ASYNCHRONOUS DECENTRALIZED LEARNING OF A NEURAL NETWORK

Xinyue Liang, Alireza M. Javid, Mikael Skoglund, Saikat Chatterjee

Division of Information Science and Engineering
School of Electrical Engineering and Computer Science, KTH Royal Institute of Technology, Sweden
xinyuel@kth.se, almj@kth.se, skoglund@kth.se, sach@kth.se

ABSTRACT

In this work, we exploit an asynchronous computing framework namely ARock to learn a deep neural network called self-size estimating feedforward neural network (SSFN) in a decentralized scenario. Using this algorithm namely asynchronous decentralized SSFN (dSSFN), we provide the centralized equivalent solution under certain technical assumptions. Asynchronous dSSFN relaxes the communication bottleneck by allowing one node activation and one side communication, which reduces the communication overhead significantly, consequently increasing the learning speed. We compare asynchronous dSSFN with traditional synchronous dSSFN in the experimental results, which shows the competitive performance of asynchronous dSSFN, especially when the communication network is sparse.

Index Terms— Asynchronous, decentralized learning, neural networks, convex optimization

1. INTRODUCTION

Decentralized learning has received a high interest in signal processing, machine learning and data analysis [1–7]. Privacy, security and unavailability of all data in a single place are primary reasons for decentralized learning where data are distributed over several places (or nodes). In a decentralized scenario, nodes are not allowed to share data and there is no master node that has access to all nodes.

An ideal objective of a decentralized learning algorithm is to provide a centralized equivalence: parameters learned in the decentralized setup should converge to the corresponding centralized setup. Besides, a decentralized algorithm should have low complexity (computational complexity and communication overhead) and work for asynchronous information exchange. It has practical advantages - processing in a node does not need to wait for all neighbor nodes to pass information, nodes can have varying processing powers, etc.

The design and learning of a neural network is a non-convex optimization problem. Gradient descent and its versions, such as stochastic gradient descent are typically applied for parameter optimization of the neural network. Gradient descent for a neural network can be realized in a decentralized setup where training data is distributed over nodes. A recent study on convergence of decentralized gradient descent for non-convex optimization problems found that it is non-trivial to establish the centralized equivalence (see proposition 1 and discussion afterward in [8]). The work [9] employs another method to train the neural networks using alternating-direction-method-of-multipliers (ADMM) which provides an approximate solution to the non-convex problem, therefore, it cannot guarantee centralized equivalence.

Our main contribution in this article is to design a multi-layer neural network with the following requirements: it can learn over a decentralized setup using asynchronous information exchange and the learning requires a low complexity. We investigate the scope of a recently proposed multi-layer feed-forward neural network [10] that is found to be suitable for satisfying all the requirements. The weight matrices of the neural network are the parameters to be learned. The weight matrices are learned using a layer-by-layer learning approach. A weight matrix is a combination of two submatrices: one submatrix is a random matrix instance and the other submatrix is an outcome of a convex optimization problem. Due to the use of convex optimization, we can employ ADMM [11,12]. ADMM is a well-known algorithm for the decentralized solution of a convex optimization problem with the centralized equivalence. ADMM can also be realized for asynchronous information exchange. An appropriate combination of decentralized ADMM and layer-by-layer learning approach leads to the design of a neural network with the centralized equivalence.

Relevant literature review. The term ‘decentralized’ often comes with the term ‘distributed’. There exists a significant amount of research on developing distributed neural networks [13–19]. They use a master-slave architecture that helps to provide the centralized equivalence. For instance, The work [13] proposes distributed neural networks for model parallelism and data parallelism scenarios where a master node coordinates information exchange in asynchronous manner.

There exist works using decentralized optimization methods for neural networks where there is no master node [9,20,22]. The work [20] uses amplified-differential compression for computing the gradient in a synchronous manner. It has the advantage of low computational complexity. In the work [21], pair-wise information exchange is required where a node has to transmit and receive information from at least one neighbor node in an iteration of the decentralized learning process.

