Abstract—Reconfigurable intelligent surfaces (RISs) can be used to establish a communication link when there is no or weak line-of-sight (LoS) path between a base station (BS) and user equipment (UE) in millimeter-wave MIMO systems. RIS comprises of many passive phase shifters, which can be controlled from the BS to focus a signal to a desired location and modify the propagation environment. Due to the passive-only elements, pilots cannot be decoded at the RIS. Thus channel estimation in RIS-assisted MIMO communication systems is challenging. Although the LoS channel between the BS and RIS can be obtained from the knowledge of their locations, the RIS-UE channel needs to be estimated. Estimating the RIS-UE channel at the BS amounts to localizing the UE. In this paper, we present a pilot-based uplink channel estimation technique to estimate the LoS channel between the RIS and UE at the BS. To do so, we assume an angular channel model. Central to the proposed algorithm is a new technique in which we observe the channel through multiple soundings with different phase shifts at the RIS. These multiple measurements allow us to resolve the underlying ambiguity in resolving the complex path gain and the angle of arrival at the RIS and to estimate the RIS-UE channel. Simulation results indicate that the performance of the proposed method is comparable to that of an oracle estimator, which assumes a partial channel state information with perfect knowledge of the locations of the BS, UE, and RIS.

Index Terms—Channel estimation, direction estimation, massive MIMO, millimeter-wave MIMO, reconfigurable intelligent surfaces.

I. INTRODUCTION

The availability of unused frequency resources at millimeter-wave (mmWave) frequencies and the ever-increasing demand for higher data rates make mmWave MIMO systems a natural choice for the next generation wireless communication systems [1], [2]. One of the major challenges in operating at mmWave frequencies is the heavy path loss with weak or no line-of-sight (LoS) components [3], [4]. Therefore ensuring a strong LoS component is necessary to establish a reliable communication link.

Recently, reconfigurable intelligent surfaces (also referred to as intelligent reflecting surfaces or large intelligent surfaces) have received significant attention as they can be used to control the propagation environment and ensure a reasonable LoS link between the transmitter and receiver [5]–[10].

Reconfigurable intelligent surface (RIS) is a two-dimensional structure consisting of many passive sub-wavelength elements, which act as diffuse scatterers. By controlling the surface impedance of these elements, we can steer and focus the energy of the electromagnetic wave impinging on the RIS to any desired direction, similar to a phased array [9]. These elements may be viewed as phase shifters, each of which can be controlled independently and remotely.

By appropriately choosing the phase shifts, we can establish a non-direct LoS path between the transmitter and receiver through the RIS. This is particularly useful in scenarios with no direct LoS path. Although RIS plays a similar role of a relay, since the elements in RIS are passive, the overall power consumption of RIS-assisted communication systems does not significantly increase when compared to using relays [8].

To leverage the advantages of a RIS-assisted communication system, we need to know the wireless channel. The channel estimation problem in a RIS-assisted MIMO system amounts to estimating the MIMO channel between the transmitter and the RIS, and the MIMO channel between the RIS and receiver. When the channel is known, the phase shifts of the RIS elements can be designed to form beams in any desired direction. However, due to the passive nature of the RIS with no symbol decoding capabilities, conventional pilot-based channel estimation techniques cannot be readily used.

A. Prior works

Most of the existing channel estimation techniques in RIS-assisted systems assume an angular channel model and are based on compressed sensing and sparse signal recovery, where they exploit the inherent sparsity in the angular domain in mmWave MIMO channels to estimate the cascaded channel, i.e., the overall channel between the transmitter and receiver via the RIS [11]–[13]. While [12] considered an LoS channel, an extension to multipath channels was proposed in [13]. These aforementioned techniques estimate the cascaded channel matrix as separately estimating the underlying angular MIMO channel matrices is difficult due to the passive nature of RIS and underlying ambiguity (see Section III for details).

In [14], bilinear matrix factorization was proposed for channel estimation, where the cascaded channel matrix was factorized into low-rank transmitter-RIS and RIS-receiver channel matrices. Using a RIS architecture with a few active elements to decode pilots, a compressed sensing based channel estimation algorithm for RIS-assisted single antenna communication systems was proposed in [15]. In contrast to the aforementioned prior works, we estimate the RIS-UE channel by proposing a technique to resolve the underlying ambiguity as discussed next.

