Abstracting Path Conditions

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Abstract

We present a symbolic execution based algorithm that for a given program and a given program location produces a nontrivial necessary condition on input values to drive the program execution to the given location. We propose a usage of the produced condition in contemporary bug finding and test generation tools based on symbolic execution. Experimental results indicate that the presented technique can significantly improve performance of the tools.

1 Introduction

Symbolic execution [5, 20, 19] is enjoying a renaissance during the last decade. The basic idea of the technique is to replace input data of a program by symbols representing arbitrary data. Executed instructions then manipulate expressions over the symbols rather than exact values. A symbolic execution produces, for each path in a program flowgraph starting in the initial location, a formula called path condition, i.e. the necessary and sufficient condition on input data to drive the execution along the path. Symbolic execution is utilized by many successful algorithms and tools for test generation and bug finding, for example Exe [7], Cute [27], Klee [6], Sage [13], or Pex [28]. These tools can relatively quickly find tests that cover vast majority of a given code. However, they usually fail to cover the code completely in a reasonable time. In this paper we suggest a method that helps the tools to cover a chosen location and hence to further improve their performance.

The core of our method and the main contribution of the paper is an algorithm that, for a given program and a given program location, produces a nontrivial necessary condition on input values to drive the program execution to the given location. An intuitive explanation of the algorithm is illustrated on the following simple C++ program, where we want to compute a necessary condition to reach the assertion on the last line.
void foo(int* A, int n) {
    int k = 3;
    for (int i = 0; i < n; ++i) {
        if (A[i] == 1)
            ++k;
    }
    if (k > 12)
        assert(false);
}

It is easy to check (for human) that the assertion is reached when there is more than twelve numbers 1 in array A. Figure 1(a) depicts a flowgraph of foo. Note that nodes and edges that are not on any path to the target location h have been removed.

As the first step of our algorithm, we find all nontrivial maximal strongly connected components in the flowgraph. For each entry node $x$ of each component (i.e., there is an edge leading to $x$ from a vertex outside the component), we compute a summary of the overall effect of iterating within the component, since the first visit of $x$ till the last visit of $x$. The summary is described by an iterated symbolic state and looping condition. An iterated symbolic state is a function that assigns to each program variable its value given by an expression over symbols and path counters. Symbols represent initial values of variables (for each variable $v$ the symbol is denoted by $γ_v$). Path counters $κ_1, κ_2, \ldots$ correspond to different acyclic paths leading from $x$ to $x$ within the component. Each path counter represents the number of iterations of the corresponding path. A looping condition is a nontrivial formula implied by any of path conditions resulting from any symbolic execution of the component.
In our example, there is only one nontrivial maximal strongly connected component \{c, d, e, f\} with one entry node \(c\). There are two acyclic paths through the component: \(\pi_1 = cdefc\) and \(\pi_2 = cdfc\). We assign path counters \(\kappa_1, \kappa_2\) to \(\pi_1, \pi_2\) respectively. The overall effect of the component with respect to the entry node \(c\) can be described by the iterated symbolic state \(\theta^c\) with only two interesting values (as the other variables are not changed in the component):

\[
\begin{align*}
\theta^c(i) &= \kappa_1 + \kappa_2 + i \\
\theta^c(k) &= \kappa_1 + k
\end{align*}
\]

In other words, by \(\kappa_1\) iterations of \(\pi_1\) and \(\kappa_2\) iterations of \(\pi_2\) executed in an arbitrary order, the values of \(i\) and \(a\) are increased by \(\kappa_1 + \kappa_2\) and \(\kappa_1\), respectively.

Further, for every component and its entry node \(x\) we compute a looping condition \(\varphi^c\). Given path counters \(\kappa_1, \kappa_2, \ldots\), formula \(\varphi^c\) describes a necessary condition to keep looping in the component for \(\sum_i \kappa_i\) iterations such that, for each \(i\), exactly \(\kappa_i\) iterations use path \(\pi_i\). More precisely, a looping condition is a conjunction of subformulae \(\varphi_i\) corresponding to the acyclic paths \(\pi_i\). Each subformula \(\varphi_i\) says that, for each of the \(\kappa_i\) iterations along the path \(\pi_i\), all tests on the path must be satisfied for some possible values of variables, i.e. for some values given by the iterated symbolic state and some admissible values of path counters.

In the example, the looping condition for the component \{c, d, e, f\} with the entry node \(c\) has the form \(\varphi^c = \varphi_1 \land \varphi_2\). We focus on the construction of \(\varphi_1\) which corresponds to path \(\pi_1 = cdefc\) with two tests: \(i < n\) and \(A[i] == 1\). The iterated symbolic state says that values of \(i\), \(n\), and \(A[i]\) in \((\tau_1 + 1)\)-st iteration of \(\pi_1\) and after \(\tau_2\) iterations of \(\pi_2\) are \(\tau_1 + \tau_2 + i, \leq n\), and \(A(\tau_1 + \tau_2 + i)\) respectively. Hence, if we want to make \(\kappa_1\) iterations of \(\pi_1\) and \(\kappa_2\) iterations of \(\pi_2\), the formula \(\varphi_1\) says that for each \(\tau_1\) satisfying \(0 \leq \tau_1 < \kappa_1\) there has to be some \(\tau_2\) satisfying \(0 \leq \tau_2 \leq \kappa_2\) such that \(\tau_1 + \tau_2 + i < n\) and \(A(\tau_1 + \tau_2 + i) = 1\).

The complete looping condition for our example is as follows:

\[
\begin{align*}
\varphi^c &\equiv \varphi_1 \land \varphi_2 \\
\varphi_1 &\equiv \forall \tau_1 (0 \leq \tau_1 < \kappa_1 \rightarrow \exists \tau_2 (0 \leq \tau_2 \leq \kappa_2 \land \tau_1 + \tau_2 + i < n \land A(\tau_1 + \tau_2 + i) = 1)) \\
\varphi_2 &\equiv \forall \tau_2 (0 \leq \tau_2 < \kappa_2 \rightarrow \exists \tau_1 (0 \leq \tau_1 < \kappa_1 \land \tau_1 + \tau_2 + i < n \land A(\tau_1 + \tau_2 + i) \neq 1))
\end{align*}
\]

The resulting summary of the component is a pair \((\theta^c, \varphi^c)\). We attach the summary at the entry node \(c\) and we can proceed to analysis of the path \(abcdefgh\) in the flowgraph. We symbolically execute the path as usual. Only at loop entry \(c\) we add the saved summary into the current symbolic state and current path condition. The abstract path condition (and thus also the final result of our technique) is the following formula \(\text{apc}\), where \(\varphi^c[0/0, 3/3]\) is the looping condition computed above with \(i\) replaced by 0 and \(a\) replaced by 3.

\[
\begin{align*}
apc &\equiv \exists \kappa_1, \kappa_2 (\kappa_1, \kappa_2 > 0 \land \varphi^c[0/0, 3/3] \land \kappa_1 + \kappa_2 \geq n \land \kappa_1 + 3 > 12)
\end{align*}
\]
To sum up, our technique produces a formula \( \text{apc} \) that has to be satisfied by all inputs driving the execution to the given location. In general, the formula is not a sufficient condition on inputs to reach the target location. This has basically two reasons.

- It is not always possible to express the overall effect of a strongly connected component to a variable in a declarative way. In such a case, the variable is assigned the special value \( \star \) with the meaning “unknown”. If we symbolically execute a test containing a variable with the value \( \star \), we do not add this test to our abstract path condition. Similarly, the tests containing \( \star \) are not added to looping conditions.

