A complete characterization of the heralded noiseless amplification of photons

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Abstract. Heralded noiseless amplification of photons has recently been shown to provide a means to overcome losses in complex quantum communication tasks. In particular, to overcome transmission losses that could allow for the violation of a Bell inequality free from the detection loophole, for device independent quantum key distribution (DI-QKD). Several implementations of a heralded photon amplifier have been proposed and the first proof of principle experiments realized. Here we present the first full characterization of such a device to test its functional limits and potential for DI-QKD. This device is tested at telecom wavelengths and is shown to be capable of overcoming losses corresponding to a transmission through 20 km of single mode telecom fibre. We demonstrate heralded photon amplifier with a gain $>100$ and a heralding probability $>83\%$, required by DI-QKD protocols that use the Clauser–Horne–Shimony–Holt inequality. The heralded photon amplifier clearly represents a key technology for the realization of DI-QKD in the real world and over typical network distances.

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1. Introduction

The concept of amplification in communication systems has long been used in the classical regime to overcome transmission loss. However, for quantum systems, amplification of quantum states is generally not possible due to the no (perfect) cloning theory—amplification will normally introduce noise, thus degrading the quality of the quantum state [1]. Heralded photon amplification can allow one to overcome these constraints as it operates in a probabilistic fashion. Importantly, while it is probabilistic in nature, when successful, it provides a heralding signal that allows one to then perform subsequent operations. This heralding signal is what makes this approach interesting for device independent quantum key distribution (DI-QKD) as it can herald the arrival of a photon (or qubit) and hence prepares the system so that the Bell test may be performed [2–4]. More recently, it has also been incorporated into a quantum repeater protocol where it is used to herald the storage of a photon in a quantum memory [5], opening the door to even greater distance for device independent operations. It is clear that such a device could find wide spread use in myriad quantum systems where one needs to overcome inefficiencies associated with loss or multiple probabilistic operations, as well as where feed-forward signals can help in scaling complex quantum systems.

Noiseless photon amplification is related to quantum scissors [6] and relies on quantum teleportation to herald the amplified state. It was first proposed by Ralph and Lund [7] and has found several different implementations [8], either exploiting polarization modes [7, 9, 10], spatial modes in fibre optics [11] or using techniques such as single photon addition and subtraction [12–15]. Also, it has shown potential application in both discrete and continuous systems [4, 16].

In this paper, we first present the principle operation of a heralded photon amplifier, then we introduce how our test device is realized. The purpose of this paper is to completely characterize the performance of the heralded photon amplifier at telecom wavelengths, independently of the source and detector characteristics. We then discuss the operational limits of such devices and give some perspectives on further improvements in the context of DI-QKD.

2. Principle of the heralded photon amplifier

The concept of a heralded photon amplifier is illustrated in figure 1(a). The incoming state that we are interested in is usually a single photon that has been mixed with some vacuum due to transmission loss and has the form \( \rho_{in} = p|0\rangle\langle 0| + (1 - p)|1\rangle\langle 1| \). An ancilla photon is first used to generate single photon entanglement [17]. The input state is then combined
on a beam splitter with one mode of the entangled state and subsequently one photon is detected which corresponds to a Bell state measurement. This requires that the two photons are indistinguishable and that the detector ($D_b$, see figure 1(a)) can resolve the number of photons. If the initial ancilla state is maximally entangled, i.e. for a transmission of $t = 0.5$, then this corresponds directly to the standard teleportation scenario. However, if we now vary this transmission we can bias the output state such that it has the form

$$\rho_{\text{out}} = \frac{1}{N(t)} |0\rangle \langle 0 | + g^2(t)(1 - p) |1\rangle \langle 1 |. \quad (1)$$

Here $g^2(t) = t / (1 - t)$ is the gain factor, while $N(t) = p + g^2(t)(1 - p)$ is a normalization factor. The renormalized gain is defined as the ratio between the probability for the single photon component before and after the amplification and is given by

$$G(t) = \frac{g^2(t)}{p + g^2(t)(1 - p)} = \frac{t}{(1 - t)p + t(1 - p)}. \quad (2)$$

We see that for $G(t = \frac{1}{2}) = 1$ the protocol reduces to a teleportation of the input state, and the gain is then greater than 1 for $t > \frac{1}{2}$.

