Self-oscillating pump in a topological dissipative atom–cavity system

Pumps are transport mechanisms in which direct currents result from a cyclic evolution of the potential\(^1\). As Thouless showed, the pumping process can have topological origins, when considering the motion of quantum particles in spatially and temporally periodic potentials\(^1\). However, the periodic evolution that drives these pumps has always been assumed to be imparted from outside, as has been the case in the experimental systems studied so far\(^4\)–\(^12\). Here we report on an emergent mechanism for pumping in a quantum gas coupled to an optical resonator, where we observe a particle current without applying a periodic drive. The pumping potential experienced by the atoms is formed by the self-consistent cavity field interfering with the static laser field driving the atoms. Owing to dissipation, the cavity field evolves between its two quadratures\(^13\), each corresponding to a different centrosymmetric crystal configuration\(^14\). This self-oscillation results in a time-periodic potential analogous to that describing the transport of electrons in topological tight-binding models, such as the paradigmatic Rice–Mele pump\(^15\). In the experiment, we directly follow the evolution by measuring the phase winding of the cavity field with respect to the driving field and observing the atomic motion in situ. The observed mechanism combines the dynamics of topological and open systems, and features characteristics of continuous dissipative time crystals.

Models for geometrical pumps in lattices show singularities in their parameter space, which lead to pumping as soon as these are encircled\(^16\)–\(^19\), with the transported charge proportional to the winding number. For filled bands, the current is quantized in this number and topologically protected, as recognized by Thouless\(^1\). Such singularities appear in lattices with bipartite unit cells, for example, in the Rice–Mele model\(^15\). This paradigmatic model describes a tight-binding lattice with alternating on-site energies and hopping integrals, where the singularity corresponds to the simultaneous degeneracies of both. If the singularity is periodically encircled by these two parameters, transport can be induced. This is the basis for topological pumps realized in cold atoms, where a periodic evolution of the polarization owing to the motion of the Wannier center\(^21\). In the case of a centrosymmetric crystal, the Wannier periodic evolution of the polarization turns into a cyclic evolution, after which the potential landscape resembles itself relative to the former along the joint lattice axis. The pumping follows the spatial period\(^9\),\(^10\),\(^17\)–\(^19\). An external drive then slowly moves the latter relative to the former along the joint lattice axis. The pumping follows a cyclic evolution, after which the potential landscape resembles itself with the centre of the Wannier functions being shifted by a unit vector.

The location of the Wannier centre in the unit cell defines the polarization of the system, which is the real space manifestation of the band topology\(^20\)–\(^22\). The pumping current can thus be understood as the periodic evolution of the polarization owing to the motion of the Wannier centre\(^22\). In the case of a centrosymmetric crystal, the Wannier centre may sit either on or inbetween the inversion centre, defining two different \(Z_2\) symmetry classes. When evolving continuously between the different centrosymmetric classes without closing the bandgap, a current must occur. This argument can be extended to pumping mechanisms in two-dimensional environments\(^23\).

In this work, we show that geometrical pumping can emerge self-consistently without applying any external time-dependent drive. We demonstrate this using a Bose–Einstein condensate (BEC) coupled to a dissipative cavity, where the dynamics is governed by the self-consistent interplay between matter and light fields\(^24\). The geometrical pumping process is set in motion by dissipation after crossing a dynamical instability point of a self-organized phase, which marks a transition from a steady state to a self-oscillating state\(^25\). Coupling internal degrees of freedom to a cavity, a self-oscillation between density and spin waves has recently been observed in a spinor BEC\(^26\). Such emerging dynamical phases and persistent oscillations are a distinctive feature of non-Hermitian and non-reciprocal physics\(^26\),\(^27\).

In our experimental set-up, a BEC of \(^{87}\)Rb atoms is placed within an optical resonator. We illuminate the atoms with a pair of counter-propagating laser beams, with different Rabi rates \(\Omega_{1,2}\), and that intersect the cavity mode at a 60° angle (Fig. 1a). Collective light scattering from the transverse beams into the cavity is accompanied by a spatial symmetry breaking in the BEC\(^28\), which forms a density wave by self-organizing in the optical lattice generated by the interference of the cavity field with the transverse beams. The resulting potential \(V_{\text{lattice}} = V_{\text{atom}} + V_{\text{long}}(\phi)\) consists of a short-space two-dimensional lattice \(V_{\text{short}}\), fixed in space, and a lattice \(V_{\text{long}}\) with doubled periodicity, whose position depends on the time phase \(\phi\) between the transverse beam and the cavity field (Fig. 1b). The total lattice has two \(Z_2\) symmetry classes, which are set by \(\phi\) for \(0\) or \(\pi\), the scattered field is in-phase with the coupling beams and for \(\pi/2\) it out of phase, that is, the atoms

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Symmetry classes, the corresponding eigenfrequencies become obtained from averaging over classes of the crystals.

After a full revolution, the wavefunction’s centre of mass (yellow crosses) has been shifted by one unit cell, closing a pump cycle. The dashed rectangle on top of the lattice potentials highlights the unit cell.

