Anomalous phase shift in a twisted quantum loop

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Abstract

The coherent motion of electrons in a twisted quantum ring is considered to explore the effect of torsion inherent to the ring. Internal torsion of the ring composed of helical atomic configuration yields a non-trivial quantum phase shift in the electrons' eigenstates. This torsion-induced phase shift causes novel kinds of persistent current flow and an Aharonov–Bohm-like conductance oscillation. The two phenomena can occur even when no magnetic flux penetrates inside the twisted ring, thus being in complete contrast with the counterparts observed in untwisted rings.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

There exist two classes of geometric effects relevant to coherent motion of electrons in low-dimensional systems. The first is the occurrence of effective potential fields originating from the geometric shape of the system [1, 2]. Suppose that electrons are confined in a low-dimensional curved geometry such as a quasi-one-dimensional (1D) twisted wire or a two-dimensional curved surface. Provided the curved shape is sufficiently thin, quantum excitation energies in the transverse direction are much larger than those in the tangential direction so that the electron's motion is described by an effective Hamiltonian of reduced dimension. It has been shown that the effective Hamiltonian involves effective scalar [1–3] and/or vector [4–6] potentials whose magnitudes depend on the local geometry of the system. The presence of the effective fields implies that, for example, electrons moving along a thin twisted wire feel an effective magnetic field; thus, the electrons will exhibit a quantum phase shift whose magnitude is determined by the internal torsion of the wire. Physical consequences of such geometry-induced potential fields have been explored with a great interest in the last decade.
[7–22], owing to the technological progress that enables us to fabricate exotic nanostructures having curved geometry [23–31].

The second geometric effect arises in the cyclic motion of electrons along a closed loop. Generally when an electron travels along a closed loop, it acquires a memory of the cyclic motion as a geometric phase in the wavefunction [32]. The most important manifestation of the geometric phase is the Aharonov–Bohm (AB) effect [33–35]; it occurs when an electron goes round in a loop encircling magnetic flux. It is known that the AB effect has many analogs both in quantum physics and beyond, which evidences the relevance of the effect to diverse fields in physics [36–46].

The existence of the above two geometric effects implies intriguing phenomena peculiar to quasi-1D twisted systems. The phase shift induced by torsion is, for instance, expected to cause a novel class of persistent current that flows along a closed loop of a twisted wire. This persistent current flow is novel in the sense that no magnetic flux needs to penetrate inside the loop; this feature is distinctive compared with usual persistent current in a non-twist quantum loop [47–54]. In addition, the torsion-induced phase shift should be manifested in the conductance of the twisted ring. Constructive and destructive interference between the electrons will give rise to a conductance modulation as similar to the ordinary AB oscillation [33–35]; nevertheless, in the twisted ring, the modulation (if it occurs) is driven by the torsion instead of the penetrating magnetic flux. To examine the possibility of these two phenomena is interesting from viewpoints of both mathematical physics and nanoscale sciences.

In this paper, we present a thorough mathematical derivation of the torsion effect on the coherent electron transport through a twisted quantum ring. We suppose a closed loop of a quantum wire having helical atomic configuration and consider how the internal torsion of the configuration results in a non-trivial phase shift in the electron’s wavefunction. Two physical consequences of the phase shift are reviewed and reconsidered: they are called the torsion-induced persistent current [55] and the flux-free AB effect [56]. Both phenomena take place in the absence of magnetic flux penetrating inside the ring, which is in complete contrast with the ordinary counterparts observed in untwisted quantum rings. A further detailed analysis is performed to make clear the similarities and differences between the phase shift behaviors of twisted and untwisted rings.

2. Schrödinger equation of a quasi-1D twisted system

The Hamiltonian of a particle moving in a twisted system can be formulated via three different approaches. The first approach established by Mitchell [5] leads us to the Hamiltonian that is applicable to arbitrary-dimensional twisted systems. The second one, suggested by Magarill and Ř in [6], assumes a quasi-1D system having a finite thickness. The thickness allows the excitation of mobile electrons in the transverse direction. The last approach, which also assumes a quasi-1D system, is based on a simplification that electrons reside in the lowest energy eigenstate of the transversal motion [4]. The last one applies to sufficiently thin quantum systems with internal torsion and is relatively simple rather than other two methods. In this section, we give a brief review of the last formulation based on which we will deduce the torsion-induced phase shift in a sufficiently thin twisted ring.

