Simple collision operators for direct Vlasov simulations of laser plasma interaction and transport

T. D. Arber and N. J. Sircombe

1Centre for Fusion, Space and Astrophysics, Department of Physics, University of Warwick, Coventry, U.K.
2AWE Plc, Aldermaston, Reading, Berkshire, U.K.

E-mail: T.D.Arber@warwick.ac.uk

Abstract. Non-local electron transport effects have a direct influence on the compression of cryogenic targets in laser driven ICF and target heating in high energy density experiments. There is a growing need for self-consistent models of laser plasma interactions coupled to non-local transport. We present a direct Vlasov solver that includes multiple species and a simple collision operator. This BGK model operator - which conserves particle density, energy and momentum – is fully implicit. For collisionless plasmas it has been shown that a double layer may be formed in which an accelerated, kinetic ion population satisfies the zero current condition. Here we extend this result to collisionalities of interest to laser driven ignition to assess the validity of nonlocal electron transport models based on fluid ions.

1. Introduction

Recent experimental campaigns on the OMEGA laser[1] have again highlighted the importance of non-local electron transport effects for direct drive of cryogenic ignition targets. There is therefore a growing need for self-consistent models of laser plasma interactions coupled to non-local transport. This is true both for long pulses where flux limiters may be employed and short pulses where fast electron transport may be inhibited. Considerable progress has been made[2,3] using traditional hydro models of the laser plasma interactions coupled to a non-local electron transport model based on a BGK[2], collision operator. However, these models cannot describe the detailed kinetics of fast electron generation and transport. To obtain a fully consistent description of the electron dynamics one must resort to a Fokker-Planck treatment. These have been successful in modelling many aspects of hot electron transport; see for example[5]. To be computationally tractable these full Fokker-Planck treatments rely on an expansion in spherical harmonics, or similar. To accurately model the laser plasma interaction region the Vlasov equation is usually solved on an Eulerian phase space grid. Currently the inclusion of the full Fokker-Planck term in such Eulerian grid based codes is often too computationally expensive. There is therefore a requirement for a simplified approach to the collision operator.

We present a direct Vlasov solver[6] that includes multiple species and a simple collision operator, based on the approach taken in reference [2], in which the collision frequency is energy dependent. This BGK model operator, which conserves particle density, energy and momentum, is fully implicit. While the fully kinetic treatment can only be applied to a smaller domain than was used in fluid models[1] the electric field is solved for self-consistently. In the fluid limit one has to determine the electric field under the assumption of zero net current for the electrons only. For collisionless plasmas
it has been shown\cite{7} that a double layer may be formed in which an accelerated, kinetic ion population satisfies the zero current condition. Here we extend this result to collisionalities of interest to laser driven ignition to assess the validity of nonlocal electron transport models based on fluid ions.

2. BGK Collision Operator

We solve the relativistic Vlasov-Poisson system with one spatial and one momentum dimension. Vlasov’s equation for the distribution function $f_i$ is given by:

$$\frac{\partial f_i}{\partial t} + \frac{u_i}{\gamma} \frac{\partial f_i}{\partial x} + \frac{q_i}{m_i} E \frac{\partial f_i}{\partial u} = C(f_i)$$

