PARTON CASCADE DESCRIPTION
OF RELATIVISTIC HEAVY ION COLLISIONS
AT CERN SPS ENERGIES

Klaus Geiger\(^1\) and Dinesh Kumar Srivastava\(^2\)

\(^1\)Physics Department, Brookhaven National Laboratory, Upton, N. Y. 11973, U. S. A.
\(^2\)Variable Energy Cyclotron Centre, 1/AF Bidhan Nagar, Calcutta 700 064, India

Abstract

We examine \(Pb + Pb\) collisions at CERN SPS energy 158 \(A\) GeV, by employing the earlier developed and recently refined parton-cascade/cluster-hadronization model and its Monte Carlo implementation. This space-time model involves the dynamical interplay of perturbative QCD parton production and evolution, with non-perturbative parton-cluster formation and hadron production through cluster decays. Using computer simulations, we are able to follow the entwined time-evolution of parton and hadron degrees of freedom in both position and momentum space, from the instant of nuclear overlap to the final yield of particles. We present and discuss results for the multiplicity distributions, which agree well with the measured data from the CERN SPS, including those for K mesons. The transverse momentum distributions of the produced hadrons are also found to be in good agreement with the preliminary data measured by the NA49 and the WA98 collaboration for the collision of lead nuclei at the CERN SPS. The analysis of the time evolution of transverse energy deposited in the collision zone and the energy density suggests an existence of partonic matter for a time of more than 5 \(fm\).

PACS numbers: 12.38.Bx, 12.38.Mh, 25.75.+r, 24.85.+p
1 INTRODUCTION

The study of relativistic heavy ion collisions over the last several years has been driven by the motivation
to discover novel phenomena associated with the collective behavior of highly compressed QCD matter. At
the forefront of this effort lies the search for the ‘notorious’ quark-gluon plasma (QGP), a thermal state
of colored partons deconfined over a macroscopic volume. A large number of experiments have already
been conducted at the CERN SPS with oxygen, sulphur, and lead beams. Some of the proposed signals
like a suppression of $J/\Psi$ production, an enhanced production of strangeness, and an excess production of
dileptons have already been observed.

A large number of papers \[1\] in the literature explicitly start with an assumption of the actual existence
of a QGP at some stage during the nuclear collision. Leaving aside the most interesting question whether
and how such a QGP may be realistically formed through the evolution of the initial nuclear state, further
assumptions about the specific quark-gluon composition, volume, density and temperature, etc., are usually
employed due to the lack of a better knowledge. Are these assumptions too “bold” or “tantalizing”? What
is the justification for such assumptions except for the plea that hadronic densities may get too large for
such collisions, and thus a treatment in terms of hadrons may get unreasonable? Is this plea too specious?
Also, have such collisions succeeded in creating a reasonably large volume over which the energy density is
large enough to admit a QCD phase transition?

Quite a few models have been used to describe the gross features of these collisions in terms of the so-called
string picture for hadronic interactions, i.e., based on modeling nuclear collisions in terms of nucleon-nucleon
collisions on a basis of a constituent valence-quark picture plus string-excitation and -fragmentation. Popular examples for these models are FRITIOF \[2\], VENUS \[3\], RQMD \[4\], DPM \[5\]. The continuous
refinements and the fine tunings of the necessarily involved parameters have resulted in development of
several generations of these ‘event generators’. In particular VENUS and RQMD are impressively fine-tuned
to describe the recent CERN SPS data. Yet, the price paid is the invention of supplemental mechanisms
(such as ‘string droplets’ in VENUS or ‘color ropes’ in RQMD) to mimic certain underlying dynamics due
to nuclear effects, which cannot be accounted for in a non-trivial manner in these models. What does the
success of VENUS and RQMD imply, in view of the fact that the ‘strings’ and the ‘ropes’ only serve to
mimic the actual interactions? These studies have also lead to a “faith” that the interactions at the SPS
energies is mostly “soft”. Should we remain complacent in this belief?

At the same time, a large body of hadronic data measured at the SPS energies has been shown \[6\] to
be fairly consistent with a thermal model, suggesting a thermal and chemical equilibrium among hadrons at
the time of freeze-out. A purely hadronic picture of the interaction is not very likely to drive the system to
the verge of chemical equilibrium \[7\], as the hadronic reactions are far too slow.

