A Study of Granular Flow in A Conical Hopper Discharge Using Discrete and Continuum Approach

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Abstract

Discrete and continuum modelling of granular flow in a conical hopper discharge is presented. The Discrete Element Method (DEM) is used for discrete modelling and Finite Element Method based on an Arbitrary Lagrangian-Eulerian (ALE) formulation for continuum modelling. The ALE model has shown its capability to model granular flow and macroscopic behaviour in silos in the authors’ previous work (Wang et al, 2013). The use of ALE overcomes mesh distortion problem due to material large deformation which is often encountered in standard finite element modelling of silo discharge. However, there is still limitation when a continuum method is used to model silo discharge, in particular, to pursue particle-scale features in such granular flow. In this study, wall pressure, flow pattern and flow rate during silo discharge are investigated using both ALE and DEM simulation. Classical theories are used as a comparison, to verify the two numerical models. By comparison, it is indicated that both FE model using ALE formulation and DEM model have the capacity to model the granular behaviour and wall pressure for granular process in silos. The conclusion that ALE has advantages over DEM in wall pressure prediction giving stable pressure profile with smooth trend; while DEM has the ability to evaluate the role of particle-scale parameters on macro-scale response in such granular flow after carefully smoothing out the results has been drawn in the present study.

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1. Introduction

Silos and hoppers are widely used for the processing of granular and powder materials which is important in many engineering applications. Their correct design requires the good understanding of the behaviour of the granular material and adequate prediction of the pressures exerted on their walls by the stored material. Significant progress has been made in these aspects. Traditionally, analytical procedures have been used to determine the values of pressures inside silos\(^1\,^2\). However, these procedures are intrinsically based on some assumptions for simplicity and not able to describe situations other than those for which they were formulated. Experimental operations are undertaken to measure the pressures and flow pattern\(^3\,^4\). However, these operations are very expensive and not repeatable. These problems lead to the use of numerical techniques for the study of the pressures and flow characteristics in silos and hoppers. One of these is the finite element method (FEM), which has gained some success in prediction of the pressures and flow pattern in silos\(^5\,^6\), particularly, it becomes powerful for dealing with large deformations in silo emptying simulations with the help of remeshing techniques\(^7\,^8\). However, this method contemplates the granular mass as a continuum, preventing it from being able to evaluate individual particles on the dynamical behaviour of particulate materials during silo discharge. Other numerical methods were therefore sought, and the discrete element method (DEM) shows a great ability to meet these requirements\(^9\).

The DEM, which was first applied to soils by Cundall and Strack\(^10\), allows one to obtain a great deal of details on the particle-scale features in granular flow in silos, which affords a great advantage over the experimental measurements and continuum simulations. In fact, many researchers are now using DEM models to investigate the pressures distributions and flow patterns produced in silos\(^11\,^12\). The DEM is still being developed and not yet able to make useful blind predictions of silo phenomena without any aids. A major shortcoming of DEM is that a realistic stress state cannot yet be predicted accurately with the large scatter in the data, due to the high computational cost and thus the limitation of the number of particles most programs can model\(^14\). Therefore, the adequate correspondence of DEM prediction with reality in silos cannot be guaranteed without a combination with other methods\(^15\).

In the present work, a conical hopper is modelled using the DEM and FE model. The FE model is formulated based on an Arbitrary Lagrangian-Eulerian (ALE) theory. The ALE model, which is able to overcome mesh distortion problem due to material large deformation often encountered in standard FE modelling of silo discharge, has shown its capability to model granular flow and macroscopic behaviour in silos in the authors’ previous work\(^16\). Meanwhile, the DEM is used to investigate normal wall pressure distribution, flow pattern and flow rate during silo discharge. Their advantages and capacity to model the granular behaviour in the hopper will be presented by a comparison with those most accepted classical theories.

2. Methodology

A brief description of FE using ALE and DEM model is given in this section. The basic theories of ALE and DEM have been covered extensively in the literature, so only the specific implementation and model parameters are described below.

