Simultaneous blockade of a photon phonon, and magnon induced by a two-level atom

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The hybrid microwave optomechanical-magnetic system has recently emerged as a promising candidate for coherent information processing because of the ultrastrong microwave photon-magnon coupling and the longlife of the magnon and phonon. As a quantum information processing device, the realization of single excitation holds special meaning for the hybrid system. In this paper, we introduce a single two-level atom into the optomechanical-magnetic system and show that an unconventional blockade due to destructive interference cannot offer a blockade of both the photon and magnon. Meanwhile under the condition of single excitation resonance, the blockade of photon, phonon, and magnon can be achieved simultaneously even in a weak optomechanical region, but the phonon blockade still requires the cryogenic temperature condition.

I. INTRODUCTION

The effect of one photon preventing the second photon entrance is called a photon blockade [1, 2], which is the pivotal effect to achieve photons at the quantum level. It is believed that photon blockade can be used as a single photon source and to process quantum information [3]. The photon blockade in the cavity-QED systems [4–6] were thoroughly investigated and have been achieved in experiments [7, 8]. Recently, the optomechanical system has attracted significant attention, such as working as a sensor to detect tiny mass and force [9–12], a platform to investigate the fundamental physics [13] and a device to processing quantum information [14–20]. The most attractive characteristic of an optomechanical system is the nonlinearity resulting from the radiation pressure, which can induce Kerr nonlinearity [21] and produce the photon blockade [22]. However, currently, the single-photon optomechanical coupling is still within a weak coupling region, which only induces only fainter Kerr nonlinearity. Therefore, some strategies were put forward to enhance the nonlinearity [23, 24]. To avoid the weakness of the delicate single-photon nonlinear coupling, the photon blockade resulting from destructive interference called unconventional blockade (UB) was proposed and thoroughly investigated [25, 26].

Most recently, the photon-magnon coupling system in the microwave [27–29] and optical frequency [30–32] regime has aroused attention. Different from the weak optomechanical coupling, the ultrastrong coupling between microwave photons and magnons [the collective spin excitation in yttrium iron garnet (YIG)] was realized [28, 33], and the magnons possess a very low damping rate. Meanwhile, the magnon excitation interacting with phonons (vibrational modes of the YIG sphere) is similar to the optomechanical interaction, [34], so, both kinds of interactions magnetic-mechanical [35] and optical-mechanical are nonlinear. The phonons and magnons posse coupling mediated by cavity fields [36], and the entanglement of a magnon, photon, and phonon in cavity magnomechanics has been investigated where photon-magnon and magnomechanical interactions were considered [34]. In Ref. [37], the supermode of a photon exhibits blockade under the Kerr effect in optomagnonic microwavities system.

The photon blockade can be generated from the destructive interference [25, 38] as well as the single excitation resonance [6, 21, 39–41]. Usually, the destructive interference and the single excitation resonance resulting from dressed states can supply a better blockade than the Kerr effect because of the weak coupling strength of the Kerr interaction. The photon blockade in an optomechanical system [25] as well as in an optomagnonic system [37] were separately thoroughly investigated. A magnon blockade via qubit-magnon coupling has been studied in Ref. [42]. However, in the hybrid optomechanical-magnetic system, the simultaneous blockade of the photon, phonon, and magnon has not been studied. Meanwhile, the hybrid system has special significance for the realization of quantum information processing, like the quantum internet [43]. If the hybrid optomechanical-magnetic system was used as a quantum device, the single excitation level is important, and the simultaneous blockade of photon, phonon, and magnon should be pivotal and deserves further investigation.

In this paper, we consider a hybrid microwave optomechanical-magnetic system aiming to generate the simultaneous photon-phonon-magnon blockade. Considering the achievement of ultrastrong microwave optical-magnetic coupling in experiments [27, 44], we derive three-partite interaction among photon, phonon, and magnon. By introducing a single two-level atom, under the condition of single excitation resonance, we show that the simultaneous blockade of photon, phonon, and magnon can be achieved with the assistance of the three-partite interaction on the condition of cryogenic temperature of the mechanical mode, while the unconventional destructive interference can not offer the simultaneously multi-modes antibunching. In our scheme the single-photon strong optomechanical coupling is not required,
The system reads

\[ H = H_{om} + H_{op} + H_{ao} + H_d, \]  

where

\[ H_{om} = \omega_j a_+^\dagger a + \omega_m m_+^\dagger m + G_m (a_+^\dagger m + am^\dagger), \]
\[ H_{op} = \omega_b b_+^\dagger b + ga_+^\dagger a(b_+ + b), \]
\[ H_{ao} = \omega_n \sigma^+ \sigma + g_\alpha (\sigma a_+^\dagger + \sigma^+ a), \]
\[ H_d = \Omega_c (\sigma e^{i\omega_L t} + \sigma^* e^{-i\omega_L t}). \]  

\( j^\dagger (j, j = a, m, b) \) is the creation (annihilation) operator of the related mode (photon, magnon, and phonon) with frequency \( \omega_c, \omega_m \) and \( \omega_b \), respectively. \( \sigma \) stands for the pseudo-spin of the two-level atom. \( H_{om} \) consists of the energy of the photon and magnon, as well as the photon-magnon interaction with the effective strength \( G_m \), which is called the cavity magnon polaritons [45]. \( H_{op} \) is composed of the energy of the photon and the optomechanical interaction with coupling strength \( g \). The first term in \( H_{ao} \) is the energy of the atom, and the second term describes the atom interacting with the cavity field. \( H_d \) denotes an atom pumped with a classical field with frequency \( \omega_L \).

