Abstract: In this paper, we study the semi-Hyers–Ulam–Rassias stability and the generalized semi-Hyers–Ulam–Rassias stability of some partial differential equations using Laplace transform. One of them is the convection partial differential equation.

Keywords: semi-Hyers–Ulam–Rassias stability; generalized semi-Hyers–Ulam–Rassias stability; Laplace transform; convection partial differential equation

MSC: 44A10; 35B35

1. Introduction

It is well known that the study of Ulam stability began in 1940, with a problem posed by Ulam concerning the stability of homomorphisms [1]. In 1941, Hyers [2] gave a partial answer in the case of the additive Cauchy equation in Banach spaces.

After that, Obloza [3] and Alsina and Ger [4] began the study of the Hyers–Ulam stability of differential equations. The field continued to develop rapidly. Linear differential equations were studied in [5–7], integral equations in [8], delay differential equations in [9], linear difference equations in [10,11], other equations in [12], and systems of differential equations in [13]. A summary of these results can be found in [14].

The Hyers–Ulam stability of linear differential equations was studied using the Laplace transform by H. Rezaei, S. M. Jung, and Th. M. Rassias [15], and by Q. H. Alqifiary and S. M. Jung [16]. This method was also used in [17–19].

The study of the stability of partial differential equations began in 2003, with the paper [20] of A. Prastaro and Th.M. Rassias. The Ulam–Hyers stability of partial differential equations was also studied in [21–26].

In [27], M. N. Qarawani used the Laplace transform to establish the Hyers–Ulam–Rassias–Gavruta stability of initial-boundary value problem for heat equations on a finite rod:

\[
\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}, \quad t > 0, 0 < x < l.
\]

In [28], D.O. Deborah and A. Moyosola studied nonlinear, nonhomogeneous partial differential equations using the Laplace differential transform method:

\[
\frac{d^2 w(x,t)}{dt^2} + a_n(x)Rw(x,t) + b_n(x)Sw(x,t) = f(x,t), \quad t > 0, x > 0, n \in \mathbb{N},
\]

where \(a_n(x), b_n(x)\) are variable coefficients, \(n \in \mathbb{N}\), \(R\) is the linear operator, \(S\) is the nonlinear operator, and \(f(x,t)\) is the source function.

In [29], E. Bicer used the Sumudu transform to study the equation:

\[
y_t - ky_{xx} = 0, \quad k \text{ a positive real constant}, \quad (x,t) \in D, \quad D = (x_0,x] \times (0,\infty).
\]
In [30], the Poisson partial differential equation
\[ u_{xx}(x, y) + u_{yy}(x, y) = g(x, y) \]
is studied via the double Laplace transform method (DLTM).

In the following sections, we will study the semi-Hyers–Ulam–Rassias stability and the generalized semi-Hyers–Ulam–Rassias stability of some partial differential equations using Laplace transform. One of them is the convection partial differential equation:
\[ \frac{\partial y}{\partial t} + a \frac{\partial y}{\partial x} = 0, \quad a > 0, \quad x > 0, \quad t > 0, \quad y(0, t) = c, \quad y(x, 0) = 0. \] (1)

A physical interpretation [31] of these equations is a river of solid goo, since we do not want anything to diffuse. The function \( y = y(x, t) \) is the concentration of some toxic substance. The variable \( x \) denotes the position where \( x = 0 \) is the location of a factory spewing the toxic substance into the river. The toxic substance flows into the river so that at \( x = 0 \), the concentration is always \( C \). We also study the semi-Hyers–Ulam–Rassias stability of the following equation:
\[ \frac{\partial y}{\partial t} + \frac{\partial y}{\partial x} - x = 0, \quad x > 0, \quad t > 0, \quad y(0, t) = 0, \quad y(x, 0) = 0. \] (2)

Our results regarding Equation (1) complete those obtained by S.-M. Jung and K.-S. Lee in [22]. In [22], the following equation:
\[ a \frac{\partial y(x, t)}{\partial x} + b \frac{\partial y(x, t)}{\partial t} + cy(x, t) + d = 0, \quad a, b \in \mathbb{R}, \quad b \neq 0, \quad c, d \in \mathbb{C}, \quad \text{with} \ \Re(c) \neq 0, \] (3)
where \( \Re(c) \) denotes the real part of \( c \), was studied. In our paper, we consider the case \( c = 0 \) in Equation (3). Moreover, we also study the generalized stability. The method used in [22] was the method of changing variables.

