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eHDECAY: an Implementation of the Higgs Effective Lagrangian into HDECAY

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Abstract

We present eHDECAY, a modified version of the program HDECAY which includes the full list of leading bosonic operators of the Higgs effective Lagrangian with a linear or non-linear realization of the electroweak symmetry and implements two benchmark composite Higgs models.
PROGRAM SUMMARY

Manuscript title: eHDECAY: an Implementation of the Higgs Effective Lagrangian into HDECAY

Authors: Roberto Contino, Margherita Ghezzi, Christophe Grojean, Margarete Mühleitner and Michael Spira

Program title: eHDECAY

Licensing provisions: None

Programming language: Fortran77

Computer(s) for which the program has been designed: Any with a Fortran77 system

Operating system(s) for which the program has been designed: Linux, Unix

RAM required to execute with typical data: 0.5MB

Has the code been vectorised or parallelized?: No

Number of processors used: 1

Supplementary material: None

Keywords: Higgs decays, loop decays, higher order corrections to decays, effective theories

Nature of problem: Numerical calculation of the decay widths and branching ratios of a Higgs-like boson within four different parametrisations: the non-linear Lagrangian, the Strongly-Interacting Light Higgs (SILH) Lagrangian and the MCHM4 and MCHM5 Lagrangians. The Fortran program eHDECAY includes the most important higher-order QCD effects and in case of the SILH and composite Higgs parametrisation also the electroweak (EW) higher order corrections. The user furthermore has the possibility to turn off these EW corrections.

Solution method: The necessary input values, the choice of the parametrisation and the values of the various couplings are set in the input file ehdecay.in. These are read in by the main routine ehdecay.f. The main routine calculates the decay widths and branching ratios through analytical formulae, by using several help routines (dmb.f, elw.f, feynhiggs.f, haber.f, hgaga.f, hgg.f, hsqsq.f, susylha.f). The calculated branching ratios and total width are given out in the files br.eff1 and br.eff2.

Restrictions: The EW corrections are included only in the SILH and the composite Higgs parametrisation in an approximate way. They are consistently not included in the non-linear case. This would require the explicit calculation of the EW higher order correctons in this framework. The program does not provide any distributions.
1 Introduction

In a companion paper [1], we gave a detailed review of the low-energy effective Lagrangian which describes a light Higgs-like boson and estimated the deviations induced by the leading operators to the Higgs decay rates. We discussed in particular how the effective Lagrangian can be used beyond the tree-level by performing a multiple perturbative expansion in the SM coupling parameter $\alpha/\pi$ and in powers of $E/M$, where $E$ is the energy of the process and $M$ is the New Physics (NP) scale at which new massive states appear. When the Higgs-like boson is part of a weak doublet, a third expansion must be performed for $v/f \ll 1$, where $f \equiv M/g_*$ and $g_*$ is the typical coupling of the NP sector.

A recent study [2] concluded that, at tree-level, there are 8 dimension-6 CP-even operators that can be constrained by Higgs physics only. It is of course essential to have automatic tools to give accurate predictions of the deviations induced by these operators to Higgs observables. These operators are all part of the Strongly Interacting Light Higgs (SILH) Lagrangian [3] that we will be dealing with (the SILH Lagrangian, Eq. 2.2 contains 12 operators but 2 combinations of them are severely constrained by electroweak (EW) precision data and two other combinations are constrained by the bounds on anomalous triple gauge couplings). These operators are also included in Monte Carlo codes recently developed [4,5].

The purpose of this note is to present the Fortran code eHDECAY, which implements the leading operators in the effective Lagrangian and gives an extension of the program

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HDECAY [6] for the automatic calculation of the Higgs decay widths and branching ratios. The program can be obtained at the URL:

http://www-itp.particle.uni-karlsruhe.de/~maggie/eHDECAY/

The organization of the paper is as follows. In Section 2, we briefly review the definition of the effective Lagrangians, with linearly and non-linearly realized electroweak symmetry breaking (EWSB), that have been implemented in the program. This is mainly to set the notation. For more details and for a discussion of the physics implications we refer the reader to Ref. [1]. Section 3 gives a detailed discussion of how the partial decay widths have been implemented into the program eHDECAY, including higher-order effects in the perturbative expansion. For issues related to the perturbative expansion and the inclusion of higher-order corrections we again refer the reader to Ref. [1]. In Section 4, we give numerically approximated results for the Higgs decay rates in the framework of linearly realized EWSB. Section 5 explains how to run eHDECAY and presents sample input and output files. We conclude in Section 6.

2 Effective Lagrangians for linearly and non-lineary realized EW symmetry

We assume for simplicity that the Higgs boson is CP-even and that baryon and lepton numbers are conserved. If the Higgs is part of a weak doublet, the leading effects beyond the Standard Model are parametrized by 53 operators with dimension-6 [7–9] (additional 6 operators must be added if the assumption of CP conservation is relaxed), when a single family of quarks and leptons is considered. In the following we will adopt the so-called SILH basis proposed in Ref. [3]:

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_i \bar{c}_i O_i \equiv \mathcal{L}_{SM} + \Delta \mathcal{L}_{SILH} + \Delta \mathcal{L}_{F_1} + \Delta \mathcal{L}_{F_2} + \Delta \mathcal{L}_V + \Delta \mathcal{L}_{4F} , \quad (2.1)$$
with

\[
\Delta \mathcal{L}_{\text{SIH}} = \frac{\bar{c}_H}{2v^2} \partial^\mu (H^\dagger H) \partial_\mu (H^\dagger H) + \frac{\bar{c}_T}{2v^2} (H^\dagger \overrightarrow{D}_\mu H) (H^\dagger \overleftarrow{D}_\mu H) - \frac{\bar{c}_\lambda}{v^2} (H^\dagger H)^3 + \left( \left( \frac{\bar{c}_u}{v^2} y_u H^\dagger H q_L H^c u_R + \frac{\bar{c}_d}{v^2} y_d H^\dagger H q_L H d_R + \frac{\bar{c}_l}{v^2} y_l H^\dagger H \bar{L}_L H l_R \right) + \text{h.c.} \right) + \frac{i \bar{c}_W}{2m_W^2} (H^\dagger \sigma^i \overrightarrow{D}_\mu H) (D^\nu W_{\mu \nu})^i + \frac{i \bar{c}_B g'}{2m_W^2} (H^\dagger \overrightarrow{D}_\mu H) (\partial^\nu B_{\mu \nu}) + \frac{i \bar{c}_{HB} g'}{m_W^2} (D^\mu H)^i (D^\nu H) B_{\mu \nu} + \frac{\bar{c}_g g_S^2}{m_W^2} (H^\dagger H B_{\mu \nu} B^{\mu \nu} + \bar{c}_g g_S^2 H^\dagger H G_{\mu \nu}^a G^{a \mu \nu},
\]

(2.2)

\[
\Delta \mathcal{L}_{F_1} = \frac{i \bar{c}_H q}{v^2} (q_L \gamma^\mu q_L) (H^\dagger \overleftarrow{D}_\mu H) + \frac{i \bar{c}'_H q}{v^2} (q_L \gamma^\mu q_L) (H^\dagger \sigma^i \overleftarrow{D}_\mu H)
\]

\[
+ \frac{i \bar{c}_H u}{v^2} (\bar{u}_R \gamma^\mu u_R) (H^\dagger \overleftarrow{D}_\mu H) + \frac{i \bar{c}_H d}{v^2} (\bar{d}_R \gamma^\mu d_R) (H^\dagger \overleftarrow{D}_\mu H) + \left( \left( \frac{i \bar{c}_H u d}{v^2} (\bar{u}_R \gamma^\mu u_R) (H^\dagger \sigma^i \overleftarrow{D}_\mu H) \right) + \text{h.c.} \right)
\]

