Comment on “Temperature range of superconducting fluctuations above $T_c$ in YBa$_2$Cu$_3$O$_{7-\delta}$ single crystals”

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In the article by Grbić et al. (Phys. Rev. B 83, 144508 (2011)), measurements of microwave absorption were used to determine the in-plane ac-fluctuation conductivity (or paraconductivity) at zero-field of YBa$_2$Cu$_3$O$_{7-\delta}$ single crystals with different doping levels, and then to establish the temperature range of the superconducting fluctuations above $T_c$. The ac-paraconductivity was obtained as the difference between the measurements at zero magnetic field and at 16 T, the maximum field amplitude used in their experiments. However, we will argue that such field amplitude is not enough to quench all superconducting fluctuations in the studied compounds and that, therefore, the authors actually determine the ac-fluctuation magnetoconductivity at 16 T. So, the temperature they propose for the onset of the superconducting fluctuations, $T'$, will correspond to the one at which the finite field effects at 16 T become measurable in their experiments, and the actual temperature of the superconducting fluctuations onset will be located well above $T'$.

These conclusions, which also concern influential recent publications on that issue, are then confirmed by analyzing some of the Grbić and coworkers data on the grounds of the Gaussian Ginzburg-Landau (GGL) approach for the finite-field (or Prange) fluctuation regime.

The starting assumption of the Grbić and coworkers analysis of their interesting microwave absorption measurements in YBa$_2$Cu$_3$O$_{7-\delta}$ (YBCO) single crystals is that the field of 16 T is well sufficient to suppress all superconducting fluctuations above the zero field $T_c$. Accordingly, these authors claim to extract the real part of the in-plane ac-paraconductivity in zero magnetic field, as the difference of the curves (of the conductivities) measured in zero field and in the field of 16 T. The temperature at which this difference becomes measurable with their experimental resolution, denoted $T'$, is then identified with the onset of the superconducting fluctuations. To further support their conclusions, Grbić and coworkers claim that the so-obtained zero field ac-paraconductivity may be quantitatively explained on the grounds of the GGL scenario.

In this Comment, we first stress that a magnetic field amplitude of 16 T is much smaller than different proposals for the upper critical magnetic field amplitude, $H_{c2}(0)$, of YBCO. As it is well known since the pioneering studies of M. Tinkham and coworkers, 2,3 such field amplitude is not enough to suppress the superconducting fluctuations above $T_c$. As a consequence, instead of the real part of the zero field in-plane ac-paraconductivity, we will argue that the measurements of Grbić and coworkers determine the real part of the total in-plane ac-magnetoconductivity at 16 T, and that the onset temperature of the superconducting fluctuations, $T_{\text{onset}}$, will be located well above $T'$. These conclusions are confirmed by analyzing, as an example, the data of Ref. 1 for the overdoped sample on the grounds of the GGL approach for the finite-field (or Prange) fluctuation regime 3,4 and by taking also into account the frequency dependence of the ac conductivity. 5 Although Ref. 1 has been published eight years ago, the interest and suitability of our present Comment is enhanced by the fact that some of the assumptions and/or conclusions of that paper that we question now are still being used in influential studies to support different, and in some cases contradictory, phenomenological descriptions of the rounding effects around $T_c$ in high temperature superconductors (HTSC). 2,3

The central aspect of the procedure used by Grbić and coworkers was to approximate the zero-field in-plane paraconductivity, $\Delta \sigma_{ab}(\varepsilon,0) = \sigma_{ab}(\varepsilon,0) - \sigma_{abB}(\varepsilon,0)$, by the in-plane magnetoconductivity, $\Delta \sigma_{ab}(\varepsilon, H) = \sigma_{ab}(\varepsilon,0) - \sigma_{ab}(\varepsilon, H)$. In these expressions, $\sigma_{ab}(\varepsilon, H)$ and $\sigma_{abB}(\varepsilon, H)$ are respectively the as-measured and background (or normal-state) in-plane electrical conductivities, at a reduced temperature $\varepsilon \equiv \ln(T/T_c)$ and at a magnetic field, $H$, applied perpendicularly to the ab planes. However, this simple and well known approximation (see e.g., Refs. 2, 3, 6, 7, and references therein), which allows to estimate the zero-field paraconductivity from two directly measurable observables, is applicable only if two conditions are fulfilled: First, if the normal state magnetoconductivity can be neglected, i.e., if $\sigma_{abB}(\varepsilon,0) \approx \sigma_{abB}(\varepsilon, H)$, and second, if it is used a field amplitude, $H_q$, large enough to quench the superconducting fluctuations at all reduced temperatures, i.e., $\sigma_{ab}(\varepsilon, H_q) = \sigma_{abB}(\varepsilon, H_q)$.

