Optimization of hanger arrangement in tied arch bridge using genetic algorithm

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Abstract. Hanger arrangement has a decisive influence on the mechanical behavior of tied arch bridge. Nielsen system and network system show good results in this respect, but the configuration of them is monotonous, which makes obstacles to the diversity of design. By using genetic algorithm with strong global searching ability, this paper combines ANSYS with MATLAB, and optimizes the hanger arrangement of tied arch bridge to obtain some layout forms of sparse and non-uniform hanger, which have good mechanical performance and varied configuration. Through the comparative analysis, it can be found that several hanger arrangement schemes obtained in this paper are very close to Nielsen system on stiffness. Even some schemes are not second to Nielsen system in the artistic respect, and show greater stiffness and better mechanical behavior. Furthermore, these different optimized schemes provide more design options.

1. Introduction
Relying on its artistic configuration and unique structural advantages, tied arch bridges have great advantages in small and medium span bridges. In the design of tied arch bridges, the arrangement of hangers is particularly important, as it directly affects the mechanical and aesthetic properties of the arch bridge.

Among the existing tied arch bridges, the hanger is usually arranged vertically. Tied arch bridge of Nielsen system is uniformly distributed with oblique hangers, and the network system adopts a more densely meshed inclined hangers. The engineering practice shows that the deflection of Nielsen system and network arch system bridge is much less than that of the arch bridge with vertical hanger arrangement. However, in both Nielsen System and network arch system, there are usually too many hangers to meet the design of special aesthetic requirements for hanger layout. More importantly, their configuration is monotonous, which makes obstacles to the diversity of design.

A few researches regarding optimization of network arch bridges have been performed in recent years. Tveit pointed the hangers should not be inclined too steeply to avoid relaxation (1987). Brunn et al. adopted a hypothesis for optimization of hanger arrangement that arch should be a part of a circle and hanger should be arranged in such a way that hanger intersections lie on the radii of the arch circle (2003). De Zotti et al. suggested that a configuration with 28° angle hanger inclination with upper nodes distributed with the same distances along the circular arch shows a good structural behavior (2007).
Although many scholars made a lot of optimizations of hanger arrangement on mechanical behavior, which are mainly about Nielsen system and network system, few of them have considered the optimization from both design richness and mechanical behavior. With the help of genetic algorithm and MATLAB GA Toolbox, this paper aims to optimize the hanger layout of tied arch bridge, and finds some layout forms of sparse and non-uniform hangers, which has good mechanical ability (especially the overall stiffness of structures under dead load and live load) and varied configuration.

Genetic algorithm (GA) is a computational model of biological evolution simulating Darwin's genetic selection and natural elimination (Goldberg 1989). It was first proposed by Professor Holland in 1975. It is particularly effective for non-differentiable, discontinuous, global, paralleled and multi-objective optimization problems and has been explored intensively in engineering optimization in recent years. The optimization of this paper involves the combination optimization of hangers, which is one of the best fields that can be solved by genetic algorithm. Therefore, the author uses genetic algorithm to optimize the hanger arrangement, and has achieved unexpected results.

2. The proposition of a tied arch bridge with sparse hanger system

Based on a large number of numerical calculations and analysis with GA, this paper proposes a new kind of hanger system for tied arch bridge (Figure 1), which has sparse hangers but shows favorable mechanical behavior. Compared with the traditional vertical hanger system, the biggest advantage of this system lies in the rich configuration changes and much better mechanical performance. According to the results calculated in this paper, there are at least hundreds of reasonable sparse hanger systems for tied arch bridge. Considering the aesthetics, a typical sparse hanger system (Figure 1, the exact hanger arrangement is showed in Figure 10) is selected to compare with the vertical hanger system (Figure 2), and the parameters of both bridges are exactly the same.

We can see the results of the two systems under the same live load: the maximum deflection of the main girder in the vertical hanger system is over 0.3m, and the maximum deflection of the new system is about 0.03m, which is lower than the former by an order of magnitude (Figure 3). It proves that the sparse hanger system obtained in this paper is superior to the traditional vertical hanger system in stiffness. At the same time, the stress envelope of the main girder shows that the stress amplitude of new system is about 50MPa, and the stress amplitude of the vertical hanger system reaches 80MPa (Figure 4). It means that the new system reduces the stress of the main girder, so that the carrying capacity and anti-fatigue performance of structure are greatly improved.

In addition, comparing the area of the stress envelope of the different systems, if the stress of the vertical hanger system is used as the control stress to implement the cross section optimization, the cross-section of the main girder of the sparse hanger system will be reduced, and the steel consumption of the main girder will be reduced by more than half of the original. Thus the construction cost will be greatly reduced.