- The looping condition is constructed as a necessary but not a sufficient condition. More precisely, it checks whether tests in each iteration are satisfied for the iterated symbolic state with some admissible values of path counters, but the consistency of these admissible values over all iterations is not checked.

In the following sections, we explain our algorithm in more detail. After providing some preliminaries (Section 2), we present the basic version of the algorithm for flowgraphs with integer arithmetic and read-only multi-dimensional arrays and without function calls (Section 3). Then we indicate necessary changes to the algorithm to work with programs that can modify arrays (Section 4). In the same way as arrays, the algorithm can also handle flowcharts manipulating content of lists (we currently do not support programs changing shape of lists). To demonstrate efficiency of our approach, we provide experimental results of a prototype implementation of our algorithm on several small examples (Section 5). The results show that in some cases, the application of our algorithm can discover a bug in a code much faster than selected bug finding tools do. Therefore we suggest possible utilization our algorithm in contemporary bug finding and test generation tools (Section 6). Finally, we discuss some related work (Section 9) and conclude the paper (Section 10).

2 Preliminaries

This section defines some terms heavily used in the rest of the paper, in particular terms related to program and symbolic state.

2.1 Program

The algorithm works with programs in the form of flowcharts. A target location is a distinguished node of the flowchart and it has no successor. Moreover, we assume that the flowchart contains only nodes from which the target node is reachable. Formally, a program is a tuple \( P = (V_P, E_P, l_s, l_t, \iota_P) \) such that \((V_P, E_P)\) is a finite connected oriented graph, nodes \( V_P \) represent program locations, edges \( E_P \subseteq V_P \times V_P \) represent control flow between them, \( l_s, l_t \in V_P \),
Figure 2: Example of nested loops.

ls ≠ lt are start and target nodes respectively. A node is branching if its out-degree is 2. All other nodes, except lt, have out-degree 1. In-degree of ls and out-degree of lt are both 0. Function \( \iota_P : E_P \rightarrow I \) assigns to each edge \( e \) an instruction \( \iota(e) \). We use two kinds of instruction: an assignment instruction \( v \leftarrow e \) for some scalar variable \( v \) and some expression \( e \), and an assumption \( \text{assume}(\gamma) \) for some quantifier-free formula \( \gamma \) over program variables. Out-edges of any branching node are labelled with instructions \( \text{assume}(\gamma) \) and \( \text{assume}(\neg\gamma) \) for some \( \gamma \). Further, we assume that all instructions in \( I \) use only linear integer arithmetic. By \( V_a \) and \( V_A \) we denote the sets of all scalar variables and array variables occurring in \( P \), respectively. And \( V = V_a \cup V_A \) is a set of all variables. When program \( P \) is clearly determined by a context, we omit the subscript \( P \) in \( V_P, E_P, \iota_P \).

A path in a program is a finite sequence \( \pi = v_1v_2 \cdots v_k \) of program nodes such that \((v_i, v_{i+1}) \in E\) for all \( 1 \leq i < k \). Paths are always denoted by greek letters. A path leading from ls to lt is called complete path.

Instead of strongly connected components, our algorithm works with loops. In contrast to components, loops can be nested. Let \( \pi \) be an acyclic path from the initial node ls and let \( \alpha \) be a prefix of \( \pi \) leading to a node \( v \). The node \( v \) on \( \pi \) is an entry node of a loop if there exists a path \( v\beta v \) such that none of the nodes on \( \beta \) appears in \( \alpha \). The entry node \( v \) on \( \pi \) enters the loop \( C \) that is the smallest set containing \( v \) and all nodes in \( \beta \) for each path \( v\beta v \) such that none of the nodes on \( \beta \) appears in \( \alpha \). For example, program in Figure 2 contains two acyclic complete paths: \( \pi_1 = l_sbdl_t \) and \( \pi_2 = l_abdl_t \). While \( \pi_1 \) contains only one entry node \( b \) associated with loop \{a, b, c, d\}, \( \pi_2 \) contains entry node \( a \) with loop \{a, b, c, d\} and entry node \( b \) with loop \{b, c\}. A node \( u \) is an exit node of \( C \), if there exists \( w \in C \), such that \((w, u) \in E \).

For a loop \( C \) with an entry node \( v \), a program induced by the loop, denoted as \( P(C, v) \), is the subgraph of the original program induced by \( C \) where \( v \) is marked as the start node, a fresh node \( v' \) is added and marked as the target node, and every edge \((u, v) \in E \) leading to \( v \) is replaced by an edge \((u, v') \).
Let \( \pi \) be a complete path. We define a backbone of \( \pi \) as the result of the following procedure: If \( \pi \) is acyclic, then the backbone is directly \( \pi \). Otherwise, \( \pi \) can be written as \( \alpha v \beta v \gamma \) where \( v \) is the first repeating node in \( \pi \) and \( \gamma \) does not contain \( v \). In this case, we set \( \pi \) to \( \alpha v \gamma \) and repeat the procedure. By \( B_P \) we denote the set of all backbones of all complete paths. Note that backbones are exactly all acyclic complete paths. Alternatively, backbone of a complete path \( \pi \) can be defined as the path from \( l_s \) where the successor of each node \( u \) is the same as the successor of the last occurrence of \( u \) in \( \pi \).

### 2.2 Symbolic State

The set \( S \) of symbolic expressions contains all expressions build with integers, standard integer operations and functions, and

- a constant symbol \( \underline{a} \) for each scalar variable \( a \in V_a \),
- a function symbol \( \underline{A} \) for each array variable \( A \in V_A \), where \( \underline{A} \) has the same arity as \( A \),
- a countable set \( \{ \kappa_1, \tau_1, \kappa_2, \tau_2, \ldots \} \) of variables called path counters,
- a special construct \( \text{ite}(\varphi, e_1, e_2) \) where \( e_1, e_2 \) are expressions and \( \varphi \) is a first order formula over the same signature extended with standard relation symbols, and
- a special constant symbol \( \star \) called unknown.

The value of \( \text{ite}(\varphi, e_1, e_2) \) is the same as \( e_1 \) if \( \varphi \) holds and the same as \( e_2 \) otherwise. The domain of integers is extended with a new special value \( \perp \). All expressions containing \( \star \) are interpreted to \( \perp \) while the other expressions are never interpreted to \( \perp \). In the following we identify every expression containing \( \star \) with \( \star \).

Let \( f, e_1, e_2, \ldots, e_n \) be symbolic expressions and \( x_1, x_2, \ldots, x_n \) be some path counters or constant symbols corresponding to scalar variables. Then \( f[x_1/e_1, x_2/e_2, \ldots, x_n/e_n] \) is an symbolic expression \( f \) where all occurrences of \( x_i \) are replaced by \( e_i \), simultaneously for all \( i \). To shorten the notation, we also write \( f[\vec{x}/\vec{e}] \) when the meaning is clearly given by a context. We also use the notation \( \varphi[\vec{x}/\vec{e}] \) with the analogous meaning.

A symbolic state is a function \( \theta : V \rightarrow S \) assigning to each variable \( a \) a symbolic expression \( \theta(a) \). We define initial symbolic state \( \theta_I \) and unknown symbolic state \( \theta_* \) as

\[
\theta_I(a) = \underline{a}, \quad \theta_I(A) = \lambda \vec{\chi}. \underline{A}(\vec{\chi}) \quad \text{and} \quad \theta_*(a) = \star, \quad \theta_*(A) = \lambda \vec{\chi}. \star
\]

for each \( a \in V_a \) and \( A \in V_A \). Note that \( \lambda \)–expressions for scalar variables can be omitted (and they actually were), since symbols \( \underline{a} \) are constants. We use the notation \( \theta(\cdot) \) in a more general way. For a given expression over program variables it always denotes the operation of replacing each variable \( a \in V \) by
the symbolic expression \( \theta(a) \). Moreover, we further extend
the operation for formulae over program variables. We note that
predicates containing \( \star \) in some of its terms are immediately
reduced to \( \text{true} \).

