Rapid coherent control of population transfer in lattice systems

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(Dated: November 11, 2013)

Abstract

We derive the driving potential that accelerates adiabatic population transfer from an initial state to a target state in a lattice system without unwanted excitation of other states by extending to discrete systems the fast-forward theory of adiabatic transfer. As an example we apply the theory to a model that describes a Bose-Einstein condensate in a quasi one-dimensional optical lattice, and show that modulation of the tilting of the lattice potential can transfer the population of the Bose-Einstein condensate from site to site with high fidelity and without unwanted excitations.

PACS numbers: 02.30.Yy, 37.10.Jk, 67.85.d

I. INTRODUCTION

During the last three decades there have been dramatic advances in both theoretical understanding of the requirements for control of quantum dynamics and the technology that is needed for the execution of proposed control paradigms\textsuperscript{1,2}. Experimental verifications of the theory for systems as diverse as control of population transfer in Bose-Einstein condensates (BECs) and in chemical reactions have been reported \textsuperscript{3–10}. A particularly useful subgroup of the proposals for control of quantum dynamics of a system rely on adiabatic transfer via the slow variation of an external field that is applied to the system. However, experimental exploitation of such control schemes can be rendered difficult by the occurrence of unwanted internal decoherence processes and by external noise; both of these difficulties can be reduced or avoided if the adiabatic transfer process can be speeded up sufficiently to permit population transfer to compete successfully with the time-dependence of the perturbations. Indeed, with this goal in mind, several methods for the acceleration of quantum dynamics, including adiabatic dynamics, have been proposed. These methods include the counter-diabatic protocol \textsuperscript{11}, frictionless quantum driving \textsuperscript{12}, invariant-based inverse engineering \textsuperscript{13}, and fast forward scaling \textsuperscript{14–}
which is also used for protection of quantum states from potential fluctuations.

Lattice models are widely used to describe quantum systems, examples of which are a BEC in an optical lattice, a network of nonlinear waveguides and optical fibers, and a superconducting ladder of Josephson junctions. For example, motivated by the potential applicability to quantum computation, and by the opportunity to simulate aspects of complex electronic behavior in crystalline matter, many remarkable features of BECs in optical lattices have been studied. The existing studies clearly reveal the value of the ability to manipulate BECs in optical lattices for the purpose of preparing well-defined quantum states. We have been stimulated by this observation to extend the theory of accelerated adiabatic transfer to lattice systems so as to determine the potential that drives specified state-to-state population transfer without excitation of unwanted quantum states. In this paper we provide a derivation of that driving potential, and we apply the theory to site-to-site population transfer of a BEC in a quasi-one-dimensional optical lattice. We show that modulation of the lattice potential can transfer the population of the BEC between sites of the lattice with high fidelity and without unwanted excitations. The theory developed is applicable to any lattice in which the on-site potential is tunable. We also demonstrate the robustness of the accelerated population transfer to variation (approximation) of the driving potential.

In Sec. II we present the framework of the theory of accelerated quantum adiabatic dynamics in a lattice system and discuss its relationship with the corresponding theory for a continuous system. In Sec. III we study accelerated population transfer in a Bose-Einstein condensate in a one-dimensional optical lattice potential. The robustness of the method with respect to approximation of the driving potential is studied in Sec. IV. An Appendix provides a brief description of the basic theory of acceleration of non-adiabatic quantum dynamics.

II. FAST-FORWARD TRANSFORMATION IN DISCRETE SYSTEMS

We consider a lattice system in which the dynamics is governed by a discrete time-dependent Schrödinger equation

$$i \frac{d \Psi(m, t)}{dt} = \sum_l \tau_{m,l} \Psi(l, t) + \frac{V_0(m, R(t))}{\hbar} \Psi(m, t), \quad (1)$$

