Charged lepton flavor violating decays in the extended BLMSSM

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Abstract

Within the extended BLMSSM, the exotic Higgs superfields ($\Phi_{NL}, \varphi_{NL}$) are added to make the exotic leptons heavy, and the superfields ($Y, Y'$) are also introduced to make exotic leptons unstable. This new model is named as the EBLMSSM. We study some charged lepton flavor violating (CLFV) processes in detail in the EBLMSSM, including $l_j \rightarrow l_i \gamma$, muon conversion to electron in nuclei and $h^0 \rightarrow l_i l_j$. Being different from BLMSSM, some particles are redefined in this new model, such as slepton, sneutrino, exotic lepton (neutrino), exotic slepton (sneutrino) and lepton neutralino. We also introduce the mass matrices of superfields $Y$ and spinor $\tilde{Y}$ in the EBLMSSM. All of these lead to new contributions to the CLFV processes. In the suitable parameter space, we obtain the reasonable numerical results. The results of this work will encourage physicists to explore new physics beyond the SM.

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The Higgs boson, an elementary particle, has been researched by the Large Hadron Collider (LHC) as one of the primary scientific goals. Combining the updated data of the ATLAS\cite{1} and CMS\cite{2} Collaborations, now its measured mass $m_{h^0} = 125.09 \pm 0.24$GeV\cite{3}, which represent that the Higgs mechanism is compellent. In the Standard Model (SM), the lepton-flavor number is conserved. However, the neutrino oscillation experiments\cite{4–12} have convinced that neutrinos possess tiny masses and mix with each other. So the individual lepton numbers $L_i = L_e, L_\mu, L_\tau$ are not exact symmetries at the electroweak scale. Furthermore, the presentation of the GIM mechanism makes the charged lepton flavor violating (CFLV) processes in the SM very tiny\cite{13–15}, such as $Br_{SM}(l_j \rightarrow l_i \gamma) \sim 10^{-55}$\cite{16}. Therefore, if we observe lepton flavor violating processes in future experiments, it is an obvious evidence of new physics beyond the SM.

Studying the CLFV processes is an effective way to explore new physics beyond the SM. MEG Collaboration gives out the current experiment upper bound of the CLFV process $\mu \rightarrow e \gamma$, which is $Br(\mu \rightarrow e \gamma) < 5.7 \times 10^{-13}$ at 90% confidence level\cite{17}. $Br(\tau \rightarrow e \gamma) < 3.3 \times 10^{-8}$ and $Br(\tau \rightarrow \mu \gamma) < 4.4 \times 10^{-8}$ are also shown in Ref.\cite{18}. SINDRUM II collaboration has updated the sensitivity of the $\mu - e$ conversion rate in Au nuclei $CR(\mu \rightarrow e : ^{197}_{79}Au) < 7 \times 10^{-13}$\cite{19}. Furthermore, a direct research for the 125.1 GeV Higgs boson decays including the CLFV, $h^0 \rightarrow l_il_j$, has been given out by the CMS Collaboration\cite{20, 21} and ATLAS Collaboration\cite{22}. We show the corresponding experiment upper bounds for processes $h^0 \rightarrow l_il_j$ in TABLE I. Physicists do more research on the CLFV processes for $l_j \rightarrow l_i \gamma$ decays, $\mu - e$ conversion rates in nuclei and $h^0 \rightarrow l_il_j$ decays in models beyond the SM\cite{23–28}. In our previous work, we have studied $l_i \rightarrow l_i \gamma$, muon conversion to electron in nuclei and $h^0 \rightarrow l_il_j$ processes in the $\mu$SSM\cite{29–31}. We also have discussed $l_j \rightarrow l_j \gamma$ processes and muon conversion to electron in nuclei in the BLMSSM\cite{32, 33}. In this work, we study the processes $l_j \rightarrow l_j \gamma$, $\mu - e$ conversion in Au nuclei and $h^0 \rightarrow l_il_j$ in the extended BLMSSM, which is named as the EBLMSSM\cite{34}.

Extending the MSSM with the introduced local gauged B and L, one obtains the so-called BLMSSM\cite{35–38}. In the BLMSSM, the exotic lepton masses are obtained from the
TABLE I: Present experiment limits for 125 GeV Higgs decays $h^0 \to l_il_j$ with the CLFV.

| CLFV process | Present limit (CMS) | Present limit (ATLAS) | confidence level (CL) |
|--------------|---------------------|-----------------------|-----------------------|
| $h^0 \to e\mu$ | $< 0.035\%[20]$ | $-$ | 95% |
| $h^0 \to e\tau$ | $< 0.61\%[21]$ | $< 1.04\%[22]$ | 95% |
| $h^0 \to \mu\tau$ | $< 0.25\%[21]$ | $< 1.43\%[22]$ | 95% |

Yukawa couplings with the two Higgs doublets $H_u$ and $H_d$. The values of these exotic lepton masses are around 100 GeV, which can well anastomose the current experiment bounds. However, with the development of high energy physics experiments, we may obtain the heavier experiment lower bounds of the exotic lepton masses in the near future, which makes the BLMSSM model do not exist. Therefore, two exotic Higgs superfields, the $SU(2)_L$ singlets $\Phi_{NL}$ and $\varphi_{NL}$, are considered to be added in the BLMSSM. With the introduced superfields $\Phi_{NL}$ and $\varphi_{NL}$, the exotic leptons can turn heavy and should be unstable. This new model is named as the extended BLMSSM (EBLMSSM)\cite{34}. In order to make the exotic leptons unstable, we add superfields $Y$ and $Y'$ in the EBLMSSM.

In the BLMSSM, the dark matter (DM) candidates include the lightest mass eigenstate of $X, X'$ mixing and a four-component spinor $\tilde{X}$ composed by the superpartners of $X, X'$. In the EBLMSSM, the DM candidates not only include above terms presented in the BLMSSM, but also contain new terms due to the new introduced superfields of $Y, Y'$. So the lighter mass eigenstates of $Y, Y'$ mixing and spinor $\tilde{Y}$ are DM candidates\cite{34,39}. In section 4.2 of our previous work\cite{34}, we suppose the lightest mass eigenstate of $Y, Y'$ mixing as a DM candidate, and calculate the relic density $\Omega_D h^2$. In the reasonable parameter space, $\Omega_D h^2$ of $Y_1$ can match the experiment results well.

The Higgs boson $h^0$ is produced chiefly from the gluon fusion ($gg \to h^0$) at the LHC. The leading order (LO) contributions originate from the one loop diagrams. In the BLMSSM, we have studied the $h^0 \to gg$ process in our previous work\cite{40}, and the virtual top quark loops play the dominate roles. The EBLMSSM results for $h^0 \to gg$ are same as those in BLMSSM, which have been discussed in Ref.\cite{34}. Being different from BLMSSM, the exotic leptons in EBLMSSM are more heavy and the exotic sleptons of the 4-th and 5-th generations mix
together to form a $4 \times 4$ mass matrices. These new parts can give new contributions to the CLFV processes. The leading order contributions for $h^0 \to \gamma \gamma$ originate from the one loop diagrams. In the EBLMSSM, we have studied the decay $h^0 \to \gamma \gamma$ in detail. The processes $h^0 \to VV, V = (Z, W)$ also have been researched in this new model. Considering the constraints from the parameter space of these researches, we study the processes $l_j \to l_i \gamma$, muon conversion to electron in Au nuclei and $h^0 \to l_i l_j$ in this work.

The outline of this paper is organized as follows. In section II, we present the ingredients of the EBLMSSM by introducing its superpotential, the general soft SUSY-breaking terms, new corrected mass matrices and couplings which are different from those in the BLMSSM. In section III, we analyze the corresponding amplitudes and the branching ratios of rare CLFV processes $l_j \to l_i \gamma$, muon conversion to electron rates in nuclei and decays $h^0 \to l_i l_j$.

The numerical analysis is discussed in section IV, and the conclusions are summarized in section V. The tedious formulæ are collected in Appendix.

II. INTRODUCTION OF THE EBLMSSM

In the EBLMSSM, the local gauge group is $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_B \otimes U(1)_L$. We introduce the exotic Higgs superfields $\Phi_{NL}$ and $\varphi_{NL}$ with nonzero VEVs $\nu_{NL}$ and $\bar{\nu}_{NL}$ to make the exotic leptons heavy. Accordingly, the superfields $Y$ and $Y'$ are introduced to avoid the heavy exotic leptons stable. In TABLE II, we show the new introduced superfields in the EBLMSSM.

