The 3.5 keV line from non-perturbative Standard Model dark matter balls

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Abstract

Our earlier put forward model of dark matter, consisting of cm-size pearls with ordinary matter inside under high pressure and with a mass of order $1.4 \times 10^8$ kg, is used to explain the mysterious 3.5 keV X-ray line from the galactic center and various galaxies and galaxy clusters. The pearls are bubbles of a new type of vacuum and thus surrounded by a surface tension providing the high pressure.

We have two rather successful order of magnitude numerical results:

1) the X-ray energy of 3.5 keV comes out as the homolumo-gap or rather as the energy due to screening of electrons in the high pressure ordinary matter inside the pearls, and is predicted correctly to within a factor 2;

2) Using the fitting of Cline and Frey for dark matter radiation arising from collisions or annihilations of dark matter particles we fit the overall intensity of the radiation in our pearl model.

Only using the old parameters from our previous work \cite{3} we obtain the right order of magnitude for the frequency 3.5 keV and the intensity would also be correct in order of magnitude, provided that all of the energy released in the collision goes into the 3.5 keV line. However, in order to achieve that we have to make a couple of improvements of the model, most importantly the pearls should be bigger than the minimum required by just stability.

1 Introduction

We have for some time worked on a model \cite{1, 2, 3, 4} for dark matter being balls of cm-size (pearls) with mass of the order of $1.4 \times 10^8$ kg $\sim$ 100000 tons, consisting of ordinary matter highly compressed in a bubble of a new type of vacuum called “condensate vacuum”. It is the purpose of the present article,
using the parameters of our earlier speculations and fits to predict the intensity and frequency of an X-ray line expected to be emitted from our dark matter pearls. This emission is then of course to be identified with the controversial 3.5 keV X-ray radiation that somewhat mysteriously has been observed by several satellites [5, 6].

Let us first list a series of remarkable characteristics of our dark matter model:

- Contrary to most other proposals for what dark matter could be, our model is in principle built into the Standard Model, needing for its realization of the new vacuum only an appropriate fine tuning of the coupling parameters of this Standard Model according to our proposed “Multiple Point Principle” [7, 8, 9, 11, 12]. So no new physics needs to be found at LHC in order that our model could be true.

- We have proposed [3] that the fall of one of our dark matter pearls on the earth is to be identified with the Tunguska event in 1908 in Siberia.

- And we suggest remnants from earlier impacts of our pearls on the earth are to be identified with kimberlite pipes [9, 13, 14], of which about 6500 have been found around the earth. Most kimberlite pipes produced through the history of the earth must of course have been covered by sediments through the earth history, and it may thus not be unexpected that the still accessible kimberlite pipes are found in old cratons, the very oldest sediments on the earth.

- An interesting property of our model, assuming that our pearls collect in the stars developing into supernovae [15], is that they can cause the major neutrino outburst from the supernova to split into two outbursts, as apparently occurred in the supernova SN1987A [16, 17, 18, 19, 20]. The idea here is that our pearls, when confronted with a high density of neutrons, absorb a lot of neutrons and heat up so as to temporarily stop the collapse of the star. That heating puts an end to the first major neutrino burst, but some several hours later the heat production has stopped and enough heat escaped by neutrino emission to let the collapse restart. Thereby comes the second major neutrino burst.

- Our pearls also help to make the supernova truly become a supernova, in the sense of emitting visible material to the outside of the star and thus becoming indeed visible from earth as a spectacular new star.

- According to our model of dark matter, the pearls form in the first tenth of a second or so of the life time of the universe. Close to the end of this process they have a fusion caused explosion transforming helium into heavier nuclei inside the pearls and thereby emitting nucleons with the fusion energy. This leads to an estimate [1, 2] of the ratio of the amount of dark matter to normal matter outside the pearls - meaning in practice ordinary matter - to be about 6, in very good agreement with observations. The idea here is that the nucleons pushed out by the He-fusion caused explosion is the matter which we today see as the ordinary matter. What remains inside the dark matter pearls is also ordinary matter, but that is not in practice observed as such today.
The observation of a new X-ray emission line at 3.5 keV was reported in 2014. It was detected in the Andromeda galaxy, the Perseus cluster and different combinations of other galaxy clusters. Later the line was detected in the Galactic Center. This line has been suggested to originate from radiatively decaying dark matter usually identified as sterile neutrinos of mass 7 keV. However this interpretation of the data is controversial, as the line was not observed in the blank sky (Milky Way halo or in dwarf spheroidal galaxies). So the expected distribution of the 3.5 keV line radiation as simply coming from decaying dark matter and therefore proportional to the amount of dark matter present is not quite supported. Rather a distribution as coming from interactions of two dark matter particles giving an emission rate proportional to the square of the dark matter density and velocity dispersion - such as from annihilation, inelastic scattering or, as we propose in the present article, the collision of two dark matter particles - could fit a bit better.

A feature that at first looks like killing the main idea that this 3.5 keV radiation should come from the dark matter is that the line shows up in the Tycho supernova remnant. However it actually supports our model for dark matter, because we namely have dark matter pearls that will radiate with this line 3.5 keV whenever they are energized some way or another, as e.g. by the supernova remnant.

In the present article we shall estimate/calculate two numbers concerning the 3.5 keV radiation and compare the results gotten from our model with the astronomical observations:

- We shall estimate the screening energy per electron in the highly compressed material in the inside of our pearls, i.e. the energy lowering achieved by the electrons around a given electron adjusting - moving away appropriately - to diminish the energy. We then expect that this screening energy lowering functions like a homolumo gap effect and causes there to be a gap of the size given by the screening energy in the electron spectrum. Using this energy gap excitons may be formed as weakly bound states of an electron above the gap and a hole below. Such excitons are now supposed to decay under emission of most of their energy as an X-ray and thus produce the mysterious 3.5 keV line.

- The major point of our first calculation is to show that we order of magnitude-wise indeed get close to 3.5 keV.

The next calculation concerns the emission rate of the 3.5 keV line expected from the dark matter pearls of ours. In order to obtain just crudely enough radiation compared to the observed rate, we need that our pearls collide with each other, rather seldomly though, forming a double as heavy pearl; there is then a very strong release of energy, due to the fact that the surface tension area around the two colliding pearls is bigger than that around the doubly as heavy pearl. The energy due to contraction of the surface/skin is supposed to be mainly radiated out as 3.5 keV radiation.

1 Recently there have been conflicting claims on observation: Boyarsky et al. see some 3.5 keV radiation in the Milky Way Halo, while Dessert et al. do not see it.

3
1.1 Comments on our “Tunguska” model

In a previous article we put forward the model that dark matter is made up from macroscopic size pearls of mass $M = m_B = 1.4 \times 10^8$ kg supposed to consist of a region of a new vacuum - another vacuum phase - surrounded by a domain wall with a surface tension $S$, of which the cubic root is $S^{1/3} = 28$ GeV. The only new physics compared to what we hope comes out of the Standard Model by just an enormously ambitious calculation is our Multiple Point Principle. For our purpose this principle tells us that the energy densities of the two vacuum phases are the same. Under the pressure from the surface tension ordinary matter like e.g. carbon or silicon,... is brought to the density of the order of $\rho_B = 10^{14}$ kg m$^{-3}$. It is the electron gas, which is highly degenerate, that keeps up the major part of this high pressure and the electrons have relativistic speeds typically. The major difference to be felt by the ordinary material inside the pearl is that the Higgs field expectation value is supposed to be smaller inside the pearl phase than outside. This in turn means that both nucleons and electrons have slightly smaller masses inside a pearl than outside, and that thus the matter is pressed into the pearl by a potential barrier at the surface.

We used the fact that about hundred years ago there was the famous Tunguska event of trees falling over a 70 km large range in Tunguska in Siberia; a rather mysterious impact in as far as no proper meteor was found in spite of the rather large effects. We identified this kind of impact with one of our pearls hitting the earth with a rate of approximately $r_B \approx \frac{1}{200 \text{ years}} = 1.5 \times 10^{-8}$ s$^{-1}$.

In the old article we made the theoretical approximation that the pearls were all of just the size that places them on the borderline of collapsing, because the pressure is just about to overcome the potential barrier across the surface wall.

With these assumptions we obtained the parameters presented in Table 1.

The Tunguska event rate $r_B$, the borderline stability hypothesis and the quark masses led to an estimate for the surface tension of $S^{1/3} = 28$ GeV. It agreed well with the theoretical estimate of $S^{1/3} = 16$ GeV, which was essentially derived by assuming the vacuum phases were caused by electroweak physics so that the tension came out to be of the order of the electroweak energy scale. This agreement is a success for our model.

The parameter $\Delta V$, which denotes the potential difference per nucleon in passing the domain wall, was crudely determined by assuming that the Higgs field inside the pearl vacuum was about half of the Higgs field expectation value in the outside (= “present vacuum”). Thus, assuming the additivity of quark mass contributions to the nucleon masses, we derived the value $\Delta V = 10 \pm 7$ MeV.

The physics behind the two vacua is that the vacuum inside the pearls, called the “condensate vacuum”, results from bose condensation of a speculated bound state of 6 top + 6 anti-top quarks. The scenario is that the top-quark Yukawa coupling to the Higgs boson $g_t = 0.935$ is actually so large - taking into account the many colors and spin states of the top-quark - that non-perturbative effects come in and e.g. cause the formation of a bound state of the mentioned 6 top and 6 anti-top quarks. We have six of each because this represents a closed shell in the atomic physics sense. One shall imagine that these bound states
Table 1: The parameters of our model picture of the Tunguska particle as a ball of a new type of vacuum with a bound state condensate, filled with ordinary white dwarf-like matter and on the borderline of stability.

| Parameter                          | Value                              |
|-----------------------------------|------------------------------------|
| Time Interval of impacts          | $r_B^{-1}$ 200 years               |
| Rate of impacts                   | $r_B 1.5 \times 10^{-8}$ s$^{-1}$  |
| Dark matter density in halo       | $\rho_{\text{halo}} 0.3$ GeV/cm$^3$|
| Dark matter near solar system     | $\approx 2\rho_{\text{halo}} 0.6$ GeV/cm$^3$ |
| Mass of the ball                  | $m_B 1.4 \times 10^8$ kg           |
| Estimated typical speed of ball   | $v 160$ km/s                       |
| Kinetic energy of ball            | $T_v 1.8 \times 10^{18}$ J         |
| Energy observed in Tunguska       | $E_{\text{Tunguska}} (4 - 13) \times 10^{16}$ J 10-30 megaton TNT |
| Potential shift between vacua     | $\Delta V 10$ MeV                  |
| Cube root of tension              | $S^{1/4} 28$ GeV                   |
| Cube root of tension              | $S^{1/3} 16$ GeV                   |
| Ball density                      | $\rho_B 10^{14}$ kg/m$^3$         |
| Radius of ball                    | $R 0.67$ cm                        |

interact with each other, mainly by Higgs boson exchange, and bind to their neighbors so as to make the energy (density) of the self-interacting condensate just zero compared to the vacuum outside, called the “present vacuum”. This is supposed to come about by our proposed Multiple Point Principle, which is supposed - in a mysterious way - to fine-tune the couplings in the Standard Model so as to precisely achieve this kind of degeneracy between several vacua.

We introduce an effective field for the bound state, which we may here call $F$, denoted $\phi_F$ and write an effective potential as a function of both this field $\phi_F$ and of the Higgs field $\phi_H$. Then the fine-tuning from the multiple point principle should organize the Standard Model couplings in such a way that this effective potential $V_{\text{eff}}(\phi_H, \phi_F)$ has two different and equally deep minima corresponding to the two degenerate vacua. In our previous work [3] we assumed that, while in the “present vacuum” the $\phi_F$ field has zero expectation value, both fields $\phi_F$ and $\phi_H$ have non-zero expectation values in the “condensate vacuum”. We then took as a crude guess that the Higgs expectation value in the “condensate vacuum” $< \phi_H > 123$ GeV equal, as already mentioned, to half the value of the one in the “present vacuum”.

However we present a theoretical investigation of the effective potential $V_{\text{eff}}(\phi_H, \phi_F)$ in the appendix, which suggests that in the “condensate vacuum” it is more natural for the Higgs field expectation value to be zero $< \phi_H >= 0$, somewhat in analogy to the assumption that the expectation value of the $\phi_F$ field is zero in the “present vacuum”. Thereby we get a bigger difference between the quark masses in the two phases and thus a larger value, by a factor of 2, for the mass or potential energy difference $\Delta V$ between the two phases for a nucleon. Therefore in the present paper we prefer to take the value $\Delta V = 20 \pm 14$ MeV rather than $\Delta V = 10 \pm 7$ MeV as used in [3].

Another modification or improvement of our model in the present article compared to the original one [3], is that we now believe it to be unrealistic to assume that the size of the pearl is just so that it is on the stability borderline, because a tiny little vibration - especially during its formation in the big bang era - would have caused it to collapse. Rather we introduce in section 7 a
parameter $\xi$ denoting the ratio of the actual (average) radius $R$ of the pearls relative to the critical radius $R_{\text{crit}}$ used in \cite{3} corresponding to the pearls being in the critical state just about to collapse.

Throughout the paper we shall present results using both the parameters in the original model \cite{3} and with the above improvements.

2 The Frequency of the 3.5 keV X-ray

2.1 Estimate of Energy Gap

Now we shall estimate the homolumo energy gap \cite{35,36}, which is supposed to appear between the filled and the empty states in the material in the interior of our dark matter pearls. Basically we shall first argue in subsection 2.2 that there will be a homolumo gap in the supposed fluid or glassy state of the highly compressed material inside our pearls. Next we shall give a very simple dimensional argument in subsection 2.3, that the order of magnitude of this gap for our relativistically compressed matter shall be given in terms of the Fermi energy/momentum $E_f = cp_f$ by

$$\text{"homolumo-gap"} \approx \frac{3}{2} \alpha \left( c^{-1/2} p_f \right) \approx \frac{3}{2} \alpha c^{-3/2} E_f.$$  \hspace{1cm} (1)

2.2 Existence of Energy Gap

It is not easy to know what sort of state, such as crystalline or fluid, that ordinary matter should take under the enormous pressure inside our pearls. So we shall allow ourselves to speculate, that it is in a fluid or glassy state in which there is a lot of irregularities.

We shall assume that in such a glass-like material it is typical to find a gap in the electron state spectrum.

If the glassy structure is very pronounced the electron energy eigenstates tend to be (Anderson) localized, and effectively the spectrum should be determined from a local region in the material. A priori, before one takes into account the back reaction from the interaction between the electrons with themselves or other degrees of freedom, the electron spectrum would be given by the band structure or let us say better by some random matrix model. In that situation all the intervals between neighboring energy levels will be similar in magnitude. But now, when back reactions from the electrons in the filled states are taken into account, the special level spacing between the highest occupied homo- and the lowest unoccupied lumo-levels would be expanded to be much larger than the other spacings. It is this expansion we call the homolumo-gap-effect \cite{36}.

In the present article we want to assume that for some reason there is an expanded (due to the interaction) homolumo gap. Then below we shall seek to estimate the size of this homolumo gap from the philosophy that it appears due to the interaction that an electron has with neighboring electrons, which consequently adjust to minimize the total energy in the presence of the first considered electron. The interaction between the electrons and other charged particles - the protons or the electrons themselves - is basically due to the Coulomb force and thus we are led to estimate the homolumo gap effect from such a Coulomb interaction. This means that the homolumo gap effect is essentially the same.
as the screening of the electric charge of the electrons in the filled states. We therefore suggest in the present article to estimate the homolumo gap effect by using an estimate of the screening based on the Thomas Fermi approximation [38].

2.3 Dimensional Estimate of Energy Gap

We shall here use units with Planck’s constant $\hbar = 1$. Note however, that we do NOT here put the light velocity to 1 as one would normally have done in high energy physics. With the Planck constant $\hbar$ put to 1, it follows that the distance $r$ multiplied by a momentum $p$ is dimensionless $[rp] = [h] = 1$. Since $\alpha/r$ has the units of energy, it then follows that the fine structure constant $\alpha$ has the dimension of $J$.

However we consider such a high density of matter that, because of the pressure, the electrons become relativistic. Consequently two velocities appear in the calculation, namely both the fine structure constant $\alpha$ and the light velocity $c$.

In order to make a dimensional argument for the homolumo gap in this relativistic case, we shall extract the main dependence on the speed of light from the Thomas Fermi based calculation in section 2.4. The crucial quantity for calculating the screening energy - which we identify energy-wise with the homolumo gap - is the derivative of the density of electrons $n_e$ with respect to the Fermi energy $E_f$ of the electrons in the material. In the units with Planck’s constant put to unity the density is given as $p_f^3$ dimensionally/order of magnitudewise. The (Fermi) energy of an electron is $E_f = c p_f$, and thus the derivative of the density of electrons w.r.t. the Fermi energy $\frac{\partial n_e}{\partial E_f}$ is given dimensionally by $p_f^3/(p_f c) = p_f^2/c$. In addition to this derivative we need no more information about the sea of electrons to calculate its screening effect, and thus the homolumo gap will only depend on this derivative $p_f^2/c$ and on the fine structure constant $\alpha$, which occurs in the driving force for screening. The homolumo-gap or the screening energy of course has dimension of energy $[J]$, and thus in our units for now we have ignoring dimensionless factors

$$E_H = \sqrt{\frac{p_f^2}{c} \alpha^{3/2}} \quad \text{(order of magnitudewise).}$$

(2)

As in [3] we at first assume that our pearls are on the borderline of stability, which leads to the following result for the Fermi energy of the relativistic degenerate electrons

$$E_f = 2 \Delta V = (20 \pm 14) \text{ MeV} \quad \text{for } \phi_H = 123 \ \text{GeV}. \quad (3)$$

or

$$E_f = 2 \Delta V = (40 \pm 28) \text{ MeV} \quad \text{for } \phi_H = 0. \quad (4)$$

Ignoring factors of order unity and thus only making a dimensional argument we get using [2]

$$E_H = \sqrt{\frac{\alpha^3}{c^3}} (20 \pm 14) \text{ MeV} \quad (5)$$
With the realistic suggested uncertainty this already agrees well with the 3.5 keV of the observed mysterious line.

2.4 Thomas Fermi Calculation

Now we want to estimate the order of unity factors ignored in the above dimensional argument. For this purpose we shall use the semiclassical Thomas Fermi approximation [38]. In this approximation we estimate the effect of an electric field, say, using macroscopic considerations of Fermi surface statistical mechanics over infinitesimally small pieces of matter.