(2.3)

\[
\Delta \mathcal{L}_{F_2} = \frac{\bar{c}_u B g'}{m_W^2} y_u q_L H^c \sigma^{\mu \nu} u_R B_{\mu \nu} + \frac{\bar{c}_u W g}{m_W^2} y_u q_L \sigma^i H^c \sigma^{\mu \nu} u_R W_{\mu \nu}^i + \frac{\bar{c}_u G g_S}{m_W^2} y_u q_L H^c \sigma^{\mu \nu} \lambda^a u_R G_{\mu \nu}^a
\]

\[
+ \frac{\bar{c}_d B g'}{m_W^2} y_d q_L \sigma^{\mu \nu} d_R B_{\mu \nu} + \frac{\bar{c}_d W g}{m_W^2} y_d q_L \sigma^i H^c \sigma^{\mu \nu} d_R W_{\mu \nu}^i + \frac{\bar{c}_d G g_S}{m_W^2} y_d q_L H^c \sigma^{\mu \nu} \lambda^a d_R G_{\mu \nu}^a
\]

\[
+ \frac{\bar{c}_l B g'}{m_W^2} y_l \bar{L}_L H \sigma^{\mu \nu} l_R B_{\mu \nu} + \frac{\bar{c}_l W g}{m_W^2} y_l \bar{L}_L \sigma^i H^c \sigma^{\mu \nu} l_R W_{\mu \nu}^i + \text{h.c.}
\]

(2.4)

\(^1\)In this paper we follow the same notation as in Ref. [1]. In particular, the expression of the SM Lagrangian \( \mathcal{L}_{SM} \) and the convention for the covariant derivatives and the gauge field strengths can be found in Appendix A of Ref. [1].
Here $\lambda$ denotes the Higgs quartic coupling which appears in the SM Lagrangian $L_{SM}$, and the weak scale is defined by

$$v \equiv \frac{1}{(\sqrt{2}G_F)^{1/2}} \simeq 246 \text{ GeV}.$$  \hspace{1cm} (2.5)

We have defined the Hermitian derivative

$$iH^\dagger D^\mu H \equiv iH^\dagger(D^\mu H) - i(D^\mu H)^\dagger H$$  \hspace{1cm} (2.6)

and $\sigma^{\mu\nu} \equiv i[\gamma^\mu, \gamma^\nu]/2$. The Yukawa couplings $y_{u,d,l}$ and the Wilson coefficients $\bar{c}_i$ are matrices in flavor space, and a summation over flavor indices has been implicitly assumed. In order to avoid large Flavor-Changing Neutral Currents (FCNC) through the tree-level exchange of the Higgs boson, we assume that each of the operators $O_{u,d,l}$ is flavor-aligned with the corresponding mass term. The coefficients $\bar{c}_{u,d,l}$ are then proportional to the identity matrix in flavor space. Furthermore, as we assume CP-invariance, they are taken to be real. A naive estimate of the Wilson coefficients $\bar{c}_i$ can be found in Eq. (2.9) of Ref. [1], following the power counting of Ref. [3].

In addition to those listed in Eqs. (2.2)-(2.4), the effective Lagrangian includes also five extra bosonic operators, $\Delta L_V$, as well as 22 four-Fermi baryon-number conserving operators, $\Delta L_{4F}$. Two of the operators in Eqs. (2.2), (2.3) are in fact redundant and can be eliminated through the equations of motion. A most convenient choice is that of eliminating two of the three operators involving leptons in $\Delta L_{F_1}$.

In the unitary gauge with canonically normalized fields, the SILH effective Lagrangian $\Delta L_{SILH}$ reads:

$$\mathcal{L} = \frac{1}{2} \partial_\mu h \partial^\mu h - \frac{1}{2} m_h^2 h^2 - c_3 \frac{1}{6} \left( \frac{3m_W^2}{v} \right) h^3 - \sum_{\psi=u,d,l} m_{\psi(i)} \bar{\psi}^{(i)}(i) \psi^{(i)} \left( 1 + c_\psi \frac{h}{v} + \ldots \right) + m_W^2 W_\mu^+ W_-^\mu \left( 1 + 2c_W \frac{h}{v} + \ldots \right) + \frac{1}{2} m_Z^2 Z_\mu Z^\mu \left( 1 + 2c_Z \frac{h}{v} + \ldots \right) + \ldots$$  \hspace{1cm} (2.7)

$$+ \left( c_W W_\mu^+ W_-^\mu + \frac{c_{\gamma Z}}{2} Z_\mu Z^\mu + c_{Z\gamma} Z_\mu \gamma^\mu \gamma^{\mu} + \frac{c_{\gamma^2}}{2} \gamma^\mu \gamma^{\mu} + \frac{c_{gg}}{2} G_\mu G^{\mu} \right) \frac{h}{v} + \ldots$$

$$+ \left( c_{W\partial W} W_\nu^+ D_\mu W_-^{\mu + \nu} + h.c. \right) + c_{Z\partial Z} Z_\nu \partial_\mu Z^\mu + c_{Z\partial \gamma} Z_\nu \partial_\mu \gamma^{\mu} \right) \frac{h}{v} + \ldots$$

where we have shown terms with up to three fields and at least one Higgs boson. The couplings $c_i$ are linear functions of the Wilson coefficients of the effective Lagrangian (2.1).
and are reported in Table 1. In particular, the following relations hold

\[ c_{WW} - c_{ZZ} \cos^2 \theta_W = c_{Z\gamma} \sin 2\theta_W + c_{\gamma\gamma} \sin^2 \theta_W \tag{2.8} \]

\[ c_{W\partial W} - c_{Z\partial Z} \cos^2 \theta_W = \frac{c_{Z\partial \gamma}}{2} \sin 2\theta_W, \tag{2.9} \]

which are a consequence of the accidental custodial invariance of the SILH Lagrangian at the level of dimension-6 operators [1]. Imposing custodial invariance for the Lagrangian (2.2), so that \( \bar{c}_T = 0 \), implies a third relation that holds for the non-derivative couplings \( c_W \) and \( c_Z \):

\[ c_W = c_Z. \tag{2.10} \]

For arbitrary values of the couplings \( c_i \), Eq. (2.7) represents the most general effective Lagrangian which can be written at \( O(p^4) \) in a derivative expansion by focusing on cubic terms with at least one Higgs boson and making the following two assumptions: i) CP is conserved; ii) vector fields couple to conserved currents. Effects which violate the second assumption, in particular, are suppressed by the fermion masses, hence they are small for all the processes of interest in this work. Such description does not require the Higgs boson to be part of an electroweak doublet, and in this sense Eq. (2.7) can be considered as a generalization of the SILH Lagrangian \( \Delta L_{SILH} \). It contains 10 couplings involving a single Higgs boson and two gauge fields (\( hVV \) couplings, with \( V = W, Z, \gamma, g \)), 3 linear combinations of which vanish if custodial symmetry is imposed [1]. This counting agrees with the complete non-linear Lagrangian at \( O(p^4) \) recently built in Refs. [10–13]. This general Lagrangian contains many more operators but it can be easily checked that only 10 independent operators remain after assuming CP invariance and the conservation of fermionic currents, and among them 3 break the custodial symmetry. If the assumption on conserved currents is relaxed, there are two more independent operators at \( O(p^4) \) that give rise to \( hVV \) couplings (they are the operators \( P_9 \) and \( P_{10} \) of Ref. [13], see also the general form factor description of Ref. [14]). These two additional couplings can only be obtained from dimension-8 operators when the Higgs boson is part of an EW doublet. In the non-linear realization of the EW symmetry, all Higgs

\footnote{Notice that the similar Table 1 in Ref. [1] contains an erroneous factor 2 in the dependence of \( c_Z \) on \( \bar{c}_T \).}

