Low Computational Complexity for Optimizing Energy Efficiency in mm-wave Hybrid Precoding System for 5G

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ABSTRACT Millimeter-wave (mm-wave) communication is the spectral frontier to meet the anticipated significant volume of high data traffic processing in next-generation systems. The primary challenges in mm-wave can be overcome by reducing complexity and power consumption by large antenna arrays for massive multiple-input multiple-output (mMIMO) systems. However, the circuit power consumption is expected to increase rapidly. The precoding in mm-wave mMIMO systems cannot be successfully achieved at baseband using digital precoders, owing to the high cost and power consumption of signal mixers and analog-to-digital converters. Nevertheless, hybrid analog–digital precoders are considered a cost-effective solution. In this work, we introduce a novel method for optimizing energy efficiency (EE) in the upper-bound multiuser (MU) - mMIMO system and the cost efficiency of quantized hybrid precoding (HP) design. We propose effective alternating minimization algorithms based on the zero gradient method to establish fully-connected structures (FCSs) and partially-connected structures (PCSs). In the alternating minimization algorithms, low complexity is proposed by enforcing an orthogonal constraint on the digital precoders to realize the joint optimization of computational complexity and communication power. Therefore, the alternating minimization algorithm enhances HP by improving the performance of the FCS through advanced phase extraction, which involves high complexity. Meanwhile, the alternating minimization algorithm develops a PCS to achieve low complexity using HP. The simulation results demonstrate that the proposed algorithm for MU - mMIMO systems improves EE. The power-saving ratio is also enhanced for PCS and FCS by 48.3% and 17.12%, respectively.

INDEX TERMS mm-wave, MIMO, energy efficiency, fully connected structure, partially connected structure.

I. INTRODUCTION

The massive multiple-input multiple-output (mMIMO) system operations on millimeter-wave (mm-wave) aims to meet the anticipated emergence as key solutions and growth of traffic demands for high-data-rate multimedia access in fifth-generation (5G) systems [1]–[3]. The growing mobile data traffic, such as those from smartphone and mobile internet users, can be supported through...
the availability of more bandwidth, which offers higher data rates for all users and capacity in a mobile cellular network. Consequently, mm-wave has a frequency that ranges between 30 and 300 GHz, which provides gigabit-per-second data rates in 5G wireless systems. High propagation loss and rain attenuation are the main limitations that cause the attenuation at transmitting signals in mm-wave frequencies. The considerable beamforming gain can be decreased by reducing the propagation loss, and the highly directional beam can be generated by deploying the small wavelength of mm-wave signals. The energy cost and hardware complexity of digital precoders (DPs) increase when the number of antennas is large. Reducing the cost, power consumption (PC), and the number of radio frequency (RF) chains cannot realize precoding only by using DPs at baseband in [4] or only using analog precoding, which uses phase shifters in [5].

The hybrid precoding (HP) architecture for mm-wave mMIMO systems is a newly emerging technique that enables directional beamforming with large antenna arrays to improve energy efficiency (EE). It jointly enhances the computation and PC by means of two different HP architectures in RF systems equipped with fully connected structures (FCSs) and partially connected structures (PCSs). The minimum data rate of users in the former can be maximized by connecting every RF chain (FCS) by phase shifters to activate the large spatial degree of freedom gain [6]–[14]. HPs based on orthogonal matching pursuit (OMP) algorithm exhibit a reasonably good performance by studying the sparsity constrained matrix reconstruction problem [5], [6]. Hybrid precoders based on phase-shifting topologies with OMP algorithm are shown to provide a decent performance by investigating the sparsity reconstruction to increase EE full resolution of quantized HP provided with FCS phase shifting in [7]. The simulation results confirm that phase shifting enhances spectral efficiency (SE) and may plausibly result in EE regression with extra bits in digital–analog converters. Tradeoffs between computational complexity and performance are demonstrated in [8] by improving four different algorithms for single-user HP and combiner. An algorithm that iteratively updates hybrid transceiver design for phases in the RF precoders or combiner is proposed in [9]. This approach achieves a near-identical rate in mm-wave between the full-baseband and the HP. A two-loop iterative algorithm and two sparse RF chain structures are also developed in [10] to decrease energy consumption by inserting auxiliary variables and employing successive convex approximation. This two-loop iterative algorithm aims to develop a sparse RF chain antenna at the base station (BS) and HP design to increase the EE of FCS/PCS. The nearly optimal low-complexity FCS RF precoding and combining in wideband are investigated in [12]. The proposed algorithm aims to attain higher EE by decreasing the correlation among different users and developing the unitary matrix quality of subarrays. Low-complexity hybrid block diagonalization is proposed for the downlink (DL) in [11] by employing the RF precoding to harvest the high array gain through phase-only RF precoding.

In harvesting the large array gain through a phase for only RF precoding, the target capacity is achieved by performing low-cost hybrid block [11]. It is shown in [13] that when the number of data streams (\(N_s\)) is more than twice the number of RF chains (\(N_{RF}\)), the energy-efficient HP achieves the global maximum EE. The FCS presented in [14], [15] involves a heuristic algorithm to solve the precoding problem, RF chains, and baseband processing to trade off the energy and cost efficiency in the RF chain circuits.

However, PCSs can reduce the hardware complexity for practical implementation by connecting every RF chain that is only connected to a limited number of antennas [14]–[19]. Corresponding to the PCS, the antenna arrays at the BS consist of multiple subarrays, which entail a lower hardware complexity than the FCS [14]. The proposed matrix of factorizing based on a near-optimal design for every subarray is investigated under the effects of the number of subarrays and RF chains. The simulation results in [14] show that the proposed algorithm reduces SE for FCS/PCS with an increased \(N_{RF}\). The \(N_{RF}\) does not need to be increased according to [15] as it will increase cost and PC. The authors in [15] proposed alternating minimization algorithms for FCS/PCS that optimize the DPs and the analog precoders. The simulation results reveal that the PCS can outperform the FCS in terms of a higher EE and lower-complexity hardware with a large \(N_{RF}\). The transmission rate can be improved by addressing the tradeoff between performance and hardware efficiency by employing the FCS/PCS with dynamic subarray and low-resolution phase shifters. This approach proposes an iterative hybrid beamformer design to mitigate the performance loss by fixed subarrays connected for every RF chain to all transmission antennas [17]. The optimal HP in [18] proposes a PCS to bypass the hardware constraint on the analog phase. This PCS jointly designs the optimal unconstrained precoders by optimizing the analog and DPs to reduce the required RF chain. Implementing signal processing in the baseband and allocating an RF chain per antenna are difficult because of the resulting high cost and PC. The energy-efficient maximization based on solving the analog precoders and combiner problem is optimized in [19] through the alternating direction optimization method in a sub-connected structure. Decreasing the circuit complexity effect on circuit PC and efficiency has not been attempted yet. EE is maximized in [20] by finding low-complexity transmission by improving the Doherty power amplifier for FCS/PCS with low complexity. This development for power amplifiers depends on studying the beam-steering codebook and baseband zero-forcing digital beamformer to maximize the EE. Motivated by the abovementioned gap, we optimize EE and offer low computational complexity based on updating a phase extraction for constraint transmission power and zero gradient-based iterative minimization algorithms, which increase the \(N_s\) compared with more RF chains in FCS and a PCS. However, from the abovementioned gap, the performance of FCS and PCS cannot be achieved compared to the \(N_s\) because the semidefinite relaxation program, alternating
TABLE 1. Summary of notations and abbreviations.

