Search for the tensor glueball

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Abstract

The tensor glueball is searched for in BESIII data on radiative $J/\psi$ decays into $\pi^0\pi^0$ and $K_sK_s$. The $\pi\pi$ invariant mass distribution exhibits an enhancement that can be described by a pole at $(2210\pm 60) - i(180\pm 60)$ MeV. We speculate if the tensor glueball could be distributed among high-mass tensor mesons.

1. Introduction

Quantum chromodynamics (QCD), the fundamental theory of strong interactions, predicts the existence of a full spectrum of glueballs, of composite particles containing gluons but no valence quarks. Their existence is a direct consequence of the nonabelian nature of QCD and of confinement. The properties of glueballs have been studied in many models since their prediction in the 1970s \cite{1,2} but experimentally, no generally accepted view had emerged. Recent reviews of glueballs and of light-quark mesons can be found elsewhere \cite{3,4}. The scalar glueball is expected in the 1500 - 2000 MeV mass range \cite{5,6}. Based on lattice calculations, we have to expect the tensor glueball mass 600 to 800 MeV above the scalar glueball. However, a small mass gap is also possible. The authors of Ref. \cite{5} use QCD sum rules and predict 1780 MeV for the scalar, and 1860 MeV for the tensor glueball.

The radiative decay branching ratios for producing glueballs were predicted by lattice gauge calculations \cite{24,25}:

\begin{equation}
\frac{\Gamma_{J/\psi\to\gamma/G_{0^+}}}{\Gamma_{\text{tot}}} = (3.8 \pm 0.9) \times 10^{-3},
\end{equation}

\begin{equation}
\frac{\Gamma_{J/\psi\to\gamma/G_{2^+}}}{\Gamma_{\text{tot}}} = (11 \pm 2) \times 10^{-3}.
\end{equation}

These are large numbers. The yield of $f_0(1270)$ in radiative $J/\psi$ decays is $(1.64 \pm 0.12) \times 10^{-3}$, about six times weaker than the predicted rate for the tensor glueball!

Little is known about the glueball width. Arguments based on the $1/N_c$ expansion (see, e.g., Ref. \cite{19}) suggest that glueballs might be narrow, 100 MeV or less. Narison \cite{20} gave a $2\pi$ partial decay width of the scalar glueball of $(119 \pm 36)$ MeV, not incompatible with a large total width. Minkowski and Ochs assume a width exceeding 1 GeV \cite{21}. The authors of Refs. \cite{22,23} reproduce the decay rates of $f_0(1710)$ assuming that this is the scalar glueball and predict the tensor glueball to be very wide.

Recently we have presented a coupled-channel analysis of the $S$-wave amplitude from BESIII data on radiative $J/\psi$ decays. The data on radiative $J/\psi$ into $\pi^0\pi^0$ and $K_sK_s$ were published including a bin-wise partial-wave decomposition into $S$-wave and $D$-wave \cite{27,28}, the data on decays into for $\eta\eta$ and $\phi\omega$ were published only in an energy-dependent amplitude analysis \cite{29,30}. A large number of further data were included in the coupled-channel analysis, references to the additional data can be found in Ref. \cite{26}. Ten scalar isoscalar resonances were required to fit the data. Five of them were interpreted as mainly singlet, five as mainly octet resonances in SU(3). The yield of resonances showed a striking peak at $(1865 \pm 25_{-15}^{+10}) - i(185 \pm 25_{-10}^{+15})$ MeV called $G_0(1865)$. In a subsequent paper we studied the decays of the scalar mesons into pairs of pseudoscalar mesons. We found that the decays can be understood only when these mesons contain an additional flavor singlet fraction beyond the one expected for any mixing angle.

