Marginal perturbations of N=4 Yang-Mills as deformations of $\text{AdS}_5 \times S^5$

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Abstract

We study constant dilaton supersymmetric solutions of type IIB Supergravity with 5-form and 3-form flux with isometry group $U(1) \times Z_3$. Some of these solutions correspond to marginal perturbations of N=4 Yang-Mills. We find one line of solutions in particular of $\text{AdS}_5$ fibred over an $S^5$. This line is described by a single complex parameter. $\text{AdS}_5 \times S^5$ is obtained when this parameter is tuned to zero.

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1 Introduction

Much progress has been made in understanding non-perturbative aspects of gauge theories in the last few years. A particularly fruitful approach to studying gauge theories was suggested by Maldacena who conjectured a duality relating quantum field theories to supergravity (and more generally string theory) in appropriate geometrical backgrounds. The conjecture is particularly powerful in the case of conformal field theories which are believed to be dual to anti-de Sitter spaces. The reason why the conjecture is most powerful in the CFT case is that to trust supergravity one normally has to take the ‘t Hooft coupling constant to be large and this leads to problems with defining the dual continuum asymptotically free gauge theory.

The greatest success story of Maldacena’s conjecture is that of N=4 Yang-Mills theory which is believed to be dual to type IIB supergravity in an AdS$_5 \times$S$^5$ background geometry. In this particular case physicists have been able to calculate Wilson loop correlators, chiral primary operator correlation functions etc$^1$. Much is also known about relevant perturbations of this gauge theory on the supergravity side$^2$.

In this paper we look for supergravity solutions dual to marginal deformations of N=4 Yang-Mills theory. These deformations were discovered in the field theory context by Leigh and Strassler in a beautiful paper$^2$. In$^2$ the authors developed non-perturbative methods to establish the existence of exactly marginal operators which preserve at least N=1 supersymmetry.

This paper is structured as follows. In section 2 we briefly describe the exactly marginal superpotentials of Leigh and Strassler. In section 3 we describe how to incorporate some of the isometries of the field theory in the Killing spinor. In section 4 we solve for the fields of IIB supergravity by solving the Killing spinor equations. In section 5 we impose source equations and Bianchi identities on the supersymmetric field configurations of section 4. In section 6 we describe a set of solutions dual to a complex line of conformal field theories. Finally, in section 7 we close with some comments.

2 Leigh-Strassler deformations of N=4 Yang-Mills theory

Using field theory techniques, marginal perturbations of $\mathcal{N} = 4$ supersymmetric gauge theory were analyzed by Leigh and Strassler in an elegant paper$^2$. The essential idea of the analysis is as follows. Using the fact that the $\beta$ functions$^3,^4$ of the couplings depend only on the anomalous dimensions and gauge group representation of the chiral fields, one can write down the equations for a fixed point as a constraint on the anomalous dimensions. These equations define a hypersurface in the space of couplings. In general the number of equations

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$^1$See$^5$ for an extensive review and exhaustive references on these topics.
equals the dimension of the space of couplings and thus the solutions are isolated points in
the space of couplings. However, in the presence of additional symmetries the number of inde-
pendent anomalous dimensions gets reduced and gives rise to a smaller number of constraints
resulting in a larger subspace of conformal field theories. Moving in this subspace corresponds
to marginal deformations of the field theory. These techniques enable one to obtain exactly
marginal perturbations of $\mathcal{N} = 4$ theory.

For the sake of completeness let us briefly review the relevant part of the analysis of Leigh
and Strassler\cite{Leigh:1995at}. We start with $\mathcal{N} = 4$ supersymmetric Yang-Mills theory with gauge group
$SU(n)$, $n > 2$ written in a $\mathcal{N} = 1$ language, which corresponds to a choice of a complex
structure in the space of scalars. The action consists of one vector and three chiral multiplets
with a superpotential, $W = g \text{Tr}([\Phi^1, \Phi^2]\Phi^3)$ where $g$ is the gauge coupling. In this form only
a $SU(3) \times U(1)$ part of the R-symmetry is manifest where $U(1)$ acts as a common
phase on the chiral fields and $SU(3)$ is the unitary rotation of the 3 chiral fields.

