Microarcsecond instability of the celestial reference frame.

M. V. Sazhin, V.E. Zharov, A.V. Volynkin, T.A. Kalinina
Sternberg Astronomical Institute, Moscow 119899, Russia

ABSTRACT
The fluctuation of the angular positions of reference extragalactic radio and optical sources under the influence of the irregular gravitational field of visible Galactic stars is considered. It is shown that these angular fluctuations range from a few up to hundreds of microarcseconds. This leads to a small rotation of the celestial reference frame. The temporal variation of these coefficients due to the proper motion of the foreground stars is of the order of one microsecond per 20 years. Therefore, the celestial reference frame can be considered inertial and homogeneous only to microarcsecond accuracy. Astrometric catalogues with microarcsecond accuracy will be unstable, and must be reestablished every 20 years.

Key words: Reference frame; weak microlensing; extragalactic radio sources.

INTRODUCTION
Modern Very Long Baseline Interferometry (VLBI) observations approach microarcsecond accuracy in the determination of the positions of extragalactic radio sources. This accuracy will soon reach the fundamental limit of positional measurements associated with the curvature of space–time. Optical astrometric experiments currently under consideration (the GAIA project) will also be able to reach microarcsecond accuracy. Here, we consider the effect of weak microlensing on the rotation of the radio celestial reference frame based on radio quasars, though this effect is general, and will also be relevant, in principle, to the corresponding optical reference frame.

Photons of a radio or optical source (a quasar, for instance) move along a path that is defined by the gravitational field of all the stars in our Galaxy. The stars in our Galaxy also move. As a result, their gravitational field is nonstationary. Thus, the paths of photons are not straight lines, but rather curves along nonstationary trajectories.

Two photons that leave the quasar at different moments of time move along different paths. Therefore, the directions in space from which an observer detects these photons will be different. This causes fluctuations in the position of the quasar. The aim of this paper is: 1) to calculate the value of variations of the angular positions of selected quasars due to this effect and 2) determine the influence of the gravitational field of visible Galactic stars on the stability of the celestial reference frame.

The difference between the real and apparent angular positions is proportional to the mass of the body that bends the trajectory of the photon. The lower the mass of a body, the less the bending of the trajectory of a test photon. The main contribution to the effect, therefore, comes from the most massive and most dense populations of stars.

Star-like objects can act on the propagation of light in two ways. The first is via microlensing, which forms two images (in place of the single true image) separated by an angular distance of the order of $10^{-3}$ arcsec. A close angular coincidence of the background quasar and deflecting body is necessary in this case. This microlensing effect can act as a space telescope with extremely high resolution [Blair & Sazhin 1993]. The microlensing effect was predicted by Paczynski (1986) and later discovered by the MACHO, EROS, and OGLE groups (Alcock et al. 1993; Aubourg et al. 1993; Udalski et al. 1994).

As an ordinary lens that has an infinite radius of gravitational action, a foreground object can change the trajectory of photons from a background object even if the microlensing effect is absent. Therefore, the second way star-like objects can act on the propagation of light is by producing a small deviation of a photon trajectory from a straight line when the deflecting body is relatively far from the unperturbed star–observer trajectory. A body with mass $M$ and impact parameter $p$ relative to the unperturbed trajectory bends a light beam by an angular amount $2M/p$. The observer measures an angular deviation $M/p$ from the unperturbed position of the quasar. This idea was considered in Sazhin (1996) in connection with the limit of astrometric positional accuracy and more theoretically in Zhdanov (1995). Let us consider a population with a local density (in the vicinity of the Sun) of the order of $n = 0.1$ pc$^{-3}$ with masses for individual objects of the order of $0.1 M_{\odot}$. If an extragalactic object is observed, one can expect that at least one deflecting object will have an impact parameter of the
order of
\[ p = (\pi R_H n)^{-\frac{1}{2}}. \]
Here, \( R_H \) is the radius of the Galactic halo. In this case, the observer expects to see an angular deviation of the apparent position of the order of 1 µarcsec = µas from the true position of this extragalactic object.

Only the second propagation effect noted above will be considered here. The first effect (microlensing) is a very rare event: it is necessary to observe one or two million stars in order to detect one microlensing effect. On the contrary, the second effect will act to some extent on every celestial source of light. One can expect a deviation of the order of 1 µas for every extragalactic source. Thus, the angular range of interest to us is from 1 µas to 1000 µas, which is well below the level of the microlensing effect, but is measurable using modern VLBI techniques. The effect of weak microlensing can already be probed at radio wavelengths due to the high accuracy of VLBI observations.

