt{\frac{2}{3}}((γ^μ - γ^ν)H^1_1 + (γ^ν - γ^μ)H^μ_1 - \frac{1}{4}(γ^μγ^ν - \frac{1}{2}γ^νγ^μ)γ_0H_0^0)
+ K_0^μνγ^5\frac{1 - \hat{γ}}{2}. $$

At the leading order of $1/mQ$ expansion, the radiative transitions between $mS$ and $nP$ states, and between $mP$ and $nD$ states can be described by the Lagrangian given in Ref. [13][14]:

$$\mathcal{L}^{SP} = \sum_{m,n} \delta^{p,mS}_Q Tr[J(mS)J_μ(nP))y_μF^{μν} + h.c., $$

and

$$\mathcal{L}^{PD} = \sum_{m,n} \delta^{p,mD}_Q Tr[J(nP)J_μ(0S))y_μF^{μν} + h.c., $$

where $\delta^{p,mS}_Q (Q = c, b)$ and $\delta^{p,mD}(Q = c, b)$ are the coupling constants, and $F^{μν}$ is the electromagnetic tensor. The Lagrangian in Eq. (7) preserves parity, charge conjugation, gauge invariance and heavy quark and antiquark spin symmetry.

The radiative decays of the low lying $S$- and $P$- wave states have been well measured. It will be interesting to do some delicate analysis beyond leading order. One important higher order contribution comes from the spin-breaking effect which is due to the spin-spin $S_0 \cdot S_0$, spin-orbit $L \cdot S$, and tensor $S_0 \times S_0$ interactions in pNRQCD [15] (for potential models), where $S_0$ and $S_0$ are the spin of the quark and anti-quark respectively, $S = S_0 + S_0$, and $L$ is the orbital angular momentum of the heavy meson. To figure out all the spin-breaking terms in the effective Lagrangian, it will be more perspicuous to construct them in the rest frame of the heavy meson, where the $4 \times 4$ dimensional space is reduced to the $2 \times 2$ dimensional space. In the two component notation, the field $J$ and $J^μ$ is simplified as:

$$ J = H \cdot \vec{σ} + K_0, J^μ = (H^μ_2 + \frac{1}{\sqrt{2}}\epsilon^{μνβγ}γ_νy_βH^γ_1y_μ + \frac{1}{\sqrt{3}}(γ^μ - γ^ν)H_0 + K^μ_1γ^5), $$

where $\vec{σ}$ is the Pauli matrix. The Lagrangian in Eq. (7a) becomes:

$$\mathcal{L}^{SP} = \sum_{m,n} \delta^{p,mS}_Q Tr[J^μ(mS)J_μ(nP))E^μ + h.c., $$

In the $2 \times 2$ dimensional space, the spin breaking terms can only be in the form of $\vec{σ} \cdot \vec{σ}$, where $\vec{σ}$ is an arbitrary three dimensional vector. After analyzing all the possible combinations of the field and $\vec{σ}$ operators, we find that there are three independent spin-breaking terms at sub-leading order in $1/mQ$, which are given by

$$\mathcal{L}_{0S}^{SP} = \delta^{p,mS}_Q (Tr[J^μσ^μJ^μσ^μ])E^μ + h.c., $$

and

$$\mathcal{L}_{0L}^{SP} = -i \frac{\delta^{p,mS}}{2} \epsilon^{μνκ} (Tr[J^μσ^νJ^κ] - Tr[J^νσ^κJ^μ])E^μ + h.c..$$
Table 1: The numerical values of the coupling constants $\delta_{Q,j}^{P_{mS}}$ (GeV$^{-1}$) determined by fitting the experimental decay widths.

