Polarized Gluon Distribution $\Delta g(x)$ in the Proton

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Abstract

We study the polarized gluon distribution $\Delta g(x)$ in a longitudinally polarized proton. We argue that the distribution can be calculated approximately from the quark color currents found in simple quark models. The first result from the MIT bag model as well as the non-relativistic quark model shows that $\Delta g(x)$ is positive at all $x$. If this feature holds in QCD, it imposes a strong constraint on phenomenological fits to experimental data. The total gluon helicity $\Delta G$ from the bag model is about $0.3\hbar$ at the scale of 1 GeV, considerably smaller than previous theoretical expectations.
The polarized gluon distribution in the proton $\Delta g(x)$ has been the focus of high-energy spin physics since the European Muon Collaboration measurement of the quark helicity distributions through polarized inclusive deep-inelastic scattering [1]. Gluons are known to play the key role in the proton’s mass and momentum [2], and it is expected that they also play a similar important role in the spin of the proton. This expectation has motivated extensive experimental programs at DESY (HERMES collaboration), CERN (COMPASS Collaboration), and polarized RHIC. Much experimental progress has been made in probing the polarized gluon distribution through leading hadron production, charm production, and neutral pions and di-jets [3]. High statistics data expected from the polarized RHIC will provide a much better picture on $\Delta g(x)$ in the near future [4].

Two fundamental questions about the gluon polarization have attracted the most attention: what is its total size $\Delta G = \int_0^1 dx \Delta g(x)$, and how does its sign vary with Feynman momentum $x$. Both questions are difficult to answer in practice. Since $\Delta G$ is not related to the matrix element of a local, gauge-invariant operator, it cannot be calculated directly in lattice QCD simulations, which have been the only non-perturbative approach to solve the fundamental theory so far. Experimentally, it is a challenge to measure $\Delta g(x)$ directly at a fixed-Feynman $x$, with the exception of few channels at tree order (e.g. direct photon production [4]). The phenomenological distributions in the literature are obtained by “educated” parametrizations and fitting of the parameters to experimental data [5]. The results depend on the functional forms adopted, sensitive to assumptions such as if $\Delta g(x)$ is allowed to change sign in $x$. Given the above situation, it is important to investigate the possibility of calculating $\Delta g(x)$ in proton models, with the hope that some key features might be shared by QCD.

Gluons are known to play the dominant role in QCD. The gluons in the QCD vacuum are responsible for, among others, color confinement and chiral symmetry breaking. Modeling these gluons is beyond the scope of this study. On the other hand, the gluons in the unpolarized proton contribute as much as 50% of its momentum and mass [2]. These gluons are generated from the valence quarks and affect strongly the dynamics of quarks in return. Therefore they must be solved self-consistently with the motion of the quarks, which is again outside the scope of this paper.

On the other hand, the polarized gluons—induced through quark polarizations in a polarized proton—is a much smaller effect. In fact, in QCD with a large number of colors $N_c$, the polarized gluons are suppressed by $1/N_c^2$ related to those in the QCD vacuum. As a consequence, the spin-dependent gluon potential $A^\mu$ may be solved from the chromodynamic Maxwell equation,

$$D_\mu F^{\mu\nu} = J^{\nu},$$

(1)

with some reasonable modeling of the spin-dependent quark color current $J^\mu$. This situation is analogous to the small-$x$ gluons which are calculable from the valence quark color charges in the saturation region [8].

SU(6) quark models have had some reasonable successes in describing the valence quark structure of the proton. For example, they give a reasonable account of the magnetic moment of the proton. In particular, the signs and magnitudes of the magnetic moments of the up and down quarks are correlated positively with the total angular momentum carried by them ($\mu_u = 3.6, \mu_d = -1.0$, vs. $J_u = 4/3$ and $J_d = -1/3$). What about the proton spin for which the naive quark model prediction seems to have failed so badly? Well, in the MIT bag model, although the spin is carried entirely by quark angular momentum in the lowest-order wave function, about 40% comes from the orbital motion of the quarks [9]. Once the gluon
contribution is taken into account by the higher Fock states, the quark contribution must be renormalized from these additional wave function components. If the gluons contribute 50% of the proton spin, for example, the renormalized singlet axial charge in the bag model will be about $\Delta \Sigma = 0.60/2 = 0.30$, roughly consistent with the current experimental data. Therefore, as a first estimate, the polarized gluons may be calculated from the quark color currents in the MIT bag model.

