Branching Fraction Measurements
of the Decays $B \rightarrow \eta_c K$, where $\eta_c \rightarrow K\bar{K}\pi$ and $\eta_c \rightarrow 4K$

The BABAR Collaboration

Abstract

In this report, we present the observation of the exclusive decays $B^0 \rightarrow \eta_c K^0$ and $B^+ \rightarrow \eta_c K^+$, and the measurement of the related branching fractions. Using a sample of $22.7 \times 10^6 \ Upsilon(4S) \rightarrow B\bar{B}$ decays collected with the BABAR detector at the SLAC PEP-II asymmetric $B$ Factory during 1999-2000, we have observed statistically significant signals in the $\eta_c \rightarrow K^0_s K^+K^-$ and $K^+K^-\pi^0$ channels and set upper limits in the $\eta_c \rightarrow K^+K^-K^+K^-$ channels. All the results presented are preliminary.

We have measured

$$B(B^+ \rightarrow \eta_c K^+) = (1.50 \pm 0.19 \pm 0.15 \pm 0.46) \times 10^{-3}$$
$$B(B^0 \rightarrow \eta_c K^0) = (1.06 \pm 0.28 \pm 0.11 \pm 0.33) \times 10^{-3}$$

where the first error is statistical, the second systematic and the last due to the uncertainty on the world average $\eta_c \rightarrow K\bar{K}\pi$ branching fraction.

Presented at the XXXVII$^{th}$ Rencontres de Moriond on QCD and Hadronic Interactions,
3/16–3/23/2002, Les Arcs, Savoie, France
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1 Introduction

We present the measurement of the branching fractions of the exclusive decays $B^0 \to \eta_c K^0$ and $B^+ \to \eta_c K^+$, with $\eta_c$ decaying into $K^0_S K^+\pi^-$, $K^0 S K^-\pi^+$, and $K^+ K^- K^+ K^- (K^0_s \to \pi^+\pi^-)$ and $\pi^0 \to \gamma\gamma$). The $\eta_c$ is a $c\bar{c}$ meson with $I_G(J^{PC}) = 0^+(0^{-}+)$. The decay $B^0 \to \eta_c K^0$ proceeds through the same $b \to c\bar{c}s$ color-suppressed quark diagram as the "golden" mode, $B^0 \to J/\psi K^0$, used to measure the CP-violating parameter $\sin2\beta$ with negligible theoretical uncertainty [1]. Up to now, experimental information on $B$ decays into $\eta_c$ has been sparse [2, 3].

The ratio of the decay rates for the exclusive charmonium decays

$$R_K \equiv \frac{\Gamma(B \to \eta_c K)}{\Gamma(B \to J/\psi K)}$$

has been calculated with different dynamical assumptions [4]–[8] including factorization [2]. The ratio is used since one expects that the corrections to the heavy quark limit, due to the relatively light $s$-quark, are likely to cancel. This leads to the following predictions for $R_K$: 1.6 $\pm$ 0.2 [4], 1.64 $\pm$ 0.55 [5], 1.8 $\sim$ 2.3 [6], 0.94 $\pm$ 0.25 [7], 1.0 $\sim$ 1.3 [8].

2 The BABAR detector and dataset

The data used in this analysis are obtained with the BABAR detector at the PEP-II asymmetric $e^+e^-$ storage ring. The BABAR detector is described elsewhere [10]. The 1.5 T superconducting solenoidal magnet, whose cylindrical volume is $\approx 1.4$ m in radius and $\approx 3$ m long, contains a charged-particle tracking system, a Cherenkov detector (DIRC) dedicated to charged particle identification and an electromagnetic calorimeter. The segmented iron flux return, including endcaps, provides identification of muons and $K_0^L$. In addition, the end of the cylindrical volume in the $e^-$ direction is instrumented with an electromagnetic calorimeter. The tracking system consists of a 5-layer double-sided silicon vertex tracker and a 40-layer drift chamber filled with a gas mixture of helium and isobutane. The calorimeter consists of 6580 CsI(Tl) crystals. The flux return is instrumented with resistive plate chambers.

