A GLOBAL ASSESSMENT OF THE STRANGENESS-INCLUDING STATISTICAL BOOTSTRAP MODEL ANALYSIS OF NUCLEUS-NUCLEUS AND p – p̅ INTERACTIONS

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Abstract

A strangeness and isospin asymmetry including statistical bootstrap model analysis of the multiparticle system produced in the Pb + Pb collision at 158 AGeV at CERN is presented. It is concluded that this interaction process has not crossed the deconfinement line. Direct comparisons with the results of similar analyses pertaining to nucleus-nucleus and p – p̅ collisions at CERN are made. The overall picture points to the S + S collision at 200 AGeV as a prime candidate of a process which has crossed the border separating the hadronic from the deconfined phase of matter.

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The quintessential issue probed by relativistic heavy ion experiments concerns whether or not a phase of matter beyond the hadronic one has been achieved during the interaction process. Any conclusive evidence, furnishing an affirmative answer to this question, is relevant not only in solidifying the status of QCD, as the microscopic theory for the strong interaction, but also in providing a short glimpse into the early cosmos.

The most tangible element of experimental input, on which one relies to infer as to what has taken place during a given collision between two heavy nuclei, is the constitution of the produced multiparticle system. One viewpoint to examine is whether such a system originates from a thermally and chemically equilibrated state, having gone through a complex stage of dynamical evolution. This is the premise upon which a number of extensive analyses have been conducted [1-3], pertaining to multihadron systems in heavy ion collisions, as well as $p - \bar{p}$ and $e^+ - e^-$. A common feature in the aforementioned studies is the adoption of an “ideal hadron gas” model to simulate such multiparticle systems. These kinds of approaches, however, can go only so far as to verify the thermalization hypothesis - which, in fact, they do - but fall short of addressing the basic question whether a given observed multiparticle system provides any signature which could trace its source inside or outside the hadronic phase of matter. This is due to the absence of interactions in the thermodynamic description of such relativistic, many-particle system in this type of calculations\(^1\).

The fundamental issue of constructing a self-consistent scheme, which furnishes a thermodynamic account of an interacting relativistic multihadron system was formulated in the 1960’s by Hagedorn [4-7] and is known as “Statistical Bootstrap Model” (SBM). In this scheme, the notion of interaction at a distance between relativistic particles is simulated by a successive organization of the system in terms of particle-like entities of increasing complexity, known as fireballs. At the lowest level enter all known hadrons, which are incorporated as input into a self-consistent (bootstrap) iterative scheme, realized in terms of an integral equation for the fireball mass spectrum function $\rho(m)$. The final construction of the SBM is accomplished by enriching the scheme, in a relativistically consistent way, via the introduction of thermodynamic variables (temperature and fugacities), which monitor its thermal and chemical equilibrium. After some interim mathematical manipulations, the

\(^1\)Only an excluded volume correction has been addressed in some of these studies.
bootstrap equation acquires the form
\begin{equation}
\varphi(T, \{\lambda\}) = 2G(T, \{\lambda\}) - \exp[G(T, \{\lambda\})] + 1 ,
\end{equation}
where \( \varphi \) is the so-called input function, whose specification is determined by masses and quantum numbers of the known hadrons, while \( G \) incorporates the fireball mass spectrum. The most notable feature of the bootstrap equation is that it displays, in the \( \varphi - G \) plane, a square-root branch point at
\begin{equation}
\varphi(T_{cr}, \{\lambda_{cr}\}) = \ln 4 - 1 , \quad G(T_{cr}, \{\lambda_{cr}\}) = \ln 2 .
\end{equation}
The physical branch serves to define a critical hyper-surface in the space of thermodynamic variables, which sets the limits of the hadronic phase. Finally, the “statistical” aspect of the SBM is encoded in a grand-partition function \( Z_{SBM} \). It is of crucial importance to note that the mass spectrum function, which incorporates the interaction dynamics among the hadrons, enters the expression for \( Z_{SBM} \).