One can notice that the gain depends on both $p$ and $t$. In particular, $G$ tends to infinity as $p \to 1$ (high losses) and $t \to 1$ (high transmission). However, a high gain does not imply a high heralding probability, which, on the contrary, is inversely proportional to the losses.

In addition, one should note that equation (2) does not take into account that, in practice, we have non-photon-number resolving detectors and non-zero losses through the components of the amplifier. This can result in a reduction in the actual experimentally achievable gain for a fixed input state and, in general, change the response of the amplifier as a function of $t$.

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**Figure 1.** (a) Standard representation of heralded photon amplifier, composed by a beam splitter with variable transmission $t$ and a balanced beam splitter followed by a photon detector. (b) Experimental setup for our test-device based on bulk optics and using, instead of spatial, polarization modes (see main text for details).
3. Experiment

In practice, our test device is of the form represented in figure 1(b), as this provides a more flexible setup for testing. The half wave plate and the polarizing beam splitter (HWP$_2$ + PBS$_1$) are used to simulate losses on the input state, while HWP$_1$ and PBS$_1$ play the role of a beam splitter with variable splitting ratio to define the transmission ($t$).

In this experiment, both the input and the ancillary photons belong to the same pair created in a type II spontaneous parametric down conversion process. For this purpose a 2 cm periodically poled lithium–niobate crystal is pumped by a mode locked Ti:sapphire laser at 780 nm, pulsed in the picosecond regime. The photons at 1560 nm are filtered down to 1 nm by an interference filter to eliminate spectral distinguishability, before being separated by a PBS. Coupling into single mode fibre with $\sim 50\%$ efficiency, ensures a well defined spatial mode. Photon counting is performed by using two gated avalanche photo-diodes (IDQ-210) with 25% detection efficiency, 3 ns gate and a noise probability of $10^{-5}$ per gate that are synchronized with the laser. In all the performed measurements the laser is used to trigger one detector at 80 MHz, which, in turn, triggers the second one.

To ensure indistinguishability between the two photons in all degrees of freedom, a Hong Ou Mandel (HOM) type interference measurement is performed [18], using the two polarization modes at PBS$_3$ [19]. Following the setup reported in figure 1(b), it can be seen that in order for the amplification to take place the two photons are required to arrive at the same time on the PBS$_1$. From this point they travel through the same optical path until they arrive at HWP$_4$ in two orthogonal polarization states: $|H\rangle|V\rangle$. Here, the polarization is rotated by $\pi/4 : (|H\rangle + |V\rangle) (|H\rangle - |V\rangle)$. If there is perfect indistinguishability the terms $|H\rangle|V\rangle$ and $|V\rangle|H\rangle$ interfere and vanish, therefore two detectors at the outputs of PBS$_3$ will not click in coincidence. The measured net HOM visibility is 0.98 ± 0.03, with a pair creation probability per pulse $p = 0.01$. The visibility is limited only by double pair emission, and is in good agreement with the theory [20], indicating that all degrees of freedom are well controlled in the experiment.

The gain is evaluated as the ratio between the probability of having a photon in the output state and the probability of having a photon in the input: $G = p_{\text{out}}/p_{\text{in}}$. We estimate the input probability $p_{\text{in}}$ as the ratio between the rate of triggers (counts in $D_a$) and the rate of coincidences with $D_b$. The input losses are varied between 0.5 and 1 by turning the wave plate HWP$_2$. In a second measurement the detector $D_b$ triggers $D_a$, and the output probability $p_{\text{out}}$ is given by the ratio between singles in $D_b$ and coincidences with $D_a$. For each value of input loss we vary the amplifier transmission $t$ between 0.5 and 1.

The gain $G$ is measured as a function of the transmission of the amplifier for eleven loss values. As shown in figure 2, the resulting gain is in agreement with the theoretical prediction taking in account losses, detection efficiency and the use of non-photon-number resolving detection. Figure 2(b) shows the curves for three fixed values of $t$ as a function of $p$. We notice that the gain is measured to be $>100$ for the limit of high losses ($p$) and high transmission (see figure 2(a), blue stars), but in this regime the performance of the heralded photon amplifier tends to be less efficient in terms of success probability, as we will see in the following analysis.

