Non-Routine Problem-Posing Skills of Prospective Mathematics Teachers*

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Purpose: Problem-posing, an important component for developing mathematical thinking, is of great interest in integrating into classroom practice. Pre-service and in-service teachers are expected to carry out high-quality problem-posing activities, and it is thought that non-routine problem-posing may be a good way to achieve this. In this context, this study focuses on non-routine problem-posing and aims to determine the characteristics of the problems that prospective mathematics teachers have posed.

Research Methods: The study was carried out with 43 middle school prospective mathematics teachers in an elective course on problem-solving and problem-solving strategies. To analyse the data, descriptive analysis was carried out on the problems posed by prospective teachers. All problems were analysed according to the five criteria; problem type, contextuality, originality, complexity, and strategy.

Findings: It has been determined that almost all of the problems have a single answer, include a context, have a low or medium level of complexity, and contain different problem-solving strategies. Although prospective teachers were asked to pose their own problems, almost half of them had posed similar, traditional problems. Implications for Research and Practice: These results show that prospective teachers can pose non-routine problems. Although this study provides some meaningful results, it is clear that it has limitations that require further investigation.

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Introduction

The role of problem-posing as an important component for learning and the potential of integrating problem-posing into various levels of formal education are recognised by an increasing number of researchers in the field of mathematics education (Singer, Ellerton & Cai, 2013). In this context, it is an undisputed fact that teachers have an important role in implementing problem-posing in the curriculum. However, many teachers avoid problem-posing and base their teaching processes on practising their routine exercises (Stickles, 2011). Moreover, it has been determined that the problems posed by teachers are not cognitively or structurally complicated, mostly similar to the problems in the textbook (Lavy & Shriki, 2007). Prospective teachers especially have difficulties in posing problems because they are not familiar with this skill neither as students nor as educators (Osana & Pelczer, 2015). For this reason, it is thought that it may be a good way for teachers and prospective teachers to pose non-routine problems to create qualified and quality problems. In this context, this study about non-routine problem-posing aims at revealing the quality of the problems posed by prospective teachers.

Problem-Posing

Freudenthal (1973), Polya (1945), and many researchers (e.g. English, 1997; Silver & Cai, 2005) emphasise the importance of problem-posing activities as part of mathematics education. Stoyanova (1998) defined problem-posing as the process in which individuals create personal comments and explain certain situations and structure them as mathematical problems. Providing students with the opportunity to pose their own problems can broaden their perceptions of mathematics, enrich and reinforce basic concepts (Brown & Walter, 1993, English, 1997), and improve mathematical thinking and creativity (Leung & Silver, 1997; Silver, 1995; Toluk-Ucar, 2009). Problem-posing activities have positive effects on students' ability to solve or analyse math problems and allow them to understand mathematical processes and concepts (Christou et al., 2005; English, 2003; Leung & Silver, 1997). Additionally, it is thought that problem-posing can help lessen students' dependence on teachers and textbooks and give students a greater sense of participation in the learning process (Lavy & Shriki, 2007).

Stoyanova (1998) also divided three categories of mathematical problem-posing situations: (a) free, (b) semi-structured, and (c) structured problem-posing situations. In free situations, which we also draw on in our study, individuals pose problems without restrictions or limitations. In semi-structured situations, individuals are given an open situation and are asked to complete it by applying knowledge, skills, and relationships from their previous mathematical experiences (Van Harpen & Presmeg, 2013). Writing problems, similar to given problems or writing problems, are based on specific pictures (images and figures can be examples of such situations). Structured problem-posing situations refer to situations in which individuals generate problems by reformulating already solved problems or through changing the conditions or questions of given problems (Christou et al., 2005).
In the literature, studies focus on problem-posing ranged from examining the cognitive processes involved in problem-solving (e.g. Harel, Koichu & Manaster, 2006), examining the role of problem-posing in problem-solving (e.g. Armstrong, 2014), identifying the problem-posing strategies of teachers and students (e.g. Silver, Mamona-Downs, Leung & Kenney, 1996), and designing pedagogical strategies that improve problem-posing skills (e.g. Crespo & Sinclair, 2008) to determining the contribution of problem-posing in developing students' mathematical knowledge and conceptual understanding (e.g. Toluk-Ucar, 2009). The conclusions drawn from this comprehensive literature can be summarised as follows: First, problem-posing supports other mathematical goals such as improving self-confidence in mathematics, deepening mathematical understanding, improving mathematical problem-solving skills, and developing mathematical abilities (Leavy & Hourigan, 2019). Second, despite all these advantages, both teachers and students cannot produce challenging problems, and most of the problems they pose are ill-formulated or poorly expressed (e.g. Silver et al., 1996). Third, the experimental studies conducted have shown that subjecting students and teachers to systematic training on problem-posing can increase their awareness of problems' consistency and significance (Singer et al., 2013). Finally, although various studies analyse different aspects of problem-posing, there are still some ambiguous points in the field, such as the advantages and disadvantages of problem-posing approaches or techniques and the dynamics of a class culture that includes problem-posing.

