Chemo-spectrophotometric evolution of spiral galaxies: V. Properties of galactic discs at high redshift

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ABSTRACT

We explore the implications for the high redshift universe of “state-of-the-art” models for the chemical and spectrophotometric evolution of spiral galaxies. The models are based on simple “scaling relations” for discs, obtained in the framework of Cold Dark Matter models for galaxy formation, and were “calibrated” as to reproduce the properties of the Milky Way and of nearby discs (at redshift \( z \sim 0 \)). In this paper, we compare the predictions of our “hybrid” approach to galaxy evolution to observations at moderate and high redshift. We find that the models are in fairly good agreement with observations up to \( z \sim 1 \), while some problems appear at higher redshift (provided there is no selection bias in the data); these discrepancies may suggest that galaxy mergers (not considered in this work) played a non negligible role at \( z > 1 \). We also predict the existence of a “universal” correlation between abundance gradients and disc scalelengths, independent of redshift.

Key words: Galaxies: general - evolution - spirals - photometry - abundances

1 INTRODUCTION

Important progress has been made in the past few years in our understanding of galaxy evolution, mainly due to the number and the quality of observations of galaxies at intermediate redshifts (up to \( z \sim 1 \)). Observations of ground-based telescopes, combined with results from the Hubble Space Telescope, provided photometric and kinematic data on the size, luminosity and rotational velocity of galactic discs as a function of redshift (Lilly et al. 1998, Schade et al. 1996, Vogt et al. 1997, Roche et al. 1998, Simmard et al. 1999). Although the question of various selection effects is not settled yet, these data allow, in principle, to probe the evolution of normal galaxies and to test theories of galaxy formation.

Several attempts have been made to interpret these data in the framework of currently popular models of hierarchical growth of structure in the Universe. Combined with simple assumptions about angular momentum conservation, such models lead to simple scaling relations as a function of redshift and may successfully reproduce some of the aforementioned high redshift data (Mo et al. 1998, van den Bosch 1998, Steinmetz and Navarro 1999, Firmani and Avila-Reese 2000, Avila-Reese and Firmani 2000). This “forwards” approach infers the structural properties of discs from those of the corresponding dark matter haloes and does not consider in detail the driving force of galaxy evolution, namely the transformation of gas to stars; for that reason, assumptions have to be made about the variation of the mass to light ratio and the colours of a disc as function of its mass and of redshift, while the associated chemical evolution is, in general, ignored.

On the other hand, several studies (Ferrini et al. 1994, Prantzos and Aubert 1995, Chiappini et al. 1997, Boissier and Prantzos 1999) focused mainly on the properties of local discs, for which a large body of observational data is available. In those studies, the observed chemical and photometric profiles of discs are used to constrain the radial variation of the Star Formation Rate (SFR) and of the infall rate. These multi-zone models naturally lead to a relation between luminosity and size (i.e. exponential scalelength) of discs. The fact that the Milky Way is a typical disc galaxy helps to “calibrate” such models, but there is no simple prescription as to how to extend them to other discs.

In principle, one can utilise a “backwards” approach and try to infer the properties of high redshift discs from the histories of the models that reproduce well the local disc population. Such an approach has been adopted in Cayon et al. (1996), Bouwens et al. (1997), Prantzos and Silk (1998) and Bouwens and Silk (2000). In the latter work, a comparison is made between simplified versions of the “forward” and “backwards” approaches.

A kind of “hybrid” approach is adopted in Jimenez et al. (1998): they relate the disc surface density profile to the properties of the associated dark matter halo, while they adopt radially varying SFR and infall timescales such as to reproduce in detail current profiles of the Milky Way disc.
While Jimenez et al. (1998) applied their model to the study of Low Surface Brightness galaxies, we developed a modified and more detailed version of this “hybrid” model and applied it extensively to the study of local discs. In a series of papers (Boissier and Prantzos 1999, 2000; Prantzos and Boissier 2000; Boissier et al. 2001) we showed that this model can readily reproduce a large number of local disc properties: Tully-Fisher relations in various wavelength bands, colour-colour and colour-magnitude relations, gas fractions vs. magnitudes and colours, abundances vs. local and integrated properties, abundance gradients, as well as integrated spectra as a function of a galaxy’s rotational velocity. A crucial ingredient for the success of the model is the assumption that the infall rate scales with mass of the galaxy, i.e. massive discs form the bulk of their stars earlier than low mass ones.

Stimulated by the success of this “hybrid” model in reproducing properties of the local disc population, we explore in this work its implications for the high redshift Universe, comparing its predictions to data of recent surveys. The plan of the paper is as follows: In Section 2, we present the models that were developed in the previous papers of this series in order to match the properties of the Milky Way and of nearby spirals; we also present the adopted probability distributions of the two main parameters of our models, namely the disc rotational velocity \( V_C \) and its spin parameter \( \lambda \). The predictions for the evolution of spirals, deduced from those models, is shown in Section 3. In Section 4 we compare the results of models with observations up to \( z = 1 \). In Section 5, we explore the implications at higher redshifts and we compare our results to scalability distributions observed up to \( z = 3.5 \); we also predict that abundance gradients and scalelengths should present at all redshifts the same correlation as the one observed locally. The conclusions of our “backwards” approach for the exploration of the high redshift Universe of disc galaxies are summarized in Section 6.

## 2 MODELS AND RESULTS FOR NEARBY DISCS

In this section we briefly describe the main ingredients and the underlying assumptions of our model for the chemical and spectro-photometric evolution of spiral galaxies. The model has been “calibrated” in order to reproduce the properties of the Milky Way disc (Sec. 2.1), then extended to other spirals with the help of simple scaling relations that allow to reproduce fairly well most of the observed properties of nearby spirals (Sec. 2.2). In Sec. 2.3 we present the adopted distribution functions concerning the two main parameters of our disc models, namely the rotational velocity \( V_C \) and the spin parameter \( \lambda \).

### 2.1 The Milky Way model

The model for the Milky Way disc is presented in detail in Boissier and Prantzos (1999, hereafter Paper I). The galactic disc is simulated as an ensemble of concentric, independently evolving rings, gradually built up by infall of primordial composition. The chemical evolution of each zone is followed by solving the appropriate set of integro-differential equations (Tinsley 1980), without the Instantaneous Recycling Approximation. Stellar yields are from Woosley and Weaver (1995) for massive stars and Renzini and Voli (1981) for intermediate mass stars. Fe producing SNIa are included, their rate being calculated with the prescription of Matteucci and Greggio (1986). The adopted stellar Initial Mass Function (IMF) is a multi-slope power-law between 0.1 M\(_{\odot}\) and 100 M\(_{\odot}\) from the work of Kroupa et al. (1993).