The adequacy of the first approximation noted above, namely the smallness of the normal state magnetoconductivity when compared with the one associated with the superconducting fluctuations, has been earlier proved in different HTSC, including YBCO, at least at a qualitative level and up to moderate reduced temperatures and magnetic fields. 3,6,7,10,11 In what concerns the amplitude of $H_q$, the pioneering measurements of Gollub and coworkers of the fluctuation induced diamagnetism in several low-$T_c$ superconductors (LTSC) suggested that it is of the order of $H_{c2}(0)$. 12 This conclusion was later confirmed by measurements in other LTSC 13 and in
a HTSC with a low \( H_{c2}(0) \). Measurements of the dc paraconductivity under relatively high magnetic fields in different HTSC also support \( H_g \sim H_{c2}(0) \). \(^{13,14}\)

The question now is, therefore, if the \( H_{c2}(0) \) values of the samples studied in Ref. \(^{1}\) are of the order or less than 16 T, the largest magnetic field used in these measurements. As already stressed in Tinkham’s textbook, \( H_{c2}(0) \) in HTSC is poorly defined, because of fluctuation rounding of the transition. In fact, even in the case of the highly studied YBCO compounds around their optimal doping the discrepancies between different determinations of \( H_{c2}(0) \) remain up to now very important, the proposals leading to field amplitudes between 100 T and 400 T. \(^{2,3,8,12,14,17}\) For the most underdoped YBCO compound studied in Ref. \(^{1}\) with \( T_c = 57 \) K, different proposals lead to \( \mu_0 H_{c2}(0) \) values between 30 T and 90 T \(^{2,3,8,12,14,17}\) in any case still much larger than the experimental parameters, in particular the assumption of the error sources affecting both Eq. (1) and the field amplitudes used in Ref. \(^{1}\). For both compounds, the largest \( H_{c2}(0) \) values are those extracted from the analysis of the superconducting fluctuations around \( T_c \) on the grounds of the GL scenario, this last procedure probably being, as suggested by the above comment in Tinkham’s textbook, the most adequate to determine \( H_{c2}(0) \) in HTSC.

The qualitative analysis summarized above already suggest that \( T' \) in Ref. \(^{1}\) actually corresponds to the temperature at which the finite-field effects at 16 T become measurable in their experiments. On the grounds of the GGL approach, these effects are expected to be appreciable when \( \varepsilon \) becomes of the order of the reduced magnetic field \( h \equiv H/H_{c2}(0) \). \(^{5,8,12,14,17}\) i.e.,

\[
T' \approx T_c(1 + h),
\]

an approximate relationship which will breakdown when \( H \approx H_{c2}(0) \). As in Ref. \(^{1}\) the measured parameters are \( T_c \) and \( T'(16 \) T), the easiest way to check the applicability of Eq. (1) is to estimate the corresponding \( H_{c2}(0) \) values. This leads to \( \mu_0 H_{c2}(0) \sim 200 \) T for sample OD89 (with \( T_c = 89.4 \) K and \( T' - T_c \approx 7 \) K), and \( \mu_0 H_{c2}(0) \sim 40 \) T for sample UD57 (with \( T_c = 57.2 \) K and \( T' - T_c \approx 23 \) K). These \( H_{c2}(0) \) values are in reasonable agreement with the ones in the literature \(^{5,8,12,14,17}\), taking into account the error sources affecting both Eq. (1) and the experimental parameters, in particular the assumption that the normal-state magnetoconductivity may be neglected. Note also that the seemingly much wider temperature range of the superconducting fluctuations observed in the underdoped YBCO (when compared to the widths observed in the almost optimally doped samples), a result claimed by Grbić and coworkers as the most intriguing in the current controversy about the nature of the pseudogap in deeply underdoped HTSC, may be easily explained by just taking into account in Eq. (1) the much lower value of the upper critical field in this deeply underdoped sample.

