Dark Revelations of the $[SU(3)]^3$ and $[SU(3)]^4$
Gauge Extensions of the Standard Model

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Abstract

Two theoretically well-motivated gauge extensions of the standard model are $SU(3)_C \times SU(3)_L \times SU(3)_R$ and $SU(3)_q \times SU(3)_L \times SU(3)_l \times SU(3)_R$, where $SU(3)_q$ is the same as $SU(3)_C$ and $SU(3)_l$ is its color leptonic counterpart. Each has three variations, according to how $SU(3)_R$ is broken. It is shown here for the first time that a built-in dark $U(1)_D$ gauge symmetry exists in all six versions, and may be broken to discrete $Z_2$ dark parity. The available dark matter candidates in each case include fermions, scalars, as well as vector gauge bosons. This work points to the unity of matter with dark matter, the origin of which is not \textit{ad hoc}. 
Introduction: To extend the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry of the standard model (SM) of quarks and leptons, there are many possibilities. We focus in this paper on two such theoretically well-motivated ideas. The first \cite{1,2} is $SU(3)_C \times SU(3)_L \times SU(3)_R$, and the second \cite{3,4,5} is $SU(3)_q \times SU(3)_L \times SU(3)_l \times SU(3)_R$, where $SU(3)_q$ is the same as $SU(3)_C$ and $SU(3)_l$ is its color leptonic counterpart. It has been known for a long time that $[SU(3)]^3$ has three distinct variations, according to how $SU(3)_R$ is broken to $SU(2)_R$.

- (A) $(u, d)_R$ is a doublet, which corresponds to the conventional left-right model.
- (B) $(u, h)_R$ is a doublet \cite{6,7,8,9,10,11}, where $h$ is an exotic quark with the same charge as $d$, which corresponds to the alternative left-right model.
- (C) $(h, d)_R$ is a doublet \cite{12,13,14,15}, which implies that the vector gauge bosons of this $SU(2)_R$ are all neutral.

Note that in the early days of flavor $SU(3)$ for the $u, d, s$ quarks, these $SU(2)$ subgroups are called $T, V, U$ spins. The same three versions are obviously also possible for $[SU(3)]^4$.

Whereas these structures have been known for a long time, an important property of these models has been overlooked, i.e. the existence of a built-in dark $U(1)_D$ gauge symmetry already present in $[SU(3)]^3$ and $[SU(3)]^4$ under which the SM particles are distinguished from those of the dark sector. We will identify this symmetry in all six cases and discuss how it may fit into a viable extension of the SM.

**Dark Symmetries in $[SU(3)]^3$:** The fermion assignments under $SU(3)_C \times SU(3)_L \times SU(3)_R$ are

\[
q \sim (3,3^*,1) \sim \begin{pmatrix}
  d & u & h \\
  d & u & h \\
  d & u & h
\end{pmatrix},
\]

\[q \sim (3,3^*,1) \sim \begin{pmatrix}
  d & u & h \\
  d & u & h \\
  d & u & h
\end{pmatrix}, \tag{1}
\]
where the $I_{3L}$ values from left to right are $(-1/2, 1/2, 0)$ and the $Y_L$ values from left to right are $(-1/3, -1/3, 2/3)$;

$$\lambda \sim (1, 3, 3^*) \sim \begin{pmatrix} N & E^c & \nu \\ E & N^c & e \\ \nu^c & e^c & S \end{pmatrix},$$

(2)

where the $I_{3L}$ values from top to bottom are now $(1/2, -1/2, 0)$ and the $Y_L$ values from top to bottom are $(1/3, 1/3, -2/3)$, the $I_{3R}$ values from left to right are $(-1/2, 1/2, 0)$ and the $Y_R$ values from left to right are $(-1/3, -1/3, 2/3)$;

$$q^c \sim (3^*, 1, 3) \sim \begin{pmatrix} d^c & d^c & d^c \\ u^c & u^c & u^c \\ h^c & h^c & h^c \end{pmatrix},$$

(3)

where the $I_{3R}$ values from top to bottom are $(1/2, -1/2, 0)$ and the $Y_R$ values from top to bottom are $(1/3, 1/3, -2/3)$. The electric charge operator is given by

$$Q = I_{3L} - \frac{Y_L}{2} + I_{3R} - \frac{Y_R}{2}.$$  

(4)

Since $(d^c, u^c)$ and $(e^c, \nu^c)$ are $SU(2)_R$ doublets, this reduces to the conventional left-right model. Consider now

$$D_A = 3(Y_L - Y_R).$$  

(5)

