Orientifold D-String in the Source of the Kerr Spinning Particle

Alexander Burinskii
Gravity Research Group, NSI Russian Academy of Sciences
B. Tulskaya 52, 115191 Moscow, Russia

The model of spinning particle, based on the Kerr-Newman solution with $|a| >> m$, is discussed. It is shown that the Kerr singular ring can be considered as a string with an orientifold world-sheet. Orientifold adds to the Kerr ring an extra peculiar point, the fixed point of the world-sheet parity operator $\Omega$. It is shown that the Kerr string represents a new type of the folded string solutions taking the form of the open D-string with joined ends which are in the circular light-like motion along the Kerr ring.

I. INTRODUCTION

The Kerr rotating black hole solution displays some remarkable relations to the spinning particles [1–9]. For parameters of elementary particles $|a| >> m$, and the black-hole horizons disappear, obtaining the source in the form of a closed singular ring of the Compton radius. In the model of the Kerr spinning particle [3] this ring was considered as a gravitational waveguide containing traveling electromagnetic (and fermionic) wave excitations. The assumption that the Kerr singular ring represents a closed relativistic string was advanced about thirty years ago, [4], which had confirmations on the level of evidences [6,10,11]. However, the attempts to show it explicitly ran into obstacle which was related with a very specific motion of the Kerr ring - the light-like sliding along itself. It could be described as a string containing the light-like modes of only one direction. However, the relativistic string equations do not admit such solutions.

In this note we resolve this problem showing that the Kerr ring is a string with orientifold structure, which represents a new type of the folded string solutions. The Kerr orientifold string takes the form of an open D-string with joined ends which propagate along the Kerr ring.

Our treatment is based on the previous papers [14,15] where the real and complex structures of the Kerr geometry were considered. For the reader convenience we describe briefly the necessary details of these structures.

II. THE STRUCTURE OF THE KERR SOURCE AND MICRIGEON WITH SPIN

We use the Kerr-Schild approach to the Kerr geometry [16], which is based on the Kerr-Schild form of metric

$$g_{\mu\nu} = \eta_{\mu\nu} + 2hk_\mu k_\nu,$$

where $\eta_{\mu\nu}$ is metric of auxiliary Minkowski space-time, $h = \frac{mr - \frac{e^2}{r^2+\alpha^2\cos^2\theta}}{r^2+\alpha^2}$, and $k_\mu$ is a twisting null field, which is tangent to the Kerr principal null congruence (PNC) and is determined by the form

$$k_\mu dx^\mu = dt + \frac{z}{r} dz + \frac{r}{r^2+a^2}(xdy-ydx) - \frac{a}{r^2+a^2}(ydx-xdy).$$

The form of the Kerr PNC is shown on Fig. 1. It follows from (1) that the field $k^\mu$ is null with respect to $\eta_{\mu\nu}$ as well as with respect to the full metric $g_{\mu\nu}$,

$$k^\mu k_\mu = k^\mu k^\nu g_{\mu\nu} = k^\mu k^\nu \eta_{\mu\nu}.\quad (3)$$

---

1. $a = J/m$ is the density of angular momentum $J$ per mass $m$. We use the units $c = \hbar = G = 1$, and signature $(-+++)$.

2. Note, that interest to the classical string solutions, and especially to the folded ones, was recently raised by the suggested gauge/string correspondence [12,13].

3. The rays of the Kerr PNC are twistors and the Kerr PNC is determined by the Kerr theorem as a quadric in projective twistor space [14].
FIG. 1. The Kerr singular ring and 3-D section of the Kerr principal null congruence. Singular ring is a branch line of space, and PNC propagates from “negative” sheet of the Kerr space to “positive” one, covering the space-time twice.

The metric is singular at the ring \( r = \cos \theta = 0 \), which is the focal region of the oblate spheroidal coordinate system \( r, \theta, \phi \).

The Kerr singular ring is the branch line of the Kerr space on two folds: positive sheet \((r > 0)\) and ‘negative’ one \((r < 0)\). Since for \(|a| >> m \) the horizons disappear, there appears the problem of the source of Kerr solution with the alternative: either to remove this twofoldedness or to give it a physical interpretation. The both approaches have paid attention, and it seems that the both are valid for different models. The most popular approach was connected with the truncation of the negative sheet of the Kerr space, which leads to the source in the form of a relativistically rotating disk [2] and to the class of the disk-like [5] or bag-like [9] models of the Kerr spinning particle.

