Denoising of SAR Images Based on Lifting Scheme Wavelet Packet Transform

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Abstract  According to the different characteristics that signal and noise exhibit during wavelet decomposition, a new denoising method based on the lifting scheme wavelet packet decomposition is presented. In this method, the SAR images are decomposed by using the best wavelet packet and the norm of each sub-band are calculated; signals and noise can be discriminated based on the norm and soft-threshold method, and the images can be denoised. Experiments show that the proposed algorithm has excellent performance in denoising SAR images, and can remove most noise of images with well-kept texture detail information. The calculating speed of the method is twice the speed of the general wavelet packet transform algorithm.

Keywords  lifting scheme; wavelet packet decomposition; SAR image; image denoising

1  Theory of lifting scheme wavelet packet

In traditional wavelet transform only scale spaces $V_j$ are decomposed, while the wavelet spaces $W_j$ are

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not decomposed. While the equation \( L^2(R) = \oplus W_j \) shows that Hilbert space \( L^2(R) \) is decomposed as sub-spaces \( W_j \) (\( j \in \mathbb{Z} \)) in multiresolution analysis. In wavelet transform, with scale \( j \), the time resolving ability increases, but the frequency resolving ability decreases. In wavelet packet decomposing, with \( j \), wavelet spaces \( W_j \) continue to be decomposed and the frequency resolving ability increases persistently. Therefore, the wavelet packet transform overcomes the main flaw of wavelet transform.

In lifting scheme wavelet transform, the original signal \( c_j \) is decomposed as low frequency coarse signal \( c_{j+1} \) and high frequency final signal \( d_{j+1} \) by splitting, predicting and updating. Similar to the traditional wavelet, the lifting scheme wavelet only continues to decompose the low-frequency signal \( c_{j+1} \), but does not continue to decompose the high-frequency signal \( d_{j+1} \). To overcome this flaw, we decompose the signal using lifting scheme wavelet packet. In this paper, we select the classic biorthogonal symmetry CDF9-7 lifting scheme wavelet.

Supposing \( \Phi(x) = (\phi(x), \phi_1(x), \cdots, \phi(J(x))) \) is a \( t \)-dimension scale function and \( \Psi(x) = (\psi_1, \psi_2, \cdots, \psi(J-1)) \) is corresponding wavelet function. They satisfy the \( \alpha \) scale relation equations \( \Phi(x) = \sum_{k \in \mathbb{Z}} P_k \phi(ax - k) \), \( \Psi(x) = \sum_{j \in \mathbb{Z}} Q_j \psi(ax - k) \), where \( P_k, Q_j \) are called \( \alpha \) scale lists. Supposing \( \Phi(x) \) and \( \Phi(x) = (\tilde{\phi}(x), \phi_1(x), \cdots, \phi(J(x))) \) are corresponding \( \alpha \) scale \( t \)-dimension biorthogonal multiscale functions, \( \Psi(x) \) and \( \Psi(x) = (\psi_1, \psi_2, \cdots, \psi(J-1)) \) are biorthogonal wavelet functions corresponding \( \Phi(x) \) and \( \Phi(x) \), where \( \Phi(x) \) and \( \Psi(x) \) satisfy the \( \alpha \) scale relation: \( \Phi(x) = \sum_{k \in \mathbb{Z}} \tilde{P}_k \Phi(ax - k), \Psi(x) = \sum_{j \in \mathbb{Z}} \tilde{Q}_j \Psi(ax - k) \).