This peak at 1865 MeV showed properties expected from a scalar glueball:

- $G_0(1865)$ is produced abundantly in radiative $J/\psi$ decays above a very low background. Its mass is 1\sigma compatible with the mass calculated in unquenched lattice QCD \cite{16} and the yield is 1.6\sigma compatible with the yield calculated in lattice QCD.
- The decay analysis of the scalar isoscalar mesons shows that the assignment of mesons to mainly-octet and mainly-singlet states is correct. Even the production of mainly-octet scalar mesons - which should be forbidden in radiative $J/\psi$ decays - peaks at 1865 MeV.
- The decay analysis requires a small glueball content in the flavor wave function of several scalar resonances. The glueball content as a function of the mass shows a peak compatible with the peak in the yield of scalar isoscalar mesons. The sum of the fractional glueball contributions is compatible with one \cite{31}.
- In the reaction $B_s \to J/\psi + K^+K^-$, a primary $ss$ couples to mesons having a strong coupling to
$K^+K^-$ [32]. Two peaks in the $K^+K^-$ mass spectrum are seen due to $\phi(1020)$ and $f_2(1525)$ (see Fig. 7 in Ref. [32]. Fig. 2b below shows a fit to the tensor wave) but there is no sign of higher-mass scalar mesons. In particular $f_0(1710)$ with its prominent $KK$ decay mode is not seen in $ss \to f_0(1710)\to KK$. It is, however, produced very strongly in the process gluon-gluon $\to f_0(1710)\to KK$:Obviously, $f_0(1710)$ is produced by two initial-state gluons but not by an $ss$ pair in the initial state. $f_0(1710)$ must have a sizable glueball fraction!

For these reasons, we are convinced that the scalar glueball is distributed among the agglomeration of scalar isoscalar mesons in the range from 1500 to 2300 MeV. Based on lattice calculations, we have to expect the tensor glueball above 2500 MeV even though smaller masses are possible as well [13].

In this Letter we search for the tensor glueball expected to be produced in radiative $J/\psi$ decays into $\pi^0\pi^0$ and $K_sK_s$. In Section 2 we compare the intensities of the $\pi\pi$ and $KK$ invariant masses in the scalar and tensor wave. The tensor wave reveals a wide high-mass resonance. Subsequently, in Section 3 we discuss if the high-mass enhancement is split into several states and if these contain a small fraction of the tensor glueball. The results are discussed and summarized in Section 4.

2. A high-mass tensor resonance from radiative $J/\psi$ decays

The $\pi^0\pi^0$ or $K_sK_s$ systems produced in radiative $J/\psi$ decays are limited to even angular momenta due to Bose symmetry. Practically, only $S$ and $D$-waves are relevant. These two partial waves can be written in the multipole basis $\perp \perp 0$. The scalar intensity originates from the electric dipole transition $E0$. Three electromagnetic amplitudes, $E1, M2$, and $E3$, lead to the production of tensor mesons where the $E1$ amplitude is the most significant one. These three amplitudes and relative phases are discussed below.

The $E0$ and $E1$ squared amplitudes lead to strikingly different mass distributions (see Fig. 1). The distributions were derived in Refs. [27, 29] by exploiting the statistical precision provided by $(1.311 \pm 0.011) \times 10^6 J/\psi$ decays collected by the BESIII collaboration. The $\pi\pi$ mass distribution in the scalar partial wave (Fig. 1a) is characterized by a mountainous landscape starting with a steady rise growing up to 1450 MeV and a rapid fall-off. After a minimum, the intensity increases due to the $f_0(1710)/f_0(1770)$ complex. After a further deep minimum, a wide and asymmetric structure due to $f_0(2020)/f_0(2100)$ follows. At the highest mass, at about 2300 MeV, a small dip-peak structure is seen. In the $KK$ distribution (Fig. 1b), the intensity rises slightly up to 1450 MeV, followed by a very significant interference pattern and then rises steeply to an asymmetric peak at about 1700 MeV that dominates the mass distribution. After a fast drop on its right side, a second peak at about 2100 MeV appears with a high-mass shoulder.