Alternative way is to retain the negative sheet treating it as the sheet of advanced fields. In this case the source of spinning particle turns out to be the Kerr singular ring with the electromagnetic excitations in the form of traveling waves, which generate spin and mass of the particle. Model of this sort was suggested in 1974 as a model of ‘microgeon with spin’ [3]. Singular ring was considered as a waveguide providing a circular propagation of an electromagnetic or fermionic wave excitation. Twofoldedness of the Kerr geometry admits the integer and half integer excitations with \( n = 2\pi a/\lambda \) wave periods on the Kerr ring of radius \( a \), which turns out to be consistent with the corresponding values of the Kerr parameters \( m = J/a \).

The light-like structure of the Kerr ring world-sheet is seen from the analysis of the Kerr null congruence near the ring. The light-like rays of the Kerr PNC are tangent to the ring (see Fig.2).

FIG. 2. The (unoriented) surface \( \phi = \text{const.} \) formed by the light-like generators of the Kerr principal null congruence (PNC). The Kerr string (fat line) is tangent to this surface.

Formally it follows from the Eq. (2). Approaching the ring \((r \to 0)\) PNC takes the form \( \tilde{k} = k|_{r=\cos \theta=0} = dt - (xdy - ydx)/a = dt - ad\phi \), and the light-like vector field \( k_\mu \) is tangent to the world sheet of the Kerr ring. One
sees that the Kerr ring is sliding along itself with the speed of the light. 4

III. STRINGY NATURE OF THE KERR SOURCE

It was recognized long ago [4] that the Kerr singular ring can be considered in the Kerr spinning particle as a string with traveling waves. We recall here one of the most convincing evidences obtained by the analysis of the axidilatonic generalization of the Kerr solution (given by [17]) near the Kerr singular ring . It was shown [6] that the fields near the Kerr ring are very similar to the field around a heterotic string.

The limiting form of the Kerr-Sen metric near the ring is

\[ ds^2_a = e^{-2(\Phi - \Phi_0)}(dv^2 + du^2) + dy^2 - dt^2 + \left(2Mr/\Sigma_d\right)(dy - dt)^2, \tag{4} \]

and the gauge field in this limit is given by the vector potential

\[ A = 2Q(r/\Sigma_d)(dt - dy). \tag{5} \]

By introducing an electric charge per unit length of the Kerr ring \( q = 2(3/2)Q/(2\pi a) \) and a two-dimensional (two-valued) Green’s function \( G_a^{(2)} \) in the \((u, v)\) complex plane near the Kerr singularity

\[ G_a^{(2)} = 2\pi a/r/\Sigma \simeq \pi \left\{ \left[ a^2(u + iv)\right]^{1/2} + \left[ a^2(u - iv)\right]^{1/2} \right\}, \tag{6} \]

the dilaton factor may be represented as

\[ e^{-2(\Phi - \Phi_0)} = 1 + NG_a^{(2)}, \tag{7} \]

where \( N = r_ - /2\pi a. \)

Then the rescaled \( \sigma \)-model metric \( ds^2_{str} = e^{2(\Phi - \Phi_0)}ds^2_a \), used in string theory may be written in the form

\[ ds^2_{str} = (dv^2 + du^2) + \frac{1}{1 + NG_a^{(2)}}(dy^2 - dt^2) + \frac{2MG_a^{(2)}(2\pi a(1 + NG_a^{(2)}))^2(dy - dt)^2}{2\pi(1 + NG_a^{(2)})} \tag{8} \]

which is exactly the form of metric obtained by Sen for a field around a fundamental heterotic string [18]. The difference is only in the form of two-dimensional Green’s function \( G_a^{(2)} \), but it is very natural and caused by the twovaluedness of the fields near the Kerr singularity.

IV. THE KERR ORIENTIFOLD WORLD-SHEET

One can see that the world-sheet of the Kerr ring satisfies the bosonic string equations and constraints, however, there appear the problems with boundary conditions.

