TORSIONAL OSCILLATIONS OF A MAGNETAR WITH A TANGLED MAGNETIC FIELD

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ABSTRACT

Motivated by stability considerations and observational evidence, we argue that magnetars possess highly tangled internal magnetic fields. We propose that the quasi-periodic oscillations (QPOs) seen to accompany giant flares can be explained as torsional modes supported by a tangled magnetic field, and we present a simple model that supports this hypothesis for SGR 1900+14. Taking the strength of the tangle as a free parameter, we find that the magnetic energy in the tangle must dominate that in the dipolar component by a factor of \( \sim 14 \) to accommodate the observed 28 Hz QPO. Our simple model provides useful scaling relations for how the QPO spectrum depends on the bulk properties of the neutron star and the tangle strength. The energy density in the tangled field inferred for SGR 1900+14 renders the crust nearly dynamically irrelevant, a significant simplification for study of the QPO problem. The predicted spectrum is about three times denser than observed, which could be explained by preferential mode excitation or beamed emission. We emphasize that field tangling is needed to stabilize the magnetic field, so should not be ignored in treatment of the QPO problem.

Key words: dense matter – magnetic fields – magnetohydrodynamics (MHD) – stars: magnetars – stars: neutron – stars: oscillations

1. INTRODUCTION

Soft-gamma repeaters (SGRs) are strongly magnetized neutron stars that produce frequent, short-duration bursts (\( \lesssim 1 \) s) of \( \lesssim 10^{44} \) erg in hard X-rays and soft gamma-rays. SGRs occasionally produce giant flares that last \( \sim 100 \) s; the first giant flare to be detected occurred in SGR 0526-66 on 1979 March 5 (Barat et al. 1979; Mazets et al. 1979; Cline et al. 1980), releasing \( \sim 2 \times 10^{45} \) erg\(^1\) (Fenimore et al. 1996). The 1998 August 27th giant flare from SGR 1900+14 liberated \( \geq 4 \times 10^{43} \) erg, with a rise time of \(< 4 \) ms (Feroci et al. 1999; Hurley et al. 1999). The duration of the initial peak was \( \sim 1 \) s (Hurley et al. 1999). On 2004 December 27, SGR 1806-20 produced the largest flare yet recorded, with a total energy yield of \( \geq 4 \times 10^{46} \) erg (Hurley et al. 2005; Palmer et al. 2005; Teresawa et al. 2005). In both short bursts and in giant flares, the peak luminosity is reached in under 10 ms. Measured spin-down parameters imply surface dipole fields of \( 6 \times 10^{14} \) G for SGR 0526-66 (Tiengo et al. 2009), 7 \( \times 10^{14} \) G for SGR 1900+14 (Mereghetti et al. 2006), and \( 2 \times 10^{15} \) G for SGR 1806-20 (Nakagawa et al. 2008), establishing these objects as magnetars.

The giant flares in SGR 1806-20 (hereafter SGR 1806) and SGR 1900+14 (hereafter SGR 1900) showed rotationally phase-dependent, quasi-periodic oscillations (QPOs). QPOs in SGR 1806 were detected at 18 \( \pm 2 \) Hz, 26 \( \pm 3 \) Hz, 30 \( \pm 4 \) Hz, 93 \( \pm 2 \) Hz, 150 \( \pm 17 \) Hz, 626 \( \pm 2 \) Hz, and 1837 \( \pm 5 \) Hz (Israel et al. 2005; Strohmayer & Watts 2006; Watts & Strohmayer 2006; Hambaryan et al. 2011). QPOs in the giant flare of SGR 1900 were detected at 28 \( \pm 2 \) Hz, 53 \( \pm 5 \) Hz, 84 Hz (width unmeasured), and 155 \( \pm 6 \) Hz (Strohmayer & Watts 2005). Recently, oscillations at 57 \( \pm 5 \) Hz were identified in the short bursts of SGR 1806 (Huppenkothen et al. 2014b), and at 93 \( \pm 12 \) Hz, 127 \( \pm 10 \) Hz, and possibly 260Hz in SGR J1550-5418 (Huppenkothen et al. 2014a).\(^2\) To summarize, SGRs 1806 and 1900 have QPOs that begin at about 20 Hz, with a spacing of some tens of Hz below 160 Hz, and that are sharp with typical widths of 2–4 Hz.

