**Plasmon Spectroscopy of Fluctuations in Superconductors**

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We propose to unleash an optical spectroscopy technique to monitor the superconductivity and properties of superconductors in the fluctuating regime. This technique is operational close to the plasmon resonance frequency of the material, and it intimately connects with the superconducting fluctuations slightly above the critical temperature \(T_c\). We find the Aslamazov-Larkin corrections to AC linear and DC nonlinear electric current in a generic two-dimensional system exposed to an external longitudinal electromagnetic field. First, we study the plasmon resonance of normal electrons near \(T_c\), taking into account their interaction with superconducting fluctuations, and show that fluctuating Cooper pairs reveal an additional mechanism of plasmon damping, which surpasses the electron-impurity scattering. Second, we demonstrate the emergence of a drag effect of superconducting fluctuations by the external field resulting in considerable, experimentally measurable corrections to the electric current in the vicinity of the plasmon resonance.

**Introduction.**—The study of fluctuating phenomena in superconductors is a wide field of modern research \(^1\)–\(^3\). At the temperatures approaching the critical point of the phase transition \(T_c\) from above, there start to emerge (and collapse) Cooper pairs even before the system reaches \(T_c\). It results in fluctuations of the Cooper pairs density, which might sufficiently modify the conductivity of the system. This effect is especially pronounced in samples of reduced dimensionality, as Aslamazov and Larkin (AL) reported in their pioneering work \(^4\). Later their theory was developed further to study high-frequency phenomena in superconductors in the fluctuating regime \(^5, 6\) and the fluctuating corrections in linear transport phenomena in superconductors, such as the Hall effect \(^3\), thermoelectric phenomena \(^7\), and the critical viscosity of electron gas \(^8\). Optical methods to superconductors are still lacking due to the generally weak interaction of superconductors with light.

In this Letter, we demonstrate that it is possible to monitor and manipulate transport of carriers of charge in superconductors using external electromagnetic (EM) waves interacting with the superconducting fluctuations (SFs). We develop a theory of linear AC and second-order DC response of a two-dimensional (2D) electron gas (2DEG) in the vicinity of the plasmon resonance and \(T_c\), where the SFs play an essential role. As a first step, we study the plasmon oscillations of normal electrons in the presence of the gas of fluctuating Copper pairs. Second, we find the fluctuating corrections to the drag effect, which consists in the emergence of a stationary electric current as the second-order response to an external alternating EM perturbation of the system \(^9\).

The most general and powerful approach to the description of fluctuations in superconductors above the transition point \(T_c\) relies on rather cumbersome methods of quantum field theory. However, as it was first pointed out by AL, it is often sufficient to use a considerably simpler approach based on the Boltzmann kinetic equations. Despite its simplicity, this approach has proved to be efficient and sufficient to study the fluctuating corrections to the Hall effect, the magnetoconductivity, high-frequency phenomena, and the transport in alternating EM fields of high intensity \(^10, 11\).

We will use the Boltzmann equations to calculate the AL corrections in the response of a 2DEG to an external longitudinal EM field \(E(r, t) = (E(r, t), 0)\), with \(E(r, t) = E_0 \cos(ikx - \omega t)\), which directs along the plane of the quantum well (xy plane), containing the electron gas. Such a setup arises (i) when studying acoustooptoelectric effects in two-dimensional systems \(^12\)–\(^13\), (ii) in photo-induced transport in two-dimensional systems (e.g., the photon drag effect) \(^14\)–\(^15\), (iii) when plasma waves are excited \(^16\), and also (iv) in ratchet effects in two-dimensional systems \(^20\)–\(^24\). In particular, it has recently been shown, that a photoinduced ratchet current can be sufficiently enhanced in the vicinity of the plasmon resonance \(^25\). This finding and the details of the approach used in Ref. \(^25\) let us hypothesize that there might be many phenomena, which become enhanced near the plasmon resonance. In this Letter, we will pay special attention to this region of frequencies when studying the behavior of fluctuating corrections to the drag electric current.