\footnote{If the assumption of CP conservation is relaxed, \( c_{W\partial W} \) can in general be complex, while the other bosonic couplings of Eq. (2.7) are real. In this case Eq. (2.9) corresponds to two real identities, respectively, on the real and on the imaginary parts, so that custodial symmetry implies \( \text{Im}(c_{W\partial W}) = 0 \).}
| Higgs couplings | $\Delta L_{SILH}$ | MCHM4 | MCHM5 |
|-----------------|-------------------|-------|-------|
| $c_W$           | $1 - \bar{c}_H/2$ | $\sqrt{1-\xi}$ | $\sqrt{1-\xi}$ |
| $c_Z$           | $1 - \bar{c}_H/2 - \bar{c}_T$ | $\sqrt{1-\xi}$ | $\sqrt{1-\xi}$ |
| $c_\psi$ ($\psi = u, d, l$) | $1 - (\bar{c}_H/2 + \bar{c}_\psi)$ | $\sqrt{1-\xi}$ | $\frac{1-2\xi}{\sqrt{1-\xi}}$ |
| $c_3$           | $1 + \bar{c}_6 - 3\bar{c}_H/2$ | $\sqrt{1-\xi}$ | $\frac{1-2\xi}{\sqrt{1-\xi}}$ |
| $c_{gg}$        | $8 (\alpha_s/\alpha_2) \bar{c}_g$ | 0 | 0 |
| $c_{\gamma\gamma}$ | $8 \sin^2\theta_W \bar{c}_\gamma$ | 0 | 0 |
| $c_{Z\gamma}$   | $(\bar{c}_{HB} - \bar{c}_{HW} - 8 \bar{c}_\gamma \sin^2\theta_W) \tan \theta_W$ | 0 | 0 |
| $c_{WW}$        | $-2 \bar{c}_{HW}$ | 0 | 0 |
| $c_{ZZ}$        | $-2 (\bar{c}_{HW} + \bar{c}_{HB} \tan^2\theta_W - 4\bar{c}_\gamma \tan^2\theta_W \sin^2\theta_W)$ | 0 | 0 |
| $c_{W\partial W}$ | $-2(\bar{c}_W + \bar{c}_{HW})$ | 0 | 0 |
| $c_{Z\partial Z}$ | $-2(\bar{c}_W + \bar{c}_{HW}) - 2 (\bar{c}_B + \bar{c}_{HB}) \tan^2\theta_W$ | 0 | 0 |
| $c_{Z\theta\gamma}$ | $2 (\bar{c}_B + \bar{c}_{HB} - \bar{c}_W - \bar{c}_{HW}) \tan \theta_W$ | 0 | 0 |

Table 1: The second column reports the values of the Higgs couplings $c_i$ defined in Eq. (2.7) in terms of the coefficients $\bar{c}_i$ of the effective Lagrangian $\Delta L_{SILH}$. The last two columns show the predictions of the MCHM4 and MCHM5 models in terms of $\xi = (v/f)^2$, see Ref. [1] for details. The auxiliary parameter $\alpha_2$ is defined by Eq. (3.12).
couplings are truly independent of other parameters that do not involve the Higgs boson, like EW oblique parameters or anomalous triple gauge couplings. In a linear realization, on the other hand, only 4 $hVV$ couplings are independent of the other EW measurements [2]. In custodial invariant scenarios, it is thus not possible to tell whether the Higgs is part of an EW doublet by focusing only on $hVV$ couplings, since their number is the same in both the linear and non-linear descriptions under our assumptions (CP and current conservation). The decorrelation between the $hVV$ couplings and the other EW data might instead be a way to disprove the doublet nature of the Higgs boson [2,13,14].

The code \texttt{eHDECAY} retains only the couplings induced by the operators of $\Delta L_{SILH}$ since the effects of the other operators with fermions are either severely constrained by non-Higgs physics or, like the top dipoles, are irrelevant for the Higgs total decay rates (they could modify in a sensible way the differential decay rates but such an analysis is beyond the scope of the present work). The CP-odd operators are not considered either since they do not interfere with the inclusive SM amplitudes and thus modify the decay rates at a subleading order in the perturbative expansion considered in this paper.

3 Implementation of the Higgs effective Lagrangian into \texttt{eHDECAY}

The program \texttt{HDECAY} [6] was originally written for the automatic computation of the Higgs partial decay widths and branching ratios in the SM and in its Minimal Supersymmetric extension (MSSM). It includes the possibility of specifying modified couplings for up-type quarks, down-type quarks, leptons and vector bosons in the parametrization of Eq. (2.7), as well as of including the effective couplings $c_{gg}$, $c_{\gamma\gamma}$ and $c_{Z\gamma}$. We present here a modified version of the program, labelled \texttt{eHDECAY}. In addition to the features already present in \texttt{HDECAY}, the new program includes the effective couplings $c_{WW}$, $c_{ZZ}$, $c_{W\gamma}$ and $c_{Z\gamma}$, and thus fully implements the non-linear Lagrangian (2.7). In fact, similarly to \texttt{HDECAY} 5.10, it also includes the possibility of choosing different couplings of the Higgs boson to each of the

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4 Notice that the operator proportional to $c_{Z\gamma}$ does not affect the decay $h \rightarrow Z\gamma$ as long as the photon is on-shell.
up and down quark flavors and lepton flavors. In this sense the program assumes neither
custodial symmetry nor flavor alignment. As explained in the text, Eq. (2.7) describes a
generic CP-even scalar $h$ at $O(p^4)$ in the derivative expansion. If $h$ forms an $SU(2)_L$ doublet
together with the longitudinal polarizations of the $W$ and the $Z$, the Lagrangian can be
expanded as in Eq. (2.2) for $(v/f) \ll 1$; in this case the values of the Higgs couplings $c_i$
are given in the second column of Table 1. The program $eHDECAY$ provides an option in its
input file where the user can switch from the non-linear parametrization of Eq. (2.7) to that
of the SILH Lagrangian Eq. (2.2). The user can also choose to set the values of the Higgs
couplings to those predicted at leading order in an expansion in powers of weak couplings
in the benchmark composite Higgs models MCHM4 [15] and MCHM5 [16], see the last two
columns of Table 1.

Similarly to the original version of $HDECAY$, all the relevant QCD corrections are included.
They generally factorize with respect to the expansion in the number of fields and deriva-
tives of the effective Lagrangian, and can thus be straightforwardly included by making
use of the existing SM computations. The inclusion of the electroweak corrections is less
straightforward and can currently be done in a consistent way only in the framework of the
Lagrangian (2.2) and up to higher orders in $(v/f)$. Going beyond such approximations would
require dedicated computations which at the moment are not available in the literature. In $eHDECAY$ the user has the option to include the one-loop EW corrections to a given decay
rate only if the parametrization of Eq. (2.2) has been chosen. The same EW scheme as used
by $HDECAY$, with $G_F$, $m_W$ and $m_Z$ taken as input parameters, is also adopted in $eHDECAY$.
The sine of the Weinberg angle is defined as

$$\sin^2 \theta_W = 1 - \frac{m_W^2}{m_Z^2}, \quad (3.11)$$

following the conventional on-shell scheme [17]. Derived quantities in this scheme are also the
electromagnetic coupling and the weak coupling. To describe the latter we have conveniently
defined the parameter

$$\alpha_2 \equiv \frac{\sqrt{2} G_F m_W^2}{\pi}. \quad (3.12)$$

The formulas implemented in the program are thus written in terms of only the input pa-
rameters or their derived quantities $\sin \theta_W$ and $\alpha_2$. The only exception to this rule is given
by the decay rates $\Gamma(h \rightarrow \gamma\gamma)$ and $\Gamma(h \rightarrow Z\gamma)$, where we use the experimental value of the electromagnetic coupling in the Thomson limit, $\alpha_{em}(q^2 = 0)$, in order to avoid large logarithms for on-shell photons.

Below a detailed discussion follows of how the New Physics corrections are incorporated for each of the Higgs decay modes. We report explicitly the formulas implemented in the code and their level of approximation in the perturbative expansion of the effective Lagrangian. In all the following expressions, as explained in the text, the coefficients of the dimension-6 operators of the SILH Lagrangian (2.2) and those of the derivative operators of Eq. (2.7) must be identified with their values at the relevant low-energy scale $\mu = m_h$.