Let \( \theta \) be a symbolic state, \( a \in \mathcal{V} \) be a variable and \( e \) be a symbolic expression.
Then \( \theta[a \rightarrow e] \) is a symbolic state equal to \( \theta \) except for variable \( a \), where \( \theta[a \rightarrow e](a) = e \). Further, \( \theta(e) \) denotes a symbolic expression derived from \( e \) by
simultaneously replacing all occurrences of each symbol \( a \) by symbolic expression \( \theta(a) \). We also extend the notation \( \theta(\cdot) \) for formulae
in natural way. We only note that predicates containing \( \star \) in some of its terms are immediately reduced to \( \text{true} \). Finally, we extend the notation \( \theta(\cdot) \) to symbolic states:
\( \theta(\theta') \) is a symbolic state satisfying \( \theta(\theta')(a) = \theta(\theta'(a)) \) for each variable \( a \in \mathcal{V} \).

For brevity of notation, we often use vector notation. Let \( \vec{u} = (u_1, \ldots, u_n) \) and \( \vec{v} = (v_1, \ldots, v_n) \) be two vectors of some symbolic expressions. We use \( \vec{u} \leq \vec{v} \) and \( \vec{u} < \vec{v} \) as abbreviations for the following formulae.

\[
\vec{u} \leq \vec{v} \equiv u_1 \leq v_1 \land \ldots \land u_n \leq v_n
\]

\[
\vec{u} < \vec{v} \equiv \vec{u} \leq \vec{v} \land \sum_{i=1}^{n} u_i < \sum_{i=1}^{n} v_i
\]

3 Algorithm for Read-only Arrays

The idea of our algorithm is relatively simple. Given a program \( P \), we compute
the set \( B_P \) of all backbones of \( P \), i.e. all acyclic complete paths. Then we
compute an abstract path condition for each backbone. To compute an abstract
path condition for a backbone \( \pi \), we perform a standard symbolic execution of
instructions along \( \pi \) (i.e. we gradually construct a path condition \( \text{apc} \) and we
maintain a symbolic state \( \theta \)) and whenever we visit an \( a \) entry node, we process
the corresponding loop and then we add the resulting summary into the current
path condition and symbolic state.

Before we explain summary computation of loops, we need to define
the several terms. Let \( v \) be an entry node of a loop \( C \). An iteration is
an arbitrary path of the form \( v \alpha v \) such that \( \alpha \) is a (possibly empty) sequence of nodes in
\( C \setminus \{v\} \). There is a clear bijection between iterations and complete paths in the
program \( P(C,v) \) induced by the loop. Hence, we do not distinguish between an
iteration and the corresponding complete path. Let \( \pi_1, \ldots, \pi_k \) be all backbones
in \( P(C,v) \). We associate a fresh path counter \( \kappa_i \) to each backbone \( \pi_i \). A looping path
is an arbitrary path over nodes of \( C \) leading from \( v \) to \( v \). Let \( \beta \) be a looping path.
Then \( \beta \) can be written as \( \beta = v \alpha_1 v \alpha_2 v \ldots v \alpha_n v \) where \( n \geq 0 \) and each
\( v \alpha_i v \) is an iteration. We define \( \vec{\kappa}(\beta) \) as vector \( (c_1, c_2, \ldots, c_k) \), where each \( c_j \) is
the number of iterations \( v \alpha_j v \) in \( \beta \) such that their backbone is \( \pi_j \).

To process a loop \( C \) with entry node \( v \) means to compute an iterated symbolic
state and looping condition for the loop. Let \( \vec{\kappa} \) be a vector of path counters
firmly associated to backbones of the loop. On intuitive level, Iterated symbolic state \( \theta^{\vec{\kappa}} \) is a symbolic state that represents values of variables after arbitrary
looping path. The values are expressions that may contain path counters of \( \vec{\kappa} \).
Further, looping condition $\varphi^{\vec{\kappa}}$ is a formula generalizing all path conditions of all looping paths. The formula $\varphi^{\vec{\kappa}}$ may contain path counters of $\vec{\kappa}$. Formally, $\theta^{\vec{\kappa}}$ and $\varphi^{\vec{\kappa}}$ have to satisfy the following condition: for each path condition $pc$ and each symbolic memory $\theta$ produced a standard symbolic execution along some looping path $\beta$, it holds that

- $pc \rightarrow \varphi^{\vec{\kappa}}[\vec{\kappa}/\vec{\kappa}(\beta)]$,
- for each scalar variable $a$, either $\theta^{\vec{\kappa}}(a)[\vec{\kappa}/\vec{\kappa}(\beta)]$ contains $\star$, or $\theta(a) = \theta^{\vec{\kappa}}(a)[\vec{\kappa}/\vec{\kappa}(\beta)]$ is a valid formula.

Hence, an iterated symbolic state and a looping condition can be seen as an abstraction (or an over-approximation) of all symbolic states and path conditions for all looping paths.

Now we return back to the symbolic execution of the backbone $\pi$. When we have $\theta^{\vec{\kappa}}$ and $\varphi^{\vec{\kappa}}$, we update path condition to $apc \land \theta(\varphi^{\vec{\kappa}})$, symbolic state to $\theta^{\vec{\kappa}}$, and we continue with standard symbolic execution along the backbone $\pi$. When the symbolic execution of the backbone $\pi$ finishes, we set $apc$ to $\exists \vec{\kappa}'(\vec{\kappa}' \geq \vec{0} \land apc)$, where $\vec{\kappa}'$ is a vector of all path counters with free occurrences in $apc$. The resulting formula $apc$ is a generalization of all (standard) path conditions for all paths along the backbone $\pi$, as $pc \rightarrow apc$ holds for each such a standard path condition $pc$.

Let $apc_{\pi}$ be an abstract path condition for each backbone $\pi \in B_P$. Then the necessary condition on input data to reach the target node and thus the final result of our algorithm is the formula

$$\bigvee_{\pi \in B_P} apc_{\pi}.$$  

The computation of abstract path condition $apc$ for a given backbone $\pi$ is precisely formulated in Algorithm 1. On Line 12 the algorithm calls function $processLoop(P(C, v_i))$ that returns an iterated symbolic state $\theta^{\vec{\kappa}}$ and a looping condition $\varphi^{\vec{\kappa}}$ (i.e. a summary) for a loop $C$ at its entry node $v_i$ represented by an induced program $P(C, v_i)$. We assume that the path counters $\vec{\kappa}$ used in $\theta^{\vec{\kappa}}$ and $\varphi^{\vec{\kappa}}$ are fresh, i.e. they do not occur in current values of $\theta$ or $apc$.

In the rest of this section, we describe two versions of the $processLoop$ procedure.

### 3.1 Loop Processing: Lightweight Version

We are given a program $P'$ induced by a loop at some entry node. We compute the set of all backbones $B_{P'} = \{\pi_1, \ldots, \pi_k\}$ and we run the function $executeBackbone(\pi_i, P')$ on each backbone $\pi_i$. Let $\theta_i$ and $apc_i$ be the returned symbolic state and abstract path condition, respectively. Further, we assign a fresh path counter $\kappa_i$ to each backbone $\pi_i$. We set $\vec{\kappa} = (\kappa_1, \ldots, \kappa_k)$.