2.1. FE continuum model

To avoid mesh distortion, in the present study the entire hopper filling and discharge processes are simulated using the FE model based on the so called uncoupled Arbitrary Lagrangian-Eulerian (ALE) formulation in the program Abaqus/Explicit\(^17\). The ALE method is a particular extension of the standard finite element method. In the ALE method, the mesh motion is taken arbitrarily from material deformation to keep element shape optimal, particularly under large material deformation. In this method, a Lagrangian phase is first performed. In the next smoothing phase, the mesh configuration is adjusted by moving element nodes in an appropriate way so as to control mesh distortion. In turn, the Eulerian phase is considered, where a remap of the solution of the Lagrangian phase onto the new mesh is carried out by introducing all the convective effect between the two phases (see Fig.1).
A conical hopper with an axisymmetrical geometry was considered in the present FE simulation, as shown in Fig.2. The height of the hopper is 2.64 m, radius at the top 1.2 m, radius at the bottom as the outlet 0.1 m and apex half-angle $\alpha = 22^\circ$. In the ALE simulation, the stored granular material was modelled as an elastic-perfectly plastic material using a Drucker-Prager failure criterion [18]. Through a mesh convergence study, a fine mesh of 8000 first-order 4-node quadrilateral elements with reduced integration was used to model the granular material. The walls were modelled using 2-node rigid elements, and their interaction with granular material were described using Coulomb type contact with a constant coefficient of wall friction. The material properties were chosen to represent some generic granular material properties. These properties resembles quite well following granular material that has low friction, such as agricultural grains [19-21]. The assumed material parameters are presented in Table 1.

An explicit time integration scheme is used to perform the non-linear dynamic analysis. A convergence study on the time increment has been carried out and a time increment of the order of $1 \times 10^{-6}$ s is required so as to ensure a stable solution. In the ALE model, both Lagrangian and Eulerian boundaries were employed. The sides and top surface of the material were defined as Lagrangian boundaries while the base (outlet) is set to be an Eulerian boundary as shown in Fig.2. As such, this configuration permits the stored material to deform and flow within the hopper while eliminating the potential mesh distortion problem at the outlet. A full description of the ALE boundary definition can be found in Ref.[17].

| Table 1 Material parameters used in FE model |
|---------------------------------------------|
| Bulk density ($\rho_b$) | 1000 kg/m$^3$ |
| Young’s modulus ($E$) | 5.5 c5 Pa |
| Poisson’s ratio ($\nu$) | 0.3 |
| Internal angle of friction ($\phi_i$), or coef.$=\tan\phi_i$ | $30^\circ$, 0.577 |
| Dilation angle ($\psi$) | $10^\circ$ |
| Coefficient of solid-wall friction ($\tan\phi_w$) | 0.267 |

The whole numerical process contains two main stages of analyses: the first is for filling and the second is for discharge. The filling state in the hopper was modelled by discretizing the final geometry of the material fill into ten layers and then activating each layer sequentially in the FE analysis starting from the bottom layer. The numerical process involved achieving equilibrium for each activated layer under the gravity before next one is laid on. The top surface of the material is at the top of the hopper. To simulate the discharge process, a gravity-driven flow is considered by removing the constraints at the hopper outlet instantaneously.
2.2. Discrete element model

The DEM model represents a slice of the hopper. The slice is considered to be a simplification of a 3D conical hopper having the same dimensions as considered in the ALE model. The model thus ignores any out of plane behaviour which is deemed an adequate approximation for silos of 2D planar cross-section. An assembly of about 8,000 spherical particles, which have a diameter varying between 8.0 and 10.0 mm, are centred within a common plane domain. The number of spheres under this configuration is appropriate to describe the microscopic behaviour in such dimension hopper. The model is bounded by physical walls in all its faces except at the upper surface. Lateral walls in the hopper are each split into 15 equal segments. For each segment, the mean contact force is calculated to evaluate the pressure distribution acting on the hopper walls. The wall representing the hopper outlet consists of a single section.

