Search for a Lorentz invariance violation contribution in atmospheric neutrino oscillations using MACRO data.

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Abstract

Neutrino-induced upward-going muons in MACRO have been analysed in terms of relativity principles violating effects, keeping standard mass-induced atmospheric neutrino oscillations as the dominant source of $\nu_{\mu} \rightarrow \nu_{\tau}$ transitions. The data disfavor these exotic possibilities even at a sub-dominant level, and stringent 90\% C.L. limits are placed on the Lorentz invariance violation parameter $|\Delta v| < 6 \times 10^{-24}$ at $\sin 2\theta_{\nu} = 0$ and $|\Delta v| < 2.5 \div 5 \times 10^{-26}$ at $\sin 2\theta_{\nu} = \pm 1$. These limits can also be re-interpreted as upper bounds on the parameters describing violation of the Equivalence Principle.

Neutrino mass-induced oscillations are the best explanation of the atmospheric neutrino problem \cite{1, 2, 3, 4}. Two flavor $\nu_{\mu} \rightarrow \nu_{\tau}$ oscillations are strongly favored over a wide range of alternative solutions such as $\nu_{\mu} \rightarrow \nu_{\text{sterile}}$ oscillations \cite{5, 6}, $\nu_{\mu} \rightarrow \nu_{e}$ oscillations \cite{3, 4} or other exotic possibilities \cite{7, 8}.

In this letter, we assume standard mass-induced neutrino oscillations as the leading mechanism for flavor transitions and we treat Lorentz invariance flavor transitions as a sub-dominant effect \cite{9}. In the literature, neutrino oscillations induced by Violation of (CPT-conserving) Lorentz Invariance (VLI) and Violation of the Equivalence
Principle (VEP) are described within the same formalism. In the following we will mention only VLI for simplicity.

In this scenario, neutrinos can be described in terms of three distinct bases: flavor eigenstates, mass eigenstates and velocity eigenstates, the latter being characterized by different Maximum Attainable Velocities (MAVs) in the limit of infinite momentum.

Both mass-induced oscillations and VLI transitions are treated in the two-family approximation and we assume that mass and velocity mixings occur inside the same families (e.g. \( |\nu_2\rangle \) and \( |\nu_3\rangle \)).

The usual interpretation of the atmospheric neutrino oscillations is \( \nu_\mu \to \nu_\tau \) induced by the mixing of the two mass eigenstates \( |\nu^m_2\rangle \) and \( |\nu^m_3\rangle \), and two weak eigenstates \( |\nu_\mu\rangle \) and \( |\nu_\tau\rangle \), i.e.

\[
|\nu_\mu\rangle = |\nu^m_2\rangle \cos \theta^m_{23} + |\nu^m_3\rangle \sin \theta^m_{23}
\]

\[
|\nu_\tau\rangle = -|\nu^m_2\rangle \sin \theta^m_{23} + |\nu^m_3\rangle \cos \theta^m_{23}
\]

(1)

where \( \theta^m_{23} (\equiv \theta^m) \) is the flavor-mass mixing angle. The survival probability of muon neutrinos at a distance \( L \) from production is

\[
P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta \sin^2 (1.27 \Delta m^2 L / E_\nu)
\]

(2)

where \( \Delta m^2 = (m^2_3 - m^2_2) \) is expressed in eV\(^2\), \( L \) in km and the neutrino energy \( E_\nu \) in GeV. Notice the dependence on \( L/E_\nu \) in the argument of the second \( \sin^2 \) term.

