Light Neutrinos from a Mini-Seesaw Mechanism in Warped Space

Kristian L. McDonald

Max-Planck-Institut für Kernphysik, Postfach 10 39 80, 69029 Heidelberg, Germany.

Email: kristian.mcdonald@mpi-hd.mpg.de

Abstract

The seesaw mechanism provides a simple explanation for the lightness of the known neutrinos. Under the standard assumption of a weak scale Dirac mass and a heavy sterile Majorana scale the neutrino mass is naturally suppressed below the weak scale. However, Nature may employ Dirac and Majorana scales that are much less than typically assumed, possibly even far below the weak scale. In this case the seesaw mechanism alone would not completely explain the lightness of the neutrinos. In this work we consider a warped framework that realizes this possibility by combining naturally suppressed Dirac and Majorana scales together in a mini-seesaw mechanism to generate light neutrino masses. Via the AdS/CFT correspondence the model is dual to a 4D theory with a hidden strongly coupled sector containing light composite right-handed neutrinos.
1 Introduction

The confirmed discovery of neutrino mass has provided further information about the flavor structure of Nature. Despite the riches afforded by these developments we still do not know the underlying mechanism responsible for neutrino mass generation (for nice reviews see [1]). The seesaw mechanism provides a simple explanation for the lightness of the known neutrinos [2]. In the standard seesaw picture one assumes $M_R \gg m_D \sim m_W$, where $m_D$ is the neutrino Dirac mass, $M_R$ is the singlet Majorana mass and $m_W$ is the $W$ boson mass. The resulting mass eigenstates include a “mostly active” light neutrino with mass $m_\nu \sim m_D^2/M_R$ and a “mostly sterile” heavy neutrino with mass $\sim M_R$. The suppressed mass of the lightest state nicely explains the existence of light neutrinos that interact weakly with standard model (SM) leptons. The existence of a large mass scale $M_R$ also marries well with our expectations for gauge unification in the ultraviolet (UV) completion of the SM.

Beyond expectations of naturalness, there is perhaps no particular reason to expect the Dirac mass to be of order $\sim m_W$. It is at least reasonable to consider Dirac masses in the range $m_e \leq m_D \leq m_t$, consistent with those observed in the charged fermion sector [3]. Unfortunately, even at the light end of this range, values of $m_\nu \sim 0.1$ eV require $M_R \sim 10$ TeV and it is difficult to probe the Majorana scale directly; a situation that becomes increasingly hopeless as $m_D$ increases. From a naturalness point of view [4], arbitrarily light Dirac mass scales are technically natural due to the restoration of a chiral symmetry in the limit $m_D \to 0$ with $M_R = 0$. Indeed, even arbitrarily light Majorana mass scales are technically natural as $M_R \to 0$ restores lepton number symmetry. However, despite being technically natural small values typically require small couplings and one hopes that the discovery of neutrino mass reveals more than a simple preference for tiny couplings in Nature.

An interesting exception to the standard seesaw picture arises if the right-handed neutrinos are composite objects of a strongly coupled hidden sector [5, 6, 7]. The resulting neutrino mass scales can be suppressed by powers of $(\Lambda_{\text{hid}}/\Lambda)$ and may be much lighter than typically assumed $(\Lambda_{\text{hid}}$ is the hidden confinement scale and $\Lambda$ is the cutoff). A related scenario is that with “late-time neutrino masses” wherein flavor symmetries ensure massless neutrinos until relatively low energies, after which symmetry breaking induces neutrino mass [8].

In this work we develop an approach to the generation of SM neutrino mass that is inspired by the notion of light, composite right-handed neutrinos [5]. The approach realizes light SM neutrinos by combining naturally suppressed Dirac and Majorana mass scales together in a low-scale or “mini” seesaw mechanism. Motivated by the AdS/CFT correspondence [9], we consider a warped extra dimension [10] with a sub-TeV infrared (IR) scale [11, 12, 13], in which the right-handed neutrinos propagate. Via the application of AdS/CFT to Randall-Sundrum (RS) models [14], this 5D model is dual to a 4D theory with a hidden strongly coupled sector of which the right-handed neutrinos are the lightest fermionic composites. The SM fields, taken localized on the UV brane, are fundamental objects in the sense that they are not part of the hidden CFT. We work with an effective theory for energies $\lesssim$ TeV, and depending on the UV completion the SM fields may remain as fundamental or may themselves be composites of a separate strongly coupled sector.
Before proceeding we note that bulk gauge-singlet neutrinos in RS models were considered in \cite{15} and bulk SM fermions in \cite{16}. Subsequent studies of neutrino mass appeared in \cite{17} and for an incomplete list of recent works in this active field see \cite{18,19}. Related work on right-handed neutrinos within a strongly coupled CFT was undertaken in Ref. \cite{20}. Our implementation within a sub-TeV scale effective theory differs from these previous works. We also note that an order GeV hidden sector has been invoked in connection with some recent experimental anomalies \cite{21,22}. A GeV scale mediator may be motivated by the leptonic cosmic ray anomalies\cite{22}, or the dark matter may itself be $\sim$ GeV (see, e.g., \cite{24}). It is sensible to ask what consequences a sub-weak hidden sector may have in the neutrino sector. If the hidden sector contains fermions they could clearly influence the mechanism of neutrino mass generation. We present a specific framework here, but this matter may be of more general interest.

2 Light Neutrinos from a Mini-Seesaw

We consider a truncated RS model with a warped extra dimension described by the coordinate $z \in [k^{-1}, R]$. A UV brane of characteristic energy scale $k$ is located at $z = k^{-1}$ and an IR brane with characteristic scale $R^{-1}$ is located at $z = R$. The metric is given by

$$ds^2 = \frac{1}{(kz)^2} \left( \eta_{\mu\nu} dx^\mu dx^\nu - dz^2 \right) = G_{MN} dx^M dx^N,$$

where $M, N, .. (\mu, \nu, ..)$ are the 5D (4D) Lorentz indices and $k$ is the $AdS_5$ curvature. The characteristic IR scale is suppressed relative to the curvature due to the warping, as is readily seen using the proper coordinate for the extra dimension\footnote{Our approach may also be of interest for models of TeV scale dark matter with a warped GeV scale mediator \cite{23}.} When sourced by a bulk cosmological constant and appropriate brane tensions the metric of Eq. (1) is a solution to the 5D Einstein equations \cite{10}. The length of the space is readily stabilized \cite{25}.

