Non-convex layup optimization of the composite laminates with fast and robust FEA model and modified SPEA-II

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Abstract. The biggest design challenge regarding composite laminates is the selection of a stacking sequence such that the cost and/or weight of the laminate to be minimized subject to constraints like failure criteria. The layup optimization of the composite laminates involves a very complex multidimensional solution space which is usually non-exhaustively explored using heuristic computational methods like genetic/evolutionary algorithms. To ensure the convergence of the preferred method it is necessary to evaluate a lot of layup configurations during the optimization process which involves that the evaluation of a single configuration should be fast enough to keep the overall optimization time to an acceptable level. On the other hand the mechanical behaviour of the composite laminates is very complex and it is analyzed with expensive computational tools such as finite element analysis. In this paper we propose a computational efficient framework which combines a very fast and robust Matlab-based FEA model with a non-convex evolutionary algorithm (strength pareto evolutionary algorithm II). We speed up the convergence by modifying the standard algorithm to generate new potentially pareto-optimal individuals. Also, a detailed numerical example is presented in order to highlight the steep computational time improvement between our method and a convex genetic algorithm.

1. Introduction

In most of the real-world applications the composite laminates are built by stacking up unidirectional or bidirectional reinforced layers at different orientations and sometimes having different mechanical properties as in the case of hybrid composite laminates where multiple reinforcing materials can be used – i.e. carbon, Kevlar or glass fibers. The orthotropic particularity exhibited by the fiber reinforced layers is crucial when we bring into question the analysis of the mechanical behavior and failure mechanism of composite laminates.

Depends on the applications, sometimes it is desirable to use a composite laminate with a small cost and/or the material weight is kept at a certain level. Acceptable costs and weights are application dependent – for example in the aeronautics industry the material weight is a crucial characteristic. The proper laminate design is usually searched using a single/multi-objective optimization process with the purpose to find the configuration with the smallest cost and with the smallest number of layers able to carry out the loads for the application where it will be used. The searching space is very complex and multidimensional and it cannot be exhaustively explored due to the computational limitations. This is the reason why in the layup optimization are usually addressed various heuristic methods of which the most reliable are the genetic/evolutionary algorithms [1-8]. To ensure the convergence of the applied method during the optimization process a lot of layup configurations are evaluated. To keep the
overall optimization time to an acceptable level it becomes mandatory that the evaluation of a single configuration to be fast.

On the other hand, the mechanical behavior and the failure mechanism of the composite laminates are complex due to the orthotropic particularity and the analysis requires time consuming numerical tools from which the most popular is the finite element analysis (FEA) [9-15]. This is the reason why a lot of layup optimization studies are restricted to simple topologies and boundary conditions for which analytical closed form solutions can be derived [16-19]. Based on the observation that a lot of calculations can be reused in the layup optimization for all the explored configurations, some studies [20] propose very fast finite element models close to the lower bound in terms of execution time, which is given by the global linear system solving.

In this paper, we propose a computational efficient framework for the layup optimization of the composite laminates. The optimization objectives are the non-convex minimization of the cost and weight constrained by the material failure. Our method takes the advantage of the FEA accuracy by using a fast and robust Matlab-based model in conjunction to a state-of-the-art evolutionary heuristic – strength pareto evolutionary algorithm II (SPEA-II). To improve the convergence of the standard algorithm it was considered a modification which forces a new generation to contain potentially pareto-optimal individuals.

The results and the applicability of our proposed method are highlighted by a detailed numerical example at the end of the paper. Also, a computation time comparison between this framework and a standard convex optimization procedure reveals a difference of two orders of magnitudes in favor of our method.

2. Method description

In this section, we present a detailed description of the proposed method by splitting the discussion between the FEA model and the modified SPEA-II.

2.1. FEA model

A complete and detailed description can be found in [20]. In this section, we only review the basics of the FEA model:

- Mathematical model based on first order shear deformation theory (FSDT);
- The equations of motion resulting from the principle of virtual displacements;
- Static equilibrium case is considered for the layup optimization procedure;
- Topology (discretization) with linear and quadratic triangular elements.