Another check of the crude conclusions summarized above may be done on the grounds of the GGL approach by analyzing, as an example, the results on the real part of the ac measurements presented in Fig. 9(a) of Ref. \(^{1}\) for the overdoped sample OD89, which is the closer to the prototypical optimally-doped YBCO. As commented above, these data actually correspond to the ac in-plane magnetoconductivity, \( \Delta \sigma_{ab}(\varepsilon, 16 \) T). By neglecting the normal state magnetoconductivity when compared with the one associated with the superconducting fluctuations, these data may be then approximated as

\[
\sigma_{ab}(\varepsilon, 0) - \sigma_{ab}(\varepsilon, 16 \) T \approx \left[ S(\omega, T) \right] \left[ \Delta \sigma_{ab}(\varepsilon, 0) - \Delta \sigma_{ab}(\varepsilon, 16 \) T) \right].
\]

For the pre-factor \( S(\omega, T) \), which takes into account the high-frequency influence on the ac conductivity, we have used the proposal of Ref. \(^{8}\) as in Ref. \(^{1}\). In turn, for the dc paraconductivity, \( \Delta \sigma_{ab}(\varepsilon, H) \), we have used Eq. (8) in Ref. \(^{5}\). In doing the comparison of Eq. (2) with the data of Fig. 9(a) of Ref. \(^{1}\), we have used \( \xi_{ab}(0) = 1.1 \) nm, \( \xi_0(0) = 0.11 \) nm, \( \tau_{rel} = 1 \) and \( \varepsilon_c = 0.55 \). These values are those recently obtained in optimally doped YBCO through measurements of the precursor diamagnetism and the dc paraconductivity and magnetoconductivity and summarized in Table 2 of Ref. \(^{5}\). However, the differences with the precise values of these parameters in the overdoped YBCO sample studied in Ref. \(^{1}\) will just.

![FIG. 1. Analyses on the grounds of the GGL approach of the data of Fig. 9(a) taken from Ref. \(^{1}\) for the overdoped sample OD89 (open circles, noted there \( \sigma_{ab} \)). These data are attributed in Ref. \(^{1}\) to the real part of the in-plane ac superconducting fluctuation conductivity in zero field, although actually they correspond to the in-plane ac magnetoconductivity at 16 T. The latter (solid line) was approximated as the difference between the fluctuation conductivities measured without field and, respectively, with 16 T (dashed lines). In this GGL scenario, the well-defined onset temperature of the superconducting fluctuations is noted \( T_{onset} \), whereas \( T' \) was proposed in Ref. \(^{1}\). However, as it may be appreciated in this figure, this last corresponds instead, well within the experimental resolution, to the onset of the field effects at 16 T (i.e., when \( \varepsilon \sim h \)). For details see the main text.](image-url)
affect somewhat the cutoff value. Note also that, as the $\varepsilon$-region where the field effects at 16 T become relevant is relatively close to $T_c$, the resulting fit will not be appreciably affected by a cutoff in the dc paraconductivity (although the total-energy cutoff is crucial to precisely locate $T_{onset}$ in the GGL scenario).

The solid line in Fig. 1 corresponds to the best fit of Eq. (2) to the $\sigma_{ab}$ data in Fig. 9(a) of Ref. [1]. Our analysis leads to $\Lambda \approx 0.19$, much closer to the unity than the value 0.031 obtained in Ref. [1] through an analysis in terms of the zero-field paraconductivity. Taking into account the crude approximation used to estimate the high-frequency effects (although similar to the one used in Ref. [4]), and that the only free parameter was the cutoff $\Lambda$ arising in $S(\omega, T)$, the resulting agreement may be considered as remarkable. Fig. 1 also illustrates the paraconductivity expected under 0 T and 16 T (dashed and dot-dashed lines, respectively) up to the actual GGL onset temperature for the superconducting fluctuations, noted $T_{onset}$. As noted before, this last corresponds to a total-energy cutoff of 0.55 in the GGL approach used here, and it is in excellent agreement with dc measurements in optimally-doped YBCO (see Ref. [6] and references therein). It may also be clearly seen in Fig. 1 that the $T'$ value proposed in Ref. [1] agrees, well within the experimental uncertainties, with the onset of the field effects on the paraconductivity at 16 T. It is worth noting that the implications of the precise location of $T_{onset}$, in particular in relation to the so-called pseudogap temperature, is still at present a debated central aspect of the phenomenological descriptions of the superconducting transition in HTSC (comments on this issue may be seen in Sections 5.2 and 5.3 of Ref. [5] and also in Refs. [7] and [8] and references therein).

In conclusion, the interesting microwave absorption measurements of Ref. [1] in an overdoped YBCO sample may be explained quantitatively, and consistently with previous dc measurements in similar samples, on the grounds of the GGL approach for conventional superconducting fluctuations. In particular, our results confirm that the reduced-temperature range of the superconducting fluctuations above $T_c$ extends well beyond the one proposed in Ref. [1] almost one order of magnitude in the case of the overdoped YBCO sample. These conclusions concern also several of the proposals, in some cases contradictory, of Refs. [7],[8], and enhance the interest of extending to the other samples measured in Ref. [1] but also to other HTSC and to other observables (particularly, the Nernst effect), quantitative analysis on the grounds of the GGL scenario.

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