| Charmion | Bottomonium |
|----------|-------------|
| Decay Width (keV) | $\delta_{Q,j}^{P_{mS}}$ (GeV$^{-1}$) | Decay Width (keV) | $\delta_{Q,j}^{P_{mS}}$ (GeV$^{-1}$) |
| $\Gamma(\chi_{c0} \rightarrow J/\psi + \gamma)$ | $121.7 \pm 10.9$ | $2.13 \pm 0.95 \times 10^{-1}$ | $\Gamma(\Upsilon(2S) \rightarrow \chi_{bc}(1P) + \gamma)$ | $1.22 \pm 0.16$ | $(9.01 \pm 0.50) \times 10^{-2}$ |
| $\Gamma(\chi_{c1} \rightarrow J/\psi + \gamma)$ | $295.8 \pm 21.5$ | $2.31 \pm 0.08 \times 10^{-1}$ | $\Gamma(\Upsilon(2S) \rightarrow \chi_{bc}(1P) + \gamma)$ | $2.21 \pm 0.22$ | $(9.89 \pm 0.50) \times 10^{-2}$ |
| $\Gamma(\chi_{c2} \rightarrow J/\psi + \gamma)$ | $384.2 \pm 26.6$ | $2.29 \pm 0.08 \times 10^{-1}$ | $\Gamma(\Upsilon(2S) \rightarrow \chi_{bc}(1P) + \gamma)$ | $2.29 \pm 0.22$ | $(9.86 \pm 0.47) \times 10^{-2}$ |
| $\Gamma(\psi' \rightarrow \chi_{c0} + \gamma)$ | $29.2 \pm 1.3$ | $2.25 \pm 0.04 \times 10^{-1}$ | $\Gamma(\Upsilon(3S) \rightarrow \chi_{bc}(2P) + \gamma)$ | $1.20 \pm 0.16$ | $(1.39 \pm 0.09) \times 10^{-1}$ |
| $\Gamma(\psi' \rightarrow \chi_{c1} + \gamma)$ | $28.0 \pm 1.5$ | $2.36 \pm 0.06 \times 10^{-1}$ | $\Gamma(\Upsilon(3S) \rightarrow \chi_{bc}(2P) + \gamma)$ | $2.56 \pm 0.34$ | $(1.57 \pm 0.10) \times 10^{-1}$ |
| $\Gamma(\psi' \rightarrow \chi_{c2} + \gamma)$ | $26.6 \pm 1.3$ | $2.74 \pm 0.07 \times 10^{-1}$ | $\Gamma(\Upsilon(3S) \rightarrow \chi_{bc}(2P) + \gamma)$ | $2.66 \pm 0.41$ | $(1.55 \pm 0.12) \times 10^{-1}$ |

$\Gamma_{PQ}^{SP} = \frac{\delta_{Q,j}^{P_{mS}}}{2}(\text{Tr}[J'\sigma^j J/\sigma^j] + \text{Tr}[\Upsilon(3S) \rightarrow \chi_{bc}(2P) + \gamma]$ ($10c$)

where $\delta_{Q,j}^{P_{mS}}$ and $\delta_{Q,j}^{P_{mS}}$ are the coupling constants that are suppressed by $1/m_Q^2$, since the spin-breaking potentials is of $O(1/m_Q^2)$ compared to the static potential $[15]$. After including the spin-breaking contribution, the formula of the $E1$ transition decay widths turn to be:

$$\Gamma(m^3S_1 \rightarrow n^3P_J) = (2J + 1) \frac{(\delta_{Q,j}^{P_{mS}})^2}{9\pi} k_\gamma^3 \frac{M_{nJ}}{M_{mS}}$$

