Emission Mechanisms in High Energy Pulsars: From Gamma Rays to Infrared

André R. Crusius–Wätzel and Harald Lesch
Institut für Astronomie und Astrophysik der Universität München, Scheinerstr. 1, 81679 München, Germany

Abstract. We present a model for the gamma-ray emission of pulsars in terms of curvature radiation by highly relativistic particles. It is shown that the injection of power-law primary particles from an outer gap and subsequent cooling by curvature radiation losses reproduces the high energy spectrum and the luminosity of pulsars. As a result a spectral break is to be expected at a photon energy of \( \sim 1 \) GeV. This value does not depend on the surface magnetic field strength or on the period of the neutron star, but only involves a geometric factor. The predicted change of the spectral index by 1 explains in particular the spectral shape of PSR B1706–44. We find that according to this model the luminosity of pulsars in the high energy band varies according to \( L \propto \gamma B_0^{5/3} \), which is in good agreement with the observations.

A model for the infrared, optical and soft X-ray emission of pulsar is also presented. It is based on anisotropic synchrotron emission by relativistic particles in an outer gap scenario with a single energy distribution \( N(\gamma) \propto \gamma^{-2} \). It is shown that this synchrotron model is able to reproduce the spectral shape between the infrared and soft X-rays and also the corresponding luminosities for the Crab pulsar. In particular, the long standing problem of the power-law energy distribution of primary particles can be understood as emission at very small pitch angles from low energy particles with \( \gamma \gtrsim 10 \). It is also shown that the scaling of the synchrotron model explains the observed correlation between the X-ray luminosity and the spin-down luminosity of the neutron star \( L_x \sim 10^{-3}E \).

1. Gamma-Ray Emission of Pulsars

There are now at least seven spin-powered pulsars known to be emitting pulsed gamma radiation. Detailed spectral information has been obtained by observations with the Compton Gamma Ray Observatory in the energy range of up to 10 GeV. Several of these pulsars have a break in the spectrum at about 1 – 2 GeV. This is also the energy where the gamma-ray pulsars emit most of the spectral power. The gamma ray luminosity is correlated to the square root of the spin-down power of these pulsars (with some scatter). For specific informations about the properties of individual gamma ray pulsars we refer to Thompson et al. (1996), Fierro et al. (1998), Thompson et al. (1999), and Kanbach (2001).

The models proposed for high-energy pulsars can be devised into polar cap models (e.g., Daugherty and Harding 1996; Sturmer and Dermer 1994, Usov and Melrose 1996) and outer gap models (e.g., Romani and Yadi-garoglu 1995; Romani 1996). In the first group the acceleration of particles and the gamma ray production is done in the open field line region above the magnetic pole of the neutron star. In the second group this takes place in the vacuum gaps between the neutral line and the last open field line in the outer magnetosphere, near the light cylinder. Depending on the distribution of the electric field parallel to the magnetic field along and across the gap, a power-law energy distribution of primary particles can be expected that emit gamma radiation. Here we discuss the effects of subsequent cooling by curvature radiation.

1.1. Primary Particles, and Particle Energy Spectrum

Particles entering the starvation gap will be accelerated by the strong electric field along the magnetic field. The accelerating electric field given in Cheng, Ho, and Ruderman (1986) can be rewritten as

\[
E_0 = \delta B ,
\]

where \( \delta \sim 0.3 \) is the ratio of the gap thickness to the light cylinder radius. The particles gain energy at a rate \( P_{\text{gain}} = cE_0 \). The energy of these primary particles is limited by curvature radiation losses, that depend on the bending of the field lines. The equilibrium between gains and losses determines the maximum energy. The curvature radius \( R_c \) of the last open field line is of the order of the light cylinder radius \( R_{lc} = c/\Omega \), with \( \Omega = 2\pi/P \), and increases towards the neutral line. This becomes even more important for a (nearly) perpendicular rotator. By setting \( R_c = kR_{lc} \), with \( k \sim 1 \), the power emitted by a single particle is then

\[
P_{ct} = \frac{2e^2c}{3R_c^2} \gamma^4 = \frac{2e^2}{3c} k^{-2} \Omega^2 \gamma^4 .
\]
The typical frequency emitted is
\[ \nu_c = \frac{3c}{4\pi R_e} \gamma^3 = \frac{3}{4\pi} k^{-1} \Omega \gamma^3 . \] (3)

