Diameter and axial position measurement of micrometric particles by in-line digital holography using wavelet transform

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Abstract. In this paper, the size and axial position of micrometric particles is obtained for an in-line Fraunhofer holography setup. The hologram reconstruction was realized using the wavelet transform. By digital image processing tools, the size distribution histogram for the particles in the sample was obtained. The contrast measurement in the amplitude reconstruction presents a peak when the axial coordinate and the register distance are equal. This fact lets the axial position in the sample be determined.

1. Introduction
In industry, science and technology, there are several processes and products that involve micrometric particles. The physical and chemical characteristics of these particles are fundamental to determine the quality and efficiency of these processes and products [1]. Actually, there are a wide range of techniques to measure the size of micrometric particles, some of which are based in digital image processing like digital holography. In digital holography, the object information is recorded on a solid state sensor as an interference pattern. The object’s optical field can be reconstructed using numerical tools. From this reconstruction it is possible to obtain some data related to size, geometry and dynamic variables. The work in particle holography dates back to 1962 with Bernard Silverman and collaborators in the Air Force Cambridge Research Laboratories in Bedford, Massachusetts [2, 3]. Later the work of Glenn Tyler and others in 1976 established the Fraunhofer holography basis [4]. Several subsequent experiments and developments in imaging analysis consolidated the particle holography as a powerful tool in the micrometric samples analysis. The wavelet transform has been used as a digital imaging processing tool to study fibers [5] and micrometric particle systems [6, 7]. This transform makes a decomposition of a signal in its space-frequency and can be used in the hologram analysis even if an object image (object reconstruction) is not obtained [8]. The hologram reconstruction using wavelet transform can be used to calculate variables related to physical properties of the analyzed objects and for filtering the virtual image in off-axis holography [9]. The wavelet functions are nowadays a useful alternative in the analysis of holograms to obtain the objects location and sizing. In this work, the size distribution and the axial location of micrometric particles in an in-line Fraunhofer holography setup are determined. The analysis is realized using the wavelet transform of the recorded holograms. First, the basic theory and analysis used in reconstructing...
the holograms is shown. Then the experimental setup used for holographic recording is described. Finally the experimental results are analyzed and some conclusions are presented.

2. In-line Fraunhofer holography

G.B. Parrent and B.J. Thomson defined the Fraunhofer holography by analyzing the Fraunhofer diffraction pattern of an object illuminated by a coherent quasi-monochromatic source [10, 11]. In the Fraunhofer approximation (Equation 1), the distance \( z \) between the object \((\xi, \eta)\) and recording \((x, y)\) planes is larger than the maximum dimension of the objects \((\xi, \eta)_{\text{max}}\) and the wavelength \( \lambda \) of the source [6, 12]:

\[
z \gg \pi (\xi^2 + \eta^2)_{\text{max}} / \lambda
\]

where \((\xi, \eta)\) are the coordinates in the object plane.

The object and recording planes are aligned as is showing in Figure 1. The recorded hologram corresponds to the interference pattern between the field diffracted by the particles and the illumination field.

![Figure 1. Coordinates planes.](image)

The optical field in the recording plane can be expressed as a convolution of the form:

\[
A(x, y) = [1 - O(x, y)] \otimes h_z(x, y),
\]

where the function \( 1 - O(x, y) \) represents the optical field amplitude in the object plane and the function \( O(x, y) \) represents the object geometry. The impulse response function \( h_z(x, y) \) in Equation (2) has the form:

\[
h_z(x, y) = \frac{1}{\lambda z} \exp \left[ i \frac{\pi}{\lambda z} (x^2 + y^2) - \frac{\pi}{2} \right],
\]

and the geometry function \( O(x, y) \) for particles with circular shape and diameter \( d \) corresponds to a circle function:

\[
O(\xi, \eta) = \text{circ} \left( \frac{2(\xi^2 + \eta^2)^{1/2}}{d} \right) = \begin{cases} 
1, & \text{for } \left| \frac{2(\xi^2 + \eta^2)^{1/2}}{d} \right| \leq 1, \\
0, & \text{in another case.}
\end{cases}
\]

In digital holography, the recorded information in the \((x, y)\) plane corresponds to the intensity \( I(x, y) \),

\[
I(x, y) = A(x, y)A^*(x, y),
\]

where \( A^*(x, y) \) represents the complex conjugate of the \( A(x, y) \) function. Using the Equation (2) and considering a real \( O(\xi, \eta) \) function, the intensity has the form:

\[
I(x, y) = 1 - O(x, y) \otimes \frac{2}{\lambda z} \sin \left[ \frac{\pi}{\lambda z} (x^2 + y^2) \right],
\]

in which the crossed terms were not considered.
The intensity function in the Equation (5) can be expressed in terms of the wavelet transform \( CTW_O \) of the function \( O(x, y) \) [7, 13, 5]:

\[
I(x, y) = 1 - \frac{2}{\pi} CTW_O(a; \psi_G),
\]

