Anisotropic vortex squeezing in Rashba superconductors: a manifestation of Lifshitz invariants

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Most of 2D superconductors are of type II, i.e., they are penetrated by quantized vortices when exposed to out-of-plane magnetic fields. In presence of a supercurrent, a Lorentz-like force acts on the vortices, leading to drift and dissipation. The current-induced vortex motion is impeded by pinning at defects, enabling the use of superconductors to generate high magnetic fields without dissipation. Usually, the pinning strength decreases upon any type of pair-breaking. Here we show that in Rashba superconductors the application of an in-plane field leads, instead, to an unexpected enhancement of pinning. When rotating the in-plane component of the field with respect to the current direction, the vortex inductance turns out to be highly anisotropic. We explain this phenomenon as a manifestation of Lifshitz invariant terms in the Ginzburg-Landau free energy, which are enabled by inversion and time-reversal symmetry breaking and lead to an elliptic squeezing of vortex cores. Our experiment provides access to a fundamental property of Rashba superconductors and offers an entirely new approach to vortex manipulation.

Breaking the inversion symmetry in superconductors has numerous important consequences [13]. Often it occurs through the Rashba spin-orbit interaction (SOI) which splits the Fermi surface and links the electron spin to the momentum. If the SOI is strong enough to compete with the superconducting pairing, it gives rise to a plenty of interesting phenomena, as e.g. singlet-triplet mixing [4], unconventional pairing [5], Ising superconductivity [9, 10], magnetochiral resistance [11,15], anomalous Josephson effect [16,26], supercurrent diode effect [27–29], topological superconductivity [30–52], and helical phases [33–35] with a spatially modulated order parameter.

One possibility to engineer synthetic 2D Rashba superconductors consists in proximitizing a 2D electron gas (2DEG) with large Rashba SOI by a standard s-wave superconductor. This can be realized, e.g., by epitaxially growing an Al film on a shallow InAs quantum well [36–59]. Owing to their non-trivial topological features [5,30–49], such hybrid 2D semiconductor-superconductor heterostructures have been intensely investigated. So far, theory and experiments mainly aimed at exploring the Majorana modes at the edge of topological superconductors, which is enabled by the bulk-boundary correspondence [41–47]. In contrast, experimental signatures of the impact of SOI on the superfluid condensate as such are rather sparse in hybrid systems [8].

Besides the microscopic description in terms of the Bogoliubov-de Gennes or Gor’kov equations, the effects of SOI and magnetic field on the superconducting condensate can be accounted for phenomenologically by adding new terms into the underlying Ginzburg-Landau free energy. Such terms—called Lifshitz invariants [33,38,49]—depend on the crystal point-group symmetry [1] and, in the simplest case, they form triple products of magnetic field, linear spatial gradient of the order parameter and a vector specified by SOI. The presence of the Lifshitz invariants leads to an anisotropic response of the superflow and gives rise to magnetoelastic effects [50,51], helical phases [52–54], anomalous magnetization [55,56] and anomalous $\phi_0$-shift [17].

Vortices can be used to probe the structure of the order parameter $\Psi(x,y)$, because $|\Psi(x,y)|^2$ near the vortex core is proportional to the potential $U(\mathbf{r})$ that confines a vortex near a point-like pinning site, with $\mathbf{r} = (x,y)$ being the vortex displacement from the pinning center at $(x_0,y_0)$, see Fig. 1 [57]. In parabolic approximation the potential $U(\mathbf{r}) \approx k r^2/2$ is characterized only by its curvature $k$ [58]. Driving vortex oscillations around the pinning centers with an AC-current leads to an inductive voltage response, i.e., a vortex inductance

$$L_v = N_\square B_z \frac{\Phi_0}{k},$$

where $B_z$ is the out-of-plane magnetic field, $\Phi_0 = h/(2e)$ the superconducting flux quantum and $N_\square = l/w$ being the ratio of length $l$ and width $w$ of the film [59,61].

In this work, we demonstrate an unusual, anisotropic decrease of the vortex inductance when varying the magnitude and spatial orientation of the in-plane magnetic field. We interpret this observation as a consequence of Lifshitz invariant terms in the Ginzburg-Landau equations for $\Psi(x,y)$ that, in presence of an in-plane field, cause an elliptic contraction of the order parameter profile. This contraction is detected in the vortex inductance. The pinning enhancement is hard to explain by other known mechanisms, and offers a direct insight into the unusual structure of the order parameter near the vortex cores of Rashba superconductors.

