Effects of Milling Feed Rate and Tool Diameter on Cutting Forces and Cutting Coefficient for Medium Carbon Steel (S45C)

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ABSTRACT
In this study, a prediction method for the milling cutting force and cutting coefficient for medium carbon steel (S45C) is developed. Two prediction methods for cutting force – the average cutting force method and the recursive least-squares method – are introduced, and then, the prediction results of cutting forces obtained by these methods are compared with experimentally measured cutting forces. Following acquisition of accurate cutting force coefficients, the cutting parameters, i.e. the friction angle and shear stress, are estimated using the oblique cutting theory. The estimation results reveal good agreement between the forces predicted by the recursive least-squares method and the experimentally measured forces. With an increase in the feed per tooth, the cutting force increases and the cutting coefficient becomes smaller in the case of shearing forces in tangential directions. Additionally, the magnitude of the friction angle is found to remain unchanged under varying feed per tooth and tool diameter.

1. Introduction
In the last few decades, improvements have been achieved in the milling process on account of the availability of new tooling systems, technologies, and control, thereby enhancing the performance of the process. Prediction of power and torque requirements, machine tool vibrations, workpiece surface quality and geometrical accuracy, and chatter-free cutting parameters necessitates reliable quantitative prediction of the cutting forces in the milling process. Therefore,
ensuring the effectiveness and accuracy of the cutting force and the cutting force model is necessary for the monitoring, planning, and control of milling operations. A mechanistic cutting force model is typically employed for this purpose, wherein the cutting force is assumed to be proportional to the cross-sectional area of the uncut chip. The proportionality constant is also known as the specific cutting force coefficient or specific cutting pressure; this constant is a function of the cutter geometry, cutting conditions, and properties of the workpiece material. Two types of mechanistic cutting force models have been reported in the literature. In one type of model, the effects of the shearing mechanism resulting from the chip-generation process on the rake face of the tool and those of the rubbing and plowing mechanisms on the flank face are combined into one specific force coefficient for each cutting force component (tangential, radial, and axial). The other type of mechanistic model separately characterizes the shearing and plowing effects by means of the respective specific cutting- and edge-force coefficients. The second type of model appears more suitable for analytical work given that the coefficients of this model are relatively independent of the average chip thickness.

Previous studies contributed to advancements in research on the cutting force model and cutting force coefficients. For example, Grossi et al. [1] performed a comprehensive evaluation of the cutting force coefficient at various spindle speeds and analyzed a wide range of spindle speeds. In their study, in order to overcome issues pertaining to transducer dynamics, the authors compensated for dynamometer signals by employing an improved technique based on the Kalman filter estimator. Further, they implemented two different coefficient identification methods: the conventional average force method and a proposed instantaneous method based on the genetic algorithm, which is capable of simultaneous estimation of cutting coefficients and tool run-out. Yoon and Kim [2] proposed a mechanistic cutting force model and predicted the results of the cutting process using this model. In their study, they developed an experimental coefficient modeling method for estimation of the theoretical cutting force by consideration of the specific cutting force coefficient. Further, for calculation of the cutting force in end milling in each direction, they used the specific cutting force, which is defined as the product of the specific cutting force coefficient and the uncut chip thickness. Tukora and Szalay [3] proposed a cutting force prediction method based on the mechanistic cutting force model, along with an algorithm for determination of the cutting force coefficients during a single experiment without any restrictions on the cutting geometry. Campatellia and Scippaa [4] examined the influence of the feed per tooth and cutting speed on the cutting coefficients with the aim of constructing a model capable of reliable prediction of the cutting force under different process parameters. Stepan et al. [5] demonstrated that cutting force coefficients show strong nonlinearity as functions of the chip load, cutting speed, and material imperfections. They revealed the relation between the sensitivity of the dynamics of regenerative cutting and the characteristic nonlinearity of the cutting force and also mathematically modeled the non-linear milling process. Dotcheva et al. [6] proposed an approach for determination of cutting force coefficients; in their proposed methodology, a practical mechanism for collection and analysis of experimental data was modeled. Gonzalo et al. [7] proposed a method for acquisition of specific cutting coefficients required for prediction of the milling forces by means of a mechanistic model of the milling process. In their work, they applied an inverse approach using instantaneous cutting force values and solved the system equation using a constrained least-squares fitting method. Srinivasa and Shunmugam [8] proposed a methodology for prediction of the cutting coefficients by taking into consideration the edge radius and material-strengthening effects.

In the present study, we develop a method for prediction of the milling cutting force and cutting coefficient for medium carbon steel (S45C). We introduce two cutting force prediction methods – the average force method and the recursive least-squares method – and compare their prediction results of cutting forces with experimentally measured cutting forces. Additionally, we examine the effects of the feed per tooth and tool diameter on the cutting force and cutting force coefficient. Finally, following acquisition of accurate cutting

Figure 1. Schematic of force analysis in a workpiece cut by a milling tool.
force coefficients, we estimate the cutting parameters, i.e. the friction angle and shear stress, using the oblique cutting theory.