| LIST OF NOTATIONS | LIST OF ABBREVIATIONS |
|-------------------|------------------------|
| $F_k^{RF}$        | mMIMO                  |
| $J_k^{RF}$        | EE                     |
| $I_k$             | HP                     |
| $h_k$             | PCS                    |
| $F_k^{BB}$        | FCS                    |
| $N_s$             | BS                     |
| $N_{RF}$          | PC                     |
| $N_{PP}$          | BBP                    |
| $N_{DP}$          | DPs                    |
| $N_{RF}$          | RF                     |
| $N_{S}$           | SD                     |
| $N_{RX}$          | QCQP                   |
| $N_{TX}$          | 5G                     |
| $N_{DL}$          | DL                     |
| $N_{SE}$          | SE                     |

The analog RF precoding vector achieved phase shifters
The BBP matrix for the $k$th UE
Unitary precoding matrix
The DL channel matrix
The optimal BBP
The complex white Gaussian noise
The number of transmit antennas array at the BS
The number of antennas array at UE
The transmitter data streams
The complex channel gain of the $k$th UE
The number of multi-paths propagation channel of the $k$th UE
The received interference and noise power of the $k$th UE
Power consumption
The communication power
The power consumed by power amplifiers
The power consumed at every RF chain
The circuit PC is affected by baseband signal processing
The bandwidth transmission of the $k$th UE
The available data rate at $k$th data stream of UE
The iterative step
Hermitian symmetric positive
A symmetric positive definite matrix

mMIMO	Massive multiple-input multiple-output
EE	Energy efficiency
HP	Hybrid precoding
PCS	Partially connected structures
FCS	Fully connected structures
BS	Base station
PC	Power consumption
BBP	Baseband precoding
DPS	digital precoders
RF	Radio frequency
5G	Fifth-generation
QCQP	Quadratic constraint quadratic programming
DL	Downlink
SE	Spectral efficiency

A. CONTRIBUTIONS

Comprehensive achievement evaluation of quantized HP design in mm-wave MIMO systems with FCS/PCS depends on proposing alternating minimization algorithms for solving the problem of HP and optimizing the digital and analog precoders. The optimal solutions for designing a PCS for HPs are offered by decreasing the Euclidean distance between analog RF and BBPs. This approach decreases the hardware complexity in the RF chains domain with low-cost phase shifters. The optimal HP design is proposed with novel PCS based on adopting the notation quadratic constraint quadratic programming (QCQP) with low-complexity HP for alternating minimization algorithm to avoid high energy consumption. In addition, an optimal HP design is proposed with novel PCS, which provides high resolution and considerable gain for analog beamforming, generates a relatively large $N_{RF}$, and maximizes EE. The contributions of this study are summarized as follows:

- The optimal HP design problem for mm-wave mMIMO systems is investigated using the orthogonal property of the DPs for an FCS and a PCS by employing fewer phase shifters in the analog RF precoders to attain high EE.
- Energy- and cost-efficient optimization solutions are proposed by formulating the design of hybrid RF chains of energy consumption and BBP matrix to adopt upper-bound energy-efficient solutions for 5G mm-wave mMIMO systems.
- Low computational complexity and small performance loss are achieved by applying columns of the unconstrained property of the mutually orthogonal optimal precoding matrix by using the square of the Frobenius norm in the upper bound. An FCS of HP design is performed by updating a phase extraction for constraint transmission power, and zero gradient-based iterative minimization algorithms are improved by enforcing an orthogonal constraint on the DPs. Thus, the joint optimization of computational complexity and communication power is realized.

II. SYSTEM MODEL

A. MULTIUSER HP FOR WIRELESS TRANSMISSION MODEL

We consider the DL transmission for the mm-wave MU-mMIMO system of FCS/PCS based on HP, as shown in Fig. 1. We assume that the HP is the perfect channel state information (CSI) known at the transmitter and receiver [20]–[24]. The transmitter sends $N_s$ data streams and collects transmission antennas $N_t$, and every user is equipped with $N_s$. Each active user (UE) has a single $N_{RF}$ connected to an antenna element. Consequently, one data stream can be supported by each user. The transmitter is equipped with $N_{RF}$ to support each antenna element data stream $N_t$ from $N_t$ transmission antennas to $N_s$ receiver antennas simultaneously as $N_t \leq N_{RF} \leq N_s$. The received signal at $Y_k \in \mathbb{C}^{N_t \times N_s}$ UE in the HP - mMIMO system can be expressed as:

$$ Y_k = h_k^H \bar{F}_k^{RF} \bar{F}_k^{BB} u_k + \delta_k $$

The vector transmitted signal $u_k$ from the BS to user $u_k = [u_1, \ldots, u_k, \ldots, u_K] \in \mathbb{C}^{N_s \times 1}$, and $h_k \in \mathbb{C}^{N_t \times N_s}$ is the channel vector matrix $h_k$ from BS to UE. $\bar{F}_k^{BB} \in \mathbb{C}^{N_{RF} \times K}$ is
the BBP matrix for the $k$th user, where the $k$th column of $F_{BB}^k$ is expressed as $F_{BB}^k \mathcal{CN} (0, \alpha^2)$ is the complex white Gaussian noise (AWGN) vector, and $\alpha^2_n$ represents the noise power. $F_{RF} \in \mathcal{C}^{N_t \times N_{RF}}$ is the analog RF precoding vector achieved by $N_t$ phase shifters. Moreover, the analog RF and BBPs can fulfill the received power constraint $\| F_{RF} F_{BB}^k \|^2 \leq P$.

B. MILLIMETER-WAVE CHANNEL MODEL

The channel modeling of the mm-wave propagation channel is supported through the extreme high antenna correlation. The channel propagation loss is extreme compared with that in the low-band channel with the fully dispersed environment to adopt the CSI with $L$ path propagation in transmitting a signal from BS to every user. The DL channel matrix can be expressed as follows:

$$h_k = \sqrt{\frac{N_t N_r}{L_k}} \sum_{l=1}^{L_k} \Omega_{k,l} \hat{a}_l(\phi_{k,l}^r) \hat{a}_l^H(\phi_{k,l}^t) \quad \forall k$$

(2)

where $N_t$ is the number of transmit antennas array at the BS, $N_r$ is the number of antennas array at UE, $\Omega_{k,l}$ is the complex channel gain of the $k$th UE over the $l$th multi-path, which includes the path loss with $\mathbb{E} \left[ |\Omega_{k,l}|^2 \right] = 1$, and $L_k$ is the number of multi-paths propagation channel of the $k$th user $l \in \{1, 2, \ldots L_k\}$. $\hat{a}_l(\phi_{k,l}^r)$ and $\hat{a}_l^H(\phi_{k,l}^t)$ represent the normalized transmit and receive array response vector for the azimuth angles of arrival $\phi_{k,l}^r$ and $\phi_{k,l}^t$ of the $l$th multi-path propagation, respectively. The array response vector for the $N$-element of a uniform linear array can be written as:

$$\hat{a}_{ULA}(\phi_{k,l}^r) = \frac{1}{\sqrt{N}} \left[ 1, e^{j2\pi \gamma \sin \phi}, \ldots, e^{-j2\pi \gamma (N-1) \sin \phi} \right]^T$$

