We give best covariant amplitude decompositions for two-body decay processes involving ground state hadrons (0\(^{-}\), 1\(^{-}\), 1\(^{+}\), 2\(^{+}\)) and show how these are simply related to helicity amplitudes. After discussing how electromagnetic interactions are incorporated via meson dominance in a relativistic supermultiplet scheme, we extend the analysis to weak flavour changing nonleptonic processes. Such weak interactions are described by three generic amplitudes, which we estimate according to the rules of calculation within covariant SU(2\(_{f}\)).

1 Introduction

The advent of heavy meson factories and the discovery of new heavy baryon states has led to a veritable explosion of new data on the decays of such hadrons. A high fraction of these decays occur through nonleptonic channels and it becomes an urgent matter to gain a full understanding of how these occur. In the first place we should try to arrive at a comprehensive and reasonably accurate description for such nonleptonic amplitudes, since a gross picture using the Fermi constant \(G_F\), the CKM elements \(V_{UD}\) and appropriate phase space factors is already able to predict the widths within a factor of about two. Several reviews of this subject have appeared in which the virtues and failings of different descriptions of nonleptonic models are discussed.

In this paper we shall exhibit a scheme for calculating the weak flavour-changing amplitudes which is founded on a relativistic supermultiplet scheme that already agrees with measured strong and electromagnetic 3-particle amplitudes within 10\% or better. We are hopeful that it will account for experimentally measured nonleptonic decays to roughly the same accuracy, since it allows for mixing between weak states (which are normally rotated away into the strong interactions by a physical state mass diagonalization) in addition to the usual W-exchange contributions.

In the next section we summarise how the weak amplitudes are best expressed, because of their simple connection with helicity amplitudes. Following this analysis we recall the main features of relativistic SU(2\(_{f}\)) and

\(^{a}\)Semileptonic amplitudes are pretty well understood in terms of the current-current picture, using a number of ‘decay constants’ and ‘form factors’ which parametrise them.
the rules for calculating strong interaction amplitudes. After a digression into how meson dominance is implemented in such a supermultiplet framework, we turn to weak interactions. We show how all processes to order $G_F$ naturally subdivide into three types and we describe how these three amplitudes may be estimated. In future research we shall apply these evaluative rules to physical decays; this is a major task as there is such a multitude of nonleptonic processes and our ambition is to predict them all to the same accuracy as strong and electromagnetic (em) processes.

2 Weak Amplitude Analysis

2.1 Counting the Amplitudes

Let us focus on two-body decays, since these must be properly comprehended before many-body decays. Consider the process $-p_1 \lambda_1 \to p_2 \lambda_2 + p_3 \lambda_3$, associated with elements, $\langle p_2 \lambda_2, p_3 \lambda_3 | S | p_1 \lambda_1 \rangle = (2\pi)^4 \sum_{\{\lambda\}} \delta^4(p_1 + p_2 + p_3) M_{\{\lambda\}}$, and leading to the decay rate

$$\Gamma = \frac{\sum_{\lambda} \Delta^2 |M_{\{\lambda\}}|^2}{16\pi m_1^4(2j_1 + 1)}; \quad \Delta^2 = m_1^4 + m_2^4 + m_3^4 - 2m_1^2m_2^2 - 2m_2^2m_3^2 - 2m_3^2m_1^2.$$

Now the number of independent couplings of these particles equals the number of ways $N$ in which the three spins can be coupled to produce an integer angular momentum. If the $j_i$ form a Euclidean triangle,

$$N = j_1(1 - j_1) + j_2(1 - j_2) + j_3(1 - j_3) + 2(j_1j_2 + j_2j_3 + j_3j_1) + 1.$$

Otherwise arrange them in decreasing order: $j_a \geq j_b \geq j_c$; with $j_a \geq j_b + j_c$; then $N = (2j_a + 1)(2j_c + 1)$. [The same result can be deduced via the standard $L - S$ coupling method or via the helicity formalism.] No assumption about parity conservation has been made since we shall largely concentrate on weak amplitudes; nor have we supposed the couplings to be real, because they are surely not in the decay region as a result of strong final state interactions. The values of the couplings will depend on the masses $m_i$ and the quantum numbers of the particles. Generally, we should expect that the larger the difference in the spin values (excitation numbers) and quantum numbers (flavour values) the smaller the couplings will be because of decreased overlap between the particle ‘wave-functions’.

