Investigation of the Prompt SNe Ia progenitor nature through the analysis of the chemical composition of globular clusters and circumgalactic clouds

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ABSTRACT
A method is proposed for determining the properties of type Ia supernovae from short-lived precursors – Prompt SNIa. This method is based on the assumption that this subtype of type Ia supernovae exploded into low-metallicity globular clusters (GCs), and is responsible for the enrichment of the high-metallicity subgroup of GCs and circumgalactic clouds (CGCs) with the iron peak elements. We justify that CGCs are the formation places of GCs of both subgroups. The accuracy of the method depends, first, on the number of GCs, the spectra of which have been studied in detail; second, on the number of chemical elements, the abundances of which have been worked out. Only those elements are of interest for this method that are produced in supernova explosions and are not produced at the previous stage of the stellar evolution. Our estimates of nucleosynthesis in low-metallicity supernova GCs are in the best agreement with the following Prompt SNIa model: Single Degenerate Pure Deflagration Models of white dwarfs (WDs) burning with masses in the range from $1.30 M_\odot$ to $1.31 M_\odot$ if carbon explodes in the centre of a WD with a low central density from $0.5 \cdot 10^9 \text{g/cm}^3$ to $10^9 \text{g/cm}^3$.

Key words: (Galaxy:) globular clusters: general – (stars:) supernovae: general – galaxies: evolution – galaxies: formation – (galaxies:) quasars: absorption lines

1 INTRODUCTION
The possibility to consider circumgalactic clouds (CGCs) as remnants of the parent clouds, in which globular clusters (GCs) have been formed, was first established by Acharova & Sharina (2018) based on the following facts. First, both types of objects show a bimodal distribution with the minimum near $[\text{Mg}/H] = -0.9$. The mean values and standard deviations of the Mg abundance in GCs and CGCs with $[\text{Mg}/H] < -0.9$ and $[\text{Mg}/H] > -0.9$ are very similar. This coincidence implies that the clouds and GCs can be evolutionary related. Second, high-metallicity CGCs are observed at redshifts smaller than $z \sim 2$. At the redshifts $2.5 < z < 4$, the metallicity distribution of clouds is presented mainly by the metal-poor component (Rafelski et al. 2012; Lehner et al. 2016). High-metallicity CGCs appeared for the first time at $z \sim 2.5$ (Rafelski et al. 2012; Berg et al. 2014; Jorgenson et al. 2013). This means that high-metallicity gas has originated in the formation epoch of GCs. Therefore, heavier-element enrichment of the fraction of metal-poor gas has taken place through thermonuclear fusion products of supernovae (SNe) of the first GC generation entering into it. Subsequently, the second generation has been formed from this enriched gas. In addition to the closeness of the metallicity distributions and the presence of GCs and CGCs at $z < 2$, we can offer more observations in support of the hypothesis of formation of GCs from CGCs. The observation facts and theoretical modelling results indicate that the mass of CGCs is proportional to the mass of the halos of the galaxies, in the vicinity of which they are observed (Perez-Rafols et al. 2007). On the other hand, according to Blakeslee (1999), the GC formation rate is universal and is $0.5 - 1$ GC per $10^9 M_\odot$ of a parent galaxy. In other words, the number of both objects we considered increases monotonically with host galaxy mass. To justify the formulation of the problem solved in our paper, it is important to note the following.

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There is no clear relationship between \( \alpha/\text{Fe} \) and the age of GCs. This issue was discussed in the papers by Carney (1996). The decrease of \( \alpha/\text{Fe} \) with the increase of metallicity in the Galactic disc stars or in dwarf galaxies with extended star-formation histories is believed to be a direct indication of the iron production enhanced by an increasing amount of type Ia SNe (SNe Ia) from old stellar populations (Carney 1996; Kirby et al. 2019). However, in the case of GCs, it should be noted that according to the statistical studies by Chattopadhyay et al. (2012), the mean age of the metal-rich Galactic and M31 GCs is 10.637 \( \pm \) 0.468 Gyr, and the mean age of the metal-poor Galactic and M31 GCs is 10.217 \( \pm \) 0.226 Gyr. The high-metallicity group of Galactic GCs contains on average fifteen times greater amount of iron and twelve times greater amount of magnesium than that of low metallicity (Acharova & Sharina 2015). These differences in the abundances cannot be explained by the predominant contribution of core-collapse SNe (SNe CC=SNe type II and SNe types Ib/c) to nucleosynthesis. These facts can be naturally explained thanks to the following recent theoretical discoveries of the last decade. These studies have shown that SNe Ia are formed not only in old stellar populations but also in young stellar populations (Bartunov et al. 1994; Mannucci et al. 2006; Li et al. 2011; Rigault et al. 2013; Kim et al. 2018). That is, not only SNe CC but also SNe Ia from short-lived progenitors explode in GCs at early evolutionary epochs. The nature of the short-lived progenitors of SNe Ia has not yet been established. A young or so-called prompt progenitor population produces SNe Ia on timescales of 100–330 Myr (e.g., Aubourg 2008; Maoz et al. 2010 and references therein).

The analysis of the SNe Ia spectra (Nomoto et al. 2013) suggests that, most likely, each such event is an explosion of a white dwarf that occurs as a result of its interaction with the companion star. Since the density of stars in globular clusters is high, it can be expected that the interaction of stars in them will occur more often than in other stellar groups. Interacting binaries experience significant deviations from a single star evolution. The evolution of multiple stars allows, at least in one of them, a carbon-oxygen core, capable of a subsequent explosion, with a mass of about 1.4\( M_\odot \) required by the theory to form faster in comparison with that of a single star (Anguiano et al. 2020). On this basis, we believe that GCs are the most appropriate laboratories for studying the Prompt SN Ia nucleosynthesis.

Analysis of the fine features in the chemical pattern of the Galactic disc in the paper by Acharova et al. (2013) has shown that there is approximately three-times difference in the mass between the synthesized Fe per a SN Ia exploding in the star-formation regions and a SN Ia exploding in the old stellar population: 0.23 \( \pm \) 0.06\( M_\odot \) and 0.6 \( \pm \) 0.2\( M_\odot \), respectively. A value of 0.6\( M_\odot \) coincides with that obtained in the calculations of nuclear burning in a white dwarf (WD) in the classical SN Ia models: W7 – pure turbulent deflagration models and WDD – delayed detonation or deflagration-detonation transition models (Nomoto et al. 1984; Thielemann et al. 1993). These models both belong to the so-called “Single Degenerate” type.

Theoretical possibility of producing a reduced number of iron peak elements during a SN Ia explosion was first found by Leung & Nomoto (2013) in the study of various burning regimes in white dwarfs. If carbon explodes in the centre of a WD with a low central density from 0.5 \( \times 10^9 \text{g/cm}^3 \) to 1.0 \( \times 10^9 \text{g/cm}^3 \), then, as calculations show, the iron mass synthesized in a two-dimensional pure turbulent deflagration model can be three times lower than that obtained with the W7 and WDD models. According to the authors, the possibility of the carbon ignition at such low densities is possible with a powerful influx of matter onto the WD surface. At a density of 0.5 \( \times 10^9 \text{g/cm}^3 \), an iron mass of 0.21\( M_\odot \) is synthesized (Leung & Nomoto 2018). With the density increase, the mass of the synthesized iron increases. At a density of 10\(^9 \text{g/cm}^3 \), an iron mass of 0.265\( M_\odot \) is synthesized (Leung & Nomoto 2018). A value of 0.23 \( \pm \) 0.06\( M_\odot \) for the iron nucleosynthesis in a Prompt SN Ia obtained in the paper by Acharova et al. (2013) is very close to their arithmetic mean of the aforementioned values.

Thus, the study of nucleosynthesis in GCs allows one to choose the Prompt SN Ia progenitor model or put independent constraints on the theory of the SN nucleosynthesis and can help one understand the mechanism leading to formation of SNe Ia from short-lived progenitors. This is the subject of the present paper.

The idea of our method can be shortly formulated as follows. We will use the method of determination of the Mg and Fe mass synthesized by SNe CC and Prompt SNe Ia explosions in the low-metallicity subgroup of GCs, described in detail by Acharova & Sharina (2018). In this way, we can estimate the corresponding number of SNe CC and SNe Ia. Then, using the determined number of SNe of different types one can explain the nucleosynthesis of other chemical elements. Finally, we choose the Prompt SN Ia progenitor model. Nucleosynthesis of several chemical elements in this model should most closely match the abundances of these elements in GCs obtained from the analysis.

