On deformation criteria of a special limit state of eccentrically compressed elements of RC building frames

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Abstract. At accidental impacts caused by the sudden removal of a load-bearing element of the building frame, the stress-strain state of the sections of eccentrically compressed structural elements such as columns, pylons, braces and belts of trusses, etc. becomes more disadvantageous than their stress-strain state at the stage of normal operation. The factors that lead to such a situation are sharp increase in the eccentricities of loads attaching, slenderness of cross sections, corrosion, initial curvature and mechanical damages. In these cases, buckling scenario for the exhaustion of their bearing capacity is a possible scenario. The paper proposes an approach to the obtaining of deformation criteria for assessing the special limiting state of eccentrically compressed bar elements of reinforced concrete frames. Such an approach is based on the use of a combination of nonlinear deformation analysis and nonlinear stability analysis. Paper shows that the limiting values of deformations for the considered reinforced concrete element can be determined by the criteria of strength or buckling, depending on the ratio of the forces acting in the sections of the structural element.

1. Introduction
Analysis of accidents that have occurred with building and structures in recent decades, as well as numerical results of study of such and hypothetical accident situations [1-4] shows that failure of a bearing structure causes restructuring of structural system that is accompanied by redistribution of power flows and strain (alternate load pathing). If the time for removing the bearing element is a hundred part of a second, then this process is accompanied by the arising of significant inertial forces, which leads to dynamic loading of the remaining elements of the structural system.

This is confirmed by the results of full-scale tests carried out by Sasani and Sagiroglu [5-7] at the demolition of buildings, as well as testing of flat and three-dimensional scale models of building frames carried out by Kolchunovym V.I. et al. [8], Fedorova N.V. and Ngoc Vu Tuen [9], Demyanov A.I. and Alcadi S.A. [10]. According to these studies, the power flow redistribution (alternate load pathing) covers in the first order the overlaps or coating above the removed structure.

However, the removal of the structural element of the building frame can lead to the buckling of the eccentrically compressed elements of the deformed structural system. Such a scenario for the propagation of progressive failures can be associated with the accumulation of damage caused by corrosion or high temperatures [11-13], by a change in the nature of the stress-strain state of eccentrically compressed elements due to an increase in the span of the floor structure or an increase in the eccentricity of the application of an axial force [14], by an increase in the effective length of the considered an eccentrically compressed element due to the removal of vertical ties or degradation of
the fixing conditions [15]. The criteria for the evaluation of the bearing capacity of the eccentrically compressed elements of building frames subjected to additional dynamic loading at the instantaneous removal one of the supporting structures as a rule, are the ultimate strains in concrete and reinforcement or the ultimate efforts perceived by sections of the elements in accordance with strength conditions.

Trekin N.N. and Kodysh E.N. [16] proposed the deformation criteria limiting the relative ultimate deflections of structures as criteria for the special limiting state of bending reinforced concrete elements of the frames of buildings and structures. Here, the special limiting state is understood as a stress-strain state of a structure in which any increment of efforts or deformations will lead to its destruction. It seems that it is also advisable to introduce similar integral criteria to assess the special limiting state of eccentrically compressed elements. In this regard, the paper proposes approach to obtain the deformation criteria using numerical simulation.

2. Materials and Methods

In the paper, we used a combination of nonlinear static analysis and nonlinear stability analysis in the form proposed in the work [17] in order to constructing the deformation criteria of specific limiting state for eccentrically compressed concrete rod elements of building frames.

When solving the stability problems of reinforced concrete frames, we accepted the hypothesis of plane sections and elastic-plastic model of construction material, assuming that features of deforming and cracks' formation can be described by a single mathematical model based on idealized stress-strain diagrams of a material.

