Theory of metascreen-based acoustic passive phased array

Yong Li\textsuperscript{1,2}, Shuibao Qi\textsuperscript{1,2} and M Badreddine Assouar\textsuperscript{1,2}

\textsuperscript{1} CNRS, Institut Jean Lamour, Vandeur-\textsc{e}uvres-\textsc{l}es-\textsc{N}ancy F-54506, France
\textsuperscript{2} Université de Lorraine, Institut Jean Lamour, Boulevard des Aiguillettes, BP: 70239, Vandeur-\textsc{e}uvres-\textsc{l}es-\textsc{N}ancy 54506, France

E-mail: Yong.Li@univ-lorraine.fr and Badreddine.Assouar@univ-lorraine.fr

Keywords: acoustic metascreen, acoustic metasurface, passive phased array, wave manipulation

Abstract

The metascreen-based acoustic passive phased array provides a new degree of freedom for manipulating acoustic waves due to their fascinating properties, such as a fully shifting phase, keeping impedance matching, and holding subwavelength spatial resolution. We develop acoustic theories to analyze the transmission/reflection spectra and the refracted pressure fields of a metascreen composed of elements with four Helmholtz resonators (HRs) in series and a straight pipe. We find that these properties are also valid under oblique incidence with large angles, with the underlying physics stemming from the hybrid resonances between the HRs and the straight pipe. By imposing the desired phase profiles, the refracted fields can be tailored in an anomalous yet controllable manner. In particular, two types of negative refraction are exhibited, based on two distinct mechanisms: one is formed from classical diffraction theory and the other is dominated by the periodicity of the metascreen. Positive (normal) and negative refractions can be converted by simply changing the incident angle, with the coexistence of two types of refraction in a certain range of incident angles.

1. Introduction

Modulating wave propagation in a desired way has always been of great interest to physicists and engineers. Optical lenses, as the most classic example, have been used to focus optical waves since the dawn of civilization. With the development of knowledge, a number of methods for the manipulation of wave fields with more complex yet fascinating patterns have emerged, such as superlenses beating the diffraction limit\textsuperscript{[1–3]}, non-diffracting self-accelerating beams in the paraxial and non-paraxial regions\textsuperscript{[4–7]}, twisting vortex beams\textsuperscript{[8–10]}, and so on. Wave fields can be regarded as the spatial interference of each elementary wave with the desired amplitude and phase profile. This notion has stimulated the concept of an active phased array in electromagnetic\textsuperscript{[11]} and acoustic waves\textsuperscript{[12–14]}, which can be utilized to reshape the radiated wave field by providing the desired amplitude and phase shift as a function of position. However, these physically distributed sources in the active way need to be driven individually using electrical techniques, inevitably leading to major drawbacks such as high cost and complexity.

In a different context, the emergence of engineered artificial metamaterials, which can provide properties unavailable in natural materials, has significantly broadened the horizon for acoustic wave manipulation\textsuperscript{[15–23]}. Acoustic metascreens/metasurfaces, a family of novel metamaterials, have attracted increasing attention recently due to the advantageous features of a planar profile and subwavelength thickness compared to bulky meta-structures\textsuperscript{[24–33]}. Considerable effort has been dedicated to exploring the possibilities for effectively molding the emerging wave field. Recent works have demonstrated both numerically and experimentally that metascreens can provide great flexibility in reshaping the transmitted wavefront in a passive manner, by providing additional momentum in the transverse direction. To form a desirable wave field with excellent performance, especially in the non-paraxial region, the passive structures should possess the ability to transmit sound effectively, shift phases with a $2\pi$ range, and hold subwavelength spatial resolution to avoid the spatial aliasing effect\textsuperscript{[34]}. A metascreen, composed of several hybrid elements with four Helmholtz resonators (HRs) and a straight pipe, has been proposed to achieve these tough goals\textsuperscript{[31]}. Unfortunately, almost
all previous designs have demonstrated the ability to shape wavefront based on numerical simulations at normal incidence, without revealing the underlying physics properly. The case of oblique incidence, containing richer physics and that has potential, is only considered in limited literature and without giving the applicable scope [28].

In this work, a complete theory is developed for the metascreen to reveal its underlying mechanism at both normal and oblique incidence. It is found that the aforementioned goals can be realized within a certain range of incident angles. Theoretical pressure fields emerging from the metascreen are also derived based on the transmission spectrum of each element. With these formulas, four distinct wave manipulation properties are demonstrated: anomalous refraction, conversion from a propagating wave to an evanescent wave, negative refraction, and focusing from a cylinder wave. In particular, two types of negative refraction are observed based on different orders of diffractions. In addition, the negative and positive (normal) refractions are interconverted by simply changing the incident angle, with the coexistence of two types of refraction in a certain range of incident angles.

2. Analytical model of the acoustic metascreen

Imagine that an incident wave impinges obliquely on an individual element of the acoustic metascreen, which is periodic in the x direction (see figure 1). Four HRs are connected in series to construct one such element with a width of w and a height of h, further forming a straight pipe with the linear combination of these elements. The geometric parameters w2, h2, w3 and h3 denote the width and height of the neck and cavity of the HRs, respectively. The HRs are formed by solid materials with identical height, h1, and the width of the straight pipe formed between adjacent elements is w1. To obtain the theoretical properties of the element simply, these HRs are usually treated as lumped elements with an effective acoustic impedance boundaries and only the plane wave component is considered to mimic the impedance boundary. This treatment inevitably leads to large deviations in the resonant states due to the fact that the geometric size of the cavity is comparable to the working wavelength in our design [31]. The effect of higher modes in the cavity needs to be considered for a corrected acoustic impedance (figure 1(b)). Furthermore, the effect of radiation impedance between the neck and the straight pipe should also be included (figure 1(c)). After obtaining these corrected impedance boundaries, the transmission and reflection spectra of the whole system can be presented under both normal and oblique incidence (figure 1(a)).

2.1. Corrected impedance of the cavity

A model of a single HR, to show the corrected acoustic impedance of the cavity, is shown in figure 1(b). The geometrical parameters stay the same as those shown in figure 1(a). The pressure field, p(x, z), in the cavity (w2 ≤ x ≤ w2 + w3) can be expanded in terms of the normal modes [35]:

\[ p(x, z) = \sum_n \phi_n(z) [A_n e^{-j\lambda(x-w_2)} + B_n e^{j\lambda(x-w_2-w_3)}], \]

where \( \phi_n(z) \) is the nth eigenmode in z direction, \( \phi_n(z) = \sqrt{2} - \delta_{0n} \cos \left( k_{zn} (z-h_s/2) \right) \), and satisfies the orthogonality, \( \int \phi_m(z) \phi_n(z) dz = \delta_{mn} \), with \( \delta_{mn} \) representing the Kronecker delta and \( \sigma \) being the cross section of the pipes and apertures. \( A_n \) and \( B_n \) refer to the propagating coefficients in the +x and −x directions. The wavenumbers in the z and x directions are \( k_{zn} = n\pi/h_3 \) and \( k_{xn} = \sqrt{k^2 - k_{zn}^2} \) with \( k \) being the wavenumber \( k = 2\pi/\lambda \). The time factor \( e^{j\omega t} \) with \( \omega = c_0k \) is always omitted as understood. The velocity, \( \nu \), and the pressure, \( p \), is connected to the momentum conservation equation,

\[ \nu = \frac{j}{k\rho_0 c_0} \nabla p, \]

with \( \rho_0 c_0 \) referring to the characteristic acoustic impedance of the fluid medium. By substituting equation (2) into equation (1), we obtain the x component of \( \nu \), \( u(x, z) \), in the cavity

\[ u(x, z) = \frac{1}{\rho_0 c_0} \sum_n \frac{k_{zn}}{k} \phi_n(z) [A_n e^{-j\lambda(x-w_2)} - B_n e^{j\lambda(x-w_2-w_3)}]. \]

