Chiral Extrapolation of Hadronic Observables

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One of the great challenges of lattice QCD is to produce unambiguous predictions for the properties of physical hadrons. We review recent progress with respect to a major barrier to achieving this goal, namely the fact that computation time currently limits us to large quark mass. Using insights from the study of the lattice data itself, together with the general constraints of chiral symmetry, we demonstrate that it is possible to extrapolate accurately and in an essentially model independent manner from the mass region where calculations will be performed within the next five years to the chiral limit.

1. INTRODUCTION

Involving as it does a finite grid of space-time points, lattice QCD requires numerous extrapolations before one can compare with any measured hadron property. The continuum limit, $a \to 0$ (with $a$ the lattice spacing), is typically under good control. With improved quark and gluon actions the $O(a)$ errors can be eliminated so that the finite-$a$ errors are quite small, even at a modest lattice spacing – say 0.1 fm.

In contrast, the infinite volume limit is much more difficult to implement as the volume, and hence the calculation time, scales like $N^4$. This limit is also inextricably linked to the third extrapolation, namely the continuation to small quark masses (the “chiral extrapolation”). The reason is, of course, that chiral symmetry is spontaneously broken in QCD, with the pion being a massless Goldstone boson in the chiral limit. As the lattice volume must contain the pion cloud of whatever hadron is under study, one expects that the box size, $L$, should be at least $4m_\pi^{-1}$. At the physical pion mass this is a box 5.6 fm on a side, or a $56^4$ lattice with $a = 0.1$ fm. This is roughly 2$^4$ times as big as the lattices currently in use.

Because the time for calculations with dynamical fermions (i.e. including quark-anti-quark creation and annihilation in the vacuum) scales as $m_q^{-3.6}$, current calculations have been limited to light quark masses 6–10 times larger than the physical ones. With the next generation of supercomputers, around 10 Teraflops, it should be possible to get as low as 2–3 times the physical quark mass, but to actually reach that goal on an acceptable volume will require at least 500 Teraflops. This is 10-20 years away.

Since a major motivation for lattice QCD must be to unambiguously compare the calculations of hadron properties with experiment, this is somewhat disappointing. The only remedy for the next decade at least is to find a way to extrapolate masses, form-factors, and so on, calculated at a range of masses considerably larger than the physical ones, to the chiral limit. In an effort to avoid theoretical bias this has usually been done through low-order polynomial fits as a function of quark mass. Unfortunately, as we discuss in sect. 2, this is incorrect and can yield quite misleading results because of the Goldstone nature of the pion.

Once chiral symmetry is spontaneously broken, as we have known for decades that it must be in QCD and as confirmed in lattice calculations, all hadron properties receive contributions involving Goldstone boson loops. These loops inevitably
lead to results that depend on either logarithms or odd powers of the pion mass. The Gell-Mann-Oakes-Renner relation, however, implies that \( m_\pi \) is proportional to the square root of \( m_q \), so logarithms and odd powers of \( m_\pi \) are non-analytic in the quark mass \( \tilde{m} \), with a branch point at \( m_q = 0 \). One simply cannot make a power series expansion about a branch point.

On totally general grounds, one is therefore compelled to incorporate the non-analyticity into any extrapolation procedure. The classical approach to this problem is chiral perturbation theory, an effective field built upon the symmetries of QCD \( \chi \). There is considerable evidence that the scale naturally associated with chiral symmetry breaking in QCD, \( \Lambda_{\chi} \), is of order \( 4\pi f_\pi \), or about 1 GeV. Chiral perturbation theory (\( \chi pt \)) then leads to an expansion in powers of \( m_\pi /\Lambda_{\chi} \) and \( p/\Lambda_{\chi} \), with \( p \) a typical momentum scale for the process under consideration. At \( \mathcal{O}(p^4) \), the corresponding effective Lagrangian has only a small number of unknown coefficients which can be determined from experiment. On the other hand, at \( \mathcal{O}(p^6) \) there are more than 100 unknown parameters \( \tilde{a} \), far too many to determine phenomenologically.

