Robust Stochastic Bayesian Games for Behavior Space Coverage

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Abstract

A key challenge in multi-agent systems is the design of intelligent agents solving real-world tasks in close interaction with other agents (e.g. humans), thereby being confronted with a variety of behavioral variations and limited knowledge about the true behaviors of observed agents. The practicability of existing works addressing this challenge is being limited due to using finite sets of hypothesis for behavior prediction, the lack of a hypothesis design process ensuring coverage over all behavioral variations and sample-inefficiency when modeling continuous behavioral variations. In this work, we present an approach to this challenge based on a new framework of Robust Stochastic Bayesian Games (RSBGs). An RSBG defines hypothesis sets by partitioning the physically feasible, continuous behavior space of the other agents. It combines the optimality criteria of the Robust Markov Decision Process (RMDP) and the Stochastic Bayesian Game (SBG) to exponentially reduce the sample complexity for planning with hypothesis sets defined over continuous behavior spaces. In an intersection crossing task with broad continuous behavioral variations, we find that our approach outperforms the state-of-the-art algorithms achieving the same performance as a planning algorithm with knowledge of the true behaviors of other agents.

1 Introduction

Autonomous agents must be able to solve complex, real-world tasks in close interaction with humans. In many tasks there remain only a few seconds of observations for the agent to adapt its plan to the behavior of the participating humans. Important examples include intersection crossing of an autonomous vehicle or robot navigation through dense pedestrian areas. Among the variety of options to model this partially-cooperative multi-agent problem [Albrecht and Stone, 2018], the Stochastic Bayesian Game (SBG) [Albrecht and Ramamoorthy, 2013] is particularly qualified: It uses a predefined finite set of behavior hypothesis for the other agents to adapt to the observed behavior of other agents during the interaction process [Albrecht and Stone, 2017; Stone et al., 2010]. Each hypothesis is commonly defined as probability distribution mapping observation histories to actions. The hypothesis set can either be learned from data of interaction histories [Barrett et al., 2013; Barrett and Stone, 2015] or defined by domain experts [Barrett et al., 2011; Ravula et al., 2019]. However, the following shortcomings exist with the SBG to deal with the prescribed problem: 1) It is unclear how to define the set of hypothesis to cover every physically feasible human behavior. Data-based methods often neglect edge-cases in human behavior as they may not be observed during the data recording process. Domain experts do not have any method at hand to design hypothesis sets covering the complete human behavior space. 2) The SBG is defined for a limited, finite number of hypothesis. However, a finite hypothesis set is unable to express the subtle, continuous variations inherent to human behavior.

To clarify the two shortcomings, we exemplarily define the hypothesis set for an autonomous vehicle having to cross an intersection. A domain expert could assume that a human has two intentions in this task with respect to other vehicles, "give way" or "take way". It would directly map these to a set of two corresponding behavioral hypothesis. This definition describes what may happen in the intersection leaving unclear how to further partition the hypothesis set to model how this physically happens. For instance, giving way can be realized at various distances to the other vehicle. One can suggest to learn a mapping from intents to physical realizations. However, edge-cases in behavior, e.g. emergency braking, are rarely recorded and may thus not be adequately represented in a learned hypothesis set. On the other hand, since human intents are not physically measurable, this impedes a definition of a ground truth label set. Learned mappings from intent models to physical behavior might thus be incorrect to a certain extent.

The goal of this work is to overcome these shortcomings. We present a design process for hypothesis sets achieving coverage over the physically feasible, continuous behavior space of other agents. The behavior space is defined such that it comprises all physically feasible behavioral variations and can straightforwardly be defined by a domain expert. To reduce the sample complexity when planning with hypothesis sets defined over a continuous space, we formulate
the Robust Stochastic Bayesian Game (RSBG). It integrates the worst-case optimality criterion of the RMDP [Nilim and El Ghaoui, 2005] into the Harsanyi-Bellman optimality equation [Albrecht, 2015] of the SBG. We present a variant of Monte Carlo Tree Search (MCTS) to solve the RSBG. Finally, in an intersection crossing task with broad behavioral variations of other agents, we find that our approach outperforms the SBG in the average number of successful trials and achieves the same performance as a planning algorithm with knowledge of the true behavior of other agents.

2 Related Work
In this section, we discuss methods of hypothesis definition for the SBG. Next, we present the RMDP and its link to our research.

2.1 Hypothesizing Behaviors
Previous works frequently use small hypothesis sets in simple domains defined by domain experts [Stone et al., 2010; Albrecht and Ramamoorthy, 2013]. Discrete sets of behavior hypothesis are also frequently employed in robotics with intention-based agent models [Bai et al., 2015; Tamura et al., 2012; Sadigh et al., 2016]. As previously discussed, we consider discrete hypothesis sets as inadequate to cover all behavioral variations emerging in real-world tasks.