Considering the fact that the normal velocity should be zero, \( u = 0 \), at the hard boundary \( (x = w_2 + w_3) \), the first relation between the coefficients \( A_n \) and \( B_n \) can be obtained as below,

\[ A_n e^{-j\lambda w_2} - B_n = 0. \]
At the junction between the neck and the cavity, \( x = w_2 \), the continuity of velocity requires

\[
u(x)|_{x=w_2} = U(x)|_{x=w_2/h_2}.
\]  

Here \( U(x) \) is the volume velocity in the neck, defined as \( U = \int u(x, z) d\sigma \). The velocity component along the \( z \) direction is treated as being identical due to the deeply subwavelength size of the neck. Replacing \( u(x) \) with equation (3), multiplying by \( \phi_m(z) \), and integrating along the boundaries in the \( z \) direction, we can obtain

\[
\int_{-h_2/2}^{h_2/2} \frac{U(x)|_{x=w_2}}{h_2} \phi_m(z) \, dz = \frac{1}{\rho_0 c_0} \int_{-h_2/2}^{h_2/2} \sum_k k \phi_n(z) \phi_m(z) [A_n - B_n e^{-jk_{w_2}}] \, dz.
\]  

**Figure 1.** (a) Schematic diagram of an individual element of the acoustic metascreen (width \( w \) and height \( h \)) consisting of four HRs connected in series and a straight pipe (width \( w_1 \) and height \( h \)). The width and height of the neck and cavity of the HRs are \( w_2, h_2 \) and \( w_3, h_3 \), respectively. The solid material forming the HRs has an identical height, \( h_1 = w_2 \). The symbols \( p_i, p_r, \) and \( p_t \) represent the obliquely incident, reflected and transmitted sound waves, respectively. (b) A single HR is illustrated for the purpose of the corrected acoustic impedance of the cavity. (c) A single HR is located in front of a hard boundary (dashed line) to obtain the corrected acoustic radiation impedance at the junction between the neck and the straight pipe.
After utilizing the orthogonality of $\phi_n(z)$, the second relation between $A_n$ and $B_n$ yields

$$A_n - B_ne^{-jk_mw_3} = \frac{kU(x)|_{x=w_2}}{k_3h_3}\Phi_n,$$

with $\Phi_n = \frac{1}{h_3} \int_{-h_3/2}^{h_3/2} \phi_n(z) \, dz$.

Combining equation (4) and equation (7), these coefficients can be solved as

$$A_n = \rho_0c_0\frac{kU(x)|_{x=w_2}}{k_3h_3(1 - e^{-2jk_mw_3})}\Phi_n,$$

$$B_n = A_ne^{-jk_mw_3}.$$ 

Substituting these coefficients into equation (1), the averaged pressure field, defined as $\bar{p}(x) = \frac{1}{h_3/2} \int_{-h_3/2}^{h_3/2} P \, dz$, at $x = w_2$ can be written as

$$\bar{p}(x)|_{x=w_2} = \sum_n \rho_0c_0\frac{kU(x)|_{x=w_2}}{k_3h_3(1 - e^{-2jk_mw_3})} \Phi_n^2,$$

After obtaining the relationship between $\bar{p}(x)$ and $U(x)$ at $x = w_2$, the acoustic impedance, defined as $Z = \bar{p}/U$, of the cavity can be expressed as

$$Z_c = \frac{\bar{p}(x)|_{x=w_2}}{U(x)|_{x=w_2}} = \sum_n \rho_0c_0\frac{k(1 + e^{-2jk_mw_3})}{k_3h_3(1 - e^{-2jk_mw_3})} \Phi_n^2.$$ 

If the geometric size of the cavity is much smaller than the working wavelength, only the plane wave component, viz. $n = 0$, needs to be considered. Considering $k_0 = k$ and $\Phi_0 = 1$, then equation (10) is degraded to

$$Z_c = \frac{\rho_0c_0}{h_3} \frac{1 + e^{-2jk_mw_3}}{1 - e^{-2jk_mw_3}} = -j\frac{\rho_0c_0}{h_3} \cot (kw_3),$$

which is identical to the traditional form [36].

### 2.2. Corrected impedance of the HRs

In our previous design, the metascreen is composed of four HRs connected to a straight pipe. These HRs can radiate energy into the straight pipe through the neck, where the radiation impedance needs to be considered. The corresponding model is shown in figure 1(c) and the dashed line referring to the boundary of the adjacent element is treated as a hard boundary. To describe the pressure/velocity field in the straight pipe, the Green function theory is employed [35]:

$$\nabla^2 G(x, z) + k^2 G(x, z) = -\delta(x-x_0)\delta(z-z_0).$$

(12)

The solution of equation (12) can be expressed as,

$$G(x, z; x_0, z_0) = \sum_n \frac{\varphi_n(x)\varphi_n(x_0)}{2jk^2\omega_1} e^{-jk\omega_1|z-z_0|},$$

(13)

where $\varphi_n(x) = \sqrt{2 - \delta_{0n}} \cos [k_n^w (x + w_1/2)]$ is the nth eigenmode with $k_n^w = n\pi/w_1$ and $k_n^z = \sqrt{k^2 - k_n^w}$ being the wavenumbers in the $x$ and $z$ directions, respectively. The eigenmodes of the $\varphi_n(x)$ satisfy the orthogonality, $\int \varphi_n(x)\varphi_m(x) \, dx = \delta_{nm}$. According to the Green theory, the pressure distribution in the straight pipe can be written as,

$$p(x, z) = p_t(x, z) + \int_{-h_2/2}^{h_2/2} G(x, z; \frac{w_1}{2}, z_0) \frac{\partial p(x_0, z_0)}{\partial x_0}|_{x_0=w_1/2} \, dz_0,$$

(14)

with $p_t(x, z)$ representing the incident pressure. Substituting equation (2) into equation (14) yields

$$p(x, z) = p_t(x, z) - \frac{jk\rho_0c_0}{h_2} \int_{-h_2/2}^{h_2/2} \int_{-h_2/2}^{h_2/2} G(x, z; \frac{w_1}{2}, z_0) \, dz_0 \, U(x_0)|_{x_0=w_1/2}. $$

(15)

Integrating equation (15) along the boundary in the $z$ direction from $-h_2/2$ to $h_2/2$ and applying the orthogonality of $\varphi_n(x)$, we can obtain the averaged pressure at $x = w_1/2$ as shown below

$$\bar{p}(x)|_{x=w_1/2} = \frac{1}{h_2} \int_{-h_2/2}^{h_2/2} p_t(x, z) \, dz$$

$$- \frac{jk\rho_0c_0}{h_2} \int_{-h_2/2}^{h_2/2} \int_{-h_2/2}^{h_2/2} G(x; \frac{w_1}{2}, z; \frac{w_1}{2}, z_0) \, dz_0 \, dz U(x)|_{x=w_1/2}. $$