The $[Q, D_A]$ assignments of $q$, $\lambda$, and $q^c$ are then given by

$$Q_q = \begin{pmatrix} -1/3 & 2/3 & -1/3 \\ -1/3 & 2/3 & -1/3 \\ -1/3 & 2/3 & -1/3 \end{pmatrix}, \quad D_q = \begin{pmatrix} -1 & -1 & 2 \\ -1 & -1 & 2 \\ -1 & -1 & 2 \end{pmatrix},$$

(6)

$$Q_\lambda = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \quad D_\lambda = \begin{pmatrix} 2 & 2 & -1 \\ 2 & 2 & -1 \\ -1 & -1 & -4 \end{pmatrix},$$

(7)

$$Q_{q^c} = \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ -2/3 & -2/3 & -2/3 \\ 1/3 & 1/3 & 1/3 \end{pmatrix}, \quad D_{q^c} = \begin{pmatrix} -1 & -1 & -1 \\ -1 & -1 & -1 \\ 2 & 2 & 2 \end{pmatrix}.$$  

(8)
This shows that \( u, u^c, d, d^c, \nu, \nu^c, e, e^c \) have \( D_A = -1 \) (odd), whereas \( h, h^c, N, N^c, E, E^c, S \) have even \( D_A \) charges, i.e. 2 and \(-4\). Let us define a parity [16] using the particle’s spin \( j \):

\[
R_A = (-1)^{D_A + 2j}. \tag{9}
\]

Since \( j = 1/2 \), \( R_A \) is even for \( u, u^c, d, d^c, \nu, \nu^c, e, e^c \) and odd for \( h, h^c, N, N^c, E, E^c, S \), thereby allowing the latter to be considered as belonging to the dark sector, as long as \( U(1)_D \) is broken only by two units, in analogy to the breaking of \( B - L \) in models of neutrino mass, where lepton parity \((-1)^L\) remains conserved.

To break \( [SU(3)]^3 \), a scalar bitriplet \( \phi \sim (1, 3, 3^*) \) is used. It transforms exactly as \( \lambda \) and has the same \([Q, D]\) assignments. Now \( \langle \phi_{33} \rangle \) breaks \( SU(3)_L \times SU(3)_R \) to \( SU(2)_L \times SU(2)_R \times U(1)' \). The \( U(1)_D \) symmetry is broken by 4 units at the same time. This gives masses to the exotic fermions \( h, N, E \). Two other neutral scalars \( \phi_{11}, \phi_{22} \) have \( D_A = 2 \).

Their vacuum expectation values would break \( SU(2)_L \times SU(2)_R \) to \( U(1)' \), and \( U(1)_D \) by 2 units, allowing mass terms for \( uu^c, dd^c, ee^c, \nu \nu^c, NS, \) and \( N^c S \). At this point, it looks like a dark residual \( Z_2 \) symmetry is still possible. However this is not a viable scenario, because the \( SU(2)_L \) and \( SU(2)_R \) breaking are now at the same scale, contrary to what is observed. Furthermore, both \( I_{3L} + I_{3R} \) and \( Y_L + Y_R \) are still unbroken. Whereas \( Q \) is a linear combination of the two, there remains another unbroken \( U(1) \) gauge symmetry. To solve these problems, the usual procedure is to allow \( \phi_{31} \) and \( \phi_{13} \) to acquire nonzero vacuum expectation values as well, thus breaking \( SU(2)_R \) and \( SU(2)_L \) separately. However, since they have \( D_A = -1 \) (odd \( R_A \)), the dark symmetry is lost.

To save the dark symmetry, we insert another bitriplet \( \eta \sim (1, 3, 3^*) \) with an extra \( Z_2 \) symmetry under which it is odd and all other fields are even. This extra symmetry prevents \( \eta \) from coupling to the quarks and leptons, so that the absolute \( R_A \) values of the \( \eta \) components are not fixed by them as in \( \phi \). However their relative \( R_A \) values are still fixed by the gauge bosons. Using Eqs. (5) and (9), we see that of the eight \( SU(3)_L \) and eight \( SU(3)_R \) gauge
bosons, the four gauge bosons which take $u$ and $d$ to $h$, and the corresponding ones which take $u^c$ and $d^c$ to $h^c$ are odd under $R_A$, and the others are even. We can now choose $\langle \eta_{31} \rangle \neq 0$ and $\langle \eta_{13} \rangle \neq 0$ to break $SU(3)_L \times SU(3)_R$ to just $U(1)_Q$ and preserve $R_A$, because $\eta_{31}, \eta_{32}, \eta_{13}, \eta_{23}$ may be defined to be even and the other components odd without breaking $R_A$.