\footnote{The choice of momentum is for ease of crossing: one simply interchanges particle labels, includes the conjugation factor $(-1)^{2j}$ and modifies the spin averaging and mass factors to derive the crossed decay rate.}
2.2 Covariant Decompositions

In Lorentz covariant descriptions regards the amplitudes as arising from an effective three-particle Lagrangian

\[ \mathcal{L} = \sum_{I=1}^{N} g_{I}^{j_1 j_2 j_3} \phi^{j_1}(-p_1)\phi^{j_2}(p_2)\phi^{j_3}(p_3), \]  

(1)

where \( \phi^j(p) \) stands for a free-field solution, namely \( 1, u^\lambda(p), \epsilon^\lambda(p), u^\mu(p) \ldots \) for particles of spin \( 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots \). We give two examples of how the couplings are best defined within the covariant formalism, as they illustrate the basic simplicity of the resulting expressions. A complete set of decompositions is provided in Ref 3.

\[ \frac{1}{2} \rightarrow 1/2 + 1 \ (e.g. \Sigma^+ \rightarrow p\gamma, \ A_b \rightarrow A\psi). \]

The couplings are most neatly expressed in the Sachs form,

\[ \mathcal{L} = \epsilon^\nu_2 \bar{u}_2[(p_2 - p_1)_\nu(f_E + g_E\gamma_5)/2 + \epsilon_{\nu\rho\sigma}p_1^\rho p_2^\sigma\gamma^\tau(f_M\gamma_5 + g_M)]u_1, \]

(2)

where the subscripts \( M, E \) refer to the ‘electric’ and ‘magnetic’ parts of the vector interaction. It is not recommended to use the more traditional decompositions \( \gamma_\nu, \gamma_\nu\gamma_5, \sigma_{\nu\rho}p_3^\rho, \sigma_{\nu\rho}\gamma_5p_3^\rho \), because they lead to complicated cross-terms in decay rates and the like; also they are more distantly related to helicity amplitudes.

However, with decomposition (2), the ensuing decay rate looks neat:

\[ \Gamma = \frac{\Delta^4}{32\pi m_1^4} \left[ ((m_1m_2 - p_1 \cdot p_2)(|f_E|^2/m_3^2 + 2|f_M|^2) - (m_1m_2 + p_1 \cdot p_2)(|g_E|^2/m_3^2 + 2|g_M|^2) \right] . \]

(3)

If the final vector particle is a photon, gauge invariance ensures that \( f_E.g_E \rightarrow 0 \) (remember that \( m_1 \neq m_2 \)), whereupon the rate simplifies further to

\[ \Gamma \rightarrow \frac{(m_1^2 - m_2^2)^3}{16\pi m_1^4} \left[ (m_1 + m_2)^2 |f_M|^2 + (m_1 - m_2)^2 |g_M|^2 \right], \]

(4)

and one of \( f_M \) or \( g_M \) must be discarded because of em parity conservation.

\[ 1 \rightarrow 1 + 1 \ (e.g. \Psi \rightarrow K^*\bar{K}^*, \gamma\gamma). \]

As far as we know, the tidiest decomposition has \( \mathcal{L} = \epsilon_2^\mu \epsilon_3^\nu \mathcal{M}_{\lambda\mu\nu} \epsilon_1^\lambda \) with

\[ \mathcal{M}_{\lambda\mu\nu} = (p_3 - p_2)\lambda(g_1 T(p_2 \cdot p_3)\eta_{\mu\nu} - p_3(\eta_{\mu\nu} + p_2\gamma_5) + g_1 M\epsilon_{\mu\nu\rho\sigma}p_3^\rho p_2^\sigma)/2 \]

