Cross sections for 2-to-1 meson-meson scattering

Wan-Xia Li\textsuperscript{1}, Xiao-Ming Xu\textsuperscript{1}, and H. J. Weber\textsuperscript{2}

\textsuperscript{1}Department of Physics, Shanghai University, Baoshan, Shanghai 200444, China
\textsuperscript{2}Department of Physics, University of Virginia, Charlottesville, VA 22904, USA

Abstract

We study the processes $K\bar{K} \rightarrow \phi$, $\pi D \rightarrow D^*$, $\pi \bar{D} \rightarrow \bar{D}^*$, and the production of $\psi(4160)$ and $\psi(4415)$ mesons in collisions of charmed mesons or charmed strange mesons. The 2-to-1 meson-meson scattering involves a process where a quark and an antiquark from the two initial mesons annihilate into a gluon and subsequently the gluon is absorbed by the spectator quark or antiquark. Transition amplitudes for the scattering process derive from the transition potential in conjunction with mesonic quark-antiquark wave functions and the relative-motion wave function of the two initial mesons. We derive these transition amplitudes in the partial wave expansion of the relative-motion wave function of the two initial mesons so that parity and total-angular-momentum conservation are maintained. We calculate flavor and spin matrix elements in accordance with the transition potential and unpolarized cross sections for the reactions using the transition amplitudes. Cross sections for the production of $\psi(4160)$ and $\psi(4415)$ generally increase as the colliding mesons go through the cases of $D\bar{D}$, $D^*\bar{D}$, and $D^*\bar{D}^*$ or the cases of $D_s^+D_s^-$, $D_s^{*+}D_s^-$, and $D_s^{*+}D_s^{*-}$. We suggest the production of $\psi(4160)$ and $\psi(4415)$ as a probe of hadronic matter that results from the quark-gluon plasma created in ultrarelativistic heavy-ion collisions.

Keywords: Inelastic meson-meson scattering, Quark-antiquark annihilation, Relativistic constituent quark potential model.

PACS: 13.75.Lb; 12.39.Jh; 12.39.Pn
I. INTRODUCTION

Elastic meson-meson scattering produces many resonances. Starting from meson-meson scattering amplitudes obtained in chiral perturbation theory [1], elastic scattering has been studied within nonperturbative schemes, for example, the inverse amplitude method [2] and the coupled-channel unitary approaches [3]. Elastic meson-meson scattering has also been studied with quark interchange in the first Born approximation in Ref. [4] and with quark-antiquark annihilation and creation in Ref. [5]. Elastic scattering reported in the literature includes \( \pi\pi [1–3], \pi K [1–3], K\bar{K} [6–9], \pi\eta [6, 7, 10–13], K\eta [6], \eta\eta [8], \pi\rho [14,15], \pi D [16], \bar{K}D [17,18], \bar{K}D^* [17], \) and \( D D^* [17] \). We know that resonances observed in the elastic scattering are usually produced by a process where two mesons scatter into one meson. The 2-to-1 meson-meson scattering includes \( \pi\pi \to \rho, \pi\pi \to f_0(980), \pi K \to K^*, K\bar{K} \to \phi, \pi\eta \to a_0(980), \pi\rho \to a_1(1260), \pi D \to D^*, \) and so on. Since some resonances like \( f_0(980), a_0(980), \) and \( a_1(1260) \) are not quark-antiquark states, we do not study \( \pi\pi \to f_0(980), \pi\eta \to a_0(980), \pi\rho \to a_1(1260), \) etc. in the present work. The reactions \( \pi\pi \to \rho \) and \( \pi K \to K^* \) have been studied in Ref. [19] via a process where a quark in an initial meson and an antiquark in another initial meson annihilate into a gluon and subsequently the gluon is absorbed by the other antiquark or quark, and the resulting cross sections in vacuum agree with empirical data. Since these two reactions also take place in hadronic matter that is created in ultrarelativistic heavy-ion collisions at the Relativistic Heavy Ion Collider and at the Large Hadron Collider, the dependence of cross sections for the two reactions on the temperature of hadronic matter has also been investigated. With increasing temperature the cross sections decrease. In the present work we consider the reactions: \( K\bar{K} \to \phi, \pi D \to D^*, \pi\bar{D} \to \bar{D}^*, D\bar{D} \to \psi(4160), D^*\bar{D} \to \psi(4160), D\bar{D}^* \to \psi(4160), D_s^+D_s^- \to \psi(4160), D_s^+D_s^- \to \psi(4160), D_s^+D_s^- \to \psi(4160), D_s^+D_s^- \to \psi(4160), D_s^+D_s^- \to \psi(4160), D_s^+D_s^- \to \psi(4160), D_s^+D_s^- \to \psi(4160), D_s^+D_s^- \to \psi(4160), D^*_s^+D^-_s \to \psi(4160), D^*_s^+D^-_s \to \psi(4160), D^*_s^+D^-_s \to \psi(4160), D^*_s^+D^-_s \to \psi(4160), D^*_s^+D^-_s \to \psi(4160), D^*_s^+D^-_s \to \psi(4160), D^*_s^+D^-_s \to \psi(4160), D^*_s^+D^-_s \to \psi(4160), D^*_s^+D^-_s \to \psi(4160), D^*_s^+D^-_s \to \psi(4160), \) and \( D^*_s^+D^-_s \to \psi(4160). \) Both \( \psi(4160) \) and \( \psi(4415) \) consist of a quark and an antiquark [20,21]. All these reactions are governed by the strong interaction.
The reaction $K\bar{K} \to \phi$ was studied in Ref. [22] in a mesonic model. The fifteen reactions that lead to $\psi(4160)$ or $\psi(4415)$ as a final state have not been studied theoretically. Now we study $K\bar{K} \to \phi$, $\pi D \to D^{*}$, $\pi \bar{D} \to \bar{D}^{*}$, and the fifteen reactions using quark degrees of freedom. The production of $J/\psi$ is a subject intensively studied in relativistic heavy-ion collisions. The $\psi(4160)$ and $\psi(4415)$ mesons may decay into the $J/\psi$ meson. Through this decay the fifteen reactions add a contribution to the $J/\psi$ production in relativistic heavy-ion collisions. This is another reason why we study the fifteen reactions here.

This paper is organized as follows. In Sect. II we consider four Feynman diagrams and the $S$-matrix element for 2-to-1 meson-meson scattering, derive transition amplitudes and provide cross-section formulas. In Sect. III we present transition potentials corresponding to the Feynman diagrams and calculate flavor matrix elements and spin matrix elements. In Sect. IV we calculate cross sections, present numerical results and give relevant discussions. In Sect. V we summarize the present work.

II. FORMALISM

Lowest-order Feynman diagrams are shown in Fig. 1 for the reaction $A(q_1\bar{q}_1) + B(q_2\bar{q}_2) \to H(q_2\bar{q}_1$ or $q_1\bar{q}_2)$. A quark in an initial meson and an antiquark in the other initial meson annihilate into a gluon, and the gluon is then absorbed by a spectator quark or antiquark. The four processes $q_1 + \bar{q}_2 + q_1 \to \bar{q}_1$, $q_1 + \bar{q}_2 + q_2 \to q_2$, $q_2 + q_1 + q_1 \to q_1$, and $q_2 + q_1 + \bar{q}_2 \to \bar{q}_2$ in Fig. 1 give rise to the four transition potentials $V_{rq_1q_2\bar{q}_1}$, $V_{rq_1q_2q_2}$, $V_{rq_2q_1q_1}$, and $V_{rq_2q_1\bar{q}_2}$, respectively. Denote by $E_i$ and $\vec{P}_i$ ($E_f$ and $\vec{P}_f$) the total energy and the total momentum of the two initial (final) mesons, respectively; let $E_A$ ($E_B$, $E_H$) be the energy of meson $A$ ($B$, $H$), and $V$ the volume where every meson wave function is normalized. The $S$-matrix element for $A + B \to H$ is

$$S_{fi} = \delta_{fi} - 2\pi i\delta(E_f - E_i)(<H \mid V_{rq_1q_2\bar{q}_1} \mid A, B > + <H \mid V_{rq_1q_2q_2} \mid A, B >$$

$$+ <H \mid V_{rq_2q_1q_1} \mid A, B > + <H \mid V_{rq_2q_1\bar{q}_2} \mid A, B >)$$

$$= \delta_{fi} - (2\pi)^4 i\delta(E_f - E_i)\delta^3(\vec{P}_f - \vec{P}_i)\frac{M_{rq_1q_2q_1} + M_{rq_1q_2q_2} + M_{rq_2q_1q_1} + M_{rq_2q_1\bar{q}_2}}{V^2 \sqrt{2E_A^2E_B^2E_H^2}}. \quad (1)$$

where in the four processes mesons $A$ and $B$ go from the state vector $| A, B >$ to the
state vector \( |H\rangle \) of meson \( H \), and \( M_{vq_1\bar{q}_2q_1} \), \( M_{vq_1\bar{q}_2q_2} \), \( M_{vq_2\bar{q}_1q_1} \), and \( M_{vq_2\bar{q}_1q_2} \) are the transition amplitudes given by

\[
M_{vq_1\bar{q}_2q_1} = \sqrt{2E_A2E_B2E_H} \int d\vec{r}_{q_1\bar{q}_1}d\vec{r}_{q_2\bar{q}_2} \psi_H^+V_{r_{q_1\bar{q}_1q_1}}\psi_{AB}\bar{\psi}_{q_1\bar{q}_2q_2}, \quad (2)
\]

\[
M_{vq_1\bar{q}_2q_2} = \sqrt{2E_A2E_B2E_H} \int d\vec{r}_{q_1\bar{q}_1}d\vec{r}_{q_2\bar{q}_2} \psi_H^+V_{r_{q_1\bar{q}_2q_2}}\psi_{AB}\bar{\psi}_{q_1\bar{q}_1q_1}, \quad (3)
\]

\[
M_{vq_2\bar{q}_1q_1} = \sqrt{2E_A2E_B2E_H} \int d\vec{r}_{q_1\bar{q}_1}d\vec{r}_{q_2\bar{q}_2} \psi_H^+V_{r_{q_2\bar{q}_1q_1}}\psi_{AB}\bar{\psi}_{q_1\bar{q}_2q_2}, \quad (4)
\]

\[
M_{vq_2\bar{q}_1q_2} = \sqrt{2E_A2E_B2E_H} \int d\vec{r}_{q_1\bar{q}_1}d\vec{r}_{q_2\bar{q}_2} \psi_H^+V_{r_{q_2\bar{q}_1q_2}}\psi_{AB}\bar{\psi}_{q_1\bar{q}_2q_2}, \quad (5)
\]

where \( \vec{r}_{ab} \) is the relative coordinate of constituents \( a \) and \( b \); \( \vec{r}_{q_1\bar{q}_1q_2} \) the relative coordinate of \( q_1\bar{q}_1 \) and \( q_2\bar{q}_2 \); \( \vec{p}_{q_1\bar{q}_1q_2} \) the relative momentum of \( q_1\bar{q}_1 \) and \( q_2\bar{q}_2 \); \( \psi_H^+ \) the Hermitean conjugate of \( \psi_H \). The wave function of mesons \( A \) and \( B \) is

\[
\psi_{AB} = \phi_{Arel}\phi_{Brel}\phi_{Acolor}\phi_{Bcolor}\chi_{S_A}S_{A_z}\chi_{S_B}S_{B_z}\varphi_{AB\text{flavor}}, \quad (6)
\]

and the wave function of meson \( H \) is

\[
\psi_H = \phi_{Hrel}\phi_{Hcolor}\chi_{S_H}S_{H_z}\phi_{H\text{flavor}}, \quad (7)
\]

where \( S_A \) (\( S_B, S_H \)) is the spin of meson \( A \) (\( B, H \)) with its magnetic projection quantum number \( S_{A_z} \) (\( S_{B_z}, S_{H_z} \)); \( \phi_{Arel} \) (\( \phi_{Brel}, \phi_{Hrel} \)), \( \phi_{Acolor} \) (\( \phi_{Bcolor}, \phi_{Hcolor} \)), and \( \chi_{S_AS_{A_z}} \) (\( \chi_{S_BS_{B_z}}, \chi_{S_HS_{H_z}} \)) are the quark-antiquark relative-motion wave function, the color wave function, and the spin wave function of meson \( A \) (\( B, H \)), respectively; \( \phi_{H\text{flavor}} \) and \( \varphi_{AB\text{flavor}} \) are the flavor wave functions of meson \( H \) and of mesons \( A \) and \( B \), respectively.

