Disentanglement as particles separate

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Abstract

Disentanglement refers to decoherence that destroys the quantum interference terms between particles as they separate. This process reduces the pure isotropic entangled EPR state to a mixed anisotropic state. Averaging over the ensemble of states leads to correlation between separated particles that satisfies Bell’s Inequalities. Applying disentanglement to EPR pairs of photons shows that entanglement is characterized by various two-particle symmetry properties. These symmetry properties are destroyed by disentanglement but photon helicity is conserved. This is sufficient to account for the correlations needed to resolve the EPR paradox. Apart from a numerical factor, the functional form for the correlation due to entanglement and disentanglement is identical, thereby making it difficult to distinguish between the two in the current experiments.

I. Introduction

Without attempting to summarize the long, controversial and well known history of the interpretation of quantum mechanic (QM) since the famous work of Einstein, Podolsky and Rosen¹ (EPR), it is sufficient to state that the EPR paradox raised fundamental questions about entanglement in separated particles. Essential to their work is the concept of Einstein locality, where it is assumed that once separated, one particle cannot influence its distant and separated partner. This notion was put to the test in 1964 with the equally famous inequalities of Bell² (BI) that show that the correlation between two separated fermions in a singlet state violate BI under some experimental arrangements. Since Einstein locality was assumed in the derivation of BI, it is usual to point to this approximation to conclude that QM is a non-local theory³.

Although in this paper, the non-locality is not questioned, it is re-interpreted. It is concluded that the non-local aspect of QM can lead to no conclusions that cannot otherwise be deduced from a local theory of QM. This is based upon a study of the correlations that can exist between separated EPR pairs⁴. In particular such correlations are a result of conservation laws and symmetry properties between the two particles rather than a result of interactions that depend on some mediating force. BI can be violated between entangled EPR pairs because of correlation whose origin lies in the quantum interference terms. Once these interference terms have undergone decoherence, the remaining correlations between the particles obeys BI.

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Although it has been concluded that QM and the theory of special relativity live in "peaceful coexistence" it is nonetheless troubling that non-locality appears to permit instantaneous influence between separated EPR pairs by the use of quantum of EPR channels. Once it is understood that the correlation due to entanglement in separated particles is simply due to symmetry, the difficulties associated with instantaneous influence over space-like distances evaporate since knowledge of the state of one particle permits one to deduce the state of its distant partner without resorting to quantum channels.

The main purpose of this paper is to discuss the concept of disentanglement, its relationship to entanglement and some consequences of disentanglement. Both terms were introduced by Schrödinger who stated, in reference to the EPR paradox:

“It seems worth noticing that the paradox could be avoided by a very simple assumption, namely if the situation after separating were described by the expression (12), but with the additional statement that the knowledge of the phase relations between the complex constants $a_k$ has been entirely lost in consequence of the process of separation. This would mean that not only the parts, but the whole system, would be in the situation of a mixture, not of a pure state. It would not preclude the possibility of determining the state of the first system by suitable measurement in the second one or vice versa. But it would utterly eliminate the experimenters influence on the state of that system which he does not touch.”

For completeness Schrödinger’s Eq.(12) is

$$\Psi(x, y) = \sum_k a_k g_k(x) f_k(y)$$

In the above quote, the words, “process of separation” were not underscored in the original paper by Schrödinger, but they are here to emphasize that he wished to distinguish the process of separation from that of measurement. According to the Copenhagen Interpretation of QM (CI), it is usually assumed that the measurement process causes a complete reduction or decoherence of the state. In contrast, the process of disentanglement is a partial reduction that destroys the interference terms between particles, leaving intact the quantum states of the separated particles that can still certainly be in a superposition of those states available to them alone.

In the following the concept of disentanglement is described by first treating a pair of spins of magnitude $\frac{1}{2}$ each. Following this, the correlations between an EPR spin pair are evaluated using both entanglement and disentanglement. The results are then compared and discussed with respect to the EPR paradox and the Aspect-type experiments. In the process, the symmetry properties of a pair of entangled photons are reviewed.

