The comparison of velaroidal shell structures of square plane load bearing properties

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Abstract. Velaroid surfaces is a class of analytical surfaces, formed by translating generatrix curves of variable curvature along the directrix curve of the same type, limited by plane contour lines. A new type of velaroidal shell –on the base of the catenary curve is introduced in this job, it’s surface equation obtained. The static structural analysis of the shells under equally distributed dead weight loading is conducted. Four types of structures of equal overall size and rising height were investigated: velaroidal shells on the base of catenary, parabola, ellipse, and sinusoid lines. The analysis is provided using the finite element method. The material has characteristics of reinforced concrete. The comparison revealed the peculiarities of each type of behavior under loading. The velaroidal shell with catenary generatrices demonstrates the most advantageous behavior under loading in the aspect of bending and membrane stresses ratio. This comparison is of interest for the potential introduction of such structures for roof elements.

1. Introduction

Modern shell and spatial structures architecture and structural mechanics are constantly in quest of minimizing of structure weight and optimizing of its behavior under load. New approaches for geometrical and strength studies are arising, like Force density methods [1, 2], R-funicularity [3, 4], e.t.c. The classical approach, supposing analytical expression for the form-finding, also has some obvious advantages.

Velaroidal surface is a translational surface on the square plane, generated by the curve, which, keeping constant its class, changes its curvature in each point, tending to zero near contour lines.

The term ‘velaroid’ is first mentioned in the paper [5]. The surface is limited by four mutually orthogonal complanar contour right lines. Such configuration has plus points for floor structures design.

Three types of velaroid shells are mentioned in the literature: parabolic, elliptical, and sinusoidal, according to the class of generator lines. The papers [6, 7] prove that the power of the velaroid surface set is precisely the power of the set of real numbers. The paper [8] extends the original concept of velaroid surface into surfaces of flat ring circuit con-tour.

The present paper introduces a new type of velaroidal surface based on catenary. Such surface resembles a square piece of clothing material, slackening under gravity being fixed on contours.
Reinforced concrete systems in the shape of draped fabrics were empirically invented by Heinz Isler [9, 10]. Catenary form of arcs is popularly known since ancient times, its features were appreciated by R. Hooke, and inspired the greatest architects like A. Gaudi.

Four types of surfaces and shells of the corresponding form are investigated in the present paper. Each of them can be fabricated industrially or in situ. Several fool-proof technologies to fabricate such shells: the bar mat can be shaped by surcharging or flexural, and after that sprayed by concrete. The concrete form can also be fabricated in a traditional way, for example like it is described for parabolic moulds in [11]. Finally, 3d-printer technologies provide powerful capabilities for mould shaping. Four lines for surface generators were chosen for their simplicity and ‘naturalness’. Theoretically, not only steel but fiber reinforcement, glass-cloth, or asbestos cloth can be used for concrete spraying.

The main purpose of this job was to compare shells of nearly equal geometric overall sizes and evaluate the difference in their behavior. The shell which is closer to pure membrane solution without bending is of the most interest, especially it reinforced concrete elements under nearly pure compression. The numerical value of deflection and stresses is estimated and the shell of minimum stresses seems to be advantageous obviously. Several criteria of optimization for shells of revolution are given in papers [12, 13] and among them, the criterion of minimal axial stresses for shells of similar overall sizes, loading, and boundary condition can be applied for translational shells too.

2. Methods
2.1. Geometrical modelling

The models for comparison were created using analytical expressions for the considered surfaces. The equations of velaroidal surfaces of three types (sinusoid, parabolic, and ellipse generatrix) are provided in [14]. The velaroid surface with catenary generatrix is established in the present job.

**Velaroidal surface on the base of catenary:**

The surface equation:

\[
    f(x, y) = (-a \cdot \cosh((x - b) / a) + a \cdot \cosh(b / a)) \cdot (-d \cdot \cosh((y - c) / d) + d \cdot \cosh(c / d))
\]

The model parameters in the present comparison \(a = d = 3.3945\ m, b = c = 3\ m\), The rise height is defined approximately because it can not be derived in an explicit form.

**Velaroidal surface on the base of parabolic curve:**

The surface equation:

\[
    z(x, y) = c \left( \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{x^2}{a^2} \frac{y^2}{b^2} \right)
\]

\(a, b\) is the half of the span along the coordinate lines \(x, y\) respectively,
\(c\) is the rising height in the center.

The model with parameters \(a = b = 3\ m, c = 2\ m\) is considered.
Velaroidal surface on the base of ellipse:
The surface equation:

\[ z(x, y) = \sqrt{f^2 - \left(\frac{f^2 - c^2}{a^2}\right)(x^2 + y^2) - \left(\frac{f^2 - c^2}{a^2}\right)x^2y^2} \]  

(3)

The shell is limited by planes \( z=c \) and \( z=f \), \( a \) – half – spans along axis x and y (for a square plane).

In the present job, the parameters are \( f=3 \) m, \( c=1 \) m, \( a=3 \) m.

