Discrete optimization model for permutation flow shop under time-of-use electricity tariffs

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Abstract. The time-of-use electricity tariffs (TOU) which is widely used at home and abroad make the timeline for scheduling to have the attribute of different electricity tariffs. In order to solve the difficulty of coordination between electricity price segmentation and product processing time, this paper proposes a discrete optimization model, which uses central moment method modelling to ensure the continuity of job-processing and the uniqueness of processing time points. A discrete optimization model with maximum completion time and total electricity cost is constructed. According to the instance of the processing workshop and the characteristics of GAMS software, a virtual small-scale case of detection is constructed. By setting 11 groups of experiments with different weights, the total electricity cost is reduced by 30\% compared with the traditional scheduling scheme after the electricity cost optimization is considered to avoid the peak arrangement of the electricity price. Although the maximum completion time is increased by nearly 30\% on the surface, the downtime of staggering the peak is removed, the actual processing time is 42 minutes, and the total completion time is only increased by 5\%.

1. Introduction

The continual increase in the world's energy demand and the serious shortage of energy reserves had led to energy tariffs and electricity costs rising [1]. And in the scheduling, the optimization targets should be thought over, and electricity cost should be taken into account seriously and formally. Considering the TOU electricity tariffs to arrange the processing machines and order for jobs rationally benefits not only reducing the production cost of plant but also helping to achieve the low-carbon economy by avoiding producing at rush period for electricity need [2].

There are relatively more research achievements about low-carbon scheduling. In the Literature [3] realized a low-carbon scheduling for the flexible flow shop of the cutting process by optimizing the machine speed of the machining parameters when the reasonable machines were chosen and the processing orders for jobs were decided. In the Literature [4] and [5] succeeded in improving the energy efficiency of the flow shop by establishing a mixed integer linear programming model. In the Literature [6] improved the energy efficiency of the flow shop considering by a multi-objective backtracking search algorithm to find out the best scheduling. In the Literature [7] studied scheduling problems that minimized energy consumption and minimized delay penalties. In the Literature [8] proposed a mechanism to enhance the energy-using efficiency by deciding to turn on/off the machines and vary the speed of machines.
In the study of scheduling optimization under time-of-use electricity tariffs, the Literature [9] proposed a new continuous-time mixed integer linear programming model based on time-of-use electricity tariffs for the single machine scheduling problem, and developed an effective greedy insertion heuristic algorithm to solve this model. And the Literature [10] further proposed an improved continuous-time mixed integer linear programming model and a two-stage heuristic algorithm to solve the model for non-correlated parallel machine scheduling. In the Literature [11] developed a polynomial time algorithm to solve the problem of constant machine processing speed, and developed several approximation algorithms to solve the problem. In the Literature [12] designed discrete-time integer programming models and developed genetic algorithms to minimize the total cost of electricity, considering time-of-use electricity tariffs and machine switching mechanisms.

This paper proposes a new discrete-time mixed integer programming linear model, which can achieve synergistic optimization of energy cost and maximum completion time, and pursue both economic and environmental benefits. By arranging the different processing sequences of the jobs and determining the start-up time of the job processing, the low-energy products are processed in a high price period, and the high-energy products are processed in a low-price period, thereby minimizing the total energy consumption cost of permutation flow shop scheduling.

2. Problem description and modelling

2.1. Problem description

The permutation flow shop scheduling is to process n jobs on m machines, the processing order for the jobs is the same on all machines, and the schedule is generated by determining the job order. In this paper, the permutation flow shop scheduling requirement of time-of-use electricity price determines the processing start processing time of the job. The job processing time spans multiple electricity price stages, and different time periods correspond to different electricity prices, as shown in Figure 1.

![Figure 1. Schematic diagram of job processing under time-of-use electricity tariffs.](image)

The model in this paper is a discrete time as a point in time, there are n time points in the entire time zone T. The interval between time points is uniform, for example, in seconds or in hours. As shown in Figure 2.

