Hidden local symmetry and the reaction $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$ at energies $\sqrt{s} \leq 1$ GeV.

N. N. Achasov

Laboratory of Theoretical Physics, S. L. Sobolev Institute for Mathematics, 630090, Novosibirsk, Russian Federation

A. A. Kozhevnikov

Laboratory of Theoretical Physics, S. L. Sobolev Institute for Mathematics, and Novosibirsk State University, 630090, Novosibirsk, Russian Federation

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Based on the generalized hidden local symmetry as the chiral model of pseudoscalar, vector, and axial vector mesons, the excitation curve of the reaction $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$ is calculated for energies in the interval $0.65 \leq \sqrt{s} \leq 1$ GeV. The theoretical predictions are compared to available data of CMD-2 and BaBar. It is shown that the inclusion of heavy isovector resonances $\rho(1450)$ and $\rho(1700)$ is necessary for reconciling calculations with the data. It is found that at $\sqrt{s} \approx 1$ GeV the contributions of the above resonances are much larger, by the factor of 30, than the $\rho(770)$ one, and are amount to a considerable fraction $\sim 0.3 - 0.6$ of the latter at $\sqrt{s} \sim m_\rho$.

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I. INTRODUCTION.

The theory aimed at describing low energy hadron processes should be formulated in terms of effective colorless degrees of freedom $[1]$. They are introduced on the basis of spontaneously broken chiral symmetry $SU(3)_L \times SU(3)_R$ which is the symmetry of QCD Lagrangian relative independent global rotations of right and left fields of approximately massless $u, d, s$ quarks. The pattern of the spontaneous breaking of the above approximate symmetry is $SU(3)_L \times SU(3)_R \rightarrow SU(3)_L + R$, where $SU(3)_L + R$ is the well known flavor $SU(3)$ symmetry. According to Goldstone theorem, spontaneous breaking of global symmetry results in appearance of massless fields. In the present case, they are light $J^P = 0^- \pi^+, \pi^-, \pi^0, K^+, K^0, K^-$, and $\eta$. The transformation law of these fields fixes the Lagrangian of interacting Goldstone mesons:

$$\mathcal{L}_{GB} = \frac{f^2_\pi}{4} \text{Tr} \left( \partial_\mu U \partial^\mu U^\dagger \right) + \cdots ,$$

where

$$U = \exp \left( i \Phi \frac{\sqrt{2}}{f_\pi} \right) ,$$

with

$$\Phi = \begin{pmatrix} \pi^0 & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \frac{\pi^0}{\sqrt{2}} - \frac{\eta}{\sqrt{6}} & -\frac{2\eta}{\sqrt{6}} \end{pmatrix} ,$$

is the matrix of pseudoscalar meson octet, and $f_\pi = 92.4$ MeV is the pion decay constant. Upon adding the term $\propto m_\rho^2 \text{Tr}(U + U^\dagger)$ which explicitly breaks chiral symmetry, the Goldstone bosons become massive.

Pseudoscalar mesons are produced via vector resonances, so the problem appears as how should one include vector mesons in a chiral invariant way? This problem was studied in a number of papers, see, for example, Ref. [1, 2] and references therein. However, among the models aimed at the description of interactions of the pseudoscalar mesons with the low lying vector and axial vector ones the most elegant is the generalized hidden local symmetry (GILS) model $[3, 4, 5, 6]$. It relates all coupling constants to only the pion decay constant $f_\pi$ and $g_{\rho\pi\pi}$, and accounts for anomalous processes in a way that does not break low energy theorems. Strikingly, this very popular model was not scrutinized in the processes with sufficiently soft pions where one can rely on the tree approximation. The fact is that testing chiral models of the vector meson interactions with Goldstone bosons is really difficult problem because in the well studied decays $\rho \rightarrow 2\pi$, $\omega \rightarrow 3\pi$ final pions are not soft enough to rely on lowest derivative tree effective Lagrangian. Multiple pion decays are most promising because pions are soft.

The aim of the present paper is to confront the generalized hidden local symmetry model $[3, 4, 5]$ with available data on the reaction $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$ taken by CMD-2 $[12]$ and BaBar $[13]$ collaborations. The final state $\pi^+\pi^-\pi^+\pi^-$ can be produced via intermediate $\rho(770)$ meson. In the framework of chiral approach the $\rho(770) \rightarrow 4\pi$ decay width was evaluated in the papers $[14, 15, 16, 17, 18, 19, 20]$.

As was emphasized in Ref. $[18, 19]$ because of a too rapid growth with energy, the $\rho(770) \rightarrow 4\pi$ decay width evaluated at $\sqrt{s} = m_\rho$ is not adequate characteristic of the chiral dynamics, and one should study the excitation curve of the process $\rho \rightarrow 4\pi$ in such reactions as $e^+e^-$ annihilation, $\tau$ decays, photoproduction etc. The corresponding excitation curves were calculated in Ref. $[18, 19]$ in the chiral model which neglects the $a_1(1260)$ contribu-
tion and under assumption of the resonant mechanism $e^+e^- \rightarrow \rho(770) \rightarrow 4\pi$ and similar in case of other mentioned reactions.