where $f = f(x,u) = f(x,u_\gamma)$, $u = u\gamma$, and $\gamma = \sqrt{1+u^2/c^2}$. Vlasov’s equation is solved using the VALIS code\cite{6} which uses a split-Eulerian conservative scheme based on the PPM method\cite{8}. For speed, simplicity and scalability the electric field is updated through Ampere’s law rather than Poisson’s equation. For the ions the collision term is given simply by $C(f_i) = -\nu_e(f_i - F_i)$ while for the electrons it is given by $C(f_i) = -\nu_e(f_i - F_{ie}) - \nu_e(f_i - F_{i2})$. Following the treatment devised by Mannheimer et al.\cite{2} we use the high and low speed collision frequencies from the NRL Plasma Formulary\cite{9} analytically matched. For example, for the electron-electron collisions we use $\nu_{ee} = 2\nu_e\left(1 + 0.75\left(m_e v/2k_B T\right)^{1/2}\right)$ and expressions for $\nu_a$ and $\nu_b$ are given in reference [2]. The Krook operator is treated implicitly so that for the ions, $f_i^{n+1} - f_i^n = \nu_i \Delta t (f_i^{n+1} - F_i)$ giving $f_i^{n+1} - f_i^n = \nu_i \Delta t \left(f_i^n - F_i\right)/(1 + \nu_i \Delta t)$. By taking velocity moments of this latter expression it is clear that the Krook operator can be made to conserve energy, mass and momentum by a suitable choice of the distribution functions $F_i$, $F_{ie}$ and $F_{i2}$. For the ions the requirement is that $F_i$ satisfies the set of equations defined by $\int \nu_a \left(1 + \Delta t \nu_a\right)^{-1/2} v^n \left(f_i^n - F_i\right) du = 0$ where $n = 0, 1$ or 2. The Krook operator must also be consistent with the relaxation towards a Maxwell-Jüttner distribution. Hence the procedure is to assume $F_i = a \exp\left(-\left(1 + (u - b)^2\right)/d\right)$ and find the set of $(a,b,d)$ which satisfy the conservation of mass, momentum and energy. This only requires knowledge of $f_i^n$ which is of course available at the start of each step so the specification of $F_i$ is complete. This is a simplification over the full Fokker-Planck treatment but has the advantage of being computationally tractable for large grids and mobile kinetic ions.

3. Test Results

An initial tanh profile for the electrons (from 400 eV down to 100eV) was used for testing against previous VFP work. These tests have stationary ions (Z=4) and an electron number density of 2.97x10^{20}m^{-3} so that there are 100 particles per Debye sphere for the 100 eV electrons. The computed heat flux compared to Spitzer-Harm is plotted in Figure 1. This is after 1000 plasma periods for the hot electrons. The figure shows the computed heat flux from the simulation divided by the free streaming flux. Since for the Spitzer-Harm conductivity this quantity is a function only of mean free path over scale length this Spitzer-Harm value is also plotted as a solid line. This shows heat flux limiting of ~0.1. The computed heat flux is multi-valued when plotted in this way. The lower section of the dashed curve corresponds to the hot side of the temperature profile and exhibits thermal flux limiting. The upper section corresponds to the cold side of the simulation and exceeds the Spitzer-Harm value due to the contribution from hot electrons from the higher temperature region. These results are in qualitative and quantitative agreement with previous simulations based on the full Fokker-Planck collision operator\cite{5}.
Figure 1: Ratio of calculated electron heat flux \((Q)\) to the free streaming limit \((Q_f)\) as a function of scale-length over electron mean free path. Solid line is the Braginskii (Spiter-Harn) predicted heat flux. The dashed line is the computed heat flux.

4. Ion Kinetic Effects on Fast Electron Transport

Previous studies of flux limited thermal transport have ignored ion kinetic effects. It is known that hot electron populations in solar coronal flares are capable of triggering thermal fronts\([7]\). In these the initial flux of hot electrons generates a polarization electric field. This draws a return current of cold electrons until there is no net current. For sufficiently large initial temperature jumps the hot electrons draw a cold electron return current which triggers Ion Acoustic Turbulence (IAT) in the hot electron region. The hot electrons must now overcome the polarization electric field and scattering off IAT to conduct energy from the hot region. It is possible that this suppression of electron current allows an ion current to flow through the polarization field region. The net effect is a zero-current double layer in which electron heat transport is completely suppressed while initially cold ions are accelerated up to hot electron energies.

