THE TRANSVERSE CROSS SECTION IN EXCLUSIVE VECTOR MESON PRODUCTION

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We show that the ratio of the transverse to the longitudinal cross sections observed at HERA cannot be reproduced by models which assume that the meson can be described by an on-shell $q \bar{q}$ pair. We explain how a simple model allowing off-shell (anti)quarks can naturally lead to agreement with experiment.

1 Introduction

Exclusive production of vector mesons at HERA has attracted a lot of attention in recent years because of the theoretical suggestion that this is a good way to measure the gluon distribution, as simple arguments suggest that the cross section behaves as the square of $xg(x)$. Recently, these arguments have been formalised and extended by Collins, Frankfurt and Strikman in the context of a factorisation theorem for diffractive processes. One outcome of this theorem is that the longitudinal cross section (describing the transition of a longitudinal photon to a longitudinal vector meson) is indeed proportional to the gluon distribution squared, and that the amplitude for the process can be described, after integration over transverse motion, by a convolution in longitudinal momentum of a meson wave function, a hard scattering amplitude, and a gluon distribution. Furthermore, the theorem implies that this convolution is correct at high $Q^2$ for any meson, irrespective of its mass.

Hence one has in principle a golden process to measure the gluon distribution directly. However, an outstanding problem has been that the theorem holds only for longitudinal vector mesons. The transverse part of the cross section can only be related to quark transversity distributions, which are so far unknown. Gauge-invariance implies that the cross section is fully transverse in $Q^2 = 0$ photoproduction, but very little else was known until now about the transverse part.

The naive use of the gluon distribution in $\sigma_L$ as in $\sigma_T$ leads to the prediction the ratio $R_{L/T} = \sigma_L/\sigma_T$ of longitudinal to transverse cross sections is proportional to $Q^2$, and hence that the transverse cross section is negligible at

\[ a \text{Presented by J.R.C. at the Fourth Workshop on Quantum Chromodynamics, 1 - 6 June 1998, The American University of Paris} \]
Figure 1: (a) Ratio of the longitudinal and transverse parts of the $\rho$ cross section as functions of $Q^2$; (b) The reduction factor in the ratio $\sqrt{\sigma_L/\sigma_T}$ as a function of the quark mass.

This hope has been contradicted by the observation by the ZEUS and H1 collaborations that $R_{L/T}$ was of the order of 4 at the highest values of $Q^2$ reachable at HERA, and that furthermore it did not seem to be linear in $Q^2$ but rather presented a high-$Q^2$ plateau, as can be seen in Fig. 1.

Our goal here is to show that despite this surprise, some theoretical handle on this process can be obtained. We shall not give all the details, which can be found in Ref. 4, but rather show what the HERA observation mean, and provide a simple model which reproduces the observed ratio.

## 2 On-shell quarks

One usually assumes that the exclusive process at high $s$ proceeds from the emission of a pair of gluons from the proton, which interact with a $q\bar{q}$ pair emerging from the photon, and which transfer momentum so that this pair can be turned into a vector meson, as shown in Fig. 2. Cutting the process, one sees that the lower part of the diagram looks exactly as the gluon structure function, albeit in all generality the off-diagonal one, as the process cannot occur at $t = 0$. The lower part of the diagram mainly controls the $w^2$ and $t$ dependences of the process, whereas the upper loop controls the helicity structure of the amplitude and its $Q^2$ dependence. Hence the anomalous behaviour of $R_{L/T}$ has to come from the behaviour of the upper loop, and we must look again in detail at the treatment of the bound state of quarks.

From previous works, we already know that neglecting Fermi motion and putting quarks on-shell lead to $R_{L/T} = Q^2/m_V^2$, i.e. about a factor 10
Figure 2: The standard DIS process, compared with exclusive vector meson production

above the data at the highest $Q^2$. One can refine the formalism further by
the introduction of Fermi momentum in the loop. A simple parton model
implementation assumes that the quark and the antiquark which make the
meson are left on-shell. To examine the consequences of this assumption, we
slightly extend our previous model\cite{5} and introduce a 3-point vertex function
$\Phi(l^2)\gamma_{\mu}$ describing the transition $q\bar{q} \rightarrow V$, $l$ being the relative 4-momentum of
the quarks. As the lower part of the diagram is not of paramount importance
for the problem at hand, we treat it as a sum of two dipole form factors, which
partially kill the infrared divergence coming from the gluon propagators of
transverse impulsion $k_T$. The amplitude is then purely imaginary, and we use
cutting rules to evaluate it.

