The Stability of the Gauge Hierarchy in $SU(5) \times SU(5)$ *

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Abstract

It has been shown that the Dimopoulos-Wilczek (or missing-VEV) mechanism for doublet-triplet splitting can be implemented in $SU(5) \times SU(5)$ models, which requires no adjoint Higgs fields. This is an advantage from the point of view of string theory construction. Here the stability of the gauge hierarchy is examined in detail, and it is shown that it can be guaranteed much more simply than in $SO(10)$. In fact a $Z_2$ symmetry ensures the stability of the DW form of the expectation values to all orders in GUT-scale VEVs. It is also shown that models based on $SO(10) \times SU(5)$ have the advantages of $SU(5) \times SU(5)$ while permitting complete quark-lepton unification as in $SO(10)$.

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1 Introduction

The impressive unification of gauge couplings in the MSSM$^1$ is regarded by many as strong evidence for supersymmetric grand unification. This has led to renewed interest in finding satisfactory unification schemes and to attempts to derive such schemes from string theory. Many gauge groups have been considered. The simplest possibilities, at least at first sight, are $SU(5)$ and $SO(10)$. An $SU(5)$-based theory is strongly suggested by the unification of couplings and mass relations such as $m_b(M_{GUT}) = m_\tau(M_{GUT})$, while $SO(10)$ gives more complete quark-lepton unification. However, both these groups require adjoint Higgs for symmetry breaking, which is a drawback from the point of view of string theory.$^2$ Moreover, there is the problem of finding a satisfactory mechanism for doublet-triplet splitting$^3$ in models based on these groups.

The only technically natural scheme for doublet-triplet splitting in $SU(5)$ is the missing-partner mechanism$^4$, but this requires the large Higgs representations $75$, and $50 + \bar{50}$. (The sliding-singlet mechanism$^5$ is unstable to radiative corrections in $SU(5)$, though it is not in certain larger groups.$^6$)

In $SO(10)$ the only available scheme for doublet-triplet splitting is the Dimopoulos-Wilczek (or “missing VEV”) mechanism.$^7$ It has been shown$^8,9$ that this can be implemented in a technically natural way in $SO(10)$, but there are difficulties which require relatively elaborate model-building to solve. The trickiest of these is breaking the rank of $SO(10)$ down to 4 at a high scale. (This is necessary if righthanded neutrinos are to have a large mass.) The problem is that the sector which breaks the rank (probably having to contain at least a $16 + \bar{16}$ of Higgs fields) will, if it couples to the Dimopoulos-Wilczek (DW) sector, generally destabilize the DW form of the VEVs needed to give the doublet-triplet splitting. On the other hand, if these two sectors do not couple — or couple only weakly — to each other, there arise goldstone or pseudo-goldstone bosons that badly affect $\sin^2\theta_W$. In Ref. 8 a way was proposed to overcome this problem, involving a totally antisymmetric interaction among three distinct adjoint Higgs fields, $\sum_{abc}(\Omega_1)_a^b(\Omega_2)_b^c(\Omega_3)_c^a$. However, having three distinct adjoint Higgs fields with different symmetry properties may be difficult or impossible to achieve in string theory.$^2$

Flipped $SU(5) \times U(1)$ can be broken to the Standard Model without adjoint Higgs, and also admits an extremely elegant implementation of the
missing-partner mechanism. However, as this is not really a grand unified group it does not explain the precise unification of gauge coupling that has been seen. Moreover, it does not give such successful relations as \( m_b = m_\tau \) at the unification scale.

A very elegant possibility that preserves the good features of \( SU(5) \) but avoids the problems mentioned above is \( SU(5) \times SU(5) \), with the Standard Model group contained in the “diagonal” \( SU(5) \) subgroup.\(^{10,11,12}\) This allows the symmetry to be broken without adjoint Higgs. Instead, there are Higgs in the \((5,\bar{5}) + \text{h.c.}\), which under the diagonal subgroup decompose into \( 24 + 1 \). Barbieri, Dvali, and Strumia\(^1\) have pointed out that the Dimopoulos-Wilczek mechanism is simply implemented in this group and in other groups of the form \( G \times G \). These have been studied in the context of string theory by a number of groups\(^2\) and seem very promising.

