Optimal Design of Creative Flat Folding Table
Based on Stability Equation

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Abstract. In this paper, we introduce a new method to design the optimum flat folding table. Using stability equation and the optimization model, we determine the range of processing parameters. Then we give the optimal algorithm of the folding table design software and the diagram of dynamic process.

Introduction

Creative flat folding tables focus on expressing the elegance of wood products and the automation and functionality designers want to emphasize. In order to increase the effective area, the designer regards the width of the rectangular board as the diameter and intercepts a circle as a desktop. Then cut the remaining area of the board into a number of different lengths of woods, cross both sides of the wood with two reinforcements respectively [1].

Folding table is shown in Fig. 1. Desktop is round, legs can be flattened into a flat with the movement of the hinge. The legs are made up of a number of woods and divided into two groups, each of which is linked by a reinforcement. Both ends of the reinforcements are fixed to the outermost two woods of the legs. The shape of the table is made up with fingerprint surface, and it is beautiful in appearance.

Figure 1. Folding table.

The company plans to develop a kind of software of flat folding table designing. According to the height of the folding table, the shape and size of the desktop’s edge line, and the approximate shape of the edge of the table, we give the shape and size of the flat material and the optimum design and processing parameters, which makes the production as close as the customer’s desired shape [2,3]. In this paper, we help designers to establish the mathematical model of this software design, and give our own creative design flat folding table according to the established model.
Use the Spatial Geometric Model to Describe the Desktop Folding Process

Look at the top view of the table, which is shown in Fig. 2.

We take the ith piece of wood, the 10th piece of wood and the desktop to form the left side of the object, which is shown in Fig. 3.

![Figure 2. The top view of the table.](image1)

![Figure 3. The left side of the object.](image2)

After folding, the table reaches steady state. At this time, the 10th piece of wood $l_{10}$ is the table leg which is on the floor. Due to the height is fixed, hence A is fixed.

And $A = \arccos \frac{h}{l_{10}}$, by cosine theorem, we have

$$l_{PO} = \sqrt{x_i^2 + \left(\frac{l_{10}}{2}\right)^2 - 2x_i \frac{l_{10}}{2} \cos B}$$

$$= \sqrt{x_i^2 + \left(\frac{l_{10}}{2}\right)^2 - 2x_i \frac{l_{10}}{2} \sin A}$$

We denote by $C_i = l_{PO} - \left(l_i - \frac{l_{10}}{2}\right)$ the slot length. Also by the geometric relationship we have $\frac{h/2}{z_i} = \frac{l_{PO}}{l_i}$, the vertical coordinate of the ith table leg $z_i = \frac{hl_i}{2l_{PO}}$. The approximate position of the slot is shown in Fig. 4.

![Figure 4. The approximate position of the slot.](image4)

Let $t = \sqrt{\frac{l_{10}^2 - h^2}{2}}$, then

$$x_i = \begin{cases} x_{10} + x_i - \sqrt{h^2 - z_i^2} & x_i \geq t \\ x_{10} + x_i + \sqrt{h^2 - z_i^2} & x_i < t \end{cases}$$

$$y_i = y_i$$
When the angle of the desktop and the outermost wood is $\theta$, the corresponding coordinates of each leg and table leg:

$$x_i = \begin{cases} \frac{L}{2} - l_i - \sqrt{l_i^2 - z_i^2} & l_{10} - l_i \geq t \\ \frac{L}{2} - l_i + \sqrt{l_i^2 - z_i^2} & l_{10} - l_i < t \end{cases}$$

$$y_i = 1.1(i-1)d$$

$$z_i = \frac{h l_i}{2\sqrt{(l_{10}-l_i)^2 + (l_{10}-x)^2 - 2(l_{10}-x)(l_{10}-l_i)\cos\theta}}$$

At this time, we can obtain the coordinates of each leg reinforcement position:

$$x_i = \frac{L}{2} - \left(1 - \frac{\cos\theta}{2}\right) l_{10}$$

$$y_i = 1.1(i-1)d$$

$$z_i = h - \frac{x}{l_{10}}h$$

Use Optimization Model to Determine the Machining Parameters

The Parameters Relationship Construction and Optimization Analysis

Let $h$ denote the height of the folding table, $2r$ denote the diameter of the desktop, $4n$ denote the number of wood, $L$ denote the plate length, $x$ denote reinforcement position of the outermost wood from the end, $C_i$ denote the slot length of the $i$th wood from the middle. Suppose the thickness of the desktop is $3cm$, the length of the $i$th piece of wood:

$$l_i = \frac{L}{2} - \sqrt{r^2 - y_i^2} = \frac{L}{2} - \sqrt{r^2 - \left[1.1(i-1)d\right]^2}.$$ 