We have used data corresponding to 20.7 fb$^{-1}$ of integrated luminosity collected at the $\Upsilon(4S)$ resonance ("on-resonance"), and 2.1 fb$^{-1}$ recorded ("off-resonance") about 40 MeV lower in energy in the $\Upsilon(4S)$ rest frame ("CM"), between October 1999 and October 2000. The asymmetric collisions produce a boost in the $e^-$ direction, with $\beta\gamma = 0.55$ in on-resonance running.  

\footnote{Throughout this paper, whenever a mode is given, the charge conjugate (c.c.) is also implied.}

\footnote{We note that we have measured a departure from the factorization hypothesis [2] in another $b \to c\bar{c}s$ color-suppressed mode, $B \to J/\psi K^*$, wherein we have made a polarization measurement, more sensitive than a measurement of $R_K$ to the existence of a factorization-violating term.}
3 Analysis method

A blind analysis is performed in which all selections are chosen to maximize $N_S/\sqrt{N_S+N_B}$ using simulated or off-resonance data, or sidebands in on-resonance data. $N_S(N_B)$ is the number of expected signal (background) events after all selection criteria have been applied.

Event selection designed to enhance the number of $B$ decays requires four or more charged tracks, the sum of all charged and neutral energies to be above 2 GeV, the sum of all the charged momenta to be above 1 GeV/c, and the normalized second Fox-Wolfram moment [1] to be less than 0.6. In addition, at least one neutral or charged kaon candidate is required to have a momentum in the $\Upsilon (4S)$ rest frame within 15 MeV/$c$.

$B^0$ or $B^\pm$ candidates are formed from an $\eta_c$ candidate and a “fast” kaon, either a charged kaon or a $K^0_S \rightarrow \pi^+\pi^-$. The $\eta_c$ candidates correspond to three different topologies: two charged tracks with either $K^0_S \rightarrow \pi^+\pi^-$ or $\pi^0 \rightarrow \gamma\gamma$, or four charged tracks. The $B$ decay vertex is calculated using the charged $\eta_c$ daughters, and the fast kaon if charged.

We require that all charged tracks be within $0.35 < \theta < 2.54$ to obtain well-reconstructed tracks, where $\theta$ is the polar angle with respect to the $e^-$ direction. An important requirement of our analysis is that charged kaon candidates from the $B \rightarrow \eta_cK$ decay are identified by the DIRC and/or by measurements of ionization energy loss $dE/dx$ in the drift chamber and silicon tracker. The momentum of each kaon from $\eta_c$ decay is required to be greater than 250 MeV/$c$.

The $K^0_S$ particles can arise from $\eta_c$ or $B$ decays. In the following, the number in parentheses corresponds to the latter $K^0_S$. The $K^0_S \rightarrow \pi^+\pi^-$ candidates are required to have a reconstructed invariant mass within 12.5 (10) MeV/$c^2$ of the nominal, i.e. world average, value [12]. Furthermore, the cosine of the opening angle between the flight direction and the momentum vector of the $K^0_S$ candidate is required to be greater than 0.990 (0.9995), and the flight distance from the $B$ vertex greater than 2 (3) times its error.

The $\pi^0 \rightarrow \gamma\gamma$ candidates are formed from pairs of photons detected in the calorimeter with a reconstructed invariant mass within 15 MeV/$c^2$ of the nominal value. We require that the cosine of the decay angle in the $\pi^0$ rest frame be less than 0.82 to avoid accidental combinations involving very soft photons. In addition, the electromagnetic showers are required to have moments of the lateral energy deposition [13] between 0.01 and 0.55. The lower energy photon has a minimum energy of 130 MeV while the minimum value for the higher energy photon is 270 MeV.

