Survey of hyperfine structure measurements in alkali atoms

Maria Allegrińi,1,2 Ennio Arimondo,1,3 and Luis A. Orozco4

1) Dipartimento di Fisica “E. Fermi”, Università di Pisa, Largo B. Pontecorvo 3, 56127 Pisa, Italy
2) Istituto Nanoscienze-CNR, Piazza S. Silvestro 12, I-56127 Pisa, Italy
3) Istituto Nazionale di Ottica-CNR, Via G. Moruzzi 1, 56124 Pisa, Italy
4) Joint Quantum Institute, Dept. Physics, Univ. Maryland and National Institute of Standards and Technology, College Park, MD 20742, USA

(Dated: September 26, 2022)

The spectroscopic hyperfine constants for all the alkali atoms are reported. For atoms from lithium to cesium, only the long lived atomic isotopes are examined. For francium, the measured data for nuclear ground states of all available isotopes are listed. All results obtained since the beginning of laser investigations are presented, while for previous works the data of Arimondo et. al. Rev. Mod. Phys. 49, 31 (1977) are recalled. Global analyses based on the scaling laws and on the hyperfine anomalies are performed.

I. NOTE

In press J. Phys. Chem. Ref. Data.

II. INTRODUCTION

Atomic spectroscopy was a key element in the original development of quantum mechanics theory at the beginning of the twentieth century. Its exploratory role continued after the Second World War, when microwave sources, very stable radiofrequency generators, optical pumping, and later on lasers, entered into the laboratories. The atomic contributions to quantum electrodynamics, parity violation, and the present searches for variation of the fundamental constants and for dark matter tests are noteworthy. In this scenery, the alkali atoms and their hyperfine splittings represent an important reference because of their rather simple energy level structure and their relatively easy laboratory exploration. Furthermore, they offer an opportunity to verify new experimental tools for a direct comparison within a wide research community. Very precise hyperfine constants of the alkalis are required for a large set of hyperfine measurements. More accurate hyperfine structure measurements have also revitalized their use in studies and tests of nuclear physics and fundamental symmetries in nature. Formidable progress achieved by atomic physics calculations supporting and also stimulating research on the above advanced topics has refined its tools on the alkali hyperfine data.

A complete collection of hyperfine constants for alkali atoms was published by Arimondo, Inguscio, and Viozino (1977) at the time when laser sources introduced high-resolution atomic spectroscopy. Since that time, new spectroscopy tools have been developed, and technological advances have produced extremely precise atomic measurements. This progress is the motivation of the present work. The most amazing example of the combination of scientific and technological progress is the atomic fountain proposed by Zacharias in 1953, unpublished but described by Ramsey (1956). Although it does not operate for room temperature atoms, it is very successful for launching ultracold ones, as exploited for hyperfine measurements based on atomic clocks, (e.g., see Guéna et al. (2014); Ovchinnikov, Szymaniec, and Edris (2015)). Using such tool, the hyperfine ground states of rubidium and cesium are presently measured with such a precision that small variations of the fundamental constants can be tested. Recently, the use of frequency combs to perform absolute optical frequency measurements has provided alkali hyperfine values with a precision increased by a factor of up one thousand. In addition, some well assessed spectroscopy tools have been refined. For instance, in Bayram et al. (2014), the detection of delayed quantum beats at the hyperfine transition frequencies is used to determine very precise hyperfine coupling constants in several cesium excited states, for which the precision of other techniques suffers from short lifetimes. These approaches have increased the precision for a large set of hyperfine measurements.

In another class of experiments, the hyperfine constants have been determined for excited states accessible only by laser sources covering new spectral regions or by multiple laser excitations. The most spectacular example is the alkali Rydberg states investigated for hyperfine structure up to levels with principal quantum number $n \approx 70$. For completeness, here we report a third class of hyperfine measurements of pre-laser times for alkali states not recently investigated. A very interesting example of this class is the ground state hyperfine structures of lithium and sodium, for which the precise atomic-beam investigations of 1973-1974 remain the reference point. Certainly, atomic fountain experiments applied to those atoms could yield a precision comparable to that of the atomic clocks.

This work presents a complete overview of the measured hyperfine constants for the alkali atoms in ground or singly excited electronic states. It enlarges or supersedes the recent reviews of Das and Natarajan (2008); Kiran Kumar and Suryanarayana (2014); Williams.
Herd, and Hawkins (2018) that report a limited data set for lighter alkalis. We also add data to the francium review of Sansonetti (2007). The main target is to provide to interested experimentalists and theoreticians the full set of hyperfine data in an easily accessible form. We have collected the hyperfine data of the stable alkali isotopes published after the review of Arimondo, Inguscio, and Violin (1977). In order to give a complete overview, our tables include measurements already reported in that review in all cases where new and more precise values are not available. We do not examine the unstable isotopes for the alkalis from lithium up to cesium as for them very limited data were recently published. Instead we review the full spectrum of the francium nuclear ground state isotopes, because of the recent interest associated with this atom as a test of the nuclear structure owing to the large number of explored isotopes for this alkali. This review intends to cover the experimental investigations, while discussing theoretical results only briefly. However, it should be mentioned here that theoretical comparison has greatly progressed, and global analyses for a given alkali atom have produced a large set of theoretical values for the hyperfine constants.

For the experimentalists, this overview might stimulate investigations of specific atomic states for which the precision remains low. As advanced spectroscopic techniques are usually tested only on a few states, a large set of high precision data might boost the theoretical effort for global analyses. Based on Kramida (2022) “NIST atomic energy levels and spectra bibliographic database” (NIST stands for National Institute of Standards and Technology, USA), we have examined all articles that to the best of our knowledge have been published so far. However, we have disregarded publications that do not target the hyperfine splittings. For publications by the same research group reporting subsequent measurements with increasing precision our tables include only the most recent value.

Section II presents the experimental tools exploited to measure the hyperfine constants. At first the atomic sample is examined from vapor cells up to ultracold atomic clouds where the spatial and velocity confinement greatly increases the spectral resolution. In the following the experiments are classified within some broad categories allowing a connection to the precision reached in ground and excited states. The core part of this review is Sec. III. It is composed of tables reporting the hyperfine constants for each alkali isotope classified on the basis of the atomic state and for a given state in chronological order. Before presenting the tables, the basic theoretical concepts of the hyperfine interaction are briefly recalled. For each atom we discuss the main results and we mention large discrepancies, if any, between the data of a given state. For Rb and Cs atoms, where more data are available, Section IV presents scaling laws vs the quantum number of the excited states. Such scaling is applied to determine or confirm the sign of the hyperfine constants for several states. The scaling law is applied also to the S states of francium isotopes for which few data are available. Hyperfine anomalies producing information on the nuclear structure are discussed, at least for the states measured with reasonable precision. A short section concludes this review.

III. SPECTROSCOPIC TOOLS

A. Samples

Atomic beam (AB). In an atomic beam atoms propagate along a given direction with a small spread in the orthogonal plane, see Ramsey (1956). Usually, an exciting laser propagates perpendicularly to the beam propagation leading to a very small Doppler broadening.

Vapor cell (VC). In a glass/quartz cell the atoms are in the vapour phase and their vapour pressure and the atomic density are controlled by the cell temperature.

Magneto-optical trap (MOT). The combined action of laser cooling and magnetic field confinement produces dense atomic samples having a greatly reduced Doppler linewidth. Such samples allow to detect weak absorption features, as for highly excited states, and to perform experiments with long interaction times leading to increased precision.

Fountain (OUNT). In a fountain, atoms from a MOT are launched vertically by radiation pressure or a moving optical lattice, see Metcalf and van der Straten (1999). Excitation and detection take place at the same vertical position, the first one at the launching time and the second after the parabolic motion. Very high parabolic evolutions are used in order to increase the interrogation time. This approach is applied to the atomic clocks.

Optical Dipole Trap (ODT) In the experiment by Neuzner et al. (2015) a single $^{87}$Rb atom is trapped by an optical dipole trap created by a 2-D optical lattice. Cavity-enhanced state detection of the optical absorption produces a good signal-to-noise ratio even for a single atom. Light-shift correction is carefully applied.

Thermoionic diode (TD). In presence of a weak electrical discharge in a VC, the light excited atoms are ionized by electron collisions. These ions diffuse into the space charge region of the diode, compensate partially the space charge, and increase the thermionic diode current, as described in Herrmann et al. (1985).

B. Spectroscopic Techniques

The experimental techniques used to measure the reported hyperfine measurements are classified in the following. Several research groups have introduced a specific name for their technique. While our classification scheme is concise, detailed presentations of the techniques can be found in textbooks, such as Ramsey (1956), Kopfermann (1958), Foot (2005), Budker, Kimball, and DeMille (2008), Inguscio and Fallani...
Cross-over resonances are induced in three-level laser is modulated at the hyperfine splitting frequency, readings is largely increased, see for instance Udem et al. (2005). The absolute precision of the frequency comb spectroscopy is based on frequency comb (FC), The atomic absorption peaks are determined with a reference to a laser frequency, as in Moon, Lee, and Suh (2009). In the copropagating geometry, one laser excites zero velocity atoms on the lower transition and a second laser is scanned in frequency, as in Moon, Lee, and Suh (2009). In the counterpropagating geometry, it is often combined with electromagnetic-induced transparency.

Delayed detection (DD). The natural linewidth of a spectroscopic resonance is reduced by monitoring the atom evolution for times longer than the spontaneous emission lifetime. This refinement was combined with other techniques, such as as laser-induced fluorescence by Shimizu et al. (1987) or hyperfine quantum-beats by Deech et al. (1977); Krist et al. (1977); Yel, Sieradzan, and Havey (1993).

Double-resonance optical pumping (DROP). A double-resonance optical excitation on a ladder three-level scheme is applied. An increased signal-to-noise ratio is obtained by detecting the population of the ground state rather than the excited state one. In the copropagating geometry, one laser excites zero velocity atoms on the lower transition and a second laser is scanned in frequency, as in Moon, Lee, and Suh (2009). In the counterpropagating geometry, it is often combined with electromagnetic-induced transparency.

Electromagnetic induced transparency (EIT). This coupling/probe spectroscopy is based on the very narrow coherent feature produced in the absorption spectrum of three-level \( \Lambda \) or ladder systems. For the \( \Lambda \) scheme a very narrow linewidth is determined by the long relaxation rate of the ground state coherence. Counterpropagating lasers and one laser locked to an atomic transition are used to produce sub-Doppler resolution, as presented in Krishna et al. (2005); Wang et al. (2013 2014a).

Frequency comb spectroscopy (FC). The atomic absorption peaks are determined with a reference to a frequency comb. The absolute precision of the frequency readings is largely increased, see for instance Udem et al. (1999); Das et al. (2006a).

Frequency modulated laser (FML). When the exciting laser is modulated at the hyperfine splitting frequency, cross-over resonances are induced in three-level \( \Lambda \) or V systems, as in Noble et al. (2006). The advantage of this technique is that only a single laser is required.

Hyperfine optical pumping and focus (HOPF). Hyperfine transitions induced by microwaves or by optical pumping change the relative populations of the hyperfine levels of the ground state. In HOPF this modification is detected by measuring the atomic beam intensity at the exit of a magnet that focuses or defocuses atoms with different magnetic quantum numbers. The focused atoms are analyzed by a mass spectrometer, as in Liberaman et al. (1980). Hyperfine quantum beats (HQB). Quantum beats are based on coherent pulsed excitation of excited hyperfine levels producing a time decay of the excited state populations modulated by the hyperfine frequency splitting. Polarized excitation and detection are required, as in Deech et al. (1977). Bellini, Bartoli, and Hänsch (1997) applied delayed pulses of a frequency comb in order to probe the coherent hyperfine superposition of excited states.

Ion detection (ION). This detection technique is very sensitive because a single ion can be detected. It is applied within different schemes, such as the resonant laser ionization (RIS), the selective electric field ionization of a Rydberg state or the thermonionic diode operation.

Laser-induced fluorescence spectroscopy (LIF). The emitted fluorescence is monitored as a function of the laser frequency. In Doppler spectroscopy, a high resolution is achieved by a careful analysis of the absorption line shapes, as in Truong et al. (2015). In an AB with the laser propagation perpendicular to the atomic motion, the resolution is limited by the natural linewidth. By applying a sudden change to the laser phase and monitoring the atomic evolution at a later time \( T \), subnatural width resolution reaching \( \approx 1/(2T) \) is achieved in Shimizu et al. (1987). For short-lived atoms with low density using a fast beam and a collinear laser propagation, LIF is combined with nuclear decay to increase the signal to noise ratio as in Duong et al. (1987); Lynch et al. (2016). Level crossing by magnetic or electric fields. An energy crossings of excited state levels vs an external parameter, either magnetic field (MLC) or electric field (ELC), is monitored, see for instance Nagourney, Happer, and Lurio (1975); Auzinsh et al. (2007). A precise determination of the applied field is required.

Maser (MA). The emission frequency of a maser operating on a hyperfine ground state transition is measured in Tetu, Fortin, and Savard (1976). Magnetic field decoupling (MFD). Starting from an initial anisotropic Zeeman sublevel population distribution, the hyperfine constants are derived from the polarization of the fluorescent emission monitored vs an applied magnetic field decoupling the nuclear and electronic angular momenta, as presented in van Wijngaarden and Sagle (1991b).

Microwave spectroscopy (MWS). The population modifications induced by transitions between hyperfine levels, mainly in the microwaves, are detected. In order to increase the signal-to-noise ratio, MWS is combined to other techniques, such as HOPF, LIF, or selective electric field ionisation for Rydberg states as in Goy et al. (1982). The FOUNT+MWS combination applied to optical clocks leads to an extremely high precision, as in Guené et al. (2014); Ovchinnikov, Szymaniec, and Edris (2015). A Ramsey optical interferometer is used for the potassium ground state measurement of Arras et al. (2019); Peper et al. (2019).
(ORFDR). The radio-frequency induced transitions between excited states are detected through the modification of the LIF, either in its spectrum or in its polarization, as in [Farley, Tsekeris, and Gupta (1977); Lam, Gupta, and Happer (1980)]. Optical pumping is applied to modify the population distribution and increase the detected signal.

**Optical-optical double-resonance (OODR).** A two-colour excitation via an intermediate step produces the population of excited states. The sub-Doppler resolution is obtained by operating in a MOT in [Fort et al. (1995a), by applying the lasers in a counterpropagating geometry in [Stalnaker et al. (2010), by a counterpropagating laser geometry selecting a single class of velocities different from zero in [Lee and Moon (2015), or by using saturated absorption to lock on the first transition and excite only the atoms at zero velocity in [Yang et al. (2016). Detection is based mainly on the spontaneous emission from the intermediate or final state. In presence of an optical pumping process, the population distribution perturbed by the second step excitation is monitored, as in [Wang et al. (2014b).

**Optical spectroscopy (OS).** Doppler limited high optical resolution spectroscopy from an alkali cell as in [Thai et al. (2015), or Doppler-free in a MOT as in [Antoni-Micollier et al. (2017); Arias et al. (2019).

**Resonant ionization spectroscopy (RIS).** Atoms in specific states are ionized by multistep laser absorption, and the ions are detected. It was introduced by [Andreev, Letokhov, and Mishin (1987) for measuring of the ground state hyperfine structure in francium. The resonant tuning of the intermediate step provides the spectroscopic resolution of its hyperfine structure. High resolution and sensitivity due to ion detection, are associated with this technique as in the MOT experiment by [Gabbani et al. (1999) or for exotic isotopes in an atomic beam of exotic isotopes using the collinear laser spectroscopy as in [Lynch et al. (2014). These last authors directed the ion to an alpha-decay detection station for clear identification in order to reduce isobaric and ground state contamination in their francium isotopes studies. In order to increase the frequency resolution, in recent francium accelerated atomic beam experiments [Neugart et al. (2017) the excitation laser is split into two beams, co-propagating and counter-propagating with the atoms in order to increase the frequency resolution.

**Saturated absorption spectroscopy (SAS).** This technique is based on a pump and probe laser applied to the same transition. It produces spectra with a natural linewidth resolution. The counter-propagating geometry compensates for the Doppler broadening. Main limitations are imposed by the laser stability, as analyzed in [Das and Natarajan (2008); Glaser et al. (2020).

**Stark spectroscopy (SS).** Stark spectroscopy is based on the electric field shift of atomic level energies. It is used mainly for Rydberg states as in [Stevens et al. (1995). Information on lower energy states may be derived by the difference in level Stark shifts.

**Two-photon sub-Doppler spectroscopy (TPSDS).** A single-colour two photon not-resonant excitation explores highly excited states. The sub-Doppler resolution is obtained by operating with counterpropagating beams in a VC as described by [Herrmann et al. (1985); Hagel et al. (1999) or in a MOT as in [Georgiades, Polzik, and Kimble (1994).

**IV. HYPERFINE THEORY**

The hyperfine structure Hamiltonian $H_{hyp}$ of an atom having a single valence electron outside the closed shells consists of the magnetic dipole $H_{dip}$, the electric quadrupole $H_{quad}$, and the octupole $H_{octup}$ terms

$$ H_{hyp} = H_{dip} + H_{quad} + H_{octup}. $$

$H_{dip}$ describes the interaction of the nuclear magnetic moment with the magnetic field generated by the electrons. For the electron angular momentum $J$ and the nuclear angular momentum $I$, it is given by

$$ H_{dip} = hA I \cdot J, $$

where $A$ is the magnetic dipole constant and $h$ is the Planck constant.

The electric quadrupole term originates from the Coulomb interaction between the electron and a non-spherically symmetric nucleus. It is given by

$$ H_{quad} = hB \frac{3(I \cdot J)^2 + \frac{3}{2} I \cdot J - I(I+1)J(J+1)}{2(2I-1)(2J-1)}, $$

where $B$ is the electric quadrupole moment coupling constant. This expression is valid for nuclear spins $I \geq 1$ and is zero otherwise.

The octupole Hamiltonian $H_{octup}$ presented in [Armenteros (1971)] depends on the electron and nuclear tensor operators of rank 3, requires an electron angular momentum at least equal to 3/2 and is characterised by the hyperfine $C$ constant. The explicit dependence on the atomic quantum numbers is detailed in [Gerginov, Derevianko, and Tanner (2003).]

Hyperfine interactions decrease rapidly for higher-lying states. [Kopfermann (1958)] showed that the hyperfine constants are proportional to the expectation value of $1/r^3$, where $r$ is the distance between the nucleus and valence electron. For highly excited electrons, the valence electron is far from the core electrons and $<1/r^3>$ is well approximated by the hydrogenic result

$$ <1/r^3> \approx \frac{1}{(n^*)^3} \left( 1 - \frac{\partial}{\partial n^*} \right), $$

where $n^*$ is the effective principal quantum number, $n$ is the principal quantum number, and the difference
\( \sigma(n) = n - n^* \) is the quantum defect. In a more refined treatment, by expressing the Schrödinger wavefunction at the nucleus position through the effective nuclear charge \( Z_1 \) in the inner region where the orbit penetrates, and setting \( Z_0 = 1 \) for the net charge of the ion around which the single electron moves, the modified Fermi-Segre formula for the dipolar constant of the state with angular momentum \( l \) is derived in Kopfermann (1958):

\[
A = \frac{8}{3} R_\infty^2 \alpha^2 g_l Z_l Z_0^2 (n^*)^3 F_{r,j}(n, l, Z) (1 - \epsilon)(1 - \delta) \tag{5}
\]

with \( R_\infty \) being the Rydberg constant, \( \alpha \) the fine structure constant, and \( g_l \) the gyromagnetic ratio of the nuclear magnetic moment. The relativistic effects are expressed through the factor \( F_{r,j}(n, l, Z) \) near unity for light atoms and different from unity for large \( Z \) numbers. The \( 1 - \epsilon \) factor is the change in the electronic wave function for distributions of the nuclear charge over its volume. The \( 1 - \delta \) factor is the change in the electron-nuclear interaction by the distribution of the magnetic moment, which is called the Breit-Weisskopf effect.

For the quadrupole coupling constant \( B \) the following expression is derived in Kopfermann (1958):

\[
B = \frac{1}{h} \frac{e^2}{4 \pi \varepsilon_0} \frac{2J + 1}{2J + 2} Q \left( \frac{1}{r^3} \right) > R_{r,j}(n, l, Z) \tag{6}
\]

with \( e \) the charge of the electron, \( \varepsilon_0 \) the vacuum permittivity, \( Q \) the quadrupole nuclear magnetic moment and \( R_{r,j}(n, l, Z) \) a relativistic correction factor. From Eq. (6) for \( 1/r^3 \), it follows that the \( B \) constant is also proportional to \( 1/(n^*)^3 \).

On the basis of the above expressions, where the nucleus is represented by a point charge, the following scaling is derived in Kopfermann (1958) for the magnetic dipole and magnetic quadrupole constants \( A \) and \( B \) associated with two different isotopes:

\[
\frac{A_i}{A_j} = \frac{g_{i1}}{g_{j1}}, \tag{7}
\]

\[
\frac{B_i}{B_j} = \frac{Q^2_{i1}}{Q^2_{j1}}. \tag{8}
\]

where \( g_{i1} \) and \( Q^2_{i1} \) represent the nuclear gyromagnetic factor \( g \) and the quadrupole moment of the isotope \( i \), respectively. Deviations from these isotopic scaling laws represent the hyperfine anomalies presented in Subsection VI.B.

The dipolar and quadrupolar hfs Hamiltonians, involving the interaction of spin and orbital angular momenta with the nuclear moments, have matrix elements diagonal over the hyperfine quantum numbers, but also off-diagonal ones \( (A_{i1,j1-1}) \) connecting fine-structure states with different \( J \) values as presented in Arimondo, Inguscio, and Violiño (1975). These off-diagonal couplings produce a mixing of the eigenstates and a shift of the energies. For fine structure states far apart in energy, a perturbation of the hyperfine constants includes the influence of the off-diagonal matrix elements, as for the \( 6^2P \) Cs doublet in Johnson et al. (2001). In the opposite case, such as the \( 2^2P \) Li doublet, the hyperfine splittings are expressed through the \( a_i \) (\( i = c, d, o \)) parameters of an effective hyperfine-splitting Hamiltonian: contact, dipolar and orbital, respectively, as derived in Lyons and Das (1970). In the first perturbation order and using the one-electron theory, the different hyperfine constants are written as

\[
A(2^2P_{1/2}) = -a_c - 10a_d + 2a_o,
\]

\[
A(2^2P_{3/2}) = a_c + a_d + a_o,
\]

\[
A(2^2P_{3/2}) = -a_c + \frac{5}{4}a_d + \frac{1}{2}a_o. \tag{9}
\]

These relations have been used by experimentalists for their data analysis. Higher perturbation order corrections to the \( Li \) constants were derived in Beloy and Derevianko (2008); Puchalski and Pachucki (2009). The mixing produced by off-diagonal hyperfine interactions plays an important role in the cesium measurements for parity nonconservation, as in the experiments of Gilbert et al. (1980); Bouchiat, M.-A. and Guéna, J. (1988), and in the theoretical analysis of Dzuba and Flambaum (2000). It is expected to be even more important in francium.

Eq. (7) predicts the following scaling law:

\[
A \propto \frac{1}{(n^*)^3}. \tag{10}
\]

A similar one applies to \( B \) on the basis of Eqs. (6) and (8).

For the \( 87^{\text{Rb}} \) low-\( n \) states, the \( A \) dependence on \( n^* \) was tested in 1976 by Belin, Holingren, and Svanberg (1976b), using the \( 1/(n^*)^2 \) dependence of the fine structure data. The \( A \) scaling law was later verified for \( 85^{\text{Rb}} \) high \( D \) states within 2 percent by van Wijngaarden, Li, and Koh (1993). More recently, with the very high-\( n^* \) values having been precisely derived from laser spectroscopy, the scaling was tested for the \( 85^{\text{Rb}} \) Rydberg states between \( n = 27 \) and \( n = 33 \) in Li et al. (2003) and Ramos, Cardman, and Raithel (2019). The \( 87^{\text{Rb}} \) data obtained by Li et al. (2003) were reexamined for the scaling in Mack et al. (2011). In Sasmannshausen, Merkt, and Deiglmayr (2013) the scaling law was verified for the \( 85^{\text{Rb}} \) high \( D \) states in the \( n = 10 - 80 \) range.

The theoretical determination of the hyperfine constants has greatly evolved within the last few years. Instead of focusing on a few atomic states, the more recent calculations target a very large number of states. While in 1999 a few hyperfine constants of different alkalis were calculated, e.g., in Safronova, Johnson, and Derevianko (1999), more recently M. Safronova and coworkers in (Johnson et al. 2008; Safronova and Safronova 2008; Auzinsh et al. 2007) have produced a global derivation of the constants for \( 7^{\text{Li}}, 39^{\text{K}}, 85^{\text{Rb}}, \) and \( 133^{\text{Cs}} \) Later, the hyperfine data for all the K isotopes were carefully examined by Singh, Nandy, and Sahoo (2012), for \( \text{Rb} \) and \( \text{Cs} \) by Grunefeld, Roberts, and Ginges (2019), for \( \text{Cs} \) by Tang, Lou, and Shi (2019), and for \( \text{Fr} \) by Sahoo et al. (2019). Lou et al. (2019).
The global analysis by Singh, Nandy, and Sahoo (2012) has derived precise values for the nuclear quadrupole moments of the potassium isotopes demonstrating a good internal consistency of the hyperfine data. The development led by Marianna Safronova to provide both experimental and theoretical energy level information for a large variety of atoms is a welcome addition to the data compilation. It is currently available online, see Barakhshan et al. (2022). The hyperfine interaction and the weak interaction that gives rise to parity-nonconservation (PNC) in atoms both happen because the electron density overlaps with the nucleus. From a particle physics point of view, the exchange of the $Z^0$ boson carries the weak interaction with its PNC. Atomic PNC interest comes from its unique possibility to test the standard model at low energy. The structure of the nucleus is key to the details of nuclear-spin-independent PNC, where the electron axial-vector-current interacts with the nucleon vector-current. The nuclear-spin-dependent PNC, where the nucleon axial-vector-current interacts with the electron vector-current also depends on nuclear structure and is primarily due to the nuclear anapole moment. As noted by Flambaum and Khriplovich (1989) and confirmed by Bouchiat and Piketty (1991); Johnson, Safronova, and Safronova (2003) the hyperfine interaction leads to the nuclear spin dependence of the matrix element in the atomic PNC. Cs measurements by Wood et al. (1997) reached enough sensitivity to measure the anapole moment in the nuclear-spin-dependent part of the PNC interaction. A new generation of atomic parity violation experiments is underway, e.g. Gwinner and Orozco (2022). These experiments are made with francium atoms. The PNC effects in Fr with respect to Cs are estimated to be 18 times larger for the nuclear-spin-independent and 11 times larger for the nuclear-spin-dependent part.