(16)
Rearranging equation (16) yields
\[ \tilde{p}_i(x) \big|_{x=w_1/2} = [Z' \big|_{x=w_1/2} + Z_d] U(x) \big|_{x=w_1/2}, \]
where \( Z' \big|_{x=w_1/2} = \tilde{p}(x) \big|_{x=w_1/2} U(x) \big|_{x=w_1/2} \) means the impedance at \( x = w_1/2 \) and \( Z_d \) refers to the corrected radiation impedance of the neck
\[ Z_d = \frac{jk \rho_0 \epsilon_0}{wh_2^2} \int_{-h_2/2}^{h_2/2} G \left( \frac{w_1}{2}, z; \frac{w_1}{2}, z_0 \right) dz_0 dx. \]
After some derivations, the resulting \( Z_d \) becomes
\[ Z_d = \frac{\rho_0 \epsilon_0}{w_1 h_2^2} \left[ \frac{1 - e^{-jkh_2} - jkh_2}{k^2} + \sum_{n=1}^{\infty} 2k \left( 1 - e^{-jk' \alpha n} - jk' \alpha n \right) \right]. \]
Only the plane wave component (\( n = 0 \)) contributes to the real part of \( Z_d \) and the imaginary part mainly stems from the influence of higher orders of evanescent modes, which can be regarded as a pure additional radiation reactance [35].

In order to obtain the whole acoustic impedance of the HRs attached to the straight pipe, \( Z_h \), three factors need to be taken into account: the corrected impedance of the cavity, \( Z_c \), the impedance of the neck, \( Z_n = \rho_0 \epsilon_0 / h_2 \), and the radiation impedance between the neck and the straight pipe, \( \text{Im}(Z_d) \). According to the impedance transfer method [36], the acoustic effective impedance of the HRs at \( x = w_1/2 \) can be expressed as
\[ Z_{\text{eff}}(x = w_1/2) = Z_c + jZ_n \tan(kw_2) \]
By adding the corrected impedance stemming from the varied cross sections between the neck and the straight pipe, the whole acoustic impedance of the HRs, \( Z_h \), can be expressed as
\[ Z_h = Z_{\text{eff}}(x = w_1/2) + j\text{Im}(Z_d). \]
It is worth emphasizing that only the imaginary part of \( Z_d \) needs to be included for the corrected radiation impedance [35]. The real part of \( Z_d \) is corrected with the impedance of HRs at \( x = w_1/2 \) (see equation (20)).

2.3. Transmission and reflection spectra

We consider a plane wave obliquely impinging on an acoustic metascreen composed of periodic element (figure 1(a)). Without loss of generality, we assume an incident plane wave with unity amplitude, \( \tilde{p}_i(x, y) = e^{-j(k \sin \theta_i x + k \cos \theta_i y)} \), with \( \theta_i \) being the angle of incidence with respect to the normal. By employing the Bloch–Floquet periodic boundary condition, the reflected (region \( z < 0 \)) and transmitted wave (region \( z > h \)) can be expressed as [37, 38]
\[ p_r = \sum_n r_n e^{-j \beta_n x + \kappa_n z} \quad (z < 0), \]
\[ p_t = \sum_n t_n e^{-j \beta_n x - \kappa_n z} \quad (z > h), \]
where the symbols \( r_n \) and \( t_n \) represent the reflection and transmission coefficients of the \( n \)th modes with the horizontal wavenumber, \( \beta_n = k \sin \theta_i + 2 \pi n / w \), and the vertical wavenumber, \(-jk \alpha n = k \sqrt{1 - \beta_n^2} \). At \( z = 0 \), the continuity of the velocity requires
\[ u_i + u_r = \begin{cases} u_a & 0 \leq x \leq w_1, \\ 0 & \text{otherwise} \end{cases} \]
where \( u_a, u_i, u_r \) represent the velocity in the straight pipe, and the velocity of the incident and reflected waves at \( z = 0 \), respectively. Considering the pressure expression in equation (22a) and the conservation of momentum equation in equation (2), equation (23) becomes
\[ -\alpha_0 e^{-j \beta_n x} + \kappa_n \sum_n r_n e^{-j \beta_n x} = -j \rho_0 \epsilon_0 \begin{cases} u_a & 0 \leq x \leq w_1, \\ 0 & \text{otherwise} \end{cases}. \]
Multiplying by \( e^{j \beta_n x} \) and integrating along the \( x \) direction, we can obtain
\[ \int_0^w -\alpha_0 e^{-j \beta_n x} e^{j \beta_n x} dx + \sum_n r_n \alpha_0 e^{-j \beta_n x} e^{j \beta_n x} dx = -j \rho_0 \epsilon_0 \int_0^{w_1} u_a e^{j \beta_n x} dx. \]
Utilizing the orthogonality of the exponential function, equation (25) can be simplified and the reflection coefficient can be expressed as shown below.
\[ r_n = \xi_{on} - (-j\alpha_n)^{-1}R_w U_n(0) \Theta_n(\theta_n), \]

where \( \Theta_n(\theta_n) = \frac{1}{\omega_n} \int_0^{w_n} e^{jkx} e^{\omega t} dx, \]

\( R_w = \rho_0 c_0/w \) and \( U_n(0) \) is the volume velocity in the straight pipe at \( z = 0 \).

Using the same procedure at \( z = h \), the transmission coefficient can be expressed as

\[ t_n = (-j\alpha_n)^{-1}\Theta_n(\theta_n)R_w U_n(l). \]

Substituting equation (26) into equation (22a), the total pressure at \( z = 0 \) yields

\[ p(x, 0) = p_1(x, 0) + p_2(x, 0) = 2e^{-jk\sin \theta x} - \sum_n (-j\alpha_n)^{-1}R_w U_n(0) \Theta_n(\theta_n)e^{-k\alpha_n x}. \]

Integrating equation (28) along the \( x \) direction from \(-w_1/2\) to \( w_1/2\), we can obtain

\[ \bar{p}_n(0) = \frac{1}{w_1} \int_{-w_1/2}^{w_1/2} 2e^{-jk\sin \theta x} dx \]

\[ - \frac{1}{w_1} \int_{-w_1/2}^{w_1/2} \sum_n (-j\alpha_n)^{-1}R_w U_n(0) \Theta_n(\theta_n)e^{-k\alpha_n x} dx, \]

with \( \bar{p}_n \) being the averaged pressure in the straight pipe. It can be further simplified as

\[ 2\Theta_n^*(\theta_n) - \bar{p}_n(0) = Z_n U_n(0), \]

with \( Z_n = \sum_n (-j\alpha_n)^{-1}R_w |\Theta_n(\theta_n)|^2 \) and the asterisk \( ^* \) on a variable denoting the conjugate complex. Similarly, the relation between the averaged pressure and volume velocity at \( z = h \) can be written as shown below.

\[ \bar{p}_n(h) = Z_n U_n(h). \]

Considering the fact that the width of the straight pipe, \( w_1 \), is much smaller than the working wavelength, only the plane wave component of the pressure and the volume velocity in the straight pipe need to be considered. Then the pressure and volume velocity in the region from the inlet to the crotch of the first HR (viz. \( 0 \leq z \leq a_1 \)) can be expressed as

\[ p_n(z) = A^+ e^{-kz} + A^- e^{kz}, \]

\[ U_n(z) = \frac{A^+ e^{-kz} - A^- e^{kz}}{R_{w_1}}, \]

where \( A^+ \) and \( A^- \) are the propagating coefficients in the +\( z \) and -\( z \) directions and \( R_{w_1} = \rho_0 c_0/w_1 \) refers to the acoustic impedance of the straight pipe. At \( z = 0 \), equation (32) yields

\[ p_n(0) = A^+ + A^-, \]

\[ U_n(0) = \frac{A^+ - A^-}{R_{w_1}}. \]

