Heavy neutrinos production and decay in future $e^+e^-$ colliders

J. Gluza\textsuperscript{1} and M. Zralek\textsuperscript{2}

Department of Field Theory and Particle Physics
Institute of Physics, University of Silesia
Uniwersytecka 4, PL-40-007 Katowice, Poland

Abstract

The production of heavy and light neutrinos in $e^+e^-$ future colliders is considered. The cross section for the process $e^+e^- \rightarrow \nu N$ and then the heavy neutrino decay $N \rightarrow W^\pm e^\mp$ is determined for experimentally possible values of mixing matrix elements. The bound on the heavy neutrino-electron mixing is estimated in models without right-handed currents. The role of neutrino CP eigenvalues and the mass of the lightest Higgs particle are investigated. The angular distribution of charged leptons in the total CM frame resulting from the heavy neutrino decay and from the main $W^+W^-$ production background process are briefly compared.

\textsuperscript{1}e-mail address: gluza@us.edu.pl
\textsuperscript{2}e-mail address: zralek@us.edu.pl
1 Introduction

Our experimental knowledge about neutrinos is still relatively small. The results of terrestrial experiments agree with the prediction of the standard model (SM) where neutrinos are massless, left current interacting particles. As a consequence we do not even know if neutrinos have Dirac or Majorana character. There are however, astrophysical observations and cosmological estimations which, most probably, require massive neutrinos [1]. There is also the first terrestrial experiment in which there is some indications that a neutrino oscillates [2] and as a consequence at least one should be massive. The existence of such small mass neutrinos is predicted by many extensions of the SM. Usually the light neutrinos are accompanied by neutrinos with large mass in such a way that the so called see-saw mechanism [3] occurs. The production of heavy neutrinos in the future linear colliders depends on their masses and couplings to known leptons and bosons. The couplings of a neutrino below the $M_Z$ mass are strongly restricted by present LEP data [4] so we will concentrate on neutrinos with masses above the $Z_0$ mass. If the explanation of small neutrino masses is given by the see-saw mechanism then the present experimental bounds for the light (eV-keV-MeV region) and the heavy neutrinos $M_N > M_Z$ give very small mixing angles. With such mixing angles the heavy neutrinos decouple from low energy physics and the cross section for their production in the future linear colliders is beyond our experimental interest. There are, however, models where light-heavy neutrino mixings are not connected with the see-saw mechanism. The general idea can be explained by an elementary example of the ‘light’ ($\nu$) and the ‘heavy’ ($N$) neutrino. Let us assume that in the $(\nu, N)^T$ basis the neutrino mass matrix is

$$ M = \begin{pmatrix} a & b \\ b & c \end{pmatrix}, $$

where for simplicity we assume that all elements $a, b, c$ are real numbers. The masses and the mixing angle are given by

$$ m_{1,2} = \frac{1}{2} \left( a + c \mp \sqrt{(a - c)^2 + 4b^2} \right), $$

and

$$ \sin 2\xi = \frac{2b}{\sqrt{(a - c)^2 + 4b^2}}. $$
There are two ways of predicting the light-heavy spectrum of neutrino masses. One is the see-saw mechanism where \( a=0, \ c \gg b \) and then

\[
\begin{align*}
| m_1 | & \simeq \frac{b^2}{c}, \\
| m_2 | & \simeq c \gg m_1,
\end{align*}
\]

and, unavoidably,

\[
\xi \simeq \frac{b}{c} \simeq \sqrt{\frac{|m_1|}{m_2}} \ll 1.
\]

The other one in which we assume that \( a \neq 0 \) and due to internal symmetry \( ac = b^2 \) gives

\[
\begin{align*}
m_1 & = 0, \\
m_2 & = a + c,
\end{align*}
\]

and

\[
\sin \xi = \frac{2\sqrt{ac}}{a + c}.
\]

If the symmetry, which at the tree level gives the relation \( ac = b^2 \), is broken we obtain

\[
m_1 \neq 0 << m_2
\]

in the higher order (see e.g. [5]). In this sort of models \( \sin 2\xi \) is not connected with the ratio \( m_1/m_2 \) and can be large (\( \sin 2\xi \simeq 1 \)) for \( a \simeq c \). Any model realizing this idea in the natural way is an alternative to the see-saw mechanism and helps to explain the spectrum of neutrino masses. Several kinds of such models were considered in literature [6]. In these scenarios the mixing angles are independent parameters not connected to the neutrino masses and are only bound by existing experimental data. In this paper we have found such boundary for mixing parameters which is model independent. We also assume that heavy neutrinos exist with such masses that they can be produced in future \( e^+e^- \) colliders [7]. With these assumptions we determined the cross section for the production of heavy and light neutrinos in future \( e^+e^- \) colliders. We have also considered the decay of heavy neutrinos \( N \rightarrow W^+l^- \) or \( W^-l^+ \) and the angular distribution of charged leptons in the total CM system. The decay channel is easily distinguished from the charged lepton production in various background processes where \( W^\pm \) pair production and decay are dominant. The effect of the lightest, SM Higgs particle on the process \( e^+e^- \rightarrow \nu N(W^+e^- \text{ or } W^-e^+) \) is also discussed. The process of production of heavy neutrinos in \( e^+e^- \) colliders has already
been considered in literature [8]. However, to our knowledge, the analysis with all
details mentioned above have not been performed.

