Does the Tachyon Matter?

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Abstract

We study time-dependent solutions of Einstein-Maxwell gravity in four dimensions coupled to tachyon matter—the Dirac-Born-Infeld Lagrangian that provides an effective description of a decaying tachyon on an unstable D-brane in string theory. Asymptotically, the solutions are similar to the recently studied space-like brane solutions and carry S-brane charge. They do not break the Lorentzian R-symmetry. We study the tachyon matter as a probe in such a background and analyze its backreaction. For early/late times, the tachyon field has a constant energy density and vanishing pressure as in flat space. On the other hand, at intermediate times, the energy density of the tachyon diverges and produces a space-like curvature singularity.

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It has been argued in [1] that string theory and related field theories have solutions corresponding to topological defects with finite temporal extent—so-called space-like (S-) branes. In string theory or supergravity, such solutions carry the same RR charge as the familiar time-like D-branes. In fact, the basic argument for existence of S-branes in string theory comes from the existence of unstable D-brane systems, for example the D3-brane in type IIA string theory. The worldvolume of such a brane has a tachyon $T$ which is coupled to a RR form by a term such as $\int dT \wedge C^{(3)}$. There are configurations of $T$ which are topologically non-trivial in time and source the corresponding magnetic flux. The S-brane is the gravitational backreaction of this time-dependent process. Explicit supergravity solutions corresponding to S-branes have been proposed in [1, 6, 7]. S-branes are likely to provide interesting time-dependent backgrounds of string theory. Moreover, due to their kinship with D-branes, they can play a role in temporal holography, see [1] and references therein.

In perturbative string theory, S-branes are described by imposing Dirichlet boundary conditions on the time coordinate of the string [1]. The identification proceeds as for time-like branes through the RR charge carried by open string boundary conditions. The relation of these space-like D-branes to tachyon decay was clarified by Sen [2], who showed that open string field theory possesses a family of solutions corresponding to the roll of the open string tachyon from its maximum down to the closed string vacuum. The rolling tachyon is peculiar for two reasons. Firstly, the energy density remains localized in the plane of the original unstable brane system. This is explained by the open string origin of the tachyon and the absence of open strings in the bulk. Secondly, when at late times the tachyon approaches the minimum of its potential, the tachyon is universally (i.e., independently of the details of the potential) characterized by a constant energy density and exponentially vanishing pressure. This state of the string field is called tachyon matter [3]. Moreover, tachyon matter admits an effective description by a Dirac-Born-Infeld type Lagrangian [16, 4]. The advantage of this field theory description is that it provides a drastic simplification in a complicated initial value problem in open string field theory, see [17, 18] for recent discussions. Tachyon matter has been derived in the context of BSFT in [5], and has also attracted considerable cosmological interest [8].

In this note, we explore the relation of charged space-like branes with tachyon decay one step further. In the context presented in [1], it is natural to ask for the inclusion of tachyon matter [2–4] into the effective dynamics. In particular, one might hope to
see explicitly the excitation (or decay) of the tachyon by incoming (or into outgoing) radiation. Furthermore, one would like to know whether the tachyon matter changes the nature of the singularities that plague the explicit supergravity solutions found in \[1,6,7\]. Here, we study the modification of these backgrounds produced by coupling tachyon matter to the supergravity fields. We find that while the asymptotics are essentially unchanged, the tachyon indeed modifies the singularity structure, producing a space-like curvature in the middle of space-time. For simplicity, we will consider Einstein-Maxwell theory in four dimensions, which yielded the simplest S-brane example in \[1\], but we expect that the qualitative picture is similar in other cases.

S-branes

We begin with a lightning review of the supergravity properties of S-branes \[1\].

S-branes are expected to exist as time-dependent solutions both in ten-dimensional IIA/IIB supergravity, and in eleven-dimensional supergravity. Conventionally, the worldvolume of an $S_p$-brane is a $p+1$-dimensional Euclidean space. The types of charge that are expected to occur are the same as for D-branes, so that in IIA/IIB supergravity, we have odd/even codimension S-branes with RR charge, as well as SNS5-branes and strings. Furthermore, we have $S_5$-brane and $S_2$-brane in $D=11$ supergravity. However, the properties of S-branes should depend drastically on the parity of the codimension. More precisely, as explained in \[1\], even codimension branes should have fluxes supported on their lightcone, while odd codimension S-branes source a flux inside their entire lightcone. In explicit supergravity solutions, the former seems much harder to realize, and unsmeared S-brane solutions in IIB supergravity might not exist\[1\]. In this paper we consider only odd codimension S-branes.

