A Game Generative Network Framework with Its Application to Relationship Inference

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ABSTRACT
A game process is a system where the decisions of one agent can influence the decisions of other agents. In the real world, social influences and relationships between agents may influence the decision makings of agents with game behaviors. And in turn, this also gives us the possibility to mine some information from such agents, such as the relationships between them, by the interactions in a game process. In this paper, we propose a Game Generative Network (GGN) framework which utilizes the deviation between the real game outcome and the ideal game model to build networks for game processes, which opens a door for understanding more about agents with game behaviors by graph mining approaches. We apply GGN to the team game as a concrete application and conduct experiments on relationship inference tasks.

KEYWORDS
social mining, game theory, signed network

1 INTRODUCTION
In the real world, there are a variety of systems with cooperation and competition behaviors among agents. Such behaviors are usually modeled as games where the decisions of one agent can influence the decisions of other agents [16, 18]. Game theory is devoted to the logic of decision making in multi-agent interactions. However, most existing game-theoretic models either ignore the social influences and relationships between agents or do not consider such information comprehensively. Besides, such information of agents is difficult to obtain and hard to characterize due to the complexity of social behaviors. If we want to better understand agents with game behaviors, it is necessary for us to mine the social influences and relationships between them. Also, by incorporating such information, it is possible for us to build game models that better characterize agents’ real game behaviors.

On the other hand, the pattern of connections and relationships between objects in a system can be represented as a network [17], where the objects correspond to nodes and the relationships between objects correspond to edges. A game is a system of multiple agents with special relationships, either explicit or implicit. Thus it is possible for us to generate a network for a game process and mine the relationships between agents on the network. Besides, although there are a large number of studies on social network mining, none of them are directly applied to social agents with game behaviors. Generating networks for game processes no doubt will open a door for mining information from agents with game behaviors.

Besides, relationships between agents in game processes are usually more complex than common relationships modeled by traditional networks. In addition to positive relationships, there may be also some negative relationships between agents in a game system, which greatly influence the strategies taken by agents. In a game process, positive and negative relationships reflect in many different ways. For instance, a positive relationship may represent friendship, cooperation, trust or win-win and a negative relationship may represent hostility, competition, distrust or loss-loss [2, 8, 19]. Compared to traditional networks that reduce the relationships between objects as simple pairwise links, signed networks [23] in which the weights of edges can be positive and negative are better to represent such positive and negative relationships between agents in game processes.

Based on the above considerations, we propose a general Game Generative Network (GGN) framework, which generates signed networks for game processes based on the deviation between the real game outcome and the ideal game model. Our assumption for such games is that if there are some relationships between agents, the relationships will affect the strategies the agents take. In general, GGN serves as an intermediary between game processes and graph mining approaches. In this paper, we mainly focus on the relationship inference tasks. We give a simple illustration of the Prisoners’ Dilemma [20]. Besides, we introduce a mining approach for GGN and apply it to infer the relationships between social agents in team games.

The main contributions of this paper are summarized as follows:

• We appear to be the first to mine positive and negative relationships between social agents with game behaviors from game processes.
• We propose a Game Generative Network framework to generate networks for game processes, which serves as an intermediary between game processes and graph mining approaches and opens a door for mining information from agents with game behaviors.
• We study the problem of team game and apply our framework to infer the relationships between agents in such games.

The remainder of this paper is structured as follows. In Section 2, we briefly review some related work. In Section 3, we describe the background and the framework of GGN. In Section 4, we introduce the relationship inference task and the mining approach for GGN. In Section 5, we give the definition of team game and conduct some experiments by using GGN. In Section 6, we conclude and highlight some promising directions for future work.

2 RELATED WORK
Our work presents a brand new idea of generating signed networks for game processes and applying graph mining to social individuals with game behaviors by using the generated networks. Basically, our work is related to social network analysis [21, 24] and game theory [16, 18]. There are a lot of studies on social network discovery, such as link analysis [7, 13] and community detection [6].
Although there are some research [3] applying game-theoretic models to graph mining, no one, in turn, applies graph mining to game systems. In addition, some work [5] conducts relationship inference tasks on various social networks and some work [1, 4] studies the cooperation and competition among agents in game theory, but none can infer the hidden relationships between agents with game behaviors. Besides, due to the framework of GGN we designed, our work has a strong connection with graph mining methods on signed networks [10–12, 23, 25].

