Numerical analysis on the performance characteristics of a new gas journal bearing by using finite difference method

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Abstract
This paper proposes a novel gas journal bearing in which orifices are different in diameter and distribute unevenly. Finite Difference Method (FDM) combined with Linear Perturbation Method (LPM) is used to solve the unsteady-state Reynolds equation of the flow field in the bearing clearance. Moreover, four types of bearing structures are used to discuss the effects of orifices different in diameter and uneven distribution on the bearing performance. The results demonstrate that the new bearing has better static and dynamic performances compared with those of traditional bearing in which orifices are equal in diameter and distribute evenly. Moreover, thin gas film thickness, high supply pressure, and large eccentricity ratio are hopeful for improving load capacity of the new bearing. Furthermore, the stability of the novel bearing is improved if eccentricity ratio is 0.25–0.3.

Keywords
Gas journal bearing, Finite Difference Method, Linear Perturbation Method, load capacity, dynamic performance

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Introduction
By using compressible gas as the lubricant for realizing non-contact rotary motion, gas journal bearing has many advantages such as approximate zero friction, high precision, high speed, no wear, and long life. It is widely used in the fields of large-scale mirror fly-cutting, microelectronics manufacturing, etc. The performance of gas journal bearing (the key component of the equipment) relates to the processing and manufacturing accuracy greatly. Although the bearing performance determined by the gas flow field (described by Navier-Stokes equation) in the bearing clearance, nearly all scholars used Reynolds equation to analyze the performance because of the difficulty of solving Navier-Stokes equations directly.

Li et al. utilized Finite Difference Method (FDM) to solve Reynolds equation and employed Successive Over Relaxation (SOR) method in the iterations. Lo et al. combined FDM and Rate Cutting Method (RCM) to solve Reynolds equation and the calculation was convergent if average gas film thickness was larger than 6 μm. In order to reduce calculation time, Zheng and Jiang proposed a novel variable step iteration method when using FDM to calculate the performance of gas journal bearing and the iteration times was reduced greatly. Li et al. concluded that FDM...
combined with flow-difference feedback was more suitable for solving Reynolds equation than FDM combined with SOR or RCM. Wang et al.\textsuperscript{5,6} combined FDM with Linear Perturbation Method (LPM) to solve unsteady-state Reynolds equation and analyze the effect of bearing parameters and manufacturing errors on the dynamic performance of gas journal bearing. Cui et al.\textsuperscript{7} applied Finite Element Method (FEM) to discuss the influence of bearing parameters and manufacturing error on angular stiffness of aerostatic bearing. Sun and Gu\textsuperscript{8} modified Reynolds equation for the gas journal bearing with misalignment caused by shaft deformation and analyzed the bearing performance by using FDM. Zhang et al.\textsuperscript{9} studied the effect of misalignment of rotor and sleeve on load capacity and stiffness of gas journal bearing under pure-static and hybrid condition by using FDM. The results demonstrated that load capacity and stiffness improved with the increase of the degree of journal misalignment under hybrid condition, while the pure static condition was opposite. Yang et al.\textsuperscript{10} discussed the effect of the numbers and locations of orifices on the performance of rotor-aerostatic bearing. They concluded that the bearing had better static and dynamic performance if there was an orifice located at the bottom of the bearing. Chen et al.\textsuperscript{11} discussed the effect of the unbalanced aerostatic spindle on the manufacturing surface. The conclusion indicated that rotation speed and unbalanced spindle directly affected manufacturing surface topography. Hargreaves\textsuperscript{12} improved load capacity of gas journal bearing by adding waviness inside sleeve surface. Moreover, they concluded that sine waviness increases load capacity significantly. Xiao et al.\textsuperscript{13} discussed the performance of aerostatic journal microbearings (bearing diameter was 3–9 mm and orifice diameter was 0.08–0.14 mm) by using FDM. They concluded that aerodynamic pressure effect was more clear at ultra-high speeds and large eccentricities. Ishibashi et al.\textsuperscript{14} concluded that gas journal bearing compensated by porous materials was helpful for improving load capacity, stiffness, and stability at high rotation speeds. Gao et al.\textsuperscript{15} concluded that gas journal bearing by using CFD method. They concluded that the bearing with cylindrical chamber shape was helpful for improving load capacity and stiffness. Chen et al.\textsuperscript{16} investigated the influence of spindle speed on the dynamic properties of aerostatic spindle systems according to CFD results. They concluded that rotational speed and rotation effects mainly impacted the tilting motion natural frequencies of the spindle. Lu et al.\textsuperscript{17} reported that structure deformation could lower load capacity of aerostatic spindle by using CFD simulations.

This paper proposes a new gas journal bearing in which orifices are different in diameter and distribute unevenly to improve the bearing performance. FDM combined with LPM are used to solve the unsteady-state Reynolds equation of the flow field in the bearing clearance. Furthermore, four types of bearing structures are used to analyze the effect of orifices different in diameter and uneven distribution on the bearing performance. This paper is organized as follows: Section 2 proposes the structure of a new gas journal bearing. Moreover, FDM combined with LPM is described for the performance calculation of the bearing. Section 3 discusses the effect of bearing parameters on its performance. Finally, the conclusions are given in the last section.

