ARE NEUTRINO OSCILLATIONS A NON-STATIONARY PHENOMENON?

S. M. Bilenky

Joint Institute for Nuclear Research, Dubna, R-141980, Russia
Physik-Department E15, Technische Universität München, D-85748 Garching, Germany

F. von Feilitzsch and W. Potzel

Physik-Department E15, Technische Universität München, D-85748 Garching, Germany

Abstract

We discuss different schemes of neutrino oscillations and the time-energy uncertainty relation. From the results of the K2K and MINOS accelerator experiments follows that neutrino oscillations are a non-stationary phenomenon and that the standard time-energy uncertainty relation is satisfied. Are these properties a general feature of neutrino oscillations? In this paper we demonstrate that a recently proposed tritium/helium-3 Mössbauer neutrino-experiment could answer this question.

1 Introduction

The observation of neutrino oscillations in the Super Kamiokande atmospheric [1], SNO solar [2], KamLAND reactor [3], and other neutrino experiments [4, 5, 6] is one of the most important recent discoveries in particle physics. It is common opinion that small neutrino masses and peculiar neutrino mixing are signatures of physics beyond the Standard Model. The principal aim of future experiments concerning the study of neutrino oscillations, the search for neutrinoless double $\beta$-decay and the direct measurement of neutrino masses is to make a further step in the investigation of neutrino properties which would allow to reveal new physics determining the nature of neutrinos and of small neutrino masses.

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In the T2K [7], Double CHOOZ [8], Daya Bay [9] and other next-generation neutrino oscillation experiments, a high accuracy in the measurement of oscillation parameters will be achieved. An even higher accuracy is planned to be reached in experiments at future Super beam, $\beta$-beam and Neutrino Factory facilities (see Ref. [10]).

We will address the following related questions:

- Are neutrino oscillations a non-stationary phenomenon?
- Are stationary neutrino oscillations also possible?

## 2 Neutrino mixing

The investigation of neutrino oscillations is based on the following assumptions (see, for example, Refs. [11, 12, 13]):

1. The neutrino processes are described by the SM charged current (CC) and neutral current (NC) interactions. The Lagrangian of the standard leptonic CC interaction is given by the expression

$$\mathcal{L}_{I}^{CC}(x) = -\frac{g}{2\sqrt{2}} j_{\alpha}^{CC}(x) W_{\alpha}(x) + \text{h.c.},$$

(1)

where

$$j_{\alpha}^{CC}(x) = 2 \sum_{l=e,\mu,\tau} \bar{\nu}_{lL}(x) \gamma_{\alpha} l_{L}(x).$$

(2)

2. The field $\nu_{lL}(x)$ is a unitary "mixture" of left-handed components of fields of Majorana (or Dirac) neutrinos $\nu_{i}$ with mass $m_{i}$:

$$\nu_{lL}(x) = \sum_{i=1}^{3} U_{li} \nu_{iL}(x).$$

(3)

Here $U$ is the unitary PMNS mixing matrix.

The relation (3) is a relation between quantum fields. There is no dispute about this relation. There have been, however, many discussions on the consequences which follow from this relation for the observable neutrino transition probabilities (see Ref. [14] and references therein).
3 Neutrino oscillation data

All existing neutrino oscillation data can be described if we assume that

1. The number of massive neutrinos is equal to the number of flavor neutrinos (three).

2. The $\nu_l \to \nu_{l'}$ transition probability is given by the expression

$$P(\nu_l \to \nu_{l'}) = |\sum_{i=1}^{3} U_{li}^* e^{-i\Delta m_{ki}^2 L/2E} U_{li}|^2 .$$  \hspace{1cm} (4)

Here $\Delta m_{ki}^2 = m_i^2 - m_k^2$, $L$ is the distance between the neutrino source and the neutrino detector, $E$ is the neutrino energy, and the index $k$ is fixed.