For $m^3P_J \rightarrow m^3S_1$ states in pNRQCD (or in any potential model, see Ref. [3] for a recent review). For some processes, such as $\chi_{cJ} \rightarrow J/\psi + \gamma$, $\psi' \rightarrow \chi_{cJ} + \gamma$, $\Upsilon(2S) \rightarrow \chi_{bc}(1P) + \gamma$, and $\Upsilon(3S) \rightarrow \chi_{bc}(2P) + \gamma$, their decay widths have been measured $[16]$, so we can obtain the values of the corresponding coupling constants $\delta_{Q,j}^{P_{mS}}$ (for $J=0,1,2$) by fitting the data, which are list in Table 1. Note, in our treatment of the uncertainties, we only take into account the uncertainties in total decay widths and the branching ratios. The formulas in Eq. (12) show that up to the sub-leading order $\delta_{Q,j}^{P_{mS}}$ (for $J=0,1,2$) only depends on $\delta_{Q,j}^{P_{mS}} - \delta_{Q,j}^{P_{mS}}$, $\delta_{Q,j}^{P_{mS}}$, and $\delta_{Q,j}^{P_{mS}}$. After resolving Eq. (12), we obtain that

$$\delta_{Q,j}^{P_{mS}} - \delta_{Q,j}^{P_{mS}} = 22.7 \times 10^{-2} \text{GeV}^{-1}, \delta_{Q,j}^{P_{mS}} - \delta_{Q,j}^{P_{mS}} = 25.6 \times 10^{-2} \text{GeV}^{-1},$$

$$\delta_{Q,j}^{P_{mS}} - \delta_{Q,j}^{P_{mS}} = 9.7 \times 10^{-2} \text{GeV}^{-1}, \delta_{Q,j}^{P_{mS}} - \delta_{Q,j}^{P_{mS}} = 15.3 \times 10^{-2} \text{GeV}^{-1},$$

(13)
Table 2: The values of the coupling constants for the spin-breaking terms $\delta_{QL}^{PnS}$ and $\delta_{QT}^{PnS}$ determined by fitting the experimental decay widths (unit $10^{-5}$ GeV$^{-1}$).

| $\delta_{JT}^{1P,1S}$ | $\delta_{JT}^{1P,1S}$ | $\delta_{JT}^{1P,2S}$ | $\delta_{JT}^{1P,2S}$ | $\delta_{JT}^{1P,2S}$ | $\delta_{JT}^{1P,2S}$ | $\delta_{JT}^{1P,2S}$ | $\delta_{JT}^{1P,2S}$ |
|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| $-0.3 \pm 0.4$       | $-0.4 \pm 0.3$       | $-1.8 \pm 0.3$       | $0.1 \pm 0.2$        | $-0.1 \pm 0.3$       | $-0.1 \pm 0.2$       | $-0.2 \pm 0.6$        | $-0.3 \pm 0.3$       |

The values of the corresponding $\delta_{QL}^{PnS}$, and $\delta_{QT}^{PnS}$ are given in Table 2. If we assume that $\delta_{QS}^{PnS}$, $\delta_{QL}^{PnS}$, and $\delta_{QT}^{PnS}$ are in the same order, the results in Table 2 and those in Eq.(13) will indicate that in these decay processes the contribution of the spin-breaking effect is less than that of the leading order term by at least a factor of 10, and furthermore comparing to the $2S \rightarrow 1P$ transition process in charmonium system the spin-breaking effect in the bottomonium system $2S \rightarrow 1P$ process is of $(m_c/m_b)^2$ suppressed, which is consistent with the power counting rule of pNRQCD [15].

3. Radiative Transitions of $\psi(1^3D_J) \rightarrow \chi_{cJ} + \gamma$

The spectrum of $D$-wave heavy quarkonia has been calculated in potential model by many groups, for example recently in Ref. [16, 18]. In $c\bar{c}$ system, the $\psi(3770)$ state is treated as a pure $3^3D_1$ state or a predominant $D$-wave state with a small admixture of $2S$ state [19, 20]. The other states $1^3D_2$, $2^3D_2$, and $1^3D_2$, whose decay widths are all expected to be narrow, have not been observed yet. The masses of $\psi(3^3D_2)$ and $\eta_{3c}(1^3D_2)$ predicted by potential models lie between $DD$ and $D^*D$ thresholds [17]. They are forbidden to decay into a pseudoscalar pair by parity. The narrowness of $\psi(3^3D_3)$ is due to that its decay into $D\bar{D}$ is a $F$-wave decay, which is highly suppressed. Hence, the branching ratios of their radiative decays are all considerable. The widths of the radiative transition $nD$ to $mP$ can be calculated straightforwardly by employing the Lagrangian in Eq. (7b):