\section*{Bearing structure and performance calculation}

\subsection*{Structure of a new gas journal bearing}

The gas journal bearing with traditional structure is shown in Figure 1, in which two rows of orifices are equal in diameter and distribute evenly in the circumferential direction inside the sleeve. Where \(d, h, L, D, e, \theta, \) and \(c\) are orifice diameter, gas film thickness, bearing length, bearing diameter, eccentricity, attitude angle, and average gas film thickness, respectively. \(y\) and \(z\) are the coordinates in the radial and axial directions, respectively. \(x\) is perpendicular to \(y\) and \(z\) simultaneously. \(L_1\) is the distance between bearing edge and orifice distribution circumference. Attitude angle \(\theta\) relates to the centroids of rotor and sleeve. External pressurized gas enters the bearing clearance through orifices and forms gas film, then exhausts to atmosphere at bearing edge. The uneven gas film induced by eccentricity leads to gas pressure difference and enables the bearing support vertical load.

A new bearing structure is proposed in order to improve the bearing performance, as shown in Figure 2. The orifice diameter decreases from \(d_1\) to \(d_n\), etc. \(d_{i+1} = d_i^* \gamma, \ i = 1, 2, \ldots, n - 1.\) \(\gamma\) is orifice diameter coefficient. Moreover, the orifice spacing in the lower part is decreased and that of in the upper part is increased. The angle between orifice \(i\) and orifice \(i + 1\) is \(\alpha_i, \ i = 1, 2, \ldots, n - 1.\)
Performance calculation

Supposed the flow field is isothermal and laminar.\textsuperscript{21} Lubricated gas is compressible and its viscosity is constant. The gas pressure distribution in the bearing clearance is governed by compressible Reynolds equation.\textsuperscript{22}

\[
\frac{\partial}{\partial x} \left( p h^3 \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left( p h^3 \frac{\partial p}{\partial z} \right) = 6 \eta u \frac{\partial (ph)}{\partial x} + 12 \eta \frac{\partial (ph)}{\partial t} \tag{1}
\]

where \( p, t, \) and \( \eta \) are gas pressure, time, and gas viscosity coefficient, respectively. \( u \) is gas velocity in the \( x \) direction. Gas film thickness \( h = c[1 - \varepsilon \cos(2\pi x/D - \theta)] \). Where \( \varepsilon \) is eccentricity ratio and \( \varepsilon = e/c \).

Given \( p = p_0, h = cH, z = LZ, x = R\varphi, t = \tau/\omega_s, \lambda = \omega_s/\omega, \Lambda = (6\eta_0 R^2)/(p_0\omega^2) \). \( p_0, \omega_s, \tau, \omega \) and \( R \) are atmospheric pressure, perturbation frequency, dimensionless time, rotation speed, and bearing radius, respectively. \( \lambda \) is perturbation frequency ratio and \( \Lambda \) is bearing number. \( P \) and \( H \) are dimensionless gas pressure and dimensionless gas film thickness, respectively. \( \varphi \) and \( Z \) are dimensionless coordinate in the circumferential and axial directions, respectively. Therefore, the dimensionless Reynolds equation can be written as

\[
\frac{\partial}{\partial \varphi} \left( PH^3 \frac{\partial P}{\partial \varphi} \right) + \frac{R^2}{L^2} \frac{\partial}{\partial Z} \left( PH^3 \frac{\partial P}{\partial Z} \right) = \Lambda \frac{\partial (PH)}{\partial \varphi} + 2\Lambda \frac{\partial (PH)}{\partial \tau} \tag{2}
\]

Supposed gas pressure is \( p_0 \) and gas film thickness is \( h_0 \) when the flow field reaches stable condition. Load capacity of the bearing is

\[
F = \sqrt{F_x^2 + F_y^2} \tag{3}
\]

where \( F_x \) and \( F_y \) are load capacity in the \( x \) and \( y \) directions, respectively.
\[ F_x = \int \frac{p_0 D}{2} \sin \varphi dx dz \]
\[ F_y = -\int \frac{p_0 D}{2} \cos \varphi dx dz \]

Attitude angle is calculated by

\[ \theta = \arctan \left( \frac{F_x}{F_y} \right) \quad (4) \]

Given a linear displacement perturbation \((\Delta X, \Delta Y)\) to the rotor, \(\Delta X = |\Delta X|e^{\psi t} = |\Delta X|e^{ir}\) and \(\Delta Y = |\Delta Y|e^{\psi t} = |\Delta Y|e^{ir}\). The perturbation velocity is \((\Delta X, \Delta Y)\). Gas pressure and film thickness can be written as following through Taylor expansion.

\[ P = P_0 + P_X \Delta X + P_Y \Delta Y + P_t \Delta Y \quad (5) \]
\[ H = H_0 + H_X \Delta X + H_Y \Delta Y + H_t \Delta Y \quad (6) \]

where \(P_0\) and \(H_0\) are dimensionless gas pressure and gas film thickness in the steady-state. \(P_X, P_Y, P_t,\) and \(P_Y\) are the perturbation pressures. \(H_X = \partial H/\partial X = -\sin \varphi, \quad H_Y = \partial H/\partial Y = 0, \quad H_t = \partial H/\partial \varphi = 0\).

