Research Article

Design of an Unmatching Observer-Based Controller for Discrete-Time Fuzzy Systems with Time Delay

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The problem of an unmatching observer-based controller design for discrete-time fuzzy systems with time delay is investigated, in which the fuzzy controller shares different membership functions from the fuzzy model. The objective is to design a state observer and unmatching fuzzy controller such that the discrete closed-loop system with time delay is asymptotically stable. A sufficient condition that contains the information of the membership functions of fuzzy model and fuzzy controller for the stabilization via an unmatching observer-based output feedback is presented. The proposed control scheme is well capable of enhancing the design flexibility, and the stability condition is less conservative. Three numerical examples are given to illustrate the effectiveness and advantages of the proposed method.

1. Introduction

Time-delay phenomena intensively exist in natural science and engineering and social life, such as nonlinear and time-delay models. For example, they are in numerous dynamical systems including biology systems, mechanics, economics, chemical systems, and network systems. Generally, time delay can often lead to instability and poor performances. Therefore, it is of great significance to investigate the issue of the stability for time-delay systems. A lot of relevant research work has been reported in [1–3]. However, the results obtained are mainly only based on the linear time-delay systems. It is indeed necessary to generalize them to the nonlinear time-delay systems. As we know, the fuzzy model proposed by Takagi and Sugeno can effectively represent many nonlinear dynamic systems [4]. During the past decade, more and more researchers paid attention to the issue of the stability for the nonlinear fuzzy systems with time delay [5, 6]. The stability problem refers to the design of a controller that can guarantee the fuzzy model stability. State feedback plays an important role in handling stabilization in the control systems. However, sometimes the system states cannot be measured directly, and there are limitations of the measurement equipment in use. It is actually impossible to get all the information of the system state variables. Under this circumstance, the physical form of state feedback is difficult to realize. Therefore, state estimation and observation of nonlinear systems are important but challenging problems in modern control theory. The design of observers and controllers for the T-S fuzzy systems is a demanding topic. For example, the stability analysis and stabilization of the T-S fuzzy models for designing the observers and control laws are addressed in [7, 8]. A single-step linear matrix inequality method is developed for the observer-based controller design for discrete-time fuzzy systems in [9], which overcomes the drawbacks induced by the conventional two-step approach, and yields less conservative results. The problem of fuzzy observer-based controller design is investigated for nonlinear networked control systems subject to imperfect communication links and parameter uncertainties in [10]. Some results are generalized to the observer-based adaptive model in [11]. A novel method based on the nonuniform delay partitioning approach is put forward to analyze the stability of the time-delay T-S fuzzy...
system, and the observer-based feedback controller design scheme via the parallel distributed compensation (PDC) scheme is also discussed in [12]. A fuzzy functional observer method is proposed to design a controller for the observer-based fuzzy model in [13]. Furthermore, some results related to the observer-based controller design problem have been extended to the MIMO time-delay systems in [14]. Recently, the observer-based controller design problem is investigated in networked control systems [15]. In the abovementioned work, the observer-based feedback controller design problems are all based on the PDC scheme, which means that the observed-based fuzzy model and the observed-based fuzzy controller share the same membership functions. As a matter of fact, if the membership functions of the fuzzy controller in the premise of the fuzzy rules are allowed to be designed arbitrarily, a greater design flexibility can be achieved. Thus, some research studies on the control problems under imperfect premise matching have been carried out in [16–20]. As the states of the systems are sometimes difficult to be obtained or the costs of measurement are too high in many practical problems, it is critical to design observers for the systems. In addition, nonlinear and time delays are inherent and not all the states are available in most practical systems. Therefore, the observer-based stabilization control for the discrete-time T-S fuzzy systems with time delay is an important topic.

In this paper, we focused on the observer-based stabilization control for discrete-time T-S fuzzy time-delay systems and a novel observer-based output feedback controller is investigated for the discrete-time T-S fuzzy time-delay systems. The main contributions of this paper can be summarized as follows:

(i) As the membership functions of the fuzzy model and the fuzzy controller are all considered in the analysis, some membership-function-dependent stability conditions are derived, which are less conservative than membership-function-independent stability conditions.

(ii) As the membership functions of the controller are distinctive from those of the fuzzy model, the membership functions can be chosen arbitrarily, which makes a higher degree of design flexibility of the fuzzy controller achieved, and its structure can be further simplified.

The remainder of this paper is organized as follows. In Section 2, the considered problem is described. An unmatched observer-based controller design method is proposed in Section 3. Three numerical examples are utilized to illustrate the conservativeness, effectiveness, and superiority of the proposed method in Section 4. Finally, Section 5 concludes this paper.