where $l, m$ denote sites and $t$ time, respectively, and $\tau_{m,l} = \tau_{l,m}^*$ is the rate of hopping between sites $m$ and $l$. The potential $V_0$ is modulated by a parameter $R$, which is a function of $t$. If the parameter $R$ changes
slowly enough from \( R_i \) to \( R_f \), and if the initial state is the \( n \)th energy eigenstate of the Hamiltonian with potential \( V_0(R_i) \), the wave function of the state on site \( m \) changes from \( \phi_n(m, R_i) \) to \( \phi_n(m, R_f) \) modulo the dynamical and adiabatic phases of the states. The wave function \( \phi_n(m, R) \) is a solution of the time-independent Schröinger equation

\[
\sum_l \hbar \tau_{m,l} \phi_n(l, R) + V_0(m, R) \phi_n(m, R) = E_n(R) \phi_n(m, R).
\]  (2)

On the other hand, when the parameter \( R \) changes at a non-zero rate, transitions occur to other levels. Our purpose is to derive a potential that drives the state from \( \phi_n(m, R_i) \) to \( \phi_n(m, R_f) \) in some short time \( T_F \) without unwanted excitations to other states. For that purpose we consider an intermediate state whose wave function is represented as

\[
\Psi_{FF}(m, t) = \phi_n(m, R(t)) \exp \left[ -i \int_0^t E_n(R(t')) dt' \right].
\] (3)

Note that Eq. (3) contains the additional phase \( f(m, t) \), and that the intermediate state connects the initial state \( \phi_n(m, R_i) \) and the target state \( \phi_n(m, R_f) \) \( \exp \left[ -i \int_0^{T_F} E_n(R(t')) dt' \right] \) in time \( T_F \). We require that this additional phase vanishes at \( t = 0 \) and at \( t = T_F \), and we assume that the intermediate state satisfies the time-dependent Schröinger equation

\[
\frac{i}{\hbar} \frac{d \Psi_{FF}(m, t)}{dt} = \sum_l \tau_{m,l} \Psi_{FF}(l, t) + \frac{V_{FF}(m, t)}{\hbar} \Psi_{FF}(m, t),
\]  (4)

in which \( V_{FF}(m, t) \) is the driving potential. We seek the driving potential that generates \( \phi_n(m, R_f) \exp \left[ -i \int_0^{T_F} E_n(R(t')) dt' \right] \) from \( \phi_n(m, R_i) \). Although we do not aim to generate the adiabatic phase, that uniform phase can be tuned by a uniform potential if necessary.

To find the forms of the driving potential and the additional phase \( f(m, t) \) we substitute Eq. (3) into the Schröinger equation (4) and we use Eq. (2) to rearrange the resulting equation. The imaginary part of the resultant equation leads to

\[
\dot{R} \text{Re} \left\{ \phi_n^*(m, R) \partial_R \phi_n(m, R) \right\} = \sum_l \text{Im} \left( \tau_{m,l} \phi_n^*(m, R) \phi_n(l, R) \right) \times \left\{ \exp \left[ i \left( f(l, t) - f(m, t) \right) \right] - 1 \right\}.
\] (5)

The solution of Eq. (5) yields the additional phase \( f(m, t) \), and the real part gives the driving potential as a functional of \( f, V_0, R \).
and \( \phi_n \):

\[
V_{FF}(m, t) = V_0(m, R(t)) + \sum_l \text{Re}\left\{ \hbar \tau_{m,l} \frac{\phi_n(l, R(t))}{\phi_n(m, R(t))} \times (1 - \exp\left[i\{f(l, t) - f(m, t)\}\right]) \right\} - \hbar \dot{f}(m, t) - \hbar \dot{R} \text{Im}\left[ \frac{\partial R \phi_n(m, R(t))}{\phi_n(m, R(t))} \right].
\]

It is necessary that \( R \) satisfies the conditions

\[
R(0) = R_i, \quad R(T_F) = R_f.
\]

If we take the boundary conditions to be

\[
\dot{R}(0) = \dot{R}(T_F) = 0, \quad f(m, t) \text{ vanishes at } t = 0 \text{ and at } t = T_F \text{ (see Eq. (5))}, \text{ and the intermediate state coincides with the target state at } T_F. \text{ The driving potential is obtained by substituting the additional phase into Eq. (6). With the boundary conditions}
\]

\[
\dot{R}(0) = \dot{R}(T_F) = 0 \quad (8)
\]

the driving potential coincides with \( V_0 \) at \( t = 0 \) and at \( t = T_F \). The time-dependence of \( R \) is arbitrary except for the requirement imposed by the above boundary conditions. The driving potential depends on the time-dependence of \( R \).

