"CHEMICAL ABUNDANCES IN H II REGIONS AND THEIR IMPLICATIONS" RETROSPECTIVE ON: M. PEIMBERT & R. COSTERO, BOLETIN DE LOS OBSERVATORIOS DE TONANTZINTLA Y TACUBAYA, VOL. 5, 3, 1969

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RESUMEN

Presentamos una revisión sobre la relevancia del artículo de Peimbert y Costero (1969), en la determinación de la composición química de regiones H II. Analizamos las evidencias observacionales en favor de la presencia de variaciones de temperatura en nebulosas gaseosas. Hacemos una breve mención a los métodos para tomar en cuenta a los iones no observados en la determinación de las abundancias de los elementos.

ABSTRACT

We present a review about the relevance of the paper by Peimbert and Costero (1969), on the chemical abundance determinations of H II regions. We analyze the observational evidence in favor of the presence of temperature variations inside gaseous nebulae. We make a brief mention of the methods used to estimate the contribution of the unobserved ions to the total chemical abundances.

Key Words: GALAXIES: ABUNDANCES — GALAXIES: EVOLUTION — H II REGIONS — H II REGIONS: INDIVIDUAL: ORION NEBULA, M8, M17 — ISM: ABUNDANCES — SUN: ABUNDANCES

1. INTRODUCTION

The paper by M. Peimbert and R. Costero (1969, hereafter PC) presents the determination of the chemical composition of three galactic H II regions, the Orion nebula, M8 and M17. To derive the chemical abundances PC took into account the presence of temperature variations inside gaseous nebulae based on the definitions of the average temperature, \( T_0 \), and the mean square temperature variation, \( t^2 \), introduced by Peimbert (1967), that are given by

\[
T_0(V, i) = \frac{\int T_e N_e N_i dV}{\int N_e N_i dV},
\]

and

\[
t^2 = \frac{\int (T_e - T_0)^2 N_e N_i dV}{T_0^2 \int N_e N_i dV},
\]

respectively, where \( T_e \), \( N_e \), and \( N_i \) are the electron temperature, the electron density, and the ion density of the volume element, and \( V \) is the observed volume.

PC includes simple equations to determine the chemical abundances in the presence of temperature variations, additional and more detailed equations to determine the chemical abundances have been presented by Peimbert et al. (2002), Ruiz et al. (2003) and Peimbert et al. (2004).

Accurate abundances are needed to test models of: gaseous nebulae, stellar evolution, galactic chemical evolution, and the observable universe as a whole. The presence of temperature variations produces biases in the abundance determinations derived from collisionally excited lines that are typically in the 0.1 to 0.5 dex range. PC provided significant advances in the procedures to determine abundances of H II regions and planetary nebulae.

We will mention the observational evidence in favor of large temperature variations based on seven independent methods, and some of the most relevant results that abundances, derived including the effect of temperature variations, have had in different areas of astronomical research. We will define those gaseous nebulae with \( t^2 \) values < 0.01 as small temperature variation objects, and those with \( t^2 \) values > 0.01 as large temperature variation objects.

Most papers dedicated to the determination of abundances in gaseous nebulae assume that temperature variations are small, and consequently that the abundances can be derived under the assumption of constant temperature. In section 2 we present evidence in favor of large temperature variations inside most gaseous nebulae. In section 3 we present abundance determinations that agree with the presence of large temperature variations based on: the comparison of stellar abundances with gaseous nebular abundances, the comparison of solar and H II region

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2. SIMULTANEOUS DETERMINATIONS OF $T_0$ AND $t^2$

To determine $T_0$ and $t^2$ we need to combine electron temperatures, $T_e$, based on two different methods: one that weighs preferentially the high temperature regions and one that weighs preferentially the low temperature regions. Photoionization models of chemically homogeneous H II regions predict $t^2$ values typically in the 0.003 to 0.01 range, while observations yield $t^2$ values typically in the 0.02 to 0.06 range.

These results imply that there are additional heating and cooling processes in H II regions that are not yet included in photoionization models. Many possible causes of temperature variations have been discussed in the literature some of them are: X-ray heating, cosmic-ray heating, density variations, deposition of mechanical energy, deposition of magnetic energy, presence of shadowed regions, chemical inhomogeneities, dust heating, and transient effects due to changes in the ionizing flux. The source of large temperature variations in gaseous nebulae is still an open problem, and the relative importance of the effects causing the variations might be different for each nebula.

In what follows we will mention some relevant papers that, based on seven independent methods, indicate the presence of large temperature variations in gaseous nebulae.