In this paper we examine in detail the question of the stability of the gauge hierarchy in \( SU(5) \times SU(5) \). We find that the DW form of the vacuum expectation values can in a simple way be rendered stable to all orders in GUT-scale VEVs by merely a \( Z_2 \) symmetry. Some of the difficulties that exist in implementing the DW mechanism in \( SO(10) \) are avoided. But \( SU(5) \times SU(5) \) still does not give the full quark-lepton unification that is the most beautiful feature of \( SO(10) \). We show that \( SO(10) \times SU(5) \) does this while still avoiding the problems of \( SO(10) \) itself.

2 The Stability of the Hierarchy in \( SU(5) \times SU(5) \)

We will discuss an \( SU(5) \times SU(5)' \) model with Higgs fields in the following representations: \( H_I = (5,\bar{5}), \quad \overline{H}_I = (\bar{5},5) \), where \( I = 1, 2 \), and \( h = (5,1), \quad \overline{h} = (\bar{5},1), \quad h' = (1,5), \quad \overline{h}' = (1,\bar{5}) \). Aside from some gauge singlets, these are the only Higgs fields that will be needed to do all the symmetry breaking.

Consider, first, the following terms in the superpotential that contain only \( H_1 \) and \( \overline{H}_1 \):

\[
W_1 = A_1 M \text{Tr}(H_1 \overline{H}_1) + B_1 M^{-1} \text{Tr}(H_1 \overline{H}_1 H_1 \overline{H}_1) + B'_1 M^{-1} [\text{Tr}(H_1 \overline{H}_1)]^2. \quad (1)
\]

Here \( \text{Tr}(H_1 \overline{H}_1 H_1 \overline{H}_1) \equiv \sum (H_1)^{\alpha \beta} (\overline{H}_1)^{\alpha'} (H_1)^{\beta \beta'} (\overline{H}_1)^{\alpha'.} \) (We use unprimed
Greek indices for $SU(5)$ and primed for $SU(5)'$. The mass scale $M$ is assumed to be of order $10^{16}$ GeV. It is trivial to see that one solution is

$$
\langle (H_1)_{\alpha}^{\prime} \rangle = \langle (\overline{H}_1)_{\alpha} \rangle = \begin{pmatrix} a_1 & a_1 \\ a_1 & 0 \\ 0 & 0 \end{pmatrix},
$$

(2)

where $a_1 = \sqrt{-\frac{A_1}{6B_1 + 2B_1^\prime}} M$. This is the form required for the Dimopoulos-Wilczek mechanism for doublet-triplet splitting. The two vanishing diagonal entries are responsible (see below) for the lightness of the two doublet Higgs fields of the Standard Model. It should be noted that the diagonal forms with $n$ vanishing diagonal entries and $5 - n$ equal to $a_1$ are also solutions.

This so far only breaks $SU(5) \times SU(5)'$ to $SU(3) \times SU(2) \times SU(2) \times U(1)$. The rest of the breaking can be done by the fields $H_2$ and $\overline{H}_2$. Assume that these have a superpotential of the same form as Eq. (1) (just replacing the index ‘1’ everywhere in Eq. (1) with a ‘2’) and from it acquire the VEVs

$$
\langle (H_2)_{\alpha}^{\prime} \rangle = \langle (\overline{H}_2)_{\alpha} \rangle = \begin{pmatrix} 0 & 0 \\ 0 & a_2 \\ a_2 & a_2 \end{pmatrix},
$$

(3)

where $a_2 = \sqrt{-\frac{A_2}{6B_2 + 2B_2^\prime}} M$. Taken together the VEVs in Eqs. (2) and (3) break $SU(5) \times SU(5)'$ all the way down to the Standard Model. Henceforth, we will call the form of the VEVs given in Eqs. (2) and (3) the “DW form”.