In the case that the hopping rate and the wave function are real, Eq. (5) and Eq. (6) simplify to

\[
\dot{R} \frac{\partial R \phi_n(m, R(t))}{\phi_n(m, R(t))} = \sum_l \tau_{m,l} \phi_n(l, R(t)) \sin[f(l, t) - f(m, t)],
\]

and

\[
V_{FF}(m, t) = V_0(m, R(t)) + \sum_l \hbar \tau_{m,l} \phi_n(l, R(t)) \times \left\{ 1 - \cos[f(l, t) - f(m, t)] \right\} - \hbar \dot{f}(m, t).
\]

We note that Eq. (6) implies that for \( \dot{R} \) sufficiently large there is no solution for \( f(m, t) \). That is, there is a lower limit to the control time \( T_F \). This property is not seen in the fast-forward theory for continuous systems \[15\]. Eqs. (5) and (6), for \( f \) and for \( V_{FF} \), reduce to the corresponding equations for continuous systems shown in Ref. \[15\] in the limit that the differences between adjacent sites of \( f \) and of \( \phi_n \) are small. The theory of acceleration of non-adiabatic quantum dynamics in a continuous system is described in Ref. \[14\]. Following the same analysis as in Ref. \[14\], the key elements of the theory of accelerated non-adiabatic quantum dynamics in a lattice system are exhibited in the Appendix.

The analysis described above can be straightforwardly extended to the case when a nonlinear Schrödinger equation is the basic
descriptor of the system dynamics. Consider

\[ i \frac{d\Psi(m, t)}{dt} = \sum_l \tau_{m,l} \Psi(l, t) + \frac{V_0(m, R(t))}{\hbar} \Psi(m, t) + \frac{c}{\hbar} |\Psi(m, t)|^2 \Psi(m, t), \]

where \( c \) is a constant. We assume the same form of the wave function of the intermediate state \( \Psi_{FF} \) as in Eq. (3). Then \( \phi_n \) is a solution of the time-independent nonlinear Schrödinger equation

\[ \sum_l \hbar \tau_{m,l} \phi_n(l, R) + V_0(m, R) \phi_n(m, R) + c |\phi_n(m, R)|^2 \phi_n(m, R) = E_n(R) \phi_n(m, R). \]

We assume that the intermediate state wave function is defined by the nonlinear Schrödinger equation

\[ i \frac{d\Psi_{FF}(m, t)}{dt} = \sum_l \tau_{m,l} \Psi_{FF}(l, t) + \frac{V_{FF}(m, t)}{\hbar} \Psi_{FF}(m, t) + \frac{c}{\hbar} |\Psi_{FF}(m, t)|^2 \Psi_{FF}(m, t). \]

We can derive the equations for the additional phase and the driving potential in the same manner as for the linear Schrödinger equation. The resultant equations are the same as Eqs. (5) and (6), respectively. The nonlinear term influences the driving potential through \( \phi_n \) in Eq. (13).

