Virial mass in DGP brane cosmology

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Abstract
We study the virial mass discrepancy in the context of a Dvali, Gabadadze and Poratti (DGP) brane-world scenario and show that such a framework can offer viable explanations to account for the mass discrepancy problem. This is done by defining a geometrical mass $N$ that we prove to be proportional to the virial mass. Estimating $N$ using observational data, we show that it behaves linearly with $r$ and has a value of the order of $M_{200}$, pointing to a possible resolution of the virial mass discrepancy. We also obtain the radial velocity dispersion of galaxy clusters and show that it is compatible with the radial velocity dispersion profile of such clusters. This velocity dispersion profile can be used to differentiate various models predicting the virial mass.

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1. Introduction
The past decade has been marked by the advent of an avant garde school of thought which has tried to address a number of important problems and observations in standard general relativity and cosmology, namely the hierarchy problem; the great disparity between the fundamental forces of nature, the observation that the universe is undergoing an accelerated expansion relating to dark energy and the problem of galaxy rotation curves relating to dark matter. This school of thought is based on the assumption that our four-dimensional observable universe, the brane, is embedded in a higher dimensional space, the bulk, which has the geometry of an AdS space and to which gravitons can escape but ordinary matter cannot. The AdS nature of the bulk space would cause gravity to become localized around the brane and would modify the gravitational potential at short distances. Such a scenario [1], proposed by Randall and Sundrum (RS), has been able to account for the hierarchy of the fundamental forces with great success and has been generating a myriad of other scenarios and variations. Shortly after, in a seminal work [2], it was shown how to project the Einstein field equations, assumed to hold in the bulk, onto the brane. An unprecedented number of works utilizing this idea have been appearing ever since [3, 4].
In an effort to relax the restriction of an AdS bulk and hence allowing gravity to penetrate large distances, Dvali, Gabadadze and Poratti (DGP) [5, 6] proposed an alternative model in which the influence of gravity on the brane is accounted for by including an induced three-dimensional Einstein–Hilbert term to the full action. In this model, in contrast to the standard RS model, gravity is modified at large distances. For a comprehensive review of this model see [7]. The cosmological implications of this model were investigated in [8] where it was shown that the Friedmann equation on the brane has two branches, both reducing to the usual FRW equation at the small Hubble radius limit. However, the important discovery was that one of these branches predicts a self-accelerating universe at late times, consistent with the observation that our universe is undergoing an accelerated expanding phase. So much for the success of the DGP model, a word of caution is in order; the theory predicts the existence of ghost-like excitations. Many scenarios have been undertaken to explain away such ghosts, but as yet no satisfactory solution exists. The interested reader should consult [9, 10] for further insight. In this paper, we do not discuss such excitations since our aim lies in studying the virial mass discrepancy in DGP models.

One of the interesting problems in cosmology is the calculation of the mass of cluster of galaxies. In recent years, our ability in performing precision measurements in observational cosmology has been developed to such an extent that we can obtain accurate values for the mass of individual galaxies and their velocities. We may therefore find the total mass of the cluster of galaxies in two ways, adding up the masses of individual galaxies, or doing it statistically and using the virial theorem. Since these methods represent two aspects of the same thing we must obtain the same result. However, almost in all clusters the virial mass is 20–30 times greater than $M$, the mass obtained by adding up the individual masses. Also by Newton’s second law we know that the mass of a galaxy is proportional to $r v t g$, where $v t g$ is the tangential velocity of a test particle located at the distance $r$ from the center of the galaxy. As observations have shown, the tangential velocity of a test particle remains nearly constant at large distances from the center of the galaxy. This is, of course, in contradiction to what Newtonian gravity predicts. One way around this is to postulate the existence of dark matter. However, there are geometrical approaches to address this problem, namely to use modified Einstein field equations, as is done in brane-world models [11, 12] or in modified gravity [13, 14].