We take the SM to be localized on the UV brane where the natural mass scale is $\sim k$, and accordingly take $k \sim$ TeV. In addition to the SM we consider three singlet fermions propagating in the bulk. We label these by $N_R$ as the zero modes will be right-chiral fields that we identify as gauge-singlet neutrinos. The IR scale is nominally taken to be $R^{-1} \sim$ GeV, but smaller values may be possible and will be considered below\footnote{This is defined by $y = k^{-1} \log(kz)$, where $y_i \in [0, L]$ and $L = k^{-1} \log(kR)$, in terms of which the IR scale is exponentially suppressed, $R^{-1} = e^{-kL} k \ll k$.} A sketch of the setup is given in Figure 1. In analogy with the Little RS model \cite{28} we can refer to this as a “Little Warped Space” (LWS) (for additional work on a truncated slice of $AdS_5$ see \cite{29}). Such a truncated spacetime may seem somewhat unusual at first sight. However, the setup can be thought of as an effective theory that describes the sub-TeV scale physics of a more complete theory, enabling one to consider the effects of a light warped/composite hidden sector without having to specify the supra-TeV physics. In general the supra-TeV effects

\footnote{Light sterile neutrinos may be of interest in relation to the observed pulsar velocities \cite{26} or as dark matter candidates (see e.g. \cite{27}).}
Figure 1: Sketch of the “Little Warped Space”. The sterile neutrino $N_R$ propagates in a hidden warped space with IR scale of order a GeV and UV scale of order a TeV. The standard model resides on the UV brane and has a suppressed coupling to the chiral zero mode neutrino, which is localized toward the IR brane.

will be encoded in UV localized effective operators. As with any sub-TeV scale effective theory, the effective operators that break approximate/exact symmetries of the SM must be adequately suppressed to ensure that problematic effects like rapid p-decay and excessive flavour violation do not occur.

To demonstrate our main points it will suffice to consider a single generation of neutrinos. The action for a bulk fermion $N_R$ in the background (1) is

$$S_{N_R} = \int d^5x \sqrt{G} \left\{ \frac{i}{2} \bar{N}_R \Gamma^B e_B^A \partial_A N_R - \frac{i}{2} (\partial_A \bar{N}_R) \Gamma^B e_B^A N_R + ck \bar{N}_R N_R \right\},$$

where $\Gamma^{\mu,5} = \{\gamma^{\mu}, i\gamma^5\}$ are the 5D Dirac-gamma matrices, $e_B^A$ is the fünfbein and we write the Dirac mass in units of the curvature $k$. We have dropped the spin-connection terms which cancel in the above. A Kaluza-Klein (KK) decomposition may be performed as

$$N_R(x,z) = (kz)^{2} \sum_n \left\{ \nu_L^{(n)}(x)f_L^{(n)}(z) + \nu_R^{(n)}(x)f_R^{(n)}(z) \right\},$$

and the bulk wavefunctions $f_{L,R}^{(n)}$ for the chiral components of $N_R$ can be readily found [15].

The boundary conditions force one chirality to be odd and without loss of generality we take this to be the left-chiral field. The single massless mode in the spectrum then has right-chirality. The KK-expanded Lagrangian is

$$S_N = \sum_n \int d^4x \left\{ \bar{\nu}^{(n)}(x)\gamma^\mu \partial_\mu \nu^{(n)}(x) - m_n \bar{\nu}^{(n)}(x)\nu^{(n)}(x) \right\},$$

where $\nu^{(n)} = \nu_L^{(n)} + \nu_R^{(n)}$ is a Dirac fermion with KK mass $m_n$ for $n > 0$ and $\nu^{(0)} = \nu_R^{(0)}$ is the massless right-chiral zero mode. Its bulk profile is

$$f_R^{(0)}(z) = \sqrt{\frac{k(1+2c)}{(kR)^{1+2c} - 1}} (kz)^c,$$

where the dimensionless mass parameter $c$ controls its localization along the extra dimension. We will be interested in IR localization with $c \simeq 1$ as this reduces the wavefunction overlap.
of the zero mode with UV localized SM neutrinos and therefore suppresses the Dirac mass below the weak scale.

At this point there is no lepton number violation and the spectrum consists of a single Weyl neutrino and a tower of Dirac neutrinos with masses \( m_n \sim n\pi/R \). We can introduce lepton number violation in the form of a marginal operator on the IR brane:

\[
S_N \to S_N - \frac{\lambda_N}{2} \int d^5x \sqrt{-g} \{ \bar{N}_N N_R + \text{H.c.} \} \delta(z - R)
\]

\[
= \sum_{m,n} \int d^4x \left\{ \bar{\nu}^{(n)} \gamma^\mu \partial_\mu \nu^{(n)} - m_n \bar{\nu}^{(n)} \nu^{(n)} - \frac{M_{mn}}{2} (\bar{\nu}_R^{(m)} \nu_R^{(n)} + \text{H.c.}) \right\},
\]

where the effective Majorana masses are

\[
M_{mn} = \lambda_N f_R^{(m)} f_R^{(n)} \bigg|_{z=R}.
\]

Note that \( \nu_L^{(n)} \) does not acquire a boundary Majorana mass as \( f_L^{(n)} \bigg|_{z=R} = 0 \). For IR localization of interest to us the zero mode Majorana mass takes a particularly simple form:

\[
M_{00} \simeq (1 + 2c) \frac{\lambda_N}{R}.
\]

Using the results in the Appendix for \( c \simeq 1 \), the Majorana masses for the \( m, n > 0 \) modes can be approximately related to that of the zero mode:

\[
|M_{0n}| \simeq \sqrt{\frac{2}{2c+1}} M_{00} \quad \text{and} \quad |M_{mn}| \simeq \frac{2M_{00}}{(2c+1)} \quad \text{for} \quad m, n > 0.
\]

We note that \( M_{mn} \sim \lambda_N/R \) for all \( m, n \), as expected for an IR localized mass. These Majorana masses mix the KK modes and the true mass eigenstates are linear combinations of \( \nu^{(n)} \). The spectrum consists of a tower of Majorana neutrinos with masses starting at \( \sim R^{-1} \). For \( \lambda_N \sim 0.1 \) one has \( M_{00} \sim R^{-1}/10 \) and the lightest mode is predominantly composed of \( \nu_R^{(0)} \). The higher modes are pseudo-Dirac neutrinos with mass splittings set by \( M_{mn} < m_n \). The Dirac mass \( m_n \) increases with \( n \) as \( m_n \sim (n + c/2)\pi/R \) while \( M_{mn} \) does not significantly change, so the Majorana masses become increasingly unimportant for the higher modes.