In the next Chapter the bounds on mixing matrix elements using the full experi-
mental information are given. In Chapter 3 the angular distribution for final electron
(positron) in the process
\[ e^+e^- \rightarrow \nu N \]
\[ \leftrightarrow e^\pm W^\mp \]  \hspace{1cm} (8)
is calculated. Conclusions are given in Chapter 4.

2 Mixing matrix elements.

The cross sections for production and decay of heavy neutrinos (Eq. (8)) are given in
the Appendix (Eqs. A.1,A.2,A.3). The mixing matrix elements \( K_{NI} \) and \( \Omega_{N\nu} \) of the
lepton sector analog of Kobayashi-Maskawa matrices [9], decide about the magnitude
of the cross section. Precisely the helicity amplitudes are proportional to
\[ (K_{Ne})^2 K_{\nu e} \] in the t and u channels,

\[ K_{Ne}\Omega_{N\nu} \text{ where } \Omega_{N\nu} = \sum_{l=e,\mu,\tau} K_{NI}K_{\rho l}^* \] in the s channel. \hspace{1cm} (9)

From the present experimental data we are not able to determine all elements of the
K matrix. Fortunately, with good approximation only one mixing matrix element
\( K_{Ne} \) between electron and the lightest heavy neutrino N will decide about the size
of the cross section and it is possible to determine the bound on it from existing
experimental data. Phases of \( K_{NI} \)'s have no influence (there is no t-u interference)
and this means that no effects of CP violation are seen in the process [10].

In the previous paper [11] we have analyzed the existing experimental data which
restrict the mixing matrix elements. Three different combinations of light and heavy
neutrino masses and their mixing with leptons are possible to be limited:

(i) from the lack of lepton number violation processes (e.g. \( \mu \rightarrow e\gamma, \mu \rightarrow 3e, \mu \rightarrow e \)
conversion in nuclei [12]) and from the number of light neutrino species \( N_\nu \), it is
possible to get
\[ \sum_{N(\text{heavy})} |K_{Ne}|^2 \leq \kappa^2, \] \hspace{1cm} (10)
and the lack of a signal in neutrinoless double-β decay \((\beta\beta)_0\nu\) gives two bounds

(ii) for the light neutrinos
\[
| \sum_{\nu(\text{light})} K_{\nu e}^2 m_{\nu} | < k_{\text{light}}^2,
\]

(11)

(iii) for the heavy neutrinos
\[
| \sum_{N(\text{heavy})} K_{Ne}^2 \frac{1}{m_N} | < \omega^2.
\]

(12)

The matrix \(K\) must be unitary and this means that

(iv)
\[
\sum_{\nu(\text{light})} | K_{\nu e} |^2 + \sum_{N(\text{heavy})} | K_{Ne} |^2 = 1.
\]

(13)

In paper [11] we have also used the constraints which follow from the lack of Higgs triplets in considered gauge models. As a result, in the first order, the mass term for left-handed neutrinos does not appear. Here we will omit this assumption. In this way the limits which we get are model independent. To find the inequalities (10)-(12) only one model assumption is made, i.e. the lack of right-handed current hence our considerations are valid for any model without right-handed charged currents. We know however, that due to large mass of the right-handed gauge boson(s) \(W_R^\pm\), the influence of right-handed current on the production of one light and one heavy neutrino is marginal [13].

Using restrictions (i)↔(iv) the upper bound on \(K_{NI}\) mixing depends on 1) the number of heavy neutrinos \(n_R\) and 2) their CP parities \(\eta_{CP}\).

- \(n_R = 1\)

   For heavy neutrino with mass less than 1 TeV \((M < 1\ TeV)\) we get from relation (12)
\[
| K_{Ne} |^2 < \omega^2 M
\]

(14)
and the total cross section is bounded by the small value of \(\omega\) (see next Chapter).

- \(n_R = 2\)

   There are two heavy neutrinos with masses \(M_1 = M\) and \(M_2 = AM\ (A \geq 1)\). The couplings depend on the CP parities of both neutrinos. If they are the same e.g.
\[ \eta_{CP}(N_1) = \eta_{CP}(N_2) = +i \text{ then mixing parameters can be treated as real } K_{N_1e} = x_1 \text{ and } K_{N_2e} = x_2. \]

The relations (10) and (11) give

\[ x_1^2 + x_2^2 \leq \kappa^2, \]
\[ |x_1^2 + \frac{x_2^2}{A}| \leq \omega^2 M, \tag{15} \]

and the situation is the same as in case \( n_R = 1 \) (Eq. (14)), the coupling of the \( N_1 \) neutrino is small \( x_1^2 \leq \omega^2 M \). If, however, heavy neutrinos have opposite CP parities \( \eta_{CP}(N_1) = -\eta_{CP}(N_2) = i \) then \( K_{N_1e} = x_1, \ K_{N_2e} = ix_2 \) and the relations (10) and (12) give

\[ x_2^2 \leq \kappa^2 - x_1^2, \]
\[ x_2^2 \geq A(x_1^2 - \omega^2 M), \]

and

\[ x_2^2 \leq A(x_1^2 + \omega^2 M). \tag{16} \]

The sketch of the region in \( x_1^2 \leftrightarrow x_2^2 \) plane of still experimentally acceptable mixing parameters is shown in Fig.1. The maximum value of \( K_{N_1e}^2 \) is equal

\[ (K_{N_1e}^2)_{\text{max}} = \frac{\kappa^2 + AM \omega^2}{A + 1}. \tag{17} \]

