Frontiers of the Universe: 
What do we know, what do we understand?

Ruth Durrer
Université de Genève
Département de Physique Théorique
24, Quai Ernest Anserment, 1211 Genève 4, Suisse

Abstract

In this talk I summarize the topics addressed during the conference. We have discussed the most relevant cosmological observations of the last say 10 years and their implications for our understanding of the Universe. My finding throughout this meeting has been that it is amazing how our knowledge of cosmological parameters has improved during the last years. It comes as a surprise to many, that the simplest inflationary model of adiabatic perturbations agrees well with present data. But it is frustrating that we have not made any progress in identifying the dark matter, in contrary, we have supplemented it by what we call the 'dark energy', a gravitationally and esthetically repulsive component which is even more mysterious than the dark matter. We have no understanding about the origin of these two most abundant energy components of the present Universe.
1 Introduction

Most of us are convinced that below a certain (very high) temperature and density and above a certain (very small) length scale, general relativity is the relevant theory for the description of the interaction of matter and spacetime. The Friedmann universe is one of the simplest solutions of Einstein’s equation and this is probably the main reason these solutions have been found and discussed originally by Friedmann. In contrast to Newtonian gravity, Einstein’s equations do allow for homogeneous and isotropic solutions, but they are generically non-static.

Nevertheless, I consider it as a big surprise that the Friedmann universe is such a good approximation to the observable universe. As I will argue below, we have several independent pieces of evidence, that the metric deviations from the Friedmann solution are of the order of

$$\Psi \approx 10^{-5} \ll 1,$$  \hspace{1cm} (1)

on all cosmological scales from about 0.1 Mpc up to the Hubble scale, $H_0^{-1} \approx 3000 h^{-1}$ Mpc. The fact that this amplitude of the gravitational potential is scale independent is equivalent to the fact that the observed fluctuations obey a Harrison Zel’dovich spectrum.

In the following sections I recollect some observational evidences and discuss the most important lessons we learn from them. At several occasions I shall also formulate some doubts, especially concerning the data analysis.

I shall finish with a brief inventory about what we know and then turn to the more interesting even if less well defined question of what we understand. Clearly, ‘understanding’ has many levels and probably means something different for all of us. In this sense I just present a personal point of view.

I shall conclude with a few words about the direction the field might develop into.

2 Big bang nucleosynthesis and Type Ia Supernovae

Big bang nucleosynthesis has been discussed by T. X. Thuan, S. Sarkar, and also A. Lubowich during this conference. The agreement of the observed light element abundances with calculations represents one of the fundamental pillars of big bang cosmology. There are two cosmological parameters which can be determined by comparing nucleosynthesis calculations with observations. The Helium abundance restricts any additional, non-standard, relativistic contribution to the energy density of the Universe. This can be cast in

$$\Delta N_{\nu} \leq 0.2 \quad \text{or} \quad \Omega_{X_{\text{rel}}} \leq 1.16 \times 10^{-6}.$$  \hspace{1cm} (2)

Here, $X_{\text{rel}}$ denotes any ‘particle’ species which is still relativistic at the time of nucleosynthesis ($T \sim 0.1$ MeV) in addition to the three species of neutrinos and the photons. Examples are a component of stochastic gravity waves or some other unknown particles, e.g., sterile neutrinos, with mass $m_X < 0.1$ MeV.

The deuterium and Lithium abundances determine the density of baryons in the universe [1],

$$\Omega_b h^2 = 0.02 \pm 0.002.$$  \hspace{1cm} (3)
Even though we do not understand type Ia supernovae in detail (see contributions by J. Niemeyer and C.R. Ghezzi), we have realized observationally, that they represent excellent 'modified standard candles'. The Supernovae type Ia measurements of the past years \[2, 3\] have led to the most surprising result that the present universe is accelerating, in other words the gravitationally active combination \(\rho + 3p\) is negative in the present universe. Casting this in terms of an equation of state, \(p = w\rho\), the data yield \(w_{\text{eff}} \sim -0.6 \pm 0.15\). Or, if we assume the dark energy of the universe to be in the form of a cosmological constant with \(w_\Lambda = -1\), which is combined with dark matter which has \(w_m = 0\), this can be cast as \(\Omega_m - 0.75\Omega_\Lambda = -0.25 \pm 0.125\). These results have been discussed by K. Schahmaneche in this meeting.

Clearly, it is still somewhat premature to exclude possibilities like 'grey dust' or 'oscillations of the photons' (see e.g. \[1\]). But also the large scale structure (LSS) data combined with cosmic microwave background anisotropies give strong evidence for a cosmological constant (or quintessence).