$$\Gamma(n^3D_J \rightarrow m^1P_J) = \frac{1}{3\pi} \left(\frac{\delta_{Q}^{nP,nP}}{M_{np}}\right)^2 \frac{M_{mp}}{M_{nD}} \quad \text{for} \quad S_{J,J} = 5/9, 5/12, 1/36, S_{2,J} = 3/4, 1/4, \text{and} \ S_{3,J} = 0, 0, 1 \text{ for } J = 0, 1, 2 \text{respectively.}$$

where the coefficients $S_{J,J}$ are $S_{1,J} = 5/9, 5/12, 1/36, S_{2,J} = 0, 3/4, 1/4, \text{and} S_{3,J} = 0, 0, 1$.

The decay widths of $\psi(3770)$ decay to $\chi_{c0} + \gamma$ and $\chi_{c1} + \gamma$ given in PDG are [16]:

$$\Gamma(\psi(3770) \rightarrow \chi_{c0}\gamma) = 199 \pm 26 \text{ keV}, \quad \Gamma(\psi(3770) \rightarrow \chi_{c1}\gamma) = 79 \pm 17 \text{ keV}.$$  

(15)

If $\psi(3770)$ is a pure $1D$ state, the values of the coupling constant $\delta_c^{1DIP}$ determined through $\Gamma(\psi(3770) \rightarrow \chi_{cJ} + \gamma$ are $\delta_c^{1DIP} = 0.31 \pm 0.02 \text{ GeV}^{-1}$, and $\delta_c^{1DIP} = 0.35 \pm 0.04 \text{ GeV}^{-1}$ for $J = 0$ and $J = 1$, respectively, which are very close to each other. In this case, it indicates that the heavy quark spin symmetry is roughly preserved in the $1D$ to $1P$ radiative transitions. The average value is

$$\delta_c^{1DIP} = 0.32 \pm 0.02 \text{ GeV}^{-1}.$$  

(16)

If we treat $\psi(3770)$ as a S-D mixing state and using the same notation in Ref.[20]:

$$\psi(3686) = \cos(\theta)|2S⟩ - \sin(\theta)|1D⟩, \quad \psi(3770) = \cos(\theta)|1D⟩ + \sin(\theta)|2S⟩,$$

(17)

For the $3^3D_1$ decay into $^1P_J + \gamma$, our results do not agree with those in Ref. [14]. After private communications, their new results in the erratum [21] now agree with ours.
the analytical formulas for \( \psi(3686) \) and \( \psi(3770) \) decay into \( \gamma + \chi_{cJ} \) turn to be

\[
\Gamma(\psi(3686) \to \chi_{c0} + \gamma) = \frac{3 M_{\chi_{c0}}}{9 \pi M_{\psi(3686)}} k_1^3 \left( \cos^2(\theta) \delta_{1c} \right)^2 \left( \cos^2(\theta) \delta_{2c} \right)^2 - 2 \sqrt{\frac{5}{3}} \sin(\theta) \cos(\theta) \delta_{1c} \delta_{2c} + \frac{5}{3} \sin^2(\theta) \delta_{1c}^2 \delta_{2c}^2 \)  
\]

\[
\Gamma(\psi(3686) \to \chi_{c1} + \gamma) = \frac{3 M_{\chi_{c1}}}{9 \pi M_{\psi(3686)}} k_1^3 \left( \cos^2(\theta) \delta_{1c} \right)^2 + \sqrt{\frac{5}{3}} \sin(\theta) \cos(\theta) \delta_{1c} \delta_{2c} + \frac{5}{12} \sin^2(\theta) \delta_{1c}^2 \delta_{2c}^2 \)  
\]