2.1. Temperatures derived from the Balmer continuum and Balmer line intensities together with temperatures derived from collisionally excited line intensities

The large temperature variations idea was based mainly on three results: a) the smaller $T$(Bac) values than those derived from $T$([O III]) and $T$([N II]) (PC and Peimbert 1967) and the $t^2$ values around 0.04 derived from the photoionization models by Hjellming (1966), that included only N, O and Ne as potential coolants of the model H II regions. In the seventies and eighties most observers obtained $t^2$ values from $T$(Bac) and $T$(O III) of about 0.02 $\pm$ 0.04 that were consistent with $t^2 = 0.00$. Moreover photoionization models that included more chemical elements than those considered by Hjellming predicted $t^2$ values of about 0.005. Probably these two results together with the simpler equations to determine chemical abundances led most workers to adopt $t^2 = 0.00$ and the temperature provided by $T$(O III) to determine the abundances of gaseous nebulae.

The accuracy of the temperature determinations improved and Liu and Danziger (1993) by combining $T$(Bac) with $T$(O III) determinations found large $t^2$ values for 14 planetary nebulae with an average value around 0.03 and errors considerably smaller than the $t^2$ values. Similarly García-Rojas, Esteban and collaborators (García-Rojas et al. 2004, 2005, 2006, 2007, Esteban et al. 2004) by combining $T$(Bac) with $T$(O III) and $T$(O II) determinations for eight galactic H II regions found $t^2$ values in the 0.01 to 0.056 range with an average value of 0.034, again with errors smaller than the $t^2$ values. Moreover González Delgado et al. (1994) studied the giant extragalactic H II region NGC 2363 and based on measurements of the Paschen discontinuity found $t^2$ values of 0.064 and 0.098 for knots A and B respectively.

2.2. Abundances derived from collisionally excited C III lines together with abundances derived from C II recombination lines

The $N$(C++/$N$(H+)) values derived for H II regions and planetary nebulae based on the C II $\lambda$ 4267 to H$\beta$ intensity ratio are, in general, higher than those derived from the C III $\lambda\lambda$ 1906 + 1909 to H$\beta$ intensity ratio; in some cases the difference reaches a factor of ten. General discussions of this problem have been given in the literature (e. g. Torres-Peimbert, Peimbert & Daltabuit 1980; Kaler 1986; Rola & Stasinska 1994, Peimbert, Torres-Peimbert, & Luridiana 1995, Lin 2006, Peimbert & Peimbert 2006, and references therein). Several ideas have been advanced to explain the discrepancy: a) errors in the atomic data, b) errors in the observations and c) the presence of spatial temperature variations.

2.3. Abundances derived from collisionally excited [O III] lines together with abundances derived from O II recombination lines

Peimbert, Storey, & Torres-Peimbert (1993) were the first to determine O/H values for gaseous nebulae based on the recombination coefficients for O II lines computed by Storey (1994). The temperature dependence of the O II lines is relatively weak and very similar to that of the H I lines, therefore the O+++/$H^+$ ratios are independent of the electron temperature. Alternatively the O+++/$H^+$ ratios derived
from collisionally excited lines do depend strongly on the average temperature, \( T_0 \), and the mean temperature square, \( t^2 \) (e.g.: Peimbert 1967, PC, Ruiz et al. 2003, Peimbert et al. 2004).

García-Rojas, Esteban and collaborators (García-Rojas et al. 2004, 2005, 2006, 2007, Esteban et al. 2004) observed 8 galactic H II regions and by combining the \([O III]\) lines with the \([O II]\) lines find \( t^2 \) values in the 0.020 to 0.046 range with an average value of 0.038. Similarly, Esteban et al. (2002), Peimbert (2003), Bresolin (2007), and Esteban et al. (2009) for 11 extragalactic H II regions find \( t^2 \) values in the 0.027 to 0.124 range with an average value of 0.048.

2.4. \( \text{He I} \) recombination lines

From a large set of \( \text{He I} \) lines it is possible to determine \( T(\text{He I}) \). This temperature combined with \( T([O III]) \) and \( T([O II]) \) yields \( T_0 \) and \( t^2 \) (Peimbert, Peimbert, & Ruiz 2000, Peimbert, Peimbert, & Peña 1990, Peimbert, Sarmiento, & Fierro 1990). This method has been applied by García-Rojas, Esteban and collaborators (García-Rojas et al. 2004, 2005, 2006, 2007, Esteban et al. 2004) to 7 galactic H II regions yielding \( t^2 \) values in the 0.017 to 0.046 range, with an average value of 0.027.

This method has also been applied by Peimbert, Luridiana, and Peimbert (2007a) to five metal poor extragalactic H II regions obtaining an average \( t^2 \) value of 0.026; and by Esteban et al. (2009) to 14 giant extragalactic H II regions obtaining \( t^2 \) values in the 0.022 to 0.125 range with an average value of 0.058.

