The mutual influence of electromagnetic and mechanical processes in dynamic modes of inertial vibrating electric drives

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Abstract. To conduct studies on the mutual influence of electromagnetic and mechanical transient processes of starting inertial vibration drives with an asynchronous motor (IVD-AD), a mathematical model is developed in this work. For this, the analytical dependence of the dynamic mechanical characteristics of the vibrator is determined for various values of the acceleration of the rotor, significantly different from the standard moments of resistance. In the mathematical model, the dynamic mechanical characteristic of an induction motor is used as a torque. Studies on a mathematical model showed that one of the disturbing effects on the dynamic mode of the IVD-AD is a change in the load parameters during operation, another disturbance is a change in the acceleration of the rotor, which leads to a shift in the resonance curves, which must be taken into account when choosing methods and means of tuning control in resonance.

Because of the simplicity of construction, high specific efficiency (per unit mass of output), therefore, low cost, convenience and low operating costs, inertial vibration motors have been widely used in the vibration machines (VM) of the construction industry and other sectors of the economy. As electric drives of VM, inertial vibration drives (IVD) with an induction motor (IM) with a squirrel-cage rotor are mainly used [1]. Expanding these positive features inertial vibration drives by introducing control of the starting modes of IM, automating tuning in the resonant mode and controlling the parameters of the output values will increase the efficiency of use of VM.

It should be noted that complex dynamic electromagnetic and mechanical processes in IM themselves, on the one hand, are not simple mechanical processes in resonant-type VM, and moreover, their mutual influence on each other greatly complicates the problem of experimental studies aimed at automating the operation of an electric drive.

Recent advances in science and technology [2, 3] make it possible to consider physical processes in IM in a generalized, and at the same time, whiter, deeper analysis of the operation of individual modes of unbalanced vibration machines, choosing a universal mathematical model and on its basis to study the fundamentals of electro-dynamic processes in IVD with IM.

The purpose of this work is to develop a mathematical model of IVD with IM, which allows model studies of the mutual influence of electromagnetic and mechanical transient starting processes.
If we approach strictly the structure of many mass vibration machines, we can see that the number of variables and the equations of oscillation depend on the number of masses linearized many mass system. Therefore, to simplify and reduce the number of differential equations, without compromising the accuracy of the mathematical model, we will make the following assumptions: the elastic elements are linear; the damping forces are concentrated and proportional to the vibration mass velocity, all masses are stationary and the elements are divided by masses; the induction motor is linearized; we assume that the vibration machines is one mass. Studies show [1, 2, 3] that in a large mass linearized vibration machine the resonant type of the oscillation amplitude of the bulk (with a frequency close to the frequency of free oscillations of the bulk) significantly exceeds the oscillation amplitudes of other masses $A_1 \gg A_N$, where $A_1$ is the amplitude of oscillations of the bulk, $A_N$ - oscillation amplitude of the $N$-noy mass of VM.

In the mathematical model, IVD with IM to determine the dependence of input-output quantities, i.e. electrical, magnetic and mechanical variables, we will use the well-known equations of equilibrium of the phase voltages of the stator windings, rotor and moment equations by IM

$$
\begin{align*}
\frac{d}{dt}u_1 &= i_1 r_1 + \frac{d\psi_1}{dt} + \omega_1 \psi_1; \\
\frac{d}{dt}u_2 &= i_2 r_2 + (d\psi_2)/dt + (\omega_1 - \omega)\psi_2; \\
M - M_R &= \frac{J}{p} \frac{d\omega}{dt}; \\
M &= \frac{p m_1 (I_2')^2 r_2'}{s} \tag{1}
\end{align*}
$$

Where $u_1$ and $u_2$ are the instantaneous voltage values of the stator and rotor windings: if the rotor winding is short-circuited, then $u_2=0$; $i_1$ and $i_2$ - instantaneous values of currents of stator and rotor windings; $m_1$, $p$, $r_1$ and $r_2$ - the number of phases of the stator winding, the number of pole pairs, the values of the active resistances of the stator and rotor windings by induction motors; $\psi_1$ and $\psi_2$ - instantaneous values of flux linkages of the stator and rotor windings; $J$ - moment of inertia of the rotating parts; $\omega_1$ and $\omega_2$ - respectively, the rotational speed of the rotating magnetic field of the stator and the rotational speed of the rotor.