ARock is an asynchronous computing algorithm that can handle decentralized convex optimization problems [23]. ARock does not require pair-wise information exchange. It allows even a relaxed condition - one side (single) activation and one side communication. That means one node can update with out-of-date information and send updates to neighbors without prior notice. ARock has low complexity. We explore the use of ARock for ADMM implementation in our development of the decentralized neural network.

2. DECENTRALIZED SSFN

We begin this section with a decentralized problem formulation for a feedforward neural network. Then we briefly explain the architecture and learning of a (centralized) specific neural network that is amenable for our requirements. Finally, we provide decentralized
and asynchronous training.

2.1. Decentralized problem
In a supervised learning problem, let \((x, t)\) be a pair-wise form of input data vector \(x\) that we observe and target vector \(t\) that we wish to infer. We assume \(x \in \mathbb{R}^D\) and \(t \in \mathbb{R}^Q\). A neural network is an inference function providing output \(t = f(x, \theta)\), where \(\theta\) contains the parameters of the neural network. Let \((x^{(j)}, t^{(j)})\) denote the \(j\)th data-and-target pair in a \(J\) sample training dataset \(D = \{(x^{(j)}, t^{(j)})\}_{j=1}^J\). We can use a cost function

\[
C(\theta) = \sum_{(x^{(j)}, t^{(j)}) \in D} \|t^{(j)} - f(x^{(j)}, \theta)\|_2^2. \tag{1}
\]

For a centralized scenario, we have all the \(J\) samples in a single node. In the training phase, we learn optimal parameters by minimizing the cost in a regularized manner, as follows

\[
\theta^*_c = \arg\min_{\theta} C(\theta) \text{ s.t. } \|\theta\|_2^2 \leq \epsilon, \tag{2}
\]

where ‘s.t.’ is the abbreviation of ‘subject to’. The above problem is non-convex for a general neural network and hence, non-trivial.

In a distributed scenario, we have \(M\) processing nodes and we assume that the training dataset \(D\) is divided into \(M\) datasets. We denote the training dataset at the \(m\)th node by \(D_m\) such that \(\cup_m D_m = D\). Let us denote the cost at the \(m\)th node by

\[
C(\theta_m) = \sum_{(x^{(j)}, t^{(j)}) \in D_m} \|t^{(j)} - f(x^{(j)}, \theta_m)\|_2^2 \tag{3}
\]

where \(\theta_m\) denotes the parameters learned at the \(m\)th node. Our interest is to learn parameters in a distributed manner as follows

\[
\theta^*_m = \arg\min_{\theta, \theta_m} \sum_m C(\theta_m) \text{ s.t. } \forall m, \theta_m = \theta, \|\theta\|_2^2 \leq \epsilon. \tag{4}
\]

The constraint \(\forall m, \theta_m = \theta\) enforces the same solution for all nodes. In the next subsection we discuss a neural network for which we can achieve \(\theta^*_m = \theta^*_c\) using ADMM under some technical conditions. This is the centralized equivalence.

2.2. A neural network
The neural network we investigate in this article is called self-size estimating feedforward neural network (SSFN) [10]. The main motivation for developing SSFN in [10] is the self-estimation of its size, which means the network automatically finds the necessary number of layers and number of nodes. The SSFN training algorithm is shown to have a low computational requirement due to its layer-by-layer learning approach. The SSFN does not use gradient search (backpropagation) and hence avoids the requirement of a high computational resource. It is also shown in [10] that the optimization of weight matrices using backpropagation does not lead to significant performance improvement. In our work in this article, we refrain from estimating the size of the decentralized setup due to a high search complexity. Instead, we design a fixed-size decentralized feedforward neural network based on the SSFN learning principle such that we retain the complexity advantage. We assume that each hidden layer has a fixed number of hidden neurons. Even if we are not estimating its size, it is doable at the expense of high complexity.

The SSFN signal flow relation is

\[
t = O_Q(W_1 g(\ldots g(W_2 g(W_1 x)) \ldots)) = O_y,
\]

where \(W_i\) are weight matrices and \(g(\cdot)\) denotes the non-linear transform comprised of rectified-linear-unit (ReLU) activation function.

The parameters to learn are \(\theta = \{\{W_i\}, O\}\). Figure [1] shows the architecture of SSFN with its weight matrices.

