Research Note on Uncertain Probabilities and Abstract Argumentation

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The sixth assessment of the international panel on climate change (IPCC) states that “cumulative net CO2 emissions over the last decade (2010-2019) are about the same size as the 11 remaining carbon budget likely to limit warming to 1.5°C (medium confidence).” Such reports directly feed the public discourse, but nuances—such as the degree of belief and of confidence—are often lost. In this paper, we propose a formal account for allowing such degrees of belief and the associated confidence to be used to label arguments in abstract argumentation settings. Differently from other proposals in probabilistic argumentation, we focus on the task of probabilistic inference over a chosen query building upon Sato’s distribution semantics which has been already shown to encompass a variety of cases including the semantics of Bayesian networks. Borrowing from the vast literature on such semantics, we examine how such tasks can be dealt with in practice when considering uncertain probabilities, and discuss the connections with existing proposals for probabilistic argumentation.

1 Introduction

The sixth assessment of the international panel on climate change (IPCC) states that “cumulative net CO2 emissions over the last decade (2010-2019) are about the same size as the 11 remaining carbon budget likely to limit warming to 1.5°C (medium confidence)” [1, p. TS-16]. Such reports directly feed the public discourse, but nuances—such as the degree of belief and of confidence—are often lost.

The degree of belief and the confidence in such a degree, however, refer to two different notions of uncertainty: aleatory (or aleatoric), and epistemic uncertainty [2]. Aleatory uncertainty refers to the variability in the outcome of an experiment which is due to inherently random effects (e.g., flipping a fair coin): no additional source of information but Laplace’s daemon3 can reduce such a variability. Epistemic uncertainty refers to the epistemic state of the agent using the model, hence its lack of knowledge that—in principle—can be reduced on the basis of additional data samples (e.g., being in the position to assess whether a coin is fair or not requires trials).

The ultimate goal of this research is to support human decision makers in their tasks of reasoning in presence of aleatory and epistemic uncertainty. Beta distributions (Section 2.1) can provide a compact representation of both aleatory and epistemic uncertainty when dealing with binary variables [3], i.e., that can be only true or false, or—in argumentation terms—arguments that can be accepted or not according to some accep-
tance criterion: in particular we focus on credulous acceptance. Moreover, beta distributions can be also translated into expressions such as *likely with some confidence* [5, p. 49].

For what concerns the reasoning under aleatory and epistemic uncertainty, most of the community’s efforts focused on Bayesian networks and probabilistic logic programming under distribution semantics [5,7], which we will illustrate in the form of algebraic model counting [8] (Section 2.2). In particular, [9] looks at aleatory and epistemic uncertainty propagation in singly-connected Bayesian networks; [10] provides efficient mechanisms for embedding aleatory and epistemic uncertainty in probabilistic logic programs; and finally [11] generalises the approach considering probabilistic circuits.

We then introduce the distribution semantics in abstract argumentation (Section 3) at first considering only probabilities, with an in-depth discussion of similarities and differences with the constellation approach to probabilistic argumentation [12,13,14]. We then show how the same semantics can handle beta distributions by leveraging recent work on probabilistic circuits [11]. Section 4 concludes the paper pencilling in a series of related research questions that should be addressed by this line of research.

2 A Formal Account of Aleatory and Epistemic Uncertainty using Beta Distributions

Scientific assessments of complex phenomena [15] require to consider not only their likelihood but also the overall confidence in the exercise. Let us suppose we know there is a coin that can be flipped, and a source of information is telling us that it is unlikely with low confidence that it will land on the head. This is a way to express uncertain probabilities, for instance because of limited observations: just observing a few times a coin flipping we can only have a low confidence on whether the coin is fair or not.

2.1 Representing Epistemic and Aleatory Uncertainty as Beta Distributions

Epistemic and aleatory uncertainty can be jointly captured by a beta distribution, namely a distribution of possible probabilities. When facing a phenomenon with just two outcomes a complete dataset \( D \) is a sequence (allowing for repetitions) of examples, each being a vector of instantiations of independent Bernoulli distributions with true but unknown parameter \( \pi \). From this, the likelihood is thus: 

\[
p(D \mid \pi) = \prod_{n=1}^{|D|} p(x_n \mid \pi) = \prod_{n=1}^{|D|} \pi^{x_n}(1 - \pi)^{1-x_n}
\]

where \( x_i \) represents the \( i \)-th example in the dataset \( D \), that is assumed to hold either the value 1 or 0.