A Fisher discriminant is used to suppress continuum backgrounds. The Fisher variable is defined as a linear combination of eighteen variables, including the energies between each of nine cones relative to the $\eta_c$ direction in the CM [14]. The most important variables are the normalized second Fox-Wolfram moment and the event thrust, constructed with all charged tracks and neutral clusters in the event. The Fisher discriminant is trained on signal, $u\overline{u}$, $d\overline{d}$, $s\overline{s}$, and $c\overline{c}$ simulated events, and tested on off-resonance data. The requirements on the Fisher discriminant depends on the decay mode.

The charmonium mass region is defined by $2.74 < m_X < 3.22$ GeV/$c^2$. After all selection criteria, the weighted double-Gaussian mass resolutions are 10, 12, and 26 MeV/$c^2$ for the $K^0_S K^\pm\pi^\mp$, $K^+K^-K^+K^-$ and $K^+K^-\pi^0$ channels, respectively, as obtained from a simulation. The $\eta_c$ signal region varies between ±55 MeV/$c^2$ and ±70 MeV/$c^2$ relative to the nominal $\eta_c$ mass (2979.8 MeV/$c^2$), depending on the $\eta_c$ decay mode.

The total energy of the $e^+e^-$ system in the $\Upsilon (4S)$ CM and laboratory frames are denoted by $\sqrt{s}$ and $E_o$, respectively. In the $e^+e^-$ laboratory frame, the candidate energy is defined as $E_B = (s/2 + \mathbf{p}_o \cdot \mathbf{p}_B)/E_o$, where $\mathbf{p}_o$ and $\mathbf{p}_B$ are the momentum vectors of the $e^+e^-$ system and
the $B$ candidate, respectively. The analysis region is defined by a rectangular area in the $\Delta E$-$m_{ES}$ plane where $\Delta E$ is the difference between the energy of the $B$ candidate in the CM frame and $\sqrt{s}/2$, and $m_{ES}$ is the beam-energy substituted mass, $\sqrt{E_{B}^{2} - p_{T}^{2}}$. For events with multiple candidates, the one with the smallest $|\Delta E|$ is retained; this choice affects only a small fraction of events, from 3.4% to 12.4% in the analysis region.

The limits of the analysis region are defined by $5.1 < m_{ES} < 5.29$ GeV/$c^{2}$ and $|\Delta E| < 0.25$ GeV. According to the full detector simulation based on GEANT3, depending on the $\eta_{c}$ decay mode, the signal is Gaussian-distributed in $\Delta E$ with a mean near zero and a resolution between 15 and 30 MeV, while it is Gaussian-distributed in $m_{ES}$ with a mean near the $B$ mass and a resolution around 2.5 MeV/$c^2$. The $\Delta E$ resolution depends on the $\eta_{c}$ decay mode, best for $K_{S}^{0}K^{\pm}\pi^{\mp}$ and worst for $K^{+}K^{-}\pi^{0}$. Note that the $\Delta E$ distribution in data is not centered at zero but rather at about $-10$ MeV; the window is shifted accordingly, leading to a contribution to the overall systematic error. The shifted-$|\Delta E|,|m_{ES} - m_{ES}^{beam}|$ signal region is $<30$ MeV, $<7$ MeV/$c^2$ for the tightest ($K_{S}^{0}K^{\pm}\pi^{\mp}$) case and $<70$ MeV, $<9$ MeV/$c^2$ for the loosest ($K^{+}K^{-}\pi^{0}$) case.