![Figure 2. Discrete time schematic diagram.](image)

2.2. Modelling

In this paper, a new MILP model is established for multi-objective optimization of makespan and minimum total electricity cost. The discrete time unit can describe the relationship between time length and processing time. Therefore, the discrete time modelling technique is a classic modelling method of scheduling problem with time-of-use electricity tariffs. Since the discrete precision is difficult to determine and the high computational complexity, it is necessary to control the discrete precision to be balanced. Set the parameters as follows.
Sets:
i, k: jobs, 1 ≤ i, k ≤ n
j: machines, 1 ≤ j ≤ m
t, tt: event point, 0 ≤ t, tt ≤ T

Parameters:
T: Maximum value of scheduling time domain, [0, T]
Max: An infinite number of positive numbers
a_{i,j}: Unit time power consumption of job i processed on machine j
f(t): Time-of-use electricity tariffs
p_{i,j}: Processing time of job i on machine j

Decision variables:
y_{i,j,t} = \begin{cases} 1, & \text{When the job i is working at the time point t of the machine j}, \\
0, & \text{other} \end{cases}

Auxiliary decision variable:
D_{i,k} = \begin{cases} 1, & \text{When the job i is processed before the job k}, \\
0, & \text{When the job i is processed after the job k} \end{cases}

Auxiliary continuous variable:
C_{\text{max}}: The end time of the last job on the last machine;
C_{\text{min}}: The start time of the first job on the first machine;

Using the symbols given above, a multi-objective optimization model with makespan and total electricity cost minimization is constructed. The problem can be expressed as the following MILP model.

\begin{align*}
\text{Min TEC} &= \sum_{j=1}^{m} \sum_{i=1}^{n} \sum_{t=0}^{T} (a_{i,j} \cdot f(t) \cdot y_{i,j,t}) \\
\text{Min Makespan} &= C_{\text{max}} - C_{\text{min}} \\
\text{s. t.} & \sum_{t=0}^{T} y_{i,j,t} = p_{i,j}, 1 ≤ i ≤ n, 1 ≤ j ≤ m \\
(1 - y_{i,j,t} + y_{i,j,t+1}) \cdot \text{Max} & ≥ \sum_{t=0}^{T} y_{i,j,t}, 1 ≤ i ≤ n, 1 ≤ j ≤ m, 0 ≤ t ≤ T - 2 \\
y_{i,j,t} + y_{k,j,t} & ≤ 1, 1 ≤ i ≤ k ≤ n, 1 ≤ j ≤ m, 0 ≤ t ≤ T \\
\left(\sum_{t=0}^{T} \frac{f \cdot y_{i,j,t}}{p_{i,j}} - \sum_{t=0}^{T} \frac{f \cdot y_{k,j,t}}{p_{k,j}}\right) & ≤ (1 - D_{i,j}) \cdot \text{Max}, 1 ≤ i ≤ k ≤ n, 1 ≤ j ≤ m \\
\left(\sum_{t=0}^{T} \frac{f \cdot y_{i,j,t}}{p_{i,j}} - \sum_{t=0}^{T} \frac{f \cdot y_{k,j,t}}{p_{k,j}}\right) & ≥ -D_{i,k} \cdot \text{Max}, 1 ≤ i ≤ k ≤ n, 1 ≤ j ≤ m \\
\sum_{t=0}^{T} \frac{f \cdot y_{i,j,t}}{p_{i,j}} & + \frac{p_{i,j} - 1}{2} - \frac{1}{2} \sum_{t=0}^{T} \frac{f \cdot y_{i,j,t+1}}{p_{i,j+1}} - \frac{p_{i,j+1} - 1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot I \leq n, 1 \leq j \leq m - 1 \\
C_{\text{max}} & ≥ \sum_{t=0}^{T} \frac{f \cdot y_{i,j,t}}{p_{i,j}} + \frac{p_{i,j} - 1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot I \leq n \\
C_{\text{min}} & ≤ \sum_{t=0}^{T} \frac{f \cdot y_{i,j,t}}{p_{i,j}} - \frac{p_{i,j} - 1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot I \leq n 
\end{align*}
Equation (1) represents minimization of electricity costs, and Equation (2) represents minimum completion time. Equation (3) indicates that the machining time of job i on machine j must be equal to \( P_{i,j} \). Equation (4) indicates that the processing time of job i on the machine j in \( P_{i,j} \) units must be continuous, that is, it must be of this form: 00011100000; but not the following form: 000100001100. Therefore, as long as 1 appears after 0, the subsequent values of 1 must be 0. Formula (5) means that the same machine can only process one job at the same time, ensuring the uniqueness of the time point. Equations (6) and (7) use the auxiliary variables \( D_{i,k} \) to represent the machining sequence of the job on the machine. Equation (8) indicates that the starting time of the job on the next machine must be greater than the completion time of the job at the previous machine. This formula also utilizes the processing time center point method. Equation (9) indicates that the completion time of each job on the last machine is less than \( C_{\text{max}} \). The Formula (10) expresses that the starting time of each job on the first machine is greater than \( C_{\text{min}} \), that is, \( C_{\text{min}} \) is the origin of the machining start. Note that this time may not be 0, because the time-of-use electricity tariffs make a batch of jobs flexible. Select the starting time of the process. At the same time, it should be noted that the model does not consider the energy consumption during idling.