Substituted equations (5) and (6) into equation (2) and neglecting high-order terms. The perturbation equations about \(P_X, P_Y, P_t,\) and \(P_Y\) are equations (7)–(10) considering \(\partial P_t/\partial t = P_X \Delta X - P_X \Delta Y + P_Y \Delta Y - P_Y \Delta Y\) and \(\partial H/\partial t = H_X \Delta X - H_X \Delta Y + H_t \Delta Y - H_t \Delta Y\).

\[ \frac{\partial}{\partial \varphi} \left( H_0^3 \frac{\partial P_X}{\partial \varphi} P_X + 3H_0^2 P_0 \frac{\partial P_0}{\partial \varphi} H_X + P_0 H_0^3 \frac{\partial P_X}{\partial \varphi} P_X \right) + \left( \frac{R}{L} \right)^2 \frac{\partial}{\partial Z} \left( H_0^3 \frac{\partial P_X}{\partial Z} P_X + 3H_0^2 P_0 \frac{\partial P_0}{\partial Z} H_X + P_0 H_0^3 \frac{\partial P_X}{\partial Z} P_X \right) = \Lambda \frac{\partial}{\partial \varphi} \left( P_0 H_X + H_0 P_X \right) - 2\Lambda \Lambda H_0 P_X \quad (7) \]

\[ \frac{\partial}{\partial \varphi} \left( H_0 \frac{\partial P_Y}{\partial \varphi} P_Y + 3H_0^2 P_0 \frac{\partial P_0}{\partial \varphi} H_Y + P_0 H_0^3 \frac{\partial P_Y}{\partial \varphi} P_X \right) + \left( \frac{R}{L} \right)^2 \frac{\partial}{\partial Z} \left( H_0 \frac{\partial P_Y}{\partial Z} P_Y + 3H_0^2 P_0 \frac{\partial P_0}{\partial Z} H_Y + P_0 H_0^3 \frac{\partial P_Y}{\partial Z} P_X \right) = \Lambda \frac{\partial}{\partial \varphi} \left( P_0 H_Y + H_0 P_Y \right) - 2\Lambda \Lambda H_0 P_Y \quad (8) \]

\[ \frac{\partial}{\partial \varphi} \left( P_0 H_0^3 \frac{\partial P_X}{\partial \varphi} + H_0 \frac{\partial P_0}{\partial \varphi} P_X \right) + \left( \frac{R}{L} \right)^2 \frac{\partial}{\partial Z} \left( P_0 H_0^3 \frac{\partial P_X}{\partial Z} + H_0 \frac{\partial P_0}{\partial Z} P_X \right) = \Lambda \frac{\partial}{\partial \varphi} H_0 P_X + 2\Lambda \Lambda (H_0 P_X + P_0 H_X) \quad (9) \]

\[ \frac{\partial}{\partial \varphi} \left( P_0 H_0^3 \frac{\partial P_Y}{\partial \varphi} + H_0 \frac{\partial P_0}{\partial \varphi} P_Y \right) + \left( \frac{R}{L} \right)^2 \frac{\partial}{\partial Z} \left( P_0 H_0^3 \frac{\partial P_Y}{\partial Z} + H_0 \frac{\partial P_0}{\partial Z} P_Y \right) = \Lambda \frac{\partial}{\partial \varphi} P_0 H_Y + 2\Lambda \Lambda (P_0 H_Y + P_0 H_Y) \quad (10) \]

Equation (7) and equations (9), (8), and (10) are coupled equations, which can be solved by using SOR method.

FDM is used to solve the dimensionless Reynolds equation. In the calculations, the rotor curvature is not considered because bearing clearance is much small compared with bearing diameter. Therefore, the rotor surface is unfolded into a plane, as shown in Figure 3. \(M + 1\) and \(N + 1\) nodes are equally distributed in the \(x\) and \(z\) directions, respectively. Therefore, the calculation domain is divided into \(M \times N\) grids. \(M\) is 64 and \(N\) is 20 in present study because the improvement on calculation accuracy is negligible as the numbers of grids increases while the calculation time increases significantly.

