Kaon Condensation in Neutron Star Matter with Hyperons

Paul J. Ellis
School of Physics and Astronomy
University of Minnesota
Minneapolis, MN 55455

Roland Knorren and Madappa Prakash
Physics Department
State University of New York at Stony Brook
Stony Brook, NY 11794

Abstract

Based on the Kaplan-Nelson Lagrangian, we investigate kaon condensation in dense neutron star matter allowing for the explicit presence of hyperons. Using various models we find that the condensate threshold is sensitive to the behavior of the scalar density; the more rapidly it increases with baryon density, the lower is the threshold for condensation. The presence of hyperons, particularly the $\Sigma^-$, shifts the threshold for $K^-$ condensation to a higher density. In the mean field approach, with hyperons, the condensate amplitude grows sufficiently rapidly that the nucleon effective mass vanishes at a finite density and a satisfactory treatment of the thermodynamics cannot be achieved. Thus, calculations of kaon-baryon interactions beyond the mean field level appear to be necessary.
The idea that, above some critical density, the ground state of baryonic matter might contain a Bose-Einstein condensate of negatively charged kaons is due to Kaplan and Nelson [1]. Subsequently, the formulation, in terms of chiral perturbation theory, was discussed [2] and the astrophysical consequences have been explored in ref. [3, 4]. Physically, the strong attraction between $K^-$ mesons and nucleons increases with density and lowers the energy of the zero-momentum state. A condensate forms when this energy becomes equal to the kaon chemical potential, $\mu$. In catalyzed (neutrino-free) dense neutron star matter, $\mu$ is related to the electron and nucleon chemical potentials by $\mu = \mu_e = \mu_n - \mu_p$ due to chemical equilibrium in the reactions $n \leftrightarrow p + e^- + \bar{\nu}_e$ and $n \leftrightarrow p + K^-$. The density at which this takes place is model and parameter dependent, but is typically $\sim 4n_0$, where $n_0$ denotes equilibrium nuclear matter density. Since this may be less than the central density in neutron stars, a $K^-$ condensate is expected to be present in the core region. Apart from the softening effect on the equation of state, which lowers the maximum mass, the proton abundance is dramatically increased resulting in a nucleon, rather than a neutron, star.

However, many calculations, e.g. [5, 6], of dense matter indicate that hyperons, starting with the $\Sigma^-$ and $\Lambda$, begin to appear at densities $\sim (2-3)n_0$, a little lower than the kaon threshold mentioned above. Once a significant number of negatively charged hyperons are present, less electrons are required for charge neutrality, so the electron chemical potential, $\mu_e$, begins to decrease with density. Since $\mu_e = \mu$ this would be expected to inhibit kaon condensation, delaying it to a higher density. However, since overall charge neutrality must be maintained in the presence of many strongly interacting particles, the issue of whether or not kaon condensation would occur in matter containing hyperons is uncertain. The only previous work [4] involving both hyperons and kaons was concerned with $p$-wave interactions which led to quasinucleons through the mixing with hyperons.

Our objective here is to explore the influence of the explicit presence of hyperons on kaon condensation in dense neutron star matter. In our model calculations, we employ the
Kaplan-Nelson Lagrangian for the kaon-baryon interaction and a relativistic field theoretical approach for the baryon-baryon interactions. Use of the relativistic approach for the baryons allows for a distinction between the scalar and vector densities, which is necessary for an adequate treatment of the kaon-baryon interactions. Ignoring this distinction results in a lower threshold density for kaon condensation.

We begin with the Kaplan-Nelson $SU(3) \times SU(3)$ chiral Lagrangian for the kaons and the $s$-wave kaon-baryon interactions. Specifically

\[ \mathcal{L}_K = \frac{1}{4} f^2 \text{Tr} \partial_\mu U \partial^\mu U + C \text{Tr} m_q (U + U^\dagger - 2) + i \text{Tr} \bar{B} \gamma^\mu [V_\mu, B] + a_1 \text{Tr} \bar{B} (\xi m_q \xi + \text{h.c.}) B \\
+a_2 \text{Tr} \bar{B} (\xi m_q \xi + \text{h.c.}) + a_3 \{ \text{Tr} \bar{B} B \} \{ \text{Tr} (m_q U + \text{h.c.}) \}. \]

