Enhanced Optomechanical Levitation of Minimally Supported Dielectrics

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Optically levitated mechanical sensors promise isolation from thermal noise far beyond what is possible using flexible materials alone. One way to access this potential is to apply a strong optical trap to a minimally supported mechanical element, thereby increasing its quality factor $Q_m$. Current schemes, however, require prohibitively high laser power ($\sim 10$ W), and the $Q_m$ enhancement is ultimately limited to a factor of $\sim 50$ by hybridization between the trapped mode and the dissipative modes of the supporting structure. Here we propose a levitation scheme taking full advantage of an optical resonator to reduce the circulating power requirements by many orders of magnitude. Applying this scheme to the case of a dielectric disk in a Fabry-Perot cavity, we find a simple tilt-based tuning mechanism for optimizing both center-of-mass and torsional mode traps. Notably, the two modes are trapped with comparable efficiency, and we estimate that (e.g.) a 10-μm-diameter, 100-nm-thick Si disc could be trapped to a frequency of 10 MHz with only 30 mW circulating in a cavity of (modest) finesse 1500. Finally, we simulate the effect such a strong trap would have on a realistic doubly-tethered disc. Of central importance, we find torsional motion is comparatively immune to $Q_m$-limiting hybridization, allowing a $Q_m$ enhancement factor of $\sim 1500$. This opens the possibility of realizing a laser-tuned 10 MHz mechanical system with a quality factor exceeding a billion.

A central theme in optomechanics is to use the forces exerted by light to enable new functionality in mechanical systems of all sizes [1, 3]. For example, laser radiation has been used to cool the motion of flexible solids to the quantum ground state [4, 5] at which point quantum motion becomes apparent in the optical spectrum [6, 8]. Equally impressively, ultrathin membrane “microphones” have been made sensitive enough to detect the “hiss” from the quantized nature of incident laser light [9], and the mechanical response to this noise has been shown to squeezed the light [10, 12]. Furthermore, it now seems a realistic goal to create optomechanical force detectors capable of “sensing” the delicate supertorsion forces from a variety of quantum systems, and faithfully imprinting this information on an arbitrary wavelength of light [13, 17], as supported by demonstrations of wavelength conversion in the classical regime [15, 20].

All of these (and other sensing) pursuits are at some level fundamentally limited by the dissipation of the mechanical element. Indeed, any channel by which energy escapes is also a channel by which the thermal environment applies unwanted force noise, limiting measurement sensitivity and destroying coherence. One approach to circumvent this limitation is to use an optically-levitated dielectric particle as the mechanical element. Because laser light can be made to exert a minuscule radiation pressure force noise, such systems are predicted to achieve an unprecedented level of coherence [21, 23], enabling ultrasensitive force / mass detection and quantum optomechanics experiments in a room temperature apparatus. Promising work toward purely levitated systems is currently underway, e.g. Refs. [21, 27].

A complementary approach is to only mostly replace the material supports with light, by applying a strong optical trap to a mechanical element fabricated with minimal material supports. By storing a large fraction of its mechanical energy in the light field, its quality factor $Q_m$ can in principle be increased beyond the limits imposed by the material [28]. The idea can be understood by imagining a simple harmonic oscillator of mass $m$ and material spring constant $K_{\text{mat}}$ stiffened by an essentially dissipationless optical spring $K_{\text{opt}}$. Assuming material dissipation enters as the imaginary component of $K_{\text{mat}}$ [29], the equation of motion is

$$m\ddot{x} + (K_{\text{mat}} + K_{\text{opt}})x = 0. \quad (1)$$

This results in an untrapped mechanical frequency $\omega_m^2 = \sqrt{K_{\text{mat}}/m}$, which can be optically tuned to

$$\omega_m^2 = \omega_{\text{mat}}^2 + \omega_{\text{opt}}^2 \quad \text{(where } \omega_{\text{opt}} = \sqrt{K_{\text{opt}}/m})$$

and a $Q_m$ enhancement by a factor $K_{\text{opt}}/K_{\text{mat}} = \omega_{\text{mat}}^2/\omega_{\text{mat}}^2$. Importantly, $Q_m \propto \omega_m^2$, meaning not only does $Q_m$ increase with frequency, but the overall dissipation rate also decreases, meaning the mass experiences a reduced thermal force noise from the material. If this noise can be made insignificant compared with that of the trapping light, such a system would essentially behave as though it is optically levitated. Note that in this work, we ignore the quantum noise contribution of the laser light, focusing instead on the elimination of coupling to the thermal bath. The effects of radiation pressure shot noise (RPSN) have already been well-analyzed in the context of levitation [21, 22, 28], and could be either nulled out with a single-port optical resonator in the “resolved sideband” regime [30] (i.e. for low-noise applications), or enhanced with an two-port resonator [31] or the “bad cavity” limit (for RPSN measurements and squeezing applications). We briefly revisit these ideas in Section V.

The primary advantage of this “partially levitated” approach is that the mechanical element can be fabricated
in a variety of shapes using standard lithography, and attached to a manageable frame (e.g. as in Fig. 1). This firstly eliminates the need for launch-and-trap techniques, and secondly enables a finer level of control over the device’s orientation with respect to the light field – this is of central importance for increasing the efficiency of the optical trap, as discussed below. Finally, a wide array of optically-incompatible probes (e.g. sharp / scattering tips, nanomagnets, etc) could be fabricated on regions of the device lying outside the optical field, for coupling to external systems such as qubits [14] or nuclear spins [32]. Needless to say, this technology holds the potential to transform how we think about engineering future mechanical sensors and optomechanics experiments.