2.5. Comparison of the \( t^2 \) values derived from different methods

In Table 1 we present \( t^2 \) values determined from six different methods for four well observed objects, two H II regions and two planetary nebulae (Esteban et al. 2004, Peimbert 2003, Peimbert et al. 2004, Georgiev et al. 2008). For each object the different methods yield \( t^2 \) values that are in very good agreement. Moreover the adopted average values show errors that are from five to twelve times smaller than the \( t^2 \) determinations. These two results imply that the temperature variations are real and very large.

To explain the presence of large spatial temperature fluctuations, considerably higher than those predicted by chemically homogeneous photoionized models, several possibilities have been proposed. Reviews on this problem have been presented by many authors see for example: Torres-Peimbert, Peimbert, & Peña (1990), Peimbert, Sarmiento, & Fierro (1991), Esteban (2002), Torres-Peimbert and Peimbert (2003), Liu (2006), Peimbert, & Peimbert (2006).

2.6. High spatial resolution map of the columnar temperature in the Orion nebula

O’Dell, Peimbert, and Peimbert (2003), based on observations with the Hubble Space Telescope, presented a high spatial resolution map of the columnar electron temperature \( T_e \) of a region to the south west of the Trapezium in the Orion Nebula. From their \( T_e(\text{O}^{++}) \) map of the Orion Nebula, that includes \( 1.5 \times 10^6 \) independent temperature determinations, they found that the observed mean square temperature variation in the plane of the sky, \( t^2_a(\text{O}^{++}) \), amounts to 0.008.

Based on their \( t^2_a(\text{O}^{++}) \) value, together with geometrical considerations and other observations in the literature, they estimated that \( t^2(\text{O}^{++}) = 0.021 \). Note that the total \( t^2(\text{O}^{++}) \) is larger than \( t^2_a(\text{O}^{++}) \) because in addition to the variations across the plane of the sky it includes the temperature variations along the line of sight. From their \( t^2(\text{O}^{++}) \) value and comparisons between the temperatures in the low- and high-ionization zones, the \( \text{O}^+ \) and \( \text{O}^{++} \) zones, they found that \( t^2(\text{H} \ II) = 0.028 \pm 0.006 \). Their derived \( t^2(\text{H} \ II) \) value is 7 times higher than

| Method | Orion Nebula | 30 Doradus | NGC 5315 | NGC 6543 |
|--------|--------------|-------------|-----------|-----------|
| \( T(\text{Bac}) \) and \( T_e([O II],[O III]) \) | 0.018±0.018 | 0.022±0.007 | 0.039±0.022 | 0.028±0.009 |
| \( T(\text{He II}) \) and \( T_e([O II],[O III]) \) | 0.022±0.002 | ... | 0.060±0.007 | 0.035±0.014 |
| \( N(C^{++})_{\text{CEL}} \) and \( N(C^{++})_{\text{REL}} \) | 0.039±0.011 | 0.056±0.040 | 0.063±0.035 | 0.036±0.010 |
| \( N(O^{++})_{\text{RL}} \) and \( N(O^{++})_{\text{CEL}} \) | 0.052±0.029 | 0.075±0.040 | ... | ... |
| \( N(\text{Ne}^{++})_{\text{RL}} \) and \( N(\text{Ne}^{++})_{\text{CEL}} \) | 0.020±0.002 | 0.038±0.005 | 0.048±0.004 | 0.024±0.008 |
| Average | 0.022±0.002 | 0.033±0.005 | 0.051±0.004 | 0.028±0.005 |
those obtained from homogeneous one-dimensional photoionization models of the Orion Nebula (e. g. Baldwin et al. 1991, Peimbert et al. 1993).

2.7. Comparison of the different ways to calibrate Pagel’s method

The most popular metallicity indicator to determine the O/H ratio in extragalactic H II regions was introduced by Pagel et al. (1979, see also Edmunds & Pagel 1984) and is indistinctly known as Pagel’s, or $R_{23}$, or $O_{23}$ indicator, where
$$O_{23} = I([\text{O} II] \lambda 3727 + [\text{O} III] \lambda\lambda 4959, 5007)/I(\text{H} \beta).$$

The $O_{23}$ indicator has been calibrated with the O/H values based on three different methods: a) by using photoionization models, that we will call PIM calibrations or PIM method, b) by using observational determinations of the O/H abundances based on the electron temperature derived from the $I(4363)/I(5007)$ ratio together with the $I(3727)/I(\text{H} \beta)$ and the $I(5007)/I(\text{H} \beta)$ line ratios, the so called $T(4363)$ method, and c) by using observational determinations of the O/H abundances based on the intensity ratio of O II recombination lines to H I recombination lines that has been called the $O_{23RL}$ method by Peimbert et al. (2007b).