Fig.1 Sketch of the region in \( x_1^2 \leftrightarrow x_2^2 \) plane of still experimentally acceptable mixing parameters for two heavy neutrinos. Maximum value of \( x_2^2 \) is equal \( (x_2^2)_{\text{max}} = \frac{\kappa^2 + AM \omega^2}{A + 1} \) and approaches \( \frac{\kappa^2 + M \omega^2}{2} \) for \( A \to 1 \).
\textbullet \ n_R = 3

If the CP parities of all neutrinos are the same \( \eta_{CP}(N_1) = \eta_{CP}(N_2) = \eta_{CP}(N_3) = \pm i \) then all couplings are small and the same inequality (14) as in the \( n_R = 1, 2 \) cases restricts the \( K_{N_1 e} \) mixing.

A more interesting situation arises if we assume that not all \( \eta_{CP} \)'s of neutrinos are the same. Let us assume that \( \eta_{CP}(N_1) = \eta_{CP}(N_2) = -\eta_{CP}(N_3) = +i \) then \( K_{N_1 e} = x_1, K_{N_2 e} = x_2 \) and \( K_{N_3 e} = i x_3 \). From relations (10) and (12) we obtain three inequalities (\( M_1 = M, M_2 = AM, M_3 = BM \))

\[
\begin{align*}
x_3^2 & \leq -x_1^2 - x_2^2 + \kappa^2, \\
x_3^2 & \geq B x_1^2 + B x_2^2 - BM \omega^2, \\
\text{and} & \\
x_3^2 & \leq B x_1^2 + B x_2^2 + BM \omega^2.
\end{align*}
\]

(18)

The region in \( (x_1^2, x_2^2, x_3^2) \) frame of still experimentally acceptable parameters is shown in Fig.2. The maximum value of \( K_{N_1 e}^2 \) is equal

\[
(K_{N_1 e}^2)_{\text{max}} = \frac{\kappa^2 + BM \omega^2}{B + 1}
\]

and can be as large as \( (\kappa^2 + M \omega^2)/2 \) for \( B \rightarrow 1 \).

The other combination of \( \eta_{CP} \)'s leads to the bound on \( K_{N_1 e}^2 \) which is the same as in the case \( n_R = 1 \) or to this given by Eq.(19). So finally we can state that regardless of the number of heavy neutrinos the most optimistic bound on \( |K_{Ne}|^2 \) is equal \( |K_{Ne}|^2 < \omega^2 M \) if there are no correlations between elements of the K matrix or \( |K_{Ne}|^2 < (\kappa^2 + \omega^2 M)/2 \) if there are correlations and some \( \eta_{CP} \)'s of heavy neutrinos are opposite.
Fig. 2 Sketch of the region in $(x_1^2, x_2^2, x_3^2)$ plane of still experimentally acceptable mixing parameters for three heavy neutrinos ($n_R = 3$). The region of acceptable parameters is bound by three reference frame planes $(x_1^2, x_2^2), (x_1^2, x_3^2), (x_2^2, x_3^2)$ and the planes indicated in the figure. The maximum value of $(x_1^2)$ is equal $(x_1^2)_{\text{max}} = \frac{\kappa^2 + BM \omega^2}{B + 1}$ and approaches $\frac{\kappa^2 + BM \omega^2}{B + 1}$ for $B \to 1$.

3 Numerical results

3.1 Production and decay of heavy neutrinos

The light neutrinos will not be detected in the process $e^+ e^- \to \nu N$ and we can only measure the sum

$$\sigma_{\text{tot}} = \sum_{i=e,\mu,\tau} \sigma(e^+ e^- \to \nu_i N), \quad (20)$$

over all light neutrinos. For $N$ we take the lightest heavy neutrino $N = N_1$. But from Eq.(A.1) (neglecting charged lepton masses)

$$\sigma_{\text{tot}} \propto |K_{N e}|^2 \left( |K_{\nu e}|^2 + |K_{\nu \mu}|^2 + |K_{\nu \tau}|^2 \right)$$
\[ |K_{Ne}|^2 (1 - \sum_N |K_{Ne}|^2)^2 \approx |K_{Ne}|^2. \]  

(21)

To calculate the cross section \( \frac{d\sigma}{d\cos \Theta_e} \) (Eq.(A.3)) we also need to know the total decay width \( \Gamma_N \) for heavy neutrino decay. From Eqs. (A.2) we can calculate the partial decay width for

\[ \Gamma(N \rightarrow W^\pm l^\mp) = \frac{|K_{Nl}|^2 G_F}{8\sqrt{2\pi} m_N^3} (m_N^2 + 2m_W^2)(m_N^2 - m_W^2)^2, \]  

(22)

and

\[ \Gamma(N \rightarrow Z\nu_l) = \frac{\Omega_{N\nu_l}^2 G_F}{8\sqrt{2\pi} m_N^3} (m_N^2 + 2m_Z^2)(m_N^2 - m_Z^2)^2. \]  

(23)

Whether the decay channels of the N into the lightest Higgs particle H and light neutrinos \( \nu_l \), \( N \rightarrow \nu_l H \) are opened depends on the relation between masses \( m_N \) and \( m_H \), if \( m_N > m_H \) the channels are opened and (see e.g. [5])

\[ \Gamma(N \rightarrow H\nu_l) = \frac{\Omega_{N\nu_l}^2 G_F}{8\sqrt{2\pi} m_N^3} (m_N^2 - m_H^2)^2. \]  

(24)

We will consider both situations where \( m_N > m_H \) and \( m_N < m_H \) when the decay channel is closed. However, since we are looking for a relatively light \( m_N \) (~ 100 ÷ 200 GeV) the situation where \( m_N < m_H \) (if \( m_H \sim 300 \) GeV) seems more plausible.