The provocative question we dare to pose then is: How can we really improve the knowledge of the
underlying properties of QCD in dense matter, when drastically different phenomenological approaches
(with their very own parametrizations of unknown physics details) describe the experimental data more or
less equally well?

We are convinced that this issue and many of the above-mentioned questions can be settled satisfactorily
in the near future, if we attempt to understand the collision among heavy ions at these energies at a more
microscopic level by borrowing and extending rigorous QCD techniques that have been developed over the
years in particle physics. Clearly, the entwined dynamics of partonic and hadronic degrees of freedom that
we have to face, requires that we consider a mixed parton-hadron system with its space-time dependent
relative proportions being determined by the multi-particle dynamics itself.

We therefore advocate that for ultra-relativistic nucleus-nucleus collisions a description based on the
pQCD interactions and cascade evolution of involved partons can and should be used, owing to the claim
that short-range parton interactions play (at least during the early and most dissipative stage of the first few
$fm$) an important role at sufficiently high beam energies (say, $\sqrt{s} > 20$ A GeV). Here copiously produced
quark-gluon mini-jets cannot be considered as isolated rare events, but must be embedded in the complex
multiple cascade-type processes. The present study is a step in this direction.

We advertize an elaborate QCD-based space-time model that allows to simulate nucleus-nucleus collisions
(among other particle collisions), which is now available \[8\] as a computer simulation program called VNI \[11\].
It is a pQCD parton cascade description \[10\], supplemented by a phenomenological hadronization model

\[1\] The program VNI-3.1 (pronounced Vinnie) is available from \[http://rhic.phys.columbia.edu/vni\].
with dynamically changing proportions of partons and hadrons, in which the evolution of a nuclear collision is traced from the first instant of overlap, via QCD parton-cascade development at the early stage, parton conversion into pre-hadronic color-singlet clusters and hadron production through the decays of the clusters, as well as the fragmentation of the beam remnants at late times. (We will in the following refer to the parton-cascade/cluster-hadronization model as PCM, and to its Monte Carlo implementation as VNI.) The PCM description has been used \[10\] to provide very useful insight into the dynamics of the evolution of the matter at energies likely to be reached at BNL RHIC and CERN LHC. The experimental data for these will, however, come only in the next millennium.

Returning to our above-posted hypothesis of the applicability of the pQCD parton-picture for high-energy nuclear collisions, we must address the question: Can we use the PCM picture already at the SPS energies \(\sqrt{s} \approx 17–20\) GeV/A? Recall that the parameters of the models were fixed by the experimental data for \(pp\) (\(p\bar{p}\)) cross-sections over \(\sqrt{s} = 10–1800\) GeV, and \(e^+e^-\) annihilation \[10\]. The nucleon-nucleon energy reached at SPS is well within this range. Indeed, as we shall show, this approach does remarkably well in comparison to the gross particle production properties observed at CERN SPS. This comes as a surprise to us, since no attempt was made to fine-tune the model to the data. In view of this success, a very good opportunity opens to understand these collisions in their entirety, where the early stages are fairly well understood in terms of the pQCD. This will allow us to draw definite conclusions about the initial energy density and the constitution of the matter produced. This will also give us confidence that projections for the RHIC and LHC energies using the PCM may be reasonable.

In Sect. 2 we briefly describe the PCM, and its Monte Carlo implementation VNI. In Sect. 3, we first discuss the general outcome of a model study for central collisions of lead nuclei as SPS energies. In particular we explore the contribution of various processes to the transverse energies deposited in the collision and examine the time-evolution and magnitude of the energy density in the central collision region. Next we present our results for the rapidity distributions for the transverse energy, multiplicities, and the transverse momentum spectra of hadrons and compare them with the available experimental data. We find a good agreement and discuss the consequences. Finally, Sect. 4 gives a summary.

2 ASPECTS OF THE MODEL

Let us recall the main aspects of the PCM, which are specific to our analysis of heavy-ion collisions afterwards. For the sake of brevity, we will not review the model in detail, but kindly refer to the extensive documentations of Refs. \[10, 11\].