The determination of the parity-conserving quantities in both high precision experiments and ab initio calculations, such as transition matrix elements, lifetimes, polarizabilities, and hyperfine constants, is essential for PNC studies. The hyperfine constants test in quantitative ways the quality of the electronic wavefunction near the nucleus. This unique combination between theory and experiment has greatly favored the heaviest alkali atoms and has stimulated a large search effort for the hyperfine structures in their isotopes as presented in Safronova et al. (2018).

V. MEASURED HYPERFINE CONSTANTS

In the following, we present several Tables for the measured $A$ and $B$ constants of the alkali atoms. In a few cases, we derive the $A$ constant from the measured hyperfine splitting reported by the authors using the formula for the hyperfine energies given by Kopfermann (1958). In each Table, the atomic states are listed in order of increasing $n$, then of increasing $L$, increasing $J$, and finally chronologically. Two columns report the acronyms determining the atomic sample and the spectroscopic technique applied in the measurement. The reference to the original publication is in the last column. The spectroscopic technique column reports the “From” notation for the data taken from Arimondo, Inguscio, and Violino (1972), for which a critical examination or a weighted averaging over several measurements was performed, leading to a recommended value. When the hyperfine value reported in that reference remains the only one available, or its error bar is smaller than later measurements, the original work is directly quoted in the Table. Within the Table’s B column the entry “0.” denotes that the authors have assumed the quadrupole constant equal to zero.

A few techniques, such as the HQB, are not able to resolve the sign of the constants for the explored state. On the basis of the above scaling laws applied to different atoms within Section VIA, we have produced a dependable sign assignment for most states. If the sign was not identified, the absolute value is reported.

The measurement uncertainties are reported in the text and Tables in parentheses after the value, in units of the last decimal place of the value. For example, 153.3(11) means 153.3 ± 1.1. Most authors specify their uncertainty on the level of one standard deviation. If a different convention is used, it is mentioned in the text. For each atomic species, we mention in the text the states where a very high precision is obtained or where a disagreement between the measured values exists. When several ($n$) measurements are associated with a single state, the Tables include a weighted average, w.a., representing a reference for further work. We follow the procedure of the Particle Data Group in Zyla et al. (2020) in the Introduction, Sec. 5.2.2. Unconstrained averaging, to find the weighted error (w.e.). We calculate it first based on the $n$ individual errors ($e_i$), w.e. = $(1/n)\sum_{i=1}^{n} e_i^2$ We also calculate the reduced $\chi$-squared ($\chi^2_{\text{red}}$) with $n-1$ degrees of freedom, to test the size of the weighted error. If $\chi^2_{\text{red}}$ is greater than unity by more than one standard deviation $(2/\chi^2_{\text{red}})^{1/2}$, then we increase the w.e. of the w.a. by the factor $(\chi^2_{\text{red}})^{1/2}$, so that the weighted enhanced error (w.e.e.) is w.e.e. = $(\chi^2_{\text{red}})^{1/2}\times\text{w.e.}$. We report in the table either the w.a. with its weighted error or the enhanced error (w.e.e.), which we explicitly state. Such averaging is not performed when the precision of one measurement is greater than all the remaining ones, which is then denoted by “Recommended” in the Table’s last column. The last column contains a “See text” statement, if one or more values are not included into the w.a.. A different way of averaging developed by Rukhin (2009, 2019) evaluates the clustering of the data and assigns individual hidden uncertainties to the measurements from different groups. These uncertainties are then added in quadrature to the stated uncertainties. Weighted averages and weighted errors calculated by this cluster maximum likelihood estimator (CMLE) are not always identical to the values reported in our Tables. For completeness, whenever a Table recommended value
includes a w.e.e. derived from the $(\chi^2_{\text{red}})^{1/2}$ analysis, we have used the CMLE method to calculate the corresponding w.a.-CMLE weighted average and w.e.e.-CMLE weighted enhanced error. The comparison between the recommended values obtained by these two approaches shows that in almost all the 28 cases the weighted average of the two analyses agrees within two times the w.e.e. There are only two cases with a greater difference, associated with a large $(\chi^2_{\text{red}})^{1/2}$ value as an indicator of an anomalous scatter of the data. The interested reader can find results and plots in the Supplementary Information (SI) of this review, reference Allegrini, Arimondo, and Orozco (2022).

A. Lithium

Data for this atom are reported in Table I. As for all alkalis, several spectroscopic investigations are stimulated by the interest in laser cooling, but only a few experiments are performed in a MOT. The recent $^6$Li MOT experiments by Wu et al. (2018); Li et al. (2020); Rut et al. (2021) at Shanghai may open a new trend. For the ground state of both lithium isotopes, the "old" atomic beam measurements based on MWS report the highest precision, not reached by the several ones based on laser spectroscopy. Otto et al. (2002) performed measurements for a few high-n states of $^7$Li, as well as for other alkalis.

For the $^6,^7$Li $^{2S}_{1/2}$ state measurements by Unfer, Windholz, and Musso (1992), the error bar is derived on the basis of Eqs. (9) from the off-diagonal constants presented in the following. For the $A$ value of the $^{3S}_{1/2}$ state of both isotopes measured by Lien et al. (2011), we estimated the error bar not reported by the authors.

A wide experimental effort concentrates on the $^{2S}_{1/2}$ state of both isotopes. Instead, the $^{2P}_{3/2}$ state of $^6$Li remains with the data obtained before laser spectroscopy. For $^7$Li, three experimental investigations of this doublet, by Orth, Ackermann, and Otten (1975), Nagourney, Happer, and Lurio (1978) and Unfer, Windholz, and Musso (1992), consider the contribution of the off-diagonal hyperfine couplings linked to the small fine-structure splitting. Orth, Ackermann, and Otten (1975) include in their analysis the previous level crossing data by Brog, Eck, and Wieder (1967). The analysis by Nagourney, Happer, and Lurio (1978) combines their own data and the previous ones. Unfer, Windholz, and Musso (1992) acquired enough data for their own analysis. Orth, Ackermann, and Otten (1975) using Eqs. (9) and their measured $A_{3/2,1/2} = 11.85(35)$ MHz found $a_c = -9.838(48)$, $a_d = -1.876(12)$, $a_o = 8.659(37)$. Nagourney, Happer, and Lurio (1978) reported $a_c = -9.838(48)$, $a_d = -1.975(22)$, $a_o = 8.659(37)$, and Unfer, Windholz, and Musso (1992) obtained $a_c = -9.93(37)$, $a_d = -1.72(20)$, $a_o = 8.69(31)$ (all in MHz). For the same doublet, Beloy and Derevianko (2008) performed an accurate evaluation of the off-diagonal hyperfine couplings. They produced corrections to the measured dipole constant in the $A(2P_{3/2})$ accurately measured by Orth, Ackermann, and Otten (1975), Walls et al. (2003), and Das and Natarajan (2008). For this last one, the 27.0 kHz correction should be compared to the author's original 3 kHz experimental uncertainty.

A disagreement exceeding the error bars exists for the $^{3S}_{1/2}$ $^7$Li hyperfine constant $A$ measured in Stevens et al. (1995), compared to those of Bushaw et al. (2003); Lien et al. (2011). We do not include that measurement in the w.a.

The discrepancy for the $^{3P}_{3/2}$ $^7$Li constants between the previous values by Budick et al. (1966); Isler, Marcus, and Novick (1969) and the Nagourney, Happer, and Lurio (1978) ones remains unexplained as pointed out by these authors. Therefore, instead of the recommended value from Arimondo, Inguscio, and Violino (1977), we consider all the previous measurements. The $\chi^2_{\text{red}}$ correction is applied to the statistical error. An analysis of the off-diagonal elements was performed by Nagourney, Happer, and Lurio (1978) for the $^{3P}_{1/2}$ $^7$Li doublet, leading to $a_c = -3.10(67)$, $a_d = -0.54(27)$, $a_o = 2.61(40)$.

\begin{table}[h]
\centering
\caption{Measured $A$ and $B$ values for Li isotopes}
\begin{tabular}{|c|c|c|c|}
\hline
State & $A$ (MHz) & $B$ (MHz) & Sample Technique Ref. \\
\hline
$^6$Li & & & \\
$^{2S}_{1/2}$ & 152.1368407(20) & AB & MWS From Arimondo, Inguscio, and Violino (1977) \\
* & 151.3(3) & AB & VC Lien and Niemax (1982) \\
* & 153.3(11) & KB & VC Lien and Niemax (1982) \\
* & 152.109(43) & AB & VC Lien and Niemax (1982) \\
* & 152.121(57) & AB & VC Lien and Niemax (1982) \\
* & 152.143(11) & AB & VC Lien and Niemax (1982) \\
* & 152.1343(9) & AB & VC Lien and Niemax (1982) \\
* & 152.1368407(20) & AB & VC Lien and Niemax (1982) \\
$^{2P}_{3/2}$ & 17.375(18) & AB & VC Lien and Niemax (1982) \\
* & 17.8(3) & AB & VC Lien and Niemax (1982) \\
* & 16.81(70) & AB & VC Lien and Niemax (1982) \\
* & 17.386(31) & AB & VC Lien and Niemax (1982) \\
* & 17.407(37) & AB & VC Lien and Niemax (1982) \\
* & 17.394(4) & AB & VC Lien and Niemax (1982) \\
* & 17.407(10) & AB & VC Lien and Niemax (1982) \\
\hline
\end{tabular}
\end{table}

Continued on next page
| State | $A(MHz)$ | $B(MHz)$ | Sample Technique | Rel. |
|-------|-------|-------|-----------------|------|
| $2^2P_{3/2}$ | 17.4021(9) | - | MOT LIF | Li et al. (2020); Rui et al. (2021) |
| $2^2P_{3/2}$ | 17.408(13) | - | MOT LIF+DD | Li et al. (2021) |
| $2^2P_{3/2}$ | 17.4017(9) | - | - | w.a. |
| $3^2S_{1/2}$ | -1.155(8) | -0.10(14) | AB ORFDR | Orth et al. (1973) |
| $3^2S_{1/2}$ | 34.13 | - | VC ION | Vadla, Obrebski, and Niemax (1987) |
| $3^2P_{3/2}$ | 35.263(15) | - | AB TPSDS+RIS | Bushaw et al. (2003) |
| $3^2P_{3/2}$ | 35.283(10) | - | AB RIS | Ewald et al. (2004) |
| $3^2P_{3/2}$ | 35.20(20) | - | AB LIF | Lien et al. (2011) |
| $3^2P_{3/2}$ | 35.267(14) | - | AB RIS | Nörtershäuser et al. (2011) |
| $3^2S_{1/2}$ | 35.274(7) | - | - | w.a. |
| $3^2P_{1/2}$ | 3.4(15) | - | VC MLC | Nagourney, Happer, and Lurio (1978) |
| $3^2P_{3/2}$ | -0.40(2) | 0 | VC MLC | Isler, Marcus, and Novick (1969) |
| $4^2S_{1/2}$ | 13.1(13) | - | AB LIF | Kowalski et al. (1978) |
| $4^2S_{1/2}$ | 15.3 | - | VC ION | Lorenzen and Niemax (1982) |
| $4^2S_{1/2}$ | 13.5(8) | - | AB LIF | DeGraffenreid and Sansonetti (2003) |
| $4^2S_{1/2}$ | 13.5(7) | - | - | w.a. |

Table I – Continued from previous page

| State | $A(MHz)$ | $B(MHz)$ | Sample Technique | Rel. |
|-------|-------|-------|-----------------|------|
| $2^2S_{1/2}$ | 401.7520433(5) | - | AB MWS | Beckmann, Böklen, and Elke (1974) |
| $2^2S_{1/2}$ | 401.5 | - | VC ION | Lorenzen and Niemax (1982) |
| $2^2S_{1/2}$ | 401.81(25) | - | AB LIF | Windholz et al. (1990) |
| $2^2S_{1/2}$ | 401.767(39) | - | AB LIF | Walls et al. (2003) |
| $2^2S_{1/2}$ | 401.772(33) | - | AB MFL | Noble et al. (2006) |
| $2^2S_{1/2}$ | 401.747(7) | - | AB LIF+FCS | Sansonetti et al. (2011) |
| $2^2S_{1/2}$ | 401.755(8) | - | AB LIF | Huang et al. (2013) |
| $2^2S_{1/2}$ | 401.7520433(5) | - | - | Recommended |

$^Li$

| State | $A(MHz)$ | $B(MHz)$ | Sample Technique | Rel. |
|-------|-------|-------|-----------------|------|
| $2^2P_{1/2}$ | 45.914(25) | - | AB ORFDR | Orth et al. (1974) |
| $2^2P_{1/2}$ | 46.05(30) | - | AB LIF | Windholz et al. (1990) |
| $2^2P_{1/2}$ | 46.175(2980) | - | AB MLC | Umler, Windholz, and Musso (1992) |
| $2^2P_{1/2}$ | 46.010(25) | - | AB LIF | Walls et al. (2003) |
| $2^2P_{1/2}$ | 45.903(26) | - | AB FML | Noble et al. (2006) |
| $2^2P_{1/2}$ | 46.024(3) | - | AB LIF | Das and Natarajan (2008) |
| $2^2P_{1/2}$ | 46.047(3) | - | AB LIF | Li et al. (2010) |
| $2^2P_{1/2}$ | 45.938(5) | - | AB LIF+FCS | Sansonetti et al. (2011) |
| $2^2P_{1/2}$ | 45.946(4) | - | AB LIF | Huang et al. (2013) |
| $2^2P_{1/2}$ | 46.005(16) | - | - | w.a. (w.e.e.) |

$^S_{1/2}$

| State | $A(MHz)$ | $B(MHz)$ | Sample Technique | Rel. |
|-------|-------|-------|-----------------|------|
| $2^2P_{1/2}$ | -3.055(14) | -0.221(29) | AB ORFDR | Orth, Ackermann, and Otten (1975) |
| $2^2P_{1/2}$ | -2.95(4) | 0 | VC MLC | Nagourney, Happer, and Lurio (1978) |
| $2^2P_{1/2}$ | -3.08(4) | -0.16(10) | AB LIF+DD | Shimizu et al. (1987) |
| $2^2P_{1/2}$ | -3.18(10) | -0.8(7) | AB LIF | Windholz et al. (1990) |
| $2^2P_{1/2}$ | -3.08(8) | -0.20(27) | AB HQB | Carlsson and Sturesson (1989) |
| $2^2P_{1/2}$ | -2.96(88) | -0.1(5) | AB MLC | Umler, Windholz, and Musso (1992) |
| $2^2P_{1/2}$ | -3.050(16) | -0.22(3) | AB MLC | Umler, Windholz, and Musso (1992) |

$^S_{1/2}$

| State | $A(MHz)$ | $B(MHz)$ | Sample Technique | Rel. |
|-------|-------|-------|-----------------|------|
| $3^2P_{3/2}$ | 95.10 | - | VC ION | Vadla, Obrebski, and Niemax (1987) |
| $3^2S_{1/2}$ | 94.68(22) | - | AB SS | Stevens et al. (1995) |
| $3^2P_{3/2}$ | 93.106(11) | - | AB TPSDS+RIS | Bushaw et al. (2003) |
| $3^2P_{3/2}$ | 93.117(25) | - | AB RIS | Ewald et al. (2004) |
| $3^2P_{3/2}$ | 93.13(20) | - | AB LIF | Lien et al. (2011) |
| $3^2P_{3/2}$ | 93.103(11) | - | AB RIS | Nörtershäuser et al. (2011) |
| $3^2P_{3/2}$ | 93.095(52) | - | AB SAS | Kumar and Natarajan (2017) |
| $3^2P_{3/2}$ | 93.105(7) | - | - | w.a. (w.e.e.) |

$^P_{3/2}$

| State | $A(MHz)$ | $B(MHz)$ | Sample Technique | Rel. |
|-------|-------|-------|-----------------|------|
| $3^2P_{3/2}$ | 13.5(2) | - | VC MLC | Budick et al. (1966) |
| $3^2P_{3/2}$ | 13.7(12) | - | VC MLC | Nagourney, Happer, and Lurio (1978) |
| $3^2P_{3/2}$ | 13.5(2) | - | - | Recommended |
| $3^2P_{3/2}$ | 0.96(13) | 0 | VC MLC | Budick et al. (1966) |
| $3^2P_{3/2}$ | 0.965(20) | 0.019(22) | VC MLC | Isler, Marcus, and Novick (1969) |
| $3^2P_{3/2}$ | -1.036(16) | -0.094(10) | VC MLC | Nagourney, Happer, and Lurio (1978) |
| $3^2P_{3/2}$ | -1.01(2) | -0.081(28) | AB | w.a. (w.e.e.) |

$^P_{3/2}$

| State | $A(MHz)$ | $B(MHz)$ | Sample Technique | Rel. |
|-------|-------|-------|-----------------|------|
| $3^2P_{3/2}$ | 0.843(41) | 0 | AB TPSDS | Burghardt, Hoffmann, and Meisel (1988) |
| $3^2P_{3/2}$ | 1.14(49) | 0 | VC TPSDS | Otto et al. (2002) |

Continued on next page
excited states. In the same year, Yei, Sieradzan, and et al. (1989) published the first hyperfine constant determination in an atomic fountain with a relative precision of 1 mHz precision, close to the previous best atomic beam value of Table II, opening the road to further amazing improvements in fountain atomic clocks. Zhu, Oates, and Hall (1993) performed the first hyperfine constant measurement in a MOT exploring the first excited state, 3P1/2, with an increasing precision and a good agreement among the different results. For the 4D states of Biraben and Beroff (1978) and for the 7, 8 and 9P3/2 ones, no competitive new data for the ground state are available. The 3P1/2, 3/2 states have been examined by several authors, with an increasing precision and a good agreement among the different results. For the 4D states of Biraben and Beroff (1978) and for the 7, 8 and 9P3/2 ones, no competitive new data for the ground state are available.

B. Sodium

The sodium results of Table II are emblematic of the progress achieved in hyperfine constant measurements. In chronological order, the two-photon sub-Doppler spectroscopy was applied in 1978 to probe the 4D excited states, by Biraben and Beroff (1978) and Burghardt et al. (1978), extended by this last research group to the 3D states in Burghardt, Hoffmann, and Meisel (1988). In 1989, Kasevich et al. (1989) published the first hyperfine ground state determination in an atomic fountain with 1 mHz precision, close to the previous best atomic beam value of Table II, opening the road to further amazing improvements in fountain atomic clocks. Zhu, Oates, and Hall (1993) performed the first hyperfine constant measurement in a MOT exploring the 5P states with a relative precision ≈ 1 × 10⁻⁴ among the best ones for excited states. In the same year, Yei, Sieradzan, and Havey (1993) performed subnatural linewidth measurements of hyperfine coupling constants using the delayed detection in polarization quantum beat spectroscopy. In 2003, the excitation of ultracold atoms in a MOT on the 3P − 4P electric quadrupole transition allowed Bhat-tacharya, Haimberger, and Bigelow (2003) to perform high resolution spectroscopy of the 4P1/2 level. Das and Natarajan (2006b) introduced the CCS approach for the first excited state, 3P1/2. However, no competitive new data for the ground state are available.

The 3P1/2, 3/2 states have been examined by several authors, with an increasing precision and a good agreement among the different results. For the 4D states of Biraben and Beroff (1978) and for the 7, 8 and 9P3/2 ones, no competitive new data for the ground state are available. The large majority of excited states above the 5P ones, no new data have been published after 1982.

### Table I – Continued from previous page

| State  | A(MHz)     | B(MHz)  | Sample | Technique | Ref. |
|--------|------------|---------|--------|-----------|------|
| 3D5/2  | 0.3436(10) | 0.      | AB     | TPSDS     | Burghardt, Hoffmann, and Meisel (1988) |
|        | 0.31(13)   | 0.      | VC     | TPSDS     | Otto et al. (2002) |
|        | 0.3426(10) | 0.      | -      | -         | Recommended |
| 4S1/2  | 36.4(40)   | -       | AB     | LIF       | Kowalski et al. (1978) |
|        | 38.3       | -       | VC     | ION       | Lorenzen and Niemax (1982) |
|        | 35.32(72)  | -       | VC     | TPSDS     | Otto et al. (2002) |
|        | 34.9(4)    | -       | AB     | LIF       | DeGraffenreid and Sansonetti (2003) |
|        | 35.05(35)  | -       | -      | -         | w.a. |
| 4P3/2  | -0.41(4)   | 0.      | VC     | MLC       | Isler, Marcus, and Novick (1969) |
| 6S1/2  | 38.0(15)   | -       | VC     | TPSDS     | Otto et al. (2002) |

### Table II – Measured A and B values for 23Na

| State  | A(MHz)     | B(MHz)  | Sample | Technique | Ref. |
|--------|------------|---------|--------|-----------|------|
| 3S1/2  | 885.8130644(5) | -       | AB     | MWS       | From Arimondo, Inguscio, and Violino (1977) |
|        | 885.70(25)  | -       | VC     | SAS       | Pesch, Gerhardt, and Matthias (1977) |
|        | 885.8130651(1) | -       | FOUNT | RIS       | Kasevich et al. (1989) |
|        | 885.8130644(4) | -       | -      | -         | Recommended |
| 3P1/2  | 94.25(15)   | -       | VC     | SAS       | Pesch, Gerhardt, and Matthias (1977) |
|        | 94.47(1)    | -       | AB     | LIF       | Grillith et al. (1977) |
|        | 94.05(20)   | -       | AB     | LIF       | Umier,Windholz,and Musso (1992) |
|        | 94.42(19)   | -       | AB     | HQB       | Carlsson et al. (1992) |
|        | 94.44(13)   | -       | AB     | LIF       | van Wijngaarden and Li (1994) |
|        | 94.7(2)     | -       | AB     | LIF       | Scherl et al. (1996) |
|        | 94.349(7)   | -       | VC     | SAS       | Das and Natarajan (2008) |
|        | 94.39(2)    | -       | -      | -         | w.a. |
| 3P3/2  | 16.84(6)    | 2.77(6) | AB     | HQB       | Krist et al. (1977) |
|        | 16.69(6)    | 2.83(10)| AB     | HQB       | Carlsson and Struresson (1989) |
|        | 16.78(8)    | 2.60(41)| AB     | HQB       | Umier, Windholz, and Musso (1992) |
|        | 16.534(15)  | 2.724(30)| VC    | HQB+DD    | Yei, Sieradzan, and Havey (1993) |
|        | 16.62(21)   | 2.11(52)| AB     | LIF       | van Wijngaarden and Li (1994) |
|        | 18.8(1)     | 2.7(2)  | AB     | LIF       | Scherl et al. (1996) |
|        | 18.79(12)   | 2.75(12)| AB     | HQB       | Voiz et al. (1996) |
|        | 18.572(24)  | 2.723(55)| AB    | LIF       | Gangersky et al. (1998) |
|        | 18.530(3)   | 2.721(8)| VC     | CCS       | Das et al. (2006b) |
|        | 18.532(6)   | 2.722(7)| -      | -         | w.a. |
| 3D3/2  | 0.527(25)   | 0.      | AB     | TPSDS     | Burghardt, Hoffmann, and Meisel (1988) |
| 3D5/2  | 0.1085(24)  | 0.      | -      | -         | -     |

Continued on next page
Continued from previous page

| State | A(MHz) | B(MHz) | Sample | Technique | Ref. |
|-------|--------|--------|--------|-----------|------|
| \(4^2S_{1/2}\) | 203.6(2) | – | VC | TPSDS+LIF | Arqueros (1988) |
| \(4^2P_{1/2}\) | 30.4(5) | – | VC | ORFDR | Grundevik and Lundberg (1978) |
| \(4^2P_{3/2}\) | 30.6(1) | – | MOT | OODR+RIS | Bhattacharya, Haumberger, and Bigelow (2003) |
| \(4^2P_{5/2}\) | 30.6(1) | – | – | – | w.a. |
| \(4^2D_{3/2}\) | 6.022(61) | 0.97(6) | VC | LC+ORFDR | From Arimondo, Inguscio, and Violino (1977) |
| \(4^2D_{5/2}\) | 0.23(12) | 0. | VC | TPSDS | Biraben and Beroff (1978) |
| \(5^2S_{1/2}\) | 77.6(2) | – | VC | ORFDR | Tsekeris, Liao, and Gupta (1976) |
| \(5^2P_{1/2}\) | 13.3(2) | – | VC | ORFDR | Grundevik and Lundberg (1978) |
| \(5^2P_{3/2}\) | 2.64(1) | 0.38(3) | AB | HQB | Grundevik et al. (1979) |
| \(6^2S_{1/2}\) | 37.5(2) | – | AB | SS | Hawkins et al. (1977) |
| \(6^2P_{3/2}\) | 34.5(45) | – | – | – | Recommended |
| \(7^2S_{1/2}\) | 37.5(2) | – | – | – | Recommended |
| \(7^2P_{1/2}\) | 1.39(1) | 0.21(2) | AB | HQB | Grundevik et al. (1979) |
| \(8^2S_{1/2}\) | 20.9(1) | – | AB | ORFDR | Grundevik, Mårtensson, and Svaneberg (1978) |
| \(8^2P_{1/2}\) | 23.3(65) | – | AB | SS | Hawkins et al. (1977) |
| \(9^2P_{3/2}\) | 29.1(1) | – | – | – | Recommended |
| \(9^2P_{5/2}\) | 0.82(1) | 0.13(3) | VC | QBS | Jing, Jönsson, and Lundberg (1982) |

C. Potassium

Our previous Li and Na remarks on the ground state values do not apply to the \(^{39}\text{K}\) ground state, as shown in Table III. For this atom, several recent measurements exist, with the frequency comb spectral resolution applied to determine the absolute transition frequencies, and from them, the fine and hyperfine splittings. A high precision is achieved, usually with a good agreement among data from different research groups, e. g. for several n = 5 – 8 \(2^2S_{1/2}\) states with measurements performed over a long time span. Otto et al. (2002) explored the \(2^2S\) states of the 39 and 41 isotopes up to n = 14 using two-photon sub-Doppler spectroscopy.

The spatial dependence of the HQB polarized fluorescence intensity in a given magnetic field was used by Głódź and Kraińska-Miszczak (1985a) to derive for the first time the signs of the A constants in the \(6^2D\) states of \(^{39}\text{K}\). For other \(2^2D\) states, the investigations by Belin et al. (1975b), Sieradzan et al. (1997) and Głódź and Kraińska-Miszczak (1985b) produced only the absolute sign of the A and B constants. Their signs are determined here on the basis of the scaling laws.

1. \(^{39}\text{K}\)

The ground \(4^2S_{1/2}\) state was measured in three MOT experiments by Antoni-Micollier et al. (2017); Arias et al. (2019); Peper et al. (2019). Peper et al. (2019) obtained a value close to the old atomic beam experiments with a difference in the 10 Hz range. Arias et al. (2019) claimed that their small discrepancy with the previous AB measurement can be accounted for by the 16(6) Hz quadratic Zeeman shift of the bias field and by the differential ac Stark shift in the optical dipole trap. The Table III value takes into account such shift. Antoni-Micollier et al. (2017) reported an all-optical measurement of the hyperfine splitting with a low statistical uncertainty, but there were uncontrolled systematical errors in their work, according to Peper et al. (2019).

