The algorithm of the impulse noise filtration in images based on an algorithm of community detection in graphs

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Abstract

This article suggests an algorithm of impulse noise filtration, based on the community detection in graphs. The image is representing as non-oriented weighted graph. Each pixel of an image is corresponding to a vertex of the graph. Community detection algorithm is running on the given graph. Assumed that communities that contain only one pixel are corresponding to noised pixels of an image. Suggested method was tested with help of computer experiment. This experiment was conducted on grayscale, and on colored images, on artificial images and on photos. It is shown that the suggested method is better than median filter by 20% regardless of noise percent. Higher efficiency is justified by the fact that most of filters are changing all of image pixels, but suggested method is finding and restoring only noised pixels. The dependence of the effectiveness of the proposed method on the percentage of noise in the image is shown.

Keywords: noise reduction in images, community detection in graphs.

1 Introduction

Impulse noise in the graphical images is looks like random change of color of some random pixels, that called damaged pixels [1] [2]. The presence of noise in the image affects not only on the visual perception, but also affects the results of image segmentation algorithms, contour highlighting, pattern recognition, etc.

There are a lot of smoothing filters are existing for reduction of noise. Most often used are Wiener filter and median filter [3]. One need to point out a non-local filtration methods [4] [5] [6], that demonstrate a better impulse noise reduction. Nevertheless, smoothing filters has a significant disadvantage: they changing the whole image and not only the damaged pixels. This changes are lead to smoothing of contours of the image and make it difficult to additional processing.
To minimize the impact of filter on the non damaged part of the image, one can use an approach based on searching pixels that has changed by the impulse noise. Objective of searching the damaged pixels is a very complex problem and solving as a rule by using Data Mining on the image. In this approach the algorithm of the impulse noise reduction is consists of two steps. The first step is a finding of damaged pixels. And the second is choosing color for each damaged pixel.

Algorithms of damaged pixel searching can be divided into 2 groups. The first group is intended to search a Solt & Pepper Noise (SPN). The feature of this type of noise is that damaged pixels can have only maximum or minimum color of the palette. So, the SPN algorithms are based on this information [7, 8, 9]. However, either this type of algorithms is not guarantee that 100% of damaged pixels will be found. The second group is oriented on the impulse noise with random values. For this group, algorithms based on scheme SD-ROM are most widely used [10, 11]. The main idea of the SD-ROM scheme is that pixels are analyzing within a sliding window with size of $3 \times 3$ and a decision is made about the corruption of the central pixel. The decision algorithm in SD-ROM is based on the threshold circuit. A decision algorithm, based on the hierarchy analysis method has suggested in the paper [12] wherein the sliding window is still using. In the paper [13] for damaged pixels searching the method of associative rules has used. In the paper [14] for damaged pixels searching, an image segmentation algorithm is using.

Algorithms of impulse noise filtration with a known list of damaged pixels are reduced to filling out a table with omissions. The easiest way is to choose color based on colors of the nearest neighbors. This method is produce good results on the areas with uniform fill, but not acceptable in case of sharp color transitions, since it leads to blurring of the boundaries. Much more acceptable approach based on the linear manifolds [15]. Additionally, the neural networks can be used for impulse noise filtration.

As shown by the analysis of previous works, methods based on the analysis of the entire image provide better impulse noise reduction then local methods. The purpose of this article is to implement and test an algorithm for impulse noise filtration based on the method of community detection in graphs, which has proved itself in the task of image segmentation [16].
2 Formulation of the problem and filtration algorithm

It is assumed that the input of the algorithm is an image damaged by impulse noise with size of $NM$ pixels. There is a pair of integers $(x, y)$ used to determine a location of pixel on the image, that represents coordinates of this pixel. The number $x$ takes integer values on the interval $[0, N1]$, $y$ takes integer values on the interval $[0, M1]$. In a case of colored image, a pixel with coordinates $(x, y)$ is characterized by 3 color components: $r(x, y)$ – intensity of a red color, $g(x, y)$ – intensity of a green color, $b(x, y)$ – intensity of a blue color.