WEAK MICROLENSING

Let us consider a situation in which there are an extragalactic source (source of a test photon \( S \)), a massive body \( B \) with mass \( M_B \) (source of a gravitational field that bends the trajectory of the test photon), and an observer who detects the test photon (bent by the gravitational field of \( B \)).

The direction of the incident photon does not coincide with the straight line connecting the extragalactic source and the observer. Therefore, the apparent celestial position of the extragalactic object does not coincide with its true position.

The effect is defined by the Einstein cone, which has the value
\[ \varphi_E^2 = \frac{4GM_B}{c^2} \frac{L_{SB}}{L_{OB}(L_{SB} + L_{OB})} \]
where \( L_{OB} \) and \( L_{SB} \) are the distances from light source \( S \) to the body \( B \) and from the observer \( O \) to the body \( B \), respectively. In the microlensing effect, two images appear, which have angular distances from the body \( B \):
\[ \varphi_1 = \varphi_E + \frac{1}{2} \sqrt{\varphi^2 + 4\varphi_E^2} \]
\[ \varphi_2 = \varphi_E - \frac{1}{2} \sqrt{\varphi^2 + 4\varphi_E^2} \]
where \( \varphi \) is the angular distance from \( B \) to the true position of the quasar, and \( \varphi_1 \) and \( \varphi_2 \) are the angular distances of the two images relative to \( B \). The brightness of the second image \( \varphi_2 \) is inversely proportional to the fourth power of \( \varphi \), and in the case \( \varphi > > \varphi_E \), the observer cannot see the second image due to its weakness. In this case, the first image has approximately the same brightness as the intrinsic brightness, without the microlensing effect. The separation of the first image from the real position of the star is
\[ \delta \varphi = \frac{1}{2} \sqrt{\varphi^2 + 4\varphi_E^2} - \varphi \sim \frac{\varphi_E^2}{\varphi}. \]

The Galactic body \( B \) moves with some angular velocity \( \Omega \) due to its peculiar motion in the Galaxy. In this case, one can write an equation for \( \varphi(t) \) as a function of time:
\[ \varphi^2(t) = \varphi_p^2 + \Omega^2 t^2 \]
where \( \varphi_p \) is the angular impact parameter of the body relative to the background quasar, and we take \( t = 0 \) to be the moment of closest approach. It is easy to recalculate this equation from the modulus of the angular separation to the spherical coordinates \( \alpha \) and \( \delta \).

VARIATIONS OF THE COORDINATES OF REFERENCE QUASARS

The question of the variations of the coordinates of selected quasars is especially important because the 23rd General Assembly of the International Astronomical Union decided that, as of 1 January, 1998, the IAU celestial reference system is the International Celestial Reference System, in replacement of the J2000 system realized by the FK5 (Perry et al. 1988). The ICRS is realized by the International Celestial Reference Frame (ICRF), defined by the equatorial coordinates of a set of selected extragalactic compact radio sources determined using VLBI. The ICRF consists of a catalogue of equatorial coordinates of these radio sources (McCarthy 1996).

To test the effect of gravitational refraction (or microlensing effect), we used the catalogue published in the 1995 International Earth Rotation Service (IERS) Annual Report (RSC(IERS)95 C02) (IERS 1995). This includes a total of 607 objects spread over the sky from declination −85° to +85°. The uncertainties in the coordinates are from 50 ± 2000 µas. There are 236 primary objects that are the most compact and best observed; 321 secondary compact sources that may have very precise coordinates in the future, when more observations are accumulated; and 50 sources that are complementary objects observed for optical frame ties or other objectives.

The stability of the frame is based on the assumption that the sources have no proper motion. The hypothesis that the sources are fixed is used in performing the coordinate transformation between the celestial and terrestrial reference systems. In reality, some of these sources have significant structure on mas scales. Changes in the source’s brightness distribution can shift the effective brightness centroid of the source and thus its coordinates. A small number of objects from the reference frame exhibit such changes (Jacobs et al. 1993), and they can reach a few tenths of mas.