To complete the characterization of the heralded photon amplifier, it is necessary to verify the coherence of the process. As we see in figure 1(b), the input state is separated into two modes after PBS$_1$, the ratio depending on the angle of HWP$_2$. The reflected and the transmitted modes correspond to the state to be ‘amplified’ and the ‘lost’ part, respectively. The latter is obviously
not present in a communication channel, where lost photons are mainly absorbed or reflected, but in our case these two modes remain in a coherent superposition, such that we can use the state in the ‘lost’ mode as a reference and interfere it with the state in the ‘amplified’ mode [11]. The visibility of the interference pattern is a signature of the coherence of the amplification process, i.e. that the teleportation protocol preserves the state. The visibility of the interference pattern is related to the coherence of the amplification process, and it allows one to complete the characterization of the device. The interference fringes are measured for each setting and the corresponding visibilities are represented in figure 3(a), and found to be consistent with the expected behaviour. In particular, it is maximal (∼94%) when \( p \) and \( t \) are complementary, i.e. the amplitudes of the two interfering modes are balanced. Changing the two parameters, the visibility inevitably decreases only because of the imbalance in the amplitudes, thus confirming the coherence of the amplification process. Figure 3(b) shows two examples of the measured interference fringes with maximal visibility.

Summarizing the result in a more intuitive way, as in figure 4, it is convenient to look at the heralding probability as a function of the losses introduced in the input state. With an amount of loss corresponding to the typical network distances, i.e. sending a photon through more than 20 km of network installed fibre, it is still possible to have a heralding probability greater than 83%. The results are renormalized taking into account the probability of pair emission and the losses before and after the device, i.e. they consider the amplifier performance only, and as such, it is limited only by its intrinsic losses.

4. Discussion and conclusion

We have fully characterized a heralded noiseless photon amplifier at telecom wavelengths and obtained a gain \( >100 \) associated with a heralding probability greater than 83% up to a distance in fibre of 20 km. Moreover, by duplicating the amplification stage it is possible to

\[ \text{Gain} \text{ vs. Transmission (t) Losses (p)} \]

\[ \text{Gain} = 0.57, 0.77, 0.99 \]

Figure 2. (a) Gain of the photon amplifier as a function of the losses introduced by rotating HWP\(_2\) and the transmission of the beam splitter tuned by HWP\(_1\). The result is in good agreement with the theoretical simulation (inset), which takes into account losses and non-photon number resolving detection. (b) Example of the gain as a function of the losses for three different transmissions.
Figure 3. (a) Single photon interference visibility measured as a function of input losses and transmission of the amplifier, the theoretical prediction is represented in the inset. (b) Interference fringes for two measurement settings such that the visibility is maximal, i.e. losses and transmission are balanced. The visibilities are 0.92 ± 0.01 and 0.94 ± 0.03 for the red (dots) and the blue (stars) curves, respectively. Visibilities are mainly limited by double pair emission in the down conversion process and polarization dependent losses in optical elements in the setup, which change the weight of the two interfering modes.

Figure 4. Heralding probability as a function of the input losses, for the eleven settings of $t$. The probability is higher than 80% when $t \geq 0.95$ even for losses of around 70%. The coloured region represents the theoretical prediction. The upper bound on the heralding probability is given by the intrinsic losses of the amplifier. On the upper axis, the equivalent transmission distance for an installed single mode fibre at telecommunication wavelength is given (losses are 0.24 dB km$^{-1}$). In the extreme regime of high losses and high transmission the performance of the amplifier no longer follows the theory, because the trigger and coincidences rates fall to the noise level and the error bars significantly increase, as expected.
have a heralded polarization qubit amplification [21], which could allow the violation of a Bell inequality without the detection loophole by compensating for the losses [4]. The heralded efficiency of such devices could be improved by reducing losses in the setup, in particular, by using anti-reflection coated and optimized optical elements [22]. However, for a more practical implementation of such a device, a fibre-based approach [11] with a fixed gain for a fixed amount of loss would provide a realistically efficient solution with even lower internal losses, i.e. a heralding efficiency $>83\%$. One of the biggest challenges, though, is the generation of pure photons and coupling them into the heralded photon amplifier [22–26]. To resolve the coupling problem, one could also think of a device completely realized with integrated optics on a chip, which could include, sources of pure photons—either from single photon emitters, or heralded single photons realized via engineering the phase-matching of photon pair sources [27, 28] as well as wavelength division multiplexers [29], variable couplers [30] and potentially even detectors [31].

At the moment, all of these devices have been shown to work independently and it remains a grand challenge to bring these together on one chip, however, the heralded photon amplifier would provide an excellent motivation.

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