A brief overview of various problems related to the description of a polarized proton in quark models is presented. Structure functions are discussed both for longitudinal and transverse polarization. A recently introduced quantity, relevant in the study of single-spin asymmetries, is shown to be in principle non-vanishing when computed in chiral models.

1 Introduction

The possibility of describing a polarized nucleon in terms of effective degrees of freedom is still an open problem. Various experiments have discovered effects that cannot be explained within perturbative QCD, e.g. the single-spin asymmetries found in proton-proton scattering. Even the so-called spin problem of the nucleon, namely the way in which the total angular momentum of the system is distributed among the various components, is not yet completely understood and substantially different mechanisms have been proposed.

In this contribution I will shortly review a few topics related to the description of a polarized nucleon in high-energy experiments. To this purpose, let me recall the expression of the three leading twist quark distribution functions: unpolarized:

\[ q(x) = \frac{\sqrt{2}}{4\pi} \int d\xi^- e^{-ixp^+\xi^-} \langle N|\psi^+_\perp(\xi)(0)|N\rangle_{\xi^+=\xi_\perp=0} \]

longitudinally polarized:

\[ \Delta q = \frac{\sqrt{2}}{4\pi} \int d\xi^- e^{-ixp^+\xi^-} \langle N|\psi^+_\perp(\xi)\gamma_5\psi^+_\perp(0)|N\rangle_{\xi^+=\xi_\perp=0} \]

transversely polarized:

\[ h_1(x) = \frac{\sqrt{2}}{4\pi} \int d\xi^- e^{-ixp^+\xi^-} \langle NS_\perp|\psi^+_\perp(\xi)\gamma_\perp\gamma_5\psi^+_\perp(0)|NS_\perp\rangle_{\xi^+=\xi_\perp=0} \]

The first two quantities are the well known distributions of the momentum and of the helicity, respectively. The third one has been introduced recently and is the distribution of the transversity in a transversely polarized proton.
Figure 1: Unpolarized non-singlet $F_2^p - F_2^n$ (left) and polarized proton $xg_1^p$ (right) structure functions. The dotted curves represent the leading-twist contributions at the model scale $Q^2_0 = 0.16 \text{ GeV}^2$. The solid curves are the evolved distributions at $Q^2 = 4 \text{ GeV}^2$ (unpolarized) and at $Q^2 = 10 \text{ GeV}^2$ (polarized), respectively. Diamonds and square are the data. For details see Ref.[2].

The previous quantities can be evaluated in any quark model which provides the wave function of the nucleon. What one obtains are the leading twist contributions to the distributions evaluated at a very low $Q^2_0$, the scale of the model. To compare with the experiments one has to compute their evolution to larger value of $Q^2$ using DGLAP equations.

2 Longitudinal polarization

In Fig.1 results for the unpolarized and for the longitudinally polarized structure functions are presented. They were obtained in Ref.[2], using the so-called chiral chromodielectric model. The latter is a non-topological soliton model in which chiral fields play a relatively minor role. This is due to the absence in the model of solutions having non-zero winding number. As it appears, the computed longitudinally polarized structure function largely overestimate the data, its first momentum being $\Gamma_1^p = 0.225$ (exp. $\sim 0.136$).

Chiral models in which the pion develops a non-trivial topology are able to reduce the valence quark polarization, converting spin into orbital angular
momentum of the sea, represented by the chiral fields. They offer therefore a possible solution of the spin problem.

It must anyway be emphasized that the analysis of the DIS data suggests a rather large polarization of the gluons, possible only if the latter are polarized already at very low $Q^2$. Moreover, QCD sum rules results indicates that roughly half of the total angular momentum is carried by the gluons. In Ref. [7] the total angular momentum carried by the gluons $J_g$ was estimated using the simple Isgur-Karl model. At the scale of the model $Q_0^2 = 0.25$ GeV$^2$ roughly half of the total angular momentum is attributed to the spin of the gluons $\Delta G \sim 0.24$. The orbital angular momentum of the gluons turns out to be negligible at $Q_0^2$. Performing a leading-order QCD evolution, $\alpha_s \Delta G$ is constant, whereas $J_g$ increases slowly.

At the moment it is impossible to decide which of the two solutions of the spin problem is realized in nature. Probably the angular momentum is carried partly by the sea and partly by the gluons. The forthcoming experiments aiming to measure $\Delta G$ should clarify the situation.

3 Transverse polarization

The transversity distribution $h_1$ cannot be measured in fully inclusive DIS, since it corresponds to a process in which the helicity of the struck quark is flipped. At the moment no experimental data is available. $h_1$ could be measured in semi-inclusive electron scattering or in transversely polarized Drell-Yan processes.

The QCD evolution of $h_1$ differs from that of $g_1$ since:

- $h_1(x, Q^2)$ does not mix with the gluon distribution;
- its first momentum (tensor charge) decreases as
  \[ \delta q(Q^2) = \delta q(Q_0^2) \left( \frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \right)^{-4/27}, \]
- at small $x$ its splitting function $P_h(x) \sim 8x/3$, whereas $\Delta P_{qq}$ and $\Delta P_{qg}$ (the splitting functions for $g_1$) behave as constant as $x \to 0$.

