Comparison of the fluctuation influence on the resistive properties of the mixed state of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+x}$ and of thin films of conventional superconductor.

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The resistive properties of layered HTCS $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+x}$ in the mixed state are compared with those of thin films of conventional superconductors with weak disorder (amorphous $\text{Nb}_{1-x}\text{O}_x$ films) and with strong disorder ($\text{Nb}_{1-x}\text{O}_x$ films with small grain structure). The excess conductivity in the mixed state is considered as a function of the superconducting electron density and the phase coherence length. It is shown that the transition into the Abrikosov state differs from the ideal case both in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+x}$ and $\text{Nb}_{1-x}\text{O}_x$ films, i.e. the appearance of the long-range phase coherence is a smooth transition in both cases. The quantitative difference between thin films with weak and strong disorder is greater than the difference between $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+x}$ and conventional superconductors, showing that the dimensionality of the system, rather than the critical temperature, is the key factor ruling fluctuation effects.

I. INTRODUCTION

One of the obstacles to the wide application of high temperature superconductors (HTSC) in a high magnetic field is the fluctuation influence on the resistive properties of the mixed state. Therefore this topic is much more popular now than it was before the discovery of HTSC. Previously, the majority of scientists thought that the fluctuation effects in superconductors are quite small, and now many are of the opinion that the fluctuation effects are big in HTSC only. Indeed, it is currently believed that higher critical temperatures enhance drastically the effects of thermal fluctuations. However, this is not right.

The fluctuation value is determined by the ratio of the thermal energy, $k_BT$, to the effective energy of a superconductor $F_{\text{eff}} = (f_n(T) - f_s(T))V_{\text{ef}}$. Here $f_n(T) - f_s(T)$ is the difference of the energy density in the normal and superconducting states; $V_{\text{ef}}$ is the effective volume. The fluctuation effects are small if $|k_BT/F_{\text{eff}}| \ll 1$. Since at $T = T_c$, $f_n(T) - f_s(T) = 0$ a critical region exists near the critical temperature where the fluctuation effects are
not small. The width of the critical region can be estimated by the relation
\[ |k_BT/(f_n(T) - f_s(T))| V_{cf}| \approx 1. \]

According to the Ginzburg-Landau theory \( f_n(T) - f_s(T) = (1 - t)|1 - t|H_c^2(0)/8\pi | \), where \( t = T/T_c \) and \( H_c(0) \) is the thermodynamic critical field.

The effective volume in bulk (three-dimensional) superconductors is \( V_{ef} = \xi^3(T) \), \( \xi(T) \) being the coherence length (the effective volume of anisotropic superconductors is \( V_{ef} = \xi_x\xi_y\xi_z \)). If one dimension of the superconductor \( d < \xi(T) \) then the effective volume is \( V_{ef} = d\xi^2(T) \) and the superconductor can be considered as two-dimensional. The effective volume of a one-dimensional superconductor (two dimensions of which are smaller than the coherence length) is \( V_{ef} = d_xd_y\xi(T) \), and for a zero-dimensional superconductor \( V_{ef} = d_xd_yd_z \).

According to the Ginzburg-Landau theory \( \xi(T) = \xi(0)[1 - t]^{-1/2} \), so that the width \( |1 - t|_c \) of the critical region is given by the following Ginzburg numbers for the relevant dimensionality of the system:

- \( G_{i3D} = (k_BT_c/H_c^2(0)\xi^3(0))^2 \)
- \( G_{i2D} = (k_BT_c/H_c^2(0)d\xi^2(0)) \)
- \( G_{i1D} = (k_BT_c/H_c^2(0)d_xd_y\xi(0))^{2/3} \)
- \( G_{i0D} = (k_BT_c/H_c^2(0)d_xd_yd_z)^{1/2} \).

The value of the Ginzburg number determines the strength of the thermal fluctuations. \( T_c/H_c^2(0) \approx (dH_c/dT)^{-2}T_c^{-1} \). The \( dH_c/dT \) value of HTSC is not bigger than the one of conventional superconductors. Consequently, higher critical temperatures can not be the reason for the enhanced thermal fluctuations in HTSC. The cause of this enhancement are the shorter coherence length and the quasi-two-dimensionality of many HTSC.

Being \( k_BT_c/H_c^2(0)\xi^3(0) < 1 \) for all superconductors, the Ginzburg number increases with decreasing dimensionality. As it was shown first in [1] the effective dimensionality of fluctuation reduces to two near the second critical field, \( H_{c2} \), in a region where the lowest Landau level (LLL) approximation is valid. (The LLL approximation is valid at \( h \gg Gi \), \( h - h_{c2} \ll 2h \) and \( h > h_{c2}/3 \), where \( h = H/H_{c2}(0) \), \( h_{c2} = H_{c2}(T)/H_{c2}(0) \), \( H_{c2}(0) = T_c(dH_{c2}/dT)|_{T=T_c} \). Therefore, the fluctuation effects increase in high magnetic field. The Ginzburg number in the LLL approximation is \( G_{i3D,H} = G_{i3D}^{1/3}(th_{c2})^{2/3} \) for three-dimensional superconductors and \( G_{i2D,H} = G_{i2D}^{1/2}(th_{c2})^{1/2} \) for two-dimensional superconductors. Consequently, the difference among the fluctuation effect values in HTSC and conventional superconductors decreases in high magnetic field.

Thus, the fluctuation values in the mixed state of HTSC and conventional superconductors do not differ very much and therefore the fluctuation effects in these superconductors can be compared. Such comparison [2] has shown that there is no reason to think that the fluctuation effects in \( YBa_2Cu_3O_{7-x} \) qualitatively differ from those in bulk conventional superconductors.

In the present work we compare the fluctuation effects in quasi-two-dimensional HTSC (\( Bi_2Sr_2Ca_2Cu_3O_{10+x} \) films) and in thin films of conventional superconductor (\( Nb_{1-x}O_x \)). One ought expect that the fluctuation ef-
fects of two-dimensional superconductors does not depend on the $\xi(0)$ value in an enough high magnetic field, because the fluctuation is zero-dimensional in the LLL approximation. $\xi_{2D,H} = (G_{i2D} th c)_{H}^{1/2} = (k_B T / H^2_{c2}(0) d \xi^2(T))^{1/2} \approx (k_B T / H^2_{c2}(0) \Phi_0)^{1/2}$ at $H \approx H_{c2}(T)$. Where $\Phi_0 = h c / 2 e$ is the flux quantum. We use the relation $\Phi_0 = 2 \pi H c^2 \xi^2$.[3]

The long-range order of a superconducting state is the long-range phase coherence. Two main fundamental aspects of superconductivity, zero electrical resistance and the Meissner effect are conditioned by it. The Abrikosov state is the mixed state with long-range phase coherence. According to the mean field approximation[7] the long-range phase coherence appears simultaneously with a non-zero density of superconducting electrons, $n_s$, at the second order phase transition taking place at $H_{c2}$.

But according to the fluctuation theory the appearance of long-range phase coherence is not connected directly with the $n_s$ value. Therefore a mixed state without long-range phase coherence can exist according to the fluctuation theory. The transition from the mixed state without phase coherence to the Abrikosov state (i.e. to the mixed state with long-range phase coherence) can have different nature in superconductors with weak and strong disorders[8]. This topical problem was investigated insufficiently. Therefore it is considered first of all in the present work.