Quite recently, Serrino et al. [22] train a multi-agent reinforcement learning agent to find friend and foe in The Resistance: Avalon, a hidden role game played by several players. But their model is just designed for similar hidden role games with several players, and can not be applied to infer the relationships between a large number of social individuals with game behaviors. Thus the problem solved in their work is somewhat narrower than the problem investigated in this paper.

3 GAME GENERATIVE NETWORK

3.1 Background

Game Theory. Game theory is a tool to analyze decision making in multi-agent interactions. Based on different analyzing methods, there are different types of game models designed for different specific games. Since game models are not the most important part in this work and describing them systematically will take up a lot of space, we aim at introducing basic concept of game theory here and extending some theories in the following sections. A game denoted as \( G(V, E, u) \) usually includes three parts: the set of agents \( V \), the strategy space \( S \), and the utility function \( u : S \to \mathbb{R} \). If the number of agents in a game is \( n \), we call it \( n \)-agent game or \( n \)-player game. A (pure) Nash equilibrium in an \( n \)-agent game is a list of strategies \( s^* = (s_1^*, s_2^*, \ldots, s_n^*) \) such that

\[
\arg \max_{s_i} \{ u_i(s_1^*, s_2^*, \ldots, s_i, \ldots, s_n^*) \}
\]

where \( s_i \) is the strategy taken by agent \( p_i \). In other words, Nash equilibrium is a stable strategy list that neither agent can increase her payoff by taking another strategy, thus no agent will try to change her strategy.

Signed Network. A signed network is defined as a graph \( G(V, E, w) \), where \( V \) is the node set, \( E \) is the edge set, and \( w : E \to \mathbb{R} \) is a weight mapping function associated with each edge and the weight can be positive or negative.

3.2 A Special Case: Dynamic Game

We first study a special type of game named dynamic game. Dynamic game is a kind of game in which decisions of agents are made at various times with some of the earlier decisions being public knowledge when the later decisions are being made. Here we consider the \( n \)-agent dynamic game with agent set \( \mathcal{P} = \{p_1, p_2, \ldots, p_n \} \), where each agent has multiple opportunities to change her strategies and the payoff updates after each strategy changes.

In order to involve the interactions between agents, we build a network with node set \( \mathcal{V} = \mathcal{P} \). The original strategy list of each agent at time \( t-1 \) is \( s = (s_1, s_2, \ldots, s_n) \). At time \( t \), agent \( p_1 \) change her strategy from \( s \) to \( s' \), which converts \( s \) to \( s' = (s_1', s_2, \ldots, s_n) \). Since the strategies made by agents in dynamic games are based on some public knowledge, agents are aware of others’ strategies and can make decisions according to such knowledge. Thus relationships between agents may be reflected dynamically such interactions, which is a key to build the edges between agents. Based on this consideration, we use the utility difference before and after the conversion of each agent to build the edges of the network, the weight of each edge is formulated as follows:

\[
w(s^{(t)}_{i,j}) = u_j(s') - u_j(s),
\]

where \( u_j(s) \) is the utility of agent \( p_j \) with strategy list \( s \), and \( e^{(t)}_{i,j} \) is the directed edge from \( p_1 \) to \( p_j \) built at time \( t \). Repeating the above process for \( T \) times, we will end up with a directed signed network containing the interactive information of agents.

3.3 General Game Generative Network Framework

For the special case in the previous section, it is natural to generate networks. However, many times we cannot get the whole game process. This is to say, in a real game, we may only get the final outcome, i.e., the real strategies taken by agents. Besides, such dynamic games are not universal. So a more general game generative network framework is needed.

Most existing game-theoretic models assume that all the agents are in selfish behaviors, where each agent aims at maximizing her own utility. Such assumptions ignore social influences and relationships between agents. Besides, sometimes relationships among agents will cause a deviation in the utility of each agent. On one hand, most game models either do not consider relationships between agents or do not consider such information comprehensively. On the other hand, such information of agents is also difficult to obtain and hard to characterize by computing models.

In order to generate networks for game processes, one practical approach is to use the deviation between the real game outcome and the ideal game model. As discussed above, an ideal game model is based on the assumptions that each agent is selfish and usually does not consider the complete relationships between agents, but the real outcomes does not follow such assumptions and reflects some unobserved relationships. For instance, from an idea game model, we may find that both agent \( p_1 \) and agent \( p_2 \) can achieve a higher payoff by reducing the other’s payoff, but the real outcome shows that they did not do so (we will use Prisoners’ Dilemma as a concrete example in the following section). This may be because there is a positive relationship between them, and such a relationship will be reflected in the deviation between the real game outcome and the ideal game model.

