The transfer of stylised artistic images in eye movement experiments based on fuzzy differential equations

Jie Zhou1†, Long Li2, Zaiyang Yu3

1School of Art Design and Media, East China University of Science and Technology, Shanghai, China
2University of Science and Technology of China, Anhui, China
3School of Mechanical Electronic and Information Engineering, China University of Mining and Technology, Beijing, China

Abstract

In many aspects of people’s production and life, artistic images have been widely used. Because the image has the function of transmitting information, it can provide necessary space environment information for people. However, there are many problems in the design of stylised art images, and hence the usability of images is affected. Due to its unique advantages, the study of artistic eye movement has gradually become a research hotspot. The fuzzy differential equation is an important branch of differential equation theory, which can be used to study eye movement experiments in the field of the art research. In the process of observation, experiment and maintenance, errors cannot be avoided, and the variables and parameters obtained are often fuzzy, incomplete and inaccurate. And fuzzy differential equations can deal with these uncertainties well. At first, this paper studies the migration-image-study-related theory and art image, with the help of eye movement experiment method to investigate the effects of two types of image on people read mechanism. This research mainly uses the fuzzy differential equation for the visual search experimental paradigm to identify the influence of the difference of the effect.

Keywords: fuzzy differential equation, eye movement experiment, artistic images, the migration research.

AMS 2010 codes: 03C62

1 Introduction

With the continuous change and development of modern information technology, in the field of art technology research, eye movement experimental research methods study the stylised art image transfer art research

†Corresponding author.
Email address: 392404215@qq.com
which has a leading role. At present, good results have been obtained in the study of the image effect. Research using eye movement experimental research methods can be more objectively recorded and studied, but little is involved. The eye-tracking device was used to record the eye-tracking track of the subjects, which was quantified, and the experimental data was used to scientifically demonstrate the reading effect of the learners on the teaching art images. China attaches great importance to the development of information technology, so the study of artistic image effect can not only promote the application of information technology in the process of artistic development but also conform to the development of the times [1]. At present, there are many methods to study the cognitive processing of artistic images in reading, and among which the most mainstream and effective method is the eye movement method [2]. Eye movement experimental method plays an important role in various fields (for example, modern psychology) and has gradually become a mainstream research method. The brain is connected to the eye, and what the eye sees responds to the brain [3]. Therefore, the learner’s mental process can be analysed by observing what people pay attention to with the help of an eye tracker. According to the way the visual system works, what you see means what you think, and getting your gaze equals attention. The more the subjects looked, the more they thought, but not that the subjects understood what they saw. The reason is that sometimes there is a lot of casual gazes or gaze that do not understand its meaning.

Eye movement technology refers to the use of eye movement equipment to collect the eye movements of the subjects, analyse and obtain such things as fixation time, fixation point, pupil size, eye jump distance, etc. so as to study the psychological process of the subjects. The line-of-sight tracking system can be divided into three parts: the centre coordinate extraction system of the pupil, the optical system, the superposition system of view and pupil coordinates, and the image and data recording and analysis system [4]. The basic states of eye movement include following the movement, fixation and saccades. It has become a scientific and rigorous quantitative research method for eye movement experiments to analyse the psychological activities of the subjects scientifically by means of fuzzy differential equations. The combination of eye movement experiments and artistic image transfer can effectively improve the readers’ artistic skills. Most of the previous image studies were qualitative and lacked quantitative analysis. The use of eye movement research technology combined with it is just to make up for the lack of quantitative analysis of research. The study of image migration uses new technical routes and research ideas to promote the application of eye movement experimental research in the field of educational technology. In this paper, eye movement experimental research is used as the experimental research method from the perspective of learners, and with the help of fuzzy differential equation theory, the eye movement data of image readers is deeply analysed and the application strategy of artistic image transfer research is obtained. The application of eye movement experiments to artistic image migration and the use of fuzzy differential equations in the field of artistic technology make the analysis of artistic image migration more scientific and intuitive [5].

