Threshold Decision Algorithm Based On The Maximum Interclass Variance For TDCS

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Abstract. Transform domain communication system (TDCS) is a new type of wireless communication system, who can take the initiative to avoid interference. In this paper, Fractional Fourier Transforms (FRFT) is introduced into Transform Domain Communication Systems (FRFT-TDCS), which can effectively target linear frequency modulation (LFM). But the multicomponent linear frequency modulation (MLFM) is difficult to effectively eliminate, when using traditional threshold decision algorithm of TDCS. A novel threshold decision algorithm for FRFT-TDCS is proposed in this paper. Because the order of FRFT has no effect on noise, the threshold decision algorithm by using the idea of maximum interclass variance means minimum misclassification probability. Simulation results show that compared with the traditional hard threshold decision algorithm, the proposed algorithm can effectively eliminate MLFM interference and negative influence of the order of FRFT. Whether bipolar modulation or cyclic shift keying (CSK) modulation, the bit error rate (BER) of this threshold decision algorithm is always better than the traditional threshold decision algorithm; With the increase of signal to noise ratio (SNR), the BER performance of the two threshold decision algorithms increase gradually.

1. Introduction
In modern electronic warfare, anti-jamming communication technology is the key to battlefield perseverance. TDCS has been widely concerned in the communication field[1-4]. TDCS not only have the characteristics of Low Probability of Interception (LPI) and Low Probability of Interference (LPD), but also have unique ideas of actively avoiding interference by using spectrum holes. Traditional TDCS based on Fourier transform can effectively remove stationary interference but not to deal with non-stationary LFM interference effectively. In recent years, FRFT has been introduced into TDCS because of its better energy focusing property for LFM interference. And the related research on FRFT-TDCS has been carried out in the paper [5-8].

The anti-interference ability of TDCS depends on the ability of removing the interference. In addition, the setting of the threshold determines whether the interference can be completely eliminated. In the increasingly complex electromagnetic environment, the threshold decision algorithm has become one of the key technologies to improve the anti-interference ability of TDCS. In the aspect of amplitude spectrum generation, FRFT-TDCS removes the interference spectrum through the preset threshold, and the effect of interference rejection directly affects the anti-interference performance of
the system. At present, there are few researches on the threshold design of FRFT-TDCS. In paper [9], the interference spectrum is removed with 40% of the maximum spectral amplitude of the interference signal. The proposed method is simple in design and low in complexity. However, the effect of threshold decision algorithm is poor. In paper [10], a double-threshold amplitude spectrum generation method is proposed. Double thresholds are deduced from the probability of missed detection and the false alarm probability. The frequency spectrum between two thresholds is marked as 0.5, which improves the system throughput. However, The low-power access to the spectrum that may be interfered between the two thresholds which still leads to an increase of the system BER. In paper [11], deduced the relationship between the BER and the threshold value, the optimal threshold set by this method is obtained through one-dimensional search when the interference spectrum is determined. However, in most cases, we do not know whether a spectrum carrier is interference, which makes the applicability of the method has been more limited. The above methods are based on FFT-TDCS. However in FRFT-TDCS, MLFM interference in different orders of FRFT has different spectrum amplitude and position, which leads to it is difficult to effectively remove out the interference spectrum by the traditional TDCS threshold decision algorithm.

In this paper, a novel threshold decision algorithm is proposed, which aiming FRFT-TDCS to deal with MLFM poorly, by using the idea of maximum interclass variance meaning minimum misclassification probability. This algorithm can adaptively determine the threshold value, which not only can effectively eliminate the interference, but also can avoid the adverse influence by the order of FRFT.

The organization of this paper is as follows. The concept of LFM interference is introduced in Section 2. In Section 3, the principle of FRFT-TDCS is given. Threshold decision algorithm and its derivarion are in 4 Section. The simulation result shows in 5 Section and paper is concluded in 6 Section.

2. LFM interference

LFM interference is a typical non-stationary interference signal whose instantaneous frequency varies linearly with time. MLFM interference can be expressed as:

\[ x_i(t) = \sum_{k=1}^{L} A_i (j\pi k_i t^2 + j2\pi f_i t) \]  

(1)

Where \( k_i, f_i, A_i \) respectively are frequency modulation rate, the initial frequency and amplitude of the MLFM interference.