The mathematics that students learn in lessons is influenced by the variety of problems posed by the teachers (Stickles, 2011). Problem-posing is a critical aspect of teachers' work, as they help students to be able to pose problems better (Cai, Hwang, Jiang & Silber, 2015). For students to have the opportunity to learn to pose a problem, first of all, teachers should have sufficient knowledge and skills about problem-posing (Li, Song, Hwang & Cai, 2020). Considering all these, teachers play an important role in implementing problem-posing in the curriculum (Crespo & Sinclair, 2008; Rizvi, 2004). Using problem-posing successfully in teaching requires teachers and prospective teachers to gain knowledge and experience related to such activities (Crespo & Sinclair, 2008; Isik, Kar, Yalcin & Zehir, 2011; Lavy & Shriki, 2007). Developing problem-posing skills from producing routine problems to revealing more complex maths problems encourages teachers and prospective teachers to think about problem-posing (Milinković, 2015). Therefore, opportunities should be offered to teachers and prospective teachers to pose their own problems (Koichu & Kontorovich, 2013; Leung & Silver, 1997). However, teachers rarely refer to problem-posing as they find it difficult to apply to the teaching process and have little opportunity to pose their own pre-service education problems (Crespo & Sinclair, 2008; Leung & Silver, 1997). Additionally, many studies have reported that students' and teachers' problems are mostly cognitively simple (low level) and textbook-like (Crespo & Sinclair, 2008; Lavy & Shriki, 2007). For this reason, this descriptive study focuses specifically on posing 'non-routine' problems.

Non-Routine Problem
There are different classifications of mathematical problems in the literature, such as routine/non-routine, word/real, well-defined /ill-defined, etc. The most common of these classifications and this study’s subject is the routine / non-routine problem distinction. The distinction between these two types of problems can be explained by the complexity of the reasoning and thinking level necessary for the problem’s solution. While routine problems can be solved using familiar methods, non-routine problems do not have a predictable and repeated approach or path for their solution (Woodward et al., 2012). The extensive literature on mathematical problem-solving shows that non-routine problems are the most appropriate problem for improving mathematical problem-solving and reasoning skills (e.g. Cai, 2003; London, 2007; Polya, 1945).

Non-routine problem-solving strategies can be defined as basic procedures that can help individuals solve such problems. In the literature, the most important and prominent non-routine problem-solving strategies are: “act it out, look for a pattern, make a systematic list, work backwards, guess and check, make a drawing or diagram, write an equation or open sentence, simplify the problem, make a table, eliminate the possibilities, use logical reasoning, matrix logic, and estimation” (Herr & Johnson, 2002; Leng, 2008; Posamentier & Krulik, 2009). The most important reason for learning non-routine problem-solving strategies is that they can help students solve unfamiliar problems and broaden their perspectives (Tiong, Hedberg, & Lioe, 2005).

Most research on non-routine problem-solving aims to examine students’ skills and attitudes on this topic at that particular moment without any intervention (e.g., Elia, Van den Heuvel-Panhuizen & Kolovou, 2009; Mabilangan, Limjap & Belecina, 2012). Additionally, some studies examine the effects of an educational intervention on students' non-routine problem-solving skills (e.g., Lee, Yeo & Hong, 2014). The results of all these studies can be summarised under four headings: (i) many students think that non-routine problems are more complex and challenging than routine problems, (ii) students’ abilities to solve non-routine problems are generally low, and low and medium level students especially have difficulties in solving non-routine problems, (iii) it is useful to provide students with a framework for implementing strategies, and (iv) only a small fraction of the problems in textbooks are non-routine problems.

Related Literature

Studies on non-routine problem-posing are scarce in the literature. In one of these studies, Kilic (2017) followed an approach to problem-posing based on problem-solving strategies. In this context, prospective teachers are expected to create a problem that can be solved by finding a pattern strategy. After analysing the data, it was seen that 55% of the problems that emerged were in accordance with the desired strategy. Unlu (2017) conducted another similar study in this category, aiming to identify prospective mathematics teachers’ knowledge of problem-solving strategies through problem-posing. In this study, prospective teachers were asked to pose problems that require the use of certain problem-solving strategies. The results showed that the problems that emerged were mostly related to daily life, were suitable for the strategy, and were solvable. Additionally, many prospective teachers stated
that instead of acting creatively, they presented problems similar to those in textbooks. Kool and Keijzer (2018) asked prospective teachers to design non-routine problems in their case study. The findings revealed that interaction with peers, feedback from experts, evaluating existing problems to find criteria, creating a repertoire, and a cyclical design process are encouraging and supportive for the generation of non-routine problems.