A flat $S_p$-brane in $D$ dimensions has a “classical” $ISO(p+1) \times SO(D−p−2,1)$ symmetry. Supergravity solutions preserving this symmetry are singular. In \[1\] it was argued that the resolution of these singularities should involve a (spontaneous) breaking of the transverse R-symmetry down to $SO(D−p−2)$. Indeed, this is precisely the breaking that is realized if one thinks of the space-like $p$-brane as a decaying unstable time-like $p+1$-brane.

Let us now turn to the simplest example of a space-like brane, the S0-brane in

\[1\text{It is possible to construct time-dependent solutions in type IIB supergravity which have an S-brane background ansatz analogous to }\[1\]. \text{ We will not discuss these solutions here.}\]
four-dimensional Einstein-Maxwell gravity \[1\]. The metric is given by
\[ ds^2 = -\frac{d\tau^2}{\lambda^2} + \lambda^2 d\sigma^2 + R^2 dH_2^2, \]  
where \( dH_2^2 \) is the normalized metric on the two-dimensional hyperbolic space \( H_2 \). All warp factors depend on \( \tau \) only, and the metric has a manifest \( SO(2,1) \times \mathbb{R} \) symmetry. Moreover, the (electric) flux is naturally given by the volume form on \( H_2 \),
\[ \star F = 2Q\epsilon_2, \]  
where \( Q \) is the charge of the brane. Integration of the equations of motion yields \[1\]
\[ R^2 = \frac{Q^2 \tau^2}{\tau_0^2}, \quad \lambda^2 = \frac{\tau_0^2}{Q^2} \frac{\tau^2 - \tau_0^2}{\tau^2}. \]  

One of the characteristics of the solution\[3\] is the vanishing of the \( \lambda \) warp factor at \( \tau = \tau_0 \) at finite curvature. This is reminiscent of a black hole horizon. Indeed, the simplest way to arrive at the solution (3) is to analytically continue the Reissner-Nordström black hole at imaginary charge and zero mass (the mass corresponds to a second integration constant in (3), which is set to zero by requiring \( \tau \to -\tau \) symmetry). This also gives an easy way to obtain the maximally extended spacetime associated with the metric (1).  

\[2\]Some of these arguments leading to the Penrose diagram in fig. 1 were given in \[9, 10\].
We have depicted (the projection onto the $\tau$-$z$-plane of) this maximally extended spacetime in fig. 1. As for a black hole, there are two asymptotic regions, separated by a region of strong curvature. Here, the two regions are causally connected. Another interesting region is close to the coordinate singularity at $\lambda = 0$, which we will refer to as the Milne region. In addition, there is a genuine curvature singularity at $\tau = 0$. This is a naked time-like (rather than space-like!) singularity associated with the vanishing of the scale-factor of the transverse spatial directions of the S-brane. It is worthwhile to stress that only one of the asymptotic regions corresponds to a piece of the metric (1), while the other is obtained by continuing past the Milne singularity. One should note that overall the diagram looks little like what one would have expected for a space-like brane.

We will now introduce the coupling of the tachyon matter and then study its effect on the background (1), focusing on the three interesting regions in turn.

**Coupling the Tachyon Matter**

In general, one should be cautious about supergravity (string theory) backgrounds containing naked singularities. Certain singularities should not be resolvable, and backgrounds containing them should simply be disallowed [19]. However, if the singularity occurs in a background for which there is a solid physical argument that it must be consistent, we expect that such singularity be resolved by incorporating the right physics. Adopting the viewpoint that an S-brane is a time-dependent description of an unstable D-brane, such branes are well-motivated physical objects, and we expect that it is possible to give a meaning to the singularities.