The value is $M_R$ the moment of resistance of the production machinery (VM) in the third equation of system. It should be pointed out that that $M_R$ in VM differs significantly figure 1 from the known static resistance moments, adopted in the general course of electric drives [4, 5, 6]. Its analytical dependence can be written as the sum of the following components where

$$M_R = M_{R,\text{init}} + M_{R,\text{com}} \left( \frac{\omega}{\omega_n} \right) + M_{R,\text{vib}} \tag{2}$$

$M_{R,\text{init}}$ - the initial moment; $M_{R,\text{com}}$ - the fan component of the moment; $M_{R,\text{vib}}$ - the vibrational moment; $\omega$ - the angular frequency of rotation of the rotor; $\omega_n$ - the nominal rotor speed.

![Figure 1. Static and dynamic moments of resistance of vibration machines.](image)

![Figure 2. Static and dynamic AFC.](image)
For clarity, the moment of resistance of VM in the figure 1 is represented as a graph of two mass vibrational systems, while the number of extremes of the vibration component is equal to the number of masses. For example, the frequency $\omega_{01}$ of free vibrations corresponds to one mass, the frequency $\omega_{02}$ - the second mass of the vibrator.

It is worth noting that at frequencies close to $\omega_{01}$, vibrations of another mass of VM are damped, and, conversely, at a frequency of $\omega_{02}$, the oscillation of the first mass is damped.

The first two components of the static moment of resistance of vibration machines are known, and the value of $M_{R\text{,init}}$ depends on the total moment of inertia of the rotor and all rotating parts of the inertial vibration drive with induction motors. The third component $M_{R\text{,vib}}(\omega)$ of dependence (2) - the vibration component of the moment of resistance expresses the inverse action of the vibrating working body of the VM on the rotating shaft and variables by IM, is determined by the following analytical expression

$$M_{R\text{,vib}}(\omega) = \frac{p(A, \omega)}{2} \frac{A(t) \omega}{\sqrt{[\Omega(A, \omega) - \omega]^2 + (\frac{r_e}{m})^2}}$$

(3)

where $p(A, \omega)$ is the stiffness of the elastic elements, depending on the amplitude $A$ and the angular frequency $\omega$ of the vibrations of the working body. In compliance with the assumptions made above, $p(A, \omega)=p$ is an element with a linear characteristic; $A(t)$ is the dynamic amplitude of oscillations of the working body, which, within the duration of the transition processes, depends on time $t$; $m=m_0+m_1, m_0$ and $m_1$ - masses of the oscillatory system, unbalance and working body.

Other distinctive features of the mechanical characteristics of VMs are that they are significantly different in static and dynamic modes. Furthermore, the extremum of the vibration component of the resistance moment $M_{R\text{,vib}}(\omega)$ change [3] depending on the acceleration of the rotor with changes in the rotor speed $\omega$ or the frequency of free vibrations $\omega_0$ of the mechanical vibration system. The different maximum values of moments of the rotor $\gamma$ correspond to different values and signs of acceleration of the rotor, on the one hand and to different shifts from the resonance point ($\omega_0$) to the right (the curves $\gamma_1, \gamma_2$ and $\gamma_3$ in figure 1) with positive acceleration, and to the left if acceleration (the curves $\gamma_4$ and $\gamma_5$) is negative, that is, when braking the rotor.

The dynamic resistance moment curves $M_R(\omega)$ shown in figure 1 are constructed according to equations (2) and (3) for various values of rotor acceleration ($\gamma = \frac{d\omega}{dt}$) for inertial vibration drives with induction motors like VIEW-2. From the calculated curves it is seen that with increasing acceleration of the rotor $\gamma_1 < \gamma_2 < \gamma_3$, the dynamic mechanical characteristic of VM shifts to the right with respect to the static mechanical characteristic ($\gamma=0$). The dynamic mechanical characteristic of the vibrating machines after the transient attenuation returns to its original state and is determined by the curve $\gamma$.

