Hawking radiation from spherically symmetrical gravitational collapse to an extremal R-N black hole for a charged scalar field

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Abstract

Sijie Gao has recently investigated Hawking radiation from spherically symmetrical gravitational collapse to an extremal R-N black hole for a real scalar field. Especially he estimated the upper bound for the expected number of particles in any wave packet belonging to \( H_{\text{out}} \) spontaneously produced from the state \(|0\rangle\), which confirms the traditional belief that extremal black holes do not radiate particles. Making some modifications, we demonstrate that the analysis can go through for a charged scalar field.

1 Introduction

Particle creation by black holes (Hawking radiation) has been widely studied since 1970s \cite{1,2}. One of the most important results is that a black hole radiates particles with the same spectrum as a black body with temperature \( T = \frac{\hbar \kappa}{2\pi k} \), where \( \kappa \) is the surface gravity of the black hole and \( k \) is Boltzmann’s constant.

Although the standard Hawking radiation is only derived for a spacetime appropriate to a collapsing body which settles down to a non-extremal black hole at the late stage, it is generally accepted that an extremal black hole has zero temperature (zero surface gravity) and consequently no particles are produced (not for superradiance modes if exist). However Liberati \textit{et al.} \cite{3} pointed out that the generalization to extremal black holes from non-extremal black holes is not trivial. One important point is that Kruskal transformation for non-extremal black holes, which plays a crucial role in calculating the particle creation, need be modified for extremal black holes.

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The modified Kruskal transformation given in \cite{3} is briefly reviewed as follows. Start with the usual form of the R-N geometry with parameters $M$ and $Q$

$$ds^2 = \frac{\Delta}{r^2} dt^2 - \frac{r^2}{\Delta} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$

(1)

where $\Delta = r^2 - 2rM + Q^2$. In the extremal case $Q = M$, the tortoise coordinate $r_*$ is given by

$$r_* = r + 2M[\ln(r - M) - \frac{M}{2(r - M)}]$$

(2)

Define the advanced time $v$ and retarded time $u$ as

$$v = t + r_*$$

$$u = t - r_*$$

(3)

the modified Kruskal transformation for extremal R-N black holes is \cite{3}

$$u = -4M[\ln(-U) + \frac{M}{2U}]$$

$$v = 4M[\ln V - \frac{M}{2V}]$$

(4)

where $U$ and $V$ are regular across the past and future horizons of the extended spacetime.

Using this modified Kruskal transformation, Liberati et al. \cite{3} find that the extremal black hole does not behave as a thermal object and can not be regarded as the thermodynamic limit of a non-extremal black hole. However, Gao \cite{4} pointed out some deficiencies in the analysis of \cite{3} and circumvented them by focusing on the wave packet solutions with unit norm. The result in \cite{4} confirms that extremal black holes do not radiate particles.

Nevertheless, \cite{3, 4} only involves the calculation for the massless real scalar field. this paper will focus on the massless charged scalar field in an extremal R-N black hole formed from a collapsing spherically symmetrical star, which makes two major differences from the massless real scalar field: One is that not only particle creation but also antiparticle creation need be calculated; the other is that the influence from the electromagnetic field need be considered. Our calculation follows the similar steps to \cite{4}.

2 Calculation of particle creation

2.1 Construction of the wave packet at future null infinity

Our purpose in this subsection is to construct the wave packet with unit norm at future null infinity $\mathcal{I}^+$. Start with the wave equation in the appendix with the massless and zero potential case\footnote{Without loss of generalization, only discuss the case of $Q > 0$.}

$$\frac{1}{\sqrt{-g}} (\partial_i + iqA_i) [\sqrt{-g} g^{ij} (\partial_j + iqA_j)] \phi = 0$$

