Theory and Practice of Harmonic Drive Mechanisms

G A Timofeyev, Yu V Kostikov, A V Yaminsky and Ye O Podchasov

Bauman Moscow State Technical University, Moscow, Russia
E-mail: timga@bmsstu.ru

Abstract. Among different mechanical drives of industrial application harmonic drive mechanisms (HDM) are distinguished by minimal materials consumption and overall dimensions, provide high efficiency, torsion rigidity and low kinematic error. Due to this they are widely used in mechanical actuators, robotics, aerial drives, supporting and turning arrangements of follow-up systems, dial indicators and other precise mechanisms. Combined design technique of different-type harmonic drive mechanisms and system concept to design process of electromechanical drives of follow-up servo is presented. Design system is made as a set of interconnected interacted computer application software which shortens design term and improves quality of the process. Drive designs based on internal and external gearing wave generating are considered, many of them are produced at the state-of-the-art level and implemented in many batch production servos.

1. Introduction

Instrument and special mechanical engineering industry requires minimal dimensions and mass of follow-up low-inertia mechanical drives for mobile devices. Multi-stage gearing drives (gear ration up to 100000 and higher) are used to reduce electrical motor high rpm (from above 10000 rpm). In these drives lesser number of reducer stages is possible due to implementation of drives distinguished by high quality characteristics of loading capability, kinematic error and efficiency. Harmonic drives are among the best in terms of these criteria [1-18]. This type drive principle of operation is based on the engagement of rigid and deformable flexible gear wheels. Deformation process is provided by wave generator. Flexible wheel is a ring or a thin shell. Variety of flexible gear geometry provides different designs of the drive which comply with different requirements in terms of configuration, motion transmission to any closed space without using any sealing units, etc. Wave generator may be positioned inside or outside relatively to the flexible wheel and deform it in one or several zones. Gear ratio of a single stage harmonic drive having steel flexible wheel ranges from 60 to 600. It provides multi-pair engagement. Total amount of teeth engaged is up to 40%. There is no engagement backlash, kinematic error decreases, efficiency and loading capability are high [1-4, 6, 8-12, 16, 17]. All this predetermines the usage of harmonic drive as the output stage of a gear train. It is also characterized by low vibration and noise [5, 13, 18].
2. Geometrical calculation of harmonic drives

Harmonic drive geometry design and computational theory is created at Theory of Machines and Mechanisms dept. of BMSTU and is based on theory of geometry analysis of involute engagement developed by V.A. Gavrilenko and his disciples research of HDM [6, 11-13, 15-18], other proceedings.

This theory is based on the assumption that wave generator of the drive provides constant radius of curvature of deformed flexible wheel mean line within the area of engagement marked by central angle $2\beta$ in Fig. 1, a, b. Outside these areas flexible wheel has natural/loose form of deformation. Within constant radius of curvature area harmonic drive engagement is considered as internal involute engagement of rigid wheel (number of teeth is $z_r$) and conditional wheel which number of teeth $z_c$ is calculated.

![Figure 1](image_url)

**Figure 1.** Harmonic drive engagement diagram: a – outside deformation, b – inside deformation by the wave generator
Calculated number of teeth is:

\[ z_c = \frac{z_f}{1 \pm k_p \left( \frac{w_0}{r_{mlf}} \right)} \]

where \( w_0 \) – radial deformation along major axis, \( r_{mlf} \) – mean radius of the flexible wheel; \( k_p = \frac{\beta}{\pi \cdot B} \) - coefficient depending of the angle of wheel generator cover;

\[ A = \frac{\pi}{2} - \beta - \sin \beta \cdot \cos \beta; \]

\[ B = \frac{4\beta}{\pi} \cdot \sin \beta + \frac{4}{\pi} \cdot \cos \beta - 2 \sin \beta \nonumber \]

where \( \beta \) – angular coordinate of the constant radius of curvature area (\( 35^\circ \leq \beta \leq 65^\circ \)).

The main input parameters for geometry characteristics calculation are: \( U \) – gear ratio of the drive; type of deformation (inside/outside); \( z_f, z_r \) – flexible and rigid wheels numbers of teeth; \( T, T_{max} \) – rated and maximal torques at the output drive shaft; \( n_h \) – rpm of the wave generator shaft; \( l_h \) – service life of the drive; \( \psi_{ring} \) – flexible wheel gear teeth tip width coefficient; flexible wheel strength parameters – hardness (HRC), endurance strength (\( \sigma_{\cdot 1} \)) of the material, torsion rigidity coefficient (C, optional parameter) [15].

The greater flexible wheel diameter the greater torsion rigidity and the less kinematic error. As the wheel diameter depends on the transmitted torque gear ratio of HDM should be the greatest providing compliance with overall dimensions of the unit.