The experimental affirmation of the above encounters difficulties associated with the inability to maintain $\gamma=const$ during transient changes in the rotor speed when starting vibrating machines. Accordingly, we continue to study the dynamic mechanical characteristics using a mathematical model of inertial vibration drives with induction motors, which allow us to more carefully study the properties of vibration machines in dynamics.

In order to develop a mathematical model of the vibrator, together with the dependences of the moments of resistance of the VM (2) - (3), we use the equations of equilibrium of the forces moving the working body — the forces of vibration excitation and the forces of resistance to motion of one mass of VM [2]

$$m\ddot{x} + \rho \dot{x} + \omega_0^2 x = F_m \cos \omega t$$

(4)

where $F_m = m_0 \cdot r_e \cdot \omega^2$

(5)

the vibration excitation force created by rotation unbalanced systems; $x$, $\dot{x}$ and $\ddot{x}$-instantaneous values of vibration, speed and acceleration during vibration of the working body; $r_e$ - radius of unbalance.
(distance between the axes of inertia and rotation); \( \omega_0 \), \( \omega \) - the angular frequency of free and the frequency of forced oscillations of the working body.

For the purpose of the determination of the analytical dependence of the amplitude of oscillations of the working body on time and build the amplitude-frequency characteristics of vibration machines, we use the dependence of the course of the theory of oscillations. [7]

\[
A = \frac{F_m}{m \sqrt{\left(\omega_0^2 - \omega^2\right)^2 + 4 \left(\frac{\rho}{m \omega_0}\right)^2}}
\]  
(6)

The displacements of the vibrational component of the moment of resistance \( M_{R, vib} \) at different accelerations of the rotor are associated with similar changes in the \( A = f(F_m) \) frequency response of the vibration machines, which are shown in figure 2. The extremum of the vibrational component of the moment of resistance \( M_{R, vib}(\omega) \) in a mechanical oscillatory system changes [3] depending on the acceleration of the rotor.

The different values and signs of acceleration of the rotor \( \gamma \) correspond to different maximum values of moments, on the one hand, and different values of displacement from the resonance point \( (\omega_0) \) to the right (the curves \( \gamma_1, \gamma_2 \) and \( \gamma_3 \) in figure 1) at positive acceleration, and to the left, if the acceleration (the curves \( \gamma_4 \) and \( \gamma_5 \)) is negative, on the other side when the rotor is decelerated.

In the third equation of the system (1), the value is the static electromagnetic moment of IM, which is determined by a well-known analytical expression from the course of electric machines [4]. It corresponds to the curve 1 in figure 3.

![Figure 3. Static and dynamic electromagnetic moments at idle by IM.](image)

![Figure 4. Static and dynamic mechanical characteristics of IM and the AFC of VM.](image)

The dynamic processes with IM are accompanied by both electromagnetic and mechanical transients. Therefore, the rotor speed, the inrush current and the duration of the transient processes are influenced by electromagnetic parameters by induction motors, and mechanical quantities associated with the load, and the initial state of the electric drive. Additionally, the figure 3 shows the dependence of the dynamic electromagnetic moment on the rotor speed when starting by asynchronous motors in idle mode [8]. As can be seen from comparing this curve with the characteristic of the static electromagnetic moment figure 3, it can be noted that they differ significantly. Accordingly, given the nature of the change in the curves of the dynamic moment of resistance of VMs and the dynamic electromagnetic moment of IMs, we take them to develop a mathematical model of inertial vibration drives with IMs.

In order to eliminate the complex electromagnetic coupling between the phases of the three-phase winding of an induction motor, its mathematical model is transformed into a biaxial coordinate system \( \alpha-\beta \), using known conversion methods [2, 4, 8]. Then the developed mathematical model consists of the equations of the equilibrium of the voltage of the two-phase winding of a stator and a rotor (\( \alpha-\beta \)), the equations of the equilibrium of the moments, electromagnetic moment, moment of resistance of VM (3), equations of motion of working body (4), (5), (6).
For the equations of the mathematical model, a solution algorithm was compiled, a program for its solution is developed, and a computational experiment was conducted.