Integrating continuity into behavior hypothesis can be broadly categorized into approaches using a parameterized set of hypothesis or approaches learning a hypothesis set on the fly during task completion. Methods in the former category either build a hypothesis set by sampling hypothesis out of a parameterized hypothesis space [Southey et al., 2005] or adapt online the parameters of a predefined set of hypothesis [Hindriks and Tykhonov, 2008; Albrecht and Stone, 2017]. However, such methods only consider a single parameter set for each hypothesis and do not model types which cover a certain part of the parameter space. For instance, instead of modeling the preferred distance of a specific agent to other vehicles in an intersection as fixed, single parameter, it should be defined as varying slightly over time to express the subtle continuous behavioral variations in human behavior. With Q-learning [Barrett and Stone, 2015] or decision trees [Barrett et al., 2011], the hypothesis set can be adapted on the fly avoiding the definition of a continuous hypothesis model. However, online adaptation of the hypothesis set is impractical when the task is characterized by short interaction times as considered in this work.

In addition to the mentioned shortcomings, all of the above works do not specify a hypothesis design process to achieve coverage over all possible agent types. In our work, a hypothesis set partitions a behavioral space, defined by a domain expert. This process may be a potential solution to this open question.

2.2 Robustness-Based Optimality
The robustness of a plan or policy to continuous modeling errors has long been studied in the control and reinforcement learning community [Bagnell et al., 2001; Nilim and El Ghaoui, 2005; Li et al., 2019; Lim et al., 2013]. The Robust Markov Decision Process (RMDP) framework searches for a solution which is optimal under the worst-case parameter realizations of a (possibly continuous) set of parameters of the transition function, denoted uncertainty set. The main challenge with the robustness criterion is finding an uncertainty set which avoids overly conservative policies [Derman et al., 2019; Petrik and Russell, 2019].

Combinations of robust optimization and Bayesian decision making have been investigated in reinforcement learning [Derman et al., 2019] and game theory [Aghassi and Bertsimas, 2006]. The latter approach, denoted Robust game theory, applies the worst-case operation over the type space to omit dependencies on posterior type-beliefs in the expected value calculation. In contrast to their work, we split the continuous parameter space into multiple uncertainty sets and apply the worst-case operation over the parameter space of each type. The outcomes are then weighted with the posterior belief of each type. This method allows to cover a continuous parameter space and to control the conservativeness of the policy via the number of defined types.

3 Preliminaries
We propose a mathematical definition of behavior spaces and present background on the SBG and RMDP.

3.1 Behavior Spaces
We consider a multi-agent environment with N interacting agents. The process starts at time $t = 0$. At time step $t$, each agent $j$ observes the joint environment state $o^t = (o^t_1, o^t_2, \ldots, o^t_N)$ and chooses an action $a^t_j$ from a continuous action space $A_j$. The environment state $o^t$ describes the current physical properties, e.g. position, velocity, etc. We assume deterministic transitions to the next environment state, $o^{t+1} = T(o^t, a^t)$ based on the agents’ joint action $a^t \in A = \times A_j$ with joint action space $A$. This process continues until some terminal criterion is satisfied.

An agent chooses an action $a^t_j$ according to its policy $a^t_j \sim \pi_j(a^t_j|H^t_j, t^t_j)$. The policy depends on the observation action history up to time $t$, $H^t_j = (o^0, o^1, o^2, \ldots, o^t)$ and a time-dependent intention state $t^t_j$. The intention state may encode long- or short-term abstract goals or a more precise plan. We leave the exact model and dynamics of the intention state open.

We control a single agent, $i$, which reasons about the behavior of the other agents $j$. We assume that $i$ knows the action space and can observe past actions of the other agents. The true policy $\pi_j$ and any intent information of the other agent are unknown to $i$. However, we assume for a specific task there exists a single hypothetical policy

$$\pi^*: \mathcal{H}_0 \times \mathcal{B}_j \rightarrow A_j$$

with $b^t_j \in \mathcal{B}_j$ being agent’s $j$ behavior state at time $t$, $\mathcal{B}_j \subset \mathbb{R}^{N_0}$ its behavior space of dimension $N_0$ and $\mathcal{H}_0$, the space of all action observation histories. The hypothetical policy is defined such that a behavior state $b^t_j$ is a physically interpretable quantity describing $j$’s behavior at the time point of interaction. Agent $j$ covers its behavior space $\mathcal{B}_j$ by sampling its behavior state $b^t_j$ uniformly from $\mathcal{B}_j$ in each
time step, \( b_j^t \sim U(B_j) \) before choosing an action according to \( \pi^* \). In our model, solely \( B_j \) depends on the intention state, whereas the policy is independent. The causal diagram in Fig. 1 illustrates the relations between the random variables in our model.