1.1. Convergence of \( \chi pt \)?

Another complication, not often discussed, is that there is yet another mass scale entering the study of nucleon (and other baryon) structure \( \tilde{a} \). This scale is the inverse of the size of the nucleon, \( \Lambda \sim R^{-1} \). Since \( \Lambda \) is naturally more like a few hundred MeV, rather than a GeV, the natural expansion parameter, \( m_\pi /\Lambda \), is of order unity for \( m_\pi \sim 2 - 3m_\pi^{\text{phys}} \) – the lowest mass scale at which lattice data exists. This is much larger than \( m_\pi /\Lambda_{\chi} \sim 0.3 - 0.4 \), which might have given one some hope for convergence. As it is, the large values of \( m_\pi /\Lambda \) at which lattice data exist make any chance of a reliable expansion in traditional (dimensionally-regulated) chiral perturbation theory (\( \chi pt \)) fairly minimal \( \tilde{a} \). Even though one has reason to doubt the practical utility of \( \chi pt \), the lattice data itself does give us some valuable hints as to how the dilemma might be resolved. The key is to realize that, even though the masses may be large, one is actually studying the properties of QCD, not a model. In particular, one can use the behaviour of hadron properties as a function of mass to obtain valuable new insights into hadron structure.

1.2. Where the Constituent Quark Picture Might Work

The first thing that stands out, once one views the data as a whole, is just how smoothly every hadron property behaves in the region of large quark mass. In fact, baryon masses behave like \( a + bm_q \), magnetic moments like \( (c + dm_q)^{-1} \), charge radii squared like \( (e + fm_q)^{-1} \) and so on. Thus, if one defined a light “constituent quark mass” as \( M \equiv M_0 + \tilde{c}m_q \) (with \( \tilde{c} \sim 1 \)), one would find baryon masses proportional to \( M \) (times the number of u and d quarks), magnetic moments proportional to \( M^{-1} \) and so on - just as in the constituent quark picture. There is little or no evidence for the rapid, non-linearity associated with the branch cuts created by Goldstone boson loops. Indeed, there is little evidence for a statistically significant difference between properties calculated in quenched versus full QCD! How can this be?

The natural answer is readily found in the additional scale, \( \Lambda \sim R^{-1} \), mentioned earlier. In QCD (and quenched QCD), Goldstone bosons are emitted and absorbed by large, composite objects built of quarks and gluons. Whenever a composite object emits or absorbs a probe with finite momentum one must have a form-factor which will suppress such processes for momenta greater than \( \Lambda \sim R^{-1} \). Indeed, for \( m_\pi > \Lambda \) we expect Goldstone boson loops to be suppressed as powers of \( \Lambda /m_\pi \), not \( m_\pi /\Lambda \) (or \( m_\pi /\Lambda_{\chi} \)). Of course, this does not necessarily mean that one cannot in principle carry through the program of \( \chi pt \). However, it does mean that there may be considerable correlations between higher order coefficients and that it may be much more efficient to adopt an approach which exploits the physical insight we just explained.

Over the past three years or so we have developed an efficient technique to extrapolate every hadron property which can be calculated on the lattice from the large mass region to the physical quark mass – while preserving the most important
non-analytic behaviour of each of those observables. This task is not trivial, in that various observables need different phenomenological treatments. On the other hand, there is a unifying theme. That is, pion loops are rapidly suppressed for pion masses larger than $\Lambda$ ($m_\pi > 0.4 - 0.5$ GeV). In this region the constituent quark model seems to represent the lattice data extremely well. However, for $m_\pi$ below 0.4–0.5 GeV the Goldstone loops lead to rapid, non-analytic variation with $m_q$ and it is crucial to preserve the correct leading non-analytic (LNA) and sometimes the next-to-leading non-analytic (NLNA) behaviour of $\chi pt$.