2.1 Theoretical framework

The central element in the space-time cascade description is the use of relativistic transport theory \[10\] in conjunction with renormalization-group improved QCD \[12\], which provides the theoretical basis to follow the QCD evolution in 7-dimensional phase-space \(d^3rd^3kdE\) of a mixed multi-particle system of partons and hadrons with dynamically changing proportions. We remark, that a theoretical basis for such a space-time cascade description of a multiparticle system in high-energy collisions can be derived systematically from quantum-kinetic theory on the basis of QCD’s first principles in a stepwise approximation scheme (see e.g., Refs. \[12\] and references therein). This framework allows to cast the time evolution of the mixed system of individual partons, composite parton-clusters, and physical hadrons in terms of a closed set of integro-differential equations for the phase-space densities of the different particle excitations. The definition of these phase-space densities, denoted by \(F_\alpha\), where \(\alpha \equiv p, c, h\) labels the species of partons, pre-hadronic clusters, or hadrons, respectively, is:

\[
F_\alpha(r, k) \equiv F_\alpha(t, r; E, k) = \frac{dN_\alpha}{d^3rd^3kdE} \quad (\alpha \equiv p, c, h),
\]

where \(r \equiv r^\mu = (t, r)\), \(k \equiv k^\mu = (E, k)\), and \(k^2 = k^\mu k^\mu = E^2 - k^2\) can be off-shell (space-like \(k^2 < m^2\), time-like \(k^2 > m^2\)) or on-shell \((k^2 = m^2)\). The densities \(F_\alpha\) measure the number of particles of type \(\alpha\)
at time $t$ with position in $r + dr$, momentum in $k + dk$, and energy in $E + dE$ (or equivalently invariant mass in $k^2 + dk^2$). The $F_\alpha$ are the quantum analogues of the classical phase-space distributions, including both off-shell and on-shell particles, and hence contain the essential microscopic information required for a statistical description of the time evolution of a many-particle system in complete 7-dimensional phase-space $d^3r d^3k dE$, thereby providing the basis for calculating macroscopic observables.

The phase-space densities are determined by the self-consistent solutions of a set of transport equations (governing the space-time change with $r^\mu$) coupled with renormalization-group-type evolution equations (controlling the change with momentum scale $k^\mu$). These equations can be generically expressed as convolutions of the densities $F_\alpha$ of particle species $\alpha$, interacting with specific cross-sections $\hat{\sigma}^{(l)}$ for the processes $l$. The resulting coupled equations for the space-time development of the densities of partons $F_p$, clusters $F_c$ and hadrons $F_h$ is a self-consistent set in which the change of the three distinct densities is governed by the balance of the various possible interaction processes among the particles. The generic form is

\[
\begin{align*}
  k_\mu \frac{\partial}{\partial r_\mu} F_\alpha(r, k) &= \sum_{\beta, \ldots} \sum_l \left\{ \hat{I}^{(l)}_{\text{gain}}[\hat{\sigma}^{(l)}, F_\alpha, F_\beta, \ldots] - \hat{I}^{(l)}_{\text{loss}}[\hat{\sigma}^{(l)}, F_\alpha, F_\beta, \ldots] \right\} \\
  k^2 \frac{\partial}{\partial k^2} F_\alpha(r, k) &= \sum_{\beta, \ldots} \sum_l \left\{ \hat{J}^{(l)}_{\text{gain}}[\hat{\sigma}^{(l)}, F_\alpha, F_\beta, \ldots] - \hat{J}^{(l)}_{\text{loss}}[\hat{\sigma}^{(l)}, F_\alpha, F_\beta, \ldots] \right\},
\end{align*}
\]

where $\alpha, \beta, \ldots \equiv p, c, h$. Equations (2) and (3) allow a probabilistic interpretation of the multi-particle evolution in space-time and momentum space in terms of sequentially-ordered interaction processes $l$, in which the rate of change of the particle distributions $F_\alpha$ ($\alpha = p, c, h$) in a phase-space element $d^3r d^3k$ is governed by the balance of gain (+) and loss (−) terms. The left-hand side describes free propagation of a quantum of species $\alpha$, whereas on the right-hand side the interaction kernels $\hat{I}$, $\hat{J}$ are integral operators that incorporate the effects of the particles' self- and mutual interactions. Explicit expressions are given in Refs. [12]. Here we merely note that the kernels $\hat{I}$ and $\hat{J}$ embody convolutions of the density of particles $F_\alpha, F_\beta, \ldots$ entering or leaving a particular vertex, and a phase-space integration weighted with the associated probability distribution, i.e. with the relevant cross-section $\hat{\sigma}^{(l)}$, of the squared amplitude.