Figure 1 illustrates the helical atomic configuration that consists of a twisted quantum wire. Internal torsion of the wire is defined by the rotation rate of the cross section along the central axis C. The axis C of the wire is parametrized by \(q_0\). In addition, we introduce curvilinear coordinates \((q_0, q_1, q_2)\) such that the \(q_1–q_2\) plane perpendicular to C rotates along the axis with the same rotation rate as that of the helical atomic configuration (see figure 1).
Figure 1. Helical atomic configuration consisting of a twisted quantum wire with a circular cross section. Internal torsion of the wire is defined by the rotation rate of the cross section along the wire axis.

A point on $C$ is represented by the position vector $r = r(q_0)$. Similarly, a point close to $C$ is given by

$$R = r(q_0) + q_1 e_1(q_0) + q_2 e_2(q_0),$$

where $e_1, e_2$ are the orthogonal unit vectors that span the cross section. $e_0$ is defined by $e_0 \equiv \partial_0 R(q_0)$ with the notation $\partial_i \equiv \partial/\partial q_i$. Since $e_0$ is tangential to $C$, the set $(e_0, e_1, e_2)$ composes an orthogonal triad. Using the helical coordinate system, the Hamiltonian of the system is written by [57]

$$H = -\frac{\hbar^2}{2m^*} \sum_{i,j=0}^{1,2} \frac{1}{\sqrt{g}} \partial_i (\sqrt{g} g^{ij} \partial_j) + V.$$ (2)

Here, $m^*$ is the effective mass of electrons and

$$g = \det [g_{ij}], \quad g_{ij} = \partial_i R \cdot \partial_j R, \quad g^{ij} = g_{ij}^{-1}.$$ (3)

The term $V$ in equation (2) represents a strong confining potential that constrains the transverse motion of electrons’ motion within the circular cross section of radius $d$.

Let us introduce an important geometric parameter, the internal torsion $\tau$, defined by

$$\tau = e_2 \cdot \partial_0 e_1.$$ (4)

It quantifies the rotation rate of $e_1$ (and $e_2$) that rotates in a helical manner along $C$. In terms of $\tau$ and $\kappa_a = e_0 \cdot \partial_0 e_a$, $g_{ij}$ is rewritten by

$$\begin{align*}
g_{00} &= \gamma^4 + \tau^2 (q_1^2 + q_2^2), \\
g_{01} &= g_{10} = -\tau q_2, \\
g_{02} &= g_{20} = \tau q_1, \\
g_{ab} &= \delta_{ab}, \quad [a, b = 1, 2],
\end{align*}$$ (5)

where $\gamma = (1 - \kappa_a q_a)^{1/2}$ (the Einstein convention for repeated indices was used). The elements $g^{ij}$ are those of the $3 \times 3$ matrix $[g^{ij}]$ inverse to $[g_{ij}]$; we can prove that [57]

$$\begin{align*}
g^{00} &= \gamma^{-4}, \\
g^{01} &= g^{10} = \gamma^{-4} \tau q_2, \\
g^{02} &= g^{20} = -\gamma^{-4} \tau q_1, \\
g^{ab} &= \delta_{ab} + \gamma^{-4} \tau^2 [(q_1^2 + q_2^2) \delta_{ab} - q_a q_b],
\end{align*}$$ (6)
Substituting the results into equation (2), we obtain

$$H = -\frac{\hbar^2}{2m^*} \left[ \frac{1}{\gamma} \partial_0^2 \frac{1}{\gamma} + \bar{\alpha}_1^2 + \bar{\alpha}_2^2 - \frac{1}{\gamma} (\bar{\alpha}_1^2 + \bar{\alpha}_2^2) \right] + \frac{1}{\gamma} \left[ \partial_0 \tau q_b \frac{1}{\gamma^2} \partial_0 + \partial_0 \tau q_b \frac{1}{\gamma^2} \partial_0 - \left( \partial_0 \tau q_a \frac{1}{\gamma^2} \partial_0 + \partial_0 \tau q_a \frac{1}{\gamma^2} \partial_0 \right) \right] + \frac{1}{\gamma} \left( \bar{\alpha}_1 \partial_0 \partial_a \right) \bar{\alpha}_{ab} - q_a q_b \right] + V. \tag{7}$$