Design calculation implies obtaining diameter \( d_{mlf} \) of the flexible wheel mean surface in non-deformed state in accordance with the following conditions:

1) Flexible wheel bending strength [13]:

\[ d_{mlf} = 220 \cdot \sqrt[k_d \cdot k_{ovl} \cdot \left( \frac{h_{ml}}{d_{mlf}} \right) \cdot \left( k_z - \frac{u}{u + 1} \right) \cdot T}, \]

where \( k_d \) - dynamism coefficient;

\( k_{ovl} \) - overload coefficient;

\( k_z \) - deformation form coefficient;

\( \frac{h_{ml}}{d_{mlf}} \) - flexible wheel relative thickness below teeth tip;

2) Endurance calculation: \( d_{mlf} = 165 \cdot \sqrt[3]{\frac{T}{(0.03 \cdot U - 1) \cdot \sigma_{F0}}} \),

3) Torque rigidity coefficient: \( d_{mlf} = \left( 1.12 - \frac{\sqrt{c}}{8000} \right) \cdot c^{(0.34 \cdot \frac{c}{\text{mm}})} \).

The greatest among these diameters is the basis for engagement module pitch calculation which is rounded up to the closest standard value \( m \).
Module pitch may be readjusted.

Pitch diameters of the wheels are:

\[ d_f = m \cdot z_f \quad ; \quad d_r = m \cdot z_r . \]

Flexible wheel rim thickness below the tooth tip:

\[ h_{ml} = \left( \frac{10.6 \cdot T \cdot 10^3}{\sigma_f \cdot d_f^3} + 0.007 \right) \cdot d_f , \]

but not greater than \(0.018 \cdot d_f\).

The main variable parameter is relative radial deformation of the flexible wheel:

\[ \frac{w_0}{r_{mlf}} = \left( \frac{z_r - z_f}{z_f} \right) \cdot \gamma , \]

where \(\gamma = 0.9...1.2\) - coefficient of relative radial deformation.

The first expression as all the rest, containing dual arithmetic signs, is combined. The upper sign relates to inner deformation of flexible wheel by disk or cam wave generator, the lower sign – to outer deformation by ring-shaped generator.

Modification factor of the basic rack is:

\[ x_f = \left( h_u^* + c^* + \frac{h_{ml}}{2m} \right) \cdot \delta , \]

where \(\delta\) - coefficient of modification factor deviation.

Based on the diagram of wave engagement the radius of deformed flexible wheel mean line may be presented as

\[ r_{mlc} = m \left( \frac{z_f}{2} \mp h_u^* \mp c^* \mp \frac{h_{ml}}{2m} + x_f \right) . \]

\(\delta\) and \(\gamma\) are variable parameters and are assigned as: \(\delta = 1,0...1,4\), \(\gamma = 0,9...1,2\) - for inside deformation, \(\delta = 0,8...1,1\), \(\gamma = 0,8...1,2\) - for outside deformation.

Quality of engagement may be improved considering possible ranges of input values. Goal function in this process is contact ratio coefficient.

3. Industrial harmonic drives samples

Geometry calculation algorithm is presented in [12, 13] and is applicable to HDM which gear ratio ranges from 60 to 100000. Principle schemes of these HDMs of inside and outside deformation are presented in [15].

Given below are some industrially implemented designs of drives based on HDM which geometry and kinematic characteristics were obtained based on the developed algorithm.
3.1. Two-stage gear transmission power drive

Fig. 2 presents the design of power drive based on two-stage harmonic drive. For unification purpose geometry parameters of gear engagement are the same: gear ratio is 60 or 80; module pitch is 0.4 or 0.3; total gear ratio is 4500 or 13000. Relatively to known designs overall dimensions of the drive developed are 14% less due to usage of narrow flexible wheel of intermediate stage and positioning supporting bearings 15 of the output shaft 14 inside flexible wheel 16.

Technical features:
- Output shaft load torque, N·m: 80
- Efficiency: 0.57
- Angular error of the output shaft at rated load, not greater than: 0.0005
- Power source voltage, V: 27 ± 2
- Power consumption, Wt: 18
- Overall dimensions, mm: 70×120×140
- Mass, kg: 2.5

3.2. High-torque drive

Fig. 3 presents output stage of power train designed for high-torque drives of automated control. Three-disk wave generator is implemented in this design. In comparison with two-disk wave generator this design excludes sideway of the flexible wheel, twisting of its end cross-sections that improves mesh of the harmonic drive teeth.

Figure 2. Two-stage gear transmission power drive

Figure 3. High-torque drive
Implementation of such design as the output stage of reducer increases its loading capability 1.5 times, efficiency – 1.4 times in comparison with the reducer containing planetary gear with five satellites as output stage.

