On the structure of the X(1835) baryonium

J.-P. Dodonder and B. Loiseau

Laboratoire de Physique Nucléaire et de Hautes Énergies, Groupe Théorie, IN2P3-CNRS,
Universités Pierre & Marie Curie et Paris Diderot, 4 Place Jussieu, 75252 Paris, Cedex, France

B. El-Bennich

Physics Division, Argonne National Laboratory, Argonne, IL 60439, USA

S. Wycech

Soltan Institute for Nuclear Studies, Warsaw, Poland

The measurement by the BES collaboration of \( J/\psi \rightarrow \gamma p\bar{p} \) decays indicates an enhancement at the \( p\bar{p} \) threshold. In another experiment BES finds a peak in the invariant mass of \( \pi \) mesons produced in the possibly related decay \( J/\psi \rightarrow \gamma \pi^+ \pi^- \eta' \). Using a semi-phenomenological potential model which describes all the \( NN \) scattering data, we show that the explanation of both effects may be given by a broad quasi-bound state in the spin and isospin singlet \( S \) wave. The structure of the observed peak is due to an interference of this quasi-bound state with a background amplitude and depends on the annihilation mechanism.

PACS numbers: 12.39.Pn, 13.20Gd, 13.60Je, 13.75Cs, 14.65Dw

I. INTRODUCTION

The search for exotic states in the \( NN \) systems has been pursued for a few decades, but significant results have only been obtained recently. An indication of such states below the \( NN \) threshold may be given by the scattering lengths for a given spin and isospin state. However, in scattering experiments, it is difficult to assess a clear separation of quantum states. Measurements of the X-ray transitions in the antiproton hydrogen atom can select some partial waves if the fine structure of atomic levels is resolved. Such resolution has been achieved for the 1S states \(^1\) and partly for the 2P states \(^2\). One can also use formation experiment methods to reach specific states. In this way, an enhancement close to the \( p\bar{p} \) threshold, has been observed by the BES Collaboration \(^3\) in the radiative decay

\[
J/\psi \rightarrow \gamma p\bar{p}.
\]  

(1)

On the other hand, a clear threshold suppression is seen in the decay channel \( J/\psi \rightarrow \pi^0 p\bar{p} \). To understand better the nature of these \( p\bar{p} \) states, one has to look directly into the subthreshold energy region. This may be achieved in the antiproton-deuteron or the antiproton-helium reactions at zero or low energies. Such atomic experiments have been performed, although the fine structure resolution has not been reached so far \(^4\). \(^5\). Another way to look below the threshold is the detection of \( NN \) decay products. Recently the reaction

\[
J/\psi \rightarrow \gamma \pi^+ \pi^- \eta'
\]  

(2)

has been studied by the BES Collaboration \(^6\). This reaction is attributed \(^6\) to an intermediate \( pp \) configuration in the \( J^{PC}(pp) = 0^{-+} \) state which corresponds to spin singlet \( S \) wave state. A peak in the invariant meson mass is observed, interpreted as a new baryon state, named X(1835). The interpretation of the peak as a new X(1835) has been questioned by the Jülich group \(^7\). The latter view is supported by our calculations, but we suggest the origin of the BES finding to differ from the possibilities presented in Ref. \(^8\). It is argued here that the peak is due to an interference of a quasi-bound, isospin 0, \( NN \) state with a background amplitude. The same quasi-bound state was found in Ref. \(^8\) to be responsible for the threshold enhancement in reaction \( \odot \).

The purpose of the present work is to discuss the physics of \( NN \) states produced in these \( J/\psi \) decays and relate it to atomic experiments. In reaction \( \odot \) only three \( p\bar{p} \) final states are possible, as a consequence of the \( J^{PC} \) conservation. These differ by the internal angular momenta and spins. Close to the \( p\bar{p} \) threshold a distinctly different behavior of scattering amplitudes is expected in different states. A further selection of states is possible, but one has to rely on the analyzes of the elastic and inelastic \( NN \) scattering experiments. This has been studied in Ref. \(^8\) within the Paris potential model \(^9\), \(^10\), \(^11\), \(^12\), which is also used in the present work.