The material is arranged as follows. Section II is devoted to the exposition of the low momentum expansion of the generalized hidden local symmetry model lagrangian necessary for obtaining the coupling constants of $\rho(770)$ meson and the virtual photon to the state $\pi^+\pi^-\pi^+\pi^-$. The amplitude of the reaction $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$ with the necessary lowest number of derivatives is given in section III. Section IV contains the results of the evaluation of the energy dependence of the reaction $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$ in GHLS and the comparison of the calculations with the data [12, 13] on the reaction $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$. The $\rho^\prime$, $\rho^\prime\prime$ contributions necessary for reconciling the calculation with the data are studied in the same section. Section V is devoted to the discussion of the results and to the comparison of GHLS approach with different models exploited by other authors in order to describe the reaction $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$. Our conclusions are stated in section VI. The divergence equation of the axial vector current allowing for external electromagnetic field and its matrix element pertinent for the reaction $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$ in the hidden local symmetry model is studied in Appendix. This is necessary for studying the Adler limit $q_{\mu\alpha} \rightarrow (0,0,0,0)$ ($q_{\mu\alpha}$ being the four-momentum of any of the final pions) [21] of the $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$ reaction amplitude.

II. FIXING COUPLING CONSTANTS IN GHLS.

The virtue of generalized hidden local symmetry (GHLS) model [9, 10, 11] is that, in the tree approximation, there are no free parameters in the non-anomalous sector except the masses of $\rho$ and $a_1$ mesons and the gauge coupling constant $g = g_{\rho\pi\pi}$ determined from the $\rho \rightarrow \pi^+\pi^-$ decay width, provided $f_\pi$ is known. All couplings including $a_1\rho\pi$ and the direct $a_1\pi\pi\pi$ are fixed by such natural requirements as vector meson dominance, absence of higher derivative $\rho\pi\pi$ coupling, the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation [22]

$$2g_{\rho\pi\pi}^2f_\pi^2 = m_\rho^2,$$

(3)

eq etc. GHLS model was used in Ref. 21 devoted, in particular, to the evaluation of the $\rho \rightarrow 4\pi$ decay width at $\sqrt{s} = m_\rho$. It is important that the electro-weak sector is included into the framework independently of the strong interacting one. This permits one to take into account contact vertices $\gamma^* \rightarrow 4\pi$, $\gamma^* \rightarrow a_1\pi$ which include the virtual photon $\gamma^*$, and the analogous ones with the replacement $\gamma^* \rightarrow W^-$ in case of $\tau^-$ decays.

In order to calculate the $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$ excitation curve in the framework of GHLS model and to compare the result with existing data of CMD-2 [12] and BaBar [13], we use recent calculations of the $\rho \rightarrow 4\pi$ decay amplitudes [20] and add them with the above mentioned contact non-resonant terms whose explicit form is found here.

In order to demonstrate the fixing of the coupling constants in GHLS model, let us give the expressions for the interaction lagrangians following the parameters choice made in Ref. 20 where the necessary notations and details can be found. The boldface characters refer hereafter to the isotopic vectors. The expressions include the following pieces.

(i) The simple hidden local symmetry (HLS) contribution arising in case of neglecting the $a_1(1260)$ contribution

$$\mathcal{L}_{\text{HLS}} = \frac{m_\pi^2}{24f_\pi^2} \pi^4 + \frac{1}{12f_\pi^2} \left[ \pi \times \partial_\mu \pi \right]^2 + g \left( 1 - \frac{\pi^2}{12f_\pi^2} \right) \left( \rho_\mu \cdot \left[ \pi \times \partial_\mu \pi \right] \right).$$

(4)

It generates the $\pi \rightarrow 3\pi$ vertices and the contact $\rho \rightarrow 4\pi$ one.

(ii) The term responsible for the decay $a_1 \rightarrow \rho\pi\pi\pi$ in $3\pi$

$$\mathcal{L}_{a_1\rho\pi\pi} = -\frac{1}{f_\pi} (\partial_\mu \rho_\nu - \partial_\nu \rho_\mu) \cdot \left[ a_\mu \times \partial_\nu \pi \right] - \frac{1}{2f_\pi} (\partial_\mu a_\nu - \partial_\nu a_\mu) \cdot \left[ \rho_\mu \times \partial_\nu \pi \right] - \frac{1}{8g f_\pi} \left[ a_\mu \times \partial_\nu \pi \right] \cdot \left[ \partial_\nu \pi \times \partial_\mu \pi \right] - \frac{1}{4g f_\pi} \partial_\mu a_\nu \cdot \left[ \pi \times \left[ \partial_\nu \pi \times \partial_\mu \pi \right] \right].$$

(5)

It is essential that both $a_1\rho\pi$ and the contact $a_1\pi\pi\pi$ terms are necessary for fulfilling the Adler condition [21] in the chiral limit $m_\pi \rightarrow 0$, that is the vanishing of the amplitude at the vanishing four-momentum of any final pion. This is the point of departure of the present consideration from that of Ref. 23 where the contact terms are absent but the $a_1\rho\pi$ interaction vertex contains additional derivative as compared to Eq. 5 and is characterized by three arbitrary parameters. To be more precise, the first two lines of Eq. 5 and the $a_1\rho\pi\pi$ lagrangian in the paper 23 can be shown to be equivalent, but only on the mass shells of both $a_1(1260)$ and $\rho(770)$ mesons. In our case these two resonances are off mass shells, so that restricting by the first two lines in Eq. 5 would result in breaking of the Adler condition for the $a_1 \rightarrow 3\pi$ decay amplitude. Non-resonant $a_1 \rightarrow 3\pi$ terms written down in Eq. 5 restore chiral symmetry and the Adler condition. The $a_1\rho\pi\pi$ coupling in the paper 23 results in $a_1 \rightarrow 3\pi$ decay amplitude which obeys the Adler condition just due to its higher derivative form.