II. Disentanglement
Although it is not ruled out that entanglement can exist between two separated particles, such as, for example using the technique of parametric down-conversion or for natural processes such as positronium annihilation, it is argued that the correlation due to entanglement is a consequence of symmetry properties that are established during the process of separation. Such symmetries permit correlations of the state of the one entangled particles to be used to deduce the state of its distant partner without the need for any mediating force. In contrast, it is more likely that any interaction specific to one or the other particle will disrupt the entanglement by destroying the symmetries between them. Although the act of measurement would do this, it is also possible, as envisioned by Schrödinger in the above quote of his that it can happen during the process of separation. We therefore define disentanglement as the destruction of quantum interference terms between particles as they separate from each other.

This definition of disentanglement can be applied to the EPR density operator for a pair of spins that display two states each to give a total of four,

\[
|+\rangle_z^1 |+\rangle_z^2 ; |+\rangle_z^1 |-\rangle_z^2 ; |-\rangle_z^1 -\rangle_z^2 ;
\]

where the axis of spin quantization is taken to be in the \(z\) direction, so that the Pauli spin matrix is an eigenoperator on these states,

\[
\sigma_z^i |\pm\rangle_z^i = \pm |\pm\rangle_z^i
\]

These can be transformed into the four Bell states,

\[
|\Phi_{12}^z\rangle = \frac{1}{\sqrt{2}} (|+\rangle_z^1 |+\rangle_z^2 \pm |\rangle_z^1 |-\rangle_z^2 )
\]

\[
|\Psi_{12}^z\rangle = \frac{1}{\sqrt{2}} (|+\rangle_z^1 |-\rangle_z^2 \pm |\rangle_z^1 |-\rangle_z^2 )
\]

The singlet EPR density operator is defined by \( \rho_{EPR}^{12} = |\Psi_{12}^z\rangle \langle \Psi_{12}^z |. \)

Disentanglement requires performing a (non-unitary) partial reduction of the state as the two particles separate while maintaining conservation of angular momentum between them,

\[
\rho_z^1 (+) \equiv Tr_z \left[ -\langle -\rangle_z | -\rangle_z^2 \rho_{EPR}^{12} \right]
\]

\[
\rho_z^2 (-) \equiv Tr_z \left[ +\langle +\rangle_z | +\rangle_z^2 \rho_{EPR}^{12} \right]
\]
These single particle density operators for the two separated EPR photons conserve angular momentum, but all phase relations between the two are lost. An alternate way of expressing the disentanglement process is to represent the EPR density operator in the basis states, Eq.(2.1)

\[
\rho_{\text{EPR}}^{12} = |\psi_{12}^+\rangle\langle\psi_{12}^+| = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}
\]  

(2.7)

and assume that the off-diagonal elements undergo decoherence as the particles separate,

\[
\rho_{\text{EPR}}^{12} \xrightarrow{\text{decoherence}} \rho_{\text{disentangled},z}^{12} = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}
\]  

(2.8)

Using the definitions in Eqs.(2.5) and (2.6) identifies the disentangled density operator as describing a maximally mixed anisotropic state of zero total angular momentum,

\[
\rho_{\text{disentangled},z}^{12} = \frac{1}{2} \left[ \rho_z^1 (+) \rho_z^2 (-) + \rho_z^1 (-) \rho_z^2 (+) \right]
\]

(2.9)

Finally, we have assumed so far that during disentanglement, the axis of quantization is defined along the $z$ direction. However there is nothing special about this direction as particles are emitted from a source. Rather as each EPR pair disentangles, it does so into any random direction $\hat{P}$ with the only proviso that the same axis of quantization exists for both particles from the same EPR pair. Whereas the entangled pair can admit only four states, Eqs.(2.3) and (2.4), upon disentanglement, an infinite number of possible states arises. Therefore an ensemble is expected to result from the process of disentanglement, with each pair being described by density operator, $\rho_{\text{disentangled},\hat{P}}^{12}$. The above equations can be thus modified for an ensemble of states that are quantized along the axis of propagation, $\hat{P}$, so that $z \rightarrow \hat{P}$ in Eqs.(2.1) to (2.9).