Ω beams (with Rabi rates $\Delta$, and the photon dissipation rate $\kappa$, as expected for an oscillator driven close to resonance. This phase shift leads to a local distortion of the effective potential minima and the equilibrium position of the atoms is shifted with respect to the situation with $\Delta > \kappa$. This distortion reflects the dissipative coupling between the cavity quadratures. If the system is prepared in vicinity to the transition point between the two $\mathbb{Z}_2$ symmetry classes, the corresponding eigenfrequencies become comparable. For sufficiently strong dissipative coupling, these modes hybridize at an exceptional point, encompassing one mode that experiences gain and thus leads to a dynamical instability$^{13,29,30}$. In our case, the system is dragged in a chiral motion in quadrature space around a singularity and simultaneously through the $\mathbb{Z}_2$ classes of the crystals.

We detect the emergence of this oscillation in the real-time recording of the quadratures of the cavity field. The cavity detuning $\Delta$, and the power of the transverse beams, measured in terms of the standing-wave potential depth $V_\text{c}$, are used as experimental control parameters. In our experimental protocol, we increase the power of the transverse beams linearly in time while keeping the detuning $\Delta$, (that is, $\delta_\kappa$) at a fixed value. Figure 2a shows a trace of a heterodyne measurement of the cavity field phase $\phi$. At the beginning of the ramp, the system is strongly coupled to the real quadrature $Q$, whereas the imaginary quadrature $P$ is prevailing at the end. In these limiting cases, the phase takes on a defined steady-state value that is determined by the dominant coupling, indicating an initially broken $\mathbb{Z}_2$ symmetry. Inbetween these steady states, the dynamical instability prevails, and we observe a monotonic increase of the phase $\phi$ in a step-like fashion. This region of self-oscillation widens and the number of windings increases as we repeat the experiment at progressively smaller $|\Delta|$, (Fig. 2). This is consistent with dissipative effects becoming more dominant as $\Delta/k \to 0$. In Fig. 2c, we have merged the data obtained for a wide range of detunings $\Delta$, to a phase diagram. The heterodyne data plotted in optical quadrature space (Fig. 2d), highlights the chiral dynamics of the intracavity coherent field $\alpha$. The ellipsoidal shape, as much as the step-like behaviour of the time evolution of $\phi$, is a direct consequence of the self-consistency, as we will detail in the following.

Each horizontal line shows the mean phase $\bar{\phi}$ obtained from averaging over 50 experimental runs. The polar angle is folded into the first quadrant, $\bar{\phi} \in [0, \pi/2]$, such that an increasing phase appears as $n/4$ on average. The horizontal dashed line indicates the value of $\Delta/k$ at which the trace in a was recorded. Polar plot of the cavity field $\alpha$ in quadrature space, same data as in a. The radii indicate the modulus of $|\alpha|$ and the colour map follows the time axis.

**Fig. 1** | Non-stationary lattice in a dissipative atom–cavity system. 
a. A synthetic crystal is realized by a $^{87}$Rb BEC self-organizing in an optical cavity. Photons scatter in the cavity from a pair of imbalanced transverse beams (with Rabi rates $\Omega_1, \Omega_2$) and leak from the cavity at a rate $\kappa$. A heterodyne set-up records in real time the coherent cavity field $\alpha = |\alpha|e^{i\phi}$. 

**Fig. 2** | Emergent dynamics of the intracavity field. 
a. Single trajectory of the time evolution of the intracavity field phase $\phi$, recorded for at $|\Delta|/\kappa = 6.94(2)$. The coupling $V_\text{c}$ ramps linearly from $12(1) E_r$ to $49(5) E_r$, where $E_r$ is the recoil energy. The insets show the calculated real space potentials associated to a purely real or imaginary steady-state field quadrature (Q and P, respectively). 

b. The same as in a, but for different dissipative coupling strengths. From top to bottom, $|\Delta|/\kappa = 5.31(2), |\Delta|/\kappa = 5.99(2)$ and $|\Delta|/\kappa = 8.50(2)$. 

c. Phase diagram.

| Time (ms) | $V_\text{c}$ ($E_r$) |
|-----------|---------------------|
| 0         | 15                   |
| 1         | 30                   |
| 2         | 45                   |

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|-----------|---------------------|
| 0         | 15                   |
| 1         | 30                   |
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|-----------|---------------------|
| 0         | 15                   |
| 1         | 30                   |
| 2         | 45                   |
An explicit connection between quadratures and crystal structures comes from a microscopic picture of the optical potentials. The short-period lattice is generated by the standing waves

$$V_{\text{short}} = V_{\text{c}} \cos^2(k_x r) + hU_0 \delta a \cos^2(k_y r),$$  

(1)

where the momenta $k_x$ and $k_y$ (with $|k| = k = 2\pi/\lambda$) are oriented along the transverse coupling beams and the cavity axis, respectively. The period of the potential is set by the wavelength $\lambda = 780$ nm of the light, which we detune by $\Delta = +2\pi \times 3.1(1)$ GHz with respect to the D$_1$ transition frequency of $^{87}$Rb. At this detuning, spontaneous scattering is largely suppressed and the potential depths can be approximated by off-resonant two-photon scattering, yielding $V_{\text{c}} = \hbar \Omega \Omega / \Delta_\perp$ for the lattice formed by the coupling beams and $\delta a \hbar \Omega / \Delta_\parallel$ for the intracavity lattice. The latter is proportional to the intracavity photon number $\delta a^\dagger \delta a$, with $\delta a^\dagger$ the photon annihilation (creation) operator. Here $\hbar$ is the reduced Planck constant and $\Omega$ is the single photon vacuum Rabi rate of the cavity. The interference lattice is given by

$$V_{\text{long}} = \hbar \eta_q \hat{q} \cos(k_x r) \cos(k_y r) + \hbar \eta_p \sin(k_x r) \cos(k_y r),$$