Let's correspond the weighted, unoriented graph $G$ to the image. Each pixel of the image is corresponding to a vertex in the graph $G$. Each vertex has edges only to all nearest neighbors. The weight of an edge is calculated based on the color components of connected vertexes. For two neighbor vertexes $v_i = (x_i, y_i)$ and $v_j = (x_j, y_j)$, the weight of the edge will be equal to:

$$d(v_i, v_j) = \exp\left(-\frac{1}{h} \sqrt{(r_i - r_j)^2 + (g_i - g_j)^2 + (b_i - b_j)^2}\right).$$

Here, $r_i = r(x_i, y_i)$, $g_i = g(x_i, y_i)$, $b_i = b(x_i, y_i)$. Parameter $h$ is used to change the difference between neighbor pixels, corresponding to moving into another segment of the image. This parameter is setting by user and used for the whole image. As it shown in paper [14, 18] this kind of weight function allows to accurately distinguish the color change corresponding to the boundaries of the areas in the image. Let's split the graph on insets with vertexes that connected much more than others. Such kind of insets are called communities. A quantitative estimation of the split can be obtained by Newmans modularity function [20, 21]. Greater value of the modularity function means more qualitatively the partition is performed. Assumed that damaged pixels are pixels that decrease value of the modularity function during connection to other communities. This way one can select communities that contain only one vertex, i.e. vertexes that different from each other a lot. Let's describe this procedure more formal.

Let's define a matrix of weights $E$ for graph $G$. The values of diagonal elements $E_{ij}$ are equal to weight of vertexes. At the first step of the algorithm assume that the weight of all vertexes are equal to 0. Other elements of the matrix $E_{ij}$ ($i \neq j$) are equal to a weight of a corresponding edge. One should point out that matrix $E_{ij}$ will contain a lot of non-zero elements because of only nearest neighbor vertexes are connected with edges in graph $G$. The matrix $E$ will be symmetric with respect to the main diagonal because of graph $G$ is non-oriented. Let's move to the reduced view of the matrix of weights $e = E/m$, where $m = \sum_{i,j=1}^{MN} E_{ij}$. The element $e_{ij}$ is...
equal to a part of edge weight in the whole graph weight. Further one can assume that matrix
of weight will have a reduced view. Its easy to see that $\sum_{i,j=1}^{MN} e_{ij} = 1$.

Modularity is defined as $[21, 22]$: $Q(G) = \frac{1}{K} \sum_{i=1}^{K} e_{ii} - \sum_{i=1}^{K} a_{i} b_{i}$,
where $K$ is a count of vertex in graph, $a_{i} -$ reduced outbound power of the vertex $v_{i}$ ($a_{i} = \sum_{j=1, j \neq i}^{K} e_{ij}$), $b_{i} -$ reduced inbound power of the vertex $v_{i}$ ($b_{i} = \sum_{j=1, j \neq i}^{K} e_{ji}$). Given graph is
non-oriented, so outbound and inbound power of all vertexes are equal ($a_{i} = b_{i}$; $i = 1, ..., K$).
Modularity function will have a more simpler representation:

$$Q(G) = \sum_{i=1}^{K} e_{ii} - \sum_{i=1}^{K} a_{i}^{2}.$$ 

Lets use a screaming procedure to find communities in the graph. Assumed that scream is a
transformation that replace some inset $H$ of graph $G$ to a some vertex $v_{H}$. If some of vertexes
of inset $H$ has been connected by edge with vertex $v$ from inset $G \setminus H$, then the vertex $v_{H}$
will be connected by edge with same weight with vertex $v$. The weight of the new edge will be
equal to a sum of weights of vertexes and edges that was included to the inset $H$. Lets name
a new graph as $G_{H}$. Lets assume that inset $H$ is a community if $Q(G_{h}) > Q(G)$. Lets point
it out that during creation of scream, the count of graph vertexes ($K$) is decreasing. The main
task is to find vertexes that not included to any big community. To find such vertexes lets use
the following algorithm:

1. Consecutive circumcision of all pixels of the image.
2. For each pixel $v$ lets consider nearest neighbor pixels $v^{(i)}$ ($i = 1, ..., 8$). Consider insets
that contains two vertexes $v$ and $v^{(i)}$ ($i = 1, ..., 8$) and try to connect them into community. For
each try lets calculate a delta of modularity function $\Delta Q_{i}$ ($i = 1, ..., 8$).
3. If delta of modularity function is negative ($\Delta Q_{i} < 0$, $i = 1, ..., 8$), assumed that corre-
sponding pixel is damaged.