In the studies of Ellerton (2013), and Leavy and Hourigan (2019), non-routine problem-posing was included indirectly and ambiguously. The prospective teachers in these studies posed non-routine problems similar to a particular problem or appropriate for a particular context. However, in this study, participants were asked to pose completely original non-routine problems, and these skills were examined in detail.

Rationale and Questions of the Research

Problem-posing is an essential component of mathematics teaching, making it an essential skill for prospective teachers and, most importantly, students. Numerous studies on problem-posing confirm its importance. On the other hand, although the need for routine problems in the curriculum is not denied, non-routine problems are those that adequately address the mathematical knowledge, processes, creative thinking, and communication skills that students need for the twenty-first century (Bonotto & Dal Santo, 2015). However, the vast majority of studies on mathematical problem-posing point to posing routine problems based on the three strategies (using free, semi-structured, and structured situations) mentioned in the Problem-posing section. At this point, it can be stated that the emphasis is especially on the use of semi-structured situations. Therefore, the literature often focuses on students’ generating a problem based on a specific problem outside of context (e.g., posing a problem with the answer 15). This approach does not consider the importance of context regarding the quality of the problems posed (Crespo & Sinclair, 2008).

Much of the research evidence suggests that most of the problems posed by teachers or prospective teachers focus on algorithmic process, rote, and procedural understanding while neglecting mathematical reasoning and conceptual understanding (e.g., Stein, Smith, Henningsen & Silver, 2000; Stevenson & Stigler, 1992). Simultaneously, as seen in related studies, there are very few studies directly examining non-routine problem-posing. However, non-routine problem-posing is well suited to the use of free situations and contexts. Therefore, it also contributes to the development of creativity. When making changes in problem-posing, teachers must have problem-posing skills, and this must begin in prospective teacher education for this to happen (Abu-Elwan, 2007). This study investigates the characteristics of non-routine problems that prospective middle school mathematics teachers pose for the reasons given. In connection with this purpose, the study’s research question is, “What are the characteristics of non-routine problems that prospective teachers pose in terms of problem type, contextuality, originality, complexity, and use of strategy?"
Method

Research Design

This research is built on the principles of a descriptive case study. Yin (2003) considered the descriptive case study as a description of a phenomenon (non-routine problem-posing in this study) in a specific and limited context (the skill of a limited number of prospective middle school mathematics teachers for this study) with an in-depth perspective. Such research designs are often used to provide researchers with a detailed description of the phenomenon under study. In this study, non-routine problem-posing skills of prospective middle school mathematics teachers are the main focus.

Research Procedure and Participants

This study was conducted in a semester of a “problem-solving strategies” elective course in mathematics teaching at a state university in Bursa, Turkey. This course is given in the third year of the 4-year teacher preparation program. A total of 43 students participating in this course were naturally considered as a participant of the research. The students in this group had not taken any other lessons on problem-solving before and were introduced to non-routine problem-solving for the first time in this lesson.

This course’s content, given by the second researcher, includes problem and problem-solving, problem-solving stages, classification of problems, routine and non-routine problem-solving. The course was held once a week for 14 weeks, for an average of 2 hours. In the first week, the concepts of problem and problem-solving were introduced to the participants, and in the second week, some problem classifications (routine-non-routine, word-real, well-ill defined, etc.) were taught over sample problems. In the third week, the educational importance and aims of problem-solving were discussed. The problem-solving stages of Polya (1945) were discussed in the fourth week. The main focus of the fifth, sixth, and seventh weeks were teaching routine problem-solving. Later, this focus left its place to non-routine problem-solving strategies, and in this context, making a systematic list and guess and check strategies were discussed in the eighth week. In the ninth week, finding a pattern and drawing a diagram strategy were examined. In the tenth week, the participants studied writing an equation, estimation, simplifying the problem, and working backwards and elimination strategies at the eleventh week. The twelfth week was devoted to making a table and logical reasoning strategies. The thirteenth and fourteenth weeks were devoted directly to non-routine problem-posing.