Modeling of changes of strength and deformation properties of a power resistance of reinforced concrete under serial static-dynamic loading of structural elements of a building frame is based on the theory of plasticity proposed by G.A. Geniev [16]. According to this theory, the dynamic tangent module under static-dynamic deformation can be written in the following form:

\[ E_{t,d}(y) = \frac{E_{t,1}}{(1 - e^{-\omega t})}, \omega = \frac{E_{sec,0}}{K}, \]

where \( E_{t,1} = \frac{d\sigma}{d\varepsilon} = E_0 - 2H_c \varepsilon \) is tangential modulus of deformed state before dynamic additional loading (before structure removal);

\( \varepsilon = \varepsilon_t + \frac{\varepsilon_c - \varepsilon_t}{h} \left( y + \frac{h}{2} \right) \) is fiber deformation in an arbitrary point of a cross section;

\( \varepsilon_c, \varepsilon_t \) are deformations in the most and the less compressed fibers taking into account physical and geometrical nonlinearity of the rod under consideration (figure 1);

![Figure 1](image_url). Scheme for determining the parameters of the stress-strain state of a compressed-bent reinforced concrete element to solve the problem of buckling

\( E_{sec,0} \) – sectorial modulus at normal operation;
$K$ – modulus of viscous resistance of an element;
$t$ – duration of dynamic impact to elements of a structural system at sudden structural transformation. If the quasi static approach is used for the purpose to determinate $t$, then $t = T/4$ in the first approximation. Here $T$ is period of the free vibration of \(n-1\) structural system with masses attached instead forces (including forces acting in the removed element).

It should be noted that a number of software packages for the calculation of building structures apply deformation parameters (tangent modulus of deformation) that correspond to penultimate step of calculation when performing the analysis of structure for design combination of full loads or for conditionally static determining design combination of dead loads [19]. Using of these values of deformability parameters in the analysis of stability leads to an overestimation of the critical forces when a small number of calculation steps is accepted (Figure 2). Therefore, when calculating the stability of deformation of the rod element we use tangent modulus of deformation for the final deformed state of the cross section. That is more consistent with the buckling problem specifics. In this case, the reduced geometric characteristic of the resistance of the cross sections, which is understood here, as the resistance of the cross sections to deformation from the increment of forces, is a function of the deformations of the edge fibers.

![Figure 2. Scheme for calculating the shape stability for a deformed state](image)

The paper [17] contains the expressions for calculation the stiffness at effort increment of finite elements of the model of an eccentrically compressed bar. In this work, we present only expressions for a particular case corresponding to the situation when the section is rectangular, and a compressive stress acts at each point of it:

\[
I_{\text{red},\theta} = I_{\text{red},\theta_1} - A_{\text{red},\theta} \cdot a_\theta^2, \\
I_{\text{red},\theta_1} = \frac{E_s}{E_0} \cdot A_s \cdot a^2 + \frac{E_s}{E_0} \cdot A'_s \cdot (h - a')^2 + \\
b \cdot \int_0^h \left(1 - \frac{\varepsilon}{\varepsilon_b}\right) y^2 dy = \frac{E_s}{E_0}(A_s \cdot a^2 + A'_s \cdot (h - a')^2) + \\
+ \frac{b h^3}{3} \left(1 - \frac{\varepsilon_t}{\varepsilon_b} - \frac{\varepsilon_c - \varepsilon_t}{12 \varepsilon_b}\right), \\
a_\theta = \frac{S_{\text{red},\theta}}{A_{\text{red},\theta}'}, \\
S_{\text{red},\theta} = \frac{E_s}{E_0} \cdot \left(A_s \cdot a + A'_s \cdot (h - a')\right) +
\]
\[ + b \cdot \int_0^h \left(1 - \frac{\varepsilon}{\hat{\varepsilon}_b}\right) dy = \frac{E_s}{E_0} \cdot \left(A_s \cdot a + A'_s \cdot (h - a')\right) + \]
\[ + \frac{bh^2}{2} \left(1 - \frac{\varepsilon_c - \varepsilon_t}{\hat{\varepsilon}_b} - \frac{\varepsilon_t}{6\hat{\varepsilon}_b}\right). \]
\[ A_{red,\theta} = b \cdot \int_0^h \left(1 - \frac{\varepsilon}{\hat{\varepsilon}_b}\right) dy + A_{s,tot} \cdot \frac{E_s}{E_0} = bh \left(1 - \frac{\varepsilon_c + \varepsilon_t}{2\hat{\varepsilon}_b}\right) + A_{s,tot} \cdot \frac{E_s}{E_0}. \]

where \( J_{red,\theta} \) – the moment of inertia of the cross section given with respect to the tangent modulus relative to its own center of gravity;

\( J_{red,\theta1} \) - the moment of inertia of the cross section given along the tangent modulus relative to the center of gravity of the non-deformed cross section;

\( A_{red,\theta} \) - the cross-sectional area given by the tangent modulus;

\( S_{red,\theta} \) - static moment of the tangential modulus of the cross section;

\( a_0 \) – the coordinate of the center of gravity of the section tangent to the module, measured from the axis relative to which \( S_{red,\theta} \);

\( b, h \) – width and cross-sectional height, respectively;

\( A_s, A'_s \) – area of compressed and stretched (least compressed) reinforcement, respectively;

\( E_s, E_0 \) – modulus of elasticity of steel and initial modulus of elasticity of concrete, respectively;

\( \hat{\varepsilon}_b \) – deformations of compressed concrete, corresponding to the normative resistance to compression of concrete.