In equation (7), we used the summation convention with respect to the repeated subscripts: $a, b = 1, 2$.

For the sake of analytic arguments, we assume that torsion and curvature of the wire is sufficiently smooth and small so that $(\kappa_1^2 + \kappa_2^2)^{1/2} d \ll 1$ and $\tau d \ll 1$. Under these conditions, the Hamiltonian (7) is reduced to

$$H = -\frac{\hbar^2}{2m^*} \left[ \left( \bar{\alpha}_1^2 + \bar{\alpha}_2^2 \right) + \left( \bar{\alpha}_0 \right)^2 + \frac{\kappa^2}{4} + V. \tag{8} \right]$$

The operator $L \equiv -i\hbar (q_1 \partial_2 - q_2 \partial_1)$ measures the angular momentum of electrons moving in the cross section. Eigenfunctions of the Hamiltonian (8) are assumed to have the form

$$\phi(q_0, q_1, q_2) = \psi(q_0) \sum_{j=1}^{N} c_j u_j(q_1, q_2). \tag{9}$$

Here, $u_j(q_1, q_2)$ are $N$-fold eigenfunctions in the cross section and $\psi(q_0)$ describes the axial motion of electrons along the twisted wire. From equations (8) and (9), we can prove that $\psi(q_0)$ obeys the effective one-dimensional Schrödinger equation such as

$$-\frac{\hbar^2}{2m^*} \left[ \left( \bar{\alpha}_0 \right)^2 + \frac{\kappa^2}{4} - \frac{\tau^2}{\hbar^2} \langle L \rangle^2 \langle L \rangle \right] \psi(q_0) = \epsilon \psi(q_0). \tag{10}$$

The angular brackets $\langle \cdots \rangle$ indicate to take an expectation value with respect to the cross-sectional eigenfunctions $u_j(q_1, q_2)$ that are degenerate in general. The product $\tau \langle L \rangle / \hbar$ appearing in equation (10) gives rise to a quantum phase shift in the wavefunction $\psi(q_0)$, as discussed in detail in the next section.

3. Torsion-induced phase shift

This section is devoted to formulate the torsion-induced phase shift in a twisted quantum ring. The ring radius $R (\gg d)$ is assumed to be constant throughout the ring. Under this assumption, the eigenfunction of the differential equation (10) is given by

$$\psi(q_0) = e^{\epsilon \tau q_0 - i \frac{1}{\hbar} \int_{q_0}^{q_0} \langle L \rangle \, dq_0} \tag{11}$$

with a normalization constant $c$. The second term in the parenthesis in equation (11), $-\langle i\tau / \hbar \rangle \int_{q_0}^{q_0} \langle L \rangle \, dq_0$, plays the role of the torsion-induced phase shift. The degree of the shift depends on the internal torsion $\tau$ of the system, as the name implies.

It follows from equation (11) that a nonzero value of $\langle L \rangle$ is necessary for the phase shift to occur. One possible setup to obtain a finite $\langle L \rangle$ is illustrated in figure 2. The ring is threaded by an external current flow $I_{ext}$ that generates a magnetic field $B$ in the direction tangential to the ring. The magnitude of the magnetic field reads $B = \mu_0 I_{ext} / \ell$ with $\ell = 2\pi R$, where $\mu_0$ is
the permeability of vacuum. Due to the presence of $B$, the cross-sectional angular momentum operator $L$ takes the form of

$$L_B = -\hbar \frac{\partial}{\partial \theta} \frac{eB r^2}{2},$$

(12)

where we used the polar coordinates $(r, \theta)$ with respect to the circular cross section. We shall see below that the expectation value $\langle L_B \rangle$ with respect to the cross-sectional eigenstates has a nonzero value when the current flow $I_{ext}$ is injected.