The two sectors, $(H_1, \overline{H}_1)$ and $(H_2, \overline{H}_2)$, must couple together if goldstone modes are to be avoided. In particular, the generators of $\frac{SU(5)}{SU(3) \times SU(2) \times SU(5)'}$ are broken in both sectors, so that there are two sets of uneaten goldstones in the representations $[(3, 2)^{-\frac{5}{2}} + (\overline{3}, 2)^{\frac{5}{2}}]$ as well as the two sets in $[(3, 2)^{-\frac{5}{2}} + (\overline{3}, 2)^{\frac{5}{2}}]$ that do get eaten (one set for each $SU(5)$).

The two sectors could be coupled together by a term like $\text{Tr}(H_2 \overline{H}_1)$, but that would clearly destabilize the DW form of the VEVs, as then, for instance, the $F_{\overline{H}_1} = 0$ equation would have a term proportional to $H_2$. Such a term must be ruled out, and this can be done by a $Z_2$ symmetry, $K$, under which
$H_2 \rightarrow -H_2$ and $\bar{H}_2 \rightarrow -\bar{H}_2$. This still allows the following mixed terms at the quartic level:

$$\text{Tr}(H_1 \bar{H}_1)\text{Tr}(H_2 \bar{H}_2), \text{ Tr}(H_1 \bar{H}_2)\text{Tr}(H_2 \bar{H}_1)$$

$$(\text{Tr}(H_1 \bar{H}_2))^2, \text{ (Tr}(H_2 \bar{H}_1))^2$$

$$\text{Tr}(H_1 \bar{H}_1 H_2 \bar{H}_2), \text{ Tr}(H_1 \bar{H}_2 H_2 \bar{H}_1)$$

$$\text{Tr}(H_1 \bar{H}_2 H_1 \bar{H}_2), \text{ Tr}(H_2 \bar{H}_1 H_2 \bar{H}_1).$$

(4)

It is easy to check that the last four terms in this list give mass to all of the would-be goldstone bosons that are not eaten by the gauge bosons. For example, the term

$$(H_1)_{i}^{a}\langle(\bar{H}_1)_{i}^{a}\rangle(H_2)_{i}^{a}\langle(\bar{H}_2)_{i}^{a}\rangle,$$

(5)

where we use $a$ for $SU(3)$ indices and $i$ for $SU(2)$ indices, couples a $(\bar{3}, 2)^{\frac{1}{2}}$ in $H_1$ to a $(3, 2)^{\frac{1}{2}}$ in $H_2$.

A crucial issue is whether the DW form is stable under the influence of these operators. It is easy to see that it is. For example, it is necessary for the stability of the DW form of the VEV of $H_1$ that there be no contributions to the $F_{\bar{H}_1}$ that are proportional to powers of $H_2$ or $\bar{H}_2$. The first term in the list gives a contribution to $F_{\bar{H}_1}$ of $H_1\text{Tr}(H_2 \bar{H}_2)$ which is proportional to $H_1$ as required. The second term in the list contributes $H_2\text{Tr}(H_1 \bar{H}_2)$ to $F_{\bar{H}_1}$, which might seem to be a problem, except that $\text{Tr}(H_1 \bar{H}_2)$ vanishes when the fields take the forms given in Eqs. (2) and (3). Similarly, the other mixed quartic terms listed above do not destabilize the DW form.

Since essential use is being made of non-renormalizable operators here (see the discussion of this below) it is important that one show that the gauge hierarchy is stable even when higher-than-quartic operators are taken into account. The zeros on the diagonal of the DW form must vanish at least to order $M_{\text{GUT}}^{3}/M_{\text{Pl}}$ and probably higher.

In fact, it is not difficult to show that the DW form given in Eqs. (2) and (3) is stable to all orders in GUT-scale VEVs because of the simple $Z_2$ symmetry that we called $K$. Let us call the value of some product of fields, $\Pi$, when these fields take the DW form $\langle \Pi \rangle_{\text{DW}}$. Suppose there is a term, $T$, in the superpotential $W$ that destabilizes the DW form of the VEV of $H_1$. 

