COMPLETE INITIAL STATE RADIATION TO OFF-SHELL $Z^0$ PAIR PRODUCTION IN $e^+e^-$ ANNIHILATION†

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Abstract
A cross-section calculation for the Standard Model reaction $e^+e^- \rightarrow (Z^0 Z^0) \rightarrow f_1 \bar{f}_1 f_2 \bar{f}_2$ including the effects of the finite $Z^0$ width and initial state radiative corrections is presented. The angular phase space integrations are performed analytically, leaving the invariant masses for numerical integration. Semi-analytical and numerical results in the energy range $\sqrt{s} = 150$ GeV to 1 TeV are reported.

1 Introduction
At LEP 2 energies and above $e^+e^-$ annihilation into four fermions is a major issue. Monte Carlo approaches to four-fermion production with and without inclusion of radiative corrections have been developed by several authors [1, 2, 3, 4]. Complementing these results we follow a program of ‘semi-analytical’ calculations [5, 6, 7, 8, 9]. These calculations comprise Initial State QED corrections [5, 6, 8] and Born computations of large sets of four-fermion Feynman diagrams [9]. In this paper we are going to present the semi-analytical results for the complete $\mathcal{O}(\alpha)$ QED Initial State Radiation (ISR) to $Z^0$ Pair production in $e^+e^-$ annihilation, also including finite $Z^0$ width effects:

$$e^+e^- \rightarrow (Z^0 Z^0) \rightarrow f_1 \bar{f}_1 f_2 \bar{f}_2 (\gamma) , \quad f_1 \neq f_2 , \quad f_1 \neq e .$$

Semi-analytical means that all angular phase space variables are integrated analytically, leaving three invariant masses for numerical integration. Quasi-experimental cuts on the latter can be easily implemented.

On-shell $Z^0$ pair production has been discussed long ago [10]. Numerical calculations including all $\mathcal{O}(\alpha)$ electroweak corrections except hard photon bremsstrahlung were reported in [11] for on-shell and in [12] for off-shell $Z^0$ bosons. Below, ISR will be treated completely, including hard bremsstrahlung corrections.

The paper is organized as follows. In section 2 semi-analytical results for the off-shell Born cross-section are presented, followed by the ISR results in section 3. The paper closes in section 4 with a summary, an outlook and some conclusions.

2 The Born Cross-Section
At Born level, process (1) is described by the two Feynman diagrams depicted in figure 1. The Born cross-section for process (1) is given by a simple double convolution formula

$$\sigma_{ZZ}^\text{Born} (s) = \int_{4m_e^2}^{(\sqrt{s}-2m_2)^2} ds_1 \rho_Z (s_1) \int_{4m_e^2}^{(\sqrt{s}-\sqrt{s_1})^2} ds_2 \rho_Z (s_2) \cdot \sigma_4^\text{ZZ} (s; s_1, s_2) \times 2 \cdot BR(1) \cdot BR(2)$$

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invoking the branching ratios \( BR(1) \) and \( BR(2) \) for the decay of the \( Z^0 \) boson into \( f_1 \bar{f}_1 \) and \( f_2 \bar{f}_2 \) respectively, the invariant \( Z^0 \) masses \( s_1 \) and \( s_2 \) and Breit-Wigner density functions for the \( s \)-channel \( Z^0 \) propagator:

\[
\rho_Z(s_i) = \frac{1}{\pi} \frac{\sqrt{s_i} \Gamma_Z(s_i)}{|s_i - M_Z^2 + i \sqrt{s_i} \Gamma_Z(s_i)|^2} \quad \text{for } s_i \to M_Z^2.
\]

The \( Z^0 \) width is given by

\[
\Gamma_Z(s_i) = \frac{G_\mu M_Z^2}{24 \pi \sqrt{2}} \sqrt{s_i} \sum_f (v_f^2 + a_f^2).
\]