(3)

where $N$ represents the number of antenna elements equipped with the BS, $\gamma$ is the wavelength of carrier frequency, and $d = \frac{\lambda}{2}$ represents the inter distance for antenna elements. Every mm-wave can save energy by controlling the beam direction at transmitting beam from $N_{RF}$ BSs to serving the $k$th UE. The sidelobe beam creates an analog beamforming vector based on a signal-to-interference noise ratio. The available data rate at $k$th data stream of UE can be written as:

$$r_k = \sigma \sum_{k=1}^{K} \log_2(1 + \frac{\| H_k F_{BB}^k (\phi_{k,l}^r)^H H_{RF} F_{BB}^k (\phi_{k,l}^t)^H h_k \|^2}{\sum_{i \neq k}^{K} \| H_k F_{BB}^i (\phi_{k,l}^r)^H H_{RF} F_{BB}^i (\phi_{k,l}^t)^H h_k \|^2 + \alpha^2_n})$$

(4)

where $\sigma$ is the bandwidth transmission of the $k$th UE. We assume that every mm-wave BS achieves the low-dimensional processing with the BBPs to cancel inter-user

FIGURE 1. Hybrid precoding in mm-wave MU - mMIMO systems.
interference by defining $h_i^H f_k^{BB} = 0$ when $i \neq k$. The achievable data rate at $k$ th UE can be written as

$$
\mathcal{R}_{sum} = \sum_{k=1}^{K} \mathcal{R}_k
$$

(5)

From (5), the BS transmitter has a perfect CSI to adopt with the precoding problem [21]–[24]. In practical scenarios, the channel reciprocities are the same through the uplink channel estimation and application of DL channels in the time division duplex [25]. The estimation of the mm-wave using pressed channel sensing is investigated in [26], [27].

III. PROBLEM FORMULATION

A. POWER MODEL

In mm-wave MU - mMIMO system, maximize the EE at the BS depend on optimizing BB $f_k^{BB}$ and RF precoding transmission systems $f_k^{RF}$ need to be computed. In this case, the consumption power cannot be ignored. The PC at the transmitter consists of two parts: communication power and circuit PC. It can be written as follows:

$$
P_t = P_{Communi} + P_C
$$

(6)

The communication power in the first term in (6) consists of two parts $P_{Communi} = P_{PA} + P_{RF}$ : $P_{PA}$ is the power consumed by power amplifiers, and $P_{RF}$ is the power consumed at every RF chain. The circuit PC $P_C$ is affected by baseband signal processing, phase shifter [28], [19], cooling, and synchronization in the BS. The consumed power to transmit signal for the $k$ th UEs by the power amplifier can be described mathematically as:

$$
P_{PA} = \frac{1}{2} \sum_{k=1}^{K} \| f_k^{RF} f_k^{BB} \|_F^2
$$

(7)

where $\mathcal{Z}$ is the efficiency of the power amplifier, and $\sum_{k=1}^{K} \| f_k^{RF} f_k^{BB} \|_F^2$ is the power consumed to transmit signal for $k$ th UE. We suppose equal power allocation between the baseband signals of different UEs. The power consumed $P_{RF}$ by converters, mixers, filters, and phase shifters at every RF chain can be written as:

$$
P_{RF} = N_{RF} P_{RF}
$$

(8)

where $P_{RF}$ is the PC by an RF chain. By substituting (7) and (8) into (6), the total PC can be expressed as follows:

$$
P_t = \frac{1}{2} \sum_{k=1}^{K} \| f_k^{RF} f_k^{BB} \|_F^2 + N_{RF} P_{RF} + P_C
$$

(9)

B. ENERGY EFFICIENCY

Maximizing EE is of prime concern while designing the mm-wave MU - mMIMO system. The optimization problem for the maximization of EE can be defined mathematically as follows, (10), as shown at the bottom of the next page.

The EE in an MU - mMIMO system can be maximized by achieving a good tradeoff between the data rate and total PC through jointly optimizing the BBP matrix $f_k^{BB}$ and the RF precoding matrix $f_k^{RF}$. This optimization is formed by

$$
\max_{f_k^{RF}, f_k^{BB}} \EE = \sum_{k=1}^{K} \frac{\| f_k^{RF} f_k^{BB} \|_F^2}{P_t}
$$

(11)

$$
\sum_{k=1}^{K} \| f_k^{RF} f_k^{BB} \|_F^2 \leq P_{max}
$$

$$
\mathcal{R}_k \geq \mathcal{R}_k, \quad k = 1, \ldots, K
$$

$$
\| f_k^{RF} \|_F^2 = \frac{1}{N_f}
$$

where $P_{max}$ is the total transmission power in the abovementioned constraints. Constraint (11) signifies that one BS should serve all UEs. $\sum_{k=1}^{K} \| f_k^{RF} f_k^{BB} \|_F^2 \leq P_{max}$ represents a non-convex constraint because of the analog precoding and digital precoding matrix connection.

The problem in (11), $\| f_k^{RF} \|_F^2 = 1/N_f$, involves difficulty while solving for antenna selection because of the matrix variables $f_k^{RF}$ and $f_k^{BB}$. We propose that the HP matrix design approach achieves an optimum solution with less computational complexity to resolve this issue. As shown in [24], minimizing the objective function in (11) nearly advances to the maximization of the EE. Moreover, the optimal HPs must be close to the unconstrained optimal BBP. Explicitly solving the baseband and RF precoding matrices is also difficult because of the complexity and non-concave properties of the optimization problem in (11). To solve this problem, we derive the upper bound on the EE.

C. UPPER BOUND OF EE

To derive the upper bound of EE, the EE optimization constraints are relaxed in (11) owing to the non-concavity of the EE, which is difficult to apply in (11) for finding the local maximum using Lagrange multipliers. Therefore, we use a zero gradient-based iterative algorithm to obtain an optimal BBP vector. Furthermore, simplifying the derivation is based on the BBP matrix and RF precoding matrix, where $f_k^{BB} \in \mathbb{C}^{N \times K}$ is related to the $N_{RF}$ and $f_k^{RF} \in \mathbb{C}^{N_f \times N_{RF}}$ is related to the fully digital precoding matrix: $f_k^{RF} \in \mathbb{C}^{N_f \times K}$, $f_k^{BB} = f_k^{RF} f_k^{BB}$ in the first term in (9), $f_k^{opt} = [f_1, f_2, \ldots, f_K]$, where the $k$ th column of $f_k^{opt}$ is expressed as $f_k^i$ representing the local optimization for BBP vector for the $k$ th UE. The optimal solution of EE uses the zero gradient method to obtain the stationary point of an optimal BBP vector for EE ($f_1, f_2, \ldots, f_K$) and $\mathcal{R}_k (f_1, f_2, \ldots, f_K)$ [29]–[31].

$$
f_k^{opt} = \arg \max_{f_k} \EE (f_1, f_2, \ldots, f_K) = \frac{1}{2} \sum_{k=1}^{K} \| f_k^{RF} f_k^{BB} \|_F^2 + N_{RF} P_{RF} + P_C
$$

(12)

