$p_t$ dependence of transverse flow in relativistic heavy-ion collisions

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The strength of transverse flow is examined as a function of transverse momentum $p_t$ using a simple, transversely moving thermal model and a more realistic, relativistic transport model (ART). It is shown that the $p_t$ dependence reveals useful information about the collective flow that is complementary to that obtained from the standard in-plane transverse momentum analysis. Interesting features of using the $p_t$ dependence to study the equation of state of the superdense hadronic matter formed in relativistic heavy-ion collisions are demonstrated.
Compressional shock waves created in relativistic heavy-ion collisions were predicted to induce collective flow effects [1]. In heavy-ion collisions at beam energies below 2 GeV/nucleon, collective flow phenomena have been firmly established by many experiments (e.g. [2–4]). In particular, the sideward deflection of matter in the reaction plane was clearly revealed in the in-plane transverse momentum analysis [5]. In this case, a typical S shape was seen for the average transverse momentum in the reaction plane as a function of rapidity. It was also found that the strength of the collective flow measured in terms of the slope of the S shaped distribution at mid-rapidity or the total in-plane transverse momentum depends on the reaction dynamics, in-medium properties of hadron-hadron interactions and the equation of state [6–13]. Furthermore, it was pointed out recently that a change in the strength of the collective flow could be a possible signature of quark-gluon plasma formation in heavy ion collisions at energies up to about 15 GeV/nucleon currently available at the AGS [14–16]. It is thus extremely interesting to test these predictions against experimental data. However, it has been difficult to perform the standard in-plane transverse momentum analysis at the AGS due to the limited angular coverage and movability of existing spectrometers. Instead, azimuthal angle distributions of transverse energy measured with calorimeters without particle identification were analyzed [17], and pronounced azimuthal anisotropies indicating directed sideward flow were clearly identified. Nevertheless, it is experimentally possible to study correlations between transverse momenta of particles detected in the forward spectrometer and the flow direction determined by the calorimeters. It was thus first suggested in ref. [18] that one can extract information about the transverse flow by studying the ratio $R(p_t) \equiv (dN^+/dp_t)/(dN^-/dp_t)$, where $N^+(N^-)$ is the number of particles in the reaction plane emitted in the same (opposite) direction of sideward flow, as a function of $p_t$ near the projectile rapidity. Although this approach is similar in spirit to that for studying the “squeeze-out” phenomenon at BEVALAC/SIS energies [19–22], it is still interesting to investigate whether this approach can reveal new features of the transverse flow and thus information about the equation of state of the superdense hadronic matter formed in relativistic heavy-ion collisions. In this paper, using both a simple, transversely

moving thermal model and a more realistic, relativistic transport model (ART) \cite{15} we show
that the ratio $R(p_t)$ at high $p_t$ is very sensitive to the equation of state in the most violent
stage of the reaction.

Although particles are continuously emitted throughout the whole reaction process as
indicated in our dynamical simulations, i.e, particles freeze-out at different temperatures, a
simple thermal model with a single temperature is useful for a qualitative discussion of the
more realistic dynamical calculations. We thus first discuss, using a transversely moving
thermal model, the transverse momentum distributions $dN_{\pm}/p_t dp_t$ in the reaction plane for
nucleons emitted in the same or opposite direction of the transverse flow. In particular, we
discuss the limiting behaviour of the transverse momentum distributions and the ratio $R(p_t)$
at high $p_t$.

Let us assume that all or a fraction of particles in a small rapidity bin around $y$ are in
local thermal equilibrium at a temperature $T$, and the center of mass of these particles are
moving with velocity $\beta$ along the direction $+x$ in the reaction plane. From the standard
transverse momentum analysis, we have

$$\beta = \sum_i \frac{(p_x)_i}{E_i} = \frac{\sum_i (p_x)_i}{\sum_i (m_i) \cosh(y)} \leq \frac{\langle p_x \rangle}{m_n \cosh(y)},$$

where $\langle p_x \rangle$ is the average transverse momentum per nucleon in the reaction plane, and $m_n$ is
the nucleon mass. For semicentral collisions of Au+Au at $P_{\text{beam}}/A = 10.8 \text{ GeV/c}$, $\langle p_x \rangle$ is
about 0.1 GeV/c \cite{15} at the projectile rapidity, $\beta$ thus has a value of about 0.05. Using the
Boltzmann distribution for the thermal source and boosting it with the transverse velocity
$\beta$, one obtains the following spectrum,