The SSFN addresses the optimization problem (2) in a suboptimal manner. The SSFN parameters \(O\) and \(\{W_i\}\) are learned layer-by-layer as a sequential forward learning approach. The feature vector of \(l\)th layer is

\[
y_l = g(W_l g(\ldots g(W_2 g(W_1 x)) \ldots)) \in \mathbb{R}^n. \tag{5}
\]

The layer-by-layer sequential learning approach starts with layer number \(l = 0\) and then the layers are increased one-by-one until we reach \(l = L\). We have \(y_0 = x\). Let us first assume that we have an \(l\)-layer network. The \((l + 1)\)-layer network will be built on the \(l\)-layer network. For designing the \((l + 1)\)-layer network given the \(l\)-layer network, the steps for finding parameter \(W_{l+1}\) at the \((l + 1)\)th layer are as follows:

1. For all the samples in the training dataset \(D\), we compute

\[
y_l^{(j)} = g(W_l g(\ldots g(W_2 g(W_1 x^{(j)})) \ldots)).
\]

2. Using the samples \(\{y_l^{(j)}\}_{j=1}^J\) we solve the following convex optimization problem

\[
O_l^* = \arg\min_{O_l} \sum_{j=1}^J \|y_l^{(j)} - O_l y_1^{(j)}\|_2 \text{ s.t. } \|O_l\|_F^2 \leq \epsilon_l. \tag{6}
\]

It is shown in [10] that we can choose the regularization parameters \(\epsilon_l = \epsilon = 2Q, l = 1, 2, \ldots, L\). Note that \(O_0\) is \(Q \times P\)-dimensional, and every \(O_l\) for \(l = 1, 2, \ldots, L\) is \(Q \times n\)-dimensional.

3. We form the weight matrix for the \((l + 1)\)th layer

\[
W_{l+1} = \begin{bmatrix} V_Q O_l^* \\ R_{l+1} \end{bmatrix}, \tag{7}
\]

where \(V_Q = [I_Q - I_Q]^T\) is a fixed matrix of dimension \(2Q \times Q\), \(O_l^*\) is learned by convex optimization (6), and \(R_{l+1}\) is an instance of random matrix. The matrix \(R_0\) is \((n - 2Q) \times P\)-dimensional, and every \(R_l\) for \(l = 1, 2, \ldots, L\) is \((n - 2Q) \times n\)-dimensional. Note that we only learn \(O_l^*\) to form \(W_l\). We do not learn \(R_l\) and it can be pre-fixed before training of SSFN. Deciding the weight matrix, the \((l + 1)\)-layer network is \(y_{l+1} = g(W_{l+1} g(\ldots g(W_2 g(W_1 x)) \ldots)) = g(W_l y_l)\).

It is shown in [10] that the three steps mentioned above guarantees monotonically decreasing cost \(\sum_j \|t^{(j)} - O_l y_l^{(j)}\|_2^2\) with increasing the layer number \(l\). The monotonically decreasing cost with increase in layer number is the key to address the optimization problem (6) as we continue to add new layers one-by-one and set the weight matrix of a layer using (7). It was experimentally shown (see Table 5 of [10]) that the use of gradient search (backpropagation) for further optimization of parameters in SSFN could not provide a reasonable performance improvement. Note that backpropagation based optimization requires a significant computational complexity.

2.3. Decentralized SSFN for Asynchronous Communication
Let us consider that the decentralized setup of \(M\) nodes is expressed by a set of undirected connections between the nodes. The connection set is \(E = \{(m, n)\} \text{ if node } m \text{ connects to node } n\). The \(m\)th node has \(J_m\) training samples, that means \(|D_m| = J_m\). Let us introduce the observation data matrix \(X_m \in \mathbb{R}^{P \times J_m}\) and the target data matrix \(T_m \in \mathbb{R}^{Q \times J_m}\) using the dataset \(D_m\). Here \(X_m\) is
formed by the $x^{(j)} \in D_m$ and $T_m$ is formed by the corresponding $t^{(j)} \in D_m$.