3.1 Decays into quarks and leptons

Upon adopting the effective description of the non-linear Lagrangian (2.7) and working at leading order in the derivative expansion, the Higgs boson partial decay width into a pair of fermions is obtained by rescaling the tree-level SM value $\Gamma_{0}^{SM}(\bar{\psi}\psi)$ by a factor $c_{\psi}^2$. The QCD corrections to the decay widths into quarks which are currently available for the SM case include fully massive next-to-leading order (NLO) corrections near threshold [18] and massless $O(\alpha_s^4)$ corrections far above threshold [19–22]. Also, large logarithms can be resummed through the running of the quark masses and of the strong coupling constant. They are evaluated at the scale given by the Higgs mass. The transition from the threshold region involving mass effects to the renormalization-group-improved large-Higgs mass regime is provided by a smooth linear interpolation. All these QCD corrections factorize with respect to the tree-level amplitude and can therefore be incorporated as done in HDECAY for the SM case. The decay rate can be written as follows:

$$\Gamma(\bar{\psi}\psi)_{NL} = c_{\psi}^2 \Gamma_0^{SM}(\bar{\psi}\psi) \left[ 1 + \delta_{\psi} \kappa_{QCD} \right],$$

(3.13)

where $\Gamma_0^{SM}$ denotes the leading-order decay width, $\delta_{\psi} = 1(0)$ for $\psi = \text{quark} (\text{lepton})$ and $\kappa_{QCD}$ encodes the QCD corrections.\footnote{This is the formula implemented by eHDECAY in the case of the non-linear Lagrangian (2.7). It is valid up to corrections of $O(m_h^2/M^2)$ in the case of decays into strange, charm or bottom quarks. There is one caveat, however. In the case of decays into strange, charm or bottom quarks there are two-loop diagrams which involve loops of top quarks coupling to the Higgs boson. They need a rescaling different from $c_{\psi}^2$. It has been correctly taken into account by the appropriate modification factor $c_{\psi}c_t$ ($\psi = c, s, b$).}
derivative expansion and of $O(\alpha_2/4\pi)$ from EW loops. These latter corrections are available in the SM but contrary to the QCD ones do not factorize. Their inclusion in the case of generic Higgs couplings thus requires a dedicated calculation, which is not available at present. The two benchmark composite Higgs models MCHM4 and MCHM5 provide a resummation of higher-order terms in $\xi = v^2/f^2$. Contrary to the SILH Lagrangian which is to be seen as an expansion in $\xi$, in these two models rather large coupling deviations can in principle be possible (eventually they are precluded due to the constraints from electroweak precision measurements). We therefore apply the formula Eq. (3.13) also for the MCHM4 and MCHM5, with $c_\psi$ given by the corresponding coupling values in columns 3 and 4 of Table 1.

In case of the SILH parametrization, where the deviations of the Higgs couplings from their SM values are assumed to be of $O(v^2/f^2)$ and small, the decay rate can be written as

$$\Gamma(\bar{\psi}\psi)_{SILH} = \Gamma_{\text{SM}}^0(\bar{\psi}\psi) \left[1 - \bar{c}_H - 2\bar{c}_\chi + \frac{2}{|A_{0,SM}^S|^2} \text{Re} \left(A_{0,SM}^S A_{1,ew}^{SM}\right) \right] [1 + \delta_\psi \kappa^{QCD}], \tag{3.14}$$

where $A_{0,SM}^S$, $A_{1,ew}^{SM}$ are, respectively, the tree-level and EW one-loop [23] amplitudes of the SM. In this case the one-loop EW corrections can be easily included if one neglects terms of $O[(\alpha_2/4\pi)(v/f)^2]$. In particular, mixed contributions up to $O[(\alpha_2/4\pi)(\alpha_s/4\pi)^4]$ have been included by assuming that the electroweak and QCD corrections factorize, as the non-factorizable contributions are small. From the viewpoint of the expansion in inverse powers of the NP scale, the formula (3.14) includes corrections of order $O(v^2/f^2)$. It neglects terms of $O(v^4/f^4)$, $O[(\alpha_2/4\pi)(v/f)^2]$, $O[(\alpha_2/4\pi)^2]$.

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6As pointed out in footnote 21 of Ref. [1], in the strict sense this equation is valid for the genuine EW corrections only, while for simplicity we include the (IR-divergent) virtual QED corrections to the SM amplitude in the same way. The corresponding real photon radiation contributions to the decay rates are treated in terms of a linear novel contribution to the Higgs coupling for the squared amplitude in order to obtain an infrared finite result. Pure QED corrections factorize as QCD corrections in general so that their amplitudes scale with the modified Higgs couplings. However, they cannot be separated from the genuine EW corrections in a simple way.
3.2 Decay into gluons

Upon selecting the Lagrangian (2.7), the rate into two gluons is computed in eHDECAY by means of the following formula:

$$\Gamma(gg)\big|_{NL} = \frac{G_F^2 \alpha_s^2 m_h^3}{4\sqrt{2}\pi^3} \left[ \sum_{q=t,b,c} \frac{c_q}{3} A_{1/2} (\tau_q) \right] c_{eff}^2 \kappa_{soft}$$

$$+ 2 \text{Re} \left( \sum_{q=t,b,c} \frac{c_q}{3} A_{1/2}^*(\tau_q) \frac{2\pi c_{gg}}{\alpha_s} \right) c_{eff} \kappa_{soft} + \left[ \frac{2\pi c_{gg}}{\alpha_s} \right]^2 \kappa_{soft}$$

$$+ \frac{1}{9} \sum_{q,q'=t,b} c_q A_{1/2}^*(\tau_q) c_{q'} A_{1/2} (\tau_{q'}) \kappa_{NLO} (\tau_q, \tau_{q'}) \right],$$

where $$\tau_q = 4m_q^2/m_h^2$$ and the loop function, normalized to $$A_{1/2}(\infty) = 1$$, is defined as

$$A_{1/2}(\tau) = \frac{3}{2} \tau [1 + (1 - \tau) f(\tau)],$$

with

$$f(\tau) = \begin{cases} \arcsin^2 \frac{1}{\sqrt{\tau}} & \tau \geq 1 \\ -\frac{1}{4} \left[ \ln \frac{1 + \sqrt{1 - \tau}}{1 - \sqrt{1 - \tau}} - i\pi \right]^2 & \tau < 1. \end{cases}$$

The first term corresponds to the one-loop contribution from the top, bottom and charm quarks, whose couplings to the Higgs boson are modified with respect to their SM values. In order to minimize the effects from higher-order QCD corrections, we use the pole masses for the top, bottom and charm quarks, $$m_t = 172.5$$ GeV, $$m_b = 4.75$$ GeV and $$m_c = 1.42$$ GeV.

The second and third terms encode the effect of the derivative interaction between the Higgs boson and two gluons generated by New Physics. Naively $$c_{gg} \approx (\alpha_s/4\pi)(g^*_v v^2/M^2)$$, so that the correction from the effective interaction can be as important as the one from the top quark if $$(g^*_v v^2/M^2) \approx 1$$. No expansion is thus possible in $$c_{gg}$$ in the general case.

The QCD corrections have been included up to N^3LO in Eq. (3.15) in the limit of large loop-particle masses, similarly to what is done in HDECAY for the SM. In this limit the effect of soft radiation factorizes and is encoded by the coefficient $$\kappa_{soft}$$. The coefficient $$c_{eff}$$, instead, takes into account the correction from the exchange of hard gluons and quarks with virtuality $$q^2 \gg m_t^2$$. More in detail, for $$m_h \ll 2m_t$$, one can integrate out the top quark and obtain the
following five-flavour effective Lagrangian

\[ L_{\text{eff}} = -2^{1/4} G_F^{1/2} C_1 G^0_{a\mu \nu} G^{a\mu \nu}_h , \]

where bare fields are labeled by the superscript 0. The renormalized coefficient function \( C_1 \) encodes the dependence on the top quark mass \( m_t \). The coefficients \( \kappa_{\text{soft}} \) and \( c_{\text{eff}} \) are thus defined as

\[ \kappa_{\text{soft}} = \frac{\pi}{2m_h^4} \text{Im} \Pi^{GG}(q^2 = m_h^2), \]
\[ c_{\text{eff}} = -\frac{12 \pi C_1}{\alpha_s(5)(m_h)}, \]

where \( \Pi^{GG}(q^2) \) is the vacuum polarization induced by the gluon operator. The N3LO expression of the coefficient function \( C_1 \) [24–27] in the on-shell scheme and that of \( \text{Im} \Pi^{GG} \) can be found in Ref. [28]. At NLO the expressions for \( \kappa_{\text{soft}} \) and \( c_{\text{eff}} \) take the well-known form [29]

\[ \kappa_{\text{NLO}}^{\text{soft}} = 1 + \frac{\alpha_s}{\pi} \left( \frac{73}{4} - \frac{7}{6} N_F \right), \quad c_{\text{NLO}}^{\text{eff}} = 1 + \frac{\alpha_s}{\pi} \frac{11}{4}, \]

where here \( \alpha_s \) is evaluated at the scale \( m_h \) and computed for \( N_F = 5 \) active flavours. In eHDECAY it is consistently computed up to N3LO. The last line in Eq. (3.15) contains the additional mass effects at NLO QCD [30] in the top and bottom loops, encoded in \( \kappa_{\text{NLO}}^{\text{soft}}(\tau_q, \tau_{q'}) \), which have been explicitly implemented in HDECAY and taken over in eHDECAY. While the mass effects for the top quark loops play only a minor role, below the percent level, for the bottom loop contribution the mass effects for a 125 GeV Higgs boson amount to about 8% relative to the approximate NLO result. Hence, formula (3.15) includes the QCD corrections at N3LO (i.e. at \( O(\alpha_s^5) \) in the decay rate), and neglects next-to-leading order terms in the derivative expansion (i.e. terms further suppressed by \( O(m_h^2/M^2) \)). The decay width within the MCHM4 and MCHM5 is calculated with the same formula (3.15) by replacing \( c_q \) with the values in column 3 and 4 of Table II and \( c_{gg} \equiv 0 \).