In the frame rotating with \( H_0 = \omega_L (a_+^\dagger a + \sigma^+ \sigma + m_+^\dagger m) \), the Hamiltonian can be changed into time-independent. For simplicity, we assume \( \omega_m = \omega_c \) then \( \delta = \omega_{(m)} - \omega_L \). We diagonalize the Hamiltonian \( H' = \delta (a_+^\dagger a + m_+^\dagger m) + G_m (a_+^\dagger m + am^\dagger) \) by introducing supermodes \( a_{\pm} = \frac{1}{\sqrt{2} g} (a_\mp + am) \). Considering photon-magnon interaction larger than the optomechanical and atom-phonon interaction, i.e., \( G_m \gg \{g, g_\alpha\} \) and choosing \( \omega_b = 2G_m \), we rewrite the Hamiltonian as

\[ H_{eff} = \Delta a_+^\dagger a_+ + (\Delta - 2G_m) a_+^\dagger a_- + \omega_b b_+^\dagger b + \Delta_\sigma \sigma^+ \sigma - \eta (a_+^\dagger a_- b + a_+ b^\dagger a_-^\dagger) + \eta_\alpha (a_+^\dagger \sigma + a_-^\dagger \sigma^+), \]  

where \( \Delta = \delta + G_m, \eta = \frac{g}{2}, \eta_\alpha = g_\alpha / \sqrt{2}, \Delta_\sigma = \omega_n - \omega_L \). The detailed deduction of Hamiltonian (3) is given in Appendix A. For simplicity, hereafter we will assume \( \Delta = \Delta_\sigma \). We see that the effective Hamiltonian contains three-partite interaction, which is similar to in Ref. [25]. Differently from their scheme, we introduce a pumped two-level atom aiming to achieve a blockade of the photon, magnon, and phonon. We also would like to compare the different effect of a blockade between the destructive interference mechanism and the single excitation resonance mechanism. Observe the last two brackets in Eq. (3): the pumped two-level atom interacts with mode \( a_+ \), which results in the blockade of mode \( a_+ \). Although the three-partite nonlinear interaction means the parametric-down conversion form between \( a_- \) and \( b \) mediated by absorption or emission of mode \( a_+ \), the blockade of the mode \( a_+ \) can not result in the amplification in mode \( a_- \) and \( b \). Instead, if there is only one excitation in the mode \( a_+ \), the transfer of the single excitation creates only one excitation in every mode of \( a_- \) and \( b \), that is to say, the blockade in mode \( a_+ \) will lead to the blockade in mode \( a_- \) and mode \( b \); therefore it is possible to generate a blockade in supermodes \( a_+, a_- \) and mode \( b \). We will

**II. THE MODEL AND THE ANALYTICAL ANALYSIS**

We consider a hybrid optomechanical-magnetic system, where a two-level atom and a YIG microsphere are contained in the microwave cavity, and one of the mirrors is movable, shown in Fig. 1(a). The magnons are sourced from a collective spins in a ferrimagnet. Here, we ignore the interaction between magnons and phonons due to deformation of the YIG sphere, because the single-magnon magnomechanical coupling rate is typically small [34, 36]. The magnetic dipole mediates the coupling between magnons and cavity photons. The Hamiltonian of the system reads

![Diagram](image-url)
show that the bare modes $a$, $b$, and $m$ can also be blocked simultaneously.

To check the validity of the approximation from Hamiltonian (1) to Hamiltonian (3), we choose $|g100\rangle$ as the initial state and plot the evolution of the probabilities of states $|g200\rangle$ and $|g100\rangle$ governed by the Hamiltonians $H$ and $H_{eff}$ respectively, shown in Fig. 2, where $|e\rangle$, $|n_{a}\rangle$, $|n_{b}\rangle$ represent a state with atom in $|e\rangle$ ($|g\rangle$), and $|n_{a}\rangle$, $|n_{b}\rangle$ are the number state for the $a_{+}$, $a_{-}$, and $b$ modes, respectively. From Fig. 2, we see clearly that the results of original Hamiltonian agree very well with that of effective Hamiltonian $H_{eff}$, which means that the effective Hamiltonian $H_{eff}$ is reliable.

Due to the limit of the weak driving field, for understanding the blockade mechanism of the photon (phonon, magnon), we temporarily ignore the pumping of the atom and derive the eigenstates and eigenvalues of $H_{eff}$ (3) in the few-photon subspace, yielding

$$\begin{align*}
|0\rangle: & \lambda_0 = 0, \\
|10\rangle: & \lambda_{10} = \Delta, \\
|1\pm\rangle: & \lambda_{1\pm} = \Delta \pm \beta_1, \\
|20\rangle: & \lambda_{20} = 2\Delta, \\
|21\pm\rangle: & \lambda_{21\pm} = 2\Delta \pm \beta_2, \\
|22\pm\rangle: & \lambda_{22\pm} = 2\Delta \pm \beta_3,
\end{align*}$$

(4)

where $\beta_1 = \sqrt{\eta_a^2 + \eta^2}$, $\beta_2 = \sqrt{3\eta_a^2 + 7\eta^2 + D}$, $\beta_3 = \sqrt{3\eta_a^2 + 7\eta^2 + D}$, $D = \sqrt{\eta_a^4 + 26\eta_a^2\eta^2 + 25\eta^4}$. The expression of the dressed states $|s\rangle$ ($s = 0, 1, 2; c = 0, \pm, 1, \pm, 2\pm$) is given in Appendix A, and the energy-levels are shown on the right side of Fig. 1(b).