The observed QPOs are generally attributed to oscillations of the star excited by an explosion of magnetic origin that creates the flare. The oscillating stellar surface should modulate the charge density in the magnetosphere, creating variations in the optical depth for resonant Compton scattering of the hard X-rays that accompany the flare (Timokhin et al. 2008; D’Angelo & Watts 2012). In this connection, the problem of finding the oscillatory modes for a strongly magnetized neutron star has received much attention, and has proven to be a formidable problem. To make the problem tractable, most theoretical treatments of the QPO problem have assumed smooth field geometries, usually dipolar or variants (e.g., Glampedakis et al. 2006; Levin 2006, 2007; Setani et al. 2008a, 2008b; Cerda-Durán et al. 2009; Colaiuda et al. 2009; Colaiuda & Kokkotas 2011; Gabler et al. 2011, 2012, 2013a, 2013b, 2014; van Hoven & Levin 2011, 2012; Passamonti & Lander 2013). Smooth field geometries support a problematic Alfvén continuum that couples to the discrete natural spectrum of the crust. As pointed out by Levin (2006), if energy is deposited in the crust at one of the natural frequencies of the crust, and this frequency lies within a portion of the core continuum, the energy is lost to the core continuum in less than 0.1 s as the entire core continuum is excited. The crust excitation is effectively damped through resonant absorption, a familiar process in MHD; see, e.g., Goedbloed & Poedts (2004). The problem has been addressed by assuming field geometries with gaps in the Alfvén continuum. Under this assumption, long-lived quasi-normal modes can exist inside the gaps or near the edges of the Alfvén continuum. van Hoven & Levin (2011) showed for a “box” neutron star that introduction of a magnetic tangle breaks the Alfvén continuum. Link & van Eysden (2015) showed that for magnetic tangling in a spherical neutron star

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\(^1\) These energy estimates assume isotropic emission.

\(^2\) El-Mezeini & Ibrahim (2010) reported evidence for oscillations in the short, recurring bursts of SGR 1806, but this analysis was shown by Huppenkothen et al. (2013) to be flawed.
the problematic Alfvén continuum disappears. They found that the star acquires discrete normal modes, and quantified the mode spacing. It is clear from these investigations that the unknown magnetic field geometry is the most important ingredient in determining the oscillation spectrum of a magnetar.

Because no model presented so far has provided good quantitative agreement with observed QPOs, we take a new direction in this Letter. We begin by arguing that stability considerations and observational evidence show that magnetars do not possess the smooth fields considered in most previous work, but rather have highly tangled fields. We propose that magnetar QPOs represent torsional normal modes that are supported by the magnetic tangle, and we present a simple model that supports this hypothesis. Keeping the energy in the magnetic tangle as a free parameter, we adjust this parameter to accommodate the 28 Hz QPO observed in SGR 1900 while maintaining consistency with QPOs observed at higher frequencies. We obtain a rough measurement of the energy density in the tangled field to be ~14 times that in the dipole field. Our model, though simple, is the first to give reasonable quantitative agreement with the data. Our model also provides useful scaling relations for the frequencies of the QPOs on bulk neutron star parameters and provides insight into the problem that might not emerge so clearly from more detailed numerical simulations. In particular, the model shows that if strong field tangling occurs, the normal-mode spectrum of a magnetar is determined principally by field tangling, and less so by crust rigidity, the dipole field, relativistic effects, and detailed stellar structure. We conclude that the effects of a tangled field cannot be neglected in the QPO problem, and we outline what we see to be interesting research directions on this issue.

2. THEORETICAL AND OBSERVATIONAL EVIDENCE FOR FIELD TANGLING

A pure dipole field is unstable, and a strong toroidal field is needed to stabilize the field (Flowers & Ruderman 1977; Braithwaite & Spruit 2006). Purely toroidal fields are also unstable (Tayler 1973; Wright 1973). There has been considerable progress recently on the identification of magnetic equilibria. Braithwaite & Nordlund (2006) found a “twisted torus” configuration, which consists of torus of flux near the magnetic equator that stabilizes the linked poloidal plus toroidal configuration. The twisted torus is topologically distinct from any poloidal field, or twisted poloidal field, in the sense that the twisted torus cannot be continuously deformed into a dipole field—the field is tangled. This topological complexity is required to establish hydromagnetic stability. Simulations by Braithwaite (2008) show that the evolution of the magnetic field from initially turbulent configurations can evolve to configurations other than the twisted torus, generally non-axisymmetric equilibria with highly tangled fields; see, e.g., Figure 12 of that paper. Braithwaite (2009) studied the relative strengths of the poloidal and toroidal components in stable, axisymmetric configurations, and found that the energy in the toroidal component typically exceeds that in the poloidal component. By what factor the toroidal energy exceeds the poloidal energy in an actual neutron star depends on initial conditions and the equation of state; Braithwaite (2009) finds examples in which this ratio is 10–20, and he argues that this ratio could plausibly be ~10^2 since a proto-magnetar should be in a highly turbulent state that winds up the natal field (Thompson & Duncan 1993; Brandenburg & Subramanian 2005). In this process, energy injected at some scale propagates down to the dissipative scale as well as up to large scales, giving a large-scale mean field with complicated structure at many scales.