Equation (33) can be revised in a matrix form as

\[ \begin{pmatrix} A^+ \\ A^- \end{pmatrix} = \mathbf{M}_1 \begin{pmatrix} \bar{p}_n(0) \\ U_n(0) \end{pmatrix}, \]

where \( \mathbf{M}_1 \) is the transfer matrix,

\[ \mathbf{M}_1 = \begin{pmatrix} 1 & R_{w_1} \\ 2 & 2 \\ 1 & -R_{w_1} \\ 2 & \end{pmatrix}. \]

The expressions of the pressure and the volume velocity in the region between the first HR and the second HR (viz., \( a_1 \leq z \leq a_1 + a_2 \)) are

\[ p_n(z) = B^+ e^{-k(z-a_1)} + B^- e^{k(z-a_1)}, \]

\[ U_n(z) = \frac{B^+ e^{-k(z-a_1)} - B^- e^{k(z-a_1)}}{R_{w_1}}. \]

At the position of the first HR (viz., \( z = a_1 \)), the continuity of the pressure and the volume velocity require

\[ p_n(a_1) = p_n(a_2), \]

\[ U_n(a_1) = U_n(a_2) + U_n(a_1), \]

where \( U_n(a_1) = p_n(a_1)/Z_n \) is the velocity component of the HR at \( z = a_1 \). Combining equations (33) and (37), the transfer matrix between \( A^+ \), \( A^- \) and \( B^+ \), \( B^- \) is shown in equation (38).
Here $M_2$ and $N_1$ are the transfer matrices,

\[
M_2 = \begin{pmatrix} \frac{2 - \alpha}{2} & \frac{-\alpha}{2} \\ \frac{\alpha}{2} & \frac{2 + \alpha}{2} \end{pmatrix},
\]

with $\alpha = R_w/Z_h$. Following the same procedure, the transfer matrices for other region of the element can be expressed as

\[
C^+ = M_2 N_2 \begin{pmatrix} B^+ \\ B^- \end{pmatrix},
\]

\[
D^+ = M_2 N_1 \begin{pmatrix} C^+ \\ C^- \end{pmatrix},
\]

\[
E^+ = M_2 N_2 \begin{pmatrix} D^+ \\ D^- \end{pmatrix},
\]

where $C^+$ ($D^+$) represent the coefficients in the region between second (third) HRs and third (fourth) HRs, $E^+$ refers to the region between fourth HRs and the outlet of the straight pipe, and $N_2$ is

\[
N_2 = \begin{pmatrix} e^{-j\alpha z_1} & 0 \\ 0 & e^{j\alpha z_1} \end{pmatrix}.
\]

At $z = h$, the relation between $E^+$ and the transmitted pressure/velocity, $\bar{p}_d(h)$ and $U_d(h)$, yields

\[
\begin{pmatrix} \bar{p}_d(h) \\ U_d(h) \end{pmatrix} = M_3 N_1 \begin{pmatrix} E^+ \\ E^- \end{pmatrix},
\]

with

\[
M_3 = \begin{pmatrix} 1 \frac{1}{R_{w3}} \\ \frac{1}{R_{v3}} \end{pmatrix}.
\]

According to these transfer matrices, we can finally obtain the relationship for the pressure and volume velocity between $z = 0$ and $z = h$,

\[
\begin{pmatrix} \bar{p}_d(h) \\ U_d(h) \end{pmatrix} = M \begin{pmatrix} \bar{p}_d(0) \\ U_d(0) \end{pmatrix},
\]

with the whole transfer matrix $M = M_3 N_1 (M_2 N_2)^3 M_2 N_2 M_1$. Combining equations (30), (31) and (44), we can obtain $U_d(0)$ and $U_d(h)$,

\[
U_d(0) = \frac{2(m_{21} Z_a - m_{11}) \Theta_d^a(\theta_i)}{m_{31} Z_a^2 - (m_{11} + m_{21}) Z_a + m_{12}},
\]

\[
U_d(h) = \frac{2(m_{11} m_{21} - m_{12}) \Theta_d^a(\theta_i)}{m_{31} Z_a^2 - (m_{11} + m_{21}) Z_a + m_{12}},
\]

with $m_i$ representing the element in $M$. Substituting equations (45a) and (45b) into equations (26) and (27) yields the coefficients of the reflection and transmission as shown below.

\[
r_0 = 1 - \frac{R_w |\Theta_d(\theta_i)|^2}{\cos \theta_i \left( m_{21} Z_a^2 - (m_{11} + m_{21}) Z_a + m_{12} \right)},
\]

\[
t_0 = \frac{R_w |\Theta_d(\theta_i)|^2}{\cos \theta_i \left( m_{21} Z_a^2 - (m_{11} + m_{21}) Z_a + m_{12} \right)},
\]

The theoretical transmission and reflection spectra (amplitude and phase shift) predicted by equations (46a) and (46b) are shown in figures 2(a) and 2(b). To evaluate the accuracy of the theoretical derivations, simulated results based on a model with the same geometrical parameters are also illustrated with dots. Commercial software based on the finite element method, COMSOL Multiphysics, is employed for the simulations. It can be found that the theoretical results agree excellently with the simulated ones, providing solid proof of the validity of the theoretical approach. One interesting phenomenon observed in figure 2 is that there are four transmission.
peaks within the domain of $0.28 < h/\lambda < 0.61$ where the local phase shift of the transmitted wave can cover the $2\pi$ range.

2.4. Physical interpretation

In order to understand the transmission peaks, we further explore the underlying physics of the acoustic metascreen. Equation (31) indicates that the transmitted wave is formed by the radiation effect of the outlets, and $Z_a$ can be regarded as the output impedance. According to the theory of the transmission line, the impedance of the pipe, $Z_p$, at $z_4 = a_1 + 3a_2$ can be obtained as

$$Z_p|_{z=z_4} = R_{w_1} Z_a + jR_{w_1} \tan k a_1,$$

(47)

Considering the fact that $Z_p|_{z=z_i}$ and $Z_h$ are in parallel connection, we can obtain the total impedance at $z = z_4$,

$$Z|_{z=z_4} = \frac{Z_p|_{z=z_4} + Z_h}{Z_p|_{z=z_4} Z_h}.$$  

(48)