In the first part of the text, we develop the plasmon response of normal 2D electrons accounting for their interaction with fluctuating Cooper pairs and analyse the influence of the AL corrections to the plasmon resonance position and the plasmon damping. The second part is devoted to the theory of drag effect of SFs due to the longitudinal EM field with the frequency close to the plas-
Plasmon resonance of normal electrons. It should be noted, that while exciting plasmons, the internal induced long-range Coulomb fields activate. They act both on the normal electrons and the fluctuating Cooper pairs. In other words, the interaction between normal electrons and fluctuating Cooper pairs cannot be disregarded, as it is usually done when considering static and dynamic corrections to the Drude conductivity due to the presence of an external uniform EM field in superconductors above $T_C$.

Plasmon resonance of 2DEG in the presence of SFs.— A standard theoretical approach to study the plasmon physics in many-component electron [26, 27] or electron-hole [28] systems starts with the consideration of the dielectric function of the 2DEG $\varepsilon(k, \omega)$ accounting for its spatial and temporal dispersion. We will follow this approach taking into account the presence of SFs and the interaction between electrons and Cooper pairs. Thus, we consider the system as a mixture of two gases, one is the degenerate gas of normal-state electrons and the other is the Bose gas of fluctuating Cooper pairs.

In the absence of external perturbations, the Cooper pairs obey the classical Rayleigh-Jeans distribution $f_0(p) = T/\varepsilon_p$, where $\varepsilon_p = \alpha T_c (\varepsilon + \xi_p^2 p^2)$ is the Cooper pair energy with $\varepsilon = (T - T_c)/T_c > 0$ the reduced temperature [3]. The parameter $\alpha$ is fixed by the relation $4\alpha T_c \xi^2 = 1$, where $m$ is an electron effective mass. The coherence length $\xi$ in 2D samples has different definitions for the cases of clean $T\tau \gg 1$ and disordered $T\tau \ll 1$ regimes, where $\tau$ is a relaxation time of a normal electron. Both the regimes are sewn in the general expression

$$\xi^2 = -\frac{v_F^2 \tau^2}{2} \left[ \psi \left( \frac{1}{2} + \frac{1}{4\pi T\tau} \right) - \psi \left( \frac{1}{2} \right) - \psi' \left( \frac{1}{2} + \frac{1}{4\pi T\tau} \right) \right]$$

where $\psi(x)$ is the digamma function and $v_F$ is the Fermi velocity. The Cooper pair energy can be written in the form $\varepsilon_p = p^2/4m + \mu$. It depends on equilibrium electron density $n$ via its “binding” energy term $\mu = \alpha T_c \varepsilon$ due to the relations [1] and $v_F^2 = 4\pi \varepsilon n/m^2$.

The internal induced electric field due to the fluctuations of the charge (electron and Cooper pairs) densities $E^i(k, \omega)$ can be found from the Poisson equation in the quasistatic limit, when we can neglect the retardation effects. Assuming that $z$ axis is directed across the 2D system, which is located on a substrate ($z < 0$) with a dielectric constant $\kappa$ (Fig. 1), and using the ansatz $\exp(ikx - i\omega t)$ for all the time- and position-dependent quantities, we find the Poisson equation for the scalar potential $\varphi(z)$ of the induced field in the form

$$\frac{\partial}{\partial z} \kappa(z) \frac{\partial}{\partial z} \varphi(z) = -4\pi (\rho_{kw} + \theta_{kw}) \delta(z),$$

where $\kappa(z) = 1$ for $z > 0$ and $\kappa(z) = \kappa$ for $z < 0$. The functions $\rho_{kw}$ and $\theta_{kw}$ in Eq. (2) are Fourier-transforms of charge densities due to the normal electrons and fluctuating Cooper pairs, respectively. To solve Eq. (2) we use its Green’s function $g(z - z') = -[(\kappa + 1)k]^{-1} \exp(-k|z - z'|)$ and find the induced potential