4 Observation of exclusive $\eta_{c}$ signals

Figure displays the mass distribution of the charmonium system in the $(\Delta E,m_{ES})$ signal region for the $K_{S}^{0}K^{\pm}\pi^{\mp}$ channel, using $B^{+}$ candidates after subtraction of the combinatorial background. We see clear $\eta_{c}$ and $J/\psi$ peaks where we have indicated the $\eta_{c}$ mass selection excluding the $J/\psi$ region. The representative curves are fits of three contributions: flat background, $J/\psi$ peak, and $\eta_{c}$ peak with two different widths. The $J/\psi$ peak is represented by a Gaussian with mean constrained at the nominal $J/\psi$ mass and a 12 MeV/$c^2$ resolution. The $\eta_{c}$ mass peak is represented by a Breit-Wigner distribution convoluted with the same Gaussian. The mean of the Breit-Wigner distribution is fixed at the nominal $\eta_{c}$ mass and the width, either to the world average or to the CLEO measurement. Since we cannot yet distinguish among the various measurements, we have used for the $\eta_{c}$ width the average value $16.7 \pm 6.0$ MeV/$c^2$; the efficiency depends on the width central value and the systematic error on its error.

In Figure we display the analysis region for the $B^{+}$ and $B^{0}$ ($\eta_{c} \rightarrow K_{S}^{0}K^{\pm}\pi^{\mp}$) channels as examples. Clear accumulations in the $(\Delta E,m_{ES})$ signal region are apparent. Figures and display projections of the analysis region for the different $\eta_{c}$ channels. The combinatorial background shape is parametrized by a linear function in $\Delta E$ and a threshold function in $m_{ES}$ with a fixed endpoint given by the average beam energy.

In addition to the combinatorial background, a background that peaks in the $(\Delta E,m_{ES})$ signal region can arise from cross-feed from other $\eta_{c}$ decay modes, from partial reconstruction and/or incorrect particle identification, or from $B$ decays into the same detected particles without an intermediate $\eta_{c}$ decay (exact matches). After study, the first two sources are found to be negligible. A quantitative evaluation of the exact matches for each mode is made using data by studying the $\eta_{c}$ mass sidebands for events in the $(\Delta E,m_{ES})$ signal region, after subtracting the combinatorial background as a function of mass. The peaking background is consistent with zero for all modes except possibly for the $K_{S}^{0}K^{\pm}\pi^{\mp}$ mode, see the flat background in Figure 3.

The raw yield and expected backgrounds in the $(\Delta E,m_{ES})$ signal region, and the probability that the background fluctuates to the observed yield are given in Table. In order to ensure the

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3 The world average width is $13.2^{+1.8}_{-1.3}$ MeV/$c^2$ while more recent results give $27.0 \pm 5.8 \pm 1.4$ MeV/$c^2$ [16], $11.0 \pm 8.1 \pm 4.1$ MeV/$c^2$ [7] and $21.4^{+5.9}_{-6.2}$ MeV/$c^2$ [18].
Figure 1: $K_S^0K^±π^±$ mass for $B^+$ candidates in the ($ΔE, m_{ES}$) signal region after subtraction of the combinatorial background. The fits are described in the text. The remaining flat background is that due to the peaking background; see text. The “thick” curve corresponds to the CLEO $η_c$ width while the “thin” curve corresponds to the world average.

Figure 2: $ΔE$ vs. $m_{ES}$ for candidate $B^+ → η_cK^+$ events (on left) and $B^0 → η_cK_S^0$ events (on right), with $η_c → K_S^0K^±π^±$. The ($ΔE,m_{ES}$) signal region is indicated. All selection criteria have been applied except for the signal region requirements.
Figure 3: The $\Delta E$ distribution relative to its mean in the $m_{ES}$ signal band for combined $B^+ \rightarrow \eta_c K^+$ and $B^0 \rightarrow \eta_c K^0_S$ candidates with $\eta_c \rightarrow K^0_S K^\pm \pi^\mp$, fitted to a double Gaussian with common mean on top of a linear background. The weighted average resolution is 16.3 MeV/$c^2$. The narrower Gaussian represents 71% of the area of the double Gaussian; its resolution is 5.9 MeV/$c^2$. All selection criteria have been applied except that for $\Delta E$.

statistical independence of the signal and background measurements, the combinatorial background is estimated here by the extrapolation into the $m_{ES}$ signal band of the threshold function fitted in the $\Delta E$ signal band below the $m_{ES}$ signal band ($m_{ES} < 5.27$ GeV/$c^2$). Because of the low statistics in the $B^0$ channels, the shape parameter of the background function is fixed to that fitted in the corresponding $B^+$ channel.