The definition of S-branes also suggests a natural way to try to make progress on understanding the above background, namely by including the decaying unstable D-brane as a source. Explicitly, we assume that the missing mode is the open string tachyon of the corresponding D-brane. The general form of the coupling of this tachyon to supergravity fields is quite well-known [11–15]. There is a DBI term containing the tachyon kinetic term and potential, and a WZ term describing the coupling of the tachyon to the RR fields. The precise expressions for the tachyon potential and tachyon dependence of the RR coupling are not known in all cases, but most of our results only depend on their generic features.

Specifically, the S0-brane that we have reviewed above should be the decay of an

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We thank Gary Horowitz for emphasizing this.
“unstable D1-brane” in 4-d Einstein-Maxwell gravity. This can be viewed as a toy model of type IIA string compactification with suppressed scalar moduli, and it is this model that we will study in the following. The action is given by

\[
S = S_{EM} + S_{branes}
= \frac{1}{16\pi G_4} \int d^4x \sqrt{-g} \left( R - \frac{1}{4} F^2 \right) + \mu_1 \int d^4x \rho(x_\perp) \left[ -V(T) \sqrt{-A + f(T)dT \wedge C^{(1)}} \right],
\]

(4)

where \( A = \det(g_{\mu\nu} + \partial_\mu T \partial_\nu T) \). In (4), \( G_4 \) is the four-dimensional Newton constant, \( \mu_1 \) is the tension of the unstable brane, \( V(T) \) is the tachyon potential, and \( f(T) \) is the tachyon coupling to the “RR” one-form \( C^{(1)} \), with flux \( F = dC^{(1)} \). Moreover, \( \rho(x_\perp) \) is the density of D1-branes in the transverse space, further discussed below. In the following, we will use Roman indices or the symbol \( \perp \) to denote directions transverse to the D1-brane, and Greek indices or the symbol \( \parallel \) for the directions parallel to it, so that for instance, \( g = \det g_{\parallel} \det g_{\perp} \equiv \det g_{\mu\nu} \det g_{ij} \).

The tachyon potential and the RR coupling is not known exactly, but we will be mostly concerned with universal features of the system that are independent of the exact form of \( V \) and \( f \). Specifically, we shall assume that \( V(T) \) is smooth at the unstable maximum, which we take to be at \( T = 0 \). We also assume symmetries, \( V(T) = V(-T), \ f(T) = -f(-T) \), and the universal asymptotics \( [3,4] \)

\[
V(|T| \to \infty) \to e^{-|T|/\sqrt{2}}, \quad f(|T| \to \infty) \to \text{sign}(T) e^{-|T|/\sqrt{2}}. \quad (5)
\]

It is straightforward to derive equations of motion for (4). We find the following Einstein equations

\[
R_{\mu\nu} = \frac{1}{4} \Lambda V(T) \rho(x_i) \sqrt{\frac{A}{g}} \left[ (A^{-1})^{\alpha\beta} g_{\alpha\beta} g_{\mu\nu} - 2(A^{-1})^{\alpha\beta} g_{\alpha\mu} g_{\beta\nu} \right] + T^{(2)}_{\mu\nu}
\]

\[
R_{ij} = \frac{1}{4} \Lambda V(T) \rho(x_i) \sqrt{\frac{A}{g}} \left[ (A^{-1})^{\alpha\beta} g_{\alpha\beta} \right] g_{ij} + T^{(2)}_{ij},
\]

(6)

where \( \Lambda \equiv 16\pi G_4 \mu_1 \), and

\[
T_{AB} = \frac{1}{2} \left[ F_{AC} F_{B}^{ \ C} - \frac{1}{4} F^2 g_{AB} \right]
\]

(7)

is the stress tensor of the 2-form flux \( F_{AB} \). We also have a Maxwell equation

\[
\partial_A (\sqrt{-g} F^{AB}) + \Lambda f \rho(x_i) \left[ \partial_\alpha T \delta^B_{\beta} - \partial_\beta T \delta^B_{\alpha} \right] = 0, \quad (8)
\]
and the tachyon equation
\[
\rho(x_i)\left(\sqrt{-A} \frac{dV}{dT} + f F_{\alpha\beta}\right) - \partial_A\left[\rho(x_i)V \sqrt{-A} (A^{-1})^{AB} \partial_B T\right] = 0 . \tag{9}
\]