The spectrophotometric evolution is followed in a self-consistent way, i.e. with the SFR \( \Psi(t) \) and metallicity \( Z(t) \) of each zone determined by the chemical evolution, and the same IMF. The stellar lifetimes, evolutionary tracks and spectra are metallicity dependent; the first two are from the Geneva library (Schaller et al. 1992, Charbonnel et al. 1996) and the latter from Lejeune et al. (1997). Dust absorption is included according to the prescriptions of Guiderdoni et al. (1998) and assuming a “sandwich” configuration for the stars and dust layers (Calzetti et al. 1994).

The star formation rate (SFR) is locally given by a Schmidt-type law, i.e proportional to some power of the gas surface density \( \Sigma_g \) and varies with galactocentric radius \( R \) as:

\[
\Psi(t, R) = \alpha \Sigma_g(t, R)^{1.5} V(R) R^{-1}
\]

where \( V(R) \) is the circular velocity at radius \( R \). This radial dependence of the SFR is suggested by the theory of star formation induced by density waves in spiral galaxies (e.g. Wyse and Silk 1989). The efficiency \( \alpha \) of the SFR (Eq. 1) is fixed by the requirement that the observed local gas fraction \( \sigma_g \left( R_0 = 8 \text{ kpc} \right) ~ 0.2 \) is reproduced at \( T = 13.5 \text{ Gyr} \).

The infall rate is assumed to be exponentially decreasing in time with a characteristic time \( \tau \). In the solar neighborhood we adopt \( \tau = 7 \text{ Gyr} \) in order to reproduce the local G-dwarf metallicity distribution (Paper I). In order to mimic the “inside out” formation of the disc, \( \tau \) is assumed to be shorter in the inner zones and larger in the outer ones.

The really “free” parameters of the model are the radial dependence of the infall timescale \( \tau(R) \) and of the SFR \( \Psi(R) \). It turns out that the number of observables explained by the model is much larger than the number of free parameters. In particular the model reproduces present day “global” properties (gas, O/H, SFR, and supernova rates), as well as the current disc luminosities in various wavelength bands and the corresponding radial profiles of gas, stars, SFR and metal abundances; moreover, the adopted inside-out star forming scheme leads to a scalelength of \( \approx 4 \text{ kpc} \) in the B-band and \( \approx 2.6 \text{ kpc} \) in the K-band, in agreement with observations (see Paper I).

### 2.2 Extension to other disc galaxies

In order to extend the model to other disc galaxies we adopt the “scaling properties” derived by Mo, Mao and White (1998, hereafter MMW98) in the framework of the Cold Dark Matter (CDM) scenario for galaxy formation. According to this scenario, primordial density fluctuations give rise to haloes of non-baryonic dark matter of mass \( M \), within which baryonic gas condenses later and forms discs of maximum circular velocity \( V_C \). It turns out that disc profiles can be expressed in terms of only two parameters: rotational velocity \( V_C \) (measuring the mass of the halo and, by assuming a constant halo/disc mass ratio, also the mass of the disc) and spin parameter \( \lambda \) (measuring the specific angular
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The momentum of the halo. If all discs are assumed to start forming their stars at the same time, the profile of a given disc can be expressed in terms of the one of our Galaxy (the parameters of which are designated hereafter by index G):

\[ \frac{R_d}{R_{dc}} = \frac{\lambda}{\lambda_G} \frac{V_C}{V_{CG}} \]  
and

\[ \frac{\Sigma_0}{\Sigma_{dc}} = \left( \frac{\lambda}{\lambda_G} \right)^{-2} \frac{V_C}{V_{CG}} \]

Eqs. 2 and 3 allow to describe the mass profile of a galactic disc in terms of the one of our Galaxy and of two parameters: \( V_C \) and \( \lambda \). The range of observed values for the former parameter is 80-360 km/s (with \( V_{CG} = 220 \) km/s), whereas for the latter numerical simulations give values in the 0.01-0.15 range (with \( \lambda_G \sim 0.03-0.06 \)). Larger values of \( V_C \) correspond to more massive discs and larger values of \( \lambda \) to more extended ones. Although it is not clear yet whether \( V_C \) and \( \lambda \) are independent quantities, we treated them as such and we constructed a grid of 25 models characterised by \( 80 < V_C < 360 \) km/s and \( 1/3 < \lambda/\lambda_G < 3 \).

As discussed in Boissier and Prantzos (2000, hereafter Paper II) the resulting disc radii and central surface brightness are in excellent agreement with observations, except for the smallest values of \( \lambda \) (\( \sim 1/3 \lambda_G \)); this “unphysical” value leads to galaxies resembling to bulges or ellipticals, rather than discs.

The two main ingredients of the model, namely the Star Formation Rate \( \Psi(R) \) and the infall time-scale \( \tau(R) \), are affected by the adopted scaling of disc properties in the following way:

- For the SFR we adopt the prescription of Eq. 1, with the same efficiency \( \alpha \) as in the case of the Milky Way (i.e. the SFR is not a free parameter of the model). In order to have an accurate evaluation of \( V(R) \) across the disc, we calculate it as the sum of the contributions of the disc and of the dark halo, the latter having a density profile of a non-singular isothermal sphere (see Paper II).

- The infall time scale is assumed to decrease with both surface density (i.e. the denser inner zones are formed more rapidly) and with galaxy’s mass, i.e. \( \tau[\mathcal{M}_d, \Sigma(R)] \). In both cases it is the larger gravitational potential that induces a more rapid infall. The radial dependence of \( \tau \) on \( \Sigma(R) \) is calibrated on the Milky Way, while the mass dependence of \( \tau \) is adjusted as to reproduce the properties of the galactic discs.

This model, “calibrated” on the Milky Way and having as main parameter the infall dependence on galaxy mass, reproduces fairly well a very large number of properties of spiral galaxies at low redshift (Paper II). Tully-Fisher relations in various wavelength bands, colour-colour and colour-magnitude relations, gas fractions vs. magnitudes and colours, abundances vs. local and integrated properties, as well as spectra for different galactic rotational velocities. The main assumption of the model is that infall onto massive discs occurred earlier than infall onto low mass galaxies. More recently, in Paper IV (Boissier et al., 2001) we used a homogeneous set of observational data (a subset of the data presented in Boselli et al., 2000) in order to test this hypothesis. We determined the gas fraction \( \sigma_G = M_{\text{gas}}/M_{\text{TOT}} \) and the star formation efficiency \( \epsilon \) in a large number of “normal” spirals as a function of circular velocity \( V_C \). We found that \( \epsilon \) is independent of \( V_C \) while \( \sigma_G \) decreases with \( V_C \). Taken at face value, those findings imply that low mass discs had not access to their gas reservoir as early as massive ones, otherwise they should have already turned most of their gas into stars (because their current star formation efficiency is similar to the one of massive discs, and we assume that this has always been the case). This crucial finding justifies the assumptions we made concerning the form of the infall.