Of the 27 fermion fields for each family, 16 are in the visible sector ($R_A$ even), i.e. $u, u^c, d, d^c, \nu, \nu^c, e, e^c$, and 11 are in the dark sector ($R_A$ odd), i.e. $h, h^c, N, N^c, E, E^c, S$. Of the 24 gauge bosons, 16 are visible, i.e. the 8 gluons, $W^+_L, W^+_R$, the photon, $Z$, and two other heavier neutral ones, a linear combination of which couples to the dark charge $D_A$, and 8 are dark, i.e. those with odd $R_A$. The scalars are also divided into sectors with even and odd $R_A$. This is thus a model with possible fermion, scalar, and vector dark-matter candidates. Their existence is not an ad hoc invention, but a possible outcome of the postulated theoretical framework beyond the standard model.

Consider next the alternative left-right model, i.e. variation (B), where $d^c$ is switched with $h^c$ and $(\nu, e, S)$ are switched with $(N, E, \nu^c)$, i.e.

$$q^c \sim \begin{pmatrix} h^c & h^c & h^c \\ u^c & u^c & u^c \\ d^c & d^c & d^c \end{pmatrix}, \quad \lambda \sim \begin{pmatrix} \nu & E^c & N \\ e & N^c & E \\ S & e^c & \nu^c \end{pmatrix}. \quad (10)$$

The electric charge is given as before by Eq. (4), but the dark charge is now

$$D_B = 3(Y_L + I_{3R} + \frac{Y_R}{2}). \quad (11)$$

Hence $D_q$ remains the same as in Eq. (6), but $D_\lambda$ and $D_{q^c}$ are now given by

$$D_\lambda = \begin{pmatrix} -1 & 2 & 2 \\ -1 & 2 & 2 \\ -4 & -1 & -1 \end{pmatrix}, \quad D_{q^c} = \begin{pmatrix} 2 & 2 & 2 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \end{pmatrix}. \quad (12)$$

Again using $R_B = (-1)^{D_B+2j}$, we find it to be even for $u, u^c, d, d^c, \nu, \nu^c, e, e^c$ and odd for $h, h^c, N, N^c, E, E^c, S$. Choosing $\phi_{13}, \phi_{22}, \phi_{31}$ to have nonzero vacuum expectation values, the symmetry breaking pattern is as in (A), only that the $SU(2)$ subgroup of $SU(3)_R$ is now
different. It suffers from the same problems as in (A), which may be solved again by adding \( \eta \), with \( \langle \eta_{33} \rangle \neq 0 \) and \( \langle \eta_{11} \rangle \neq 0 \).

In the third variation (C), \( u^c \) is switched with \( h^c \), and \( (\nu, e, S) \) are switched with \( (E^c, N^c, e^c) \), i.e.

\[
q^c \sim \begin{pmatrix}
d^c & d^c & d^c \\
h^c & h^c & h^c \\
u^c & u^c & u^c
\end{pmatrix}, \quad 
\lambda \sim \begin{pmatrix}
N & \nu & E^c \\
E & e & N^c \\
\nu^c & S & e^c
\end{pmatrix}.
\]

(13)

The electric charge and dark charge are now given by

\[
Q = I_{3L} - \frac{Y_L}{2} + Y_R, \quad D_C = 3(Y_L - I_{3R} + \frac{Y_R}{2}).
\]

(14)

Hence

\[
Q_\lambda = \begin{pmatrix}
0 & 0 & 1 \\
-1 & -1 & 0 \\
0 & 0 & 1
\end{pmatrix}, \quad D_\lambda = \begin{pmatrix}
2 & -1 & 2 \\
2 & -1 & 2 \\
-1 & -4 & -1
\end{pmatrix},
\]

(15)

\[
Q_{qe} = \begin{pmatrix}
1/3 & 1/3 & 1/3 \\
-2/3 & -2/3 & -2/3
\end{pmatrix}, \quad D_{qe} = \begin{pmatrix}
-1 & -1 & -1 \\
2 & 2 & 2 \\
-1 & -1 & -1
\end{pmatrix}.
\]

(16)