(2)

where $\eta_q = g(\Omega_1 \pm \Omega_2) / \Delta_\perp$ are the two-photon scattering rates between the transverse beams and the cavity mode. The operators of the quadratures of the cavity field are given by $\hat{q} = (\hat{a} + \hat{a}^\dagger) / 2$ and $\hat{p} = (\hat{a} - \hat{a}^\dagger) / 2i$. A finite cavity field $\langle \hat{q} \rangle = 0$ is the signature of a self-organized phase, in which the atomic density is periodically modulated in space, such that the Bragg condition for the scattered light is fulfilled. This density wave breaks the discrete $\mathbb{Z}_2$ symmetry of the Hamiltonian, which is invariant under the operation $(\hat{a}, \hat{p}) \rightarrow (\hat{a}^\dagger, \hat{p} + \hbar \Omega / \Delta_\parallel)$. The phase transition happens above a critical coupling strength of $\Delta_\perp$.

From equation (2), it is evident that $\langle \hat{q} \rangle = Q = 0$ or $\langle \hat{p} \rangle = P = 0$ result in different lattice structures. Couplings to both optical quadratures is facilitated by imbalanced Rabi rates ($\Omega_1 \pm \Omega_2$, Methods). The resulting symmetry between $\eta_q$ explains the difference in cavity-field amplitude in the two crystalline phases, leading to the ellipsoidal trajectory in Fig. 2d. For the chosen atomic detuning $\Delta_\parallel > 0$, the self-organized phase corresponding to the real quadrature $\hat{q}$ vanishes for increasing transverse lattice depth. Therefore, increasing the coupling strength results in lowering (closing) and leads to a competition with the crystal structure with $\langle \hat{p} \rangle \neq 0$.

Photons leaking out of the cavity provide real-time access to the atomic distribution, probing the phase transitions and the atomic currents. In our experiment, the cavity-field phase is given by (Methods)

$$\phi = \phi_k + \phi_{\text{ap}} = \tan^{-1} \left( \frac{\Delta \eta_q \Theta_y - \eta_p \Theta_y}{\eta_q \Theta_x + \Delta \eta_p \Theta_x} \right),$$

(3)

containing the information on the atomic distribution through the expectation values of the overlap integrals $\Theta_x = \langle \cos(k_x r) \cos(k_y r) \rangle$ and $\Theta_y = \langle \sin(k_y r) \cos(k_x r) \rangle$. Here $r = (x,y)$ is the spatial coordinate in the lattice plane and the bare cavity detuning $\Delta_\perp$ has been replaced by $\Delta_\parallel$, which is the dispersive shifted resonant frequency of the cavity (Methods). Therefore, the data in Fig. 2 provide an indirect measurement of the atomic displacement through the overlap integrals in equation (3), that is, a change in the phase of the light field signals the position of the atomic cloud has shifted in real space.

The chiral evolution of $\phi$ (for example, Fig. 2d) demonstrates the presence of a pumping mechanism, as it implies a monotonous displacement of the atoms in a non-vanishing optical potential. As further confirmation, we perform in situ measurements of the centre-of-mass position of the atomic cloud. We observe that the cloud moves in real space, mainly in the direction orthogonal to the transverse beams, with a displacement that correlates with the observed windings of the photon phase $\phi$ (Fig. 3). We observe a displacement of less than one lattice site per pump cycle. We attribute this to the inhomogeneous filling of the lowest band and to reduced adiabaticity for increasing pump speeds. However, the movement fully correlates with the number of windings, demonstrating the robustness of the transport (Methods).

The angle between the transverse beams and the cavity is crucial for pumping. For deep $V_{\text{c}}$, the atomic distribution resembles a collection of one-dimensional bipartite chains along the $y$ direction, which is reminiscent of the one-dimensional Rice–Mele pump, although the measured lattice depths do not fully justify a tight-binding approximation. In this analogy, the angle between the beams breaks the inversion symmetry of the unit cell, contrary to the case of orthogonal interference.

The direction of the atomic current can, in principle, be inverted by winding around the singularity with opposite chirality in parameter space. In our system, the chiral evolution of $\phi$ is fixed by the intrinsic non-Hermitian origin of the dynamics. We can, however, change the sign of the singularity by inverting the system in real space. We use a second cavity that is mounted realizing a chiral evolution of $\phi$, with $\Theta_x$ and $\Theta_y$ alternating (Methods). By coupling the atoms to this cavity, we indeed observe an inversion of the pumping direction (Fig. 3b).

We have furthermore performed numerical simulations of the open-system Hamiltonian. We integrate the self-consistent equations for the light field and the BEC (Methods), which also gives access to experimentally inaccessible observables such as the overlap integrals of equation (3) (Fig. 3d). The numerical results qualitatively match the experimental data, confirming both the field dynamics and the atomic current associated with every phase step (theory lines in Fig. 3b). The global nature of the
The emerging motion, together with the self-organized lattice, breaks time and space translation symmetry simultaneously. The former can be linked to the concept of continuous dissipative time crystal in contrast to discrete realizations in systems with a time-dependent parametric drive. However, the mechanism presented in this work features a topological gap. We therefore expect the superfluid system to be protected from energy absorption in the oscillating steady state.

