Principles of the Field Theory of High Temperature Superconductivity in Underdoped Copper-Oxides

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Key Words: High Temperature Superconductivity.
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Abstract

Here I extend my last work about the origin of the pseudo-gaps in underdoped cuprates (arXiv: cond-mat. 1011.3206), to include the mechanism of superconductivity. This is done by adapting the formalism of the double correlations in systems with nested Fermi surfaces to the semi one dimensional system of strings of holes. It is proposed that magnetic interaction is crucial for the establishment of the pseudogap and the high temperature superconductivity. It is shown that superconductivity disturbs the completeness of the strings of holes, and creates fluctuations in them. This, in turn, reduces the magnetic interaction and the pseudogap order.
1. INTRODUCTION

The undoped edge of the Copper-oxide HTSC are Mott insulators. They exhibit anti-ferromagnetism (AFM) that is basically two dimensional, but becomes three dimensional by means of (weaker) inter-layer coupling. When very lightly hole-doped, these materials are still insulators. The conducting regions of their phase diagrams start usually with small doping, of the order of 5 percent. Assuming that uncorrelated itinerant quasiparticles maintain the electrical conductivity, would lead to the breakdown of AFM order, with an energy cost of $J \approx 125meV$ for every lattice site, whereas the compensation by kinetic energy applies only to a few percents of the sites- the doped sites. Consequently, the system "makes its best" to preserve regional AFM order over some doping range. This is done by correlating the movement of the holes, and it is the origin of the pseudogap phase. Indeed, Neutron scattering measurements (NSM) have proved that AFM regional order still exists in the underdoped regime [3-7], and some investigators linked this magnetic order to the correlation of the holes [5-7].

In a recent paper I proposed that holes doped in underdoped HTSC cuprates aggregate into linear rows and columns to produce arrays of checkerboard geometry [1]. These arrays were shown to result in the modulated anti-ferromagnetic (AFM) structures that had been observed by NSM, which contains four peaks, two at $(1 \pm 2\delta,1)\pi a^{-1}$, and two at $(1,1 \pm 2\delta)\pi a^{-1}$. The movements of the holes of such a column (row) are correlated to preserve their linear aggregation. The spins on both sides of any string of holes are of the opposite kind, so that they fit the local AFM phase when the string of holes moves [1,8]. The movement of the holes is one dimensional, where columns move in the x direction, and rows in the y direction. It was argued that this one dimensionality (of the columns and rows of holes) should necessarily be reflected in the arrangement of the electronic states in the reciprocal space. This led to propose the Fermi arrangement as in Fig.1 of Ref. [1], which includes the four sections in the anti-nodal directions, and the four diagonal sections in the nodal directions. It was proposed (in accord with experiment) that the one dimensional aggregated holes are related to the anti-nodal parts. Since the width of the anti-nodal parts is $\pi a^{-1}/2$ (which includes $N/2$ spins, where $N$ is the number of sites along one direction), a simple electron state counting leads to the conclusion that...
the annihilation of two columns of states in the reciprocal space is needed to produce one column of holes in the real space.

In order to examine the considerations that are presented above, about the preservation of AFM order, the application of perturbation theory on the $t$-$J$ Hamiltonian was attempted in [1]. This led to the conclusion that the system should be unstable if the hopping parameter $\tau$ is larger than the magnetic parameter $J$, since $\tau/J$ is the natural perturbation parameter for the movement of a string of holes as a unit. This, in turn, suggests the formation of a new phase— the pseudogap phase, since it is well known that one cannot bridge between two phases with different symmetry by the regular perturbation theory. Instead, the symmetry of the ordered phase has to be built into the unperturbed Hamiltonian, and eventually justified when obtained from the interaction part of the Hamiltonian [2,9,10]. This procedure was indeed exercised in [1], where an internal field with modulated AFM order was obtained from the suggested ground state which, in turn, has the symmetry of the ordered phase.

In the present paper I wish to apply the theory of the double correlations (DC) that was formulated for electronic systems with nested Fermi surfaces [2], to the present problem. There are questions to be addressed before this attempt. These questions are related to the basic difference between the species with which the two formalisms deal. While the formalism of DC assumes independent Fermionic species, in two dimensions, the system under consideration assumes one-dimensional aggregated strings. Moreover, the formalism of DC did not deal with charge and magnetic phase coherence of the individual holes, as we assume here. A common feature, though, is nesting. Nesting presents one dimensionality in the minimal sense of one-dimensional dispersion relations, independent of the direction lateral to the nesting.

We start the adaptation to the DC model (DCM) by presenting the columns (and rows) of holes as products of $N$ holes, residing on the same column parameter for columns (same row parameter for rows). This step resolves the problem of the dependence of the anti-commutation relations of the $C_j^+$’s (and their conjugates) on the arbitrary choice of odd or even number of holes per string [1]. The spin order that is related to any string of holes should be reflected by its associated magnetic energy. This is clear in [1], where AFM order is taken as the vacuum state on which the holes string operator $C_j^+$ acts upon. On the other hand, the DCM as presented in [2] does
not include any built-in magnetic order. In the present work, we correct this fault by defining holes states that are magnetically coherent. As a result, it is shown that the strings of holes- \( C_j^+ \) interact magnetically with each other, and that this interaction is crucial to superconductivity as well as to the formation of the pseudogap.

The DCM uses four dimensional matrices for the Hamiltonian and the propagators \([2]\). Consequently, it uses the four dimensional Dirac matrices. This dimensional extension was introduced for the simultaneous treatment of two order parameters, \( \Delta \) and \( \Lambda \), which express superconductivity and pseudogap, respectively. However, it seems that this extension has an extra benefit, which will be clear from the following analysis. That is the obtainment of an excitation spectrum with no energy gap even without resorting to the explicit definition of the magnetic phase B, which is complementary to phase A \([1]\).  

2. ADAPTING THE DC THEORY TO THE PRESENT PROBLEM

Without meaning to ignore the strings that are aligned horizontally in the checkerboard lattice, here we deal only with columns of holes just for a pedagogical convenience. Such an entity was defined as \( C_j^+ |0\rangle \), where \( C_j^+ \) creates a string of holes at the column \( j \), while reversing all the spins beyond \( j \). The vacuum state \( |0\rangle \) is an AFM state of \( N \times N \) sites. We assume \( n \) columns of holes, so that only \( N-n \) columns are occupied with spins, and the AFM phases A and B alternate when passing each column of holes. The Fourier transform of \( C_j^+ |0\rangle \) is

\[
C_k^+ |0\rangle = N^{-1/2} \sum_{j=1}^{N} C_j^+ e^{-ikaj} |0\rangle
\]

The ground state of the uncondensed phase of columns is

\[
|\Phi_0\rangle = \prod_{k=-\pi a^{-1} \delta}^{\pi a^{-1} \delta} C_k^+ |0\rangle
\]
The energy dispersion for the movement of a column of holes in the direction normal to its length was found to be

\[ e_k = -2J \cos(ak), \]  

where \( E_F \) is the Fermi energy, and the zero of \( k \) is defined to be on the boundary of the Brillouin Zone (BZ) of the occupied states [1]. In Eqs. (3) \( k \) denotes only the longitudinal component of the wave-vector. In fact, the wavevector has also a "transversal" component normal to it. The nature of the movements of the holes that constitute a column (or a row), as portrayed by Eqs. (13-18) of [1], suggests that only the longitudinal component is dispersive. However, I am still not certain about the dispersion of the transversal component, and it is treated here as non-dispersive only because of a lack of former investigation. The checkerboard geometry of the arrangement of the strings of holes suggests that the regions in the anti-nodal directions of the "Fermi surface" occupy only the central quarter of the BZ. It has been suggested [1], that this feature gives rise to the pattern of 4a periodicity that had been observed in STM experiments [11-13]. I believe that such a feature of the electronic system should invoke lattice re-adjustments via the electron-phonon interaction. However, since to my knowledge this problem has never been treated, it will still be bypassed in the present analysis. Consequently, the transversal component of \( k \) in the anti-nodal regions is still considered dispersion-less in the present paper.