The task of employing the SBM to perform non-trivial (in the sense of including interactions) thermal-type analysis of multiparticle yields from heavy ion collisions has been undertaken by the present authors in recent publications [8-11]. To this end, we extended the original Hagedorn scheme, in which a single fugacity variable for baryon number was employed, so as to include fugacities relating to strangeness numbers, both net (strangeness minus anti-strangeness) \( \lambda_s \) and absolute (strangeness plus anti-strangeness) \( \lambda_{|S|} \) [9-10], as well as a “net charge” fugacity [11]. The latter, in combination with the baryon number fugacity, accounts for isospin asymmetry in the colliding system. We formulated in this way the Strangeness-including SBM (SSBM). Moreover, we departed [8-9] from the usual choice of a certain parameter \( \alpha \) entering the bootstrap scheme, which determines the particular partitioning between a kinematical and a dynamical factor composing the mass spectrum function \( G \). In this way, a direct relationship arises between the maximum value of the temperature, \( T_0 \), on the critical surface and the MIT bag constant. The SSBM is analytically applicable only within and up to the limit of the hadronic phase, defining in a precise way this limit. For our purposes the boundary of the hadronic domain will be designated by its projection on the 2-dimensional \( (T, \mu_a) \) phase diagramme. This projection is close to the intersection of the critical surface with the \( \mu_s = 0 \) plane.
Given the wide interest generated by the \( Pb + Pb \) experiments at SPS, we shall present, in this letter, results stemming from an SSBM analysis of data at 158 A GeV. We shall also make a global assessment of all results we have so far obtained via the SSBM approach, which include \( S + S \), \( S + Ag \) and \( p + \bar{p} \) collisions at SPS. Such an analysis is provided by the space of thermodynamical variables, a comprehensive form of which is represented by the set \( (V, T, \lambda_u, \lambda_d, \lambda_s, \gamma_s) \), where \( \gamma_s = \lambda_{|S|} \). Two constraints are imposed on these variables, corresponding to zero strangeness and the connection between total baryon number and total charge of the colliding system

\[
\begin{align*}
\langle S \rangle &= 0, \\
\frac{\langle B \rangle}{\langle Q \rangle} &= \frac{N_{p}^{in} + N_{n}^{in}}{N_{p}^{in}},
\end{align*}
\tag{3}
\]

where \( N_{p}^{in} (N_{n}^{in}) \) is the total number of participant protons (neutrons).

Referring to the first column of Table 1, we consider the full phase space yields of particles produced in the \( Pb + Pb \) NA49-experiment at CERN \([12,13]\) and perform a \( \chi^2 \)-fit in relation to the theoretical expressions furnishing the corresponding particle multiplicities in the SSBM

\[
N_{i}^{th} = \lambda_{i} \frac{\partial \ln Z_{SSBM}}{\partial \lambda_{i}} \bigg|_{\{\lambda_{i}=1\}}
\tag{4}
\]

through which an optimum value for each one of the thermodynamical variables is determined\[^{[9]}\].

Columns 3 and 4 of Table 1 present the SSBM calculated particle yields corresponding to optimum values of the thermodynamical variables, obtained with and without the inclusion of pions, respectively. The corresponding optimum values of these variables are entered in Table 2. Finally, in Table 3 we list the experimentally measured particle ratios, upon which we shall base the presentation of our results. In Fig. 1 we depict the location of the multiparticle emitting source on the \( T - \mu_u \) plane, resulting from the SSBM analysis with the inclusion of pion multiplicities and the fugacity variable \( \gamma_s \) fixed to its optimal value 0.76 (see second column of Table 2). For each particle ratio we have formed corresponding bands, as determined by experimental uncertainties. The source coordinates, obtained from the fit to the particle yields, are located at the point which is nearer to all the particle-ratio bands,

\[^{2}\text{Actually, along with the thermodynamical variables, two additional ones enter the } \chi^2 \text{-fit, namely the two Lagrange multipliers that serve to impose the constraints.}\]
identifying the region of the original achievement of thermal and chemical equilibrium by the multiparticle system. The most significant feature in the plot is the thick solid line, the SSBM (de)confinement line for the value of $\gamma_s$ corresponding to the $Pb + Pb$ interaction. It is an indigenous characteristic of the SSBM and its particular specification in the plot involves the choice for the critical temperature, $T_0$, at zero chemical potentials $\mu_u$, $\mu_d$ and $\mu_s$ ($\gamma_s$ is fixed). The reasoning behind our particular choice: $T_0 \simeq 183$ MeV, corresponding to maximum MIT bag constant value $B^{1/4} = 235$ MeV [12] is that it allows as much as possible space to the hadronic phase [9]. Thus no doubt can be left that an interaction point is outside the hadronic domain if it lies beyond the surface for which the hadronic domain is maximally extended.

