Parametrizations of three-body hadronic $B$- and $D$-decay amplitudes

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Abstract. A short review of our recent work on amplitude parametrizations of three-body hadronic weak $B$ and $D$ decays is presented. The final states are here composed of three light mesons, namely the various charge $\pi\pi\pi$, $K\pi\pi$ and $KK\bar{K}$ states. These parametrizations are derived from previous calculations based on a quasi-two-body factorization approach where the two-body hadronic final state interactions are fully taken into account in terms of unitary $S$- and $P$-wave $\pi\pi$, $\pi K$ and $KK$ form factors. They are an alternative to the isobar-model description and can be useful in the interpretation of CP asymmetries.

1 Introduction

1.1 Motivations: why study three-body hadronic $B$ and $D$ decays?

Three-body hadronic $B$ and $D$ decays provide a rich tool to study not only the Standard Model, QCD, CP violation [1] but also hadron physics. The hadron physics, often characterized by two-body resonances and their interferences, affect weak observables and any reliable determination of the later will require a good knowledge of the final state meson-meson interactions. This can be realized by introducing theoretical constraints such as unitarity, analyticity, chiral symmetry and the use of data from reactions other than $B$ and $D$ decays. Basic Dalitz-plot analyzes rely on sums of relativistic Breit-Wigner amplitudes representing the different possible implied resonances to which some non resonant background amplitude is added. The $S$-wave resonance contributions are often difficult to fit. Can one go beyond this isobar model approach?

One can replace the sums of relativistic Breit-Wigner components by parametrizations [2] in terms of unitary two-meson form factors keeping the weak-interaction dynamics governing the flavor-changing process via $W$-meson exchange. These parametrizations are based on published results and motivated by analyzes of high-statistics present and forthcoming data at BES III, LHCb, Belle II, Super c-tau factory .... Up to now there is no three-body decay factorization theorem but major contributions arise from intermediate resonances such

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as $\rho(770)$, $K^*(892)$, $\phi(1020)$ which allows to describe three-body decays as quasi-two-body ones. For instance, for the three-meson final state of the $D^0 \rightarrow K_S^0 \pi^- \pi^+$ decay, one can introduce quasi-two-body pairs, $[K_S^0 \pi^+]_L \pi^-$, $[K_S^0 \pi^-]_L \pi^+$, $K_S^0 [\pi^+ \pi^-]_L$, two of the three mesons forming a state of angular momentum 0 or 1 with $L = S$ or $P$, respectively.

### 1.2 QCD quasi-two-body factorization

Decays are mediated by local four-quark operators $O_i(\mu)$ forming the weak effective non-renormalizable Hamiltonian $\mathcal{H}_{\text{eff}}$. Schematically for $B \rightarrow M_1 M_2^* (M_2^* \rightarrow M_3 M_4)$ one has

$$\langle M_1 M_2^* | \mathcal{H}_{\text{eff}} | B \rangle = \frac{G_F}{\sqrt{2}} V_{\text{CKM}} \sum_i C_i(\mu) \langle M_1 M_2^* \rangle O_i(\mu) | B \rangle,$$

(1)

where $G_F$ is the Fermi decay constant, $V_{\text{CKM}}$ the product of Cabibbo-Kobayashi-Maskawa matrix elements and $C_i(\mu)$ Wilson coefficients renormalized at scale $\mu \sim m_b$ (or $m_c$ in $D$ decays). In the factorization approach [3] with the strong coupling $\alpha_s(\mu)$, i.e. at scale $\mu$,

$$\langle M_1 M_2^* | O_i(\mu) | B \rangle = \left( \langle M_1 | J^\nu_1 | B \rangle \langle M_2^* | J_{2\nu} | 0 \rangle \right)$$

$$+ \langle M_1 | J_3^\nu | 0 \rangle \langle M_2^* | J_{4\nu} | B \rangle \left[ 1 + \sum_n r_n \alpha_s^n(\mu) + O \left( \frac{\Lambda_{\text{QCD}}}{m_b} \right) \right],$$

(2)

where $r_n$ are strong interaction constant factors and $| 0 \rangle$ the vacuum state. For the leading order the factorization takes place with either weak quark currents $J_1$, $J_2$ or $J_3$, $J_4$. The radiative corrections can be evaluated to a given order $\alpha_s^n(\mu)$. The nonperturbative corrections to the heavy-quark limit $O \left( \frac{\Lambda_{\text{QCD}}}{m_b} \right)$ are less reliable for $D$ decays as $m_c \sim m_b/3$; therefore, even though the factorization is more phenomenological for charmed mesons, it can still represent a good starting point.

