Magnetothermal force effect on natural convection through a partially-heated vertical channel

Kengo WADA*, Hiroaki FUJIWARA*, Masayuki KANEDA* and Kazuhiko SUGA*
*Department of Mechanical Engineering, Osaka Prefecture University
1-1 Gakuuencho, Naka-ku, Sakai 599-8531, Japan
E-mail: mkaneda@me.osakafu-u.ac.jp

Received: 12 December 2019; Revised: 13 February 2020; Accepted: 31 March 2020

Abstract
Natural convection of a paramagnetic fluid through a partially-heated vertical channel is numerically studied in the presence of a magnetic field from two block magnets placed behind the heated wall. Magnets are aligned with opposite orientations. This magnet orientation induces strong magnetic force normal to the magnet at the magnet junction due to short distance between poles. When the temperature-dependent magnetic susceptibility changes due to wall heating near the magnet, the magnetothermal force is induced remarkably near the magnet junction. This additional force overlaps to the natural convection along the heated wall and results in the changes of heat and fluid flow along the heated wall. It is found that, flow becomes slow and the local heat transfer is suppressed below the elevation of the magnet junction, and the flow acceleration and heat transfer enhancement are observed above the junction elevation, of which effects depend on the relative magnet elevation to the heated wall. It is also found that the transition to vibrating flow occurs at the specific magnet elevation. The time-averaged Nusselt number suggests this vibrating convection has the potential to enhance the heat transfer remarkably because this magnetically-induced flow vibration continues along the heated wall up to the outlet.

Keywords : Natural convection, Paramagnetic fluid, Magnetic field, Magnetothermal force, Heat transfer enhancement/suppression, Vibrating flow

1. Introduction

The paramagnetic fluid such as oxygen and air has a positive magnetic susceptibility thus is attracted to the magnet. Since the magnetic susceptibility $\chi$ of the paramagnetic fluid is quite small (O$_2$ gas: $106.2 \times 10^{-6}$ cm$^3$/g) rather than that of ferromagnetic materials (iron: $218 \times 10^3$ cm$^3$/g), the attracting magnetic force can reveal only when the strong magnetic field is applied. To be accurate, a steep gradient of the magnetic flux density $b$ is required because the magnetic force is defined by the equation below.

$$F_m = \frac{\rho \chi \mu_m}{2\mu_0} \nabla b^2$$

(1)

The magnetic susceptibility of the paramagnetic material depends on the temperature (Curie’s law). This suggests that the magnetic force is induced depending on the temperature like the density-driven buoyant force due to the temperature. This thermally-dependent magnetic force has been known in the past, and intensively studied since late 1990s at which strong magnetic fields have become available by the presence of the superconducting magnets.

In terms of the temperature-dependent characteristic of the paramagnetic fluid, this force is recognized by magnetothermal force and the application to the convection control has been studied. Braithwaite et al. (1991) performed the experiment of Rayleigh-Bénard convection inside a bore of superconducting magnet and they showed the change of heat transfer according to the magnetic strength. Kenjereš et al. (2012) imposed on the magnetic field effect on the natural convection in a cubic enclosure heated from bottom. Ozoe’s group conducted numerical simulations and simple visualization experiments and showed that the magnetothermal force can induce the flow in temperature stratified air in a cube (Kaneda et al., 2002) and the force overlaps to the natural convection (Kaneda et al., 2004). These studies employ
the tesla order and stronger magnetic induction.

However, the superconducting magnet requires cryogenic devices to achieve the superconducting state of electric wire. The running cost becomes expensive for the convection control if the unit price of resulting product from the working fluid is cheap. As aforementioned, the magnetic or magnetothermal force depends both on the magnetic susceptibility and a gradient of the magnetic induction. The magnetic susceptibility is the physical property of the working fluid. Thus, the magnetic field to induce steep magnetic gradient has a key to pronounce the force and resulting effect at lower cost by using normal electric magnets (Zhang, et al., 2016) and permanent magnets (Song and Tagawa, 2014).