We are looking for solutions of (6), (8) and (9) that in the asymptotic regions look like the solutions without tachyon, i.e., eq. (1). As discussed in [1], it is likely that in order to resolve the singularity, one must violate the $SO(2,1)$ R-symmetry of this solution in the intermediate regions. Indeed, the most natural way to achieve this in our framework is to localize the decaying D1-branes at a point in the transverse space, i.e., to put $\rho(x_\perp) = \delta(x_\perp)$. However, doing so makes the equations intractable. We shall instead preserve the R-symmetry by smearing the branes and tachyon matter uniformly along the transverse space. Explicitly, we take
\[
\rho(x_i) = \rho \sqrt{g_\perp} , \tag{10}
\]
where $\rho$ is a constant number density of D1-branes. With the ansatz
\[
ds^2 = -c_1^2 d\tau^2 + c_2^2 dz^2 + c_3^2 dH_2^2 \tag{11}
\]
for the metric, the two-form flux
\[
F_{\tau z} = A c_1 c_2 , \tag{12}
\]
and with only time-dependent warp-factors $c_i = c_i(\tau)$, $A = A(\tau)$, and tachyon $T = T(\tau)$, we obtain the following differential equations
\[
R_{\tau\tau} = \frac{1}{c_1 c_2 c_3} \left( c_3 \left( c_2^' \right)' + 2 c_2 \left( c_3^c_2 \right) c_1^' \right) g_{\tau\tau} = \left( \frac{AV \rho}{4c_3^2} \left( \triangle^{1/2} - \triangle^{-1/2} \right) - \frac{A^2}{4} \right) g_{\tau\tau} \tag{13}
\]
\[
R_{z z} = \frac{1}{c_1 c_2 c_3} \left( c_3^c_2 \right) c_1^' g_{z z} = \left( \frac{AV \rho}{4c_3^2} \left( \triangle^{1/2} - \triangle^{-1/2} \right) + \frac{A^2}{4} \right) g_{z z}
\]
\[
R_{i j} = \frac{1}{c_1 c_2 c_3^2} \left( c_3 \left( c_2^c_3 \right) c_1^' - c_1 c_2 \right) g_{i j} = \left( \frac{AV \rho}{4c_3^2} \left( \triangle^{1/2} + \triangle^{-1/2} \right) + \frac{A^2}{4} \right) g_{i j}
\]
\[
\left( \frac{dV}{dT} \triangle^{1/2} + \frac{dF}{dT} A \right) c_1 c_2 \left[ T' V \triangle^{-1/2} c_2 \right] = 0 , \tag{14}
\]
where we introduced
\[
\triangle \equiv 1 - \frac{(T')^2}{c_1^2} , \quad \frac{dF(T)}{dT} \equiv f(T) . \tag{15}
\]
\[7\]
The Maxwell equation (8) can be explicitly integrated with the result

\[ A = \frac{2Q + \Lambda \rho F}{c^2}, \]  

where the integration constant \( Q \) has been chosen in such a way as to reproduce the S0-brane solution of (1)-(3) as \( \Lambda = 0 \).

We now proceed to solving eqs. (13) and (14), using (16). We will first discuss asymptotics at late times, then close to a Milne coordinate singularity and finally near a curvature singularity. Both types of singularities appear in (8), and we wish to see whether including the tachyon changes their structure. In what follows, we will usually only need the generic behavior (5). For some purposes, such as numerical integration, for example, it is useful to have an explicit expression for \( V \) and \( f \). Whenever appropriate, we have used

\[ V(T) = \frac{1}{\cosh T/\sqrt{2}}, \quad F(T) = -\frac{\sqrt{2}}{\cosh T/\sqrt{2}}. \]  

Asymptotic Region

Tachyon condensation in a flat background produces pressureless gas with constant energy density [2–4]. This non-zero energy density is the basic reason for asking for the backreaction of the tachyon on S-brane backgrounds. One might expect in particular that the asymptotic flatness of the space is modified. In our model, the tachyon still produces a pressureless gas at late times, but the energy density is “time-diluted” because of the smearing of the branes. We find that the backreaction produces a logarithmic correction to flat space.