In order to guide the construction of an effective, phenomenological extrapolation formula for each hadron property, we have found it extremely valuable to study the behaviour in a particular chiral quark model – the cloudy bag model (CBM) \cite{12,15,16}. Built in the early 80’s it combined a simple model for quark confinement (the MIT bag) with a perturbative treatment of the pion cloud necessary to ensure chiral symmetry. The consistency of the perturbative treatment was, not surprisingly in view of our earlier discussion, a consequence of the suppression of high momenta by the finite size of the pion source (in this case the bag). Certainly the bag model, with its sharp, static surface, has its quantitative defects. Yet the model can be solved in closed form and all hadron properties studied carefully over the full range of masses needed in lattice QCD. Provided one works to the appropriate order the CBM preserves the exact LNA and NLNA behaviour of QCD in the low mass region while naturally suppressing the Goldstone boson loops for $m_\pi > \Lambda \sim R^{-1}$. Finally, it actually yields quite a good description of lattice data in the large mass region.

We now summarise the particular situation with respect to the chiral extrapolation of lattice data for some phenomenologically significant baryon properties.

2. HADRON MASSES

By far the most extensive and accurate data for hadron properties concerns their masses. At present the dynamical quark data is limited to quark masses a little over 5 times larger than the physical light quark masses, with most of the data at masses at least 10 times larger. One can expect to have data at 2–3 times the physical quark mass within 5 years and hence the crucial issues for chiral extrapolation are:

- a) How accurately can one extract physical hadron masses from current data?
- b) How accurately can one extract physical hadron masses from the next generation of data.
- c) How model dependent are the results?

Until the developments in Adelaide in the last few years\cite{17}, the usual approach to chiral extrapolation was to draw a straight line through the data. That is, taking the nucleon as the classic example:

$$m_N(m_\pi) = c_0 + c_2m_\pi^2.$$ \hspace{1cm} (1)

For pion masses above 1 GeV or so this does not give such a good fit. However, this can be attributed to the fact that $m_\pi^2$ is no longer proportional to $m_q$ in this region \cite{14,19,20}. We are not concerned with such large quark masses, instead we concentrate on the region $m_\pi < 1$ GeV. Only recently has data in this region exhibited significant deviation from Eq. (1) \cite{21,22} and this led to the more sophisticated extrapolation function:

$$m_N(m_\pi) = c_0 + c_2m_\pi^2 + c_3m_\pi^3,$$ \hspace{1cm} (2)

where the presence of the $m_\pi^3$ term is motivated by chiral symmetry. (In terms of $m_q$ this term is non-analytic, being proportional $m_q^{3/2}$, and is the LNA term in the formal expansion of the nucleon mass.) Indeed, it is relatively simple to see that $c_3$ comes from the pion pole term in the $\pi N$ self-energy process and hence that it is model independent \cite{7}:

$$c_3 \equiv c_{LNA} = -\frac{3g_A^2}{32\pi f_\pi^2},$$ \hspace{1cm} (3)

with $g_A$ and $f_\pi$ the axial charge of the nucleon and the pion decay constant, respectively, in the chiral limit.
The first empirical indication of serious problems in this approach came with the realization that a fit to lattice data gives \( c_3 \sim -0.76 \text{ GeV}^{-2} \), whereas the model independent value given by \( \chi \text{pt} \), Eq. (3), is \(-5.6 \text{ GeV}^{-2} \) — a factor of 8 larger! This tells us immediately that either there are serious convergence problems with Eq. (3) or lattice QCD is in error. Clearly most readers would opt for the first possibility and so do we.