(5)
In the region outside the collapsing star, the spacetime is described by the extremal R-N metric. Let \( \phi = \frac{1}{r} f(t, r) Y_{lm}(\theta, \varphi) \), where \( Y_{lm}(\theta, \varphi) \) is spherical harmonic. Thus if choose \( A_a = -\frac{Q}{r} \frac{\partial}{\partial t} \), then outside the collapsing star

\[
\frac{\partial^2 f}{\partial^2 t} - \frac{\partial^2 f}{\partial^2 r} = \left[ \frac{2qQ}{r} \frac{\partial}{\partial t} f + \frac{(qQ)^2}{r^2} f \right] + V(r) f = 0
\]

where

\[
V(r) = \left( \frac{M - r}{r^6} \right) \left[ 2M(M - r) + l(l + 1)r^2 \right]
\]

It is easy to know

\[
F_{\omega_0 lm} = \frac{1}{r} e^{i\omega_0 u} Y_{lm}(\theta, \varphi)
\]

is a solution near \( \mathcal{I}^+ \). Then following the procedure in [4], construct the wave packet with frequency around \( \omega_0 \) and centered on the retarded time \( u = 2\pi n \) as

\[
P_{n\omega_0 lm} = \frac{1}{r} z_{n\omega_0} Y_{lm}(\theta, \varphi)
\]

where

\[
z_{n\omega_0} = N \int_{-\infty}^{\infty} g(\omega - \omega_0) e^{i\omega(u - 2\pi n)} d\omega
\]

The normalization constant \( N \) can be determined by the inner product in the appendix

\[
\langle \Omega^{-1} P, \Omega^{-1} P \rangle = (P, P) = 1
\]

It is convenient to make the integral on \( \mathcal{I}^+ \), and accordingly

\[
sign(-\omega_0) \bar{z}(u) z'(u) - z(u) \bar{z}'(u) du = 1
\]

Then the straightforward calculation gives [4]

\[
N = \frac{1}{\sqrt{\text{sign}(\omega_0)(\beta \epsilon \omega_0 + \gamma \epsilon^2)}}
\]

where

\[
\beta = 2 \int_{-\infty}^{\infty} |g(\tilde{\omega})|^2 d\tilde{\omega}
\]

\[
\gamma = 2 \int_{-\infty}^{\infty} \tilde{\omega} |g(\tilde{\omega})|^2 d\tilde{\omega}
\]
Thus (11) becomes
\[ z(u) = \frac{\sqrt{\epsilon}}{\sqrt{\text{sign}(\omega_0)(\beta \omega_0 + \gamma \epsilon)}} e^{i\omega_0 \hat{u} \hat{g}(\epsilon \hat{u})} \] (17)

2.2 Geometrical optics approximation

To calculate the particle production at late times, the wave packet (9) need be propagated backwards from \( \mathcal{I}^+ \) to the past null infinity \( \mathcal{I}^- \). For simplicity, first investigate the propagation of wave (8); then the propagation of (9) can be got by superposition. A part of wave (8) will be scattered by the gravitational field and electromagnetic field outside the collapsing star and will end up on \( \mathcal{I}^- \) with the same frequency, which indicates this part will not contribute the particle production. So we will only care the remaining part which will propagate through the collapsing star, eventually emerging to \( \mathcal{I}^- \).

Note geometrical optics approximation holds for the propagation of the remaining part from near the future event horizon to \( \mathcal{I}^- \). Thus if set \( v = 0 \) for the points where the null curves generating the future event horizon intersects \( \mathcal{I}^- \); for \( -v \) positive and small
\[ U = \alpha v \] (18)

Therefore near \( \mathcal{I}^- \), the remaining part takes the form as
\[ F_{\text{rem}} = \begin{cases} t(\omega_0) & v < 0 \\ 0 & v > 0 \end{cases} \] (19)

where \( t(\omega_0) \) is the transmission amplitude and
\[ S(v) = (\omega_0 - \frac{qQ}{M})\{-4M\ln(\alpha v) + \frac{M}{2\alpha v}\} \] (20)