Assume the real strategy list is \( s_r = (s'_1, s'_2, \ldots, s'_n) \). For agent \( p_1 \), we can predict the strategy \( s'_1 \) taken by \( p_1 \) from an ideal game model. By replacing \( s'_1 \) with \( s_1 \), we get a new strategy list \( s' = (s'_1, s'_2, \ldots, s'_n) \). And then we use the utility difference before and after the conversion of each agent to build the edges of the network, the weight of each edge is formulated as follows:

\[
w(e_{i,j}) = u_j(s_r) - u_j(s').
\]

There are two ways to choose \( s'_1 \); one way is to use the strategy in a pure strategy Nash equilibrium. If there are multiple pure
strategy Nash equilibria, we can also build multiple networks. But the concern is that the Nash equilibrium may be hard to find and pure strategy Nash equilibrium does not exist in some cases. The second way is to choose the strategy that maximizes the utility of \( p_1 \) as follows:

\[
s_i^* = \text{argmax} u_i(s_1', s_2', \ldots, s_i', \ldots, s_n').
\]

Based on the above framework, we can generate signed networks for most game processes, which are named as Game Generative Networks (GGNs) in this paper.

4 RELATIONSHIP INFERENCE ON GAME GENERATIVE NETWORK

Relationship inference aims to infer the relationships between agents. And specifically, in this paper, we aim at judging whether the relationship between two agents in a game is positive or negative. In some games, the relationships between agents may influence agents’ decision makings. For instance, some positive relationships (e.g., friendship, cooperation, trust, and win-win) and negative relationships (e.g., hostility, competition, distrust or loss-loss) may be involved in game processes, and the utilities of agents are affected by such relationships. But most of the time, we have no knowledge about relationships between agents or the signs of such relationships. Thus relationship inference on social agents with game behaviors is quite interesting and useful if we want to understand more about agents’ behaviors and the relationships between agents.

4.1 Prisoners’ Dilemma

Here we give a simple illustration of relationship inference based on GGN, we study the famous problem of Prisoners’ Dilemma [20]. The problem is defined as follows: two criminals (\( p_1 \) and \( p_2 \)) are imprisoned and they cannot talk to each other. The police offer a bargain. Each prisoner is given the opportunity either to keep quiet (Q) or to squeal (S). The payoff table of these two criminals is shown in Fig. 1 (left). For instance, if \( p_1 \) chooses strategy S and \( p_2 \) chooses strategy Q, \( p_1 \) will be released and \( p_2 \) will be imprisoned for 4 years.

In the ideal game model where the relationship between these two agents is ignored, Nash equilibrium of Prisoners’ Dilemma is that both criminals choose to squeal thus both of them will be imprisoned for 4 years. But if we consider the relationship between \( p_1 \) and \( p_2 \), the real strategies they take may not be consistent with the Nash equilibrium of an ideal model.

For example, suppose that the real outcome of the Prisoners’ Dilemma game happens as follows: \( p_1 \) chooses to keep quiet and \( p_2 \) chooses to squeal. Based on the framework of GGN, we can generate a network of \( p_1 \) and \( p_2 \) in Fig. 1 (upper right). For example, \( w_{(p_1, S)} = u_4(Q, S) = -4 \). From a network, we can conclude that \( p_1 \) has a positive relationship to \( p_2 \), that is \( p_1 \) is doing favor towards \( p_2 \). Another possible situation is that both \( p_1 \) and \( p_2 \) keep quiet. From the GGN illustrated in Fig. 1 (lower right), we can conclude that there is a strong relationship between \( p_1 \) and \( p_2 \) so that they believe in each other. These examples illustrate that GGNs can be useful to capture the relationships between agents.

4.2 Mining Approach

For small GGNs (e.g., \( n \leq 10 \)), we can mine the relationships between agents by observation or simple statistical analysis. For big GGNs, graph mining approaches are needed. From our definition above, a GGN is essentially a signed network. We can use such a relationship between agents. For signed networks, graph mining approaches are needed. In this paper, we design a relatively simple and intuitive method to mine for GGN and observe the power of GGN by using this method.

The most direct relationship between agents in GGNs is the first-order proximity. For each pair linked by an edge \( e_{u,v} \) with weight \( w_{u,v} \), if \( w_{u,v} > 0 \), \( e_{u,v} \) represents a positive relationship from \( u \) to \( v \); if \( w_{u,v} < 0 \), \( e_{u,v} \) represents a negative relationship from \( u \) to \( v \); otherwise, there is no first-order proximity between \( u \) and \( v \). For example, given a relationship network of agents, we have the prior knowledge about whether two agents know each other, thus the edges beyond the edges in the relationship network, including self-loops, should be filtered out.