2.2 Practical application

In practice, the above formalism allows one to trace the microscopic history of the dynamically-evolving particle system in space-time and momentum space, so that the correlations of particles in space, time, color and flavor can be taken into account systematically. The interplay between perturbative and non-perturbative regimes is controlled locally by the space-time evolution of the mixed parton-cluster-hadron system itself (i.e., the time-dependent local particle densities).

As mentioned, these concepts are implemented in the Monte Carlo program VNI which simulates high-energy particle collisions on the basis of the PCM concepts. For nucleus-nucleus collisions, there are three main building-blocks, describing the collision dynamics from the initial beam/target collision system upon collisional contact, through the QCD-evolution of parton distributions, hadron formation, up to the emergence of final hadronic states:

(i) The initial state associated with the incoming nuclei involves their decomposition into nucleons, and, of the nucleons into partons on the basis of the experimentally measured nucleon structure functions and elastic form-factors. This procedure translates the initial nucleus-nucleus system into two colliding clouds of virtual partons according to the well-established parton decomposition of the nuclear wave functions at high energy [14].

(ii) The parton cascade development starts from the initial interpenetrating parton clouds, and involving the space-time development with mutual- and self-interactions of the system of quarks and gluons. Included are multiple elastic and inelastic interaction processes, described as sequences of elementary $2 \to 2$ scatterings, $1 \to 2$ emissions and $2 \to 1$ fusions. Moreover, correlations are accounted for between primary virtual partons, emerging as unscathed remainders from the initial state, and secondary real partons, materialized or produced through the partonic interactions.
The hadronization dynamics of the evolving system in terms of parton-coalescence to color-neutral clusters is described as a local, statistical process that depends on the spatial separation and color of nearest-neighbor partons. Each pre-hadronic parton-cluster fragments through isotropic two-body decay into primary hadrons, according to the density of states, followed by the decay of the latter into final stable hadrons.

2.3 Simulation procedure

The simulation of the time development of the mixed system of partons, clusters, and hadrons in position and momentum space on the basis of eqs. (2)-(3) emerges then from following each individual particle through its history with the various probabilities and time scales of interactions sampled stochastically from the relevant probability distributions in the kernels $\hat{I}$ and $\hat{J}$. The microscopic history of the system can thus be traced by evolving the phase-space distributions of particles are evolved in small time steps ($\Delta t \approx 10^{-3} \text{ fm}$) and 7-dimensional phase-space $d^3r d^3kdE$ throughout the stages of parton cascade, parton-cluster formation, cluster-hadron decays, until stable final-state hadrons and other particles are left as freely-streaming particles. The essential ingredients in this Monte-Carlo procedure are summarized as follows [11]:

(i) The initial state is constructed in three steps. First, the nuclei are decomposed into the nucleons with an appropriate Fermi-distribution. Second, the nucleons are in turn decomposed into their parton substructure according to proton/neutron structure functions with a spatial distribution given by the Fourier transform of the nucleon elastic form-factor. Third, the so-initialized phase-space densities of (off-shell) partons are then boosted with the proper Lorentz factor to the center-of-mass frame of the colliding nuclei.

(ii) The parton cascade development proceeds then by propagating the partons along classical trajectories until they interact, i.e., collide (scattering or fusion process), decay (emission process) or coalesce to pre-hadronic composite states (cluster formation). Both space-like and time-like radiative corrections are included within the Leading-Log approximation. The relevant interaction probabilities, entering in the kernels $\hat{I}$, $\hat{J}$ in (2) and (3), are obtained from the well-known perturbative QCD cross-sections, and the coalescence probabilities of the Ellis-Geiger model [9], respectively. Both the production of partons and the emergence of pre-hadronic clusters through their coalescence are subject to an individually specific formation time $\Delta t_{p,c} = \gamma/M_{p,c}$ where $1/M_{p,c} = 1/\sqrt{k^2}$ is the proper decay time of off-shell partons or clusters with invariant mass $M_p$, respectively $M_c$, and $\gamma = E/M_{p,c}$ is the Lorentz factor.

(iii) The cluster-decay and hadron formation; the decay of the pre-hadronic clusters and the decays of excited hadrons and resonances which are sampled from the particle data tables [15]. Again, each newly produced hadron becomes a ‘real’ particle only after a characteristic formation time $\Delta t_h = \gamma/M_h$ depending on their invariant mass $M_h$ and their energy through $\gamma = E/M_h$. Before that time has passed, a hadron must be considered as a still virtual object that cannot interact incoherently until it has formed according to the uncertainty principle.