3. Simulation

3.1. Case design

In this paper, this mathematical model is validated by a small-size case from the actual production shop. The small-size case has 5 jobs and 3 machines, and the specific parameter setting is shown in Table 1.

**Table 1.** Processing time and unit power consumption of jobs on machines (unit: minute; kw).

| Processing time | J1 | J2 | J3 | J4 | J5 | Electricity consumption rate | J1 | J2 | J3 | J4 | J5 |
|----------------|----|----|----|----|----|-----------------------------|----|----|----|----|----|
| Machine 1      | 4  | 6  | 2  | 8  | 10 | Machine 1                   | 3  | 4  | 2  | 5  | 4  |
| Machine 2      | 6  | 4  | 10 | 8  | 2  | Machine 2                   | 2  | 8  | 2  | 4  | 7  |
| Machine 3      | 2  | 8  | 4  | 2  | 6  | Machine 3                   | 3  | 5  | 3  | 3  | 9  |

Referring to the actual data, the time-of-use electricity tariffs function \( f(t) \) is designed. As shown in Table 2, it is assumed that there are 60 minutes in total and the unit time is 2 minutes, so there are 30 time points. Among them, the on-peak period is 21 minutes to 40 minutes, the off-peak period is 1 minute to 20 minutes, the flat period is 41 to 60 minutes; at the starting point of 0 seconds, the electricity price is 0.

**Table 2.** Time-of-use electricity tariffs.

| Time (minutes) | 0 < t ≤ 20 | 20 < t ≤ 40 | 40 < t ≤ 60 |
|----------------|------------|-------------|-------------|
| Corresponding price (yuan/kw*min) | 1          | 3           | 2           |

3.2. Experimental result

This paper uses linear weighting to transform the problem into a single target. When considering only the best total electricity cost, a computer with Intel (R) i5-4210U CPU @ 1.70G Hz 2.40G Hz and 4GB memory runs on GAMS23.0 software for 24 hours, and calculates the optimal job order as \{2, 3,1,4,5\}, and calculated the optimal power cost is 528 yuan, and the corresponding completion time is 60 minutes, as shown in Figure 3. The key to optimizing power costs is to make most machines try to avoid processing at high power prices. Since the electricity consumption rate of job 1 processed on machine 2 is the lowest, compared with other jobs, the electricity cost of job 1 processed in the peak time of 38 to 40 minutes of machine 2 is the key to reducing the total electricity consumption cost.
which reflects the optimization idea of the model. When considering the optimal completion time of all the jobs, this paper uses the same computer equipment to run about 36 hours on the GAMS23.0 software. The optimal completion time is calculated to be 42 minutes, and the job sequence is [3, 5, 2, 4, 1], as shown in Figure 4. Figure 5 is a comparison of the total electricity cost changes in the two cases. In the former case, the increase in electricity tariffs is stable and remains flat during peak electricity prices, which is because the machines use low power during peak electricity prices. Obviously, the latter is that most studies consider the minimization of the traditional target completion time and ignore the variable electricity price, which will cause the cost of electricity to increase sharply, especially during peak electricity prices.