Considering the complexity and non-concave properties of the problem, the constraint of EE and optimization in (11) are relaxed. The EE is a non-concave function and difficult to apply using the traditional method for obtaining its
The global solution, such as the Lagrange multiplier, EE can be maximized by optimizing the BBP matrix, where \( F_{\text{opt}} \) is the global solution. To solve the constrained by the received interference and noise power of the \( k \)th user [31] is

\[
\Gamma_i = \sum_{j \neq i}^K h_j H_f(j_i)^H h_i + \alpha_n^2 \tag{13}
\]

Based on (12), the data rate in the numerator and the transmission PC in the denominator are shown in (14) and (15), respectively.

\[
\bar{\tau}_k = \log_2 \left( 1 + \frac{\sum_{i=1}^K h_k^H t_k^BB(\bar{t}_k^B)^H h_k}{\sum_{i \neq k}^K \sum_{i=1}^K h_k^H t_k^BB(\bar{t}_k^B)^H h_k + \alpha_n^2} \right) \tag{14}
\]

\[
(P) = 2 \sum_{k=1}^K \| f_k \|^2 + N_{\text{RF}} P_{\text{RF}} + P_C \tag{15}
\]

From the first partial derivation of (13) and data rate of the baseband beamforming \( F_{\text{opt}} \) is (11), where the EE is not convex/concave in the data rate

\[
\frac{\partial \text{EE} (f_k)}{\partial (f_k)} = 2 \left( Q_k - \mathcal{T}_k \right) f_k \tag{16}
\]

The regulated beamforming vectors and the transmitted powers for all users can be improved by developing a zero gradient-based energy-efficient strategy. From an unconstrained optimization problem, the zero gradient condition \( f_k \) must be satisfied when the partial derivation of

\[
\frac{\partial \text{EE} (f_k)}{\partial (f_k)} = 0; \quad 1 \leq k \leq K \tag{19}
\]

The global optimal solution is difficult to obtain, as illustrated in (11), where the EE is not convex/concave in the weighting vector. A zero gradient-based iterative algorithm is developed to obtain the optimal local solution. Therefore, the partial derivation of EE \( f_k \) is achieved using the iterative algorithm as follows:

**Algorithm 1 Energy Efficiency on Upper Bound of HP Design**

1. Assuming \( F_{\text{opt}}^{(0)} \) as an arbitrary initial value of the digital precoding matrix, \( Q_k^{(0)} \) and \( T_k^{(0)} \) are computed by (16).
2. A superscript \( (i) \) is considered the \( i \)th iteration for all user \( k = 1, \ldots, K \).
3. Repeat
4. Calculate \( T_k^{(i)}, Q_k^{(i)}, k = 1, \ldots, K \) based on (13), (17), and (18).
5. Given that \( Q_k^{(i)} \) is a Hermitian symmetric positive definite matrix, \( Q_k^{(i)} \) can be denoted by \( Q_k^{(i)} = \mathbf{C}_k^{(i)} \mathbf{C}_k^{(i)*} \), where \( \mathbf{C} \) represents a symmetric positive definite matrix, if \( \mathbf{C} = Q_k^{(i)} [T_k^{(i)}]^{-1} f_k^{(i)} \).
6. Compute

\[
\left( \frac{\partial \text{EE} (f_k)}{\partial (f_k)} \right)_{f_k^{(i)}} = \begin{bmatrix} 2 \left( Q_k^{(i)} - T_k^{(i)} \right) f_k^{(i)} \end{bmatrix} = \begin{bmatrix} 2 \mathbf{Q}_k^{(i)} - \mathbf{T}_k^{(i)} \end{bmatrix} f_k^{(i)}
\]

7. For \( \gamma = 0 : \mathbf{C} : \gamma \)
8. For \( \gamma = 1 : K \)
9. \( \text{Temp}_{f_k^{(i)}} = f_k^{(i+1)}(\gamma) = \gamma \left[ T_k^{(i)} \right] f_k^{(i)} \)
10. End for \( k \)
11. End for \( \gamma \)
12. Set \( \text{temp}_{f_k^{(i+1)}}(\gamma) \) as the \( k \)th column to form the matrix \( \text{temp}_{f_k^{(i+1)}}(\gamma) \)
13. Compute the maximum EE \( \left( \text{temp}_{f_k^{(i+1)}}(\gamma) \right) \) and let \( f_k^{(i+1)} = \text{temp}_{f_k^{(i+1)}}(\gamma) \)
14. \( i = i + 1 \)
15. Until
16. \( \left\| F^{(i+1)} - F^{(0)} \right\|_F \leq \varepsilon \) // a stopping criterion is triggered
17. \( F_{\text{opt}} = f_k^{(i+1)} \)

where \( \gamma \) is the iterative step, \( \mathcal{C} \) is the step length interval, and \( \varepsilon \) is the stopping trigger. Step (6) is Hermitian symmetric
positive low-complexity HP and \( \left| \mathbb{E}(J_{k}^{\text{opt}})_{k} \right| H \left( \mathbf{c} - J_{k}^{(i)} \right) \) for all \( i_{k}^{(i)} \). The algorithm is formulated and satisfies all the values when the \( \left| \mathbb{E}(J_{k}^{\text{opt}})_{k} \right| H \left( \mathbf{c} - J_{k}^{(i)} \right) \geq 0 \), while the left term in step (6) \( \frac{2}{R} \left| J_{k}^{(i)} \right| H \left( \mathbb{C} - Q_{k}^{(i)} \right) \left( \mathbb{C} - Q_{k}^{(i)} \right)^{H} J_{k}^{(i)} \) is extremum point by starting from any \( i_{k}^{(i)} \) and going in the direction of \( \mathbf{c} - J_{k}^{(i)} \). The \( \mathbb{E}(J_{k}^{(i)}) \) becomes a non-decreasing function for all \( i_{k}^{(i)} \) and shifts to \( \mathbf{c} \). If the fully digital precoding matrix \( I_{k} \) is assumed to be a stationary point according to [32], then the upper bound of \( \mathbb{E}(J_{k}^{(i)}) \) can be determined. Moreover, the upper bound of \( \mathbb{E}(J_{k}^{(i)}) \) is convergent for MU-mMIMO, and the upper bound of \( \mathbb{E} \) is reached. We present the HP problem and solve the same using the FCS/PCS as follows.

\section*{D. Phase Extraction Alternating Minimization for the FCS}

Developing a HP matrix design can achieve low computational complexity and small performance loss. The interference between the multi-path streams can be mitigated by applying columns of the unconstrained property of the mutually orthogonal optimal precoding matrix \( \mathbf{J}_{k}^{\text{opt}} \). To help greatly simplify the design of the analog precoders, this orthogonal restriction depends on the ability of the analog precoders \( \mathbf{J}_{k}^{\text{RF}} \) to get rid of the product form with the \( \mathbf{J}_{k}^{BB} \). The digital precoding matrix should be mutually orthogonal by using a unitary matrix \( \sigma \mathbf{J}_{k}^{SS} \) with the same dimension as \( \mathbf{J}_{k}^{BB} \) as \( \mathbf{J}_{k}^{BB} \). The mutual orthogonal property of a digital precoding matrix \( \mathbf{J}_{k}^{BB} \) in HP is replaced with \( \sigma \mathbf{J}_{k}^{SS} \) in (11).