Notation 1. In this paper, if not explicitly stated, matrices are assumed to have compatible dimensions. The notation $M > (\geq, < , \leq) 0$ is used to denote a symmetric positive-definite (positive semidefinite, negative, and negative semidefinite, respectively) matrix. $\| \cdot \|$ denotes the Euclidean norm for vector or the spectral norm of matrix. For convenience, we use $u_i, m_i$ instead of $u_i(z(t)), m_i(z(t))$.

2. Problem Formulation and Preliminaries

Let $r$ be the number of the fuzzy rules describing the time-delay nonlinear plant. The $i$th rule can be represented as follows.

If $z_i(t) = M_i^1$ and ... and $z_p(t) = M_p^1$, then

$$x(t + 1) = A_{i1}x(t) + A_{i2}x(t - \tau) + B_iu(t), \quad y(t) = C_{i1}x(t) + C_{i2}x(t - \tau), \quad t \geq 0,$$

where $r$ is the number of the fuzzy rules and $M_a^1$, $a = 1, 2, \ldots, p$ and $i = 1, 2, \ldots, r$, denotes the fuzzy set. $z_k(t), k = 1, 2, \ldots, r$, are the known premise variables which do not depend on the input variables. $x(t) \in R^m$ is the state vector, $u(t) \in R^m$ is the input vector, $y(t) \in R^l$ is the output vector, and $A_{i1}, A_{i2}, B_i, C_{1i},$ and $C_{2i}$ are some constant matrices of compatible dimensions. Here, we assume that $\tau > 0$ is a constant.

With the fuzzy inference methods, the final outputs of the fuzzy time-delay model can be formalized as follows:

$$x(t + 1) = \sum_{i=1}^{r} w_i(z(t))[A_{i1}x(t) + A_{i2}x(t - \tau) + B_iu(t)],$$

$$y(t) = \sum_{i=1}^{r} w_i(z(t))[C_{i1}x(t) + C_{i2}x(t - \tau)],$$

where

$$\sum_{i=1}^{r} w_i(z(t)) = 1, \quad w_i(z(t)) \geq 0, i = 1, 2, \ldots r \forall t,$$

$$w_i(z(t)) = \frac{\mu_i(z(t))}{\sum_{i=1}^{r} \mu_i(z(t))}, \quad \mu_i(z(t)) = \prod_{k=1}^{p} \mu_{M^1_k}z_k(t),$$

in which $z_i(t), \ldots, z_p(t)$ are the premise variable and $\mu_{M^1_k}(z_k(t))$ is the grade of membership of $z_k(t)$ in $M_k^1$.

In engineering practices, the state information of system equation (1) is often not available. Therefore, it is necessary to construct a state observer satisfying the following formal equation to estimate the state of the discrete system.

Observer rule $i$: if $z_i(t)$ is $M_i^1$ and ... and $z_p(t)$ is $M_p^1$, then

$$\hat{x}(t + 1) = A_{i1}\hat{x}(t) + A_{i2}\hat{x}(t - \tau) + B_iu(t) + L_i(y(t) - \hat{y}(t)), \quad i = 1, 2, \ldots, r,$$

$$\hat{y}(t) = C_{i1}\hat{x}(t) + C_{i2}\hat{x}(t - \tau),$$
where $L_i$ is the fuzzy observer gain for the $i$th subsystem. The overall fuzzy observer is represented by
\[
\hat{x}(t+1) = \sum_{i=1}^{r} \sum_{j=1}^{r} w_{ij} (z(t))[A_{ii} \hat{x}(t) + A_{2i} \hat{x}(t-\tau) + B_i u(t) + L_i(y(t) - \tilde{y}(t))],
\]
\[
y(t) = \sum_{i=1}^{r} w_{ij} (z(t))[C_{ij} x(t) + C_{2i} x(t-\tau)],
\]
\[
\tilde{y}(t) = \sum_{i=1}^{r} w_{ij} (z(t))[C_{ij} \hat{x}(t) + C_{2ij} \hat{x}(t-\tau)].
\]

With the aforementioned fuzzy observer, an unmatching fuzzy observer control law is defined as follows. If $z_i(t)$ is $N_i^1$ and ... and $z_i(t)$ is $N_i^n$, then
\[
u(t) = F_i \hat{x}(t), \quad j = 1, 2, \ldots, r.
\]

The overall observer-based fuzzy control law is represented by
\[
u(t) = \sum_{j=1}^{r} m_{ij} (z(t))F_i \hat{x}(t).
\]