2. Preliminaries

We first recall some notions and results regarding the Laplace transform.

Let \( f : (0, \infty) \rightarrow \mathbb{R} \) be a piecewise differentiable and of exponential order, that is \( \exists M > 0 \) and \( a_0 \geq 0 \) such that
\[ |f(t)| \leq M \cdot e^{a_0 t}, \quad \forall t > 0. \]

We denote by \( \mathcal{L}[f] \) the Laplace transform of the function \( f \), defined by
\[ \mathcal{L}[f](s) = F(s) = \int_{0}^{\infty} f(t)e^{-st}dt. \]

Let
\[ u(t) = \begin{cases} 0, & \text{if } t \leq 0 \\ 1, & \text{if } t > 0 \end{cases} \]
be the unit step function of Heaviside. We write \( f(0) \) instead of the lateral limit \( f(0^+) \). The following properties are used in the paper:
\[ \mathcal{L}[t^n](s) = \frac{n!}{s^{n+1}}, \quad s > 0, \quad n \in \mathbb{N}, \]
\[ \mathcal{L}^{-1}\left[ \frac{1}{s^n} \right](t) = \frac{t^{n-1}}{(n-1)!}u(t), \]
\[ \mathcal{L}[f'](s) = s\mathcal{L}[f](s) - f(0), \]
\[ \mathcal{L}[f(t-a)u(t-a)](s) = e^{-as}F(s), \quad a > 0, \]
hence,
\[ L^{-1}[e^{-as}F(s)](t) = f(t - a)u(t - a). \]

We now consider the function \( y : (0, \infty) \times (0, \infty) \to \mathbb{R}, y = y(x, t), \) a piecewise differentiable and of exponential order with respect to \( t \). The Laplace transform of \( y \) with respect to \( t \) is as follows:
\[ L[y(x, t)] = \int_0^\infty y(x, t)e^{-st}dt, \]
where \( x \) is treated as a constant. We also denote the following:
\[ L[y(x, t)] = Y(x, s) = Y(x) = Y. \]

We treat \( Y \) as a function of \( x \), leaving \( s \) as a parameter. We then have the following:
\[ L[\frac{\partial y}{\partial t}] = sy(x, s) - y(x, 0), \]
\[ L[\frac{\partial^2 y}{\partial t^2}] = s^2y(x, s) - sy(x, 0) - \frac{\partial y}{\partial t}(x, 0). \]

Since we transform with respect to \( t \), we can move \( \frac{\partial}{\partial x} \) to the front of the integral; hence, we have:
\[ L[\frac{\partial y}{\partial x}] = \frac{dY}{dx} = Y'(x). \]

Similarly,
\[ L[\frac{\partial^2 y}{\partial x^2}] = \int_0^\infty \frac{\partial^2 y}{\partial x^2}e^{-st}dt = \frac{d}{dx} \int_0^\infty y(x, t)e^{-st}dt = \frac{dY}{dx} = Y''(x). \]

For the Laplace transform properties and applications, see [31,32].