The objective function in (11) and (20) is obtained by minimizing the Euclidean distance between \( \mathbf{J}_{k}^{RF} \mathbf{J}_{k}^{BB} \) and \( \mathbf{J}_{k}^{\text{opt}} \).

\[
\| \mathbf{J}_{k}^{\text{opt}} - \mathbf{J}_{k}^{RF} \mathbf{J}_{k}^{BB} \|_{F}^{2} = \text{Tr} \left( \mathbf{J}_{k}^{\text{opt}} \mathbf{J}_{k}^{\text{opt}}^{H} \right) - \text{Tr} \left( \mathbf{J}_{k}^{RF} \mathbf{J}_{k}^{BB} \mathbf{J}_{k}^{RF} \mathbf{J}_{k}^{BB}^{H} \right)
\]

\[
\| \mathbf{J}_{k}^{\text{opt}} - \mathbf{J}_{k}^{RF} \mathbf{J}_{k}^{BB} \|_{F}^{2} = \| \mathbf{J}_{k}^{RF} \mathbf{J}_{k}^{BB} \|_{F}^{2} = 2 \sigma \text{Re} \left( \mathbf{J}_{k}^{SS} \mathbf{J}_{k}^{\text{opt}}^{H} \mathbf{J}_{k}^{RF} \right) + \sigma^{2} \| \mathbf{J}_{k}^{RF} \mathbf{J}_{k}^{SS} \|_{F}^{2}
\]

The partial derivatives in (21) can be computed as:

\[
\frac{\partial}{\partial \sigma} \| \mathbf{J}_{k}^{\text{opt}} - \mathbf{J}_{k}^{RF} \mathbf{J}_{k}^{BB} \|_{F}^{2} = 2 \sigma \| \mathbf{J}_{k}^{RF} \mathbf{J}_{k}^{SS} \|_{F}^{2} - 2 \text{Re} \left( \mathbf{J}_{k}^{SS} \mathbf{J}_{k}^{\text{opt}}^{H} \mathbf{J}_{k}^{RF} \right)
\]

\[
\| \mathbf{J}_{k}^{RF} \mathbf{J}_{k}^{SS} \|_{F}^{2} = \frac{\text{Tr} \left( \mathbf{J}_{k}^{SS} \mathbf{J}_{k}^{\text{opt}}^{H} \mathbf{J}_{k}^{RF} \right) + \| \mathbf{J}_{k}^{RF} \mathbf{J}_{k}^{SS} \|_{F}^{2}}{\text{Re} \left( \mathbf{J}_{k}^{SS} \mathbf{J}_{k}^{\text{opt}}^{H} \mathbf{J}_{k}^{RF} \right)}
\]

The smallest value obtained in (21) when the objective function \( \sigma = \frac{\| \mathbf{J}_{k}^{RF} \mathbf{J}_{k}^{SS} \|_{F}^{2}}{\| \mathbf{J}_{k}^{RF} \mathbf{J}_{k}^{SS} \|_{F}^{2}} \) and the optimal fully DPs loss and reduction in computational complexity are obtained using the square of the Frobenius norm in the denominator \( \| \mathbf{J}_{k}^{RF} \mathbf{J}_{k}^{SS} \|_{F}^{2} \) of the upper bound as:

\[
\| \mathbf{J}_{k}^{RF} \mathbf{J}_{k}^{SS} \|_{F}^{2} = \left( \mathbf{J}_{k}^{SS} \mathbf{J}_{k}^{SS}^{H} \mathbf{J}_{k}^{RF} \mathbf{J}_{k}^{RF} \mathbf{J}_{k}^{RF} \right)^{H} = \left( \mathbf{I}_{N_{k}} \right) \mathcal{M}^{H} \mathbf{J}_{k}^{RF} \mathbf{J}_{k}^{RF} \mathcal{M} \leq \left( \mathbf{J}_{k}^{RF} \mathbf{J}_{k}^{SS}^{H} \mathbf{J}_{k}^{RF} \mathcal{M} \right) = \| \mathbf{J}_{k}^{RF} \|_{F}^{2}
\]

where \( \mathbf{J}_{k}^{SS} \mathbf{J}_{k}^{SS}^{H} = \left( \mathbf{I}_{N_{k}} \right) \mathcal{M}^{H} \mathcal{M} \) represents the singular value decomposition (SVD) of \( \mathbf{J}_{k}^{SS} \mathbf{J}_{k}^{SS}^{H} \) and the different powers when \( \mathbf{J}_{k}^{RF} = \mathbf{I}_{N_{k}} \mathbf{J}_{k}^{SS} \) represents the square matrix. The RF precoding transmission system \( \mathbf{J}_{k}^{RF} \) finds a free product with the BBP matrix \( \mathbf{J}_{k}^{BB} \) based on selecting the constant \( \left( \frac{1}{2} \| \mathbf{J}_{k}^{RF} \|_{F}^{2} - 1 \right) \) plus to the compulsion term and multiplying it by the positive constant term \( \| \mathbf{J}_{k}^{SS} \|_{F}^{2} \). The objective function \( \| \mathbf{J}_{k}^{RF} - \mathbf{J}_{k}^{RF} \mathbf{J}_{k}^{BB} \|_{F}^{2} \) in (21) maximizes EE at the BS without enhancing the loss by adding the Frobenius norm in the upper bound \( \| \mathbf{J}_{k}^{RF} \mathbf{J}_{k}^{SS} \|_{F}^{2} = \| \mathbf{J}_{k}^{RF} \|_{F}^{2} \) and reducing the complexity.

\[
\| \mathbf{J}_{k}^{RF} \|_{F}^{2} = 2 \text{Re} \left( \mathbf{J}_{k}^{ss} \mathbf{J}_{k}^{\text{opt}}^{H} \mathbf{J}_{k}^{RF} \right) + \| \mathbf{J}_{k}^{RF} \|_{F}^{2}
\]

\[
\| \mathbf{J}_{k}^{SS} \|_{F}^{2} = 2 \text{Re} \left( \mathbf{J}_{k}^{ss} \mathbf{J}_{k}^{\text{opt}}^{H} \mathbf{J}_{k}^{RF} \right) + \| \mathbf{J}_{k}^{RF} \|_{F}^{2}
\]

\[
\| \mathbf{J}_{k}^{RF} \mathbf{J}_{k}^{SS} \|_{F}^{2} = \| \mathbf{J}_{k}^{RF} \mathbf{J}_{k}^{SS} \|_{F}^{2}
\]
As shown in (21), the computational complexity is still high. The upper bound adopted in (24) satisfies the constraint transmission power by updating a phase extraction-based alternating minimization algorithm. The updating in (24) as the objective function for the HP $\|F^{\text{opt}} - F^{\text{RF}} J^{\text{BB}}\|_F^2$ is given as:

$$
\min_{F^{\text{RF}} \in C_{N^R \times N_t}} \left\| F^{\text{opt}} J^{\text{SS}} - F^{\text{RF}} \right\|_F^2 \\
\text{s.t.} \quad (F^{\text{SS}})^H J^{\text{SS}} k(:, :, i) = I_{N_t} \\
\quad (F^{\text{RF}})^H J^{\text{RF}} k(:, :, j) = 1 \quad \forall i, j
$$