Notice that the energy in Eqs. (3) includes only the kinematical part. The much larger magnetic part is the vacuum energy \( E_J = -N(N - 3n/2)J \), which is the eigenvalue of \( |\Phi_0 > \) on the magnetic Hamiltonian \( H_J \). Since \( H_J \) was taken as the unperturbed Hamiltonian, on which \( H_i \) acts as a perturbation, it is essential that the energy \( E_J \) is conserved as the column \( C_j^+ \) propagates. This could be achieved only when the two complimentary AFM phases A and B reside on the two different sides of the column of holes. When several columns of holes exist (but rows are disregarded) the magnetic energy is \( E_J \), as long as the columns do not touch each
other. Since the holes occupancy on a site is never larger than one, a column \( C^+_j \) is considered as "touching" another column \( C^+_i \), if and only if \( j = i \pm 1 \). In this case of two columns that are "touching" each other, the magnetic energy is lowered by \( NJ / 2 \), which suggests that the magnetic interaction energy between two columns is

\[
V(i, j) = -\frac{1}{2} N J \delta(i, j \pm 1).
\]  

(4)

When this interaction is Fourier transformed, one gets

\[
V(\Delta k) = -J \cos(\Delta k \cdot a).
\]  

(5)

The magnetic interaction between two strings of holes is short ranged, attractive, and its Fourier transform is of the order \( J \).

The DCM incorporates two kinds of correlations, which result in superconductivity and pseudogap, the correlations of the Cooper pairs and the correlations of the electron-hole pairs. The application of the DCM to the present model of aggregates of holes has to be carried out with caution. The problem is that the physical entity \( C^+_k |0> \) is a collective entity, whereas the DCM deals with electrons or holes as its basic quasiparticles components. In the following I make the first step to resolve this problem by expressing \( C^+_k |0> \) as a product of its components, while insuring that these components maintain their magnetic coherence. Thus, we define the AFM vacuum phase A as

\[
|0>_A = \prod_{i,j} a^+_{ij,s_{ijA}} |100>.
\]  

(6a)

\[
s_{ijA} = (-1)^{i+j} \uparrow.
\]  

(6b)

In Eqs. (6) \( a^+_{ij,s_{ijA}} \) creates an electron with spin projection \( s_{ijA} \) at the lattice site \((ij)\), and \( |100> \) is the vacuum state without electrons. For phase B the spins are reversed, \( s_{ijB} = (-1)^{i+j+1} \uparrow \). A column of holes is defined as
\[ C_j^+ |0 >_{\{j_{n-1}\}} = \prod_{i=1}^{N} a_{ij,s_{ij}} \prod_{l=N}^{j+1} a_{il,s_{id}}^+ a_{il,s_{dl}}^+ |0 >_{\{j_{n-1}\}} \]  
\hspace{1cm} (7a)

\[ = \prod_{i=1}^{N} c_{ij,x_{ij}}^+ |0 >_{\{j_{n-1}\}} \]  
\hspace{1cm} (7b)

\[ = \prod_{k_i,s} c_{k_i,j,s}^+ |0 >_{\{j_{n-1}\}} \]  
\hspace{1cm} (7c)

In Eqs. (7) \(|0 >_{\{j_{n-1}\}}\) indicates that the vacuum already includes \((n-1)\) hole columns apart from \(C_j^+\), and \(\{j_{n-1}\}\) is the set of indexes of the columns of holes that are higher than \(j\). The operators are defined implicitly for the phases A and B. Eq. (7b) defines the operators \(c_{ij,x_{ij}}^+\). With this definition it is evident that \(c_{ij,x_{ij}}^+\) obeys Fermionic anticommutation relations, as does \(a_{ij,x_{ij}}\). The operator \(c_{k_i,j,s}^+\), is defined as the Fourier transform over the transversal sites, so that \(k_i\) is the transversal component of \(k\), and the spin \(s\) must be defined in accordance with \(s_{ij}\).

\[ c_{k_i,j,s}^+ (A,B) = N^{-1/2} \sum_{i=1}^{N} c_{ij,s_{ij}A,B}^+ e^{-ik_i(ia)} \delta(s-s_{ijA,B}) . \]  
\hspace{1cm} (8a)

Therefore, Eq. (1) may also be written as

\[ C_k^+ = N^{-1/2} \sum_{j=1}^{N} e^{-ik_{ij}a} \prod_{k_i,s} c_{k_i,j,s}^+ |0 >_{\{j_{n-1}\}} = \prod_{k_i,s} c_{k_i,k_i,s}^+ = C_k^+ C_k^+. \]  
\hspace{1cm} (8b)

The aggregate \(C_k^+\) is defined as a product of the \(c_k^+\)'s, because the \(c_k^+\)'s were defined to include the proper spin order. As in [2], we define the Nambu-like field
\[
\Psi_{k_j,k_i} = \begin{pmatrix}
    c_{k_i,k_i,s} \\
    c_{-k_i,-k_i,s} \\
    c_{k_i,k_j,s} \\
    c_{-k_i,-k_j,s}
\end{pmatrix}
\]

and the ground state of the condensed phase

\[
|\Psi_0 \rangle = \prod_{k \leq k_F} |\Psi_k \rangle = \prod_{k \leq k_F} \left[ v_k + u_k \bar{\psi}_k^+ \alpha_1 \bar{\psi}_k + w_k \bar{\psi}_k^+ \alpha_3 \bar{\psi}_k + \theta_k c_{k,j,s}^+ c_{-k,-j,s} + c_{-k,j,s}^+ c_{k,-j,s} \right] |\Phi_0 \rangle
\]

In Eq.(10), \(\alpha_{1,3}\) are Dirac matrices, \(k_j\) is not shown for the sake of simplicity, so that it is implicitly included in \(k\). Notice that \(k\) is within the Fermi surface, and that there is no multiplication over the spin states because both are included within each \(|\Psi_k \rangle\). The order of the multipliers \(|\Psi_k \rangle\) in Eq.(10) is not important since each is made of an even number of operators, so that they commute with each other.