2 Methods

Fractional order fuzzy differential equation is a fractional order differential equation defined on a specific fuzzy set, also known as a fractional order differential equation with uncertainty. Fractional fuzzy differential equations have the advantages of nonlocal and memory of fractional derivatives and are more widely used. In the field of art, the application of fuzzy differential equations can reflect the subjectivity or uncertainty in the law of determination, and it is more appropriate to describe the problem. The initial value problem of a fractional fuzzy differential equation is one of the basic research objects of qualitative theory of fractional fuzzy differential equation. In this paper, we discuss the existence, non-existence, uniqueness or multiplicity, boundedness, periodicity, vibration, stability and branching of the initial value problem of fractional equations by means of nonlinear analysis [6].
The initial value problem of fractional fuzzy differential equations:

\[
\begin{align*}
\begin{cases}
{}^{RL} D^q u &= f(t, u(t)), q \in (0, 1) \\
\lim_{t \to 0^+} t^{1-q} u(t) &= u_0 \in E^1
\end{cases}
\end{align*}
\] (1)

The RLqD is based on the Riemann-Liouville H-derivative, and the first value problem is solved by studying the \( E^1 \) corresponding fuzzy integral equation in one-dimensional fuzzy number space.

\[
\begin{align*}
\begin{cases}
{}^{RL}_{GH} D^q_u u(t) &= f(t, u(t)), q \in (0, 1) \\
{}^{RL}_{GH} D^{q-1}_u u(t_0) &= u_0 \in E^1
\end{cases}
\end{align*}
\] (2)

Two kinds of nonlinear fractional-order fuzzy integral equations are obtained by using the up-down solution method:

\[
\begin{align*}
u(t) &= g(t) \oplus \frac{1}{\Gamma(q)} \int_0^t (t-s)^{q-1} f(t, u(s), Tu(s)) ds \\
u(t) &= g(t) - \frac{1}{\Gamma(q)} \int_0^t (t-s)^{q-1} f(t, u(s), Tu(s)) ds
\end{align*}
\] (3)

The results of the existence and uniqueness of the solution of the fuzzy integral differential equation is

\[
\begin{align*}
\begin{cases}
{}^{RL}_{GH} D^q_u u(t) &= f(t, u(t), Tu(t)), q \in (0, 1) \\
\lim_{t \to 0^+} t^{1-q} u(t) &= u_0 \in E^1
\end{cases}
\end{align*}
\] (4)

Among them,

\[
Tu(t) = \int_0^t T(t,s)u(s)ds
\] (5)

Based on the application of the eye movement experiment in this paper, the definition of the solution of two set marks and initial value problems is given, namely

\[
\Omega = \{ u : u_{(-\infty, 0]} \in C^E(-\infty, 0], u_{[0,a]} \in C^E[0,a] \}
\] (6)

Along with,

\[
E = \{ u \in C^E(-\infty, T] : u(t) = \phi(t) (-\infty, 0], (u(t), \phi(t)) \leq \rho [0,T] \}
\] (7)

Suppose \( u(t) [0, T] \) as a differential.

Then the upper formula of the initial value problem is equivalent to the fuzzy differential equation

\[
\begin{align*}
u(t) &= \phi(t) , \\
\phi(0) \oplus \frac{1}{\Gamma(q)} \int_0^t (t-s)^{q-1} r(s) f(t, u_s) ds, & t \in (-\infty, 0] \\
\phi(0) \oplus \frac{1}{\Gamma(q)} \int_0^t (t-s)^{q-1} r(s) f(t, u_s) ds, & t \in [0,T]
\end{align*}
\] (8)

In fact, because it is the solution of the initial value problem, it can be \( u(t) \) expressed as:

\[
u(t) = u(0) \oplus \frac{1}{\Gamma(q)} \int_0^t (t-s)^{q-1} r(s) f(t, u_s) ds
\] (9)

Combined with initial conditions:

\[
u(t) = \phi(0) \oplus \frac{1}{\Gamma(q)} \int_0^t (t-s)^{q-1} r(s) f(t, u_s) ds
\] (10)
In addition, for arbitrary cases, \( t \in [0, T] \) according to:

\[
d(u(t), \phi(0)) = d\left( \frac{1}{\Gamma(q)} \int_0^t (t-s)^{q-1} r(s)f(t, u_s) \, ds, 0 \right) \leq \frac{1}{\Gamma(q)} \int_0^t (t-s)^{q-1} r(s)df(s, u_s), 0) \, ds
\]

\[
\leq \frac{Kr}{\Gamma(q)} \int_0^t (t-s)^{q-1} ds = \frac{KrT^q}{\Gamma(q+1)} \leq \rho
\]

\( u \) belongs to \( E \).

