Electroweak Sudakov Corrections and the Top Quark Forward-Backward Asymmetry

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The Standard Model (SM) prediction of the top quark forward backward asymmetry is shown to be slightly enhanced by a correction factor of 1.05 due to electroweak Sudakov (EWS) logarithms of the form \((\alpha/\sin^2 \theta_W)^n \log^{m\leq 2n} (s/M_{W,Z}^2)\). The EWS effect on the dijet and \(t\bar{t}\) invariant mass spectra is significant, reducing the SM prediction by \(
\sim 20\%\) respectively for the highest invariant masses measured at the LHC, and changing the shape of the high-mass tail of the spectrum. These corrections significantly affect measurements of the top quark invariant mass spectrum and the search for an excess of events related to \(A_{FB}^{tt}\).

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I. Introduction: In high-energy scattering processes at the LHC, with partonic center-of-mass energies \(\sqrt{s}\) much larger than the electroweak scale, the \(W\) and \(Z\) bosons act as massless particles in an unbroken gauge theory. The wide separation of scales, \(s \gg M_{W,Z}^2\) leads to Sudakov double logarithms \(\alpha_W L^2, L = \log s/M_{W,Z}^2\), \(\alpha_W = \alpha/\sin^2 \theta_W\), at each order in perturbation theory, which can be substantial (e.g. \(\sim 37\%\) for \(WW\) production at 2 TeV). While QCD Sudakov corrections cancel for inclusive processes, the electroweak ones do not, because the incident beams are not electroweak singlets\(^1\). Recently\(^2\) effective field theory (EFT) methods have been used to systematically sum the electroweak Sudakov (EWS) corrections using renormalization group methods. The EFT result is naturally given in terms of \(\log \sigma\), and has the schematic form

\[
\log \sigma = L f_0 (\alpha_W L) + f_1 (\alpha_W L) + \ldots
\]  

(1)

in terms of the leading-log series \(f_0\), the next-to-leading-log series \(f_1\), etc. The EFT computation neglects power corrections of the form \(M_{Z}^2/s\), but includes the complete dependence on electroweak scale mass-ratios such as \(M_W/M_Z, m_t/M_Z\) and \(M_H/M_Z\). The results in Ref.\(^2\) include the complete NNLL series including Higgs effects, and the most important terms in the NNLL series. The omitted NNLL corrections are Higgs effects in the three-loop cusp anomalous dimension and a two-loop matching correction which are not known. The EWS resummation cannot be performed simply by exponentiating a fixed order result, because there are different color and flavor structures which mix under renormalization group evolution in the EFT. While the EFT is formally not valid near threshold, numerically, the results are still quite accurate because the EWS corrections are not log-enhanced in this region (see Ref.\(^2\)).

The EWS logs grow with energy, and are important for large invariant mass measurements, such as the recent CDF measurement of the top-quark forward-backward asymmetry, \(A_{FB}^{tt}(M_{t\bar{t}})\)\(^4\), which has a \(\sim 3\sigma\) deviation from the SM prediction\(^4\) at \(M_{t\bar{t}} > 450\) GeV. In this paper, we study EWS effects on observables needed to study \(A_{FB}^{tt}(M_{t\bar{t}})\), as a function of the \(t\bar{t}\) invariant mass \(M_{t\bar{t}}\). Since the EWS effects are a multiplicative correction, we present them as rescaling factors, by taking the ratio of \(A_{FB}^{tt}\) computed with and without the EWS effect. This greatly reduces the sensitivity of our results to the choice of parton distribution functions (PDF) or QCD corrections.

We find that the SM EWS corrections enhance \(A_{FB}^{tt}\) by a factor 1.05.\(^1\) They also suppress the \(d\sigma/dM_{t\bar{t}}\) spectrum at large invariant mass, which is crucial in attempts to understand if the \(A_{FB}^{tt}\) anomaly is a sign of new physics or not. We emphasize that neglected SM electroweak Sudakov corrections can cancel a t-channel driven rise in \(d\sigma/dM_{t\bar{t}}\) at \(M_{t\bar{t}} > 500\) GeV due to possible new physics associated with the \(A_{FB}^{tt}\) anomaly.