The development in spherical harmonics of the relative-motion wave function of mesons \( A \) and \( B \) (aside from a normalization constant) is given by

\[
e^{i\vec{p}_{q_1\bar{q}_1q_2}\vec{r}_{q_1\bar{q}_1q_2}} = 4\pi \sum_{L_i=0}^{\infty} \sum_{M_i=-L_i}^{L_i} j_{L_i}\left(\frac{\vec{p}_{q_1\bar{q}_1q_2}}{r_{q_1\bar{q}_1q_2}}\right) Y_{L_iM_i}^{*}(\vec{r}_{q_1\bar{q}_1q_2}) Y_{L_iM_i}(\vec{r}_{q_1\bar{q}_1q_2}), \quad (8)
\]

where \( Y_{L_iM_i} \) are the spherical harmonics with the orbital-angular-momentum quantum number \( L_i \) and the magnetic projection quantum number \( M_i \); \( j_{L_i} \) are the spherical Bessel functions, and \( \vec{p}_{q_1\bar{q}_1q_2} \) (\( \vec{r}_{q_1\bar{q}_1q_2} \)) denote the polar angles of \( \vec{p}_{q_1\bar{q}_1q_2} \) (\( \vec{r}_{q_1\bar{q}_1q_2} \)). Let \( \chi_{SS_z} \),
stand for the spin wave function of mesons A and B, which has the total spin S and its z component $S_z$. The Clebsch-Gordan coefficients $(S_A S_A S_B S_B | S S_z)$ couple $\chi_{SS_z}$ to $\chi_{S_A S_A S_B S_B}$,

$$\chi_{S_A S_A S_B S_B} = \sum_{S = S_{\text{min}}}^{S_{\text{max}}} \sum_{S_z = -S}^{S} (S_A S_A S_B S_B | S S_z) \chi_{SS_z},$$  \hspace{1cm} (9)

where $S_{\text{min}} = |S_A - S_B|$ and $S_{\text{max}} = S_A + S_B$. $Y_{l_i m_i}$ and $\chi_{SS_z}$ are coupled to the wave function $\phi_{J J_z}$ which has the total angular momentum $J$ of mesons A and B and its z component $J_z$,

$$Y_{l_i m_i} \chi_{SS_z} = \sum_{J = J_{\text{min}}}^{J_{\text{max}}} \sum_{J_z = -J}^{J} (L_i M_i S S_z | J J_z) \phi_{J J_z}^{\text{in}},$$  \hspace{1cm} (10)

where $J_{\text{min}} = |L_i - S|$, $J_{\text{max}} = L_i + S$, and $(L_i M_i S S_z | J J_z)$ are the Clebsch-Gordan coefficients. It follows from Eqs. (8)-(10) that the transition amplitude given in Eq. (2) becomes

$$M_{r_{q1} q_{21}} = \sqrt{2E_A 2E_B 2E_H} 4\pi \sum_{L_i = 0}^{L_i} \sum_{M_i = -L_i}^{L_i} i^{L_i} Y_{L_i M_i}^* (\hat{p}_{r_{q1} q_{21}}) \phi_{H \text{color}}^+ \phi_{H \text{flavor}}^+$$

$$\int d^3 r_{q1} d^3 r_{q2} \phi_{J H J_z}^+ V_{r_{q1} q_{21}} \sum_{S = S_{\text{min}}}^{S_{\text{max}}} \sum_{S_z = -S}^{S} (S_A S_A S_B S_B | S S_z)$$

$$\sum_{J = J_{\text{min}}}^{J_{\text{max}}} \sum_{J_z = -J}^{J} (L_i M_i S S_z | J J_z) \phi_{J J_z}^{\text{in}} (\bar{p}_{r_{q1} q_{21}}, r_{q1} q_{22}) \phi_{A \text{rel}} \phi_{B \text{rel}} \phi_{A \text{color}} \phi_{B \text{color}} \phi_{A \text{flavor}} \phi_{B \text{flavor}},$$  \hspace{1cm} (11)

where $\phi_{J H J_z}^+ = \phi_{H \text{rel}} \chi_{SH H_{S_z}}$. Conservation of total angular momentum implies that $J$ equals the total angular momentum $J_H$ of meson H and $J_z$ equals the z component $J_{H_z}$ of $J_H$. This leads to

$$M_{r_{q1} q_{21}} = \sqrt{2E_A 2E_B 2E_H} 4\pi \sum_{L_i = 0}^{L_i} \sum_{M_i = -L_i}^{L_i} i^{L_i} Y_{L_i M_i}^* (\hat{p}_{r_{q1} q_{21}}) \phi_{H \text{color}}^+ \phi_{H \text{flavor}}^+$$

$$\int d^3 r_{q1} d^3 r_{q2} \phi_{J H J_z}^+ V_{r_{q1} q_{21}} \sum_{S = S_{\text{min}}}^{S_{\text{max}}} \sum_{S_z = -S}^{S} (S_A S_A S_B S_B | S S_z)$$

$$(L_i M_i S S_z | J H J_{H_z}) \phi_{J H J_{H_z} J_z}^{\text{in}} (\bar{p}_{r_{q1} q_{22}}, r_{r_{q1} q_{22}}) \phi_{A \text{rel}} \phi_{B \text{rel}} \phi_{A \text{color}} \phi_{B \text{color}} \phi_{A \text{flavor}} \phi_{B \text{flavor}},$$  \hspace{1cm} (12)
Using the relation

\[
\phi_{IJ_{H\bar{z}}}^{\text{in}} = \sum_{L_{i}} \sum_{S} \left( L_{i} M_{i} S S_{\bar{z}} | J_{H} J_{H\bar{z}} \right) Y_{L_{i} M_{i}} \chi S S_{\bar{z}},
\]

where \((L_{i} M_{i} S S_{\bar{z}} | J_{H} J_{H\bar{z}})\) are the Clebsch-Gordan coefficients, we get

\[
\mathcal{M}_{r_{q_{1}q_{2}\bar{q}_{1}}} = \sqrt{2} A_{2} B_{2} E_{H} 4 \pi \sum_{S_{\text{max}}} \sum_{S_{z}=-S} \left( S_{A} S_{A_{2}} S_{B_{2}} S_{S_{\bar{z}}} | S_{S_{\bar{z}}} \right) \sum_{L_{i}=0}^{\infty} \sum_{M_{i}=-L_{i}}^{L_{i}} i^{L_{i}}
\]

\[
Y_{L_{i} M_{i}}^{*} (\hat{p}_{q_{1} q_{2} \bar{q}_{1}}) (L_{i} M_{i} S S_{\bar{z}} | J_{H} J_{H\bar{z}}) \sum_{M_{i}=-L_{i}}^{L_{i}} \sum_{S_{z}=-S} \left( L_{i} M_{i} S S_{\bar{z}} | J_{H} J_{H\bar{z}} \right)
\]

\[
\phi_{H\text{color}}^{+} \phi_{H\text{flavor}}^{+} \int d^{3} r_{q_{1} q_{2} \bar{q}_{1}} d^{3} r_{q_{2} \bar{q}_{2}} \phi_{J_{H} J_{H\bar{z}}}^{+} V_{r_{q_{1} q_{2} \bar{q}_{1}}} J_{L_{i}} (| \vec{p}_{q_{1} q_{2} \bar{q}_{1}} |) r_{q_{1} q_{2} \bar{q}_{1}}
\]

\[
Y_{L_{i} M_{i}}^{*} (\hat{r}_{q_{1} q_{2} \bar{q}_{1}}) \phi_{\text{Arel}} \phi_{\text{Brel}} \chi S S_{\bar{z}} \phi_{\text{Acolor}} \phi_{\text{Bcolor}} \varphi_{AB\text{flavor}}.
\]

Furthermore, we need the identity

\[
 j_{i}(p r) Y_{lm}(\hat{r}) = \int \frac{d^{3} p'}{(2\pi)^3} \frac{2\pi^{2}}{p^{2}} (p - p') i^{l} (-1)^{l} Y_{lm}(\hat{p'}) e^{ip' \cdot \hat{r}},
\]

which is obtained with the help of \(\int_{0}^{\infty} j_{i}(p r) j_{i}(p' r) r^{2} dr = \frac{\pi}{2p^{2}} \delta(p - p')^{[23][24]}\). Substituting Eq. (15) in Eq. (14), we get

\[
\mathcal{M}_{r_{q_{1}q_{2}\bar{q}_{1}}} = \sqrt{2} A_{2} B_{2} E_{H} 4 \pi \sum_{S_{\text{max}}} \sum_{S_{z}=-S} \left( S_{A} S_{A_{2}} S_{B_{2}} S_{S_{\bar{z}}} | S_{S_{\bar{z}}} \right) \sum_{L_{i}=0}^{\infty} \sum_{M_{i}=-L_{i}}^{L_{i}} i^{L_{i}}
\]

\[
Y_{L_{i} M_{i}}^{*} (\hat{p}_{q_{1} q_{2} \bar{q}_{1}}) (L_{i} M_{i} S S_{\bar{z}} | J_{H} J_{H\bar{z}}) \sum_{M_{i}=-L_{i}}^{L_{i}} \sum_{S_{z}=-S} \left( L_{i} M_{i} S S_{\bar{z}} | J_{H} J_{H\bar{z}} \right)
\]

\[
\phi_{H\text{color}}^{+} \phi_{H\text{flavor}}^{+} \int d^{3} r_{p_{\text{firm}}} \frac{2\pi^{2}}{(2\pi)^3} \delta(| \vec{p}_{q_{1} q_{2} \bar{q}_{1}} | - | \vec{p}_{\text{firm}} |) i^{L_{i}} (-1)^{L_{i}}
\]

\[
Y_{L_{i} M_{i}}^{*} (\hat{p}_{\text{firm}}) \int d^{3} r_{q_{1} q_{2} \bar{q}_{1}} d^{3} r_{q_{2} \bar{q}_{2}} \phi_{J_{H} J_{H\bar{z}}}^{+} V_{r_{q_{1} q_{2} \bar{q}_{1}}} e^{i\vec{p}_{\text{firm}} \cdot \vec{r}_{q_{1} q_{2} \bar{q}_{1}}}
\]

\[
\phi_{\text{Arel}} \phi_{\text{Brel}} \chi S S_{\bar{z}} \phi_{\text{Acolor}} \phi_{\text{Bcolor}} \varphi_{AB\text{flavor}}.
\]

Let \(\vec{r}_{c}\) and \(m_{c}\) be the position vector and the mass of constituent \(c\), respectively. Then \(\phi_{\text{Arel}}, \phi_{\text{Brel}},\) and \(\phi_{\text{Hrel}}\) are functions of the relative coordinate of the quark and the antiquark. We take the Fourier transform of \(V_{r_{q_{1}q_{2}\bar{q}_{1}}}\) and the mesonic quark-antiquark relative-motion wave functions:

\[
V_{r_{q_{1}q_{2}\bar{q}_{1}}}(\vec{r}_{q_{1}} - \vec{r}_{\bar{q}_{1}}) = \int \frac{d^{3} k}{(2\pi)^3} V_{r_{q_{1}q_{2}\bar{q}_{1}}}(\vec{k}) e^{i \vec{k} \cdot (\vec{r}_{q_{1}} - \vec{r}_{\bar{q}_{1}})},
\]
\[ \phi_{Arel}(\vec{r}_{q_1q_1}) = \int \frac{d^3p_{q_1q_1}}{(2\pi)^3} \phi_{Arel}(\vec{p}_{q_1q_1}) e^{ip_{q_1q_1}\cdot\vec{r}_{q_1q_1}}, \quad (18) \]

\[ \phi_{Brel}(\vec{r}_{q_2q_2}) = \int \frac{d^3p_{q_2q_2}}{(2\pi)^3} \phi_{Brel}(\vec{p}_{q_2q_2}) e^{ip_{q_2q_2}\cdot\vec{r}_{q_2q_2}}, \quad (19) \]

\[ \phi_{JH_{JH}}(\vec{r}_{q_2q_2}) = \int \frac{d^3p_{q_2q_2}}{(2\pi)^3} \phi_{JH_{JH}}(\vec{p}_{q_2q_2}) e^{ip_{q_2q_2}\cdot\vec{r}_{q_2q_2}}, \quad (20) \]

for the two upper diagrams in Fig. 1, and

\[ \phi_{JH_{JH}}(\vec{r}_{q_1q_1}) = \int \frac{d^3p_{q_1q_1}}{(2\pi)^3} \phi_{JH_{JH}}(\vec{p}_{q_1q_1}) e^{ip_{q_1q_1}\cdot\vec{r}_{q_1q_1}}, \quad (21) \]

for the two lower diagrams. In Eq. (17) \( \vec{k} \) is the gluon momentum, and in Eqs. (18)-(21) \( \vec{p}_{ab} \) is the relative momentum of constituents \( a \) and \( b \). The spherical polar coordinates of \( \vec{p}_{irm} \) are expressed as \( (|\vec{p}_{irm}|, \theta_{irm}, \phi_{irm}) \). The mesonic quark-antiquark relative-motion wave functions in momentum space are normalized as

\[ \int \frac{d^3p_{q_1q_1}}{(2\pi)^3} \phi_{Arel}^*(\vec{p}_{q_1q_1}) \phi_{Arel}(\vec{p}_{q_1q_1}) = 1, \]

\[ \int \frac{d^3p_{q_2q_2}}{(2\pi)^3} \phi_{Brel}^*(\vec{p}_{q_2q_2}) \phi_{Brel}(\vec{p}_{q_2q_2}) = 1, \]

\[ \int \frac{d^3p_{q_2q_1}}{(2\pi)^3} \phi_{Hrel}^*(\vec{p}_{q_2q_1}) \phi_{Hrel}(\vec{p}_{q_2q_1}) = 1, \]

\[ \int \frac{d^3p_{q_1q_2}}{(2\pi)^3} \phi_{Hrel}^*(\vec{p}_{q_1q_2}) \phi_{Hrel}(\vec{p}_{q_1q_2}) = 1. \]

Integration over \( |\vec{p}_{irm}|, \vec{r}_{q_1q_1}, \) and \( \vec{r}_{q_2q_2} \) yields

\[ M_{r_1q_2q_1} = \sqrt{2E_A E_B E_H} \sum_{S_{max}} \sum_{S_{min}} \sum_{S_{z}=S} (S_A S_{A_{\pi}} S_{B_{\pi}} | SS_z) \sum_{L_i=0}^{L_i} \sum_{L_i=-L_i}^{L_i} Y_{L_i M_i}^*(\vec{p}_{q_1q_1,q_2q_2})(L_i M_i S_{S_{\pi}} | J_{H^*} J_{H^*}) \sum_{M_i=-L_i}^{L_i} \sum_{S_{z}=-S} (L_i M_i S S_z | J_{H} J_{H}) \]

\[ \phi_{H^{color}}^* \phi_{H^{flavor}}^* \int d\theta_{irm} d\phi_{irm} \sin \theta_{irm} Y_{L_i M_i} (\vec{p}_{irm}) \]

\[ \int \frac{d^3p_{q_1q_1}}{(2\pi)^3} \int \frac{d^3p_{q_2q_2}}{(2\pi)^3} \phi_{JH_{JH}}^* (\vec{p}_{q_2q_2} - \frac{m_{q_2}}{m_{q_2} + m_{q_1}} \vec{p}_{irm}) V_{r_1q_2q_1} \]

\[ [\vec{p}_{q_1q_1} - \vec{p}_{q_2q_2} + (\frac{m_{q_2}}{m_{q_2} + m_{q_1}} - \frac{m_{q_1}}{m_{q_1} + m_{q_1}}) \vec{p}_{irm}] \]

\[ \phi_{Arel}(\vec{p}_{q_1q_1}) \phi_{Brel} (\vec{p}_{q_2q_2}) \chi_{SS_z} \phi_{A^{color}} \phi_{B^{color}} \varphi_{AB^{flavor}}. \quad (22) \]
in which \(|\vec{p}_{\text{irm}}| = |\vec{p}_{q_1q_1q_2q_2}|\). So far, we have obtained a new expression of the transition amplitude from Eq. (2).