+ cyclic + \( g_L(p_3 - p_2)(p_2 - p_1)_{\lambda}(p_1 - p_3)_{\mu}(p_2 - p_1)_{\nu}/8. \)

(5)
The expression for the decay rate substantiates our claim:

\[
\Gamma = \frac{\Delta^3}{192\pi m_1^3} \left[ \frac{\Delta^4}{16m_1^2 m_2^2 m_3^2} |g_L|^2 + \frac{1}{2} \Re(g_L^* \sum_i \frac{g_{iT}}{m_i^2}) + \sum_{i \neq j \neq k} \left( 2\Re(m_i^2 g_{jT} g_{kT}^*) + \frac{(\Delta^2 + m_j^2 m_k^2)|g_{jT}|^2 + \Delta^2 |g_{kM}|^2}{2m_i^2} \right) \right].
\]  

(6)

Once again, simplifications arise when one vector meson is a photon, say particle 2; in that case, \(g_L, g_{2M}, g_{2T} \to 0\), and

\[
\Gamma \to \frac{\Delta^5}{384\pi m_1^3} \left[ \frac{|g_{1T}|^2 + |g_{1M}|^2}{m_1^2} + \frac{|g_{3T}|^2 + |g_{3M}|^2}{m_3^2} \right].
\]  

(7)

These examples are but two cases of the entire set involving ground state hadrons, which are listed in Ref 3; they nicely illustrate the advantages of an elegant covariant decomposition.

2.3 Helicity Amplitudes

As further confirmation of the ‘best coupling’ covariants we may work out the helicity amplitudes \(M_{\lambda_2, \lambda_3}\). The idea is that they should be very simple linear combinations of the chosen couplings \(g_{j1, j2, j3}\); and indeed they are. With our two examples one finds

\[
\begin{align*}
1/2 \to 1/2 + 1 & \\
M_{1/2, 0} + M_{-1/2, 0} = \sqrt{2(m_1 m_2 - p_1 \cdot p_2)} \Delta f_E / m_3, \\
M_{1/2, 0} - M_{-1/2, 0} = \sqrt{2(m_1 m_2 + p_1 \cdot p_2)} \Delta g_E / m_3, \\
M_{1/2, 1} + M_{-1/2, 1} = -2\sqrt{m_1 m_2 - p_1 \cdot p_2} \Delta f_M, \\
M_{1/2, 1} - M_{-1/2, 1} = -2\sqrt{m_1 m_2 + p_1 \cdot p_2} \Delta g_M.
\end{align*}
\]

Notice how the threshold factors of relative momentum turn up automatically and how simple it is to continue from coupling \(f\) to its parity opposite \(g\). Of course the helicity state \(\lambda_3 = 0\) and amplitude must be ignored for photons.

\[
\begin{align*}
1 \to 1 + 1 & \\
M_{0, 0} = -\frac{1}{2} \Delta m_1 m_2 m_3 \left( \frac{g_{1T}}{m_1^2} + \frac{g_{2T}}{m_2^2} + \frac{g_{3T}}{m_3^2} + \frac{\Delta^2 g_L}{4m_1^2 m_2^2 m_3^2} \right),
\end{align*}
\]

Recall that parity conservation, encapsulated by \(M_{\lambda_2, \lambda_3} = \eta_1 \eta_2 \eta_3 (-1)^{j_2 + j_3 - j_1} M_{-\lambda_2, -\lambda_3}\), will roughly halve the number of couplings.
while the other well-defined parity combinations are

\[ M_{1,1} + M_{-1,1} = p_2 \cdot p_3 \Delta g_{1T}/m_1, \quad M_{1,1} - M_{-1,1} = i \Delta^2 g_{1M}/2m_1, \]

\[ M_{0,1} + M_{0,-1} = i \Delta^2 g_{2M}/2m_2, \quad M_{0,1} - M_{0,-1} = p_2 \cdot p_3 \Delta g_{2T}/m_2, \]

\[ M_{1,0} + M_{-1,0} = i \Delta^2 g_{3M}/2m_3, \quad M_{1,0} - M_{-1,0} = -p_2 \cdot p_3 \Delta g_{3T}/m_3, \]

whereupon we see why \( g_{2T}, g_{2M} \) and \( g_L \) have to discarded when 2 is a photon.

