Fortified quantum mass function utilizing ordinal pictorial check based on time interval analysis and expertise

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Abstract

Information management has enter a completely new era, quantum era. However, there exists a lack of sufficient theory to extract truly useful quantum information and transfer it to a form which is intuitive and straightforward for decision making. Therefore, based on the quantum model of mass function, a fortified dual check system is proposed to ensure the judgment generated retains enough high accuracy. Moreover, considering the situations in real life, everything takes place in an observable time interval, then the concept of time interval is introduced into the frame of the check system. The proposed model is very helpful in disposing uncertain quantum information in this paper. And some applications are provided to verify the rationality and correctness of the proposed method.

Keywords: Quantum model of mass function Dual check system Time interval Uncertain quantum information

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1. Introduction

In recent years, the management of information has become a very hot field. A lot of relevant works have been completed to provide different kinds of methods to properly handle information offered which promotes the development of information industry. The representatives of the corresponding theories are soft theory [1-5], Z-numbers [6-9], D-numbers [10-14], fuzzy theory [15-18], Dempster-Shafer evidence theory [19-23] and some other mixed theories [24-26]. And the effectiveness of these theories are verified in many practical applications, like risk evaluation [27-29], pattern classification [30], optimization [31-34] and decision making [35-38]. Moreover, due to the rapid progress of quantum computing, some researchers come up with the idea that traditional information management can be transferred to the level of quantum. Some meaningful works about the topic are complex mass function [39-43] and quantum information theory [44-47]. In this paper, the proposed method is based on the quantum model of mass function [47]. In order to avoid the deviation which may caused by the original quantum evidences, a dual check system is designed to ensure the authenticity of the original judgments which utilizes the concept of Z-number [9]. Besides, because of the introduction of the time interval, a specially devised rule is proposed to appropriately decide the importance of different relationships of incidents, which is a kind of expert system under some restrictions. The contributions of the proposed method can be listed as:

(1) The second dual check system can help avoid the deviation produced by the original evidences to help provide more effective results.

(2) Introduction of time interval make the frame of discernment more adap-
tive to the real life.

(3) The fortified quantum mass function is able to produce intuitive and reasonable judgments about current situations compared with traditional rule of combination.

The rest of this paper is organized as follows. The part of preliminaries generally introduces some basic concepts of the proposed method. Then, the following section provides details of the fortified mass function. Besides, in the part of application, two applications are provided to prove the superiority and validity of the method in this paper. In the last part, some conclusive opinions are made to summarise advantages of the method proposed in this paper.

2. Preliminaries

In this part, some related concepts are briefly introduced. And there exists lots of works which solves problems in relative fields [48–51].

2.1. Quantum model of mass function [47]

**Definition 2.1.** *(Quantum mass function)*

Assume there exists a quantum frame of discernment, in which a quantum mass function $Q$ can be defined as:

$$Q(\mid A\rangle) = \psi e^{i\theta}$$  \hspace{1cm} (1)

The quantum mass function is also named as quantum basic probability assignment (QBPA), which is a mapping of $Q$ from 0 to 1 and the properties it satisfies are given as:

$$Q(\phi) = 0$$  \hspace{1cm} (2)
The value of $|Q(|A\rangle)|$ equals to $\psi^2$, which is regarded as the degree of belief to $|A\rangle$. Besides, the phase angle of $|A\rangle$ is represented by $\theta$.

**Remark 1:** The quantum mass function can degenerate to classical mass function when the phase angle of quantum mass function equals $0^\circ$.

**Remark 2:** Additivity is not satisfied in the quantum mass function, which is defined as:

$$|Q(|A\rangle) + Q(|B\rangle)| \neq |Q(|A\rangle)| + |Q(|B\rangle)| \quad (4)$$

### 2.2. Z-numbers

Z-number is a relatively new instrument to measure the reliability of the information. The credibility of information gotten plays an significant part in information extraction and disposal.

**Definition 2.2. (Z-numbers)**

A Z-number is a kind of group of fuzzy numbers, which is expressed as $Z = (A, B)$. A Z-number is bound up with $X$, which is an indeterminate variable with real value. The first constituent part $A$ is sort of constraint of $Z$. Besides, the second component $B$ is an instrument of measure of the degree of reliability of the first constituent part $A$.

### 2.3. Picture fuzzy set (PFS)

A picture fuzzy set $A$ on the finite universe of discourse $X$ whose mathematical form is represented as:

$$A = \{ (x, u_A(x), e_A(x), v_A(x)) | x \in X \} \quad (5)$$
with the condition:

\[ u_A(x) : X \to [0, 1] \]
\[ \epsilon_A(x) : X \to [0, 1] \]
\[ v_A(x) : X \to [0, 1] \]
\[ 0 \leq u_A(x) + \epsilon_A(x) + v_A(x) \leq 1 \] (6)

The \( u_A(x) \) represents the membership degree of \( x \in X \). Similarly, \( v_A(x) \) is the non-membership degree of \( x \in X \). Besides, \( \epsilon_A(x) \) is called the degree of hesitancy \( x \in X \).

For a PFS \( A \) in \( X \), a refusal function of \( x \in X \) which represents the degree of refusal can be defined as:

\[ \S_A(x) = 1 - (u_A(x) + \epsilon_A(x) + v_A(x)), \forall x \in X \] (7)

Largely, picture fuzzy set is appropriate for these situations when decision makers face their opinions involving with their determination making as follows: support(yes), neutrality(hesitancy), oppose(no) and refusal. For example, four probable circumstances people may encounter in voting are: "vote for", "abstain", "vote against" and "refusal of the voting". The intentions of group "vote for" and "vote against" are apparent. Group "abstain" means that voters hesitate between "vote for" and "vote against". As for group "refusal of voting", which denotes either invalid voting papers or abstention.