In the weak driving limit, to analytically derive the equal-time second-order correlation function, the state of the system can be truncated in few excitation subspace and approximately expressed as

$$|\psi\rangle = C_{g000}|g000\rangle + C_{g100}|g100\rangle + C_{g011}|g011\rangle + C_{e000}|e000\rangle + C_{g200}|g200\rangle + C_{g111}|g111\rangle + C_{e100}|e100\rangle + C_{g022}|g022\rangle + C_{e011}|e011\rangle.$$

(5)

Under the action of the non-Hermite Hamiltonian $\tilde{H} = H_{eff} - i(\kappa_{c}a_{+}^{\dagger}a_{+} + \kappa_{a}a_{-}^{\dagger}a_{-} + \kappa_{0}a_{+}a_{-}\sigma + \kappa_{\sigma}\sigma^{\dagger}\sigma)$ with the decay rate $\kappa_{c}$ ($j = +, -, a$), the probability amplitude in $|\psi\rangle$ can be obtained by solving the Schrödinger equation $i\partial|\psi\rangle/\partial t = \tilde{H}|\psi\rangle$. The detail of the deduction and the steady-state solution can be found in Appendix B.

To characterize nonclassical photon (magnon, phonon) statistics, we employ and equal-time second-order correlation function defined by

$$g_{i}^{2}(0) = \frac{\text{Tr}(c_i^\dagger c_i \rho)}{\text{Tr}(c_i^\dagger c_i)^2},$$

(6)

where $i = a, m, b, a_{+}, a_{-}$. The steady-state correlation functions of our system can be analytically obtained via the steady-state wave function (5) as

$$g_{a_{+}}^{2}(0) = \frac{2|C_{g200}\rangle^2}{(|C_{g100}|^2 + u_1)^2} \approx \frac{2|C_{g200}\rangle^2}{|C_{g100}|^4},$$

(7)

$$g_{a_{-}}^{2}(0) = \frac{2|C_{g022}\rangle^2}{(|C_{g011}|^2 + u_2)^2} \approx \frac{2|C_{g022}\rangle^2}{|C_{g011}|^4},$$

with $u_1 = 2|C_{g200}\rangle^2 + |C_{g111}\rangle^2 + |C_{g100}\rangle^2$, and $u_2 = 2|C_{g022}\rangle^2 + |C_{g111}\rangle^2 + |C_{e011}\rangle^2$ where the second approximate equals in Eq. (7) are obtained under the conditions $|C_{g000}\rangle \gg \{ |C_{g100}\rangle, |C_{g011}\rangle, |C_{e000}\rangle \} \gg \{ |C_{g200}\rangle, |C_{g111}\rangle, |C_{g022}\rangle, |C_{e100}\rangle, |C_{e011}\rangle \}$. For mode $b$, it is not reasonable to obtain $g_{b}^{2}(0)$ with the analytical solution (5) because its decay has been ignored. We will directly calculate it from the master equation. The correlation function $g_{b}^{2}(0) \geq 1$ is referred to as Poissonian and super-Poissonian. The correlation function $g_{b}^{2}(0) < 1$ indicates sub-Poissonian, and the limit $g_{b}^{2}(0) \rightarrow 0$ corresponds to the complete blockade. Remarkably, the single-photon regime is usually characterized by $g_{b}^{2}(0) < 0.5$ [38]. From the expression Eq. (7) and Eq. (B2), one can see that the blockade in mode $a_{+}$ ($a_{-}$) is possible only if the population $C_{g200}C_{g022} \approx 0$. We will plot second-order correlation function and discuss it further in the next section.

Although the polariton modes [45–47] consisting of optical mode and magnetic mode can be indirectly derived by directly detecting the output spectrum of photons, the blockade of the photon, phonon, and magnon still deserve our investigation. Due to the combination of the optical mode and magnetic mode, the statistical properties of supermodes $a_{\pm}$ and bare modes $a$ and $m$ are different. In order to see clearly the difference, we derive
the relations between the two bases

\[
\begin{align*}
|00\rangle_d &= |00\rangle, \\
|10\rangle_d &= \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle), \\
|01\rangle_d &= \frac{1}{\sqrt{2}}(|10\rangle - |01\rangle), \\
|02\rangle_d &= \frac{1}{2}(|20\rangle - \sqrt{2}|11\rangle + |02\rangle), \\
|20\rangle_d &= \frac{1}{2}(|20\rangle + \sqrt{2}|11\rangle + |02\rangle), \\
|11\rangle_d &= \frac{1}{\sqrt{2}}(|20\rangle - |02\rangle),
\end{align*}
\]  

where the left side states are labeled by \(|n_+, n_-\rangle_d\) \((n_+\) and \(n_-\) correspond to the Fock state of mode \(a_+\) and \(a_-\) and \(n_+ \) and \(n_-\) denote the Fock state of mode \(m\) and \(a\)). The derivation of Eq. (8) is given in Appendix C. See the last line in Eq. (8), where the state \(|11\rangle_d\) means only one excitation in mode \(a_+\) and \(a_-\), however for the modes \(m\) and \(a\), they might be populated in two excitations. That is to say, the blockade of supermodes \(a_+\) and \(a_-\) does not mean the blockade of bare modes \(a\) and \(m\). Therefore, we need to calculate the second-order correlation of the mode \(m\) and \(a\):

\[
\begin{align*}
g_m^2(0) &= \frac{2(|C_{g200}|^2 + |C_{g022}|^2 + 2|C_{g111}|^2)}{(|C_{g100}|^2 + |C_{g011}|^2)^2}, \\
g_a^2(0) &= \frac{2(|C_{g200}|^2 + |C_{g022}|^2 + 2|C_{g111}|^2)}{(|C_{g100}|^2 + |C_{g011}|^2)^2}.
\end{align*}
\]  

We can see that the correlation functions for the optical and magnetic mode are the same. The blockades in the modes \(m\) and \(a\) require that \(C_{g200}\), \(C_{g022}\) and \(C_{g111}\) reach zero simultaneously. Fortunately, as one can observe from Eq. (B2), when \(C_{g022}\) equals zero, \(C_{g111}\) is equal to zero too. That is to say, when both \(a_+\) and \(a_-\) modes are a blockade, the photon and magnon modes \(a\) and \(m\) are both a blockade too.

