Improved analysis of converging shock radiographs

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We previously reported an experimental platform to induce a spherically-convergent shock in a sample using laser-driven ablation, probed with time-resolved x-ray radiography, and an analysis method to deduce states along the principal shock Hugoniot simultaneously with the x-ray opacity. We have now developed a modified method of analysis that is numerically better-conditioned and faster, and usually provides a better representation of the radiograph with correspondingly lower uncertainties. The previous approach was based on optimizing parameters in a model of the density distribution as a function of radius and time, warped to follow loci such as the shock and the outside of the sample. The converging shock configuration can be described more efficiently in terms of the shocked density and sound speed, expressed as functions of the shock speed. Studies of the Hugoniot from various theoretical equations of state (EOS) indicate that, in the typical range of states explored by these experiments, these functions can be described by low-order polynomials. Similarly, few-parameter functions were found suitable for representing the variation of x-ray opacity with shock pressure. This approach was found to perform better in most cases than an alternative method based on parameterization of the EOS.

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INTRODUCTION

Simultaneous compression and heating of matter by a shock wave has been used for over seven decades to induce and study the highest pressures accessible in bulk matter [1]. We have developed an experimental platform using pulsed lasers to generate a spherically-converging shock [2–6]. The speed and compression of the shock are inferred from x-ray radiography. Because these are simultaneous measurements of two parameters of the shock in the sample material itself, the measurement is absolute as opposed to being with respect to a reference material. Even if driven by a constant ablation pressure, the shock becomes stronger as it propagates radially inward, because of isentropic compression in the flow behind which causes a progressive rise in shock pressure. Thus each experiment can sample a range of shock pressures, at the cost of a more complicated analysis.

Because, along the x-ray trajectories forming the radiograph, matter immediately behind the shock is obscured by matter in other states, the profile of mass density must be reconstructed in order to determine the shock state. Although this reconstruction can in principle be performed in a deterministic way by working radially inward by the process of Abel inversion [13], this approach accumulates noise. We have found it advantageous to make more direct use of the known state ahead of the shock in an iterative analysis in which the profile is represented by analytic functions of radius and time, whose parameters are adjusted until the inferred radiograph best matches the observation. In consequence, an obvious concern is the degree to which the choice of function imposes bias or structure on the model and the inferred shock states. We investigated this sensitivity by performing analyses with different choices of function, including tabulations, and by analyzing the radiograph by dividing it into subsets of different length in the temporal domain, and found that the variation in shock Hugoniot from the choice of functional form was several times smaller than the uncertainty from noise in the radiograph and from systematic uncertainties in the time and space scales. However, we were not able to deduce the shock Hugoniot to the precision expected from our original assessments of the experimental uncertainties, suggesting inaccuracy or redundancy in the underlying structure of the radiographic model. Furthermore, given more recent experimental campaigns in which we collected data from more than one shot on the same type of sample, we started to consider how we might infer the best-fitting shock Hugoniot to measurements from multiple experiments, for which the space-time object model is inherently unsuitable.

A different strategy would be to parameterize the equation of state (EOS), and perform simulations of each converging shock experiment, adjusting the parameters until the simulated radiograph matches the measured record. We originally disfavored this approach because it relies on additional numerical parameters whose value is not closely constrained, such as the treatment of the shock discontinuity (usually via an arbitrary artificial viscosity) and the resolution in the radial direction. More fundamentally, current capabilities to predict ablative loading are not sufficiently accurate, and therefore the drive would also have to be expressed in parameterized form and adjusted simultaneously with the EOS. This additional degree of freedom makes iterative optimization over a hydrocode model computationally much slower.
and more expensive than the approach via an object model.

Here we report different approaches to constructing object models for analyzing radiographic shock data, showing example data from experiments on carbon (initially solid density in the diamond structure).

**EXPERIMENTAL CONFIGURATION**

The experiments were performed at the National Ignition Facility (NIF). The experimental configuration was as described previously [2–4], and is summarized here for convenience. The sample material was prepared in the form of a sphere. A uniform layer of a polymer was deposited over the surface, to act as an ablator during the experiment. The coated sphere was mounted within a Au hohlraum [7], 30 μm thick, 5.75 mm diameter and 9.42 mm high, with a gas fill of 0.03 mg/cm³ He to impede filling of the hohlraum by ablated Au. 168 beams of the NIF laser were used to heat the hohlraum. The resulting soft x-ray field within the hohlraum induced a pressure pulse in the ablator, driving a shock into the sample. The overall configuration and laser pulses were based on ICF designs, to take advantage of synergies in fabrication and also the large development effort performed to give uniform drive conditions over the surface of the bead [8]. (Fig. 1)

We consider data from two experiments, N161016-3 and N140529-2, in which the laser drive was 300 and 800 kJ respectively. The latter was based on the ‘high foot’ ICF drive [8, 9]; the former was the foot of the drive continued for 5 ns. The temperature history of soft x-rays in the hohlraum was calculated using radiation hydrodynamics and measured by the DANTE filtered diode system [10]. The peak temperature was around 205 eV for the low drive and 275 eV for the high.