Construct the iterative sequence \( u \). Let the initial function be:

\[
u^{(0)}(t) = \begin{cases} 
\phi(t), & t \in (-\infty, 0] \\
\phi(0), & t \in [0, T]
\end{cases}
\]

Clearly, the \( d\left( u^{(0)}(t), 0 \right) \leq \rho, t \in [0, T]\) definition:

\[
u^{m+1}(t) = \begin{cases} 
\phi(t), & t \in (-\infty, 0] \\
\phi(0) + \frac{1}{\Gamma(q)} \int_0^t (t-s)^{q-1} r(s)f(s, u_s^m) \, ds, & t \in [0, T]
\end{cases}
\]

Among them \( m = 0, 1, \ldots \), here \( t \in [0, T] \):

\[
d\left( u^{(1)}(t), u^{(0)}(t) \right) = d\left( \frac{1}{\Gamma(q)} \int_0^t (t-s)^{q-1} r(s) f\left( t, u_s^{(0)} \right) \, ds, 0 \right)
\]

\[
\leq \frac{1}{\Gamma(q)} \int_0^t (t-s)^{q-1} r(s) \sup_{\tau \in (-\infty, 0]} d(u_s^{(1)}(\tau), u_s^{(0)}(\tau)) \, ds \leq \frac{Kr^q}{\Gamma(q+1)} \int_0^t (t-s)^{q-1} ds = \frac{K\rho^q}{\Gamma(q+1)}
\]

Because the fuzzy numerical function \( f \) satisfies the local conditions, there are \( t \in [0, T] \):

\[
d(u^{(2)}(t), u^{(1)}(t)) \leq \frac{1}{\Gamma(q)} \int_0^t (t-s)^{q-1} r(s)df(s, u_s^{(1)}), f(s, u_s^{(0)})ds
\]

\[
\leq \frac{L}{\Gamma(q)} \int_0^t (t-s)^{q-1} r(s)D(u_s^{(1)}, u_s^{(0)})ds
\]

\[
\leq \frac{L}{\Gamma(q)} \int_0^t (t-s)^{q-1} r(s) \sup_{\tau \in (-\infty, 0]} d(u_s^{(1)}(\tau), u_s^{(0)}(\tau))ds
\]

\[
\leq \frac{L}{\Gamma(q)} \int_0^t (t-s)^{q-1} r(s) \sup_{\tau \in (-\infty, 0]} d(u_s^{(1)}(s+\tau), u_s^{(0)}(s+\tau))ds
\]

\[
\leq \frac{L^2}{\Gamma(q)} \int_0^t (t-s)^{q-1} r(s) \sup_{\theta \in (-\infty, 0]} d(u_s^{(1)}(\theta), u_s^{(0)}(\theta))ds
\]

\[
\leq \frac{K\rho^2}{\Gamma(q+1)\Gamma(q)} \frac{L}{\Gamma(q)} \int_0^t (t-s)^{q-1} s^4 ds = \frac{Kr^2 L^2}{\Gamma(2q+1)}
\]