(iii) There are also the $\rho \rightarrow \rho\pi\pi\pi$ and the higher derivative contact $\rho \rightarrow 4\pi$ terms arising due to the procedure of diagonalization of the axial vector-pseudoscalar mixing added with the counter terms 10(11). They are 20

$$\mathcal{L}_{\rho\rho\pi\pi\pi} = -\frac{1}{16f_\pi^2} \left( \rho_\mu \times \partial_\mu \pi \right)^2.$$
\[
\frac{1}{8g_f^2} [\rho_\mu \times \partial_\nu \pi] \cdot [\pi \times [\partial_\mu \pi \times \partial_\nu \pi]].
\]

(6)

Again, the contact term is necessary for fulfilling the Adler condition of the corresponding contribution to the \(\rho \rightarrow 4\pi\) decay amplitude [20].

(iv) The terms due to the direct photon coupling (\(A_\mu, a_\mu\) stand for the photon four-vector potential and \(a_1(1260)\) meson field, respectively), the Adler condition of the corresponding contribution to the final state, and the contributions of the second order in electric charge \(e\) are neglected. The first, second, third, and fourth terms in Eq. (7) describe, respectively, the \(\gamma^* \rightarrow \rho^0\) transition, the contact \(\gamma^* \rightarrow \pi^+\pi^+\pi^-\), \(\gamma^* \rightarrow \rho^0\pi^+\pi^-\), and \(\gamma^* \rightarrow a_1^+\pi^-\pi^-\) vertices. One should have in mind that the contact \(\gamma^* \rightarrow \pi^+\pi^-\) and \(\gamma^* \rightarrow \pi^+\pi^-\pi^-\) vertices cannot be simultaneously eliminated in HLS [24]. See Appendix for the discussion of the details of the direct pointlike contribution in the hidden local symmetry model.

III. THE AMPLITUDE OF THE REACTION \(e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-\).

Let us represent the energy dependence of the \(e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-\) reaction cross section in the form

\[
\sigma_{e^+e^- \rightarrow 4\pi}(s) = \frac{12\pi m_\pi^3 \Gamma_{\rho e^+e^-}(m_\rho) \Gamma_{\rho \rightarrow 4\pi}(s)}{s^{3/2}|D_\rho(q)|^2},
\]

where the leptonic width of the vector meson \(V\) on the mass shell looks as

\[
\Gamma_{\rho \rightarrow 4\pi}(s) = \frac{4\pi a^2 m_V}{3f_V^3},
\]

and \(s = q^2\) is the total energy squared in the center-of-mass system. The function \(\Gamma_{\rho \rightarrow 4\pi}(s)\) in Eq. (8) can be evaluated with the help of Eq. (5.1) in Ref. [20]. For the purposes of the present work it should be evaluated with the effective \(\rho \rightarrow 4\pi\) decay amplitude \(M_{\rho \rightarrow 4\pi}\) which includes both the resonant contribution \(\rho \rightarrow \pi^+\pi^-\pi^+\pi^-\) due to Eq. (1), (3), and (5) side by side with the contact one \(e^+e^- \rightarrow \gamma^* \rightarrow \rho \rightarrow \pi^+\pi^-\pi^-\) due to the terms Eq. (7).

In the lowest order in electromagnetic coupling constant this amplitude is given by the expression

\[
M_{\rho \rightarrow 4\pi} = \frac{g_\rho \pi}{f_\pi^2} \epsilon_\mu (A_1 q_1 + A_2 q_2 + A_3 q_3 + A_4 q_4),
\]

where \(\epsilon_\mu\) stands for the polarization four-vector of the virtual \(\rho\) meson, and \(A_a \equiv A_a(q_1, q_2, q_3, q_4)\), \(a = 1, 2, 3, 4\) are dimensionless invariant functions. Then, say, \(A_1 \equiv A_1(q_1, q_2, q_3, q_4)\) is given by the expression

\[
A_1 = -1 + (1 + \tilde{P}_{34}) B_1,
\]

\[
B_1 = \frac{2}{D_\pi(q - q_1)} \left[ \frac{m_\rho^2}{D_{\rho 23}} (q_4, q_2, q_3) - (q_2, q_3) \right] - D_\rho(q) \left[ \frac{1}{D_{\rho 14}} - \frac{1}{2 m_\rho^2} \right] - \frac{(1 - \tilde{P}_{23})}{4 D_{a_1}(q - q_1)} \times
\]

\[
\left\{ \frac{1}{D_{\rho 23}} [4(q_2, q_4)(2q_1 - q_2 - q_2) - (2q_2 - q_2)(q_1 + q_1) + (2q_2 - q_1 + q_1)] - \frac{1}{2 m_\rho^2} (q_2 - 2q_2 + q_1 + q_4)(2q_1 - q_1) - \frac{3(q, q_2) - m_\rho^2 - D_\rho(q)}{4 D_{a_1}(q - q_2)} \times \left[ \frac{1}{D_{\rho 13}^2} + \frac{1}{2 m_\rho^2} \right] \right\} - \frac{3(q, q_4) - m_\rho^2 - D_\rho(q)}{4 D_{a_1}(q - q_3)} \left[ \frac{q_2, q_3 - q_2 + q_3}{D_{\rho 23}} - \frac{(q_1 + q_3 - q_2 - q_3)}{2 m_\rho^2} \right] - \frac{3(q, q_4) - m_\rho^2 - D_\rho(q)}{4 D_{a_1}(q - q_3)} \left[ \frac{q_2, q_3 - q_2 + q_3}{D_{\rho 23}} - \frac{(q_1 + q_3 - q_2 - q_3)}{2 m_\rho^2} \right] - \frac{3(q, q_4) - m_\rho^2 - D_\rho(q)}{4 D_{a_1}(q - q_3)} \left[ \frac{q_2, q_3 - q_2 + q_3}{D_{\rho 23}} - \frac{(q_1 + q_3 - q_2 - q_3)}{2 m_\rho^2} \right].
\]