3 Strong and Electromagnetic Interactions of Supermultiplets

3.1 Supermultiplet Wavefunctions

We now turn to the strong hadronic interactions, which are believed to be described by the fundamental chromodynamic Lagrangian,

\[ \mathcal{L} = \sum_{f, \text{colour}} \bar{\psi}_f(i \not\! \partial - g A.\lambda/2 - M_f)\psi_f - F_{\mu\nu}F^{\mu\nu}/4. \]  \hspace{1cm} (8)

This possesses a heavy quark symmetry \(^{\ddagger}\) for \( M_f \gg \Lambda_{\text{QCD}} \) corresponding to ‘flavour-blindness’ of quarks moving with equal velocity \( v \). In fact, if one neglects the gluon field altogether (by doing a Foldy-Wouthuysen transformation \( ^\dagger \) to leading order) one finds that for hadrons bound by \textit{constituent} quarks, the Lagrangian has \( ^{\ddagger} \text{a} [U(2N_f) \otimes U(2N_f)]_v \) symmetry, regardless of \( m_q \). In this traditional picture of hadrons, the baryons and mesons are composed of quark moving in tandem with the \textit{same} velocity \( v \) and all gluon effects are taken into account in dressing the quarks from ‘current’ to ‘constituent’ with little or no binding energy.

For instance, mesons are described by the wavefunction \( \Phi_A \equiv u_A(p_1)\bar{v}B(p_2) \) which can be pictured as two parallel quark lines with symbols \( A = \alpha \alpha, B = \beta \beta \) carrying the Dirac spinor labels \( \alpha, \beta \) and flavour labels \( a, b \). Since the quarks have equal velocity \( v = p/(m_1 + m_2) \equiv p/\mu \), we may write \( p_i = m_i v \), where \( p \) is the total momentum of the meson, whereupon we see that, \textit{even for unequal mass quarks} the meson wavefunction obeys the Bargmann-Wigner equations for a bispinor, \( (\hat{p} - \mu)\Phi = \Phi(\hat{p} + \mu) = 0 \), with \( \Phi_A^B(p) \equiv u_A(p)v_B(p) \). It follows that

\[ \Phi_A^B(p) = [(\hat{p} + \mu)(\gamma_5 \phi^b_{a_5} - \gamma^\mu \phi^b_{a_\mu})]_\alpha^\beta \equiv 2\mu P_{+v}(\gamma_5 \phi_5 - \gamma \cdot \phi), \]  \hspace{1cm} (9)

where \( P_{+v} \equiv (1 + \gamma_v)/2 \). Similarly, for baryons one finds

\[ \Psi_{(ABC)}(p) = [P_{+v} \gamma_i C]_{\alpha\beta} u_{(abc)\gamma}^\mu + \frac{\sqrt{2}}{3} \left( [P_{+v} \gamma_5 C]_{\alpha\beta} u_{[ab]c\gamma} + [P_{+v} \gamma_5 C]_{\beta\gamma} u_{[bc]a\alpha} + [P_{+v} \gamma_5 C]_{\gamma\alpha} u_{[ca]b\beta}. \]  \hspace{1cm} (10)
The basic three-point vertices between ground state mesons and baryons, maintaining the maximal symmetry $U_w(2N_f)$ are governed by just two couplings $F$ and $G$:

$$\mathcal{L} = F \Phi^A_B(p_1) \left( \Phi^C_B(p_2) \Phi^A_C(p_3) + \Phi^C_B(p_3) \Phi^A_C(p_2) \right) + G \bar{\Psi}^{ABC}(p_2) \Phi^D_C(-p_3) \Psi_{DAB}(p_1).$$

(11)