Combining the fuzzy controller in equation (7) and the fuzzy observer in equation (5) and denoting $e(t) = x(t) - \hat{x}(t)$, we obtain the following system representations:
\[
x(t+1) = \sum_{i=1}^{r} w_{ij} \sum_{k=1}^{n} m_{ij} k \left[ (A_{ii} - L_i C_{ik}) e(t) + (A_{2i} - L_i C_{2k}) e(t-\tau) \right],
\]
\[
e(t+1) = \sum_{i=1}^{r} w_{ij} \sum_{k=1}^{n} m_{ij} k \left[ (A_{ii} - L_i C_{ik}) e(t) + (A_{2i} - L_i C_{2k}) e(t-\tau) \right].
\]

Therefore, the augmented system can be written as
\[
\bar{x}(t+1) = \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{k=1}^{n} w_{ij} m_{ij} k \left[ G_{ijk} \bar{x}(t) + (\bar{M}_i + \bar{N}_{ik}) \bar{x}(t-\tau) \right],
\]

where
\[
\bar{x}(t) = \begin{bmatrix} x(t) \\ e(t) \end{bmatrix},
\]
\[
G_{ijk} = \begin{bmatrix} A_{ii} + B_i F_j & -B_i F_j \\ 0 & A_{ii} - L_i C_{ik} \end{bmatrix},
\]
\[
\bar{M}_i = \begin{bmatrix} A_{2i} & 0 \\ 0 & A_{2i} \end{bmatrix},
\]
\[
\bar{N}_{ik} = \begin{bmatrix} 0 & 0 \\ 0 & -L_i C_{2k} \end{bmatrix}.
\]
is the equilibrium of the closed-loop discrete fuzzy

\[
\Omega_{ij} - \Lambda_i < 0,
\]

\[
\rho_i \Omega_{ii} - \rho_i \Lambda_i + \Lambda_i < H_{ii},
\]

\[
\rho_j \Omega_{ij} + \rho_i \Omega_{ji} - \rho_i \Lambda_i - \rho_i \Lambda_j + \Lambda_i + \Lambda_j \leq H_{ij} + H_{ji},
\]

\[
\Psi_{ii} < M_{ii},
\]

\[
\Psi_{ik} + \Psi_{ki} < M_{ik} + M_{ik}^T, i < k,
\]

where

\[
\Omega_{ij} = \begin{bmatrix}
-X_1 + \tilde{S}_i & 0 & * \\
A_i X_1 - B_i Y_j & A_2 X_1 - X_1 & \\
0 & -\tilde{S}_i & *
\end{bmatrix},
\]

\[
\Psi_{ik} = \begin{bmatrix}
-X_2 + S_2 & 0 & * \\
X_2 A_{2i} - R_i C_{ik} & X_2 A_{2i} - R_i C_{ik} - X_2 & \\
0 & -S_2 & *
\end{bmatrix}.
\]

The state feedback gains and observer gains can be constructed as \( F_i = Y_i X_1^{-1} \) and \( L_i = X_2^{-1} R_i \).

\section{Main Result}

\textbf{Theorem 1.} The equilibrium of the closed-loop discrete fuzzy

time-delay system in equation (10) with an unmatching observer-based control law in equation (7) is asymptotically stable, if the membership functions of the fuzzy model and fuzzy controller satisfy

\[
m_j (z(t)) - \rho_j w_j (z(t)) \geq 0 \quad \text{for all } j \quad \text{and} \quad z(t), \text{ where } 0 < \rho_j < 1. \text{ There exist } X_1 > 0, X_2 > 0, \tilde{S}_i > 0, S_i > 0, \quad Y_i, \quad \Lambda_i = \Lambda_i^T \in \mathbb{R}^{n \times n},
\]

\[
H_{ij} = H_{ij}^T \in \mathbb{R}^{n \times n}, M_{ik} = M_{ik}^T \in \mathbb{R}^{n \times n}, i, j = 1, 2, \ldots, r, \text{ and } \rho_i \text{ satisfying}
\]

\[
\rho_i \Omega_{ii} - \rho_i \Lambda_i + \Lambda_i < H_{ii},
\]

\[
\rho_j \Omega_{ij} + \rho_i \Omega_{ji} - \rho_i \Lambda_i - \rho_i \Lambda_j + \Lambda_i + \Lambda_j \leq H_{ij} + H_{ji},
\]

\[
\Psi_{ii} < M_{ii},
\]

\[
\Psi_{ik} + \Psi_{ki} < M_{ik} + M_{ik}^T, i < k,
\]

\[
\Omega_{ij} - \Lambda_i < 0,
\]