A large disagreement exists between the measured A values of the \(4^2P_{1/2}\) states, some of them having been reported with high precision. The data are centred around two separate values 27.78(4) and 28.849(5) MHz with a separation larger than the reported precision. The lower values are by Bendali, Duong, and Vialle (1981); Touchard et al. (1982); Duong (1982); Papuga et al. (2014), (all of them with low precision) and by Falke et al. (2006) with higher precision. The greater values

10
were obtained in an AB magnetic resonance experiment by Buck and Rabi [1957] with a low precision and in two recent measurements by the Bangalore research group Banerjee, Das, and Natarajan (2004); Das and Natarajan (2008) used the accurate CCS technique. The $\chi^2_{\text{red}}$ for the full data set leads to a very large error bar. Light shifts corrections, an important issue for several recent publications, were taken into account by the Bangalore group. They stated, “we do not have a satisfactory explanation for such a large discrepancy”. Such a discrepancy does not exist for the $4^2P_{3/2}$ values determined at the same time by Falke et al. (2006) and Das and Natarajan (2008). Falke et al. (2006) compared their $^6$Li $D$ optical transition values to those by Banerjee, Das, and Natarajan (2004) and attributed the discrepancies to systematic errors in the laser calibration, more precisely to phase shifts in the wavelength (not frequency) comparison of the atomic excitation lasers. In Das and Natarajan (2008), the driving frequency of an acousto-optical modulator gives a direct measurement of the hyperfine interval and the calibration issue should have been resolved. For the discrepancies in those optical frequencies, Brown et al. (2013) pointed out the important role of quantum interference and light polarization effects. For the $4^2P_{3/2}$ state the Table III reports a w.a. excluding the Banerjee, Das, and Natarajan (2004); Das and Natarajan (2008) values.

For the $6^2S_{1/2}$ state, the $A$ value referred to Thompson et al. (1983) in the Table III was derived from their hyperfine splitting in Stalnaker et al. (2017).

2. $^{41}$K

The $7^2S_{1/2}$ values by Thompson et al. (1983) and by Otto et al. (2002) have a large disagreement. The first value is not consistent with the $1/(n^*)^3$ scaling law applied to the $n^2S_{1/2}$ states of this isotope. The Otto et al. (2002) value is the “Recommended” one.

Table III: Measured $A$ and $B$ values for K isotopes

| State | $A(MHz)$ | $B(MHz)$ | Sample | Technique | Ref. |
|-------|----------|----------|--------|-----------|------|
| $^{40}$K | | | | | |
| $3^2D_{3/2}$ | < 1.6 | 0.0 | VC | ORFDR | Lam, Gupta, and Happer (1980) |
| | 0.96(4) | 0.37(8) | VC | HQBD+DD | Sieradzki et al. (1997) |
| | 0.96(4) | 0.37(8) | - | - | Recommended |
| $3^2D_{5/2}$ | < 2.2 | 0.0 | VC | ORFDR | Lam, Gupta, and Happer (1980) |
| | -0.62(4) | 0.0 | VC | HQBD+DD | Sieradzki et al. (1997) |
| | -0.62(4) | 0.0 | - | - | Recommended |
| $4^2S_{1/2}$ | 230.8598601(3) | - | AB | MWS | From Arimondo, Inguscio, and Violino (1977) |
| | 231.0(3) | - | AB | LIF | Touchard et al. (1982) |
| | 230.85990(1) | - | AB | MWS | Duong et al. (1993) |
| | 231.0(3) | - | AB | LIF | Papaga et al. (2014) |
| | 230.859859(6) | - | MOT | MWS | Arias et al. (2019) |
| | 230.859850(3) | - | MOT | MWS | Peper et al. (2019) |
| | 231.1(3) | - | AB | RIS | Koszoruz et al. (2019) |
| | 230.8598601(3) | - | - | - | Recommended |
| $4^2P_{3/2}$ | 28.85(30) | - | AB | MWS | Buck and Rabi (1957) |
| | 27.80(15) | - | AB | FLM | Wendel, Duong, and Vialle (1981) ; Duong (1982) |
| | 27.5(4) | - | AB | LIF | Touchard et al. (1982) |
| | 28.859(15) | - | VC | SAS | Banerjee, Das, and Natarajan (2004) |
| | 27.775(42) | - | AB | LIF+FC | Falke et al. (2006) |
| | 28.848(5) | - | VC | CCS | Das and Natarajan (2008) |
| | 27.8(2) | - | AB | LIF | Papaga et al. (2014) |
| | 27.793(71) | - | - | - | See Text. w.a. (w.e.e.) |
| $4^2P_{1/2}$ | 6.903(25) | 2.786(71) | AB | LIF+FC | Falke et al. (2006) |
| | 6.077(23) | 2.875(55) | VC | CCS | Das and Natarajan (2008) |
| | 6.084(17) | 2.842(43) | - | - | w.a. |
| $5^2S_{1/2}$ | 55.50(60) | - | VC | ORFDR | Gupta et al. (1973) |
| $5^2P_{1/2}$ | 9.02(17) | - | VC | ORFDR | From Arimondo, Inguscio, and Violino (1977) |
| | 8.93(69) | - | VC | SAS | Halloran et al. (2009) |
| | 9.01(17) | - | - | - | w.a. |
| $5^2P_{3/2}$ | 1.969(13) | 0.870(22) | VC | MLC | From Arimondo, Inguscio, and Violino (1977) |
| $5^2D_{1/2}$ | 0.44(10) | - | VC | ORFDR | Belin et al. (1975b) |
| $5^2D_{3/2}$ | -0.24(7) | 0.0 | - | - | |
| $6^2S_{1/2}$ | 21.81(18) | - | VC | ORFDR | Gupta et al. (1973) |
| | 20.4(23) | - | TD | TPSDS | Thompson et al. (1983) |

Continued on next page
| State | \(A(MHz)\) | \(B(MHz)\) | Sample | Technique | Reference |
|-------|-------------|-------------|--------|-----------|-----------|
| \(3^2\) \(D_3/2\) | 0.07(2) | 0.4(1) | AB | HQB+DD | Sieradzan et al. (1997) |
| \(3^2\) \(D_3/2\) | 0.71(4) | 0.8(8) | • | • | • |
| \(4^2\) \(S_1/2\) | -285.7308(24) | - | AB | MWS | From Arimondo, Inguscio, and Violino (1977) |
| \(4^2\) \(P_1/2\) | 126.9(8) | - | AB | LIF | Touchard et al. (1982) |
| \(4^2\) \(P_3/2\) | 15.19(21) | - | AB | LIF | Touchard et al. (1982) |
| \(5^2\) \(P_1/2\) | 15.1(8) | - | AB | LIF | Touchard et al. (1982) |
| \(5^2\) \(P_3/2\) | 15.245(42) | - | AB | LIF+FC | Falke et al. (2006) |
| \(6^2\) \(P_1/2\) | 3.43(5) | 0. | AB | MFD | Ney (1969) |
| \(6^2\) \(P_3/2\) | 3.325(15) | 3.230(23) | AB | HQB | Sieradzan, Kutiatunga, and Havey (1995) |
| \(7^2\) \(S_1/2\) | 3.363(25) | 3.351(71) | AB | LIF+FC | Falke et al. (2006) |
| \(7^2\) \(P_1/2\) | 3.342(12) | 3.242(22) | • | • | • |
| \(8^2\) \(S_1/2\) | 30.75(75) | - | VC | ORFDR | Gupta et al. (1973) |
| \(8^2\) \(P_1/2\) | 4.96(17) | - | VC | MLC | Ney (1969) |
| \(9^2\) \(S_1/2\) | 1.08(2) | 1.06(4) | AB | LIF | Ney (1969) |
| \(9^2\) \(S_2/2\) | 12.03(40) | - | VC | TPSDS | Kiran Kumar and Suryanarayana (2011) |
| \(10^2\) \(S_1/2\) | 1.18(16) | - | • | • | • |

**K**

| State | \(A(MHz)\) | \(B(MHz)\) | Sample | Technique | Reference |
|-------|-------------|-------------|--------|-----------|-----------|
| \(3^2\) \(D_3/2\) | 0.55(3) | 0.51(8) | AB | HQB+DD | Sieradzan et al. (1997) |
| \(3^2\) \(D_3/2\) | 0.40(2) | < 0.2 | • | • | • |
| \(4^2\) \(S_1/2\) | 127.0069352(6) | - | AB | MWS | From Arimondo, Inguscio, and Violino (1977) |
| \(4^2\) \(P_1/2\) | 127.0069352(6) | - | AB | LIF | Touchard et al. (1982) |
| \(4^2\) \(P_3/2\) | 15.1(8) | - | AB | LIF | Touchard et al. (1982) |
| \(4^2\) \(P_3/2\) | 15.245(42) | - | AB | LIF+FC | Falke et al. (2006) |
| \(5^2\) \(S_1/2\) | 3.43(5) | 3.24(24) | AB | MFD | Ney (1969) |
| \(5^2\) \(P_1/2\) | 3.325(15) | 3.230(23) | AB | HQB | Sieradzan, Kutiatunga, and Havey (1995) |
| \(5^2\) \(P_3/2\) | 3.363(25) | 3.351(71) | AB | LIF+FC | Falke et al. (2006) |
| \(6^2\) \(P_1/2\) | 3.342(12) | 3.242(22) | • | • | • |
| \(5^2\) \(S_1/2\) | 30.75(75) | - | VC | ORFDR | Gupta et al. (1973) |
| \(5^2\) \(S_2/2\) | 4.96(17) | - | VC | MLC | Ney (1969) |
| \(6^2\) \(S_2/2\) | 1.08(2) | 1.06(4) | AB | LIF | Ney (1969) |
| \(6^2\) \(P_1/2\) | 12.03(40) | - | VC | TPSDS | Kiran Kumar and Suryanarayana (2011) |
| \(7^2\) \(S_1/2\) | 11.8(13) | - | VC | TPSDS | Kiran Kumar and Suryanarayana (2011) |

Continued on next page
Table III – Continued from previous page

| State | \( A(MHz) \) | \( B(MHz) \) | Sample | Technique | Ref. |
|-------|-------------|-------------|--------|-----------|-----|
| \( 6^2D_{1/2} \) | 12.03(40) | - | - | - | Recommended |
| \( 7^2S_{1/2} \) | 9.0(9) | - | - | - | Thompson et al. (1983) |
| \( 8^2S_{1/2} \) | 2.98(8) | - | - | - | Thompson et al. (1983) |
| \( 9^2S_{1/2} \) | 3.5(13) | - | - | - | Thompson et al. (2002) |
| \( 10^2S_{1/2} \) | 3.1(7) | - | - | - | Thompson et al. (2002) |
| \( 11^2S_{1/2} \) | 2.6(13) | - | - | - | Thompson et al. (2002) |

D. Rubidium

The long list of recent spectroscopic data for this atom is the result of the laser cooling research in worldwide spread laboratories. For both the 85 and 87 isotopes, the ultracold atomic samples have produced very precise measurements of several hyperfine splittings, in particular for the ground state and the Rydberg ones from \( n=26 \) to \( n=46 \). Note that \(^{87}\text{Rb}\) has 50 neutrons, the exact magic number that makes it a closed shell.

Entries with very high precision and good agreement are presented in the Tables IV and V for \(^{85}\text{Rb}\) and \(^{87}\text{Rb}\), respectively. The Tables report the weighted average of the two published measured values for the \(^{87}\text{Rb} 6^2P_{1/2}\) state by Nyakang’o et al. (2020), while for the \(^{85,87}\text{Rb} 5^2S_{1/2}\), and \(^{6^2P_{1/2}}\) states, the reported average was communicated by Shiner et al. (2007). The error bars of the \(^{6^2S_{1/2}}\) hyperfine constants for both isotopes measured by Orson et al. (2021) and the measurements with a 30 kHz precision of McLaughlin et al. (2022) were communicated privately by M. Lindsay. For the \(^{A}\) constants of the 57 and 58 \(^2S_{1/2}\) states measured by Meschede (1987) for the \(^{87}\text{Rb}\) the missing error bar is assumed equal to the \(^{85}\text{Rb}\) ones.

For \( n^2D \) (\( n \geq 6 \)) states the investigations by Svanberg and Tsekeris (1975), van Wijngaarden, Li, and Koh (1993) and by Glötz and Krajinska-Miszczak (1987, 1990, 1991); Krajinska-Miszczak (1994) produced only the relative signs of the \( A \) and \( B \) constants. Their signs are determined here on the basis of the scaling laws presented in Fig. 3 of Sec. VTA.

For the 85 isotope in Table IV the ground state hyperfine measurement in a maser by Teut, Fortin, and Savard (1976) agrees with and supersedes the atomic beam MWS results reported in Arimondo, Inguscio, and Violino (1977). The following Table entries based on saturated absorption spectroscopy by Barwood, Gill, and Rowley (1991); Shiner et al. (2007) or atomic beam laser spectroscopy by Duong et al. (1993) have a lower precision. They are superseded by the clock measurements in an optical fountain by Wang et al. (2019). The \(^{87}\text{Rb}\) fountain progress was summarized in 2012 by the International Committee for Weights and Measures with their recommended Table V value. Later observations by Guéna et al. (2014); Ovchinnikov, Szymaniec, and Edris (2015) improved its precision.

Data for the \(^5^2P_{1/2}\) level of both isotopes are classified in two groups. The first one includes Barwood, Gill, and Rowley (1991), and Maric, McFerran, and Luiten (2008) while the second one includes the optical spectroscopy determination by Beacham and Andrew (1971) recommended in Arimondo, Inguscio, and Violino (1977), the Banerjee, Das, and Natarajan (2004) and Das and Natarajan (2006a) data from the Bangalore research team, and the Rupasinghe et al. (2022) value. The agreement of the results within each group is good. However, the first group compared to the second one derives the \(^{86}\text{Rb} A \) constant lower by 0.146(13) MHz, and the \(^{85}\text{Rb} A \) value higher by 2.02(33) MHz. The data of both groups lead to peculiar values for the hyperfine anomaly. The higher measured values for \(^{87}\text{Rb}\) are closer to the theoretical predictions by Safanova and Safanova (2011) of 408.53 MHz and by Grunefeld, Roberts, and Ginges (2019) of 410.06 MHz. The only theoretical prediction for \(^{85}\text{Rb}\) by Pal et al. (2007) of 119.192 MHz is off by a few MHz above both group results. The search for similar systematic errors as discussed for the case of the \(^{39}\text{K} 4^2P_{1/2}\) state combined with the present restricted data set does not resolve the discrepancy. For this state the weighted average entry in both Tables IV and V is reported with a large error bar determined from the \( \chi^2_{\text{rel}} \) approach.

For the \(^{85}\text{Rb} 5^2P_{3/2}\) state, excellent agreement exists for the \( A \) value. This is not the case for the \( B \) constant, where the SAS measurement by Barwood, Gill, and Rowley (1991) considered as a reference point increases greatly the \( \chi^2 \) and as a consequence the error bar. An agreement within less than 30 kHz is reached for most data of the \(^{87}\text{Rb} 5^2P_{3/2}\) state. An excellent relative precision of \( 2 \times 10^{-5} \) was reached by two separate measurements on the \(^{87}\text{Rb}\) isotope by Ye et al. (1996): Das and Natarajan (2008). In Table VI the Ye et al. (1996) data reexamined by Gerginov, Tanner, and Johnson (2009) bring evidence of the octupole contribution to the \( \text{Rb} \) hyperfine interactions.

Another example of large discrepancies is found for the \(^{85}\text{Rb} 6^2P_{1/2}\) state. The 39.470(32) MHz SAS+FC measurement by Glaser et al. (2020) presents a difference exceeding the error bar, compared to the values
reported by Shiner et al. (2007) based on the same SAS+FC technique, and also the ORFDR measurement by Feiertag and zu Putlitz (1973) and the OODR-EIT measurement by Nyakang'o et al. (2020). Our derivation of the hyperfine splitting from the measured optical frequencies of Glaser et al. (2020) leads to the 39.11(22) MHz value in good agreement with other ones. Therefore, the Glaser et al. (2020) entry is not included in the weighted average.

In contrast, the $^6D_{3/2}$ Glaser et al. (2020) data for both isotopes are in very good agreement with the earlier radiofrequency and level-crossing data. For that state in $^{87}$Rb, the $A$ value derived by Zhang et al. (2017) on the basis of saturated absorption and EIT measurements is close to those of other references. However, that experiment produced a $B$ constant with a large deviation from other values, probably because the hyperfine lines were not well resolved. Both their $A$ and $B$ values are not included into the w.a..

The $^7S_{1/2}$ state of both isotopes has received a wide attention because of the large probability for the two-photon excitation from the ground state. Precision at the $1 \times 10^{-6}$ level is reached in the $87$ isotope and at the $2 \times 10^{-5}$ level in the $85$ one, limited by the 2 kHz resolution of the frequency comb in Krishna et al. (2005); Chui et al. (2005); Barmes, Witte, and Eikema (2013); Morzyński et al. (2013). However, in both isotopes the EIT results by Krishna et al. (2005) are lower than the other ones by $\approx 400$ kHz, while their claimed precision is $\approx 20$ kHz precision. The presence of light shifts originated by the intense control laser producing the EIT signal was not tested. The weighted average are performed excluding the Krishna et al. (2005) values.

For the $n = (9 - 13) ^2S_{1/2}$ states, the optical spectra recorded by Stoicheff and Weinberger (1979) produce hyperfine constants in good agreement with more recent ones, but only for the $85$ isotope. Their $n = (9 - 11)$ data for the $87$ isotope are very far off and not included in the w.a..

The $^7S_{1/2}$ state of both isotopes has received a wide

| State   | $A(MHz)$ | $B(MHz)$ | Sample | Technique | Ref.                          |
|---------|----------|----------|--------|-----------|------------------------------|
| $^4D_{3/2}$ | 7.3(5)   | 0.       | VC     | ORFDR     | Lam, Gupta, and Happer (1980) |
|         | 7.329(35)| 4.52(23) | VC     | OODR      | Moon, Lee, and Suhl (2009)   |
| $^4D_{5/2}$ | -5.06(10)| 7.42(15) | MOT    | OODR      | Sinclair et al. (1994)       |
|         | -4.978(4)| 6.560(52)| VC     | EIT+ODR   | Wang et al. (2014b)          |
|         | -5.008(9)| 7.15(15) | VC     | OODR+FC   | Lee and Moon (2015)          |
|         | -4.983(7)| 6.70(21) | -      | -         | w.a. (w.e.e.)                |

$^5S_{1/2}$

| $^5P_{1/2}$ | 120.72(25) | -        | VC     | OS        | Beacham and Andrew (1971)    |
|             | 120.499(10)| -        | VC     | SAS       | Barwood, Gill, and Rowley (1991) |
|             | 120.64(2)  | -        | VC     | SAS       | Barwood, Gill, and Rowley (1991) |
|             | 120.645(5) | -        | VC     | SAS       | Das and Natarajan (2006a)    |
|             | 120.500(13)| -        | MOT    | LIF+FC    | Maric, McFerran, and Luiten (2008) |
|             | 120.79(29) | -        | VC     | SAS       | Rupasinghe et al. (2022)     |
|             | 120.605(29)| -        | -      | -         | w.a. (w.e.e.)                |

| $^5P_{3/2}$ | 25.009(22) | 25.88(3) | VC     | ORFDR     | From Arimondo, Inguscio, and Violino (1977) |
|             | 25.3(4)    | 21.4(40)| AB     | LIF       | Thibault et al. (1981b)      |
|             | 24.988(31)| 25.693(31)| VC   | SAS       | Barwood, Gill, and Rowley (1991) |
|             | 25.038(5) | 26.011(22)| VC   | SAS       | Rapol, Krishna, and Natarajan (2003) |
|             | 25.041(6) | 26.013(25)| VC   | SAS       | Barnerjee, Das, and Natarajan (2003) |
|             | 25.0403(11)| 26.0084(49)| VC   | SAS       | Das and Natarajan (2008)     |
|             | 25.0401(11)| 26.00004(22)| -    | -         | w.a.; a.a. (w.e.e.).         |

| $^5D_{5/2}$ | 4.2609(2) | 1.9106(8)| VC     | TPSDS     | Nez et al. (1993)            |
|             | 4.43(28)  | 1.7(24)  | MOT    | OODR+RIS  | Gabbannin et al. (1999)      |
|             | 4.2609(2) | 1.9106(8)| -      | -         | Recommended.                 |

| $^5D_{3/2}$ | -2.2112(12)| 2.6804(200)| VC   | TPSDS     | Nez et al. (1993)            |
|             | -2.196(52) | 2.51(53) | VC   | TPSDS     | Gagabannin et al. (1999)     |
|             | -2.31(23)  | 2.7(27)  | MOT   | TPSDS     | Grove et al. (1995)          |
|             | -2.222(19) | 2.664(130)| VC   | EIT       | Yang, Wang, and Wang (2017)  |
|             | -2.2112(12)| 2.6804(200)| -    | -         | Recommended.                 |

| $^6S_{1/2}$ | 239.18(3) | -        | VC     | FML       | Pérez Galván et al. (2007)   |
|             |           |          |        |           | in Pérez Galván, Zhao, and Orozco (2008) |
| State | $A(MHz)$ | $B(MHz)$ | Sample | Technique | Ref. |
|-------|----------|----------|--------|-----------|------|
| $^4S_{1/2}$ | 17.68(8) | – | VC | ORFDR | Feiertag and zu Putlitz [1973] |
| $^4S_{1/2}$ | 3.71(1) | 3.68(8) | MOT | TPSDS | Snadden et al. [1996] |
| $^4D_{5/2}$ | 1.34(1) | 0.152(6) | VC | MLC | Hermann et al. [1994]; W. et al. (w.e.e.) |
| $^4D_{5/2}$ | 1.04(10) | 0.31(6) | VC | TPSDS | van Wijngaarden and Sagle [1991a] |
| $^4F_{7/2}$ | 1.05(2) | 0.31(6) | – | – | – |
| $^4F_{7/2}$ | 0.35(7) | 0.152(6) | VC | MLC | Hermann et al. [1994]; W. et al. (w.e.e.) |
| $^4I_{9/2}$ | 3.0(2) | – | TD | TPSDS | Stoicheff and Weinberger [1979] |
| $^4I_{9/2}$ | 0.56(11) | 0.20(3) | VC | TPSDS | Stoicheff and Weinberger [1979] |
| $^4I_{11/2}$ | 7.3(2) | – | TD | TPSDS | Li et al. [2003] |
| $^4I_{1/2}$ | 8.3(4) | – | MOT | MWS | Li et al. [2003] |
| $^4I_{1/2}$ | 0.32(26) | – | MOT | MWS | Li et al. [2003] |
| $^4I_{1/2}$ | 0.28(26) | – | MOT | MWS | Li et al. [2003] |
| $^4I_{1/2}$ | 0.22(20) | – | MOT | MWS | Li et al. [2003] |
| $^4I_{1/2}$ | 0.25(31) | – | MOT | MWS | Li et al. [2003] |
| $^4I_{1/2}$ | 0.20(22) | – | MOT | MWS | Li et al. [2003] |
| $^4I_{1/2}$ | 0.18(21) | – | MOT | MWS | Li et al. [2003] |
| $^4I_{1/2}$ | 0.08(4) | – | MOT | MWS | Li et al. [2003] |
| $^4I_{1/2}$ | 0.0743(3) | – | – | – | – |
### Table IV – Continued from previous page

| State   | $A(MHz)$  | $B(MHz)$ | Sample | Technique | Ref.                  |
|---------|------------|----------|--------|-----------|-----------------------|
| $4^2S_{1/2}$ | 0.0703(7) | —        | AB     | MWS       | Meschede (1987)        |
| $4^2D_{5/2}$ | 0.0653(1) | —        | —      | —         | —                     |
| $5^2S_{1/2}$ | 0.057(13) | —        | —      | —         | —                     |
| $5^2D_{5/2}$ | 0.057(12) | —        | —      | —         | —                     |
| $5^2S_{1/2}$ | 0.053(10) | —        | —      | —         | —                     |
| $5^2D_{5/2}$ | 0.050(10) | —        | —      | —         | —                     |
| $5^2S_{1/2}$ | 0.047(10) | —        | —      | —         | —                     |
| $5^2D_{5/2}$ | 0.041(8)  | —        | —      | —         | —                     |
| $5^2S_{1/2}$ | 0.040(7)  | —        | —      | —         | —                     |
| $5^2D_{5/2}$ | 0.036(7)  | —        | —      | —         | —                     |
| $5^2S_{1/2}$ | 0.036(3)  | —        | —      | —         | —                     |