Some of superconductor characteristics (for example such thermodynamic properties as specific heat[9] and magnetization[10]) depend only on the $n_s$ value near $H_{c2}$. Whereas the transport properties strongly depend on the length of the phase coherence. The thermodynamic average value of $n_s$ depends very weakly on the length of phase coherence. Therefore the resistive properties of $Bi_2Sr_2Ca_2Cu_3O_{10+x}$ films and $Nb_{1-x}O_x$ films with weak and strong disorder are studied in our paper.

II. THEORETICAL CONSIDERATION

The resistivity has different nature in the mixed states with and without long-range phase coherence. A direct voltage can be in a state with long-range phase coherence only if the phase difference changes in time. This means that the resistivity in the Abrikosov state, $\rho_f$, is caused by the vortex flow. The $\rho_f$ is called flux flow resistivity[3]. This denomination is not really correct because it is obvious that magnetic flux does not flow in a superconductor. We will use a more correct denomination - vortex flow resistivity. It is obvious also that the Lorentz force can not be the driving force on an Abrikosov vortex[11] because the vortex is a singularity but not a magnetic flux.

A resistivity value in the mixed state without phase coherence decreases in the consequence of the paraconductivity (i.e. excess conductivity induced by superconducting fluctuation) but the cause of the resistivity does not differ qualitatively from the one in the normal state. A resistive transition from the paraconductivity regime to the vortex flow regime should take place at the
appearance of long-range phase coherence. This transition is sharp if the length of phase coherence changes by jump and smooth if it changes continuously.

There is important a definition of the phase coherence. Commonly a coherence length (length of phase coherence) is defined by way of an correlation function. Behaviour of the length of phase coherence near $H_{c2}$ differs qualitatively from the one near $T_c$ (i.e. in zero magnetic field) in a consequence of the reduction of the effective dimensionality on two near $H_{c2}$. The coherence length increases up to infinity at the second order phase transition occurred in $T_c$. Whereas the transversal (across magnetic field) coherence length changes little near $H_{c2}$ (see for example [12]). In the LLL region it is equal approximately $(\Phi_0/B)^{1/2}$ [12]. Here $B$ is the magnetic induction. Consequently, if we define the phase coherence by way of the correlation function we can conclude that the long-range phase coherence can not be in the mixed state of a type II superconductor.

On other hand it is obvious that the Abrikosov state is the mixed state with long-range phase coherence. The Abrikosov vortices appears because according to the relation

$$\int_l d\lambda L^2 j_s = \frac{\Phi_0}{2\pi} \int_l d\phi \frac{d\phi}{dR} - \Phi$$

(1)

magnetic flux $\Phi$ can not penetrate in a superconductor with long-range phase coherence and without singularities. Here $\lambda_L = (mc/e^2n_s)^{0.5}$ is the London penetration depth; $j_s$ is the superconducting current density; $l$ is a closed path of integration; $\Phi$ is the magnetic flux contained within the closed path of integration $l$. Consequently, the definition of phase coherence by way of the correlation function is unsuited for the mixed state. It should follow from a right definition that the existence of Abrikosov vortices (as singularities in the mixed state with long-range phase coherence) is evidence of phase coherence.

Such right definition is proposed in the work [13]. It is proposed in [13] to use the relation (1) for the definition of phase coherence. The phase coherence exists on a length $D$ if the relation (1) is valid for closed path of integration $l \simeq \pi D$ with diameter equal $D$. A maximum value of $D$ may be considered as the length of phase coherence. If the length of phase coherence is not greater than $(\Phi_0/B)^{1/2}$ then a magnetic flux penetrate within superconductor without singularities (i.e. without Abrikosov vortices). The absence of a singularity means that $\int_l dR \frac{d\Phi}{dR} = 0$ for all $l$. We call this state without vortices as mixed state without phase coherence.

If the relation (1) is valid on a length considerably greater than $(\Phi_0/B)^{1/2}$ then singularities appear inside a superconducting region. $\int_l dR \frac{d\phi}{dR} = 2\pi N$ if $l$ is a closed path of integration around $N$ singularities. A minimum value of the energy is corresponded to a minimum possible value of the superconducting current density $j_s$. Therefore according to (1) $N\Phi_0 \simeq \Phi$, or $n\Phi_0 \simeq B$, here $n$ is the density of the Abrikosov vortices.
Because the transversal coherence length changes little near $H_{c2}$, only two characteristic lengths ($\Phi_0/B)^{1/2}$ and the sample size $L$) exist in an ideal (without pinning disorder) superconductor. Therefore, the length of phase coherence can change only by jump from the small value $(\Phi_0/B)^{1/2}$ to the big value $L$ and the transition to the Abrikosov state should be first order in superconductors without pinning disorders. Sharp changes of resistive properties should be observed at this transition.

Such sharp change is indeed observed in bulk superconductors with weak disorder at $H_{c4} < H_{c2}$ [14, 15]. ($H_{c4}$ marks the position of the transition to the Abrikosov state [16].) The correct interpretation of this transition was proposed first in paper [14]. But in HTSC [15] this sharp change was interpreted as the vortex lattice melting. This popular interpretation can not be right [17] because no transition from the vortex liquid to the mixed state without phase coherence is observed above $H_{c4}$. Results of [18] allow to suppose that the transition to the Abrikosov state of bulk superconductors with weak disorders is indeed the first order phase transition.

Pinning disorders can change the type of the transition to the Abrikosov state, because a distance between the pinning centers is an additional characteristic length. The length of phase coherence changes not by jump but continuously if a density of pinning centers is big. Therefore the resistive properties change smoothly at the transition to the Abrikosov state of superconductors with strong pinning disorders.

Qualitative explanation of continuous increase of the length of phase coherence (and smooth resistive transition to the Abrikosov state) in superconductors with strong pinning disorders is proposed in work [8]. It is proposed to return to the Mendelssohn’s model [19] and to consider a real superconductor with pinning disorders as an intermediate case between the Mendelssohn’s and Abrikosov’s models. The Mendelssohn’s model is a limit case of strong pinning. The Abrikosov’s model is a limit case of weak pinning. The transition into the Abrikosov state has different nature in the Abrikosov’s model and the Mendelssohn’s model.

The length of phase coherence can change only by jump from $(\Phi_0/B)^{1/2}$ to a sample size $L$ in the Abrikosov’s model, because the effective dimensionality of the fluctuations is reduced on two (from two to zero in a two-dimensional superconductor) in a high magnetic field. The Mendelssohn’s sponge [19] is a system of one-dimensional superconductors. Magnetic field does not change the fluctuation dimensionality in a one-dimensional superconductor. Therefore we may say that pinning disorders increase the effective dimensionality of a two-dimensional superconductor near $H_{c2}$ from zero to one. The appearance of phase coherence is smooth transition in one-dimensional superconductor [20].

Thus, the transition into the Abrikosov state of a real superconductor can be both sharp (if this superconductor has weak pinning and therefore is close to the Abrikosov’s model) and smooth (if it has strong pinning and therefore is close to the Mendelssohn’s model). It is easy to find experimentally the position of the
transition to the Abrikosov state, $H_{c4}$, in a superconductors with weak pinning, because the sharp change of the resistive properties is observed in this case.[14, 15] But it is not so easy to determine a $H_{c4}$ position in a superconductor with strong disorder. Because the transition to the Abrikosov state is continuous in the Mendelssohn’s model we ought consider the $H_{c4}$ in a superconductor with strong pinning as a magnetic field (or temperature) region (the $H_{c4}$ region) in which the length of phase coherence changes from $(\Phi_0/B)^{1/2}$ to a sample size $L$.