Let us point out that the recent data [14] give smaller values for the $4\pi$ multiplicities of $\Xi^-$, $\Xi^+$ and for the $\bar{\Lambda}/\Lambda$ ratio. This leads to the lowering of the fitted temperature as well as the lowering of $\gamma_s$. If we had used the older values $\Xi^- + \Xi^+ = 8.19 \pm 1.06$, $\Xi^- = 7.23 \pm 0.88$ [15] and $\bar{\Lambda}/\Lambda = 0.20 \pm 0.04$ [16] we would get $T = 172.3 \pm 9.3$ MeV, $\mu_u = 70.4 \pm 7.4$ MeV, $\mu_d = 80.3 \pm 7.8$ MeV, $\mu_s = 10.4 \pm 9.7$ MeV and $\gamma_s = 0.827 \pm 0.088$ with $\chi^2/dof = 22.8/7$ and even larger values for $T$ and $\gamma_s$ if the pions were excluded. These results are compatible with the results of Analysis B for the $Pb + Pb$ case in [16].

From Fig. 1 it is evident that, the source of the produced multiparticle system in the $Pb + Pb$ interaction at 158 AGeV lies well within the hadronic domain, which equivalently means that the deconfined, partonic phase has not been attained in this system.

It is evident when comparing the last columns of Table 2 that neither a notable improvement of the fit (e.g. the value of $\chi^2/dof$) nor a remarkable change in the fitted thermodynamic variables occurs when the $< \pi >$ multiplicity (which contains pions) is excluded from the data, as is the case with the fits in the $S + S$ [10] and $S + Ag$ [11] data. The same conclusion can be inferred from Fig. 2, where the fittings of particle multiplicities in $4\pi$ phase space, with and without the inclusion of the pion multiplicities, are compared to the experimental values. The overall picture is that there is no notable difference between the two cases, meaning that the produced entropy (mainly associated with the pion production)
is well accounted for by the hadronic thermal model.

A summary of results attained through the SSBM analysis of \( S + S \) [10] and \( S + Ag \) [11] interactions at 200 AGeV (NA35), as well as the \( p + \bar{p} \) (UA5) at several energies [10], is exhibited in Fig. 3. It should be pointed out that the surface setting boundaries on the hadronic domain is a constraint among the thermodynamic variables \((T, \lambda_u, \lambda_d, \lambda_s, \gamma_s)\). The projection of this surface on the \((T, \mu_B)\) plane which corresponds to the particular value of \(\gamma_s\) resulting from the fit to the experimental multiplicities is plotted for every interaction. Thus every point of interaction should be compared with the relevant projection. The analysis shows that the \( S + S \) source is situated mostly outside the hadronic phase (with probability of being outside 74%), whilst the \( S + Ag \) is located just beyond the deconfinement line (with probability of being outside 52%). In addition, the analysis exhibits a large (\(\sim 30\%\)) entropy enhancement of the experimental negative-hadron yields compared to the model, an effect observed also by other calculations [1-3]. This enhancement may be attributed to contributions from the deconfined quark phase with many libarated new partonic degrees of freedom. Also, the thermal-statistical models [1-3], which do not posses inherently any bounds for the hadronic phase, give fitted source temperature beyond those of the corresponding SSBM analysis: \( T \simeq 180 - 200 \) MeV, which is about \( 5 - 25 \) MeV higher.

We conclude that all these results are corroborating our position that the \( Pb + Pb \) interaction at the maximum energy of 158 AGeV is well within the hadronic phase. The \( S \)-induced interactions and in particular the \( S + S \) one are located beyond the HG phase, in the deconfined quark-gluon domain. For the \( p + \bar{p} \) collision at \( \sqrt{s} = 200 \) to 900 GeV, the SSBM analysis clearly locates this system within the hadronic domain\(^5\).

In summary, the potential of SSBM to perform not only thermal-statistical fits to multiparticle systems, but also to define the limits of the hadronic phase has been employed to

\(^4\)It should be noted that, as far as experiments whose multiparticle data flirt with the critical curve, special considerations of computational nature are called for. Such issues have been discussed at length in Refs [10,11]. Moreover, as an additional aid to probing the region beyond the critical surface we conducted relevant analyses for an increased value of \( T_0 \).

\(^5\)The \( p + \bar{p} \) thermodynamic variables have been obtained through a grand-canonical analysis. It is known that canonical suppression is relevant to the \( p + \bar{p} \) interaction and it may affect the extraction of parameters. It cannot, however, move the point outside the critical surface since it lies well inside the hadronic domain.
analyze and assess several nucleus-nucleus interactions. On the basis of these analyses we reach the conclusion that the \( Pb + Pb \) interaction at 158 AGeV has not crossed the deconfinement line, remaining within the hadronic phase. On the other hand, the \( S + S \) interaction at 200 AGeV appears to have produced a deconfined partonic state, for the first time.