In FSDT, the displacements $(\Delta_x, \Delta_y, \Delta_z)$ are assumed to have the following form:

\[
\begin{align*}
\Delta_x(x, y, z, t) &= \Delta^m_x(x, y, t) + z \cdot \phi_x(x, y, t) \\
\Delta_y(x, y, z, t) &= \Delta^m_y(x, y, t) + z \cdot \phi_y(x, y, t) \\
\Delta_z(x, y, z, t) &= \Delta^m_z(x, y, t)
\end{align*}
\]  

(1)

where $(\Delta^m_x, \Delta^m_y, \Delta^m_z, \phi_x, \phi_y)$ represents the displacements and rotations around $O_x$ and $O_y$ axes of the midplane points in the composite laminate.

The FSDT finite element model can be written in the compact form as:

\[
[K^e][\Delta^e] - [F^e] = \{0\}
\]  

(2)

where:

- $(K^e)$ represents the element stiffness matrix;
- $(\Delta^e)$ represents the displacements and rotations for the element nodes at the midplane. Also, $\Delta^e$ are the unknowns in the linear system (2);
- $(F^e)$ represents the force vector associated with the element and is computed by applying the Neumann boundary conditions – the loads which the laminate is constrained to carry out.
The expanded form of the system (2) is:

\[ \sum_{\beta=1}^{5} \sum_{j=1}^{n} K_{ij}^{\alpha} \Delta_j^{\beta} - F_i^{\alpha} = 0, \quad (\alpha = 1, 5 \text{ and } i = 1, n) \]  

(3)

where:
- \((n)\) is the number of nodes of the element – 3 for linear and 6 for quadratic triangular elements;
- \(K_{ij}^{\alpha}\) represents the blocks of the elemental stiffness matrix:

\[
\begin{align*}
K_{1j}^{\alpha} & = \int_{\Omega^e} \left( \frac{\partial \psi_i}{\partial x} N_{1j}^{\alpha} + \frac{\partial \psi_i}{\partial y} N_{2j}^{\alpha} \right) dx \, dy \\
K_{ij}^{2\alpha} & = \int_{\Omega^e} \left( \frac{\partial \psi_i}{\partial x} N_{3j}^{\alpha} + \frac{\partial \psi_i}{\partial y} N_{4j}^{\alpha} \right) dx \, dy \\
K_{ij}^{3\alpha} & = \int_{\Omega^e} \left( \frac{\partial \psi_i}{\partial x} Q_{ij}^{\alpha} + \frac{\partial \psi_i}{\partial y} Q_{2j}^{\alpha} \right) dx \, dy \\
K_{ij}^{4\alpha} & = \int_{\Omega^e} \left( \frac{\partial \psi_i}{\partial x} M_{ij}^{\alpha} + \frac{\partial \psi_i}{\partial y} M_{2j}^{\alpha} + \psi_i Q_{ij}^{\alpha} \right) dx \, dy \\
K_{ij}^{5\alpha} & = \int_{\Omega^e} \left( \frac{\partial \psi_i}{\partial x} M_{6j}^{\alpha} + \frac{\partial \psi_i}{\partial y} M_{2j}^{\alpha} + \psi_i Q_{ij}^{\alpha} \right) dx \, dy
\end{align*}
\]  

(4)

where:
- \((\psi_i)\) represents the Lagrange shape functions;
- \((N_{ij}^{\alpha})\) are defined as:

\[
\begin{align*}
N_{1j}^{1} & = A_{11} \frac{\partial \psi_j}{\partial x} + A_{16} \frac{\partial \psi_j}{\partial y}, \quad N_{1j}^{2} = A_{16} \frac{\partial \psi_j}{\partial x} + A_{12} \frac{\partial \psi_j}{\partial y} \\
N_{1j}^{4} & = B_{11} \frac{\partial \psi_j}{\partial x} + B_{16} \frac{\partial \psi_j}{\partial y}, \quad N_{1j}^{5} = B_{16} \frac{\partial \psi_j}{\partial x} + B_{12} \frac{\partial \psi_j}{\partial y} \\
N_{2j}^{1} & = A_{12} \frac{\partial \psi_j}{\partial x} + A_{26} \frac{\partial \psi_j}{\partial y}, \quad N_{2j}^{2} = A_{26} \frac{\partial \psi_j}{\partial x} + A_{22} \frac{\partial \psi_j}{\partial y} \\
N_{2j}^{4} & = B_{12} \frac{\partial \psi_j}{\partial x} + B_{26} \frac{\partial \psi_j}{\partial y}, \quad N_{2j}^{5} = B_{26} \frac{\partial \psi_j}{\partial x} + B_{22} \frac{\partial \psi_j}{\partial y} \\
N_{6j}^{1} & = A_{16} \frac{\partial \psi_j}{\partial x} + A_{66} \frac{\partial \psi_j}{\partial y}, \quad N_{6j}^{2} = A_{66} \frac{\partial \psi_j}{\partial x} + A_{26} \frac{\partial \psi_j}{\partial y} \\
N_{6j}^{4} & = B_{16} \frac{\partial \psi_j}{\partial x} + B_{66} \frac{\partial \psi_j}{\partial y}, \quad N_{6j}^{5} = B_{66} \frac{\partial \psi_j}{\partial x} + B_{26} \frac{\partial \psi_j}{\partial y}
\end{align*}
\]  

(5)

- \((M_{ij}^{\alpha})\) are defined as:

\[
\begin{align*}
M_{1j}^{1} & = B_{11} \frac{\partial \psi_j}{\partial x} + B_{16} \frac{\partial \psi_j}{\partial y}, \quad M_{1j}^{2} = B_{16} \frac{\partial \psi_j}{\partial x} + B_{12} \frac{\partial \psi_j}{\partial y} \\
M_{1j}^{4} & = D_{11} \frac{\partial \psi_j}{\partial x} + D_{16} \frac{\partial \psi_j}{\partial y}, \quad M_{1j}^{5} = D_{16} \frac{\partial \psi_j}{\partial x} + D_{12} \frac{\partial \psi_j}{\partial y} \\
M_{2j}^{1} & = B_{12} \frac{\partial \psi_j}{\partial x} + B_{26} \frac{\partial \psi_j}{\partial y}, \quad M_{2j}^{2} = B_{26} \frac{\partial \psi_j}{\partial x} + B_{22} \frac{\partial \psi_j}{\partial y}
\end{align*}
\]  

(6)
where \( \psi_j \) denotes the components of the extensional stiffness matrix, bending stiffness matrix and bending-extensional coupling stiffness matrix, respectively.

The elemental force vector is computed using:

\[
\begin{align*}
F_i^1 &= \int_{\Gamma_0} \psi_i N_n \, ds \\
F_i^2 &= \int_{\Gamma_0} \psi_i N_{ns} \, ds \\
F_i^3 &= \int_{\Gamma_0} \psi_i Q_n \, ds + \int_{\Gamma_{\alpha \beta}} \psi_i \, q \, dx \, dy \\
F_i^4 &= \int_{\Gamma_0} \psi_i M_n \, ds \\
F_i^5 &= \int_{\Gamma_0} \psi_i M_{ns} \, ds
\end{align*}
\]

where \( (N_n, N_{ns}, M_n, M_{ns}, Q_n, q) \) represent the corresponding edge normal and tangential forces and moments, transverse force and transverse distributed load, respectively.

After global linear system assembling and solving the solutions for \( (\Delta_{x}^m, \Delta_{y}^m, \Delta_{z}^m, \phi_x, \phi_y) \) are obtained at each node at the laminate midplane. Using the interpolation functions, equation (1), von Karman strain-displacements and Hooke’s law it can be computed the displacements, strains, and stresses at any point from the entire laminate. The solutions from the laminate midplane can be extrapolated to the entire laminate using FSDT assumptions.

We are using this FEA model to quantify how close to failure are the configurations generated during the layup optimization procedure. In this respect, we define \( (\lambda) \) to be the failure factor which can be calculated using various failure criteria such as maximum strain, maximum stress, Tsai-Hill, or Tsai-Wu.