2. Cutting force model and shear stress formulation

Figure 1 shows the geometric locations of the cutter and workpiece during end milling. Figure 2 shows the primary areas of material deformation with a shear zone and edge zone, as well as the uncut chip thickness. The Altintas model [9] is typically employed for the evaluation and calculation of machining forces in the milling process. In

Figure 3. Experimentally measured cutting forces and cutting forces predicted using Altintas method and recursive least-squares method under slot milling and half-slot milling operations with tool diameter D12 and feed rate of 200 mm/min.
Here, $f_t$ and $\phi$ denote the feed rate per tooth and the instantaneous phase angle of the cutter edge, respectively. The tangential cutting force $dF_t$, radial cutting force $dF_r$, and axial cutting force $dF_a$ can be represented using the specific cutting force coefficients $K$ in each direction as follows:

$$dF_t = K_{tc} a_c db + K_{te} dl$$
$$dF_r = K_{rc} a_c db + K_{re} dl$$
$$dF_a = K_{ac} a_c db + K_{ae} dl$$

Here, $db$, $dl$, and $a_c$ denote the chip width and length for an infinitesimal section of the chip and the cutting thickness, respectively.

With rotation of the end mill, the chip thickness $a_c$ may vary and the instantaneous chip thickness at a particular rotation angle $\phi$ in end milling can be represented as

$$a_c = f_t \sin \phi$$  \hspace{1cm} (1)

The uncut chip area is required to be known beforehand in order to identify the cutting force in ‘up- and down-milling processes’ with different conditions of the start and exit immersion angles. Therefore, the uncut chip thickness must be verified at a specific edge point to calculate the cutting force at a particular section.

**Figure 4.** Experimentally measured cutting forces and cutting forces predicted using Altintas method and recursive least-squares method under slot milling and half-slot milling operations with tool diameter D16 and feed rate of 200 mm/min.
According to the oblique cutting theory proposed by Amarego and Whitfield [11], the shear stress can be further calculated from the cutting force coefficients ($K_{tc}$, $K_{rc}$, $K_{ac}$), $\beta_n$, $\alpha_n$, and $\phi_n$ as follows:

$$\tau_s = \frac{F_{yc}}{N_{b}} \sin \phi_i \frac{r_c \cos i}{\sin \phi_i} + \frac{F_{xc}}{N_{b}} \tan \eta \sin \beta_n tan i$$

$$K_{tc} = \frac{F_{yc}}{N_{b}} \sin \phi_i \frac{r_c \cos i}{\sin \phi_i} \frac{F_{xc}}{N_{b}} \tan \eta \sin \beta_n tan i$$

$$K_{rc} = \frac{F_{xc}}{N_{b}} \tan \eta \sin \beta_n tan i$$

$$K_{ac} = \frac{F_{zc}}{N_{b}}$$

$$K_{te} = \frac{F_{ye}}{N_{b}}$$

$$K_{re} = \frac{F_{xe}}{N_{b}}$$

$$K_{ae} = \frac{F_{ze}}{N_{b}}$$

respectively; $K_{tc}$, $K_{rc}$, and $K_{ac}$ denote the tangential, radial, and axial cutting force coefficients, respectively; and $K_{te}$, $K_{re}$, and $K_{ae}$ denote the tangential, radial, and axial edge-force coefficients, respectively [10]. For prediction of the specific cutting force coefficients, experimental forces were considered to act under full-immersion conditions of slot milling. If we define the phases of the entry and exit points as $\phi_{st} = 0$ and $\phi_{sx} = 180$, respectively, the above equations become

\[ K_{tc} = \frac{4F_{yc}}{N_{b}} \sin \phi_i \frac{r_c \cos i}{\sin \phi_i} \frac{F_{xc}}{N_{b}} \tan \eta \sin \beta_n tan i \]

\[ K_{rc} = \frac{4F_{xc}}{N_{b}} \tan \eta \sin \beta_n tan i \]

\[ K_{ac} = \frac{F_{zc}}{N_{b}} \]

\[ K_{te} = \frac{F_{ye}}{N_{b}} \]

\[ K_{re} = \frac{F_{xe}}{N_{b}} \]

\[ K_{ae} = \frac{F_{ze}}{N_{b}} \]

\[ c = \sqrt{\cos^2(\phi_n + \beta_n - \alpha_n) + \tan^2 \eta \sin^2 \beta_n} \]

\[ \tan \phi_n = \frac{r_c \cos \alpha_n}{1 - r_c \sin \alpha_n} \]

where $\tau_s$ is the shear stress and $\beta_n$, $\alpha_n$, $i$, and $\phi_n$ are the friction angle, rake angle, helix angle, and shear angle, respectively. Additionally, for determination of $\phi_n$, experimental

**Figure 5.** Experimentally measured cutting forces and cutting forces predicted using Altintas method and recursive least-squares method under slot milling and half-slot milling operations with tool diameter D20 and feed rate of 200 mm/min.
results of orthogonal turning based on the chip ratio $r_c$ are required.