The ablation-induced shock wave strengthened as it propagated toward the center of the sample. X-ray radiography was used to measure the variation of attenuation across the diameter of the sample, from which the shock trajectory, mass distribution and opacity in the sample could be deduced as described below. The x-ray source was a Zn foil, heated by eight beams to produce a plasma that emitted strong He-like radiation (i.e. from atoms stripped of all but two electrons). Slits were cut in the hohlraum wall to enable transmission of the x-rays through the sample; diamond wedges were mounted in the slits to impede their closure by ablated Au. The transmitted x-rays were imaged through a slit in a Ta foil onto an x-ray streak camera (Fig. 2).

The streak radiograph was used to reconstruct the radial distribution of mass density, as a function of time. As described previously [4], the presence of undisturbed material ahead of the shock provided a strong constraint on the inference of the change in attenuation across the shock front. In order to take advantage of this constraint, the analysis was performed by adjusting a parameterized representation of the distribution of mass density until the corresponding simulated radiograph matched the measured radiograph.

In shot N140529-2, the shock became strong enough that the opacity of shocked material to 9 keV x-rays decreased significantly. The outer edge of the diamond sphere was clearly visible on the radiograph, so the variation in apparent mass from the profile-matching fit could be used to infer the change in opacity as the shock passed, as was demonstrated previously for polystyrene [6].
ANALYSIS

As discussed previously [4], the reconstructed radius-time distribution of mass density $\rho(r, t)$ gives an absolute measurement of the shock Hugoniot over a range of pressures, from the position of the shock $r_s(t)$ and hence its speed $u_s(t)$, and the mass density immediately behind the shock, $\rho_s(t) = \rho(r_s(t), t)$. Simultaneous knowledge of $u_s$ and $\rho_s$ gives the complete mechanical state behind the shock by solving the Rankine-Hugoniot relations [1] representing the conservation of mass, momentum, and energy across the shock,

$$u_s^2 = \frac{v_0^2 p - p_0}{v_0 - v} \quad (1)$$
$$u_p = \sqrt{(p - p_0)(v_0 - v)} \quad (2)$$
$$e = e_0 + \frac{1}{2}(p + p_0)(v_0 - v) \quad (3)$$

where $v$ is the reciprocal of the mass density $\rho$, $e$ is the specific internal energy, $p$ the pressure, and subscript ‘0’ denotes material ahead of the shock (with $u_p = 0$). The state ahead of the shock is known, leaving five quantities to be determined ($v, p, e, u_p, u_s$). If any two of these quantities are measured, the Rankine-Hugoniot equations determine the rest. In particular,

$$p = p_0 + \frac{u_s^2}{v_0^2} (v_0 - v). \quad (4)$$

Thus the mechanical state on the Hugoniot can be deduced directly from the distribution of mass density, without reference to any other material used as a standard, as is the case in some other experimental configurations, and so the measurement is absolute.

We previously reported analyses using a direct parameterization of the density distribution $\rho(r, t)$, and the opacity either assumed constant or inferred along the Hugoniot by the variation in apparent mass enclosed within a Lagrangian marker [4, 6]. One complication is that it is not immediately clear what functional forms to use to represent $\rho(r, t)$, and so it was necessary to choose forms with a high degree of generality and thus a large number of parameters, and to investigate the sensitivity of the deduced Hugoniot to the forms used. The analysis thus became relatively ill-conditioned and computationally intensive, and we did not achieve the precision expected from our original assessments of uncertainty. The opposite extreme would be to perform an analysis by parameterizing models in forward, hydrocode, simulations of the experiment. We considered this approach undesirable partly because of potential concerns about the accuracy of the simulations, for example from the use of artificial viscosity to stabilize shocks by smearing them spatially and temporally, but mostly because our ability to predict the drive from the fundamental experimental input of the laser power history is not adequately accurate, and therefore the drive itself would also require parameterization, introducing an additional family of parameters and computational expense.

