An Improved Method for Two-UAV Trajectory Planning for Cooperative Target Locating Based on Airborne Visual Tracking Platform

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SUMMARY The reconnaissance mode with the cooperation of two unmanned aerial vehicles (UAVs) equipped with airborne visual tracking platforms is a common practice for localizing a target. Apart from the random noises from sensors, the localization performance is much dependent on their cooperative trajectories. In our previous work, we have proposed a cooperative trajectory generating method that proves better than EKF based method. In this letter, an improved online trajectory generating method is proposed to enhance the previous one. First, the least square estimation method has been replaced with a geometric-optimization based estimation method, which can obtain a better estimation performance than the least square method proposed in our previous work; second, in the trajectory optimization phase, the position error caused by estimation method is also considered, which can further improve the optimization performance of the next way points of the two UAVs. The improved method can well be applied to the two-UAV trajectory planning for corporate target localization, and the simulation results confirm that the improved method achieves an obviously better localization performance than our previous method and the EKF-based method.

key words: unmanned aerial vehicle, target localization, trajectory optimization

1. Introduction

In recent years, unmanned aerial vehicle (UAV) have played an important role in many modern applications such as surveillance, tracking, mapping and transportation. This letter focuses on the problem of planning the trajectory for two UAVs to cooperatively localize a moving vehicle. The referred UAVs are both equipped with an inertial navigation system (INS), a global positioning system (GPS) and a visual tracking platform [1]. In the process, video algorithms are utilized to determine the pixel centroid of the target, and the onboard control algorithm is also used to adjust the camera platform to fix the target centroid pixel at the centre of the field of view (FOV). Based on this setting, the target position in the geographic coordinate system can be estimated using the GPS data, UAV attitude angles and angles of the camera platforms for the two UAVs. The localization performance of the above localization process is not only corrupted by the measurement noises, but also significantly influenced by the trajectories of the two UAVs. Bishop et al. [2] studied the influence of the geometric configuration between multiple stationary observation stations and a target on localization performance. Bai et al. [3] established a localization model based two-UAV intersection with the airborne optoelectronic platform, and studied the influence of UAV position on the localization performance. Ponda et al. [4] presented a trajectory optimization scheme for 3D target localization problem, they used the Fisher information based EKF method, but when the target has a highly non-linear dynamic model or the measurements are corrupted by flicker noises, the EKF based method will diverge. To tackle this problem, we have proposed a cooperative two-UAV target localizing scheme by planning the trajectories of the two UAVs [5], and the method has a better localization performance for target with unknown dynamic model than EKF based method because the target estimation is independent from previous measurement. However, the proposed method in our previous work can not estimate the target position well, and has neglected the errors caused by the target estimation in the trajectory planning objective. To compensate these two problems, we propose an improved method in this letter.

2. Proposed Method

Using video algorithm, the offset from the target centroid pixel to the centre of camera FOV can be obtained, and using onboard control algorithm, the camera control module drives the pan/tilt servo motor to adjust its elevation angle and azimuth angle through pulse width modulation such that the target centroid pixel can be permanently locked in the centre of the camera’s FOV. Given the measurement data such as GPS and IMU data and the output angle data from the visual tracking platform, the line of sight (LoS) with respect to the target in the geographic coordinate system can be obtained. Therefore, the target localization problem can essentially be attributed to a 3D bearings-only estimation problem, as is shown in Fig. 1. The two-UAV trajectory planning approach proposed in [5] is a method based on the simultaneous target localization and way-points optimization scheme. To optimize the two-UAV trajectory, we first analyze the localization principle.
the horizontal plane (assuming the two UAVs have the same height), and point A is also the projection of the target onto the horizontal plane. From the figure it can be seen that $\phi_1 = \alpha_1 - \gamma$, $\phi_2 = \alpha_2 - \gamma$, $\theta_1 = -\beta_1$, and $\theta_2 = -\beta_2$, where $\gamma$ denotes the angle between the vector pointing from UAV1 to UAV2 and the positive direction of $X$ in geographic coordinate system, which can be computed by using the currently known positions of the two UAVs. Based on the theorem 3 in [6], the geometrically consistent measurement error should satisfy the following equation:

$$
\cot(\hat{\theta}_1 - \tilde{\alpha}_1) \sin(\hat{\phi}_1 + \tilde{\beta}_1) - \cot(\hat{\theta}_2 - \tilde{\alpha}_2) \sin(\hat{\phi}_2 + \tilde{\beta}_2) = 0
$$