III. THE STATISTICAL PROPERTIES OF THE MULTIMODE FIELD

In the above analytical calculation of \(g_i^2(0)\) \((i = a_+, b, a, m)\), we have made some approximations. We now show the correction of the approximations and investigate the statistical properties of the multimode field. For simplicity, we assume that the decay rates of the optical mode, magnetic mode, and atom are equal, and then we can derive the master equation as

\[
\dot{\rho} = -i[H_{\text{eff}}, \rho] + \kappa(D[a_+] + D[a_-] + D[\sigma])\rho \\
+ (n_{th} + 1)\kappa_b D[b]\rho + n_{th}\kappa_b D[b^\dagger]\rho,
\]  

where \(\rho\) is the density matrix of the hybrid system, \(D[\sigma] \rho = 2\rho o^\dagger - o^\dagger o \rho - \rho o o^\dagger\), and \(n_{th}\) is the thermal phonon population. We assume that the average particle numbers of photons (magnons) in thermal equilibrium are zero because of their high frequencies.

We truncate the Fock space up to \(|5\rangle\) for modes \(a_+\) and \(b\). Based on the subspace consisting of the two-level atom and the modes \(a_+\) and \(b\), we numerically solve Eq. (10) and calculate the second-order correlation function of mode \(a_+\) and \(b\). In Fig. 3(a), we plot \(g_{a_+}^2(0)\) with an analytical solution of Eqs. (7) and numerical results of Eq. (10), respectively. We see that they agree well, which means that we can understand the second order correlation with the analytic analysis. In order to make clear the relation between the mechanism of blockade and the probability distribution, we define the function \(y(N) = \log_{10} \frac{P(N)}{P_{\text{Poission}}(N)}\), where \(P(N)\) is the probability in \(|N\rangle\), and \(P_{\text{Poission}}(N)\) is Poissonian distribution; thus the value of \(y\) reveals the relative difference between the population and Poissonian distribution. In Fig. 3(c) to 3(f), we plot \(y(N)\) corresponding to point A to D respectively. If \(y\) is positive, population at \(N\) excitation is higher than Poissonian distribution, or otherwise it is lower than the Poissonian distribution.

For the mark point A in Fig. 3(a), \(\Delta = \beta_1, \lambda_{1+} = 0\), which means the single excitation resonance. Then \(|1_-\rangle\) can be easily populated (for the symmetry point of A, \(\Delta = -\beta_1, \lambda_{1+} = 0\), then \(|1_+\rangle\) is easy to be populated). Notice the expression \(|1_-\rangle\) in Eq. (A6), where there
is only one excitation in modes $a_\pm$ and $b$, so we can see strong blockade in $a_+$, $a_-$, and $b$ modes under the same condition. Meanwhile the average numbers $n_{a_+}$, $n_{a_-}$, and $n_b$ reach their local maximum of $n_{a_+}(0)$[see Fig. 3(b)]. All of the probability at $N > 1$ is less than Poissonian distribution due to the resonance mechanism, shown in Fig. 3(c).

For the mark point B in Fig. 3(a), $g^2_{a_+}(0)$ achieves a local minimum value where the real part of numerator of $C_{g200}$ is zero. By observing Fig. 1(b), the two jumps $|e100\rangle \rightarrow |g200\rangle$ and $|g111\rangle \rightarrow |g200\rangle$ destructively interfere each other, such that the population in $|g200\rangle$ is low, so the mode $a_+$ is blockade. That is the so-called UB. However, under this condition, the $a_-$ mode is super-Poissonian because there is a population in $|g111\rangle$, resulting in population $|g022\rangle$. By observing Fig. 3 (d), the destructive interference only decreases the probability in $N = 2$ for the mode $a_+$, while for the mode $a_-$ the probability for $N > 1$ is higher than Poissonian distribution. This result indicates that the destructive interference cannot offer blockade for both supermodes $a_+$ and $a_-$.

For the point C in Fig. 3(a), $g^2_{a_+}(0)$ achieves a local minimum value. As one can observe from Eq. (B2), the requirement for $C_{g022} \approx 0$ is the same as that for $C_{g111} \approx 0$, if $\langle \eta, \eta_a \rangle \neq 0$. As seen in Fig. 1(b), there are two jumps $|g200\rangle \rightarrow |g111\rangle$ and $|e011\rangle \rightarrow |g111\rangle$. Their destructive interference results in blockade in mode $a_-$. Meanwhile, there is a population in the state $|g200\rangle$, which means the super-Poissonian in mode $a_+$. Correspondingly, in Fig. 3(e), the population of mode $a_+$ is still higher than the Poissonian distribution, while for the mode $a_-$, the destructive interference only decrease the probability in only $N = 2$. This result is similar to what we have pointed in the analysis of point B, i.e., the destructive interference cannot offer us a simultaneous blockade in supermode $a_+$ and $a_-$.

For the point D, in Fig. 3(a), $\Delta = 0$, $\lambda_{10} = \lambda_{20} = 0$, which means that the single excitation resonance $|1_0\rangle$ and double resonant excitation $|2_0\rangle$ are both satisfied. Observing Eq. (A6), the resonance between state $|1_0\rangle$ and state $|0\rangle$ can lead to the populations in the states $|g011\rangle$ and $|e000\rangle$. Likewise, the population in $|2_0\rangle$ means that the states $|g200\rangle$ and $|e011\rangle$ are easily populated, while the state $|g022\rangle$ is not so easily populated because of the mutual cancellation between $\eta$ and $\eta_a$ [the factor $\frac{n^2_{\eta} - n^2_{\eta_a}}{\sqrt{2}n_{\eta}A_{\eta}}$, is smaller than $\frac{4n}{\beta_1}$, see Eq. (A6)]. Therefore, the mode $a_+$ will be strong super-Poissonian, and the mode $a_-$ is sub-Poissonian. The results are corresponding to Fig. 3(f), where the population for mode $a_+$ is higher than the Poissonian distribution, and the probabilities distribution for mode $a_-$ are less than Poissonian.

