Residual translation compensations in radar target narrowband imaging based on trajectory information

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Abstract. High velocity translation will result in defocusing scattering centers in radar imaging. In this paper, we propose a Residual Translation Compensations (RTC) method based on target trajectory information to eliminate the translation effects in radar imaging. Translation could not be simply regarded as a uniformly accelerated motion in reality. So the prior knowledge of the target trajectory is introduced to enhance compensation precision. First we use the two-body orbit model to figure out the radial distance. Then, stepwise compensations are applied to eliminate residual propagation delay based on conjugate multiplication method. Finally, tomography is used to confirm the validity of the method. Compare with translation parameters estimation method based on the spectral peak of the conjugate multiplied signal, RTC method in this paper enjoys a better tomography result. When the Signal Noise Ratio (SNR) of the radar echo signal is 4dB, the scattering centers can also be extracted clearly.

1. Introduction
High velocity translation induces propagation delay of the echo, which subject micro-Doppler to translation, slope and leads to defocusing phenomena in radar imaging [1]. Thus, translational compensation is a crucial step in micro-Doppler extraction and radar imaging processing. The existing researches on translation compensation can be divided into two aspects: (1) Conduct translation compensation according to the Doppler spectrum. Y. C. Yang proposes a translation parameter estimation method based on the Doppler information of the strongest scattering point [2]. S. S. He proposes a translation parameters estimation method based on the spectral peak information [3]. He designs a quadratic equation to model the translation, and then use the spectral peak position to estimate the velocity and acceleration based on the conjugated multiplied signal. X. Q. Luo proposes a compensation method for ISAR imaging based on short time adaptive Gaussian chirplet decomposition [4]. (2) Estimate the translational parameters of the target. Reference [1] and [5] have analyzed the acceleration effects on the micro-Doppler. They proposed a compensation method based on spectral rearrangement, which can only be used for uniform motion and uniformly accelerated motion. However, most of the researches are based on the assumption that the translation is simple motion, which is difficult to achieve in reality.

Focus on the translation compensation problem, we introduce a priori information of the target trajectory, and then use the two-body orbit model to enhance the compensation precision. The tomography results are used to confirm the validity of the method. The paper is organized as follows: In section II, we figure out the trajectory of target, and then build the radar echo model of the micro-
motion target. In section III, the estimated trajectory information is applied to deduce the propagation delay of the high velocity translation. Then, we conduct stepwise compensations on the residual translation until it achieves the required precision. Finally, we use tomography to evaluate the compensation results. Experimental results are exhibited in Section IV to demonstrate the necessity of the proposed method. We conclude this paper in Section V.

2. Signal model

2.1. Trajectory of the target in two-body orbit model

Suggest that earth and the radar target are regarded as the two-body system, the orbit model of the target is referred to as a two-body orbit model. During its flight, only gravity is taken into consideration, the radar target is doing non-distribution motion. As is shown in Figure 1, the trajectory of the radar target is part of an ellipse [6]. The trajectory can be represented by six integrated constants, which is called orbital element. The most frequently used orbital element is \( \mathbf{r} = (a, e, i, \Omega, \omega, \tau) \) [6]. The orbital element solving process is as follows.

![Figure 1. Geometric relation of orbit parameters.](image)

In Earth Centered Inertial (ECI), we suggest that the position vector of point \( k \) at time \( t \) is \( \mathbf{r}_k = (p_x, p_y, p_z) \), the velocity vector is \( \mathbf{v}_k = (v_x, v_y, v_z) \), the radar target flies at the optimal angle. The geocentric distance is \( r_k = \sqrt{p_x^2 + p_y^2 + p_z^2} \), the velocity is \( v_k = \sqrt{v_x^2 + v_y^2 + v_z^2} \) [7]. The semi-major axis of the elliptic orbit can be written as,

\[
a = \left( \frac{2}{r_k} \frac{v_k^2}{\mu} \right)^{-1}
\]

where \( \mu = 3.9860044 \times 10^{14} \text{m}^3 / \text{s}^2 \), it is the geocentric gravitational constant. The eccentricity of the elliptic orbit can be written as,

\[
e = \left[ \left( \frac{r_k v_k}{\sqrt{a \mu}} \right)^2 + (1 - \frac{r_k}{a})^2 \right]^{1/2}
\]

The time of passing through the perigee \( \tau \) can be expressed as,

\[
\tau = t_k - \frac{E_k - e \sin E_k}{n}
\]

where \( E_k = \arctan \left( \frac{\sqrt{a r_k v_k}}{\sqrt{\mu(a - r_k)}} \right) \) is the eccentric anomaly of point \( k \), \( n = \frac{\sqrt{\mu}}{a} \) is the mean angular velocity.

