Resource Allocation and Outage Analysis for An Adaptive Cognitive Two-Way Relay Network

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Abstract—In this paper, an adaptive two-way relay cooperation scheme is studied for multiple-relay cognitive radio networks to improve the performance of secondary transmissions. The power allocation and relay selection schemes are derived to minimize the secondary outage probability where only statistical channel information is needed. Exact closed-form expressions for secondary outage probability are derived under a constraint on the quality of service of primary transmissions in terms of the required primary outage probability. To better understand the impact of primary user interference on secondary transmissions, we further investigate the asymptotic behaviors of the secondary relay network including power allocation and outage probability, when the primary signal-to-noise ratio goes to infinity. Simulation results are provided to illustrate the performance of the proposed schemes.

Index Terms—Two-way relay, cognitive radio networks, outage probability, power allocation, relay selection.

I. INTRODUCTION

COGNITIVE radio techniques enable secondary users (SUs) to access the frequency bands originally licensed to primary users (PUs) while ensuring that the quality of service (QoS) of primary transmissions is not affected, which can improve spectral efficiency significantly [1]. However, the SUs often operate with constrained transmit power to guarantee the QoS of PUs in terms of interference temperature, thus limiting the throughput and coverage of the secondary system. To combat this problem, cooperative diversity systems involving scattering relay networks have recently been researched to exploit the spatial diversity gain and to enhance the secondary channel performance [2], [3]. It has also been shown that cooperative diversity with relay selection can achieve the same diversity-multiplexing tradeoff as achieved by the traditional cooperation protocols where all relays are involved in forwarding the signals from source nodes [4], [5].

The conventional one-way relay scheme suffers from a loss in spectral efficiency because of half-duplex transmission [6]. To circumvent this disadvantage, a two-way relay system was proposed in [7]. A two-way relay system has two transmission phases. During the first phase, two secondary transceivers (STs) simultaneously broadcast their signals. After successfully receiving the combined signals, the relay node forwards the signals to the two STs during the second phase. Since there are two different relaying paths, the total spectral efficiency of a two-way relay system can be doubled compared with a conventional one-way relay system. Two protocols for two-way relay networks, commonly known as decode-and-forward (DF) and amplify-and-forward (AF) relaying, were proposed in [7]. Based on these, several cooperative diversity schemes for two-way relay networks with relay selection have been proposed [8], [9], [10], [11]. Note that all the aforementioned works studied non-cognitive radio networks. However, in practical cognitive radio systems, PUs and SUs can simultaneously transmit signals by sharing the same spectrum resources. As a result, the relays and secondary receivers inevitably suffer interference from PUs. From the viewpoint of SUs, these interferences come in the form of co-channel interference (CCI) and it is important to analyze their effect on system performance.

A. Related Work

So far, the literature that studies outage performance and resource allocation in cognitive two-way relaying networks with CCI is relatively scarce. Interference was considered only during the second transmission phase in [12], where exact outage probability was obtained while ignoring the noise at the receivers. In [13], the exact outage probability was derived under a cognitive two-way relay network setting. However, the system outage event was defined as having either one of the two STs in outage, which simplifies the derivation but does not represent system outage correctly. In [14], a max-min strategy over instantaneous achievable channel rates was employed to address relay selection and power allocation for cognitive two-way AF relaying networks. The CCI from the PUs was modeled as Gaussian noise, which does not characterize the practical cognitive radio communication appropriately. Relay selection and power allocation schemes in cognitive two-way DF relaying network were studied for the first time to maximize the achieved sum rate in [15]. However, the CCI was considered at primary nodes whereas the interference resulting from primary transmission in secondary receivers was not considered. In [16], the power allocation problem in the cognitive two-way relay network with amplify-and-forward strategy was studied and the secondary sum rate was maximized whereas the optimization problem dealt with the terminal side without any control on relay parameters. Instantaneous secrecy rate was maximized in [17] for relay selection, which is the same as maximizing signal-to-noise ratio (SNR). Besides the inappropriate system modeling of CCI, the resource allocation schemes in the aforementioned works lead to the maximization of the instantaneous SNR. These resulting resource allocation schemes require perfect knowledge of instantaneous channel.
state information (CSI) between the nodes in the cognitive network. In fact, it is highly computationally complex and also sometimes impossible to accurately learn the knowledge of instantaneous CSI in the network. Moreover, in the cognitive radio network setting, the knowledge of instantaneous CSI for the primary interference transmitted from the primary network to the secondary network is required if those schemes are to be implemented, which is extremely difficult if there are no pilot symbols specifically designed for the secondary nodes in the primary signal. Therefore, optimal power allocation for outage probability minimization comes into consideration in such a scenario, which only requires the knowledge of statistical CSI [18], [19], [20].

B. Main Contributions

In this paper, we investigate an adaptive cooperative diversity scheme in cognitive two-way relay networks using the DF protocol, where mutual interference between PUs and SUs is considered. The STs broadcast their signals to the relays and to each other through the direct link during the first phase. During the second phase, if the relays can decode the signals received during the first phase, the best relay is chosen to forward the signals to the STs; otherwise, the STs adaptively repeat the same transmission to each other through the direct link as during the first phase. Then, the STs combine the two copies of the received signals after the two transmission phases.

The main contributions of this paper are as follows:

1) We explore the adaptive use of the direct link and the relay link to achieve higher system performance in cognitive DF two-way relaying networks. Our analysis can also be used for the scenario where only relay link is available.

2) For the first time, a power allocation scheme for STs and the relays is developed that minimizes the secondary outage probability under a QoS constraint from the primary network, requiring no instantaneous CSI of the transmission links.

3) The optimal relay selection approach for this two-way system is also provided to minimize outage probability, which requires only statistical CSI. To the best of our knowledge, this is the first work to study the resource allocation problem using statistical CSI information for the proposed general framework. An exact closed-form expression for the secondary outage probability is also derived in this paper. Asymptotic behavior of the secondary system is analyzed given that the primary user SNR goes to infinity.

The rest of the paper is organized as follows: we give the system model in Section II. Section III provides the outage analysis of the relaying network. Based on the outage analysis, we address the power allocation and relay selection problems in Section IV. We analyze the asymptotic behavior of the system in Section V. Numerical simulation results and conclusion are given in Section VI and Section VII.

II. PROPOSED ADAPTIVE COOPERATION SCHEME

A. System Model

Consider a general spectrum-sharing cognitive two-way relaying network as shown in Fig. 1. In the primary network, a primary transmitter $u$ sends data to a primary destination $v$. Meanwhile, in the secondary relay network, STs $s$ and $d$ exchange information with each other. Secondary relays $r_i, i = 1, 2, 3, \ldots, M$, are available to assist secondary data transmissions using the DF protocol. We assume that the channel link from $k$ to $j$ ($k, j \in \{u, v, s, r_i, d\}$) undergoes Rayleigh fading with instantaneous coefficient $h_{k,j}$. Therefore, the channel gain $|h_{k,j}|^2$ is exponentially distributed with mean $\frac{1}{\sigma_{k,j}^2}$. We also assume reciprocity of all the channels and zero-mean additive white Gaussian noise (AWGN) with variance $N_0$ at each receiver.

