Deterministic Factors of Stock Networks based on Cross-correlation in Financial Market

Cheoljun Eom\textsuperscript{1}, Gabjin Oh\textsuperscript{2} and Seunghwan Kim\textsuperscript{2}

\textsuperscript{1}Division of Business Administration, Pusan National University, Busan 609-735, Korea and
\textsuperscript{2}Asia Pacific Center for Theoretical Physics \& NCSL, Department of Physics,
Pohang University of Science and Technology, Pohang, Gyeongbuk, 790-784, Korea

(Received 10 01 2007)

The stock market has been known to form homogeneous stock groups with a higher correlation among different stocks according to common economic factors that influence individual stocks. We investigate the role of common economic factors in the market in the formation of stock networks, using the arbitrage pricing model reflecting essential properties of common economic factors. We find that the degree of consistency between real and model stock networks increases as additional common economic factors are incorporated into our model. Furthermore, we find that individual stocks with a large number of links to other stocks in a network are more highly correlated with common economic factors than those with a small number of links. This suggests that common economic factors in the stock market can be understood in terms of deterministic factors.

PACS numbers: 89.65.Gh, 89.75.Fb, 89.75.Hc

Keywords: stock network, minimal spanning tree, multi-factor model, econophysics

I. INTRODUCTION

Stock markets have long been known to be extremely complex systems, evolving through interactions between heterogeneous units. Hence, the attempts to study and understand the nature of interactions between stocks have been important in understanding the pricing mechanism in the stock market. The cross-correlation matrix has been widely used to quantify interaction among stocks. If we can classify and use significant information included in the cross-correlation matrix between stocks, we will better understand the stock market. However, the extraction of significant information from the cross-correlation matrix has been quite difficult. In finance, researchers have usually used the methods of multivariate statistical analysis, such as principal component analysis, factor analysis, and cluster analysis. In econophysics, for instance, a stock network is proposed by Mantegna \textit{et al.} for investigating the interaction between stocks using the minimal spanning tree (MST) method \cite{1}.

The stock network visually constructs the relationship between stocks, which is extracted by the MST based on the cross-correlations between stock returns. Our work is based on the arbitrage pricing model (APM), widely acknowledged in financial literature \cite{2}. That is, there are many common economic factors in the stock market, which influence all stocks traded \cite{3}. Common economic factors include the industrial product, the risk premium, the term structure of interest, and inflation. The stocks with the same common economic factors are highly correlated with each other and tend to be grouped into a community. That is, individual stocks are divided into small homogeneous stock groups depending on the tendency of stock price changes in correlation with common economic factors \cite{4-6}. Accordingly, the pricing mechanism of individual stocks might be explained by the common economic factors in the stock markets. These observations in the financial sector are similar to the results derived from the MST that in a stock network individual stocks belonging to the same industry from groups with linking relations \cite{7-10}.

In previous studies, much focus is made on the topological properties and the formation principles of stock networks, which revealed that the degree distribution of a stock network follows a power law \cite{11-12}. This implies that most individual stocks in a network have a small number of links with other stocks, while a few stocks, so called hub stocks, have a large number of links. Eom \textit{et al.} suggested that the larger the degree of a stock is in the stock network, the more it is correlated with the market index empirically \cite{13}. Therefore, some stocks acting as a hub for each cluster in a network are affected much more by the market index.

In a recent work, Bonanno \textit{et al.} investigated the degree of consistency between networks using estimated returns from the pricing model and networks from real stock returns \cite{14}. The purpose of this study is to investigate whether stock returns from pricing models can explain interactions between stocks. This study may reveal the deterministic
factors that significantly affect the formation process of a stock network. The widely accepted model in the previous study has been the capital asset pricing model (CAPM) [15-16]. In the CAPM, the prices of individual assets are determined by the market portfolio including all risk assets. In addition, the one-factor model (or the market model), which is an empirical model of the CAPM, uses a market index as a proxy for the market portfolio. However, stock networks from the estimated returns of the one-factor model show a very different structure from the original stock network [9-10, 14]. That is, even if the market index as a representative factor is important for determining the stock price, the market index alone cannot completely explain interactions among stocks.