$$\varphi(z) = \frac{4\pi}{(\kappa + 1)k} e^{-k|z|} (\rho_{kw} + \theta_{kw}).$$

Furthermore, using the continuity equation for both the components of the charge density and expressing the currents via conductivities, we come down to the system of equations

$$\rho_{kw} = -\frac{i k^2 \sigma_{kw}^D}{\omega} \varphi(0),$$

$$\theta_{kw} = -\frac{i k^2 \sigma_{kw}^{AL}}{\omega} \varphi(0),$$

where $\sigma_{kw}^D$ is a Drude conductivity, whereas $\sigma_{kw}^{AL}$ is the Aslamazov-Larkin conductivity of fluctuating Cooper pairs. The determinant of the system (4) gives the dispersion relation of collective modes

$$\varepsilon(k, \omega) = 1 + i \frac{4\pi k}{(\kappa + 1)\omega} \left( \sigma_{kw}^D + \sigma_{kw}^{AL} \right) = 0.$$
be solved by the iteration procedure and it allows us to calculate the correction to the plasmon dispersion,

$$\frac{\delta \omega_p}{\omega_p} = \frac{1}{2} \text{Re} \left( \frac{\sigma^{AL}_\omega}{\sigma^{D}_\omega} \right) \omega=\omega_p,$$

and its damping (additional to the conventional damping due to the electron-impurity scattering, $\Gamma^0 = -1/2\tau$),

$$\frac{\delta \Gamma_p}{\omega_p} = \frac{1}{2} \text{Im} \left( \frac{\sigma^{AL}_\omega}{\sigma^{D}_\omega} \right) \omega=\omega_p. \quad (8)$$

Thus one can estimate the shift of the plasmon frequency and damping due to the interaction of normal electrons with fluctuating Cooper pairs. To observe the plasmon resonance the following inequality $\omega_p\tau \gg 1$ should be fulfilled. Thus, we assume that $\omega \tau \gg 1$ in Drude conductivity expression in Eq. (6). As concerns the AL conductivity, in principle, one may have $\omega \tau_p \ll 1$ or $\omega \tau_p \gg 1$. Let us consider both the regimes.

In the limit $\omega \tau_p \ll 1$ we find

$$\frac{\delta \omega_p}{\omega_p} \approx \frac{1}{2} \frac{\sigma^{AL}_\omega}{\sigma^{D}_\omega}, \quad \frac{\delta \Gamma_p}{\omega_p} \approx -\omega_p \frac{\sigma^{AL}_\omega}{\sigma^{D}_\omega}, \quad (9)$$

where $\sigma^{AL}_\omega = e^2/16c$ and $\sigma^{D}_\omega = e^2 n \tau/m$ are corresponding conductivities in the static limit. Thus, the plasmon shift is rather small due to the smallness of the ratio $\sigma^{AL}/\sigma^{D}$. In contrast, the plasmon damping due to the SFs in the system, the drag current of normal electrons as a nonlinear response of the system to the external EM perturbation in the simple case of longitudinal EM waves reads [29] (see Supplemental Material [32] for the details of derivations)

$$\tilde{f}(\omega) = \frac{k}{2c\omega n} \left| \frac{\sigma^D_{\omega} E_0}{\varepsilon(k,\omega)} \right|^2, \quad \text{where} \quad \sigma^D_{\omega} = \frac{\sigma^{D}_\omega}{1 - i\omega \tau}. \quad (10)$$

The presence of the function $\varepsilon(k,\omega)$ in the denominator here reflects the screening of the external field by the carriers of charge. It should be noted, that in the presence of the SFs in the system, the drag current of normal electrons is affected by them at plasmon frequencies via their contribution to the dielectric function $\varepsilon(k,\omega)$, as becomes evident from Eq. (5).