The corresponding superpotential of the EBLMSSM is shown here

$$ W_{EBLMSSM} = W_{MSSM} + W_B + W_L + W_X + W_Y, $$

$$ W_L = \lambda_L \hat{L}_4 \hat{\nu}^c_{NL} + \lambda_E \hat{E}^c_4 \hat{E}_5 \hat{\Phi}_{NL} + \lambda_{NL} \hat{N}^c_4 \hat{\bar{N}}_5 \hat{\bar{\Phi}}_{NL} + \mu_{NL} \hat{\Phi}_{NL} \hat{\bar{\varphi}}_{NL} + $$

$$ + Y_{e_4} \hat{L}_4 \hat{H}_u \hat{\nu}^c_4 + Y_{\nu_4} \hat{L}_4 \hat{H}_u \hat{N}^c_4 + Y_{e_5} \hat{L}_5 \hat{H}_u \hat{E}_5 + Y_{\nu_5} \hat{L}_5 \hat{H}_u \hat{N}_5 $$

$$ + Y_L \hat{L} \hat{H}_u \hat{\nu}^c + \lambda_{Nc} \hat{N}^c \hat{\bar{N}}^c \hat{\varphi}_L + \mu_L \hat{\phi}_L \hat{\phi}_L, $$

$$ W_Y = \lambda_Y \hat{L}^c_5 \hat{\bar{Y}} + \lambda_{\bar{N}c} \hat{\bar{N}}^c \hat{\bar{Y}}' + \lambda_6 \hat{E}^c \hat{E}^c_5 \hat{\bar{Y}}' + \mu_Y \hat{\bar{Y}} \hat{\bar{Y}}'. $$ (1)

$W_{MSSM}$ is the superpotential of the MSSM. $W_B$ and $W_X$ are same as the terms in the BLMSSM. The new terms $\lambda_L \hat{L}_4 \hat{L}^c_{5} \hat{\varphi}_{NL} + \lambda_E \hat{E}^c_4 \hat{E}_5 \hat{\Phi}_{NL} + \lambda_{NL} \hat{N}^c_4 \hat{\bar{N}}_5 \hat{\bar{\Phi}}_{NL} + \mu_{NL} \hat{\Phi}_{NL} \hat{\bar{\varphi}}_{NL}$...
are added to \( \mathcal{W}_L \) based on the original BLMSSM. Comparing with the \( \mathcal{W}_X \) in the BLMSSM, \( \mathcal{W}_Y \) is introduced in the EBLMSSM, which includes the lepton-exotic lepton-\( Y \) coupling and lepton-exotic slepton-\( \tilde{Y} \) coupling. These new couplings can produce one loop diagrams influencing the CLFV decays. These new couplings can also produce one loop diagrams contributing to the lepton electric dipole moment(EDM) and lepton magnetic dipole moment(MDM), which will be discussed in our next work. With the 4-th and 5-th generation exotic sleptons mixing together, the \( h^0 \)-exotic slepton-exotic slepton (\( h^0 - \tilde{E} - \tilde{E} \)) coupling is deduced in the EBLMSSM. In the EBLMSSM, the couplings for lepton-slepton-lepton neutralino, \( h^0 \)-slepton-slepton, \( h^0 \)-sneutrino-sneutrino and \( h^0 \)-exotic lepton-exotic lepton also have new contributions to CLFV processes. In the whole, the new couplings in the EBLMSSM enrich the lepton physics in a certain degree.

In the EBLMSSM, \( \mathcal{W}_Y \) are the new terms in the superpotential. In \( \mathcal{W}_Y \), \( \lambda_4(\lambda_6) \) is the coupling coefficient of \( Y \)-lepton-exotic lepton and \( \tilde{Y} \)-slepton-exotic slepton couplings. We consider \( \lambda_4^2(\lambda_6^2) \) is a \( 3 \times 3 \) matrix and has non-zero elements relating with the CLFV. In our following numerical analysis, we assume that \( (\lambda_4^2)^{IJ} = (\lambda_6^2)^{IJ} = (Lm^2)^{IJ} \), \( I(J) \) represents the \( I \)-th (\( J \)-th) generation charged lepton. When \( I = J \), there is no CLFV, which has no contributions to our researched decay processes. So, only the non-diagonal elements \( (Lm^2)^{IJ}(I \neq J) \) influence the numerical results of the CLFV processes. Therefore, we should take into account the effects from \( \mathcal{W}_Y \) in this work.

Based on the new introduced superfields \( \Phi_{NL}, \varphi_{NL}, Y \) and \( Y' \) in the EBLMSSM, the soft breaking terms are given out

\[
\mathcal{L}_{\text{soft}}^{EBLMSSM} = \mathcal{L}_{\text{soft}}^{BLMSSM} - m_{\Phi_{NL}}^2 \Phi_{NL}^\dagger \Phi_{NL} - m_{\varphi_{NL}}^2 \varphi_{NL}^\dagger \varphi_{NL} + (A_{LL}\lambda_4 \tilde{L}_4 \tilde{L}_5^c \varphi_{NL})
\]
\[ L_{\text{soft}}^{\text{BLMSSM}} \] is the soft breaking terms of the BLMSSM discussed in our previous work [40, 44]. Here, corresponding to the \( SU(2)_L \) singlets \( \Phi_{NL} \) and \( \varphi_{NL} \), we obtain the nonzero VEVs \( v_{NL} \) and \( \bar{v}_{NL} \) respectively. Generally, the values of these two parameters are at TeV scale. The exotic Higgs \( \Phi_{NL} \) and \( \varphi_{NL} \) can be written as

\[
\Phi_{NL} = \frac{1}{\sqrt{2}} (v_{NL} + \Phi_{NL}^0 + iP_{NL}^0), \quad \varphi_{NL} = \frac{1}{\sqrt{2}} (\bar{v}_{NL} + \varphi_{NL}^0 + i\bar{P}_{NL}^0),
\]

where \( \tan \beta_{NL} = \bar{v}_{NL}/v_{NL} \) and \( v_{NL} = \sqrt{\bar{v}_{NL}^2 + v_{NL}^2} \).

Comparing with the BLMSSM, the introduced superfields \( \Phi_{NL} \) and \( \varphi_{NL} \) in the EBLMSSM can give corrections to the mass matrices of the slepton, sneutrino, exotic lepton, exotic neutrino, exotic slepton, exotic sneutrino and lepton neutralino. However, the mass matrices of squark, exotic quark, exotic squark used in this work are same as those in the BLMSSM [40, 45]. We deduce the adjusted mass matrices in the EBLMSSM as follows.

**A. The mass matrices of slepton and sneutrino in the EBLMSSM**

In our previous work, we can easily obtain the slepton and sneutrino mass squared matrices of the BLMSSM [32]. Using the replacement \( V_L^2 - v_L^2 \rightarrow V_{L}'^2 \) (here \( V_{L}'^2 = V_L^2 - v_L^2 + \frac{3}{2}(v_{NL}^2 - v_{NL}^2) \)) for the BLMSSM results, we acquire the mass squared matrices of slepton and sneutrino in the EBLMSSM.

**B. The mass matrices of exotic lepton and exotic neutrino in the EBLMSSM**

Being different from the exotic lepton masses in the BLMSSM, the EBLMSSM exotic leptons will be heavier than those in the BLMSSM due to the introduction of large parameters \( v_{NL} \) and \( \bar{v}_{NL} \). One can obtain the mass matrix of exotic lepton in the Lagrangian:

\[
-L_{L'}^{\text{mass}} = \begin{pmatrix}
\tilde{e}_{4R}^\prime & \tilde{e}_{5R}^\prime
\end{pmatrix}
\begin{pmatrix}
\frac{1}{\sqrt{2}} \lambda_L v_{NL} & \frac{1}{\sqrt{2}} \tilde{e}_5 v_u \\
-\frac{1}{\sqrt{2}} Y_{e_4} v_d & \frac{1}{\sqrt{2}} \lambda_{EUNL}
\end{pmatrix}
\begin{pmatrix}
\tilde{e}_{4L}^\prime \\
\tilde{e}_{5L}^\prime
\end{pmatrix}
+ \text{h.c.}
\]
Similarly, the mass matrix of the exotic neutrinos in the EBLMSSM can be given through the Lagrangian:

$$-\mathcal{L}_{N^r}^{\text{mass}} = \left( \tilde{\nu}_{4R} \tilde{\nu}_{5R}^* \right) \begin{pmatrix} \frac{1}{\sqrt{2}} \lambda_L \mathbf{T}_{NL} & - \frac{1}{\sqrt{2}} Y_{\nu} v_d \\ \frac{1}{\sqrt{2}} Y_{\nu} v_u & \frac{1}{\sqrt{2}} \lambda_{NL} U_{NL} \end{pmatrix} \begin{pmatrix} \nu_{4L}^* \\ \nu_{5L}^* \end{pmatrix} + \text{h.c.} \quad (5)$$

C. The mass matrices of exotic slepton and exotic sneutrino in the EBLMSSM

In the EBLMSSM, the exotic slepton of 4-th generation and 5-th generation mix together, and its mass matrix is $4 \times 4$, which is different from that in the BLMSSM. Using the superpotential in Eq. (1) and the soft breaking terms in Eq. (2), the mass squared matrix for exotic slepton can be obtained through Lagrangian:

$$-\mathcal{L}_{E}^{\text{mass}} = \tilde{E}^T \cdot \mathcal{M}_{E}^2 \cdot \tilde{E}. \quad (6)$$