\[
\rho_i \Omega_{ii} - \rho_i \Lambda_i + \Lambda_i < H_{ii},
\]

\[
\rho_j \Omega_{ij} + \rho_i \Omega_{ji} - \rho_i \Lambda_i - \rho_i \Lambda_j + \Lambda_i + \Lambda_j \leq H_{ij} + H_{ji},
\]

\[
\Psi_{ii} < M_{ii},
\]

\[
\Psi_{ik} + \Psi_{ki} < M_{ik} + M_{ik}^T, i < k,
\]

\[
\Omega_{ij} = \begin{bmatrix}
-X_1 + \tilde{S}_i & 0 & * \\
A_i X_1 - B_i Y_j & A_2 X_1 - X_1 & \\
0 & -\tilde{S}_i & *
\end{bmatrix},
\]

\[
\Psi_{ik} = \begin{bmatrix}
-X_2 + S_2 & 0 & * \\
X_2 A_{2i} - R_i C_{ik} & X_2 A_{2i} - R_i C_{ik} - X_2 & \\
0 & -S_2 & *
\end{bmatrix}.
\]

Proof. Select a Lyapunov function as

\[
V (\tilde{x}(t)) = \tilde{x}^T (t) P \tilde{x}(t) + \sum_{\sigma = t-r}^{t-1} \tilde{x}^T (\sigma) S \tilde{x}(\sigma),
\]

where \( P > 0 \) and \( S \geq 0 \). By Lemma 2, there exist \( \sigma_1 \) and \( \sigma_2 \) such that

\[
\sigma_1 \| \tilde{x}(t) \|^2 \leq V (\tilde{x}(t)) \leq \sigma_2 \| \tilde{x}(t) \|^2.
\]

In this section, we will prove the asymptotic stability of the discrete time-delay system in equation (10) based on the Krasovskii theorem [22].
\[ \Delta V(\bar{x}(t)) = V(\bar{x}(t + 1)) - V(\bar{x}(t)) = \bar{x}^T(t + 1)P\bar{x}(t + 1) - \bar{x}^T(t)P\bar{x}(t) + \bar{x}^T(t)S\bar{x}(t) - \bar{x}^T(t - \tau)S\bar{x}(t - \tau) \]
\[
= \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{k=1}^{r} w_i m_j w_k \left[ \bar{x}^T(t) \left( G_{ijk}^TPG_{ijk} - P + S \right) \bar{x}(t) + \bar{x}^T(t)G_{ijk}^TP \left( \bar{M}_i + \bar{N}_ik \right) \bar{x}(t) + \bar{x}^T(t - \tau) \left( \bar{M}_i + \bar{N}_ik \right)^T \bar{x}(t - \tau) \right] \\
= \bar{x}^T(t) \left( \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{k=1}^{r} w_i m_j w_k \left( Q_{ijk}^TPQ_{ijk} - \Theta \right) \right) \bar{x}^T(t), \tag{21} \]

where
\[
\bar{x}(t) = \begin{bmatrix} \bar{x}(t) \\ \bar{x}(t - \tau) \end{bmatrix}, \\
\Theta = \begin{bmatrix} P & -S & 0 \\ 0 & S & 0 \end{bmatrix}, \\
Q_{ijk} = \left[ G_{ijk} \bar{M}_i + \bar{N}_ik \right].
\]

If
\[
\sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{k=1}^{r} w_i m_j w_k (Q_{ijk}^TPQ_{ijk} - \Theta) < 0, \tag{23} \]
then \( \Delta V(x(t)) < 0. \)

By Schur complement,
\[
\sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{k=1}^{r} w_i m_j w_k \left( Q_{ijk}^TPQ_{ijk} - \Theta \right) = \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{k=1}^{r} \sum_{k=1}^{r} w_i m_j w_k \begin{bmatrix} -\Theta & Q_{ijk}^TP \\ Q_{ijk}^TP & -P \end{bmatrix} = \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{k=1}^{r} \sum_{k=1}^{r} w_i m_j w_k \begin{bmatrix} -P + S & 0 \\ 0 & -S \end{bmatrix} \begin{bmatrix} G_{ijk}^TP \\ P \left( \bar{M}_i + \bar{N}_ik \right)^T \end{bmatrix} \tag{24} \]

If
\[
\sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{k=1}^{r} w_i m_j w_k \begin{bmatrix} -P + S & 0 \\ 0 & -S \end{bmatrix} \begin{bmatrix} G_{ijk}^TP \\ P \left( \bar{M}_i + \bar{N}_ik \right)^T \end{bmatrix} < 0, \tag{25} \]
then \( \Delta V(x(t)) < 0. \)