In order to study marginal perturbations the superpotential is perturbed by special ad-
ditional terms which generically break the R-symmetry down to $U(1)$ and reduce the super-
symmetry to $\mathcal{N} = 1$. The general form for the perturbed superpotential considered in \cite{Leigh:1995at}
is
\[
W = \lambda_1 \text{Tr}([\Phi^1, \Phi^2]\Phi^3) + \lambda_2 \text{Tr}(\{\Phi^1, \Phi^2\}\Phi^3) + \lambda_3 \sum_{i=1}^3 \Phi_i^3. 
\] (1)
For generic couplings in (1), the R-symmetry is reduced to $U(1) \times Z_3 \times Z_3$. The first $Z_3$ acts as
the group of cyclic permutations on the $\Phi_i$'s which implies that the anomalous dimensions of all
three chiral fields are equal. The second $Z_3$ acts as follows: $(\Phi^1, \Phi^2, \Phi^3) \rightarrow (\omega \Phi^1, \omega^2 \Phi^2, \omega^3 \Phi^3)$,
with $\omega^3 = 1$, it prevents the mixing of different fields \cite{Leigh:1995at}.

The general expressions of the $\beta$ functions for the couplings in the superpotential and for
the gauge coupling is obtained explicitly \cite{Leigh:1995at} as
\[
\beta_{\lambda_i} = \lambda_i(\mu)(-d_w + \sum_{i=1}^3 [d_i + \frac{1}{2}\gamma_i]), \\
\beta_g = -f(g[\mu]) \left( [3C_2(G) - T_i] + \sum_i T(R_i)\gamma_i \right). 
\] (2)
Where $d_w, d_i$ are canonical dimensions of the superpotential and the field $\phi$, $\gamma_i$ is the anomalous
dimension of the chiral fields $\phi_i$. $C_2(G)$ and $T_i$ are the quadratic Casimirs of the adjoint
representation and the representation of $\Phi_i$ respectively. $f(g)$ is a function of the gauge coupling
which can have a pole at large $g$.

In the present case \cite{Leigh:1995at}, due to the $Z_3$ symmetry the $\beta$-functions are reduced to the common
anomalous dimension $\gamma_i = \gamma(g, \lambda_1, \lambda_2, \lambda_3)$ with scale dependent multiplicative prefactors. As a
result vanishing of the $\beta$ functions implies a single equation
\[
\gamma(g, \{\lambda_i\}) = 0, \quad (3)
\]
which corresponds to a complex codimension one hypersurface in the four dimensional space of couplings. The origin in \( \{ \lambda_i \} \) is \( \mathcal{N} = 4 \) supersymmetric Yang-Mills theory. A generic point, however, corresponds to an \( \mathcal{N} = 1 \) superconformal theory.

As a consequence of the AdS/CFT correspondence there must exist a family of gravity duals corresponding to the marginally perturbed conformal theories which are continuously connected to \( \text{AdS}_5 \times S_5 \) with isometry same as that of the superconformal group. In the following sections we will construct gravity duals of marginal perturbations of \( \mathcal{N}=4 \) Yang-Mills theory.

3 Constraints on Killing spinors with \( U(1) \times Z_3 \) isometry

As indicated in the previous section, for generic values of the coupling constants, \( \lambda_i \), the global symmetry is broken from \( \text{SO}(6) \) down to \( U(1) \times Z_3 \times Z_3 \). When using \( \mathcal{N}=1 \) superspace, the maximal manifest global symmetry is \( U(1) \times \text{SU}(3) \) where the \( \text{SU}(3) \) rotates the three chiral multiplets into each other while the \( U(1) \) multiplies them all by the same phase. To have a particular \( \mathcal{N}=1 \) description of the system one picks a complex structure on the scalars so that they can be organized into chiral multiplets. The \( \text{SU}(3) \) acts in a way which preserves that complex structure.

\( \mathcal{N}=4 \) Yang-Mills in the AdS/CFT description is realized on the supergravity side by the near-horizon geometry of D3-branes spanning the directions 0123. The transverse directions 456789 are in one-to-one correspondence with the scalars of the Yang-Mills theory. To give an \( \mathcal{N}=1 \) description we define a complex structure on the transverse coordinates as follows:

\[
 z^m = x^{m+3} + i x^{m+6}
\]  

where the label \( m \) takes values in 1, 2, 3. Now the \( z^m \) form a triplet under the \( \text{SU}(3) \) subgroup of \( \text{SO}(6) \), and they have the same charge under the \( U(1) \).