In such relativistic supermultiplet schemes, all scattering amplitudes are obtained as tree diagrams by sewing together the multispinors and vertices. No loop corrections are to be evaluated since those quantum loops really correspond to QCD gluon effects and, by assumption, those have already been largely taken into account into dressing the constituent quarks in the hadrons. For example the process $\pi(k)N(p) \rightarrow \rho(k')N(p')$ due to $\omega$ and $\pi$ exchange is contained within the expression,

$$GF \bar{\Psi}^{ABC}(p') \Psi_{ABD}(p) \Delta^{DE}_{CE}(p - p') \Phi^G_E(k) \Phi^F_C(-k'); \quad \Delta = \text{meson propagator},$$

or as a duality diagram drawn in Figure 1. The constants $F$ and $G$ are related to the $\rho NN$ and $\rho\pi\pi$ couplings, and can be further related by invoking Sakurai’s isospin universality hypothesis. One can write a similar expression for the baryon pole term and even add a four-point contact term to represent non-resonant contributions to the process. A large body of data on strong and em interactions (via vector dominance) can be tied within such a higher symmetry scheme and it is a source of wonder—especially for the lighter hadrons where the symmetry is on shakier ground—that the theoretical predictions agree\(^d\)

\(^d\)Actually the dimensionless couplings to be used are $g = 3G\Sigma/4$ and $f = F\mu\Sigma$, where $\Sigma$ is the sum of the participating masses.
Table 1: Experimental values of vector meson decay constants $f_V$, in MeV as determined from leptonic decay widths, via $(e^2 f_V)^2 = 12 \pi m_V \Gamma_V$, compared with the theoretical predictions, $(g_V f_V)^2 = \Lambda m_V$. Experimental errors hover around 5%. Heavier mesons values are inferred as there is no direct experimental data for them.

| Vector Meson | $\rho$ | $\omega$ | $\phi$ | $\psi$ | $\Upsilon^*$ | $K^*$ | $D^*$ | $D_s^*$ | $B^*$ |
|--------------|-------|---------|--------|-------|-----------|------|------|-------|------|
| $|f_V| (\text{expt.})$ | 154   | 47      | 80     | 274   | 240       | ?    | ?    | ?     | ?    |
| $|f_V| (\text{theory})$  | 154   | 51      | 83     | 281   | 252       | 232  | 347  | 355   | 565  |

with the experimental facts to within 10% and generally much better, using as inputs $g_{\pi NN}$ and the average supermultiplet constituent masses $m_u \approx 350$, $m_s \approx 450$, $m_c \approx 1500$, $m_b \approx 4700$ (in MeV).

3.2 Vector Dominance

There exist a number of implementations of vector dominance. The version we shall adopt has a photon ($A$) vector meson ($V$) coupling $-e A \cdot F_V (\partial^2) V / g_V m_V^2$, where $F_V$ is a ‘form-factor’ describing the off-shell mixing. By convention, the normalization $F(0) = m_V^2$ ensures that the quark charges come out correctly at soft photon momentum, provided that the vector coupling $g_V$ to the quarks is normalized by the appropriate CG coefficient; specifically,

$$g_\rho : g_\omega : g_\phi : g_\psi : g_\Upsilon = \sqrt{2} : 3 \sqrt{2} : -3 : 3/2 : -3; \quad g_\rho = 3.54 \pm 14.1.$$  (12)

One can get a good idea of how $F_V$ varies with $m_V$ by examining the leptonic decay widths of the vector mesons, $\Gamma_{V \to l \bar{l}} \approx \frac{m_V}{12 \pi} \left( \frac{e^2 f_V}{g_V} \right)^2$. The experimental results strongly suggest that, once the CG factor is subsumed in $g_V$ as above, the widths vary little. Thus one is led to assume that $F_V^2 \propto 1/m_V$, although more sophisticated extrapolations, based on the renormalization group, exist. Thus we take $(0)_{\mu\nu}^{em} |V| = e m_V^2 \epsilon_{\mu\nu}, F_{\mu\nu}^V \epsilon_\mu / g_V \equiv e m_V f_V \epsilon_\mu$, where the ‘meson decay constant’ $f_V$ equals $m_V F_V / g_V \approx \sqrt{\Lambda m_V / g_\rho}$ with $\Lambda = m_\rho$. A comparison between these crude theoretical predictions and the experimental values of $f_V$, derived from leptonic decay widths, is provided in Table 1. In supermultiplet terms one is led to introduce the em field multispinor $A^\mu = e(\gamma \cdot A)_{\beta} Q^\mu_{\beta}$ ($Q$ is the flavour charge matrix) and the contact interaction