To develop a Bayesian analysis of the phenomenon, we can choose as prior the beta distribution, with parameters \( \alpha = (\alpha_x, \alpha_{\pi}) \), \( \alpha_x > 0 \) and \( \alpha_{\pi} > 0 \), that is conjugate to the Bernoulli: 

\[
\text{Beta}(\pi \mid \alpha) = \frac{\Gamma(\alpha_x + \alpha_{\pi})}{\Gamma(\alpha_x)\Gamma(\alpha_{\pi})} \pi^{\alpha_x - 1}(1 - \pi)^{\alpha_{\pi} - 1}
\]

where \( \Gamma(t) \equiv \int_0^\infty u^{t-1} e^{-u} du \) is the gamma function.

Considering a beta distributed prior Beta(\( \pi \mid \alpha^0 \)) and the Bernoulli likelihood function, and given \(|D|\) observations \( m = \langle m_x, m_{\pi} \rangle \) of \( x \), viz.,
$m_x$ observations of $x = 1$, $m_\tau$ observations of $x = 0$, and $m_x + m_\tau = |\mathcal{D}|$:  
\[
p(\pi \mid \mathcal{D}, \alpha^0) = \text{Beta}(\pi \mid \alpha^0 + m).
\]

Thus, the parameters of a beta distribution can be considered pseudo-counts \cite{16} of pieces of evidence for the two outcomes of a phenomenon, and the beta distribution itself can be seen as a representation of the uncertain probability associated with the phenomenon. Among the various priors, using $\alpha^0 = 1 = (1,1)$ is equivalent to using the uniform distribution, which represents a non-informative prior that maximises entropy.

Given a beta-distributed random variable $X$, $s_X = \alpha_x + \alpha_\tau$ is its Dirichlet strength and $\mu_X = \frac{s_\tau}{s_X}$ is its mean, which relates to the aleatory uncertainty. The variance of a beta-distributed random variable—which associates to the epistemic uncertainty—$X$ is $\sigma^2_X = \frac{\mu_X(1-\mu_X)}{s_X+1}$.

Following \cite{5, p. 49} we can represent fuzzy labels\footnote{The list of fuzzy labels for aleatory uncertainty is \cite{8, p. 49}: absolutely not likely, very unlikely, unlikely, somewhat unlikely, chances about even, somewhat likely, likely, very likely, absolutely likely. The list of fuzzy labels for epistemic uncertainty is \cite{8, p. 49}: no confidence, low confidence, some confidence, high confidence, total confidence.} such as likely with low confidence by means of beta distributions, by associating the first label to ranges of expected values (aleatory uncertainty) and the latter to ranges of variance values (epistemic uncertainty): the lower the confidence, the higher the variance. For instance, a proposition that is likely true with some confidence could be associated to a beta distribution Beta(5,00,1,50) as its expected value (0.7692) sits at the centre of the chosen range of values for the fuzzy label likely, and analogously the variance (0.0237) for the fuzzy label some confidence.

### 2.2 Logical Reasoning with Beta Distributions

In \cite{11} the authors provide a formal account for reasoning over probabilistic circuits which can be derived from propositional theories through knowledge compilation. In this setting, probabilistic inference is a special case of algebraic model counting (AMC) \cite{8}, which generalises weighted model counting (WMC) to the semiring setting and supports various types of labels, including numerical ones as used in WMC, but also sets, polynomials, Boolean formulae, and many more. The underlying mathematical structure is that of a commutative semiring.

A semiring is a structure $\langle \mathcal{A}, \oplus, \otimes, e^\oplus, e^\otimes \rangle$, where addition $\oplus$ and multiplication $\otimes$ are associative binary operations over the set $\mathcal{A}$, $\oplus$ is commutative, $\otimes$ distributes over $\oplus$, $e^\oplus \in \mathcal{A}$ is the neutral element of $\oplus$, $e^\otimes \in \mathcal{A}$ that of $\otimes$, and for all $a \in \mathcal{A}$, $e^\oplus \oplus a = a \otimes e^\otimes = e^\oplus$. In a commutative semiring, $\otimes$ is commutative as well.