Here $U$ is the non-linear field involving the pseudoscalar meson octet from which we retain only the $K^\pm$ contributions, $\xi^2 = U$ and $B$ represents the baryon octet – nucleons plus hyperons. The quark mass matrix $m_q = \text{diag}(0, 0, m_s)$. For the mesonic vector current, $V_\mu$, only the time component survives in an infinite system with $V_0 = \frac{1}{2} (\xi^\dagger \partial_0 \xi + \xi \partial_0 \xi^\dagger)$. The pion decay constant $f = 93$ MeV and $C, a_1, a_2$ and $a_3$ are constants. After some algebra, the relevant part of $\mathcal{L}_K$ takes the form

\[ \mathcal{L}_K = \left( \frac{\sin \chi}{\chi} \right)^2 \left\{ \partial_\mu K^+ \partial^\mu K^- + \frac{i}{4 f^2} \frac{(K^+ \partial_0 K^- - K^- \partial_0 K^+)}{\cos^2 \frac{\chi}{2}} \sum_B (Y_B + q_B) B^\dagger B \\
- \left( \frac{m_K^2 + m_s^2}{2 f^2} \sum_B \left[ (a_1 + a_2)(1 + Y_B q_B) + (a_1 - a_2)(q_B - Y_B) + 4 a_3 \right] B B \\
+ \frac{m_s^2}{6 f^2} (a_1 + a_2) \left( 2 \bar{\Lambda} \Lambda + \sqrt{3} \bar{\Sigma}^0 \Lambda + \bar{\Lambda} \Sigma^0 \right) \right\} \frac{K^+ K^-}{\cos^2 \frac{\chi}{2}}, \]

where $\chi^2 = 2 K^+ K^- f^2$, $q_B$ and $Y_B$ are the baryon charge and hypercharge, respectively, and the kaon mass is given by $m_K^2 = 2 C m_s / f^2$. We have not included in Eq. (2) terms which simply give a constant shift to the baryon masses; they indicate that $a_1 m_s = -67$ MeV and $a_2 m_s = 134$ MeV using the hyperon–nucleon mass differences. The remaining constant $a_3 m_s$ is not accurately known and we shall use values in the range $-134$ to $-310$ MeV.
corresponding to 0 to 20% strangeness content for the proton. Some guidance is provided by recent lattice gauge simulations [8] which find that the strange quark condensate in the nucleon is large, i.e. \(\langle N|\bar{s}s|N\rangle = 1.16 \pm 0.54\). From the relation \(m_s(\bar{s}s)_p = -2(a_2 + a_3)m_s\) and using \(m_s = 150\) MeV, we obtain \(a_3m_s = -(220 \pm 40)\) MeV, which is in the middle of our range of values.

In the baryon sector, we employ a relativistic field theory model in which baryons interact via the exchange of \(\sigma\)-, \(\rho\)- and \(\omega\)-mesons. In the case that only nucleons are considered, \(B = n, p\), this is the well-known Walecka model [9] and we allow for possible “non-linear” \(\sigma^3\) and \(\sigma^4\) terms. The Lagrangian takes the form

\[
\mathcal{L}_B = \sum_B \left( i\gamma^\mu\partial_\mu - g_{\omega B}\gamma^\mu\omega_\mu - g_{\rho B}\gamma^\mu b_\mu \cdot \mathbf{t} - M_B + g_{\sigma B}\sigma \right) B - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu \\
- \frac{1}{4}B_{\mu\nu} \cdot B^{\mu\nu} + \frac{3}{2}m_\rho^2b_\mu \cdot b^\mu + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{4}bM(g_{\sigma n}\sigma)^3 - \frac{1}{4}c(g_{\sigma n}\sigma)^4. \tag{3}
\]

Here \(M_B\) is the vacuum baryon mass, the \(\rho\)-meson field is denoted by \(b_\mu\), the quantity \(\mathbf{t}\) denotes the isospin operator which acts on the baryons and the field strength tensors for the vector mesons are given by the usual expressions: \(F_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu\), \(B_{\mu\nu} = \partial_\mu b_\nu - \partial_\nu b_\mu\). The nucleon mass, \(M = 939\) MeV, is included in the penultimate term so that \(b\) is dimensionless.