The electrons’ motion in the cross section subjected to $B$ is described by

$$\frac{1}{2m^*} \left\{ \frac{1}{r} \left( -i\hbar \frac{\partial}{\partial r} \right) \left[ r \left( -i\hbar \frac{\partial}{\partial r} \right) + eB r^2 \right] + \frac{1}{r} \left( -i\hbar \frac{\partial}{\partial \theta} \right) + \frac{eB r^2}{2} \right\} u(r, \theta) + V(r) u(r, \theta) = E_\perp u(r, \theta)$$

(13)

with the transversal energy $E_\perp$. It is reasonable to set the confining potential $V(r)$ be a parabolic well centered at $r = 0$, $V(r) = m^* \omega_p^2 r^2 / 2$ [58, 59], where $\omega_p$ characterizes the steepness of the potential. Then equation (13) is rewritten as

$$-\frac{\hbar^2}{2m^*} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \right] u(r, \theta) + \left( -\frac{i\hbar \omega_c}{2} + \frac{m^*}{8} \omega_p^2 r^2 + \frac{m^*}{2} \omega_c^2 r^2 \right) u(r, \theta) = E_\perp u(r, \theta),$$

(14)

where $\omega_c = eB / m^*$ is the cyclotron frequency. Substituting $u(r, \theta) = e^{im\theta} t_m(r) / \sqrt{2\pi}$ ($m = 0, \pm 1, \pm 2, \ldots$) into equation (14), we obtain

$$-\frac{\hbar^2}{2m^*} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) - \frac{m^2}{r^2} \right] t_m(r) + \left[ \frac{\hbar \omega_c}{2} + \frac{m^*}{2} \left( \omega_p^2 + \frac{\omega_c^2}{4} \right) \right] t_m(r) = E_m t_m(r).$$

(15)

To solve equation (15), we introduce a new variable defined by

$$\rho = \frac{r^2 m^* \Omega}{\hbar}, \quad \text{where} \quad \Omega = \sqrt{\omega_p^2 + \left( \frac{\omega_c}{2} \right)^2}.$$
Using the relation
\[ \frac{\partial}{\partial r} = 2r m^* \frac{\partial}{\partial \rho}, \]
we can simplify equation (15) as
\[ \frac{d^2 t_m}{d \rho^2} + \frac{d t_m}{d \rho} - \left[ \rho^4 + \frac{1}{4\Omega} \left( m\omega_c - \frac{2E_m}{h} \right) + \frac{m^2}{4\rho} \right] t_m = 0. \]
(18)

Its solution is given by \[60, 61\]
\[ t_m(r) = \sqrt{\frac{2m^*\Omega n!}{\hbar \left( |m| + n \right)!}} \exp \left( - \frac{m^*\Omega}{2\hbar} r^2 \right) \left( \frac{m^*\Omega}{\hbar} r \right)^{|m|} L_n^{m^*}(\frac{m^*\Omega}{\hbar} r), \]
(19)

Here, \( L_n^{m^*}(x) \) is the associated Laguerre polynomial defined by \[57\]
\[ L_n^{m^*}(x) = \frac{d^n}{dx^n} \sum_{a=0}^{n} (-1)^a \frac{(n!)^2}{(a!)^2(n-a)!} x^a. \]
(20)

We are ready to evaluate the explicit form of \( \langle L_B \rangle \). From equation (19), we see that all eigenstates in the cross section are labeled by the integers \( n \) and \( m \). We assume for simplicity that electrons lie at the lowest energy eigenstate characterized by \( n = m = 0 \). Using the notation \( t \equiv t_m(r, n = 0) \), we have
\[ t(r) = \sqrt{\frac{m^*\Omega}{\pi\hbar}} \exp \left( - \frac{m^*\Omega}{2\hbar} r^2 \right). \]
(21)