2.4. Ordinal Quantum Frame of Discernment (OQFD)

The ordinal quantum frame of discernment is a set whose propositions are relational with a certain order, which is defined as:
\( \Theta_{\text{ordinal}} = \{M_1, M_2, \ldots, M_n\} \)  

The sequence of proposition is expressed by subscripts. The propositions in the ordinal frame of discernment satisfy the following characters:

- For any element with subscript \( i \), it is supposed to be confirmed before the ones with subscripts \( i + n (n \geq 1) \).

- The definition of proposition in an ordinal frame of discernment is the same as the ones defined in the traditional quantum frame of discernment besides order of elements.

- The degree of uncertainty of whole evidence system can be further ascertained.

3. Proposed method

In the traditional ordinal quantum frame of discernment, propositions are associated in a certain order. The subscript \( i \) represents the order of them. Traditional ordinal quantum frame of discernment takes the influence of propositions which take place before its later ones preliminarily. However, previous frame of discernment does not take the detail of true situation so alleged ordinal relationship is opinionated and does not accord with the actual condition. As a result, the consequence of prediction based on traditional ordinal quantum frame of discernment do not possess enough accuracy and could not reflect the true circumstance. Hence, in order to describe the connection between sequential propositions appropriately, time interval is added into the \( OQFD \).
3.1. Ordinal Quantum Frame of Discernment with Time Interval (OQFDTI)

The ordinal quantum frame of discernment with time interval is a set whose elements are related in a certain order, which is defined as:

\[ \Theta_{OQFDTI} = \begin{cases} 
M^\alpha_1 \rightarrow M^\beta_2 & \text{Time} \in [\theta, \nu] \\
M^\beta_2 \rightarrow M^\gamma_3 & \text{Time} \in [\kappa, \xi] \\
M^\gamma_3 \rightarrow M^\delta_4 & \text{Time} \in [\rho, \varsigma] \\
. & . \\
. & . \\
. & . \\
M^n_{n-1} \Rightarrow M^n_n & \text{Time} \in [\tau, \omega] 
\end{cases} \]  

The sequence of propositions is denoted by subscripts. The order of propositions is stipulated by time sequence, the propositions which possess smaller subscripts are proposed earlier. Superscripts represent the time when the propositions are raised. Different time intervals between sequential two propositions contained in the frame of discernment signify the degree of closeness of connection. Time interval is shorter, the relation between two propositions is closer, which means that the influence of the antecedent proposition on the subsequent proposition is bigger. In OQFDTI, time interval given by sensors and the order of incident occurred ae taken into consideration which make the forecast reflect the essence of the objects further. As a result, the frame mentioned above possesses more accuracy and rationality.

The following properties of elements in OQFDTI satisfied are listed:

- Diverse styles of arrows represent different degree of relation between any two sequential propositions in corresponding time interval.
• The degree of uncertainty of the whole quantum system can be further ascertained in the course of determining one more proposition.

• For any element with a subscripts $i$, it is supposed to be confirmed before the proposition with subscripts $i + n$ ($n \geq 1$).

• The definition of proposition in an ordinal frame of discernment is exactly the same as the ones defined in traditional ordinal quantum frame of discernment except time interval.

3.2. Time quantitative rule

Since the $OQFDTI$ is ordinal, there exists a decisive relationship between the uncertainty of evidence system and the time intervals of two sequential propositions. First of all, in $OQFDTI$, the propositions which occur in the first place are considered to have a crucial effect on the propositions which occur after them. Besides, $OQFDTI$ mentioned, different time intervals between sequential two propositions contained in the frame of discernment signify the degree of the closeness of connection of them. Because the multifarious time intervals given by sensors are fluctuating, how to confirm three disparate degree of connection of every time interval ($\Upsilon_{strong}, \Upsilon_{moderate}, \Upsilon_{weak}$) should be taken into account which signify strong, moderate and weak connection of two sequential propositions in according time intervals. On the basis of the definition of $OQFDTI$, the detailed process of getting modified values of each proposition is defined as:

(1) According to the weights denoted by $Wgt$ given by sensors, the modified weights of proposition $i$ can be expressed specifically as:
$$\text{Weights}_{Time}^i = \begin{cases} 
Wgt \times Y_{\text{strong}} & \kappa \leq \text{Time} < \frac{\kappa + \xi}{2} \\
Wgt \times Y_{\text{moderate}} & \text{Time} = \frac{\kappa + \xi}{2}, \quad \text{Time} \in [\kappa, \xi] \\
Wgt \times Y_{\text{weak}} & \frac{\kappa + \xi}{2} < \text{Time} \leq F 
\end{cases}$$

(10)

Note: The weight of the first proposition is 1.

(2) For OQFDTI, mass of proposition with subscript $i$ is assembled into a group with proposition whose position is $i-1$ except $i = 1$. Original mass of every proposition is denoted by $\text{Mass}_i$ and the process of obtaining modified intermediate values of each proposition of an evidence is defined as:

$$\begin{bmatrix} \text{Value}_{\text{inter}}^1 & \text{Value}_{\text{inter}}^2 & \cdots & \text{Value}_{\text{inter}}^i 
\end{bmatrix} = \begin{bmatrix} \text{Mass}_1 & \text{Mass}_2 & \cdots & \text{Mass}_i 
\end{bmatrix} \times \begin{bmatrix} \text{Weights}_{\text{Time}}^1 \\
\text{Weights}_{\text{Time}}^2 \\
\vdots \\
\text{Weights}_{\text{Time}}^i 
\end{bmatrix}$$

(11)

Note: For mass of each proposition with subscript $h$ must be confirmed earlier than the ones with subscript $i (i > h)$.