Having determined the spectrum of sterile neutrinos we can proceed to consider their coupling to the SM. This occurs via a UV localized Yukawa interaction

\[
S \supset -\frac{\lambda}{\sqrt{M_x}} \int d^5x \sqrt{-g_{uv}} L H N_R \delta(z - k^{-1}),
\]

where \( L \) is a lepton doublet and \( H \) is the SM scalar doublet. After integrating out the extra dimension this generates Dirac mass terms coupling the KK neutrinos to the SM:

\[
S \supset -\sum_n \int d^4x \ m_n^D \bar{\nu}_L \nu_R^{(n)},
\]

\footnote{One could instead include the boundary mass in the IR boundary conditions and obtain the full KK spectrum directly \cite{17}. However our main points are easily seen treating the boundary terms as perturbations.}
where \( m_n^D = \lambda \langle H \rangle f_R^{(n)}(k^{-1})/\sqrt{M_\ast} \) and \( \langle H \rangle \simeq 174 \text{ GeV} \) is the vacuum value of the SM scalar. For the zero mode this gives

\[
m_0^D \simeq \lambda \sqrt{k(1 + 2c)/M_\ast} (kR)^{-c-1/2} \langle H \rangle,
\]

and the results from the Appendix can be used to find \( m_n^D \) for \( n > 0 \).

Including the boundary coupling to SM neutrinos produces a somewhat complicated mass matrix describing both the SM and KK neutrinos (similar to that in [17]). Despite this the hierarchy of scales generated by the KK profiles allows the basic spectrum to be readily understood. For \( n > 0 \) the previously pseudo-Dirac neutrinos now have Dirac masses coupling them to the SM, which are given by \( (m_n^D/m_n) \sim \lambda (\langle H \rangle/k)\sqrt{2/M_\ast R} \ll 1 \) for \( c \simeq 1 \). This coupling can essentially be neglected to leading order and therefore the \( n > 0 \) modes remain as pseudo-Dirac neutrinos comprised of predominantly sterile KK modes.

The coupling of the SM to the zero mode is more important as this mode is not strongly coupled to a KK partner. To leading order the zero mode and the SM neutrino essentially form a standard seesaw pair. The heavy mode has mass \( \simeq M_{00} \) and is mostly comprised of \( \nu^{(0)} \), while the mass of the light SM neutrino is of the usual seesaw form,

\[
m_\nu \simeq \frac{(m_0^D)^2}{M_{00}} \simeq \frac{\lambda^2}{\lambda_N} \frac{\langle H \rangle^2}{M_\ast} (kR)^{-2c}.
\]

To get a feeling for the scales involved we plot the inverse radius \( R^{-1} \) as a function of \( c \) for fixed values of \( m_\nu = 1 \text{ eV} \) and \( m_\nu = 10^{-2} \text{ eV} \) in Figure 2. The following set of parameter values is used for the plot: \( k = 1.5 \text{ TeV}, \lambda/\sqrt{2} = \lambda_N = 0.1 \) and \( k/M_\ast = 1/6 \). We also restrict \( |c| \) to be no larger than the values employed in [28]. It is clear that the SM neutrino mass is readily suppressed below the weak scale to within the range of interest for the solar and atmospheric neutrino data.

We observe that this approach generates naturally suppressed neutrino masses in a two-fold process. Firstly, the effective 4D Dirac and Majorana mass scales are suppressed; the sub-TeV Majorana scale (\( M_{00} \ll m_W \)) is generated by warping while the Dirac mass is suppressed by a small wavefunction overlap (\( m_0^D \ll M_{00} \ll m_W \)). Secondly, a low-scale or “mini” seesaw mechanism operates between the lightest KK mode and the SM neutrino, serving to further suppress the SM neutrino mass. Together these elements realize the order eV neutrino masses. We emphasize that, unlike most seesaw models with light sterile Majorana mass scales, the small 4D masses are generated naturally and do not require tiny couplings.

Note that Figure 2 includes values of \( R^{-1} \) much less than a GeV, showing that naturally light neutrino masses can be generated for a range of IR scales. Clearly, the phenomenological consequences of generating neutrino mass in this way depend on the specific value of \( R^{-1} \), and one cannot make general statements over such a wide range of energies. A detailed study would be required to determine the relevant limits on \( R \), which is beyond the scope of this work. However, we will offer some comments on aspects of the hidden sector for parts of this range in Section 4 focusing on the case of \( R^{-1} \sim \text{GeV} \). We emphasize that while light
neutrino masses can be obtained with $R^{-1} \ll \text{GeV}$, models with light values of $R^{-1}$ may face severe bounds from experimental data. These require further detailed investigation to determine their viability.

3 4D Gravity and AdS/CFT

The particle spectrum will contain additional sub-TeV modes in the form of metric fluctuations. These are localized towards the IR and their precise coupling to the UV localized SM fields depends on the UV action [30]. If one seeks to include 4D gravity then a UV localized Einstein-Hilbert term with strength $M_{UV}$ will appear to reproduce the 4D Planck mass as $M_{Pl}^2 \sim M_{UV}^2 + M_3^2/k$. Here $M_3^2/k$ is the usual bulk contribution to the 4D Planck mass in RS models but is suppressed relative to the 4D Planck scale for the LWS. Including this boundary term is akin to the 4D approach of retaining a ~ TeV scale cutoff for the SM while including Einstein gravity. For $M_{UV} \to \infty$ gravity decouples and the KK gravitons acquire a Dirichlet UV boundary condition as in Ref. [28]. In this case they do not directly couple to the SM. Alternatively, 4D gravity could be included by lowering the fundamental gravity scale to ~ TeV. This requires large extra dimensions transverse to the warping but has the advantage of potentially solving the SM hierarchy problem [31].