The right ascension of ascending node \( \Omega \) can be expressed as [8],

\[
\Omega = \arccos \left( \frac{-H_{jk}}{\sqrt{H_{jk}^2 + H_{sk}^2 + H_{sk}^2}} \right)
\]
where the moment of orbital momentum \( \vec{r}_k \times \vec{v}_k = \begin{bmatrix} H_{sk} \\ H_{sk} \\ H_{sk} \end{bmatrix} \).

The included angle of orbital plane and equatorial plane \( i \) can be expressed as,
\[
i = \arccos\left( \frac{H_{sk}}{\sqrt{H_{sk}^2 + H_{sk}^2 + H_{sk}^2}} \right)
\]

The included angle of perigee and ascending node \( \omega \) can be expressed as,
\[
\omega = \arctan\left( \frac{p_{e} \sin i}{p_{e} \cos \Omega + p_{e} \sin \Omega} \right) - f_k
\]

where \( f_k = 2 \arctan\left( \frac{1+e}{1-e} \tan \frac{E_k}{2} \right) \).

To summarize, \( \sigma = (a,e,i,\Omega,\omega,\varepsilon) \) is obtained.

The Kepler’s equation can be expressed as,
\[
M = E - e \sin E
\]

Equation (7) is a transcendental equation of \( E \), it is hard to figure out \( E \) accurately. So we use Newton iterative method to solve Kepler’s equation,
\[
E_{i+1} = E_i - \frac{E_i - e \sin E_i - M}{1 - e \cos E_i} \quad (i = 0,1,2 \ldots)
\]

where \( E_0 \) is the eccentric anomaly at time \( t_0 \), the condition of convergence is \( |E_{i+1} - E_i| < \varepsilon \). From equation (7) and (8) we can get the relationship between \( E \) and \( t \).

In the two-body model, the position vector of the radar target can be expressed as,
\[
\vec{r}_t = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = a (\cos E - e) \cdot \vec{P} + a \sqrt{1-e^2} \sin E \cdot \vec{Q}
\]

where
\[
\vec{P} = \begin{bmatrix} \cos \Omega \cos \omega - \sin \Omega \sin \omega \cos i \\ \sin \Omega \cos \omega + \cos \Omega \sin \omega \cos i \\ \sin \omega \sin i \end{bmatrix}
\]
\[
\vec{Q} = \begin{bmatrix} -\cos \Omega \cos \omega - \sin \Omega \sin \omega \cos i \\ -\sin \Omega \cos \omega + \cos \Omega \sin \omega \cos i \\ \cos \omega \sin i \end{bmatrix}
\]

We can figure out the relationship between the position vector \( \vec{r}_t \) and \( t \) from equation (9).
\[
\vec{r}_{\text{LOS}}(t) = \vec{r}_t - \vec{r}_{\text{radar}}
\]

where \( \vec{r}_{\text{radar}} \) is the position vector of the radar.

2.2. Radar echo signal model of the micro-motion target

Micro-motion includes vibration, rotation, precession, nutation [9]. In Figure 2, we take precession for an example. Suggest that there is a micro-motion target in the reference coordinate system \( o-xyz \), \( o \) is the origin. \( P_1 \) is the scattering center of the conic node. \( P_2 \) and \( P_3 \) are the scattering centers of the conic bottom. Precession angular velocity is \( \omega \), precession angle is \( \theta \), the included angle between radar line of sight (LOS) and the precession axis is \( \gamma \), initial phase is \( \phi_0 \). Distance between \( P \) and the radar is \( R_0 \). The aspect angle of the target is \( \beta(t) \).
\( \beta(t) \) can be expressed as,
\[
\beta(t) = \arccos \left( \cos \gamma \cos \theta + \sin \gamma \sin \theta \cdot \cos (\omega t + \phi_b) \right)
\] (12)

Owing to the precession, the distance changes of the target scattering center can be expressed as,
\[
r_s(t) = x_m \cos \beta(t) + y_m \sin \beta(t)
\] (13)

where \((x_m, y_m)\) is the position of the scattering center \(m\).