The final \( p\bar{p} \) states allowed by \( P \) and \( C \) conservation in the \( \gamma p\bar{p} \) channel are specified in Table I. These are denoted as \( 2S+1L_J \) or \( 2I+12S+1L_J \), where \( S, L \) and \( J \) are the spin, angular momentum and total momentum of the pair, respectively, while \( I \) denotes the isospin. A unified picture and a better specification in the radiative decays is achieved, semi-quantitatively, with an effective three-gluon exchange model \(^8\). This description indicates the final \( \gamma p\bar{p} \) state to be dominated by the \( p\bar{p} \) \(^{11}S_0 \) partial wave. In this wave the Paris potential generates a 52 MeV broad quasi-bound state at 4.8 MeV below threshold. This state is named \( NN\gamma(1870) \). A similar conclusion has been reached by the Jülich group.
TABLE I: The states of the low-energy $p\bar{p}$ pairs allowed in the $J/\psi \to \gamma p\bar{p}$ decays. The first column gives the decay modes to the specified internal states of the $p\bar{p}$ pair. The $J^{PC}$ for the photon is $1^{--}$. The second column gives the $J^{PC}$ for the internal $p\bar{p}$ system, the last column gives the relative angular momentum of the photon vs. the pair. $J^{PC} = 1^{--}$ for $J/\psi$.

| Decay mode          | $J^{PC}(p\bar{p})$ | relative 1 |
|---------------------|---------------------|-------------|
| $\gamma p\bar{p}(^3S_0)$ | $0^{--}$           | 1           |
| $\gamma p\bar{p}(^3P_0)$ | $0^{++}$           | 0           |
| $\gamma p\bar{p}(^3P_1)$ | $1^{++}$           | 0           |

Although the Bonn-Jülich potential does not generate a bound state in the $p\bar{p}$ $^{11}S_0$ partial wave [7].

Under the assumption that the $\pi^+$, $\pi^-$ and $\eta'$ are produced in relative $S$ waves, the reaction (2), if attributed to an intermediate $p\bar{p}$ as suggested by the BES group, is even more restrictive than the reaction (1). It allows only one intermediate state the $p\bar{p}$ $^{1}S_{0}$, which coincides with the previous findings. The presence of an intermediate $p\bar{p}$ state in reaction (2) is possible but not granted. We show below that a more consistent interpretation is obtained with the dominance of the $N\bar{N}(1870)$ state which is a mixture of $p\bar{p}$ and $n\bar{n}$ pairs.

The content of this work is as follows. Sec. II contains a description of the final state $p\bar{p}$ interaction and is included here for completeness. The subthreshold $N\bar{N}$ scattering amplitude, needed to describe reaction (2), is defined in Sec. III. Sec. IV gives the equation to be solved to calculate the amplitude of the meson formation through the intermediate $N\bar{N}$ interaction. The results are presented and discussed in Sec. V together with some concluding remarks.

II. FINAL STATE INTERACTIONS

For any multichannel system at low energies, described by an $S$ wave $K$-matrix, the transition amplitude from an initial channel $i$ to a final channel $f$ may be described by

$$T_{if} = \frac{A_{if}}{1 + iq_f A_{ff}},$$  \hspace{1cm} (3)

where $A_{if}$ is a transition length, $A_{ff}$ is the scattering length in the channel $f$ and $q_f$ is the momentum in this channel [13]. In the following, channel $f$ is understood to be the $p\bar{p}$ channel. Within the same formalism the scattering amplitude in channel $f$ reads

$$T_{ff} = \frac{A_{ff}}{1 + iq_f A_{ff}}.$$  \hspace{1cm} (4)

For $S$ waves at low energies $A_{if}, A_{ff}$ are functions of $q_f^2$ and the main energy dependence of the amplitudes comes from the denominators in Eqs. (3) and (4). With large values of $\text{Re } A_{if} > 0$ one may expect a bound (quasi-bound) state. For large $\text{Re } A_{if} < 0$ a virtual-state is likely, but one cannot determine these properties with absolute certainty unless a method to extrapolate below the threshold exists. This is particularly true in the $p\bar{p}$ case where the absorptive part $\text{Im } A_{if}$ is large because of the presence of many open annihilation channels. Since the final photon interactions are believed to be negligible, the energy dependence observed in the $J/\Psi \to \gamma p\bar{p}$ decay rate reflects the energy dependence in $q_f |T_{if}|^2$.

Practical calculations also indicate an energy dependence in $A_{if}$ and the Watson approximation, i.e., the constant $A_{if}$ is not applicable in a broader energy range. One needs to use Eq. (3) and a weakly energy dependent formation amplitude $A_{if} \sim 1/(1 + q_f^2 r_i^2)$ as explained in Ref. [8], where a best fit value $r_i = 0.55$ fm was found.