Suppose we have a Killing spinor \( \eta \) for a geometry dual to a marginal deformation. Then clearly a \( U(1) \times Z_3 \) transform of \( \eta \) must also be a Killing spinor. While generic elements of \( \text{SU}(3) \) would transform \( \eta \) into spinors which do not satisfy the Killing spinor equations. To investigate the constraints imposed by this simple requirement consider the generators of \( \text{SO}(6) \) in the \( 4 \oplus \mathbf{T} \) representation:

\[
 J_{ij} = i/2 [\hat{\gamma}_i, \hat{\gamma}_j], \quad i, j = 4, \ldots, 9
\]  

where \( \hat{\gamma}_i \) are flat space \( \gamma \)-matrices:

\[
 \{ \hat{\gamma}_i, \hat{\gamma}_j \} = 2 \delta_{ij}.
\]  

\(^2\)In all that follows we will ignore the second \( Z_3 \) simply because we donot understand how to incorporate it without requiring more symmetry than we want. Our explicit solutions do not respect it either. We are presently investigating Killing spinors which are invariant under \( U(1) \times \text{SU}(3) \), these would be invariant under both the \( Z_3 \)s.
Now consider defining $\gamma$-matrices with holomorphic indices corresponding to the complex structure defined above:

$$\hat{\Gamma}_m = \frac{1}{2}(\hat{\gamma}_{m+3} + i\hat{\gamma}_{m+6}),$$

$$\{\hat{\Gamma}_m, \hat{\Gamma}_n\} = 0,$$

$$\{\hat{\Gamma}_m, \hat{\Gamma}_\pi\} = \delta_{mn},$$

$$\hat{\Gamma}_\pi = (\hat{\Gamma}_n)^\dagger.$$  \hspace{2cm} (7)

Now consider the following commuting subset of the generators $J_{ij}$:

$$Q = \frac{1}{3}(\hat{\Gamma}_1\hat{\Gamma}_1 + \hat{\Gamma}_2\hat{\Gamma}_2 + \hat{\Gamma}_3\hat{\Gamma}_3) - 1,$$

$$H_1 = \hat{\Gamma}_1\hat{\Gamma}_2 - \hat{\Gamma}_2\hat{\Gamma}_1,$$

$$H_2 = \frac{1}{2}(\hat{\Gamma}_1\hat{\Gamma}_1 + \hat{\Gamma}_2\hat{\Gamma}_2 - 2\hat{\Gamma}_3\hat{\Gamma}_3).$$  \hspace{2cm} (8)

We will take $Q$ to generate the $U(1)$ and $H_i$ to be the two Cartan subalgebra generators of $SU(3)$. We now decompose the $4 \oplus \overline{4}$ representation of the spinors of $SO(6)$ into eigenstates of $Q$. There are 4 distinct eigenvalues: $-1, -1/3, 1/3, 1$, the subspaces corresponding to these eigenvalues are given in terms of states built on the spinor $\chi$ which satisfies:

$$\hat{\Gamma}_m \chi = 0.$$  \hspace{2cm} (9)

The subspaces are:

$$\epsilon_{-1} = a_0^0 \chi$$

$$\epsilon_{-1/3} = a_i^i \hat{\Gamma}_i \chi$$

$$\epsilon_{1/3} = b_j^j \hat{\Gamma}_1 \hat{\Gamma}_2 \hat{\Gamma}_3 \chi$$

$$\epsilon_1 = b_0^0 \hat{\Gamma}_1 \hat{\Gamma}_2 \hat{\Gamma}_3 \chi.$$  \hspace{2cm} (10)

The $a, b$ coefficients are complex numbers. The subspaces $\epsilon_1, \epsilon_{-1/3}$ together span the $4$ of $SO(6)$ while the remaining two span the $\overline{4}$. Under $SU(3)$ $\epsilon_{\pm 1}$ are singlets, $\epsilon_{-1/3}$ is in the $3$ and $\epsilon_{1/3}$ in the $\overline{3}$.

Now we must find an $\eta$ which picks up a constant phase under the $U(1)$, but is not invariant under $SU(3)$ except under its discrete $\mathbb{Z}_3$ subgroup of cyclic permutations. Since $\epsilon_{\pm 1}$ are $SU(3)$ singlets neither one of them is a possibility. Instead we must pick a spinor either from $\epsilon_{-1/3}$ or $\epsilon_{1/3}$. Since either one will do (upto a redefinition of complex structure $z^m \leftrightarrow z^\overline{m}$ they are the same) we pick the following combination from $\epsilon_{-1/3}$:

$$\eta = \sum_{m=1}^{3} \hat{\Gamma}_m \chi.$$  \hspace{2cm} (11)

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Clearly $\eta$ is symmetric under $S_3$ (which contains $Z_3$) and picks up a phase under the $U(1)$, while under generic $SU(3)$ rotations $\eta$ is not invariant. Note that since $\eta$ is a chiral spinor in 10d it satisfies the following chiral property:

$$\gamma^{0123}\eta = i\eta.$$  

4 Solving the Killing spinor equations

In this section we solve the Killing spinor equations assuming that the dilaton is constant. The reason why we hold the dilaton constant is that the dilaton is related to the Yang-Mills coupling constant and should not run because of conformal symmetry.