During the first phase, STs $s$ and $d$ simultaneously broadcast their signals to the relay $r_i$ and to the corresponding receiver, i.e., $s \rightarrow r_i \leftarrow d$, $s \leftrightarrow d$. By employing multiple antennas and self-interference cancelation (SIC), the STs can send and receive at the same time [21]. Thus, considering coexistence of primary transmission, the received signal at the primary receiver can be expressed as

$$y_u = \sqrt{P_u} h_{u,u} x_u + \sqrt{P_s} h_{s,u} x_s + \sqrt{P_d} h_{d,u} x_d + n_u, \quad (1)$$

where $P_u$, $P_s$, and $P_d$ are the transmit powers of $u$, $s$ and $d$ respectively, $x_u$, $x_s$, and $x_d$ denote the unit-mean-energy symbols transmitted respectively by $u$, $s$, and $d$, and $n_u$ is the AWGN. The QoS of primary transmissions is quantified by the outage probability in this paper. The primary QoS guarantee is represented by the inequality that the outage probability of primary transmission $P_{uv}$ does not exceed a predefined outage probability threshold $P_{th}$, which is expressed as

$$P_{uv} = P \left( \log_2 \left( 1 + \frac{\frac{P_u}{\sigma_{u,u}^2} |h_{u,u}|^2}{\frac{P_s}{\sigma_{s,u}^2} |h_{s,u}|^2 + \frac{P_d}{\sigma_{d,u}^2} |h_{d,u}|^2 + N_0} \right) < R_u \right)$$

$$= P \left( \frac{\frac{P_s}{\sigma_{s,u}^2} |h_{s,u}|^2}{\frac{P_s}{\sigma_{s,u}^2} |h_{s,u}|^2 + \frac{P_d}{\sigma_{d,u}^2} |h_{d,u}|^2 + N_0} < \Delta_u \right) \leq P_{th}, \quad (2)$$

where $\Delta_u = 2^{R_u} - 1$ with $R_u$ being the primary data rate. We calculate $P_{uv}$ and write the primary QoS guarantee during
the direct transmission between STs is needed or not. If it is not, the decode the received signals constitute a set of all the relays and those relays that are able to successfully and SIC and the two STs. Finally, STs combine the two copies of the received signals using SIC and MRC methods. Therefore, the respective SINR is given as

\[
SINR_s(D = \emptyset) = \frac{2 \| h_{d,s} \|^2}{\| h_{u,s} \|^2 + N_0}, \quad (6)
\]

\[
SINR_d(D = \emptyset) = \frac{2 \| h_{d,d} \|^2}{\| h_{u,d} \|^2 + N_0}. \quad (7)
\]

Otherwise, if \( D \) is not empty, where \( D = D_S \), the relay \( r_i \) chosen within \( D_S \) will transmit its decoded data stream to the two STs. Finally, STs combine the two copies of the received signals using SIC and MRC methods. Therefore, the respective SINR is given as

\[
SINR_s = \frac{P_d | h_{d,s} |^2}{\| h_{u,s} \|^2 + N_0} + \beta_s P_r | h_{r_i,s} |^2, \quad (8)
\]

\[
SINR_d = \frac{P_s | h_{d,d} |^2}{\| h_{u,d} \|^2 + N_0} + \alpha_d P_r | h_{r_i,d} |^2. \quad (9)
\]

where \( P_r \) is the transmit power of \( r_i \), and \( \alpha_d \) and \( \beta_s \) are the ratios of total transmit power at \( r_i \) for the transmission of original signals from \( s \) and \( d \) to \( d \) and \( s \), respectively.

### III. OUTAGE PERFORMANCE ANALYSIS

In this section, we give the analysis of the outage probability of the proposed adaptive relay cooperation scheme. The exact results of secondary outage probability are derived. Based on the results, we shall provide the resource allocation schemes.

We first study the outage in the relay nodes as defined in (10). According to the achievable rate region as discussed in [22], [12], the event of each \( r_i \) failing to decode the received signals and resulting in outage is denoted as \( O(r_i) \) and can be expressed as

\[
O(r_i) = \{ \gamma_{s,r_i} + \gamma_{d,r_i} < \Delta \text{ or } \gamma_{s,r_i} < \Delta_s \text{ or } \gamma_{d,r_i} < \Delta_d \}. \quad (10)
\]

where \( \Delta = 2(\Delta_s + \Delta_d) - 1 \), \( \Delta_s = 2(\Delta_s - 1) \), \( \Delta_d = 2(\Delta_d - 1) \) with \( R_s \) and \( R_d \) being the data rates at STs \( s \) and \( d \), respectively, \( \gamma_{s,r_i} = \gamma_{s} | h_{s,r_i} |^2 / (\gamma_{u,s}| h_{u,r_i} |^2 + 1) \) and \( \gamma_{d,r_i} = \gamma_{d} | h_{d,r_i} |^2 / (\gamma_{u,d}| h_{u,r_i} |^2 + 1) \) are correlated and represent the signal-to-interference-plus-noise ratio (SINR) at \( r_i \) with respect to signals from \( s \) and \( d \) respectively.

**Proposition 1:** The outage probability of each relay \( r_i \) is given as

\[
P(O(r_i)) = \left\{ 1 - \frac{T \exp \left( \frac{-\gamma_{s,r_i}}{\sigma_{s,r_i}^2} \right)}{\gamma_{s,r_i} \sigma_{s,r_i}^2 - 1} \right. \left[ \frac{1 + \Delta_s \Delta_d (1 + T)}{\gamma_{s,r_i} \sigma_{s,r_i}^2 + 1} \right], \quad \text{if } \gamma_{s,r_i} \sigma_{s,r_i}^2 > \Delta_s \Delta_d, \quad (11)
\]

\[
\frac{1 - C \exp(-A)}{1 - (1-C) \exp(-B)} - \frac{1 - C \exp(-A)}{1 - (1-C) \exp(-B)}, \quad \text{otherwise}
\]

where \( T = \left( \frac{1 - \Delta_s \Delta_d}{\gamma_{s,r_i} \sigma_{s,r_i}^2} \right)^{-1} \), \( A = \frac{\Delta_d - \Delta_s}{\gamma_{d,r_i} \sigma_{d,r_i}^2}, \) and \( B = \frac{T - \Delta_d}{\gamma_{d,r_i} \sigma_{d,r_i}^2} + \frac{\Delta_s}{\gamma_{s,r_i} \sigma_{s,r_i}^2} \) and \( C = \frac{\gamma_{s,r_i} \sigma_{s,r_i}^2}{\gamma_{d,r_i} \sigma_{d,r_i}^2} \).

**Proof:** See Appendix A.

As we can see from the proposition, the outage probability \( P(O(r_i)) \) is dependent on the transmit powers of the networks, data rates, and the statistical conditions of the channels linked to the relay \( r_i \). Note that only the relays that are not in outage can be chosen to forward the signals to the STs. Depending on the channel coefficient \( \sigma_{s,r_i}^2 \), the outage probability \( P(O(r_i)) \) takes different forms of expressions. Thus, further analysis in this paper that is based on \( P(O(r_i)) \) is conducted in a case-by-case fashion.

