Application of Fractal Theory in Concrete Research

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Abstract. Fractal theory is a powerful tool to deal with chaotic and disordered phenomena and behaviors in nature. In recent years, it has been widely used in the research of concrete internal structure, and has achieved a lot of remarkable results. This paper introduces the origin and definition of fractal theory, summarizes the application status of fractal theory in the research of concrete pores, cracks and aggregates, analyzes the existing problems in the current research, and looks forward to the future development trend.

Keywords: fractal theory, concrete research, pore, crack.

1. Introduction

Fractal theory is a new nonlinear subject, which is a powerful tool to deal with the complex and fragmentary things and phenomena in nature and life. The concept of fractal was first proposed by Mandelbrot, an American scientist, in 1983, and words were created on the basis of the Latin root fractus [1]. In the same year, he published his book the fractal geometry of nature, which laid a mathematical foundation for fractal theory and other nonlinear related disciplines [2]. Fractal theory has not only profound theoretical significance, but also great practical value. Its appearance opens up a new way for people to solve the extremely irregular geometric problems that classical geometry can't deal with, and also changes the way and method of people's understanding of nature.

At present, there is no strict definition of fractal in the academic circle, which is mainly because the scope of fractal is extremely wide, and it is difficult to describe the scope of fractal completely. British mathematician Kenneth Falcone [3] proposed in his book the mathematical basis and application of fractal geometry that fractal can be defined in a way similar to the definition of "life" in biology, that is, not to pursue a clear definition of fractal, but to determine the characteristics of fractal. According to this point of view, Ma Shuo [4] summarized that when the set F satisfies the following characteristics, it satisfies fractal: set F has extremely fine details; set F is extremely irregular; set F has self-similar characteristics; generally, the fractal dimension of set F is greater than its topological dimension; in most cases, set F can be determined by simple methods.

Concrete is a kind of heterogeneous, isotropic, multi-phase mixed material which changes with time and environmental conditions. No matter the size and distribution of aggregate, mortar and pore in the forming process, or the deformation and cracks in the service process, the internal structure of concrete becomes extremely chaotic and complex. It is difficult to accurately describe the internal structure of concrete by using the rules of point, line, plane and three-dimensional composition of traditional European geometry. As a new theoretical tool to study the irregular phenomena in nature, fractal theory...
has been widely used in the field of civil engineering. Many studies show that concrete shows certain fractal characteristics in the process of forming and service. Fractal theory can quantitatively describe the geometric characteristics of the internal structure of concrete, and provide a new basis for the study of the relationship between the internal structure and macro performance of concrete. This paper summarizes the application progress of fractal theory in concrete research, summarizes the existing problems, and looks forward to the future development direction of concrete research based on fractal theory.