Example: Assume there exists three propositions $\{M_1^a, M_2^b, M_3^c\}$. When you try to ascertain $\text{Weights}_{\text{Time}}^3$ for $M_3^c$ in the corresponding interval $M_2^b \rightarrow M_3^c$ $\text{Time} \in [\kappa, \xi]$, the $\text{Weights}_{\text{Time}}^2$ for $M_2^b$ must be determined in terms of $M_1^a$ in the interval $M_1^a \rightarrow M_2^b$ $\text{Time} \in [\vartheta, \upsilon]$. 

(3) The step of normalization of modified intermediate value of proposition $i$ is defined as:
\[
\begin{bmatrix}
Value_{1}^{\text{inter}} & Value_{2}^{\text{inter}} & \ldots & Value_{i}^{\text{inter}}
\end{bmatrix}
\rightarrow
\begin{bmatrix}
Value_{1}^{\text{nor}} & Value_{2}^{\text{nor}} & \ldots & Value_{i}^{\text{nor}}
\end{bmatrix}
\]

(12)

**Note:** The step of normalization is obtaining the quotient respectively of each intermediate value over the sum of all intermediate values.

### 3.3. TDQBF: TWO-DIMENSIONAL QUANTUM BELIEF FUNCTION

**Definition 3.1.** A TDQBF, \( M=(m_{1}, m_{2}) \). \( m_{1} \) and \( m_{2} \) are also QBPAs and \( m_{2} \) is a measure of reliability of \( m_{1} \).

The elements of \( m_{1} \) and \( m_{2} \) are consist of quantum probability assignment. The quantum frame of discernment of \( m_{2} \) can be represented by \( \Theta=\{Y, N, H, R\} \).

Assume there is a voting, the tickets for support, opposition, waiver neutral(hesitating), abstention are denoted by \( \{Y\}, \{N\}, \{H\} \) and \( \{R\} \) respectively.

### 3.4. DHDF: Dynamic Hesitation Distribution Formula

Assume there exists three QBPAs, which are \( m_{2}(Y) \), \( m_{2}(N) \) and \( m_{2}(H) \), DHDF can be shown as follow:

\[
m_{Y}(H) = \sqrt{\frac{|m_{2}(Y)|^{2}}{(|m_{2}(Y)| + |m_{2}(N)| + 2|m_{2}(Y)||m_{2}(N)|)^{2}}} \times m_{2}(H) \quad (13)
\]

\[
m_{N}(H) = \sqrt{\frac{|m_{2}(N)|^{2}}{(|m_{2}(Y)| + |m_{2}(N)| + 2|m_{2}(Y)||m_{2}(N)|)^{2}}} \times m_{2}(H) \quad (14)
\]

Because the degree of uncertainty should be reduced to make the results more deterministic, a part of hesitance should be distributed to corresponding propositions which means a part of mass of \( m_{2}(H) \) should be
allocated to $m_2(Y)$ and $m_2(N)$. And the mass assigned to $m_2(Y)$ and $m_2(N)$ are represented by $m_Y(H)$ and $m_N(N)$ respectively.

3.5. QPDR: Quantum Pignistic Distribution rule

Assume there exists three QBPAs, which are $m_2(Y)$, $m_2(N)$ and $m(H)$, QPDR can be defined as:

$$m_Z(Y) = \sqrt{\frac{|m_2(Y)|^2}{|m_2(Y)|^2 + |m_2(N)|^2}} \times m_Y(H) + m_2(Y)$$

$$m_Z(N) = \sqrt{\frac{|m_2(N)|^2}{|m_2(Y)|^2 + |m_2(N)|^2}} \times m_N(H) + m_2(N)$$

where the results of two QBPAs after distribution are expressed as $m_Z(Y)$ and $m_Z(N)$. $m_Y(H)$ and $m_N(N)$ represent the mass of $m_2(H)$ distributed to $m_2(Y)$ and $m_2(N)$ respectively. The variables $\sqrt{\frac{|m_2(Y)|^2}{|m_2(Y)|^2 + |m_2(N)|^2}}$ and $\sqrt{\frac{|m_2(N)|^2}{|m_2(Y)|^2 + |m_2(N)|^2}}$ aim to solve the problem about the loss when transferring the quantum into the form of classic probability. The method in this part decrease degree of uncertainty and conform to the actuality and reflect the internal connections of thighs.

**Example 3.1** Assume there are three QBPAs, which are be defined as:

$m_2(Y) = \frac{\sqrt{5}}{4} + \frac{\sqrt{5}}{4}i$, $m_2(N) = \frac{\sqrt{5}}{4} + \frac{\sqrt{5}}{4}i$, and $m_2(H) = \frac{1}{2} + \frac{1}{2}i$.

If the mass of $m_2(H)$ assigns to $m_2(Y)$ and $m_2(N)$ totally. The value of $m_2(H)$ distributed to $m_2(Y)$ and $m_2(N)$ by classic method are: $m_Y(H) = \frac{1}{4} + \frac{1}{4}i$, $m_N(H) = \frac{1}{4} + \frac{1}{4}i$. And the sum of classic probability of $m_Y(H)$ and $m_N(H)$ are $\frac{1}{4}$.

$$m_Y(H) = \frac{|m_2(Y)|^2}{|m_2(Y)|^2 + |m_2(N)|^2} \times m_2(H) = \frac{1}{4} + \frac{1}{4}i$$

$$P1 = |m_Y(H)|^2 = \frac{1}{8}$$
$m_n(H) = \frac{|m_2(N)|^2}{|m_2(Y)|^2 + |m_2(N)|^2} \times m_2(H) = \frac{1}{4} + \frac{1}{4}i$

$P_2 = |m_n(H)|^2 = \frac{1}{8}$

**classic probability** $= P_1 + P_2 = \frac{1}{4}$

The value obtained by formula $\sqrt{\frac{|m_2(Y)|^2}{|m_2(Y)|^2 + |m_2(N)|^2}}$ and $\sqrt{\frac{|m_2(N)|^2}{|m_2(Y)|^2 + |m_2(N)|^2}}$ are: $m_y(H) = \frac{\sqrt{2}}{4} + \frac{\sqrt{2}}{4}i$, $m_n(H) = \frac{\sqrt{2}}{4} + \frac{\sqrt{2}}{4}i$. And the sum of classic probability of $m_y(H)$ and $m_n(H)$ are $\frac{1}{2}$.