Gravitational refraction leads to changes in the coordinates of all sources with time, and to a small rotation of the frame. Here, we do not calculate the effect of gravitational refraction from dark deflecting bodies and faint stars, but only from visible stars. The influence of dark bodies on the stochastic motion of apparent extragalactic source positions was considered in (Sazhin 1996), and is of the order of 1 µas. The rotation of the celestial reference frame due to the influence of dark bodies and fainter stars will be considered in a later study.

We used the Guide Star Catalogue (Lasker et al. 1990; Russel et al. 1990; Jenkner et al. 1990) and the HIPPARCOS catalogue (Perryman et al. 1997) to find stars whose angular distances from the extragalactic reference sources do not exceed 2°. A total of 170 quasars have a total of 313 nearby stars. 73 quasars have 86 neighboring stars in a circle with radius 1°. For most of these stars, we know neither their mass nor their distance. Two bright stars were found in the
HIPPARCOS catalogue. The proper motion, trigonometric parallax, and spectral type of both stars are known, so that it is possible to determine their masses and distances.

To estimate the rotation of the true ICRF relative to the observed ICRF, we used only the primary reference sources. 27 sources have nearby stars, and 5 have two stars within 1'. The rotation vector $\Theta = (\theta_1, \theta_2, \theta_3)$ is defined by the matrix

$$
\mathbf{r}_i' = \begin{pmatrix} 1 & -\theta_3 & \theta_2 \\ \theta_3 & 1 & -\theta_1 \\ -\theta_2 & \theta_1 & 1 \end{pmatrix} \mathbf{r}_i,
$$

where $\mathbf{r}_i$ and $\mathbf{r}_i'$ are the radius vector of the $i$th source before and after the deflection of the light from this source. We estimated the distance to the star $L_{OB}$ using the mass-luminosity relation (Allen 1973):

$$
\log \frac{L}{L_\odot} = 3.5 \log \frac{M}{M_\odot},
$$

where $L_\odot$, $L$ are the luminosities and $M_\odot$, $M$ the masses of the star and the Sun, respectively. The mass of each star was determined in the range from $M_\odot$ to 30$M_\odot$ using the Salpeter mass function $dn/dM \sim M^{-2.35}$. The distance $L_{OB}$ can be found from the equation

$$
\log L_{OB} + 0.2L_{OB} = 0.2(m + 0.26) + 1.75 \log \frac{M_\odot}{M_\star}. 
$$

The apparent magnitudes $m$ of the stars lie in the range from 11.3 – 15.8. The distances vary from 100 to 550 pc. We assume here that the total absorption $A$ in the vicinity of the Sun is $A = 1.9''$ kpc$^{-1}$ (Kaplan & Pikelner 1979).

We find that $\Theta = (-0.08 \pm 0.02, -0.47 \pm 0.03, -0.36 \pm 0.03)$ mas for various realizations of the Salpeter mass distribution (the variances of $\Theta$ correspond to the ranges for various mass distributions). The change of $\Theta$ with time was estimated using a uniform distribution of possible proper motions of the stars in the range $–100 \div 100$ mas/year. The value of the time derivative $\Theta$ (or the angular rotation rate of the ICRF) is of the order of 1 mas per 20 years, but can reach 1 mas per year. The number of primary reference sources in the catalogue is 236. The resulting value of $\Theta$ is defined by the ~10% of the sources that have nearby stars. The stochastic process of ray deflection has an unknown distribution. If we increase the number of primary reference sources, the value of $\Theta$ will be decreased, but the law by which $\Theta$ diminishes is unknown. Since this law has a non-Gaussian distribution, the diminishing of $\Theta$ will not be simply proportional to the inverse square root of the size of the catalogue.

The largest effects of gravitational refraction were displayed by several of the extragalactic sources (Table 1). The first row of Table 1 contains the name of the source, its right ascension and declination, and their uncertainties as reported by the IERS. The second row shows the magnitude and coordinates of the nearby star. In the third row, the angular distance (in arcsec) and corrections for the quasar’s coordinates (in mas) are shown. These coordinate corrections were calculated assuming that the mass and distance of each star are $M_\star = 5M_\odot$ and $L_{OB} = 500$ pc.