First, we compute an iterated symbolic state $\theta^{\vec{\kappa}}$. In other words, for each scalar variable $a$ we construct a symbolic expression over symbols and path
Algorithm 1: executeBackbone($\pi, P$)

Input:
\[ \pi // a backbone of P \]
\[ P // a program \]

Output:
\[ \theta // symbolic state \]
\[ apc // abstracted path condition \]

1. $\theta \leftarrow \theta_I$
2. $apc \leftarrow true$
3. Let $\pi$ has the form $v_0v_1 \ldots v_n$
4. for $i \leftarrow 1$ to $n$ do
5. if $\iota_P((v_{i-1}, v_i))$ has the form $assume(\gamma)$ then
6. \[ apc \leftarrow apc \land \theta(\gamma) \]
7. if $\iota_P((v_{i-1}, v_i))$ has the form $\nu \leftarrow e$ then
8. \[ \theta \leftarrow \theta[\nu \rightarrow \theta(e)] \]
9. if $v_i$ is an entry node on $\pi$ then
10. Let $C$ be the loop at entry $v_i$ on the backbone $\pi$
11. Compute induced program $P(C, v_i)$
12. \[ (\theta^k, \varphi^k) \leftarrow processLoop(P(C, v_i)) \]
13. \[ apc \leftarrow apc \land \theta(\varphi^k) \]
14. \[ \theta \leftarrow \theta(\theta^k) \]
15. \[ apc \leftarrow \exists \vec{\kappa}'(\kappa' \geq \vec{0} \land apc) \] where $\kappa'$ are all path counters
with free occurrences in $apc$
16. return $(\theta, apc)$

counters of $\vec{\kappa}$ describing the value of $a$ after arbitrary $\sum_{1 \leq m \leq k} \kappa_i$ successive executions of program $P'$ such that exactly $\kappa_i$ executions took backbone $\pi_i$ for each $\pi_i \in B_{P'}$. In general, this is a very hard task. To be on the safe side, we start with $\theta_\vec{\kappa}$ set to $\theta_\star$ and we gradually improve its precision. More precisely, we change the value of $\theta_\vec{\kappa}(a)$ in one of the following four cases:

1. For each backbone $\pi_i \in B_{P'}$, $\theta_i(a) = a$. In other words, the value of $a$ is not changed on any complete path in $P'$. This case is trivial. We set $\theta_\vec{\kappa}(a) = a$.

2. For each backbone $\pi_i \in B_{P'}$, either $\theta_i(a) = a$ or $\theta_i(a) = a + d_i$ for some symbolic expression $d_i$ such that $\theta_\vec{\kappa}(d_i)$ contains neither $\star$ nor any path counters. Let us assume that the latter possibility holds for $\pi_1, \ldots, \pi_{k'}$ and the former one for $\pi_{k'+1}, \ldots, \pi_k$. The condition on $\theta_\vec{\kappa}(d_i)$ guarantees that the value of $d_i$ is constant during all iterations over the loop. In this case, we set $\theta_\vec{\kappa}(a) = a + \sum_{1 \leq i \leq k'} d_i \cdot \kappa_i$.

3. There exists a symbolic expression $d$ such that $\theta_\vec{\kappa}(d)$ contains neither $\star$ nor any path counters, and for each backbone $\pi_i \in B_{P'}$, either $\theta_i(a) = a$
or $\theta_i(a) = d$. Let us assume that the latter possibility holds for $\pi_1, \ldots, \pi_{k'}$ and the former one for $\pi_{k'+1}, \ldots, \pi_k$. In other words, the value of $a$ is set to $d$ along each backbone $\pi_j$ for $1 \leq j \leq k'$, while it is unchanged on any other complete path. Hence, we set $\theta^a_i(a) = \text{ite}(\sum_{1 \leq j \leq k'} \kappa_i > 0, d, a)$.

4. For one backbone, say $\pi_i$, $\theta_i(a) = d$ for some symbolic expression $d$ such that $\theta^\kappa_i(d)$ contains neither $*$ nor any path counters except $\kappa_i$. Further, for each backbone $\pi_j$ such that $i \neq j$, $\theta_j(a) = a$. That is, only the complete paths with backbone $\pi_i$ modify $a$ and they set it to a value independent on other path counters than $\kappa_i$. Note that if we assign $d$ to $a$ in the $\kappa_i$-th iteration along the complete paths with backbone $\pi_i$, then the actual assigned value of $d$ is the value after $\kappa_i - 1$ iterations along the paths.

Hence, we set $\theta^a_i(a) = \text{ite}(\kappa_i > 0, \theta^\kappa_i(d)[\kappa_i/\kappa_i - 1], \varnothing)$.

We apply these rules repeatedly until no other precise value of $\theta^a_i(a)$ can be derived.

Computation of an looping condition $\varphi^\kappa_i$ is straightforward. The intuition has been already given in the introduction. We set

$$\varphi^\kappa_i \equiv \bigwedge_{i=1}^{k} \forall \tau_i \left( 0 \leq \tau_i < \kappa_i \rightarrow \right. \left. \exists \bar{\tau}_i \left( 0 \leq \bar{\tau}_i \leq \bar{\kappa}_i \land \theta^\kappa_i(\text{apc}_i)[\bar{\kappa}_i/\bar{\tau}_i]) \right) \right.$$

where

$$\bar{\tau}_i = (\tau_1, \ldots, \tau_{i-1}, \tau_{i+1}, \ldots, \tau_k),$$

$$\bar{\kappa}_i = (\kappa_1, \ldots, \kappa_{i-1}, \kappa_{i+1}, \ldots, \kappa_k).$$

Let us note that the program $P'$ induced by a loop can again contain loops. Hence, a symbolic state $\theta_i$ and an abstract path condition $\text{apc}_i$ for a backbone $\pi_i$ can contain path counters $\bar{\kappa}'$ corresponding to some loops inside $P'$. As the number of iterations of the inner loop can be different in each iteration of the outer loop, the meaning of $\bar{\kappa}'$ is different in each iteration of $P'$. Our construction of $\theta^\kappa_i$ handles this situation correctly: if some $\theta_i(a)$ contains a path counter of $\bar{\kappa}'$, then $\theta^\kappa_i(a) = *$. Further, the path counters of $\bar{\kappa}'$ may occur in the looping condition $\varphi^\kappa_i$, but all their occurrences are in subformulae of the form $\text{apc}_i$ talking about a single iteration of $P'$, and they are bound there by an existential quantifier.

### 3.2 Improving Precision of Lightweight Version

The loop processing procedure described in the previous subsection is correct, but not very precise when program $P'$ contains nested loops. We illustrate it on the following program.
for (i = 0; i < m; ++i) {
    j = i;
    while (j < n) {
        ++j;
    }
}

The corresponding flowgraph is depicted in Figure 3 (upper). The program contains one backbone \( l_a a l_b \) with entry node \( a \) and the corresponding loop \( C = \{a, b, c, d, e\} \). The induced program \( P' = P(C,a) \) contains again one backbone \( abcea' \) with entry node \( c \) and the corresponding loop \( C' = \{c, d\} \). The induced programs \( P' \) and \( P'' = P(C', c) \) are depicted in Figure 3 (lower left and lower right respectively).