In Fig. 2 estimates of $h_1$ made using the chiral chromodielectric model are presented. As it can be seen, $h_1$ is not substantially different from $g_1$ at the scale of the model, but due to the linear behaviour in $x$ of its splitting function as $x \to 0$, $h_1$ is considerably smaller than $g_1$ at small $x$ after being evolved to large $Q^2$. As a consequence, the double transverse Drell-Yan asymmetries turn out to be considerably smaller than the longitudinal ones, making it very difficult to measure $h_1$ through this process. The previous statements have been confirmed by a model independent analysis.
4 Single spin asymmetries and chiral lagrangians

Consider the plane defined by the momentum of the proton and its transverse spin. Recently a new quantity has been introduced \(^1\). It has the following partonic interpretation: it gives the asymmetry in the distribution of the momenta perpendicular to the above defined plane.

\[
\Delta^N f_{a/p \uparrow} (x_a, k_{\perp a}) = \sum_{\lambda_a} \left[ \hat{f}_{a,\lambda_a/p \uparrow} (x_a, k_{\perp a}) - \hat{f}_{a,\lambda_a/p \uparrow} (x_a, -k_{\perp a}) \right]
\] (4)

This quantity can be regarded as a single spin asymmetry for the \( p \uparrow \rightarrow a + X \) process and was introduced to describe the process \( pp \uparrow \rightarrow \pi X \). It can be written in terms of matrix elements of quark operators as:

\[
\Im \int \frac{dy^- dy_\perp}{(2\pi)^3} e^{-ixp^+ y^- + ik_{\perp} \cdot y_\perp} \langle p, -|\bar{\psi}_a(0, y^-, y_\perp) \gamma^+ \psi_a(0)|p, + \rangle
\] (5)

It has been shown by Collins \(^3\) that, if the flavor \( a \) is not touched by time reversal, the previous quantity vanishes due to time reversal invariance of QCD. Indeed, no time-reversal even observable can be constructed with two independent momenta and one spin vector.
In Weinberg’s book on field theory\textsuperscript{14} it is shown that a field can transform under time reversal in a more complicated way than just getting a phase. In particular a doublet of fields can mix under time reversal. Now, in chiral lagrangians the flavor is not a dummy index. Let me consider the single-quark Dirac equation in the presence of static chiral fields:

\[
[k_\mu \gamma^\mu - g(\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi})]u(k) = 0.
\]

Seeking the time reversed solution of the same equation \((k \rightarrow -k)\) we get:

\[
[k_\mu \gamma^\mu - g(\sigma - i\gamma_5 \vec{\tau}^T \cdot \vec{\pi})]\gamma_5 C u^*(\tilde{k}) = 0
\]

where \(\tilde{k} = (k_0, -\vec{k})\) and \(C = i\gamma_0\gamma_2\). The term containing the pion has been modified by the previous transformation. Since \((-i\tau_2)(-\vec{\tau})^T(i\tau_2) = \vec{\tau}\), the time reversed solution reads \((-i\tau_2)\gamma_0 C u^*(\tilde{k})\). This means that quark states of fixed isospin are in general not eigenstates of the chiral hamiltonian. For instance, taking the hedgehog form for the pion field, \(\vec{\pi} = \hat{r}\phi(r)\), the spin-isospin wave function is given by \(|h\rangle = (|u\rangle + |d\rangle)/\sqrt{2}\).

If time reversal mixes up and down quarks the asymmetry function \(\Delta^N f_{g/p}^{\uparrow}\) need not be zero and single spin asymmetries in inclusive DIS are allowed:

\[
\frac{d\sigma^\ell p^{\uparrow} \rightarrow \ell X}{dx dQ^2} - \frac{d\sigma^\ell p^{\downarrow} \rightarrow \ell X}{dx dQ^2} = \sum_q \int d\vec{k}_\perp \Delta^N f_{q/p}^{\uparrow}(x, \vec{k}_\perp) \frac{d\hat{\sigma}_Q^{\ell q \rightarrow \ell q}}{dQ^2}(x, \vec{k}_\perp)
\]

It would be extremely interesting to test this asymmetry with an experiment.

5 Conclusions

- Longitudinal Polarization
  - Chiral fields alone could be not sufficient to solve the spin problem
  - The spin carried by polarized gluons can be computed in quark models
  - The gluon contribution to the spin turns out to be of the right order of magnitude and of the right sign

- Transverse Polarization
  - The transverse distribution \(h_1(x)\) evolves differently from the longitudinal one
  - At small \(x\) \(h_1(x)\) is considerably smaller than \(g_1(x)\)
– It is therefore difficult to measure $h_1(x)$

- Single-spin asymmetries
  - They are not allowed in DIS if flavor is untouched by time reversal
  - They are possible in chiral models and could be tested experimentally.

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