2 OBSERVED DATA ANALYSIS

Looking ahead, it should be noted that to study the properties of supernovae using our method, described in detail below in Sec. 3, accurate estimates of the abundances of several chemical elements are needed. Such data available today only for GCs of our Galaxy will be analysed in this section. We will begin our consideration of the observed data with the metallicity distributions of CGCs in order to emphasize the similarities in the metallicity distributions of CGCs and GCs and to argue for the scenario of CGC evolution associated with the formation of GCs. The proposed scenario is described in Sec. 2.3.

Supernovae that exploded in low-metallicity GCs enriched the surrounding gas with metals. In order to correctly estimate the number of supernovae formed, it is necessary
2.1 Analysis of metallicities of circumgalactic clouds

Acharova & Sharina (2018) discussed the possibility to consider partial Lyman limit systems and Lyman limit systems as the residual parts of clouds, in which GCs were formed. The conclusion was drawn from the statistical analysis of the Mg abundances in GCs (Dias et al. 2016; Carretta et al. 2014; Pritzi et al. 2003) and in CGCs (Wotta et al. 2016), and on the spatial location of these objects. The analysis was carried out for the CGCs located at the redshifts $z \lesssim 1$. Let us consider the metallicity distribution of CGCs at the epoch of the formation of the major amount of GCs, that is at $2 < z < 4$ which corresponds to a time interval of about 9 to 12 Gyr ago.

Gaseous clouds within the virial radii of galaxies are usually classified according to their neutral hydrogen column densities ($\log N_{\text{HI}}$) as follows: damped Lyman limit systems (DLAs) with $\log N_{\text{HI}} \geq 20.3$, sub-damped Lyman limit systems (sub-DLAs) with $19 \leq \log N_{\text{HI}} < 20.3$, Lyman limit systems (LLSs) with $17.2 \leq \log N_{\text{HI}} < 19$, and partial Lyman limit systems (pLLSs) with $16.1 \leq \log N_{\text{HI}} < 17.2$ (Wotta et al. 2013; Quiret et al. 2016). The lower $N_{\text{HI}}$ is, the higher the gas ionization rate is. DLAs and sub-DLAs are mainly composed of neutral gas. pLLSs and LLSs are almost completely ionized (Lehner et al. 2010). While studying the location of CGCs relative to galaxies, one should take into account that only most luminous galaxies can be observed at redshifts higher than $z \sim 2$. In relation to such galaxies, pLLSs and LLSs are always found within a virial radius of $\lesssim 300$ kpc with more than a half found in the circumgalactic medium of massive galaxies (Rudie et al. 2012). It cannot be ruled out that all pLLSs and LLSs may belong to the circum-
are usually determined from the lines of singly ionized ions. We will start our analysis of the metallicity distribution with the densest cloud types: DLAs and sub-DLAs that consist mainly of neutral gas (see Fig. 1). Their metallicities are usually determined from the lines of singly ionized ions (Rafelski et al. 2012; Quiret et al. 2016; Wotta et al. 2019). In Rafelski et al. (2012), Fe II ions and the ions of the α-process elements (S II, Si II) were used to determine metallicity for 2 < z < 4. Sulphur is known to be a volatile element with negligible depletion. This element is considered the best option for determining the metallicity composition. Therefore, if it is possible to determine the composition from this ion, then it is considered that [X/H] = [S/H], and other indicators are not used in Rafelski et al. (2012). Si is the second choice element, it is used if the metallicity cannot be determined from the lines of S II. It is assumed that [X/H] = [Si/H]. For all the clouds, the metallicity was determined by Rafelski et al. (2012) simultaneously on the basis of Fe II ions. [X/H] = [Fe/H] + 0.3. Note that the histograms obtained from different metallicity indicators are different.

The metallicity distribution, built from S II, shows a division into high- and low-metallicity subgroups, with two well-defined maxima and cloud deficit near ([X/H] ∼ −1) (Fig. 1, panel a). The high-metallicity subgroup has half the amount of clouds than the low-metallicity subgroup (see Table 1). The average metallicity of the low-metallicity clouds ([X/H] < −1) is ⟨[X/H]⟩ = −1.53 ± 0.26, and for the high-metallicity clouds ([X/H] > −1), it is ⟨[X/H]⟩ = −0.61 ± 0.19. The metallicity of the same clouds, determined from the Fe II ion abundance, shows a less unambiguous division into high- and low-metallicity subgroups (Fig. 1, panel b). The high-metallicity peak is much weaker than that obtained in the previous case. The average metallicity of the low-metallicity clouds ([X/H] < −1) is ⟨[X/H]⟩ = −1.47 ± 0.22, and for the high-metallicity clouds ([X/H] > −1), it is ⟨[X/H]⟩ = −0.64 ± 0.31. 90% of other 69 clouds from the Rafelski et al. (2012) sample in the same redshift range, the metallicities of which were determined based on Si II ions (Fig. 1, panel c) belong to low-metallicity subgroups with ⟨[X/H]⟩ = −1.72 ± 0.42.

Figure 1 shows a noticeable difference in the distributions of [X/H] in S II and Si II ions with respect to the dividing line [X/H] = −1. The minimum between the high- and low-metallicity components is distinct only in the distribution based on S II ions, which most reliably reflects the metallicity (Rafelski et al. 2012).

In Rafelski et al. (2012), [X/H] for z < 1 was determined, almost completely, from Fe II ions. As can be seen from (Fig. 1, panel d), this study presents clouds mainly from the high-metallicity subgroup. The average metallicity of the high-metallicity clouds ([X/H] > −1) is ⟨[X/H]⟩ = −0.74 ± 0.13 (see Table 1). As will be seen from further discussion (see also Fig. 3), for pLLSs and LLSs at z < 1, on the contrary, there are more low-metallicity clouds than high-metallicity ones. Therefore, the profile observed by Rafelski et al. (2012) for DLAs metallicity requires further careful thought which is out of the scope of this study.

Next, let us consider the metallicity measurements of sub-DLAs located at 1.8 < z < 4.0 (see, please, Table 1, Fig. 3). These data were compiled from the literature by Quiret et al. (2016). The metallicity was determined using ions of different metals and also represented as a histogram with the averaging bin of 0.25 dex. We have extended the redshift range to z = 1.8, since half of the high-metallicity clouds, whose metallicity was measured based on S II ions, were in the range of 1.8 < z < 2.0. Let us discuss the resulting distributions.

The metallicity distribution of 51 clouds obtained from the analysis of the S II ion lines has the minimum near [X/H] = −1 and is divided into two components: the high-metallicity component (about 30% of clouds) with...
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\langle [X/H] \rangle = -0.54 \pm 0.26 \quad \text{and the low-metallicity component with} \quad \langle [X/H] \rangle = -1.53 \pm 0.33 \quad \text{(Fig. 2 panel a)}.
\]

The metallicity distribution of the other 86 clouds, obtained from the analysis of the Si II ion lines, is represented mainly by the low-metallicity component. The high-metallicity component (about 20% of clouds) is characterized by \( \langle [X/H] \rangle = -0.55 \pm 0.27 \quad \text{and the low-metallicity component} \quad \langle [X/H] \rangle = -1.73 \pm 0.42 \quad \text{(Fig. 2 panel b)}.

The metallicity distribution of 33 clouds, obtained from the analysis of the Fe II ion lines, is represented by a low-metallicity component (97% of clouds) with \( \langle [X/H] \rangle = -1.79 \pm 0.40 \quad \text{(Fig. 2 panel c)}.

In contrast to the result based on Si II and Fe II ions, 94 clouds, the metallicity of which was determined based on the analysis of the abundance of Zn II ions, appeared to be mainly high-metallicity (about 63% of clouds) with \( \langle [X/H] \rangle = -0.54 \pm 0.33 \). The low-metallicity component is characterized by \( \langle [X/H] \rangle = -1.35 \pm 0.24 \quad \text{(Fig. 2 panel d)}.

As well as in the case with the \( \text{(Rafelski et al. 2012)} \) data, only the distribution based on S II ions demonstrates the minimum near \( [X/H] = -1 \).

Figures 1 and 2 demonstrate that finding the dependence of the average metallicity of CGCs on redshift can give a deliberately false result if the determination of metallicity for different ranges of redshifts is based on different ions. For example, Fig. 6 from \( \text{Rafelski et al. 2012} \) shows that the metallicity in the range of \( 2 < z < 4 \) was determined mainly from Si II ions, and for \( z < 1 \), in most cases, from Zn II ions. Figures 1 and 2 demonstrate that the distribution determined from Si II ions shows mainly a low-metallicity component of clouds, while that determined from Zn – a high-metallicity component.