The variation of the dilatino and gravitino under supersymmetry with the dilaton held constant are as follows $[6]$:

$$\delta \lambda = -\frac{i}{24} \gamma^{ijk} \eta G_{ijk},$$

$$\delta \psi_i = D_i \eta + \frac{i}{480} \gamma^{i_1 i_2 i_3 i_4 i_5} \gamma_i \eta F_{i_1 i_2 i_3 i_4 i_5} + \frac{1}{96} (\gamma_i \gamma^{jkl} G_{jkl} - 9 \gamma^{jk} G_{ijk}) \eta^*.$$  

We take the following metric ansatz:

$$ds^2 = \Omega^2 \eta_{\mu\nu} dx^\mu dx^\nu + 2 g_{m\bar{n}} dz^m dz^{\bar{n}},$$  

and assume that $F_5 = \mathcal{F}_5 + *\mathcal{F}_5$, with

$$\mathcal{F}_5 = \mathcal{F}_{0123m} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge dz^m + \mathcal{F}_{0123\bar{m}} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge dz^{\bar{m}}.$$  

Using the explicit form of the Killing spinor $\eta$ given in (11) one finds that the equations (13) are satisfied if the following equations hold:

$$F_5 = \frac{1}{4} \partial_m \Omega^4 dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge dz^m + \frac{1}{4} \partial_m \Omega^4 dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge dz^{\bar{m}}$$

$$- \frac{1}{12} \partial_n \ln \Omega^4 \sqrt{\det g} \epsilon_{mnpqrs} g^{pq} dz^m \wedge dz^p \wedge dz^q \wedge dz^{\bar{r}} \wedge dz^{\bar{s}}$$

$$- \frac{1}{12} \partial_{\bar{m}} \ln \Omega^4 \sqrt{\det g} \epsilon_{mnpqrs} g^{\bar{m}\bar{p}} dz^{\bar{m}} \wedge dz^{\bar{r}} \wedge dz^{\bar{s}} \wedge dz^{\bar{r}} \wedge dz^{\bar{s}}$$

$$G = K dz^1 \wedge dz^2 \wedge dz^3$$

$$\partial_m (\Omega^2 g_{n\bar{p}}) = \partial_{n\bar{m}} (\Omega^2 g_{m\bar{p}}).$$  

Here $\epsilon_{mnpqrs}$ is a completely anti-symmetric symbol with $\epsilon_{123123} = -1$.

3We take this ansatz so as to preserve $SO(1,3)$ invariance in the 0123 directions.
It is convenient to define the rescaled metric \( \hat{g}_{\mu_\nu} = \Omega^2 g_{\mu_\nu} \). This re-scaled metric is Kahler according to (16). \( \hat{g} \) satisfies an additional constraint which comes from the differential equation for the spinor \( \chi \) imposed by the equations in (13). This differential equation can be expressed in terms of a function \( f \) defined by

\[
0 = \partial_m \ln f - \frac{1}{2} \partial_m \ln \Omega + \frac{1}{2} \sum_b \hat{e}_b^\nu \hat{e}_b^\rho \partial_m \hat{g}_{\rho\nu} - \frac{1}{8} \partial_m \ln \det \hat{g}
\]

(17)

\[
0 = \partial_m \ln f - \frac{1}{2} \partial_m \ln \Omega - \frac{1}{2} \sum_b \hat{e}_b^\nu \hat{e}_b^\rho \partial_m \hat{g}_{\rho\nu} + \frac{1}{8} \partial_m \ln \det \hat{g}.
\]

The indices \( a, b \) are tangent space indices and the \( \hat{e} \) are vierbein for the \( \hat{g} \) metric. Notice that the index \( a \) is uncontracted in the two equations above. Thus the equations must be independent of \( a \) (this is related to the \( \mathbb{Z}_3 \) symmetry), and \( \partial_m \ln \det \hat{g} = 0 \). These two constraints are solved by constant metrics \( \hat{g} \) symmetric under cyclic interchange of indices. There may be more general solutions.