To derive the drag current of fluctuating Cooper pairs, we use the Boltzmann equation

$$\frac{df}{dt} + u \partial f + 2e \left[ E(r,t) + E^*(r,t) \right] \partial_p f = -\frac{f - \langle f \rangle}{\tau_p}. \quad (11)$$

To simplify the formulas, we will omit the induced field $E^*(r,t)$ in the derivations. Later its influence will be restored by the replacement of the EM field amplitude $E_0$ by $E_0/\varepsilon(k,\omega)$ [33].

To find the SFs distribution function, we assume that a weak external probe field causes small perturbation over the homogeneous case, and thus we expand the distribution function of SFs $f$ and the normal electron density $N$ in powers of external field [16]: $f = f_0 + f_1 + f_2 + O(f_3)$, $N = n_1 + n_2 + O(n_3)$, and $\langle f \rangle = f_0 + \partial_n f_0(n_1 + n_2) + \partial^2_{n^2} f_0(n_1 + n_2)^2/2$. The latter expansion holds since the equilibrium distribution of fluctuating Cooper pairs depends on the density of normal electrons, as it has been mentioned above, after Eq. (1).

Furthermore, due to the dependence of the Cooper pairs lifetime $\tau_p$ on normal electron density, it also expands as $\tau_{p}^{-1} + \partial_n \tau_{p}^{-1}(n_1 + n_2 + O(n_3))$.
Decomposing the first-order corrections as plane waves, \( f_1(\mathbf{r}, t) = \frac{1}{2} \left[ f_1 \exp(i k x - i \omega t) + f_1^* \exp(-i k x + i \omega t) \right] \), and combining all the first-order terms in the Boltzmann equation \([16]\), we find the first-order correction to the distribution function of fluctuating Cooper pairs,

\[
f_1 = -\frac{2 e \tau_p \mathbf{E}_0 \cdot \partial_p f_0 + n_1 \partial_t f_0}{1 - i(\omega - k \cdot \mathbf{u}) \tau_p}.
\]

Obviously, \( f_1 \) is determined not only by the direct action of the external EM field (the term \( \mathbf{E}_0 \cdot \partial_p f_0 \)), but also by the normal electron density fluctuations directly (\( n_1 \)-containing term). To find the expression for the first-order correction \( n_1 \) to normal electron density, one can use the continuity equation, \( n_1 = \frac{\partial \rho_D}{\partial t} \).

Onwards, we consider the second-order terms in Eq. \([16]\) and find

\[
\text{Re} \left[ \mathbf{E}_0 \frac{\partial f_1}{\partial \mathbf{p}} \right] = -\frac{1}{\tau_p} \left( f_2 - \pi_2 \right) - \frac{n_1 n_1^*}{2} \frac{\partial^2 f_0}{\partial \mathbf{p}^2} - \frac{1}{\partial t} \left( f_1 - n_1 \frac{\partial f_0}{\partial t} \right) \frac{n_1^*}{2},
\]

where the bar sign stands for the time averaging. This equation determines the stationary part of the second-order correction \( f_2 \), which determines the drag current

\[
j^{AL} = (2e) \int \frac{d \mathbf{p}}{(2\pi)^2} u_x f_2.
\]

Due to the integration over the angle in this expression (while taking the 2D integral over \( d \mathbf{p} \)), all the terms in Eq. \([18]\) containing the derivative(s) of \( f_0 \) over \( n \) do not contribute to the current \([19]\). The remaining terms give the final expression for the second-order correction to the distribution function,

\[
f_2 = -e \tau_p \text{Re} \left[ \mathbf{E}_0 \frac{\partial f_1}{\partial \mathbf{p}} - \frac{\tau_p}{2} \frac{\partial \tau_p^{-1}}{\partial n} \text{Re} (f_1 n_1^*) \right].
\]