Equating \( P_{\text{gain}} \) with \( P_{\text{loss}} = P_{\text{cr}} \), this results in an equilibrium Lorentz factor for the primaries of
\[ \gamma_{\text{max}} = 4.6 \times 10^7 (k\delta)^{1/2} \kappa^{-3/4} \left( \frac{B_{012}}{P_{-1}} \right)^{1/4} , \] (4)

where \( \kappa \sim 0.5 \) is the ratio of the inner starting radius of the gap to the light cylinder radius.

The emission of gamma ray photons by the primaries leads to a pair creation cascade. These pairs, if produced inside the gap, will also be accelerated, start to shield the electric potential in the gap. As their density increases and becomes of the order of the Goldreich-Julian density \( n_{\text{GJ}} = \Omega B/2\pi e c \) this finally closes the gap. The outward moving particles are no longer accelerated and only loose energy via curvature radiation.

The pairs generated outside the gap with a finite pitch angle due to aberration effects will emit synchrotron radiation that can be seen in the MeV, X-ray and optical bands (Romani, 1996; Crusius-Wätzel, Kunzl, and Lesch, 2001). The electric field available for the acceleration of particles depends on the details of the cascade process, which leads to a quasi stationary state where the electric field is a function of the location along and across the gap. Although attempts have been made to understand the cascade and outer gap closure, no solid model exists. Therefore we just assume a power-law energy distribution of the accelerated particles,
\[ Q(\gamma) \propto \gamma^{-s}, \quad \gamma_{\text{min}} < \gamma < \gamma_{\text{max}} , \] (5)

with an index \( s \), that must be determined from the observations, rather than from theory in the light of the considerations given above. Since \( \nu \propto \gamma^3 \) the ratio \( \gamma_{\text{max}}/\gamma_{\text{min}} \) needs to be only about 5 to cover a frequency range of more than two decades.

1.2. Curvature Radiation Cooling

The timescale for curvature cooling of relativistic electrons or positrons near the light cylinder of the pulsar magnetosphere is given by
\[ \tau = 4.1 \times 10^{-2} \kappa^2 P_{-1}^{-1} \gamma^{-3} s , \] (6)

where \( \gamma = P_{\text{cr}}/mc^2 \) and equation (2) has been used. We estimate the Lorentz factor \( \gamma_b \) above which we expect a spectral break by setting the cooling timescale equal to the time needed for the particle to move out of the magnetosphere into the wind region, \( t_{\text{esc}} = f R_{\text{lc}}/c = f/\Omega \), where \( f \sim 0.1 \) is the fraction of the light cylinder radius, corresponding to the outer gap region in which the particles are not accelerated anymore but only radiate. From this consideration we find that cooling will be important for particles with \( \gamma > \gamma_b \), where
\[ \gamma_b = \left( \frac{3mc^3k^2}{2e^2f\Omega} \right)^{1/3} = 2.9 \times 10^7 k^{2/3} f_{-1}^{-1/3} P_{-1}^{1/3} . \] (7)

The frequency at which the spectral break occurs is then
\[ \nu_b = \frac{3\Omega}{4\pi k} \gamma_b^3 = \frac{9mc^4}{8\pi e^2} \frac{k}{f} , \] (8)

which corresponds to a frequency and photon energy of \( \nu_b = 3.8 \times 10^{23} (k/f_{-1}) \text{Hz} \) and \( E_b = h\nu_b = 1.6 \text{GeV} \), respectively, depending only on the geometrical factors \( k \) and \( f \). In particular, the break energy is independent of the magnetic field strength and period.

