A novel Local Correction-based (LC-b) approach for low-order control-oriented models

Nunzio Cotrufo¹, José Candanedo¹

¹CanmetENERGY, NRCan, 1615 Lionel-Boulet, Varennes J3X 1P7, Quebec, Canada
nunzio.cotrufo@canada.ca

Abstract. The development of a model is a crucial step for Model-based Predictive Control (MPC) strategies. Modelling is also the phase involving the most time and effort. Low-order models represent a practical solution to reduce time, information and technical expertise required to develop a control-oriented approach. However, it is essential to guarantee a reasonable prediction accuracy while keeping the model simple. In this paper, a novel Local Correction-based (LC-b) approach, which improves the prediction accuracy of low-order models of thermal zones, is presented. The LC-b approach enables accounting for local thermal effects, thus improving the predictions of the indoor air temperature and heating load. The proposed approach was used to model a two-storey building using operation measurements; the accuracy of predictions was evaluated in terms of statistical indices. Results show that the proposed LC-b approach improved the predictions from a conventional low-order model by up to 68%.

1. Introduction

Large opportunities to reduce building operational costs and energy may be attained through advanced building controls [1]. Within the context of Model-based Predictive Control (MPC) of buildings, low-order, simplified models are of significant interest. However, the development of an effective control-oriented model is the most time-consuming, labour-intensive phase, and it represents an obstacle for the deployment of MPC [2]. Low-order models are easier and faster to develop when compared to detailed physical, numerical or computational models[3]. For instance, lumped Resistance-Capacitance (RC) is a commonly used, grey-box technique for the thermal modelling of buildings [4, 5]. Picard et al. [6] investigated the minimum order of a RC model to guarantee prediction accuracy. Black-box techniques have been used as well for simplified building modelling [7-10]. An exhaustive review of modelling techniques for building advanced control is given in [1]. Although they are practical solutions for control-oriented modelling of buildings, low-order models fail to account for local effects. This study introduces a novel Local Correction-based (LC-b) approach for the improvement of low-order models, which enables accounting for different boundary conditions (e.g. weather conditions) of each room within the same thermal zone. The proposed LC-b approach was used to improve a conventional RC model and to predict the heating demand of a two-storey experimental building.

2. Case study building

The building modelled in this study is a two-storey, 250 m² research facility located in Ottawa (ON), Canada. The building includes 16 rooms, each one equipped with a thermostat and electrical baseboard heaters ranging between 300 and 1000 W, for a total installed heating power of 11.25 kW. All the rooms are controlled with the same set-point profile; therefore, the whole building was modelled as a single thermal zone. The facility can be operated under two modes: Mode #1, a PID control, and Mode #2, an
ON/OFF control rule with dead-band of ±0.1ºC. Under Mode #1, operation data for 14 days (January 19 to February 2, 2018) were collected and used for model training, while the following 8 days (February 3-10, 2018) were used for validation. Similarly, under Mode #2, measurements for 14 days (March 17-30, 2015) were used for model training, and the following 8 days (March 31 to April 7, 2015) were used for validation. Outdoor air temperature (OAT) measurements and horizontal solar radiation (HSR) data were retrieved from the Simeb database available online [11].

3. Methodology

The LC-b approach intends to improve existing low-order control-oriented models for building applications. Statistical indices such as maximum Absolute Error (AE\text{max}), Mean Absolute Error (MAE), Coefficient of Variation of the Root Mean Square Error (CV-RMSE), and coefficient of determination (R^2) were used to evaluate the accuracy of the model. An RC model with a 30-min simulation time-step was used as reference to evaluate the improvement of the heating load predictions due to the proposed LC-b approach.