The metallicities of DLAs at \( z < 1 \) were determined by \( \text{Wotta et al. 2014} \) (see please, Fig. 3 panels a). The metallicity indicators chosen by \( \text{Wotta et al. 2019} \) are as follows: Zn II for nineteen CGCs, Fe II for nine CGCs, S II for five CGCs, and Si II for two CGCs. The authors used the following abundance ratios: \( [\alpha/Zn] \approx 0.0 \pm 0.2 \), that is \( [X/H] = [Zn/H] \), \( [X/H] = [\alpha/H] \), and \( [X/H] = [Fe/H] + 0.5 \). Only one component with \( [X/H] > -1 \) can be seen, that is, 29 of 35 clouds (see Fig. 3). The value \( \langle [X/H] \rangle = -0.57 \pm 0.26 \) coincides within the measurement errors with the mean value of the high-metallicity component of the distribution from \( \text{Rafelski et al. 2012} \) at \( 2 < z < 4 \), and from \( \text{Quiret et al. 2016} \) at \( 1.8 < z < 4 \). It likely means (see Table 1, sixth column) that the changes of the metallicities of clouds during the evolution from \( z = 4 \) up to present time are not detected for the high-metallicity subgroup.

### 2.1.2 LLSs and pLLSs

LLSs and pLLSs are almost completely ionized. The gas in LLSs and pLLSs is often multiphase with the absorption seen in different ionization stages.

For pLLSs and LLSs at \( z < 1 \), the resulting sample of CGCs in \( \text{Wotta et al. 2019} \) is the following: 82 pLLSs, 29 LLSs. The metallicity of each cloud was determined by analysing the abundance of several ions: Mg II, O I, O II, S III, Fe II, Si III and Si IV. Using the observed data, we excluded the objects with uncertain abundances from the sample according to the data from Table 6 in \( \text{Wotta et al. 2019} \). The excluded clouds have the following metallicities: \( [X/H] < -3 \) dex and \( [X/H] > 1 \) dex. These are the objects having only lower or upper limit abundance estimates. The resulting sample of pLLSs and LLSs consists of 60 and 20 members respectively (see Fig. 3 panels b and c). For the average metallicity of the low-metallicity and high-metallicity clouds obtained by \( \text{Wotta et al. 2019} \) see please, third and sixth columns in Table 1.

\( \text{Lehner et al. 2016} \) measured metallicities for sixteen pLLSs and seven LLSs at \( 2.3 < z < 3.0 \). The metallicity was determined mainly by ions at a high ionization level, Si III and Si IV. The data obtained by \( \text{Lehner et al. 2016} \) contain a very metal-poor component with \( [X/H] \lesssim -2.3 \) for LLSs (see Fig. 3 panel d). This feature only slightly manifests itself in the histogram for pLLSs at \( z < 1 \) \( \text{Wotta et al. 2019} \).

We do not consider it in this paper, because the GC samples used in this paper (see Sec. 2.2) do not show any analogue of this feature in their metallicity distribution. \( \text{Cooke et al. 2017} \) suggest that most metal-poor CGCs are progenitors...
of the lowest-mass galaxies. The study of this hypothesis is beyond the scope of this study. We only note that several GCs in this metallicity range are also known in our Galaxy and other galaxies (Harris 1996; Larsen et al. 2021).

The average metallicity of pLLSs is $\langle [X/H] \rangle = -1.69 \pm 0.20$ (Lehner et al. 2016). The high metallicity component is not pronounced.

It is instructive to compare the $[X/H]$ distribution for DLAs at high redshifts measured by Rafelski et al. (2012) and Quiret et al. (2014) and the corresponding distribution for pLLSs at $z < 1$ from Wotta et al. (2019). The comparison shows that they have similar mean $[X/H]$ values for the high- and low-metallicity components (see Table 1, third and sixth columns).

We can summarize the properties of the clouds as follows. All types of CGCs (DLAs, sub-DLAs, LLSs and pLLSs) are observed at the redshifts $z < 4$. The average metallicity within each subgroup (high- and low-metallicity) coincides for pLLSs, LLSs, and DLAs within the estimated errors for the redshift intervals considered in Table 1. This implies that the evolution of the metallicity in the high- and low-metallicity components of CGCs, separately, is not evident starting from $z \sim 4$, and different types of clouds for a given metallicity show similar mean $[X/H]$ values within the corresponding errors of their determination.

2.2 Comparison of the metallicities of GCs and CGCs

In this section, we use the results of the statistical study by Acharova & Sharina (2018), who have analysed the [Mg/H] and [Fe/H] distributions of Galactic GCs.

As in the case of the clouds (Sec. 2.1), the $[X/H]$ values for GCs were determined in the literature by analysing different elemental abundances. Figure 1 shows the distributions of $\alpha$-element abundances in GCs built using the data of the integrated-light spectroscopy from Dias et al. (2016) for the objects in three subsystems of the Galaxy: the disc, the inner, and outer halos. Acharova & Sharina (2018) plotted the [Fe/H] and [Mg/H] distributions using the data from the MILES library Sánchez-Blázquez et al. (2004). Since the $[X/H]$ metallicity is determined using $\alpha$-elements Mg, Ca, and Ti, we need to know the $[\alpha/Fe]$ values for GCs. For this, we use the data from Dias et al. (2016) obtained from the Coelho library (Coelho et al. 2005). This is the largest homogeneous data sample of $\alpha$-element abundances to date. We avoid to compile abundances from various literature sources, because they may lead to wrong conclusions due to possible systematic deviations in the data of different authors (see, e.g., Schiavon et al. 2012). The distributions of $[\alpha/Fe]$ and [Mg/H] Dias et al. (2016) look very similar (Fig. 3).

Let us consider the value $[X/H] = -1$ as the boundary between the low- and high-metallicity subgroups of GCs as in the case of CGCs. Hereafter, we will provide statistical analysis based on this division. Table 2 shows the number of GCs in each subgroup, the average $[X/H]$ values and the root-mean-square deviations according to Dias et al. (2016) obtained from the Coelho library.

The coincidence within the errors of the $[Mg/H]$ mean values and the root-mean-square deviations for low- and high-metallicity subgroups of CGCs and GCs has allowed Acharova & Sharina (2018) to hypothesize that these clouds can be the residual parts of the clouds, in which GCs have been formed. The considered here $[X/H]$ data by Wotta et al. (2016); Rafelski et al. (2012); Quiret et al. (2016); Lehner et al. (2016) argue in favour of the conclusions by Acharova & Sharina (2018). The average metallicity of the low-metallicity Galactic GC subgroup $\langle [X/H] \rangle = -1.67 \pm 0.35$, whereas the average metallicity of the high-metallicity Galactic GC subgroup $\langle [X/H] \rangle = -0.62 \pm 0.25$. The average metallicity of the subgroup of low-metallicity CGCs $\langle [X/H] \rangle = -1.61 \pm 0.39$, whereas the average metallicity of the high-metallicity CGC subgroup $\langle [X/H] \rangle = -0.52 \pm 0.28$.

In this regard, it is important to quote the studies of metallicity distributions for GCs in other galaxies. By combining the information of the LAMOST spectra and the multi-band photometry, Wang et al. (2021) determined the...
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Table 1. Metallicities of circumgalactic clouds used in the statistical analysis

| Galactic subsystem      | Low-metallicity subgroup ([X/H] < −1) | High-metallicity subgroup ([X/H] ≥ −1) |
|-------------------------|--------------------------------------|----------------------------------------|
|                         | objects                              | redshifts | < [X/H] | σ([X/H]) | number of clouds | < [X/H] | σ([X/H]) | number of clouds | reference          |
|                         | DLAs, S II                           | 2 < z < 4 | -1.53   | 0.26     | 34            | -0.61   | 0.19     | 15            | Rafelski et al. (2012) |
|                         | DLAs, Fe II                          | 2 < z < 4 | -1.47   | 0.22     | 35            | -0.64   | 0.31     | 13            | Rafelski et al. (2012) |
|                         | DLAs, Si II                          | 2 < z < 4 | -1.72   | 0.42     | 57            | -0.81   | 0.16     | 12            | Rafelski et al. (2012) |
|                         | DLAs, Fe II                          | z < 1     | -0.74   | 0.13     | 13            |         |          |               | Rafelski et al. (2012) |
|                         | sub-DLAs, S II                       | 1.8 < z < 4 | -1.53 | 0.33     | 37            | -0.54   | 0.26     | 14            | Quiret et al. (2016) |
|                         | sub-DLAs, Si II                      | 1.8 < z < 4 | -1.73 | 0.42     | 68            | -0.55   | 0.27     | 18            | Quiret et al. (2016) |
|                         | sub-DLAs, Zn II                      | 1.8 < z < 4 | -1.35 | 0.24     | 35            | -0.54   | 0.33     | 59            | Quiret et al. (2016) |
|                         | sub-DLAs, Fe II                      | 1.8 < z < 4 | -1.79 | 0.40     | 32            |         |          |               | Quiret et al. (2016) |
|                         | DLAs                                 | z < 1     |         |          |               | -0.57   | 0.26     | 29            | Wotta et al. (2019) |
|                         | pLLSs                                | z < 1     | -1.75   | 0.46     | 37            | -0.50   | 0.27     | 23            | Wotta et al. (2019) |
|                         | LLSs                                 | z < 1     | -1.60   | 0.42     | 11            | -0.45   | 0.19     | 9             | Wotta et al. (2019) |
|                         | pLLSs                                | 2.3 < z < 3 | -1.69 | 0.20     | 8             |         |          |               | Lehner et al. (2016) |