\[
\Gamma(\psi(3686) \to \chi_{c2} + \gamma) = \frac{5 M_{\chi_{c2}}}{9 \pi M_{\psi(3686)}} k_1^3 \left( \cos^2(\theta) \delta_{1c} \right)^2 - \frac{1}{15} \sin(\theta) \cos(\theta) \delta_{1c} \delta_{2c} + \frac{1}{60} \sin^2(\theta) \delta_{1c}^2 \delta_{2c}^2 \)  
\]

\[
\Gamma(\psi(3770) \to \chi_{c0} + \gamma) = \frac{5 M_{\chi_{c0}}}{27 \pi M_{\psi(3770)}} k_2^3 \left( \cos^2(\theta) \delta_{1c} \right)^2 + 2 \sqrt{\frac{5}{3}} \sin(\theta) \cos(\theta) \delta_{1c} \delta_{2c} + \frac{3}{5} \sin^2(\theta) \delta_{1c}^2 \delta_{2c}^2 \)  
\]

\[
\Gamma(\psi(3770) \to \chi_{c1} + \gamma) = \frac{5 M_{\chi_{c1}}}{56 \pi M_{\psi(3770)}} k_2^3 \left( \cos^2(\theta) \delta_{1c} \right)^2 - 4 \sqrt{\frac{5}{3}} \sin(\theta) \cos(\theta) \delta_{1c} \delta_{2c} + \frac{12}{5} \sin^2(\theta) \delta_{1c}^2 \delta_{2c}^2 \)  
\]

\[
\Gamma(\psi(3770) \to \chi_{c2} + \gamma) = \frac{M_{\chi_{c2}}}{108 \pi M_{\psi(3770)}} k_2^3 \left( \cos^2(\theta) \delta_{1c} \right)^2 + 4 \sqrt{\frac{5}{3}} \sin(\theta) \cos(\theta) \delta_{1c} \delta_{2c} + 60 \sin^2(\theta) \delta_{1c}^2 \delta_{2c}^2 \)  
\]

The mixing angle \( \theta = (12 \pm 2)^\circ \) that is determined from the leptonic decays of \( \psi(3686) \) and \( \psi(3770) \) also is favored by some other considerations[22]. If we fit \( \psi(3770) \) and \( \psi(3686) \) decays into \( \chi_{c0} + \gamma \) with \( \theta = 12^\circ \), we obtain two set solutions, which are labeled by subscripts 1 and 2, respectively,

\[
\delta_{1c}^{1P,2S} = 0.30 \text{GeV}^{-1}, \quad \delta_{1c}^{1D,1P} = 0.27 \text{GeV}^{-1}, \quad \delta_{2c}^{1P,2S} = 0.14 \text{GeV}^{-1}, \quad \delta_{2c}^{1D,1P} = -0.34 \text{GeV}^{-1};  
\]

If we fit their decay into \( \chi_{c1} + \gamma \) the results are:

\[
\delta_{1c}^{1P,2S} = 0.18 \text{GeV}^{-1}, \quad \delta_{1c}^{1D,1P} = 0.42 \text{GeV}^{-1}, \quad \delta_{2c}^{1P,2S} = 0.28 \text{GeV}^{-1}, \quad \delta_{2c}^{1D,1P} = -0.27 \text{GeV}^{-1};  
\]

The difference between the results in Eq. (18) and those in Eq. (20) shows that the heavy quark spin symmetry is largely violated if \( \psi(3686) \) and \( \psi(3770) \) are assigned as two S-D mixing sates as given in Eq. (17) with mixing angle \( \theta = (12 \pm 2)^\circ \). Consequently, to understand the radiative decays of \( \psi(3770) \) and \( \psi(3686) \) in the S - D mixing picture, some other effects like the relativistic corrections[23] or the couple channels effect[24] should be taken into account.