To construct the biorthogonal wavelet packet, \( (\alpha-1) \)-dimension vector function \( \Psi(x) \) is divided into \( (\alpha-1) \) vector functions \( \Psi_i(x) \) where \( \Psi_i(x) = (\psi_{i-1}(ax-1)+1), \psi_{i-1}(ax-1)+2, \cdots, \psi_{i-1}(ax-1)+(J-1)) \), and \( \Psi_i(x) \) is divided correspondingly into \( \tilde{\Psi}_i(x) \) where \( \tilde{\Psi}_i(x) = (\tilde{\psi}_{i-1}(ax-1)+1), \tilde{\psi}_{i-1}(ax-1)+2, \cdots, \tilde{\psi}_{i-1}(ax-1)+J) \). Corresponding to \( \Psi_i(x) \) and \( \tilde{\Psi}_i(x) \), \( Q_i \) and \( \tilde{Q}_i \) are divided into \( (\alpha-1) \) matrix \( Q_i = \{Q_{i0}, Q_{i1}, \cdots, Q_{i-1} \} \) and \( \tilde{Q}_i = \{\tilde{Q}_{i0}, \tilde{Q}_{i1}, \cdots, \tilde{Q}_{i-1} \} \), where \( Q_i \) and \( \tilde{Q}_i \) are \( t \)-dimension. Supposing \( P^{(0)} = R \), \( \tilde{P}^{(0)} = \tilde{R} \), \( P^{(0)} = Q^{(0)} \), \( \tilde{P}^{(0)} = \tilde{Q}^{(0)} \), the biorthogonal wavelet packet based on scale function \( \Phi(x) \) and \( \Phi(x) \) can be defined as:

\[
\Psi^\alpha_{i,j}(ax-k) = \sum_{k \in \mathbb{Z}} P_{i,j}(ax-k),
\tilde{\Psi}^\alpha_{i,j}(ax-k) = \sum_{k \in \mathbb{Z}} \tilde{P}_{i,j}(ax-k)
\]

Here, \( i = 0,1, \cdots, \alpha - 1; j = 0,1, \cdots, k \). From reference [5], we know the \( \alpha \) scale biorthogonal wavelet packet transform formula is:

\[
\Psi^\alpha(x-k) = \frac{1}{\alpha} \sum_{j=0}^{\alpha-1} \tilde{P}_{\alpha,j}^\alpha(x-l),
\tilde{\Psi}^\alpha(x-k) = \frac{1}{\alpha} \sum_{j=0}^{\alpha-1} \tilde{P}_{\alpha,j}^\alpha(x-l)
\] (1)

where \( \tilde{P}^\alpha \) is the conjugated transpose of \( P^\alpha \). When \( \alpha = 2 \), applying Eq(1) to the constructing formula of CDF9-7 lifting scheme wavelet [5, 6], we can calculate the filter coefficients of the CDF9-7 wavelet packet decomposing.

The filter coefficients of CDF9-7 are irrational numbers, so we only use the approximate value of the filter coefficient in calculating. Because the precision of filter coefficients is not enough, we cannot get the completing reconstructing filter coefficients of CDF9-7 wavelet. In this paper, we make use of this rational number to replace the filter coefficients of CDF9-7:

\[
\{h(0), h(1), h(2), h(3), h(4)\} = \begin{cases}
10, 16, 24, 6, 9, \\
16, 24, 16, 16, 16
\end{cases},
\{\tilde{h}(0), \tilde{h}(1), \tilde{h}(2), \tilde{h}(3)\} = \begin{cases}
18, 19, 32, 64, \\
32, 64, 1, 3, 16
\end{cases},
\]

\[
h(-k) = h(k), \tilde{h}(-k) = \tilde{h}(k)
\] (2)

Reference [6] has proved that the image decomposing effect using the rational number filter coefficients in (2) is almost the same to that using the filter coefficients of CDF9-7.