Using \( f_1 \) from Eq. \([17]\) and an explicit expression for \( n_1 \), and substituting them into Eq. \([18]\), we can find the drag current density, which consists of four contributions (see \([32]\) for the full formula and derivations). Keeping only significant terms and restoring \( \varepsilon(k, \omega) \) to account for the induced field (screening the external perturbation), we find

\[
j^{AL} = \frac{k}{2e \alpha n} \frac{\sigma_0^{AL} \sigma_0^D}{\varepsilon(k, \omega)} G(\beta_\omega),
\]

where \( \beta_\omega = \pi \omega/16 T_c \varepsilon \) and

\[
G(\beta_\omega) = \frac{1}{\beta_\omega} \left[ 2 \beta_\omega \frac{\beta_\omega}{\omega} \tau - (\beta_\omega + 2 \omega \tau) \arctan(\beta_\omega) \right] + (2 \beta_\omega - \beta_\omega^2 \omega \tau + 2 \omega \tau) \ln(1 + \beta_\omega^2).
\]

Formula \([21]\) represents the second central result of this Letter.

Results and discussion. — We can compare the magnitude of the SFs drag current \([21]\) with Eq. \([15]\) describing the drag current of normal electrons,

\[
\frac{j^{AL}}{j^{(c)}} = \frac{\sigma_0^{AL}}{\sigma_0} G(\beta_\omega).
\]

The dependence of \( G(\beta_\omega) \) on frequency \( \omega \) in the general case is depicted in Fig. 2. To build the curves, we expressed \( \omega_\tau = \gamma \beta_\omega \) in Eq. \([22]\), where \( \gamma = 16 c (T_c \tau) / \pi \).

In the vicinity of the plasmon resonance \( \omega_p = \omega_p \), \( \omega_p \tau \gg 1 \), the ratio in Eq. \([23]\) depends on the value of \( \beta = \beta_\omega = \omega_p \). Let us discuss two limiting cases (see Fig. 2).

\[
\frac{j^{AL}}{j^{(c)}} = \frac{\sigma_0^{AL}}{\sigma_0} \begin{cases} -\frac{3}{2} \frac{\omega_p \gamma \beta}{\beta_\omega} & \beta \ll 1; \\ -\frac{2}{\beta} \frac{\omega_p \gamma \beta}{\beta_\omega} \ln \beta & \beta \gg 1. \end{cases}
\]

Substituting here the plasmon frequency \( \omega_p = (4 \pi e^2 n k / (\kappa + 1) m) ^{1/2} \), we find for \( \beta \ll 1 \),

\[
\frac{j^{AL}}{j^{(c)}} = -\frac{\pi^2}{96} \frac{c^2 k}{(\kappa + 1) T_c e^2},
\]

and for \( \beta \gg 1 \),

\[
\frac{j^{AL}}{j^{(c)}} = -\frac{4}{\zeta} T_c \ln \left( \frac{2 c^2 k}{(\kappa + 1) T_c e^2} \right).
\]

The behavior of these current ratios over temperature is dramatically different. Equation \([25]\) has a strong singularity \( \epsilon^{-2} \) at \( T \to T_c \), whereas the ratio \([26]\) logarithmically increases by absolute value at \( \epsilon \to 0 \) \([34]\).
In the regime (25), the smallness of \( \beta \) can be compensated by the large parameter \( \omega_p \tau \gg 1 \), resulting in an experimentally measurable value of SFs drag current. Indeed, at \( n \sim 10^{11} \text{ cm}^{-2} \), \( \omega_p \sim 10^{10} \text{ c}^{-1} \). At the same time, the electron density \( n \sim 10^{14} \text{ cm}^{-2} \) has been recently created in MoS\(_2\) material to study the superconducting fluctuations [36]. Since \( \omega_p \propto \sqrt{n} \), we estimate \( \omega_p \sim 3.2 \cdot 10^{12} \text{ c}^{-1} \). Thus, at \( \epsilon = 0.1 \), we find \( \beta \sim (0.01 \div 0.4) \). Taking \( \omega_p \tau \sim 10 \), we estimate the drag current \( j^p = j^p / j^e \sim (0.1 \div 4) \sigma_0^{AL} / \sigma_0^D \).