Table 2. Average [X/H] values, root-mean-square deviations, and the number of GCs according to Dias et al. (2016) for our Galaxy and Wang et al. (2021) for the M31 galaxy

| Galactic subsystem      | Low-metallicity subgroup ([X/H] < −1) | High-metallicity subgroup ([X/H] ≥ −1) |
|-------------------------|--------------------------------------|----------------------------------------|
|                         | objects                              | [X/H] | σ([X/H]) | number of GCs | [X/H] | σ([X/H]) | number of GCs |
| disc                    | disc                                 | -1.65 | 0.30     | 6             | -0.59 | 0.17     | 12             |
| inner halo              | inner halo                           | -1.74 | 0.43     | 8             | -0.69 | 0.24     | 5              |
| outer halo              | outer halo                           | -1.62 | 0.28     | 8             |         |          |               |
| M31 galaxy              | M31 galaxy                           | -1.44 | 0.25     | 145           | -0.63 | 0.19     | 148            |

ages of GCs and derived parameters of 53 young and 293 old clusters in the Andromeda galaxy. Most old clusters have the ages approximately equal to 10 Gyr. The metallicity distribution of [X/H] for old GCs in the M31 galaxy is shown in Fig. 2. It is similar to the metallicity distribution for GCs of our Galaxy. The high- and low-metallicity peaks of the distribution are less pronounced, but there is a natural explanation for this fact. The uncertainties of the metallicity estimates and the metallicities of GCs originated in the processes of active galactic interactions can wash away sharp features. The average metallicity of the subgroup of the low-metallicity GCs ([X/H] = −1.44±0.25) is −1.62±0.28 for the high-metallicity subgroup of GCs ([X/H] = −0.63±0.19). The number of GCs in each group is given in Table 2.

Large galaxies in the centres of galactic clusters have experienced more complex star-formation histories with violent star-forming events than faint isolated galaxies (e.g., Kroupa et al. 2019, Villaume et al. 2020, Longobardi et al. 2015). Interestingly, even in such massive galaxies, statistical analysis is able to reveal ~10-Gyr old metal-rich and metal-poor GC populations with the corresponding metallicity peaks similar to the ones considered in this section (Dias et al. 2013). For example, Peng et al. (2006) and Harris (2009) have studied the extragalactic GC systems in giant elliptical galaxies and argued that the metallicity distributions of GCs in them are bimodal.

Abundances of chemical elements in extragalactic GCs obtained using their integrated-light spectra are determined together with their ages using stellar population models. Even in the galaxies and galaxy subsystems with prevailing old stellar populations, there may be a significant population of intermediate age GCs (1 ≤ T ≤ 7 Gyr). For example, about 15% of GCs are ≤ 3 Gyr old in M31 (Wang et al. 2021). These abundances can also be biased (see, e.g., Schiavon et al. 2012) due to the differences in the methods of the observed data reduction and analysis used by different authors and due to the low signal-to-noise ratios in the spectra of distant objects. We, therefore, used only the data for Galactic GCs from Dias et al. (2016) for the analysis in this section. Dias et al. (2016) demonstrated the agreement of their estimates with the literature data.

2.3 Possible scenario of CGC evolution associated with the formation of GCs

Before we consider the properties of supernovae responsible for the enrichment of GCs and their parent CGCs with chemical elements, let us discuss formation of GCs. Since the average ages of the high- and low-metallicity subgroups of GCs discussed above are similar (Chattopadhyay et al. 2012, VandenBerg et al. 2013), the metal enrichment of the low-metallicity clouds has happened fast as a result of explosions of massive stars or rapidly evolving binary stellar systems. As it was shown in Acharova & Sharina (2018), the fraction of mass of the enriched part of the cloud is from 20% to 50% of the initial cloud mass depending on the fraction of the cloud gas transformed into stars. A high-metallicity...
group of GCs is formed from this enriched gas. The analysis of the observed data for CGCs and GCs, performed in this paper, identified several phenomena related to our study, the further study of which can add important refinements to the theory of galaxy formation and evolution.

There are very metal-poor clouds shown in Fig. 3 with \( < [X/H] < -2.3 \). There are relatively few such clouds found. Some authors considered them as the gaseous clouds left over from the explosion of the first generation of stars (Welsh et al. 2019; Kulkarni et al. 2013). Can we associate these CGCs with GCs using the criterion for the closeness of their metallicities? Yes, we can. Although, we should admit, such CGCs are very rare. Beasley et al. (2019) compiled spectroscopic metallicity data for the GCs of 28 nearby galaxies and found 1–2 GCs with \( [X/H] < -2.3 \) only in 5 galaxies. In the paper by Kruijssen (2019), a possible explanation for this phenomenon is proposed, which is that the galaxies with metallicities of \([Fe/H] \leq -2.5\) have too small masses to form GCs with initial masses greater than \(10^8 M_\odot\) and needed to survive for the Hubble time. The processes of enrichment with chemical elements of most metal-poor clouds require separate careful consideration.

The next special feature worth noting is as follows. Note that both on histograms for \( 2 < z < 4 \) and on histograms for \( z < 1 \) (Fig. 4 and Fig. 2), there are practically no CGCs with \( [X/H] > -0.3 \). Figures 3 and 5 show that a similar situation is observed for GCs. GCs formed from high-metallicity clouds, in turn, should have contained supernovae that enriched the surrounding gas. Therefore, the absence of very high-metallicity GCs and CGCs requires an explanation. With care, one can tentatively assume that cloud fragments enriched to metallicity \( [X/H] > -0.3 \) pass into another phase due to rapid cooling on metals; and the conditions are created in them for the clusters with \( M < 10^5 M_\odot \) to form, which do not survive for the Hubble time according to Kruijssen (2019).

A natural question is why we do not observe the formation of GCs in the near-galactic clouds in the present epoch. Mandelker et al. (2018) presented a new model of the GC formation and showed that extremely turbulent conditions are required for cold filamentary accretion to lead to the formation of star-forming clumps. According to calculations, the densities required for the formation of GCs are achieved with the collision of counter-rotating streams of very massive clouds. Such conditions were likely to occur at redshifts of \( z > 4.5 \) (Mandelker et al. 2018). Therefore, it must be assumed that GCs, which formed from CGCs, are of about 10 billion years old or older. GCs can also form in dwarf galaxies. In this case, it is natural to assume that the metallicity of these GCs will correspond to the metallicity of the CGCs. Therefore, it is of interest to study the chemical composition of GCs in isolated low-mass dwarf galaxies which probably did not undergo merger processes with other galaxies (Sharina et al. 2017).

At present, the collision of clouds resulting in the formation of star clusters, the masses of which are comparable to the initial mass of GCs, \(10^6 - 10^7 M_\odot\), can be found only in the processes of major merging. Antennae Galaxies NGC 4038/NGC 4039 (Tsuge et al. 2021) are a good example. The collision relative velocity of the clouds in these galaxies, according to estimates, is about 100km/s. It is natural to expect that the metallicity of GCs formed in such processes will be equal to the metallicity of gas, that has undergone several stages of transformation by stars, and will differ from the metallicity of CGCs that existed at the stage of the initial formation of galaxies.