Table area $S = 2 \sum_{i=1}^{n} (L-2l_i)d = 4d \sum_{i=1}^{n} \sqrt{r^2 - \left[1.1(i-1)d\right]^2}$, while the area of the ideal desktop $S' = \pi r^2$, they satisfy $\frac{S}{S'} \approx 1$. We constraint $\left|1 - \frac{S}{S}\right| \leq 0.1$, take $n$ as the minimum value under the constraint condition, then we obtain the image of $\left|1 - \frac{S}{S}\right|$, which is shown in Fig. 5.

![Figure 5. The image of $\left|1 - \frac{S}{S}\right|$](image.png)
It is clear that $\left| 1 - \frac{S}{S} \right|$ converges to a value near 0.096, that is, as $n$ increases, the change of $\left| 1 - \frac{S}{S} \right|$ tends to be stable, thus the minimum $n$ which satisfies $\left| 1 - \frac{S}{S} \right| \geq 0.096$ and $\left| 1 - \frac{S}{S} \right| - \left| 1 - \frac{S}{S} \right|_{n+1} \leq 0.0001$ makes it easy to process.

After the table is folded, the angle between the table and the outermost wood is $B = \arcsin \frac{h}{l_n}$, the projection length of the outermost wood on the ground is $d = l_n \cos B$ (That is $d = \sqrt{l_n^2 - h^2}$). In order to make the table stable, $d$ should satisfy $d \geq r$. Square on both sides then we have $l_n \geq \sqrt{r^2 + h^2}$, thereby 

$$L \geq 2\left[ \sqrt{r^2 + h^2} + \sqrt{r^2 - \left[ 1.1(i-1)d \right]^2} \right].$$

In order to use the least material, we take $L = 2\left[ \sqrt{r^2 + h^2} + \sqrt{r^2 - \left[ 1.1(i-1)d \right]^2} \right]$.

Next, we consider the slot length, 

$$l_{p0} = \sqrt{(l_n - l)^2 + (l_n - x)^2 - 2(l_n - x)(l_n - l)\cos B}$$ 

$$C_i = l_{p0} - (l - x)$$

For processing convenience, the slot length should be as short as possible, that is, $\sum C_i$ should be as small as possible, then we obtain the following formula:

$$\min \sum_{i=1}^{n} C_i = \sum_{i=1}^{n} \left[ l_{p0} - (l - x) \right],$$

where $l_{p0} = \sqrt{(l_n - l)^2 + (l_n - x)^2 - 2(l_n - x)(l_n - l)\cos B}$. That is $\min \sum_{i=1}^{n} C_i = \sum_{i=1}^{n} \left[ \sqrt{(l_n - l)^2 + (l_n - x)^2 - 2(l_n - x)(l_n - l)\cos B} - (l - x) \right]$. Only $\sum_{i=1}^{n} C_i$ and $x$ in the above formula are the variable, so we can obtain the corresponding reinforcement position when $\sum_{i=1}^{n} C_i$ take the minimum value. At this time, because the slot length is the shortest, processing is relatively convenient [4].

**Use the Stability Equation to Determine the Processing Parameters**

Suppose the height of the table $h = 70cm$, the radius of the desktop $r = 40cm$, next we determine the optimum machining parameters [2,3].

$$S = 4d \sum_{i=1}^{n} \sqrt{r^2 - \left[ 1.1(i-1)d \right]^2} = 4 \sum_{i=1}^{n} \sqrt{r^2 - \left[ 1.1(i-1)\frac{r}{n} \right]^2}$$

$$S' = \pi r^2, 1 - \frac{S}{S'} = 1 - \frac{4}{\pi r n} \sum_{i=1}^{n} \sqrt{r^2 - \left[ 1.1(i-1)\frac{r}{n} \right]^2}$$

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By programming, $n = 10$. That is, when the number of wood is 40, the design is the most reasonable. According to $L = 2 \left[ \sqrt{r^2 + h^2} - \sqrt{r^2 - \left[ 1.1(i-1)d \right]^2} \right]$, $r = 40\text{cm}$, $h = 70\text{cm}$, $n = 10$, $d = \frac{r}{n} = 4.0\text{cm}$, by programming, $L = 172.530$. For processing convenience, we take $L = 172\text{cm}$.