For the maximum strain criteria \( \lambda \) is computed as:

\[
\lambda = \max_i \lambda_i^t
\]  

Where \( \lambda_i^t \) indicates the maximum failure factor at layer \((i)\) and is computed as follow:

\[
\lambda_i^t = \max_j \lambda_i^{t,n}
\]  

Where \( \lambda_i^{t,n} \) indicates the value of the failure factor at layer \((i)\) and node \((j)\).
In the case of maximum strain criteria, the laminate failure occurs when the value of $\lambda$ is bigger than 1.

2.2 Modified SPEA-II

Due to its huge complexity, the solution space of the layup optimization problem cannot be explored by a brute force algorithm because of the computational limitations. Instead, heuristic methods are successfully addressed for solution space exploration from which the most reliable are genetic/evolutionary algorithms.

One of the most stable multi-objective algorithms which provides particularly good performance is strength pareto evolutionary algorithm II (SPEA-II) [21-22]. The algorithm makes use of an external set named archive with a fixed cardinality to retain the non-dominated individuals. The concept of dominance arises in the multi-objective optimization problems where there exist some conflicting objectives in the sense that optimizing one objective usually involve weaken other objectives. An individual dominates another if it is no worse regarding all the considered objectives and there exist at least one objective where it outperforms. In case of layup optimization there are two conflicting objectives one representing the weight of the material ($f^W$) and another representing the material cost ($f^C$). We say that individual $i$ dominates individual $j$ (individual $j$ is dominated by individual $i$) if $(f^W_i < f^W_j$ and $f^C_i < f^C_j$) or $(f^W_i = f^W_j$ and $f^C_i < f^C_j$) or $(f^W_i < f^W_j$ and $f^C_i = f^C_j$). Instead, neither individual dominates the other if $(f^W_i < f^W_j$ and $f^C_i > f^C_j$).

To integrate the material failure constraint, we modified the basic concept of pure-objectives dominance described above by adding the notion of feasibility. One individual is feasible if it does not violate the failure criteria. In case of maximum strain criteria, feasible individuals have the value of failure factor $\lambda < 1$. We define the constrained dominance as follow, and we say that individual $i$ dominates individual $j$ if:

- Individual $i$ is feasible and individual $j$ is not feasible: ($\lambda_i < 1$ and $\lambda_j \geq 1$);
- Both individuals are unfeasible ($\lambda_i \geq 1$ and $\lambda_j \geq 1$) but individual $i$ has less constrain violation than individual $j$ ($\lambda_i < \lambda_j$);
- Both individuals are feasible and individual $i$ dominates individual $j$ in the pure-objectives sense.

We denote that individual $i$ dominates individual $j$ as $i > j$.

Next, we describe the algorithm adapted to the layup optimization problem.

Input:
- $|P|$ - population P size
- $|A|$ - archive A size
- $t_{\text{max}}$ - maximum number of generations

Output:
- $A_{\text{pareto}}$ - pareto front

1. $t = 0$ - Generate random initial population $P_t$ and initial archive $A_t$

2. Compute the fitness value for $P_t$ and $A_t$

   2.1 Compute the strength $s(i)$ for each of individuals in the combined population $P_t + A_t$ as the number of individuals it dominates:
      $$s(i) = |\{j | j \in P_t + A_t \text{ and } i > j\}|$$

   2.2 Compute the raw fitness $r(i)$ for each of individuals in the combined population $P_t + A_t$:
      $$r(i) = \sum_{j \in P_t + A_t | j > i} s(j)$$

   2.3 Compute the density $\rho(i)$ for each of individuals in the combined population $P_t + A_t$:
      $$\rho(i) = (\alpha_i^R + 2)^{-1}$$
where \( k = \sqrt{|P| + |A|} \) and \( a_i^k \) is the distance of individual \( i \) from the \( k \)-th nearest neighbor in the normalized objective space.

