STATISTICAL EARTHQUAKE FOCAL MECHANISM FORECASTS

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SUMMARY

Forecasts of the focal mechanisms of future earthquakes are important for seismic hazard estimates and Coulomb stress and other models of earthquake occurrence. Here we report on a high-resolution global forecast of earthquake rate density as a function of location, magnitude, and focal mechanism. In previous publications we reported forecasts of 0.5 degree spatial resolution, covering the latitude range from \(-75\) to \(+75\) degrees, based on the Global Central Moment Tensor earthquake catalog. In the new forecasts we’ve improved the spatial resolution to 0.1 degree and the latitude range from pole to pole. Our focal mechanism estimates require distance-weighted combinations of observed focal mechanisms within 1000 km of each grid point. Simultaneously we calculate an average rotation angle between the forecasted mechanism and all the surrounding mechanisms, using the method of Kagan & Jackson proposed in 1994. This average angle reveals the level of tectonic complexity of a region and indicates the accuracy of the prediction. The procedure becomes problematical where longitude lines are not approximately parallel, and where earthquakes are so sparse that an adequate sample spans very large distances. North or south of 75 degrees, the azimuths of points 1000 km away may vary by about 35 degrees. We solved this problem by calculating focal mechanisms on a plane tangent to the earth’s surface at each forecast point, correcting for the rotation of the longitude lines at the locations of earthquakes included in the averaging. The corrections are negligible between \(-30\) and \(+30\) degrees latitude, but outside that band uncorrected rotations can be significantly off. Improved forecasts at 0.5 and 0.1 degree resolution are posted at http://eq.ess.ucla.edu/~kagan/glob_gcmt_index.html.

Key words:

Probabilistic forecasting; Earthquake interaction, forecasting, and prediction; Seismicity and tectonics; Theoretical seismology; Statistical seismology; Dynamics: seismotectonics.
1 Introduction

This paper addresses two problems: forecasting earthquake focal mechanisms and evaluating forecast skill. Properties of earthquake focal mechanisms and methods for their determination are considered by Snoke (2003) and Gasperini & Vannucci (2003).

The focal mechanism forecast method was originally developed by Kagan & Jackson (1994). Kagan & Jackson (2000, 2011) applied this method to regional and global seismicity forecasts inside the latitude band $[75^\circ S - 75^\circ N]$. In the present forecast, the weighted sum of normalized seismic moment tensors within 1000 km radius is calculated and the T- and P-axes for the predicted focal mechanism are evaluated by calculating summed tensor eigenvectors. We also calculate an average rotation angle between the forecasted mechanism and all the surrounding mechanisms. This average angle shows tectonic complexity of a region and indicates the accuracy of the prediction.

Recent interest by CSEP (Collaboratory for the Study of Earthquake Predictability) and GEM (Global Earthquake Model) has motivated some improvements, particularly to extend the previous forecast to polar and near-polar regions. For more information on CSEP see Eberhard et al. (2012), Zechar et al. (2013 and references therein). The GEM project is briefly described by Storchak et al. (2013).

The major difficulty in extending the forecast beyond the $[75^\circ S - 75^\circ N]$ latitude band is convergence of longitude lines in polar areas. To take it into account we need to account for bearing (azimuth) difference within the 1000 km circle that we used for averaging seismic moment tensors. We consider the bearing correction and apply it in averaging seismic moment tensors. In most situations a forecast point where we calculate an average focal mechanism is surrounded by earthquakes, so a bias should not be strong due to the difference effect cancellation. We show that such a correction improves the forecast in near-polar areas.
The skill of a focal mechanism forecast can be measured by studying the distribution of orientation difference between predicted and measured focal mechanisms. We investigate this difference by measuring a 3-D rotation angle (Kagan 1991) between those earthquake sources. In particular, a predicted focal mechanism is evaluated on the basis of the GCMT catalog for 1977-2007, and we measure the angle for earthquake measurements during 2008-2012. Thus, this is a pseudo-prospective test, and we plan to carry out real-time prospective test.