Since in the S-D mixing picture the heavy quark spin symmetry does not hold anymore, thereafter we will adopt that \( \psi(3770) \) is a pure D-wave state and choose the value of the coupling constant to be that in Eq. (16) to study the radiative decay of the D-wave states. The upper limit of \( \psi(3770) \to \chi_{c2} + \gamma \) is \( \mathcal{B}(\psi(3770) \to \chi_{c2} + \gamma) < 9 \times 10^{-4} \) \cite{17}. Using the result in Eq. (14a), we predict that

\[
\Gamma(\psi(3770) \to \chi_{c2} + \gamma) = 2.55 \pm 0.28 \text{ keV}, \quad \mathcal{B}(\psi(3770) \to \chi_{c2} + \gamma) = (9.4 \pm 1.0) \times 10^{-5}.  
\]

which is compatible with the experimental data and is about 4 times larger than those in Ref. [14]. The \( \psi(3770) \) decay into \( \chi_{cJ} + \gamma \) has also been studied by potential model. For comparison, we choose two potential models calculations \cite{19,17}, in which the predictions of \( \psi(3770) \) decay into \( \chi_{c0,1} + \gamma \) agree well with the experimental data after including relativistic corrections. Their predictions of \( \Gamma(\psi(3770) \to \chi_{cJ} + \gamma) \) are 3.0 \cite{19} or 3.3 \cite{17} keV. Both of them are consistent with our results.

As mentioned above, the other 1D states are all expected to be narrow. Their spectrum and the E1 transition decay widths have also been calculated in Ref. [17]. Their results are:

\[
M(1^D_2) = 3.838 \text{GeV}, \quad \Gamma(1^D_2 \to \chi_{c1}(1^P_2) + \gamma) = 268(66) \text{ keV}  
\]

\[
M(1^D_3) = 3.849 \text{GeV}, \quad \Gamma(1^D_3 \to \chi_{c2} + \gamma) = 296 \text{ keV}  
\]

\[
M(1^D_2) = 3.837 \text{GeV}, \quad \Gamma(1^D_2 \to \h_2 + \gamma) = 344 \text{ keV}.  
\]
If we choose the same mass values, our predictions are

\[
\Gamma(1^3D_2 \rightarrow \chi_{c1} + \gamma) = (288 \pm 25) \text{ keV}, \quad \Gamma(1^3D_2 \rightarrow \chi_{c2} + \gamma) = (50.3 \pm 5.5) \text{ keV}, \quad (23a)
\]

\[
\Gamma(1^3D_3 \rightarrow \chi_{c2} + \gamma) = 224 \pm 25 \text{ keV}, \quad \Gamma(1^3D_2 \rightarrow h_c + \gamma) = 267 \pm 29 \text{ keV}. \quad (23b)
\]

which agree with the potential model results.

Recently, the \(X(3872)\) state has received much attention since it was first discovered by Belle Collaboration [25], and then was confirmed in \(p\bar{p}\) collision at Tevatron [26]. It was also observed by Babar Collaboration [27]. Until now, there is not a convincing explanation about its nature yet. Only the charge parity \(C = +\) is established from its decay into \(J/\psi + \gamma\) [28]. After analyzing \(B \rightarrow J/\psi + \omega + K\), Babar Collaboration found its \(J^{PC}\) favors \(2^{−}\) [29]. If it is a pure charmonium D-wave state, the only assignment will be the \(\eta_{c2}(1^3D_2)\). It then should have a sizeable decay into \(\gamma + h_c\). Evaluating in a similar way, we obtain

\[
\Gamma(X(3872) \rightarrow h_c + \gamma) = 359 \pm 59 \text{ keV}, \quad (24)
\]

which is very large. So studying \(X(3872)\) decay into \(\gamma + h_c\) will be helpful to understand its nature.

The \(D\)-wave bottomonium states were observed from the cascade of \(\Upsilon(3S)\) [30], however no other further information is known yet. We cannot make any prediction about their radiative decay with the effective Lagrangian method at present.