Now we study the outage behavior of secondary system under the condition that the relay node \( r_i \) is chosen and...
Similarly, the occurrence probability of the case and the outage probability of the secondary network given this
where \( SNRs \) and \( SINR_d \) are defined in (8) and (9), respectively. The STs are in outage if they cannot receive and decode the received signal. Let \( O(ST|r_i) \) denote the corresponding outage event.

\[
P(O(ST|r_i)) = P(SINRs < \Delta_d \text{ or } SINR_d < \Delta_s),
\]
where \( SINRs \) and \( SINR_d \) are defined in (8) and (9), respectively. The STs are in outage if they cannot receive and decode the signals as implied by (12).

**Proposition 2:** In the high SNR regime, i.e., when \( N_0 \rightarrow 0 \), the probability that the STs are in outage is given as

\[
P(O(ST|r_i)) = A + B - A B,
\]

where \( A = \frac{1}{1 + \frac{\Delta_d\sigma^2_{d,s}}{\Delta_s\sigma^2_{s,d}}} \), and \( B = \frac{1}{1 + \frac{\Delta_d\sigma^2_{d,s}}{\Delta_s\sigma^2_{s,d}}} \).

**Proof:** See Appendix B.

The probability \( P(O(ST|r_i)) \) characterizes the outage property of the secondary system when the relay node \( r_i \) is chosen to forward the signals to the STs. From the expressions of \( A \) and \( B \), we observe that the choice of relay has an influence on the secondary system outage performance by the following means: the forward power ratios \( \alpha_i \) and \( \beta_i \), transmit power of the relay \( P_{r_i} \), and the channel conditions of the links between the relay and the transceivers \( \sigma^2_{r,s} \) and \( \sigma^2_{r,d} \).

Here, we provide the exact probability that the secondary system is in outage.

With (11), the probability of the case \( D = \emptyset \) can be simply given as

\[
P(D = \emptyset) = \prod_{i=1}^{M} P(O(r_i)),
\]
and the outage probability of the secondary network given this case is expressed as

\[
P(\text{out} | D = \emptyset) = P \left( \frac{2\gamma_d|h_{d,s}|^s}{\gamma_u|h_{u,s}|^s + 1} < \Delta_d \text{ or } \frac{2\gamma_s|h_{u,d}|^s}{\gamma_u|h_{u,d}|^s + 1} < \Delta_s \right)
\]=\[
1 - \frac{2\gamma_d\sigma^2_{d,s}}{\Delta_d\gamma_u\sigma^2_{u,s} + \Delta_d\gamma_u\sigma^2_{u,s}} \exp \left(\frac{-\Delta_d}{\gamma_u\sigma^2_{u,s}}\right) - \frac{2\gamma_s\sigma^2_{s,d}}{\Delta_s\gamma_u\sigma^2_{u,s} + \Delta_s\gamma_u\sigma^2_{u,s}} \exp \left(\frac{-\Delta_s}{\gamma_u\sigma^2_{u,s}}\right).
\]

Similarly, the occurrence probability of the case \( D = D_s \) is

\[
P(D = D_S) = \prod_{r_i \in D_S} [1 - P(O(r_i))] \prod_{r_i \in D_S} P(O(r_i)),
\]
where \( D_S = D_M - D_S \) is the complementary set to \( D_S \). Based on (29), we can also derive the conditional secondary outage probability in this case as

\[
P(\text{out} | D = D_S)
\]=\[
\int \int P(\text{out} | D = D_S, X, Y) f(X) f(Y) dX dY,
\]

with \( P(\text{out} | D = D_S, X, Y) = \prod_{r_i \in D_S} (1 - \Phi) \) where \( \Phi = [1 - P(\beta_i\gamma_r|h_{r,s}|^2 < X)][1 - P(\alpha_i\gamma_r|h_{r,d}|^2 < Y)] \) and \( X = \Delta_d\gamma_u|h_{u,s}|^s + \Delta_d - \Delta_s|h_{u,d}|^s \), \( Y = \Delta_s\gamma_u|h_{u,d}|^s + \Delta_s - \Delta_u|h_{u,s}|^s \). The PDFs \( f(X) \) and \( f(Y) \) are given in Appendix C. Then, taking account of various integral intervals and binomial expansion, we derive the expression of \( P(\text{out} | D = D_S, X, Y) \) as in (18).

Substituting (18) into (17), we have

\[
P(\text{out} | D = D_S) = 1
\]+\[
\sum_{D_S \in D_M} \left( (-1)^E \left( \Omega \Xi + \Omega \Psi + \Xi \Lambda - \Lambda \Psi \right) \right)
\]

\[
(\Delta_d\gamma_u\sigma^2_{u,s} + \gamma_d\sigma^2_{d,s}) \left( \Delta_s\gamma_u\sigma^2_{u,d} + \gamma_s\sigma^2_{s,d} \right),
\]

where

\[
\Omega = \sum_{r_i \in D_C} \frac{1}{\alpha_i\gamma_r\sigma^2_{r,d} + \gamma_s\sigma^2_{s,d}} \exp \left(\frac{-\Delta_d}{\alpha_i\gamma_r\sigma^2_{r,d} + \gamma_s\sigma^2_{s,d}}\right)
\]

\[
+ \sum_{r_i \in D_C} \frac{1}{\alpha_i\gamma_r\sigma^2_{r,d} + \gamma_s\sigma^2_{s,d}} \exp \left(\frac{-\Delta_s}{\alpha_i\gamma_r\sigma^2_{r,d} + \gamma_s\sigma^2_{s,d}}\right),
\]

\[
\Xi = \sum_{r_i \in D_C} \frac{1}{\alpha_i\gamma_r\sigma^2_{r,d} + \gamma_s\sigma^2_{s,d}} \exp \left(\frac{-\Delta_d}{\alpha_i\gamma_r\sigma^2_{r,d} + \gamma_s\sigma^2_{s,d}}\right)
\]

\[
+ \sum_{r_i \in D_C} \frac{1}{\alpha_i\gamma_r\sigma^2_{r,d} + \gamma_s\sigma^2_{s,d}} \exp \left(\frac{-\Delta_s}{\alpha_i\gamma_r\sigma^2_{r,d} + \gamma_s\sigma^2_{s,d}}\right),
\]

\[
\Lambda = \gamma_d\sigma^2_{d,s} \exp \left(\frac{-\Delta_d}{\gamma_d\sigma^2_{d,s}}\right), \quad \Psi = \gamma_s\sigma^2_{s,d} \exp \left(\frac{-\Delta_s}{\gamma_s\sigma^2_{s,d}}\right).
\]

Finally, we derive the outage probability of the secondary two-way relay network as

\[
P_{out} = P(\text{out} | D = \emptyset) P(D = \emptyset)
\]+\[
\sum_{D_S \in D_M} P(\text{out} | D = D_S) P(D = D_S),
\]

where \( P(D = \emptyset) \), \( P(\text{out} | D = \emptyset) \), \( P(D = D_S) \) and \( P(\text{out} | D = D_S) \) are given in (14), (15), (16) and (19), respectively.