### Table V: Measured $A$ and $B$ values for $^{87}$Rb isotope

| State   | $A(MHz)$  | $B(MHz)$ | Sample | Technique | Ref.                  |
|---------|------------|----------|--------|-----------|-----------------------|
| $4^2D_{5/2}$ | 25.1(9)   | 0.1     | VC     | ORFDR     | Lam, Gupta, and Happer (1980) |
|         | 24.75(12) | 2.19(11)| VC     | OODR      | Moon, Lee, and Sult (2009)  |
|         | 24.75(12) | 2.19(11)| -      | -         | Recommended            |
| $4^2D_{1/2}$ | -16.9(6)  | 0.0     | VC     | ORFDR     | Lam, Gupta, and Happer (1980) |
|         | -16.747(10)| 4.149(59)| VC | SAS+FC    | Lee, Moon, and Sult (2007, 2015) |
|         | -16.801(5)  | 3.645(30)| VC | OODR+EIT  | Wang et al. (2014b)      |
|         | -16.779(6)  | 4.112(52)| VC | OODR+FC   | Lee and Moon (2015)      |
|         | -16.786(10)| 3.82(16)  | -     | -         | w.a.                   |
| $5^2S_{1/2}$ | 3417.330(7)| -       | VC     | SAS       | Barwood, Gill, and Rowley (1991) |
|         | 3417.3415(5)| -    | AB     | LIF       | Duong et al. (1993)      |
|         | 3417.341305452156(4) | - | VC | SAS+FC | CCTF (2012) |
|         | 3417.353(19)| -     | -      | MOT       | Hiner et al. (2007)      |
|         | 3417.341305452156(3) | - | MOT | FOUNT    | Guena et al. (2014)      |
|         | 3417.341305452154(2) | - | MOT | FOUNT    | Ovchinnikov, Szymaniec, and Edris (2015) |
|         | 3417.341305452154(15) | - | -    | -        | w.a.                   |
| $5^2P_{1/2}$ | 406.2(8)  | -       | VC     | OS        | Beacham and Andrew (1971) |
|         | 406.328(15) | - | VC     | SAS       | Barwood, Gill, and Rowley (1991) |
|         | 406.147(15) | - | VC     | SAS       | Banerjee, Das, and Natarajan (2004) |
|         | 406.119(7)  | -       | VC     | SAS       | Das and Natarajan (2006a) |
|         | 408.330(56) | - | MOT | LIF+FC   | Maric, McFerran, and Luiten (2008) |
|         | 408.3(1)   | -       | ODT    | OS        | Neuner et al. (2015)     |
|         | 407.75(50)  | -       | VC     | SAS       | Rupasinghe et al. (2022)  |
|         | 406.48(33)  | -       | -      | -         | w.a. (w.e.e.)            |
| $5^2P_{3/2}$ | 84.29(50)  | 12.2(20)| VC     | SAS       | Thibault et al. (1981b)  |
|         | 84.676(28)  | 12.475(28)| VC | SAS       | Barwood, Gill, and Rowley (1991) |
|         | 84.7185(20) | 12.4965(37) | MOT | SAS       | Ye et al. (1996)         |
|         | 84.7189(22) | 12.4942(43) | MOT | SAS       | Gergov, Tanner, and Johnson (2009) |
|         | 84.7200(16) | 12.4970(35) | VC | SAS       | Das and Natarajan (2008) |
|         | 84.745(6)   | 12.528(10)| VC | SAS       | Chang et al. (2017)      |
|         | 84.720(3)   | 12.497(2) | -    | -         | w.a. (w.e.e.)            |
| $5^2D_{3/2}$ | 14.4303(5) | 0.9320(17)| VC | TPSDS     | Nez et al. (1993)        |
|         | 14.6430(30) | 0.8(8) | MOT | OODR+RIS | Gabbanini et al. (1999)  |
|         | 14.4303(5)  | 0.9320(17)| -    | -         | Recommended             |
| $5^2D_{5/2}$ | -7.4605(3) | 1.2713(20)| VC | TPSDS     | Nez et al. (1993)        |
|         | -7.45(21)   | 0.462(1088) | MOT | OODR     | Gabbanini et al. (1999)  |
|         | -7.51(28)   | 2.7(24)  | MOT | OODR+RIS | Gabbanini et al. (1999)  |
|         | -7.4605(3)  | 1.2713(20)| -    | -         | Recommended             |
| $6^2S_{1/2}$ | 807.66(8)  | -       | VC    | FML       | Perez Galvan et al. (2007) also in Perez Galvan, Zhao, and Orozco (2008) |
|         | 797.3(30)   | -       | VC    | TPSDS     | Orson et al. (2021)      |
|         | 807.341(15) | -      | -     | -         | McLaughlin et al. (2022) |
|         | 807.354(4)  | -      | -     | -         | w.a. (w.e.e.)            |
| $6^2P_{1/2}$ | 132.56(3)  | -       | VC    | ORFDR     | Feiertag and zu Putlitz (1973) |
|         | 132.559(13) | -      | VC    | SAS+FC    | Shiner et al. (2007)     |

Continued on next page
### Table V – Continued from previous page

| State | $A(MHz)$ | $B(MHz)$ | Sample | Technique | Ref. |
|-------|----------|----------|--------|-----------|------|
| $^72S_{1/2}$ | 319.7(1) | – | MOT | TPSDS | | |
| $^72S_{1/2}$ | 319.714(45) | – | VC | EIT | | |
| $^72S_{1/2}$ | 319.702(65) | – | MOT | TPSDS+FC | | |
| $^72S_{1/2}$ | 319.759(28) | – | VC | * | | |
| $^72S_{1/2}$ | 319.7518(51) | – | VC | * | | |
| $^72S_{1/2}$ | 319.7479(23) | – | VC | * | | |
| $^72S_{1/2}$ | 319.762(6) | – | VC | * | | |
| $^72S_{1/2}$ | 319.756(20) | – | – | – | | |
| $^72P_{1/2}$ | 59.92(9) | – | VC | ORFDR | | |
| $^72P_{1/2}$ | 12.57(1) | 1.762(16) | VC | ORFDR+MLC | From Arimondo, Inguscio, and Violino (1974) |
| $^72P_{1/2}$ | 4.53(3) | 0.26(4) | VC | MLC | Svanberg and Tsekieris (1975) |
| $^72P_{1/2}$ | 4.69(23) | 0.26(4) | VC | TPSDS | Otto et al. (2002) |
| $^72P_{1/2}$ | 4.53(3) | 0.26(4) | VC | TPSDS | Otto et al. (2002) |
| $^72P_{1/2}$ | -2.0(3) | -1.85(80) | VC | ORFDR+MLC | Hogervorst and Svanberg (1975) |
| $^72P_{1/2}$ | -1.98(28) | 0. | VC | TPSDS | Otto et al. (2002) |
| $^82S_{1/2}$ | 159.2(15) | – | VC | ORFDR | Tsekieris and Gupta (1975) |
| $^82S_{1/2}$ | 159.3(30) | – | VC | TPSDS | Otto et al. (2002) |
| $^82S_{1/2}$ | 159.2(13) | – | – | w.a. | Feiertag and zu Putlitz (1973) |
| $^82P_{1/2}$ | 32.12(11) | – | VC | ORFDR | Tsekieris, Farley, and Gupta (1975) |
| $^82P_{1/2}$ | 6.739(15) | 0.935(22) | VC | ORFDR+MLC | From Arimondo, Inguscio, and Violino (1974) |
| $^82P_{1/2}$ | 2.840(15) | 0.17(2) | VC | MLC+ORFDR | Belin, Holmgren, and Svanberg (1976) |
| $^82P_{1/2}$ | -1.20(15) | -1.00(13) | VC | ORFDR+MLC | Hogervorst and Svanberg (1975) |
| $^82P_{1/2}$ | -1.09(10) | -1.09(10) | VC | TPSDS | Otto et al. (2002) |
| $^92S_{1/2}$ | 90.9(8) | – | VC | ORFDR | Tsekieris and Gupta (1975) |
| $^92S_{1/2}$ | 106.3(3) | – | TD | TPSDS | Stoicheff and Weinberger (1979) |
| $^92S_{1/2}$ | 91.6(47) | – | VC | TPSDS | Otto et al. (2002) |
| $^92S_{1/2}$ | 90.9(8) | – | – | w.a. | Feiertag and zu Putlitz (1973) |
| $^92P_{1/2}$ | 4.05(3) | 0.55(3) | VC | ORFDR | Belin, Holmgren, and Svanberg (1976) |
| $^92P_{1/2}$ | 1.90(1) | 0.11(3) | VC | MLC+ORFDR | Belin, Holmgren, and Svanberg (1976) |
| $^92P_{1/2}$ | 2.01(17) | 0.11(3) | VC | TPSDS | Otto et al. (2002) |
| $^92P_{1/2}$ | 1.90(1) | - | VC | ORFDR+MLC | Belin, Holmgren, and Svanberg (1976) |
| $^92P_{1/2}$ | -0.80(15) | 0.16(10) | VC | HQB | Głódź and Kraińska-Miszczak (1990) |
| $^92P_{1/2}$ | -0.740(12) | 0.16(15) | VC | HQB | Głódź and Kraińska-Miszczak (1990) |
| $^102P_{1/2}$ | 56.3(2) | – | VC | ORFDR | Farley, Tsekieris, and Gupta (1977) |
| $^102P_{1/2}$ | 70.3(3) | TD | TPSDS | Stoicheff and Weinberger (1979) |
| $^102P_{1/2}$ | 56.3(2) | – | – | w.a. | Belin, Holmgren, and Svanberg (1976) |
| $^102P_{1/2}$ | 2.60(8) | 0. | VC | ORFDR | Belin, Holmgren, and Svanberg (1976) |
| $^102P_{1/2}$ | 1.315(17) | 0.070(11) | VC | HQB | Głódź and Kraińska-Miszczak (1991) |
| $^102P_{1/2}$ | -0.510(10) | 0.098(11) | * | * | Głódź and Kraińska-Miszczak (1987) |
| $^112S_{1/2}$ | 37.4(3) | – | VC | ORFDR | Farley, Tsekieris, and Gupta (1977) |
| $^112S_{1/2}$ | 54.10 | – | TD | TPSDS | Stoicheff and Weinberger (1979) |
| $^112S_{1/2}$ | 37.2(35) | – | VC | TPSDS | Otto et al. (2002) |
| $^112S_{1/2}$ | 37.4(3) | – | – | w.a. | Belin, Holmgren, and Svanberg (1976) |
| $^112P_{1/2}$ | 0.955(11) | 0.049(6) | VC | HQB | Głódź and Kraińska-Miszczak (1991) |
| $^112P_{1/2}$ | -0.361(7) | 0.071(11) | * | * | Głódź and Kraińska-Miszczak (1989) |

Continued on next page.
E. Cesium

Considering the large set of measured values that cover up Rydberg states with high $n$ numbers, cesium is a favourite atom for hyperfine spectroscopy. In addition, there is a good (or very good) agreement for the large majority of states.

The use of frequency combs in order to perform absolute optical frequency measurements produces very precise values for the explored states. For instance, the lute optical frequency measurements produces very precise values by the second reference and by Tanner and Wieman (1991) produced only the absolute value of the $A$ constant. Their signs are determined here on the basis of the scaling laws. All $B$ values for those states were assumed equal zero.

The inversion of the $2D_3/2$ hyperfine states is basically due to core-polarization and electron-correlation effects induced by the valence electron, as initially pointed out by Fredriksson, Lundberg, and Svanberg (1980) and carefully examined recently in Auzinsh et al. (2007); Grunefeld, Roberts, and Ginges (2019); Tang, Lou, and Shi (2019).

The $6^2S_{1/2}$ A coefficient corresponding to the ground state hyperfine splitting is not listed in the Table VII because it is related, as the 2298.157943 MHz frequency, to the Bureau International des Poids et Mesures definition of the second.

For the $6^2D_{3/2}$ state, the Table VII reports the value by Kortyna, Malsuk, and Bragdon (2006), because the remeasured value in Kortyna et al. (2011) using a different method is less precise. The $A$ and $B$ values measured by Cheng et al. (2017) are not included into the calculation of the weighted average because their fit does not reproduce all the measured hyperfine frequencies within the reported error bars.

| State         | $A(MHz)$ | $B(MHz)$ | Sample | Technique | Ref.               |
|---------------|----------|----------|--------|-----------|-------------------|
| $12^2S_{1/2}$ | 27.8(8)  | –        | TD     | TPSDS     | Stieche and Weinberger (1979) |
| $12^2D_{3/2}$ | 0.715(12)| 0.037(8) | VC     | HQB       | Głoźd and Krańska-Miszczak (1991) |
| $12^2D_{5/2}$ | –0.266(9)| 0.063(14)| *      | *         | Głoźd and Krańska-Miszczak (1989) |
| $13^2S_{1/2}$ | 23.8(8)  | –        | TD     | TPSDS     | Stieche and Weinberger (1979) |
| $13^2D_{5/2}$ | –0.20(1) | 0.05(2)  | VC     | HQB       | Głoźd and Krańska-Miszczak (1989) |
| $20^2S_{1/2}$ | 3.891(2) | –        | VC     | EIT       | Tauschinsky et al. (2013) |
| $21^2S_{1/2}$ | 3.249(2) | –        | *      | *         |                   |
| $22^2S_{1/2}$ | 2.721(3) | –        | *      | *         |                   |
| $23^2S_{1/2}$ | 2.390(4) | –        | *      | *         |                   |
| $24^2S_{1/2}$ | 2.115(5) | –        | *      | *         |                   |
| $28^2S_{1/2}$ | 1.07(5)  | –        | MOT    | MWS       | Li et al. (2003)  |
| $29^2S_{1/2}$ | 0.97(5)  | –        | *      | *         |                   |
| $30^2S_{1/2}$ | 0.78(4)  | –        | *      | *         |                   |
| $31^2S_{1/2}$ | 0.81(7)  | –        | *      | *         |                   |
| $32^2S_{1/2}$ | 0.71(5)  | –        | *      | *         |                   |
| $33^2S_{1/2}$ | 0.63(4)  | –        | *      | *         |                   |
| $50^2S_{1/2}$ | 0.185(20)| –        | AB     | MWS       | Meschede (1987)   |
| $51^2S_{1/2}$ | 0.170(18)| –        | *      | *         |                   |
| $52^2S_{1/2}$ | 0.165(18)| –        | *      | *         |                   |
| $53^2S_{1/2}$ | 0.160(10)| –        | *      | *         |                   |
| $54^2S_{1/2}$ | 0.145(18)| –        | *      | *         |                   |
| $55^2S_{1/2}$ | 0.145(15)| –        | *      | *         |                   |
| $56^2S_{1/2}$ | 0.142(13)| –        | *      | *         |                   |
| $57^2S_{1/2}$ | 0.135(13)| –        | *      | *         |                   |
| $58^2S_{1/2}$ | 0.111(13)| –        | *      | *         |                   |
| $59^2S_{1/2}$ | 0.105(10)| –        | *      | *         |                   |
The small values of the hyperfine constants limit the frequency resolution. Three experiments Lee et al. (2011); Wang et al. (2020b, 2021) measured a restricted number of hyperfine splittings with a limited agreement of their frequency. Stalnaker et al. (2010) reported a full high-resolution spectrum and a careful study of systematic errors. Having observed a dependence of the B value on the applied magnetic field, they increase the error bar of their B measurement in order to cover both negative and positive values. Their A and B values are recommended in Table VII.

For the 7^2D_{3/2} state, the A and B values examined by Arimondo, Inguscio, and Violino (1977) were based on early ORFDR investigations by Barbey and Geneux (1962); Bucka and von Oppen (1962); Faist, Geneux, and Koide (1964), all suffering from radiofrequency shifts as discussed in this last reference. All of them are reported in Table VII. Arimondo, Inguscio, and Violino (1977) recommended the Faist, Geneux, and Koide (1964) values, where the shift corrections were included. That A value and the one by Bayram et al. (2014) agree at 0.15 MHz level. Table VII reports the w.a. of Faist, Geneux, and Koide (1964) and Bayram et al. (2014) with \chi^2_{red} correction. The B values by Faist, Geneux, and Koide (1964) and Bayram et al. (2014) are identical except for their sign. Bayram et al. (2014) defended their positive value on the basis of their earlier Na 3P_{3/2} hyperfine constants by Yei, Sieradzan, and Havey (1993) using a similar technique and agreeing with the best other measurements as in Table II. A negative B value is confirmed by the scaling law of Fig. 1. Despite weighted average being -0.12(5), Table VII recommends the value of Faist, Geneux, and Koide (1964).

For the measured constant of 9^2S_1/2, the high reported precision of Jin et al. (2013) leads to an anomalously large contribution to \chi^2_{red}. When that value is excluded from the w.a., the error bar is greatly reduced. It is worth noting that the 8^2S_1/2 and 7^2D_{3/2} values presented by the same authors with a similar precision match very well those by other authors. The 9^2S_1/2 recommended value is the Morgenweg, Barms, and Eikema (2014) measurement having a 2 · 10^-8 precision.

The 9 and 10^2P_{3/2} values by Rydberg and Svanberg (1972) are corrected for the g_j value in Arimondo, Inguscio, and Violino (1977). Rydberg and Svanberg (1972) derive the 10^0P_{3/2} B value on the basis of the average measured ratio B/A = -0.010(3) in the lower n^2P_{3/2} levels.

We have received privately from J. Deiglmayr the A values for the 2S_{1/2} · 2P_{1/2} states with n between 43 and 81 measured by Saßmannshausen, Merkt, and Deiglmayr (2013).

Table VI: Measured C values for ^87Rb and ^133Cs

| State | C(kHz) | Sample | Technique | Ref. |
|-------|--------|--------|-----------|------|
| 5^2P_{3/2} | -0.12(9) | MOT | SAS | Gerginov, Tanner, and Johnson (2009) |
| 6^2P_{3/2} | 0.56(7) | AB | LIF | Gerginov, Derevianko, and Tanner (2003) |
| 5^2D_{3/2} | 0.87(32) | VC | CCS | Das and Natarajan (2005) |
| 6^2D_{3/2} | 4.3(10) | VC | TPSDS | Chen et al. (2018) |

Figure 1. For Cs 2P_{3/2}B(n^*)^3 scaling test, with B in MHz, vs. the n number in logarithmic scale states. For the n = 8 state one negative and one positive value are reported here from Table VII. The quantum defect parameters are derived by Lorenzen and Niemax (1984). The continuous horizontal line represents a fit of the negative values based on the 1/(n^*)^3 scaling.

Table VII: Measured A and B values for ^133Cs

| State | A(MHz) | B(MHz) | Sample | Technique | Ref. |
|-------|--------|--------|--------|-----------|------|
| 5^2D_{3/2} | 48.6(2) | 0.0(8) | AB | LIF | Fredriksson, Lundberg, and Svanberg (1980) |
| 48.8(3) | 0. | VC | HQB | Ryschka and Marek (1981) |

Continued on next page
| State | $A (MHz)$ | $B (MHz)$ | Sample | Technique       | Ref.                                    |
|-------|----------|----------|--------|-----------------|-----------------------------------------|
|       | 48.78(7) | 0.1(7)   | VC     | HQB+DD          | Yei et al. (1998)                       |
|       | 48.78(7) | 0.7(1)   | -      | -               | Recommended                            |
| $5^2 D_{5/2}$ | -21.2(2) | 0.0(10)  | AB     | LIF             | Fredriksson, Lundberg, and Svanberg (1980) |
|       | -22.1(5) | 0.0      | VC     | ORFDR           | Lam, Gupta, and Happer (1980)           |
|       | -21.24(5)| 0.2(5)   | VC     | HQB+DD          | Yei et al. (1998)                       |
|       | -21.24(5)| 0.2(5)   | -      | -               | Recommended                            |
| $5^2 F_{7/2}$ | < 0.7    | 0.0      | VC     | ORFDR           | Svanberg, Tsekeris, and Happer (1973)   |
| $5^2 F_{7/2}$ | < 1.0    | -       | -      | -               | w.a. (w.e.e.)                          |
| $6^2 P_{1/2}$ | 291.90(12)| -       | VC     | ORFDR           | Abele (1975a)                          |
|       | 291.3(7) | -       | AB     | ORFDR           | Coc et al. (1987)                      |
|       | 291.885(80) | -     | AB     | LIF             | Rafa and Tanner (1997)                  |
|       | 291.922(20) | -     | VC     | SAS+FCS         | Udem et al. (1999)                     |
|       | 291.918(8) | -     | VC     | SAS             | Das et al. (2006)                      |
|       | 291.9135(15)| -    | VC     | SAS             | Das and Natarajan (2006b)              |
|       | 291.939(12)| -     | AB     | LIF+FC          | Gerginov et al. (2006)                 |
|       | 291.929(1) | -     | VC     | OS              | Truong et al. (2015)                   |
|       | 291.9263(25)| -    | -      | -               | w.a. (w.e.e.)                          |
| $6^2 P_{3/2}$ | 50.15(8) | -1.35(80) | AB     | HOPF            | Thibault et al. (1981a)                |
|       | 50.275(3) | -0.53(2) | AB     | LIF             | Tanner and Wieman (1988)               |
|       | 50.28827(23)| -0.4934(17) | AB     | LIF             | Gerginov, Derevianko, and Tanner (2003) |
|       | 50.28163(86)| -0.5266(57) | VC     | CCS             | Das and Natarajan (2006)               |
|       | 50.2878(11) | -0.496(6) | -      | -               | w.a. (w.e.e.)                          |
| $6^2 D_{3/2}$ | 16.30(15)| 0.0      | VC     | TPSDS           | Tai, Happer, and Gupta (1975)          |
|       | 16.17(17)| -0.11(127)| VC     | TPSDS           | Ohtsuki et al. (2005)                  |
|       | 16.34(3) | -0.1(2)  | VC     | OODR            | Kortyna, Maslak, and Bragdon (2006)     |
|       | 16.3331(80)| -0.36(1) | VC     | OODR            | Cheng et al. (2017)                    |
|       | 16.338(3) | -0.136(24)| VC     | TPSDS           | Chen et al. (2018)                     |
|       | 16.338(3) | -0.136(24)| -      | -               | See text. Recommended                  |
| $6^2 D_{5/2}$ | -4.69(4)| 0.18(73) | MOT    | TPSDS           | Georgiades, Polzik, and Kimble (1994)  |
|       | -4.56(9) | -0.35(183)| VC     | TPSDS           | Ohtsuki et al. (2005)                  |
|       | -4.66(4) | 0.9(8)   | VC     | OODR            | Kortyna, Maslak, and Bragdon (2006)     |
|       | -4.50(6) | -0.78(66)| VC     | OODR            | Wang et al. (2020a)                    |
|       | -4.629(14)| -0.10(15)| VC     | TPSDS           | Herd, Cook, and Williams (2021)        |
|       | -4.629(11)| -0.10(15)| -      | -               | Recommended                            |
| $7^2 S_{1/2}$ | 545.90(9)| -       | AB     | LIF             | Gilbert, Watts, and Wieman (1983)      |
|       | 545.81(16)| -       | VC     | OODR            | Yang et al. (2015); Ren et al. (2016)  |
|       | 545.90(32)| -       | MOT    | TPSDS           | Tian et al. (2019)                     |
|       | 545.87(1) | -       | VC     | OODR+ETT        | He et al. (2020)                       |
|       | 545.856(14)| -      | -      | -               | w.a. (w.e.e.)                          |
| $7^2 P_{1/2}$ | 94.35(4) | -       | VC     | ORFDR           | Feiertag, Sahm, and zu Putlitz (1972)   |
|       | 94.2(5) | -       | VC     | SAS             | Gerhardt et al. (1978)                 |
|       | 94.40(5) | -       | VC     | SAS             | Williams, Herd, and Hawkins (2018)     |
|       | 94.37(3) | -       | -      | -               | w.a.                                   |
| $7^2 P_{3/2}$ | 16.605(6)| -0.15(3) | VC     | VR              | From Arimondo, Inguscio, and Violino (1977) |
|       | 16.6(3) | 0.0      | VC     | HQB             | Deech et al. (1997)                    |
|       | 16.605(6)| -0.19(5) | VC     | SAS             | Williams, Herd, and Hawkins (2018)     |
|       | 16.605(4)| -0.16(3) | -      | -               | w.a.                                   |
| $7^2 D_{3/2}$ | 7.36(3) | -0.1(2)  | AB     | OODR            | Kortyna, Fiore, and Farrara (2008)      |
|       | 7.386(15)| -0.18(16)| VC     | OODR+FC         | Stainaker et al. (2010)                |
|       | 7.36(7) | -0.88(87)| VC     | TPSDS           | Lee et al. (2011)                      |
|       | 7.38(1) | -0.18(10)| VC     | TPSDS           | Kiran Kumar, Sankari, and Suryanarayana (2013) |
|       | 7.38(19)| -0.15(21)| VC     | OODR+FC         | Jin et al. (2013)                      |
|       | 7.39(6) | -0.19(18)| VC     | TPSDS           | Wang et al. (2021)                     |
|       | 7.380(8) | -0.17(7) | -      | -               | w.a.                                   |
| $7^2 D_{5/2}$ | -1.56(9)| 0.0      | VC     | ELC             | Auzinsh et al. (2007)                  |
|       | -1.717(15)| -0.18(52)| VC     | OODR+FC         | Stainaker et al. (2010)                |
|       | -1.81(5) | 1.01(106)| VC     | TPSDS           | Lee et al. (2011)                      |
|       | -1.729(9)| -0.77(58)| VC     | OODR+ETT        | Wang et al. (2020a)                    |
|       | -1.719(5)| 1.05(29) | VC     | TPSDS           | Wang et al. (2021)                     |
|       | -1.717(15)| -0.18(52)| -      | -               | See text. Recommended                  |