In this paper, we use the DGP model discussed above to explain the virial mass discrepancy in a geometric manner. As we shall see later, the linearly increasing behavior of the virial mass with distance can be explained by taking into account the extra terms that appear in the field equations, which in turn originate from the bulk geometry. In order to do this we must have a procedure to obtain the total mass of the cluster. We use Jean’s equation and the observational data to obtain the total mass distribution of clusters. We also obtain the radial velocity dispersion in this model which suggests an alternative way to obtain the virial mass of clusters and can be used to explain the observational data. The question of the flat rotation curves of individual galaxies requires a separate undertaking and will be dealt with in a future work.

2. Einstein equations on the brane

Let us start with the standard DGP action [6]:

$$S = \frac{m_3^4}{2} \int_M d^5 x \sqrt{-g} R + \frac{m_3^2}{2} \int_{\partial M} d^4 x \sqrt{-q} (R - 2\Lambda)$$
\[ -m_3^4 \int_{\partial M} d^4 x \sqrt{-q} K + S_m(q_{\mu \nu}) + S_B(g_{AB}), \]  

where \( g_{AB}, R \) and \( S_B \) are the metric, Ricci scalar and matter action of the bulk and \( q_{\mu \nu} \), \( R \) and \( S_m \) are those of the brane with \( \Lambda \) being the brane cosmological constant, and \( m_3^4 \) is the bulk (brane) Planck scale. The third term is the Hawking–Gibbons boundary term \([15]\) and \( K \) is the extrinsic curvature. After varying the action and denoting the extra dimension by \( y \), we obtain the Einstein field equations in the bulk:

\[ m_3^4 \left( R_{AB} - \frac{1}{2} g_{AB} R \right) + m_3^2 \delta A^A_{\delta y} \left( R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R \right) \delta(y) = \delta A^A_{\delta y} \left( R_{\mu \nu} - m_3^2 \Lambda q_{\mu \nu} \right) \delta(y) + \hat{T}_{AB}, \]

where \( \hat{T}_{AB} \) is the bulk (brane) energy momentum tensor for which a perfect fluid form is assumed:

\[ \hat{T}^A_B = \text{diag}(-\rho_x, p_x, p_x, p_x, p_5), \]

\[ T^\mu_\nu = \text{diag}(-\rho_b, p_b, p_b, p_b), \]

and for the bulk metric we take

\[ ds^2 = -e^{\nu(r)} dt^2 + e^{\mu(r)} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) + e^\lambda(r) dy^2. \]

The induced metric on the brane is simply

\[ ds^2 = -e^{\nu_0(r)} dr^2 + e^{\mu_0(r)} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \]

where \( \nu_0(r) = \nu(r, 0) \), etc. Assuming that the brane is at \( y = 0 \) and noting that \([16]\)

\[ \frac{df}{dy} = f \frac{d|y|}{dy} = f \left[ 2\delta(y) - 1 \right], \]

\[ \frac{d^2 f}{dy^2} = \ddot{f} + 2\dot{f} \delta(y), \]

\[ \left( \frac{df}{dy} \right) \left( \frac{dh}{dy} \right) = \dot{f} \dot{h}, \]

where \( \dot{f} = \frac{df}{dy} \) and \( f, h \) are arbitrary functions of \( |y| \), we obtain the following field equations in the bulk:

\[ m_3^4 \left( 4 + 4r \lambda' + 2 \lambda' r^2 + \lambda^2 r^2 - 4r \mu' - \mu \lambda' r^2 \right) e^{-\mu} \]

\[ + \left( 2 \dot{\mu} r^2 + 4 \mu r^2 \delta(y) + \mu^2 r^2 - \mu \lambda r^2 \right) e^{-\lambda} - 4 \]

\[- \frac{m_3^2}{r^2} e^{-\mu_0} \left[ r \mu'_0 + e^{\mu_0} - 1 \right] \delta(y) = - \left( \rho_b + m_3^2 \Lambda \right) \delta(y) - \rho_x, \]

\[ m_3^4 \left( 4 + 4r \nu' + 4r \lambda' + \nu \lambda' r^2 \right) e^{-\nu} + \left( 2 \nu \dot{r}^2 + 4 \nu r^2 \delta(y) + \nu^2 r^2 - \nu \lambda r^2 \right) e^{-\lambda} - 4 \]