The squared amplitudes that describe production of tensor mesons do not show such rich structures. Below 1 GeV, the $\pi\pi - D$-wave (Fig. 1b) shows only a tail of the $f_2(1270)$ and no entries in $KK$ mass distribution (Fig. 1c). Above 1 GeV the $\pi\pi\perp$ intensity exhibits only one strong peak due to $f_2(1270)$ production and a wide enhancement that reaches a maximum at about 2200 MeV. The $KK$ intensity exhibits only one peak due to $f_2(1525)$. Above, only little intensity is seen.

In a first fit, we describe the high-mass region by one additional resonance. Neither the mass distribution nor the phase difference are well reproduced. The $\chi^2/N_{data} = 1088/765$ for the mass distributions and 2584/677 for the phase differences. The mass distribution is reasonably described, the phase differences qualitatively only. However, apparent discrepancies are often enforced by adjacent high-statistics points, and some structures are limited to a narrow mass window.

Alternatively, we allow for one further resonance $f_2(1640)$, the fit improves only marginally. The fit with or without $f_2(1640)$ gives a narrower or wider high-mass tensor resonance. Since we do not know if $f_2(1640)$ participates in the reaction, we increase the errors correspondingly:

$$M = (2210 \pm 60)\text{ MeV}; \quad \Gamma = (360 \pm 120) \text{ MeV}. \quad (3)$$

The error does not contain the possibility that the production amplitude of tensor mesons may be reduced dy-
namically with decreasing photon energy. Only the phase space is taken into account. Tentatively, we call these resonances \(X_2(2210)\) (not \(f_2(2210)\) since it might be a cluster of resonances).

An energy-dependent partial-wave analysis of the same data on was reported by the JPAC Collaboration\[14\]. Four scalar and three tensor mesons were identified. The tensor amplitude was described by \(f_2(1270)\), \(f_2^\prime(1525)\), and a further state at about \(f_2(1590)\) and a width of 700 MeV. A possible tensor glueball was not discussed. The difference in mass might be due a different choice of the ambiguous solutions of the energy-independent partial wave analysis. With our choice, the \(E1^\pm\) distribution shows a clear peak at about 2200 MeV.

In our fit, \(X_2(2210)\) was parameterized by a three-channel relativistic Breit-Wigner amplitude with \(\pi\pi\), \(K\bar{K}\), and \(\rho\rho\) as decay channels. The ratio of the frequencies of \(X_{2+}(2210)\) decays into \(K\bar{K}\) and \(\pi\pi\) is

\[
BR_{K\bar{K}/\pi\pi} = 0.23 \pm 0.05.
\]

In Table 1, the properties of \(f_2(1270)\) and \(f_2(1525)\) are compared with values given in the Review of Particle Physics (RPP)\[57\] and with other determinations using radiative \(J/\psi\) decay.

The \(f_2(1270)\) mass found here is incompatible with the RPP value. We note that in an analysis of BESIII data on \(J/\psi \to \gamma\pi\pi\), the \(f_2(1270)\) mass was determined to \((1262^{+1}_{-2} \pm 8)\) MeV\[38\], and from CLEO data on this reaction, \((1259\pm4\pm4)\) MeV was deduced\[36\]. JPAC finds masses between 1262 and 1282 MeV\[31\].

The \(K\bar{K}/\pi\pi\) ratio for the \(f_2(1270)\) could be determined with a large uncertainty from the faint peak at about 1270 MeV in the \(K\bar{K}\) mass distribution. Here, we fix the ratio to the RPP value. Also, the small \(\pi\pi\) decay mode of \(f_2^\prime(1525)\) is fixed to the RPP value. Our radiative yields of \(f_2(1270)\) and \(f_2^\prime(1525)\) yields are fully compatible with RPP values.