3. Semi-Hyers–Ulam–Rassias Stability of the Convection Partial Differential Equation

Let \( \epsilon > 0 \). We also consider the following inequality:
\[ \left| \frac{\partial y}{\partial t} + a \frac{\partial y}{\partial x} \right| \leq \epsilon, \quad (4) \]
or the equivalent
\[ -\epsilon \leq \frac{\partial y}{\partial t} + a \frac{\partial y}{\partial x} \leq \epsilon. \quad (5) \]

Analogous to [33], we give the following definition:

**Definition 1.** The Equation (1) is called semi-Hyers–Ulam–Rassias stable if there exists a function \( \varphi : (0, \infty) \times (0, \infty) \to (0, \infty), \) such that for each solution \( y \) of the inequality (4), there exists a solution \( y_0 \) for the Equation (1) with
\[ |y(x, t) - y_0(x, t)| \leq \varphi(x, t), \quad \forall x > 0, t > 0. \]

**Theorem 1.** If a function \( y : (0, \infty) \times (0, \infty) \to \mathbb{R} \) satisfies the inequality (4), then there exists a solution \( y_0 : (0, \infty) \times (0, \infty) \to \mathbb{R} \) for (1), such that
\[ |y(x, t) - y_0(x, t)| \leq \begin{cases} \epsilon t, & t < \frac{x}{a} \\ \epsilon \frac{x}{a}, & t \geq \frac{x}{a} \end{cases}, \quad (6) \]
that is, the Equation (1) is considered semi-Ulam–Hyers–Rassias stable.
**Proof.** We apply the Laplace transform with respect to $t$ in (5); thus, we have the following:

$$-\frac{\varepsilon}{s} \leq sY(x) - y(x, 0) + aY'(x) \leq \frac{\varepsilon}{s}.$$  

Since $y(x, 0) = 0$, dividing by $a$ we get the following:

$$-\frac{\varepsilon}{as} \leq Y'(x) + \frac{s}{a}Y(x) \leq \frac{\varepsilon}{as}.$$  

We now multiply by $e^{\frac{x}{a}}$ and we obtain this equation:

$$-\frac{\varepsilon}{as} e^{\frac{x}{a}} \leq e^{\frac{x}{a}}Y'(x) + \frac{s}{a} e^{\frac{x}{a}}Y(x) \leq \frac{\varepsilon}{as} e^{\frac{x}{a}},$$  

hence,

$$-\frac{\varepsilon}{as} e^{\frac{x}{a}} \leq \frac{d}{dx} \left( e^{\frac{x}{a}}Y(x) \right) \leq \frac{\varepsilon}{as} e^{\frac{x}{a}}.$$  

Integrating from 0 to $x$ we get the following:

$$-\frac{\varepsilon}{as} e^{\frac{x}{a}} \frac{x}{a} \leq e^{\frac{x}{a}}Y(x) - e^{\frac{x}{a}}Y(0) \leq \frac{\varepsilon}{as} e^{\frac{x}{a}} \frac{x}{a},$$  

that is,

$$-\varepsilon \left( \frac{e^{\frac{x}{a}}}{s^2} - \frac{1}{s^2} \right) \leq e^{\frac{x}{a}}Y(x) - Y(0) \leq \varepsilon \left( \frac{e^{\frac{x}{a}}}{s^2} - \frac{1}{s^2} \right).$$

But $Y(0) = \mathcal{L}[y(0, t)] = \mathcal{L}[c] = \frac{\varepsilon}{s}$, so we obtain:

$$-\varepsilon \left( \frac{e^{\frac{x}{a}}}{s^2} - \frac{1}{s^2} \right) \leq e^{\frac{x}{a}}Y(x) - \frac{c}{s} \leq \varepsilon \left( \frac{e^{\frac{x}{a}}}{s^2} - \frac{1}{s^2} \right).$$

We now multiply by $e^{-\frac{x}{a}}$ and we obtain the equation below:

$$-\varepsilon \left( \frac{1}{s^2} - \frac{e^{-\frac{x}{a}}}{s^2} \right) \leq Y(x) - \frac{e^{-\frac{x}{a}}}{s} \leq \varepsilon \left( \frac{1}{s^2} - \frac{e^{-\frac{x}{a}}}{s^2} \right).$$