**IV. POWER ALLOCATION AND RELAY SELECTION**

In the following, we optimize the outage performance of the secondary receivers in the relay network to address the problems of power allocation and relay selection. In the context of power allocation for the DF relaying network, we have to determine the powers of STs \( s \) and \( d \), represented by \( P_s \) and \( P_d \), power of the relay \( r_i \), represented by \( P_{r_i} \), and the
integrated power allocation strategy of

\[ P (\text{out}|D = D_S, X, Y) = \begin{cases} 0 \\ 1 + \sum_{D_C \in D_S} (-1)^E \exp \left( \sum_{r_i \in D_C} \frac{-Y}{\alpha_i \gamma_{r_i} \sigma_{r_i, d}} \right) \\ 1 + \sum_{D_C \in D_S} (-1)^E \exp \left( \sum_{r_i \in D_C} \frac{-X}{\beta_i \gamma_{r_i} \sigma_{r_i, s}} \right) \\ 1 + \sum_{D_C \in D_S} (-1)^E \exp \left[ \sum_{r_i \in D_C} \left( \frac{-X}{\beta_i \gamma_{r_i} \sigma_{r_i, s}} - \frac{Y}{\alpha_i \gamma_{r_i} \sigma_{r_i, d}} \right) \right] \end{cases} \]

where \( D_C \) is the non-empty subset of \( D_S \) with \( E \) elements.

Next, we determine the power allocation for the STs, i.e., \( P_s \) and \( P_d \). First, note that the quality of the direct link between the STs may be severely affected due to long distance. This also partially constitutes the reason to employ relays since the links between the STs and the relays are relatively of much higher quality as well as providing diversity. To effectively make use of the relay channel diversity to enhance system performance, we maximize the minimum probability that the link between the STs and a relay is connected, while \( P_s \) and \( P_d \) satisfy constraint \( C \), which can be expressed as

\[ \{P_s, P_d\} = \arg \max_{\text{subject to } C} \{1 - P(O(r_i))\} \]

\[ = \arg \max_{\text{subject to } C} \{1 - P(O(r_{\min}))\} \]

\[ = \arg \min_{\text{subject to } C} P(O(r_{\min})) \]

\[ \text{(24)} \]

where \( r_{\min} = \arg \min_{r_i \in D_s} \{1 - P(O(r_i))\} \).

Recalling (11), we give the optimal power allocation of \( \{P_s, P_d\} \) to minimize \( P(O(r_{\min})) \) while satisfying the constraint \( C \).

Let \( P(O(r_{\min}), P_s, P_d) \) represent the corresponding \( P(O(r_{\min})) \) with respect to \( \{P_s, P_d\} \). We provide the integrated power allocation strategy of \( \{P_s, P_d\} \) in the following lemma.

**Lemma 1:** The optimal power allocation \( \{P_s, P_d\} \) to minimize \( P(O(r_{\min})) \) is given by

\[ \{P_s, P_d\} = \arg \min_{\{P_s, P_d\}} \left\{ P(O(r_{\min}), P_s, P_d) \right\} \]

\[ \text{(25)} \]

where \( P' = \sqrt{\left( \frac{\sigma_{r_{\min}}^2}{2B \sigma_{r_{\min}}^2} \right)^2 + \frac{g \sigma_{r_{\min}}^2}{AB \sigma_{r_{\min}}^2} - \frac{\sigma_{r_{\min}}^2}{2B \sigma_{r_{\min}}^2}} \]

\[ \frac{1}{2A} \times P' = \frac{\sigma_{r_{\min}}^2}{\sigma_{r_{\min}}^2} \]

\[ \text{(26)} \]

\[ P' = \left( \frac{\sigma_{r_{\min}}^2}{2B \sigma_{r_{\min}}^2} \right)^2 + \frac{g \sigma_{r_{\min}}^2}{AB \sigma_{r_{\min}}^2} - \frac{\sigma_{r_{\min}}^2}{2B \sigma_{r_{\min}}^2} \]

\[ \frac{1}{2A} \times P' = \frac{\sigma_{r_{\min}}^2}{\sigma_{r_{\min}}^2} \]

\[ \text{(27)} \]

\[ g - 1 \]

\[ \sqrt{\frac{\Delta \sigma_{r_{\min}}^2 \cdot AB \sigma_{r_{\min}}^2 + B}{\Delta \sigma_{r_{\min}}^2 \cdot AB \sigma_{r_{\min}}^2 + A}} \]

\[ \text{with } A = \frac{\Delta \sigma_{r_{\min}}^2}{P_s \sigma_{r_{\min}}^2}, \text{ and } B = \frac{\Delta \sigma_{r_{\min}}^2}{P_d \sigma_{r_{\min}}^2}. \]

**Proof:** See Appendix 1.

Then, let us look at the allocation schemes at the relay, i.e., the power allocation for the relay node and relay selection. We give the power of the relay, i.e., \( P_{r_i} \) in the following lemma.

**Lemma 2:** The optimal transmit power of the chosen best relay \( P_{r_i} \) is given by

\[ P_{r_i} = \frac{P_o \sigma_{u,v}^2}{\Delta \sigma_{u,v}^2} (g - 1). \]

**Proof:** See Appendix 1.

During the second phase, the selected relay \( r_i \) forwards the combined data streams to the STs with power ratios \( \alpha_i \) and \( \beta_i \). Here, we address the relay selection and provide the power allocation for optimal \( \alpha_i \) and \( \beta_i \) to minimize overall secondary system outage probability. Note that when \( r_i \) is chosen for relaying, the secondary outage probability is \( P(O(ST|r_i)) \), which is hereby to be minimized.

**Lemma 3:** The optimal power ratios of \( \alpha_i \) and \( \beta_i \) are given by

\[ \alpha_i = \begin{cases} \frac{a b + c - 1}{a b - c d} & \text{if } a b = c d \\ \frac{b d + d - b}{2 b d} & \text{if } a b \neq c d \end{cases} \]

\[ \text{and} \]

\[ \beta_i = 1 - \alpha_i \]

\[ \text{(28)} \]

where \( a = 1 + \frac{P_o \sigma_{u,v}^2}{P_s \Delta \sigma_{u,v}^2}, b = \frac{P_r \sigma_{u,v}^2}{P_s \Delta \sigma_{u,v}^2}, c = 1 + \frac{P_r \sigma_{u,v}^2}{P_d \Delta \sigma_{u,v}^2}, d = \frac{P_r \sigma_{u,v}^2}{P_d \Delta \sigma_{u,v}^2}. \]

**Proof:** See Appendix 1.

Therefore, these three lemmas constitute the power allocation scheme including all the transmission powers of the secondary nodes. Note that the derived optimal \( \{P_s, P_d\}, P_{r_i} \) also apply in the case where there is no direct link between the STs.