$m_y(H) = \frac{|m_2(Y)|^2}{|m_2(Y)|^2 + |m_2(N)|^2} \times m_2(H) = \frac{\sqrt{5}}{4} + \frac{\sqrt{5}}{4}i$

$P_1 = |m_y(H)|^2 = \frac{1}{4}$

$m_n(H) = \frac{|m_2(N)|^2}{|m_2(Y)|^2 + |m_2(N)|^2} \times m_2(H) = \frac{\sqrt{2}}{4} + \frac{\sqrt{2}}{4}i$

$P_2 = |m_n(H)|^2 = \frac{1}{4}$

**classic probability** $= P_1 + P_2 = \frac{1}{2}$

The classic probability of $m_2(H) = \frac{1}{2} + \frac{1}{2}i$ is $\frac{1}{2}$. So, the loss of quantum probability by classic methods can be solved by the formula in this part.

**classic probability** $= |m_2(H)|^2 = \frac{1}{2}$

**Example 3.2** Assume there are three QBPAs, which are $m_2(Y) = \frac{2\sqrt{7}}{5} + \frac{2\sqrt{7}}{5}i$, $m_2(N) = \frac{2\sqrt{7}}{5} + \frac{2\sqrt{7}}{5}i$, and $m_2(H) = \frac{1}{2} + \frac{1}{2}i$.

The results are listed as follows: $m_Z(Y) = \frac{61\sqrt{7}}{165} + \frac{61\sqrt{7}}{165}i$, $m_Z(N) = \frac{61\sqrt{7}}{165} + \frac{61\sqrt{7}}{165}i$

$$m_Z(Y) = \sqrt{\frac{|m_2(Y)|^2}{|m_2(Y)|^2 + |m_2(N)|^2}} \times m_Y(H) + m_2(Y) = \frac{61\sqrt{7}}{165} + \frac{61\sqrt{7}}{165}i$$

$$m_Z(N) = \sqrt{\frac{|m_2(N)|^2}{|m_2(Y)|^2 + |m_2(N)|^2}} \times m_Y(H) + m_2(N) = \frac{61\sqrt{7}}{165} + \frac{61\sqrt{7}}{165}i$$
3.6. The combination of TDQBF

TDQBF can be defined as:

\[ m_X(x_i) = m_1(\{x_i\}) \times m_Z(\{Y\}) + (1 - m_1(\{x_i\})) \times m_Z(\{N\}) \]
\[ + \sum_{x_i \subseteq x_a} m_1(x_a) \times m_Z(\{Y\}) + \sum_{x_i \not\subseteq x_\beta} m_1(x_\beta) \times m_Z(\{N\}) \quad \forall x_i \subset 2^\Theta \]  
(17)

\[ m_X(A_i) = m_1(\{A_i\}) \times m_Z(\{Y\}) + m_1(\Theta) \times m_Z(\{Y\}) \quad \forall A_i \subset 2^\Theta \]  
(18)

\[ m_X(\Theta) = m_1(\Theta) \times m_Z(\{R\}) \]  
(19)

where \( x_i \) is a single subset of \( 2^\Theta \), \( A_i \) is multisubset of \( 2^\Theta \), \( i = 1, 2, 3, 4, \ldots, n \).

\( x_i \) which is a single subset included in multisubset \( A_a \) and not included in multisubset \( A_\beta \). After distributing \( m_2(\{H\}) \) based on QPDR to \( m_2(\{Y\}) \) and \( m_2(\{N\}) \), the two elements are denoted by \( m(Z(\{Y\})) \) and \( m(Z(\{N\})) \).

In equation 17 the mass of \( m(Z(\{Y\})) \) is distributed to the \( m_1(\{x_i\}) \) proportionally. Besides, mass of \( m(Z(\{N\})) \) is distributed to the reverses of single subsets proportionally. Because single proposition is a part of multiple propositions, \( \sum_{x_i \subseteq x_a} m_1(x_a) \times m(Z(\{Y\})) \) takes this effect of this situation into the formula, which means the mass of multiple propositions is divided into single proposition. In the same way, the case that single propositions calculated are not an element of multiple propositions is reflected in the part of formula, \( \sum_{x_i \not\subseteq x_\beta} m_1(x_\beta) \times m(Z(\{N\})) \).

In equation 18 the mass of \( m(Z(\{Y\})) \) is distributed to the \( m_1(\{A_i\}) \). Since the multisubset \( A_i \) is also included in \( \Theta \), \( m_1(\Theta) \times m(Z(\{Y\})) \) is utilized for dividing multisubset into universal set.

In equation 19 Because \( \Theta \) is symbol of uncertainty. \( m_1(\{\Theta\}) \times m_2(\{R\}) \)
is defined to decrease degree of uncertainty.

According to the three cases of sets, the synthesis of propositions in different cases respectively from single subset, multsubset and universal set propositions discussed by TDQBF are converted from uncertain propositions into certain propositions to guarantee the actuality.