The amplitude $\langle M_1 | J^\nu_1 | B \rangle$ is a heavy-to-light transition form factor which can be evaluated within light-front and relativistic constituent quark models, light-cone sum rules, continuum functional QCD and lattice QCD (see Appendix A4 of Ref. [2]). Semi-leptonic decay measurements like $D^0 \rightarrow \pi^- e^+ \nu_e$ can also allow a phenomenological determination of these form factors.

The matrix element $\langle M_2^* | J_{2\nu} | 0 \rangle \propto \langle M_3 M_4 | J_{2\nu} | 0 \rangle$, where the $M_3 M_4$ resonance, $M_2^*$, originates from a $\bar{q}q$ pair, corresponds to the $M_3 M_4$ form factor. It has been shown, in Ref. [4], that, using dispersion relations and field theory, this form factor can be fully determined, if the $M_3 M_4$ strong interaction is known at all energies. These form factors are calculated from Muskhelishvili-Omnès equations [5] using two-body data, unitarity, asymptotic QCD and chiral symmetry constraints at low energies.

The term $\langle M_1 | J^\nu_0 | 0 \rangle$, related to the $M_1$ weak decay constant, is known from experiment, e.g. the pion decay constant, $f_\pi$ or that of the kaon, $f_K$. It can also be evaluated with lattice-regularized QCD and other nonperturbative approaches.

The matrix element $\langle M_2^* | J_{3\nu} | B \rangle \propto \langle M_3 M_4 | J_{3\nu} | B \rangle$ corresponding to $B$ meson transitions to two-meson pairs via the $M_2^*$ resonance is the biggest uncertainty in our approach. It could be evaluated from semi-leptonic processes: like $B^0 \rightarrow K^+ \pi^- \mu^+ \mu^-$ or $D^0 \rightarrow K^- \pi^+ \mu^+ \mu^-$. In the derivation of the amplitude presented here it will be related to the $\langle M_2^* | \rightarrow M_3 M_4 | J_{2\nu} | 0 \rangle$ form factor. Within the soft-collinear effective theory, the amplitude can be factorized in terms of generalized $B$-to-two-body form factor and two-hadron light-cone distribution amplitude [6].
1.3 Application to the \(D^+ \rightarrow K^-\pi^+\pi^+\) decay

In this process, studied in Ref. [7], the final state \(\pi^+\pi^+\) interaction can be neglected and the quasi-two-body \([K^-\pi^+]_{S,P}\) \(\pi^+\) can be introduced. There is no penguin contribution (loop with \(W\) meson) and only the effective Wilson-coefficients \(a_{1(2)}\) appear in the quasi-two-body factorized amplitude,

\[
\langle [K^-\pi^+]_{S,P} \pi^+|\mathcal{H}_{\text{eff}}|D^+\rangle = \frac{G_F}{\sqrt{2}} \cos^2 \theta_C \left[ a_1 \langle [K^-\pi^+]_{S,P} |\bar{\psi}\gamma^5(1 - \gamma_5)c|D^+\rangle \langle \pi^+_2 | \bar{u}\gamma_5(u - \gamma_5)c|D^+\rangle \right] + a_2 \langle [K^-\pi^+]_{S,P} |\bar{\psi}\gamma^5(1 - \gamma_5)d|0\rangle \langle \pi^+_2 | \bar{u}\gamma_5(u - \gamma_5)c|D^+\rangle \right] + (\pi^+_1 \leftrightarrow \pi^+_2),
\]

(3)

\(\theta_C\) being the Cabibbo angle. The matrix element \(\langle [K^-\pi^+]_{S,P} |\bar{\psi}\gamma^5(1 - \gamma_5)d|0\rangle\) is given by the \(K\pi\) form factors. The term \(\langle [K^-\pi^+]_{S,P} |\bar{\psi}\gamma^5(1 - \gamma_5)c|D^+\rangle\) is less straightforward to evaluate. Assuming a dominant intermediate resonance \(R\), it can be written as being proportional to the \(D\) to \(R [R \rightarrow K\pi]\) transition form factor multiplied by the \(K\pi\) form factors. This description is a feature of crucial importance to our proposed parametrizations. In Eq. (3), \(\langle \pi^+_1(p) |\bar{u}\gamma_5(u - \gamma_5)c|D^+\rangle\) is the \(D\pi\) transition form factor.