(iv) The beam remnants, being the unsathed remainders of the initial nuclei, emerge from reassembling all those remnant primary partons that have been spectators without interactions throughout the evolution. The recollection of those yields two corresponding beam clusters with definite charge, baryon number, energy-momentum and position, as given by the sum of their constituents. These beam clusters decay into final-state hadrons which recede along the beam direction at large rapidities of the beam/target fragmentation regions. Again, individual formation times of the produced hadrons are accounted for.

We note that the spatial density and the momentum distribution of the particles are intimately connected: The momentum distribution continuously changes through the interactions and determines how the quanta propagate in coordinate space. In turn, the probability for subsequent interactions depends on the resulting local particle density. Consequently, the development of the phase-space densities is a complex interplay, which - at a given point of time - contains implicitly the complete preceding history of the system.
3 RESULTS FOR Pb + Pb Collisions at 158 A GeV

We apply now the PCM to collisions involving lead nuclei \( (E_{\text{beam}} = 158 \cdot A \text{ GeV}) \) studied in detail at the CERN SPS by a number of experiments. It is important to stress again, that no attempt was made to fine-tune the model to the data, that is, we used VNI with all its default settings, which are based on \( e^+e^- \) and \( pp (p\bar{p}) \) physics [10].

3.1 ‘Hard’ versus ‘soft’ physics

The creation of a deconfined strongly interacting matter is crucially dependent on the energy deposited in the interaction zone by the colliding nuclei. We have plotted the transverse energy distribution for a central collision in Fig. 1. The individual contributions to the transverse energy from parton cascades and the beam fragmentation are self-evident. We see that at the SPS energies, the ‘beam remnants’, i.e., the primary partons which remained spectators throughout the evolution account for about 50% of the transverse energy, at central rapidities. This partonic fraction is seen to damp out quickly as we move away from the central rapidity and approach the beam/target rapidities. We have additionally estimated the contribution of multiple scatterings among partons by switching off all but the collision between the primary partons. This reduced the \( dE_T/dy \) at \( y = 0 \) by about 10%, implying that up to 20% of the energy deposited in partonic collisions originates from multiple scatterings. This will certainly alter at higher energies.

We emphasize that the pQCD parton cascading is treated strictly within perturbative QCD, whereas the non-perturbative hadronization dynamics is a phenomenological prescription that has been shown to work equally well for \( e^+e^- \), \( pp \) and AA collisions [10]. The rationale is keeping these two elements rigorously separate and not entangling them in a foggy manner. However, there is an additional important aspect which is inherent to any pQCD description of parton evolution, namely, the well-known defect that - for massless quarks and gluons - the interaction integrals \( \hat{I}^{(0)} \) and \( \hat{J}^{(0)} \) in eqs. (2) and (3) are plagued by divergences due to the singular behavior of the embodied perturbative QCD cross-sections \( \sigma^{(0)}(q_\perp^2) \) as the momentum transfer \( q_\perp^2 \rightarrow 0 \).

However, since the partons during the cascade evolution are generally off-shell, they are not massless but carry an invariant mass \( |k^2| \geq \max(\mu^2, m^2) \) where \( \mu \approx 500 \text{ MeV} \) and \( m \) is the flavor-dependent mass (equal to zero only for gluons). This feature makes it possible to employ in VNI a dynamical regularization of the divergent pQCD cross-sections (rather than resorting to the common recipe of cutting out all parton interactions involving momentum transfers \( q_\perp^2 < p_0^2 \) below some fixed \( p_0 \)). Combining the uncertainty principle \( b_\perp q_\perp \lesssim 1 \) where \( b_\perp \) and \( q_\perp \) is the impact parameter and momentum transfer of two colliding partons, respectively, with the resolution requirement \( q_\perp^2 \geq \max(k_{1,2}^2) \), where \( k_{1,2} \) are the partons invariant masses, one obtains a distinct constraint \( p_0 \rightarrow p_0(b_\perp, q_\perp) \geq p_0 \) for each individual parton collision. This procedure hence resolves the divergence problem, except for the special case of gluons that are almost on-shell and undergo a collision with very small \( q_\perp \), which are cut-off if \( q_\perp^2 < p_0^2 \). Specifically, we use \( p_0 = 1.1 \text{ GeV} \) for the present analysis, as is the default value in VNI, which provides agreement with \( pp \) data at \( \sqrt{s} \approx 20 \text{ GeV} \).