**Velaroidal surface on the base of sinusoid:**
The surface equation:

\[ z(x, y) = f \cdot \sin \frac{x \pi a}{a} \cdot \sin \frac{y \pi b}{b} \]  

(4)

where \( a, b \) are the overall dimensions of flat contour in a plan, \( f \) - rising height.

Geometrical parameters of the model in the present job: \( f=2 \) m, \( a=6 \) m, \( b=6 \) m.

2.2. **Finite element analysis**

Each structure element has an overall size 6x6 m, the rising height of 2 m, the thickness of 8 cm. The material characteristics \( E=325 \) MPa, \( \nu = 0.17 \) are equal to average reinforced concrete characteristics. The shells are supported by rigid membranes that are fixed support of all edges. The dead load of 10000 N/m² is applied.

The static structural analysis using the finite element method was conducted. The FEM was realized by Ansys APDL. The model surfaces are generated and meshed directly using APDL. The type of finite element shell 181 (quadrilateral) corresponds to the thin elastic shell model. The length of the side (the element size) of 0.25 m was applied, the model contains 729 nodes and 676 elements, that is sufficient for the analysis purposes. The isofields of displacements and inner moments and forces were got, and also epures along the line through the centers of the opposite sides and the center of the surface. The equivalent stress isofields (von Mises stress) were obtained for the abstract material model without taking into account any armouring scheme, such an approach is reasonable for the approximate estimation of ‘problem’ zones, which need extra stiffening. The figures below contain epures in halves of structural elements because the loading scheme is symmetric.

3. **Results**

3.1. **Velaroidal shell on the base of catenary**
Figure 5. Axial forces

Fig. 5. Bending moments

Figure 7. The vertical deflection (a) and von Mises stress (b)
3.2. *Velaroidal shell on the base of parabola*

![Figure 8. Axial and shear forces](image1)

![Figure 9. Bending moments](image2)

![Figure 10. The vertical deflection (a) and von Mises stress (b)](image3)
3.3. **Volaroidal shell on the base of ellipse**

![Figure 11. Axial forces](image1)

![Figure 12. Axial and shear forces](image2)

![Figure 13. Bending moments](image3)
3.4. Velaroidal shell on the base of sinusoid

Figure 14. The vertical deflection (a) and von Mises stress (b)

Figure 15. Axial and shear forces

Figure 16. Bending moments
The maximum results are shown in the summary table below:

**Table 1.** Maximum inner forces and moments, deflection, and equivalent stress.

| Generatrix curve type | catenary | parabolic | ellipse | sinusoid |
|-----------------------|----------|-----------|---------|----------|
| \(N_{11}, \text{N/m}\) | -1093    | -1195     | -2061   | -3323    |
| \(N_{22}, \text{N/m}\) | -2930    | -3025     | -1386   | -1670    |
| \(N_{12}, \text{N/m}\) | -33      | -        | -43     | -        |
| \(Q_{23}, \text{N/m}\) | -80      | 111       | -       | -        |
| \(Q_{13}, \text{N/m}\) | -        | -         | -120    | 262      |
| \(M_{11}, \text{N·m/m}\) | 3.3       | 4.795     | -18.2   | 48       |
| \(M_{22}, \text{N·m/m}\) | -18      | -24.7     | -3.03   | 0        |
| \(u_z, 10^{-6} \text{m}\) | -2.84    | -4.06     | -4.08   | -5.2     |

4. Discussion

The stress-strain state of velaroid shell on the base of catenary demonstrates the least bending moments, mostly the forces of compression appear in it; its deflection is minimal among the other shells considered;

The supporting contour of all the structures of this kind is a ‘problem zone’, while fixed supported edges, the stresses far outweigh the stresses in other points, but is within one order. Obviously, the supporting contour needs extra reinforcements.

The most disadvantageous behavior sinusoid velaroid shell has demonstrated: it has the maximum deflection, maximum stresses, bending moments and shear forces;

Velaroidal shells on the base of parabolic curve demonstrate larger values of bending moment than shells of catenary form, this can be explained by the fact that catenary curve closer approximate the form of hanging chain, which is used to get compression-only arches; the shell of such form can be introduced into practice because it has an advanced ratio of bending and compression stresses.

Shell on the base of the ellipse curve shows the minimum (among the others) value of equivalent stresses on the supporting contour.
5. Conclusions
A lot of up-to-date investigations concern form-finding for advanced roof design [15-20]. Most of them are devoted to domes, their strength and stability, and also behavior under dynamic loadings. Buildings on the square and rectangular plan are more widespread and common than those of circular plan, and usually, they are most handy for the range of different purposes. So, finding new shapes for square and rectangular plans can be considered relevant. Such disadvantages of thin shell structures, as manufacturing complexity, difficulties while electrical and plumbing systems installation, low insolation of enclosed volumes, limited by shells, are usually mentioned by the researchers [21]. Search for solutions, which can help to minimize these factors, has potential because the positive points of thin shell structures appliance can give high-cost impact, and also such structures can be aesthetically attractive for architects. Particularly, according to the results of this paper, shells of velaroid shape should be investigated more carefully. In prospects, the stability and dynamic behavior of such types of shells should be investigated.

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