In addition to constructing a focal mechanism forecast and studying its performance, we need to evaluate the complexity of the forecasted moment tensor. For this purpose we measure the non-double-couple or CLVD component of the tensor. The Γ-index (Kagan & Knopoff 1985a) is the best method to accomplish this. A non-zero index indicates that earthquake focal mechanisms around the forecast point have different orientations. We compute the index and analyze its correlation with the rotation angle for all the predicted points. Thus deformation complexity displays itself in the average rotation angle and in the index.

As the final result of these investigations we construct the whole Earth forecast in two formats: medium- and high-resolution. Both new $0.5 \times 0.5^\circ$ and $0.1 \times 0.1^\circ$ forecasts are posted at [http://eq.ess.ucla.edu/~kagan/glob_gcmt_index.html](http://eq.ess.ucla.edu/~kagan/glob_gcmt_index.html).

Input catalog data are described in Section 2. Section 3 considers methods used in creating focal mechanism forecasts. In Section 4 we discuss a method for computation of bearing difference between two points on a sphere. Section 5 describes methods for measuring the focal mechanism forecast performance or skill. Section 6 is dedicated to the statistical analysis of 3-D rotation angle and the Γ-index for characterization of focal mechanism complexity. Section 7 summarizes our results and suggests techniques for improving focal mechanism forecasts.
2 Data

In Fig. 1, we display a map of earthquake centroids in the global CMT (Centroid-Moment-Tensor) catalog (Ekström et al. 2012, and its references). The earthquakes in the catalog are mostly concentrated at tectonic plate boundaries. Each earthquake is characterized by a centroid moment tensor solution.

The present catalog contains more than 38,000 earthquake entries for the period 1976/1/1 to 2012/12/31. Earthquake size is characterized by a scalar seismic moment $M$. Earthquake moment magnitude $m_W$ is related to the scalar seismic moment $M$ via (Kanamori 1977)

$$m_W = \frac{2}{3} \log_{10} M - C,$$

where seismic moment $M$ is measured in Newton-m, and $C$ is usually taken to be between 6.0 and 6.1. Below we use $C = 6.0$ (Hanks 1992). We consider the full catalog to be complete above our lower threshold magnitude $m_t = 5.8$ (Kagan 2003).

Fig. 2 displays the distribution of earthquakes in the 1977-2012 GCMT catalog by latitude. Earthquakes are concentrated more in equatorial areas compared to an equal-area distribution, and there are more events in the northern hemisphere (compare Fig. 1). There are few earthquakes (around 5%) below 50°S, but in the northern polar area the number is greater, close to 8%. As we will see later, these near polar focal mechanisms would need a special correction to calculate the forecasted mechanism.

3 Long-term focal mechanism estimates

Kagan & Jackson (1994) present long-term earthquake forecasts in several regions using the GCMT catalog. They use a spatial smoothing fixed kernel proportional to inverse epicentroid distance $1/r$, except that it was truncated at short and very long distances (see their Fig. 4).
Jackson & Kagan (1999) and Kagan & Jackson (2000; 2011) use a different power-law kernel

\[ f(r) = \frac{1}{\pi} \times \frac{1}{r^2 + r_s^2}, \]  

(2)

where \( r \) is epicentroid distance, \( r_s \) is the scale parameter, taken here \( r_s = 2.5 \) km, and we select \( r \leq 1000 \) km to make the kernel function integrable. Kagan & Jackson (2012) consider several kernel functions including the adaptive Fisher kernel specifically conformed to the spherical surface. Our procedure (Kagan & Jackson 1994, 2000, 2011) allows us to optimize the parameters by choosing those \( r_s \) values which best predict the second part of a catalog (test or validation period), using a maximum likelihood criterion, from the first part (training or learning period).