where \(\tilde{P}_{ab}\) is the operator interchanging the pion momenta \(q_a \leftrightarrow q_b\), and \(D_{\rho ab} \equiv D_{\rho}(q_a + q_b)\) is the inverse propagator of \(\rho\) meson with the invariant mass squared \((q_a + q_b)^2\). The inverse propagator of \(\rho\) meson with the four-momentum \(q\) and the invariant mass \(\sqrt{q^2}\) is taken in the form

\[
D_\rho(q) = m_\rho^2 - q^2 - i\sqrt{q^2} \Gamma_\rho(\sqrt{q^2}),
\]

see Eqs. (3.3)–(3.5) of Ref. [20] for explicit expression of \(\Gamma_\rho\). Notice, that the terms \(\propto D_\rho(q)\) in Eq. (11) refer to the contact terms generated by Eq. (7). The remaining
notations are as follows. \((P, Q)\) stands for invariant scalar product of two four-vectors \(P\) and \(Q\). \(D\) is the energy dependent width \(\Gamma\) which interpolates the form of the energy dependent width \(\Gamma\) in the generalized hidden local symmetry model. The data are CMD-2 \cite{12} and BaBar \cite{13}. "HLS" refers to the lagrangian Eq. (4). "GHLS" refers to the model based on lagrangian Eq. (4), (5), (6), and (7). "GHLS, no contact terms" refers to the model without contact terms Eq. (7).

The details of evaluation of \(\Gamma\) are taken from Ref. \cite{25}. The invariant amplitudes \(A_{2,3,4}\) are obtained from \(A_1\) in accord with the relations

\[
A_2 = A_2(q_1, q_2, q_3, q_4) = A_1(q_2, q_1, q_3, q_4),
\]

\[
A_3 = A_3(q_1, q_2, q_3, q_4) = -A_1(q_3, q_4, q_1, q_2),
\]

\[
A_4 = A_4(q_1, q_2, q_3, q_4) = -A_1(q_4, q_3, q_1, q_2).
\]  

The results of evaluation of the \(e^+ e^- \rightarrow \pi^+ \pi^- \pi^+ \pi^-\) reaction cross section in the model specified by the lagrangians Eq. (4), (5), (6), and (7) are shown in Fig. 1. The curves are obtained in the case \(m_{\rho} = 1.23\) GeV; the results for the mass \(m_{\rho} = 1.09\) GeV look qualitatively the same. One can see that the model is unable to reproduce the magnitude of the cross section at energies \(\sqrt{s} > 0.8\) GeV.

Let us include the contributions of heavier resonances \(\rho' \equiv \rho(1450)\) and \(\rho'' \equiv \rho(1700)\) trying to explain the cross section magnitude at \(\sqrt{s} > 0.8\) GeV. Since the far left shoulder of the resonance \(\rho'\) is not a proper place to their study, we choose the simplest parametrization consisting of the Breit-Wigner resonance shape with the constant widths and masses \(m_{\rho'} = 1.149\) GeV, \(\Gamma_{\rho'} = 0.147\) GeV, \(m_{\rho''} = 1.72\) GeV, \(\Gamma_{\rho''} = 0.25\) GeV taken from Ref. \cite{25} and neglect the contribution due to their common decay modes \(\rho(1600)\) trying to explain the cross section magnitude at \(\sqrt{s} > 0.8\) GeV. In addition, we take into account the model of \(a_1\) dominance in the \(\rho', \rho'' \rightarrow 4\pi\) decay dynamics proposed in Ref. \cite{25}, but modify in it in order to make the corresponding terms to obey the Adler condition, see Eq. (9) and Ref. \cite{26}. Then taking into account the \(\rho', \rho''\) resonance contributions results in the factor

\[
R(s) = \left[1 + \frac{D_{\rho'}(q)}{1 + r(s)} \left(\frac{x_{\rho'}^2}{x_{\rho'}^2} + \frac{x_{\rho''}^2}{x_{\rho''}^2}\right)\right]^2,
\]  

multiplying the right hand side of Eq. (8), where \(D_{\rho'}(q) = m_{\rho'}^2 - s - i m_{\rho'} \Gamma_{\rho'}, V = \rho', s = q^2; x_{\rho'}\) and \(x_{\rho''}\) are free parameters to be determined from fitting the data. The meaning of \(x_{\rho'}\) is that

\[
x_{\rho'} = \frac{g_{\rho'} g_{\rho} g_{\rho'} - a_1 \pi \rightarrow 4\pi}{g_{\rho} g_{\rho} g_{\rho'} - a_1 \pi \rightarrow 4\pi},
\]  