Parametrized amplitudes based on quasi-two-body factorization have been given in Ref. [2] in terms of analytic and unitary meson-meson form factors for final states composed of three light mesons, namely the various charge \(\pi\pi\pi\), \(K\pi\pi\) and \(KK\bar{K}\) states. For these hadronic three-body decays we have shown, in previous studies, that this approach is phenomenologically successful. Below, we illustrate these parametrizations for the \(B \rightarrow K\pi^+\pi^-\) [8–10] and \(D^0 \rightarrow K_S^0\pi^+\pi^-\) [11] for meson-meson final states in \(S\) wave. Formulae for meson-meson final states in \(P\) wave are given in Ref. [2].

2 Parametrized amplitudes for the \(B \rightarrow K\pi^+\pi^-\) decays

2.1 Parametrization of the \(B \rightarrow K[\pi^+\pi^-]_S\) amplitudes

Let us label the momenta as \(B(p_B) \rightarrow K(p_1)\pi^+(p_2)\pi^-(p_3)\) with \(s_{12} = (p_1 + p_2)^2\), \(s_{13} = (p_1 + p_3)^2\), \(s_{23} = (p_2 + p_3)^2\) and \(s_{12} + s_{13} + s_{23} = m_B^2 + m_K^2 + 2m_\pi^2\). As can be seen from Eq. (1) of Ref. [8] the \(B \rightarrow K[\pi^+\pi^-]_S\) amplitude can be parametrized in terms of three complex parameters, \(b_i^S, i = 1, 2, 3\), for the different charged states \(B = B^+, K = K^\pm\) and \(B = B^0(\bar{B}^0), K = K^0(\bar{K}^0)\) or \(K_S^0\). For the \(B^-\) decays one has

\[
\mathcal{A}_S(s_{23}) \equiv \langle K^- [\pi^+\pi^-]_S |\mathcal{H}_{\text{eff}}|B^-\rangle = b_1^S \left( M_B^2 - s_{23} \right) F_{00}^{\pi\pi}(s_{23}) + \left( b_2^S F_0^{BK}(s_{23}) + b_3^S \right) F_{0\pi}^{\pi\pi}(s_{23}),
\]

(4)

where \(F_{00}^{BK}(s)\) is the \(B\) to \(K\) transition form factor (see Refs. [2, 8]). The non-strange scalar form factor \(F_{0\pi}^{\pi\pi}(s)\) contains the contributions of \(f_0(500), f_0(980)\) and \(f_0(1400)\). Several models are compared in Fig. 8 of Ref. [12]. Although there are large differences, it has been checked by the authors that, with the fitted form factor to obtain the lowest \(\chi^2\) for \(D^0 \rightarrow K_S^0\pi^+\pi^-\), the main conclusions achieved for the \(B^+ \rightarrow \pi^+\pi^-\pi^+\pi^-\) in Ref. [13] were unchanged (see Ref. [12] for explanations). The modulus of the Moussallam pion scalar form factor [14], calculated by solving the Muskhelishvili-Omnès equation [5], is close to that of the form factor obtained in Ref. [12], notably below 1 GeV. A plot of the strange scalar form factor \(F_{0\pi}^{\pi\pi}(s)\), which receives the contribution of the \(f_0(980)\) and \(f_0(1400)\), can be found in Fig. 6 of Ref. [15]. It has been calculated using the Muskhelishvili-Omnès approach.
In terms of the original amplitude [8] one has\(^1\), \(F_{0}^{B\rightarrow f_{0}(980)}(m_{K}^{2})\) being the \(B\) to \(f_{0}(980)\) transition form factor evaluated at \(m_{K}^{2}\) [2],

\[
b_{1}^{S} = \frac{G_{F}}{\sqrt{2}} \left[ \chi f_{K} F_{0}^{B\rightarrow f_{0}(980)}(m_{K}^{2}) U - \tilde{C} \right],
\]