Addressing the decentralized problem \(\text{[4]}\) that has centralized equivalence to \(\text{[2]}\), the SSFN design boils down to efficient decentralized solution of \(\text{[6]}\) as follows

$$\min_{O_{i,m}} \sum_{m=1}^{M} \|T_m - O_{i,m} Y_{i,m}\|_F^2, \quad \text{s.t.} \forall (m, n) \in E, O_{i,m} = O_{i,n}, \forall m, \|O_{i,m}\|_F^2 \leq \epsilon,$$  \(\text{[8]}\)

where $Y_{i,m}$ is the signal matrix at the \(l\)th layer of SSFN. Note that $Y_{i,m}$ is generated by feeding the observation data matrix $X_{k,l}$ to the \(l\)th layer of SSFN. To solve this optimization problem, auxiliary variable $\Lambda_{mn}$ is introduced associated with each connected link $(m, n)$. Then the original problem can be rewritten as

$$\min_{O_{i,m}} \sum_{m=1}^{M} \|T_m - O_{i,m} Y_{i,m}\|_F^2, \quad \text{s.t.} \forall (m, n) \in E, O_{i,m} = \Lambda_{mn}, O_{i,n} = \Lambda_{mn}, \forall m, \|O_{i,m}\|_F^2 \leq \epsilon.$$  \(\text{[9]}\)

For every pair of constraints $O_{i,m} = \Lambda_{mn}$ and $O_{i,n} = \Lambda_{mn}$, \(\forall (m, n) \in E, \) dual variables $Z_{mn}$ and $Z_{nm}$ are associated to denote the information sent from the \(m\)th node to \(n\)th node, and vice versa. Applying ARock framework for ADMM iterations, ADMM get simplified as

$$\Lambda_{mn}^{(k+1)} = -\frac{Z_{mn}^{(k)} + Z_{nm}^{(k)}}{2}, \quad O_{i,m}^{(k+1)} = \arg \min_{O} \|T_m - O Y_{m} \|_F^2 + \sum_{n \in N(m)} Z_{mn}^{(k)} O_{i,m} + \frac{\gamma}{2} \|E(m)\|_F^2, \quad \text{s.t.} \|O_{i,m}\|_F^2 \leq \epsilon,$$  \(\text{[10]}\)

$$Z_{mn}^{(k+1)} = Z_{mn}^{(k)} - \frac{\eta}{2} (Z_{mn}^{(k)} + Z_{nm}^{(k)}) + \gamma O_{i,m}^{(k+1)}), \forall n \in N(m),$$  \(\text{[11]}\)

where $\gamma \in (0, \frac{\eta}{2})$ is a scalar and $\eta$ is the step size, both of them needed to be chosen properly. $E(m)$ is the set of links connected with node $m$, and $N(m)$ denotes the set of neighbor nodes of the \(m\)th node. Note that $\Lambda_{mn}$ vanishes, \(m\)th processing node only updates $O_{i,m}$ and $Z_{mn}$. Therefore when a processing node $m$ is activated, the ADMM iterations are

$$O_{i,m}^{(k+1)} = P_r((2T_m Y_{i,m}^{F} - \sum_{n \in N(m)} Z_{mn}^{(k)}) (2Y_{i,m} Y_{i,m}^{F} + \gamma |E(m)| I)^{-1}),$$

$$Z_{mn}^{(k+1)} = Z_{mn}^{(k)} - \frac{\eta}{2} (Z_{mn}^{(k)} + Z_{nm}^{(k)}) + \gamma O_{i,m}^{(k+1)}), \forall n \in N(m),$$  \(\text{[12]}\)

where $|E(m)|$ is the cardinality of $E(m)$, i.e., the number of connected links associated with node $m$. $Z_{mn}$ is the previous received dual variable sent from neighbors $n \in N(m)$ to node $m$. Here $P_r$ is the projection onto the space of matrices with $|| \cdot ||_F^2 \leq \epsilon$ to avoid overfitting, which is

$$P_r(O_{i,m}) = \begin{cases} O_{i,m} - \frac{\epsilon^2}{\|O_{i,m}\|_F^2} O_{i,m} : & \|O_{i,m}\|_F^2 > \epsilon \\ O_{i,m} : & \text{otherwise}. \end{cases}$$  \(\text{[13]}\)

These two steps of ADMM updates would only be executed when some processing node $m$ is activated. This implies that instead of waiting for neighbors to be activated and communicate at the same time, this asynchronous ADMM only requires one side communication during the update. We show the algorithm for this decentralized problem in Algorithm 1.