### 2 Denoising of lifting scheme wavelet packet

#### 2.1 Detecting of signal singularity

In 1992, Mallat combined Lipschitz exponential with local model maximum of wavelet coefficients to estimate the local singularity of signal by the attenuation speed of wavelet coefficient local maximum in each scale.
Suppose \( f(x,y) \in L^2(\mathbb{R}^2) \) and \( \delta(x_0,y_0) \) is an open region including \((x_0,y_0)\), if random \((x,y)\) \( \in \delta(x_0,y_0) \) then

\[
| f(x,y) - f(x_0,y_0) | = k | (x - x_0)^2 + (y - y_0)^2 |^{p/2} .
\]

The singularity of \( f(x,y) \) is \( \alpha \) in \((x_0,y_0)\), where \( \alpha \) is the Lipschitz exponential. If wavelet function \( \varphi(x,y) \in L^2(\mathbb{R}^2) \) has \( n \) vanishing moment, then

\[
| W_j f(x,y) | \leq k 2^j \alpha
\]

If point \((x_0,y_0)\) is the local breaking point of \( f(x,y) \), then the wavelet coefficient \( W_j f(x_0,y_0) \) is the local maximum of \( W_j f(x,y) \) in point \((x_0,y_0)\).

Eq.(3) shows that, if the singularity of one point is positive, its wavelet coefficient will become big when adding scale \( j \), but if the point singularity is negative, its wavelet coefficient will become small when adding scale \( j \). Because the Lipschitz exponential of noise is negative, the wavelet coefficients of noise will become small rapidly with scale \( j \), and because the Lipschitz exponential of signal is positive, the wavelet coefficients of signal will not change evidently with scale \( j \). References [3,4,7] have proved that the coefficients of lifting scheme wavelet also possess this character.

### 2.2 Selecting of wavelet packet best basis

The calculation of the signal decomposed by wavelet packet is very complex. The sub-trees is \( 2^n \) for an \( N \) depth bintree, where \( N = 2^l \) is the length of the signal. The image complete bintree decomposing is more complex. Therefore, we must select the best basis of wavelet packet decomposing. The decomposing criterion is the least entropy rule. The Shannon entropy and Threshold entropy are applied widely [8]. The steps of least entropy rule decomposing are: first, the image is decomposed to most level; second, compute the entropy value of each node, if the entropy value of the father node is bigger than that of sub-node, then keep this effective decomposing, and use the sum of sub-nodes entropy value to replace the father node entropy value, else this decomposing is ineffective and merge the sub-nodes. From references [8,9] we know the threshold decomposing rule is more effective than Shanno rule. The decomposing result of Shanno rule is almost a complete tree. This result does not accord with the variety characters of the image. The best tree of wavelet packet decomposing by threshold rule can change correspondingly when the threshold is changed. If the threshold is very big, the best tree will approach the complete tree of the wavelet packet. This character shows that the sub-nodes’ coefficients more than threshold will descend with threshold adding.

Therefore, the decomposing condition of the wavelet packet satisfies that the entropy value of the father node is more than the sum of sub-nodes entropy value. This condition is the same to the character of wavelet decomposing. In the paper, we select the threshold entropy rule in wavelet packet decomposing.

#### 2.3 Denoising method based on lifting scheme wavelet packet

From the character of noise wavelet coefficients and signal wavelet coefficients, we know that the model maximum of signal wavelet coefficients changes very little and model maximum of noise wavelet coefficients declines fast when decomposing with scale \( j \).

Suppose the energy norm of the sub-band is \( || u_{-d} || = max \{ || d_j ||, 1 \leq j \leq M \} \), where \( d_j \) is the wavelet packet decomposing coefficients of \( j \)th sub-band, and \( M \) is the number of coefficients in this sub-band.

The idea behind the denoising algorithm based on wavelet is that the noise exists in the high-frequency wavelet coefficient. The high-frequency wavelet coefficients is kept or replaced by zero based on threshold, then the noise is removed. The common denoising algorithms are the hard-threshold method and soft-threshold method, which were presented by Donoho. The soft-threshold method is selected to remove noise in this paper. The formula of the soft-threshold method is

\[
d_j = \begin{cases} 
\text{sign}(d(n))(|d(n) - \tau|, |d(n)| > \tau) \\
0, |d(n)| < \tau
\end{cases}
\]

where \( \tau = \sigma \sqrt{2 \ln N} \), \( \sigma = \frac{1}{0.6745 \text{med}(|d|)} \).

