INTRODUCTION TO THE DISTRIBUTION OF EARNINGS AND OF INDIVIDUAL OUTPUT, BY A.D. ROY*

James J. Heckman and Michael Sattinger

This article summarises the lasting contribution of Roy’s (1950) fundamental paper.

A.D. Roy (28 June 1920–12 March 2003) had a brief but illustrious career as an academic economist (see Sullivan, 2011 for a biography). A Cambridge-educated economist who won the distinguished Tripos award in 1939 in mathematics, he served with the British Army in India and fought in the Battle of Imphal which stemmed the Japanese invasion of India in 1944. After the war, he returned to Cambridge to study economics and history and was awarded a second Tripos award in economics in 1948.

Roy was appointed Faculty Assistant Lecturer in Statistics at Cambridge University in 1949. He completed his Master of Arts in Economics at Cambridge in 1950, reading under Professor Edward Austin Gossage Robinson, who served as editor of this JOURNAL. In the same year, he was appointed Director of Economic Studies at Cambridge and then University Lecturer in Economics and Politics at Sidney Sussex College, Cambridge, in 1951. While at Cambridge, Roy wrote the 1950 paper republished here as well as his better known 1951 Oxford Economic Papers article. In addition to the 1950 and 1951 papers reviewed here, Roy made a key contribution to portfolio theory in his paper ‘Safety First and the Holding of Assets’ (1952).

Roy’s (1952) paper is widely regarded as a contribution to portfolio theory co-equal with that of the Nobel-Prize winning analysis of Harry Markowitz (1952). Both developed the mean-variance trade-off for a portfolio of correlated assets (Markowitz, 1990). Roy goes one step further than Markowitz in analysing portfolio strategies for worst case situations. He presents methods for estimating the probability of a disaster (i.e. the failure of a portfolio to deliver some minimum rate of return).

Reflecting on his 1952 work and Roy’s (1952) paper, Markowitz wrote: ‘On the basis of Markowitz (1952), I am often called the father of modern portfolio theory (MPT),

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The research was supported in part by the American Bar Foundation, the Pritzker Children’s Initiative, the Buffett Early Childhood Fund, Eunice Kennedy Shriver National Institute of Child Health and Human Development of the National Institutes of Health under award numbers NICHD R37HD065072 and R01HD54702, an anonymous funder, a European Research Council grant (DEVHEALTH 269874), and a grant from the Institute for New Economic Thinking (INET) to the Human Capital and Economic Opportunity Global Working Group (HCEO)—an initiative of the Becker Friedman Institute for Research in Economics (BFI). The views expressed in this article are those of the authors and not necessarily those of the funders or persons named here. We thank Kenneth Mayhew and Nicholas Rogers for providing valuable biographical information on A.D. Roy.

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but Roy can claim an equal share of this honor’. (Markowitz, 1999, p. 6) For a more extensive discussion of Roy’s contribution to portfolio theory, see Bernstein (1992) and Sullivan (2011).

On secondment from the Economics Faculty, Roy left Sidney Sussex College in 1962 to serve as Economic Consultant and later Senior Economic Advisor to the Treasury. After 1964, Roy held a number of positions as a civil servant at the Department of Trade and Industry, Ministry of Defence, and Department of Health and Social Security. In these positions, Roy addressed issues more closely related to macroeconomics such as labour productivity (Roy, 1982; Roy and Wenban-Smith, 1983).

Roy’s papers published in 1950 and 1951 address issues in income distribution that are as relevant now as they were when he wrote them. At the time of their publication, there was a perception—called ‘Pigou’s Paradox’ (Pigou, 1948)—of an apparent conflict between the normal, symmetrical distribution of income that one would expect if income were proportional to ability (assumed normally distributed), and the skewed distributions of income that were observed.\(^1\) Observed distributions of income are better described by the lognormal distribution than by the normal distribution. Pigou’s explanation was that inherited wealth was highly skewed and was an important component of income. Roy observed that labour earnings, excluding income from capital, were themselves right skewed, so Pigou’s explanation did not go far enough.

Roy’s (1950) paper seeks to explain why the distribution of earnings might follow a lognormal distribution, generating skewed distributions without invoking an unequal distribution of property income that could separately contribute to skewness and inequality. The basic explanation, related to Gibrat’s Law of Proportionate Growth, is that the output of a worker in a given period is not the outcome of a normally distributed attribute like that assumed for ability but was the product of several attributes that combine multiplicatively to determine the worker’s output. Under conditions more general than the ones assumed in his paper (including correlation among components, Cramér, 1946), the Central Limit Theorem implies that the sum of the logarithms of the several attributes will tend to be normally distributed and the product of the attributes and worker output will be lognormally distributed.

Roy considers reasons why this simple mechanism may not operate. His first explanation is that the attributes or factors contributing to a worker’s output may be correlated. Unfortunately, this reasoning is incorrect. The Central Limit Theorem continues to hold even with strongly correlated components.\(^2\) His second explanation plants the seeds for his 1951 paper. Roy suggests that the economy could produce a large number of different goods, requiring employment in many occupations. With two occupations, wages would need to be adjusted so that the required number of workers in each occupation equals the number of workers choosing that occupation. Roy comments as follows on the consequences of this equilibrium adjustment:

\(^1\) Roy (1950) correctly observes that the normal distribution of ability was a convention of psychology and not a fundamental law of nature. He thus implicitly questions whether Pigou’s paradox was paradoxical.

\(^2\) Gary Becker’s resolution of Pigou’s Paradox is also based on the skew-inducing effect of multiplying random variables. In his case human capital multiplied by the rate of return, both assumed to be random variables, produce skewed earnings distributions (Becker, 1975).

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The shape of the distribution of earnings that results from this adjustment depends on two considerations, the proportions of the labour force required in each occupation and the association existing between any individual’s performance in the two occupations.