The cutting rules do not demand that we put on-shell both $q$ and $\bar{q}$ from
the meson. In fact, a kinematical argument shows that such a contribution can
only be present in the case of exclusive production of an object with positive
off-shellness, and not in the standard DIS case of Fig. 2.a. In exclusive meson
production we can follow the usual rules to calculate the discontinuity due
to the pole in the quark propagator, and the answer will then match the one
usually calculated in a wavefunction formalism. Putting $q$ and $\bar{q}$ on-shell selects
a particular value for the quark relative momentum $l$ and for $\Phi(l^2) = \Phi_0$, and
hence the exact form of the wave-function does not enter the ratio. At $t = 0,
and for large $Q^2$, we obtain the simple expressions:

$$dA_{L(disc)} \approx \frac{-8m_V\beta_{\rho}\Phi_0}{k_T^2Q^3} \frac{dk_t^2}{(2\pi)^3}$$  \hspace{1cm} (1)
\[ dA_T(disc) \approx \frac{-4m_V^2\beta_d + 2(\mu_q^2 + 2m_V^2) \log(\frac{1+\beta_d}{1-\beta_d})\Phi_0}{k_t^2Q^4} \frac{d k_t^2}{(2\pi)^3} \quad (2) \]

with \( \beta_d^2 = \frac{M_V^2 - 4m_q^2}{M_V^2} \). These amplitudes still need to be convoluted in \( k_T \) with the proton form factor, which will remove the remaining infrared pole.

We see that the ratio \( A_L/A_T \) is still linear, but depends on the quark mass chosen. In the limit \( \beta_d \to 0 \), we recover our previous results\(^4\) for the ratio, and in general the ratio \( Q/M_V \) gets multiplied by a constant, which depends on the quark mass. Unfortunately, this constant is always between 0.5 and 1 for reasonable values of the (constituent) quark mass, as shown in Fig. 1.b. The argument can be extended for nonzero \( t \), with the same conclusion. We also tried to introduce an effective (momentum-dependent) quark mass, so that we could keep it “on-shell” while performing a true convolution with the meson wavefunction. This still leads to a linear behaviour, and does not limit the rise enough to make it compatible with the data.

3 Off-shell quarks

The only possibility left is to extend the model by allowing one quark to be off-shell (the other is kept on shell by the cutting rules). This seems to be reasonable, as it is indeed the case for the usual DIS case of Fig. 2. There, the quark has always a negative off-shellness of order \( Q^2 \). In the meson case, the situation is a little more complicated due to the imbalanced kinematics, and one in general has a contribution to the amplitude both from the discontinuity discussed in the previous section, and from the principal part integral. The behaviour of the amplitudes is as follows, again at \( t = 0 \) and large \( Q^2 \):

\[
\begin{align*}
    dA_{L\,(pp)} &= \frac{\Phi(l^2)dl^2 \, dk_t^2}{(2\pi)^3 \, k_t^2 \, m_V Q^4} \times \left\{ \frac{2m_V^2(1 + \beta_+)}{P} \right\} \\
    &\quad + \log \left[ \frac{(4P - k_t^2)}{(1 + \beta_+)^2Q^4} \right] - 4 \frac{P}{k_t^2} \log \left[ \frac{4P - k_t^2}{2P} \right] \\
    dA_{T\,(pp)} &= \frac{-2 \log \left( \frac{4P - k_t^2}{4P} \right) \Phi(l^2) \, dl^2 \, dk_t^2}{k_t^4 \, Q^2 \, (2\pi)^3} \\
\end{align*}
\]

with \( \beta_+ = \frac{2\sqrt{\lambda^2 + 4m_M \lambda - 2\lambda^2 - 4m_M \lambda + M_V^2}}{M_V^2} \) and

\[
    P = 2l^2 + \frac{m_V^2 - \mu_q^2}{2} + i\epsilon \quad (4)
\]
We see that the principal parts behave like $A_L \propto \frac{1}{Q^3}$, and $A_T \propto \frac{1}{Q^2}$ at high $Q^2$, plus logarithmic corrections, whereas the imaginary parts behaved like $A_L \propto \frac{1}{Q^4}$, and $A_T \propto \frac{1}{Q^3}$. Because this effect is suppressed by the fall-off of the vertex function, it sets in only at relatively large $Q^2$.

The leading behaviour of the principal parts integrals comes from quark off-shellnesses bigger than the constituent quark mass. Whereas in the massless case an increase in the cross section can only come from extremely small off-shellnesses, we find that the dominant region in the massive case is shifted by the quark mass: the principal part integral cancels as long as one is very close to the pole, and the kinematics is such that this cancellation is not present anymore once the off-shellness is of the order of the quark mass squared.

Hence the source of the plateau observed at HERA is the interplay between the principal part and the pole singularity of the amplitude. Our model in fact predicts that asymptotically the transverse and the longitudinal cross sections first become equal, and that ultimately the process is dominated by the transverse cross section. This prediction is however driven by the details of the quark propagators, which could be modified by confinement effects.