These three methods have been discussed by several authors, (e. g. McGaugh 1991, Pilyugin & Thuan 2005, Peimbert et al. 2007b, Kewley & Ellison 2009, and references therein). These three methods produce very different O/H calibrations, in extreme cases the differences in the inferred O/H abundances among these calibrations reach values of 0.7 dex.

Peimbert et al. (2007b) reach the following conclusions: a) the $O_{23RL}$ method supports the suggestion that the controversy produced by the relatively high O/H values predicted by the PIM calibrations and the relatively low O/H values predicted by the $T(4363)$ calibrations are mainly due to temperature variations; b) the best way to calibrate the $O_{23}$ indicator is to use the $O_{23RL}$ method to obtain the O/H values because it is independent of the temperature structure of the H II regions; c) the use of $T(4363)$ values to derive O/H, under the assumption of constant temperature, provides a lower limit to the O/H abundance ratios; d) since the nebular lines are less sensitive to $T_0$ and $t^2$ than the auroral lines, the model calibrations that adjust the nebular lines are closer to the $O_{23RL}$ calibration than those derived using the observed $T(4363)$ values; and e) the presence of temperature variations affects strongly the $T(4363)$ method, weakly the PIM method, and leave the $O_{23RL}$ method unaffected, or

| Element | Stellar | Nebular(RL) | Nebular(CL) |
|---------|---------|-------------|-------------|
| He      | 11.00±0.04 | 11.05±0.01 | ···          |
| C       | 9.03±0.10  | 8.90±0.10  | 8.40±0.10   |
| N       | 8.36±0.10  | 8.83±0.20  | 8.43±0.20   |
| O       | 9.02±0.10  | 9.19±0.12  | 8.79±0.06   |
| S       | 7.57±0.10  | ···        | 7.08±0.06   |
| Ne      | ···       | 8.67±0.10  | 8.25±0.06   |

*In units of 12 + Log $N(X)/N(H)$.

in other words the O II$_{RL}$ method is independent of the temperature structure of the nebula.

3. ADDITIONAL SUPPORT IN FAVOR OF LARGE TEMPERATURE VARIATIONS

3.1. Comparison of the abundances of the nebula of NGC 6543 with those of its central star

Table 2 presents the stellar abundances of the planetary nebula NGC 6543 together with the nebular envelope abundances derived from recombination lines (RC) that do not depend on the temperature structure of the nebula, and nebular abundances derived from collisionally excited lines (CL) under the assumption that $t^2 = 0.00$ determined by Georgiev et al. (2008).

The stellar abundances of He, C and O are in excellent agreement with the nebular abundances derived from recombination lines. In particular the similar He/H values imply that there are no He rich inclusions present in the nebula. On the other hand the C, O and S abundances derived from nebular collisionally excited lines (under the assumption of $t^2 = 0.00$) are considerably smaller than the stellar abundances. Table 1 shows the $t^2$ values needed to reach agreement between the recombination and the collisional abundance determinations. These results imply that indeed large temperature variations are present NGC 6543 and that the nebula of this object is chemically homogeneous.

In addition a $t^2$ value of about 0.028 permits to reach agreement between the nebular determination of S and the stellar one. Similarly a $t^2$ value of about 0.028 also permits to reach agreement between the Ne abundances derived from recombination and collisionally excited lines.
3.2. Comparison of the oxygen abundances of the Orion nebula with those of B stars of the solar vicinity

Przybilla, et al. (2008) based on the study of B stars of the solar vicinity have found that $12 + \log O/H = 8.76 \pm 0.03$, while Simón-Díaz (2009 private communication) finds that $12 + \log O/H = 8.76 \pm 0.04$ for the Orion association B stars, these results are in excellent agreement with that derived by Esteban et al. (2004) based on the O recombination lines, that amounts to $8.73 \pm 0.03$. The result by Esteban et al. (2004) includes a correction of $0.08$ dex due to the fraction of O tied up in dust grains estimated by Esteban et al. (1998). By adopting the O dust correction of $0.12 \pm 0.03$ dex estimated by Mesa-Delgado et al. (2009), the Orion nebula O abundance derived by Esteban et al. (2004) becomes $8.77 \pm 0.04$, also in excellent agreement with those derived from B stars.