(Roy, 1950, p. 494)

To avoid this complication, Roy assumes that a worker’s outputs in different occupations are highly correlated, an assumption he would relax in his 1951 paper.

Roy tests the hypothesis of lognormality of earnings by calculating measures of ‘humpedness’ (the mean deviation divided by the standard deviation) and skewness for a collection of different samples and comparing the measures to what one would expect from a normal versus a lognormal distribution. Criteria for the samples are that workers should be engaged in the same activity and output should depend only on the worker and not on the operation of machines or flow of material. Additionally, each worker in the population should have the same likelihood of choosing the occupation.

Commenting on this criterion, Roy observes:

The workers engaged on a particular job have not an equal chance of being drawn from all strata of the working population. They are probably selected from a rather restricted field. If a certain minimum of intelligence, skill, etc., is required, workers below this standard will be rejected by the management at some stage, while workers, whose capacity is not fully used in this activity, will either be promoted or will themselves undertake more suitable sorts of employment.

(Roy, 1950, p. 495)

Roy recognises the seriousness of this point but at the time lacked the tools to do anything but assume that the factors determining a worker’s occupation are not correlated with output. He returns to this problem in his more famous 1951 paper.

The activities for the samples collected by Roy include packing chocolates in boxes, packing cigarettes in tins, making springs for electrical equipment, bank wiring, top-stitching shoes, winding armatures, operating phonographic record-presses, packing a variety of objects into cartons and cases, and a psychological test related to work in light industry. Roy shows that the evidence from his tests is not decisive. There is no clear domination of one distributional form over another in the output data for the samples.

The continuing significance of Roy’s (1950) paper lies in his response to this empirical evidence. He re-examines the reasons why one would not obtain the expected lognormal result even if the underlying argument, that factors apply multiplicatively to determine worker outputs, is valid. Primary among the reasons that worker outputs do not always follow the lognormal distribution is that worker selection operates to modify the distribution of outputs for workers observed in a given occupation from the distribution of outputs for all workers. This insight, explicitly posed by Roy in the above quotations, is developed systematically in his 1951 paper.

Selection among alternative occupations, as described by Roy, draws on notions of potential outcomes used in the literature on the design of experiments (Neyman,
1923; Fisher, 1952), later applied in Rubin (1974). Like later work by Quandt (1972), Gronau (1974) and Heckman (1974, 1976), he adds to the models of potential outcomes in the statistical literature a choice mechanism determining allocations of individuals across sectors (Heckman, 2008 for discussion of the contribution of economists to the Neyman–Fisher–Rubin literature on potential outcomes). Roy’s insight introduces economic decision-making into the determination of occupations and earnings and fundamentally modifies the procedures that should be applied to the analysis of earnings data (see French and Taber (2011) for an up-to-date survey).

In the field of income distribution, consideration of why particular individuals are found in particular occupations is related to the assignment problem in operations research, analysed in an economic context by Koopmans and Beckmann (1957). In this view, a basic problem for any economy is to determine how workers are assigned to jobs. From the perspective of individual workers and firms, the phenomenon of assignment is now more commonly described by matching, a term introduced by Jovanovic (1979) to describe selection through job turnover from the point of view of workers and firms.

A major mechanism guiding assignment is David Ricardo’s principle of comparative advantage, recognised by Roy in his 1951 paper. Sattinger (1975) and Rosen (1978) extend the analysis of comparative advantage in international trade to the labour market setting. A worker choosing an occupation will compare the income from one occupation with the income from another occupation. If workers have comparative advantages at different occupations, workers will not all make the same decisions. Using the two occupations of trout fishing and rabbit catching whimsically analysed in his 1951 paper, an individual choosing trout fishing would have a comparative advantage at that activity compared to an individual choosing rabbit catching.

A second consequence of the choice of occupations is that the distribution of earnings is built up from the distributions of earnings in individual occupations. Within an occupation, the distribution of earnings is affected by the selection of workers into that occupation, as determined by ‘the association existing between any individual’s performance in the two occupations’ (Roy, 1951, p. 494). Roy considers both positive and negative correlation between performances. As a result of the selection, the aggregate distribution of earnings will not resemble any distribution of abilities and instead will depend on the occupations available to workers and their performances in different occupations. In a review of assignment models of income distribution, Sattinger (1993) shows graphically how the aggregate distribution of earnings is constructed from underlying distributions of earnings within occupations.

An alternative characterisation of Roy’s view of earnings as generated by occupational choice is that the earnings of workers are order statistics, which are used to describe the distribution of the n-th largest (e.g. the maximum or minimum) of a set of random variables. In this view, a worker considers the earnings available in different occupations and chooses the occupation that maximises the worker’s earnings. The opportunity to choose occupations will in general raise the earnings of the lowest income recipients and reduce inequality. A worker with a low outcome in one occupation will not be forced to accept that income but instead will have a chance to get a higher income in some other occupation. Houthakker (1974) cites Roy’s paper in developing a model of the distribution of earnings based on order statistics.
Roy’s selection phenomenon has been applied and extended to a wide range of other contexts, beginning with choice of market versus non-market work and wage comparisons (Gronau, 1974; Heckman, 1974, 1976; Lewis, 1974). Decision rules more general than simple income maximisation have been developed and applied under the rubric of the generalised Roy model (Heckman and Vytlacil, 2007a). Applications of the Roy and generalised Roy models include choice of union versus non-union sectors (Lee, 1978), levels of education (Willis and Rosen, 1979), geographical region (Robinson and Tomes, 1982), marital status (McElroy and Horney, 1981), occupational choice (Miller, 1984), piece rate versus salary pay structures (Lazear, 1986), industry changers (Solon, 1988; Gibbons and Katz, 1992), immigration (Borjas, 1990), army retention (Brown, 1980) and segmented labour markets (Magnac, 1991). Flinn and Heckman (1982) and Moscarini (2001) extend Roy’s selection model by incorporating choices based on search.