The available permanent magnets such as neodymium ones have the magnetic induction of 0.5T and more. Although this induction is much smaller than that of the superconducting magnets, authors found by experiment that the natural convection of paramagnetic liquid ( gadolinium nitrate hexahydrate aqueous solution, $\chi_{\text{mm}} = 120.5 \times 10^{-6} \text{ cm}^3/\text{g}$) can be controlled by a permanent magnet of 573 mT (Kaneda et al., 2019). Additionally, this effect becomes remarkable by employing multiple magnets with opposite orientations. Therefore, based on the experimental results, the mechanism of the magnetothermal force effect from multiple magnets on the heat and fluid flow of natural convection is numerically studied. To simulate the experimental conditions, a partially-heated vertical channel is employed.

Nomenclature

- $b$: magnetic induction, T
- $B$: normalized magnetic induction, $B=b/b_0$
- $Cc$: coefficient in Eq.(3) = $1 + 1/\beta \theta_0$
- $g$: gravity acceleration
- $\ell$: characteristic length (= channel width, m)
- $P$: dimensionless pressure
- $Pr$: Prandtl number
- $q$: heat flux from the wall, W/m$^2$
- $Ra^*$: modified Rayleigh number, $Ra^* = g\beta q \ell^4/\alpha \lambda V$
- $T$: dimensionless temperature
- $U$: dimensionless velocity
- $Z_{\text{mag}}$: magnet elevation from the bottom edge of heated plate
- $\alpha$: thermal diffusivity, m$^2$/s
- $\beta$: volumetric expansion ratio due to temperature, 1/K
- $\gamma$: dimensionless magnetic induction, $\gamma = \chi_{\text{mm}} b_0^3/\mu_m g \ell$
- $\theta_0$: reference temperature, K
- $\mu_m$: magnetic permeability, H/m
- $\rho$: density of the fluid, g/cm$^3$
- $\chi_{\text{mm}}$: mass magnetic susceptibility of the fluid, cm$^3$/g

2. Computed model

2.1 Schematic model

The schematic model of the computation is shown in Fig. 1. A two-dimensional vertical channel that width of 1.0 and height of 6.0 is filled with the paramagnetic liquid. Note that the length is non-dimensionalized by the channel width $\ell$. Top and bottom of the computational domain are open boundaries. The left wall is heated at a constant heat flux at $Z \geq 0$. To avoid the unsteady natural convection induced in the computational domain, the heated area is extended to the top of the computational domain. The rest of the left wall ($Z < 0$) and the opposite sidewall are thermally-insulated. Since the experiment suggested that the most effective magnet location is near the bottom edge of the heating wall, the adiabatic wall is presumed at the bottom edge to avoid the interception of flow depending on the magnet elevation. Additionally, the facing adiabatic wall is to obtain the smooth flow. Due to the wall heating, the buoyant flow is induced thus the fluid comes in from the bottom and goes out from the top. Two block magnets with opposite orientations are placed behind the heated wall at an arbitrary elevation of $Z_{\text{mag}}$. The physical properties of the working fluid are referred from our experimental work (Kaneda et al., 2019) where the computational domain is referred to.
2.2 Governing equations

The governing equations consist of the continuity, momentum, and energy equations. The buoyant and the magnetothermal forces due to temperature difference are considered in the momentum equation. The fluid is presumed as incompressible Newtonian. By applying the Boussinesq approximation to external forces, the dimensionless governing equations are yielded as follows.

\[
\nabla \cdot \mathbf{U} = 0 \tag{2}
\]

\[
\frac{D\mathbf{U}}{D\tau} = -\nabla P + \Pr \nabla^2 \mathbf{U} - \frac{Pr Ra^* Cc}{2} \nabla B^2 \gamma (T - T_0) + Pr Ra^* (T - T_0) \mathbf{e}_z \tag{3}
\]

\[
\frac{D\mathbf{T}}{D\tau} = \nabla^2 T \tag{4}
\]