The treatment here, in comparison with the DCM, needs some clarifications. Eqs.(9) and (10) are formally not different from their counterparts of the DCM. Moreover, both models are based on electronic systems that occupy states with nesting features. However, Eqs.(6-8) portray a system with AFM coherence, which does not exists in the DCM. This should not violate the validity of the application of the DCM to the present case. This is so because the term \(w_k \bar{\psi}_k^+ \alpha_3 \bar{\psi}_k\) in Eq.(10) only redistributes the unoccupied electronic states while keeping the symmetry with regard to the transversal direction. The concept "redistribution" here means that the Fourier transform of \(c_{k,j,s}^+\) includes \(k_j\) component, in addition to the \(k_i\) component. The extent to which it acts as the term \(w_k C^+_k C_k\) of Eq.(22) in [1] will be evaluated in the following. Notice that \(\Lambda_k = -2E_k w_k\), which is the parameter that controls the electron state redistribution rate, is independent of the transversal component of \(k\). The redistribution of all the transversal components by the same rate does not necessarily exclude deviations and fluctuations from the perfect linear order of \(C^+_j\). This issue is
semi-quantitatively analyzed in the following. We start this analysis with the simple case where only the pseudogap order is present, and the superconductive parameter is set equal to zero. In doing so, it is simpler to switch from 4-dimensional matrix system to two dimensional one. This is so because the 4-dimensional analysis still treats the combination of the two time reversal operators $c_{k,s}$ and $c_{-k,-s}$, despite the removal of superconductivity. The two dimensional version of the $k$ component of the ground state is

$$\ket{\Psi_k} = \prod_{k_{r,s}} \left[ v_{k_l} + w_{k_l} c_{k_{r,s}}^{+} c_{k_{l,s}} \right] \ket{\Phi_0}.$$ 

$$= \prod_s v_{k_l}^{N/2} \sum_{i=0}^{N/2} f_k \sum_j \left\{ c_{k_{r,s}}^{+} c_{k_{l,s}}^{+} \cdots c_{k_{l,s}}^{+} c_{k_{l,s}} \right\} \ket{\Phi_0}.$$  

(11)

In Eq. (11), $f_k = \left( w_{k_i} / v_{k_i} \right)$, and $\left\{ c_{k_{r,s}}^{+} c_{k_{l,s}}^{+} \cdots c_{k_{l,s}}^{+} c_{k_{l,s}} \right\}$ is the $j$-th combination of $i$ pairs of operators $c_{k_{r,s}}^{+} c_{k_{l,s}}$, out of the total $N/2$ pairs. The sub-subscripts in the combination $\left\{ c_{k_{r,s}}^{+} c_{k_{l,s}}^{+} \cdots c_{k_{l,s}}^{+} c_{k_{l,s}} \right\}$ denote the transversal components $k_l$. At the Fermi level, $v_{k_l} = w_{k_l} = 1/\sqrt{2}$, and $f_k = 1$. The numerical quantity that determines the redistribution is the number of combinations, which is given by the Binomial-

$$\binom{N/2}{i}.$$ 

It has its maximum at $i = N/4$, and its width is of order $\sqrt{N/2}$. Recalling that the index $i$ denotes the number of $k_l$ states whose longitudinal component transformed from $k_l$ to $\overline{k_l}$, means that half of the states undergo this redistribution. When the products $\left\{ c_{k_{r,s}}^{+} c_{k_{l,s}}^{+} \cdots c_{k_{l,s}}^{+} c_{k_{l,s}} \right\}$ are written as products of Fourier transforms, terms with mixed column indexes in the real space should be eliminated, because they are not magnetically coherent. This elimination is justifiable, according to the perception of [1], due to the large magnetic energy of such terms, which is unacceptable for the ground state. Since $H_j$ is not explicitly incorporated into our treatment, the elimination of these high energy terms has to be done formally. Thus, we get
\[ |\Psi_k| = C_1 \sum_{i,j} C_j^+ \left( \binom{N/2}{i} \right) [(N/2 - i) \exp(-ik_j ja) + i \exp(-ik_j ja)] \]

\[ = \frac{1}{\sqrt{2}} \sum_j C_j^+ \left[ \exp(-iajk_j) + \exp(-iajk_j) \right], \quad (12a) \]

where \( C_1 \) is a normalization constant, and the pre-factors of the exponentials give the relative weights that result from the number of \( k_i \) states. Away from the Fermi level, \( f_k \neq 1 \), and the numerical quantity that determines the redistribution is \( f^I_k \left( \binom{N/2}{i} \right) \). In these conditions we have

\[ |\Psi_k| = C_1 \sum_{i,j} C_j^+ \left( \binom{N/2}{i} \right) [(N/2 - i) \overline{\nu}_k \exp(-ik_j ja) + i \overline{\nu}_k \exp(-ik_j ja)] \]

\[ = \sum_j C_j^+ \left[ \overline{\nu}_k \exp(-ik_j ja) + \overline{\nu}_k \exp(-ik_j ja) \right]. \quad (12b) \]

The right hand sides of Eqs. (12a,b) have been summed over \( i \), and yielded the constants \( \overline{\nu}_k \) and \( \overline{\nu}_k \) as the pre-factors of the exponentials, while keeping \( C_j^+ \) intact. Therefore, the DCM seems to be compatible with magnetically coherent columns of holes, as long as one considers only the pseudogap phase.

When superconductivity is incorporated into the analysis we write

\[ |\Psi_k| = \prod_{k_i} \left[ v_k + u_k (c_{k \downarrow} \downarrow c_{k \uparrow} \uparrow + c_{k \uparrow} \uparrow c_{k \downarrow} \downarrow) + w_k (c_{k \uparrow} \uparrow c_{k \downarrow} \downarrow + c_{-k \downarrow} \downarrow c_{-k \uparrow} \uparrow) + \theta_k (c_{k \uparrow} \uparrow c_{k \uparrow} \downarrow + c_{-k \downarrow} \downarrow c_{-k \downarrow} \downarrow) \right] \prod_{k_i} c_{k \uparrow}^+ c_{k \downarrow}^+ > \]

\[ = v_k^{N/2} \sum_{i,n,m=0} \left\{ \frac{u_k}{v_k} \right\}^i \sum_{b_i \neq b_n \neq b_m} [(c_{k_1 \downarrow} \downarrow c_{k_1 \uparrow} \uparrow + c_{k_1 \uparrow} \uparrow c_{-k_1 \downarrow} \downarrow) \ldots (c_{k_n \downarrow} \downarrow c_{k_n \uparrow} \uparrow + c_{-k_n \downarrow} \downarrow c_{-k_n \downarrow} \downarrow) \right \} b_i \]

\[ \times \left( \frac{W_k}{v_k} \right)^n [(c_{k_1 \uparrow} \uparrow c_{k_1 \downarrow} \downarrow + c_{k_1 \downarrow} \downarrow c_{-k_1 \uparrow} \uparrow) \ldots (c_{k_n \uparrow} \uparrow c_{k_n \downarrow} \downarrow + c_{-k_n \downarrow} \downarrow c_{-k_n \uparrow} \uparrow) \right] b_n \]
In Eq. (13), each square bracket in the sums over $b_i, b_n, b_m$ represent a combination of their respective $i,n,m$ pairs of operators (in addition to their time reversals), as indicated. The combinations $b_i, b_n, b_m$ are different in each component. The number of combinations is given by the 3-dimensional Binomial $\binom{N/2}{i,n,m}$ whose maximum is at $i = n = m = N/8$, and its width with respect to each degree of freedom is of the order $\sqrt{N/2}$. The state $|\Psi_k\rangle$ may also be written as