Continued on next page
| State   | A(MHz) | B(MHz) | Sample Technique | Rel.                          |
|---------|--------|--------|------------------|------------------------------|
| $^8S_{1/2}$ | 225.(15) | – | VC LOF | Campani et al. (1978) |
| * $^8S_{1/2}$ | 219.3(2) | – | TD TPSDS | Herrmann et al. (1985) |
| * $^8S_{1/2}$ | 219.1(1) | – | MOT OODR | Fort et al. 1995a |
| * $^8S_{1/2}$ | 220.1 | – | VC TPSDS | Fort et al. 1995b |
| * $^8S_{1/2}$ | 205.15 | – | VC FC+QBS | Bellini, Bartoli, and Hänsch (1997) |
| * $^8S_{1/2}$ | 219.12(1) | – | VC TPSDS | Hagel et al. (1999) |
| * $^8S_{1/2}$ | 219.125(4) | – | VC TPSDS+FC | Fendel et al. (2001) |
| * $^8S_{1/2}$ | 219.14(11) | – | VC OODR+FC | Stainaker et al. (2010) |
| * $^8S_{1/2}$ | 219.124(7) | – | VC SAS | Wu et al. (2013) |
| * $^8S_{1/2}$ | 219.08(12) | – | VC OODR+ET | Wang et al. 2013, 2014a |
| * $^8S_{1/2}$ | 219.137(11) | – | VC OODR+FC | Jin et al. (2013) |
| * $^8S_{1/2}$ | 219.125(3) | – | – | w.a. |
| $^8P_{3/2}$ | 42.97(10) | – | VC ORFDR+MFD | Tait, Gupta, and Happer (1973) |
| * $^8P_{3/2}$ | 42.9(3) | – | VC SAS | Cataliotti et al. (1996) |
| * $^8P_{3/2}$ | 42.96(9) | – | – | w.a. |
| $^8P_{3/2}$ | 7.55(5) | 0.63(35) | VC ORFDR | Barbev and Geneux (1962) |
| * $^8P_{3/2}$ | 7.626(5) | –0.049(42) | VC ORFDR | Bucka and von Oppen (1962) |
| * $^8P_{3/2}$ | 7.58(1) | –0.14(5) | VC ORFDR | Faist, Geneux, and Koide (1964) |
| * $^8P_{3/2}$ | 7.644(25) | 0. | VC ORFDR | Abele (1976b) |
| * $^8P_{3/2}$ | 7.42(6) | 0.14(29) | VC HQB+DD | Bayram et al. (2014) |
| * $^8P_{3/2}$ | 7.58(5) | 0.14(5) | – | w.a. |
| $^8D_{3/2}$ | 3.94(8) | 0. | VC MLC+HQB | From Arimondo, Inguscio, and Violino (1977) |
| * $^8D_{3/2}$ | 3.92(7) | 0. | VC HQB | Deech et al. (1977) |
| * $^8D_{3/2}$ | 3.92(10) | 0. | VC MFD | van Wijngaarden and Sagle (1991b) |
| * $^8D_{3/2}$ | 3.95(1) | 0. | VC HQB | Sagle and van Wijngaarden (1991) |
| * $^8D_{3/2}$ | 3.95(1) | 0. | – | Recommended |
| $^8D_{3/2}$ | –0.85(20) | 0. | VC ORFDR+MLC | From Arimondo, Inguscio, and Violino (1977) |
| $^9S_{1/2}$ | 110.1(5) | – | VC ORFDR | Farley, Tsekeris, and Gupta (1977) |
| * $^9S_{1/2}$ | 109.93(9) | – | VC OODR+FC | Stainaker et al. (2010) |
| * $^9S_{1/2}$ | 109.7(3) | – | VC TPSDS | Kisan Kumar and Suryanarayana (2012) |
| * $^9S_{1/2}$ | 110.15(13) | – | VC OODR+FC | Jin et al. (2013) |
| * $^9S_{1/2}$ | 109.999(3) | – | VC TPSDS+FC | Morgenweg, Barmes, and Eikema (2014) |
| * $^9S_{1/2}$ | 110.999(3) | – | – | w.a. |
| $^9P_{1/2}$ | 23.19(15) | – | VC ORFDR | Tsekeris, Farley, and Gupta (1975) |
| $^9P_{1/2}$ | 4.123(3) | –0.051(25) | VC MLC | Rydberg and Svanberg (1972) |
| $^9D_{3/2}$ | 2.35(4) | 0. | VC MLC+HQB | From Arimondo, Inguscio, and Violino (1977) |
| * $^9D_{3/2}$ | 2.32(4) | 0. | VC HQB | Deech et al. (1977) |
| * $^9D_{3/2}$ | 2.38(1) | 0. | VC HQB | Sagle and van Wijngaarden (1991) |
| * $^9D_{3/2}$ | 2.375(10) | 0. | – | w.a. |
| $^9D_{3/2}$ | –0.43(4) | 0. | VC ELC | Auzinsh et al. (2007) |
| $^{10}S_{1/2}$ | 63.2(3) | – | VC ORFDR | Tsekeris et al. (1974) |
| $^{10}P_{1/2}$ | 13.9(2) | – | VC ORFDR | Farley, Tsekeris, and Gupta (1977) |
| $^{10}P_{1/2}$ | 2.481(9) | –0.025(8) | VC MLC | Rydberg and Svanberg (1972) |
| $^{10}D_{3/2}$ | 1.52(3) | 0. | VC MLC | Svanberg and Tsekeris (1975) |
| * $^{10}D_{3/2}$ | 1.51(2) | 0. | VC HQB | Deech et al. (1977) |
| * $^{10}D_{3/2}$ | 1.54(2) | 0. | VC HQB | Sagle and van Wijngaarden (1991) |
| * $^{10}D_{3/2}$ | 1.503(91) | 0. | VC TPSDS | Otto et al. (2002) |
| * $^{10}D_{3/2}$ | 1.524(13) | 0. | – | w.a. |
| $^{10}D_{3/2}$ | –0.34(3) | 0. | VC ELC | Auzinsh et al. (2007) |
| $^{11}S_{1/2}$ | 39.4(2) | – | VC ORFDR | Tsekeris et al. (1974) |
| * $^{11}S_{1/2}$ | 39.4(17) | – | VC TPSDS | Otto et al. (2002) |
| * $^{11}S_{1/2}$ | 38.81(23) | – | VC OODR+ET | He et al. (2012) |
| * $^{11}S_{1/2}$ | 39.15(15) | – | – | w.a. (w.e.c.) |
| $^{11}P_{1/2}$ | 1.600(15) | 0. | VC ORFDR | Belin and Svanberg (1974) |
| $^{11}D_{3/2}$ | 1.055(15) | 0. | VC MLC | Svanberg and Belin (1974) |
| * $^{11}D_{3/2}$ | 1.05(4) | 0. | VC HQB | Deech et al. (1977) |
| * $^{11}D_{3/2}$ | 1.11(11) | 0. | VC TPSDS | Otto et al. (2002) |
| * $^{11}D_{3/2}$ | 1.0530(69) | 0. | VC TPSDS+FC | Quirk et al. (2012) |
| * $^{11}D_{5/2}$ | 1.0530(69) | 0. | – | Recommended |
| $^{11}D_{5/2}$ | –0.24(6) | 0. | VC ORFDR | Svanberg and Belin (1974) |
| State | $A(MHz)$  | $B(MHz)$  | Sample | Technique | Ref.            |
|-------|-----------|-----------|--------|-----------|-----------------|
| 12$^2S_{1/2}$ | 26.31(10) | - | VC | ORFDR | Tsekeris and Gupta (1975) |
| 12$^2D_{3/2}$ | 0.758(12) | 0. | VC | MLC | Svansberg and Belin (1974) |
| 12$^2D_{5/2}$ | -0.19(5) | 0. | VC | ORFDR | Svansberg and Belin (1974) |
| 13$^2S_{1/2}$ | 18.41(1) | - | VC | ORFDR | Farley, Tsekeris, and Gupta (1977) |
| 13$^2D_{3/2}$ | 0.556(8) | 0. | VC | MLC | Svansberg and Belin (1974) |
| 13$^2D_{5/2}$ | -0.14(4) | 0. | VC | ORFDR | Svansberg and Belin (1974) |
| 14$^2S_{1/2}$ | 13.9(15) | - | VC | TPSDS | Otto et al. (2002) |
| 14$^2D_{3/2}$ | 0.425(7) | 0. | VC | MLC | Belin, Holmgren, and Svansberg (1976a) |
| 15$^2S_{1/2}$ | 10.1(1) | - | VC | ORFDR | Farley, Tsekeris, and Gupta (1977) |
| 15$^2D_{3/2}$ | 0.352(8) | 0. | VC | MLC | Belin, Holmgren, and Svansberg (1976a) |
| 16$^2S_{1/2}$ | 7.73(5) | - | VC | ORFDR | Farley, Tsekeris, and Gupta (1977) |
| 16$^2D_{3/2}$ | 0.255(12) | 0. | VC | MLC | Belin, Holmgren, and Svansberg (1976a) |
| 17$^2S_{1/2}$ | 6.06(10) | - | VC | ORFDR | Farley, Tsekeris, and Gupta (1977) |
| 17$^2D_{3/2}$ | 0.190(12) | 0. | VC | MLC | Belin, Holmgren, and Svansberg (1976a) |
| 18$^2D_{3/2}$ | 0.160(10) | 0. | * |  |  |
| 23$^2S_{1/2}$ | 2.3(2) | - | AB | OODR+MWS | Raimond et al. (1981), also in Goy et al. (1982) |
| 23$^2P_{1/2}$ | 0.56(5) | - | * | * | * |
| 23$^2P_{3/2}$ | 1.5(2) | - | * | * | * |
| 25$^2S_{1/2}$ | 0.40(5) | - | * | * | * |
| 25$^2P_{1/2}$ | 1.4(2) | - | * | * | * |
| 26$^2S_{1/2}$ | 0.31(5) | - | * | * | * |
| 26$^2P_{1/2}$ | 1.2(2) | - | * | * | * |
| 28$^2S_{1/2}$ | 0.28(5) | - | * | * | * |
| 45$^2P_{3/2}$ | 0.0103(27) | 0. | MOT | MWS | Saffmannshausen, Merkt, and Deiglmayr (2013) |
| 49$^2S_{1/2}$ | 0.147(4) | - | * | * | * |
| 59$^2P_{3/2}$ | 0.0047(10) | 0. | * | * | * |
| 67$^2P_{3/2}$ | 0.0030(17) | - | * | * | * |
| 68$^2S_{1/2}$ | 0.0520(13) | - | * | * | * |
| 72$^2P_{3/2}$ | 0.0021(36) | 0. | * | * | * |
| 81$^2S_{1/2}$ | 0.0318(19) | - | * | * | * |
| 90$^2S_{1/2}$ | 0.0227(28) | - | * | * | * |
| 66$^2D_{3/2}$ | 0.0026(5) | 0. | * | * | * |
| 66$^2D_{5/2}$ | 0.00010(45) | 0. | * | * | * |

F. Francium

In the 1980-87 years, the Orsay group at the ISOLDE atomic beam facility in CERN measured several hyperfine constants of different Fr isotopes reported in Liber-man et al. (1980); Coc et al. (1985, 1987); Duong et al. (1987). Francium spectroscopy has reached a higher precision level with the preparation of a cold atom MOT.
in 1996-97 by Simsarian et al. (1996); Lu et al. (1997). Two years later, the important information on the nuclear structure associated with the $2P_{1/2}$ hyperfine structure pointed out by Grossman et al. (1999) has triggered a new interest leading to several more recent experimental investigations.

Table VIII reports all results for the Fr nuclear ground state configuration, avoiding duplicates when the same value was published more than once. The Table does not include the francium data published by Voss et al. (2013), where systematic errors are unaccounted, since updates and corrections of these measurements were presented by Voss et al. (2015); see Table I of that paper. Table VIII evidences that a very large set of isotopes was investigated, all of them targeted at the nuclear structure exploration. The agreement between different hyperfine values is not exceptional, in several cases the differences being greater than the error bars. Because for most isotopes the explored energy levels are limited in number, usually only the lower levels for each $L$ series, global analyses are not efficient. A partial spectroscopic information is associated with the $S$ series in the 210 and 212 isotopes composed by three explored states, with both $n = 8$ and $n = 9$ hyperfine values missed out in the second isotope. The inconsistency in Gomez et al. (2008) between the number in Table I and that reported in the text and the abstract, is resolved by using the number reported in the text and the abstract as checked against the original data by one of us (LAO).

Table VIII: Measured $A$ and $B$ values for Fr isotopes. We use g next to the isotope to indicate the ground state of the nucleus.

| State | $A(MHz)$  | $B(MHz)$  | Sample  | Technique | Ref.          |
|-------|-----------|-----------|---------|-----------|--------------|
| $^{202}\text{Fr}$ | $^{2S1/2}$ | 12800.50  | –       | AB        | LIF          | Flanagan et al. (2013), also in Lynch et al. (2014) |
| $^{202}\text{Fr}$ | $^{2S1/2}$ | 8180.30   | –       | AB        | RIS          | Lynch et al. (2014) |
| $^{202}\text{Fr}$ | $^{2S1/2}$ | 8187.2    | –       | AB        | LIF          | Voss et al. (2015) |
| $^{202}\text{Fr}$ | $^{2S1/2}$ | 8187.2    | –       | –         | –            | w.a. (w.e.e) |
| $^{202}\text{Fr}$ | $^{2P3/2}$ | 29.5(2)   | –93.1(20)| –         | –            | Wilkins et al. (2017) |
| $^{202}\text{Fr}$ | $^{2S1/2}$ | 12990.30  | –       | AB        | RIS          | Lynch et al. (2014) |
| $^{202}\text{Fr}$ | $^{2S1/2}$ | 13146.7(36)| –       | AB        | LIF          | Voss et al. (2015) |
| $^{202}\text{Fr}$ | $^{2P3/2}$ | 1417.3(3) | –36.1(19)| –         | –            | Voss et al. (2015) |
| $^{202}\text{Fr}$ | $^{2S1/2}$ | 8350.0(11)| –       | AB        | LIF          | Voss et al. (2013, 2015) |
| $^{202}\text{Fr}$ | $^{2S1/2}$ | 8400.30   | –       | AB        | RIS          | Lynch et al. (2014) |
| $^{202}\text{Fr}$ | $^{2S1/2}$ | 8350.0(11)| –       | –         | –            | Recommended |
| $^{202}\text{Fr}$ | $^{2P3/2}$ | 89.7(4)   | –81.0(48)| AB        | LIF          | Voss et al. (2015) |
| $^{203}\text{Fr}$ | $^{2S1/2}$ | 13052.2(18)| –       | –         | –            | Voss et al. (2015) |
| $^{203}\text{Fr}$ | $^{2S1/2}$ | 13057.8(10)| –       | AB        | RIS          | Lynch et al. (2016) |
| $^{203}\text{Fr}$ | $^{2S1/2}$ | 13056.5(24)| –       | –         | –            | w.a. (w.e.e) |
| $^{203}\text{Fr}$ | $^{2P1/2}$ | 1716.90(16)| –       | MOT       | FML          | Zhang et al. (2015) |
| $^{203}\text{Fr}$ | $^{2P3/2}$ | 139.1(8)  | –66.8(50)| AB        | LIF          | Voss et al. (2015) |
| $^{203}\text{Fr}$ | $^{2P3/2}$ | 47.5(10)  | –29.8(10)| AB        | LIF          | Lynch et al. (2016) |
| $^{204}\text{Fr}$ | $^{2S1/2}$ | 8484.1(1) | –       | AB        | HOPF         | Coc et al. (1985) |
| $^{204}\text{Fr}$ | $^{2S1/2}$ | 8480.30   | –       | AB        | LIF          | Lynch et al. (2014) |
| $^{204}\text{Fr}$ | $^{2S1/2}$ | 8482.2    | –       | AB        | LIF          | Wilkins et al. (2017) |
| $^{204}\text{Fr}$ | $^{2S1/2}$ | 8483.6(9) | –       | –         | –            | w.a. (w.e.e) |
| $^{204}\text{Fr}$ | $^{2P1/2}$ | 1111.81(11)| –       | MOT       | FML          | Zhang et al. (2015) |
| $^{204}\text{Fr}$ | $^{2P3/2}$ | 90.7(6)   | –42.1(13)| AB        | HOPF         | Coc et al. (1985) |
| $^{204}\text{Fr}$ | $^{2P3/2}$ | 30.4(2)   | –20.0(16)| AB        | LIF          | Wilkins et al. (2017) |
| $^{205}\text{Fr}$ | $^{2S1/2}$ | 6639.7(70)| –       | AB        | HOPF         | Liberman et al. (1980) |
| $^{205}\text{Fr}$ | $^{2S1/2}$ | 6650.5(7) | –       | AB        | HOPF         | Coc et al. (1985) |
| $^{205}\text{Fr}$ | $^{2S1/2}$ | 6653.7(4) | –       | AB        | LIF          | Voss et al. (2015) |
| $^{205}\text{Fr}$ | $^{2S1/2}$ | 6653.1(10)| –       | –         | –            | w.a. (w.e.e) |
| $^{205}\text{Fr}$ | $^{2P1/2}$ | 874.83    | –       | MOT       | FML          | Grossman et al. (1999) |
| $^{205}\text{Fr}$ | $^{2P3/2}$ | 72.8(5)   | 8.9(75) | AB        | HOPF         | Liberman et al. (1980) |
| $^{205}\text{Fr}$ | $^{2P3/2}$ | 72.4(5)   | 1.10   | AB        | HOPF         | Coc et al. (1985) |
| $^{205}\text{Fr}$ | $^{2P3/2}$ | 71.9(2)   | 13.6(29)| AB        | LIF          | Voss et al. (2015) |

Continued on next page
| State  | A(MHz)      | B(MHz)  | Sample Technique | Ref.                        |
|--------|-------------|---------|-----------------|-----------------------------|
| $^72S_{1/2}$ | 8590.5(105) | -       | AB HOPF          | Liberman et al. [1980]      |
|        | 8606.7(9)   | -       | AB HOPF          | Coc et al. [1985]           |
|        | 8606.6(9)   | -       | -                | w.a.                        |
| $^72P_{1/2}$ | 1127.9(2)   | -       | MOT FML          | Grossman et al. [1999]      |
|        | 1127.67(11)| -       | MOT FML          | Zhang et al. [2015]         |
|        | 1127.72(10)| -       | -                | w.a.                        |
| $^72P_{3/2}$ | 93.1(6)     | -61.0(58)| AB HOPF          | Liberman et al. [1980]      |
|        | 93.3(5)     | -62.5   | AB HOPF          | Coc et al. [1985]           |
|        | 93.2(4)     | -62.4   | -                | w.a.                        |
| $^72D_{5/2}$ | -21.1(1)    | -81.22  | MOT LIF          | Agustsson et al. [2017]     |

| State  | A(MHz)      | B(MHz)  | Sample Technique | Ref.                        |
|--------|-------------|---------|-----------------|-----------------------------|
| $^72S_{1/2}$ | 7182.4(81)  | -       | AB HOPF          | Liberman et al. [1980]      |
|        | 7195.1(4)   | -       | AB HOPF          | Coc et al. [1985]           |
|        | 7195.1(6)   | -       | -                | w.a.                        |
| $^72P_{1/2}$ | 945.6(58)   | -       | AB HOPF          | Coc et al. [1987]           |
|        | 946.3(2)    | -       | MOT FML          | Grossman et al. [1999]      |
|        | 946.3(2)    | -       | -                | w.a.                        |
| $^72P_{3/2}$ | 77.9(2)     | 47.6(22)| AB HOPF          | Liberman et al. [1980]      |
|        | 78.0(2)     | 51.4    | AB HOPF          | Coc et al. [1985]           |
|        | 77.95(14)   | 48.4(19)| -                | w.a.                        |
| $^72D_{3/2}$ | 22.3(5)     | 0.      | MOT OODR         | Grossman et al. [2000]      |

| State  | A(MHz)      | B(MHz)  | Sample Technique | Ref.                        |
|--------|-------------|---------|-----------------|-----------------------------|
| $^82S_{1/2}$ | 1577.8(11)  | -       | MOT TCSDS       | Simsarian et al. [1999]     |
| $^92S_{1/2}$ | 622.29(36)  | -       | MOT TCSDS       | Gomez et al. [2005]         |

| State  | A(MHz)      | B(MHz)  | Sample Technique | Ref.                        |
|--------|-------------|---------|-----------------|-----------------------------|
| $^72S_{1/2}$ | 8698.2(105)| -       | AB HOPF          | Liberman et al. [1980]      |
|        | 8713.9(8)   | -       | AB HOPF          | Coc et al. [1985]           |
|        | 8700.60     | -       | AB RIS           | Lynch et al. [2014]         |
|        | 8713.8(8)   | -       | -                | w.a.                        |
| $^72P_{1/2}$ | 1142.1(2)   | -       | MOT FML          | Grossman et al. [1999]      |
|        | 1187.1(68)  | -       | AB LIF           | Duong et al. [1987]         |
|        | 1192.0(2)   | -       | MOT FML          | Grossman et al. [1999]      |

| State  | A(MHz)      | B(MHz)  | Sample Technique | Ref.                        |
|--------|-------------|---------|-----------------|-----------------------------|
| $^72S_{1/2}$ | 9051.3(95)  | -       | AB HOPF          | Liberman et al. [1980]      |
|        | 9064.2(2)   | -       | AB HOPF          | Coc et al. [1985]           |
|        | 9064.4(15)  | -       | AB LIF           | Duong et al. [1987]         |
|        | 9064.2(2)   | -       | -                | w.a.                        |
| $^72P_{1/2}$ | 1189.1(46)  | -       | AB HOPF          | Coc et al. [1987]           |
|        | 1187.1(68)  | -       | AB LIF           | Duong et al. [1987]         |
|        | 1192.0(2)   | -       | MOT FML          | Grossman et al. [1999]      |

| State  | A(MHz)      | B(MHz)  | Sample Technique | Ref.                        |
|--------|-------------|---------|-----------------|-----------------------------|
| $^72P_{3/2}$ | 99.1(9)     | -35.3(155)| AB HOPF         | Liberman et al. [1980]      |
|        | 97.2(1)     | -26.2   | AB LIF           | Coc et al. [1985]           |
|        | 97.2(1)     | -26.0(2)| AB LIF           | Duong et al. [1987]         |
|        | 97.21(10)   | -26.0(2)| -                | w.a.(w.e.c.); w.a.          |
| $^82P_{1/2}$ | 373.0(1)    | -       | AB LIF           | Duong et al. [1987]         |
| $^82P_{3/2}$ | 32.8(1)     | -7.7(9) | AB               |                            |
| $^82D_{3/2}$ | 13.0(6)     | 0.      | AB LIF           | Arnold et al. [1990]        |
| $^82D_{5/2}$ | -7.2(6)     | 0.      | AB               |                            |
| $^92D_{3/2}$ | 7.1(7)      | 0.      | AB               |                            |
| $^92D_{5/2}$ | -3.6(4)     | 0.      | AB               |                            |
| $^{102}S_{1/2}$ | 401.5       | -       | AB               |                            |
| $^{112}S_{1/2}$ | 225.3       | -       | AB               |                            |

| State  | A(MHz)      | B(MHz)  | Sample Technique | Ref.                        |
|--------|-------------|---------|-----------------|-----------------------------|
| $^72S_{1/2}$ | 8744.9(105)| -       | AB HOPF          | Liberman et al. [1980]      |
|        | 8759.9(6)   | -       | AB HOPF          | Coc et al. [1985]           |

Continued on next page
## Table VIII – Continued from previous page

| State   | $A(MHz)$ | $B(MHz)$ | Sample Technique | Ref.          |
|---------|----------|----------|------------------|---------------|
| $^7S_{1/2}$ | 2370.150 | –        | AB RIS           | Farooq-Smith et al. [2016a,b] |
| $^7S_{1/2}$ | 6820.30 | –        | AB RIS           | Budinčević et al. [2014] |
| $^8P_{3/2}$ | 24.7(5) | –104.1(1)| –                | de Groote et al. [2015] |
| $^7S_{1/2}$ | –6549.4(9) | –    | AB HOPF         | Coc et al. [1985] |
| $^7S_{1/2}$ | –6549.2(12) | –   | AB LIF          | Duong et al. [1987] |
| $^7S_{1/2}$ | –6500.40 | –  | AB RIS          | Lynch et al. [2014] |
| $^7S_{1/2}$ | –6549.3(7) | –  | –                | w.a.          |
| $^7P_{3/2}$ | –73.2(5) | 126.8(5) | AB HOPF         | Coc et al. [1985] |
| $^7P_{3/2}$ | –68.5(62) | 123.9(44) | AB HOPF         | Coc et al. [1987] |
| $^8P_{3/2}$ | –23.3(1) | 41.4(14) | AB LIF          | Duong et al. [1987] |
| $^7S_{1/2}$ | 6204.6(8) | –  | AB HOPF         | Coc et al. [1985] |
| $^7S_{1/2}$ | 6100.200 | –  | AB HOPF         | Andreev, Letokhov, and Mishin [1986] |
| $^7S_{1/2}$ | 6205.6(17) | –  | AB HOPF         | Coc et al. [1987] |
| $^7S_{1/2}$ | 6209.9(10) | –  | AB LIF          | Duong et al. [1987] |
| $^7S_{1/2}$ | 6200.30 | –  | AB RIS          | Lynch et al. [2014] |
| $^7S_{1/2}$ | 6209.1 | –  | AB RIS          | de Groote et al. [2015] |
| $^7S_{1/2}$ | 6207.2(11) | –  | –                | w.a.(w.e.e.)  |
| $^7P_{1/2}$ | 808.12 | –  | AB HOPF         | Coc et al. [1987] |
| $^7P_{3/2}$ | 801.0(13) | –  | MOT LIF         | Lu et al. [1997] |
| $^7P_{3/2}$ | 810.3(18) | –  | MOT FML         | Zhang et al. [2015] |
| $^7P_{3/2}$ | 810.7(10) | –  | –                | –              |
| $^7P_{3/2}$ | 65.5(6) | –264.3(1) | AB HOPF         | Coc et al. [1985] |
| $^7P_{3/2}$ | 65.4(29) | –259.1(6) | AB HOPF         | Coc et al. [1987] |
| $^7P_{3/2}$ | 66.5(9) | –260.4(8) | MOT LIF         | Lu et al. [1997] |
| $^7P_{3/2}$ | 65.8(49) | –264.3(1) | –                | w.a.          |
| $^8P_{3/2}$ | 22.4(1) | –85.7(8)  | AB LIF          | Duong et al. [1987] |
| $^8P_{3/2}$ | 22.3(5) | –87.2(2)  | AB RIS          | de Groote et al. [2015] |
| $^8P_{3/2}$ | 22.4(10) | –86.9(3)  | –                | w.a.; w.a.(w.e.e.) |
| $^7S_{1/2}$ | 3070.3 | –  | AB HOPF         | Coc et al. [1985] |
| $^7P_{3/2}$ | 33.1 | 133.9(1) | –                | –             |
| $^7S_{1/2}$ | 7654.2 | –  | –                | –             |
| $^7P_{3/2}$ | 83.3(9) | 308.3(3) | –                | –             |
| $^7S_{1/2}$ | 3876.1 | –  | –                | –             |
| $^7P_{3/2}$ | 42.1(7) | 136.1(1) | –                | –             |
| $^7S_{1/2}$ | 6980.1 | –  | AB HOPF         | Coc et al. [1987] |
| $^7S_{1/2}$ | 6980.1(75) | –  | AB HOPF         | Coc et al. [1987] |
| $^7P_{3/2}$ | 77.1(5) | 347.2(1) | AB HOPF         | Coc et al. [1985] |

Continued on next page
VI. DATA ANALYSIS

A. Quantum number scaling law

The present level of high precision for the theoretical computations leads to agreement with selected experimental results up to 0.1 percent, also for high quantum numbers. Even at that precision level, semi-empirical laws, such as the scaling ones, remain useful for verifying numbers. Even at that precision level, semi-empirical mental results up to 0.1 percent, also for high quantum computations leads to agreement with selected experimental and theoretical data.

We have tested the scaling laws of Eq. (10) for A and similar one of B, for the overall S, P and D states of potassium, rubidium, cesium and francium, using the quantum defect parameters from [Lorenzen and Niemax (1983); Li et al. (2003); Lorenzen and Niemax (1984); Simsarian et al. (1999); Peper et al. (2019)]. As examples, we report in Fig. 2 the A dipole constant results for both Rb isotopes, in (a) for 2S states and in (b), (c) for the 2D ones. Panel (d) of that figure reports the data for the 2S and 2P states of Cs, and for the 2D ones.

The 2S n = (12 − 13) 85Rb data by Stoicheff and Weinberger (1979) with large error bars are not plotted. Note that the A(n*)3 values are plotted vs. n enhancing the deviations from the scaling law. The validity of the scaling law is tested by the horizontal lines derived from data fits. For the 2D states in (b) and (c), the 85Rb data have been scaled to the 87Rb ones by supposing the validity of the gJ scaling of Eq. (7), leading to a precise superposition of the two isotope results. Similar results are obtained for all the 39K states and the 2S states of 41K.

For the 2S states, the 87Rb theoretical results by [Grunefeld, Roberts, and Ginges (2019)] reproduce very closely the experimental A values for all quantum numbers, as shown by the black dots in the (a) panel of the figure. The A scaling law was tested theoretically for the 2S, 2P and 2D Rb states in the log/log plot of [Safronova and Safronova (2011)]. In panels (b) and (c) of Fig. 2, the black dots depict those theoretical predictions for the 2D3/2 states. For the values up to n ≈ 9, the differences between theoretical and experimental results are small. For higher n values, the differences are significant, because of the limitation in the computer codes at that time. A good agreement with experimental data exists for the [Grunefeld, Roberts, and Ginges (2019) predictions of the Cs 2S1/2 and 2P1/2 states, as shown in (d). For Cs, there are additional theoretical results for the 2P3/2 states by [Tang, Lou, and Shi (2019)], for 2D3/2 by [Auzinsh et al. (2007)], and for 2D5/2 by [Tang, Lou, and Shi (2019)]. For the 2D states, the data by [Auzinsh et al. (2007)] cannot be distinguished on the figure scale.

In addition, they cover a short range of n values.