3.1 RC modelling

A schematic of the second-order RC network considered in this study is shown in Figure 1-a. The order of a model corresponds to the number of differential equations needed to describe the model itself. Equations 1 and 2 are the discretized representation of two differential equations describing the thermal behavior of the building. Equation 3 represents the control rule, which derives the heating supplied by the heating system to the buildings ($Q_{aux}$ in figure 1). Equations 1, 2 and 3 are applied at each time-step $t$ within the prediction horizon, to derive the building dynamic and the heating system output at time (e.g. the output from eq. 3 is used as input to eq. 1 at the following time-step). The RC network from Figure 1 has two capacitances and three resistances (2C3R). Node 1, 2 and 3 represent the indoor air mass, the envelope, and the outdoor air, respectively. The resistances are equivalent thermal resistances between the indoor air and the building envelope ($R_{12}$), between the envelope and the outdoors ($R_{23}$), and the infiltration rate ($R_{13}$). The terms $Q_{sol}$ and $Q_{aux}$ stand for the solar radiation and the heat supplied by the heating system. The coefficients R and C were calibrated using the MATLAB \texttt{fmincon} function [12] and measurements from the training datasets. The Root Mean Squared Error (RMSE) of the predicted supplied heat ($Q_{aux}$) was used as cost function in the calibration routine.

$$T_{1(t+1)} = \frac{\Delta t}{C_1} \left[ Q_{aux}(t) + \frac{T_{2(t)} - T_{1(t)}}{R_{12}} + \frac{T_{3(t)} - T_{1(t)}}{R_{13}} \right] + T_{1(t)} \quad (1)$$

$$T_{2(t+1)} = \frac{\Delta t}{C_2} \left[ Q_{sol}(t) + \frac{T_{1(t)} - T_{2(t)}}{R_{12}} + \frac{T_{3(t)} - T_{2(t)}}{R_{23}} \right] + T_{2(t)} \quad (2)$$

$$Q_{aux(t+1)} = f_{CTRL}(T_{sp(t)} - T_{1(t)}) \quad (3)$$

where $T_i$ is the temperature at node $i$ (ºC), $\Delta t$ is the model time-step (1,800 s), $f_{CTRL}$ is the model control rule, and $T_{sp}$ is the indoor air temperature set-point (ºC).

3.1.1 Control rules. Although the indoor temperature set-point is the same at every room, the rooms air temperatures are slightly different among them, because of boundary effects. The control rules applied at each room under modes #1 and #2 cannot be directly applied to $T_1$ (Eq. 1), which is an average value of the indoor air temperature. Adjustments are needed:

- Under Mode #1, the control rule $f_{CTRL}$ from Eq. 3 is the same PID (same coefficients) applied at each room in the real building. The discrepancy between the average $T_i$ and the air temperature at each room is absorbed (indirectly taken into account) by the RC coefficients during calibration.
- Under Mode #2, the ON/OFF control rule applied in each room cannot be directly applied to the predicted average indoor air temperature $T_i$. Therefore, an “equivalent” proportional control rule was derived from measurements and used as $f_{CTRL}$ in Eq. 3; it uses a piece-wise linear regression model, to derive the heating load from the indoor air temperature set-point residual (Figure 1-b).
3.2 Local Correction–based (LC-b) approach

Modelling the heat transfer phenomena of each single room would require far more time, information and technical expertise than a conventional 2C3R model for the entire building. The goal of the proposed LC-b approach is to improve the model prediction accuracy by accounting for the air temperature and the local control rules in each room, without increasing the RC model order. In this novel approach the “equivalent” control rule $f_{CTRL}$ from Eq. 3 is replaced by a Local Correction Module (LCM), for each room, which includes: i) a correlation-based model, and ii) the actual local control rule implemented in the room. The correlation-based model is used to derive the temperature of each room from the predicted average indoor air temperature $T_1$ (as derived by Eq. 1) and the room boundary conditions (e.g. weather variables, adjacent room temperature, etc.). A schematic of the RC model modified with the LC-b approach is given in Figure 2. Several machine learning techniques were considered for the development of the correlation-based models. The selection of the most appropriate model is part of the LCM development. Once the LCM is developed, the coefficients R and C can be re-calibrated.

Figure 2. Schematic of the proposed LC-b approach applied at each simulation time-step.

The steps implemented at each time-step by the LC-b modified low-order model are listed below:

1. The RC model simulates the building thermal dynamics and predicts $T_1$ (eqs. 1 and 2);
2. The predicted $T_1$ is then used by the LCM to derive the air temperature $T_i$ in each $i$-room;
3. The $i$-room local control rule (e.g. PID or ON/OFF) is applied to the residual between the room set-point and air temperature $T_{room,i}$ to derive the $i$-room heating load ($Q_{room,i}$);
4. The total building heating load at time $t$ is derived by the aggregation of the heating load contributions from all rooms.