Again using \( R_C = (-1)^{D_C+2j} \), we find it to be even for \( u, u^c, d, d^c, \nu, \nu^c, e, e^c \) and odd for \( h, h^c, N, N^c, E, E^c, S \). Choosing \( \phi_{11}, \phi_{23}, \phi_{32} \) to have nonzero vacuum expectation values, the pattern of symmetry breaking is the same as in (A) and (B), but the \( SU(2)_R \) subgroup is different from either. It suffers from the same problems as the two previous cases, and they are again solved by adding \( \eta \), with \( \langle \eta_{31} \rangle \neq 0 \) and \( \langle \eta_{12} \rangle \neq 0 \). However, in contrast to the variations (A) and (B), the \( \phi_{33} \) and \( \eta_{33} \) entries are not neutral, so it is not possible to preserve \( SU(2)_L \times SU(2)_R \) as a low-energy subgroup.

**Gauge Boson Masses in (B)**: Consider the breaking of \( SU(3)_L \times SU(3)_R \) by a very large \( \langle \eta_{33} \rangle = v_{33} \). Of the 8 vector gauge bosons \( W_i^L \) of \( SU(3)_L \) and the 8 vector gauge bosons \( W_i^R \) of \( SU(3)_R \), 9 become very heavy. The remaining 7 are the 3 of \( SU(2)_L \), the 3 of \( SU(2)_R \), and the one linear combination \( W_8^V = (W_8^L + W_8^R)/\sqrt{2} \). We assume that they survive to just
above the electroweak scale with equal couplings \((g)\) for \(SU(2)_L\) and \(SU(2)_R\) and a different one \((g')\) for \(Y_L + Y_R\). Let \(\langle \eta_{11} \rangle = v_{11}, \langle \phi_{22} \rangle = v_{22}, \langle \phi_{13} \rangle = v_{13}, \langle \phi_{31} \rangle = v_{31}\), then

\[
M^2(W^R_{1\,2}) = \frac{g^2}{2} [v^2_{11} + v^2_{22} + v^2_{31}], \tag{17}
\]

where \((W^R_1 + iW^R_2)/\sqrt{2} = W^+_R\) are the charged \(SU(2)_R\) gauge bosons with odd \(R_B\). The other gauge bosons have even \(R_B\) with

\[
M^2(W^L_{1\,2}) = \frac{g^2}{2} [v^2_{11} + v^2_{22} + v^2_{13}], \tag{18}
\]

and the massless photon given by

\[
A = \frac{e}{g} (W^L_3 + W^R_3) - \frac{e}{g'} \frac{\sqrt{2}}{3} W^V_8. \tag{19}
\]

This implies

\[
\frac{e^2}{g'^2} = \frac{3}{2} (1 - 2 \sin^2 \theta_W). \tag{20}
\]

If \(g' = g\) (which is valid at the unification scale), then \(\sin^2 \theta_W = 3/8\) as expected. Now \(v_{31}\) breaks \(SU(2)_R\) without breaking \(SU(2)_L\), so its value may be greater than the electroweak scale. Its associated gauge boson \(Z'\) is given by

\[
Z' = \frac{\sqrt{2}gW^R_3 + \sqrt{3}g'W^V_8}{\sqrt{2}g^2 + 3g'^2} = \frac{1}{\cos \theta_W} [\sqrt{1 - 2 \sin^2 \theta_W} W^R_3 + \sin \theta_W W^V_8]. \tag{21}
\]

Hence the SM \(Z\) boson is now

\[
Z = \cos \theta_W W^L_3 - \tan \theta_W [\sin \theta_W W^R_3 - \sqrt{1 - 2 \sin^2 \theta_W} W^V_8]. \tag{22}
\]

The \((Z, Z')\) mass-squared matrix is given by

\[
M^2_{ZZ} = \frac{g^2}{2 \cos^2 \theta_W} [v^2_{11} + v^2_{22} + v^2_{13}], \tag{23}
\]

\[
M^2_{Z'Z'} = \frac{g^2}{2} \left[ \frac{\cos^2 \theta_W}{1 - 2 \sin^2 \theta_W} v^2_{31} + \frac{1 - 2 \sin^2 \theta_W - \cos^2 \theta_W (v^2_{11} + v^2_{22}) + 2 \tan^2 \theta_W v^2_{13}}{\cos^2 \theta_W v^2_{13} - (1 - 2 \sin^2 \theta_W)(v^2_{11} + v^2_{22})} \right], \tag{24}
\]

\[
M^2_{ZZ'} = \frac{g^2 \tan^2 \theta_W}{2 \sqrt{1 - 2 \sin^2 \theta_W}} [\sin^2 \theta_W v^2_{13} - (1 - 2 \sin^2 \theta_W)(v^2_{11} + v^2_{22})]. \tag{25}
\]
To avoid $Z - Z'$ mixing so as not to upset precision electroweak measurements, $M_{ZZ'}^2$ may be chosen to be negligible in the above.