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Then this term forces \( \langle (H_1)_4^4 \rangle = \langle (H_1)_5^5 \rangle \neq 0 \). (Given the forms of \( \langle H_2 \rangle \) and \( \langle \overline{H}_2 \rangle \), this is the only potential instability of \( H_1 \) which we need consider.) This could happen only if \( \langle (\partial T/\partial \overline{H}_1)^{\alpha \alpha'}_{\omega \omega'} \rangle_{\text{DW}} = t(\delta_4^\alpha \delta_4^{\alpha'} + \delta_5^\alpha \delta_5^{\alpha'}) \neq 0 \). But that would imply that \( \Sigma_{\alpha, \alpha'} \langle (\overline{H}_2)_\omega^{\alpha'} (\partial T/\partial \overline{H}_1)^{\gamma \gamma'}_{\gamma' \omega} \rangle \neq 0 \). In other words, replacing a factor of \( \overline{H}_1 \) by \( \overline{H}_2 \) in \( T \) leads to a term \( T' \) which has the property that \( \langle T' \rangle_{\text{DW}} \neq 0 \). In the same way if the DW form of \( H_2 \) is unstable, there must be a term which after a factor of \( \overline{H}_2 \) is replaced by \( \overline{H}_1 \) does not vanish when the fields take the DW form.

If we can prove, then, that for every possible term in \( W \), changing one factor of \( H_1 \) (or \( \overline{H}_1 \)) to \( H_2 \) (or \( \overline{H}_2 \)) or vice versa leads to a term \( T' \) such that \( \langle T' \rangle_{\text{DW}} = 0 \), we shall have shown that the DW form is stable. But the terms in \( W \) all have, by the symmetry \( K \), an even number of factors of \( H_2 \) or \( \overline{H}_2 \). Thus replacing a field with label 1 by one with label 2, or vice versa, always produces a terms with an odd number of factors of \( H_2 \) or \( \overline{H}_2 \). All that needs to be shown, then, is that for any expression, \( T' \), with an odd number of factors of \( H_2 \) or \( \overline{H}_2 \), \( \langle T' \rangle_{\text{DW}} = 0 \). This is easy to do.

Consider, first, a term, \( T' \), with no \( SU(5) \) or \( SU(5)' \) \( \epsilon \)-symbols. Such a term must be a product of factors of the form \( \text{Tr}(H_1 \overline{H}_j H_K \cdots \overline{H}_L) \). But if \( \langle \text{Tr}(H_1 \overline{H}_j H_K \cdots \overline{H}_L) \rangle_{\text{DW}} \neq 0 \), then all of the labels \( I, J, K, ..., L \) must be the same, all 1's or all 2's — and there are an even number of such labels. Thus, if \( \langle T' \rangle_{\text{DW}} \neq 0 \) it must have an even number of factors of \( H_2 \) or \( \overline{H}_2 \).

The argument is almost as simple for terms which contain \( SU(5) \) and \( SU(5)' \) \( \epsilon \)-symbols. Consider a term, \( T' \) for which \( \langle T' \rangle_{\text{DW}} \neq 0 \). The simplest thing is to keep track of the \( SU(5) \) (\( SU(5)' \)) indices that take values in the \( SU(2) \) (\( SU(2)' \)) subgroup. Each \( \epsilon \)-symbol has two such indices, and three that take values in the \( SU(3) \) (\( SU(3)' \)) group. Because \( \langle T' \rangle_{\text{DW}} \neq 0 \) it must be that each factor of \( H_2 \) or \( \overline{H}_2 \) has one \( SU(2) \) and one \( SU(2)' \) index. And each factor of \( H_1 \) or \( \overline{H}_1 \) has no \( SU(2) \) or \( SU(2)' \) indices. Thus if we think of an \( SU(2) \) or \( SU(2)' \) index as being a line, we can think of \( \epsilon \)-symbols and factors of \( H_2 \) and \( \overline{H}_2 \) as being vertices into which precisely two lines enter, and there are no \( n \)-vertices with \( n \neq 2 \). Thus the lines representing the \( SU(2)' \) indices go around in a loop, and the term \( T' \) must in such a diagram be represented by a set of disconnected single loops. Now in each such loop \( SU(2) \) indices are converted into \( SU(2)' \) indices, or vice versa, by the factors of \( H_2 \) and \( \overline{H}_2 \), but not by the \( \epsilon \)-symbols. Thus each loop must contain an even number of factors of \( H_2 \) or \( \overline{H}_2 \) and so also, therefore, must the term \( T' \). But this is what we had to prove. Thus the VEVs given in Eqs. (2) and (3) are stable.
solutions to all orders because of the simple $Z_2$ symmetry, $K$.