The exact effect of gravitational refraction can be calculated for the two stars from the HIPPARCOS catalogue (Table 2). The change of the stars’ coordinates due to their proper motion leads to a change of the angular distance to the nearby quasar. As a result, a secular variation of the coordinates of the quasar will be observed. Figure 1 shows the change of the coordinates of the quasar 0459-753 over 100 years, from 1952 to 2050. During the period from 1986–1996 (when the observations were carried out) the right ascension and declination of the quasar changed by up to 7.5 mas and 1 mas, respectively. The first star (with number 23106) has spectral type K1 IIIp. It is a star with mass approximately $4M_\odot$ (Allen 1973). The parallax of this star is 3.43 ± 0.61 mas and its proper motion is $\mu_\alpha = -4.00 \pm 0.57$ mas/year, $\mu_\delta = -2.55 \pm 0.78$ mas/year.

We suggest that the large uncertainties of the coordinates reported by IERS (Table 2) could be connected with the apparent motion of the quasar due to gravitational refraction. The calculated values of $\Delta \alpha$ and $\Delta \delta$ are less than the observed values. The path of the quasar (Fig. 1) depends on the star’s parameters, including its coordinates and mass. If the mass is underestimated, the true values of $\Delta \alpha$ and $\Delta \delta$ should be larger. In the new version of the ICRF (available at IERS under the label RSC(WGRF)95 R 01), the quasar 0459-753 is omitted (perhaps due to large coordinate uncertainties due to the unidentified effect of gravitational refraction).

Figure 2 shows the change of the coordinates of the quasar 1213-172 over 100 years. The second star (with number 59803) has spectral type B8 III. It is a normal giant with mass approximately $10M_\odot$ (Allen 1973). The parallax of this star is 19.78 ± 0.81 mas and its proper motion is $\mu_\alpha = -159.58 \pm 0.66$ mas/year, $\mu_\delta = 22.31 \pm 0.54$ mas/year. The effect of refraction is smaller in this case, though the star is very close to the Sun (50 pc).
Table 1. Coordinates of quasars and nearby stars and the effect of gravitational refraction. $\Delta \alpha$, $\Delta \delta$ are measured in $\mu$as.

| Source/Magnitude/ Distance (") | Right ascension | Declination | Uncertainty |
|--------------------------------|-----------------|-------------|-------------|
|                                | h m s           | ° ' "       | s           |
| 0007+106                       | 0 10 31.005871  | 10 58 29.50408 | 0.000018    | 0.00042     |
| 14.7                            | 0 10 31.01     | 10 58 29.9   |            |            |
| 0.358                           | $\Delta \alpha = 39.8$ | $\Delta \delta = 219.6$ |            |            |
| 0111+021                        | 1 13 43.144949 | 2 22 17.31639 | 0.000014    | 0.00038     |
| 13.8                            | 1 13 43.13     | 2 22 17.8    |            |            |
| 0.509                           | $\Delta \alpha = -82.4$ | $\Delta \delta = 133.7$ |            |            |
| 0735+178                        | 7 38 7.393743  | 17 42 18.99868 | 0.000003    | 0.00005     |
| 4.3                             | 7 38 7.37      | 17 42 19.0   |            |            |
| 0.284                           | $\Delta \alpha = -293.6$ | $\Delta \delta = -30.5$ |            |            |
| 0912+297                        | 9 15 52.401619 | 29 33 24.04293 | 0.000017    | 0.00034     |
| 15.3                            | 9 15 52.40     | 29 33 23.5   |            |            |
| 0.496                           | $\Delta \alpha = 18.1$ | $\Delta \delta = -160.5$ |            |            |
| 1101+384                        | 11 4 27.313911 | 38 12 31.79962 | 0.000026    | 0.00038     |
| 12.8                            | 11 4 27.31     | 38 12 31.8   |            |            |
| 0.054                           | $\Delta \alpha = 776.7$ | $\Delta \delta = 1329.0$ |            |            |
| 1302-102                        | 13 5 33.015008 | -10 33 19.42722 | 0.000018    | 0.00021     |
| 14.8                            | 13 5 32.98     | -10 33 20.1  |            |            |
| 0.812                           | $\Delta \alpha = -62.6$ | $\Delta \delta = -77.0$ |            |            |
| 1514-241                        | 15 17 41.813132 | -24 22 19.47552 | 0.000019    | 0.00031     |
| 13.3                            | 15 17 41.84    | -24 22 19.8  |            |            |
| 0.536                           | $\Delta \alpha = 120.8$ | $\Delta \delta = -100.9$ |            |            |