We notice that in another recent work, Brinchmann and Ellis (2000) found that the total stellar mass density in massive galaxies is constant over the redshift range \( 0.2 < z < 1 \). This implies that the stellar content of these galaxies must have formed at higher redshifts, which is in qualitative agreement with our assumptions.

Finally, it is worth noticing that the properties of abundance and colour gradients among local spirals are readily reproduced by the generalization of the Milky Way model and the assumed radial dependence of the SFR and infall timescales (Prantzos & Boissier 2000, hereafter Paper III).

2.3 Distributions of the parameters \( V_C \) and \( \lambda \)

Our models presented in Sec. 2.2 simulate the evolution of individual disc galaxies, characterised by \( V_C \) and \( \lambda \). In order to treat the evolution of the disc population, we need to make some assumptions about the probability distributions of the relevant parameters. In this paper we assume that the distributions \( F_\lambda(\lambda) \) and \( F_{V_C}(V_C) \) are independent and do not evolve in time. We adopt the velocity distribution suggested in Gonzalez et al. (2000):

\[ F_{V}(V) dV = \tilde{\Psi}_* \left( \frac{V}{V_*} \right)^\beta exp \left[ - \left( \frac{V}{V_*} \right)^\alpha \right] dV. \]  

The parameters \( \Psi_* \), \( V_* \), \( \beta \) and \( n \) are determined in Gonzalez et al. (2000) on the basis of observed Tully-Fisher relationships and luminosity (Schechter-type) functions. We adopt here the set of parameters of their Table 4 (fifth row, LCRS-Courteau data) corresponding to the velocity interval covered by our models. By construction, the distribution of Eq. (4) is normalised to the local luminosity function for spirals.

The distribution of the spin parameter is given by

\[ F_\lambda(\lambda) d\lambda = \frac{1}{\sqrt{2\pi}\sigma_\lambda} exp \left[ -\frac{\ln^2(\lambda/\lambda)}{2\sigma_\lambda^2} \right] d\lambda \]  

with \( \lambda = 0.05 \) and \( \sigma_\lambda = 0.5 \), according to numerical simulations (see e.g. MMW98). The distribution \( F_\lambda(\lambda) \) is normalised to unity.

Distributions \( F_\lambda(\lambda) \) and \( F_{V_C}(V_C) \) are shown in Fig. 1. Low mass discs are more numerous than high mass ones, while the spin parameter distribution favours discs with intermediate \( \lambda \) values. Notice that in this work we study discs with \( V_C > 80 \) km/s and \( \lambda < 0.1 \), since discs with larger \( \lambda \) values lead to Low Surface Brightness galaxies (e.g. Jimenez et al. 1998), which will be the subject of a forthcoming paper (Boissier and Prantzos, in preparation).

In the following, we assume that the number of discs with velocity \( V_C \) and spin parameter \( \lambda \) is \( dN = F_{V_C}(V_C) F_\lambda(\lambda) dV_C d\lambda \); we compute then the distribution \( \phi(Q) \) of various quantities \( Q \) (magnitude, radius, etc), defined by \( dN = \phi(Q) dQ \).
merge (into e.g. ellipticals), modifying F is an (important) oversimplification, since some spirals may according to our models. It is clear that this assumption from changes in intrinscic properties of individual galaxies, M has been a small but steady evolution in the distribution of Q that we obtain. As can be seen from Fig. 2, there has been a small but steady evolution in the distribution of R_B, with all discs becoming progressively larger. Disc scalelengths span today the range 0.8-11 kpc, compared to 0.3-6 kpc at z = 4.2. This is due to the inside-out star formation scheme adopted in our models (see Sec. 2.1 and Paper II). The most probable value has increased only slightly between z = 0 and z = 2.8, 1.7, 1.2, 0.7, 0.5, 0.2 (dotted curves) and z = 0 (solid curves). E Burgasser et al. 2000). See text (Sec. 2.3) for details.

We make the assumption that the circular velocity and spin parameter of a galaxy are determined at its formation and do not evolve in time. Since the distributions \( F_V(V) \) and \( F_\lambda(\lambda) \) are always the same, any evolution in \( \phi(Q) \) results from changes in intrinsic properties of individual galaxies, according to our models. It is clear that this assumption is an (important) oversimplification, since some spirals may merge (into e.g. ellipticals), modifying \( F_V(V) \). The history of spirals that we present concerns only fairly isolated discs, that have not suffered major merger episodes.

### 3 MODEL PREDICTIONS FOR THE EVOLUTION OF SPIRALS

In Fig. 2 we present the results of our models concerning the evolution of the distribution functions of various quantities: disc scalelengths \( R_B \) in the B-band (top left), central surface brightness \( \mu_0(B) \) in the B-band (top right), B-magnitude \( M_B \) (middle left), colour index \( (U-V)_{AB} \) (middle right), average oxygen abundance in the gas \([O/H]\) (bottom left) and star formation rate \( \psi \) (bottom right). The corresponding distributions are shown at various redshifts, starting at \( z=4.2 \) (dashed curves) and ending at \( z=0 \) (solid curves). A cosmological model with matter density \( \Omega_M=0.3, \Omega_\Lambda=0.7 \) and \( H_0=65 \text{ km/s/Mpc} \) is adopted throughout this work.

The comparison of our results to observations of local spirals has been done in previous papers (Papers II and III). Here we comment on the evolution of the distribution functions \( \phi(Q) \) that we obtain. As can be seen from Fig. 2, there has been a small but steady evolution in the distribution of \( R_B \), with all discs becoming progressively larger. Disc scalelengths span today the range 0.8-11 kpc, compared to 0.3-6 kpc at \( z = 4.2 \). This is due to the inside-out star formation scheme adopted in our models (see Sec. 2.1 and Paper II). The most probable value has increased only slightly between \( z=4.2 \) and \( z=0 \), from \( \sim 1 \text{ kpc} \) to \( \sim 1.5 \text{ kpc} \).

The central surface brightness \( \mu_0(B) \) is found to span a rather narrow range of values today (20-22.5 mag arcsec\(^{-2}\)). Notice, however, that our models were developed to reproduce properties of High Surface Brightness discs, while Low Surface Brightness ones (i.e. with \( \mu_0(B) > 22.5 \text{ mag arcsec}^{-2} \)) could be obtained by larger values of the spin par-
The most probable $M_B$ value shifted by $\sim 3$ mag in the past $\sim 10$ Gyr.