We apply the inverse Laplace transform and we obtain the following:

$$-\varepsilon \left[ t - \left( t - \frac{x}{a} \right) u \left( t - \frac{x}{a} \right) \right] \leq y(x, t) - c \cdot u \left( t - \frac{x}{a} \right) \leq \varepsilon \left[ t - \left( t - \frac{x}{a} \right) u \left( t - \frac{x}{a} \right) \right],$$  

that is,

$$\left| y(x, t) - c \cdot u \left( t - \frac{x}{a} \right) \right| \leq \varepsilon \left[ t - \left( t - \frac{x}{a} \right) u \left( t - \frac{x}{a} \right) \right].$$

We then put

$$y_0(x, t) = c \cdot u \left( t - \frac{x}{a} \right) = \begin{cases} 
0, & t < \frac{x}{a} \\
c, & t \geq \frac{x}{a}
\end{cases}.$$  

This is the solution of (1) and the equation below:

$$\left| y(x, t) - y_0(x, t) \right| \leq \begin{cases} 
\varepsilon t, & t < \frac{x}{a} \\
\frac{\varepsilon x}{a}, & t \geq \frac{x}{a}
\end{cases}.$$  

\qed
4. Generalized Semi-Hyers–Ulam–Rassias Stability of the Convection Partial Differential Equation

Let \( \phi : (0, \infty) \times \mathbb{R} \rightarrow (0, \infty) \), and \( \mathcal{L}[\phi(x,t)] = \Phi(x,s) \). We consider the following inequality:

\[
\left| \frac{\partial y}{\partial t} + a \frac{\partial y}{\partial x} \right| \leq \phi(x,t),
\]

or the equivalent

\[
-\phi(x,t) \leq \frac{\partial y}{\partial t} + a \frac{\partial y}{\partial x} \leq \phi(x,t), \quad \forall x > 0, t > 0.
\]

**Definition 2.** The Equation (1) is called generalized semi-Hyers–Ulam–Rassias stable if there exists a function \( \varphi : (0, \infty) \times (0, \infty) \rightarrow (0, \infty) \), such that for each solution \( y \) of the inequality (7), there exists a solution \( y_0 \) for the Equation (1) with

\[
|y(x,t) - y_0(x,t)| \leq \varphi(x,t), \quad \forall x > 0, t > 0.
\]

**Theorem 2.** Assume that

\[
\int_0^x e^{-s} \Phi(x,s) dx \leq \Phi(x,s), \quad \forall x > 0, s > 0.
\]

If a function \( y : (0, \infty) \times (0, \infty) \rightarrow \mathbb{R} \) satisfies the inequality (7), then there exists a solution \( y_0 : (0, \infty) \times (0, \infty) \rightarrow \mathbb{R} \) for (1), such that

\[
|y(x,t) - y_0(x,t)| \leq \frac{1}{a} \varphi \left( x, t - \frac{x}{a} \right), \quad \forall x > 0, t > 0,
\]

that is, the Equation (1) is considered generalized semi-Hyers–Ulam–Rassias stable.

**Proof.** We apply the Laplace transform with respect to \( t \) in (8), so we have the following:

\[
-\Phi(x,s) \leq sY(x) - y(x,0) + aY'(x) \leq \Phi(x,s).
\]

Since \( y(x,0) = 0 \), dividing by \( a \) we get the equation below:

\[
-\frac{1}{a} \Phi(x,s) \leq Y'(x) + \frac{s}{a} Y(x) \leq \frac{1}{a} \Phi(x,s).
\]

We now multiply by \( e^{\frac{x}{a}} \) and we obtain the following:

\[
-\frac{e^{\frac{x}{a}}}{a} \Phi(x,s) \leq e^{\frac{x}{a}} Y'(x) + \frac{s}{a} e^{\frac{x}{a}} Y(x) \leq \frac{e^{\frac{x}{a}}}{a} \Phi(x,s),
\]

hence,

\[
-\frac{e^{\frac{x}{a}}}{a} \Phi(x,s) \leq \frac{d}{dx} \left( e^{\frac{x}{a}} Y(x) \right) \leq \frac{e^{\frac{x}{a}}}{a} \Phi(x,s).
\]

