Disordered Weyl semimetals and their topological family

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We develop a topological theory for disordered Weyl semimetals in the framework of boundary-bulk correspondence of Chern insulators and the gauge invariance of replica formalism. An anisotropic topological $\theta$-term is analytically derived for the effective non-linear sigma model, which corresponds to the anisotropic Chern-Simons term of the anomalous electromagnetic response theory of Weyl semimetals. Moreover, we establish a general diagram that reveals the intrinsic relations among topological terms in the non-linear sigma models and gauge response theories respectively for $(2n+2)$-dimensional topological insulators, $(2n+1)$-dimensional chiral fermions, $(2n+1)$-dimensional chiral semimetals, and $(2n)$-dimensional topological insulators with $n$ being a positive integer.

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Introduction Recently, Weyl semimetal (WSM) has been attracting more and more attention due to its interdisciplinary interest from anomalous transport in condensed matter physics [1–8] and quantum field theory [9–18]. According to the no-go theorem [19], anomalies as well as a topological character of its gapless modes [19–21]. According to the no-go theorem [19], gapless Weyl points in a WSM appear as left- and right-handed pairs in the momentum space. Since all the energy bands of a system with both time-reversal symmetry (TRS) and/or inversion symmetry (IS) are doubly degenerate for both left- and right-handed modes, a WSM can only be realized in a system breaking TRS and/or IS symmetry. Here we focus essentially on the simplest model of WSM with two Weyl points, whose low-energy effective Hamiltonian is given by

$$\mathcal{H}_{WSM}(k, b) = \left( -\sigma \cdot (k - b(x)) \quad \sigma \cdot (k + b(x)) \right), \quad (1)$$

where $\sigma$’s are Pauli matrices and $b$ is the displacement of Weyl points from the origin of the momentum space. Under semi-classical approximation, the spatial dependence of $b(x)$ is assumed to be adiabatic. The model does not have TRS [22]. Each Weyl point has a nontrivial topological charge [11–21], namely the Chern number on the gapped two-dimensional sphere enclosing a single Weyl point in $k$ space is $\pm 1$, which leads to topological terms in the $U(1)$ response of this model [12–18]. For real materials, disorders are normally unavoidable. As is known, for topological matter, topological terms in the non-linear sigma model (NL$\sigma$M) are significantly important for describing physics of the system under disorders [22–24], which highly motivates us to develop a general theory for disordered Weyl semimetals.

In this Letter, for the model of Eq. (1) under disorders and preserving no discrete symmetry, we derive for the first time an anisotropic topological term in the action of the NL$\sigma$M as

$$S_{A\theta} = -\frac{1}{16\pi^2} \int d^3 x \, \epsilon^{ijk} b_i \text{tr}(Q \partial_j Q \partial_k Q), \quad (2)$$

where $\epsilon^{ijk}$ is the total anti-symmetry tensor and $Q$ stands for a sigma field to be specified later. We refer to this term as an anisotropic $\theta$-term, as it is just a usual $\theta$-term on the plane perpendicular to $b$ for a constant $b$ [22]. Actually this term can be generalized to any chiral semimetal (CSM) with two chiral points separated in the $k$ space of odd dimensions [26]. We also reveal that this new anisotropic $\theta$-term is associated with the so-called opposite coupling in CSM, which plays a key role in a relationship diagram of topological terms for a family of topological matter, as illustrated in Fig. (1), where each topological term in the NL$\sigma$M has a counterpart in the $U(1)$ gauge response theory.

Boundary-bulk correspondence Let us start with an introduction to the Wess-Zumino-Witten term (WZW-term) for a single Weyl point with disorders but without any anti-unitary symmetry, from a viewpoint of boundary-bulk correspondence (BBC) [27]. For a $(2n+2)$-