With the base $\tilde{E}^T = (\tilde{e}_4, \tilde{e}_4^c, \tilde{e}_5, \tilde{e}_5^c)$, we show the concrete elements of exotic slepton mass matrix $\mathcal{M}_{E}^2$ in the following form

$$\begin{align*}
\mathcal{M}_{E}^2(\tilde{e}_5^c \tilde{e}_5^c) &= \lambda_L^2 \frac{v_u^2}{2} + \frac{v_u^2}{2} |Y_{\nu}|^2 + M_{L_\alpha}^2 - \frac{g_1^2 - g_2^2}{8} (v_d^2 - v_u^2) - g_L^2 (3 + L_4) V_L^2, \\
\mathcal{M}_{E}^2(\tilde{e}_5^c \tilde{e}_5) &= \lambda_L^2 \frac{v_u^2}{2} + \frac{v_u^2}{2} |Y_{\nu}|^2 + M_{L_\alpha}^2 + \frac{g_1^2}{4} (v_d^2 - v_u^2) + g_L^2 (3 + L_4) V_L^2, \\
\mathcal{M}_{E}^2(\tilde{e}_4^c \tilde{e}_4^c) &= \lambda_L^2 \frac{v_u^2}{2} - \frac{g_1^2 - g_2^2}{8} (v_d^2 - v_u^2) + \frac{v_u^2}{2} |Y_{\nu}|^2 + M_{L_4}^2 + g_L^2 L_4 V_L^2, \\
\mathcal{M}_{E}^2(\tilde{e}_4^c \tilde{e}_5^c) &= \lambda_L^2 \frac{v_u^2}{2} - \frac{g_1^2}{4} (v_d^2 - v_u^2) + \frac{v_u^2}{2} |Y_{\nu}|^2 + M_{L_4}^2 - g_L^2 L_4 V_L^2, \\
\mathcal{M}_{E}^2(\tilde{e}_4^c \tilde{e}_4^c) &= v_d Y_{\nu} \lambda_{E} \frac{v_u}{2} + \lambda_L Y_{\nu} \frac{\bar{v}_{UNL} v_u}{2}, \quad \mathcal{M}_{E}^2(\tilde{e}_5^c \tilde{e}_5^c) = \mu^* \frac{v_u}{\sqrt{2}} Y_{\nu} + A_{e_5} Y_{e_5} \frac{v_u}{\sqrt{2}}, \\
\mathcal{M}_{E}^2(\tilde{e}_4^c \tilde{e}_5^c) &= \mu^* \frac{\bar{v}_{UNL}}{\sqrt{2}} - A_{LE} \lambda_{E} \frac{v_u}{\sqrt{2}}, \quad \mathcal{M}_{E}^2(\tilde{e}_4^c \tilde{e}_4^c) = -\mu^* \frac{\bar{v}_{UNL}}{\sqrt{2}} \lambda_{L} + A_{LL} \lambda_{L} \frac{\bar{v}_{UNL}}{\sqrt{2}}, \\
\mathcal{M}_{E}^2(\tilde{e}_4 \tilde{e}_4^c) &= \mu^* \frac{v_u}{\sqrt{2}} Y_{e_4} + A_{e_4} Y_{e_4} \frac{v_u}{\sqrt{2}}, \quad \mathcal{M}_{E}^2(\tilde{e}_5 \tilde{e}_5^c) = -Y_{e_5} \lambda_{E} \frac{v_u}{\sqrt{2}} - \lambda_L \frac{\bar{v}_{UNL} v_d}{2}. \quad (7)
\end{align*}$$

The matrix $Z_{E}$ is used to rotate exotic slepton mass matrix to mass eigenstates, which is $Z_{E}^\dagger \mathcal{M}_{E}^2 Z_{E} = \text{diag}(m_{E_1}^2, m_{E_2}^2, m_{E_3}^2, m_{E_4}^2)$.

In the same way, the exotic sneutrino mass squared matrix is also obtained through the Lagrangian:

$$-\mathcal{L}_{N}^{\text{mass}} = \tilde{N}^\dagger \cdot \mathcal{M}_{N}^2 \cdot \tilde{N}, \quad (8)$$
where the corresponding elements of the matrix $M_N$ are

\[
M_N^2(\tilde{\nu}_5^e \tilde{\nu}_5^e) = \frac{\lambda_L^2 v_{NL}^2}{2} - \frac{g_1^2 + g_2^2}{8} (v_d^2 - v_u^2) + \frac{v_d^2}{2} |Y_{\nu_3}|^2 + M_{L_5}^2 - g_L^2 (3 + L_4) V_L^2,
\]

\[
M_N^2(\tilde{\nu}_4^e \tilde{\nu}_4^e) = \frac{\lambda_L^2 v_{NL}^2}{2} - \frac{g_1^2 + g_2^2}{8} (v_d^2 - v_u^2) + \frac{v_d^2}{2} |Y_{\nu_4}|^2 + M_{L_4}^2 + g_L^2 L_4 V_L^2,
\]

\[
M_N^2(\tilde{\nu}_5^e \tilde{\nu}_5) = \frac{\lambda_L^2 v_{NL}^2}{2} + g_1^2 (3 + L_4) V_L^2 + \frac{v_d^2}{2} |Y_{\nu_5}|^2 + M_{\nu_5}^2,
\]

\[
M_N^2(\tilde{\nu}_4^e \tilde{\nu}_4) = \frac{\lambda_L^2 v_{NL}^2}{2} - g_2^2 L_4 V_L^2 + \frac{v_u^2}{2} |Y_{\nu_4}|^2 + M_{\nu_4}^2,
\]

\[
M_N^2(\tilde{\nu}_5^e \tilde{\nu}_4^e) = \frac{\lambda_L^2 v_{NL}^2}{2} + g_2^2 (3 + L_4) V_L^2 + \frac{v_u^2}{2} |Y_{\nu_5}|^2 + M_{\nu_5}^2,
\]

In the base $(\tilde{\nu}_4, \tilde{\nu}_4^e, \tilde{\nu}_5, \tilde{\nu}_5^e)$, we can diagonalize the mass squared matrix $M_N^2$ by $Z_N$.

**D. The lepton neutralino mass matrix in the EBLMSSM**

In the EBLMSSM, $\lambda_L$, the superpartner of the new lepton type gauge boson $Z_L^\mu$, mixes with the SUSY superpartners $(\psi_{\Phi_L}, \psi_{\varphi_L}, \psi_{\Phi_{NL}}, \psi_{\varphi_{NL}})$ of the superfields $(\Phi_L, \varphi_L, \Phi_{NL}, \varphi_{NL})$. So the lepton neutralino mass matrix is obtained in the base $(i\lambda_L, \psi_{\Phi_L}, \psi_{\varphi_L}, \psi_{\Phi_{NL}}, \psi_{\varphi_{NL}})$:

\[
M_L = \begin{pmatrix}
2M_L & 2v_{gL} & -2\bar{v}_{gL} & 3v_{NL}g_L & -3\bar{v}_{NL}g_L \\
2v_{gL} & 0 & -\mu_L & 0 & 0 \\
-2\bar{v}_{gL} & -\mu_L & 0 & 0 & 0 \\
3v_{NL}g_L & 0 & 0 & -\mu_{NL} & 0 \\
-3\bar{v}_{NL}g_L & 0 & 0 & 0 & -\mu_{NL}
\end{pmatrix}.
\]  

(10)

The mass matrix $M_L$ can be diagonalized by the rotation matrix $Z_{NL}$. Then, we can have

\[
i\lambda_L = Z_{NL}^{1i} K_{Li}, \quad \psi_{\Phi_L} = Z_{NL}^{2i} K_{Li}, \quad \psi_{\varphi_L} = Z_{NL}^{3i} K_{Li},
\]

\[
\psi_{\Phi_{NL}} = Z_{NL}^{4i} K_{Li}, \quad \psi_{\varphi_{NL}} = Z_{NL}^{5i} K_{Li}.
\]  

(11)

Here, $X_{Li}^0 = (K_{Li}^0, \bar{K}_{Li}^0)^T$ represent the mass eigenstates of the lepton neutralino.
E. The superfields $Y$ in the EBLMSSM

The scalar superfields $Y$ and $Y'$ mix. Adopting the unitary transformation,

$$
\begin{pmatrix}
Y_1 \\
Y_2
\end{pmatrix}
= Z_Y^\dagger
\begin{pmatrix}
Y \\
Y'
\end{pmatrix},
$$

(12)

the mass squared matrix for the superfield $Y$ is deduced. With $S_Y = g_L^2(2 + L_4)V_L^2$, the concrete form for the $Y$ mass squared matrix is shown here

$$
\mathcal{M}_Y^2 = \begin{pmatrix}
|\mu_Y|^2 + S_Y & -\mu_Y B_Y \\
-\mu_Y^* B_Y^* & |\mu_Y|^2 - S_Y
\end{pmatrix}.
$$

(13)

The matrix $Z_Y$ is used to diagonalize the matrix to the mass eigenstates:

$$
Z_Y^\dagger
\begin{pmatrix}
|\mu_Y|^2 + S_Y & -\mu_Y B_Y \\
-\mu_Y^* B_Y^* & |\mu_Y|^2 - S_Y
\end{pmatrix}
Z_Y = \begin{pmatrix}
m_{Y_1}^2 & 0 \\
0 & m_{Y_2}^2
\end{pmatrix}.
$$

(14)

We suppose $m_{Y_1}^2 < m_{Y_2}^2$. The superpartners of $Y$ and $Y'$ form a four-component Dirac spinor $\tilde{Y}$, and the mass term for superfield $\tilde{Y}$ in the Lagrangian is given out

$$
-\mathcal{L}_Y^{mass} = \mu_Y \bar{\tilde{Y}} \tilde{Y}, \quad \tilde{Y} = \begin{pmatrix}
\psi_Y \\
\bar{\psi}_Y
\end{pmatrix}.
$$

(15)

F. The mass of $h^0$ in the EBLMSSM

In the basis $(H^0_d, H^0_u)$, the mass squared matrix for the neutral CP even Higgs boson is studied in the EBLMSSM, which possesses similar form as that in the BLMSSM.

$$
\mathcal{M}_{even}^2 = \begin{pmatrix}
M_{11}^2 + \Delta_{11} & M_{12}^2 + \Delta_{12} \\
M_{12}^2 + \Delta_{12} & M_{22}^2 + \Delta_{22}
\end{pmatrix},
$$

(16)

where $M_{11,12,22}^2$ are the tree level results, which can be found in Ref. [40]. The one-loop corrections are $\Delta_{11,12,22} = \Delta_{11,12,22}^{MSSM} + \Delta_{11,12,22}^{B} + \Delta_{11,12,22}^{L}$. Accordingly, $\Delta_{11,12,22}^{MSSM}$ represent the MSSM contributions at one-loop level [46, 47]. The contributions from exotic quark (squark) fields expressed by $\Delta_{11,12,22}^{B}$ in the EBLMSSM are same as those in the BLMSSM [40]. $\Delta_{11,12,22}^{L}$, the one-loop exotic lepton (slepton) corrections, are influenced by the new corrected exotic
lepton (slepton) and exotic neutrino (sneutrino) in the EBLMSSM. The concrete expressions for those $\Delta L_{11,12,22}$ are summarized in our previous work[34].