### III. SITE-TO-SITE POPULATION TRANSFER OF A BEC IN AN OPTICAL LATTICE

As an example, we now consider site-to-site population transfer of a BEC in an optical lattice. The lattice is defined by an external potential that is the sum of a spatially linear potential, which is tunable, and a stationary periodic potential

\[ V_{ext}(r, t) = \xi(t) z + U_L(x, y) \sin^2(2\pi z/\lambda), \]

where \( \lambda/2 \) is the wavelength (period) of the potential. We consider the case that the mean field condensate interaction is negligible. A discrete model of the BEC in a tilted trap was introduced in Ref. [21], using the tight binding approximation. In the tight binding approximation the condensate order parameter is written as

\[ \Phi(r, t) = \sqrt{N_T} \sum_m \Psi(m, t) \varphi(r - r_m), \]

where \( N_T \) is the total number of atoms and \( \varphi(m, r) = \varphi(r - r_m) \) is the condensate wave function localized in the \( m \)th trap with location \( r_m \). We assume that \( \int \varphi(m, r) \varphi(m + 1, r) dr = 0 \) and \( \int \varphi^2(m, r) dr = 1 \). Using Eq. (16), the Gross-Pitaevskii equation can be rewritten to read

\[ i \hbar \frac{\partial \Psi(m, t)}{\partial t} = -K \left[ \Psi(m - 1, t) + \Psi(m + 1, t) \right] + \frac{\xi(t) \lambda m}{2 \Psi(m, t)}, \]
where

\[ K \simeq -\int dr \left[ \frac{\hbar^2}{2m_0} \nabla \varphi(m, r) \cdot \nabla \varphi(m + 1, r) \right. \\
+ \left. \varphi(m, r)V_{\text{ext}}(r)\varphi(m + 1, r) \right], \quad (18) \]

with \( m_0 \) the mass of an atom. \( K \) is independent of \( m \) because of the orthogonality \( \int \varphi(m, r)\varphi(m + 1, r)dr = 0 \). Equation (17) then can be rewritten as

\[ i \frac{\partial}{\partial t} \Psi(m, t) = \tau \left[ \Psi(m - 1, t) + \Psi(m + 1, t) \right] \\
+ \frac{V(m, t)}{\hbar} \Psi(m, t), \quad (19) \]

with

\[ \tau = -\frac{K}{\hbar}, \quad (20) \]

and

\[ V(m, t) = \frac{1}{2} \xi(t) \lambda m. \quad (21) \]

We demonstrate the acceleration of population transfer for a BEC in a lattice with this model. Our goal is the transfer of population to the ground state of the linear potential with \( \xi = \xi_f \) from the ground state of the linear potential with \( \xi = \xi_i \). We take \( \xi_i = -\xi_f \) so that the population is transferred from one side of the lattice to the opposite side of the lattice.

### A. A three-site model

We consider first a three-site model with site potential

\[ V_0(m, R(t)) = \hbar \omega R(t)m. \quad (22) \]

In Eq. (22), the constant frequency \( \omega \) is defined by

\[ \omega = -\frac{\xi_i \lambda}{2\hbar} = \frac{\xi_f \lambda}{2\hbar}, \quad (23) \]

and the time-dependence of \( R(t) \) is chosen to be

\[ R(t) = R_0 + \frac{2}{T_F} \left[ t - \frac{T_F}{2\pi} \sin \left( \frac{2\pi t}{T_F} \right) \right]. \quad (24) \]

We take \( R_0 = -1 \), so that \( V_0(m, R(t)) \) changes from \( \xi_i \lambda m/2 \) to \( \xi_f \lambda m/2 \) in time \( T_F \), and take the hopping rate in Eq. (1) to be

\[ \tau_{m,t} = \tau(\delta_{m,t-1} + \delta_{m,t+1}). \quad (25) \]

We calculated the additional phase and driving potential for this model system using Eqs. (10) and (11), respectively, with the parameter set \( T_F = 4.2 \text{ ms}, \omega = 2.14 /\text{ms}, \hbar/2K = 0.35 \text{ ms} \) and \( \lambda = 850 \text{ nm} \ [21] \). The time-dependence of the additional phase is shown in Fig. 1, where we choose \( f(1, t) = 0 \). The driving potential \( V_{\text{FF}}(m, t) \), shown in Fig. 2, differs from \( V_0(m, R(t)) \) for \( 0 < t < T_F \), and is equal to \( V_0 \) at \( t = 0 \) and \( t = T_F \). We have simulated the evolution of the model system driven by \( V_{\text{FF}}(m, t) \) from the ground state corresponding to \( V_0(m, R(0)) \). That evolution is monitored by the fidelity