In the present work, we improve the analysis in essence by considering a hierarchy of models with different degrees of physical constraint. The previous approach assumed only that mass was conserved and that the state ahead of the shock was known, with further implicit constraints (intended to be as few as possible and motivated by the goodness of fit) from the choice of $\rho(r, t)$ functions. Here we investigate the effect of assuming in addition...
that the experiments probe matter with a unique shock Hugoniot, but no further hydrodynamic constraints from the large-scale flow. A hydrocode-based analysis is equivalent to applying further constraints on the nature of the flow behind the shock.

An important addition to the analysis presented here is the use of the density gradient immediately behind the shock, which can be related to its acceleration. This is a key aspect of converging shock experiments. The acceleration of the shock depends on the spatial gradient of density behind the shock, \( \partial p/\partial r_s \), the variation of pressure with density variation along isentrope \( \partial p/\partial \rho_s \), the rate at which characteristics catch up with shock (its sonicity) \( c(u_s) + u_p(u_s) - u_s \), and the variation of shock speed with pressure \( \partial u_s/\partial p_s \). The acceleration of the shock can be expressed with respect to density gradient, the Hugoniot and its derivative, and the sound speed on the Hugoniot:

\[
\dot{u}_s = \frac{\partial \rho}{\partial r} \left|_{r, t} \right| \rho_s \left[ c(u_s) + u_p(u_s) - u_s \right] \frac{\partial u_s}{\partial p_s} \tag{5}
\]

where \( \partial p/\partial \rho_s = c^2 \). Given an analytic EOS, the sound speed can be calculated. The use of a functional fit for \( r_s(t) \) gives analytic expressions for \( u_s(t) \) and \( \dot{u}_s(t) \). We have generally found that \( r_s(t) \) can be expressed as a power law, also giving power laws for \( u_s(t) \) and \( \dot{u}_s(t) \).

This connection between the density gradient and the acceleration can be regarded as employing some of the apparent self-similarity of converging shocks, in local and differential form as opposed to seeking a global solution. In particular, the analysis can take advantage of the local consistency of acceleration with speed.

Another important change was the use of an algebraic function to describe the x-ray opacity along the shock Hugoniot. We investigated the use of several functions, but eventually settled on an expression derived from the Fermi occupation function, which captured the essential behavior of an initially constant opacity followed by a smooth decrease. Temperature was not determined in these experiments: the Hugoniot relations allow mechanical quantities to be determined, including the internal energy, but not the temperature. However, \( p \sim T \) in the regime where k-shell ionization occurs, and we found it possible represent the variation in theoretical opacity with a Fermi-like function in pressure,

\[
\mu = \frac{\mu_0}{1 + \exp \left[ (\log_{10} \rho - \log_{10} \rho_c) / w \right]} \tag{6}
\]

This function was not able to reproduce the variation of theoretical opacity accurately over the full range of its decrease, but captured it to better than 1% for the variation from cold to around a 50% decrease. Low-order polynomials, fitted to the theoretical opacity, were used to represent the subsequent variation \( \partial \mu/\partial p \), along isentropes from states along the Hugoniot. (Fig. 5)

With a model-based opacity as opposed to an unfold, the mass constraint used to break the redundancy in seeking to deduce density as well as opacity becomes implicit instead of explicit. The use of the model provides a constraint of uniqueness and also makes the opacity much less sensitive to noise in the radiograph.

**Parameterize density field \( \rho(r, t) \)**

In the approach applied previously [3, 4], the mass density was parameterized directly as a function of radius and time, \( \rho(r, t) \). The radial variation was defined in terms of a scaled radius between the time-varying shock and marker loci. As discussed above, the density model is inherently specific to each experiment, does not naturally admit guidance from theoretical models, and does not lend itself to deducing Hugoniot data from multiple experiments simultaneously.

The approach chosen previously [3, 4] to account for the varying opacity was, at each instant of time, to integrate inward from the marker layer to the shock, and infer the change in opacity from the apparent mass enclosed. As with Abel inversion, this unfolding technique accumulates error. It would be possible to apply the model-based technique described above with a parameterized \( \rho(r, t) \) field, but we did not explore this additional permutation of approaches.

**Parameterize natural Hugoniot functions**

The inherent Hugoniot measurement from the radiographic, converging shock experiments is the mass density as a function of the shock speed, \( \rho(u_s) \). In the density field analysis above, the shock trajectory \( r_s(t) \) was represented independently of the density along the shock locus, \( \rho(r_s(t), t) \), and neither are inherent properties of
the EOS. \(\rho_s(u_s)\) is however a property of the EOS (from the Hugoniot, which is unique for a given initial state.) Represented in this form, it is readily possible to analyze multiple experiments simultaneously.