The overall effect of EWS effects on phenomenology related to \(A_{FB}^{tt}\) can be even more significant when they also impact attempts to measure the top quark invariant mass spectrum indirectly. For example, in highly boosted top studies\(^5\) a precise understanding of the normalization and shape of the SM dijet invariant mass spectrum is essential, and we will show there is also a suppression of \(\sim 10 - 20\%\) due to EWS logs for large invariant mass dijet events. These SM effects are unaccounted for in current Monte Carlo tools,\(^6\)

II. \(t\bar{t}, b\bar{b}, c\bar{c}\) Phenomenology: In\(^7\) the LO SM asymmetry \(A_{FB}^{tt}\) was computed from the \(\mathcal{O}(\alpha^2)\) cross-section, and a subset of the fixed order \(\mathcal{O}(\alpha^2)\) terms were also determined. These calculations are based on earlier results\(^7\) on the \(e^+e^- \rightarrow \mu^+\mu^-\gamma\) asymmetry, and \(q\bar{q} \rightarrow Q\bar{Q}\sigma\)\(^8\). A recent SM calculation\(^9\) extended the calculation of \(\mathcal{O}(\alpha^2)\) terms and included \(\mathcal{O}(\alpha^2)\) corrections from photon radiation. The effect of next-to-leading

\(^1\) The correction can be applied to partonic calculations (even those including non SM interactions) when calculating if the process has the same SU(2) × U(1) gauge flow as the SM. Note, however, that our final results are given averaging over quark spins, and SM corrections are different for left and right-handed quarks. We restrict ourself to \(0.1 \leq \theta_{CM} \leq 0.9\pi\) to avoid soft scattering. This cut is less restrictive than experimental cuts.

\(^2\) Herwig and Sherpa include pure QED soft and collinear photon re-summations\(^10\), but not these EWS corrections.
as well as next-to-next-to leading logarithmic QCD corrections have been studied in [10]. An interesting discrepancy remains between the SM prediction of \( A_{FB}^\mu \) at large invariant mass \((M_\ell > 450 \text{ GeV})\), and the CDF measurement [3].

The EFT method we use can be illustrated using the process \( qq \rightarrow t\bar{t} \) for left-handed quarks. At the high scale \( \mu = Q = \sqrt{s} \), the scattering is given by an effective Lagrangian

\[
L = C_{11} q^a T^A \tau_i Q^b T^A \tau_j Q + C_{12} q^a T^A \tau_i Q^b T \tau_j Q + C_{21} q^a T^A \tau_i Q^b T^A \tau_j Q + C_{22} \bar{q}^a \tau_i \bar{Q}^b \tau_j \bar{Q},
\]

where \( q = (u, d)_L \) or \((c, s)_L\) are light quark doublets, and \( Q = (t, b)_L\) is the heavy quark doublet. \( T^A \) are color matrices, \( t^a\) are \( SU(2) \) matrices and \( C_{ij}(\mu)\) are hard-matching coefficients. At tree-level, \( c(Q)\) is given by single gauge boson exchange. Gluon exchange gives \( C_{12} = 4\pi \alpha_s/Q^2 \), \( W\) exchange gives \( C_{12} = 4\pi \alpha_s W^2/Q^2 \), and \( B\) exchange gives \( C_{22} = 4\pi \alpha_s W^2/(1/6)^2 \). At one-loop, \( C_{ij}(\mu)\) are given by computing the finite part of one-loop graphs such as box-graphs with all low scales such as \( M_Z\) set to zero. The hard-matching \( C_{ij}(\mu)\) is computed at the scale \( \mu = Q\), and does not contain any large logarithms. The Lagrangian is evolved in the EFT to a low-scale of order \( M_Z\), and then the scattering cross-section is taken by squaring the EFT amplitude and integrating with PDFs. The EWS terms arise from the renormalization group evolution of the coefficients \( C_{ij}\) from \( \mu = Q \) down to \( \mu \sim M_Z\). This method has been checked against fixed order computations up to two-loop order, and details can be found in Ref. [2].