Section 4 reviews the general features of this quadratic coupling within the context of levitation, and discusses how the quality factor (or finesse) of an optical resonator can be exploited beyond the simple amplification of input light. Sections 5-11 then apply the scheme to the case of a dielectric disc in a Fabry-Perot cavity, and we derive expressions for both the center-of-mass and torsional trap efficiencies. Of note, we find the efficiencies are comparable, and that they can be increased by a factor of order the cavity finesse by properly orienting the disc. Finally, in section 11 we simulate the response of a readily-fabricated device to this newly accessible trap strength. In particular, we find the $Q_m$ associated with torsional motion can be increased by more than three orders of magnitude, meaning a “commonplace” $Q_m \sim 10^6$, 100 kHz oscillator could be optically tuned to a $Q_m$ of a billion at 10 MHz.

I. QUADRATIC COUPLING AND TRAPPING

When two optical modes having opposed linear (dispersive) optomechanical coupling are nearly degenerate, it is possible to create a purely-quadratic readout of a mechanical element’s motion. Here we review this type of coupling within the context of levitation. As discussed below, whether on chip [37, 38], in a fiber cavity [39], or in a macroscopic cavity [30, 34–36, 39–41], the key to fully utilizing the quality factor $Q_\gamma$ (or finesse $F$) of an optical resonator lies in the ability to control the scattering rate between the two optical modes.

![FIG. 1. A proposed levitation geometry, comprising a doubly-tethered dielectric disc immersed in a cavity optical field. Inset shows the intensity profile of the cavity’s TEM$_{00}$ and TEM$_{10}$ modes relative to the structure.](image)

![FIG. 2. Quadratic optomechanical coupling. When two optical modes of linear coupling constants $G_1$ (red) and $G_2$ (green) are degenerate (at $\xi = 0$ and $\omega_1 = \omega_0$), an inter-mode scattering rate $\Gamma$ leads to adiabatic frequencies $\omega_{\pm}$ (black dashed curves). The underlying blue gradient represents the optical (amplitude) susceptibility of the mixed modes, with linewidth $\kappa$ set by the decay rate of the cavity. Holly leaf indicates an optical mode with positive quadratic coupling $\partial^2 \omega_{\pm} > 0$, which can be used to generate a stable optical trap.](image)
To illustrate, consider the generic cavity spectrum drawn in Fig. 2. A mechanical element’s displacement coordinate $\xi$ linearly changes the frequency of two optical modes such that their frequencies are $\omega_1 = \omega_0 + G_1 \xi$ (red line) and $\omega_2 = \omega_0 + G_2 \xi$ (green line), with constants $G_1$, $G_2$, and degenerate frequency $\omega_0$ at $\xi = 0$. If the optical modes also scatter into one another at a rate $\Gamma$ (note this can be via any mechanism, not necessarily the mechanical element itself), the optical eigenmodes will have $\xi$-dependent frequencies (dashed curves) given by

$$\omega_{\pm} = \omega_0 + G_+ \xi \pm \sqrt{G^2 \xi^2 + \Gamma^2} \approx \omega_0 + G_+ \xi \pm (G^2 / 2\Gamma) \xi^2$$

(near $\xi = 0$) with $G_+ \equiv (G_1 \pm G_2) / 2$. Assuming the optical mode responds adiabatically at the mechanical frequency $\omega_m$ (i.e. $\omega_m \ll \Gamma$), each photon populating the upper (+) branch will have an energy $U_+(x) = h\omega_+(\xi)$, thereby exerting a static force $-hG_+$ and an optical spring constant $K_+ \approx hG^2 / \Gamma$. To maximize this per-photon restoring force, one therefore engineers for (or tunes) $G_1$ and $G_2$ to be of opposite sign and as large as possible, and for the scattering rate $\Gamma$ to be as small as possible.

Of course, $\Gamma$ cannot be made arbitrarily small. First, the adiabatic assumption breaks down when $\Gamma \sim \omega_m$, leading to (i) an appreciable lag in the restoring force and (ii) a larger fraction of the cavity light responding linearly, rather than quadratically. Bounding $\Gamma \gtrsim \omega_m$ correspondingly bounds the per-photon trap efficiency to

$$K_+ \lesssim hG^2 / \omega_m$$

where $\omega_m$ is the trapped mechanical frequency. Near and beyond this limit, interesting new effects arise, such as enhanced quadratic readout and St"uckelberg dynamics.