On the other hand, based on the $T(4363)$ method with $t^2 = 0.00$ Deharveng et al. (2000), Pilyugin, Ferrini, & Shkvarun (2003), and Esteban et al. (2004) obtain for the Orion nebula $12 + \log O/H$ values of $8.51$, $8.49$, and $8.51$ respectively, values that after adding the fraction of dust tied up in dust grains are still smaller than those derived from B stars.

3.3. Comparison of the oxygen abundances of the ISM of the solar vicinity with those of the Sun and F and G stars of the solar vicinity

In addition to the evidence presented in section 2 in favor of large $t^2$ values, and consequently in favor of the O $\Pi_{RL}$ method, there is another independent test that can be used to discriminate between the $T(4363)$ method and the O $\Pi_{RL}$ method that consists in the comparison of stellar and H II region abundances of the solar vicinity.

Esteban et al. (2005) determined that the gaseous O/H value derived from H II regions of the solar vicinity amounts to $12 + \log (O/H) = 8.69$, and including the fraction of O atoms tied up in dust grains it is obtained that $12 + \log (O/H) = 8.81 \pm 0.04$ for the O/H value of the ISM of the solar vicinity. Alternatively from the protosolar value by Asplund et al. (2009), that amounts to $12 + \log(O/H) = 8.71$, and taking into account the increase of the O/H ratio due to galactic chemical evolution since the Sun was formed, then according to the chemical evolution model of the Galaxy by Carigi et al. (2005) amounts to $0.13$ dex, we obtain an O/H value of $8.84 \pm 0.04$ dex, in very good agreement with the value based on the O $\Pi_{RL}$ method.

In this comparison we are assuming that the solar abundances are representative of the abundances of the solar vicinity ISM when the Sun was formed.

There are two other determinations of the present O/H value in the ISM that can be made from observations of F and G stars of the solar vicinity. According to Allende-Prieto et al. (2004) the Sun appears deficient by roughly $0.1$ dex in O, Si, Ca, Sc, Ti, Y, Ce, Nd, and Eu, compared with its immediate neighbors with similar iron abundances, by adding this $0.1$ dex difference to the solar value by Asplund et al. (2009) we obtain a lower limit of $12 + \log O/H = 8.81$ for the local interstellar medium. A similar result is obtained from the data by Bensby & Feltzing (2006) who obtain for the six most O-rich thin-disk F and G dwarfs of the solar vicinity an average $[O/H] = 0.16$; by adopting their value as representative of the present day ISM of the solar vicinity we find $12 + \log O/H = 8.87$. Both results are in very good agreement with the O/H value derived from the O $\Pi_{RL}$ method.

3.4. Comparison of the heavy element and helium abundances of M17 with those of K dwarf stars and with models of galactic chemical evolution

The best Galactic H II region to determine the He/H ratio is M17 because it contains a very small fraction of neutral helium and the error introduced by correcting for its presence is very small. Carigi & Peimbert (2008) obtained for M17 a value of $\Delta Y/\Delta Z = 1.97 \pm 0.41$ for $t^2 = 0.036 \pm 0.013$, where $Y$ and $Z$ are the helium and heavy elements by unit mass and $t^2$ was determined observationally. By correcting this value considering that the fraction of O trapped in dust amounts to $0.12$ dex instead of $0.08$ dex (Mesa-Delgado et al. 2009) we obtain $\Delta Y/\Delta Z = 1.77 \pm 0.37$. This $\Delta Y/\Delta Z$ value is in very good agreement with three independent determinations: the one derived from the chemical evolution of the Galaxy for the galactocentric distance of M17 that amounts to $\Delta Y/\Delta Z = 1.70 \pm 0.4$, and two $\Delta Y/\Delta Z$ determinations derived from K dwarf stars of the solar vicinity that amount to $2.1 \pm 0.4$ (Jiménez et al. 2003) and to $2.1 \pm 0.9$ (Casagrande et al. 2007). On the other hand the value $\Delta Y/\Delta Z = 3.60 \pm 0.68$ derived from collisionally excited lines of M17 under the assumption of $t^2 = 0.00$ is not in agreement with the chemical evolution models by Carigi & Peimbert (2008) nor with the values derived from K dwarf stars of the solar neighborhood.
3.5. Comparison of the primordial helium abundance derived from H II regions together with that derived from WMAP and SBBN

In Table 3 we present the determination of the primordial helium abundance, $Y_p$, based on two different methods: a) that based on determination of He/H ratios of metal poor H II regions and their extrapolation to the He/H value for the case of no heavy elements, and b) that based on the barion to photon ratio derived from WMAP observations together with the assumption of Standard Big Bang Nucleosynthesis, SBBN. Note that these two values might be different if SBBN does not apply to the primordial nucleosynthesis stage of the universe (e.g. different number of neutrino families, varying gravitational constant, etc.).