Distance \( d(i,j) \) between two individuals \( i \) and \( j \) is defined as:

\[
d(i,j) = \sqrt{ \left( \frac{f_i^w - f_j^w}{f^w_{\max} - f^w_{\min}} \right)^2 + \left( \frac{f_i^c - f_j^c}{f^c_{\max} - f^c_{\min}} \right)^2 }
\]

where \( f^w_{\max}, f^w_{\min}, f^c_{\max}, f^c_{\min} \) are the updated upper and lower bound for the weight and cost objectives, respectively.

2.4 Compute the fitness \( F(i) \) for each of individuals in the combined population \( P_t + A_t \):

\[
F(i) = r(i) + \rho(i)
\]

3. Complete the archive \( A_{t+1} \) with all non-dominated individuals (with fitness values smaller than 1) from \( P_t + A_t \). Ensure that the size of the new archive has the predefined value \( |A| \). Three scenarios are possible:

- **Scenario 1:** \( |A_{t+1}| = |A| \) - no modifications should be considered;
- **Scenario 2:** \( |A_{t+1}| < |A| \) - \( A_{t+1} \) is completed with the best (in terms of fitness \( F \)) \( |A| - |A_{t+1}| \) dominated individuals from \( P_t + A_t \);
- **Scenario 3:** \( |A_{t+1}| > |A| \) - \( A_{t+1} \) is truncated by iteratively eliminating \( |A_{t+1}| - |A| \) individuals. At each iteration, an individual \( i \) is eliminated if \( \forall j \in A_{t+1}: \forall i < j \).

where:

\[
i < j \Rightarrow \forall 0 < k < |A_{t+1}|: a_j^k = a_i^k \land \exists 0 < k < |A_{t+1}|: (\forall 0 < l < k: a_i^l < a_j^l)
\]

This truncation procedure ensures that individuals from objective space areas with a big density are eliminated.

4. If \( t = t_{\max} \) then the non-dominated individuals from archive are copied to the output \( A_{\text{pareto}} \).

5. Mating pool is filled with individuals from \( A_{t+1} \) by performing binary tournament selection.

6. Generate the new population \( P_{t+1} \) by applying genetic operators to the parents selected from the mating pool then increment the value of \( t \): \( t = t + 1 \).

We added a modification to step 6 by forcing a subset of predefined size of the new population individuals to be potential-pareto optimal. This means that this subset contains individuals which are not dominated by the current pareto front in the objective space - in the pure-objectives dominance sense.

This modification forces the algorithm to explore potential-pareto optimal areas of the solution space. If individuals from this subset are also feasible, they will be included in the current pareto front. This modification decreases the convergence time of the numerical example considered in the next section by 65%.

3. Numerical example

In this section, we present a detailed numerical example performed with the proposed framework. Also, we show the results in comparison with a convex optimization method proposed in [23]. We used the same encoding/decoding scheme, penalties, and genetic operators in order for the comparison to be more relevant.

3.1 Geometry, boundary conditions and materials

We simulate a scenario with a simply supported square plate illustrated in figure 1. The layup consists of a mixture between carbon fiber and glass fiber plies symmetrically distributed around the laminate.
midplane having discrete orientations from -90° to +90° with increments of 15°. We used a cost factor of 10 for carbon and 1 for E glass layers.

![Figure 1](image)

**Figure 1.** (a) geometry and boundary condition, (b) mesh.

The mechanical properties of the layers used in the layup optimization are presented in Table 1.

|               | $E_1$ (GPa) | $E_2$ (GPa) | $G_{12}$ (GPa) | $G_{23}$ (GPa) | $G_{13}$ (GPa) | $\theta_{12}$ (°) | $\theta_{23}$ (°) |
|---------------|-------------|-------------|----------------|----------------|----------------|------------------|------------------|
| Carbon        | 135         | 10          | 5              | 3              | 5              | 0.3              | 0.0222           |
| E glass       | 40          | 8           | 4              | 2.2            | 4              | 0.25             | 0.05             |
| $h$ (mm)      |             |             | $UTS^1_t$ (%)  | $UTS^1_c$ (%)  | $UTS^2_t$ (%)  | $UTS^2_c$ (%)    | $UTS^{12}$ (%)   |
| Carbon        | 0.5         | 1600        | 1.05e-03       | 0.85e-03       | 0.5e-03        | 2.5e-03          | 1.4e-03          |
| E glass       | 0.5         | 1900        | 2.5e-03        | 1.5e-03        | 0.35e-03       | 1.4e-03          | 1e-03            |

