Symmetry Imbalance in a Shaking Triangular Optical Lattice

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Lots of interesting phenomena can be investigated under symmetry imbalance condition, such as symmetry protected topological orders, simulation of frustrated quantum anti-ferromagnetism and so on. In this work, we demonstrated a simple but effective method to realize desired symmetry imbalance of momentum distribution of the atoms in an optical triangular lattice, by shaking the lattice and controlling the driven signal. A quasi-momentum oscillation along the shaking direction in lattice frame of reference causes the formation of the mix of ground energy band and first excited band in laboratory frame of reference and thus the symmetry imbalance, within the regime that the shaking frequency is far less than the coupling frequency between ground band and higher energy bands. We experimentally investigate the influence of the initial phase, frequency, amplitude of the shaking signal on the symmetry imbalance, and observe good agreement with our theoretical model. This supplies a useful tool to study atomic behavior in the non-zero quasi-momentum.

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I. INTRODUCTION

Quantum simulation with cold atoms in optical lattices has provided much insight into the behavior of electrons in condensed matter theory [1], as well as the quantum many-body problem. In the ground state, they represent nearly ideal emulations of single-flavour Hubbard models [2–6]. Numerous work has been carried out to manipulate the atoms in optical lattices [7–8], investigate phase transition [9] and dynamics properties [10]. However, the ground state, i.e. the S-band in optical lattices, are highly symmetric, which limited their usefulness in investigating a large variety of interesting phenomena related to symmetry protected properties, such as the emerging of Dirac points [11–12], symmetry protected topological orders [13] and so on. Therefore, the frustration in symmetry of the quantum states of cold atoms in optical lattices raised much attention of the community [14]. Many interesting investigation has been carried out under symmetry breaking conditions, such as frustrated quantum anti-ferromagnetism [15].

To break the symmetry, different techniques were proposed, sharing the idea of exciting atoms into higher energy bands or shaking the lattice. A number of research are done into P-band [10–11] and F-band [12]. Others consider shaking lattice, which is also a useful tool to create symmetry imbalance condition. With shaking lattice, Mott insulator to superfluid (MI-SF) phase transition can be realized [13–21], opening up a new direction for cold atom simulation that is unique under symmetry breaking condition [21]. For instance, generating gauge potentials [22–23], the realization of effective flux and various frustrations [24], artificial gauge fields [21–23], engineering Ising XY models [25], and the realization of Hal-dane model in ultra-cold atoms [26] which leads to further research of topological bands and states [27–28].

In this work, we demonstrated a simple but effective method to realize desired symmetry imbalance of S-band bosons in a triangular optical lattice. The idea is to apply shaking on the lattice, where the shaking signal is designed as a biased sinusoidal function with linearly ramped amplitude. Within the lattice frame of reference, depending on the chosen value of initial phase, amplitude and frequency of the shaking signal, the atoms experience an oscillation of quasi-momentum, and could be transferred to different desired quasi-momentum states. In laboratory frame of reference, a small fraction of atoms are populated onto the P-band so that a mixed S- and P-orbital is formed, resulting the symmetry imbalance in the laboratory frame. Here the initial phase \( \varphi \) (ranging from 0 to \( 2\pi \)) plays the critical role. We experimentally investigate the relation between the symmetry imbalance and shaking initial phase \( \varphi \), which shows good agreement with the quasi-momentum-oscillation model, within the regime that the amplitude is less than one lattice constant and the frequency is far less than the coupling frequency between S-band and higher energy bands.