The 4D Planck mass would then be given by $M_{Pl}^2 \sim M_3^{3+d}V_d/k$, where $V_d$ is the volume of the $d$-dimensional transverse space. We focus on the case of a large UV boundary term in the remainder of this work.

It is helpful to comment on the dual 4D description of the LWS, obtained via the application of the AdS/CFT correspondence to RS models [14], before discussing the hidden-sector particle spectrum further. The present model would be considered dual to a 4D theory in which the UV localized SM fields are treated as fundamental objects external to a hidden strongly coupled sector. The composite objects correspond to IR localized fields in the 5D picture (roughly; see e.g. [32]) and the lightest fermionic composite is the right-handed neutrino (or neutrinos in a three generation model). Note that the IR localization of the
Majorana mass term indicates that lepton number is broken within the strongly coupled sector, distinct to [18] in which lepton number is broken in the UV.

A discussion of the modified application of AdS/CFT for truncated warped spaces is given in [28] and similar points remain valid in the present work. The dual theory is conformal for energies $\text{TeV} \gtrsim E \gtrsim R^{-1}$, with the conformal symmetry broken spontaneously in the IR (dual to the IR brane) and explicitly in the UV (dual to the TeV brane, itself associated with the scale of electroweak symmetry breaking). For every bulk field in the 5D picture there exists a CFT operator in the dual 4D theory that is sourced by a fundamental field. The source field corresponds to the UV-brane value of the given bulk field in the 5D theory. If the bulk field does not possess a UV-localized brane kinetic term, the source field has no bare kinetic term in the 4D theory and is seemingly non-dynamical. However, due to the coupling to the CFT, an induced kinetic term arises, so the “source” is in fact dynamical [33].

The dual description can shed light on the above discussion of the 4D Planck mass. The dual 4D theory for RS1 corresponds to a CFT plus 4D Einstein gravity [14]. In the absence of a UV localized Einstein-Hilbert term, the dual 4D theory does not contain a bare Einstein-Hilbert term. However, the coupling of the 4D graviton to the CFT induces an Einstein-Hilbert term so that gravity acquires its usual Einstein dynamics. Thus the 4D Planck mass obtained in RS1, namely $M_{Pl}^2 \approx M_*^2/k$, is predominantly induced by the CFT [14]. Adding a UV localized kinetic term is dual to including a bare Einstein-Hilbert term in the 4D theory. The 4D Planck mass then becomes $M_{Pl}^2 \approx M_{UV}^2 + M_*^2/k$, where the first (second) term is a bare (CFT-induced) contribution. Thus the 4D Planck mass is no longer completely induced by the CFT. In the RS model the CFT induced part of the Planck mass is typically taken to dominate (or is the same order as) the UV piece, so the CFT-induced part of the Planck mass plays an essential role in determining the dynamics of the 4D graviton.

One can consider a 4D CFT, however, with a UV cutoff much less than the Planck scale, that does not completely induce the Planck mass. In the 5D picture this corresponds to a slice of $AdS_5$ with a UV cutoff much less than the Planck scale, $M_* \ll M_{Pl}$. This is the case considered in the Little RS model [28], where the UV cutoff is of order $10^3$ TeV. In Ref. [28] the bulk graviton was given a Dirichlet UV BC, thereby removing the zero-mode graviton and projecting 4D gravity out of the theory. The Dirichlet UV BC can be realized by sending the coefficient of the UV Einstein-Hilbert term to infinity, $M_{UV} \rightarrow \infty$. In the dual picture Poincaré symmetry is no longer a gauged dynamical symmetry but rather a global symmetry. Though not essential for the TeV-scale physics of interest in Ref. [28], to be a completely realistic low-energy theory the Little RS model should of course include 4D Einstein gravity. One can ask how 4D gravity can be included in the model. The answer is to simply include a large (but finite) UV localized Einstein-Hilbert term such that $M_{Pl}^2 \approx M_{UV}^2 \gg M_*^2/k$. In this case the CFT-induced part of the Planck mass is subdominant and the 4D Planck mass is essentially an input parameter, separate from the scale at which the CFT behaviour

---

5Strictly speaking, a brane kinetic term is necessary to regulate the 5D theory [34], so this discussion corresponds to the case where, after divergences are removed, the contribution to the renormalized kinetic term from the bare kinetic term is dominated by the CFT-induced kinetic term.

6See Ref. [30] for a detailed discussion of the modified UV BC for gravity with brane localized curvature.
breaks down. The particle physics cut-off remains at $M_s \ll M_{Pl}$ and the theory requires some UV completion at energies $E > M_s$, but the low-energy theory now includes the usual coupling to 4D Einstein gravity in addition to the (broken) CFT.

A similar discussion carries over for the dual description of the LWS. Whether or not the dual theory contains Einstein gravity depends on the UV action. With an infinite UV localized Einstein-Hilbert term, the 4D graviton is projected out and the dual theory is purely that of fundamental SM fields and a hidden CFT, both possessing a global Poincaré symmetry. Einstein gravity is included in the dual 4D theory by retaining a finite value for $M_{UV}$ in the 5D theory. Provided $M_{UV}^2 \simeq M_{Pl}^2 \gg M_s^3/k$ the Planck mass is an input parameter whose origin is separate from the CFT dynamics (though the CFT induces a subdominant contribution of order $M_s^3/k$). Note that the UV cutoff for the theory remains at the TeV scale, $M_s \sim \text{TeV}$, but the low-energy theory also includes 4D Einstein gravity. This is akin to cutting off the SM (or any other 4D theory) at the TeV scale while retaining 4D gravity, despite the fact that $M_{Pl} \gg \text{TeV}$.