\[ + \frac{m_3^2}{r^2} e^{-\mu_0} \left[ r \nu'_0 - e^{\mu_0} + 1 \right] \delta(y) = \left( p_b - m_3^2 \Lambda \right) \delta(y) + p_x, \]
\[
\begin{align*}
&\frac{m_3^2}{4r} \left[ 2v' - 2\mu' + 2\lambda' + 2v''r + v'^2r + 2\lambda''r + \lambda'^2r - \mu'v' + v'\lambda'r - \mu'\lambda'r \right] e^{-\mu} \\
&\quad + \left( 2v + 4v\delta(y) + v'^2r + 2\mu r + 4\mu r\delta(y) + \mu^2 r + v\mu r - \mu\lambda r \right) e^{-\lambda}
\end{align*}
\]
\[
+ \frac{m_3^2}{4r} \left[ 2v_0' - 2\mu_0' - v_0\mu_0' + 2v_0''r + v^2r \right] \delta(y) = (p_b - m_3^2\Lambda)\delta(y) + p_a,
\]
\[
\frac{m_3^2}{4r} \left[ 4 - 4r\lambda' + 4v\mu'r^2 + v'^2r^2 + 2v''r^2 \right] e^{-\mu} + \mu\lambda r e^{-\lambda} = 4
\]
\[
= p_5,
\]
where a prime represents derivative with respect to \( r \). Since only \( \delta(y) \) can contribute to the brane equations, we obtain the Einstein field equations on the brane:
\[
\begin{align*}
m_3^2e^{-\mu_0} \left( \mu_0' - \frac{1}{r^2} + \frac{\rho_0}{r^2} \right) &= p_b(r) + \mathcal{U}(r) + m_3^2\Lambda, \\
m_3^2e^{-\mu_0} \left( \frac{v_0'}{r} + \frac{1}{r^2} - \frac{\rho_0}{r^2} \right) &= p_b(r) + \mathcal{P}(r) - m_3^2\Lambda,
\end{align*}
\]
where we have defined the induced energy density \( \mathcal{U}(r) \) and pressure \( \mathcal{P}(r) \) as
\[
\mathcal{U}(r) = m_3^2\mu \bigg|_{y=0} e^{-\lambda_0},
\]
\[
\mathcal{P}(r) = -m_3^2v \bigg|_{y=0} e^{-\lambda_0}.
\]

### 3. The virial theorem

To obtain the virial theorem in the context of the model discussed above, we use the tetrad formalism by defining the following frame of orthonormal vectors [17]:
\[
e^{(0)}_\rho = e^\rho_\rho, \quad e^{(1)}_\rho = e^\rho_\lambda \delta_\rho^\lambda, \quad e^{(2)}_\rho = r e^\rho_\rho, \quad e^{(3)}_\rho = r \sin \theta \delta^3_\rho,
\]
where \( \delta^{\alpha\beta} e_\beta^{(a)} e_\gamma^{(b)} = \eta^{(a)(b)} \) and the tetrad indices are surrounded by parentheses. The 4-velocity \( v^\mu \) of a typical galaxy with \( v^\mu v_\mu = -1 \) is written as
\[
v^{(a)} = v^\mu e^{(a)}_\mu, \quad a = 0, 1, 2, 3,
\]
Let us start with the Boltzmann equation in tetrad formalism. If \( f(x^\mu, v^{(a)}) \) represents the distribution function of galaxies, supposed to be made of identical and collisionless point particles, we have [17, 18]
\[
v^{(a)} v^\rho \frac{\partial f}{\partial x^\rho} + y^{(a)}_{(b)(c)} v^{(b)} v^{(c)} \frac{\partial f}{\partial v^{(a)}} = 0,
\]
Multiplying equation (23) by set contribute. Also, the velocity of galaxies is much smaller than the speed of light, so we can \( \gamma(a) \) where \( \gamma(\theta) \) is the speed of light in the velocity space and \( \langle v^2 \rangle \) represents the average values. Multiplication of equation (25) by \( 4\pi r^2 \) and integration over the cluster of galaxies yield