The remainder of this manuscript is organized as follows. In Sec. [1] we discussed the more complicated circumstances. In Sec. [11] briefly introduce the application on our experimental triangular lattice and control sequence. The results and analysis are given in Sec. [1V] Then we discuss several potential application of this method in Sec. [V] before the conclusions are given in Sec. [VI].
II. THEORETICAL MODEL FOR REALIZING
SYMMETRY IMBALANCE BY SHAKING

We first consider the general situation for the motion of atoms in a shaking optical lattice. As shown in Fig. 1(a), an optical lattice is formed as the blue line shows, with lattice constant \( a \), namely the distance between two nearest neighbor sites. When shaking signal is applied, the lattice moves a displacement \( \mathbf{S} \) of its nearest neighbor sites. When the shaking signal is applied, the lattice constant \( a \) is transformed into a shaking optical lattice. As shown in Fig. 1(a), where \( \mathbf{S} \) shows the non-zero dashed line shows the position of the shaking lattice. The blue line denotes the initial position of the lattice. The red dashed line shows the real-time position of the shaking lattice. The loading time sequence, after which the atoms are populated into the lattice. (b) The time sequence. (b1) shows the adiabatic situation. (b2) shows the shaking time sequence. The red dashed line denotes the real-time position of the shaking lattice, while term \( 2\pi - \mathbf{S} \cdot \mathbf{r} \) denotes the real-time position of the shaking lattice. The blue line denotes the initial position of the lattice. The following discussions in this section are all in the lattice frame of reference. For convenience, we leave out the subscript \( \text{lab} \). What does matter is the presence of the lattice, whose effect could be simply described by using quasi-momentum \( \mathbf{q} \) and effective mass \( m^* \) in place of momentum \( p \) and mass \( m \), respectively. The motion equation reads

\[
h \frac{d}{dt} \mathbf{q} = \mathbf{F}(t)
\]
or in the integral form

$$\vec{q} = \frac{1}{\hbar} \int_{0}^{t_{\text{total}}} \vec{F}(t) dt = -\frac{m^*}{\hbar} \frac{d}{dt} \vec{S} \bigg|_{0}^{t_{\text{total}}}$$  \hspace{1cm} (7)$$

where $t_{\text{total}}$ is the total time as defined in Fig.1(b). Here we don’t take the force of lattice into consideration, since under the near-free-electron model the effect of lattice has been described as distortion of electrons’ dispersion relation. A simple sinusoidal function lasts for integer periods won’t work obviously, since the integral is zero. Considering the driving signal mentioned above, the total cumulated quasi-momentum is given by

$$q = \frac{m^* \omega A}{\hbar} \sin \varphi = Q \sin \varphi$$  \hspace{1cm} (8)$$

where $Q \equiv \frac{m^* \omega A}{\hbar}$ is defined as the oscillation amplitude of quasi-momentum.

III. EXPERIMENTAL SETUP AND THEORETICAL PREDICTION

Compared to square lattice and honeycomb lattice, the symmetry imbalance in a triangular lattice could be observed more clearly. The experimental setup is similar to our previous work \[9\], a nearly pure condensate of about $1.5 \times 10^5$ $^8$Rb atoms is obtained in a hybrid optical-magnetic trap with harmonic trapping frequencies $(\omega_x, \omega_y, \omega_z) = 2\pi \times (28, 55, 65)$Hz in three directions respectively. The lattice is constructed with laser beams of wavelength $\lambda = 1064$ nm and lattice constant approximately $a = 709$ nm. We adiabatically load the atoms into the S-band in $t_{\text{rise}} = 60$ ms. The lattice is formed by three linearly polarized laser beams intersecting at one point with enclosing angle of 120°. The polarization directions of the three lasers are perpendicular to the lattice plane. So $\vec{k}_i$ in Eq.(1) become $\vec{k}_1 = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right) k$, $\vec{k}_2 = \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right) k$, $\vec{k}_3 = (0, 1)k$. The shaking is realized with the help of a PZT, with direction along direction $\vec{k}_1$, as shown in Fig.2(a). The probe is taken perpendicular to the lattice plane, namely along $z$-axis.

The whole process can be well understood as an oscillation of quasi-momentum in the lattice frame on the S-band, as shown in Fig2(b). In this frame, the quasi-momentum of atoms follows a certain path determined by our shaking signal and periodically go around the designed path.