The chief conclusion of these theoretical studies is that a topologically distinct tangle is needed to stabilize the dipolar component. Most theoretical work on QPOs has assumed simple field geometries that are demonstrably unstable.

Observational evidence that the internal fields of neutron stars are highly tangled can be found in the “low-field SGRs.” In these objects, the interior fields must be stronger than the inferred dipole fields in order to power observed burst activity. SGR 0418+5729 has a dipole field inferred from spin-down of ~6 × 10^{12} G (Esposito et al. 2010; Rea et al. 2010; van der Horst et al. 2010; Turolla et al. 2011). Two other examples are Swift J1822-1606, with an inferred dipole field of ~3 × 10^{13} G (Rea et al. 2012) and 3XMM J1852+0033 (Rea et al. 2014), with an inferred dipole field of less than 4 × 10^{13} G. Energetics indicate that the interior field consists of strong multipolar components, while stability considerations require these components to be tangled (Braithwaite 2008, 2009).

3. A SIMPLE MODEL OF QPOs

Much of the work cited above on the QPO problem has included realistic stellar structure, specific magnetic field geometries, and the effects of general relativity. The normal-mode frequencies are determined principally by the strength of the tangled field and bulk stellar properties, with realistic stellar structure and general relativity coming in as secondary effects. In Section 5, of the supplementary materials (Link & van Eysden 2016) we show that realistic structure has a relatively small effect on the normal-mode spectrum for an isotropic tangle. Our approach, therefore, is to proceed with a very simple model that elucidates the consequences of a tangled field. We do not expect refinements of the simple model given here to alter our chief conclusions.

We treat the magnetic field as consisting of a smooth dipolar contribution \( B_d \), plus a tangled component \( B_t \), that stabilizes the field. At location \( r \), the field is

\[
B(r) = B_d(r) + B_t(r) .
\]

We assume that \( B_t(r) \) averages to nearly zero over a dimension of the order of the stellar radius or smaller, that is, \( \langle B_t(r) \rangle \approx 0 \) where \( \langle ... \rangle \) denotes a volume average; see the supplementary materials (Link & van Eysden 2016) for details. The magnetic energy density in the tangle is \( \langle B_t^2 \rangle / 8\pi \). We define a dimensionless measure of the strength of the tangle as the ratio of the energy density in the tangle to that in the dipolar component:

\[
b_t^2 = \frac{\langle B_t^2 \rangle}{B_d^2} .
\]

We regard the magnetic tangle as approximately isotropic, and the dominant source of magnetic stress, so that \( b_t^2 \approx 1 \), as supported by the simulations and arguments of Braithwaite (2009). In this limit, the fluid core acquires an effective shear
Example Eigenfrequencies for SGR 1900 with \( b_i^2 = 14.4 \), Corresponding to \( (B_i^2)^{1/2} = 2.7 \times 10^{15} \) G

| \( n \) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| \( \nu_{\text{obs}} \) | 28 ± 2 | 43(2) | 57(4) | 70(5) | 83(7) | 95(8) | 108(10) | 120(12) | 132(14) | 144(16) | 156(18) | 155 ± 6 |

Note. All frequencies are in Hz. Numbers in parentheses indicate the factor of energy required to excite the mode to the same amplitude as the \( l = 2 \) fundamental. Numbers in boldface lie close to observed QPOs (third row), and represent plausible mode identifications. The fourth, fifth, and sixth rows show the \( n = 1, n = 2, \) and \( n = 3 \) overtones. The overtones begin at higher frequencies than do the fundamentals, and require more energy to excite to the same amplitude. We only list frequencies below 163 Hz.