As we have mentioned before, for mode $b$, $g^2_b(0)$ should not be calculated from an analytical solution Eq. (5). We directly calculate $g^2_b(0)$ with the master equation (10), shown in Fig. 3(a). We see that around point A, we can also achieve blockade in mode $b$. Therefore, under single excitation resonance, all of the modes $a_+, a_-$, and $b$ exhibit the blockade phenomenon. In addition, the parameters, in Fig. 3, $\frac{g^2_{\omega_{\kappa}}}{\omega_{\kappa}} = 9/16 < 1$ means that weak photon nonlinearity from radiation pressure in an optomechanical system could generate a photon, magnon, and phonon blockade, in our system. But the single-photon optomechanical coupling $g$ is still larger than the damping rate $\kappa$. We will show that the single excitation resonant does not require $g > \kappa$; that is to say, even under the condition $g < \kappa$, we still can obtain the simultaneous blockade for the three modes.

In Fig. 4, we plot $g^2_a(0)$ (solid) and $g^2_{m}(0)$ (dots), where, obviously, they are the same and agree well with Eq. (9). As we have analyzed before, the blockade of mode $a_+$ means $|C_{g200}|^2 \approx 0$, and the $a_-$ mode blockade corresponds to $|C_{g022}|^2 \approx 0$ (also $|C_{g111}|^2 \approx 0$). When both $a_+$ and $a_-$ modes are a blockade, from the expression Eq (9), the photon $a$ and the magnon $m$ are both blockade; therefore at point A [see Fig. 4(a)], the optical mode and magnetic mode are both a blockade. However, around $\Delta = 0$ (point D), the statistical property of $g^2_{a_+}(0)$ is different from that of $g^2_{a_-}(0)$, $g^2_{a_+}(0)$ still showing sub-Poissonian. From Eqs. (8), (9), and (B2), we obtain

$$g^2_a(0) \approx \frac{1}{F_1^2} g^2_{a_+}(0) + \left(\frac{2}{F_1^2} - 1\right) g^2_{a_-}(0), \quad (11)$$

where $F_1 = \frac{\Delta}{\eta}^2 + 1$, $F_2 = \frac{\Delta}{\eta}^2 + 1$, and $\tilde{\Delta} = \Delta - i\kappa$. So, when $\Delta$ is extremely small, $F_1 \rightarrow 1$ and $F_2 \rightarrow \infty$, then $g^2_{a_+}(0)$ is dominated by $g^2_{a_+}(0)$. Therefore, we can observe a sub-Poissonian around $\Delta = 0$ regime. Comparing the value of $g^2_{a_+}(0)$ around point B with that around point A, we see that the sub-Poissonian resulting from destructive interference (point B) does not exist, but the blockade resulting from single excitation resonance (point A) still exists.

To further characterize the blockade of modes $a_\pm$, $b$, $a$, and $m$, choosing a single excitation resonance condition $\Delta = \beta_i$, we plot a second-order delay correlation function defined by $g^{(2)}(\tau) = \frac{(\langle \eta(t)\eta(t + \tau)\rangle - \langle \eta(t)\rangle^2)}{(\langle \eta(t)\rangle^2)}$ in Fig. 5. $g^{(2)}(\tau) \leq g^2_a(0)$ is called bunching, and $g^{(2)}(\tau) > g^2_a(0)$ is called antibunching which is also the quantum signature [48]. Meanwhile, $g^{(2)}(\tau)$ is proportional...
to the condition probability for detecting a second photon (magnon, phonon) at \( t = \tau \), given that a photon (magnon, phonon) has been detected earlier at \( t = 0 \) [49]. Observing Figs. 5(a) and (b), because of the single excitation resonance, the time-delay correction functions for supermodes \( a_\pm \) and optical, magnetic, or mechanical mode are all antibunching even in the weak photon nonlinear region. \( g_m^{(2)}(\tau) \) agrees well with \( g_a^{(2)}(\tau) \) which is just like the equal-time second-order correlation function. Comparing Figs. 5(a) and (b), the time-delay correction function of supermodes \( a_\pm \) has no quick oscillations, but that of the optical and magnetic mode exhibits quick oscillations. The quick local oscillations in the time-delay second-order function for optical and magnetic mode results from the interference between supermodes \( a_+ \) and \( a_- \) and the frequency of mechanical mode \( b \) [39].

We now investigate the second-order correlation function \( g_\alpha^{(2)} \) affected by the coupling strength \( \eta_a \) shown in Fig. 6. From Figs. 6(a) and (b), we observe that with the increasing of \( \eta_a \), the low value \( \log_{10} g_a^{(2)}(0) \) points (single excitation resonance) in terms of \( \Delta \) are increased, which is because the resonant condition \( \Delta = \beta_1 \) is increased with \( \eta_a \). In Fig. 6(b), interestingly, the minimum value of \( g_a^{2}\) is not monotonous decreasing with increasing \( \eta_a \). When \( \eta_a \approx 17.7 \kappa \), \( g_a^{2}\) is abnormal where the effect of the single excitation resonance does not result in a blockade as in the other case. See the mark point P in Fig. 6(a), where there is a cross where the \( \Delta = \beta_2 \) (the single excitation resonance) and \( \Delta = \beta_2/2 \) (two excitation resonance) are both satisfied, so, \( g_a^{2}\) can not show a blockade. Except for the cross point, the larger value of \( \eta_a \), the better the blockade.