Figure 3: Example of nested path counter dependency (upper). Program \( P' \) induced by loop \( C = \{a, b, c, d, e\} \) with entry node \( a \) (lower left). Program \( P'' \) induced by loop \( C' = \{c, d\} \) with entry node \( c \) (lower right).
We can easily compute the iterated symbolic state $\theta^\kappa'$ and looping condition $\varphi^\kappa'$ for $P''$:

$$
\theta^\kappa'(j) = j + \kappa' \\
\varphi^\kappa' = \forall \tau'(0 \leq \tau' < \kappa' \rightarrow j + \tau' < n)
$$

With this information, one can compute symbolic state $\theta'$ for backbone $abcea'$ of $P'$. As there is only one backbone in $P'$, the iterated symbolic state $\theta^\kappa$ can be computed directly from $\theta'$.

$$
\begin{align*}
\theta'(1) & = i + 1 \\
\theta'(j) & = i + \kappa' \\
\theta'(m) & = m \\
\theta'(n) & = n
\end{align*}
$$

In fact, the value of $j$ after one iteration of $P'$ can be expressed without $\kappa'$ as $\theta'(j) = \max(n, i)$. If we modify $\theta'$ in this way, the algorithm presented in the previous section computes more precise iterated symbolic state $\theta^\kappa$, namely it returns

$$
\theta^\kappa'(j) = \text{ite}(\kappa > 0, \max(n, i + \kappa - 1), j).
$$

The crucial step towards higher precision of iterated symbolic state is detection of dependencies between path counters of an outer loop and path counters of its nested loops. In the example, we would like to detect the fact that in $(\kappa + 1)$-st iteration of $P'$, the nested loop is iterated $\kappa' = \max(0, n - (i + \kappa))$ times. In the next section we show how we detect these dependencies between path counters.

### 3.3 Loop Processing: Heavyweight Version

Intuitively, our heavyweight loop processing algorithm is looking for linear dependency of the sum of all path counters of a nested loop on path counters of the outer loop and on scalar program variables. We are asking an SMT solver to infer dependencies from adjusted abstract path conditions. If such a dependency is found, it is used to eliminate path counters of nested loop in computation of iterated symbolic state corresponding to the outer loop.

The heavyweight loop processing procedure is given in Algorithm 2. The algorithm works with the set of artificial program variables $V_s$, which is empty at the beginning. The algorithm starts similarly as the lightweight one: it computes all backbones of $P'$ and for each backbone $\pi_i$ it computes the corresponding symbolic state $\theta_i$ and abstract path condition $apc_i$. Further, for each loop on the backbone $\pi_i$ it performs the following three steps, where $\vec{\kappa}_j$ are path counters of the loop.

- A fresh artificial variable $s_{i,j}$ is added to $V_s$. In each iteration with backbone $\pi_i$, the variable $s_{i,j}$ represents the sum of path counters $\vec{\kappa}_j$. 

12
Algorithm 2: processLoop($P'$)

Input:
$P'$ // an induced program of a loop at an entry node

Output:
$\theta^\kappa$ // iterated symbolic state
$\varphi^\kappa$ // looping condition

1. $V_s \leftarrow \emptyset$
2. Compute backbones $B_{P'} = \{\pi_1, \pi_2, \ldots, \pi_k\}$
3. Let $\kappa = (\kappa_1, \kappa_2, \ldots, \kappa_k)$ be fresh path counters
4. foreach $\pi_i \in B_{P'}$ do
5. $(\theta_i, apc_i) \leftarrow \text{execute Backbone} (\pi_i, P')$
6. foreach entry node $v_j$ on $\pi_i$ do
7. Let $\kappa_j$ be path counters of the loop entered by $v_j$
8. Add a fresh variable $s_{i,j}$ to $V_s$
9. Replace $\sum \kappa_j$ in $\theta_i$ by $s_{i,j}$ and remove remaining $\kappa_j$
10. Construct a weakened looping condition $\delta_{i,j}$ on $s_{i,j}$
11. Let $E$ be a set of all loop entry nodes along backbones in $B_{P'}$
12. foreach entry node $v_{i,j} \in E$ do
13. Construct a exit condition $\delta_{i,j}$ from formulae $apc_i$
14. Extend $\theta^\kappa$ to $V \cup V_s$
15. $\theta^\kappa \leftarrow \theta_s$
16. repeat
17. change $\leftarrow$ false
18. foreach $a \in V$ do
19. $e \leftarrow$ infer $\theta^\kappa (a)$ from $\theta_1, \theta_2, \ldots, \theta_k$ and $\theta^\kappa$
20. if $\theta^\kappa (a) = * \land e \neq *$ then
21. $\theta^\kappa (a) \leftarrow e$
22. change $\leftarrow$ true
23. foreach $s_{i,j} \in V_s$ do
24. $e \leftarrow$ infer $s_{i,j}$ from $\delta_{i,j}$ and $\theta^\kappa$
25. if $\theta^\kappa (s_{i,j}) = * \land e \neq *$ then
26. $\theta^\kappa (s_{i,j}) \leftarrow e$
27. change $\leftarrow$ true
28. until change = false
29. $\theta^\kappa \leftarrow \theta^\kappa | V$
30. Construct $\varphi^\kappa$ from $apc_1, \ldots, apc_k$ and $\theta^\kappa$
31. return ($\theta^\kappa, \varphi^\kappa$)

- We change $\theta_i$ to $\theta_i[\sum \kappa_j/s_{i,j}[\kappa_j/]$. Hence, we replace each sum of all path counters $\kappa_j$ by $s_{i,j}$ and we replace all other occurrences of path counters $\kappa_j$ by $*$.
- Since we are only interested in the sum $\sum \kappa_j$, we can weaken the looping condition.
condition of the inner loop. We denote this weakened formula as $\sigma_{i,j}$ and compute it as follows. Let $apc_1', \ldots, apc_k'$ be abstracted path conditions for backbones of the inner loop and let $\theta^j_y$ be the iterated symbolic state computed for the inner loop. Then we set

$$
\sigma_{i,j} \equiv \begin{cases} 0 \leq s_{i,j} - 1 \rightarrow \\ \theta^j_y \langle apc_1' \lor \cdots \lor apc_k' \rangle \left[ \sum \vec{\kappa}_j/s_{i,j} - 1 \right] \left[ \vec{\kappa}_j/\vec{\kappa}^j \right].
\end{cases}
$$

Next we compute exit conditions $\delta_{i,j}$ from inner loops at loop entries $v_{i,j}$. For each exit node $x$ from an inner loop at entry node $v_{i,j}$, there is a backbone $\pi_l, l \leq k, \ldots v_{i,j} \alpha x \ldots$. To leave the loop, all the conditions along the path $\alpha x$ must by satisfiable. Therefore, if we denote by $apc_{v_{i,j},x}(\alpha x)$ the conjunction of these conditions and $x_1, \ldots, x_r$ are all exits from the inner loop at entry $v_{i,j}$, then we can express $\delta_{i,j}$ as a formula:

$$
\delta_{i,j} \equiv (apc_{v_{i,j},x_1}(\alpha_1 x_1) \lor \cdots \lor apc_{v_{i,j},x_r}(\alpha_r x_r)) \\
\left[ \sum \vec{\kappa}_j/s_{i,j} - 1 \right] \left[ \vec{\kappa}_j/\vec{\kappa}^j \right].
$$

In the second half of the algorithm, we extend $\theta^\vec{\kappa}$ to the artificial program variables and we compute iterated symbolic state $\theta^\vec{\kappa}$. We alternately try to infer more precise information for standard and artificial program variables. The inference on line 19 employs the four conditions formulated in Subsection 3.1, while the inference on line 24 executes an SMT solver. The solver decides whether there exists a linear expression over path counters $\vec{\kappa}$ and constant symbols corresponding to scalar program variables equivalent to $s_{i,j}$ for each $s_{i,j}$ that satisfies the necessary condition given by $\sigma_{i,j} \land \delta_{i,j}$. Hence, an SMT solver asked for satisfiability of the formula

$$
\forall \vec{a}, \vec{\kappa}, s_{i,j} \left( (\vec{\kappa} \geq \vec{0} \land s_{i,j} \geq 0 \land \theta^\vec{\kappa} (\sigma_{i,j} \land \delta_{i,j})) \rightarrow \\ s_{i,j} = \max \{0, (\vec{\kappa} \cdot M + \vec{w}) \cdot (\vec{a}, 1)^T \} \right),
$$

where $M$ is a matrix and $\vec{w}$ is a vector of constant symbols and of an appropriate sizes, $\vec{a}$ is a vector of variables containing a variable $a$ for each scalar program variable $a$, and $(a, 1)$ is the same vector prolonged with constant 1. Note that it is possible to formulate stronger necessary conditions on $s_{i,j}$ than $\rho_{i,j}$. A stronger condition can lead to more discovered dependencies. One can also look for more complex dependencies (for example dependencies involving arrays). We chose a simple and relatively weak conditions $\rho_{i,j}$ to get quick reactions of an SMT solver.