Here we report on the numerical computation of EWS corrections to dijet and \( t\bar{t} \) production. These corrections are defined as

\[
R_{FB}(t) = \frac{\sigma_{QCD+EWS}(tt)}{\sigma_{QCD}(tt)} , \quad \tau_t = \frac{\sigma_{QCD+EWS}(t\bar{t})}{\sigma_{QCD}(t\bar{t})}.
\]

\( \sigma_{FB} \) and \( \sigma_{t\bar{t}} \) are the forward-backward asymmetry and the total cross-section. The superscript \( QCD + EW \) means that the EFT calculation is done using the full standard model, and QCD means that QCD alone has been used. The cross-sections include virtual electroweak effects, but not real radiation of additional EW bosons. In dijet production, for example, such events would be part of the \( W, Z + \text{jets} \) signal. With this definition, multiplying by \( \tau_t \) converts a QCD computation into one including EWS corrections as well. The QCD computation can be done using an EFT, or by any other method. The ratios are insensitive to the choice of PDF.

We incorporate EWS corrections by modifying the analytic results of [3] using the results of Ref. [2]. The asymmetry is defined as the ratio \( A = (F - B)/(F + B) \), where \( F \) and \( B \) are the cross-section in the forward and backward hemisphere. In QCD, \( F \) and \( B \) are order \( \alpha_s^2 \), whereas \( F - B \) is order \( \alpha_s^3 \) because the order \( \alpha_s^2 \) cross-section is \( FB \) symmetric. The EWS corrections are not \( FB \) symmetric. There are three contributions that we include that have been previously neglected: (a) the change in the normalization of the LO cross section of order \( \alpha_s^2 \alpha_W^2 L^{m \leq 2n} \) given by \( R_t \) which multiplies the denominator in \( A \). (b) a new term in the numerator of \( A \) of order \( \alpha_s^2 \alpha_W^2 L^{m \leq 2n} \) from multiplying the \( FB \) symmetric QCD cross-section by the EWS corrections \( (\tau_t^F)^2(t) \) in Table I. (c) the effect of EWS corrections on the leading QCD FB asymmetry of order \( \alpha_s^3 \alpha_W^2 L^{m \leq 2n} \). The sum of (b) and (c) is \( R_{FB}(t) \) in Eq. (3), and is the total rescaling of the numerator of \( A \).

We use NLO MSTW PDFs [11] with the LO QCD results and \( \mu = m_t = 173.1 \text{ GeV} \) for the factorization and renormalization scales. \( \alpha_s \) is set by the MSTW fit value: \( \alpha_s(M_Z) = 0.1208 \). Numerical values are given in Table I. We find \( A_{FB}^\mu = 7.4\% \) and \( A_{FB}^\mu(m_{t\bar{t}} < 450) = 5.3\% \), \( A_{FB}^\mu(m_{t\bar{t}} > 450) = 10.7\% \) for the purely QCD asymmetry, in good agreement with other determinations [3, 4, 9, 10]. We find that (a) and (c) essentially

\[
\begin{array}{cccccc}
\text{Bin [GeV]} & A_{FB}^\mu(\%) & R_{FB}(b) & R_\alpha & A_{FB}^\mu(\%) & R_{FB}(c) \\
[2 m_{t\bar{t}}, 1600] & 7.7 & 1.6 & 1.02 & 1.03 & 0.98 \\
[2 m_{t\bar{t}}, 450] & 5.6 & 5.4 & - & 1.02 & 1.03 & 0.98 \\
[450, 900] & 11 & 12 & 2.3 & 1.02 & 1.04 & 0.97 \\
\end{array}
\]

\[
\begin{array}{cccccc}
\text{Bin[GeV]} & A_{FB}^\mu(\%) & R_{FB}(b) & R_\alpha & A_{FB}^\mu(\%) & R_{FB}(c) \\
[50, 1960] & 0.4 & 0.99 & 0.99 & 0.99 & 0.99 \\
[50, 350] & 0.4 & 0.96 & 0.93 & 0.98 & 0.99 \\
[350, 650] & 8.1 & 1.00 & 6.7 & 6.6 & 1.04 & 1.00 \\
[650, 950] & 20 & 0.97 & 0.98 & 18 & 16 & 1.06 & 0.99 \\
\end{array}
\]
The overall rescaling of $A$ is $R_{t\bar{f}}(t)/R_t$. In all the mass bins that we have considered, the net EWS effect is an enhancement of $A^t_{FB}$ by a factor of 1.05.