By using a computational experiment and solving the equations of motion of a working body of a single-mass vibration machine, amplitude-frequency characteristics (AFC) and graphs of the dynamic moment of resistance $M_R(t)$ were constructed for inertial vibration drives with IMs, which have a resonant nature of change.

$$u_{1\alpha} = \left[ r_1 + \frac{dL_{\alpha_1}(t)}{dt} \right] i_{1\alpha} + \frac{dM_{i_2\alpha}}{dt},$$

$$u_{1\beta} = \left[ r_1 + \frac{dL_{\alpha_1}(t)}{dt} \right] i_{1\beta} + \frac{dM_{i_2\beta}}{dt},$$

$$0 = \left[ r_2 + \frac{dL_{\alpha_2}(t)}{dt} \right] i_{2\alpha} + \frac{dM_{i_2\alpha}}{dt} + \omega [L_2 i_{2\beta} + M i_{1\beta}],$$

$$0 = \left[ r_2 + \frac{dL_{\alpha_2}(t)}{dt} \right] i_{2\beta} + \frac{dM_{i_2\beta}}{dt} - \omega [L_2 i_{2\alpha} + M i_{1\alpha}],$$

$$M(t) = pM[i_{2\alpha} i_{1\beta} + i_{1\alpha} i_{2\beta}],$$

$$\omega = \frac{p}{j} [M(t) - M_R(t)],$$

(7)

The work in the pre-resonance zone on the static mechanical characteristic of $M_R$ VM (the curve 1 or the line О1C, figure 4) is statically stable. On the other hand, the zone of stable operation by induction motors is also in a limited small segment of the static mechanical characteristic in the range of the rotational speed of the rotor $\Delta \omega = \omega_1 - \omega_k$ within the working zone of the mechanical characteristic $M(\omega)$ by the induction motor (the straightened part of the curve 2, figure 4). Thus, the operation of vibrating machines is statically stable at the point $N$ of intersection of curve 1 and line 2.

The conducted experimental studies on a mathematical model showed that due to the fact that the areas of stable operation, both by induction motors and vibration machines, are limited by small ranges of variation of the rotor speed. Therefore, in order to achieve the required value of the amplitude of the working body vibrations of the vibration machines and the stability of the electric drive under various external disturbing influences, the control of the tuning of inertial vibration drives with induction motors in the resonance mode (or near resonance mode) at point $N$ must be performed using a frequency converter (IC).

If we take into account that with a slight change in the load parameters of $m$ vibration machines, the frequency of free vibrations

$$\omega_0 = \sqrt{\frac{p}{m}}$$

(8)

changes (increases or decreases), it can be noted that the maxima of the amplitude-frequency characteristics and $M_R(\omega)$ shift from the point $\omega_0$ to the right or left (the curve 3 with a frequency $\omega_{01}$ figure 4). At the same time, in order to maintain the most effective mode of vibration machines with the maximum amplitude of oscillations of the working body, it is necessary to control the rotor speed by induction motors $\omega$ to set point $N$ closer to the resonant frequency $\omega_0$ or $\omega_{01}$.

Therefore, one of the disturbing effects on the change in the stability of work at point $N$ is the change in the parameters of the vibrated load during operation of the installation according to (8). The displacement of the resonant curves $(y_1$ or $y_2$ in figure 4 and 2) itself changes the rotor speed $\omega$ with acceleration $y_1$ or $y_2$ in the process of controlling the vibration machines. Both of these factors negatively affect the process of controlling the setting of inertial vibration drives with the IM in resonant mode.
In the loaded mode of the vibration machines, the electromagnetic moment at low rotor speeds will also change with pulsation, since the component $M_{R,0}$ of the vibration moment of resistance in equation (2) has a minimum value. With an increase in the rotor speed and approach to the vibration zone, to the static mechanical characteristic of the vibration machines (the curve 3) and to the point of statically stable operation, the start-up process of inertial vibration drives with induction motors ends with vibrational attenuation of the moment around point $N_1$ (the curve 3).

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