analogously for \(x_{\rho''}\), where \(g_{\rho'} - a_1 \pi \rightarrow 4\pi\) etc. means the amplitude corresponding to the specific intermediate state \(a_1 \pi\) followed by both the resonant \(a_1 \rightarrow 3\pi\) and the direct transition \(a_1 \rightarrow 3\pi\) of the intermediate \(a_1\) meson. Since \(\rho\) and \(\rho'\) are assumed here to have the similar coupling to the state \(a_1\pi\), the ratio Eq. (15) is constant. As usual, \(g_{\rho V} = e m_{\rho}^2 / f_V\) stand for the amplitude of the photon-vector meson \(V\) transition, and \(f_V\)
is related with the leptonic width Eq. (9). The complex function \( r(s) \) in Eq. (14) is the ratio of the amplitude with the intermediate \( a_1 \) meson to one with no \( a_1 \) contribution. It approximately takes into account the \( a_1 \pi \) dominance in the four pion decay of heavy isovector resonances. We precalculate it for the fitting purposes for the CMD-2 \[12\] and BaBaR \[13\] data points \( \sqrt{s} \leq 1 \text{ GeV} \) in accord with the relations

\[
\begin{align*}
    r(s) &= \left[ \frac{\Gamma_{\text{eff},\rho_{\pi\to\pi^+\pi^-}}}{\Gamma_{\rho_{a_1\pi\to\pi^+\pi^-}}} \right]^{1/2} \exp(i\chi), \\
    \chi &= \cos^{-1} \left( \frac{\Gamma_{\text{eff},\rho_{\pi\to\pi^+\pi^-}} - \Gamma_{\text{eff},\rho_{a_1\pi\to\pi^+\pi^-}}}{2\sqrt{\Gamma_{\rho_{a_1\pi\to\pi^+\pi^-}} \Gamma_{\rho_{\pi\to\pi^+\pi^-}}}} \right),
\end{align*}
\]

where \( \Gamma_{\rho_{a_1\pi\to\pi^+\pi^-}} \equiv \Gamma_{\rho_{a_1\pi\to\pi^+\pi^-}}(s) \) is the \( \rho^0 \to \pi^+\pi^-\pi^+\pi^- \) decay width due to the intermediate \( a_1 \pi \) state only, while \( \Gamma_{\text{eff},\rho_{\pi\to\pi^+\pi^-}} \equiv \Gamma_{\rho_{\pi\to\pi^+\pi^-}}(s) \) is the effective width of the same decay including all the contribution mentioned above except the \( a_1 \pi \) one. Hence, the approximation of Eq. (16) corresponds to the averaging over four pion phase space. The approximation is necessary, because the direct evaluation would take unacceptable long time for numerical calculations in the fitting procedure.

The CMD-2 \[12\] and BaBaR \[13\] data are taken at different apparatus, with different methods. The systematic uncertainties are usually estimated rather subjectively and are naturally different on each detector. So it is more correct to treat different data sets separately. Although, at first sight, two data sets seem to be compatible, fitting them in the framework of the single model gives different central values of the fitted parameters and \( \chi^2/n\text{d.f.} \), see below.

The results of fitting the CMD-2 data are given in Table I. The curves corresponding to the fit variant 3 are shown in Fig. 2. This is the variant with two additional heavy resonances \( \rho' \) and \( \rho'' \), and it is indistinguishable from the variants with the single resonance \( \rho' \) (variant 1) or \( \rho'' \) (variant 2), both resulting in the same curves as the dashed one shown in Fig. 2. However, variant 3 is based on the destructively interfering large contributions of \( \rho' \) and \( \rho'' \), so that each of the above (not shown) is large as compared to their sum. Variants 4–6 correspond to the fits with the mass of \( a_1 \) meson \( m_{a_1} = m_\rho \sqrt{2} = 1.09 \text{ GeV} \) as given by Weinberg’s relation and result in the same corresponding curves not shown here. One can see that all the fitting variants are not quite good. Nevertheless, we quote the contribution of the sum \( \rho' + \rho'' \) (in variant 3) or \( \rho' \) (variant 1) and \( \rho'' \) (variant 2) relative to the case of pure GHLS contribution (dotted line in Fig. 3) to be 0.3 at \( \sqrt{s} \approx m_\rho \) and 32 at \( \sqrt{s} = 1 \text{ GeV} \). These numbers refer to the case \( m_{a_1} = 1.23 \text{ GeV} \). The case \( m_{a_1} = 1.09 \text{ GeV} \) results in almost the same figures for above ratios.

The results of fitting the CMD-2 data are presented in Table II. Contrary to the previous case, here the variants with the single additional heavy resonance give a bad description. The fit chooses two destructively interfering \( \rho' \) and \( \rho'' \) resonances each coupled to \( a_1 \pi \) much strongly than in the variants of the single heavy resonance. The curves shown in Fig. 3 refer to variant 3 in Table II with \( m_{a_1} = 1.23 \text{ GeV} \). The contribution of the sum \( \rho' + \rho'' \) (in variant 3) or \( \rho' \) (variant 1) and \( \rho'' \) (variant 2) relative to the case of pure GHLS contribution (dotted line in Fig. 3) to be 0.6 at \( \sqrt{s} \approx m_\rho \) and 30 at \( \sqrt{s} = 1 \text{ GeV} \). As in the case of the CMD-2 data, here the variant 6 with \( m_{a_1} = 1.09 \text{ GeV} \) results in practically the same corresponding curves and ratios.