The kernel (2) is elongated along the fault-plane, which is estimated from the available focal mechanism solutions. To accomplish this we multiply the kernel by an orientation function \( D(\varphi) \) depending on the angle \( \varphi \) between the assumed fault plane of an earthquake and the direction to a map point

\[ D(\varphi) = 1 + \Theta \times \cos^2(\varphi). \]  

(3)

The parameter \( \Theta \) above controls the degree of azimuthal concentration (Kagan & Jackson 1994, their Fig. 2), we take \( \Theta = 25 \). In smoothing we weigh each earthquake according to its moment magnitude

\[ w = m/m_t, \]  

(4)

where \( m_t \) is a magnitude threshold, \( m_t = 5.8 \). Fig. 3 demonstrates such a forecast of the global long-term earthquake rate density for magnitudes of 5.8 and above, based on the GCMT catalog.

We forecast the focal mechanism of a predicted earthquake following Kostrov’s (1974) suggestion: first we predict the focal mechanism of an earthquake by summing up the past
events, with a weighting as given above:

\[ M_{pq} = \sum_{i=1}^{n} (M_{pq})_i f(r_i) D(\varphi_i) w_i, \]  

where \((M_{pq})_i\) is the normalized seismic moment tensor of the \(i\)-th earthquake in the catalog.

Then we calculate the eigenvectors of the sum \(M_{pq}\) and assign the eigenvector corresponding to the largest eigenvalue as the \(T\)-axis, and the eigenvector corresponding to the smallest eigenvalue as the \(P\)-axis of a forecasted event.

Thus, although \(M_{pq}\) in general is not a double-couple, we take the normalized double-couple (DC) component of the tensor as the forecasted mechanism. The angular density function of \(M_{pq}\) is a function of the \(n\) minimum 3-D rotation angles \(0^\circ \leq \Phi_1 \leq 120^\circ\) (Kagan 1991) necessary to transform each of the observed focal mechanisms into the predicted one \(\Phi_1\). The weighted average rotation angle \(\Phi_1\) shows the degree of tectonic complexity at this point. Our forecast tables show the plunge and azimuth of the \(T\)- and \(P\)-axes as well as the rotation angle \(\Phi_1\) (see [http://eq.ess.ucla.edu/~kagan/glob_gfmt_index.html](http://eq.ess.ucla.edu/~kagan/glob_gfmt_index.html)). An example of the forecast table is shown by Kagan & Jackson (2000, their table 1).

### 4 Bearing difference correction

The bearing (or azimuth) of a tangent vector through a point on a sphere (Richardus & Adler 1972) is the angle between that vector and the longitude line through the point. Near the equator, the bearing \(\beta_{12}\) from point 1 to point 2 is different by about \(180^\circ\) from the bearing \(\beta_{21}\) from point 2 to point 1, because the longitude lines at the two points are nearly parallel. However, near the poles the longitude lines could be far from parallel, and we need to correct for the bearing difference \(\Delta \beta\).
To compute the bearing difference at two points on the Earth surface, we calculate

\[ S_1 = \cos(\phi_2) \times \sin(\psi_2 - \psi_1), \]  

(6)

where \( \phi_i \) is latitude and \( \psi_i \) is longitude of the points,

\[ C_1 = \cos(\phi_1) \times \sin(\phi_2) - \sin(\phi_1) \times \cos(\phi_2) \times \cos(\psi_2 - \psi_1), \]  

(7)

\[ \beta_{12} = \text{mod} \left[ \arctan \left( \frac{S_1}{C_1} \right) + 360^\circ, 360^\circ \right], \]  

(8)

where \( \arctan \left( \frac{S_1}{C_1} \right) \) returns angle values in the range \([ -180^\circ ... +180^\circ ]\), and \( \text{mod} (x_1, x_2) \) is the remainder when \( x_1 \) is divided by \( x_2 \), i.e., \( x_1 \) modulo \( x_2 \). Similarly

\[ S_2 = \cos(\phi_1) \times \sin(\psi_1 - \psi_2), \]  

(9)

\[ C_2 = \cos(\phi_2) \times \sin(\phi_1) - \sin(\phi_2) \times \cos(\phi_1) \times \cos(\psi_1 - \psi_2), \]  

(10)

\[ \beta_{21} = \text{mod} \left[ \arctan \left( \frac{S_2}{C_2} \right) + 180^\circ, 360^\circ \right], \]  

(11)

and

\[ \Delta \beta = \beta_{21} - \beta_{12}. \]  

(12)

To calculate the focal mechanism parameters in the tangent plane we add \( \Delta \beta \) to azimuths of \( T \)- and \( P \)-axes, and recompute the seismic moment tensor.