\[
- \int_0^R 4\pi \rho \left[ \langle v_r^2 \rangle + \langle v_\theta^2 \rangle + \langle v_\phi^2 \rangle \right] r^2 dr + \frac{1}{2} \int_0^R 4\pi r^3 \rho \left[ \langle v_r^2 \rangle + \langle v_\theta^2 \rangle + \langle v_\phi^2 \rangle \right] \frac{\partial \nu}{\partial r} dr = 0.
\]

We can also write equation (26) in the form

\[
2K = \frac{1}{2} \int_0^R 4\pi r^3 \rho \left[ \langle v_r^2 \rangle + \langle v_\theta^2 \rangle + \langle v_\phi^2 \rangle \right] \frac{\partial \nu}{\partial r} dr,
\]

since the total kinetic energy of galaxies is defined as

\[
K = \int_0^R 2\pi \rho \left[ \langle v_r^2 \rangle + \langle v_\theta^2 \rangle + \langle v_\phi^2 \rangle \right] r^2 dr.
\]

To obtain the virial theorem in our model we must express the energy–momentum tensor components in the terms of the distribution function. This is done according to

\[
T_{\mu\nu} = \int f m v_\mu v_\nu dv,
\]

which leads to

\[
\rho_0 = \rho \langle v_r^2 \rangle, \quad \rho_p = \rho \langle v_\theta^2 \rangle = \rho \langle v_\phi^2 \rangle.
\]

Adding equations (15), (16) and twice of (17) yields

\[
m_3 e^{-\mu_0} \left( \frac{v_0^2}{r} - \frac{v_0^2 v_0^2}{4} + \frac{v_0^2}{2} + \frac{v_0^2}{4} \right) = \frac{1}{2} \rho \langle v^2 \rangle + \frac{1}{2} [3P(r) - U(r)] - m^2 \Lambda,
\]

where we have defined \( \langle v^2 \rangle = \langle v_r^2 \rangle + \langle v_\theta^2 \rangle + \langle v_\phi^2 \rangle \). For the cluster of galaxies we may assume that \( \mu(r) \) and \( v(r) \) are small so that the quadratic terms in equation (31) do not contribute. Also, the velocity of galaxies is much smaller than the speed of light, so we can set \( \langle v_r^2 \rangle, \langle v_\theta^2 \rangle, \langle v_\phi^2 \rangle \ll \langle v_r^2 \rangle \approx 1 \) [11]. Taking these approximations into consideration, equation (31) is reduced to

\[
\rho = m_3^2 \frac{1}{r^2} \frac{d}{dr} \left( r^2 v_0^2 \right) + 2m^2 \Lambda - [3P(r) - U(r)].
\]
Multiplying equation (32) by \( r^2 \) and integrating from 0 to \( r \) yields
\[
m^2 r^2 v_0 = \frac{1}{4 \pi} M(r) + \frac{2}{3} m^2 \Lambda r^3 - \frac{1}{4 \pi} N(r) = 0,
\]
where
\[
M(r) = 4 \pi \int_0^r \rho r' \, dr',
\]
and
\[
N(r) = 4 \pi \int_0^r \left[ 3 \mathcal{P}(r') - U(r') \right] r'^2 \, dr'.
\]
Again, multiplying equation (33) by \( \frac{dM(r)}{r} \) and integrating from 0 to \( R \), we finally obtain the generalized virial theorem in a DGP scenario:
\[
W + 2K + \frac{1}{3} \Lambda I + W_B = 0,
\]
where
\[
W = -\frac{1}{8 \pi m_3^2} \int_0^R \frac{M(r)}{r} \, dM(r),
\]
\[
W_B = -\frac{1}{2 m_3^2} \int_0^R \rho r N(r) \, dr
\]
and
\[
I = \int_0^R r^2 dM(r),
\]
is the moment of inertia of the system. Without the last term, this would constitute the usual virial theorem with a cosmological constant, first derived by Jackson [17] using the Boltzmann equation (22) into which the metric of the spacetime is substituted. In order to obtain a relation between the virial mass and the extra term \( N(r) \) which has its origins in the bulk, we define the following radii [17]:
\[
R_v = \frac{M^2}{\int_0^R \frac{M(r)}{r} \, dM(r)},
\]
\[
R^2 = \frac{\int_0^R r^2 dM(r)}{M(r)},
\]
\[
R = -\frac{1}{8 \pi m_3^2} \frac{N^2}{W_B}
\]
where \( R_v \) is the virial radius and \( R \) is the radius defined by the extra term \( N \). By defining the virial mass as
\[
2K = \frac{1}{8 \pi m_3^2} \frac{M^2}{R_v},
\]
and using the relations
\[
W = -\frac{1}{8 \pi m_3^2} M^2 \frac{R^2}{R_v}, \quad I = MR^2,
\]