Based on the analysis above, the radar echo signal of the target can be expressed as,
\[
s(t) = \sum_{m=1}^{M} \sigma_m \exp \left( -j \frac{4\pi f_c}{c} (r_{LOS}(t) + r_s(t)) \right) + w[n]
\] (14)

where \(f_c\) is the radar center frequency, \(c\) is the velocity of light, \(\sigma_m\) is the scattering coefficient of the scattering center, \(r_{LOS}(t)\) is the position change of the target caused by high velocity translation, \(r_s(t)\) is the position change of the scattering center caused by micro-motion, \(w[n]\) is the white Gaussian noise.

3. RTC method

In order to revise the time-frequency spectrums, we need to eliminate the effects caused by the high velocity translation. Therefore, a Residual Translation Compensations (RTC) method is proposed in this section. The steps are as follows.

- **Step 1**: Setup parameters of the target to figure out the trajectory and the radial distance \(r_{LOS}(t)\) according to section II A.
- **Step 2**: Then we use the estimated radial distance \(r_{LOS}(t)\) to eliminate the effects of translation by conjugate multiplication method as follows.

\[
s'_{LOS}(t) = \exp \left( \frac{j 4\pi f_c}{c} r_{LOS}(t) \right)
\] (15)

\[
s'(t) = s(t) \cdot s'_{LOS}(t)
\] (16)

where \(s'(t)\) is the radar echo signal after the first time compensation by using the estimated trajectory.

- **Step 3**: The word “data” is plural, not singular. Use stepwise compensations method to deal with the residual translation. Suggest that the time delay before being compensated is \(t_i(i)\), the time delay after being compensated is \(t_i(i)\), the deviation is,

\[
\Delta_{i+1} = t_i(i) - t_i(i) - \Delta_i, \quad i = 1, 2, \ldots, n
\]

\[
\Delta_0 = 0
\] (17)

Suggest that \(\Delta\) is a quadratic equation, then we can figure out the parameters of equation.
• Step 4: If \( \max \left( \frac{\Delta_{\text{LOS}}}{\Delta_{\text{LOS}}(t)} \right) > 10^{-5} \), repeat step 3. If \( \max \left( \frac{\Delta_{\text{LOS}}}{\Delta_{\text{LOS}}(t)} \right) \leq 10^{-5} \), the required precision is obtained after \( i \) times compensation. Finally, the radar echo signal can be expressed as,

\[
s'_{\text{final}}(t) = s(t) \cdot s_{\text{LOS}}^*(t) \cdot \prod_{j=r}^{n} \exp(j2\pi f_c \cdot \Delta_j)
\]  

(18)

• Step 5: The time-frequency spectrum of micro-Doppler can be obtained by using STFT on \( s'_{\text{final}}(t) \).

\[
[mD] = \text{STFT}(s'_{\text{final}}(t))
\]  

(19)

The flow chart is as follows (Figure 3).

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4. Simulation and analysis

4.1. Simulation on trajectory of the target

When \( t_0 = 0 \), suggest that the launching position is \((-180340.978 m, 2786423.914 m, 5727756.069 m)\), the velocity is \((-1935.96 m/s, 1300.56 m/s, 4644.84 m/s)\), the target flies in the optimum trajectory, it flies for 1128.4s. Radar is close to the landing position. The three-dimensional trajectory of the target in ECI is shown in Figure 4.

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Figure 3. Flow chat of the algorithm.

Figure 4. The three-dimensional trajectory of the target in ECI.
The changes of height and the actual velocity are shown in Figure 5. The radial distance between radar and the target is shown in Figure 6.

Figure 5. The changes of height and the actual velocity.

Figure 6. The radial distance between radar and the target.