The general solution of the string wave equation \( (\frac{\partial^2}{\partial \tau^2} - \frac{\partial^2}{\partial \sigma^2})X^\mu = 0 \) can be represented as the sum of the ‘left’ and ‘right’ modes: \( X^\mu(\sigma, \tau) = X^\mu_R(\tau - \sigma) + X^\mu_L(\tau + \sigma) \), and the oscillator expansion is

\[ X^\mu_R(\tau - \sigma) = \frac{1}{2}[x^\mu + l^2p^\mu(\tau - \sigma)] + il \sum_{n \neq 0} \frac{1}{n} \alpha_n^\mu e^{-2in(\tau - \sigma)}, \tag{9} \]

\[ X^\mu_L(\tau + \sigma) = \frac{1}{2}[x^\mu + l^2p^\mu(\tau + \sigma)] + il \sum_{n \neq 0} \frac{1}{n} \tilde{\alpha}_n^\mu e^{-2in(\tau + \sigma)}, \tag{10} \]

It follows also from all the previous treatments of the Kerr source, for example in [2,3,5]. In spite of the fact that function \( h \) is singular at the ring, one can check that it is true also with the respect to the auxiliary Minkowski metric, since in the limit \( r \to 0, k^\mu k_\nu g_{\mu\nu} = k^\mu k_\nu \eta_{\mu\nu} \to 0. \)
where \( l = \sqrt{2a^2} = \frac{1}{\sqrt{T}} \), \( T \) is tension, \( x^\mu \) is position of center of mass, and \( p^\mu \) is momentum of string.

The string constraints \( \dot{X}_\mu \ddot{X}^\mu + X_\mu' X''^\mu = 0, \quad \dot{X}_\mu' X''^\mu = 0 \), are satisfied if the modes are light-like \((\dot{\tau}' \equiv \partial_\tau())\),

\[
(\partial_\tau X_{L(R)})(\partial_\tau X_{L(R)}) = 0. \tag{11}
\]

Setting \( 2\sigma = a\phi \) one can describe the light-like world-sheet of the Kerr ring (in the rest frame of the Kerr particle) by the surface

\[
X^\mu_L(t, \sigma) = x^\mu + \frac{1}{\pi T} \delta_\mu^0 p^0 (t + \sigma) + \frac{a}{2} [(m^\mu + im^n)e^{-i(\tau + \sigma)} + (m^\mu - im^n)e^{i(\tau + \sigma)}], \tag{12}
\]

where \( m^\mu \) and \( n^\mu \) are two space-like basis vectors lying in the plane of the Kerr ring. One can see, that

\[
X^\mu_L = \frac{1}{\pi T} \delta_\mu^0 p^0 + 2a[-m^\mu \sin 2(\tau + \sigma) + n^\mu \cos 2(\tau + \sigma)] \tag{13}
\]

will be a light-like vector if one sets \( p^0 = 2\pi a T \). It shows that the Kerr world-sheet could be described by modes of one (say ‘left’) null direction. The solution \( X(\tau, \sigma) = X_L(\tau + \sigma) \) satisfies the string wave equation and constraints, but there appears the problem with boundary conditions. The closed string boundary condition

\[
X^\mu(\tau, \sigma) = X^\mu(\tau, \sigma + \pi) \tag{14}
\]

will not be satisfied since the time component \( X^\mu_L(t, \sigma + \pi) \) acquires contribution from the second term in (12), which is usually compensated by this term from the ‘right’ mode. The familiar boundary conditions for the open strings follow from the condition of canceling of the surface term \(-T \int d\tau [X^\mu_L X^\mu |_{\sigma=\pi} \mp X^\mu_L X^\mu |_{\sigma=0}] \) in the string action [19], and are

\[
X'^\mu(\tau, 0) = X'^\mu(\tau, \pi) = 0, \tag{15}
\]

which also demand the both types of modes to form standing waves. However, this demand can be weakened to

\[
X'^\mu(\tau, 0) = X'^\mu(\tau, \pi). \tag{16}
\]

It seems that the light-like oriented string can contain traveling waves of only one direction if we assume that it is open, but has the joined ends. However, the ends \( \sigma = 0 \), and \( \sigma = \pi \) are not joined indeed.

These difficulties can be removed by the formation of the world-sheet orientifold.

It is well known [19] that the interval of an open string \( \sigma \in [0, \pi] \) can be formally extended to \([0, 2\pi]\), setting

\[
X_R(\sigma + \pi) = X_L(\sigma), \quad X_L(\sigma + \pi) = X_R(\sigma). \tag{17}
\]

By such an extension, the both types of modes, ‘right’ and ‘left’, will appear in our case since the ‘left’ modes will play the role of ‘right’ ones on the extended piece of interval. If the extension is completed by the changing of orientation on the extended piece, \( \sigma' = \pi - \sigma \), with a subsequent identification of \( \sigma \) and \( \sigma' \), then one obtains the closed string on the interval \([0, 2\pi]\) which is folded and takes the form of the initial open string.

