Mathematical design theory of screw extruder used for additive manufacturing

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Abstract

Additive manufacturing as the technology of the future brings new challenges. One of them is the economic efficiency of production. This paper focuses on the mathematical analysis and structural design of extrusion screw used for additive manufacturing. The primary objective is to analyze the screw tool geometry and determine a procedure for its design, specifically the theory involved with the pressing tool and force relations which are necessary for the verification of the proposed tool geometry and its strength analysis. Procedures for determining frictional performance of the screw press are used in designing the drive of the screw extruder of 3D printer. Familiarity with the above mentioned procedures forms the basis for research into new tool - screw that will improve the service life and competitiveness of the technology.

Keywords: Additive manufacturing; Screw extruder; Extrusion, 3D printer; 3D printing; Screw design

1. Introduction

The global trend of additive manufacturing in industry brings new challenges. One of them is primarily the efficiency of the production process. Production costs significantly affect the application of additive manufacturing. Relatively advanced Fused Deposition Modelling technologies are experiencing a worldwide boom in the manufacture of products from polymers. The basic building material is a polymer in the form of a treated and processed filament. However, in some cases, this technology could be replaced by a more efficient production method, where the building material would not have to be in the form of a modified filament, but only in the form of a granular polymer. This granulate is melted in the screw extruder and extruded from the 3D printer head by means of an extrusion screw. The basis for the correct function and efficient work of the extrusion head is, above all, a suitable design of the screw geometry with regard to the polymer used and its properties. Based on the geometric optimization of the screw, it is possible to achieve high efficiency of 3D printing at low energy costs.

The correct design of the construction and geometry of the extrusion screw must be based on knowledge of the technology of screw extrusion and knowledge of the properties of the extruded polymer. However, fundamentals in engineering design of a screw is its detailed analysis and generally applicable mathematical description of: the volume of the screw profiles, speed and power ratios, friction in the screw, and subsequent statement of torque on the screw necessary for scaling the power of the machine itself.

Additive manufacturing based on the polymer extrusion by means of screw pressing represents a progressive technology in this field of production. Due to the great advantages of the screw extrusion in producing the part of high quality, it represents the most effective and promising technology in the field of additive manufacturing. Thus further
research to advance this technology is only logical. Increasing the tool efficiency and tool life will reduce operating costs and will be reflected in a lower price for the final product, making it a more viable application.

2. Screw extrusion process

Controlling the polymer extrusion process by means of a screw extruder is relatively complicated. The parameters of this process tend to be unstable due to external influences and faults entering the process [1]. These effects of instability of the extrusion process are described in detail in the works [2–6]. In order to achieve fast and continuous extrusion and a quality final product, it is necessary to harmonize the control of the speed ratios on the screw and the temperature ratios in the extruder. Several physical models have been proposed in [7–15], which mathematically describe this complex extrusion process and result in very complex methodologies for designing its control. The model proposed in the work [5] is focused on control and describes the outlet flow at the extruder nozzle as a function of screw speed and temperature distribution of the melting material along the extruder barrel.

The present study deals with the mathematical theory of geometry design for screw extruder as a new research avenue for 3D printing processes. The screw ensures a continuous feed of material and at the same time creates a sufficiently high pressure in the chamber of the extruder so that the speed and resolution of 3D printing at the exit of the nozzle is comparable to the processes of filament-based [16–18] and syringe-based 3D printing [19, 20]. The advantage of the screw extruder is also the homogenization of the molten material, which allows processing of a wider range of raw materials [21–24].

The building materials for 3D printing using a screw extruder are various types of polymers in the form of pellets. These pellets enter from the hopper into the working space of the screw, which transports this material to the nozzle opening (Figure 1). The screw extruder is divided into three zones: (i) conveying zone where pellets are transported through the extruder chamber, (ii) melting zone which is heated by an external heated source to melt the pellets, (iii) mixing zone where the extruded melt is homogenized and exposed to high pressure. After the liquefied material exits the screw channel, it is compressed into the nozzle and extruded through the small hole in the nozzle. [5, 13–15]

The key parameters of this technology are the temperature and heating time of the material. The temperature distribution in the extruder as well as its source have a fundamental influence on the melting mechanism of the material, its viscosity, extrusion, flow and printing speed. The heating time of the material depends on the geometry and speed of the screw. At high temperature resp. slow convey, the material at the inlet can melt and seal the system. Excessive overheating of the material will reduce the viscosity at the outlet and seal the nozzle. Too low temperature can limit the extrusion and cause system overload due to high pressures. Friction in the screw is a significant source of heat in the extruder. So sometimes not only external heating, but also external cooling of the extruder chamber is needed to achieve optimal temperature conditions [25, 26].