The steady-state Reynolds equation can be discretized as following by using the second-order difference method.

\[ A_{i,j} f_{i+1,j} + B_{i,j} f_{i-1,j} + C_{i,j} f_{i,j} + D_{i,j} f_{i,j-1} + E_{i,j} f_{i,j+1} = 2\Lambda P_{0(i,j)} \sin \left( \frac{2\psi_{i,j}}{D} \right) + \Lambda H_{0(i,j)} \frac{P_{0(i,j+1)} - P_{0(i,j-1)}}{2(\Delta \varphi)} \quad (11) \]

where

\[ A_{i,j} = \frac{H_{0(i,j)}^3}{(\Delta \varphi)^2} + \frac{3H_{0(i,j)}^2 \sin \left( \frac{2\psi_{i,j}}{D} \right)}{2(\Delta \varphi)} \]
\[ B_{i,j} = \frac{H_{0(i,j)}^3}{(\Delta \varphi)^2} + \frac{3H_{0(i,j)}^2 \sin \left( \frac{2\psi_{i,j}}{D} \right)}{2(\Delta \varphi)} \]

**Figure 3.** Unfolded rotor surface and mesh grid for FDM calculation.
\[
C_{i,j} = -2 \left[ H^3_{0(i,j)} / (\Delta \varphi)^2 + H^3_{0(i,j)} / (\Delta Z)^2 \right]
\]
\[
D_{i,j} = H^3_{0(i,j)} / (\Delta Z)^2
\]
\[
E_{i,j} = H^3_{0(i,j)} / (\Delta Z)^2
\]

The boundary conditions are as follows:

(1) Atmospheric boundaries, \( f_i - 0 = f_i - N = (p_{0(i,j)} / p_a)^2 = 1 \).

(2) Periodic boundaries, \( f_i - 0 = f_i - M \).

(3) The square of dimensionless gas pressure at node \((i, j)\) where orifice located is

\[
f_{i,j} = \left( p_{0(i,j)} / p_a \right)^2 = (\beta_{i,j} p_s / p_a)^2 = \beta_{i,j}^2 (p_s / p_a)^2
\]

where \( p_{0(i,j)} \) is the gas pressure at orifice outlet. \( p_s \) is supply pressure and \( \beta_{i,j} \) is the ratio of the gas pressure at orifice outlet to supply pressure.

The partial differential terms of equation (7) at each grid satisfy the following equations:

\[
\frac{\partial}{\partial \varphi} \left( p_a H_0 \frac{\partial P_a}{\partial \varphi} \right) = \frac{1}{\Delta \varphi} \left[ \left( p_a H_0 \left( \frac{\partial P_a}{\partial \varphi} \right) \right)_{i+1/2,j} - \left( p_a H_0 \left( \frac{\partial P_a}{\partial \varphi} \right) \right)_{i-1/2,j} \right]
\]

\[
= \frac{1}{2(\Delta \varphi)^2} \left[ (p_{0(i,j+1)} + p_{0(i,j)}) H^3_{0(i,j+1/2)} (P_{X(i,j+1)} - P_{X(i,j)}) - (p_{0(i,j)} + p_{0(i,j-1)}) H^3_{0(i,j-1/2)} (P_{X(i,j)} - P_{X(i,j-1)}) \right]
\]

\[
\frac{\partial}{\partial \varphi} \left( H^3_0 \frac{\partial P_a}{\partial \varphi} \right) = \frac{1}{\Delta \varphi} \left[ \left( H^3_0 \left( \frac{\partial P_a}{\partial \varphi} \right) \right)_{i+1/2,j} - \left( H^3_0 \left( \frac{\partial P_a}{\partial \varphi} \right) \right)_{i-1/2,j} \right]
\]

\[
= \frac{1}{2(\Delta \varphi)^2} \left[ (p_{0(i,j+1)} - p_{0(i,j)}) H^3_{0(i,j+1/2)} (P_{X(i,j+1)} + P_{X(i,j)}) - (p_{0(i,j)} - p_{0(i,j-1)}) H^3_{0(i,j-1/2)} (P_{X(i,j)} + P_{X(i,j-1)}) \right]
\]

\[
= \frac{\partial}{\partial \varphi} \left( 3 p_a H_0^2 \frac{\partial P_a}{\partial \varphi} \right) = \frac{\partial}{\partial \varphi} \left( -3 p_a H_0^2 \frac{\partial P_a}{\partial \varphi} \sin \varphi \right)
\]

\[
= -\frac{3}{2 \Delta \varphi^2} \left[ (P_{X(i,j+1)} - P_{X(i,j)}) H^2_{0(i,j+1/2)} (\sin \varphi)_{i,j+1} + (P_{X(i,j)} - P_{X(i,j-1)}) H^2_{0(i,j-1/2)} (\sin \varphi)_{i,j-1/2} \right]
\]

Therefore,

\[
P_{X(i,j)} = (A'_{i,j} P_{X(i,j+1)} + B'_{i,j} P_{X(i,j-1)} + C'_{i,j} P_{X(i+1,j)} + D'_{i,j} P_{X(i-1,j)} - F'_{i,j}) / E'_{i,j}
\]

where
\( A'_{i,j} = H^3_{0(i,j+1)}P_{0(i,j+1)} - H^2_{0(i,j+1/2)} \Lambda(\Delta \phi)/2 \)

\( B'_{i,j} = H^2_{0(i,j-1/2)}P_{0(i,j-1)} + H^2_{0(i,j-1)} \Lambda(\Delta \phi)/2 \)

\( C'_{i,j} = (R/L)^2(\Delta \phi/\Delta Z)^2H^3_{0(i+1/2,j)}P_{0(i+1,j)} \)