In the VLI case, the two flavor eigenstates \( |\nu_\mu\rangle \), \( |\nu_\tau\rangle \) and the two velocity eigenstates \( |\nu^v_2\rangle \), \( |\nu^v_3\rangle \) are connected through the mixing angle \( \theta^v_{23} (\equiv \theta^v) \) in analogy with mass-induced oscillations:

\[
|\nu_\mu\rangle = |\nu^v_2\rangle \cos \theta^v_{23} + |\nu^v_3\rangle \sin \theta^v_{23}
\]

\[
|\nu_\tau\rangle = -|\nu^v_2\rangle \sin \theta^v_{23} + |\nu^v_3\rangle \cos \theta^v_{23}
\]

(3)

In this case, the \( \nu_\mu \) survival probability is

\[
P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta \sin^2 (2.54 \cdot 10^{18} \Delta v \ LE_\nu)
\]

(4)

where \( \Delta v = (v_{\nu_\mu} - v_{\nu_\tau}) \) is the neutrino MAV difference in units of \( c \). Notice that neutrino flavor oscillations induced by VLI are characterized by an \( LE_\nu \) dependence of the oscillation probability (Eq. 4), to be compared with the \( L/E_\nu \) behavior of mass-induced oscillations (Eq. 2).

When both mass-induced transitions and VLI-induced transitions are considered simultaneously, the muon neutrino survival probability can be expressed as

\[
P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\Theta \sin^2 \Omega
\]

(5)

where the global mixing angle \( \Theta \) and the term \( \Omega \) are given by:

\[
2\Theta = \tan^{-1}(a_2/a_1)
\]

\[
\Omega = \sqrt{(a_1^2 + a_2^2)}
\]

(6)
with

\[ a_1 = 1.27 \left| \Delta m^2 \sin 2\theta_m L/E_\nu + 2 \cdot 10^{18} \Delta v \sin 2\theta_v \ E_\nu \ e^{i\eta} \right| \]

\[ a_2 = 1.27 \left( \Delta m^2 \cos 2\theta_m L/E_\nu + 2 \cdot 10^{18} \Delta v \cos 2\theta_v \ E_\nu \right) \]  \hspace{1cm} (7)

Here \( \Delta m^2, L \) and \( E_\nu \) are expressed, as in Eq. 2 and Eq. 4, in eV\(^2\), km and GeV, respectively. The additional factor \( e^{i\eta} \) connects the mass and velocity eigenstates, and for the moment it is assumed to be real (\( \eta = 0 \) or \( \pi \)). Note that formulae 2 and 5 do not depend on the sign of the mixing angle and/or on the sign of the \( \Delta v \) and \( \Delta m^2 \) parameters; this is not so in the case of mixed oscillations, where the relative sign between the mass-induced and VLI-induced oscillation terms is important. The whole domain of variability of the parameters can be accessed with the requirements \( \Delta m^2 \geq 0, 0 \leq \theta_m \leq \pi/2, \Delta v \geq 0 \) and \( -\pi/4 \leq \theta_v \leq \pi/4 \).

The same formalism also applies to violation of the equivalence principle, after substituting \( \Delta v/2 \) with the adimensional product \( |\phi| \Delta \gamma \); \( \Delta \gamma \) is the difference of the coupling constants for neutrinos of different types to the gravitational potential \( \phi \) [12].

As shown in [10], and more recently in [11], the most sensitive tests of VLI can be made by analysing the high energy tail of atmospheric neutrinos at large pathlength values. As an example, Fig. 1 shows the energy dependence of the \( \nu_\mu \rightarrow \nu_\mu \) survival probability as a function of the neutrino energy, for neutrino mass-induced oscillations alone and for both mass and VLI-induced oscillations for \( \Delta v = 2 \times 10^{-25} \) and different values of \( \sin 2\theta_v \) parameter. Note the large sensitivity for large neutrino energies and large mixing angles. Given the very small neutrino mass (\( m_\nu \lesssim 1 \) eV), neutrinos with energies larger than 100 GeV are extremely relativistic, with Lorentz \( \gamma \) factors larger than 10\(^{11}\).