By moving beyond choice rules based on simple comparisons of outcomes across occupations, Roy’s selection insight has generated progressively wider ranging results in econometrics, beginning with self-selection biases and extending to evaluation of social policies, treatment effects, and causality. Implications of selection for econometrics have been reviewed by Heckman and Vytlacil (2007a, b), Heckman and Taber (2010), Heckman (2010) and French and Taber (2011). Heckman (1976, 1979) and Heckman and Sedlacek (1985) describe the statistical problems that arise from the selection problem identified by Roy. In a simple regression model of a behavioural relationship using ordinary least squares applied to a random sample from a larger population, the error terms are assumed to be randomly distributed with an expected value of zero. With selection, not all potential observations have the same likelihood of appearing in the data set, with the result that the expected value of the error terms for the included observations may not be zero. For example, in an early application, women participate in the paid labour force if their market wage exceeds what they could earn in home production. Observations of market wages for working women would only be observed for those women whose market wages were higher than their alternatives out of the labour force and would therefore tend to be higher than the market wage that would be offered to all women (Gronau, 1974; Heckman, 1974). Alternatively, data analysts may apply some selection criteria to exclude incomplete observations or panel data that do not extend over the full sample period. Heckman provides methods to eliminate the bias generated by selection so that simple regression techniques can be employed. These methods have been substantially extended in the later literature in econometrics (French and Taber, 2011).

Heckman and Sedlacek (1985) estimate a Roy model of earnings in manufacturing and non-manufacturing incorporating selection of workers into the two sectors. In the initial specification of the model, workers choose between the two sectors based on income maximisation. Since the results for the initial specification fail test criteria that they develop, they extend the model by adding a third sector for non-market or household production and assume that individuals choose sectors on the basis of utility maximisation instead of income maximisation. The results explain the distribution of earnings within the manufacturing and non-manufacturing sectors, aggregation bias in wages, and the effect of selection on earnings inequality. The contributions of the

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non-market sector and utility maximisation to improvement in goodness-of-fit tests are analysed further in Heckman and Sedlacek (1990).

Heckman and Honore (1990) derive further statistical implications of Roy’s selection process. They demonstrate that worker selection of sectors based on comparative advantage reduces inequality compared to random assignment to sectors. Further, the aggregate distribution of the logarithms of earnings will be skewed to the right even when one of the sectors is skewed to the left. They also consider conditions which would permit identification of parameters in the model.

Analysis of selection bias has led to improvement in methods used to evaluate social programs (Heckman et al., 1996, 1997; Abbring and Heckman, 2007; Heckman and Vytlacil, 2007a, b). A further extension considers general equilibrium models of income distribution and treatment effects (Heckman et al., 1998; Abbring and Heckman, 2007).

Roy’s 1950 paper has received steady citations from the time of its publication. Although early citations arose from its results for earnings distributions, the increase in citations in recent decades arises from its relevance to fundamental problems in observational analyses and the econometrics that addresses them. In empirical social sciences, including sociology, political science and criminal justice, current trends towards increasing the validity of causal inferences are rooted in Roy’s problem of explaining why the workers observed in an occupation are not representative of the population as a whole.

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Appendix A. Roy, A.D. (1950). ‘The distribution of earnings and of individual output’, Economic Journal, vol. 60(239), pp. 489–505.
THE DISTRIBUTION OF EARNINGS AND OF INDIVIDUAL OUTPUT

I

There appears to exist to-day a fairly widespread conviction that if each individual were paid what he is worth, the distribution of incomes in any community would look like the distribution of heights of adult males instead of resembling the size distribution, by numbers employed, of firms in industry. This view is based upon the contention that it is the unequal distribution of property which is the root cause of gross inequalities of income. While accepting the fact that the property distribution is a most important aggravating factor, this article will attempt to argue that the distribution of earned income alone is itself of much the same form as the distribution of total income, and an endeavour will be made to discover the reasons for this. The claim that the distribution of output of a homogeneous commodity among individual workers is likely to be similar to the distribution of earnings itself will be subjected to statistical tests.

It is considered that the examination of the distribution of earned income alone will avoid the difficulties that are encountered when using distributions of total income derived from taxation statistics. Ambiguity exists over the precise meaning of "an income" as assessed for taxation and also with regard to the total number of such incomes. To some extent such problems can be shelved by fitting a Pareto distribution to the upper tail of distributions of taxable income, but in no sense can such a distribution be said to "explain" the distribution found to exist.

The assumption will be made throughout the discussion that the reader is familiar with the properties of the probability distribution known variously as the "normal distribution," the "Gaussian distribution" or the "normal curve of error."

I should like to take the opportunity of thanking the members of the various bodies who took so much trouble in giving me the information for which I asked and who have consented to the presentation of the data in this paper.

I should also like to express my debt to J. R. N. Stone, Esq., C.B.E., Director of the Department of Applied Economics, University of Cambridge, and to various members of the Faculty of Economics and Politics and of the Department of Applied Economics for their encouragement and help in preparing this paper; also to the computing and secretarial staff of the Department of Applied Economics for their assistance.

Finally, I take full responsibility for the views expressed and for the errors perpetrated in this work.
II

If the distributions of earnings of workers in industry or of professional men, both in this country and in the United States, are examined, it is found that they can be reasonably approximated by fitting a theoretical probability distribution, in which the logarithms of earnings conform to the normal curve of error. Such a distribution will be called in what follows a logarithmic normal ("log-normal" for short) distribution.

This fact can be confirmed by studying such earnings distributions as exist, e.g., those given in the results of the Wages and Earnings Enquiry of 1906, in S. Kuznets' and M. Friedman's *Income from Independent Professional Practice* and, in respect of British Dentists, in *The Times*, of May 20, 1949.

There must be some rational explanation of the fact that all these earnings' distributions have such similar shapes. The reasons may be highly complex, and if so it is unlikely that they can be found but, on the other hand, there may be a relatively simple solution to the problem. Can such a simple hypothesis be found?

