12C spectroscopy above the 3α threshold

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Abstract. We investigate three-α continuum states in the hyperspherical formalism for \( J = 0^+ \) and \( J = 2^+ \). We use the shallow Ali-Bodmer potential and the deep potential of Buck et al. We determine the 3α phase shifts up to \( E = 6 \text{ MeV} \), in parallel with an analysis of resonances in the framework of the Complex Scaling method. We show that shallow potentials provide additional narrow resonances, in contrast with experimental data. Deep potentials, however, only give rise to broad resonances, and are more consistent with the data.

1. Introduction
The 12C spectroscopy below the 3α threshold is well known since many years (see Ref. [1] and references therein). The 0+ ground state and the 2+ first excited state (\( E_x = 4.44 \text{ MeV} \)) have been investigated in many experimental and theoretical works, and their properties are now well established. The situation, however, is very different above the 3α threshold. In the continuum region, the second 0+ state at \( E_x = 7.65 \text{ MeV} \) plays a very important role in nuclear physics and in astrophysics. This state cannot be reproduced in standard [2, 3] or in no-core [4] shell model calculations. The 02+ state is known to have a well-developed cluster structure and has been successfully described by α cluster models [5, 6, 7, 8, 9]. Fermionic molecular dynamics (FMD) calculations [10] also reproduce the 02+ resonance provided that α-cluster structures are introduced in the FMD basis.

Historically, the interest for the 02+ state started with the prediction of Hoyle [11] that a resonance with zero angular momentum should exist just above the α+8Be threshold to explain the 12C abundance in the Universe. In nuclear physics the interest for the 02+ structure was recently revived by its interpretation as a dilute cluster gas state of three weakly interacting α particles [12].

The experimental search for a 2+ broad resonance is recent. Due to its specific structure, the observation of such resonance is extremely difficult. Some evidence for a 2+ broad level was reported by Itoh et al. [13] near \( E_\alpha \approx 9 - 10 \text{ MeV} \), though not widely accepted. More recently, the situation was clarified, and a possible 2+ resonance at \( E_\alpha = 9.6 \pm 0.1 \text{ MeV} \) with \( \Gamma = 0.6 \text{ MeV} \) was observed by Freer et al. [14]. These properties are in fair agreement with predictions of microscopic cluster models [9, 15].

The aim of the present work (see Ref. [16] for details) is to investigate the 3α continuum in a non-microscopic approach. We use the Complex Scaling Method (CSM, see Ref. [17]) complemented by the triple α phase shifts. Recently, we have extended the hyperspherical formalism [18] to three-body continuum states by implementing the R-matrix theory with a
Lagrange basis [19]. Although the hyperspherical theory is known to converge slowly in the continuum [20, 21, 19], it provides genuine 3-body phase shifts.

2. Theoretical framework
For 3-body systems, the Schrödinger equation can be solved with the hyperspherical approach [18]. The hyperradius \( \rho \) and hyperangle \( \alpha \) are deduced from the scaled Jacobi coordinates \( x \) and \( y \). The wave function in partial wave \( J\pi \) is then expanded over hyperspherical harmonics as

\[
\Psi^{J\pi}(\rho, \Omega_5) = \rho^{-5/2} \sum_{\gamma K} \chi^{J\pi}_{\gamma K}(\rho) \gamma^{JM}_{\gamma K}(\Omega_5),
\]

where the hyperradial functions \( \chi^{J\pi}_{\gamma K}(\rho) \) have to be determined, and where \( \gamma^{JM}_{\gamma K}(\Omega_5) \) are known functions, depending on the five angles \( \Omega_5 = (\Omega_x, \Omega_y, \alpha) \). The Schrödinger equation is replaced by a system of coupled differential equations involving matrix elements of the 2-body potentials. In Eq. (1), a truncation must be done in the summation over \( K \); the maximum \( K \) value is denoted as \( K_{\text{max}} \). The number of \( \gamma K \) components increases rapidly when \( K_{\text{max}} \) increases.

The treatment of three-body continuum states (\( E > 0 \)), with exact three-body asymptotic conditions, is recent [22, 19]. It is performed with the \( R \)-matrix method [23, 24], where the configuration space is divided into two regions: the internal region (with radius \( a \)) where the nuclear force must be taken into account, and the external region where the potentials have reached their asymptotic (Coulomb) behaviour. Accordingly, the external solution reads, for an entrance channel \( \gamma' \),

\[
\chi^{J\pi}_{\gamma K,\text{ext}}(\rho) = C^{J\pi}_{\gamma K} \left[ H^{+}_{\gamma K}(k\rho) \delta_{\gamma'\gamma} \delta_{KK'} - U^{J\pi}_{\gamma K,\gamma' K'} H^{+}_{\gamma' K'}(k\rho) \right],
\]

where \( k = \sqrt{2m_N E/h^2} \) is the three-body wave number (\( m_N \) is the nucleon mass). In Eq. (2), \( C^{J\pi}_{\gamma K} \) is a normalization coefficient, \( U^{J\pi} \) is the three-body collision matrix and the incoming and outgoing functions \( H^{\pm}_{\gamma K}(x) \) are defined as

\[
H^{\pm}_{\gamma K}(x) = G^{\pm}_{K+\frac{1}{2}}(\eta_{\gamma K}, x) \pm iF^{\pm}_{K+\frac{1}{2}}(\eta_{\gamma K}, x),
\]

where \( \eta_{\gamma K} \) is the Sommerfeld parameters in channel \( \gamma K \), and \( F \) and \( G \) are the regular and irregular Coulomb functions. The internal components \( \chi^{J\pi}_{\gamma K,\text{int}}(\rho) \) are expanded over a Lagrange mesh \( \varphi_i(\rho) \).