(6)

The function \( CTW_O \) is calculated as the correlation between \( O(x, y) \) and a family of wavelet functions \( \Psi^G(x, y) \) whose functional form is given in Equation (7)

\[
\Psi^G_a(x, y) = \frac{1}{a^2} \left[ \sin \left( \frac{x^2 + y^2}{a^2} \right) - M_\psi \right] \exp \left[ -\frac{(x^2 + y^2)}{a^2 \sigma^2} \right],
\]

(7)

This family of wavelet functions is constructed from a \( \sin(x, y) \) function modified by a Gaussian envelope with a width determined by the parameters \( a \) and \( \sigma \). This allows a wavelet transform and its inverse can be defined. The recording distance \( z \) determines the value of the parameter \( a \): \( a = \sqrt{\frac{2\lambda}{\pi}} \), and \( \sigma \) gives the value of \( M_\psi \): \( M_\psi = \frac{a^2}{1+\sigma^2} \).

The functional dependence of \( a \) allows the wavelet transform behavior to be related to with the axial location of the particles. The relationship between the intensity \( I(x, y) \) and the wavelet transform \( CTW_O \) permits the information in the object plane to be recovered by applying the inverse wavelet transform to the recorded hologram.

3. Experimental Setup

The hologram recording was made using the experimental setup for in-line holography shown in Figure 2(a). This setup was composed of a 35 mW He-Ne laser (1) with a wavelength \( \lambda = 0.633 \mu m \). To avoid the sensor saturation, a variable density filter was used. Then, the beam was spatially filtered and collimated using a positive lens (2). The resultant plane wave was reflected vertically by a plane mirror (3) tilted 45\(^\circ\) to the incident direction. The vertical beam illuminated the particles on a thin glass sheet (4). The hologram was recorded using the CMOS sensor Lumenera L 120 with a pixel size of 6.7\(\mu\)m in length (5). Finally, the intensity information was saved and processed by computer (6).

The sample was composed of particles between 30\(\mu\)m – 250 \(\mu\)m in size. Initially, a recording of a sample of particles with the glass sheet located at the distance \( z = 17.70 \) cm was made. In a second experiment, the particles were distributed on three glass sheets located in \( z_1 = 14.50 \) cm, \( z_2 = 15.50 \) cm and \( z_3 = 16.00 \) cm, as shown in Figure 2(b).

4. Results and analysis

From the recorded holograms the information on the particles plane using its inverse wavelet transform was recovered. The symmetry and parity of the wavelet functions in Equation (7) allows the wavelet transform \( CTW_O \) to be calculate as a convolution between \( O(x, y) \) and \( \Psi^G_a(x, y) \). In practice, this is reduced to the application of two Fourier transforms. From the wavelet transform of the hologram, the digital calculation of the axial coordinate \( z \) was made by finding the maximum value for the contrast \( \alpha_{RMS} \) when the \( z \) value was changed. The contrast is defined in the Equation 8, where \( I_{ij} \) represents the normalized intensity value for the \( ij \) element in a \( M \times N \) image, and \( \bar{I} \) is the mean intensity of the image.

\[
\alpha_{RMS} = \left[ \frac{1}{MN} \sum_{i=0}^{N-1} \sum_{j=0}^{M-1} (I_{ij} - \bar{I})^2 \right]^{\frac{1}{2}}.
\]

(8)

Due to the importance of the \( \sigma \) value in the wavelet functions defined in (7), it was necessary to apply the sampling and admissibility conditions on \( \Psi^G_a(x, y) \) [12]. The value used for \( \sigma \) is defined in Equation 9:
Figure 2. (a) Experimental setup for the recording of Fraunhofer in-line holograms. (b) Modification of the setup for the recording of a hologram of particles distributed on three glass sheets.

\[
\sigma \ll \frac{N \delta_x}{2a} \left[ \frac{1}{\ln(\epsilon^{-1})} \right]^{\frac{1}{2}},
\]

where \( N \) is the sensor size, \( \delta_x \) the pixel size and \( \epsilon \) a constant value that guarantees a value close to zero for \( \Psi^G(x, y) \) when \( x = \pm N \delta_x \). The experimental results for a simple glass sheet located in \( z = 17.70 \) cm are shown in Figure 3. In 3(a) the recorded hologram is shown. Using Equation 8, the contrast for the hologram’s wavelet transform was calculated. The plot of Contrast value vs. \( z \) coordinate is shown in Figure 3(b) where there is a maximum of \( 4.17 \times 10^{-3} \) when \( z = 18.30 \) cm. The Figure 3(c) displays the wavelet transform for the \( z \) value with the maximum contrast. In this image the areas that correspond to the particles in the sample are defined as clear regions. In this way, the axial location of the particles plane was found with a deviation of 3.4%. The particles size histogram distribution shown in Figure 3(b) was made by applying an intensity threshold on the wavelet transform and numerically finding the radius of the circle with the same area for each clear region on Figure 3(c).