$$\left|\Psi_k\right\rangle = \sum_{i=0}^{N/2} \frac{W_k}{v_k} \sum_{b_i} [(1 + c_{k_1}^+ c_{-k_1}^+ c_{k_1}^- c_{-k_1}^-) \ldots (1 + c_{k_m}^+ c_{-k_m}^+ c_{k_m}^- c_{-k_m}^-)] b_i C_{\pm k}^\dagger \downarrow (b_i)$$

$$= \sum_{i=0}^{N/2} \frac{W_k}{v_k} \sum_{b_i,b_j} [(c_{k_1}^+ c_{-k_1}^+ c_{k_1}^- c_{-k_1}^-) \ldots (c_{k_m}^+ c_{-k_m}^+ c_{k_m}^- c_{-k_m}^-)] b_i C_{\pm k}^\dagger \downarrow (b_i) \quad \text{(14a)}$$

$$c_{k_1}^+ c_{-k_1}^- \downarrow (b_i) = v_k^{N/2-i} \sum_{n,m=0}^{N/2-i} \frac{W_k}{v_k} \sum_{b_n \neq b_m \neq b_j} \left[(c_{k_1}^+ c_{-k_1}^- + c_{k_1}^+ c_{-k_1}^-) \ldots (c_{k_n}^+ c_{-k_n}^- + c_{k_n}^+ c_{-k_n}^-)] b_n \right. \left. \times \left(\frac{\theta_k}{v_k}\right)^{2l} \left[ c_{k_1}^+ c_{-k_1}^+ \ldots c_{k_m}^+ c_{-k_m}^+ \right] b_m \left[ c_{k_1}^+ c_{-k_1}^- \ldots c_{k_i}^+ c_{-k_i}^- \right] b_i 10 \rangle \quad \text{(14b)}$$

In Eqs. (14), $l = \frac{N}{2} - i - n - m$, $10 \rangle$ is the vacuum state of holes, and $(c_{k_1}^+ c_{-k_1}^- c_{k_1}^+ c_{-k_1}^-) \ldots (c_{k_i}^+ c_{-k_i}^- c_{k_i}^+ c_{-k_i}^-) b_j$ denotes a combination of $2j$ pairs of operators out of the $2i$ pairs of the combination $[\ldots]_{b_j}$. The number of these combinations is the Binomial $\binom{i}{j}$. The superconductive operators in Eq. (14a) do not conserve the number operator. Consequently, we are unable to construct a modified
Fourier transform, such as the ones in Eqs. (12), which keep the column operator $C^+_j$ intact. Such a Fourier transform could be defined for $C^+_\\pm_k \uparrow \downarrow (b_i)$. If so, then $C^+_\\pm_k \uparrow \downarrow (b_i)$ should represent a column in the real space weighted by some number smaller than unity. This is so because $C^+_\\pm_k \uparrow \downarrow (b_i)$ includes only $2N$ states per spin.

The $[(...)_{b_j}]_{b_i}$ combinations of Eq. (14a) create between zero and $2i$ state operators per spin. As a result, the column can no longer be intact in the regular space. It is replaced by a linear combination of columns, each weighted by a different number, which may be considered as fluctuations of the holes population in the string. Superconductivity tends to disturb the one dimensional order that characterized the pseudogap phase. The scheme of DCM can still be applied with the notion that the column in the real space should be replaced by a fluctuating column. These fluctuations should reduce the pseudogap via the reduction of the effective magnetic interaction.

The latter conclusion is in agreement with experiment. The well known phase diagram of Copper-oxide HTSC shows that electric conductivity and superconductivity start at small doping levels (of order 5%). At this level of doping superconductivity is minimal, but pseudogap is maximal. Increasing doping increases the density of the carriers, which enhances superconductivity. This enhancement is correlated with a steep reduction of the pseudogap, until the two order parameters become equal around the optimum doping of about 15%-20%. Beyond this level both order parameters reduce fast to zero.

We continue the treatment of Eq. (14) according to the principles that led to Eq. (12b), namely, we demand magnetic coherence for the selected terms which make up $C^+_\\pm_k \uparrow \downarrow (b_i)$. Technically this amounts to discarding terms that are not magnetically coherent, and including only terms that result from columns of holes. These columns are weighted by $(1-2i/N)$. The missing weight is compensated by the superconducting term. The problem is that the "compensating part" is not fixed to the right value. It varies between under compensation to over compensation. One could fix the index $j$ in the combination $(c^+_{k_1}c^+_{-k_1}c^+_{k_1}c^+_{-k_1}\ldots c^+_{k_j}c^+_{-k_j}c^+_{k_j}c^+_{-k_j})_j(b_i)$ to be $j=i/2$, and get the proper number of holes in each string. The attempted
justification for such an arbitrary fixing could be similar to the one we have just used for the other (non-superconducting) components, namely- to reduce magnetic energy. Here however, such a number fixing is not justifiable since it implies a frozen fixed collective state with no room for the variations that enable Cooper pair correlations.

The experimental evidence is that Cooper pairing occurs between separate holes, and not between strings [14]. It was observed that the magnetic flux through a ring of cupper oxide HTSC (YBCO- for example) is quantized in multiples of \( \Phi_{cp} = h/2e \approx 2 \times 10^{-15} \text{Wb} \), which demonstrates Cooper pairing by two units of electron charge [14,15]. There is also an experimental evidence for singlet (even parity) pairing [15-17]. On the other hand Neutron scattering experiments indicate the agglomeration of holes into linear strings [3-7]. These two perceptions may be reconciled by the following approximation

\[
|\Psi_\uparrow(i)\rangle = C_2[\bar{\psi}_{\uparrow}^c \bar{C}_{k \uparrow}^+ C_{k \downarrow}^+ \bar{\psi}_{\uparrow} - \bar{\psi}_{\downarrow} \bar{C}_{k \downarrow}^+ C_{k \uparrow}^+ \bar{\psi}_{\downarrow} + \bar{\psi}_{\downarrow} \bar{C}_{k \downarrow}^+ C_{k \uparrow}^+ \bar{\psi}_{\uparrow} + \bar{\psi}_{\uparrow} \bar{C}_{k \uparrow}^+ C_{k \downarrow}^+ \bar{\psi}_{\downarrow} ] + \bar{\psi}_{\downarrow} \bar{C}_{k \downarrow}^+ C_{k \uparrow}^+ \bar{\psi}_{\downarrow} ] |0\rangle , \quad \text{(15a)}
\]

\[
C_{sc}(i) = (N/2i) \sum_{b_j,b_j} [(c_{k \uparrow}^c c_{-k \uparrow}^c c_{k \downarrow}^+ c_{-k \downarrow}^+ ... c_{k_j \uparrow} c_{-k_j \uparrow} c_{k_j \downarrow} c_{-k_j \downarrow}) b_j b_j] . \quad \text{(15b)}
\]

In Eq. (15a) \( C_2 \) is a normalization constant. The superconducting term is scaled by \( N/2i \) because otherwise \( C_{sc}(i) \) comprises an average of \( i \) states per spin, whereas each of the other terms comprises \( N/2 \) states per spin. Eq. (15) assumes a magnetically coherent average of all the terms of Eq. 14, except for the superconductive term- \( C_{sc}(i) \). Notice that \( C_{sc}(i) \) is written in terms of operators of single holes, whereas the other components are written in terms of operators of strings. The multiple values of \( j \) (for each \( i \)) in Eq. (15b) suggest that the column occupancy in the regular space should fluctuate, and that the strength of the fluctuations is controlled by \( \bar{\psi}_k \).