In most cases, the A scaling law is verified in both experimental and theoretical data, and its validity is used to assign the sign of the A values reported in the Tables of the previous Section. For the Cs 2P1/2,3/2 and 2D3/2,5/2 states in a large range of n values, the horizontal fits are good. The scaling validity applies to the high-n states, as for the (n = 40, 90) 2S1/2 and 2P3/2 data of [Safranrvansh] as seen in panel (d). For the 85Rb and Cs 2S states, the scaling does not apply precisely to low-n values because of additional contributions to A in Eq. (5). In contrast, the 85Rb values satisfy the 1/(n*)3 scaling. The low-n difference between the two isotopes produces the deviation from the gJ scaling, linked to the hyperfine anomalies discussed in the following subsection. For the low-n 2D states, large deviations from the scaling lead to lower A values in Rb and higher A ones in Cs for both experimental and theoretical data. These deviations are equivalent for the two Rb isotopes. They originate from the pair-correlation and core-polarization, as explained in [Auzinsh et al. (2007); Tang, Lou, and Shi (2019)].

We have verified the validity of the 1/(n*)3 scaling law also for the B constants. Fig. 8 reports the B(n*)3 constants of both Rb isotopes for the 2P3/2 and 2D3/2 states. The continuous horizontal lines indicate that the scaling law is valid for the 87Rb B constants of both states. For

| State | A(MHz) | B(MHz) | Sample Technique Ref. |
|-------|--------|--------|-----------------------|
| 72S1/2 | 699.4  | AB HOPF Coc et al. (1985) |
| 72P3/2 | 7.1    | -      | w.a.                  |
| 107S1/2 | 698.107(20) | * | * | Duong et al. (1986) |
| 107P3/2 | 627.123(12) | - | - | w.a. |
| 228S1/2 | 30080.110(10) | AB | RIS | Budinčević et al. (2014) |
| 211S1/2 | 30770.130 | * | * | * |

Table VIII – Continued from previous page
Figure 2. $A(n^*)^3$ scaling test, with $A$ in MHz, vs. $n$ number. Experimental result with their error bars are shown in colors, while black dots represent the theoretical predictions. Experimental and theoretical data: (a) data for Rb $^2S_{1/2}$ states, and (b) and (c) for $^2D_{3/2,5/2}$ states, with red open squares for $^{85}$Rb and blue circles for $^{87}$Rb, (d) data for Cs $^2S_{1/2}$ and $^2P_{1/2,3/2}$, (e) and (f) for Cs $^2D_{3/2}$ and $^2D_{5/2}$, respectively. Panel (f) plots do not include the $n = 66$ value of Table VII owing to its large error bar. $^{85}$Rb $^2D$ states data are scaled to the $^{87}$Rb ones by assuming the validity of the isotope $g_I$ scaling of Eq. (10). Note the logarithmic horizontal scale in (a), (d), and (e). The continuous horizontal lines represent fits based on the $1/(n^*)^3$ scaling. Theoretical predictions are for $^{87}$Rb $^2S_{1/2}$ by Grunefeld, Roberts, and Ginges [2019] and $^2D_{3/2,5/2}$ by Safronova and Safronova [2011]; for Cs $^2S_{1/2}$ and $^2P_{1/2} \text{ states by Grunefeld, Roberts, and Ginges }$ [2019], $^2P_{3/2}$ and $^2D_{3/2,5/2}$ by Tang, Lou, and Shi [2019]; for the Cs $^2D_{5/2}$ states, the data by Auzinsh et al. [2007] appear superimposed.

Figure 3. $B(n^*)^3$ scaling, with $B$ in MHz, for Rb isotopes versus $n$, with open blue circles for the $^{87}$Rb data, and open red squares for the $^{85}$Rb data, with error bars for the experiments. (a) $^2P_{3/2}$ data; (b) $^2D_{3/2}$ data. The continuous horizontal lines represent fits based on the $(n^*)^{-3}$ scaling law. For the $^{85}$Rb $^2D_{3/2}$ data, the fit does not include the $n = 6,7$ states. The black dots joined by a line represent the theoretical predictions by Safronova and Safronova [2011] for $^{87}$Rb. $B$ data are precisely scaled to the $^{87}$Rb ones by assuming the validity of the quadrupole moment dependence of Eq. (8), with the $Q$ values of Raghavan [1989]. This $Q$ proportionality applies also to the $^2D_{3/2}$ data, where the $n^*$ scaling is valid.

Since the data for each Fr isotope are very limited in number, the application of the quantum number scaling law is not very efficient. Nevertheless, its validity test is useful to verify the present level of knowledge. The law is also useful for the experimental determination of the hyperfine constants in states not yet explored. While the francium effective quantum numbers may be derived from the quantum defects of Arnold et al. [1990], Simsaarian et al. [1999]; Huang and Sun [2010], we rely on those of Simsarian et al. [1999] based on the full spectrum of the francium absorption lines compiled by Sansonetti [2007]. The francium Table evidences that hyperfine sequences are available only for the $^4S$ states of the 210 and 212 isotopes, each sequence limited to three entries. Fig. 4 presents those experimental results in an $A(n^*)^3$ plot. The dipole constant for the 211 isotope $^5^2S$ state is also plotted. The dots joined by continuous lines represent the theoretical data for $^{210}$Fr by Sahoo et al. [2015], for $^{211}$Fr by Grunefeld, Roberts, and Ginges [2019], and for $^{212}$Fr by Lou et al. [2019]. On the figure scale, equivalent results are obtained using the quantum defect numbers of all the above references. For the experimental and theoretical data on the $^{210,212}$Fr isotopes, the deviations from the scaling law for the lowest $n = 7$ number are similar to those presented for the Rb and Cs low-$n$ $^4S$ and $^2D$ states in Fig. 2. Such deviation from the scaling law does not appear in the $^{211}$Fr theoretical data by Grunefeld, Roberts, and Ginges [2019], while their prediction for the $^4S$ Cs and Rb states match very
closely the experimental data as shown in Fig. 4. An acquisition of more experimental data is required in order to progress with this exploration. The program highlighted by Grunefeld, Roberts, and Ginges (2019) and by Roberts and Ginges (2020) focuses on the importance of having more hyperfine splitting information for higher excited levels to better understand not only the Fr atom, but the properties of the different nuclear isotopes.

### B. Anomalies

The hyperfine anomalies, first introduced by Bohr and Weisskopf (1950), are defined as the $A$ deviations from Eq. (7) produced by the finite size of the nucleus. As a measure of the finite structure influence on the dipole constants of isotopes 1 and 2, following Persson (2013), the expression for the $^1\Delta^2$ hyperfine anomaly is:

$$^1\Delta^2 = \frac{A^1}{A^2} g_i^2 - 1,$$

(11)

where $(A^i, g_i)$ are the hyperfine magnetic dipole constant and the nuclear gyromagnetic ratio, respectively, of the $i = (1, 2)$ isotopes. For a point-like nucleus, the hyperfine anomaly is null. The article of Persson (2013) represents the most recent review of the atomic anomalies. Those of the $^6S_{1/2}$ Fr states are examined in Zhang et al. (2015). Recent theoretical studies of Fr anomalies can be found in Konovalova et al. (2018); Konovalova, Demidov, and Kozlov (2020); Roberts and Ginges (2020).

Anomalies can be derived from measured $A$ constants and accurate values of the nuclear $g$-ratio. This is the case for light alkali atoms with their precise values in the gas phase determined in the sixties and seventies, as discussed in Arimondo, Inguscio, and Violino (1977). Anomalies for those atoms are reported in Table IX derived from the weighted mean values and variances of the dipole constants reported in the Tables of the previous Section. Only anomalies with a value significantly different from zero are presented in Table IX. For the $^5P_{1/2}$ state of the Rb isotopes, where a very large discrepancy between the measured values was noted in subsection V the value is missed because none reasonable anomaly is associated to the different set of data. The Rb values of Table IX are in good agreement with those derived by Pérez Galván et al. (2007); Pérez Galván, Zhao, and Orozco (2008); Wang et al. (2014b) for $^2S$ and $^2D$ states. For the Rydberg $S$ states, enormous anomalies are obtained, $\approx -50(5)$ percent, a quite surprising result, because the interaction of a Rydberg electron with the nucleus should be comparable to that of low orbitals. The fairly constant anomaly for all the $^2S$ states of the Rb isotopes was pointed out by Pérez Galván et al. (2007). For Rb $^2P$ states the situation is not well defined, with the the $^5P_{1/2}$ value reflecting the discording results associated to this state, and for the $^2P_{3/2}$ states the quadrupole interaction playing an important role. The constant value of the $^4P_{1/2}$ anomaly applies also to the K isotopes.

The francium case is different, because the list of isotopes data is quite long and therefore, interesting information about the nuclear structures could be derived from the anomaly determinations. However, for francium an important element is missing in Eq. (11) because a direct measurement of the nuclear gyromagnetic ratio is available only for $^{211}$Fr in Stone (2005). For the remaining isotopes, the $g_i$ ratios are derived by assuming a zero anomaly: see Kozlov (1989). In the recent

| Element | Isotope 1 | Isotope 2 | State | $^1\Delta^2$(%) |
|---------|-----------|-----------|-------|-----------------|
| Li      | 6         | 7         |       |                 |
| K       | 39        | 40        | $^5S_{1/2}$ | 0.0068067(8)    |
|         |           |           | $2P_{1/2}$  | -0.1734(2)      |
|         |           |           | $2P_{3/2}$  | -0.155(8)       |
| Rb      | 85        | 87        | $^5S_{1/2}$ | 0.35141(2)      |
|         |           |           | $6S_{1/2}$  | 0.361(19)       |
|         |           |           | $7S_{1/2}$  | 0.342(3)        |
|         |           |           | $5P_{1/2}$  | 0.55(8)         |
|         |           |           | $5P_{3/2}$  | 0.168(5)        |
|         |           |           | $6P_{1/2}$  | 0.31(7)         |
|         |           |           | $6P_{3/2}$  | 0.46(5)         |
|         |           |           | $4D_{5/2}$  | 0.60(15)        |
|         |           |           | $5D_{3/2}$  | 0.270(6)        |
|         |           |           | $5D_{5/2}$  | 0.44(5)         |

Table IX. Light alkali hyperfine anomalies $^1\Delta^2$ with states listed in order of increasing $L$, then of increasing $n$ and finally of increasing $J$. |
theoretical studies of Fr anomalies by Konovalova et al. (2018); Roberts and Ginges (2020); Konovalova, Demidov, and Kozlov (2020), the information on the nuclear structure is replaced by derivations of the radial nuclear structure and of the nuclear radius. In order to obtain the anomalies without relying on such theoretical analyses, Grossman et al. (1999), following Persson (1998), concentrated their attention on the \( ^2S_{1/2} \) and \( ^2P_{1/2} \) states. The \(^2P_{1/2}\) electron probes the nucleus with a more uniform radial dependence of the interaction than does the \(^2S_{1/2}\) electron. They introduced the following \( R(S/P) \) ratio of their hyperfine constants:

\[
R(S/P) = \frac{A(S_{1/2})}{A(P_{1/2})}
\]

as a probe of the nuclear magnetization distribution. Since both states are spin-1/2, these ratios are independent of quadrupole effects that complicate the extraction of the nuclear structure information. For several francium isotopes, the ratio is presented in Fig. 5. Notice the staggered isotope behaviour of \( R(S/P) \) vs. the even/odd isotope number. Such behavior evidences the different radial distributions of the nuclear magnetization for the even/odd number of neutrons. The ratios of the hyperfine constants of the \(^7^2S_{1/2}\) and \(^7^2P_{3/2}\) states, as well as those of the \(^7^2P_{1/2}\) and \(^7^2P_{3/2}\) ones, do not exhibit such a clear staggered dependence as the \( R(S/P) \) ratio plotted in the figure. We have tested the presence of a \( R(S/P) \) staggered dependence also for the Rb isotopes using the 85 and 87 data in Table VII and the 82 isotope ones from Zhao et al. (1999). This limited data set appears to confirm the \( R(S/P) \) staggered behavior.

The ratio of the hyperfine constants of the \(^2P_{1/2}\) and \(^2P_{3/2}\) states has been proposed in Konovalova, Demidov, and Kozlov (2020) as an additional test of the nuclear structure, even if the nuclear quadrupole coupling is important for the latter state. This ratio calculated from the francium Table VIII data does not exhibit a clear dependence on the isotope number.

VII. CONCLUSIONS

The previously published experimental values for the stable isotopes of the light alkali atoms and for all the nuclear-ground-configuration francium isotopes have been compiled. The Tables report the most accurate data obtained before 1977 and all those published after that time. For each measured hyperfine constant we present a recommended value. A critical examination of the most interesting cases, or of the most discordant ones, is presented. For the discordant cases we have calculated a weighted enhanced error following the procedure of the Particle Data Group in Zyla et al. (2020). For those cases a comparison of our data analysis to the cluster maximum likelihood estimator introduced by Rukhin (2009), is presented in the supplementary information (SI) of this review Allegrini, Arimondo, and Orozco (2022).

We encourage future reviewers of large sets of data to look into these more recent methods, that are currently gaining adepts in the community, but are not followed by those in charge of the recent redefinition of the fundamental constants of physics and chemistry Tiesinga et al. (2019). The experimental interest in measuring hyperfine constants is renewed in the recent years. Today, the laser sources required to excite energy levels not easily assessed at the optical pumping time are available on the market.

Those sources and the associated atomic species can be important for quantum simulation and computational investigations. As in the past, the quest for higher precision spectroscopic measurement could represent an additional step for refining the experimental tools required in those areas.

Instead of concentrating the attention on a specific atomic state, the recent global theoretical analyses cover all the atomic states of a single species, possibly of all the isotopes. For this global approach, precise experimental data are required for a large set of states, stimulating the hyperfine data search in several directions.

Spectroscopic investigations of alkali atoms are also associated with the theoretical progress on the electron-nucleus hyperfine coupling. The hyperfine coupling being an important probe of the nuclear structure, the Grossman et al. (1999) francium research has introduced the ratio of the \( S \) and \( P \) state hyperfine couplings as a new tool for studying nuclear properties. This has stimulated a large theoretical effort, but it has pointed out the need for more precise experimental data. The application of this ratio to lighter alkalis could be an interesting exploration direction.

The electron-nucleus interaction contains a term characterized by the nuclear magnetic octupole moment. In recent years three different measures of that moment are...
reported for $^{133}$Cs $6^2P_{3/2}$ by Gerginov, Derevianko, and Tanner (2003), for $^{87}$Rb $5^2P_{3/2}$ in the experimental data of Ye et al. (1996) reexamined by Gerginov, Tanner, and Johnson (2009), and more recently for the $^{133}$Cs $6^2D_{3/2}$ state by Chen et al. (2018). The parity violation search in alkali atoms may also find new life. Up to now, the most accurate data on the atomic parity-non-conserving interaction was derived from the $6^2S_{1/2} \rightarrow 7^2S_{1/2}$ transition in cesium by Wood et al. (1997). To obtain a more accurate value of the nuclear weak charge producing the parity-violating Hamiltonian, it would be desirable to consider other candidates. Gwinner and Orozco (2022) with their collaborators are pursuing its measurement in the $7^2S_{1/2} \rightarrow 8^2S_{1/2}$ transition in a variety of Fr isotopes, while Aoki et al. (2017) have proposed to search for that violation operating on the $7^1S_{1/2} \rightarrow 6^2D_{3/2}$ electric quadrupole transition in $^{210}$Fr. It is expected that advances in the understanding of hyperfine interactions will continue to illuminate atomic parity nonconversion and vice versa.

As a final expectation, in the near future, a few totally new entries will be added to the above Tables, counter-balanced by several refined entries.

VIII. ACKNOWLEDGMENTS

The authors are grateful to Marianna Safronova for a guide through the recent progress in the theoretical analyses of the hyperfine structures and a careful reading of the manuscript, and to Roberto Calabrese for hints on the francium spectroscopy. The authors are grateful and indebted to the referee for ample comments that have made this a much better review and for providing the algorithm of the CMLE approach. Hassan Jawahery, Peter J. Mohr and William D. Phillips have helped elucidate the evaluation of the errors to report recommended values. The authors acknowledge Wang-Yau Cheng, Pierre Dinh, Johannes Deiglmayr, Randy J. Kuize, Mark Lindsley, Frederic Merkt, Dieter Meschede, Priyanka Rupasinghe and Sun Svanberg for communicating data of various atomic states. The very kind help of the librarians Armelle Michetti and Odile Richaud in Grenoble, and Massimiliano Bertelli in Pisa is also acknowledged.

REFERENCES

Abele, J., “Bestimmung der $g_i$-faktore in den zutänden $6^2P_{3/2}$-62p3/2 und $8^2P_{3/2}$-82p3/2 von $^{133}$Cs,” Zeitschrift für Physik A 274, 179–184 (1975a).

Abele, J., “Untersuchung der hyperfeinstruktur des $6^2P_{1/2}$-zustandes von $^{133}$Cs im starken magnetfeld mit der optischen doppelresonanzmethode,” Zeitschrift für Physik A 274, 185–190 (1975b).

Agustsson, S., Bianchi, G., Calabrese, R., Corradi, L., Dainelli, A., Khanbekyan, A., Marinelli, C., Mariotti, E., Marmugi, L., Mazzocca, G., Moi, L., Ricci, L., Stiaccini, L., and Tomassetti, L., “Observation of $7p^2P_{3/2} \rightarrow 7d^2D$ optical transitions in 209 and 210 francium isotopes,” Opt. Lett. 42, 3682–3685 (2017).

Allegrini, M., Arimondo, E., and Orozco, L. A., “Supplementary information: Survey of hyperfine structure measurements in alkali atoms,” J. Phys. Chem. Ref. Data ??, ?? (2022).

Andreev, S. V., Letokhov, V. S., and Mishin, V. I., “Laser resonant photoionization detection of traces of the radioactive isotope $^{221}$Fr in a sample,” Sov. Phys. JEPT Lett. 43, 736–740 (1986).

Andreev, S. V., Letokhov, V. S., and Mishin, V. I., “Laser resonance photoionization spectroscopy of Rydberg levels in Fr,” Phys. Rev. Lett. 59, 1274–1276 (1987).

Antoni-Micollier, L., Barrett, B., Chichet, L., Condon, G., Battelier, B., Landragin, A., and Bouyer, P., “Generation of high-purity low-temperature samples of $^{39}$K for applications in metrology,” Phys. Rev. A 96, 023608 (2017).

Aoki, T., Torii, Y., Sahoo, B., Das, B., Harada, K., Hayamizu, T., Sakamoto, K., Kawamura, H., Inoue, T., Uchiyama, A., Ito, S., Yoshioka, R., Tanaka, K., Itoh, M., Hatakeyama, A., and Sakemi, Y., “Parity-nonconsuming interaction-induced light shifts in the $7S_{1/2} - 6D_{3/2}$ transition of the ultracold $^{210}$Fr atoms to probe new physics beyond the standard model,” Applied Physics B: Lasers and Optics 123 (2017), 10.1007/s00304-017-6673-3.

Arias, A., Lochead, G., Winternantel, T. M., Helmrich, S., and Whitlock, M., “Realization of a Rydberg-dressed Ramsey interferometer and electrometer,” Phys. Rev. Lett. 122, 053601 (2019).

Arimondo, E., Inguscio, M., and Violino, P., “Experimental determinations of the hyperfine structure in the alkali atoms,” Rev. Mod. Phys. 49, 31–75 (1977).

Armstrong, J. L., Theory of the Hyperfine Structure of Free Atoms, 1st ed. (Wiley-Interscience, New York, 1971).

Arnold, E., Borchers, W., Duong, H. T., Juncar, P., Lerman, J., Lievens, P., Neugart, R., Pellarin, M., Pinard, J., Vialle, J. L., and Wendt, K. (ISOLDE Collaboration), “Optical laser spectroscopy and hyperfine structure investigation of the $10^2S, 11^2S, 8^2D$, and $9^2D$ excited levels in francium,” Journal of Physics B: Atomic, Molecular and Optical Physics 23, 3511–3520 (1990).

Arqueros, F., “Doppler-free two-photon spectroscopy of the 4S state of Na,” Optics Communications 67, 341–342 (1988).

Auzinsh, M., Bluss, K., Ferber, R., Gahlbauer, F., Jarmola, A., Safronova, M. S., Safronova, U. I., and Tamanis, M., “Level-crossing spectroscopy of the $7, 9,$ and $10D_{5/2}$ states of $^{133}$Cs and validation of relativistic many-body calculations of the polarizabilities and hyperfine constants,” Phys. Rev. A 75, 022502 (2007).

Banerjee, A., Das, D., and Natarajan, V., “Precise frequency measurements of atomic transitions by use of
a Rb-stabilized resonator,” Opt. Lett. 28, 1579–1581 (2003).

Banerjee, A., Das, D., and Natarajan, V., “Absolute frequency measurements of the D1 lines in 39K, 85Rb, and 87Rb with ~ 0.1 ppb uncertainty,” Europhysics Letters 65, 172–178 (2004).

Barakhshan, P., Marrs, A., Bhosale, A., Arora, B., Eigenmann, R., and Safonova, M. S., “Portal for High-Precision Atomic Data and Computation (version 2.0),” University of Delaware, Newark, DE, USA. (2022).

Barbey, P. and Genoux, E., “Structure hyperfine de l’état 82P3/2 du césium 133,” Helv. Phys. Acta 35, 561–562 (1962).

Barnes, I., Witte, S., and Eikema, K. S. E., “High-resolution laser spectroscopy on the D1 and D2 lines of 39,40,41K using RF modulated laser light,” Journal of Physics B: Atomic and Molecular Physics 14, 4231–4240 (1981).

Bhattacharya, M., Haimberger, C., and Bigelow, N. P., “Forbidden transitions in a magneto-optical trap,” Phys. Rev. Lett. 91, 213004 (2003).

Biraben, F. and Beroff, K., “Hyperfine interaction in the 4D3/2 and the 4F5/2 levels of sodium,” Physics Letters A 65, 209–212 (1978).

Bohr, A. and Weisskopf, V. F., “The influence of nuclear structure on the hyperfine structure of heavy elements,” Phys. Rev. 77, 94–98 (1950).

Bouchiat, C. and Piketty, C., “Nuclear spin dependent parity violating electron-nucleus interaction in heavy atoms: the anapole moment and the perturbation of the hadronic vector neutral current by the hyperfine interaction,” Physics Letters B 269, 195–200 (1991).

Bouchiat, M.-A., and Guéna, J.,” “The e2s6-7s amplitude in cesium and its importance in a precise calibration of epv1,” J. Phys. France 49, 2037–2044 (1988).

Brandenberger, J. R. and Lindley, R. E., “Hyperfine structure in the 6D3/2 and 6D5/2 states of 85Rb and 87Rb,” Phys. Rev. A 91, 062505 (2015).

Brog, K. C., Eck, T. G., and Wieder, H., “Fine and Hyperfine Structure of the 2P Term of Li6 and Li7,” Phys. Rev. 153, 91–103 (1967).

Brown, R. C., Wu, S.-J., Porto, J. V., Sansonetti, C. J., Simien, C. E., Brewer, S. M., Tan, J. N., and Gillaspy, J. D., “Quantum interference and light polarization effects in unresolvable atomic lines: Application to a precise measurement of the 57Li D2 lines,” Phys. Rev. A 87, 032504 (2013).

Buck, F. and Rabi, I. I., “Hyperfine Structure of K39 in the 4P State,” Phys. Rev. 107, 1291–1294 (1957).

Bucka, H., Kopfermann, H., and Minor, A., “Präzisionsmessung der Hyperfeinstruktur des 62P3/2 Terms des Rb I-Spektums,” Zeitschrift für Physik 161, 123–131 (1961).

Bucka, H. and von Oppen, G., “Hyperfeinstruktur und Lebensdauer des 82P3/2-Terms im Cs I-Spektrum,” Ann. Phys. 456, 119–120 (1962).

Budick, B., Bucka, H., Goshen, R. J., Landman, A., and Novick, R., “Fine and hyperfine structure of the 3S P term in lithium,” Phys. Rev. 147, 1–5 (1966).

Budinčević, I., Bilowos, J., Bissell, M. L., Cocolios, T. E., de Groote, R. P., De Schepper, S., Fedosseev, V. N., Flanagan, K. T., Franchoo, S., Garcia Ruiz, R. F., Heylen, H., Lynch, K. M., Marsh, B. A., Neyens, G., Procter, T. J., Rossel, R. E., Rothe, S., Strashnov, I., Stroke, H. H., and Wendt, K. D. A., “Laser spectroscopy of francium isotopes at the borders of the region of reflection asymmetry,” Phys. Rev. C 90, 014317 (2014).
Budker, D., Kimball, D., and DeMille, D., Atomic physics (Oxford University Press, 2008).

Burghardt, B., Dubke, M., Jitschin, W., and Meisel, G., “Sub-natural linewidth laser spectroscopy of Doppler-free two-photon resonances,” Physics Letters A 69, 93–96 (1978).

Burghardt, B., Hoffmann, B., and Meisel, G., “The 3d states of $^{23}$Na and $^7$Li: determination of unresolved hyperfine splittings and radiative lifetimes,” Zeitschrift für Physik D Atoms, Molecules and Clusters 8, 109–118 (1988).

Bushaw, B. A., Nörtershäuser, W., Ewald, G., Dax, A., and Drake, G. W. F., “Hyperfine splitting, isotope shift, and level energy of the 3S states of $^6$Li,” Phys. Rev. Lett. 91, 043004 (2003).

Campani, E., Degn, G., Gorini, G., and Polacco, E., “Measurement of the 85Sr hyperfine splitting in cesium,” Optics Communications 24, 203–206 (1978).

Carlsson, J., Jönsson, P., Sturesson, L., and Froese Fischer, C., “Multi-configuration Hartree-Fock calculations and time-resolved laser spectroscopy studies of hyperfine structure constants in sodium,” Physica Scripta 46, 394–398 (1992).

Carlsson, J. and Sturesson, L., “Accurate time-resolved laser spectroscopy on lithium atoms,” Zeitschrift für Physik D Atoms, Molecules and Clusters 14, 281–287 (1989).

Cataliotti, F. S., Fort, C., Pavone, F. S., and Inguscio, M., “Doppler-free excitation of the weak $6S_{1/2} - 6P_{1/2}$ cesium transition at 389 nm,” Zeitschrift für Physik D Atoms, Molecules and Clusters 38, 31–33 (1996).

CCTF, Consultative Committee for Time and Frequency, Recommendation CCTF 1 (2012). Report of the 19th meeting (19-14 September 2012) to the International Committee for Weights and Measures (BIPM, Sèvres, 2012) p. 59.

Chang, H., Myneni, K., Smith, D. D., and Liaghati-Mobarhan, H. R., “High-precision, accurate optical frequency reference using a Fabry-Perot diode laser,” Review of Scientific Instruments 86, 063101 (2017).