Table 1

\[ \begin{array}{c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c} \hline \( n \) & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ \hline \nu_{\text{obs}} & 28(1) & 43(2) & 57(4) & 70(5) & 83(7) & 95(8) & 108(10) & 120(12) & 132(14) & 144(16) & 156(18) & 155 ± 6 \\ \hline \end{array} \]

Torsional modes have the form \( u = u_\phi(r, \theta, \phi) \) in spherical coordinates \( (r, \theta, \phi) \) with the origin at the center of the star. The solutions to Equation (5) are (see Section 3 of the supplementary materials (Link & van Eysden 2016) for further details):

\[ u_\phi(r) = \tilde{A}_l(\nu) \frac{dP_\lambda(\theta)}{d\theta}, \]

where \( \nu \equiv \omega/c_\nu \) and \( A \) is normalization. The eigenfunctions and associated eigenfrequencies are determined by the boundary condition that the traction vanishes at the stellar surface:

\[ \left[ \frac{dj_l}{dr} - \frac{j_l}{r} \right]_{r=R} = 0, \]

where \( R \) is stellar radius.

For each value of \( l \), Equation (7) has solutions \( x_\nu \equiv k_\nu R \), where \( n = 0, 1, 2, ... \), the overtone number, gives the number of nodes in \( j_l(\nu) \). The eigenfrequencies are

\[ \omega_{\nu n} = \kappa \left( \frac{B_x^2}{3\chi_p M} \right)^{1/2} x_{\nu n}, \]

where a redshift factor \( \kappa \equiv (1 - R_s/M)/(R_s)^{3/2} \) has been introduced; \( R_s \) is the Schwarzschild radius and \( M \) is the stellar mass. In terms of fiducial values

\[ \nu_{\nu n} = \frac{\omega_{\nu n}}{2\pi} = 4.3 \left( \frac{\kappa}{0.77} \right) \left( \frac{R}{10 \text{ km}} \right)^{1/2} \left( \frac{M}{1.4M_\odot} \right)^{-1/2} \times \left( \frac{x_p}{0.1} \right)^{-1/2} \left( \frac{B_x^{2.5}}{10^{15} \text{ G}} \right) \lambda_{\nu n}, \]

The energy in a torsional mode \( (l, n) \) is

\[ E_{\nu n} = \frac{1}{2} \int d^3x x_p \rho \omega^2 (u_\phi)^2. \]

For the purpose of comparing the energies of different normal modes of the same amplitude, we choose the normalization \( A \) in Equation (6) so that

\[ \tilde{u}^2 = \int d\Omega u_\phi(R)^2, \]

\[ \text{Note.} \quad \text{All frequencies are in Hz. Numbers in parentheses indicate the factor of energy required to excite the mode to the same amplitude as the } l = 2 \text{ fundamental. Numbers in boldface lie close to observed QPOs (third row), and represent plausible mode identifications. The fourth, fifth, and sixth rows show the } n = 1, n = 2, \text{ and } n = 3 \text{ overtones. The overtones begin at higher frequencies than do the fundamentals, and require more energy to excite to the same amplitude. We only list frequencies below 163 Hz.} \]
where $\tilde{n}^2$ is the square of the mode amplitude averaged over the stellar surface. We evaluate the energy in a mode $(l \approx n \approx 0)$, normalized by the $l = 2, n = 0$ mode energy, with $\tilde{n}^2$ set equal for both modes.

In the second row of Table 1, we give example frequencies for the fundamental of each $l$ for $M = 1.4M_\odot$, $R = 10$ km, and $x_p = 0.1$. Numbers in parenthesis give the normalized energy in the mode. By tuning the parameter $b_l^2 = (B_l^2)/B_0^2$ to 14.4, the spectrum of the fundamentals for each $l$ agrees with the QPOs seen in SGR 1900. A more accurate analysis given in Section 4 of the supplementary materials (Link & van Eysden 2016) changes $b_l^2$ slightly to 13.3. Taking $B_l$ equal to the inferred dipole field for SGR 1900 of $7 \times 10^{15}$ G, this value of $b_l^2$ corresponds to $(B_l^2)^{1/2} = 2.7 \times 10^{15}$ G. We also show some of the eigenfrequencies for the first three overtones. The overtones begin at higher frequencies; they also require more energy to excite to the same root-mean-square amplitude (Equation (11)) than the fundamental modes and so are energetically suppressed. To the extent that the surface amplitude of a mode determines its observability through variations of magnetospheric emission, the overtones might not be as important as the fundamental modes. The amplitude of a given mode depends on the excitation process, and we note that overtones could prove relevant in a more detailed treatment that addresses the initial-value problem of mode excitation; we discuss this issue further below.