Let it be assumed that workers' earnings are related to the output that they produce of a commodity. This is not as restrictive an assumption as it might first appear, for even when workers are paid day wages the rate of payment is related to the real output that is expected from workers of the particular class in question. If in the long run such expectations are falsified the wages will be revised appropriately. The assumption is, then, that all workers tend to be rewarded in direct proportion to the output that they produce or, in other words, that the ruling method of wage-payment is considered to be straight piece-work.

Suppose now that out of the entire working population, a large sample of workers is selected and then trained and set to work producing the same homogeneous commodity. The product that they make is one that can be made by hand, and thus the time that any individual takes to produce one article depends solely on his own skill and training and is not affected by the running time of any machine. Let the group of work-people continue at this job for some length of time, such as a month or a year; at the end of the time period, how will the output of the commodity be distributed amongst the various operators?

Investigations have shown that in nature many characteristics tend to be distributed approximately normally, and thus, perhaps, the distribution of output of human beings will have similar...
properties. For after all the adult human is himself a fairly standard product, whose attributes do not appear to be capable of very wide variation. Our heights are normally distributed, and so, psychologists tell us, are our intelligence quotients. The latter claim should not be taken too seriously, since it is more informative about psychologists' systems of measurement than about anything else.

Weights and volumes, however, are not, in general, normally distributed, because as can be easily understood their distributions tend to the distributions of the product of three linear measurements which are each normally distributed. These distributions have in some measure the characteristic form which is, in economics, associated with income distribution.

The reason for the natural distribution of weights may well provide the key to the distribution of output. Suppose that the sample of workers has been selected at random, i.e., each worker in the whole working population has an equal chance of being chosen. Assume further, that workers differ in speed of working, in accuracy and in the numbers of hours that they work. For each of these properties there exists a probability distribution which for the sake of simplicity will be assumed to be a binomial one. Further, each distribution will be deemed to be independent in the statistical sense of that word, i.e., a high (or low) speed for a worker is not, in general, associated with long (or short) hours of work or with meticulous accuracy (or the reverse). In other words, in the entire working population these three characteristics are uncorrelated.

On this foundation, consider the following numerical example:

**Table I**

| Speed (units produced per hour) | Accuracy (proportion of production approved) | Hours of work per day | Number of workers (per thousand all workers) to which figures in columns 1, 2 and 3 apply |
|----------------------------------|---------------------------------------------|-----------------------|-------------------------------------------------------------------------------------|
| (1)                              | (2)                                         | (3)                   | (4)                                                                                 |
| 2                                | 0·50                                        | 4                     | 250                                                                                 |
| 3                                | 0·75                                        | 6                     | 500                                                                                 |
| 4                                | 1·00                                        | 8                     | 250                                                                                 |
|                                  |                                             |                       | 1,000                                                                               |

At the end of a day the output of any worker will equal the product of his speed, accuracy and hours of work, and will therefore be distributed as in Table II.
A cursory examination of Table II shows that the distribution of output is skew (or asymmetrical), although the three distributions of speed, accuracy and hours of work were all symmetrical. This phenomenon is a particular manifestation of a very general theorem in statistics known as the Central Limit Theorem.\footnote{See, for instance, H. Cramer's \textit{Mathematical Methods of Statistics}, pp. 213 \textit{et seq.}} Broadly speaking, this theorem states that the distribution of the sum of variates, which themselves have independent distributions of a very general kind, tends to normality as the number of such variates is increased. Applying this theorem to the logarithms of variates, it follows that the distribution of the product of independently distributed variates tends to log-normality as the number of such variates increases, provided that the logarithms of the variates have probability distributions which fulfil the not very restrictive conditions of the Central Limit Theorem.

For simplicity we have considered the distribution of the product of only three variates, but the three factors considered may themselves be derived from the multiplication of a large number of other basic variates. In other words, perhaps it is more realistic to consider the distributions of speed, accuracy and hours of work to be asymmetrical in the same direction as the distribution of output which was obtained from their multiplication. For speed alone perhaps it is true that, although most people have a relatively low speed of working, yet there is a long upper tail to the distribution containing individuals capable of remarkably high speeds. If this is so, then the distribution of output would approximate far more closely to the log-normal distribution than does the distribution obtained in the example given here.
The proposition is, then, that the output of an individual depends on a great number of different factors which may conveniently be considered to act together in a multiplicative rather than in an additive way. As far as simplicity is concerned there is no reason to place multiplication lower in the scale than addition. All this means is that any one factor, such as health, exercises its effect proportionately, and work-people with the same degree of ill health tend to have their output reduced in the same ratio rather than by the same absolute amount. Similarly, women's output will tend to be a certain fraction of the output of men of similar skill, age, education and so on.

So far it has been assumed that the various factors on which output depends are uncorrelated, e.g., skill is not, in general, associated with age or with education. This is obviously a very unreal assumption which ought to be removed, but unfortunately this cannot be done in an entirely satisfactory way. Nevertheless, there do exist some indications of the effect of dispensing with the condition of independence. Professor Haldane has shown that distribution of the product of three correlated normally distributed variates does not differ markedly from the distribution when the variates are independent, provided that the ratios of the standard deviation to the mean (coefficients of variation) of each of the three variates are approximately equal.1 If, however, these ratios are unequal the existence of an association between the variates tends to exaggerate the asymmetry and the humpedness of the distribution of their product. This would mean that more individuals would have very high outputs than would be indicated by a log-normal distribution. Examination of the distributions of the earnings of professional men show that this systematic departure from log-normality at the upper tail of the distribution does in fact exist.