References

[1] Sollfrank J 1997 \textit{J. Phys.} G \textbf{23} 1903

[2] Becattini F, Gaździcki M and Sollfrank J 1998 \textit{Eur. Phys. J.} C \textbf{5} 143

[3] Becattini F 1997 \textit{J. Phys.} G \textbf{23} 1933

[4] Hagedorn R 1965 \textit{Suppl. Nuovo Cimento} \textbf{III} 147

[5] Hagedorn R and Ranft J 1968 \textit{Suppl. Nuovo Cimento} \textbf{VI} 169
   Hagedorn R 1968 \textit{Suppl. Nuovo Cimento} \textbf{VI} 311

[6] Hagedorn R 1968 \textit{Nuovo Cimento} \textbf{LVI A} 1027

[7] Hagedorn R, Rafelski J 1980 \textit{Phys. Lett.} \textbf{97B} 136

[8] Kapoyannis A S, Ktorides C N and Panagiotou A D 1997 \textit{J. Phys.} G \textbf{23} 1921

[9] Kapoyannis A S, Ktorides C N and Panagiotou A D 1998 \textit{Phys. Rev.} D \textbf{58} 034009

[10] Kapoyannis A S, Ktorides C N and Panagiotou A D 1998 \textit{Phys. Rev.} C \textbf{58} 2879

[11] Kapoyannis A S, Ktorides C N and Panagiotou A D 2000 \textit{Eur. Phys. J.} C \textbf{14} 299

[12] Haxton W C, Heller L 1980 \textit{Phys. Rev.} D \textbf{22} 1198
   Hasenfratz P, Horgan R R, Kuti J and Richard J M 1981 \textit{Phys. Lett.} \textbf{95B} 199

[13] Karsch F 2002 \textit{Nucl. Phys.} A \textbf{698} 199

[14] Sikler F et al, NA49 Collaboration 1999 \textit{Nucl. Phys.} A \textbf{661} 45c
   Afanasiev S V et al., NA49 Collaboration 2000 \textit{Phys. Lett.} \textbf{491B} 59
Barton R A et al, NA49 Collaboration) 2001 J. Phys. G 27 367
Barnby L S June 1999 “Measurements of $\Lambda$, $\bar{\Lambda}$ and $K_0^*$ from Pb-Pb Collisions at 158 GeV per nucleon in a Large Acceptance Experiment”, Phd. Thesis, University of Birmingham

[15] Gabler F et al., NA49 Collaboration 1999 J. Phys. G 25 199

[16] Becattini F, Cleymans J, Keränen A, Suhonen E and Redlich K 2001 Phys. Rev. C 64 024901
Table Captions

Table 1 Experimentally measured full phase space multiplicities in the NA49 \( Pb + Pb \) experiment at 158 AGeV and their theoretically fitted values by SSBM, with the inclusion of the \(< \pi >\) multiplicity and without it.

Table 2 Results of the analysis by SSBM of the experimental data from \( Pb + Pb \) experiment (\( 4\pi \) phase space), with the inclusion of the \(< \pi >\) multiplicity and without it.

Table 3 Particle ratios from the experimentally measured full phase space multiplicities in the \( Pb + Pb \) experiment at 158 AGeV, used in the analysis.

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Particles} & \text{Experimental Data} & \text{Calculated with} < \pi > & \text{Calculated without} < \pi > \\
\hline
< \pi >^a & 600 \pm 30 & 570.51^b & 527.05^c \\
K^+ & 95 \pm 10 & 96.583 & 93.470 \\
K^- & 50 \pm 5 & 58.168 & 54.528 \\
K_{s0} & 60 \pm 12 & 76.110 & 72.800 \\
p & 140 \pm 12 & 150.73 & 148.01 \\
\bar{p} & 10 \pm 1.7 & 8.1956 & 7.2435 \\
\phi & 7.6 \pm 1.1 & 7.4101 & 7.3914 \\
\Xi^- & 4.42 \pm 0.31 & 4.0838 & 4.1886 \\
\Xi^+ & 0.74 \pm 0.04 & 0.76299 & 0.77076 \\
B - \bar{B} & 362 \pm 12 & 366.65 & 364.43 \\
\Lambda & 5.14 \pm 0.6 & 4.8591 & 4.6047 \\
\Lambda & 52 \pm 3 & 48.263 & 48.472 \\
\hline
\end{array}
\]

\(^a\) \(< \pi > = (\pi^+ + \pi^-)/2\)

\(^b\) A correction factor 1.03272 has been included for the effect of Bose statistics.