A continuous transition from the paraconductivity regime to the vortex flow regime takes place in this region. The vortex pinning exists in the vortex flow regime both of a real type II superconductor and a Mendelssohn’s sponge.[8] As a consequence of pinning the vortices can not flow when the transport current is lower than a critical current $j_{cs}$, sometimes called static critical current, and practically measured through a small but finite voltage level. At $j \gg j_{cs}$ the current voltage characteristics can be described in any case [21] by the equation

$$E = V/l = \rho_f (j - j_{cd})$$

(2)

where $j_{cd}$ is called dynamic critical current.

The resistivity at a small ($j < j_{cd}$) current can have a finite value in a consequence of the thermally activated vortex creep. Therefore $j_{cs}$ can be zero when $j_{cd} > 0$. According to the Kim-Anderson vortex creep theory (see [22])

$$E = E_0 \sinh(j/j_0)$$

(3)

where $j_0 = k_B T/BV_j l_j$; $V_j$ is jumping volume and $l_j$ is the jump width.

Majority of real non-Ohmic current-voltage characteristic in the vortex flow regime can not be described completely by the relation (2) and (3). But we can state that non-Ohmic current-voltage characteristic is evidence of the vortex flow regime, because they are Ohmic in the paraconductivity regime. (We do not considered here the overheating influence and nonlinear effects, which can be observed at a high measuring current.) We will consider the appearance of the non-Ohmic current-voltage characteristic as the end of the transition to the vortex flow regime. The onset of this transition is the appearance of vortices, which takes place when the length of phase coherence surpasses $(\Phi_0/B)^{1/2}$.

The onset of the transition to the Abrikosov state can be detect experimentally by means of a comparison of experimental dependence of excess conductivity and paraconductivity dependence of the mixed state without phase coherence. The paraconductivity of two-dimensional superconductors in the linear (Gaussian) approximation region above $H_{c2}$ can be expressed by the relation

$$\sigma_{fl,2D} = \frac{\sigma_0}{d} F_{2D}(t, h)$$

(4)
where \( d \) is the film thickness; \( \sigma_0 = e^2/h = 0.00024\Omega^{-1} \). The exact \( F_{2D}(t, h) \) dependencies can be calculated from the results of the Ami-Maki work \[24\]. Near \( H_{c2} \), at \( h - h_{c2} \ll h_{c2} \), \( F_{2D}(t, h) \approx F(t, h - h_{c2}) \). For example the Aslamasov-Larkin contribution near \( H_{c2} \) is equal

\[
\sigma_{AL,2D} \simeq \frac{\sigma_0 t}{4d h - h_{c2}} \quad (5)
\]

The linear approximation is valid for \( h - h_{c2} \gg G_{iH} \). Near \( H_{c2} \), in the critical region, the fluctuation interaction must be taken into account. The Aslamasov-Larkin contribution prevails in high magnetic field in the critical region. In the mixed state without phase coherence \( \sigma_{AL,2D} \) is determined only by the thermodynamic average density of superconducting electrons

\[
\left< n_s \right> = \frac{\sum n_s \exp(-F_{GL}/k_B T)}{\sum \exp(-F_{GL}/k_B T)} \quad (6)
\]

because the length of phase coherence is approximately constant. Here

\[
\frac{F_{GL}}{k_B T} = \sum q \epsilon_n |\Psi_q|^2 + \frac{1}{2S} \sum q_i V_{q_1,q_2,q_3,q_4} \Psi_{q_1}^* \Psi_{q_2}^* \Psi_{q_3} \Psi_{q_4} \quad (7)
\]

is the relation of the Ginzburg-Landau free energy of two-dimensional superconductor, \( F_{GL} \), to the thermal energy, \( k_B T \). We use the expansion \( \Psi(r) = V^{-1/2} \sum_q \Psi_q(r) \) and a dimensionless unit system. \( \epsilon_n = (t + h - 1 + 2nhh)/G_{i2D,H} \); \( V_{q_1,q_2,q_3,q_4} = V^{-1} \int V dV \Psi_{q_1}^* \Psi_{q_2}^* \Psi_{q_3} \Psi_{q_4} \); \( q = (n, l) \); \( n \) is a number of a Landau level; \( l \) is the index of degenerate eigenfunctions. We used the formula \( mc_0/\hbar = h_{c2} \). \( \alpha = \alpha_0(t - 1) \) is the coefficient of the Ginzburg-Landau theory.

Only the lowest \((n=0)\) Landau level is taken into account in the LLL approximation. In this approximation

\[
\frac{F_{GL}}{k_B T} = \epsilon \bar{n_s} + 0.5\beta_0 \bar{n_s^2} \quad (8)
\]

where \( \epsilon = \epsilon_0 \) is the distance from \( H_{c2} \) \((T_{c2}) \); \( t + h - 1 = t + c_2 = h - h_{c2} \); \( \bar{n_s} = (\int_V dV n_s)/V = \sum |\Psi|^2 \) is the spatial average density of the superconducting pairs, \( \beta_0 = \bar{n_s^2}/\bar{n_s}^2 = S^{-1} \sum q_i V_{q_1,q_2,q_3,q_4} \Psi_{q_1}^* \Psi_{q_2}^* \Psi_{q_3} \Psi_{q_4}/(\sum |\Psi|^2)^2 \) is the generalized Abrikosov parameter.

The expression (8) is similar to the one for zero-dimensional superconductors. A main difference consists in a difference of the Abrikosov parameter value. The thermodynamic average of the Abrikosov parameter \( \left< \beta_0 \right> \) changes from 2 at \( \epsilon \gg 1 \) to the minimum possible value \( \beta_A \approx 1.16 \) \([26]\) at \( -\epsilon \gg 1 \) in two-dimensional superconductor \([24, 27]\) and from 2 to 1 in zero-dimensional superconductors. This difference is small. Therefore a D-2 model can be used for description of some properties in the LLL approximation region: theoretical results obtained for zero-dimensional superconductors can be used to describe the experimental dependencies of two-dimensional superconductors and results.
for one-dimensional superconductors can be used for the description of bulk superconductors [28].

It is obvious that the thermodynamic average of $\bar{n}$ and $\beta_a$ depend only on the $\epsilon$ value in this (D-2 model) approximation, and therefore the density of the superconducting pairs is a universal function of $(t + h - 1)/Gi_{2D,H} = (t + h - 1)/(thGi_{2D})^{1/2}$. This is a scaling law of the LLL approximation.

According to the D-2 model the paraconductivity dependence of a two-dimensional superconductor can be described by the relation

$$\sigma_{fl,2D} = \frac{\sigma_0 t}{dGi_{2D,H}} F(\epsilon) = \frac{\sigma_0 t}{dGi_{2D,H}} F((h + t - 1)/Gi_{2D,H})$$  (9)

where $F(\epsilon)$ is a universal function. This relation is a consequence of the LLL scaling law. It is valid if the length of phase coherence does not exceed $(\Phi_0/B)^{1/2}$. Therefore in order to detect the onset of the transition to the Abrikosov state in the LLL region we can use the scaling law (9). The experimental dependencies of excess conductivity deviate from the universal dependence (9) when the length of phase coherence begins to increase (i.e. becomes larger than $(\Phi_0/B)^{1/2}$). This method is enough substantial because the scaling law is a general consequence of the fluctuation Ginzburg-Landau theory in the LLL approximation [6]. The scaling of the paraconductivity dependencies is observed both in bulk superconductors [29, 30] and in thin films [31, 32] with weak disorders not only above $H_{c2}$ but also appreciably lower than $H_{c2}$.