It is obvious that equation (25) is equivalent to
\[
\sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{k=1}^{r} \sum_{k=1}^{r} w_i m_j w_k \begin{bmatrix} -X + \bar{S} & 0 \\ 0 & \bar{S} \end{bmatrix} \begin{bmatrix} XG_{ijk}^TP \\ P \left( \bar{M}_i + \bar{N}_ik \right)^T \end{bmatrix} < 0, \tag{26} \]

where \( X = P^{-1} \) and \( \bar{S} = XSX. \) Let
\[
X = \begin{bmatrix} X_1 & 0 \\ 0 & X_2^{-1} \end{bmatrix}, \tag{27} \]
and by substituting equation (27) into equation (26) and defining \( Y_j = F_jX_1, \) we can show equation (26) is equivalent to the following LMIs:
\[
\sum_{i=1}^{r} \sum_{j=1}^{r} w_i m_j \begin{bmatrix} -X_1 + \bar{S}_1 & 0 & * \\ 0 & -\bar{S}_1 & * \\ \bar{S}_1 & 0 & X_2^{-1}S_2X_2^{-1} \end{bmatrix} < 0, \tag{28} \]
\[
\sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{k=1}^{r} \sum_{k=1}^{r} w_i m_j w_k \begin{bmatrix} -X_2 + S_2 & 0 & * \\ 0 & -S_2 & * \\ X_2A_2 + R_iC_{1i}X_2A_2 - R_iC_{2i}X_2 - X_2 \end{bmatrix} < 0. \tag{29} \]

Premultiplying and postmultiplying equation (29) with \( \text{diag}(X_2, X_2, X_2) \) and denoting \( R_i = X_2L_i, \) we have
\[
\sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{k=1}^{r} w_i m_j \begin{bmatrix} -X_2 + S_2 & 0 & * \\ 0 & -S_2 & * \\ X_2A_2 + R_iC_{1i}X_2A_2 - R_iC_{2i}X_2 - X_2 \end{bmatrix} < 0. \tag{30} \]

For arbitrary \( A_i = A_i^T \in \mathbb{R}^{3n \times 3n} > 0, i = 1, 2, \ldots, r, \) we have
\[
\sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{k=1}^{r} \sum_{k=1}^{r} w_i m_j \begin{bmatrix} -X_2 + S_2 & 0 & * \\ 0 & -S_2 & * \\ X_2A_2 + R_iC_{1i}X_2A_2 - R_iC_{2i}X_2 - X_2 \end{bmatrix} \tag{31} \]

These terms are introduced to equation (28) to alleviate the conservativeness. From equation (28), we obtain
\[
\Omega = \sum_{i=1}^{r} \sum_{j=1}^{r} w_i m_{ij} \begin{bmatrix} -X_i + \bar{S}_i & 0 & * \\ 0 & -\bar{S}_i & * \\ A_{ij}X_i - B_iY_j & A_{ii}X_i - X_i & 0 \end{bmatrix} = \sum_{i=1}^{r} \sum_{j=1}^{r} w_i m_{ij} \Omega_{ij}
\]

\[
= \sum_{i=1}^{r} \sum_{j=1}^{r} w_i m_{ij} \Omega_{ij} + \sum_{i=1}^{r} \sum_{j=1}^{r} w_i \left( w_j - m_j + \rho_j w_j - \rho_j w_j \right) \Lambda_i
\]

\[
= \sum_{i=1}^{r} \sum_{j=1}^{r} w_i \left( m_j + \rho_j w_j - \rho_j w_j \right) \Omega_{ij} + \sum_{i=1}^{r} \sum_{j=1}^{r} w_i \left( w_j - \rho_j w_j \right) \Lambda_i - \sum_{i=1}^{r} \sum_{j=1}^{r} w_i \left( m_j - \rho_j w_j \right) \Lambda_i
\]

\[
= \sum_{i=1}^{r} \sum_{j=1}^{r} \left( m_j w_j \left( \rho_j \Omega_{ij} - \rho_j \Lambda_i + \Lambda_i \right) + \sum_{i=1}^{r} \sum_{j=1}^{r} w_i \left( m_j - \rho_j w_j \right) \left( \Omega_{ij} - \Lambda_i \right) \right)
\]

\[
\leq \sum_{i=1}^{r} \sum_{j=1}^{r} w_i \left( \rho_j \Omega_{ij} + \rho_j \Omega_{ji} - \rho_j \Lambda_i - \rho_j \Lambda_i + \Lambda_i + \Lambda_j \right)
\]

\[
\Psi_k < M_{ik},
\]

\[
\Psi_{ik} + \Psi_{ki} < M_{ik} + M_{ik}^T, \quad i < k,
\]

\[
M = \begin{bmatrix} M_{11} & M_{12} & \cdots & M_{1r} \\
M_{21} & M_{22} & \cdots & M_{2r} \\
\vdots & \vdots & \ddots & \vdots \\
M_{r1} & M_{r2} & \cdots & M_{rr} \end{bmatrix} < 0,
\]

for all \( i, k = 1, 2, \ldots, r \), then \( \Psi < 0 \).