The AL correction gives an increase of conductivity when the system approaches \( T_c \). In contrast, the AL correction to the drag effect has negative sign, as it follows from Eq. (24). If the drag current of normal electrons is given by Eq. (15), SFs give a decrease of the total drag current of the system in the vicinity of \( T_c \). However, if we account for the dependence of the normal electron relaxation time on electron energy, the drag current Eq. (15) might also have negative sign or even change it with frequency [9]. In this case, the SFs can increase the overall magnitude of the total drag current.

An important and essential feature of Eq. (24) is that the effect is stronger at bigger \( \omega_p \tau \). It makes us envisage that from the experimental point of view, the photon and acoustic drag effects seem not the best candidates to observe the plasmon amplification of SFs drag current. Indeed, the acoustic frequencies are much smaller than \( \omega_p \), whereas in the photon drag effect the in-plane projection of the photon wavevector is too small to excite plasmons. Thus, probably, the most prominent configuration can be the ratchet [20, 21], when an asymmetric grating structure is deposited above the 2DEG. Recently, it has been reported that the ratchet current of normal electrons is enhanced at plasmon frequencies [24]. Therefore, our calculations suggest the plasmon enhancement of SFs in such structures.

In addition, we want to mention that our formalism allows considering one-dimensional systems, where the AL conductivity scales as \( \epsilon^{-3/2} \) [3], as compared to the 2D case, where it is \( \sim \epsilon^{-1} \), but we leave it beyond the scope of this Letter.

Conclusions.— We have considered a two-dimensional material in the vicinity of the transition temperature to a superconducting state, where the superconducting fluctuations are weak. Using the Boltzmann transport equations, we have studied the dynamics of fluctuations, taking into account the interaction between the Cooper pairs and the normal electron gas within the mean-field random phase approximation approach, and analysed the plasmon resonance phenomenon, showing that it experiences an anomalously large broadening caused by the presence of fluctuations in the system. This broadening of the plasmon resonance has strong sensitivity to the temperature, and it substantially increases when the temperature approaches \( T_c \). Furthermore, we have studied the drag effect of fluctuating Cooper pairs and shown that the drag electric current magnitude is measurable in an experiment. Our findings open a way for the plasmon spectroscopy (a well-established experimental technique) to serve as an effective tool to test fluctuating phenomena and thus optically explore the properties of superconductors.

Outlook.— In this Letter, we have considered one particular type of fluctuation corrections: the Aslamazov-Larkin corrections. In certain situations, other contributions, in particular, the Maki-Thompson [37, 38] and the “density of states” [39] can play a significant role. The Boltzmann equations approach cannot be adapted to these corrections, and to study them a microscopic examination is required, which is beyond the scope of this Letter. Nevertheless, the dispersion relation describing the combined action of normal electrons and SFs given in Eq. (5) has a general form independent of the type of contribution to conductivity caused by the SFs. The other corrections will just appear as additional terms in \( \alpha_{\omega_c} \). In the mean time, this simple modification of the theory is not applicable to the calculations of the drag current. There one is bound to utilize the quantum approaches instead of the semiclassical Boltzmann equations.

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SUPPLEMENTAL MATERIAL

In this Supplemental Material, we provide the details of derivations of the Aslamazov-Larkin (AL) corrections. We find (i) the drag current of normal electrons in the presence of superconducting fluctuations (SFs) and (ii) the drag current of SFs themselves.
1. Drag current of normal electrons

Here we derive the expressions describing the linear and second-order responses of normal (non-superconducting) degenerate electron gas. We will use the Boltzmann transport equation \[ \partial_t F + \mathbf{v} \cdot \partial_p F + \mathbf{F} \cdot \partial_p \mathbf{F} = I\{F\}, \tag{27} \]
where \( F \) is the distribution function of normal electrons and \( I\{F\} \) is the collision integral, for which we use the model of single-\( \tau \) approximation \[51\], which means that \( \tau \) is energy-independent and \( I\{F\} = -(F - \langle F \rangle)/\tau \). Here \( \langle F \rangle \) is a locally-equilibrium Fermi-Dirac electron distribution function, which depends on electron density \( n \). We can expand the electron density in series: \( N(r,t) = n_n(r,t) + n_s(r,t) + O(n_3) \), where \( n \) is the equilibrium electron density and \( n_i \) are the corrections to the electron density due to external EM field perturbation.