Figure 1: Relationship diagram of topological terms and materials, where $n$ is a positive integer. For every node, the upper, below-left and below-right boxes indicate the model, the topological terms in NL$\sigma$M and gauge theory, respectively. The dashed arrow is composed by the three successive solid arrows. Nodes: TI(topological insulator), $\Theta$($\theta$-term), CS(Chern-Simons term), CF(chiral fermion), WZW(WZW-term), CC(Chern character), CSM(chiral semimetal), $A$-$\Theta$(anisotropic $\Theta$) and $A$-$CS$(anisotropic CS). Arrows: BBC(boundary-bulk correspondence), OC(opposite coupling) and DR(dimension reduction).
dimensional (D) TI in the Altland-Zirnbauer class A with a nontrivial Chern number $C$ in its bulk [28, 29], there are $C$ flavors of chiral fermions with the same chirality on each of its $(2n+1)$D boundaries [30, 31]. The NLσM of the $(2n+2)$D TI under disorders has a $\theta$-term [22]. In the infrared limit the behavior of the TI is entirely determined by its gapless boundary, and the coupling constant of the $\theta$-term becomes the Chern number $C$ under the renormalization flow [22]. To be concrete, for a 4D TI, the $\theta$-term of the NLσM is given by

$$S^{\theta}_{\text{NL}} = \frac{iC}{256\pi^2} \int d^4x \, e^{\mu\nu\rho\lambda} \text{tr} Q\partial_\mu Q\partial_\nu Q\partial_\rho Q\partial_\lambda Q.$$  

(3)

Viewing from the boundary of the TI, this implies that there exists a WZW-term at level $C$ in the NLσM of the chiral fermions on the boundary [22, 23]. For the case of Eq. (4), the corresponding WZW-term is found to be

$$S_{WZW} = \frac{i\nu_b}{256\pi^2} \int d\tau d^3x \, e^{\mu\nu\rho\lambda} \text{tr} Q\partial_\mu Q\partial_\nu Q\partial_\rho Q\partial_\lambda Q,$$

where $\tau \in [0, 1]$ is the extending parameter, and $Q(x)$ on the $S^3$ is extended continuously to $Q(x, \tau)$ with $Q(x, 0) = Q(x)$ and $Q(x, 1)$ being constant. In this case, $\tau$ can be regarded as the parameter of the radial direction if the geometry of the 4D TI is a disc $D^4$. Here $\nu_b$ is the total Z-type topological charge of the boundary gapless modes, which is defined as the Chern number on the gapped sphere enclosing these gapless points in momentum space, and it is equal to the bulk Chern number $C$ according to an established index theorem [11, 21, 32]. Inherently, the Z-type topological charge $\nu_b = C$ also implies the WZW-term to be at level $\nu_b$ in the NLσM. Since this WZW-term has a topological character with a discrete coupling constant, gapless chiral modes on the boundary is very robust, free from the Anderson localization [23]. As the counterpart of the above results for NLσM, the BBC can also be used to deduce the Chern character (CC) term in the gauge response of $(2n+1)$D chiral fermions from the Chern-Simons (CS) term of the gauge response of $(2n+2)$D TIs, since the CS term is not gauge invariant on a manifold with boundary, leading to the boundary CC terms.

**Disordered model** We now consider that the WSM, Eq. (1), is subjected to a white-noise and random scalar potential $V(r)$, namely

$$\mathcal{H} = \mathcal{H}_{WSM}(b) + V(r).$$  

(5)

Assuming that the probability density of $V$ is Gaussian, we have $\langle V(r) \rangle = 0$ and $\langle V(r)V(r') \rangle = g^2 \delta^3(r-r')$ with $g^2$ indicating the strength of the random potential. After applying the replica scenario [33, 35] and averaging over the random potential, the disordered system may be described by a Lagrangian

$$\mathcal{L} = \psi_\alpha^\dagger G^{-1} \psi_\alpha - \frac{g^2}{2} \psi_\alpha^\dagger \psi_\alpha \psi_b^\dagger \psi_b,$$

where the spinor is decomposed into retarded and advanced space, namely $\psi = (\psi^r, \psi^a)^T$, and $G^{-1} = \omega - \mathcal{H}_{WSM} + i\eta \tau_3^a$. The Pauli matrix $\tau_3^a$ acts in this space, and $\eta$ is an infinitesimal positive number. The subscript as the replica index ranges from 1 to $N$ with repeated one being summed over. The four-operator term comes from the averaging over the random potential. Since our system is in the Altland-Zirnbauer class A, we introduce the sigma field

$$Q \in \frac{U(2N)}{U(N) \times U(N)},$$  

(6)

to decouple the four-operator term [22, 28, 29],

$$e^{-\frac{\Delta^2}{4g^2} \int \text{tr} Q^2 + \Delta \int \psi_\alpha^\dagger Q_{\alpha\beta} \psi_\beta,}$$

where the Greek subscripts indicate both replica and retarded-advanced indexes. Accordingly the partition function can be transformed to be in the form,