### 3 SELECTION OF THE PROPER SN Ia MODEL

#### 3.1 Details of the method

To select the proper Prompt SN Ia model from the analysis of nucleosynthesis presented in Sec. 2, we need accurate data on the abundances of chemical elements, the main production channels of which are two subtypes of SNe: SNe CC and SNe Ia. Note that the contribution of these elements from other stages of the stellar evolution should be negligible. The idea of our method can be shortly formulated as follows (please, see also Fig. 6). We will use the method of determination of the Mg and Fe mass, which has been synthesized by SNe CC and Prompt SNe Ia explosions in the

| Chemical element | Low-metallicity subgroup \( [X/H] < -1 \) | High-metallicity subgroup \( [X/H] \geq -1 \) |
|------------------|---------------------------------------------|---------------------------------------------|
|                  | \( \langle [X/H] \rangle \), \( \sigma([X/H]) \), \( N_{GC} \) | \( \langle [X/H] \rangle \), \( \sigma([X/H]) \), \( N_{GC} \) |
| \( ^{24}\text{Mg} \) | \(-1.57\), \( 0.40 \), \( 4 \) | \(-0.38\), \( 0.26 \), \( 7 \) |
| \( ^{28}\text{Si} \) | \(-1.14\), \( 0.25 \), \( 3 \) | \(-0.18\), \( 0.19 \), \( 7 \) |
| \( ^{40}\text{Ca} \) | \(-1.34\), \( 0.35 \), \( 4 \) | \(-0.32\), \( 0.21 \), \( 7 \) |
| \( ^{52}\text{Cr} \) | \(-1.68\), \( 0.41 \), \( 4 \) | \(-0.57\), \( 0.25 \), \( 7 \) |
| \( ^{56}\text{Fe} \) | \(-1.63\), \( 0.35 \), \( 4 \) | \(-0.48\), \( 0.21 \), \( 7 \) |
low-metal subgroup of GCs, which is described in detail by Acharova & Sharina (2018), section 4. In this way, we can estimate the corresponding number of SNe CC and SNe Ia. Then, using the determined number of SNe of different types, one can explain the nucleosynthesis of other chemical elements. Acharova & Sharina (2018) show and we demonstrate in this paper that our calculations are independent of the fraction of the enriched cloud mass. We consider that the average mass of an element ejected into the interstellar medium during a single burst of a SN CC as known. This assumption is justified, because theoretical models of the SN CC nucleosynthesis are confirmed by direct observations of massive stellar progenitors of SNe CC and by the analysis of the spectra of their expanding shells of different masses (Smartt et al. 2006 and references therein). The average mass of the element synthesized during the SN CC explosion is determined using formula (1) by Tsujimoto et al. (1993) with the calculations by Nomoto et al. (2006). The results of Tsujimoto et al. (1995) and Nomoto et al. (2006) almost coincide for the chemical elements under consideration. It is necessary to clarify the maximum mass of stars involved in the enrichment process. Studies of the SN progenitors and their stellar remnants in nearby galaxies argue that the maximum mass of the progenitor of a SN CC is \( \sim 20 M_{\odot} \). Stars with masses higher than this limit experience implosion and form a black hole without any release of the enriched substance (Auchettl et al. 2018 and references therein). At the same time, the authors conclude that there may be “islands of explodability” for stars heavier than the limiting mass. They used the processes in the Small and Large Magellanic Clouds as the example. In this case, even high-mass progenitors up to \( \sim 40 M_{\odot} \) may explode in the case of some specially selected physical parameters of stars and their multiplicity (Auchettl et al. 2018). Since the maximum mass of stars depends on the mass of the parent molecular cloud (Larson 1981) and metallicity (Nomoto et al. 1982), it would be naturally expected that stars of such great masses form in GCs. When calculating the average masses of newly synthesized chemical elements, we will take into account the contribution from the progenitors of SNe CC with masses from 8 to 40\( M_{\odot} \). The average mass of the synthesized oxygen significantly depends on the maximum mass of a star, but it is not used in this study. The average masses of considered Mg, Si, Ca, Cr, and Fe are negligibly dependent on whether we use 20\( M_{\odot} \) or 40\( M_{\odot} \) as the maximum mass of SNe CC.

Let us discuss a series of SN Ia models, with which we will compare the results of our calculations. As mentioned in the Introduction, they differ, first, in the explosion mechanism: pure turbulent deflagration models and models with deflagration-detonation transition. These two explosion mechanisms comprise the basis of all single degenerate models of SNe Ia. Second, the models differ in the mass of a degenerate carbon-oxygen WD. The W7 and WDD models are calculated for a dwarf mass equal to the Chandrasekhar limit of 1.38\( M_{\odot} \) (Nomoto et al. 1984). In Leung & Nomoto (2018), the models for the WD masses from 1.30\( M_{\odot} \) to 1.38\( M_{\odot} \) are also considered.

Let us note that the average mass of a chemical element released during a Prompt SN Ia explosion is a free parameter of our theory for all elements except for iron, because there are independent studies consistent with theoretical calculations of the explosive nucleosynthesis of SNe Ia (Leung & Nomoto 2018) only for iron (Acharova et al. 2013; Childress et al. 2015).

### 3.2 Analysis of abundances of chemical elements in GCs

To find observation limitations on the nucleosynthesis, we use the results of studies of GCs, in which the abundances of several chemical elements have been determined. Among them, those have been selected that are produced during SN bursts, while their nucleosynthesis does not depend on the

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3 The words cited from the paper by Auchettl et al. (2018) mean that the masses of exploding stars (\( \sim 20 M_{\odot} \)) can be described by a discrete mass function. This assumption is introduced for setting the theoretical limit on the maximum mass of a star that can participate in the enrichment processes (Heger et al. 2003; Sukhbold et al. 2016) in agreement with the observation estimates obtained from studies of supernova remnants and their environments in star-forming regions (Smartt et al. 2006; Sarbadhicary et al. 2013). According to Auchettl et al. (2018), stars with masses up to 40\( M_{\odot} \) can explode in these environments.

4 It is believed that the masses of the first stars were even greater than 40\( M_{\odot} \).

5 Another SN Ia explosion channel is possible – double degenerate – the merger of two white dwarfs with masses close to that of the Sun. The double-degenerate scenario is possible in old stellar populations with ages larger than \( \sim 2.4 \) Gyr. Therefore, we do not take it into account in this paper.
metallicities of SNe. The following chemical elements meet our aims: Mg, Si, Ca, Cr, and Fe. Among the listed chemical elements, Mg shows the manifestation of chemical evolution in GCs (Gratton et al. 2013) and references therein). The currently observed low-mass stars cannot reach the temperature threshold required for activating the Mg-Al conversion, thus, this nuclear burning must have occurred in more massive stars, already evolved and dead (Gratton et al. 2019). Magnesium and oxygen (oxygen also experiences depletion) are indicators of the amount of SNe CC in chemical evolution models, because about 98% of Mg and O are produced by SNe CC (McWilliam et al. 2008). At the same time, according to the method described in Acharova & Sharina (2018), the depletion of magnesium in a certain fraction of stars does not influence the results of determination of the number of SNe CC. The magnesium abundances in the low- and high-metallicity groups decrease by the same value.

Colucci et al. (2017) measured the abundances of Mg, Si, Ca, Cr, and Fe in eleven GCs using one method and the integrated-light high-resolution spectra of GCs. Seven of these GCs are in the metal-rich group and four GCs are in the metal-poor group. According to the classification of Carretta et al. (2010), nine of these GCs belong to the disc or bulge and two GCs belong to the inner halo.

In the Pritzl et al. (2005) sample, thirty-five low-metallicity GCs belong mainly to the outer halo and six metal-rich GCs belong to the disc or bulge. However, this sample is not suitable for our method (see Subsec. 3.8 for details). Apparently, the abundances of the brightest stars in GCs, given by Pritzl et al. (2005) cannot be representative for all their stars.

There are several other studies in the literature, in which abundances of chemical elements in GCs were determined using the integrated-light spectra. Abundances of several chemical elements were determined by Larsen et al. (2018) using the integrated-light spectra of seven GCs. However, only two of them have high metallicity. Likewise, in Sharina et al. (2020), all GCs considered are of the low metallicity only.

Conroy et al. (2013) measured abundances of several chemical elements using the medium-resolution integrated-light spectra of 41 GCs from Schiavon et al. (2005) and their original method. We cannot use their results, because the distribution of [Fe/H] determined by Conroy et al. (2013) do not show a separation into low- and high-metallicity components, while the library of Schiavon et al. (2005) contains GCs with the [Fe/H] values higher and lower than −1.35. The mean value ([Fe/H]) = −1.35 (Conroy et al. 2013) is higher by ∼0.3 dex for low-metallicity GCs and ([Fe/H]) = −0.68 (Conroy et al. 2013) is lower by ∼0.2 dex for high-metallicity GCs than the corresponding value in Dias et al. (2010), Carretta et al. (2010), and Pritzl et al. (2003) (see, please, Acharova & Sharina (2018)).

In the following, we will provide the analysis of nucleosynthesis using the data from Colucci et al. (2017). The statistical analysis of chemical abundances in the low- and high-metallicity groups of GCs from the paper by Colucci et al. (2017) is given in Table 3.