The detailed derivations and definitions of dimensionless numbers can be found in Ozoe (2005) and Maki et al. (2005). Fundamentally, dimensional value is normalized by reference values such as \( X=x/\ell \), \( P=p/p_0 \), \( \mathbf{U}=u/u_0 \), \( B=b/b_0 \), etc. The temperature dependency on the magnetic susceptibility is translated to temperature difference from the reference temperature by Boussinesq approximation. The computational dimensionless parameters are the Prandtl number for the working fluid, the modified Rayleigh number for the heat flux from the heated wall and the magnetic strength \( \gamma \). The governing equations are discretized into finite-difference equations in the staggered uniform mesh and solved numerically. The third-order upwind scheme is employed for the inertial term and the pressure term in the momentum equation is analytically-computed by the HSMAC method by Hirt et al. (1975). The second-order central finite-difference approximation is applied to the diffusion terms. The grid number is 102 and 602 which is determined by the evaluation of the local Nusselt number without a magnetic field performed at \( Ra^*=10^{4}-10^{7} \). The magnetic field from two block magnets with opposite orientations is obtained by a free software FEMM. This can solve the two-dimensional Maxwell equations by finite element method. The obtained magnetic field is remapped in the Cartesian coordinate and \( \nabla B^2 \) is solved to compute the magnetothermal force in Eq.(3).

2.3 Computational conditions

The computational conditions are as follows. The Prandtl number is 13.1 to presume the fluid used in the experiment by Kaneda et al. (2019). The modified Rayleigh number is fixed at \( 4.41 \times 10^7 \). This corresponds to the heat flux of 1240
W/\text{m}^2\) when the characteristic length \(\ell\) is presumed at 30mm. The dimensionless magnetic induction \(\gamma\) is 0.0631 which corresponds to \(b_0 = 0.440\) T. The elevation of the magnet \(Z_{\text{mag}}\) is varied from -0.32 to 1.0 of which are partially indicated in Fig. 2.

The initial condition is that, the flow is static and uniform temperature of \(T = 0\) is applied. The computations are performed until the steady state convection is attained.

3. Results and discussions
3.1 Magnetic field

The obtained magnetic field from two block magnets is shown in Fig. 3(a). Since the FEMM calculates the dimensional magnetic induction, the magnitude in the figure is colored by dimensional value. It is confirmed that, the magnetic induction becomes locally large at the magnet edge. Especially, since the N pole and S pole are close at the magnet junction of two magnets, the magnetic field line becomes dense and the obtained magnetic induction is remarkably strong. This results in the locally large value of \(\mathbf{\nabla} \cdot \mathbf{b}\) as shown in Fig. 3(b), and the strong magnetic force in Eq. (3) is expected.

Fig. 2. Magnets elevations. At \(Z_{\text{mag}} = -0.32\), the upper edge of the upper magnet corresponds to \(Z = 0\). At \(Z_{\text{mag}} = 0.32\), the lower edge of the lower magnet corresponds to \(Z = 0\).

Fig. 3. Magnetic field and obtained \(\mathbf{\nabla} \cdot \mathbf{b}\) by two block magnets with opposite orientations.
3.2 Heat and fluid flow with and without magnetic field

The buoyant force is induced due to the temperature and the magnetothermal force occurs near the magnets. Fig.4(a) shows the magnetothermal force at $Z_{mag} = 1.0$ where the heated wall fully overlaps the magnets. In the force term in Eq.(3), the magnetothermal force is defined by the derivation from the reference temperature ($T=0$). Therefore, the force is induced where the temperature difference exits and in the opposite direction to the $\nabla B^2$. As shown in Fig.4(a), the strong magnetothermal force is observed near the magnet junction and is normal to the heated wall.

Fig.4(b) shows the velocity distribution in $Z$ direction at $Ra^* = 4.41 \times 10^7$ and $\gamma = 0.0631$. The dashed line shows the case without the magnetic field. The velocity boundary layer develops along the heated wall without the magnetic field. In case with the magnetic field, below the magnet junction ($Z < 1.0$), the development of the boundary layer is interrupted and upward flow slows down. Additionally, the velocity peak is shifted away from the heated wall. In contrast, above the magnet junction ($Z > 1.0$), the velocity peak overtakes the flow without magnets and the peak shifts near the hot wall. Therefore, the magnetothermal force works like a bump for the uprising buoyant flow. This is a similar tendency observed in the forced convection in a heated pipe under a magnetic field by Kaneda et al. (2018).