Fig. 4 displays forecasted focal mechanisms, similarly as was done by Kagan & Jackson (1994, their Figs. 6a,b). To avoid the figure congestion the mechanisms are shown at \( 5^\circ \times 5^\circ \) grid, but they are calculated at \( 0.5^\circ \times 0.5^\circ \) or \( 0.1^\circ \times 0.1^\circ \) spatial resolution. We exclude from the display the areas where no earthquake was registered within 1000 km distance. These regions are shown by the greenish-gray color in Fig. 3. In these areas our forecast tables specify a “default” focal mechanism output

\[ T = 0^\circ, 180^\circ; \ P = 90^\circ, 90^\circ; \ \text{and} \ \Phi_1 = 0.0; \ \Gamma = 0.0, \]  

(13)
where the first item in $T$- and $P$-axes is a plunge and the second one is an azimuth (Aki & Richards 2002, Figs. 4.13 and 4.20).

Comparing these predicted mechanisms with their actual distribution in Fig. I demonstrates that our forecast reasonably reproduces earthquake sources properties. The forecast advantage is that the prediction accuracy is evaluated.

5 Focal mechanism forecast skill

To evaluate the skill of the focal mechanism forecast we subdivide the GCMT catalog into two parts: 1977-2007 and 2008-2012. The first part was used to calculate the expected focal mechanism at all the epicentroids of 1977-2012 earthquakes, then we estimate how the observed mechanisms of 2008-2012 period differ from the prediction. To accomplish this we measure the minimum 3-D rotation angle $\Phi_2$ between these double-couples (Kagan 1991).

Fig. 5 shows the cumulative distribution of the $\Phi_1$ angle which is the average rotation angle between the weighted focal mechanisms and mechanisms of the 1977-2007 earthquakes in a 1000 km circle surrounding this forecasted event. For about 90% of the forecasts the average angle $\Phi_1$ is less than 45°. The average angle ($<\Phi_1>$) and its standard deviation ($\sigma_{\Phi}$) are also shown.

For comparison in Fig. 5 we display two theoretical angle distributions: the rotational Cauchy distribution (Kagan 2000) and the purely random rotation of a $DC$ source (Kagan 1991). Kagan (2000) argues that in the presence of random defects in solids, rotation angles should follow the Cauchy law. The Cauchy distribution has only one parameter ($\kappa$), and it approximates the $\Phi_1$ curve reasonably well up to about 20°. If the observed $\Phi_1$ curve were close to the random rotation distribution for the $DC$ source, this would mean that there is no useful information in the forecast. The former curve is significantly different from the
latter one, demonstrating good forecast skill.

In Fig. 6 the distribution for the $\Phi_2$ angle is displayed; this is the angle between the predicted mechanism and the $DC$ mechanism of the observed events in the 2008-2012 period. Angle $\Phi_1$ in Fig. 5 is usually smaller than the observed angle $\Phi_2$, because the former angle is an average of many disorientation angles, whereas the latter angle corresponds to just one observation. The distribution difference of two angles can be seen in their averages ($<\Phi>$) and standard deviations ($\sigma_\phi$). Both are significantly larger for the $\Phi_2$ angle compared to $\Phi_1$. As in Fig. 5 for comparison we display two theoretical angle distributions, Cauchy and uniform. The $\Phi_2$ distribution for smaller angles is shifted toward zero, compared to that in Fig. 5; this effect may be caused by higher random fluctuations of the observed angle.