MACRO [13] was a multipurpose large area detector (~10000 m\(^2\) sr acceptance for an isotropic flux) located in the Gran Sasso underground Lab, shielded by a minimum rock overburden of 3150 hg/cm\(^2\). The detector had global dimensions of 76.6 \times 12 \times 9.3 m\(^3\) and used limited streamer tubes and scintillation counters to detect muons. \( \nu_\mu \)'s were detected via charged current interactions \( \nu_\mu + N \rightarrow \mu + X \); upgoing muons were identified with the streamer tube system (for tracking) and the scintillator system (for time-of-flight measurement). Early results concerning atmospheric neutrinos were published in [14] and in [1] for the upthroughgoing muon sample and in [15] for the low energy semi-contained and upgoing-stopping muon events. Matter effects in the \( \nu_\mu \rightarrow \nu_{\text{sterile}} \) channel were presented in [5] and a global analysis of all MACRO neutrino data in [2].

In order to analyse the MACRO data in terms of VLI, we used a subsample of 300 upthroughgoing muons whose energies were estimated via Multiple Coulomb Scattering in the 7 horizontal rock absorbers in the lower apparatus [16] [17]. The energy estimate was obtained using the streamer tubes in drift mode, which allowed to considerably improve the spatial resolution of the detector (~ 3 mm). The
overall neutrino energy resolution was of the order of 100%, mainly dominated by muon energy losses in the rock below the detector (note that $\langle E_\mu \rangle \simeq 0.4 \, \langle E_\nu \rangle$). Upgoing muon neutrinos of this sample have large zenith angles ($> 120^\circ$) and the median value of neutrino path-lengths is slightly larger than 10000 km.

Following the analysis in Ref. [17], we selected a low and a high energy sample by requiring that the reconstructed neutrino energy $E_{\nu}^{\text{rec}}$ should be $E_{\nu}^{\text{rec}} < 30$ GeV and $E_{\nu}^{\text{rec}} > 130$ GeV. The number of events surviving these cuts is $N_{\text{low}} = 49$ and $N_{\text{high}} = 58$, respectively; their median energies, estimated via Monte Carlo, are 13 GeV and 204 GeV (assuming mass-induced oscillations).

The analysis then proceeds by fixing the neutrino mass oscillation parameters at the values obtained with the global analysis of all MACRO neutrino data [2]: $\Delta m^2 = 0.0023$ eV$^2$, $\sin^2 2\theta_m = 1$. Then, we scanned the plane of the two free parameters ($\Delta v$, $\theta_v$) using the function

$$\chi^2 = \sum_{i=\text{low}}^{\text{high}} \left( \frac{N_i - \alpha N_i^{\text{MC}}(\Delta v, \theta_v; \Delta m^2, \theta_m)}{\sigma_i} \right)^2$$

where $N_i^{\text{MC}}$ is the number of events predicted by Monte Carlo, $\alpha$ is a constant which normalizes the number of Monte Carlo events to the number of observed events and $\sigma_i$ is the overall error comprehensive of statistical and systematic uncertainties.

We used the Monte Carlo simulation described in [17] with different neutrino fluxes in input [18, 19, 20, 21]. The largest relative difference of the extreme values of the MC expected ratio $N_{\text{low}}/N_{\text{high}}$ is 13%. However, in the evaluation of the systematic error, the main sources of uncertainties for this ratio (namely the primary cosmic ray spectral index and neutrino cross sections) have been separately estimated and their effects added in quadrature (see [17] for details): in this work, we use a conservative 16% theoretical systematic error on the ratio $N_{\text{low}}/N_{\text{high}}$. The experimental systematic error on the ratio was estimated to be 6%. In the following, we show the results obtained with the computation in [21].

The inclusion of the VLI effect does not improve the $\chi^2$ in any point of the ($\Delta v$, $\theta_v$) plane, compared to mass-induced oscillations stand-alone, and proper upper limits on VLI parameters were obtained. The 90% C.L. limits on $\Delta v$ and $\theta_v$, computed with the Feldman and Cousins prescription [22], are shown by the dashed line in Fig. 2.

The energy cuts described above (the same used in Ref. [17]), were optimized for mass-induced neutrino oscillations. In order to maximize the sensitivity of the analysis for VLI induced oscillations, we performed a blind analysis, based only on Monte Carlo events, to determine the energy cuts which yield the best performances. The results of this study suggest the cuts $E_{\nu}^{\text{rec}} < 28$ GeV and $E_{\nu}^{\text{rec}} > 142$ GeV; with these cuts the number of events in the real data are $N'_{\text{low}} = 44$ events and $N'_{\text{high}} = 35$ events. The limits obtained with this selection are shown in Fig. 2 by the continuous line. As expected, the
limits are now more stringent than for the previous choice.