3. THE APPLICATION OF THE DOUBLE CORRELATION MODEL.

The ground state of Eq. (15) assumes column operators for the \( \bar{\psi}_k, \bar{\psi}_k, \bar{\psi}_k \) terms, while preserving single holes operators for the superconducting term. This treatment
inherently assumes that $C_{sc}(i)$ is not consistent with a fixed number of Cooper pairs, which in turn, results in columns of fluctuating occupancy. These fluctuations demand the inclusion of the magnetic Hamiltonian $H_J$ into the analysis to establish a formalism of a grand perturbation scheme. This scheme should include perturbations with respect to magnetic interactions as well as perturbations with respect to the off-diagonal elements that are typical to DCM. This in turn, would present difficulties that are beyond the present analysis, where it is assumed that the holes aggregate in columns. The approximation here is consistent with fixing $j$ at its most probable value: $j = i/2$, which should result in exact full occupancy of the columns in the regular space. It is a reasonable approximation only for the highly underdoped regime. Accordingly we define the string field operator as

$$\hat{\Psi}_k = \prod_{k_i} \begin{pmatrix}
c_{k_i, k_j, s} \\
c_{k_i, -k_j, s}^+
\end{pmatrix}.$$  \hspace{1cm} (16)$$

In Eq. (16), the multiplication operator operates separately on each component of the vector. The order of the multiplied terms is reversed in the transposed (conjugated) row operator $\hat{\Psi}_k^+$. Then the discussed approximation for the ground state is

$$|\Psi_0> = \prod_{k \leq k_F} |\Psi_k> = \prod_{k \leq k_F} [v_k + u_k \hat{\Psi}_k^+ \alpha_1 \hat{\Psi}_k + w_k \hat{\Psi}_k^+ \alpha_3 \hat{\Psi}_k + \theta_k C_{-kF}^+ C_{-k} + C_{k}^+ C_{-kF} + C_{-k}^+ C_{-k}] |\Phi_0>.$$  \hspace{1cm} (17)$$

The parameters $v_k, u_k, w_k, \theta_k$ are redefined in Eq. (17), and are not necessarily equal to the ones defined in Eq. (10). We demonstrate the consistency of Eq. (17) with complete strings of holes in the regular space by comparing its resultant spatial charge density with Eq. (25) in Ref. [1].

$$\rho(a_j) = \langle 0 | C_j^+ C_j | 0 > = (2N)^{-1} \langle 0 | \sum_{q \neq q'} C_q^+ C_{q'} e^{ia_j(q-q')}$$
Straightforward calculations yield

$$\rho(aj) = N^{-1}[1 + 2w_k \cos(2k_F aj)] + \prod_{k \neq k_1} \Psi_k | C_j^+ C_j | \prod_{k \neq k_1} \Psi_k >$$

$$= N^{-1}[n + \sum_k 2w_k \cos(2k_F aj)]$$

(19)

The agreement with Eq. (25) of [1] suggests that the ground state of Eq. (17) is indeed consistent with full strings of holes in the regular space. As in [1], the implied CDW of Eq. (19) may be neutralized by defining a complimentary AFM phase B, whose $w_k$ is of reversed sign. It is shown in [1] that this eliminates the CDW, while enhancing the SDW.

In Ref. [2] a special effort was made to define the four basic excitations, so that all should have the same excitation energy of $E_k = \sqrt{\epsilon_k^2 + \Lambda_k^2 + \Lambda_k^2}$. The main problem for doing so was that there is an additive energy term that comes in different signs for two different pairs of the excitations. The effort was technically successful, and the final set of the four excitations in [2] has the same eigenvalue. However, this "success" has an inherent flaw since it conceals an important physical feature of the system, a feature that is related to its electrical conductivity. It turns out that electrical conductivity is more apparent with the other set of excitations. The operators of these other set were denoted in [2] by, $\hat{\gamma}_k, \hat{\eta}_k, \hat{\rho}_k, \hat{\sigma}_k$. This set of excitations is found, as in [2,9], by requiring that the ground state yields zero when operated upon by each one of these annihilation operators, namely

$$\hat{\gamma}_k, \hat{\eta}_k, \hat{\rho}_k, \hat{\sigma}_k | \Psi_0 > = 0.$$  

(20)
Eqs. (20) make one set of four equations which do not yield immediately the basic four excitations. However, following the procedure in [2], we get

\[ \hat{O}_k = \begin{pmatrix} \hat{\eta}_k \\ \hat{\gamma}_k \\ \hat{\rho}_k^+ \\ \hat{\delta}_k^+ \end{pmatrix} = \hat{P}_k \hat{\Psi}_k, \quad (21) \]

\[ \hat{P}_k = (1 + 2w)^{-1/2} [I(\theta_k + w_k) + u_k \alpha_1 \beta + (v_k + w_k) \alpha_3 \beta + iu_k \tau_2]. \quad (22) \]

In Eq. (22) \( \beta \) is the Dirac matrix, \( I \) is the unity matrix, and \( \tau_2 \) is the four dimensional Pauli matrix, as described in [2]. We also get the following relations between the parameters

\[ v_k^2 + 2u_k^2 + 2w_k^2 + \theta_k^2 = 1, \quad (23a) \]

\[ \theta_k v_k = u_k^2 + w_k^2, \quad (23b) \]

\[ \theta_k + v_k = \pm 1. \quad (23c) \]

We arbitrarily assume the plus sign in Eq. (23c). Eqs. (21-23) yield

\[ \{ \hat{\eta}_k, \hat{\rho}_k^+ \} = \{ \hat{\delta}_k, \hat{\delta}_k^+ \} = \delta_{k,k'}, -\delta_{k,k}. \quad (24a) \]

\[ \{ \hat{\gamma}_k, \hat{\rho}_k^+ \} = \{ \hat{\rho}_k, \hat{\delta}_k^+ \} = \delta_{k,k'} + \delta_{k,k}. \quad (24b) \]