Formally, the world-sheet orientifold represents a doubling of the world-sheet with the orientation reversal on the second sheet. The fundamental domain \([0, \pi]\) is extended to \( \Sigma = [0, 2\pi] \) with formation of folds at the ends of the interval \([0, \pi] \).

![FIG. 3. Formation of orientifold: a) the initial string interval, b) extension of the interval and formation of the both side movers, c) formation of the orientifold.](image-url)
The parity operator $\Omega : \sigma \rightarrow 2\pi - \sigma$ changes the layers of the world-sheet. The ends of the fundamental domain $\sigma = 0$ and $\sigma = \pi$ are the fixed points of $\Omega$. Applying the operator $\Omega$ to (17), one sees that $\Omega$ acts on the string by changing the right and left modes, and as a result the solution $X = X_R + X_L$ turns out to be parity invariant. Orientifold is the factor space $\Sigma/\Omega$ setting the equivalence of the original interval and the extended piece.

Orientifolding the Kerr light-like world-sheet $X = X_L(\tau + \sigma)$, containing only the left modes on the fundamental interval $[0, \pi]$, one obtains the open string which can be represented as closed one on the covering space, but folded taking the form of open string. Solution $X(\tau, \sigma) = X_L(\tau + \sigma) + X_R(\tau - \sigma)$ contains the both types of modes, and the discussed above linear in $\sigma$ term cancels. As a result the ends of the open string turns out to be joined.

The resulting orientifold string retains the form of the Kerr light-like string which is covered twice in opposite directions. The joined ends of the string are in the light-like circular motion along the string.

V. SOME RELATIONS TO D-BRANES AND SUPERSTRING THEORY

By orientifolding the Kerr string, it acquires simultaneously the properties of the open and closed strings. Being formally closed, it forms a configuration with open ends joined to the fixed points of the operator $\Omega$. With the respect to the initial model of the Kerr geon, the Kerr spinning particle acquires some new features connected with the light-like orientifold fixed points, which are the peculiar points of the model. The orientifold is closely related to D-branes which are carriers of the RR-charges [20]. The ends of the open string has to be stuck to D-branes and supplied by Chan-Paton factors corresponding to some gauge group of symmetry G [20,21,19]. In superstring theories p-brane is the brane with $p+1$ Neymann boundary conditions [20] and Dirichlet conditions on the remaining 9-p of the 10 dimensional space-time. Looking on the boundary conditions (16) one sees that they are Neumann conditions along the directions tangent to the Kerr string (including time), and Dirichlet conditions along directions transverse to the string. Therefore, the Kerr string can be considered as a Dp-brane with $p=1$, or the D-string of the type I string theory. It was shown in [23–25] that worldsheet of the D-string has the structure of a heterotic string, which is responsible for the considered in sec.III heterotic properties of the Kerr string.

On the other hand, the type I string theory can be obtained orientifolding the type IIB string theory [25,22], which requires the presence of Dirichlet 9-branes as the end points of the string [26,27]. Since D-branes are oriented, and the D-branes of opposite orientation, anti-D-branes [31], carry opposite charge, there appears the possibility to form the D-string anti-D-string pairs which are not BPS states [32], but are the lightest stable states [33,30,32]. It represents especial interest since the Kerr solution with the parameters of the elementary particles is very far from the BPS one.

---

5 Nice introduction is given in [22].
6 A 9-brane fills 9 dimensional space of 10 dimensional space-time, so they do not restrict the motion of the end points of the string, but form the Chan-Paton degrees of freedom [20]. The type I theory by compactification on K3 contains D5-branes. A pair of D5-branes carry SU(2) Chan-Paton factors [28,29,34,21,30].
The obtained orientifold structure of the Kerr string displays some new features of the Kerr spinning particle. In particular, the light-like open Kerr string with the Chan-Paton factors resembles the well known model of the light-like quarks sitting on the ends of a gluonic string.