It should be noted that the mean values of [Mg/H] for the low- and high-metallicity groups of GCs according to the data obtained by Colucci et al. (2017) are close to those obtained by Wotta et al. (2016) for CGCs but systemati-
fractions $lgZ_{\text{El}}$ of the chemical elements Mg, Fe, Si, Cr, and Ca in GCs together with the values $lgZ_{\text{El}}(\odot)$ and $A_{\text{El}}$.

In the next sections, we will calculate the number of SNe CC and SNe Ia that have exploded during the formation of the first generation of GCs from the known masses of the synthesized Mg and Fe. Then, using the determined number of SNe, we will calculate the mass of Si, Cr, and Ca synthesized in the explosion of a single SN Ia. Each time we will consider the situation that 20% of the cloud mass was enriched by SN explosions during the formation of low-metallicity GCs. It will be shown below in the paragraphs 3.3–3.5 that the predictions of supernova nucleosynthesis do not depend on the fraction of the enriched cloud mass. The results of the analysis of the Si, Cr, and Ca production in GCs are presented in Tables 5 and 6. These tables are organized as follows. The considered isotopes are indicated in the first row. The first column indicates what kind of data are considered: observed (values obtained in this paper) or theoretical (the values obtained using the supernova nucleosynthesis models). In the case of the observed data, the number of SNe is given, for which the mass of the synthesized chemical elements will be calculated. If the theoretical data are considered, the reference and the name of the model used are given.

### 3.4 Number of SNe Ia responsible for the iron production in circumgalactic clouds

If 20% of the cloud mass was enriched by SN explosions during the formation of low-metallicity GCs, the cloud acquired the Mg mass $M_{\text{Mg}} = 0.2 \cdot 10^{9} \cdot (10^{-3.32} - 10^{-4.47}) \approx 10^{4.95} M_{\odot}$.

(If 50% of the cloud mass was enriched, then the number of SNe Ia increased 2.5 times: $M_{\text{Fe}} = 0.5 \cdot 10^{9} \cdot (10^{-3.32} - 10^{-4.47}) \approx 2.5 \cdot 10^{4.95} M_{\odot}$.)

Since SNe CC produce the negligible Fe mass (about 6%: see, please, the reasoning by Acharova & Sharina (2018)), then one could expect the number of SNe Ia depending on the iron mass ejected during the explosion of one SN Ia: $10^{4.95} / 10^{5.6} = 10^{5.05}$. (If 50% of the cloud mass was enriched, then the number of SNe Ia will increase 2.5 times: $2.5 \cdot 10^{4.95} / 2.5 \cdot 10^{5.6} = 2.5 \cdot 10^{5.3}$.)

The value 0.23 was obtained for the first time in the paper by Acharova et al. (2013) employing the developed theory of the iron synthesis in the galactic disc, which was able to explain the subtle features in its distribution.

This value agrees with the conclusions drawn by Childress et al. (2013) and is the mean value for pure deflagration models 050 − 1 − c3 − 1P and 100 − 1 − c3 − 1P, in which carbon ignites at a lower central stellar density of the stellar remnant Leung & Nomoto (2018). This situation is possible in the single degenerate scenario at a high accretion rate onto the carbon-oxygen WD that leads to a rapid temperature increase in the centre of a WD until it reaches the Chandrasekhar limit. An iron mass of 0.67$M_{\odot}$, first proposed by Nomoto et al. (1984), was obtained with the W7 and WDD2 models and reconsidered for the updated nuclear reaction network by Leung & Nomoto (2018).

It is important to note that due to the insufficient completeness of the samples of GCs, the obtained average values can be biased from the values that would be obtained for more complete samples. But, since there are no more complete samples, we will continue the study using the available material.

Using the determined number of SNe of different types on the basis of the magnesium and iron abundances, we will analyse the enrichment of the cloud with other chemical elements: silicon, chromium, and calcium.

### 3.5 Analysis of silicon abundances in GCs

If 20% of the cloud mass was enriched by SN explosions during the formation of low-metallicity GCs, the cloud has acquired the mass $M_{\text{Si}} = 0.2 \cdot 10^{9} \cdot (10^{-3.35} - 10^{-4.35}) \approx 10^{4.93} M_{\odot}$. Accordingly, if 50% of the cloud mass was enriched

| Element (El) | values for Sun (lgZ_{El (odot)}, A_{El}) | Low-metallicity subgroup (lgZ_{El (H)}, lgZ_{El (H/H)}) | High-metallicity subgroup (lgZ_{El (H)}, lgZ_{El (H/H)}) |
|-------------|----------------------------------------|------------------------------------------------------|-----------------------------------------------------|
| Mg          | -3.1, 7.64                             | -1.57, -4.67                                         | -0.38, -3.48                                         |
| Fe          | -2.84, 7.54                            | -1.63, -4.47                                         | -0.48, -3.32                                         |
| Si          | -3.172, 7.51                           | -1.14, -4.31                                         | -0.18, -3.35                                         |
| Cr          | -4.75, 5.64                            | -1.68, -6.43                                         | -0.57, -5.32                                         |
| Ca          | -4.192, 6.34                           | -1.34, -5.53                                         | -0.32, -4.51                                         |
by SN explosions, the cloud acquired the mass: \( M_{\text{Si}} \approx 2.5 \cdot 10^{-5} M_\odot \).

We will consider the mass of \( ^{28}\text{Si} \) produced during a SN CC explosion (Tables 4 and 5 and the number of SNe CC and pSNe Ia as known.

Hence, it is obvious that the mass of silicon produced during the pSN Ia explosion does not depend on what fraction of the cloud is enriched. For example, if 50\% of the cloud mass was enriched by SN explosions, \( 2.5 \cdot 10^{-5} \) SNe CC will produce \( 2.5 \cdot 10^{-5} \cdot 0.1M_\odot \) of silicon, then the remaining mass of silicon falling onto pSNe Ia \( M_{\text{Si}} \approx 2.5 \cdot 10^{-5} - 2.5 \cdot 10^{-5} \cdot 0.1(M_\odot) \). This is 2.5 times larger than the mass produced under the assumption that 20\% of the cloud is enriched. However, this number is divided by the number of the pSN Ia supernovae that is also 2.5 times greater: \( 2.5 \cdot 10^{-5} \). Thus, the coefficient of 2.5 is reduced. Therefore, in Tables 5 and 6, all the average masses of chemical elements are given under the assumption that 20\% of the cloud has become enriched, because the conclusions about nucleosynthesis in pSNe Ia remain valid for any arbitrary fraction of the enriched part of the cloud. Furthermore, the analysis of the nucleosynthesis of the remaining chemical elements will be carried out under the assumption that 20\% of the cloud has been enriched. Tables 4 and 5 present comparison between the masses of \( ^{28}\text{Si} \) synthesized in a SN burst and calculated from the analysis of the corresponding elemental abundances in GCs from the paper by Colucci et al. (2017) with the results of the theoretical studies of nucleosynthesis yields. The second row of the observation section in Table 5 shows the amount of SNe Ia determined for the mass of iron \( M_{\text{Fe}} = 0.23M_\odot \) synthesized during the explosion of a single SN. This mass of iron, as was already mentioned, is the average value for two-dimensional pure deflagration models \( 050-1-c3-1P \) (it produces \( M_{\text{Fe}} = 0.21M_\odot \)) and \( 100-1-c3-1P \) (it produces \( M_{\text{Fe}} = 0.265M_\odot \)) (Leung & Nomoto 2018). Since we have determined that SNe CC produce \( 10^{-4.9}M_\odot \) of Si, SNe Ia produce \( M_{\text{Si}} = 10^{-4.9} - 10^{-3} = 10^{-5}M_{\odot} \). The mass of Si produced during one SN explosion with this amount of SNe Ia \( 4.1 \cdot 10^{-5}M_\odot \) is shown in the second row of the observation section of Table 5. Four rows of the theoretical data section of Table 5 list the versions of the two-dimensional pure deflagration models for different values of the central density of a WD by Leung & Nomoto (2018) and the corresponding mass of \( ^{28}\text{Si} \) synthesized during the explosion of a WD.

The data in the first column of the observation section in Table 5 indicate the amount of SNe Ia determined for the mass of iron \( M_{\text{Fe}} = 0.67M_\odot \) synthesized during the explosion of a single SN. This mass of iron corresponds to the one-dimensional Chandrasekhar mass deflagration model W7 (Nomoto et al. 1984) and the model with deflagration-detonation transition WDD taking into account the updated nuclear reaction network Leung & Nomoto 2018 (Mori et al. 2018). The first column of the observation section of Table 5 shows the mass of \( ^{28}\text{Si} \) in the explosion of one SN Ia which we obtained based on the analysis of GCs with this number of SNe Ia. While the four rows of the first column in the theoretical data section of Table 5 show the mass of \( ^{28}\text{Si} \) synthesized during the explosion of a WD corresponding to the indicated models (Leung & Nomoto 2018; Mori et al. 2018).