Finally we obtain \( \langle L_B \rangle \) in regard to \( t \) as
\[ \langle L_B \rangle = \int_0^{\infty} r \, dr \int_0^{2\pi} d\theta \, t^*(r) L_B^t(r) = -\frac{\hbar eB}{2m^*\Omega}, \]
(22)
or equivalently,
\[ \langle L_B \rangle = -\frac{I_{\text{ext}}/I_0}{\sqrt{4 + (I_{\text{ext}}/I_0)^2}}, \quad I_0 = \frac{m^*\omega_p \ell}{e\mu_0}. \]
(23)

From equation (23), we see that \( \langle L_B \rangle \neq 0 \) if \( I_{\text{ext}} \neq 0 \). This means that a sizeable amount of quantum phase shift occurs by introducing an appropriate magnitude of \( I_{\text{ext}} \).

4. Physical consequences

4.1. Persistent current

An important consequence of the torsion-induced phase shift is the occurrence of persistent current in the absence of penetrating magnetic flux. To derive it, we recall that in a coherent ring (both twisted and untwisted) the \( \alpha \)th one-particle eigenstate carries the current \[62\]
\[ I_\alpha = \frac{\hbar k_\alpha}{m^*}. \]
(24)
The wavenumber \( k_\alpha \) is determined by considering the periodic boundary condition \( \psi(q_0 + \ell) = \psi(q_0) \), i.e.
\[ \exp(-ik\ell) \exp \left( -\frac{i}{\hbar} r \langle L_B \rangle \ell \right) = 1, \]
(25)
Figure 3. Torsion-induced persistent current $I$ as a function of the external current $I_{\text{ext}}$ for three different values of torsion $\tau$. Open and solid circles represent discrete jumps of $I$ from $-1$ to $+1$. which results in

$$ k = \frac{2\pi}{\ell} \alpha - \frac{\tau \langle L_B \rangle}{\hbar} \equiv k_\alpha \quad (\alpha = 0, \pm1, \pm2, \ldots). \quad (26) $$

The total persistent current $I$ carried by the whole electrons in a ring is the sum of the contributions from all eigenstates. From equations (24) and (26), we can prove that [55, 63]

$$ I = I(p) = \begin{cases} 0 & \text{for } p = 0, \\ \frac{e v_F}{\ell} (1 - p) & \text{for } 0 < p < 2, \end{cases} \quad (27) $$

where $p = 4\tau \langle L_B \rangle / \hbar$ and $I(p) = I(p + 2)$. The result (27) indicates that the persistent current takes place when $p \neq 0$, i.e. $\langle L_B \rangle \neq 0$ that is realized by introducing a nonzero $I_{\text{ext}}$ as discussed earlier.

Figure 3 shows the normalized persistent current $I/(e v_F / \ell)$ as a function of $I_{\text{ext}}$. Three different values of the torsion $\tau$ are chosen as indicated in the figure. With increasing $I_{\text{ext}}$, the persistent current $I$ jumps from $-1$ to $0$ and then $+1$ at several discrete points. The jump frequently occurs in the region $|I_{\text{ext}}/I_0| < 4$, while it disappears outside the region. This non-uniform distribution of discrete jumps is understood by equation (23); we see from the equation that $\langle L_B \rangle$ becomes almost independent of $I_{\text{ext}}$ in the region $|I_{\text{ext}}/I_0| \gg 4$.

The position of $I_{\text{ext}}$ at which a jump occurs strongly depends on $\tau$. Figures 4(a) and (b) show a drastic change in the curves of $I$ across a critical value of $\tau_c$. At $\tau < \tau_c$ ($\tau \ell = 0.50$), only one jump of $I$ is found at $I_{\text{ext}} = 0$. Contrariwise, at $\tau > \tau_c$ ($\tau \ell = 0.51$), two additional jumps emerge at $I_{\text{ext}}/I_0 = \pm 10$. The occurrence of the two jumps at $\tau \ell = 0.51$ results from the fact that $p = 4\tau \ell \langle L_B \rangle / \hbar$ exceeds the threshold $p = 2$. A further increase in $\tau$ makes the positions of the two jumps approach the central one at $I_{\text{ext}} = 0$; it then engenders additional two jumps newly at symmetric positions that are located outside the existing three jumps. It should be emphasized that the discrete jumps in the torsion-induced persistent current are distributed densely around $I_{\text{ext}} = 0$, whereas those in an ordinary persistent current observed in untwisted rings occur periodically in response to an increase in the penetrating magnetic flux.
4.2. Aharonov–Bohm effect