Figure 1 displays sizable model dependence of $q_f |T_{if}|^2$ for the $^{1}S_{0}$ calculated for four versions of the Paris potential model [8, 10, 11, 12]. These versions followed the increasing data basis which, for the most recent case, includes antineutron scattering and antiprotonic hydrogen data. The threshold enhancement is attributed to a strong attraction in this partial wave. It does not prove the existence of a quasi-bound state, but such a state is indeed generated by the model in the $^{11}S_{0}$ wave [9].

There are additional arguments to support this result which follow from light $\bar{p}$ atoms. The absorptive amplitudes can be extracted from the atomic level widths. With the data from Refs. [4, 5] such an extraction was described in Refs. [14] and [15]. The data allow to obtain only an isospin-spin average but as indicated in Fig. 2 the existence of a quasi-bound state is consistent with the atomic data. The increase of subthreshold absorption is also supported by the atomic level widths in heavy $\bar{p}$ atoms [15]. In addition to the broad $S$-wave state.
the Paris potentials generate a narrow $^{33}P_1$ quasi-bound state which arises in Paris 08 and Paris 99 potentials. It gains some support from widths in the antiprotonic deuterium as indicated in Fig. 2.

The procedure outlined above is based on a simple form of the low-energy final state wave function $\Psi_{p\bar{p}}$. At large distances and for small $q_f r$, it becomes

$$\Psi(r)_{p\bar{p}} \sim 1 - \frac{T_{ff} \exp(iq_f r)}{r} \approx [1 - A_{ff}/r] \frac{1}{1 + iq_f A_{ff}}. \quad (5)$$

The right side of this relation expands the wave up to $q_f^2$ terms. At shorter distance the wave function is no more directly related to the scattering matrix and depends on details of the interaction in channel $f$. Integrated over an unknown transition potential $V_{ff}$ it generates the formation amplitude $A_{ff}$ in the transition amplitude $T_{ff}$. Eq. (5) was used with the Paris [8] and Jülich [16] potentials. These potentials also generated the $A_{ff}$. Formulas (3) or (5) are useful above the threshold but cannot be simply extrapolated to the subthreshold region. The difficulty is related to the momentum $q_f = \sqrt{2\mu_{NN}E_{NN}}$ where $\mu_{NN}$ is the reduced mass. Above, the threshold $E_{NN}$ is the kinetic energy in the CM system, below the threshold $E_{NN}$ is negative and $q_f$ becomes imaginary. The outgoing wave $\exp(iq_f r)/r$ becomes $\exp(-|q_f| r)/r$.

III. OFF-SHELL $NN$ INTERACTIONS

For further calculations one needs the off-shell extension of the scattering amplitude in the energy as well as in the momentum variables. The most general extension for $S$ waves is given by
\[ f(k, E, k') = \frac{\mu_{NN}}{2\pi} \int \psi_o(r, k)V_{NN}(r, E)\Psi^+(r, E, k') r^2 \, dr, \]

where \( \Psi^+(r, E, k') \) is the full outgoing wave calculated with the regular free wave \( \psi_o(r, k') = \sin(\kappa r')/\kappa r' \). In this equation the momentum \( k' \) is not related to the energy \( E \). The Fourier-Bessel double transform of \( f(k, E, k') \) would generate a nonlocal \( T(r, E) \) in the coordinate representation. So involved calculations do not seem necessary as the experimental data are rather crude. We resort to a simpler procedure, standard in nuclear physics (for an application in the antiproton physics see Ref. [13]). The subthreshold scattering amplitudes are calculated in terms of \( T \) matrix defined in the coordinate representation by

\[ \bar{T}(r, E) = \frac{\mu_{NN}}{2\pi} V_{NN}(r, E) \frac{\Psi^+(r, E, k')}{\psi_o(r, k')}, \]

with \( k'(E) = \sqrt{2\mu_{NN}E} \). The \( \bar{T}(r, E) \) is a local equivalent of the nonlocal \( T \) matrix in the sense that matrix elements in the \( S \) waves fulfill the relation \( f(k, E, k') = \int dr \, r^2 \psi_o(r, k)\bar{T}(r, E)\psi_o(r, k') \) valid in a narrow subthreshold region where the last integral is convergent. The \( V_{NN}(r, E) \) is the recent Paris interaction potential [8] which is used in the Schrödinger equation to calculate \( \Psi^+(r, E, k') \). The potentials used are plotted in Figs. 3 and 4 and the resulting scattering amplitude is given in Fig. 5.