In this study, the average milling forces were measured via repeated experiments with varying feeds per tooth during the milling process. A MATLAB (MathWorks, Inc., MA, USA) routine was executed to evaluate the cutting force, given the geometrical parameters of the tool and the workpiece. The recursive least-squares method was used to modify the cutting coefficients with the aim of predicting the cutting forces and cutting coefficients more accurately. The recursive least-squares method is an adaptive filter that recursively determines the coefficients that minimize the weighted linear least-squares cost function related to the input signals. Recursive least-squares algorithms have a faster convergence speed, and they do not suffer from the eigenvalue spread problem.

3. Experimental details

A VMP-40A three-dimensional (3D) vertical machining center (Feeler Machine Tools, Ltd.), with a maximum spindle speed of 10,000 rpm, was selected as the machine tool for the experiments. The cutters employed in the experiments were two-tooth cemented carbide flat-end milling cutters with diameters of 12 mm (D12), 16 mm (D16), and 20 mm (D20), with helix angles of 45° and rake angles of 0°. All the workpieces used in this study had dimensions of 100 × 100 × 60 mm and were made of medium carbon steel (S45C). Dry milling was performed. The experimental milling parameters were set such that the spindle speed (1000 rpm), axial depth of cut (1 mm), and radial width of cut were fixed; however, the feed rate was varied linearly to values of 200, 260, 300, 360, and 400 mm/min.

The flat-end mill was clamped on the spindle of the VMP-40A machining center. To prevent any interference during the measurement, a Kistler-9257B piezoelectric milling dynamometer (load range of 3D force: 5–5000 N) was mounted on a clean, smooth worktable by means of outer hexagonal M8 screws. To enable measurement of the dynamic milling forces in the $X$, $Y$, and $Z$-directions, the workpiece for milling was fixed through application of a 10-N load on top of the dynamometer. The dynamometer charge signal was converted into a voltage signal by means of a Kistler-5018A three-channel charge amplifier; this amplifier was attached to a connecting box through its output line on both ends. Further, the Kistler-9257B terminal of the connecting box was linked to a Kistler-5018A multi-function data acquisition card via a Kistler-5697A connecting cable to ensure that the data acquisition card received an analog signal. The data acquisition card was inserted into the Kistler-5697A expansion slot of a PC, and the analog signal was digitized for input to the PC. Finally, the collected data were analyzed and processed using the dynamic cutting force measurement system software programmed in MATLAB R2010b. The signals were sampled at a frequency of 5 kHz. Before the measured signals of the cutting force could be used, they were required to be filtered to prevent the generation of noisy signals by the use of software.

4. Results and discussion

Cutting force is divided into shearing force generated by shearing in the shear zone and edge force resulting from the flank surface friction of the cutting edge. The shearing force can be expressed as a product of the tangential force coefficient $K_{tc}$, radial force coefficient $K_{rc}$, axial force coefficient $K_{ac}$, and shear area. The edge force can be expressed as a product of the tangential edge-force coefficient $K_{te}$, radial edge-force coefficient $K_{re}$, axial edge-force coefficient $K_{ae}$, and cutting width. The coefficients $K_{tc}$, $K_{rc}$, $K_{ac}$, $K_{te}$, $K_{re}$, and $K_{ae}$ are collectively referred to as the cutting force coefficients.[10] Figures 3–5, respectively, show examples of the cutting forces predicted by the average cutting force method (Avg) and the recursive least-squares method (ID) and the experimentally measured cutting force data.

![Figure 6. Plot of shear angle as a function of feed per tooth.](image)

![Table 1. Milling force coefficients for different milling parameters and tool diameter D12.](table)

| Run number | Cutter diameter (mm) | Feed per tooth (mm/z) | $K_{tc}$ | $K_{rc}$ | $K_{ac}$ | $K_{te}$ | $K_{re}$ | $K_{ae}$ |
|------------|----------------------|-----------------------|----------|----------|----------|----------|----------|----------|
| 1          | 12                   | 0.1                   | 1820     | 774      | 125      | 40.6     | 25.6     | 1.5      |
| 2          | 12                   | 0.13                  | 1816     | 608      | 96.9     | 46.1     | 27.1     | 1.98     |
| 3          | 12                   | 0.15                  | 1997     | 537      | 61.1     | 50       | 25.8     | 13.8     |
| 4          | 12                   | 0.18                  | 1692     | 292      | 54.8     | 57.8     | 32.6     | 7.8      |
| 5          | 12                   | 0.2                   | 1600     | 222      | 91       | 61.8     | 35.4     | 4.8      |
agreement between the forces predicted by the recursive least-squares method and the experimentally measured forces. Conversely, the peak and valley forces predicted