III. Disentanglement and the EPR paradox.

In this section, using the density operator for both an entangled EPR pair, $\rho_{\text{EPR}}^{12}$, and for a disentangled EPR pair $\rho_{\text{disentangled},\hat{P}}^{12}$, the various single spin probabilities are calculated. If the particles are disentangled, then the phase coherence that existed due to entanglement is destroyed. This results in an ensemble of particles that all have the same axis of spin quantization. Since this axis is random, the two spin states are defined by
The angles $\theta, \phi$ orient $\hat{P}$. In order to emphasize the phase decoherence between the particle moving left from that of its partner moving right, the azimuthal angles in Eq.(3.1) are maintained different, $\phi_1 \neq \phi_2$.

To evaluate the consequences of disentanglement for the EPR paradox, a singlet state is disentangled so that one spin moving left is in a state with opposite orientation from its partner moving right. That is, assuming that the spin moving left towards a Stern-Gerlach detector is in the $|+\rangle_P^1$ state, then to conserve angular momentum, its partner must be in the $|+\rangle_P^2$ state. The density operator for this case is therefore given by,

$$\rho_{\text{disentangled}, P}^{12} = \rho_P^1(+)\rho_P^2(-)$$ (3.2)

The usual experimental arrangement consists of a Stern-Gerlach (or polarizing) filter on the left oriented in the direction of the unit vector, $a$, while that on the right is oriented in the direction $b$. An EPR source is placed equidistant between the two filters. The probabilities for detecting a spin being in either the $|+\rangle_a$ or $|\rangle_a$ state if it were initially in the $|+\rangle_P^1$ state are:

$$P_+^1(+, a) \equiv \left|\langle + | + \rangle_a^1\right|^2 = \cos^2\left(\frac{\theta_a}{2}\right)$$ and $$P_-^1(-, a) \equiv \left|\langle + | - \rangle_a^1\right|^2 = \sin^2\left(\frac{\theta_a}{2}\right)$$ (3.3)

Likewise, for its partner moving left towards the detector oriented in the $b$ direction, the probabilities are

$$P_+^2(+, b) \equiv \left|\langle + | + \rangle_b^2\right|^2 = \sin^2\left(\frac{\theta_b}{2}\right)$$ and $$P_-^2(-, b) \equiv \left|\langle + | - \rangle_b^2\right|^2 = \cos^2\left(\frac{\theta_b}{2}\right)$$ (3.4)

The angles are those between the direction of spin quantization axis and the direction of the magnetic fields,

$$\cos \theta_a = a \cdot \hat{P} \quad \text{and} \quad \cos \theta_b = \hat{P} \cdot b$$ (3.5)

These results are independent of the choice of phase, $\phi_1$ and $\phi_2$ in Eq.(3.1).

A pair of disentangled spins, Eqs.(3.3) and (3.4), displays the correct conservation of angular momentum no matter what orientation is chosen for the two analyzing filters. That is the angular momentum of the spin moving left is always correlated with the spin moving to the right. For this it is essential that the two spins originate from the same
EPR pair so they share the same axis of quantization \( \hat{P} \). In contrast, if the entangled isotropic state did not undergo disentanglement, it would appear that choosing the orientation of one polarizer would serve to determine the outcome at the other, distant, polarizer. In this sense, the process of disentanglement resolves the EPR paradox by virtue of the anisotropy of the state due to the common axis \( \hat{P} \) for the two. Therefore measuring the angular momentum of one spin can be used to predict the angular momentum of its distant particle due only to the correlation from conservation of angular momentum. The other correlations that characterize entanglement and which arise from the quantum interference terms, play no role in determining the probabilities in Eqs.(3.3) and (3.4).

In general, since the common quantization axis, \( \hat{P} \), is completely random, with all possible directions possible, it provides an “element of reality” for each disentangled EPR pair within the ensemble.

If the entangled EPR density operator, Eq.(2.7) is used to calculate the four probabilities in Eqs.(3.3) and (3.4), all four are zero. In order to satisfy the EPR paradox, it therefore appears necessary to assume that the entangled EPR state disentangles either at the time of separation, or some time up to the point of measurement. The disentanglement not only causes decoherence of the quantum interference terms between the two spins, but must also conserve angular momentum as expressed in Eqs.(2.5) and (2.6). In such a treatment, there is no conflict with Einstein locality.