Making use of the Fourier transform of \(V_{q_1q_2q_2}, V_{q_2q_1q_1},\) and \(V_{q_2q_1q_2},\)

\[
V_{q_1q_2q_2}(\vec{r}_{q_2} - \vec{r}_{q_2}) = \int \frac{d^3k}{(2\pi)^3} V_{q_1q_2q_2}(\vec{k}) e^{i\vec{k} \cdot (\vec{r}_{q_2} - \vec{r}_{q_2})},
\]

\[
V_{q_2q_1q_1}(\vec{r}_{q_1} - \vec{r}_{q_1}) = \int \frac{d^3k}{(2\pi)^3} V_{q_2q_1q_1}(\vec{k}) e^{i\vec{k} \cdot (\vec{r}_{q_1} - \vec{r}_{q_1})},
\]

\[
V_{q_2q_1q_2}(\vec{r}_{q_2} - \vec{r}_{q_2}) = \int \frac{d^3k}{(2\pi)^3} V_{q_2q_1q_2}(\vec{k}) e^{i\vec{k} \cdot (\vec{r}_{q_2} - \vec{r}_{q_2})},
\]

from Eqs. (3)-(5) we obtain

\[
\mathcal{M}_{q_1q_2q_2} = \sqrt{2E_A2E_B2E_H} \sum_{S=S_{\text{min}}}^{S_{\text{max}}} \sum_{S_z=-S}^{S} (S_A S_A S_B S_B | SS_z) \sum_{L_i=0}^{\infty} \sum_{M_i=-L_i}^{L_i} \int d\theta_{\text{irm}} d\phi_{\text{irm}} \sin \theta_{\text{irm}} Y_{L_i M_i}^* (\vec{r}_{q_1 q_2}) \left( (L_i M_i S S_z | J_H J_H) \sum_{M_i=-L_i}^{L_i} \sum_{S_z=-S}^{S} (L_i M_i S S_z | J_H J_H) \phi_{\text{color}}^+ (\vec{p}_{q_1 q_2}) \phi_{\text{flavor}}^+ (\vec{p}_{q_1 q_2}) \chi S S_z \phi_{\text{color}} A \phi_{\text{flavor}} A \right) \int d^3p_{q_1 q_2} (\vec{p}_{q_1 q_2} - \vec{p}_{q_1 q_2}) \phi_{\text{reel}} (\vec{p}_{q_1 q_2}) \chi S S_z \phi_{\text{color}} A \phi_{\text{flavor}} A \phi_{\text{color}} A \phi_{\text{flavor}} A).
\]

\[
\mathcal{M}_{q_2q_1q_1} = \sqrt{2E_A2E_B2E_H} \sum_{S=S_{\text{min}}}^{S_{\text{max}}} \sum_{S_z=-S}^{S} (S_A S_A S_B S_B | SS_z) \sum_{L_i=0}^{\infty} \sum_{M_i=-L_i}^{L_i} \int d\theta_{\text{irm}} d\phi_{\text{irm}} \sin \theta_{\text{irm}} Y_{L_i M_i}^* (\vec{r}_{q_1 q_1}) \left( (L_i M_i S S_z | J_H J_H) \sum_{M_i=-L_i}^{L_i} \sum_{S_z=-S}^{S} (L_i M_i S S_z | J_H J_H) \phi_{\text{color}}^+ (\vec{p}_{q_1 q_2}) \phi_{\text{flavor}}^+ (\vec{p}_{q_1 q_2}) \chi S S_z \phi_{\text{color}} A \phi_{\text{flavor}} A \phi_{\text{color}} A \phi_{\text{flavor}} A \right) \int d^3p_{q_1 q_2} (\vec{p}_{q_1 q_2} - \vec{p}_{q_1 q_2}) \phi_{\text{reel}} (\vec{p}_{q_1 q_2}) \chi S S_z \phi_{\text{color}} A \phi_{\text{flavor}} A \phi_{\text{color}} A \phi_{\text{flavor}} A).
\]

\[
\mathcal{M}_{q_2q_1q_2} = \sqrt{2E_A2E_B2E_H} \sum_{S=S_{\text{min}}}^{S_{\text{max}}} \sum_{S_z=-S}^{S} (S_A S_A S_B S_B | SS_z) \sum_{L_i=0}^{\infty} \sum_{M_i=-L_i}^{L_i} \int d^3p_{q_1 q_2} (\vec{p}_{q_1 q_2} - \vec{p}_{q_1 q_2}) \phi_{\text{reel}} (\vec{p}_{q_1 q_2}) \chi S S_z \phi_{\text{color}} A \phi_{\text{flavor}} A \phi_{\text{color}} A \phi_{\text{flavor}} A.
\]
III. FLAVOR AND SPIN MATRIX ELEMENTS

Let $p_c$ be the four-momentum of constituent $c$. The two upper diagrams give $q_1(p_{q_1}) + \bar{q}_1(p_{\bar{q}_1}) + q_2(p_{q_2}) + \bar{q}_2(p_{\bar{q}_2}) \rightarrow q_1(p'_{q_1}) + q_2(p'_{q_2})$, and the two lower diagrams $q_1(p_{q_1}) + \bar{q}_1(p_{\bar{q}_1}) + q_2(p_{q_2}) + \bar{q}_2(p_{\bar{q}_2}) \rightarrow q_1(p'_{q_1}) + \bar{q}_2(p'_{\bar{q}_2})$. The transition potentials $V_{q_1\bar{q}_1q_2\bar{q}_2}$, $V_{q_1\bar{q}_1q_2\bar{q}_2}$, $V_{q_2\bar{q}_2q_1\bar{q}_1}$, and $V_{q_2\bar{q}_2q_1\bar{q}_1}$ are expressed as

\[
V_{q_1\bar{q}_1q_2\bar{q}_2}(\vec{k}) = -\frac{\tilde{X}(1)}{2} \cdot \frac{\tilde{X}(21) g_s^2}{2 k^2} \left( \frac{\vec{\sigma}(21) \cdot \vec{k}}{2m_{q_1}} - \frac{\vec{\sigma}(1) \cdot \vec{p}_{\bar{q}_1} \vec{\sigma}(1) \cdot \vec{\sigma}(21) + \vec{\sigma}(1) \cdot \vec{\sigma}(21) \vec{\sigma}(1) \cdot \vec{p}_{q_1}}{2m_{\bar{q}_1}} \right),
\]

\[
V_{q_2\bar{q}_2q_1\bar{q}_1}(\vec{k}) = -\frac{\tilde{X}(1)}{2} \cdot \frac{\tilde{X}(12) g_s^2}{2 k^2} \left( \frac{\vec{\sigma}(12) \cdot \vec{k}}{2m_{q_2}} - \frac{\vec{\sigma}(2) \cdot \vec{\sigma}(21) \vec{\sigma}(2) \cdot \vec{p}_{q_2} + \vec{\sigma}(2) \cdot \vec{p}_{q_2} \cdot \vec{\sigma}(2) \cdot \vec{\sigma}(21)}{2m_{q_2}} \right),
\]

\[
Y_{L,M}(\vec{p}_{q_1\bar{q}_1q_2\bar{q}_2})(L_1 M_1 S S_z | J_H J_{H'}) = \sum_{L_1=-L_1}^{L_1} \sum_{S_z=-S_z}^{S_z} (L_1 M_1 S S_z | J_H J_{H'})
\]

\[
\phi_{H_\text{color}} \phi_{H_\text{flavor}} \int d\theta_{\text{irm}} d\phi_{\text{irm}} \sin \theta_{\text{irm}} Y_{L,M}(\hat{p}_{\text{irm}})
\]

\[
\int \frac{d^3 p_{q_1\bar{q}_1}}{(2\pi)^3} \int \frac{d^3 p_{q_2\bar{q}_2}}{(2\pi)^3} \phi_{H_\text{flavor}}(\vec{p}_{q_1\bar{q}_1} + \frac{m_{q_1}}{m_{q_1} + m_{\bar{q}_1}} \vec{p}_{\text{irm}}) V_{q_2\bar{q}_2q_1\bar{q}_2}
\]

\[
\phi_\text{Arel}(\vec{p}_{q_1\bar{q}_1}) \phi_\text{Brel}(\vec{p}_{q_2\bar{q}_2}) \chi S S_z \phi_{\text{Acolor}} \phi_{\text{Bcolor}} \phi_{\text{Aflavor}}.
\]

With these transition amplitudes the unpolarized cross section for $A + B \rightarrow H$ is

\[
\sigma_{\text{unpol}} = \frac{\pi \delta(E_i - E_f)}{4 \sqrt{(P_A \cdot P_B)^2 - m_A^2 m_B^2 E_H}} \frac{1}{(2J_A + 1)(2J_B + 1)} \sum |\mathcal{M}_{q_1\bar{q}_1q_2\bar{q}_2} + \mathcal{M}_{q_1\bar{q}_1q_2\bar{q}_2} + \mathcal{M}_{q_2\bar{q}_2q_1\bar{q}_1} + \mathcal{M}_{q_2\bar{q}_2q_1\bar{q}_1}|^2,
\]

where $P_A$, $m_A$, and $J_A$ ($P_B$, $m_B$, and $J_B$) are the four-momentum, the mass, and the total angular momentum of meson $A$ ($B$), respectively. We calculate the cross section in the center-of-mass frame of the two initial mesons, i.e., with meson $H$ at rest.
and initial quark \( q \) elements between the color (spin) wave functions of initial antiquark \( \bar{q}_2 \) and initial quark \( q_1 \). They have matrix elements between the color (spin) wave functions of initial antiquark \( \bar{q}_2 \) and initial quark \( q_1 \). In Eqs. (32) and (33), \( \bar{\sigma}(12) \) (\( \bar{\sigma}(12) \)) mean that they have matrix elements between the color (spin) wave functions of the initial antiquark \( \bar{q}_1 \) and initial quark \( q_2 \). In Eqs. (30) and (33), \( \bar{\lambda}(1) \) (\( \bar{\lambda}(2) \)) mean that they have matrix elements between the color (spin) wave functions of the initial antiquark \( \bar{q}_1 \) and the final quark. In Eqs. (31) and (32), \( \bar{\lambda}(1) \) and \( \bar{\lambda}(2) \) (\( \bar{\sigma}(1) \) and \( \bar{\sigma}(2) \)) mean that they have matrix elements between the color (spin) wave functions of the initial antiquark and the final antiquark. In Eqs. (31) and (32), \( \bar{\lambda}(2) \) and \( \bar{\lambda}(1) \) (\( \bar{\sigma}(2) \) and \( \bar{\sigma}(1) \)) mean that they have matrix elements between the color (spin) wave functions of the final quark and the initial quark.