According to the theory mentioned above, we can predict the momentum distribution as shown in Fig3(a1), by calculating the accumulated quasi-momentum of the atoms and transfer the TOF image back to the laboratory frame of reference, given the initial phase $\varphi$. shaking amplitude $A$ and the frequency $f$. The atomic density peak marked by the red dashed circles are sharper than that marked by the green dashed circles, i.e. the so-called unbalanced symmetry condition. If we take integral along the shaking direction $\vec{k}_1$, we get a one-dimensional side view image (Fig3(a2)), where the green (red) circles denote the addition of the two points in green (red) circles in Fig3(a1), and its corresponding atomic density distribution (Fig3(a3)). Clearly, the right peak (number of atoms being $N_2$) is higher than the left one (number of atoms being $N_1$). In order to quantify the symmetry imbalance, we define imbalance $I$ as

$$I = \frac{N_1 - N_2}{N_1 + N_2}$$  \hspace{1cm} (9)$$

where $N_1$ and $N_2$ are shown in Fig3(a3).

Given the value of initial phase $\varphi$, amplitude $A$ and frequency $f$ of the shaking signal, we could calculate the quasi-momentum accumulated by the atoms with the assumption $m^* = m$. By solving a set of central equations, atomic density distribution in reciprocal space is obtained. Therefore, we could give the numerical simulation of imbalance $I$ (defined in Eq.(9) with respect to $\varphi$.

FIG. 2. (color online) (a) The construction of the triangular lattice. Three linear perpendicular-to-plane polarized laser beam intersect at one point in x-y plane, where the triangular lattice are formed. The three laser beams are denoted as $k_1$, $k_2$ and $k_3$. The initial phase $\theta_1$ of laser beam $k_1$ is controlled with a PZT. The shaking driving signal is applied to this PZT to create shaking along direction $k_1$. The probe is set to perpendicular to the lattice plane (axis z). (b) In the lattice frame of reference, this whole manipulation process of shaking lattice can be understood as an oscillation of the quasi-momentum on S-band.
A smaller than \( \varphi \) first and then rise up as \( \varphi \) is fixed (red dashed line), the imbalance \( I \) depending on whether the chosen value of \( \varphi \) just needs to determine the value of \( Q \) and \( \varphi \) chosen value of \( Q \) shows \( t_{\text{total}} + \delta t \), resulting in a bias on the accumulated quasi-momentum

\[
q = Q \sin \varphi + \omega^2 A \delta t \cos \varphi
\]

Taking this bias into consideration, this whole image shows a periodic change, allowing us to manipulate the symmetry imbalance, or the quasi-momentum in the lattice frame of reference.

In Fig. 3(b) the momentum distribution is shown for \( \varphi \) ranging from 0 to \( 4\pi \) with step of 0.25\( \pi \) (each point is an average of five times of TOF images). Due to the imperfection in our phase control system, atoms accumulated an extra quasi-momentum in the shaking process, presented as a bias of the symmetry imbalance shown in Fig. 4. The momentum distribution of atoms shows a significant deviation to the lower right at phase being larger than 0.5\( \pi \), and a deviation to the upper left at phase being 0 and 0.25\( \pi \). The \( \varphi = 0 \) case isn’t necessarily symmetric due to the imperfections in our experiment — the shaking signal can’t be switched off immediately, but with a little delay \( \delta t \). As a result, the integration time in Eq. (6) becomes \( t_{\text{total}} + \delta t \), resulting in a bias on the accumulated quasi-momentum

\[
q = Q \sin \varphi + \omega^2 A \delta t \cos \varphi
\]

and \( Q \) (defined in Eq. 5) as shown in Fig. 3(b). When \( Q \) is fixed (red dashed line), the imbalance \( I \) will fall down first and then rise up as \( \varphi \) increases from 0 to 2\( \pi \). When phase \( \varphi \) is fixed, the imbalance \( I \) will either rise or fall depending on whether the chosen value of \( \varphi \) is larger or smaller than \( \pi \). To achieve desired symmetry imbalance, one just needs to choose appropriate value of \( A \) and \( \varphi \) according to Fig. 3(b). Practically, \( Q \) is fixed and one just needs to determine the value of \( \varphi \). Our experimental data is obtained along this red and yellow dashed line.