In Fig. 7 we display a scatterplot of two angles $\Phi_1$ and $\Phi_2$. A relatively high correlation coefficient ($\rho = 0.44$) implies that the focal mechanism forecast performs reasonably well, and its uncertainty is reasonably well evaluated by the angle $\Phi_1$. However, the distribution of either angles is not Gaussian, hence the correlation coefficient and regression parameters should be considered with a certain caution. However, at least the distribution offers some quantitative measure of angles mutual dependence. Therefore, we need to carry out additional testing with modified forecast parameters to determine appropriate measure of the forecast skill. This topic needs to be investigated in future studies.

Figs. 8 and 9 show the result of applying the bearing angle correction (Eqs. 6–12) to estimate the $\Phi_1$ angle. The corrected angle is slightly larger for latitudes approaching polar areas. As we mentioned earlier, this angle depends on earthquake spatial distribution around a forecast point, as well as point’s proximity to the pole. Additional studies, perhaps involving simulated earthquake spatial distribution, need to be carried out to understand these features of the $\Phi_1$ angle distribution. On the other hand, for the $\Phi_2$ angle the bearing cor-
rection result is opposite (see Figs. 10 and 11): after the correction the observed mechanism is in a better agreement with the predicted focal mechanism.

After solving the problem of bearing correction for polar areas, we extend our focal mechanism forecast to \([90^\circ S - 90^\circ N]\) latitude range, i.e., the whole Earth. Close to the poles we need to use the exact spherical distance formula (for example, Turner 1914; Bullen 1979, Eq. 5, p. 155) which requires about twice the computation time.

Kagan & Jackson (2012) performed such whole Earth forecast based on the PDE (Preliminary determinations of epicenters) catalog (PDE 2012). No prediction of the focal mechanism was done in this work, because the PDE catalog lacks focal mechanism estimate for many earthquakes. However, the PDE catalog contains many smaller earthquakes, and its magnitude threshold is \(m_t = 5.0\), so the forecast has a better spatial resolution.

6 Source complexity

There are several ways to measure earthquake source complexity. The simplest method, which can be applied to a single earthquake seismic moment tensor, is the CLVD \(\Gamma\)-index (Kagan & Knopoff 1985a). The \(\Gamma\)-index equals zero for a double-couple source, and its value +1 or −1 corresponds to the pure CLVD mechanism of the opposite sign.

In Fig. 12 we display the index distribution for the GCMT catalog. The distribution is concentrated around the \(\Gamma\)-value close to zero, i.e., most earthquakes have a double-couple focal mechanism or a focal mechanism that is close to double-couple. Kagan (2002, his Fig. 6) obtained a similar estimate of the \(\Gamma\)-index standard deviation (\(\sigma_{\Gamma}\)) dependence on magnitude.

The \(\Gamma\)-index standard deviation (0.39) shown in Fig. 12 is significantly smaller than that
of the uniform distribution:

$$\sigma_\Gamma^u = \frac{2}{\sqrt{3}} = 1.155.$$  \hspace{1cm} (14)

Kagan & Knopoff (1985a) showed that for the sum of randomly rotated focal mechanisms the distribution of the $\Gamma$-index is uniform at the range $[-1, 1]$. However, for tectonic events non-$DC$ mechanisms like the CLVD are likely due to various systematic and random errors in determining the mechanism (Frohlich & Davis 1999; Kagan 2003). These results suggest that routinely determined CLVD values would not reliably show the deviation of earthquake focal mechanisms from a standard $DC$ model.

Kagan (2000, Figs. 4a and 5a) simulated the effect of reported moment inversion errors in the GCMT catalog on possible values of the $\Gamma$-index and found that these inversion errors may cause significant standard errors, up to 0.2, in the $\Gamma$-index. Kagan (2000, 2003) suggests that the internal uncertainties in the GCMT data may be only a part of the total random and systematic errors. Therefore, it is quite feasible that $\sigma_\Gamma = 0.39$ shown in Fig. 12 is due to these systematic and random errors.