Assumption:

\[
d\left( u^{(m)}(t), u^{(m-1)}(t) \right) \leq \frac{K \rho^m L^m \rho^m}{L \Gamma(mq+1)}
\]
The establishment of \( t \in [0, T] \), there are

\[
d \left( u^{(m+1)}(t), u^{(m)}(t) \right) \leq \frac{1}{\Gamma(q)} \int_0^t (t-s)^{q-1} r(s) d \left( f \left( s, u_s^{(m)} \right), f \left( s, u_s^{(m-1)} \right) \right) ds
\]

\[
\leq \frac{L}{\Gamma(q)} \int_0^t (t-s)^{q-1} r(s) D^m_u \left( u_s^{(m)}, u_s^{(m-1)} \right) ds
\]

\[
= \frac{L}{\Gamma(q)} \int_0^t (t-s)^{q-1} r(s) \sup_{\tau \in (-\infty, 0]} \left( u_s^{(m)}(\tau), u_s^{(m-1)}(\tau) \right) ds
\]

\[
= \frac{L}{\Gamma(q)} \int_0^t (t-s)^{q-1} r(s) \sup_{\theta \in (-\infty, 0]} d \left( u_s^{(m)}(\theta), u_s^{(m-1)}(\theta) \right) ds
\]

\[
\leq \frac{K}{\Gamma(mq+1)} \frac{L}{\Gamma(q)} \int_0^t (t-s)^{mq} ds = \frac{K}{\Gamma(mq+1)} \frac{L}{\Gamma(q+1)}
\]

According to the mathematical induction method, the inequality holds for all \( m = 1 \), and it is proved that the fuzzy differential equation can be applied to this art eye movement experiment [7].

3 Empirical analysis

The purpose of this study is to explore how to maximise the effect of artistic images in artistic image migration. To understand the cognitive process of learners in reading artistic images, the following research hypotheses are proposed based on the theoretical review: (1) There is no significant difference between cognitive style and fixation times of artistic images. (2) There was no significant difference between cognitive style and fixation time of artistic images. In this study, an eye movement experiment was used to study the effect of teaching art images. With the help of the eye movement index, the cognitive process of learners is explored, and the eye movement data such as the number of fixation times, fixation time and hot spot chart of learners are recorded by eye movement instrument so as to analyse the eye movement track and the fixation points of learners during the learning process. The research process includes the training process and the experimental process.

During the training, before all the work was started, the experimenter needed to check the eyes and the blue dots showed on the screen. The subjects had to keep their eyes fixed on the blue dots and keep them for a period of time so as to check the eye movement device. The blue dots would appear five times in a row. After verification, the subject maintains his or her posture, and eye movements are recorded by the eye movement device [8]. At this point, the subjects are introduced to the pre-experiment program, and the space bar will control the beginning and end of the experiment. After the subjects fully understood the experiment process, the training process was over. During the experiment, the eyes of the subjects were checked according to the training procedure. After the verification, the subjects were informed to start the experiment officially. When the subject is ready, press the space bar to start. At the end of the experiment, press the space bar again. After the experiment, the observers interviewed the subjects and collected experimental data. In this study, the eye movement analysis software of an eye movement instrument was used to extract the eye movement data of the subjects. Based on the theory of fuzzy differential equations, SPSS statistical software was used to analyse the collected eye movement data. In the eye movement test, the fixation time of the target object is determined by the fixation time of each fixation point [9]. At a single fixation point, the duration of the subject’s gaze can be seen in the line of sight as shown in Figure 1. The circle represents each fixation point, and the size of the circle represents the fixation time. The larger the area, the longer the fixation; the smaller the area, the shorter the gaze time. Each circle represents the subject’s eye movement fixation point. According to the circular area, the fixation time of each fixation point can be inferred, and the sum of each fixation point time is the total fixation time of the experiment. The first time it reaches the target region is shown in Figure 2. In the experiment, the experimental materials of eye movement were divided into several regions of interest. After the target interest area is specified, the time when the line of sight reaches the specified interest area for the first time represents the time when the target area is first reached. This time is a point in time, not a time period.
4 Results

4.1 Eye movement data analysis

In the study of fixation count, an independent sample T test is used to analyse whether cognitive style has an influence on the fixation count of artistic images. The null hypothesis H was put forward, and it was concluded that there was no significant difference between cognitive style and fixation number of artistic images. The detailed results are obtained from Table 1. The variance does not need to be corrected, and the difference is significant.

In the study of fixation time, the results are shown in Table 2. Independent sample T test was used to analyse whether cognitive style has influence on fixation time of artistic images. Null hypothesis H is proposed, in which cognitive mode has no significant difference in fixation time of artistic images.