Tachyon matter in asymptotic S0-brane background

We begin by treating the tachyon matter as a probe in the background (8) in the region \( \tau \to \infty \). We will see that \( T \to \infty \), so we can use the asymptotics (5) for the tachyon potential and the coupling to the RR potential. The tachyon equation becomes

\[ 0 \approx 2Q \Delta(\tau) \sqrt{1 - \Delta(\tau)} + \sqrt{2\tau_0} \frac{d\Delta(\tau)}{d\tau}, \]  

with the solution

\[ \Delta = \cosh^{-2} \left( \frac{Q\tau}{\sqrt{2\tau_0}} + \beta \right), \]  

8
where $\beta$ is an integration constant. From (19) and (15) it follows that asymptotically the tachyon probe behaves as

$$T = \frac{Q \tau}{\tau_0} + \frac{1}{2^{3/2}} e^{-2\left(\frac{Q \tau}{\tau_0} + \beta\right)} + a(e^{-\frac{\sqrt{2}Q \tau}{\tau_0}}).$$

(20)

Notice that at infinity, $V \Delta^{-1/2} \to \text{const}$, rather similar to the flat space result [4]. The only difference is that, because of the smearing, the energy density is here diluted, $\rho_{\text{tachyon}} \sim \tau^{-2}$. The pressure still decays exponentially $P_{\text{tachyon}} \sim e^{-\sqrt{2}Q \tau/\tau_0}/\tau^2$. Since the flux contribution to the stress tensor behaves asymptotically like $T^{(2)} \sim \tau^{-4}$, the tachyon seems to dominate at late times. We now show that this basic conclusion is unchanged if we include the backreaction.

**Backreaction**

In eqs. (13) and (14), let us neglect all exponentially suppressed terms, i.e., we neglect $\Delta^{1/2}$ over $\Delta^{-1/2}$, and $F$ over $Q$ for the gauge potential (16). Choosing the gauge where $c_3 = Q \tau/\tau_0$, a certain combination of Einstein equations reads

$$0 = Q^2 \tau^2 (2\tau c_1' - c_1) + \tau_0^2 c_1^3 (\tau^2 + \tau_0^2),$$

(21)

which can be readily integrated to yield

$$c_1^2 = \frac{Q^2}{\tau_0^2} \frac{\tau^2}{\tau^2 - \tau_0^2 + \beta \tau},$$

(22)

where $\beta$ is an integration constant. Notice that this warp factor is the same as in the original S0-brane solution (3), where the integration constant $\beta$ was set to zero. Using this, we can write the Einstein equations in terms of $\epsilon(\tau) \equiv V \Delta^{-1/2}$,

$$\epsilon' = -\frac{\epsilon(\Lambda \rho \tau^2 \epsilon + 4\tau_0^2 - 2\beta \tau)}{4\tau (\tau^2 - \tau_0^2 + \beta \tau)}$$

$$0 = (\epsilon c_2)' \quad \text{.}$$

(23)

The latter equations can be solved analytically with the result

$$\epsilon^{-1} = \frac{1}{4\tau} \left\{ \Lambda \rho \sqrt{\tau^2 - \tau_0^2 + \beta \tau} \ln \left[ \beta_1 (\beta + 2\tau + 2\sqrt{\tau^2 - \tau_0^2 + \beta \tau}) \right] + \frac{2\Lambda \rho (\tau_0^2 \beta - 2\tau \tau_0^2 - \tau \beta^2)}{4\tau_0^2 + \beta^2} \right\}$$

$$c_2 = \beta_2 / \epsilon, \quad \text{where } \beta_1, \beta_2 \text{ are additional integration constants.}$$

We see that at late times, we have $\epsilon = V \Delta^{-1/2} \sim 1/\ln \tau$. Therefore, $\rho_{\text{tachyon}} \sim 1/(\tau^2 \ln \tau)$, and still dominates the asymptotic flux stress tensor. And indeed, the warp factor $c_2 \sim \epsilon^{-1} \sim \ln \tau$ as $\tau \to +\infty$. The curvature still vanishes asymptotically.
We now consider the effect of tachyon matter on the Milne coordinate singularity of the S0-brane solution, which is near $\tau = \tau_0$ in eq. (3). We will find that in general the Milne singularity is in fact replaced with a genuine space-like curvature singularity.