Putting on one side for the moment the question of testing statistically the hypothesis that output of any one commodity is distributed log-normally, let it be assumed that such is indeed the case. Then, before the theory so far outlined can be regarded as a satisfactory explanation of the distribution of earnings, the effect of introducing a large number of different goods into the economy must be considered. Any one member of the working population is potentially capable of producing some output (however small) of every commodity, but in fact he will produce only one. If all the working population were employed in one

1 J. B. S. Haldane, “Moments of the distribution of powers and products of normal variates,” Biometrika (1942), 32 pp. 226 et seq.
occupation, the distribution of output would be log-normal. Suppose there are two possible occupations and that demand conditions require a certain proportion of the labour force to be in each occupation. Then the piece-rates in the two occupations must be varied until the chance of an individual obtaining higher earnings in one of the two occupations is equal to the proportion of the labour force required in that occupation, assuming that labour is perfectly mobile. The shape of the distribution of earnings that results from this adjustment depends on two considerations, the proportions of the labour force required in each occupation and the association existing between any individual's performance in the two occupations.\(^1\) Provided that those people with high outputs in one occupation would, in general, have high outputs in the other, the distribution of earnings will approximate closely to log-normality. Even if the reverse held, the effect would be unimportant if only a small proportion of the labour force were required in one of the occupations. In the extreme case where the best performers in one occupation were the worst in the other and where the labour force was equally divided between both occupations, the distribution of earnings would tend to a log-normal distribution with the part containing the lower half of the total frequency removed.

This argument can be reasonably easily extended to the case where many occupations exist. In order that a log-normal distribution of output of any given commodity should give rise to a log-normal distribution of earnings for all occupations taken together, the assumption must be made that a high degree of association exists between any individual's possible outputs in the major occupations of the economy. Major, in this context, meaning the employment of a substantial proportion of the working population.

Even if this assumption is valid, labour is not highly mobile in the short run, and consequently an additional factor is operating to distort the distribution of earnings away from log-normality. The theory, then, even if it is acceptable on other grounds, can only be regarded as describing the causes that make the distribution of earnings tend to log-normality in the long run. The existence of machines having fixed running times will only be a disturbing factor in the short period.

\(^1\) This statement and what follows are only strictly true if the number of persons, whose potential output is more than \(x\) times the geometric mean output, is the same in both occupations for all values of \(x\).
III

The next task must be an endeavour to provide some statistical verification or disproof of the abstract argument which has been given at some length. This attempt gives rise to various difficulties of a statistical character. The requirement is to find a sample, preferably large, of workers engaged in producing exactly similar objects. Such a sample ought to be random, i.e., each member of the working population should have an equal chance of being chosen. The work done must depend solely on the worker, and must not be limited by the time of operation of a machine or by an inadequate flow of materials.

In modern industry very few people can be found who are doing exactly the same job. A larger number are doing relatively similar tasks, but the assessment of their outputs on a comparable basis involves value judgments which would seriously affect the validity of any deductions that might be drawn. The samples are thus inevitably small.

The workers engaged on a particular job have not an equal chance of being drawn from all strata of the working population. They are probably selected from a rather restricted field. If a certain minimum of intelligence, skill, etc., is required, workers below this standard will be rejected by the management at some stage, while the workers, whose capacity is not fully used in this activity, will either be promoted or will themselves undertake more suitable sorts of employment. On the whole, it may be expected that the inferior workers will be eliminated more thoroughly than the high-grade ones, since the interests of the management demand this. In so far as the employing company carries out careful and systematic personnel selection, so will the field from which the workers come be narrowed still further.

It might so happen that a group of workers in industry, of which the variations in individual output are studied, had decided to do a "fair" day's work for the management and no more. In this case, the distribution of daily output is likely to be concentrated about their estimate of the work required, but it must be hoped that this state of affairs will be comparatively rare.

Obviously, we are powerless to do anything about these possible difficulties. It must be assumed that the factors determining any worker's occupation are not associated in any marked degree with the size of his output in any period. If this assumption is not justified, the statistical tests used will be invalid.

If fortune had provided large samples of workers doing the same thing, the testing process would be fairly simple. Two
hypotheses could be set up; (i) that the sample had been drawn from a population of workers in which individual output was distributed normally; (ii) that the distribution of output in the population was log-normal. These two possible distributions could then be fitted to the sample data, and the probability of each hypothesis could be calculated. It might be found possible to reject one or other hypothesis, or simply to say that one appeared more probable than the other.

Although the procedure adopted is roughly equivalent to that sketched out, the fact that the samples are small means that the fitting of log-normal and normal distributions to the data would waste information owing to the necessity of grouping the observations in classes. Instead, therefore, measures of the asymmetry and humpedness of the sample distributions of individual output and of the logarithms of output are calculated. Geary and Pearson have tabulated the values of these measures which will turn up by chance 1 in 20 and 1 in 100 times in samples drawn at random from normal populations for different sizes of sample. The mean or expected values of the measures for different sample sizes are also known. Thus it may be possible to reject an hypothesis on some ground or to say that under the circumstances it is more probable than another. The method of procedure will be clarified by discussing particular applications.

IV

In this section, an account will be given of the samples obtained and of the deductions that can be drawn therefrom. Each sample is denoted by a letter of the alphabet.

Sample A. In a certain factory thirty-five girls were engaged in packing boxes with chocolates. This they did entirely by hand, but the supply of chocolates was delivered to them on a moving belt. The girls' positions astride the belt were changed each day, since the end positions were thought to receive a restricted supply of chocolates. The extent of the control over

\[ a(1) = \text{mean deviation} = \frac{1}{n} \sum_{i=1}^{n} \left| x_i - \bar{x} \right| \]

and

\[ g_1 = \frac{1}{n(n-1)(n-2)} \left( n^2 \sum_{i=1}^{n} x_i^3 - 3n \sum_{i=1}^{n} x_i \sum_{i=1}^{n} x_i^2 + 2 \left( \sum_{i=1}^{n} x_i \right)^3 \right) \]

\[ \left( \frac{n \sum_{i=1}^{n} x_i^2 - \left( \frac{1}{n} \sum_{i=1}^{n} x_i \right)^2}{n(n-1)} \right) \]

1 If the sample consists of \( n \) values \( x_1, x_2, \ldots x_n \), the two following measures are calculated:

2 R. C. Geary and E. G. Pearson, Tests of Normality, Biometrika Office, 1938 (or Biometrika (1938)).
variations in output among the girls exercised by the uniform speed of the belt is difficult to estimate.