Table 1: Raw yield, extrapolated combinatorial (see text) and peaking backgrounds in the $(\Delta E, m_{ES})$ signal region, and Poisson probability that the combined background fluctuates to the number of events found in the signal region (called “Prob$_{fluct}$”). Due to the limited data sample, the fitted combinatorial background estimate for $B^0 (\eta_c \rightarrow K^+ K^- K^+ K^-)$ comes from the $\Delta E$ sidebands.

| mode | Yield | Fitted combinatorial background | Peaking background | Prob$_{fluct}$ |
|------|-------|--------------------------------|--------------------|---------------|
| $B^+ (\eta_c \rightarrow K^0_S K^\pm \pi^\mp)$ | 72 | $6.08 \pm 1.39$ | $6.12 \pm 2.61$ | $2 \times 10^{-16}$ |
| $B^+ (\eta_c \rightarrow K^+ K^- \pi^0)$ | 25 | $2.92 \pm 0.92$ | $0.58 \pm 0.58$ | $3 \times 10^{-13}$ |
| $B^+ (\eta_c \rightarrow K^+ K^- K^+ K^-)$ | 17 | $7.41 \pm 1.78$ | $1.72 \pm 2.75$ | $2 \times 10^{-3}$ |
| $B^0 (\eta_c \rightarrow K^0_S K^\pm \pi^\mp)$ | 19 | $1.18 \pm 0.38$ | $1.48 \pm 1.08$ | $3 \times 10^{-13}$ |
| $B^0 (\eta_c \rightarrow K^+ K^- \pi^0)$ | 8 | $1.73 \pm 0.38$ | 0 | $4 \times 10^{-4}$ |
| $B^0 (\eta_c \rightarrow K^+ K^- K^+ K^-)$ | 1 | $1.01 \pm 0.25$ | - | - |
5 Branching fraction determination

The measured $B^+$ or $B^0$ branching fraction ($\mathcal{B}$) is given by

$$\mathcal{B} = \frac{N_{\text{yield}}}{N_{\mathcal{B}} \times \epsilon}, \tag{2}$$
where \( N_{\text{yield}} \) is the net yield in the \( (\Delta E, m_{ES}) \) signal region, extracted from fits to the \( m_{ES} \) distributions in the \( \Delta E \) signal region (Figure 3), and corrected for the peaking background contributions listed in Table 3; \( \epsilon \) is the signal efficiency determined by applying the same analysis chain to signal Monte Carlo (MC) samples and correcting for data-MC differences; and \( N_{\text{BB}} \) is the number of produced \( B\bar{B} \) pairs, \((22.73 \pm 0.36) \times 10^{6}\), determined by a comparison of the rate of multihadron events taken on-resonance to that off-resonance.

5.1 Determination of signal efficiency

The efficiency for reconstructing \( B \rightarrow \eta_c K \) candidates for each \( \eta_c \) decay mode is given by the fraction of generated signal events that are reconstructed in the appropriate mode. We have compared simulations with real data, using for example \( \tau^+\tau^- \) and \( D^{*\pm} \) control samples. There are small differences in reconstruction efficiency for charged particles, \( K^0_S \) and \( \pi^0 \) mesons, vertexing efficiency, resolution and absolute scale of charged particle momentum and photon energies, and charged kaon identification and pion misidentification probabilities. These effects have been measured and corrected. The resulting efficiencies are given in the first line of Table 2.