The only other Higgs fields in the model, $h$, $\wedge$, $h'$, and $\vec{h}$, have either vanishing or Weak-scale VEVs, so that the stability of the gauge hierarchy is guaranteed by what has been said already.

In the foregoing, we have made essential use of non-renormalizable operators, and the question arises whether these could have been the result of integrating out fields of mass $M \sim M_{GUT}$. The answer is yes if the fields integrated out include adjoints of $SU(5)$ and $SU(5)'$. For example, if one had $H_1 \overline{H}_2 X + M X^2$, integrating out $X$ would give $(H_1 \overline{H}_2)(H_1 \overline{H}_2)$. But if $X$ is a singlet, then only the contraction $[\text{Tr}(H_1 \overline{H}_2)]^2$ results. This does not give mass to the uneaten would-be goldstone bosons. If $X$ is a $(24, 24)$ then one obtains $\text{Tr}(H_1 \overline{H}_2 H_1 \overline{H}_2)$ as required to give mass to the goldstones. Note as well that $\langle X \rangle$ is determined by the equation $F_X = 0$ to be $\langle X \rangle = \frac{1}{2} \langle H_1 \rangle \langle \overline{H}_2 \rangle = 0$. Thus the desired form of Eqs. (2) and (3) is not destabilized by $\langle X \rangle$. The choice would therefore seem to be between having adjoint representations of $SU(5)$ or having higher-dimensional operators as in Eq. (1).

The $K$-invariant operators involving $H_1$, $\wedge_1$, $H_2$, and $\overline{H}_2$ up to fourth order have an accidental $U(1)^2$ symmetry. The charges of $(H_1, \wedge_1, H_2, \overline{H}_2)$ under these $U(1)$'s are $(1, 1, -1, -1)$ and $(1, -1, 1, -1)$. One might worry, therefore, about axions or goldstone bosons. But at quintic and higher order there are $K$-invariant operators that explicitly break these accidental $U(1)$'s. For example, there is $(H_1)^\alpha_\gamma (H_1)^{\beta_\gamma}(H_1)^{\gamma_\delta}(H_1)^{\delta_\epsilon}(H_1)^{\epsilon_\delta} \epsilon_\alpha \beta \gamma \delta \epsilon'$ and the same structure with two or four factors of $H_1$ replaced by $H_2$.

It is an interesting question whether other possibilities exist for breaking down to the Standard Model gauge group besides the pattern of VEVs given in Eqs. (2) and (3). It was noted in Ref. 9 that any two of the three forms, diag$(a, a, a, 0, 0)$, diag$(0, 0, a, a, a)$, and diag$(a, a, a, a, a)$, will achieve this breaking. However, the inclusion of VEVs in the $SU(5)_V$-singlet direction diag$(a, a, a, a, a)$ seems to greatly complicate the problem of achieving a stable DW mechanism. For example let there be a set of Higgs, $H_3$ and $\overline{H}_3$, with VEVs in this direction as well as the set $H_1$ and $\overline{H}_1$. In this case the generators in $\overline{SU(3) \times SU(3)'}_{SU(3)_V}$ are broken in both the $(H_1, \overline{H}_1)$ and $(H_3, \overline{H}_3)$ sectors. To avoid these (and other) consequent goldstone modes these two sectors must be coupled. However, the method that worked above does not work here. A $Z_2$ symmetry under which $H_3$ and $\overline{H}_3$ are odd will indeed
forbid the dangerous term $\text{Tr}(H_3\overline{H}_1)$, but terms like $[\text{Tr}(H_3\overline{H}_1)]^2$ are no less dangerous, since $\text{Tr}(H_3\overline{H}_1)$ does not vanish, and therefore all such mixed terms destabilize the hierarchy. It may be possible to stabilize the hierarchy in the presence of such $SU(5)_V$-singlet VEVs, but it would doubtless be complicated. This problem is reminiscent of the difficulty of stabilizing the DW form in $SO(10)$ in the presence of $SU(5)$-singlet VEVs which are needed there to break the rank of the group and make righthanded neutrinos superheavy.$^8$,$^9$