FIG. 5: The real Re \( T(E) \) and imaginary Im \( T(E) \) parts for \( N\bar{N} \) scattering amplitudes in the \( ^{11}\text{S}_0 \) state.

To describe the intermediate \( N\bar{N} \) state we need also the Fourier transform of \( T(r, E) \)

\[ T(\kappa, E) = \int dr \, \bar{T}(r, E) \frac{\sin(\kappa r)}{\kappa r}. \]

In the next step, Eq. (3) is used at negative energies \( E = E_{NN} \). For positive energies this equation is not practical due to zeros in the denominator that occur in \( \bar{T}(r, E) \) [see Eq. (7)] at multiplicities of \( k' = \pi/r \). One could nevertheless use it for \( k' < \pi/r_{\text{max}} \) where \( r_{\text{max}} \) is the distance at which the potential is cut-off. One has to set \( r_{\text{max}} \approx 2 \text{ fm} \) if one wants to extend the calculations up to energies of \( \approx 20 \text{ MeV} \) above the \( N\bar{N} \) threshold.

IV. THE INTERMEDIATE \( pp \) STATES

We assume that the photon in reaction (2) is emitted before the annihilation into mesons has taken place, as it happens in reaction (1). A specific model for that process was suggested in Ref. [8] but it will not be needed here. We assume however that the formation of the \( N\bar{N} \) pair is described by a source function \( F_{i,f} \) and the annihilation by another function \( F_{f,\text{mes}} \). In this way the effect of
the intermediate $N\bar{N}$ interactions can be described by an amplitude for the meson formation

$$T_{i,mes} = \int dp \, dp' \, F_{i,f}(p) \, G(p, p', E_{N\bar{N}}) \, F_{f,mes}(p', Q),$$

(10)

where $G(p, p', E_{N\bar{N}})$ is the full Green’s function for the intermediate $N\bar{N}$ system. The form assumed for the annihilation amplitude is

$$F_{f,mes}(p', Q) = \langle \exp(-Q - p')^2 r_f^2 \rangle,$$

(11)

where the angular average over $Q$ is indicated by the brackets. This choice is motivated by simple model considerations and the simplest possible assumption that the two $\pi$ mesons in reaction (2) are correlated to the $f_0(600)$ (also named $\sigma$ meson). The mass of the latter is assumed to be 500 MeV in our calculations. The relative momentum $\mathbf{p'}$ of the final $\eta'$ and $\sigma$ mesons is denoted by $Q$ while the Gaussian profile comes from quark rearrangement models of annihilation which operate Gaussian wave functions.

The Green’s function in Eq. (10) may be expressed in terms of the free Green’s function $G_o$ and the $N\bar{N}$ scattering amplitude $T$ as

$$G = G_o + G_o \, T \, G_o.$$  

(12)

Now, with the scattering amplitude defined by Eq. (7) and Eq. (5) one obtains

$$T_{i,mes} = \int dp \, dp' \, F_{i,f}(p)G_o(p, E_{N\bar{N}}) \, |\delta(p - p')|$$

$$+ T(p - p', E_{N\bar{N}}) G_o(p', E_{N\bar{N}}) \, F_{f,mes}(p').$$

(13)

The first term in Eq. (13) corresponds to a background amplitude with a non-interacting $N\bar{N}$ pair. The second one describes intermediate state interactions. The Green’s function is $G_o(p, E) = 4\pi/[(2\pi)^3(q_f^2 - p^2)]$ and the normalization is chosen such that $T$ in Eq. (13) has dimension of length. Let us notice that below the $N\bar{N}$ threshold both $q_f^2$ and $G_o$ are negative. Below the quasibound state $T$ is attractive (negative) and the interference in Eq. (13) becomes constructive. This effect extends the peak structure to lower energies. We assume the formation amplitude to be described by

$$F_{i,f}(p) = \frac{1}{1 + r_i^2 p^2}$$

(14)

with the range parameter $r_i = 0.55$ fm determined before from the final interactions above the threshold [8]. The normalization is arbitrary. The angular integrations in Eq. (13) generate an amplitude which depends only on $|Q|$; that is due to the momentum dependence of the half-off shell $T$ matrix and to the absence of any preferred direction in the initial $N\bar{N}$ state.