Table 2: Relative systematic errors on efficiency. All values are expressed in percentage relative to the efficiency, which is given in the first line as a fraction. The last line gives the total relative systematic error obtained as a sum in quadrature of the individual contributions. The 1.6% error from the determination of the number of \( B\bar{B} \) events, common to all modes, is not listed but is included in the total as is the statistical error on the efficiency determination.

| \( \eta_c \) decay | \( B^+ \rightarrow \eta_c K^0_S \) | \( B^0 \rightarrow \eta_c K^0_L \) | \( B^+ \rightarrow \eta_c K^+ \) |
|-------------------|-----------------|-----------------|-----------------|
| Efficiency        | 0.111           | 0.148           | 0.0733          |
| Rel. stat. err.   | 0.004           | 0.003           | 0.0027          |
| Tracking eff.     | 9.5             | 7.7             | 5.8             |
| \( K^0_L \) eff. and cuts | 5.9           | 12.2            | 5.3             |
| \( \gamma \) eff. and \( \pi^0 \) cuts | -              | -               | 3.5             |
| Vertexing eff.    | 1.3             | 1.0             | 1.0             |
| Kaon ident. eff.  | 10.5            | 2.9             | 5.6             |
| Fisher cut eff.   | 2.3             | 2.2             | 4.0             |
| \( \eta_c \) width uncert. | 2.9           | 3.2             | 3.1             |
| \( \Delta E \) centroid shift | 0.47          | 3.3             | 0.86            |
| \( \Delta E \) resolution | 3.6           | 3.8             | 4.4             |
| \( \Sigma \)      | 16.2            | 16.1            | 12.3            |

5.2 Determination of systematic errors

We have evaluated the systematic errors on the yield, \( B \) counting and efficiency determination. The systematic error on the yield comes from a comparison of the combinatorial background estimations from the \( \Delta E \) side and signal bands while that on \( B \) counting comes principally from the uncertainty on the efficiency due to small differences between data and simulation.

Each of the efficiency corrections, as well as our knowledge of the \( \eta_c \) width, has a corresponding systematic uncertainty. In addition, each requirement in the analysis method has been studied to evaluate any systematic differences between simulation and data. The dominant systematic errors
on the signal efficiency are due to kaon identification, tracking efficiency, and $K^0_S$ reconstruction as can be seen in Table 2.

### 5.3 Results

Our results for the product of the branching fractions for each mode are listed below. We have used the nominal values for the $K^0_S \to \pi^+\pi^-$ and $\pi^0 \to \gamma\gamma$ branching fractions. The $B \to \eta\,K$ branching-fraction determinations assume that the branching fraction of the $\Upsilon(4S)$ into $B\bar{B}$ is 100%, with an equal admixture of charged and neutral $B$ final states, and similarly for $K^0$ relative to $K^0_L$.

\[
\begin{align*}
B(B^+ \to \eta\,K^+)B(\eta_c \to K^0K^-\pi^+ + \text{c.c.}) &= (52.8 \pm 7.9 \pm 7.3) \times 10^{-6} \\
B(B^+ \to \eta\,K^+)B(\eta_c \to K^+K^-\pi^0) &= (15.5 \pm 3.6 \pm 2.5) \times 10^{-6} \\
B(B^+ \to \eta\,K^+)B(\eta_c \to K^+K^-K^+K^-) &< 5.6 \times 10^{-6} \ (90\% \ \text{CL}) \\
B(B^0 \to \eta\,K^0)B(\eta_c \to K^0K^-\pi^+ + \text{c.c.}) &= (36.8 \pm 11.6 \pm 6.0) \times 10^{-6} \\
B(B^0 \to \eta\,K^0)B(\eta_c \to K^+K^-\pi^0) &= (11.3 \pm 5.1 \pm 2.4) \times 10^{-6} \\
B(B^0 \to \eta\,K^0)B(\eta_c \to K^+K^-K^+K^-) &< 2.3 \times 10^{-6} \ (90\% \ \text{CL})
\end{align*}
\]

The first error is statistical and the second systematic. The central value for $B(B^+ \to \eta\,K^+)B(\eta_c \to K^+K^-K^+K^-)$ is $3.2 \times 10^{-6}$, while the two-sided 68\% CL varies from $2.6 \times 10^{-6}$ to $4.1 \times 10^{-6}$. No correction is made for any potential $\phi\phi$ contribution to the $K^+K^-K^+K^-$ channels.