Therefore, the interactions between stocks may be explained better by the multi-factor model than the one-factor model. As the APM proposed by Ross suggests, stock prices are mostly determined by common economic factors observed in stock markets. These results show that the grouping process of stocks can be observed systematically by the cross-correlation matrix between stocks [17]. The previous study also found that an estimated stock network with returns generated by the stochastic dynamics model (with control parameters included in the market, group and individual stock properties) is very similar to the original stock network with real return data of the stock market [18].

In order to find deterministic factors of the stock network, we propose a model based on common economic factors and investigate whether these common economic factors in the stock market play an important role in determining the stock network. We used the APM, extensively acknowledged as the multi-factor model in financial literature, so that the estimated stock returns reflect the properties of common economic factors. In addition, we investigate the degree of consistency between the original stock networks with real return data and the estimated network with returns based on the multi-factor model. To quantify the strength of consistency between stock networks, we use the survivor ratios suggested in the previous studies [19]. This ratio measures whether stocks that are linked directly to specific stocks in the original stock network have the same links to those in the estimated stock network. We found that the estimated stock networks with more common economic factors in the stock market, give a higher consistency with the original stock network of real returns. In particular, in stock networks, stocks with a large number of links to other stocks in the original stock network have a higher consistency than those with a small number of links. Therefore, these results suggest that common economic factors in the stock market may help to explain the formation principles of stock networks.

In the next section, we describe the data and methods used in this paper. In Section III, we present the results obtained from the APM. Finally, we end with a summary and conclusion.

II. DATA AND METHODS

We used the daily prices of \( N = 400 \) individual stocks in the S&P 500 index of the American stock market from January 1993 to May 2005. The test procedure in this paper can be explained by the following three steps. First, we determine input data that is needed to create the stock network. Second, we create a stock network using the MST method. Third, we calculate the survivor ratio to compare stock networks.

In the first step, the return from input data is significant in that we use real stock returns \( R_i = \ln(P_{t+1}) - \ln(P_t) \) and estimated stock returns \( \hat{R} \) by using the multi-factor model, respectively. The correlation matrix \( \rho_{ij} \) using real and estimated returns is calculated by

\[
\rho_{ij} = \frac{\langle R_i R_j \rangle - \langle R_i \rangle \langle R_j \rangle}{\sqrt{\langle R_i^2 \rangle - \langle R_i \rangle^2} \sqrt{\langle R_j^2 \rangle - \langle R_j \rangle^2}},
\]

where the notation \( \langle \cdots \rangle \) means an average over time. The cross-correlation coefficient can lie in the range of \(-1 \leq \rho_{ij} \leq +1\), where \( \rho_{ij} = -1 \) denotes completely anti-correlated stocks and \( \rho_{ij} = 1 \), completely correlated stocks. In the case of \( \rho_{ij} = 0 \), the stocks \( i \) and \( j \) are uncorrelated.

In the second step, we create a stock network with a significant relationship between stocks using the MST method. The MST, a theoretical concept in graph theory [20], is the spanning tree of the shortest length using the Kruskal or Prim algorithm [21-22]. Therefore, it is a graph without a cycle connecting all nodes with links. This method is also known as the single linkage method of cluster analysis in multivariate statistics [23]. The metric distance \( d_{ij} \), introduced by Mantegna, relates the distance between two stocks to their cross-correlation coefficient [24], and is defined as

\[
d_{ij} = \sqrt{2(1 - \rho_{ij})},
\]
where the distance \( d_{ij} \) can lie in \( 0 \leq d_{ij} \leq 2 \), where the small distance implies strong cross-correlation between stocks. In addition, this distance matrix can be used to construct stock networks using essential information of the market.