\[ F(t) = | < \phi_0 | \Psi > |, \quad (26) \]

where \( |\phi_0> \) is the ground state of the instantaneous Hamiltonian \( H_0(R(t)) \) and \( |\Psi> \) is the state driven by the potential \( V_{\text{FF}}(m, t) \).
FIG. 2: (Color online) Time-dependence of $V_{FF}(m,t)/\hbar$. The unit of time is 1 ms. The inset shows the time-dependence of the fidelity, defined by $F(t) = |<\phi_0|\Psi>|$.

The time-dependence of the fidelity is shown in the inset to Fig. 2; it is equal to unity at $T_F$. A comparison of the population evolution under $V_0(m,R(t))$ and under $V_{FF}(m,t)$ is shown in Fig. 3. We note that the non-adiabatic transfer generates unwanted excitations, with the population of each site deviating from that evolving under the instantaneous Hamiltonian (dotted lines in Fig. 3). The fidelity of the population evolution driven by $V_0(m,R(t))$ is 0.938 at $T_F$.

FIG. 3: (Color online) Time-dependence of the population evolution under $V_0(m,R(t))$ (dashed and solid lines) and $V_{FF}(m,t)$ (dotted lines). The evolution under the instantaneous Hamiltonian is also shown with dotted lines. The notation is $\Psi_m = \Psi(m,t)$.

B. A four-site model

We have also examined accelerated population transfer of a BEC in a four-site model. The parameters used for these calculations are the same as for the three-site model except that $\omega = 0.714$ /ms. The population of the ground state of the instantaneous Hamil-
Hamiltonian for each site is shown in Fig. 4. The initial state is located mainly at sites 3 and 4, while the target state is located mainly at sites 1 and 2. The time-dependence of the driving potential is shown in Fig. 5. The time-dependence of the fidelity are compared in the inset to Fig. 5. The solid curve and the broken curve correspond to the dynamics with $V_{FF}$ and $V_0$, respectively. We note that the fidelity decreases and does not recover at $T_F$ in the $V_0$ generated dynamics because of unwanted excitations whereas for the $V_{FF}$ generated dynamics the fidelity becomes unity at $t = T_F$.

FIG. 4: (Color online) Population of the ground state of the instantaneous Hamiltonian in the four-site model system.

FIG. 5: (Color online) Time-dependence of $V_{FF}(i, t)/\hbar$. The unit of time is 1 ms. The inset shows the time-dependence of the fidelity.

IV. COMMENTS

It is one matter to calculate the exact driving potential required to transfer the BEC population between sites with perfect fidelity, but it is another matter to generate that potential in a real experiment. It is usually the case that in real experiments we cannot generate a perfect rendition of a specified potential. Then, the robustness of the proposed population transfer method to variation of the driving potential is important. We can test the efficiency of our proposed transfer process to approximation of the driving potential by considering population transfer under a driving potential that is proportional to the site number:

$$V_{app}(j, t) = \mathcal{V}(t) j.$$  \hspace{1cm} (27)
In Eq. (27), \( V(t) \) is a function designed so that \( V_{\text{app}} \) approximates the exact driving potential. For the three-site model, for transfers between ground states, \( V_{\text{app}} \) coincides with \( V_{FF} \) because

\[
\phi_n(1, R) \left[ 2\phi_n^2(3, R) - \phi_n^2(2, R) \right] \\
= \phi_n(3, R) \left[ 2\phi_n^2(1, R) - \phi_n^2(2, R) \right],
\]

(28)

for any \( R \). This property also holds for second and third eigenstates of the instantaneous Hamiltonian, although the driving potential depends on the level \( n \). Thus the simple potential defined in Eq. (27) can transfer population in the three-site model without unwanted excitation. The approximation \( V_{\text{app}}(j, t) = V(t)j \) is not exact for the four-site model, but it is a good approximation to \( V_{FF} \) for that model. We show the difference between \( V_{\text{app}} \) and \( V_{FF} \) for the four-site model in Fig. 6. In general, \( V_{FF} \) is well approximated by \( V_{\text{app}} \) with a larger deviation near \( t = T_F/2 \) than in other time domains (Fig. 7). The fidelity of the population transfer in the four-site system driven by \( V_{\text{app}} \) is 0.997 at \( T_F \) whilst the fidelity of the population transfer driven by \( V_0 \) is 0.916.