2. Application of fractal theory in concrete pore research

The pore shape of concrete is different, the structure is complex, the pore size distribution range is wide, across the macro, meso, micro three scales, is extremely chaotic, disordered, in order to better quantify the impact of concrete pore on the macro performance, many scholars introduce the fractal theory into the research of concrete pore. Xie Chao et al. [5] obtained the fractal dimension of concrete pore structure by using mercury porosimetry, and found that with the increase of pore fractal dimension, the pore of concrete first decreases and then increases, while the specific surface area first increases and then decreases, and with the increase of pore fractal dimension, the pressure resistance of concrete gradually decreases. Zhou Jinghai et al. [6] used mercury intrusion method to analyze the pore structure of fiber reinforced concrete, and investigated the relationship between pore fractal dimension and concrete carbonation depth. It was found that the larger the pore fractal dimension of concrete is, the more complex the pore distribution is, and the smaller the carbonation depth is. Zarnaghi et al. [7] measured the pore structure characteristics of self-compacting concrete by mercury intrusion method, analyzed the fractal dimension of pore structure, and found that the compressive strength increased with the increase of fractal dimension, with good correlation. Wang Jianchao et al. [8] combined fractal theory to study the carbonation depth of waste fiber recycled concrete. The results show that the smaller the fractal dimension of pore volume is, the greater the carbonation depth is. Based on the relationship between the fractal dimension of pore volume and carbonation depth, the carbonation depth prediction model of recycled fiber reinforced concrete is established. Wang Wanping [9] showed that the pore shape and pore size distribution can be comprehensively characterized by the pore integral dimension, which provides an important support for the comprehensive evaluation of pore structure. When the temperature is low, the diversity of pore structure is greater, the pore surface is rough, the complexity is large, and the micro pores are widely distributed, so the mechanical properties of concrete are better at this stage; when the fractal dimension is small, the pore surface area, porosity and average pore size are deteriorated to varying degrees, and the micro pores are greatly reduced. Liu et al. [10] used pumice and aeolian sand to improve the preparation of aeolian sand lightweight aggregate concrete, analyzed the micro pore structure characteristics of concrete, and quantitatively analyzed the pore radius distribution of concrete using nuclear magnetic resonance detection technology combined with fractal theory. The results show that when the aperture is 0-0.1 μ m, the fractal dimension is 1.2467 to 1.6835. When the aperture is 0.1 μ M-10 μ m, the fractal dimension is 2.8066 to 2.8899. When the aperture is larger than 10 μ m, the fractal dimension is 2.8653 to 2.9715. Tian Wei et al. [11] used CT technology to scan the freeze-thaw cycle concrete blocks, and reconstructed the spatial distribution and evolution law of internal pores in three dimensions. After comparing the fractal dimension of internal pore structure, it was found that the evolution law of fractal dimension can better characterize the development and expansion characteristics of internal pore structure of concrete under freeze-thaw environment and the evolution law of freeze-thaw damage of materials. Chen Wei et al. [12] calculated the fractal dimension of pores in high-strength concrete by box dimension method, and found that the change of fractal dimension of pores in high-strength concrete under high temperature is consistent with the deterioration of pores. Fu et al. [13] used CT technology to scan aerated concrete blocks, and used box dimension method to analyze and calculate the pore fractal dimension of 1205 slices. The results show that the pore fractal dimension of aerated concrete block is between 1.775 and 1.805. There is a strong correlation between pore fractal dimension and porosity, pore surface area and pore shape factor. Pore fractal dimension can effectively characterize the surface roughness and pore size distribution of pores.
3. Application of fractal theory in concrete crack research

As a kind of multiphase composite material, concrete will produce a lot of cracks at the interface of each phase during its service. These cracks have strong nonlinear characteristics, so it is very difficult to analyze them. Fractal theory can solve this problem well. By using fractal dimension, the development process of cracks can be quantified, so as to establish the relationship between crack development and concrete macro performance. Liu Jinghong et al. [14] carried out the 3D reconstruction of concrete blocks under uniaxial compression, and the fractal dimension of each model was calculated by using the fractal theory. The results show that the fractal dimension can better reflect the development of internal cracks in concrete, which provides a new method to study the damage evolution and failure process of concrete. Dang Funing et al. [15] carried out uniaxial static and dynamic tensile CT tests on concrete specimens, analyzed the fractal characteristics of cracks, and calculated the differential box dimension of CT scanning section. Zhang et al. [16] proposed an improved differential box counting method to calculate the fractal dimension of concrete CT image. The fitting error of the obtained fractal dimension is reduced by nearly 50% compared with the traditional DBC method. Jiang Shang et al. [17] carried out the bending test on the compression zone of the beam after the freeze-thaw cycle, and calculated the fractal dimension of the crack. The results show that the fractal dimension is proportional to the load. In the process of increasing the load, the fractal dimension becomes larger and the crack evolution becomes more complex. The beam shows that the fractal dimension of cracks can be used as an index to predict the safety performance of reinforced concrete members. Fan Xiaochun et al. [18] used fractal theory to study the flexural behavior of basalt reinforced waste steel fiber reinforced concrete beams. The results show that the surface cracks of basalt reinforced concrete beams with waste steel fiber meet the fractal characteristics in the loading process and limit state, and the fractal dimension is between 0.89 and 1.07. Adding appropriate amount of waste steel fiber into basalt reinforced concrete beams can limit the crack development. Yu Jiang et al. [19] carried out four point bending tests on concrete beams with different replacement rates of recycled aggregate, and calculated and analyzed the fractal dimension of surface cracks. The results show that, with the increase of replacement rate of recycled aggregate, the fractal dimension of cracks first decreases and then increases; when the concrete reaches the ultimate load, the fractal dimension of pure bending section is smaller than that of bending shear section, which indicates that the cracks in bending shear section are relatively dense. Ding Yining et al. [20] tested the permeability coefficient and calculated the section fractal dimension of fiber reinforced concrete specimens with different crack width and different crack roughness. The results show that the fractal dimension increases with the increase of fiber content, which can better characterize the roughness of fracture section. There is a strong functional relationship between fractal dimension, fracture width and mixed permeability coefficient. Liu Pan et al. [21] used steel slag powder to partially replace cement to prepare steel slag concrete, and used box counting method to calculate the fractal dimension of steel slag concrete cracks. The results show that with the increase of steel slag powder content, the fractal dimension of cracks decreases, the cracking of concrete tends to be simplified, and the crack resistance of concrete is improved.