IV. Symmetry between a separated pair of entangled EPR photons.

Since the experiments use photon pairs, rather than spin angular momentum, the above analysis must be modified. Although the mathematical treatment of a pair of entangled spins of magnitude \( \frac{1}{2} \) in a singlet state is isomorphic to that for the helicity of a pair of entangled photons, in fact the two systems are different. Apart from doubling the angles from the treatment of singlet fermion pair to obtain the photon results, it is usually assumed that the angular momentum states and the linear momentum states are not coupled. However, for photons, the direction of linear momentum, \( \kappa \), defines the quantization axis about which the left and right helicities are defined for the \( i \)th photon. These helicity states are eigenfunctions of the Pauli spin operator, \( \sigma^i_z \), with helicity eigenvalues of \( \lambda = \pm 1 \). In the following, the helicity states are associated with the spin states with the linear momentum included, \( \left| + \right>_z^i \rightarrow |\kappa, R\rangle \) and \( \left| - \right>_z^i \rightarrow |\kappa, L\rangle \).

Although a proper QM treatment of an EPR photon pair involves raising their states from the vacuum state \(^{14}\), a classical approach also shows that their trajectories reveal considerable symmetry. Figure (1) depicts this for the \( |\kappa, R\rangle \leftarrow |\kappa, L\rangle \) and \( |\kappa, L\rangle \leftarrow |\kappa, R\rangle \) states.
Figure 1. Classical illustration of a pure-state entangled pairs of photons moving from an EPR source towards the detectors. One “screw” denotes states $|\kappa, R\rangle -|\kappa, L\rangle$ and the other $|\kappa, L\rangle -|\kappa, R\rangle$ (Use the left and right hand rules). The source is not indicated so as to more clearly show the symmetry properties.

Defining the two-photon parity operator, $\Pi$, as an operation on a photon pair by inversion of the coordinates through the source, the four states transform as\textsuperscript{14,15},

$$\Pi |RR\rangle = |LL\rangle; \quad \Pi |LL\rangle = |RR\rangle;$$
$$\Pi |LR\rangle = |RL\rangle; \quad \Pi |RL\rangle = |LR\rangle;$$ \hfill (4.1)

Eigenfunctions of this parity operation are therefore: $(\Pi \psi = \pm \psi)$

**Parity +1:**
$$|RL\rangle \quad \begin{cases} |\Phi_{12}^+\rangle = \frac{1}{\sqrt{2}} (|RR\rangle + |LL\rangle) \\ |LR\rangle \end{cases}$$ \hfill (4.2)

**Parity -1:**
$$|RL\rangle \quad \begin{cases} |\Phi_{12}^-\rangle = \frac{1}{\sqrt{2}} (|RR\rangle - |LL\rangle) \\ |LR\rangle \end{cases}$$ \hfill (4.3)

A second symmetry property of the photon pair can be identified as a rotation by $\pi$ perpendicular to the direction of propagation, $R_\perp$, and passing through the source, $R_\perp \psi = \pm \psi$ obtaining (see e.g. Figure 1):

$$R_\perp |RR\rangle = |RR\rangle; \quad R_\perp |LL\rangle = |LL\rangle;$$
$$R_\perp |LR\rangle = |RL\rangle; \quad R_\perp |RL\rangle = |LR\rangle;$$ \hfill (4.4)