When the SILH Lagrangian (2.2) is selected, on the other hand, eHDECAY computes the
The decay rate into gluons by means of the following approximate formula:

\[
\Gamma(gg)|_{\text{SILH}} = \frac{G_F \alpha_s^2 m_h^2}{4\sqrt{2} \pi^3} \left[ \frac{1}{9} \sum_{q,q'=t,b,c} (1 - \bar{c}_H - \bar{c}_q - \bar{c}_{q'}) A^*_{1/2}(\tau_{q'}) A_{1/2}(\tau_q) \, c_{eff}^2 \kappa_{soft} \right.
\]

\[
+ 2 \Re \left( \sum_{q=t,b,c} \frac{1}{3} A^*_{1/2}(\tau_q) \frac{16\pi}{\alpha_s} \bar{c}_q \right) c_{eff} \kappa_{soft}
\]

\[
+ \left| \sum_{q=t,b,c} \frac{1}{3} A_{1/2}(\tau_q) \right|^2 c_{eff}^2 \kappa_{ew} \kappa_{soft}
\]

\[
+ \frac{1}{9} \sum_{q,q'=t,b} (1 - \bar{c}_H - \bar{c}_q - \bar{c}_{q'}) A^*_{1/2}(\tau_{q'}) A_{1/2}(\tau_q) \left( \kappa_{NLO}^{\tau_{q}, \tau_{q'}} \right) .
\]

(3.21)

The last line contains the mass effects at NLO QCD for the top and bottom quark loops. The NLO electroweak corrections [31, 32] are included through the coefficient \( \kappa_{ew} \) and by neglecting terms of \( O[(\alpha_s/4\pi)^5(\alpha_s/4\pi)] \). The above formula thus includes the leading \( O(v^2/f^2) \) corrections, as well as mixed \( O[(\alpha_s/4\pi)^5(\alpha_s/4\pi)] \) ones. Indeed, we assume factorization of the QCD and EW corrections. Since QCD corrections are dominated by soft gluon radiation, in which QCD and EW effects completely factorize, this is a good approximation. It neglects terms of \( O[(\alpha_s/4\pi)^2] \) and \( O(v^4/f^4) \).

### 3.3 Decay into photons

In the SM the decay of the Higgs boson into a pair of photons is mediated by \( W \) and heavy fermion loops. According to the chiral Lagrangian (2.7), these two contributions to the total amplitude are rescaled, respectively, by the parameters \( c_W \) and \( c_\psi \). Similarly to \( h \to gg \), the contact interaction proportional to \( c_{\gamma\gamma} \) can also contribute significantly. With \( c_{\gamma\gamma} \approx (\alpha_{em}/4\pi)(g^2 v^2/M^2) \), the contribution due to the effective interaction becomes comparable to the loop induced contributions if \( (g^2 v^2/M^2) \approx 1 \). The partial width for a Higgs boson decaying into two photons implemented in eHDECAY in the framework of the

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7Bottom loops contribute \( O(10\%) \) to the SM decay rate and are well approximated by an effective coupling at the 10%-level thus leading to negligible non-factorizing contributions at the percent level.
non-linear Lagrangian is thus given by

\[
\Gamma(\gamma\gamma)|_{NL} = \frac{G_F O_{em}^2 m_h^3}{128 \sqrt{2} \pi^3} \sum_{q=t,b,c} \frac{4}{3} c_q 3 Q_q^2 A_{1/2}^{NLO}(\tau_q) + \frac{4}{3} c_\tau Q_\tau^2 A_{1/2}(\tau_\tau) + c_W A_1(\tau_W) + \frac{4\pi}{\alpha_{em}} c_{\gamma\gamma} \right|^2,
\]

(3.22)

which is approximate at leading order in the derivative expansion, i.e. it neglects terms further suppressed by \(O(m_h^2/M^2)\). By \(Q_{q,\tau}\) we denote the electric charge of the quarks and the \(\tau\) lepton, respectively. Note that \(\alpha_{em}\) is the electromagnetic coupling in the Thomson limit, in order to avoid large logarithms for on-shell photons. We have defined \(\tau_i = 4m_i^2/m_h^2\) \((i = q, \tau, W)\) and the form factor

\[
A_1(\tau) = -[2 + 3\tau + 3\tau(2 - \tau)f(\tau)]
\]

(3.23)

normalized to \(A_1(\infty) = -7\). The top, bottom and charm quark loops receive NLO QCD corrections, while the effective contact interaction does not. The NLO QCD corrected quark form factor is denoted in Eq. (3.22) by

\[
A_{1/2}^{NLO}(\tau_q) = A_{1/2}(\tau_q)(1 + \kappa_{QCD}),
\]

(3.24)

where \(\kappa_{QCD}\) encodes the \(O(\alpha_s/4\pi)\) QCD corrections \([30, 33, 34]\) and \(A_{1/2}(\tau)\) is given in Eq. (3.16). In the MCHM4 and MCHM5 we use the same formula for the decay width with \(c_q\) and \(c_V\) replaced appropriately and \(c_{\gamma\gamma} \equiv 0\).

In order to improve the perturbative behaviour of the QCD-corrected quark loop contributions, they are expressed in terms of the running quark masses \(m_Q(\mu_Q^2)\) \([30, 33]\). These are related to the pole masses \(M_Q\) through

\[
m_Q(\mu_Q^2) = M_Q \left[ \frac{\alpha_s(\mu_Q^2)}{\alpha_s(M_Q^2)} \right]^{12/(33 - 2N_F)} (1 + O(\alpha_s^2))
\]

(3.25)

at the mass renormalization point \(\mu_Q\) with \(N_F = 5\) active flavours. Their scale is identified with \(\mu_Q = M_H/2\). This ensures a proper definition of the \(QQ\) thresholds \(M_H = 2M_Q\) without artificial displacements due to finite shifts between the pole and the running quark masses, as is the case for the running \(\overline{\text{MS}}\) masses. Note, that the same running quark mass \(m_Q(\mu_Q^2)\),
at the renormalization scale \( \mu_Q = M_H/2 \), enters in the lowest order amplitude \( A^{LO}_{1/2} \), which is used in the SILH parametrization hereafter.