A lag in the restoring force represents a particularly important concern, because it leads to instability via anti-damping. To give a sense of scale, for a spring delay $t_d$, the equation of motion is $\frac{d^2 \xi(t)}{dt^2} = -\omega_m^2 \xi(t - t_d)$, leading to a parasitic anti-damping rate $\approx \omega_m^2 t_d / 2$ in the high-$Q_m$ limit. For practical systems this can be quite significant, especially when compared with the intrinsic damping of a modern mechanical element: even if $t_d$ is of order of the round trip time for light in a 3-cm cavity, the anti-damping rate exceeds 1 kHz for a levitated 1 MHz oscillator; this is many orders of magnitude greater than the linewidth associated with a typical high-$Q_m$ oscillator. However, for quadratic coupling, as is apparent in Ref. 20 (Figs. 2 and 3), this anti-damping can be mitigated while still achieving a high $K_+$ by slightly detuning the laser and/or moving away from the purely-quadratic point. In other words, a small amount of laser cooling can prevent this lag from increasing the effective temperature. With this in mind, a significant advantage of quadratic levitation may be that it does not require a second stabilizing laser, as is the case for linear optomechanical traps.

A second lower bound on $\Gamma$ is the cavity decay rate $\kappa$ (labeled in Fig. 2). If $\Gamma \sim \kappa$, the upper and lower branches are no longer distinct and the system reduces to a mechanical element linearly coupled to two independent optical modes. Bounding $\Gamma \gtrsim \kappa$ places a second upper bound on the per-photon trap efficiency

$$K_+ \lesssim hG^2 Q_\kappa / \omega_0 = hG^2 F / \omega_{FSR}$$

where $\omega_{FSR} = \pi c / L$ is the free spectral range for the case of a Fabry-Perot resonator of length $L$. This illustrates how an optimized scattering rate can utilize the quality factor or finesse of an optical resonator to improve trap efficiency. We emphasize the upper bound is proportional to the power stored inside the cavity, so the enhancement is in addition to the “usual” resonant amplification of the input field. This contrasts the result obtained for thin dielectrics in a standing wave, be it retro-reflected or contained within a cavity. Both schemes generate the same per-watt trap as a free-space standing wave, even if the cavity in the latter case has a very high finesse.

II. OPTIMAL LEVITATION OF A DIELECTRIC DISC

We now describe how a dielectric disc positioned within a Fabry-Perot optical cavity (Fig. 1) can generate an optimal trap for both its center-of-mass (CM) and torsional mode (TM) motion. The Hermite-Gaussian modes $\phi_{nm}(x, y, z)$ provide a natural basis for the electric field in an optical cavity with curved end mirrors. These modes are indexed by one longitudinal ($n$) and two transverse ($\mu$ and $\nu$) integers counting the number of nodes along the $\hat{x}$, $\hat{y}$, and $\hat{z}$ axes respectively. Following first-order optical perturbation theory, the disc’s refractive index $n$ results in a modified Helmholtz equation $\nabla^2 \psi + (\omega^2 / c^2)(1 + V) \psi = 0$, with the perturbation “potential” $V(x, y, z) = n^2 - 1$ that is non-zero only inside the disc. If $V$ perturbs two of the basis modes $\phi_1$ and $\phi_2$ (subscript $i$ corresponds to indices $\eta_\mu \nu_\nu$, $i$ for brevity) so that they become nearly degenerate, the resulting eigenmodes $\psi \approx a_1 \phi_1 + a_2 \phi_2$ and eigenfrequencies $\omega$ satisfy

$$\begin{pmatrix} V_{11} + 1 - \omega_1^2 / \omega^2 & V_{12} \\ V_{21} & V_{22} + 1 - \omega_2^2 / \omega^2 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = 0,$$

where

$$V_{ij} = \iint V \phi_i \phi_j dxdydz,$$

and $\omega_i$ are the unperturbed cavity mode frequencies. The matrix elements $V_{11}$ and $V_{22}$ describe the frequency change of the modes, while the quantity $V_{12} = V_{21}$ determines the size of the gap and mediates hybridization.
where we have defined normalized angles $\Theta$ and $\hat{\Theta}$.

In general, the integrals $V_j$ must be solved numerically, but a simple solution containing all of the relevant physics is possible with a few simplifying assumptions. First, near the cavity mode waist, the optical wavefronts are approximately parallel, and hence the $x$-dependence of the field is approximately sinusoidal:

$$\phi_{\eta\nu z} \approx \frac{H_\mu(Y)H_\nu(Z)e^{-\sqrt{y^2+z^2}/2}}{\sigma_{\mu\nu} e^{i\mu\nu/2}} \cos(k_{\mu\nu}x + \pi\eta/2).$$ (8)

Here, $H_j$ is the $j^{th}$ Hermite polynomial, $Y = \sqrt{2y}/\sigma_{\mu\nu}$ and $Z = \sqrt{2z}/\sigma_{\mu\nu}$ are the transverse coordinates normalized by the cavity mode radius $\sigma_{\mu\nu}$, $L$ is the cavity length, and $k_{\mu\nu} \approx 2\pi/\lambda$ is the effective longitudinal wave number including the (approximately linear) Gouy phase at wavelength $\lambda$.

Second, if the disc tilt angles $\theta_y$ and $\theta_z$ about the $\hat{y}$ and $\hat{z}$ axes (respectively) are small, the bounds of the $x$-integral in $V_j$ are approximately $x_0 + \theta_y y + \theta_z z \pm \theta_\tau/2$, where $x_0$ is the position of the disc center, and $t_\tau = t/\cos(\theta_y) \cos(\theta_z)$ is the tilt-corrected thickness along $\hat{x}$ (given actual thickness $t$).