B. Main results and contributions

In this work, we consider a RIS-assisted mmWave MIMO communication system with a multi-antenna base station (BS),
multi-antenna user equipment (UE), and RIS with multiple passive phase shifters. We assume the following: (a) there is no direct path between the BS and UE, (b) the BS-RIS and RIS-UE MIMO channels with only LoS paths admit an angular parameterization, and (c) locations of the BS and RIS are known. Consequently, the angles associated with the RIS-BS link are much larger than the inter-element spacings at the antenna matrix between the RIS and BS by

Let us denote the MIMO channel matrix between the UE and BS as $\mathbf{H}_{br}$, the RIS as $\mathbf{H}_{ru}$, and the overall complex path gain of the UE-RIS-BS link as $g_{bru}$. We further assume that the UE-RIS link is also LoS, which is a reasonable assumption owing to the excessive pathloss exhibited at mmWave frequencies [3, 4]. Under these assumptions, both $\mathbf{H}_{br}$ and $\mathbf{H}_{ru}$ are rank-1 matrices. Specifically, the MIMO channel matrix between the UE and RIS is expressed as

$$\mathbf{H}_{ru} = g_{ru} \mathbf{a}_r(\phi_u, \psi_u) \mathbf{a}_u^H(\theta_u),$$

where $g_{ru}$ denotes the complex path gain, $\theta_u$ is the AoD at the BS, and $\phi_u$ and $\psi_u$ are the elevation and azimuth AoA at the RIS made by the LoS path coming from the UE, respectively.

The rest of the paper is organized as follows: In Section II we describe the problem model. Section III discusses the ambiguity in channel estimation in RIS-assisted MIMO systems. The proposed channel estimation technique based on multiple channel soundings is presented in Section IV, followed by the numerical simulations in Section V. We conclude the paper in Section VI.

II. PROBLEM MODELING

In this paper, we consider a MIMO communication system having a BS with $N_b$ antennas, a UE with $N_u$ antennas, and one RIS consisting of $N_r$ passive elements (i.e., phase shifters). The antennas at the BS and UE form uniform linear arrays (ULAs) and the elements in the RIS form a uniform planar array (UPA) having $N_x$ and $N_y$ elements (with $N_r = N_x N_y$) along the horizontal and vertical directions, respectively. In this paper, we consider that the phase shift of each reflecting element in the RIS can be independently controlled via a low-rate backhaul link connecting the BS and RIS [5].

A. Channel model

We focus on pilot-based uplink channel estimation, in which the UE transmits known pilot symbols to the BS via the RIS. Let us denote the MIMO channel matrix between the UE and RIS by $\mathbf{H}_{ru} \in \mathbb{C}^{N_r \times N_u}$, and denote the MIMO channel matrix between the RIS and BS by $\mathbf{H}_{br} \in \mathbb{C}^{N_b \times N_r}$. The elements of the RIS may be indexed linearly (say in a column-major format) and the phase shifts may be collected in a diagonal matrix $\Omega \in \mathbb{C}^{N_r \times N_r}$.

Then the cascaded MIMO channel matrix $\mathbf{H} \in \mathbb{C}^{N_b \times N_u}$ can be expressed as

$$\mathbf{H} = \mathbf{H}_{br} \Omega \mathbf{H}_{ru}. \quad (1)$$

We assume that the distances between the BS, UE, and RIS are much larger than the inter-element spacings at the antenna elements. Hence we can express the channel matrices $\mathbf{H}_{br}$ and $\mathbf{H}_{ru}$ using a parametric angular channel model, where the corresponding AoA, AoD, and complex path gain describe the MIMO channel matrix [6]. Typically, BS and RIS are considered to be situated in environments with limited local scattering, resulting in $\mathbf{H}_{br}$ to be an LoS channel matrix [3]. We further assume that the UE-RIS link is also LoS, which is a reasonable assumption owing to the excessive pathloss exhibited at mmWave frequencies [3, 4]. Under these assumptions, both $\mathbf{H}_{br}$ and $\mathbf{H}_{ru}$ are rank-1 matrices. Specifically, the MIMO channel matrix between the UE and RIS is expressed as

$$\mathbf{H}_{ru} = g_{ru} \mathbf{a}_r(\phi_u, \psi_u) \mathbf{a}_u^H(\theta_u), \quad (2)$$