### IV. EXPERIMENTAL DEMONSTRATION FOR SYMMETRY IMBALANCE

In the experiment, the driven signal is chosen as \( t_{\text{ramp}} = 20 \text{ms} \) and \( t_{\text{hold}} = 5 \text{ms} \). We first investigated the influence of the initial phase \( \varphi \) of the shaking signal, with frequency and amplitude fixed as \( f = 3 \text{kHz} \) and \( A = 532 \text{nm} \) and well depth fixed as \( V_0 = 4E_r \), where \( E_r = (\hbar k)^2/(2m) \) is the recoil energy of an atom, to realize good visibility. With various initial phase \( \varphi \) (as the red dashed line shown in Fig. 3(b)), we take TOF images of the atoms.

In Fig. 4 the momentum distribution is shown for \( \varphi \) ranging from 0 to 4\( \pi \) with step of 0.25\( \pi \) (each point is an average of five times of TOF images). Due to the imperfection in our phase control system, atoms accumulated an extra quasi-momentum in the shaking process, presented as a bias of the symmetry imbalance shown in Fig. 4. The momentum distribution of atoms shows a significant deviation to the lower right at phase being larger than 0.5\( \pi \), and a deviation to the upper left at phase being 0 and 0.25\( \pi \). The \( \varphi = 0 \) case isn’t necessarily symmetric due to the imperfections in our experiment — the shaking signal can’t be switched off immediately, but with a little delay \( \delta t \). As a result, the integration time in Eq. (6) becomes \( t_{\text{total}} + \delta t \), resulting in a bias on the accumulated quasi-momentum

\[
q = Q \sin \varphi + \omega^2 A \delta t \cos \varphi
\]

These images can be understood in another perspective. In addition to our initial idea to realize the symmetry imbalance by manipulating the quasi-momentum in the lattice frame of reference, a pronounced interpretation in the laboratory frame of reference as shown in Fig. 6. In the Bloch band picture, the observed states can be projected onto a superposition as

\[
\Psi = \sum_{n,q} c_{n,q} \cdot \psi_{n,q}
\]

where \( \Psi \) is our observed state, \( c_{n,q} \) is the superposition coefficients, \( \psi_{n,q} \) is the Bloch states, \( n \) and \( q \) denotes the band index and quasi-momentum, respectively. We find that \( \psi_{1,0} \) and \( \psi_{2,0} \) (denoted as \( |s\rangle \) and \( |p\rangle \) respectively in context below) make up about 99.6% of our observed state, which means the symmetry-unbalanced state we realize is a mix up of the first two Bloch bands at zero quasi-momentum. In Fig. 6(a), (a1) shows the real space
FIG. 4. (color online) Unbalanced momentum distribution at different shaking initial phase $\varphi$. As $\varphi$ increases, the momentum distribution shows a periodic change. Due to the imperfections in our experiment procedure, the imbalance $I$ experience a bias, which is demonstrated more clearly in Fig.5(a).

Therefore, an unbalanced picture is formed when the mixed state emerges (Fig 5(b2)). In addition, the proportion of $|s\rangle$ and $|p\rangle$ state varies as $\varphi$ changes, as shown in Fig 5(a). As expected, more unbalanced TOF images show larger proportion of $|p\rangle$ component (Fig 5(c)).