Several techniques have been proposed for measuring the complexity of focal mechanism distribution in a region. Kagan & Knopoff (1985b) suggested measuring irregularity of the earthquake focal mechanism distribution by calculating three scalar invariants of a moment tensor set. The simplest invariant is

$$J_3 = \langle m_{ij} n_{ij} \rangle,$$  \hspace{1cm} (15)

where $\langle \rangle$ is the averaging symbol; $m_{ij}$ and $n_{ij}$ are earthquake moment tensors. The standard index summation is assumed. For normalized tensors $-2 \leq J_3 \leq 2$. The former equality characterizes the oppositely rotated tensors and the latter equality the equally oriented tensors (see also Alberti 2010).

Apperson (1991, Eq. 4) recommends characterizing complexity for a group of earthquakes
by a “seismic consistency” index \((C_s)\) which is the ratio of the summed seismic moment tensor for \(n\) earthquakes to a sum of their scalar moments:

\[
C_s = \frac{\sum_1^n |m|}{\sum_1^n M},
\]

where \(M\) is the scalar moment, \(m\) is the tensor, and \(||\) means that the scalar moment of the tensor sum is taken. For earthquakes with identically oriented focal mechanisms \(C_s = 1\); for randomly disoriented sources \(C_s = 0\). Bailey et al. (2010) used \(C_s\) to investigate the complexity of the focal mechanism distribution in California.

In our forecast of focal mechanisms we calculate the seismic moment tensor for all events within a 1000 km circular area around each forecast point by using Eq. 5. For this averaged moment tensor, we calculate the \(\Gamma\)-index as well as the average rotation angle \(\Phi_1\) between the forecasted double-couple and all other earthquakes in the 1000 km circular area. These two variables indicate the complexity of the focal mechanism distribution around the forecast point. The advantage of these two complexity measures is that the angle \(\Phi_1\) has a clear geometrical meaning and can be roughly evaluated by inspection of focal mechanism maps. The \(\Gamma\)-index general and statistical properties are known (see above).

Fig. 13 displays a two-dimensional distribution of the \(\Gamma\)-index vs the forecasted angle \(\Phi_1\). For large \(\Gamma\)-values the angle is also large, whereas for small \(\Gamma\) the angle can be practically of any value \((0^\circ \leq \Phi_1 \leq 120^\circ)\). In Fig. 14 a two-dimensional distribution is shown for predicted 2008-2012 earthquakes. The distributions in Figs. 13 and 14 can be expected to be similar. They are obtained by the same computational procedure. A similar picture for the “seismic consistency” index \((16)\) is shown by Bailey et al. (2010, their Fig. 3). However, Fig. 15 displays a different behaviour: the \(\Phi_2\) angle points are scattered over the diagram: the observed focal mechanisms are less correlated with the CLVD component of the predicted moment tensor.
In Figs. 16 and 17 we show marginal distributions of the angle $\Phi_1$ and $\Gamma$-index for all forecasted cells. About 42% of the cells have these variables equal to zero. These cell centers do not have any earthquake centroid within 1000 km distance. To avoid future ‘surprises’, we assume 1% of all earthquakes to occur uniformly over the Globe (Jackson & Kagan 1999; Kagan & Jackson 2000). These places can be identified in Fig. 3 by the greenish-gray color. In Fig. 13 these zero values of $\Phi_1$ and $\Gamma$ are all plotted at the point $[0.0, 0.0]$ (see Eq. 13) and thus are not visible.

The CLVD source can be decomposed in various ways (Wallace 1985; Julian et al. 1998). If we arrange the moment tensor eigenvalues in their absolute values as $|\lambda_1| \geq |\lambda_2| \geq |\lambda_3|$, then in our forecast we use $|\lambda_1|, |\lambda_2|$, depending on their sign, as the $T$- or $P$-axes of a double-couple, and we assign $|\lambda_1|$ eigenvalue with an opposite sign to these axes. This corresponds to the selection of the major double-couple as a representative of the predicted focal mechanism (Wallace 1985, his Eq. 3; Julian et al. 1998, their Fig. 3b). The remaining part of the moment tensor is called a minor double-couple. Though we do not list the minor double-couple in our forecast table, it can be easily done. On the other hand, we could explore other decompositions (Wallace 1985; Julian et al. 1998) of the predicted seismic moment tensor, and, in particular, we can study what predictive skill they may have.