If \( \Omega < 0 \) and \( \Psi < 0 \), then equation (28) and equation (30) hold, so that equation (26) holds, then we have \( \Delta V(x(t)) < 0 \), which means system (10) is asymptotic stable.

4. Numerical Examples

In this section, three examples will be given to illustrate the less conservativeness and the effectiveness of the proposed results.

Example 1. From the proposed results, we can see the stability conditions are connected with the values of \( \rho_j \). In this example, different values of \( \rho_j \) will be testified using the LMI based on the stability conditions in Theorem 1 and then we will illustrate the less conservativeness of the proposed results. Consider the following fuzzy time-delay model [6]:

Rule i: if \( x_i(t) \) is \( M^i_1 \), then

\[
\sum_{i=1}^{r} \sum_{j=1}^{r} w_i \Psi_{ik} = \sum_{i=1}^{r} w_i \Psi_{ii} + \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{j < i} w_i w_k \left( \Psi_{ik} + \Psi_{ki} \right).
\]
\[ x(t + 1) = A_{ij}x(t) + A_{2i}x(t - \tau) + B_iu(t), \quad i = 1, 2, 3, \]

where

\[
A_{11} = \begin{bmatrix} 1 & 0.5 \\ 0.51 & -0.1 \end{bmatrix}, \\
A_{12} = \begin{bmatrix} 0.1 & 0.25 \\ 0.15 & -0.5 \end{bmatrix}, \\
B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\
A_{13} = \begin{bmatrix} a & 0.75 \\ 0.25 & -0.8 \end{bmatrix}, \\
A_{21} = \begin{bmatrix} 0.1 & 0 \\ -0.25 & 0 \end{bmatrix}, \\
B_2 = \begin{bmatrix} 1 \\ -0.5 \end{bmatrix}, \\
A_{22} = \begin{bmatrix} 0.25 & 0 \\ 0.15 & 0 \end{bmatrix}, \\
A_{23} = \begin{bmatrix} 0.19 & 0 \\ 0.06 & 0 \end{bmatrix}, \\
B_3 = \begin{bmatrix} b \\ 0.5 \end{bmatrix}, \\
0 \leq a \leq 2, 0 \leq b \leq 2.
\]

It is assumed that the membership functions of the discrete fuzzy model with time delay and fuzzy controller are different and satisfy \( m_i(z(t)) - \rho_i \omega_j(z(t)) \geq 0 \) for all \( j \) and \( z(t) \). The feedback gains of \( F_j, j = 1, 2, 3 \), are determined such that the eigenvalues of \( A_{ij} + B_iF_j \), \( i, j = 1, 2, 3 \), are located at \(-5\). The stability region given by stability conditions in Theorem 1 with \( \rho_1 = 0.45, \rho_2 = 0.5, \) and \( \rho_3 = 0.75 \) (denoted by “o”) and \( \rho_1 = 0.5, \rho_2 = 0.75, \) and \( \rho_3 = 0.85 \) (denoted by “x”) is shown in Figure 1. For comparison purposes, the stability region given by the stability conditions in [5, 6] (denoted by “x” and denoted by “o”, respectively) is shown in Figure 2. From Figure 1, it can be seen that the proposed stability conditions in Theorem 1 become larger with the increase in \( \rho_1, \rho_2, \) and \( \rho_3 \). Meanwhile, the proposed stability conditions offer a larger stability region than in [5, 6], as can be seen in Figure 2. Based on the simulation, it can also be observed that when \( a \) and \( b \) take certain values, the stability conditions in [5, 6] cannot provide feasible solutions.