In the reaction \(J/\psi \to \gamma\) plus a tensor meson, the production process couples to the mesonic flavor-singlet component only. The ratio \(R\) of \(f_2^\prime(1525)/f_2(1270)\) production is related to the mixing angle via

\[
\tan^2\theta_{\text{tens}} = \frac{1}{\lambda} \cdot R \cdot \frac{q_{f_2}}{q_{f_2^\prime}}
\]

from which we find a tensor mixing angle \(\theta_{\text{tens}} = (35\pm2)\)°. The mixing angle identifies the tensor meson nonet as ideally mixed but is inconsistent with 29.8(28.0)° derived from the quadratic (linear) GMD formula.

We now need to ask: Is it plausible that just one tensor resonance above 1700 MeV is produced in radiative \(J/\psi\) decays? There is at most marginal evidence for \(f_2(1640)\), and no evidence at all for \(f_2(1910/1950)\). Both states are seen in several experiments\[57\]. But these states are at most very weakly produced in radiative \(J/\psi\) decays. Above these two states, a tensor meson at 2210 MeV suddenly appears. Could \(X_2(2210)\) contain a fraction of the tensor glueball? And why is the fit to the phases bad with a single-resonance fit? Are several tensor resonances hidden in \(X_2(2210)\)?

### 3. Could \(X_2(2210)\) be the tensor glueball?

#### 3.1. The tensor wave in \(B^0(\psi) \to J/\psi + f_2\)

Figure 2 shows a comparison of the contribution of the \(E1\) amplitude in radiative \(J/\psi\) decays and a fit to LHCb data on \(B^0\) decays. In Fig. 2a,b the data from Fig. 1b are reproduced, in Fig. 2c the data from Fig. 1c. Superimposed is a fit to the \(\pi\pi\) and \(K\bar{K}\) D-wave contributions to \(B^0 \to J/\psi + (\pi\pi)\[35\](a), B^0 \to J/\psi + (\pi\pi)\[35\](b), and \(B^0 \to J/\psi + (KK)\[32\](c). In \(B^0 \to J/\psi + \text{hadrons}\), the \(b\) quark converts into a \(c\) quark radiating off a \(W^-\) boson. The \(W^-\) boson decays into a \(c\) plus a \(d\) quark. The \(c\) is seen as \(J/\psi\), the \(dd\) forms a light-quark meson. In \(B^0 \to J/\psi + \text{hadrons}\), the \(W^-\) boson decays into a \(c\) plus a \(s\) quark, and an \(ss\) pair creates the final state. The phase space of the BESIII data on radiative \(J/\psi\) extends up to 3.1 GeV, the phase space in the reaction \(B^0 \to J/\psi + \text{hadrons}\) is limited to 2180 MeV, and to 2270 MeV in the case of \(B_s \to J/\psi + \text{hadrons}\).

The main objective of the LHCb collaboration was the study of CP violation through the interference of \(B^0(\psi)\) and its decay amplitudes. But the resonant structures in the \(\pi\pi\) and \(K\bar{K}\) system were studied as well. The angular distributions were presented in the form of spherical harmonic moments. We have included these spherical harmonic moments into the data set described above for a joint coupled-channel analysis. The main results are presented elsewhere\[11\]. The fit returns the \(\pi\pi\) and \(K\bar{K}\) S, P and D-wave amplitudes recoiling against the \(J/\psi\).

| \(f_2(1270)\) | \(f_2^\prime(1525)\) | \(X_2(2210)\) |
|---|---|---|
| \(M\) (MeV) | 1257±6.0 | 1518±4.3 | 2210±60 |
| \(\Gamma\) (MeV) | 1532±8.7 | 1517±4.25 | 2212±8.8 |
| \(R_{K\bar{K}/\pi\pi}\) | 0.054±0.006 | 0.0094±0.00018 |
| \(Y(\times10^3)\) | 1.69±0.07 | 0.61±0.06 | 0.35±0.10 |

| \(f_2(1270)\) | \(f_2^\prime(1525)\) | \(X_2(2210)\) |
|---|---|---|
| \(M\) (MeV) | 1257±6.0 | 1518±4.3 | 2210±60 |
| \(\Gamma\) (MeV) | 1532±8.7 | 1517±4.25 | 2212±8.8 |
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| \(Y(\times10^3)\) | 1.69±0.07 | 0.61±0.06 | 0.35±0.10 |