And then by the length of each piece of wood $l_i = \frac{L}{2} - \sqrt{r^2 - \left[ 1.1(i-1)d \right]^2}$, we obtain the relationship between $i$ and $l_i$ which is shown in Table 1.

| $i$ | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 |
|-----|----|----|----|----|----|----|----|----|----|----|
| $l_i$ | 46 | 46.24 | 46.98 | 48.24 | 50.08 | 52.59 | 55.94 | 60.47 | 67.00 | 80.35 |

As shown in Table 1, the first line is the number of wood; the second line is the corresponding length of wood.

**Optimal Design Algorithm**

According to known conditions, the machining parameters can be calculated using the following algorithm:

**Step 1** Let $n = 1$, calculate $a_i = \frac{2}{\pi r} \sqrt{r^2 - \left[ 1.1(i-1)\frac{r}{n} \right]^2} - \frac{2}{\pi r} \sqrt{r^2 - \left[ 1.1(i-1)r \right]^2}$, if $a_i \leq 0.0001$, then go to step 3; Otherwise $n = n + 1$, and go to step 2.

**Step 2** Calculate $a_n = \frac{4}{\pi r(n+1)} \sum_{i=1}^{n} \sqrt{r^2 - \left[ 1.1(i-1)\frac{r}{n+1} \right]^2} - \frac{4}{\pi r n} \sum_{i=1}^{n} \sqrt{r^2 - \left[ 1.1(i-1)\frac{r}{n} \right]^2}$, if $a_i \leq 0.0001$, go to step 3; Otherwise $n = n + 1$, and go to step 2.

**Step 3** Calculate $d = \frac{r}{n}$, then go to step 4.

**Step 4** Calculate $L = 2 \left[ \sqrt{r^2 + h^2} - \sqrt{r^2 - \left[ 1.1(n-1)\frac{r}{n} \right]^2} \right]$, then go to step 5.

**Step 5** Let $i = 1$, go to step 6.

**Step 6** If $i \leq n$, calculate $l_i = \frac{L}{2} - \sqrt{r^2 - \left[ 1.1(i-1)d \right]^2}$, let $i = i + 1$, and go to step 6; Otherwise, go to step 7.

**Step 7** Find the root of the quadratic equation

$$(l_i - l_i) + (l_n - x)^2 - 2(l_n - x)(l_i - l_i)\cos B - l_i^2 = 0.$$ 

Take the root of the interval $\left(0, l_n\right)$, and denote by $x_0$, then $x = x_0$. Go to step 8.

**Step 8** Calculate $B = \arcsin \frac{h}{l_n}$, let $i = 1$, go to step 9.

**Step 9** If $i \leq n$, calculate $C_i = \sqrt{(l_i - l_i)^2 + (l_n - x)^2 - 2(l_n - x)(l_i - l_i)\cos B - l_i^2}$, let $i = i + 1$, go to step 9; Otherwise, the end.
For any given requirement, the optimal design parameters can be obtained by the above algorithm. Here we consider the dynamic change process of table in folding process when desktop radius \( r = 25cm \), the height of the table \( h = 50cm \). By the above algorithm, we obtain the plate length \( L = 120cm \), the radius \( r = 25cm \), the height \( h = 50cm \), the number of wood \( n = 10 \), the width of the wood \( d = 2.5cm \), the reinforcement position from the end \( x = 28.2cm \), the length of wood and the slot length are shown in Table 2:

Table 2. The length of wood and the slot length.

| wood | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  |
|------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Slot length | 19.6 | 19.4 | 18.8 | 17.8 | 16.3 | 14.5 | 12.1 | 9.2  | 5.6  | 0   |
| Length of wood | 35.0 | 35.2 | 35.6 | 36.4 | 37.6 | 39.1 | 41.2 | 44.0 | 48.1 | 56.5 |

**Conclusions**

By programming, the dynamic change process of the table machining process is shown in Fig. 6. (The table has symmetry, so only half of the diagram)
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