7 Discussion

Although in our previous forecasts (Kagan & Jackson 1994, 2000; Jackson & Kagan 1999) we optimized the smoothing kernel to obtain a better prediction of the future seismicity rate, focal mechanism forecasts were not specifically optimized. In Fig. 7 we showed how this optimization can be accomplished. As Eq. 5 indicates, several parameters can be involved in the optimization, making it a time-consuming task.
Until now in our forecasts we have predicted only the long-term focal mechanisms. However, a similar technique can be applied to the short-term forecast prediction. Kagan (2000) investigated the temporal correlations of earthquake focal mechanisms and showed that at short time intervals future focal mechanisms closely follow the mechanisms of recent earthquakes. These results can be applied to evaluate short-term forecasts of focal mechanisms.

Moreover, our forecasts of short-term seismicity rates are largely based on the results of the likelihood analysis of earthquake catalogs (Jackson & Kagan 1999) and Kagan & Jackson (2000; 2011). In such analysis only space-time patterns of earthquake occurrence have been investigated; focal mechanisms were not included. Both long- and short-term forecasts can certainly be improved by a method which would incorporate focal mechanism similarity in the likelihood calculation.

In our forecasts of focal mechanisms we need to resolve the fault-plane ambiguity, i.e., to decide which of two focal planes in the GCMT moment tensor solution is a fault-plane. This information is necessary to extend our rate forecast along the fault-plane (Eq. 3). This equation also governs the selection of earthquake focal mechanisms to infer predicted moment tensor (Eq. 5). In the GCMT catalog such a decision is based on a statistical guess (Kagan & Jackson, 1994, their Fig. 3) which is correct only in about 75% cases in subduction zones. In other tectonic regions it is likely that the fault-plane guess selection is correct only in about 50% cases.

In many cases, especially in continental areas, additional geological and seismic information exists which might help resolve the fault-plane ambiguity (Kagan et al. 2006; Wang et al. 2009). Moreover, aftershock pattern (Kagan 2002) and surface deformation measurements can also supply necessary information. Such a program would require a significant but feasible work.

Bird et al. (2010a) used a global strain rate model (GSRM) to construct a high-resolution
forecast based on moment-rates inferred from geodetic strain rate data. Focal mechanisms can also be estimated from the observed geodetic strain rate tensor. In most cases the geodetic data constrain only the horizontal components, so that some additional data or assumptions regarding the dip angle are needed to estimate the full strain rate tensor. As for seismic moment tensors, the major and minor double-couples each have two nodal planes, and additional data or assumptions are needed to determine which corresponds to the fault plane. In developing work, Bird et al. (2010b) have found that hybrid forecasts combining smoothed seismicity and strain rate forecasts performed better than either one by itself. We anticipate that the same will hold for focal mechanism forecasts.

Quasi-static Coulomb stress provides another possible tool for earthquake forecasting (Stein, 1999; King et al., 1994; Toda & Stein 2003). The primary hypothesis is that changes in Coulomb stress caused by a “source” earthquake, resolved onto the rupture plane of a future “receiver” earthquake, brings that fault closer to failure. In retrospective testing, one can know the rupture plane, but for prospective forecasting the eventual rupture plane, onto which the tensor stress should be resolved, is unknown. Focal mechanism forecasts, as described above, can provide most probable options in the form of the two nodal planes of the double-couple moment tensors. Additional data may in some cases indicate which is more likely to be the fault rupture plane.

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Figure 1: Location of shallow (depth 0–70 km) earthquakes in the Global Centroid Moment Tensor (GCMT) catalog, 1976–2012. Earthquake focal mechanisms are shown by stereographic projecting of the lower focal hemisphere (Aki & Richards 2002). The size of a symbol is proportional to earthquake magnitude. (Courtesy of Göran Ekström and the GCMT project).
Figure 2: GCMT catalog, 1977–2012.