So far it has been shown that one can achieve the DW form (Eq. (2)), have it stable to all orders in the GUT-scale VEVs, completely break to the Standard Model gauge group, and avoid pernicious goldstone modes. All of this we have achieved with a simple $Z_2$ discrete symmetry and a small set of Higgs representations. This is quite simple compared to what was shown to be necessary in $SO(10)$ in Refs. 8 and 9.

The doublet-triplet splitting itself can be achieved in a way closely analogous to what was suggested in Refs. 8 and 9 for $SO(10)$. Gauge symmetry allows the terms $M\overline{h}h$ and $M'\overline{h}'h'$. If both of these are present with $M \sim M' \sim M_{\text{GUT}}$, then all of the Weak-doublet Higgs become superheavy and the gauge hierarchy is destroyed. Suppose, then, that a discrete symmetry, $K_M$, forbids the $M\overline{h}h$ term, but allows $M'\overline{h}'h'$. Then the pair of doublets in $\overline{h} + h$ would remain light, to play the role of the Higgs of the MSSM. The color-triplets in $\overline{h} + h$ become heavy from the terms $\overline{h}\langle H_1 \rangle h'$ and $\overline{h'}\langle \overline{H}_1 \rangle h$. The triplets have a $2 \times 2$ mass matrix of the form

$$\langle \overline{h}_a, \overline{h}_a' \rangle \begin{pmatrix} 0 & \langle (H_1)_a \rangle \\ \langle (\overline{H}_1)_a \rangle & M' \end{pmatrix} \begin{pmatrix} h^a \\ h'^a \end{pmatrix}, \quad (6)$$

while the doublets have the mass matrix

$$\langle \overline{h}_i, \overline{h}_i' \rangle \begin{pmatrix} 0 & \langle (H_1)_i \rangle \\ \langle (\overline{H}_1)_i \rangle & M' \end{pmatrix} \begin{pmatrix} h^i \\ h'^i \end{pmatrix}. \quad (7)$$

In order for the doublets in $\overline{h} + h$ to be light, there must be no couplings $\overline{h}H_2h'$ and $\overline{H}_2h$. But this is insured by just the $Z_2$ symmetry $K$ which reflects $H_2$ and $\overline{H}_2$, as long as $h$, $\overline{h}$, $h'$, and $\overline{h}'$ transform trivially under it. Moreover, such allowed higher-dimension operators as $(\overline{h}H_2h')\text{Tr}(H_1\overline{H}_2)$ are not dangerous because of the vanishing of $\text{Tr}(H_1\overline{H}_2)$ at the DW minimum.
It is clear that the symmetry $K$ together with the DW forms of the VEVs protect the hierarchy from such operators to all orders.

The only non-trivial problem is to ensure to sufficiently high order to preserve the gauge hierarchy that there is no term that effectively gives $M_{h\bar{h}}$. However, it was shown how to solve this problem for $SO(10)$ in Ref. 9, and the same kinds of solutions work here as well. A simple possibility is the following. Let $K_M = Z_n$. Suppose there is a singlet Higgs field, $S$, which under $K_M$ transforms as $S \rightarrow z^\ast S$ (but that there is no singlet filed $S$ that transforms as $S \rightarrow z^\ast S$). And suppose that under $K_M$ one has $h \rightarrow z h$, $h' \rightarrow z^\ast h'$, with all other fields transforming trivially. Then $K_M$ allows $S h h'$, $h H_1 h'$, and $h h' H_1 h$, but forbids $h h'$ and $S h h$. The lowest dimension operator that gives mass to the light doublets is then $S h h'/(n-1)h/M_{n-2}$. Thus the hierarchy can be made stable enough by making $n$ large. Of course, there are other, and perhaps more elegant, possibilities for $K_M$, but this example is enough to show that the dangerous term $h h'$ can be sufficiently suppressed.