$$Z = \int DQ \int D\psi^\dagger D\psi \exp(- \int d^3x L_f + \frac{\Delta^2}{4g^2} \text{tr} Q^2),$$

where

$$L_f = \psi_\alpha^\dagger (G^{-1} - \Delta Q_{\alpha\beta}) \psi_\beta.$$  

$\Delta$ is determined by the consistence equation $\pi/g^2 = \ln[1 + (a_0\Delta)^{-1}]$ with $a_0$ being the short-distance cutoff. Assuming that $\omega$ is small and $\Delta > 0$, the dynamic terms of the effective NLσM are given by

$$S_{\text{eff}} = \ln \text{Det}(\mathcal{H}_{WSM} + \Delta Q).$$

It is observed that the operator in the determinant is diagonal in the chirality space, recalling that $\psi_\alpha = (\xi_\alpha, \chi_\alpha)^T$, where $\xi$ and $\chi$ are Weyl spinors with opposite chirality. This means the effective NLσM $S_{\text{eff}}$ for $\mathcal{H}_{WSM}$ is a summation of the NLσMs for the left and right-handed chiral components, namely

$$S_{\text{eff}}[Q] = S_{-\text{eff}}[Q, -b] + S_{+\text{eff}}[Q, b]$$  

(7)

with signs in front of $b$ indicating it couples oppositely to the left and right-handed ones. We below derive $S_{+\text{eff}}$ separately.

**The coupling of disordered Weyl fermions with a gauge field** If $b = 0$, $S_{+\text{eff}}[Q]$ just corresponds to chiral fermions under disorders. According to the boundary-bulk correspondence, the NLσM for a model $H$ with only one gapless point contains a WZW-term,

$$\Gamma_m[Q] = -\frac{im}{256\pi^2} \int d^3x \, e^{\mu\nu\rho\lambda} \text{tr} Q\partial_\mu Q\partial_\nu Q\partial_\rho Q\partial_\lambda Q,$$

where $m$ is the topological charge of the gapless modes in the model, raising from

$$S[\chi, \chi^\dagger] = \int d^3x \, \chi^\dagger (\omega - H + i\eta \tau_3^a - V(r)) \chi.$$
In the present case, \( m = \pm \) correspond to the left and right-handed fermions, respectively. So it is clear that the WZW-terms are cancelled if ever \( b = 0 \), leading to vanishing topological terms and corresponding to a Dirac Hamiltonian under disorders.

When \( b \) is nonzero, we can regard it as a chiral gauge field coupling oppositely to the left- and right-handed fermions. Let us consider \( H_{W,+} = \sigma \cdot k \) coupled with a \( U(1) \) gauge field \( A \) under disorders, noting that \( H_{W,-} = -\sigma \cdot k \) can be treated similarly. After application of the replica method, the Lagrangian reads

\[
\mathcal{L}_+ = \chi_0^a (\omega + i\sigma \cdot (\nabla + iA)\delta_{ab} + i\eta \sigma^a_3) \chi_b - \frac{g}{2} \chi_0^a \chi_0^b \chi_0^c,
\]

It is found that the \( U(1) \) gauge transformation can be made for each component of \( \chi \) independently, namely the action is invariant under the transformation,

\[
\chi^{a,s} \rightarrow \chi^{a,s} e^{-ia_{a,s}(x)}
\]

and

\[
A(x) \rightarrow A(x) - \nabla \alpha_{a,s}.
\]

Accordingly in the non-linear sigma version, \( Q^{(a,s)(a')^s} \sim \psi_{\bar{a}}|^{a,a'} \psi_{a,a'}^{s} \) transforms under the gauge transformation as

\[
Q \rightarrow e^{i\alpha_{a,a'} Q^{(a,s)(a')^s}} e^{-i\alpha_{a,a'}}.
\]

In the absence of gauge field, the NL\( \sigma \)M of the Weyl semimetal is

\[
S_{+,eff}[Q] = \frac{1}{\lambda_+} \int d^2x \, (\partial_\perp Q \partial_\perp J) + \Gamma_+[Q]
\]