\( D'_{i,j} = (R/L)^2(\Delta \phi/\Delta Z)^2H^3_{0(i-1/2,j)}P_{0(i-1,j)} \)

\( E'_{i,j} = (H^3_{0(i,j+1/2)} + H^2_{0(i,j-1/2)})P_{0(i,j)} + (R/L)^2(\Delta \phi/\Delta Z)^2\left( H^3_{0(i+1/2,j)} + H^3_{0(i-1/2,j)} \right)P_{0(i,j)} + (H^2_{0(i,j+1/2)} - H^2_{0(i,j-1/2)}) \Lambda(\Delta \phi)/2 \)

The boundary conditions of equations (7)–(10) are

\( \begin{align*}
& \sin \phi \quad d\phi dZ, \\
& \sin \phi \quad d\phi dZ, \\
& \sin \phi \quad d\phi dZ, \\
& \sin \phi \quad d\phi dZ.
\end{align*} \)

Therefore, the coefficients of dynamic stiffness and dynamic damping are \( \left( k_{XX} \right) = \frac{p_{RL}}{c} \left( K_{XX} \right), \quad \left( k_{YY} \right) = \frac{p_{RL}}{c} \left( K_{YY} \right), \quad \left( d_{XX} \right) = \frac{p_{RL}}{w_{c}c} \left( D_{XX} \right), \quad \left( d_{YY} \right) = \frac{p_{RL}}{w_{c}c} \left( D_{YY} \right). \)

\( \lambda_{c} = \left( K_{XX} - \left( K_{eq} \right) \right) \left( K_{YY} - \left( K_{eq} \right) \right) - K_{XY} \)

The calculation procedure is shown in Figure 4. Finite Difference Method (FDM) combined with Linear Perturbation Method (LPM) is used to solve equation (2). Firstly, set bearing parameters and boundary conditions. Then, solve the steady-state Reynolds equation by using FDM and \( P_{0} \) and \( H_{0} \) are obtained. Thirdly, resolve the unsteady-state Reynolds equation by combining FDM with LPM and calculate load capacity, dynamic stiffness coefficients, and dynamic
damping coefficients. Finally, calculate critical inertial force and critical whirl ratio.

Results and discussion

In order to verify the correctness of the numerical method, the calculation results are compared with those of in reference 4. It indicates that gas pressure distribution and load capacity of present study are in good agreement with those of in reference 4, as shown in Figure 5.

The performance of four types of gas journal bearing listed in Table 1 is calculated. Structure I is the bearing with traditional structure whose orifices are equal in diameter and distribute evenly. The orifices of structure II distribute evenly while their diameters decrease from $d_1$ to $d_5$ gradually. The orifices of structure III are equal in diameter while distribute unevenly. Structure IV is the new bearing proposed in this paper in which orifices are different in diameter and distribute unevenly.

The bearings use gas as the lubricant. The gas characteristics and other parameters of the bearing are listed in Table 2.

The working parameters of the bearings are listed in Table 3. Case 1 analyzes gas pressure distribution on the rotor surface and load capacity of the four types of bearing structures. Cases 2–5 discuss the influence of eccentricity ratio, rotating speed, orifice diameter, and supply pressure on load capacity of the four types of bearing structures, respectively. Cases 6 and 7 analyze the effect of eccentricity ratio on the dynamic stiffness coefficients and dynamic damping coefficients of the bearings, respectively.

Gas pressure distribution and load capacity of different bearing structures

Figure 6 demonstrates the calculated results of case 1. Gas pressure distribution is symmetrical in the axial direction because of the symmetry of bearing structure while it is asymmetrical in the circumferential direction due to the aerodynamic effect when the rotor rotates. The maximum gas pressure is near the outlet of orifice 1 at where the bearing clearance is minimum. Moreover, gas pressure decreases from orifice 1 to orifice 5 gradually which generates load capacity because of the pressure difference in the bearing clearance. The pressure difference between adjacent orifices is enlarged in structure II, structure III, and structure–IV compared with structure I.
Load capacities of structure I, structure II, structure III, and structure–IV are 535, 582, 596, and 651 N, respectively. Compared with structure I, orifice diameter gradually decreases from \(d_1\) to \(d_5\) in structure II improves load capacity 9%, from 535 to 582 N. Similarly, load capacity of structure IV is 9% bigger than that of structure III because orifices are different in diameter, from 596 to 651 N. Compared with Structure I, load capacity of structure III increases 11%, from 535 to 596 N because orifices distribute

![Figure 5. Comparison of the results of present study and those of in Li et al.4 (\(\omega = 0 \text{ krpm}, D = 100 \text{ mm}, L = 100 \text{ mm}, c = 20 \mu \text{m}, p_s = 0.5 \text{ MPa})\): (a) Present study (\(c = 0.05, d_1 = 0.2 \text{ mm}\)), (b) Li et al.4 (\(c = 0.05, d_1 = 0.2 \text{ mm}\)), (c) Present study (\(c = 0.25, d_1 = 0.2 \text{ mm}\)), (d) Li et al.4 (\(c = 0.25, d_1 = 0.2 \text{ mm}\)), (e) Present study (\(c = 0.05\)), and (f) Li et al.4 (\(c = 0.05\)).](image-url)
unevenly. Similarly, load capacity of structure IV grows 12% compared with structure II as orifices distribute unevenly, from 582 to 651 N. Therefore, the gas journal bearing in which orifices are different in diameter and distribute unevenly is helpful for improving load capacity. Compared with structure I, load capacity of structure IV increases 22%, from 535 to 651 N.