4. Relation Between Radiative Decay and LH Decay of \(\chi_{bJ}(nP)\)

The total decay widths of the \(P\)-wave bottomonium states \(\chi_{bJ}(nP)\) \((n = 1, 2)\) have not been measured yet, so we can not compute \(\alpha_{bJ}^{P\to S}\) by fitting the data. Besides the radiative decay, the LH decay is also an important decay mode for the \(P\)-wave quarkonium states. One remarkable success of NRQCD is that it can systematically resolve the infrared divergence problem in the CS model (CSM) calculation for the LH decays of \(P\)-wave states by introducing the CO contribution [4, 31]. For the states in strong coupling region, most of the heavy quarkonium states below threshold are expected to belong to, further study of pNRQCD shows that the CO matrix elements can be related to the wave function of the bound states [10, 11]. In particular, in the strong coupling region the ratio \(\rho_b(nP) = m_b^2\mathcal{H}_b(nP)/\mathcal{H}_1(nP)\) does not depend very much on the radial quantum number \(n\) [10]. By fitting the open charm decays of \(\chi_{bJ}(1P)\) and \(\chi_{bJ}(2P)\), CLEO collaboration obtained that \(\rho_b(1P)\) = 0.160^{+0.071}_{-0.047}\) and \(\rho_b(2P) = 0.074^{+0.040}_{-0.006}\) [32], which is a little different from pNRQCD prediction.

Next, we will show that the relation between the radiative decay and the LH decay could provide another way to extract the values of \(\rho_b(1,2P)\). According to NRQCD approach, at \(v_b\) leading order, the LH decay width for \(P\)-wave states is given by:

\[
\Gamma(\chi_{bJ}(nP) \rightarrow \Upsilon(mS) + \gamma, \gamma) = \frac{C_1(\mu)\mathcal{H}_1}{m_b^4} + \frac{C_2(\mu)\mathcal{H}_8}{m_b^6}, \quad (25)
\]

where \(C_1\) and \(C_2\) are the \(J\)-dependent short distance coefficients and have been calculated up to \(\alpha_s^3\) order [34]. Although neither of the radiative and LH decay widths have been measured, their branching ratios are known. Recently Babar Collaboration update the branching ratios of \(\chi_{bJ}(nP) \rightarrow \Upsilon(mS) + \gamma\), their latest results are [33]:

\[
\mathcal{B}(\chi_{bJ}(1P) \rightarrow \Upsilon(1S) + \gamma) = (2.2 \pm 1.5^{+1.0}_{-0.7} \pm 0.2, 34.9 \pm 0.8 \pm 2.2 \pm 2.0, 19.5 \pm 0.7^{+1.3}_{-1.5} \pm 1.0)% \quad \text{for} \quad (J = 0, 1, 2) \quad (26a)
\]

\[
\mathcal{B}(\chi_{bJ}(2P) \rightarrow \Upsilon(2S) + \gamma) = (-4.7 \pm 2.8^{+0.7}_{-0.8} \pm 0.5, 18.9 \pm 1.1 \pm 1.2 \pm 1.8, 8.3 \pm 0.8 \pm 0.6 \pm 1.0)% \quad \text{for} \quad (J = 0, 1, 2) \quad (26b)
\]

\[
\mathcal{B}(\chi_{bJ}(2P) \rightarrow \Upsilon(1S) + \gamma) = (0.7 \pm 0.4^{+0.3}_{-0.2} \pm 0.1, 9.9 \pm 0.3^{+0.3}_{-0.2} \pm 0.9, 7.0 \pm 0.2 \pm 0.3 \pm 0.9)\% \quad \text{for} \quad (J = 0, 1, 2) \quad (26c)
\]