We now turn to the dual description of the bulk neutrino. In the absence of the IR localized Majorana mass term, the dual 4D theory for $c > 1/2$ is given schematically by the Lagrangian [36]

$$\mathcal{L}_{4D}^N \sim \mathcal{L}_{\text{CFT}} + g_N k^{1/2-c} \psi_R \mathcal{O}_N + \ldots \ldots \ldots$$  

(14)

Here $\psi_R$ is a fundamental source whose dynamics are induced by the CFT (absent a UV localized kinetic term), $g_N$ is a dimensionless coupling, and the fermionic composite CFT operator $\mathcal{O}_N$ has dimension $3/2 + |c + 1/2|$. Observe that the mixing operator between the fundamental field and the CFT operator in Eq. (14) is irrelevant. Pulling a factor of $\mu^{1/2+c}$ out of $\mathcal{O}_N$ to give the fermionic operator a canonical dimension at the scale $\mu$, we see that the mixing becomes tiny in the IR, $\mu \ll k$. The source $\psi_R$ is determined by the UV value of the bulk field $N_R$, and the relationship between $\psi_R$ and the chiral mode depends on the localization parameter $c$. For the values of interest here, the zero-mode is localized towards the IR brane and has very little overlap with the UV brane. Therefore the source contains only a tiny admixture of the zero-mode and the physical chiral-mode corresponds predominantly to a composite CFT state. The tower of Dirac fermions consists mostly of composites but contains an admixture of $\psi_R$.

The small Yukawa coupling between the SM and the lightest right-chiral neutrino is also understood in the dual picture. The SM is external to the CFT, but couples directly to the source field $\psi_R$. Writing this field in terms of the physical mass eigenstates introduces the aforementioned tiny mixing angle, the dependence on which ensures the effective Yukawa coupling is highly suppressed. Turning on the IR Majorana mass gives the chiral mode a mass and splits the Dirac fermions. This IR term only affects CFT correlation functions at distances larger than the conformal length of the space, $\Delta x \gtrsim R$. The Majorana mass should therefore be on the order of $\sim R^{-1}$, in agreement with Eqs. (8) and (9).
4 Comments on the Hidden Sector Spectrum

The hidden sector will contain a tower of KK gravitons with mass splittings of order GeV (or less for smaller $R^{-1}$). On the surface it might seem that the light mass of these KK gravitons could be phenomenologically troublesome. However, as mentioned above, the strength with which these modes couple to the SM depends on the UV action. Consider the case of a large UV localized Planck mass $M_{Pl} \simeq M_{UV}$. Then, by construction, the UV values of the graviton wavefunctions are highly suppressed, such that the coupling between the UV localized SM and the KK gravitons is of order $M_{Pl}^{-1}$. Much like the KK gravitons with masses below the weak scale in RS2 [35], the KK gravitons are not phenomenologically worrisome as their direct coupling to the SM simply produces subdominant corrections to the Newtonian potential. Indeed, for $M_{Pl} \simeq M_{UV}$ the coupling of KK gravitons to the SM will essentially match those in Ref. [13], where a detailed analysis of a warped hidden sector containing order GeV KK gravitons found them to be viable. The similarity to RS2 makes obvious the fact that even sub-GeV IR scales are viable as far as the KK gravitons are concerned. In the limit $M_{UV} \to \infty$ the graviton wavefunctions will be banished from the UV brane and the KK gravitons have no direct coupling to the SM Then the model is purely a particle physics framework, involving no scales beyond the TeV scale, and does not include low-energy gravity effects (as in Ref. [28]). Note that one cannot consider a purely Neumann UV BC for the graviton as then the 4D graviton would couple with strength $M_{Pl}^{3}/k \sim$ TeV. The UV BC must be either Dirichlet or include the effects of a large UV term.

Let us emphasize that, even with a large UV Einstein-Hilbert term, the particle physics cutoff on the warped space remains at the TeV scale, in connection with the scale of electroweak symmetry breaking. The most general UV action will contain the usual series of non-renormalizable operators involving SM fields, which encode the details of the UV completion. These operators can induce unwanted effects (like rapid p-decay) and experimental bounds require the dimensionless coefficients of such operators to be sufficiently small. This is true of any sub-TeV scale effective theory. The inclusion of a large UV Einstein-Hilbert term is not inconsistent with retaining a TeV-scale UV cutoff as this is to be viewed purely as a phenomenological tool with which to include the low-energy effects of gravity in the theory. The cutoff remains at the TeV scale, where new physics associated with, e.g., stabilization of the weak scale, should appear. As mentioned earlier, this approach, motivated by AdS/CFT considerations, is akin to including low-energy gravity effects in particle physics models like the SM, even though new physics is likely to appear well before the Planck scale to solve, e.g., the hierarchy problem. The key particle physics point we seek to make in this work, regarding a sub-TeV seesaw mechanism, does not require the inclusion of low-energy 4D gravity. We have discussed a way to include 4D gravity in the theory for completeness, but as far as the neutrino sector is concerned, our key points are seen simply by taking a Dirichlet

7Note that taking $M_{UV} \to \infty$ is not the same as taking the UV cutoff to infinity: the UV cutoff for the truncated warped space ($M_{c}$) remains at the TeV scale throughout this discussion. $M_{UV}$ refers only to the coefficient of the UV-localized Einstein-Hilbert term. The use of large boundary terms to interpolate between mixed and Dirichlet BCs in models on an interval is discussed in, e.g., [37]. In our case the large UV term provides a convenient way to interpolate between a finite 4D Planck scale and a Dirichlet UV BC (or, equivalently, an infinite 4D Planck mass).
UV BC for gravity and decoupling the 4D graviton. In this case the KK gravitons do not couple directly to the SM.

Independent of their coupling strength to the SM the KK gravitons possess sizable couplings to the KK neutrinos. The KK neutrinos and gravitons are both localized towards the IR brane and in general have large wavefunction overlaps, so the relevant coupling strength is of order $R^{-1} \gg M_p^{-1}$. This coupling allows a given KK graviton $h_a$ to decay to lighter KK neutrinos $h_a \rightarrow \nu^{(m)}\nu^{(n)}$, with width $\Gamma_a \sim (k/M_*)^3 m_a^2 R^2$ (similar to the case for IR localized fields [38]; also see Ref. [13]). Absent hierarchically small values of $k/M_*$ these decays will be prompt for $R^{-1} \sim \text{GeV}$. The radion $r$ will also decay via $r \rightarrow \nu^{(0)}\nu^{(0)}$, provided $m_r$ adequately exceeds the zero mode mass, $m_r \gtrsim 2M_{00}$, with a typical coupling strength set by the IR scale $R^{-1}$.