Our strategy is to find the minimal coupling of the NL\( \sigma \)M with the \( U(1) \) gauge field, which is invariant under the above gauge transformation. To simplify our calculation but without loss of generality, we consider a specific gauge transformation as

\[
A(x) \rightarrow A(x) - \nabla \alpha_{a,s}^a,
\]

and

\[
Q \rightarrow e^{i\alpha_{a,a'} Q} e^{-i\alpha_{a,a'}},
\]

which are the opposite constants in advanced and retarded spaces, respectively. Following a seminar work on current algebra of Witten [27], we highlight our derivations below. The infinitesimal variation of \( Q \) is

\[
Q \rightarrow Q + i\alpha [\tau_3, Q].
\]

\( \tau_3 \) operates on retarded and advanced spaces, and hereafter we drop the superscript for simplicity. Since the minimal coupling for the ordinary term is readily obtained by the substitution, \( \partial_\perp \rightarrow \partial_\perp = \partial_\perp + [iA_\perp, ] \), we thus focus only on the WZW-term. The variation of \( \Gamma_+ \) under the infinitesimal local gauge transformation is

\[
\delta \Gamma_+ = -\frac{1}{32\pi^2} \int_{S_3} \text{tr}(d\alpha \tau_3 Q, dQ L dQ).
\]

Thus we have

\[
J_{WZW} = -\frac{1}{32\pi^2} Q dQ L dQ.
\]

We may expect that the coupling takes the form \( \int \text{tr}(A_j J) \). However, since the current \( J_{WZW} \) is not gauge invariant, additional terms are needed to cancel its variation under gauge transformation, which turns out to be

\[
\frac{1}{32\pi^2} \int \text{tr}(A_j dA \partial_j dQ).
\]

As a result, the total gauge invariant action is given by

\[
S_{+,eff}[Q] = \frac{1}{\lambda_+} \int d^2x \, (\partial_\perp Q \partial_\perp J) + \Gamma_+[Q]
\]

with \( 1/\lambda_+ > 1/\lambda_- + 1/\lambda_- \), recalling that \( S_{A_0} \), as one of our main results, is given by Eq.[3].

A physical meaning of the anisotropic \( \theta \)-term becomes clear if \( b \) is constant. Intuitively, a 2D slice at any \( k \in (-, b) \) along \( b \) direction may be viewed as a 2D TI with unit Chern number, which is accompanied with the bulk transverse conductivity and the edge chiral gapless modes (see the latter discussion around Eq.[10]). The anisotropic \( \theta \)-term just indicates that there are stable gapless modes in the bulk with topological protection, free from Anderson localization. It is remarkable that the transverse conductivity in the plane perpendicular to \( b \) is proportional to the magnitude of \( b \). The anisotropic form in the bulk is also consistent with the edge currents traveling perpendicular to \( b \), giving them topological protection as a WZW-term. On the other hand, it is seen from the derivation that the WZW-terms of two Weyl points are cancelled, which relies only on the fact that the two Weyl points have opposite topological charges, independent of the locations of the two Weyl points in \( k \) space. When the WZW-terms is absent after the opposite coupling, its corresponding topological protection against disorders is lost, in consistence with the fact that the chiral gapless modes in the WSM can be localized in the presence of inter-Weyl-point scattering. From this point of view, it is the new anisotropic \( \theta \)-term that embodies the remaining anisotropic topological protection from topological charges after the global cancellation, analogous to the net electric field from an electric dipole.

So far, we have established over a half of the relationship diagram Fig.[1], namely, (i) starting from the \( \theta \)-term
of NLSM and the CS term of $U(1)$ gauge response theory for $(2n + 2)$D TIs in class A, we have respectively derived the WZW and CC terms for $(2n + 1)$D chiral fermions through the boundary-bulk correspondence (the BBC arrow); and (ii) through technically treating $b(x)$ as a gauge field coupled oppositely to two chiral Fermi points with opposite topological charge, we have also derived the anisotropic $\theta$ term of NLSM for CS from the WZW term for chiral fermions (the OC for NLSM).

We now elaborate how to derive the A-CS term in Fig. (1) from a trick of opposite coupling. Treating $b$ as a gauge field coupling oppositely to the two Weyl points, the corresponding Lagrangian reads $\mathcal{L} = \bar{\psi}(i\partial^\mu - A - \gamma^5 b)\psi$, where $b$ is promoted to be a space-time vector $b_\mu$ with $b_0$ corresponding to deviation in energy, and notations about Dirac matrices are consistent with those in [36]. There are two types of vertexes, $\mathbb{B} = -\int d^4x \bar{\psi}\gamma^5 b\psi$ and $\bullet = -\int d^4x \bar{\psi}\gamma^5 b\psi$. The A-CS term is given by the three-vertex diagrams with two $\bullet$’s and one $\mathbb{B}$, shown in Fig. (2).

