Students’ Covariational Reasoning in Fraction Compare Problem

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Abstract. Covariational reasoning plays important role to mathematics education such as fraction compare word problem. This study investigates students’ covariational reasoning after their studies concerning fraction word problem in first semester on fifth grade. Three students who had high mathematics ability were chosen to solve problem that involved how quantities change with focused on operation (addition, subtraction, multiplication, and division) and compare. Interviews were conducted to reveal the students’ reasoning while solving covariational problems. The results emphasizes that fifth grade students were nearly unable to construct the relation of quantity that changes with the other quantity. It cause students faced difficulty in determining the operation of the problem solving, could not operation the fraction with different denumerator, and could not apply the concept of fractions. These findings suggest that learning fraction should be increased emphasis on coordinating two quantities changing about compare magnitude and operation of fraction of change and to improve the knowledge in fraction compare.

1. Introduction
Mathematics word problem are very interesting in the research world. Several studies [1-7] showed that all level students assume that compare word problem are difficult. One of the factor causing students’ difficulties in solving compare word problem is the lack of the school learning that concerns the students’ reasoning. Schools place greater emphasis on procedures rather than reasoning [8]. Reasoning is one of the important components on every mathematics learning. Reasoning is the process of thinking in drawing the conclusions based on logical analysis and argument. Compare word problem needs students’ reasoning to solve the problem. Covariational reasoning is one of the reasoning that students need to solve compare word problem. Covariational reasoning to be the cognitive activities involved in coordinating two varying quantities while attending to the ways in which they change in relation to each other [9,10]. For example elementary students that given questions, “if You got 6 score for the mathematics test, then you don’t get additional money. If you got 7 score, then you get additional money amount Rp 1.000,00. If you got 8 score, then you get additional money amount Rp 2.000,00.” It showed the students know two quantities was changes. Thats students can be said thinking by covariational reasoning.

Moreover, many researchers [9-12] had investigated about covariational reasoning, but none of them exposed covariational reasoning that related with fractions. The fraction is one of the main concepts in studying mathematics especially algebra [13-17]. In addition none of researchers exposed covariational...
reasoning on elementary students. Therefore this study focuses to investigation on how students’ reason during their studies concerning two covarying quantities in fractions on elementary students.

2. Method
This study was conducted on the five grade students of the elementary school at Mojokerto, in academic year 2017/2018 which was chosen by purposively sampling. The fifth grade class consists of 12 male and 12 female. All students were given the fraction ability test and the fraction compare problem, which was focused on arithmetic’s operations (addition, subtraction, multiplication, and division). The results of fraction ability test were ranked into three levels, i.e. high, medium, and low mathematical abilities respectively.

The instrument consists of three items, and had validated by two senior lecturers of the mathematics’ department of Unesa. The six students invited to complete three written problems that involve interpreting and representing how quantities vary in tandem, which were identically designed in context. Three of these students (labeling with A, B, and C) were volunteerly invited to participate in the interview sessions that concentrated on how students reason during solving covariational problems. The selection of the interview subjects was also based on diversity responses on their written test. The analysis focused on students’ use of reasoning covariationally while responses to the written test, which related with five mental actions of covariational reasoning were adapted from [9]. Five mental action of covariational reasoning on compare problem were modified by researcher which appropriate with students elementary (see Table 1).

| Mental Action | Description of Mental Action | Behavior |
|---------------|------------------------------|----------|
| Mental Action 1 (MA 1) | Coordinating the dependence of one quantity on an ongoing basis. | • Determining the value of two interrelated quantities  
• Verbalizing that the value of two quantities are interrelated. |
| Mental Action 2 (MA 2) | Coordinating the direction of change of one quantity with changes in the other quantity. | • Using the countdown to determine another quantity change  
• Verbalizing that the quantity of one changes depends on the other quantity. |
| Mental Action 3 (MA 3) | Coordinating the amount of change of one quantity with changes in the other | • Determining the results of the second change quantity  
• Verbalizing that the occurrence of quantity changes affects other quantity changes. |
| Mental Action 4 (MA 4) | Coordinating the average rate-of-change of the quantity with uniform increments of change in the input quantity | • Determining changes in amount of quantity and change of quantity with same increasement.  
• Verbalizing that the change in quantity (with same increasement) affects other quantity change. |
| Mental Action 5 (MA 5) | Coordinating the momentary changes of the quantity that constantly changes in both quantities | • Determining changes in amount of quantity and changes of quantity with changes increasement (up or down).  
• Verbalizing that quantity change affects other quantity changes. |

Adopted from [9]

3. Results and Discussion
The fraction ability test were given by students on fifth grade. The result of fraction ability are shown in Table 2. The results of fraction ability test were ranked into three levels, i.e. high, medium, and low mathematical abilities respectively.

|   | score > 80 | 80 ≥ score ≥70 | score < 70 |
|---|------------|----------------|------------|
| Boys | 3          | 6              | 3          |
| Girls | 4          | 5              | 4          |

Table 2. Students' Fraction Test Results

Based on fraction ability test were chosen students with high mathematics ability to solve fraction compare tests. Fraction compare tests were given to know how students’ covariational reasoning on compare problem. Table 3 shows the kinds of responses that seven students provided on the written test. Only two students of these high fraction ability given an acceptable solution, while four students had some missed calculation, and one other had some missed problem. During the follow up interview, the three interview subjects provided varied responses.