In the EBLMSSM, the 4-th and 5-th generation exotic sleptons mix together. So the exotic slepton couplings in this new model are different from those in the BLMSSM. We deduce the $h^0$-exotic slepton-exotic slepton ($h^0 - \tilde{E} - \tilde{E}$) coupling as follows

$$
\mathcal{L}_{h^0\tilde{E}\tilde{E}} = \sum_{i,j=1}^{4} \bar{E}_i^{* \pm} \tilde{E}_j^\mp h^0 \left[ (e^2 v \sin \beta \frac{1-4 s_w^2}{4 s_w^2 c_w^2} (Z_E^{4i*} Z_E^{4j}) - \frac{\mu^*}{\sqrt{2} Y_{4s}} Z_E^{2i*} Z_E^{1j})
\right.
\left. - v \sin \beta |Y_{4s}|^2 \delta_{ij} - \frac{A_{E_i}}{\sqrt{2}} Z_E^{4i*} Z_E^{3j} + \frac{1}{2} \lambda L Y_{4s} Z_E^{3j} Z_E^{2i*} \bar{v}_{NL} - \frac{1}{2} Y_{4s} Z_E^{2j} \lambda E E^*_L Y_{4s} v_{NL} \right] \cos \alpha
\right.
\left. - \left( e^2 v \cos \beta \frac{1-4 s_w^2}{4 s_w^2 c_w^2} (Z_E^{1i*} Z_E^{1j} - Z_E^{4i*} Z_E^{4j}) - v \cos \beta |Y_{4s}|^2 \delta_{ij} - \frac{A_{E_i}}{\sqrt{2}} Z_E^{2i*} Z_E^{1j}
\right.
\left. - \frac{\mu^*}{\sqrt{2}} Y_{4s} Z_E^{4i*} Z_E^{3j} - \frac{1}{2} Y_{4s} Z_E^{2j} \lambda L Z_E^{4i*} \bar{v}_{NL} + \frac{1}{2} Z_E^{2i*} Y_{4s} \lambda E E^*_L v_{NL} \right] \sin \alpha
\right].
(17)

As the new introduced superfield in the EBLMSSM, $Y$ leads to new couplings. The lepton-exotic lepton-$Y$ coupling used in this work is shown here

$$
\mathcal{L}_{\ell Y} = \sum_{i,j=1}^{2} \bar{\ell} \left( \lambda_4 W_L^{1i} Z_Y^{1j*} P_R - \lambda_6 U_L^{2i} Z_Y^{2j*} P_L \right) \ell L_{i+3} Y_j^* + h.c.
(18)
$$

Superfield $\tilde{Y}$ is also a new term beyond the BLMSSM. We deduce the lepton-exotic slepton-$\tilde{Y}$ coupling as

$$
\mathcal{L}_{\ell \tilde{Y}} = \bar{\ell} \left( \lambda_4 Z_E^{4i*} P_L - \lambda_6 Z_E^{3i*} P_R \right) \ell \tilde{Y}_i + h.c.
(19)
$$

In the EBLMSSM, though the effects from the couplings of lepton-slepton-lepton neutralino, $h^0$-slepton-slepton, $h^0$-sneutrino-sneutrino and $h^0$-exotic lepton-exotic lepton are different from those in the BLMSSM, these couplings possess the same forms as those in the BLMSSM.

III. THE PROCESSES $l_j \to l_i \gamma$, MUON CONVERSION TO ELECTRON IN NUCLEI AND $h^0 \to l_i l_j$ IN THE EBLMSSM

In this section, we analyze the branching ratios of CLFV processes $l_j \to l_i \gamma$, muon conversion rates to electron in Au nuclei and branching ratios of $h^0 \to l_i l_j$ in the EBLMSSM.
A. Rare decay $l_j \rightarrow l_i \gamma$

Generally, the corresponding effective amplitude for processes $l_j \rightarrow l_i \gamma$ can be written as [48]

$$
M = e \epsilon \bar{u}_i(p + q)[g^2 \gamma_\mu(C^L \gamma \mu P_L + C^R \gamma \mu P_R) + m_{l_j} i \sigma_{\mu \nu} q^\nu(C^L_2 P_L + C^R_2 P_R)]u_j(p),
$$

$$
C^L_R = C^L_R(n) + C^L_R(c) + C^L_R(W), \alpha = 1, 2,
$$

where $p (q)$ represents the injecting lepton (photon) momentum. $m_{l_j}$ is the j-th generation lepton mass. $\epsilon$ is the photon polarization vector and $u_i(p) (v_i(p))$ is the lepton (antilepton) wave function. In FIG.1 we show the relevant Feynman diagrams corresponding to above amplitude. The Wilson coefficients $C^L_R(\alpha = 1, 2)$ are discussed as follows.

$C^L_R(n)(\alpha = 1, 2)$, the virtual neutral fermion contributions corresponding to FIG.1(a), are deduced in the following form,

$$
C^L_1(n) = \sum_{F=\chi^0/\chi^0_L, \nu, \tilde{Y}} \sum_{S=\tilde{L}, \tilde{Y}} \frac{1}{6 m_\Lambda^2} H^S_{F \tilde{l}_i} H^{S^{*} l_j F} I_4(x_F, x_S),
$$

$$
C^L_2(n) = \sum_{F=\chi^0/\chi^0_L, \nu, \tilde{Y}} \sum_{S=\tilde{L}, \tilde{Y}} \frac{m_F}{m_{l_j} m_\Lambda^2} H^S_{F \tilde{l}_i} H^{S^{*} l_j F} [I_2(x_F, x_S) - I_3(x_F, x_S)],
$$

$$
C^R(\alpha = 1, 2),
$$

where $x_i = m_i^2/m_\Lambda^2$, $m_i$ is the corresponding particle mass and $m_\Lambda$ is the new physics energy scale. $H^S_{L,R}$ represent the left (right)-hand part of the coupling vertex. The concrete expressions for one-loop functions $I_i(x_1, x_2)(i = 1, 2, 3, 4, 5)$ are collected in Appendix.

Then, we discuss the virtual charged fermion contributions $C^L_R(c)(\alpha = 1, 2)$ corresponding to FIG.1(b)

$$
C^L_1(c) = \sum_{F=\chi^0, L'} \sum_{S=\nu, \tilde{Y}} \frac{1}{6 m_\Lambda^2} H^S_{F \tilde{l}_i} H^{S^{*} l_j F} [3 I_2(x_S, x_F) - I_4(x_S, x_F)],
$$

FIG. 1: The triangle type diagrams for decays $l_j \rightarrow l_i \gamma$. 
\[ C_L^2(c) = \sum_{F=\pm, L, S=\pm, Y} \sum_{m_F} \frac{m_F}{m_L} H_L^{SF} H_L^{S*I} I_2(x_S, x_F), \]
\[ C_R^\alpha(c) = C_L^\alpha(c)|_{L\leftrightarrow R}, \alpha = 1, 2 \]  
(22)

Furthermore, the corrections from FIG.1(c) are denoted by \( C_L^{{L,R}}(W) (\alpha = 1, 2) \)
\[ C_L^1(W) = \sum_{F=\nu} \frac{1}{m^2_L} H_L^{WF} H_L^{W*I} [\frac{1}{3} I_2(x_F, x_W) + \frac{1}{3} I_4(x_F, x_W)], \]
\[ C_L^2(W) = \sum_{F=\nu} \frac{1}{m^2_L} H_L^{WF} H_L^{W*I} \frac{m_F}{m_{l_j}} [I_2(x_F, x_W) + I_4(x_F, x_W)], \]
\[ C_R^1(W) = 0, \]
\[ C_R^2(W) = \sum_{F=\nu} \frac{1}{m^2_L} H_L^{WF} H_L^{W*I} [I_2(x_F, x_W) + I_4(x_F, x_W)]. \]  
(23)

However, the contributions from \( W-W \)-neutrino diagram can be ignored due to the tiny neutrino mass.

We deduce the decay widths for processes \( l_j \rightarrow l_i\gamma \) as
\[ \Gamma (l_j \rightarrow l_i\gamma) = \frac{e^2}{16\pi m^5_{l_j}} \left( |C_L^2|^2 + |C_R^2|^2 \right). \]  
(24)

Then, the concrete branching ratios of \( l_j \rightarrow l_i\gamma \) can be expressed as
\[ Br (l_j \rightarrow l_i\gamma) = \Gamma (l_j \rightarrow l_i\gamma) / \Gamma_{l_j}. \]  
(25)

Here, \( \Gamma_{l_j} \) represent the total decay widths of the charged leptons \( l_j \). We take \( \Gamma_\mu \simeq 2.996 \times 10^{-19} \text{ GeV} \) and \( \Gamma_\tau \simeq 2.265 \times 10^{-12} \text{ GeV} \) in our latter numerical calculations.

B. \( \mu - e \) conversion in nuclei within the EBLMSSM

In this section, we give out the figures for \( \mu - e \) conversion in nuclei at the quark level within the EBLMSSM, which are shown in FIG.2 and FIG.3. The theoretical results for muon conversion to electron rates in nuclei are discussed specifically in our previous work. In our latter work, we will analyze this \( \mu - e \) conversion rates in nuclei in detail.