Similarly, the impedance at $z = z_5 = a_1 + 2a_2$, $z = z_2 = a_1 + a_2$, and $z = z_1 = a_1$ can be expressed as

$$Z|_{z=z_5} = \frac{Z_p|_{z=z_5} + Z_h}{Z_p|_{z=z_5} Z_h},$$

(49)

with

$$Z_p|_{z=z_5} = R_{w_1} Z|_{z=z_{i+1}} + jR_{w_1} \tan k a_2,$$

(50)
The transfer impedance of the output load at \( z = 0 \), \( Z_{e} \), can be finally obtained
\[
Z_{e} = R_{w} \frac{Z_{i} |_{z=0} + jR_{w} \tan k_{h}}{R_{w} + jZ_{i} |_{z=0} \tan k_{h}}. \tag{51}
\]
The averaged pressure, \( \tilde{p}_{a}(0) \) and volume velocity, \( U_{a}(0) \), at \( z = 0 \) in the straight pipe can be correlated as
\[
\tilde{p}_{a}(0) = Z_{e} U_{a}(0). \tag{52}
\]
Combining equations (52) and (30), we can obtain
\[
\tilde{p}_{a}(0) = \frac{2\Phi_{y}(\theta)}{Z_{a} + Z_{e}}, \tag{53a}
\]
\[
U_{a}(0) = \frac{2\Phi_{y}(\theta)}{Z_{a} + Z_{e}}, \tag{53b}
\]
where \( Z_{e} + Z_{a} \) refers to the total impedance of the metascreen. Then the power transmitted into the metascreen at \( z = 0 \) is
\[
W_{m} = \frac{1}{2} \text{Re} [\tilde{p}_{a}^{*}(0) U_{a}(0)] = \frac{1}{2} \left| \frac{2\Phi_{y}(\theta)}{Z_{a} + Z_{e}} \right|^{2} \text{Re}(Z_{e}). \tag{54}
\]
The incident sound power is
\[
W_{i} = \frac{1}{2} \text{Re} [\tilde{p}_{a}^{*}(0) U_{i}(0)] = \frac{\cos \theta_{i}}{2R_{w}}. \tag{55}
\]
Then the power transmission of the metascreen is \( t_{t} = W_{m}/W_{i} \). From equation (54), the resonant condition is
\[
\text{Im} (Z_{a} + Z_{e}) = 0. \tag{56}
\]
Meanwhile, it can be verified that
\[
\text{Re}(Z_{a}) = \text{Re}(Z_{e}). \tag{57}
\]
Combining equations (56) and (57) yields
\[
Z_{a} = Z_{e}^{*}. \tag{58}
\]
The conjugation matching of \( Z_{a} \) and \( Z_{e} \) indicates that the incident energy can penetrate the metascreen at the resonant states, resulting in the total transmission, \( t_{t} = 1 \).

Figure 3 illustrates the real part of the acoustic impedance of effective load, \( \text{Re}(Z_{e})/R_{w} \), and the imaginary part of the total acoustic impedance of the system, \( \text{Im}(Z_{e} + Z_{a})/R_{w} \). It can be observed that the resonant conditions and the impedance matching can be simultaneously achieved at four specific positions (black arrows), resulting in the total transmission. The positions agree excellently with the four peaks in the transmission amplitude (figure 2(a)). By employing these hybrid resonances, induced by the HRs and the straight pipe, the individual element can achieve a high transmission and fully controlled phase shifts. With these properties, the elements can be employed to construct an acoustic metascreen. Although only the frequency
response is investigated in figure 2, it is expected to achieve similar behaviors by tuning some geometrical parameters, such as \( w_1 \), at a fixed working frequency.

### 3. Elements of the acoustic metascreen

The amplitude and phase shift of the transmitted coefficient varying with the width of the straight pipe, \( w_1/w \), and the incident angle, \( \theta_i \), is shown in figure 4. The black contour line demonstrates the region with transmission amplitude higher than 0.9 (figure 4(a)). Fully controlled phase shift can be achieved in the same region (figure 4(b)). By tuning \( w_1/w \), viz. \( w_3 \), the reactance provided by the HRs is effectively tailored, resulting in the phase shift covering a \( 2\pi \) span. Eight elements with steps of \( \pi/4 \) can be readily selected to implement the desired phase profile, while keeping the impedance matching. Interestingly, fully controlled phase shift and high transmission can be achieved even for oblique incidence with an incident angle smaller than 56° if the threshold of the transmission amplitude of each element is set to 0.9. The angle increases to 75° if we set the transmission amplitude of each element larger than \( -3 \) dB, which means that 50% of the incident power penetrates the element. Furthermore, with fixed geometries, the phase shift is almost independent of the incident angle. These tremendous properties, i.e. fully tuned phase and high transmission within wide incident angles, guarantee excellent performance of the metascreen composed of these elements even for oblique incidence. For instance, in figure 4, eight elements with different geometries are identified from the case with incident angle of 30° (white circles). Full phase coverage can be realized by these elements with steps of \( \pi/4 \), meanwhile holding high transmission amplitudes (>0.93). After discretizing the desired profile with finite steps and carefully choosing the corresponding geometries along the transverse direction \( (x) \), the acoustic metascreen is constructed to tailor the phase in the desired manner. With the selected geometries, the phase shifts caused by the elements are almost independent of the incident angle. It should be emphasized that the spatial resolution of the formed array is as small as \( \lambda/10 \), allowing sufficient control of the sound field even beyond the paraxial region.

To verify the phase shift and the amplitude predicted by the theoretical derivations, figure 5 shows the simulated pressure fields of the eight elements with the geometries identified from figure 4 at normal incidence. A plane wave with unity amplitude impinges on each structure. The incident wave penetrates the elements with little reflection and the phase of the transmitted wave can be effectively tuned to cover a \( 2\pi \) span.

Figure 6 illustrates the effective acoustic impedance of the metascreen normalized to \( R_w' = R_w/\cos \theta_i \) with incident angles of 0° and 80°. The real part of the impedance of the effective load, \( \text{Re}(Z_e)/R_w' \), varied

![Figure 4: Maps of (a) transmitted amplitude, \(|p/p_1|\), and (b) phase shift, \(\phi/2\pi\), of the acoustic metascreen as functions of the width of the straight pipe, \(w_1/w\), and the incident angle, \(\theta_i\). The working wavelength is fixed at \(\lambda = 0.1\) m. The solid and dashed lines enclose the region where amplitudes are larger than 90% and \(-3\) dB, respectively. White circles show eight elements selected to cover a \(2\pi\) phase shift with steps of \(\pi/4\) at 30°.](image-url)
Figure 5. Simulated pressure fields passed through the eight elements of the metascreen at normal incidence. Each case is calculated independently by impinging a plane wave with a wavelength ($\lambda$) of $2h$, clearly showing that the transmitted phase can be shifted from 0 to $2\pi$. The parameters of $w_i/w$ are 0.299, 0.349, 0.428, 0.562, 0.189, 0.206, 0.226, 0.258, respectively.

Figure 6. Real part of the acoustic impedance of the effective load (red line), $\text{Re}(Z_e)/R_e^\infty$, and imaginary part of the total acoustic impedance of the whole system (blue dotted line), $\text{Im}(Z_e + Z_o)/R_e^\infty$, with an incident angle of (a) $0^\circ$ and (b) $80^\circ$. The resonant states are illustrated (a) with black arrows with $w_i/w = (0.171, 0.205, 0.310)$ and (b) with $w_i/w = (0.182, 0.283)$. 
around 1 from $w/w > 0.165$, indicating that the impedance is well matched, and resulting in high transmission (figure 4). With an increase in the incident angle, the impedance, mainly $\text{Re}(Z_\rho)/R_{\rho\rho}$, becomes mismatched, leading to low transmission with large incident angles. Interestingly, hybrid resonances, $\text{Im}(Z_\rho + Z_\delta)/R_{\rho\rho} = 0$, are also supported under oblique incidence [39].

4. Shaped beams with acoustic metascreens

The great properties of the high power transmission, fully controlled phase shift, and deeply subwavelength spatial resolution, $w = \lambda/10$, guarantee that the acoustic metascreen-based passive phased array can be employed to realize a wide range of wave manipulations. In this section, we theoretically and numerically investigate several interesting wave manipulations using the proposed acoustic metascreen, such as anomalous refraction, negative refraction, and focusing.