The probabilities $P(\nu_l \to \nu_{l'})$ depend on six parameters (two mass-squared differences $\Delta m_{12}^2$ and $\Delta m_{23}^2$, three mixing angles $\theta_{12}, \theta_{23}$ and $\theta_{13}$, one phase $\delta$) and have rather complex forms. However, we can take into account that two parameters are small:

$$\frac{\Delta m_{12}^2}{\Delta m_{23}^2} \simeq 3 \cdot 10^{-2}, \quad \sin^2 \theta_{13} \lesssim 5 \cdot 10^{-2}. \hspace{1cm} (5)$$

If we neglect the contributions of these small parameters to the transition probabilities and consider the range of $L/E$ values characteristic for LBL atmospheric-neutrino experiments ($\Delta m_{23}^2 L/E \gtrsim 1$), the leading oscillations are, in this case, $\nu_\mu \leftrightarrow \nu_\tau$ (see, for example, Ref. [13]). For the probability of $\nu_\mu$ ($\bar{\nu}_\mu$) to survive we find the following expression

$$P(\nu_\mu \to \nu_\mu) = P(\bar{\nu}_\mu \to \bar{\nu}_\mu) \simeq 1 - \frac{1}{2} \sin^2 2\theta_{23}(1 - \cos \Delta m_{23}^2 L/2E) . \hspace{1cm} (6)$$

For the values of $L/E$ which satisfy the condition $\Delta m_{12}^2 L/E \gtrsim 1$ (KamLAND range), in leading approximation the probability of $\bar{\nu}_e$ to survive takes the two-neutrino form

$$P(\bar{\nu}_e \to \bar{\nu}_e) \simeq 1 - \frac{1}{2} \sin^2 2\theta_{12}(1 - \cos \Delta m_{12}^2 L/2E) . \hspace{1cm} (7)$$

Notice that both $\bar{\nu}_e \to \bar{\nu}_\mu$ and $\bar{\nu}_e \to \bar{\nu}_\tau$ transitions contribute to $P(\bar{\nu}_e \to \bar{\nu}_e)$. 

3
In leading approximation, also the probability of the solar $\nu_e$'s to survive in matter has the two-neutrino form and depends on the parameters $\Delta m^2_{12}$ and $\sin^2 \theta_{12}$.

From the analysis of the data of the S-K atmospheric neutrino experiment, the following 90% CL ranges were obtained \[1\] for the parameters $\Delta m^2_{23}$ and $\sin^2 2\theta_{23}$
\[
1.9 \cdot 10^{-3} \leq \Delta m^2_{23} \leq 3.1 \cdot 10^{-3} \text{eV}^2, \quad \sin^2 2\theta_{23} > 0.9. \tag{8}
\]
The best-fit values of the parameters are \[1\]
\[
\Delta m^2_{23} = 2.5 \cdot 10^{-3} \text{eV}^2, \quad \sin^2 2\theta_{23} = 1. \tag{9}
\]
The results of the S-K atmospheric neutrino experiment have been confirmed by the K2K \[5\] and MINOS \[6\] accelerator long-baseline experiments. From the analysis of the MINOS data was obtained \[6\]:
\[
\Delta m^2_{23} = (2.38^{+0.20}_{-0.16}) \cdot 10^{-3} \text{eV}^2, \quad \sin^2 2\theta_{23} > 0.84 \ (90\% \ CL). \tag{10}
\]
From the global analysis of the data of the KamLAND reactor experiment and the data of the solar neutrino experiments the following values of the parameters $\Delta m^2_{12}$ and $\tan^2 \theta_{12}$ were found \[3\]:
\[
\Delta m^2_{12} = (7.59^{+0.21}_{-0.21}) \cdot 10^{-5} \text{eV}^2, \quad \tan^2 \theta_{12} = 0.47^{+0.06}_{-0.05}. \tag{11}
\]

4 Neutrino production and detection.

Schrödinger equation for neutrino states

Let us consider the decay
\[
a \rightarrow b + l^+ + \nu_i. \tag{12}
\]
For the state of the final particles we have
\[
|f\rangle = \sum_i |\nu_i\rangle |l^+\rangle |b\rangle \langle \nu_i l^+ b |S| a\rangle. \tag{13}
\]
Here $|\nu_i\rangle = c_i^\dagger (p_i) |0\rangle$ is the state of a neutrino with four-momentum $p_i^0$ and helicity equal to -1 and $\langle \nu_i l^+ b |S| a\rangle$ is the matrix element of the process.
We are interested in neutrino energies $\gtrsim 1$ MeV. At such energies, the neutrino mass-squared differences can safely be neglected in the matrix elements $\langle \nu_l^+b | S | a \rangle$ since $\frac{\Delta m^2}{E^2} \lesssim 10^{-16}$. Thus, we have

$$\langle \nu_l^+b | S | a \rangle \approx U_{li}^* \langle \nu_l l^+_b | S | a \rangle_{SM}$$