**Example 2.** An unmatched observer-based output feedback controller is designed to backing up control of a computer-simulated truck-trailer [23]. Consider the following modified truck-trailer model with time delay:

\[ x_1(t + 1) = \left( 1 - \frac{v_T}{L} \right) \left[ ax_1(t) + \left( \frac{v_T}{L} \right) + (1 - a)x_1(t - \tau) \right] u(t), \]

\[ x_2(t + 1) = x_2(t) + \left( \frac{v_T}{L} \right) \left[ ax_1(t) + (1 - a)x_1(t - \tau) \right], \]

\[ x_3(t + 1) = x_3(t) + v_T \sin \left[ x_2(t) + \left( \frac{v_T}{2L} \right) ax_1(t) \right. \]

\[ \left. + (1 - a)x_1(t - \tau) \right] \]

\[ y(t) = x_2(t) + \left( \frac{v_T}{2L} \right) \left[ ax_1(t) + (1 - a)x_1(t - \tau) \right], \]

where \( x_1(t) \) is the angle difference between the truck and trailer, \( x_2(t) \) is the angle of trailer, \( x_3(t) \) is the vertical position of the rear end of trailer, and \( u(t) \) is the control
input. The model parameters are given as \( T = 2.8, L = 5.5, v = -1.0, \) \( \bar{T} = 2.0, \) and \( a = 0.9. \)

The fuzzy truck-trailer model can be modeled by a two-rule fuzzy model.

Rule \( i, i = 1, 2: \) if \( \theta(t) = x_2(t) + (\nu T/2L)[ax_1(t) + (1 - a)x_1(t - \tau)] \) is about 0, then

\[
x(t + 1) = A_{1i}x(t) + A_{2i}x(t - \tau) + B_1u(t),
\]

\[
y(t) = C_{1i}x(t) + C_{2i}x(t - \tau).
\]

If \( \theta(t) = x_2(t) + (\nu T/2L)[ax_1(t) + (1 - a)x_1(t - \tau)] \) is about \( \pm \pi, \) then

\[
x(t + 1) = A_{12}x(t) + A_{22}x(t - \tau) + B_1u(t),
\]

\[
y(t) = C_{12}x(t) + C_{22}x(t - \tau),
\]

where

\[
A_{1i} = \begin{bmatrix} a & 1 - \frac{\nu T}{L} & 0 & 0 \\ \frac{\nu T}{L} & 1 & 0 & 0 \\ a & \frac{\nu^2 T^2}{2L} & \nu T & 1 \end{bmatrix},
\]

\[
A_{2i} = \begin{bmatrix} (1 - a) & (1 - \frac{\nu T}{L}) & 0 & 0 \\ (1 - a) & \frac{\nu T}{L} & 0 & 0 \\ (1 - a) & \frac{\nu^2 T^2}{2L} & \nu T & 1 \end{bmatrix},
\]

\[
A_{12} = \begin{bmatrix} a & 1 - \frac{\nu T}{L} & 0 & 0 \\ \frac{\nu T}{L} & 1 & 0 & 0 \\ a & \frac{\nu^2 T^2}{2L} & \nu T & 1 \end{bmatrix},
\]

\[
A_{22} = \begin{bmatrix} (1 - a) & (1 - \frac{\nu T}{L}) & 0 & 0 \\ (1 - a) & \frac{\nu T}{L} & 0 & 0 \\ (1 - a) & \frac{\nu^2 T^2}{2L} & \nu T & 1 \end{bmatrix},
\]

\[
B_i = B_2 = \begin{bmatrix} \frac{\nu T}{L} & 0 & 0 & T \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},
\]

\[
C_{1i} = C_{12} = \begin{bmatrix} a & \frac{\nu T}{L} & 1 & 0 \\ \frac{\nu T}{L} & \nu T & 1 & 0 \end{bmatrix},
\]

\[
C_{2i} = C_{22} = \begin{bmatrix} (1 - a) & \frac{\nu T}{L} & 0 & 0 \\ \frac{\nu T}{L} & \nu T & 1 & 0 \end{bmatrix}.
\]

The fuzzy state observer for the T-S fuzzy model in equations (39) and (40) is formulated as follows.

Observer rule \( i: \) if \( \theta(t) = x_2(t) + (\nu T/2L)[ax_1(t) + (1 - a)x_1(t - \tau)] \) is \( M_{ij}, i = 1, 2, \) then

\[
\hat{x}(t + 1) = A_{1i}\hat{x}(t) + A_{2i}\hat{x}(t - \tau) + B_iu(t) + L_i(y(t) - \hat{y}(t)),
\]

\[
\hat{y}(t) = C_{1i}\hat{x}(t) + C_{2i}\hat{x}(t - \tau).
\]

The output of equation (42) is represented as follows:

\[
\hat{x}(t + 1) = \sum_{i=1}^{2} w_i(\theta(t))[A_{1i}x(t) + A_{2i}x(t - \tau_i(t)) + B_iu(t)],
\]

\[
\hat{y}(t) = \sum_{i=1}^{2} w_i(\theta(t))[C_{1i}x(t) + C_{2i}x(t - \tau)].
\]