4. Results and discussion

To develop the LC-b model, the first step was to develop the correlation-based models used by the LCMs to predict the room air temperature. Measurements from the training datasets were used for this purpose. Secondly, the RC coefficients were calibrated. Finally, the prediction accuracy of the LC-b model was compared to the results from the reference RC model (Eqs. 1-3).

4.1 Correlation-based models

Four modelling techniques were considered for the development of the correlation-based models: i) a linear regression; ii) a Gauss Process Regression (GPR) with exponential Kernel function; iii) Gauss Process Regression (GPR) with Matérn Kernel function with a 3/2 coefficient; and iv) and Artificial Neural Network (ANN). Two different datasets were considered: dataset (a) including only $T_i$, and dataset (b) including $T_i$ and the outdoor air temperature (OAT). Results for the four considered techniques are given in Figure 3 in terms of average MAE between measured and predicted rooms air temperature. The GPR-based model with an exponential Kernel function and input dataset (b) provided the best results under both Mode #1 (MAE = 0.05, Figure 3-a) and #2 (MAE = 0.094, Figure 3-b).

![Figure 3. Results from correlation-based model development: a) Mode #1, and b) Mode #2.](image)

4.2 Prediction results

The predictions of the heating load from the RC reference and LC-b models were compared to the measurements. Results are given in Table 1 along with their variation from the reference RC to the LC-b modified model. The variations indicating a degradation of the prediction performance are marked with an asterix (*). Figure 4 shows measurements of the building heating load during few days under operation Mode #1, along with the predictions from the RC reference and LC-b models. From Figure 4, the predictions provided by the LC-b model appear to better capture the heating load daily fluctuations.

Indices from Table 1 indicate that the LC-b approach yielded major improvements of the heating load predictions from a second-order RC model. Under operation Mode #1, the statistical indices considered were improved by 38% in average (up to 68.6% in the case of $R^2$). Under Mode #2 the improvement due to the LC-b approach is also evident (e.g. 34.6% for the $AEmax$), although the MAE indicates a slight degradation (6.8% and 14.9%).

The selection of the appropriate statistical indices to be used for evaluating the performance of a control-oriented model should take into account the objective of the control strategy for which the model is developed. For instance, if the objective is peak demand reduction, the maximum absolute error...
\((AEmax)\) is a more appropriate index than the MAE. On the other hand, if the control strategy targets the reduction of the total heating demand, the model evaluation will account for the mean bias error, eventually integrated by the CV-RMSE and/or the \(R^2\). The MAE should be considered when the most accurate prediction of the heating load at each time-step is needed. This is the case when, for instance, the control strategy objective is the reduction of the energy cost with a time-of-use energy rate structure. In this study the RMSE was used as cost function for the calibration of the RC coefficients, which led to the improvement of most of the statistical indices considered. The improvement of a specific statistical index can be addressed by setting the appropriate cost function along model calibration.

In this study an RC model was considered. However, the proposed LC - b approach is intended to improve the prediction accuracy of low-order models regardless of the modelling technique used to develop it. However, the level of accuracy will vary depending on the modelling approach that is adopted. The schema from Figure 2 can be implemented with any model that predicts the average indoor air temperature of a thermal zone. The correlation-based models can be developed quickly, as long as room measurements are available, and do not imply increase of required time or expertise.

### Table 1. Heating load prediction accuracy of the RC reference model and LC-b model along training (train.) and validation (valid.) periods.

| Mode | Model   | \(AEmax\) (W) | MAE (W) | CV-RMSE (%) | \(R^2\) |
|------|---------|---------------|---------|-------------|--------|
|      |         | train.        | valid.  | train.      | valid. | train.    | valid. |
| #1   | RC (ref.) | 6,940         | 8,770   | 1,067       | 1,702  | 32.4      | 59.5   |
|      | LC-b     | 5,191         | 4,561   | 657         | 1,425  | 18.9      | 45.5   |
|      | Variation| -25.2%        | -48.0%  | -38.4%      | -16.3% | -42.7     | -23.5% |
| #2   | RC (ref.) | 10,435        | 10,070  | 1,146       | 938    | 51.8      | 75.0   |
|      | LC-b     | 7,999         | 6,583   | 1,224       | 1,078  | 48.0      | 68.7   |
|      | Variation| -23.4%        | -34.6%  | +6.8%*      | +14.9%*| -9.3%     | -8.4%  | +16.0% +16.0% |