Here $\langle \nu_l l^+_b | S | a \rangle_{SM}$ is the SM matrix element of the production of the flavor neutrino $\nu_l$ in the decay

$$a \rightarrow b + l^+ + \nu_l .$$

From (13) and (14) we obtain

$$|f\rangle \approx |\nu_l\rangle |l^+_b\rangle |b\rangle \langle \nu_l l^+_b | S | a \rangle_{SM} ,$$

where

$$|\nu_l\rangle = \sum_{i=1}^3 U_{li}^* |\nu_i\rangle$$

is the state of a flavor neutrino $\nu_l$ which is produced together with the lepton $l^+$ in CC weak decays.

We now will consider the propagation of the flavor neutrino states. In quantum field theory, the evolution equation is the Schrödinger equation

$$\frac{d}{dt} |\Psi(t)\rangle = H |\Psi(t)\rangle ,$$

where $H$ is the total Hamiltonian.

Let us assume that at $t = 0$ the flavor neutrino $\nu_l$ was produced. Thus, we have $|\Psi(0)\rangle = |\nu_l\rangle$. For the neutrino state in vacuum at time $t$, we obtain the following expression

$$|\Psi(t)\rangle = \sum_i |\nu_i\rangle e^{-iE_i t} U_{li}^* , \quad E_i = \sqrt{p_i^2 + m_i^2} .$$

If we develop the state $|\Psi(t)\rangle$ on the flavor states $|\nu_{l'}\rangle$ we have

$$|\Psi(t)\rangle = \sum_{l'} |\nu_{l'}\rangle \left( \sum_i U_{l'i} e^{-iE_i t} U_{li}^* \right) .$$

Neutrinos are detected via the observation of weak processes. Let us consider the process

$$\nu_{l'} + N \rightarrow l^' + X .$$
Neglecting extremely small terms of the order $\frac{\Delta m^2_{ik}}{E^2}$ for the matrix element of the process $\nu_i + N \rightarrow l' + X$ we obtain the following relation

$$\langle l'X | S | \nu_i N \rangle \simeq \langle l'X | S | \nu_{l'} N \rangle_{SM} U_{l'i} ,$$

(22)

where $\langle l'X | S | \nu_{l'} N \rangle_{SM}$ is the SM matrix element of the process (21). Taking into account the unitarity of the mixing matrix we find

$$\langle l'X | S | \nu_{l'} N \rangle = \sum_i \langle l'X | S | \nu_i N \rangle U_{l'i}^* \simeq \langle l'X | S | \nu_{l'} N \rangle_{SM} .$$

(23)

Thus, the normalized $\nu_i \rightarrow \nu_{l'}$ transition probability is given by the relation

$$P(\nu_i \rightarrow \nu_{l'}) = | \sum_i U_{l'i} e^{-i(E_i - E_{l'})t} U_{l'i}^* |^2 , \quad \sum_{l'} P(\nu_i \rightarrow \nu_{l'}) = 1 .$$

(24)

Summarizing, if we neglect terms of the order $\frac{\Delta m^2_{ik}}{E^2} \ll 1$ in the $S$-matrix elements, we come to the conclusion that

- in weak processes, flavor neutrinos $\nu_e$, $\nu_\mu$ and $\nu_\tau$ are produced. The states of the flavor neutrinos do not depend on the neutrino production (and detection) processes and are given by the relation (17).

- the matrix elements of the neutrino production and detection processes are given by the Standard Model.

- neutrino oscillations are taking place if the neutrino state $|\Psi(t)\rangle$ is a superposition of states of neutrinos with different energies (non-stationary states).

The non-stationary nature of neutrino oscillations was advocated by B. Pontecorvo and his collaborators in early neutrino oscillation papers [16, 11].

In the K2K [5] and MINOS [6] accelerator experiments, the time of neutrino production and the time of neutrino detection were measured. In the K2K experiment, $\nu_\mu$'s were produced in 1.1 $\mu$s spills. After the time $t \simeq L/c \simeq 0.8 \cdot 10^3 \mu$s, muon neutrinos were observed in the Super Kamiokande detector. Agreement with the S-K results for atmospheric neutrino oscillations was found. Thus, it was proven that neutrino oscillations observed in long-baseline neutrino experiments are a non-stationary phenomenon.