Each lesson usually started with a group activity in which participants explored problem-solving concepts and principles or worked on a problem, resulting in a review of the content’s educational results. During the lesson, the researcher always supported discussions during problem-solving and sharing the various strategies and ways to reach solutions (Leavy & Houigan, 2019). While the participants were working in groups, the researcher first supported group work one-on-one and then conducted whole-class discussions. There was not a separate title for problem-posing in the course. The main reason for this situation is not to hinder students’ creative attempts.
while posing problems. However, the participants were implicitly busy with problem-posing in two ways: (i) working on simpler versions of a problem to see a model or generalisation that would solve the original problem, and (ii) seeing how the solution might be affected by various changes in the problem by looking back (Kilpatrick, 1987).

Data Collection Tools

The data collected for this study consists of written studies of prospective teachers. At the end of the term, all participants were asked to pose a non-routine problem and submit the problems and their solutions to the researchers in writing. Considering the difficulty of posing non-routine problems, only one problem was requested to be posed. Participants were given a week of problem-posing as homework. They were asked to construct problems that were as original as possible and target problem-solving strategies. Since it is a free problem-posing situation, there is no limitation in terms of a subject or level, etc., for the problems they would pose.

Data Analysis

The nature of non-routine problems posed by the participants was evaluated through the problems they posed. For the analysis of the problems, problem-posing frameworks in the literature were examined, and the criteria developed by Crespo (2003) and Silver and Cai (2005) were adopted. All problems were analysed according to the five criteria in Table 1 below; problem type, contextuality, originality, complexity, and strategy use.

The first criterion, which is expressed as the problem type, examined whether the problem has an open-ended character, with a single answer or multiple answers. Another dimension of the problem type is related to computational and multiple methods. So, it was examined whether it was limited to only four operations. The second criterion, contextuality, is defined as the real-life situation in which the subject is dressed (Altun, 2017). Problems are considered in three categories as having a contextually poor, medium, or high level. Third, the originality of the problem is determined by looking at the source of the problem produced. By looking at how different the problems were from known problems posed or previously solved by other students, it was decided whether the generated problems would fall into self-generated or similar problem categories. The fourth criterion is the complexity of the problem. Problem complexity can be examined from various perspectives, including mathematical relationships embedded in problems, problem difficulty, linguistic complexity, and mathematical complexity (Silver & Cai, 2005). This study especially focuses on linguistic-syntactic structures embedded in posed problems. The linguistic-syntactic structure of mathematical problems focuses on the presence of assignment, relational and conditional expressions (Mayer, Lewis & Hegarty, 1992). Among these three criteria, there is an increase in difficulty from assignment to conditional. Therefore, the presence of conditional or relational expressions are considered indicators of problem complexity. Finally, in the fifth criterion, problem-solving strategies that can be chosen to solve the posed problems are predicted. Accordingly, it is classified as using single or multiple strategies.
Table 1
Evaluation of Non-Routine Problems Posed

| Problem type          | Contextuality | Originality | Complexity (Linguistic-syntactic) | Strategy Use |
|-----------------------|---------------|-------------|-----------------------------------|--------------|
| Single-answer problem | Poor          |             |                                    | Single Use   |
| Which number should replace the question mark? |
| Multiple-answer methods | Medium       |             |                                    | Multiple Use |
| How can we find the result of \(25 \times 20\)? operation with a broken multiplication key? |
| High                   |               |             |                                    | Conditional  |
| Must want to build a square pyramid-shaped apartment with other-shaped Lego in his hand. One edge of the base of the pyramid should consist of the same number of Lego as the number of floors in the apartment building. How many floors can be build the tallest apartment out of 305 Lego he has? |
| Low                    |               |             |                                    | Single Use   |
| How many of which package do you get to meet your needs at the best price? |

As an example, analysis, a problem that one of the prospective teachers posed is given in Figure 1 above. In terms of the type of problem, it has a single answer and can only be solved based on calculation. The problem, which did not contain a context for life, was considered poor in contextuality. The posed problem was evaluated in the similar questions category since it was quite similar to one of the problems discussed in the course. The text of the problem has been created in accordance with certain instructions, and therefore it is considered ‘conditional’ in terms of complexity. Limited to the reasoning strategy, this problem is in the single strategy use category.
The first researcher evaluated all problems that were posed. Afterwards, the second researcher made a second score on the problems posed by ten randomly selected participants. Thus, it was determined that there were no inconsistencies between the two researchers' scores and the evaluation was concluded to be reliable.

**Results**

The participants posed a total of 43 non-routine problems. In this section, the analysis of these problems based on the five criteria is given. Analysis of the established problems according to their type is given in Table 2 below.