We substitute the derived optimal \( \alpha_i \) and \( \beta_i \) back into \( P(O(ST|r_i)) \). The relay selection scheme selects \( r_i \in D_S \) such that the system outage probability \( P(O(ST|r_i)) \), given that \( r_i \) is selected, is minimized, which can be written as

\[ r_i = \arg \min_{r_i \in D_S} P(O(ST|r_i)). \]

\[ \text{(29)} \]

It indicates that the proposed relay selection criterion considers the statistical instead of instantaneous CSI of the primary and secondary networks. The benefit of this criterion is prominent since the instantaneous CSI of the networks is typically difficult to obtain. However, the statistical CSI of the primary and secondary networks are much easier for the relays to obtain. Thus, in the particular settings of cognitive
V. ASYMPTOTIC BEHAVIOR ANALYSIS

In a cognitive radio setting, the primary users are licensed to access the channel with QoS guarantee, and the power of primary transmitter is rather high, comparing to secondary transmit power and interference. In order to have a better understanding of the impact of primary interference on secondary network performance, we analyze the asymptotic behaviors of the derived power allocation and the secondary outage probability when the primary SNR $\gamma_u$ approaches infinity.

First, let us look at the asymptotic behavior of the power allocation. To make it compact and consistent, the power allocation scheme is expressed with respect to SNRs $\{\gamma_s, \gamma_d, \gamma_r\}$ as well.

When $\gamma_u \to \infty$, we have $g = \max \left\{ \frac{1}{1-\beta_u}, 1 \right\} = \frac{1}{1-\beta_u} \triangleq g'. \quad \text{Let } P(O(r_{\min})), \gamma_u, \gamma_d \quad \text{represent the corresponding asymptotic } P(O(r_{\min})) \text{with respect to the } \{\gamma_s, \gamma_d\}. \quad \text{We provide the integrated asymptotic power allocation strategy of } \{\gamma_s, \gamma_d\} \text{ in the following corollary.}

**Corollary 1:** The optimal power allocation $\{\gamma_s, \gamma_d\}$ when $\gamma_u \to \infty$ is given by

$$\{\gamma_s, \gamma_d\} = \arg \min_{\{\gamma_s', \gamma_d'\} \in \{\gamma_s, \gamma_d\}} \{P(O(r_{\min})), \gamma_s, \gamma_d\}$$

$$\triangleq \{\rho_s \gamma_u, \rho_d \gamma_u\}, \quad (30)$$

with $\rho_s, \rho_d \in \{(\rho_s', \rho_d'), (\rho_s', \rho_d')\}$, where $\gamma_s = \frac{2\Delta_u}{\Delta_u + \Delta_d - \Delta_u \gamma_u}$, $\gamma_d = \frac{\Delta_d}{\Delta_d + \Delta_u - \Delta_d \gamma_u}$, and $\gamma_s'' = \frac{\Delta_u}{\Delta_u + \Delta_d - \Delta_u \gamma_u}$.

Proof: See Appendix [C]

**Corollary 2:** The optimal power allocation for the relay $\gamma_r$, when $\gamma_u \to \infty$ is given by $\gamma_r = \frac{\Delta_u}{\Delta_u + \Delta_d - \Delta_u \gamma_u}$.

Proof: It is straightforward and follows from (26).

**Corollary 3:** The optimal power ratios of $\alpha_i$ and $\beta_i$ when $\gamma_u \to \infty$ are given by

$$\alpha_i = \frac{ab + d - b}{ab - cd} \quad \sqrt{\frac{bd + d - b}{ab - cd}} \quad \text{if } ab = cd \quad \text{or} \quad \beta_i = 1 - \alpha_i, \quad \text{where } a = 1 + \frac{\rho_s \sigma_{r_u}^2}{\Delta_u \sigma_{d_u}^2}, \quad b = \frac{\rho_d \sigma_{r_u}^2}{\Delta_u \sigma_{d_u}^2}, \quad c = 1 + \frac{\rho_s \sigma_{r_r}^2}{\Delta_u \sigma_{d_r}^2}, \quad d = \frac{\rho_d \sigma_{r_r}^2}{\Delta_u \sigma_{d_r}^2}, \quad \Delta_u = \frac{\Delta_u^2}{\Delta_u + \Delta_d - \Delta_u \gamma_u}, \quad \Delta_d = \frac{\Delta_u^2}{\Delta_u + \Delta_d - \Delta_d \gamma_u}, \quad \beta_i = 1 - \alpha_i.$$
Since the secondary outage probability is written as

\[ P_{\text{out}} = P(\text{out}|D = \emptyset) P(D = \emptyset) + \sum_{D_S \in D_M} P(\text{out}|D = D_S) P(D = D_S), \]

(36)

the asymptotic behavior of \( P_{\text{out}} \) when \( \gamma_u \to \infty \) is therefore derived.

Note that the asymptotic secondary outage probability when \( \gamma_u \to \infty \) is only associated with statistical channel coefficients whereas it is independent of \( \gamma_u \). Hence, if we characterize the exact secondary outage probability in terms of \( \gamma_u \), a horizontal performance floor is expected in the high \( \gamma_u \) regime. The underlying reason is that when \( \gamma_u \) is large, the secondary transmit SNRs can be expressed linearly of \( \gamma_u \). Therefore, in the high \( \gamma_u \) regime, secondary signal-to-interference-plus-noise ratios are parameters independent of \( \gamma_u \).

VI. SIMULATION RESULTS

In this section, we provide simulation results to validate the analysis and to show the improvement brought by the proposed cooperative diversity scheme. Referring to the system model in Fig. 1, the simulation setup is: data rate \( R_u = 0.6 \) bits/s/Hz, \( R_d = 0.3 \) bits/s/Hz, \( R_s = 0.2 \) bits/s/Hz, channel coefficients \( \sigma_{u,v}^2 = \sigma_{s,d}^2 = \sigma_{s,r_i}^2 = 5 \) dB, \( \sigma_{u,s}^2 = \sigma_{u,d}^2 = \sigma_{s,v}^2 = \sigma_{d,v}^2 = \sigma_{u,r_i}^2 = \sigma_{r_i,v}^2 = -5 \) dB. The case \( M = 0 \) indicates non-cooperation scheme.

Fig. 2. Secondary outage probability versus primary transmit SNR \( \gamma_u \).

First, we set \( P_{th} = 0.02 \) and demonstrate in Fig. 2 the secondary outage probability versus primary SNR \( \gamma_u \) of the proposed cooperation and non-cooperation schemes with uniform power allocation, i.e., \( P_s = P_d \) and \( \alpha_i = \beta_i \). It is observed that the proposed scheme outperforms the non-cooperation scheme with lower outage probability, which is also improved as the number of relays increases. We notice that the two schemes share the same cutoff value, and secondary transmission is forbidden when \( \gamma_u \) is smaller than the cutoff value because no extra interference is allowed in order to achieve the pre-defined primary QoS. A higher \( \gamma_u \) results in greater secondary transmit power, and then lower secondary outage probability. As is expected, we can also see a performance floor occurs in high \( \gamma_u \) regime, which is due to the fact that the interference from the primary transmitter dominates the secondary outage rather than noise in this case. This also validates the asymptotic outage probability analysis when \( \gamma_u \to \infty \).