Combining the symmetry properties from both parity, $\Pi$ and rotation, $R_\perp$, shows that the four Bell states for photons, $|\Phi_{12}^\pm\rangle; |\Psi_{12}^\pm\rangle$, are simultaneously eigenfuntions of the two:

$$|\Phi_{12}^+\rangle = \frac{1}{\sqrt{2}} (|RR\rangle + |LL\rangle); \text{even parity}$$
$$|\Psi_{12}^+\rangle = \frac{1}{\sqrt{2}} (|RL\rangle + |LR\rangle); \text{even parity}$$
$$|\Phi_{12}^-\rangle = \frac{1}{\sqrt{2}} (|RR\rangle - |LL\rangle); \text{odd parity}$$ \hfill (4.5)
These two discrete symmetry operators, \( \Pi \) and \( R_\perp \), commute. For EPR photon pairs we are interested in the singlet state \( |\Psi_{12}^-\rangle \), which is even to parity and odd to the rotation. The singlet state is also isotropic and has total helicity of zero. Therefore as an entangled pair of photons separate the state is characterized by the symmetry properties of \( \Pi \) and \( R_\perp \) as well as by conservation of their total helicity. As seen from figure 2, if the relative phases of the two photons are not maintained, then the pair loses their symmetry under \( \Pi \) and \( R_\perp \), yet still conserves helicity. Additionally, of course, total kinetic energy and total linear momentum are conserved for separated pairs.

These symmetry properties, \( \Pi \) and \( R_\perp \), do not depend upon the presence of any mediating force. Nor can these symmetry properties be used by one photon to influence the other. The symmetries simply exist between the pair until some interaction is introduced that destroys them. In other words, there are two types of correlation that can be identified in an entangled EPR pair of photons: correlation due to the symmetries of \( R_\perp \) and \( \Pi \), and correlation due to the conservation of helicity (angular momentum).

\[
|\Psi_{12}^-\rangle = \frac{1}{\sqrt{2}}(|RL\rangle - |LR\rangle); \text{even parity odd } R_\perp
\]

Figure 2. Classical depiction of disentanglement to contrast figure 1. Here it is shown that the photons moving left and moving right have lost phase coherence between them and are therefore no longer entangled. The center dot represents the photon source.

V. Coincidence detection

The calculations presented in section III cannot yet be tested experimentally because it is not possible to know if the spins moving left and right originate from the same EPR pair. Also the calculations are presented for an sub-ensemble of spins that all have the same axis of quantization, \( \hat{P} \). Coincident measurement techniques\(^{10,11} \), however, can be used to determine if two spins that originate from the same EPR pair are correlated. If the two remain entangled, the calculations are well known\(^2,16 \) and are given here for completeness. Since the entangled EPR density operator, \( \rho_{EPR}^{12} \), is isotropic, it is not necessary to perform an ensemble average. The results are,

\[
\begin{align*}
\langle a \cdot \sigma^1 \sigma^2 \cdot b \rangle_{\text{Entangled}} &= a \cdot \left[ Tr_{12} \rho_{EPR}^{12} \sigma^1 \sigma^2 \right] \cdot b = -a \cdot b = -\cos \theta_{ab} . \\
\langle a \cdot \sigma^1 \rangle_{\text{Entangled}} &= a \cdot \left[ Tr_{12} \rho_{EPR}^{12} \sigma^1 \right] = 0 . \\
\langle \sigma^2 \cdot b \rangle_{\text{Entangled}} &= \left[ Tr_{12} \rho_{EPR}^{12} \sigma^2 \right] \cdot b = 0 .
\end{align*}
\]

(5.1)
so the correlation function is defined by

\[ E(a, b) = \langle a \cdot \sigma^z \sigma^z \cdot b \rangle - \langle a \cdot \sigma^z \rangle \langle \sigma^z \cdot b \rangle = -\cos \theta_{ab} \ . \] (5.2)

The angle is that between the two filters pointing in the \(a\) and \(b\) directions, \(a \cdot b = \cos \theta_{ab}\).

The four probabilities for coincident detection are:

\[ P_{\pm\pm}(a, b) = \frac{1}{4} (1 + \cos \theta_{ab}) = \frac{1}{2} \cos^2 \frac{\theta_{ab}}{2} \] (5.3)

and

\[ P_{\pm\mp}(a, b) = \frac{1}{4} (1 - \cos \theta_{ab}) = \frac{1}{2} \sin^2 \frac{\theta_{ab}}{2} \] (5.4)

so that:

\[ E(a, b) = P_{\mp\mp}(a, b) - P_{\mp\pm}(a, b) - P_{\pm\mp}(a, b) + P_{\pm\pm}(a, b) = -\cos \theta_{ab} \ . \] (5.5)

The sum of the four probabilities is unity.