4.1. Theoretical pressure fields

The total transmitted pressure field emerging from the metascreen can be regarded as the synthesis of a radiated wave from the elements. The radiated fields can be obtained using the Green function theory by treating the outlets of the elements as line sources with uniform volume velocities. The Green function of a line source located on a hard boundary can be written as [35]

$$G(x, y; x', y') = -\frac{j}{2} H_{1}^{(1)}(kR),$$

where $R$ refers to the distance between the measured point $(x, y)$ and the source $(x', y')$.

$$R \equiv \sqrt{(x - x')^2 + (y - y')^2},$$

and $H_{1}^{(1)}(\cdot)$ represents the first type of Hankel function of the zeroth order. The total pressure field is the integration of the Green function over the boundaries of metascreen (3)

$$p(x, z) = \int_{\Sigma} \left[ G(x, z, x', h) \frac{\partial p(x, z)}{\partial z} + p \frac{\partial G(x, z, z', z')}{\partial z} \right] d\sigma.$$  

(60)

Considering the fact that $\partial G/\partial z = 0$ and equation (2), equation (60) becomes

$$p(x, z) = -\frac{j \rho_0 c_0 k}{w_1} \sum_{n=1}^{w_1} \int_{(n-1)\lambda}^{(n-1/2)\lambda} G(x, x', z, h) U_d(x', h) dx',$n

(61)

with $n$ referring to the $n$th individual element.

4.2. Anomalous refractions at normal incidence

Under the guidance of generalized Snell’s law [40, 41], the relation between the incident angle, $\theta_i$, and refracted angle, $\theta_r$, should be revised as

$$k \sin \theta_i = k \sin \theta_r + \xi(x),$$

(62)

with $\xi(x) = d\phi_i/dx$ referring to the phase gradient.

In order to realize the anomalous refraction at normal incidence, $\theta_i = 0$, a linear phase profile should be yielded, indicating a constant value of $\xi(x)$. The relationship between the transmitted angle, $\theta_r$, and the constant phase gradient, $\xi$, under normal incidence is illustrated in figure 7. The theoretical and simulated results, obtained six wavelengths away from the center of the metascreen with a plane wave of $5\lambda$ width incidence, are also plotted for comparison and an excellent agreement is observed.

After the desired profile, $\xi$, is selected, the metascreen can be constructed by suitably discretizing the profile in $\pi/4$ steps and arranging these elements in the transverse direction, $x$. Additional phase shift is provided by the metascreen so that the transmitted propagation is inevitably redirected to the desired angle due to the conservation of the transverse momentum. Figure 8 illustrates the theoretical and simulated pressure fields of a metascreen with $\xi/(2\pi) = 50/8$, clearly showing that the metascreen generates extra transverse momentum on the normally incident wave, resulting in a bending transmitted wave propagating with predicted desired angles (black arrows in the output region).

Equation (62) implies that there is a critical value with $\xi/(2\pi) = 10$. No transmitted angle is allowed when $\xi/(2\pi) > 10$, indicating the conversion from propagating wave to evanescent wave in the $z$ direction with energy concentrated near the metascreen. Figure 9 illustrates the theoretical and simulated normalized sound pressure level distributions of an acoustic metascreen with $\xi/(2\pi) = 100/9$ at normal incidence with a width of $8\lambda$. The acoustic energy is confined near the metascreen and rapidly attenuated in the $z$ direction. The proposed metascreen can provide an extra momentum profile along the $x$ direction for the conversion from propagating wave to evanescent wave with the energy concentrated near the surface. In the simulations, the existing interaction between the elements and other orders of diffractions enlarges the reflected energy, resulting in
a slight difference between the theoretical and simulated results. The energy concentration near the surface may provide an alternative method for generating surface acoustic waves with some structures supporting its propagation, such as the corrugated structure [28, 42].

4.3. Negative refractions with oblique incidence
The individual element possesses the capability of shifting an almost identical phase within a certain range of incident angles, while maintaining high transmission, as shown in figure 4(b). This indicates that our proposed metascreen can effectively tune the wave propagation even for oblique incidence. The scalar diffraction theory, however, is limited and higher orders of diffractions need to be counted when the incident angle becomes larger [43]. Then the non-local effect from the periodicity will play a crucial role in wave shaping when the wavelength is comparable to the period of the metascreen. The generalized Snell’s law should be revised as [28]
where \( G = 2\pi / \Lambda \) is the amplitude of the reciprocal lattice vector with \( \Lambda \) being the periodicity of the phase profile of the metascreen.

The formula of the theoretical pressure field (equation (61)) needs to be revisited under oblique incidence

\[
p(x, z) = - \frac{j\mu_0 c_0 k}{w_1} \sum_{n=-1}^{\infty} G(x, x', z, h) e^{i\phi(x')} U_n(x', h) dx',
\]

with \( \phi(x) = kx \sin \theta_i + n_G xG \) revealing the incident phase difference at the boundary of the metascreen (\( z = 0 \)). In the following calculations, the metascreen designed for normal incidence is employed to challenge the performance for oblique incidence.

In order to investigate the behaviors at oblique incidence, \( \xi \) is fixed to 50/8 to construct the metascreen, indicating that the bending angle at normal incidence should be 38.68°. The black solid line in figure 10 shows the predicted relationship between the transmitted angle and the incident angle. The theoretical and simulated ones are also plotted with red dots and blue rectangles for comparison, and excellent agreement is obtained. For a negative incident angle defined in the clockwise direction, \(-50^\circ \leq \theta_i \leq 0^\circ\), classical diffraction plays a dominant role in the transmitted wave formation, indicating \( n_x = 0 \). For the positive incident angle defined

\[ n_G = 1 \] is another permitted value, however, the amplitude of the transmitted wave of this branch is much smaller than that of \( n_G = 0 \), therefore, this branch is neglected in figure 10.
in the anti-clockwise direction, \( n_G = 0 \) is also valid for the cases with small incident angle (\( \theta_i < 22^\circ \)) and \( n_G \) becomes non-zero values (in our case, \( n_G = -3 \)) when \( \theta_i > 14.5^\circ \), governed by the surface momentum matching condition. As a result, the curve of \( \theta_t \) versus \( \theta_i \) exhibits four interesting features: (i) negative refraction with \( n_G = 0 \) (the region \( -38.7^\circ < \theta_i < 0^\circ \)), (ii) positive (normal) refraction (region \( 0^\circ < \theta_i < 22^\circ \)), (iii) negative refraction with \( n_G = -3 \) (region \( 14.5^\circ < \theta_i < 22^\circ \)), and (iv) coexistence of the normal and negative refractions (region \( 22^\circ \leq \theta_i \leq 50^\circ \)). By modulating the spatial resolution, \( w \), and the phase gradient, \( \xi(x) \), the conversion from a propagating wave to an evanescent wave can also be achieved within a certain range of incident angles [28].

Theoretical and simulated pressure fields in figure 11 clearly illustrate the negative refraction. The oblique incidence with small negative angle (\( \theta_i > -38.7^\circ \)) cannot provide enough momentum to cancel the additional transverse momentum generated by the metascreen. Therefore the wave vector of the transmitted wave always lies on the same side of the surface normal as the incident wave vector, namely, negative refraction. The transmitted wave emerges from the metascreen with an angle of 21.5° when an incident plane wave with an angle of –15° impinges on the structure (figures 11(a) and (b)). The plane wave has unity amplitude and a width of 5λ. For incidence with a positive angle of 35°, the theoretical and simulated pressure fields demonstrate that the metascreen redirects the transmitted waves with an angle of –42.6°, situated on the same side of the incidence (figures 11(c) and (d)). The transmitted angles of the negative refraction agree well with the predicted ones (black arrows in the output region). Even though the negative refractions are realized with both positive and negative incident angles, the physical mechanisms are totally different. For the incidence with negative angle, the transmitted field is formed following classical diffraction theory, \( n_G = 0 \). However, for the incidence with positive angle, the periodicity of the metascreen plays a dominant role in the field shaping with \( n_G = -3 \). The asymmetric property for positive and negative incidence may be used to realize some novel functions, such as asymmetric acoustic transmission [44–48].