**Table 2**

| Problem type          | Single answer | Multiple answers | Total |
|-----------------------|---------------|------------------|-------|
| Computational         | 9             | -                | 9     |
| Multiple methods      | 33            | 1                | 34    |
| Total                 | 42            | 1                | 43    |

Examining the problems posed in terms of their types was carried out as a single answer-multiple answer and computational-multiple method. Only one question with multiple answers was written by the prospective teachers, other than that, they preferred to write closed-ended questions with a single answer. In terms of solution, about 79% of the posed problems are designed to use multiple methods.

**Table 3**

| Contextuality | Originality | Similar Questions | Self-generated | Total |
|---------------|-------------|-------------------|----------------|-------|
| Poor          | 5           | 1                 | 6              |
| Medium        | 11          | 9                 | 20             |
| High          | 3           | 14                | 17             |
| Total         | 19          | 24                | 43             |
Analysis of the posed non-routine problems regarding their contextuality and originality is given in Table 3 above. In terms of contextuality, only six problems were determined as poor. Besides, it was observed that prospective teachers generally tend to pose contextual problems. When the preferred contexts are examined, it is determined that similar contexts are used, which are generally found in textbooks (for example, sharing materials or products such as walnuts or pencils within a group). Figure 2 below gives an example of a problem at a high contextual level.

![Figure 2](image)

**Figure 2. The Problem Classified as High in Contextuality**

In evaluating the problems in terms of originality, a classification was made between self-generated and similar questions. Approximately 44% of the established problems are similar to problems in textbooks and posed by taking advantage of the problems encountered in the teaching processes. Most of the questions similar in nature were medium in respect of context (58%). In the examinations made, it was determined that structurally similar problems were created in three different ways:

- Transforming a routine problem into a non-routine one
- Using a non-routine problem with partial changes (such as changing numbers)
- Putting a context-free non-routine problem into context
An example of the problems posed related to the third of the methods mentioned above are given in Figure 3 below. Here, the classic problem, the "handshake problem", has been revised in context.

Figure 3. Contextualised Handshaking Problem

Approximately half of the problems (56%) posed are problems created by prospective teachers themselves. It was noted that the expression of the problem in the questions they produced was a bit troublesome; there were incomprehensibilities and some expressions that affect the understanding of the sentence. It was determined that the problems they posed were created at medium and high levels (96%) concerning context.

Table 4
Analysis of Problems by Complexity

| Linguistic-syntactic | Similar Questions | Self-Generated | Total |
|-----------------------|------------------|----------------|-------|
| Assignment            | 7                | 11             | 18    |
| Relational            | 12               | 7              | 19    |
| Conditional           | –                | 6              | 6     |
| Total                 | 19               | 24             | 43    |

Analyses of the posed non-routine problems in terms of their complexity are given in Table 4 above.

Evaluation of the complexities of the problem has been handled in terms of linguistic-syntactic, and classifications have been made as assignment, relational, and conditional. It was determined that the least of the posed problems were conditional problems (14%), and all of these problems consisted of self-generated problems. It was observed respectively that mostly assignment (46%), relational (29%), and conditional (25%) types of problems were posed in self-generated problems. All of the posed problems used similar questions as those at the assignment and relational level.
Examples of the complexity dimension of the prospective teachers' problems are given in Figures 4, 5, and 6 above. The problems given in the figures are respectively classified as assignment, relational, and conditional types.
The prospective teachers, who planned using strategies to form solutions to the problems they posed, made their own solutions and expressed the appropriate strategy or strategies to be used. The types of strategies preferred by 24 prospective teachers using a single strategy are given in Table 5 above. Making a systematic list, guess and check, and drawing a diagram have been the most preferred strategies. In contrast, the least used strategy is working backwards. Whereas making a table and simplifying the problem strategies are not at all included.

### Table 5

**Preferred Strategies Based on Single Strategy Use**

| Strategy names                                      | n  |
|-----------------------------------------------------|----|
| Making a systematic list                            | 5  |
| Guess and check                                     | 5  |
| Drawing a diagram                                   | 5  |
| Finding correlation                                 | 3  |
| Reasoning                                           | 3  |
| Writing equations and equations                     | 2  |
| Working backwards                                   | 1  |
| **Total**                                           | **24** |

The strategies preferred by prospective teachers who posed problems requiring multiple strategies in solution are given in Table 6 above. 17 of the prospective teachers posed problems requiring the use of multiple problem-solving strategies. Finding a pattern, reasoning, and making a systematic list have especially been the most preferred strategies in this process. It was determined that the strategies of working backwards...
and simplifying the problem are, again, not preferred in problems that require multiple strategies.