Using disentanglement, the calculation has two steps. First, the disentangled density operator, Eq.(2.9) is used to evaluate the same expectation values and probabilities as for entanglement, Eq.(5.1) to (5.4). Following this, the results are ensemble averaged over all random values of \(\hat{P}\). The results from disentanglement for the four probabilities before performing the ensemble average are (superscript D for disentanglement),

\[ P_{\pm\pm}^D(a, b) = \frac{1}{4} (1 - \cos \theta_a \cos \theta_b) \] (5.6)

\[ P_{\pm\mp}^D(a, b) = \frac{1}{4} (1 + \cos \theta_a \cos \theta_b) \] (5.7)

The product of angles occurring in Eqs.(5.6) and (5.7) can be expressed in terms of the axis of quantization, \(\hat{P}\). For coincidence detection, this axis must be the same for both spins,

\[ \cos \theta_a \cos \theta_b = a \cdot \hat{P}\hat{P} \cdot b \] (5.8)

The ensemble average over all possible quantization axes depends upon the system under study. If the system is composed of two singlet state fermions of \(\frac{1}{2}\) magnitude each, the system is isotropic in all three Cartesian directions so the ensemble average gives,

\[ \langle a \cdot \sigma^z \sigma^z \cdot b \rangle_{\text{disentangled}} = -a \cdot \hat{P}\hat{P} \cdot b = -\frac{1}{3} a \cdot b = -\frac{1}{3} \cos \theta_{ab} \ . \] (5.9)

In contrast, photon helicity is quantized along the direction of propagation. If this axis is defined as \(\hat{z}\), then the ensemble average is performed in the \(\hat{x}\hat{y}\) plane perpendicular to the
direction of propagation. In that plane, isotropy implies that $x^2 = y^2$ leading to an ensemble averaged result appropriate for photons (except that the angle must be doubled),

$$
\overline{P_{\pm}^{D}(a, b)} = \frac{1}{4} \left(1 - a \cdot \mathbf{P} \cdot b\right) = \frac{1}{4} \left(1 - \frac{1}{2} a \cdot \left(\hat{x}\hat{x} + \hat{y}\hat{y}\right) \cdot b\right)
$$

$$
= \frac{1}{4} \left(1 - \frac{1}{2} a \cdot b + \frac{1}{2} a \cdot \hat{z}\hat{z} \cdot b\right) = \frac{1}{4} \left(1 - \frac{1}{2} \cos \theta_{ab}\right)
$$

(5.10)

The same result can be obtained by first changing from polar to cylindrical coordinates $(r, z, \chi)$ and then performing the ensemble average over random values of $r$ and $\chi$. The ensemble averages over the individual spins is the same as from entanglement, cf. Eq.(5.1),

$$
\langle a \cdot \sigma^1 \rangle \hat{p} = 0
$$

(5.11)

$$
\langle b \cdot \sigma^2 \rangle \hat{p} = 0
$$

(5.12)

The final results that can be compared with those from entanglement, Eqs.(5.2) to (5.5) are

$$
\overline{P_{\pm}^{D}(a, b)} = \frac{1}{4} \left(1 - \frac{1}{2} \cos \theta_{ab}\right)
$$

(5.13)

$$
\overline{P_{\pm}^{D}(a, b)} = \frac{1}{4} \left(1 + \frac{1}{2} \cos \theta_{ab}\right)
$$

(5.14)

and from Eq.(5.5), the disentangled correlation function is

$$
E(a, b)_{\text{disentanglement}} = -\frac{1}{2} \cos \theta_{ab}
$$

(5.15)

VI. Discussion

Comparing the correlation function from entanglement, Eq.(5.5) and disentanglement, Eq.(5.15), it is immediately evident that the functional form of the two is the same yet they differ by the prefactor of $\frac{1}{2}$. The correlations that can be present between an EPR pair are identified as due to conservation of angular momentum (helicity), parity and rotation, $R_\perp$. All three of these correlations can exist between a separated EPR pair without the presence of a mediating force. In the angular momentum representation of the spin pair, Eq.(2.7) shows that conservation of angular momentum results from the diagonal or “classical” terms of the density operator. In contrast, the off-diagonal quantum interference terms are responsible for maintaining the phase relations between the two separated entangled spins that are necessary for the conservation of parity and the rotation, cf. Eq.(4.5). If the EPR entangled pair undergoes disentanglement, correlation from phase symmetry is destroyed leaving only the correlation due to conservation of angular momentum. The correlation function for
disentangled photons is therefore reduced to \(1/2\) of that of entangled photons, cf. Eqs.(5.5) and (5.15).