The observation of a self-consistent pump raises interesting questions that call for future investigations. Besides the concept of a topological time crystal, a natural extension would be a generalization to insulating systems. Coupling the cavity field to a self-organized Mott insulator or Fermi gas, the transported charge would be quantized and the cavity phase would directly reflect the topological index. In addition, applying our scheme to a spinor quantum gas may generate self-driven spin pumps.

Online content
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Methods

Preparation of the BEC
We prepared a gas of $^{87}$Rb atoms in an optical dipole trap. We reached degeneracy of the bosonic gas through optical evaporation, preparing an almost pure BEC of $N = 2.5(4) \times 10^{10}$ Rb atoms in the ground state hyperfine level $|F = 1, m_F = -1\rangle$. The final crossed optical dipole trap consisted of two laser beams at a far-detuned wavelength of 1,064 nm that created an ellipsoid-shaped trap with the three trap frequencies $\omega_x [\text{rad s}^{-1}], \omega_y [\text{rad s}^{-1}], \omega_z [\text{rad s}^{-1}] = 2\pi \times [89(3), 74(1), 224(2)] \text{Hz}$. These trap frequencies set a time frame of around 10 ms in which we can observe displacements of the cloud without relaxation owing to the harmonic confinement.

A magnetic offset field of about 25 G was applied to avoid spin-dependent effects on self-organization as, for example, superradiance to the orthogonally polarized cavity mode that is significantly detuned owing to the birefringence of our cavity as studied in ref. 48.

Imbalanced transverse beams
To engineer the couplings $n_{\theta, \phi}$ to the two different quadratures of the light field, we used imbalanced transverse beams as explained and used in ref. 44. The BEC was placed at a distance of around 4 mm from an in vacuo retro-reflecting mirror. Tuning of the imbalance was achieved by mechanically shifting the position of the focusing lens along the optical axis of the pump beam. We extracted the imbalance from the experimental data by comparing the threshold of the two distinct self-organized phases (coupling either to the field quadrature $P$ or $Q$) with numerical calculations14. The imbalance was quantified by the imbalance parameter

$$y = \sqrt{\frac{\Omega_1}{\Omega_2}},$$

with $\Omega_{1,2}$ being the Rabi frequencies of the two plane waves counter-propagating along $x$. The lattice depth $V_{\text{L}}$ was calibrated using Raman–Nath diffraction at the standing-wave component (that couples to the $P$ quadrature of the cavity field) together with the extracted imbalance. For the data in Figs. 2 and 3, we used an imbalance parameter of $y = 1.36(5)$.

Heterodyne measurement
Balanced optical heterodyne detection was used to record the amplitude of the intracavity field and its phase with respect to the transverse beam lattice. A local oscillator (LO) laser field was combined on a beam splitter with the light field leaking from one cavity mirror and guided together onto the two photodiodes of a balanced photodetector. The signal was afterwards mixed down to a moderate radio frequency of 300 kHz with a home-build-in-phase and quadrature (IQ) filter device. The LO was generated by shifting the frequency of the same laser source as the transverse beam with an acousto-optical modulator by 50 MHz. To compensate for phase drifts introduced by sending the light fields through separate optical fibres, we locked their relative phase after passing the fibres. The 300-kHz signal data were recorded with a computer-interfaced oscilloscope (PicoScope 5444B). The signal was then processed by a software that extracts the quadratures through a digital IQ mixer and applies a low-pass filter using a binning window of $1 \times 10^{-5}$ s. All related radio-frequency signal sources were phase locked to a 10-MHz global positioning system (GPS) frequency standard that has a fractional stability better than $10^{-15}$ to minimize the technical phase noise in the heterodyne detection system. More technical details and design considerations are described in detail in ref. 44.

The measured absolute value of the phase $\phi$ of the intracavity field is dominated by technical fluctuations inbetween experimental repetitions like, for example, fibre-length drifts, air-density changes and unsynchronized start of experimental sequences. Nevertheless, the value relative to the pump field is fixed. When the system enters the superradiant phases coupled to the quadrature $Q(P)$, the phase $\phi$ is locked to either 0 or $\pi$ ($\pi/2$ or $-\pi/2$). However, as the lines in the phase diagrams consist of independent measurements, we plot $\phi$ modulo $\pi/2$ in Extended Data Fig. 1. In the main text, we focus on the dynamics between the quadratures $P$ and $Q$ where we set the starting point for the physical processes to the positive $Q$ quadrature.

To calibrate the amplitude of the intracavity light field, we carried out Raman–Nath diffraction analogous to the calibration of the transverse beam by applying to the cavity a short coherent on-axis probe.