Next we discuss an AB-like conductance modulation in the electron interferometer based on a twisted quantum ring. The interferometer consists of one ring of perimeter $\ell$ and two infinitely long leads attached to the opposite sides of the ring. Two semicircular arcs of the ring have the same length of $\ell/2$, and the incident electron is set to be a plane wave having the wavenumber $k$. Using the two-terminal Landauer formula [64], we obtain the conductance $G$ of the system as

$$G = \frac{2e^2}{h} \left| 1 + \frac{\theta_1}{\theta_1} \frac{\alpha' \gamma - \beta' \gamma'}{\alpha \alpha' - \beta \beta'} + 2\theta_1 \left( \frac{\alpha' \gamma - \beta' \gamma'}{\alpha \alpha' - \beta \beta'} - 1 \right) \right|^2,$$

where

$$
\alpha(k, a) = k \left[ 2(2k + a) - (k + a)\frac{\theta_2}{\theta_1} + (k - a)\theta_2 \right], \\
\beta(k, a) = k \left[ 2k + a + \frac{\theta_2}{\theta_1} + 2(k - a)\theta_2 \right], \\
\gamma(k, a) = 2k[2k + a + (k - a)\theta_2], \\
\theta_1 = \exp(-ik\ell), \quad \theta_2 = \exp(ia\ell), \quad a = \frac{\tau (L_B)}{\hbar},
$$

with a notation $\xi'(k, a) = \xi(k, -a)$ for $\xi = \alpha, \beta, \gamma$. Note that $G$ is dependent on the dimensionless parameters $k\ell$ and $a\ell$, or equivalently, $k$, $\tau$ and $I_{\text{ext}}$. Among choices, we consider the behavior of $G$ in the following two cases: the dependence of $G$ on $\tau$ and $I_{\text{ext}}$ ($k$ is fixed), and that of $k$ and $I_{\text{ext}}$ ($\tau$ is fixed).

Figures 5(a) and (b) show three-dimensional plots of the dimensionless conductance $\tilde{G} \equiv G/(2e^2/h)$. The two plots exhibit the modulation of $\tilde{G}$ in response to changes in the parameters $k$, $\tau$ and $I_{\text{ext}}$. We emphasize that the modulation of $G$ requires no magnetic flux threading the twisted ring. The degree of modulation is pronounced close to the line of $I_{\text{ext}} = 0$ around which the surface of $\tilde{G}$ is corrugated steeply. This pronounced modulation close to $I_{\text{ext}} = 0$ is similar to the dense population of jumps observed in the persistent current around $I_{\text{ext}} = 0$. In fact, both features are attributed to the nonlinear behavior of $\langle L_B \rangle$ as a function of $I_{\text{ext}}$ (see equation (23)).
5. Discussion

We remark another possible way to obtain a nonzero value of $\langle L_B \rangle$. Instead of the introduction of $I_{\text{ext}}$, we may directly apply an external magnetic field in a direction tangential to a twisted structure. Such a tangential field plays a similar role to $I_{\text{ext}}$ depicted in figure 2, and therefore it causes torsion-induced current flow and conductance modulation in the twisted ring.