![Figure 1. A tied arch bridge with sparse hanger system](image)
Figure 2. A tied arch bridge with vertical hanger system

Figure 3. Deflections of main girder for sparse hanger system and vertical hanger system

Figure 4. Stress envelope of main girder for sparse hanger system and vertical hanger system

3. Genetic algorithm process

3.1. Determination and calculation of objective function
The overall stiffness of the structure determines the mechanical behavior of the structure to a great extent, especially for the tied arch bridge. The study shows that the tied arch bridge of greater stiffness is more reasonable in the mechanical performance. Therefore, according to the principle of stiffness optimization, the objective function is defined as follows.
\begin{align}
\left\{ \begin{array}{l}
f = \min_{N_c} g = \sum_\omega(x) \\
\text{with } [N^c]^T = \{N^A_1, \ldots, N^A_n, N^G_1, \ldots, N^G_{0.5n}\}
\end{array} \right.
\text{s.t} \ 1 \leq N^A_i \leq a, \ 1 \leq N^G_j \leq 0.5b \quad i=1\ldots n, j=1\ldots 0.5n
\end{align}

Among them, \( f \) is objective function, the sum of the vertical deflection values of the main girder points. \( \omega(x) \) is the vertical deflection of the point at the main girder whose coordinate is \( x \) in longitudinal direction. \( N^A_i \) is the hanging point number on the arch. \( N^G_j \) is the hanging point number on the main girder. \( a \) is the number of possible hanging points on the arch. \( b \) is the number of possible hanging points on the main girder. \( n \) is the number of hangers, which is defined as 12.

Then the optimization process is translated into a problem to solve the minimum value of the objective function \( f \). In consideration of the powerful modeling and solving functions of ANSYS, the genetic operation uses ANSYS to model the tied arch bridge and solve the objective function \( f \), which greatly simplifies the process of objective function calculation.

3.2. Chromosome coding

Genetic manipulation involves simultaneous processing of a large number of individuals, and these individuals make up groups. In this paper, a string of fixed length binary symbols is used to represent individuals in a population, whose allelic genes are composed of two values \{0, 1\}. The individual genes in an initial population can be generated by randomly distributed numbers. First, in MATLAB, a binary matrix consisting of 0 and 1 is randomly generated, and then it will be transformed into a decimal matrix that ANSYS can read into by mathematical transformation. Each row of the matrix represents an individual.

Chromosome specific encoding rules are as follows. As shown in Figure 5, the arch and the main girder are divided into 20 equal parts. According to symmetrical characteristic, the arch will generate 19 possible hanging points \((a=19)\), while the main girder will generate 10 possible hanging points \((b=10)\), and each hanging point is followed by the corresponding number 1-29. Finally, an integer string composed of hanging point numbers makes the individual chromosome. Figure 6 shows a randomly generated individual whose chromosome is \{2 5 5 7 10 15 22 22 25 27 25 27\}.

Figure 5. The possible hanging points on the arch and the main girder

Figure 6. A randomly generated individual
3.3. Detection and evaluation of individual fitness

After the model calculation of ANSYS, the objective function value will be output into a readable text file, so that MATLAB can read it smoothly. Then, based on the linear scale principle, the objective function $f$ will be translated into fitness function $\text{Fit}(f)$:

$$\text{Fit}(f) = -f$$  \hfill (2)

According to the law of survival of the fittest in natural selection, the higher the fitness value, the probability that it inherits to the next generation is greater.

3.4. Genetic manipulation and generation of progeny populations

After encoding and generating the initial population, the task of genetic manipulation is to do some operations in groups according to their individual degree of adaptation to the environment (fitness evaluation), generating the progeny population, so as to realize the evolutionary process of survival of the fittest.

Genetic manipulation involves the following three basic genetic operators: selection, crossover, and mutation. In this paper, the fitness proportional model is used to establish the selection operator, that is, the selection probability of each individual is proportional to its fitness value in the selection operation. If the population size is $M$, and the fitness value of individual $i$ is $y_i$, the probability that $i$ is chosen will be $P_i$:

$$P_i = \frac{y_i}{\sum_{k=1}^{M} y_k}$$  \hfill (3)

The crossover refers that some parts of the structure of two parent individuals recombine and generates new individuals. Similar to the natural evolution process, the crossover operation is also playing a key role in the genetic algorithm. This paper adopts the method of single point crossover to do crossover operation. In order to improve the local random search ability of genetic algorithm and maintain the diversity of population, mutation operator is introduced to simulate the gene mutation in nature.

4. Finite element analysis of a tied arch bridge

The tied arch bridge considered as a reference in this paper is a scheme with span of 100 m, arch-rise of 17 m, $L_{\text{arch}}/L_{\text{girder}} = 0.14$ in the vertical plane, width of 4 m, and 12 hangers per arch. The arches are circular; the bridge girder is a steel box beam.

Regarding loads, the total dead load is 5381.6KN and the pedestrian load is 4KN/m². Since the bridge considered is a pedestrian landscape bridge, there is only one traffic lane on the bridge and the lane load is defined according to General Specifications for Design of Highway Bridges and Culverts in China.

The analysis is performed with the FEM program ANSYS, considering 100 different possible positions (100 load positions in longitudinal direction are defined as 100 steps of 1 m) of the most unfavorable live load, according to the specifications mentioned above.

5. Optimization results and discussion

First, a typical optimization process will be presented. Then some of the better sparse hanger system with 12 hangers obtained by genetic algorithms will be introduced in the following paragraph. Finally, the results of this paper are compared to the results of Nielsen system.