In this alternative left-right model, $(u, h)_R$ and $(S, e)_R$ are $SU(2)_R$ doublets with $h$ and $S$ odd under $R_B$. The mass terms for $u$ and $v$ come from $v_{22}$, those for $d$ and $e$ from $v_{13}$, those for $h$, $E$ from $v_{31}$, and the $3 \times 3$ matrix spanning $(N, N^c, S)$ from all three. As such, it contains the necessary ingredients for a consistent model of built-in dark matter. In variation (C), it has already been noted that $SU(2)_L \times SU(2)_R$ cannot be maintained as a low-energy subgroup. Hence the associated dark sector must be very heavy and does not lead to a realistic model. In variation (A), whereas $SU(2)_L \times SU(2)_R$ may emerge as a low-energy subgroup, the dark sector consists of singlets under this symmetry and must also be very heavy.

\textit{Dark Symmetries in $[SU(3)]^4$}: The notion of leptonic color \cite{17, 18} is based on quark-lepton interchange symmetry. Postulating $SU(3)_l$ to go with $SU(3)_q$, leptons have three color components to begin with, but $SU(3)_l$ is broken to $SU(2)_l$ which remains exact, so that two of these leptonic color fields are confined in analogy to the three color quarks being confined. The third unconfined component is the observed lepton of the SM. The new particles of this model are not easily produced and observed at the Large Hadron Collider, but will have unique signatures in a future lepton collider, as recently discussed \cite{5}. Under $SU(3)_q \times SU(3)_L \times SU(3)_l \times SU(3)_R$, $q \sim (3, 3^*, 1, 1)$ as in Eq. (1) and $q^c \sim (3^*, 1, 1, 3)$ as in Eqs. (3), (10), and (13) for the three variations (A,B,C) in parallel to what has been discussed for $[SU(3)]^3$. As for the leptonic sector,

\begin{equation}
l \sim (1, 3, 3^*, 1) \sim \begin{pmatrix} x_1 & x_2 & \nu \\ y_1 & y_2 & e \\ z_1 & z_2 & n \end{pmatrix}
\end{equation}

is the same in all three variations, in analogy to $q$, whereas $l^c$ has three variations to match

8
\( q^c \), i.e.

\[
l^c \sim (1, 1, 3, 3^*) \sim \begin{pmatrix} x_1^c & y_1^c & z_1^c \\ x_2^c & y_2^c & z_2^c \\ \nu^c & e^c & n^c \end{pmatrix}, \quad \begin{pmatrix} z_1^c & y_1^c & x_1^c \\ z_2^c & y_2^c & x_2^c \\ n^c & e^c & \nu^c \end{pmatrix}, \quad \begin{pmatrix} x_1^c & z_1^c & y_1^c \\ x_2^c & z_2^c & y_2^c \\ \nu^c & n^c & e^c \end{pmatrix}.
\] (27)

The electric charge and dark charge in (A) are given by

\[
Q = I_{3L} - \frac{Y_L}{2} + I_{3R} - \frac{Y_R}{2}, \quad D_A = 3(Y_L - Y_R).
\] (28)

Hence

\[
Q_l = \begin{pmatrix} 1/2 & 1/2 & 0 \\ -1/2 & -1/2 & -1 \end{pmatrix}, \quad Q_{l^c} = \begin{pmatrix} -1/2 & 1/2 & -1/2 \\ -1/2 & 1/2 & -1/2 \end{pmatrix}, \quad D_l = -D_{q^c}
\]

and \( D_l = -D_{q^c} \) of Eq. (8), \( D_{l^c} = -D_q \) of Eq. (6), i.e. \( u, u^c, d, d^c, \nu, \nu^c, e, e^c, x, x^c, y, y^c \) have \( D_A = 1 \) (odd), whereas \( h, h^c, n, n^c, z, z^c \) have \( D_A = -2 \) (even). Again let \( R_A = (-1)^{D_A+2} \), then the former group of fermions is even and the latter odd, i.e. belonging to the dark sector if \( U(1)_D \) is broken only by two units.