The total decay width we calculate from

\[ \Gamma_N = \sum_l \left( 2\Gamma(N \rightarrow l^+W^-) + \Gamma(N \rightarrow \nu_lZ) + \Gamma(N \rightarrow \nu_lH)\Theta(m_N - m_H) \right) \]  

(25)

where

\[ \sum_l \Gamma(N \rightarrow l^+W^-) \propto \sum_{l=e,\mu,\tau} |K_{Nl}|^2 \approx |K_{Ne}|^2, \]  

(26)

\[ \sum_l \Gamma(N \rightarrow \nu_lH), \sum_l \Gamma(N \rightarrow \nu_lZ) \propto \sum_l |\Omega_{N\nu_l}|^2 \approx \sum_l |K_{Nl}|^2 \approx |K_{Ne}|^2. \]  

(27)

In the approximations made in Eqs. (21, 26 and 27) we assume that in each column of K matrix \( (l = e, \mu, \tau) \)

\[ (K_{e\nu_l}, K_{\nu_\mu l}, K_{\nu_\tau l}, K_{N_l1}, K_{N_l2}, \ldots)^T \]
one element $K_{\nu l} \simeq 1$ (lepton universality) and only one coupling between heavy neutrinos and lepton is visible $K_{Nl} \simeq x$. All other couplings are very small and we neglect them.

The calculated decay width $\Gamma_N$ normalised to the factor $|K_{Ne}|^2$ for various masses $m_N$ is given in the Table 1.

Now we have all the ingredients to calculate the electron angular distribution in the process

$$e^+e^- \rightarrow \nu N \rightarrow \nu e^-W^+.$$ 

In our approximation only one parameter $|K_{Ne}|^2$ decides about the value of the cross section. For $n_R = 1$, regardless of the $\eta_{CP}$ of the heavy neutrino, and for $n_R > 1$ with the assumption that $\eta_{CP}$’s of all neutrinos are the same, $|K_{Ne}|^2$ is bounded by the lack of neutrinoless double $\beta$ decay (Eq.(14)). There are problems with estimating the role of heavy neutrinos in the $(\beta\beta)_{0v}$ process as the nuclear structure matrix elements are calculated with limited accuracy [14]. The best limit is found from absence of neutrinoless double beta decay in $^{76}Ge$ by Heidelberg-Moscow collaboration [15]

$$\omega^2 < 2 \cdot 10^{-5} \text{ TeV}^{-1}.$$ 

There are also other estimations of $\omega^2$. In paper [16] it was found that

$$\omega^2 < 2.8 \cdot 10^{-5} \text{ TeV}^{-1}.$$ 

In Table II we give the maximum values of $\sigma_{tot}(e^+e^- \rightarrow \nu N)$ (Eq.20) for various heavy neutrino masses $m_N$ and different total energies $\sqrt{s}$. The value of $\omega^2$ decides about $\sigma_{tot}(\text{max})$: $\sigma_{tot} \propto \omega^2$ and the values of the total cross section for various $\omega^2$ can be easily obtained from the Table. As the maximum value of $|K_{Ne}|^2$ is proportional to $m_N$ (see Eq.(14)) the cross section (Eq.(20)) increases with the heavy neutrino mass with the exception when $m_N \rightarrow \sqrt{s}$ at the end of the phase space.

For $n_R > 1$ and for different values of $\eta_{CP}$ of heavy neutrinos the bound from $(\beta\beta)_{0v}$ (Eq.14) is not so crucial and $|K_{Ne}|^2$ can be much larger (Eqs. (17) and (19)). In the both considered cases $n_R = 2$ and $n_R = 3$ the largest possible value is

$$|K_{Ne}|^2_{\text{max}} \rightarrow \frac{\kappa^2 + M[\text{TeV}]^2 \omega^2}{2} \frac{\omega^2}{M_{\leq 1\text{TeV}}} \frac{\kappa^2}{2} \quad (28)$$

for almost degenerate heavy neutrinos ($A \rightarrow 1$ for n=2, $A \gg B, B \rightarrow 1$ for n=3). In the case B=1 there are two Majorana neutrinos with the same masses and opposite CP parities which form the Dirac neutrino. In our studies, however, calculation of the
cross section for Dirac neutrino production is not performed. Different values of $\kappa^2$ are found for the model with singlet neutrinos: $\kappa^2 < 0.015$ [17] and the more recent one $\kappa^2 < 0.0054$ [18]. If we use the recent LEP result for the number of light neutrino species $N_\nu = 2.989 \pm 0.012$ [19] we obtain $\kappa^2 < 0.0055$, a value very close to the global fit given in [18]. In Table III the total cross section $\sigma(e^+e^- \to \nu N)$ for various $m_N$ and $\sqrt{s}$ is presented. Results are given for $\kappa^2 = 0.0054$. Since $\sigma_{tot} \propto |K_{Ne}|^2$, values of the $\sigma_{tot}$ for various $K_{Ne}$ can be easily obtained from this Table.