the generalized virial theorem (36) can be written as
\[
\left( \frac{M}{M} \right)^2 = 1 - \frac{8\pi M_\Lambda^2 R^2}{3 M} + \left( \frac{N}{M} \right)^2 \left( \frac{R}{M} \right).
\] (45)

The contribution of \( \Lambda \) to the mass of the galaxy is several orders of magnitude smaller than the observed mass. Also \( M_\Lambda \) is much larger than \( M \) for most galaxies. Therefore, we can neglect the unity and the term involving the cosmological constant in equation (45). The virial mass in our model is then given by
\[
M_v(r) \simeq N(r) \sqrt{R_v/M}.
\] (46)
As can be seen, the virial mass is proportional to an extra term stemming from the global bulk effects.

4. Estimating \( N(r) \)

In order to estimate \( N(r) \), we must solve the Einstein equations for \( P(r) \) and \( U(r) \) and obtain \( N(r) \) from (35). However there is a simpler way of doing this which we will follow. First, consider the conservation of the right-hand side of Einstein equations (15)–(17):
\[
\nu_0' = \frac{2}{\rho_b + p_b} + \frac{M}{\rho_b + p_b} + (p + U).
\] (47)
\[ U = 0. \] (48)
This means that we only need to calculate \( P(r) \). In most clusters the majority of the baryonic mass is in the form of intra-cluster gas. Taking this assumption into consideration and using equations (34) and (35), we obtain an expression for the total mass of the cluster:
\[
\frac{dM_{tot}}{dr} = 4\pi \rho_g r^2 + \frac{12\pi M_{tot}}{r}.
\] (49)
Another expression can be obtained from Jean’s equation,
\[
\frac{d}{dr} \left[ \rho_g \sigma^2 r \right] + \rho_g(r) \frac{d\Phi}{dr} = 0,
\] (50)
where \( \Phi(r) \) is the gravitational potential. We also assume that the gas is isotropically distributed inside the cluster so that the mass-weighted velocity dispersions in the radial and tangential directions are equal; \( \sigma_r = \sigma_{\theta,\phi} \). Assuming that the gravitational field is weak so that \( \Phi(r) \) satisfies the Poisson equation \( 2\pi m^2 \nabla^2 \Phi \approx \rho_{tot} \), Jean’s equation is reduced to
\[
\frac{d\rho_g(r)}{dr} = -\frac{1}{8\pi m^2 \rho_{tot}} \frac{M_{tot}}{r^2} \rho_g(r),
\] (51)
where we have used the relation \( p_g = \rho_g \sigma^2 r \). As has been shown in [19], the gas density \( \rho_g \) can be fitted to the observational data by the following radial distribution:
\[
\rho_g(r) = \rho_0 \left(1 + \frac{r^2}{r_c^2} \right)^{-\frac{\beta}{2}},
\] (52)
where \( r_c \) is the core radius and \( \rho_0 \) and \( \beta \) are cluster independent constants. For most clusters \( \beta \geq \frac{2}{3} \) [19] and therefore, in the limit \( r \gg r_c \) considered here, the gas density distribution can be written as
\[
\rho_g(r) = \rho_0 \left( \frac{r}{r_c} \right)^{-\beta}, \quad \beta \geq \frac{2}{3}.
\] (53)
Moreover, we choose the following equation of state for the intra-cluster gas [19]:  
\[ p_g(r) = \frac{k_B T_g}{\mu m_p} \rho_g(r), \tag{54} \]
where \( \mu = 0.61 \) is the mean atomic weight of the particles in the cluster gas and \( m_p \) is the mass of proton. With these assumptions, equation (51) reduces to  
\[ M_{\text{tot}}(r) = 8\pi m_1^3 \frac{k_B T_g}{\mu m_p} \beta r. \tag{55} \]
The contribution of the gas density to the total mass of the cluster is very small at the boundary of the cluster, i.e. where \( r \gg r_c \), as can be seen from equations (53) and (49). We may now calculate \( \mathcal{P}(r) \) from equations (49) and (55), taking the above approximation,  
\[ \mathcal{P}(r) = 2m_2^2 \frac{k_B T_g}{\mu m_p} \beta \frac{1}{r^2}. \tag{56} \]
and using equation (35) to obtain  
\[ N(r) = M_{\text{tot}}(r) = 8\pi m_1^3 \frac{k_B T_g}{\mu m_p} \beta r. \tag{57} \]
This equation is obviously an approximation since the contribution of the baryonic mass is neglected. However this is no cause for concern since the value of the baryonic mass of the clusters is about three orders of magnitude smaller than its total mass. As can be seen, \( N(r) \propto r \) and since the virial mass is proportional to \( N \) and the latter is proportional to \( r \), this could offer a possible resolution to the virial mass discrepancy in the context of DGP brane worlds.