The first nonzero correction to the electric drag current should be found as the second-order response to the external EM field. Therefore we expand the distribution function in series: \( F = F_0 + F_1 + F_2 + O(F_3) \), where \( F_0 = \exp\left\{ \varepsilon_p - \zeta(n) \right\}/T + 1 \) is the equilibrium Fermi-Dirac distribution. We will also need the expansion of locally-equilibrium electron density \( n \), which has been found from the Maxwell’s equation \( \text{div} \mathbf{D} = 4\pi \rho \), where \( \mathbf{D} = \kappa(z) \mathbf{E} \), \( \kappa(z) \) is the dielectric function, and \( \rho = \epsilon(N(r,t) - n) \delta(z) \) is the charge density. We find \( \mathbf{E}^i = -\frac{e}{c} \mathbf{k} (N - n)(\mathbf{k}, \omega) / \left[ \left( \kappa + 1 \right) \mathbf{k} \right] \), where \( \kappa \) is the dielectric constant of the media.

For an EM perturbation with the momentum \( \mathbf{k} \) and assuming that the phase velocity of the wave significantly exceeds the electron velocity \( \omega/k \gg v_F \), for a degenerate electron gas at zero temperature [which means here \( \partial \zeta / \partial n = 2\pi / m \) and \( \int d\mathbf{p} \partial F_0(\mathbf{p})/(2\pi)^2 = m/(2\pi) \)], we find the electric drag current density (for the normal electrons):
\[
 j^{(e)} = \frac{2e^{3} \kappa \sigma_D^0}{\varepsilon_{k\omega}} \int \frac{E_0}{(2\pi)^2} \partial \left( u_{\mathbf{r}} \tau_{\mathbf{p}} \right) \partial f_0 \frac{1}{1 - i(\omega - k_\mathbf{r})\tau_{\mathbf{p}}}.
\]

This expression can also be found from the simple consideration of Newton’s equations of motion, as it has been reported in Ref.\[21\]. The expression in work mentioned above differs from our result \[25\], first, by the numerical factor since we do not take into account the electron spin and, second, by the factor \( \varepsilon(k, \omega) \) describing the dynamical screening of external EM perturbation.

2. Drag current of superconducting fluctuations

Performing the algorithm of analytical derivations discussed in the main text, we find that the drag current of fluctuating Copper pairs consists of four contributions:
\[
j_1 = -4e^3 \frac{E_0}{\varepsilon_{k\omega}} \int \frac{(\tau_p \partial u_{x} \tau_p)}{(2\pi)^2} \partial f_0 \frac{1}{1 - i(\omega - k_\mathbf{r})\tau_{\mathbf{p}}} \tag{29},
\]
\[
j_2 = e^2 k \frac{E_0}{\varepsilon_{k\omega}} \int \frac{\partial u_{x} \tau_p}{(2\pi)^2} \partial f_0 \frac{\sigma_\omega}{\partial n} \frac{1}{1 - i(\omega - k_\mathbf{r})\tau_{\mathbf{p}}} \tag{29},
\]
\[
j_3 = e^2 k \frac{E_0}{\varepsilon_{k\omega}} \int \frac{u_{x} \tau_p u_{x} \tau_p}{(2\pi)^2} \partial f_0 \frac{\sigma_\omega}{\partial n} \frac{1}{1 - i(\omega - k_\mathbf{r})\tau_{\mathbf{p}}} \tag{29},
\]
\[
j_4 = -e^2 k \frac{\sigma_\omega^D}{(2\pi)^2} \int \frac{(\tau_p - 1)}{\partial n} \partial f_0 \frac{\sigma_\omega^D}{\partial n} \frac{1}{1 - i(\omega - k_\mathbf{r})\tau_{\mathbf{p}}} \tag{29}.
\]
Not all these terms are equivalent in the order of magnitude of the resulting electric current density. Our calculations show that the leading contribution comes from \( j_2 \) and \( j_3 \) terms (see below).