Considering the constraints from \( \mu \rightarrow e\gamma \), we study \( \mu - e \) conversion in Au nuclei in this part. And the corresponding numerical results will be discussed in subsection B of section IV.
FIG. 2: The triangle type diagrams for the $\mu - e$ conversion processes at the quark level.

FIG. 3: The box type diagrams for the $\mu - e$ conversion processes at the quark level.

C. Rare decay $h^0 \to l_il_j$

The corresponding effective amplitude for $h^0 \to \bar{l}_il_j$ can be summarized as

$$A = \bar{u}_i(q)(N_{L}P_{L} + N_{R}P_{R})v_j(p),$$

$$N_{L,R} = N_{L,R}(S_1) + N_{L,R}(S_2) + N_{L,R}(W) + A_{L,R}(S_1) + A_{L,R}(S_2) + A_{L,R}(W_1) + A_{L,R}(W_2).$$

(26)

Here $N_{L,R}(S_1)$ are the coupling coefficients corresponding to triangle diagrams in FIG 4(a), $N_{L,R}(S_2)$ denote the contributions from FIG 4(b). The effects from FIG 4(c) and FIG 4(d) can be shown by $N_{L,R}(W)$. $A_{L,R}(S_1)$ and $A_{L,R}(S_2)$ represent the contributions from self-energy diagrams FIG 5(a) and FIG 4(b) respectively. The effects from FIG 5(c) and FIG 5(d) can be summarized by $A_{L,R}(W_1)$ and $A_{L,R}(W_2)$. We give out the concrete expressions for these contributions as follows.

The contributions from triangle diagrams in FIG 4:

$$N_L(S_1) = \sum_{F=\chi^\pm,\chi^0,\tilde{\chi}^0,\tilde{\chi}_L^0,\nu,\tilde{\nu}} \sum_{s=\tilde{r},\tilde{l}} \frac{m_F}{m_X^2} H_L^{S_1 F L} H_{h^0}^{S_1 S_2^*} H_L^{S_1^* S_2} G_1(x_F, x_{S_1}, x_{S_2}),$$
The contributions from self-energy type diagrams correspond to FIG. 5:

\[ N_R(S_1) = N_L(S_1)|_{L \leftrightarrow R}, \]

\[ N_L(S_2) = \sum_{F=\chi,\tilde{\chi},\nu,\nu'} \sum_{S=\tilde{L},L,H^+} \left[ H_L^{S,F,2l} H_R^{h_0,F_1,F_2} H_L^{S^*l,F_1} G_2(x_S,x_{F_1},x_{F_2}) + \sum_{F_1,F_2=\nu} \frac{m_{F_1} m_{F_2}}{m_\Lambda^2} H_L^{H,F_1,F_2} H_L^{W,F_1,F_2} G_1(x_S,x_{F_1},x_{F_2}) \right], \]

\[ N_R(S_2) = N_L(S_2)|_{L \leftrightarrow R} \]

\[ N_L(W) = -\sum_{F=\nu} \sum_{F_1,F_2=\nu} \frac{m_{F_1}}{m_\Lambda^2} H_L^{H,F_1,F_2} H_L^{F_1,F_2} H_L^{h_0,W_+,W_+} I_2(x_F,x_W) + \sum_{F_1,F_2=\nu} \sqrt{x_{F_1} x_{F_2}} H_L^{W,F_1,F_2} H_L^{F_1,F_2} H_L^{h_0,W_+,W_+} I_2(x_F,x_W) \]

The contributions from self-energy type diagrams correspond to FIG. 5:

FIG. 4: The triangle type diagrams for decays \( h^0 \to \tilde{l}_i l_j \).

FIG. 5: The self-energy type diagrams for decays \( h^0 \to \tilde{l}_i l_j \).

\[ A_L(S_1) = \sum_{F=\chi,\tilde{\chi},\nu,\nu'} \sum_{S=\tilde{L},L,H^+} \frac{1}{m_l^2 - m_i^2} H_L^{h_0,l,F} \left\{ m_{F_l} H_R^{l,F} H_R^{l,F} + m_{F_{l'}} H_L^{l,F} H_L^{l,F} [I_1(x_F,x_S) + m_{l'}^2 (I_2(x_F,x_S) - I_3(x_F,x_S))] - \frac{1}{2} (m_{l'}^2 H_R^{l,F} H_R^{l,F} + m_{l'} m_{l''} H_L^{l,F} H_L^{l,F} ) I_5(x_F,x_S) \right\}, \]

\[ A_R(S_1) = A_L(S_1)|_{L \leftrightarrow R}, \]
\[ A_L(S_2) = \sum_{F=\chi^0,\chi^{\pm},\chi^{\mp},\ell^-,\ell^+} \sum_{S=L,\nu,Y,E} \frac{1}{m_{i_i}^2 - m_{j_j}^2} H^{h_{ij}l_{ij}} \{ m_F(m_iH^lFS H^lF^SS) + m_jH^lFS H^lF^SS \} \]

\[ A_R(S_2) = A_L(S_2)|_{L \leftrightarrow R}, \]

\[ A_L(W_1) = - \sum_{F=\nu} \frac{m_{i_i}^2}{m_{j_j}^2 - m_{i_i}^2} H^{h_{iij}l_{iij}} H^{l_{iij}FW} H^{l_{jjj}FW^*} I_5(x_F, x_W), \]

\[ A_R(W_1) = - \sum_{F=\nu} \frac{m_{i_i}m_{j_j}}{m_{i_i}^2 - m_{j_j}^2} H^{h_{iij}l_{iij}} H^{l_{jjj}FW} H^{l_{jjj}FW^*} I_5(x_F, x_W). \]

where, the one-loop functions \( G_i(x_1, x_2, x_3)(i = 1, 2) \) are collected in Appendix.

The decay widths for processes \( h^0 \to l_il_j \) are deduced here

\[ \Gamma(h^0 \to l_il_j) = (h^0 \to \bar{l}_i\bar{l}_j) + (h^0 \to l_i\bar{l}_j), \]

where \( \Gamma(h^0 \to \bar{l}_i\bar{l}_j) = \frac{1}{16\pi} m_{h^0} (|N_L|^2 + |N_R|^2) \). Correspondingly, the calculations for \( h^0 \to l_i\bar{l}_j \) are same as those for \( h^0 \to \bar{l}_i\bar{l}_j \).

Above all, the branching ratios of \( h^0 \to l_i l_j \) can be summarized as

\[ Br(h^0 \to l_i l_j) = \Gamma(h^0 \to l_i l_j)/\Gamma_{h^0}. \]

Here, the total decay width of the 125.1 GeV Higgs boson \( \Gamma_{h^0} \simeq 4.1 \times 10^{-3} \) GeV [18].

IV. NUMERICAL RESULTS

In this section, we discuss the numerical results. In our previous work [34], we research the processes \( h^0 \to \gamma\gamma, h^0 \to VV, V = (Z, W) \) in the EBLMSSM, and the corresponding numerical results are discussed in section 5.1 of work [34]. The CP even Higgs masses \( m_{h^0}, m_{h^0} \)
and CP odd Higgs mass $m_A^0$ are also analyzed. It implies that the new added parameters in the EBLMSSM play important roles in our numerical results. In the reasonable parameter space, the values of branching ratios for $h^0 \to \gamma\gamma(R_{\gamma\gamma})$ and $h^0 \to VV(R_{VV})$ both meet the experiment limits. Therefore, the Higgs decays in the EBLMSSM play important roles to promote physicists to explore new physics. And the corresponding constraints are also considered in our work. For the parameter space, the most strict constraint is the mass squared matrix in Eq. (16), whose mass of the lightest Higgs boson is around 125.1 GeV. Therefore, the CP-odd Higgs mass should satisfy the following relation with the input parameter $m_{h^0} = 125.1$ GeV:

$$m_{A_0}^2 = \frac{m_{h^0}^2 (m_Z^2 - m_{h^0}^2 + \Delta_{11} + \Delta_{22}) - m_Z^2 \Delta_{1} - \Delta_{11} \Delta_{22}}{-m_{h^0}^2 + m_Z^2 \cos^2 2\beta + \Delta_B},$$

$$\Delta_A = \sin^2 \beta \Delta_{11} + \cos^2 \beta \Delta_{22} + \sin 2\beta \Delta_{12},$$

$$\Delta_B = \cos^2 \beta \Delta_{11} + \sin^2 \beta \Delta_{22} + \sin 2\beta \Delta_{12}.$$  \hspace{1cm} (36)

In the EBLMSSM, to obtain a more transparent numerical results, we adopt the following assumptions on parameter space:

$$Y_{u4} = 1.2Y_t, \ Y_{u5} = 0.6Y_t, \ Y_{d4} = Y_{d5} = 2Y_b, \ Y_{v4} = Y_{v5} = 0.8, \ \mu_B = \mu_L = 0.5\text{TeV},$$

$$m_{\tilde{Q}_4} = m_{\tilde{Q}_5} = m_{\tilde{U}_4} = m_{\tilde{U}_5} = m_{\tilde{D}_4} = m_{\tilde{D}_5} = m_{\tilde{e}_4} = m_{\tilde{e}_5} = 1\text{TeV}, \ B_4 = L_4 = 1.5,$$

$$A_{u4} = A_{u5} = A_{d4} = A_{d5} = A_{v4} = A_{v5} = 1\text{TeV}, \ \lambda_Q = 0.4, \ \lambda_u = \lambda_d = 0.5, (\lambda_{Nc})_{ij} = 1,$$

$$A_{BQ} = A_{BU} = A_{BD} = 1\text{TeV}, \ g_B = 1/3, \ g_L = 1/6, \ \tan \beta_B = 1.5, \ \tan \beta_L = 2,$$

$$m_{\tilde{Q}_3} = m_{\tilde{U}_3} = 1.2\text{TeV}, \ m_{\tilde{D}_3} = 1.5\text{TeV}, \ A_t = 1.7\text{TeV}, \ A_b = 3\text{TeV}, \ M_L = 1\text{TeV},$$

$$(m_{\tilde{v}})_{ii} = 1\text{TeV}, \ (A_N)_{ii} = (A_{Nc})_{ii} = 0.5\text{TeV}, \ m_A = 1\text{TeV}$$  \hspace{1cm} (37)

where $i = 1, 2, 3, \ Y_t (Y_b)$ corresponds to the Yukawa coupling constant of top (bottom) quark, whose concrete form can be written as $Y_t = \sqrt{2}m_t/(v \sin \beta)$ ($Y_b = \sqrt{2}m_b/(v \cos \beta)$).