Algebraic model counting is now defined \cite{8} as follows. Given: a propositional logic theory $T$ over a set of variables $\mathcal{Y}$, a commutative semiring $\langle \mathcal{A}, \oplus, \otimes, e^\oplus, e^\otimes \rangle$, and a labelling function $\rho : \mathcal{L} \to \mathcal{A}$, mapping literals $\mathcal{L}$ of the variables in $\mathcal{Y}$ to elements of the semiring set $\mathcal{A}$, compute

$$A(T) = \bigoplus_{l \in \mathcal{L}} \bigotimes_{\mathcal{M}(T) \models l} \rho(l),$$  \hspace{1cm} (1)

where $\mathcal{M}(T)$ denotes the set of models of $T$.

### Probabilistic Inferences

Among others, AMC generalises the task of probabilistic inference according to \cite{6}'s semantics (PROB). \cite{8} Thm. 1]. A query $q$ is a finite set of algebraic literals $q \subseteq \mathcal{L}$. We denote the set of
interpretations where the query is true by $\mathcal{I}(q)$,
\[
\mathcal{I}(q) = \{ I \mid I \in \mathcal{M}(T) \land q \subseteq I \}. \tag{2}
\]

The label of a query $q$ is defined\(^5\) as the label of $\mathcal{I}(q)$,
\[
\Lambda(q) = \Lambda(\mathcal{I}(q)) = \bigoplus_{I \in \mathcal{I}(q)} \bigotimes_{l \in I} \rho(l).
\tag{3}
\]

As both operators are commutative and associative, the label is independent
of the order of both literals and interpretations.

In the case of probabilities as labels, i.e., $\rho(\cdot) \in [0,1]$, \(^4\) presents the
AMC parametrisation for handling PROB of queries:
\[
\begin{align*}
\mathcal{A} &= \mathbb{R}_{>0} \\
\rho_{\ominus} &= 0 \\
\rho_{\otimes} &= 1 \\
\rho(f) &\in [0,1] \quad \text{and} \quad \rho(f) = \rho(\neg f) = 1 - \rho(f)
\end{align*}
\tag{4}
\]

A naive implementation of \(^{[1]}\) is clearly exponential: \(^{[17]}\) introduced
the first method for deriving tractable circuits (d-$\text{DNNFs}$) that allow poly-
time algorithms for clausal entailment, model counting and enumeration.
In particular, we can exploit the succinctness results of the knowledge
compilation map by \(^{[13]}\), where an overview of succinctness relationships
between various types of circuits is provided. Instead of focusing on classical,
flat target compilation languages based on conjunctive or disjunctive
normal forms, \(^{[13]}\) considers a richer, nested class based on representing
propositional sentences using directed acyclic graphs: NNFs. A sentence in
\textit{negation normal form (NNF)} over a set of propositional variables $\mathcal{Y}$ is
a rooted, directed acyclic graph where each leaf node is labeled with true
($\top$), false ($\bot$), or a literal of a variable in $\mathcal{Y}$, and each internal node with
disjunction ($\lor$) or conjunction ($\land$).

An NNF is \textit{decomposable} if for each conjunction node $\bigwedge_{i=1}^{n} \phi_i$, no two
children $\phi_i$ and $\phi_j$ share any variable. An NNF is \textit{deterministic} if for each
disjunction node $\bigvee_{i=1}^{m} \phi_i$, each pair of different children $\phi_i$ and $\phi_j$ is logi-
cally contradictory, that is $\phi_i \land \phi_j = \bot$ for $i \neq j$. In other terms, only one
child can be true at any time. An NNF is \textit{smooth} when $\text{vars}(\phi_i) = \text{vars}(\phi_j)$
for any two children $\phi_i$ and $\phi_j$ of a disjunction node, where $\text{vars}(x)$ denotes
all propositional variables that appear in the subgraph rooted at $x$.

The function $\text{EVAL}$ specified in Algorithm\(^{[1]}\) \textit{evaluates} an NNF circuit
for a commutative semiring $(\mathcal{A}, \oplus, \otimes, e^\oplus, e^\otimes)$ and labelling function $\rho$.
Evaluating an NNF representation $N_T$ of a propositional theory $T$ for a
semiring $(\mathcal{A}, \oplus, \otimes, e^\oplus, e^\otimes)$ and labelling function $\rho$ is a sound AMC com-
putation iff $\text{EVAL}(N_T, \oplus, \otimes, e^\oplus, e^\otimes, \rho) = \Lambda(T)$.