The formal series for \( m_N \) about the chiral SU(2) limit is usually written as

\[
m_N = m_0 + c_3 m_\pi^2 + c_{\text{LNA}} m_\pi^3 + c_4 m_\pi^4 + \cdots
\]

where the next-to-leading non-analytic term, \( m_\pi^4 \ln m_\pi \), is dominated by the \( N \to \Delta \pi \to N \) loop. However, for any meaningful extrapolation of lattice data in the next decade this expression is essentially useless. It is derived in the limit \( m_\pi \ll \Delta (\equiv m_\Delta - m_N) \), whereas the lowest lattice data with dynamical fermions that one can expect in the next decade is perhaps 200-250 MeV — c.f. \( \Delta = 292 \) MeV. In fact, most lattice data will still lie above \( \Delta \). Mathematically the region around \( m_\pi \approx \Delta \) is dominated by a square root branch cut which starts at \( m_\pi = \Delta \). Using dimensional regularization this takes the form [18]:

\[
\frac{6g_A^2}{25\pi^2 f_\pi^2} \left[ (\Delta^2 - m_\pi^2) \frac{3}{2} \ln(\Delta + m_\pi - \sqrt{\Delta^2 - m_\pi^2}) - \frac{\Delta}{2} (2\Delta^2 - 3m_\pi^2) \ln m_\pi \right],
\]

for \( m_\pi < \Delta \), while for \( m_\pi > \Delta \) the first logarithm becomes an arctangent. No serious attempt has been made to extend the formal expansion in Eq. (4) to incorporate this cut in an analysis of lattice data and, given the number of parameters to be determined if one works to order \( m_\pi^6 \), it is not likely that it will be done in the next decade.

Even ignoring the \( \Delta \pi \) cut for a short time, studies of the formal expansion of the \( N \to N \pi \to N \) self-energy integral (\( \sigma_{N\pi} \)), suggest that it has abysmal convergence properties. Using a sharp, ultra-violet cut-off, Wright showed [8] that the series diverged for \( m_\pi > 0.4 \) GeV. If one instead uses a dipole cut-off, which in view of the phenomenological shape of the nucleon’s axial form-factor is much more realistic, it is worse — with the radius of convergence being around 0.25 GeV.

In summary, the lattice data itself, formal studies in the NJL model and phenomenological studies of the nucleon based on long-distance regularization all suggest that the radius of convergence of the formal chiral expansion of the nucleon mass barely touches the lowest data point that we will get from lattice QCD in the next decade. This is not a productive approach to the problem.

2.1. The Solution

On the other hand, there is a better way. One can directly use the form [7]:

\[
m_N = \alpha + \beta m_\pi^2 + \sigma_{N\pi}(m_\pi, \Lambda) + \sigma_{\Delta\pi}(m_\pi, \Lambda),
\]

where \( \sigma_{B\pi} \) is the self-energy arising from a \( B \pi \) loop (\( B = N \) or \( \Delta \)) and \( \alpha, \beta \) and \( \Lambda \) are determined by fitting lattice data. For the reasons outlined, it is essential that the self-energies are evaluated using some ultra-violet regulator — a sharp cut-off or a dipole form, for example. Whatever is chosen does not effect the non-analytic structure which is guaranteed correct. The branch points at \( m_\pi \) equals zero and \( \Delta \) are incorporated naturally. While \( \alpha \) and \( \beta \) will depend on the regulator, one can easily expand the self-energy terms to order \( m_\pi^6 \) (or higher) to obtain the chiral coefficients at the appropriate order to compare with effective field theory — e.g. see Ref. [13] for a full discussion of this issue. The essential point is that studies of the nucleon, the \( \Delta \) (c.f. Fig. 4 of Leinweber et al. [17]) and the \( \rho \) meson [23] suggest that this procedure will result in little or no model dependence in the extrapolation to the physical pion mass once there is accurate lattice data for \( m_\pi \sim 0.3 \) GeV or less. Physically this is possible because the self-energy loops are rapidly suppressed in the region \( m_\pi > 0.4 \) GeV. Thus, an extrapolation based on Eq. (6) allows one to respect all the chiral constraints, keep the number of fitting parameters low and yield essentially model independent results at the physical pion mass. No other approach can do this.