The hopper is initially filled by generating all particles at one instant. once generated, all these particles are allowed to fall under gravity until reaching a static state, when hopper filling is deemed complete. The hopper discharge process is initiated by removing the wall closing the hopper outlet after the completion of filling.

All DEM simulations are performed using the commercial PFC 2D software\textsuperscript{[22]}. A linear contact model is employed to describe the force-displacement relation in normal and tangential directions between two particles or particle and wall in contact. A slip behaviour between them, which is defined by the Coulomb law with a constant coefficient of friction, is allowed to occur when the maximum allowable shear contact force is reached. All the contacts are assumed to have a normal contact stiffness \( k_n \), and tangential contact stiffness \( k_s=0.1k_n \). The contact stiffness and Coulomb friction coefficient are determined by a calibration process. A series of DEM numerical biaxial compression tests with various contact stiffness and Coulomb friction coefficient are performed to correlate with the macroscopic parameters of the Young’s modulus and internal friction angle used in the FE simulation. The remaining microscopic material parameters used in the DEM model are directly obtained from the literature\textsuperscript{[21]}. The parameters used in the DEM simulations are summarized in Table 2.

| Table 2 Parameters used in DEM model |
|--------------------------------------|
| Particle material                    | Particle density | 1250 kg/m\(^3\) |
| Normal contact stiffness, \( k_n \)  | 1.1e6 Pa         |
| Shear contact stiffness, \( k_s \)   | 1.1e5 Pa         |
| Friction coefficient, \( \mu \)      | 0.577            |
| Interface                            | Normal contact stiffness, \( k_n \)  | 1.1e6 Pa         |
| Shear contact stiffness, \( k_s \)   | 1.1e5 Pa         |
| Friction coefficient, \( \mu w \)    | 0.267            |

3. Results and discussion

3.1. Normal wall pressure

The wall pressures on the hopper walls have been investigated. Figure 3 shows the predicted normal wall pressure distributions on the walls for filling and discharge. The normal wall pressure in DEM simulations are computed from the mean resultant forces applied on wall segments of 22-com length. In the FE simulations, the pressure are calculated directed on a series of wall nodes. For both FEM and DEM, the filling wall pressure are temporally averaged over the last 0.5 s of the filling process, whilst the discharge results are averaged over the first 10 s of the discharge process. The pressure at representative locations on the hopper wall is extracted at every time point over an interval of 10 s with a sampling rate of 50 Hz. The average pressure is determined by a simple arithmetic mean via the following equation:
where \( p_{wn(i)} \) represents the normal pressure at a specified wall point at the \( i \)th time point and \( p_{wn} \) is the average normal pressure at the corresponding wall point from the \( N \) time points within the period of interest.

Walker’s theory\(^{[2]}\), which has been extensively used as a classical theory for the analysis of wall pressures in conical hoppers, is employed to compare with the present numerical predictions. A detailed description of comparison of wall pressure in a hopper with Walker’s solution can be found in Ref\(^{[16]}\). Through Fig. 3, it is found that a good agreement is reached for normal wall pressures for filling process given by the classical theory and numerical methods (DEM, FEM). For the discharge pressure, it is clearly seen that the numerically calculated pressure pattern also follows the theoretical solution well. Namely, the pressure peaks move from the lower part to the higher part of the hopper. Some discrepancy of discharge pressure distributions between the numerical and theoretical prediction can be accepted since Walker’s solution for the discharge state is complying with the static equilibrium in the hopper, whilst the numerical simulations are considering the falling filling level of the stored solid under the quasi-static equilibrium condition. For the discrete calculation, there exists the spatial scatter of pressure distribution, it may be alleviated by refining the wall division. The results provide a verification of the FEM and DEM models in the sense of producing a reliable wall pressure prediction which is consistent with the classical theory.