![Figure 1 Screw extrusion process for 3D printing. [5]](image_url)

Based on the above facts, it is clear what important role the optimal geometry of the extrusion screw plays in the whole process. The geometry of these screws (Figure 2) varies depending on the physical properties of the polymer used, which affect the individual levels of temperature, friction, pressure, drive parameters, etc.
3. Analysis of the extrusion screw

3.1. Volume of the screw profiles

Assuming that the screw thread area is packed 100% with material, the volume of the screw profile represents for us the amount of material being transported. The filling volume of the feed screw is designed to work coaxially with the pressing screw, to create a single unit [28, 29]. The main task of the feed screw is the homogenization of the input material throughout the whole area of the screw profile.

![Screw profile](image)

**Figure 3** Screw profile - meridian section (α1 – face angle, α2 – back angle, s – screw pitch, e – guide surface, h – profile depth, D – outer diameter of the screw)

The volume of the screw profile (Figure 3) and screw run can be expressed as follows, (where i – number of threads):

\[
\frac{V'}{\pi D^2} = \left(1 - \frac{h}{D}\right) \frac{h}{D} \left[ \frac{s}{D} \left(1 - \frac{i \cdot e}{s}\right) \right]
\]

This expression can be significantly simplified if we apply some assumptions applicable to special types of structures, such as.

\[
\alpha_1 = \alpha_2 = \frac{\pi}{2}, \text{or} \quad \frac{r_1}{D} = \frac{r_2}{D} \to 0
\]

\[
\frac{V}{\pi D^3} = \left(1 - \frac{h}{D}\right) \frac{h}{D} \left[ \frac{s}{D} \left(1 - \frac{i \cdot e}{s}\right) \right] \quad \Rightarrow \quad V = \pi h(D - h)(s - i e)
\]
The profile volume for one thread revolution can be changed over the screw’s length. Such changes will achieve:

- Change of the outer diameter (conical screw)
- Change of the profile depth (screw with tapered core)
- Change of the pitch angle (screw with progressive pitch)
- Several parameters can be changed simultaneously. Then the volume ratios in areas of interest on the screw are referred to as the compression ratio, that is: \( k = \frac{V}{V_i} \).

### 3.2. Speed and power ratios

The material, in this case polymer pellets, is fed into the screw profile in loose form. The feeding screw is located under a hopper which is smoothly and continuously filled with material, thus the screw profile is completely packed and feeds a consistent amount of material. The advantage of feeding the material with a screw is the partial compaction of the material before it enters the actual pressing screw compartment.

To describe the movement of material within the screw, we will introduce the following assumptions:

- The thread profile is completely filled with material
- The material moves in the direction of the screws pitch
- We ignore the effects of screw profile curvature
- The coefficient of friction between the material and its corresponding surface is constant
- The processes material doesn’t transfer shear stresses.

The validity of these assumptions, coupled with steady state conditions, enables us to track any particle with respect to Figure 4, where:

- \( \alpha \) – pitch angle of screw
- \( \varphi \) – feed angle
- \( u_0 \) – circumferential velocity of screw
- \( v_r \) – relative circumferential velocity of the screw
- \( v_a \) – absolute circumferential velocity of the screw

The velocity \( v_a \) moves the material relative to the screws (in moveable coordinates). The velocity \( v_r \) moves the material in relation to the nozzle (at fixed coordinates). From Figure 4 can be derived:

\[
\begin{align*}
v_a &= v_r \sin \alpha = u_0 \frac{\sin \alpha}{\sin(\alpha + \varphi)} = u_0 \frac{\sin \alpha}{\sin \alpha \cos \varphi + \cos \alpha \sin \varphi} = u_0 \frac{\sin \alpha}{\tan \alpha \cos \varphi + \sin \varphi} = u_0 \frac{\sin \alpha}{\tan \alpha \cos \varphi + \sin \varphi} \\
v_r &= \frac{v_a}{\sin(\alpha + \varphi)} = u_0 \frac{\sin \alpha}{\sin \alpha \cos \varphi + \cos \alpha \sin \varphi} = u_0 \frac{\sin \alpha}{\tan \alpha \cos \varphi + \sin \varphi} \\
v_{ax} &= v_a \sin \alpha = u_0 \frac{\tan \alpha \sin \varphi}{\tan \alpha \cos \varphi + \sin \varphi} = u_0 \frac{\tan \alpha \sin \varphi}{\tan \alpha \cos \varphi + \sin \varphi} \quad (3)
\end{align*}
\]