The method of paper [12] is used to estimate the parameters of the MLFM interference signal in the electromagnetic environment. The optimal order of the \( i \) component LFM interference signal can be calculated as:

\[ p_i = \frac{2}{\pi} \arccot (-k_i) \]  

(2)

Figure.1 shows the time-frequency distribution of LFM interference. Figure.2 shows the LFM interference spectral distribution of the FRFT domain.
We can see that the optimal orders of different components of are not the same. Different components of the LFM interference only in its optimal order reach the maximum value. But other components are as far away from its optimal order, which resulted in decreased peak in FRFT domain, and tends to be stable.

3. Principle of FRFT-TDCS
FRFT-TDCS is a kind of multi-carrier communication system with strongly anti-jamming capability. It uses the "clean" spectrum to join the pseudo-random phase in the FRFT domain to generate the basis function with noise-like performance and multiple access performance. Working principle of FRFT-TDCS is shown in Figure.3.
multiplies the pseudo. The basis function \( \theta \). Then the \( L \).

\[
\sum_{n=0}^{L-1} a_k n n n = \sum_{n=0}^{L-1} b_k n n n
\]

Threshold decision algorithm

Optimal order selection

Transmission signal

Amplitude adjustment

Generation of basis functions

Spectrum mask

(a) Principle block diagram of transmitter

(b) Principle block diagram of receiver

**Figure 3.** Principle block diagram of FRFT-TDCS

The transmitter take sample the electromagnetic environment in the FRFT domain to estimate the interference signal parameters; FRFT-TDCS removes the interference according to the threshold setting and selects the optimal order \( p \) to obtain the formed amplitude spectrum \( A'(k) \); Then the amplitude spectrum \( A'(k) \) multiplies the pseudo-random phase \( e^{ip} \) to get \( b'(k) \). The basis function \( b_k \) which take amplitude adjustment should be obtained by inverse fractional Fourier transform (IFRFT); The time domain waveform of the basis function \( b(n) \) can be obtained by the above steps; The information is modulated to produce a signal waveform. It is assumed that there are in the same electromagnetic environment for transmitter and receiver (e.g. Formation flying communication). Receiver will get the same basis function as transmitter, which can demodulate the received signal waveform to get the data.

FRFT-TDCS can eliminate the interference spectrum in the FRFT domain, so that the spectrum of the generated base function is orthogonal to the interference signal spectrum. Threshold decision algorithm can judge the interference spectrum; the spectrum carrier which is less than the threshold value \( T \) is retained. In contrast, the spectrum carrier is higher than the threshold value \( T \) is removed. Then the amplitude spectrum \( A(k) \) composed of 0 and 1 sequences is obtained.

\[
A(k) = \begin{cases} 
1 & \text{if } X(k) < T \\
0 & \text{if } X(k) > T 
\end{cases}
\]  \hspace{1cm} (3)

Therefore, the ability to eliminate the interference spectrum directly determines the anti-interference ability of FRFT-TDCS. If the threshold value is too high, the interference may not be eliminated, resulting in an increase in BER; if the threshold is too low, it may lead to eliminate the frequency band without interference, which can result in a reduction in the spectrum efficiency [13].

4. **Adaptive threshold decision algorithm**

First, it is proved in [14] that FRFT does not change the statistical properties of noise. Therefore, it can be considered that the amplitude of noise in the FRFT domain will not change with the change of transform order.

Then, let the phase points be \( N \) and FRFT transform order search points be \( M \cdot [0,1,2,\cdots,L-1] \) represent the amplitude of the FRFT domain. \( n_i \) stands for the number that amplitude is equal to \( i \). \( MN = n_0 + n_1 + n_2 + \cdots + n_{L-1} \). The probability of that normalized amplitude is equal to \( i \) is \( p_i = n_i / MN \).

It can be drawn that:

\[
\sum_{i=0}^{N-1} p_i = 1, \quad p_i \gg 0
\]  \hspace{1cm} (4)
Assuming there is a threshold value \( T(k) = k, \quad 0 < k < L - 1 \), the amplitude values are divided into two categories \( L_1 \) and \( L_2 \). The amplitude value \( L_1 \) consists of all search points \([0, k]\) and the amplitude value \( L_2 \) consists of all the search points \([k, L - 1]\). If \( L_1 \) is a part of noise, but \( L_2 \) contains only the part of interference, \( k \) is considered as the optimal threshold \( k^* \).