Our derivation of the driving potential that accelerates adiabatic population transfer in a lattice reveals a striking difference between a lattice system and a continuous system. Specifically, in the lattice system there is lower limit to \( T_F \). This limit derives from the condition for the additional phase in Eq. (5), which gives the lower limit for \( \dot{R} \) for each \( R \) depending on \( \phi_n(R) \), that is, trajectory of the evolution of the system. We believe that the accelerated population transfer scheme described in this paper can be used for the
coherent control of many quantum systems which are described by chain or lattice models.

Appendix A: Acceleration of non-adiabatic dynamics

We consider the acceleration of non-adiabatic quantum dynamics. Consider the wave function $\Psi(m, t)$, which is a solution of a discrete time-dependent Schrödinger equation:

$$i\frac{d\Psi(m, t)}{dt} = \sum_l \tau_{m,l} \Psi(l, t) + \frac{V(m, t)}{\hbar} \Psi(m, t). \quad (A1)$$

We seek a driving potential that generates the target state $\Psi(m, T)$ at $t = T_F (< T)$. We assume that the wave function of the intermediate state is

$$\Psi_{FF}(m, t) = \Psi(m, \Lambda(t)) e^{i f(m, t)}, \quad (A2)$$

where $f(m, t)$ is the additional phase and

$$\Lambda(t) = \int_0^t \alpha(t') dt'. \quad (A3)$$

$\alpha$ is a real function of time called magnification factor [14]. The time-dependence of $\alpha$ is chosen so that it satisfies

$$\Lambda(T_F) = T. \quad (A4)$$

We assume that $\Psi_{FF}(m, t)$ is a solution of the Schrödinger equation:

$$i\frac{d\Psi_{FF}(m, t)}{dt} = \sum_l \tau_{m,l} \Psi_{FF}(m, t)$$

$$+ \frac{V_{FF}(m, t)}{\hbar} \Psi_{FF}(m, t), \quad (A5)$$

where $V_{FF}$ is the driving potential. Following the same analysis as in Sec. II we find

$$\alpha(t) \sum_l \text{Im} [\tau_{m,l} \Psi^*_m \Psi_l]$$

$$= \sum_l \text{Im} \left\{ \tau_{m,l} \Psi^*_m \Psi_l \exp [i (f_l - f_m)] \right\}, \quad (A6)$$

and

$$V_{FF}(m, t) =$$

$$\sum_l \text{Re} \left\{ \frac{\hbar \tau_{m,l} \Psi_l}{m} \left[ \alpha(t) - e^{i (f_l - f_m)} \right] \right\}$$

$$+ \alpha(t) V(m, \Lambda(t)) - \hbar \partial_t f_m, \quad (A7)$$

where $f_m$ and $\Psi_m$ are abbreviations for $f(m, t)$ and $\Psi(m, \Lambda(t))$, respectively. Equation (A6) is used to obtain the additional phase. The driving potential is obtained by substitution of $f_m$ into Eq. (A7). As in the case of acceleration of adiabatic population transfer there is a lower limit to $T_F$ because Eq. (A6) gives the upper limit of $\alpha(t)$ for each $t$. The equations for $f$ and $V_{FF}$ in Eqs. (A6) and (A7) reduce to those for continuous systems in Ref. [14] in the limit that the differences in $f(m, t)$ and $\Psi(m, t)$ between adjacent sites are small.
Acknowledgments

SM thanks Grants-in-Aid for Centric Research of Japan Society for Promotion of Science and JSPS Postdoctoral Fellowships for Research Abroad for its financial support.
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