(For a more complete discussion of the problem see Ref. 9.)

In the scheme for doublet-triplet splitting just described, the amplitude for Higgsino-mediated proton decay is suppressed by a factor of $M'_{GUT}/M'_{GUT}$. As noted in Ref. 8 it would be possible with a VEV of the form given in Eq. (3) replacing the mass $M'$ to suppress proton decay completely. However, it does not seem possible in the present context to have a term like $h' H_2 H_2 h'$ (which respects $K$) without also having $M'_{GUT} h'$. The light quarks and leptons most simply come from $\psi^A_{10} = (10, 1)$ and $\psi^A_{5} = (5, 1)$ of $SU(5) \times SU(5)'$, with $A = 1, 2, 3$ being the family index. Then $\psi^A_{10} \psi^B_{10} h$ and $\psi^A_{10} \psi^B_{5} h$ give quark and lepton masses just as in minimal $SU(5)$.

If $a_1$ and $a_2$ (in the VEVs of $H_1$ and $H_2$) are very different, then the value of $\sin^2 \theta_W$ would differ substantially from the minimal SUSY $SU(5)$ value. For example, if $a_1 \gg a_2$, one would have $SU(5) \times SU(5)' \rightarrow SU(3)_V \times SU(2) \times SU(2)' \times U(1)_V \rightarrow SU(3)_V \times SU(2)_V \times U(1)_V$, and corrections to $\sin^2 \theta_W$ would be of order $\frac{a_1}{30\pi} \ln(a_1/a_2)$. We can crudely estimate the effect on $\sin^2 \theta_W$ as follows. The largest threshold corrections come from multiplets in $(8, 1)^0$ and $(1, 3)^0$ of the Standard Model group, which contribute at one-loop $\Delta \sin^2 \theta_W(M_Z) = \frac{a_8}{30\pi}[21g_8 \ln M_8 - 24g_3 \ln M_3]$. ($g_8$ and $g_3$ represent the effective number of Higgs multiplets in the representations $(8, 1)^0$ and $(1, 3)^0$, and $M_8$ and $M_3$ represent their masses.) It is easily seen that $M_8 \sim a_1$ and $M_3 \sim a_2$. Moreover, $g_8 = g_3$. Thus one expects an effect
\[ \Delta \sin^2 \theta_W \sim g^2 \frac{\alpha}{4 \pi} 21 \ln(a_1/a_2) \sim 10^{-2} \ln(a_1/a_2), \]
if \( a_1/a_2 \) is very different from unity. However, one expects \( a_1/a_2 \) to be of order unity, and for that case the different multiplets will contribute with varying signs to the threshold corrections, which one would therefore expect to be somewhat less than \( 10^{-2} \).

### 3 \ SO(10) \times SU(5)

The group \( SU(5) \times SU(5) \) has been shown to have two main advantages over \( SO(10) \), namely the possibility of breaking symmetry all the way to \( SU(3) \times SU(2) \times U(1) \) without adjoint Higgs fields, and the simplicity of preserving the hierarchy while doing so. However, \( SO(10) \) has at least one greatly attractive feature, which is that it achieves complete quark-lepton unification. All the fermions of a generation are unified into one irreducible representation, and therefore the up-quark masses are related to the masses of the down-quarks and charged leptons. Moreover, righthanded neutrinos are predicted to exist.

If one generalizes the discussion given in the previous section to the group \( SO(10) \times SU(5) \) one easily sees that one can have all the advantages of \( SU(5) \times SU(5) \) combined with those of \( SO(10) \).