A semi-free parameter $r_f$ is related to the radius parameter in the quark models for the nucleon and mesons. The range of allowed $r_f$ values is limited. The upper limit $r_f \approx 0.55$ fm is obtained assuming the r.m.s. radii of the quark densities to be equal to the electromagnetic radii (0.8 fm for baryons and 0.6 fm for mesons). A lower limit $r_f \approx 0.25$ fm is obtained with the radii used in $NN$ interaction models based on quark approaches [17] and quark rearrangement models of $NN$ annihilation [18]. These rely on r.m.s radii in the range 0.5 – 0.6 fm for baryons and 0.4 – 0.6 fm for mesons.

The last factor needed in this calculation involves the four body phase space for $J/\psi \rightarrow \gamma \pi^+\pi^-\eta'$. We follow Ref. [13] to find

$$dL_4(M_f^2; P_\gamma, P_{\pi^+}, P_{\pi^-}, P_{\eta'}) = \frac{(M_f^2 - S_M)}{(2\pi)^2 4M_f^2}dL_3(S_M; P_{\pi^+}, P_{\pi^-}, P_{\eta'})dS_M,$$

(15)

where $M_f$ is the mass of $J/\psi$ and $dL_3$ is the invariant phase space for the three meson system of invariant mass squared $S_M$. The $dL_3$ may be found in Ref. [20] and it generates only a weak energy dependence. The full phase space is used but one finds a simple approximation $dL_4 \sim \varepsilon/(m_{\eta'} + 2m_\pi + \varepsilon)^2$ with $\varepsilon = \sqrt{S_M} - m_{\eta'} - 2m_\pi$ to work well in the whole region of interest.

All together the spectral function representing the $X(1835)$ is given by

$$X_S = |T_{i,mes}|^2 \, dL_4/dS_M.$$  

(16)

V. RESULTS AND CONCLUDING REMARKS

The best description of the BES data is obtained with $r_f \approx 0.4 - 0.5$ fm, and the shape of $X(1835)$ calculated in this way is given in Fig. 4. The intermediate state is the isospin 0 state. The data are reproduced fairly well despite the fact that the bound state itself occurs at 1871.7 MeV, i.e., 4.8 MeV below the threshold. This shape is determined by the interference effect of the two terms in Eq. (13) describing the decay process. Within the Paris potential model and within a broad range of semi-free $r_i, r_f$ parameters one finds no peak structure with the intermediate $pp$ state. This result is consistent with the observation that isospin 1 for the final mesons is not allowed and the decay $NN(T = 1) \rightarrow \pi^+\pi^-\eta'$ is not permitted by the isospin conservation [18].

Other contributions, possible improvements. Above the $N\bar{N}$ threshold our estimation of $X_S$ from our model Eq. (13) using Eq. (8) with Eq. (7) where $\psi_0(r, k'(E))$ is replaced by unity [see our discussion in Sec. II] just below Eq. (8) generate a minimum (see Fig. 6) that is deeper than the minimum indicated by the data. Below we indicate several possible explanations.
interference with the main mode and could contribute a term

$$X_P = \frac{|C_P Q|}{E_{NN} - E_p + i\Gamma_P/2}^2 dL_4/dS_M$$

(17)

to be added to the main expression for $X_S$ given in Eq. (16). The $X_P$ possibility is a speculative one and the relative strength $C_P$ would be very hard to predict. Also, with the recent update of the Paris potential the position of $P$ wave resonance is not generated at "the proper" position.

- In a more complete study, outside the scope of the present work, one could extend our $S$-wave equations (Eqs. (7), (8) and (13)) to the $P$-wave case. Here we illustrate in Fig. 7 the possible effect of an effective $P$ wave represented by a resonant term given by Eq. (17). It can be seen that such an effective resonance with $E_p = 1900$ MeV and $\Gamma_p = 200$ MeV can fill up the above threshold dip of $X_S$.

- The off-shell extension in terms of Eq. (7) cannot be fully trusted and the procedure of Eq. (9) should be used.

- The final state factor given by Eq. (11) is perhaps too simple to be used above the $NN$ threshold. In some decay models an energy dependent phase factor $F_{mes}$ is expected [13]. This would have very limited effect below the threshold since the loop integrals - over $G_o$ - generate real functions. However, above the threshold the loop integral becomes complex and the interference pattern seen in figure (6) might be changed.