We use the notation

\[
K = \left( \begin{array}{c} K^+ \\ K^0 \\ K^- \end{array} \right), \quad \bar{K} = \left( \begin{array}{c} \bar{K}^0 \\ K^- \end{array} \right), \quad K^* = \left( \begin{array}{c} K^{*+} \\ K^{*0} \end{array} \right), \quad \bar{K}^* = \left( \begin{array}{c} \bar{K}^{*0} \\ K^{*-} \end{array} \right), \quad D = \left( \begin{array}{c} D^+ \\ D^0 \\ D^- \end{array} \right), \quad \bar{D} = \left( \begin{array}{c} \bar{D}^0 \\ D^- \end{array} \right), \quad D^* = \left( \begin{array}{c} D^{*+} \\ D^{*0} \end{array} \right), \quad \text{and} \quad \bar{D}^* = \left( \begin{array}{c} \bar{D}^{*0} \\ D^{*-} \end{array} \right).
\]

Based on the formulas in Sect. II, we study the following reactions:

\[
K\bar{K} \rightarrow \phi, \quad \pi D \rightarrow D^*, \quad \pi\bar{D} \rightarrow \bar{D}^*,
\]

\[
D\bar{D} \rightarrow \psi(4160), \quad D^*\bar{D} \rightarrow \psi(4160), \quad D\bar{D}^* \rightarrow \psi(4160), \quad D^*\bar{D}^* \rightarrow \psi(4160),
\]

\[
D_s^+D_s^- \rightarrow \psi(4160), \quad D_s^{*+}D_s^- \rightarrow \psi(4160), \quad D_s^+D_s^{*-} \rightarrow \psi(4160), \quad D_s^{*+}D_s^{*-} \rightarrow \psi(4160),
\]

\[
D\bar{D} \rightarrow \psi(4415), \quad D^*\bar{D} \rightarrow \psi(4415), \quad D\bar{D}^* \rightarrow \psi(4415), \quad D^*\bar{D}^* \rightarrow \psi(4415),
\]

\[
D_s^+D_s^- \rightarrow \psi(4415), \quad D_s^{*+}D_s^- \rightarrow \psi(4415), \quad D_s^+D_s^{*-} \rightarrow \psi(4415), \quad D_s^{*+}D_s^{*-} \rightarrow \psi(4415).
\]

From the Gell-Mann matrices and the Pauli matrices in the transition potentials, the expressions of the transition amplitudes in Eqs. (22) and (26)-(28) involve color matrix
elements, flavor matrix elements, and spin matrix elements for the above reactions. The color matrix elements in $\mathcal{M}_{r_1q_1q_2}$, $\mathcal{M}_{r_1q_1q_2}$, $\mathcal{M}_{r_2q_1q_1}$, and $\mathcal{M}_{r_2q_1q_1}$ are $-\frac{4}{3\sqrt{3}}$, $\frac{4}{3\sqrt{3}}$, and $\frac{4}{3\sqrt{3}}$, respectively. While we calculate the flavor matrix elements, we keep the total isospin of the two initial mesons the same as the isospin of the final meson. The flavor matrix element $\mathcal{M}_{fK\to \phi} (\mathcal{M}_{fD\to D^*}, \mathcal{M}_{fD\to \psi(4160)}, \mathcal{M}_{fD^*\to D^-\to \psi(4160)})$ for $K\bar{K} \to \phi (\pi D \to D^*, D\bar{D} \to \psi(4160), D_s^+ D_s^- \to \psi(4160))$ is shown in Table 1. The flavor matrix element for $\pi \bar{D} \to \bar{D}^*$ equals $\mathcal{M}_{fD\to D^*}$. The flavor matrix elements for $D^* \bar{D} \to \psi(4160), D\bar{D} \to \psi(4160), D_s^* D_s \to \psi(4160), D^* \bar{D} \to \psi(4165), D\bar{D} \to \psi(4415),$ and $D^* \bar{D}^* \to \psi(4415)$ are the same as $\mathcal{M}_{fDD\to \psi(4160)}$. The flavor matrix elements for $D_s^+ D_s^- \to \psi(4160), D_s^+ D_s^- \to \psi(4160), D_s^+ D_s^- \to \psi(4165), D_s^+ D_s^- \to \psi(4415)$, and $D_s^+ D_s^- \to \psi(4415)$ equal $\mathcal{M}_{fD_s^+ D_s^- \to \psi(4160)}$. The flavor matrix elements for $K\bar{K} \to \phi, \pi D \to D^*$, and $\pi \bar{D} \to \bar{D}^*$ are zero for the two lower diagrams, and the ones for the production of $\psi(4160)$ and $\psi(4415)$ are zero for the two upper diagrams. Hence, every reaction receives contributions only from two Feynman diagrams.

Now we give the spin matrix elements. Let $P_{r_1q_1q_2}$ with $i = 0, \cdots, 15$ stand for $1, \sigma_1(21), \sigma_2(21), \sigma_3(21), \sigma_4(1), \sigma_2(1), \sigma_3(1), \sigma_1(21)\sigma_1(1), \sigma_1(21)\sigma_2(1), \sigma_1(21)\sigma_3(1), \sigma_1(21)\sigma_4(1), \sigma_2(21)\sigma_2(1), \sigma_2(21)\sigma_3(1), \sigma_3(21)\sigma_1(1), \sigma_3(21)\sigma_2(1), \sigma_3(21)\sigma_3(1)$, and $\sigma_3(21)\sigma_4(1)$, respectively. Let $P_{r_1q_2q_2}$ with $i = 0, \cdots, 15$ correspond to $1, \sigma_1(21), \sigma_2(21), \sigma_3(21), \sigma_4(1), \sigma_2(1), \sigma_3(1), \sigma_1(21)\sigma_1(1), \sigma_1(21)\sigma_2(1), \sigma_1(21)\sigma_3(1), \sigma_1(21)\sigma_4(1), \sigma_2(21)\sigma_2(1), \sigma_3(21)\sigma_1(1), \sigma_3(21)\sigma_2(1), \sigma_3(21)\sigma_3(1), \sigma_3(21)\sigma_4(1)$, and $\sigma_3(21)\sigma_4(1)$, respectively. Let $P_{r_2q_1q_2}$ with $i = 0, \cdots, 15$ represent $1, \sigma_1(12), \sigma_2(12), \sigma_3(12), \sigma_4(1), \sigma_2(1), \sigma_3(1), \sigma_1(12)\sigma_1(1), \sigma_1(12)\sigma_2(1), \sigma_1(12)\sigma_3(1), \sigma_1(12)\sigma_4(1), \sigma_2(12)\sigma_2(1), \sigma_3(12)\sigma_1(1), \sigma_3(12)\sigma_2(1), \sigma_3(12)\sigma_3(1), \sigma_3(12)\sigma_4(1), \sigma_2(12)\sigma_2(1), \sigma_3(12)\sigma_1(1), \sigma_3(12)\sigma_2(1), \sigma_3(12)\sigma_3(1), \sigma_3(12)\sigma_4(1), \sigma_2(12)\sigma_2(1), \sigma_3(12)\sigma_1(1), \sigma_3(12)\sigma_2(1), \sigma_3(12)\sigma_3(1), \sigma_3(12)\sigma_4(1)$, and $\sigma_3(12)\sigma_4(1)$, respectively. Let $P_{r_2q_1q_2}$ with $i = 0, \cdots, 15$ denote $1, \sigma_1(12), \sigma_2(12), \sigma_3(12), \sigma_4(1), \sigma_2(1), \sigma_3(1), \sigma_1(12)\sigma_1(1), \sigma_1(12)\sigma_2(1), \sigma_1(12)\sigma_3(1), \sigma_1(12)\sigma_4(1), \sigma_2(12)\sigma_2(1), \sigma_3(12)\sigma_1(1), \sigma_3(12)\sigma_2(1), \sigma_3(12)\sigma_3(1), \sigma_3(12)\sigma_4(1), \sigma_2(12)\sigma_2(1), \sigma_3(12)\sigma_1(1), \sigma_3(12)\sigma_2(1), \sigma_3(12)\sigma_3(1), \sigma_3(12)\sigma_4(1), \sigma_2(12)\sigma_2(1), \sigma_3(12)\sigma_1(1), \sigma_3(12)\sigma_2(1), \sigma_3(12)\sigma_3(1), \sigma_3(12)\sigma_4(1)$, and $\sigma_3(12)\sigma_4(1)$, respectively. Set $n_A$ as $-1, 0$, or $1$, and $n_B$ as $-1$, $0$, or $1$. In order to easily tabulate values of the spin matrix elements, we define $\phi_{fss}(S_A, S_{A\bar{A}}; S_B, S_{B\bar{B}}) \equiv \chi_{S_A S_{A\bar{A}}} \chi_{S_B S_{B\bar{B}}}$ and $\phi_{fss}(S_H, S_{H\bar{H}}) \equiv \chi_{S_H S_{H\bar{H}}}$. The spin matrix elements $\phi_{fss}^+(S_A, S_{A\bar{A}}; S_B, S_{B\bar{B}}) = P_{r_1q_1q_2} \phi_{fss}(S_A, S_{A\bar{A}}; S_B, S_{B\bar{B}})$ are shown in Tables 2-6. Other spin matrix elements, $\phi_{fss}^+(S_H, S_{H\bar{H}}; P_{r_1q_1q_1}) = P_{r_1q_1q_1} \phi_{fss}(S_A, S_{A\bar{A}}; S_B, S_{B\bar{B}})$, $\phi_{fss}^+(S_H, S_{H\bar{H}}; P_{r_1q_1q_1})$,
\( \phi_{\text{iss}}(S_A, S_A, S_B, S_B) \) and \( \phi^+_{\text{iss}}(S_H, S_H) \) \( P_{r_{q_2 q_1 i}} \phi_{\text{iss}}(S_A, S_A, S_B, S_B) \), are related to \( \phi^+_{\text{iss}}(S_H, S_H) P_{r_{q_2 q_1 i}} \phi_{\text{iss}}(S_A, S_A, S_B, S_B) \) by the following equations:

\[
\phi^+_{\text{iss}}(S_H = 1, S_H) P_{r_{q_1 q_2 q_1 i}} \phi_{\text{iss}}(S_A = 1, S_A = n_A; S_B = 0, S_B = 0) \\
= \phi^+_{\text{iss}}(S_H = 1, S_H) P_{r_{q_1 q_2 q_1 i}} \phi_{\text{iss}}(S_A = 0, S_A = 0; S_B = 1, S_B = n_A), \tag{34}
\]

with \( i = 2, 5, 8, 10, 12, \) and 14;

\[
\phi^+_{\text{iss}}(S_H = 1, S_H) P_{r_{q_1 q_2 q_1 i}} \phi_{\text{iss}}(S_A = 1, S_A = n_A; S_B = 0, S_B = 0) \\
= -\phi^+_{\text{iss}}(S_H = 1, S_H) P_{r_{q_1 q_2 q_1 i}} \phi_{\text{iss}}(S_A = 0, S_A = 0; S_B = 1, S_B = n_A), \tag{35}
\]

with \( i = 0, 1, 3, 4, 6, 7, 9, 11, 13, \) and 15;

\[
\phi^+_{\text{iss}}(S_H = 1, S_H) P_{r_{q_1 q_2 q_1 i}} \phi_{\text{iss}}(S_A = 0, S_A = 0; S_B = 1, S_B = n_B) \\
= \phi^+_{\text{iss}}(S_H = 1, S_H) P_{r_{q_1 q_2 q_1 i}} \phi_{\text{iss}}(S_A = 1, S_A = n_B; S_B = 0, S_B = 0), \tag{36}
\]

with \( i = 2, 5, 8, 10, 12, \) and 14;

\[
\phi^+_{\text{iss}}(S_H = 1, S_H) P_{r_{q_1 q_2 q_1 i}} \phi_{\text{iss}}(S_A = 1, S_A = 0; S_B = 1, S_B = n_B) \\
= -\phi^+_{\text{iss}}(S_H = 1, S_H) P_{r_{q_1 q_2 q_1 i}} \phi_{\text{iss}}(S_A = 1, S_A = n_B; S_B = 0, S_B = 0), \tag{37}
\]

with \( i = 0, 1, 3, 4, 6, 7, 9, 11, 13, \) and 15;

\[
\phi^+_{\text{iss}}(S_H = 1, S_H) P_{r_{q_1 q_2 q_1 i}} \phi_{\text{iss}}(S_A = 1, S_A = n_A; S_B = 1, S_B = n_B) \\
= \phi^+_{\text{iss}}(S_H = 1, S_H) P_{r_{q_1 q_2 q_1 i}} \phi_{\text{iss}}(S_A = 1, S_A = n_B; S_B = 1, S_B = n_A), \tag{38}
\]

with \( i = 0, 1, 3, 4, 6, 7, 9, 11, 13, \) and 15;

\[
\phi^+_{\text{iss}}(S_H = 1, S_H) P_{r_{q_1 q_2 q_1 i}} \phi_{\text{iss}}(S_A = 1, S_A = n_A; S_B = 1, S_B = n_B) \\
= -\phi^+_{\text{iss}}(S_H = 1, S_H) P_{r_{q_1 q_2 q_1 i}} \phi_{\text{iss}}(S_A = 1, S_A = n_B; S_B = 1, S_B = n_A), \tag{39}
\]

with \( i = 2, 5, 8, 10, 12, \) and 14;

\[
\phi^+_{\text{iss}}(S_H = 1, S_H) P_{r_{q_1 q_2 q_1 i}} \phi_{\text{iss}}(S_A = 1, S_A = n_A; S_B = 0, S_B = 0) \\
= \phi^+_{\text{iss}}(S_H = 1, S_H) P_{r_{q_1 q_2 q_1 i}} \phi_{\text{iss}}(S_A = 1, S_A = n_A; S_B = 0, S_B = 0), \tag{40}
\]
with $i = 2, 5, 8, 10, 12,$ and $14$;

$$\phi_{\text{iss}}^+(S_H = 1, S_{Hz}) P_{rq_2 q_1 i} \phi_{\text{iss}}(S_A = 1, S_{Az} = n_A; S_B = 0, S_{Bz} = 0)$$