We can draw the following conclusion based on the comparison of the data in Tables 5 and 6: If in the analysis of iron enrichment we take the mass corresponding to its synthesis in pure turbulent deflagration models, that is, \( 0.23M_\odot \), then we get \( 10^{-5} \) SNe Ia. This amount of SNe will lead to the mass of \( ^{28}\text{Si} \) ejected during the explosion of a single SN \( 4.1 \cdot 10^{-5}M_\odot \) which corresponds to the theoretically predicted value for the same pure turbulent deflagration models. If we use the \( ^{56}\text{Fe} \) mass corresponding to the W7 or WDD2 models, then the \( ^{28}\text{Si} \) mass corresponds to the W7 model. As can be seen from Table 9 of Leung & Nomoto (2018), the W7 and WDD2 models provide close mass estimates only in the case of the Fe production. The WDD2 model provides the mass 1.5 – 2 times higher than that from the W7 model for other chemical elements.

To summarise, the \( ^{28}\text{Si} \) masses synthesized during a single SN Ia explosion are fully consistent with the theoretical nucleosynthesis calculations of the pure turbulent deflagra-

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**Table 5. Comparison between the masses of silicon \(^{28}\text{Si} \), chromium \(^{56}\text{Cr} \), and calcium \(^{40}\text{Ca} \) synthesized in a SN burst and calculated from the analysis of the corresponding elemental abundances in GCs (Colucci et al. 2017) and the results of theoretical studies of nucleosynthesis yields of two-dimensional pure turbulent deflagration (PTD) models for various values of the central density of a WD by Leung & Nomoto (2018) (LN18)**

| Isotope/Source | \(^{28}\text{Si} \) | \(^{56}\text{Cr} \) | \(^{40}\text{Ca} \) |
|---------------|--------------|--------------|--------------|
| Observed data | \( N_{\text{SN Ia}} > 10^{5.8} \) | 1.2 \cdot 10^{-3} | 5.6 \cdot 10^{-3} |
| Theoretical data | LN18 | | |
| 050-1-c3-1P | 3.5 \cdot 10^{-2} | 7.5 \cdot 10^{-4} | 1.7 \cdot 10^{-3} |
| 100-1-c3-1P | 3.8 \cdot 10^{-2} | 1.1 \cdot 10^{-3} | 1.8 \cdot 10^{-3} |
| 300-1-c3-1P | 4.4 \cdot 10^{-2} | 1.5 \cdot 10^{-3} |
| 500-1-c3-1P | 2.3 \cdot 10^{-2} | 1.5 \cdot 10^{-3} |

**Table 6. Comparison between the masses of silicon \(^{28}\text{Si} \), chromium \(^{56}\text{Cr} \), and calcium \(^{40}\text{Ca} \) synthesized in a SN burst and calculated from the analysis of the corresponding elemental abundances in GCs (Colucci et al. 2017) and the results of theoretical studies of nucleosynthesis yields of the one-dimensional Chandrasekhar mass deflagration model W7 (for \( Z = 0.1Z_\odot \)) and the model with deflagration-detonation transition WDD by Leung & Nomoto (2018) and Mori et al. (2018)**

| Isotope/Source | \(^{28}\text{Si} \) | \(^{56}\text{Cr} \) | \(^{40}\text{Ca} \) |
|---------------|--------------|--------------|--------------|
| Observed data | \( N_{\text{SN Ia}} > 10^{5.1} \) | 1.3 \cdot 10^{-1} | 1.6 \cdot 10^{-2} |
| Theoretical data | LN18 | | |
| W7 | 1.3 \cdot 10^{-1} | 8.3 \cdot 10^{-4} | 1.4 \cdot 10^{-2} |
| WDD2 | 2.2 \cdot 10^{-1} | 1.5 \cdot 10^{-2} | 2.4 \cdot 10^{-2} |
| Mori+2018 | 1.7 \cdot 10^{-1} | 7.2 \cdot 10^{-3} | 1.1 \cdot 10^{-2} |
| WDD2 | 2.3 \cdot 10^{-1} | 1.35 \cdot 10^{-2} | 2.4 \cdot 10^{-2} |
tion models of nucleosynthesis, both under the assumption of the reduced central density and for the W7 model.

### 3.6 Analysis of chromium abundances in GCs

If 20% of the cloud mass was enriched by SN explosions during the formation of low-metallicity GCs, then the cloud acquired the mass $M_{\text{Cr}} = 0.2 \cdot 10^9 \cdot (10^{-5.32} - 10^{-6.43}) \approx 10^{7.91} \, M_\odot$.

Further, we will consider the mass of $^{52}$Cr produced during a SN CC explosion (Tables 5 and 6) and the number of SNe CC and pSNe Ia as known. Our reasoning will be analogous to that in the case of $^{28}$Si considered in the previous section (see also Tables 3 and 5). We find that SNe CC produce $10^2.87 \, M_\odot$ of $^{52}$Cr. Therefore, SNe Ia produce $M_{\text{Cr}} = 10^{2.91} - 10^{2.87} = 10^{1.9} \, M_\odot$. Depending on the number of SNe Ia, the chromium mass produced during a single SN explosion can be equal to $2.5 \cdot 10^{-4} \, M_\odot$ or $8 \cdot 10^{-4} \, M_\odot$.

The mean chromium mass produced by a single SN Ia basing on the assumption that $10^{-6} \, M_\odot$ SNe Ia exploded in the clouds is equal to $2.0 \cdot 10^{-4} \, M_\odot$. It is 3.5 times lower than the minimum mass predicted by the theoretical models, as it follows from the pure turbulent deflagration model 050 – 1 – c3 – 1P from Leung & Nomoto (2012). The mean chromium mass during $10^{5.2}$ SN Ia explosions is an order of magnitude lower than the value predicted with W7 and WDD2.

To summarize, additional studies are necessary in the case of chromium. So, due to a small amount of GCs in the Colucci et al. (2017) sample, the average values of magnesium and iron in the high- and low-metallicity groups of the considered chemical elements can be biased, one can assume that the amounts of SNe CC and SNe Ia may differ from the calculated ones. However, in this case, first, the simultaneous coincidence of theory and observations for iron and silicon production is violated, and second, in order to harmonize the theory and observations for calcium (see below), the amount of SNe Ia should not be 3.5 time reduced, but vice versa, three times increased. If we assume that the difference between the average values of the chromium abundance of the low- and high-metallicity subgroups is by 0.1 dex greater than it follows from the Colucci et al. (2017) data; then we can reconcile the result of the SN Ia nucleosynthesis with the predictions of pure turbulent deflagration models for lower central density values. The value of 0.1 dex sufficient for the agreement is several times smaller than the standard deviation $\sigma([\text{Cr/H}])$ as can be seen from Table 5.

### 3.7 Analysis of calcium abundances in GCs

If 20% of the cloud mass was enriched by SN explosions during the formation of low-metallicity GCs, the cloud acquired the mass of $^{40}$Ca: $M_{\text{Ca}} = 0.2 \cdot 10^9 \cdot (10^{-4.51} - 10^{-5.53}) \approx 10^{7.75} \, M_\odot$.

Further, we will consider the $^{40}$Ca mass produced during a SN CC explosion (Tables 5 and 6) and the number of SNe CC and pSNe Ia as known. Our reasoning will be analogous to that in the case of $^{28}$Si considered in Sec. 3.5 (see also Tables 3 and 5). We obtain that SNe CC produce $10^{3.55} \, M_\odot$ of $^{40}$Ca. Therefore, SNe Ia produce $M_{\text{Ca}} = 10^{7.75} - 10^{7.55} = 10^{7.4} \, M_\odot$. Depending on the number of SNe Ia, the calcium mass produced during a single SN explosion can be equal to $5.0 \cdot 10^{-2} \, M_\odot$ or $1.6 \cdot 10^{-2} \, M_\odot$.

A good agreement can be reached for the calcium mass with the W7 results (Table 5). The mass of calcium produced by $10^{-6}$ SNe Ia is equal to $5.0 \cdot 10^{-2} \, M_\odot$ that is 3 times larger than the value predicted in the two-dimensional pure turbulent deflagration models.

However, one can notice the following trend: the increase in the synthesized calcium mass and at the same time reducing the synthesized chromium mass can be achieved with the decrease in the burning WD density (see, please, Table 5).