Our purpose is to calculate the homolumo gap expected to occur as a gap in the single electron spectrum due to back reaction from the electrons themselves. We plan to do that by calculating the decrease in energy of the system around an inserted charged particle (e.g. an electron) due to screening. That is to say we consider the decrease in energy as being due to the displacement of the charged particles in the medium adjusting to the inserted charged particle, and that is the screening. The potential around the inserted charged particle is of course \( \alpha/r \) where \( r \) is the distance to this inserted charge. The important thing to estimate is thus the screening-length for this inserted charge. The potential really means that at the distance \( r \) the Fermi-energy gets shifted by the amount \( \alpha/r \), and thus the density of the charged particles in the medium (the electrons) get changed by \( \alpha/r* \frac{\partial n_e}{\partial E_f} \). The derivative \( \frac{\partial n_e}{\partial E_f} \) of the electron density \( n_e = \frac{2}{3} \frac{4\pi}{(2\pi)^3} \frac{4\pi p_f^3}{3\pi^2 p_f^3} \) w.r.t. the Fermi energy \( E_f = cp_f \) in the ultra relativistic limit of the electron moving almost with speed of light, becomes

\[
\frac{\partial n_e}{\partial E_f} = \frac{p_f^2}{\pi^2 c^2} \tag{8}
\]

Close to the inserted particle, i.e. for small \( r \), the change in the density due to the screening is simply \( \frac{\alpha}{r} \frac{\partial n_e}{\partial E_f} \), but as we go to large \( r \) part of the inserted charge has already been screened and thus we should rather use a diminished charge \( \approx 1 - \int_0^r 4\pi \frac{\alpha}{r} \frac{\partial n_e}{\partial E_f} r^2 dr \). More precisely we shall calculate the unscreened charge \( Q(r) \) measured in Millikan charge quanta at a distance \( r \) from the inserted point charge using the differential equation

\[
\frac{dQ(r)}{dr} = -4\pi r^2 Q(r) \frac{\alpha \frac{\partial n_e}{\partial E_f}}{r} \tag{9}
\]

\[
= -k_0^2 r Q(r) \tag{10}
\]

where \( k_0^2 = \frac{4\alpha p_f^2}{\pi c} \) and \( Q(0) = 1 \) \( \tag{11} \)

It is easy to see that the solution of the differential equation is given by the Gaussian

\[
Q(r) = \exp\left(-\frac{k_0^2}{2} r^2\right) \tag{12}
\]
The screening energy is thus in first approximation given by an integral over the energy per charge \( \frac{\alpha Q(r)}{r} \) multiplied by the induced charge density distribution \( \alpha Q(r) \) divided by the energy per charge \( \frac{\partial n_n}{\partial E_f} \) times \( \frac{4\pi r^2}{\alpha} \)dr and thus becomes

\[
E_{\text{HW}} = \int_0^\infty \frac{\alpha}{r} \frac{\alpha Q(r)}{r} \frac{\partial n_n}{\partial E_f} 4\pi r^2 dr \quad (13)
\]

\[
= \alpha k_0^2 \int_0^\infty \exp\left(-\frac{k_0^2}{2} r^2\right) dr \quad (14)
\]

\[
= \alpha k_0 \sqrt{\frac{\pi}{2}} \quad (15)
\]

\[
= \sqrt{2} \alpha^{3/2} c^{-1/2} p_f \quad (16)
\]

2.4.1 Energy of medium due to the screening change

The quantity \( E_{\text{HW}} \) calculated just above was the energy decrease of the Coulomb interaction of the inserted charge with the charges in the medium being pushed somewhat away in the screening. We, however, did not yet take into account that these particles in the medium would thereby increase their energy in their interaction with each other or something else than just this inserted particle. Thus the above calculated decrease in energy has to be diminished by this increase in medium energy caused by the screening disturbing the medium. We have estimated the effect of this disturbance by imagining that the electrons in the medium function like harmonic oscillators and that the charge of the inserted particle is gradually increased to its actual value. In this way we find that the disturbance increases the energy of the medium by just \( E_{\text{HW}}/2 \). So overall our estimate of the decrease in energy due to the screening is

\[
E_{\text{decrease}} = E_{\text{HW}} - E_{\text{HW}}/2 = E_{\text{HW}}/2. \quad (17)
\]

Now, however, what we are really interested in is the homolumo gap, in the sense of asking for the energy increase when an electron is moved from a homo-state to a lumo-state. The moved electron then leaves behind a hole with an anti-screening energy of the opposite sign \( E_{\text{hole}} = -E_{\text{HW}}/2 \), due to the interaction energy of the electrons surrounding the hole. Hence the homolumo gap ends up being just

\[
E_H = E_{\text{decrease}} - E_{\text{hole}} = E_{\text{HW}} = \sqrt{2} \left(\frac{\alpha}{c}\right)^{3/2} E_f. \quad (18)
\]

Inserting \( E_f = (20 \pm 14) \text{ MeV} \) and \( \alpha/c = 1/137 \) into (18), we obtain for our estimated homolumo gap

\[
E_H = (20 \text{ MeV} \pm 14 \text{ MeV}) \times 137^{-3/2} \sqrt{2} \quad (19)
\]

\[
= (8.8 \pm 6.2) \text{ keV} \quad \text{for } \phi_H = 123 \text{ GeV} \quad (20)
\]

or

\[
E_H = (40 \text{ MeV} \pm 28 \text{ MeV}) \times 137^{-3/2} \sqrt{2} \quad (21)
\]

\[
= (17.6 \pm 12.4) \text{ keV} \quad \text{for } \phi_H = 0 \quad (22)
\]
3 Line Intensity

The model proposed for how the 3.5 keV line gets energized is that pairs of our pearls meet each other, very seldomly of course. In the collision of two pearls the skin surrounding the two pearls contract to only one skin surrounding a united bubble. But now since the skin has a very high tension the energy released by this skin contraction is of the order of 1% of the Einstein energy of a whole pearl.

3.1 Ratio of Gain of Energy by Pearl Collapse to Mass

In this subsection we shall calculate the ratio of the energy $E_R$ released, when two of our pearls unite to the mass of one such pearl $M$:

We shall do that calculation of the ratio $\frac{E_R}{Mc^2}$ in terms of the fermi-momentum $p_f$ of the degenerate electrons in the pearl. This Fermi-momentum cannot be larger than $2\Delta V$, because if so the electrons would pull the nucleons out of the pearl [3],

$$p_f \leq 2\Delta V.$$ (23)

However, we shall at first assume - as in our earlier paper [3] - that the pearls are just on the borderline of stability, so that we indeed have equality in the above inequality (23). We take it that there are 2 nucleons per electron, but really we expect the ordinary matter in the pearl to be slightly over rich in neutrons so this 2 might go up a bit. Thus, using (7) for the electron density, the mass-density $\rho_B$ of pearls becomes

$$\rho_B = 2m_N * n_e = \frac{2m_N p_f^3}{3\pi^2}.$$ (24)

The radius $R$ of a pearl is related to its mass by $M = \frac{4\pi}{3}R^3 \rho_B$ and hence

$$R = \frac{1}{p_f} \left( \frac{9\pi M}{8m_N} \right)^{1/3}.$$ (25)

When a collision occurs between two dark matter balls, an energy $E_R$ of the order of the energy in the bubble surface $E_S$ of a single pearl is released. Denoting the surface tension, the tension of the skin around the pearl, by $S$ the pressure from this skin is $2S/R$. This pressure must equal the pressure from the relativistic electrons, which dominates the pressure of the material in the pearl,

$$\frac{2S}{R} = P = \frac{cp_f^4}{12\pi^2}.$$ (26)

Thus the energy of the skin, the surface energy, is

$$E_S = S * 4\pi R^2 = R^3 \frac{cp_f^4}{6\pi} = \frac{3}{16} \frac{Mc^2}{m_N}.$$ (27)

So the fraction of the Einstein energy $Mc^2$ of a pearl that is emitted in some way, which we suggest to be mainly in the form of the 3.5 keV X-ray line, when
two particles collide, is
\[
\frac{E_R}{M c^2} \sim \frac{E_S}{M c^2} = \frac{3}{16} \frac{\epsilon p_f}{m N c^2} \tag{29}
\]

With \( p_f = 2\Delta V = 20 \text{ MeV/c} \) and \( m N c^2 = 1 \text{ GeV} \), we thus find
\[
\frac{E_S}{M c^2} = 0.38\% \tag{30}
\]

With \( p_f = 2\Delta V = 40 \text{ MeV/c} \) (as for \( \phi_H = 0 \)) we get
\[
\frac{E_S}{M c^2} = 0.76\%. \tag{31}
\]

4 Fit by \( \rho^2 \), Cline Frey

4.1 Cline Frey work

In the article [34] Cline and Frey fit the observations of the 3.5 keV line from the point of view that the line is due to inelastic scattering of dark matter to an excited state that subsequently decays - the mechanism of excited dark matter (XDM). The speciality of such an XDM model is that the intensity of radiation coming from a region in space is proportional to the square of the density \( \rho_D \text{M} \) i.e. to \( \rho_D^2 \) rather than to the first power only. In a model like ours, in which the production of the X-ray line 3.5 keV comes from collisions of our dark matter pearls, of course the production rate of these X-rays also goes proportionally to the square of the density.

4.2 Their analysis

Table 2: This table is based on the table 1 in reference [34].

| Name       | \( N < \sigma_{CFV} > \) \( \times 10^2 \) | \( v \) | boost \( \times 10^{-22} \text{cm}^3 \text{s}^{-1} \) | \( \frac{2 \times 10^{21} \text{cm}^2}{\text{MeV}} \) | Remark |
|------------|---------------------------------|--------|---------------------------------|---------------------------------|--------|
| Clusters 5 | 480 ± 250                      | 975    | 30                              | 0.016 ± 0.008                   |        |
| Perseus 4  | 1400 - 3400                    | 1280   | 30                              | 0.037 - 0.09                    |        |
| Perseus 6  | (1 - 2) \( \times 10^5 \)     | 1280   | 30                              | 2.7 - 5.3                       | ignored|
| Perseus 41 | 2600 - 4100                    | 1280   | 30                              | 0.07 - 0.11                     |        |
| CCO 5      | 1200 - 2000                    | 926    | 30                              | 0.04 - 0.07                     |        |
| M31 6      | 10 - 30 (NFW)                  | 116    | 10                              | 0.0086 - 0.026                  |        |
|            | 30 - 50 (Burkert)              |        |                                 | 0.026 - 0.043                   |        |
| MW 21      | 0.1 - 0.7 (NFW)                | 118    | 5                               | 0.00017 - 0.0012                | ignored|
|            | 50 - 550 (Burkert)             |        |                                 | 0.084 - 0.93                    | in average|
| Average    |                                 |        |                                 | 0.032 ± 0.006                   |        |

The main result of the analysis of Cline and Frey is reproduced by their table given just above. The notation used is as follows:

- \( N \) is the number of 3.5 keV photons emitted per collision.
• $\sigma_{CF}$ is the cross section as calculated assuming that there is no boost (see below) for the particles of the dark matter to collide. That is to say $\sigma_{CF}$ is calculated as if the density $\rho$ of dark matter were given just by the model for dark matter distribution used by Cline and Frey, without any clumping further than the galaxies themselves having been included. The estimated true cross section is thus rather

$$\sigma = \sigma_{CF}/\text{boost}. \quad (32)$$

• $v$ is the average velocity dispersion.

• $\text{boost}$ is the increase in collision rate due to the clumping of the dark matter into sub-halos, which we crudely estimate from references [42], [43] and [44]. These papers contain computer simulations of gravitational interactions between dark matter constituents in galactic halos forming clumps of much higher density than the average density. Such clumping obviously enhances the true average square density compared to the square of the average density.

• $M$ is the mass of the dark matter particles.

• Since it looks hopeless to get a fit with values for the same quantity differing by a factor 50 or 100, we left out the Boyarsky measurement for the Perseus cluster. It really means that the model used by Cline and Frey with the intensity proportional to the square of the dark matter density is disfavored by the Perseus cluster analysis. In the section 4.3 below we mention other troubles for precisely the Perseus Cluster, possibly supporting the idea that something there goes on which we do not understand well.

• Because of the uncertainty in the dark matter distribution in the Milky Way center (MW) - whether it be NFW or Burkert - we left the Milky Way center out of the averaging; but within uncertainties it is quite consistent with the average.

• The notations “NFW” [45] and “Burkert” [46] stand for a couple of different models for the distribution of dark matter in a galaxy mainly deviating by “NFW” having a strong peak at the center of the galaxy.

• MW stands for our Milky Way galactic center.

• M31 is the Andromeda Galaxy.

• CCO stands for a combination of Coma + Centaurus + Ophiuchus clusters.

Using the average value for $\left(\frac{N<\sigma v^{\nu}_{\text{boost}}}{v^{\ast}}\right)$ from the table and (32), we obtain

$$\left(\frac{N\sigma}{M^2}\right)_{\text{exp}} = \left(0.032 \pm 0.006\right) \times 10^{-27} \text{cm}^2/(10 \text{ GeV})^2 \quad (33)$$

$$= \left(1.0 \pm 0.2\right) \times 10^{23} \text{cm}^2/\text{kg}^2 \quad (34)$$
4.3 Trouble of Perseus Cluster

The amount of 3.5 keV radiation from the Perseus Cluster is controversial: At first it seemed to be a significant source \cite{5,6} suggesting a lifetime of the order of $3 \times 10^{27}$ s in the sterile neutrino model. But then the Hitomi satellite \cite{47} did not see any 3.5 keV signal from the Perseus Cluster. Now a possible way out of the controversy would be \cite{48} that there is a 3.5 keV absorption line which for Hitomi (which had less angular resolution and thereby included the active galactic nucleus in their observations) would compensate the diffuse cluster emission line. Such a story about an absorption line is, however, totally unacceptable in our model. Our pearls would certainly not be able to absorb radiation of any significance from Perseus.

Now, however, this absorption picture has severe problems by itself: In fact Conlon et al. \cite{48} have suggested a fluorescent dark matter model to solve the Perseus Cluster problem, in which a 3.5 keV absorption line results from resonant excitation of a dark matter particle $\chi_1$ of mass $m_{DM}$ much greater than 3.5 keV. The excited dark matter particle $\chi_2$ then drops back to its ground state $\chi_1$, providing the 3.5 keV emission line seen in the diffuse cluster. Making an ansatz for the fluorescent dark matter particle interaction with the photon of the form

$$\mathcal{L} \supset \frac{1}{M} \bar{\chi}_2 \sigma_{\mu \nu} \chi_1 F^{\mu \nu}.$$  \hspace{1cm} (35)

(In this subsection $M$ is the inverse of the effective coupling.) Conlon et al derive a lower bound for the $\chi_2 \rightarrow \chi_1 + \gamma$ decay width

$$\Gamma \geq \left( \frac{m_{DM}}{\text{GeV}} \right) \times (1 \text{ to } 10) \times 10^{-10} \text{ keV.}$$  \hspace{1cm} (36)

This relation leads to

$$\frac{m_{DM}}{\eta^{2/3}} \lesssim 10^6 \text{ keV,}$$  \hspace{1cm} (37)

where $\eta = m_{DM}/M$.

But now earlier Profumo and Sigurdson, see fig 2 in their paper \cite{49}, had investigated the experimental constraints on the parameters $m_{DM}$ and $\eta$ in such a resonant absorption model of dark matter. The region corresponding to \cite{37} in their figure is incompatible with the allowed range for the parameters. Thus the fluorescent model does not seem tenable.

Taking the intensity of the 3.5 keV radiation to be proportional to the square of the dark matter density (as in the analysis of Cline and Frey), the data at larger angles by Boyarsky et al. \cite{6} are far too high compared to the data from Bulbul et al. \cite{5} at lower angles from the direction to the center of the Perseus cluster. This could mean a severe disfavoring of square density models. In order to avoid having this discrepancy entering into our fit, we simply leave out the Perseus results of Boyarsky et al., because it is the Bulbul et al. results which agree best with the other data in the table.

4.4 Prediction

We found (see \cite{25} and \cite{25}):

$$E_S = \frac{3}{16} \frac{M_{Pl}}{m_N}$$  \hspace{1cm} (38)
\[ R = \frac{1}{p_f} \left( \frac{9\pi M}{8m_N} \right)^{1/3} \]  
(we use c=1 units, except in sections 2 and 5) and so
\[ \sigma = \pi (2R)^2 = 4\pi \star \frac{1}{p_f} \left( \frac{9\pi M}{8m_N} \right)^{2/3}. \]  
(40)

Assuming 100% efficiency in converting the energy released by the surface contraction into the 3.5 keV radiation, we obtain
\[ N = \frac{E_S}{(3.5 \text{ keV})} = \frac{3}{16} \frac{M p_f}{m_N (3.5 \text{ keV})} \]  
(41)
and, using the parameters \( M = 1.4 \times 10^8 \text{ kg} \) and \( p_f = 20 \text{ MeV} \) from our previous article [3], we find
\[ \frac{N \sigma M^2}{M^2} = \frac{3}{16} \frac{M p_f}{m_N (3.5 \text{ keV})} \star \frac{4\pi}{p_f} \left( \frac{9\pi M}{8m_N} \right)^{2/3} \frac{1}{M^2} \]  
\[ = 5.5 \left( \frac{m_N}{M} \right)^{1/3} m_N^{-2} (p_f * 3.5 \text{ keV})^{-1} \]  
\[ = 2.6 \times 10^{22} \text{cm}^2/\text{kg}^2, \]  
(43)
for which the above data analysis gave \((1.0 \pm 0.2) \times 10^{23} \text{cm}^2/\text{kg}^2\).

This means we predict just a factor of four too little production of 3.5 keV radiation, which is already a remarkably good agreement! However, this simple result (44) was made assuming that all the released energy went into the 3.5 keV X-ray radiation. We want to argue that this is likely to be true, but that is far from obvious at first.

5 Efficiency of Sending Energy to the 3.5 keV line

The heat energy of the electrons [38] in the pearl at a temperature \( T \) would be
\[ \text{“heat energy (electrons)”} = \frac{\pi^2 T^2 M}{4cp_f^2 m_N} \]  
(45)
and the heat energy of the nucleons which is essentially that of the phonons would be
\[ \text{“heat energy (nuclei)”} = \frac{3TM}{m_N} \text{ (above the Debye temperature).} \]  
(46)
If the nucleons dominate the heat energy after the collision we have
\[ \text{“heat energy (nuclei)”} = E_S. \]  
(47)
Using the estimate [28] \( E_S = \frac{3Mc_p}{16m_N} \) for the energy released by the contraction of the skins of two colliding pearls, we then obtain the following equation for the temperature after the collision
\[ \frac{3TM}{m_N} = \frac{3Mc_p}{16m_N} \]  
(48)
\( T = \frac{c p_f}{16} \) (49)

In the case the electrons should have dominated the heat energy we would have got

\[ \frac{T^2 M \pi^2}{4 p_f c^2 2 m_N} = \frac{3 M c p_f}{16 m_N} \] (50)

or

\[ T = \sqrt{\frac{3}{2}} \frac{c p_f}{\pi} \] (51)

If we take into account that the nucleons are not oscillating separately, because they are bound into nuclei of say \( N \) nucleons per nucleus, the temperature which we found at first to be \( T = \frac{c p_f}{16} \) is replaced by

\[ T = \frac{N c p_f}{16} \] (52)

\[ \approx \frac{3}{4} c p_f \) (for C, Ne say, assuming C dominates). (53)

But if so then the take up of heat by the electrons and the nuclei become similar. Thus we must add their contributions and the temperature will be about the half of each of them separately. In fact combining the two heat capacity sources, electrons and nuclei, we get (via a second order equation):

\[ T \approx 0.3 c p_f = 0.6 \Delta V \] (54)

\[ = 6 \text{ MeV if } \phi_{H \text{ condensate}} = 123 \text{ GeV} \] (55)

\[ = 12 \text{ MeV if } \phi_{H \text{ condensate}} = 0 \] (56)

This \( T \) is thus the temperature of the pearl if the energy from the contraction of the surface has spread throughout the pearl. We shall consider it to be the temperature in the central region during the cooling off of the pearl, suggested by us to go by emission of the 3.5 keV radiation. But now the problem is whether the pearl material has sufficiently low heat conductivity to allow the surface to maintain a temperature just of the order of magnitude of 3.5 keV with such an enormously hot center.

### 5.1 The heat conductivity.

In this subsection we shall by dimensional analysis estimate the heat conductivity \( k \) for our pearl material as well as in parallel for ordinary matter.

We shall assume that our material is in the high temperature range which corresponds to what in ordinary metals leads to a temperature independent conductivity \( k \). This means that we take it that the temperature is higher than the Debye temperature, as discussed below.

The velocity of the sound is crudely estimated as the square root of the pressure over the density,

\[ \text{“sound velocity” } \approx \sqrt{\frac{\text{“pressure”}}{\text{“density”}}} \] (57)

\[ \approx \sqrt{\frac{c p_f^4}{12(2\pi)^2}} = \sqrt{\frac{c p_f}{8 m_N}} \] (58)

\[ = 0.05 c. \] (59)
Here we took $p_f = 20 \text{ MeV/c}$.