$$= -\phi_{\text{iss}}^+(S_H = 1, S_{Hz}) P_{rq_2 q_1 i} \phi_{\text{iss}}(S_A = 1, S_{Az} = n_A; S_B = 0, S_{Bz} = 0), \quad (41)$$

with $i = 0, 1, 3, 4, 6, 7, 9, 11, 13,$ and $15$;

$$\phi_{\text{iss}}^+(S_H = 1, S_{Hz}) P_{rq_2 q_1 i} \phi_{\text{iss}}(S_A = 0, S_{Az} = 0; S_B = 1, S_{Bz} = n_B)$$

$$= \phi_{\text{iss}}^+(S_H = 1, S_{Hz}) P_{rq_2 q_1 i} \phi_{\text{iss}}(S_A = 0, S_{Az} = 0; S_B = 1, S_{Bz} = n_B), \quad (42)$$

with $i = 2, 5, 8, 10, 12,$ and $14$;

$$\phi_{\text{iss}}^+(S_H = 1, S_{Hz}) P_{rq_2 q_1 i} \phi_{\text{iss}}(S_A = 0, S_{Az} = 0; S_B = 1, S_{Bz} = n_B)$$

$$= -\phi_{\text{iss}}^+(S_H = 1, S_{Hz}) P_{rq_2 q_1 i} \phi_{\text{iss}}(S_A = 0, S_{Az} = 0; S_B = 1, S_{Bz} = n_B), \quad (43)$$

with $i = 0, 1, 3, 4, 6, 7, 9, 11, 13,$ and $15$;

$$\phi_{\text{iss}}^+(S_H = 1, S_{Hz}) P_{rq_2 q_1 i} \phi_{\text{iss}}(S_A = 0, S_{Az} = 0; S_B = 1, S_{Bz} = n_B)$$

$$= \phi_{\text{iss}}^+(S_H = 1, S_{Hz}) P_{rq_2 q_1 i} \phi_{\text{iss}}(S_A = 0, S_{Az} = 0; S_B = 1, S_{Bz} = n_B), \quad (44)$$

with $i = 0, 1, 3, 4, 6, 7, 9, 11, 13,$ and $15$;

$$\phi_{\text{iss}}^+(S_H = 1, S_{Hz}) P_{rq_2 q_1 i} \phi_{\text{iss}}(S_A = 0, S_{Az} = 0; S_B = 1, S_{Bz} = n_B)$$

$$= -\phi_{\text{iss}}^+(S_H = 1, S_{Hz}) P_{rq_2 q_1 i} \phi_{\text{iss}}(S_A = 0, S_{Az} = 0; S_B = 1, S_{Bz} = n_B), \quad (45)$$

with $i = 0, 2, 5, 8, 10, 12,$ and $14$;

$$\phi_{\text{iss}}^+(S_H = 1, S_{Hz}) P_{rq_2 q_1 i} \phi_{\text{iss}}(S_A = 0, S_{Az} = 0; S_B = 0, S_{Bz} = 0)$$

$$= \phi_{\text{iss}}^+(S_H = 1, S_{Hz}) P_{rq_2 q_1 i} \phi_{\text{iss}}(S_A = 0, S_{Az} = 0; S_B = 1, S_{Bz} = n_A), \quad (46)$$

with $i = 0, \ldots, 15$;

$$\phi_{\text{iss}}^+(S_H = 1, S_{Hz}) P_{rq_2 q_1 i} \phi_{\text{iss}}(S_A = 0, S_{Az} = 0; S_B = 1, S_{Bz} = n_B)$$

$$= \phi_{\text{iss}}^+(S_H = 1, S_{Hz}) P_{rq_2 q_1 i} \phi_{\text{iss}}(S_A = 0, S_{Az} = 0; S_B = 1, S_{Bz} = n_B), \quad (47)$$
with \( i = 0, \cdots, \text{and} \ 15; \)

\[
\phi^+_{\text{iss}}(S_H = 1, S_{Hz}) P_{q_2 \bar{q}_1 \bar{q}_2} \phi_{\text{iss}}(S_A = 1, S_{Az} = n_A; S_B = 1, S_{Bz} = n_B) \\
= \phi^+_{\text{iss}}(S_H = 1, S_{Hz}) P_{q_1 \bar{q}_1 \bar{q}_1} \phi_{\text{iss}}(S_A = 1, S_{Az} = n_B; S_B = 1, S_{Bz} = n_A),
\]

(48)

with \( i = 0, \cdots, \text{and} \ 15. \)

**IV. NUMERICAL CROSS SECTIONS AND DISCUSSIONS**

The mesonic quark-antiquark relative-motion wave functions \( \phi_{A_{\text{rel}}}, \phi_{B_{\text{rel}}}, \) and \( \phi_{H_{\text{rel}}} \) in Eqs. (6) and (7) are solutions of the Schrödinger equation with the potential between constituents \( a \) and \( b \) in coordinate space \[25\],

\[
V_{ab}(\vec{r}_{ab}) = -\frac{\tilde{\lambda}_a}{2} \frac{\tilde{\lambda}_b}{2} \left[ \frac{3}{4} + 1.3 - \left( \frac{T}{T_c} \right)^4 \right] \tanh(\lambda \vec{r}_{ab}) + \frac{\tilde{\lambda}_a}{2} \frac{6\pi v(\lambda \vec{r}_{ab})}{25} \exp(-E \vec{r}_{ab})
\]

\[
-\frac{\tilde{\lambda}_a}{2} \frac{\tilde{\lambda}_b}{2} \frac{16\pi^2}{25} \frac{d^3}{\pi^{3/2}} \exp(-d^2 \vec{r}_{ab}^2) \frac{\vec{s}_a \cdot \vec{s}_b}{m_a m_b} + \frac{\tilde{\lambda}_a}{2} \frac{4\pi}{25} \frac{1}{r_{ab}} \frac{d^2 \nu(\lambda r_{ab})}{dr_{ab}^2} \frac{\vec{s}_a \cdot \vec{s}_b}{m_a m_b},
\]

(49)

where \( D = 0.7 \) GeV, \( E = 0.6 \) GeV, \( T_c = 0.175 \) GeV, \( A = 1.5[0.75 + 0.25(T/T_c)^{10}]^6 \) GeV, and \( \lambda = \sqrt{25/16\pi^2\alpha'} \) with \( \alpha' = 1.04 \) GeV\(^{-2} \); \( T \) is the temperature; \( \vec{s}_a \) is the spin of constituent \( a \); the quantity \( d \) is given in Ref. \[25\]; the function \( v \) is given by Buchmüller and Tye in Ref. \[20\]. The potential is obtained from perturbative QCD \[20\] and lattice QCD \[26\]. The masses of the up quark, the down quark, the strange quark, and the charm quark are 0.32 GeV, 0.32 GeV, 0.5 GeV, and 1.51 GeV, respectively. Solving the Schrödinger equation with the potential at zero temperature, we obtain meson masses that are close to the experimental masses of \( \pi, \rho, K, K^*, J/\psi, \chi_c, \psi(4160), \psi(4415), D, D^*, D_s, \) and \( D_s^* \) mesons \[27\]. Moreover, the experimental data of \( S \)-wave and \( P \)-wave elastic phase shifts for \( \pi \pi \) scattering in vacuum \[28, 29\] are reproduced in the Born approximation \[5, 25\].

From the transition potentials, the color matrix elements, the flavor matrix elements, the spin matrix elements, and the mesonic quark-antiquark relative-motion wave functions, we calculate the transition amplitudes. As seen in Eq. (8), the development in spherical harmonics contains the summation over the orbital-angular-momentum quantum number \( L_i \) that labels the relative motion between mesons \( A \) and \( B \). However,
not all orbital-angular-momentum quantum numbers are allowed. The orbital-angular-momentum quantum numbers are selected to satisfy parity conservation and $J = J_H$, i.e., the total angular momentum of the two initial mesons equals the total angular momentum of meson $H$. The choice of $L_i$ thus depends on the total spin $S$ of the two initial mesons. From the transition amplitudes we get unpolarized cross sections at zero temperature. The selected orbital-angular-momentum quantum numbers and the cross sections are shown in Table 7. $D^*\bar{D}^* \to \psi(4160)$, $D^*\bar{D}^* \to \psi(4415)$, and $D_s^+D_s^- \to \psi(4415)$ allow $S = 0$, $S = 1$, and $S = 2$. Including contributions from $S = 0$, $S = 1$, and $S = 2$, the cross sections for the three reactions are 19.83 mb, 0.14 mb, and 1.8 mb, respectively.

In Table 7 the cross section for $K\bar{K} \to \phi$ equals 5.96 mb. The magnitude 5.96 mb is slightly larger than the peak cross section of $K\bar{K} \to K^*\bar{K}^*$ for total isospin $I = 0$ at zero temperature, and is roughly 5 times the peak cross section of $K\bar{K} \to K^*\bar{K}^*$ for $I = 1$ in Ref. [5]. The case $K\bar{K} \to K^*\bar{K}^*$ may be caused by a process where a quark in an initial meson and an antiquark in another initial meson annihilate into a gluon and subsequently the gluon creates another quark-antiquark pair. The magnitude is much larger than the peak cross sections of $K\bar{K} \to \pi K\bar{K}$ for $I=1$ and $I_{\pi\bar{K}} = 3/2$ and for $I=1$ and $I_{\pi\bar{K}} = 1/2$ at zero temperature in Ref. [30], where $I_{\pi\bar{K}}$ is the total isospin of the final $\pi$ and $\bar{K}$ mesons. The case $K\bar{K} \to \pi K\bar{K}$ is governed by a process where a gluon is emitted by a constituent quark or antiquark in the initial mesons and subsequently the gluon creates a quark-antiquark pair. The magnitude is also much larger than the peak cross section of $KK \to K^*K^*$ for $I = 1$ at zero temperature in Ref. [31]. The case $KK \to K^*K^*$ for $I = 1$ can be caused by quark interchange between the two colliding mesons. The cross section for $\pi D \to D^*$ is particularly large. This means that the reaction easily happens. The large cross section is caused by the very small difference between the $D^*$ mass and the sum of the $\pi$ and $D$ masses.

The transition potentials involve quark masses. The charm-quark mass is larger than the strange-quark mass, and the transition potentials with the charm quark are smaller than the ones with the strange quark. The cross section for $D\bar{D} \to \psi(4160)$ is thus smaller than the one for $K\bar{K} \to \phi$. Because the $D^*$ radius is larger than the $D$ radius,
the cross section for \( D^* \bar{D}^* \rightarrow \psi(4160) \) is larger than the one for \( D^* \bar{D} \rightarrow \psi(4160) \), and the cross section for \( D^* \bar{D} \rightarrow \psi(4160) \) is larger than the one for \( D \bar{D} \rightarrow \psi(4160) \). Since \( D_s^+ \) (the antiparticle of \( D_s^- \)) consists of a charm quark and a strange antiquark, the cross section for \( D_s^+ \bar{D}_s^- \rightarrow \psi(4160) \) is smaller than the one for \( D \bar{D} \rightarrow \psi(4160) \). Since the \( D_s^{\pm} \) radii are larger than the \( D_s^{\pm} \) radii, the cross section for \( D_s^{\pm} \bar{D}_s^{\mp} \rightarrow \psi(4160) \) is larger than the one for \( D_s^{\mp} \bar{D}_s^{\pm} \rightarrow \psi(4160) \). As seen in Table 7, we have the increasing order of the cross sections for \( D_s^{\pm} \bar{D}_s^{\mp} \rightarrow \psi(4415) \), \( D_s^{\pm} \bar{D}_s^{\pm} \rightarrow \psi(4415) \), and \( D_s^{\mp} \bar{D}_s^{\pm} \rightarrow \psi(4415) \). The radial part of the quark-antiquark relative-motion wave function of \( \psi(4415) \) has three nodes. The radial wave function on the left of a node has a sign different from the one on the right of the node. The nodes lead to cancellation between the positive radial wave function and the negative radial wave function in the integration involved in the transition amplitudes. Consequently, the cross section for \( D^* \bar{D}^* \rightarrow \psi(4415) \) is much smaller than that for \( D^* \bar{D} \rightarrow \psi(4415) \).

The cross sections for \( A + B \rightarrow \psi(4160) \) and for \( A + B \rightarrow \psi(4415) \) are compared in Table 7. Since the \( \psi(4160) \) mass is smaller than the \( \psi(4415) \) mass, the production of \( \psi(4160) \) is easier than the production of \( \psi(4415) \). The cross section for the former [for example, \( D \bar{D} \rightarrow \psi(4160) \)] is larger than the cross section for the latter [for example, \( D \bar{D} \rightarrow \psi(4415) \)].