The predicted curve in figure 10 also implies that the normal refraction and negative refraction can be achieved simultaneously within a special range of incident angles. The theoretical pressure field of the metascreen with an incident angle of 21° is shown in figure 12. It can be observed that the transmitted wave is split into two main lobes with an angle of 79.5° (normal refraction) and –63° (negative refraction), showing good agreement with the predicted ones.

---

4. Theoretical and simulated pressure fields are not comparable because the role of \( n_G \) in the theoretical derivations is implemented in the incident part.
4.4. Acoustic focusing from a cylinder wave

To further demonstrate the great capability of the proposed metascreen to control wave propagation flexibly, we also implement an acoustic focusing effect with the desired quadratic phase profile. A line source located at \((x, z) = (x_s, z_s) = (6\lambda, -5\lambda)\) is employed to radiate an incident cylinder wave considering that each element with a fixed geometry generates almost identical phase shifts within a certain range of incident angles (see figure 4). To form a focus spot with a focal length of \(f = 5\lambda\), the phase profile provided by the metascreen can be written as

\[
\phi(x) = k\left[\sqrt{(x-x_s)^2 + f^2} - f\right] + \phi_c(x),
\]

where the first term is the desired profile to focus a plane wave, and the second term, \(\phi_c(x) = k\left[\sqrt{(x-x_s)^2 + z_s^2} - z_s\right]\), compensates the arrival phase difference along the boundary of the metascreen \((z = 0)\).

The theoretical pressure field from a line source is

\[
p(x, z) = -j\rho_0 c_0 k G(x, y, x_s, z_s),
\]

with \(G(x, z, x_s, z_s)\) being the Green function of a line source in free space,

\[
G(x, z, x_s, z_s) = -\frac{j}{4} H^{(1)}_0\left\{[(x-x_s)^2 + (z-z_s)^2]\right\}.
\]

Then the emergent pressure field from the metascreen can be expressed as

\[
p(x, z) = -\frac{j\rho_0 c_0 k}{w_1} \sum_{n=1}^{(n-1)w_2+w_1} A_c(x) G(x, z', z, h) e^{i\phi_c(x')} U_d(x', h) dx',
\]

where \(A_c(x) = -j\rho_0 c_0 k G(x, 6\lambda, 0, -5\lambda)\) and \(\phi_c\) refer to the amplitude and phase difference at the boundary of the metascreen \((z = 0)\) from the line source. In the derivations, the elements are selected according to equation (65) and then the amplitude and the phase difference are compensated in the incident part to ensure the identical geometries in theoretical and simulated cases.

The theoretical and simulated normalized pressure fields are illustrated in figure 13. The corresponding distributions clearly demonstrate the focusing effect at the designed focal plane (dashed lines). The excellent agreement between the theoretical/simulated results and the predicted ones demonstrates the abilities of metascreen even for the cylinder wave from the line source. Due to the fully controllable phase pattern, acoustic lenses can be designed to focus energy effectively at an arbitrary position [26, 49, 50].

5. Summary and discussion

In summary, we develop a complete theory to analyze the transmission/reflection spectra of the individual element of the acoustic metascreen. The great properties of a shifting phase with \(2\pi\) span, keeping well matched impedance, and holding deeply subwavelength spatial resolution, are well achieved even for oblique incidence within a certain range of incident angles. The underlying physics has been well revealed in that the desired phase pattern can be fully tuned by changing the reactance provided by the HRs, while keeping impedance matching to the surrounding medium with hybrid resonances.
By carefully discretizing the desired phase profile and arranging eight types of elements, the acoustic metascreen was constructed to yield additional transverse momentum. Anomalous refraction and conversion from propagating wave to evanescent wave were demonstrated at normal incidence. Under oblique incidence, two types of negative refraction were observed stemming from distinct mechanisms. One is based on the classical diffraction theory, and the other is dominated by the periodicity of the metascreen. Significantly, negative and positive (normal) refractions can be inter-converted through simply changing the incident angle, with the existence of two types of refraction in a certain range of incident angles. An acoustic lens is also demonstrated to focus the acoustic energy with a line source rather than normal plane wave, validating the wide angle feature of the metascreen. Both the theoretical and simulated results agree excellently with the predicted ones, providing solid evidence that the metascreen can control wave propagation flexibly and concisely.

Our planar passive phased array composed of an acoustic metascreen, overcoming the drawbacks of an active phased array, takes advantage of the simple configuration and extreme acoustic performance, and hence shows great potential in diverse wave-shaping applications, such as non-diffracting beams, vortex beams, acoustic focusing, acoustic holography, and so on. The concept of a passive phased array can also be extended into underwater, nondestructive evaluation, and micro-fluidic systems.

Acknowledgments

This work was supported by the FEDER ‘Fonds Européen de Développement Régional’ (project ‘MASTER’) and the ‘Région Lorraine’. YL wishes to thank Dr Gaokun Yu and Mr Xishan Yang for fruitful discussions.
References