The effective lattice constants would be $p_f^{-1}$ if we took it that there were equally many nuclei as electrons. But first there are both a proton and a neutron for each electron, and secondly these nucleons are collected into nuclei. So, taking $N = 12$ nucleons per nucleus, the effective lattice constant becomes

\[
\text{“effective lattice constant”} = p_f^{-1} \sqrt[3]{N/2} \quad (60)
\]

\[
\approx 0.1c/\text{MeV.} \quad (61)
\]

Thus the Debye frequency becomes of the order

\[
\text{“Debye frequency”} \approx \frac{\text{“sound velocity”}}{\text{“effective lattice constant”}} = 0.5 \text{ MeV} \quad (62)
\]

The temperature in the interior in the beginning $\sim 6 \text{ MeV} \quad (55)$ or $\sim 12 \text{ MeV} \quad (56)$ is significantly higher than this Debye temperature of $0.5 \text{ MeV}$. So we are in the high temperature regime where the heat conduction should be roughly temperature independent.

5.1.1 The power of $\alpha$ in the pearl matter conductivity.

A priori one thinks that the mean free path is determined from interactions and that each time we have an interaction in a material this interaction must be electromagnetic. Thus such interactions will always be proportional to the electromagnetic coupling constant $\alpha$, which in our units here is a velocity.

However when we consider phonons or equivalently interactions with the vibrating nuclei, there is an interesting cancelation preventing the coupling constant $\alpha$ from coming into the calculation, except as the electron velocity in the non-relativistic materials:

The point is that the vibration of the whole crystal - or better above the Debye temperature the vibration of the atomic nuclei - is only driven back to its equilibrium state by forces that are themselves proportional to the fine structure constant $\alpha$. This means that the back driving force is the weaker the smaller this constant $\alpha$. Thus the displacement of the nuclei or equivalently the phonon amplitudes are the bigger the smaller is $\alpha$.

For instance a nucleus being displaced a distance $r$ from its equilibrium position feels a potential proportional to $\alpha$. Strictly speaking $\alpha/r$ say, but we could approximate it by an harmonic potential $\alpha * r^2$ corrected by some constant of the order of the “lattice constant” $a$ say to get the right dimension; then the potential energy is approximated as $\alpha * r^2 / a^3$. In any case the distance the atomic nucleus gets displaced by its thermal motion will be so as to increase its potential energy by of order the temperature $\sim T$. With the harmonic approximation the distance squared for its typical deviation then becomes $r^2 \sim T a^3 / \alpha$. Now the electron being stopped by interacting with such a nucleus has an interaction that must be proportional to $\alpha$. So it gets an amplitude for its scattering proportional to $r \alpha$, provided the screened field of the electron hits the oscillating atom. Most of the scattering of the electrons by phonons turns out to have such small impact parameters that the screening of the electric field around the electron essentially does not prevent the scatterings. So the cross section for an electron hitting a phonon or equivalently a proton the displacement of
which represents the phonons goes as \( (\alpha r)^2 \propto \alpha \). So this cross section is a linear function of \( \alpha \), i.e. it is proportional to \( \alpha \) to the first power (only). This in turn means that the mean free part \( a \) must behave as \( \alpha^{-1} \). The other factors \( v, c_p, \rho \) entering the thermal conductivity \( k = \frac{1}{3} v * a * c_p * \rho \) have no \( \alpha \)-dependence in our pearl material. But in ordinary materials the typical electron velocity at the fermi surface is \( \alpha \) (using units in which \( \alpha \) is a velocity).

Having argued for how many factors of \( \alpha \) must occur in the expression for the conductivity \( k \) also for the pearl matter we can by dimensional analysis derive the order of magnitude expression for this conductivity. In fact we simply work as if there was only the velocity \( c \) at our disposal and then multiply by the extra dimensionless factor \( \frac{c}{\alpha} \), because we decided that \( \alpha \) should come in to the power \(-1\) in \( k \). Thus, provided we exclude the possibility that the velocity of sound can come in, we obtain the result from dimensional analysis:

\[
k = \frac{c}{\alpha} * c * p^2_f = \frac{c^2 p^2_f}{\alpha}.
\] (64)

Had we considered ordinary matter at usual pressures, where the electrons and nuclei remain non-relativistic so that there is only one relevant velocity \( \alpha \), the only possibility by dimensional analysis is

\[
k = \alpha p^2_f.
\] (65)

### 5.2 More accurate heat conductivity

To do a little bit better than the above only dimensional or order of magnitude derivation of the heat conductivity \( k \) for our pearls, we now consider the supposed main effect of scattering the electrons - namely scattering by the emission of phonons - a bit more accurately. We start from the transition rate as given in formula (26.40) in the book “Solid State Physics” by Ashcroft and Mermin [38]:

\[
|g_{k, k'}|^2 = \frac{V}{|k - k'|^2 + k_0^2} \frac{1}{2} \hbar \omega_{k - k'}.
\] (66)

\[
= \frac{1}{V} \frac{4\pi e^2}{|k - k'|^2 + k_0^2} \frac{1}{2} \hbar \omega_{k - k'}.
\] (67)

The notation here is

- \( g_{k, k'} \) is the transition amplitude for an electron going from having wave number \( k \) to wave number \( k' \) by emission (or absorption) of a phonon of energy \( \hbar \omega_{k - k'} \).

- The quantization volume \( V \) is used to discretize the wave numbers \( k \) and \( k' \).

- The charge \( e \) in the first line is, following Ashcroft and Mermin, in e.s.u. units. In our notation the potential between two elementary charges separated by a distance \( r \) is \( e/\alpha \). Thus we have \( \alpha = e^2 \), where \( e \) is the one in the first line [66].
\[ k_0 = 2\sqrt{\frac{\pi}{\alpha}} \delta_f \]

We may consider the transition amplitude \( g_{\vec{k}\vec{k}'} \) as a matrix element of the perturbation energy \( V_{int} \), due to the interaction with the phonon between the electron state \( \vec{k} \) and the electron state \( \vec{k}' \). Counting the dimensionality of the \( \vec{k} - \vec{k}' \) say as momentum, it is indeed seen that the quantity \( |g_{\vec{k}\vec{k}'}|^2 \) (in the equation (66)) has dimensionality of energy squared.

The Fermi golden rule says that the decay rate \( \Gamma_{\vec{k}f} \) of say the electron in the state \( \vec{k} \) decaying into a continuum of states \( f = (\vec{k}', \gamma_f) \) of a phonon marked \( \gamma_f \) and an electron with wave number \( \vec{k}' \) is

\[ \Gamma_{\vec{k}f} = 2\pi \rho_{DOS}|g_{\vec{k}\vec{k}'}|^2 \]

Here \( \rho_{DOS} \) is the density of states as a function of energy of the combined electron phonon states \( f \). This factor \( \rho_{DOS} \) will come out effectively, if instead we insert an energy conservation delta-function \( \delta(E_{\vec{k}} - E_f) \) and integrate or sum over all the possible combined states \( f \). Thus the total decay rate due to the interaction ending with the phonon having the momentum equal to \( \vec{k} - \vec{k}' \) is (ignoring at first the boson statistical enhancement by the number of phonons already in the state):

\[ \Gamma_{\vec{k}f} = \sum_{\vec{k}'} 2\pi \delta(E_{\vec{k}} - E_f)|g_{\vec{k}\vec{k}'}|^2 \]

Here we have used the replacement of the sum over \( \vec{k}' \) by an integration

\[ \sum_{\vec{k}'} \rightarrow \int \ldots \frac{V d^3 \vec{k}'}{(2\pi)^3}. \]

If the final state for the electron \( \vec{k}' \) is not empty, of course its contribution will be missing. We are concerned with electrons \( \vec{k} \) near the fermi surface where they have the possibility to decay into empty states \( \vec{k}' \). Near the fermi surface we have \( \frac{dE_{\vec{k}'}|}{d|\vec{k}'|} = v_{\text{fermi}} = c \) in our pearls, and thus

\[ \delta(E_{\vec{k}} - E_{\vec{k}'}) = \frac{1}{v_{\text{fermi}}} \delta(|\vec{k}| - |\vec{k}'|). \]

The phonon velocity - i.e. the speed of sound \( c_{\text{sound}} \) - is of the order \( v_{\text{fermi}} \sqrt{\frac{m_{\text{rel}}}{M_N}} \) as compared to the electron velocity \( v_{\text{fermi}} \). Thus the sound velocity is relatively very low and the phonon energy \( \hbar \omega_{\vec{k} - \vec{k}'} \) becomes small compared to the energies of the electrons. Consequently the phonon energy is not so important in the energy conservation delta-function.

Luckily we do not have to calculate the phonon energy \( \hbar \omega_{\vec{k} - \vec{k}'} \) in the high temperature regime we have in mind, because it is cancelled out by the boson enhancement effect. This effect enhances the decay rate into a boson state by a factor equal to the number of bosons already present in that state. It namely
happens that, since the number of bosons/phonons at temperature $T$ in a state with energy $\hbar \omega_{\vec{k}, \vec{k}'}$ is just $T/(\hbar \omega_{\vec{k}, \vec{k}'}$), the phonon energy $\hbar \omega_{\vec{k}, \vec{k}'}$ drops out of the expression for the decay rate:

The decay rate of an electron state with momentum/wave-number $\vec{k}$ under the temperature $T$ and including the boson-enhancement effect becomes

$$\Gamma_{\text{bose-enhanced}} = \int 2\pi \delta(E_\vec{k} - E_f)|g_{\vec{k}\vec{k}'}|^2 \frac{T}{\hbar \omega_{\vec{k}, \vec{k}'}^2} \frac{d^3\vec{k}'}{(2\pi)^3} V$$ (73)

$$= \int 2\pi \delta(E_\vec{k} - E_f) * \frac{4\pi \alpha}{|\vec{k} - \vec{k}'|^2 + k_0^2} \frac{1}{2T} \frac{d^3\vec{k}'}{(2\pi)^3}$$ (74)

$$= \frac{\alpha T}{2\pi v_{\text{fermi}}} \int \frac{\delta(\vec{k} - \vec{k}'|)}{|\vec{k} - \vec{k}'|^2 + k_0^2} d^3\vec{k}'$$ (75)

$$\approx \frac{\alpha T}{2\pi v_{\text{fermi}}} \int \approx 4p_f^2 \frac{1}{|\vec{k} - \vec{k}'|^2 + k_0^2} d(\Delta \vec{k}'|^2)$$ (76)

$$\approx \frac{\alpha T}{2c} \ln \left( \frac{4p_f^2}{k_0^2} \right) = \frac{\alpha T}{2c} \ln \left( \frac{\pi c}{\alpha} \right).$$ (77)

Let us now resume the expressions for the quantities $c_p$, $v$, $\rho$ and $a$ to be used to construct the conductivity $k = \frac{c_p a \rho v}{3}$. We have

- The average heat capacity of an electron $c_V \approx c_p = \frac{n_e^2}{2} * \frac{T}{E_f}$,
- The mean free path - here taken with phonons dominating - $a \approx \frac{2c^2}{\alpha T \ln \left( \frac{\pi c}{\alpha} \right)}$,
- The density of electrons in the material $\rho = \frac{p_f^2}{3\pi^2}$,
- The electron velocity $\approx c$ in our pearls.

Thus we obtain the conductivity estimate for our pearl material:

$$k = \frac{c_p a \rho v}{3}$$ (81)

$$\approx \frac{c^2 p_f^2}{9 \alpha \ln \left( \frac{\pi c}{\alpha} \right)} = \frac{c^2 p_f^2}{550}.$$ (82)

Compared to the expression $k \sim \frac{c^2 p_f^2}{\alpha}$ obtained above just by dimensional and physical arguments, the present supposedly more accurate expression deviates
from it by a factor 55 in the denominator. This factor in the denominator arose
from the order unity numerical numbers included giving 9 multiplied by an extra
logarithm essentially of the inverse $\alpha$.

Let us now compare our calculational method to the non-relativistic physics
of ordinary metals:

In ordinary metals the fermi velocity is no longer $c$ as in the interior of the
dark matter pearls, but rather of the order of $\alpha = \frac{c}{137}$. So we should just
replace the $c$ in our formula (82) for $k$ with true fermi velocity, approximately $\alpha$.

Thus our expectation for the non-relativistic material thermal conductivity
is $k = \frac{\alpha v_f^2}{m_e^2} = \frac{\alpha v_f^2}{10}$. Assuming that the fermi momentum is given by a fermi velocity $v_f$ as $m_e v_f$
our dimensional analysis formula means that we predict the ratio $k/v_f^2$ to be

$$\frac{k}{v_f^2} = \frac{\alpha m_e^2}{v_f^2} = \alpha m_e^2.$$ (83)

In rather strange units we have

$$\alpha m_e^2 = 1.6 \times 10^{14} m^{-3} s,$$ (84)

while

$$\frac{W/(mK)}{(10^6m/s)^2} = \frac{1}{1.38} \times 10^{14} m^{-3} s$$ (85)

so that

$$\alpha m_e^2 = 2250 \frac{W/(mK)}{(10^6m/s)^2},$$ (86)

If we take our best estimate for $k$, which is 10 times smaller than the dimensional estimate for non-relativistic ordinary materials, we get the prediction

$$\frac{k}{v_f^2} = \frac{\alpha m_e^2}{10} = 225 \frac{W/(mK)}{(10^6m/s)^2}.$$ (87)

For a series of metals, copper, gold, silver, iron, lead, lithium, mercury, we find
respectively the thermal conductivity values $k = 385$(Cu), 314(Au), 406(Ag),
79.5(Fe), 34.7(Pb), 85(Li), 8.3(Hg), in the unit $W/(mK)$ and the fermi velocities,
1.57(Cu), 1.40(Au), 1.39(Ag), 1.98(Fe), 1.83(Pb), 1.29(Li), 1.58(Hg), in the unit
$10^6m/s$. This means that they have the values, 156(Cu), 160(Au), 210(Ag),
20(Fe), 13(Pb), 51(Li), 3.3(Hg) respectively for the ratio $k/v_f^2$.

The variation of the ratio $k/v_f^2$, which is a constant in our calculation, suggests
an uncertainty of this calculation of the order of a factor 5 up or down. This
means our value of $k$ comes with an uncertainty $\exp(\pm 150\%)$. But actually
all the mentioned metals except silver have a lower conductivity $k$ than our estimate. So presumably also our pearls have a bit lower conductivity than our
calculational estimate $\frac{c^2 p_f^2}{50a}$. This means that presumably the true $\frac{t_{spread}}{t_{equation}}$ to be
discussed later becomes bigger than our immediate estimate. Taking the
above metals as representing a mean value for $k/v_f^2 \sim 50$, we would assume that
our a priori estimates should be decreased by a factor $\frac{225}{50} \sim 5$. So, in this way,
the corrected estimate for the relativistic interior of the dark matter would be

$$k_{empirically corrected} = \frac{c^2 p_f^2}{250a}$$ (88)
6 Cooling of Pearl

Let us form ourselves a picture of how we imagine the cooling goes when two pearls have collided:

- First notice that the skin of the two pearls having collided will typically deliver the energy/heat in a highly non-uniform way. It is basically that some very thin region near the part of the skin that gets most contracted takes up almost all the heat $\sim E_S$. In the very first moment before heat has had time to spread therefore of course the rest of the pearl is cold.

- Next the heat spreads according to heat diffusion. The easiest situation for calculation might be the special case where only one point was heated up, while a more realistic picture may be that it is a smaller part of the skin that gets heated. If a unit amount of heat were sitting at the origin, with delta-function density $u = \delta(\vec{r})$, say at time $t = 0$, it is well-known and easy to see that this situation corresponds to the solution of the heat diffusion equation

$$ \rho c_p \dot{u} - k \Delta u = 0, \quad (89) $$

where we have no extra addition of heat and $\dot{u} = \frac{\partial u}{\partial t}$ is the time derivative of the energy density $u$, while $\Delta$ is the Laplacian. The solution of the heat diffusion equation with the initial condition (at time $t = 0$)

$$ u = \delta(\vec{r}) \quad (90) $$

leads to the heat-kernel:

$$ u = \left( \frac{4\pi k t}{\rho c_p} \right)^{-3/2} \exp \left( -\frac{\vec{r}^2}{4kt} \right). \quad (91) $$

From this solution one sees that the size of the region, which gets hot, spreads - defining its border say from where the expression in the exponent is just unity - with

$$ |\vec{r}_{\text{border}}| \propto \sqrt{t}. \quad (92) $$

That is to say the size $|\vec{r}_{\text{border}}|$ of this hot region grows at first very fast, and then slows down.

- The time it takes for the “hot” blob of temperature $T = 6$ MeV or $T = 12$ MeV to reach the size of the pearl, i.e. to reach $|\vec{r}_{\text{border}}| \sim R = 0.67\text{cm}$, or $\sim R = 0.34\text{cm}$ will be of the order

$$ t_{\text{spread}} \approx \frac{\rho c_p * R^2}{4k} \quad (93) $$

$$ \approx \frac{\rho c_p}{4\pi^2} * \frac{T^2}{2E} * \frac{R^2}{4 * c^2 \phi_H^{1/2}} = \frac{a55R^2T}{24c^3} \quad (94) $$

$$ \approx \frac{55(0.67 \text{ cm})^2(6 \text{ MeV})}{137 * 24c^2} \quad (\phi_H=123 \text{ GeV}) \quad (95) $$

$$ = 7.1 * 10^{-2} s \quad (\phi_H=123 \text{ GeV}) \quad (96) $$
During a time of the order of $t_{\text{spread}}$ there will be a hot spot spreading. Since this spot has a Gaussian temperature profile \cite{91}, the temperature in the outskirts of it as a function of the distance will quickly drop down to be almost as cold as the initial cold pearl, which had been in balance with the 3 K outer space for milliards of years. We shall thus imagine an intermediate situation in which part of the pearl is still exceedingly cold, while there is a hot spot with temperature close to the average temperature of the pearl of the order 6 MeV or 12 MeV.

Where this hot spot reaches the surface of the pearl, there will be radiation into space with frequencies up to the average temperature of order, say, 6 MeV or 12 MeV. That will however then quickly lower the temperature just near the surface and the hot spot will be effectively pushed a bit closer to the center of the pearl.

During this period of there being a rather isolated hot spot there will from the border of this hot spot, where the temperature is just of the order of 3.5 keV, still exist to the cold side an undisturbed homolumo gap. Thus only radiation of frequencies lower than the homolumo gap size can penetrate through the cold part of the pearl out into outer space. Just where the temperature passes order of magnitude-wise the 3.5 keV, there will be some excited electrons in the material able to interact so strongly with the photons with frequencies below the gap value that there will be emitted radiation in these frequencies with approximately the strength of a black body.

Provided the surface specified by having just the suitable temperature close to 3.5 keV, from which the 3.5 keV photons just could reach optically from the outside, dominates the surface around the hot spot, there will be a dominantly 3.5 keV radiation cooling off of the hot spot in such an era wherein part of the pearl is still very cold.

In order for such a dominating 3.5 keV emission, it is needed that there is no hot edge of the hot spot reaching the surface of the pearl. At such an edge there would namely be radiated extremely much energy because of the $T^4$ proportionality of the emission. But one could hope that such a hot edge region would be cooled off and the hot spot go into a state with colder and thus 3.5 keV emitting surface almost all around it.

But even if the temperature at the pearl surface does not get down to the 3.5 keV order completely, it would be enough to get a major part of the energy out as 3.5 keV radiation provided the hot surface pieces are rather small and the temperature not much bigger than the 3.5 keV.

As time passes there is now the possibility that the hot spot spreads all over the pearl and at the end there will no longer be any surface where the temperature is just around 3.5 keV. In such a case the radiation in this frequency will stop or rather there will no longer be any peak at this frequency in the radiation from pearl. This kind of stopping happens roughly after the time $t_{\text{spread}}$. 

or

$$t_{\text{spread}} = 3.5 \times 10^{-2} \text{s \hspace{1cm} } (\phi_H=0 \hspace{1cm} (97)$$
But there is another time parameter which we call $t_{\text{radiation}}$ which we define to be the time it takes to radiate out - say into 3.5 keV - essentially all the energy in the heat. In the case that this radiation time should happen to be smaller than the $t_{\text{spread}}$, the energy would run out before the hot spot has spread all over the pearl. Thus it could be that the 3.5 keV radiation would never be stopped before all the energy was essentially used up.