Next assume that \( t(\omega) \) varies negligibly over the frequency interval \( 2\epsilon \), thus near \( \mathcal{I}^- \)
\[ P_{\text{rem}} = \begin{cases} t(\omega_0) & v < 0 \\ 0 & v > 0 \end{cases} \] (21)

where
\[ z(v) = \frac{\sqrt{\epsilon}}{\sqrt{\text{sign}(\omega_0)(\beta \omega_0 + \gamma \epsilon)}} e^{i(\omega_0 - \frac{4M}{\alpha v} + 2\pi)n} \] (22)

and
\[ \hat{u}(v) = -4M \ln(-\alpha v) - \frac{2M^2}{\alpha v} - 2n\pi \] (23)

2.3 Calculation of particle creation

According to the formulae in the appendix, the expected number of particles spontaneously produced in the state represented by \( P_{\omega_0 lm} \) is given by
\[ N_{\omega_0 lm} = 2|t(\omega_0)|^2 \int_0^\infty \omega' |z[\text{sign}(\omega_0)\omega']|^2 d\omega' \] (24)
where
\[
\hat{z}(\omega') = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0} z(v)e^{i\omega'v}dv = \frac{\sqrt{\epsilon}}{\sqrt{\text{sign}(\omega_0)}2\pi(\beta\omega_0 + \gamma\epsilon)}Z(\omega')
\]
(25)

where
\[
Z(\omega') = \int_{-\infty}^{0} e^{i\omega_0'\hat{u}(v)}e^{i\omega'v}\hat{g}[\epsilon\hat{u}(v)]dv
\]
(26)

where
\[
\omega_0' = \omega_0 - \frac{qQ}{M}
\]
(27)

In addition, by a simple rescaling, It is easy to know that \(N_{n\omega_0lm}\) is independent of the choice of \(\alpha\) in \(\hat{u}(v)\). So without loss of generality, from now on let
\[
\hat{u}(v) = -4M\ln(-v) - \frac{2M^2}{v} - 2n\pi
\]
(28)

Since the traditional belief that extremal black holes do not radiate particles is only for non-superradiance modes, next the creation of particles will be calculated in the following two cases: (a)\(\omega_0 > \frac{qQ}{M}, \omega' > 0\) ‡ or (b)\(\omega_0 < 0, \omega' < 0\). The calculation here follows Gao’s [4] by some simple modifications.

According to the theorem and the lemma in the appendix, make a Wick rotation (See Figure 1). Thus if let \(x = -i\omega'v\), then
\[
Z(\omega') = \frac{e^{-i\omega_0'M\pi}}{i\omega'} \int_{0}^{\infty} e^{-\pi}e^{-i\omega_0'M\ln(x)}e^{-\frac{2M^2\omega_0'}{x}}\hat{g}[\epsilon\hat{u}(x)]dx
\]
(29)

‡Require \(0 < \epsilon \ll \omega_0'\) to guarantee \(P_{n\omega_0lm}\) only includes non-superradiance modes.

Figure 1: The Wick rotation (a) for \(\omega_0 > \frac{qQ}{M}, \omega' > 0\) and (b) for \(\omega_0 < 0, \omega' < 0\)
where
\[
\hat{u}(x) = -4M \ln \left( \frac{x}{\omega} \right) + \frac{i2M^2 \omega'}{x} - 2n\pi \tag{30}
\]

Using the fact \( \text{Im}[\hat{u}(x)] = \text{sign}(\omega_0)2M\pi + \frac{2M^2\omega'}{x} \) and \( e^{-i\omega_04M\ln \left( \frac{x}{\omega} \right)} = e^{i\text{sign}(\omega_0)2M\pi\omega_0} \) when \( x \) is real,

\[
|Z(\omega')| \leq \frac{C_k e^{i\text{sign}(\omega_0)2M\pi(\omega_0'-\epsilon)}}{\text{sign}(\omega_0)\omega'} \int_0^\infty e^{-\frac{x}{2}} e^{-\frac{2M^2\omega'(\omega_0'-\epsilon)}{x}} \frac{1}{\left[ 1 + |\hat{u}(x)|^2 \right]^k} dx \tag{31}
\]