By using threshold \( k \), the magnitude assigned to \( L_1 \) has probability of \( P_1 \):

\[
P_1(k) = \frac{k}{\sum_{i=0}^{k} p_i}
\]

Then we can calculate the value of the amplitude assigned to \( L_1 \):

\[
m_1 = \sum_{i=0}^{k} iP(i/L_1)
\]

According to Bayesian formula:

\[
m_1 = \sum_{i=0}^{k} iP(L_i/i) P(i)/P(L_1)
\]

\[
m_1 = \frac{1}{P_1(k)} \sum_{i=0}^{k} iP_i
\]

get the value of the amplitude assigned to \( L_2 \):

\[
m_2 = \sum_{i=k+1}^{L-1} iP(i/L_2)
\]

\[
m_2 = \frac{1}{P_2(k)} \sum_{i=k+1}^{L-1} iP_i
\]

The average amplitude value of the entire fractional domain three-dimensional distribution is given:

\[
m_g = \sum_{i=0}^{L-1} iP_i
\]

Based on the above formula, we can calculate the following conclusions:

\[
P_1m_1 + P_2m_2 = m_g
\]

\[
P_1 + P_2 = 1
\]

We define the interclass variance as follows:

\[
\sigma_B^2 = P_1(m_1 - m_g)^2 + P_2(m_2 - m_g)^2
\]

Bringing equations (11) and (12) into (13), we can get the following equation:

\[
\sigma_B^2 = P_1P_2(m_1 - m_2)^2
\]

It can be seen from the formula (14) that the bigger the difference between \( m_1 \) and \( m_2 \) is, the larger the value of \( \sigma_B^2 \) is. This phenomenon indicates that the interclass variance represents the measure of separability between classes [15]. Therefore, the optimal threshold is maximize the value \( k \).

5. Simulation

Simulation parameter setting: the time value of LFM interference is \([-2, 2]\); the sampling frequency \( f_s \) is 128Hz; the number of sampling points is 512; the dimension of FRFT domain coordinate is normalized to 4 (the same as the signal duration); the noise is 4dB. The interference signal is three-
component LFM interference, whose frequency modulation rates are respectively 8, 12 and 20 Hz/s and the initial frequencies are 40, 15 and 5 Hz respectively. In addition, the initial phases of LFM interference are 0 rad. Corresponding to the order of 1.33, the interference power spectrum is showed in Figure 4.

As shown in Figure 4, traditional threshold decision algorithm is set based on the 40% of maximum value of power spectrum [16]. For MLFM interference, the ability of the traditional threshold decision algorithm to remove interference is terrible. The traditional threshold algorithm leads to the decrease of FRFT-TDCS anti-jamming performance. However, the adaptive threshold decision algorithm in paper can ignore the influence of the transform order on the threshold setting, and can effectively eliminate the interference spectrum under different transformation orders.

(a) Traditional threshold decision algorithm  (b) Adaptive threshold decision algorithm

Figure 5. Amplitude spectrum of three component LFM interference under different threshold decision algorithms
The simulation results in Figure 5 show that the adaptive threshold decision algorithm is more accurate than the traditional threshold decision algorithm. The adaptive threshold decision algorithm proposed in this paper is suitable for the processing of MLFM interference.

![BER performance of different threshold decision algorithms under bipolar modulation](image1)

**Figure 6.** BER performance of different threshold decision algorithms under bipolar modulation

![BER performance of different threshold decision algorithms under BCSK modulation](image2)

**Figure 7.** BER performance of different threshold decision algorithms under BCSK modulation

The BER performance of different threshold decision algorithms is shown in Figure 6 and Figure 7. Whether it is bipolar modulation or BCSK modulation, the improvement of BER is still very obvious. The BER of the adaptive threshold decision algorithm is closer to the ideal value (BER of the system without interference). With the increase of signal-to-noise (SNR), the difference of BER between the two threshold decision methods gradually increases.

6. Conclusion
Aiming that the MLFM interference is difficult to effectively eliminate for FRFT-TDCS. This paper proposes an adaptive threshold decision algorithm based on the maximum interclass variance. The algorithm adaptively determines threshold value in the three-dimensional FRFT domain. Simulation results show that compared with the traditional threshold decision algorithm, this algorithm can not only effectively eliminate the interference, but also avoid the influence of FRFT order on the threshold decision. The FRFT-TDCS anti-jamming performance has been greatly improved.

7. Appendices
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