In the case of the SILH parametrization, the EW corrections have been incorporated as well. It is useful to define the SM amplitude at leading order (LO) and NLO QCD level as

\[
A_X^{SM} (\gamma\gamma) = \sum_{q=t,b,c} \frac{4}{3} 3 Q_q^2 A_{1/2}^X (\tau_q) + \frac{4}{3} Q_{\tau}^2 A_{1/2} (\tau_{\tau}) + A_{1/2} (\tau_{W}) , \quad X = LO, NLO ,
\]

and the deviation from the SM amplitude as

\[
\Delta A (\gamma\gamma) = - \sum_{q=t,b,c} \frac{4}{3} \left( \frac{\bar{c}_H}{2} + \bar{c}_q \right) 3 Q_q^2 A_{NLO}^{LO} (\tau_q) - \left( \frac{\bar{c}_H}{2} + \bar{c}_{\tau} \right) \frac{4}{3} Q_{\tau}^2 A_{1/2} (\tau_{\tau})
\]

\[
- \left( \frac{\bar{c}_H}{2} - 2 \bar{c}_W \right) A_{1/2} (\tau_{W}) .
\]

The decay width implemented in eHDECAY in the SILH case is thus the following

\[
\Gamma (\gamma\gamma) \big|_{SILH} = \frac{G_F \alpha_{em} m_h^3}{128 \sqrt{2} \pi^3} \left\{ |A_{NLO}^{SM} (\gamma\gamma)|^2 + 2 \text{Re} \left( A_{LO}^{SM*} (\gamma\gamma) A_{ew}^{SM} (\gamma\gamma) \right) \right.
\]

\[
+ 2 \text{Re} \left[ A_{NLO}^{SM*} (\gamma\gamma) \left( \Delta A (\gamma\gamma) + \frac{32 \pi \sin^2 \theta_W \bar{c}_\gamma}{\alpha_{em}} \right) \right] \right\} ,
\]

where \( A_{ew}^{SM} (\gamma\gamma) \) denotes the SM amplitude which comprises the NLO electroweak corrections [31, 35]. Equation (3.28) includes the leading \( O(v^2/f^2) \) and \( O(m_h^2/M^2) \) corrections, while it neglects terms of order \( O(v^4/f^4) \). The electroweak corrections are implemented up to NLO, neglecting corrections of \( O[(\alpha_2/4\pi)(v^2/f^2)] \) and of \( O[(\alpha_2/4\pi)^2] \). Finally, the QCD corrections are included up to NLO, and mixed terms of \( O[(\alpha_2/4\pi)(\alpha_s/4\pi)] \) are neglected.

### 3.4 Decay into \( Z\gamma \)

In the SM the Higgs boson decay into a \( Z \) boson and a photon is mediated by \( W \) boson and heavy fermion loops. Adopting the parametrization of the non-linear Lagrangian, the correction from the effective interaction due to the coupling \( c_{Z\gamma} \) has to be considered, too, and it can become as important as the loop contributions for \( (g_s^2 v^2/M^2) \approx 1 \). The decay

\^8\text{For a Higgs mass value of } M_H = 125 \text{ GeV the running top, bottom and charm quark masses are given by } m_t = 188.03 \text{ GeV, } m_b = 3.44 \text{ GeV and } m_c = 0.76 \text{ GeV. They differ from the running MS masses.}
width is therefore given by (here also $\alpha_{em} \equiv \alpha_{em}(0)$):

$$\Gamma(Z\gamma)|_{NL} = \frac{G_F^2 \alpha_{em} m_W^2 m_h^2}{64\pi^4} \left(1 - \frac{m_Z^2}{m_h^2}\right)^3$$

$$\times \sum_{\psi} c_\psi N_c Q_\psi \hat{v}_\psi A_{1/2}^{Z\gamma}(\tau_\psi, \lambda_\psi) + c_W A_1^{Z\gamma}(\tau_W, \lambda_W) - \frac{4\pi}{\sqrt{\alpha_{em}\alpha_2}} c_{Z\gamma} \right|_2^{2},$$  \hspace{1cm} (3.29)

with $\tau_i = 4m_i^2/m_h^2$, $\lambda_i = 4m_i^2/m_Z^2$ and $\hat{v}_\psi = 2I_3^\psi - 4Q_\psi \sin^2 \theta_W$ ($\psi = t, b, c, \tau$) in terms of the third component of the weak isospin $I_3^\psi$ and the electric charge $Q_\psi$. The form factors are defined by [36]

$$A_{1/2}^{Z\gamma}(\tau, \lambda) = [I_1(\tau, \lambda) - I_2(\tau, \lambda)],$$

$$A_1^{Z\gamma}(\tau, \lambda) = \cos \theta_W \left\{ 4 \left(3 - \tan^2 \theta_W \right) I_2(\tau, \lambda) + \left[ (1 + \frac{2\tau}{\lambda}) \tan^2 \theta_W - \left(5 + \frac{2\tau}{\lambda}\right) \right] I_1(\tau, \lambda) \right\}. $$  \hspace{1cm} (3.30)

The functions $I_1$ and $I_2$ can be cast into the form

$$I_1(\tau, \lambda) = \frac{\tau \lambda}{2(\tau - \lambda)} + \frac{\tau^2 \lambda^2}{2(\tau - \lambda)^2} \left[ f(\tau) - f(\lambda) \right] + \frac{\tau^2 \lambda}{(\tau - \lambda)^2} \left[ g(\tau) - g(\lambda) \right]$$

$$I_2(\tau, \lambda) = -\frac{\tau \lambda}{2(\tau - \lambda)} \left[ f(\tau) - f(\lambda) \right],$$  \hspace{1cm} (3.31)

where $f(\tau)$ is defined in Eq. (3.17) and $g(\tau)$ reads

$$g(\tau) = \begin{cases} 
\sqrt{\tau - 1} \arcsin \frac{1}{\sqrt{\tau}} & \tau \geq 1 \\
\frac{\sqrt{1 - \tau}}{2} \left[ \ln \frac{1 + \sqrt{1 - \tau}}{1 - \sqrt{1 - \tau}} - i\pi \right] & \tau < 1.
\end{cases} $$  \hspace{1cm} (3.32)

The QCD radiative corrections [37] are small and thus have been neglected, while the NLO EW corrections are unknown. Because of the smallness of the QCD corrections, there is no relevant issue arising from the intrinsic uncertainty due to the unknown higher-order corrections, so that the choice of the scheme in which the quark masses are calculated does not play any role. In eHDECAY we use the pole masses for the quarks. Finally, Eq. (3.29) neglects terms further suppressed by $O(m_h^2/M^2)$, which are of higher-order in the derivative expansion. The decay width for the MCHM4 and MCHM5 is obtained by replacing $c_\psi$ and $c_W$ with the coupling values of column 3 and 4 of Table I and setting $c_{Z\gamma} \equiv 0$.  

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In the SILH parametrization the decay width is computed by \texttt{eHDECAY} according to the
formula
\[ \Gamma(Z\gamma)|_{SILH} = \frac{G_F^2 \alpha_{em} m_W^2 m_h^3}{64 \pi^4} \left( 1 - \frac{m_Z^2}{m_h^2} \right)^3 \]
\[ \times \left\{ \left| A^{SM}(Z\gamma) \right|^2 + 2 \Re \left( A^{SM*}(Z\gamma) \Delta A(Z\gamma) \right) \right. \]
\[ + \left. 2 \Re \left[ -\frac{4 \pi \tan \theta_W}{\sqrt{\alpha_{em} \alpha_2}} (\tilde{c}_{HB} - \tilde{c}_{HW} - 8 \tilde{c}_\gamma \sin^2 \theta_W) A^{SM*}(Z\gamma) \right] \right\}, \]

where we have defined the LO SM amplitude
\[ A^{SM}(Z\gamma) = \sum_\psi \frac{N_\psi Q_\psi \hat{\psi}}{\cos \theta_W} A^{Z\gamma}_{1/2}(\tau_\psi, \lambda_\psi) + A^{Z\gamma}_1(\tau_W, \lambda_W) \] (3.34)
and the deviation from the SM amplitude
\[ \Delta A(Z\gamma) = -\sum_\psi \left( \frac{\tilde{c}_H}{2} + \tilde{c}_\psi \right) \frac{N_\psi Q_\psi \hat{\psi}}{\cos \theta_W} A^{Z\gamma}_{1/2}(\tau_\psi, \lambda_\psi) - \left( \frac{\tilde{c}_H}{2} - 2 \tilde{c}_W \right) A^{Z\gamma}_1(\tau_W, \lambda_W). \] (3.35)

Equation (3.33) includes corrections of \(O(v^2/f^2)\) and \(O(m_h^2/M^2)\). The EW corrections are unknown, and small QCD radiative corrections have been neglected.