For the transport of material, only the axial velocity components \( v_{ax} \) are applied, given by:

\[
v_{ax} = v_a \sin \alpha = u_0 \frac{\tan \alpha \sin \varphi}{\tan \alpha \cos \varphi + \sin \varphi} \quad (4)
\]

In order for \( 0 < v_{ax} \) to apply, the numerator must be significantly larger than zero. Thus, the feed angle \( \varphi \) must belong to the interval \( 0 < \varphi < \pi/2 \).

If \( \varphi = 0 \), then \( \tan \varphi = 0 \), as well as \( v_{ax} = 0 \). This is the first limiting condition. In other words, we can say that the material is not transported in the direction of the axis of the screw, but will rotate with it. The second limiting condition occurs when \( \varphi = \pi/2 \). This value cannot be directly substituted into the above relationship, because it would produce an indefinite term. Therefore, it is necessary to modify the original relationship:

\[
v_{ax} = v_a \sin \alpha = u_0 \frac{\tan \alpha \sin \varphi}{\tan \alpha \cos \varphi + \sin \varphi} \quad (5)
\]

Substituting \( \varphi = \pi/2 \) we obtain:

\[
v_{ax} = u_0 \frac{\tan \alpha \sin 90^\circ}{\tan \alpha \cos 90^\circ + \sin 90^\circ} = u_0 \tan \alpha \quad \Rightarrow \quad u_0 = v_a \quad (6)
\]
In this case the material moves only in the direction of the screw axis without circumferential velocity components. It follows that the actual value of the angle $\phi$ will lie somewhere between the above mentioned extremes. Using the expressions mentioned above, we can determine the quantity of material extruded. For simplicity, we will assume a quadratic profile with negligible fillet.

$$V_s = v_{as} \pi (D-h) h \left( 1 - \frac{ie}{s} \right) \rho_{sl}$$  \hspace{1cm} (7)

After substitution:

$$V_s = u_0 \cdot \frac{tg \alpha tg \phi}{tg \alpha + tg \phi} v_{as} \pi (D-h) h \left( 1 - \frac{ie}{s} \right) \rho_{sl}$$  \hspace{1cm} (8)

For circumferential velocity $u_0$ the following applies:

$$u_0 = \pi D n$$

Where $D$ – screw diameter, $n$ – number of screw revolutions. Thus we can write:

$$V_s = \pi^2 D (D-h) h \left( 1 - \frac{ie}{s} \right) n \frac{tg \alpha tg \phi}{tg \alpha + tg \phi} \rho_{sl}$$  \hspace{1cm} (3)

If the ratios change along the length $l$, the feed angle will also change. For compaction of loose material, the feed angle $\phi$ decreases along the length. In a packed screw profile, the following forces act on a defined particle, Figure 5.
Figure 5 Power ratios in screw profile

The forces in the space are converted into a plane, such that we reduce all forces on the outer diameter of the screw. An equal slope increase was respected by means of introducing relevant pitch angles, see Figure 6.

Figure 6 Pitch angles on the screw profile

\[
\tan \alpha_r = \frac{s}{\pi D}, \quad \tan \alpha_0 = \frac{s}{\pi(D-h)} = \tan \alpha_r \frac{1}{1 - \frac{h}{D}}, \quad \tan \alpha_z = \frac{s}{\pi(D-2h)} = \tan \alpha_r \frac{1}{1 - \frac{2h}{D}} \tag{10}
\]