In an \( SO(10) \times SU(5)' \) model let there be the following Higgs fields: \( H_I = (10, 5), \overline{H}_I = (10, 5), I = 1, 2, \) and \( h = (1, 5), h' = (1, 5), \) and \( h'= (1, 5) \). The discussion closely parallels that in the last section. \( SO(10) \times SU(5)' \) contains \( SU(5) \times SU(5)' \) as a subgroup, under which \( H_I \) decomposes to a \( (5, 5) \) (which just corresponds to the \( H_I \) of the last section) and a \( (1, 5) \). With a superpotential that contains at least up to quartic terms, one can ensure that the \( (5, 5) \) parts of \( H_I \) and \( H_I \) get no VEVs they

For example, in \( SU(5) \times SU(5)' \) notation, terms like \( H_{\alpha 
abla} (H_{\alpha 
abla})^\dagger H_{\beta 
abla} (H_{\beta 
abla})^\dagger \) give mass to goldstones which are in \( 10 + \overline{10} \) representation of \( SU(5) \), that is, the ones from the coset \( \frac{SO(10)}{SU(5)} \).

The DW forms in Eqs. (2) and (3) can be guaranteed to all orders in GUT-scale VEVs by the same \( Z_2 \) symmetry as in the last section. Since the \( SU(5) \times SU(5)' \) \( (5, 5) \) and \( (5, 5) \) parts of \( H_I \) and \( \overline{H}_I \) get no VEVs they
can be ignored, and the proof of stability reduces exactly to that in the last section.

The doublet-triplet splitting is achieved just as in the last section, the only difference being that what we called $h$ and $\bar{h}$ there are both contained in the ten-dimensional $h$ in the present case. Thus the term that must be forbidden by “$K_M$” is $h^2$.

The quark-lepton unification is achieved simply by putting the known fermions in $(16,1)$ representations, which we shall call $\psi^A_{16}$, where $A$ is the family index. Then all the Dirac mass matrices come from the coupling $\psi^A_{16}\psi^B_{16}h$, and one has the possibility of relating the masses of the up quarks to those of the down quarks and leptons that is an important feature of $SO(10)$.

A possible difficulty is the generation of the Majorana masses of the righthanded neutrinos. These require a product of VEVs that is in a $(126,1)$. A $126$ field cannot be constructed in $SO(10)$ string models, but one can get this effectively from a product of two spinor Higgs fields: $(\mathbf{16},1)^2$. The problem, however, is that that raises all the difficulties in preserving the stability of the gauge hierarchy that one encounters in $SO(10)$ models. The $SU(5)$-singlet VEV will either lead to unwanted goldstones (if the spinor-Higgs decouples from the DW sector) or will destabilize the DW form (if the sectors are coupled). The trick that allowed a resolution of this difficulty in Ref. 8 involved the existence of three adjoint Higgs fields, which we are eschewing in the present approach.

Fortunately, however, one need introduce no additional fields in this $SO(10) \times SU(5)$ model to generate large righthanded neutrino masses. The following term suffices:

$$\mathcal{O} = \psi^A \Gamma^{abcde} \psi^B (H_1)_{\alpha}^a (H_1)_{\beta}^b (H_2)_{\gamma}^c (H_2)_{\delta}^d (H_2)_{\epsilon}^e \epsilon_{\alpha \beta \gamma \delta \epsilon} / M^4.$$  \hspace{1cm} (8)

Note that the product of $H_I$ fields is totally antisymmetric under both $SO(10)$ and $SU(5)'$, and so symmetric in Higgs “flavor”. Note also that this term is even under the $Z_2$ symmetry. The five Higgs are contracted precisely into a $(126,1)$ and, with the VEVs in Eqs. (2) and (3), give a mass of order $M_{GUT}/M^4$ to the righthanded neutrinos. If we assume that $\mathcal{M}$ is about 10 to 20 times $M_{GUT}$ this gives $M_R$ at an intermediate scale of $10^{11}$ to $10^{12}$ GeV.
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