Numerical modeling of the geomassif stress-strain state with account of different rock resistance to tension or compression

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Abstract. The results of numerical modeling of the stress-strain state of the geomassif under the influence of natural technogenic forces are presented. To carry out computational experiments, a mathematical model was developed, in the constitutive relations of which the nonlinear relationship between stresses and deformations of rocks and their different resistance to tension or compression are taken into account. A computer program was developed, and an example is given in which the results of a nonlinear solution and a boundary value problem of the theory of elasticity are compared.

1. Introduction
As a result of mining operations under the influence of technogenic factors, zones of tension and compression are formed in the vicinity of mine workings. The stresses in these zones may differ several times from natural ones. The tensile strength of rocks is an order of magnitude lower than the compressive strength, which is sufficient for the occurrence of plastic deformations under the influence of the own weight of rocks [1].

It should be noted that the results of numerous experiments [1-3] confirmed that geomaterials are characterized by different resistance to compressive and tensile forces. S.A. Ambratsumyan developed a theoretical approach that allows the stress-strain state of a multi-modulus elastic body to be evaluated. As a direction for the subsequent development of his approach, he notes that in the mechanics of solid deformable bodies, as a rule, inelastic problems are solved in the elastic approximation, and the solution of nonlinear problems in a multi-modular formulation can lead to significant refinements of the results obtained [4].

In the development of the approach proposed by S.A. Ambratsumyan, the author of the paper developed a family of mathematical models, in the constitutive relations of which the nonlinear relationship between stresses and deformations of rocks and their different resistance to compressive and tensile forces are taken into account [5, 6].

A computer program was developed to study the constructed models in the Mathematica computer algebra system.

2. Formulation of the problem
The mathematical model of the geomassif stress-strain state is formulated in the form of a nonlinear boundary value problem: for a rectangular region to find the displacement vector \( \vec{U} = (u_x, u_y) \), the coordinates of which \( u_x, u_y \) satisfy the system of differential equations (1) and homogeneous boundary conditions.

The system of differential equations is as follows:
where

$$\mu^* = \frac{E^*}{2(1+\nu^*)}; \nu^* = \frac{\nu^* E^*}{(1+\nu^*)(1-2\nu^*)}; E^* = \frac{3E}{2E+1-\nu^*}; \nu^* = \frac{\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2}}{\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2}}$$

- variable parameters; $E$ is the modulus of elasticity; $\nu$ – Poisson’s ratio; $\gamma$ – density, $g$ – gravitational constant; $\psi$ is the function that determines the nonlinear relationship between stresses and strains.

The boundary conditions are set as follows:
- vertical boundaries, upper and lower bases $u_x = 0$;
- vertical boundaries $(u_y)'_x = 0$;
- lower base $u_y = 0$;
- vertical stresses on the upper base $\sigma_y = 0$.

The boundary value problem was solved under the condition that the mass forces are directed along the vertical axis and were created by the own weight of the rocks. The numerical solution was obtained by the finite element method [7, 8].

As an example, figure 1 shows a computational domain for a section of a layered geomassif $\Omega$, consisting of eight sub-areas, each of which characterizes rocks of a certain type: $\Omega_1$ – coarse-grained siltstone; $\Omega_2$ – fine-grained siltstone; $\Omega_3$ – satellite reservoir; $\Omega_4$ – interbedding of siltstones of different grain size; $\Omega_5$ – coal seam, including a mine, the boundaries of which are marked by points A1 and A2 in figure 1; $\Omega_6$ – medium-grained siltstone; $\Omega_7$ – mudstone; $\Omega_8$ – sandstone.

![Figure 1. Estimated area of a layered geomassif section that includes a mine working.](image-url)
Figure 2. Zone of unloading of rocks in the area of influence of a mine working.

Inside the unloading zone, marked by hatching in figure 2, plastic deformation of rocks is observed, where the nonlinear relationship between stresses and strains, as well as their different resistance to tensile and compressive forces, is taken into account. Outside the unloading zone, the numerical solution satisfies the constitutive relations of the theory of elasticity.

The results of numerical simulation of vertical stresses, obtained by solving the boundary value problem of the theory of elasticity and the nonlinear problem, are compared in Figure 3 (a and b) (1).

Based on the quantitative analysis of the distribution of vertical compressive stresses, acting in the underworked satellite layer, it follows that their maximum values when solving the boundary value problem of the theory of elasticity are 11 MPa, which is 9.1% greater than in the solution of a nonlinear problem.

Figure 3. Distribution of vertical stresses, MPa: (a) solution of the boundary value problem of the elasticity theory; (b) nonlinear solution obtained taking into account different resistance of rocks to tensile or compressive stresses.

From the comparison of the tensile vertical stresses distribution over the gob, presented in figures 3a and 3b, it follows that the stoping leads to the formation of a tension zone in the vicinity of mining
operations, the height of which, with a nonlinear solution, is significantly greater than it can be determined based on the boundary value problem of the theory elasticity. When solving a nonlinear problem, the height of the tension zone formed above the gob was 132 m, with an elastic solution it is 2.5 times less and does not exceed 53 m.

As a result of technogenic impacts in the unloading zone, conditions are created for the formation of zones of increased fracturing, in which an increase in the volume of pores and cracks occurs. That leads to the formation of a gas reservoir in the zone of displacement. When stresses caused by gas pressure and gravitational forces in the gas reservoir exceed the ultimate strength of rocks in the vicinity of the mined-out space, a dangerous situation will arise when the methane-air mixture is squeezed out into the mine workings, which can lead to accidents.

4. Conclusion
The study of the developed nonlinear mathematical model of the stress-strain state of the geomassif allows hazardous zones to be identified in the vicinity of the working face, which are formed in the displacement zone as a result of anthropogenic influences. Accounting for such zones in the design documentation will improve the safety of mining operations.

Acknowledgments
The study was carried out with the financial support of the Russian Foundation for Basic Research and Kemerovo Region within the framework of the scientific project No. 20-41-420004.

References
[1] Stumpf G G et al 1994 Physical and Technical Properties of Rocks and Coals of the Kuznetsk Basin: Handbook (Moscow: Nedra) p 447
[2] Egorov P V et al 2011 Geomechanics (Kemerovo: KuzSTU) p 325
[3] Ilintskaia E I et al 1969 Properties of Rocks and Methods for Their Determination (Moscow: Nedra) p 392
[4] Ambartsumyan S A 1982 Multimodular Theory of Elasticity (Moscow: Nauka) p 320
[5] Tsvetkov A B and Pavlova L D 2018 IOP Conference Series: EES 206 012008
[6] Tsvetkov A B, Pavlova L D and Korneva A V 2020 Scientific and Technical Bulletin of the Volga Region 1 142–145
[7] Zenkevich O K 1975 The Finite Element Method in Engineering (Moscow: Mir) p 541
[8] Tsvetkov A B and Pavlova L D 2015 Proc. of Int. Sci. and Practical Conf. Science-Intensive Technologies for the Development and Use of Mineral Resources p 121–125