In Fig. 3 we present the angular distribution for the final electron $e^-e^+ \to \nu(N \to e^-W^+)$ for various masses of heavy neutrino $m_N = 100, 150$ and 200 GeV calculated for the maximum possible value of $|K_{Ne}|^2 \approx \frac{\kappa^2}{2}$. For $\kappa^2$ we take the value $\kappa^2 = 0.0054$. Results are given for the Next Linear Collider with CM energy $\sqrt{s} = 500$ GeV. This distribution has forward-backward symmetry. To show the influence of Higgs particle we present results for $m_H = 100$ GeV on the left side of the Figure ($-1 \leq \cos \Theta_e \leq 0$) and on the right side ($0 \leq \cos \Theta_e \leq 1$) the Higgs decay channels are excluded. For higher Higgs mass the total width $\Gamma_N$ is smaller and, due to the greater value of the branching ratio for the $N \to lW$ decay, the cross section $\frac{d\sigma}{d\cos \Theta_e}$ is larger. Numerically, Higgs has no influence on the cross section for $m_N = 100$ GeV (for $m_H \geq 100$ GeV the $N \to \nu H$ decay channel is closed) and the influence of the Higgs particle ($m_H = 100$ GeV) is approximately equal 10 %, 15% for $m_N = 150, 200$ GeV, respectively. For higher energies the final electron distribution is more peaked in the forward-backward direction ($\cos \Theta_e = \pm 1$). This is the result of $W^\pm$ exchange in t and u channels and small contribution of the s channel $Z^0$ exchange. For $\sqrt{s} = 0.5$ TeV the $Z^0$ exchange mechanism gives only 2% contribution to the total cross section [12] and is smaller for higher energies. As an example we compared final electron distribution produced by the decay of a heavy neutrino with mass $M_N = 100$ GeV for $\sqrt{s} = 500$ and 1000 GeV (Fig. 4).
Fig. 3 Distribution of the final electron from a heavy neutrino decay for $\sqrt{s} = 500$ GeV collider with $M_N = 100$ GeV (solid line), $M_N = 150$ GeV (long-dashed line) and $M_N = 200$ GeV (short-dashed line). Left half of the Figure gives results for $m_H = 100$ GeV. On the right-hand side the Higgs decay channels are excluded.
Fig. 4 Backward distribution of the final electron coming from a heavy neutrino decay ($M_N = 100$ GeV) for two different energies: $\sqrt{s} = 500$ GeV (dashed line) and $\sqrt{s} = 1000$ GeV (solid line). Forward distribution is the same.

Finally in Fig. 5 we present the angular distribution $d\sigma/d\cos(\Theta_e)$ for various masses of heavy neutrino $M_N = 100$, 300 and 500 GeV (for $m_H = 100$ GeV). The cross section becomes higher and more peaked in the forward-backward direction for smaller mass of heavy neutrinos. The effect of growing $d\sigma/d\cos(\Theta_e)$ is the result of increasing $BR(N \rightarrow lW)$ and increasing of $\sigma_{tot}(e^+e^- \rightarrow \nu N)$ (Table III) for smaller $m_N$. The effect of slope reducing with $m_N$ mass in the forward-backward direction is also kinematically understandable.
Fig. 5 Backward distribution of the final electron coming from a heavy neutrino decay with mass $M_N = 100$ GeV (solid line), $M_N = 300$ GeV (short-dashed line) and $M_N = 500$ GeV (long-dashed line) for $\sqrt{s} = 1$ TeV. Forward distribution is the same.

The main background process is the production of $W^+W^-$ pair and then the $W^\pm \rightarrow e^\pm \nu$ decay. The distribution of charged lepton coming from the heavy neutrino decay (N) already mentioned in this paper and from $W$’s decays by $e^+e^- \rightarrow W^+W^-$ process differs very much in forward-backward direction. For high energy ($\sqrt{s} > 0.5$ TeV) angular distribution of electrons coming from the $W^-$ decay is peaked in the forward direction and has reducing slope in background direction. On the contrary, the $e^-$ coming from N decay will travel equally well both in the forward and the backward direction with increasing slope of angular distribution for $|\cos \Theta_e| \rightarrow 1$ (Figs.3-5).
4 Conclusions

We have found the cross section for heavy and light neutrino production in future electron-positron colliders with energy $\sqrt{s} \geq 0.5$ TeV. The bounds on mixing matrix element $K_{Ne}$ between heavy neutrino and electron are found from existing experimental data in models without right-handed currents. The maximum possible value of the $K_{Ne}$ is very small if there is only one heavy neutrino ($n_R = 1$) or, in the case of a larger number of heavy neutrinos ($n_R > 1$), if their CP eigenvalues are the same. This small bound results from the lack of a signal in neutrinoless double-$\beta$ decay. In this case the cross section for production of light and heavy neutrinos ($e^+e^- \rightarrow \nu N$) is very small from 0.16 fb for $\sqrt{s} = 0.5$ TeV and $m_N = 100$ GeV up to 1.6 fb for $\sqrt{s} = 2$ TeV and $m_N = 1$ TeV.