Here we will address the following questions: Are neutrino oscillations, in general, a non-stationary phenomenon? Are stationary neutrino oscillations also possible?
The oscillation phase in equation (24) is given by the following expression

$$
\phi_{ki} = (E_i - E_k) t \simeq (p_i - p_k) t + \frac{\Delta m^2_{ki}}{2E_k} t.
$$

(25)

Generally, we have for the difference of neutrino momenta

$$
(p_i - p_k) \simeq a \frac{\Delta m^2_{ki}}{2E},
$$

(26)

where $a \lesssim 1$. Thus, the first term in (25) could be comparable to the second one. We know, however, that all existing neutrino oscillation data are described by the expression (4) in which the oscillation phase is given by

$$
\phi_{ki} = \frac{\Delta m^2_{ki}}{2E} L
$$

(27)

Comparing (25) and (27) we come to the conclusion that in the case of ultrarelativistic neutrinos with $t \simeq L$ and the Schrödinger evolution equation, we need to assume that neutrinos with different masses in the mixed flavor state must have the same momenta ($p_i = p_k = p$). In this case, the flavor neutrino state $|\nu_l\rangle$ is characterized by the momentum $\vec{p}$.

5 Wave-function approach to neutrino propagation

We now will consider another approach to neutrino propagation (see, for example Ref. [14] and references therein). We will assume that the propagation of $\nu_i$ with four-momentum $p_\alpha^i$ is determined by the Dirac equation and that the wave function of the neutrino is the coherent superposition

$$
\Psi_{\nu_i}(x) = \sum_i e^{-ip_\alpha^i x_\alpha} U_{li}^* |i\rangle.
$$

(28)

According to the Dirac equation, $E_i = \sqrt{p_\alpha^i + m_i^2}$ and the vector $|i\rangle$ describes a neutrino with mass $m_i$ and helicity equal to -1.

In this case we find the following expression for the normalized probability of the transition $\nu_i \rightarrow \nu_{\nu'}$

$$
P(\nu_i \rightarrow \nu_{\nu'}) = |\sum_i U_{\nu_i} e^{-i(p_\alpha^i - p_\alpha^{\nu})x_\alpha} U_{\nu'}^*|^2.
$$

(29)
It is evident from (29) that transitions of flavor neutrinos are due to the fact that wave functions of neutrinos with different masses at the distance $\vec{x}$ and after the time $t$ gain different phases.

Let us consider the oscillation phase. We have

$$\phi_{ik} = (E_i - E_k)t - (p_i - p_k)x$$

(30)

For $E_i \neq E_k$, $p_i \neq p_k$, we obtain

$$\phi_{ik} \simeq \frac{\Delta m^2_{ik}}{2E}t - (p_i - p_k)(x - t)$$

(31)

The second term disappears because $x \simeq t$. Thus, if the neutrino evolution is described by a superposition of plane waves, the oscillation phase is given by the standard expression ($\phi_{ik} \simeq \frac{\Delta m^2}{2E}L$) in the general case that $E_i \neq E_k$, $p_i \neq p_k$.

Let us now consider the stationary case $E_i = E_k$. For the oscillation phase we obviously obtain the standard expression

$$\dot{\phi}_{ik} = -(p_i - p_k)x = \frac{\Delta m^2_{ik}}{2E}L.$$ 

(32)

Thus, if propagating neutrinos are described by a coherent superposition of plane waves, oscillations are also possible in the stationary case.

The two principally different possibilities we have discussed above (evolution of the neutrino state in time or evolution of the neutrino wave function in space and time) can not be distinguished by standard neutrino oscillation experiments. Special neutrino experiments are necessary. Such an experiment we will discuss in the last part of this paper.