The total gluon polarization $\Delta G$ has been studied before in quark models in Refs. [10, 11]. In Ref. [10], the calculation is incomplete because only the interference diagram has been included, and the contribution from a single quark intermediate states has been ignored. The result is a negative $\Delta G$. In Ref. [11], the single quark contribution was taken into account in the non-relativistic quark model and $\Delta G$ is found to be positive after canceling the negative interference contribution. As we shall see, a direct calculation of $\Delta g(x)$ in non-local operator form produces actually a different total $\Delta G$ that is consistent with parton physics.

The polarized gluon distribution $\Delta g(x)$ can be calculated as a matrix element of the non-local operator [12]

$$\Delta g(x) = -\frac{i}{x} \int d\lambda \frac{d\lambda}{2\pi} e^{-i\lambda x} \langle P| F^{+\alpha}(\lambda n) W_{\alpha}^{+}(0)|P \rangle ,$$  

where $|P\rangle$ is the proton state normalized covariantly, $n$ is a light-like vector conjugating to an infinite momentum frame $P$. $F^{\mu\nu}$ is the gluon field tensor and $W$ is a gauge link along the direction $n$ connecting the two gluon field tensors, making the whole operator gauge invariant. In this first calculation, we neglect the effects of the nonlinear interactions, and then the gluons fields behave as 8 independent Abelian fields. Under this approximation, the gauge link can be ignored and an equivalent expression is obtained by inserting a complete set of intermediate states between the gluon field tensors,

$$\Delta g(x) = -i \int \frac{d^4k}{(2\pi)^4} \frac{1}{V_4} \epsilon^{\alpha\beta}(k^+g^{\alpha\mu}g^{\beta\nu} - k^\alpha g^{+\mu}g^{\beta\nu} - k^\beta g^{+\nu}g^{\alpha\mu}) \delta(x - k \cdot n) \times \sum_m \langle \tilde{P}| A^*_\mu(k)|m\rangle \langle m| A_\nu(k)|\tilde{P}\rangle (V_3 \cdot 2P^+),$$  

where $m$ sums over all possible intermediate states, and $V_3$ and $V_4$ are the space and space-time volumes, respectively. The rescaled state $|\tilde{P}\rangle$ is normalized to 1.

![FIG. 1: One-body and two-body contributions to the matrix element of the polarized gluon operator in the quark models of the proton.](image)

In the quark models, the above matrix element receives contributions from one-body and two-body terms, shown schematically in Fig. 1. It is easy to show that the one-body term
with the same quark intermediate state as the initial one cancels the two-body contribution. This cancellation is due to the color structure of the states as well as the spin property of the operator. Therefore, the net contribution arises from one-body term with excited intermediate quark states. The matrix element in the single quark state is

\[ \langle m | A_\nu(k) | \bar{P} \rangle = 2\pi\delta(k^0 - (\epsilon_f - \epsilon_i)) \frac{-i}{k^2} (-igt^a) \langle m | j_\nu(k) | \bar{P} \rangle, \tag{4} \]

where the \( \delta \)-function comes from the energy conservation and \( j_\nu \) is the color current. For simplicity we have used the free-space gluon propagator.

The sum over all intermediate quark states produces a divergent result. This divergence is the usual ultra-violet divergence in field theory and must be regulated by cut-offs. In our case, the cut-off may be taken as the excitation energy of the intermediate states.