According to (25), a unitary precoding matrix $J^{\text{SS}}$ can be obtained by applying the precoding matrix $J^{\text{BB}}$ with orthogonal columns. The analog precoding $F^{\text{RF}}$ in (25) obtains free product form with mutual orthogonal digital precoding matrix $J^{\text{BB}}$, which can assist and simplify the analog precoder design by using phase extraction alternating minimization algorithm. The closed-form solution can be expressed as follows:

$$
\arg F^{\text{RF}} = \arg \left( F^{\text{opt}} J^{\text{SS}} k(:, :, i) \right) 
$$

where $\arg F^{\text{RF}}$ represents the phase of $F^{\text{RF}}$ obtained from the phases of a corresponding precoder $F^{\text{opt}} J^{\text{SS}}$ if $F^{\text{RF}}$ is fixed for the DPs. Therefore, the analog $F^{\text{RF}}$ phases can be derived from the phases of an analogous $F^{\text{opt}} J^{\text{SS}}$ precoder. When $F^{\text{RF}}$ is fixed, the optimization solution for DPs can be solved as:

$$
\min_{F^{\text{SS}} \in C_{N^R \times N_t}} \left\| F^{\text{opt}} J^{\text{SS}} - F^{\text{RF}} \right\|_F^2 \\
\text{s.t.} \quad (F^{\text{SS}})^H J^{\text{SS}} k(:, :, i) = I_{N_t}
$$

On the basis of (27), only one optimization variable $J^{\text{SS}}$ is obtained, and it is equal to:

$$
\max_{F^{\text{SS}} \in C_{N^R \times N_t}} \text{Tr} \left( (J^{\text{SS}})^H \left( F^{\text{opt}} \right) F^{\text{RF}} \right) \\
\text{s.t.} \quad (J^{\text{SS}})^H J^{\text{SS}} k(:, :, i) = I_{N_t}
$$

In accordance with (28), we use the dual norm for the phases of an equivalent precoder $F^{\text{opt}} J^{\text{SS}}$ and phases of $F^{\text{RF}}$ as:

$$
\text{Tr} \left( (J^{\text{SS}})^H \left( F^{\text{opt}} \right) F^{\text{RF}} \right) \\
\leq \left\| (J^{\text{SS}})^H \left( F^{\text{opt}} \right) F^{\text{RF}} \right\|_\infty \\
\leq \left\| F^{\text{opt}} \right\|_1 \left\| F^{\text{RF}} \right\|_F
$$

where $\| \cdot \|_\infty$ represents the stand infinite norm and $\| \cdot \|_1$ represents one Schatten norm. The equality is determined only according to (29) when $J^{\text{SS}} = \nu y^H$, where $\left( \left( F^{\text{opt}} \right)^H F^{\text{RF}} \right) = \nu \sum y^H$ is FCS in HP. Moreover, the optimal HP $\left( F^{\text{opt}} \right)^H F^{\text{RF}} = \nu \mathbf{s} y_1^H$, which is the SVD of $\left( F^{\text{opt}} \right)^H F^{\text{RF}}$, and $\mathbf{s}$ represents diagonal matrix with elements that are the first $N_t$ non-zero singular values according to (33) $\alpha_1, \ldots, \alpha_{N_t}$. As per (29), the value of $\alpha$ is determined according to the construct a matrix $J^{\text{RF}}$ related to every $J^{\text{SS}}$, where the exact normalization $J^{\text{SS}}$ fulfills the power constraint by reducing the complexity of the phase extraction for alternating minimization algorithm.

**Algorithm 2 Optimized HP-Based Phase Extraction Alternating Minimization for the FCS**

1. **Input:** $F^{\text{opt}}$
2. Establish $F^{\text{RF}}$ and set $i = 0$
3. **Repeat**
   4. Fix $F^{\text{RF}}$, and calculate the SVD of $\left( F^{\text{opt}} \right)^H F^{\text{RF}} = \nu \mathbf{s} y_1^H$;
   5. Establish $J^{\text{SS}}$ and optimize the HP matrices according to (27) $\arg F^{\text{RF}} = \arg \left( F^{\text{opt}} J^{\text{SS}} k(:, :, i) \right)$;
   6. $i = i + 1$
8. **Until** stopping criterion is triggered;
9. For a DP at the end of the transmission, $F^{\text{BB}} = J^{\text{SS}} \sqrt{N_t} / \left\| F^{\text{RF}} J^{\text{SS}} \right\|_F$

**E. LOW-COMPLEXITY HP ALTERNATING MINIMIZATION FOR THE PCS**

This section proposes the PCS–HP matrix design to optimize EE and provide low hardware complexity in mm-wave by employing fewer phase shifters in the analog RF precoders. The EE in MU-mMIMO system will conduct optimization which are used to solve and optimize baseband and RF precoding matrices $F^{\text{RF}}$ and $J^{\text{BB}}$ according to [8], [20], [22].

$$
\max_{F^{\text{RF}} \in C_{N^R \times N_t}} \left\| F^{\text{opt}} - F^{\text{RF}} J^{\text{BB}} \right\|_F^2 \\
\text{s.t.} \quad \left\| F^{\text{RF}} J^{\text{BB}} \right\|_F^2 = p_{\text{max}}
$$
where $F^{opt}$ is the optimal BBP for the $k$th user. The hardware complexity in the RF chains domain can be decreased using every RF chain only connected with $N_i/N_{RF}$. According to (30), every RF chain is supplied with an antenna subarray to guarantee the total PC and multiplexing gain determined by the active RF chains and the dimension of PCS $F^{RF}$ as:

$$F^{RF} = \begin{bmatrix} i^{RF} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & i^{RF} \end{bmatrix}_{N_{RF}}$$

where $i_i = [i = 1, 2, \ldots, N_{RF}] \in [N_{RF} \times 1]$ represents the analog precoding vector $F^{RF}$ to the $i$th subarray. The block diagonal $i_i$ represents the $i$th block matrix, which matches the precoding matrix between the $i$th RF chain and PCS $N_i/N_{RF}$ antennas. Optimizing the BBP matrix $F^{RF}$ and minimizing the number of RF chain precoding matrix $N_{RF}$ can maximize EE at the BS without any loss. In this section, we propose the alternating minimization method for communication power to optimize baseband because of the different structures of the constraint on $F^{RF}$ in the product $F^{RF}F^{BB}$ by fixing the $F^{RF}$ and obtaining a good solution for $F^{BB}$ as [33], [34].

$$\min_{F^{RF}F^{BB}} \left\| F^{opt} - F^{RF}F^{BB} \right\|_F^2$$

s.t. $\left\| F^{RF}F^{BB} \right\|_F \leq p_{max}$

(32)

We presume that (32) is a non-convex constraint according to channel in BS antenna arrays in (2) and from [15], [35] QCQP problem. Let $D = \text{vec}(F^{BB})$ and $d^{opt} = \text{vec}(F^{opt})$ and $\gamma = I_{N_i} \otimes F^{RF}$. From the vectorization property, the objective function

$$\left\| F^{opt} - F^{RF}F^{BB} \right\|_F = \left\| \text{vec}(F^{opt}) - F^{RF}F^{BB} \right\|_2^2 = \left\| \text{vec}(F^{opt}) - (I_{N_i} \otimes F^{RF})F^{BB} \right\|_2^2$$

A difficulty in (32) is that the rank constraint to transfer (32) can be formulated in standard QCQP form.

$$\min d \left\| t - F^{opt} - \gamma d \right\|_2^2$$

s.t. $\left\| d \right\|_2 \leq \frac{p_{max}}{N_i} N_{RF}$

(33)