To proceed, first give a lower bound for \( |\hat{u}(x)| \). Start with

\[
|\hat{u}(x)|^2 = (2n\pi + 4M \ln y)^2 + \left( 2M\pi + \frac{2M^2}{y} \right)^2 \tag{32}
\]

where \( y = \frac{x}{\text{sign}(\omega_0)\omega} \). Thus it is not difficult to find

\[
|\hat{u}(x)| \geq c_1 2n\pi \tag{33}
\]

where \( c_1 \) is a positive constant. Therefore

\[
|Z(\omega')| \leq \frac{C_k e^{i\text{sign}(\omega_0)2M\pi(\omega_0'-\epsilon)}}{\text{sign}(\omega_0)\omega'(c_1)^k(2\pi)^k} \int_0^\infty e^{-\frac{x}{2}} e^{-\frac{2M^2\omega'(\omega_0'-\epsilon)}{x}} \frac{1}{\left[ 1 + |\hat{u}(x)|^2 \right]^k} dx \tag{34}
\]

where \( K_1(z) \) is modified Bessel function \([5]\).

In order to investigate the bound in \((31)\) for \( \text{sign}(\omega_0)\omega' \leq \Omega \ll 1 \) more carefully, denote by \( G(\omega') \) the integral in \((31)\) for \( \text{sign}(\omega_0)\omega' \leq \Omega \ll 1 \), i.e.

\[
G(\omega') = \int_0^\infty e^{-\frac{x}{2}} e^{-\frac{2M^2\omega'(\omega_0'-\epsilon)}{x}} \frac{1}{|\hat{u}(x)|^k} dx \tag{35}
\]

and rewrite \( |\hat{u}(x)|^2 \) as

\[
|\hat{u}(x)|^2 = [F_n(\omega') + 4M \ln x]^2 + \left( 2M\pi + \frac{\text{sign}(\omega_0)2M^2\omega'}{x} \right)^2 \tag{36}
\]

where

\[
F_n(\omega') = 2n\pi - 4M \ln[\text{sign}(\omega_0)\omega'] \tag{37}
\]

In addition, in the following discussion, assume \( a \ll 1 \) but \( an \gg 1 \).
Therefore

$$G(\omega', D_1) \leq \frac{1}{|c_2 F_n(\omega')|^k} \int_0^\infty e^{-x} e^{-\frac{2M^2(\omega'_{0} - \epsilon)}{\omega'}} dx = 2\sqrt{2M^2(\omega'_{0} - \epsilon)}K_1 \left[ 2\sqrt{2M^2(\omega'_{0} - \epsilon)\omega'} \right]$$

where \( c_2 \) is a positive constant. For \( z \to 0, K_1(z) \sim \frac{1}{z} \). Therefore

$$G(\omega', D_1) \leq \frac{1}{|c_2 F_n(\omega')|^k}$$

(b) \( D_2 = \{ x \leq \frac{\text{sign}(\omega_0)2M^2\omega'}{a F_n(\omega')} \} \)

$$G(\omega', D_2) \leq \frac{\text{sign}(\omega_0)2M^2\omega' e^{\text{sign}(-\omega_0)a F_n(\omega')}(\omega'_{0} - \epsilon)}{a F_n(\omega')(\epsilon c_1)^k(2n\pi)^k}$$

(c) \( D_3 = \{ x \in (0, \infty) | x \notin D_1 \cup D_2 \} \)

Thus

$$G(\omega', D_3) \leq \frac{1}{|c_3(\omega_0)F_n(\omega')|^k}$$

where \( c_3(\omega_0) \) is a positive constant.

Thus

$$|\hat{\varepsilon}[\text{sign}(\omega_0)\omega'] \geq \Omega| \leq \frac{\sqrt{7}C_k e^{\text{sign}(\omega_0)2M\pi(\omega'_{0} - \epsilon)}}{\text{sign}(\omega_0)2\pi(\beta\omega_0 + \gamma\epsilon)\omega'\epsilon c_3(\omega_0)F_n(\omega')^k}$$

and

$$|\hat{\varepsilon}[\text{sign}(\omega_0)\omega'] > \Omega| \leq \frac{\sqrt{7}C_k e^{\text{sign}(\omega_0)2M\pi(\omega'_{0} - \epsilon)}2\sqrt{2M^2(\omega'_{0} - \epsilon)}\omega'K_1 \left[ 2\sqrt{2M^2(\omega'_{0} - \epsilon)\omega'} \right]}{\text{sign}(\omega_0)\sqrt{\text{sign}(\omega_0)2\pi(\beta\omega_0 + \gamma\epsilon)\omega'}\epsilon c_3(\omega_0)F_n(\omega')^k(2n\pi)^k}$$