3. Could \(X_2(2210)\) be the tensor glueball?
The $\pi\pi$ $D$-wave from LHCb in Fig. 2a, shows a peak due to $f_2(1270)$ production, the $KK$ $D$-wave Fig. 2b: a peak due to $f_2'(1525)$. The intensities were multiplied by factors given in the subfigures. The factors are chosen to match the LHCb data [27, 29]. The solid (red) line represents the $\pi\pi$ mass distribution in the tensor wave derived from data of the LHCb collaboration [32, 39, 40] on $B^0 \rightarrow J/\psi + (\pi\pi)$ (a), $B^0_s \rightarrow J/\psi + (\pi\pi)$ (b), and on $B^0 \rightarrow J/\psi + (KK)$. The intensities from the $B^0_s$ decays are chosen to match the $f_2(1270)$ and $f_2'(1525)$ intensities from radiative $J/\psi$ decays.

The $\pi\pi$ $D$-wave from LHCb in Fig. 2a, shows a peak due to $f_2(1270)$ production, the $KK$ $D$-wave Fig. 2b: a peak due to $f_2'(1525)$. The intensities were multiplied by factors given in the subfigures. The factors are chosen to match the $f_2(1270)$ or $f_2'(1525)$ peak heights of the results from radiative $J/\psi$ decays. Masses and widths are well compatible. The solid curve in Fig. 2b exhibits a double-peak structure; both, $f_2(1270)$ and $f_2'(1525)$, contribute to this reaction.

First, we discuss the overall intensities of the LHCb data. The strongest reaction, $B^0_s \rightarrow J/\psi + f_2(1525)$, is about 12 times stronger than $B^0 \rightarrow J/\psi + f_2(1270)$, $f_2'(1270) \rightarrow KK$, is about 12 times stronger than $B^0 \rightarrow J/\psi + f_2(1270)$, $f_2'(1270) \rightarrow KK$. In the former reaction, the intermediate $W$ boson converts into a $c$ and an $s$ quark, in the latter reaction into a $c$ and a $d$ quark. From the ratio of the CKM matrix elements $|V_{cd}|/|V_{cs}|$, we expect a large reduction.

There is very weak intensity only in the LHCb data above the $f_2(1270)$ or $f_2'(1525)$. If the peak at 2210 MeV in Fig. 2c were due to a regular $q\bar{q}$ state, we would expect an onset of the tensor intensity in the LHCb data, in particular in Fig. 2a. This is not the case. As in the case of scalar mesons, the high-mass enhancement is not produced by $q\bar{q}$ in the initial state but by gluon-gluon interactions. The enhancement at 2210 MeV seems to contain a significant fraction of the tensor glueball in its wave function. Due to the limited phase space, this argument is, however, suggestive only and not really enforcing.

3.2. $\phi\phi$ decays of tensor mesons

The scalar glueball was distributed among several scalar isoscalar resonances. Hence we expect that also the tensor glueball might not be concentrated in a single resonance. Etkin et al. [42] at BNL observed a strikingly high intensity above 2000 MeV in the reaction $\pi^- p \rightarrow \phi\phi n$. The intensity was fully ascribed to the $J^{PC} = 2^{++}$ wave and was described by three tensor resonances with masses and widths of about $(M, \Gamma) = (2010, 200)$ MeV, $(2300, 150)$ MeV, and $(2340, 320)$ MeV. The unusual production characteristics were interpreted in Ref. [42] as evidence that these states are produced by $1 - 3$ glueballs. The BESIII collaboration studied the process $J/\psi \rightarrow \gamma\phi\phi$ and found that the tensor wave of this reaction can be described well with these three tensor mesons $X^{2+}$. The mean mass of the three $\phi\phi$ resonances is 2215 MeV. This mass agrees perfectly well with the mass of $X^{2+}(2210)$. The BNL experiment may thus have revealed the tensor glueball and its splitting into several tensor mesons 40 years ago [41].