The progress of combination is shown in Figure 1.
3.7. XG Rules

On the basis of the definition of OQFDTI, for describing the true situation of incident more accurately, the degree of urgency of the incident should be taken into account reasonably. For an evidence system, the weights for the same incident or proposition told by different quantum evidences are disparate to accord with the consideration about practical condition. For example, assume there exists a health assessment system, fever, trauma, internal injury and healthy are four status of it. For this system, OQFDTI can be defined as: $\Theta = \{\text{fever, trauma, internal injury, healthy}\}$. Apparently, the importance of propositions given in disparate conditions is supposed to be distinguished. For instance, in one group of evidences given during the period of COVID – 19, fever should be given more weights. Even though the quantum basic probability of assignment of fever is relatively low, it is worth a great concern because it is the most central cardinal symptom of COVID – 19. Correspondingly, the attention of trauma, internal injury and healthy are supposed to be cut down. Hence, confirmation of the extent of urgency of the incidents of the evidence system is of great significance and XG Rules is proposed to solve the problem about how to measure the urgency of incidents. The particular process of obtaining the results from the combination of each ordinal quantum evidence can be expressed as:

(1) In accordance with XG Rules, incidents are divided into several degrees of urgency and corresponding symbolic expressions $U^e$ are designed. The specific intervals of divisions are defined as:

- $U^{0<e<1/2}$ considered as "Ignorable event";
- $U^{1/2\leq e<1}$ considered as "More or less ignorable event";
- $U^{e=1}$ considered as "Normal event";
\[ U^{1 \leq e < 2} \text{ considered as “More or less urgent event”;} \]
\[ U^{e=2} \text{ considered as “Urgent event”;} \]
\[ U^{2 \leq e < 5/2} \text{ considered as “Quite urgent event”;} \]
\[ U^{5/2 \leq e < 3} \text{ considered as “Very urgent event”;} \]
\[ U^{3 \leq e < 5} \text{ considered as “Quite very urgent event”;} \]
\[ U^{5 \leq e} \text{ considered as “Very very urgent event.”} \]

\textbf{Note:} \( U^e \) represents the extent of importance of each piece of quantum evidence.

(2) The process of getting modified value for proposition with subscript \( p \) is defined as:
\[
MID(p) = \sum_{i=1}^{n} (U^e \times m_X(p)) \tag{20}
\]

(3) The step of normalization of \( MID(p) \) is defined as:
\[
NOR(p) = \frac{MID(p)}{\sum_{p=1}^{k} MID(p)} \tag{21}
\]

(4) The step of employing classical Dempster’s rule of combination to combine the \( NOR(p) \) for \( n-1 \) times is defined as:
\[
\text{Combine \ (n-1) times} \left\{ \begin{array}{c} \{\text{NOR}(1), \text{NOR}(2), \ldots, \text{NOR}(p)\} \\ \{\text{NOR}(1), \text{NOR}(2), \ldots, \text{NOR}(p)\} \\ \{\text{NOR}(1), \text{NOR}(2), \ldots, \text{NOR}(p)\} \end{array} \right\} \rightarrow \left\{ \begin{array}{c} \{\text{FIN}(1), \text{FIN}(2), \ldots, \text{FIN}(p)\} \end{array} \right\} \tag{22}
\]

4. Application 1

\textbf{Example 4.1. Application of virtual currency}
Assume there exists a financial company which makes judgment about a kind of virtual currency. The OQFDTI of it is given as: \( \Theta = \{R, S, D, SD\} \). 

\( R, S \) and \( D \) represent the development of the virtual currency are Raised, Steady and Decreasing respectively. Analogously, hesitation in Smooth and Decreasing is indicated by SD. Besides, the predictions given by sensors in four pieces of evidences \( m_1 \) are listed in Table 1 and corresponding degree of reliability \( m_2 \) of the mass of \( m_1 \) is shown in Table 2. Then the moments given to propositions are listed in Table 3 and the unit of moment of the begin to forecast is seconds. Besides, modified \( m_1 \) after process with time interval are exhibited in Table 4 and the value of \( m_2 \) whose hesitation is reassigned is displayed in Table 5. In addition, the initial results of combination of TDQBF is enumerated in Table 6 and the degree of urgency of the incident is exhibited in Table 7. Finally, the comparison of final results using proposed method and traditional rule of combination in quantum field and in the form of classic probability assignment are listed respectively in Table 8 and Table 9.

**Table 1: Quantum evidences given by sensors**

| Evidences | Values of propositions |
|-----------|------------------------|
|           | \{R\}                  | \{S\}                  | \{D\}                  | \{SD\}                  |
| Evidence1 | \(0.7141e^{0.9294j}\) | \(0.33176e^{1.0206j}\)| \(0.4796e^{0.7399j}\)| \(0.3873e^{1.0965j}\) |
|           | \{SD\}                 | \{S\}                  | \{R\}                  | \{D\}                  |
| Evidence2 | \(0.6481e^{0.6594j}\) | \(0.4899e^{0.9376j}\) | \(0.5568e^{1.0414j}\)| \(0.1732e^{0.1023j}\) |
|           | \{D\}                  | \{R\}                  | \{SD\}                 | \{S\}                  |
| Evidence3 | \(0.3873e^{0.9872j}\) | \(0.6245e^{0.3164j}\) | \(0.1732e^{0.9260j}\)| \(0.6633e^{0.9643j}\) |
|           | \{D\}                  | \{SD\}                 | \{S\}                  | \{R\}                  |
| Evidence4 | \(0.7141e^{0.2656j}\) | \(0.2449e^{1.4837j}\) | \(0.4000e^{1.0661j}\)| \(0.5196e^{0.7514j}\) |
Table 2: The value of the two-dimensional judgment given by sensors

| Evidences | Values of propositions |
|-----------|------------------------|
|           | {Y} | {N} | {H} | {R} |
| Evidence1 | 0.5657e^{0.9738} | 0.4243e^{1.2265} | 0.6245e^{1.3738} | 0.3317e^{1.2518} |
|           | {Y} | {N} | {H} | {R} |
| Evidence2 | 0.4899e^{1.2874} | 0.5196e^{0.9994} | 0.5916e^{0.8465} | 0.3742e^{1.3030} |
|           | {Y} | {N} | {H} | {R} |
| Evidence3 | 0.4123e^{1.2475} | 0.3742e^{1.2780} | 0.7141e^{0.7366} | 0.4243e^{1.3231} |
|           | {Y} | {N} | {H} | {R} |
| Evidence4 | 0.5099e^{1.0785} | 0.4583e^{0.6236} | 0.5385e^{1.3802} | 0.4899e^{1.1747} |