When finally the energy of the heat from the collision is about to run out, the hot spot will of course be colder and there will (again) appear a surface around it where the temperature is down to about the 3.5 keV and 3.5 keV radiation in a peak will be re-established for a time.

To get an idea as to whether there is any chance that a scenario like the above could provide enough 3.5 keV radiation so as to dominate the radiation of the energy from the collision of two pearls, we shall now estimate the time scale $t_{\text{radiation}}$ for the energy to be used up by this radiation.

Remembering that the energy deposited as heat by the skin contraction from a collision of two pearls was estimated to be

$$E_S = \frac{3}{16} \frac{M_{pH}}{m_N}$$

it follows that

$$E_S = 0.38\% M c^2 \quad \text{(with } \phi_H = 123 \text{ GeV}) \quad (99)$$

$$E_S = 0.76\% M c^2 \quad \text{(with } \phi_H = 0) \quad (100)$$

Now the mass of the pearl is

$$M c^2 = 1.4 \times 10^8 \text{ kg} = 0.8 \times 10^{41} \text{ keV}.$$  

(101)

So we get the deposited energy to be

$$E_S = 3.0 \times 10^{38} \text{ keV} \quad \text{(with } \phi_H = 123 \text{ GeV}) \quad (102)$$

$$E_S = 6.0 \times 10^{38} \text{ keV} \quad \text{(with } \phi_H = 0). \quad (103)$$

The typical area through which this heat energy hopefully gets radiated out with a temperature close to the 3.5 keV - if we shall be successful to have most of the energy expended in this way - is the surface area of the pearl

$$4\pi R^2 = 4\pi (0.67 \text{ cm})^2 \quad \text{(for } \phi_H = 123 \text{ GeV}) \quad (104)$$

$$= 1.4 \times 10^{16} \text{ (keV/c)}^{-2} \quad \text{(for } \phi_H = 123 \text{ GeV}) \quad (105)$$

while

$$4\pi R^2 = 0.35 \times 10^{16} \text{ (keV/c)}^{-2} \quad \text{(for } \phi_H = 0). \quad (106)$$

Taking it that the temperature at the emission layer is just 3.5 keV and the Stefan constant $\sigma_{St} = \frac{\pi^2}{60\pi}$ the radiation of $E_S$ through the $4\pi R^2$ takes the time

$$t_{\text{radiation}} = \frac{E_S}{4\pi R^2 \sigma_{St} T^4}$$

$$= \frac{3 \times 10^{38}\text{keV}}{1.4 \times 10^{16} \text{ (keV/c)}^{-2} \times \frac{\pi^2}{60\pi} \times (3.5 \text{ keV})^4} \quad (108)$$

$$= 570 \text{ s} \quad \text{(for } \phi_H = 123 \text{ GeV}). \quad (109)$$

23
while

\[ t_{\text{radiation}} = 4600 \text{ s} \quad (\text{for } \phi_H = 0). \]  

(110)

As a comment on the uncertainty of the here evaluated \( t_{\text{radiation}} \) we especially worry about the factor of \((3.5 \text{ keV})^4\). This is the fourth power of the supposed temperature at the place from which the 3.5 keV radiation is sent out and it is, of course, sensible to take this temperature at emission of the line to be close to 3.5 keV. But really the line is supposed to be emitted by exciton decay (see section 6.1) and that emission could in principle be done at e.g. a higher temperature. If in some layer the temperature \( T \) is higher, the emission will be favoured by a factor \( T^4 \). Thus the effective or dominant temperature is expected to be bigger than 3.5 keV. We have only very crude speculative estimates for how much bigger this could be. However let us say that the truly relevant emission temperature to be used in evaluating \( t_{\text{radiation}} \) is 1.5 *3.5 keV rather than just 3.5 keV:

\[ t_{\text{radiation}} \propto \frac{1}{(3.5 \text{ keV})^4} \quad \Rightarrow \quad t_{\text{radiation}} \propto \frac{1}{(1.5 \times 3.5 \text{ keV})^4} \]  

(111)

This temperature is rather uncertain since it will depend on how quickly as a function of temperature the homolumo gap gets washed out. We therefore assign an uncertainty of at least \( \exp(\pm 100\%) \) to the value of the temperature of emission. Because this emission temperature comes in to the fourth power, it follows that \( t_{\text{radiation}} \) itself has a rather large uncertainty of \( \exp(\pm 400\%) \).

After this discussion we write the final values

\[ t_{\text{radiation}} = 570 s \times 1.5^{-4} \exp(\pm 400\%) \]  

(112)

\[ = 110 \exp(\pm 400\%) s \quad (\text{for } \phi_H = 123 \text{ GeV}). \]  

(113)

\[ t_{\text{radiation}} = 910 \exp(\pm 400\%) s \quad (\text{for } \phi_H = 0) \]  

(114)

In the light of great arbitrariness of our guessed correction by a factor 1.5 in the emission temperature, we do not think it is reasonable to make the following calculations depend on it. We shall thus go on with the more well-defined values where we use precisely 3.5 keV for the emission temperature, (although we do believe that inclusion of the 1.5 factor is more likely to be right). In the light of the \( \exp(\pm 400\%) \) uncertainty however this makes little difference to our results.

So the time to spread the heat over the whole pearl relative to the time for emitting the whole energy (if it were done by 3.5 keV radiation with a temperature of that order too) using the critical size ball parameters \[3\] becomes

\[ \frac{t_{\text{spread}}}{t_{\text{radiation}}} = \frac{7.1 \times 10^{-2} s}{570 s} \]  

(115)

\[ = \frac{1}{8000} \times \exp(\pm 400\%) \quad (\text{for } \phi_H = 123 \text{ GeV}). \]  

(116)

while

\[ \frac{t_{\text{spread}}}{t_{\text{radiation}}} = \frac{0.35 \times 10^{-1} s}{4600 s} \]  

(117)

\[ = \frac{1}{132000} \times \exp(\pm 400\%) \quad (\text{for } \phi_H = 0). \]  

(118)
After the time $t_{\text{spread}}$ the hot spot heated up by the collision will have spread to the whole pearl and no regions with the original low temperature would be left. Thus after this time $t_{\text{spread}}$ the emission frequencies would be appreciably larger than 3.5 keV and from that time on no more 3.5 keV peak radiation can be radiated. This means that the energy fraction emitted in the line 3.5 keV can at the most be of the order of $t_{\text{spread}}/t_{\text{radiation}}$ times the total emitted energy. If the total energy $E_S$ was emitted only as 3.5 keV radiation, the number of photons emitted would be given by

$$\frac{E_S}{3.5 \text{ keV}} = N_{\text{all} \rightarrow 3.5}.$$  (119)

So the actual number $N$ of 3.5 keV photons emitted could at most be

$$N = \frac{t_{\text{spread}}}{t_{\text{radiation}}} \times \frac{E_S}{3.5 \text{ keV}}.$$  (120)

This in turn would reduce the intensity or say $N\sigma_M^2$, by being multiplied by the factor $t_{\text{spread}}/t_{\text{radiation}} = 1/8000$ (for $\phi_H = 123$ GeV) and $1/132000$ (for $\phi_H = 0$). That would make our prediction too small by a factor even a bit bigger than 8000 or 132000.

At first it seems that our fit, with the parameters corresponding to a pearl of the critical size just needed for stability against collapse, is not so good. However we should stress that, with our accuracy, our prediction only being off by 8000 or 132000 is already promising.

### 6.1 Why a line?

In the picture just sketched we at least see that as long as only part of the pearl has been heated up there is a cold part, through which the X-ray radiation from the hot spot has to penetrate to reach the outer space and thereby us. Through this cold part of the pearl there will for radiation with frequencies less than the homolumo gap $E_H$ of eq. (18) be essentially free passage as if it were glass, while for radiation of frequency bigger than $E_H$ the pearl will be highly non-transparent. If for instance there were at the hot spot surface produced radiation with a normal black body radiation with the frequency distribution of the Planck radiation being proportional to $\nu^3$ (where $\nu$ is the frequency), then after the passage of the cold region of the pearl the spectrum would look like such a $\nu^3$ spectrum chopped off at the homolumo gap frequency $E_H$. Since 3 is a rather high power, this would actually mean a spectrum with a strong maximum approximately at $E_H$ already not completely far away from what is seemingly seen by the X-ray spectrometers. However, the peak would not be a very sharp peak in as far as it would have fall off to the left only as a third power, which is somewhat sharp but not extremely so.

However, we have already mentioned that there is the possibility of obtaining X-ray radiation at the frequency of the homolumo gap value $E_H$, or almost so, by excitons - bound states of a hole and an (excited quasi) electron - decaying into a photon and a phonon. Now the question of course is whether a large part of the heat energy being converted to radiation will be produced via such excitons or at least by hole electron annihilation.

In order to investigate this question as to how big a fraction of the heat energy comes out from exciton or hole electron annihilation relative to the part
coming out just from thermal radiation we shall first look at in what ratio the heat energy is transported along in the pearl-material in the presence of a temperature gradient $\nabla T$ by ordinary heat conduction

$$\vec{J}_{\text{heat}} = k \nabla T$$

or by means of the holes and electrons moving along in the material, which also gives effectively an energy current

$$\vec{J}_{\text{exc}} = \frac{E_H}{-e} \frac{1}{2}(\vec{J}_{el} + \vec{J}_{holes}).$$

Here $\vec{J}_{el}$ and $\vec{J}_{holes}$ are the currents of electron excitations and of holes respectively. Since each pair of an electron and a hole brings along an energy equal to the homolumo gap $E_H$ the current called $\vec{J}_{exc}$ is indeed the current of energy carried along in the form of electron hole excitations. Both currents $\vec{J}_{el}$ and $\vec{J}_{holes}$ are basically Seebeck or thermoelectric currents and as such proportional to the temperature gradient $\nabla T$ and thus also proportional w.r.t. direction and strength of flow to heat current $\vec{J}_{heat}$ although with a temperature dependent coefficient. The denominator $-e$ were inserted because the usual Seebeck coefficients are normalized to give the current including the charge factor $-e$ for an electron, but we have to count simply the number of electrons, if we want to get the energy of annihilation be just $E_H$.

### 6.1.1 Mott Formulas

The current, which we are interested in is not exactly the electric Seebeck current because we want to add up the flow of holes and electron excitation, whereas the true electric current is rather the difference between the two because holes and electrons have opposite electric charge. So if we use the Seebeck coefficients for contribution from the bottom of the conduction band $S_C$ and from the top of the valence band $S_V$ as separate Seebeck coefficients we should combine them with an opposite sign relative to the one used when one wants to make the electric full Seebeck coefficient. Indeed while the usual Seebeck coefficient is:

$$S = \frac{\sigma_C S_C + \sigma_V S_V}{\sigma_C + \sigma_V}$$

the coefficient important for us is:

$$S_{our} = \frac{\sigma_C S_C - \sigma_V S_V}{\sigma_C + \sigma_V}. \quad (124)$$

Indeed the flow of energy in the form of excitations - electrons and holes - formula \[122\] becomes

$$\vec{J}_{exc} = \frac{E_H}{-e} \frac{1}{2}(\sigma_C S_C - \sigma_V S_V)S_{our} \nabla T$$

where $S_C$ and $S_V$ are the Seebeck coefficients as if there were respectively only the conduction band or only the valence band.
Now we insert the Mott-formulas for these Seebeck coefficients and conductivities $\sigma_C$ and $\sigma_V$ as found in the Wikipedia article on “Seebeck coefficients”,

\begin{align}
S_C &= \frac{k_B}{e} \left[ \frac{E_C - \mu}{k_B T} + a_C + 1 \right] \quad (127) \\
\sigma_C &= A_C(k_B T)^{a_C} \exp\left(-\frac{E_C - \mu}{k_B T}\right) \Gamma(a_C + 1) \quad (128) \\
S_V &= \frac{k_B}{e} \left[ \frac{\mu - E_V}{k_B T} + a_V + 1 \right] \quad (129) \\
\sigma_V &= A_V(k_B T)^{a_V} \exp\left(-\frac{\mu - E_V}{k_B T}\right) \Gamma(a_V + 1). \quad (130)
\end{align}

Here $E_C$ is the lowest energy point of the conduction band and $E_V$ the highest point of the valence band, so that the homolumo gap is

\[ E_H = E_C - E_V, \quad (131) \]

and $\mu$ is the chemical potential for electrons. The conductivity band levels’ conductivity function $c_C(E)$ has been put to the approximate form

\[ c_C(E) = A_C(E - E_C)^{a_C} \quad (132) \]

and analogously for the valence band

\[ c_V(E) = A_V(E_V - E)^{a_V}. \quad (133) \]

The parameters $A_C$, $A_V$, $a_C$, and $a_V$ are material dependent ansatz parameters with the small $a$’s in the range 1 to 3. The conductivity functions may be defined as

\[ c(E) = e^2 D(E) \nu(E), \quad (134) \]

where $D(E)$ is diffusion constant for electrons of energy $E$ and $\nu(E)$ the level density at $E$. This $c(E)$ represents the density on the energy axis of conductivity.

For our estimation we shall like to make a symmetric approximation because we believe that the homolumo gap $E_H$ is so narrow compared to the fermi energy $E_f$ that very little asymmetry will appear. That is to say we take $a_C = a_V = a$ and $A_C = A_V = A$, and in addition the chemical potential $\mu$ in the middle of the homolumo gap. Then we get

\begin{align}
S_{our} &= \frac{\sigma_C S_C - \sigma_V S_V}{\sigma_C + \sigma_V} \\
&= \frac{1}{2} (S_C - S_V) \quad (135) \\
&= \frac{k_B}{e} \left[ \frac{E_H}{2k_B T} + a + 1 \right]. \quad (136)
\end{align}

and now we shall use the Wiedemann Franz law relating the conductivity $\sigma = \sigma_C + \sigma_V$ to the heat conductivity $k$ by

\[ \frac{k}{\sigma} = LT \quad (137) \]
where
\[ L = \frac{\pi^2}{3} \left( \frac{k_B}{e} \right)^2. \]  
(138)

The point now is that since we have two types of energy flow both flowing proportionally to the temperature gradient \( \nabla T \), we can consider the ratio of the coefficients to these gradients. We have
\[
\begin{align*}
\vec{J}_{\text{heat}} &= k \nabla T \quad (139) \\
\vec{J}_{\text{exc}} &= \frac{1}{\varepsilon} \sigma * E_H * S_{\text{our}} \nabla T \quad (140) \\
&= \frac{3k}{\pi^2} \left[ \frac{1}{2} \left( \frac{E_H}{k_B T} \right)^2 + (a + 1) \frac{E_H}{k_B T} \right] \nabla T. \quad (141)
\end{align*}
\]

This means that the ratio of the part of energy carried via the excitation electrons and the holes relative to the normal heat conduction heat flow is
\[
\frac{\text{“electron and hole carried”}}{\text{“normal heat flow”}} = \frac{3}{\pi^2} \left[ \frac{1}{2} \left( \frac{E_H}{k_B T} \right)^2 + (a + 1) \frac{E_H}{k_B T} \right].
\]

We see that for high temperature compared to the homolumo gap the normal heat conduction dominates, but that when temperature goes small, the energy transport by means of excited electrons and holes which finally get radiated out as 3.5 keV radiation takes over.

Now the situation during the time when the pearl is still cold around the hot spot is such that only electromagnetic radiation with lower or equal frequency with the homolumo gap can escape to the outside.

But if it comes as it must from the neighboring region with temperature approaching the homolumo gap the mechanism with the annihilating excited electrons and holes take over.

So it seems that indeed during the spreading time \( t_{\text{spread}} \) of the hot spot during which it grows up to cover the whole pearl, the 3.5 keV radiation peak must be the dominant emission process. So if just this grow up of the hot spot time \( t_{\text{spread}} \) were long enough for the energy to be emitted the main energy would be emitted into the line 3.5 keV.

However, with the parameters of a critical size pearl, it seems that the time to emit the energy \( t_{\text{radiation}} \) is longer than the spreading time \( t_{\text{spread}} \) by a factor of 4800 or 33000.

7 Dropping critical size of pearls.

Now we must remember that in our previous article [3], we made the assumption that the size of the pearls was just so that they were on the borderline of collapsing by squeezing out the contained nuclei or nucleons. Of course it would be highly unlikely that they should be exactly on this border. They would have to be at least a little bit bigger than that. Instead of really making this assumption, we shall now use the experimentally determined quantity \( \left( \frac{N}{M^2} \right)_{\text{exp}} \) given in eq. (34) to fit the ratio
\[
\xi = \frac{R_{\text{actual}}}{R_{\text{crit}}} \quad (142)
\]
of the actual average radius of the pearls \( R_{\text{actual}} \) compared to the radius \( R_{\text{crit}} \) that corresponds to the pearls being in the critical state just about to collapse. This borderline to collapse corresponds to taking the Fermi momentum

\[ p_f = 2\Delta V \]  \hspace{1cm} (143)

now written as:

\[ p_f \text{ crit} = 2\Delta V \]  \hspace{1cm} (144)

while the actual Fermi momentum becomes

\[ p_f \text{ actual} = \xi^{-1} p_f \text{ crit}. \]  \hspace{1cm} (145)

Here we have chosen to change \( p_f \) by the inverse factor \( \xi^{-1} \) to that for \( R \), because we want to keep fixed the mass \( M \) of the pearl, which is essentially given by the rate of Tunguska-events. In addition we can and do keep \( \Delta V \) fixed under the \( \xi \) modification. The surface tension, however, we have to change according to the formula

\[ S_{\text{actual}}^{1/3} = S_{\text{crit}}^{1/3}/\xi. \]  \hspace{1cm} (146)

It is remarkable that the parameter \( \frac{N\sigma}{M^2} \) describing the overall scale of the intensity of the 3.5 keV line radiation is very sensitive to this here introduced parameter \( \xi \). In fact it is proportional to \( \xi^6 \) as we show below, provided the time ratio \( \frac{t_{\text{spread}}}{t_{\text{radiation}}} \leq 1 \) (if not it is simply proportional to \( \xi \)).

The \( \Delta V \) and \( \xi \) dependence of some of our quantities for a fixed pearl mass \( M \) are as follows:

\begin{align*}
\text{Pearl radius } R &\propto \frac{\xi}{\Delta V} \hspace{1cm} (147) \\
\text{Cubic root of tension } S^{1/3} &\propto \frac{\Delta V}{\xi} \hspace{1cm} (148) \\
\text{Fermi momentum } p_f &\propto \frac{\Delta V}{\xi} \hspace{1cm} (149) \\
\text{Energy release by collision } E_S &\propto \frac{\Delta V}{\xi} \hspace{1cm} (150) \\
\text{Collision cross section } \sigma &\propto \left( \frac{\xi}{\Delta V} \right)^2 \hspace{1cm} (151) \\
\frac{t_{\text{spread}}}{t_{\text{radiation}}} &\propto \frac{\xi^2}{\Delta V} \hspace{1cm} (152) \\
\frac{t_{\text{spread}}}{t_{\text{radiation}}} &\propto \left( \frac{\Delta V}{\xi} \right)^3 \hspace{1cm} (153) \\
\frac{t_{\text{spread}}}{t_{\text{radiation}}} &\propto \frac{\xi^5}{\Delta V^4} \hspace{1cm} (154) \\
\frac{N\sigma}{M^2} \bigg|_{\text{all } E_S \rightarrow 3.5 \text{ keV}} &\propto \frac{\xi}{\Delta V} \hspace{1cm} (155) \\
\frac{t_{\text{spread}}}{t_{\text{radiation}}} \bigg|_{\text{all } E_S \rightarrow 3.5 \text{ keV}} &\propto \frac{\xi^6}{\Delta V^5} \hspace{1cm} (156)
\end{align*}
In fact our true prediction for this cross section for pearl collision $\sigma$ times the number $N$ of 3.5 keV photons per collision divided by the pearl mass squared $M^2$ is given by

$$\frac{N\sigma}{M^2}_{\text{pred}} = \min \left\{ 1, \left( \frac{t_{\text{spread}}}{t_{\text{radiation}}} \right) \right\} \times \frac{N\sigma}{M^2}_{\text{as if } E_S \to 3.5\text{keV's}}$$ (157)

Here for a critical size ball we have (28)

$$E_S = \frac{M p_{\text{crit}} m}{m_N} \times \frac{3}{16},$$ (158)

and we define $\frac{N\sigma}{M^2}_{\text{as if } E_S \to 3.5\text{keV's}}$ as the value for the ratio $\frac{N\sigma}{M^2}$ calculated as if all the energy $E_S$ from the surface contraction went into 3.5 keV photons.