We considered the principal shock Hugoniot from multiple EOS. Irrespective of the underlying models and of the actual loci, \(\rho_s(u_s)\) was found to vary smoothly and slowly in the regime explored by these experiments (Fig. 6). We therefore used low-order polynomials to represent \(\rho_s(u_s)\) in parameterized form.

Similarly, for the density variation behind the shock, Eq. 5, we considered the variation of sound speed with shock speed, \(c(u_s)\), from example EOS. In the regime of these experiments, \(c(u_s)\) was predicted to be almost indistinguishable from a straight line (Fig. 7), and thus low-order polynomials also appear suitable for representing this Hugoniot function in parameterized form.

We note that, even if any given functional forms for \(\rho(u_s)\) and \(c(u_s)\) are unsuitable for describing these parameters over the full Hugoniot, they are always able to represent a sufficiently small portion of the Hugoniot as a Taylor expansion about any arbitrary pressure. This result is hydrodynamically rigorous, i.e. it does not introduce any approximations.

If shock experiments are affected by non-hydrodynamic effects such as preheat from the hohlraum, the deduced \(\rho(u_s)\) relation would not be unique. Thus, preheat may be identified by comparing experiments with different drive pressures and thus different levels of preheat at a given location along the Hugoniot, such as from the low and high drive shots considered here.

Parameterize EOS

In moving from an explicitly density field reconstruction \(\rho(r, t)\) to a reconstruction via Hugoniot functions \(\rho(u_s)\) and \(c(u_s)\), we are replacing an experiment-specific model with a model defined via functions related to the EOS. A further step is to define the model directly via analytic EOS functions. We investigated this approach with Grüneisen EOS defined in a way commonly used in the past for data obtained from shock experiments on solids up to a few hundred gigapascals [24], through the relation between shock and particle speeds \(u_s(u_p)\), and the Grüneisen parameter expressed as a function of mass density \(\Gamma(\rho)\),

\[
p(\rho, e) = p_r(\rho) + \Gamma(\rho) [e - e_r(\rho)]
\]

where \(p_r(\rho)\) and \(e_r(\rho)\) constitute a reference locus, in this case the Hugoniot inferred from \(u_s(u_p)\) and the Rankine-Hugoniot relations. We implemented this approach as a further step in the Hugoniot function method above. The Rankine-Hugoniot relations were solved to deduce \(\rho(u_s)\) from \(u_s(u_p)\), and the derivatives of the Grüneisen EOS were used to calculate \(c(u_s)\). When this procedure is valid, it has the advantage of producing EOS that may be used directly in many hydrocodes, which often have versions of the Grüneisen EOS based on a polynomial \(u_s(u_p)\) relation and variants of a power law form for \(\Gamma(\rho)\).

As for the Hugoniot functions, we investigated the behavior of these functions using EOS constructed with different physical assumptions. In the regime of our experiments, the \(u_s(u_p)\) relation was found to be very close to a straight line (Fig. 8). The Grüneisen parameter \(\Gamma\) was found to have a more complicated variation than found for the other functions (Fig. 9). In particular, it
became multivalued at densities around peak compression, where the simplification that \( \Gamma \) is a function of \( \rho \) only (and not temperature, or energy deviation from the reference) breaks down. In any case, any EOS using the principal Hugoniot as a reference must break down by peak compression as the Hugoniot cannot be used as a reference for states of higher density than peak compression.

As with the Hugoniot function approach, even if the Grüneisen EOS cannot capture the wide-range structure of the Hugoniot, it still represents a Taylor expansion about a state on the Hugoniot, and can thus be used in a local sense, for instance by splitting the radiograph into multiple sections. Similarly, the use of an EOS imposes uniqueness on the deduced Hugoniot, which may be incorrect if different experiments are affected by different levels of preheat, but this can again be assessed by the quality of fit to the radiographs.