Integrating from 0 to \( x \) we get the following equation:

\[
-\frac{1}{a} \int_0^x e^{\frac{s}{a}} \Phi(x,s) dx \leq e^{\frac{x}{a}} Y(x) \bigg|_0^x \leq \int_0^x \frac{1}{a} e^{\frac{x}{a}} \Phi(x,s) dx.
\]

Using (9), we have

\[
-\frac{1}{a} \Phi(x,s) \leq e^{\frac{s}{a}} Y(x) - Y(0) \leq \frac{1}{a} \Phi(x,s).
\]
But $Y(0) = L[y(0, t)] = L[c] = \frac{c}{a}$, so we obtain

$$-\frac{1}{a} \Phi(x, s) \leq e^{\frac{-s}{a}x} Y(x) - \frac{c}{s} \leq \frac{1}{a} \Phi(x, s).$$

We now multiply by $e^{-\frac{s}{a}x}$ and we obtain the following equation:

$$-\frac{1}{a} e^{-\frac{s}{a}x} \Phi(x, s) \leq Y(x) - \frac{c}{s} e^{-\frac{s}{a}x} \leq \frac{1}{a} e^{-\frac{s}{a}x} \Phi(x, s).$$

We apply the inverse Laplace transform and we obtain:

$$-\frac{1}{a} \phi(x, t - \frac{x}{a}) \leq y(x, t) - c \cdot u(t - \frac{x}{a}) \leq \frac{1}{a} \phi(x, t - \frac{x}{a}).$$

that is,

$$|y(x, t) - c \cdot u(t - \frac{x}{a})| \leq \frac{1}{a} \phi(x, t - \frac{x}{a}).$$

We then put the following:

$$y_0(x, t) = c \cdot u(t - \frac{x}{a}) = \begin{cases} 0, & t < \frac{x}{a} \\ c, & t \geq \frac{x}{a} \end{cases}.$$

This is the solution of Equation (1) and the equation below:

$$|y(x, t) - cy_0(x, t)| \leq \frac{1}{a} \phi(x, t - \frac{x}{a}).$$

\[\square\]

5. Semi-Hyers–Ulam–Rassias Stability of Equation (2)

Let $\epsilon > 0$. We also consider the following inequality:

$$\left| \frac{\partial y}{\partial t} + \frac{\partial y}{\partial x} - x \right| \leq \epsilon,$$  \hspace{1cm} (10)

or the equivalent

$$-\epsilon \leq \frac{\partial y}{\partial t} + \frac{\partial y}{\partial x} - x \leq \epsilon.$$  \hspace{1cm} (11)

Definition 3. The Equation (2) is called semi-Hyers–Ulam–Rassias stable if there exists a function $\varphi : (0, \infty) \times (0, \infty) \rightarrow (0, \infty)$, such that for each solution $y$ of the inequality (10), there exists a solution $y_0$ for the Equation (2) with the following:

$$|y(x, t) - y_0(x, t)| \leq \varphi(x, t), \quad \forall x > 0, t > 0.$$

Theorem 3. If a function $y : (0, \infty) \times (0, \infty) \rightarrow \mathbb{R}$ satisfies the inequality (10), then there exists a solution $y_0 : (0, \infty) \times (0, \infty) \rightarrow \mathbb{R}$ for (2), such that

$$|y(x, t) - y_0(x, t)| \leq \begin{cases} \epsilon t, & t < x \\ \epsilon x, & t \geq x \end{cases},$$

that is, the Equation (2) is considered semi-Hyers–Ulam–Rassias stable.