The factor $\min \left\{ 1, \left( \frac{t_{\text{spread}}}{t_{\text{radiation}}} \right) \right\}$ gives the ratio of the energy emitted as 3.5 keV radiation compared to the total energy released $E_S$. For $t_{\text{spread}} \geq t_{\text{radiation}}$ all the energy gets radiated from a surface cold enough that it all becomes 3.5 keV radiation.

Now we introduce the $\xi$ parameter to allow for the radius $R_{\text{actual}}$ not being the critical one just corresponding to collapse limit and obtain

$$\frac{N\sigma}{M^2}_{\text{actual}} = \min \left\{ 1, \left( \frac{t_{\text{spread}}}{t_{\text{radiation}}} \right)_{\text{actual}} \right\} \times \frac{N\sigma}{M^2}_{\text{actual, as if } E_S \to 3.5\text{keV's}}$$

$$= \min \left\{ 1, \left( \frac{t_{\text{spread}}}{t_{\text{radiation}}} \right)_{\text{crit}} \right\} \times \frac{N\sigma}{M^2}_{\text{crit, as if } E_S \to 3.5\text{keV's}} \xi^6$$

(provided $t_{\text{spread}}/t_{\text{radiation}} \leq 1$)

$$= \frac{1}{8000} \times 2.6 \times 10^{-22} \xi^6 \text{ cm}^2/\text{kg}^2 \quad (\text{for } \phi_H = 123 \text{ GeV})$$ (159)

or

$$\frac{N\sigma}{M^2}_{\text{actual}} = \min \left\{ 1, \left( \frac{\xi^6}{152000} \right) \right\} \times 1.3 \times 10^{22} \xi \text{ cm}^2/\text{kg}^2 \quad (\text{for } \phi_H = 0)$$ (160)

Here we have used (44), (116) and (118) for the critical size ball quantities.

These predictions (159) and (160) should be compared with the experimental value (34):

$$\frac{N\sigma}{M^2}_{\text{exp}} = (1.0 \pm 0.2) \times 10^{23} \text{ cm}^2/\text{kg}^2.$$ (161)

So requiring $\frac{N\sigma}{M^2}_{\text{actual}} = \frac{N\sigma}{M^2}_{\text{exp}}$ gives

$$\xi = \sqrt[6]{31000} = 5.6 \exp(\pm 70\%) \quad (\text{for } \phi_H = 123 \text{ GeV})$$ (162)

or

$$\xi = \sqrt[6]{1000000} = 10 \exp(\pm 70\%) \quad (\text{for } \phi_H = 0)$$ (163)
We remark that the both of these $\xi$-values still leave the ratio $\left| \frac{t_{\text{spread}}}{t_{\text{radiation}}} \right|_{\text{actual}} \leq 1$ so that the simple $\xi^6$-corrections (159, 160) are valid. But note that the ratio is remarkably close to unity in both cases within our accuracy.

With the corrections of section 9 these $\xi$-values become

$$\xi = 1.3 \times 5.6 = 7.3 \exp(\pm 70\%) \quad (\text{for } \phi_H = 123 \text{ GeV}) \quad (164)$$

or

$$\xi = 1.4 \times 10 = 14 \exp(\pm 70\%) \quad (\text{for } \phi_H = 0). \quad (165)$$

Including the dominant uncertainty of 70% coming from our estimate (3) of $\Delta V$ and the uncertainty from the emission temperature not being exactly 3.5 keV, the energy gap is now predicted to be

$$E_H = \frac{E_{H \text{ old}}}{\xi} = \frac{8.8 \text{ keV}}{7.3} = (1.2 \pm 1.1) \text{ keV} \quad (\text{for } \phi_H = 123 \text{ GeV}) \quad (166)$$

or

$$E_H = \frac{17.6 \text{ keV}}{14} = (1.3 \pm 1.2) \text{ keV} \quad (\text{for } \phi_H = 0). \quad (167)$$

The $\xi$-correction shifts the prediction for $E_H$ from the previous values (20, 22), $E_{H \text{ old}} = 8.8 \text{ keV}$ or 17.6 keV for critical size pearls to the opposite side of the experimental value 3.5 keV. Contrary to the situation before this $\xi$-correction it is now the $\phi_H = 0$ value which is marginally closer to the experimental 3.5 keV, namely the 1.3 keV.

The corresponding correction to the surface tension $S$, which was determined from the mass $M$ and thereby the rate of Tunguska falls to be $S^{1/3} = 28 \text{ GeV}$ for the critical sized ball in [3], gives $S^{1/3} = 28 \text{ GeV}/7.3 = 3.8 \text{ GeV}$ for $\phi_H = 123 \text{ GeV}$ and $S^{1/3} = 56 \text{ GeV}/14 = 4.0 \text{ GeV}$ for $\phi_H = 0$.

In the appendix A we shall investigate some theoretical expectations for this tension $S$ from the effective potential as a function of the relevant scalar fields, the Higgs field $\phi_H$ and the effective field $\phi_F$ for the bound state supposed to be the main constituent in the “condensate” phase. The a priori value from this theory turns out to be $S^{1/3}_{\text{theory}} \approx 100 \text{ GeV}$, which is embarrassingly much larger than these fitted values. However, we shall put forward the idea that there is yet another vacuum phase, which enforces the effective potential to be flatter than we assumed when obtaining the above estimate $S^{1/3}_{\text{theory}} \approx 100 \text{ GeV}$. In this way we get what we call “theory 2)” for the tension $S$, for which our estimate becomes $S^{1/3} \approx 30 \text{ GeV}$. Even this value is large compared to the above fitted values around 4 GeV. However in the following section we investigate a more detailed fitting procedure using the only relevant combination of parameters $\xi^{10/\text{MeV}}$.

8 Fitting with $\left| \frac{t_{\text{spread}}}{t_{\text{radiation}}} \right|_{\text{actual}} \geq 1$.

We remarked above that actually the ratio $\frac{t_{\text{spread}}}{t_{\text{radiation}}}$, giving the fraction of the energy from the contraction of the surface ending up in the 3.5 keV radiation,
turned out to be very close to unity in our fitting. However the calculation of $t_{\text{radiation}}$ is very sensitive to the emission temperature and we have assigned an uncertainty of $\exp \pm 100\%$ to the emission temperature. So the uncertainty on $t_{\text{radiation}}$ becomes $\exp \pm 400\%$. Also the calculation of $t_{\text{spread}}$ depends on the hard to calculate heat conductivity. The time ratio $t_{\text{spread}}/t_{\text{radiation}}$ is therefore very uncertain (by $\exp(\pm 400\%)$). However, if this ratio happens to be bigger than unity the expression $\min \left\{ 1, t_{\text{spread}}/t_{\text{radiation}} \right\}$ simply becomes unity. In this case the uncertainty from the calculation of the ratio disappears from the calculation and we get a much slower variation with the parameter $\xi$ of the predicted value of $\frac{N\sigma}{M^2}$, which is to be compared to (34). Thus, in this case, there is a much bigger range of $\xi$ available for fitting than in the range of $\xi$ where $t_{\text{spread}}/t_{\text{radiation}}$ ratio is smaller than unity. From the point of view that the true value of $\xi$ can be considered random, there is therefore an appreciably higher chance for $\xi$ to lie in the range where $t_{\text{spread}}/t_{\text{radiation}} \geq 1$

According to (116,118), under the assumption of exactly critical size pearls (i.e. for $\xi = 1$), the ratio $t_{\text{spread}}/t_{\text{radiation}}$ takes the values $\frac{1}{8000}$ and $\frac{1}{132000}$ for $\phi_H = 123$ GeV and $\phi_H = 0$ respectively. This means that this ratio becomes just unity for 

$$\xi = \sqrt{8000} = 6.0 \quad (\text{for } \phi_H = 123 \text{ GeV}) \quad (168)$$

$$\xi = \sqrt{132000} = 10.6 \quad (\text{for } \phi_H = 0). \quad (169)$$

Once we look for $\xi$-values larger than $\xi_1$, the sensitivity of the observable quantity $\frac{N\sigma}{M^2}$ to $\xi$ becomes much weaker (only depending on $\xi$ to the first power). In practice we could for the matter of fitting the 3.5 keV rate simply take $\xi = \xi_1$.

Statistically it is very possible that our estimates of say the emission temperature and of the conductivity $k$ were so inaccurate that actually the ratio $t_{\text{spread}}/t_{\text{radiation}}$ was bigger than unity. In such a case a fit to $\xi$ would be independent of the details of the calculation of this ratio and be given alone as the correction factor to make the value of $\frac{N\sigma}{M^2}$ in the critical pearl case be corrected to the experimental value $\frac{N\sigma}{M^2} = (1.0 \pm 0.2) \times 10^{23} \text{cm}^2/\text{kg}^2$

$$= \frac{(1.0 \pm 0.2) \times 10^{23} \text{cm}^2/\text{kg}^2}{2.6 \times 10^{22} \text{cm}^2/\text{kg}^2} = 3.8 \quad (\text{for } \phi_H = 123 \text{ GeV}) \quad (170)$$

and

$$\xi = \frac{(1.0 \pm 0.2) \times 10^{23} \text{cm}^2/\text{kg}^2}{1.3 \times 10^{22} \text{cm}^2/\text{kg}^2} = 7.7 \quad (\text{for } \phi_H = 0) \quad (171)$$

Actually for $\xi = 3.8$ the time ratio is

$$\frac{t_{\text{spread}}}{t_{\text{radiation}}} = \frac{3.8^5}{8000} = \frac{1}{10.1} \quad (\text{for } \phi_H = 123 \text{ GeV}) \quad (172)$$

and for $\xi = 7.7$ it is

$$\frac{t_{\text{spread}}}{t_{\text{radiation}}} = \frac{7.7^5}{132000} = \frac{1}{4.9} \quad (\text{for } \phi_H = 0). \quad (173)$$
So the factors by which we should have miscomputed e.g. the heat conductivity $k$ or more likely the fourth power of the emission temperature, in order for this “the ratio being bigger than unity” situation to be realized, should be numbers like 10.1 or 4.9 above. The emission temperature entered $t_{\text{radiation}}$ to the fourth power and gave rise to an uncertainty $\exp(\pm 400\%)$ in the ratio $t_{\text{spread}} / t_{\text{radiation}}$, so indeed this ratio is compatible with 1 for $\xi = 3.8$ or $\xi = 7.7$ in the respective cases.

We suggested above (111) that the true emission temperature was about 1.5 times larger than the line frequency 3.5 keV. With this crudely estimated improvement the $\xi$-values making the ratio $t_{\text{spread}} / t_{\text{radiation}}$ just unity would be shifted from the $\xi_1$ values above (168, 169) to

$$\xi = \xi_1 = \sqrt[3]{\frac{8000}{1.5^4}} = 4.3 \quad \text{(for } \phi_H = 123 \text{ GeV)}$$

(174)

$$\xi = \xi_1 = \sqrt[3]{\frac{132000}{1.5^4}} = 7.7 \quad \text{(for } \phi_H = 0).$$

(175)

Written for the variable $\frac{\xi \times 10^{10}\text{MeV}}{\Delta V}$ stressed in the next section, these values giving the time ratio to be just unity are $\frac{\xi \times 10^{10}\text{MeV}}{\Delta V} = 4.3$ and $\frac{\xi \times 10^{10}\text{MeV}}{\Delta V} = 3.9$ respectively. So, in the following section, we shall take $\frac{\xi \times 10^{10}\text{MeV}}{\Delta V} > 4$ as the condition for $t_{\text{spread}} / t_{\text{radiation}}$ to be bigger than or equal to unity.

8.1 Quantities to fit

It turns out that all of the four quantities whose values we would like to use as tests of our model depend on the same ratio $\frac{\xi \times 10^{10}\text{MeV}}{\Delta V}$ rather than on $\xi$ and $\Delta V$ separately, provided $t_{\text{spread}} / t_{\text{radiation}} \geq 1$ as assumed here. These quantities are:

- The frequency of the radiation 3.5 keV,
- The intensity of the 3.5 keV radiation as given in (34),
- The cube root of the surface tension $S^{\frac{1}{3}}$,
- Theoretical values of both $\xi$ and $\Delta V$.

We therefore plot the predictions for the ratio $\frac{\xi \times 10^{10}\text{MeV}}{\Delta V}$ from these four quantities in Figure 1 with logarithmic uncertainties estimated crudely as seen in Table 3. The entries in this table and figure were estimated as follows:

- In our model the frequency of the radiation is equal to the energy gap $E_H$, which is inversely proportional to our parameter $\frac{\xi \times 10^{10}\text{MeV}}{\Delta V}$. Under the assumption that $\Delta V = 10$ MeV and the pearl being of the smallest possible size for stability, so that $\xi = 1$, the homolumo gap (20) is $E_H = 8.8$ keV. So our parameter is required to take the value

$$\frac{\xi \times 10^{10}\text{MeV}}{\Delta V} = \frac{8.8 \text{ keV}}{3.5 \text{ keV}} = 2.5$$

(176)

in order to predict a frequency of 3.5 keV.
Figure 1: The values of the ratio $\frac{\xi_{10}}{\Delta V}$ as needed for four constraints. There are two experimental constraints from the frequency and intensity of the 3.5 keV radiation respectively and two theoretical constraints in two versions corresponding to taking theory 1 or theory 2 for the tension. We make the simplifying assumption that all energy from the surface contraction in a collision gets emitted as 3.5 keV X-rays. The sixth line “Ratio $\frac{\text{spread}}{\text{radiation}} = 1$” represents the condition for all the energy actually going to 3.5 keV radiation. The dashed line indicates that the required value $\frac{\xi_{10}}{\Delta V} = 4$ is only a lower limit, but we need to stress the uncertainty on this lower limit.
Our calculations are almost just order of magnitude estimates and so crudely we take the logarithm of the above determined value of our parameter to have an uncertainty of order unity so that \( \frac{\xi}{10\text{ MeV}} = 2.5^{\pm 4.3} \). The uncertainty here is likely to be dominated by the lack of good knowledge as to the clumpiness of the dark matter, see the column “boost” in the table in section 2. Including also the uncertainty in our crude estimate of the energy released from the contraction of the collision surface etc., we estimate a total uncertainty of about 90% for this restriction on \( \ln \frac{\xi}{10\text{ MeV}} \).

- The intensity-related quantity \( \frac{N\sigma}{M^2} \) is proportional to our parameter \( \frac{\xi}{10\text{ MeV}} \).

  This leads to a critical size pearl with \( \Delta V = 10\text{ MeV} \) and all the energy released from the contraction of the collision surface going into the 3.5 keV line.

  \( \frac{\xi}{10\text{ MeV}} \) is inversely proportional to the parameter \( \frac{\Delta V}{\Delta V} \) and its value for a critical sized ball with \( \Delta V = 10\text{ MeV} \) was determined to be 28 GeV in [3]. This leads to

\[ (177) \]
the requirement

\[
\frac{\xi \times 10\text{MeV}}{\Delta V} = \frac{28\text{ GeV}}{100\text{ GeV}} = 0.28.
\]  (178)

One here a priori calculates \( S \) but the parameter \( \frac{\xi \times 10\text{MeV}}{\Delta V} \) is inversely linearly related to the cubic root of this quantity, namely to \( S^{\frac{1}{3}} \). So the a priori typical uncertainty gets reduced by a factor 3, and we end up estimating the uncertainty on the extracted value of \( \ln \frac{\xi \times 10\text{MeV}}{\Delta V} \) to be rather about one third of the “usual” 100%, taken to be 40%.

- Tension theory 2.)

In the appendix A.6 we propose the existence of a third low energy vacuum which, according to the multiple point principle, is degenerate with the other two vacua. In this way we are naturally led to a smaller theoretical value \( S \sim (30\text{ GeV})^{\frac{3}{2}} \). This value \( S \sim 30\text{ GeV} \) leads to the requirement

\[
\frac{\xi \times 10\text{MeV}}{\Delta V} = \frac{28\text{ GeV}}{30\text{ GeV}} \approx 1.
\]  (179)

with a similar uncertainty to that of Tension theory 1.

- In the fifth row of Table 3 we give the value of this parameter \( \frac{\xi \times 10\text{MeV}}{\Delta V} \) simply obtained from our best theoretical ideas for \( \xi \) and \( \Delta V \) separately:
  - \( \xi = \sqrt{4\pi} \times 2^{\frac{1}{2}} = 4.8, \) see (197) below.
  - In the appendix A we argue for \( \Delta V = 20\text{ MeV} \) being the most likely value.

Combining these separate values leads to \( \frac{\xi \times 10\text{MeV}}{\Delta V} = 2.4 \). Taking the logarithmic uncertainty on each of these values to be 70%, we obtain an overall uncertainty of 100% on \( \ln \frac{\xi \times 10\text{MeV}}{\Delta V} \).

- In the last row in the table -separated by a line - we present the condition that the ratio \( \frac{\text{r}_{\text{spread}}}{\text{r}_{\text{radiation}}} \) be bigger than or equal to unity, in order that the fitting used for the other quantities becomes relevant. The value \( \frac{\xi \times 10\text{MeV}}{\Delta V} = 4 \) is in fact the lower limit needed to ensure that \( \frac{\text{r}_{\text{spread}}}{\text{r}_{\text{radiation}}} \geq 1 \). However it turns out that the fits considered below tend to violate this lower limit. So, in practice, we replace the inequality by an equality with an estimated uncertainty of 80% on \( \ln \frac{\xi \times 10\text{MeV}}{\Delta V} \).

However, this last lower limit is not a genuine prediction, but rather a condition it turned out we needed to impose and presumably represents a warning that something is unexpected in our model.

8.2 The Various Fits

Using the data put forward in the table we here present three fits using selected amounts of these data:

- Tension theory 1)
The a priori simplest fit to make would be to use for the $S^{\frac{1}{2}}$, the theory
1), which just use the simplest assumption about field expectation for the vacua of the two phases “present” and “condensate” vacua.

The average of the parameter fitted becomes

$$< \ln \frac{\xi * 10 \text{ MeV}}{\Delta V} > = -0.23 \pm 0.3$$  \hspace{1cm} (180)

$$\Rightarrow < \frac{\xi * 10 \text{ MeV}}{\Delta V} > = \exp(-.23) = 0.79^{+0.3}_{-0.2}$$  \hspace{1cm} (181)

$$\chi^2 = 15.9 \text{ (for 4 degrees of freedom)}$$  \hspace{1cm} (182)

But this fit is very bad in as far as e.g. the tension deviates from its fitted value by 2.75 standard deviations.