From information giving the total weekly output and hours worked in four successive weeks, the average hourly output over the whole period was calculated for each operator. The two measures described above relating to the asymmetry and humpedness of the sample distributions of output and of the logarithms of output were obtained.

**TABLE III**

| Variate.                  | Humpedness (mean deviation/standard deviation). | Asymmetry (skewness). |
|---------------------------|-----------------------------------------------|------------------------|
| Hourly output             | 0·858                                         | +0·365                 |
| Logarithm of hourly output| 0·853                                         | -0·067                 |

Geary and Pearson's tables disclose that a humpedness, in this sense, of 0·878 or higher would occur by chance only 1 in 100 times if the sample had been drawn at random from a normal population. A value of 0·859 or higher would turn up 1 in 20 times, and a value of 0·848 or higher would turn up 1 in 10 times. The expected value of the humpedness is 0·835. Similarly, a measure of asymmetry as high as ± 0·921 would appear only once in a 100 times, and a value as high as ± 0·621 once in 20 times; the expected value is zero.

An hypothesis will be rejected only if the chance is less than 1 in 20, and so we cannot in this example reject either hypothesis. On the other hand, the two measures are nearer their expected values for the logarithms of hourly output, and consequently, the hypothesis that individual output is distributed log-normally is the more probable of the two.

**Sample B.** In the same factory as for A sixty-six girls packed chocolates in a different way which was easier than the method used in A. The data was supplied for the same period, and was treated in the same manner.

**TABLE IV**

| Variate.                  | Humpedness (mean deviation/standard deviation). | Asymmetry (skewness). |
|---------------------------|-----------------------------------------------|------------------------|
| Hourly output             | 0·796                                         | +0·027                 |
| Logarithm of hourly output| 0·793                                         | -0·029                 |
Neither hypothesis can be rejected, but the normal distribution is more probable than the log-normal one.

*Sample C.* In a tobacco factory, eleven girls packed round tins with fifty cigarettes in each tin. Their average output per hour was calculated and the customary measures obtained.

### Table VI

| Variate.                  | Humpedness (mean deviation/standard deviation) | Asymmetry (skewness) |
|---------------------------|-----------------------------------------------|----------------------|
| Hourly output             | :                                             | : 0.822              |
| Logarithm of hourly output| :                                             | : 0.844              |

### Table VII

| Value of humpedness (expected value = 0.818) | Value of asymmetry (expected value zero) | Chance of such values occurring in a sample drawn at random from a normal population. |
|--------------------------------------------|----------------------------------------|----------------------------------------------------------------------------------|
| 0.936 or more                              | Not tabulated                          | 1 in 100                                                                         |
| 0.907 or more                              | Not tabulated                          | 1 in 20                                                                          |
| 0.890 or more                              | Not tabulated                          | 1 in 10                                                                          |

The above table and extrapolation by eye for asymmetry do not enable either hypothesis to be discarded. From the point of view of humpedness the normal distribution is more likely, but asymmetry favours the log-normal distribution. The test is thus inconclusive.

*Sample D.* In the same works as C, twenty girls packed cigarettes by hand into flat tins holding twenty-five cigarettes.

### Table IX

| Variate.                  | Humpedness (mean deviation/standard deviation) | Asymmetry (skewness) |
|---------------------------|-----------------------------------------------|----------------------|
| Hourly output             | :                                             | : 0.758              |
| Logarithm of hourly output| :                                             | : 0.749              |
The average number of cigarettes packed per hour by each operator was computed and the results treated in the usual way.

| Value of humpedness (expected value = 0.809) | Value of asymmetry (expected value zero) | Chance of such values occurring in a sample drawn at random from a normal population. |
|---------------------------------------------|-----------------------------------------|-----------------------------------------------------------------------------------|
| 0.698 or less                               | Not tabulated                           | 1 in 100                                                                          |
| 0.729 or less                               |                                        | 1 in 20                                                                           |
| 0.749 or less                               |                                        | 1 in 10                                                                           |

On the grounds of excessive asymmetry, which can be detected by extrapolation by eye, the hypothesis of a log-normal distribution can be rejected, and thus the test decides in favour of a normal distribution.

Sample E. Twenty-one men and twenty-four women were employed in making springs in a company manufacturing electrical equipment. The exclusively male and female groups were first

**Table XI**

| Variate.                           | Group. | Humpedness (mean deviation/standard deviation) | Asymmetry (skewness) |
|------------------------------------|--------|-----------------------------------------------|----------------------|
| Hourly output                      | Male   | 0.859                                         | +0.136               |
|                                    | Female | 0.780                                         | +0.547               |
|                                    | Whole  | 0.789                                         | +0.214               |
| Logarithm of hourly output         | Male   | 0.850                                         | -0.482               |
|                                    | Female | 0.792                                         | -0.014               |
|                                    | Whole  | 0.807                                         | -0.371               |

**Table XII**

| Value of humpedness. | Value of asymmetry. | Chance of such values occurring in a sample drawn at random from a normal population. |
|----------------------|---------------------|-----------------------------------------------------------------------------------|
| 21 24 45             | 21 24 45            |                                                                                  |
| 0.808 0.807 0.802    | 0 0 0              | 1 in 100                                                                          |
| 0.905 0.701 0.724    | +1.075 +0.825       | 1 in 100                                                                          |
| or more or less or less | or more or more or less | 1 in 20                                                                           |
| 0.877 0.733 0.749    | -1.075 -0.815       |                                                                                  |
| or more or less or less | or more or more or less | 1 in 10                                                                           |
| 0.863 0.752 0.762    | -0.720 -0.560       |                                                                                  |
| or more or less or less | or more or more or less | 1 in 10                                                                           |
treated separately, and then, since the average output per hour in the week concerned was found to be significantly different for men and women, the sample was taken as a whole, with the differences in output due to sex eliminated. In other words, the distribution of variations of output about the appropriate sex mean was examined.