4. Conclusion

It has been more than 20 years since fractal theory was applied to the research of concrete. It has achieved fruitful research results in a number of subdivision fields, especially in the quantitative analysis of the internal structure of concrete, which plays a vital role and provides a new method for people to understand and explore concrete. At present, there are still some problems in the application of fractal theory in concrete:

(1) At present, most of the fractal research is based on the experimental data, but due to the limitations of objective conditions, it is difficult to make enough specimens for comparison, so that most of the experimental conclusions are limited in certain conditions, the application range is very narrow, and it does not have universality and reference. Slightly changing the experimental parameters, the obtained functional equations will be very different.
(2) Many scholars focus on the function fitting of fractal dimension and macro performance of concrete internal structure, and get a simple change trend. However, there are few researches on how to parameterize the change process of concrete internal structure and how to establish a deeper mathematical model or constitutive equation.

(3) The determination of fractal dimension depends on the establishment of fractal model. Different fractal models have different definitions of fractal dimension, so the calculation methods and results are not the same. In the process of fractal calculation, some researches have the problem of unclear concept, which leads to the inaccuracy of the fractal model and the error of some conclusions.

(4) At present, the research on the fractal characteristics of concrete is still in its infancy. Many fractal dimension models come from other research fields. The different construction ideas directly determine that it is difficult to accurately describe the fractal characteristics of the internal structure of concrete, resulting in the deviation of the subsequent fractal dimension in the calculation results.

It is a main development direction in the future to overcome the above problems and solve the breadth and accuracy of the application of fractal theory in concrete. The future development trend can be summarized as follows:

(1) This paper explores the method of using finite element software to establish a random numerical model of concrete, and replaces the traditional experiment with numerical simulation, which gets rid of the dependence on experimental conditions and provides more possibilities for the research of concrete fractal.

(2) This paper explores the concrete fractal research by numerical simulation and experimental auxiliary verification, establishes the appropriate concrete numerical model, compares a small amount of experimental data with the numerical model, and verifies the accuracy of the numerical model. By changing the initial parameters, the concrete numerical model under different conditions can be constructed, which greatly reduces the experimental workload.

(3) In depth study on the fractal characteristics and macro properties of concrete, on the basis of fitting data, try to establish a mathematical model or constitutive equation.

(4) Based on the fractal theory, this paper explores a fractal model suitable for analyzing the internal structure of concrete.

Acknowledgements
The research work in this paper was supported by National Natural Science Foundation of China (51678374, 51808351) and Project of Education Department of Liaoning Province (lnqn2020004).

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