**Algorithm 1**: Algorithm for Asynchronous learning decentralized SSFN (dSSFN)

**Input:**
1. Training dataset $D_m$ for the \(m\)th node
2. Parameters to set: $L$, number of hidden neurons, $\gamma, \eta$
3. Set of random matrices $\{R_l\}_{l=1}^{L}$ are generated and shared between all nodes

**Initialization:**
1. $l = -1$ (Index for $l$th layer)

**Progressive growth of layers:**
1. **repeat**
   2. $l \leftarrow l + 1$ (Increase layer number)
   3. $k = 0$ (Iteration index of ADMM)
   4. Compute $Y_{i,m} = g(W_{l,m}X_{l,m})$ = $g(W_{l-1,m} Y_{l-1,m})$
   5. Solve $\text{[8]}$ using ARock framework to find $O_{i,m}^{(k)}$.
   6. Each node $m$ initializes the dual variables $Z_{mn}^{(0)} = 0$ for \(m, n) \in E(m)$, $K > 0$
    7. while $k < K$, any activated processing node $m$ do
    8. $k \leftarrow k + 1$
    9. Solve $O_{i,m}^{(k+1)}$ using $\text{[10]}$
   10. Calculate all the $Z_{mn}^{(k+1)}$ $n \in N(m)$ using $\text{[11]}$
   11. Send $Z_{mn}^{(k+1)}$ to all neighbors
   12. **end while**
   13. Form weight matrix $W_{l+1} = \left[ V_{l} O_{l}^{T} R_{l+1} \right]$
   14. **until** $l = L$

3. EXPERIMENTAL RESULTS

We perform experimental simulations to evaluate the performance of the dSSFN on a circular network structure with undirected connections for classification tasks. The datasets that we use are briefly described in Table 1. The datasets used for the experiments can be found at [24, 27]. We use the $Q$-dimensional target vector $t$ in a classification task as a discrete variable with an indexed representation of 1-out-of-$Q$-classes. A target variable (vector) instance has

\[\text{All the Matlab code for experiments are available at https://sites.google.com/site/saitkatchatt/} \]
Table 1: Databases for multi-class classification

| Database     | # of train data | # of test data | Input dimension (P) | # of classes (Q) |
|--------------|-----------------|----------------|---------------------|-----------------|
| Vowel        | 528             | 462            | 10                  | 11              |
| Satimage     | 4435            | 2000           | 36                  | 6               |
| Caltech101   | 6000            | 3000           | 3000               | 102             |
| Letter       | 13333           | 6667           | 16                  | 26              |
| NORB         | 24300           | 24300          | 2048                | 5               |
| MNIST        | 60000           | 10000          | 784                 | 10              |

Table 2: Classification performance comparison between centralized SSFN, synchronous dSSFN, and asynchronous dSSFN on a circular graph where \( M = 20, d = 8, K = 200 \)

| Dataset       | central SSFN | sync dSSFN | async dSSFN |
|---------------|--------------|------------|-------------|
|               | Test Accuracy| Test Accuracy| Training Time(s) | Test Accuracy| Training Time(s) |
| Vowel         | 61.9±1.7     | 61.3±1.6   | 34.9         | 61.2±1.7 | 7.8          |
| Satimage      | 88.7±0.5     | 89.2±0.4   | 16.3         | 89.9±0.2 | 4.2          |
| Caltech101    | 74.6±0.7     | 74.2±0.6   | 427.5        | 75.4±0.9 | 27.4         |
| Letter        | 95.7±0.2     | 93.1±0.4   | 43.6         | 95.8±0.5 | 9.9          |
| NORB          | 86.2±1.1     | 83.6±0.9   | 21.7         | 86.0±0.5 | 11.9         |
| MNIST         | 95.8±0.2     | 94.9±0.2   | 43.5         | 95.0±0.3 | 11.7         |