$$L = \text{Tr}[A \sqrt{\Lambda m_V} \Phi / g_V] / 4 = e m_V f_V A \cdot \phi,$$

as required. Note that as before, we do not rotate the em and vector field in order to diagonalize the masses, according to $V' = (V + eA/g_V) / \sqrt{1 + e^2 / g_V^2}$. 

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and $A' = (A - eV/g_V)/\sqrt{1 + e^2/g_V^2}$. This would otherwise muddy the leptonic interaction,

$$L_A = e\bar{l}A' = e\bar{l}(A' + e V'/g_V)l/\sqrt{1 + e^2/g_V^2},$$

by introducing an intrinsic interaction of the lepton with the strong vector meson, and complicating the picture. It is much easier to use the original $A$ and $V$ fields, which mix through quark interactions, and thereby allow a $A-V$ contact term as is done in most interpretations of vector dominance. We will adopt exactly the same approach for weak interactions.

4 Weak Flavour-changing Nonleptonic Interactions

4.1 The electroweak boson multispinor

An examination of the form of the weak interactions according to the standard model, shows that the electroweak multiplet can be collected into a bispinor

$$W \equiv -eQ + g \sqrt{2} Z + gW \left(T_3(1 - i\gamma_5) - 2q^2 \sin^2 \theta_W\right)/2 \cos \theta_W,$$

where $U$ stands for the CKM matrix acting between ‘up’ and ‘down’ quarks, which obeys the unitarity property,

$$U_{ij}U_{kj} = \delta_{ik}; \quad U_{ik} \equiv (U_{ik}^*)^*,$$

guaranteeing that the neutral weak interaction is flavour diagonal. It is worth pointing out that the left-handed coupling of the weak bosons strictly applies to current quarks. There is reason to suspect that $V-A$ is not quite correct for constituent quarks, since it leads to an excessive $g_A/g_V$ ratio for the nucleon; but for the purposes of this paper we shall adhere to the picture of purely left-handed constituent quarks with the bonus of projection and simple Fierz reshuffling.

One of the primary elements of weak interactions is the pseudoscalar meson decay constant for charged leptonic channels, which arises via

$$\langle 0|J_{\mu5}P^0|P^0\rangle \equiv ifPP_{\mu}, \quad \langle 0|J_{\mu5}^+P^-\rangle \equiv i\sqrt{2}U_{\mu\nu}P_{\nu}; \quad J_{\mu5} = \bar{\psi}i\gamma_\mu\gamma_5\psi. \quad (14)$$

We can think of this as a contact interaction between the weak bosons and the pseudoscalar fields, in analogy to the vector dominance model:

$$L = f_P(W^0 \cdot \partial P^0 + U\sqrt{2}W^+ \cdot \partial P^- + iU^*\sqrt{2}W^- \cdot \partial P^+). \quad (15)$$

This is apparent on two counts: (i) the D/F ratio of 5/3 is rather larger than the experimental value $g_A/g_V \sim 1.25$ for nucleons, (ii) in supermultiplet theory the axial mesons represent the first orbital excitation of the ground state mesons and have independent couplings to the quarks in principle. Really we should be writing the weak couplings more accurately as the renormalized expression $\bar{\psi}W(1 - 3i\gamma_5/4)\psi$, for constituent fields $\psi$. 