Table 3: The value of moment of each proposition

| Evidences | The moment of occurrence |
|-----------|--------------------------|
|           | {R} | {S} | {D} | {SD} |
| Evidence1 | 3.1752 | 8.4246 | 40.6898 | 51.2317 |
|           | {SD} | {S} | {R} | {D} |
| Evidence2 | 10.2351 | 11.2819 | 15.3474 | 121.413 |
|           | {D} | {R} | {SD} | {S} |
| Evidence3 | 34.7341 | 90.2663 | 150.3377 | 157.4826 |
|           | {D} | {SD} | {S} | {R} |
| Evidence4 | 181.4964 | 197.2876 | 314.9728 | 528.7392 |
Table 4: The value of first-dimensional judgment processed by time quantitative rule

| Evidences     | Values of propositions |
|---------------|------------------------|
|               | {R}                    |
| Evidence_1    | {S}                    |
|               | {D}                    |
|               | {SD}                   |
| 0.850e^{0.9294j} | 0.1821e^{1.0206j} | 0.4710e^{0.7399j} | 0.1503e^{1.0965j} |
| (SD)          | {S}                    |
| Evidence_2    | {R}                    |
|               | {D}                    |
|               | {SD}                   |
| 0.7973e^{0.6594j} | 0.5890e^{0.9376j} | 0.1300e^{1.0414j} | 0.0202e^{0.1023j} |
| (D)           | {R}                    |
| Evidence_3    | {SD}                   |
|               | {S}                    |
|               | {R}                    |
| 0.6224e^{0.9872j} | 0.2394e^{0.3164j} | 0.1944e^{0.9260j} | 0.7193e^{0.9643j} |
| (D)           | {SD}                   |
| Evidence_4    | {S}                    |
|               | {R}                    |
| 0.9118e^{0.2656j} | 0.2456e^{1.4837j} | 0.2966e^{1.0661j} | 0.1425e^{0.7514j} |

Table 5: The value of two-dimensional judgment redistributed by DHDF and QPDR

| Evidences     | Values of propositions |
|---------------|------------------------|
|               | {Y}                    |
| Evidence_1    | {N}                    |
|               | {H}                    |
|               | {R}                    |
| 0.8739e^{1.1198j} | 0.6056e^{1.2735j} | 0.1169e^{1.3758j} | 0.3317e^{1.2518j} |
| (Y)           | {N}                    |
| Evidence_2    | {H}                    |
|               | {R}                    |
| 0.6971e^{1.1511j} | 0.7674e^{0.9498j} | 0.1199e^{0.8465j} | 0.3742e^{1.3030j} |
| (Y)           | {N}                    |
| Evidence_3    | {H}                    |
|               | {R}                    |
| 0.7276e^{1.0173j} | 0.6305e^{1.0474j} | 0.0951e^{0.7366j} | 0.4243e^{1.3231} |
| (Y)           | {N}                    |
| Evidence_4    | {H}                    |
|               | {R}                    |
| 0.7442e^{1.1752j} | 0.6150e^{0.8433j} | 0.1015e^{1.3802j} | 0.4899e^{1.1747j} |
Table 6: The initial combination of result of TDQBF

| Evidence | Values of propositions |
|----------|------------------------|
|          | {R}                    | {S}                  | {D}                  | {SD}                 |
| Evidence1| 0.8898e^{1.4871}       | 0.7403e^{1.4550}     | 0.8273e^{1.4456}     | 0.1314e^{2.2162}     |
|          | {R}                    | {S}                  | {D}                  | {SD}                 |
| Evidence2| 1.2856e^{1.2455}       | 1.1201e^{1.3439}     | 1.2049e^{1.3095}     | 0.5559e^{1.8105}     |
|          | {R}                    | {S}                  | {D}                  | {SD}                 |
| Evidence3| 0.7347e^{1.1842}       | 0.7877e^{1.2503}     | 0.7787e^{1.2455}     | 0.1415e^{1.9433}     |
|          | {R}                    | {S}                  | {D}                  | {SD}                 |
| Evidence4| 0.6409e^{1.1351}       | 0.5711e^{1.2676}     | 0.7200e^{1.4523}     | 0.1828e^{2.6589}     |

Table 7: The degree of urgency of each piece of evidence $U_e$ given by XG Rules

| Quantum evidence | Evidence1 | Evidence2 | Evidence3 | Evidence4 |
|------------------|-----------|-----------|-----------|-----------|
| Degree of ergency| $U_1^{3.1741}$ | $U_1^{1.3627}$ | $U_1^{1.0183}$ | $U_1^{2.7549}$ |