3.3. Fits with $X^{2+}(2210)$ as cluster of resonances

We fitted high-mass enhancement at 2210 MeV with these three resonances. Figure 8 shows for the reaction $J/\psi \rightarrow \gamma\pi^0\pi^0$ and $K_s K_s$.

1. the magnitudes of the three amplitudes $E1; M2$, and $E3$, and
2. the phase difference between the $E0$ and $E1$, $M2$ and $E1$, $E3$ and $E1$ amplitudes.

The amplitudes - moduli and phases - are shown here in a mass region limited to $0.75 - 2.75$ GeV. The amplitudes were determined in slices of the invariant mass in a “mass-independent fit”. It was shown that at each invariant mass, two solutions exist. Assuming continuity of the amplitude, the full mass range could be described by four different solutions. One of the solutions gave the best energy-dependent fit for the scalar wave [26]. This solution also defines unambiguously the tensor waves.

The tensor intensities $E1$, $M2$, and $E3$ and the phase differences of this solution are shown as histograms. The solid curve represents our fit to the $S$ and $D$-waves. The $S$-wave was refit; the changes of $S$-wave parameters compared to Ref. [26] are marginal only. The $\chi^2$ of the overall fit is now $\chi^2/N_{\text{data}} = 890/765$ for the mass distributions
Figure 3: D-wave intensities and phases for radiative $J/\psi$ decays into $\pi^0\pi^0$ (top subfigures) and $K_sK_s$ (bottom subfigures) from Ref. [27, 28]. The subfigures show the $E1$ (a), $M2$ (b) and $E3$ (c) squared amplitudes and the phase differences between the $E0$ and $E1$ (d) amplitudes, the $M2$ and $E1$ (e) amplitudes, and the $E3$ and $E1$ (f) amplitudes as functions of the meson-meson invariant mass. The phase of the $E0$ amplitude is set to zero. The curve represents our best fit.
and 1716/677 for the phase differences. Adding further high-mass tensor resonances improves the fit only slightly.

A few regions need to be discussed. The M2 yields for \( \pi^0 \pi^0 \) and \( K_s K_s \) yields above the \( f_2^0(1525) \) are underestimated by our fit. Larger yields are, however, incompatible with the phase motions. The M2 – E1 and E3 – E1 phases have data points with very small errors and large deviations from their neighbors. The most important phase difference \( E0 – E1 \) is described by \( \chi^2/N_{\text{data}} = 298/245 \).

4. Discussion and Summary

The total observed yield in \( \pi \pi \) and \( \bar{K}K \) is

\[
M=2.5 \text{ GeV} \sum_{M=1.9 \text{ GeV}} Y_{J/\psi \rightarrow \gamma f_2, f_2 \rightarrow \pi \pi, \bar{K}K} = (0.35 \pm 0.15) \times 10^{-3}.
\]

Data on \( \pi \pi \) elastic D-wave scattering in this mass range do not exist. The missing intensity cannot be determined from the data included in our fits. An estimate can be obtained from tensor states reported to be seen in radiative \( J/\psi \) decays. The reactions and their contributions to the high-mass region are listed in Table 2. Summation yields

\[
M=2.5 \text{ GeV} \sum_{M=1.9 \text{ GeV}} Y_{J/\psi \rightarrow \gamma f_2} = (3.1 \pm 0.6) \times 10^{-3}.
\]

This is a substantial yield even though still smaller than the observed yield of the scalar glueball. We note that the 4\( \pi \) tensor contribution is rather small when compared to the \( K^*(892) \bar{K}^*(892) \) and \( \phi \phi \) contributions. The 6\( \pi \) tensor contribution is completely unknown. Hence there may still be missing intensity. Here we emphasize the importance of further studies of these channels with the much larger statistics taken by the BESIII Collaboration.