The results for particles distributed on three glass sheets located at \( z_1, z_2 \) and \( z_3 \) (Figure 2(b)) are shown in Figure 4. In this case, the contrast calculation was made on square regions of 128 pixels on the wavelet transform of the hologram shown in 4(a). This procedure allowed the influence of the wavelet transform noise on the maximum contrast calculation to be reduced. The comparison of the contrast values for all square regions showed contrast maximums in the axial coordinates \( z_1 = 14.90 \) cm, \( z_2 = 15.90 \) cm and \( z_3 = 16.20 \) cm with a deviation of 2.7% for the firsts and 1.3% for the last. Figure 4(b) shows a 3D plot with the axial planes locations \( z_1, z_2 \) and \( z_3 \). The wavelet transform for each axial coordinate is shown in the figures 4 (c), (d) and (e) respectively. For each coordinate found, the particles located are enclosed in square and circular regions. Following a procedure similar to the one sheet case, the particles size distribution was found. The histogram is shown in Figure 4 (f).
Figure 3. Experimental results for a sample of particles located on a glass sheet with $z = 17.70 \text{ cm}$ (a) Recorded Hologram. (b) Contrast vs. $z$ graphics. The maximum value is located at $z = 18.30 \text{ cm}$ (c) Wavelet transform of the recorded hologram when $z = 18.30 \text{ cm}$. (d) Size distribution histogram for the particles sample.

Figure 4. Experimental results for a sample of particles distributed on three glass sheets with $z_1 = 14.50 \text{ cm}$, $z_2 = 15.50 \text{ cm}$ and $z_3 = 16.00 \text{ cm}$. (a) Recorded Hologram. (b) 3D location of the particles. (c) Wavelet transform of the recorded hologram when $z = 14.90 \text{ cm}$. (d) Wavelet transform of the recorded hologram when $z = 15.90 \text{ cm}$. (e) Wavelet transform of the recorded hologram when $z = 16.20 \text{ cm}$. (f) Size distribution histogram for the particles sample.
In Figure 4 (c) three particles located at \(z_1 = 14.90\) cm are numbered. These particles are in different square areas selected to contrast calculation. In Table 1 the variations of the contrast value for these particles in some axial coordinates near to \(z_1\) are showed. These contrast values present some fluctuations but for the three regions analyzed, the contrast has a maximum in \(z_1 = 14.90\) cm.

In Table 2 the variations of the contrast value for the regions containing the particles numbered as 4 (figure 4(d)) and 5 (figure 4(e)) are showed. The contrast has a maximum value in \(z_2 = 15.90\) cm for the particle 4 and a maximum value in \(z_2 = 16.20\) cm for the particle 5 as it was expected.

| \(Z\) (cm) | Particle 1 | Particle 2 | Particle 3 |
|------------|------------|------------|------------|
| 14.50      | 6.93       | 6.57       | 5.80       |
| 14.60      | 6.94       | 6.59       | 5.81       |
| 14.70      | 7.03       | 6.68       | 5.88       |
| 14.80      | 6.84       | 6.51       | 5.72       |
| **14.90**  | **7.11**   | **6.76**   | **5.94**   |
| 15.00      | 6.79       | 6.48       | 5.66       |
| 15.10      | 6.96       | 6.63       | 5.80       |
| 15.20      | 6.89       | 6.56       | 5.70       |
| 15.30      | 6.70       | 6.39       | 5.55       |

**Table 1.** Contrast values variation with the axial coordinate for the regions that corresponds to particles 1, 2 and 3 in the figure 4(c). The contrast values are multiplied by \(10^{-2}\)

| \(Z\) (cm) | Particle 4 | \(Z\) (cm) | Particle 5 |
|------------|------------|------------|------------|
| 15.50      | 16.61      | 15.8       | 7.07       |
| 15.60      | 16.76      | 15.9       | 7.45       |
| 15.70      | 16.80      | 16.0       | 7.27       |
| 15.80      | 16.65      | 16.1       | 7.26       |
| **15.90**  | **16.85**  | **16.2**   | **7.53**   |
| 16.00      | 16.67      | 16.3       | 7.27       |
| 16.10      | 16.58      | 16.4       | 7.32       |
| 16.20      | 16.67      | 16.5       | 7.40       |
| 16.30      | 16.41      | 16.6       | 7.17       |

**Table 2.** Contrast values variation with the axial coordinate for the regions that corresponds to particle 4 in figure 4(d) and particle 5 in figure 4(e). The contrast values are multiplied by \(10^{-2}\)

5. Conclusions

This paper shows that it is possible to make a digital focusing of a sample of particles using the contrast value of the wavelet transform for an in-line digital hologram. The contrast value is maximum when the axial coordinate value in the wavelet function corresponds to the recording distance. Additionally, from the wavelet transform of the digital holograms recorded, it is possible to obtain the particle size histogram distribution using basic digital image processing tools.
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