From Fig. 6 and with force equilibrium in the direction of relative velocity, the following applies:

\[
F \cdot \frac{D-h}{D} = p f_s s \left(1 - \frac{ie}{s}\right) \cos \alpha_r \frac{dl}{\sin \alpha_r} \sin(\alpha_r + \varphi)
\]

\[
2 p f_s h \cdot \frac{dl}{\sin \alpha_r} \frac{D-h}{D} + p f_s s \left(1 - \frac{ie}{s}\right) \cos \alpha_r \frac{dl}{\sin \alpha_r} \frac{D-2h}{D} + dp h s \left(1 - \frac{ie}{s}\right) \cos \alpha_r \frac{D-h}{D} + F \cdot \frac{D-h}{D} f_s = \tag{11}
\]

\[
= p f_s s \left(1 - \frac{ie}{s}\right) \cos \alpha_r \frac{dl}{\sin \alpha_r} \cos(\alpha_r + \varphi)
\]

Where \(f_s\) – is the friction coefficient between the material and screw, \(f_r\) – is the friction coefficient between the material and nozzle.

By eliminating \(F\), we obtain:
The above relationship shows that the pressure in the screw profile is exponentially dependent on the length of the screw. The constant of proportionality \( A \) depends on the geometry of the screw profile and the friction between the material and the nozzle \( fp \) as well as the friction between the material and the screw \( fz \). The condition of sharp pressure increase requires that the friction coefficient \( fp \) be as large as possible, while the coefficient \( fz \) be as small as possible. The coefficient \( fz \) can greatly affect the surface quality of the screw. The goal is to achieve the lowest surface roughness.

Increasing coefficient \( fp \) can be achieved by increasing the nozzle roughness or increasing the machining grooves on the surface of the nozzles in the direction of the screw axis. Grooving not only increases friction, but also prevents rotation of the material invoking the so-called axial block flow.
3.3. Friction in the screw

Movement of the material in the packed profile is associated with the friction performance, and this energy is converted to heat. Elementary friction performance of the nozzle is given by:

\[ dp_k = \frac{k_a f_p s}{s - 1} \cos \alpha_p \cdot \frac{dl}{\sin \alpha_p} \cdot v_a \]  

(18)

Then the elementary friction of the screw is given by:

\[ dp_z = ka \left[ p_s f_s \left(1 - \frac{i_e}{s}\right) \cos \alpha_s v_s + 2p_s f_s h_s \frac{dl}{\sin \alpha_a} v_s + p_s f_p s \left(1 - \frac{i_e}{s}\right) \cos \alpha_p \cdot \frac{dl}{\sin \alpha_p} v_o \sin (\alpha + p) \right] \]  

(19)

Using the above relations (18), (19) and integrating across the limits from 0 to l we obtain:

\[ P_p = k_a f_p s \left(1 - \frac{i_e}{s}\right) \cos \alpha_p \cdot \frac{u_o \cdot (p_l - p_0)}{\ln \left(\frac{p_l}{p_0}\right) \sin \phi} \]  

(20)

\[ P_z = u_o \sin \phi l \cdot \frac{f_s}{\sin (\alpha_a + \phi)} \frac{p_l - p_0}{\ln \left(\frac{p_l}{p_0}\right)} k_p s \left(1 - \frac{i_e}{s}\right) \cos \alpha_s \sin (\alpha + p) + 2l_h \frac{1}{\sin \alpha_s} \frac{D}{D - h} - k_a f_p s \left(1 - \frac{i_e}{s}\right) \cos \alpha_p \cdot \frac{dl}{\sin \alpha_p} \sin (\alpha + p) \]  

(21)

The resulting friction performance for the screw with packed profile is:

\[ P_v = P_p + P_z \]  

(22)

The torque on the screw:

\[ Mk = \frac{P_v}{2\pi n} \]  

(23)

Pv and Mk are necessary for scaling the power of the machine itself.

4. Conclusion

The additive manufacturing by screw extruder used in 3D printing is a relatively complicated process. Therefore the correct design of the screw geometry is an essential precondition for successful implementation of this extruding technology. This study describes the complex parameters of the screw. The above procedure can be directly applied in the design and optimization of the geometry of the extrusion screw intended for additive production. An analysis of the screw geometry can help in designing new and innovative tools. However it should also be noted that the complexity of working tools design is not completely determined by the optimization of the geometry itself. The quality of the screw is a symbiosis of screw geometry, materials used, chemical and heat treatments, surface finish, and price. Optimization of each of these criteria can significantly increase the service life of the screw.

Compliance with ethical standards

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Disclosure of conflict of interest

There is no conflict of interest.
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