| variant | \( x_{\rho'} \) | \( x_{\rho''} \) | \( \chi^2/N_{\text{d.f.}} \) | \( m_{a_1} \) [GeV] |
|---------|----------------|----------------|-----------------|--------------|
| 1       | \(-27.5 \pm 1.5\) | \(\equiv 0\) | 15.4/10 | 1.23 |
| 2       | \(\equiv 0\) | \(-46.2 \pm 2.5\) | 15.4/10 | 1.23 |
| 3       | 96.8 \pm 1.5 | \(-208.7 \pm 2.5\) | 14.5/9 | 1.23 |
| 4       | \(-17.8 \pm 1.0\) | \(\equiv 0\) | 15.7/10 | 1.09 |
| 5       | \(\equiv 0\) | \(-30.1 \pm 1.5\) | 15.4/10 | 1.09 |
| 6       | 72.5 \pm 1.0 | \(-151.9 \pm 1.6\) | 14.7/9 | 1.09 |

\[ \text{TABLE I: The results of fitting CMD-2 data [12].} \]

\[ \text{TABLE II: The results of fitting BaBaR data [13].} \]

V. DISCUSSION.
As is found in the present paper, the $\rho'$, $\rho''$ contributions are large even at $\sqrt{s} \lesssim 1$ GeV. So, our conclusions differ from the results presented in Ref. [12, 23, 29]. Indeed, the contribution of heavy resonances in Ref. [12] is rather small: the ratio of the $\rho'$ to $\rho$ contributions is 0.02 at $\sqrt{s} = 0.8$ GeV and grows to the figure of 0.15 at $\sqrt{s} = 1$ GeV [12]. Ref. [23] also point to the possibility of describing the low energy data without the $\rho'$ contribution [23] or with the small one [24]. We attribute this disagreement to the difference among the models used in the present analysis and in that of Ref. [12, 23, 28, 29]. Indeed, the parametrization used by CMD-2 [12] is based on effective vertices provided by the isobar model, where one introduces all possible effective terms allowed by Lorentz invariance and G-parity. The amplitude does not obey the demands of chiral symmetry expressed in the property of the divergence of the axial vector current. In contrast, our parametrization is much more restrictive since satisfies requirements of chiral symmetry. Hence, the strong chiral cancellations among different terms in the amplitude take place. This results in much stronger $\rho'$, $\rho''$ contributions. The model in Ref. [28] is also of the kind of effective isobar model. It is purely phenomenological chiral-non-invariant model in that part which concerns $a_1\rho\pi$ coupling. The contact $\gamma^* \rightarrow \pi^+\pi^-\pi^+\pi^-$ is omitted in Ref. [28]. The advantage of the version exploited in our paper is that it is chiral invariant in all sectors and hence is justified from the point of view of basic principles. As compared to Ref. [23] based on chiral amplitude, the present analysis is alternative in that invoked are heavier resonances characterized by two arbitrary parameters $x_{\rho'}, x_{\rho''}$ instead of the higher derivative $a_1\rho\pi$ vertex of Ref. [23] characterized by three arbitrary parameters $c_{1,2,3}$. The authors of Ref. [23] presented only one specific choice of three free parameters without justifying it. They did not give the bounds for variation of their results against going beyond the specific choice made. Even with this single choice, they did not give uncertainties nor $\chi^2/n.d.f.$ necessary for assessment of quality of their approach. In contrast, we clearly state our all assumptions and give the information necessary for assessment of quality of the fits. Taken literally, the amplitude in the paper [23] contains the direct $\gamma^* \rightarrow \pi^+\pi^-$ vertex which breaks vector dominance of the pion form factor. In the effective chiral model used by these authors, such breaking (which is allowed by them) can be in principle avoided by adjusting arbitrary parameters. The necessary adjustment can be implemented only in zero $\rho(770)$ width approximation. As opposed to [23], we include the demand of the vector dominance. It is important that vector dominance can be implemented in HLS without demanding the vanishing $\Gamma_\rho$.

One can show that HLS parameter $a$ enters the $\rho^0 \rightarrow 2\pi^+2\pi^-$ decay amplitude as $a^{1/2}$. This is due to the fact that the above amplitudes contains the factor

$$\frac{g_{\rho\pi\pi}}{f_\pi^2} = \sqrt{\frac{m_\rho}{2f_\pi^2}},$$

provided $g_{\rho\pi\pi}$ is expressed through $m_\rho$ and $f_\pi$ [11]. Hence the variation around $a = 2$ within 20 percent results in variation of overall factor in cross section within the same limits, while the difference between the measures cross section and the calculation in HLS is clearly dynamical effect of a stronger energy dependence than predicted in GHLS with lowest number of derivatives. Hence, the inclusion of HLS parameter $a$ into fits will not result in any appreciable shift of the fitted $\rho'$, $\rho''$ couplings $x_{\rho',\rho''}$.

We intentionally limit ourselves by $s^{1/2} < 1$ GeV because our goal is testing GHLS as specific chiral model, not the study of the $\rho$ excitations. Such rather low energy is necessary in order to rely on the tree chiral amplitudes for vertices but allowing for finite widths for vector mesons. In fact, the relevant invariant masses of the pion pairs are such that the effects of the finite widths of vector resonances in intermediate states are small, hence the loop effects due to finite width are also effectively suppressed in the chosen energy range. Hence, upon choosing above energy range we are in almost pure situation when the tree contribution is dominant. Extending the consideration to higher energies in the framework of chiral models demands inclusion of higher derivatives in effective chiral lagrangian and adding chiral loops. This goes far beyond the scope of our study. At present, the hadron physics community is only at the start of this very difficult road.