The fundamental modes are nearly evenly spaced in $l$, with frequencies given by

$$\nu_l(\text{Hz}) \approx 0.5 l \nu_2, \quad (12)$$

where $\nu_2$ is the frequency of the $l = 2$ fundamental. The mode spacing is about half of $\nu_2$. From Equation (9), the lowest-frequency mode and the mode spacing both scale as $z(M/R)^{-1/2}$. The observed QPO spacing follows Equation (12), though only four of the 12 modes in the range $2 \leq l \leq 12$ are seen; we discuss this point further below.

The highest fundamental frequency given in Table 1 is 156 Hz for $l = 12$. The wavelength of this mode is $\approx 0.5R$. Hence, for modes in the 28–156 Hz range, the approximation of an isotropic tangle is required to hold over stellar dimensions, as supported by the simulations of Braithwaite (2009).

This model of a tangle that dominates the dipole field does not apply to SGR 1806. If we attempt to explain the lowest-frequency QPO observed (18 Hz) as an $l = 2$ fundamental mode for the inferred dipole field strength of $2 \times 10^{15}$ G, Equation (9) gives $b_l^2 \approx 0.5$, which is inconsistent with the approximation of a strong, nearly isotropic tangle. For this case, the magnetic stress of the smooth field must be included. This problem is solved in Section 4 in the supplementary materials (Link & van Eysden 2016). A match to the 18 Hz QPO implies $b_l^2 = 0.17$. The spectrum is very dense with a spacing of about two Hz. If the dipole field has been overestimated by a factor of several for this object, not implausible, then the predicted spectrum is much less dense, and more similar to SGR 1900. For example, taking $B_l$ equal to 0.62 of the reported value, a value of $b_l^2 = 1.0$ matches the 18 Hz QPO, and the predicted spectrum is less dense, with a spacing of about 7 Hz in the fundamentals.

4. EFFECTS OF THE CRUST ARE NEGLIGIBLE

So far we have ignored the crust under the assumption that magnetic stresses throughout the star dominate material stresses in the crust. Here we show that crust rigidity increases the eigenfrequencies calculated above for SGR 1900 by 3% or less. A more detailed treatment is given in the Section 4 of the supplementary materials (Link & van Eysden 2016).

To estimate the effects of the crust, we use a two-zone crust plus core model, assuming a nearly isotropic tangle ($b_l^2 \approx 1$). The core has constant density $\rho$ and constant effective shear modulus $\mu_B$. The crust has inner radius $R_c$, outer radius $R$, thickness $\Delta R$, average density $\bar{\rho}$, average material shear modulus $\bar{\mu}_c$, and average effective shear modulus $\bar{\mu}_{\text{crust}} = \bar{\rho} + \mu_B$. Chamel (2005, 2012) finds that the neutron fluid is largely entrained by ions in the inner crust; in evaluating the wave speed in the crust, we use the total mass density, so that the wave propagation speed in the crust is $v_{\text{crust}} = \sqrt{\bar{\rho}/\bar{\mu}}$. In the core, the propagation speed is $v_\text{crust} = \sqrt{\rho_0/\mu_0}$. We take the proton mass fraction to be $x_p = 0.1$ (see discussion after Equation (5)).

In the core, the solution to the mode problem is $u_{\text{core}} = \hat{k}(kr)$. The solution in the crust is $u_{\text{crust}} = \hat{a}_l(kr'') + b_{l1}k(r'')$, where $n_i$ are spherical Neumann functions, and $a$ and $b$ are constants. The wavenumbers are related through $\omega = c_i k = \epsilon_{\text{crust}} k$. The boundary conditions are continuity in value and traction at $r = R_c$, and vanishing traction at $r = Rd$:

$$u_{\text{core}}(R_c) = u_{\text{crust}}(R_c). \quad (13)$$

$$\mu_B \frac{d u_{\text{core}}}{dr} \bigg|_{r=R_c} = \mu_{\text{crust}} \frac{d u_{\text{crust}}}{dr} \bigg|_{r=R_c}, \quad (14)$$

$$\left[ \frac{d u_{\text{crust}}}{dr} \right]_r = 0. \quad (15)$$

For $\epsilon_{\text{crust}}$ and $\mu_{\text{crust}}$, we use volume-averaged quantities obtained in the following way. The shear modulus in the crust as a function of density, ignoring magnetic effects, is in cgs units (Strohmayer et al. 1991)