Proof. We apply the Laplace transform with respect to $t$ in (11), so we have the equation below:

$$-\frac{\epsilon}{s} \leq sY(x) - y(x, 0) + Y'(x) - \frac{x}{s} \leq \frac{\epsilon}{s}.$$
Since \( y(x,0) = 0 \), we get the following:
\[
-\frac{\varepsilon}{s} \leq Y'(x) + sY(x) - x\frac{1}{s} \leq \frac{\varepsilon}{s}.
\]

We now multiply by \( e^{sx} \) and we obtain the following equation:
\[
-\frac{\varepsilon}{s} e^{sx} \leq e^{sx}Y'(x) + se^{sx}Y(x) - \frac{e^{sx}}{s} \leq \frac{\varepsilon}{s} e^{sx}.
\]
hence,
\[
-\frac{\varepsilon}{s} e^{sx} \leq \frac{d}{dx}(e^{sx}Y(x)) - \frac{e^{sx}}{s} \leq \frac{\varepsilon}{s} e^{sx}.
\]
Integrating from 0 to \( x \), we get the following:
\[
-\frac{\varepsilon}{s} e^{sx} \bigg|_0^x \leq e^{sx}Y(x) \bigg|_0^x - \frac{1}{s} \int_0^x e^{sx}dx \leq \frac{\varepsilon}{s} e^{sx} \bigg|_0^x.
\]
Integrating by parts, we get the equation below:
\[
\int_0^x e^{sx}dx = \frac{(xs - 1)e^{sx}}{s^2} + \frac{1}{s^2},
\]
hence,
\[
-\varepsilon \left( \frac{e^{sx}}{s^2} - \frac{1}{s^2} \right) \leq e^{sx}Y(x) - Y(0) - \frac{1}{s} \left[ \frac{(xs - 1)e^{sx}}{s^2} + \frac{1}{s^2} \right] \leq \varepsilon \left( \frac{e^{sx}}{s^2} - \frac{1}{s^2} \right).
\]

But \( Y(0) = \mathcal{L}[y(0, t)] = 0 \), so we obtain the following:
\[
-\varepsilon \left( \frac{e^{sx}}{s^2} - \frac{1}{s^2} \right) \leq e^{sx}Y(x) - \frac{1}{s} \left[ \frac{(xs - 1)e^{sx}}{s^2} + \frac{1}{s^2} \right] \leq \varepsilon \left( \frac{e^{sx}}{s^2} - \frac{1}{s^2} \right).
\]

We now multiply by \( e^{-sx} \) and we obtain the following:
\[
-\varepsilon \left( \frac{1}{s^2} - \frac{e^{-sx}}{s^2} \right) \leq Y(x) - \frac{1}{s} \left[ \frac{xs - 1}{s^2} + \frac{e^{-sx}}{s^2} \right] \leq \varepsilon \left( \frac{1}{s^2} - \frac{e^{-sx}}{s^2} \right),
\]
hence,
\[
-\varepsilon \left( \frac{1}{s^2} - \frac{e^{-sx}}{s^2} \right) \leq Y(x) - \frac{x}{s^2} + \frac{1}{s^2} \frac{e^{-sx}}{s^2} \leq \varepsilon \left( \frac{1}{s^2} - \frac{e^{-sx}}{s^2} \right).
\]

We apply the inverse Laplace transform and we obtain the following equation:
\[
-\varepsilon [t - (t - x)u(t - x)] \leq y(x, t) - xt + \frac{1}{2}t^2 - \frac{1}{2}(t - x)^2u(t - x) \leq \varepsilon [t - (t - x)u(t - x)].
\]

We then put the following:
\[
y_0(x, t) = xt - \frac{1}{2}t^2 + \frac{1}{2}(t - x)^2u(t - x) = \begin{cases} xt - \frac{1}{2}t^2, & t < x \\ \frac{1}{2}x^2, & t \geq x \end{cases}.
\]

This is the solution of (2) and the equation below:
\[
|y(x, t) - y_0(x, t)| \leq \begin{cases} \varepsilon t, & t < x \\ \varepsilon x, & t \geq x \end{cases}.
\]
6. Conclusions