The only form of energy which leads to a 'cosmological constant' is a constant potential energy (in classical physics) or vacuum energy (in quantum physics). But exactly this form of energy has no dynamical consequences in all known physical interactions except gravity. Only differences in the vacuum energy are dynamically meaningful in non-gravitational interactions. Once gravity is taken into account, this otherwise arbitrary integration constant suddenly becomes dynamically important.

Since in supersymmetric theories the vacuum energy from fermions and bosons exactly cancels, the typical value of the vacuum energy expected from modern particle physics is

\[
\rho_\Lambda = \frac{\Lambda}{8\pi G} \sim E_{\text{SUSY}}^4 \gtrsim 10^{12}\text{GeV}^4 \simeq 10^{59}\rho_c .
\]

Here \(E_{\text{SUSY}}\) denotes the supersymmetry breaking scale which we know to be at least 1TeV or larger. The above mentioned observations find a non-vanishing value which is at least 59 orders of magnitude smaller than what one naively expects. This finding represents probably the most amazing puzzle in physics. Clearly, we are lacking a basic piece in our understanding and interpretation of the cosmological constant and/or of vacuum energy.

Actually, we can also face this result in a positive way: nobody (except perhaps Prof. Burbidge) has expected it! This is the clearest evidence that cosmology, which for a long time lived from 'theoretical speculations' has become data dominated.

I believe that the accelerating universe and the cosmological constant represents the biggest puzzle in present physics. There are divers attempts to address it, but none of them so far could convince a majority (see contributions from M. Turner and F. Piazza). More observational constraints on this mysterious dark energy are desperately needed. Various attempts in this direction have been addressed in the contributions by J. Newman, R. Bean and J. Weller.
The cosmic microwave background

The Universe is permeated by an isotropic thermal radiation at $T = (2.728 \pm 0.01)\,\text{K}$, the cosmic microwave background (CMB). Anisotropies in this background radiation (a part from a dipole of $C_1 \sim 10^{-3}$ which is due to our motion with respect to the CMB rest frame) are of the order of

$$\frac{\Delta T}{T} \simeq 10^{-5} \simeq \Phi.$$  

(5)

The last $\simeq$ indicates that CMB anisotropies are of the same order of magnitude as the gravitational potential (Bardeen potential) $\Phi$. This isotropy, together with the cosmological principle (we are not sitting in a special place in the Universe), proves that the Universe is close to Friedmann on large scales.

The accurate mapping of CMB anisotropies, especially by the three latest experiments Boomerang [5], MAXIMA [6] and DASI [7] teaches us much more than this: the simplest model of a Harrison Zel’dovich (scale invariant) spectrum of purely adiabatic scalar fluctuations with reasonable cosmological parameters fits the data very well (see Fig. 1).

![Figure 1: The CMB anisotropy power spectrum measurements from Boomerang (solid,red), MAXIMA (dashed, green) and DASI (dotted,blue) are shown. A typical inflationary model with scale invariant scalar perturbations is indicated to guide the eye (solid line). The CMB anisotropy spectrum for causal global defects (texture) is also shown (dashed line).](image)

This result is by no means trivial: the simplest worked out models with ’causal’ initial perturbations, i.e. initial conditions where all correlations on super horizon scales vanish, do not fit the data (dashed line in Fig. 1). The reason for this is relatively subtle: these models, where fluctuations are induced by topological defects, actually
also produce a Harrison Zel’dovich spectrum of fluctuations. But the fluctuations are
not ‘in phase’, i.e. at the moment of recombination not all fluctuations of a given
wavelength are at a maximum/minimum/zero respectively. The peak structure which
is so characteristic of fluctuations created at a very early initial time and which have a
fixed amplitude at horizon crossing, is therefore not expected from topological defects.
For topological defects the physical mechanism which creates the fluctuations shortly
after horizon crossing is non-linear. The non-linearities imply a coupling of modes which
smears out the acoustic peaks at least to some extend. Furthermore, tensor and especially
vector perturbations contribute significantly (more than 50%) to the CMB anisotropies
at large scales, leading to a suppression of the purely scalar acoustic peaks in a COBE
normalized spectrum.

This is the main lesson which we have learned so far from these beautiful experiments:
the cosmological initial fluctuations stem most probably from an inflationary phase or
some equivalent, which produced fluctuations on scales larger than the Hubble horizon.
I consider this data as ‘evidence for inflation’ of a completely different quality than
the homogeneity and isotropy of the Universe and the flatness problem. For the latter
inflation justifies highly improbable initial conditions. In contrary, the acoustic peaks
have been predicted from inflation. This actually even before the development of inflation
by Doroshkevich, Zeldovich and Sunyaev (1978) [9]. These authors have used an initial
spectrum of fluctuations with correlations on super Hubble scales which can only be
generated during an inflation-like process. Here we use the term ‘inflation’ in its most
generic sense as a period where the co-moving size of the Hubble scale is increasing.