Fig. 3. Secondary outage probability versus primary QoS constraint \( P_{th} \).

In Fig. 3 we present the secondary outage probability for different values of \( P_{th} \). When the QoS requirement of the primary system is too stringent, no secondary transmission is allowed. When the QoS requirement loosens, there begins the secondary transmission and the proposed adaptive cooperation diversity scheme achieves lower outage probability than the non-cooperation scheme. Higher \( P_{th} \) allows greater secondary transmit power and then the secondary outage probability is consequently reduced.

Next, we show the simulation results regarding power allocation. The simulation setup is: data rate \( R_u = 0.6 \) bits/s/Hz, \( R_d = 0.3 \) bits/s/Hz, \( R_s = 0.2 \) bits/s/Hz, channel coefficients \( \sigma_{u,v}^2 = \sigma_{s,r_i}^2 = 5 \) dB, \( \sigma_{d,r_i}^2 = 8 \) dB, \( \sigma_{s,d}^2 = 0 \) dB, \( \sigma_{u,s}^2 = \sigma_{s,v}^2 = \sigma_{d,v}^2 = \sigma_{u,r_i}^2 = \sigma_{r_i,v}^2 = -5 \) dB, \( \sigma_{u,d}^2 = -8 \) dB.

Fig. 4. Power allocation comparison with various noise intensity \( N_0 \).

In Fig. 4 we plot the power allocation comparison in different noise intensity regimes. Here, the exhaustive optima is obtained by multiple exhaustive search to achieve the minimum overall secondary outage probability, \( P_d \) and \( \alpha \) are given in the figure to conduct the comparison. First, we can see the allocated power values given by these two schemes are tightly matched, which indicates the significance of the proposed power allocation scheme. Second, the reason that the scenario \( N_0 > 3 \) dB is not given is due to the fact that secondary
transmission is switched off to prevent interference to primary users in low SNR regime with the given system parameters. In cognitive relaying networks with high noise intensity, it is highly possible that the secondary transmission is turned down to provide protection to the primary transmission. By this, the high SNR approximation in the derivation of optimal \( \{ P_s, P_d \} \) is reasonable.

Fig. 5. Secondary outage probability versus noise intensity \( N_0 \).

Fig. 5 shows the secondary outage probability versus noise intensity \( N_0 \) corresponding to number of relays \( M = 2 \) and \( M = 4 \), respectively. It is observed that the outage performance worsens as noise gets more intense. Also, as the number of relays increases, the outage performance improves. We notice that for all the cases considered, the secondary outage probability approaches 1. This is because in the high \( N_0 \) scenario, \( g = \max\{\exp \left( -\frac{\Delta}{\gamma_d \sigma_w^2} \right) / (1 - P_{th}), 1 \} \rightarrow 1 \). Then we will find that all the powers of the nodes in the secondary network approach 0. This means that the secondary network is transmitting data with extremely low power in a high noise environment and consequently the outage probability approaches 1. We also plot the performance of the secondary network with uniform-power scheme where \( P_x = P_d \) and \( \alpha_i = \beta_i \), and this uniform power allocation scheme is widely adopted in two-way relay network literatures. We can see that the proposed power allocation scheme clearly leads to performance improvement compared with the uniform-power scheme, even though the power of relay nodes is both maximized in these two schemes. Another interesting finding is that the proposed power allocation scheme can result in higher relative performance when the secondary network has more relay nodes. Since the proposed scheme is designed to optimize the outage performance and power allocation of \( P_x \) and \( P_d \) considers all the relay channels, more relay nodes enhance the possibility that the given \( P_x \) and \( P_d \) can result in lower outage probability. Although we employ high SNR with asymptotic analysis as a part of our power allocation scheme, the performance improvement can also be seen in the high noise level regime.

VII. CONCLUSION

In this paper, we proposed an adaptive cooperative diversity scheme with power allocation and relay selection in cognitive two-way relay networks. The QoS of the primary network is given by the primary outage probability, which is guaranteed during the transmissions between the secondary users. The closed form of secondary outage probability was derived using the decode-and-forward protocol. To better understand the impact of primary interference on the secondary transmissions, we also investigated the asymptotic behaviors of secondary network when the primary SNR goes to infinity, including power allocation and outage probability. We have presented various simulation results to show the validation of the proposed cooperation scheme.

The model and the analysis in this paper also suggest potential topics for future research, which include the resource allocation problem in multi-pair cognitive two-way relay networks with imperfect CSI, multiple and paired secondary transceivers, and so on.

VIII. ACKNOWLEDGEMENT

We thank Dr. Thakshila Wimalajeewa for helpful discussions and insightful comments on the paper.

APPENDIX A

We rewrite (10) using Bayes rule as

\[
P(O(r_i)) = \int P'(O(r_i)) \cdot f(z) \, dz, \tag{37}
\]

where \( P'(O(r_i)) \triangleq P(O(r_i) | z) \), which represents the conditional probability of \( P(O(r_i)) \) given that \( \gamma_u | h_{u,r_i} |^2 + 1 = z \). Thus, we can further express the conditional probability of \( P'(O(r_i)) \) given that \( \gamma_s | h_{s,r_i} |^2 = x \) as

\[
P'(O(r_i) | \gamma_s = x) = \left\{ \begin{array}{ll} 1, & \text{if } x < \Delta_s z \\ P(\gamma_d < \Delta z - x \text{ or } \gamma_d < \Delta d z), & \text{if } x > \Delta_s z \\ 1 - \exp \left( -\frac{(\Delta z - x)}{\gamma_u \sigma_w^2} \right) \\ 1 - \exp \left( -\frac{(\Delta d z)}{\gamma_d \sigma_w^2} \right), & \text{if } x > (\Delta - \Delta d) z \end{array} \right.
\]

(38)

with \( \gamma_s = \gamma_u | h_{s,r_i} |^2 \) and \( \gamma_d = \gamma_d | h_{d,r_i} |^2 \). Consequently, \( P'(O(r_i)) \) can be obtained by the following integration

\[
P'(O(r_i)) = \int P'(O(r_i) | \gamma_s = x) f(x) \, dx
\]

\[
= \left\{ \begin{array}{ll} 1 - \frac{1 + \Delta \Delta d z}{\gamma_u \sigma_w^2} \exp \left( -\frac{\Delta z}{\gamma_u \sigma_w^2} \right), & \text{if } \frac{\Delta d}{\gamma_u} = \frac{\sigma_w^2}{\sigma_d^2} \\ 1 - C \exp (-Az) - (1 - C) \exp (-Bz), & \text{otherwise} \end{array} \right.
\]

(39)

Substituting (39) into (37) and with the PDF \( f(z) = \frac{1}{\gamma_u \sigma_w^2} \exp \left( -\frac{(z-1)}{\gamma_u \sigma_w^2} \right) \), we get (11).
APPENDIX B