### 3.5 Decays into \(WW\) and \(ZZ\) boson pairs

The Higgs boson decay into a pair of massive vector bosons is important not only above the threshold, but also below. For example, in the SM with \(m_h = 125\) GeV the branching ratio of \(h \to WW\) is about 20%. In \texttt{HDECAY} various options are present to compute the partial decay widths with on-shell or off-shell bosons, controlled by the ON-SH-WZ input parameter. In \texttt{eHDECAY} we have implemented the case ON-SH-WZ=0, which includes the double off-shell decays \(h \to W^*W^*, Z^*Z^*\). For this case, which is obviously the most complete as it takes into account both on-shell and off-shell contributions, the partial decay width \(h \to V^*V^*\) \((V = W, Z)\) can be written in the following compact form [38]:

\[ \Gamma(V^*V^*) = \frac{1}{\pi^2} \int_0^{m_h^2} \frac{dQ_1^2}{(Q_1^2 - m_V^2)^2 + m_V^2 \Gamma_V^2} \int_0^{(m_h - Q_1)^2} \frac{dQ_2^2}{(Q_2^2 - m_V^2)^2 + m_V^2 \Gamma_V^2} \Gamma(VV), \] (3.36)
where $Q^2_1$, $Q^2_2$ are the squared invariant masses of the virtual gauge bosons and $m_V$ and $\Gamma_V$ their masses and total decay widths. In the parametrization of Eq. (2.7), by defining
\[
a_{VV} = c_{VV} \frac{m_h^2}{m_V^2}, \quad a_{V\partial V} = \frac{c_{V\partial V} m_h^2}{2 m_V^2}, \quad (3.37)
\]
the squared matrix element $\Gamma(VV)$ reads
\[
\Gamma(VV)\big|_{NL} = \Gamma^{SM}(VV) \times \left\{ c_V^2 - 2c_V \left[ \frac{a_{VV}}{2} \left( 1 - \frac{Q^2_1 + Q^2_2}{m_h^2} \right) + a_{V\partial V} \frac{Q^2_1 + Q^2_2}{m_h^2} \right] + c_V a_{VV} \frac{\lambda(Q^2_1, Q^2_2, m_h^2)}{\lambda(Q^2_1, Q^2_2, m_h^2) + 12 Q^2_1 Q^2_2/m_h^4} \right\}, \quad (3.38)
\]
with \[38\]
\[
\Gamma^{SM}(VV) = \frac{\delta_V G_F m_h^3}{16\sqrt{2}\pi} \sqrt{\lambda(Q^2_1, Q^2_2, m_h^2)} \left( \lambda(Q^2_1, Q^2_2, m_h^2) + \frac{12Q^2_1 Q^2_2}{m_h^4} \right), \quad (3.39)
\]
where $\delta_V = 2(1)$ for $V = W(Z)$ and $\lambda(x, y, z) \equiv (1 - x/z - y/z)^2 - 4xy/z^2$. The second and third term in Eq. (3.38) represent the interference between the tree-level contribution and the one from the derivative operators. They are of order $O(m_h^4/M^2)$, hence next-to-leading in the chiral expansion compared to the tree-level contribution; we have consistently neglected terms quadratic in $a_{VV}$ and $a_{V\partial V}$, since they are of $O(m_h^4/M^4)$, which is beyond the accuracy of the effective Lagrangian (2.7). Setting $a_{VV} = a_{V\partial V} = 0$ and $c_V = \sqrt{1 - \xi}$ we obtain the decay formula for the MCHM4 and MCHM5.

In the SILH parametrization the squared matrix element $\Gamma(VV)$ implemented in eHDECAY reads
\[
\Gamma(VV)\big|_{SILH} = \Gamma^{SILH}(VV) + \Gamma^{SM}(VV) \frac{2}{|A_0^{SM}|^2} \text{Re} \left( A_0^{SM} A_{ew}^{SM} \right), \quad (3.40)
\]
where $A_0^{SM}$ denotes the SM LO amplitude and $A_{ew}^{SM}$ is the SM amplitude which comprises the NLO EW corrections [39] (the same remark as in footnote 6 applies). Furthermore,
\[
\Gamma^{SILH}(VV) = \Gamma^{SM}(VV) \times \left\{ 1 - \bar{c}_{H} - 2\bar{c}_{T}\delta_V Z - 2 \left[ \frac{\bar{a}_{VV}}{2} \left( 1 - \frac{Q^2_1 + Q^2_2}{m_h^2} \right) + \bar{a}_{V\partial V} \frac{Q^2_1 + Q^2_2}{m_h^2} \right] + \bar{a}_{VV} \frac{\lambda(Q^2_1, Q^2_2, m_h^2)}{\lambda(Q^2_1, Q^2_2, m_h^2) + 12 Q^2_1 Q^2_2/m_h^4} \right\}, \quad (3.41)
\]
with $\delta_{VZ} = 0(1)$ for $V = W(Z)$ and where we have defined,

\[
\bar{a}_{WW} = -2 \frac{m_h^2}{m_W^2} \bar{c}_{HW}
\]

\[
\bar{a}_{W\partial W} = -2 \frac{m_h^2}{2m_W^2} (\bar{c}_W + \bar{c}_{HW})
\]

\[
\bar{a}_{ZZ} = -2 \frac{m_h^2}{m_Z^2} (\bar{c}_{HW} + \bar{c}_{HB} \tan^2 \theta_W - 4\bar{c}_i \tan^2 \theta_W \sin^2 \theta_W)
\]

\[
\bar{a}_{Z\partial Z} = -2 \frac{m_h^2}{2m_Z^2} (\bar{c}_W + \bar{c}_{HW} + (\bar{c}_B + \bar{c}_{HB}) \tan^2 \theta_W)
\]  \hspace{1cm} (3.43)

The decay width (3.40) includes terms of $O(v^2/f^2)$, $O(m_h^2/M^2)$ and $O(\alpha_2/4\pi)$, while it neglects contributions of $O(v^4/f^4)$ and $O[(\alpha_2/4\pi)^2]$. The corrections of $O[(\alpha_2/4\pi)(v^2/f^2)]$ are only partly included through the terms proportional to $\bar{c}_{HW}$ and $\bar{c}_{HB}$, cf. [1].

4 Numerical formulas for the decay rates in the SILH Lagrangian

We display here numerically approximated formulas of the Higgs decay rates valid at linear order in the effective coefficients $\bar{c}_i$ of the SILH Lagrangian (2.2) for $m_h = 125$ GeV. All the ratios $\Gamma/\Gamma_{SM}$ have been computed by switching off the EW corrections, since their effect on the numerical prefactors appearing in front of the coefficients $\bar{c}_i$ is of order $(v^2/f^2)(\alpha_2/4\pi)$ and thus beyond the accuracy of the formulas implemented in eHDECAY. Conversely, we have fully included the QCD corrections, as they multiply both the SM and the NP terms. The numerical results are thus the following:

\[
\frac{\Gamma(\bar{\psi}\psi)}{\Gamma(\psi\psi)_{SM}} \approx 1 - \bar{c}_H - 2 \bar{c}_i, \quad \text{for } \psi = \text{leptons, top-quark}.
\]  \hspace{1cm} (4.44)

The QCD corrections to the decays decays into charm, strange or bottom quark pairs involve two-loop diagrams with top quarks loops that are rescaled differently [20]. Taking this into
account, we have the numerical results
\[ \frac{\Gamma(\bar{c}c)}{\Gamma(\bar{c}c)_{SM}} \simeq 1 - \bar{c}_H - 1.985 \bar{c}_c - 0.015 \bar{c}_t, \quad (4.45) \]
\[ \frac{\Gamma(\bar{s}s)}{\Gamma(\bar{s}s)_{SM}} \simeq 1 - \bar{c}_H - 1.971 \bar{c}_s - 0.029 \bar{c}_t, \quad (4.46) \]
\[ \frac{\Gamma(\bar{b}b)}{\Gamma(\bar{b}b)_{SM}} \simeq 1 - \bar{c}_H - 1.992 \bar{c}_b - 0.0085 \bar{c}_t. \quad (4.47) \]