**Tension theory 2)**

If we accept the story that there are more degenerate vacua, which by their presence enforce some almost with the two minima associated to “present” and “condensate” vacua degenerate minimum of the effective potential as function of the fields for the bound state $\phi_F$ and the Higgs field $\phi_H$, then we get to an estimate of the tension and thereby $S^{\frac{1}{2}}$ which is lower. This we call theory 2) and it comes about because the intermediate almost minimum degenerate with the two first ones tend to keep the effective potential from having a big mountain between the two vacua we know. A smaller $S$ means it can be fitted by a bigger - more positive - $\ln \frac{\xi * 10 \text{ MeV}}{\Delta V}$. But if we must go to this choice - theory 2) - it means that we predict yet a new phase which is related to our two field $\phi_F$ and $\phi_H$. In this theory 2) we get :

$$< \ln \frac{\xi * 10 \text{ MeV}}{\Delta V} > = 0.51 \pm 0.3$$  \hspace{1cm} (183)

$$\Rightarrow < \frac{\xi * 10 \text{ MeV}}{\Delta V} > = \exp(0.51) = 1.7^{+0.6}_{-0.3}$$  \hspace{1cm} (184)

$$\chi^2 = 3.80 \text{ (for 4 degrees of freedom)}$$  \hspace{1cm} (185)

This fit is much more satisfactory, since even the most deviating quantity - still the tension - only deviates by 1.3 standard deviations.

This our best fit including everything gives the following overall fitted values for the experimental quantities

The frequency predicted $^{3.5keV}_{}|_{theory} = 5.2^{+0.6}_{-3.4}$ keV  \hspace{1cm} (186)

and

The intensity predicted $N\sigma M^2 |_{theory} = 4.4^{+7.0}_{-2.7} * 10^{22}$ cm$^2$/kg$^2$  \hspace{1cm} (187)

which is to be compared with

$$N\sigma M^2 |_{exp} = (10 \pm 2) * 10^{22}$ cm$^2$/kg$^2$$  \hspace{1cm} (188)
• Theory prediction

Using only the theoretically predicted quantities - the tension theory 2), the combined theory $\xi$ and $\Delta V$, and the restriction $\frac{t_{spread}}{t_{formation}} = 1$ - so that we get what can be considered as a purely theoretical fitting, we obtain

$$\left< \frac{\xi \cdot 10 \text{ MeV}}{\Delta V} \right>_{\text{theory}} = 0.35 \pm 0.34 \quad (189)$$

$$\Rightarrow \left< \frac{\xi \cdot 10 \text{ MeV}}{\Delta V} \right>_{\text{theory}} = \exp(0.35) = 1.4^{+0.6}_{-0.4} \quad (190)$$

$$\chi^2 = 2.00 \text{ (for 2 degrees of freedom)} \quad (191)$$

Using our estimated logarithmic uncertainties and the purely theoretical value of $\ln \left( \frac{\xi \cdot 10 \text{ MeV}}{\Delta V} \right)_{\text{theory}} = 0.35$ we obtain the following values for the experimental quantities:

The frequency predicted $\left. 3.5 \text{ keV} \right|_{\text{theory}} = 6.3^{+11.4}_{-4.1} \text{ keV} \quad (192)$

and

The intensity predicted $\left. \frac{N\sigma}{M^2} \right|_{\text{theory}} = 3.6^{+5.8}_{-2.2} \times 10^{22} \text{ cm}^2/\text{kg}^2 \quad (193)$

which is to be compared with

$$\left. \frac{N\sigma}{M^2} \right|_{\text{exp}} = (10 \pm 2) \times 10^{22} \text{ cm}^2/\text{kg}^2 \quad (194)$$

The errors in the above predictions are dominated by the uncertainties estimated for the calculations of the homolumo gap and the rate of X-ray production respectively, so that the uncertainty of only 35 % in the parameter $\frac{\xi \cdot 10 \text{ MeV}}{\Delta V}$ is almost negligible. But we did indeed include even this little uncertainty in the results presented in (192, 193).

These numbers (192) and (193) can be considered theoretical predictions only based on the rate of Tunguska-like events in our model. They agree wonderfully to the expected accuracy. So we can claim that of course a posteriori we predict the two overall features of the 3.5 keV radiation!

With theory 2) for the tension - the one with an extra suppression of the effective potential in terms of the fields $\phi_H$ and $\phi_F$ due to a speculated extra vacuum phase also degenerate with the other ones - we actually achieve very good fitting. We could even fit alone to the theoretical constraints and then get - as a prediction - good values for the two experimental quantities concerning the 3.5 keV X-ray radiation: the frequency and the intensity fitted to the various clusters and galactic center etc.

Thus our model should be considered extremely successful in the version with theory 2) for the tension $S$. This means that, via the fitting, we were driven towards the suggestion of yet another vacuum degenerate with the other two and at a similar energy scale to the present vacuum and the condensate vacuum.
9 Some Further Corrections

There are two smaller improvements which we might include, but which were left out of the discussion above so as not to overcomplicate it:

- Provided that $t_{\text{spread}} < 1$ then after the time $t_{\text{spread}}$ the hot spot has spread to the whole pearl and the 3.5 keV radiation stops and gets replaced by the thermal radiation from the now getting hotter and hotter surface of the pearl. However, after some further time the whole pearl gets so cold that the temperature at the surface of the pearl again reaches down into the 3.5 keV range, and then the emission of 3.5 keV radiation is reinstated. Now the hot spot, which is not so hot as the original one but still much hotter than 3.5 keV in the interior, begins to contract while emitting 3.5 keV radiation from the boundary along which the temperature is of that order. This boundary contracts as time then goes on more and more; first at the end, when even the temperature in the center of the hot spot has cooled, the emission of 3.5 keV radiation stops forever.

At most we should expect that this effect of the contraction era for the hot spot should give an amount of 3.5 keV radiation about the same as the expansion era. So a factor of 2 increase in the amount of 3.5 keV radiation would be an upper limit for this effect of a re-contraction. In spite of the average temperature being smaller at the contraction of the 3.5 keV emitting surface, the temperature at the emitting surface remains the same and thus the emission per unit time only depends on the area of the emitting surface, which we just approximate by the surface area of the pearl.

We conclude that the fraction of the emitted energy appearing as 3.5 keV radiation should, because of this correction, be increased by a factor between 1 and 2.

- The border of the hot spot at which the 3.5 keV radiation is emitted and which thus defines the size of this hot spot which really counts for our calculation is not, as we used it, the radius $|\vec{r}|$ at which the expression in the exponent of equation (91) is just unity. It should really be the distance at which the temperature has got a value of the order of the homolumo gap $\sim 3.5$ keV. This means that the exponent should rather than just be of the order $e^{-1}$ be of the order of the ratio of this "low" 3.5 keV temperature divided by the typical temperature from the start of the spread of the hot spot, or by some average of that temperature. Now the initial temperature is of the order of say [53, 56] 6 MeV (for $\phi_H = 123$ GeV), or 12 MeV (for $\phi_H = 0$) or we use some average a bit smaller, and the low temperature at which the 3.5 keV radiation gets emitted is of the order of 3.5 keV. So the value of the quantity in the exponent should rather be taken to be $\ln\left(\frac{6 \text{ MeV}}{3.5 \text{ keV}}\right) = 7.4$ (for $\phi_H = 123$ GeV) or $\ln\left(\frac{12 \text{ MeV}}{3.5 \text{ keV}}\right) = 8.1$ (for $\phi_H = 0$). This means that the true value for the spreading time $t_{\text{spread}}$, and hence for the ratio $\frac{t_{\text{spread}}}{t_{\text{radiation}}}$, should be reduced by a factor of 7.4 or 8.1, from the above estimated values.

Taking say for the first of the above corrections an increase by a factor 1.3 in the amount of 3.5 keV radiation per collision, we get a total decrease from our
two corrections here by a factor of $\frac{74}{73} = 5.7$ for $\phi_H = 123$ GeV and $\frac{84}{83} = 6.2$ for $\phi_H = 0$. So to compensate for such an extra factor we must increase our fitted $\xi$ values [(162) and (163)] by a factor of $\sqrt{5.7} = 1.3$ for $\phi_H = 123$ GeV and $\sqrt{6.2} = 1.4$ for $\phi_H = 0$.

10 Speculative estimate of $\frac{R}{R_{crit}}$.

In our previous article [3] we took it as a likely hypothesis that the size of the pearls would be close to the critical size at which the pressure would be so big that the nucleons inside the pearl would just be about to be spit out. It is of course likely that, on the average, the actual size of the pearls produced in the early Universe would be close to this critical value. Pearls of smaller size will of course disappear because they will collapse. If thus the production of the pearls is somehow biased towards rather small pearls the average size will come close to the critical one.

Now, however, we must imagine that the pearls in the creation era are not yet perfectly spherical but rather have highly deformed shapes. In such a situation the curvature of the skin around the pearl is not the same all around, but varies from place to place. Now whenever the curvature reaches the value of the critical size spherical pearl the nuclei may start to be spit out. Once the spit out starts the pearl gets volume-wise smaller and thus once such a collapse has started it may very easily come to continue. Thus for a pearl of the formally just barely stable size, the smallest fluctuation in the surface relative to sphericity would cause its collapse. Thus the minimally required size of a pearl would be such that the fluctuations in the curvature as you go around the pearl surface never cause the curvature to reach the true critical value, so that the curvature remains less than the critical one all around the pearl. A situation in which the curvature varies statistically around the pearl and does not meet the critical value in more than one point should effectively replace the critical radius pearl in our earlier considerations. That is to say that the statistical distribution of pearl sizes should begin rather from this size in which the critical curvature is only met in one point around the surface. Then from there on the distribution should fall off in some way hoped to be biased towards small pearls.

Can we obtain at least a very crude estimate of how big compared to a true critical size spherical pearl the statistical non-spherical randomly shaped pearl should be? We could say that if the random variation of the curvature varies on a scale given by the radius of the pearl over the pearl surface, then it would be effectively as if there were $4\pi$ approximately independent regions of the surface. Each of these regions might take its own curvature - say by dimensional argument of the same order as the pearl average. This would mean that we would expect the spread in the curvature, as you go around the pearl, to be of a similar order as the average say of $4\pi$ independent variables with each having a spread of one unit in terms of the pearl average curvature. To look for the effective collapse size, which we now estimate, we may take the unit curvature here to be the absolute collapse curvature for the exactly spherical pearl.

Now we simply say: The average of $4\pi$ independent variables with spread 1 has spread $\sqrt{4\pi}$. We must ensure that this fluctuating curvature shall not reach the critical size, and that we do by making the pearl $\sqrt{4\pi}$ times larger than the
genuine critical size. Thereby namely the curvatures in the fluctuation around the surface get scaled down by a factor $\sqrt{\frac{4}{\pi}}$ and just becomes unity.

So we argue that the scaling ratio $\xi$ of the average pearl size relative to the genuine critical size should be at least $\sqrt{\frac{4}{\pi}}$ to just avoid collapse.

However, as we estimated in appendix B2 of [3], there will in addition be some excess in size over the effectively critical size because of fluctuations in the formation of the pearls. The latter increase in the expected size depends on how biased the size is towards small sizes. But it means that we expect at least

$$\xi \geq \xi_{\text{crit}} = \sqrt{\frac{4}{\pi}}.$$  \hspace{1cm} (195)

In fact we predicted that the statistical distribution of pearl sizes would give

$$\frac{R_{\text{median}}}{R_{\text{crit}}} = 2^{4/9} \approx 1.4$$  \hspace{1cm} (196)

for the ratio of the median radius $R_{\text{median}}$ to the critical radius $R_{\text{crit}}$. This value should be multiplied by the critical size parameter $\xi_{\text{crit}}$ above (195). Thus the final prediction for the ratio $\frac{R}{R_{\text{crit}}}$ becomes

$$\xi = 2^{4/9} \times \sqrt{\frac{4}{\pi}} \approx 5$$  \hspace{1cm} (197)

in reasonable agreement with the values 7.3 or 14, given their uncertainty, which we obtained from our fit above (164, 165). It should be considered successful for our model.

11 Conclusion

We have developed our earlier published model [1,2,3,4] for dark matter being balls or pearls of centimeter size and a mass of the order of 140000 ton so as to investigate whether our pearls can deliver the controversial 3.5 keV X-ray line. Our picture is that this 3.5 keV radiation appears from collisions of pairs of our dark matter pearls; the pearls unite in the collision and by the contraction of their skin liberate so much energy $E_S$ as heat in the ball that it can produce sufficient 3.5 keV X-rays to reproduce the observed radiation in this line. The two main successes of our model w.r.t. the 3.5 keV radiation are the following:

- **Frequency**

  The very value of the photon energy 3.5 keV we reproduce by identifying it with the homolumo energy gap in pearl material.

  We take it, that there is very generally a homolumo gap effect in for instance glassy materials - as we can suspect our pearl material to be - consisting in that the nuclei and the electrons will adjust to arrange the empty (single) electron states to increase in energy, while the filled ones will sink in energy. By such an arrangement the single electron energy of the Fermi sea is lowered. Thereby a gap appears between the empty and the filled levels and that is called the homolumo gap.

  The existence of a homolumo gap means that strictly speaking the material is an insulator or a semi-conductor in spite of it being in first approximation, when the homolumo gap is ignored, a metal. With such an
energy gap, it can easily happen that excited electrons get collected just above this gap and holes of missing electrons just below. When such holes annihilate with these excited electrons, radiation with energy essentially equal to the homolumo gap will be emitted. We can therefore expect radiation from say our dark matter pearls with a frequency just equal to the homolumo gap in frequency.

It is thus a great success for our model that we estimated the size of the homolumo gap and found a value close in order of magnitude to the observed 3.5 keV energy of the mentioned controversial radiation suspected to arise from the dark matter.

Indeed we found:

Using the old numbers as determined from the Tunguska event and estimates performed in earlier works, before we studied the 3.5 keV line, we calculated by use of the Thomas Fermi approximation or just by dimensional arguments the homolumo gap value to be

\[ E_H = (8.8 \pm 6.2) \text{ keV}. \]  

(198)

In the old theory used in our Tunguska-paper [3] we used the approximation or theoretical assumption that the pearls had just such a size that they were on the borderline of collapsing, because the the material was just about to be pressed out of the pearls across the \( \Delta V \) potential per nucleon.

However this hypothesis of exact borderline stability is unrealistic. So we introduced a parameter \( \xi = \frac{R}{R_{\text{crit}}} \), denoting the radius of the actual pearl relative to the radius of a critical size pearl which is just barely stable. We then discovered that all the important features of our pearls depended only on the ratio \( \frac{10\text{MeV}}{\Delta V} \) of this new parameter relative to the already used parameter \( \Delta V \) (denoting the potential barrier for a nucleon to leave the pearl). Since in [3] we used \( \Delta V = 10 \text{ MeV} \), this composed parameter \( \frac{10\text{MeV}}{\Delta V} \) was equal to unity in the old calculations.

In the present work we calculated a value for this parameter by fitting the frequency and observed intensity of the 3.5 keV line together with a few theoretical constraints and our tension theory [2] for the surface tension of the skin of the pearls. We obtained the value \( \frac{10\text{MeV}}{\Delta V} = 1.7 \pm 30\% \), which corresponds to a homolumo gap of

\[ E_H = \text{frequency} = 5.2^{+9.6}_{-3.4} \text{ keV} \]  

(199)

which is to be identified with the frequency of the 3.5 keV radiation.

The above was a fit, but had we used only our theoretical guesses and the rate of Tunguska-like impacts we would have predicted the X-ray line to be

\[ E_H = \text{frequency} = 6.3^{+11.4}_{-4.1} \text{ keV} \]  

(200)

Within our order of magnitude accuracy both the above values for \( E_H \) agree well with the experimental frequency of 3.5 keV.
• **Intensity of radiation**

The parameters in the Tunguska article \[3\] fit very badly with the requirement that the time ratio \( \frac{t_{\text{spread}}}{t_{\text{radiation}}} \) should be bigger than unity. This time ratio means the time \( t_{\text{spread}} \) it takes for the heating blob to spread out to the whole pearl divided by the time \( t_{\text{radiation}} \) it takes to radiate out all the energy from the collision into 3.5 keV X-rays. So if, as our fit using the “old” Tunguska numbers suggests, this ratio is less than unity, it means that a lot of the energy is still present when the heat blob has spread all over the pearl. Consequently the temperature at the surface of the pearl grows much higher than 3.5 keV. Thus with these old parameters we end up obtaining only the fraction

\[
\frac{t_{\text{spread}}}{t_{\text{radiation}}} = \frac{1}{8000} \ast \exp(\pm400\%) \quad \text{(for } \phi_H = 123 \text{ GeV)} \tag{201}
\]

of the energy being emitted as 3.5 keV X-rays.

Taking that seriously our prediction of the intensity related expression - the number \( N \) of 3.5 keV photons, multiplied by the cross-section for pearls colliding \( \sigma \) and divided by the square of the pearl mass \( M \) - becomes

\[
\frac{N\sigma}{M^2} = 3.25 \ast 10^{18} \exp(\pm400\%) \text{ cm}^2/\text{kg}^2, \tag{202}
\]

which is to be compared with the experimental value

\[
\frac{N\sigma}{M^2} = (1.0 \pm 0.2) \ast 10^{23} \text{ cm}^2/\text{kg}^2. \tag{203}
\]

This result gotten from our old numbers, assuming that the pearl has the critical radius so that it is just barely stable against the interior ordinary matter being pressed out against the potential \( \Delta V = 10 \text{ MeV} \), does not fit well! However, the ratio \( \frac{t_{\text{spread}}}{t_{\text{radiation}}} \) giving the fraction of the energy appearing as the X-ray 3.5 keV radiation is very sensitive to the new parameter \( \xi \) or \( \xi_{10\text{MeV}} \Delta V \). So we can only hope for a good fit, if this parameter is chosen to make the time ratio \( \frac{t_{\text{spread}}}{t_{\text{radiation}}} \) very close to unity. Assuming that it is indeed unity, we then have this restriction effectively as a further (theoretical) constraint on our fit.

We made what we would call our main fit by indeed imposing this constraint that we get essentially all the energy released out as 3.5 keV radiation. A good fit is then obtained by taking the surface tension of the pearl to be \( S = (30 \text{ GeV})^3 \) from tension theory 2) in appendix A.6 and A.7. This tension theory 2) has a special suppression of the effective potential \( V_{\text{eff}}(|\phi_H|^2, \phi_F) \) due to the existence of a speculated extra vacuum.

At first we used only the theory constraints in our fit, taken to include the rate of Tunguska-like impacts, and obtained \( \xi_{10\text{MeV}} = 1.4 \) predicting the value of the intensity related expression to be

\[
\frac{N\sigma}{M^2} = 3.6^{+5.8}_{-2.2} \ast 10^{22} \text{ cm}^2/\text{kg}^2 \tag{204}
\]
which is to be compared to the experimental value \(203\).

Fitting also to the known experimental results we obtained our overall best fit to everything \(\frac{\xi_{10\text{MeV}}}{\Delta V} = 1.7\) and thus

\[
\frac{N\sigma}{M^2}_{\text{overall}} = 4.4^{+7.0}_{-2.7} \times 10^{22} \text{ cm}^2/\text{kg}^2 \tag{205}
\]

which again should be compared to \(203\).