The colour distribution function $\phi(U-V)$ shows considerable evolution in both its shape and average value. As time goes on, galaxies become (obviously) redder, while the increasing difference in effective ages between massive discs (formed early on) and low mass ones (formed much later) contributes to broaden the $\phi(U-V)$ function.

The average metallicity is defined as the amount of oxygen in the gas divided by the gaseous mass of the disc. It increases steadily with time in all discs. At early times, massive discs have already developed relatively high O/H values, while low mass ones are essentially unevolved; the $\phi(O/H)$ distribution is then quite broad, as in the case of $\phi(M_B)$. At late times, massive discs show little evolution while the metallicity of low mass ones increases rapidly, and the $\phi(O/H)$ distribution becomes narrower.

Similar arguments hold for the distribution of the SFR $\phi(\Psi)$, which spans a large range of values at early times, from $10^{-3}$ $M_\odot$ yr$^{-1}$ to $60$ $M_\odot$ yr$^{-1}$. At late times, SFR values range from $10^{-1}$ $M_\odot$ yr$^{-1}$ to $10$ $M_\odot$ yr$^{-1}$, with an average around $1$ $M_\odot$ yr$^{-1}$.

An interesting consequence of our models is that the average Fe/O ratio becomes slightly higher than solar at $z \sim 0$, since the rate of SNIa (major Fe producers) is such that their contribution slightly exceeds the one of O from SNII.

In summary: the distributions of $\mu_0(B)$, $M_B$, $O/H$ and $\Psi$ become narrower with time and shift to higher average values. Both effects result from the rapid late evolution of the numerous low mass discs; because of this late evolution, the large differences created initially by the rapid early evolution of massive discs are reduced at late times. The distribution of $R_B$ keeps its overall shape, but its peak value slightly increases. Finally, the distribution of $(U-V)$ is the only one that broadens with time, because of the increasing difference between the effective ages of massive and low mass discs.

The results of Fig. 2 are presented in Fig. 3 in a different form. The evolution of the average values of $R_B$, $\mu_0(B)$, $M_B$, $(U-V)$, $O/H$ and $\Psi$ (solid curves) is shown as a function of redshift $z$, along with the $\pm 1\sigma$ values of the same quantities (inside dark shaded areas). There is little increase in $R_B$ for $z < 1$ (induced mostly by low mass discs), while $\mu_0$ and $\Psi$ decrease slightly at $z < 2$. Other average quantities ($< M_B >$, $< U-V >$, $< O/H >$) increase constantly, from $z \sim 4$ to $z=0$. In most cases, the corresponding values for the Milky Way (dotted curves) are larger than the average ones (since the latter are dominated by low mass discs).

In fact, except for the case of $R_B$ and $\Psi$, the Milky Way values are higher than the average ones by more than $1\sigma$ during most of the evolution. The Milky Way is a large spiral and its evolution is different from the average one not only quantitatively, but also qualitatively in some cases (e.g. in the case of $M_B$ and $\mu_0$, where the Milky Way values decline for $z < 2$ while the average ones continue to increase or remain constant).

At this point, we notice that most (but not all) semi-analytical models of galaxy evolution today use the Salpeter IMF (a power-law with a unique slope $-1.35$ over the whole stellar mass range), which is flatter than the one used here.

![Figure 3. Evolution of several quantities computed with our models as a function of redshift: scalelength $R_B$ (top left), B-magnitude $M_{AB}(B)$ (middle left), average oxygen abundance (bottom left), central surface brightness $\mu_0_{AB}(U)$ (top right), colour index $(U-V)_{AB}$ (middle right) and star formation rate $\Psi$ (bottom right). In all panels the solid curve indicates the average value of the probability distributions in Fig. 2, while the borders of the dark shaded areae indicate the evolution of the corresponding $\pm 1\sigma$ values and the light shaded areae represents the remaining discs. The dotted curves indicate the corresponding quantities for the Milky Way disc. The dashed curve in the upper left figure indicates the predictions of hierarchical models by Mao et al. (1998), according to which $R_B \propto (1+z)^{-3.5}$.](image)

In Paper I we discussed at length the reasons of our choice and the current observational evidence against a unique slope IMF. We also mentioned briefly the main differences resulting from its use, mainly the fact that it produces a larger effective yield of metals and a smaller number of stars of 1-2 $M_\odot$, which dominate the galactic luminosity at late times. Since the effective yield of a stellar generation depends not only on the IMF but also on the individual stellar yields, which are still uncertain by at least a factor of two (see Prantzos 2000 and references therein), the former effect of the choice of the IMF cannot be tested against observations today. As for the latter, we find that it results in differences of $\sim 30\%$ in luminosity at late times; i.e. $\sim 0.15$ magnitudes in the B-band; this is also comparable to other theoretical uncertainties entering our calculation (neglect of late AGB phase in the adopted stellar tracks, crude treatment of dust etc.)

4 EVOLUTION UP TO Z = 1: MODELS VS. OBSERVATIONS

In this section we compare our results to recent observations concerning the evolution of discs at moderate redshifts, for $z < 1$. A nice recent overview of the relevant observations can be found in Hammer (1999).

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Figure 4. Evolution of scalelength $R_B$, magnitude $M_{AB}(B)$, central surface brightness $\mu_{0,AB}(B)$ and colour index $(U-V)_{AB}$ in the range $0 < z < 1.3$. The presentation of our results is similar to the one of Fig. 3, but the average (solid curves) and $\pm1\sigma$ values (borders of the shaded areas) were calculated taking into account only the portion of the distribution function concerning large discs ($R_B > 4h_{50}^{-1}$ kpc). Observations of large discs by Lilly et al. (1998) are shown as circles. Observations of the local sample of de Jong (1996) are inside error bars. The Milky Way evolution is shown with dotted curves.

4.1 Large vs. small discs

Lilly et al. (1998) used two-dimensional surface brightness profiles extracted from HST images of galaxies selected from the CFRS and LDSS redshift survey to study the evolution of several properties of star-forming galaxies between $z = 0$ and $z = 1.3$. Their observations concern the disc scalelength $R_B$, magnitude $M_{AB}(B)$, central surface brightness $\mu_{AB,0}(B)$ and colour index $(U-V)_{AB}$.

The data of Lilly et al. (1998) concern galaxies of various inclinations, and are not corrected to the face-on case; however, those data do not show any trend with inclination. We checked that the scatter produced by varying the inclination in our models from 0 to 50 degrees is smaller than the scatter obtained via the variation of the parameters $V_c$ and $\Lambda$. We thus present our results for galaxies seen face-on. The comparison to the data of Lilly et al. (1998) with our “face-on” models serves merely to check whether the “average” history of our models is compatible with the observational trends.