Effects of bearing parameters on load capacity

Figure 7(a) to (d) demonstrate the calculated results of cases 2–5, respectively. The figures indicate that load capacity of structure IV is obviously higher than those of other three structures. The order of load capacity from large to small of the four types of bearing structures is: structure IV, structure III, structure II, and structure I with the same eccentricity ratio.

Effects of bearing parameters on load capacity

Figure 7(a) indicates that eccentricity ratio affects load capacity greatly. The larger eccentricity ratio is, the higher load capacity is. The improvement of bearing structure on load capacity is obvious at thin average gas film thickness. However, it is negligible if average gas film thickness is larger than 22.5 μm at large eccentric ratio.

Figure 7(b) demonstrates that bearing structure has obvious effect on load capacity at rotation speed of 5 krpm while the effect can be neglected at rotation speed of 20 krpm. Moreover, load capacity increases firstly and then decreases after it reaches the maximum value with the growth of average gas film thickness at low rotation speed while it decreases with increasing average gas film thickness at high rotation speed.

Figure 7(c) indicates that orifice diameter has little effect on the maximum value of load capacity. Moreover, the average gas film thickness corresponding to the maximum load capacity increases with the growth of orifice diameter.

Figure 7(d) shows that supply pressure influences load capacity significantly. The larger the supply pressure is, the larger load capacity is and the greater the influence of bearing structure on load capacity is.

Effects of bearing parameters on dynamic stiffness coefficients

The calculated results of case 6 indicate that eccentricity ratio influences dynamic stiffness coefficients greatly, as shown in Figure 8. Principal stiffness coefficients $k_{xx}$ and $k_{yy}$ increase with the growth of eccentricity ratio while cross-couple stiffness coefficient $k_{xy}$ is on the

Table 1. Bearing structures.

| Structure | $d_1$/mm | $d_2$/mm | $d_3$/mm | $d_4$/mm | $d_5$/mm | $\alpha_1$ | $\alpha_2$ | $\alpha_3$ | $\alpha_4$ |
|-----------|----------|----------|----------|----------|----------|---------|---------|---------|---------|
| I         | 0.2      | 0.2      | 0.2      | 0.2      | $\pi$    | $\frac{\pi}{4}$ | $\frac{\pi}{4}$ | $\frac{\pi}{4}$ | $\frac{\pi}{4}$ |
| II        | 0.2      | $0.95 \times d_1$ | $0.95 \times d_2$ | $0.95 \times d_3$ | $0.95 \times d_4$ | $\pi$ | $\frac{\pi}{4}$ | $\frac{\pi}{4}$ | $\frac{\pi}{4}$ |
| III       | 0.2      | 0.2      | 0.2      | 0.2      | $\frac{\pi}{8}$ | $\frac{\pi}{4}$ | $\frac{\pi}{4}$ | $\pi$ | $\frac{\pi}{4}$ |
| IV        | 0.2      | $0.95 \times d_1$ | $0.95 \times d_2$ | $0.95 \times d_3$ | $0.95 \times d_4$ | $\frac{\pi}{8}$ | $\frac{\pi}{4}$ | $\frac{\pi}{4}$ | $\pi$ |

Table 2. Bearing parameters and gas characteristics.

| Parameter | Value |
|-----------|-------|
| Bearing diameter, $D$ | 100 mm |
| Bearing length, $L$ | 100 mm |
| Axial distance between orifice and bearing edge, $L_1$ | 25 mm |
| Atmospheric pressure, $p_a$ | 0.101325 MPa |
| Gas viscosity, $\eta$ | $1.82 \times 10^{-5}$ Ns/m² |
| Temperature | 298 K |
| Column numbers of orifices | 2 |
| Numbers of orifices on each column | 8 |

Table 3. Bearing working parameters.

| Case | Case 2 | Case 3 | Case 4 | Case 5 | Case 6 | Case 7 |
|------|--------|--------|--------|--------|--------|--------|
| $c$/μm | 15 | 12.5–30 | 12.5–30 | 12.5–30 | 12.5–30 | 20, 30 |
| $\omega$/krpm | 15 | 5 | 5, 20 | 5 | 5 | 10 |
| $p_s$/MPa | 0.5 | 0.5 | 0.5 | 0.5 | 0.4, 0.6 | 0.5 |
| $d_1$/mm | 0.2 | 0.15 | 0.15 | 0.15, 0.2 | 0.15 | 0.15 |
| $\varepsilon$ | 0.15 | 0.15, 0.35 | 0.15 | 0.15 | 0.25 | 0.15-0.45 |

$c = 12.5, 15, 17.5, 20, 22.5, 25, 27.5$, and $30$ μm in cases 2–5. $\varepsilon = 0.15, 0.2, 0.25, 0.3, 0.35, 0.4,$ and $0.45$ in cases 6 and 7.
contrary. Furthermore, $k_{yy}$ changes more clearly than other dynamic stiffness coefficients with the increase of eccentricity ratio.