Since the details of the acoustic oscillations depend sensitively on cosmological pa-
rameters like
\[
\Omega_m, \quad \Omega_\Lambda, \quad \Omega_b h^2, \quad h, \quad r = T/S, \quad n_s, \quad n_T, \quad \tau_c, \quad \cdots, \quad (6)
\]
many of us have used them to determine them. Progress in the determination of cos-

mological parameters has been reported here by S. Sarkar, J. Silk, M. Tegmark and C.
Pryke. A. Melchiorri has discussed to which extend present results will improve with
new data from the MAP and PLANCK satellite experiments.

Even though this parameter estimation is potentially a very powerful method, es-
pecially when combined with other datasets, which are desperately needed since the CMB
alone has important degeneracies, I think we must be careful not to over-interpret the
data. Simply consider the number of papers written by us theoreticians (I, myself wrote
one of them...) about the missing second peak after the first Boomerang-98 results [10]
which have now turned out to be due to a systematic error in the data analysis (a
miss-estimated beam size). The person who warned me most not to over-interpret this
marginal feature in the Boomerang power spectrum was Paolo de Bernardis himself. As
Zwicky [11] put it: “If only theorists would know what goes into an experimental data
point and if only experimenters would know what goes into a theoretical calculation,
they both would take each other much less serious”.

Therefore, I guess we may be confident that the cosmological parameters as deter-
mined so far seem to lie in the right bulk part, but I do by no means take seriously the
formal errors quoted in the published papers.
An additional point to be aware of is that these parameter estimations always assume that the correct model is in the family of models being parameterized. If one allows for more generic models (e.g. relaxing the requirement of adiabaticity) one can find very different cosmological parameters which also fit the present CMB data (see Fig. 2).

Figure 2: The CMB anisotropy power spectrum measurements from Boomerang are compared with flat models with $\Omega_b h^2 = 0.042$ with cosmological constant, $\Omega_\Lambda = 0.7$. The model with mixed isocurvature and adiabatic components is a good fit (solid line), while the adiabatic model does not fit the data with this parameter choice. Figure from Trotta et al. [12], where one can find more details.

Much more remarkable than the precision of cosmological parameters obtained from CMB measurements so far is their concordance with independent determinations from other data like LSS, peculiar velocities, lensing etc., under the simplest possible model assumptions. Let us therefore summarize results from other datasets.

4 Peculiar velocities

Apart from the CMB (and lensing), peculiar velocities represent one of the most straightforward ways to measure the gravitational potential and hence the metric perturbations. Unfortunately, they are notoriously difficult to measure, since

$$v_{r,pec}^2 = cz - rH_0$$

is the small difference of two large numbers, of which the second one ($r$) is very difficult to measure. Therefore, one has to consider carefully how to use this noisy data at its best. A first order of magnitude result is that peculiar bulk velocities are of the order
of $v^{pec}(r) \sim 300\text{km/s}$ on scales of $r \sim 50h^{-1}\text{Mpc}$. Using the scaling relation from linear perturbation theory,

$$v(r) \sim \Phi \times \frac{c^2}{rH_0}, \quad \text{this implies} \quad \Phi \sim 10^{-5}, \quad (8)$$

where $\Phi$ is again the (dimensionless) gravitational potential.

But also on smaller scales, $0.1\text{Mpc} \lesssim r \lesssim 1\text{Mpc}$, where structures are virialized and we expect $\Phi \sim (v/c)^2$, we obtain from the velocity dispersion in clusters, $v \sim 1000\text{km/s}$ again $\Phi \sim 10^{-5}$.

Together with the CMB anisotropies on large scales, this gives $\Phi \sim 10^{-5}$ on all cosmologically relevant scales, from the size of galaxies up to the Hubble scale, $0.1\text{Mpc} \lesssim r \lesssim 6000\text{Mpc}$.

Clearly, to determine bulk velocity flows, a precise measurement of the Hubble parameter is desperately needed (see contribution by J. Beckman). All the results discussed here are not very precise but they are nevertheless very interesting and might be telling us something which we have overlooked so far...

Finally I want to mention that during the last couple of years, a new technique, namely the statistics of pairwise velocities of galaxies has been developed and it has been shown that it can lead to a relatively precise measurement of $\sigma_8$ and $\Omega_m$ as it does not suffer from many of the problems of bulk velocities \[\text{[13]}\].