1.3. Gamma-Ray Spectrum

For frequencies below the break \( \nu_b \) the spectrum can be calculated from the particle energy distribution according to equation (9). By using the monochromatic approximation for the spectral power of a single lepton, \( P_{\nu}(\gamma) = P_{\nu}(\gamma)\delta[\nu - \nu(\gamma)] \), it follows that
\[ I_{\nu} \propto \int d\gamma Q(\gamma)P_{\nu}(\gamma) , \] (10)

\[ \propto \int d\gamma \gamma^{-s} \left| \frac{d(\nu - \nu_c)}{d\gamma} \right|^{-1} \delta(\gamma - \gamma_0) , \]

where \( \gamma_0 \propto \nu^{1/3} \) is the Lorentz factor at which the argument of the \( \delta \)-function becomes zero. We then find that
\[ I_{\nu} \propto \nu^{-(s-2)/3} \left( \nu < \nu_b \right) . \] (11)

This corresponds to a photon energy index of \( \phi(\nu < \nu_b) = -(s+1)/3 \), where \( N_{\nu} \propto E^{\phi} \). The Crab, Geminga, and Vela pulsar have photon indices -2, -1.4, and -1.6, respectively, below the spectral break (Fierro, et al., 1998). This then gives the particle energy indices \( s \) of the primaries below the break energy as 5, 3.2, and 3.8, respectively.

For frequencies above the break we first have to find the distribution function modified by radiation losses. Assuming that a spectrum \( Q(\gamma) \) is injected at a rate
\[ Q(\gamma) = Q(\gamma)/t_{\text{esc}} , \]

the cooled spectrum is given by the following expression
\[ N(\gamma) = \frac{1}{|s|} \int_{\gamma}^{\gamma_{\text{max}}} d\gamma' \dot{Q}(\gamma') , \] (12)

from which we find \( N(\gamma) \propto \gamma^{-(s+3)} \). By using the monochromatic approximation again, we finally get
\[ I_{\nu} \propto \nu^{-(s+1)/3} \left( \nu > \nu_b \right) , \] (13)
corresponding to a photon index of \( \phi(\nu > \nu_0) = -(s + 4)/3 \). Therefore the spectral index of the gamma radiation changes by

\[
\Delta \phi = -1
\]

at the break frequency (eqn. 8) in the direction of increasing frequencies.

The pulsar PSR B1706–44 shows exactly this behavior (Thompson et al., 1996). The gamma ray photon spectrum changes by \( \Delta \phi = -1 \), from \( \phi = -1.25 \) below 1 GeV to \( \phi = -2.25 \) above 1 GeV, with the spectrum extending for more than a decade in energy, both, below and above the break. From this we find \( s = 2.75 \) for this pulsar.

The emitted power per logarithmic frequency band scales as \( \nu I_\nu \propto \nu^{(5-s)/3} \) below the break and \( \nu I_\nu \propto \nu^{(2-s)/3} \) above the break. For \( 2 < s < 5 \) the gamma ray energy distribution thus is rising below and is falling above the break energy. The maximum power is then emitted at the break energy.

The highest frequencies are produced by the primary particles that feel the vacuum potential. The spectrum of the radiation emitted by them cuts off exponentially above

\[
\nu_{\text{max}} = \frac{3 \Omega \gamma^3}{4 \pi k} \propto \frac{1}{\nu_0}
\]

The change in the spectrum is dominated by cooling only if the exponential cutoff is at higher energies than the break, \( \nu_{\text{max}} > \nu_0 \). The cooled spectrum is nicely seen in PSR B1706–44, where the cutoff must be above 10 GeV. The situation is less clear for the other gamma ray pulsars, where the energy range between the break and the cutoff is either smaller or not existent. It can be expected that with the GLAST mission more pulsars with a clear break will be found and also that a smaller range of the cooled spectrum can be detected.