Following the intuition in \(^{[19]}\), the same circuit can be reused for mul-
tiple queries. For instance, from \(^{(2)}\) and \(^{(3)}\), the very same circuit used for
computing $\Lambda(T)$ can be used for computing the probability of a query $q$
by enforcing that the leaf associated to $\neg q$ carries no weight in the final
computation. For further details, see \(^{[19]}\).\(^{[7]}\).

\textbf{Inferences with Beta Distributions} In order to use beta distributions as labels, in \(^{[11]}\), the authors provide operators that expand on \(^{[4]}\)
for what concerns expected values of the random variables, and approxi-
mating the resulting variance using Taylor approximations. They require as
Algorithm 1: Evaluating an NNF circuit \( N \) for a commutative semiring \((A, \oplus, \otimes, e^\oplus, e^\otimes)\) and labelling function \( \rho \).

1: procedure \textsc{Eval}(\( N, \oplus, \otimes, e^\oplus, e^\otimes, \rho \))
2: \hspace{1em} \textbf{if} \( N \) is a true node \( \top \) \textbf{then return} \( e^\otimes \)
3: \hspace{1em} \textbf{if} \( N \) is a false node \( \bot \) \textbf{then return} \( e^\oplus \)
4: \hspace{1em} \textbf{if} \( N \) is a literal node \( l \) \textbf{then return} \( \rho(l) \)
5: \hspace{1em} \textbf{if} \( N \) is a disjunction \( \bigvee_i N_i \) \textbf{then}
6: \hspace{2em} return \( \bigoplus_i \text{\textsc{Eval}}(N_i, \oplus, \otimes, e^\oplus, e^\otimes, \rho) \)
7: \hspace{1em} \textbf{if} \( N \) is a conjunction \( \bigwedge_i N_i \) \textbf{then}
8: \hspace{2em} return \( \bigotimes_i \text{\textsc{Eval}}(N_i, \oplus, \otimes, e^\oplus, e^\otimes, \rho) \)

input also the matrix of covariance between such random variables, which captures their second-order dependencies. In their proposal, \( e^\oplus \) is a beta distribution with expected value 0 and 0 variance. \( e^\otimes \) is a beta distribution with expected value 1 and 0 variance.

Looking at Algorithm 1, there is then the need to introduce specific operators for \( \bigoplus \) and \( \bigotimes \), which are used when facing respectively a disjunction or a conjunction node. In the case of a disjunction \( n \) over \( C \) nodes—its children—the operators proposed in [11] return a beta distribution whose expected value is the sum of the expected values of its children, hence \( \mathbb{E}[X_n] = \sum_{c \in C} \mathbb{E}[X_c] \). In the case of a conjunction, the operator returns a beta distribution whose expected value is approximated to the product of the expected values of its children, \( \mathbb{E}[X_n] \simeq \prod(\mathbb{E}[X_C]) \), as it results from a Taylor approximation. The operators proposed in [11] also try to faithfully approximate the variance of the resulting beta distributions, for both disjunction and conjunction nodes, by manipulating the covariances between each of their children and the other nodes in the circuit.

3 Distribution Semantics in Abstract Argumentation

In Section 2 we briefly recalled techniques for reasoning about epistemic and aleatory uncertainty represented through beta distributions. We now turn our attention to the issue of supporting argumentative-probabilistic reasoning.

3.1 Background in Abstract Argumentation

An argumentation framework (AF) [20] is a pair \( \Gamma = (\mathcal{A}, \mathcal{R}) \) where \( \mathcal{A} \) is a set of arguments and \( \mathcal{R} \subseteq \mathcal{A} \times \mathcal{A} \). We say that \( b \) attacks \( a \) iff \( (b, a) \in \mathcal{R} \), also denoted as \( b \rightarrow a \). The set of attackers of an argument \( a \) is denoted as \( a^- \triangleq \{b : b \rightarrow a\} \), the set of arguments attacked by \( a \) is denoted as \( a^+ \triangleq \{b : a \rightarrow b\} \). Analogously, we can define the set of arguments attacked by a set of arguments \( E \subseteq \mathcal{A} \) as \( E^+ \triangleq \{b \mid \exists a \in E, a \rightarrow b\} \).