VI. THE KERR SPINNING PARTICLE AND THE COMPLEX KERR STRING

We have to discuss excitations of the Kerr D-string. The mixed type of the boundary conditions (Dirichlet-D and Neumann-N) yield three sectors of the excitation spectrum of the D-string. It was studied in [24,23,25] and shown to reproduce the world sheet structure of the heterotic string. The DD-sector of excitations yields the right-handed world-sheet fermions and scalars leading to oscillations in position of the D-string [25]. With application to the Kerr string it corresponds to oscillations in the form and position of the string. The general dynamics of the Kerr singular ring is determined by the Kerr theorem [14]. The recent progress in the obtaining of the nonstationary Kerr solutions is connected with a complex representation of the Kerr geometry [14]. The nonstationary solutions can be represented in this approach as the retarded-time fields. However, in the Kerr case they are generated by a complex source moving along a complex world line $x_0(\tau)$ in complex Minkowski space-time $CM^4$. The real part of the complex world line controls the position of the Kerr ring, and the imaginary part determines orientation of the Kerr singular ring, connected with spin [14,15].

The objects described by the complex world lines occupy an intermediate position between particle and string. Like the string they form the two-dimensional surfaces or the world-sheets in the space-time [35,7]. The corresponding complex Kerr string in many respects is similar to the "mysterious" $N = 2$ complex string of superstring theory [35], but it has the euclidean world sheet. It was shown in [7] that the boundary conditions of the complex Kerr string demand also the orientifold structure, similar to the Kerr singular ring. Moreover, the corresponding complex string equations admit the analytic in $\tau$ solutions which are analogues to the above one side stringy movers, and this complex orientifold restores the movers of both sides.\footnote{Note, that orientifold was introduced by Sagnotti and Horava in 1989 [36], and was called by Horava the world-sheet orbifold. In the papers [7] it was independently considered by analysis of the complex Kerr string. After electronic publication [7], Horava informed us on his works. The term orientifold has gained the broad currency after the well-known works by Polchinski and Witten [20,25].}

Let us now discuss the origin of the NN and DN excitations. The NN-sector in which both ends satisfy the Neumann boundary conditions carries the Chan-Paton factors leading to the usual open strings of type I string theory. This sector can be interpreted in the Kerr case as a fundamental (1,0) string coupled to the (0,1) D-string, forming a (1,1) system. It corresponds in the microgeon model to the role of the Kerr D-string as a waveguide keeping the traveling e.m. waves.

Excitations of DN sector are responsible for the current algebra modes. The structure of this sector is represented as a cloud of the fundamental DN-strings surrounding the D-string [27]. The D-ends of these strings are stuck to the D-string and the N-ends are free (coupled to D9-branes) and carry the Chan-Paton factors. This sector acquires a semiclassical explanation in the model of Kerr spinning particle. The oscillating Kerr solutions are classically radiative [14,15], and the null rays of the Kerr PNC are the lines of propagation of the electromagnetic radiation (pp-waves in the Penrose limit). The stress energy tensor of the radiation has the form $T^\mu_\nu = \Phi k^\mu k^\nu$, where $\Phi \sim 1/r^2$, and $k^\mu$ is tangent to the Kerr PNC. Therefore, radiation leads to an infrared divergence of the total mass. However, the Kerr radiation has the remarkable property that its flow is conserved $\nabla_\mu T^\mu_\nu = 0$, and can be decoupled from the Kerr singular source. Therefore, the divergence can be 'regularized' by the simple subtraction of the radiative terms [14,15]. It can also be seen from the Fig.1 since the flow of the out-going radiation has extension to the Kerr negative sheet of advanced fields where it turns into in-going field. A very small part of the rays of PNC lying in the equatorial Kerr plane $\theta = \pi/2$ reaches the Kerr D-string, but it does not lead to the above divergence. These rays are carriers of the fundamental pp-waves which are responsible for the DN-sector of the Kerr D-string.\footnote{From this point of view the DN excitations of the Kerr D-string represent a resonance of the e.m. zero-point field on the Kerr D-string, which is an one dimensional version of the Casimir effect.}
preliminary attempts to get selfconsistent solutions [15] have showed that oscillations lead to the appearance of the imaginary mass term (dual mass), which is similar to the NUT parameter [37], but is also oscillating in this case.

ACKNOWLEDGMENTS

We are thankful to the participants of the Workshop “Supersymmetry and Quantum Symmetry” (JINR, Dubna, July 2003) for very useful discussion, in particular to A. Zheltukhin, I. Bandos, A. Tseytlin and G. Alekseev.