$$\mu_c = 0.1194 \frac{n_i(Ze)^2}{a}, \quad (16)$$

where $n_i$ is the number density of ions of charge $Ze$, and $a$ is the Wigner–Seitz cell radius given by $n_i \pi a^2/3 = 1$. For the composition of the inner crust, we use the results of Douchin & Haensel (2001), conveniently expressed analytically by Haensel & Potekhin (2004). We solve for crust structure using the Newtonian equation for hydrostatic equilibrium for a neutron star of $1.4M_\odot$. The volume-averaged density and shear modulus in the crust are $\bar{\rho} = 0.06\bar{\rho}$ and $\bar{\mu}_c = 2.5 \times 10^{29}$ erg cm$^{-3}$, respectively. The crust thickness is $\Delta R = 0.1R$. We take $(B_0^2)^{1/2} = 2.7 \times 10^{15}$ G assumed above for SGR 1900, corresponding to $\mu_B = 5.8 \times 10^{29}$ erg cm$^{-3}$.

To evaluate the effects of the crust, we solve the two-zone model for a particular eigenmode first taking $\mu_{\text{crust}} = \mu_c + \mu_B$, then $\mu_{\text{crust}} = \mu_B$, and evaluating the difference between the two eigenfrequencies. We find that the finite shear modulus of the crust increases the frequencies of the fundamental normal modes by only about 3% for $l = 2, 3, 4,$ and 2% for $l = 12$. We have confirmed that the effects are even smaller for overtones. The crust is nearly dynamically irrelevant in this
limit of a strong magnetic tangle, and neglect of the crust is a good approximation.

5. DISCUSSION AND CONCLUSIONS

Theoretical study of magnetar QPOs is seriously hampered by the fact that we do not know the detailed magnetic structure within a magnetar. Even if the magnetic structure were known or is assumed, the solution of the full problem is very difficult. Motivated by stability considerations and observational evidence, we have argued that the magnetic field should be highly tangled; significant tangling of the magnetic field is needed to stabilize the linked poloidal and toroidal fields (Braithwaite & Nordlund 2006; Braithwaite 2008). We have shown with a simple model that a highly tangled field, under the assumption that the tangle is approximately isotropic, supports normal modes with frequencies consistent with the QPOs observed in SGR 1900. In comparison with data, we have obtained a rough measurement of the ratio of the energy density in the tangle to that in the dipolar field of ~14. Given the approximations we have made, this model should not be taken as quantitatively very accurate, but what is significant is that the assumption of a strong magnetic tangle leads naturally to a normal mode spectrum with frequencies that lie in the range of observed QPOs. Our model predicts about three times as many modes below 160 Hz than are observed. That the normal mode spectrum is more dense than the observed QPO spectrum is crucial, for if the opposite were true, we would be forced to abandon the model as unviable.

The main point that we would like to emphasize is that field tangling has important effects that cannot be ignored in the QPO problem, and that the magnetic tangle is likely to be the principal factor that determines the normal-mode spectrum. Use of dipolar magnetic geometries and their variants is not adequate, and we point out that such magnetic geometries are unstable and therefore unphysical starting points for the study of normal modes of neutron stars. One simplifying feature of magnetic tangling is that for a tangle of magnetar-scale strength, the crust becomes dynamically unimportant.

We conclude by briefly describing several directions of future research that we consider to be interesting and important. One issue is the prediction that the normal-mode spectrum is denser than the observed QPO spectrum. In general, any excitation mechanism will give preferential mode excitation. The determination of which modes are excited requires the solution of the initial-value problem. Also, it is unknown at present if the QPO emission is beamed or not. If the emission has a beaming fraction less than unity, only that fraction of modes would be potentially observable. We are currently studying the mode excitation problem; we find that excitation in a localized region of the star can lead to excitation of separate groups of modes.

Further development of the model into a quantitative tool will require inclusion of realistic stellar structure and treatment of the case of comparable energy densities in the magnetic tangle and the dipolar component. For SGR 1806, the energy densities in the tangle and the dipolar component appear to be comparable within our interpretation; see Section 4 of the supplementary materials (Link & van Eysden 2016). In this case, the spectrum of normal modes could be very dense, with a mode spacing of about 2–7 Hz. If the spectrum is so dense that it begins to assume the qualities of a quasi-continuum, resonant absorption might be important as has been considered in previous work with smooth field geometries. We will study this interesting problem in future work. The magnetic field that occurs in nature may not be a nearly isotropic tangle as we have assumed. It would be interesting to explore how spatial variations in the magnetic tangle affect the spectrum of normal modes.

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