There is some interest in measuring $A^t_{FB}$, $A^{\bar{f}Z}$ to investigate the possible origin of the $A^t_{FB}$ anomaly [13–15]. The EWS corrections for these observables are given in Table III. We have normalized the $A_{FB}$ calculations by the LO QCD cross section, as no complete NLO correction of the asymmetric cross section is known. This approach leads to the estimate of $A^t_{FB}$ being larger (by about 1.3) than the results when normalized by the NLO cross section, such as with MCFM [4]. Normalizing by inclusive NLO cross sections $\sigma_f fX$ ($f = b, c$) will lead to an even larger reduction for $A^t_{FB}$, due to the $t$-channel singularity enhancing production via $gg \to gg \to f f X$. For this, and other reasons [13–15], the reported asymmetries are extremely challenging to probe experimentally.

EWS corrections only make a small change to the total cross section, since $t$ is dominated by low invariant mass events because of the PDFs, where the EWS correction is small. However, the tails of the invariant mass distributions have significant EWS corrections that grow in importance with invariant mass. For the $t\bar{t}$ mass bins reported by CDF [16], the correction factors $R_t$ are {0.99, 0.98, 0.98, 0.97, 0.97, 0.96, 0.96, 0.95}. The $R_{b,c}$ corrections are less than 2% in this region. At the LHC, there are larger effects due to EWS corrections. Some values of $R_{t,b,c}$ are given in Table III.

Preliminary measurements of the reconstructed $d\sigma/dM_{t\bar{t}}$ spectrum have been reported by ATLAS [17] and CMS [3], and no large deviation from the SM has been found. At the large invariant masses studied in [3], both the $t\bar{t}$ production rate and the subtracted dijet background rate receive large EWS corrections.\(^5\) Both corrections act to increase the possibility for a non-resonant excess of large $M_{t\bar{t}}$ events in this study, since they suppress the SM rate, and should be taken into account before any precise conclusions can be drawn. A data driven normalization of the Monte Carlo estimation of the $d\sigma/dM_{t\bar{t}}$ spectrum, that is subsequently extrapolated to large $s$ to search for deviations in the tail from the SM expectation, is also susceptible to large EWS corrections.

As a specific example of the importance of these corrections, note that some plausible flavor symmetric models of new physics that could marginally explain the $A^t_{FB}$ anomaly can cause a rise in the tail of $d\sigma/dM_{t\bar{t}}$ by ~10% [18]. Such an effect could be completely canceled by SM EWS corrections currently unaccounted for in MC simulation tools.

### III. Dijets: EWS corrections are also important for dijet studies at the Tevatron and LHC.\(^6\) We evaluate EWS corrections for all partonic LO $2 \to 2$ QCD dijet processes with a rapidity cut, $|y| < 1$, implemented as described in [24] for the Tevatron dijet corrections. The quark flavours ($u, c, s, d, b$) and gluon initial and final partonic states are summed over. We average over the bin mass range, which is taken to be 10% of the central value of the bin as in [25]. The renormalization scale is $\mu = M_Z$. Varying the scale in the range $\mu = (M_Z/2, 2M_Z)$, or the

\(^4\) Cancelations of some EWS corrections in $A_{FB}$, were also noted in [12], which studied an SU(2) theory.

\(^5\) We thank Gilad Perez for discussion on this point.