In order to simplify the numerical analysis, we use the following assumptions:

$$m_{\tilde{l}_4} = m_{\tilde{l}_5} = m_{\tilde{e}_4} = m_{\tilde{e}_5} = M_{\tilde{E}}, \ A_{e4} = A_{e5} = A_{\tilde{E}}, \ \lambda_L = \lambda_E = \lambda_{N_L} = L_l,$$

$$A_{LL} = A_{LE} = A_{LN} = A_E, \ \lambda_i^2 = \lambda_6^2 = (L m^2)^{IJ}, \ I, J = 1, 2, 3, \ v_{NHint} = v_N,$$

$$(m_{\tilde{l}}^2)_{ii} = (M_{Ls})_{ii}^2, \ (m_{\tilde{l}}^2)_{ij} = M_{LJ}^2, \ (A_{i})_{ii} = Al, (A_{i})_{ij} = A'l, \ i, j = 1.2.3, i \neq j.$$  \hspace{1cm} (38)

We take $\sqrt{(L m^2)^{12}} = L_F$ and $\sqrt{(L m^2)^{13}} = \sqrt{(L m^2)^{23}} = L_f.$
A. \( l_j \rightarrow l_i \gamma \)

1. \( \mu \rightarrow e \gamma \)

CLFV process \( \mu \rightarrow e \gamma \) contributes to explore the new physics, whose experiment upper bound of the branching ratio is around \( 5.7 \times 10^{-13} \) at 90\% confidence level. In this part, we discuss the effects on process \( \mu \rightarrow e \gamma \) from some new introduced parameters in the EBLMSSM.

Parameter \( A_\tilde{E} \) presents in the non-diagonal parts of the exotic slepton mass matrix. Parameter \( Y_{e5} \) is not only related to the non-diagonal parts of the exotic lepton and exotic slepton mass matrices, but also connected with the diagonal exotic slepton elements. In the EBLMSSM, exotic lepton and exotic slepton are both different from those in the BLMSSM. We assume that \( M_\tilde{E} = \mu_Y = 1.5\text{TeV}, A_\tilde{E} = \mu_{NL} = 1\text{TeV}, L_l = 1, L_F = 10^{-3}, \mu = 0.7\text{TeV}, B_Y = 0.94\text{TeV}, Y_{e4} = 3, (M_{Ls})_{11}^2 = 6\text{TeV}^2, (M_{Ls})_{22}^2 = 4\text{TeV}^2, (M_{Ls})_{33}^2 = 1\text{TeV}^2, M_{Lf}^2 = 10^{-3}\text{TeV}^2, Al = 2\text{TeV}, A'l = 0.3\text{TeV}, m_1 = m_2 = 1.5\text{TeV}, \tan \beta = 6 \) and \( \tan \beta_{NL} = 2 \).

With \( Y_{e5} = 0.2(0.7,1.2) \), the branching ratios of \( \mu \rightarrow e \gamma \) versus parameter \( A_\tilde{E} \) are studied, which are shown in FIG.6. When \( A_\tilde{E} \) is in the region \( 0.1 \sim 2.5\text{TeV} \), the numerical results change from \( 5 \times 10^{-15} \) to \( 5 \times 10^{-13} \). These three lines all increase quickly and approach to the experiment upper bound. Furthermore, corresponding to same \( A_\tilde{E} \), the solid line results are about 2 times as the dashed line results, and the dashed line results are about 2 times as the dotted line results. Larger \( Y_{e5} \) can lead to larger numerical results. Therefore, \( A_\tilde{E} \) and \( Y_{e5} \) both affect the numerical results strongly.

With the introduced superfields \( Y \) and \( Y' \) in the EBLMSSM, we deduce the \( Y \) and \( \tilde{Y} \) mass matrices. Parameters \( \mu_Y \) and \( B_Y \) respectively present in the diagonal and non-diagonal terms of the \( Y \) mass matrix. And the mass of \( \tilde{Y} \) possesses the same value as that \( \mu_Y \). So these two parameters affect the \( Y \)-lepton-exotic lepton and \( \tilde{Y} \)-lepton-exotic slepton couplings. Furthermore, these new couplings make contributions to the numerical results. Using \( Y_{e5} = 0.8 \) and \( A_\tilde{E} = 1\text{TeV} \), we plot the branching ratios changing with \( \mu_Y \) in FIG.7. The dotted (dashed, solid) line represents \( B_Y = 0.4(0.8,1.2)\text{TeV} \). We find that the branching ratios decrease quickly with the increasing \( \mu_Y \), which indicates that the large \( \mu_Y \) can restrain
FIG. 6: With $Y_{e5} = 0.2(0.7, 1.2)$, the branching ratios of $\mu \to e\gamma$ versus parameter $A_E$ are plotted by the dotted, dashed and solid lines respectively.

FIG. 7: With $B_Y = 0.5(0.8, 1.2)$ TeV, the branching ratios of $\mu \to e\gamma$ versus parameter $\mu_Y$ are plotted by the dotted, dashed and solid lines respectively.

Then, we study effects from the parameters $A_E$ and $\mu_{NL}$ on our numerical results. In EBLMSSM, parameters $A_E$ and $\mu_{NL}$ are both the non-diagonal elements in the exotic slepton and exotic sneutrino mass matrices. $\mu_{NL}$ is also the non-diagonal element of lepton neutralino mass matrix. In FIG. 8 we present the branching ratios of $\mu \to e\gamma$ versus $A_E$ with $\mu_{NL} = 0.3(1.0, 1.7)$ TeV, and the concrete results are plotted by dotted (dashed, solid) line. These three lines all increase smartly when $A_E$ ranges from 0.1 to 2 TeV. The larger $\mu_{NL}$ leads to the larger numerical result with the unchanging $A_E$. Therefore, as the sensitive
FIG. 8: With $\mu_{NL} = 0.3(1.0, 1.7)\,\text{TeV}$, the branching ratios of $\mu \rightarrow e\gamma$ versus parameter $A_E$ are plotted by the dotted, dashed and solid lines respectively.

parameters in the EBLMSSM, the large $A_E$ and $\mu_{NL}$ both produce the large contributions on the results.

2. $\tau \rightarrow \mu\gamma$ ($\tau \rightarrow e\gamma$)

In a similar way, the CLFV processes $\tau \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ are studied. The corresponding experimental upper bounds of the branching ratios are $Br(\tau \rightarrow e\gamma) < 3.3 \times 10^{-8}$ and $Br(\tau \rightarrow \mu\gamma) < 4.4 \times 10^{-8}$.

As a new introduced parameter in the EBLMSSM, parameter $v_N$ is present in the mass matrices of slepton, sneutrino, exotic lepton (neutrino), exotic slepton (sneutrino) and lepton neutralino. In this part, we research the branching ratios of $\tau \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ changing with $v_N$. Supposing $M_{\tilde{E}} = \mu_Y = 1.5\,\text{TeV}$, $A_E = \mu_{NL} = 1\,\text{TeV}$, $A_{\tilde{E}} = 1\,\text{TeV}$, $L_l = 1$, $L_f = 0.08$, $\mu = 0.7\,\text{TeV}$, $B_Y = 0.94\,\text{TeV}$, $Y_{e4} = 3$, $Y_{e5} = 0.8$, $(M_{Ls})^2_{ii} = S_m^2 = 1\,\text{TeV}^2$, $i=1,2,3$, $M_{L_f}^2 = 10^{-2}\,\text{TeV}^2$, $A l = 2\,\text{TeV}$, $A' l = 0.3\,\text{TeV}$, $m_1 = m_2 = 1.5\,\text{TeV}$, $\tan \beta = 6$ and $\tan \beta_{NL} = 2$, we plot the allowed results with $v_N$ in FIG.9 by dashed line and solid line respectively. Obviously, when the value of $v_N$ increases in the suitable scope, the results of $Br(\tau \rightarrow e\gamma)$ and $Br(\tau \rightarrow \mu\gamma)$ both shrink quickly. This implies that $v_N$ is a sensitive parameter to the numerical results. Though the figure of process $\tau \rightarrow e\gamma$ is under that of $\tau \rightarrow \mu\gamma$, the both lines possess almost the same results when $v_N$ takes same value. So we only study the branching ratios of process $\tau \rightarrow \mu\gamma$ in following discussion.
Multiplying 10
Minus 9

1.5

\(N/\text{GeV}\)

\(\text{Br}(\tau \rightarrow e\gamma)\)
\(\text{Br}(\tau \rightarrow \mu\gamma)\)

FIG. 9: The branching ratios of \(\tau \rightarrow e\gamma\) and \(\tau \rightarrow \mu\gamma\) versus parameter \(v_N\) are plotted by the dashed line and solid line respectively.