4.2 The image transfer

In this paper, the image transfer case is as follows: given some specific scene photos (castle images), the photos are seamlessly integrated into the photography art. Image migration is an important image editing technique. The photos were imported into the desktop of the test machine, the program was run in the experiment folder of the test machine, and the observation time of 60 s was given for the two images each. It mainly analyses two eye movement trajectory images generated by the eye tracker, examines the staring time, spatial density of fixation
and length of the scanning path. It imbedded the source image object into the target background image and then generates a new seamless composite image. The method of image migration is mainly to solve the problem of colour mismatch between the source image and target image in the migration boundary, and it can also process natural photos. Take Figure 3 for example. It shows the frame diagram of style-aware image migration.

4.3 Accuracy analysis

In the experiment, the accuracy rate mainly depends on whether the last eye-tracking point click is correct or not. Research accuracy is helpful to judge the overall effectiveness of artistic image tracking under different conditions. The final accuracy rate of each level is shown in Figure 4. The results showed that the main effect of image type was significant, but the main effect of colour relation was not. The interaction between type and colour relationship is not significant. The interaction between type and subject is not significant, the interaction between colour relation and the subject is not significant and the interaction between type, colour relation and
the subject is not significant [10]. The significant difference of artistic images indicates that the search accuracy of artistic images is significant, and the migration accuracy is high, indicating that this method can improve the image effect and enhance attention.

Fig. 4 The right line diagram of eye movement test.

5 Conclusion

This study uses the eye movement experiment research method, with the help of fuzzy differential equations, and scientifically analyses the influence of art image application on learners’ learning effect. In art image making, we should follow the learner’s learning characteristics, evaluate the quality of art image by ourselves and take eye movement instruments as the assistant. Using line-of-sight tracking technology to accurately record eye movement data can evaluate the level of artistic image production from the perspective of learners and will be a trend of evaluation methods. In this paper, the method of eye movement experiment is related to the effect of stylised art image migration, and the scientific quantitative analysis is carried out so as to explore the deeper meaning and function of art image from a scientific point of view.

References

[1] Allahviranloo, T., Kiani, N. A., & Motamedi, N.. (2009). Solving fuzzy differential equations by differential transformation method. Information Encyc., 179(7), 956-966.
[2] Nnolim, U. A.. (2017). Improved partial differential equation-based enhancement for underwater images using local–global contrast operators and fuzzy homomorphic processes. Iet Image Processing, 11(11), 1059-1067.
[3] Chen, K., Hou, J., Huang, Z., Cao, T., Zhang, J., & Yu, Y., et al. (2015). All-optical 1st- and 2nd-order differential equation solvers with large tuning ranges using fabry-pérot semiconductor optical amplifiers. Optics Express, 23(3), 3784-94.
[4] Martin, J. L., Maconochie, D. J., & Knight, D. E.. (1994). A novel use of differential equations to fit exponential functions to experimental data. Journal of Neuroscience Methods, 51(2), 135.
[5] Cresson, J., & Pierret, Frédéric. (2016). Multiscale functions, scale dynamics and applications to partial differential equations. Journal of Mathematical Physics, 57(5), 4907-4938.
[6] SUZUKI, & Otohiko. (1962). Experimental verification for differential equations of sweep line. Nsugaf, 28(10), 988-991.
[7] Schittkowski, & K. (2007). Experimental design tools for ordinary and algebraic differential equations. Industrial & Engineering Chemistry Research, 79(26), 521-538.
[8] Reynolds, J. S., Przadka, A., Yeung, S. P., & Webb, K. J.. (1996). Optical diffusion imaging: a comparative numerical and experimental study, Applied.
[9] Kaur, A., Takhar, P. S., Smith, D. M., Mann, J. E., & Brashears, M. M.. (2008). Fractional differential equations based modeling of microbial survival and growth curves: model development and experimental validation. Journal of Food Science.
[10] Phair, R. D., & Lippincott-Schwartz, J.. (2018). Differential equation methods of simulation of gfp kinetics in non-steady state experiments. Molecular Biology of the Cell, mbc.E17-06-0396.