Table 2. The coordinates of the quasars and HIPPARCOS stars

| Source/Number | Right ascension | Declination | Uncertainty |
|---------------|-----------------|-------------|-------------|
|               | h m s           | ° ' "       | s           |
| 0459-753      | 4 58 17.945614  | -75 16 37.95439 | 0.000923    | 0.00315     |
| 23106         | 4 58 17.95     | -75 16 38.0  | 0.000037    | 0.00060     |
| 1213-172      | 12 15 46.751743 | -17 31 45.40314 | 0.000043    | 0.00041     |
| 50803         | 12 15 48.47    | -17 32 31.1  | 0.000042    | 0.00049     |

CONCLUSION

There are two important conclusions from our investigation. First, we have shown that variations of the angular positions of reference quasars range from a few microarcseconds up to hundreds of microarcseconds. The effect of weak microlensing leads to a small rotation of the celestial reference frame. The nondiagonal coefficients of the rotation matrix are of the order of microarcsecond. The temporal variation of these coefficients due to stars’ proper motions is of the order of one microarcsecond per 20 years. Because the proper motions of the majority of Galactic stars are unknown, the extragalactic reference frame can be considered inertial and homogeneous only to microarcsecond accuracy. The effect considered here should be investigated using current VLBI catalogues; however, this effect is general, and astrometric optical catalogues with microarcsecond accuracy will also be unstable, and must be reestablished about every 20 years. It is possible that one example of weak microlensing has been found: the source 0459-753, which has a nearby HIPPARCOS star and shows large uncertainties in its coordinates.

Acknowledgments

We thank Dr. A. Kuzmin, V. Sementsov, K. Kuimov, and M. Prokhorov for a critical reading of the original version of the paper and many helpful suggestions. We would also like to acknowledge an anonymous referee whose comments and remarks substantially improved our paper. This work was supported in part by the “Cosmion” Center for Cosmo-Particle Physics, the Russian Foundation for Basic Research (grants NN 97-02-17434, 97-05-64342, 98-05-64797), and Russian Federal Programme "Integration" (project K0641).
Microarcsecond instability of the celestial reference frame.

Figure 2. The variation of the coordinates of the quasar 1213-172 due to the proper motion of the star 59803.

REFERENCES
Alcock C., Akerlof C.W., Allsman R.A. et al., 1993, Nature, 365, 621
Allen C.W., ed., 1973, Astrophysical Quantities. Univ. of London, The Athlone Press
Aubourg E., Barette P., Brehin S. et al., 1993, Nature, 365, 623
Blair D.G., Sazhin M.V., 1993, Astron. and Astrophys. Trans., 3, 191
Fricke W., Schwan H., Lederle T., 1988, Fifth Fundamental Catalogue, Part I, Veroff. Astron. Rechen Inst., Heidelberg
Hog E., Novikov I.D., Polnarev A.G., 1994, Nordita Preprint, Macho Photometry and Astrometry, Nordita – 94/26 A
IERS, 1995, 1994 International Earth Rotation Service Annual report, Observatoire de Paris
Jacobs C.S., Soevers O.J., Williams J.G., Standish E.M., 1993, Advances in Space Research, 13, N 11, 161
Jenker H., Lasker B.M., Struch C.R., McLean B.J., Shara M.M., Russel J.L., 1990, AJ, 99, 2082
Kaplan S.A., Pikelner S.B., 1979, Physics of Interstellar Medium, Moscow, Nauka (in russian)
Lasker B.M., Struch C.R., McLean B.J., Russel J.L., Jenker H., Shara M.M., 1990, AJ, 99, 2019
McCarthy D.D., 1996, ed., IERS Conventions. IERS Technical Note 21, Observatoire de Paris
Pacinsky B., 1986, ApJ, 304, 1
Perryman M.A.C., Lindegren L., Kovalevsky J., Hog E., et al., 1997, A& A, 323, L49
Russel J.L., Lasker B.M., McLean B.J., Struch C.R., Jenkner H., 1990, AJ, 99, 2059
Sazhin M.V., 1996, Pisma v Astron. Zhurn., 22, 643 (in russian)
Udalski A., Szynamski M., Kaluzny J., et al., 1994, ApJ Lett., 426, L69
Zhadanov I.I, 1995, A& A, 299, 321