In Table 3 we present two values derived from H II regions (Peimbert, Luridiana & Peimbert 2007a): one for constant temperature inside the H II regions, that based on the $T(4363)$ value ($t^2 \neq 0.00$), and the other based on the assumption that the temperature varies over the observed volume ($t^2 \neq 0.00$). The adopted atomic physics for the helium recombination line is that presented by Porter et al. (2005, 2007, 2009). We also present three determinations based on the WMAP observations by Dunkley et al. (2009), and three neutron lifetimes, $\tau_n$, those by: a) Arzumanov et al. 2000, that amounts to $885.7 \pm 0.8$ s, b) Mathews et al. (2005) that amounts to $881.9 \pm 1.6$ s, and is based mainly on the average of the results by Arzumanov et al. (2000) and Serebrov et al. (2005), and c) Serebrov et al. (2005, 2008) that amounts to $878.5 \pm 0.8$ s. From Table 3 it is clear that the main source of error in the WMAP + SBBN method to determine $Y_p$ is due to the neutron lifetime.

Moreover it follows from Table 3 that the $Y_p$ value for ($t^2 \neq 0.00$) is closer to the $Y_p$ value based on WMAP and SBBN than the $Y_p$ value for ($t^2 = 0.00$). Additional discussion on $Y_p$ is presented by Peimbert (2008) and by Peimbert et al. (2010).

4. THE IONIZATION CORRECTION FACTOR

To derive the total gaseous abundances of a given element it is necessary to observe the relative amounts of all the ionization stages of that element present in a given nebula. Very often this is not possible and one has to correct for the presence of the unobserved ions. This correction can be done by three different methods: a) an empirical ionization correction curve, b) the use of equations, where the correction for the unseen ions is estimated by an ionization correction factor based on ratios of other observed ions, and c) from tailor made photoionization models.

Bowen and Wyse (1939) were the first to propose the use of what has been called the empirical ionization distribution curve (e.g. Seaton 1960, Aller, 1961, Aller and Liller 1968). While PC together with other authors started to use equations with ionization correction factors (e.g. Seaton 1968, Rubin 1969, Peimbert and Torres-Peimbert 1971).

The calibration of the empirical ionization correction factors, $ICF$’s, has been obtained by using photoionization models and by observing a given object at different lines of sight with different degrees of ionization under the assumption of chemical homogeneity.

The ionization correction factors of N and Ne presented by PC have been used widely and are given by:

$$\frac{N(N)}{N(H)} = ICF(N) \frac{N(N^+)}{N(H^+)} = \frac{N(O)}{N(O^+)} \frac{N(N^+)}{N(H^+)},$$

and

$$\frac{N(Ne)}{N(H)} = ICF(Ne) \left( \frac{N(Ne^{++})}{N(H^+)} \right)$$
\[
\frac{N(O)}{N(O^{++})} = \left( \frac{N(Ne^{++})}{N(H^+)} \right) \quad (4)
\]

where \(N(O) = N(O^+) + N(O^{++})\).

These two equations present very good approximations to the total abundance ratios for those cases where the ICF is small, for large values of the ICF they should be taken with caution.

For example in Figure 3 of Peimbert, Torres-Peimbert and Luridiana (1995) it can be seen that Ne\textsuperscript{++}/O\textsuperscript{++} in planetary nebulae increases with decreasing density, indicating that for objects with \(N_e \leq 1000 \text{ cm}^{-3}\) equation (8) is a poor approximation to the Ne/O value; this result probably is due to the presence of the charge exchange reaction

\[
\text{O}^{+2} + \text{H}_0 \rightarrow \text{O}^+ + \text{H}^+
\]

that permits the coexistence of Ne\textsuperscript{++} with O\textsuperscript{+} (e.g. Hawley & Miller 1977, 1978; Hawley 1978; Pequignot, Aldrovandi & Staśińska 1978; Butler, Bender & Dalgarno 1979; Pequignot 1980). Ionization structure models predict that the lower the density the higher the H\textsuperscript{0}/H\textsuperscript{+} ratio, in agreement with the charge exchange suggestion and Figure 3.

Moreover for PNe and H II regions of low degree of ionization, those with a substantial fraction of once ionized O, equation 4 only presents a lower limit to the total Ne/H abundance (e. g. Figure 4 of Torres-Peimbert and Peimbert, 1977; and Figure 9 of Peimbert, Torres-Peimbert, & Ruiz, 1992).

Sets of useful ionization correction factor equations for gaseous nebulae have been presented by many authors (e. g. Kingsburgh and Barlow 1994, Izotov et al. 2006).