The objective function in (33) can be expressed as follows:

$$\left\| t - F^{opt} - \gamma d \right\|_2^2 = \left[ b^H t \right]_2^2$$

$$= \left[ \begin{bmatrix} b^H \end{bmatrix} \right] F^{opt} H (I_{N_i} \otimes F^{RF}) - (I_{N_i} \otimes F^{RF})^{opt} - (I_{N_i} \otimes F^{RF})^{opt} H (I_{N_i} \otimes F^{RF})^{opt}$$

$$\times \left[ \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right]$$

$$\leq \frac{p_{max}}{N_i} N_{RF}$$

(35)

From (34), the objective function develops in real QCQP. The special characteristic of HP $N_{RF}/N_{i}$ can find the best $N_{RF}$ vectors or global optimal solution of the precoder design problem in (33). The digital precoding design problem in (32) is solved using the low-complexity HP for alternating minimization for the PCS structure to satisfy the power constraint and considerable gain for analog beamforming [20]. $b_n$ represents the values of $a, \Sigma = \left[ \begin{bmatrix} b \end{bmatrix} \right]$, and $t = \Sigma^H \Sigma$. The original problem and each difference constraint can be rewritten $\left\| t_n \right\|_2^2$ as:

$$\min_{t \in \mathbb{C}} \left\| F^{opt} - F^{RF}F^{BB} \right\|_F^2$$

s.t. $\left\| t_n \right\|_2 = 1$

$$\leq \frac{p_{max}}{N_i} N_{RF}$$

$$\text{rank} \left( t \right) = 1$$

(37)

Subsequently, we identify:

$$\Sigma = \left[ \begin{bmatrix} b \end{bmatrix} \right], t = \Sigma^H \Sigma, \in \epsilon$$

$$= \left( I_{N_i} \otimes F^{RF} \right) H (I_{N_i} \otimes F^{RF}) - (I_{N_i} \otimes F^{RF})^{opt} - (I_{N_i} \otimes F^{RF})^{opt} H (I_{N_i} \otimes F^{RF})^{opt}$$

$$\leq \frac{p_{max}}{N_i} N_{RF}$$

(38)

We assume $\Sigma^H \Sigma = \text{Tr} \left( \epsilon \Sigma^H \Sigma \right)$. The most complex portion in (37) is the rank constraint, which is non-convex.
with recognized $f$. The original problem (32) is non-convex and can be rewritten in (37) by using QCQP to obtain the relaxed version by using the dimension Hermitian matrices. We first drop (37) for the rank constraint to obtain the relaxed version by using low-complexity HP for alternating minimization as follows:

$$\begin{aligned}
\min_{f = \mathbf{a}^*} & \quad \text{Tr} (\mathbf{E} f) \\
\text{s.t.} & \quad \text{Tr} (\mathbf{A}_1 f) = \frac{p_{\text{max}}}{N_t} N_{\text{RF}} \\
& \quad \text{Tr} (\mathbf{A}_2 f) = 1 \\
& \quad f \geq 0 \\
\end{aligned} \tag{39}$$

where $\mathbf{a}^*$ is the set of $n = N_{\text{RF}} N_t + 1$ dimension complex Hermitian matrices. The constraint in (39) becomes convex based on the initial relaxation of the constraint condition rank $(\mathbf{f}) = 1$ [35] if the optimization solution $\mathbf{f}^{\text{opt}}$ does not satisfy the constraint condition according to rank $(\mathbf{f}) = 1$ in (37). By contrast, the optimization solution $\mathbf{f}^{\text{opt}}$ cannot be satisfied using the decomposed product between two vectors $\mathbf{f}^{\text{opt}} = \mathbf{X}^H \mathbf{X}$. Therefore, the BBP $\mathbf{f}^{\text{BB}}_k$ is solved using the approximate method in [36]–[38] by computing the unconstrained optimal precoders of the first $N_t$ column of $\mathbf{X}$ and $\mathbf{X}^H$.

They are computed using unitary matrices grown from the channel’s SVD to be decomposed as $\mathbf{f}^{\text{opt}} = \mathbf{X} \sum \mathbf{Y}^H$. The approximate solution is obtained for optimization solution $\mathbf{f}^{\text{opt}} = \mathbb{E} (\mathbf{X}^H \mathbf{X})$ for each column of $\mathbf{X}$ to be the eigenvalues of $\mathbf{f}^{\text{opt}}$, and $\sum$ is a diagonal matrix for eigenvalues of $\mathbf{f}^{\text{opt}}$. The corresponding solution is obtained by selecting the random vector $\mathbf{X}$, which satisfies $\left\| \mathbf{f}^{\text{RF}} \mathbf{f}^{\text{BB}}_k \right\|_F \leq p_{\text{max}}$ by approximate solution (32). The signal phase is affected by the change in RF precoding matrix, and high EE is obtained with a relatively approximate solution is obtained for optimization solution $\mathbf{f}^{\text{opt}}$.

The value range of the element in the $i$th column of $\mathbf{f}^{\text{RF}}$ to be continuous depend on applying the specified minimization to ensure continuity (40). The optimal phase precoders Phase $(\mathbf{f}^{\text{opt}})$ operation is obtained with low-cost phase shifters by minimizing the Euclidean distance or a limited number of lossy connections between analog RF and BBPs of each element of the matrix.

$$\text{Phase} \left( \mathbf{f}^{\text{RF}}_{(i,j)} \right) = \text{Phase} \left( \mathbf{f}^{\text{opt}}_{(i,j)} \mathbf{f}^{\text{BB}}_k^H \right)$$

$$1 \leq i \leq N_t, \ j = \frac{i}{N_t}$$ \tag{41}

Algorithm 3 Optimized HP-Based Alternating Minimization With Low Complexity for PCS

1. **Input**: $\mathbf{f}^{\text{opt}}$
2. **Output**: $\mathbf{f}^{\text{opt}}_{\text{RF}}, \mathbf{f}^{\text{opt}}_{\text{BB}}$
3. Create $\mathbf{f}^{\text{RF}}_i$ with random phase, and adjust $i = 0$;
4. Repeat
5. Fix $\mathbf{f}^{\text{RF}}_i$ compute $\mathbf{f}^{\text{BB}}_i$ using (40);
6. Fix $\mathbf{f}^{\text{RF}}_i$ and update $\mathbf{f}^{\text{RF}}_i$ by (41);
7. Until
8. $i = i + 1$;
9. $\left\| \mathbf{f}^{\text{opt}} - \mathbf{f}^{\text{RF}}_i \mathbf{f}^{\text{BB}}_i \right\|_F^2 < \varepsilon$ a stopping criterion is triggered;
10. $\mathbf{f}^{\text{RF}}_i = \mathbf{f}^{\text{opt}}_i, \mathbf{f}^{\text{BB}}_i = \mathbf{f}^{\text{RF}}_i$;