In the third step, we measure the degree of consistency between stock networks generated by models based on common economic factors and those for the original stock markets. The stock networks studied are the original stock network \( G^0 \), created using the real returns and the estimated stock networks \( G^E \), using returns from the multi-factor model. To quantify the consistency between networks, we use the survivor ratio. This is a ratio of frequency \( FQ \) measuring that stocks that are directly linked with a specific stock, \( j \), in the original stock networks \( FQ_j(G^L_1) \) have the same links with one in the estimated stock network \( FQ_j(G^E_1) \) among the number of all possible connections between stocks in the stock network (\( N - 1 \)). The survivor ratio is defined by

\[
\rho_{sL \geq i} = \frac{1}{N-1} \sum_{j=1}^{N-1} \frac{FQ_j[\mathcal{G}^0 L \cap \mathcal{G}^E L]}{FQ_j[\mathcal{G}^L L]}, \quad (i = 1, 2, \ldots, M),
\]

where \( N \) is the number of stocks with more than the degree \( i \) in the original stock network and the survivor ratio can vary between \( 0 \leq \rho_s \leq 1 \). If \( \rho_s = 0 \), two stock networks have a completely different structure, and if \( \rho_s = 1 \), they have the same structure. We calculated the survivor ratio according to the various numbers of links with other stocks in the original stock network. The number of links with other stocks, \( i \), is from over one \( L \geq 1 \) to over maximum \( L \geq M \). That is, the survivor ratio of \( L \geq 1 \) targets stocks which have one or more links. Therefore, it represents a degree of consistency for all stocks which exist in a given network. The survivor ratio of \( L \geq M \) targets stocks that have the largest number of links; therefore, it shows a degree of consistency for stocks having the greatest number of links in a network (so-called hub stocks).

### III. RESULTS

In this section, using the cross-correlation-based MST method, we investigated the strength of consistency between the original and estimated stock networks. The independent variables used in the multi-factor model are common economic factors that are estimated through the factor analysis method in multivariate statistics [25]. Factor analysis widely used in the field of social science can reduce many variables in a given data set to a few factors.

Using factor analysis, we calculated the eigenvalues for returns, and chose significant factors. We also made new time series called factor scores in statistics, which use weights with the elements of an eigenvector. To give economic meaning to significant factors, regression analysis was conducted with a calculated factor scores set as dependent variables and the financial market and economic data as independent variables. After confirming statistically significant independent variables, factor scores are regarded as having the attributes of significant independent variables. Through this process, factor scores can be interpreted as common economic factors. The common economic factors generated by factor analysis are used as independent variables in the multi-factor model. Therefore, the stock returns, \( R_j(t), j = 1, 2, \ldots, N \), can be explained by common economic factors, \( F_k(t), k = 1, 2, \ldots, K \), and the multi-factor model is defined by

\[
R_j(t) = \alpha_j + \beta_{j,1}F_1(t) + \beta_{j,2}F_2(t) + \ldots + \beta_{j,k}F_k(t) + \epsilon_j(t),
\]

where \( \alpha_j \) is an expected return on the stock, \( \beta_{j,k} \) are the sensitivity of the stock to changes in common economic factors, and \( \epsilon_j(t) \) are the residuals (\( E(\epsilon_j) \approx 0 \), \( E(\epsilon_j, \epsilon_m) \approx 0 \), and \( E(\epsilon_j, F_k) \approx 0 \)). To establish the multi-factor model in Eq. 4, we have to determine a number of common economic factors \( K \) and control the problem of multicolinearity between common economic factors \( Cor(F_j(t), F_k(t)) \approx 0 \) [26].