We begin by exhibiting an analytical solution with static tachyon $dT/d\tau \equiv 0$. It is convenient to choose the gauge $c_1 = 1/c_2 \equiv 1/\lambda$. The coordinate singularity then occurs at $\tau = \tau_*$, where $\lambda(\tau_*) = 0$, provided that $c_3 \equiv R(\tau_*) \neq 0$. Note that for a stationary tachyon, which is a solution of (14) only if tachyon sits at the top of the potential $T(\tau) \equiv 0$, we have $\Delta \equiv 1$. Denoting $F \equiv F|_{\tau=\tau_*} = F_0$, $V \equiv V|_{\tau=\tau_*} = V_0$, we find an exact analytical solution of the Einstein equations (13)

$$R^2 = \frac{Q^2 \tau^2}{\tau_0^2}$$

$$\lambda^2 = \frac{\tau^2}{Q^2} \left[ \frac{\Lambda V_0 \rho}{2} + \left( 1 - \frac{\tau_0^2}{\tau^2} \left( 1 + \frac{\Lambda \rho F_0}{2Q} \right)^2 \right) + \frac{\beta}{\tau} \right],$$

where $\beta$ is an integration constant. We have chosen the other two integration constants such as to reproduce solution (3) as $\Lambda = 0$. From (25), it is easy to see that $\lambda^2$ always has a simple zero for a certain $\tau = \tau_* > 0$, and everything is perfectly smooth. Note that although (25) was obtained for a stationary tachyon, it is clearly valid whenever the tachyon evolution across the Milne singularity satisfies $|T'(\tau_*)| < \infty$. Indeed, in this case $\Delta = 1$ at $\lambda = 1$, and thus (25) is an approximate solution. The tachyon acts somewhat like a cosmological constant along the decaying D1-brane.

We now argue that finiteness of $T'$ is in fact not generic, and that we rather have $T'|_{\tau\to\tau_*} \to \infty$. To see this we take $\lambda^2 \approx \lambda_0^2(\tau - \tau_*)$ and $R \approx Q\tau_*/\tau_0$. Eq. (14) then yields

$$0 = \Delta^2 - \Delta + x \Delta',$$

where we introduced $x \equiv \tau - \tau_*$, and neglected subdominant terms as $x \to 0$. The derivative in eq. (23) is with respect to $x$. The general solution to this equation is

$$\Delta = \frac{x}{x + \beta_1} = \frac{\tau - \tau_*}{\tau - \tau_* + \beta_1},$$

where $\beta_1$ is a new integration constant. Note that, unless $\beta_1 = 0$, (27) implies that $T' \sim 1/\sqrt{\tau - \tau_*}$ and $T|_{\tau\to\tau_*} \to \text{const.}$ Thus, in this case the energy density of the tachyon itself has a finite expectation value at the Milne coordinate singularity. This is confirmed by numerical computation and asymptotic analysis.
tachyon matter diverges as $\rho_{\text{tachyon}} \sim V\Delta^{-1/2} \sim V_0/\sqrt{\tau - \tau_*}$, producing a space-like curvature singularity in the metric. The warp factors can be solved asymptotically to yield

$$
c_2 \approx \lambda_0 \sqrt{\tau - \tau_*} \\
c_1 \approx c_2^{-1} \left(1 + \frac{\Lambda V_0 \rho \tau_0^2 \sqrt{\tau - \tau_*}}{Q^2 \tau_*^2 \lambda_0^2} + O(\tau - \tau_*)\right) \\
c_3 \approx \frac{Q\tau_*}{\tau_0} + \frac{V_0 \rho \tau_0 \sqrt{\tau - \tau_*}}{Q \tau_* \lambda_0^2} + O(\tau - \tau_*) \cdot
$$

Notice that to obtain (28) as a solution of Einstein equations (13), we had to relax the gauge condition $c_1 c_2 \equiv 1$. We also note that the special case $\beta_1 = 0$ in (27) corresponds to a smooth evolution of the tachyon across the Milne coordinate singularity. The local geometry there is analogous to (25).

Another special class of interesting solutions can be obtained (numerically) by tuning the tachyon so that it ends up precisely at the top of its potential at $\tau = \tau_*$. However, because $T'$ (and the curvature) still diverges there, it is not clear whether these solutions can be continued by “time-reversal” to the other side. If this were true, the tachyonic kink corresponding to the S-brane would have infinite slope at the origin.