6 Time-energy uncertainty relation for neutrino oscillations

We now discuss the time-energy uncertainty relation for neutrino oscillations[17]. All uncertainty relations are based on the Cauchy-Schwarz inequality

$$\Delta A \Delta B \geq \frac{1}{2} |\langle \Psi | [A, B] | \Psi \rangle|.$$ 

(33)
Here \( A \) and \( B \) are two Hermitian operators, \( |\Psi\rangle \) is some state and
\[
\Delta A = \sqrt{A^2 - \langle A \rangle^2}, \quad \langle A \rangle = \langle \Psi | A | \Psi \rangle.
\] (34)
The Heisenberg uncertainty relations are direct consequences of the commutation relations for \( A \) and \( B \). For example, from the commutation relation \([p, q] = \frac{1}{i}\) follows that
\[
\Delta p \Delta q \geq \frac{1}{2}.
\] (35)
Let us stress that the right-hand side of the Heisenberg uncertainty relations for canonically conjugated quantities does not depend on the state \( |\Psi\rangle \).

The time-energy uncertainty relation has a completely different character. This is connected with the fact that in quantum theory, time is a parameter and there is no operator of time.

The time-energy uncertainty relations for different systems are based on the fact that the evolution of the quantum systems is determined by the Hamiltonian. A general method of deriving the time-energy uncertainty relations was proposed by Mandelstam and Tamm [13].

Let us consider an operator \( O_H(t) \) in the Heisenberg representation. We have
\[
i \frac{d}{dt} O_H(t) = [O_H(t), H],
\] (36)
where \( H \) is the total Hamiltonian. From the Cauchy-Schwarz inequality (33) and relation (36) we find
\[
\Delta E \Delta O_H(t) \geq \frac{1}{2} \left| \frac{d}{dt} \overline{O}_H(t) \right|.
\] (37)
Here
\[
\overline{O}_H(t) = \langle \Psi_H | e^{iHt} O e^{-iHt} | \Psi_H \rangle = \langle \Psi(t) | O | \Psi(t) \rangle,
\] (38)
where \( O \) and \( |\Psi(t)\rangle = e^{-iHt} |\Psi_H\rangle \) are the operator and the vector of the state, respectively, in the Schrödinger representation. It is evident that from (37) nontrivial constraints can be obtained only for non-stationary processes.

In order to obtain the time-energy uncertainty relation for neutrino oscillations we will choose for \( O \) the operator of the projection on the flavor state \( |\nu_t\rangle \):
\[
O = |\nu_t\rangle \langle \nu_t |.
\] (39)
Assuming that $|\Psi(0)\rangle = |\nu_l\rangle$ we find

$$\overline{O}_H(t) = P_{\nu_l\rightarrow\nu_l}(t) ,$$

(40)

where $P_{\nu_l\rightarrow\nu_l}(t)$ is the $\nu_l$ survival probability.

If we take into account that $O^2 = O$, the Mandelstam-Tamm inequality (37) takes the form

$$\Delta E \geq \frac{1}{2} \frac{|d\nu_l\rightarrow\nu_l(t)|}{\sqrt{P_{\nu_l\rightarrow\nu_l}(t) - P_{\nu_l\rightarrow\nu_l}^2(t)}} .$$

(41)

Let us integrate (41) over the time $t$ from $t = 0$ to $t = t_{1\text{min}}$, where $t_{1\text{min}}$ is the time at which the survival probability $P_{\nu_l\rightarrow\nu_l}(t)$ reaches its first minimum. The time-energy uncertainty relation takes the form

$$\Delta E \Delta t \geq \frac{1}{2} \left( \frac{\pi}{2} - \arcsin(2 P_{\nu_l\rightarrow\nu_l}(t_{1\text{min}}) - 1) \right) ,$$

(42)

where $\Delta t = t_{1\text{min}}$.

We will first consider the $\nu_\mu \rightarrow \nu_\mu$ transition driven by $\Delta m_{23}^2$. We have in this case

$$\Delta t_{23} = t_{1\text{min}}^{(23)} = 2\pi \frac{E}{\Delta m_{23}^2} .$$

(43)

Taking into account that $P_{\nu_\mu\rightarrow\nu_\mu}(t_{1\text{min}}) \simeq 0$ we find

$$\Delta E \Delta t_{23} \geq \frac{\pi}{2} .$$

(44)

From this relation we obtain the following constraint on $\Delta E$:

$$\Delta E \geq \frac{\Delta m_{23}^2}{4E} .$$

(45)

This relation can easily be fulfilled in the atmospheric and long-baseline accelerator neutrino experiments.