Third, if we assume mode 1 is an even function of $x$, $y$, and $z$ (e.g. the TEM$_{00}$ mode with $\eta$ even; see Fig. 1), while mode 2 is odd along $x$ and $y$ but even along $z$ (e.g. the TEM$_{10}$ mode with $\eta$ odd), completing the integral over $x$ in Eq. [8] and keeping only second-order terms in the small quantities $\theta_y, \theta_z, t/\lambda$ yields:

$$V_{11} \approx \alpha \{ 1 + \tau C_{\mu\mu_1}(\theta_z)C_{v_1v_1}(\theta_y) \cos 2kx_0 \}$$
$$V_{22} \approx \alpha \{ 1 - \tau C_{\mu\mu_2}(\theta_z)C_{v_2v_2}(\theta_y) \cos 2kx_0 \}$$
$$V_{12} \approx \alpha \tau S_{\mu_1\mu_2}(\theta_z)C_{v_1v_2}(\theta_y) \cos 2kx_0,$$

where we have defined normalized angles $\theta_{\mu\nu} = \sqrt{2k}\theta_{\mu\nu}$, perturbation strength $\alpha = (n^2 - 1)/\theta_\tau L$, thickness correction $\tau = 1 - k^2\theta_\tau/6$, and unitless transverse overlap integrals:

$$C_{ij}(\Theta) = \int_{-\infty}^{\infty} \frac{H_i(\chi)H_j(\chi)}{2\pi i^{1/2}} e^{-\chi^2/2} \cos(\Theta\chi) d\chi$$
$$S_{ij}(\Theta) = \int_{-\infty}^{\infty} \frac{H_i(\chi)H_j(\chi)}{2\pi i^{1/2}} e^{-\chi^2/2} \sin(\Theta\chi) d\chi.$$

Since the modes are nearly degenerate, we approximate $k_{\mu\mu_1v_1} \approx k_{\mu\mu_2v_2} \equiv k$ and $\sigma_{\mu\mu_1v_1} \approx \sigma_{\mu\mu_2v_2} \equiv \sigma$. Finally, we assume the disc radius $r > \sigma$ and approximate the transverse integrals by taking their limits to infinity.

Relaxing this last approximation adds additional prefactors (involving error functions) that reduce the perturbation (especially if $\sigma > r$), but this does not alter the symmetry of the problem or gap-tuning mechanism discussed below.

Without specifying a particular pair of optical modes, we can make basic arguments about how this system will behave by inspecting the symmetry of the integrals $C$ and $S$. For example, if $\theta_y = 0$, the wavefronts along $\hat{y}$ are parallel to the disc surfaces, meaning $C_{ij} \rightarrow \delta_{ij}$ by orthogonality of the Hermite-Gaussian modes, and hence $V_{12} \propto S_{ij} = 0$. In other words, the assumed symmetry along $\hat{y}$ eliminates the possibility of the modes scattering into one another, even if $\theta_y \neq 0$. As a result, when $\theta_z = 0$, the eigenmodes can change frequency, but cannot hybridize, and the avoided gap is zero.

This symmetry is broken when $\theta_z \neq 0$, and we can derive the scaling of the gap $\Gamma$ with $\theta_z$ by inspecting the quantity $\mathcal{C} + i\mathcal{S}$, which can be written:

$$\mathcal{C} + i\mathcal{S} = \sum_m A_m \int \chi^m e^{-\chi^2} e^{i\Theta\chi} d\chi$$
$$= \sum_m A_m e^{-\Theta^2/4} \int (\chi + i\Theta/2)^m e^{-\chi^2} d\chi$$

where the constant coefficients $A_m$ are determined by the Hermite polynomials of Eqs. [10]. Since each real term in the expansion of $(\chi + i\Theta/2)^m$ contains only even powers of $\Theta$ and each imaginary term contains only odd powers, $\mathcal{C}$ and $\mathcal{S}$ must have the form:

$$C_{ij}(\Theta) = e^{-\Theta^2/4} C_{ij}(\Theta)$$
$$S_{ij}(\Theta) = e^{-\Theta^2/4} S_{ij}(\Theta)$$

where $C_{ij}$ and $S_{ij}$ are even and odd polynomials. Hence to lowest order $C_{ij}(\Theta) = (\text{const}) + \Theta^2$ and $S_{ij}(\Theta) \propto \Theta + \Theta^3$. As a result, the scattering rate $\Gamma$ between any modes of this symmetry is expected to be linearly tunable via tilt about the $\hat{z}$ axis. This motivates the incorporation of a second tether along $\hat{y}$ as shown in Fig. 1.