In the present work the \( \psi(4160) \) and \( \psi(4415) \) mesons come from fusion of \( D, \bar{D}, D^*, \bar{D}^*, D_s, \) and \( D_s^* \) mesons. The reason why we are interested in the reactions is that \( \psi(4160) \) and \( \psi(4415) \) may decay into \( J/\psi \) which is an important probe of the quark-gluon plasma produced in ultrarelativistic heavy-ion collisions. We do not investigate the \( \chi_{c0}(2P) \) and \( \chi_{c2}(2P) \) mesons because they cannot decay into the \( J/\psi \) meson. The \( \chi_{c1}(2P) \) meson may decay into the \( J/\psi \) meson, but \( D \bar{D} \rightarrow \chi_{c1}(2P) \) allowed by energy conservation does not simultaneously satisfy the parity conservation and the conservation of the total angular momentum. Therefore, we do not consider \( D \bar{D} \rightarrow \chi_{c1}(2P) \). The production of \( \chi_{c1}(2P) \) from fusion of other charmed mesons is forbidden by energy conservation. \( \pi D_s^{\pm} \rightarrow D_s^{*\pm} \) is not allowed because of violation of isospin conservation, and \( K D \rightarrow D_s^{*+} \) and \( \bar{K} \bar{D} \rightarrow D_s^{*-} \) because of violation of energy conservation.
As seen in Eq. (49), the potential between two constituents depends on temperature. The meson mass obtained from the Schrödinger equation with the potential thus depends on temperature. The temperature dependence of meson masses is shown in Figs. 2-4. In vacuum the \( \phi \) mass is larger than two times the kaon mass, and so \( K\bar{K} \rightarrow \phi \) takes place. Since the \( \phi \) mass in Fig. 2 decreases faster than the kaon mass with increasing temperature, the \( \phi \) mass turns smaller than two times the kaon mass. The reaction thus does not occur in the temperature region \( 0.6T_c \leq T < T_c \). In Fig. 3 the \( D^* \) mass decreases faster than the pion and \( D \) masses, and the \( D^* \) mass is smaller than the sum of the pion mass and the \( D \) mass. \( \pi D \rightarrow D^* \) and \( \pi \bar{D} \rightarrow \bar{D}^* \) also do not occur for \( 0.6T_c \leq T < T_c \). It is shown in Refs. [20,21] that \( \psi(4160) \) and \( \psi(4415) \) can be individually interpreted as the \( 3^3S_1 \) and \( 4^3S_1 \) quark-antiquark states, but they are dissolved in hadronic matter when the temperature is larger than \( 0.97T_c \) and \( 0.87T_c \), respectively [32]. Their masses are thus plotted only for \( 0.6T_c \leq T < 0.97T_c \) and for \( 0.6T_c \leq T < 0.87T_c \) in Fig. 4, and are smaller than the sum of the masses of the two initial mesons that yield them. Therefore, in hadronic matter where the temperature is constrained by \( 0.6T_c \leq T < T_c \), we cannot see the production of \( \psi(4160) \) and \( \psi(4415) \) from the fusion of two charmed mesons and of two charmed strange mesons. \( T_c \) is the critical temperature at which the phase transition between the quark-gluon plasma and hadronic matter takes place. Since \( \psi(4160) \) and \( \psi(4415) \) are dissolved in hadronic matter when the temperature is larger than \( 0.97T_c \) and \( 0.87T_c \), respectively, they cannot be produced in the phase transition, but they can be produced in the following reactions in hadronic matter:

\[
D\bar{D} \rightarrow \rho\psi(4160), \rho\psi(4415);
\]

\[
D\bar{D}^* \rightarrow \pi\psi(4160), \rho\psi(4160), \eta\psi(4160), \pi\psi(4415), \rho\psi(4415), \eta\psi(4415);
\]

\[
D^*\bar{D} \rightarrow \pi\psi(4160), \rho\psi(4160), \eta\psi(4160), \pi\psi(4415), \rho\psi(4415), \eta\psi(4415);
\]

\[
D^*\bar{D}^* \rightarrow \pi\psi(4160), \rho\psi(4160), \eta\psi(4160), \pi\psi(4415), \rho\psi(4415), \eta\psi(4415);
\]

\[
D_{s}^{+}\bar{D} \rightarrow K^*\psi(4160), K^*\psi(4415);
\]

\[
D_{s}^{-}D \rightarrow \bar{K}^*\psi(4160), \bar{K}^*\psi(4415);
\]

17
\[ D_s^+ \bar{D}^* \rightarrow K\psi(4160), K^*\psi(4160), K\psi(4415), K^*\psi(4415); \]
\[ D_s^- D^* \rightarrow \bar{K}\psi(4160), \bar{K}^*\psi(4160), \bar{K}\psi(4415), \bar{K}^*\psi(4415); \]
\[ D_s^{*-} \bar{D} \rightarrow \bar{K}\psi(4160), \bar{K}^*\psi(4160), \bar{K}\psi(4415), \bar{K}^*\psi(4415); \]
\[ D_s^{*+} \bar{D} \rightarrow K\psi(4160), K^*\psi(4160), K\psi(4415), K^*\psi(4415); \]
\[ D_s^{*0} D \rightarrow \bar{K}\psi(4160), \bar{K}^*\psi(4160), \bar{K}\psi(4415), \bar{K}^*\psi(4415); \]
\[ D_s^{*+} \bar{D}^* \rightarrow K\psi(4160), K^*\psi(4160), K\psi(4415), K^*\psi(4415); \]
\[ D_s^{*-} D^* \rightarrow \bar{K}\psi(4160), \bar{K}^*\psi(4160), \bar{K}\psi(4415), \bar{K}^*\psi(4415); \]
\[ D_s^+ D_s^- \rightarrow \phi\psi(4160), \phi\psi(4415); \]
\[ D_s^+ D_s^{*-} \rightarrow \eta\psi(4160), \phi\psi(4160), \eta\psi(4415), \phi\psi(4415); \]
\[ D_s^{*-} D_s^- \rightarrow \eta\psi(4160), \phi\psi(4160), \eta\psi(4415), \phi\psi(4415); \]
\[ D_s^{*+} D_s^{*-} \rightarrow \eta\psi(4160), \phi\psi(4160), \eta\psi(4415), \phi\psi(4415). \]

Therefore, \( \psi(4160) \) and \( \psi(4415) \) may provide us with information on hadronic matter, and are a probe of hadronic matter that results from the quark-gluon plasma.

V. SUMMARY

With the process where one quark annihilates with one antiquark to create a gluon and subsequently the gluon is absorbed by a spectator quark or antiquark, we have studied 2-to-1 meson-meson scattering. In the partial wave expansion of the relative-motion wave function of the two initial mesons, we have obtained the new expressions of the transition amplitudes. The orbital-angular-momentum quantum number corresponding to the total spin of the two initial mesons is selected to satisfy parity conservation and conservation of the total angular momentum. The flavor and spin matrix elements have been calculated. The spin matrix elements corresponding to the four transition potentials are presented. The mesonic quark-antiquark relative-motion wave functions are given by the Schrödinger equation with the temperature-dependent potential. From the transition amplitudes we have obtained the cross sections for the reactions: \( K\bar{K} \rightarrow \phi, \pi D \rightarrow D^*, \pi \bar{D} \rightarrow \bar{D}^*, D\bar{D} \rightarrow \psi(4160), D^*\bar{D} \rightarrow \psi(4160), D\bar{D}^* \rightarrow \psi(4160), D^*\bar{D}^* \rightarrow \psi(4160), \)
\[ D_s^+ D_s^- \rightarrow \psi(4160), \quad D_s^+ D_s^- \rightarrow \psi(4160), \quad D_s^+ D_s^- \rightarrow \psi(4160), \quad D \bar{D} \rightarrow \psi(4415), \quad D^* \bar{D} \rightarrow \psi(4415), \quad D \bar{D}^* \rightarrow \psi(4415), \quad D^* \bar{D}^* \rightarrow \psi(4415), \quad D^* \bar{D}^* \rightarrow \psi(4415), \quad D^* \bar{D}^* \rightarrow \psi(4415), \quad D^* \bar{D}^* \rightarrow \psi(4415). \]

The cross sections are affected by radii of initial mesons and quark masses that enter the transition potentials. The cross section for \( K \bar{K} \rightarrow \phi \) is larger than the one for \( D \bar{D} \rightarrow \psi(4160) \), but smaller than the one for \( \pi D \rightarrow D^* \). Excluding \( D^* \bar{D}^* \rightarrow \psi(4415) \), the cross section for the production of \( \psi(4160) \) or \( \psi(4415) \) increases with the initial mesons going through the cases of \( D \bar{D} \ (D_s^+ D_s^-) \), \( D^* \bar{D} \ (D_s^+ D_s^-) \), and \( D^* \bar{D}^* \ (D_s^+ D_s^-) \). The \( \psi(4160) \) and \( \psi(4415) \) mesons resulting from ultrarelativistic heavy-ion collisions have been shown to be a probe of hadronic matter that is produced in the phase transition of the quark-gluon plasma.

ACKNOWLEDGEMENTS

This work was supported by the National Natural Science Foundation of China under Grant No. 11175111.

References

[1] J. Gasser and H. Leutwyler, Ann. Phys. (N.Y.) 158, 142 (1984); J. Gasser and H. Leutwyler, Nucl. Phys. B 250, 465 (1985); J. Gasser and U. G. Meissner, Phys. Lett. B 258, 219 (1991); J. Gasser and U. G. Meissner, Nucl. Phys. B 357, 90 (1991); I. Bijnens, G. Colangelo, G. Ecker, J. Gasser, and M. E. Sainio, Nucl. Phys. B 508, 263 (1997); F. Guerrero and J. A. Oller, Nucl. Phys. B 537, 459 (1999); A. G. Nicola and J. Peláez, Phys. Rev. D 65, 054009 (2002).

[2] T. N. Truong, Phys. Rev. Lett. 67, 2260 (1991); T. Hannah, Phys. Rev. D 55, 5613 (1997); A. Dobado and J. R. Peláez, Phys. Rev. D 56, 3057 (1997); M. Boglione and M. R. Pennington, Z. Phys. C 75, 113 (1997); J. A. Oller, E. Oset, and J. R. Peláez, Phys. Rev. D 59, 074001 (1999); J. Nieves, M. P. Valderrama, and E. R. Arriola,
Phys. Rev. D 65, 036002 (2002); J. Nebreda and J. R. Peláez, Phys. Rev. D 81, 054035 (2010); M. Döring and U.-G. Meißner, JHEP 01, 009 (2012).

[3] J. A. Oller and E. Oset, Nucl. Phys. A 620, 438 (1997); F.-K. Guo, R.-G. Ping, P.-N. Shen, H.-C. Chiang, and B. S. Zou, Nucl. Phys. A 773, 78 (2006); I. V. Danilkin, L. I. R. Gil, and M. F. M. Lutz, Phys. Lett. B 703, 504 (2011); Z.-H. Guo, L. Liu, U.-G. Meißner, J. A. Oller, and A. Rusetsky, Phys. Rev. D 95, 054004 (2017); I. V. Danilkin and M. Vanderhaeghen, Phys. Lett. B 789, 366 (2019).

[4] T. Barnes and E. S. Swanson, Phys. Rev. D 46, 131 (1992); E. S. Swanson, Ann. Phys. (N.Y.) 220, 73 (1992); T. Barnes, E. S. Swanson, and J. Weinstein, Phys. Rev. D 46, 4868 (1992); T. Barnes, N. Black, and E. S. Swanson, Phys. Rev. C 63, 025204 (2001).

[5] Z.-Y. Shen, X.-M. Xu, and H. J. Weber, Phys. Rev. D 94, 034030 (2016).

[6] J. A. Oller, E. Oset, and J. R. Peláez, Phys. Rev. D 59, 074001 (1999).

[7] J. J. Dudek, R. G. Edwards, and D. J. Wilson, Phys. Rev. D 93, 094506 (2016).

[8] Y. S. Surovtsev, P. Bydžovský, T. Gutsche, R. Kamiński, V. E. Lyubovitskij, and M. Nagy, Phys. Rev. D 97, 014009 (2018).

[9] A. T. Aoude, P. C. Magalhães, A. C. dos Reis, and M. R. Robilotta, Phys. Rev. D 98, 056021 (2018).

[10] D. Black, A. H. Fariborz, and J. Schechter, Phys. Rev. D 61, 074030 (2000).

[11] Z.-H. Guo, L. Liu, U.-G. Meißner, J. A. Oller, and A. Rusetsky, Phys. Rev. D 95, 054004 (2017).

[12] V. Baru, J. Haidenbauer, C. Hanhart, A. Kudryavtsev, and U.-G. Meißner, Eur. Phys. J. A 23, 523 (2005).

[13] B. Borasoy and R. Nißler, Eur. Phys. J. A 26, 383 (2005).
[14] C. B. Lang, L. Leskovec, D. Mohler, and S. Prelovsek, JHEP 04, 162 (2014).

[15] L. Roca and E. Oset, Phys. Rev. D 85, 054507 (2012).

[16] J. M. Flynn and J. Nieves, Phys. Rev. D 75, 074024 (2007).

[17] Y. Ikeda, B. Charron, S. Aoki, T. Doi, T. Hatsuda, T. Inoue, N. Ishii, K. Murano, H. Nemura, and K. Sasaki, Phys. Lett. B 729, 85 (2014).

[18] A. M. Torres, L. R. Dai, C. Koren, D. Jido, and E. Oset, Phys. Rev. D 85, 014027 (2012).

[19] K. Yang, X.-M. Xu, and H. J. Weber, Phys. Rev. D 96, 114025 (2017).

[20] W. Buchmüller and S.-H. H. Tye, Phys. Rev. D 24, 132 (1981).

[21] S. Godfrey and N. Isgur, Phys. Rev. D 32, 189 (1985).

[22] G. Q. Li and C. M. Ko, Nucl. Phys. A 582, 731 (1995); W. S. Chung, G. Q. Li, and C. M. Ko, Nucl. Phys. A 625, 347 (1997).

[23] G. B. Arfken and H. J. Weber, Mathematical Methods for Physicists (Elsevier, Amsterdam, 2006).