Figure 1a shows a schematic of the pinning landscape $U(x,y)$ for a pinned vortex together with the directions of in-plane magnetic field and the AC drive current. A supercurrent
in $x$-direction generates a Lorentz force that displaces vortices in $y$-direction from their equilibrium positions. This increases the free energy, producing a restoring force. For small displacements and low frequencies, pinned vortices thus behave as underdamped harmonic oscillators [59–61].

Our synthetic Rashba-superconductor is fabricated starting from a InAs/InGaAs quantum well capped by an epitaxial Al film of thickness $d = 7$ nm [62]. The Al film induces superconducting correlations in the shallow 2DEG by proximity effect. The penetration depth $\lambda$ and the coherence length $\xi$ for the Al/2DEG system at 100 mK are 227 nm and 73 nm, respectively [63]. Using optical lithography and wet etching, we pattern a meander structure, as depicted in Fig. 1b. The meander is 24 $\mu$m-wide, which is larger than the Pearl penetration depth $\lambda_\perp = 2\lambda^2/d = 8$ $\mu$m, and a total length of 7.3 cm, resulting in $N_\square = 3042$ squares. These dimensions are motivated by the need of having at the same time a device in the 2D regime and a large number of squares to increase the measured vortex and kinetic inductance.

The sample holder is mounted on a piezo rotator, whose rotation axis is parallel to the $\hat{z}$ axis, i.e., perpendicular to the film, see Fig. 1a. A superconducting coil provides an in-plane magnetic field parallel to the $\hat{x}$ axis, while an orthogonal pair of coils provides a small out-of-plane field in the $\hat{z}$ direction. The device under study is embedded in a RLC resonant circuit located on the sample holder, see Fig. 1d. The circuit, described in Ref. [62], allows us to simultaneously measure DC transport characteristics and sample inductance. The latter is deduced from the center frequency shift of the RLC resonance spectrum, which is measured by lock-in detection in

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**FIG. 1. Vortex inductance as a probe of the pinning potential.** a, Sketch of the device under study. An epitaxial Al film (light blue) proximitizes from the top a shallow InAs quantum well (yellow). The sample is patterned as a 24 µm-wide and 7.3 cm-long meander (see micrograph in panel b). The current flows mainly along the $\hat{x}$ direction, and is subjected to a vortex-generating out-of-plane magnetic field $B_z$ and an in-plane field $B_{ip} \equiv B_z \hat{x} + B_y \hat{y}$ at a variable angle $\theta$ with respect to the $\hat{x}$-axis, which can be controlled. The grid represents the vortex free energy $U(x, y)$ for displacement from the pinning centers. An AC current $I \parallel \hat{x}$ exerts Lorentz force $F \parallel \hat{y}$. The restoring potential in harmonic approximation (small oscillations) is $U(x_0, y) = k_y (y - y_0)^2/2$ (red parabola), with $k_y = \partial^2 U(x_0, y_0)/\partial y^2$. b, Optical micrograph of the sample. Light grey area corresponds to the Al film, while the dark green ones are etched down to the mesa. c, Vortex ($L_v$) plus kinetic ($L_k$) inductance as a function of $B_y$. By increasing the vortex density, the inductance increases. At low fields (inset) the increase is linear Eq. (1). At larger vortex densities, the increase is super-linear owing to pair-breaking, which leads to divergence at $B_{y,c} = 61$ mT. d, Measurement scheme: the sample is embedded in a RLC circuit at low temperature and can be rotated with respect to $B_{ip}$ by means of a piezo rotator. e, Measured inductance for $B_z = 6.4$ mT as a function of temperature. f, Kinetic inductance vs. $B_y$ for $B_z = B_{ip} = 0$. g, $R(T)$ curves measured at $B_z = B_{ip} = 0$ for (right to left) $B_y = 0, 0.5, 1.0, 1.5, 2.0, 2.25, 2.5, 2.6$ T.
the few MHz regime. This is far below the characteristic frequency \( \omega_0/2\pi = R_N/2\pi L_n \approx 4.6 \text{ GHz} \) (\( R_N \) being the normal state resistance) \(^{60}^{61} \) that separates inductive and dissipative regimes \(^64\).