The data of Lilly et al. (1988) concern only large discs, with scalelengths $R_B > 4h_{50}^{-1}$ kpc. We selected then in our models discs satisfying this criterion and we plotted in Fig. 4 the results concerning the evolution of $R_B$, $\mu_{0}(B)$, $M_B$ and $(U-V)_{AB}$ in a way similar to the one used in Fig. 3; this time, only the part of the distribution functions of Fig. 2 concerning large discs is used to derive average and $\pm1\sigma$ values, and results are shown only in the $z < 1.3$ range, where the measurements of Lilly et al. (1998) were made.

As can be seen in Fig. 4 (left top panel) the data show no evolution of $R_B$ up to $z \sim 1$. The behaviour of our models is compatible with this trend, since most large discs are already formed at $z \sim 1$ and undergo very little increase in size afterwards. Note that our models do not reproduce the largest galaxies of the Lilly et al. (1998) sample (those with $R_B > 10$ kpc). Such discs are never obtained in our models and we consider that they are not representative of “normal” spirals like those of the local sample of de Jong (1996) on which we based our modelisation in Paper II. We notice that the Milky Way has a more rapid late evolution than an average disc: its $R_B$ (dotted curve in Fig. 3) decreases by a factor of $\sim 2$, from $\sim 4$ kpc at $z = 0$ to $\sim 2$ kpc at $z \sim 1.2$. In this redshift range the Milky Way behaves almost as expected from the hierarchical scenario of Mao et al. (1998), i.e. with $R_B \propto (1 + z)^{-1}$.

Observed B-magnitudes (left-bottom panel) exhibit some decline since $z \sim 1$. This is in qualitative agreement with our models and is related to the decrease of star formation with time in massive galaxies (Paper II). The range of $M_B$ predicted by the models is also in fair agreement with the observed one. A comparison with Fig. 3 shows that the situation differs a lot from the case where all discs are taken into account: in that case, the average $M_B$ increases lately, because of the late brightening of the numerous small discs.

There is also a slight decline of the observed central surface brightness of discs between $z = 0$ and $z = 1$ (right-top panel), in agreement again with the evolution of our model $\phi(\mu_{0,AB}(B))$ for large discs.

The right-bottom panel of Fig. 4 shows the evolution of the $(U-V)_{AB}$ colour index. The data seem to indicate a strong increase of that colour index with redshift (albeit with a large scatter). This is quite at odds with the results of our models that predict a slow evolution in this redshift range. However, as noticed by Lilly et al. (1998) the colour of discs in the local sample of de Jong (1996) has a mean value of $(U-V)_{AB} \sim 1.5$, considerably smaller than their observations at moderate redshift (which suggest a mean value of 2 at $z \sim 0.3$). Notice that our models reproduce correctly colours at redshift $z = 0$, since they were constructed as to match local observations.

The local values of de Jong (1996), combined to the observations of Lilly et al. (1998) point towards little colour evolution on average, as in our models. However, the large scatter in the $(U-V)$ data can not be explained by our histories of spiral galaxies. Brinchmann and Ellis (2000) notice that colour is a transient property. It could be affected by small episodes of interaction, or of enhancement of star formation, effects that are not taken into account in our models (which give an “average” history of galaxies) and could perhaps explain part of the observed scatter.

The observations of Lilly et al. (1998) suggest a modest amount of evolution of large discs in the redshift range $0 < z < 1$, in agreement with the results of our models. This is also compatible with the conclusion of Brinchmann and Ellis (2000), who found that massive galaxies achieved the bulk of their star formation before $z \sim 1$.

Another conclusion of Lilly et al. (1998) is that the main evolutionary changes between $z = 1$ and $z = 0$ among spirals concern small (i.e. with half-light radii $R_{eff} < 5h_{50}^{-1}$ kpc) rather than large discs. Their Fig. 17 illustrates the existence of small and bright discs at redshift $z \sim 1$, with no local counterparts. Our model also predicts a stronger evolution of properties among small galaxies than among large ones, and the existence of small and luminous spirals at high redshift.

This can be seen in Fig. 5, where we present the luminosity function $\phi(M_B)$ computed at three redshifts ($z = 0$ on the top, $z = 0.7$ in the middle, $z = 1.2$ at the bottom). The first
Figure 5. Evolution of the luminosity function of discs as a function of size, up to $z \sim 1.2$. The luminosity function for all discs is shown in the first column, while in the second and third columns we present the luminosity function of “large” discs and “small” discs, respectively. The limit between large and small discs is $R_B > 4h_{50}$ kpc, adopted from Lilly et al. (1998). In the top panels, the functions are shown at redshift $z=0$ (galactic age $T=13.5$ Gyr), in the middle panels at redshift $z \sim 0.7$ ($T=6.5$ Gyr) and in the bottom panels at $z \sim 1.2$ ($T=4.5$ Gyr). The local ($z=0$) functions of the top panels (solid curves) are repeated as dotted curves in the lower panels, in order to show the evolution of $\phi(M_B)$ between low and high redshifts. The local luminosity function for all discs of our models (top left panel) is compared with two Schechter-type fits to observational data (grey curves), obtained for spiral galaxies with the APM survey (Loveday et al., 1992) and the SSRS2 survey (Marzke et al., 1998). According to our models, large discs were brighter in the past, while small discs were both brighter and fainter than today.

Figure 6. Evolution of the distribution of $Q = \log(R_B/V_C)$, where $R_B$ is in kpc and $V_C$ in km/s. Top-left panel: histograms derived by Mao et al. (1998) for a local sample (Courteau 1996, 1997, solid), and for the high-redshift ($< z > 0.7$) sample of Vogt et al. (1996, 1997; dotted). The comparison of the two samples suggests that discs at high redshift were smaller for a given velocity. Bottom-left panel: Distribution computed with our models at redshift $z=0$ (solid curve) and at redshift $z \sim 0.7$ (dotted). We find a trend similar to the the observationally inferred one (albeit of slightly smaller magnitude), due to the “inside-out” formation of discs in our models. Top-right panel: Same as bottom-left panel, for bright discs only ($M_B < -20$). Bottom-right panel: Same as bottom-left panel, for large discs only ($R_B > 4h_{50}$ kpc).

| Column | Results for All Discs | Results for Large Discs | Results for Small Discs |
|--------|-----------------------|-------------------------|------------------------|
| First  | Show all discs        | Show all discs          | Show all discs         |
| Second | Show large discs only | Show large discs only   |                        |
| Third  | Show small discs only | Show small discs only   |                        |

4.2 The $R_d - V_C$ relationship

The variation of disc size, surface brightness, magnitude etc. with redshift can help to test theories of galaxy formation and evolution. Mao et al. (1998) used data at low and high redshift to deduce that $R_d \propto (1+z)^{-1}$, in agreement with hierarchical models of galaxy formation.