$$\frac{d^3N}{p_t dp_t d\phi dy} = C \gamma (E - \beta p_t \cos(\phi)) e^{-\gamma (E - \beta p_t \cos(\phi))/T},$$

where $C$ is the normalization constant and $\phi$ is the azimuthal angle with respect to the
reaction plane. The transverse momentum spectra in a small rapidity bin around $y$ in
the reaction plane for particles emitted in the same ($dN_+/p_t dp_t$) and opposite ($dN_-/p_t dp_t$)
directions of the transverse flow are therefore given by
\[
\frac{dN_{\pm}}{p_t dp_t} = C_{\pm} e^{-\gamma E/T} (\gamma E \mp T \alpha) e^{\pm \alpha},
\]
where \( \alpha \equiv \gamma p_t/T \). These distributions reduce to simple exponentials at high transverse momenta

\[
(dN_{\pm}/p_t dp_t)_{\infty} \propto \exp(-p_t/T_{eff}^{\pm}),
\]
where the inverse slopes or effective temperatures \( T_{eff}^{\pm} \) in the semilogarithmic plot of the spectra at high \( p_t \) are obtained as

\[
\frac{1}{T_{eff}^{\pm}} = -\lim_{p_t \to \infty} \left[ \frac{d}{dp_t} \ln \left( \frac{dN_{\pm}}{p_t dp_t} \right) \right] = \frac{\gamma}{T} (\cosh(y) \mp \beta).
\]

In general, the inverse slope \( 1/T_{eff}^{\pm} \) reflects combined effects of the temperature \( T \) and the transverse flow velocity \( \beta \). For the special case of particles at midrapidity, \( \beta \) is zero and the effective temperatures \( T_{eff}^{\pm} \) equal to the local temperature \( T \) of the thermal source. Otherwise, one expects \( T_{eff}^{+} > T_{eff}^{-} \). For high energy heavy-ion collisions such as those at AGS energies, \( \beta \) is much smaller than \( \cosh(y) \) around the projectile rapidity, and one again expects \( T_{eff}^{+} \approx T_{eff}^{-} \), i.e. two approximately parallel spectra \( dN_{+}/p_t dp_t \) and \( dN_{-}/p_t dp_t \) at high transverse momenta. This fact also indicates that it is difficult to extract the transverse flow velocity by studying the transverse momentum spectra alone.

The degree of azimuthal asymmetry or the strength of transverse flow can be expressed in terms of the ratio

\[
R(p_t) \equiv \frac{dN_{+}/dp_t}{dN_{-}/dp_t} = \frac{C_{+}}{C_{-}} \cdot \frac{1 - \beta p_t^2}{1 + \beta p_t^2} \cdot e^{2p_t \gamma T}.
\]

To see how \( R(p_t) \) might be sensitive to the flow velocity \( \beta \) and/or the temperature \( T \), we illustrate in Fig. 1 its variation with \( p_t \) for four sets of \( \beta \) and \( T \). It is seen that the ratio increases as a function of \( p_t \) for any fixed values of \( T \) and \( \beta \). A slight increase in \( \beta \) or decrease in \( T \) results in a dramatic increase in the ratio \( R(p_t) \). These results indicate that the \( p_t \) dependence of the ratio \( R(p_t) \) indeed carries interesting information about the strength of transverse flow and the freeze-out temperature. It is, however, important to stress that
in reality particles are emitted continuously at different freeze-out temperatures during the whole reaction process. In particular, particles with high $p_t$ are mostly emitted from the most violent space-time regions where the local temperatures are high. Since the ratio $R(p_t)$ varies slowly with the transverse momentum for low $\beta/T$, one thus expects in dynamical model calculations a weaker $p_t$ dependence of the ratio $R(p_t)$ at high transverse momenta.

We now turn to our study using the relativistic transport model (ART 1.) [15]. This model was developed by including more baryon and meson resonances as well as their interactions in the BUU transport model (e.g. [8,23,24]). More specifically, we have included in ART 1.0 the following baryons: $N$, $\Delta(1232)$, $N^*(1440)$, $N^*(1535)$, $\Lambda$, $\Sigma$, and mesons: $\pi$, $\rho$, $\omega$, $\eta$, $K$, as well as their explicit isospin degrees of freedom. Both elastic and inelastic collisions among most of these particles are included by using as inputs either the available experimental data or results from one-boson-exchange and resonance models. An optional, self-consistent mean field for baryons is also included. We refer the reader to ref. [15] for more details of the model and its applications in studying various aspects of heavy-ion collisions at AGS energies.