The branching ratio of \(\chi_{bJ}(nP)\) decay into LH can be obtained by subtracting its all the known transitions to other bottomonium states, which can be read out directly from PDG [10]. The ratio of the two branching ratios can be expressed as

\[
R_f(nP) = \frac{\mathcal{B}(\chi_{bJ}(nP) \rightarrow \Upsilon(nS) + \gamma) \times \Gamma(\chi_{bJ}(nP) \rightarrow \gamma + \Upsilon(nS))}{\mathcal{B}(\chi_{bJ}(nP) \rightarrow \Upsilon(nS)) \times \Gamma(\chi_{bJ}(nP) \rightarrow \gamma + \Upsilon(nS))} = \frac{3\pi M_{b\Upsilon}^2}{M_{\Upsilon(nS)}^2 \alpha_{bJ}^{P\to S}} \left( C_1(\mu)\mathcal{H}_1(nP) + C_2(\mu)\mathcal{H}_8(nP) \right) \quad (27)
\]
$R_i(nP)$ only depends on three unknown parameters $\mathcal{H}_i(nP)$, $\mathcal{H}_b(nP)$ and $\delta_{ib}^{PSF}$. Therefore, to compute the ratio $\rho_b(nP)$, we only need two independent inputs. Since the uncertainties of the $\chi_{bJ}(1,2P)$ radiative decays are large, we choose the data of $\chi_{bJ}(1,2P)$ decays. Using the $a_i^J$ order short-distances coefficients listed in Ref. [34] and setting $\mu_b = 2m_b$, $\alpha_s(2m_b) = 0.18$, and the number of light flavor quark $N_f = 4$, we obtain $\rho_b(1P) = 0.150^{+0.036}_{-0.037}$ and $\rho_b(2P) = 0.110 \pm 0.030$. Our value of $\rho_b(1P)$ is a little smaller than that of CLEO, while our value of $\rho_b(2P)$ is about 1.5 times larger than that of CLEO, which makes $\rho_b(1P)$ close to $\rho_b(2P)$. This indicates that the pNRQCD assumptions is reasonable to study the LH decays of $\chi_{bJ}(1P)$ and $\chi_{bJ}(2P)$ states [10].

5. Summary and Conclusion

In summary, the radiative decays of the heavy quarkonia are studied with the help of the effective Lagrangian. To have a better understanding of the radiative transitions among $S$- and $P$-wave states, we take into account the contribution that is due to the spin-breaking interactions. By fitting the experimental data, the coupling constants of the spin-breaking terms in $\psi(2S)$, $\chi_{PJ}$, $\Upsilon(2S)$ and $\Upsilon(3S)$ radiative are obtained, which are listed in Table 2. We find that the values of the coupling constants in the spin-breaking terms are less than those in the leading order terms by at least a factor of 10 and that the spin-breaking terms in $\psi(2S)$ and $\Upsilon(2S)$ indicate that the spin-breaking contribution is suppressed by $1/m_c^2$, which agrees with pNRQCD power counting rule. We also calculate the radiative decays of the $c\bar{c}$($D^0$) states, whose total decay widths are expected to be narrow. Based on that $\psi(3770)$ is a pure $D$-wave state, our predictions of the radiative decay widths of the other $D$-wave states are consistent with the potential model results. Furthermore, we predict that $\Gamma(X(3872) \to h_c + \gamma) = 359 \pm 59$keV, if $X(3872)$ is the $2^{++}$ state. We also study the S-D mixing effect in $\psi(3770)$ radiative decay and find that there is no heavy quark spin-symmetry if the mixing angle $\delta$ is 12$^\circ$. As an useful application, we find that relating the radiative decay of $\chi_{bJ}(nP)$ to their LH decays can provide another way to estimate the ratios of $\rho_b(nP) = m_{bJ}^2\mathcal{H}_b(nP)/\mathcal{H}_i(nP)$. By fitting the data of $\chi_{bJ}(1P,2P)$ and that of $\chi_{b1}(1P,2P)$, we get that $\rho_b(1P) = 0.150^{+0.036}_{-0.037}$ and $\rho_b(2P) = 0.110 \pm 0.030$, which approximately equal to each other. Our result provide an evidence on pNRQCD assumptions [10].

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