Now we need to choose the right field of the system- \( \psi_k \). So far we have presented two possible fields, the Nambu-like field- \( \hat{\Psi}_k \) of Eq. (16), and \( \hat{O}_k \) of Eq. (21). They transform to each other by the unitary transformation \( \hat{P}_k \). The fact that they transform by a unitary transformation does not mean that they yield the same Hamiltonian. This is so because, as is shown in the following, the Hamiltonian
contains the term: $\sum_k E_k (\psi^+_k \psi_k^+ + \psi^+_k \psi_k^+)$, which is dependent upon the chosen field. Therefore, we must adopt a suitable criterion for choosing the right field. Such a criterion may be that the chosen field should yield an internal space dependent field that is consistent with the pseudogap phase, namely that $< \Psi_0 | \psi^+(x) \psi(x) | \Psi_0 >$ includes a term of the form $\cos(2k_F x)$, as in [1]. It turns out that either $\bar{\Psi}_k$ or $\hat{O}_k$ yields zero for an internal field. However, for either $\bar{\Psi}_k$ or $\hat{O}_k$, the zero is a result of two equal finite parts that cancel each other. This reminds us of the similar internal charge cancellation that exists between phases A and B in Ref. [1]. The oscillating internal fields there result from the products between $v_k C_k^+$ and $\pm w_k C_k^+$, which yield $\pm 2 v_k w_k \cos(2k_F x)$, which out-cancelled when added. In the present analysis, the internal fields created by $\hat{\rho}_k^+$ and $\hat{\sigma}_k^+$ out-cancel each other, for either $\bar{\Psi}_k$ or $\hat{O}_k$. This is so because $\hat{\rho}_k^+ = \hat{\sigma}_k^+$, while $\hat{\sigma}_k^+ = -\hat{\sigma}_k^+$, so that the products $\hat{\rho}_k \hat{\rho}_k^+ | \Psi_0 >$, and $\hat{\sigma}_k \hat{\sigma}_k^+ | \Psi_0 >$, cancel each other. However, while the pre-factor for these products is unity, when the field is $\hat{O}_k$, it is of the proper size when the field is $\bar{\Psi}_k$, as one finds in the following:

$$< \Psi_0 | \bar{\Psi}^+(x) \bar{\Psi}(x) | \Psi_0 > = \frac{1}{2N} < \Psi_0 | \sum_{k,k'} \bar{\Psi}^+_k \bar{\Psi}_k^+ \exp[i(k-k')x] | \Psi_0 >$$

$$= \frac{n}{N} + \frac{1}{N} \sum_k < \Psi_k | \cos 2k_F x (\hat{P}_{k,\rho} \hat{P}_{k,\rho}^{-1} - \hat{P}_{k,\sigma} \hat{P}_{k,\sigma}^{-1}) | \Psi_k > = \frac{n}{N} . \quad (25)$$

However, each of the two terms in the sum is equal to

$$\frac{1}{N} \sum_k < \Psi_k | \cos 2k_F x \hat{P}_{k,\rho} \hat{P}_{k,\rho}^{-1} | \Psi_k > = \frac{1}{N} \sum_k (2w_k + \frac{4\mu_k^2}{1+2w_k}) \cos 2k_F x \quad (26)$$
Notice that by the Fermi level- \(|\varepsilon_k| \ll |\Lambda_k|\), and in the underdoped regime- \(|u_k| \ll |w_k|\). Consequently, \(2\sqrt{w_k^2 + u_k^2} \approx 2w_k + \frac{u_k^2}{w_k} \approx 2w_k + \frac{4u_k^2}{1 + 2w_k}\), because in this limit \(2w_k = \frac{|\Lambda_k|}{E_k} \approx 1\). This suggests that the field \(\hat{\psi}_k\) yields an internal field which is in agreement with the one obtained in [1], for \(u_k = 0\).

The conclusion from the last paragraph is that the unperturbed Hamiltonian density should be calculated by means of \(H_0(x) = i\hat{\bar{\psi}}^+(x,t) \frac{d}{dt} \hat{\psi}(x,t)\), which yields,

\[
H_0 = \frac{i}{2} \sum_{k,k'} \hat{\Delta}_k^+ \hat{\Delta}_k \hat{P}_k^{-1} \frac{d}{dt} \hat{O}_k(t) \tag{27a}
\]

\[
= \frac{1}{2} \sum_k E_k \hat{O}_k^+ \{\beta - \cos(2k_F x) [\tau_3 \xi_k + \tau_1 \xi_k - \alpha_0 \beta (\theta_k - v_k)]\} \hat{O}_k \tag{27b}
\]

In Eq. (27), \(\xi_k = (2w_k + \frac{4u_k^2}{1 + 2w_k})\), \(\xi_k = \frac{2u_k (\theta_k - v_k)}{1 + 2w_k}\), \(\tau_1\) and \(\tau_3\) are the four dimensional Pauli matrices, \(\alpha_0\) contains two 2x2 unit matrices in the cross diagonal, and we have assumed that the time dependence of the basic excitation (creation) operators is given by \(\exp(iE_k t)\). The Hamiltonian of Eq. (27) may be diagonalized by the usual method of solving the characteristic equation \(|H_0 - \lambda I| = 0\). We easily find that, \(\xi_k^2 + \xi_k^2 = 4(w_k^2 + u_k^2) = \frac{F_k^2}{E_k}\), and \(\lambda_{k \pm}^2 = \{E_k \pm \cos(2k_F x)F_k\}^2\). We have neglected the \(\alpha_0 \beta\) term in the characteristic equation since it scales by \((\theta_k - v_k)^4\), and we assume \(\varepsilon_k \ll |\Lambda_k|\). Thus, we get

\[
H_0 = \frac{1}{2} \sum_k O_k^+ (\beta E_k - \tau_3 F_k \cos 2k_F x) O_k \tag{28}
\]
The diagonalization matrix is \( Q_k = \frac{1}{\sqrt{1+ d_k^2}} (I + id_k \tau_2) \), where \( d_k = \frac{\xi_k}{2 \zeta_k} \ll 1 \), so that

\[
O_k = Q_k \hat{O}_k = P_k \hat{\Psi}_k
\]

\[
P_k = C_3 \left\{ I(\theta_k + w_k - d_k u_k) + \alpha_1 \beta[u_k - d_k (v_k + w_k)] + \alpha_3 \beta(v_k + w_k + d_k u_k) + i \tau_2[u_k + d_k (\theta_k + w_k)] \right\}
\]

(29a)

(29b)

where \( C_3 \) is a normalization constant.