Table 2. Milling force coefficients for different milling parameters and tool diameter D16.

| Run number | Cutter diameter (mm) | Feed per tooth (mm/z) | $K_{tc}$ | $K_{rc}$ | $K_{ac}$ | $K_{te}$ | $K_{re}$ | $K_{ae}$ |
|------------|----------------------|-----------------------|---------|---------|---------|---------|---------|---------|
| 1          | 16                   | 0.1                   | 2238    | 903     | 223     | 28      | 16.7    | 4.9     |
| 2          | 16                   | 0.13                  | 2074    | 757     | 235     | 34      | 20      | 6.1     |
| 3          | 16                   | 0.15                  | 2106    | 754     | 152     | 34.5    | 28.6    | 5.6     |
| 4          | 16                   | 0.18                  | 2036    | 754     | 152     | 44.5    | 28.6    | 7.7     |
| 5          | 16                   | 0.2                   | 2009    | 595     | 136.6   | 47.7    | 30.5    | 9.2     |

Table 3. Milling force coefficients at different milling parameters and tool diameter D20.

| Run number | Cutter diameter (mm) | Feed per tooth (mm/z) | $K_{tc}$ | $K_{rc}$ | $K_{ac}$ | $K_{te}$ | $K_{re}$ | $K_{ae}$ |
|------------|----------------------|-----------------------|---------|---------|---------|---------|---------|---------|
| 1          | 20                   | 0.1                   | 2045    | 966     | 172     | 24.4    | 14.9    | 2.1     |
| 2          | 20                   | 0.13                  | 2095    | 791     | 216     | 29.9    | 19      | 3.7     |
| 3          | 20                   | 0.15                  | 2117    | 732     | 184     | 30      | 17.5    | 5.07    |
| 4          | 20                   | 0.18                  | 2055    | 797     | 139.9   | 39.5    | 25      | 6.2     |
| 5          | 20                   | 0.2                   | 1908    | 758     | 138     | 51.2    | 34.4    | 6.6     |

Figure 7. Plot of shear stress vs. feed per tooth for tool diameters (a) D12, (b) D16, and (c) D20.

Figure 8. Plot of friction angle vs. feed per tooth for tool diameters (a) D12, (b) D16, and (c) D20.
by the average cutting force method do not agree with the experimentally measured forces. Then, it can be said that the cutting force predicted by the recursive least-squares method is highly consistent with the measured force in terms of shape. Some fluctuations are added to the predicted force owing to the effects of rubbing and vibration; however, the global shape of the predicted force well matches that of the experimentally measured force. Two sets of cutting force data – measured by half-slot milling and slot milling – were required to determine the cutting force coefficients. Tables 1–3 present the determined cutting force coefficients for different feeds per tooth and tool diameters. It can be seen that a cutting coefficient for the feed per tooth (uncut chip thickness) with a negative power function was obtained for the tool diameters D12, D16, and D20.

Figure 6 shows the results of shear angle estimation by orthogonal experiments under different feeds per tooth. It is seen that the shear angle increased with increasing feed per tooth. Figure 7 shows the predicted shear stress as a function of the feed per tooth for tool diameters D12, D16, and D20. The results plotted in this figure indicate that the shear stress did not change with an increase in the feed per tooth. Further, it can be seen from Figure 8 that the magnitude of the friction angle did not change with varying feed per tooth and tool diameter.

5. Conclusions

In this study, we developed a prediction method for the milling cutting force and cutting coefficient for medium carbon steel (S45C). We introduced two predictive cutting force methods – the average cutting force method and the recursive least-squares method – and compared their prediction results of cutting forces with experimentally measured cutting forces. We also investigated the influence of the feed rate and tool diameter on the cutting force and cutting force coefficient. Finally, following the acquisition of accurate cutting force coefficients, we estimated the cutting parameters, i.e. the friction angle and shear stress, using the oblique cutting theory. The results of the study showed good agreement between the forces predicted by the recursive least-squares method and the experimentally measured forces. With an increase in the feed rate, the cutting force increased and the cutting coefficient became smaller in the case of shearing forces in tangential directions. Further, an increase in the feed per tooth resulted in an increase in the shear angle. It was also found that the magnitude of the friction angle did not increase with varying feed per tooth and tool diameter. The findings of the study demonstrate the effectiveness and accuracy of the proposed method, as well as the good agreement between the predicted force and the experimentally measured force.

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Disclosure statement

No potential conflict of interest was reported by the authors.

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