Thus, we can detect the onset of the transition to the Abrikosov state in the LLL region by means of the transgression of the scaling law (9) and the end of this transition by means of the non-Ohmic current-voltage characteristic. This method can be especially useful for the investigation of the phase coherence appearance in superconductors with strong disorders, where the transition into the Abrikosov state is smooth [8].

III. SAMPLE PREPARATION AND CHARACTERISTICS

The $Bi_2Sr_2CaCu_2O_{8+x}$ films used in this paper were grown by liquid phase epitaxy (LPE) on $NdGaO_3$ substrates [33], have a thickness of 0.5 $\mu$m, a room temperature resistivity $\rho_{ab}(300K) = 250 \mu\Omega cm = 25 \times 10^{-7} \Omega m$, and an extrapolated resistance at 0 K of less than 50 $\mu\Omega cm = 5 \times 10^{-7} \Omega m$. The films are epitaxial, with the c-axis perpendicular to the substrate and a mosaic spread slightly larger than 0.1°. Magnetic fields up to about 1 T were applied perpendicular to the film surface by a conventional electromagnet.

The $Nb_{1-x}O_x$ films were produced by magnetron sputtering of Nb in an atmosphere of argon and oxygen. Changing the oxygen we produced films with different oxygen contents. The transmission electron microscopy investigation shown that the films with small oxygen contents have small grain structure whereas the films with greater oxygen contents are amorphous. The temperature
of the superconducting transition, $T_c$, of the films decreases with increasing oxygen content. For an oxygen content $x > 0.2$ the critical temperature $T_c < 2$ K. In the present work the amorphous films with $x \approx 0.2$ and the films with small ($\approx 10$ nm) grain structure with $x \approx 0.08$ were used. The oxygen content was determined by Auger analysis with a relative error 0.3. The critical temperature of the used amorphous films is $T_c = 1.8 - 3$ K and that of the used films with small grain structure is $T_c = 5.7$ K. $(dH_{c2}/dT)_{T=T_c} = -2.2$ $T/K$ for the amorphous films and $(dH_{c2}/dT)_{T=T_c} = -0.6$ $T/K$ for the crystalline films. The temperature dependence of normal resistivity of both films is weak, the normal resistivity $\rho_n = 4 \times 10^{-7} \Omega m$ of the films with small grain structure weakly decreasing as temperature decreases, and the resistivity $\rho_n = 20 \times 10^{-7} \Omega m$ of the amorphous films increasing with decreasing temperature. This change can be connected with weak localization.

A perpendicular magnetic field up to 5 T produced by a superconducting solenoid was used for the measurements. It was measured with a relative error 0.0005. The temperature was measured with a relative error 0.001. The resistivity was measured with a relative error 0.0001.

IV. EXPERIMENTAL RESULTS AND DISCUSSION

The in-plane resistive dependencies of $Bi_{2}Sr_{2}CaCu_{2}O_{8+x}$ film, amorphous and small grain structure $Nb_{1-x}O_x$ films in perpendicular magnetic fields are compared in Fig.1. In all cases no sharp feature is observed. The resistivity value decreases smoothly with decreasing temperature. The magnetic field has a different influence on the resistive transition of $Bi_{2}Sr_{2}CaCu_{2}O_{8+x}$ and $Nb_{1-x}O_x$ films. The resistive transition of $Bi_{2}Sr_{2}CaCu_{2}O_{8+x}$ widens only whereas the resistive transitions of $Nb_{1-x}O_x$ films are displaced to lower temperature at the increase of the magnetic field value from 0 to 1 T (Fig.1).

The transition from Ohmic to non-Ohmic behavior of the current-voltage characteristics is also smooth. The observed change in the current-voltage characteristics is qualitatively similar in all films (Fig.2). From the current-voltage characteristics three different H-T regimes can be distinguished [34]. 1) A high-field regime with zero critical current and Ohmic current-voltage characteristics ($j_{cs} = 0$ and $j_{cd} = 0$). 2) An intermediate field regime in which the critical current is zero, but the current-voltage characteristics are non-Ohmic ($j_{cs} = 0$ and $j_{cd} > 0$). 3) A low-field regime with a nondetectable voltage below a finite static critical current ($j_{cs} > 0$ and $j_{cd} > 0$). These three regimes are observed both in the $Bi_{2}Sr_{2}CaCu_{2}O_{8+x}$ films and in the $Nb_{1-x}O_x$ films (see Fig.2). This change in the current-voltage characteristics observed in the films and in layered HTCS differs from the one observed in bulk superconductors with weak disorder, where the current-voltage characteristics change by jump from Ohmic to zero resistivity at a current value smaller than the critical current [14].

The $H/H_{c2}$ values corresponding to the intermediate field regime are different in the $Bi_{2}Sr_{2}CaCu_{2}O_{8+x}$ films, in amorphous and small grain structure
$Nb_{1-x}O_x$ films. This value is highest in $Nb_{1-x}O_x$ films with small grain structure and lowest in amorphous $Nb_{1-x}O_x$ films (Fig. 2). For example, at a reduced temperature value $t = T/T_c \simeq 0.75$, the $H/H_{c2}$ values corresponding to lower and higher boundaries of the intermediate regime are respectively < 0.0009 and 0.008 for the amorphous $Nb_{1-x}O_x$ film, 0.001 and 0.02 for $Bi_2Sr_2CaCu_2O_{8+x}$, 0.35 and 0.75 for $Nb_{1-x}O_x$ film with small grain structure. The higher boundary of the intermediate regime corresponds to the $H/H_{c2}$ value at which the current-voltage characteristics become non-Ohmic: the resistivity value $\rho = dE/dj$ at $j = 0$, $\rho_{j=0}$, is lower than 0.95$\rho_{j>0}$ at a high current value. The lower boundary of the intermediate regime corresponds to the $H/H_{c2}$ value at which $\rho_{j=0} < 10^{-11}\Omega$.m.

In the intermediate field regime the $\rho_{j=0}$ value decreases more rapidly with the decrease of temperature or magnetic field than the $\rho_{j>0}$ value. For example, the $\rho_{j=0}$ value decreases from $3 \times 10^{-9}\Omega m$ at $H = 0.01$ T to $10^{-11}\Omega m$ at $H = 0.001$ T in the amorphous $Nb_{1-x}O_x$ film at $T = 1.75$ K; from $4 \times 10^{-8}\Omega m$ at $H = 0.4$ T to $10^{-11}\Omega m$ at $H = 0.02$ T in the $Bi_2Sr_2CaCu_2O_{8+x}$ film at $T = 60$ K and from $1.4 \times 10^{-8}\Omega m$ at $H = 7$ kOe to $10^{-11}\Omega m$ at $H = 3$ T in the $Nb_{1-x}O_x$ film with small grain structure at $T = 4.2$ K (Fig. 2). Whereas the $\rho_{j>0}$ value changes in these cases from $3 \times 10^{-9}\Omega m$ to $3 \times 10^{-10}\Omega m$ in the amorphous $Nb_{1-x}O_x$ film (at $j = 2 \times 10^8$ A/m$^2$), from $4 \times 10^{-8}\Omega m$ to $6 \times 10^{-9}\Omega m$ in $Bi_2Sr_2CaCu_2O_{8+x}$ film (at $j = 10^8$ A/m$^2$) and from $1.4 \times 10^{-8}\Omega m$ to $1.3 \times 10^{-9}\Omega m$ in the $Nb_{1-x}O_x$ film with small grain structure (at $j = 6 \times 10^8$ A/m$^2$). The current-voltage characteristics of the amorphous $Nb_{1-x}O_x$ film at high current values can be described by the relation (2). Whereas the current-voltage characteristics of the $Bi_2Sr_2CaCu_2O_{8+x}$ film and of the $Nb_{1-x}O_x$ film with small grain structure are non-linear at high j.