[1] B. Carter, Phys. Rev. 174 (1968) 1559.
[2] W. Israel, Phys. Rev. D2 (1970) 641.
[3] A.Ya. Burinskii, Sov. Phys. JETP, 39(1974)193.
[4] D. Ivanenko and A.Ya. Burinskii, Izvestiya Vuz. Fiz. n.5 (1975) 135 (in russian).
[5] C.A. López, Phys. Rev. D30 (1984) 313.
[6] A. Burinskii, Phys.Rev. D 52 (1995)5826.
[7] A.Ya. Burinskii, Phys.Lett. A 185 (1994) 441; String-like Structures in Complex Kerr Geometry. In: “Relativity Today”, Edited by R.P.Kerr and Z.Perjés, Akadémiai Kiadó, Budapest, 1994, p.149.
[8] A. Burinskii, Phys.Rev. D 57 (1998)2392. Class. Quant. Grav. v.16, n. 11 (1999)3497.
[9] A. Burinskii, Grav.& Cosmology. 8 (2002) 261.
[10] D. Ivanenko and A.Ya. Burinskii, Izvestiya Vuzov Fiz. n.7 (1978) 113 (in russian).
[11] A.Ya. Burinskii, Strings in the Kerr-Schild metrics In: “Problems of theory of gravitation and elementary particles”,11(1980), Moscow, Atomizdat, (in russian).
[12] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, Nucl. Phys. B636 (2002) 99.
[13] S. Frolov and A. A. Tseytlin, JHEP 0206, 007 (2002); ‘Multi-spin string solution in $AdS_5 \times S^5$’, hep-th/0304255.
[14] A. Burinskii, Class. Quant. Grav. 20 (2003)905; Phys.Rev. D 67 (2003) 124024.
[15] A. Burinskii, ‘Alice’ String as Source of the Kerr Spinning Particle, In: Proc. of the XXV Intern. Workshop on Fundamental Problems of High Energy Physics and Field Theory. Ed. V.A. Petrov (IHEP, Protvino, June 2002), p.263, Protvino 2002.
[16] G.C. Dehney, R.P. Kerr, A.Schild, J. Math. Phys. 10(1969) 1842.
[17] A. Sen, Phys. Rev. Lett. 69 (1992) 1006.
[18] A. Sen, Nucl.Phys.,B 388 (1992) 457.
[19] M.B. Green, J.h. Schwarz, and E. Witten, ‘Superstring Theory’, V. I, II, Cambridge University Press, 1987.
[20] J. Polchinski, Phys. Rev. Lett. 75 (1995) 4724.
[21] E. Gimon and J. Polchinski, Phys. Rev. D54 (1996) 1667.
[22] C. Johnson, Introduction to D-Branes, with Applications, hep-th/9606196.
[23] A. Dabholkar, Phys. Lett. B357 (1995) 307.
[24] C. Hull, Phys. Lett. B357 (1995) 545.
[25] J. Polchinski and E. Witten, Nucl.Phys. B460 (1996) 525.
[26] C. Hull, Nucl. Phys. B509 (1998) 216.
[27] C. Hull, Phys. Lett. B462 (1999) 271.
[28] E. Witten, Nucl. Phys. B460(1996) 541.
[29] A. Dabholkar and J. Park, Nucl.Phys. B472 (1996) 207, hep-th/9602030.
[30] A. Sen, JHEP9808 (1998) 010, hep-th/9805019.
[31] T. Banks and L. Susskind, Brane – Anti-Brane Forces, hep-th/9511194.
[32] A. Sen, Non-BPS States and Branes in String Theory, APCTP winter school lectures, hep-th/9805019.
[33] A. Sen, JHEP9806 (1998) 007, hep-th/9803194.
[34] E. G. Gimon and C. V. Johnson, Nucl. Phys. B477 (1996) 715, hep-th/9604129.
[35] H. Ooguri and C. Vafa, Nucl. Phys. B 361 (1991) 469; ibid.B 361 (1991) 83.
[36] A. Sagnotti, “Open Strings and Their Symmetry Groups”, in Cargesse’87, “Non-perturbative Quantum Field Theory”, ed. G. Mack et al. (Pergamon Press, 1988) p.521.
P. Horava, Nucl.Phys. B327 (1989) 461.
[37] D.Kramer, H.Stephani, E. Herlt, M.MacCallum, “Exact Solutions of Einstein’s Field Equations’, Cambridge Univ. Press, 1980.