The other agents’ behavior spaces \( B_j^t \) and their current behavior state \( b_j^t \) are not observable. However, using the property of physical interpretability of \( b_j^t \), an expert can define a full behavior space \( B \), comprising the individual behavior spaces \( B_j \) (\( B_j \subset B \)), by looking at the physically realistic situations. For instance, it is straightforward to define the physical boundaries of a behavior state modeling the desired gap between agent \( j \) and \( i \) at the time point of crossing an intersection with the one-dimensional behavior space \( B = \{ b | b \in [-d_{\max}, d_{\max}] \} \) where \( d_{\max} \) is the maximum sensor range.

In the remainder of this paper, we design a decision model enabling sample-efficient planning for agent \( i \) based on the hypothetical policy \( \pi^* \) and hypothesis sets defined over the full behavior space \( B \).

### 3.2 Harsanyi-Bellman Ad Hoc Algorithm

The type-based approach [Albrecht et al., 2016] uses a predefined set of behavior types \( \theta_k \in \Theta \) and hypothetical behavior policies \( a_j^t \sim \pi_{\theta_k}(a_j^t|H_j^t) \) for the other agents \( j \). Given the action-observation history of an agent one can track a posterior belief \( \Pr(\theta_j|H_j^t) \sim L(H_j^t|\theta_j^k)P(\theta_j) \) over hypothesized types over time with \( P(\theta_j) \) being the prior of a type. Depending on the calculation of the likelihood \( L(\cdot) \), one obtains either a product or sum posterior.

In the remainder of this paper, an index \(-i\) denotes all agents except \( i \), giving for the joint action \( a=a_{i,-i} \) and the joint type space of other agents \( \Theta_{-i}^t = \times_{j \neq i} \Theta \). The Harsanyi Bellman Ad Hoc (HBA) algorithm [Albrecht and Ramamoorthy, 2013] plans an optimal action for agent \( i \) according to the optimality criterion \( a_j^t \sim \text{argmax}_{a_j} E_{o^i}(H_j^t) \), where \( E_{o^i}(H_j^t) = \sum_{\theta_j \in \Theta_{-i}} \Pr(\theta_j|H_j^t) \sum_{a_j \in A_{-j}} Q_{o^i}^{\pi_i}(H_j^t) \prod_{j \neq i} \pi_{\theta_j}(H_j^t, a_j) \) is the expected cumulative reward for agent \( i \) taking action \( a_j \) in state \( o \) and history \( H_j^t \). The Bellman part of HBA is \( Q_{o^i}^{\pi_i}(H_j^t) = r(o, a) + \gamma \max_{a_j \in A_j} E_{o^j}^{\pi_j}(\langle H_j^t, a, o' \rangle) \) and defines the expected cumulative future reward of agent \( i \) when joint action \( a \) is executed in observation state \( o \) after history \( H_j^t \). Future rewards are discounted by \( \gamma \). Monte Carlo Tree Search (MCTS) can be used to find approximate solutions to this equation [Barrett et al., 2013].

### 3.3 Adversarial Reasoning

A Robust Markov Decision Process (RMDP) models uncertainty about the parameters of the transition function \( p \) in an Markov Decision Process (MDP) [Nilim and El Ghaoui, 2005]. Its optimality criterion \( a_j^t \sim \text{argmax}_{a_j} Q_o^{\pi_j}(\langle H_j^t, a_j, o' \rangle) \) can be seen as two-agent stochastic game where an adversary tries to minimize the expected cumulative future reward of the controlled agent by picking the transition function \( p \) inducing the worst-case outcome. The robust Bellman equation [Tamar et al., 2014] is defined as \( Q_o^\pi = r(o, a) + \gamma \max_{a_j \in A_j} \inf_{p \in P} [Q_o^{\pi_j}|o, a] \).

In the multi-agent case, with limited knowledge about the policies of other agents, we apply the worst-case assumption over other agents’ actions to get the robust Bellman equation \( Q_o^\pi = r(o, a) + \gamma \max_{a_j \in A_j} \min_{a_i \in A_i} Q_o^{\pi_j, a_i} \) with minimax learning objective [Li et al., 2019].

### 4 Method

In this section, we first present a design process for hypothesis sets to achieve behavior space coverage based on our environment model from sec. 3.1 Next, we discuss the RSBG and our variant of MCTS to enable sample-efficient planning with our hypothesis definition.

#### 4.1 Hypothesis Sets for Behavior Space Coverage

The standard type-based method tries to define each hypothetical type \( \theta_j^k \) such that it can closely match a single unknown policy \( \pi_j \) of another agent \( j \). In contrast, we define a collection of hypothesis each covering a certain part of the continuous behavior space \( B \). Thus, multiple hypothesis equally participate in representing an unknown policy \( \pi_j \).

Specifically, we split the full behavior space \( B \) into \( K \) non-overlapping subspaces \( B = B_1 \times B_2 \times \ldots \times B_K \) to form \( K \) hypothesis \( \pi_{\theta_j^k} : H_o \times A^k \rightarrow [0, 1], k \in \{1, \ldots, K\}. \)

\( ^* \)Causal models define an interventional type of conditional distribution instead of the observational variant [Pearl, 2000]. We are interested in \( \pi(\cdot|H_o, \text{do}(b_j)) \) and not in \( \pi(\cdot|H_o, b_j) \). For the latter definition the joint distribution \( p(a_j^t, H_o, b_j^t) \) must exist which is not the case as intents cannot be measured.