## 5 Large scale structure

New galaxy redshift surveys are being completed or under way. The number of published galaxy redshifts is growing very fast. The galaxy redshift catalogs are mainly used to determine the galaxy power spectrum which we hope to be closely related to the matter power spectrum (see contribution of S. Zaroubi).

Most recently, the 2dF power spectrum has been interpreted to show evidence for oscillations or at least structure, which indicate a relatively high ratio $\Omega_b/\Omega_m \sim 0.15$ (see Ref. [14] and the contribution by C. Frenk). It is really amazing to which extent big bang nucleosynthesis, CMB, LSS, SNIa and also cluster data \[\text{[13]}\] come together and favor $h^2\Omega_b \sim 0.02$, $\Omega_m \sim 0.3$ and $\Omega_\Lambda \sim 0.7$.

I admit, however, that I do have reservations to interpret the oscillations in the 2dF power spectrum as acoustic oscillations of baryons (this concern is partially shared with C. Frenk, see his contribution). First of all, according to my naive estimate they are not quite at the right position. Secondly, they could very well mimic some effect from a finite size filter, the Fourier transform of a top-hat filter simply gives a Bessel function.

I am also a bit reluctant to take seriously the interpretation of the 'bend in the power spectrum'. Since the mean density of the universe is set equal to the mean density of galaxies in the catalog at hand, the power spectrum at the scale of the survey, $L$, vanishes by construction $P(k = 2\pi/L) \equiv 0$. It is therefore not surprising that all observed galaxy power spectra show a bend in the last few points, \textit{i.e.} for the smallest values of $k$.

Moreover, a volume limited sample out to the largest scale $k \sim 0.02h/\text{Mpc}$ can be estimated to contain roughly 1% of all the galaxies in the 2dF catalog leading to
\( N \sim 1000 \). But the Poisson noise due to this finite number is just of the order of the square of the fluctuation amplitude, \( \Delta^2(k = 0.02h/\text{Mpc}) = k^3 P(k) \sim 10^{-3} \sim 1/N \).

Therefore, and also due to a 'tradition of sloppiness' in the statistical treatment of galaxy redshift data, I am not completely convinced that these data is correctly interpreted in the present literature.

6 Weak lensing

The deflection of light is fully determined by the gravitational potential, i.e. by the clustering properties of matter. The statistical distribution of the shear, the direction of elongation of galaxies, measures the gravitational field along the line of sight. We define the amplification matrix \( A \),

\[
\begin{pmatrix}
  x \\
  y 
\end{pmatrix}_{\text{image}} = A \begin{pmatrix}
  x \\
  y 
\end{pmatrix}_{\text{object}} \quad \text{with} \quad A = \begin{pmatrix}
  \mu + \gamma_1 & \gamma_2 \\
  \gamma_1 & \mu - \gamma_1 
\end{pmatrix}.
\]

(9)

The expectation value of \( \gamma \) is related to the convergence power spectrum \( P_\kappa \) via

\[
\langle \gamma^2 \rangle = \frac{2}{\pi \theta_e^2} \int_0^\infty \frac{dk}{k} P_\kappa(k) J_1^2(k \theta_e),
\]

(10)

where \( J_1 \) is the Bessel function of order 1. More details can be found in the contribution by F. Bernardeau and in Ref. [17]. Observations of the shear of galaxies finally lead to constraints in the \( \Omega_m, \sigma_8 \) plane. For present surveys, the best results are those of Ref. [17] shown in Fig. 3, but this method is just coming up and will hopefully lead to improved results, once the errors are better under control.

Figure 3: The 1,2 and 3 \( \sigma \) contours for \( \Omega_m \) and \( \sigma_8 \) from the weak lensing analysis of the VIRMOS deep imaging survey. Figure from Werbaeke et al. [17], where one can find more details.
Conclusion

We have seen in this conference that cosmology is harvesting an enormous amount of data leading to ever better measurements of the cosmological parameters. At present it seems that the Universe is flat and $\Lambda$-dominated with purely adiabatic scalar perturbations which are responsible at the same time for the CMB anisotropies and for large scale structure. Astrophysical observations presently undergo amazing breakthroughs in almost all wavebands of the electromagnetic spectrum. This has been impressively demonstrated in the talks by A. Quirrenbach, A. Omont, A. Watson, H. Meyer, C. Cesarsky and A. Melchiorri. Also the hope that we may finally be able to detect gravitational radiation directly has become realistic (see contributions by T. Damour and M. Huber). Observational cosmology is clearly in its most exciting phase, new discoveries and challenges for theorists are to be expected every day.