These effects go beyond the technique used in this paper.

- To confirm experimentally a direct link between the $pp$ system and the $X(1835)$, authors of Ref. [7] have suggested a search at the future GSI Facility for Antiproton and Ion Research (FAIR) project using the proton antiproton detector array (PANDA) in reactions such as $pp \rightarrow \pi^+\pi^-X$ and $X \rightarrow \pi^+\pi^-\eta'$. Another possible reaction would be $pp \rightarrow \gamma X(1835)$. It could be performed with the PAX apparatus [21] with $\sim 50$ MeV polarized antiprotons on polarized protons at CERN antiproton decelerator (AD) Ring. The shape of the $X(1835)$ could be tested by the photon energy distribution. Of special value would be the comparison of two measurements obtained with the parallel and anti-parallel initial spin configurations. That could give an information on the mechanism of the $X(1835)$ formation. In particular, it would check the simple model presented in Ref. [8], where the initial state (in the $J/\psi$ case the intermediate) of the $pp$ system is the spin triplet, which, after the emission of a magnetic photon, turns into the final spin singlet.

In summary: it is shown that the $X(1835)$ structure can be generated by a conventional $NN$ potential model. Such a structure stems from a broad and weakly bound state, the $N\bar{N}_S(1870)$ that exists in the $^{11}S_0$ wave. The existence of a quasi-bound $S$-wave state receives an additional confirmation from the level widths of antiprotonic atoms.

---

**FIG. 6:** The spectral function $X_S$ representing the $X(1835)$ shape. Here the range parameter of the annihilation amplitude [Eq. (11)] is $r_f = 0.45$ fm. This $S$-wave contribution has been normalized to reproduce the data close to the $X(1835)$ peak. The experimental points are from Ref. [6]. Above the $NN$ threshold the calculation is performed replacing $\psi_\sigma(r,k'(E))$ by unity in Eq. (7).

**FIG. 7:** The spectral functions $X_S$ (calculated as for Fig. 6) and $X_P$ (Eq. (17) with $E_p = 1900$ MeV and $\Gamma_p = 200$ MeV) and their sum representing the $X(1835)$ shape. Here The $S$- and $P$-wave contribution have been normalized to reproduced the data [6].

- The $J^{PC}$ conservation allows the $NN$ pair in $^{11}S_0$ state to decay into the $f_0(600)$ $\eta'$ pair in a relative $S$ wave. This case has been discussed so far. In addition, with the baryons in $^{13}P_1$ states another decay mode to the $f_0(600)$ $\eta'$ pair is possible. It requires the two final mesons to be in the relative $P$-wave state. In the recent Paris 08 [9] as well as in the former Paris 99 potential [10] a close to threshold resonance is generated in the related $^{13}P_1$ state. With the energy $E_p = 1872$ MeV and width $\Gamma_p = 20$ MeV it may contribute a spike to the spectral distribution in Fig. 6. Since two different partial waves are involved in the final states such a decay produces no
Acknowledgements: We thank M. Lacombe for useful discussions. This research was performed in the framework of the IN2P3-Polish Laboratory Convention (collaboration No. 05-115). One of us (S.W.) was supported by EC 6-TH Program MRTN-CT-206-03502 (FLAVIA network). This work was also supported in part by the Department of Energy, Office of Nuclear Physics, Contract No. DE-AC02-06CH11357.

[1] M. Augsburger, D. F. Anagnostopoulos, G. Borchert, D. Chatellard, J.-P. Egger, P. El-Khouri, H. Goerke, D. Gotta, P. Houser, P. Indelicato, K. Kirch, S. Lenz, K. Rashid, Th. Siems, and L. M. Simons, Nucl. Phys. A361, 417 (1999), Measurements of the strong interaction parameters in antiprotonic deuterium.

[2] M. Augsburger, D. F. Anagnostopoulos, G. Borchert, D. Chatellard, J.-P. Egger, P. El-Khouri, H. Goerke, D. Gotta, P. Houser, P. Indelicato, K. Kirch, S. Lenz, K. Rashid, Th. Siems, and L. M. Simons, Nucl. Phys. A658, 149 (1999), Measurements of the strong interaction parameters in antiprotonic hydrogen and probable evidence for an interference with inner bremsstrahlung.

[3] J. Z. Bai et al. (BES Collaboration), Phys. Rev. Lett. 91, 022001 (2003), Observation of a near-threshold enhancement in the $pp$ mass spectrum from radiative $J/\psi \rightarrow \gamma pp$ decays.