In the purely male group humpedness favours a log-normal distribution and asymmetry a normal one. In the female group both measures favour the hypothesis of selection from a log-normal distribution. Taking the whole sample together, the test is inconclusive in the same way as for the male part alone.

**Sample F.** In the same firm as that for E, seventy-three operators were engaged on bank wiring. Seventy-two of the workers were women, and one was a man. Probably the man should have been excluded from consideration in order to achieve homogeneity, but in the event he was not. The information obtained related to a single week, and the average hourly output of each operator was calculated. The customary analysis yielded the following results:

| TABLE XIII |
|------------|
| Variate. | Humpedness (mean deviation/standard deviation) | Asymmetry (skewness). |
| Hourly output | 0.828 | +0.480 |
| Logarithm of hourly output | 0.818 | -0.471 |

| TABLE XIV |
|-----------|
| (Sample size = 73.) |
| Value of humpedness | Value of asymmetry | Chance of such values occurring in a sample drawn at random from a normal population. |
| (expected value = 0.800). | (expected value zero). | |
| 0.854 or more | +0.650 or more | 1 in 100 |
| 0.840 or more | -0.650 or less | 1 in 20 |
| 0.832 or more | +0.448 or more | 1 in 10 |
| | -0.448 or less | |
| | — | |

Both hypotheses can be rejected on grounds of excessive asymmetry, but that of a log-normal distribution is more probable than the other.

**Sample G.** In the closing room of a shoe factory, the output of twelve female top-stitchers was recorded over a period of six weeks. In this instance the ages of the operators were also known, and, to prevent the age distribution of the workers in the
sample producing a peculiar result, that part of the variation in output due to age differences was eliminated. Although in this particular sample the association between age and output would not on its own be adjudged significant, other evidence suggests that in the working population as a whole such an association exists.

Output per hour was not measured directly, but as average piece-rate earnings in pence per hour. With an operator of twenty years of age earning on the average 2s. 4d. per hour, the elapse of each year would be expected to reduce her earnings by 0.023d. per hour. An independent calculation using the logarithms of output shows that each year the average operator’s output per hour may be expected to fall by less than \( \frac{1}{10} \% \).

Having disposed of age differences, the measures calculated were:

| TABLE XV |
|----------|
| Variate. | Humpedness (mean deviation/standard deviation). | Asymmetry (skewness). |
|----------|-----------------------------------------------|----------------------|
| Hourly output | : : | 0.786 | -1.668 |
| Logarithm of hourly output | : : | 0.758 | -1.791 |

| TABLE XVI |
|----------|
| (Sample size = 12.) |
| Value of humpedness (expected value = 0.816). | Value of asymmetry (expected value = zero). | Chance of such values occurring in a sample drawn at random from a normal population. |
|----------|------------------|-----------------------------------------------|
| 0.671 or less | Not tabulated | 1 in 100 |
| 0.717 or less |                   | 1 in 20 |
| 0.742 or less |                   | 1 in 10 |

Extrapolation by eye suggests that both hypotheses can be discarded owing to the extent of the asymmetry, but of the two distributions the normal one is the more probable.

*Sample H.* Nine women operators were employed winding armatures of identical pattern. Their average hourly output over a four-week period was obtained and their ages were known. Despite the very small size of the sample, there was an almost significant association between age and hourly output. From the age of twenty the average hourly output of about 9.5 armatures would be reduced on the average by 0.037 of an armature every year. Or, looking at the matter proportionately, hourly output would fall each year by about \( \frac{3}{4} \% \) on the average.
After the effects of differing ages had been eliminated, the results were as follows:

**Table XVII.**

| Variate                      | Humpedness (mean deviation/standard deviation) | Asymmetry (skewness) |
|------------------------------|-----------------------------------------------|----------------------|
| Hourly output               | 0.890                                         | +0.513               |
| Logarithm of hourly output  | 0.885                                         | +0.506               |

In Geary’s and Pearson’s tables no values are given for this size of sample, but extrapolation by eye shows that neither hypothesis can be rejected. The log-normal distribution is the more likely one.

**Sample I.** In a gramophone-record factory twenty-four men were operating record presses. This sample does not fulfil the requirements laid down, since each record must remain in the press for a fixed curing time, and the speed of work is thus not wholly under the operator’s control.

Average outputs per hour over a period of four weeks were calculated and treated in the usual way.

**Table XVIII.**

| Variate                      | Humpedness (mean deviation/standard deviation) | Asymmetry (skewness) |
|------------------------------|-----------------------------------------------|----------------------|
| Hourly output               | 0.869                                         | +0.369               |
| Logarithm of hourly output  | 0.878                                         | +0.240               |

**Table XIX.**

| (Sample size = 24.) |
|---------------------|

| Value of humpedness (expected value = 0.807). | Value of asymmetry (expected value zero). | Chance of such values occurring in a sample drawn at random from a normal population. |
|----------------------------------------------|-------------------------------------------|----------------------------------------------------------------------------------|
| 0.894 or more                               | Not tabulated                             | 1 in 100                                                                         |
| 0.871 or more                               |                                           | 1 in 20                                                                          |
| 0.859 or more                               |                                           | 1 in 10                                                                          |

Extrapolation by eye must again be used and the test is inconclusive, humpedness favours a normal distribution, but asymmetry a log-normal distribution.