In this paper, we studied the semi-Hyers–Ulam–Rassias stability of Equations (1) and (2) and the generalized semi-Hyers–Ulam–Rassias stability of Equation (1) using the Laplace transform. To the best of our knowledge, the Hyers-Ulam-Rassias stability of Equations (1) and (2) has not been discussed in the literature with the use of the Laplace transform method. Our results complete those of Jung and Lee [22]. In [22], the Equation (3) was studied for \( \Re(c) \neq 0 \). We considered the case \( c = 0 \) in Equation (3). We can apply our results to the convection equation in the sense that for every solution \( y_0 \) of (4), which is called an approximate solution, there exists an exact solution \( y_0 \) of (1), such that the relation (6) is satisfied. From a different perspective, the approximate solution can be viewed in relation to the perturbation theory, as any approximate solution of (4) is an exact solution of the perturbed equation \( \frac{\partial y}{\partial t} + a \frac{\partial y}{\partial x} = h(x, t), |h(x, t)| \leq \epsilon, a > 0, x > 0, t > 0, y(0, t) = c, y(x, 0) = 0. \)

We intend to study other partial differential equations as well as other integro-differential equations using this method. We have already applied this method to [34], where we investigated the semi-Hyers–Ulam–Rassias stability of a Volterra integro-differential equation of order 1 with a convolution-type kernel.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Conflicts of Interest: The author declares no conflict of interest.

References

1. Ulam, S.M. *A Collection of Mathematical Problems*; Interscience: New York, NY, USA, 1960.
2. Hyers, D.H. On the stability of the linear functional equation. *Proc. Natl. Acad. Sci. USA* 1941, 27, 222–224. [CrossRef]
3. Obloza, M. Hyers stability of the linear differential equation. *Rocznik Nauk-Dydakt. Prace Mat.* 1993, 13, 259–270.
4. Alsina, C.; Ger, R. On some inequalities and stability results related to exponential function. *J. Inequ. Appl.* 1998, 2, 373–380. [CrossRef]
5. Cimpean, D.S.; Popa, D. On the stability of the linear differential equation of higher order with constant coefficients. *Appl. Math. Comput.* 2010, 217, 4141–4146. [CrossRef]
6. Jung, S.-M. Hyers-Ulam stability of linear differential equations of first order, III. *J. Math. Anal. Appl.* 2005, 311, 139–146. [CrossRef]
7. Takahasi, S.E.; Takagi, H.; Miura, T.; Miyajima, S. The Hyers-Ulam stability constant of first order linear differential operators. *J. Math. Anal. Appl.* 2004, 296, 403–409. [CrossRef]
8. Marian, D.; Ciplea, S.A.; Lungu, N. On a functional integral equation. *Symmetry* 2021, 13, 1321. [CrossRef]
9. Otrocol, D.; Ilea, V.A. Ulam stability for a delay differential equation. *Cent. Eur. J. Math.* 2013, 11, 1296–1303. [CrossRef]
10. Bălăs, A.R.; Popa, D. On Ulam stability of a linear difference equation in Banach spaces. *Bull. Malaysian Math. Sci. Soc.* 2020, 43, 1357–1371. [CrossRef]
11. Novac, A.; Otrocol, D.; Popa, D. Ulam stability of a linear difference equation in locally convex spaces. *Results Math.* 2021, 76, 1–13. [CrossRef]
12. Otrocol, D. Ulam stabilities of differential equation with abstract Volterra operator in a Banach space. *Nonlinear Funct. Anal. Appl.* 2010, 15, 613–619.
13. Marian, D.; Ciplea, S.A.; Lungu, N. On Ulam-Hyers stability for a system of partial differential equations of first order. *Symmetry* 2020, 12, 1060. [CrossRef]
14. Brzdek, J.; Popa, D.; Rasa, I.; Xu, B. *Ulam Stability of Operators*; Elsevier: Amsterdam, The Netherlands, 2018.
15. Rezaei, H.; Jung, S.-M.; Rassias, T. Laplace transform and Hyers-Ulam stability of linear differential equations. *J. Math. Anal. Appl.* 2013, 403, 244–251. [CrossRef]
16. Aqlifiary, Q.; Jung, S.-M. Laplace transform and generalized Hyers-Ulam stability of linear differential equations. *Electron. J. Differ. Equ.* 2014, 2014, 1–11.
17. Bicer, E.; Tunc, C. On the Hyers-Ulam-Stability of Laguerre and Bessel Equations by Laplace Transform Method. *Nonlinear Dyn. Syst. Syst.* 2017, 17, 340–346.
18. Murali, R.; Ponmana Selvan, A. Mittag-Leffler-Hyers-Ulam stability of a linear differential equation of first order using Laplace transforms. *Canad. J. Appl. Math.* 2020, 2, 47–59.
19. Shen, Y.; Chen, W. Laplace Transform Method for the Ulam Stability of Linear Fractional Differential Equations with Constant Coefficients. *Mediterr. J. Math.* 2017, 14, 25. [CrossRef]
20. Prastaro, A.; Rassias, T.M. Ulam stability in geometry of PDE’s. *Nonlinear Funct. Anal. Appl.* 2003, 8, 259–278.