Although this dynamical regularization eliminates to large extent the arbitrariness of a fixed cut-off \( p_0 = \text{const} \), it still ignores non-perturbative contributions to parton-parton collisions. Neither the detailed form, nor the significance of these latter interactions, are known, notwithstanding their possible impact on collective behavior under extreme high-temperature/density conditions. Therefore it may be worth while to attempt accounting for non-perturbative soft parton cascading \( (p_0^2 < p_0^2) \) in addition to the truly perturbative hard parton evolution \( (p_0^2 > p_0^2) \), and to study the consequences phenomenologically. In the simulation program VNI, this option [10] is offered, and we have examined the non-perturbative effects by comparing our results with and without inclusion of soft parton collisions. The crucial observation was that switching off the soft-scatterings among the partons resulted in no noticeable difference in the energy deposited in the system or in any of the other observables examined in the present work. This is easily understood, because the ‘soft’ scatterings by definition \( (q_\perp^2 < p_0^2 = 1.1 \text{ GeV}) \) involve momentum transfers which are on the average a couple of hundred MeV only. As a consequence, neither do these ‘soft’ collisions

\[ \alpha_s \equiv \left( \frac{\mu^2}{\mu^2} \right) \]

\[ q_\perp^2 \]

Notice that the applicability of pQCD down to \( p_0 = 1.1 \text{ GeV} \) may be justified by observing that with \( \alpha_s(p_0^2) \approx 0.45 \).

\[ ^3 \text{The default setting in VNI is not to include soft parton collisions, but only the perturbatively calculable hard interactions.} \]
generate significant transverse energy, nor do they cause a notable deflection of the involved partons - contrary to the ‘hard’ collisions.

Thus, as the contribution of soft scatterings is negligible at CERN SPS energy, the parton cascade development can be described completely within pQCD without any further model assumptions, i.e., by considering hard parton collisions only, and avoiding the phenomenological inclusion of soft collisions.

3.2 Time-evolution of energy density and $E_T$-production

The magnitude of transverse energy production and of the energy density achieved in the central region of the nuclear collision are most crucial quantities for the detection of a QGP formation. The transverse energy deposited in the collision can be used to estimate the time-development of the energy density in the central collisions region, using the Bjorken relation \( \epsilon(\tau) \approx \frac{1}{\pi R^2 \tau} \left( \frac{dE_T}{dy} \right) \mid y - y_{cm} \mid < 0.5 \ | z | < 0.5 \text{ fm} \), \( 4 \)

where \( R < R_A \approx 7 \text{ fm} \) for Pb. The simulation with VNI permits us to obtain the dynamic evolution of this density from the first parton collision, through the cascading stage with re-interactions and gluon emission, the parton recombination to pre-hadronic clusters, and the emergence of final hadrons and resonances. The top panel of Fig. 2 shows the evolution of the total transverse energy and also the energy density resident in partons and clusters and hadrons, with the proper time \( \tau \) in the central rapidity region. We see a rapid build-up of the transverse energy carried by the partons, which reaches a peak around 0.7 \( \text{ fm} \), and then starts dropping as more and partons form clusters and hadrons. The bottom panel displays the evolution of the energy density \( \epsilon \) contributed by the partons and the clusters also provides a beautiful insight. The energy density is rather large \( \sim 5 \text{ GeV/fm}^3 \) at very early times, when the primary hard scatterings take place. After about 0.2 \( \text{ fm} \), the increase in the transverse energy deposited due to scatterings and radiations is matched by the increase in the co-moving four-volume \( \pi R^2 \tau \Delta y \), where \( R \) is the transverse size of the volume and the energy density will not change. Once the partonic processes stop depositing transverse energy in the system, the energy density carried by the partons will start decreasing, due to the increase in volume. As more and more partons coalesce into clusters or hadrons, the energy density of the partonic matter decreases and that of the hadronic matter increases. Note the long period of time over which the hadronic matter is created. Note also that the energy density contained by the hadronic matter never exceeds 0.5 \( \text{ GeV/fm}^3 \). This analysis suggests the existence of partonic matter for a period of almost 7 \( \text{ fm} \). Note that in order to get the energy density of the matter (partonic or otherwise) at \( \tau = 1 \text{ fm} \) we have to weight the respective densities with the volume fractions occupied by the partonic and the hadronic matter. Such details will be the subject of a future publication.