In statistics, there are two methods that determine the number of significant factors. First, the eigenvalues with the modulus larger than one determine the significant factors according to the Kaiser rule [27]. Second, these eigenvalues are arranged in the order of their size; that is, from the largest value to the smallest one. After this, the changing slopes between eigenvalues are observed, and then the point where the changing slopes abruptly become smooth determine the significant factors. [28]. While the previous work in the financial field chose the number of common economic factors from the second method, we will use the first method in order to investigate results more extensively. Next, when we use the multiple regression model, there is a problem of multicolinearity among independent variables. That is, the results may be distorted due to the higher correlation between independent variables. Therefore, in order to minimize the correlation between common economic factors, we created a new time series according to factor analysis controlled by the rotated varimax method. Here, the maximum number of common economic factors chosen by the Kaiser rule is 47. That is, we consider the number of common economic factors, \( F_{k(i)}^{(i)} \) with \( i = 1, 2, \ldots, 47 \) where \( k(i) \) is a number of factors used. To apply different common economic factors to Eq. 4, we generate new time series (factor
FIG. 1: The strength of consistency between the original stock networks with real returns and estimated stock networks with (a) estimated returns and (b) residual returns by the multi-factor model, respectively. Each cell denotes the average of the survival ratio for the number of common economic factors and the number of links, respectively. Fig. 1(c) indicates the changes in the average values of the survivor ratios of all the link numbers depending on the increase in the number of common economic factors, irrespective of the link numbers, from the verification results of Fig. 1(a) of estimated returns and Fig. 1(b) of residual returns. Fig. 1(d) shows the changes in the average value of the survivor ratios depending on the increase in the link number, irrespective of the number of common economic factors, from the verification results of Fig. 1(a) and Fig. 1(b). In the figure, the circles (red) and boxes (blue) denote the survival ratio for stock networks with estimated returns and residual returns, respectively.

scores) by optimizing through factor analysis using the method mentioned above. We find that for all cases studied in our paper the correlations between common economic factors are very small with the mean correlation of 0.69 %. That is, there is no problem with multicollinearity that may occur in multiple regression models.

We investigate stock networks with returns estimated by the multi-factor model reflecting various common economic factors. The stock returns, $\hat{R}_{ij}(t)$, estimated by the multi-factor model consist of the returns, $\hat{R}_{ij}^C(t)$, described by common economic factors and the returns, $\hat{R}_{ij}^R(t)$, following the random process that cannot be explained by common economic factors. The process of creating stock returns can be explained by the following five steps. First, we determined the specific number of common economic factors, $F_{k(i)}$. Second, we estimated the coefficients $\hat{\alpha}_j$ and $\hat{\beta}_{j,k}$ of a multi-factor model for all individual stocks as dependent variables, using common economic factors ($F_k$) as independent variables. Third, we calculated the returns, $\hat{R}_{ij}^C(t) = \hat{\alpha}_j + \sum_{k=1}^{K} \hat{\beta}_{j,k} F_k(t)$, that can be explained by common economic factors for all individual stocks using the coefficients estimated in the second step. Fourth,
we created the random returns, $R^R_j(t)$, with a calculation of the mean and standard deviation of individual stock returns. Fifth, the returns, $R^R_j(t) = R^C_j(t) + R^R_j(t)$, can be calculated from the addition of two returns created in the above process. After these processes are completed, we created the stock network, $G^E$, with the estimated return of individual stocks by the MST method. Additionally, we measured the strength of consistency between the original stock network, $G^O$, with real returns and the estimated stock network, $G^E$, with returns created by the multi-factor model, using the survivor ratio $r_k(G^O, G^E)$. The above processes are repeated for all of the common economic factors from 1, $F_{k=1}^{(1)}$, to 47, $F_{k=47}^{(47)}$, respectively. Also, as in the above process, we investigated stock networks with residual returns, with $\epsilon_j(t) = R^R_j(t) - (\alpha_j + \sum_{k=1}^{K} \beta_{j,k} F_k(t))$ eliminating the properties of common economic factors.