The basic properties of conflict-freeness, acceptability, and admissibility of a set of arguments are fundamental for the definition of argumentation semantics. Given an AF \( \Gamma = (\mathcal{A}, \mathcal{R}) \):

- a set \( S \subseteq \mathcal{A} \) is a conflict-free set of \( \Gamma \) if \( \nexists a, b \in S \text{ s.t. } a \rightarrow b \);
• an argument $a \in \mathcal{A}$ is acceptable in $\Gamma$ with respect to a set $S \subseteq \mathcal{A}$ if
  \[ \forall b \in \mathcal{A} \text{ s.t. } b \rightarrow a, \exists c \in S \text{ s.t. } c \rightarrow b; \]
• the function $\mathcal{F}_\Gamma : 2^\mathcal{A} \rightarrow 2^\mathcal{A}$ such that $\mathcal{F}_\Gamma (S) = \{ a \mid a \text{ is acceptable in } \Gamma \text{ w.r.t. } S \}$
is called the characteristic function of $\Gamma$.

An argumentation semantics $\sigma$ prescribes for any $\mathcal{AF} \Gamma$ a set of extensions, denoted as $\mathcal{E}_\sigma (\Gamma)$, i.e., a set of sets of arguments satisfying the conditions dictated by $\sigma$.
Given an $\mathcal{AF} \Gamma = \langle \mathcal{A} \Gamma, \mathcal{R} \Gamma \rangle$, a set $S \subseteq \mathcal{A}$ is:
• an admissible extension of $\Gamma$, i.e., $S \in \mathcal{E}_{AD} (\Gamma)$, iff $S$ is an admissible set of $\Gamma$, i.e., $S \subseteq \mathcal{F}_\Gamma (S)$;
• a complete extension of $\Gamma$, i.e., $S \in \mathcal{E}_{CO} (\Gamma)$, iff $S \in \mathcal{E}_{CF} (\Gamma)$ and $S = \mathcal{F}_\Gamma (S)$;
• the grounded extension of $\Gamma$, i.e., $S \in \mathcal{E}_{GR} (\Gamma)$, iff $S$ is the minimal (w.r.t. set inclusion) complete extension of $\Gamma$;
• a stable extension of $\Gamma$, i.e., $S \in \mathcal{E}_{ST} (\Gamma)$, iff $S$ is a conflict-free set of $\Gamma$ and $S \cup S^+ = \mathcal{A}$;
• a preferred extension of $\Gamma$, i.e., $S \in \mathcal{E}_{PR} (\Gamma)$, iff $S$ is a maximal (w.r.t. set inclusion) admissible set of $\Gamma$.

Given an argumentation framework $\Gamma$, an argument $a$, and a semantics $\sigma$, $a$ is said to be credulously accepted according to $\sigma$ if $\exists S \in \mathcal{E}_\sigma (\Gamma)$ such that $a \in S$.

In the following we make use of the fact that given an argumentation framework $\Gamma$ and a semantics $\sigma$, it is possible to derive a propositional theory whose models are isomorphic to $\mathcal{E}_\sigma (\Gamma)$. For instance, from [21] an argumentation framework can be transformed into a propositional theory whose models are isomorphic to the admissible sets. In particular, from [21 Prop. 6], such a formula can be:

\[
\bigwedge_{a \in \mathcal{A}} \left( a \supset \bigwedge_{b \in \{ a \}^-} \neg b \right) \land \left( a \supset \bigwedge_{b \in \{ a \}^-} \bigvee_{c \in \{ b \}^-} c \right)
\]

Given an argumentation framework $\Gamma$ and a semantics $\sigma$, let us denote with $\Phi (\Gamma, \sigma)$ an arbitrary propositional theory whose models are isomorphic to $\mathcal{E}_\sigma (\Gamma)$. Please note that we do not claim that such a theory can always be derived in polynomial time from the definition of the argumentation framework.

To illustrate our proposal, let us consider the following running example loosely connected to the IPCC sixth’s assessment [1, p. TS-16] (cf. Section 1).