The low-field regime of the $Bi_2Sr_2CaCu_2O_{8+x}$ and the amorphous $Nb_{1-x}O_x$ films corresponds to very low reduced magnetic field $H/H_{c2}$, whereas in the $Nb_{1-x}O_x$ films with small grain structure $j_{cs} > 0$ already at a high $H/H_{c2}$ value (Fig. 3). The static critical current has most value in the $Nb_{1-x}O_x$ film with small grain structure Fig.3. The $j_{cs}$ values of $Bi_2Sr_2CaCu_2O_{8+x}$ and the amorphous $Nb_{1-x}O_x$ film are close but their dependencies on the magnetic field differ qualitatively Fig.3. The critical current of the amorphous $Nb_{1-x}O_x$ film decreases sharply in low magnetic field, whereas the one of $Bi_2Sr_2CaCu_2O_{8+x}$ changes weakly. For example, the critical current of the amorphous $Nb_{1-x}O_x$ film at $T = 1.75$ K ($T/T_c = 1.77K$) exceeds $2 \times 10^8A/m^2$ at $H = 0$ and is lower than $10^4A/m^2$ at $H = 0.002$ T, whereas the critical current of $Bi_2Sr_2CaCu_2O_{8+x}$ at $T = 60$ K ($T/T_c = 0.75K$) changes from $10^7A/m^2$ at $H = 0$ to $0.4 \times 10^7A/m^2$ at $H = 0.02$ T (see Fig.3 and also Fig.2).

We interpret the higher boundary of the intermediate regime as the end of the transition to the Abrikosov state. The length of phase coherence exceeds a sample size in the intermediate regime, but the $\rho_{j=0}$ value is not equal zero in a consequence of the thermally assisted vortex flow resistivity (the vortex creep) $\rho_{TAFF}j$ at a low current $j \ll j_0$. According to Eq. (3) $E \simeq E_0j/j_0 = \rho_{TAFF}j$ at a low current $j \ll j_0$. 

\[ \]
Where \( \rho_{T \text{AFF}} = E_0/j_0 \) is a thermally activated linear resistivity.

The current-voltage characteristics of the \( Nb_{1-x}O_x \) film with small grain structure are described very well by Eq. (3) in a wide region of magnetic field and temperature values as shown in Fig.4. For example at \( T = 4.2 \) K Eq. (3) describes the current-voltage characteristics in the region from 0.2 T to 0.6 T \( (H_{c2} = 0.9 T; \text{ at } H = 0.7 \) T the current-voltage characteristic is Ohmic). The \( E_0 \) value changes in this region from 0.27 V/m at \( H = 0.6 \) T to \( 4 \mu V/m \) at \( H = 0.2 \) T. The \( j_0 \) value changes from \( 0.90 \times 10^8 A/m^2 \) at \( H = 0.6 \) T to \( 0.37 \times 10^8 A/m^2 \) at \( H = 0.2 \) T.

Eq. (3) describes a crossover from the thermally activated linear resistivity to the vortex flow resistivity \( \rho_f \). It is obvious that this relation can be valid at \( E \ll \rho_f j \) only. Therefore a deviation of the experimental data from the (3) dependence is observed at high current values (Fig.4). If \( \rho_{T \text{AFF}} \approx \rho_f \) Eq. (3) can not have a validity region. The current-voltage characteristics is close to Ohmic in this case.

We can estimate the \( V_j l_j = k_B T/j_0 B d \) value from the comparison of the experimental dependencies in Fig.4 with the dependence (3). Since the films are thin (the film thickness \( d \approx \xi \) \( V_j = d S_j \), where \( S_j \) is the jumping area. The comparison shows that the \( (S_j l_j)^{1/3} \) value of the \( Nb_{1-x}O_x \) films with small grain structure is smaller than the distance between the Abrikosov vortex in the triangular lattice \( (2\Phi_0/3^{1/2} B)^{1/2} \). The \( (S_j l_j)^{1/3} \) value increases with decreasing magnetic field as well as the \( (2\Phi_0/3^{1/2} B)^{1/2} \) value. This means that individual vortex creep is observed in the \( Nb_{1-x}O_x \) films with small grain structure.

The vortex creep in the \( Nb_{1-x}O_x \) films with small grain structure can be described also as the creep in the Mendelssohn sponge with variable width of superconducting threads. The vortex creep in the Mendelssohn sponge is described by Eq.(3) at low currents \( E_0 = B l_j \omega_0 \exp(\xi dw f_{GL}/k_B T) \) and \( j_0 \approx 8\pi^2 k_B T/\xi d \Phi_0 \) in this model. Here \( \omega_0 \) is an attempt frequency; \( f_{GL} \) is the difference of the free-energy density in the normal and the superconducting phase; \( d \) is the film thickness; \( w \) is the width of superconducting threads across magnetic field.

We considered a distance between the normal vortex cores in superconductors as the width of superconducting threads across magnetic field. Because the radius of the normal vortex core is equal approximately \( \xi(T) \) and the distances between vortex centers is equal \( a(\Phi_0/H)^{0.5} \) then \( w(T, H) \approx a(\Phi_0/H)^{0.5} - 2\xi(T) \approx 2\xi(T)((H_{c2}/H)^{0.5} - 1) \), a being a number of order 1 (in the triangular lattice \( a = (2/3)^{0.5} \)). Consequently, according to our model \( \ln(E_0/E_{0,H_{c2}}) = A((H_{c2}/H)^{1/2} - 1) \). Where \( E_{0,H_{c2}} = B l_j \omega_0 \) is a fit parameter, because we do not know the \( \omega_0 \); the \( A = 2\xi^2 d f_{GL}/k_B T = \xi^2 d H_{c2}^2(0)/4\pi k_B T \) value can be evaluated from known parameters of the superconducting film. The experimental dependencies \( E_0(H) \) of the \( Nb_{1-x}O_x \) films with small grain structure can be described qualitatively by this relation: the \( \ln(E_0) \) value is proportional to \( ((H_{c2}/H)^{1/2} - 1) \) in a wide region of the \( H \) values. But the experimental \( A \) value differs from the one evaluated from the parameter values of the superconducting
film. For example at $T = 4.2$ K the experimental value $A \approx 16$ whereas the theoretical one is equal 100.