In order to understand the dependence of this result with respect to the choice of the $\Delta m^2$ parameter, we varied the $\Delta m^2$ values around the best-fit point. We found that a variation of $\Delta m^2$ of $\pm 30\%$ moves up/down the upper limit of VLI parameters by at most a factor $2$.

Finally, we computed the limit on $\Delta v$ marginalized with respect to all the other parameters left free to variate inside the intervals: $\Delta m^2 = \Delta m^2 \pm 30\%, \theta_m = \theta_m \pm 20\%, -\pi/4 \leq \theta_v \leq \pi/4$ and any value of the phase $\eta$. We obtained the 90\% C.L. upper limit $|\Delta v| < 3 \times 10^{-25}$.

An independent and complementary analysis was performed on a sample of events with a reconstructed neutrino energy $25 \text{ GeV} < E^\nu_{\text{rec}} < 75 \text{ GeV}$. The number of events satisfying this condition is $106$. A negative log-likelihood function was built event by event and then fitted to the data. We allowed mass-induced oscillation parameters to vary inside the MACRO 90\% C.L. region and we left VLI parameters free in the whole $(\Delta v, \theta_v)$ plane. The upper limit on the $\Delta v$ parameter resulting from this analysis is slowly varying with $\Delta m^2$ and is of the order of $\approx 10^{-25}$.

In conclusion: we have searched for “exotic” contributions to standard mass-induced atmospheric neutrino oscillations arising from a possible violation of Lorentz invariance. We used a subsample of MACRO upthroughgoing muon events for which an energy measurement was made via multiple Coulomb scattering. The inclusion of VLI effects does not improve the fit to the data, and we conclude that these effects are disfavored even at the sub-dominant level.

The 90\% C.L. limits of VLI parameters are $|\Delta v| < 6 \times 10^{-24}$ at $\sin 2\theta_v = 0$ and $|\Delta v| < 2.5 \div 5 \times 10^{-26}$ at $\sin 2\theta_v = \pm 1$, see Fig. 2. In terms of the parameter $\Delta v$ alone (marginalization with respect to all the other parameters), the VLI parameter bound is (at 90\% C.L.) $|\Delta v| < 3 \times 10^{-25}$.

These results may be reinterpreted in terms of 90\% C.L. limits of parameters connected with violation of the equivalence principle, giving the limit $|\phi \Delta \gamma| < 1.5 \times 10^{-25}$.

These limits are comparable or better to those estimated using K2K and Super-Kamiokande data 4.

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Figure 1: Energy dependence of the $\nu_\mu \rightarrow \nu_\mu$ survival probability for mass-induced oscillations alone (continuous line) and mass-induced + VLI oscillations for $\Delta v = 2 \cdot 10^{-25}$ and $\sin 2 \theta_v = 0, \pm 0.3, \pm 0.7$ and $\pm 1$ (dashed lines for positive values, dotted lines for negative values). The neutrino pathlength was fixed at $L = 10000$ km and we assumed $\Delta m^2 = 0.0023$ eV$^2$, $\theta_m = \pi/4$. 

![Graphs showing survival probability vs. neutrino energy for different values of $\sin 2 \theta_v$.](image-url)
Figure 2: 90% C.L. upper limits on the Lorentz invariance violation parameter $\Delta v$ versus $\sin 2\theta_v$. Standard mass induced oscillations are assumed in the two-flavor $\nu_\mu \rightarrow \nu_\tau$ approximation, with $\Delta m^2 = 0.0023$ eV$^2$ and $\theta_m = \pi/4$. The dashed line shows the limit obtained with the same selection criteria of Ref. [17] to define the low and high energy samples; the continuous line is the final result obtained with the selection criteria optimized for the present analysis (see text).
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