Using \( a_e=1, \ v_e=1-4 \sin^2 \theta_W, \ L_e=(a_e+v_e)/2 \) and \( R_e=(a_e-v_e)/2 \) , \( \sigma_{4Z}Z(Z; s_1, s_2) \) is obtained after fivefold analytical integration over the angular phase space variables:

\[
\sigma_{4Z}^Z(s; s_1, s_2) = \frac{\lambda^{1/2}}{s} \left( \frac{G_\mu M_Z^2}{8 \pi s} \right)^2 \left( L^4_e + R^4_e \right) G_{4}^{4}(s; s_1, s_2).
\]

Algebraic manipulations were carried out with the help of FORM [13]. The sub-index 4 indicates that the underlying matrix element squared contains four resonant propagators. If considered separately, the contributions \( G_{4}^{t} \) from the t-channel, \( G_{4}^{u} \) from the u-channel, and \( G_{4}^{t u} \) from the t-u interference violate unitarity. Due to so-called unitarity cancellations, their sum \( G_{4}^{t u} \) exhibits proper unitarity behavior and can be very compactly written as

\[
G_{4}^{t u}(s; s_1, s_2) = \frac{s^2 + (s_1 + s_2)^2}{s - s_1 - s_2 - \Delta_4}.
\]

This formula was derived in [3] and agrees with earlier results [10, 14, 15]. We use \( \lambda \equiv s^2 + s_1^2 + s_2^2 - 2s_1s_2 - 2s_2s \) and

\[
\Delta_4 = \frac{1}{\sqrt{\lambda}} \ln \frac{s - s_1 - s_2 + \sqrt{\lambda}}{s - s_1 - s_2 - \sqrt{\lambda}}.
\]

The effect of the finite \( Z^0 \) width can be seen from figure 2, as the characteristic smearing of the peak. It should be mentioned that gauge violation problems arise with the introduction of finite boson widths. It is, however, generally agreed that these are not critical for process (3). We only peripherically mention that a scheme to avoid these gauge violations was proposed, but gives incorrect results around threshold [11].

Up to now no treatment of finite widths in boson pair processes seems theoretically satisfactory.

Furtheron we neglect fermion masses wherever sensible and acceptable for numerical stability.
Figure 2: The total cross-section $\sigma_{\text{tot}}^{ZZ}(s)$ for process (1). The numerical input for the figures is: $\alpha = 1/137.0359895$, $G_\mu = 1.16639 \times 10^{-5}$ GeV$^{-2}$, $m_e = 0.51099906$ MeV, $M_Z = 91.173$ GeV, $\Gamma_Z = 2.487$ GeV, $\sin^2 \theta = 0.2325$. The numerical precision is estimated to be better than 0.01 permille.

In order to prevent potential confusion we mention that the convention used in [12] has been adopted, namely to present the total cross-section $\sigma_{\text{tot}}^{ZZ} \equiv \sigma^{ZZ}/(2 \cdot BR(1) \cdot BR(2))$ in the plots. This is adequate in the sense that in the narrow width approximation $\sigma_{\text{tot}}^{ZZ}$ recovers the cross-section for on-shell $Z^0$ pair production which is seen if a summation over the decay channels $f_i \bar{f}_i$ of the $Z^0$ bosons is performed. Using the result for on-shell $Z^0$ pair production $\sigma_{\text{on-shell}}^{ZZ} = \sigma_{\text{on-shell}}^{ZZ}(s, s_1, s_2)$ [8,10] we obtain:

\[
\sigma_{\text{on-shell}}^{ZZ} = \sigma_{\text{on-shell}}^{ZZ} \times 1^2 = \sigma_{\text{on-shell}}^{ZZ} \left[ \sum_i BR(i) \right]^2 = \sigma_{\text{on-shell}}^{ZZ} \left[ \sum_i BR(i)^2 + \sum_{i<j} 2 \cdot BR(i) \cdot BR(j) \right] \approx \sum_{i \leq j} \sigma_{\text{narrow width}}^{ZZ}
\]

where $i \leq j$ in the summation over the narrow width four-fermion cross-section is necessary to avoid double counting. For $i \neq j$, the terms $\sigma_{\text{narrow width}}^{ZZ}$ are given by the narrow width approximation of the right hand side of eq. (2). This convention of ours was not emphasized in earlier publications [3,8].
\[ e^+ e^- \rightarrow \gamma(p) \]

Figure 3: The amputated ISB diagrams for \( Z^0 \) pair production.

\[ e^+ e^- \rightarrow \gamma(p) \]

Figure 4: The amputated virtual ISR diagrams for \( Z^0 \) pair production.