The channels $\eta_c \to K^0_SK^+\pi^+$ and $\eta_c \to K^+K^-\pi^0$ are manifestations of the general decay $\eta_c \to K\bar{K}\pi$. From isospin symmetry, the corresponding rates are related by simple Clebsch-Gordon coefficients: $B(\eta_c \to K^0K^-\pi^+ + \text{c.c.}) = 2/3 \ B(\eta_c \to K\bar{K}\pi)$ and $B(\eta_c \to K^+K^-\pi^0) = 1/6 \ B(\eta_c \to K\bar{K}\pi)$. Therefore the ratio of branching fractions, $B(\eta_c \to K^+K^-\pi^0)/B(\eta_c \to K^0K^-\pi^+ + \text{c.c.})$, should be 0.25. Our measurements are consistent with this value for $B^+ \ (0.29 \pm 0.08 \pm 0.04)$ and $B^0 \ (0.31 \pm 0.17 \pm 0.05)$.

We therefore combine our two results, taking into account common systematic errors, to obtain the values for the general decay:

\[
\begin{align*}
B(B^+ \to \eta\,K^+)B(\eta_c \to K\bar{K}\pi) &= (82.5 \pm 10.4 \pm 8.3) \times 10^{-6} \\
B(B^0 \to \eta\,K^0)B(\eta_c \to K\bar{K}\pi) &= (58.1 \pm 15.2 \pm 6.3) \times 10^{-6}
\end{align*}
\]

The first error is statistical and the second systematic. We deduce the branching fraction ratio from our measurements of the $K\bar{K}\pi$ channel: $B(B^0 \to \eta\,K^0)/B(B^+ \to \eta\,K^+) = 0.71 \pm 0.20 \pm 0.08$. We have not used the $\eta_c \to K^+K^-K^+K^-$ results since their statistical weight would be marginal.

Using the world average for the $\eta_c \to K\bar{K}\pi$ branching fraction, $0.055 \pm 0.017$ [12], our results become

\[
\begin{align*}
B(B^+ \to \eta\,K^+) &= (1.50 \pm 0.19 \pm 0.15 \pm 0.46) \times 10^{-3} \\
B(B^0 \to \eta\,K^0) &= (1.06 \pm 0.28 \pm 0.11 \pm 0.33) \times 10^{-3}
\end{align*}
\]

where the last error is due to the $\eta_c \to K\bar{K}\pi$ branching fraction. We have not used the $\eta_c \to K^+K^-K^+K^-$ results since the $\eta_c$ branching fraction is not very well known. We compare these results to the exclusive branching fractions measured by CLEO [3]: $B(B^+ \to \eta\,K^+) = (0.69_{-0.21}^{+0.26} \pm 0.08 \pm 0.20) \times 10^{-3}$ and $B(B^0 \to \eta\,K^0) = (1.09_{-0.42}^{+0.55} \pm 0.12 \pm 0.31) \times 10^{-3}$. The third error is that due to the nominal $J/\psi \to \gamma\eta_c$ branching fraction. Assuming that the errors due to the nominal branching fractions cancel, our results differ by a factor $2.2 \pm 0.9$ for the $B^+$ channel, combining statistical and systematic errors in quadrature. The $B^0$ channel results are consistent.
To determine $R_K$, we have used our measurements\(^\text{[20]}\) of the branching fractions, $\mathcal{B}(B^+ \to J/\psi K^+) = (10.1 \pm 0.3 \pm 0.5) \times 10^{-4}$ and $\mathcal{B}(B^0 \to J/\psi K^0) = (8.5 \pm 0.5 \pm 0.6) \times 10^{-4}$, taking into account common systematic errors, to obtain