Cavities and detunings
We can reverse the displacement of the atomic cloud by performing the experiment with two physically distinct cavities that set different directions for the cavity wave vector (Fig. 3) and also show different rates of dissipation. Both cavities are near-planar with a vacuum Rabi coupling of $|g_1, g_2| = 2\pi \times [1.95, 1.77] \text{MHz}$ and a mode waist of $[48.7, 50.4] \mu$m, which is much larger than the atomic cloud itself. This also justifies modelling the system with a transversally constant cavity lattice field within the atomic cloud. Cavity 1 with dissipation rate $\kappa_1 = 2\pi \times 147(4) \text{kHz}$ was used for most experiments demonstrated in the main text (Figs. 2–4). Cavity 2 was used for the orange data points in Fig. 3 and has a higher dissipation rate of $\kappa_2 = 2\pi \times 800(11) \text{kHz}$.

All measurements were done far in the dispersive regime where the cavity resonance is detuned by $\omega_0 = +2\pi \times 70 \text{GHz}$ with respect to the rubidium $D_2$ line. The detuning between the frequency of the transverse beams and the cavity is tunable within tens of megahertz by an electro-optical modulator. The length of the cavity was stabilized using a low amplitude, far-detuned additional laser field at 830 nm, allowing a constant feedback on the cavity length while having a negligible effect on the atomic cloud.

Imaging of the atomic cloud
Besides the detection of the light field leaking out of the cavities, we used resonant absorption imaging to measure the atom number, temperature, momentum distribution and centre-of-mass position of the atomic cloud. The physical imaging was done through the viewport below the vacuum chamber by an imaging objective built of two spherical lenses with an effective numerical aperture of 0.15. This limited the resolution at our imaging wavelength of $\lambda_{\text{D}} = 780 \text{nm}$ to $\Delta x_{\text{res}} = 2.5 \mu$m. The final image of the atoms was recorded on a charge-coupled device (CCD) camera (Pointgrey Grassembler CCD) with a pixel size of 4.50 $\mu$m and $1,452 \times 1,932$ pixels. Together with the magnification of the imaging system, this resulted in an effective pixel size of 2.25 $\mu$m, which allows for precise centre-of-mass detection in situ, while still allowing for time-of-flight measurements with the same imaging system.

Temperature, number of atoms and the momentum-state populations were extracted from absorption images taken after 25 ms of ballistic expansion of the atomic cloud. The displacement measurements of the cloud’s centre-of-mass position shown in Fig. 3 were instead extracted by fitting a two-dimensional Gaussian to in situ absorption images taken within the dipole trap. Shot-to-shot fluctuations of the cloud position (standard error of 4.54 $\mu$m) were dominated by position fluctuations of the dipole trap. However, by independently imaging the position of the dipole beam and correlating the trap position with the cloud position, we reduced the error by a factor of about four. This procedure, together with the large number of measurements, resulted in the error bars shown in Fig. 3.

Measurement protocol and data selection
The data shown were taken by setting the cavity detuning $\omega_0$ to a fixed value and ramping the transverse beam within 5 ms linearly from zero to a desired transverse beam lattice depth $V_0$.

Whereas Fig. 2 shows a phase diagram resulting from averaging 50 experimental repetitions, in Extended Data Fig. 1a we show a phase
diagram constructed from a single experimental run at each \( \Delta \). Extended Data Fig. 1b shows the consecutive repetitions of such experimental runs at a fixed detuning \( \Delta _{1} \), from which the averaged phase diagram can be derived. We attribute the observed variations between the different runs to fluctuations in the atom number and in the relative alignment between the cavity mode and the transverse beams.

The displacement data in Fig. 3 show the correlations between the number of phase windings and the displacement of the cloud. As the duration of a single phase winding is short (<0.1 ms) compared with the whole ramp time, the fluctuations shown in Extended Data Fig. 1 make it impossible to deterministically choose a number of phase windings. Instead, we chose a final transverse beam lattice depth such that the ramp ends within the dynamic phase, where we perform the in situ absorption imaging to determine the position of the cloud. After recording a large number of repetitions, we selected a posteriori the desired number of phase windings and plot the according displacements in Fig. 3b. One data point displayed for cavity 1 (orange data) contains on average 34 experimental repetitions (ranging between 8 and 60, depending on the number of phase windings). Whereas one data point displayed for cavity 2 (blue data points) contains on average 34 experimental repetitions (ranging between 15 and 1,008, depending on the number of phase windings).

**Many-body Hamiltonian**

In the following, we derive a model that describes the self-organization within the quadratures \( P \) and \( Q \) and shows how including the dissipation channel \( K \) the non-Hermitian dynamics occurs. With the two counter-propagating plane waves (with the strength \( \Omega _{1} \) and \( \Omega _{2} \) of the transverse beam, we define all effective two-photon Rabi frequencies in the dispersive coupling regime and rotating-wave approximation. The transverse beam lattice \( \hat{V} _{0} = h \hat{k} \cdot \hat{r} \), the dispersive shift per cavity photon \( \Delta _{2} = \frac{\Delta _{2}}{\Delta _{1}} \), the coupling to the quadrature \( \hat{Q} \), \( \eta _{p} = \frac{\eta _{p}}{\Delta _{1}} \) and the coupling to the quadrature \( \hat{P} \), \( \eta _{p} = \frac{\eta _{p}}{\Delta _{1}} \). The Hamiltonian in the standard form of ref. \( ^{11} \) is:

\[
\hat{\mathcal{H}} = -\hbar \Delta _{2} \hat{a}^\dagger \hat{a} + \hat{H}_0 + hUa^\dagger \hat{b} \hat{b}^\dagger + \hbar \eta _{q} \frac{(\hat{a} + \hat{a}^\dagger)}{2} \hat{\theta} _{q} - i \hbar \eta _{p} \frac{(\hat{a} - \hat{a}^\dagger)}{2} \hat{\theta} _{p} \tag{5}
\]

where \( \hat{a}^\dagger \) (\( \hat{a} \)) are the creation (destruction) operators for the cavity photon field and having defined:

\[
\hat{H}_0 = \int d\mathbf{r} \left[ \frac{\mathbf{p}^2}{2m} + V_{\text{trap}}(\mathbf{r}) \right] \psi (\mathbf{r})^\dagger \psi (\mathbf{r}) + \frac{\mathbf{g}^2}{2} \left[ \psi (\mathbf{r})^\dagger \psi (\mathbf{r}) \right]^2 \tag{6}
\]

\[
\hat{B} = \int d\mathbf{r} \psi (\mathbf{r}) \cos (\mathbf{k}_{r} \cdot \hat{r}) \psi (\mathbf{r}) \tag{7}
\]

\[
\hat{\theta} _{q} = \int d\mathbf{r} \psi (\mathbf{r}) \cos (\mathbf{k}_{r} \cdot \hat{r}) \psi (\mathbf{r}) \tag{8}
\]

\[
\hat{\theta} _{p} = \int d\mathbf{r} \psi (\mathbf{r}) \sin (\mathbf{k}_{r} \cdot \hat{r}) \psi (\mathbf{r}) \tag{9}
\]

\( \mathbf{k} _{r} \) and \( \mathbf{k} \) are the pump and cavity wave vectors respectively (with \( | \mathbf{k} | = k = \frac{2\pi}{\Delta _{1}} \)) and \( \psi (\mathbf{r}) \) is the atomic field operator that creates (annihilates) a particle at the position \( \mathbf{r} \). The direction of the transverse beam is chosen such that the field with strength \( \Omega _{2} \) is along the \( x \) axis and the field with strength \( \Omega _{1} \) is counter-propagating. The atomic cloud is small enough to neglect the transverse spatial component of the electromagnetic modes. The physical interpretation of the integrals is: \( \hat{H}_0 \) is describing the BEC with collisional interaction \( \hat{g} \) and trapped in the harmonic potential \( V_{\text{trap}} \) and in the optical lattice of the transverse beam, \( \hat{B} \) is the overlap with the cavity mode, and \( \hat{\theta} _{q} \) and \( \hat{\theta} _{p} \) the overlaps with the two patterns coupling to the different quadratures. These integrals are our choice of order parameter in a phenomenological theory of the self-organization phase transition. Note that changing sign \( (\hat{Q} - \hat{P}) \) from a Hamiltonian point of view corresponds to flipping \( \hat{x} \rightarrow -\hat{x} \), and the same is valid for changing the direction of \( \mathbf{k} _{r} \).

**Heisenberg equation of motion and dissipation**

To observe the dynamics of the photonic part of the Hamiltonian, we look at the Heisenberg equation of motion of the field operator of the light, incorporating also the cavity dissipation \( \kappa \):

\[
\frac{d\hat{a}}{dt} = -\frac{i}{\hbar} (\langle \hat{a} \rangle \hat{\theta} _{q} ) - \kappa \langle \hat{a} \rangle \tag{10}
\]

where we omitted quantum fluctuations as they equal zero at the mean-field level for \( \alpha = \langle \hat{a} \rangle \). We assume a quasi-stationary light field \( \delta \alpha = 0 \) as the timescales of the light field separate from the motional timescales of the atomic wavefunction. With this we get for the light field:

\[
\alpha = e^{i\phi} = \frac{\eta _{q} \hat{Q}^2 + \eta _{p} \hat{P}^2}{\Delta _{2} + \kappa _{2} + \kappa _{1} + \kappa _{0}} \tag{11}
\]

The complex light phase \( \phi \) is defined in equation (3) and can be separated into \( \phi _{q} = \tan ^{-1} \frac{\Delta _{2}}{\eta _{q} \Delta _{1}} \) and \( \phi _{p} = \tan ^{-1} \frac{\Delta _{2}}{\eta _{p} \Delta _{1}} \). For simpler notation, we define \( \Omega _{2} = \langle \hat{Q} \rangle , \Delta _{2} = \frac{\eta _{q} \Delta _{1}}{\eta _{p} \Delta _{1}} \), and \( \Delta _{2} = \Delta _{2} - \eta _{q} \Delta _{1} B \), which is the dynamical dispersive cavity detuning owing to the changing overlap of the atomic wavefunction with the cavity field. Equations (11) and (5) thus form a self-consistent loop governing the dynamics of the matter and the light fields (Extended Data Fig. 2a).