4 A model

Once the cure of the problem has been identified, one can try to make a complete model for the process. Clearly, the vertex function is not known, and we assume a simple behaviour, similar to that of a 1s wavefunction:

$$\Phi(l) = N e^{-\frac{L^2}{2p_F^2}}$$  \hspace{1cm} (5)

where $L^2$ is the quark 3-momentum in the meson rest frame, equal to $L^2 = (\frac{4}{3}l M)^2 - l. l$, and where the Fermi momentum $p_F$ is 0.3 GeV in the $\rho$ and $\Phi$ cases, and 0.6 GeV in the $J/\psi$ case. Furthermore, we choose the following values for the quark masses: $m_u = m_d = 0.3$ GeV, $m_s = 0.45$ GeV and $m_c = 1.5$ GeV. One can then normalise this vertex to reproduce the leptonic decay widths of the various mesons. Using our previous model for the proton form factor, we have shown that the following features are reproduced up to an $s$-dependent factor:

- The helicity dependence of the process. This is a general feature of all the models which use a meson vertex proportional to $\gamma_\mu$.
- The vector-meson-mass dependence of the process, in particular the model naturally predicts the cross-over of the $J/\psi$ cross section with that of the $\rho$ at large $Q^2$. This feature is true whether one includes the off-shell contribution or not.
Figure 3: (a) Ratio of the $\rho'$ production cross section to that of $\rho$ as a function of $Q^2$; (b) Dependence of the ratio of the $J/\psi$ to the $\rho$ photoproduction cross sections as a function of $t$.

- The $Q^2$ dependence both of the longitudinal and transverse cross sections is reproduced once the off-shell quark contribution is included (See Fig. 1.a). In particular, the ratio $R_{L/T}$ is easily reproduced for the typical values of $m_q$ and $p_F$ mentioned above.
- The higher-state cross sections ($\rho'$, $\psi'$) can be reproduced by taking a $2s$ form for the vertex. The rapid rise of the ratio of the $2s$ to the $1s$ cross sections with $Q^2$ is predicted, as can be seen in Fig. 3.a.
- The model unexpectedly works in photoproduction. Despite the fact that there is no hard scale present there, the shape of the $t$ distribution comes out correctly, and the (Regge) factor multiplying the cross section is the same as in the high-$Q^2$ case. Similarly, the ratios of the $Q^2=0$ cross sections $\sigma_{J/\psi}/\sigma_{\rho}$ and $\sigma_{J/\psi}/\sigma_{\rho}$ as functions of $t$ turn out to be compatible with the data, as shown in Fig. 3.b.
- The $Q^2$ dependence of the $t$ slopes of the $\rho$ exclusive production cross sections is well reproduced.

5 Conclusion
We have shown that most of the salient features of vector meson exclusive production can be understood, provided one takes into account both the on-shell and the off-shell contributions to the production amplitude.

One property which is not included in this model is the energy dependence. We find that all data are reproduced within a factor, and that this factor does
not depend on $Q^2$, on $t$ or on the meson mass. Hence on the one hand, no BFKL enhancement at large scale is needed to agree with experiment. On the other hand, we also find that no pomeron slope is needed either, and hence we seem to contradict H1 measurements.\(^3\) It must be kept in mind however that the prediction of the $t$ dependence is the weakest point of the model so far. It depends both on the proton form factor, and on the details of the infrared behaviour of the gluon propagator\(^6\) and the uncertainties affecting these may be large enough to allow for a standard soft pomeron slope.

Nevertheless, such modifications will not affect the behaviour of the upper quark bubble, and hence the ratio $R_{L/T}$ will remain stable. Hence, we believe that it is now possible to estimate reliably the contribution of the transverse cross section and to use the total cross section measurements directly to extract the off-diagonal gluon density.

References

1. M.G. Ryskin, *Z. Phys.* C57 (1993) 89.
2. John C. Collins, Leonid Frankfurt, and Mark Strikman, *Phys. Rev.* D56 (1997) 2982.
3. H1 collaboration, S. Aid et al., preliminary results presented at the International Conference on High Energy Physics, Jerusalem, August 1997; *Z. Phys.* C75 (1997) 607-618, *Nucl. Phys.* B468 (1996) 3; *Nucl. Phys.* B472 (1996) 3; *Nucl. Phys.* B463 (1996) 3; H1 collaboration, T. Ahmed et al., *Phys. Lett.* B338 (1994) 507.ZEUS collaboration, M. Derrick et al., preliminary results presented presented at the XXVIII International Conference on High Energy Physics, Warsaw, July 25-31 1996 and at the International Conference on High Energy Physics, Jerusalem, 19-26 August 1997, e-Print Archive: hep-ex/9712020, and *Eur. Phys. J.* C2 (1998) 247-267; M. Derrick et al., *Phys. Lett.* B356 (1995) 601; *Phys. Lett.* B380 (1996) 220; *Phys. Lett.* B350 (1995) 120; *Phys. Lett.* B377 (1996) 259-272; *Z. Phys.* C69 (1995) 39, *Z. Phys.* C73 (1997) 253-268.
4. I. Royen and J.R. Cudell, Nucl. Phys. B545 (1999) 505; e-Print Archive: hep-ph/9807294.
5. J.R. Cudell and I. Royen, *Phys. Lett.* B397 (1997) 317; e-Print Archive: hep-ph 9609490.
6. J.R. Cudell, *Nucl. Phys.* B336 (1990) 1.