Therefore

$$N_{n_0lm} \leq 2t(\omega_0)^2 \left\{ \frac{\epsilon C_k^2 e^{\text{sign}(\omega_0)4M\pi(\omega'_{0} - \epsilon)}}{\text{sign}(\omega_0)2\pi(\beta\omega_0 + \gamma\epsilon)\epsilon c_3(\omega_0)2k} \int_0^\Omega \frac{1}{\omega'(2n\pi - 4M \ln \omega') \omega'} d\omega' \right. + \left. 8\epsilon C_k^2 e^{\text{sign}(\omega_0)4M\pi(\omega'_{0} - \epsilon)} \frac{M^2(\omega'_{0} - \epsilon)}{\text{sign}(\omega_0)2\pi(\beta\omega_0 + \gamma\epsilon)\epsilon c_3(\omega_0)2k(2n\pi)^2} \int_\Omega^{\infty} K_1^2 \left[ 2\sqrt{2M^2(\omega'_{0} - \epsilon)\omega'} \right] d\omega' \right\}$$
For $z \to \infty$, $k_1(z) \sim \sqrt{\frac{\pi}{2}} e^{-z}$ \[45\]. Thus

$$N_{n\omega_0 tm} \leq \frac{2c_4(\omega_0)|l(\omega_0)|^2}{(2n\pi)^{2k-1}} \tag{46}$$

where $c_4(\omega_0)$ is a positive constant.

### 3 Conclusion and discussion

Since $2n\pi$ represents time, \[46\] shows that for any non-superradiance mode associated with a wave packet, the rate of particle and antiparticle creation drops off to zero faster than any inverse power of time at late times. This result confirms that extremal black holes do not radiate particles as \[4\]. In addition, it is obvious that our result is immediately applicable to the case of a charged scalar field in an extremal K-N black hole; since the modified Kruskal transformation here is exactly the same as the extremal R-N case \[7\].

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### Appendices

#### A The charged (complex) scalar quantum field in curved spacetimes

Here the charged scalar quantum field in curved spacetimes is briefly reworked \[8\]. Start with the gauge-invariant\(^8\) wave equation

$$\left(\nabla_a + i q A_a\right)\left(\nabla^a + i q A^a\right)\phi - \frac{1}{6} R\phi + m^2 \phi + V\phi = 0 \tag{1}$$

The Klein-Gordon inner product

$$\langle \phi, \psi \rangle = i \int_{\Sigma} \bar{\phi} \left(\nabla_a + i q A_a\right) \psi - \psi \left(\nabla_a - i q A_a\right) \bar{\phi} |n^a dV \tag{2}$$

is gauge-invariant and independent of choice of Cauchy surface $\Sigma$.

For the free field, we define the one-particle Hilbert space $\mathcal{H}$ as

$$\mathcal{H} = \mathcal{H}^+ \bigoplus \mathcal{H}^- \tag{3}$$

\(^8\)Under the transformations: $\phi' = e^{-iq\tau} \phi$, $A'_a = A_a + \nabla_a \tau$
where $\mathcal{H}^+$ is the Hilbert space of positive frequency solutions of the Klein-Gordon equation with the above inner product; $\mathcal{H}^-$ is the Hilbert space of negative frequency solutions with the inner product given by minus the above Klein-Gordon inner product, and the bar denotes the complex conjugate (dual) operation.