Figure 1 shows how the sample inductance depends on the out-of-plane magnetic field \( B_z \). We notice that the function \( L_v(B_z) \) is nearly linear up to 20 mT (corresponding to \( B_z \approx B_c/3 \)), indicating that the inductance per added vortex is approximately constant. This means that the interaction between vortices is not relevant in this regime. Importantly, the measured ratio \( L_v/B_z = 118 \text{ nH/mT} \) is of the same order as the value \( L_v/B_z = N(0)\Phi_0/k = 36 \text{ nH/mT} \) expected from Eq. (1), if we estimate \( k \approx 0.25d_B^2/\mu_0 \) \(^{61} \). Here \( B_c = 13.9 \text{ mT} \) is the thermodynamic critical field and \( d \approx 4.5 \text{ nm} \) \(^65\) the effective Al thickness, i.e. the nominal one minus 2.5 nm of oxide. The factor three discrepancy between the expected and the measured value of \( L_v/B_z \) is acceptable, in particular when considering the large theoretical uncertainty for the numerical prefactor 0.25 \(^{61} \).

At fields higher than 20 mT, the \( B_z \)-dependence of the vortex inductance increases faster than linear (Fig. 1b). In this regime, the order parameter in between the close-packed vortices is reduced compared to unperturbed value far from an isolated vortex. Hence, the bottom curvature \( k \) of \( U(x,y) \) is reduced as well, leading to a super-linear dependence of \( L_v \) on \( B_z \).

More generally, any pair-breaking mechanism tends to reduce the bottom curvature \( k \). As another example, Fig. 1b shows the temperature dependence of the vortex inductance at \( B_z = 6.4 \text{ mT} \) (linear low-field regime in Fig. 1b). A pronounced increase of the vortex inductance is observed, which becomes very steep when the critical temperature is approached. Finally, pair-breaking by a purely in-plane magnetic field \( B_{ip} \) must be reflected in the pure kinetic inductance \( L_k \) too. This measurement is shown in Fig. 1c. Notice the scale of the vertical axis: the kinetic inductance is 25 times smaller than the vortex inductance at 10 mT, and it varies only by few nH for applied fields of the order of 1 T \(^{66} \). The sharp minimum for \( |B_{ip}| < 100 \text{ mT} \) is likely due to the suppressed contribution of the Al wires used to bond the sample on the chipcarrier. Orbital pair-breaking is also seen in Fig. 1c, which displays a reduction of \( T_c(B_{ip}) \) in \( R(T,B_{ip}) \).

Having established the vortex inductance as a sensitive probe of the pinning strength \( k \), we now come to our main observation, namely, an entirely unexpected increase \( \text{of the pinning strength controlled by the in-plane magnetic field. Figure 2a shows } L_v \text{ vs. } B_{ip} \text{ at } B_z = 10 \text{ mT} \text{ (linear regime in Fig. 1c)} \text{, for both } B_{ip} \text{ parallel (blue) and perpendicular (red) with respect to the drive current } I = dwJ, \text{ where the current density vector } \mathbf{j} \text{ is oriented along } \hat{x}, \text{ corresponding to the } [110]- \text{direction of InAs (Fig. 1). In stark contrast to the behavior at } B_{ip} = 0, \text{ a drastic and surprising suppression of the vortex inductance is seen for both orientations. At very high magnetic fields exceeding 2 T, the inductance reaches a minimum and increases again near } B_{c2} = 61 \text{ mT} \text{ where it is expected to diverge. The full angle dependence of } L_v(\theta) \text{ is displayed in Fig. 2b for } B_z = 0, 2, 5 \text{ and 10 mT. The blue curve in Fig. 2b corresponds to the absence of vortices, i.e., the measured inductance is the kinetic inductance of the superfluid. The red curve in Fig. 2b corresponds to the same vortex density as in Fig. 2a. While the kinetic inductance is nearly isotropic at } B_{ip} = 0 \text{ \(^63\)}, \text{ the vortex inductance shows a pronounced } \theta- \text{dependence with a two-fold symmetry. In order to check whether the effect results from SOI in the InAs quantum well, we have performed a control measurement on an Al film grown epitaxially on GaAs. There is no 2DEG in GaAs and hence superconductivity is confined to the Al film. Moreover, in GaAs SOI is much smaller than in InAs, even when considering the effect of the Al/GaAs interface. For this device, the measured vortex inductance gradually increases with increasing in-plane field, see gray symbols in Fig. 2a. Importantly, this increase is almost perfectly isotropic \(^63\). Panels c-f of Fig. 2 illustrate the order parameter profiles \( |\Psi(x,y)|^2 \) near the vortex cores as inferred from the measured reduction of the vortex inductance. Panel c shows \( |\Psi(x,y)|^2 \approx U(x,y) \text{ in the vicinity } (x^2 + y^2 \ll \xi^2) \) of the vortex center for \( B_{ip} = 0 \). The vortex is assumed to be pinned at a point-defect located at the center of the figure. Since nothing breaks isotropy, the contours of constant \( |\Psi(x,y)|^2 \) are circles. As discussed above, the corresponding inductive voltage response reflects the curvature \( k_r \) of \( |\Psi(x,y)|^2 \) along \( \hat{y} \).