**Equation of motion and low-energy momentum expansion**

To get further intuition for the dynamics of the photon field and the atomic wavefunction, we can construct an atomic field operator from eigenstates \( \psi _{i} \) of the momentum operator:

\[
\psi = \sum _{i} c_{i} \psi _{i} \tag{12}
\]

where \( c_{i} \) annihilates a particle in the state \( \psi _{i} \). After substitution in the many-body Hamiltonian (5) one gets

\[
\hat{\mathcal{H}} = -\hbar \Delta _{2} \hat{a}^\dagger \hat{a} + \sum _{ij} (\hat{H}_0 + hUa^\dagger \hat{b} \hat{b}^\dagger + \hbar \eta _{q} \frac{(\hat{a} + \hat{a}^\dagger)}{2} \hat{\theta} _{q} - i \hbar \eta _{p} \frac{(\hat{a} - \hat{a}^\dagger)}{2} \hat{\theta} _{p} ) \psi _{i} \psi _{j} \tag{13}
\]

\[
+ (\hat{a} + \hat{a}^\dagger ) \eta _{q} \hat{Q}^2 - i (\hat{a} - \hat{a}^\dagger ) \eta _{p} \hat{P}^2 - c_{i} c_{j} \hat{\epsilon}_{ij} \psi _{i} \psi _{j} \tag{14}
\]

that only uses the three momentum modes \( \psi _{i} = \frac{1}{\sqrt{A}} \cos (\mathbf{k}_{r} \cdot \mathbf{r}) \cos (\mathbf{k} \cdot \mathbf{r}) \), \( \psi _{i} = \frac{2}{\sqrt{A}} \sin (\mathbf{k}_{r} \cdot \mathbf{r}) \cos (\mathbf{k} \cdot \mathbf{r}) \) and \( \psi _{0} = \frac{1}{\sqrt{A}} \), where \( A \) defines the area of the Wigner–Seitz cell. In the definition of equation (14), each mode is normalized to unity, \( \int \psi _{i}^\dagger \psi _{i} = 1 \) and the field operator is normalized to the total particle number \( \int \psi \psi = \frac{\sqrt{N}}{\sqrt{A}} \), and \( \hat{\epsilon}_{ij} = \hat{\epsilon}_{ij} \psi _{i} \psi _{j} \). Under this approximation, and neglecting the harmonic trapping potential the
collisional interactions, we arrive at the following mean-field equations of motion for the three-level system (Extended Data Fig. 2b):

\[
\begin{align*}
\dot{c}_q &= i\Delta c_q - \alpha c_q - \frac{i\hbar}{4}(c_q c_0 + c_0 c_q) + \frac{n_\eta}{2}\text{Re}(\eta a c_q) - \frac{n_\eta}{2}\text{Im}(\eta a c_q) \\
\dot{c}_p &= -\frac{i\hbar}{4}(c_p c_0 + c_0 c_p) \\
\dot{c}_0 &= -\frac{i\hbar}{2}\text{Re}(\alpha a c_q) - \frac{n_\eta}{2}\text{Im}(\eta a c_0) \\
\dot{c}_r &= -\frac{i\hbar}{2}\text{Re}(\phi a c_0) \\
\dot{c}_p &= -\frac{i\hbar}{2}\text{Re}(\phi a c_p)
\end{align*}
\]

(15)

Minimal model and non-Hermitian dynamics

To demonstrate the instability, it is sufficient to reduce the equations of motion further by keeping the $c_0$ population constant ($c_0 = \sqrt{N}$). This is an appropriate approximation as the relative depletion of the zero momentum state is small. As a result of this approximation, the number of coupled equations is further reduced. Applying another time derivative to $\dot{c}_q$ and using the steady-state value for $\alpha$, the differential equations for the populations of $\psi_{q,p}$ can be written as

\[
\begin{pmatrix}
\dot{c}_q \\
\dot{c}_p
\end{pmatrix} =
\begin{pmatrix}
\Omega_q^2 - K_r & -K_r \\
K_r & \Omega_p^2
\end{pmatrix}
\begin{pmatrix}
c_q \\
c_p
\end{pmatrix}
\]

(16)

In the above matrix notation, we have defined

\[
\Omega_i^2 = -\left(\omega_i \pm \frac{V_0}{4\hbar}\right)^2 - N\eta_i^2\left(\omega_i \pm \frac{V_0}{4\hbar}\right)\frac{\Delta_c}{\Delta_c + K_i^2}
\]

(17)

\[
K_i = N\eta_i^2\left(\omega_i \pm \frac{V_0}{4\hbar}\right)\frac{\kappa}{\Delta_c + K_i^2}
\]

(18)

with $i \in \{q,p\}$ and $\pm$ is positive for $i = q$ and negative for $i = p$. The eigenenergies $\omega_i$ of the $q$ and $p$ modes can be obtained from the free-particle dispersion as $\omega_i = 2\omega_0$, with $\omega_0$ being the recoil frequency. The two-photon coupling strength $\eta_{q,p}$ are defined in the main text by the Rabi rates of the two transverse beams and can be rewritten as $\eta_{q,p} = \frac{1}{\sqrt{2}}(\Omega_q \pm \Omega_p) = \sqrt{\gamma} \sqrt{\frac{\gamma}{2}}\sqrt{\frac{\gamma}{2}}\frac{\Delta_c}{\Delta_c + K_i^2} \frac{\omega_0}{\hbar}$ to express the explicit dependences on the imbalance parameter $\gamma$, transverse beam lattice $V_0$, and dispersive shift per cavity photon $\Delta_c$.

It is noted that in equation (17), the coupling to the $Q$ quadrature becomes weaker for strong enough transverse beam lattice strengths $V_0$, which makes the energies of the $P$ and $Q$ modes cross eventually. For more details of the differences of the two couplings see ref. 34.