To estimate the value of \( N \), we first note that \( 8\pi m_2^3 = G^{-1} = \frac{4}{3} G_{x}^{-1} \) where \( G_{x} \) is the gravitational constant [8]. A typical value of the temperature of a cluster gas is \( k_B T_g \approx 5 \) keV [19]. The virial radius of the cluster of galaxies is usually assumed to be \( r_{200} \), indicating the radius for which the mass density of the cluster is about \( \rho_{200} = 200\rho_c \), where \( \rho_c = 4.6975 \times 10^{-27} h_{50}^2 \) kg m\(^{-3}\). The virial mass of the cluster is then estimated as \( M_V = M_{200} = M(r < r_{200}) \). We can therefore define the maximum extension of the \( N(r) \) mass to be the radius at which \( \mathcal{P} = \rho_{200} \). From equation (56) we have  
\[ r_{\text{max}} = 4.28 \beta^2 h_{50}^{-1} \left( \frac{k_B T_g}{5 \text{ keV}} \right)^{\frac{1}{2}} \text{ Mpc}. \tag{58} \]
Finally, we can estimate \( N(r) \) from (57):  
\[ N(r) = 32.72 \times 10^{14} \beta^2 \left( \frac{k_B T_g}{5 \text{ keV}} \right)^{3} M_\odot. \tag{59} \]
This is in agreement with observational values for the virial mass of clusters [19].

5. Radial velocity dispersion

Another important observational quantity is the radial velocity dispersion which plays an important role in estimating the virial mass of the clusters. As is well known, the simple form  
\[ \sigma^2 = B/(r + b) \]
can be used to fit the observational data [20]. The DGP model can provide an expression for the radial velocity dispersion which we will derive in this section.