![Fig.3 Comparison between numerical and theoretical predictions of wall normal pressure distributions along the hopper wall for end of filling and beginning of discharge (discharge pressure averaged over the first 10 s of numerical simulations of discharge)](image)

### 3.2. Flow pattern

It is possible to visually assess the predicted flow pattern within the hopper. To facilitate this task, in the FE model with the ALE approach, tracers are used to monitor the material movement. Herein, in order to save the computational cost, a limited number of tracers, as indicated by red square-shaped symbols, are attached to material points along a few horizontal layers, as shown as Fig.4. In the DEM model, all particles in the hopper are divided into different horizontal layers at the beginning of the discharge process. These layers are coloured alternatively with two contrasting colours so that particle movement can be easily identified (see Fig.5).
Figure 4 and 5 respectively shows three different stage snapshots of the discharge process for FE and DEM model. Both of the two models are likely to develop a mass flow as all the material particles are in motion and no significant internal channel along the hopper height is present during most of the discharge process. The mass flow predicted is expected as the apex half-angle and wall friction coefficient of the hopper considered in the present numerical models lie within the threshold of mass flow according to Eurocode[23]. Through the result shown in Fig. 5 for the DEM simulation, particle-scale behaviour in the flow is easily detected and it enables one to examine the local deformation in the particle assembly for granular processes. A further quantitative analysis of flow pattern can be pursued through the investigation into cross-section velocity profile and particle residence time.

### 3.3. Flow rate

A comparison of mass flow rate predicted between the numerical models and the extended Beverloo equation[24] has been made to further verify the present numerical models. There exists a number of analytical solutions to predict the mass flow rate in silo discharge. Among those, Beverloo equation, which was derived from the results of a great deal of experiments, is most trustable. Nedderman has extended the Beverloo equation and applied to the flow prediction in a mass flow conical hopper[24]. The relevant parameters, namely, the modification factor C, the outlet diameter of the hopper D₀, the particle shape coefficient of \( \lambda = 1.5 \) for spherical particles, the particle size \( d_p \),
the apex half-angle of the hopper $\alpha$, the angle between the stagnant zone boundary and the horizontal in the hopper $\phi_0$, and the material bulk density $\rho$, involved in the extended Beverloo equation are chosen according to published experimental results\cite{24} and listed in Table 3.

| C  | $D_0$ (m) | $\lambda$ | $d_p$ | $\alpha$ (°) | $\phi_0$ (°) | $\rho$ (kg/m$^3$) |
|----|-----------|-----------|-------|--------------|--------------|-----------------|
| 0.58 | 0.2       | 1.5       | 0     | 22           | 45           | 1000            |

Figure 6 shows time histories of mass flow rate obtained by the numerical simulations and empirical solution. Whilst the extended Beverloo equation gives a constant mass flow rate, a progressively increase flow rate is predicted by the FE model until a steady state is achieved about a few seconds after discharge starts. DEM model predicts a flow with a sudden increase in flow rate before reducing gradually to a relatively constant value during discharge. From Fig. 6, it is clearly seen that the fully developed flow rate predicted by the DEM and FE simulation matches the extended Beverloo empirical equation very well.

4. Conclusions

The DEM and FE model have been used to study the granular flow and wall pressure in a conical hopper. The mass flow pattern, the mass flow rate and wall pressure distribution, which are predicted by the numerical models, are generally in line with those classical theoretical solutions.

The FE model using the AEL formulation, as a continuum method, has shown to be an effective technique for the simulation of silo discharge process without mesh distortion and it could be used to facilitate the prediction of macroscopic behaviour of particle assemblies using a discrete method. DEM, as a discrete method, naturally has the ability to model the particle assembly and its flowing behaviour for granular processes in silos; however, there are still a few aspects needed to be undertaken in future research: firstly, the wall pressure distribution with spatial scatter may be smoothed out with clearer trends by refining wall division; secondly, the effect of microscopic material parameters on the macroscopic material parameters and further on the macro-scale response of particle assembly in the silo remains indefinite and needs a further study; finally, rigorous procedures for verification and validation in DEM are still required to develop the computational framework and the experimental procedures for granular processes in silos.
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