Summarizing, we have presented a coupled-channel analysis of BESIII data on \( J/\psi \) decays into \( \pi^0 \pi^0 \) and \( K_s K_s \). The data are dominated by S-wave and D-wave contributions. This fit is important since it does not only provide the tensor wave but also shows that S and D-waves are both consistently described.

In the tensor wave we find an enhancement at \( M = (2210 \pm 60) \text{ MeV}, \Gamma = 360 \pm 120 \text{ MeV} \) and a yield (in \( \pi \pi \) and \( K\bar{K} \)) of \( (0.35 \pm 0.10) \times 10^{-3} \) called \( X_2(2210) \). There are arguments speaking in favor and against a glueball interpretation.

Prop.: \( X_2(2210) \) is produced as a high-mass tensor resonance in radiative \( J/\psi \) decays, a process in which glueballs are supposed to be produced. It is the only tensor meson seen clearly above \( f_2^0(1525) \). This suggests that \( X_2(2210) \) could contain a contribution from the tensor glueball. The improvement of the fit with three resonances instead of one only points to the possibility that \( X_2(2210) \) is composed of several resonances. In particular, \( X_2(2210) \) is consistent with early results in \( \phi \phi \) production in \( \pi N \) scattering and supports the claim that this could be the tensor glueball.

Table 2: Yield of tensor mesons above 1900 MeV in radiative \( J/\psi \) decays in units of \( 10^{-5} \). The 4\( \pi \) yield is calculated from the \( J/\psi \rightarrow \pi^+ \pi^- \pi^+ \pi^- \) yield by multiplication with the factor 9/4. The sum of all measured yields is \( (3.1 \pm 0.6) \times 10^{-3} \).

| \( \pi \pi \) | \( f_2(2210) \) | This work |
| \( K\bar{K} \) | \( f_2(2210) \) | \( 6 \pm 3 \) |
| \( \eta \eta \) | \( f_2(2340) \) | \( 7.6^{+3}_{-2.1} \) |
| \( \eta' \eta' \) | \( f_2(2340) \) | \( 8.7^{+0.9}_{-1.3} \) |
| \( 4\pi \) | \( f_2(1950) \) | \( 1244 \pm 43 \) |
| \( \omega \omega \) | \( f_2(1910) \) | \( 298 \pm 19 \) |
| \( K^* K^* \) | \( f_2(2300), f_2(2340) \) | \( 57^{+24}_{-18}, 47^{+8}_{-5} \) |
| \( \phi \phi \) | \( f_2(2300), f_2(2340) \) | \( 9^{+4}_{-3}, 298 \pm 19 \) |

There is no evidence for a similar enhancement in the reactions \( B^0 \rightarrow J/\psi^+ (\pi\pi) \) or \( B^0 \rightarrow J/\psi^+ (\pi\pi) \) studied by the LHCb collaboration where resonances in the \( (\pi\pi) \) are formed by a \( dd \) pair in the initial state. The absence of a structure can serve as additional evidence for the glueball interpretation.

Contra.: In calculation on a lattice, mass and yield of the scalar glueball are predicted which agree well with the result of a coupled-channel analysis of the same data as discussed here. These calculations predict a tensor glueball mass considerably above \( X_2(2210) \). Also the yield of the tensor glueball should be substantially larger than the \( X_2(2210) \) yield. The LHCb data are limited in phase space, and the non-observation of a signal could be a phase-space effect.

Further studies of radiative \( J/\psi \) decays are certainly required to support or to reject the possibility that \( X_2(2210) \) is the tensor glueball. Possibly, \( X_2(2210) \) is only the low-energy tail of a tensor glueball centered at a higher mass. This conjecture could be tested by analyzing data on radiative \( \psi(2S) \) decays.

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