REFERENCES

Allende-Prieto, C., Barklem, P. S., Lambert, D. L., & Kunha, K. 2004, A&A, 420, 183

Aller, L. H. 1961, in: The Abundance of the Elements (New York, Interscience Publishers)

Aller, L. H. & Miller, W. 1968, in: Nebulae and Interstellar Matter, Barbara M. Middlehurst (ed.), (Chicago: University of Chicago Press), p.483

Arzumanov, S., Bondarenko, L. Chernyavsky, S., et al. 2000, Physics Letters B, 483, 15

Asplund, M., Grevesse, N., Sauval, A. J., & Scott, P. 2009, RevMexAA, SC, 12, 56

Bresolin, F. 2007, ApJ, 656, 186

Butler, S. E., Bender, C. F., & Dalgarno, A. 1979, ApJ, 230, L59

Carigi, L. & Peimbert, M. 2008, RevMexAA, 44, 341

Casagrande, L., Flynn, C., Portinary, L., Girardi, L., & Jiménez, R. 2007, MNRAS, 382, 1516

Deharveng, L., Peña, M., Caplan, J., & Costero, R. 2000, MNRAS, 311, 329

Dunkley, J., Komatsu, E., Nolta, M. R., et al., 2009, ApJS, 180, 306

Edmunds, M. G. & Pagel, B. E. J. 1984, MNRAS, 211, 507

Esteban, C. 2002, RevMexAA, SC, 12, 56

Esteban, C., Bresolin, F., Peimbert, M., García-Rojas, J., Peimbert, A., & Mesa-Delgado, A. 2009, ApJ, 700, 654

Esteban, C., García-Rojas, J., Peimbert, M., Peimbert, A., Ruiz, M. T., Rodríguez, M., & Carigi, L. 2005, ApJ, 618, L95

Esteban, C., Peimbert, M., García-Rojas, J., Ruiz, M. T., Peimbert, A. & Rodríguez, M. 2004, MNRAS, 355, 229

Esteban, C., Peimbert, M., Torres-Peimbert, S., & Escalante V. 1998, MNRAS, 295, 401

Esteban, C., Peimbert, M., Torres-Peimbert, S., & Rodríguez, M. 2002, ApJ, 581, 241

García-Rojas, J. & Esteban, C. 2007, ApJ, 670, 457

García-Rojas, J., Esteban, C., Peimbert, A., Peimbert, M., Rodríguez, M., & Ruiz, M. T. 2005, MNRAS, 362, 301

García-Rojas, J., Esteban, C., Peimbert, A., Rodríguez, M., Peimbert, M., & Ruiz, M. T. 2007, RevMexAA, 43, 3

García-Rojas, J., Esteban, C., Peimbert, M., Costado, M. T., Rodríguez, M., Peimbert, A., & Ruiz, M. T. 2006, MNRAS, 368, 253

García-Rojas, J., Esteban, C., Peimbert, M., Rodríguez, M., Ruiz, M. T., & Peimbert, A. 2004, ApJS, 153, 501

Georgiev, L. N., Peimbert, M., Hillier, D. J., Richer, M. G., Arrieta, A., & Peimbert, A. 2008, ApJ, 681, 333

González-Delgado, R. M., Pérez, E., Tenorio-Tagle, G., et al. 1994, ApJ, 437, 239

Hawley, S. A. 1978, PASP, 90, 370

Hawley, S. A. & Miller, J. S. 1977, ApJ, 212, 94

Hawley, S. A. & Miller, J. S. 1978, PASP, 90, 39

Hjellming, R. M. 1966, ApJ, 143, 420

Izotov, Y. I., Stasinska, G., Meynet, G., Guseva N. G., & Thuan, T. X. 2006, A&A, 448, 955