By comparing and analyzing the Table 9, clear conclusion can be obtained. Traditional OQFOD can process elementary predictive parsing. However, traditional frame does not take the closeness of degree of any two sequential propositions and the urgency of incidents into consideration which cause the outcomes of prediction which does not have higher accuracy and accords with the situation of true world. The quantum probability assignment of proposition $S$ told by proposed method goes down dramatically which manifests that proposed method exists a clearer judgment. Besides, the quantum probability assignment of proposition $D$ increases which illustrates that the development of this virtual currency is with a high possibility to decline under the consideration of all influence factors.
factors. In addition, the basic probability assignment of $SD$ and $RSD$ are so small that it can be ignored, which also illustrates that proposed method is extremely advantageous in eliminating uncertainty and provide explicit indicator of the true situation. The reason to observe such a big difference in the results obtained by the proposed and traditional method is that the value of quantum basic probability assignment of multiset proposition is allocated suitably to single subset proposition to reduce the uncertainty contained in $OQFDTI$. What’s more, the degree of hesitation of the dual judgment system is also distributed to the membership and non-membership in a similar way to reflect information exerted by multi-source sensors. Hence, the prominent positive performance is possessed by method proposed.
Example 5.1. Application of meteorological disaster prediction

How to make correct precise judgment to meteorological disaster for preventing a large amount of economic losses is still an urgent issue. Assume there exists a quantum frame of discernment about meteorological disaster which is denoted as $\Theta = \{H, D, S, HS, HDS\}$. The quantum evidences under the quantum frame of discernment are given in Table 10. Besides, the belief of propositions are listed in Table 11. And the moment given of each propositions of quantum evidences is shown in Table 12. Then the more precise data processed by Time quantitative rule is shown in Table 13 and 14 and the results combined using them by TDQBF are listed in Table 15. The quantum and classic probability form of comparison of final results are represented in Table 17 and 18 respectively.

Table 10: Quantum evidences given by sensors

| Evidence   | $\{H\}$     | $\{D\}$     | $\{S\}$     | $\{HS\}$   | $\{HDS\}$  |
|------------|--------------|--------------|--------------|-------------|-------------|
| Evidence 1 | 0.6083$e^{0.8916j}$ | 0.3742$e^{1.1948j}$ | 0.5292$e^{1.1009j}$ | 0.2828$e^{1.5301j}$ | 0.3606$e^{1.2776j}$ |
| Evidence 2 | 0.2000$e^{1.4842j}$ | 0.4123$e^{1.2508j}$ | 0.5099$e^{1.0976j}$ | 0.2646$e^{1.4625j}$ | 0.6782$e^{0.8703j}$ |
| Evidence 3 | 0.2449$e^{1.5099j}$ | 0.3317$e^{1.1739j}$ | 0.3742$e^{1.2794j}$ | 0.4123$e^{1.1101j}$ | 0.7211$e^{0.5107j}$ |
| Evidence 4 | 0.5831$e^{1.1545j}$ | 0.4243$e^{1.2250j}$ | 0.4359$e^{1.2319j}$ | 0.1414$e^{1.4310j}$ | 0.5196$e^{0.9987j}$ |
| Evidence 5 | 0.1414$e^{1.2899j}$ | 0.1732$e^{1.4951j}$ | 0.5657$e^{0.8162j}$ | 0.5916$e^{1.0703j}$ | 0.5292$e^{1.0643j}$ |
Table 11: The value of the two-dimensional judgment given by sensors

| Evidences  | Values of propositions |
|------------|------------------------|
|            | {Y} | {N} | {H} | {R}     |
| Evidence_1 | 0.6083 | 0.3317 | 0.5385 | 0.4796 |
| Evidence_2 | 0.6481 | 0.4243 | 0.3742 | 0.5099 |
| Evidence_3 | 0.4583 | 0.5831 | 0.5745 | 0.3464 |
| Evidence_4 | 0.7280 | 0.3606 | 0.4359 | 0.3873 |
| Evidence_5 | 0.7810 | 0.3873 | 0.4123 | 0.2646 |

Table 12: The value of moment of each proposition

| Evidences  | The moment of occurrence |
|------------|--------------------------|
|            | {H} | {D} | {S} | {HS} | {HDS} |
| Evidence_1 | 17.9022 | 58.4711 | 162.4293 | 174.1377 | 199.3269 |
| Evidence_2 | 41.3827 | 152.5734 | 244.3795 | 377.9305 | 445.1812 |
| Evidence_3 | 159.0479 | 177.2146 | 194.4576 | 247.5243 | 307.2954 |
| Evidence_4 | 1047.2412 | 1372.5814 | 1779.4607 | 1993.2938 | 2451.3926 |
| Evidence_5 | 1973.4528 | 1324.5271 | 1473.3968 | 1709.4353 | 1877.2964 |
Table 13: The value of first-dimensional judgment processed by time quantitative rule

| Evidence  | Values of propositions |
|-----------|------------------------|
|           | \{H\} | \{D\} | \{S\} | \{HS\} | \{HDS\} |
| Evidence1 | 0.8755, \(e^{0.8916}\) | 0.1996, \(e^{1.1949}\) | 0.1956, \(e^{1.1009}\) | 0.3077, \(e^{1.5301}\) | 0.246, \(e^{1.2276}\) |
| Evidence2 | 0.3673, \(e^{1.4842}\) | 0.3106, \(e^{1.2508}\) | 0.5373, \(e^{1.0976}\) | 0.2319, \(e^{1.4625}\) | 0.6528, \(e^{0.8703}\) |
| Evidence3 | 0.5026, \(e^{1.5099}\) | 0.4333, \(e^{1.1739}\) | 0.4595, \(e^{1.2794}\) | 0.2867, \(e^{1.1101}\) | 0.5160, \(e^{0.5107}\) |
| Evidence4 | 0.8280, \(e^{1.1545}\) | 0.3441, \(e^{1.2250}\) | 0.2416, \(e^{1.2319}\) | 0.1081, \(e^{1.4310}\) | 0.3550, \(e^{0.9987}\) |
| Evidence5 | 0.2694, \(e^{1.2899}\) | 0.1423, \(e^{1.4951}\) | 0.5506, \(e^{0.8162}\) | 0.4576, \(e^{1.0703}\) | 0.6282, \(e^{1.0643}\) |