After the image is decomposed by wavelet packet, the noise in each \( d_j \) can be removed by soft threshold algorithm. With wavelet packet decomposing level addition, the wavelet coefficients add gradually in sub-band \( d_j \), but the wavelet coefficients of the
signal change little in high-frequency sub-band. Therefore, the high-frequency signal coefficients can be preserved when most noise is removed. The wavelet coefficient whose absolute value is maximum is called the energy norm in each sub-band. To further remove noise, comparing the energy norms of each father node and its sub-node, if the absolute value of the father node energy norm subtracting sub-node energy norm is bigger, we know that the noise is dominant in this node by the wavelet coefficients characters of signal and noise. Therefore, the coefficient of the sub-node ought to be replaced by zero. If the absolute value is smaller, the coefficient of sub-node ought to be kept.

In the wavelet packet decomposing of low SNR images, we begin to calculate their sub-band energy norms from $j = 2$, because the wavelet coefficients of noise are dominant when $j$ is smaller, and the wavelet coefficients of signal are dominant when $j$ is bigger.

3 Denoising experiment of SAR image

Based on the above analysis the rich texture SAR images can remove noise by these steps.

1) Decomposing image by using lifting scheme wavelet packet based on the filter coefficients Eq.(2);
2) Computing the best wavelet packet basis by using the threshold entropy rule and decomposing the image by the best wavelet packet.
3) Removing the noise of each sub-band by soft-threshold method and adjusting the wavelet packet coefficients.
4) Computing the energy norm $\| u_c \|$ of sub-band and continuing to adjust the wavelet packet coefficients by comparing the $\| u_c \|$ of father node and sub-nodes.
5) Synthesizing the adjusted coefficients by the reconstructed formula of lifting scheme wavelet packet and getting the removed image.

In this paper, we have a $400 \times 388$ JERS-1 SAR image of Putian in Fujian province. We remove the noise of the image by db7 wavelet, db7 wavelet packet and the proposed method. The image is $4^{th}$ decomposed during the denoising. Fig.1 illustrates the experimental results.

Fig.1(b) shows that the wavelet denoising method removes the most noise and makes the main objects become clear, but much texture in high-frequency coefficients has been removed, and many rich texture areas become blurry. Fig.1(d) is the denoising experimental result by using the lifting scheme wavelet packet. The texture details in Fig.1(d) are well kept because the high-frequency coefficients do not remove all and most coefficients of texture details are presented by filtering the high-frequency coefficients.

Comparing the image Fig.1(b) and image Fig.1(d) we know that the lifting scheme wavelet packet,
method can keep the texture detail well and has better denoising effect than the wavelet denoising method.

The denoising effect in Fig.1(c) and Fig.1(d) is similar, but the calculating time by db7 wavelet packet method is much more than that by the lifting scheme wavelet packet method. After the predicting and updating coefficients are given, the lifting scheme wavelet transform becomes a simple arithmetic operation and the calculating time is a direct ratio of \( \log_2 (N - 1) / (M_{\max} - 1) \). The calculating time of Mallat algorithm is a direct ratio of \( N \log_2 N \), where \( N \) is the length of signal, \( M_{\max} = \max(M, \hat{M}) \) (\( M \) and \( \hat{M} \) are the vanishing moment of \( CDF(M, \hat{M}) \)). Daubechies and Sweldens researches show that the calculating speed of lifting scheme wavelet is twice than that of Mallat algorithm\[2\]. The programmes of denoising experiment run in Matlab 7.0. The programme running time of db7 wavelet method is 0.4430s, the running time of db7 wavelet packet method is 5.695 7s and the running time of CDF9-7 lifting scheme wavelet packet method is 2.633 2s.

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- GPS
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