\( ^1 \)As we consider deterministic joint transition functions, we can neglect the expectation over potential subsequent states \( s' \).
To define the hypothesis, we need a probability distribution over actions. We define this distribution in terms of the hypo-

thesis policy \( \pi^* \) and the subspaces \( B^k \). In sec. 3.1 we define that an agent covers its behavior space \( B_j \) by sampling a behavior state \( b_j \) from a uniform distribution in each time step \( t \). Therefore, we can use a uniform density over behavior states \( f(b) = \frac{1}{|B^t|} \) to define the hypothesis set (with \(| \cdot | \) measuring the volume of a space), and obtain

\[
\pi_\theta^t(a_j^t|H_o^t) = \Pr\{(b) \forall b \in B^k, \pi^t_{\text{real}}(b, H_o^t) = a_j^t\} \tag{5}
\]

with action space \( A^k = \{a|\forall b \in B^k, \pi^t_{\text{real}}(b, H_o^t) = a\} \). The action space \( A^k \) becomes continuous, since different behavior states typically imply different actions and we have \(|A^k| \approx |B^k|\) where \(| \cdot | \) is an abstract measure of how many samples sufficiently represent the underlying continuous space.

### 4.2 Robust Stochastic Bayesian Games

Finding an approximate solution to eq. 2 with MCTS becomes computationally demanding for a continuous space of joint actions \( A_j = \times_{k \in \Theta} A^k \).

To get further insight into the problem, we calculate the sample complexity of eq. 2 for our hypothesis definition: Using equal-sized partitions of \( B \), we get \(|A^k| \approx |B|/K \) and obtain for the size of the joint action space \( A_{-i} = \prod_{j \neq i} A^k \approx (|B|/K)^N \) with \( N = N-1 \) being the number of other agents. Sampling over a joint action space \( A_{-i} \) occurs for all combinations of types \( \theta_{-i} \in \Theta_{-i} \) with \(|\Theta_{-i}| = K^{N_{-i}} \) whereas \( \theta_{-i} \) is sampled once in each iteration [Barrett et al., 2013]. Since different joint actions can occur at each prediction step time \( t \), this introduces an additional exponent and we get a sample complexity \( O(|\Theta_{-i}| \cdot |A_{-i}|^t) = O_{SBG}(|B|^{N_{-i}} K^{N_{-i}})^t \) for solving eq. 3 with MCTS.

By increasing \( K \), we can reduce the sample complexity. Yet, it is mainly dominated by the non-controllable variables and exponentially depends on \( t \) and \( N \) over the sample size of the behavior space \(|B|\).

To overcome this problem, we propose a different optimality criterion achieving reduced sample complexity. We combine the optimality criteria of the RMDP defined in eq. 4 and SBG defined in eq. 2. We let other agents act adversarially only within a hypothesis by defining the worst-case action over the respective hypothesis action space \( A^h \).

We call this decision model the Robust Stochastic Bayesian Game (RSBG). Specifically, it uses the formal definition of the SBG, but with \( E^{{>}\pi^t}(H_o^t) = \sum_{\theta_{-i} \in \Theta_{-i}} \Pr(\theta_{-i}|H_o^t) \min_{a_{-i} \in A_{-i}} Q_{{>}\pi^t}(H_o^t) \tag{6} \)

and eq. 3 remaining unchanged. In the next section, we show that with this criterion, we can reduce the sample complexity exponentially compared to the SBG for planning over continuous behavior spaces.

### 4.3 Monte Carlo Tree Search for the RSBG

Planning algorithms incorporating posterior beliefs over types or transition functions are commonly based on variants of MCTS [Guez et al., 2012] [Barrett et al., 2013]. We extend the Bayes-adaptive Monte Carlo Planning algorithm [Guez et al., 2012] to the SBG: At the beginning of each search iteration, we sample a type for each of the other agents \( j \) from the posterior belief over types \( \theta_j^t \sim \Pr(\theta_j^t|H_o^t) \) and use it in expansion and rollout steps.

To solve the RSBG, we implement the minimum operation in eq. 6 over \( A_{-i} \) sample-efficiently by letting each other agent \( j \) subjectively choose a worst-case action at history node \((H_o)\) within the hypothesis action space \( A^h \).

For this, during back-propagation steps, we maintain expected action-values with respect to agent \( i \)'s reward function, \( Q_i((H_o), \theta_j^t, a_j) \) separately for each hypothesis and other agent \( j \). We argue that in tasks with short interaction times a joint action of other agents, consisting of the subjective worst-case actions, is close to their global worst-case action. Such a decoupled action selection results in a sample complexity for the minimum operation equal to the size of only the hypothesis action space \( |A^h| \). We then get \( O(|\Theta_{-i}| \cdot |A_{-i}|^t) = O_{RSBG}(|B|^{N_{-i}} K^{N_{-i}})^t \) for the SBG.