**Example 1.** Let consider the argument $d$ stating that “from the collected evidence, we conclude that cumulative net CO2 emissions over the last decade (2010-2019) are about the same size as the 11 remaining carbon budget to limit warming to 1.5C.” Let us assume that such a conclusion involved assessing three further arguments, thus giving rise to the $\Gamma_E = \langle \mathcal{A}_E, \mathcal{R}_E \rangle$ argumentation framework where $\mathcal{A}_E = \{ a, b, c, d \}$ and $\mathcal{R}_E =$...
\{a \rightarrow c, b \rightarrow c, c \rightarrow d\}. Given the sensitivity and complexity of the topic, we refrain from instantiating such additional arguments with natural language text for avoiding unnecessary debate over what should be just an illustrative example.

From (5), \( \Phi(\Gamma_E, \mathcal{A}D) = (\neg a) \land (\neg b) \land (a \lor b) \land (\neg c) \).

Figure 1 shows a smooth, deterministic, and decomposable NNF equivalent to \( \Phi(\Gamma_E, \mathcal{A}D) \) generated using the method proposed in [22].

3.2 Algebraic Model Counting and Abstract Argumentation

Before addressing the case of uncertain probabilities represented through beta distributions, let us discuss probabilistic inferences over argumentation frameworks considering simple probabilities and thus the very same setting illustrated in (4).

A probabilistic graph is a tuple \( \langle \mathcal{A}, \mathcal{R}, \rho \rangle \) where \( \mathcal{A} \) is an abstract argumentation framework and \( \rho : \mathcal{A} \rightarrow [0, 1] \). Given a probabilistic graph \( \langle \mathcal{A}, \mathcal{R}, \rho \rangle \) and a set of arguments \( S \subseteq \mathcal{A} \), the probability of such a set of arguments is:

\[
\rho(S) = \bigotimes_{a \in S} \rho(a) \otimes \bigotimes_{a \not\in \mathcal{A}' \cap S} \rho(a)^c
\]

While so far we share definitions with the constellation approach to probabilistic argumentation [12, 13, 14] the probabilistic inference according to [6]’s semantics of a queried argument \( a \) as the sum of the probabilities of \( \sigma \)’s extensions in which \( a \) is present.

Definition 2. Given a probabilistic graph \( \langle \mathcal{A}, \mathcal{R}, \rho \rangle \) over an argumentation framework \( \Gamma = \langle \mathcal{A}, \mathcal{R} \rangle \), an argument \( a \in \mathcal{A} \), and a semantics

\footnote{While in this paper we focus on the semantics introduced in Dung’s paper [20], Definition 2 can apply to any semantics introduced over abstract argumentation framework that returns set(s) of acceptable arguments.}

\footnote{Definition 2 generalises the semantics provided in [13], where the authors restrict to the CF case only.}
Definition 4. Given a probabilistic graph $\Gamma$, let us recall its main definitions.

AMC and weighted MaxSAT, see for instance [25].

In future work we will explore further its connection with MaxSAT problems, already investigated, among others, in [24] and the connection between AMC and weighted MaxSAT, see for instance [25].

To show how Definition 2 diverges from the constellation approach to probabilistic argumentation framework $\Gamma = \langle A, \mathcal{R}, \rho \rangle$, let us recall its main definitions.

Given $\Gamma = \langle A, \mathcal{R} \rangle$ and $\Gamma' = \langle A', \mathcal{R}' \rangle$, $\Gamma'$ is a subgraph of $\Gamma$, i.e., $\Gamma' \subseteq \Gamma$, iff $\mathcal{R}' \subseteq \mathcal{R}$ and $\mathcal{R}' = \{ (a, b) \in \mathcal{R} | a, b \in A' \}$. Let $\Phi(\Gamma) = \{ \Gamma' \subseteq \Gamma \}$. Under independence assumptions, we can compute the probability of subgraphs as follows. Let $\Gamma = \langle A, \mathcal{R} \rangle$, $\Gamma' \subseteq \Gamma = \langle A, \mathcal{R} \rangle$,

$$\rho(\Gamma') = \prod_{a \in A'} \rho(a) \otimes \prod_{a \in A \setminus A'} \rho(a)$$

The constellation probability of an argument $a$ is then defined as follows.