The breaking of \( SU(3)_L \times SU(3)_R \) by a scalar bitriplet \( \phi \sim (1, 3, 1, 3^*) \), which couples also to the fermions, proceeds as before. It has the same problems as discussed in the \([SU(3)]^3\) case. However, there are now two additional scalar bitriplets \([4]\) in \([SU(3)]^4\) with nonzero vacuum expectation values, i.e.

\[
\phi^L \sim (1, 3, 3^*, 1) \sim l, \quad \phi^R \sim (1, 1, 3, 3^*) \sim l^c.
\] (30)

They have thus the same would-be \([Q, D]\) assignments. They are not responsible for fermion masses, but are required to break leptonic color \( SU(3)_l \) to \( SU(2)_l \). Now \( \phi^L_{33} \) has \( D_A = 2 \) which may be used to break \( SU(3)_l \times SU(2)_L \) to \( SU(2)_l \times SU(2)_L \times U(1)_{Y_l+Y_L} \). To break \( SU(2)_R \) as well without breaking \( R_A \), we use the same trick as before by assigning \( \phi^R \) an odd parity under \( Z_2 \) as in \([SU(3)]^3\) for \( \eta \). To preserve the \( R_A \) parity for the gauge bosons, we may again define \( \phi^R_{11}, \phi^R_{12} \) to be even, and \( \phi^R_{33} \) to be odd. Now \( \langle \phi^R_{33} \rangle \) breaks \( SU(3)_l \) to \( SU(2)_l \),

\[\]
but it also breaks $SU(2)_R$ without breaking $SU(2)_L$. It allows thus the separation of the $SU(2)_R$ scale without breaking the dark parity $R_A$.

In the second variation (B), the electric charge is again the same as in (A) and the dark charge is the same as in (B) of $[SU(3)]^3$, i.e. Eq. (11). Using the same changes in the pattern of symmetry breaking as discussed before, a model with dark $Z_2$ symmetry is again achieved. Here $\langle \phi_{33}^R \rangle$ breaks $SU(3)_l \times SU(3)_R$ to $SU(2)_l \times SU(2)_R \times U(1)_{Y_l+Y_R}$ and separates the $SU(2)_l$ scale from the breaking of $SU(2)_R$ by $\langle \phi_{31} \rangle$. This is the analog of the alternative left-right model in the $[SU(3)]^3$ case. Applying $\langle \phi_{33}^L \rangle$ as well, the residual $U(1)$ symmetry is now $Y_L + Y_R + Y_l$, exactly as needed for the electric charge of Eq. (28). In the third variation (C), the electric charge is

$$Q = I_{3L} - \frac{Y_L}{2} + Y_R - \frac{Y_l}{2},$$

and the dark charge is the same as $D_C$ of Eq. (14). It also results in a model with dark $Z_2$ symmetry. However, as with its $[SU(3)]^3$ analog, it is not possible to preserve $SU(2)_L \times SU(2)_R$ as a low-energy subgroup. Note that $\sin^2 \theta_W = 1/3$ at the unification scale for $[SU(3)]^4$ which is of order $10^{11}$ GeV for a nonsupersymmetric model [4, 5].

**Concluding Remarks**: The existence of a dark sector is easily implemented by adding a new symmetry and new particles to the standard model. There are indeed numerous such proposals. As a guiding principle, supersymmetry is a well-known and perhaps the only example, where superpartners of all SM particles belong to the dark sector. In this paper, we suggest another, i.e. that such a dark symmetry may have a gauge origin buried inside a complete extended theoretical framework for the understanding of quarks and leptons. The inevitable consequence of this hypothesis is to divide all fermions, scalars, as well as *vector gauge bosons* into two categories. One includes all known particles of the SM and some new ones; the other is the dark sector. They are however intrinsically linked to each other as essential components of the unifying framework.
We consider as first examples $[SU(3)]^3$ and $[SU(3)]^4$, and identified the exact nature of this dark symmetry in three variations of the above two unified symmetries. We have shown how this dark gauge symmetry is broken to the discrete $Z_2$ dark parity which stabilizes dark matter. Whereas all these models contain dark matter, only variation (B) in either $[SU(3)]^3$ or $[SU(3)]^4$ allows it to be such that it exists at or near the electroweak scale. They may serve as the prototypes for a deeper understanding of the origin of dark matter as a built-in symmetry of a theoretically motivated extension of the Standard Model. Our study points to the unity of matter with dark matter, the origin of which is not \textit{ad hoc}. Other possible candidates are $SU(6)$ \cite{19,20} and $SU(7)$ \cite{20}. Future more detailed explorations are called for.

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