The diagonal elements of the matrix describe the soft mode energies. Above a critical pump strength, these matrix elements give rise to growing populations $c_{q,p}$. The off-diagonal elements describe the coupling rate of the two modes through the dissipation.

From diagonalizing the non-Hermitian matrix in equation (16), we get an analytical expression of the eigenvalues

\[
\epsilon_i^2 = \frac{\Omega_q^2 + \Omega_p^2}{2} \pm \frac{1}{2} \sqrt{(\Omega_q^2 - \Omega_p^2)^2 - 4K_r^2K_p^2}.
\]

(19)

The real and the imaginary parts of $\epsilon_i$ describe the growth rate and the oscillation frequency of the decoupled eigenvectors $c_{q,p}$, from which the evolution of $c_{q,p}$ can be derived. The populations start oscillating between each other when $(\Omega_q^2 - \Omega_p^2)^2 < 4K_rK_p$, that is, the dynamics emerges once the dissipative coupling strength $K_i^2 \propto \kappa$ is larger than the energy gap between the two energies. It is noted that the term $4K_rK_p^2 \propto \Delta_c^4$ whereas $\Omega_i^4 \propto \Delta_c^2$, such that the dissipation can only overcome the energy gap in the regime of $\kappa/\Delta_c = 1$. This can be seen in Fig. 4b, when the real part of the eigenvalues coalesce as they are close enough together.

Numerical simulations

As an alternative to the simplified analytical model described above, we numerically integrate the self-consistent many-body Hamiltonian equation (5) using a standard symmetrized split-step Fourier method. This allows to propagate the equations in time and calculate the dynamics of the system, as well as to obtain the ground state by first performing a Wick rotation. The cavity field is assumed to be in its steady state at each integration time step, so that it can be numerically calculated from the wavefunction using equation (11). All theoretical simulations reproduced in the figures throughout the paper include the trap and interparticle collisional interactions. In the calculations, the transverse extent of the coupling beams and the cavity mode are neglected, such that the initial harmonic confinement is not modified. We checked the validity of the simulation by comparing the numerical phase diagram to the experimental one (Extended Data Fig. 3), which show qualitative agreement.

The current shown in Fig. 3d is calculated from the probability current

\[
J(t) = -\frac{\hbar}{2m}\left(\nabla^2 \psi - \psi \nabla^2 \psi\right)
\]

(20)

integrated over the whole system.

The polarization of Fig. 3a,c is calculated by obtaining the ground-state wavefunction in the unit cell at different choices of $\alpha$. As this calculation was performed in periodic boundary conditions, the centre of mass has to be extracted using ref. 30. The polarization

\[
P_e = \frac{e}{2mL^2} \text{Im}(\psi^\dagger \nabla \psi - \psi \nabla^2 \psi)
\]

(21)

is the phase of the expectation value of $\hat{X}$ (the position operator) in periodic boundary conditions, which, in the words of ref. 30 is "just a Berry's phase in disguise". Here, $L$ is the linear system size and the charge is set to $e = 1$ for neutral atoms.

Data availability

The data to reproduce the figures of this study are available in the data repository of ETH Zurich’s Research Collection (http://www.research-collection.ethz.ch) at https://doi.org/10.3929/ethz-b-000547966.

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Author contributions

D.D., A.B. and X.L. prepared the experiment, D.D., A.B., X.L. and S.H. took and analysed the data. D.D. performed the numerical simulations. T.D. and T.E. supervised the work. All authors contributed to discussions of the manuscript.

Competing interests

The authors declare no competing interests.

Additional information

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Extended Data Fig. 1 | Non-averaged phase diagram and repeated measurement results. a, Phase diagram using the phase data $\phi(t)$ from the heterodyne detector by varying cavity detuning $\Delta_c$ of cavity 1. The two different self-organized phases can be well observed for values around $\phi = 0$ and $\phi = \pi/2$. At low transverse beam fields the system shows no self-organization and $\phi$ is not well defined. Between the two self-organized phases the dynamical phase with varying $\phi(t)$ is visible. b, Many repetitions of the same trace with constant $\Delta_c = -1.1$ MHz. The extent of the instability region varies slightly at each repetition. c, Same data as in b, but shifted in time such that the onset of pumping coincides for all traces. The dashed line as guide to the eye illustrates that the rate at which the phase evolves is robust.
Extended Data Fig. 2 | Schematic representation of the theoretical models.
a, Self-consistent loop between cavity field $\alpha$, optical lattice $V_{\text{lattice}}$, and wavefunction $\psi$ as described by the set of equations Eq. (5) and Eq. (11).
b, Minimal model given by the three-level momentum expansion of Eq. (14). The coherent coupling (solid arrows) mixes the condensate mode $\psi_0$ with the spatially modulated $\psi_{p,q}$, which are then mutually coupled by dissipation (dashed arrows).
Extended Data Fig. 3 | Comparison of experimental and numerical phase diagrams. Figures show the amplitude of the intracavity light field dependent on $V_0$ and the cavity detuning $\Delta_c$ for cavity 2. a, Dataset showing in each row a trace of single experimental realization for the given cavity detuning $\Delta_c$. The transverse beam lattice is linearly ramped to the final transverse lattice strength $V_0 = 40 \, E_r$ within 5 ms. b, Corresponding simulation of the experimental results with GPE simulation.