Technical features of the drive:
- Output shaft torque, kN·m: 80
- Efficiency: 0.63
- Overall dimensions, mm: 800×800×1 000
- Mass, kg: 2 880
- Gear ratio:
  - total: 18 000
  - harmonic drive stage: 120
  - Harmonic stage module pitch, mm: 2

3.3. Discrete positioning electromechanical drive

The design of discrete positioning electromechanical drive based on HDM, presented in Fig. 4, provides new functional possibilities for transmitting continuous electrical motor shaft rotation to discrete translational motion of the output link of servo mechanism. This is three-stage drive. The first stage is cylindrical gearing of outer engagement, the second – combined harmonic drive providing discrete motion, the third, output one, is screw-and-nut drive.

![Figure 4. Discrete positioning electromechanical drive](image)

Introduction of discrete motion harmonic drive decreased production cost of a single unit by 40% due to the 2.5 times decrease of the number of parts, metal consumption decrease (3 times) and production labor intensity reduction.

Technical features of the drive:
- Linear displacement of the output link per one step, micrometer: 16
- Output shaft displacement accuracy per one step, micrometer: 1
- Number of steps per second: 25
- Axial force at output shaft, N: 300
- Overall dimensions, mm: 110x115x120
- Mass, kg: 1.1

Significant variety of different drive designs is presented in scientific report [12].
4. Conclusion
In some cases harmonic gear drive has no competitors. It’s also clear that new HDM designs and rigid wheel gear trains are no rivals, both contribute to new decisions. HDM has some shortcomings: inability to provide low gear ratio (less than 50), rather high losses at idle, it is sensitive to lubrication and types of lubricants.
Nevertheless, the area of HDM will broaden, enriching modern servo engineering.

References
[1] Volkov D P, Krainev A F 1976 Harmonic drives. (Kiev, Technika) p 222.
[2] 1975, Harmonic drives. ed. N.I.Zeitlin, V.N.Tatischev (Moscow: Stankin) p 243.
[3] Ginzburg E G 1969 Harmonic drives (Leningrad, Mashinostroenie) p 169.
[4] Ivanov M N 1981 Harmonic drives (Moscow, High School) p 183.
[5] Koval'ev N A 1979 Transmissions with flexible gears (Moscow, Mashinostroenie) p 200
[6] Gavril’enko V A, Skvortsova N A, Semin Yu. Let al 1978 Works of MVTU. vol 8. (Moscow: BMSTU) pp 22-33.
[7] Schuvalov S A 1986 Theory and automated design of harmonic drives PhD theses 05.02.02 (Moscow, BMSTU) p 28
[8] Krainev A F 2000 Mechanics of machines. Fundamental vocabulary (Moscow: Mashinostroenie) p 904
[9] Popov P K 1997 Computational and experimental approval of gearing accuracy D.Sc theses 05.02.02 (Moscow, BMSTU) p 269
[10] Bleys E S, Zimin A V, Ivanov E S 1999 Servo drives ed. B.K.Chemodanov (Moscow: BMSTU) p 904
[11] Timofeev G A, Tarabarin W B, Yaminski A V 1988 Construction and CAD of harmonic drives with internal and external deformation (Moscow, VINITI) p 71
[12] Timofeev G A, Kostikov Yu V, Barbashov N N, Podchasov E O 2016 Scientific report NIR 986.16 Development of calculation and design methods of harmonic drives for servo systems ed. G.A. Timofeev (Moscow, BMSTU) p 78.
[13] Timofeev G A. 1997 Development of calculation and design methods of harmonic drives for servo systems D.Sc theses 05.02.02 (Moscow, IMASH RAN) 352 p.
[14] Timofeev G A, Kostikov Yu V, Barbashov N N 2015 Scientific report NIR 986.15 Development of calculation and design methods of harmonic drives for servo systems ed. G.A. Timofeev (Moscow, BMSTU) p 106.
[15] Timofeev G.A., Samoilova M.V. 2015 Comparing analysis of harmonic drives schemes for servo systems Herald of BSTU, mashinostroenie ser. 4 109.
[16] Frolov K V, Popov S A, Musatov A K et al 2017 Theory of mechanism and mechanics of machines ed. G.A. Timofeev (Moscow, BMSTU) p 654.
[17] Kostikov Yu V, Timofeev G A, Fursiak F I 2016 The degree of influence of errors in manufacturing the details of the wave transfer on its kinematic accuracy Machine drives and parts 3 pp 10
[18] Timofeev G A, Kuzenkov V V 2015 Features of the servo drive dynamics with wave gearing Problems of machine Problems of machine building and machines reliability 6 pp 34.