III. EXAMPLE: A TEM$_{00}$ - TEM$_{10}$ CROSSING

To see how this level of control can be achieved in practice, we apply the above formalism to the simplest cavity modes of the appropriate symmetry: TEM$_{00}$ and TEM$_{10}$ (see inset of Fig. 1). In this case, $C_{00} = 1$ and $C_{11} = 1 - 4k^2\sigma^2/6$, and Eqs. [9] reduce to:

$$V_{11} = \alpha \{ 1 + \tau e^{-2k^2\sigma^2(\Theta_\mu_v^2 + \Theta_\nu_z^2)} \cos 2kx_0 \}$$
$$V_{22} = \alpha \{ 1 - \tau e^{-2k^2\sigma^2(\Theta_\mu_v^2 + \Theta_\nu_z^2)} \cos 2kx_0 \}$$
$$V_{12} = 2\alpha \tau k\sigma_\theta e^{-2k^2\sigma^2(\Theta_\mu_v^2 + \Theta_\nu_z^2)} \cos 2kx_0.$$

Figures 3 (a) and (b) show the resulting eigenfrequencies versus $x$ and $\theta_y$ for a 50-nm-thick Si$_3$N$_4$ disc ($n = 2$)
FIG. 3. Single mode and enhanced cavity traps. The cavity comprises two mirrors with radius of curvature 2.5 cm separated by a length $L = 4.9$ cm, and a 50-nm-thick Si$_3$N$_4$ membrane ($n = 2$) positioned near the waist ($x_0 = 0$). Seasonal decorations (holly and baubles) indicate stable cavity traps, solid curves show the analytical theory, and diamonds show a numerical solution with no approximations. (a) Detuning of TEM$_{00}$ and TEM$_{10}$ cavity modes versus displacement $x_0$ for an aligned ($\theta_z = \theta_y = 0$) membrane. (b) Detuning of the same modes versus tilt about the $\hat{y}$ (tether) axis for $x_0 = 0$. (c) and (d) show refined plots of the regions indicated by dotted boxes in (a) and (b) for fixed tilts $\theta_z = 0, 0.1, 0.2, 0.3$ mrad about $\hat{z}$. In both cases the gap is tuned linearly with $\theta_z$.

Aligned ($\theta_z = 0$) in a cavity of length $L = 4.9$ cm and mirror radius of curvature 2.5 cm. Lines show the analytical solution, and diamonds show a “sanity check” numerical solution from Eq. 5 with none of the subsequent approximations. Note these expressions are valid over a much wider range of tilts than those of Ref. [35], sufficient to not only describe the torsional trap of interest, but also how a purely-quartic coupling [36] can be generated with just two cavity modes (see Appendix I).

Seasonal decorations indicate a positive quadratic dependence on $x$ or $\theta_y$, which can be used to generate stable optical traps for CM or TM motion. At these points, the “single-mode” (TEM$_{00}$) spring constants are

$$K_{\text{CM},1} = 4LP\alpha\tau k^2/c$$

$$K_{\text{TM},1} = 4LP\alpha\tau k^2\sigma^2/c,$$

where $P$ is the circulating power in the cavity. We reiterate that for a thin, weakly-perturbing disc, Eq. 14 is identical to the expression derived from a 1D scattering / transfer matrix approach [33], though a 1D theory can of course not describe torsional motion. Also notice that $K_{\text{CM},1}$ and $K_{\text{TM},1}$ differ only by a factor $\sigma^2$, which, together with the disc mass $m$ and moment of inertia $I \approx \frac{1}{4}mr^2$, results in a ratio of TM and CM optical trap frequencies

$$\frac{\omega_{\text{TM}}}{\omega_{\text{CM}}} = \frac{2\sigma}{r}.$$

Importantly, this factor is of order unity when $\sigma \sim r$ (though it never exceeds unity due to the breakdown of approximated integral bounds), so torsional motion can be trapped with an efficiency comparable to the center of mass. The reduction in TM trap efficiency for smaller $\sigma$ can be understood as arising from the non-uniform spring constant density across the disc: mechanical modes having more displacement near the center of the disc (where
the intensity is higher) experience a stronger integrated restoring force. In the opposite “large spot” limit $\sigma \gg r$ this factor approaches unity at the expense of a reduced trapping efficiency for both CM and TM. The effect of finite spot size on the resulting $Q_m$-enhancement is addressed in Section IV.

Figures 2(c) and (d) show the crossings in more detail, for $\theta_z$ between 0 to 0.3 milliradians. In both cases the avoided gap can be tuned linearly with $\theta_z$, as expected.

In principle, the crossing points can also be tuned (e.g. via cavity length) to occur where the slope of the uncoupled optical modes is nearly maximal. In Fig. 3 for example, $L$ was chosen because the CM and TM crossings simultaneously occur near their maximal slopes, and varying $L$ allows one to shift TEM$_{10}$ vertically with respect to TEM$_{00}$. Assuming the crossings have been tuned to the optimal points, the maximum enhancement of the spring constants $K_{CM,2}$ and $K_{TM,2}$ for this 2-mode scheme is

$$\frac{K_{CM,2}}{K_{CM,1}} = \frac{\omega_{FSR}(n^2 - 1)\tau}{\lambda \tau} \quad (17)$$

$$\frac{K_{TM,2}}{K_{TM,1}} = \frac{\omega_{FSR}(n^2 - 1)\tau}{\lambda \tau} \quad (18)$$

where $\omega_{FSR} = \pi c/L$. For the present example, a gap $\Gamma/2\pi \sim 1$ MHz corresponds to a factor of order $\sim 200$ increase in the per-photon restoring force. Since this enhancement scales as the ratio $\omega_{FSR}/\Gamma$, it is apparently (perhaps not surprisingly) beneficial to use a shorter cavity at fixed $\Gamma$. A 1 mm cavity, for example, could achieve an enhancement factor of order $\sim 30,000$.