The total hadron Lagrangian is then \(\mathcal{L}_{\text{tot}} = \mathcal{L}_K + \mathcal{L}_B\). We shall treat the kaons in the mean field approximation. For the baryons, we shall consider calculations at the mean field level (with the “non-linear” terms so as to obtain a reasonable compression modulus for nuclear matter) and at the one-loop Hartree level (without the “non-linear” terms). As we shall see, there is a qualitative difference between the results. We need to calculate the potential, \(\Omega\), of the grand canonical ensemble at zero temperature. Notice first that the \(\Lambda - \Sigma^0\) mass matrix needs to be diagonalized (for details see ref. [8]) producing eigenstates

\[
H_1 = \frac{\Sigma^0 - \delta\Lambda}{(1 + \delta^2)^2}, \quad H_2 = \frac{\Lambda + \delta\Sigma^0}{(1 + \delta^2)^2}.
\tag{4}
\]
with corresponding masses $M^*_H$, and $M^*_H$. These are henceforth included in the sum over baryon states $B$, along with the $n, p, \Sigma^{\pm, -}, \Xi^{0, -}$. The kaon-baryon interactions can be included in the baryon effective masses and chemical potentials. The remaining kaon terms are then easily treated by writing the time dependence of the fields $K^\pm = \frac{1}{\sqrt{2}} f \theta e^{\pm i \mu t}$; thus, $\theta$ gives the condensate amplitude. Then it is straightforward to obtain
\[
\frac{\Omega}{V} = f^2 (2m_K^2 \sin^2 \frac{1}{2} \theta - \frac{1}{2} \mu^2 \sin^2 \theta) + \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{3} b M (g_{\sigma n} \sigma)^3 + \frac{1}{4} c (g_{\sigma n} \sigma)^4
\]
\[
- \frac{1}{2} m_\omega^2 \omega_0^2 - \frac{1}{2} m_\rho^2 \rho_0^2 + \sum_B \left[ \frac{1}{\pi^2} \int_0^{k_{FB}} dk k^2 (E_B^* - \nu_B) + \Delta E(M_B^*) \right].
\]
(5)

Here $V$ is the volume, $E_B^* = \sqrt{k^2 + M_B^*}$ and the effective masses, $M_B^*$ for $H_1$ and $H_2$ are obtained by diagonalizing the mass matrix, as we have mentioned, while the remaining cases are given by
\[
M_B^* = M_B - g_{\sigma B} \sigma + [(a_1 + a_2)(1 + Y_{BqB}) + (a_1 - a_2)(q_B - Y_B)] + 4a_3 m_s \sin^2 \frac{1}{2} \theta.
\]
(6)
The chemical potentials $\mu_B$ are given in terms of the effective chemical potentials, $\nu_B$, by
\[
\mu_B = \nu_B + g_{B\omega} \omega_0 + g_{B\rho} t_{3B} b_0 - (Y_B + q_B) \mu \sin^2 \frac{1}{2} \theta,
\]
(7)
where $t_{3B}$ is the $z$-component of the isospin of the baryon and the relation to the Fermi momentum $k_{FB}$ is provided by $\nu_B = \sqrt{k_{FB}^2 + M_B^*}$.

In the one loop Hartree approximation (with $b = c = 0$) , an additional contribution $\Delta E(M_B^*)$ to the energy density is introduced due to the shift in the single-particle energies caused by the negative energy baryon states. After removing divergences, $\Delta E(M_B^*)$ can be written in the form
\[
\Delta E(M_B^*) = -\frac{1}{8\pi^2} \left[ 4 \left( 1 - \frac{\mu_r}{M} + \ln \frac{\mu_r}{M} \right) M_B (M_B - M_B^*)^3 - \ln \frac{\mu_r}{M} (M_B - M_B^*)^4 
\]
\[
+ M_B^* \ln \frac{M_B^*}{M_B} + \frac{1}{2} M_B^* (M_B - M_B^*) - \frac{7}{2} M_B^2 (M_B - M_B^*)^2
\]
\[
+ \frac{13}{8} M_B (M_B - M_B^*)^3 - \frac{25}{12} (M_B - M_B^*)^4 \right].
\]
(8)
Here the necessary renormalization introduces a scale parameter, $\mu_r$. For the standard choice $[9]$ of $\mu_r/M=1$, the first two terms in Eq. (8) vanish. Other choices of $\mu_r/M$ introduce explicit $\sigma^3$ and $\sigma^4$ contributions. The freedom to vary $\mu_r/M$ allows the density dependence of the energy to be modified, which makes it possible [10] to explore equations of state with different stiffnesses. We refer to this approach as the modified relativistic Hartree approximation (MRHA). In previous work [11], without hyperons, we found that while neutron star masses do not significantly constrain $\mu_r/M$, finite nuclei favor a value of 0.79.