where $g_{ru}$ denotes the complex path gain, $\theta_u$ is the AoD at the BS, and $\phi_u$ and $\psi_u$ are the elevation and azimuth AoA at the RIS made by the LoS path coming from the UE, respectively. Here, $\mathbf{a}_U(\theta) \in \mathbb{C}^{N_u}$ is the array steering vector at the UE, and is defined as

$$\mathbf{a}_U(\theta) = \left[ 1 \quad e^{-j2\pi \frac{\lambda}{\lambda} \sin(\theta)} \quad \ldots \quad e^{-j(N_u-1)2\pi \frac{\lambda}{\lambda} \sin(\theta)} \right], \quad (3)$$

where $\lambda$ is the signal wavelength. The array response vector of the RIS is denoted by $\mathbf{a}_R(\phi, \psi) \in \mathbb{C}^{N_r}$ and is given by [7]

$$\mathbf{a}_R(\phi, \psi) = \mathbf{a}_x(u) \otimes \mathbf{a}_y(v), \quad (4)$$

where $u = \sin(\phi)\sin(\psi)$ and $v = \sin(\phi)\cos(\psi)$ being the direction cosines, and

$$\mathbf{a}_x(u) = \left[ 1 \quad e^{-j2\pi \frac{\lambda}{\lambda} u} \quad \ldots \quad e^{-j(N_r-1)2\pi \frac{\lambda}{\lambda} u} \right], \quad (5)$$

$$\mathbf{a}_y(v) = \left[ 1 \quad e^{-j2\pi \frac{\lambda}{\lambda} v} \quad \ldots \quad e^{-j(N_r-1)2\pi \frac{\lambda}{\lambda} v} \right], \quad (6)$$

being the array response vectors. Here, $\otimes$ denotes the Kronecker product. The angle $\theta_u$ is interpreted as the angle made by the LoS path from the UE to the RIS with the line perpendicular to the axis of the ULA at the UE along the plane containing the LoS path from the UE to RIS.

Similarly, we can express the channel between the RIS and BS as

$$\mathbf{H}_{br} = g_{br} \mathbf{a}_B(\theta_b) \mathbf{a}_R^H(\phi_b, \psi_b), \quad (7)$$

where $g_{br}$ is the complex path gain, $\theta_b$ is the AoA at the BS (which is interpreted similar to $\theta_u$), and $\phi_b$ and $\psi_b$ are the angles made by the LoS path departing from the RIS towards the BS in the elevation and azimuth directions, respectively. Here, $\mathbf{a}_B(\theta) \in \mathbb{C}^{N_b}$ denotes the array steering vector at the BS and is defined similar to (3) with $N_u$ replaced by $N_b$. Thus, the cascaded MIMO channel matrix is given by

$$\mathbf{H} = g \mathbf{a}_B(\theta_b) \mathbf{a}_R^H(\phi_b, \psi_b) \Omega \mathbf{a}_U(\theta_u), \quad (8)$$

where $g = g_{br}g_{ru}$ is the overall complex path gain.
B. Problem statement

Consider a scenario with no direct path between the BS and UE. Hence we use the RIS to establish a communication link between the BS and UE. To fully utilize the energy focusing capabilities of the RIS by appropriately selecting the phase shift matrix \( \Omega \) and to obtain the full transmit/receive diversity at the UE/BS by appropriately designing the precoder and combiner, we need to estimate the channel matrix \( \mathbf{H} \).

Let \( \mathbf{S} \in \mathbb{C}^{N_u \times M} \) be the pilot matrix transmitted from the UE. Then the signal received at the BS, \( \mathbf{X} \in \mathbb{C}^{N_u \times M} \), is given by

\[
\mathbf{X} = \mathbf{H}\mathbf{S} + \mathbf{N},
\]

where \( \mathbf{N} \in \mathbb{C}^{N_u \times M} \) is the noise matrix with each entry following a complex Gaussian distribution as \( n_{ij} \sim \mathcal{CN}(0,1) \).