Other simple materials exerting a larger perturbation, e.g. a 110-nm-thick Si disc (see Appendix II) or a high-reflectivity structured dielectric [28], readily achieve a linear coupling that is very near the maximum possible for a 100%-reflective disc, $G_{max} = 4\pi c/\lambda L$. In this case, for scattering rate $\Gamma$, the best possible per-photon CM trap efficiency would be

$$K_{\text{ultimate,CM}} = 16\pi P/\lambda \Gamma L. \quad (19)$$

For these larger perturbations, however, additional cavity modes must be included in the theory (see Appendix II) and it becomes impossible to derive simple analytical expressions describing the avoided crossings. However, the symmetry arguments of Section II remain valid and can be generalized to more modes, meaning the CM and TM gaps will still be linearly tunable via $\theta$. As shown in Appendix II, the CM and TM traps should also still have comparable strength. Equation 19 can therefore be used to roughly estimate the expected trap strengths for a variety of geometries. For example, a 110-nm-thick Si disc of radius $r = 5$ $\mu$m in fiber cavity of length $L = 100$ $\mu$m and finesse $F = 1500$, should achieve (for a finesse-limited gap $\Gamma = 2\pi \times 500$ MHz) levitated frequencies $\sim 10$ MHz with only 30 $mW$ of circulating power at $\lambda = 1550$ nm. This represents a significantly stronger trap requiring much less power than for a single-mode or retro-reflected approach.

As discussed in Section II, the finesse-limited upper bound (Eq. 4) can be written

$$K_{\text{ultimate,F}} = 32PF/\lambda c. \quad (20)$$

Stated briefly, the tilt-control afforded by a second tether provides a means to fully utilize the cavity finesse, without the requirement of engineering a highly-reflective disc.

IV. TORSIONAL LEVITATION

To investigate how a realistic mechanical element might react to these strong traps, we now discuss a finite-element simulation (COMSOL) of a doubly-tethered disc such as the one in Fig. 4, the disc and tethers are patterned from a single-crystal silicon sheet of thickness $t = 110$ nm, with disc radius $r = 5$ $\mu$m, tether length $l = 45$ $\mu$m, and tether width $w = 100$ nm. This mechanical element is chosen for its relative ease of fabrication (standard lithography with a silicon-on-insulator wafer), low optical absorption at telecom wavelengths $\lambda = 1550$ nm, high reflectivity, low internal stress, and high power handling via a thermal conductivity two orders of magnitude above that of SiO$_2$. The optical trap is modeled as a restoring force along $\hat{x}$ with a spring constant density $K_{\text{opt}}(y, z) \propto e^{-(y^2+z^2)/2\sigma^2}$, where $\sigma$ is the width of the Gaussian beam [28].

Figure 4(a) shows the relevant (untrapped) mechanical modes. The center of mass ($s_1$), torsional ($t_1$) and antisymmetric ($a_1$, $a_2$) modes all involve displacement along $\hat{x}$ and are optically trapped, as shown in Fig. 4(b). Here, $\sigma = 5$ $\mu$m, and the trap strength is normalized by the response of a perfectly-rigid, tether-free disc in free space (solid black line in Fig. 4(b)) to remove the dependence of the result on trap efficiency. Consistent with the aforementioned geometrical considerations, the torsional mode is only slightly less responsive than the CM mode (from the slopes of the linear regions at higher trap strength, $K_{TM}$ is estimated to be within 10% of $K_{CM}$).

To describe the effect of the optical trap on $Q_m$, we follow Ref. [28] and parameterize the $Q_m$ enhancement in terms of the potential energy stored in the light field $U_{\text{opt}} \propto K_{\text{opt}}$ and material $U_{\text{mat}} \propto K_{\text{mat}}$. Equation 4 then predicts $Q_m = Q_{\text{mat}}(K_{\text{opt}} + K_{\text{mat}})/K_{\text{max}} \approx Q_{\text{mat}}(U_{\text{opt}}/U_{\text{mat}})$ for $U_{\text{opt}} \gg U_{\text{mat}}$. This enhancement is shown in Fig. 4(c). In agreement with Ref. [28], we find that the achievable enhancement depends strongly on the symmetry of the involved mechanical modes. The optical trap hybridizes nominally orthogonal modes of the same symmetry when they are brought into degeneracy, appearing as an avoided crossings in the mechanical frequencies and a quenching of $U_{\text{opt}}/U_{\text{mat}}$. For example, the enhancement of the CM (“symmetric”) mode $s_1$ plummets near 500 kHz due to hybridization with the symmetric tether mode $a_2$. A similar avoided crossing between the antisymmetric modes $a_1$ and $a_2$ can also be seen near 600 kHz, where the concavity of the frequency...
These values of $\sigma/rQ$ of the disc (inset iii). Shown for comparison is the torsional $\omega$ its higher initial increases monotonically in Fig. 4(c) (red curve). Due to with any of these low-frequency modes, and $U_{\text{opt}}/U_{\text{mat}}$ plummets from hybridization with “violin” modes such as $s_2$. (d) Enhancement for the CM-like (blue) and torsional (red) modes at higher trap strength. For each group, the lightest, intermediate, and darkest shades correspond to $\sigma/r = 0.5, 0.75$ and 1. The CM mode $Q_m$-enhancement is continuously limited by hybridization with tether modes, (e.g. inset i) whereas the torsional hybridization is suppressed (inset ii) until the finite spot size of the trap causes coupling with the “flappy” mode of the disc (inset iii). Shown for comparison is the torsional $Q_m$-enhancement for an infinitely stiff structure (black curve). These values of $\sigma/r$ are compatible with high finesse cavities: even $\sigma/r = 1$ could achieve $F > 10^4$ for a properly manufactured structure [28].