At the end of algorithm, we restrict the iterated symbolic state obtained $\theta^\vec{\kappa}$ back to standard program variables. Finally, we compute a looping condition in the way described in Subsection 3.1.
4 Extension for Array-manipulating Programs

This section sketches necessary steps to extension of our algorithm to programs that modify arrays.

First, we extend our instruction set with an assignment instruction of the form \( A[e_1, e_2, \ldots, e_n] \leftarrow e \), where \( e, e_1, e_2, \ldots, e_n \) are program expressions of integer type and \( n \geq 1 \) is the arity of \( A \). Further, we have to define symbolic expressions of types \( \text{Int}^k \rightarrow \text{Int} \) for every arity \( k \). For expressions of such a type, we use the notation \( \lambda x_1 x_2 \ldots x_k.e \) or \( \lambda x_1 x_2 \ldots x_k.e(x_1, x_2, \ldots, x_k) \) if we want to emphasize that \( e \) is a function symbol of type \( \text{Int}^k \rightarrow \text{Int} \). We often use vector notation \( \lambda \vec{x}.e(\vec{x}) \) instead of \( \lambda x_1 x_2 \ldots x_k.e(x_1, x_2, \ldots, x_k) \), where \( k \) is determined by a context. A symbolic execution has to be extended as well in order to handle assignment instructions modifying an array.

The most interesting of the extension for full array support are the rules that allow us to compute values of array variables in and iterated symbolic state. We mention two rules we have designed for arrays.

Assume that we are given a program \( P' \) with backbones \( \pi_1, \ldots, \pi_k \) and symbolic state \( \theta_i \) and abstract path condition \( \text{apc}_i \) for each \( \pi_i \). The iterated symbolic value of an array variable \( A \) can be precisely computed if some of the following two cases happen.

- If \( \theta_i(A) = \lambda \vec{x}.A(\vec{x}) \) for all backbones \( \pi_i \), then the array is not changed along any complete path in \( P' \). Hence we set \( \theta^\vec{\kappa}(A) = \lambda \vec{x}.A(\vec{x}) \).

- If there exists one backbone, say \( \pi_1 \) such that

\[
\theta_1(A) = \lambda \vec{x}.\text{ite}(\varphi_1, t_1(\vec{x}), \text{ite}(\varphi_2, t_2(\vec{x}), \ldots, \text{ite}(\varphi_n, t_n(\vec{x}), A(\vec{x})) \ldots))
\]

and \( \theta_i(A) = \lambda \vec{x}.A(\vec{x}) \) for all other backbones \( \pi_i \), then we set

\[
\theta^\vec{\kappa}(A) = \lambda \vec{x}.\text{ite}(\kappa_1 = 0, A(\vec{x}), \text{ite}(\varphi_1, t_1(\vec{x}), \text{ite}(\varphi_2, t_2(\vec{x}), \ldots, \text{ite}(\varphi_n, t_n(\vec{x}), A(\vec{x})) \ldots))).
\]

Other rules are written in similar style.

5 Soundness and Incompleteness

In this section we formulate and prove soundness and incompleteness theorems for our algorithm.

**Theorem 1** (Soundness). Let \( \text{apc} \) be the necessary condition computed by our algorithm for a given target program location. If \( \text{apc} \) is not satisfiable, then the target location is not reachable in that program.

*Informal proof.* We build any looping condition \( \varphi^\vec{x} \) such that it is implied by all path conditions of an analysed loop. And each formula \( \text{apc}_\pi \) constructed
in Algorithm 1 collects all the predicated along passed backbone $\pi$ and it also collects looping conditions at loop entry nodes along the backbone. Therefore, $apc_\pi$ must be implied by any path condition of any symbolic execution along $\pi$. We compute final $apc$ as a disjunction of formulae $apc_\pi$ for all backbones. Since any program path leading to the target location must follow some backbone (with possible temporary escapes into loops along the backbone), its path condition exists (i.e. it is satisfiable formula) only if $apc$ is satisfiable.

**Theorem 2 (Incompleteness).** There is a program and an unreachable target location in it for which the formula $apc$ computed by our algorithm is satisfiable.

**Proof.** Let us consider the following C code:

```c
int i = 1;
while (i < 3) {
    if (i == 2)
        i = 1;
    else
        i = 2;
}
```

The loop never terminates. Therefore, a program location below it is not reachable. But $apc$ computed for that location is equal to $true$, since variable $i$ does not follow a monotone progression.

6 Dealing with Quantifiers

We can ask an SMT solver whether a computed necessary condition $apc$ is satisfiable or not. And if it is, we may further ask for some its model. As we will see in Section 7 such queries to a solver should be fast. Unfortunately, our experience with solvers shows that presence of quantifiers in $apc$ usually causes performance issues. Although SMT technology evolves quickly, we show in this section how to overcome this issue now by unfolding universally quantified formulae the looping conditions $\varphi^\kappa$ are made of.

Universally quantified variables $\tau_i$ in formulae $\varphi^\kappa$ are always restricted from above by path counters $\kappa_i$ counting iterations of backbones $\pi_i'$ of analysed loop. Let us choose some upper limits $K_i > 0$ for the path counters $\kappa_i$. Since each $\tau_i$ ranges over a finite set of integers $\{0, \ldots, K_i - 1\}$ now, we can unfold each universally quantified formula in $\varphi^\kappa$ for each possible value of $\tau_i$. Having eliminated the universal quantification, we can also eliminate existential quantification of all $\kappa_i$ and all $\tau_i'$ in $\varphi^\kappa$ and whole $apc$ by redefining them as uninterpreted integer constants. Let us see an unfolded necessary condition $apc$, denoted by $apc^K$, of our running example, when we choose upper limits $\bar{K} = (K_1, K_2)$ for the path counters $K = (\kappa_1, \kappa_2)$:

$$apc^K = 0 \leq \kappa_1 \land 0 \leq \kappa_2 \land \bigwedge_{i=0}^{K_1} (0 \leq i \leq \kappa_1 \rightarrow (0 \leq \tau_{2,i} \leq \kappa_2 \land$$
\[ i + \tau_{2,i} < n \land A(i + \tau_{2,i}) = 1) \land \\
\bigwedge_{i=0}^{K_2} (0 \leq i < K_2 \rightarrow (0 \leq \tau_{1,i} \leq \kappa_1 \land \\
\tau_{1,i} + i < n \land A(\tau_{1,i} + i) \neq 1)) \land \\
\kappa_1 + \kappa_2 \geq n \land \kappa_1 + 3 > 12, \]

where \( \kappa_1, \kappa_2, \tau_1, \ldots, \tau_{1,K_2}, \tau_{2,0}, \ldots, \tau_{2,K_2} \) are uninterpreted integer constants.