(2)

Also, according to [6], when error variables based function:

$$
f(x) = \tilde{\phi}_1^2 + \tilde{\phi}_2^2 + \tilde{\theta}_1^2 + \tilde{\theta}_2^2
$$

(3)

where $\hat{\phi}_i = \tilde{\alpha}_i$, $\tilde{\theta}_i = -\tilde{\beta}_i$, achieves it minimum, the target position has a geometrically consistent estimation. That is, the problem of estimating the target position with two cooperative UAVs can be attributed to the following constraint optimization problem:

$$
P_0 : \min f(x) \quad \text{s.t. (2)}
$$

the objective function is quadratic, and constraint is continuous, differentiable and nonlinear. To solve the objective problem, we can employ the sequential quadratic programming technique [7]. Thus, after the error computation, the angle measurements can be corrected by subtracting the obtained errors, and eventually, a geometrically consistent estimate of the target position can be obtained.

The estimation mechanism requires that the two UAVs have the same flight altitude. Under the minimum altitude restriction, according to [5], the optimal waypoints of the two UAVs have the same heights, i.e., the minimum relative height required between UAVs and the target, hence the above geometrical relation is valid.

2.2 Waypoints Optimization of UAVs

Ideally, the target observation model can be expressed as (1) if the target position is exactly known. However, the real target position is unknown, and only an estimate of the target position can be achieved using the estimation method based on geometric constraint. In our previous work, we have utilized the least square method to estimate the target position, and in the waypoints optimization phase, we have directly neglected the error caused by target position estimation, and only considered the random measurement noises. To optimize the waypoints of the UAVs more effectively, the estimation error of the target position in the observation model should also be taken into consideration, and thus, the actual observation equation for UAV $i$ based on the estimated position of target can be written as follows:

In Fig. 2, point A and the two UAV positions make up the horizontal plane (assuming the two UAVs have the same height), and point A is also the projection of the target onto the horizontal plane. From the figure it can be seen that $\phi_1 = \alpha_1 - \gamma$, $\phi_2 = \alpha_2 - \gamma$, $\theta_1 = -\beta_1$, and $\theta_2 = -\beta_2$, where $\gamma$ denotes the angle between the vector pointing from UAV1 to UAV2 and the positive direction of $X$ in geographic coordinate system, which can be computed by using the currently known positions of the two UAVs. Based on the theorem 3 in [6], the geometrically consistent measurement error should satisfy the following equation:

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where, \( \hat{\alpha}_i \) and \( \hat{\beta}_i \) denote the errors of azimuth and elevation caused by the estimation error of the target position, respectively. According to [6], the estimation error of target position is roughly determined by simulation procedure as in [6]. Note that \( x_i \) can be written as:

\[
\hat{x}_i = \hat{\alpha}_i \alpha_i + \hat{\beta}_i \beta_i
\]

where \( \hat{\alpha}_i, \hat{\beta}_i \) are Gaussian with variances as:

\[
\begin{align*}
\sigma_{\alpha,i}^2 &= c_1(\sigma_{\alpha,i} + \sigma_{\beta,i})|\hat{P}_{fi}| \\
\sigma_{\beta,i}^2 &= c_2(\sigma_{\alpha,i} + \sigma_{\beta,i})|\hat{P}_{fi}|
\end{align*}
\]

(6)

where \( \sigma_{\alpha,i}, \sigma_{\beta,i} \) are measurement variances of azimuth and elevation with respect to UAV \( i \); \( \| \cdot \| \) denotes the Euclidean measure, \( c_1, c_2 \) are some constant factors, which can be roughly determined by simulation procedure as in [6]. Note that \( x_i \) is the position of UAV \( i \) at current time, which is known.