The Hilbert space of free-field states is taken to be $\mathcal{F}_s(\mathcal{H})$. Annihilation and creation operators are be defined as usual. Then the field operator $\hat{\phi}$ is defined as

$$\hat{\phi} = g_i a(\bar{\phi}^i) + c^\dagger(\lambda_i) \bar{\lambda}^i \quad (4)$$

where $\{\phi_i\}, \{\lambda_i\}$ are respectively the orthonormal bases of $\mathcal{H}^+, \bar{\mathcal{H}}^-$.

Then according to the usual procedure, the scattering matrix can be constructed. Here only give the expected number of particles in states belonging to $\mathcal{H}^+_\text{out}$ spontaneously produced from the state $|0\rangle_{\text{in}}$

$$\langle \hat{N}(\phi_i)\rangle_{\text{out}} = |B\phi_i|^2 \quad (5)$$
$$\langle \hat{N}(\lambda_i)\rangle_{\text{out}} = |A\bar{\lambda}_i|^2$$

where $B$ denotes taking the past negative part of solutions which are purely positive frequency in the future, and $A$ denotes taking the past positive part of solutions which are purely negative frequency in the future.

Now consider some conformally invariant properties for the charged scalar field. Make the conformal transformation $\tilde{g}_{ab} = \Omega^2 g_{ab}$, then

$$[g^{ab}(\nabla_a + iqA_a)(\nabla_b + iqA_b) - \frac{1}{6} \tilde{R} + \Omega^{-2}(m^2 + V)](\Omega^{-1}\phi) = \Omega^{-3}g^{ab}(\nabla_a + iqA_a)(\nabla_b + iqA_b) - \frac{1}{6} \tilde{R} + m^2 + V\phi \quad (6)$$

Thus the conformal weight for the charged scalar field is -1. Furthermore

$$\langle \Omega^{-1}\phi, \Omega^{-1}\psi \rangle = (\phi, \psi) \quad (7)$$

In order to consider the null infinity $\mathcal{I}^-$ of R-N black hole, select the conformal factor $\Omega = \frac{1}{r}$. Thus

$$d\tilde{s}^2 = \Omega^4 \Delta dv^2 + 2vdv\Omega - (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (8)$$

$$d\tilde{s}^2 = \Omega^4 \Delta du^2 - 2udu\Omega - (d\theta^2 + \sin^2 \theta d\varphi^2)$$

Obviously, the above metrics are non-degenerate at $\Omega = 0$. Furthermore, $\mathcal{I}^-$ can be defined as $\Omega = 0, -\infty < v < \infty$; $\mathcal{I}^+$ as $\Omega = 0, -\infty < u < \infty$. In addition, if the Cauchy surface chosen includes either $\mathcal{I}^-$ or $\mathcal{I}^+$, choose $\tilde{\Omega}^\alpha$

$$n^a = \left(\frac{\partial}{\partial v}\right)^a, \quad dV = \sin \theta dv d\theta d\varphi \quad (9)$$

for $\mathcal{I}^-$;

$$n^a = \left(\frac{\partial}{\partial u}\right)^a, \quad dV = \sin \theta du d\theta d\varphi \quad (10)$$

for $\mathcal{I}^+$. 

9
B One theorem and one lemma

Here the theorem and lemma useful to our calculation of the creation of particles are briefly reviewed.

**Theorem** [4, 10] Let \( g : \mathbb{R} \rightarrow \mathbb{R} \) be a \( C^\infty \) function with compact support in \([-1, 1]\). Then, the Fourier transform, \( \hat{g}(\zeta) \) is an entire analytic function of \( \zeta \) such that for all \( k > 0 \)

\[
|\hat{g}(\zeta)| \leq \frac{C_k e^{\varepsilon \|\zeta\|}}{(1 + |\zeta|)^k}
\]

(11)

for all \( \zeta \in \mathbb{C} \), where \( C_k > 0 \) is a constant which depends on \( k \) and \( g \).

**Lemma** [4, 6] If \( F(z) \) satisfies \( \lim_{|z| \to \infty} |F(z)| = 0 \) in the second (third) quadrant, then

\[
\lim_{R \to \infty} \int_{C_R} e^{icz} F(z) = 0
\]

(12)

where \( c > 0 (c < 0) \) is a constant.

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