The metric fluctuations play an important role in the phenomenology of the KK neutrinos. The $n > 0$ KK neutrinos will promptly decay via graviton production, $\nu^{(n)} \rightarrow h_a \nu^{(m)}$. The widths go like $\Gamma_n \sim (k/M_*)^3 m_n^2 R^2$ and are increasingly broad as one goes up the KK tower. For a given value of $n$, decays with $n \sim a+m$ are preferred. Decays to the SM are also possible but these have to go through the Dirac Yukawa-coupling, which is necessarily suppressed by a small wavefunction overlap to ensure light SM neutrino masses. Therefore the SM decays are highly suppressed and, when available, decays to lighter KK neutrinos will dominate. The preference for decays with $n \sim a+m$ means production of a large-$n$ KK neutrino will create a cascade decay down the KK tower.

On the other hand, the zero mode neutrino can only decay to the SM and must therefore decay through the Dirac Yukawa-coupling. These decays will be much slower than the hidden sector KK decays. Clearly, if the light neutrinos are very long lived and are thermalized in the early universe they could present cosmological difficulties, depending on the IR scale. At energies above the IR scale the RS geometry is replaced by an AdS-Schwarschild space [39], with the phase transition to the RS-phase occurring roughly at the IR scale, though the precise behaviour is sensitive to the details of the stabilization mechanism [40]. If the IR scale is of order GeV, this phase transition would occur prior to BBN. In this case the hidden spectrum would not contain any fields lighter than $\sim \text{GeV}$, except possibly the right-chiral neutrinos $\nu^{(0)}$, depending on the value of the localization parameter $c$. Order GeV sterile neutrinos are a common feature of, e.g., the $\nu$MSM [41], and interesting scenarios are known to be viable. The decay of sterile neutrinos can produce interesting consequences and, in particular, admit non-standard leptogenesis scenarios [41].

For IR scales much less than $\mathcal{O}(\text{MeV})$ the phase transition would occur after BBN and there would be many light degrees of freedom in the spectrum. An important point to note is that any interactions between the UV localized SM fields and the IR localized KK/CFT-modes must proceed through the propagator of a bulk field. The propagating field can be either a bulk neutrino or the graviton, but the tiny coupling of the latter to the SM suppresses the graviton effects. A key point is that the UV-to-IR propagator for a bulk field

---

8 Note that provided the IR scale adequately exceeds about 4 MeV to allow BBN, it is possible that primordial inflation never reheated beyond the IR scale, in which case the details of the phase transition are irrelevant.
on a slice of $AdS_5$ essentially cuts off for distances $z^{-1} \gtrsim E$ when the injection energy $E$ greatly exceeds the IR scale, $E \gg R^{-1}$ [13]. Interactions of the CFT that could potentially bring the the light KK modes into thermal equilibrium for $R^{-1} \ll \text{MeV}$ therefore turn out to be unimportant at energies much larger than the IR scale, $E \gg R^{-1}$, as would be relevant for BBN. Thus the light KK modes need not be brought into equilibrium prior to BBN by CFT interactions.

Oscillations between light right-handed neutrinos and SM neutrinos, on the other hand, have the potential to bring the lighter modes into equilibrium. A detailed numerical analysis including the effects of mixing, and the tower of thresholds, would be required to determine the extent to which equilibrium is reached. We note that composite right-handed neutrinos with very light seesaw scales can imprint signals in the CMB which can be in conflict with the data [9], though the case of a hidden CFT with low-scale neutrino composites has not been studied in detail. With very light IR scales one could abandon the seesaw and simply consider light Dirac neutrinos, as in Ref. [6]. The neutrino mass would then be $m_\nu \equiv m_D^0 \sim \langle H \rangle/(kR)^{c+1/2}$, and taking, e.g., $R^{-1} \sim 10 \text{ eV}$ and $c \simeq 0.8$ gives $m_\nu \sim 0.1 \text{ eV}$, which is in the interesting range.

5 Conclusion

We have developed a mini-seesaw mechanism in which light neutrino masses are achieved by combining naturally suppressed Dirac and (sterile) Majorana mass scales together in a low-scale seesaw mechanism. The model is motivated by the AdS/CFT correspondence [9] and the notion of light composite right-handed neutrinos [5], and is dual to a 4D theory with a strongly coupled hidden sector whose lightest fermionic composites are right-handed neutrinos. The model employs a truncated (“little”) warped space that is dual to a 4D theory possessing conformal symmetry in some window, $M_* > E > R^{-1}$ with $M_* \ll M_{Pl}$. Depending on the UV completion the theory may or may not be conformal at higher energies $E > M_*$. It would be interesting to investigate these ideas further to consider the viability of full three-family models and perform a detailed study of the bounds on, and phenomenology of, the light KK neutrinos. As a speculation, it may be possible to construct theories of flavor using these ideas, either by considering flavor structures for sterile neutrinos within the LWS or along the lines of [42]. We will investigate some of these matters in a future work.

The author thanks E. Akhmedov, T. Frossard, W. Rodejohann, R. Takahashi and J. Ward.
Appendix

The bulk profiles for the $n > 0$ KK fermions are \[ f_L^{(n)}(z) = -\sqrt{kz} N_n \{ J_{\alpha_L}(m_n z) + \beta_n Y_{\alpha_L}(m_n z) \} , \] \[ f_R^{(n)}(z) = \sqrt{kz} N_n \{ J_{\alpha_R}(m_n z) + \beta_n Y_{\alpha_R}(m_n z) \} , \]

with the order of the Bessel functions being $\alpha_L = |c \pm 1/2|$ with $c$ defined in the text. The KK masses are determined by $J_{\alpha_L}(m_n R) \simeq 0$, giving $m_n \simeq (n + c/2)\pi R^{-1}$ for $n > 0$. For $c \simeq 1$ these masses are $m_n R \simeq 4.6, 7.7, 10.8 \ldots$. When $f_L^{(n)}(z)$ has Dirichlet BCs the $n > 0$ right-chiral modes take the following boundary values for $c \simeq 1$ and $m_n < k$:

\[ f_R^{(n)}(k^{-1}) \approx \frac{1}{\Gamma(c + 1/2)} \sqrt{\frac{2\pi}{R}} \left( \frac{m_n}{2k} \right)^c , \]

\[ f_R^{(n)}(R) \approx (-1)^n \sqrt{\frac{2}{R}} . \]  