Let \( P \) be the total transmit power at the UE. Since we assume unit variance additive white Gaussian noise (AWGN), \( P \) also denotes the signal-to-noise ratio (SNR). Without loss of generality, we consider the pilot matrix to be orthogonal with \( M = N_u \) and \( \mathbf{S}\mathbf{S}^H = \mathbf{I}_{N_u} \). Let \( \mathbf{H} \) be the channel matrix. We assume that the locations of the BS and RIS are known and the path between the BS and RIS is LoS. This implies that the angles \( \theta_b, \phi_b, \) and \( \psi_b \) are completely determined by the geometric placement of the BS and RIS. However, the complex gain \( g_{br} \) is unknown. Similarly, the angles \( \theta_u, \phi_u, \) and \( \psi_u \) are also completely determined by the geometric placement of the UE and RIS because of the assumption that the RIS-UE path is LoS. However, the location of UE is unknown. Therefore, to estimate the MIMO channel, we estimate the angles \( \theta_u, \phi_u \) and \( \psi_u \), and the combined complex path gain \( g \). In other words, we localize the UE by finding the directions \( \theta_u, \phi_u \), and \( \psi_u \).

III. Uniqueness

In order to estimate the unknown parameters, we begin by obtaining a noisy estimate of the overall channel matrix, denoted as \( \hat{\mathbf{H}} \in \mathbb{C}^{N_u \times N_u} \), at the BS by removing the pilots from the received signal \( \mathbf{X} \) as

\[
\hat{\mathbf{H}} = \frac{N_u}{P} \mathbf{X}\mathbf{S}^H = \mathbf{H} + \mathbf{W},
\]

where \( \mathbf{W} = N_u P^{-1} \mathbf{N}\mathbf{S}^H \) is the Gaussian noise term after pilot removal. The AoA at the UE, i.e., \( \theta_u \) can be estimated using any one of the standard direction estimation algorithms such as subspace-based or sparse recovery methods. However, the lack of active elements in the RIS makes the estimation of the angles of arrival at the RIS \( \{\phi_u, \psi_u\} \) challenging. In the rest of the section, we consider a noiseless setting for the ease of exposition.

Let us assume that we have the estimate of \( \theta_u \) available. Then, we are now left with the task of estimating \( \{\phi_u, \psi_u\} \), and \( g \). Ignoring the noise \( \mathbf{W} \), we can rewrite (10) as

\[
\hat{\mathbf{H}} = c\mathbf{a}_B(\theta_u)\mathbf{a}_L^H(\theta_u)
\]

with

\[
c = g \mathbf{a}_B^H(\phi_u, \psi_u)\mathbf{\Omega}_R(\phi_u, \psi_u).
\]

The scalar parameter \( c \) is a product of two complex numbers \( \mathbf{a}_B(\phi_u, \psi_u)\mathbf{\Omega}_R(\phi_u, \psi_u) \) and \( g \). It is not possible to uniquely identify \( \{\phi_u, \psi_u\} \) and \( g \) from \( \hat{\mathbf{H}} \) as there are different set of angles \( \{\phi_u, \psi_u\} \) and \( g \) that result in the same product \( c \). Next, we describe the proposed multiple channel sounding scheme to resolve this ambiguity, where we observe the channel with different RIS phase shift matrices.

To uniquely identify \( \{\phi_u, \psi_u\} \) and \( g \) from \( \hat{\mathbf{H}} \), we sound the channel with two different RIS phase shift matrices and eliminate \( g \) from the resulting observations. Specifically, let \( \hat{\mathbf{H}}_1 \) and \( \hat{\mathbf{H}}_2 \) denote the channel matrices corresponding to the phase shift matrices \( \Omega_1 \) and \( \Omega_2 \), respectively. Let us denote the measurements corresponding to the phase shift angles \( \Omega_1 \) and \( \Omega_2 \), respectively, as \( c_1 \) and \( c_2 \), which are defined as

\[
c_i \triangleq \frac{1}{N_u N_b} \mathbf{a}_B^H(\theta_u)\hat{\mathbf{H}}_i \mathbf{a}_U(\theta_u) = g \mathbf{a}_L^H(\phi_u, \psi_u)\mathbf{\Omega}_R(\phi_u, \psi_u), \quad i = 1, 2.
\]

Computing \( c_1/c_2 \) and rearranging yields

\[
\mathbf{a}_R^H(\phi_u, \psi_u)(c_1 \Omega_2 - c_2 \Omega_1) \mathbf{a}_R(\phi_u, \psi_u) = 0,
\]

where the quantities \( \Omega_1, \Omega_2, \phi_u, \) and \( \psi_u \) are known. Thus given at least two measurements from the proposed multiple channel sounding scheme, we can resolve the ambiguity, and as a consequence (13) now depends only on the angles at the RIS.