The results in Eqs.(27) and (28) seem to be inconsistent with the assumption leading to them. In calculating Eq. (27) we have assumed that the energy eigenvalues of the four basic excitations are \( E_k \), whereas Eq. (28) suggests that they are \( E_k \pm F_k \cos 2k_F x \). This seemingly inconsistency is resolved when we notice that the extra terms of \( \pm F_k \cos 2k_F x \) resulted from:

\[
\hat{\Psi}_k^\pm(t) \left. \frac{d}{dt} \hat{\Psi}_k(t) + \hat{\Psi}_k^\pm(t) \left. \frac{d}{dt} \hat{\Psi}_k(t) \right) \right|_2
\]

This terms connect between the fields with the momenta: \( k \) and \( \vec{k} \) that differ by \( 2k_F \), which is just the amount by which momentum is not conserved in the discussed system. The sum in Eq. (28) has only one momentum parameter, and the energy that results from the mixture between \( \hat{\Psi}_k \) and \( \hat{\Psi}_{\vec{k}} \) is already built in a single momentum presentation. It is equivalent (but not identical) to the uncertainty of the lattice momentum by reciprocal lattice vectors. This perception suggests that the transition from Eq. (27a) to Eq. (28) is equivalent to the transition from the extended Brillouin zone to the reduced Brillouin zone. To be more specific, one can define two fields: \( \hat{\Psi}_k \) and \( \hat{\Psi}_{\vec{k}} \), for \( |k| < |k_F| \), and associate the energies \( \pm F_k \cos 2k_F x \) to the interaction between them. Notice that an additional Fourier transformation on Eqs. (26), and (28) replaces the explicit dependence upon \( x \) by two momentum modulated satellites, namely:

\[
\pm F_k \cos 2k_F x \rightarrow \pm \frac{1}{2} (F_{k+2k_F} + F_{k-2k_F})
\]

We immediately notice the relationship to the satellites observed by Neutron scattering experiments. We also note that the second term in Eq. (28) is dependent upon \( x \), and it vanishes when averaged over \( x \). Moreover, the spatial dependence of \( \eta_k \) is anti-phased relative to that of \( \gamma_k \). So are the spatial dependences of \( \rho_k \) and \( \sigma_k \). Consequently, the application of \( H_0 \)
on the ground state yields x-independent result, because the spatial dependences of \( \rho_k \) and \( \sigma_k \) cancel each other.

The Hamiltonian \( H_0 \) may be expressed in terms of \( \tilde{\Psi}_k \). This is obtained by means of the relations: 

\[
P_k^{-1} \beta P_k = \beta(\theta_k - v_k) - 2u_k \alpha_1 - 2w_k \alpha_3, \quad \text{and} \quad P_k^{-1} \tau_3 P_k \approx \tau_3 + 2u_k \tau_1.
\]

Thus, we get

\[
H_0 = \frac{1}{2} \sum_k E_k \tilde{\Psi}_k^+ \beta(\theta_k - v_k) - 2u_k \alpha_1 - 2w_k \alpha_3 \frac{F_k(x)}{E_k}(\tau_3 + 2u_k \tau_1) \tilde{\Psi}_k
\]

Eq. (30b) is obtained from Eq. (30a) by means of the relations: 

\[
\varepsilon_k = E_k(\theta_k - v_k), \quad \Lambda_k = -2E_k w_k, \quad \Delta_k = -2E_k u_k.
\]

In Eq. (30b) the x-dependence of \( F_k(x) \) should be interpreted as \( F_k(x) = F_k \cos(2k_F x) \). One immediately notices that the x-dependence of the diagonal terms, that are related to \( c_{k,s} \) and \( c_{\mp k,s} \), are in phase with each other.

Those that are related to \( c_{-k,-s}^+ \) and \( c_{-\mp k,-s}^+ \) are also in phase with each other, but anti-phased with respect to the former ones. Thus, the reversed phases of the internal CDW fields, due to time reversal states, come out automatically in the present four dimensional DCM.

The unperturbed propagator is

\[
G_0(k,t) = -i < \Psi_0 | T \{ \tilde{\Psi}_k(t) \tilde{\Psi}_k^+(0) \} | \Psi_0 > = -i < \Psi_0 | T \{ P_k^{-1} O_k(t) O_k^+(0) P_k \} | \Psi_0 > .
\]

The time dependence of the field \( O_k(t) \) is obtained from Eq. (28),
\[ O_k(t) = \begin{pmatrix} \eta_k \exp\{-i[E_k - F_k(x)]t\} \\ \gamma_k \exp\{-i[E_k + F_k(x)]t\} \\ \rho_k \exp\{i[E_k + F_k(x)]t\} \\ \sigma_k \exp\{i[E_k - F_k(x)]t\} \end{pmatrix}, \] (32)

which yields

\[
G_0 = -i < \Psi_0 | \Theta(t) \begin{pmatrix} p_{K_\eta}^{-1} p_{K_\eta} \exp\{-i[E_k - F_k(x)]t\} \\ 0 \\ p_{K_\gamma}^{-1} p_{K_\gamma} \exp\{-i[E_k + F_k(x)]t\} \end{pmatrix} 
- \Theta(-t) \begin{pmatrix} p_{K_\rho}^{-1} p_{K_\rho} \exp\{i[E_k + F_k(x)]t\} \\ 0 \\ p_{K_\sigma}^{-1} p_{K_\sigma} \exp\{i[E_k - F_k(x)]t\} \end{pmatrix} | \Psi_0 >. \] (33)

In Eq. (33), \( \Theta(t) \) and \( \Theta(-t) \) are the known step-functions. The matrix elements \( p_{K_\eta, \gamma, \rho, \sigma}^{-1} \) are products between the columns of \( P_k^{-1} \) with the rows of \( p_k \) that correspond to the respective excitations \( \eta, \gamma, \rho, \sigma \). Straightforward calculations yield

\[
p_{K_\eta}^{-1} p_{K_\eta} + p_{K_\gamma}^{-1} p_{K_\gamma} = M_k = \frac{1}{2} [I + (\theta_k - v_k) \beta - 2w_k \alpha_3 - 2u_k \alpha_1]. \] (34a)

\[
p_{K_\rho}^{-1} p_{K_\rho} + p_{K_\sigma}^{-1} p_{K_\sigma} = N_k = \frac{1}{2} [I - (\theta_k - v_k) \beta + 2w_k \alpha_3 + 2u_k \alpha_1]. \] (34b)

\[
p_{K_\eta}^{-1} p_{K_\eta} - p_{K_\gamma}^{-1} p_{K_\gamma} = R_k \approx \frac{1}{2} \{ \tau_3 [1 + \beta(\theta_k - v_k)] + \tau_1 2u_k - \alpha_0 2w_k \}, \] (34c)

\[
p_{K_\rho}^{-1} p_{K_\rho} - p_{K_\sigma}^{-1} p_{K_\sigma} = S_k \approx \frac{1}{2} \{ \tau_3 [1 - \beta(\theta_k - v_k)] + \tau_1 2u_k + \alpha_0 2w_k \}. \] (34d)

The approximations in Eqs. (34) are valid for \( \theta_k - v_k, 2u_k \ll 2w_k \). Time Fourier transformation of Eq. (33) yields
\[ G_0(k, \omega, x) = \frac{M_k(\omega - E_k) - R_k F_k(x)}{[\omega(1 + 2\beta) - E_k]^2 - F_k^2(x)} + \frac{N_k(\omega + E_k) - S_k F_k(x)}{[\omega(1 + 2\beta) + E_k]^2 - F_k^2(x)}. \] (35)

One may also define the propagators which express probability amplitudes for shifting the momentum of the field, from \( k \) into \( \tilde{k} \), and wise versa. These are denoted by \( G_0(k, \tilde{k}, \omega, x) \), and \( G_0(\tilde{k}, k, \omega, x) \). Based on the definition of \( \tilde{\Psi}_k \) in Eq. (9), one easily finds

\[ G_0(k, \tilde{k}, \omega, x) = G_0(k, \omega, x) \alpha_0, \quad (36a) \]

\[ G_0(\tilde{k}, k, \omega, x) = \alpha_0 G_0(k, \omega, x). \quad (36b) \]