Fig. 1 shows the strength of consistency between the original stock network, $G^O$, with real returns and the estimated stock network, $G^E$, with returns created by the multi-factor model using the measurement of survivor ratio. Fig. 1(a) indicates the survivor ratios which show the strength of consistency between a stock network $G^E$ derived from estimated returns $R^E_j(t)$ and a stock network $G^O$ derived from real returns $R_j(t)$. In the figure, each cell represents the average of the survivor ratios as the number of common economic factors vary for each number of links between stocks confirmed in the original stock network. That is, the $x$-axis denotes the number of links with other stocks $L \geq i$, $i = 1, 2, \ldots, 49$, in the original stock network. The largest number of links confirmed in the original stock network was 49 ($i = 49$) and the second largest number of links was 25 ($i = 25$). As we measured the survivor ratios of stocks with $i$ or more links, we discovered that the survivor ratio of stocks with 26 or more links is the same as that of stock with 49 or more links. Accordingly, the links of $x$-axis have been presented from $i = 1$ ($L \geq 1$) to $i = 26$ ($L \geq 26$). The $y$-axis denotes the number of common economic factors, $F_k^{(j)}$ ($j = 1, 2, \ldots, 47$) in the multi-factor model. In the same way as in Fig. 1(a), Fig. 1(b) shows the strength of consistency between a stock network derived from residual returns ($\epsilon_j(t)$) and the original stock network. Fig. 1(c) is used to confirm changes in the average values of the survivor ratios of the all links depending on the increase in the number of common economic factors, irrespective of the link number, from the verification results of Fig. 1(a) using estimated returns and Fig. 1(b) using residual returns. Meanwhile, Fig. 1(d) is used to confirm the changes in the average values of the survivor ratios depending on the increase in link number, irrespective of the number of common economic factors, from the verification results of Fig. 1(a) and Fig. 1(b). In addition, in Figs. 1(c) and (d), the circles (red) and boxes (blue) denote the survival ratio for stock networks with estimated returns, Fig. 1(a), and residual returns, Fig. 1(b), respectively.

According to the results, as the number of common economic factors increased, the survivor ratio depending on estimated returns also increased but the survivor ratio depending on residual returns clearly decreased. Therefore, if we use estimated returns fully reflecting the attributes of common economic factors from the multi-factor model, we could predict the behavior of the original stock network. However, we cannot find the rising patterns of the survivor ratio after the number of common economic factors exceed certain constant levels ($F_{k=1}^{(i)}$, $10 \leq i \leq 12$). Interestingly, in the original stock network, stocks with a large number of links to other stocks have a higher survivor ratio than those with a smaller number of links. These finding are confirmed in Fig. 1(d), which presents changes in the survivor ratio with estimated returns and residual returns, respectively, as the number of links with other stocks increase. In stock networks with estimated returns, the survivor ratio of stocks with a large number of links with other stocks have a higher value than those with a smaller number of links. Meanwhile, in stock networks with residual returns removing the properties of common economic factors, the survivor ratio of stocks with a large number of links have a lower value and zero value. That is, common economic factors have a major and direct influence on stocks with a large number of links to other stocks. These results suggest that common economic factors in the stock market are the deterministic factors of the formation of the stock network. Furthermore, stocks that act as a hub for clusters in a network are affected more by common economic factors, while some stocks with a small number of links, located on the outskirts of the stock network, are affected less by common economic factors.
IV. CONCLUSIONS

Stock markets have long been considered to be extremely complex systems, which evolve through interactions between units. They have also been known to form homogeneous stock groups with a higher correlation among different stocks according to common economic factors that influence individual stocks. We investigated whether common economic factors can be deterministic factors in the formation of a stock network using the multi-factor model to reflect the properties of common economic factors sufficiently, using the daily prices of 400 individual stocks in the S&P 500 index of the American stock market from January 1993 to May 2005.