The current-voltage characteristics of the amorphous $Nb_{1-x}O_x$ films can be described by Eq. (3) at the higher boundary of the intermediate regime (Fig.4). The current-voltage characteristics are similar to the ones of the $Nb_{1-x}O_x$ film with small grain structure (Fig.4), although they are observed for magnetic field values differed more than 100 times lower. For example the current-voltage characteristics of the amorphous $Nb_{1-x}O_x$ film at $H = 0.004$ T and $T = 1.75$ K ($H_{c2} = 1.14T$) is close to the one of the $Nb_{1-x}O_x$ film with small grain structure at $H = 0.5$ T and $T = 4.2$ K ($H_{c2} = 0.9T$). The current-voltage characteristics of $Bi_2Sr_2CaCu_2O_{8+x}$ considerably differ from the (3) dependence (Fig.4).

According to the classical result the current-voltage characteristics at a enough high current value should be described by the relation (2) in the Abrikosov state. But we observed a linear part at high current value on the $H_{c2}/\rho_H$ curves of $Bi_{2}Sr_{2}CaCu_{2}O_{8+x}$ films (Fig.2). The observed $\rho_j>0$ dependencies do not change appreciably at the transition to the Abrikosov state. The absence any sharp change of the resistive properties in thin film with very weak pinning disorders allows to suppose that whole the high-field regime of the amorphous $Nb_{1-x}O_x$ films corresponds to the mixed state without phase coherence.

The resistivity $\rho = dE/dj$ values of the $Bi_2Sr_2CaCu_2O_{8+x}$ film with small grain structure in the high-field regime differ from the one of the amorphous $Nb_{1-x}O_x$ films. For example at $t = T/T_c \approx 0.75$ and at the lower boundary of the high-field regime the relation $\rho/\rho_n$ is equal 0.0015 in the amorphous $Nb_{1-x}O_x$ at $H = 0.01$ T ($H_{c2} = 0.0088$); 0.064 in the $Bi_2Sr_2CaCu_2O_{8+x}$ film at $H = 0.4$ T ($H_{c2} = 0.02$) and 0.036 in the $Nb_{1-x}O_x$ film with small grain structure at $H = 0.7$ T ($H_{c2} = 0.78$).

These $\rho$ values of the $Bi_2Sr_2CaCu_2O_{8+x}$ exceeds the $\rho_f$ value calculated in the mean field approximation. According to theories the $\rho_fH_{c2}/\rho_nH$ value...
can not exceed 1 \[33\] 1, whereas the \( \rho_{H,c2}/\rho_n H \) values of the \( Bi_2Sr_2CaCu_2O_{8+x} \) are larger than 1 in the high-field regime. For example at \( t = 0.75 \) and \( H = 0.4 \) T, \( \rho_{H,c2}/\rho_n H = 3 \). The \( \rho \) values of the \( Nb_{1-x}O_x \) film with small grain structure are smaller than the \( \rho_f \) values. For example at \( t = 0.75 \) and \( H = 0.7 \) T \( (H/H_{c2} = 0.78) \rho/\rho_n = 0.036H/H_{c2} \), whereas according to the theoretical extrapolation \[35\] the \( \rho_f/\rho_n \) value at \( H/H_{c2} = 0.78 \) can not be smaller than \( 0.3H/H_{c2} \). The large \( \rho \) values of the \( Bi_2Sr_2CaCu_2O_{8+x} \) may be connected with a strong contribution of the higher Landau levels to the order parameter. The small \( \rho \) values of the \( Nb_{1-x}O_x \) film with small grain structure may be connected with the large length of phase coherence below \( H_{c2} \).

In order to detect the onset of the increase of the phase coherence length the experimental dependencies of the excess conductivity should be comparison with the D-2 model paraconductivity dependence described by the relation (9). According to the relation (9) the function \( \Delta \sigma Gi_{2D,H}/\sigma_0 t = F(\epsilon) = F((h+t-1)/Gi_{2D,H}) \) should be universal in the LLL region for different magnetic field values and different superconductors if the length of phase coherence does not exceed \((\Phi_0/B)^{1/2}\). This function is plotted in Fig.5. There are used the experimental dependencies of the excess conductivity of the \( Bi_2Sr_2CaCu_2O_{8+x} \) film, amorphous and small grain structure \( Nb_{1-x}O_x \) films in different magnetic fields. Here \( \Delta \sigma = R^{-1}(H,T) - R_n^{-1} \), \( R(H,T) = \rho(H,T)/d \) is the resistance on a square at given \( H \) and \( T \), \( R_n = \rho_n/d \) is the normal resistance on a square and \( d \) is the film thickness for \( Nb_{1-x}O_x \) \( (d = 20 \text{ nm in both cases}) \) and the spacing between \( CuO_2 \) planes for \( Bi_2Sr_2CaCu_2O_{8+x} \) \( (d = 1.5 \text{ nm in this case} [36]) \).

The \( H_{c2} \) value of \( Nb_{1-x}O_x \) films is determined by comparison of the experimental and theoretical paraconductivity dependencies in the linear approximation region. The reliability of this method has been demonstrated by the investigation of paraconductivity in bulk superconductors. The non coincidence of the transition into the Abrikosov state and the \( H_{c2} \) line has been discovered for the first time using this method \[37\]. Later \[10\] this non coincidence was confirmed by determination of the \( H_{c2} \) value from magnetization measurement.

The \( H_{c2} \) value of the \( Bi_2Sr_2CaCu_2O_{8+x} \) film is determined from the results of \[36\].

The scaling (9) is observed for the amorphous \( Nb_{1-x}O_x \) films. We compared the \( \Delta \sigma Gi_{2D,H}/\sigma_0 t = F(\epsilon) = F((h+t-1)/Gi_{2D,H}) \) dependencies for different magnetic fields in the region \( t/t_{c2} > 0.7 \). In this whole region this dependencies are close (Fig.5). The same scaling dependence was observed in a wider region in amorphous \( a-NbGe \) films with an intermediate level of disorder \[2\]. Therefore this \( F((h+t-1)/Gi_{2D,H}) \) dependence can be considered as universal for the mixed state without phase coherence of two-dimensional superconductors in the LLL region.

The \( \Delta \sigma Gi_{2D,H}/\sigma_0 t = F(\epsilon) = F((h+t-1)/Gi_{2D,H}) \) dependencies both of the \( Nb_{1-x}O_x \) film with small grain structure and of \( Bi_2Sr_2CaCu_2O_{8+x} \) differ for different magnetic fields (Fig.5). These dependencies deviate from the universal dependence in different directions. Consequently, different causes upset the scal-
ing in the Nb$_{1-x}$O$_x$ film with small grain structure and in Bi$_2$Sr$_2$CaCu$_2$O$_{8+x}$.

The excess conductivity dependencies of Bi$_2$Sr$_2$CaCu$_2$O$_{8+x}$ deviate from the scaling one because the LLL approximation is not valid for HTSC at the magnetic field values used. The LLL approximation is valid at $H \gg H_{LLL} = G_i H_c^2(0)$. For conventional superconductors $H_{LLL} \simeq 10^{-4} T$ is a very small value, whereas for Bi$_2$Sr$_2$CaCu$_2$O$_{8+x}$ $H_{LLL} \simeq 10T$ \cite{6}. Therefore for the magnetic field values used in our work $H = 0.01 - 1 T$ the LLL approximation is valid for the Nb$_{1-x}$O$_x$ films whereas for Bi$_2$Sr$_2$CaCu$_2$O$_{8+x}$ the higher Landau levels must be taken into account.