Here, we set \( d = 10^{-2}/\pi \) and the membership functions are defined as

\[
w_1(\theta(t)) = \left(1 - \frac{1 + \exp(-3(\theta(t) - 0.5\pi))}{1 + \exp(-3(\theta(t) + 0.5\pi))}\right),
\]

\[
w_2(\theta(t)) = 1 - w_1(\theta(t)).
\]

From Theorem 1, according to the abovementioned fuzzy state observer model, an unmatched fuzzy controller with two rules is proposed to control the observer model so that the dynamic system equation (43) is asymptotically stable.

Rule \( j, j = 1, 2: \) if \( \theta(t) = x_2(t) + (\nu T/2L)[ax_1(t) + (1 - a)x_1(t - \tau)] \) is \( N_{ij}, \) then

\[
u(t) = F_j\tilde{x}(t).
\]

The fuzzy controller is defined as

\[
u(t) = \sum_{j=1}^{2} m_j(\theta(t))F_j\tilde{x}(t).
\]

The membership functions are selected as

\[
m_1(\theta(t)) = 0.93 \exp\left(-\frac{\theta(t)}{4 \times 1.5^2}\right),
\]

\[
m_2(\theta(t)) = 1 - m_1(\theta(t)).
\]

We assume that \( \rho_1 = \rho_2 = 0.85 \) such that \( m_j(\theta(t)) - \rho_jw_j(\theta(t)) > 0 \) for all \( j \) and \( \theta(t). \)

Assuming \( \tau = 2.0 \) and applying the MATLAB LMI toolbox to solve the LMIs in Theorem 1, we have
With the initial condition $x(0) = [ -0.25\pi \ 0.5\pi \ -10 ]^T$ and $\hat{x}(0) = [ 0 \ 0 \ 0 ]^T$, the simulation results under Theorem 1 are shown in Figure 3, which illustrate the effectiveness of Theorem 1.

**Example 3.** The same fuzzy state observer model equation (43) is also considered here. Some advantages of the proposed method are shown in this example. Suppose the membership function $w_j(\theta(t))$ is chosen as follows:

$$w_1(\theta(t)) = \frac{1 - \frac{c(t)\sin\left(|\theta(t)|^{-4}\right)^5}{1 + \exp^{-100(1-\theta(t))}}}{1 + \exp^{-2.5(\theta(t) + 0.42)^2}}$$

$$w_2(\theta(t)) = 1 - w_1(\theta(t)),$$

where $c(t) = ((\sin(\theta(t)) + 1)/40) \in [-0.05, 0.05]$ and $\theta(t) \in [-\pi/2, \pi/2]$, in which $c(t)$ is an uncertain variable.

Comparing with $w_j(\theta(t))$ (equation (44)) in Example 2, we can observe that the structure of the membership functions $w_j(\theta(t))$ (equation (49)) is not only complex but also contains an uncertain variable. By equation (46), under the PDC controller design framework, the membership functions of the fuzzy controller must be selected as the same.
as those of the fuzzy model, which will lead to the fuzzy controller unrealized. However, based on Theorem 1, some simple and specific membership functions $m_j(\theta(t))$ (equation (47)) can be selected instead of the complex and uncertain functions $w_j(\theta(t))$ (equation (49)) as the membership functions of the fuzzy controller, which can ensure the closed-loop system (10) is asymptotically stable (refer to Figure 4).

Remark 2. Comparing with the PDC scheme, as the membership functions of the controller are distinctive from those of the fuzzy model in our paper, the membership functions can be chosen arbitrarily. Therefore, the proposed method can enhance the design flexibility of the fuzzy controller as well as retain the robustness property of the T-S fuzzy control systems.

5. Conclusions

In this paper, an unmatched observer-based stabilization control for discrete-time T-S fuzzy systems with time delay is investigated, in which the discrete fuzzy observer model and fuzzy controller do not share the same membership functions. The information of the membership functions of the fuzzy model and controller is considered in the observer design scheme. Furthermore, the design flexibility can be enhanced by arbitrarily selecting simple membership functions for the observer-based controller. The advantages and effectiveness of the proposed method have been illustrated using simulation examples. According to the obtained results, the proposed method only fits for the systems with constant delay. Therefore, how to extend the abovementioned method to time-varying delay systems will be considered in the future.

Data Availability

The process data used to support the findings of this study have not been made available because they form part of an ongoing study.

Conflicts of Interest

The authors declare that they have no conflicts of interest to report regarding the present study.

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