Appearing in the diagonal terms of the exotic slepton and exotic sneutrino mass squared matrices, \(M_{\tilde{E}}\) affects the \(\tilde{Y}\)-lepton-exotic slepton coupling. Parameter \(L_l\), not only in the exotic lepton (neutrino) but also in exotic slepton (sneutrino) mass matrices, produces contributions to the numerical results through \(Y\)-lepton-exotic lepton and \(\tilde{Y}\)-lepton-exotic slepton couplings. With \((M_{Ls})_{11}^2 = 6\text{TeV}^2\), \((M_{Ls})_{22}^2 = 4\text{TeV}^2\), \((M_{Ls})_{33}^2 = 1\text{TeV}^2\) and \(v_N = 4\text{TeV}\), the numerical results versus \(M_{\tilde{E}}^2\) are plotted in FIG. 10. The dotted (dashed, solid) line corresponds to \(L_l = 0.7(1.0, 1.3)\). The figure shows that these three lines all decrease quickly when \(M_{\tilde{E}}^2\) varies from \(1 \times 10^6\) to \(8 \times 10^6\text{GeV}^2\). With the same \(M_{\tilde{E}}^2\), the branching ratio decreases remarkably when \(L_l\) increases. Especially, the line is much steeper with \(L_l = 0.7\) than that \(L_l = 1.0(1.3)\). Obviously, both \(M_{\tilde{E}}\) and \(L_l\) are sensitive parameters to our numerical results.

Parameter \(L_f\) contributes to the numerical results through \(Y\)-lepton-exotic lepton and \(\tilde{Y}\)-slepton-exotic slepton couplings. And parameter \(Y_{e4}\) affects our numerical results through exotic lepton and exotic slepton. We discuss the numerical results with \(L_f\) varying from 0.01 to 0.3 in FIG. 11. The dotted (dashed, solid) line corresponds to \(Y_{e4} = 1.0(2.0, 3.0)\). The branching ratios possess slight variations when \(Y_{e4}\) takes different values for the unchanged \(L_f\), which indicates the effects from \(Y_{e4}\) can be ignored in our following discussion. It is easy to see the numerical results increase sharply with the enlarging \(L_f\). So \(L_f\) should be around or smaller than 0.3. The non-diagonal elements of parameters \(\lambda_3^2\) and \(\lambda_6^2\) play important roles in our numerical studies.
FIG. 10: With $L_l = 0.7(1.0, 1.3)$, the branching ratios of $\tau \to \mu \gamma$ versus parameter $M_E^2$ are plotted by the dotted, dashed and solid lines respectively.

![Graph](image1.png)

FIG. 11: With $Y_{e4} = 1.0(2.0, 3.0)$, the branching ratios of $\tau \to \mu \gamma$ versus parameter $L_f$ are plotted by the dotted, dashed and solid lines respectively.

![Graph](image2.png)

B. $\mu - e$ conversion rates in nuclei Au

The present sensitivity for the muon conversion rates to electron in Au nuclei is $CR(\mu \to e : ^{197}_{79}Au) < 7 \times 10^{-13}$. Considering the parameter constrains from $\mu \to e\gamma$, we analyze the numerical results for this $\mu - e$ conversion in Au nuclei.

As the non-diagonal elements of matrix $(Lm^2)^{IJ}$ in the EBLMSSM, $\sqrt{(Lm^2)^{12}} = L_F$ affects the numerical results through exotic lepton and exotic slepton. Choosing $M_E = A_E = \mu_{NL} = \mu = 1\text{TeV}$, $\mu_Y = 2\text{TeV}$, $L_l = 0.8$, $B_Y = 1.5\text{TeV}$, $Y_{e4} = 3$, $Y_{e5} = 1$, $S_m^2 = 6\text{TeV}^2$, $M_{L_f}^2 = 500\text{GeV}^2$, $m_1 = m_2 = 3\text{TeV}$, $A_l = 2\text{TeV}$, $A' l = 0.3\text{TeV}$ and $\tan \beta = 6$, we analyze the $\mu - e$ conversion rates in Au nuclei with $L_F$ in FIG[12] $\tan \beta_{NL} = 1.8, 2.2, 2.6$ correspond
1. Multiply 10 / Minus 13

2. Multiply 10 / Minus 13

3. Multiply 10 / Minus 13

4. Multiply 10 / Minus 13

5. Multiply 10 / Minus 13

6. Multiply 10 / Minus 13

7. Multiply 10 / Minus 13

8. Multiply 10 / Minus 13

FIG. 12: With $\tan \beta_{NL} = 1.8, 2.2, 2.6$, the $\mu - e$ conversion rates in Au nuclei versus parameter $L_F$ are plotted by the dotted, dashed and solid lines respectively.

to the dotted, dashed and solid lines respectively. When $L_F$ changes from 0.001 to 0.006, These three lines all enlarge quickly and can easily reach the present sensitivity. So $L_F$ greatly contributes to the numerical results. Generally, we take $L_F$ around or smaller than 0.006 in our letter calculations. Furthermore, the larger $\tan \beta_{NL}$, the smaller numerical results it is. The above analyses indicate $L_F$ and $\tan \beta_{NL}$ are both sensitive parameters.

As the non-diagonal elements of slepton and sneutrino mass matrices, $M_{L_f}$ lead to strong mixing for slepton (sneutrino) with different generations. The parameter $\mu$ presents in the mass matrices of slepton, sneutrino, exotic slepton and exotic sneutrino. So we study the $\mu - e$ conversion rates in Au nuclei versus parameters $M_{L_f}^2$. As $M_\tilde{E} = 2\text{TeV}$, $\mu_Y = 3\text{TeV}$, $B_Y = 2.5\text{TeV}$, $S_m^2 = 12\text{TeV}^2$, $m_1 = m_2 = 4\text{TeV}$, $A_l = 1.9\text{TeV}$, $\tan \beta_{NL} = 2$ and $L_F = 0.001$, we show the numerical results changing with $M_{L_f}^2$, which are given in FIG.13. The dotted, dashed and solid lines respectively correspond to $\mu = 0.4, 0.5, 0.6\text{TeV}$. With the enlarging $M_{L_f}^2$, the $\mu - e$ conversion rates in Au nuclei increase quickly. However, the value of $M_{L_f}^2$ should be around or smaller than 12000 GeV$^2$. The three lines almost possess similar variation trend, which shows that the effects from parameter $\mu$ can be ignore.

$S_m$ are the diagonal elements of slepton and sneutrino mass matrices. We study the muon conversion rates to electron in Au nuclei with parameters $S_m$ and $M_{L_f}$. Choosing $\mu = 1.1\text{TeV}$, $A_l = 2\text{TeV}$ and $M_{L_f}^2 = 1500\text{(2000, 2500)}\text{GeV}^2$, we scan the numerical results changing with $S_m^2$, which are given in FIG.14. As $1\text{TeV}^2 < S_m^2 < 4\text{TeV}^2$, the values have obvious decrease, but all of the three lines are stable with $S_m^2 > 4\text{TeV}^2$. In the range of TeV
FIG. 13: With $\mu = 400(500, 600)\text{GeV}$, the $\mu - e$ conversion rates in Au nuclei versus parameter $M_{Lf}^2$ are plotted by the dotted, dashed and solid lines respectively.

FIG. 14: With $M_{Lf}^2 = 1500(2000, 2500)\text{GeV}^2$, the $\mu - e$ conversion rates in Au nuclei versus parameter $S_m^2$ are plotted by the dotted, dashed and solid lines respectively.

scale, the smaller $S_m$ can produce larger contributions to the numerical results. And the large $M_{Lf}$ can promote the $\mu - e$ conversion rates.

C. $h^0 \to l_i l_j$

In this part, we study the CLFV processes $h^0 \to l_i l_j$. The most strict constraint $m_{h^0} = 125.1\text{GeV}$ is discussed in Section III. We also consider the limits from processes $l_j \to l_i \gamma$ and the muon conversion to electron in Au nuclei discussed above.
FIG. 15: The branching ratios of $h^0 \rightarrow \mu\tau$ and $h^0 \rightarrow e\tau$ versus parameter $A_E$ are plotted by the solid line and dashed line respectively.

FIG. 16: With $\tan\beta_{NL} = 1.8(2.1, 2.4)$, the branching ratios of $h^0 \rightarrow \mu\tau$ versus parameter $\tan\beta$ are plotted by the dotted, dashed and solid lines respectively.

1. $h^0 \rightarrow \mu\tau \ (h^0 \rightarrow e\tau)$

At first, we picture the branching ratios of decays $h^0 \rightarrow \mu\tau$ and $h^0 \rightarrow e\tau$ versus $A_E$ in FIG.15 We choose the relevant parameters as $\mu_Y = 1.5\text{TeV}$, $M_E = A_E = \mu_{NL} = m_1 = 1\text{TeV}$, $L_l = 0.8$, $(Lm^2)^{13}$ = $(Lm^2)^{23} = L_f = 0.3^2$, $\mu = 0.7\text{TeV}$, $B_Y = 0.94\text{TeV}$, $Y_{e4} = 3$, $Y_{e5} = 0.5$, $S_{\mu}^2 = 1\text{TeV}^2, M_{L_f}^2 = 12000\text{GeV}^2$, $A_l = 1\text{TeV}$, $A'l = 3\text{TeV}$, $m_2 = 0.5\text{TeV}$, $\tan\beta = 6$ and $\tan\beta_{NL} = 2$. Although the line of $h^0 \rightarrow e\tau$ is under that of $h^0 \rightarrow \mu\tau$, these two processes almost have the same variation trend. With the enlarging $A_E$, the numerical results increase quickly.