The RIS phase shift matrix is designed to focus signal energy from the BS to UE (and vice versa) to ensure maximum SNR at the receiver. However, the direction of the UE is not known during the channel estimation phase. Therefore, we select the phase shifts such that all the directions are excited without any bias towards a specific direction by choosing \( \{\Omega_i\} \) as random diagonal matrices, where each entry is obtained by uniformly sampling the unit circle.

IV. PROPOSED CHANNEL ESTIMATION ALGORITHM

In this section, we present the proposed channel estimation technique, where we extend the idea of using two different phase shift matrices to that of sounding the channel with \( L \geq 2 \) different phase shift matrices corresponding to \( L \) blocks of transmitted pilot symbols. Although multiple channel soundings increase the overhead, the channel estimation performance improves as we now have access to a larger number of snapshots for estimating the underlying parameters. To perform the channel estimation, we transmit the same orthogonal pilot matrix \( \mathbf{S} \in \mathbb{C}^{N_u \times N_u} \) over \( L \) blocks, but we vary the RIS phase shift matrix for each block. More specifically, we use the RIS phase shift matrix \( \Omega_i \) during the \( i^{th} \) training block for \( i = 1, 2, \ldots, L \). We assume that the underlying channel parameters are fixed throughout the entire training duration. The signal received at the BS during each training block can be expressed as

\[
\mathbf{X}_i = \mathbf{H}_i\mathbf{S} + \mathbf{N}_i, \quad i = 1, 2, \ldots, L,
\]
where \( H_i = g a_B(\theta_i) a_R^H(\phi_i, \psi_i) \Omega_i a_R(\phi_i, \psi_i) a_R^H(\theta_i) \). Then the noisy estimate of the channel matrix in each block is given as

\[
\hat{H}_i = \frac{N_u}{P} X_i S_i^H = H_i + W_i, \quad i = 1, 2, \ldots, L, \tag{15}
\]

where \( W_i = N_u P^{-1} N_i S_i^H \) is the Gaussian noise term after pilot removal. Now, to estimate the channel matrix, we estimate \( \theta_u, \{ \phi_u, \psi_u \} \), and \( g \) using a three-step process as described next.

A. Estimation of AoD at the UE

To estimate the AoD \( \theta_u \) at the UE, we use MUSIC [17], which is a subspace-based direction finding method. In the absence of noise, we have \( \mathcal{R}(H_i^H) = \mathcal{R}(a_U(\theta_u)) \) for all \( i = 1, 2, \ldots, L \). Here, \( \mathcal{R}(H) \) denotes the range space of \( H \). Thus, to estimate \( \theta_u \), we proceed by estimating the common range space of \( \langle H_i^H \rangle_i \) as follows. Let us define the matrix \( G \in \mathbb{C}^{N_u \times L N_R} \) as

\[
G = \begin{bmatrix}
H_1^H & H_2^H & \ldots & H_L^H
\end{bmatrix}, \tag{16}
\]

where \( \mathcal{R}(G) = \mathcal{R}(a_U(\theta_u)) \) and the rank of \( G \) is one. We can compute the noise subspace, which is orthogonal to the one-dimensional signal subspace \( \mathcal{R}(a_U(\theta_u)) \) using a singular value decomposition (SVD) of \( G \) and by considering the left singular vectors corresponding to all but the largest singular value. Let us denote this noise subspace by \( U_n \), where \( U_n^H a_U(\theta_u) = 0 \). Therefore, in the presence of noise, we minimize \( ||U_n^H a_U(\theta)||^2 \) with respect to \( \theta \), and the estimate of the AoD at the UE \( \hat{\theta}_u \) can be obtained from the peak of the pseudo spectrum

\[
P_{\text{UE}}(\theta) = \frac{1}{||U_n^H a_U(\theta)||^2}. \tag{17}
\]

Next we estimate the angles \( \{ \phi_u, \psi_u \} \) at the RIS using \( \hat{\theta}_u \).