2.2. A Possible Connection to QQCD

The study of baryon spectroscopy in quenched lattice QCD (QQCD) has recently made great
progress. We have already noted that the lattice data behaves like a constituent quark model for quark masses above 50–60 MeV because Goldstone boson loops are strongly suppressed in this region. This not only provides a very natural explanation of the similarity of quenched and full data in this region but it also suggests a much more ambitious approach to hadron spectra. It suggests that one might remove the small effects of Goldstone boson loops in QQCD (including the \( \eta \)') and then estimate the hadron masses in full QCD by introducing the Goldstone loops which yield the LNA and NLNA behaviour in full QCD.

As a first test of this idea, Young et al. [24] recently analysed the MILC data [22] for the N and \( \Delta \), using Eq. (1) for full QCD and the appropriate generalization for QQCD – i.e. using quenched pion couplings as well as the single and double \( \eta' \) loops [25,26]. The results were remarkable, with the values of \( \alpha \) and \( \beta \) for the N (or the \( \Delta \)) obtained in QQCD agreeing within statistical errors with those obtained in full QCD. Certainly this result is somewhat dependent on the shape of the ultra-violet cut-off chosen – although the extent of that is yet to be studied in detail. Nevertheless, given that the study involved the phenomenologically favoured dipole form, it is a remarkable result and merits further investigation.

3. ELECTROMAGNETIC PROPERTIES OF HADRONS

Although there is only limited lattice data for hadron charge radii, recent experimental progress in the determination of hyperon charge radii [27], has led us to examine the extrapolation procedure for extracting charge radii from the lattice simulations. Figure 1 shows the extrapolation of the lattice data for the charge radius of the proton, including the \( \ln m_\pi \) (LNA) term in a generalised Padé approximant [28,29]:

\[
<r^2>_\text{ch} = c_1 + \chi \ln \frac{m_\pi^2}{m_\pi^2 + m} + \frac{m_\pi^2}{1 + c_2 m_\pi^2}.
\] (7)

Here \( c_1 \) and \( c_2 \) are parameters determined by fitting the lattice data in the large mass region \( m_\pi^2 > 0.4 \text{ GeV}^2 \), while \( \mu \), the scale at which the effects of pion loops are suppressed, is not yet determined by the data but is simply set to 0.5 GeV.\(^1\) The coefficient \( \chi \) is model independent and determined by chiral perturbation theory. Clearly the agreement with experiment is much better if, as shown, the logarithm required by chiral symmetry is correctly included – rather than simply making a linear extrapolation in the quark mass (or \( m_\pi^2 \)). Full details of the results for all the octet baryons may be found in Ref. [28].

The situation for baryon magnetic moments is also very interesting. The LNA contribution in this case arises from the diagram where the photon couples to the pion loop. As this involves two pion propagators the expansion of the proton and neutron moments is:

\[
\mu_p^{(n)} = \mu_0^{(n)} + \alpha m_\pi + \mathcal{O}(m_\pi^2).
\] (8)

Here \( \mu_0^{(n)} \) is the value in the chiral limit and the linear term in \( m_\pi \) is proportional to \( m_\pi^4 \), a branch point at \( m_\pi = 0 \). The coefficient of the LNA term is \( \alpha = 4.4 \mu_N \text{GeV}^{-1} \). At the physical pion mass this LNA contribution is 0.6\( \mu_N \), which is almost a third of the neutron magnetic moment [30].

As for the charge radii, the chiral behaviour of \( \mu_p^{(n)} \) is vital for a correct extrapolation of lattice data. From Fig. 2 we see that one can obtain a very satisfactory fit to some rather old data (which happens to be the best available) using the simple Padé approximant [31]:

\[
\mu_p^{(n)}(m) = \frac{\mu_0^{(n)} - \alpha m_\pi^2}{1 + \frac{\alpha}{\mu_0} m_\pi^2 + \beta m_\pi^4}. \] (9)