Can we can trust these results and conclusions? We believe that if the parton cascade model provides a reasonable description to the data which have already started arriving from the measurements of \( Pb + Pb \) collisions at the CERN SPS, then we can be very confident about the above discussion. Indeed, as we shall address now, we find very decent agreement with data from the CERN SPS.

3.3 Comparison with CERN SPS data

In Fig. 3 we have plotted our results for the transverse energy distributions, averaged over the appropriate impact parameter range, which corresponds to 2% $\sigma_{\text{min, bias}}$. The experimental data are taken from the measurements reported by the NA49 [16] and the WA98 [17] collaborations. A nice description is obtained without any adjustments of parameters.

The rapidity distribution of negatively charged hadrons are considered to provide a sensitive test of the particle production mechanism in such collisions, and we give our results in Fig. 4. They are seen to be in good agreement with the preliminary data.

An enhanced production of strange particles has long been considered an important manifestation of the quark-hadron phase transition. In the PCM strange quarks are produced from flavor creation as well as flavor excitation processes [19]. As the final-state interactions among the produced hadrons is not yet implemented (see however Ref. [21]), there is no additional production of strangeness, after the hadronization. In view
of this, the results shown in Fig. 5 are most interesting, as we again find a quantitative agreement with the data obtained on the rapidity distribution of strange mesons.

The transverse momentum distribution of hadrons are considered to provide information about the temperature at the time of freeze-out and the flow velocity, if any, of the matter produced in such collisions. In view of the absence of final state interaction among the hadrons in the PCM used here, we find reasonable description of the transverse mass distribution of negative hadrons (Fig. 6) and of neutral pions (Fig. 7). The results in both the figures are normalized at 0.9 GeV. In a recent paper it has been shown that this agreement is improved considerably after a final state hadron interaction is introduced [20]. These results confirm that the hadrons acquire a large part of their transverse momenta at the time of hadronization itself. The deviation of our results from the final data (when available) will help to provide a quantitative estimate of the importance of the final state interaction among hadrons.

4 SUMMARY

We have analysed Pb + Pb collisions at the CERN SPS with beam energy per nucleon 158 GeV (\(\sqrt{s}/A = 17 \text{ GeV}\)) by performing Monte Carlo simulations within the framework of the parton-cascade/cluster-hadronization model PCM that involves the dynamical interplay between parton production, evolution, parton-cluster formation, and hadron production through cluster decay. We see this study as a step to describe high-energy nuclear collisions on the microscopical level of the space-time history of parton and hadron degrees of freedom based on QCD and supplemented by phenomenology. Our conclusions from this study are as follows:

(i) The simulations of Pb + Pb collisions give a very good overall description of the CERN SPS data on transverse energy distribution, the multiplicity distribution, including those for \(K\) mesons, and a good description of the transverse momentum distribution of hadrons. This success may be taken as a ‘post-hum’ justification of the applicability of a pQCD parton description even at CERN SPS energies - especially in view of the fact that neither additional model assumptions, nor tuning of parameters were involved.

(ii) Contrary to wide-spread belief, at CERN SPS energy the ‘hard’ (perturbative) parton production and parton cascading is an important element for particle production at central rapidities - at least in Pb + Pb collisions. In effect it provides almost 50 % of final particles around mid-rapidity. ‘Soft’ (non-perturbative) parton interactions on the other hand are found to be insignificant in their effect on final-state particle distributions.

(iii) The maximum achieved energy density in the partonic phase can reach \(\epsilon \gtrsim 5 \text{ GeV/fm}^3\) in the central region, thereby providing favorable conditions for a partonic plasma formation. It drops below a ‘critical’ density \(\epsilon_c \approx 2 \text{ GeV/fm}^3\) only after about 2 fm. On the other hand, the energy density of produced hadrons is always substantially lower than 1 GeV/fm³, making a hadronic plasma formation unlikely.

Especially the latter two points would have important consequences: If both these effects indeed turn out so drastic at RHIC as compared to CERN SPS, then this would imply very favorable conditions for the formation of a baryon-free quark-gluon plasma in the central rapidity region, and possibly even a baryon-rich plasma in the nucleon-dense fragmentation regions.