B. Estimation of AoA from UE at RIS

Let us now extend the procedure in Section III to multiple channel soundings. Given \( \hat{\theta}_u \), we can obtain the measurements \( c_i \) corresponding to the \( i \)th block using (12), and can be rearranged as

\[
a_R^H(\phi_i, \psi_i)(c_i \Omega_j - c_j \Omega_i)a_R(\phi_u, \psi_u) = e_i, \tag{18}
\]

for \( i, j = 1, 2, \ldots, L \). Here, \( e_i \) is the error term. Using these equations, we pose the problem of estimating \( \phi_u \) and \( \psi_u \) as a problem of computing the solution to a system of non-linear equations. There are multiple ways to form the required system of equations. One such option is to consider the combination where we take all equations with \( i = 1 \) and \( j = 2, 3, \ldots, L \) to form the required system. By doing so, we get

\[
\begin{bmatrix}
a_R^H(\phi_1, \psi_1)(c_1 \Omega_2 - c_2 \Omega_1) \\
a_R^H(\phi_2, \psi_2)(c_2 \Omega_3 - c_3 \Omega_2) \\
\vdots \\
a_R^H(\phi_L, \psi_L)(c_L \Omega_1 - c_1 \Omega_L)
\end{bmatrix}
= \mathbf{e}, \tag{19}
\]

where \( e \) denotes the error vector. In the noiseless setting, \( e \) will be the all-zero vector and \( a_R(\phi_u, \psi_u) \) lies in the null space of \( A \). This implies that \( a_R(\phi_u, \psi_u) \) is orthogonal to \( \mathcal{R}(A^H) \). Since the matrices \( \Omega_i \) are random, the matrix \( \hat{A} \) has full row rank. In the absence of noise, the basis for \( \mathcal{R}(A^H) \) can be obtained using the SVD of \( \hat{A}^H \) and by considering the left singular vectors corresponding to the \( (L - 1) \) largest singular values of \( A^H \). Let us denote this subspace by \( U_s \in \mathbb{C}^{N_s \times (L - 1)} \). Since this space is orthogonal to \( a_R(\phi_u, \psi_u) \), we can conclude that \( ||U_s^H a_R(\phi_u, \psi_u)||^2 = 0 \) in the noiseless case. In presence of noise, minimizing \( ||U_s^H a_R(\phi, \psi)||^2 \) with respect to \( \{ \phi, \psi \} \) amounts to solving to the system of non-linear equations in (19) in the least squares sense. Thus, we can obtain the estimates of the AoA at RIS, \( \{ \hat{\phi}_u, \hat{\psi}_u \} \), by computing the locations of the peaks of the pseudo spectrum

\[
P(\phi, \psi) = \frac{1}{||U_s^H a_R(\phi, \psi)||^2}. \tag{20}
\]

We remark that the above method is not a high-resolution direction finding method as MUSIC, but results in reasonable estimates as can be seen in Section V. Next, we estimate the complex path gain using the estimates \( \hat{\theta}_u \) and \( \{ \hat{\phi}_u, \hat{\psi}_u \} \).

C. Estimation of the path gain

Once we have the estimates of the AoA and AoD, estimating the path gain can be done using least squares. Let us define \( b_i \in \mathbb{C}^{N_u \times 1} \) as

\[
b_i = \hat{H}_i a_u(\hat{\theta}_u), \quad i = 1, 2, \ldots, L. \tag{21}
\]

If we assume that \( \hat{\theta}_u \approx \theta_u \), then using (15) and (8), we can observe that

\[
b_i = g N_u a_B(\theta_i) a_R^H(\phi_i, \psi_i) \Omega_i a_R(\phi_u, \psi_u) + \nu_i,
\]

\[
= g v_i + \nu_i, \quad i = 1, 2, \ldots, L, \tag{22}
\]

where \( \nu_i = W_i a_u(\hat{\theta}_u) \) is the noise term. Using the estimates \( \{ \hat{\phi}_i, \hat{\psi}_i \} \), we can obtain an estimated version of \( v_i \), say \( \hat{v}_i \in \mathbb{C}^{N_u \times 1} \) as

\[
\hat{v}_i = N_u a_B(\theta_i) a_R^H(\phi_i, \psi_i) \Omega_i a_R(\hat{\phi}_u, \hat{\psi}_u), \quad i = 1, 2, \ldots, L.
\]

To estimate \( g \), we begin by defining length-\( LN_b \) vectors \( \mathbf{b} = [b_1^T, \ldots, b_L^T]^T, \hat{\nu} = [\hat{\nu}_1^T, \ldots, \hat{\nu}_L^T]^T \), and \( \tilde{\nu} = [\tilde{\nu}_1^T, \ldots, \tilde{\nu}_L^T]^T \) to obtain the combined system of equations

\[
\mathbf{b} = g \hat{\nu} + \tilde{\nu}, \tag{23}
\]

where \( \tilde{\nu} \) corresponds to the error term in (22) that is obtained when we use \( \hat{v}_i \) instead of \( v_i \). The least squares estimate of \( g \) is then given by

\[
\hat{g} = \frac{\hat{\nu}^H \mathbf{b}}{||\hat{\nu}||^2}. \tag{24}
\]

When \( \hat{v}_i = v_i, \nu_i \) is white Gaussian, making the least squares estimator (23) the optimal estimator for the complex path gain. This concludes the proposed channel estimation algorithm.
MIMO communication system, where the precoders at the transmit diversity and best possible reflection, respectively.