Let us consider these terms first. Taking into account the relations
\[
\frac{\partial f_0}{\partial n} = \frac{\partial \mu}{\partial n} \frac{\partial f_0}{\partial \varepsilon_p}, \quad \frac{\partial (u_{x} \tau_p)}{\partial \varepsilon_p} = \frac{\tau_p}{2m} \left( 1 - \frac{2mu^2}{\varepsilon_p} \right), \tag{30}
\]
and
\[ \text{Re} \frac{\sigma_0^D}{1 - i\omega \tau_p} = \sigma_0^D \frac{1 - \omega^2 \tau_p^2}{(1 + \omega^2 \tau_p^2)(1 + \omega^2 \tau_p^2)}, \]

we find
\[ j_2 = \frac{k}{e\omega n} \sigma_0^\text{AL} \sigma_0^D \left| \frac{E_0}{\epsilon_{k\omega}} \right|^2 \int_1^\infty \frac{dx x - \omega \tau \beta \omega}{x^3 x^2 + \beta \omega^2}, \]

where \( x = \epsilon_p/\mu \) and we have disregarded the spatial dispersion of this expression, implying \( \omega \gg ku_x \). The derivative \( \partial_n \mu = -\mu/n \) follows directly from the relations \( \mu = \alpha T c, 4\alpha_{\text{ct}} T c \xi^2 = 1 \) and Eq. (1) from the main text, describing the coherence length \( \xi \).

To find \( j_3 \) contribution, we are using the relation
\[ \frac{\partial T_{e}^{-1}}{\partial n} = 16T_c (p\xi)^2/\pi n \]

and find
\[ j_3 = -\frac{k}{e\omega n} \sigma_0^\text{AL} \sigma_0^D \left| \frac{E_0}{\epsilon_{k\omega}} \right|^2 \int_1^\infty \frac{(x-1)^2 dx x + \omega \tau \beta \omega}{x^3 x^2 + \beta \omega^2} \]

Let us further consider the remaining \( j_1 \) and \( j_4 \) terms in Eq. \( \text{[29]} \). If we disregard the terms \( ku_x \) in the denominators (like we did for the second and the third terms assuming \( \omega \gg ku_x \)), these contributions vanish. In order to get a nonzero result, one has to keep \( ku_x \) and we can neglect them.

This simple argument is supported by the direct calculations of these terms. The results (after analytical integrations) read (\( \beta \omega \ll 1 \))
\[ j_1/j^{(c)} \sim \frac{T_c}{\xi} \beta \omega^4 (1 + \frac{1}{\omega^2 \tau_p^2}), \]
\[ j_4/j^{(c)} \sim \left( \frac{k}{k_F} \right) \frac{1}{\epsilon}. \]

Obviously, both of these terms are small due to the factors \( T_c/\xi \ll 1 \) and \( k/k_F \ll 1 \) and we can neglect them.

In the mean time, it turns out possible to calculate the integrals in Eqs. \( \text{[32]} \) and \( \text{[34]} \) analytically. This integration yields the final result \( j^{\text{AL}} \equiv j_2 + j_3 \) given in the main text [Eqs. \( \text{(21)} \)-\( \text{(22)} \)].

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[32] See the Supplemental Material at [URL] for the details of derivations of the AL corrections to the drag current of normal electrons in the presence of SFs and the drag current of SFs themselves.
[33] The validity of such a trick is obvious from Eq. (15).
[34] We want to mention here, that the regime (26) is most probably inaccessible in experiments due to the small factor $T_e/\zeta \ll 1$.
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