We see these encouraging results as a motivation to extend and deepen this study by looking for instance at nuclear collisions with other beams and different energy, or at proton-nucleus collisions. We intend to pursue this project in the near future. In perspective of heavy-ion collisions at the BNL RHIC or the CERN LHC, the advocated PCM description should provide a smooth extrapolation to these future nuclear collisions beyond the CERN SPS, since the relative importance of partonic and hadronic degrees are regulated by the multi-particle dynamics itself.
ACKNOWLEDGEMENTS

We acknowledge useful comments by Miklos Gyulassy, Ron Longrace and Bikash Sinha. This work was supported in part by the D.O.E under contract no. DE-AC02-76H00016.
References

[1] See e.g., Proceedings of Quark Matter ’96, Nucl. Phys. A 610 (1996).

[2] B. Andersson, G. Gustafson and B. Nilsson-Almquist, Nucl. Phys. B281, 289 (1987); B. Nilsson-Almquist and E. Stenlund, Computer Phys. Comm. 43 (1987), 387; B. Lörstad, Int. J. Mod. Phys. A12, 2861 (1989).

[3] K. Werner, Z. Phys. C42, 85 (1989); Phys. Rep. 232, 87 (1993).

[4] H. Sorge, H. Stöcker, and W. Greiner, Nucl. Phys. A498, 567c (1989); Ann. Phys. 192, 266 (1989).

[5] A. Capella, U. Sukhatme, C.-I. Tan, and J. Tran Tran Van, Phys. Rep. 236, 225 (1994)

[6] P. Braun-Munzinger, J. Stachel, J. P. Wessels, and N. Xu, Phys. Lett. B 344, 43 (1995).

[7] H. Satz, Summary talk given at the 3rd International Conference on Physics and Astrophysics of Quark Gluon Plasma, Jaipur, India, March 1997, and to be published.

[8] K. Geiger and B. Müller, Nucl. Phys. B369 (1992) 600; K. Geiger, Phys. Rev. D47 (1993) 133.

[9] J. Ellis and K. Geiger, Phys. Rev. D 54, 949 (1996); J. Ellis and K. Geiger, Phys. Rev. D 54, 1755 (1996).

[10] K. Geiger, Phys. Rep. 258,376 (1995).

[11] K. Geiger, hep-ph/9701226; to appear in Comp. Phys. Com.

[12] K. Geiger, Phys. Rev. D 54, 949 (1996); preprint BNL-63232, hep-ph/9611400.

[13] J. Ellis and K. Geiger, Phys. Rev. D54, 949 (1996).

[14] L. V. Gribov, E. M. Levin, and M. G. Ryskin, Phys. Rep. 100, 1 (1983); E. M. Levin, and M. G. Ryskin, Phys. Rep. 189, 267 (1990).

[15] See e.g.: Particle Data Group, Review of Particle Properties, Phys. Rev. D 50, 1173 (1994).

[16] T. Alber et al. NA49 Collaboration, Phys. Rev. Lett. 75, 3814 (1995).

[17] M. Aggarwal et al., WA98 Collaboration, Nucl. Phys. A 610, 200c (1996).

[18] S. V. Afanasiev et al., NA49 Collaboration, Nucl. Phys. A 610, 188c (1996).

[19] K. Geiger, Phys.. Rev. D 48, 4129 (1993).

[20] K. Geiger and R. Longrace, nucl-th/9705041.

[21] J. D. Bjorken, Phys. Rev. D 27, 140 (1983).
Figure 1: Transverse energy distribution in central collision of lead nuclei at CERN SPS. The dashed curve gives the contribution of only the remnant partons, which have not undergone scattering, while the solid curve shows the sum of contributions from parton cascade and the fragmentation of beam remnants.
**Pb+Pb at $E_{cm} = 17$ A GeV**

Time evolution of $dE_{T}/dy$ and $\varepsilon$

Figure 2: The evolution of the energy density of the partonic matter in the central slice of the colliding lead nuclei at SPS energies.
Figure 3: Transverse energy distribution in central collision of lead nuclei at CERN SPS.
Figure 4: The rapidity density distribution of negatively charged hadrons ($\pi^-$, $K^-$, and $\bar{p}$) in central collisions of lead nuclei at CERN SPS.
Figure 5: The rapidity density distribution of K mesons in central collisions of lead nuclei at CERN SPS.
Figure 6: The transverse mass spectra of negative hadrons ($\pi^-$, $K^-$, and $p$) successively scaled down by an order of magnitude. The predictions are normalized to the data at $m_T - m_\pi = 0.9$ GeV.
Figure 7: The transverse momentum distribution of neutral pions from central collisions of lead nuclei at the CERN SPS. The predictions are normalized to the data at 0.9 GeV.