For OracleLS, true angles are used. While NMSE captures the estimation error of the path gains, average SE in (26) gives us an indication about the achievable SE when the precoder at the UE and the phase shift matrix at the RIS are designed based on the estimated angles. In contrast to NMSE, the average SE as defined in (26) does not capture the impact of the error in estimating the path gain on the actual data detection.

B. Results

We consider $N_u = 8$ and $N_b = 12$ throughout the simulations and the results are obtained by averaging over 1000 independent realizations of the receiver noise and unit-modulus complex path gain. An inter-element spacing of $\frac{\lambda}{2}$ is considered for the ULAs at the BS and UE. Since the RIS is generally assumed to be composed of sub-wavelength elements [9], we have selected an inter-element spacing of $\frac{\lambda}{4}$ for the UPA at the RIS. We considered a setup with $\theta_b = 40^\circ$, $\phi_b = 50^\circ$, $\psi_b = 65^\circ$, $\phi_u = 50^\circ$, and $\psi_u = 30^\circ$. The angles $\theta_b$, $\phi_b$, and $\psi_b$ are considered to be known from the knowledge of the positions of the RIS and BS. The remaining angles as well as the complex path gain are estimated. We choose the search grids for (17) and (20) such that the true angle lies on the grid. We have considered a search grid with a range from $0^\circ$ to $90^\circ$ having a spacing of $1^\circ$ for estimating $\theta_u$ from (17) and $5^\circ$ for estimating $\phi_u$ and $\psi_u$ from (20).

In Fig. 1, we illustrate the pseudo spectrum $P(\phi, \psi)$ in (20) for an SNR of $-10$ dB and for $L = 5$. We can see that with multiple soundings, the pseudo spectrum results in a reasonable estimate of $\{\phi_u, \psi_u\}$ with a peak at the true location. In Fig. 2a, we show NMSE for different SNRs and for different number of elements in the RIS, where we have used $L = 5$. We can see that NMSE reduces with an increase in the number of RIS elements since the direction estimates improve with an increase in the array aperture, thus leading to a sharper pseudo-spectrum. In Fig. 2b, we show NMSE for different number of channel soundings $L$, where we fix the SNR to $-10$ dB. We can see that the channel estimation performance improves as $L$ increases. More importantly, for the considered setup, we can see that the algorithm performs similar to OracleLS for SNR above $-15$ dB and for $L > 4$.

In Fig. 2c, we show average SE of OracleLS for different SNRs and different values of $N_r$. Since the design of an optimal precoder, combiner, and phase shift matrix depend only on the angles and not on the path gains, the SE of the OracleLS estimator is the same as the maximum spectral efficiency (as defined in (25)) that can be achieved using a perfect channel state information (CSI). The proposed scheme...
achieves average SE as that of OracleLS for low SNRs (around 0 dB) even when $L = 2$. This means that the error in estimating the angles $\{\theta_u, \phi_u, \psi_u\}$ is very less.

VI. CONCLUSIONS

In this paper, we have proposed a channel estimation algorithm for RIS-assisted mmWave MIMO systems. In this work, using the knowledge of the BS to RIS channel, we have proposed a method inspired by the standard direction of arrival estimation techniques to perform a pilot-based estimation of the MIMO channel between the RIS and UE. We have considered an LoS channel model for the RIS to UE link, which is parameterized by the angle of arrival, angle of departure, and the complex path gain. To resolve the ambiguity and uniquely estimate complex path gain and angle of arrival at the RIS, we have proposed a multiple channel sounding technique in which we observe the channel through different RIS phase shifts. We have estimated the channel parameters in a three-step process, wherein we estimate the angle of departure at the UE, followed by the estimation of the azimuth and elevation angle of arrivals at the RIS. Using the estimated angles, we compute the complex gain of the cascaded MIMO channel. Through numerical simulations, we have demonstrated that the proposed algorithm performs on par with a method that perfectly knows all the underlying angles. We emphasize that our proposed algorithm can be directly extended to the case where there is an array of RIS to enable multi-stream transmission and reception link between the BS and UE by switching on one RIS in the array at a time. However, more investigation is needed for channel estimation in RIS-array assisted MIMO communication systems.

