Implications of the Measurements of $U_{e3}$ to Theory

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Abstract

The predictions of $U_{e3}$ are discussed. Typical models which lead to the large, the seizable and the tiny $U_{e3}$ are also studied.

1 Introduction

The measurement of the neutrino mixing $U_{e3}$ is the main target in the long baseline experiments of the neutrino oscillations. The magnitude of the $U_{e3}$ will give a big impact on the model building. Furthermore, the CP violation measure $J_{CP}$ and the lepton flavor violation $Br(\mu \to e\gamma)$ depend on $U_{e3}$.

We assume that oscillations need only account for the solar neutrinos and atmospheric neutrinos data in the three families. Then, the atomospheric neutrino data constrain $U_{\mu 3} \simeq U_{\tau 3} \simeq 1/\sqrt{2}$, on the other hand, there are a few options of the mixing: $U_{\nu 2} \simeq 1/\sqrt{2}$, $\lambda^{2\sim 3}$ ($\lambda \simeq 0.2$), which depend on the solar neutrino solutions. Therefore, there are two possible mixing matrices: the bi-maximal and the single maximal ones.

How large is $U_{e3}$? The Choose experiments constrain $U_{e3} < 0.16$ [1]. Atmospheric neutrino and solar neutrino data also constrain $U_{e3}$ [2]. Let us consider the case of the quark mixings. CKM mixings are given in terms of $\lambda$ in the Wolfenstein parametrization as follows: $|V_{ub}| \simeq \lambda^3$, $|V_{cb}| \simeq \lambda^2$, $|V_{us}| \simeq \lambda$, which are related with quark mass ratios. Since the MNS [3] mixings are much different from CKM mixings, it is very difficult to a reliable prediction of $U_{e3}$.

* Talk at the International Workshop NuFACT’01, Tsukuba, Japan (May 2001)
2 How to predict $U_{e3}$

Since there is a preferred basis given by the underlying theory of the model, the MNS mixing matrix is given as follows:

$$U_{MNS} = L_E L_\nu, \quad L^\dagger_E M_E R_E = M_E^{\text{diag}}, \quad L^\dagger_\nu M_\nu L_\nu = M_\nu^{\text{diag}}. \quad (1)$$

Taking $(L_E)_{ij} = L_{ij}$ and $(L_\nu)_{ij} = N_{ij}$, we get $U_{e2}, U_{\mu3}$ and $U_{e3}$ as

$$U_{e2} = L_{11}^* N_{12} + L_{21}^* N_{22} + L_{31}^* N_{32}, \quad U_{\mu3} = L_{12}^* N_{13} + L_{22}^* N_{23} + L_{32}^* N_{33}, \quad U_{e3} = L_{11}^* N_{13} + L_{21}^* N_{23} + L_{31}^* N_{33}. \quad (2)$$

We show a few examples with $U_{\mu3} \simeq U_{\mu3} \simeq 1/\sqrt{2}$. The first case is (1) : $N_{23} \simeq N_{33} \simeq 1/\sqrt{2}, \quad L_{ij} \ll 1 (i \neq j)$. Then, we get $U_{e2} = N_{12} + (L_{21}^* - L_{31}^*)/\sqrt{2}$ and $U_{e3} = N_{13} + (L_{21}^* + L_{31}^*)/\sqrt{2}$. If $L_{12} \gg L_{21}, L_{31}, N_{13}$, we get $U_{e2} \simeq U_{e3}$, on the other hand, if $N_{12} \gg L_{21}, L_{31}, N_{13}$, we get $U_{e2} \gg U_{e3}$.

The second case is (2) : $L_{22} \simeq L_{32} \simeq 1/\sqrt{2}, \quad N_{ij} \ll 1 (i \neq j)$. Then, $U_{e2} = N_{12} + L_{21}^*$ and $U_{e3} = N_{13} + L_{31}^* N_{23} + L_{31}^*$ are obtained. If $N_{12} \simeq N_{23} \simeq L_{21} \simeq \lambda$, we get $U_{e2} \simeq 2\lambda$ (LMA-MSW) and $U_{e3} \simeq \lambda^2$. The last case is (3) : NNI-form texture [4] of $M_E$. In this basis, we can give

$$M_U = m \begin{pmatrix} 0 & a_u & 0 \\ a_u & 0 & b_u \\ 0 & b_u & c_u \end{pmatrix}, \quad M_D = m \begin{pmatrix} 0 & a_d & 0 \\ c_d & 0 & b_d \\ 0 & d_d & e_d \end{pmatrix}. \quad (3)$$