\(^6\) Our results are consistent with previous results using infrared evolution equations [20] or a SU(2) theory to sum Sudakov logarithms. Jets studies based on these techniques include [12, 21, 22]. Fixed order EW corrections to dijet rates also give large corrections [23]. Our results are for the full SU(2) × U(1) theory including $\gamma - Z$ mixing and Higgs effects.
A recent ATLAS study [28] has reported dijet events out to $M_{jj} \sim 5$ TeV with the 2011 data set, where the effects of EWS corrections on the QCD partonic 2 → 2 dijet processes are significant, $\sim -20\%$. These corrections can act to cancel a t-channel driven rise in the dijet invariant mass spectrum in models attempting to explain the $A_{FB}^{\ell\bar{\ell}}$ excess that involve new light quark interactions.

IV. Conclusions: We have determined the EWS corrections for a number of observables of current interest. EWS corrections enhance $A_{FB}^{\ell\bar{\ell}}$ by a factor of 1.05 and slightly reduce the tension between the SM prediction and the measurement of $A_{FB}^{\ell\bar{\ell}}(M_{t\bar{t}} > 450$ GeV) reported in [8]. They give a significant correction to the multi-TeV dijet spectrum at the LHC and are important for determining the tail of the dijet spectrum. Many models constructed to explain $A_{FB}^{\ell\bar{\ell}}$ introduce new interactions that increase the tail of this spectrum. This can be compensated for by the EWS corrections which are not included in MC simulation tools. Similarly, constraints based on the extracted tail of the $d\sigma/dm_{t\bar{t}}$ spectra are important when considering the $A_{FB}^{\ell\bar{\ell}}$ anomaly; the SM prediction of the tail of this spectra also receives large EWS corrections.

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![Graph](image)

**FIG. 2:** Same as Fig. 1 for the LHC at $\sqrt{s} = 7$ TeV (lower curves) and 14 TeV (upper curves).

choice of MSTW PDF eigenvalues used, changes the results by less than 1%. The EWS correction factor for the Tevatron is shown in Fig. 1.

The total dijet rate involves partonic processes with and without external gluons, which cannot be separated experimentally. The EWS corrections are very small for gluonic processes, since the gluon is an EW singlet. This dilutes the overall importance of EWS effects for inclusive dijet production for low invariant mass events. To illustrate this, we have also shown in Fig. 1 the EWS corrections to dijet processes involving and not involving external gluons, as well as the ratio of these contributions to the total dijet rate.

The LHC results are shown in Fig. 2. We have imposed typical central jet rapidity cuts ($|y| < 2.8$) and cuts on the separation of the rapidity of the two leading jets ($|\Delta y| < 1.2$) consistent with the ATLAS study [28]. The results are insensitive to the particular rapidity cuts made. For example, varying the rapidity cut from $2.8 \rightarrow 2$ leads to a variation in the total EWS correction of less than 1%. We have also determined the EWS correction to the angular distribution measure $F_{\chi}[M_{jj}]$ as defined in [27]. EWS corrections suppress this ratio by $\sim 2(5)\%$ for 2(4) TeV dijet masses, the correction factor to apply to the SM calculation of $F_{\chi}[M_{jj}]$ is well approximated by $1 - 0.125M_{jj}^{0.258} + 0.143M_{jj}^{0.239}$ for the mass range 0.5 – 5 TeV. This correction slightly relaxes angular distribution constraints on new physics.

We find large corrections that must be included for precise studies of multi-TeV dijet events at both $\sqrt{s} = 7$ and 14 TeV. The importance of EWS corrections in dijet studies changes with the LHC operating energy, since the relative importance of gluonic and non-gluonic processes is largely driven by the PDF’s. Gluonic dijet events become more important as the operating energy increases. A recent ATLAS study [28] has reported dijet events out to $M_{jj} \sim 5$ TeV with the 2011 data set, where the effects of EWS corrections on the QCD partonic 2 → 2 dijet processes are significant, $\sim -20\%$. These corrections can act to cancel a t-channel driven rise in the dijet invariant mass spectrum in models attempting to explain the $A_{FB}^{\ell\bar{\ell}}$ excess that involve new light quark interactions.

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