The thermal boundary layer is accordingly affected by the magnetothermal force as shown in Fig.4(c). Below the magnet junction ($Z < 1.0$), wall temperature becomes higher than the case without magnet, and the thermal boundary layer becomes thick. Above the junction, the temperature profile is not similar to the no-magnet case. The wall temperature is a bit smaller. By considering with the velocity profile, they are due to the reattachment flow above the junction. Since the local wall temperature is changed by the magnetothermal force, the local heat transfer coefficient along the heated wall is also affected, which is estimated in following section.

![Fig. 4. Dimensionless magnetothermal force, resulted velocity profile along the wall, and isotherms at $Z_{mag} = 1.0$. Magnets cover $0.68 \leq Z \leq 1.32$.](image)

3.3 Effect on the local heat transfer coefficient

The local Nusselt number $Nu_B$ at various magnet elevation $Z_{mag}$ is estimated with and without the magnetic field. The definition of $Nu_B$ is as follows.

$$Nu_B = \frac{q^*}{\lambda} = \frac{W}{W_{wall}}$$

Fig.5 shows the local Nusselt number profiles along the heated wall at various magnet elevations. Due to increased and decreased wall temperature as shown in Fig.4(c), $Nu_B$ decreases below the magnet junction and increases above it.
When the magnets are below the bottom edge of the heated wall ($Z_{mag} = -0.32$, Fig.2(a)), a bit enhancement effect is observed since the induced magnetothermal exists only above the upper edge of upper magnet. The enhancement effect becomes more remarkable at $Z_{mag} = 0$ (Fig.2(b)). In the case, the strongest magnetic field area (=magnet junction) corresponds to the bottom edge of heated area. As aforementioned, the heat transfer enhancement effect is expected above the magnet junction. Therefore, the maximum enhancement is obtained at this magnet elevation. As the magnet elevation becomes higher, the local Nusselt number profile has both the suppression and enhancement since the suppression effect emerges below the magnet junction. For example, at $Z_{mag} = 1.0$, the suppression effect surpasses the enhancement effect. These results correspond to our experimental results in terms of the local heat transfer enhancement/suppression near the magnets shown in Fig.12 in Kaneda et al., 2019.

Amongst the computed cases, case at $Z_{mag} = 0.32$ shows outstanding performance of the heat transfer enhancement. The heat transfer is suppressed below the magnet junction, however, the heat transfer enhancement is extended from the magnet junction to the end of the heated area. This case has a periodic oscillating flow and the time-averaged local Nusselt number is shown herein. This result suggests that, the oscillating flow induced by the magnetic field has a potential to enhance the heat transfer characteristic.

![Fig. 5. Local Nusselt number along the heated wall with various magnet elevations.](image)

### 3.4 Vibrating flow by the magnetic field

As aforementioned, a periodic oscillation flow is observed only at $Z_{mag} = 0.32$. The oscillating Nusselt number by a strong magnetic field has been observed in the convection inside vertical cylinder reported by Filar et al. (2005), however, the oscillation damps out at the sufficient time. Kenjereš, et al. (2012) also reported the oscillatory and turbulent convections by the magnetic field, but they are not periodic. Therefore, the transient temperature profile in one period is observed in detail.

The transient kinetic energy at $(X, Z) = (0.1, 0.64)$ and local wall temperature at $Z = 0.64$ is shown in Fig.6. It is confirmed both components have periodic oscillations. At the sufficient time passed, these oscillations continue. Next, the snapshots of temperature profile at indicated dimensionless time (a)-(e) is shown in Fig.7. As Fig.5 suggests, in the lower area than the magnet junction, a locally hot spot is generated due to the competition of buoyant and magnetothermal force. According to the force shown in Fig.4(a), the magnetothermal force repels the rising hot fluid normal to the wall. The repelled hot fluid becomes a plume and it grows at a certain size. The plume receives more buoyant force than surrounding fluid since the local temperature is higher. This rising plume disturbs the flow. Consequently, these occur continuously with periodicity, resulting in a wavy flow in the channel.
Fig. 6. Local kinetic energy and wall temperature at $Z_{mag} = 0.32$.