The higher Landau levels were taken into account in the calculation of the paraconductivity of Bi$_2$Sr$_2$CaCu$_2$O$_{8+x}$ in the Hartree approximation \cite{36}. The obtained theoretical dependencies describe well enough the experimental dependencies in the high temperature region of the resistive transition \cite{36}. This means that the phase coherence length does not exceed considerably $(\Phi_0/H)^{1/2}$ because the paraconductivity dependence can be valid in the mixed state without phase coherence only. It is obvious that the phase coherence length increases in the lower region of the resistive transition where the current-voltage characteristics become non-Ohmic. We can not determine more exactly the onset of this transition from the comparison of the experimental dependencies with the paraconductivity dependence because the Hartree approximation do not give accurate enough result.

The LLL approximation is valid for the Nb$_{1-x}$O$_x$ films at the magnetic field values used in our work: $H = 0.1T \gg H_{LLL}(Nb_{1-x}O_x) \simeq 10^{-4} T$. Therefore the deviation of the excess conductivity of the Nb$_{1-x}$O$_x$ films with strong disorder from the scaling dependence (Fig.5) is evidence of the increase of the phase coherence length. This deviation is smooth because the film with strong pinning disorder is close to the Mendellsohn’s model. In our model, in which thin film with strong disorder is considered as the Mendelsohn sponge with variable width of superconducting threads $w(T,H) \simeq a(\Phi_0/B)^{0.5} - \xi(T)$, the length of phase coherence increases primarily as a consequence of the $w(T,H)$ increase. Therefore it depends not only on the temperature but also of the H value, begins to increase at $H \simeq H_{c2}$ and increases with T more quickly than in one-dimensional superconductors with a constant $w$.

It is interesting that the difference of the excess conductivity dependence of the film with small grain structure from the scaling law is visible already above $H_{c2}$ (Fig.5). This means that the length of phase coherence can exceed $(\Phi_0/H)^{1/2}$ already above $H_{c2}$ if pinning disorder is strong enough. The Mendellsohn model can be valid near $H_{c2}$ in superconductors with strong pinning disorders. The mean field critical field of a one-dimensional superconductor (with width $w$), as well as the one of a thin film in parallel magnetic field \cite{12}, $H_{c,sp} = 3^{0.5}\Phi_0/\pi\xi w$ can exceed $H_{c2} = \Phi_0/2\pi\xi^2$. Therefore the phase coherence length of a thin film with strong disorder can exceed $(\Phi_0/H)^{1/2}$ already above $H_{c2}$. 

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V. CONCLUSIONS

The influence of fluctuations on the resistive properties of the mixed state of layered HTSC $Bi_2Sr_2CaCu_2O_{8+x}$ does not differ qualitatively from the one of thin films of conventional superconductors. The appearance of phase coherence is smooth in both cases. Moreover, the quantitative difference between the behavior of thin films with weak and strong disorder is greater than the difference between $Bi_2Sr_2CaCu_2O_{8+x}$ and conventional superconductors. Therefore the amount of disorder must taken into account in the analysis of experiments concerning fluctuation effects in thin films and layered superconductors.

The main difference between the behavior of layered HTSC $Bi_2Sr_2CaCu_2O_{8+x}$ and of thin films of conventional superconductors is connected with the used values of magnetic field. The LLL approximation is valid in this magnetic field region for conventional superconductors but not for layered HTSC. Strictly speaking, the second critical field, $H_{c2}$, has a sense only in the LLL approximation, because the $H_{c2}$ is the field value at which the linearized energy of the lowest Landau level (but not higher Landau levels) is equal zero [3]. Therefore, one may say that the magnetic field with the values used in our work, as well as in majority of other works, decreases the $T_{c2}$ value of conventional superconductors and influences weakly on the $T_c$ value of layered HTSC. The observed difference of the resistive transition is conditioned by this circumstance: the magnetic field is enough high in order to displace the onset of the resistive transition of films of conventional superconductors but not of layered HTSC. It ought be supposed that this difference will disappear when using higher magnetic field for $Bi_2Sr_2CaCu_2O_{8+x}$ investigation.

This supposition is confirmed partly by the observed likeness of the qualitative change of the resistive characteristics. The transition from the Ohmic current-voltage characteristics to the one with a finite critical current takes place in the wide intermediate regime both in the $Bi_2Sr_2CaCu_2O_{8+x}$ and in thin films of conventional superconductors. This behaviour differs qualitatively from the one observed in some bulk superconductors with weak pinning disorders where this transition occurs in a narrow region [30]. This means that the jumping volume of the vortex creep, $V_j$, changes by jump in bulk superconductors with weak pinning and changes weakly with a change of temperature or magnetic field value in two dimensional superconductors.

The observed changes of the resistive properties in all type II superconductors placed in a magnetic field can be connected with the change of the phase coherence length. Four region may be distinguished: 1) the mixed state without the phase coherence; 2) the transition region (the $H_{c4}$ region) where the phase coherence length increase to a sample size; 3) the Abrikosov state with visible thermally activated linear resistivity; 4) the Abrikosov state with a finite static critical current. Width and situation of these regions are different in different superconductors. First of all the superconductor dimensionality and the amount of pinning disorders determine the character of the phase coherence
Experimental investigations show that in bulk superconductors with weak disorder \[14, 15\] the sharp deviation of the excess conductivity value from the scaling law and the appearance of the non-Ohmic current-voltage characteristic are observed simultaneously, at the same magnetic field value \(H_{c4}\) \[29, 30\]. This means that the length of phase coherence changes by jump from \((\Phi_0/B)^{1/2}\) to a sample size \(L\) as well as in an ideal superconductor. The regime of the Abrikosov state with visible vortex creep is observed in some bulk samples and is absent in others \[30\]. But the \(H_{c4}\) region is narrow only in few bulk samples. It broadens with the pinning increase \[30, 38\]. Therefore the Mendelssohn’s model is more suitable for the description the long-range phase coherence appearance in the majority of cases.

The Mendelssohn’s model described enough well the changes of the resistive properties observed in thin films of conventional superconductors with strong disorders. All four region can be marked out in this case. For example, in our \(Nb_{1-x}O_x\) film with small grain structure at \(T = 4.2\) K: 1) the mixed state without the phase coherence exists at \(H > 1.05H_{c2}\), 2) the \(H_{c4}\) transition takes place at \(0.75H_{c2} < H < 1.05H_{c2}\), 3) the thermally activated linear resistivity is observed at \(0.2H_{c2} < H < 0.75H_{c2}\) and 4) at \(H < 0.2H_{c2}\) the static critical current is finite. In the wide \(H_{c4}\) region the excess conductivity dependencies deviate from the scaling law but the current-voltage characteristics remain Ohmic. Below the \(H_{c4}\) region the resistive properties at low current can be described by the Kim-Anderson vortex creep theory and by the theory of the vortex creep in the Mendelssohn’s sponge. The Abrikosov state with a finite static critical current ought be considered as the region where the \(E_0\) value is too small (in the consequence of the \(w(T, H) \approx 2\xi(T)((H_{c2}/H) - 1)\) increase) in order to a resistance can be measured at a low current.

The experimental data obtained in our work do not allow to mark out completely all four regions in thin films with weak disorders. The scaling of the excess conductivity dependencies shows that the mixed state without phase coherence exists at least down to \(0.77T_{c2}\). The current-voltage characteristics become non-Ohmic in a much lower temperature (magnetic field value), for example at \(0.008H_{c2}\) and \(t = 0.75\). The \(H_{c4}\) transition can take place in the wide region below \(0.7T_{c2}\). We suppose that the mixed state without phase coherence exists down to very low magnetic fields.