It is also noteworthy that our results of the persistent current (figure 4) and conductance oscillation (figure 5) should be dependent on the ring’s parameters: the ring’s perimeter (= $\ell$) and its cross-sectional area (characterized by $\omega_p$) are cases in point. These dependences are partly described by equation (23); it shows that a sizable amount of the $I_{\text{ext}}$-driven phase shift is obtained only when $|I_{\text{ext}}/I_0| < 4$, in which $I_0 \propto \ell \omega_p$. Hence, a smaller $\ell$ or $\omega_p$ requires a larger $I_{\text{ext}}$ for obtaining the same torsion-induced phase shift. In addition, if $\omega_p$ is too small, the cross-sectional area becomes so large, and thus the system can be no longer considered as 1D, since quantum excitations in the transverse directions are allowed. The effect of thickness would be trivial if the ring were untwisted; the amplitudes of the conductance [65] and persistent current [66] will increase since the number of conducting channels increases and all the channels assume the same amount of phase shift. Contrariwise, in a twisted ring with finite thickness, each conducting channel assumes different values of the phase shift [6]. Therefore, further study is needed to solve the thickness effect on the persistent current and conductance modulation in twisted rings.

6. Summary

We have demonstrated a novel type of quantum phase shift induced by internal torsion of quasi-1D twisted rings. The degree of phase shift is proportional to torsion $\tau$ and ring perimeter $\ell$, and shows a nonlinear response to the external current flow $I_{\text{ext}}$ threads inside the ring. This phase shift drives the persistent current $I$ and the AB-like oscillation in the conductance $G$. Detailed analyses of the non-periodic responses of $I$ and $G$ to $I_{\text{ext}}$ proved the distinct properties of the two phenomena from the ordinary counterparts appearing in untwisted rings.
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References