In Fig. 2, we show the nucleon spectra, $dN_{\pm}/m_t\,dm_t$ versus $m_t - m$, for both central and peripheral collisions. To increase the statistics of our analysis particles with azimuthal angles smaller than 20° with respect to the reaction plane are included. The chosen rapidity range $|y - 2.95| \leq 0.35$ is very close to the projectile rapidity of 3.1. In this rapidity range particles with high transverse momenta must have suffered very violent collisions and thus originate mostly from the very hot and dense participant region where large density gradients exist. On the other hand, particles with low transverse momentum are mostly from the cold spectators. In both central and peripheral collisions the spectra show typical exponential shapes for $m_t - m \geq 0.2$ GeV (or $p_t \geq 0.64$ GeV/c). Moreover, the spectra at high $p_t$ for particles moving in the same and opposite directions of the transverse flow are approximately parallel to each other. These features are what we have expected from the schematic discussions based on the transversely moving thermal model. We notice that the transverse momentum distribution in peripheral collisions is dominated by low
transverse momentum particles from the spectators. From the spectra at high $p_t$, effective
temperatures of about 150 MeV and 105 MeV are found for the central and peripheral
collisions, respectively. However, one needs to be cautious in interpreting these effective
temperatures as particles are continuously freezed out at different temperatures during the
whole reaction time. The effective temperatures determined from the spectra at high $p_t$ are
therefore related through Eq. (5) to the most probable or average local temperatures in the
most violent regions of the reaction.

To study the strength of the transverse flow, we show in Fig. 3 the ratio $R(p_t)$ from our
model calculations for the reaction of Au+Au at $p_{beam}/A = 10.8$ GeV/c. As expected, the
ratio increases gradually at low $p_t$ and reaches a limiting value at high $p_t$ in both central and
peripheral collisions. Moreover, it is seen that the limiting value in peripheral collisions is
much larger than that in central collisions. The reason for this behaviour is that even though
the number of particles with high $p_t$ is less in peripheral collisions, these particles have a
relatively higher value of $\beta/T$ due to a lower temperature. As the transverse momentum
decreases $R(p_t)$ is increasingly more affected by particles from the cold spectators, and the
ratio approaches one as $p_t$ goes to zero. This is most obvious for peripheral collisions where
the isotropic Fermi motion of spectator nucleons dominates and leads to $R(p_t) \approx 1$ for $p_t \leq
0.2$ GeV/c.

Since $R(p_t)$ at high $p_t$ is a very sensitive measure of the quantity $\beta/T$, and the local
temperatures during the reaction process using the cascade and the soft equation of state
are almost identical [13], one expects that $R(p_t)$ depends strongly on the equation of state
of the superdense hadronic matter formed in the most violent stage of the reaction. This is
clearly seen also in Fig. 3 by comparing results obtained using the pure cascade and the soft
equation of state for central collisions. Although there is no significant difference between
the two values of $R(p_t)$’s for $p_t \leq 0.2$ GeV/c, they differ by about a factor of 1.5 at higher
$p_t$. Moreover, the ratios are constants within a large range of high transverse momenta, and
it can thus be relatively easily measured in experiments.

In summary, we have investigated theoretically a new approach for measuring the
strength of transverse flow in terms of the number of particles in the reaction plane emitted in the same and opposite directions of the flow. The variation of the strength of flow has been examined as a function of transverse momentum using both a simple, transversely moving thermal model and a more realistic, transport model. It is shown that the strength reaches a limiting value at high $p_t$, and this value is very sensitive to the equation of state. This new approach provides new information about the collective flow that is complementary to what one learns from the standard in-plane transverse momentum analysis but has the advantage of requiring particle identification only at a single rapidity.

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**FIGURE CAPTIONS**

**Fig. 1** The strength of transverse flow as a function of transverse momentum predicted by a transversely moving thermal model. The temperature $T$ and velocity $\beta$ are in units of GeV and $c$, respectively.

**Fig. 2** Transverse mass distributions in the reaction plane for particles emitted in the same/opposite side of the transverse flow in reactions of Au+Au at $p_{\text{beam}}/A = 10.8$ GeV/c and at impact parameters less then 4 fm (upper window) and between 8 and 10 fm (lower window).

**Fig. 3** Transverse momentum dependence of the strength of transverse flow in the reaction plane for the reactions described in Fig. 2. Results using the soft equation of state and the pure cascade mode of ART are compared for central collisions.
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