Using the SU(5) relation $M_E = M_D^T$, we obtain

$$L_E^\dagger = \begin{pmatrix} 1 & -\sqrt{m_e y^2} & \sqrt{m_e \frac{1-y^2}{y^2}} \\ -\frac{1}{y^2} \sqrt{m_\mu y z} & \frac{m_\mu}{m_e} \sqrt{1-y^2} & -\sqrt{1-y^2} \\ -\frac{1}{y^2} \sqrt{m_\tau y z} & \frac{m_\tau}{m_e} \sqrt{1-y^2} & y^2 \end{pmatrix}, \quad (4)$$

where $y, z$ are free parameters. Assuming $U_{MNS} \simeq L_E^\dagger$, which corresponds to $N_{ij} \ll 1 (i \neq j)$, we predict

$$\cot \theta_{\mu\tau} \equiv \frac{U_{e3}}{U_{\mu3}} = \left| \frac{U_{e2}}{U_{\mu3}} \right|, \quad \cot \theta_{\mu\tau} = \frac{m_\mu}{m_\tau} \sqrt{\frac{m_c}{m_t}} e^{-i\theta}, \quad (5)$$

which is testable in the future.
3 Large $U_{e3}$ or Tiny $U_{e3}$

Let us consider models which predict the large $U_{e3} \simeq O(\lambda \sim 1)$. The typical model is the anarchy [5]. In this model, there is no suppression in the Yukawa couplings, and so all elements are the same order. Then, one predicts $U_{e3} \sim O(1)$, which should be observed very soon.

Next, consider models with seizable $U_{e3} \simeq O(\lambda^3 \sim \lambda)$. The typical one is models with U(1) flavor symmetry [6] or the conformal fixed point (CFT) theory [7]. In these models, the Yukawa couplings are given as $Y_{ij} = \epsilon L_{i} \epsilon R_{j}$ with $\epsilon L_{i} (R_{j}) = (M_{C}/M_{X})^{a_{L(R)}^i}$. Since $\theta_{Lij} \simeq \epsilon_{Li}/\epsilon_{Lj}$ and $m_{i}/m_{j} = (\epsilon_{Li}\epsilon_{Ri})/(\epsilon_{Lj}\epsilon_{Rj})$, we get $U_{e3} = U_{e2} \times U_{\tau 2} \simeq (1/\sqrt{2})U_{e2}$. Models with the non-abelian flavor symmetry also predict $U_{e3} = O(\lambda \sim \lambda^2)$ [8].

Let us consider the case of the tiny $U_{e3} \ll O(\lambda^3)$. Typical one is the generation of radiative neutrino masses [9] as follows:

$$M_{\nu} \sim \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & \epsilon \\ \epsilon & 0 & 0 \end{pmatrix}, \quad U_{\nu} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$  \hspace{1cm} (6)

In this model, one gets the inverse mass hierarchy $|m_{1}| \simeq |m_{2}| \gg |m_{3}|$ and $U_{e3} \leq 10^{-4}$.

4 Summary

There are many models which predict $U_{e3}$. The large $U_{e3}$ and the tiny one can be predicted as well as the seizable one. In order to test the models precisely, we need testable sum rules among $V_{CKM}$ and $U_{MNS}$ elements [4,10]. For example, there are beautiful relations [10] as follows:

$$\cot \theta_{\mu\tau} \equiv \frac{|U_{\tau 3}|}{|U_{\mu 3}|} = \frac{|U_{\mu 2}|}{|U_{\mu 3}|} = \frac{|U_{e 2}|}{|U_{e 3}|}, \quad \cot \theta_{e\tau} = \frac{m_{b}}{m_{s}} \cdot \frac{V_{cb}}{|V_{cb}|},$$  \hspace{1cm} (7)

which can be tested by LBL experiments in the future. The measurement of $U_{e3}$ will give a big impact on the model building.

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