4.2. Translation compensation and micro-Doppler extraction

Suggest that the carrier frequency is 10GHz, the pulse recurrence frequency is 1000Hz. The white Gaussian noise is 10dB. The precession angular velocity is $\omega_\psi = 2\pi$, the precession angle is $\theta_\psi = 15^\circ$, the included angle between LOS and the precession axis is $\gamma = 130^\circ$, the initial phase of the target is $\phi_0 = \frac{\pi}{2}$. The coordinate of the three scatters are $P_1 = (0, 0, 1.6)$, $P_2 = (0.2, 0.2, -0.4)$, $P_3 = (0.2, 0, -0.4)$. The scattering coefficient of the three scatters are $\sigma_1 = \sigma_2 = \sigma_3 = 1$. According to equation (17) and (18), we do stepwise compensations on residual translation, until $\max \left( \frac{\Delta_{\text{res}}(t)}{f_i(t)} \right) \leq 10^{-5}$. The simulation experiment shows that, the required precision is obtained after three times compensations.

After removing the residual translation by RTC method, the micro-Doppler time-frequency spectrum can be obtained by using STFT on the final compensated radar echo. The spectrums before using RTC method are shown in Figure 7(a). The spectrums after using RTC method are shown in Figure 7(b). Compared to Figure 7(a) we can find that the micro-Doppler spectrums have been recovered from the effects of the high velocity translation.
Figure 7. Micro-Doppler spectrums: (a) with the high speed velocity; (b) after using RTC method.

4.3. Tomography results

Figure 8(a) shows the tomography result of the radar echo without high speed translation. The tomography result of the radar echo with high speed translation is shown in Figure 8(b), translation causes the scattering centers defocusing. The tomography after using RTC method is shown in Figure 8(c). There are three scattering centers in Figure 8(c), which are corresponding to $P_1$, $P_2$ and $P_3$. Compare with the result in Figure 8(a) and (b), we find that this method can eliminate the translation effects. The coordinate of the scattering center is three-dimensional, so that the scattering centers in the planar tomography are the projection of the three scattering centers in radar LOS. The tomography result confirms the validity of the proposed algorithm.
Figure 8. The tomography results of the radar echo with a SNR of 10dB: (a) without the high speed velocity; (b) with the high speed velocity; (c) after using RTC method.

Take $P_1$ as an example, the micro-Doppler amplitude without translation is 132.8797Hz. When the estimated micro-Doppler amplitude is between $(-133Hz, 133Hz)$, the micro-Doppler can be estimated exactly. Figure 9 shows the accuracy changes with SNR. When SNR=20 dB, the accuracy of the estimated micro-Doppler frequency reaches up to 90.40%.

Figure 9. The accuracy of estimated micro-Doppler changes with SNR.

4.4. Comparison
The spectral peak based translation parameters estimation method designs a quadratic equation to model the translation, and then uses the spectral information of the conjugated multiplied signal to figure out the velocity and the acceleration [3]. The translation can’t be simply modeled as a quadratic equation, so that the estimated translation parameters have large deviation. As is shown in Figure. 10(a), when SNR=4dB, the scattering centers can be extracted clearly by RTC method. However, the scattering center in Figure. 10(b) is defocusing by method in reference [3]. So we can conclude that, RTC method in this paper performs better than method in reference [3].

Figure 10. The tomography of the radar echo with a SNR of 4dB: (a) RTC method; (b) Method in reference [3].
5. Conclusion
Aiming at eliminating the influence caused by the high velocity translation, this paper proposes a Residual Translation Compensations (RTC) method based on trajectory information. We establish the two-body orbit model of the radar target, and then build radar echo signal model of the micro-motion target. After that we use the trajectory information to conduct stepwise compensations on residual translation until the required precision is achieved. The uncontaminated micro-Doppler spectrum can be obtained by using Short-time Fourier Transform on the compensated radar echo. Finally, the tomography results confirm the validity of the proposed algorithm. Simulation results demonstrate that the micro-Doppler signature can be estimated accurately, and the scattering centers can be extracted clearly in tomography results. When SNR of the radar signal is 20dB, the accuracy of the estimated micro-Doppler frequency reaches up to 90.40%. Moreover, the tomography result shows that the scattering centers can be exact clearly with a low SNR of 4dB.

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7. References
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