Blue solid line is cumulative latitudinal distribution of earthquakes. Red dotted line corresponds to spherical equal-area earthquake distribution.
Figure 3: Global earthquake long-term potential based on smoothed seismicity, latitude range \([75^\circ S - 75^\circ N]\) at 0.1\(\times\)0.1° spatial resolution. Earthquakes from the GCMT catalog since 1977 are used. Earthquake occurrence is modelled by a time-independent (Poisson) process. Colors show the long-term probability of earthquake occurrence.
Figure 4: Global earthquake long-term focal mechanism forecast based on smoothed seismicity, latitude range $[90^\circ S - 90^\circ N]$. Focal mechanisms are shown on $5^\circ \times 5^\circ$ grid.
Figure 5: GCMT catalog, 2008–2012, earthquake number $n = 1069$. Blue curve is cumulative distribution of predicted rotation angle $\Phi_1$ at earthquake centroids. The red dashed line is for the Cauchy rotation with $\kappa = 0.075$. Right green solid line is for the random rotation.
Figure 6: GCMT catalog, 2008–2012, earthquake number $n = 1069$.

Blue curve is cumulative distribution of observed rotation angle $\Phi_2$. The red dashed line is for the Cauchy rotation with $\kappa = 0.075$. Right green solid line is for the random rotation.
in the Global Centroid Moment Tensor (GCMT) catalog, 1977–2012, earthquake number $n = 1069$. We calculate two regression lines approximating the interdependence of the predicted $\Phi_1$ and observed $\Phi_2$ angles, the linear and quadratic curves. The curves overlap, testifying that the linear regression fits. The coefficient of correlation between the angles is 0.44, indicating that the $\Phi_1$ estimate forecasts the uncertainty for future mechanisms reasonably well.
Figure 8: Distribution of difference of predicted rotation angles $\Phi_1$ in the original program (Kagan & Jackson, 2011) and with the angle corrected according to Eqs. 6–12.

Figure 9: Same as Fig. 8 with the vertical scale expanded.
Figure 10: Distribution of difference of observed rotation angles $\Phi_2$ in the original program (Kagan & Jackson, 2011) and the angle corrected according to Eqs. 6–12.

Figure 11: Same as Fig. 10 with the vertical scale expanded.
Figure 12: Blue solid line – cumulative distribution of $\Gamma$ index for seismic moment tensor of shallow earthquakes in the GCMT catalog, 1977-2012, $m_w \geq 5.8$. Red dotted line is the uniform distribution of $\Gamma$. 

\[ \langle \Gamma \rangle = -0.0115, \quad \sigma_\Gamma = 0.3899; \quad C_v = -0.0756, \quad n = 30950 \]
Figure 13: Scatterplot of Γ index vs forecasted angle $\Phi_1$ for all cells in 90°S – 90°N forecast.

Figure 14: Scatterplot of Γ index vs forecasted angle $\Phi_1$ for 2008-2012 earthquakes.
GCMT 2008/1/1—2012/12/31 (shallow 0–70km), rotation angles $\Phi_2$ corrected vs $\Gamma$

$\rho = -0.16, \Gamma = 31.5 + -15.6\Phi_2, \sigma = 29.2, \varepsilon_{\text{max}} = 82.2, n = 1069$

$\Gamma = 28.3 + -4.75\Phi_2 + 38.6\Phi_2^2, \sigma = 28.2, \varepsilon_{\text{max}} = 83.7, n = 1069$

Figure 15: Scatterplot of $\Gamma$ index vs observed angle $\Phi_2$ for 2008-2012 earthquakes.

GCMT catalog 1977–2012, angle distribution frequency plot

$\langle \Phi \rangle = 17.79, \sigma_{\Phi} = 20.2, C_v = 0.879, n = 259200, nx = 250$

Figure 16: Cumulative distribution of forecasted angle $\Phi_1$ for all cells in $90^\circ$S – $90^\circ$N forecast.
Figure 17: Cumulative distribution of $\Gamma$-index for all cells in $90^\circ$S – $90^\circ$N forecast.