### ANALYSIS OF CARBON EXPERIMENTS

The outer surface of the diamond was evident in both experiments, so the opacity could be inferred simultaneously with the density. The streak radiograph was analyzed to deduce the radius-time distribution of mass density, represented as discussed above. The locus of the shock was represented by the function

\[
  r_s(t) = \alpha (t_c - t)^\beta
\]

(8)

where \( \alpha, \beta, \) and \( t_c \) were fitting parameters. The locus of the marker layer was represented by the function

\[
  r_m(t) = r_{\text{min}} + \alpha e^{-\beta t}
\]

(9)

where \( \alpha, \beta, \) and \( r_{\text{min}} \) were fitting parameters. Quadratics were found adequate to represent \( \rho(u_s) \), and straight lines for \( c(u_s) \). The locus and value of the maximum density, and the value at the outer surface, were represented as before. These functions were able to capture these loci over the full range of the record.

Analyses were performed for each shot separately, and also for both shots together. The opacity from shot N161016-3 was consistent with it remaining cold, and so the Hugoniot states from this shot were determined assuming the cold opacity, as this gave a smaller uncertainty. The analysis was performed using the density field reconstruction (Figs 10 and 11), the Hugoniot functions (Figs 12 to 15), and the EOS functions (Figs 16 to 19). None of the \( \Gamma(\rho) \) functions tried was a good fit to the full range of the data: a linear variation gave the best match to the low drive shot; a quadratic variation to the high drive, and a power law to the simultaneous fit to both shots. Using the present method of deducing the best fit and uncertainties, the nominal uncertainty in the deduced shock Hugoniot was lowest for the Grüneisen EOS fit. However, the absolute goodness of fit along the best solution was significantly lower than for Hugoniot function fit, meaning that the best-fitting residual difference between the simulated and measured radiographs was larger. In other words, the Grüneisen EOS was less able to fit the overall flow distribution probed by the radiograph. This deficiency seems to be related to the use of a Grüneisen parameter varying with \( \rho \) only, as the deduced \( u_s - u_p \) was indistinguishable from linear. The overall goodness of fit from the nominal best Grüneisen EOS fit was consistent with the Hugoniot function fit in the same area of the \( p - \rho \) plane. The main difference between the two analysis approaches is in the use
FIG. 10: Hugoniot for carbon (initially diamond) deduced using the density field model. Thin lines are 1σ contours for the corresponding thick line.

FIG. 11: Opacity of carbon (initially diamond) to 9 keV x-rays, along the principal Hugoniot using the density field model. Thin lines are 1σ contours for the corresponding thick line.

FIG. 12: Hugoniot function ρ(𝑢𝑠) fit for carbon (initially diamond). Thin lines are 1σ contours for the corresponding thick line.

FIG. 13: Fit for sound speed along the Hugoniot for carbon (initially diamond). Thin lines are 1σ contours for the corresponding thick line.

CONCLUSIONS

By modifying the structure of the model used to describe the distribution of mass density when reconstructing time-resolved radiographs of converging shocks, so that the parameters were in functions relating to the EOS rather than experiment-specific spatial and temporal functions, we were able to significantly increase the precision and amount of EOS data deduced from these experiments while also improving the numerical stability and efficiency of the analysis, without loss of generality in the reconstruction of density from a given experiment. This approach also enabled the shock Hugoniot to be inferred using data from multiple experiments. The approach also enabled the shock Hugoniot to be inferred using data from multiple experiments. The approach also enabled the shock Hugoniot to be inferred using data from multiple experiments.
precision was increased further through the use of a parameterized function for the x-ray opacity, instead of the unfolding reconstruction used previously.

For carbon (initially diamond), the nominal uncertainty in the deduced Hugoniot was lower when analyzed using an analytic Grüneisen EOS with the Hugoniot as reference rather than when using functions for $\rho(u_s)$ and $c(u_s)$. The opposite was true for the opacity. Both techniques had significantly lower uncertainty than the previous density field model and an opacity deduced directly from the change in apparent mass. However, the analysis using the Hugoniot functions $\rho(u_s)$ and $c(u_s)$ gave a significantly better goodness of fit than the analytic Grüneisen EOS, which may be because a Grüneisen parameter $\Gamma$ varying with $\rho$ only was insufficient to represent the EOS of carbon over the range of states explored in even the lower-drive experiment.

Hugoniot data for carbon were obtained to $\sim 80$ TPa, by which point the opacity had fallen to a level implying a $k$-shell ionization of around 30%. The uncertainty was closer to what had originally been predicted for these experiments from an assessment of the expected sources of error.
FIG. 18: Hugoniot for carbon (initially diamond) deduced using the EOS function model. Thin lines are $1\sigma$ contours for the corresponding thick line.

FIG. 19: Opacity of carbon (initially diamond) to 9 keV x-rays, along the principal Hugoniot using the EOS function model. Thin lines are $1\sigma$ contours for the corresponding thick line.

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