Considering the use of strategy in terms of originality, 61% of the self-generated problems require a single strategy use, and 39% require multiple strategies. The problems created by using similar questions include 56% using a single strategy and 44% using multiple strategies. Two prospective teachers did not specify any strategies as problem solutions and made a solution based on basic operations.

Discussion, Conclusion and Recommendations

This study aimed to determine the characteristics of non-routine problems that prospective mathematics teachers pose. What was expected of prospective teachers was to pose a non-routine problem. These problems were examined through the five characteristics determined within the study’s scope, which are problem type, contextuality, originality, complexity, and use of strategy.

Most of the problems posed regarding the type of problem, which is the first criterion, can be solved with multiple methods. However, it was determined that only one multi-answer open-ended problem was posed. In the literature, when asked to establish a problem for the first time, it is more likely that participants present single-answer problems classified as word problems (Lowrie, 2002).

The problems posed are generally at medium and high levels in terms of contextuality (second criterion). It has been observed that prospective teachers tend to present the problem in a context. It is also seen that important mathematical problems are related to real-life (Isik & Kar, 2012), and contextuality is a way to do this. While Crespo and Sinclair (2008) emphasised that the importance of context in problem-posing was neglected, the opposite conclusion was reached in this study, although it was not specifically requested. Additionally, Kilic (2015) and Unlu (2017) revealed that pre-service teachers could pose contextual problems by taking their real-life situations into account.

Regarding the originality of the problem, prospective teachers were asked to pose non-routine problems that were self-generated. However, about half of them created the same or similar to existing problems. In other words, consistent with the result found by Bonotto (2013), in his study, it was observed that prospective teachers tend to create similar problems. Similarly, in various studies, it was stated that many prospective teachers, rather than being creative, posed problems similar to the problems they saw in textbooks and lessons (Crespo & Sinclair, 2008; Stickles 2006; Unlu, 2017). This was an expected situation, given that the participants did not have much experience with non-routine problems before. According to Leavy and Hourigan (2019), reshaping a given problem instead of writing a completely new non-routine problem may be the first step towards posing such problems. Some of the participants were able to write original problems regarding this criterion. This result shows that prospective teachers can cope with the situation of creating their own non-routine problems.
Another criterion for the classification of problems is the complexity dimension. When considering the increase in complexity from assignment to conditional expression, it was determined that the least number of conditional problems were established as they consisted of self-generated problems. That is, similar problems remained at low and medium level in terms of complexity. Students do not always try to reveal difficult problems but tend to be satisfied with simple complexity levels (Chen, van Dooren & Verschaffel, 2015). Therefore, a warning, instruction, or more attractive scoring systems can be presented to prospective teachers to help them create complex problems in future studies.

The fifth criterion is related to the use of strategy. The prospective teachers were able to understand the logic of the strategies and posed problems following those strategies. This result coincides with the results found by Kilic (2017) and Unlu (2017). Some strategies (such as finding a pattern) were found to be more preferred for problem-posing. Unlu (2017) put forward a similar result and found that prospective teachers were more successful in posing problems following the finding a pattern strategy. Additionally, participants set up one or fewer problems with relatively difficult and unfamiliar strategies such as working backwards and simplifying the problem. For future studies, it may be suggested to keep prospective teachers away from spending more time on non-routine problems and get them to complete problem-posing exercises for each strategy.

The prospective teachers themselves determined the use of strategies, and they solved the problems they established and stated which strategy or strategies could be used. The researchers, on the other hand, only made grouping over these strategy names. There are cases where some strategies are not expressed, although the participants used them. This was especially the case for the reasoning strategy; although the solution included using this strategy, it was determined that they did not notice it or neglected it.

Prospective teachers rarely experienced even routine problem-posing during their school years, let alone non-routine problems. Moreover, they had a teaching process that was limited to solving routine problems. Therefore, it has been observed that finding and formulating original non-routine problems is a challenging experience for them (Kool & Keijzer, 2018). Based on the results of this study, it is thought that prospective teachers have the potential to pose non-routine problems. However, it is predicted that this potential can be developed by spending more time on such problems in mathematics teachers’ education so that prospective teachers can become better at posing non-routine problems in the future.

Although providing some meaningful results, it is clear that this study has limitations that warrant further investigations. For example, this study was based on the written documents of the participants. Therefore, it is not understood which ways or strategies the participants followed in problem-posing. Thus, data triangulation with other sources such as observations, interviews, and field notes will aid a deeper analysis in this context. Another limitation of this study is that the sample is limited to prospective middle school mathematics teachers. For this reason, repeating this study
with prospective teachers attending a different program or with students from different grade levels will allow researchers to define the characteristics of non-routine problems and the strategies used in problem-posing. Finally, in this study, the participants did not have the opportunity to apply the problems they posed to their relevant class level. A more comprehensive future study involving observing the participants while implementing their problems can be of great importance to ensure the quality of the posed problems and the quality of the criteria used to evaluate them.