If an in-plane field is applied, e.g. along \( \hat{y} \) (\( \theta = 90^\circ \), Fig. 2d), the vortex core will be squeezed in both the \( \hat{x} \) and the \( \hat{y} \) direction. However, the effect is more pronounced for the direction along \( B_{ip} \) (in this case \( \hat{y} \)), i.e. \( \alpha^2 U(x,y) > \alpha^2 U(x,y) \). Thus, for \( B_{ip} > 0 \) the contour lines of constant \( |\Psi(x,y)|^2 \) become ellipses with minor axis directed along \( B_{ip} \). By rotating \( B_{ip} \), the elliptic core will rotate accordingly. Since in our device vortices oscillate along the \( \hat{y} \) direction, the largest curvature \( k_{\perp} \) (i.e., the smallest inductance) is probed for \( B_{ip} \parallel I \) (\( \theta = 90^\circ \), Fig. 2d) while the smallest curvature \( k_{\parallel} \) (largest inductance) is probed for \( B_{ip} \parallel I \) (\( \theta = 0^\circ \), Fig. 2a). As \( B_{ip} \) is continuously rotated, inductance measurements provide a tomography of the order parameter in the vicinity of the vortex center, as shown in Fig. 2a. The effect is remarkably robust: for \( B_{ip} = 1 \text{ T} \), \( k_{\perp} (k_{\parallel} \) increases by a factor 7.66 (1.64) compared to the \( B_{ip} = 0 \) case, as deduced from the corresponding reduction of \( L_v \) in Fig. 2a, red (blue) curve.

Now we turn to possible explanations for the striking observations in Figure 2. The key findings, that must be captured by a theoretical model, are: (i) the vortex inductance anomalously decreases with the applied in-plane field; (ii) the decrease is anisotropic, with a two-fold symmetry; (iii) it is maximal (minimal) when the field is perpendicular (parallel) to the current density; (iv) the effect is visible only in epitaxial Al/InAs 2DEG devices, while it is absent in the control Al/GaAs samples without 2DEG and with largely reduced SOI.

The non-centrosymmetry of the quasi-2D film is captured on the microscopic level by the isotropic Rashba Hamiltonian \( H_R = \alpha_s (\mathbf{k} \times \mathbf{n}) \cdot \sigma \), where the unit vector \( \mathbf{n} \) (along the polar axis) is normal to the plane of the superconducting film.
We also report the results (grey symbols) of the control measurement performed on a sample with epitaxially grown Al on intrinsic GaAs. The empty and full symbols refer to two measurement sessions with higher resolution at low fields and lower resolution at high fields, respectively. We estimate the Rashba coupling $\alpha_R$ and the g-factor in the Zeeman Hamiltonian $H_Z = g\mu_B B \cdot \sigma$ to be of the order of $\alpha_R = 15$ meV-nm and $g = -10$, respectively.