Before presenting our results and their comparison to observations, we notice that the size of a disc is usually measured by its exponential scalelength. But a disc can also grow by simply increasing its central surface brightness and keeping the same scalelength: it is difficult to claim that its “size” remains constant in that case (indeed, its $R_{25}$ radius increases). The use of scalelength to measure size may create some problems in that context.

With this caveat in mind, we proceed to a comparison between our models and observation, concerning the quantity $Q=\log(R_B/V_C)$. The two histograms in the top-left panel of Fig. 6 are adopted from Mao et al. (1998). One was computed for a local sample of discs (Courteau, 1996, 1997, solid), and the other for the high-redshift ($z < 1$) galaxies brighter by $\sim 2$ mag on average in the past. In our models, those are moderately massive discs, the external parts of which are formed lately. At $z \sim 1$ they are already bright, but not large enough. This population of “small” and bright discs at high redshift has no counterpart today and could well constitute the galactic population detected by Lilly et al. (1998). We further discuss this point in the end of Sec. 4.3.
served high-

Compatible with the observationally inferred one. However, the observed distributions (especially the high-redshift one) are narrower than the observed ones and slightly shifted to larger Q values. The reason for this small discrepancy may be the fact that we consider discs with $V_C > 80$ km/s, i.e. slightly brighter than the lower limit considered in the observed samples.

The right panels of Fig. 7 show the same quantity for bright discs (top) and large discs (bottom). We find again the same effect, i.e. a shift of parameter Q towards higher values with time. Selection criteria do not affect then this result, namely that discs of a given size were brighter in the past. In fact, this is another way of presenting the result of Fig. 4, showing that large discs were slightly brighter in the past.

More recently, Simard et al. (1999) analysed 190 field galaxies of the DEEP survey in order to determine the magnitude-size relationship of discs in several redshift bins, up to $z \sim 1.1$. Their survey is statistically complete for magnitudes $I_{B} < 23.5$. The envelope of their observations at a given redshift (taken from their paper) is shown in Fig. 8 along with a few other magnitude-size samples (Schade et al. 1996, Roche et al. 1998, and Phillips et al. 1997), including the one of Schade et al. (1996) that was discussed in Fig. 7. The solid diagonal line in each panel is the locus of the local Freeman relation $\mu_B = 21.65$ mag arcsec$^{-2}$ (Freeman 1970). To a first approximation, the data follow this relation at all redshifts, with some increase of the size dispersion at a given magnitude. Also, in the highest redshift bins, the average $\mu_B$ value of the observed samples is higher than the Freeman value. This is another way of presenting the results of Fig. 7, namely that at high $z$ discs of a given magnitude were smaller, on average.

Our models of $R_B$ vs $M_B$ are also plotted as a function of redshift in the various panels of Fig. 8. We show the isodensity contours, as indicated in the bottom-right panel of the figure. Our results compare fairly well to the observations of Simard et al. (1999) in the redshifts $z \sim 0.2$ and $z \sim 0.4$. At higher redshifts, the dispersion of $R_B$ for a given $M_B$ increases, while the average $R_B$ value decreases, both in agreement with the observations. In fact, in the highest redshift bins, the largest part of our model discs is fainter than the cut-off magnitude $M_B$ of the Simard et al. (1999) survey; only our brightest discs (the luminous “tip of the iceberg”) are seen in the observations, and their behaviour is entirely compatible with the data. The size dispersion of our discs is due to the effect of the spin parameter $\lambda$ on their structure and evolution (through the adopted scaling relations and the SFR and infall prescriptions, see Sec. 2.2). For all $\lambda$ values, massive discs attain their final size early on (before $z \sim 1$) and evolve little at late times, as discussed in Sec. 4.1. But, depending on the $\lambda$ value, low mass discs may have very different histories: compact ones (low $\lambda$) evolve rapidly.
Figure 8. Disc scalelength $R_B$ vs disc rest frame absolute magnitude $M_B$ as a function of redshift $z$. The grey boxes indicate various observed samples, with the one of Simard et al. (1999) shaded; detection limits are displaced to lower magnitudes as one moves to higher redshifts. Our model results, convolved with the shaded panels indicates the Freeman value of central surface brightness:

$$m_{U.B,0} = 21.65 \text{ mag arcsec}^{-2}$$

(Freeman, 1970).

i.e. before $z \sim 1$, while extended ones (large $\lambda$) evolve more slowly. This explains the large dispersion in the $R_B$ values of faint discs at high redshift.

Simard et al. (1999) draw attention to the fact that the apparent increase of mean surface brightness with $z$ in their data may be entirely due to selection effects; if the survey selection functions are used to correct the surface brightness distributions in the different redshift bins, no significant evolution is found in their data up to $z \sim 1$ for discs brighter than $M_B = -19$.

This is at odds with the results of our models, which generically predict an increase of the mean surface brightness with redshift up to $z \sim 1$. This increase is due to the fact that most of our discs form their central regions long before $z \sim 1$; the SFR and the corresponding surface brightness of those regions can only decline at late times, i.e. increase with redshift.

4.4 Tully-Fisher relation

The Tully-Fisher (TF) relation is a strong correlation exhibited by the whole population of disc galaxies. It relates their luminosity $L$ or magnitude $M$ and their circular velocity $V_c$, through $L \propto V_c^{a_1}$ or $M = -a[\log(2V_c) - 2.5] + b$.

The exact slope of the relation and its zero-point are still the subject of considerable debate (see e.g. Giovannelli et al. 1997).

In Paper II we presented a detailed comparison of our model results to local ($z \sim 0$) TF relations obtained by different groups in the $I$ and $B$ bands. We noticed that in the $I$ band (which presumably reflects better the underlying stellar population) the TF slopes of different groups differ by more than $\sim 20\%$ (i.e. from $a=6.80$ in Mathewson et al. (1992) to 8.17 in Tully et al. 1998).

Our results are in much better agreement with the data of Han and Mould (the corresponding TF relationship is given in Willick et al. 1996) or of Mathewson et al. (1992), concerning field spirals, than with those of Tully et al. (1998) or Giovannelli et al. (1997), which concern spirals in clusters. We notice that our models correspond better to the former case than to the latter, since star formation in cluster galaxies may be affected by tidal interactions which are not taken into account in our SFR prescription.