Tuning for $a_1$ inverts. Tether mode hybridization represents a fundamental limitation of CM trapping, placing the ceiling $U_{\text{opt}}/U_{\text{mat}} < 50$ as shown in Fig. 4(c) (blue curves).

Most significantly, the torsional mode does not couple with any of these low-frequency modes, and $U_{\text{opt}}/U_{\text{mat}}$ increases monotonically in Fig. 4(c) (red curve). Due to its higher initial $\omega_{\text{mat}}$, the overall enhancement is not as immediately large. However, with access to a MHz-scale optical trap, its enhancement can continue far beyond what is possible with the CM mode (or equally any of the other modes), as shown in Fig. 4(d): while the CM mode repeatedly hybridizes with tether modes (e.g. inset i), the torsional mode remains essentially unchanged to a much higher frequency (ii).

The eventual ceiling on this enhancement does not necessarily come from the (torsional) tether modes, because (unlike “violin string” modes) their frequencies can be made arbitrarily high by decreasing the tether width; in this case the first torsional tether mode occurs around 57 MHz. Instead, the torsional mode hybridizes with the first “flappy” mode of the disc (inset iii), which is not as efficiently trapped. In other words, the laser spot strongly pins the center of the disc, and the edges are then relatively free to flap. Figure 4(d) shows simulations performed with $\sigma/r = 0.5, 0.75$, and 1 (red curves). As the trap is made more uniform across the disc, the flappy modes are trapped more efficiently, and the onset of hybridization occurs at higher frequencies. We find this enhancement factor is somewhat insensitive to device thickness, diameter, or choice of material, since these changes equally affect all modes.

In the end, the maximum obtainable $Q_m$ increase for a given geometry involves a trade-off between the trap uniformity, the trap strength per photon, and the degradation of finesse associated with diffraction from the disc. In spite of this, the ratio $\sigma/r = 1$ used here is in principle compatible with a cavity finesse of $10^4$ or larger for a properly engineered disc [28]. As shown in Fig. 4(d), this corresponds to a $Q_m$ enhancement factor of $\sim 1500$. Assuming an untrapped $Q_m \sim 10^6$, the torsional mode could therefore achieve a $Q_m$ of a billion and frequency of order 10 MHz. Further improvements can be made at the expense of cavity finesse by further increasing the spot size, and as discussed in Section III the finesse requirements are not necessarily stringent.
V. SUMMARY AND DISCUSSION

We have described an efficient optical levitation scheme based on quadratic cavity optomechanical coupling, and described how to realize it with a dielectric disc in a Fabry-Perot cavity. This scheme leads to a strongly enhanced trap for both the center-of-mass and torsional motion of the disc, and simulations suggest that these traps allow the torsional mode to achieve a $Q_m$-enhancement factor far exceeding what is possible with any other mode. Using a trap geometry compatible with a finesse $> 10,000$, we predict a $Q_m$-increase of more than three orders of magnitude for a silicon device that can be readily fabricated with standard techniques. This scheme therefore presents a practical platform for a variety of applications ranging from frequency-tuned high-resolution force sensing to quantum optomechanics experiments.

In the above analysis, we focused primarily on how to generate an efficient trap, and how this in turn produces a mechanical element that is highly isolated from the thermal environment of the material. We reiterate that some care must be taken in the cavity design, depending on whether the goal is to enhance the effect of the trap’s quantum noise (i.e. in the “unresolved sideband” limit $\omega_m \ll \kappa$), or to suppress it ($\omega_m \gg \kappa$). It is also worth noting that while purely quadratic optomechanical coupling can be realized with an ideal single-port cavity (see theory of Ref. [30]), the two optical modes discussed in Section II ensure the presence of a second port, even if one cavity mirror is perfectly reflective. This can lead to a significant linear contribution to the RPSN [31]. On the other hand, cavity light landing on the disc’s flat surface will tend to scatter back into the cavity mode, which can then be collected and manipulated. It remains an interesting question to what extent the effect of a second port could be interferometrically suppressed, or how RPSN might be further controlled by pre-squeezing the trap light.

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APPENDIX I: QUARTIC TRAPS

As mentioned in Section III, the expressions for $V_{ij}$ in Eq. [13] can make predictions for a much wider range of tilts than those of previous work [35, 36]. As shown in Figure 5, this analytical solution describes how a purely-quartic optomechanical coupling such as the one observed in Ref. [36] could be generated with two transverse modes of the cavity. Panel (a) shows the detuning of the TEM00 and TEM10 cavity modes versus displacement of a 50-nm-thick, Si3N4 disc for a cavity of length $L = 4.7$ cm and $\theta_z$ between 0 and 6 milliradians. (b) Detuning versus tilt $\theta_y$ under the same conditions. Both (a) and (b) exhibit a transition from quadratic (6 mrad) to double-well (2 mrad) potentials. In between (near 4 mrad), the optomechanical coupling is purely quartic.