Definition 4. Given a probabilistic graph $\langle A, \mathcal{R}, \rho \rangle$ over an argumentation framework $\Gamma = \langle A, \mathcal{R}, \rho \rangle$, an argument $a \in A$, a semantics $\sigma \in \{ CF, AD, CO, GR, ST, PR \}$, and a probability distribution over subgraphs $\rho : \Phi(\Gamma) \rightarrow [0,1]$$

$$\text{PROB}(a, \sigma, \Gamma) = \sum_{\{\Gamma' \in \Phi(\Gamma) | E \in \mathcal{E}(\Gamma') \text{ and } a \in \mathcal{E}\}} \rho(\Gamma').$$

In particular, let us consider the subclass $\text{PROB-C}^{\text{IND}}(a, \sigma, \Gamma)$ where $\rho : \Phi(\Gamma) \rightarrow [0,1]$ is [8]. For it, we can prove that $\text{PROB-C}^{\text{IND}}(a, \sigma, \Gamma)$ is an instance of AMC.\footnote{This result has been—implicitly—already provided in [23] as the authors used ProbLog to solve $\text{PROB-C}^{\text{IND}}(a, \sigma, \Gamma)$ and, from [18], it is known that AMC generalises ProbLog inferences.}
siders the fact that admissible sets are not complete, hence there are cases
its probabilistic assessment is absolutely not likely

e (equivalent to 

during the argumentation graph heavily affects the result of probabilis-
 machinery provided by [11], recalled in Section 2.2.

Let us consider again our running example (Example 1) and let us suppose
3.3 Argumentative Reasoning over Beta Distributions

| q   | ρ(q)                          | PROB(q, AD, ΓE) | PROB-CIND(q, AD, ΓE) |
|-----|-------------------------------|----------------|----------------------|
| a   | Beta(1.00, 1.00)              | Beta(1.35, 2.07) | Beta(1.14, 1.35)    |
| b   | Beta(17.00, 2.00)             | Beta(14.57, 6.06) | Beta(2.41, 1.97)    |
| c   | Beta(4.00, 15.00)             | Beta(1.00, +∞)   | Beta(1.03, 92.30)   |
| d   | Beta(5.00, 1.50)              | Beta(7.05, 5.21)  | Beta(5.20, 1.64)    |

|       | Chances about even            | Likely with high confidence | Absolutely not likely with total confidence |
|-------|-------------------------------|-------------------------------|---------------------------------------------|
|       | with no confidence            |                               |                                              |
|       | Very likely with high confidence |                             |                                              |
|       | Unlikely with high confidence |                               |                                              |
|       | Likely with some confidence   |                               |                                              |

Theorem 5. Given a probabilistic graph ⟨A, R, ρ⟩ over an argumentation framework Γ = ⟨A, R⟩, and given a ∈ A, AMC generalises PROB-CIND(a, σ, Γ).

Proof. (Sketch.) From [5], given Γ′ = ⟨A′, R⟩ ⊆ Γ = ⟨A, R⟩, we need propositional theories with models of the form Φ(Γ′) = Λa∈R′ a ∧ Λa∈A/R′ ¬a. 9 then requires to consider models Φ(Γ′) for which the query argument belong to an extension, i.e., ΦC(Γ, σ, a) = VΓ (Γ′ ∈ Φ(Γ) | E ∈ E(Γ′) and a ∈ E) Φ(Γ′).

Taking into account Theorems 3 and 5 it follows that—in general—PROB(a, σ, Γ) is not equivalent to PROB-CIND(a, σ, Γ).

Proposition 6. There are probabilistic graphs ⟨A, R, ρ⟩ over an argumentation framework Γ = ⟨A, R⟩, for which given a ∈ A and a semantics σ, PROB(a, σ, Γ) ≠ PROB-CIND(a, σ, Γ).

Proof. Let us consider Γ = ⟨{a, b, c}, {{a, b}, {b, c}}⟩, ρ(a) = w_a ∈ [0, 1], ρ(b) = w_b ∈ [0, 1], ρ(c) = w_c ∈ [0, 1], and w = (1 - w).

PROB(c, GR, Γ) = w_a · w_b · w_c while PROB-CIND(c, GR, Γ) = (w_a · w_b · w_c) + (w_a · w_b · w_c) + (w_a · w_b · w_c).