### 1.4. Luminosity Estimates for the Gamma-Ray Emission

We now estimate the power \( L_\gamma \) emitted by pulsars in the gamma-ray region at the break frequency very roughly as the product of the volume, the Goldreich-Julian number density near the light cylinder \( n_{\text{GJ}} \propto B_0/P^4 \), and the radiation power of a single particle \( P_{\text{cr}} \propto \gamma^4/P^2 \),

\[
L_\gamma = V n_{\text{GJ}} P_{\text{cr}} ,
\]

where the volume is assumed to be a shell near the light cylinder with \( V = 4\pi f R_{lc}^3 \propto P^3 \). From this we find

\[
L_\gamma = 4.2 \times 10^{34} \frac{k^{2/3} B_{0,12}}{\kappa^{1/3} P_{-1}^{2/3}} \text{ erg s}^{-1} ,
\]

where the break Lorentz factor from equation (8) has been used in the curvature radiation power (2). Since the spin-down luminosity \( \dot{E} \) of a pulsar is proportional to \( B_0^2/P^4 \), the gamma luminosity thus scales according to \( L_\gamma \propto \dot{E}^{1/2} P^{1/3} \). The observed high-energy luminosities of the seven gamma ray pulsars indicate a trend \( L_\gamma \propto \dot{E}^{1/2} \), and the small additional dependence on \( P^{1/3} \) is still in agreement with this correlation. If we instead insert the maximum Lorentz factor (4) into the power formula, the result is

\[
L_\gamma = 7.9 \times 10^{34} \frac{k^{2/3} f_{-1} B_{0,12}^2}{\kappa^6 P_{1}^{4/3}} \text{ erg s}^{-1} ,
\]

and is thus linearly dependent on \( \dot{E} \).

In Figures (1) and (2) we compare the model results with the data points. The latter have been taken from Thompson et al. (1999). These observed luminosities are integrated fluxes above 1 eV. It is clearly seen in Figure (1) that if the maximum of the luminosity is determined by the break, a reasonable correlation is found, not only for the slope but also for the absolute values. However, the observed luminosity of Geminga is smaller by about a factor of 5, which might be due to the fact that this pulsar emits 20% of its spin-down power in gamma rays. This means that the model curve would give an efficiency of 100%, which then causes a significant nonlinear back-reaction on the rotation of the pulsar and on the formation of outer gaps, with the effect that the gamma luminosity of Geminga is reduced below the expectation from the linear model. Another point worth to note is that the Crab pulsar emits it maximum spectral power at \( \sim 100 \) keV.
Therefore the data point of the Crab pulsar that gives the luminosity in the GeV range is actually somewhat lower than plotted here. With these remarks in mind our simple model is in good agreement with the data. In Figure (2) the luminosity is plotted versus $B_0^3/P^4$. The solid line represents the dependence $\log L_\gamma = 20 + 0.5 \log (B_0^3/P^4)$, and the dotted line is the relation found, if the peak luminosity would be reached at the cutoff energy. The latter clearly overpredicts the observed values, especially for the high $P$ – low $P$ pulsars, since the slope of the curve is too steep.

2. Synchrotron Model for the Infrared, Optical and X-Ray Emission of the Crab pulsar

The Crab pulsar has a continuous spectrum from the optical to X-rays and $\gamma$-rays with different power-laws (Lyne & Graham-Smith 1990). The spectral index $\alpha$, defined as $I_\nu \propto \nu^{-\alpha}$, varies from $\alpha = 1.1$ in the $\gamma$-region via 0.7 in the hard X-ray region to 0.5 at soft X-rays (Toor & Seward 1977) and zero at optical frequencies (Percival et al. 1993). In the far infrared region the spectrum is inverted $\alpha = -2$ and cuts off sharply towards lower frequencies. This behavior is not accompanied by dramatic pulse profile changes, as one would expect from saturation or self-absorption effects. Self-absorption should first influence the peak intensity. The infrared spectrum measured by Middleditch, PENNYPACKER, & BURNS (1983) is still not explained. In the following it is shown that the theory of optically thin synchrotron radiation at very small pitch angles gives a possible solution to this problem.