Specifically, we select actions for each agent \( j \) with the function ACTIONOTHERAGENT in Algorithm 1 called once for each other agent \( \neq i \) in the expansion step. Line 8 implements the minimum operation of eq. 6: returning the worst-case action with respect to agent \( i \) among the set of previously expanded actions \( A_j((H_o^t), \theta_j^t) \) from node \((H_o^t)\) under type \( \theta_j^t \). We propose hypothesis-based progressive widening [Couétoix et al., 2011] in Lines 2-6: Depending on the number of expanded actions \( A_j((H_o^t), \theta_j^t) \) and the node visit count \( N_i((H_o^t), \theta_j^t) \) under hypothesis \( \theta_j^t \), we sample a new action from the hypothesis. This approach ensures sufficient exploration of \( A^h \) to discover the subjective worst case action while guaranteeing a sufficient depth of the search tree. During roll-out, we only use Line 3 to sample actions for each other agent \( j \) according to their currently sampled types \( \theta_j^t \). For the controlled agent \( i \), action selection during expansion and roll-out uses the standard UCB formula [Auer, 2002].

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1BAMCP converges for discrete action spaces. In continuous action spaces, it may only find a QMDP policy without information gathering behavior [Sunberg and Kochenderfer, 2017]. We neglect this deficiency since it affects both SBG and RSBG equally.
Algorithm 2: Hypothetical behavior policy for intersection crossing task

1: Output: \( a_j^t = \pi^*(H_o, b_j^t) \)
2: \( \text{GAPERROR} = x_j^t + a_j^{t-1} - x_j^t - d_j^t \)
3: if \( d_j^t > 0 \) then
4:  if \( \text{GAPERROR} < 0 \) then
5:   return \( \max(\text{GAPERROR}, \text{MINVELOCITY}) \)
6:  else
7:   return \( \min(\text{GAPERROR}, \text{MAXVELOCITY}) \)
8: else
9:  return \( \max(\min(\text{GAPERROR}, \text{MAXVELOCITY}), a_j^{t-1}) \)

5 Experiment

We evaluate the proposed method in a multi-agent chain-environment simulating intersection crossing of multiple agents.

5.1 Domain Description

The environment is depicted in Fig. 2. Each of the \( N = 9 \) agents moves along its chain with current state \( o_j^t = x_j^t \) and initial state \( o_j^0 = 5 \). The transition model is \( o_j^{t+1} = (x_j^{t+1} + a_j^t) \). State and action space are continuous with \( o_j^t \in [0, 17] \), and \( a_j^t \in [-5, 5] \). The agents’ chains intersect at a common point \( x_{\text{intersect}} = 15 \) which each agent must cross to reach its goal point \( x_{\text{goal}} = 17 > x_{\text{intersect}} \). Two agents collide when they cross \( x_{\text{intersect}} \) at the same time point \( t \).

For the prescribed domain, we define a hypothetical policy \( \pi^*(H_o, b_j^t) \) using a 1-dimensional behavior state \( b_j^t = d_j^t \). It models the desired gap \( d_j^t \) of agent \( i \) to \( j \) with respect to the crossing point: \( x_j^t - x_{\text{intersect}} \geq x_j^t - x_{\text{intersect}} - d_j^t \). For positive \( d_j^t \), agent \( j \) aims to be ahead of agent \( i \). Parameters \( \text{MIN/MAXVELOCITY} = -5/5 \) define other agent’s maximum and minimum action values. The behavioral states of the other agents change randomly over time being sampled from uniform distributions \( d_j^t \leftarrow \text{SAMPLE}(U([d_{ij,j}, d_{ij,j}]) \) at each time step. In simulation, we want to model that \( \pi_j(a_j^t | H_o, b_j^t), i_j^t \) and \( B_j \) are unknown to agent \( i \). To avoid the definition of a simulation model based on intention states \( i^t \), we apply the hypothetical policy \( \pi^* \) also in simulation: We draw unknown boundaries of behavioral variations \( B_j = [d_{ij,j}, d_{ij,j}] \) uniformly from a simulated true behavior space \( B^* (B_j \subseteq B^*) \) for each agent and trial. This simulates clearly reasonable intents, such as “give way” (\( d_{ij,j} \gg 0, d_{ij,j} \gg 0 \)) and “take way” (\( d_{ij,j} \ll 0, d_{ij,j} \ll 0 \)) and vague intents changing over time \( i_j^t \neq i_j^{t+1} (d_{ij,j} < 0, d_{ij,j} > 0) \).