3.3 Argumentative Reasoning over Beta Distributions

Let us consider a running example (Example 1) and let us suppose to label arguments with the beta distributions as listed in the second column of Table 1. Table 1 also shows the results of probabilistic inferences over ΓE when considering both PROB (Definition 2) and PROB-CIND (Definition 4) using AD in both cases as argumentation semantics, and the machinery provided by (11), recalled in Section 2.2.

When using PROB (Definition 2), the knowledge we inject through the structure of the argumentation graph heavily affects the result of probabilistic query. Because there is no admissible extension in which c is accepted, its probabilistic assessment is absolutely not likely with total confidence (equivalent to e0). Moreover, the probabilistic assessment of a and b considers the fact that admissible sets are not complete, hence there are cases

Table 1: Probabilistic inferences over ΓE, with chosen ρ(q) labels, using Definitions 2 and 4

9 In [27], the authors also label arguments with beta distributions in the form of subjective logic opinions [5] for representing their level of trust.
where only one of the two (alternating) are in such a set. Finally, we began our example from $d$ and its uncertainty assessment as *likely* with *some confidence*, and post-hoc we assumed both $\Gamma_E$ as an argumentation graph representing our understanding of the relationships between possible relevant arguments, and the labels listed in the second column of Table 1 which complete the label for $d$ we already had. Such assumptions affects $d$ which now becomes *somewhat likely* with *some confidence*. The aleatory evaluation of $d$ has been affected by the uncanny uninformative $\rho(a)$, despite being unattacked in $\Gamma_E$. In future work, we will characterise possible *rationality constraints* building upon the extensive literature on epistemic approach to probabilistic argumentation [28] and ranking semantics, e.g., [29]. PROB can thus become a useful test for assessing the consistency of both probabilistic assessments and the logical dependencies between arguments.

When using PROB-C\textsuperscript{IND} (Definition 4), instead, the assessment considers whether a rational agent is aware of the presence of arguments. The most striking difference with PROB concerns $b$ which, from *very likely* with *high confidence* then becomes *somewhat likely* with *some confidence*. This is once again due to the fact that $a$, despite being unattacked, has such an uncanny uninformative assessment which affects the overall computation irrespectively whether $a$ is considered in the subgraphs or not. Indeed, $\text{Beta}(1.00,1.00) = \text{Beta}(1.00,1.00)$.

4 Conclusions

In this paper, we show a machinery for effective evaluation of argumentation frameworks with both epistemic and aleatory uncertainty, represented through beta distributions, which are used in intelligence analysis and scientific assessments. In particular, we focused on the machinery provided by (algebraic) model counting, which led us to introduce a novel probabilistic evaluation (PROB, Definition 2) which differ from existing probabilistic evaluations based upon the constellation approach (PROB-C\textsuperscript{IND} Definition 4). We nevertheless show how both probabilistic evaluations are instances of the model counting problem, and illustrated their difference in our running example.

The paper provides some basic preliminary results which may enable further developments and raises more questions than provides solutions. In particular, while we prove the connection between probabilistic logical inferences and probabilistic argumentation, the results of our running example (Table 1) invite discussions on a wide range of topics.

First of all, there is an underlying independence assumption between the probabilistic labels associated to each argument, e.g., $\rho(a | b) = \rho(a)$ for any $a, b$. From one hand this is unrealistic: there is always the possibility of having confounding variables. At the same time, experimental analysis on synthetic data [30] show that dependencies often do not change significantly a probabilistic assessment. Deciding whether the independence assumption between random variables is reasonable or not appears thus to be debatable, and it should be represented in an argumentative assessment of a phenomenon, thus adding a meta-level.
Definition 2 can also be seen as a way to weight argumentation extensions. From this perspective, it goes in the direction presented, for instance, in [31 § 3.2] and [32 § 3]. While our focus is on supporting probabilistic reasoning, we will analyse the connection further in future work.

In this paper, we limited ourselves to consider labels for arguments only. We, however, can think of extending the approach considering uncertain probabilities labelling for attacks too thanks to formal results provided in [33]. In that paper, the authors introduced the argumentation framework with recursive attacks and, among other results, [33 Def. 19] shows how to represent attacks as arguments while ensuring [33 Prop. 6–12] semantic correspondence.

Finally, given the connection between probabilistic inferences and Bayesian networks [19], an interesting future research question concerns the syntactic relationship—if possible—between Bayesian networks with binary random variables and argumentation frameworks which ensures semantic correspondence too.

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