The thermodynamic quantities can be obtained from the grand potential in Eq. (5) in the standard way, thus the baryon number density $n_B = (3\pi^2)^{-1}k_F^3$, while for kaons

$$n_K = f^2(\mu \sin^2 \theta + 4e \sin^2 \frac{\theta}{2}) \quad \text{with} \quad e = \sum_B (Y_B + q_B)n_B/(4f^2) . \quad (9)$$

The pressure $P = -\Omega/V$ and the energy density $\varepsilon = -P + \sum_B \mu_B n_B + \mu n_K$. The meson fields are obtained by extremizing $\Omega$, giving

$$m_\omega^2 \omega_0 = \sum_B g_\omega B n_B \quad , \quad m_\rho^2 b_0 = \sum_B g_\rho B t_3 B n_B \quad \text{and}$$

$$m_\sigma^2 = -b M g_\sigma^3 \sigma^2 - c g_\sigma^4 \sigma^3 + \sum_B g_\sigma B n_B^s . \quad (10)$$

Here $n_B^s$ denotes the baryon scalar density

$$n_B^s = \frac{1}{\pi^2} \int_0^{k_F B} dk k^2 \frac{M_B^s}{E_B^*} + \frac{\partial \Delta E(M_B^s)}{\partial M_B^s} . \quad (11)$$

The condensate amplitude, $\theta$, is also found by extremizing $\Omega$. This yields the solutions

$$\theta = 0 \quad (\text{no condensate}), \quad \text{or, if a condensate exists, the equation}$$

$$\mu^2 \cos \theta + 2e\mu - m_K^2 - d_1 - d_2 = 0 \quad \text{where},$$

$$2f^2 d_1 = \sum_{B \neq H_1, H_2} \left[ (a_1 + a_2)(1 + Y_B q_B) + (a_1 - a_2)(q_B - Y_B) + 4a_3 \right] m_s n_B^s ,$$

$$2f^2 d_2 \sin^2 \frac{\theta}{2} = \sum_{B = H_1, H_2} \left( M_B^s + g_\sigma \Lambda \sigma \right) n_B^s - M_\Lambda n_{H_1}^s - M_\Sigma n_{H_2}^s$$

$$+ \frac{1}{1 + \delta^2}(M_\Lambda - M_\Sigma)(n_{H_1}^s - n_{H_2}^s) , \quad (12)$$

6
where we have taken $g_{\sigma \Lambda} = g_{\sigma \Sigma}$.

Finally we consider the leptons. The electrons and muons are governed by chemical potentials $\mu_e = \mu_\mu = \mu$. (In the cold catalyzed state, the antineutrinos would have left the star.) Their contributions to the total energy density and pressure are given adequately by the free gas expressions. The requirement of chemical equilibrium in the weak processes yields

$$\mu_{H_1} = \mu_{H_2} = \mu_{\Xi^0} = \mu_n ; \quad \mu_{\Sigma^-} = \mu_{\Xi^-} = \mu_n + \mu_e ; \quad \mu_p = \mu_{\Sigma^+} = \mu_n - \mu_e .$$

The remaining condition is that of overall charge neutrality, namely

$$\sum_B q_B n_B - n_K - n_e - n_\mu = 0 .$$

The couplings $g_{\sigma n}$ and $g_{\omega n}$ are determined by fitting the equilibrium empirical properties of nuclear matter, while the coupling $g_{\rho n}$ is obtained by fitting the nuclear symmetry energy. For the mean field model, we use the last two sets of values in table 2 of ref. [12]. These models, termed models A and B hereafter, have compression moduli of 300 and 240 MeV, respectively, and therefore exhibit different stiffnesses for the high density equation of state. The couplings employed for the MRHA model are shown in table 1.