3 \( \mathcal{O}(\alpha) \) Initial State Radiation

In \( e^+ e^- \) annihilation, ISR is known to represent the bulk of the radiative corrections. The \( \mathcal{O}(\alpha) \) ‘amputated’ Feynman diagrams for Initial State Bremsstrahlung (ISB) to process (1) are shown in figure (3). The corresponding virtual ISR diagrams are given in figure (4). External leg self energies are absorbed into the on-shell renormalization. The double-differential cross-section for off-shell \( Z^0 \) pair production including \( \mathcal{O}(\alpha) \) ISR with soft photon exponentiation can be presented as

\[
\frac{d^2 \sigma^{ZZ}}{ds_1 ds_2} = \int \frac{ds'}{s} \rho(s_1) \rho(s_2) \left[ \beta_e v^{\beta_e - 1} S + H \right]
\]

(9)

with \( \beta_e = 2\alpha/\pi [\ln(s/m_e^2) - 1] \) and \( v = 1 - s'/s \). The soft+virtual and hard photonic parts \( S \) and \( H \) are calculated analytically, requiring seven angular integrations. Both separate into a universal part with the Born cross-section factorizing and a nonuniversal part:

\[
\begin{align*}
S(s, s'; s_1, s_2) &= \left[ 1 + \tilde{S}(s) \right] \sigma_0(s'; s_1, s_2) + \sigma_{\tilde{S}}(s'; s_1, s_2) \\
H(s, s'; s_1, s_2) &= \tilde{H}(s, s') \sigma_0(s'; s_1, s_2) + \sigma_{\tilde{H}}(s, s'; s_1, s_2).
\end{align*}
\]

(10)

An explicit derivation proved that \( \tilde{S} \) and \( \tilde{H} \) are identical to the radiators known from s-channel fermion pair production [6, 8, 17]:

\[
\begin{align*}
\tilde{S}(s) &= \frac{\alpha}{\pi} \left[ \frac{\pi^2}{3} - \frac{1}{2} \right] + \frac{3}{4} \beta_e + \mathcal{O}(\alpha^2), \\
\tilde{H}(s, s') &= -\frac{1}{2} \left( \frac{s'}{s} \right) \beta_e + \mathcal{O}(\alpha^2).
\end{align*}
\]

(11)
Figure 5: The effect of universal and nonuniversal ISR on process \((e^+e^- \rightarrow Z^0Z^0 \rightarrow f_1\bar{f}_1f_2\bar{f}_2)\) for off-shell \(Z^0\) bosons. The inlay in the upper right corner exhibits the \(\sqrt{s}\)-dependence of the ratio of the universally (dash-dotted line) and nonuniversally (dotted line) ISR corrected over the Born total cross-section. The numerical precision is estimated to be better than 0.1 permille.

The analytical formulae for the nonuniversal contributions are very involved as they contain many Dilogarithm and Trilogarithm functions and will therefore be published elsewhere. Compared to the universal corrections, nonuniversal corrections are small, because they do not contain the mass singularity \(\beta_e\). It is seen from the numerical results presented in figure 5 that the relative contribution of nonuniversal corrections is negligible around threshold and increases to 3\% at 1 TeV. As in the nonuniversal corrections to W-pair production we observe the so-called screening property, i.e. the nonuniversal corrections are damped by a factor \(s_1s_2/s^2\) \cite{5}. It is seen from both figure 4 and figure 5 that ISR plays an important rôle in almost the whole energy range under consideration. Around threshold, in the energy range of LEP 2, a peak in the ISR corrections’ relative importance can be seen.