where \(\tilde{C} = f_{\pi} F_{\pi} \left( \lambda_{u} P_{1}^{GIM} + \lambda_{t} P_{1}^{GIM} \right)\), \(\lambda_{u} = V_{ub} V_{us}^{\ast}\), \(\lambda_{t} = V_{tb} V_{ts}^{\ast}\), \(F_{\pi}\) is the \(B\pi\) form factor at \(m_{\pi}^{2} = 0\), \(P_{1}^{GIM}\), \(P_{1}\) complex charm ing penguin parameters and \(U\) is a short-distance contribution given in terms of CKM matrix element multiplied by effective Wilson coefficients. The fitted parameter \(\chi\) represents the strength of the non-strange pion form factor contribution, furthermore. Its value can be estimated from the \(f_{0}(980)\) decay properties [8]. A summary of the models for the scalar-isoscalar pion form factor can be found in Appendix A4 of Ref. [2] and, as just noted above, see also the recent determination of Ref. [15] and the review talk [16].

### 2.2 Parametrization of the \(B \rightarrow [K\pi^{\pm}]_{S}\pi^{\mp}\) amplitudes

In terms of the two complex parameters \(c_{1}^{S}, c_{2}^{S}\) (see Eq. (68) of Ref. [10]) one has

\[
\mathcal{A}_{S}(s_{12}) \equiv \langle \pi^{-} [K^{-}\pi^{+}]_{S} | \mathcal{H}_{\text{eff}} | B^{-} \rangle = (c_{1}^{S} + c_{2}^{S} s_{12}) \frac{F_{0}^{B\pi}(s_{12}) F_{0}^{K\pi}(s_{12})}{s_{12}},
\]

where \(F_{0}^{K\pi}(s)\) (contribution of \(K_{0}^{p}(800)\) or \(\kappa\) and of \(K_{0}^{s}(1430)\), see e.g. Fig. 7 of Ref. [12]) and \(F_{0}^{B\pi}(s)\) are the \(K\pi\) and \(B\pi\) scalar form factors, respectively. This parametrization has been used with success in the amplitude analysis [17] of the Dalitz-plot distribution of the LHCb \(B \rightarrow K_{S}^{0}\pi^{+}\pi^{-}\) data. One has [10]

\[
c_{1}^{S} = \frac{G_{F}}{\sqrt{2}} (M_{B}^{2} - m_{\pi}^{2})(m_{K}^{2} - m_{\pi}^{2}) \left[ \lambda_{u} \left( a_{4}^{u}(S) - \frac{a_{10}^{u}(S)}{2} + c_{4}^{u} \right) + \lambda_{c} \left( a_{4}^{c}(S) - \frac{a_{10}^{c}(S)}{2} + c_{4}^{c} \right) \right],
\]

where \(\lambda_{c} = V_{cb} V_{cs}\). The \(a_{i}^{(u,c)}(S), i = 4, 10\) are the leading order effective Wilson coefficients including vertex and penguin corrections. The \(c_{i}^{(u,c)}\) are free fitted parameters simulating non-perturbative and higher order contributions to the penguin diagrams. Models for the \(F_{0}^{K\pi}(s)\) form factor are described in Ref. [2], see also some complementary aspects in Ref. [16].

### 3 \(D^{0} \rightarrow K_{S}^{0}[K^{+}K^{-}]_{S}\) and \(D^{0} \rightarrow [K_{S}^{0}K^{\pm}]_{S}K^{\mp}\) parametrized amplitudes

The \([K^{+}K^{-}]\) pairs can have isospin 0 or 1 but the \([K_{S}^{0}K^{\pm}]\) ones have isospin 1. The \(f_{0}(980), f_{0}(1400), a_{0}(980)\) and \(a_{0}(1450)\) contribute to the following parametrized amplitude

\[
\mathcal{A}_{S,0}(s_{23}) = h_{4}^{S} (m_{B}^{2} - s_{23}) F_{0}^{KK}(s_{23}) + h_{5}^{S} (m_{K}^{2} - s_{23}) F_{0}^{K0}(s_{23}) + h_{6}^{S} (m_{B}^{2} - s_{23}) G_{0}^{KK}(s_{23}),
\]

where \(s_{23}\) is the energy squared of the \(K^{+}K^{-}\) pair while \(s_{12}\) is associated to the \(K_{S}^{0}K^{-}\) pair and \(s_{13}\) to the \(K_{S}^{0}K^{+}\) one. The decay amplitude associated with the \(a_{0}(980)^{-}\) and \(a_{0}(1450)^{-}\) resonances can be parametrized as:

\[
\mathcal{A}_{S,-}(s_{12}) = (h_{4}^{S} + h_{5}^{S} s_{12}) G_{0}^{KK}(s_{12}).
\]