Large uncertainties exist in the values of the hyperon–meson coupling constants. We reduce the number of parameters by making the reasonable assumption that all the hyperon–meson coupling constants are the same as those of the $\Lambda$. Defining $x_\sigma = g_{\sigma \Lambda}/g_{\sigma n}$, with analogous definitions for the $\omega$– and $\rho$–couplings, the binding energy of the lowest $\Lambda$ level in nuclear matter at saturation yields [12]

$$-28 = x_\omega g_{\omega n} \omega_0 - x_\sigma g_{\sigma n} \sigma ,$$

in units of MeV. In ref. [12], on the basis of fits to hypernuclear levels and neutron star properties, the value $x_\sigma = 0.6$ was suggested, $x_\omega$ is then determined and the choice $x_\rho = x_\sigma$...
was made. We adopt this procedure, since we have examined the effect of varying $x_\sigma$ and setting $x_\rho = x_\omega$ and found similar qualitative behavior.

To highlight the influence of hyperons on kaon condensation, it is useful to first consider the case where they are absent, i.e. nucleons-only matter. Results for this case are contained in table 2, and, in figures 1 and 2. The threshold density $n_c$ for condensation is determined from

$$
\mu^2 + \left(\frac{2n_p + n_n}{2 f^2}\right) - m_K^2 - \left[2a_1 n_p^s + (2a_2 + 4a_3)(n_p^s + n_n^s)\right] \frac{m_s}{2 f^2} = 0,
$$

which is obtained by setting $\theta = 0$ in Eq. (12) and restricting the sum over $B$ to $n$ and $p$ only. Expressing $n_c$ as a ratio to equilibrium nuclear matter density, $n_0$, we show in table 2 results for $u_c = n_c/n_0$. The larger the magnitude of $a_3 m_s$, the lower is the value of $u_c$. In addition, $u_c$ is systematically lower for the MFT models than for the MRHA models. This latter feature is a consequence of the more rapid increase of the scalar densities in the mean field model than in the MRHA model (contrast panel (c) in fig. 1 and fig. 2).

Due to the rapidly growing condensate amplitude (see panel (e)), the net negative charge carried by the kaon fields grows rapidly with density so that the leptons play only a minor role in maintaining charge neutrality (the partial fractions are shown in panel (a)). At high density, matter contains nearly equal abundances of neutrons and protons. The softening induced by the presence of kaons has the consequence that the maximum masses of stars are reduced from the case without kaon condensation \[3,4\]. It is worthwhile to point out that if the distinction between the scalar and vector densities is ignored in Eq. (16), condensation sets in at somewhat lower densities than those shown in table 2.

To summarize the case of nucleons-only matter: the onset of kaon condensation is controlled by two factors, the first being the behavior of the scalar density in dense matter and the second being the magnitude of $a_3 m_s$ which is related to the strangeness content of the nucleon. Additional work is needed to further pin down these quantities.
We turn now to the role of hyperons in matter, results for which are shown in fig. 3 for the mean field model B with $a_3 m_s = -222$ MeV. Panel (a) of this figure shows the composition. One expects that the Λ and the Σ$^-$ first appear at roughly the same density, because the somewhat higher mass of the Σ$^-$ is compensated by the presence of the electron chemical potential in the equilibrium condition of the Σ$^-$ (see Eq. (13)). More massive and more positively charged particles than these appear at higher densities. Use of Eq. (15) to constrain the hyperon couplings has the consequence that the first strange particle to appear is the negatively charged Σ$^-$ hyperon. The Σ$^-$ competes with the leptons in maintaining charge neutrality, so the lepton concentrations begin to fall. This is reflected in the electron chemical potential $\mu_e$ (see panel (b)) which saturates at about 200 MeV and begins to decrease from this value with increasing density. Since $\mu_e = \mu$, this has the consequence that kaon condensation now takes place, i.e. Eq. (12) is satisfied, at a higher density than in nucleons-only matter. This effect is seen for all the cases listed in table 2.