In the $B$-band, the available local TF relation of Tully et al. (1998) concerns again spirals in clusters. Our model $B$-band TF relation is slightly flatter than the observed one, but in view of the uncertainties and the caveats mentioned in the previous paragraphs, we do not think that this is a serious drawback of the models. We note that other models (e.g. Mo et al. 1998, Heavens and Jimenez 1999, Bouwens and Silk 2000) apparently reproduce better the local $B$-band TF relation. However, the mass to light ratio $M/L$ in those models is either taken as constant or it is adjusted as a function of $V_c$ in order to reproduce the TF relation. Our own $M/L$ vs. $V_c$ relation (presented in Fig. 8 of Paper II) is computed from the fully self-consistent chemospectrophotometric evolution models, which reproduce all the main properties of local discs without any adjustment except for the initial scalings (see Sec. 2.2 and Paper II).

Using spectroscopic observations of the Keck telescope and high-resolution images from the Hubble deep field, Vogt et al. (1996, 1997) determined a Tully-Fisher (TF) relation for 16 galaxies lying between redshifts $z=0.15$ and 1 in the rest-frame $B$-band. They find no obvious change in the shape or slope of the TF relation with respect to the local one. Assuming the same slope, they derive a modest brightening of $0.36\pm0.13$ mag between redshifts $z \sim 0$ and
5 EVOLUTION AT REDSHIFT Z > 1.

5.1 ScaIenlength distributions

The size-luminosity relation at redshifts z > 1 has been recently investigated by Giallongo et al. (2000). They used data from the Hubble Deep Field-North, where morphological information is available for all galaxies up to apparent magnitude I ∼ 26. Using colour estimated redshifts and assuming exponential scalelengths, they derived absolute blue magnitudes M_B and disc scalelengths R_d as a function of redshift. They compared these data, concerning the redshift range 1 < z < 3.5, to those of the ESO-NTT deep field, concerning the redshift range 0.4 < z < 1 (Fontana et al. 1999). The results of their study appear in Fig. 10 (histograms) for “bright” and “faint” discs (upper and lower panels, respectively). Comparing to predictions of “standard” semi-analytical models, including merging histories for dark haloes, Giallongo et al. (2000) found a significant excess (by a factor of ∼3) in both redshift ranges of bright and small discs (i.e. discs with R_d < 2 kpc in the low redshift range and with R_d < 1.5 kpc in the high redshift range); they also found a smaller excess (by ∼50%) of “faint” discs in both redshift ranges.

Our results are also plotted in Fig. 10. In each panel, dotted and dashed curves indicate scalelength distributions obtained at the two extremes of the corresponding redshift range. When adding the model distributions of the upper and lower panels at a given z one obtains the total distribution of R_d for that z, shown in Fig. 2. We notice that the observed and model distributions are normalised differently (corresponding scales are indicated on the left and right axis of each panel), but the comparison of their shapes allows to draw some interesting conclusions.

In the low redshift range (left panels), our model distributions evolve very little between z = 1 and z = 0.4, with both “bright” and “faint” discs becoming larger with time. In both cases, distributions at z = 1 compare slightly better to the data than distributions at z = 0.4. The overall agreement with the data is excellent. We find no deficiency of small discs (either bright or faint ones), contrary to Giallongo et al. (2000). We notice that we always use the probability distributions F_V and F_λ of Fig. 1, assuming no merging. This assumption turns out to be crucial in the agreement of models with the data. Indeed, taken at face value, the results on the left panel of Fig. 10 imply that the adopted prescriptions for the scaling relations, infall, SFR, F_V and F_λ are sufficient to make discs with the correct distributions of scalelengths at intermediate redshifts.

In the high redshift range (right panels of Fig. 10) we also find little evolution and excellent agreement with observations for the “faint” discs (M_B > -20). We notice that, by construction, our models predict little early evolution for discs of low mass and luminosity. For “bright” discs, we find significant evolution in the R_d distribution, in the sense that this distribution is significantly shifted to larger and more...
Figure 10. Distribution functions of the scalelength in the $B-$ band $R_B$ in two different redshift ranges: $0.4 < z < 1.$ (left) and $1. < z < 3.5$ (right), respectively. In both redshift ranges, distributions concern “bright” discs (upper panels) and “faint” discs (lower panels), respectively; notice that the magnitude limits are different in the two cases ($M_B=-19$ on the left and $M_B=-20$ on the right). Observed distributions (histograms) are from the surveys of the ESO-NTT deep field at low $z$ and of the Hubble Deep Field at high $z$ (Giallongo et al. 2000); the corresponding scales on the vertical axis are displayed on the right of each panel. In each panel we show two model distributions (in dotted and dashed curves, respectively) corresponding to the two extremes of the corresponding redshift range, i.e. at $z=0.4$ and $1$ in the left panels and at $z=1$ and $3.5$ in the right panels, respectively. Our model distributions are not normalised to the observed ones and the corresponding scales are displayed on the left of the vertical axes.

numerous discs when the redshift decreases from $z=3.5$ to $z=1$: in that redshift range a large number of initially small and “faint” discs (belonging to the lower right panel of Fig. 10 at $z=3.5$) become brighter and larger at $z=1$ (and “migrate” to the upper panel), due to the adopted inside-out formation scheme. However, none of the two model distributions compares well to the observed histogram, and this will obviously be true for any other model distribution at redshift $1 < z < 3.5$: we find then (as Giallongo et al. 2000) a serious deficiency (factor $\sim 3$) of small and bright discs at high redshift. We notice, though, that this is the only discrepancy of our models with respect to the observations, while Giallongo et al. (2000) found discrepancies in all four panels of Fig. 10.

The deficiency of our models concerns bright and small discs, i.e. discs with high surface brightness. Selection effects may play some role in that case, since they obviously favour the detection of such discs, at the expense of more extended ones, having lower surface brightness. If there is no selection bias in the data, then it is the first hint that our models (based on the simple prescriptions of Sec. 2) may have some difficulties in reproducing observables of the high redshift Universe. In fact, our models are “calibrated” on the Milky Way disc and the infall timescale adjusted as to reproduce all major observables of discs in the local Universe (Papers II, III and IV). The discrepancy encountered in Fig. 10 may imply that mergers between small discs (not taken into account here) did play some role in that redshift range: such mergers would affect only a small fraction of the population of the lower right panel of Fig. 10, but would contribute significantly in populating the deficient part of the curves in the upper right panel.
5.2 Abundance gradients vs. scalelengths

In Paper III of this series (Prantzos and Boissier 2000) we showed that a remarkable correlation exists between the abundance gradient of a disc (expressed in dex/kpc) and its scalelength \( R_B \); small discs have large (in absolute value) abundance gradients and vice versa. This is easy to understand qualitatively, since abundance gradients are created by radially varying quantities, like the SFR \( \Psi \) and \( F_\lambda \), and do not evolve in time, i.e. we assume that mergers never play a significant role in the overall evolution of disc galaxies. We follow then the evolution of the distribution functions of various quantities and we compare the results with available observations concerning discs at high redshifts. Our results are summarised as follows:

1) Most large discs (\( R_B > 4 h_{100}^{-1} \) kpc) have basically completed their evolution already by \( z \sim 0.5 \). Subsequently, they evolve at \( z < 0.5 \) range. For that same reason, the dispersion around this value increases at low \( z \). Both the stability of the relation of \( d \log(O/H)/dR_B \) with the decrease in the dispersion of \( d \log(O/H)/dR_B \) with redshift are novel and important predictions of our model. If true, they will allow to infer something about the chemistry of high redshift discs from their morphology.