The lack of any signal from neutrinoless double-$\beta$ decay does not give such a restrictive bound if the CP eigenvalues of two or more heavy neutrinos are not the same. Now the $e^+e^- \rightarrow \nu N$ cross section can be larger and equals $\sigma = 240(287)$ fb for $\sqrt{s} = 0.5(2)$ TeV and $m_N = 100$ GeV. We have also found angular distribution of the final charged lepton in the total CM frame resulting from the heavy neutrino decay. The angular distribution has forward-backward symmetry, contrary to background process e.g. $e^+e^- \rightarrow W^+W^- (\rightarrow e^-\nu_e)$. This property could point to the existence of a heavy neutrino. The charged lepton angular distribution depends on CM energy, mass of the heavy neutrino and mass of the lightest Higgs boson.

Appendix

We would like to present the cross section for production ($e^+e^- \rightarrow \nu N$) and decay of Majorana neutrino $N \rightarrow l^{\pm}W^{\mp}, \nu'Z$ processes which are very useful in practical application. We consider the $Ne^-W^+$ interaction without the right-handed coupling and neglected the electron mass ($m_e = 0$).

Production process $e^+e^- \rightarrow \nu N$

The production process $e^-(\bar{\sigma}) + e^+(\bar{\sigma}) \rightarrow \nu(\lambda) + N(\bar{\lambda})$ is described by 8 helicity amplitudes ($\Delta\sigma = \sigma - \bar{\sigma}, \Delta\lambda = \bar{\lambda} - \lambda$)

$$M(\Delta\sigma; \lambda, \bar{\lambda}) = (\sqrt{2})^{1+|\Delta\lambda|} \left\{ \frac{A_t}{t-M_W^2} - \frac{A_u}{u-M_W^2} + \frac{A_s}{s-M_Z^2 + iM_Z\Gamma_Z} \right\} D_{\Delta\sigma,\Delta\lambda}^1(\phi, \Theta, 0),$$  \hspace{1cm} (A.1)
where $A_{t,u,s}$ are functions of fermion helicities

$$
A_t(\Delta \sigma, \lambda, \bar{\lambda}) = K^*_N K_{\nu e} \sqrt{1 + 2 \bar{\lambda} \beta \delta \lambda_{-1/2} \delta \Delta \sigma_{-1}},
$$

$$
A_u(\Delta \sigma, \lambda, \bar{\lambda}) = K^*_N K_{\nu e} \sqrt{1 - 2 \bar{\lambda} \beta \delta \lambda_{+1/2} \delta \Delta \sigma_{-1}},
$$

$$
A_s(\Delta \sigma, \lambda, \bar{\lambda}) = \left[ \frac{1}{2} (-1 + 2 \tan^2 \Theta_W) \delta \Delta \sigma_{-1} + \tan^2 \Theta_W \delta \Delta \sigma_{+1} \right]
\times \left[ \Omega_{N\nu} \sqrt{1 + 2 \bar{\lambda} \beta \delta \lambda_{-1/2} \delta \Delta \sigma_{+1}} - \Omega^*_{N\nu} \sqrt{1 - 2 \bar{\lambda} \beta \delta \lambda_{+1/2} \delta \Delta \sigma_{+1}} \right],
$$

and

$$
\beta = \frac{s - s_N}{s + s_N},
$$

s,t,u are ordinary Mandelstam variables; $\Theta$ and $\phi$ are CM azimuthal and polar angles of the heavy Majorana neutrino $N$ with respect to the initial electron, $\sqrt{s_N}$ is the invariant mass of the heavy neutrino, $\Theta_W$ is the Weinberg angle.

Decay process $N \rightarrow l^\pm W^\mp, \nu Z$

In the helicity rest frame of $N$ ($\Theta_e^*$ and $\phi_e^*$ are $l^\pm$ or $\nu$'s azimuthal and polar angles respectively) the decay process $N(\bar{\lambda}) \rightarrow V(\lambda_V) + f(\lambda_f)$ is described by 4 helicity amplitudes (the final fermion mass is neglected, $M_V$ is the gauge boson mass)

$$
T(\bar{\lambda}; \lambda_V; \lambda_f) = \sqrt{s_N - M^2_V} F_{\lambda_V \lambda_f} D^{1/2}_{\lambda_V \lambda_f - \lambda_V} (\phi^*_f, \Theta^*_f, 0) \quad (A.2)
$$

where

$$
F_{++} = \sqrt{2} X, \quad F_{--} = \sqrt{2} Y, \quad F_{0+} = \frac{\sqrt{s_N}}{M_V} X,
\quad F_{0-} = \frac{\sqrt{s_N}}{M_V} Y, \quad F_{+-} = F_{-+} = 0
$$

and

$$
\begin{cases}
X = -\frac{e}{\sqrt{2} \sin \Theta_W} K_{Ne}, & Y = 0 \\
X = 0, & Y = \frac{e}{\sqrt{2} \sin \Theta_W} K^*_{Ne}, \quad \text{for } N \rightarrow W^- l^+, \\
X = -\frac{g}{2 \sin \Theta_W \cos \Theta_W} \Omega_{N\nu}, & Y = \frac{g}{2 \sin \Theta_W \cos \Theta_W} \Omega^*_{N\nu}, \quad \text{for } N \rightarrow \nu Z.
\end{cases}
$$