As the $U(1)$ gauge symmetry is fundamental, we adopt the dimensional regularization scheme for the internal momentum $l$ in the loop. Accordingly $l$ is decomposed as $l = l_\parallel + l_\perp$. In the above notations, $l_\parallel$ is still in the four-dimensional spacetime, but $l_\perp$ is in the extended infinitesimal dimensions, anti-commuting with $\gamma^\mu$ and commuting with $\gamma^5$ [36] [37]. The three diagrams contribute equally, and after a dramatic cancellation only a finite term is left similar to the derivation of the axial anomaly [9] [10] [36], leading to the anisotropic CS term,

$$S_{\text{ACS}} = -\frac{1}{4\pi^2} \int d^4x \epsilon^{\mu\nu\rho\sigma} b_\mu A_\nu \partial_\rho A_\sigma(x).$$  

Thus we establish the arrow of opposite coupling (OS) from chiral fermions to CSM.

**Dimension reduction** The last arrow process to be established is the dimension reduction from $(2n + 1)$D CSM to $(2n)$D TI in class A. First it is observed that if $b$ is a constant vector in the model [1], along the direction of $b$, we may regard the 3D system as a collection of 2D systems perpendicular to $b$ in $k$ space, and thus the 2D systems gapped over $k \in (-b, b)$ are 2D TIs with unit Chern number and the others outside this range are trivial, except at the two gapless points. Setting $b = (0, 0, b)$ constant and all fields independent of $z$ in Eqs. (2) and (9), the dimension reduction gives $S_{A_0} = -\frac{b_0}{16\pi} \int d^2x \epsilon^{jk}\text{tr}(Q\partial_j Q\partial_k Q)$ and $S_{\text{ACS}} = -\frac{b_0}{32\pi} \int d^2x \epsilon^{\rho\sigma} A_\mu \partial_\rho A_\sigma$. Since $k_2 \in (-b, b)$ correspond to a collection of unit topological insulators (Chern insulators), divided by the dimension constant $2b \times \frac{2\pi}{2\pi}$, the above equations give the well-known $\theta$-term and CS term for 2D TIs,

$$S_{\text{ACS}}^{2D} = -\frac{1}{16\pi} \int d^2x \epsilon^{jk}\text{tr}(Q\partial_j Q\partial_k Q)$$  \hspace{1cm} (10)

$$S_{\text{CS}}^{2D} = -\frac{1}{4\pi} \int d^2x \epsilon^{\rho\sigma} A_\mu \partial_\rho A_\sigma(x).$$  \hspace{1cm} (11)

Since the above treatment is applicable for any integer $n > 0$, the dimension reduction(DR) from $(2n + 1)$D CSM to $(2n)$D TI in Fig. (1) has been completed. The dimension reduction actually illustrates the correspondence between the anisotropic $\theta$-term of Eq. (4), and the anisotropic CS term of Eq. (9), recalling that the CS term for 2D TI indicates the transverse conductivity and the $\theta$-term implies the stability of this transverse transportation under disorders.

**Remark** At this stage, we would like to make several additional comments about the essence of the emergence of topological terms. In quantum field theory, this emergence of topological terms is usually related to some quantum anomaly, where two regularization schemes with different symmetries are contradictory [9] [10] [22]. From the viewpoint of energy band theory in condensed matter physics, these contradictions are usually originated from nontrivial topological configurations of the Berry fiber bundle. Formally the coupling constant of a topological term can be expressed as a topological invariant of the Berry fiber bundle, as pointed out by Volovik [11]. Actually the Berry fiber bundle is a more direct physical quantity affecting the transport properties of a physical system [38], and thus its nontrivial topological configurations may correspond to anomalous transport currents appearing in quantum anomalies. In this sense, nontrivial topological configurations in the energy bands are essential to quantum anomalies. The correspondence of topological terms in the NLSM and gauge response theory in Fig. (1) for a given material actually supports this statement, as the topologically nontrivial Berry fiber bundle in each material leads to topological terms in both NLSM and gauge theory. Finally, we have also found that our results can be generalized to the half integer $n$, where the symmetry class becomes an AII type.

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