The total Hamiltonian is given by

\[ H = H_{00} + H_1 = H_0 + [H_1 - (H_0 - H_{00})] = H_0 + \tilde{H}_1, \quad (37) \]

where \( H_{00} \) is the kinematical part without any condensation energies. From Eqs. (30) we find

\[ H_0 - H_{00} = \frac{1}{2} \sum_k E_k \tilde{\Psi}_k^* [-2u_k \alpha_1 - 2w_k \alpha_3 - \frac{F_k(x)}{E_k} (\tau_3 + 2u_k \tau_1)] \tilde{\Psi}_k. \quad (38) \]

The interaction Hamiltonian is

\[ H_i = \frac{1}{8} \sum_{k, k', q} V_q^I (\tilde{\Psi}_{k'q}^- \tau_3 \tilde{\Psi}_k ) (\tilde{\Psi}_{k+q}^+ \tau_3 \tilde{\Psi}_k^+ ). \quad (39) \]

In Eq. (39), \( V_q^I \) is the total interaction, which includes the Coulomb, the phonon mediated, and the magnetic interactions. Eq. (37) is written this way in order to incorporate the condensation into the unperturbed Hamiltonian, and consequently, to
facilitate the use of the perturbation theory. $\tilde{H}_i$ is defined so that double counting of the condensed parts of $H_0 - H_{00}$ is eliminated.

$$\tilde{H}_i = H_i - \frac{1}{2} \sum_k E_k \tilde{\Psi}_k^+ [-2u_k \alpha_1 - 2w_k \alpha_3 - \frac{F_k(x)}{E_k} (\tau_3 + 2u_k \tau_1)] \tilde{\Psi}_k$$  (40)

The various components of the self-energy are obtained by means of the Dyson's equation and the Wick's theorem. The lowest order self-energies are of second order in the interaction, and two types are known: the Hartree type, and the Fock type. In the present problem the Hartree diagram is easier to analyze, and it was also speculated to be the main contributor [2,8,9]. It is produced when the two field operators of one vertex in Eq. (39) contract with each other, and the two field operators of the other vertex contract with other operators to produce the propagators which take part in the Dyson's equation. This is done for $q = 0, \pm (k - \bar{k})$. Hartree diagrams scale with the trace of the product of the interaction vertices with $G_0$. Consequently, Eqs. (39) and (36) suggest that the vertices that yield finite Hartree diagrams are the $\tau_3$ and $\alpha_3$ vertices. Here we point out that the problem is characterized by the four parameters: $\theta_k, \nu_k, w_k, u_k$, which are related by two independent equations out of the three equations of Eqs. (23). The energy scale is determined by Eqs. (3). Thus, in principle, solving the Hartree diagrams for the vertices of $\tau_3$ and $\alpha_3$ should supply the first approximation for $F_k$ (Hartree), and $w_k$ (Hartree), which may be further improved by adding the Fock diagrams. This translates to four equations for the four quantities: $F_k$ (Hartree, Fock), and $w_k$ (Hartree, Fock), from which $u_k$ may be found.

4. CONCLUDING REMARKS.

The work about DCM showed that double correlations that produce the pseudogap and the superconductive symmetries can co-exist in principle. That work, though, has some features that are inconsistent with the experimental observations on the HTSC cuprates. There is, on the other hand, the more recent analysis which accentuates the
semi one dimensional nature of the underdoped cuprates and their magnetic nature [1], which is consistent with some experimental observations. This later analysis assumes the existence of columns and rows of holes in a somewhat idealistic manner that is hard to reconcile with superconductivity, which is based on Cooper-pairing of otherwise "independent" quasiparticles. The present work is the first step to bridge the gap between these two analyses. By doing so it removes some of the idealistic nature of Ref. [1], while showing that Ref. [2] may be compatible with the experimental realities of the cuprates. The adaptation of the two works has revealed the statistical nature of the strings of holes, and the deterioration effect that superconductivity has on them. It has been found that the strings of holes fluctuate. The nature of the fluctuations in the regular space is not fully clear, consequently, they have been ignored in the present paper. The perception here is that in the highly underdoped regime the fluctuations are so small that their effect could be incorporated into the renormalization of the relevant physical quantities. They renormalize, for example, the magnetic energy of the string.

The results have an intriguing feature that is apparent from the Hamiltonian of Eq. (28), which exhibits spatial modulation of the excitation energy, with points in space which correspond to zero excitation energy. Although the consequences of this feature are not fully clear at this moment, one may speculate that it should have a crucial effect on the electron transport of the system. Let us examine the various excitation energies in Eq. (28), which correspond to \( \eta^+, \gamma^+, \rho^+, \) and \( \sigma^+ \). The latter two excitations are created when one of the quasi-holes is removed from the ground state. They may be considered as anti-particles. Two kinds of particle-antiparticle pairs may be excited at certain places with zero excitation energy: \( \eta^+\sigma^+ \) at \( x = 0 + n\pi k_F^{-1} \), and \( \gamma^+\rho^+ \) at \( x = \pi k_F^{-1} / 2 + n\pi k_F^{-1} \), where \( n \) is an integer. Our basic notions are that super-currents and the superconductive energy gap at the Fermi level are consistent with each other, because super-currents are maintained by the correlated condensed electronic system in its ground state. On the other hand, the normal state of these systems has been considered metallic in character, namely, with continuous density of states at the Fermi level. Although the present analysis is a zero temperature analysis, one can eliminate superconductivity by setting: \( u_k = 0 \). Then, we have the metallic condition fulfilled only in certain regions in space, whereas in the rest there is a
pseudogap. Experiments suggest that electron conductivity exists in the pseudogap phase, although with unusual characteristics. To reconcile this perception with our model, we speculate that electron conductivity occurs because of a drift of the position of the zero gap points, due to an applied electric field. If superconductivity is turned on, one had to investigate the implications of these zero gap points on super-currents. A thorough study of the electro-dynamic properties of the system is needed.

During the last decade there has been an accumulation of experimental evidence for the partial and regional diamagnetism of the cuprates at temperatures well above the superconductive critical temperature, in the pseudogap phase. Most of the experimental data come from Nernst experiments [18,19]. The measured Nernst signals in these experiments are so large to suggest the existence of superconductive currents even in the pseudogap phase (and well above the superconductive $T_c$). Recently, diamagnetism has been observed more directly by magnetic torque measurements [20]. The investigators usually concluded that their results indicate the existence of fluctuating and isolated regions with local superconductive order but with no phase coherence between them [18-21]. It is premature to speculate about the relation between this phenomenon and the present analysis, before attempting a full investigation of the electro-dynamics of the system. The first question to be addressed is about the electron transport basic character of the pseudogap phase. Could the discussed phenomenon be attributed directly to the pseudogap condensate, due to its self-consistent and correlated character? More experimental and theoretical studies are needed to support such a perception. At present we should rather take a more conservative view and attribute the observed partial diamagnetism to superconductive fluctuations.
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