[1] Pendry JB 2000 Negative refraction makes a perfect lens Phys. Rev. Lett. 85 3966–9
[2] Zhang X and Liu Z 2008 Superlenses to overcome the diffraction limit Nat. Mater. 7 435–41
[3] Kaina N, Lemoult F, Fink M and Lerosey G 2015 Negative refractive index and acoustic superlens from multiple single negative metamaterials Nature 523 77–81
[4] Durbin J, Mielczarek J and Eberly J H 1987 Diffraction-free beams Phys. Rev. Lett. 58 1499
[5] Li L, Li T, Wang S M, Zhang C and Zhu S N 2011 Plasmonic airy beam generated by in-plane diffraction Phys. Rev. Lett. 107 126804
[6] Zhang P, Hu Y, Li T, Cannan D, Yin X, Morandotti R, Chen Z and Zhang X 2012 Nonparaxial mathieu and weber accelerating beams Phys. Rev. Lett. 109 193901
[7] Epstein I and Arie A 2014 Arbitrary bending plasmonic light waves Phys. Rev. Lett. 112 023903
[8] Ng J, Lin Z and Chan C T 2010 Theory of optical trapping by an optical vortex beam Phys. Rev. Lett. 104 103601
[9] Zhang L and Marston P L 2011 Angular momentum flux of nonparaxial acoustic vortex beams and torques on axisymmetric objects Phys. Rev. E 84 066601
[10] Demore C E M, Dahl P M, Yang Z, Glynne-Jones P, Melzer A, Cochran S, MacDonald M P and Spalding G C 2014 Acoustic tractor beam Phys. Rev. Lett. 112 174302
[11] McGrath D 1986 Planar three-dimensional constrained lenses IEEE Trans. Antennas Propag. 34 46–50
[12] Ahrens J, Rabenstein R and Spors S 2014 Sound field synthesis for audio presentation Acoustics Today 10 15
[13] Zhang P, Li T, Zhu J, Zuo Y, Yang S, Wang Y, Yin X and Zhang X 2014 Generation of acoustic self-bending and bottle beams by phase engineering Nat. Commun. 5 4316
[14] Zhao S, Hu Y, Lu J, Qiu X, Cheng J and Burnett I 2014 Delivering sound energy along an arbitrary convex trajectory Sci. Rep. 4 6628
[15] Liu Z, Zhang X, Mao Y, Zuo Y Y, Yang Z, Chan C T and Sheng P 2000 Locally resonant sonic materials Science 289 1734
[16] Fang N, Xi D, Xu J, Ambati M, Srituravanich W, Sun C and Zhang X 2006 Ultrasonic metamaterials with negative modulus Nature Mater. 5 452
[17] Yang Z, Mei J, Yang M, Chan N H and Sheng P 2008 Membrane-type acoustic metamaterial with negative dynamic mass Phys. Rev. Lett. 101 204301
[18] Torrent D and Sánchez-Dehesa I 2009 Radial wave crystals: Radially periodic structures from anisotropic metamaterials for engineering acoustic or electromagnetic waves Phys. Rev. Lett. 103 064301
[19] Lemoult F, Fink M and Lerosey G 2011 Acoustic resonators for far-field control of sound on a subwavelength scale Phys. Rev. Lett. 107 064301
[20] Lee S H, Park C M, Seo Y M, Wang Z G and Kim C K 2010 Composite acoustic medium with simultaneously negative density and modulus Phys. Rev. Lett. 104 054301
[21] Liang Z and Li J 2012 Extreme acoustic metamaterial by coupling up space Phys. Rev. Lett. 108 114301
[22] Shen C, Xu J, Fang N X and Jing Y 2014 Anisotropic complementary acoustic metamaterial for canceling out aberrating layers Phys. Rev. X 4 041033
[23] Xiao S, Ma G, Li Y, Yang Z and Sheng P 2015 Active control of membrane-type acoustic metamaterial by electric field Appl. Phys. Lett. 106 091904
[24] Li Y, Liang B, Gu Z M, Zou Y Y and Cheng J C 2013 Reflected wavefront manipulation based on ultrathin planar acoustic metasurfaces Sci. Rep. 3 2546
[25] Zhao J, Li B, Chen Z and Qiu C W 2013 Manipulating acoustic wavefront by inhomogeneous impedance and steerable extraordinary reflection Sci. Rep. 3 2537
[26] Li Y, Jiang X, Li R Q, Liang B, Zou X Y, Yin L L and Cheng J C 2014 Experimental realization of full control of reflected waves with subwavelength acoustic metasurfaces Phys. Rev. Appl. 2 064002
[27] Tang K, Qiu C, Ke M, Lu J, Ye Y and Liu Z 2014 Anomalous refraction of airborne sound through ultrathin metasurfaces Sci. Rep. 5 6517
[28] Xie Y, Wang W, Chen H, Konneker A, Popa B I and Cummer S A 2014 Wavefront modulation and subwavelength diffractive acoustics with an acoustic metasurface New. Commun. 5 5533
[29] Mei J and Wu Y 2014 Controllable transmission and total reflection through an impedance-matched acoustic metasurface New J. Phys. 16 123007
[30] Ma G, Yang M, Xiao S, Yang Z and Sheng P 2014 Acoustic metasurface with hybrid resonances Nat. Mater. 13 873–8
[31] Li Y, Jiang X, Liang B, Cheng J C and Zhang L K 2015 Metascreen-based acoustic passive phased array Phys. Rev. Appl. 4 024003
[32] Li Y and Assouar M B 2015 Three-dimensional collimated self-accelerating beam through acoustic metascreen Sci. Rep. 5 76712
[33] Li Y and Assouar M B 2016 Acoustic metasurface-based perfect absorber with deep subwavelength thickness Appl. Phys. Lett. 108 063502
[34] Johnson D H and Dudgeon D E 1993 Array Signal Processing: Concepts and Techniques (Englewood Cliffs, NJ: Prentice-Hall)
[35] Morse P M and Ingard K U 1987 Theoretical Acoustics (Princeton: Princeton University Press)
[36] Blackstock D T 2000 Fundamentals of Physical Acoustics (Hoboken, NJ: Wiley)
[37] Wang X 2010 Theory of resonant sound transmission through small apertures on periodically perforated slabs J. Appl. Phys. 108 064903
[38] Qi L, Yu G, Wang X, Wang G and Wang N 2014 Interference-induced angle-independent acoustical transparency J. Appl. Phys. 116 234506
[39] Yang X, Yin J, Yu G, Peng L and Wang N 2015 Acoustic superlens using helmholtz–resonator-based metamaterials Appl. Phys. Lett. 107 193505
[40] Yu N, Genov P, Kats M A, Aïta F, Tétienne J P, Capasso F and Gaburro Z 2011 Light propagation with phase discontinuities: generalized laws of reflection and refraction Science 334 335–7
[41] Monticone F, Estakhri N M and Alù A 2013 Full control of nanoscale optical transmission with a composite metascreen Phys. Rev. Lett. 110 203903
[42] Sun S, He Q, Xiao S, Xu Q, Li X and Zhou L 2012 Gradient-index meta-surfaces as a bridge linking propagating waves and surface waves Nat. Mater. 11 426–31
[43] Popov E K, Tsoniev L V and Loewen E G 1991 Scalar theory of transmission relief gratings Opt. Commun. 80 307
[44] Liang B, Yuan B and Cheng J C 2009 Acoustic diode: rectification of acoustic energy flux in one-dimensional systems Phys. Rev. Lett. 103 104301
[45] Liang B, Guo X S, Xu T, Zhu J, Zhang D and Cheng J C 2010 An acoustic rectifier Nat. Mater. 9 989–92
[46] Boechler N, Theocharis G and Daraio C 2011 Rifraction-based acoustic switching and rectification Nature Mater. 10 665–8
[47] Li Y, Liang B, Gu Z M, Zou X Y and Cheng J C 2013 Unidirectional acoustic transmission through a prism with near-zero refractive index Appl. Phys. Lett. 103 053505
[48] Li Y, Tu J, Liang B, Guo X S, Zhang D and Cheng J C 2012 Unidirectional acoustic transmission based on source pattern reconstruction J. Appl. Phys. 112 064504
[49] Li Y, Liang B, Tao X, Zhu X F, Zou X Y and Cheng J C 2012 Acoustic focusing by coiling up space Appl. Phys. Lett. 101 233508
[50] Li Y, Yu G K, Liang B, Zou X Y, Li G Y, Cheng S and Cheng J C 2014 Three-dimensional ultrathin planar lenses by acoustic metamaterials Sci. Rep. 4 6830
[51] Cervera F, Sanchis I, Sánchez-Pérez J V, Martínez-Sala R, Rubio C, Meseguer F, López C, Caballero D and Sánchez-Dehesa J 2001 Refractive acoustic devices for airborne sound Phys. Rev. Lett. 88 023902
[52] Marzo A, Seah S A, Drinkwater B W, Sahoo D R, Long B and Subramanian S 2015 Holographic acoustic elements for manipulation of levitated objects Nat. Commun. 6 8661
[53] Sherman C H and Butler J L 2007 Transducers and Arrays for Underwater Sound (Berlin: Springer)
[54] Drinkwater B W and Wilcox P D 2006 Ultrasonic arrays for non-destructive evaluation: A review NDT & E International 39 525
[55] Friend J and Yeo L Y 2011 Microscale acoustofluidics: Microfluidics driven via acoustics and ultrasonics Rev. Mod. Phys. 83 647–704