If average film thickness is 20 $\mu$m, as shown in Figure 8(a), (c), (e) and (g). $k_{xx}$ and $k_{xy}$ of structure II and structure III are higher than that of structure I while $k_{yx}$ is on the contrary. The orifice difference in diameter and uneven distribution induce the growth of $k_{xx}$ and $k_{xy}$. $k_{yy}$ of structure II is smaller than that of structure I while $k_{yy}$ of structure III approximates to structure I. Therefore, the orifice difference in diameter leads to the decrease of $k_{yy}$ while the effect of orifice uneven distribution on $k_{yy}$ is negligible. $k_{xx}$ and $k_{xy}$ of structure IV are larger than those of other three structures while $k_{yx}$ is on the contrary. Moreover, $k_{yy}$ of structure IV is smaller than those of other three structures at small eccentricity ratio. $k_{xx}$, $k_{xy}$, and $k_{yy}$ of four structures are positive while $k_{yx}$ is negative except the eccentricity ratio of structure IV is smaller than 0.3 at average gas film thickness of 20 $\mu$m. Therefore, the stability of structure IV is improved at small eccentricity ratio compared with that of other three structures because the bearing may be whirling unstable if $k_{xy}k_{yx}<0$, as listed in Table 4.

If average gas film thickness is 30 $\mu$m, as shown in Figure 8(b), (d), (f), and (h). $k_{yx}$ of structure I is positive while those of structure II and structure IV is negative. $k_{xy}$ of structure III increases from negative to positive and decreases to negative again with growth of eccentricity ratio. $k_{xy}$ of structure I is negative while those of other structures decreases from positive to negative with the increase of eccentricity ratio. Therefore, the stability of structure IV and structure II is improved compared with that of structure III and structure I if eccentricity ratio is larger than 0.25.

**Effects of bearing parameters on dynamic damping coefficients**

The calculated results of case 7 demonstrate that eccentricity ratio influences dynamic damping coefficients significantly, as shown in Figure 9. The principal damping coefficients $d_{xx}$ and cross-couple damping coefficients $d_{xy}$ of structure II and structure III are larger
Figure 7. Effect of bearing parameters on load capacity: (a) effect of eccentricity ratio, (b) effect of rotation speed, (c) effect of orifice diameter, and (d) effect of supply pressure.

Table 4. The sign of $k_{xy}k_{yx}$.

| Average gas film thickness | Structure | Eccentricity ratio |
|---------------------------|-----------|--------------------|
|                           |           | 0.15   | 0.2    | 0.25   | 0.3    | 0.35   | 0.4    | 0.45   |
| $k_{xy}k_{yx}$ 20 μm     | I         | –      | –      | –      | –      | –      | –      | –      |
|                           | II        | –      | –      | –      | –      | –      | –      | –      |
|                           | III       | –      | +      | +      | –      | –      | –      | –      |
|                           | IV        | +      | –      | –      | –      | –      | –      | –      |
|                           |           | 30 μm  | –      | –      | +      | +      | +      | +      |
|                           |           |        | –      | +      | –      | –      | –      | +      |
|                           |           |        | –      | +      | +      | –      | –      | +      |
|                           |           |        | –      | +      | +      | –      | –      | +      |
|                           |           |        | –      | +      | +      | –      | –      | +      |
Figure 8. Dynamic stiffness coefficients with different eccentricity ratios: (a) effect of eccentricity ratio on $k_{xx}, c = 20\mu m$, (b) effect of eccentricity ratio on $k_{xx}, c = 30\mu m$, (c) effect of eccentricity ratio on $k_{xy}, c = 20\mu m$, (d) effect of eccentricity ratio on $k_{xy}, c = 30\mu m$, (e) effect of eccentricity ratio on $k_{yy}, c = 20\mu m$, (f) effect of eccentricity ratio on $k_{yy}, c = 30\mu m$, (g) effect of eccentricity ratio on $k_{yx}, c = 20\mu m$, and (h) effect of eccentricity ratio on $k_{yx}, c = 30\mu m$. 
Figure 9. Dynamic damping coefficients with different eccentricity ratios: (a) effect of eccentricity ratio on \(d_{xx}, c = 20\mu m\), (b) effect of eccentricity ratio on \(d_{xx}, c = 30\mu m\), (c) effect of eccentricity ratio on \(d_{yx}, c = 20\mu m\), (d) effect of eccentricity ratio on \(d_{yx}, c = 30\mu m\), (e) effect of eccentricity ratio on \(d_{xy}, c = 20\mu m\), (f) effect of eccentricity ratio on \(d_{xy}, c = 30\mu m\), (g) effect of eccentricity ratio on \(d_{yy}, c = 20\mu m\), and (h) effect of eccentricity ratio on \(d_{yy}, c = 30\mu m\).
than those of structure I at average gas film thickness of 20 μm. However, $d_{x,1}$ and $d_{y,1}$ of structure II and structure III are smaller than those of structure I. The orifice different in diameter and uneven distribution leads to the growth of $d_{x,1}$ and $d_{y,1}$ while $d_{x,4}$ and $d_{y,4}$ are on the contrary. Therefore, $d_{x,4}$ of structure IV is larger than those of other three structures while $d_{y,4}$ is on the contrary. Moreover, $d_{y,4}$ of structure IV is smaller than those of other three structures if eccentricity ratio is small.