Let $\mathcal{A}$ and $\mathcal{B}$ respectively represent the probabilities as

$$
\mathcal{A} = P \left( \frac{P_d [h_{d,s}]^2 + \beta_i P_r_i [h_{r_i,d}]^2}{P_u [h_{u,s}]^2} > \Delta_d \right)
$$

$$
= \frac{1}{1 + \frac{P_d \sigma_{d,s}^2}{P_u \Delta_d \sigma_{u,s}^2}} + \frac{\beta_i P_r_i \sigma_{r_i,d}^2}{P_u \Delta_d \sigma_{u,s}^2},
$$
and

$$
\mathcal{B} = P \left( \frac{P_s [h_{s,d}]^2 + \alpha_i P_r_i [h_{r_i,d}]^2}{P_u [h_{u,s}]^2} > \Delta_s \right)
$$

$$
= \frac{1}{1 + \frac{P_s \sigma_{s,d}^2}{P_u \Delta_d \sigma_{u,s}^2}} + \frac{\alpha_i P_r_i \sigma_{r_i,d}^2}{P_u \Delta_d \sigma_{u,s}^2}.
$$

In the high SNR regime, i.e., $N_0 \to 0$, it is straightforward to see that $P(O(S(T(r_i)))) = \mathcal{A} + \mathcal{B} - \mathcal{A} \mathcal{B}$ from [3], [9], and [12], and we have the desired result.

APPENDIX C

The PDF of variable $X$ can be expressed as

$$
f(X) = \frac{\partial F(X)}{\partial X} = \partial P \left( \Delta_d \gamma_{u} [h_{u,s}]^2 + \Delta_d - \gamma_{d} [h_{d,s}]^2 < X \right)
$$

$$
= \partial \int_{-\infty}^{\infty} P \left( \Delta_d \gamma_{u} [h_{u,s}]^2 < X + z - \Delta_d \right) f(z) dz,
$$

where $z = \gamma_{d} [h_{d,s}]^2$ and

$$
P \left( \Delta_d \gamma_{u} [h_{u,s}]^2 < X + z - \Delta_d \right) = \left\{ \begin{array}{ll}
0 & \text{if } z < \Delta_d - X \\
1 - \exp \left( -\frac{X+z-\Delta_d}{\Delta_d \gamma_{u} \sigma_{u,s}^2} \right) & \text{if } z > 0, X > \Delta_d \\
1 - \exp \left( -\frac{z - \Delta_d}{\Delta_d \gamma_{u} \sigma_{u,s}^2} \right) & \text{if } z > \Delta_d - X, X < \Delta_d.
\end{array} \right.
$$

Thus, the cumulative distribution function is written as

$$
F(X) = \left\{ \begin{array}{ll}
1 - \frac{\Delta_d \gamma_{u} \sigma_{u,s}^2}{\Delta_d \gamma_{u} \sigma_{u,s}^2 + \gamma_{d} \sigma_{d,s}^2} \exp \left( \frac{-X}{\Delta_d \gamma_{u} \sigma_{u,s}^2} \right) & \text{if } X > \Delta_d \\
\frac{\Delta_d \gamma_{u} \sigma_{u,s}^2}{\Delta_d \gamma_{u} \sigma_{u,s}^2 + \gamma_{d} \sigma_{d,s}^2} \exp \left( \frac{-X}{\Delta_d \gamma_{u} \sigma_{u,s}^2} \right) & \text{if } X < \Delta_d.
\end{array} \right.
$$

Therefore, we can have the PDF as

$$
f(X) = \left\{ \begin{array}{ll}
\frac{\Delta_d \gamma_{u} \sigma_{u,s}^2}{\Delta_d \gamma_{u} \sigma_{u,s}^2 + \gamma_{d} \sigma_{d,s}^2} \exp \left( \frac{-X}{\Delta_d \gamma_{u} \sigma_{u,s}^2} \right) & \text{if } X > \Delta_d \\
\frac{\Delta_d \gamma_{u} \sigma_{u,s}^2}{\Delta_d \gamma_{u} \sigma_{u,s}^2 + \gamma_{d} \sigma_{d,s}^2} \exp \left( \frac{X - \Delta_d}{\gamma_{d} \sigma_{d,s}^2} \right) & \text{if } X < \Delta_d.
\end{array} \right.
$$

Similarly, the PDF of variable $Y$ can also be addressed as:

$$
f(Y) = \left\{ \begin{array}{ll}
\frac{1}{\Delta_d \gamma_{u} \sigma_{u,s}^2 + \gamma_{d} \sigma_{d,s}^2} \exp \left( \frac{Y}{\Delta_d \gamma_{u} \sigma_{u,s}^2} \right) & \text{if } Y > \Delta_d, \\
\frac{1}{\Delta_d \gamma_{u} \sigma_{u,s}^2 + \gamma_{d} \sigma_{d,s}^2} \exp \left( \frac{Y - \Delta_d}{\gamma_{d} \sigma_{d,s}^2} \right) & \text{if } Y < \Delta_d.
\end{array} \right.
$$

APPENDIX D

To solve the optimization problem in (44), we discuss the solutions in the following two cases.

Case 1: $\frac{\gamma_d}{\gamma_s} = \frac{\sigma_{d,rmin}}{\sigma_{s,rmin}}$.

In this case, $P(O(r_i))$ decreases monotonically as $\gamma_d$ increases. Substituting $\frac{\gamma_d}{\gamma_s} = \frac{\sigma_{d,rmin}}{\sigma_{s,rmin}}$ into primary QoS constraint $C$ and we have $\{P_s, P_d\}$ expressed by $\{P_s', P_d'\}$ as

$$
P_s' = \left( \frac{\sigma_{s,rmin}^2}{\Delta_s 2B A} \right)^{\frac{1}{2}} \frac{\Delta_d}{\Delta_s} \left( \frac{\sigma_{d,rmin}^2}{\sigma_{s,rmin}^2} \right) \left( \frac{1}{2A} \right) \left( \frac{\Delta_d}{\Delta_s} \right) + \frac{\Delta_d}{\Delta_s} \left( \frac{\sigma_{d,rmin}^2}{\sigma_{s,rmin}^2} \right) \left( \frac{1}{2A} \right)
$$

and $P_d' = \frac{\sigma_{s,rmin}^2}{\sigma_{d,rmin}^2}$, where $A = \frac{\Delta_s \sigma_{s,rmin}^2}{\sigma_{u,s}^2}$ and $B = \frac{\Delta_s \sigma_{s,rmin}^2}{\sigma_{u,s}^2}$.

Case 2: $\frac{\gamma_d}{\gamma_s} \neq \frac{\sigma_{d,rmin}}{\sigma_{s,rmin}}$.