The inclusion of scalar resonances whose contributions may be essential [23, 30, 31, 32], deserves another study in the chiral framework, because canonical hidden local symmetry model is based on nonlinear realization of chiral symmetry which does not include scalar mesons.
VI. CONCLUSION.

To conclude, GHLS model which includes the ground state vector and axial vector resonances with the minimal number of derivatives fails to explain the cross section of the reaction $e^+e^- \rightarrow \pi^+\pi^- \pi^+\pi^-$ at energies $0.8 < \sqrt{s} \leq 1$ GeV. One possible way out this difficulty by including heavy resonances $\rho', \rho''$ is studied here. It is found that the contribution of these resonances is much grater than the $\rho(770)$ contribution at $\sqrt{s} \sim 1$ GeV, and comparable with it at $\sqrt{s} \sim m_\rho$. For the sake of simplicity, the assumption Ref. [28] of the $a_1$ dominance in the $\rho', \rho'' \rightarrow \pi^+\pi^-\pi^+\pi^-$ decays is supposed in the present analysis. The model of similar couplings of $\rho(770), \rho', \rho''$ results in qualitatively same conclusions about the fraction of $\rho', \rho''$ resonances. The GHLS chiral model used in the present work is based on the assumption of the nonlinear realization of chiral symmetry. It would be desirable to readdress the present issues in the framework of the chiral model of the vector and axial vector mesons based on the linear $\sigma$-model. This task is necessary in order to evaluate the robustness of the figures characterizing the contributions of heavier resonances towards various model assumptions and to reveal the role of the intermediate states which include the widely discussed scalar $\sigma$ meson. We hope to return to this problem in near future.

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APPENDIX: THE DIVERGENCE OF THE AXIAL VECTOR CURRENT IN HLS MODEL AND THE ADLER CONDITION.

The amplitude $M(\gamma) \equiv M_{a_1}^{\gamma^+\rightarrow \pi^+\pi^-\pi^+\pi^-}$ of direct transition $\gamma^+ \rightarrow \pi^+\pi^-\pi^+\pi^-$ obtained from the lagrangian Eq. (7) upon neglecting (for the technical convenience) of the $a_1$ contribution looks as

$$M(\gamma) = \frac{e}{f_\pi} \left(\epsilon, q_3 + q_4 - q_1 - q_2\right) + 2eg^2 \times$$

$$\left[\frac{(\epsilon, q_1 - q_3)}{D_\rho(q_1 + q_3)} + \frac{(\epsilon, q_2 - q_3)}{D_\rho(q_2 + q_3)} + \frac{(\epsilon, q_1 - q_4)}{D_\rho(q_1 + q_4)} + \frac{(\epsilon, q_2 - q_4)}{D_\rho(q_2 + q_4)}\right], \quad (A.1)$$

where $\epsilon$ stands for the polarization four-vector of the virtual photon. Just this expression is used in that part of Eq. (11) which does not refer to the intermediate $a_1$ meson. The limiting expression of the above amplitude at $m_\rho^2 \rightarrow \infty$, having in mind KSRF relation Eq. (9), is

$$M(\gamma) \approx \frac{e}{f_\pi} \left(\epsilon, q_1 + q_2 - q_3 - q_4\right). \quad (A.2)$$

Notice that

$$(\epsilon, q_1 + q_2 + q_3 + q_4) = 0 \quad (A.3)$$

as the consequence of the transverse character of the (virtual) photon. Setting, say, $q_4 \rightarrow 0$ results in the expression

$$M(\gamma)_{q_4=0} = \frac{2e}{f_\pi} (\epsilon, q_3) \neq 0, \quad (A.4)$$

in contradiction with the Adler condition. Let us show that this breaking of the Adler condition by the point-like $\gamma^+ \rightarrow \pi^+\pi^-\pi^+\pi^-$ contribution is the direct consequence of the breaking of the axial current conservation by electromagnetic field. We perform this task in the simple HLS model neglecting the $a_1$ contribution and in the limit of infinite $\rho$ meson mass $m_\rho \rightarrow \infty$. The inclusion of the intermediate $\rho$ and $a_1$ resonances is straightforward (however, technically cumbersome in case of the $a_1$ contribution) and does not alter the above conclusion.

The hidden local symmetry lagrangian [9, 10, 11] looks like $L_{HLS} = f_\pi^2 \text{Tr} \left(\alpha_{\perp\mu}^2 + a\alpha_{\parallel\mu}^2\right)$, where

$$\alpha_{\perp\mu} = \left(\frac{\partial_\mu \xi_R \xi_L^\dagger - \partial_\mu \xi_L \xi_R^\dagger}{2i}\right),$$

$$\alpha_{\parallel\mu} = \left(\frac{\partial_\mu \xi_R \xi_L^\dagger + \partial_\mu \xi_L \xi_R^\dagger}{2i}\right) - gV_\mu, \quad (A.5)$$

and the kinetic energy of all vector fields $V_\mu, R_\mu$, and $L_\mu$ are omitted because they are irrelevant for the present discussion. We also assume here that the explicit breaking of chiral symmetry necessary to make nonzero masses of the Goldstone bosons is absent. The HLS parameter $a$ is arbitrary, however, the convenient choice $a = 2$ [11] results in KSRF relation Eq. (3) and the vector meson dominance of the pion form factor. The lagrangian is invariant under the transformations