On the other hand, as shown by Edelstein [48], the joint effect of the Rashba SOI, in-plane magnetic field and superconducting pairing can be captured, within the Ginzburg-Landau approach, by adding a new term to the free energy—the so-called Lifshitz invariant. As discussed below, it is the Lifshitz term which can explain the anisotropic vortex squeezing in combination with the in-plane field. The Ginzburg-Landau free energy density in question has the following form:

$$F[\Psi, A] = a(T)|\Psi|^2 + \frac{b}{2}|\Psi|^4 + \frac{|\nabla \Psi|^2}{4m} + \frac{B^2}{2\mu_0} + F_L[\Psi, A],$$

where the last two terms correspond to the magnetic energy density and to the (isotropic) Lifshitz invariant [48]:

$$F_L[\Psi, A] = -\frac{1}{2} \kappa (n \times B) \cdot \left[ (\Psi^*)^T D \Psi + \Psi (D \Psi)^* \right].$$

In the above expressions $\Psi$ stands for the condensate wave function, $A$ for the vector potential, $B = \text{rot}A$ for the corresponding (in-plane + out-of-plane) magnetic field, and $D = \frac{\nabla}{2V} - 2eA$ for the covariant momentum operator ($|e|$ is the el-
eminary variation of the extended Ginzburg-Landau free energy $F$ in the Supplementary Information [63]. Upon functional variation of the extended Ginzburg-Landau free energy density $\mathcal{F}[\Psi, A]$, one obtains the first and second Ginzburg-Landau equation for 2D Rashba-superconductor, as discussed in the Supplementary Information [63].

The goal of our analytical calculation is to describe the wave function of the order parameter in the vicinity of the vortex core center at $(x_0 = 0, y_0 = 0)$, which we assume to be of the following form

$$\Psi_k(x, y) = K \cdot (x - i \delta y) \exp \left[ -\alpha \frac{x^2}{2} + i \beta xy - \gamma \frac{y^2}{2} \right],$$

where the real parameters $\alpha, \beta, \gamma, \delta$ and $K$ can be determined [63] from the Ginzburg-Landau equations including the Lifshitz term [67]. When discussing the model, we shall assume $B_{ip} \parallel \hat{y}$. In the limit of a point-like pinning defect at $(x_0 = 0, y_0 = 0)$, the effective vortex potential $U(x, y)$ mirrors $|\Psi_k(x, y)|^2$, hence

$$U(x, y) \simeq |\Psi(x, y)|^2 \simeq K^2 x^2 + K^2 \delta^2 y^2 \equiv k_x x^2 + k_y y^2.$$  

The theoretical values of $k_x$ and $k_y$ can be directly linked to the experimentally determined $k_i$ and $k_\perp$ since, for $B_{ip} \perp \mathbf{I}$, $k_x \equiv k_i$ and $k_y \equiv k_\perp$. The input parameters for our model are $\xi = 73$ nm, $\lambda = 227$ nm and $B_c = 10$ mT. Theory provides a set of algebraic equations for $k_x$ and $k_y$ as functions of $B_{ip}$. The equations contain the Lifshitz-Edelstein length $\ell_k$ and the effective thermodynamic critical field $B_c$ as parameters that can be determined by fitting data in Fig. 2a using Eq. (1) to link curvature to inductance. Restricting the fit to the range of $[-0.1 \, \text{T}, 0.1 \, \text{T}]$ we obtain $B_c = 96$ mT and $\ell_k = 590$ nm. The resulting fitting curves are shown as solid lines in Fig. 2a. Despite the simplified phenomenological approach, our model quantitatively captures (i) the increase of both curvatures, $k_i$ and $k_\perp$, upon application of an in-plane field, as well as (ii) the anisotropy ratio $k_\perp/k_i > 1$ of the two curvatures. For $B_{ip} > 0.1 \, \text{T}$ the fits underestimate $L_v$, most probably because the quadratic approximation of $\Psi(r)$ at the vortex cores is no longer valid. As a refinement, for crystals with $C_{4v}$ symmetry as for InAs, theory admits an anisotropic Lifshitz term with two different $k$ values (or, equivalently, two Lifshitz-Edelstein lengths) $\frac{1}{2} \kappa_{B_c}(\Psi^* \mathbf{D} \Psi + c.c.) \neq \frac{1}{2} \kappa_{B_c}(\Psi^* \mathbf{D} \Psi + c.c.)$. For simplicity, fitting the measured inductances we assumed an isotropic, i.e. $C_{4v}$-symmetric, Lifshitz invariant, obtaining nevertheless a good agreement with the experiment, at least for small or moderate magnetic fields, see Fig. 2a.