Chen, T.-J., Chen, J.-E., Yu, H.-H., Liu, T.-W., Hsiao, Y.-F., Chen, Y.-C., Chang, M.-S., and Cheng, W.-Y., “Absolute frequency of cesium $6S_{1/2} - 6D_{3/2}$ hyperfine transition with a precision to nuclear magnetic octupole interaction,” Opt. Lett. 43, 1954–1957 (2018).

Cheng, W.-Y., Chen, T.-J., Lin, C.-W., Chen, B.-W., Yang, Y.-P., and Hu, H. Y., “Robust sub-milliHertz-level offset locking for transferring optical frequency accuracy and for atomic two-photon spectroscopy,” Opt. Express 25, 2752–2762 (2017).

Chui, H.-C., Ko, M.-S., Liu, Y.-W., Shy, J.-T., Peng, J.-L., and Ahn, H., “Absolute frequency measurement of rubidium $5S - 7S$ two-photon transitions with a femtosecond laser comb,” Opt. Lett. 30, 842–844 (2005).

Coc, A., Thibault, C., Touchard, F., Duong, H., Juncar, P., Liberman, S., Pinard, J., Carre, M., Lerne, J., Vialle, J., Büttgenbach, S., Mueller, A., and Pesnelle, A. (ISOLDE Collaboration), “Isotope shifts, spins and hyperfine structures of $^{118,146}$Cs and of some francium isotopes,” Nuclear Physics A 468, 1–10 (1987).

Coc, A., Thibault, C., Touchard, F., Duong, H., Juncar, P., Liberman, S., Pinard, J., Lerne, J., Vialle, J., Büttgenbach, S., Mueller, A., and Pesnelle, A. (ISOLDE Collaboration), “Hyperfine structures and isotope shifts of $^{207–213}$Fr; Possible evidence of octupolar deformation,” Physics Letters B 163, 66–70 (1985).

Das, D., Banerjee, A., Barthwal, S., and Natarajan, V., “A rubidium-stabilized ring-cavity resonator for optical frequency metrology: precise measurement of the $D_1$ line in $^{133}$Cs,” The European Physical Journal D - Atomic, Molecular, Optical and Plasma Physics 38, 545–552 (2006a).

Das, D. and Natarajan, V., “Hyperfine spectroscopy on the $6P_{1/2}$ state of $^{133}$Cs using coherent control,” Europhysics Letters (EPL) 72, 740–746 (2005).

Das, D. and Natarajan, V., “Precise measurement of hyperfine structure in the $5P_{1/2}$ state of Rb,” The European Physical Journal D - Atomic, Molecular, Optical and Plasma Physics 37, 313–317 (2006a).

Das, D. and Natarajan, V., “Precise measurement of hyperfine structure in the $6P_{1/2}$ state of $^{133}$Cs,” Journal of Physics B: Atomic, Molecular and Optical Physics 39, 2013–2019 (2006b).

Das, D., Pandey, K., Wasan, A., and Natarajan, V., “Resolving closely spaced hyperfine levels in the $3P_{3/2}$ state of $^{23}$Na,” Journal of Physics B: Atomic, Molecular and Optical Physics 39, 3111–3119 (2006b).

de Groote, R. P., Budincević, I., Billowes, J., Bissell, M. L., Cocolios, T. E., Farooq-Smith, G. J., Fedoseev, V. N., Flanagan, K. T., Franchoo, S., Garcia Ruiz, R. F., Heylen, H., Li, R., Lynch, K. M., Marsh, B. A., Neyens, G., Rossel, R. E., Rothe, S., Stroke, H. H., Wendt, K. D. A., Wilkins, S. G., and Yang, X., “Use of a continuous wave laser and Pockels cell for sensitive high-resolution collinear resonance ionization spectroscopy,” Phys. Rev. Lett. 115, 132501 (2015).

Deech, J. S., Luy Copenhagen, R., Pendrill, L. R., and Series, G. W., “Lifetimes, depopulation cross sections and hyperfine structures of some Rydberg S and D states of $^{133}$Cs,” Journal of Physics B: Atomic and Molecular Physics 10, L137–L141 (1977).

DeGraffenreid, W. and Sansonetti, C. J., “$^{2S_{1/2} - 4S_{1/2}}$ transition of atomic lithium by Doppler-free two-photon spectroscopy,” Phys. Rev. A 67, 012509 (2003).

Duong, H.-T., “Hyperfine structure and isotope shifts of $^{39}$K by rf modulated laser light,” Nuclear Instruments and Methods in Physics Research 202, 341–342 (1982).

Duong, H.-T., Ekström, C., Gustafsson, M., Inamura, T. T., Juncar, P., Lievens, P., Lindgren, I., Matsuki, S., Murayama, T., Neugart, R., Nilsson, T., Nomura, T.,
Pellarin, M., Penselin, S., Persson, J., Pinard, J., Ragnarsson, I., Redi, O., Stroke, H. H., and Vialle, J. L. (ISOLDE Collaboration), “Atomic beam magnetic resonance apparatus for systematic measurement of hyperfine structure anomalies (Bohr-Weisskopf effect),” Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment 325, 465–474 (1993).

Duong, H.-T., Juncar, P., Liberman, S., Mueller, A. C., Neugart, R., Otten, E. W., Peuse, B., Pinard, J., Stroke, H. H., Thibault, C., Touchard, F., Vialle, J. L., and Wendt, K. (ISOLDE Collaboration), “First observation of the blue optical lines of francia,” Europhysics Letters (EPL) 3, 175–182 (1987).

Duong, H. T., Liberman, S., Pinard, J., Coc, A., Thibault, C., Touchard, F., Carré, M., Lerné, J., Vialle, J. L., Juncar, P., Büttgenbach, S., and Pesselle, A. (ISOLDE Collaboration), “Accurate determination of ground state hyperfine structures of some radioactive alkali isotopes by r.f. magnetic resonance and laser optical pumping,” J. Phys. France 47, 1903–1908 (1986).

Dzuba, V. A. and Flambaum, V. V., “Off-diagonal hyperfine interaction and parity nonconservation in cesium,” Phys. Rev. A 62, 052101 (2000).

Ewald, G., Nörtershäuser, W., Dax, A., Götte, S., Kirchner, R., Kluge, H.-J., Kühl, T., Sanchez, R., Wotatszek, A., Bushaw, B. A., Drake, G. W. F., Yan, Z.-C., and Zimmermann, C., “Nuclear charge radii of 6Li determined by laser spectroscopy,” Phys. Rev. Lett. 93, 113002 (2004).

Faist, A., Geneux, E., and Koide, S., “Frequency shift in magnetic transitions between hyperfine levels of 2P3/2 states of Cs133,” Journal of the Physical Society of Japan 19, 2299–2305 (1964).

Falke, S., Tiemann, E., Lisdat, C., Schnatz, H., and Grosche, G., “Transition frequencies of the D lines of 39K, 40K, and 41K measured with a femtosecond laser frequency comb,” Phys. Rev. A 74, 032503 (2006).

Farley, J., Tsekeris, P., and Gupta, R., “Hyperfine-structure measurements in the Rydberg S and P states of rubidium and cesium,” Phys. Rev. A 15, 1530–1536 (1977).

Farooq-Smith, G. J., Cocolios, T. E., Billowes, J., Bissell, M. L., Budinčević, I., Day Goodacre, T., de Groote, R. P., Fedosseev, V. N., Flanagan, K. T., Franchoo, S., Garcia Ruiz, R. F., Heylen, H., Marsh, B. A., Neyens, G., Rossel, R. E., Rothe, S., Stroke, H. H., Wendt, K. D. A., Wilkins, S. G., and Yang, X. F., “Publisher’s note: Laser and decay spectroscopy of the short-lived isotope 214Fr in the vicinity of the n = 126 shell closure [Phys. Rev. C 94, 054305 (2016)],” Phys. Rev. C 94, 059903 (2016b).

Feiertag, D. and zu Putlitz, G., “Hyperfine structure, g_{f} factors and lifetimes of excited 2P_{1/2} states of Rb,” Zeitschrift für Physik A 261, 1–12 (1973).

Feiertag, D., Sahn, A., and zu Putlitz, G., “Core polarization of the 133Cs atom by the 7p electron,” Zeitschrift für Physik A Hadrons and nuclei 255, 93–96 (1972).

Fendel, P., Bergeson, S. D., Udem, T., and Hänsch, T. W., “Two-photon frequency comb spectroscopy of the 6s – 8s transition in cesium,” Opt. Lett. 32, 701–703 (2007).

Flambaum, V. V. and Khrilovich, I. B., “New bounds on the electric dipole moment of the electron and on T-odd electron-nucleon coupling,” Sov. Phys. - JETP (Engl. Transl.) 62, 872 (1985).

Flanagan, K. T., Lynch, K. M., Billowes, J., Bissell, M. L., Budinčević, I., Cocolios, T. E., de Groote, R. P., De Schepper, S., Fedosseev, V. N., Franchoo, S., Garcia Ruiz, R. F., Heylen, H., Marsh, B. A., Neyens, G., Procter, T. J., Rossel, R. E., Rothe, S., Strashnov, I., Stroke, H. H., and Wendt, K. D. A., “Collinear resonance ionization spectroscopy of neutron-deficient francium isotopes,” Phys. Rev. Lett. 111, 212501 (2013).

Foot, C. J., Atomic Physics (Oxford University Press, 2005).

Fort, C., Cataliotti, F. S., Raspolini, P., Tino, G. M., and Inguscio, M., “Optical double-resonance spectroscopy of trapped Cs atoms: hyperfine structure of the 8s and 6d excited states,” Zeitschrift für Physik D Atoms, Molecules and Clusters 34, 91–95 (1995a).

Fort, C., Inguscio, M., Raspolini, P., Bakdes, F., and Sasso, A., “Doppler-free two-color spectroscopy of the 651/2 – 851/2 cesium transition using semiconductor diode lasers,” Appl. Phys. B 61, 462–471 (1995b).

Fredrikkson, K., Lundberg, H., and Svanberg, S., “Fine-and hyperfine-structure investigation in the 5^2D – n^2F series of cesium,” Phys. Rev. A 21, 241–247 (1980).

Gabbannin, C., Ceccherini, F., Gozzini, S., and Luchesini, A., “Resonance-enhanced ionization spectroscopy of laser-cooled rubidium atoms,” Measurement Science and Technology 10, 772–776 (1999).

Gangsky, Y. P., Karavaev, D. V., Marinova, K. P., Markov, B. N., Melnikova, L. M., Mishinsky, G. V., Zemlyanoi, S. G., and Zhemenik, V. I., “Hyperfine splitting and isotope shift in the atomic D5 line of 22Na and the quadrupole moment of 22Na,” The European Physical Journal A - Hadrons and Nuclei 3, 313–318 (1998).

Georgiades, N. P., Polzik, E. S., and Kimble, H. J., “Two-photon spectroscopy of the 6S_{1/2} → 6D_{5/2} transition of trapped atomic cesium,” Opt. Lett. 19, 1474–1476 (1994).
Głódź, M. and Kraińska-Miszczak, M., "Hyperfine structure constants of the 10, 11 and 12 states of ¹³³Cs and their impact on the fine-structure constant," Phys. Rev. A 73, 032504 (2006).

Gerginov, V., Derevianko, A., and Tanner, C. E., "Observation of the nuclear magnetic octupole moment of ¹³³Cs," Phys. Rev. Lett. 91, 072501 (2003).

Gerginov, V., Tanner, C. E., and Johnson, W. R., "Observation of the nuclear magnetic octupole moment of ⁸⁷Rb from spectroscopic measurements of hyperfine intervals," Canadian Journal of Physics 87, 101–104 (2009).

Gerhardt, H., Matthias, E., Schneider, F., and Timmermann, A., "Isotope shifts and hyperfine structure of the 6s-7p transitions in the cesium isotopes ¹³³, ¹³⁷, and ¹³⁷," Zeitschrift für Physik A Atoms and Nuclei 288, 327–333 (1978).

Gilbert, S. L., Masterson, B. P., Noecker, M. C., and Wieman, C. E., "Precision measurement of the off-diagonal hyperfine interaction," Phys. Rev. A 34, 3509–3512 (1986).

Gilbert, S. L., Watts, R. N., and Wieman, C. E., "Hyperfine-structure measurement of the 7S state of cesium," Phys. Rev. A 27, 581–582 (1983).

Glaser, C., Karlewski, F., Klie, J., Grimmel, J., Kaiser, M., Günther, A., Hattermann, H., Krutzik, M., and Fortágh, J., "Absolute frequency measurement of rubidium 5S–6P transitions," Phys. Rev. A 102, 012804 (2020).

Glódź, M. and Kraińska-Miszczak, M., "Hyperfine interaction constants and lifetime of the ⁶²D₃/₂ and ⁶²D₅/₂ states of ⁹⁹K measured by the quantum beam method," Journal of Physics B: Atomic and Molecular Physics 18, 1515–1522 (1985a).

Glódź, M. and Kraińska-Miszczak, M., "Lifetime and hyperfine structure constants of the ⁶²D₃/₂ state in ⁴¹K," Physics Letters A 110, 203–205 (1985b).

Glódź, M. and Kraińska-Miszczak, M., "Magnetic-dipole and electric-quadrupole interaction constants in the ¹⁰⁷⁵D₅/₂ state of ²⁶⁰Rb," Journal of Physics B: Atomic and Molecular Physics 20, L541–L545 (1987).

Glódź, M. and Kraińska-Miszczak, M., "Measurements of magnetic-dipole and electric-quadrupole interaction constants of the 11, 12 and 13⁵D₃/₂ states in ⁸⁷Rb by the quantum-beat method," Journal of Physics B: Atomic, Molecular and Optical Physics 22, 3109–3117 (1989).

Glódź, M. and Kraińska-Miszczak, M., "Magnetic-dipole and electric-quadrupole interaction constants and lifetime of the ⁹²D₅/₂ state in Rb-87," Acta Phys. Pol. 78, 317–322 (1990).

Glódź, M. and Kraińska-Miszczak, M., "Hyperfine interaction constants of the 10, 11 and 12⁵D₃/₂ states in ⁸⁷Rb measured by the quantum beat method," Physics Letters A 160, 85–89 (1991).

Glódź, M. and Kraińska-Miszczak, M., "Hyperfine structure constants in the ¹⁰⁷²D₃/₂ and ¹¹²D₅/₂ states of ⁸⁵Rb," Acta Phys. Pol. 83, 161–165 (1993).

Gomez, E., Aubin, S., Orozco, L. A., and Sprouse, G. D., "Lifetime and hyperfine splitting measurements on the 7s and 6p levels in rubidium," J. Opt. Soc. Am. B 21, 2058–2067 (2004).

Gomez, E., Aubin, S., Orozco, L. A., Sprouse, G. D., Iskrenova-Tchoukova, E., and Safronova, M. S., "Nuclear magnetic moment of ²¹⁰Fr: A combined theoretical and experimental approach," Phys. Rev. Lett. 100, 172502 (2008).

Goy, P., Raimond, J. M., Vitrant, G., and Haroche, S., "Millimeter-wave spectroscopy in cesium Rydberg states. Quantum defects, fine- and hyperfine-structure measurements," Phys. Rev. A 26, 2733–2742 (1982).

Griffith, J. A. R., Isaak, G. R., New, R., Ralls, M. P., and van Zyl, C. P., "Optical heterodyne spectroscopy using tunable dye lasers: hyperfine structure of sodium," Journal of Physics B: Atomic and Molecular Physics 10, L91–L95 (1977).

Grossman, J. M., Fillner III, R. P., Mehlstäubler, T. E., Orozco, L. A., Pearson, M. R., Sprouse, G. D., and Zhao, W. Z., "Energies and hyperfine splittings of the 7D states of atomic francium," Phys. Rev. A 62, 052507 (2000).

Grossman, J. S., Orozco, L. A., Pearson, M. R., Simsarian, J. E., Sprouse, G. D., and Zhao, W. Z., "Hyperfine anomaly measurements in francium isotopes and the radial distribution of neutrons," Phys. Rev. Lett. 83, 935–938 (1999).

Grove, T. T., Sanchez-Villicana, V., Duncan, B. C., Maleki, S., and Gould, P. L., "Two-photon two-color diode laser spectroscopy of the Rb 5⁵D₅/₂ state," Physica Scripta 52, 271–276 (1995).

Grundevik, P. and Lundberg, H., "Measurement of the hyperfine structure splitting for the 4 and 5⁵P₁/₂ states of sodium using radio-frequency spectroscopy," Zeitschrift für Physik A Atoms and Nuclei 285, 231–233 (1978).

Grundevik, P., Lundberg, H., Martensson, A. M., Nystrom, K., and Svanberg, S., "Hyperfine-structure study in the P sequence of ²³Na using quantum-beam spectroscopy," Journal of Physics B: Atomic and Molecular Physics 12, 2645–2654 (1979).

Grunefeld, S. J., Roberts, B. M., and Ginges, J. S. M., "Correlation trends in the hyperfine structure for Rb, Cs, and Fr, and high-accuracy predictions for hyperfine constants," Phys. Rev. A 100, 042506 (2019).

Guéna, J., Abgrall, M., Clairon, A., and Bize, S., "Contributing to TAI with a secondary representation of the SI second," Metrologia 51, 108–120 (2014).

Gupta, R., Happer, W., Lam, L. K., and Svanberg, S., "Hyperfine-structure measurements of excited s states of the stable isotopes of potassium, rubidium, and cesium by cascade radio-frequency spectroscopy," Phys. Rev. A 8, 2792–2810 (1973).

Gwinner, G. and Orozco, L. A., “Studies of the weak interaction in atomic systems: towards measurements of atomic parity non-conservation in francium,” Quan-
atum Science and Technology 7, 024001 (2022)

Hagel, G., Nesi, C., Jozefowski, L., Schwob, C., Nez, F., and Biraben, F., “Accurate measurement of the frequency of the 6S-8S two-photon transitions in cesium,” Optics Communications 160, 1–4 (1999).

Halloran, L., Fostner, S., Paradis, E., and Behr, J., “Specific mass shift of potassium $5P_{1/2}$ state,” Optics Communications 282, 554–557 (2009).

Hawkins, R. T., Hill, W. T., Kowalski, F. V., Schawlow, A. L., and Svanberg, B., “Stark-effect study of excited states in sodium using two-photon spectroscopy,” Phys. Rev. A 15, 967–974 (1977).

He, Y.-H., Fan, J.-B., Hao, L.-P., Jiao, Y.-C., and Zhao, J.-M., “Precise measurement of hyperfine structure of cesium $7S_{1/2}$ excited state,” Applied Sciences 10 (2020), 10.3390/app10020525.

He, Z.-S., Tsai, J.-H., Lee, M.-T., Chang, Y.-Y., Tsai, C.-C., and Whang, T.-J., “Determination of the cesium $11S_{2}S_{1/2}$ hyperfine magnetic coupling constant using electromagnetically induced transparency,” Journal of the Physical Society of Japan 81, 124302 (2012).

Herd, M. T., Cook, E. C., and Williams, W. D., “Absolute frequency measurement of the $6D_{5/2}$ level of neutral $^{133}$Cs using two-photon spectroscopy,” Phys. Rev. A 104, 042812 (2021).

Herrmann, P., Hoffnagle, J., Pedroni, A., Schlumpf, N., and Weis, A., “Doppler-free spectroscopy of the $8S$ state of Cs,” Optics Communications 56, 22–24 (1985).

Hogervorst, W. and Svanberg, S., “Stark effect investigation of $D$ states in $^{85}$Rb and $^{133}$Cs using level crossing spectroscopy with a CW dye laser,” Physica Scripta 12, 67–74 (1975).

Huang, S.-H. and Sun, Q.-F., “Calculation of the Rydberg energy levels for francium atom,” Physics Research International 2010, 203497 (2010).

Huang, Y.-C., Luo, W.-J., Kuo, Y.-T., and Wang, L.-B., “Precision measurement of hyperfine intervals in the $\Delta_{l}$ lines of atomic $^7$Li,” Journal of Physics B: Atomic, Molecular and Optical Physics 46, 075004 (2013).

Inguscio, M. and Fallani, L., Atomic Physics: Precise Measurements and Ultracold Matter (Oxford University Press, 2013).

Isler, R. C., Marcus, S., and Novick, R., “Hyperfine structure of the $3P$ and $4P$ states of lithium and lifetime of the $3P$ state,” Phys. Rev. 187, 76–84 (1969).

Jiang, Z.-K., Jönsson, G., and Lundberg, H., “Hyperfine-structure study of the 7, 8 and $9P_{3/2}$ states in $^{23}$Na using quantum-beat spectroscopy,” Physica Scripta 26, 459–461 (1982).

Jin, L., Zhang, Y.-C., Xiang, S.-S., Wang, L.-R., Ma, J., Zhao, Y.-T., Xiao, L.-T., and Jia, S.-T., “Experimental measurement of the absolute frequencies and hyperfine coupling constants of $^{133}$Cs using a femtosecond optical frequency comb,” Chinese Physics Letters 30, 103201 (2013).

Johnson, W. R., Ho, H. C., Tanner, C. E., and Derevianko, A., “Off-diagonal hyperfine interaction between the $6p_{1/2}$ and $6p_{3/2}$ levels in $^{133}$Cs,” Phys. Rev. A 70, 014501 (2004).

Johnson, W. R., Safronova, M. S., and Safronova, U. I., “Combined effect of coherent $Z$ exchange and the hyperfine interaction in the atomic parity-nonconserving interaction,” Phys. Rev. A 67, 062106 (2003).

Johnson, W. R., Safronova, U. I., Derevianko, A., and Safronova, M. S., “Relativistic many-body calculation of energies, lifetimes, hyperfine constants, and polarizabilities in $^7$Li,” Phys. Rev. A 77, 022510 (2008).

Kasevich, M. A., Riis, E., Chu, S., and DeVoe, R. G., “rf spectroscopy in an atomic fountain,” Phys. Rev. Lett. 63, 612–615 (1989).

Kiran Kumar, P. V., Nisheeth, B., Sankari, M., and Suryanarayana, M. V., “Precision measurement of the hyperfine structure of $8D_{3/2}$ state of $^{133}$Cs by the radio-frequency phase modulation technique,” Optics Communications 320, 77–83 (2014).

Kiran Kumar, P. V., Sankari, M., and Suryanarayana, M. V., “Hyperfine structure of the $7D_{3/2}$ level in cesium measured by Doppler-free two-photon spectroscopy,” Phys. Rev. A 87, 012503 (2013).

Kiran Kumar, P. V. and Suryanarayana, M. V., “Isotope shift and hyperfine structure measurements of $4S_{1/2} \rightarrow 6S_{1/2}$ two-photon transition of potassium isotopes,” Journal of Physics B: Atomic, Molecular and Optical Physics 44, 055003 (2011).

Kiran Kumar, P. V. and Suryanarayana, M. V., “Measurement of the hyperfine splitting of the $9S_{1/2}$ level in Cesium by Doppler-free two-photon spectroscopy,” Optics Communications 285, 1838–1842 (2012).

Kiran Kumar, P. V. and Suryanarayana, M. V., “Precision two-photon spectroscopy of alkali elements,” Pramana 83, 189–219 (2014).

Konovalova, E. A., Demidov, Y. A., and Kozlov, M. G., “Calculation of the hyperfine magnetic anomaly in many-electron atoms,” Optics and Spectroscopy 128, 1530–1536 (2020).

Konovalova, E. A., Demidov, Y. A., Kozlov, M. G., and Barzakh, A. E., “Calculation of francium hyperfine anomaly,” Atoms 6, 39 (2018).

Kopfermann, H., Nuclear Moments (Academic, New York, 1958).

Kortyna, A., Fiore, V., and Farrar, J., “Measurement of the cesium $7d_{2}D_{3/2}$ hyperfine coupling constants in a thermal beam using two-photon fluorescence spectroscopy,” Phys. Rev. A 77, 062505 (2008).

Kortyna, A., Masluk, N. A., and Bragdon, T., “Measurement of the $6d_{3/2}$ hyperfine structure of cesium using resonant two-photon sub-Doppler spectroscopy,” Phys. Rev. A 74, 022503 (2006).

Kortyna, A., Tinsman, C., Grab, J., Safronova, M. S., and Safronova, U. I., “Experimental and theoretical study of the $6d_{3/2}$ polarizability of cesium,” Phys. Rev. A 83, 042511 (2011).

Koszorús, A., Yang, X. F., Billowes, J., Binnerley, C. L., Bissell, M. L., Coccolis, T. E., Farooq-Smith, G. J., de Groote, R. P., Flanagan, K. T., Franchoo, S., Garcia Ruiz, R. F., Geldhof, S., Gins, W., Kanellakopoulos, I., and Weis, A., “Off-diagonal hyperfine interaction be-
los, A., Lynch, K. M., Neyens, G., Stroke, H. H., Vernon, A. R., Wendt, K. D. A., and Wilkins, S. G., “Precision measurements of the charge radii of potassium isotopes,” Phys. Rev. C 100, 034304 (2019).

Kowalski, J., Neumann, R., Suhr, H., Winkler, K., and zu Putlitz, G., “Two-photon intracavity dye laser spectroscopy of the 4S and 3D term in \(^6\)Li,” Zeitschrift für Physik A Atoms and Nuclei 287, 247–253 (1978).

Krańska-Miszczak, M., “Decoupling in the \(P_{3/2}\) state and \(D_2\) fluorescence of alkali metal atoms,” Optics Communications 38, 255–258 (1981).

Krańska-Miszczak, M., “Hyperfine interaction constants in the \(9^2D_{9/2}\) state of \(^{86}\)Rb measured by the beat beam method,” Acta Phys. Pol. 86, 343–347 (1994).

Kramida, A., “NIST atomic energy levels and spectra bibliographic database,” (2022).

https://doi.org/10.1016/0030-4018(94)90278-X.

Krishna, A., Pandey, K., Wasan, A., and Natarajan, V., “High-resolution hyperfine spectroscopy of excited states using electromagnetically induced transparency,” Europhysics Letters 72, 221–227 (2005).

Krist, T., Kuske, P., Gaupp, A., Wittmann, W., and Andrä, H., “Improved \(^{23}\)Na \(3^3P_{3/2}\) HFS measurement beyond the natural linewidth by beam laser quantum beats,” Physics Letters A 61, 94–96 (1977).

Kumar, P. and Natarajan, V., “Precise measurement of hyperfine structure in the \(3S_{1/2}\) state of \(^7\)Li,” Scientific Reports 7, 13204 (2017).

Lam, L. K., Gupta, R., and Happer, W., “Hyperfine-structure measurements in the first excited \(D\) levels of potassium, rubidium, and cesium by cascade-fluorescence spectroscopy,” Phys. Rev. A 21, 1225–1234 (1980).

Lee, W.-K. and Moon, H. S., “Measurement of absolute frequencies and hyperfine structure constants of \(4D_{5/2}\) and \(4D_{3/2}\) levels of \(^{87}\)Rb and \(^{85}\)Rb using an optical frequency comb,” Phys. Rev. A 92, 012501 (2015).