The consequence of disentanglement on BI is not surprising. The set of conditions for maximum violation of the CHSH\(^{17}\) form of BI give a violation from entanglement of \(2 > 2\sqrt{2}\), yet no violation from disentanglement, \(2 > \sqrt{2}\). The difference clearly arises due to the loss of the quantum interference terms due to disentanglement. BI can therefore be interpreted as being useful for determining if quantum interference terms are present or not. Although the correlation due to the quantum interference terms is non-local, by the same definition, so is the correlation due to conservation of angular momentum. The difference is that conservation of angular momentum has a classical analogue, whereas the parity and rotation symmetries are purely quantum in origin and arise from the presence of quantum interference terms. From this viewpoint, non-locality is simply a result of conservation of various properties between distant particles that shared a common origin. In this interpretation of the violation of BI between separated particles, since one particle cannot influence its distant partner, there is no conflict with Einstein locality.

It is not ruled out that particles can remain entangled after they separate\(^{12,13}\). However it would be expected that passing photon pairs by mirrors, polarizers, beam splitters and fiber optics would make it more difficult to conserve the phase relationships between them than it is to conserve helicity. Moreover, it is experimentally difficult to distinguish between Eq.(5.5) for entanglement and, Eq.(5.15) for disentanglement since they both display the same functional form of \(\cos \theta_{ab}\). The Aspect\(^{10}\) experiments confirm the \(\cos \theta_{ab}\) dependence to a high level of accuracy, yet by normalizing the coincidence counts, \(N^{(+)}_{\pm\pm}(\pm, a, b)\) to the total number of counts,

\[
E(a, b) = \frac{N_{++}(a, b) - N_{--}(a, b) - N_{+-}(a, b) + N_{-+}(a, b)}{N_{++}(a, b) + N_{--}(a, b) + N_{+-}(a, b) + N_{-+}(a, b)}
\] (6.1)

the experiments are not sensitive to the prefactor of \(1/2\). Consequently these experiments cannot distinguish between Eq. (5.5) and Eq.(5.15). This is commonly referred to as the detection loophole, but in fact, as shown here, is a critical element in testing BI. In the Weils\(^{11}\) et al. repeat of the Aspect experiments, the same difficulty arises. They report, however, a detection rate of about 5\%, which is consistent with that predicted from disentanglement\(^{3}\), being 6.25\%. In similar experiments by Gisin\(^{18}\), the empirical parameter is introduced to the entangled probabilities, Eqs.(5.3) and (5.4) called the visibility, \(V\),

\[
P_{\pm\pm}(a, b) = \frac{1}{4}(1 - V \cos \theta_{ab})
\] (6.2)

where \(V=1\) for pure entanglement and is predicted to be \(V=1/2\) from disentanglement. Although Gisin includes experimental corrections that raises his visibility to 87\%, his uncorrected value is \(V=0.46\), consistent with disentanglement.
In the calculations presented here for the single and coincidence probabilities from disentanglement, Eqs. (3.3), (3.4), (5.13), (5.14) and (5.15), there are no consequences arising from the different phases between the two spins as shown in Eq. (3.1). In contrast, “teleportation” experiments require the presence of entangled photons to give the expected results. If the two photons are disentangled, then in general $\phi \neq \phi_2$, Eq. (3.1), leading to an ensemble with an infinite number of possible states rather than the four available to entangled photon pairs. Only those members of the ensemble that have $\phi_1 = \phi_2$ will lead to the detectable results in the current experiments and this is primarily responsible for the abysmally low detection rates reported.

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