5.1. A typical optimization process

An optimization process usually involves 100 generations of genetic operations, which leads to better results. Figure 7 shows the three stages of the optimum solution in an optimization process. Stage 1 marks the emergence of genetic operation began to accelerate. Due to the strong selection mechanism of genetic operation, the optimal solution will be inherited in the next generation, and through the crossover and mutation operation of several generations, more excellent individual in stage 2 will be got. Experiencing the same process, an individual with good mechanical ability and artistic configuration finally shows in stage 3.
Figure 8 shows the comparison of the main girder deflection curve in the case of different hanger arrangements during three stages, on the whole, the deflection of each point of the main girder is falling steadily from stage 1 to stage 3. Certainly there are exceptions: the deflection near the midspan of the bridge in stage 2 is smaller than that of stage 3. But this does not affect us to draw the conclusion that in this genetic operation, with the optimization process carried out, the stiffness of tied arch bridge increases and the configuration is more artistic.

![Figure 7. Three stages of the genetic process](image)

![Figure 8. Comparison of deflection curves of the main girder at three stages](image)

5.2. Several optimization results
A genetic process can be done within a day, but it's still not easy to get a lot of results. After more than half a year's efforts, the author has obtained hundreds kinds of sparse hanger system for tied arch bridge with good mechanical performance. After artificial screening based on artistic criterion, 10 kinds of ideal sparse hanger system are obtained (Figure 9).

5.3. Comparison with Nielsen system
According to the research of De Zotti et al. (2007), using the same parameters as this paper, a better Nielsen system arch bridge model of sparse hangers is set up with a configuration with 28° angle hanger inclination (Figure 10). Compare the calculation results of Nielsen system and 10 optimized schemes in this paper, and get the reasonable evaluation of the optimized schemes.
The objective functions of different schemes are shown in Figure 11. The objective function directly reflects the mechanical behavior of the structure. It can be easily found that 10 hanger arrangement schemes obtained in this paper are very close to Nielsen system on structural performance, while scheme 1 and scheme 2 are better.

These schemes are obtained in a large number of numerical calculations, artificially based on aesthetic criteria. Therefore, although they are still unable to achieve perfection in aesthetics (scheme 10), they already have certain aesthetics, and the aesthetic performance of scheme 1 and scheme 4 is even better than that of traditional system. More importantly, when designing a tied arch bridge, there are now 10 systems for designers to choose, not just the monotonous Nelson system. This greatly increases the richness of the design, which has profound significance for the application of tied arch bridge.

| Scheme | Objective Function | Chromosome Coding | Arrangement Diagram |
|--------|--------------------|--------------------|---------------------|
| 1      | f=0.682            | [10 2 3 6 5 8 26 24 23 22 29 27] |
| 2      | f=0.685            | [7 5 5 10 9 3 6 9 2 6 10 4]    |
| 3      | f=0.709            | [7 3 5 1 9 7 10 4 4 3 7 3]     |
| 4      | f=0.714            | [3 11 5 8 5 7 3 8 8 6 3 10]    |
| 5      | f=0.733            | [2 8 9 4 9 6 5 9 5 4 10 7]     |
| 6      | f=0.742            | [4 13 10 3 6 8 3 9 7 6 9 6]    |
| 7      | f=0.747            | [4 2 4 6 9 12 23 22 25 28 24 27]|
| 8      | f=0.748            | [7 3 10 2 6 3 28 22 27 22 25 28]|
| 9      | f=0.756            | [6 9 9 3 3 6 5 10 3 2 7]       |
| 10     | f=0.759            | [5 4 6 10 8 17 29 22 24 25 28 28]|

Figure 9. 10 kinds of ideal sparse hanger system
6. Conclusion

Based on a large number of numerical calculations and analysis, the conclusions obtained in this paper are as follows:

1. Based on a large number of numerical calculations and analysis with GA, this paper proposes a new kind of tied arch bridge system, which has sparse hangers but shows good mechanical behavior. With the help of genetic algorithm, this paper obtains hundreds kinds of sparse hanger system for tied arch bridge, which all have good mechanical performance. Compared with the traditional vertical hanger system, this system has big advantages of design richness and much better mechanical performance.

2. In order to obtain accurate results, the number of hanging points can’t be divided too few, which in turn leads to the exponential explosion of the number of hanger arrangement schemes. In this paper, there are 29 hanger points and 12 hangers and the total number of arrangement schemes is 190⁶, which takes an ordinary computer 3 billion years to finish all these calculations. It is obviously that the exhaustive method is unable to achieve this optimization process. However, the genetic algorithm has very strong global search ability. Through selection, crossover and mutation operation simulating the genetic processes in nature, this paper completes an optimization process of hanger arrangement in a short span of one day, and get the ideal results through multiple processes.

3. Although Nielsen system and network system perform well on mechanical behavior of tied arch bridge, their forms are monotonous, which is not conducive to the richness and flexibility of design. 10 optimized hanger arrangement schemes obtained in this paper not only approach Nielsen system on mechanical behavior, but also possess more competitive hanger configuration, which make it possible for the design of special requirements for hanger configuration.

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