In this section we explore whether this correlation is valid at higher redshifts. In Fig. 11 we plot the relation of \( d \log(O/H)/dR_B \) for our model discs at redshifts \( z=0 \) (where they compare quite favourably with observations), \( z=1.5 \) and \( z=3 \). At all redshifts we find the relation obtained locally. The “homologous evolution” of spirals, invoked by Garnett et al. (1997) for local discs, is also found in the high redshift Universe.

In Paper III (Prantzos and Boissier 2000) we have shown that, when observational bias are taken into account, the observations of \( Zn \) abundance in such systems (DLAs) may be (proto-)galactic discs such as those considered here: in a previous paper (Prantzos and Boissier 2000) we have shown that, when observational bias are taken into account, the observations of \( Zn \) abundance in such systems as a function of their redshift \( (0.5 < z < 3.5) \) are nicely explained by our models. The apparent “no chemical evolution” picture suggested by the observations of those systems (e.g. Pettini et al. 1999) is quite compatible with our understanding of the chemical evolution of discs.

6 SUMMARY

In this paper we explore the implications of our disc galaxy models for the high redshift Universe. Our models (presented in detail in Papers I, II, III and IV) are “calibrated” on the Milky Way, utilise simple prescriptions for the radial variation of the SFR and infall rate and simple scaling relations (involving rotational velocity \( V_C \) and spin parameter \( \lambda \)) and reproduce all major observations concerning discs in the local Universe. A crucial ingredient for the success of our models is the assumption that massive discs form the bulk of their stars earlier than their low mass counterparts.

We assume here that the probability functions \( F_V \) and \( F_\lambda \) of the two main parameters of our models are independent and do not evolve in time, i.e. we assume that mergers never play a significant role in the overall evolution of disc galaxies. We follow then the evolution of the distribution functions of various quantities and we compare the results with available observations concerning discs at high redshifts. Our results are summarised as follows:

1) Most large discs (\( R_B > 4 h_{100}^{-1} \) kpc) have basically completed their evolution already by \( z \sim 0.5 \). Subsequently, they evolve at \( z < 0.5 \) range. For that same reason, the dispersion around this value increases at low \( z \). Both the stability of the relation of \( d \log(O/H)/dR_B \) with the decrease in the dispersion of \( d \log(O/H)/dR_B \) with redshift are novel and important predictions of our model. If true, they will allow to infer something about the chemistry of high redshift discs from their morphology.

At this point, we notice that Damped Luman-\( \alpha \) systems (DLAs) may be (proto-)galactic discs such as those considered here: in a previous paper (Prantzos and Boissier 2000) we have shown that, when observational bias are taken into account, the observations of \( Zn \) abundance in such systems as a function of their redshift \( (0.5 < z < 3.5) \) are nicely explained by our models. The apparent “no chemical evolution” picture suggested by the observations of those systems (e.g. Pettini et al. 1999) is quite compatible with our understanding of the chemical evolution of discs.

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predict a mild evolution of $U-V$ colour of large discs in that redshift range, while the Lilly et al. (1998) data suggest a slightly stronger evolution; but their data, extrapolated to $z \sim 0$, are in disagreement with the local values derived by de Jong (1996), while our models reproduce fairly well the observations of the local sample. Since $U-V$ colours can be strongly affected by “transient” phenomena (e.g. short “bursts” of star formation), we do not think that this discrepancy is a real problem for the models.

2) Because of the “inside-out” formation scheme of discs adopted in our models, we predict that discs were, on average, smaller in the past for a given rotational velocity or for a given magnitude $M_B$. Both results are in qualitative agreement with recent samples of discs in redshifts up to $z \sim 1$ (Sec. 4.2 and 4.3). Our models reproduce fairly well the data of Simmard et al. (1999) on the $R_B$ vs. $M_B$ relation as a function of redshift, up to $z \sim 1$. In particular, our models produce some compact and bright discs that Simmard et al. (1999) find at $z \sim 1$ and which have no counterparts in the local Universe (Sec. 4.3).

3) We find that the Tully-Fisher relation was steeper in the past, with massive discs been, on average, brighter, and low mass discs been fainter than today. Our models are in agreement with observations of the TF relation in the $B-$ band obtained by Vogt et al. (1997) at intermediate redshifts ($z \sim 0.4$ and 0.8), taking into account their error bars (Sec. 4.4). However, these data are insufficient to probe the evolution of the TF relation, which requires data on fainter discs (down to $M_B \sim -18$ in that redshift range).

4) Our models reproduce fairly well the distribution of disc scalelengths in the ESO-NTT survey, concerning both bright and faint discs in the 0.4-1 redshift range (Sec. 5.1). They also reproduce well the scalelength distribution of faint discs in the 1-3.5 redshift range obtained in the Hubble Deep Field (Giallongo et al. 2000). On the contrary, we fail to reproduce the corresponding distribution of bright discs in that same redshift range: our models show a deficiency of small ($R_B \sim 1$ kpc) and bright ($M_B < 20$) discs with respect to the Hubble Deep Field data. If this discrepancy is not explained in terms of selection effects (favouring the detection of those high surface brightness discs) it could be a strong indication that merging of small discs played indeed an important role in that redshift range.

5)  The anticorrelation between the abundance gradient $\log(O/H)/dR$ (expressed in dex/kpc) and the scalelength $R_B$, that we found in Paper III for local discs, is found to be valid also at higher redshifts (Sec. 5.2). We find that the abundance gradient of discs, expressed in dex/$R_B$, varies very little with redshift and presents a smaller dispersion at high redshifts. These findings establish a powerful link between the optical morphology and the chemical properties of disc galaxies, valid in both the local and the distant Universe.

In summary, we explored the consequences of our “backwards” model of disc galaxy evolution for the high redshift Universe. We found no serious discrepancies with currently available data up to redshift $z=1$. At higher redshifts, we find hints that the model may require some modifications, provided that currently available data are not affected by selection biases.

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