For any \( \vec{K} \) the formula \( apc^{\vec{K}} \) represents wakened \( apc \). Higher values we choose, then we get closer to the precision of \( apc \). In practice we must choose moderate values \( \vec{K} \), since the unfolding process makes \( apc^{\vec{K}} \) much longer than \( apc \).

In some cases an SMT solver is able to quickly decide satisfiability of \( apc \). Therefore, we ask the solver for satisfiability of \( apc \) in parallel with the unfolding procedure described above. And there is a common timeout for both queries. We take the fastest answer. In case both queries exceeds the timeout, the condition \( apc \) cannot help a tool to cover given target location.

### 7 Integration into Tools

Tools based on symbolic execution typically explore program paths iteratively. At each iteration there is a set of program locations \( \{v_1, \ldots, v_k\} \), from which the symbolic execution may continue further. At the beginning the set contains only program entry location. In each iteration of the symbolic execution the set is updated such that actions of program edges going out from some locations \( v_i \) are symbolically executed. Different tools use different systematic and heuristic strategies for selecting locations \( v_i \) to be processed in the current iteration. It is also important to note that for each \( v_i \) there is available an actual path condition \( \varphi_i \) capturing already taken symbolic execution from the entry location up to \( v_i \).

When a tool detects difficulties in some iteration to cover a particular program location, then using \( apc \) it can restrict selection from the whole set \( \{v_1, \ldots, v_k\} \) to only those locations \( v_i \), for which a formula \( \varphi_i \land apc \) is satisfiable. In other words, if for some \( v_i \) the formula \( \varphi_i \land apc \) is not satisfiable, then we are guaranteed there is no real path from \( v_i \) to the target location. And therefore, \( v_i \) can safely be removed from the consideration.

Tools like SAGE, PEX or CUTE combine symbolic execution with concrete one. Let us assume that a location \( v_i \), for which the formula \( \varphi_i \land apc \) is satisfiable, was selected in a current iteration. These tools require a concrete input to the program to proceed further from \( v_i \). Such an input can directly be extracted from any model of the formula \( \varphi_i \land apc \).

### 8 Experimental Results

We implemented the algorithm in an experimental program, which we call Apc. We also prepared a small set of benchmark programs mostly taken from other papers. In each benchmark we marked a single location as the target one. All
the benchmarks have a huge number of paths, so it is difficult to reach the target. We run Pex and Apc on the benchmarks and we measured times till the target locations were reached. This measurement is obviously unfair from Pex perspective, since its task is to cover an analysed benchmark by tests and not to reach a single particular location in it. Therefore, we clarify the right meaning of the measurement now.

Our only goal here is to show, that Pex could benefit from our algorithm. Typical scenario when running Pex on a benchmark is that all the code except the target location is covered in few seconds (typically up to three). Then Pex keeps searching space of program paths for a longer time without covering the target location. This is exactly the situation when our algorithm should be activated. We of course do not know the exact moment, when Pex would activate it. Therefore, we can only provide running times of our algorithm as it was activated at the beginning of the analysis.

Before we present the results, we discuss the benchmarks. Benchmark HWM checks whether an input string contains four substrings Hello, world, at and Microsoft! It does not matter at which position and in which order the words occur in the string. The target location can be reached only when all the words are presented in the string. This benchmark was introduced in [1]. The benchmark consists of four loops in a sequence, where each loop searches for a single of the four words mentioned above. Each loop checks for an occurrence of a related word at each position in the input string starting from the beginning. Benchmark HWM is the most complicated one from our set of benchmarks. We also took its two lightened versions presented in [22]: Benchmark HW consists of two loops searching the input string for the first two words above. And benchmark Hello searches only for the first one.

Benchmark MatrIR scans upper triangle of an input matrix. The matrix can be of any rank bigger then $20 \times 20$. In each row we count a number of elements inside a fixed range $(10, 100)$. When a count for any row exceeds a fixed limit 15, then the target location is reached.

Benchmarks OneLoop and TwoLoops originate from [22]. They are designed such that their target locations are not reachable. Both benchmarks contain a loop in which the variable $i$ (initially set to 0) is increased by 4 in each iteration. The target location is then guarded by an assertion $i==15$ in OneLoop benchmark and by a loop while ($i != j + 7$) $j += 2$ in the second one. We note that $j$ is initialized to 0 before the loop.

The last benchmark WinDriver comes from a practice and we took it from [14]. It is a part of a Windows driver processing a stream of network packets. It reads an input stream and decomposes it into a two dimensional array of packets. A position in the array where the data from the stream are copied into are encoded in the input stream itself. We marked the target location as a failure branch of a consistency check of the filled in array. It was discussed in the paper [14] the consistency check can indeed be broken.

The experimental results are depicted in Table 1. They show running times in seconds of Pex and Apc on the benchmarks. We did all the measurements on
### Table 1: Running times of Pex and APC on benchmarks.

| Benchmark | \(\text{Pex}^{\text{Total}}\) | \(\text{APC}^{\text{Total}}\) | \(\text{Bld}^{\text{apc}}\) | \(\text{Unf/SMT}^{\text{apc}}\) | \(\text{SMT}^{\text{apc}}\) |
|-----------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Hello     | 5.257           | 0.181           | 0.021           | 0.290 / S 0.060 | S 0.160         |
| HW        | 25.05           | 0.941           | 0.073           | 0.698 / S 0.170 | S 13.84         |
| HWM       | T/O             | 4.660           | 1.715           | 2.135 / S 0.810 | X M/O           |
| MatrIR    | 95.00           | 0.035           | 0.015           | 0.491 / S 70.80 | S 0.020         |
| WinDriver | 28.39           | 0.627           | 0.178           | 0.369 / S 0.080 | X 4.860         |
| OneLoop   | 134.0           | 0.003           | 0.001           | 0.001 / U 0.001 | U 0.010         |
| TwoLoops  | 64.00           | 0.003           | 0.002           | 0.004 / U 0.010 | U 0.001         |

A single common desktop computer\(^1\) The mark T/O in Pex column indicates that it failed to reach the target location within an hour. For APC we provide the total running times and also time profiles of different paths of the computation. In sub-column 'Bld \(\text{apc}\)' there are times required to build the necessary condition \(\text{apc}\). In sub-column 'Unf/SMT \(\text{apc}^{\tilde{K}}\)' there are two times for each benchmark. The first number identifies a time spent by unfolding the formula \(\text{apc}\) into \(\text{apc}^{\tilde{K}}\). We use a fixed number 25 for all the counters and benchmarks. The second number represents a time spent by Z3 SMT solver \[^{31}\] to decide satisfiability of the unfolded formula \(\text{apc}^{\tilde{K}}\). Characters in front of these times identify results of the queries: S for satisfiable, U for unsatisfiable and X for unknown. And the last sub-column 'SMT \(\text{apc}\)' contains running times of Z3 SMT solver directly on formulae \(\text{apc}\). The mark M/O means that Z3 went out of memory. As we explained in Section \[^6\] the construction and satisfiability checking of \(\text{apc}^{\tilde{K}}\) runs in parallel with satisfiability checking of \(\text{apc}\). Therefore, we take the minimum of the times to compute the total running time of APC.

### 9 Related Work

Early work on symbolic execution \[^{20}, 5, 19\] showed its effectiveness in test generation. King further showed that symbolic execution can bring more automation into Floyd’s inductive proving method \[^{20}, 8\]. Nevertheless, loops as the source of the path explosion problem were not in the center of interest.