We would like to emphasize that our result about destabilization of the Milne coordinate singularity to a genuine space-like singularity is rather generic and does not depend on the specific form of the tachyon potential or the RR coupling. Physically, this is a very satisfying result as S-branes should really be space-like defects, and thus their gravity description should have a space-like singularity. We should also point out that the divergence of the $\rho_{\text{tachyon}}$ is not due to the smearing of the branes. This could matter only if $c_3 \to 0$.

**Tachyon near time-like singularity**

The original S-brane solution (3) had a time-like singularity at $\tau = 0$ in the Rindler wedges of the Milne coordinate singularity (the wavy lines in fig. [1]). In the previous section we argued that the coordinate singularity is generically destabilized to a cosmological singularity by the tachyon matter. It thus seems really unjustified to continue past it. Nevertheless, in order to complete the analysis of eqs. (13), (14), we have investigated how the tachyon matter would modify this time-like singularity. This might also be relevant if for very “finely-tuned” tachyon matter the Milne coordinate singularity survives.
The basic result is that the tachyon matter does not help to remove or significantly modify the time-like singularity associated with vanishing transverse scale factor. More precisely, using only the universal properties of the tachyon potential and the RR coupling, we have analyzed the asymptotics “inside” of the Milne region, where \( \tau \) is really a space-like coordinate. Taking the gauge \( c_2 = 1/c_1 \equiv \lambda \) and denoting the transverse scale factor by \( R \equiv c_3 \), and using the properties of \( V \) and \( f \) discussed above, a power series analysis near \( T(\tau = 0) = 0 \), shows the following facts.

- There is no smooth solution with the parity properties \( R(-\tau) = R(\tau), R(\tau = 0) \neq 0, \lambda(-\tau) = \lambda(\tau), \lambda(\tau = 0) \neq 0, \) and \( T(\tau) = -T(-\tau) \). In other words, there is no smooth cosmological bounce solution.

- There is no smooth solution with the parity properties \( R(-\tau) = -R(\tau), \lambda(-\tau) = \lambda(\tau), \lambda(\tau = 0) \neq 0, \) and \( T(\tau) = -T(-\tau) \).

- There is no smooth solution with the parity properties \( R(-\tau) = -R(\tau), \lambda(-\tau) = -\lambda(\tau), \) and \( T(\tau) = -T(-\tau) \). In other words, the time-like singularity can not coincide with the Milne coordinate singularity.

We have confirmed the above results by numerical integration. We have also studied numerically a few other solutions of the equations (13) and (14) that do not fit with any asymptotics expected from S-brane solutions. For example, one can look for solutions that do not respect any \( \tau \to -\tau \) symmetry, but where the tachyon reaches the top of its potential before the scale factor \( R \) vanishes. We found that there are no smooth solutions of this type. Instead, the curvature always diverges, due to diverging stress tensor of either the flux or the tachyon.

**Discussion and Final Comments**

In this paper, we have analyzed a toy model for understanding the effect of the decaying tachyon on the supergravity background corresponding to space-like branes in string/M-theory. The model was obtained by coupling tachyon matter to Einstein-Maxwell gravity in four dimensions. Without breaking the R-symmetry, we have studied the backreaction of tachyon matter on the simplest S0-brane model. We have found that the tachyon matter modifies slightly the asymptotics at late/early times, and has a rather drastic effect on the behavior at intermediate times, where the tachyon matter produces a space-like singularity which was absent in the original background. The tachyon does not help to resolve the time-like singularity.

Our results are rather generic. They do not depend, for the most part, on the
precise details of the tachyon couplings to the gravity fields. Furthermore, it is not unreasonable to expect the qualitative picture to be similar for other space-time and brane dimensionalities. We have also argued that the preservation of the R-symmetry should not be crucial for our results, although more detailed investigations would be needed to confirm this.

To complement the picture, it would be interesting to repeat the computations for other dimensions and more general brane systems. In particular, in higher dimensions, it will be interesting to see how much the coupling to the dilaton changes the details of the picture. Furthermore, one can imagine similar computations for tachyons in D$\overline{D}$-systems. In this context, S-branes correspond to space-like tachyonic vortices in higher-dimensional branes. Since the general form of the DBI action and RR coupling are also known for such systems \[1, 13\], it should be possible to study the backreaction of tachyon matter also on these S-branes.

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