Then the effects from the parameters $\tan\beta$ and $\tan\beta_{NL}$ are studied. $\tan\beta$ is related to
FIG. 17: With $A_l = 1.5(2, 2.5)$ TeV, the branching ratios of $h^0 \to e\mu$ versus parameter $A'l$ are plotted by the dotted (dashed, solid) line.

$v_u$ and $v_d$, and appears in almost all mass matrices of CLFV processes. $\tan \beta_{NL}$ is related to $v_{NL}$ and $\bar{v}_{NL}$ (the nonzero VEVs of the $SU(2)_L$ singlets $\Phi_{NL}$ and $\varphi_{NL}$). With $A_E = 2.4$ TeV, $L_f = 0.25$, FIG. 16 shows the branching fractions of $h^0 \to \mu\tau$ varying with the parameter $\tan \beta$. $\tan \beta_{NL} = 1.8, 2.1, 2.4$ correspond to the dotted, dashed, solid lines respectively. These three lines almost overlap, so the effect from $\tan \beta_{NL}$ is small. $\tan \beta$ varies from 6 to 12, and it makes the numerical results decrease obviously. So $\tan \beta$ plays very important roles to CLFV processes.

2. $h^0 \to e\mu$

The latest experiment upper bound of decay $h^0 \to e\mu$ is smaller than 0.035% at 95% confidence level, which is detected by the CMS Collaboration. $A_l$ and $A'l$ both appear in the non-diagonal terms of the slepton mass matrix. Considering the constraints from $\mu \to e\gamma$ and $\mu - e$ conversion in Au nuclei, we take $M_{\tilde{E}} = A_E = \mu_{NL} = 1$ TeV, $A_E = \mu_Y = 2.5$ TeV $L_l = 0.8$, $L_f = 0.006$, $B_Y = 0.94$ TeV, $Y_{e4} = 3$, $Y_{e5} = 0.8$, $S_{m}^2 = 1$ TeV$^2$, $M_{L_f}^2 = 12000$ GeV$^2$, $m_1 = m_2 = 0.5$ TeV, $\mu = 0.7$ TeV, $\tan \beta = 6$ and $\tan \beta_{NL} = 2$. The dotted (dashed, solid) line in FIG. 17 denotes the branching ratios of $h^0 \to e\mu$ versus $A'l$ with $A_l = 1.5(2, 2.5)$ TeV. These three lines all increase mildly with the enlarging $A'l$. The larger $A_l$, the smaller numerical results it is. Therefore, the contributions from $A_l$ are very weak.

In this part, supposing $Y_{e5} = 2.6$ $A_E = 2$ TeV, $m_2 = 1$ TeV, $A_l = 1$ TeV and $A'l = 3$ TeV,
we picture the branching ratios of $h^0 \to e\mu$ versus parameter $\mu$. In FIG. 18, the dotted (dashed, solid) line corresponds to $m_1 = 1(2, 3)$TeV. The values of $Br(h^0 \to e\mu)$ decrease quickly when $\mu$ changes from 0.4TeV to 1TeV. As $\mu > 1$TeV, the numerical results almost do not change. Furthermore, $Br(h^0 \to e\mu)$ possess obvious change with the different $m_1$ as $\mu$ around 0.6TeV. Therefore, both $\mu$ and $m_1$ contribute to decay $h^0 \to e\mu$.

At last, we discuss the effects from parameters $Y_{e5}$ and $M_{\tilde{E}}$. With $m_1 = m_2 = 0.5$TeV, $A_l = 2.5$TeV and $L_F = 0.004$, the branching ratios varying with $Y_{e5}$ are painted in FIG. 19. The dotted, dashed and solid lines respectively correspond to $M_{\tilde{E}} = 1.1, 1.3, 1.5$TeV. These three lines all slightly increase when $Y_{e5}$ varies from 0.1 to 1.5. As $Y_{e5}$ still increases from
1.5, the results have much more conspicuous enlargement. However, the total contributions from $Y_{e5}$ are not so obvious. With the enlarging $M_{E}$, the numerical results reduce more and more slowly.

V. DISCUSSION AND CONCLUSION

We add exotic superfields $\Phi_{NL}$, $\varphi_{NL}$, $Y$ and $Y'$ to the BLMSSM, and this new model is named as the EBLMSSM. In $\mathcal{W}_Y$, $\lambda_4(\lambda_6)$ is the coupling coefficient of $Y$-lepton-exotic lepton and $\tilde{Y}$-lepton-exotic slepton. We assume $\lambda_4^2 = \lambda_6^2$ is a $3 \times 3$ squared matrix and its non-diagonal elements are related with the CLFV. Being different from the BLMSSM, the exotic slepton (sneutrino) of 4-th and 5-th generations mix together and form a $4 \times 4$ matrix. The Majorana particle, lepton neutralino $\chi_{L}^0$, is corrected to be a $5 \times 5$ matrix due to the introduction of superpartners $\psi_{\Phi_{NL}}$ and $\psi_{\varphi_{NL}}$. The terms relating with exotic lepton (neutrino) and slepton (sneutrino) are also adjusted. In Section III, we show the corresponding mass matrices and couplings of the EBLMSSM. The EBLMSSM has more abundant contents than BLMSSM for the lepton physics.

Considering the constraints from decays $h^0 \to \gamma \gamma$ and $h^0 \to VV, V = (Z,W)$, we study the CLFV processes $l_j \to l_i \gamma$ and muon conversion to electron in Au nuclei in the framework of the EBLMSSM. Parameters $Y_{e4}, Y_{e5}, S_m$ and $M_{LF}$ contribute to the numerical results very obviously. As the new introduced parameters in the EBLMSSM, $B_Y, A_E, L_l, \mu_{NL}, M_{E}$ play important roles. Especially parameters $A_E, \mu_Y, L_f, L_F$ are all sensitive parameters, which affect the numerical results very remarkably. FIG[11] and FIG[12] indicate that the enlarging $L_f$ and $L_F$ can easily improve the numerical results. So the values of $L_f$ and $L_F$ should be around or smaller than 0.3 and 0.006 respectively. Parameter $M_{LF}^2$ also should not possess a large value, which is around or smaller than 12000 GeV$^2$. Then, the 125.1 GeV Higgs boson decays with CLFV $h^0 \to l_i l_j$ are discussed. As an import constraint, $m_{h^0} = 125.1$GeV is regarded as an input parameter. Taking into account the constraints from the parameter space of decays $l_j \to l_i \gamma$ and muon conversion to electron in Au nuclei, we analyze the branching ratios for $h^0 \to l_i l_j$ in EBLMSSM. We find that the effects from $\tan \beta$ are very obvious. So $\tan \beta$ is also a sensitive parameter. Above all, due to the new
particles introduced in the EBLMSSM, the numerical results can easily reach or approach to the present experiment upper bounds.

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VI. APPENDIX

In this section, we give out the corresponding one-loop integral functions, which are read as:

\[
I_1(x_1, x_2) = \frac{1}{16\pi^2} \left[ -\left(\Delta + 1 + \ln x_\mu\right) + \frac{x_2 \ln x_2 - x_1 \ln x_1}{x_2 - x_1} \right],
\]

\[
I_2(x_1, x_2) = \frac{1}{32\pi^2} \left[ \frac{3 + 2 \ln x_2}{x_2 - x_1} - \frac{2x_2 + 4x_2 \ln x_2}{(x_2 - x_1)^2} + \frac{2x_2^2 \ln x_2 - 2x_1^2 \ln x_1}{(x_2 - x_1)^3} \right],
\]

\[
I_3(x_1, x_2) = \frac{1}{16\pi^2} \left[ \frac{1 + \ln x_2}{x_2 - x_1} + \frac{x_1 \ln x_1 - x_2 \ln x_2}{(x_2 - x_1)^2} \right],
\]

\[
I_4(x_1, x_2) = \frac{1}{96\pi^2} \left[ \frac{11 + 6 \ln x_2}{(x_2 - x_1)} - \frac{15x_2 + 18x_2 \ln x_2}{(x_2 - x_1)^2} + \frac{6x_2^2 + 18x_2^2 \ln x_2}{(x_2 - x_1)^3} \right.
\]

\[
+ \frac{6x_1^3 \ln x_1 - 6x_2^3 \ln x_2}{(x_2 - x_1)^4} \right],
\]

\[
I_5(x_1, x_2) = \frac{1}{16\pi^2} \left[ (\Delta + 1 + \ln x_\mu) + \frac{x_2 + 2x_2 \ln x_2}{x_1 - x_2} + \frac{x_2^2 \ln x_2 - x_1^2 \ln x_1}{(x_2 - x_1)^2} \right],
\]

\[
G_1(x_1, x_2, x_3) = \frac{1}{16\pi^2} \left[ \frac{x_1 \ln x_1}{(x_1 - x_2)(x_1 - x_3)} + \frac{x_2 \ln x_2}{(x_2 - x_1)(x_2 - x_3)} + \frac{x_3 \ln x_3}{(x_3 - x_1)(x_3 - x_2)} \right],
\]

\[
G_2(x_1, x_2, x_3) = \frac{1}{16\pi^2} \left[ -(\Delta + 1 + \ln x_\mu) + \frac{x_1^2 \ln x_1}{(x_1 - x_2)(x_1 - x_3)} \right.
\]

\[
+ \frac{x_2^2 \ln x_2}{(x_2 - x_1)(x_2 - x_3)} + \frac{x_3^2 \ln x_3}{(x_3 - x_1)(x_3 - x_2)} \right].
\]
with $\Delta = \frac{1}{\epsilon} - r_\epsilon + \ln 4\pi$.

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