References

[1] J. W. F. Valle, \texttt{arXiv:1001.5189} [hep-ph]. A. Strumia and F. Vissani, \texttt{arXiv:hep-ph/0606054}; R. N. Mohapatra and A. Y. Smirnov, Ann. Rev. Nucl. Part. Sci. 56, 569 (2006) [arXiv:hep-ph/0603118]; R. N. Mohapatra et al., Rept. Prog. Phys. 70, 1757 (2007) [arXiv:hep-ph/0510213]; G. Altarelli and F. Feruglio, New J. Phys. 6, 106 (2004) [arXiv:hep-ph/0405048]; S. F. King, Rept. Prog. Phys. 67, 107 (2004) [arXiv:hep-ph/0310204].

[2] T. Yanagida, in Proceedings of the “Workshop on the Unified Theory and the Baryon Number in the Universe”, Tsukuba, Japan, Feb. 13-14, 1979, edited by O. Sawada and A. Sugamoto, KEK report KEK-79-18, p. 95, and “Horizontal Symmetry And Masses Of Neutrinos”, Prog. Theor. Phys. 64 (1980) 1103; M. Gell-Mann, P. Ramond and R. Slansky, in “Supergravity” (North-Holland, Amsterdam, 1979) eds. D. Z. Freedman and P. van Nieuwenhuizen, Print-80-0576 (CERN); P. Minkowski, Phys. Lett. B 67, 421 (1977); S. L. Glashow, The future of elementary particle physics, in Proceedings of the 1979 Cargèse Summer Institute on Quarks and Leptons (M. Lévy et al. eds.), Plenum Press, New York, 1980, p. 687; R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980).

[3] A. de Gouvea, Phys. Rev. D 72 (2005) 033005 [arXiv:hep-ph/0501039]; T. Asaka, S. Blanchet and M. Shaposhnikov, Phys. Lett. B 631 (2005) 151 [arXiv:hep-ph/0503065]; E. K. Akhmedov, V. A. Rubakov and A. Y. Smirnov, Phys. Rev. Lett. 81, 1359 (1998) [arXiv:hep-ph/9803255]; T. Asaka and M. Shaposhnikov, Phys. Lett. B 620, 17 (2005) [arXiv:hep-ph/0505013]. For related work see P. H. Gu, M. Hirsch, U. Sarkar
and J. W. F. Valle, Phys. Rev. D 79, 033010 (2009) [arXiv:0811.0953 [hep-ph]]; A. Adulpravitchai, P. H. Gu and M. Lindner, [arXiv:1007.1115 [hep-ph]].

[4] G. 't Hooft, NATO Adv. Study Inst. Ser. B Phys. 59, 135 (1980). Also see K. Fujikawa, Prog. Theor. Phys. 113, 1065 (2005) [arXiv:hep-ph/0407331].

[5] N. Arkani-Hamed and Y. Grossman, Phys. Lett. B 459, 179 (1999) [arXiv:hep-ph/9806223].

[6] T. Okui, JHEP 0509, 017 (2005) [arXiv:hep-ph/0405083].

[7] Y. Grossman and Y. Tsai, JHEP 0812, 016 (2008) [arXiv:0811.0871 [hep-ph]]; Y. Grossman and D. J. Robinson, [arXiv:1009.2781 [hep-ph]].

[8] Z. Chacko, L. J. Hall, T. Okui and S. J. Oliver, Phys. Rev. D 70, 085008 (2004) [arXiv:hep-ph/0312267]; L. J. Hall, H. Murayama and G. Perez, Phys. Rev. Lett. 95, 111301 (2005) [arXiv:hep-ph/0504248]; H. Goldberg, G. Perez and I. Sarcevic, JHEP 0611, 023 (2006) [arXiv:hep-ph/0505221]; Also see M. Cirelli and A. Strumia, JCAP 0612, 013 (2006) [arXiv:astro-ph/0607086].

[9] J. M. Maldacena, Adv. Theor. Math. Phys. 2, 231 (1998) [Int. J. Theor. Phys. 38, 1113 (1999)] [arXiv:hep-th/9711200]; S. S. Gubser, I. R. Klebanov and A. M. Polyakov, Phys. Lett. B 428, 105 (1998) [arXiv:hep-th/9802109]; E. Witten, Adv. Theor. Math. Phys. 2, 253 (1998) [arXiv:hep-th/9802150].

[10] L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999) [arXiv:hep-ph/9905221].

[11] B. Gripaios, Nucl. Phys. B 768, 157 (2007) [Erratum-ibid. 830, 390 (2010)] [arXiv:hep-ph/0611218]; T. Flacke and D. Maybury, JHEP 0703 (2007) 007 [arXiv:hep-ph/0612126].

[12] K. L. McDonald and D. E. Morrissey, JHEP 1005, 056 (2010) [arXiv:1002.3361 [hep-ph]].

[13] K. L. McDonald, D. E. Morrissey, [arXiv:1010.5999 [hep-ph]].

[14] N. Arkani-Hamed, M. Porrati and L. Randall, JHEP 0108 (2001) 017 [arXiv:hep-th/0012148]; R. Rattazzi and A. Zaffaroni, JHEP 0104, 021 (2001) [arXiv:hep-th/0012248].

[15] Y. Grossman and M. Neubert, Phys. Lett. B 474, 361 (2000) [arXiv:hep-ph/9912408].

[16] T. Gherghetta and A. Pomarol, Nucl. Phys. B 586, 141 (2000) [arXiv:hep-ph/0003129]; S. J. Huber and Q. Shafi, Phys. Lett. B 498, 256 (2001) [arXiv:hep-ph/0010195].

[17] S. J. Huber and Q. Shafi, Phys. Lett. B 544, 295 (2002) [arXiv:hep-ph/0205327]; S. J. Huber and Q. Shafi, Phys. Lett. B 583, 293 (2004) [arXiv:hep-ph/0309252].