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Matematik Öğretmen Adaylarının Rutin Olmayan Problem Kurma Becerileri

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Özet

*Problem Durumu:* Problem kurma, matematik öğretiminin vazgeçilmez bir bileşenidir, bu da onu öğretmen adayları ve en önemlisi öğrenciler için zorunlu bir yetenek haline getirir. Öte yandan, müfredatta rutin sorunlara duyulan gereksinim reddedilmese de rutin olmayan problemler, öğrencilerimizin yirmi birinci yüzyıl için ihtiyaç duyduğu matematiksel bilgi, süreçler, temsili akılçılık ve iletişim becerilerini yeterince ele alan problemlerdir. Bununla birlikte, matematiksel problem durumu üzerine yapılan çalışmaların büyük çoğunluğu, rutin problemleri üretmeye işaret etmektedir. Araştırma kanıtlarının çoğu, öğretmenler/öğretmen adaylarının problem kurma durumunda bu sorunların matematiksel muhakeme ve kavramsal anlayıştan ziyade ezer ve prosedürel anlayışa odaklandığını göstermektedir. Aynı zamanda ilgili araştırmalar arasında görüldüğü üzere, rutin olmayan problem kurmayı doğrudan inceleyen çalışmalar çok az sayıdadır.

*Araştırmanın Amacı:* Bu çalışma ortaokul matematik öğretmen adaylarının kurdukları rutin olmayan problemlerin özelliklerini araştırmayı amaçlamaktadır. Bu amaçla başlangıç olarak çalışmanın araştırma sorusu “Öğreten adaylarının kurdukları rutin olmayan problemlerin tür, bağlam, orijinal, karmaşık ve strateji kullanma bakımdan karakteristikleri nezere nelerdir?” olarak belirlenmiştir.

*Yöntem:* Bu araştırma betimleyici durum çalışması prensipleri üzerine inşa edilmiştir. Çalışma, problem çözme ile ilgili bir seçmeli derste 43 ortaokul matematik öğretmen adayı ile yürütülmüştür. Bu dersin içeriği, problem ve problem çözme aşamaları, problemlerin sınıflandırılması, rutin ve rutin olmayan problem çözme konularını içermektedir. Ders 14 hafta boyuncahaftada bir kez ortalamda 2 saat süreyle
Bu çalışma için toplanan veriler öğretmen adaylarının yazılı çalışanlarından oluşmaktadır. Dönem sonunda tüm katılımcılardan rutin olmayan bir problem kurmaları ve problemleri çözümleri ile birliktede araştırılacakları yazılı olarak teslim etmeleri istenmiştir. Kurulan tüm problemler beş kritere göre analiz edilmiştir: tür, bağlamallaştırma, orijinallik, karmaşıklık ve strateji kullanım.

Bulgular: Katılımcılar tarafından toplamda 43 rutin olmayan problem kurulmuştur. Türü bakımından kurulan problemlerin incelemesi tek cevaplı - çok cevaplı ve hesaba dayalı çoklu yöntem olmak üzere yapılmıştır. Öğretmen adayları tarafından birden fazla cevabi olan sadece bir soru yazılmış, bunun dışında tek cevabı olan kapalı uçlu sorular yazmayı tercih etmişlerdir. Bağlamsallık açısından zayıf olan sadece 6 problem belirlenmişken, öğretmen adaylarının genellikle bağlamalı problem kurma eğiliminde oldukları görülmüştür. Tercih edilen bağlamlar ise genellikle ders kitaplarında yer alan benzer bağlamlardır. Problemlerin orijinalliği açısından değerlendirildiğinde kendi üretimi ve benzer sorular olmak üzere sınıflandırılmıştır. Kurulan soruların yaklaşık %44’ü ders kitaplarında yer alan ve öğretim süreçlerinde karşılaşılan benzer soruların ve öğretim süreçlerinde karşılaşılan benzer soruların kendi üretimi olan sorulardır. Benzer yapida olan soruların çoğu, bağlamsal açıdan orta seviyede kalmıştır. Kurulan problemlerin yaklaşık %36’ı ders kitaplarına göre kendi üretimi olan sorulardır. Kendi üretimi olan soruların %66’ında ders kitaplarından alınmış ve tekrar oluşturulan sorulardır. Kendi üretimi olan sorularların %33’ü ders kitapları ve öğretim süreçlerinde karşılaşılan benzer sorulardır.