### 5 Bianchi identities and source equations

Once we have solved the Killing spinor equations we must impose Bianchi identities as well as the source equations determining the various arbitrary functions appearing in the above supersymmetric ansatz. The equations remaining to be imposed or checked are:

\[
dF_5 = \frac{i}{8} G \wedge \overline{G} + \rho_{\text{local}}
\]

\[
dG = 0
\]

\[
D^{i_3}G_{i_1i_2i_3} = -\frac{2i}{3} F_{i_1i_2i_3i_4i_5} G^{i_4i_5}
\]

The third equation in (18) is automatically satisfied. The second equation gives the constraint:

\[
\partial_m K = 0.
\]

(19)

In other words \( K \) is a holomorphic function of the \( z^m \). Finally, the first equation imposes:

\[
\partial_m (\sqrt{\det \hat{g}} \hat{g}^{\mu_\rho} \partial_\rho \ln \Omega^4) + \partial_m (\sqrt{\det \hat{g}} \hat{g}^{\mu_\rho} \partial_\rho \ln \Omega^4) = -\frac{1}{8} |K|^2 + \rho_{\text{local}}
\]

(20)

where \( \rho_{\text{local}} \) collects sources for any D3-branes in the problem. Expressing this last equation in terms of the rescaled metric \( \hat{g} \) one can put it in a somewhat simpler form:

\[
\partial_m (\sqrt{\det \hat{g}} \hat{g}^{\mu_\rho} \partial_\rho \Omega^{-4}) + \partial_m (\sqrt{\det \hat{g}} \hat{g}^{\mu_\rho} \partial_\rho \Omega^{-4}) = -\frac{i}{8} |K|^2 + \rho_{\text{local}}
\]

(21)
Finally using the fact that \( \hat{g} \) is Kahler one can further simplify the equation to:

\[
2\sqrt{\det \hat{g}\hat{g}^{\overline{m}\overline{n}} \partial_m \partial_n \Omega^{-4}} = -\frac{i}{8} |K|^2 + \rho_{\text{local}} \tag{22}
\]

This completes the description of a general supersymmetric solution with \( U(1) \times \mathbb{Z}_3 \) isometry.

6 A one-parameter line of solutions

In this section we discuss a class of solutions describing a one-parameter family of fibered AdS

\[ 5 \] spaces which satisfy the above equations. Consider the simplest case where the metric:

\[
\hat{g}_{m\overline{n}} = \frac{1}{2} \delta_{m\overline{n}}. \tag{23}
\]

This metric satisfies all the constraints. The only equation we need to solve is (22), which is an ordinary Laplace equation with a source. If we have \( N \) D3-branes localized at the origin of \( (z^1, z^2, z^3) \) then consider the following solution:

\[
\Omega^{-4} = 1 + \frac{4\pi g_s s}{r^4} + \frac{1}{12} |F(z^1 + z^2 + z^3)|^2. \tag{24}
\]

where \( r^2 = \sum_{m=1}^{3} |z^m|^2 \). We are assuming an asymptotically flat solution, hence \( F \) must vanish at large \( r \). Notice that \( F \) is a holomorphic function of a single variable: \( z^1 + z^2 + z^3 \). This function satisfies (22) with \( K = F' \).

We will consider a particular form for \( F \) so that it survives in the near-horizon limit without dominating over the term describing the localized D3-branes. Consider then:

\[
F = -\frac{d\alpha'}{2(z^1 + z^2 + z^3)^2}. \tag{25}
\]

We can now take the near-horizon limit of (22): \( \alpha' \to 0 \) while keeping \( w^m = z^m/\alpha' \) fixed, we also define \( u = r/\alpha' \) and \( w^m = uf_m(y_i) \), where the \( y_i \) are coordinates on \( S^5 \) and \( \sum_{m=1}^{3} |f_m|^2 = 1 \). The metric in these new coordinates is given by

\[
ds^2 = \alpha' \left[ (u^2 \Lambda^2 \eta_{\mu\nu} dx^\mu dx^\nu + \frac{du^2}{\Lambda^2 u^2}) + \Lambda^{-2} d\Omega^2_{S^5} \right] \tag{26}
\]

where \( \Omega^2 \equiv \alpha' \Lambda^2 u^2 \) and \( d\Omega^2_{S^5} \) is the metric on \( S^5 \). The metric is an AdS_5 space fibered over a transverse space which is conformal to an \( S^5 \). The radius of curvature at fixed \( S^5 \) coordinates is:

\[
R^2_{\text{AdS}} = \Lambda^{-2} = \sqrt{4\pi g_s N + \frac{1}{48} |d|^2 / (f_1 + f_2 + f_3)^4}. \tag{27}
\]
The 3-form $G$ can be written as:

$$G = \alpha' \frac{d}{(z^1 + z^2 + z^3)^3} dz^1 \wedge dz^2 \wedge dz^3$$

$$= \alpha' \frac{d}{(w^1 + w^2 + w^3)^3} dw^1 \wedge dw^2 \wedge dw^3$$

$$= \alpha' \frac{d}{(f_1 + f_2 + f_3)^3} [df_1 \wedge df_2 \wedge df_3 + \frac{du}{u} \wedge (f_1 df_2 \wedge df_3 - f_2 df_1 \wedge df_3 + f_3 df_1 \wedge df_2)]. \quad (28)$$

Both the metric and the $G$-field has an isometry group $SO(4, 2) \times U(1) \times Z_3$ where the $Z_3$ acts as cyclic permutation of the labels of $z^m$. The symmetry of the solution under the last two factors is obvious. In order to check the invariance under SO(4,2), recall that it consists of the following generators: the generators of Poincare transformation acting on $x$'s, scale transformations which act as $(x, u) \rightarrow (cx, c^{-1} u)$ and special conformal transformations which act on both $x$ and $u$. The isometry under the Poincare and scale transformations is obvious. The killing vector for the special conformal transformations in our coordinates is given by

$$\xi = \epsilon^\sigma \left[ - \frac{1}{\Lambda^4 u^2} \partial_\sigma + x^2 \partial_\sigma - 2x_\sigma (u \partial_u - x^\lambda \partial_\lambda) \right] \quad (29)$$

A straightforward evaluation of the Lie derivative on the metric shows it is invariant while for the $G$-field it is sufficient to check that the Lie derivative on the 1-form $\frac{du}{u}$ vanishes.

Notice that this solution is a one complex parameter family of solutions labeled by $d$. When $d$ vanishes the space becomes AdS$_5 \times S^5$ which is dual to N=4 Yang-Mills theory. Thus this solution represents a supergravity dual of a continuous conformal deformation of N=4 Yang-Mills theory. It would be interesting to understand its relation to the Leigh-Strassler deformations and to try to recover the remaining direction of marginal perturbations predicted in[2].

7 Conclusions

In this paper we studied new supersymmetry preserving solutions of type IIB supergravity with a constant dilaton. Our aim was to discover backgrounds dual to certain marginal perturbations discovered by Leigh and Strassler[2] of N=4 Yang-Mills theory.

Under assumptions of preserving certain isometries we were able to solve the Killing spinor equations. The solutions of the Killing spinor equations were given in terms of a constrained Kahler metric, a holomorphic 3-form and a function $\Omega$. These were related to each other by a source equation.

N=4 Yang-Mills has a single marginal coupling: the Yang-Mills coupling. In this paper we found a solution dual to a gauge theory preserving N=1 supersymmetry with two marginal
couplings. These form a new continuous line of supersymmetry preserving solutions parameterized by two complex parameters: \( \tau = C_0 + i/g_s \) and \( d \) whose gauge theory interpretation we defer for the moment. For a single value of the parameter \( (d = 0) \) one recovers the dual of N=4 Yang-Mills theory. At other values there are new gauge theory duals.

There are a number of open questions. Although we set out to find the duals of Leigh-Strassler marginal deformations we have not established the correspondence by relating our \( d \) to the \( \lambda_i \) couplings of the superpotential. According to Leigh and Strassler the dimensionality of the space of marginal couplings is 3. It would be very interesting to find the remaining marginal direction. We hope that more general solutions to our equations will yield the full space of theories dual to the Leigh-Strassler deformations. In particular there are more general ansätze for constant metrics which we have not investigated and which may yield new marginal directions. We leave this to future work. Another question is related to the second \( Z_3 \) symmetry group which is not preserved by our solutions. We have no understanding of why this symmetry is not present in our solutions. One possibility is that the supergravity solution is hinting at a discrete anomaly in the field theory which prevents the \( Z_3 \) from being realized. By acting on our solution with this \( Z_3 \) one can generate new solutions, however, the resulting solutions are not invariant under the first \( Z_3 \) (the group of cyclic permutations).

**Note Added:** As we were completing the paper for submission we received [7, 8] which discuss marginal perturbations of theories arising from orbifolds of N=4 Yang-Mills theory.

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