[24] C. J. Joachain, Quantum Collision Theory (North-Holland Publishing Company, Amsterdam, 1983).

[25] S.-T. Ji, Z.-Y. Shen, and X.-M. Xu, J. Phys. G 42, 095110 (2015).

[26] F. Karsch, E. Laermann, and A. Peikert, Nucl. Phys. B 605, 579 (2001).

[27] M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018) and 2019 update.

[28] E. Colton et al., Phys. Rev. D 3, 2028 (1971); N. B. Durusoy et al., Phys. Lett. B 45, 517 (1973); M. J. Losty et al., Nucl. Phys. B 69, 185 (1974); W. Hoogland et al., Nucl. Phys. B 126, 109 (1977).
[29] S. D. Protopopescu et al., Phys. Rev. D 7, 1279 (1973); B. Hyams et al., Nucl. Phys. B 64, 134 (1973); P. Estabrooks and A. D. Martin, Nucl. Phys. B 79, 301 (1974); V. Srinivasan et al., Phys. Rev. D 12, 681 (1975); L. Rosselet et al., Phys. Rev. D 15, 574 (1977); C. D. Froggatt and J. L. Petersen, Nucl. Phys. B 129, 89 (1977); A. A. Bel’kov et al., JETP Lett. 29, 597 (1979); E. A. Alekseeva et al., Sov. Phys. JETP 55, 591 (1982); R. García-Martín, R. Kamiński, J. R. Peláez, J. R. de Elvira, and F. J. Ynduráin, Phys. Rev. D 83, 074004 (2011).

[30] W.-X. Li, X.-M. Xu, and H. J. Weber, Phys. Rev. D 101, 014025 (2020).

[31] Z.-Y. Shen and X.-M. Xu, J. Korean Phys. Soc. 66, 754 (2015).

[32] S.-T. Ji, X.-M. Xu, and H. J. Weber, Nucl. Phys. A 966, 224 (2017).
Figure 1: Reaction $A + B \rightarrow H$. Solid lines with right (left) triangles stand for quarks (antiquarks). Wavy lines stand for gluons.
Figure 2: $K$ and $\phi$ masses as functions of $T/T_c$. 
Figure 3: $\pi$, $D$, and $D^*$ masses as functions of $T/T_c$. 
Figure 4: $D$, $D^*$, $D_s$, $D_s^*$, $\psi(4160)$, and $\psi(4415)$ masses as functions of $T/T_c$. 
| Diagram in Fig. 1 | Left upper | Right upper | Left lower | Right lower |
|-------------------|------------|-------------|------------|-------------|
| $\mathcal{M}_{f K \bar{K} \rightarrow \phi}$ | $\sqrt{2}$ | $\sqrt{2}$ | 0 | 0 |
| $\mathcal{M}_{f \pi D \rightarrow D^*}$ | $\frac{3}{\sqrt{6}}$ | $\frac{3}{\sqrt{6}}$ | 0 | 0 |
| $\mathcal{M}_{f D D \rightarrow \psi(4160)}$ | 0 | 0 | $-\sqrt{2}$ | $-\sqrt{2}$ |
| $\mathcal{M}_{f D^+_s D^-_s \rightarrow \psi(4160)}$ | 0 | 0 | 1 | 1 |
Table 2: Spin matrix elements in $\mathcal{M}_{q_0, q_1}$ for $A(S_A = 1) + B(S_B = 0) \rightarrow H(S_H = 1)$.

|          | $S_{Hz}$ | -1 | -1 | -1 | 0 | 0 | 0 | 1 | 1 | 1 |
|----------|----------|----|----|----|---|---|---|---|---|---|
| $S_{Az}$ |          | -1 | 0  | 1  | -1| 0 | 1 | -1| 0 | 1 |
| $S_{Bz}$ |          | 0  | 0  | 0  | 0 | 0 | 0 | 0 | 0 | 0 |

| $\phi_{fss}^+ \phi_{iss}$ | $\phi_{fss}^+ \sigma_1(21) \phi_{iss}$ | $\phi_{fss}^+ \sigma_2(21) \phi_{iss}$ | $\phi_{fss}^+ \sigma_3(21) \phi_{iss}$ | $\phi_{fss}^+ \sigma_1(1) \phi_{iss}$ | $\phi_{fss}^+ \sigma_2(1) \phi_{iss}$ | $\phi_{fss}^+ \sigma_3(1) \phi_{iss}$ |
|----------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| 0                    | $-\frac{1}{\sqrt{2}}$          | $\frac{i}{\sqrt{2}}$           | 0                               | $0$                             | $-\frac{i}{\sqrt{2}}$          | $\frac{1}{\sqrt{2}}$           |
| $\phi_{fss}^+ \sigma_1(21) \sigma_1(1) \phi_{iss}$ | $0$                             | $-\frac{1}{\sqrt{2}}$          | $-\frac{1}{\sqrt{2}}$          | $\frac{i}{\sqrt{2}}$           | $0$                             | $\frac{i}{\sqrt{2}}$           |
| $\phi_{fss}^+ \sigma_1(21) \sigma_2(1) \phi_{iss}$ | $0$                             | $\frac{i}{\sqrt{2}}$           | $0$                             | $0$                             | $-\frac{i}{\sqrt{2}}$          | $\frac{1}{\sqrt{2}}$           |
| $\phi_{fss}^+ \sigma_1(21) \sigma_3(1) \phi_{iss}$ | $\frac{i}{\sqrt{2}}$           | $0$                             | $0$                             | $-\frac{i}{\sqrt{2}}$          | $0$                             | $\frac{i}{\sqrt{2}}$           |
| $\phi_{fss}^+ \sigma_2(21) \sigma_1(1) \phi_{iss}$ | $0$                             | $\frac{i}{\sqrt{2}}$           | $\frac{i}{\sqrt{2}}$           | $\frac{i}{\sqrt{2}}$           | $0$                             | $\frac{i}{\sqrt{2}}$           |
| $\phi_{fss}^+ \sigma_2(21) \sigma_2(1) \phi_{iss}$ | $0$                             | $\frac{i}{\sqrt{2}}$           | $0$                             | $\frac{i}{\sqrt{2}}$           | $0$                             | $\frac{i}{\sqrt{2}}$           |
| $\phi_{fss}^+ \sigma_2(21) \sigma_3(1) \phi_{iss}$ | $\frac{i}{\sqrt{2}}$           | $0$                             | $0$                             | $0$                             | $0$                             | $\frac{i}{\sqrt{2}}$           |
| $\phi_{fss}^+ \sigma_3(21) \sigma_1(1) \phi_{iss}$ | $0$                             | $0$                             | $-\frac{1}{\sqrt{2}}$          | $0$                             | $\frac{1}{\sqrt{2}}$           | $0$                             |
| $\phi_{fss}^+ \sigma_3(21) \sigma_2(1) \phi_{iss}$ | $0$                             | $\frac{i}{\sqrt{2}}$           | $0$                             | $0$                             | $0$                             | $-\frac{i}{\sqrt{2}}$          |
| $\phi_{fss}^+ \sigma_3(21) \sigma_3(1) \phi_{iss}$ | $0$                             | $\frac{i}{\sqrt{2}}$           | $0$                             | $0$                             | $-\frac{i}{\sqrt{2}}$          | $0$                             |

28
Table 3: Spin matrix elements in $\mathcal{M}_{q_1q_2}$ for $A(S_A = 0) + B(S_B = 1) \rightarrow H(S_H = 1)$.

| $S_{Hz}$ | -1 | -1 | -1 | 0 | 0 | 0 | 1 | 1 | 1 |
|----------|----|----|----|---|---|---|---|---|---|
| $S_{Az}$ | 0  | 0  | 0  | 0 | 0 | 0 | 0 | 0 | 0 |
| $S_{Bz}$ | -1 | 0  | -1 | 0 | 1 | -1 | 0 | 0 | 1 |

| $\phi_{\text{fss}}^+\phi_{\text{iss}}$ | 0 | $\frac{i}{\sqrt{2}}$ | 0 | $\frac{i}{\sqrt{2}}$ | 0 | $\frac{i}{\sqrt{2}}$ | 0 | $\frac{i}{\sqrt{2}}$ | 0 |
| $\phi_{\text{fss}}^+\sigma_1(21)\phi_{\text{iss}}$ | $\frac{1}{\sqrt{2}}$ | 0 | 0 | 0 | 0 | 0 | 0 | $-\frac{1}{\sqrt{2}}$ |
| $\phi_{\text{fss}}^+\sigma_2(21)\phi_{\text{iss}}$ | $\frac{i}{\sqrt{2}}$ | 0 | 0 | $\frac{i}{\sqrt{2}}$ | 0 | 0 | 0 | $\frac{i}{\sqrt{2}}$ |
| $\phi_{\text{fss}}^+\sigma_3(21)\phi_{\text{iss}}$ | 0 | $\frac{i}{\sqrt{2}}$ | 0 | $\frac{i}{\sqrt{2}}$ | 0 | $\frac{i}{\sqrt{2}}$ | 0 | $\frac{i}{\sqrt{2}}$ |
| $\phi_{\text{fss}}^+\sigma_1(1)\phi_{\text{iss}}$ | $-\frac{1}{\sqrt{2}}$ | 0 | 0 | 0 | 0 | 0 | 0 | $\frac{1}{\sqrt{2}}$ |
| $\phi_{\text{fss}}^+\sigma_2(1)\phi_{\text{iss}}$ | $\frac{i}{\sqrt{2}}$ | 0 | 0 | $\frac{i}{\sqrt{2}}$ | 0 | 0 | 0 | $\frac{i}{\sqrt{2}}$ |
| $\phi_{\text{fss}}^+\sigma_3(1)\phi_{\text{iss}}$ | 0 | $-\frac{1}{\sqrt{2}}$ | 0 | $-\frac{1}{\sqrt{2}}$ | 0 | $-\frac{1}{\sqrt{2}}$ | 0 | $-\frac{1}{\sqrt{2}}$ |

| $\phi_{\text{fss}}^+\sigma_1(21)\sigma_1(1)\phi_{\text{iss}}$ | 0 | $-\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 |
| $\phi_{\text{fss}}^+\sigma_1(21)\sigma_2(1)\phi_{\text{iss}}$ | 0 | $\frac{i}{2}$ | 0 | $\frac{i}{2}$ | 0 | $\frac{i}{2}$ | 0 | $\frac{i}{2}$ | 0 |
| $\phi_{\text{fss}}^+\sigma_1(21)\sigma_3(1)\phi_{\text{iss}}$ | $-\frac{1}{\sqrt{2}}$ | 0 | 0 | 0 | $-\frac{1}{\sqrt{2}}$ | 0 | 0 | 0 | $-\frac{1}{\sqrt{2}}$ |
| $\phi_{\text{fss}}^+\sigma_2(21)\sigma_1(1)\phi_{\text{iss}}$ | 0 | $\frac{i}{2}$ | 0 | $\frac{i}{2}$ | 0 | $\frac{i}{2}$ | 0 | $\frac{i}{2}$ | 0 |
| $\phi_{\text{fss}}^+\sigma_2(21)\sigma_2(1)\phi_{\text{iss}}$ | 0 | $\frac{i}{2}$ | 0 | $\frac{i}{2}$ | 0 | $\frac{i}{2}$ | 0 | $\frac{i}{2}$ | 0 |
| $\phi_{\text{fss}}^+\sigma_2(21)\sigma_3(1)\phi_{\text{iss}}$ | $\frac{i}{\sqrt{2}}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $\frac{i}{\sqrt{2}}$ |
| $\phi_{\text{fss}}^+\sigma_3(21)\sigma_1(1)\phi_{\text{iss}}$ | $\frac{1}{\sqrt{2}}$ | 0 | 0 | $\frac{1}{\sqrt{2}}$ | 0 | 0 | 0 | $\frac{1}{\sqrt{2}}$ |
| $\phi_{\text{fss}}^+\sigma_3(21)\sigma_2(1)\phi_{\text{iss}}$ | $-\frac{i}{\sqrt{2}}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-\frac{i}{\sqrt{2}}$ |
| $\phi_{\text{fss}}^+\sigma_3(21)\sigma_3(1)\phi_{\text{iss}}$ | 0 | $-\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 |
Table 4: Spin matrix elements in $\mathcal{M}_{\tilde{q}_1\tilde{q}_2}\tilde{q}_1$ for $A(S_A = 1) + B(S_B = 1) \rightarrow H(S_H = 1)$ with $S_{Hz} = -1$.