Karmaşıklık açısından problemlerin değerlendirilmesi dilsel-söz dizimsel açıdan ele alınmış ve belirlenen, ilişkisel ve koşulsal olarak sınıflandırılmıştır. Kurulan problemlerde en az koşulsal problemin olduğu, bu problemlerin tamamının ise kendi üretimi olan problemlerden oluştuğu belirlenmiştir. Kendi üretimi problemlerinde sırasıyla en fazla belirme (%46), ilişkisel (%29) ve koşulsal (%25) türünde problemler kurulmuştur. Benzer soruların tamamı kendi üretimi olan sorulardır. Kendi üretimi olan soruların tamamı dilsel-seviyede (%96) oluşturulmuştur. Bağlamsal açıdan ise kendi üretimi olan problemlerin %96’ında dilsel seviyede kalmıştır. Kendi üretimi olan problemlerin %4, dilsel-seviyede (%22) ve ilişkisel (%28) türünde optimizasyon amaçları olmak üzere benzer soruların tamamı kendi üretimi olan problemlerdir. Kendi üretimi olan soruların tamamı ise ilişkisel ve dilsel-seviyede oluşturulmuştur. Kendi üretimi olan soruların tamamı ise ilişkisel ve dilsel-seviyede oluşturulmuştur. Kendi üretimi olan soruların tamamı ise ilişkisel ve dilsel-seviyede oluşturulmuştur. Kendi üretimi olan soruların tamamı ise ilişkisel ve dilsel-seviyede oluşturulmuştur. Kendi üretimi olan soruların tamamı ise ilişkisel ve dilsel-seviyede oluşturulmuştur.
Sonuç ve Öneriler: İlk özellik olan problemin türü ile ilgili olarak kurulan problemlerin büyük bir kısmı çoklu yöntem ile çözülebilecektir. Ancak çok cevaplı yani açık uçlu sadece 1 problem kurulmuş olduğu tespit edilmiştir. Literatürde de ilk kez bir problem kurulması istendiğinde katılımcıların kelime problemleri olarak sınıflandıralabileceği tek cevaplı problemler ortaya koymaları daha olması görülükmektedir. İkinci özellik olan bağlamı açısından katılımcıların, literatürden farklı olarak çokcevaplı problemi bir bağlam içinde vermenin tercih ettiği görülmüştür. Problemin orijinali ile ilgili öğretmen adaylarının bilindik problemler oluştururma eğiliminde oldukları görülükmüştür. Katılımcıların daha önce rutin olmayan problemlerle ilgili çok fazla deneyimleri olmadığını göz önünde alındığında bu, beklenmeyen bir durumdur. Bu kriter ile ilgili olarak orijinal problemler yayan diğer bir kısmın katılımcı için kendi rutin olmayan problemlerini oluştururma durumunda ise pajamaksideki belirlenmiş. Karşılaştırma boyutunda belirlenen koşulsal ifadeye doğru karmaşıklığın arttığı dikkate alındığında sayıca en az koşulsal problemin kuruluğu tespit edilmiştir. Öğrenciler her zaman zor problemler ortaya çıkarma eğilimi göstermektedirler. Strateji kullanan öğretmen adaylarının kendi tarafından belirlenenlerin çözümündeki belirgin bir kusurluğudan sonra belirlenen problemler ve stratejilerin kullanılabileceğini belirtmişlerdirler. Araştırmacılar ise yazılan bu strateji isimleri üzerinden sadece gruplandırılmayı yapmışlardır. Bu noktada bazı stratejilerin katılımcı tarafından kullanılmaması durumları ifade edilmediğini duyar. Bu sonuçlar, öğretmen adaylarının rutin olmayan problem üretme potansiyeline sahip olduklarını göstermektedir. Bu çalışma bazı anlamlı sonuçlar vermesine rağmen, daha ileri incelemeleri gerektiren sınırlamalara sahiptir. Bu nedenle, çalışmanın bir diğer sınırlılığı ise orneklememizin matematik öğretmen adaylarını temsil etmediğini belirtmektedir. Bu nedenle, çalışma sonucunda farklı bir programda devam eden aday öğretmenlerle veya farklı sınıf seviyelerinde öğretmenlerle tekrarlamak, araçtırmacıların kurulan rutin olmayan problemlerinin niteliklerini tanımlamalarına ve hatta problem kurmada izlenen stratejilerin belirlenmesine imkan tanıyacaktır.

Anahtar Sözcükler: Öğretmen adayları, problem kurma, rutin olmayan problemler.