These good fits with \(\frac{\xi_{10\text{MeV}}}{\Delta V} = 1.7\) or 1.4 correspond to the radius of the pearls being (on average of course)

\[
\begin{align*}
R &= 1.1 \text{ cm (fit including experiment)} \tag{206} \\
R &= 0.94 \text{ cm (fit only “theory” requirements).} \tag{207}
\end{align*}
\]

12 Résumé of Present Fit Parameters

In the table below we now present the parameters of our model picture of the Tunguska particle as a pearl of a new type of vacuum with a bound state condensate, filled with ordinary white dwarf-like matter. In column 4 we give the results for a critical pearl on the borderline of stability as used in our previous work \[3\]. In column 5 we present a fit with one more parameter i.e. the ratio of the parameter \(\xi\), denoting the pearl’s radius divided by the radius of a critical pearl, and the potential difference \(\Delta V\) for a nucleon to pass through the skin of the pearl (multiplied by 10 MeV) \(\frac{\xi_{10\text{MeV}}}{\Delta V}\). The values in column 5 correspond to the best fit of \(\frac{\xi_{10\text{MeV}}}{\Delta V} = 1.7\) fitting both experimental and our theoretical constraints.

| Nr. | Name                        | symbol | old \(\frac{\xi_{10\text{MeV}}}{\Delta V} = 1\) | new \(\frac{\xi_{10\text{MeV}}}{\Delta V} = 1.7\) |
|-----|-----------------------------|--------|-----------------------------------------------|-----------------------------------------------|
| 1.  | Time Interval of impacts    | \(r_B^{-1}\) | 200 years                                    | kept                                         |
| 2.  | Rate of impacts             | \(r_B\)  | \(1.5 \times 10^{-10} \text{ s}^{-1}\)       | kept                                         |
| 3.  | Dark matter density in halo | \(\rho_{\text{halo}}\) | 0.3 GeV/cm\(^3\)                             | kept                                         |
| 4.  | Dark matter solar system    | \(\approx 2\rho_{\text{halo}}\) | 0.6 GeV/cm\(^3\)                             | kept                                         |
| 5.  | Typical speed of ball       | \(v\)  | 160 km/s                                      | kept                                         |
| 6.  | Mass of the ball            | \(m_B\) | \(1.4 \times 10^8 \text{ kg}\) = 140000 ton  | \(7.9 \times 10^{40} \text{ keV}\) = 16 GeV |
| 7.  | Kinetic energy of ball      | \(T_v\) | \(1.8 \times 10^{18} \text{ J} = 430 \text{ Mton TNT}\) |                                               |
| 8.  | Energy observed, Tunguska   | \(E_{\text{Tunguska}}\) | \((4 - 13) \times 10^{16} \text{ J} = 10 - 30 \text{ Mton TNT}\) |                                               |
| 9.  | Potential shift between vacua| \(\Delta V\) | 10 MeV                                       | (20 MeV)                                    |
| 10. | \(\sqrt{\text{tension(fit)}}\) | \(S^{1/3}\) | 28 GeV                                       | 16 GeV                                      |
| 11.1| \(\sqrt{\text{tension(condensate) 1}}} \) | \(S^{1/3}\) | 16 GeV                                       | 100 GeV                                     |
| 11.2| \(\sqrt{\text{tension(condensate) 2}}} \) | \(S^{1/3}\) | 16 GeV                                       | 30 GeV                                      |
| 12. | Ball density                | \(\rho_B\) | \(1.0 \times 10^{14} \frac{\text{kg}}{\text{m}^3}\) | \(2.0 \times 10^{13} \frac{\text{kg}}{\text{m}^3}\) |
13. Radius of ball \( R \) & 0.67 cm & 1.1 cm \\
14. homolumo gap \( E_H \) & 8.8 \pm 6.2 \text{ keV} & 5.2^{+2.4}_{-1} \text{ keV} \\
15. Frequency(obs.) & 3.5 \text{ keV} & 3.5 \text{ keV} & - \\
16. Released energy \( E_S \) & 0.38\%Mc^2 = 3.0 \times 10^{38} \text{ keV} & 0.22 \%Mc^2 = 1.8 \times 10^{38} \text{ keV} \\
17. # 3.5’s if all→3.5 \( N_{all→3.5} \) & 8.6 \times 10^{37} & 5.1 \times 10^{37} \\
18. Spreading time \( t_{spread} \) & 7.1 \times 10^{-2} \text{ s} & 0.21 \text{ s} \\
19. Radiation time with 1.5: \( t_{radiation} \) & 570 s & 120 s \\
20. Ratio with 1.5: \( \frac{t_{spread}}{t_{radiation}} \) & 1096 & 115 \\
21. # 3.5’s with 1.5: \( N = \frac{t_{spread}}{t_{radiation}} * N_{all→3.5} \) & 1.1 \times 10^{34} & 9.2 \times 10^{34} \\
22. cross section, balls \( \sigma = \pi (2R)^2 \) & 5.6 \text{ cm}^2 & 16 \text{ cm}^2 \\
23. cross section per \( \gamma \) as if all→3.5 \( N_{all→3.5} \sigma \) & 4.8 \times 10^{38} \text{ cm}^2 & 8.2 \times 10^{38} \text{ cm}^2 \\
24. cross section per \( \gamma \) (with time ratio) with 1.5: \( N \sigma \) & 6.2 \times 10^{34} \text{ cm}^2 & 1.5 \times 10^{36} \text{ cm}^2 \\
25. \sigma per \( \gamma \) per \( M^2 \) (as if all → 3.5) \( \frac{N \sigma}{M^2} \) & 3.1 \times 10^{35} \text{ cm}^2 \text{ kg}^{-2} & 7.6 \times 10^{35} \text{ cm}^2 \text{ kg}^{-2} \\
26. \sigma per \( \gamma \) per \( M^2 \) (with time ratio) with 1.5: \( \frac{N \sigma}{M^2} \) & 2.4 \times 10^{32} \text{ cm}^2 \text{ kg}^{-2} & 4.2 \times 10^{32} \text{ cm}^2 \text{ kg}^{-2} \\
27. Fit to Cline and Frey (including Boost corr.) \( \frac{N \sigma}{M^2} \) & (1.0 \pm 0.2) \times 10^{33} \text{ cm}^2 \text{ kg}^{-2} & - \\
28. Radius/critical R \( R \) (fitted) with \( \phi_H \geq 0 \) \( \xi = \frac{R}{R_{crit}} \) & 1 & 1.7 \\
29. Radius/ critical R \( R \) (speculated) \( \xi = \frac{2^{4/9} \sqrt{4\pi}}{\sqrt{\phi_H}} \) & - & 3.4 \\
30. Heat conductivity \( k \) & \( \frac{e^2 \rho_f}{\alpha_0} \) & 1000 \text{ MeV}^2 \text{ cm}^{-1} & 350 \text{ MeV}^2 \text{ cm}^{-1} \\

12.1 Review of Definitions and Explanation of the Table

Let us here shortly review the concepts given and explain the table above: The second column contains the short name of the quantity given in our model, and the third column is the formula expression for it. The fourth and the fifth columns contain suggested numerical order of magnitude values for the quantity in question: The fourth column gives the value obtained with the old numbers from our previous publication [3], so that what could be gotten from these numbers could be considered in some sense “pre”diction. These numbers were
based in some cases on the hypothesis that the size of the typical pearl is such that it is just on the borderline of stability towards collapsing by the matter/nuclei inside being spit out under the pressure. However further investigation suggests this hypothesis is not realistic and the actual radius $R_{\text{actual}}$ of a pearl is instead taken to be a fitting parameter $\xi$ times larger than the borderline radius $R_{\text{crit}}$. Then we actually use the parameter $\frac{\xi \times 10 \text{ MeV}}{\Delta V}$ to fit both the theoretical and the experimental constraints. The fitted value $\frac{\xi \times 10 \text{ MeV}}{\Delta V} = 1.7^{+0.6}_{-0.5}$ is used in the fifth column of the table.

The numbers in the fifth column are obtained by fitting the ratio $\frac{\xi \times 10 \text{ MeV}}{\Delta V}$ to fit both the theoretical and the experimental constraints giving in fact the value 1.7.

Now a short review of the rows in the table:

- 1. The interval $r_B^{-1}$ between successive impacts of our pearls on the earth as estimated from the fact that so far only one “Tunguska impact” event has been observed a hundred years ago, except perhaps the Sodom and Gomorrah event in biblical times.
- 2. Just the inverse of $r_B^{-1} = r_B$.
- 3. The dark matter mass density $\rho_{\text{halo}}$ in the halo in the neighborhood of our sun on a kpc scale but away from us on a scale of the order of the solar system. It is such densities, that determine the influence of the dark matter on the motion of the stars and galaxies and it is thus an astronomically measured quantity. We use it together with the rate $r_B$ to determine - after minor corrections using the speed $v$ of the pearls - the average or median mass $m_B$ of the pearls.
- 4. Using $\rho_{\text{halo}}$ as input we estimate the mass density of dark matter in the solar system near the earth to be $\approx 2 \rho_{\text{halo}}$.
- 5. Typical speed of the pearl in the region of the earth, where about half the pearls are supposed to be linked to the solar system and thus having lower speed, while about half come from the far out regions of the galactic halo. This speed is of relevance for determining how often a pearl hits the earth and thus for how to get the mass by means of the rate of impacts $r_B$.
- 6. The mass $m_B$ of a single pearl determined from the estimates (1.,4.,5.) above. (this was already done in our earlier work \[3\].)
- 7. The kinetic energy $\frac{1}{2} m_B v^2$ of the pearl responsible for the impact and release of energy in Tunguska.
- 8. The energy observed as the visible explosion in Tunguska $E_{\text{Tunguska}}$. This of course should at least be smaller than the kinetic energy of the pearl available for making explosion, since an appreciable part of the energy will be deposited deeply inside the earth.
- 9. Potential shift for a nucleon in passing through the skin of the pearl $\Delta V \approx 10$ MeV. We presume that the potential felt by a neutron or a proton inside the pearl is $\Delta V$ lower than outside due to a lower Higgs field inside the pearl. Since in appendix A.1 we argue for now believing
that $\Delta V = 20$ MeV rather than the previous 10 MeV we have put “(20 MeV)” in brackets in column 5.

- 10. The force per unit length or equivalently the energy per unit area of the pearl surface/skin is denoted $S$. In column 4 the value has been fitted to the hypothesis that the typical pearl size is just on the borderline of stability against the nuclei being spit out. Whereas in column five, this value is corrected by the fitted value of the parameter $\xi 10^4 MeV$. In both columns it is the third root of the tension $S^{1/3}$ which is given.

- 11. Here we then give the same third root but now estimated from theoretical considerations about the Higgs field $\phi_H$ and the effective field $\phi_F$ for the bound state of $6t + 6\bar{t}$ introduced in our work. Basically it means that $S^{1/3}$ is given by dimensional arguments from the Higgs mass and Higgs field expectation value.

We have two different hypotheses about estimating the effective potential from which the surface tension $S$ is obtained from a soliton. “Theory 1)” involves only the present and the condensate vacuum, while “theory 2)” is based on the assumption of yet one more vacuum phase.

- 12. Ball density or pearl density $\rho_B$ is the specific density of the bulk of the pearl, i.e. simply the ratio of the mass to the volume.

- 13. The radius $R$ of the ball, mainly thought of as the radius of the skin sphere.

- 14. The homolumo gap calculation is the first main point of the present article where we obtain the value for the gap between the lowest unoccupied and the highest occupied electronic orbits. This energy gap gives rise to radiation from the dark matter with the frequency essentially equal to the homolumo gap. It is our first and most important success that this homolumo gap turns out to be order of magnitude-wise equal to 3.5 keV.

- 15. In this line we just note down the observed X-ray frequency of 3.5 keV, supposedly emitted from dark matter.

- 16. The released energy $E_S$ stands for the energy released when two pearls collide and their common surface contracts so as to have one combined pearl instead of the previous two. This released energy is estimated as the fraction of the surface area contracted away multiplied by the surface tension $S$. It is written relative to the Einstein energy of the whole pearl $m_Bc^2$.

- 17. “# 3.5’s as if all $\rightarrow$ 3.5” means the number of photons of energy 3.5 keV, which could be produced from the released energy $E_S$ under the perhaps not realistic assumption that all the energy went into such 3.5 keV photons. I.e. it is simply $E_S/(3.5keV)$.

- 18. The spreading time $t_{spread}$ for the hot spot produced in the collision to spread over the whole pearl, so that its surface gets heated and energy escapes via higher frequencies than just the 3.5 keV line.
19. The radiation time $t_{\text{radiation}}$ is defined as the time it would take for the released energy $E_S$ to be emitted, if it was all emitted as black body radiation at the temperature $T = 3.5$ keV. Since this is what is expected to happen during the time interval $t_{\text{spread}}$, the fraction of radiation sent out as 3.5 keV radiation is estimated as the ratio $t_{\text{spread}}/t_{\text{radiation}}$. Of course the emitted number of photons with 3.5 keV cannot be bigger than the number estimated assuming all the energy goes to 3.5 keV’s. Thus, if this ratio $t_{\text{spread}}/t_{\text{radiation}} \geq 1$ we replace the ratio by 1.

The line “with 1.5” means that we took the effective temperature for the amount of 3.5 keV radiation emitted to be $1.5 \times 3.5$ keV instead of just 3.5 keV (see (111)).

20. The ratio $t_{\text{spread}}/t_{\text{radiation}}$, relevant for the amount of radiation 3.5 keV emitted.

The line “with 1.5” means that we took the effective temperature for the amount of 3.5 keV radiation emitted to be $1.5 \times 3.5$ keV instead of just 3.5 keV (see (111)).

21. “# 3.5’s” then means the estimate of how many 3.5 keV photons are truly produced in one collision. The correction is that only the fraction $\min(1, \frac{t_{\text{spread}}}{t_{\text{radiation}}})$ of the item 17.: “# 3.5’s as if all $\rightarrow 3.5$” comes out as 3.5 keV radiation.

The line “with 1.5” means that we took the effective temperature for the amount of 3.5 keV radiation emitted to be $1.5 \times 3.5$ keV instead of just 3.5 keV (see (111)).

22. The cross section $\sigma = \pi (2R)^2$ for the pearls colliding is supposed to be the geometrical cross section just given by the radii of the pearls colliding. It is of course $\pi$ times the square of the sum of the radii of the two pearls.

23. “cross section per $\gamma$ (all $\rightarrow 3.5$)” means the cross section that a pearl should have for hitting another pearl if only one photon (with energy 3.5 keV) was produced per collision and under the assumption that all energy goes to the 3.5 keV line.

24. “cross section per $\gamma$ (with time ratio)” means the cross section needed for the collision, if we have to have again one collision for each photon emitted, but this time taking the more realistic amount of 3.5 keV photons by them only being produced during the time $t_{\text{spread}}$.

25. Here we simply divide the “cross section per $\gamma$ (all $\rightarrow 3.5$)” by the mass square of the pearl $M^2 = m_B^2$.

26. Similarly we divide the number “cross section per $\gamma$ (with time ratio)” by $M^2 = m_B^2$. This is now the quantity, which determines the rate of 3.5 keV radiation from various objects, provided one can estimate the square of the density of dark matter in those astronomical objects.

The line “with 1.5” means that we took the effective temperature for the amount of 3.5 keV radiation emitted to be $1.5 \times 3.5$ keV instead of just 3.5 keV (see (111)).
• 27. The quantity $N_{\sigma}$ extracted from the observations by Cline and Frey [34]. This number should be considered as the experimental value corresponding to the theoretical value in item 25 or item 26.

• 28. The ratio of the actual radius to the “critical” radius of the pearls (our parameter $\xi$), assuming $\Delta V = 10$ MeV. The row 28b denotes the value under the assumption that $\Delta V = 20$ MeV, as will happen for zero Higgs field $\phi_H$ in the condensate vacuum.

• 29. A theoretical estimate of what this $\xi$ ratio should be provides $\xi = \frac{2^{4/3}}{\sqrt{\pi}}$, which we actually use as a theoretical restriction in our fits.

• 30. This is our estimated value for the heat conductivity $k$.

13 Outlook.

Let us at the end point to that now we have such a promising model for the dark matter we should of course as soon as possible study what our model, with its parameters getting more and more fixed, will predict for the phenomena suspected to come from dark matter as:

• The positron excess: Actually our pearls in the situation when even the surface is very hot - something happening shortly after the collision - will emit a lot of electrons, which sit more loosely than the nucleons. So presumably a turbulent plasma with strong fields (electric or/and magnetic) will appear around the exploding pearl (pair). This “little supernova remnant” could easily be imagined to send out all sorts of cosmic radiation, including positrons.

• Broad spectrum gamma-rays: Although we hope for an of order unity part of the energy coming out as the line 3.5 keV, it would of course be almost impossible that there should not come radiation of other frequencies too.

It may be hard to know if such radiation really comes form dark matter. A mark of our model should be that it like radiation from annihilation rather goes proportional to the square of the dark matter density than just proportional to the density itself.

• The supernova remnant: We really in the near future to estimate the amount of 3.5 keV radiation that could appear from a supernova remnant because of being energized by the cosmic rays in this remnant rather than by our collision of pearls, which has no reason to be especially often in supernova remnants. (Remember that indeed such radiation from a supernova remnant was observed)

A Appendix, Higgs field

A.1 Smooth realization of MPP

In previous work on this model we made the assumption that in the condensate vacuum the Higgs field vacuum expectation value was decreased compared to
the value in the present vacuum by some factor of order unity, which we in the
calculations took to be a factor 1/2. Actually some estimate in the appendices
of an earlier paper \[3\] happened to give us that the Higgs field expectation value
was indeed close to 1/2 of the one in the present vacuum.

In this appendix we shall argue that it is more natural to take the Higgs
field expectation value in the condensate phase/vacuum to actually be zero.
This argument is based on the hypothesis that there should be a minimal
amount of fine tuning to implement MPP. In fact we require the effective po-
tential, expressed in terms of the Higgs field $\phi_H$ and an effective field $\phi_F$ for the
bound state of six top and six anti-top quarks $F$ making the condensate, should
be given as a polynomial of lowest possible order. Since weak isospin symmetry
ensures that the effective potential can only depend on the Higgs field through
its square $|\phi_H|^2$, we may think of this effective potential as being a function
$V_{eff}(|\phi_H|^2, \phi_F)$.

For simplicity we shall only show how the expectation values in the MPP
model tend to go to the boundary under an extra assumption. Namely we as-
sume that even for the bound state $F$ the field $\phi_F$ only occurs, approximately
at least, in the combination $\phi_F^2$ (since the bound state $F$ is its own antiparticle,
we suppose its field is “real” in the sense that it is Hermitean). In this case we
can consider the effective potential a function $V_{eff}(|\phi_H|^2, \phi_F^2)$ and by “renor-
amalizability” it would be required to at most second order in these squares
$|\phi_H|^2$ and $\phi_F^2$.

In order to make this effective potential be as smooth as possible we now
postulate that, in terms of these variables $|\phi_H|^2$ and $\phi_F^2$, we can approximate it
by an as low order polynomial as possible.

One should have in mind of course that the variable $|\phi_H|^2$ as well as
$\phi_F^2$ must always be positive or zero

$$|\phi_H^2| \geq 0, \text{ and } \phi_F^2 \geq 0. \quad (208)$$

This opens up the possibility for a minimum in this effective potential $V_{eff}(|\phi_H|^2, \phi_F^2)$
to occur on the boundary of the allowed region, i.e. where $|\phi_H|^2=0$ or where
$\phi_F^2 = 0$. Then it is no longer needed for there to be zero derivative w.r.t. both
variables. One derivative being zero would do.

One easily sees that if one does not make use of this option of having a
minimum on the boundary of the allowed region, then drawing a straight line
through two degenerate minima as required by MPP would imply that the
effective potential restricted to this line would have two degenerate minima on
the line. Two (degenerate) minima on a line enforces a maximum in between
and the derivative of the effective potential restricted to the line to have three
separate zeros and thus be at least a third order polynomial on this line. So
without using the possibility of a minimum on the boundary, the restriction to
the line and thus even more the whole effective potential would have to be at
least of fourth order as a function of the variables $|\phi_H|^2$ and $\phi_F^2$. If we, however,
use the option of having one minimum - actually the one corresponding to the
“condensate vacuum” - on the border where $|\phi_H|^2=0$, then the derivative only
needs two zeroes. Thus the effective potential itself $V_{eff}(|\phi_H|^2, \phi_F^2)$ restricted
to the line drawn through the two MPP-minima would only need to be at most
of third order in the variables $|\phi_H|^2$, and $\phi_F^2$. But if we let both of the MPP-
minima be on border lines then we need only one maximum of the derivative
of the potential restricted to the line. This would allow the potential itself to be only of second order in the two squares. This realization of MPP by two minima on the boundaries is the only way to realize two minima with a dimensionality-wise renormalizable Lagrangian.