More recent approaches dealt mostly with limitations of SMT solvers and the environment problem by combining the symbolic execution with the concrete one \[^{11}, 12, 27, 9, 13, 10, 28, 13, 23\]. Although practical usability of the symbolic execution improved, these approaches still suffer from the path explosion problem. An interesting idea is to combine the symbolic execution with a complementary technique \[^{16}, 18, 2, 21, 17\]. Complementary techniques typi-
cally perform differently on different parts of the analysed program. Therefore, an information exchange between the techniques leads to a mutual improvement of their performance. There are also techniques based on saving of already observed program behaviour and early terminating those executions, whose further progress will not explore a new one. Compositional approaches are typically based on computation of function summaries. A function summary often consists of pre and post condition. Preconditions identify paths through the function and postconditions capture effects of the function along those paths. Reusing these summaries at call sites typically leads to an interesting performance improvement. In addition the summaries may insert additional symbolic values into the path condition which causes another improvement. And there are also techniques partitioning program paths into separate classes according to similarities in program states. Values of output variables of a program or function are typically considered as a partitioning criteria. A search strategy Fitnex implemented in PEX uses state-dependent fitness values computed through a fitness function to guide a path exploration. The function measures how close an already discovered feasible path is to a particular target location (to be covered by a test). The fitness function computes the fitness value for each occurrence of a predicate related to a chosen program branching along the path. The minimum value is the resulting one. There are also orthogonal approaches dealing with the path explosion problem by introducing some assumptions about program input. There are, for example, specialized techniques for programs manipulating strings, and techniques reducing input space by a given grammar.

Although the techniques above showed performance improvements when dealing with the path explosion problem, they do not focus directly on loops. The LESE approach introduces symbolic variables for the number of times each loop was executed and links these with features of a known input grammar such as variable-length or repeating fields. This allows the symbolic constraints to cover a class of paths that includes different number of loop iterations, expressing loop-dependent program values in terms of the input. A technique presented in analyses loops on-the-fly, i.e. during simultaneous concrete and symbolic execution of a program for a concrete input. The loop analysis infers inductive variables. A variable is inductive if it is modified by a constant value in each loop iteration. These variables are used to build loop summaries expressed in a form of pre a post conditions. The summaries are derived from the partial loop invariants synthesized dynamically using pattern matching rules on the loop guards and induction variables. In our previous work we introduced an algorithm sharing the same goal as one presented here. Nevertheless, in we transform an analysed program into chains and we do the remaining analysis there. For each chain with sub-chains we build a constraint system serving as an oracle for steering the symbolic execution in the path space towards the target location.
10 Conclusion

We presented an algorithm computing for a given target program location the necessary condition \( apc \) representing an over-approximated set of real program paths leading to the target. We proposed the use of \( apc \) in tests generation tools based on symbolic execution. Having \( apc \) such a tool can cover the target location faster by exploring only program paths in the over-approximated set. We also showed that \( apc \) can be used in the tools very easily and naturally. And we finally showed by the experimental results that PEx could benefit from our algorithm.

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A  Listing of Benchmarks in C#

public static void Hello(string A)
{
    string H = "Hello\0"; int h = 0;
    for (int i = 0; A[i] != 0; ++i)
    {
        int j = i, k = 0;
        while (H[k] != 0 && A[j] != 0 &&
            A[j] == H[k])
            { ++j; ++k; }
        if (H[k] == 0) { h = 1; break; }
        if (A[j] == 0) break;
    }
    if (h == 1)
        throw new Exception("Hello - reached!!");
}

public static void HW(string A)
{
    string H = "Hello\0"; int h = 0;
    for (int i = 0; A[i] != 0; ++i)
    {
        int j = i, k = 0;
        while (H[k] != 0 && A[j] != 0 &&
            A[j] == H[k])
            { ++j; ++k; }
        if (H[k] == 0) { h = 1; break; }
        if (A[j] == 0) break;
    }
    string W = "World\0"; int w = 0;
    for (int i = 0; A[i] != 0; ++i)
    {
        int j = i, k = 0;
        while (W[k] != 0 && A[j] != 0 &&
            A[j] == W[k])
            { ++j; ++k; }
        if (W[k] == 0) { w = 1; break; }
        if (A[j] == 0) break;
    }
    if (h == 1 && w == 1)
        throw new Exception("HW - reached!!");
}

public static void HWM(string A)
{
    string H = "Hello\0"; int h = 0;

for (int i = 0; A[i] != 0; ++i)
{
    int j = i, k = 0;
    while (H[k] != 0 && A[j] != 0 &&
        A[j] == H[k])
    {
        ++j; ++k;
    }
    if (H[k] == 0) { h = 1; break; }
    if (A[j] == 0) break;
}

string W = "World\0"; int w = 0;
for (int i = 0; A[i] != 0; ++i)
{
    int j = i, k = 0;
    while (W[k] != 0 && A[j] != 0 &&
        A[j] == W[k])
    {
        ++j; ++k;
    }
    if (W[k] == 0) { w = 1; break; }
    if (A[j] == 0) break;
}

string T = "At\0"; int t = 0;
for (int i = 0; A[i] != 0; ++i)
{
    int j = i, k = 0;
    while (T[k] != 0 && A[j] != 0 &&
        A[j] == T[k])
    {
        ++j; ++k;
    }
    if (T[k] == 0) { t = 1; break; }
    if (A[j] == 0) break;
}

string M = "Microsoft!\0"; int m = 0;
for (int i = 0; A[i] != 0; ++i)
{
    int j = i, k = 0;
    while (M[k] != 0 && A[j] != 0 &&
        A[j] == M[k])
    {
        ++j; ++k;
    }
    if (M[k] == 0) { m = 1; break; }
    if (A[j] == 0) break;
}

if (h == 1 && w == 1 && t == 1 && m == 1)
    throw new Exception("HWM - reached!!");

public static void MatrIR(int[,] A, int m, int n)
{
    int w = 0;
for (int i = 0; i < m; ++i)
{
    int k = 0;
    for (int j = i; j < n; ++j)
        if (A[i, j] > 10 && A[i, j] < 100)
            ++k;
    if (k > 15)
    {
        w = 1;
        break;
    }
}
if (m > 20 && n > 20 && w == 1)
    throw new Exception("MatrIR - reached!!");
}

public static void OneLoop(int n)
{
    int i = 0;
    while (i < n) i += 4;
    if (i == 15)
        throw new Exception("OneLoop - reached!!");
}

public static void TwoLoops(int n)
{
    int i = 0, j = 0;
    while (i < n) i += 4;
    while (i != j + 7) j += 2;
    throw new Exception("TwoLoops - reached!!");
}

public static void WinDriver(int[,] multi_array,
    int[] buffer, int MAX_PACKET, int PACKET_SIZE)
{
    for (int i = 0; i < MAX_PACKET; ++i)
        for (int j = 0; j < PACKET_SIZE; j++)
            multi_array[i, j] = 0;
    int number_of_packets;
    int packet_id;
    number_of_packets = (int)buffer[0];
    if ((number_of_packets > MAX_PACKET) ||
        (number_of_packets < 0))
        return;
    for (int i = 0; i < number_of_packets; i++)
    {
        packet_id =
(int)buffer[(i * (PACKET_SIZE + 1)) + 1];
if ((packet_id >= MAX_PACKET) ||
    (packet_id < 0))
    return;
for (int j = 0; j < PACKET_SIZE; j++)
    multi_array[packet_id,j] =
        buffer[(i * (PACKET_SIZE + 1)) + j + 2];
}
if ((number_of_packets < MAX_PACKET) &&
    (multi_array[number_of_packets,0] != 0) &&
    PACKET_SIZE > 20)
    throw new Exception("winDrw - reached!!");
}