We now show that it is possible to generate a photon, magnon, and phonon blockade without a strong optomechanical coupling coefficient. In Fig. 7, both \( \frac{g^2}{\omega_a^2} \ll 1 \) and \( g < \kappa \) are satisfied, and we plot the equal-time second-order correlation function for modes \( b, a, \) and \( m \). In Fig. 7(a), due to single excitation resonance, the strong sub-Poissonian for modes \( a, m, \) and \( b \) can be observed, while the destructive interference resulting in a blockade is not observed in the weak coupling regime. Here, although the single-photon optomechanical coupling is small, the large atom-photon interaction \( g_a \) makes \( \beta_1 \) larger than \( \kappa \), which ensures the blockade of the photon, magnon, and phonon. We can understand it from Eq. (2). In order to keep single excitation, the denominator of \( C_{g100} (C_{g011}, C_{e00}) \) should be as low as possible, i.e., \( \min |\Delta^2 - \eta^2 - \eta_a^2| \), then we deduce the condition \( \Delta = \sqrt{\eta^2 + \eta_a^2} \). Therefore, even \( \eta < \kappa \), the relative large value of \( \eta_a \) still can make \( \eta^2 + \eta_a^2 > \kappa^2 \), and then the single excitation will dominate the wave function, and the blockade can be obtained. We can conclude that the single excitation resonance can result in a multimode blockade even in a weak optomechanical coupling region while the destructive interference can not offer us multimode antibunching.

In Fig. 7(b), we plot the equal-time second-order correlation functions of a photon, magnon, and phonon affected by thermal phonon number. As we can observe the blockade of a photon and magnon under a weak coupling regime still exists after considering the thermal environment of a phonon, but the phonon blockade disappears.
and the correlation function approaches 2 with increasing $n_{th}$. When the thermal phonon population is taken into account, the state of the system truncated the in few excitation subspace can be expressed as mixed a state of $|\psi_n\rangle$ [25] where

$$
|\psi_n\rangle = C_{g00n}|g00n\rangle + C_{g10n}|g10n\rangle + C_{g0n+1}|g01n+1\rangle \\
+ C_{c00n}|e00n\rangle + C_{c20n}|e20n\rangle + C_{g11n+1}|g11n+1\rangle \\
+ C_{c10n}|e10n\rangle + C_{c02n+2}|e02n+2\rangle \\
+ C_{c01n+1}|e01n+1\rangle.
$$

(12)

Because of the three-partite interaction $a_a, a_a^\dagger, b^\dagger + h.c.$, the thermal phonon cannot be converted into a photon and magnon. From Eq. (12), although the thermal phonon can be in the state $|n\rangle$, the states of photon and magnon still can be in $|0\rangle$ or $|1\rangle$, which means the blockade of modes $a$ and $n$ still exists, but phonon blockade will be destroyed ($n > 1$) [42, 50–52], and the correlation function of the phonon will close to the that of thermal field. But, the blockade of the photon and magnon is affected slightly by the thermal environment because of the change in the single excitation resonance for $|\psi_n\rangle$ [25]. Therefore, to generate simultaneous blockade of a photon, phonon, and magnon, the small thermal phonon population is necessary.

IV. DISCUSSION AND CONCLUSION

When the single excitation resonance condition is satisfied, The simultaneous blockade of a photon, phonon and magnon can offer us some potential applications. The usual hybrid system mainly contains two different physical systems, but the quantum internet may require more complex quantum information processing, like the processing and storing of information while simultaneously updating the information in a quantum information circuit and network [43]. The simultaneous blockade of a multimode field could be used in this process and be more powerful than the usual single mode blockade. If we realize the single excitation, from Eq. (8), the photon and magnon will be a in Bell state $1/\sqrt{2}|10\rangle \pm |01\rangle$, which is useful in quantum information processing.

From Fig. 3 to Fig. 7, the parameter $G_m$ is seemingly not important in numerical simulation, but we do need strong magnon-phonon coupling, because we require the condition $G_m \gg \{\eta, \eta_a\}$ to achieve the effective Hamiltonian, and the three-partite interaction is true only under this condition. Recently, strong and even ultrastrong coupling between photons and magnons at microwave frequencies, using of a YIG sphere, has been reported [27, 28]. For instance, in Ref. [27], the magnon-phonon coupling strength was achieved as high as $g = 2\pi \times 2.5\text{GHz}$, and dissipation rates of the microwave photon and the magnon resonance are $\kappa_c = 2\pi \times 33\text{MHz}$ and $\kappa_m = 2\pi \times 15\text{MHz}$, respectively. Currently, the optomechanical single-photon strong-coupling condition $g > \kappa$ is still a challenge. Most of the experiments of the optomechanical system are still within the single-photon weak coupling regime [17, 53, 54]. In our scheme, the three-partite interaction is results from the optomechanical interaction, but the single-photon strong coupling is not necessary.

In this paper, we put forward a scheme to generate a photon, phonon and magnon blockade in a hybrid microwave optomechanical-magnetic system. By introducing a two-level atom interacting with the cavity field, we carefully compare the blockade resulting from destructive interference and that resulting from single excitation resonance. We find that the blockade resulting from single excitation resonance is much better than that resulting from destructive interference. Most importantly, under the same detuning condition, the photon, phonon and magnon can be blockade simultaneously. Furthermore, we find that the phonon blockade is easy to be destroyed by thermal excitation, while the blockade of the photon and magnon are affected slightly by the thermal environment. To generate simultaneous blockade of the photon, phonon and magnon, the small thermal phonon population is necessary.

In our system, the multipartite interaction results from optomechanical coupling, which is the key factor to obtain the simultaneous blockade of the photon, phonon, magnon. However, the single excitation is the condition of the simultaneous blockade, and the single-photon strong optomechanical coupling condition is not required. Therefore, the present scheme is feasible in experiment, which is a guideline for hybrid optomechanical-magnetic experiments nearing the regime of single-photon nonlinearity, and for potential quantum information processing applications with photons, magnons and phonons.