Table 3: The corresponding parameters of Table 2

| Dataset       | central SSFN | sync dSSFN | async dSSFN |
|---------------|--------------|------------|-------------|
|               | \( \mu_0 \) | \( \mu_1 \) | \( \mu_0 \) | \( \mu_1 \) | \( \gamma_0 \) | \( \gamma_1 \) |
| Vowel         | \( 10^{-6} \) | \( 10^{7} \) | \( 10^{-4} \) | \( 10^{9} \) | \( 10^{1} \) | \( 10^{-1} \) |
| Satimage      | \( 10^{6} \) | \( 10^{7} \) | \( 10^{-4} \) | \( 10^{9} \) | \( 10^{1} \) | \( 10^{-1} \) |
| Caltech101    | \( 10^{7} \) | \( 10^{7} \) | \( 10^{-4} \) | \( 10^{9} \) | \( 10^{1} \) | \( 10^{-1} \) |
| Letter        | \( 10^{7} \) | \( 10^{7} \) | \( 10^{-6} \) | \( 10^{9} \) | \( 10^{1} \) | \( 10^{-2} \) |
| NORB          | \( 10^{-6} \) | \( 10^{7} \) | \( 10^{-4} \) | \( 10^{9} \) | \( 10^{1} \) | \( 10^{-2} \) |
| MNIST         | \( 10^{7} \) | \( 10^{7} \) | \( 10^{-6} \) | \( 10^{9} \) | \( 10^{1} \) | \( 10^{-1} \) |

only one scalar component that is 1, and the other scalar components are zero. For the local SSFN, we set the maximum number of layers \( L = 20 \), the number of hidden neurons \( 2Q + 1000 \), and \( \epsilon = 2Q \) for each layer. We uniformly divide the training dataset between the processing nodes. We test the algorithm with a different number of processing nodes, and different network degree \( d \), i.e., each processing node has \( d \) neighbors. The classification performance results are reported in Table 2; the corresponding parameters we used is shown in Table 3. Our interest is to compare the testing performance of asynchronous dSSFN with traditional synchronous dSSFN and centralized SSFN.

Centralized equivalent performance. Based on the numerical results which are provided in Table 2 one can observe that the synchronous dSSFN and asynchronous dSSFN both provide centralized equivalence solution, while asynchronous dSSFN outperforms traditional synchronous dSSFN in training time. Note that the step size is fixed to be \( \eta = 0.5 \) for every ADMM iteration.

Effect of network degree. We fix the number of nodes \( M = 20 \), and test the performance of synchronous dSSFN and asynchronous dSSFN on different network degree \( d \). The training time for different network degree is shown in Figure 2. When the connections in the network increases, the traditional synchronous dSSFN speeds up because of the heavily reduced network consensus times in every ADMM iteration. While asynchronous dSSFN is not affected as much as synchronous dSSFN. This is consistent with the intuition that ARock based ADMM allows one side activation and communication (relaxed communication condition) so that the connectivity of network does not affect asynchronous dSSFN so much.

Efficient computation and communication. Figure 2 shows the training time versus number of processing nodes when the network degree is fixed to \( d = 2 \) (for two-node network). This kind of network is also called a ring network. While the number of processing nodes increases, the training time of synchronous dSSFN increases monotonically due to the massive information exchange for network consensus in every ADMM iteration. But asynchronous dSSFN speeds up slightly when processing nodes increases. This implies that there is a trade-off between parallel computation benefit and communication overhead. Compared with synchronous dSSFN, asynchronous dSSFN enjoys a relaxed condition of communication by allowing one side communication and releasing nodes from waiting for up-to-date information from neighbors.

4. CONCLUSION

We have proposed an ARock based ADMM for solving the decentralized convex optimization in progressively learning the weight matrices of SSFN. We conclude that the proposed asynchronous dSSFN is efficient in computation and communication, which relaxes the condition of activation and communication. Therefore asynchronous dSSFN speeds up the learning process and remains competitive performance for sparse communication networks where the number of connected links is small.

Fig. 2: Training time versus network degree, \( M = 20 \).

Fig. 3: Training time versus node number, ring network.
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