V. DISCUSSION

As previously demonstrated, this method of creating symmetry imbalance by shaking lattice can be perfectly applied to the triangular lattice, in the regime of appropriate shaking frequency $f$ and hold time $t_{\text{hold}}$. The chosen value of $f$ and $t_{\text{hold}}$ do have impact on the perfectness of this method, which are discussed as follows.

a. Effect of frequency Although the experimental results are well demonstrated by our theoretical model in the regime of $f = 2.3$ Hz, there does exist some discord in the influence of shaking frequency if frequency goes higher than 2.3 Hz. As shown in Fig 7(a), the relation between imbalance and frequency seems irregular. In this regime, our theoretical model is no longer suitable. In the derivation we directly neglect the force originated from the lattice but consider only the inertial force, which is only reasonable when the frequency is much larger than the harmonic frequency of the lattice potential well, and, at the same time, small enough compared to the corresponding frequency of the energy difference between S-band and higher energy bands. Therefore in other frequencies, either the lattice is too powerful, preventing the atoms from move in response to shaking, or the atoms could be excited to higher energy bands, where more complicated theoretical model is needed to consider.

b. Stability in long time In the above investigation, the platform period of the shaking signal are chosen as 5 ms to achieve good visibility of experiment results. We would like to emphasize that symmetry imbalance created by this method is quite stable. As shown in Fig 7(b), when changing platform time from 0 to 15 ms, the im-

atom distribution of pure $|s\rangle$ state and (a2) the real space atom distribution of a mixed state of 94% $|s\rangle + 6\% |p\rangle$ which is exactly the case in our experiment at $\varphi = 0.75\pi$. The arrows and the red or blue color denote the phase of the wave function. In reciprocal space, Fig 6(b) provides a clear image of this mixed state. Because the wave function $|p\rangle$ is anti-symmetric, the upper left two points are enhanced, while the lower right two points are weaken.
FIG. 6. (color online) Explanation for the symmetry imbalance. The atomic distribution of pure $S$-orbital and the mixed $S$- and $P$-orbital in real space are shown in (a1) and (a2), respectively. The red and blue color or the arrows denote the local phase distribution. Momentum distribution of pure $S$-orbital (symmetric), pure $P$-orbital (anti-symmetric), and the mixed $S$- and $P$-orbitals are shown in (b1), (b2) and (b3). In (b3) the upper-left two points are enhanced and the lower-right two points are weakened. (c) The fraction of $S$- (blue) and $P$-orbital (red) corresponding to the same condition shown in 5(a). The square and diamond show the band fraction calculated from experimental data, while the dashed lines are the theoretical prediction.

Balance $I$ remains a relatively constant value. Only after 10 ms can we observe unexpectedly shift in symmetry imbalance. This shift is probably due to the de-coherence. In the regime of $t_{\text{hold}} > 5$ ms, the atom gas has actually shown indications of de-coherence and become heated due to the imperfections of other parts of our experiment system.

c. Further application This kind of method could be significantly useful in quantum manipulation. Although we take the triangular lattice as an example, this method could be applied to any kind of lattice. Here we present two examples. First, by choosing appropriate value of initial phase $\varphi$ we could realize a path that the atoms would follow. Fig.8(a), showed a path called Lissajous-Figure. Second, more realistically, we present a method to control the atoms to follow a certain path that surrounds the high-symmetry points, as shown in Fig.8(a). In triangular lattice this provides no non-trivial information. However, this path in honeycomb lattice, whose reciprocal lattice and the Brillouin Zones are identical to triangular lattice, would surround two Dirac-points in opposite orientation. Therefore, it provides a probability to investigate Berry’s phase.

VI. CONCLUSION

In summary, we demonstrated a method for effective manipulation of the symmetry imbalance of the a BEC in the optical triangular lattice. This method applies
shaking to the lattice after the atoms are adiabatically loaded into the S-band. By shaking, the S-band and P-band could be correlated together and the atoms could be transferred to different quasi-momentum state in the lattice frame of reference. The initial phase $\varphi$ of the shaking signal is chosen to achieve the designed symmetry imbalance, or the designed quasi-momentum in the lattice frame of reference. The influence of shaking amplitude are also investigated. Furthermore, we show this method is also useful in the cases of two-direction shaking and other kind of lattices such as honeycomb lattice and square lattices. The experimental and theoretical results are well consistent with each other. This efficient method provides a probability to manipulate the symmetry imbalance of the momentum distribution and the mix of different energy bands.

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