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Appendix A: The derivation of an effective Hamiltonian and its eigenstates

In this appendix, we give the detailed derivation for Hamiltonian (3). In the frame rotating with $H_0 = \omega_L(a^\dagger a + \sigma^\dagger \sigma + m^\dagger m)$, the Hamiltonian (1) can be written as

$$
H = \delta_c a^\dagger a + \delta_m m^\dagger m + G_m (a^\dagger m + am^\dagger) + \omega_b b^\dagger b + ga^\dagger a(b^\dagger + b) \\
+ \Delta_a \sigma^\dagger \sigma + g_o (\sigma a^\dagger + \sigma^\dagger a) + \Omega_e (\sigma + \sigma^\dagger),
$$

(A1)

with $\delta_{c(m)} = \omega_{c(m)} - \omega_L$. For simplicity, we assume $\omega_m = \omega_c$, then $\delta_c = \delta_m = \delta$. We diagonalize the Hami-
tonian $H_0 = \delta(a^\dagger a + m^\dagger m) + G_m(a^\dagger m + am^\dagger)$ by introducing $a_{±} = \frac{1}{\sqrt{2}}(a ± m)$, then $H_0 = (\delta + G_m) a_{+} a_{+} + (\delta - G_m) a_{-} a_{-}$. Choosing $H_f = \Delta a_{+}^\dagger a_{+} + (\Delta - 2G_m) a_{-} a_{-} + \omega_b b^\dagger b + \Delta_o \sigma^I \sigma$ and assuming $\omega_b = 2G_m$, $\Delta = \Delta_o$, we switch into the interaction picture and obtain

$$H_I = \eta(a_{+}^\dagger a_{+} + a_{-}^\dagger a_{-} - a_{+}^\dagger a_{+} e^{i2G_m t} - a_{-}^\dagger a_{-} e^{-i2G_m t}) + \eta a_{+}^\dagger \sigma^I a_{+}^\dagger \sigma^I + \eta a_{-}^\dagger \sigma^I a_{-}^\dagger \sigma^I,$$

where $\eta = \frac{2a_{\omega}}{\sqrt{2}}, \eta = \frac{\eta_o}{2}$. The detuning $\Delta$ can be arbitrary value. Considering $G_m \gg \{\eta, \eta_o\}$, we take rotating wave approximation and ignore high frequency terms, then the Hamiltonian could be written as

$$H_I = -\eta(a_{+}^\dagger a_{-} b + a_{+}^\dagger a_{+} b^\dagger) + \eta a_{+}^\dagger \sigma^I a_{+}^\dagger \sigma^I + \Omega_c (\sigma^I e^{i\Delta t} + \sigma^I e^{-i\Delta t}).$$

We would like to rewrite the Hamiltonian into time-independent form by switching back into original picture, then we have

$$H_{eff} = \Delta a_{+}^\dagger a_{+} + (\Delta - 2G_m) a_{-}^\dagger a_{-} + \omega_b b^\dagger b + \Delta_o \sigma^I \sigma - g_a/2(a_{+}^\dagger a_{-} b + a_{+}^\dagger a_{+} b^\dagger)
+ g/\sqrt{2}(a_{+}^\dagger \sigma^I a_{+}^\dagger \sigma^I) + \Omega_c (\sigma^I + \sigma^I).$$

It is exactly the effective Hamiltonian (3).

In the limit of a weak driving field, we temporarily forget the pumping of the atom and derive the eigenstates and eigenvalues of $H_{eff}$ in the few-photon subspace, yielding corresponding eigenstates are

$$|0\rangle = |g000\rangle,$$

$$|1_0\rangle = \frac{1}{\beta_1}(|\eta_o| |g011\rangle + |\eta_o| e^{000\rangle}),$$

$$|1_-\rangle = \frac{1}{\sqrt{2}}(|g100\rangle + \eta_1 |g011\rangle - \eta_1 |e000\rangle),$$

$$|1_+\rangle = \frac{1}{\sqrt{2}}(|g100\rangle - \eta_1 |g011\rangle + \eta_1 |e000\rangle),$$

$$|2_0\rangle = \frac{1}{A_1}(|g200\rangle + \eta_2^2 - \eta_2^2 |g022\rangle + \sqrt{2} \eta_2 |e011\rangle),$$

$$|2_+\rangle = \frac{1}{A_2}(|d11|g200\rangle + d12|g111\rangle + d13|e100\rangle
+ d14|g022\rangle + |e011\rangle),$$

$$|2_-\rangle = \frac{1}{A_2}(|d11|g200\rangle - d12|g111\rangle - d13|e100\rangle
+ d14|g022\rangle + |e011\rangle),$$

$$|2_2\rangle = \frac{1}{A_3}(|21|g200\rangle + 22|g111\rangle + 23|e100\rangle
+ 24|g022\rangle + |e011\rangle),$$

with the coefficients: $A_1 = \frac{\sqrt{\beta_1^2 + 2\eta_2^2}}{\sqrt{\eta_2^2}}$, $d_{11} = \frac{\beta_2(D - 5\eta_2^2 - \eta_2^2)}{2\eta_o M_1}$, $d_{12} = \frac{\beta_2(-5\beta_2^2 + D)}{2\eta_o M_1}$, $d_{13} = \frac{\beta_2(\beta_2^2 - D)}{2\eta_o M_1}$, $d_{14} = \frac{\beta_2(5\beta_2^2 + D)}{\eta_o M_2}$, $d_{21} = \frac{-\beta_2(D + 5\eta_2^2 + \eta_2^2)}{2\eta_o M_2}$, $d_{22} = \frac{-\beta_2(\beta_2^2 + D)}{2\eta_o M_2}$, $d_{23} = \frac{\beta_2(\beta_2^2 + D)}{\eta_o M_2}$, $d_{24} = \frac{-\eta(D + 5\beta_2^2)}{\eta_o M_2}$, $M_1 = 3\beta_2^2 - D_1$, $M_2 = 3\beta_2^2 + D$, and $A_{23} = \sqrt{|d_{12}^2| + |d_{12}^2| + |d_{12}^2| + |d_{12}^2| + 1}.$