2.1. Synchrotron Emission at Small Pitch Angles

The emissivity at very low pitch angles ($\Psi < 1/\gamma$) of a single particle is given by (Epstein 1973)

$$\epsilon_\nu(\theta, \gamma) = \frac{\pi e^2 \gamma \Psi^2 \nu^3}{\nu_B c} \left[ 1 - \frac{\nu}{\gamma \nu_B} + \frac{\nu^2}{2 \gamma^2 \nu_B^2} \right] \times \delta \left( \nu - \frac{2 \gamma \nu_B}{1 + \theta^2 \gamma^2} \right)$$

with $\nu_B = eB/2\pi mc = 2.80 \times 10^{12} B_6 \text{Hz}$. This formula has to be applied when angles $\theta < 1/\gamma$ are resolved in the observations, i.e. when the pulse width $\Delta \phi$ becomes comparable to the emission cone angle of the particles, $1/\gamma > 2\pi \Delta \phi$. Since the maximum of the emission is in the forward direction ($\theta = 0$) the emission is dominated by the field lines that point towards the observer at each phase of the pulse. When the typical angle of emission $\theta$ is small compared to the pulse width it is useful to integrate the emissivity over all angles (averaging):

$$\epsilon_\nu(\gamma) = 2\pi \int \epsilon_\nu \sin \theta \, d\theta = \frac{2\pi^2 e^2 \Psi^2}{c} \nu \left[ 1 - \frac{\nu}{\gamma \nu_B} + \frac{\nu^2}{2 \gamma^2 \nu_B^2} \right].$$

2.2. Infrared Spectrum

The emission from a power law distribution of relativistic particles, $N(\gamma) = N_0 \gamma^{-s}$, in the direction along the magnetic field $\theta = 0$ is given by

$$I_\nu \propto \int \epsilon_\nu(0, \gamma) N(\gamma) \, d\gamma \propto \frac{\pi e^2 \Psi^2 N_0 \nu^3}{2\nu_B^2 c} \int \delta \left( \gamma - \frac{\nu}{2\nu_B} \right) \gamma^{1-s} \, d\gamma$$

so that

$$I_\nu \propto \frac{4N_0 \pi e^2 \Psi^2 \nu_B}{c} \left( \frac{\nu}{2\nu_B} \right)^{4-s}.$$
approximation is used:
\[ \varepsilon_{\nu} \approx -\frac{dE}{d\nu} \delta(\nu - 2\gamma \nu_B). \] (23)

This is a good approximation since the emission is sharply peaked at \( 2\gamma \nu_B \) and one may just put all the emission at this frequency. The result for the optical emission is then
\[ I_\nu \propto \int \varepsilon_\nu N(\gamma) \, d\gamma = \frac{4\pi^2c^2N_{\text{gles}} B^2}{3c} \left( \frac{\nu}{2\nu_B} \right)^{2-s}. \] (24)

In the case of \( s = 2 \) this gives the observed flat, \( \alpha \approx 0 \), optical/UV spectrum of the Crab pulsar, with \( \gamma \sim 10^2 \).

2.4. Soft X-ray Spectrum

Soft X-ray emission in this model comes from the particles with larger pitch angles (\( \Psi \gg 1/\gamma \)), radiating at a typical frequency \( \nu_c = \frac{2}{3} \nu_B \gamma^2 \). The spectrum in this large pitch angle limit is given by
\[ I_\nu \propto \nu^{-(s-1)/2}. \] This yields the observed \( I_\nu \propto \nu^{-0.5} \) at soft X-rays for \( s = 2 \).

2.5. Luminosity Estimates for the Optical and X-Ray Emission

The energy radiated by the pulsar in the optical and in the X-ray band can be estimated simply as the number of particles times the energy radiated by one particle \( I \propto \nu^{-(s-1)/2} \). This yields the observed \( I_\nu \propto \nu^{-0.5} \) at soft X-rays for \( s = 2 \).