### 3.8 Chemical abundances according to the Pritzl et al. (2005) data

In order to check the dependence of the estimates obtained in Secs. 3.5, 3.7, on the results of observations, we will take similar reasoning for the data by Pritzl et al. (2003). The number of GCs in this study is larger, but we cannot completely rely on them for the following reason.

Using the same algorithm as that described in Sec. 3.2, we conclude that the cloud has acquired the magnesium mass $M_{\text{Mg}} = 10^{4.93} \, M_\odot$. Therefore, $10^{5.33}$ SNe CC have exploded. We can conclude that the cloud has acquired the iron mass $M_{\text{Fe}} = 10^{4.88} \, M_\odot$. If the mass of the synthesized iron in a single SN explosion is equal to $0.23 \, M_\odot$, then $10^{3.52}$ SNe Ia have exploded. If the mass of the synthesized iron in a single SN explosion is equal to $0.67 \, M_\odot$, then $10^{5.05}$ SNe Ia have exploded.

Then, using the analysis analogous to that described in Sec. 3.5, we conclude that the cloud has acquired $M_{\text{Si}} = 10^{4.88} \, M_\odot$. One can see that a smaller amount of Mg than Si was produced. Considering that the average masses of magnesium and silicon during a single SN CC explosion are equal to $0.1 \, M_\odot$, the silicon production cannot be explained, because there is no substance left for the contribution from a SN Ia. For example, if we increase the average value of the magnesium in the low-metallicity subgroup by 0.4 dex, we will reduce the mass of the synthesized magnesium and there will be no conflict with the mass of the synthesized silicon.

Let us analyse the possible reasons for this situation. Pritzl et al. (2003) considered mostly remote low-metallicity GCs. Additionally, as was mentioned in Sec. 3.2, Pritzl et al. (2003) considered the abundances of bright red giants in the clusters which cannot be representative for all stars in GCs at all evolutionary stages. High- and low-metallicity subgroups fall into different subsystems of the Galaxy: several disc and bulge GCs are in the high-metallicity subgroup (see Table 4), halo GCs are in the low-metallicity subgroup.

The situation appeared to be better for calcium (Table 5 and 6). Then, using the analysis analogous to that described in Sec. 3.5, we conclude that the cloud has acquired $M_{\text{Ca}} = 10^{4.77} \, M_\odot$, and SNe CC produced the mass of Ca $10^{3.5} \, M_\odot$. It follows from here that SNe Ia produce $M_{\text{Ca}} = 10^{3.8} - 10^{3.7} = 10^{3.11} \, M_\odot$. Depending on the number of SNe Ia, the calcium mass ejected in a single SN explosion can be equal to $3.9 \cdot 10^{-3} \, M_\odot$ or $1.15 \cdot 10^{-2} \, M_\odot$. The resulting masses coincide with the theoretical ones only by an order of magnitude. One can achieve either full agreement with the results of the two-dimensional pure turbulent deflagration (PTD) models,
High-metallicity subgroup ([X/H] < −1.0)

| Chemical element | Low-metallicity subgroup ([X/H] < −1.0) | High-metallicity subgroup ([X/H] ≥ −1.0) |
|------------------|-----------------------------------------|------------------------------------------|
| 24Mg             | ([Mg/H] < −1.0) | 0.42 | 33 | 0.23 | 6 |
| 28Si             | ([Si/H] < −0.7) | 0.25 | 26 | -0.22 | 0.27 | 6 |
| 40Ca             | ([Ca/H] < −0.8) | -1.42 | 0.28 | 27 | -0.29 | 0.27 | 6 |
| 56Fe             | ([Fe/H] < −1.0) | -1.73 | 0.37 | 32 | -0.55 | 0.29 | 6 |

4 CONCLUSIONS

In this paper, we have proposed a method for determining the properties of type Ia supernovae from short-lived precursors – Prompt SN Ia. This method is based on the assumption that this very subtype of type Ia supernovae is responsible for the enrichment of the high-metallicity subgroup of globular clusters and circumgalactic clouds with the iron peak elements. We believe that GCs are the most suitable laboratories to study the Prompt SN Ia nucleosynthesis. First, the average age of the metal-rich and metal-poor GCs are about the same (Chattopadhyay et al. 2012; VandenBerg et al. 2013). Second, the density of stars in globular clusters is high. Hence, it can be expected that the interaction of stars in them will occur more often than in other stellar groups. The interaction of a white dwarf with its companion’s matter is an essential condition for all SN Ia explosion models existing in the literature (e.g., Leung & Nomoto 2018 and references therein). As was shown by Acharova & Sharina (2018), the occurrence of Prompt SN Ia bursts in GCs is 1.25 − 2.5 times higher than that in the disc. In addition, the evolution of multiple stars allows, at least for one of them, a carbon-oxygen core to form faster with a mass of about 1.4M⊙ required by theory and capable of a subsequent explosion in comparison with a single star (Anguiano et al. 2020). This condition is important because a young progenitor population produces SNe Ia on timescales of 100 – 330 Myr (e.g., Aubourg 2008; Msez 2011 and references therein).

The accuracy of the method depends on the number of globular clusters, in which one method has determined all the assumptions underlying the explosive nucleosynthesis models, which are produced only during supernova explosions.

The analysis of the GC chemical enrichment, together with the characteristics of the parent clouds, allowed us to draw conclusions about the nucleosynthesis in the SNe Ia concentrated in the star-formation regions, namely Prompt SNe Ia. There is no generally accepted precursor model for this SN Ia subtype. It turns out that the nucleosynthesis in globular clusters agrees with the pure turbulent deflagration models, both two-dimensional with a central density of 0.5 − 1 g/cm³ and one-dimensional W7. Based on the assumptions underlying the explosive nucleosynthesis models, it can be concluded that the progenitors of Prompt SNe Ia

or with the W7 results within error of the average value determination.

Table 7. Abundances of chemical elements in GCs according to Pritzl et al. (2005) and the results of theoretical studies of nucleosynthesis yields of the two-dimensional pure turbulent deflagration (PTD) models for various values of the central density of a WD by Leung & Nomoto (2018) (LN18)

| Isotope/Source | 40Ca | < Mass > (M⊙) |
|----------------|------|---------------|
| Observed data  | 5.8 · 10^{-3} |
| N_{SN CC} = 10^5.8 |
| N_{SN Ia} = 10^5.6 |

Theoretical data

PTD LN18 models

| Mass | Model |
|------|-------|
| 050-1-c3-1P | 1.71 · 10^{-3} |
| 100-1-c3-1P | 1.82 · 10^{-3} |
| 300-1-c3-1P | 1.50 · 10^{-3} |
| 500-1-c3-1P | 1.50 · 10^{-3} |

Table 8. Comparison between the mass of calcium 40Ca produced during a SN burst and calculated from the analysis of the corresponding elemental abundances in GCs (Pritzl et al. 2005) and the results of theoretical studies of nucleosynthesis yields of the one-dimensional Chandrasekhar mass deflagration model W7 and the model with deflagration-detonation transition WDD by Leung & Nomoto (2018) and Mori et al. (2018)

| Isotope/Source | 40Ca | < Mass > (M⊙) |
|----------------|------|---------------|
| Observed data  | 1.15 · 10^{-2} |
| N_{SN Ia} = 10^5.1 |

Theoretical data

LN18

W7 | 1.42 · 10^{-2} |
WDD2 | 2.5 · 10^{-2} |
Mori+2018 | 1.13 · 10^{-2} |
W7 | 2.48 · 10^{-2} |
WDD2 |
are best described by the Single Degenerate scenario, in which a degenerate carbon-oxygen stellar remnant accretes the matter from the companion – a Main Sequence or red giant star.

For more accurate conclusions, in order to choose between the two pure turbulent deflagration models, homogeneous data are necessary on various chemical elemental abundances in high- and low-metallicity groups of GCs. However, it is important to emphasize that estimates of the production of Mg, Si, Ca, Cr, and Fe obtained from analysis of the chemical composition of GCs and theoretical predictions of nucleosynthesis models in supernovae are consistent. This is an independent confirmation of the correct approach to modelling nucleosynthesis.

We find it important to note that the model of Single Degenerate progenitors meets another difficulty: a hydrogen mass of about 0.01M⊙ detected using the Hα emission line in the spectra of several SNe Ia is much lower than that expected for the Main Sequence+WD or Red Giant+WD progenitors.

The result allows us to understand in a new way the absence of changes of the [α/Fe] values with time during the evolution of a GC. Only SNe Ia producing a small amount of iron are responsible for the elemental enrichment in GCs.

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DATA AVAILABILITY

The data underlying this article are available in the article.

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