Besides, the first-order proximity cannot represent all relationships between agents, and because the behaviors of agents are not fully reflected in a game, some edges may be missed. Based on the multiplicative transitivity [10] of signed networks, if there is an edge with weight \( w_{u,v} \) between \( u \) and \( v \), and there is an edge with weight \( w_{v,x} \) between \( v \) and \( x \), then there may be a hidden third edge with weight \( w_{u,x} \) between \( u \) and \( x \). Multiplicative transitivity is proved by the fact that two agents connected by an even number of negative edges can be considered balance [9].

Based on the multiplicative transitivity, we introduce the exponential kernel to evaluate the kth-order relationships between agents. The kth-order exponential kernel is defined as the weighted sum of matrix powers, where the weight decays with the inverse factorial:

\[
\exp_k(A) = \sum_{i=0}^{k} \frac{1}{i!} A^i.
\]

The sign of the \((i, j)\)-th entry of the exponential kernel indicates the positive or negative relationship from \( i \) to \( j \). The greater the absolute value, the stronger the relationship. Combining with the generative process of GGN, the process of relationship inference by GGN is illustrated in Fig. 2.

![Figure 1: The illustration of Prisoners’ Dilemma.](image-url)
5 GAME GENERATIVE NETWORK FOR TEAM GAME

In this section, we will show how to apply GGN to the team game as a concrete application. The goal of a team game is to divide $n$ agents into small groups. The utility of each agent in a team game is affected by the relationships between agents. Relationships between agents in team games have a broad definition. For instance, a positive or negative relationship can represent a friend or a foe relation between two agents. In this case, everyone wants to team up with her best friends and tries not to team up with those she dislikes. One the other hand, relationships in team games can evaluate whether the cooperation between two agents will bring positive (win-win) or negative (loss-loss) effects. And in this case, everyone wants to team up with a group of agents that make her obtain the biggest profits.

5.1 Team Game Definition

In this section, we design an ideal game model for the team game. Consider an undirected signed network $G(V,E,w)$ with the node (agent) set $V$, the edge set $E$ and the weight $w(e)$ associated with each edge $e$. In our settings here, team game aims to find a set of teams and each node has one and only one team, and we use $t(i)$ to denote the team which node $i$ belongs to. The strategy of agent $i$ is to quit the current team $t(i)$ and join a new team $t'(i)$.

From the above analysis we can conclude that each agent wants to team with those agents who have positive relationships with her, thus we define the gain function as follows:

$$ g_i(s) = \sum_{j=1, j\neq i}^{n} A_{ij} \delta(i,j), \quad \text{(6)} $$

where $n$ is the number of nodes, $A_{ij}$ is the $(i,j)$-th entry of the weighted adjacent matrix $A$ of the signed network $G(V,E,w)$ above, and $\delta(i,j)$ is an indicator function which is equal to 1 when $t(i) = t(j)$ and 0 otherwise.

However, when most of the relationships of agents are positive, such gain function will lead to a trivial solution where all the agents form a single team. To solve this problem, we consider that the number of members in each group should be as balanced as possible, which is consistent with the fact that a team with a large size will weaken the relationships. To this end, we define the loss function as follows:

$$ l_i(s) = c|t(i)| - \sum_{j=1, j\neq i}^{n} A_{ij} \delta(i,j) \text{ for } |t(i)| \geq 2, \quad (7) $$

as $l_i(s) = c|t(i)|$, where $|t(i)|$ is the size of the team which agent $i$ belongs to and $c$ is a parameter to balance the gain and the loss.

Finally, the utility of agent $i$ with strategy list $s$ is calculated as follows:

$$ u_i(s) = g_i(s) - l_i(s) = \sum_{j=1, j\neq i}^{n} A_{ij} \delta(i,j) - c|t(i)|. \quad (7) $$