[4] D. Gotta, D. F. Anagnostopoulos, M. Augsburger, C. Castelli, G. Borchert, D. Chatellard, J.-P. Egger, P. El-Khouri, H. Goerke, D. Gotta, P. Houser, P. Indelicato, K. Kirch, S. Lenz, N. Nelms, K. Rashid, Th. Siems, and L. M. Simons, Nucl. Phys. A660, 283 (1999), Balmer $\alpha$ transitions in antiprotonic hydrogen and deuterium.

[5] M. Schneider, R. Bacher, P. Blum, D. Gotta, K. Heitinger, W. Kunold, D. Rohmann, J.-P. Egger, L. M. Simmons and K. Elsner, Zeit. Phys. A338, 217 (1991), X-rays from antiprotonic $^3$He and $^4$He.

[6] M. Ablikim et al. (BES Collaboration), Phys. Rev. Lett. 95, 262001 (2005), Observation of a resonance $X(1835)$ in $J/\psi \rightarrow \gamma \pi^+ \pi^- \eta'$.

[7] J. Haidenbauer, Ulf-G. Meißner and A. Sibirtsev, Phys. Rev. D 74, 017501 (2006), Near threshold $pp$ enhancement in $B$ and $J/\psi$ decays.

[8] B. Loiseau and S. Wycech, Phys. Rev. C 72 011001 (2005), Antiproton-proton channels in $J/\psi$ Decays.

[9] B. El-Bennich, M. Lacombe, B. Loiseau and S. Wycech, arXiv:0807.3454 [nucl-th], to appear in Phys. Rev. C, Paris $N\bar{N}$ potential constrained by recent antiprotonic-atom data and $\bar{n}p$ total cross sections.

[10] B. El-Bennich, M. Lacombe, B. Loiseau and R. Vinh Mau, Phys. Rev. C 59 2313 (1999), Refining the inner core of the Paris potential $N\bar{N}$.

[11] M. Pignone, M. Lacombe, B. Loiseau and R. Vinh Mau, Phys. Rev. C 50, 2710 (1994), Paris $N\bar{N}$ potential and recent proton-antiproton low energy data.

[12] J. Côté, M. Lacombe, B. Loiseau, B. Moussalam and R. Vinh Mau, Phys. Rev. Lett. 48 1319 (1982), On the nucleon-antinucleon optical potential.

[13] H. Pilkuhn, The Interaction of Hadrons (North Holland, Amsterdam, 1967), p. 167.

[14] S. Wycech and B. Loiseau, AIP. Conf. Pro. 796, 131 (2005), The $N\bar{N}$ quasi-bound states, atomic and $J/\psi$ evidence.

[15] S. Wycech, F. J. Hartmann, J. Jastrzebski, B. Klos, A. Trzcińska, and T. von Egidy, Phys. Rev. C 76, 034316 (2007), Nuclear surface studies with antiprotonic atoms $X$-rays.

[16] A. Sibirtsev, J. Haidenbauer, S. Krewald, Ulf-G. Meißner, and A. W. Thomas, Phys. Rev. D 71, 054010 (2005), Near-threshold enhancement in the $pp$ mass spectrum in $J/\psi$ decays.

[17] M. Lacombe, B. Loiseau, B. Moussalam and R. Vinh Mau, Phys. Rev. Lett. 91, 054010 (2003), Measurements of the strong interaction parameters in antiprotonic hydrogen and probable evidence for an interference with inner bremsstrahlung.

[18] A. Sibirtsev, J. Haidenbauer, S. Krewald, Ulf-G. Meißner, and A. W. Thomas, Phys. Rev. D 71, 054010 (2005), Near-threshold enhancement in the $pp$ mass spectrum in $J/\psi$ decays.

[19] M. Lacombe, B. Loiseau, B. Moussalam and R. Vinh Mau, Phys. Rev. Lett. 91, 054010 (2003), Measurements of the strong interaction parameters in antiprotonic hydrogen and probable evidence for an interference with inner bremsstrahlung.

[20] G. Källén, Elementary Particle Physics, (Addison-Wesley, Reading, MA, 1964), p. 433.

[21] P. Lenisa and F. Rathmann for P AX collaboration. This work was also supported in part by the Department of Energy, Office of Nuclear Physics, Contract No. DE-AC02-06CH11357.