21. Jung, S.-M. Hyers-Ulam stability of linear partial differential equations of first order. *Appl. Math. Lett.* 2009, 22, 70–74. [CrossRef]

22. Jung, S.-M.; Lee, K.-S. Hyers-Ulam stability of first order linear partial differential equations with constant coefficients. *Math. Inequal. Appl.* 2007, 10, 261–266. [CrossRef]

23. Lungu, N.; Ciplea, S. Ulam-Hyers-Rassias stability of pseudoparabolic partial differential equations. *Carpathian J. Math.* 2015, 31, 233–240. [CrossRef]

24. Jung, S.-M.; Lee, K.-S. Hyers-Ulam stability of first order linear partial differential equations with constant coefficients. *Carpathian J. Math.* 2019, 35, 165–170. [CrossRef]

25. Lungu, N.; Marian, D. Ulam-Hyers-Rassias stability of some quasilinear partial differential equations of first order. *Carpathian J. Math.* 2007, 10, 261–266. [CrossRef]

26. Marian, D.; Ciplea, S.A.; Lungu, N. Ulam-Hyers stability of Darboux-Ionescu problem. *Carpathian J. Math.* 2021, 37, 211–216. [CrossRef]

27. Qarawani, M.N. Hyers-Ulam-Rassias Stability for the Heat Equation. *Appl. Math.* 2013, 4, 1001–1008. [CrossRef]

28. Deborah, D.O.; Moyosola, A. Laplace differential transform method for solving nonlinear nonhomogeneous partial differential equations. *Turk. J. Anal. Number Theory* 2020, 8, 91–96. [CrossRef]

29. Bicer, E. Applications of Sumudu transform method for Hyers-Ulam stability of partial differential equations. *J. Appl. Math. Inform.* 2021, 39, 267–275.

30. Abdulah, A.A.; Ahmad, A. The solution of Poisson partial differential equations via double Laplace transform method. *Partial Differ. Equ. Appl. Math.* 2021, 4, 100058.

31. Lebl, J. *Notes on Diffy Qs: Differential Equations for Engineers*; CreateSpace Independent Publishing Platform: Charleston, SC, USA, 2021.

32. Cohen, A.M. *Numerical Methods for Laplace Transform Inversion (Numerical Methods and Algorithms, 5)*; Springer: Berlin/Heidelberg, Germany, 2007.

33. Castro, L.P.; Simões, A.M. Different Types of Hyers-Ulam-Rassias Stabilities for a Class of Integro-Differential Equations. *Filomat* 2017, 31, 5379–5390. [CrossRef]

34. Inoan, D.; Marian, D. Semi-Hyers-Ulam-Rassias stability of a Volterra integro-differential equation of order I with a convolution type kernel via Laplace transform. *Symmetry* 2021, 13, 2181. [CrossRef]