Shown on the left panel in Fig. 2 is the MIT bag result for \( \Delta g(x) \) with different intermediate state cut-offs. The bag radius \( R \) is chosen to be 1.18 fm by fitting the nucleon charge r.m.s. radius. The dotted curve corresponds to the s-wave contribution (\( \kappa = -1 \)) with zero and one node wave functions included. The dash-dotted curve includes in addition the p wave contribution (\( \kappa = -2, 1 \)); the dashed curve contains the d wave contribution (\( \kappa = -3, 2 \)); and finally the solid curve includes up to the f wave contribution (\( \kappa = -4, 3 \)).

Two features of the result are immediately obvious. First, \( \Delta g(x) \) is positive everywhere as a function of \( x \), and vanishes quickly as \( x \to 1 \). Second, as more intermediate states are included, \( \Delta g \) gets uniformly larger. In fact, for higher intermediate states, the result shall change with the cut-off following the perturbative QCD evolution. Therefore, \( \Delta g(x) \) at different scales can be obtained approximately by limiting the excitation energy of the intermediate quarks. The solid-line result roughly corresponds to a cut-off at 1 GeV. Of course, the present cut-off scheme is different from that of the perturbative dimensional regularization and minimal subtraction. The difference can in principle be calculated in perturbation theory.

![Graph showing \( \Delta g(x) \) and \( x\Delta g(x) \) calculated in the MIT bag model.](image)

**FIG. 2:** \( \Delta g(x) \) and \( x\Delta g(x) \) calculated in the MIT bag model. On the left panel, the results show successive additions of s, p, d, f wave contributions. In the second panel, the result (red solid line) is compared with that from phenomenological fit (dashed line surrounded by an uncertainty band).

The phenomenological gluon distributions have been obtained by fitting the \( Q^2 \) evolution of the spin-dependent structure function \( g_1(x) \) \[5\]. The result depends on the functional
form assumed for $\Delta g(x)$. In a recent study, the double spin asymmetries for pion production was also included in the fits \cite{13}. If one allows $\Delta g(x)$ to change sign, the fit generates a distribution with very large error bars. On the other hand, if one assumes that $\Delta g(x)$ is positive everywhere, the error becomes much smaller. On the right panel of Fig. 2, we have shown such a fit (dashed line with error band) together with our bag model result (solid line). The bag calculation is consistent with the fit, with significant strength at large and small $x$ where the model might not be trustable.

To see that the positive $\Delta g(x)$ is a generic feature of quark models, we have shown in Fig. 3 the result from a non-relativistic quark model. The different curves again show successive inclusion of higher intermediate excited states. The general shape is similar to that from the MIT bag, which is shown in the thin solid line. The model artifacts at small and large $x$ are clearly stronger.

![Graph showing $x\Delta g(x)$ calculated in non-relativistic quark model by summing contributions from $s, p, d,$ and $f$ waves. The thin solid line is the MIT bag model result.](image)

FIG. 3: $x\Delta g(x)$ calculated in non-relativistic quark model by summing contributions from $s, p, d,$ and $f$ waves. The thin solid line is the MIT bag model result.

Integrating over $x$, the $\kappa = -1$ intermediate state produces a result $\Delta G = 0.23 \bar{h}$ (with $\alpha_s = 2.55$ obtained by fitting $N - \Delta$ mass splitting). Including higher states up to $\kappa = 3, -4$, we find $\Delta G = 0.32 \bar{h}$, from which a smaller $\alpha_s$ at higher excitation energy is used. Therefore, at low-energy scales, $\Delta G$ is on the order of $0.2$ to $0.3\bar{h}$, which is considerably smaller than previous expectations \cite{14}. Indeed, the anomaly contribution from this $\Delta G$ is negligibly small, $(\alpha_s/2\pi)\Delta G \sim 0.01$.

To summarize, we have argued that the polarized gluon distribution $\Delta g(x)$ is calculable in quark models. We have carried out a first such calculation in the MIT bag, and the result shows that it is positive definite at all $x$. The total gluon helicity in the bag is on the order of $0.2 - 0.3\bar{h}$, which is substantially smaller than what has been expected in the past.

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