REFERENCES

[1] T. S. Rappaport, S. Sun, R. Mayzus, H. Zhao, Y. Azar, K. Wang, G. N. Wong, J. K. Schulz, M. Samimi, and F. Gutierrez, “Millimeter wave mobile communications for 5G cellular: It will work!” IEEE Access, vol. 1, pp. 335–349, May 2013.

[2] A. Ghosh, T. A. Thomas, M. C. Cudak, R. Ratrasuk, P. Moorut, F. W. Vook, T. S. Rappaport, G. R. MacCartney, S. Sun, and S. Nie, “Millimeter-wave-enhanced local area systems: A high-data-rate approach for future wireless networks,” IEEE J. Sel. Areas Commun., vol. 32, no. 6, pp. 1152–1163, June 2014.

[3] Z. Wan, Z. Gao, and M.-S. Alouini, “Broadband channel estimation for intelligent reflecting surface aided mmWave massive MIMO systems,” arXiv preprint arXiv:2002.01629, Feb. 2020.

[4] J. Mo, P. Schniter, N. G. Peclic, and R. W. Heath, “Channel estimation in millimeter wave MIMO systems with one-bit quantization,” in Proc. of the Asilomar Conference on Signals, Systems and Computers, Pacific Grove, USA, Nov. 2014.

[5] E. Basar, M. Di Renzo, J. De Rosny, M. Debbah, M. Alouini, and R. Zhang, “Wireless communications through reconfigurable intelligent surfaces,” IEEE Access, vol. 7, pp. 116753–116773, Aug. 2019.

[6] O. Ozdogan, E. Björnson, and E. G. Larsson, “Intelligent reflecting surfaces: Physics, propagation, and pathloss modeling,” IEEE Wireless Commun. Lett., vol. 9, no. 5, pp. 581–585, May 2019.

[7] ——, “Using intelligent reflecting surfaces for rank improvement in MIMO communications,” in Proc. of the IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP), Barcelona, Spain, May 2020.

[8] E. Björnson, O. Ozdogan, and E. G. Larsson, “Intelligent reflecting surface versus decode-and-forward: How large surfaces are needed to beat relaying?” IEEE Wireless Commun. Lett., vol. 9, no. 2, pp. 244–248, Feb. 2020.

[9] M. Najafi, V. Jamali, R. Schober, and H. V. Poor, “Physics-based modeling and scalable optimization of large intelligent reflecting surfaces,” arXiv preprint arXiv:2004.12957, Apr. 2020.

[10] V. Arun and H. Balakrishnan, “Rfocus: Practical beamforming for small devices,” arXiv preprint arXiv:1905.05130, May 2019.

[11] P. Wang, J. Fang, H. Duan, and H. Li, “Compressed channel estimation for intelligent reflecting surface-aided millimeter wave systems,” IEEE Signal Process. Lett., vol. 27, pp. 905–909, May 2020.

[12] J. He, M. Leinonen, H. Wymeersch, and M. Juntti, “Channel estimation for RIS-aided mmWave MIMO channels,” arXiv preprint arXiv:2002.06453, Feb. 2020.

[13] J. He, H. Wymeersch, and M. Juntti, “Channel estimation for RIS-aided mmWave MIMO systems via atomic norm minimization,” arXiv preprint arXiv:2007.08158, July 2020.

[14] Z.-Q. He and X. Yuan, “Cascaded channel estimation for large intelligent metasurface assisted massive MIMO,” IEEE Wireless Commun. Lett., vol. 9, no. 2, pp. 210–214, Feb. 2020.

[15] A. Taha, M. Airabehia, and A. Alkhateeb, “Enabling large intelligent surfaces with compressive sensing and deep learning,” arXiv preprint arXiv:1904.10156, Apr. 2019.

[16] D. Tse and P. Viswanath, Fundamentals of Wireless Communication. USA: Cambridge University Press, 2005.

[17] H. L. Van Trees, Optimum Array Processing: Part IV of Detection, Estimation, and Modulation Theory. USA: John Wiley & Sons, Ltd, 2002.