For the $\bar{\nu}_e \rightarrow \bar{\nu}_e$ transition driven by $\Delta m_{23}^2$ we find

$$\Delta E \geq \sin 2\theta_{13} \frac{\Delta m_{23}^2}{2\pi E} .$$

(46)

Because $\sin^2 2\theta_{13} \lesssim 2 \cdot 10^{-1}$, relation (46) gives a much weaker constraint on $\Delta E$ than (45).
7 Mössbauer neutrino experiment

In Refs. [19, 20, 21], possibilities have been considered to perform an experiment on the detection of the tritium electron antineutrino with energy $\simeq 18.6$ keV in the recoilless (Mössbauer) transitions
\[ ^3\text{H} \rightarrow ^3\text{He} + \bar{\nu}_e, \quad \bar{\nu}_e + ^3\text{He} \rightarrow ^3\text{H}. \] (47)

In the case of oscillations driven by $\Delta m^2_{23}$ the oscillation length in such an experiment is equal to $L^{(23)}_{\text{osc}} \simeq 18.6$ m.

It was estimated in Ref. [19] that
\[ \Delta E \simeq 8.4 \times 10^{-12} \text{ eV} \] (48)

and the cross section of the resonance absorption of $\bar{\nu}_e$ in the process $\bar{\nu}_e + ^3\text{He} \rightarrow ^3\text{H}$ is equal to $\sigma_R \simeq 3 \times 10^{-33} \text{ cm}^2$.\(^2\)

Let us apply the time-energy uncertainty relation (46) to the Mössbauer neutrino experiment. For $\sin^2 2\theta_{13} = 2 \cdot 10^{-1}$ (the CHOOZ bound) we find that the energy uncertainty $\Delta E$ must satisfy the condition
\[ \Delta E \geq 9 \times 10^{-9} \text{ eV}. \] (49)

If $\sin^2 2\theta_{13} = 10^{-2}$ (this value corresponds to the sensitivity strived for with the future T2K [7] and Daya Bay [9] experiments) we have
\[ \Delta E \geq 2 \times 10^{-9} \text{ eV}. \] (50)

Thus, the time-energy uncertainty relation (46) is not satisfied in the case of a Mössbauer neutrino experiment with an uncertainty of the neutrino energy given by (48) (see Ref. [19]) and even less so with the much smaller energy uncertainty $\Delta E \simeq 1 \times 10^{-24}$ eV proposed in Ref. [20] (see, however, critical review in Ref. [22]). If neutrino oscillations will be observed in such an experiment it would mean that the Mandelstam-Tamm time-energy uncertainty relation is not a characteristic feature of neutrino oscillations.

8 Conclusion

In spite of neutrino oscillations having been observed in atmospheric, solar, reactor and long-baseline accelerator experiments there are several open

\[ ^2\text{For a more critical review, see Ref. [22].} \]
problems concerning the theory of neutrino oscillations. In this paper we addressed the question: is non-stationarity a characteristic feature of neutrino oscillations?

In reality, this is a question about the propagation of neutrinos from a source to a detector. We considered here two fundamentally different options. If the evolution of the state of flavor neutrinos \( |\nu_l> \) is determined by the Schrödinger equation for quantum states, neutrino oscillations are a non-stationary phenomenon. If the evolution of the emitted neutrinos \( \nu_i \) with momenta \( p_i \) is determined by the Dirac equation and the propagating neutrino is described by a coherent wave function which depends on \( \vec{x} \) and \( t \), both non-stationary and stationary neutrino oscillations are possible.

The standard neutrino-oscillation experiments cannot distinguish between these two possibilities. Special neutrino experiments are necessary to achieve this. We discussed a recently proposed tritium/helium-3 Mössbauer neutrino experiment with practically monochromatic neutrinos.\(^3\) Such an experiment would be able to decide which assumption on the neutrino propagation is correct.

Following the Mandelstam-Tamm method we derived the time-energy uncertainty relation for neutrino oscillations. In the standard neutrino-oscillation experiments this relation is satisfied. The Mössbauer neutrino experiment could answer the question whether the Mandelstam-Tamm time-energy uncertainty relation is universally applicable to neutrino oscillations.

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\(^3\)For another discussion of this Mössbauer neutrino experiment, see Ref. [23].
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