5.2 Existence of Nash Equilibria

In this section, we will prove that the team game we designed is a finite exact potential game which always possesses pure strategy Nash equilibria [15]. Firstly, let us recall the definition of the exact potential game. A game is an exact potential game if there exists an associated potential function $\Phi(\cdot)$ defined on the strategy profiles that satisfies $\Phi(s'_i,s_{-i}) - \Phi(s_i,s_{-i}) = u_i(s'_i,s_{-i}) - u_i(s_i,s_{-i})$ for every strategy profile $s_{-i}$ of all agents except agent $i$ and every strategy $s_i$ of agent $i$. In an exact potential game that contains a finite number of strategy profiles, Nash equilibria always exist. And every better response in which each agent sequentially changes her strategy to improve her own utility, will finally converge to a Nash equilibrium [15].
we propose an algorithm for computing the Nash equilibrium of a game on \( G \) as follows: For a real network \( G \) without \( G' \) to generate GGN and calculate the \( k \)-th-order exponential kernel of the GGN to infer the relationships. Since \( G' \) is the real network, the weighted adjacency matrix of \( G' \) represents the real relationships between agents, which gives us a way to evaluate the accuracy of the relationship inference.

Here we should clarify that in the real world, the strategies taken by agents are influenced by very complicated factors, which are impossible to be modeled perfectly. Besides, not all agents will take the optimal strategy in a game. So in our experiments, we simplify the real-world situations, but this does not make a difference in our evaluation, for the purpose of this work is to propose a game generative network framework and see how powerful it is.

We extract four datasets from Slashdot and Epinions \([10, 12, 14]\) (two from each one). Slashdot is a technology news website that lets users tag other users as friends and foes and Epinions is a product review website where users build links that indicate trust or distrust about other users. In order to make the team game converge, we build the network with undirected edges. The statistics of these datasets are listed in Table 1.

5.3.2 Case Study. We use Zachary’s Karate Club network \([26]\) as a toy example in our experiments. Zachary’s Karate Club is a well-known social network of a university karate club with 34 nodes and 78 edges. We assume that node 23 has negative relationships with all her neighbors and all other relationships are positive, which leads to the outcome that no one wants to team with node 23. We stimulate the game process and generate the GGN. The structure of the real network and the generated GGN are shown in Fig. 3.

From the snapshot of the GGN in Fig. 3(b), we find that node 25, 27, 29 have first-order negative relationships to node 23, and node 24, 26 have second-order negative relationships to node 23, which is consistent with the real network.

5.3.3 Relationship Inference for Team Game. In this section, we evaluate the performance of mining on GGN in the relationship inference tasks. Since the number of positive edges and negative edges is not balanced in our datasets, we use average accuracy (the average accuracy of each class) as the evaluation metric. We set \( c = 0.2 \) for the team game process and \( k = 2 \) for the \( k \)-th-order exponential kernel in our graph mining approach. We stimulate
the team game for 5 times, the average accuracy values are shown in Fig. 4. Here we should mention that our task is brand new and there is no baseline that uses the game outcome, i.e., the strategies taken by agents, as input to infer the relationships between agents, so only random guessing can be compared in our experiments.

From the results shown in Fig. 4, we have several observations. First, the prediction based on the GGN framework far outperforms random guessing, which indicates the feasibility of our model for mining the hidden relationships among agents with game behaviors. Second, comparing with Slashdot1 and Epinions1, Slashdot2 and Epinions2 are sparser, and the average accuracies of Slashdot2 and Epinions2 are higher than those of Slashdot1 and Epinions1 respectively. This is because, in the team game, the complexity of strategies taken by users is related to the complexity of their relationships, a sparser network means that users’ strategies are easier to understand, which is compatible with our results.

We also conduct experiments on team game with different $c$ values by different $k$th-order exponential kernels. The results are shown in Fig. 5 and Fig. 6, where predictive percentage is the proportion of non-zero values in the predicted edges. From the results, we find that $k = 2$ is a good trade-off point with relatively high average accuracy and prediction percentage in most cases. Besides, according to the predictive percentages, we find that only a relatively small percentage of relationships could be predicted in a game process. This is because only a small part of relationships is reflected in a single game, and it is still very valuable to infer these relationships to understand this part of agents. In conclusion, the results demonstrate the feasibility of our GGN framework on mining relationships between agents with game behaviors.

6 DISCUSSION AND CONCLUSION

In this paper, we propose a novel game generative network framework to build networks for game systems, which is a brand new combination of game theory and graph mining. In general, GGN severs as an intermediary between game processes and graph mining approaches, which provides a new way to model and mine relationships between social agents with game behaviors. To show how to use GGN to mine the information of the hidden real networks, we introduce the team game as a concrete application and conduct experiments on it. Indeed, there are some more possible applications of the game generative network, such as mining the features of agents by using advanced technologies, e.g., graph neural network (GNN) based deep learning models. For future work, we plan to further improve the game generative network framework and apply it to more real applications.

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