![Figure 4. Heating load measured and predicted from the RC reference and LC-b models (Mode #1).](image)

### 5. Conclusions

A novel LC-b approach was presented to improve low-order control-oriented models of thermal zones. The proposed LC-b approach has been validated using operational data from a two-storey building in Ottawa, ON, under two operation modes. The model developed with the LC-b approach was used to predict the building heating load, and the accuracy of these predictions was compared to that of a conventional RC model using statistical indices. The LC-b approach improved almost all the considered statistical indices by up to 68.6% (Table 2). Only the MAE under Mode #2 showed a slight degradation. It is worth mentioning that only commonly available measurements are needed (e.g. indoor air temperature, power input, outdoor conditions), and the presented approach can be applied despite of the thermal zone modelling technique used (e.g. RC, black-box, etc.). Finally, when compared with a traditional low-order model, its ability to represent the building dynamics was higher, while preserving the typical advantages associated with simplified models (e.g. reduced modelling effort, time, and required information). In conclusion, results suggest that the LC-b approach can be used to develop...
more accurate, yet simple, low-order control-oriented models. However, the objective of the control strategy should be taken into account when calibrating the LC\textsubscript{-}b control-oriented model. Further developments of this study should investigate other modelling techniques for the development of the correlation-based models, and include additional variables (i.e. solar radiation, adjacent room temperature) to the inputs of those models. Those additional variables are expected to provide additional information about the room’s boundary conditions, and thus enabling more accurate local corrections.

Acknowledgments
We would like to thank the Office of the Research and Development of Natural Resources Canada for the financial support of this study through the PERD program, and the National Research Council Canada for providing technical support and access to the facilities.

References
[1] X. Li and J. Wen, "Review of building energy modeling for control and operation.," *Renewable Energy Review*, vol. 37, pp. 517-537, 2014.
[2] M. Killian and M. Kozek, "Ten questions concerning model predictive control for energy efficient buildings," *Building and Environment*, vol. 105, pp. 403-412, 2016.
[3] S. Pivara, J. Cigler, Z. Vana, F. Oldewurtel, C. Sagerschnig and E. Zacekova, "Building modeling as a crucial part off building predictive control," *Energy and building*, vol. 56, pp. 8-22, 2013.
[4] J. Date, J. Candanedo, A. Athienitis and K. Lavigne, "Predictive setpoint optimization of a commercial building subject to a winter demand penalty affecting 12 months of utility bills," 2017.
[5] J. Vivian, A. Zarrella, G. Emmi and M. De Carli, "An evaluation of the sustainability of lumped-capacitance models in calculating energy needs and thermal behaviour of buildings," *Energy and Buildings*, vol. 150, pp. 447-465, 2017.
[6] D. Picard, J. Drgoňa, M. Kvasnica and L. Helsen, "Impact of the controller model complexity on model predictive control performance for buildings," *Energy and Buildings*, vol. 152, pp. 739-751, 2017.
[7] F. M. Gray and M. Schmid, "Thermal building modelling using Gaussian processes," *Energy and Buildings*, vol. 119, pp. 119-128, 2016.
[8] P. D. Morosan, R. Bourdais, D. Dumur and J. Buisson, "Building temperature regulation using a distributed model predictive control," *Energy and Buildings*, vol. 42, pp. 1445-1452, 2010.
[9] F. Smarra, A. Jain, T. de Rubeis, D. Ambrosini, A. D'I\'nocenzo and R. Mangharam, "Data-driven model predictive control using random forests for building energy optimization and climate control," *Applied Energy*, vol. 226, pp. 1252-1272, 2018.
[10] A. Jain, F. Smarra, M. Behl and R. Mangharam, "Data-driven model predictive control with regression trees - An application to building energy management.," vol. 1, no. 2, p. 4, 2018.
[11] Simeb, "Weather data," [Online]. Available: https://www.simeb.ca:8443/index_fr.jsp (last access 05/13/2019).
[12] Mathworks, "fmincon," [Online]. Available: https://www.mathworks.com/help/optim/ug/fmincon.html (last access 05/13/2019).