By looking at (44) and constraint $C$, it is analytically intractable to derive an explicit and closed form expression for optimized $P_s$ and $P_d$. Herein, we seek an asymptotic solution in the high SNR regime as $N_0 \to 0$. Recall from (39) that in this case the conditional probability $P'(O(r_i))$ is given as

$$
P'(O(r_i)) = 1 - C \exp(-A z) - (1 - C) \exp(-B z)
$$

where $A = \frac{N_0 \Delta_d}{\sigma_{s,rmin}^2}$, $B = \frac{N_0 \Delta_d}{\sigma_{d,rmin}^2}$, $C = \frac{\sigma_{s,rmin}^2}{\sigma_{d,rmin}^2}$, and $[h_{u,s}]^2 + 1 = z$. Therefore, we have the following approximation when $N_0 \to 0$ as

$$
P'(O(r_i)) \approx 1 - C(1 - A z) - (1 - C)(1 - B z)
$$

$$
= A C z + B (1 - C) z.
$$

Consequently, we can obtain the approximate outage performance in high SNR regime as

$$
P(O(r_{min}) = \int_{z} f(z) dz
$$

$$
\approx \int_{z} (A C z + B (1 - C) z) \cdot f(z) dz
$$

$$
= (N_0 + \frac{\sigma_{u,s}^2}{\sigma_{d,rmin}^2}) \left( \frac{\Delta_s}{P_s \sigma_{s,rmin}^2} + \frac{\Delta_d}{P_d \sigma_{d,rmin}^2} \right).
$$

To find the optimal $P_s$ and $P_d$ in this case, we rewrite the constraint $C$ in the following form $AB P_s P_d + A P_s + B P_d + 1 = g$. To find the optimal $P_s$ and $P_d$ that minimize $P(O(r_{min})$ while satisfying constraint $C$, we constitute the Lagrange function as

$$
L = (N_0 + \frac{\sigma_{u,s}^2}{\sigma_{d,rmin}^2}) \left( \frac{\Delta_s}{P_s \sigma_{s,rmin}^2} + \frac{\Delta_d}{P_d \sigma_{d,rmin}^2} \right) + \lambda (AB P_s P_d + A P_s + B P_d + 1 - g),
$$

with $\lambda$ being the Lagrange multiplier. By solving

$$
\frac{\partial L}{\partial \gamma_s} = 0, \quad \frac{\partial L}{\partial \gamma_d} = 0,
$$

we can obtain the optimal $P_s$ and $P_d$.
and considering C, we can obtain the power allocation \( \{P_s, P_d\} \) expressed by \( \{P'_s, P'_d\} \) in this case as \( P'_s = \frac{\Delta_s \sigma^2_{d,\text{min}} AB + A}{\Delta_s \sigma^2_{s,\text{min}} AB + B} \), and \( P'_d = \frac{\Delta_d \sigma^2_{s,\text{min}} AB + B}{\Delta_d \sigma^2_{d,\text{min}} AB + B} \).

**APPENDIX E**

During the second transmission phase, the best relay forwards the signals to the STs \( s \) and \( d \). At the same time, this transmission also causes interference at the primary receivers and the corresponding received signal is expressed as

\[
y_r = \sqrt{P_d} h_{u,v} x_u + \sqrt{P_r} h_{r_i,v} x_{r_i} + n_v.
\]

Thus, the primary QoS guarantee with respect to outage probability constraint can be written as

\[
P_{uv} = P \left( \frac{|P_u h_{u,v}|^2}{|P_r h_{r_i,v}|^2 + N_0} < \Delta_u \right) \leq P_{th}.
\]

which can calculated and expressed as \( P_{uv} = 1 - \exp\left( -\frac{\Delta_u N_0}{\sigma^2_{u,v} \Delta_d} \right) \leq P_{th} \). Therefore, we obtain the transmit power limit of the best relay \( r_i \) as \( P_{r_i} \leq \frac{\Delta_d \sigma^2_{d,\text{min}}}{\Delta_s \sigma^2_{s,\text{min}}}(g - 1) \). Note that this is the limit for the transmit power of the best relay and, therefore, we have the optimal \( P_{r_i} \) when equality is attained, leading to the desired result.

**APPENDIX F**

In order to find the optimal values of \( \alpha_i \) and \( \beta_i \) that minimize \( P(O(\text{ST}|r_i)) \) in (13), we construct the Lagrange function as \( \mathcal{L} = A + B - \Delta B + \lambda (\alpha_i + \beta_i - 1) \), where \( A \) and \( B \) are given in (13).

The optimal values of \( \alpha_i \) and \( \beta_i \) satisfy the equations below

\[
\frac{\partial \mathcal{L}}{\partial \alpha_i} = 0, \quad \frac{\partial \mathcal{L}}{\partial \beta_i} = 0, \quad \alpha_i + \beta_i = 1,
\]

which result in the given expressions in the lemma.

**APPENDIX G**

Following the derivation of Lemma 1, we begin the asymptotic power allocation of \( \{\gamma_s, \gamma_d\} \).

**Case 1**: \( \gamma_s = \frac{\sigma^2_{s,\text{min}}}{\sigma^2_{d,\text{min}}} \).

Let \( \{\gamma'_s, \gamma'_d\} \) represent the power allocation scheme in this case. Rewrite (42) using the first order Taylor expansion, \( \gamma'_s \) is expressed as

\[
\gamma'_s = \frac{(g' - 1) \sigma^2_{s,\text{min}}}{\gamma_u \sigma^2_{u,v}} \gamma_u \leq \Delta_s \sigma^2_{s,\text{min}} + \frac{\Delta_u \sigma^2_{u,v}}{\gamma_u \sigma^2_{u,v}} \Delta_d \sigma^2_{d,\text{min}} = \frac{\Delta_u \sigma^2_{u,v}}{\gamma_u \sigma^2_{u,v}} \Delta_d \sigma^2_{d,\text{min}}.
\]

Accordingly, \( \gamma'_d \) can also be given \( \gamma'_d = \gamma'_s \frac{\sigma^2_{s,\text{min}}}{\sigma^2_{d,\text{min}}} \leq \rho'_d \gamma_u \).

Note that \( \rho'_s \) and \( \rho'_d \) are only associated with statistical channel conditions.

**Case 2**: \( \frac{\rho}{\rho} \neq \frac{\sigma^2_{s,\text{min}}}{\sigma^2_{d,\text{min}}} \).

In this case, the power allocation is represented by \( \{\gamma''_s, \gamma''_d\} \), which is expressed based on the expressions of \( \{\gamma'_s, \gamma'_d\} \) as

\[
\gamma''_s = \frac{(g' - 1) \sigma^2_{s,\text{min}}}{\gamma_u \sigma^2_{u,v}} \gamma_u \leq \Delta_s \sigma^2_{s,\text{min}} + \frac{\Delta_u \sigma^2_{u,v}}{\gamma_u \sigma^2_{u,v}} \Delta_d \sigma^2_{d,\text{min}} + 1
\]

and

\[
\gamma''_d = \frac{(g' - 1) \sigma^2_{s,\text{min}}}{\gamma_u \sigma^2_{u,v}} \gamma_u \leq \Delta_s \sigma^2_{s,\text{min}} + \frac{\Delta_u \sigma^2_{u,v}}{\gamma_u \sigma^2_{u,v}} \Delta_d \sigma^2_{d,\text{min}} + 1
\]

where \( \rho'_s \) and \( \rho'_d \) are also associated with statistical channel conditions.

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