Fig. 7. Snapshots of temperature field at $Z_{mag} = 0.32$.

4. Conclusion

The effect of the magnetic force on the uprising natural convection of a paramagnetic fluid is numerically studied in a partially-heated vertical channel. It is found that, two magnets with opposite orientation induces much stronger magnetic field than a single magnet, and corresponding magnetothermal force is observed where the fluid is heated. The magnetothermal force behaves like a bump thus the hot fluid is repelled and reattached to the heated wall as it goes up by buoyancy. The local heat transfer is thus suppressed below the magnet junction and enhanced above the junction where the strongest magnetothermal force exists. Therefore, the overall heat transfer depends on the relative magnet
elevation to the heated wall. A periodic oscillatory characteristic can be observed at a specific magnet elevation. This is due to the periodic behavior of repelled hot plume from the wall. This has a possibility to enhance the heat transfer drastically.

Acknowledgement

This work was partially supported by MEXT and a Grant-in-aid for Scientific Research (C) No. 18K03988.

References

Braithwaite, D., Beaugnon, E., Tourniew, R., Magnetically controlled convection in a paramagnetic fluid, Nature, 354(14), (1991), pp. 134-136.
Filar, P., Fornalik, E., Kaneda, M., Tagawa, T., Ozoe, H., Szmyd, J.S., Three-dimensional numerical computation for magnetic convection of air inside a cylinder heated and cooled isothermally from a side wall, International Journal of Heat and Mass Transfer, 48,9, (2005), 1858-1867.
Hirt, C. W., Nichols, B. D., Romero, N. C., Technical Report, Los Alamos Scientific Laboratory, (1975), LA-5852.
Kenjereš, S., Pyrda, L., Wrobel, W., Fornalik-Wajs, E., Szmyd, J.S., Oscillatory states in thermal convection of a paramagnetic fluid in a cubical enclosure subjected to a magnetic field gradient, Phys. Rev. E., 85, (2012), 046312.
Kaneda, M., Tagawa, T., Ozoe, H., Convection induced by a cusp-shaped magnetic field for air in a cube heated from above and cooled from below, Journal of Heat Transfer, 124, (2002), 17-25.
Kaneda, M., Noda, R., Tagawa, T., Ozoe, H., Lu, S.-S., Hua, B., Effect of inclination on the convection of air in a cubical enclosure under both magnetic and gravitational fields with flow visualization, Journal of Chemical Engineering of Japan, 37, 2, (2004), 338-346.
Kaneda, M., Suga, K., Magnetothermal force on heated or cooled pipe flow, International Journal of Heat and Fluid Flow, 69 (2018), 1-8.
Kaneda, M., Nazato, K., Fujiwara, H., Wada, K., Suga, K., Effect of magnetic field on natural convection inside a partially-heated vertical duct: Experimental study, International Journal of Heat and Mass Transfer, 132, (2019), 1231-1238.
Ozoe, H., Magnetic Convection, Imperial College Press, London, 2005
Maki, S., Ataka, M., Tagawa, T., Ozoe, H., Mori, W., Natural convection of a paramagnetic liquid controlled by magnetization force, AIChE Journal, 51, 4, (2005), 1096-1103.
Song, K., Tagawa, T., Wang, L.B., Ozoe, H., Numerical investigation for the modeling of the magnetic buoyancy force during the natural convection of air in a square enclosure, Advances in Mechanical Engineering, 2014, 873260 (2014), 11 pages.
Zhang, X.Y., Zhang, J.L. Song, K.W., Wang, L.B., Numerical study of the sensing mechanism of the oxygen concentration sensor based on thermal magnet convection, International Journal of Thermal Sciences, 99 (2016), pp. 71-84.