3.2 Encoding/decoding scheme

The population used in this example is triploid with fixed length chromosomes:

- One chromosome encoding the layers orientation from an orientation alphabet $O_a=\{0°, 15°, 30°, 45°, 60°, 75°, 90°\}$. Each gene from the orientation chromosome has an integer value between 0 and $|O_a|$ with 0 corresponding to empty plies. Values different from 0 associates a gene with the orientation found in the above alphabet at the index equal to the gene value. For example, a gene with a value of 3 is associated with an orientation of 30°. Even the chromosome length is fixed, the number of plies in a configuration varies due to empty plies encoding.

- One chromosome encoding the layers material from an material alphabet $M_a=\{\text{Carbon, E glass}\}$. The material chromosome has the same length and the same number of empty plies as the orientation chromosome. Also, the encoding procedure is similar with that described in the
orientation chromosome. For example, a gene with value of 2 is associated with *E glass* lamina.

- One chromosome encoding if the laminate has an odd or even number of plies. In this numerical example, the population consists of symmetrical laminates. For this reason, the individuals encode only half of the stacking sequence. This chromosome with only one gene from the alphabet $E_a = \{\text{odd, even}\}$ specify if the symmetrization should include or not the last gene from the orientations and materials chromosomes.

During the optimization procedure, all empty plies are grouped to the left side of orientations and materials chromosomes. The decoding procedure is straightforward with the only mention that to maintain a good balance in the laminate the orientation sign of identical laminas alternates - excepting $0^\circ$ and $90^\circ$ orientations. A decoding example is illustrated below:

**Code:**

| Orientations: | [0/0/0/1/1/2/1/2/1] |
| Materials:   | [0/0/0/2/1/1/2/2] |
| Even:        | 2 |

**Decode – stacking sequence:**

| Orientations: | [0°/15°/0°/−15°/90°/0°/90°/−15°/15°/0°] |
| Materials:    | [C / E / C / C / E / E / E / E / C / C / C / E] |

Where C and E represent carbon and E glass layers, respectively.

### 3.3 Genetic operators

**Crossover** – one-point crossover is performed with the crossover point forced in a non-empty ply position of at least one parent otherwise crossover operation will generate same individuals. Also, if necessary, all empty plies are grouped to the left side of the orientation and material chromosomes.

**Ply addition mutation** – if there are empty plies then add a random new ply to the right side of the orientations and materials chromosomes.

**Ply deletion mutation** – delete a random non-empty ply.

**Ply swap mutation** – swap two different random plies.

**Ply alteration** – change the values of orientation and material genes with different values from the alphabet.

**Odd/Even switch** – change the value of the chromosome encoding the odd/even number of plies.

### 3.4 Balance and continuity penalties

In order to include balance and continuity design recommendations we introduced penalties in the weight and cost objectives.

When an individual has an even number of plies with same orientation then the balance is induced by alternating orientation sign in the decoding scheme. However, if the number of plies with same orientation is odd then one ply remains unbalanced. We count all the unbalanced plies from an individual and penalize its objectives by adding an extra weight and cost proportional to the weight and cost of the unbalanced plies.

Also, we want to penalize individuals that have identical successive plies. Similarly, to unbalance penalty, we penalize these individuals by adding an extra weight and cost proportional to the weight and cost of identical successive plies.

### 3.5 Parameters and settings

The following parameters were used for the convex algorithm: population size – 50, crossover probability – 1, orientation alteration probability – 0.05, material alteration probability – 0.05, ply addition probability – 0.05, ply deletion probability – 0.1, ply swap probability – 0.75, odd/even switch probability – 0.05. The stopping limit was set to 60 generations without best individual improvement. *Variable elitist selection 1* [23] scheme was used with the number of top laminates from
the combined population which will be carried to the next generation set to $N_k=10$. Also, the penalization of unfeasible individuals is done according to [23]. The fitness of individuals is computed as a convex combination between weight and cost:

$$ F = \alpha f^w + (1 - \alpha) f^c, \alpha \in [0, 1] $$

In order to obtain the pareto front we carried out multiple optimization procedures varying the value of $\alpha$ from 0 to 1 in increments of 0.01.