[1] Jensen H and Koppe H 1971 Ann. Phys. 63 586
[2] da Costa R C T 1981 Phys. Rev. A 23 1982
[3] Schuster P C and Jaffe R L 2003 Ann. Phys. 307 132
[4] Takagi S and Tanzawa T 1992 Prog. Theor. Phys. 87 561
[5] Mitchell K A 2001 Phys. Rev. A 63 042112
[6] Magarill L I and Entin M V 2003 J. Exp. Theor. Phys. 96 766
[7] Cantele G, Ninno D and Iadonisi G 2000 Phys. Rev. B 61 13730
[8] Entin M V and Magarill L I 2002 Phys. Rev. B 66 205308
[9] Aoki H, Koshino M, Takeda D, Morise H and Kuroki K 2001 Phys. Rev. B 65 035102
[10] Bulanov V D, Geyer V A and Margulis V A 2004 Phys. Rev. B 70 195313
[11] Chaplik A V and Blick R H 2004 New J. Phys. 6 33
[12] Gravesen J and Willatzen M 2005 Phys. Rev. A 72 032108
[13] Encinosa M 2006 Phys. Rev. A 73 012102
[14] Zhang E, Zhang S and Wang Q 2007 Phys. Rev. B 75 085308
[15] Taira H and Shima H 2007 Surf. Sci. 601 5270
[16] Ferrari G and Cuoghi G 2008 Phys. Rev. Lett. 100 230403
[17] Atanasov V, Dandoloff R and Saxena A 2009 Phys. Rev. B 79 033404
[18] Cuoghi G, Ferrari G and Bertoni A 2009 Phys. Rev. B 79 073410
[19] Shima H, Yoshioka H and Onoe J 2009 Phys. Rev. B 79 201401
[20] Shima H, Yoshioka H and Onoe J 2010 Physica E 42 1151
[21] Ono S and Shima H 2009 Phys. Rev. B 79 235407
[22] Ono S and Shima H 2010 Physica E 42 1224
[23] Dandeloff R, Saxena A and Jensen B 2010 Phys. Rev. A 81 044102
[24] Shea H R, Martel R and Avouris P 2000 Phys. Rev. Lett. 84 4441
[25] Schmidt O G and Eberl K 2001 Nature 410 168
[26] Lorké A, Bolm S and Wegscheider W 2003 Superlatt. Microstruct. 33 347
[27] Ajami D, Oeckler O, Simon A and Herges R 2003 Nature 426 819
[28] Ono J, Nakayama T, Aono M and Harita T 2003 Appl. Phys. Lett. 82 595
[29] McLroy D N, Alkhateeb A, Zhang D, Aston D E, Marcy A C and Norton M G 2004 J. Phys.: Condens. Matter 16 R415
[30] Yu H, Zhang Q, Luo G and Wei F 2006 Appl. Phys. Lett. 89 223106
[31] Gupta S and Saxena A 2009 J. Raman Spectrosc. 40 1127
[32] Bohm A, Mostafazadeh A, Koszumi H, Niu Q and Zwanziger J 2003 The Geometric Phase in Quantum Systems (Berlin: Springer)
[33] Aharanov Y and Bohm D 1959 Phys. Rev. 115 485
[34] Tomomura A, Osakabe N, Matsuda T, Kawasaki T, Endo J, Yano S and Yamada H 1986 Phys. Rev. Lett. 56 792
[35] Caprez A, Barwick B and Batelaan H 2007 Phys. Rev. Lett. 99 210401
[36] Aharanov Y and Casher A 1984 Phys. Rev. Lett. 53 319
[37] Halvane F D M and Wu Y S 1985 Phys. Rev. Lett. 55 2887
[38] Cimmino A, Opat G I, Klein A G, Kaiser H, Werner S A, Arif M and Clothier R 1989 Phys. Rev. Lett. 63 380
[39] Sangster K, Hinds E A, Barnett S M and Rütt E 1993 Phys. Rev. Lett. 71 3641
[40] Wilkens M 1994 Phys. Rev. Lett. 72 5
[41] Sonin E B 1997 Phys. Rev. B 55 485
[42] Mel'nikov A S 2001 Phys. Rev. Lett. 86 4108
[43] Cotaescu I I and Papp E 2007 J. Phys.: Condens. Matter 19 242206
[44] Barwick B and Batelaan H 2008 New J. Phys. 10 083036
[45] Schwingenschlögl U and Schuster C 2008 J. Phys.: Condens. Matter 20 383201
[46] Balke K and Furtado C 2009 Phys. Rev. D 80 024033
[47] Lévy L P, Dolan G, Dunsmuir J and Bouchiat H 1990 Phys. Rev. Lett. 64 2074
[48] Chandrasekhar V, Webb R A, Brady M J, Ketchen M B, Gallagher W J and Kleinsasser A 1991 Phys. Rev. Lett. 67 3578
[49] Mailly D, Chapelier C and Benoit A 1993 Phys. Rev. Lett. 70 2020
[50] Fuhrer A, Lüscher S, Ihn T, Heinzell T, Ensollin K, Wegscheider W and Bichler M 2001 Nature 413 822
[51] Imry Y 2002 Introduction to Mesoscopic Physics (Oxford: Oxford University Press)
[52] Kleemans N A J M et al 2007 Phys. Rev. Lett. 99 146808
[53] Bluhm H, Koshnick N C, Bert J A, Huber M E and Moler K A 2009 Phys. Rev. Lett. 102 136802
[54] Bleszynski-Jayich A C, Shanks W E, Peaudecerf V, Ginossar E, von Oppen F, Glazman L and Harris J G E 2009 Science 326 272
[55] Taira H and Shima H 2010 J. Phys.: Condens. Matter 22 075301
[56] Taira H and Shima H 2010 J. Phys.: Condens. Matter 22 245302
[57] Shima H and Nakayama T 2010 Higher Mathematics for Physics and Engineering (Berlin: Springer)
[58] Chen H Y, Apalkov V and Chakraborty T 2007 Phys. Rev. Lett. 98 186803
[59] Tölo E and Harju A 2009 Phys. Rev. B 80 045303
[60] Fock V 1928 Z. Phys. 47 446
[61] Darwin C G 1930 Proc. Camb. Phil. Soc. 27 86
[62] Imry Y 2002 Introduction to Mesoscopic Physics (Oxford: Oxford University Press)
[63] Cheung H F, Gefen Y, Riedel E K and Shih W H 1988 Phys. Rev. B 37 6050
[64] Büttiker M, Imry Y, Landauer R and Pinhas S 1985 Phys. Rev. B 31 6207
[65] Stone A D 1985 Phys. Rev. Lett. 54 2692
[66] Bluhm H, Koshnick N C, Bert J A, Huber M E and Moler K A 2009 Phys. Rev. Lett. 102 136802