Full cross section
The angular distribution of the final lepton in the $e^+e^- \rightarrow \nu N(\rightarrow e^\pm W^\mp)$ is given by (where $\Theta_e, \phi_e$ are the CM azimuthal and polar angles of final $e^\pm$ with respect to the initial electron ($e^-$))

$$
\frac{d\sigma}{d\cos\Theta_e} = \frac{G_F^2 M_W^2}{2^{14} s^2 \pi^5} \int_0^{2\pi} d\phi \int_{-1}^1 d\cos \Theta \int_0^{2\pi} d\phi_e \int_{M_W^2}^s ds_N
J \frac{(s_N - M_W^2)(s - s_N)}{s_N \left[(s_N - m_N^2)^2 + M_W^2 \Gamma_N^2\right]}
\sum_{\Delta\sigma, \lambda, \lambda_V, \lambda_f} \sum_{\lambda} M \left(\Delta\sigma; \lambda, \tilde{\lambda}\right) T \left(\tilde{\lambda}; \lambda_V, \lambda_f\right) |^2
$$

(A.3)

where $J$ is the Jacobian of the transformation between the $e^\pm$ angles in the CM frame of the decaying neutrino and the CM frame of initial colliding leptons

$$
J = \frac{1 - \beta^2}{(1 - \beta z)^2 w} \left\{ \sin^2 \Theta \sin^2 (\phi_e + \phi) \\
+ \cos \Theta \sin \Theta_e - \sin \Theta \cos \Theta_e \cos (\phi_e + \phi) \right\}^2
$$

(A.4)

where

$$
w = \sin^2 \Theta_e \sin^2 (\phi_e + \phi) \\
+ \cos \Theta \sin \Theta_e \cos (\phi_e + \phi) - \sin \Theta \cos \Theta_e \right)^2,
$$

(A.5)

and

$$
z = \sin \Theta \sin \Theta_e \cos (\phi_e + \phi) + \cos \Theta \cos \Theta_e.
$$

(A.6)

The amplitude $T(\tilde{\lambda}, \lambda_V, \lambda_f)$ in Eq.(A.2) is introduced in the CM frame of the decaying neutrino. We need to determine the exact dependence between $\Theta_e, \phi_e$ and $\Theta_e^*, \phi_e^*$ variables. They are given by the relations

$$
\cos \Theta_e^* = \frac{-\beta + z}{1 - \beta z},
$$

(A.7)

$$
\tan \phi_e^* = \frac{\sin \Theta_e \sin (\phi_e - \phi)}{\cos \Theta \sin \Theta_e \cos (\phi_e - \phi) - \sin \Theta \cos \Theta_e}
$$

(A.8)

and

$$
sign(\sin \phi_e^*) = sign(\sin (\phi_e + \phi)),
$$

(A.9)

$$
(tan \phi_e^* and sign(\sin \phi_e^*) describe the $\phi_e^*$ univocally in the region $0 < \phi_e^* < 2\pi$).

16
Acknowledgements

This work was partly supported by the Polish Committee for Scientific Research under Grant No. PB 659/P03/95/08 and by the Curie Skłodowska grant MEN/NSF 93-145.

References

[1] G.Gelmini, E.Roulet, Rep. Prog. Phys. 58 (1995) 1207.

[2] The LSND Collaboration, C. Athanassopoulos et al., Phys.Rev.Lett. 75(1995)2650; ibid. 77(1996)3082.

[3] T. Yanagida, Prog. Theor. Phys. B135 (1978) 66; M. Gell-Mann, P. Ramond and R. Slansky, in ‘Supergravity’, eds. P. Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam, 1979) p.315.

[4] L3 Collaboration, O. Adriani et al., Phys. Lett. B295 (1992) 371 and B316 (1993) 427.

[5] R.N. Mohapatra, P.B. Pal, ”Massive neutrinos in physics and astrophysics”, World Scientific, 1991.

[6] D. Wyler and L. Wolfenstein, Nucl. Phys. B218 (1983) 205; R.N. Mohapatra and J.W.F. Valle, Phys. Rev. D34 (1986) 1642; E. Witten, Nucl. Phys. B268 (1986) 79; J. Bernabeu et al., Phys. Lett. B187 (1987) 303; J.L. Hewett and T.G. Rizzo, Phys. Rep. 183 (1989) 193; P. Langacker and D. London, Phys. Rev. D38 (1988) 907; E. Nardi, Phys. Rev. D48 (1993) 3277; D. Tommasini, G. Barenboim, J. Bernabeu and C. Jarlskog, Nucl. Phys. B444 (1995) 451.

[7] R. Palmer, ”Future accelerators”, plenary talk at ICHEP, Warsaw 1996.

[8] (a): For LEPI energy and below the process $e^+e^- \rightarrow \nu N$ was studied before, see e.g. A. Ali, Phys.Rev.D10 (1974) 2801; M. Gourdin, X.Y. Pham, Nucl. Phys. B164 (1980) 387; J.L. Rosner, Nucl. Phys. B248 (1984) 503; M. Ditmar, A. Santamaria, M.C. Gonzales-Garcia and J.W.F. Valle, Nucl. Phys. B332 (1990)1; M.C. Gonzales-Garcia, A. Santamaria and J.W.F. Valle, Nucl. Phys. B342 (1990) 108; J. Kugo and S.Y. Tsai Prog. Theor. Phys. 86 (1991) 183; J.W.F. Valle Nucl.Phys.Proc.Suppl.
(b): above LEPI energy, see e.g. F. del Aguila, E. Laermann and P. Zerwas, Nucl. Phys. **B297** (1988) 1; E. Ma and J. Pantaleone, Phys. Rev. **D40** (1989) 2172; W. Buchm"uller and C. Greub, Nucl. Phys. **B363** (1991) 349 and **B381** (1992) 109; J. Maalampi, K. Mursula and R. Vuopionper"a, Nucl. Phys. **B372** (1992) 23; M.C. Gonzales-Garcia, O.J.P. Eboli, F. Halzen and S.F. Noaves Phys. Lett. **B280** (1992) 313; R. Vuopionper"a Z. Phys. **C65** (1995) 311.