![Gantt chart for total electricity cost](image)

**Figure 3.** Gantt chart for total electricity cost.

![Gantt chart for completion time](image)

**Figure 4.** Gantt chart for completion time.

In this section, 11 sets of experiments with different weight combinations are designed based on the linear weighting method. The same computer equipment is used to run on the GAMS23.0 software to explore the changes in electricity costs and completion time under different weight coefficients. Among them, TEC is the total electricity costs, and Cmax is the makespan. Through the solution results obtained in Table 3, a line chart showing the change in electricity cost and makespan with the weight coefficient is shown in Figure 6.

**Table 3.** Solving results under different weights.

| weights | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
|---------|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|---|
| TEC     | 802 | 640 | 548 | 531.3 | 530 | 536 | 526.6 | 528 | 527.5 | 527.7 | 528 |
| Cmax    | 42  | 46  | 60  | 58  | 58  | 60  | 60  | 60  | 60  | 60  | 60 |

By designing 11 sets of experiments with different weight ratios, it is found that the optimization scheme considering only the electricity cost reduces the total electricity cost by 30% compared with the traditional scheduling scheme after avoiding the high peak tariffs of electricity. Although the makespan is increased by nearly 30% on the surface, the downtime of staggering the peak is removed, the actual processing time is 42 minutes, and the total completion time is only increased by 5%. In
multi-objective planning, there is a phenomenon that cannot be compared between targets, so the non-dominated solution (Pareto optimal solution) is often used to determine the decision-making scheme. And the non-dominated solution is defined as: Suppose that any two solutions \( S_1 \) and \( S_2 \) are superior to \( S_2 \) for all targets, then \( S_1 \) is said to dominate \( S_2 \). Therefore, the non-dominated solution set of the electricity cost weighting coefficient is: \([0, 0.1, 0.6]\); the dominant solution set is: \([0.2, 0.3, 0.4, 0.5, 0.7, 0.8, 0.9, 1]\).

![Figure 5. Comparison of total electricity costs.](image)

In the non-dominated solution set, companies can refer to the following situations when making production plans.

1. When the production efficiency is the primary reference index of the enterprise, it is suitable to select the scheme that the weight coefficient of electricity cost is 0 and the weight coefficient completion time is 1.

2. When considering the delivery time required by the customer, the enterprise can choose the scheme for the weighting coefficient of 0.1. Because it not only meets customer needs, but also reduces electricity costs. However, when the delivery time is less than the completion time under the scheme, in order to improve production efficiency and reduce the completion time, the enterprise should select the case where the weight coefficient is 0.

3. Under normal circumstances, regardless of delivery time and large-scale production, it is optimal for the enterprise to choose the weight coefficient of 0.6. At the same time as the production plan is completed, the electricity cost is minimal.

4. Conclusions
This paper studies the scheduling problem of minimizing the electricity cost in the permutation flow shop under time-of-use electricity tariffs. The main contributions to this paper are as follows. A
discrete-time mixed integer programming model is established for the problem of permutation flow shop scheduling for time-of-use electricity tariffs. This paper establishes a discrete-time mixed integer linear programming model. In order to make the model better solve this problem, a central moment method is used to determine the central value of the processing time of job on the machine, which indicates the processing sequence of all jobs on the machine. The model designed in this paper can solve the problem of effectively reducing the electricity cost for enterprises within the specified delivery period. In this paper, the solution solved by the GAMS software belongs to the optimal solution, but its operation requires a lot of time, and there may be cases where there is no optimal solution. The model established in this paper is based on discrete time and has a large amount of computation. In the future, we can design corresponding algorithms based on the continuous time model, so as to solve large-scale cases and solve practical problems. The model designed in this paper does not consider the energy consumption of the machine when it is idling, because considering the machine idling will add complexity of the model design. Therefore, considering the problem of machine idling on this model is also the direction of future research.

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