Existing lattice data can only determine two parameters and Eq.(8) has just two free parameters while guaranteeing the correct LNA behaviour (i.e. the correct value of \( \alpha \)) as \( m_\pi \to 0 \) as well as the correct behaviour of HQET at large \( m_\pi^2 \). The extrapolated values of \( \mu_p \) and \( \mu_n \) at the physical pion mass, \( 2.85 \pm 0.22 \mu_N \) and \( -1.90 \pm 0.15 \mu_N \) are currently the best estimates from non-perturbative QCD [33]. (Similar results, including NLNA terms in chiral perturbation theory, have been reported recently by Hemmert and Weise [32].) For the application of similar ideas to other members of the nucleon octet
Figure 1. Chiral extrapolation of lattice results for the squared electric charge radius of the proton (solid line) – from Ref. [28]. Fits to the contributions from individual quark flavors are also shown (the $u$-quark results are indicated by open triangles and the $d$-quark results by open squares). The physical value predicted by the fit is indicated at the physical pion mass, while the experimental value is denoted by an asterisk.

Figure 2. Chiral extrapolations of lattice QCD magnetic moments for the proton (upper) and neutron (lower). The curves illustrate a two parameter fit, Eq. (9), to the simulation data, using a Padé approximant, in which the one-loop corrected chiral coefficient of $m_\pi$ is taken from $\chi pt$. The experimentally measured moments are indicated by asterisks. The figure is taken from Ref. [31].

we refer to Ref. [33], while for the strangeness magnetic moment of the nucleon we refer to Ref. [34]. The last example is another case where there have been tremendous improvements in the experimental capabilities. Specifically, the accurate measurement of parity violation in $ep$ scattering [35] is giving us vital information on hadron structure.

In concluding this section, we note that the observation that chiral corrections are totally suppressed for $m_\pi$ above about 0.4 GeV and that the lattice data looks very like a constituent quark picture there suggests a novel approach to modelling hadron structure. It seems that one might avoid many of the complications of the chiral quark models, as well as many of the obvious failures of constituent quark models by building a new constituent quark model with $u$ and $d$ masses in the region of the strange quark – where SU(3) symmetry should be exact. Comparison with data could then be made after the same sort of chiral extrapolation procedure that has been applied to the lattice data. Initial results obtained by Cloet et al. for the octet baryon magnetic moments using this approach are very promising indeed [36]. We note also the extension to $\Delta$-baryons, including the NLNA behaviour, reported for the first time in Ref. [37].

4. MOMENTS OF STRUCTURE FUNCTIONS

The moments of the parton distributions measured in lepton-nucleon deep inelastic scattering are related, through the operator product expansion, to the forward nucleon matrix elements of certain local twist-2 operators which can be accessed in lattice simulations [6]. The more recent data, used in the present analysis, are taken from the QCDSF [38] and MIT [39] groups and shown.
Figure 3. Moments of the $u - d$ quark distribution from various lattice simulations. The straight (long-dashed) lines are linear fits to this data, while the curves have the correct LNA behaviour in the chiral limit – see the text for details. The small squares are the results of the meson cloud model and the dashed curve through them best fits using Eq. (10). The stars represent the phenomenological values taken from NLO fits in the MS scheme. The figure is taken from Ref. [42].

in Fig. 3 for the $n = 1, 2$ and $3$ moments of the $u - d$ difference at NLO in the $\overline{\text{MS}}$ scheme.

To compare the lattice results with the experimentally measured moments, one must extrapolate in quark mass from about 50 MeV to the physical value. Naively this is done by assuming that the moments depend linearly on the quark mass. However, as shown in Fig. 3 (long dashed lines), a linear extrapolation of the world lattice data for the $u - d$ moments typically overestimates the experimental values by 50%. This suggests that important physics is still being omitted from the lattice calculations and their extrapolations.