Some insight into the role of the scalar densities may be gained by examining the threshold condition Eq. (12) in the presence of the Σ$^-$ and Λ hyperons only:

$$\mu^2 + \frac{(2n_p + n_n - n_{\Sigma^-})}{2 f^2} \mu - m^2_K - \left[2a_1 n_p^s + (2a_2 + 4a_3)(n_p^s + n_n^s + n_{\Sigma^-}^s)\right] \frac{m_s}{2 f^2} = 0 .$$

(17)

The first two terms in this equation are smaller than in the nucleons-only case and this has to be compensated by the last term, which requires a higher density. The increase in the critical density is less in the MFT models, since the scalar densities of the various particles increase more rapidly than in the MRHA models. In fact, for the MRHA the central density of a star containing all the baryons, but no kaons, ($u_{\text{cent}}$ in table 2) is less than the critical density $u_c$ in almost all cases, which implies that kaons will not be present. This is also the case for the MFT when the magnitude of $a_3 m_s$ is the smallest; however for larger values, which we favor, a kaon condensate will be present in the star.
Returning to fig. 3, which corresponds to the MFT with $a_3m_s = -222$ MeV, we see that in the presence of hyperons, the condensate amplitude increases rapidly with density (panel (d)); so rapidly in fact that large changes are induced in the scalar densities (panel (c)) and the Dirac effective masses of all the particles (panel (b)). The kaon contribution to the Dirac effective masses of the nucleons

\begin{align}
M_p^* &= M - g_{\sigma n}\sigma + (2a_1 + 2a_2 + 4a_3)m_s \sin^2 \frac{1}{2}\theta , \\
M_n^* &= M - g_{\sigma n}\sigma + (2a_2 + 4a_3)m_s \sin^2 \frac{1}{2}\theta ,
\end{align}

is negative and this causes the effective masses to vanish at a finite baryon density. Thus, within the mean field scheme adopted here and without an explicit treatment of a hadron to quark matter transition, a calculation of the thermodynamics to encompass the full range of densities in a neutron star is precluded. This indicates the need to consider several improvements: (i) inclusion of additional terms in the effective Lagrangian, such as the $p$-wave interactions, (ii) inclusion of loop corrections in kaon-baryon interactions, in particular an exact evaluation of the zero-point energy with the non-linear Kaplan-Nelson Lagrangian.

In conclusion, we have shown that in dense matter the presence of hyperons leads to kaon condensation at higher densities than would otherwise be the case. The critical densities depend sensitively on the behavior of the scalar densities or, equivalently, the baryon Dirac effective masses in dense matter. A more rapid variation in these quantities results in lower thresholds for condensation. A mean field treatment of the kaon-baryon interactions produces rather large effects so that the nucleon effective masses vanish at a finite baryon density, indicating unphysical behavior. Clearly further work is needed to fully understand the role of the hyperon degrees of freedom on kaon condensation in dense stellar matter.

We thank Gerry Brown for much encouragement and for valuable discussions. We are grateful to Norman Glendenning for much helpful correspondence. This work was supported
in part by the U. S. Department of Energy under grant numbers DE-FG02-88ER40328 (PJE) and DE-FG02-88ER40388 (MP and RK).

References

[1] D. B. Kaplan and A. E. Nelson, Phys. Lett. B175 (1986) 57.

[2] H. D. Politzer and M. B. Wise, Phys. Lett. B273 (1991) 156; G. E. Brown, K. Kubodera, M. Rho and V. Thorsson, Phys. Lett. B291 (1992) 355.

[3] V. Thorsson, M. Prakash and J. M. Lattimer, Nucl. Phys. A572 (1994) 693.

[4] G. E. Brown, C-H. Lee, M. Rho and V. Thorsson, Nucl. Phys. A567 (1994) 937; T. Maruyama, H. Fujii, T. Muto and T. Tatsumi, Phys. Lett. B337 (1994) 19, and references therein.

[5] N. K. Glendenning, Ap. J. 293 (1985) 470; Nucl. Phys. A493 (1989) 521.

[6] M. Prakash et al., Stony Brook preprint, SUNY-NTG-94-32 (1994).

[7] T. Muto, Prog. Theor. Phys. 89 (1993) 415.

[8] S-J. Dong and K-F. Liu, University of Kentucky preprint UK/94-07, Lattice 94, Nucl. Phys. B (Proc. Suppl.), To be published.

[9] B. D. Serot, Rep. Prog. Phys. 55 (1992) 1855.

[10] E. K. Heide and S. Rudaz, Phys. Lett. B262 (1991) 375.

[11] M. Prakash, P. J. Ellis, E. K. Heide and S. Rudaz, Nucl. Phys. A575 (1994) 583.