APPENDIX II: STRONGER PERTURBATIONS

An ideal structure for trapping has low residual stress, high thermal conductivity, and is maximally reflective. As a test material, we selected 110-nm-thick single-crystal silicon, which has an index of $n = 3.48$ and is approximately a quarter wavelength thick at $\lambda = 1550$ nm. However, such a strongly-perturbing dielectric slab can no longer be treated using a simple first order perturbation theory with a small number of modes. If we naively solve perturbation theory for such a structure including only one longitudinal TEM00 mode (i.e. the dashed red line in Figure 4(a)), the problem is immediately apparent: the perturbation is larger than the free spectral range, and inspecting Eq. [6] we expect different longitudinal modes with the same transverse profile to strongly scatter into one another. This intuition is confirmed by the complete lack of agreement with a 1D transfer matrix approach (underlying gradient scale). However, this simplified few-mode perturbation theory still quantitatively agrees with the transfer matrix approach so long as the perturbation is sufficiently small compared to the free spectral range. This is why only a

![Figure 5. Purely quartic CM and TM coupling. (a) Detuning of the TEM00 and TEM10 cavity modes versus displacement of a 50-nm-thick, Si3N4 disc for a cavity of length $L = 4.7$ cm and $\theta_z$ between 0 and 6 milliradians. (b) Detuning versus tilt $\theta_y$ under the same conditions. Both (a) and (b) exhibit a transition from quadratic (6 mrad) to double-well (2 mrad) potentials. In between (near 4 mrad), the optomechanical coupling is purely quartic.]
FIG. 6. Limits of perturbation theory. (a) Detuning of the TEM$_{00}$ cavity resonances (near wavelength $\lambda = 1.55$ $\mu$m) for a cavity of length $L = 3.5$ cm and mirror radius of curvature 2.5 cm, as a function of disc displacement $x_0$ from the waist. The disc is 110 nm thick, and made of single-crystal silicon (index $n = 3.48$), and aligned with the cavity mode ($\theta_z = \theta_y = 0$). Curves show the results of first-order degenerate perturbation theory, including (i) a single TEM$_{00}$ mode, (ii) $\pm 2$ TEM$_{00}$ modes (i.e. 5 in total), and (iii) $\pm 100$ (201 total) TEM$_{00}$ modes. The gradient scale shows transmission through the cavity calculated using transfer matrices in 1D for comparison (end mirror amplitude transmission coefficients set to 0.3 for visibility). The qualitative behavior of perturbation theory approaches that of the transfer matrices, however including $\pm 1000$ modes does not noticeably change the result from the solid black curve, hinting that this perturbation is too large to be quantitatively captured by a first-order theory. (b) Displacement-mediated avoided crossings between the TEM$_{00}$ and TEM$_{10}$ modes, including $\pm 100$ of each type (402 in total), for $\theta_z = 0, 0.1, 0.2$, and 0.3 milliradians. The gap tuning mechanism is roughly linear in $\theta_z$, though a shift in the crossing point is introduced via interactions with adjacent longitudinal modes. (c) Tilt-mediated avoided crossings under the same conditions as (b). The gap tuning is again linear in $\theta_z$, but the crossing modes are no longer symmetrically tuned by $\theta_z$, leading to a skewed crossing. Using this numerically-determined quadratic coupling, a disc of radius $r = 75$ $\mu$m (i.e. equal to the TEM$_{00}$ spot size) experiences a trap frequency ratio $\omega_{\text{TM}}/\omega_{\text{CM}} = 0.43$.

Despite the requirement of additional modes, the tilt-based gap-tuning mechanism, which is based on symmetry, should fundamentally remain the same. Figures 6(b) and (c) show the behavior of avoided gaps in both displacement and tilt including 201 TEM$_{00}$ and 201 TEM$_{10}$ modes. The additional longitudinal modes introduce some drift in the location of the crossing in (b) and some skew to the linear coupling of the underlying modes in (c), but the overall behavior is otherwise identical: the gap scales roughly linearly with $\theta_z$, and the ratio of trap frequencies $\omega_{\text{TM}}/\omega_{\text{CM}} = 0.43$ for $r = \sigma$ (determined numerically from the shown curves) is of order unity for a given gap. Scattering between the TEM$_{00}$ and higher-order modes of the correct symmetry should be smaller than for the TEM$_{10}$ mode, but they should probably still be considered in a more careful (future) analysis.

If we include additional longitudinal modes in the “nearly degenerate” manifold, the agreement with the 1D transfer matrix result improves. The blue dashed curve in Fig. 6(a) shows the result of perturbation theory including 5 adjacent longitudinal modes ($\pm 2$ modes, plus the original), and the black curve shows the result for 201 modes ($\pm 100$). At first glance the results seem to be converging to the 1D model, but including, say, 2001 or more modes adds CPU cycles without causing a significant change, implying that this level of perturbation is likely just beyond the grasp of a first-order theory (perhaps not surprisingly). Note that in this calculation we made a point of including the minute differences in mode waist for the longitudinal modes so that the solution could actually converge (albeit incredibly slowly).

few modes are required for the 50-nm-thick silicon nitride disc discussed above.

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