We have discovered empirically that the process of formation in a stock network is significantly affected by common economic factors. That is, the survivor ratio between original stock networks with real returns and estimated stock networks with estimated returns had a higher value as the number of common factors increased. Furthermore, we found that stocks with a large number of links to other stocks are affected more by common economic factors. That is, the survivor ratio of stocks with a large number of links to other stocks in the original stock network have a higher value than those with a smaller number of links. Also, the highest consistency is in stocks with the largest number of links to other stocks. These results suggest that common economic factors in the stock market are significantly deterministic factors in terms of formation principle of a stock network. Additionally, stocks that act as a hub in a network are affected by common economic factors, especially by the market index, while stocks with a small number of links, located on the outskirts of the stock network, are affected less by common economic factors. Therefore, common economic factors in the stock market are important deterministic factors of a stock network.

Acknowledgments

This work was supported by the Korea Research Foundation funded by the Korean Government (MOEHRD) (KRF-2006-332-B00152), the MOST/KOSEF to the National Core Research Center for Systems Bio-Dynamics (R15-2004-033), the Korea Research Foundation (KRF-2005-042-B0075), the Ministry of Science and Technology through the National Research Laboratory Project, and the Ministry of Education and Human Resources Development through the program BK21.

[1] R. N. Mantegna, Eur. Phys. J. B. 11 (1999) 193.
[2] S. A. Ross, Journal of Economic Theory 13 (1976) 343.
[3] N. Chen, R. Roll, and S. A. Ross, Journal of Business 59 (1986) 383.
[4] B. F. King, The Journal of Business 39(1) (1966) 139.
[5] J. L. Farrell, Jr., The Journal of Business 47(2) (1974) 186.
[6] J. D. Martin and R. C. Klemkosky, The Journal of Business 49(3) (1976) 339.
[7] G. Bonanno, F. Lillo, and R. N. Mantegna, Quant. Financ. 1 (2001) 96.
[8] S. Micciche, G. Bonanno, F. Lillo, and R. N. Mantegna, Physica A 324 (2003) 66.
[9] G. Bonanno, G. Caldarelli, F. Lillo, S. Micciche, N. Vandewalle, and R. N. Mantegna, Eur. Phys. J. B. 38 (2004) 363.
[10] R. Coelho, S. Hutzler, P. Repetowicz, and P. Richmond, Physica A 373 (2007) 615.
[11] N. Vandewalle, F. Brisbois, and X. Tordoir, Quant. Financ. 1 (2001) 372.
[12] H. J. Kim, I. M. Kim, Y. Lee, and B. Kahng, J. Korean Phys. Soc. 40(6) (2002) 1105.
[13] C. Eom, G. Oh, and S. Kim, preprint available at arxiv.org, physics/0612068 (2006).
[14] G. Bonanno, G. Caldarelli, F. Lillo and R. N. Mantegna, Phys. Rev. E 68 (2003) 046130.
[15] W. F. Sharpe, Journal of Finance 19 (1964) 425.
[16] J. Lintner, Review of Economics and Statistics 47 (1965) 13.
[17] D. Kim, and H. Jeong, Phys. Rev. E 72 (2005) 046133.
[18] W. J. Ma, C. K. Hu, and R. E. Amritkar, Phys. Rev. E 70 (2004) 026101.
[19] U. Lee, S. Kim, and K.-Y. Jeong, Physical Review E 73 (2006) 041920.
[20] D. B. West, Introduction to Graph Theory, Prentice-Hall, Englewood Cliffs NJ, 1996.
[21] J. B. Kruskal, Proceedings of the American Mathematical Society 7 (1956) 48.
[22] R. C. Prim, Bell System Technical Journal 36 (1957) 1389.
[23] B. S. Everitt, Cluster Analysis, London: Heinemann Educational Books, 1974.
[24] J. C. Gower, and G. J. S. Ross, Applied Statistics 18(1) (1969) 54.
[25] H. H. Harman, Modern Factor Analysis: The University of Chicago Press 1976.
[26] D. N. Gujarati, Basic Econometrics: McGraw-Hill International Editions 1988.
[27] H. F. Kaiser, Psychometrika 23 (1958) 187.
[28] L. S. Cartell and A. J. Harman, Multivariate Behavioral Research 1 (1966) 245.