The amplitude carrying contributions from \(a_{0}(980)^{+}\) and \(a_{0}(1450)^{+}\) reads

\(^{1}\)The interested reader will find, in Appendix B of Ref. [2], the corresponding relations for the other parameters.
Models for the $F^{KK}_{0(s),(0)}(s)$ form factors entering Eq. (8) have been derived in Ref. [18, 19] (see their Figs. 1) solving three coupled channels viz. $\pi\pi, K\bar{K}$ and $4\pi$ (effective $2\pi-2\pi$ or $\sigma\sigma$ or $\rho\rho$ ...) and imposing chiral symmetry constraints. The $F^{KK}_{0(s),(0)}(s)$ form factor has also been calculated in a dispersive approach in Ref. [15] (see their Fig. 7).

In Eqs. (9) and (10), the scalar-isovector $G^{KK}_{0(0)}(s)$ form factor, built in Ref. [20] from a unitary $S$-wave coupled channel $(\eta\pi, K\bar{K})$ model, is plotted in their Fig. 7. This model, derived from the Muskheilishvili-Omnès equation [5], imposes the presence of the $a_0(980)$ and $a_0(1450)$ and includes asymptotic QCD and chiral symmetry constraints. Models for the transition form factor $F^{DK}_{0}(s)$ in Eq. (10) can be found in Ref. [2]. The above complex $h_i^S$ coefficients are given in terms of the original amplitudes in Appendix B of Ref. [2].

4 Concluding remarks

Alternatives to isobar Dalitz-plot model for weak $D, B$ decays into various $\pi\pi\pi, K\pi\pi$ and $KK\bar{K}$ charge states have been presented in Ref. [2]. Let us recall that isobar parametrizations do not respect unitarity and extraction of strong CP phases should be taken with caution. Furthermore $S$-wave resonance contributions are hard to fit.

Our parametrizations, although not fully three-body unitary, are based on a sound theoretical application of QCD factorization to a hadronic quasi-two-body decay. They assume that final three-meson state are preceded by intermediate resonant states which is justified by phenomenological and experimental evidence. Analyticity, unitarity, chiral symmetry plus correct asymptotic behavior of the two-meson scattering amplitude in $S$ and $P$ waves are implemented via analytical and unitary $S$- and $P$-wave $\pi\pi, \pi K$ and $K\bar{K}$ form factors entering in hadronic final states of our amplitude parametrizations.

These parametrized amplitudes can be readily used adjusting parameters in a least-square fit to the Dalitz plot for a given decay channel and employing tabulated form factors as functions of momentum squared or energy. The reproduction of the Dalitz-plot data might require some adjustment of the meson-meson form factors. The addition of phenomenological amplitudes (contributions of higher interacting waves, in particular $D$ waves or $J=2$ resonances), and possible three-body rescattering effects may be necessary.

We have exemplified here expressions for the $B \to K\pi^+\pi^-$ [8–10] and $D^0 \to K^0_S K^+ K^-$ [11] for meson-meson final states in $S$ wave. In Ref. [2] one can find other explicit amplitude expressions for meson-meson final states in $S$ and $P$ wave for $B^\pm \to \pi^+\pi^-\pi^\pm$, $B \to K \pi^+\pi^-$, $B^\pm \to K^+K^- K^\pm$, $D^+ \to \pi^-\pi^+\pi^+$, $D^+ \to K^-\pi^+\pi^+$, $D^0 \to K^0_S \pi^+\pi^-$. Previous studies have shown that this approach is successful. In addition, expressions for $D^0 \to K^0_S K^+ K^-$ are also given in Ref. [2]. We have derived preliminary parametrized amplitudes for the $B^\pm \to K^+K^-\pi^\pm$ decays [1, 21] and for the $B^0 \to K^0_S K^+ K^-$ process presently analyzed by the LHCb collaboration.

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