**Sample I.** Twenty-five male packers were engaged in packing toilet requisites, photographic goods, stationery, surgical dressings, etc., into cartons and cases. The cases were collected from a central point in the department and then filled with goods and
packing straw. They were fastened and labelled ready for transport outward.

This sample does not satisfy the requirement of homogeneity of output, but a system of points was used based on an estimate of the average time for each kind of job. Output in points was calculated for each operator for a standard 8½-hour day based on information collected over a four-week period. The sample differs from the previous ones in that the men were paid time-and not piece-rates.

The age of each operator was known, and the effects of differing ages were removed. With the average output per standard day of about 148 points for a worker of twenty, the average fell by 0·55 of a point per day for each additional year of age. Using proportions instead of absolute changes, average output per standard day fell by 5% a year.

**Table XX**

| Variate.                  | Humpedness (mean deviation/standard deviation) | Asymmetry (skewness) |
|---------------------------|-----------------------------------------------|----------------------|
| Daily output              | 0·787                                         | +0·610               |
| Logarithm of daily output | 0·803                                         | +0·235               |

**Table XXI**

| Value of humpedness (expected value = 0·806) | Value of asymmetry (expected value zero) | Chance of such values occurring in a sample drawn at random from a normal population. |
|---------------------------------------------|-----------------------------------------|----------------------------------------------------------------------------------|
| 0·704 or less                               | { +0·061 or more [-0·061 or less] }     | 1 in 100                                                                         |
| 0·735 or less                               | { +0·711 or more [-0·711 or less] }     | 1 in 20                                                                          |
| 0·752 or less                               | --                                      | 1 in 10                                                                          |

Neither hypothesis can be discarded, but a log-normal distribution is the more likely one.

**Sample K.** The details concerning this sample were kindly supplied by Professor Sir F. C. Bartlett, C.B.E., F.R.S., Director of the Psychological Laboratory, University of Cambridge.¹ A test was devised in which a person had to repeat a cycle of operations similar to those performed by workers in light industry. Each person who underwent the test had no practice, and did not know that it was to last 30 minutes. The number of operations completed in this time by the hundred and seven airmen, sailors,

¹ The data were collected experimentally by Miss Ruth Brown for purposes which were not concerned with the particular aims of my investigation.
undergraduates, B.O.A.C. employees, etc., who did the test varied from 28 to 342.

The age-distribution of the sample was peculiar, and it was therefore essential that variations due to this should be removed. There was a significant association in the sample between age and the number of operations completed. With the average number of operations for a man of twenty approximately equal to 142, each year of age above this reduced the average by 0.62 of an operation. Alternatively, the passage of one year reduced the average number of operations completed by $\frac{3}{8}$%.

**Table XXII**

| Variate                      | Humpedness (mean deviation/standard deviation) | Asymmetry (skewness) |
|-----------------------------|-----------------------------------------------|----------------------|
| Output in 30 minutes        | 0.749                                         | +0.903               |
| Logarithm of output in 30 minutes | 0.781                                   | -0.537               |

**Table XXIII**

| Value of humpedness (expected value = 0.800) | Value of asymmetry (expected value zero) | Chance of such values occurring in a sample drawn at random from a normal population |
|---------------------------------------------|------------------------------------------|-------------------------------------------------------------------------------------|
| 0.749 or less                               | { +0.550 or more, -0.550 or less }       | 1 in 100                                                                            |
| 0.765 or less                               | { +0.377 or more, -0.377 or less }       | 1 in 20                                                                             |
| 0.773 or less                               |                                          | 1 in 10                                                                             |

Here there are very strong grounds for rejecting the hypothesis of a normal distribution, since the values of both measures could occur by chance less than once in a hundred times. The hypothesis of a log-normal distribution could also be rejected, although less emphatically, because of excessive asymmetry.

V

The evidence is certainly confusing, with varying results being given by the analysis of the different samples. It should, however, be emphasised that the sample $K$ differs in important respects from all the others. No selection, such as is probably at work in industry, operated in the laboratory test. There is no reason to believe that there was any systematic restriction of output. In the samples drawn from industry the selective process is likely to exaggerate asymmetry, whereas the motivation towards the production of a given output by all workers will
diminish it. It seems reasonable that the evidence of sample $K$
should be treated with more respect than that of the other samples.

Theoretically it is possible to combine all the samples so that
the two hypotheses can be tested on all the evidence at once, but
in practice such an attempt would entail an enormous burden of
computation. We must rest content with less sophisticated
methods of analysis.

Out of the twelve different samples (counting $E$ as two) in
three cases the hypothesis of a normal distribution can be rejected,
while that of a log-normal distribution could be discarded in four
instances. Judging by the criterion of humpedness alone, the
normal hypothesis is more probable on five occasions and the log-
normal hypothesis on seven. Using asymmetry alone as the
test, the normal hypothesis is more likely in four cases, and the
log-normal hypothesis in eight. Possibly this suggests that the
evidence is slightly more in favour of the log-normal distribution.

Considering the distribution of hourly output (or its equivalent),
in ten cases out of twelve the asymmetry is in the same direction,
\textit{i.e.}, the longer tail is at the upper end of the distribution. With
the logarithms of hourly output, the asymmetry is in the same
direction in nine instances but the longer tail is the lower one.
This would seem to suggest that the speed of output is distributed
asymmetrically, but not to the extent that applies to a log-normal
distribution.

Before, however, output per hour can be related to earnings
in a more lengthy period, the distribution of such things as hours
worked, different variations in speeds over time for different
workers, the incidence of ill health and of many other factors
must be brought into the picture. Even if output per hour does
not approximate very closely to the distribution of yearly earnings
the distribution of output per year may well do so.

Undoubtedly few positive deductions can be made as a result
of this investigation, but it is clear that empirical evidence of the
tendency for linear measurements in nature and Intelligence
Quotients to be distributed normally forms an inadequate
platform from which to attack the present (or the past) inequality
of incomes.

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