The solution to the stability problem for the rod system is reduced to finding the parameter \( k \), which characterizes the critical force and can be obtained if we equate to zero the determinant of the matrix, composed of variable parameters at unknowns. The form of buckling obtained from the results of such a calculation is dimensionless, which does not allow us to estimate the limiting deformations of the rod element when the critical state is reached. In this regard, we find the value of the transverse deflection of the bar element in the critical state by combining deformation analysis (with verification of strength criteria) based on a nonlinear model with nonlinear stability analysis (with verification of stability criteria). Let us consider reinforced concrete truss rod and estimate its bearing capacity in order to illustrate an approach for constructing deformation criteria of specific limit state for eccentrically compressed RC elements (Figure 3). For this purpose, the Lira - CAD Software was applied.

![Figure 3. Cross-section (a) and design model (b) of an eccentrically compressed RC bar](image)
3. Results and discussion

Let us perform static and buckling analysis and study the behavior of the critical forces and deflections during deformation of the bar presented in the figure 3. We have considered two combinations of efforts: $N_1, M_1$ for $M_1/N_1 = 5$ mm and $N_2, M_2$ for $M_2/N_2 = 58$ mm. These combinations correspond to the case of small and large eccentricities, respectively. At the same time, we took into account requirement of Russian Building Code SP 63.13330.2018 [20], according to which the margin of stability should be twice or more. The results of the performed numerical analysis are presented in the form of deformed states, buckling modes and ‘$N–f$’ graphs in Figures 4 and 5.

Figure 4. Deformed state of the bar (a), buckling mode at a margin of stability 1.78 – less than 2 (b), graph ‘$N–f$’ (c) for the case of small eccentricities:

1 - deformation calculation; 2 - critical forces; 3 - critical forces, taking into account the requirements of the Building Code [20] for at least a two-fold margin for stability

Figure 5. Deformed state of the bar (a), buckling mode at a margin of stability 3.08 – more than 2 (b), graph ‘$N–f$’ (c) for the case of large eccentricities: 1 - deformation calculation; 2 - critical forces

Analysis of the graphs presented in Figures 4 and 5 shows that for eccentrically compressed reinforced concrete elements with small eccentricities, the limiting deflection $f_c$ upon reaching the
critical state (the margin for stability is less than 2) turned out to be less than the limiting deflection obtained from the condition of the strength of normal sections. Due to the fact that we take into account all the possible reserves of load-bearing capacity as much as possible when testing criteria of the specific limit state and taking into account the lack of exhaustive data on the parameters of the specific effects and the deformation modes of structures it is advisable to use for progressive collapse the least of the values of the obtained ultimate forces and deformations. For a bar element subjected to the action of an axial force applied with large eccentricities (dominance of bending), the limiting parameters of forces and deformations were determined by the criteria for the strength of the sections (Figure 5).

Using the approach presented in this work, we can obtain the boundary values of the $M/N$ ratios, the flexibilities of the rods as well as deformation criteria (deflections, angles of rotation, curvature) of the special limiting state of such elements. These parameters determine the fracture resistance mechanism under eccentric compression.

4. Conclusions

In this paper, the analytical dependences are obtained for the stiffnesses of the reduced sections of reinforced concrete bar elements compressed with small eccentricities, when all points of the section are subject to compression.

An approach to obtain the deformation criteria for a special limiting state of eccentrically compressed bar elements of reinforced concrete building frames is proposed. This approach is based on the use of a combination of nonlinear deformation analysis and nonlinear buckling analysis.

It is shown that the limiting values of deformations for the considered reinforced concrete element can be determined by the criteria of strength or stability, depending on the ratio of the forces acting in the sections of the structural element.

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