Furthermore,
\[ \frac{\Gamma(h \rightarrow W^*(W^*)^*)}{\Gamma(h \rightarrow W^*(W^*)^*)_{SM}} \simeq 1 - \bar{c}_H + 2.2 \bar{c}_W + 3.7 \bar{c}_{HW}, \quad (4.48) \]
\[ \frac{\Gamma(h \rightarrow Z^*(Z^*)^*)}{\Gamma(h \rightarrow Z^*(Z^*)^*)_{SM}} \simeq 1 - \bar{c}_H - 2 \bar{c}_T + 2.0 (\bar{c}_W + \tan^2\theta W \bar{c}_B) 
+ 3.0 (\bar{c}_{HW} + \tan^2\theta W \bar{c}_{HB}) - 0.26 \bar{c}_\gamma, \quad (4.49) \]
\[ \frac{\Gamma(h \rightarrow Z\gamma)}{\Gamma(h \rightarrow Z\gamma)_{SM}} \simeq 1 - \bar{c}_H + 0.12 \bar{c}_t - 5 \cdot 10^{-4} \bar{c}_c - 0.003 \bar{c}_b - 9 \cdot 10^{-5} \bar{c}_\tau 
+ 4.2 \bar{c}_W + 0.19 (\bar{c}_{HW} - \bar{c}_{HB} + 8 \bar{c}_\gamma \sin^2\theta W) \frac{4\pi}{\sqrt{\alpha_2} \alpha_{em}}, \quad (4.50) \]
\[ \frac{\Gamma(h \rightarrow \gamma\gamma)}{\Gamma(h \rightarrow \gamma\gamma)_{SM}} \simeq 1 - \bar{c}_H + 0.54 \bar{c}_t - 0.003 \bar{c}_c - 0.007 \bar{c}_b - 0.007 \bar{c}_\tau 
+ 5.04 \bar{c}_W - 0.54 \bar{c}_\gamma \frac{4\pi}{\alpha_{em}}, \quad (4.51) \]
\[ \frac{\Gamma(h \rightarrow gg)}{\Gamma(h \rightarrow gg)_{SM}} \simeq 1 - \bar{c}_H - 2.12 \bar{c}_t + 0.024 \bar{c}_c + 0.1 \bar{c}_b + 22.2 \bar{c}_g \frac{4\pi}{\alpha_2}. \quad (4.52) \]

5 How to run eHDECAY: Input/Output Files

All files necessary for compiling and running eHDECAY can be obtained at the URL:
http://www-itp.particle.uni-karlsruhe.de/~maggie/eHDECAY/

The program eHDECAY is self-contained, like the original code HDECAY on which it is based.

The new features related to the Lagrangian parametrizations proposed in this paper are encoded in the main source file, ehdecay.f, while other linked routines (called dmb.f, elw.f,
feynhiggs.f, haber.f, hgaga.f, hgg.f, hsqsq.f and susylha.f) are taken over from the original version. Of course \texttt{eHDECAY}, besides calculating Higgs branching ratios and decay widths according to the non-linear, SILH or MCHM4/5 Lagrangians, also calculates the SM and MSSM ones, exactly as \texttt{HDECAY} 5.10 does. The choice can be done through the flags HIGGS and COUPVAR set in the input file. The input file for \texttt{eHDECAY} has been called ehdecay.in and is based on the file hdecay.in of the official version 5.10, supplemented by further input values. Thus, with the flag LAGPARAM the user can choose between the general SILH parametrization Eq. (2.2), the model-specific parametrizations MCHM4 and MCHM5 and the general non-linear Lagrangian parametrization Eq. (2.7). Furthermore, the various related couplings can be set. The input values are explained in the following:

**COUPVAR, HIGGS:** If HIGGS=0 and COUPVAR=1, then the Higgs decay widths and branching ratios are calculated within the parametrization chosen by:

**LAGPARAM:**
- 0: Non-linear Lagrangian parametrization Eq. (2.7)
- 1: SILH parametrization Eq. (2.2)
- 2: MCHM4/5 parametrization (cf. Table 1)

**IELW:** Turn off (0) or on (1) the electroweak corrections for the SILH parametrization.\footnote{Note, that this parameter IELW has nothing to do with the parameter ELWK in the input file of \texttt{HDECAY}, where the meaning of this flag is different.}

For the non-linear Lagrangian the following parameters have to be set for the couplings of the various vertices\footnote{We explain them here all, although they are in part already present in the input file for \texttt{HDECAY} 5.10.}
In case of the SILH parametrization the input values to be set in order to calculate the various couplings are:

\[
\begin{align*}
    \text{CV: } hVV & \quad \text{vertex, } (V=W, Z) \\
    \text{Ct: } h\ell\ell & \quad \text{vertex} \\
    \text{Cs: } hs\bar{s} & \quad \text{vertex} \\
    \text{CZga: } & \quad \text{coupling } c_{\gamma\gamma} \\
    \text{CWdW: } & \quad \text{coupling } c_{W\partial W} \\
    \text{Cmubar: } & \quad \bar{c}_\mu \\
    \text{Ctbar: } & \quad \bar{c}_t \\
    \text{Cbbar: } & \quad \bar{c}_b \\
    \text{Ccbar: } & \quad \bar{c}_c \\
    \text{Cgaga: } & \quad \text{coupling } c_{\gamma\gamma} \\
    \text{CWbar: } & \quad \bar{c}_W \\
    \text{CBbar: } & \quad \bar{c}_B \\
    \text{CHWbar: } & \quad \bar{c}_{HW} \\
    \text{CHBbar: } & \quad \bar{c}_{HB} \\
    \text{Cgambar: } & \quad \bar{c}_\gamma \\
    \text{Cgbar: } & \quad \bar{c}_g
\end{align*}
\]

In the MCHM4/5 parametrization we have the input values:

**FERMREPR:**

1: MCHM4
2: MCHM5

**XI:** the value for \( \xi \)

For example:

```
COUPVAR = 1
HIGGS = 0
: 
************** LAGRANGIAN 0 - chiral 1 - SILH 2 - MCHM4/5 **************
LAGPARAM = 0
**** Turn off (0) or on (1) the elw corrections for LAGPARAM = 1 or 2 ****
IELW = 1
```
computes the branching ratios for $c_V = 1$, $c_\psi = 0.95$ ($\psi = t, b, c, s, \tau, \mu$), $c_{\gamma\gamma} = 0.005$, $c_{gg} = 0.001$ and $c_{Z\gamma} = c_{WW} = c_{ZZ} = c_{W\delta W} = c_{Z\delta Z} = 0$ in the general parametrization Eq. (2.7).

The program is compiled with the help of a makefile. With the command 'make' an executable is created, called run. Upon running the program by typing './run', the input values are read in from ehdecay.in and the calculation of the decay widths and branching ratios is performed. The output is written into the files br.eff1 and br.eff2, where the Higgs mass, branching ratios and total width are reported. For the previous example, at $m_h = 125$ GeV and for all the other parameters set at their standard values, the output reads
All the input parameters of the corresponding run are printed out in the file br.input. Otherwise, setting COUPVAR=0, the program produces the usual output files with SM or MSSM results according to the HDECAY 5.10 version.

6 Conclusions

We have described the Fortran code eHDECAY, which calculates the partial widths and the branching fractions of the decays of the Higgs boson in the Standard Model and its extension by the dimension-6 operators of the SILH Lagrangian \(^{(2.2)}\). The program also implements the more general non-linear effective Lagrangian \(^{(2.7)}\), which does not rely on assuming the Higgs boson to be part of an \(SU(2)\)_L doublet. In the SM, all decay modes are included as in the original version of HDECAY. The corrections due to the effective operators have been included consistently with the multiple perturbative expansion in the number of derivatives, fields and SM couplings. The level of approximation of the formulas implemented in eHDECAY has been discussed in detail for each decay final state. The QCD corrections to the hadronic decays as well as the possibility of virtual intermediate states have been incorporated according to the present state of the art. The QCD corrections are assumed to factorize also in the presence of higher-dimension operators, so that they are included in factorized form in all extensions of the SM. For the SILH case we have added the electroweak corrections to the SM part only and left the dimension-6 contributions at LO in the context of electroweak corrections, since deviations from the SM case are assumed to be small. In the case of the non-linear Lagrangian however, deviations can be large so that the non-factorizing
electroweak corrections to the SM part are subleading and thus have not been taken into account for consistency.

The program is fast and can be used easily. The basic SM and SILH/non-linear input parameters can be chosen from an input file. Examples of output files for the decay branching ratios have been given.

Since electroweak corrections involving the novel operators have not been calculated yet, the treatment of this type of corrections is not complete. During the coming years one may expect that these electroweak corrections will be determined so that the existing code eHDECAY can be extended to incorporate them. For the moment the present version of eHDECAY provides the state-of-the art for the partial Higgs decay widths and branching ratios in extensions of the SM by a SILH or a non-linear effective Lagrangian.

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