Adopting the approximations used after equation (31), we may write equation (25) as  
\[ \frac{d}{dr}(\rho \sigma^2) + \frac{1}{2} \rho v_0^2 = 0, \tag{60} \]
where we have assumed that the velocity distribution in the cluster is isotropic, so that $\langle v^2 \rangle = \langle v_r^2 \rangle + \langle v_\theta^2 \rangle = 3 \langle v_r^2 \rangle = 3 \sigma_r^2$. Moreover, from the Einstein field equations we have

$$m_3^2 \left( \frac{2\nu'_0}{r} + \nu''_0 \right) = 3 P(r) + \rho(r).$$  \hspace{1cm} (61)

Integrating, we obtain

$$m_3^2 v'_0 = \frac{1}{4\pi} N(r) + \frac{1}{4\pi} M(r) + C_1,$$  \hspace{1cm} (62)

where $C_1$ is some constant. The differential form of the radial velocity dispersion can be obtained from equations (60) and (62) as

$$2m_3^2 \frac{d}{dr} (\rho \sigma_r^2) = -\frac{N(r)}{4\pi r^2} \rho(r) - \frac{M(r)}{4\pi r^2} \rho(r) - \frac{C_1}{r^2} \rho(r).$$  \hspace{1cm} (63)

Now, using expressions (53) and (57) for $\rho(r)$ and $N(r)$ and by virtue of equation (34) we obtain

$$\sigma_r^2 = \frac{k_B T_e}{\mu m_p} = \frac{\rho_0 r^2}{12(\beta - 1)(3\beta - 1)m_3^2} \left( \frac{r}{r_c} \right)^{-3\beta} + \frac{C_1}{2(3\beta + 1)m_3^2} \frac{1}{r} + \frac{C_2}{2m_3^2 \rho_0} \left( \frac{r}{r_c} \right)^{3\beta} \quad \beta \neq 1.$$  \hspace{1cm} (64)

For $\beta = 1$, we have

$$\sigma_r^2 = \frac{k_B T_e}{\mu m_p} = \frac{\rho_0 r^3}{8m_3^2} \ln \frac{r}{r_c} + \frac{C_3}{r} + \frac{C_2}{m_3^2 \rho_0} \left( \frac{r}{r_c} \right)^{3}.$$  \hspace{1cm} (65)
Our expression for $\sigma^2_r$ can therefore be used to fit the observational data. In figure 1, we have plotted the radial velocity dispersion for the cluster NGC5813. This cluster has $\beta = 0.766$ and $k_B T_g = 0.52 \text{ keV}$ [19], and the radial velocity dispersion is about $240 \text{ km s}^{-1}$ [21]. We see that the radial velocity dispersion (64) is compatible with the observed profiles [21, 20]. Since an expression for the radial velocity dispersion can be obtained by other theoretical methods which explain cluster discrepancies, such a relation can be used to differentiate them [11, 22].

6. Discussion

In this paper, we have considered the virial mass in the framework of DGP brane worlds. The resulting field equations on the brane have an additional term which is due to the geometry of the extra dimension and can be associated with a geometrical mass. The virial theorem was obtained by the use of the Boltzmann equation, assuming that galaxies in the cluster are point-like, non-interacting particles. We showed that the resulting virial theorem has an additional potential term due to the extra dimension. The virial theorem has also been exploited in other brane-world models [11, 12] to explain the virial mass discrepancy. The advantage of the DGP model however is in the explanation of the self-accelerating phase of the universe without resorting to dark energy in a consistent manner. We obtained the virial mass of clusters from the virial theorem and showed that it is proportional to the geometrical mass of the model. The behavior of the virial mass was investigated in section 4 and shown to be a linear function of the distance. To estimate the geometrical mass $\mathcal{N}$ we needed another relation in addition to the Einstein field equations to close the system of equations. Such a relation was obtained by using Jean’s equation for clusters, assuming that clusters have spherical symmetry and are in thermodynamic equilibrium.

The solution presented in this work offers a possible explanation to the question of the virial mass discrepancy. The radial velocity dispersion profile of clusters was also obtained, having two arbitrary constants, one with a coefficient decreasing with $r$ and the other increasing with $r$. This allowed us to use this expression for any cluster with $\beta \geq 2$. We used the observed radial velocity dispersion for the cluster NGC5813 as an example to show that our model can account for the velocity dispersion of clusters. In addition to explaining the observational data, the velocity dispersion profile can be used to study the various aspects of models predicting the virial mass.

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