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Table 2: Experimental values of pseudoscalar decay constants $f_P$, in MeV as determined from leptonic decay widths, compared with the theoretical predictions, $f_P^2 \propto m_1 + m_2$, where $m_i$ are the constituent quark masses; note, for pseudoscalars we take $m_{ud} \simeq 200, m_s \simeq 300$, and of course $f_{\pi^0} = 93$ as our input.

| Pseudoscalar | $\pi^+$ | $K^+$ | $D^+$ | $D_s^+$ | $B^-$ |
|--------------|--------|-------|-------|--------|------|
| $f_P$ (expt.) | 131    | 160   | <310  | $240 \pm 40$ | ?    |
| $f_P$ (theory) | 131    | 146   | 270   | 278    | 458  |

If we assume that these $f$ have the same mass dependence as in the vector case, or simply for uniformity, we would conclude that $f_P \simeq g_W \sqrt{\lambda m_P}/4g$; this then leads to the predictions listed in Table 2. At any rate, it is perfectly possible to reproduce the weak bosons through the multispinor contact term

$$L_W = \frac{g_W}{4g} \text{Tr}[W^\dagger \sqrt{\lambda m_P}] \supset \frac{g_W}{8g\sqrt{2}} \text{Tr}[W(1 - i\gamma_5)U(\bar{\phi} + m_P)\gamma_5 \phi_5]$$

$$= \frac{ig_W}{\sqrt{8}} U W^\dagger p_\mu \phi_5 \sqrt{\lambda m_P} \equiv f_P \sqrt{2} U W . \partial \phi_5 . \quad (16)$$

This shows that we can regard the weak bosons as part of a bispinor field; by coupling them to two pairs of quarks one arrives at the flavour-changing nonleptonic weak interaction to order $G_F/\sqrt{2} \equiv g_W^2/8m_W^2$.

4.2 Nonleptonic amplitudes

The standard treatment is based on a current-current picture,

$$H_W = 4G_F U_{ud} U_{cd}^* \left[ c_1 (\bar{d}_L \gamma u)_L \cdot (\bar{U}_L \gamma D)_L + c_2 (\bar{d}_L \gamma D)_L \cdot (\bar{\gamma} \gamma u)_L \right] / \sqrt{2},$$

where $c_i$ are Wilson coefficients. The next step is to invoke a factorization hypothesis and evaluate this as sums of products, $\langle h|J_L|h'\rangle \cdot \langle (PorV)|J_L|0\rangle$, with hadronic $h$ elements extracted from semileptonic decays; this despite the fact that factorization does not work at all well for charmed meson decays and does not properly apply to baryonic decays.

Using first principles and permitting particle-mixing, like vector dominance and the old pole-model descriptions, we will exhibit a supermultiplet scheme for calculating nonleptonic amplitudes more completely. Basically, we find that there are just three types of diagram which require evaluation to order $G_F$, as drawn in Figures 2,3. Fig 2 corresponds to a flavour-changing charge-conserving transition from one quark to another, Fig 3 to a neutral current
transition from $d \bar{D}$ to $u \bar{U}$ and a charged transition from $d \bar{u}$ to $D \bar{U}$. Let us show how these may be estimated now (in Feynman gauge).

4.3 Quark-line transition

The first part of Fig 2 includes a self-energy flavour change, e.g. $u \leftrightarrow c$, which produces the matrix element

$$
\sum_q U^*_{uq} U_{cq} \Sigma_q(p) \text{ with}
$$

$$
\Sigma_q(p) = \frac{i g^2_W}{2} \int \gamma_L \frac{1}{p + k - m_q} \gamma_L \frac{d^4 k}{k^2 - m_q^2} \approx \left( \frac{g_W m_q}{4 \pi m_W} \right)^2 \frac{1}{2} \hat{p}_L. \quad (17)
$$

Note that a potential infinity in $\Sigma_q$ disappears by the unitarity of $U$. Summing over $q$ and using known CKM elements, we obtain for $u \leftrightarrow c$ say (in GeV), the total

$$
\frac{\hat{p}_L G_F}{\pi^2 \sqrt{8}} [U_{ud}^* U_{cd} m_u^2 + U_{uc}^* U_{cs} m_c^2 + U_{ub}^* U_{cb} m_b^2] \approx \frac{\hat{p}_L G_F}{\pi^2 \sqrt{8}} \times .02 \sim 8 \times 10^{-9} \hat{p}_L.
$$