$$L_\mu \rightarrow g_L L_\mu g_L^\dagger - i\partial_\mu g_L g_L^\dagger,$$

$$R_\mu \rightarrow g_R R_\mu g_R^\dagger - i\partial_\mu g_R g_R^\dagger,$$

$$V_\mu \rightarrow h V_\mu h^\dagger - i\partial_\mu h h^\dagger,$$

$$\xi_{L,R} \rightarrow h \xi_{L,R} g_{L,R}^\dagger, \quad (A.6)$$

where $g_{L,R}$ refers to the chiral transformation, while $h$ does to the hidden gauge one. The matrix $U$ in Eq. (1) is expressed as $U = \xi_L^\dagger \xi_L$, and is transformed according to the law $U \rightarrow g_L^\dagger U g_R$. The vector fields $V_\mu$ (corresponding to the resonances $\rho, \omega$, etc) are introduced on the basis of the invariance under the truly local hidden gauge transformation $h$, while and external vector fields $R_\mu$ and $L_\mu$
where $\epsilon$ is the isovector component of the electromagnetic field, that is, the isotopic space) of the axial current vector around the vector found in this way is $j_{\mu} = \partial L_{\mu} / \partial x_{\mu}$, then the axial vector current $j_{\mu, A} = j_{\mu, R} - j_{\mu, L}$ is

$$
j_{\mu, A} = -f_{\pi}^2 \text{Tr} \left( \frac{\partial \mu_{R} \epsilon_{L}^i - \partial \mu_{L} \epsilon_{R}^i}{2} + \epsilon_{A} \left( \frac{\epsilon_{R}^i}{2} \right) \right) \left( \epsilon_{R}^i + \frac{\epsilon_{L}^i}{2} \right) + \epsilon_{A} \left( \frac{\epsilon_{L}^i}{2} \right) - \epsilon_{A} \left( \frac{\epsilon_{R}^i}{2} \right)
$$

where $Q = \frac{\tau^3}{2} + \frac{1}{2}$. Then, taking into account the fact that chiral symmetry is global (constant $\epsilon_{L, R}$), one finds the divergence of the right and left currents as $\partial \mu j_{\mu, R(L)} = \partial L_{\mu} / \partial x_{\mu, R(L)}$. The divergence of the axial vector found in this way is

$$
\partial \mu j_{\mu, A} = \epsilon_{A} \epsilon_{3ab} j_{\mu, A}, \quad (A.8)
$$

where $\epsilon_{abc}$ is totally antisymmetric, and $\epsilon_{123} = 1$. The divergence equation (A.8) looks like the precession (in the isotopic space) of the axial vector current around the isovector component of the electromagnetic field, that is, $j_{3, A}$ is conserved, while $j_{\pm, A}$ is not. Choosing the gauge $\xi_{L} = \xi_{R} = \exp(\imath \sigma \cdot \vec{m})$, and setting $V_{\mu} = \rho_{\mu} \cdot \vec{m}$ one can obtain the soft pion expansion of the axial current to the necessary order (up to three pions):

$$
j_{\mu, A} \approx -f_{\pi} \left\{ \partial \mu \pi - \frac{3a - 4}{2} \pi \times \{ \partial \mu \pi \} \right\} + e(a - 1)A_{\mu} \epsilon_{3ab} \pi^{b} \left( \frac{1 - 2\pi^{2}}{3f_{\pi}^{2}} \right) - a g [\pi \times \rho_{\mu}]^a.
$$

In the limit of heavy $\rho$ meson its field can be replaced by the combination $\rho_{\mu} = -\frac{1}{2g f_{\pi}} [\pi \times \partial \mu \pi]$, resulting from the field equations, so that the axial current in this limit becomes

$$
j_{\mu, A} \approx -f_{\pi} \left\{ \partial \mu \pi + \frac{2}{3f_{\pi}^{2}} [\pi \times \partial \mu \pi] \right\} + e(a - 1)A_{\mu} \epsilon_{3ab} \pi^{b} \left( \frac{1 - 2\pi^{2}}{3f_{\pi}^{2}} \right).
$$

Taking the matrix element of the divergence equation (A.3) relevant for $M(\gamma)_{q=0}$ and setting the HLS parameter $a = 2$, one obtains with the help of Eq. (A.10) the equation

$$
\langle \pi_{q_1}^{+} \pi_{q_2}^{+} \pi_{q_3}^{+} | \partial \mu j_{\mu, A} | \pi_{q_0}^{-} \rangle = \frac{2e}{3f_{\pi}} (\epsilon, 2q_3 - q_1 - q_2) = \frac{2e}{f_{\pi}} (\epsilon, q_3).
$$

[One should kept in mind Eq. (A.3) taken at $q_1 \neq 0$.]

Allowing for the fact that to the leading order the axial current and the gradient of the pion field are related by the factor $-f_{\pi}$ [see Eq. (A.11)], one can see that the breaking of the Adler condition expressed by Eq. (A.4) is the direct consequence of non-conservation of the axial current by external electromagnetic field expressed by Eq. (A.11). The cases of vanishing of other three pion momenta $q_1$, $q_2$, or $q_3$ are treated in the same manner, by taking the relevant matrix elements of the axial current divergence equation.

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