| $S_{Hz}$ | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
|----------|----|----|----|----|----|----|----|----|
| $S_{Az}$ | -1 | -1 | -1 | 0  | 0  | 0  | 1  | 1  |
| $S_{Bz}$ | -1 | 0  | 1  | -1 | 0  | 1  | -1 | 0  |

| $\phi_{\text{fss}}^+ \phi_{\text{iss}}$ | 1  | 0  | 0  | 0  | $\frac{1}{2}$ | 0  | 0  | 0  | 0  |
| $\phi_{\text{fss}}^+ \sigma_1(21) \phi_{\text{iss}}$ | 0  | $\frac{1}{\sqrt{2}}$ | 0  | $\frac{1}{\sqrt{2}}$ | 0  | 0  | 0  | 0  | 0  |
| $\phi_{\text{fss}}^+ \sigma_2(21) \phi_{\text{iss}}$ | 0  | $-\frac{i}{\sqrt{2}}$ | 0  | $\frac{i}{\sqrt{2}}$ | 0  | 0  | 0  | 0  | 0  |
| $\phi_{\text{fss}}^+ \sigma_3(21) \phi_{\text{iss}}$ | -1 | 0  | 0  | 0  | $\frac{1}{2}$ | 0  | 0  | 0  | 0  |
| $\phi_{\text{fss}}^+ \sigma_1(1) \phi_{\text{iss}}$ | 0  | 0  | 0  | $\frac{1}{\sqrt{2}}$ | 0  | 0  | 0  | $\frac{1}{\sqrt{2}}$ | 0  |
| $\phi_{\text{fss}}^+ \sigma_2(1) \phi_{\text{iss}}$ | 0  | 0  | 0  | $-\frac{i}{\sqrt{2}}$ | 0  | 0  | 0  | $-\frac{i}{\sqrt{2}}$ | 0  |
| $\phi_{\text{fss}}^+ \sigma_3(1) \phi_{\text{iss}}$ | -1 | 0  | 0  | 0  | $\frac{1}{2}$ | 0  | 0  | 0  | 0  |

| $\phi_{\text{fss}}^+ \sigma_1(21) \sigma_1(1) \phi_{\text{iss}}$ | 0  | 0  | 0  | 0  | $\frac{1}{2}$ | 0  | 1  | 0  | 0  |
| $\phi_{\text{fss}}^+ \sigma_1(21) \sigma_2(1) \phi_{\text{iss}}$ | 0  | 0  | 0  | 0  | $\frac{1}{2}$ | 0  | $-i$ | 0  | 0  |
| $\phi_{\text{fss}}^+ \sigma_1(21) \sigma_3(1) \phi_{\text{iss}}$ | 0  | $-\frac{1}{\sqrt{2}}$ | 0  | $\frac{1}{\sqrt{2}}$ | 0  | 0  | 0  | 0  | 0  |
| $\phi_{\text{fss}}^+ \sigma_2(21) \sigma_1(1) \phi_{\text{iss}}$ | 0  | 0  | 0  | 0  | $\frac{1}{2}$ | 0  | 0  | $i$ | 0  |
| $\phi_{\text{fss}}^+ \sigma_2(21) \sigma_2(1) \phi_{\text{iss}}$ | 0  | 0  | 0  | 0  | $\frac{1}{2}$ | 0  | 1  | 0  | 0  |
| $\phi_{\text{fss}}^+ \sigma_2(21) \sigma_3(1) \phi_{\text{iss}}$ | 0  | $\frac{i}{\sqrt{2}}$ | 0  | $-\frac{i}{\sqrt{2}}$ | 0  | 0  | 0  | 0  | 0  |
| $\phi_{\text{fss}}^+ \sigma_3(21) \sigma_1(1) \phi_{\text{iss}}$ | 0  | 0  | 0  | $\frac{1}{\sqrt{2}}$ | 0  | 0  | 0  | $\frac{1}{\sqrt{2}}$ | 0  |
| $\phi_{\text{fss}}^+ \sigma_3(21) \sigma_2(1) \phi_{\text{iss}}$ | 0  | 0  | 0  | $\frac{i}{\sqrt{2}}$ | 0  | 0  | 0  | $-\frac{i}{\sqrt{2}}$ | 0  |
| $\phi_{\text{fss}}^+ \sigma_3(21) \sigma_3(1) \phi_{\text{iss}}$ | 1  | 0  | 0  | 0  | $\frac{1}{2}$ | 0  | 0  | 0  | 0  |
Table 5: Spin matrix elements in $\mathcal{M}_{q_1\bar{q}_2\bar{q}_1}$ for $A(S_A = 1) + B(S_B = 1) \rightarrow H(S_H = 1)$ with $S_{Hz} = 0$.

| $S_{Hz}$ | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     |
|----------|-------|-------|-------|-------|-------|-------|-------|-------|
| $S_{Az}$ | -1    | -1    | -1    | 0     | 0     | 0     | 1     | 1     |
| $S_{Bz}$ | -1    | 0     | 1     | -1    | 0     | 1     | -1    | 0     |

| $\phi_{\text{iss}}^+ \phi_{\text{iss}}$ | 0 | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ |
| $\phi_{\text{iss}}^+ \sigma_1(21) \phi_{\text{iss}}$ | 0 | 0 | $\frac{1}{\sqrt{2}}$ | 0 | $\frac{1}{\sqrt{2}}$ | 0 | $\frac{1}{\sqrt{2}}$ | 0 |
| $\phi_{\text{iss}}^+ \sigma_2(21) \phi_{\text{iss}}$ | 0 | 0 | $-\frac{i}{\sqrt{2}}$ | 0 | 0 | 0 | $\frac{i}{\sqrt{2}}$ | 0 |
| $\phi_{\text{iss}}^+ \sigma_3(21) \phi_{\text{iss}}$ | 0 | $-\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ |
| $\phi_{\text{iss}}^+ \sigma_1(1) \phi_{\text{iss}}$ | $\frac{1}{\sqrt{2}}$ | 0 | 0 | 0 | $\frac{1}{\sqrt{2}}$ | 0 | 0 | 0 | $\frac{1}{\sqrt{2}}$ |
| $\phi_{\text{iss}}^+ \sigma_2(1) \phi_{\text{iss}}$ | $\frac{i}{\sqrt{2}}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-\frac{i}{\sqrt{2}}$ |
| $\phi_{\text{iss}}^+ \sigma_3(1) \phi_{\text{iss}}$ | 0 | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | 0 | $\frac{1}{2}$ |

| $\phi_{\text{iss}}^+ \sigma_1(21) \sigma_1(1) \phi_{\text{iss}}$ | 0 | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ |
| $\phi_{\text{iss}}^+ \sigma_1(21) \sigma_2(1) \phi_{\text{iss}}$ | 0 | $\frac{i}{2}$ | 0 | $\frac{i}{2}$ | 0 | $-\frac{i}{2}$ | 0 | $-\frac{i}{2}$ |
| $\phi_{\text{iss}}^+ \sigma_1(21) \sigma_3(1) \phi_{\text{iss}}$ | 0 | 0 | $-\frac{1}{\sqrt{2}}$ | 0 | 0 | 0 | $\frac{1}{\sqrt{2}}$ | 0 | 0 |
| $\phi_{\text{iss}}^+ \sigma_2(21) \sigma_1(1) \phi_{\text{iss}}$ | $-\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 |
| $\phi_{\text{iss}}^+ \sigma_2(21) \sigma_2(1) \phi_{\text{iss}}$ | $\frac{i}{2}$ | 0 | $\frac{i}{2}$ | 0 | $-\frac{i}{2}$ | 0 | $\frac{i}{2}$ | 0 |
| $\phi_{\text{iss}}^+ \sigma_2(21) \sigma_3(1) \phi_{\text{iss}}$ | 0 | 0 | $\frac{i}{\sqrt{2}}$ | 0 | $-\frac{i}{\sqrt{2}}$ | 0 | $\frac{i}{\sqrt{2}}$ | 0 | 0 |
| $\phi_{\text{iss}}^+ \sigma_3(21) \sigma_1(1) \phi_{\text{iss}}$ | $-\frac{1}{\sqrt{2}}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $\frac{1}{\sqrt{2}}$ |
| $\phi_{\text{iss}}^+ \sigma_3(21) \sigma_2(1) \phi_{\text{iss}}$ | $-\frac{i}{\sqrt{2}}$ | 0 | 0 | 0 | $\frac{i}{\sqrt{2}}$ | 0 | 0 | 0 | $-\frac{i}{\sqrt{2}}$ |
| $\phi_{\text{iss}}^+ \sigma_3(21) \sigma_3(1) \phi_{\text{iss}}$ | 0 | $\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ |

31
Table 6: Spin matrix elements in $\mathcal{M}_{q_1\bar{q}_2\bar{q}_1}$ for $A(S_A = 1) + B(S_B = 1) \rightarrow H(S_H = 1)$ with $S_{Hz} = 1$.

| $S_{Hz}$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
|----------|---|---|---|---|---|---|---|---|
| $S_{Az}$ | -1 | -1 | -1 | 0 | 0 | 0 | 1 | 1 |
| $S_{Bz}$ | -1 | 0 | 1 | -1 | 0 | 1 | -1 | 0 |

\[
\begin{align*}
\phi_{\text{iss}}^{+}\phi_{\text{iss}} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 1 \\
\phi_{\text{iss}}^{+}\sigma_1(21)\phi_{\text{iss}} & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\
\phi_{\text{iss}}^{+}\sigma_2(21)\phi_{\text{iss}} & 0 & 0 & 0 & 0 & \frac{-i}{\sqrt{2}} & 0 & \frac{i}{\sqrt{2}} & 0 & 0 \\
\phi_{\text{iss}}^{+}\sigma_3(21)\phi_{\text{iss}} & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 1 \\
\phi_{\text{iss}}^{+}\sigma_1(1)\phi_{\text{iss}} & 0 & \frac{i}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\
\phi_{\text{iss}}^{+}\sigma_2(1)\phi_{\text{iss}} & 0 & \frac{i}{\sqrt{2}} & 0 & 0 & 0 & \frac{i}{\sqrt{2}} & 0 & 0 & 0 \\
\phi_{\text{iss}}^{+}\sigma_3(1)\phi_{\text{iss}} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 1 \\
\phi_{\text{iss}}^{+}\sigma_1(21)\sigma_1(1)\phi_{\text{iss}} & 0 & 0 & 1 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\
\phi_{\text{iss}}^{+}\sigma_1(21)\sigma_2(1)\phi_{\text{iss}} & 0 & 0 & i & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 \\
\phi_{\text{iss}}^{+}\sigma_1(21)\sigma_3(1)\phi_{\text{iss}} & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\
\phi_{\text{iss}}^{+}\sigma_2(21)\sigma_1(1)\phi_{\text{iss}} & 0 & 0 & -i & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 \\
\phi_{\text{iss}}^{+}\sigma_2(21)\sigma_2(1)\phi_{\text{iss}} & 0 & 0 & 1 & 0 & \frac{-i}{2} & 0 & 0 & 0 & 0 \\
\phi_{\text{iss}}^{+}\sigma_2(21)\sigma_3(1)\phi_{\text{iss}} & 0 & 0 & 0 & 0 & \frac{-i}{\sqrt{2}} & 0 & \frac{i}{\sqrt{2}} & 0 & 0 \\
\phi_{\text{iss}}^{+}\sigma_3(21)\sigma_1(1)\phi_{\text{iss}} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\
\phi_{\text{iss}}^{+}\sigma_3(21)\sigma_2(1)\phi_{\text{iss}} & 0 & \frac{i}{\sqrt{2}} & 0 & 0 & 0 & \frac{i}{\sqrt{2}} & 0 & 0 & 0 \\
\phi_{\text{iss}}^{+}\sigma_3(21)\sigma_3(1)\phi_{\text{iss}} & 0 & 0 & 0 & 0 & \frac{-1}{2} & 0 & 0 & 0 & 1 \\
\end{align*}
\]
Table 7: Total spin, orbital-angular-momentum quantum number, and cross section.

| reaction                      | $S$ | $L_i$ | $\sigma^{\text{unpol}}$ (mb) |
|-------------------------------|-----|-------|-------------------------------|
| $K\bar{K} \rightarrow \phi$  | 0   | 1     | 5.96                          |
| $\pi D \rightarrow D^*$       | 0   | 1     | 98.9                          |
| $\pi\bar{D} \rightarrow \bar{D}^*$ | 0   | 1     | 98.9                          |
| $D\bar{D} \rightarrow \psi(4160)$ | 0   | 1     | 3.26                          |
| $D^*\bar{D} \rightarrow \psi(4160)$ | 1   | 1     | 10.53                         |
| $DD^* \rightarrow \psi(4160)$  | 1   | 1     | 10.53                         |
| $D^*\bar{D}^* \rightarrow \psi(4160)$ | 0   | 1     | 19.83                         |
|                               | 1   | 1     |                               |
|                               | 2   | 1,3   |                               |
| $D_s^+D_s^- \rightarrow \psi(4160)$ | 0   | 1     | 0.76                          |
| $D_s^{*+}D_s^- \rightarrow \psi(4160)$ | 1   | 1     | 1.71                          |
| $D_s^+D_s^{*-} \rightarrow \psi(4160)$ | 1   | 1     | 1.71                          |
| $D\bar{D} \rightarrow \psi(4415)$ | 0   | 1     | 1.83                          |
| $D^*\bar{D} \rightarrow \psi(4415)$ | 1   | 1     | 2.42                          |
| $DD^* \rightarrow \psi(4415)$  | 1   | 1     | 2.42                          |
| $D^*\bar{D}^* \rightarrow \psi(4415)$ | 0   | 1     | 0.14                          |
|                               | 1   | 1     |                               |
|                               | 2   | 1,3   |                               |
| $D_s^+D_s^- \rightarrow \psi(4415)$ | 0   | 1     | 0.041                         |
| $D_s^{*+}D_s^- \rightarrow \psi(4415)$ | 1   | 1     | 0.29                          |
| $D_s^+D_s^{*-} \rightarrow \psi(4415)$ | 1   | 1     | 0.29                          |
| $D_s^{*+}D_s^{*-} \rightarrow \psi(4415)$ | 0   | 1     | 1.8                           |
|                               | 1   | 1     |                               |
|                               | 2   | 1,3   |                               |