The following parameters were used for the SPEA-II algorithm: population size – 50, archive size - 25, crossover probability – 1, orientation alteration probability – 0.05, material alteration probability – 0.05, ply addition probability – 0.05, ply deletion probability – 0.1, ply swap probability – 0.75, odd/even switch probability – 0.05. The stopping limit was set to 1000 generations.

### 3.6 Results

The optimization procedures were performed on a PC with i7-6700(3.2 GHz) CPU and 8 GB RAM (DDR4).

In figure 2 is shown the convergence of the modified SPEA-II to the pareto front obtained with the convex algorithm. The magenta circular marks represent the optimal individuals obtained with the convex algorithm by varying the value of $\alpha$ and the magenta line connecting these points represent the almost-pareto front associated with the convex algorithm. We used the phrase “almost-pareto” because the front also contains some dominated individuals which are kept in the representation to show the diversity of the convex algorithm. All the other graphical entities are associated with SPEA-II: circular marks (blue – feasible, red – infeasible) represent archive individuals and diamond marks (blue – feasible, red – infeasible) represent population individuals. The black line connecting the feasible archive individuals represent the current pareto front which contains only the non-dominated individuals.

Also, from figure 2 it can be easily observed that:
- All the optimal individuals found with the convex algorithm are also found with SPEA-II;
- With SPEA-II there are found some individuals which dominates optimal individuals from the convex algorithm;
- SPEA-II pareto front contains non-convex sections with individuals missed by the convex algorithm.

To highlight the importance of the pareto-optimal individuals from the design point of view let us consider the two selected areas (figure 3). Area 1 contains three pareto-optimal individuals with a very small weight difference but a very steep cost difference. It is likely that the preferred individual from this area is that having the smallest cost. On the other hand, area 2 contains four pareto-optimal individuals with a small cost difference but a big weight difference which means that probably the preferred individual is that with the smallest weight.

In terms of execution time there is a huge difference between the two algorithms: the entire convex optimization procedure had a duration of **148.5 h** while the duration of modified SPEA-II was only **2.8 h**. The biggest impact on the duration of both algorithms was due to the finite element analysis.

Without the modification which is forcing a subset of predefined size of the new population individuals to be potential-pareto optimal the convergence is achieved in about 2900 generations. This means that the improvement of the convergence speed of the modified SPEA-II is about 65% for this numerical example. In this simulation, the potential-pareto optimal subset has a size of half of the population size.

To further improve the execution time some other strategies can be easily integrated:
- Parallel computing;
- GPU computing;
- Keeping an archive with the layup configurations evaluated during the optimization process. When an offspring should be evaluated first it will be searched in this archive. If it’s found then the evaluation results saved in the archive will be used without a reevaluation with the FEA model.
Figure 2a. Modified SPEA-II convergence.
Figure 2b. Modified SPEA-II convergence.
Figure 2c. Modified SPEA-II convergence.
Figure 2d. Modified SPEA-II convergence.
Figure 3. Pareto front – design considerations.

4. Conclusions
In this paper, we proposed an accurate and time-efficient framework for the layup optimization of the composite laminates. Both the accuracy and convergence speed were achieved by the integration of a very fast and robust FEA model - specially designed for the layup optimization problem - and a modified version of SPEA-II. The performance of the proposed method was highlighted with a detailed numerical example. The optimization time was about 2.8 h in comparison with a convex algorithm with optimization duration of 148.5 h. The convergence speed was improved with 65% by a modification of the basic SPEA-II which is forcing a subset of predefined size of the new population individuals to be potential-pareto optimal. The obtained results are important because it demonstrates that the layup optimization procedure can run in a very reasonable time while taking the advantage of the accuracy of the finite element analysis.

5. References
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