Jiménez, R., Flynn, C. MacDonald, J., & Gibson, B. K. 2005, Science, 299, 1552

Kaler, J. B. 1986, ApJ, 308, 337

Kewley, L. J. & Ellison, S. L. 2009, ApJ, 700, 654

Kingsburgh, R. L. & Barlow, M. J. 1994, MNRAS, 271, 257

Liu, X.-W. 2006, in: Planetary Nebulae in our Galaxy and Beyond, IAU Symposium 234, M. J. Barlow & R. H. Méndez (eds.), (Cambridge: Cambridge Univ. Press), p. 219
Liu, X.-W. & Danziger, J. 1993, MNRAS, 263, 256
Mathews, G. J., Kajino, T., & Shima, T. 2005, Phys. Rev. D, 71, 021302
McGaugh, S. S. 1991, ApJ, 380, 140
Mesa-Delgado, A., Esteban, C., García-Rojas, J., Luridiana, V., Bautista, M., Rodríguez, M. López-Martin, L., & Peimbert, M. 2009, MNRAS, 395, 855
O'Dell, C. R., Peimbert, M., & Peimbert, A., 2003, AJ, 125, 2590,
Pagel, B. E. J., Edmunds, M. G., Blackwell, D. E., Chun, M. S., & Smith G. 1979, MNRAS, 189, 95
Peimbert, A. 2003, ApJ, 584, 735
Peimbert, A., Peimbert, M., & Luridiana, V. 2002, ApJ, 565, 668
Peimbert, M. 1967, ApJ, 150, 825
Peimbert, M. 2008, Current Science, 95, 1165
Peimbert, M. & Costero, R. 1969, Bol. Obs. Tonantzintla y Tacubaya, 5, 3
Peimbert, M., Luridiana, V., & Peimbert, A. 2007a, ApJ, 666, 636
Peimbert, M. & Peimbert, A. 2006, in: Planetary Nebulae in our Galaxy and Beyond, IAU Symposium 234, M. J. Barlow & R. H. Méndez (eds.), (Cambridge: Cambridge Univ. Press), p. 227
Peimbert, M., Peimbert, A., Carigi, L., & Luridiana, V. 2010, in: Light elements in the Universe, IAU Symposium 268, C. Charbonnel, M. Tosi, F. Primas, & C. Chiappini, (eds.), (Cambridge: Cambridge Univ. Press), in press
Peimbert, M., Peimbert, A., Esteban, C., García-Rojas, J., Bresolin, F., Carigi, L., Ruiz, M. T., & López-Sánchez, A. R. 2007b, RevMexAA, SC, 29, 72
Peimbert, M., Peimbert, A., & Ruiz M. T. 2000, ApJ, 541, 688
Peimbert, M., Peimbert, A., Ruiz, M. T., & Esteban, C. 2004, ApJS, 150, 431, 2004
Peimbert, M., Sarmiento, A., & Fierro, J. 1991, PASP, 103, 815
Peimbert, M., Storey, P. J., & Torres-Peimbert, S. 1993, ApJ, 414, 626
Peimbert, M. & Torres-Peimbert, S. 1971, ApJ, 168, 413
Peimbert, M., Torres-Peimbert, S., & Luridiana, V. 1995, RevMexAA, 31, 131
Peimbert, M., Torres-Peimbert, S., & Ruiz, M. T. 1992, RevMexAA, 24, 155
Pequignot, D. 1980, A&A, 81, 356
Pequignot, D., Aldrovandi, S. M. V., & Stasińska, G. 1978, A&A, 63, 313
Pilyugin, L. S., Ferrini, F., & Shkvarun, R. V. 2003, A&A, 401, 557
Pilyugin, L. S. & Thuan, T. X. 2005, ApJ, 631, 231
Porter, R. L., Bauman, R. P., Ferland, G. J., & MacAdam, K. B. 2005, ApJ, 622, L73
Porter, R. L., Ferland, G. J., & MacAdam, K. B. 2007, ApJ, 657, 327
Porter, R. L., Ferland, G. J., MacAdam, K. B., & Storey, P. J. 2009, MNRAS, 393, L36
Przybilla, N., Nieva, M. F., & Butler, K. 2008, ApJ, 688, L103
Rola, C. & Stasinska, G. 1994, A&A, 282, 199
Rubin, R. H. 1969, ApJ, 155, 841
Ruiz, M. T., Peimbert, A., Peimbert, M., & Esteban, C. 2003, ApJ, 595, 247
Seaton, M. J. 1960, Reports on Progress in Physics, 23, 313
Seaton, M. J. 1968, MNRAS, 139, 129
Serebrov, A. P., Varlamov, V. E., Kharitonov, A. G. et al. 2005, Physics Letters B, 605, 72
Serebrov, A. P., Varlamov, V. E., Kharitonov, A. G. et al. 2008, Phys. Rev. C, 78, 035505
Storey, P. J. 1994, A&A, 282, 999
Torres-Peimbert, S. & Peimbert, M. 1977, RevMexAA, 2, 181
Torres-Peimbert, S. & Peimbert, M. 2003, in: Planetary Nebulae and Their Role in the Universe, IAU Symposium 209, S. Kwok, M. Dopita, & R. Sutherland (eds.), (San Francisco: Astronomical Society of the Pacific), 363, 2003.
Torres-Peimbert, S., Peimbert, M., & Daltaubet, E. 1980, ApJ, 238, 133
Torres-Peimbert, S., Peimbert, M., & Peña, M. 1990, A&A, 233, 540
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