4 Summary, Outlook and Conclusions

We have presented finite width and initial state QED corrections to \(e^+e^- \rightarrow (Z^0Z^0) \rightarrow f_1\bar{f}_1f_2\bar{f}_2\) in a semi-analytical approach. It was shown that both yield important corrections to the total cross-section.
The inclusion of photons replacing $Z^0$ bosons in the crayb Feynman diagrams of figure 1 is necessary to endow our computations with experimental relevance. This inclusion is straightforward, but will also require a refined treatment of final state fermion masses. In a second step it is intended to merge the results of the multi-diagram, semi-analytical Born calculation given in reference 9 with ISR calculations as performed above to yield a precise result for some channels of the process $e^+e^- \rightarrow l\bar{l}q\bar{q}$, probably already matching the feasible experimental precision. The set of Feynman diagrams for such a calculation is given in 9. It is noteworthy that the crayb diagrams, constitute the by far most important contribution to the cross-section if $\sqrt{s}$ is far above $M_Z$. An extension of the presented analysis to QCD initial state gluon radiation in $Z^0$ pair production at LHC is also within reach. The authors acknowledge discussions with Arnd Leike and Uwe Müller.

References

[1] F.A. Berends, P.H. Daverveldt and R. Kleiss, Nucl. Phys. B253 (1985) 441; Comp. Phys. Commun. 40 (1986) 285; J. Hilgart, R. Kleiss and F. le Diberder, Comp. Phys. Commun. 75 (1993) 191.

[2] E.W.N. Glover, R. Kleiss and J.J. van der Bij, Z. Phys. C47 (1990) 435;

[3] E. Boos, M. Sachwitz, H. Schreiber and S. Shichanin, Z. Phys. C61 (1994) 675; M. Dubinin, V. Edneral, Y. Kurihara and Y. Shimizu, Phys. Lett. B329 (1994) 379; E. Boos, these proceedings; Y. Kurihara, these proceedings.

[4] F.A. Berends, R. Kleiss and R. Pittau, Nucl. Phys. B424 (1994) 308; Nucl. Phys. B426 (1994) 344; R. Pittau, Phys. Lett. B335 (1994) 490.

[5] D. Bardin, M. Bilenky, A. Olchevski and T. Riemann, Phys. Lett. B308 (1993) 403.

[6] D. Bardin, M. Bilenky, D. Lehner, A. Olchevski and T. Riemann, Contribution to the Zeuthen Workshop on Elementary Particle Theory – Physics at LEP200 and Beyond, Teupitz, Germany, April 1994, preprint CERN-TH. 7295/94 (1994) (hep-ph/9406340); to appear in the proceedings.

[7] D. Bardin, A. Leike and T. Riemann, Contribution to the Zeuthen Workshop on Elementary Particle Theory – Physics at LEP200 and Beyond, Teupitz, Germany, April 1994, preprint DESY 94-097 (1994) (hep-ph/9406273); to appear in the proceedings.

[8] D. Lehner, Contribution to the XVII Kazimierz Meeting on Elementary Particle Physics, Kazimierz, Poland, May 1994, preprint DESY 94-105 (1994) (hep-ph/9407253); Mod. Phys. Lett. A, in print.

[9] D. Bardin, A. Leike and T. Riemann, preprint DESY 94-185 (1994) (hep-ph/9410361); submitted for publication in Phys. Lett. B.

[10] R.W. Brown and K.O. Mikaelian, Phys. Rev. D19 (1979) 922.

[11] A. Denner and T. Sack, Nucl. Phys. B306 (1988) 221.

[12] A. Denner and T. Sack, Z. Phys. C45 (1990) 439.

[13] J.A.M. Vermaseren, Symbolic Manipulation with FORM, Computer Algebra Netherlands (CAN), Amsterdam, 1991.

[14] V. Baier, V. Fadin and V. Khoze, Sov. Phys. JETP 23 (1966) 104.
[15] M. Cvetić and P. Langacker, *Phys. Rev.* **D46** (1992) 4943; E: ibid. **D48** (1993) 4484.

[16] A. Aeppli, F. Cuypers and G.J. van Oldenborgh, *Phys. Lett.* **B314** (1993) 413; W. Beenakker and A. Denner, DESY 94-051 (1994).

[17] F.A. Berends, G. Burgers and W.L. van Neerven, *Nucl. Phys.* **B297** (1988) 429.