**Appendix B: The dynamic equation and steady states solution**

In this appendix, we derive probability amplitude for a steady state. Substitute the $|\psi\rangle$ expressed by Eq. (5) into the Schrödinger equation:

$$i \frac{\partial}{\partial t} |\psi\rangle = H_{eff} |\psi\rangle,$$
and we obtain the differential equations as

\begin{align}
C_{g000} &= 0, \\
i\dot{C}_{g100} &= \Delta C_{g100} - \eta C_{g111} + \eta_a C_{e000}, \\
i\dot{C}_{g101} &= -\eta C_{g100} + \Delta C_{g111}, \\
i\dot{C}_{g111} &= \eta_a C_{g100} + 2\Delta C_{g100} - \eta C_{e011}, \\
i\dot{C}_{g200} &= 2\Delta C_{g200} - 2\eta C_{g111} + \sqrt{2}\eta_a C_{g022}, \\
i\dot{C}_{g111} &= -2\eta C_{g111} + 2\Delta C_{g022}, \\
i\dot{C}_{e011} &= \Omega_a C_{g111} - \eta C_{e100} + 2\Delta C_{e011},
\end{align}

where for simplicity, we set \( \kappa_+ = \kappa_- = \kappa_e = \kappa \), \( \Delta = \Delta - i\kappa \) and temporarily ignore the small mechanical decay rate \( \kappa_b \ll \kappa \), and the jumping from high level to low level is ignored as it is done in Ref. [25].

The steady-state solution of Eq. (B1) is derived as

\begin{align}
C_{g000} &= 1, \\
C_{g100} &= \frac{\eta_a\Omega_e}{\Delta^2 - \eta_a^2 - \eta^2}, \\
C_{g101} &= \frac{\eta_a\Omega_e}{\Delta(\Delta^2 - \eta_a^2 - \eta^2)}, \\
C_{g111} &= \frac{\eta_a^2(\Delta^2 - \eta_a^2 + \eta^2)\Omega_e}{\Delta B}, \\
C_{g200} &= \frac{\eta_a^2(4\Delta^4 + \Delta^2(\eta_a^2 - \eta^2)) - 2\eta^4\Omega_e^2}{\sqrt{2}\Delta^2 B}, \\
C_{e100} &= \frac{\eta_a(4\Delta^4 - \Delta^2(\eta_a^2 + 4\eta^2) + \eta^2\eta_a^2 - 3\eta^4)\Omega_e^2}{\Delta B}, \\
C_{g022} &= \frac{\eta_a^2(5\Delta^2 - \eta_a^2 + \eta^2)\Omega_e^2}{\Delta^2 B}, \\
C_{e111} &= \frac{-\eta_a\eta(6\Delta^4 - \Delta^2(\eta_a^2 + 9\eta^2) + 2\eta^2\eta_a^2)\Omega_e^2}{\Delta^2 B},
\end{align}

where \( B = \frac{1}{2}(\Delta^2 - \beta_1^2)(4\Delta^2 - \beta_2^2)(4\Delta^2 - \beta_3^2) \).

**Appendix C: The deduction of the relations between two bases**

In this Appendix, we provide the certification of Eq. (8). We define the Fock basis of the supermode \( a_{\pm} \) as \( |n_+,n_-\rangle_d \) and the bare modes of \( a \) and \( m \) as \( |nm\rangle \). For the supermodes, we have

\begin{align}
a_{+}|n_+,n_-\rangle_d &= \sqrt{n_+ + 1}|n_+ + 1n_-\rangle_d, \\
a_{-}|n_+,n_-\rangle_d &= \sqrt{n_- + 1}|n_+ + 1n_-\rangle_d, \\
a_{+}|n_+,n_-\rangle_d &= \sqrt{n_+ - 1}|n_+ - 1n_-\rangle_d, \\
a_{-}|n_+,n_-\rangle_d &= \sqrt{n_- - 1}|n_+ - 1n_-\rangle_d.
\end{align}

Specifically, for \( n_+,n_- = 0 \), we have the relation of the annihilation operator

\( a_{\pm}|00\rangle_d = 0. \)

Since \( a_{\pm} = \frac{1}{\sqrt{2}}(a \pm m) \), we have \( a|00\rangle_d = 0, m|00\rangle_d = 0. \) We expand the state \( |00\rangle_d \) by using the bare basis \( |n,m\rangle \) of mode \( a \) and \( m \) as

\( |00\rangle_d = \sum_{n,m} C_{nm}|nm\rangle \)

Thus

\( C_{nm} = \langle nm|00\rangle_d, \)

For example \( C_{10} = \langle 10|00\rangle_d = \langle 00|a|00\rangle_d = 0. \) Finally, we have

\( |00\rangle_d = |00\rangle. \)

In addition, we can write \( a_{+}|00\rangle_d = |10\rangle_d, i.e., 1/\sqrt{2}(a^\dagger + m^\dagger)|00\rangle = 1/\sqrt{2}(|10\rangle + |01\rangle) \). Then, we can obtain

\( |10\rangle_d = \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle). \)

Similarly, we can have

\( |01\rangle_d = \frac{1}{\sqrt{2}}(|10\rangle - |01\rangle). \)

Taking action \( a_{+} = 1/\sqrt{2}(a_{+}^\dagger + m^\dagger) \) further on the right and left sides of Eq. (C4) and Eq. (C5), we can reach the other relations.
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