The delta of modularity function can be computed fast using current characteristics of the
graph that correspond to an image. For connect to vertexes lets use the following representation
of the delta of modularity function:

$$\Delta Q = 2(e_{ij} - a_{i} a_{j}).$$ 

Its obviously that suggested algorithm has linear complexity depending on pixel count.
After finding of damaged pixels one need to choose a color for all of found pixels depending on neighbor pixels analysis. Assume that minimal value of the color component of the neighbor pixels is $m_1$, and maximum value is $m_2$. Lets carry out a sequential search all of the values of damaged pixel color from $m_1$ to $m_2$. For each value lets calculate delta of the modularity function $\Delta Q$. As a result color lets use the one that takes a maximum value of the modularity function delta.

3 Computer experiment

Computer experiment has carried out on artificial images of geometric objects and on color photos. The value of the impulse noise has been characterized by $p$, which shows a percent of damaged pixels related to a whole pixel count. Impulse noise was generating with help of linear congruent generator of pseudo-random numbers. This generator was used both to generate colors and coordinates. During computer experiment the percent of damaged pixels has changed from 10% to 70%. Image filtration has carried out using suggested method and additionally using a well-known median filter.

To compare the proximity of images there is a Minkovskiy metric [22, 23] has been used according to which the proximity between images $A$ and $C$ is calculating using the following formula:

$$d(A, C) = \max_{n,m} \sum_{k=1}^{N} \frac{1}{N} |A_{nm}^{(K)} - C_{nm}^{(K)}|,$$

where $A_{nm}$ and $C_{nm}$ – values of $A$ and image pixel colors, $N$ – count of pixels.

Relative image enhancement has been calculated based on distance $d(\text{orig}, r)$ from reconstructed image $r$ to original image $\text{orig}$ and distance $d(\text{orig}, pf)$ from damaged image $pf$ to original image $\text{orig}$:

$$\delta = \frac{d(\text{orig}, pf) - d(\text{orig}, r)}{d(\text{orig}, pf)} \cdot 100\%.$$

Experiment for a rectangular area with uniform filling showed that the proposed filter allows to significantly improve the image. The dependence of the relative improvement on the percentage of corrupted pixels for the proposed filter and the median filter is shown in Fig. 1.

The results of applying the proposed filter and the median filter to improve the artificial image with the presence of a solid and gradient fill are shown in Fig. 2.

As can be clearly seen from Fig. 2, the results of the proposed filter are more advantageous
Figure 1: Dependence of the relative image improvement on the percentage of noise for the proposed filter (solid line) and the median filter (dashed line).

![Figure 1](image)

Figure 2: The results of applying the filter to an artificial image with a noise level of $p = 20\%$: a) the original image, b) a noised image, c) the image reconstructed by the proposed filter, d) the image reconstructed by the median filter.

![Figure 2](image)

for dark areas of the image, whereas the median filter gives the best visual result for the light part of the image. This effect is related to that new color is choosing for the damaged pixel. When using the median filter, the colors of the restored pixels are shifted to a white area. In this case, the colors of the surrounding pixels also change their color. Numerical comparison of the results of the work shows a significant advantage of the proposed algorithm in front of the median filter.

The dependence of the relative improvement on the percentage of noised artificial image is shown in Figure 3.

Additionally the proposed filter allows to get much better results for photographic images.
Figure 3: Dependence of the relative improvement on the percentage of noised artificial image for the proposed filter (solid line) and the median filter (dashed line).

The results for the well-known image Lena with a $p = 20\%$ are shown in Fig. 4.

Figure 4: The results of application of the filter to the Lena image with a noise level of $p = 20\%$: a) the original image, b) noised image, c) the image reconstructed by the proposed filter, d) the image reconstructed by the median filter.

It is well known that the Lena image is characterized by a large number of small details that create difficulties for all filters. As can be seen from Fig. 4, the proposed filter gives significantly better results even in a visual comparison. The dependence of the relative improvement on the percentage of noise is shown in Fig. 5.
4 Conclusion

Thus, the proposed filter has good characteristics with linear labor input. As can be seen from the graphs presented in Figures 1, 3 and 5, the efficiency of this filter is approximately 20% higher than the median for any percentage of damaged pixels. This noticeable advantage is due to the fact that conventional filters change all pixels of the image. Correcting the damaged pixels brings the image closer to the original, but changing the undamaged pixels increases the distance to the original. This property is inherent not only to the median filter, but also to all traditional filters.

The filter proposed in this article acts selectively and changes only those pixels that differ significantly from those around them. With a high probability, such pixels will be damaged by impulse noise. The choice of a new color on the basis of attaching to one of the neighboring communities of pixels allows to form communities of pixels close in characteristics. It should also be noted the high speed of the filter, due to the linear complexity of the algorithm underlying it. The processing time of one image within the error is the same as the median filter.

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