Algorithm 2 gives the implementation of the hypothetical policy \( \pi^*(H_o, b_j^t) \) realizing a desired gap \( d_j^t \). Line 2 calculates the difference between desired gap and current gap (GAPERROR) by predicting the position of agent \( i \) one time step ahead using its last action. If agent \( j \) aims to drive behind agent \( i \) (\( d_j^t > 0 \)), the agent chooses an action exactly the size of the GAPERROR limited by the maximum or minimum velocity. If agent \( j \) aims to drive ahead of agent \( i \) (\( d_j^t < 0 \)), the agent additionally avoids to decelerate again, when its last action was larger.

5.2 Planning Algorithms

Now, we take the role of a domain expert with knowledge of \( \pi^*(H_o, b_j^t) \) which must define the full behavioral space \( B \) by analyzing possible physical situations at the time point of interaction: If agent \( i \) is close to the crossing point (\( 10 < d_j^t < 15 \)), the desired gaps \( d_j^t \in B = [-10, 10] \) describe all possible behaviors of other agents with respect to agent \( i \) at the time point of interaction. We then use equal-sized partitions of \( B \) to define the hypothesis set for the RSBG planner following our methodology from sec. 4.1. We will study the influence of the parameter \( K \) in our experiments.

Based on the MCTS defined in sec. 4.3 and this hypothesis set, we define the baselines

- **SBG** replacing Line 8 in Algorithm 1 with random selection among \( A_j(H_o, b_j^t) \),
- **RMDP** using a single hypothesis equivalent to the full behavioral space, \( K = 1 \) and \( B^1 = B \),
- **MDP** using a single hypothesis as with RMDP and random action selection as with SBG and
- **SBGFullInfo/RSBGFullInfo** being equal to the SBG, respectively RSBG planners, but having access to the true behavior policies to apply these as hypothesis.

Planners RSBG and SBG use the sum posterior defined in [Albrecht et al., 2016] to track the posterior belief over hypothesis. It can deal with zero-probability actions which occur in our hypothesis definition. All planners use the reward function \( R(\cdot) = -1000 \cdot \text{COLLIDED} + 100 \cdot \text{GOAL REACHED} \) and a discrete action space \( a_i \in A_i = \{-1, 0, 1, 2\} \) for agent \( i \), and perform 10000 search iterations in each time step. Progressive widening parameters, \( k_0 = 4 \) and \( \alpha_0 = 0.25 \), discount factor \( \gamma = 0.9 \) and all other parameters are kept equal for all planners.

5.3 Results

In our experiment, we simulate the other agents \( j \) by sampling a new behavior parameter \( b_j^t \) at every time step from \( B_j \) and chose their actions with Algorithm 2, respectively. Agent \( i \) applies one of the planning algorithms to chose an action. Each planner must perform 200 trials. Fixing the random seeds for all sampling operations ensures equal conditions
for all planners. We measure the percentage of trials where the agent achieves the goal, collides, or exceeds a maximum number of time steps ($t_{\text{max}} > 50$). For successful trials, we calculate the average number of time steps to reach the goal.

Fig. 3 depicts these metrics for the different planners, for SBG and RSBG planners over increasing number of hypothesis $K$, and for the case where the true behavior space is symmetric $B^* = [-5, 5]$ (left) and unsymmetric $B^* = [-2.5, 5]$ (right). We leave out the percentage of maximum steps since the percentages sum up to one. In both settings, the RSBG planner achieves a significantly higher percentage of successful trials for $K \geq 8$ than the SBG planner. In the case of a symmetric true behavior space, for $K = 16$ and $K = 32$, RSBG achieves equal performance as SBGFullInfo knowing about the true behavior of other agents. The unsymmetric case is more demanding since other agents desire a closer gap to the controlled agent decreasing performance for all planners. The RSBG and SBG planners achieve nearly equal average of time steps than SBGFullInfo for larger $K$. In contrast, the RMDP and RSBGFullInfo planners, purely relying on the worst-case criterion, are overly conservative and mostly exceed the maximum number of allowed time steps. The RSBG planner shows no collisions in contrast to a percentage of collisions for the SBG planner, and larger percentages resulting with MDP and SBGFullInfo planners. The results demonstrate that RSBGs provide a meaningful compromise between conservative planning with RMDPs and riskier planning with SBGs and MDPs.

We calculate the ratio of sample complexities $O_{\text{SBG}}/O_{\text{RSBG}}$ to clarify the advantages of RSBGs for behavior space coverage in our experiment. We set $N = 9$, and the prediction time equal to the average number of time steps, $t \approx 20$, and get $O_{\text{SBG}}/O_{\text{RSBG}} = (|B|/K)^{160}$. Defining the required number of samples to cover the full behavior space $|B|$ is unclear, but there should be at least one sample for each hypothesis, giving $|B| \gg K$ and thus for fixed $K$, $O_{\text{SBG}} \gg O_{\text{RSBG}}$. Since both SBG and RSBG planners have the same number of iterations available, RSBG can achieve better performance due to lower sample complexity for same $K$. It seems that there is an optimal setting of $K=16$ for the RSBG planner. For larger $K$, the performance of RSBG decreases. Fig. 4 shows the normalized standard deviation of the posterior belief over agents, hypothesis and ten trials for different $K$ at initial time steps. With $K=16$, the normalized standard deviation stabilizes to the lowest value, indicating a more stable posterior belief. Larger variations in the posterior belief occur at $K=8$ or $K=16$. We assume that these instabilities counteract a reduction of sample complexity with increasing $K$, explaining the observed performance decline for $K>16$, but also the sudden performance increase from $K=8$ to $K=16$.