[12] N. K. Glendenning and S.A. Moszkowski, Phys. Rev. Lett. 67 (1991) 2414.
Figure Captions

Fig. 1. Results are for nucleons-only matter in the mean field model B with $a_3 m_s = -222$ MeV. (a) Relative fractions $Y_i = n_i/(n_n + n_p)$. (b) Nucleon Dirac effective masses, the kaon chemical potential $\mu = \mu_e$ and the scalar field $\sigma$. (c) Scalar densities with $n^s = n^n_s + n^p_s$. (d) Pressure with and without kaon condensation. (e) Condensate amplitude $\theta$ in degrees.

Fig. 2. Results are for nucleons-only matter in the Hartree MRHA model with $\mu/M = 0.79 a_3 m_s = -222$ MeV. (a) Relative fractions $Y_i = n_i/(n_n + n_p)$. (b) Nucleon Dirac effective masses, the kaon chemical potential $\mu = \mu_e$ and the scalar field $\sigma$. (c) Scalar densities with $n^s = n^n_s + n^p_s$. (d) Pressure with and without kaon condensation. (e) Condensate amplitude $\theta$ in degrees.

Fig. 3. Results are for matter with hyperons in the mean field model B with $a_3 m_s = -222$ MeV. (a) Relative fractions $Y_i = n_i/(\sum_B n_B)$. (b) Baryon Dirac effective masses, the kaon chemical potential $\mu = \mu_e$ and the scalar field $\sigma$. (c) Baryon scalar densities. (d) Condensate amplitude $\theta$ in degrees.
Table 1
MRHA coupling constants

| $\frac{\mu_r}{M}$ | Without hyperons | With hyperons |
|-------------------|-----------------|---------------|
| $K_0$             | $C_\omega^2$    | $C_\sigma^2$  | $C_\rho^2$ | $K_0$ | $C_\omega^2$ | $C_\sigma^2$ | $C_\rho^2$ | $x_\omega$ |
| 0.79              | 354             | 180.6         | 317.5       | 73.5  | 177           | 118.7         | 258.1       | 84.8       | 0.66     |
| 1.00              | 461             | 137.7         | 215.0       | 81.6  | 455           | 133.1         | 210.3       | 82.4       | 0.66     |
| 1.25              | 264             | 78.6          | 178.6       | 90.8  | 228           | 64.9          | 174.0       | 92.6       | 0.68     |

Here $C_i^2 = (g_{iN}M/m_i)^2$ and $K_0$ (in MeV) is the compression modulus. The parameters are fitted to a nuclear matter binding energy of 16 MeV and a symmetry energy of 30 MeV at a density of $0.16 \text{ fm}^{-3}$. For the hyperon case we take $x_\sigma = x_\rho = 0.6$.

Table 2
Critical density ratio, $u_c = n_c/n_0$, for kaon condensation

| $a_3 m_s$ | Without hyperons | With hyperons |
|-----------|-----------------|---------------|
|           | $u_c$           | $u_c$         | $u_{cent}^{a)}$ |
| MFT       |                 |               |               |
| A         | 4.14            | 3.14          | 2.49          | 8.97           | 4.20          | 2.74          | 6.72          |
| B         | 4.15            | 3.15          | 2.49          | 9.46           | 4.22          | 2.73          | 7.66          |
| $\mu_r/M = 0.79$ | 5.22     | 4.54          | 3.89          | 15.3           | 9.87          | 7.32          | 7.81          |
| MRHA      | $\mu_r/M = 1.00$ | 5.12          | 4.33          | 3.61          | 13.9          | 8.64          | 6.05          | 5.64          |
|           | $\mu_r/M = 1.25$ | 5.24          | 4.55          | 3.93          | 20.8          | 13.4          | 9.57          | 8.44          |

$^{a)} u_{cent} = n_{cent}/n_0$ is the central density ratio of a neutron star containing hyperons and nucleons in the absence of kaons. It can be compared with the threshold density $u_c$ for kaon condensation.
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/nucl-th/9502033v1
This figure "fig1-2.png" is available in "png" format from:

http://arxiv.org/ps/nucl-th/9502033v1
This figure "fig1-3.png" is available in "png" format from:

http://arxiv.org/ps/nucl-th/9502033v1