|   | True | False |
|---|------|-------|
| Boys 1 | 1    | 5     |
| Boys 2 | 4    | 2     |
| Girls 1 | 1    | 5     |
| Girls 2 | 2    | 4     |
| Girls 3 | 3    | 3     |
| Girls 4 | 4    | 2     |
| Girls 5 | 2    | 4     |

Table 3. Students’ Fraction Compare Test Results

In this study, we only found three from five mental action that can be achieved by students. Students A provided responses that indicated three mental actions that linear with framework of covariational reasoning. He could be verbalizing that two different fraction are interrelated, one fraction of one changes depends on the other fraction, and the occurrence of fraction changes affects other fraction changes. He had accepted answer (see Figure 2). Students A stated the following statements:

*I understand how to know the difference of their money (problem at test). It needs the price of their paid. So, the first, I must known how many chicken egg they will buy for each (MA 1). Then, I could find the price of their paid for each by multiplying how many chicken egg they will buy and the price of chicken egg per kg (MA 2). After that, I could find the difference of their money (MA 3).*

Student B had same reasoning with students A. He had finished with accepted answer (see Figure 3). He could states that two difference fraction are interrelated (MA1). He explained why he could find the price of chicken egg for each person (MA 2) until he got the difference of their money (MA 3), he stated as follows

*I must find the price of chicken egg for each to hunt for the difference of their money (problem test). I did for multiplying the price of chicken egg per kg and the amount of chicken egg(kg), and I get that.*

Student C had differ reasoning from student A and student B. Student C suppose that the difference in the problem are the difference with proportion (see Figure 4), she stated as follow.

*Since the price of the chicken egg per person had been find, I can found the difference of the chicken egg per person. But after that I must subtract the price of chicken egg per person by the price of chicken egg per kg. So, I found the price of Dian is Rp 6.000,- and the price of Ani is Rp10.000,-. In this item, I had hesitant. Therefore, I had two answer. The second answer of the end question is the difference of the price of chicken egg per person.*

She appeared to have difficulty in determining the results of the change quantity. She could be verbalizing that two different fraction are interrelated (MA 1) and one fraction of one changes depends
on the other fraction (MA 2). But she almost missed for verbalizing the occurrence of fraction changes affects other fraction changes. She indicated that she appeared to have difficulty to determine the results of the change quantity. In order studies, students like that is defined pseudo [10].

The data analysis was focused on students’ covariational reasoning, which related to three mental actions of covariational reasoning. Students were given fraction compare test. The first item test on fraction compare test as follow.

Ani buy \(\frac{5}{4}\) kg chicken eggs and Dian buy \(\frac{1}{2}\) kg chicken eggs. The price of the eggs is Rp 12,000,-. They have \(\frac{1}{4}\) kg from the shop because its celebrate shop’s birthday for each.

a. How many chicken egg which bought by Ani and Dian for each before addition?
b. Which one of the chicken egg have more after addition by shop?
c. How much the difference of their money?

1. Ani membeli \(\frac{5}{4}\) kg telur ayam dan Dian membeli \(\frac{1}{2}\) kg telur ayam. Harga per kilogram telur adalah Rp 12,000,-. Masing-masing dari mereka mendapat tambahan \(\frac{1}{4}\) kg dari toko karena toko merayakan hari ulang tahunnya

a. Berapa kg telur yang dibeli Ani dan Dian masing-masing sebelum ditambah?
b. Telur supaya yang lebih banyak setelah ditambah oleh toko? Jelaskan! (Bisa dengan menggunakan garis bilangan).
c. Berapa selisih uang yang harus dibayar oleh Dian dibandingkan dengan uang yang dibayar oleh Ani?

**Figure 1.** Question 1 in Fraction Compare Test

**Figure 2.** Subject A Answers

**Figure 3.** Subject B Answers
Based on these cases, it can be seen that students’ covariational reasoning didn’t occurred on all mental action. It caused the students (elementary students) did not use the fully knowlege. In this case the students did not really understand compare problem. It seen that one students missed on her answer. She had two answer because she had hesitant. Not only one students who didn’t understand compare problem. But the other students didn’t fully understand compare problem. It caused the students rarely get compare problem on mathematics class on daily activities. The teacher approved it too. The teacher recognized that the students had given more conceptual knowledge for the first. She expected the students should understand compare problem if she will be given the compare problem.

4. Conclusion
Based on results, students were nearly unable to construct the relation of the quantities that change with the other quantities and worked on three mental actions. Those mental action are: coordinating the changing dependence of one quantity on an ongoing basis, coordinating the direction of change of one quantity with changes in the other quantity, and coordinating the amount of change of one quantity with changes in the other. However, students appeared to have difficulty in determining the results of the second change quantity and have difficulty on compare problem. This findings suggest that learning in fraction and compare problem should placed increased emphasis on coordinating two quantities changing in tandem and conceptual knowledge in fraction. Further studies are needed to investigate how to develop students’ reasoning in coordinating the average rate-of-change of the quantity with uniform increments of change in the input quantity and coordinating the momentary changes of the quantity that constantly changes in both quantity.

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