Lee, W.-K., Moon, H. S., and Suh, H. S., “Measurement of the absolute energy level and hyperfine structure of the \(^{85}\)Rb \(4D_{5/2}\) state,” Opt. Lett. 32, 2810–2812 (2007).

Lee, W.-K., Moon, H. S., and Suh, H. S., “Measurement of the absolute energy level and hyperfine structure of the \(^{85}\)Rb \(4D_{5/2}\) state: erratum,” Opt. Lett. 40, 2111–2111 (2015).

Lee, Y.-C., Chang, Y.-H., Chang, Y.-Y., Chen, Y.-Y., Tsai, C.-C., and Chui, H.-C., “Hyperfine coupling constants of cesium \(7\) D states using two-photon spectroscopy,” Applied Physics B 105, 391–397 (2011).

Li, R., Wu, Y.-L., Rui, Y., Li, B., Jiang, Y.-Y., Ma, L.-S., and Wu, H.-B., “Absolute Frequency Measurement of \(^{6}\)Li \(D\) Lines with kHz-Level Uncertainty,” Phys. Rev. Lett. 124, 063002 (2020).

Li, R., Wu, Y.-L., Rui, Y., and Wu, H.-B., “Observation of subnatural-linewidth spectra in cold \(^6\)Li atoms,” Phys. Rev. A 103, 032823 (2021).

Li, W.-H., Mourachko, I., Noel, M. W., and Gallagher, T. F., “Millimeter-wave spectroscopy of cold Rb Rydberg atoms in a magneto-optical trap: Quantum defects of the ns, np, and nd series,” Phys. Rev. A 67, 052502 (2003).

Liberman, S., Pinard, J., Duong, H. T., Juncar, P., Pillet, P., Vialle, J.-L., Jacquinot, P., Touchard, F., Büttgenbach, S., Thibault, C., de Saint-Simon, M., Klapisch, R., Pesnele, A., and Huber, G., “Laser optical spectroscopy on francium \(D_2\) resonance line,” Phys. Rev. A 22, 2732–2737 (1980).

Lien, Y.-H., Lo, K.-J., Chen, H.-C., Chen, J.-R., Tian, J.-Y., Shi, J.-T., and Liu, Y.-W., “Absolute frequencies of the \(^6\)Li \(2S^2S_1/2\) \rightarrow \(3S^2S_1/2\) transitions,” Phys. Rev. A 84, 042511 (2011).

Lorenzen, C. J. and Niemax, K., “Level isotope shifts of \(^{6,7}\)Li,” Journal of Physics B: Atomic and Molecular Physics 15, L139–L145 (1982).

Lorenzen, C. J. and Niemax, K., “Quantum Defects of the \(n^2P_{3/2,3/2}\) Levels in \(^{39}\)K I and \(^{85}\)Rb I,” Physica Scripta 27, 300–305 (1983).

Lorenzen, C. J. and Niemax, K., “Precise quantum defects of \(nS, nP\) and \(nD\) Levels in Cs I,” Zeitschrift für Physik A Atoms and Nuclei 315, 127–133 (1984).

Lou, B.-Q., Li, F., Wan, P.-Y., Wang, L.-M., and Tang, Y.-B., “Ab initio calculation of hyperfine-structure constant \(A\) of Fr and evaluation of magnetic dipole moments of Fr isotopes,” Acta Phys. Sin. 68, 093101 (2019).

Lu, Z.-T., Corwin, K. L., Vogel, K. R., Wieman, C. E., Dinneen, T. P., Maddi, J., and Gould, H., “Efficient collection of \(^{223}\)Fr into a vapor cell magneto-optical trap,” Phys. Rev. Lett. 79, 994–997 (1997).

Lundberg, H., Mårtensson, A. M., and Svanberg, S., “Hyperfine structure in the sequence of sodium \(S\) states,” Journal of Physics B: Atomic and Molecular Physics 10, 1971–1978 (1977).

Lynch, K. M., Billowes, J., Bissell, M. L., Budinčević, I., Cocolios, T. E., de Groote, R. P., De Schepper, S., Fedosseev, V. N., Flanagan, K. T., Franchoo, S., Garcia Ruiz, R. F., Heylen, H., Marsh, B. A., Neyens, G., Procter, T. J., Rossel, R. E., Rothe, S., Strashnov, I., Stroke, H. H., and Wendt, K. D. A., “Decay-assisted laser spectroscopy of neutron-deficient francium,” Phys. Rev. X 4, 011055 (2014).

Lynch, K. M., Cocolios, T. E., Billowes, J., Bissell, M. L., Budinčević, I., Day Goodacre, T., de Groote, R. P., Farooq-Smith, G. J., Fedosseev, V. N., Flanagan, K. T., Franchoo, S., Garcia Ruiz, R. F., Heylen, H., Li, R., Marsh, B. A., Neyens, G., Rossel, R. E., Rothe, S., Stroke, H. H., Wendt, K. D. A., Wilkins, S. G., and Yang, X., “Combined high-resolution laser spectroscopy and nuclear decay spectroscopy for the study of the low-lying states in \(^{206}\)Fr, \(^{202}\)At, and \(^{198}\)Bi,” Phys. Rev. C 93, 014319 (2016).

Lyons, J. D. and Das, T. P., “Theoretical analysis of level crossing in a \(2P\) atomic state,” Phys. Rev. A 2, 2250–2259 (1970).

Mack, M., Karlefski, F., Hattermann, H., Höckh, S., Jessen, F., Cano, D., and Fortágh, J., “Measurement

of absolute transition frequencies of $^{87}$Rb to $nS$ and $nD$ Rydberg states by means of electromagnetically induced transparency, Phys. Rev. A 83, 062515 (2011).

Marcassa, L., Muniz, S., Telles, G., Zilio, S., and Bagno, V., “Measurement of Na 5S1/2 hyperfine splitting by ionization using a sample of cold atoms,” Optics Communications 155, 38–42 (1998).

Marian, A., Stowe, M. C., Felinto, D., and Ye, J., “Direct frequency comb measurements of absolute optical frequencies and population transfer dynamics,” Phys. Rev. Lett. 95, 023001 (2005).

Maric, M., McFerran, J. J., and Luiten, A. N., “Frequency-comb spectroscopy of the D1 line in laser-cooled rubidium,” Phys. Rev. A 77, 032502 (2008).

McLaughlin, C., Ayachitula, R., Lindsay, M., and Knize, R., “Hyperfine splittings and isotope shift of the Rb 5S - 6S two-photon transition,” Bulletin of the American Physical Society 53rd Annual Meeting of the APS Division of Atomic, Molecular and Optical Physics, Abstract: N01.00124 (2022).

Meschede, D., “Centimeter-wave spectroscopy of highly excited rubidium atoms,” J. Opt. Soc. Am. B 4, 413-419 (1987).

Metcalf, H. J. and van der Straten, P., Laser Cooling and Trapping of Neutral Atoms (Springer-Verlag, New York, 1999).

Moon, H. S., Lee, W.-K., and Suh, H. S., “Hyperfine-structure-constant determination and absolute-frequency measurement of the Rb 4D3/2 state,” Phys. Rev. A 79, 062503 (2009).

Morgenweg, J., Barnes, I., and Eikema, K. S. E., “Ramsey-comb spectroscopy with intense ultrashort laser pulses,” Nature Physics 10, 30–33 (2014).

Morzyński, P., Wcisło, P., Ablewski, P., Gartman, R., Gawlik, W., Masłowski, P., Nagórny, B., Oźimek, F., Radzewicz, C., Witkowski, M., Ciurylo, R., and Zawada, M., “Absolute frequency measurement of rubidium 5S – 7S two-photon transitions,” Opt. Lett. 38, 4581–4584 (2013).

Morzyński, P., Wcisło, P., Ablewski, P., Gartman, R., Gawlik, W., Masłowski, P., Nagórny, B., Oźimek, F., Radzewicz, C., Witkowski, M., Ciurylo, R., and Zawada, M., “Line shape measurements of rubidium 5S – 7S two-photon transition,” Journal of Physics: Conference Series 548, 012023 (2014).

Nagourney, W., Harper, W., and Lurio, A., “Level-crossing study of the hyperfine structure of lithium,” Phys. Rev. A 17, 1394–1407 (1978).

Nakayama, S., Kelly, F. M., and Series, G. W., “Hyperfine structures, lifetimes and collisional cross sections of some Rydberg D states of $^{133}$Cs,” Journal of Physics B: Atomic and Molecular Physics 14, 835–838 (1981).

Neugart, R., Billowes, J., Bissell, M. L., Blaum, K., Cheal, B., Flanagan, K. T., Neyens, G., Nörtershäuser, W., and Yordanov, D. T., “Collinear laser spectroscopy at ISOLDE: new methods and highlights,” Journal of Physics G: Nuclear and Particle Physics 44, 064002 (2017).
“Level magnetizabilities of the alkaline metal atoms,” Chemical Physics 282, 289–304 (2002)
Ovhinnikov, Y. B., Szymaniec, K., and Edris, S., “Measurement of rubidium ground-state hyperfine transition frequency using atomic fountains,” Metrologia 52, 595–599 (2015)
Pal, R., Safronova, M. S., Johnson, W. R., Derevianko, A., and Porsev, S. G., “Relativistic coupled-cluster single-double method applied to alkali-metal atoms,” Phys. Rev. A 75, 042515 (2007)
Papuga, J., Bissell, M. L., Kreim, K., Barbieri, C., Blaum, K., De Rydt, M., Duguet, T., García Ruiz, R. F., Heylen, H., Kowalska, M., Neugart, R., Neyens, G., Nörtershäuser, W., Rajabali, M. M., Sánchez, R., Smirnova, N., Somà, V., and Yordanov, D. T., “Shell structure of potassium isotopes deduced from their magnetic moments,” Phys. Rev. C 90, 034321 (2014)
Peper, M., Helmrich, F., Butscher, J., Agner, J. A., Schmutz, H., Merkt, F., and Deiglmayr, J., “Precision measurement of the ionization energy and quantum defects of $^{39}$K I,” Phys. Rev. A 100, 012501 (2019)
Pérez Galván, Á., Zhao, Y., Orozco, L., Gómez, E., Lange, A., Baumer, F., and Sprouse, G., “Comparison of hyperfine anomalies in the $^5S_{1/2}$ and $^6S_{1/2}$ levels of $^{85}$Rb and $^{87}$Rb,” Physics Letters B 655, 114–118 (2007)
Pérez Galván, Á., Zhao, Y., and Orozco, L. A., “Measurement of the hyperfine splitting of the $^6S_{1/2}$ level in rubidium,” Phys. Rev. A 78, 012502 (2008)
Persson, J. R., “Extraction of hyperfine anomalies without precise values of the nuclear magnetic dipole moment,” The European Physical Journal A - Hadrons and Nuclei 2, 3–4 (1998)
Persson, J. R., “Table of hyperfine anomaly in atomic systems,” Atomic Data and Nuclear Data Tables 99, 62–68 (2013)
Pescht, K., Gerhardt, H., and Matthias, E., “Isotope shift and HFS of $D_2$ lines in Na-22 and 23 measured by saturation spectroscopy,” Zeitschrift für Physik A Atoms and Nuclei 281, 199–204 (1977)
Puchalski, M. and Puchucki, K., “Fine and hyperfine splitting of the $^{2}$P state in Li and Be$^+”,$ Phys. Rev. A 79, 032510 (2009)
Quirk, J. A., Damitz, A., Tanner, C. E., and Elliott, D. S., “Measurement of the hyperfine coupling constants and absolute energies of the $12s^2S_{1/2}, 13s^2S_{1/2},$ and $11d^2D_{3}$ levels in atomic cesium,” Phys. Rev. A 105, 022819 (2022)
Rafac, R. J. and Tanner, C. E., “Measurement of the $^{133}$Cs $6p^2P_{1/2}$ state hyperfine structure,” Phys. Rev. A 56, 1027–1030 (1997)
Raghavan, P., “Table of nuclear moments,” Atomic Data and Nuclear Data Tables 42, 189–291 (1989)
Raimond, J. M., Goy, P., Vitrantr, G., and Haroche, S., “Millimeter-wave spectroscopy of cesium Rydberg states and possible applications to frequency metrology,” J. Phys. Colloques 42, 37–43 (1981)
Ramos, A., Cardman, R., and Raithel, G., “Measurement of the hyperfine coupling constant for $n^5S_{1/2}$ Rydberg states of $^{85}$Rb,” Phys. Rev. A 100, 062515 (2019)
Ramsey, N. F., Molecular Beams (Oxford University Press, New York, 1956)
Rapol, U. D., Krishna, A., and Natarajan, V., “Precise measurement of hyperfine structure in the $^5F_{3/2}$ state of $^{86}$Rb,” The European Physical Journal D - Atomic, Molecular, Optical and Plasma Physics 23, 185–188 (2003)
Ren, Y.-N., Yang, B.-D., Wang, J., Yang, G., and Wang, J.-M., “Measurement of the magnetic dipole hyperfine constant $A_{D_{5}}$ of cesium $7S_{1/2}$ state,” Act. Phys. Sin. 65, 073103 (2016)
Roberts, B. M. and Ginges, J. S. M., “Nuclear magnetic moments of francium-207–213 from precision hyperfine comparisons,” Phys. Rev. Lett. 125, 063002 (2020)
Rui, Y., Wu, Y.-L., Li, R., and Wu, H.-B., “Precise absolute frequency measurement of three-electron Li$^6$ atom D-line transitions,” Scientia Sinica 51, 074209 (2021)
Rukhin, A. L., “Weighted means statistics in interlaboratory studies,” Metrologia 46, 323–331 (2009)
Rukhin, A. L., “Homogeneous data clusters in interlaboratory studies,” Metrologia 56, 035002 (2019)
Rupasinghe, P., Pee, F., Bullock, T., and Liu, J.-X., “Measurement of hyperfine constants and the isotope shift of rubidium $5^{P}_{3/2}$ excited-state using saturated absorption spectroscopy,” ArXiv 2208.04244 (2022)
Rydberg, S. and Svanberg, S., “Investigation of the $^2P_{3/2}$ level sequence in the Cs I spectrum by level crossing spectroscopy,” Physica Scripta 5, 209–212 (1972)
Ryschka, M. and Marek, J., “Observation of quantum beats in superradiance on the $5^2D_{3/2} − 6^2P_{1/2}$ transition in cesium vapours,” Physics Letters A 86, 98–100 (1981)
Safronova, M. S., Budker, D., DeMille, D., Kimball, D. F. J., Derevianko, A., and Clark, C. W., “Search for new physics with atoms and molecules,” Rev. Mod. Phys. 90, 025008 (2018)
Safronova, M. S., Johnson, W. R., and Derevianko, A., “Relativistic many-body calculations of energy levels, hyperfine constants, electric-dipole matrix elements, and static polarizabilities for alkali-metal atoms,” Phys. Rev. A 60, 4476–4487 (1999)
Safronova, M. S. and Safronova, U. I., “Critically evaluated theoretical energies, lifetimes, hyperfine constants, and multipole polarizabilities in $^{87}$Rb,” Phys. Rev. A 83, 052508 (2011)
Safronova, U. I. and Safronova, M. S., “High-accuracy calculation of energies, lifetimes, hyperfine constants, multipole polarizabilities, and blackbody radiation shift in $^{39}$K,” Phys. Rev. A 78, 052504 (2008)
Sagle, J. and van Wijngaarden, W. A., “Excited state hyperfine structures in $^{133}$Cs using quantum beat spectroscopy,” Canadian Journal of Physics 69, 808–812 (1991)
Shano, B. K., Nandy, D. K., Das, B. P., and Sakemi, Y., “Correlation trends in the hyperfine structures of $^{210,212}$Fr,” Phys. Rev. A 91, 042507 (2015).

Sansonetti, C. J., Simien, C. E., Gillaspy, J. D., Tan, J. N., Brewer, S. M., Brown, R. C., Wu, S.-J., and Porto, J. V., “Absolute transition frequencies and quantum interference in a frequency comb based measurement of the $^6$Li D lines,” Phys. Rev. Lett. 107, 023001 (2011).

Sansonetti, J. E., “Spectroscopic data for neutral francium (Fr I),” Journal of Physical and Chemical Reference Data 36, 497–507 (2007).

Sättmannshausen, H., Merkt, F., and Deiglmayr, J., “High-resolution spectroscopy of Rydberg states in an ultracold cesium gas,” Phys. Rev. A 87, 032519 (2013).

Scherf, W., Khait, O., Jäger, H., and Windholz, L., “Measurement of hyperfine coupling constants in the $^4p_{3/2}$ state of $^7$Li measured using Stark spectroscopy of Rydberg states,” Phys. Rev. A 51, 2866–2869 (1995).

Stoicheff, B. P. and Weinberger, E., “Doppler-free two-photon absorption spectrum of rubidium,” Canadian Journal of Physics 57, 2143–2154 (1979).

Stone, N. J., “Table of nuclear magnetic dipole and electric quadrupole moments,” Atomic Data and Nuclear Data Tables 90, 75–176 (2005).

Svanberg, S. and Belin, G., “Determination of hyperfine structure and $g_j$ factors in the sequences of $^2D$ states in alkali atoms using a tunable dye laser,” Journal of Physics B: Atomic and Molecular Physics 7, L82–L86 (1974).

Svanberg, S. and Tsekiris, P., “Hyperfine-structure investigation of highly excited $^2D$ levels in $^{87}$Rb and $^{133}$Cs using a cw tunable dye laser in a two-step excitation scheme,” Phys. Rev. A 11, 1125–1137 (1975).

Tai, C., Happer, W., and Gupta, R., “Hyperfine-structure studies of highly excited $D$ and $F$ levels in alkali atoms using a cw tunable dye laser,” Phys. Rev. Lett. 30, 817–820 (1973).

Tai, C., Gupta, R., and Happer, W., “Hyperfine structure of the $^8P_{1/2}$ state of $^{133}$Cs,” Phys. Rev. A 8, 1661–1665 (1973).

Tai, C., Happer, W., and Gupta, R., “Hyperfine structure and lifetime measurements of the second-excited $d$ states of rubidium and cesium by cascade fluorescence spectroscopy,” Phys. Rev. A 12, 736–747 (1975).

Tang, Y.-B., Lou, B.-Q., and Shi, T.-Y., “Ab initio studies of electron correlation effects in magnetic dipolar hyperfine interaction of Cs,” Journal of Physics B: Atomic, Molecular and Optical Physics 52, 055020 (2019).

Tanner, C. E. and Wieman, C., “Precision measurement of the hyperfine structure of the $^{133}$Cs $6P_{3/2}$ state,” Phys. Rev. A 38, 1616–1617 (1988).

Tauschinsky, A., Newell, R., van Linden van den Heuvell, H. B., and Spreeuw, R. J. C., “Measurement of $^{87}$Rb Rydberg-state hyperfine splitting in a room-temperature vapor cell,” Phys. Rev. A 87, 042522 (2013).

Tetu, M., Fortin, R., and Savard, J.-Y., “A new determination of the Rb 85 unperturbed hyperfine transition frequency,” IEEE Transactions on Instrumentation and Measurement IM-25, 477–480 (1976).
Thibault, C., Touchard, F., Büttgenbach, S., Klapisch, R., de Saint Simon, M., Duong, H., Jacquotin, P., Juncar, P., Libernan, S., Pillet, P., Pinard, J., Vialle, J., Penselle, A., and Huber, G., “Hyperfine structure and isotope shift of the D2 line of 118–145Cs and some of their isomers,” Nuclear Physics A 367, 1–12 (1981a)

Thibault, C., Touchard, F., Büttgenbach, S., Klapisch, R., De Saint Simon, M., Duong, H. T., Jacquotin, P., Juncar, P., Libernan, S., Pillet, P., Pinard, J., Vialle, J. L., Penselle, A., and Huber, G., “Hyperfine structure and isotope shift of the D2 line of 76–98Rb and some of their isomers,” Phys. Rev. C 23, 2720–2729 (1981b)

Thompson, D. C., O’Sullivan, M. S., Stoicheff, B. P., and Xu, G.-X., “Doppler-free two-photon absorption spectrum of potassium,” Canadian Journal of Physics 61, 949–955 (1983)

Tian, Y.-L., Yang, P.-F., Wu, W., Li, S.-K., Li, G., Zhang, P.-F., and Zhang, T.-C., “Determination of hyperfine structure of the 3s2S1/2 and 3p2P3/2 levels in 67Li,” Optics Communications 63, 288–292 (1987)

Volz, U., Majerus, M., Liebel, H., Schmitt, A., and Schmoranzer, H., “Precision lifetime measurements on NaI 3p2P1/2 and 3p2P3/2 by beam-gas-laser spectroscopy,” Phys. Rev. Lett. 76, 2862–2865 (1996)

Voss, A., Buchinger, F., Cheal, B., Crawford, J. E., Dilling, J., Kortelainen, M., Kwiatkowski, A. A., Leary, A., Levy, C. D. P., Mooshammer, F., Ojeda, M. L., Pearson, M. R., Procter, T. J., and Tamimi, W. A., “Nuclear moments and charge radii of neutron-deficient francium isotopes and isomers,” Phys. Rev. C 91, 044307 (2015)

Walls, J., Ashby, R., Clarke, J. J., Lu, B., and van Wijngaarden, W. A., “Measurement of isotope shifts, fine and hyperfine structure splittings of the lithium D lines,” The European Physical Journal D - Atomic, Molecular, Optical and Plasma Physics 22, 159–162 (2003)

Wang, J., Liu, H.-F., Yang, B.-D., He, J., and Wang, J.-M., “Determining the hyperfine structure constants of caesium 88S1/2 state aided by atomic coherence,” Measurement Science and Technology 25, 035501 (2014a)

Wang, J., Liu, H.-F., Yang, G., Yang, B.-D., and Wang, J.-M., “Determination of the hyperfine structure constants of the 87Rb and 85Rb 4D5/2 state and the isotope hyperfine anomaly,” Phys. Rev. A 90, 052505 (2014b)

Wang, J., Wang, J.-M., Liu, H.-F., Yang, B.-D., and He, J., “Measurement of hyperfine splitting and determination of hyperfine structure constant of caesium 88S1/2 state by using of ladder-type EIT,” in Photon Counting Applications IV; and Quantum Optics and Quantum Information Transfer and Processing Vol. 8773, edited by J. Flurášek, I. Procházka, and R. Sobolewski, International Society for Optics and Photonics (SPIE, 2013) pp. 159 – 165.

Wang, Q., Zhang, N., Guan, W., Zhang, S.-G., Wang, W.-L., Wei, R., and Wang, Y.-Z., “Precision measurements of the ground-state hyperfine splitting of 85Rb using an atomic fountain clock,” Phys. Rev. A 100, 022510 (2019)

Wang, S.-D., Yuan, J.-P., Wang, L.-R., Xiao, L.-T., and Jia, S.-T., “Investigation on the Cs 6S1/2 to 7D electric quadrupole transition via monochromatic two-photon process at 767 nm,” Frontiers of Physics 16, 12502 (2021)
Wang, Z., Zhang, J., Zeng, Z., Yan, K., Zhou, H., and Yang, B., “Hyperfine energy level splitting structure measurement of the excited state 6D_{3/2} for cesium atom,” Laser and Optoelectronics Progress 57, 030202 (2020a).

Wang, Z.-R., Hou, X.-K., Bai, J.-D., and Wang, J.-M., “Measuring the hyperfine splitting and deriving the hyperfine interaction constants of the cesium 5p^6 7d^2D_{5/2} excited state,” Applied Sciences 10, 10.3390/app10228178 (2020b).

van Wijngaarden, W. A., Bonin, K. D., and Happer, W., Wu, C.-M., Liu, T.-W., Wu, M.-H., Lee, R.-K., and Bennett, S. C., Cho, D., Masterson, Windholz, L., Jäger, H., Musso, M., and Zerza, G., “Magnetic-field decoupling of an alkali-metal excited-state hyperfine structure,” Opt. Lett. 33, 77–81 (1986).

van Wijngaarden, W. A. and Sagle, J., “Measurement of the excited state 6p\textsuperscript{5}D_{5/2} for cesium,” Zeitschrift für Physik A Hadrons and nuclei 297, 275–282 (1984a).

van Wijngaarden, W. A. and Li, J., “Measurement of parity nonconservation and an anapole moment in cesium,” Science 275, 1759–1763 (1997).

Wu, Y.-L., Li, R., Rui, Y., Jiang, H.-F., and Wu, H.-B., “Precise measurement of Li transition frequencies and hyperfine splitting,” Acta Physica Sinica 67, 163201 (2018).

Ye, J., Swartz, S., Jungner, P., and Hall, J. L., “Hyperfine structure and absolute frequency of the 87Rb 5P_{1/2} state,” Phys. Rev. A 48, 829–831 (1993).

Williams, W. D., Herd, M. T., and Hawkins, W. B., “Spectroscopic study of the 7p_{1/2} and 7p_{3/2} states in cesium-133,” Laser Physics Letters 15, 095702 (2018).

Yang, G., Wang, J., and Wang, J.-M., “Determination of the hyperfine coupling constants of the 5D_{5/2} of 85Rb atoms by using high signal-to-noise ratio electromagnetically-induced transparency spectra,” Acta Physica Sinica 66, 103201 (2017).

Yang, G., Wang, J., Yang, B.-D., and Wang, J.-M., “Measurement of the hyperfine coupling constant of the cesium 7S_{1/2} state,” Laser Physics Letters 13, 085702 (2016).

Ye, W., Sieradzan, A., Cerasulo, E., and Havey, M. D., “Measurement of hyperfine coupling constants of the 5p\textsuperscript{2}D\textsubscript{1} levels in Cs using polarization quantum-beat spectroscopy,” Phys. Rev. A 57, 3419–3424 (1998).

Zhang, J., Tandecki, M., Collister, R., Aubin, S., Behr, J. A., Gomez, E., Gwinner, G., Orozco, L. A., Pearson, M. R., and Sprouse, G. D. (FrPNC Collaboration), “Hyperfine anomalies in Fr: Boundaries of the spherical single particle model,” Phys. Rev. Lett. 115, 042501 (2015).

Zhang, S.-N., Zhang, X.-G., Jiang, Z.-J., Shang, H.-S., Chang, P.-Y., Chen, J.-B., Lian, J.-Q., Tu, J.-H., and Yang, S.-Y., “Precision measurement of the hyperfine structure of the 85Rb 6P_{3/2} state,” in 2017 Joint Conference of the European Frequency and Time Forum and IEEE International Frequency Control Symposium (EFTF/IFCS) (2017) pp. 762–764.

Zhu, M., Oates, C. W., and Hall, J. L., “Improved hyperfine measurements of the Na 5p\textsuperscript{2}P\textsubscript{3/2} excited state through frequency-controlled Dopplerless spectroscopy in a Zeeman magneto-optic laser trap,” Opt. Lett. 18, 1186–1188 (1993).

Zyla, P. et al. (Particle Data Group), “Review of Particle Physics,” PTEP 2020, 083C01 (2020), and 2021 update.