A sharp feature of the resistive properties should be observed at the transition to the Abrikosov state in a superconductor with weak pinning disorders. The feature of the vortex flow resistivity below the transition is observed both in bulk superconductors \[37\] and in thin films \[22\]. The amount of pinning disorders in our amorphous \(Nb_{1-x}O_x\) is smaller than in the \(\alpha-NbGe\) films used in \[32\]. Therefore the feature of the resistive dependence should be more sharp in our films. But no feature is observed. Therefore it is supposed in \[31\] that the situation of the transition to the Abrikosov state is not universal in two-dimensional
superconductors, but that it depends on the amount of pinning disorders and
that the Abrikosov state is does not exist down to very low fields in thin films 
with very weak disorders. This supposition is confirmed by a theoretical result 
obtained in work [27]. 

We can not detect the onset of the transition to the Abrikosov state in 
$Bi_2Sr_2CaCu_2O_{8+x}$ from the comparison of the excess conductivity 
dependencies with the LLL scaling law, because the LLL approximation is not valid at 
$H < GiH_C(0)$. Results of work [36] allow to suppose that the mixed state 
without phase coherence exists in a wide region below $H_C$. The Abrikosov 
state exists only at low reduced magnetic fields. Comparison of our results and 
results of similar investigations in other works shows that the situation of the 
transition to the Abrikosov state depend on the amount of pinning disorder in 
$Bi_2Sr_2CaCu_2O_{8+x}$ as well as in thin films of conventional superconductors. The 
appearance of pinning in $Bi_2Sr_2CaCu_2O_{8+x}$ single crystals with weak disorder 
is observed in lower magnetic fields than it is observed in our $Bi_2Sr_2CaCu_2O_{8+x}$ 
film.

A magnetization step was observed in $Bi_2Sr_2CaCu_2O_{8+x}$ single crystal in 
some papers [39]. This step is interpreted as a consequence of the vortex lattice 
melting. This interpretation can not be right. But this transition can not be inter-
preted also as the long-range phase coherence appearance in two-dimensional 
superconductors because no sharp transition is observed in thin films [31]. This 
step can be connected with a transition from two- to three-dimensional behavior in 
the layered superconductor. This transition causes the appearance of the 
long-range phase coherence.

The strong increase of the critical current value in low magnetic fields 
oberved in the amorphous $Nb_{1-x}O_x$ is typical for a superconductor with weak 
pinning and without weak links. The large value of the critical current in zero 
magnetic field is evidence of the absence of weak links. The small $j_c$ value in 
zero magnetic field and its weak change in low magnetic fields observed in the 
$Bi_2Sr_2CaCu_2O_{8+x}$ film can be connected with presence of weak links.

ACKNOWLEDGMENTS

This work was made in the framework of the Project INTAS-96-0452. We 
thank the International Association for the Promotion of Co-operation with 
Scientists from the New Independent States for financial support. A.V.N. thanks 
also the Russian National Scientific Council on "Superconductivity" of SSTP 
"ADPCM" (Project 98013) for financial support.

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Figure Captions

Fig.1. Resistive transitions in perpendicular magnetic field of a) the amorphous Nb$_{1-x}$O$_x$ film (d = 20 nm) at 1 - H = 0, 2 - H = 0.2 T, 3 - H = 0.4 T, 4 - H = 0.8 T, 5 - H = 1.2 T; b) the Bi$_2$Sr$_2$CaCu$_2$O$_{8+x}$ in-plane (along ab) at 1 - H = 0, 2 - H = 0.044 T, 3 - H = 0.2 T, 4 - H = 0.5 T, 5 - H = 1 T; and c) the Nb$_{1-x}$O$_x$ film with small grain structure (d = 20 nm) at 1 - H = 0, 2 - H = 0.4 T, 3 - H = 1.2 T.

Fig.2. Current-voltage characteristics of a) the amorphous Nb$_{1-x}$O$_x$ film (d = 20 nm) at T = 1.75 K (T/T$_c$ = 0.77, $H_c2$ = 1.14 T) and 1 - H = 0, 2 - H = 0.001 T, 3 - H = 0.002 T, 4 - H = 0.004 T, 5 - H = 0.006 T, 6 - H = 0.01 T; b) the Bi$_2$Sr$_2$CaCu$_2$O$_{8+x}$ in-plane (along ab) at T = 60 K (T/T$_c$ = 0.75, $H_c2$ = 20 T) and 1 - H = 0, 2 - H = 0.02 T, 3 - H = 0.05 T, 4 - H = 0.1 T, 5 - H = 0.2 T, 6 - H = 0.4 T; c) the Nb$_{1-x}$O$_x$ film with small grain structure (with d = 20 nm) at T = 4.2 K (T/T$_c$ = 0.74, $H_c2$ = 0.9 T) and 1 - H = 0.005, 2 - H = 0.1 T, 3 - H = 0.2 T, 4 - H = 0.4 T, 5 - H = 0.5 T, 6 - H = 0.7 T.

Fig.3. The static critical current $j_{cs}$ dependencies on the reduced magnetic field $H/H_c2$ of the amorphous Nb$_{1-x}$O$_x$ film at T = 1.75 K (t = 0.76) (curve 1); the Bi$_2$Sr$_2$CaCu$_2$O$_{8+x}$ at T = 60 K (t = 0.75) (curve 2); the Nb$_{1-x}$O$_x$ film with small grain structure at T = 4.2 K (t = 0.74) (curve 3). The $j_{cs}$ values were determined by the voltage level 0.0001 V/m. $H/H_c2 = 10^{-4}$ is corresponded to $j_c \approx 10^4$ A/m$^2$.

Fig.4. Current-voltage characteristics of the Nb$_{1-x}$O$_x$ film with small grain structure (with d = 20 nm) at T = 4.2 K and H = 0.6 T (curve 1) and H = 0.4 T (curve 2), the amorphous Nb$_{1-x}$O$_x$ film at T = 1.75 K and H = 0.004 T (curve 3) and the Bi$_2$Sr$_2$CaCu$_2$O$_{8+x}$ at T = 60 K and H = 0.05 T (curve 4). The lines 1*, 2* and 3* denote the $E = E_0 sinh(j/j_0)$ dependencies with 1* - $E_0 = 0.27$ V/m and $j_0 = 0.90 \times 10^8$ A/m$^2$; 2* - $E_0 = 0.0015$ V/m and $j_0 = 0.43 \times 10^8$ A/m$^2$; 3* - $E_0 = 0.050$ V/m and $j_0 = 0.86 \times 10^8$ A/m$^2$.

Fig.5. The $\Delta\sigma/(Gi2Dht)^{0.5}$ dependencies of the amorphous Nb$_{1-x}$O$_x$ film (T$_c$ = 2.37 K; $Gi2D \approx 0.0005$) at H = 0.1 T (curve 1), H = 0.4 T (curve 2), H = 1.2 T (curve 3); the Nb$_{1-x}$O$_x$ film with small grain structure (T$_c$ = 5.7 K; $Gi2D \approx 0.0001$) at H = 0.4 T (h = 0.11) (curve 4), H = 1.2 T (h = 0.33) (curve 5) and the Bi$_2$Sr$_2$CaCu$_2$O$_{8+x}$ (T$_c$ = 79 K; $Gi2D \approx 0.02$) at H = 0.1 T (h = 0.0012) (curve 6), H = 1 T (h = 0.012) (curve 7).