Table 14: The value of two-dimensional judgment redistributed by DHDF and QPDR

| Evidence  | Values of propositions |
|-----------|------------------------|
|           | \{Y\} | \{N\} | \{H\} | \{R\} |
| Evidence1 | 0.9568, \(e^{0.9646}\) | 0.4368, \(e^{1.1873}\) | 0.0781, \(e^{1.1151}\) | 0.4796, \(e^{1.1314}\) |
| Evidence2 | 0.8529, \(e^{0.8879}\) | 0.5132, \(e^{1.1803}\) | 0.0753, \(e^{1.0659}\) | 0.5099, \(e^{0.9490}\) |
| Evidence3 | 0.6315, \(e^{1.0082}\) | 0.8645, \(e^{0.9727}\) | 0.1184, \(e^{0.9244}\) | 0.3464, \(e^{1.2263}\) |
| Evidence4 | 0.9945, \(e^{0.7614}\) | 0.4308, \(e^{1.2371}\) | 0.0753, \(e^{1.1018}\) | 0.3873, \(e^{1.0191}\) |
| Evidence5 | 1.0240, \(e^{0.7402}\) | 0.4525, \(e^{1.0058}\) | 0.0800, \(e^{1.1082}\) | 0.2646, \(e^{1.4800}\) |
Table 15: The initial combination of result of TDQBF

| Evidence | Values of propositions |
|----------|------------------------|
| Evidence1 | \{H\} 0.6325e^{1.7068}\j 0.9030e^{1.8523}\j 0.5260e^{2.3825}\j 0.1181e^{2.4090}\j |
|          | \{D\} 1.2655e^{1.7841} |
| Evidence2 | \{HDS\} 0.9338e^{1.7453}\j 1.2125e^{1.7084}\j 0.7212e^{2.0416}\j 0.1584e^{2.1998}\j |
|          | \{HS\} 1.3695e^{1.6299}\j |
| Evidence3 | \{S\} 0.9973e^{1.3551}\j 1.1733e^{1.3923}\j 0.4545e^{2.1567}\j 0.1501e^{2.4002}\j |
|          | \{HS\} 0.6765e^{1.6193}\j |
| Evidence4 | \{HDS\} 1.0344e^{1.6015}\j 1.3415e^{1.6284}\j 0.4480e^{2.0356}\j 0.1333e^{2.2441}\j |
|          | \{D\} 1.4469e^{1.5199}\j |
| Evidence5 | \{S\} 0.7779e^{1.8828}\j 1.2791e^{1.6005}\j 0.1662e^{2.5443}\j |

Table 16: The degree of urgency of each piece of evidence $U^e$ given by XG Rules

| Quantum evidence | Evidence1 | Evidence2 | Evidence3 | Evidence4 | Evidence5 |
|------------------|-----------|-----------|-----------|-----------|-----------|
| Degree of urgency| $U^{2.4342}$ | $U^{1.5325}$ | $U^{4.1342}$ | $U^{1.0537}$ | $U^{0.8321}$ |

The advantages of the method in this paper can be represented by taking belief of propositions, time interval and urgency of evidence into consideration in this paper. The data in Table 17 disposed by new method proposed in this paper can be analyzed that the probability of hail occurring occupies the biggest part of the whole probability distribution of meteorological disaster in this city, and then drought and dusk storms almost take up the rest of probability, at last it is difficult to observe the phenomenon of the occurrence of the propositions. And the data in Table 18 managed by traditional method can be read that hail is the most possible to occur and
Table 17: The comparison of results combined using proposed method and traditional rule of combination in quantum field

| Proposition | \{H\} | \{D\} | \{S\} | \{HS\} | \{HDS\} |
|-------------|------|------|------|------|------|
| The improved combined values | 0.7810$e^{-2.6678}$ | 0.0428$e^{-2.0078}$ | 0.6238$e^{-2.7406}$ | 0.0066$e^{-1.7475}$ | 0.0000$e^{-0.7723}$ |
| Combined values | 0.9320$e^{-2.1675}$ | 0.0664$e^{-1.1650}$ | 0.3536$e^{-1.1911}$ | 0.0426$e^{-1.0823}$ | 0.0025$e^{-0.9505}$ |

Table 18: The comparison of results combined using proposed method and traditional rule of combination in the form of classic probability assignment

| Proposition | \{H\} | \{D\} | \{S\} | \{HS\} | \{HDS\} |
|-------------|------|------|------|------|------|
| The improved combined values Values | 0.6100 | 0.0018 | 0.3882 | 0.0000 | 0.0000 |
| Combined values | 0.8686 | 0.0044 | 0.1252 | 0.0018 | 0.0000 |

almost nothing else happened. In the actual life, more than one meteorological disasters can occur in a similar period time. So, the prediction shown by traditional methods is against real life and the superiority of method mentioned in this paper can be fully demonstrated for the non-uniqueness of the possibly predictive disasters. The concrete preponderances can be discovered in Table 17 and 18.

6. Conclusion

In this paper, a completely new method in alleviating uncertainties in quantum information is proposed. To achieve the established goal, a dual check system and a rule revised for expert system are designed. The given applications proves that the proposed method successfully extracts truly useful information from quantum evidences. Compared with the traditional method, the proposed method reduces the degree of hesitancy and produce clear judgment on the current situations. All in all, the proposed
method offers a completely new vision on the uncertainty management of quantum information and can produce intuitive and reasonable results.

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