The main parameters of the cosmological model may very well be determined with sufficient accuracy within a couple of years. Then we will know the parameters of the Universe we are living in.

But do we understand cosmology? I think there we are still very far from our goal: of the two main constituents of the present Universe, we have no information other than their cosmological activity. Despite intensive searches (see contributions from T. Sumner, L. Baudis and others), we have not observed the particle giving rise to $\Omega_m$ yet. There are many dark matter candidates (see contribution from S. Scopel), but clearly not all of them can be realized in nature. Concerning the dominant term in the cosmic density, $\Omega_\Lambda$, the situation is even worse. There we have no convincing theory giving a result even in the right ballpark. Interesting ideas are being developed (see e.g. the contribution by T. Padmanabhan), but so far the problem remains unsolved.

Also inflation, the 'explanation' of the flatness and homogeneity of the universe, and of the adiabatic initial fluctuations, is more on the level of a paradigm than of a theory of the early universe, motivated by high energy physics (see contribution by S. Dodelson).

These are, as I see it, the main problems which concern the early universe. Most probably, they cannot be solved without simultaneous progress in the theory of high energy physics. They may actually provide our only clues to the physics at very high energies, like string theory, which are probably not accessible also in future accelerators (see the contributions from G. Veneziano and J. Magueijo). Cosmology may turn out as the ultimate tool to study the fundamental laws of physics.

At the redshifts $10^{10} \lesssim z \lesssim 10$, i.e. from about $T \sim 1\text{MeV}$ until cosmic structures become non-linear and the first objects form, we understand the cosmic evolution rather well. We can provide good, successful calculations of primordial nucleosynthesis and recombination, as well as integrate the linear perturbation equations to determine CMB anisotropies. These calculations are in good agreement with observations and, together with other measurements, they allow us to determine the cosmological parameters.

At low redshift, $z < 10$, when structure develops, radiation processes and chemical evolution take place together with gravitational clustering. The physics becomes complex. Many different effects have to be taken into account simultaneously, if we want to obtain a consistent picture. Complex numerical simulations combining gravity with
hydrodynamics are presently being developed (see contribution from T. Abel). But also
new observations focusing on small scales in different bands of the electromagnetic spec-
trum are underway (see contributions from S. Bridle, I. Lehmann, A. Bunker, E. de
Gouveia Dal Pino N. Gruel and others).

I believe that, within a couple years when the important work of determining cosmo-
logical parameters to good accuracy will have been solved, there will remain two major
directions for cosmological research:

- $z \gg 10^{10}$ Early universe, towards the fundamental laws of physics.
- $z \lesssim 10$ The formation of galaxies and stars, complexity.

Acknowledgment: I thank the participants of this 'Rencontre de Blois' and Filippo
Vernizzi for interesting and stimulating discussions. My participation at the conference
was supported by the Swiss National Science Foundation.

References

[1] S. Burles, K. M. Nollet, M. S. Turner, Astrophys. J. 552, L1 (2001).
[2] S. Perlmutter et al., Astrophys. J. 517, 565 (1999).
[3] A. Riess et al. Astron. J. 116, 1009 (1998).
[4] C. Csaki, N. Kaloper and J. Terning, Phys. Rev. Lett. 88 161302 (2002).
[5] Netterfield, C.B. et al., 2001, astro-ph/0104460
[6] Lee, A.T. et al., Astrophys. J. 561, L1 (2001).
[7] Halverson, N.W. et al., Astropjys. J. 568, 38 (2002).
[8] G.F. Smoot et al. Astrophys. J. 396, L1 (1992).
[9] A.G. Doroshkevich, Ya.B. Zeldovich, & R.A. Sunyaev, Sov. Astron. 22, 523 (1978).
[10] P. de Bernardis et al., Nature 404, 495 (2000).
[11] F. Zwicky, Morphological Astronomy, Berlin (1957).
[12] R. Trotta, A. Riazuelo and R. Durrer, Phys. Rev. Lett. 87, 231301 (2001)
[13] R. Juszkiewicz et al., Science 287, 109 (2000).
[14] W.J. Percival et al., Mont. Not. Roy. Ast. Soc., submitted
[15] Z. Haiman, J.J. Mohr and G.P. Holder, Astrophys. J. Lett. \textbf{560}, L111 (2001).

[16] J.A. Peacock et al., Nature \textbf{410}, 169 (2001).

[17] L. Van Waerbeke et al., Astron. Astrophys \textbf{374}, 757 (2001).