**F. COMPLEXITY**

The hardware has the objective of computation of high-dimensional optimal HP matrix design. The complexity for FCS based on using closed-form solutions and phase extraction for alternating minimization algorithm is equivalent to the complexity for the updating procedures of the DP parts. Furthermore, the dimension of the analog precoders is greater than that of the DP parts in the HP design. This solves the complexity of the algorithms based on the analog part. The updating analog precoders need to use the zero gradient method to obtain the stationary point of optimal BBP for every iteration of the phase extraction for the alternating minimization algorithm. Every iteration is realized to provide low complexity by a phase extraction process of the matrix $\mathbf{f}^{\text{opt}} (\mathbf{f}^{\text{opt}}_{(i,i)}^H)$ for alternating minimization by updating the analog precoders with dimension $N_t \times N_{\text{RF}}$. From (23), minimizing the effectiveness of the limit of the upper bound is $N_t \leq N_{\text{RF}} \leq 2N_t$, and the upper bound becomes difficult when $N_{\text{RF}} = N_t$. A small performance loss is obtained when $N_{\text{RF}}$ is increased from $N_t$ to $2N_t$ by computing the gap between $\left\| \mathbf{f}^{\text{RF}}_i \mathbf{f}^{\text{SS}}_{(i,i)} \right\|_F^2$ and $\left\| \mathbf{f}^{\text{RF}}_i \mathbf{M}_1 \right\|_F^2$, as shown as in (23). $\left\| \mathbf{f}^{\text{RF}}_i \mathbf{f}^{\text{SS}}_{(i,i)} \right\|_F^2$ is substituted to $\left\| \mathbf{f}^{\text{RF}}_i \mathbf{M}_1 \right\|_F^2$, where $\mathbf{M}_1$ represents the rightmost $N_{\text{RF}} - N_t$ of $\mathbf{M}$ to yield a small performance loss and reduce the computational complexity. When $N_{\text{RF}} \leq 2N_t$, the energy-efficient HP achieves the maximum EE.
TABLE 2. Simulation parameters.

| Parameter                  | Value  |
|----------------------------|--------|
| $N_s$                      | 144    |
| $N_o$                      | 36     |
| $N_{RF}$                   | 5      |
| $K$                        | 5      |
| $P_{max}$                  | 35 dBm |
| Power amplifier efficiency $E_{PA}$ | 0.38 |
| $P_c$                      | 20 W   |
| $P_{RF}$                   | 48 mW  |
| Carrier frequency $f_c$    | 28 MHz |
| Bandwidth $f_b$            | 20 MHz |
| The number of multi-paths propagation channel $L_k$ | 30 |
| Noise power density $\sigma$ | $-174$ dBm/Hz |
| Uniform distribution       | $[0, 2\pi)$ |

IV. SIMULATION RESULTS

This section presents simulation results to establish EE with several RF chains, the number of antennas, and some UEs. Simulation parameters are encapsulated in Table 2.

Fig. 2 illustrates the different structures for FCS/PCS. The performance of EE decreases when the PC increases with a greater number of phase shifters connected with several RF chains in the FCS structure. The proposed phase extraction alternating minimization algorithm has much lower complexity as it does not allow the $N_{RF}$ to increase. In particular, high EE provides a small $N_{RF}$. In PCS, the EE will be increased and produces the same values to increase the $N_{RF}$. Minimizing the Euclidean distance in (41) optimizes BBP and decreases the hardware complexity in the RF chains domain by employing fewer phase shifters in the analog RF precoders. Fig. 2 shows that at an intersection point for a two-HP structure, the FCS provides high EE when the $N_{RF}$ is less than the intersection point. Meanwhile, the PCS yields high EE when the $N_{RF}$ is implemented at transceivers. Fig. 3 shows the performance of SE with the effect of the $N_{RF}$ evaluated by comparing different algorithms with the assumption that 6 data streams are transmitted.

The SE is achieved by introducing phase extraction as an alternative minimization when $N_{RF} \geq 2N_s$, and it will gradually approach the sufficiently close value of the optimal BBP. As depicted in Fig. 3, the SE improves in the PCS for the region $N_{RF} \in [6, 11]$. When $N_{RF}$ increases, the PC remains unchanged because the low-complexity algorithm reduces an upper bound.

Fig. 4 shows that EE starts to decrease when the $N_{RF}$ for FCS/PCS is increased. The EE with FCS/PCS algorithms can always maintain the best performance with a large $N_{RF}$. The phase extraction of an alternative minimization in FCS provides the best EE performance when the $N_{RF}$ is between 4 and 6. It sufficiently approaches the close value of the optimal BBP. However, the gains over analog beamforming are obtained to avoid the high energy consumption in low-complexity HP for alternating minimization algorithm. The proposed optimal BBP algorithms provide higher EE than the two other proposed algorithms when the $N_{RF}$ is between 1 and 6.
Fig. 5 presents the performance of EE concerning the number of transmission antennas ($N_t$). The EE for different precoding algorithms decreases with the increase in the $N_t$ according to the computation power in an mMIMO system. When $N_t$ is fixed, the optimal BBP provides larger EE than FCS and PCS. The FCS provides a higher EE in terms of hardware complexity and communication power than PCS. Fig. 6 shows the related power-saving ratio concerning the $N_{RF}$. The PC of the phase extraction FCS increases when the $N_{RF}$ is large. When the $N_{RF}$ is 12, the power-saving ratio is 48.3% in low-complexity HP for alternating minimization algorithm. The power-saving ratio is 17.12% for the FCS of phase extraction as an alternative minimization at the same $N_{RF}$ of 12. The high power-saving ratio is more difficult to obtain when the system employs power gain for interference control. The PCS can provide good performance with higher EE. The low-complexity HP for alternating minimization algorithm assigns power to different RF chains and breaches the per RF power limit. As shown in Fig. 7, the EE with different precoding algorithms increases when the number of UEs is small. Thereafter, the EE starts to decrease with the increase in UEs. The optimal BBPs provide higher EE than the two proposed algorithms when the $N_{RF}$ is between 1 and 12. When the number of UEs is more than 6, PCS’s low-complexity HP can achieve fairly satisfactory performance and provides a larger EE than phase extraction in FCS. Moreover, when the number of UEs is small, the phase extraction in FCS gradually approaches the sufficiently close value of the optimal BBPs.

Therefore, the FCS/PCS architectures with dynamic sub-array and low-resolution phase shifters effectively reduce the number of phase shifters and improve the EE. Fig. 8 shows that the cost efficiency decreases when the $N_t$ increases in MU - mMIMO systems. The cost efficiency of optimal BBPs also improves when the $N_t$ is increased. Meanwhile, the phase extraction in FCS and low-complexity HP for alternating minimization algorithm decreases with the increase in the $N_t$. The maximum cost efficiency improved with FCS structures during each iteration is achieved to deliver the low complexity by a phase extraction process of the matrix $F_k^{opt} (SS^H_k)_{i,:}$.  

V. CONCLUSION
In this paper, we propose alternating minimization for a HP- mm-wave mMIMO system. We jointly optimize the communication power of RF systems to minimize the cost and complexity of MU- mMIMO systems. The performance of EE decreases when the $N_{RF}$ increases. The EE with FCS/PCS algorithms can always maintain the best performance of $EE = 3.7$ Mbits/J and decreases the EE.
to 2.9 Mbits/J with increases the $N_{RF}$. Moreover, when the $N_{RF}$ is greater than the $N_c$, the HPs with the FCS can move toward the optimal BBPs. The PCS improves EE and SE with a relatively large $N_{RF}$. The proposed algorithms for FCS/PCS are developed to optimize the performance of the MU - mMIMO communication system. The simulation results show that the proposed low-complexity HP for alternating minimization algorithm in FCS and PCS achieves 48.3% and 17.12%, respectively. The PCS structure becomes better than the phase extraction for alternating achieves 48.3% and 17.12%, respectively. The PCS structure becomes better than the phase extraction for alternating.

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VOLUME 10, 2022
4727