Define \( \hat{\lambda}_i := [\hat{\alpha}_i + \hat{\beta}_i, \hat{\beta}_i + \hat{\beta}_i]^T \). Assuming the estimation errors and the angle measurement noises are Gaussian and mutually independent, and based on (6) and the Fisher information theory, the Fisher information matrix (FIM) of \( \lambda_i \) after considering the estimation error can be approximated by:

\[
I_{\lambda,i} = \begin{bmatrix}
\frac{1}{\sigma_{\alpha,i}^2} & \frac{1}{\sigma_{\beta,i}^2} \\
\frac{1}{\sigma_{\beta,i}^2} & \frac{1}{\sigma_{\beta,i}^2}
\end{bmatrix}
\]

(7)

which is in fact the inverse of covariance matrix of \( \hat{\lambda}_i \). Hence, according to (5) and the FIM relation, we have

\[
I_{\lambda,i} = \nabla_{\lambda}^T \lambda_i I_{\lambda,i} \nabla_{\lambda} \lambda_i
\]

(8)

where, \( I_{\lambda,i} \) denotes the FIM of \( x_i \) with respect to UAV \( i \), \( \nabla_{\lambda} \lambda_i \) represents the Jacobian matrix of \( \lambda_i \) with respect to \( x_i \) at the current estimation \( \hat{x}_i \). It can be found that \( I_{\lambda,i} \) is singular, i.e. zero information, which means that observation by a single UAV can not localize the position of a target.

Based on the knowledge about the Fisher information, the FIM of \( x_i \) with respect to measurements of the two UAVs can be written as:

\[
I_i = I_{x,1} + I_{x,2}
\]

(9)

Hence, if the two UAVs and the target are not co-linear, \( I_i \) is nonsingular, and the target position can be estimated. Further, to optimize the waypoints of the two UAVs at next time instant such that the target position can be estimated accurately is equivalent to maximizing the FIM of \( x_i \) as [2], [8]. Additionally, consider the vehicle constraints, this can be depicted as solving the following constrained optimization problem:

\[
P1 : \max T(I_i) \quad \text{s.t.} \quad x_i(k) = \Phi x_i(k-1) + U_i(k-1)
\]

\[
\|x_i(k) - x_i^c(k)\| \geq D_{min}
\]

\[
z_i^f - z_i^d \geq h_{min}
\]

\[
|U_i(k-1)| \leq \Delta v_{max}
\]

(10)

where, \( T \) denotes a matrix measure, which can be D-criterion, which leads to minimizing the determinant of inverse of the FIM, A-criterion, which leads to minimizing the trace of the inverse of FIM and E-criterion, which results in minimizing maximum eigenvalue of the inverse of the FIM. For convenience and efficiency, a suitable measure can be selected as A-criterion [4]. \( x_i(k) \) denote the position of UAV \( i \) to be optimized at time instant \( k \).

\[
\Phi = I_{3 \times 3}
\]

denotes the transition matrix of UAV dynamic model, \( U_i(k-1) \) is the manipulating vector; \( \Delta \) is the measurement time interval; \( v_{max} \) represents the maximum velocity vector; The second constraint is to constrain the safety distance of the two UAVs; the third constraint is used to constrain the relative height between the UAVs and the target; the last constraint exists by considering the maximum velocity.

The above constraint optimization problem is nonlinear and non-convex; we resort to exploiting the grey wolf optimizer [9] to solve this optimization problem.

3. Simulation Result

We set the initial position of the two UAVs and the target as follows: \( x_1 = (-600 \, m, 300 \, m, 500 \, m) \), \( x_2 = (700 \, m, -200 \, m, 500 \, m) \), \( x_0 = (0 \, m, 0 \, m, 0 \, m) \). We designed the simulation where target sequentially makes constant velocity, constant turning and constant acceleration manoeuvres. The target moves in \( XOY \) plane, and has the initial velocity of 25 m/s and 30 m/s in \( X \) and \( Y \) axes, respectively, and has a turning rate with 2 rad/s in the second phase, acceleration 0.6 m/s² in the third phase. The two UAVs have the maximum velocity 50 m/s. The standard deviation of the observation angle errors \( \bar{\alpha} \) and \( \bar{\beta} \) are both set as 1.5 degree. The constraints are set as: the safety distance between the two UAVs is restricted with 50 m, the minimum relative flight height from the UAVs to the target is 300 m; the frequency of the measurement is 1 measurement per second; the adjusting factors \( c_1, c_2 \) are tested and set as \( c_1 = c_2 = 3 \times 10^{-4} \).