[9] see e.g. J. Gluza and M. Zralek, Phys. Lett. **B362** (1995) 148.

[10] J. Gluza and M. Zralek, Phys. Rev. **D51** (1995) 4707.

[11] J. Gluza and M. Zralek, Phys. Lett. **B372** (1996) 259.

[12] B.W. Lee, R. Shrock, Phys.Rev. **D16** (1977) 1444; B.W. Lee, S. Pakvasa, R. Shrock, H. Sugawara, Phys.Rev.Lett. **38** (1977) 937; W. Marciano, A.I. Sanda, Phys.Lett. **B37** (1977) 303; T.P. Cheng, L.F. Li, Phys.Rev. **D44** (1991) 1502.

[13] J.Gluza and M.Zralek, Phys.Rev. **D48** (1993) 5093.

[14] C.A. Heusch and P. Minkowski, [hep-ph/9611333](https://arxiv.org/abs/hep-ph/9611333).

[15] A. Balyshev et. al., Phys. Lett. **B356** (1995) 450.

[16] T. Bernatowicz et. al., Phys. Rev. Lett. **69** (1992) 2341.

[17] E.Nardi, E.Roulet and D.Tommasini, Nucl. Phys. **B386** (1992) 239; A. Ilakovac and A. Pilaftsis, Nucl. Phys. **B437** (1995) 491.

[18] A.Djoudi, J.Ng and T.G.Rizzo, [hep-ph/9504210](https://arxiv.org/abs/hep-ph/9504210).

[19] A. Blondel, ”Status of the electroweak interactions”, plenary talk at ICHEP, Warsaw 1996.
| $M_N$ [GeV] | $\Gamma^\text{total} / |K_{Ne}|^2$ [GeV] |
|----------------|-------------------|
|                | $m_H = 100$ GeV    | $m_H \geq m_N$ GeV |
| 100            | 0.22              | 0.22               |
| 150            | 2.9               | 2.6                |
| 200            | 8.7               | 7.2                |
| 300            | 33.1              | 26.1               |
| 500            | 160.2             | 143                |
| 700            | 445.5             | 337.5              |
| 1000           | 1306              | 984                |

**Table 1.** The total width for a heavy neutrino decay divided by mixing matrix element $|K_{Ne}|^2$ with the decay channels $\Gamma(N \to \nu_H)$ (second column) and without these channels (third column) for various heavy neutrino masses $m_N$. 
Table 2. Total cross section $\sigma_{tot}(e^+e^- \rightarrow \nu N)$ in $n_R = 1$ case (see Eq.(14) with $\omega^2 = 2 \cdot 10^{-5} \text{TeV}^{-1}$) for various heavy neutrino masses and three different total energies $\sqrt{s} = 0.5, 1, 2 \text{ TeV}$. If $\omega^2 \simeq 80 \cdot 10^{-5} \text{TeV}^{-1}$ [13] all numbers in the Table should be multiplied by 40.

| $M_N$ [GeV] | $\sigma_{max}^{total}$ [fb], $n_R = 1$ |
|-------------|-------------------------------------|
| $\sqrt{s} = 0.5 \text{ TeV}$ | $\sqrt{s} = 1 \text{ TeV}$ | $\sqrt{s} = 2 \text{ TeV}$ |
| 100         | 0.18                                | 0.2                          | 0.2                          |
| 150         | 0.25                                | 0.3                          | 0.3                          |
| 200         | 0.31                                | 0.4                          | 0.4                          |
| 300         | 0.34                                | 0.6                          | 0.6                          |
| 500         | -                                   | 0.8                          | 1.0                          |
| 700         | -                                   | 0.7                          | 1.3                          |
| 1000        | -                                   | -                            | 1.6                          |
| $M_N$ [GeV] | $\sigma_{\text{tot}}^\text{max}$ [fb] $n_R > 1$ |
|---|---|---|
| $\sqrt{s} = 0.5$ TeV | $\sqrt{s} = 1$ TeV | $\sqrt{s} = 2$ TeV |
| 100 | 240 | 275 | 287 |
| 150 | 227 | 271 | 286 |
| 200 | 209 | 267 | 285 |
| 300 | 155 | 252 | 281 |
| 500 | - | 207 | 270 |
| 700 | - | 138 | 252 |
| 1000 | - | - | 216 |

**Table 3** Total cross section $\sigma_{\text{tot}} (e^+e^- \rightarrow \nu N)$ for various heavy neutrino masses and total energies $\sqrt{s}$ calculated with largest possible value of $|K_{Ne}|^2$ ($n_R > 1$ case, see Eq.(28)). Result is given for $\kappa^2 = 0.0054$. 