Here, as for all other hadron properties, a linear extrapolation in $\tilde{m} \sim m_\pi^2$ must fail as it omits crucial nonanalytic structure associated with chiral symmetry breaking. The leading nonanalytic (LNA) term for the $u$ and $d$ distributions in the physical nucleon arises from the single pion loop dressing of the bare nucleon and has been shown \cite{40,41} to behave as $m_\pi^2 \log m_\pi$. Experience with the chiral behaviour of masses and magnetic moments shows that the LNA terms alone are not sufficient to describe lattice data for $m_\pi > 0.2$ GeV. Thus, in order to fit the lattice data at larger $m_\pi$, while preserving the correct chiral behaviour of moments as $m_\pi \to 0$, a low order, analytic expansion in $m_\pi^2$ is also included in the extrapolation and the moments of $u - d$ are fitted with the form \cite{42}:

$$
\langle x^n \rangle_{u-d} = a_n + b_n \frac{m_\pi^2}{m_\pi^2 + \mu^2} + a_n \, c_{\text{LNA}} \, m_\pi^2 \ln \left( \frac{m_\pi^2}{m_\pi^2 + \mu^2} \right),
$$

where the coefficient, $c_{\text{LNA}} = -(3g_A^2 + 1)/(4\pi f_\pi)^2 \cite{41}$. The parameters $a_n, b_n$ are determined by fitting the lattice data. The mass $\mu$ determines the scale above which pion loops no longer yield rapid variation and corresponds to the upper limit of the momentum integration if one applies a sharp cut-off in the pion loop integral. Consistent with our earlier discussion of this scale it is taken to be 0.55 GeV. Multi-meson loops and other contributions cannot give rise to LNA behaviour and thus, near the chiral limit, Eq. (10) is the most general form for moments of the PDFs at $\mathcal{O}(m_\pi^2)$ which is consistent with chiral symmetry.

We stress that $\mu$ is not yet determined by the lattice data and it is indeed possible to consistently fit both the lattice data and the experimental values with $\mu$ ranging from 0.4 GeV to 0.7 GeV. This dependence on $\mu$ is illustrated in Fig. 3 by the difference between the inner and outer envelopes on the fits. Data at smaller quark masses, ideally $m_\pi^2 \sim 0.05-0.10$ GeV$^2$, are therefore crucial to constrain this parameter in order to perform an accurate extrapolation based solely on lattice data.

Although we do not have sufficient space to explore the matter in detail, we note that once
we have a parametrization of the first four moments as a function of pion mass it is possible to study the behaviour of the PDFs as a function of \( x \) and \( m_\pi \). Remarkably, for \( m_\pi > 0.5 \text{ GeV} \) the distribution tends to peak at \( x \sim 1/3 \), resembling very much what one would expect in a constituent quark model. This further reinforces the discussion in sect. 1.2 concerning the behaviour of hadron structure within QCD as the light quark mass exceeds 50–60 MeV.

4.1. Spin–Dependent Moments

There has been considerable interest in spin-dependent structure functions since the discovery of the spin crisis by EMC. As a result we now have a great deal of experimental information on spin-dependent PDFs \[44\]. More recently studies of the axial charge of the nucleon within lattice QCD have revealed a surprisingly strong dependence on the volume of the lattice \[45\]. Of course, interest is not limited to the first moment of the usual PDFs. One can calculate higher moments on the lattice, as well as moments of the transverse structure functions and experiments are also planned to explore the latter.

The analysis of the chiral extrapolation of moments of the unpolarized PDFs in the previous subsection has recently been extended to the polarized sector by Detmold et al. \[46\]. Years of phenomenological experience suggest that the \( \Delta \) resonance might be important for spin-dependent observables. Within the CBM the explicit appearance of the \( \Delta \) played a vital role in the convergence of a perturbative expansion of nucleon properties, including its axial charge \[48\]. Thus it is not surprising that the explicit inclusion of the \( \Delta \) in the extrapolation of the spin-dependent moments has a very significant effect. Whereas the rapid variation of the unpolarized moments is unaffected, the inclusion of the \( \Delta \) almost totally removes the non-linearity for the polarized moments. While Detmold et al. suggest a new extrapolation formula which accurately approximates the non-analytic behaviour in this case, one could fairly reliably use a linear chiral extrapolation for the spin-dependent moments. For a full discussion of the results we refer to Ref. \[10\], simply noting here that with a careful extrapolation of data taken on a relatively large lattice \[14\] the agreement between theory and experiment for \( g_A \) is now quite reasonable: \( g_A(\text{Latt}) = 1.12 \pm 0.05 \) compared with \( g_A(\text{Expt.}) = 1.267 \).