Dynamic damping coefficients can dissipate whirling energy. Compared with the effect of principal damping coefficients on whirling stable, the influences of cross-couple damping coefficients can be ignored. The whirling stability of structure IV is improved at small eccentricity ratio because the superposition of the effect of orifice different in diameter and uneven distribution. Therefore, the stability of structure IV is improved compared with that of other three structures especially at small eccentricity ratio.

The critical whirl ratio and critical inertial force of the four types of bearing structures are shown in Table 5 ($\varepsilon = 0.25$, 0.3). Compared with structure I, structure IV improves critical whirl ratio 10.11% with $\varepsilon = 0.25$ and improves critical whirl ratio 12.9% with $\varepsilon = 0.3$. Critical inertial force of structure IV is 5.9% reduction with $\varepsilon = 0.25$ and 5.48% reduction with $\varepsilon = 0.3$ compared with structure I. Therefore, structure IV is more stable than structure I.

Conclusions

In this article, a new gas journal bearing in which orifices are different in diameter and distribute unevenly is proposed to improve the bearing performance. FDM combined with LPM is used to resolve the unsteady-state Reynolds equations numerically. Moreover, four types of bearing structures are used to discuss the effects of orifices different in diameter and uneven distribution on bearing performance. The conclusions can be summarized as following.

1. The small the orifice diameter is, the thinner average gas film thickness corresponding to the maximum load capacity is. Moreover, load capacity can be improved clearly by increase eccentricity ratio, rotation speed, and supply pressure at thin average gas film thickness. Further, the thinner average gas film thickness is, the more obvious the effect of rotation speed on load capacity.

2. The difference in orifice diameter and the uneven distribution of orifice has a superimposed effect on the static and dynamic performance of the bearing. In addition, the uneven distribution of orifice improves load capacity more than the difference in orifice diameter in small eccentricity and low rotation speed.

3. Compared with the bearing with traditional structure, the gas journal bearing compensated by orifices with different diameters and by orifices uneven distribution can improve load capacity 9% and 12%, respectively. The new gas journal bearing proposed in this paper increases load capacity 22%.

4. Average gas film thickness and eccentricity ratio influence the dynamic performance of the four types of bearing structures significantly. Moreover, principal stiffness coefficients and principal damping coefficients increase significantly with the growth of average gas film thickness. For the bearing with thin average gas film thickness, the smaller eccentricity ratio is, the better dynamic performance of the bearing is. However, the dynamic performance can be improved by increasing eccentricity ratio at thick average gas film thickness.

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**Supplemental material**

Supplemental material for this article is available online.

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**Appendix**

**Notations**

c: average gas film thickness

d: orifice diameter

D: bearing diameter

e: eccentricity

f: square of gas pressure

F: load capacity

F_c: load capacity in the x direction

F_y: load capacity in the y direction

h: gas film thickness

h_0: steady-state gas film thickness

H: dimensionless gas film thickness

H_0: dimensionless steady-state gas film thickness

K_{eq}: critical inertial force

L: bearing length

L_1: distance between bearing edge and orifice
distribution circumference

M: numbers of grids in the x direction

N: numbers of grids in the y direction

p: gas pressure

p_0: steady-state gas pressure

P: dimensionless gas pressure

P_0: dimensionless steady-state gas pressure
| Symbol | Definition                                      |
|--------|------------------------------------------------|
| $p_a$  | atmosphere pressure                           |
| $p_s$  | supply pressure                               |
| $R$    | bearing radius                                |
| $t$    | time                                          |
| $T$    | temperature                                   |
| $\alpha_i$ | the angle between orifice $i$ and orifice $i+1$, $i = 1, 2, ..., n - 1$ |
| $\eta$ | gas viscosity                                 |
| $\nu$  | eccentricity ratio                            |
| $\theta$ | attitude angle                              |
| $\lambda$ | perturbation frequency ratio                  |
| $\lambda_c$ | critical whirl ratio                         |
| $\gamma$ | orifice diameter coefficient                  |
| $\Lambda$ | bearing number                               |
| $\tau$  | dimensionless time                            |
| $\omega$ | rotating speed                                |
| $\omega_s$ | perturbation frequency                       |
| $\kappa$ | isentropic expansion index of air             |
| $\beta_{i,j}$ | pressure ratio at the orifice outlet to supply pressure |