3. Discussion and Conclusions

We have found that curvature radiation cooling results in a universal spectral break energy in the photon spectrum at \( \sim 1 \) GeV in gamma ray pulsars. This break depends only on geometric factors, but does not depend on other pulsar parameters as the magnetic field strength or the rotation period. The maximum power is emitted at this energy. In fact in several of the gamma-ray pulsars a spectral break or cut-off can be observed at this energy. A photon spectral index change of \( \Delta \phi = -1 \) is expected, when going from lower to higher energies. This is clearly seen in PSR B1706–44 (Thompson et al., 1996) and we expect that during the GLAST mission more pulsars with this spectral feature will be detected.

The high energy luminosity calculated with cooling is proportional to \( B_0/P^{5/3} \), which compares well with the observed values. Romani (1996) has obtained cutoff energies from a curvature radiation model without cooling that also lie in the observed range, but they depend on the pulsar parameters \( B_0 \) and \( P \), since they are connected to the maximum energy of the primaries. Using the cut-off particle energies for the luminosity estimate we find that the power output of pulsars in high energy photons scales as \( \propto B_0^{2/3}P^{-2} \), i.e. as a constant fraction of the spin-down power \( \dot{E} \), which overpredicts the \( \gamma \)-ray luminosity for young pulsars. But note that in deriving this result it was assumed that the density of the emitting particles is of the order of the Goldreich-Julian density, which might not be true in general. Harding (1981) estimated the gamma ray luminosity from a polar cap model by assuming that the power put into particles is completely converted into radiation and found a scaling \( \propto B_0^{-2} \). In the outer gap model with a power-law energy distribution of relativistic particles (due to gap closure), the “luminosity” in particles is dominated by the density at the lowest energy. On the other hand the maximum spectral power is dominated by the high energy particles with \( \gamma = \gamma_b \) or \( \gamma = \gamma_{\text{max}} \). So it is not immediately clear what fraction of the particle luminosity is converted into radiation.

According to Kanbach (2001) the observed luminosity trend \( L_\gamma \propto E^{1/2} \) can be caused either by a proportionality to the number of particles, which would all be accelerated to the same energy, or by the same particle flow from different pulsars with correspondingly different acceleration energies. In our opinion the reality is somewhere in between, depending on the details of the pair cascade and the gap closure. Only when the energy (depending on the locally available electric field and curvature radius) and the density of the relativistic particles, including its spatial dependence is known, we can get more insight into the spectrum and the luminosity of gamma-ray pulsars.

Synchrotron emission of particles with small pitch angles with a single power-law energy distribution, \( N(\gamma) \propto \gamma^{-2} \) reproduces the key properties of the Crab pulsar spec-
trum and the luminosity from the infrared up to soft X-rays.

Acknowledgements. This work was supported by the Deutsche Forschungsgemeinschaft with the grant LE 1039/6 (C.-W.).

References

Becker, W., Trümper, J. 1997, A&A, 326, 682
Crusius-Wätzel, A.R., Kunzl, T., Lesch, H. 2001, ApJ, 546, 401
Cheng, K.S., Ho, C., Ruderman, M. 1986, ApJ, 300, 500
Daugherty, J.K., Harding A.K. 1996, ApJ, 458, 278
Epstein, R.I. 1973, ApJ, 183, 593
Fierro, J.M. et al. 1998, ApJ, 494, 743
Harding, A.K. 1981, ApJ, 245, 267
Kanbach, G. 2001, The Universe in Gamma Rays, V. Schönfelder, Berlin: Springer, 127
Lyne, A.G., Graham-Smith, F. 1990, Pulsar Astronomy (Cambridge University Press)
Middleditch, J., Pennypacker, C., Burns, M.S. 1983, ApJ, 273, 261
Percival, J.W., et al. 1993, ApJ, 407, 276
Romani, R.W. 1996, ApJ, 470, 469
Romani, R.W., Yadigaroglu, I.-A. 1995, ApJ, 438, 314
Sturner, S.J., Dermer, C.D. 1994, ApJ, 420, L79
Thompson, D.J. et al. 1996, ApJ, 465, 385
Thompson, D.J. et al. 1999, ApJ, 516, 279
Toor, A., Seward, F.D. 1977, ApJ, 216, 560
Usov, V.V., Melrose, D.B. 1996, ApJ, 464, 306