5. CONCLUSION

At the present time we have a wonderful conjunction of opportunities. Modern accelerator facilities are providing data of unprecedented precision over a tremendous kinematic range at the same time as numerical simulations of lattice QCD are delivering results of impressive accuracy. It is therefore timely to ask how to use these advances to develop a new and deeper understanding of hadron structure and dynamics.

In combination with the very successful techniques for chiral extrapolation, which we have illustrated by just a few examples, lattice QCD will finally yield accurate data on the consequences of non-perturbative QCD. Furthermore, the physical insights obtained from the study of hadron properties as a function of quark mass will guide the development of new quark models and hence a much more realistic picture of hadron structure.

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REFERENCES

1. H. J. Rothe, World Sci. Lect. Notes Phys. 59 (1997) 1.
2. A. W. Thomas and W. Weise, “The Structure of the Nucleon,” 289 pages. Hardcover ISBN 3-527-40297-7 Wiley-VCH, Berlin 2001.
3. D. G. Richards et al., Nucl. Phys. Proc. Suppl. 109, 89 (2002).
4. J. M. Zanotti et al., Nucl. Phys. Proc. Suppl. 109, 101 (2002).
5. M. G. Alford, T. R. Klassen and G. P. Lepage, Nucl. Phys. Proc. Suppl. 53, 861 (1997).
6. T. Lippert, S. Gusken and K. Schilling, Nucl. Phys. Proc. Suppl. 83, 182 (2000).
7. L. F. Li and H. Pagels, Phys. Rev. Lett. 26, 1204 (1971).
8. S. V. Wright, Ph. D. thesis (The University of Adelaide, 2002).
9. J. Gasser and H. Leutwyler, Annals Phys. 158, 142 (1984).
10. H. W. Fearing and S. Scherer, Phys. Rev. D 53, 315 (1996) [arXiv:hep-ph/9408346].
11. W. Detmold et al., Pramana 57, 251 (2001) [arXiv:nucl-th/0104043].
12. A. W. Thomas, Adv. Nucl. Phys. 13, 1 (1984).
13. J. F. Donoghue, B. R. Holstein and B. Borasoy, Phys. Rev. D 59, 036002 (1999).
14. T. Hatsuda, Phys. Rev. Lett. 65, 543 (1990).
15. G. A. Miller, A. W. Thomas and S. Theberge, Phys. Lett. B 91, 192 (1980).
16. S. Theberge, G. A. Miller and A. W. Thomas, Can. J. Phys. 60, 59 (1982).
17. D. B. Leinweber, A. W. Thomas, K. Tsushima and S. V. Wright, Phys. Rev. D 61, 074502 (2000) [arXiv:hep-lat/9906027].
18. M. K. Banerjee and J. Milana, Nucl. Phys. Proc. Suppl. 53, 81 (1997).
19. D. B. Leinweber et al., Phys. Rev. Lett. 86, 5011 (2001) [arXiv:hep-ph/0101211].
20. D. B. Leinweber et al., Phys. Rev. D 60, 034014 (1999).
21. W. Detmold, W. Melnitchouk and A. W. Thomas, arXiv:hep-lat/0204003.
22. E. J. Hackett-Jones et al., Phys. Lett. B 489, 143 (2000).
23. M. Gockeler et al., Nucl. Phys. Proc. Suppl. 109A (2002) 55.
24. M. I. Adamovich et al. [WA89 Collaboration], Eur. Phys. J. C 8 (1999) 59.
25. G. V. Dunne et al., Phys. Lett. B 531, 77 (2002) [hep-th/0110155].