Thus we see that requiring the minimal order of the Taylor expansion approximation for the effective potential leads to letting both minima be at a border. That is to say, this smoothness requirement leads to the Higgs field expectation value in the “condensate vacuum” being zero, and the expectation value of the bound state field $\phi_F$ being zero in the present vacuum. Physically of course this means that the “condensate vacuum” has no weak hypercharge nor weak isospin spontaneous breaking. Especially the quarks are therefore massless in this vacuum-phase.

In earlier work we could only assume that the Higgs field expectation value in the condensate vacuum was smaller than in the present vacuum and of order unity compared to the latter. Now with the expectation that the Higgs field in the condensate vacuum is zero, in principle we obtain a more accurate prediction for the nucleon potential difference $\Delta V$ on passing the border of our pearls. In previous work [3] we got the estimate $\Delta V = 10$ MeV, by making the assumption that the Higgs field in the condensate vacuum were just half of that in the present vacuum $\phi_H = 246$ GeV. So with the zero Higgs field instead, we will get

$$\Delta V = 20 \text{ MeV}. \quad (209)$$

### A.2 An MPP-relation.

We write our polynomial ansatz for the effective potential $V_{eff}(|\phi_H|^2, \phi_F^2)$ with both $\phi_H$ and $\phi_F$ occurring only to even powers in the explicit form

$$V_{eff}(|\phi_H|^2, \phi_F^2) = V_H(|\phi_H|^2) + V_F(\phi_F^2) + \lambda_{mix} |\phi_H|^2 \phi_F^2 \quad (210)$$

where

$$V_H(|\phi_H|^2) = \lambda_H (|\phi_H|^2 - v_H^2)^2 \quad (211)$$

and

$$V_F(\phi_F^2) = \lambda_F (\phi_F^2 - v_F^2)^2 \quad (212)$$

In the model suggested above the degenerate minima occur on the axes where $\phi_H = 0$ and $\phi_F = 0$ respectively. Hence the term $\lambda_{mix} |\phi_H|^2 \phi_F^2$ representing the interaction between the two fields is zero at these minima. The energy density for the two minima are easily seen to be

$$V_{eff}(\text{“min at present vacuum”}) = V_{eff}(v_H^2, 0) = \lambda_F v_F^4 \quad (213)$$

and

$$V_{eff}(\text{“min at condensate vacuum”}) = V_{eff}(0, v_F^2) = \lambda_H v_H^4 \quad (214)$$

respectively. So the multiple point principle requirement that these two minima be equally deep means that we must have

$$\lambda_F v_F^4 = \lambda_H v_H^4. \quad (215)$$

Denoting the mass of the $F$-particle in the condensate vacuum by $m_F|_{\text{condensate}}$ we have

$$m_F^2|_{\text{condensate}} = 8\lambda_F v_F^2. \quad (216)$$
Similarly the Higgs mass in the present vacuum is

\[ m_H^2 |_{\text{present}} = 4 \lambda_H v_H^2. \] (217)

Thus our just derived MPP-relation becomes

\[ m_F |_{\text{condensate}} = \sqrt{2 m_H |_{\text{present}}} v_H. \] (218)

This equation is interesting because it tells that if the bound state \( F \) is heavy in the condensate phase compared to the Higgs mass, then its vacuum expectation value in the condensate phase will have to be correspondingly small.

### A.3 Minimal \( \lambda_{mix} \)

In order to ensure that the assumed two degenerate minima are indeed the lowest energy states of the system, we have to require that the effective potential does not fall to an even lower value than these minima in some other place \((\phi_H, \phi_F)\). If we did not have the \( \lambda_{mix} |\phi_H|^2 \phi_F^2 \) term, there would indeed be a minimum deeper than the two minima at the borders, which would actually not be true minima themselves anymore but only saddle points in \( V_{eff} \). In this case, with \( \lambda_{mix} = 0 \), the true minimum would be of course be at the point \((\phi_H, \phi_F) = (v_H, v_F)\). In order to fill up this minimum so that the value of \( V_{eff} \) becomes at least \( \lambda_H v_H^4 = \lambda_F v_F^4 \), we need the mixing term to satisfy

\[ \lambda_{mix} v_H^2 v_F^2 \geq \lambda_H v_H^4 = \lambda_F v_F^4. \] (219)

\[ \Rightarrow \lambda_{mix} \geq \sqrt{\lambda_H \lambda_F}. \] (220)

But we also have to make the slope of \( V_{eff} \) at the present vacuum minimum be non-negative in the increasing \( \phi_F \) direction. Since near the present vacuum the dominant term from \( V_F \) is \(-2\lambda_F v_F^2 \phi_F^2\), we in fact require

\[ \lambda_{mix} v_H^2 v_F^2 \phi_F^2 \geq 2 \lambda_F v_F^2 \phi_F^2. \] (221)

\[ \Rightarrow \lambda_{mix} v_H^2 \geq 2 \lambda_F v_F^2 \] (222)

and analogously:

\[ \lambda_{mix} v_F^2 \geq 2 \lambda_H v_H^2. \] (223)

Then multiplication gives:

\[ \lambda_{mix}^2 v_H^2 v_F^2 \geq 4 \lambda_H \lambda_F v_H^2 v_F^2 \] (224)

\[ \Rightarrow \lambda_{mix} \geq 2 \sqrt{\lambda_H \lambda_F}. \] (225)

We now show that the inequality (225) ensures that the effective potential is higher than or equal to the two hoped for MPP-minima, by writing it in the form

\[ V_{eff}(|\phi_H|^2, |\phi_F|^2) - \lambda_H v_H^4 - \lambda_F v_F^4 = \begin{align*}
\lambda_{mix} - 2 \sqrt{\lambda_H \lambda_F} |\phi_F|^2 \phi_H^2 + (\sqrt{\lambda_H} |\phi_H|^2 + \sqrt{\lambda_F} |\phi_F|^2)^2 + m_F^2 \text{ tac} |\phi_H|^2 + \frac{1}{2} m_F^2 \text{ tac} |\phi_F|^2 & = (\lambda_{mix} - 2 \sqrt{\lambda_H \lambda_F} |\phi_F|^2 \phi_H^2 + (\sqrt{\lambda_H} |\phi_H|^2 + \sqrt{\lambda_F} |\phi_F|^2)^2 - 2 \lambda_H v_H^2 |\phi_H|^2 - 2 \lambda_F v_F^2 |\phi_F|^2 \end{align*} \] (226)

\[ \lambda_{mix} - 2 \sqrt{\lambda_H \lambda_F} |\phi_H|^2 \phi_F^2 + (\sqrt{\lambda_H} |\phi_H|^2 + \sqrt{\lambda_F} |\phi_F|^2)^2 - 2 \lambda_H v_H^2 |\phi_H|^2 - 2 \lambda_F v_F^2 |\phi_F|^2 \] (227)

\[ \lambda_{mix} - 2 \sqrt{\lambda_H \lambda_F} |\phi_H|^2 \phi_F^2 + (\sqrt{\lambda_H} |\phi_H|^2 + \sqrt{\lambda_F} |\phi_F|^2)^2 - 2 \lambda_H v_H^2 (\sqrt{\lambda_H} |\phi_H|^2 + \sqrt{\lambda_F} |\phi_F|^2) \] (228)

\[ \lambda_{mix} - 2 \sqrt{\lambda_H \lambda_F} |\phi_H|^2 \phi_F^2 + (\sqrt{\lambda_H} |\phi_H|^2 - v_H^2) + \sqrt{\lambda_F} |\phi_F|^2)^2 - \lambda_H v_H^4 \] (229)
Here we had to have in mind equation (215) under the assumption of our MPP. This form of the effective potential contains

- the term \((\lambda_{\text{mix}} - 2\sqrt{\lambda_H \lambda_F})|\phi_H|^2 \phi_F^2\), which is non-negative for \(\lambda_{\text{mix}} \geq 2\sqrt{\lambda_H \lambda_F}\).

- The total square term that can be written both as \((\sqrt{\lambda_H}|\phi_H|^2 + \sqrt{\lambda_F}(\phi_F^2 - v_F^2))^2\) and as \((\sqrt{\lambda_H}|\phi_H|^2 - v_H^2) + \sqrt{\lambda_F}\phi_F^2)^2\), which is zero in the minima corresponding to the present and condensate vacua.

- Finally there is the not so important cosmological constant term, which in \(V_{\text{eff}}\) is \(\lambda_H v_H^4 = \lambda_F v_F^4\).

So from this form one sees that

\[
V_{\text{eff}}(|\phi_H|^2, \phi_F) \geq (\lambda_{\text{mix}} - 2\sqrt{\lambda_H \lambda_F})|\phi_H|^2 \phi_F^2 + \lambda_H v_H^4 \tag{230}
\]

and thus

\[
V_{\text{eff}}(|\phi_H|^2, \phi_F) \geq V_{\text{eff}}(v_H^2, 0) = V_{\text{eff}}(0, v_F^2)
\]

provided that

\[
\lambda_{\text{mix}} \geq 2\sqrt{\lambda_H \lambda_F}. \tag{231}
\]

In the case when we have equality in the inequality

\[
\lambda_{\text{mix}} \geq 2\sqrt{\lambda_H \lambda_F} \tag{232}
\]

the effective potential takes the form

\[
V_{\text{eff}}(|\phi_H|^2, \phi_F^2) = (\sqrt{\lambda_H}|\phi_H|^2 + \sqrt{\lambda_F}\phi_F^2)^2 + m_{\text{H tac}}^2|\phi_H|^2 + \frac{1}{2}m_{\text{F tac}}^2\phi_F^2. \tag{233}
\]

Here the masses in the Lagrangian are written in the usual form and carry the index \(\text{tac}\) standing for “tachyonic”. They are given by the expressions

\[
m_{\text{H tac}}^2 = -2\lambda_H v_H^2 \tag{234}
\]

and

\[
m_{\text{F tac}}^2 = -4\lambda_F v_F^2. \tag{235}
\]

In fact these tachyonic masses are proportional to the masses of the bound state \(F\) \([216]\) and the Higgs \(H\) \([217]\) in the “condensate” phase and the “present” phase respectively. If we now impose the MPP-relation \([218]\) we get that the combination of mass terms only depends on the quantity \(\sqrt{\lambda_H}|\phi_H|^2 + \sqrt{\lambda_F}\phi_F^2\), in terms of which we have written the fourth order part. In other words with MPP and \(\lambda_{\text{mix}}\) taken minimal, the whole effective potential only depends on the quantity \(\sqrt{\lambda_H}|\phi_H|^2 + \sqrt{\lambda_F}\phi_F^2\). So under these two assumptions the effective potential is constant along contour curves for this quantity. Especially one such contour curve connects the two degenerate minima. This then implies that a field can vary smoothly along this curve from one minimum to the other one. And that in turn implies that a soliton solution giving zero tension \(S\) can be found in this special case with minimum \(\lambda_{\text{mix}}\).
A.4 The Surface tension $S$, Introduction.

We already found above that, for the coupling constant $\lambda_{mix}$ taking its minimally allowed value $2\sqrt{\lambda_H\lambda_F}$, there is a flat direction from the one minimum to the other one. Thus the surface tension for the wall separating the “present” and the “condensate” vacua would be $S = 0$ in this minimum case.

Increasing $\lambda_{mix}$ from this minimum value, the surface tension $S$ will of course increase. But, however high $\lambda_{mix}$ would be, it can never lead to a higher $S$ than the value obtained when the route of the field combination $(\phi_H, \phi_F)$ in the soliton, as one passes the wall, only goes through combinations wherein one of the two fields is zero. In fact one could consider a potential soliton having the field combination following the axes in the $(\phi_H, \phi_F)$-plane from the one minimum via the origin in the field combination space to the other minimum. But if $\lambda_{mix}$ is not too large and positive, then there is the possibility of finding a “lower” path for the soliton solution, which would give a lower tension $S$. Thus the tension estimated taking the path of the soliton to be along the axes would become an upper limit for $S$.

In order to estimate the size of this upper limit we could consider a couple of helpful auxiliary problems:

Formally we can take a single field theory, say the Higgs theory with $V_H(\phi_H) = \lambda_H(\phi_H^2 - v_H^2)^2$ as if the Higgs field was real but could be both positive and negative. We would then have a model with the symmetry $\phi_H \rightarrow -\phi_H$, and two degenerate vacua because of this symmetry. (We would have MPP by this symmetry). We can now calculate the energy per unit area of a soliton in which the field $\phi_H$ goes from one of these minima to the other one. We call the energy per unit area or the tension of such a wall $S_H$. For the analogous problem, obtained by using the $\phi_F$ field and the $V_F(\phi_F)$ potential given by (212) and letting the soliton field go from $v_F$ to $-v_F$, we call the tension $S_F$. Then the tension for the soliton field going via the origin $(0,0)$ would be

$$S = \frac{1}{2}(S_H + S_F).$$  \hspace{1cm} (236)

The factor $\frac{1}{2}$ comes in because, in going from the one minimum to the other one in the full model with two fields via the origin, one only goes half of the distance one goes in our two auxiliary models.

A.5 Calculating the tension $S$.

In this subsection we shall first compute the tension or energy per unit area for a solitonic transition from a region with $\phi_H = -v_H$ to one with $\phi_H = v_H$ with the potential $V_H(\phi_H) = \lambda_H(\phi_H^2 - v_H^2)^2$. Since the potential energy density at the start and end points is just zero, the equation for the derivative of the field $\phi_H$ with respect to the coordinate $x$ in the direction perpendicular to the surface between the two phases is

$$\left(\frac{\partial \phi_H}{\partial x}\right)^2 = V_H(\phi_H) = \lambda_H(\phi_H^2 - v_H^2)^2. \hspace{1cm} (237)$$

$$\hspace{1cm} (238)$$
The first step is to find $\phi_H$ as a function of the perpendicular coordinate $x$. Taking the square root etc. of this equation gives

$$\pm \frac{\partial \phi_H}{\partial x} = \sqrt{\lambda_H(v_H^2 - \phi_H^2)}$$

(239)

$$\Rightarrow \frac{d\phi_H}{\sqrt{\lambda_H(v_H^2 - \phi_H^2)}} = dx$$

(240)

$$\Rightarrow \tanh^{-1} \left( \frac{\phi_H}{v_H} \right) = \sqrt{\lambda_H} v_H x$$

(241)

with the appropriate choice of origin. Thus one obtains

$$\phi_H = v_H \tanh(\sqrt{\lambda_H} v_H x)$$

(242)

The energy per unit area of the wall is then

$$S_H = \int_{-\infty}^{\infty} 2V_{eff} dx$$

(243)

$$= \int_{-\infty}^{\infty} 2\lambda_H(v_H^2 - \phi_H^2)^2 dx$$

(244)

$$= \int_{-\infty}^{\infty} 2\lambda_H(v_H^2 - v_H^2 \tanh^2(\sqrt{\lambda_H} v_H x))^2 dx$$

(245)

$$= 2\sqrt{\lambda_H} v_H^3 \int_{-\infty}^{\infty} \frac{1}{\cosh^4(u)} du$$

(246)

$$= \frac{8\sqrt{\lambda_H} v_H^3}{3}$$

(247)

Using (217) this means

$$S_H = \frac{4m_H v_H^2}{3}.$$  

(248)

Similarly we find

$$S_F = \frac{2m_F v_F^2}{3} = \frac{m_H}{m_F} S_H \approx \frac{1}{6} S_H,$$

(249)

where we have taken $m_F \sim 750$ GeV.

So the upper limit on the tension becomes

$$S = \frac{1}{2}(S_H + S_F) \approx \frac{7}{12} S_H = \frac{7}{9} m_H v_H^2,$$

(250)

and the upper limit for $S^{\frac{1}{2}}$ is

$$S^{\frac{1}{2}} = 140 \text{ GeV}.$$  

(251)

Hence the true tension $S$ must satisfy

$$S^{\frac{1}{2}} \leq 140 \text{ GeV}.$$  

(252)
A.6 Rescue Attempt

There is one rather natural story that can provide a better fit with the theoretical cubic root of the surface tension $S^{1/3}$ not deviating so much from the fitted value.

This proposal is based on saying that, since we anyway have assumed the multiple point principle, we can easily expect even more than two degenerate vacua (namely the present vacuum and the condensate vacuum) at low energy. Although we do not have any explicit third vacuum in mind, we could just abstractly claim that there could exist another vacuum degenerate with the other two vacua. With say three vacua at a low energy scale, it is natural to consider that we have three effective fields $\phi_H$, $\phi_F$, and $\phi_F'$ instead of only the first two as we used above. Also we shall keep our assumption from appendix A.1 that we approximate the effective potential $V_{\text{eff}}$ by a polynomial only up to fourth order terms in the fields $\phi_H$, $\phi_F$, and $\phi_F'$ and only with even powers of these fields. Then the effective potential would be a second order polynomial in the three square variables $|\phi_H|^2$, $\phi_F^2$, and $\phi_F'^2$. Now to have three degenerate minima - as required by MPP with three vacua - this polynomial in the three variables has to have three equally deep minima.

We shall now see that having the three assumed degenerate minima leads to the effective potential being completely constant/flat along the planar surface in the squared fields space spanned by the positions of these three minima. We first remark that along any line in the squared field space the effective potential is still of course also a polynomial of the second order in the squared fields. But now, if you consider the line spanned by two of the minimum positions, the effective potential restricted to such a line would have to have two (degenerate) minima. It follows that the derivative of the effective potential $\frac{dV_{\text{eff}}}{dl}$ (where $l$ parameterizes this line) would have to have at least two zeros, namely one at each of the minimum-positions. The effective potential itself would even have to have a maximum in between these two minima and thus the derivative would have at least three zeros. But the existence of three zeros is impossible for a first order polynomial - the derivative is only first order - on a line except if the polynomial is totally zero. By this argument there have to be flat directions for the effective potential along all lines passing through just two of the minima positions.

Let us consider the set of second derivatives of the effective potential $\frac{\partial^2 V_{\text{eff}}}{\partial \phi_i \partial \phi_j}$ forming a $(3 \times 3)$ matrix. We see, from the just derived flat directions, that this matrix taken at one of the minimum points must have a zero eigenvalue corresponding to each of the flat directions extending to the other two minima. But having now two zero eigenvalues the whole plane spanned by the two eigenvectors would give us a flat direction, i.e. a whole flat plane along which the effective potential would be degenerate with the MPP-minima.

Note that, in the here described approximation, the surface tensions $S$ for domain walls separating any two of the three phases/vacua would be zero. So only if the effective potential is after all not a second order polynomial in the square fields but e.g. of third order (meaning of sixth order in the fields themselves) would we obtain non-zero tensions.

This means that, to the degree that our approximation was a good one, the tensions would be appreciably smaller than first estimated, which could greatly improve the quality of our fit.
A.7 Order of magnitude for the Tension $S$

If we can give a dimensional argument for the degree of suppression of the sixth order terms compared to the fourth order or lower terms, we could deliver an order of magnitude estimate for the tension $S$ for the domain walls between the phases.

Let us adopt the philosophy that we are concerned with phases lying at a scale of the Higgs mass $M_H = 125$ GeV while the physics relevant for the effective couplings $\lambda_H, \lambda_F, \lambda_F', m_F^2, \ldots$ is rather the physics scale of the bound state $m_F$, which we have previously estimated to have a mass of order 750 GeV. If so we could claim that the Taylor expansion, which we terminated with a fourth order polynomial above, effectively has its terms go down by a factor $(\frac{125}{750})^2 = \frac{1}{36} = 0.028$ for each pair of mass dimensions. So, for example, the sixth order terms that alone contribute to the tensions in the present three vacuum model will be reduced compared to the typical order of magnitude expected had the up to fourth order terms indeed contributed - as happens with only two vacua - by a factor of 36. In the two degenerate vacua case we obtained an $S^{1/2}$-upper limit $\frac{252}{140}$ of the order 140 GeV or a typical value say of 100 GeV. This value would go down by a factor of $\sqrt{36} = 3.3$, meaning to $S^{1/3} = (100 \text{ GeV})/3.3 = 30$ GeV.

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