[18] T. Gherghetta, Phys. Rev. Lett. 92, 161601 (2004) [arXiv:hep-ph/0312392].
[[19] T. Gherghetta, K. Kadota and M. Yamaguchi, Phys. Rev. D **76**, 023516 (2007) [arXiv:0705.1749 [hep-ph]]; G. Perez and L. Randall, JHEP **0901** (2009) 077 [arXiv:0805.4652 [hep-ph]]; C. Csaki, C. Delaunay, C. Grojean and Y. Grossman, JHEP **0810** (2008) 055 [arXiv:0806.0356 [hep-ph]]; F. del Aguila, A. Carmona and J. Santiago, JHEP **1008** (2010) 127 [arXiv:1001.5151 [hep-ph]]; A. Kadosh and E. Pallante, JHEP **1008**, 115 (2010) [arXiv:1004.0321 [hep-ph]]; A. Watanabe and K. Yoshioka, arXiv:1007.1527 [hep-ph].

[20] G. von Gersdorff and M. Quiros, Phys. Lett. B **678**, 317 (2009) [arXiv:0901.0006 [hep-ph]].

[21] M. Pospelov, A. Ritz and M. B. Voloshin, Phys. Lett. B **662**, 53 (2008) [arXiv:0711.4866 [hep-ph]]; M. Pospelov and A. Ritz, Phys. Lett. B **671**, 391 (2009) [arXiv:0810.1502 [hep-ph]].

[22] N. Arkani-Hamed, D. P. Finkbeiner, T. R. Slatyer and N. Weiner, Phys. Rev. D **79**, 015014 (2009) [arXiv:0810.0713 [hep-ph]].

[23] B. v. Harling and K. L. McDonald, in preparation.

[24] S. Andreas, C. Arina, T. Hambye, F. S. Ling and M. H. G. Tytgat, Phys. Rev. D **82** (2010) 043522 [arXiv:1003.2595 [hep-ph]]; R. Foot, Phys. Lett. B **692** (2010) 65 [arXiv:1004.1424 [hep-ph]]; D. Hooper, J. I. Collar, J. Hall and D. McKinsey, arXiv:1007.1005 [hep-ph].

[25] W. D. Goldberger and M. B. Wise, Phys. Rev. Lett. **83**, 4922 (1999) [hep-ph/9907447]; J. Garriga and A. Pomarol, Phys. Lett. B **560**, 91 (2003) [arXiv:hep-th/0212227].

[26] G. M. Fuller, A. Kusenko, I. Mocioiu and S. Pascoli, Phys. Rev. D **68**, 103002 (2003) [arXiv:astro-ph/0307267]; A. Kusenko, Phys. Rept. **481**, 1 (2009) [arXiv:0906.2968 [hep-ph]].

[27] A. Kusenko, F. Takahashi and T. T. Yanagida, Phys. Lett. B **693**, 144 (2010) [arXiv:1006.1731 [hep-ph]].

[28] H. Davoudiasl, G. Perez and A. Soni, Phys. Lett. B **665**, 67 (2008) [arXiv:0802.0203 [hep-ph]].

[29] T. Fukuyama, T. Kikuchi and N. Okada, Phys. Rev. D **75**, 075020 (2007) [arXiv:hep-ph/0702048]; R. N. Mohapatra, N. Okada and H. B. Yu, Phys. Rev. D **76**, 015013 (2007) [arXiv:0704.3258 [hep-ph]].

[30] H. Davoudiasl, J. L. Hewett, T. G. Rizzo, JHEP **0308**, 034 (2003) [hep-ph/0305086].

[31] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B **429**, 263 (1998) [arXiv:hep-ph/9803315]; I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B **436**, 257 (1998) [arXiv:hep-ph/9804398].

[32] B. Batell, T. Gherghetta, Phys. Rev. D**76**, 045017 (2007). arXiv:0706.0890 [hep-th]].
[33] K. Agashe, A. Delgado, Phys. Rev. D67, 046003 (2003). [hep-th/0209212].

[34] H. Georgi, A. K. Grant, G. Hailu, Phys. Lett. B506, 207-214 (2001). [hep-ph/0012379].

[35] L. Randall, R. Sundrum, Phys. Rev. Lett. 83, 4690-4693 (1999). [hep-th/9906064].

[36] R. Contino, A. Pomarol, JHEP 0411, 058 (2004). [hep-th/0406257]; R. Contino, Y. Nomura, A. Pomarol, Nucl. Phys. B671, 148-174 (2003). [hep-ph/0306259].

[37] C. Csaki, J. Hubisz, P. Meade, [hep-ph/0510275].

[38] H. Davoudiasl, J. L. Hewett, T. G. Rizzo, Phys. Rev. Lett. 84 (2000) 2080. [hep-ph/9909255]. T. G. Rizzo, AIP Conf. Proc. 1256, 27-50 (2010). [arXiv:1003.1698 [hep-ph]].

[39] E. Witten, Adv. Theor. Math. Phys. 2, 505 (1998) [hep-th/9803131].

[40] P. Creminelli, A. Nicolis and R. Rattazzi, JHEP 0203, 051 (2002) [hep-th/0107141]; L. Randall and G. Servant, JHEP 0705, 054 (2007) [hep-ph/0607158]; J. Kaplan, P. C. Schuster and N. Toro, [hep-ph/0609012]; B. Hassanain, J. March-Russell and M. Schvellinger, JHEP 0710, 089 (2007) [0708.2060 [hep-th]].

[41] D. Gorbunov, M. Shaposhnikov, JHEP 0710, 015 (2007). [arXiv:0705.1729 [hep-ph]]; M. Shaposhnikov, JHEP 0808, 008 (2008). [arXiv:0804.4542 [hep-ph]]. Also see the second and fourth papers of Ref. [3].

[42] G. Cacciapaglia, C. Csaki, C. Grojean and J. Terning, Phys. Rev. D 74, 045019 (2006) [arXiv:hep-ph/0604218]; S. Abel and J. Barnard, JHEP 1008, 039 (2010) [arXiv:1005.1668 [hep-ph]]; S. S. C. Law and K. L. McDonald, [arXiv:1008.4336 [hep-ph]].