Overall, our results indicate that the RSBG decision model performs better than the existing alternatives for planning in continuous behavior spaces. It can plan sample-efficiently at a low number of hypothesis to avoid instabilities in the posterior belief at larger hypothesis sets.

### 6 Conclusion

This work proposes a novel prediction model for self-interested agents in multi-agent systems based on physically interpretable behavior spaces, and an accompanying hypothesis design process ensuring that a set of behavior hypothesis covers all physically realistic behavioral variations. We propose a novel decision-theoretic framework under this paradigm, the RSBG, combining RMDPs [Nilim and El Ghaoui, 2005] and SBGs [Albrecht and Ramamoorthy, 2013], and theoretically identify that, compared to SBGs, the sample complexity of RSBGs for planning with MCTS is exponentially reduced under our behavior space model. In an intersection crossing task, we empirically demonstrate that the RSBG planner outperforms the state-of-the-art planners.
by a large margin, achieving the same performance as a planner knowing of other agents’ true behavior.

In future, we plan to evaluate how the applicability of the hypothesis design process, and the RSBG planning performance and sample complexity behave in tasks with larger, multidimensional behavior spaces.

References

[Aghassi and Bertsimas, 2006] Michele Aghassi and Dimitris Bertsimas. Robust game theory. *Math. Program.*, 107(1-2):231–273, June 2006.

[Albrecht and Ramamoorthy, 2013] Stefan Albrecht and Subramanian Ramamoorthy. A Game-theoretic Model and Best-response Learning Method for Ad Hoc Coordination in Multiagent Systems. In *Proceedings of the 2013 International Conference on Autonomous Agents and Multi-Agent Systems*, AAMAS ’13, pages 1155–1156, St. Paul, MN, USA, 2013.

[Albrecht and Stone, 2017] Stefano V. Albrecht and Peter Stone. Reasoning about hypothetical agent behaviours and their parameters. In *Proceedings of the 16th Conference on Autonomous Agents and Multiagent Systems*, AAMAS ’17, pages 547–555, São Paulo, Brazil, 2017.

[Albrecht and Stone, 2018] Stefano V. Albrecht and Peter Stone. Autonomous Agents Modelling Other Agents: A Comprehensive Survey and Open Problems. *Artificial Intelligence*, 258:66–95, May 2018.

[Albrecht et al., 2016] Stefano V. Albrecht, Jacob W. Crandall, and Subramanian Ramamoorthy. Belief and Truth in Hypothesised Behaviours. *Artificial Intelligence*, 235:63–94, June 2016.

[Albrecht, 2015] Stefano Albrecht. *Utilising Policy Types for Effective Ad Hoc Coordination in Multiagent Systems*. PhD, The University of Edinburgh, November 2015.

[Auer, 2002] Peter Auer. Using Confidence Bounds for Exploitation-Exploration Trade-offs. *Journal of Machine Learning Research*, 3:397–422, January 2002.

[Bagnell et al., 2001] J. Andrew Bagnell, Andrew Y. Ng, and Je G. Schneider. Solving uncertain Markov decision processes. Technical report, 2001.

[Bai et al., 2015] Haoyu Bai, Shaojun Cai, Nan Ye, David Hsu, and Wee Sun Lee. Intention-aware online POMDP planning for autonomous driving in a crowd. In *Robotics and Automation (ICRA)*, 2015.

[Barrett and Stone, 2015] Samuel Barrett and Peter Stone. Cooperating with unknown teammates in complex domains: A robot soccer case study of ad hoc teamwork. In *Proceedings of the Twenty-Ninth AAAI Conference on Artificial Intelligence*, AAAI’15, pages 2010–2016, Austin, Texas, 2015.

[Barrett et al., 2011] Samuel Barrett, Peter Stone, and Sarit Kraus. Empirical Evaluation of Ad Hoc Teamwork in the Pursuit Domain. In *The 10th International Conference on Autonomous Agents and Multiagent Systems*, volume 2 of AAMAS ’11, pages 567–574, Taipei, Taiwan, 2011.

[Barrett et al., 2013] Samuel Barrett, Peter Stone, Sarit Kraus, and Avi Rosenfeld. Teamwork with limited knowledge of teammates. In *Proceedings of the Twenty-Seventh AAAI Conference on Artificial Intelligence*, AAAI’13, pages 102–108, Bellevue, Washington, 2013.