Figure 3 shows the results of trajectories planning. In Fig. 3(a), the planned trajectories in 3D illustration, the square, diamond and circle represent the start points of constraints, respectively. Accordingly, the improved method is similar in that the two UAVs decrease their heights rapidly until the minimum heights are reached.
The horizontal angles between the two UAVs and target are shown in Fig. 4 (a), and from the figure, we can see that the horizontal angles of the two methods have the same variation trend; besides, the improved method has more stable angles than that of the previous method, and Fig. 4 (b) also shows that the UAV-target distance variations of the previous method are more severe than the improved method. The angle and the distance amplitudes are influenced by the maneuverabilities of the UAVs, the target motion and the initial states of UAVs and target.

We compare the localization performance of the improved method with that of our previous method and the EKF based method in [4], by using 100 Monte Carlo trials. The root mean squared target localization error for each method is shown in Fig. 5. In the figure, the time intervals with different target manoeuvres, are also shown, where $\Delta t_1$ is the time interval with the constant velocity, $\Delta t_2$ constant turn, $\Delta t_3$ with constant acceleration. From the results, it can be seen that both our previous method and the improved method are robust to turning and acceleration motion. The errors caused by the position estimation of the target are taken into account to optimize the UAV trajectories. The final numerical experiment shows that the improved method can improve the localization performance by about 20%, compared with our previous method, which may contribute to engineering applications. The generated trajectories by this approach can be used indexes for controlling UAVs to track and localize a target.

### 4. Conclusions

This letter presents an improved method for our previous work. First, the least square estimation method is replaced with the geometric optimization method, second, the errors caused by the position estimation of the target are taken into account to optimize the UAV trajectories. The final numerical experiment shows that the improved method can improve the localization performance by about 20%, compared with our previous method, which may contribute to engineering applications. The generated trajectories by this approach can be used indexes for controlling UAVs to track and localize a target.

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### References

[1] L. Zhang, F. Deng, J. Chen, Y. Bi, S.K. Phang, and X. Chen, “Trajectory planning for improving vision-based target geolocation performance using a quad-rotor UAV,” IEEE Trans. Aerosp. Electron. Syst., vol.55, no.5, pp.2382–2394, 2019.
[2] A.N. Bishop, B. Fidan, B.D.O. Anderson, K. Doğançay, and P.N. Pathirana, “Optimality analysis of sensor-target localization geometries,” Automatica, vol.46, no.3, pp.479–492, 2010.
[3] G. Bai, J. Liu, Y. Song, and Y. Zuo, “Two-UAV intersection localization system based on the airborne optoelectronic platform,” Sensors, vol.17, no.1, 98, 2017.
[4] S. Ponda, R. Kolacinski, and E. Frazzoli, “Trajectory optimization for target localization using small unmanned aerial vehicles,” AIAA Guidance, Navigation, and Control Conference, 2009.
[5] C. Xu, C. Yin, W. Han, and D. Wang, “Two-UAV trajectory planning for cooperative target locating based on airborne visual tracking platform,” Electron. Lett., vol.56, no.6, pp.301–303, 2020.

[6] A.N. Bishop, B.D.O. Anderson, B. Fidan, P.N. Pathirana, and G. Mao, “Bearing-only localization using geometrically constrained optimization,” IEEE Trans. Aerosp. Electron. Syst., vol.45, no.1, pp.308–320, 2009.

[7] R. Fletcher, Practical Methods of Optimization, Wiley, 1987.

[8] Y. Zhong, X. Wu, S. Huang, C. Li, and J. Wu, “Optimality analysis of sensor-target geometries for bearing-only passive localization in three dimensional space,” Chinese Journal of Electronics, vol.25, no.2, pp.391–396, 2016.

[9] S. Mirjalili, S.M. Mirjalili, and A. Lewis, “Grey wolf optimizer,” Advances in Engineering Software, vol.69, pp.46–61, 2014.