\(^c\) A correction factor 1.03107 has been included for the effect of Bose statistics. This multiplicity is not included in the fit.
### Table 1

**Pb + Pb (NA49) Full phase space**

| Fitted Parameters | Fitted with $<\pi>$ | Fitted without $<\pi>$ |
|-------------------|---------------------|------------------------|
| $T$ (MeV)         | $156.3 \pm 4.2$     | $157.4 \pm 4.3$        |
| $\lambda_u$       | $1.606 \pm 0.051$   | $1.633 \pm 0.064$      |
| $\lambda_d$       | $1.686 \pm 0.059$   | $1.721 \pm 0.077$      |
| $\lambda_s$       | $1.178 \pm 0.030$   | $1.171 \pm 0.031$      |
| $\gamma_s$        | $0.758 \pm 0.068$   | $0.790 \pm 0.081$      |
| $VT^3/4\pi^3$     | $15.7 \pm 2.8$      | $13.8 \pm 3.4$         |
| $\chi^2/dof$      | $10.85 / 8$         | $8.46 / 7$             |
| $\mu_u$ (MeV)     | $74.0 \pm 5.3$      | $77.2 \pm 6.6$         |
| $\mu_d$ (MeV)     | $81.7 \pm 5.9$      | $85.5 \pm 7.5$         |
| $\mu_s$ (MeV)     | $25.6 \pm 4.0$      | $24.9 \pm 4.2$         |

### Table 2

**Pb + Pb (NA49) Full phase space**

| Particle Ratios used for the plots | Experimental Values |
|------------------------------------|---------------------|
| $<\pi>/ (B - \overline{B})$       | $1.66 \pm 0.10$     |
| $K^+/(B - \overline{B})$          | $0.262 \pm 0.029$   |
| $K^-/(B - \overline{B})$          | $0.138 \pm 0.015$   |
| $K^0_s/(B - \overline{B})$        | $0.166 \pm 0.034$   |
| $p/(B - \overline{B})$            | $0.387 \pm 0.036$   |
| $\overline{p}/(B - \overline{B})$| $0.0276 \pm 0.0048$ |
| $\phi/(B - \overline{B})$         | $0.0210 \pm 0.0031$ |
| $\Xi^-/(B - \overline{B})$        | $0.0122 \pm 0.0009$ |
| $\Xi^+/(B - \overline{B})$        | $0.00204 \pm 0.00013$ |
| $\Lambda/(B - \overline{B})$      | $0.0142 \pm 0.0017$ |

### Table 3
Figure Captions

**Figure 1** Experimental particle ratios in the $(\mu_u, T)$-plane for the Pb+Pb interaction measured in $4\pi$ phase space with $\gamma_s$ set to 0.76. The point and the cross correspond to the $\chi^2$ fit with the $<\pi>$. The thick solid line represent the limits of the hadronic phase (HG) as set by the SSBM. The smallest experimental value of the ratio $p/(B - \bar{B}) = 0.351$ cannot be depicted on the $(\mu_u - T)$ plane because it has no solution for the given variables. Thus the space which is compatible with the experimental values is the one which is enclosed by the ratio with the largest experimental value.

**Figure 2** Comparison between the experimentally measured multiplicities in $4\pi$ phase space and the theoretically calculated values in the fit with $<\pi>$ and without $<\pi>$ for the $Pb + Pb$ interaction. The difference is measured in units of the relevant experimental error.

**Figure 3** $(\mu_B, T)$-phase diagramme with points obtained from fits to $p + \bar{p}$, $S + S$, $S + Ag$ and $Pb + Pb$ data and corresponding critical curves given by SSBM.
Pb + Pb, full phase space, $\gamma_s = 0.76$
Pb + Pb, full phase space

\[
\frac{N_{ex} - N_{th}}{\sigma_{ex}}
\]

- Experimental Data
- Fitted with \(\langle \pi \rangle\)
- Fitted without \(\langle \pi \rangle\)

\(\langle \pi \rangle, K^+, K^-, K^0_S, p, \bar{p}, \phi, \Xi^-, \Xi^+, B-B, \Lambda, \Lambda\)
