Characterizations of almost greedy and partially greedy bases†

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Abstract

We shall present new characterizations of partially greedy and almost greedy bases. A new class of basis (which we call reverse partially greedy basis) arises naturally from these characterizations of partially greedy bases.

Keywords: almost greedy basis, partially greedy basis, reverse partially greedy basis.

MSC: 46B15, 41A65.

§1. Introduction

Let $X$ be a real Banach space with a seminormalized (Schauder) basis $(e_n)$ and biorthogonal functionals $(e^*_n)$. $X$ is allowed to be finite-dimensional, in which case $(e_n)$ is a finite algebraic basis for $X$. For any $x \in X$, set $\text{supp}(x) = \{n : e^*_n(x) \neq 0\}$. For a given finite set $A \subset \mathbb{N}$, let $|A|$ denote the cardinality of $A$, $1_A = \sum_{i \in A} e_i$, and $P_A(x) = \sum_{i \in A} e_i^*(x)e_i$. For finite sets $A, B \subset \mathbb{N}$ we write $A < B$ if $\max A < \min B$.

In [8] Konyagin and Temlyakov introduced the thresholding greedy algorithm (TGA) $(G_m)$, where $G_m(x)$ is obtained by taking $m$ of the largest coefficients in the

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