The theory and simulations\([7]\) of solar coronal thermal fronts are based on collisionless ions and electrons. Here we assess whether a similar process might occur in plasma conditions relevant to laboratory laser plasma experiments by using the BGK model collisions described above. Initial conditions are a 300eV background ion distribution and density of \(10^{23} \text{ cm}^{-3}\). These initial tests are for a hydrogen plasma. The electron temperature drops from a hot population down to 300eV. We vary the hot electron temperature from 3keV up to 300 keV and also vary the scale-length of the temperature drop so that a range of mean free path over scale lengths are covered.

Figure 2: Hot electrons at 3 keV with typical mean free path over temperature scale length of \(\sim 0.4\). The solid black lines are the simulated electron and ion kinetic energies after 5000 plasma periods and the red line is the same simulation with immobile ions.

When the hot electrons are at 3keV and the ratio of mean free path to scale length is \(\sim 0.05\) the ions have no effect on the electron temperature profile. At 3keV with a mean free path to scale length ratio of \(\sim 0.4\) the ion motion is sufficient to establish a small temperature jump in the electron profiles. These results are shown in Figure 2. In all figures temperatures are in units of the initial hot electron temperature and lengths are in Debye lengths. At higher hot electron temperatures (300 keV) the cold electron return current is sufficient to trigger IAT despite the presence of collisions. When this...
happens the electron heat flux becomes almost completely suppressed and the ions begin to be accelerated through the polarization field. This can be seen in Figure 3, which repeats Figure 2, but for hotter electrons. At the thermal front the electron flux is orders of magnitude less than the free streaming limit and ions are accelerated to 40% of the initial hot electron energy.

![Figure 3: Hot electrons at 300 keV with typical mean free path over temperature scale length of ~1000. The solid black lines are the simulated electron and ion kinetic energies after 5000 plasma periods and the red line is the same simulation with immobile ions.](image)

5. Conclusions
We have solved the 1D1P Vlasov equation with a BGK operator. The Maxwellian to which the BGK operator will relax is determined on each time-step so as to conserve number density, momentum and energy. This is therefore a highly simplified model but it does have relaxation on the correct time-scale; conserves all fluid quantities required for collisions; satisfies the H-theorem; and has been shown to accurately reproduce flux limited thermal conduction calculations from full Fokker-Planck treatments.

Applying this code to the possibility of thermal fronts in laser plasmas showed that for mean free paths of ~0.05 the ion motion was irrelevant. For a mean free paths of ~0.4 ion motion does introduce a step in the electron temperature profile but collisions are still sufficient to suppress a thermal front. With hot electrons of 300 keV and large mean free paths the collisions are no longer sufficient to suppress kinetic effects and the cold electron return current triggers IAT. This in turn inhibits further hot electron conduction and allows ion acceleration through the polarization field. The final state is a current free double layer with completely suppressed electron thermal transport and accelerated ions. These simulations are of course highly idealized but do suggest that thermal fronts may play a role in short pulse laser heating. To verify if this is indeed true will require modelling laser electron heat input, increased ion mass and charge and possibly two spatial dimensions.

British Crown Owned Copyright 2010 / MOD.

6. References
[1] Goncharov V N et al., *Phys. Plasmas*, 15, 056310 (2008)
[2] Manheimer W et al., *Phys. Plasmas*, 15, 083103 (2008)
[3] Colombant D and Manheimer W, *Phys. Plasmas*, 15, 083104 (2008)
[4] Bhatnagar P L, Gross E P, and Krook M, *Phys. Rev*. 94, 511 (1954)
[5] Bell A Ret al., *Plasma Phys. Contr. Fusion* 48, R37 (2006)
[6] Sircombe N J and Arber T D, *J. Comp. Phys.*, 228, 4773 (2009)
[7] Arber T D and Melnikov V F, *Astrophysical Journal*, 690, 238 (2008)
[8] Colella P and Woodward P R, *J. Comp. Phys.*, 54, 174 (1984)
[9] Huba J D 2006 NRL Plasma Formulary (Washington D. C. : Naval Research Laboratory)