[Couëtoux et al., 2011] Adrien Couëtoux, Jean-Baptiste Hoock, Nataliya Sokolovska, Olivier Teytaud, and Nicolas Bonnard. Continuous Upper Confidence Trees. In Carlos A. Coello Coello, editor, *Learning and Intelligent Optimization*, pages 433–445, Berlin, Heidelberg, 2011.

[Derman et al., 2019] Esther Derman, Daniel J. Mankowitz, Timothy A. Mann, and Shie Mannor. A bayesian approach to robust reinforcement learning. In *Proceedings of the 35th Conference on Uncertainty in Artificial Intelligence (UAI)*, Tel Aviv, Israel, 2019.

[Guez et al., 2012] Arthur Guez, David Silver, and Peter Dayan. Efficient Bayes-adaptive Reinforcement Learning Using Sample-based Search. In *Proceedings of the 25th International Conference on Neural Information Processing Systems*, volume 1 of *NIPS’12*, pages 1025–1033, Lake Tahoe, Nevada, 2012.

[Hindriks and Tykhonov, 2008] Koen Hindriks and Dmytro Tykhonov. Opponent modelling in automated multi-issue negotiation using Bayesian learning. In *Proceedings of the 7th International Joint Conference on Autonomous Agents and Multiagent Systems*, volume 1 of AAMAS’08, May 2008.

[Li et al., 2019] Shihui Li, Yi Wu, Xinyue Cui, Honghua Dong, Fei Fang, and Stuart Russell. Robust multi-agent reinforcement learning via minimax deep deterministic policy gradient. *Proceedings of the AAAI Conference on Artificial Intelligence*, 33:4213–4220, July 2019.

[Lim et al., 2013] Shiau Hong Lim, Huan Xu, and Shie Mannor. Reinforcement Learning in Robust Markov Decision Processes. In C. J. C. Burges, L. Bottou, M. Welling, Z. Ghahramani, and K. Q. Weinberger, editors, *Advances in Neural Information Processing Systems 26*, pages 701–709, 2013.

[Nilim and El Ghaoui, 2005] Arnab Nilim and Laurent El Ghaoui. Robust control of markov decision processes with uncertain transition matrices. *Operations Research*, 53(5):780–798, 2005.

[Pearl, 2000] Judea Pearl. *Causality: Models, Reasoning, and Inference*. New York, NY, USA, 2000.

[Petrík and Russel, 2019] Marek Petrík and Reazul Hasan Russel. Beyond confidence regions: Tight bayesian ambiguity sets for robust MDPs. In H. Wallach, H. Larochelle, A. Beygelzimer, F. dAlché-Buc, E. Fox, and R. Garnett, editors, *Advances in Neural Information Processing Systems 32*, pages 7047–7056, 2019.

[Ravula et al., 2019] Manish Ravula, Shani Alkoby, and Peter Stone. Ad hoc teamwork with behavior switching agents. In *Proceedings of the Twenty-Eighth International Joint Conference on Artificial Intelligence, IJCAI-19*, July 2019.

[Sadigh et al., 2016] Dorsa Sadigh, S. Shankar Sastry, Sanjit A. Seshia, and Anca D. Dragan. Information gathering actions over human internal state. *2016 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, pages 66–73, 2016.

[Southey et al., 2005] Finnegan Southey, Michael Bowリング, Bryce Larson, Carmelo Piccione, Neil Burch, Darse Billings, and Chris Rayner. Bayes’ bluff: Opponent modelling in poker. In *In Proceedings of the 21st Annual Conference on Uncertainty in Artificial Intelligence (UAI)*, 2005.

[Stone et al., 2010] Peter Stone, Gal A. Kaminka, Sarit Kraus, and Jeffrey S. Rosenschein. Ad hoc autonomous agent teams: Collaboration without pre-coordination. In *Proceedings of the Twenty-Fourth AAAI Conference on Artificial Intelligence*, AAAI’10, pages 1504–1509, Atlanta, Georgia, 2010.
[Sunberg and Kochenderfer, 2017] Zachary Sunberg and Mykel J. Kochenderfer. Online Algorithms for POMDPs with Continuous State, Action, and Observation Spaces. In Twenty-Eighth International Conference on Automated Planning and Scheduling, 2017.

[Tamar et al., 2014] Aviv Tamar, Shie Mannor, and Huan Xu. Scaling Up Robust MDPs using Function Approximation. In Eric P. Xing and Tony Jebara, editors, Proceedings of the 31st International Conference on Machine Learning, volume 32 of Proceedings of Machine Learning Research, pages 181–189, Beijing, China, June 2014.

[Tamura et al., 2012] Yusuke Tamura, Phuoc Dai Le, Kentarou Hitomi, Naiwala P. Chandrasiri, Takashi Bando, Atsushi Yamashita, and Hajime Asama. Development of pedestrian behavior model taking account of intention. In 2012 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), 2012.