In order to further substantiate our interpretation of the reduced vortex inductance as an enhanced pinning strength, we investigate an entirely different signature of pinning, i.e., the depinning critical current. If the local minima of $U(r)$ become sharper in in-plane field, then one would expect that not only its bottom curvature will increase, but also its maximal slope, i.e. $\max(|\partial_k U(r)|)$. This corresponds to the maximal restoring force that pinning centers can exert before the depinning point. On the basis of the vortex inductance measurements just discussed, the depinning current is expected to display a similar peculiar increase with the in-plane field. We performed such measurements on a separate sample from the same wafer that was designed in a standard Hall bar geometry. The width was reduced to 2.3 $\mu$m, leading to a smaller critical current and thus less Joule heating. The results of the DC measurements are shown in Fig. 5a, where the voltage-current characteristics averaged over 50 sweeps are plotted for different in-plane fields $B_{ip} = B_x \hat{x} + B_y \hat{y}$, we vary $B_y$ while keeping $B_x = 0$ and $B_z = 2$ mT. Starting from $B_y = 0$, we notice, Fig. 5b, an increase of the depinning current by increasing $|B_y|$, followed by a decrease already at $|B_y| < 0.5$ T. On the one hand, these results confirm the outcome of the vortex inductance measurements, i.e., the pinning interaction is enhanced by a moderate magnetic field. On the other hand, this data indicates that bottom curvature and the maximal slope of $U(r)$ have different field dependencies. While $L_v$ stays low from 0.5 up to 2 T, the depinning current rapidly decreases above 0.5 T. Hence, the depinning current provides a further independent evidence of the apparently anomalous shrinking of the vortex cores.

Before concluding we want to briefly mention alternative (or additional) mechanisms that might be in act for the anisotropic vortex inductance in 2D Rashba-superconductors. The simplest possible origin of anisotropy in $U(r)$ is through a magnetic field-induced anisotropy in the coherence length $\xi$, e.g., by an anisotropic Fermi velocity. The Rashba SOI spin-splits the Fermi surface while preserving its circular symmetry. An applied in-plane magnetic field shifts and distorts the two circular Fermi surfaces. However, for a realistic Rashba coefficient $\alpha_R = 15$ meV-nm [28], we obtain a relative anisotropy in $v_F$ of the order of just $10^{-3}$, too small to explain the marked anisotropy observed in the experiment. Moreover, the measured anisotropy of the in-plane critical field [63] is also much smaller than that of $L_v$.

Another interesting possibility is that in an InAs 2DEG proximitized by an epitaxial Al layer the pairing function is not purely of $s$-wave type, but rather admixed of $p_x + ip_y$-wave. Without an in-plane field, the modulus of the pairing function is still isotropic both in the reciprocal and in the real space. The application of an in-plane field gradually projects the $p_x + ip_y$-wave pairing into its $p_y$-wave component. The anisotropic $\Delta(k)$ produces, upon Fourier transform, an anisotropic coherence length $\xi$. This argument does not explain, per se, the increase in the pinning force with the magnetic field. However, Hayashi and Kato [68, 69] have found that for sharp defects—in the sense discussed in Ref. [70]— and for $p_x + ip_y$-wave pairing, the pinning potential becomes
FIG. 3. **DC transport signature of the field-enhanced pinning.**

**a,** Current-voltage characteristics measured on a Hall-bar device for different values of the in-plane field $B_{ip}$ directed perpendicular to the current direction. The measurement is performed under an out-of-plane field $B_z = 5$ mT and at temperature $T = 0.1$ K. As depinning current, we take here the current producing a threshold voltage of 2.5 mV. This threshold is indicated with an arrow and a dashed line. **b,** Depinning current as a function of $B_{ip}$. Notice the clear local minimum at zero field, indicated by the arrow. It demonstrates that depinning becomes initially harder with increasing field.

![Graph](image)

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