$$R_K^+ = \frac{\Gamma(B^+ \to \eta_c K^+)}{\Gamma(B^+ \to J/\psi K^+)} = 1.48 \pm 0.19 \pm 0.17 \pm 0.46$$
$$R_K^0 = \frac{\Gamma(B^0 \to \eta_c K^0)}{\Gamma(B^0 \to J/\psi K^0)} = 1.24 \pm 0.33 \pm 0.16 \pm 0.38$$

where the first error is statistical, the second systematic and the third due to the $\eta_c \to K\overline{K}\pi$ branching fraction. Our results agree with the theoretical predictions listed at the end of Section 1.

6 Acknowledgments

We are grateful for the extraordinary contributions of our PEP-II colleagues in achieving the excellent luminosity and machine conditions that have made this work possible. The success of this project also relies critically on the expertise and dedication of the computing organizations that support BABAR. The collaborating institutions wish to thank SLAC for its support and the kind hospitality extended to them. This work is supported by the US Department of Energy and National Science Foundation, the Natural Sciences and Engineering Research Council (Canada), Institute of High Energy Physics (China), the Commissariat à l’Energie Atomique and Institut National de Physique Nucléaire et de Physique des Particules (France), the Bundesministerium für Bildung und Forschung (Germany), the Istituto Nazionale di Fisica Nucleare (Italy), the Research Council of Norway, the Ministry of Science and Technology of the Russian Federation, and the Particle Physics and Astronomy Research Council (United Kingdom). Individuals have received support from the A. P. Sloan Foundation, the Research Corporation, and the Alexander von Humboldt Foundation.

References

[1] BABAR Collaboration, B. Aubert et al., Phys. Rev. Lett. 87, 091801 (2001); Belle Collaboration, K. Abe et al., Phys. Rev. Lett. 87, 091802 (2001).
[2] CLEO Collaboration, R. Balest et al., Phys. Rev. D 52, 2661 (1995).
[3] CLEO Collaboration, K. W. Edwards et al., Phys. Rev. Lett. 86, 30 (2001).
[4] M. R. Ahmady and R. R. Mendel, Z. Phys. C 65, 263 (1995).
[5] N. G. Deshpande and J. Trampetic, Phys. Lett. B 339, 270 (1994).
[6] M. Gourdin et al., Phys. Rev. D 52, 1597 (1995).
[7] P. Colangelo et al., Phys. Lett. B 352, 134 (1995).
[8] D. S. Hwang and G.-H. Kim, Z. Phys. C 76, 107 (1997).
[9] BABAR Collaboration, B. Aubert et al., Phys. Rev. D 65, 032001 (2002).
[10] BABAR Collaboration, B. Aubert et al., Nucl. Instr. and Methods A 479, 1 (2002).
[11] G. C. Fox and S. Wolfram, Phys. Rev. Lett. 41, 1581 (1978).

[12] Particle Data Group, D. E. Groom et al., Eur. Phys. Jour. C 15, 1 (2000).

[13] “GEANT, Detector Description and Simulation Tool,” CERN Program Library Long Writeup W5013 (1994).

[14] CLEO Collaboration, D. M. Asner et al., Phys. Rev. D 53, 1039 (1996).

[15] A. Drescher et al., Nucl. Instr. and Methods A 237, 464 (1985).

[16] CLEO Collaboration, G. Brandenburg et al., Phys. Rev. Lett. 85, 3095 (2000).

[17] BES Collaboration, J. Z. Bai et al., hep-ex/0002006 (2000).

[18] E835 Collaboration, M. Ambrogiani et al., Nucl. Phys. A 692, 308 (2001).

[19] ARGUS Collaboration, H. Albrecht et al., Z. Phys. C 48, 543 (1990).

[20] BABAR Collaboration, B. Aubert et al., Phys. Rev. D 65, 032001 (2002).