Similarly, for the $d \leftrightarrow s$ transition (relevant for $K$ decays), one estimates the self-energy to be

$$
\frac{\hat{p}_L G_F}{\pi^2 \sqrt{8}} [U_{us}^* U_{ud} m_u^2 + U_{uc}^* U_{cd} m_c^2 + U_{ub}^* U_{cb} m_b^2/4] \approx -\frac{\hat{p}_L G_F}{\pi^2 \sqrt{8}} \times .14 \sim -6 \times 10^{-8} \hat{p}_L.
$$

The second part of Fig 2 is a vertex integral that reduces (for $d \leftrightarrow s$ say) to

$$
\sum_q U_{qd} U_{qs} I_q, \quad \text{where}
$$

$$
I_q = \frac{i g^2_W}{4 m_q} \int \frac{(\hat{p}_d + \hat{p}_s - 2 \hat{k}) L_d^4 k}{(k^2 - m_d^2)^2 (k^2 - m_W^2)} \approx \left( \frac{g_W}{8 \pi m_W} \right)^2 \frac{1}{2} m_q (\hat{p}_s + \hat{p}_d)_L. \quad (18)
$$
Figure 3: W-exchange diagrams. The first is for a neutral quark combination and the second is for a charged combination.

Observe that the internal quark mass weighting of $m_q$ is weaker than the $m_q^2$ factor in $\Sigma_q$, significantly so for the top quark. The third part is another self-energy at the other momentum leg $p'$.

Combining the three terms, we find that the combination is dominated by the pole part associated with $\Sigma_q$, giving the total

$$\Gamma_W = \frac{1}{2} \sum_q U_{qs} U_{qd}^* \left( \frac{g_W m_q}{8 \pi m_W} \right)^2 \frac{m_s(1 - i \gamma_5) - m_d(1 + i \gamma_5)}{m_s - m_d},$$

which has to be traced out against the other quark line indices. For instance if we are studying purely pseudoscalar decay, the trace to be performed is

$$F \text{Tr}[\not p + m_2)\gamma_5 \Gamma_W(\not p_1 + m_1)\gamma_5(\not p_3 + m_3)\gamma_5].$$

4.4 W-exchange diagrams

The other two types of diagrams (see Fig 3) are essentially equal by Fierz reshuffling and, apart from differing mass terms, give the typical pseudoscalar amplitude,

$$\frac{F g_A^2}{16 m_W^2} U_{cs} U_{ud}^* \text{Tr}[(\not p_c + m_c)\gamma_L(\not p_1 + m_1)\gamma_5(\not p_u - m_u)(\not p_2 + m_2)\gamma_5(\not p_3 + m_3)\gamma_5]$$

$$= \frac{F g_A^2}{4 m_W^2} U_{cs} U_{ud}^* m_c m_u \left[1 - \left(\frac{m_c + m_u}{m_c - m_u}\right)^2\right] (m_2 - m_3)(p_2 \cdot p_3 - m_2 m_3).$$

There are numerous processes to which we may apply these calculational methods and ideas, but space restrictions do not permit us to expose any details. Work is currently in progress to evaluate reliably such weak decay elements and we expect to present some of our results in the near future. At this stage all we can venture to say is that we estimate the self-energy transition is an order of magnitude (about 20 times) bigger than the W-exchange amplitude—automatically explaining the validity of the $\Delta I = 1/2$ or ‘octet dominance’ rule for strange particle decays. It is an auspicious sign.

But if one takes $g_A/g_V \simeq 3/4$ for constituent quarks their contributions become distinct.
Acknowledgements

We thank the Australian Research Council for its support under grant number A69800907. Also, we are grateful to the CSSM for their hospitality during the QFT98 conference, when this manuscript was partially prepared.

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