According to the \( R \)-matrix formalism, matrix elements between basis functions \( \varphi_i(\rho) \) must be computed over the internal region. Then the use of the Bloch operator [24] makes the kinetic energy Hermitian, and ensures the continuity of the derivative of the wave function at \( \rho = a \). The matching between the internal and external solutions provides the collision matrix \( U^{J\pi} \). A strong test of the calculation is that the collision matrix should not depend on the channel radius \( a \) and on the number \( N \) of basis functions, provided that they are large enough to meet the \( R \)-matrix requirements. The collision matrix may involve many channels, in particular for large \( K_{\text{max}} \) values, and is in general analyzed through its eigenvalues. The number of eigenphases is of course equal to the number of channels \( \gamma K \).

The \( R \)-matrix theory provides 3-body wave functions at any scattering energy. The calculation of the phase shifts is obviously the optimal way to investigate the continuum, and in particular broad resonances. In practice, however, dealing with the continuum is an heavy task, especially for three-body systems. In particular, only shallow local potentials can be currently employed. For these reasons a number of approximate methods have been developed. The common idea is to derive resonance properties (energy and width) from bound-state calculations, which only requires slight modifications of well mastered techniques. In the CSM [25, 26, 17],
the space radial coordinate $r$ and the momentum $p$ are rotated by a scaling angle $\theta$. The diagonalization of $H(\theta)$ with square-integrable functions can be done in various bases, such as Gaussian [27] or Lagrange basis [28]. The ABC theorem [25, 26] demonstrates that this diagonalization provides continuum states along straight lines in the complex plane, rotated by an angle $2\theta$ from the positive real axis. On the contrary, resonant states are not modified under variations of $\theta$. The resonance energy $E_R$ and width $\Gamma$ are determined from

$$E(\theta) = E_R - i\Gamma/2,$$

and are independent (at the numerical accuracy of the calculation) of $\theta$.

### 3. Application to $3\alpha$ continuum states

The shallow Ali-Bodmer (AB) potential [29] ($\ell$-dependent potential $d$), and the deep potential of Buck et al. (BFW) [30] reproduce fairly well the $\alpha + \alpha$ phase shifts up to $E \approx 20$ MeV, i.e. below the proton threshold in $^8\text{Be}$. It is well known that a $3\alpha$ potential must be introduced to reproduce the ground-state energy of $^{12}\text{C}$. According to Ref. [31], we choose the three-body interaction as $V_{3\alpha\alpha}(\rho) = v_3 \exp(-(\rho/\rho_3)^2)$, with the range $\rho_3 = 6$ fm. The amplitude $v_3$ is adjusted, for each potential and angular momentum, on the experimental energies.

![Figure 1.](image1.png) ![Figure 2.](image2.png)

**Figure 1.** Three-$\alpha$ phase shifts (upper panel) and complex energies (lower panel) for the Ali-Bodmer potential ($J = 0^+$), with a 3-body potential $v_3 = -22.0$ MeV. Complex energies are displayed for two angles: $\theta = 0.25$ (crosses), and $\theta = 0.30$ (circles).

**Figure 2.** Complex energies with the BFW potential for $J = 0^+, 2^+$ with and without a $3\alpha$ potential. Two scaling angles are used: $\theta = 0.25$ and $\theta = 0.30$.

The first three $J = 0^+$ eigenphases (labeled by index $i$) and complex energies are displayed in Fig. 1, for $K_{\text{max}} = 44$, and for two scaling angles: $\theta = 0.25$ and $\theta = 0.30$. Several states are stable and can also be observed in the eigenphases. The second $0^+$ state is located at $-0.12$ MeV but, as for $v_3 = 0$, several additional resonances between 0.5 and 2 MeV are observed in the eigenphases, and confirmed by the CSM method. A further narrow state near 3 MeV ($\Gamma \approx 0.4$ MeV) is obtained.
Figure 2 presents the resonance energies with the BFW potential. With this potential, a small number of resonances is found, in agreement with Ref. [32]. Without the repulsive 3-body potential, the first 2\(^+\) excited state is unbound, and shows up as a sharp resonance. A resonance is found near \(E_x \approx 10.5\) MeV.

4. Conclusion

We have shown that the 3\(\alpha\) continuum is very sensitive to the choice of the \(\alpha + \alpha\) potential. This result is not surprising as the bound-state spectrum is also dependent on the potential (see for example Ref. [5]). The AB shallow potential provides several narrow resonances, which do not correspond to experimentally known states. Calculations with the deep BFW potential are currently limited to the CSM method, but provide resonance energies which significantly differ from the AB interaction. In addition to the 0\(^+\), 0\(^2\) and 2\(^+\) states, only broad resonances beyond 3 MeV are found. The 2\(^2\) resonance might correspond to the new state observed in recent experiments [14, 33].

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