Finite Unification: phenomenology

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Abstract. We study the phenomenological implications of Finite Unified Theories (FUTs). In particular we look at the predictions for the lightest Higgs mass and the s-spectra of two all-loop finite models with $SU(5)$ as gauge group. We also consider a two-loop finite model with gauge group $SU(3)^3$, which is finite if and only if there are exactly three generations. In this latter model we concentrate here only on the predictions for the third generation of quark masses.

1. Introduction

The traditional way to reduce the independent parameters of a theory is the introduction of a symmetry. Supersymmetric Grand Unified Theories (GUTs) are representative examples of such attempts. Finite Unified Theories (FUTs) are $N = 1$ supersymmetric Grand Unified Theories (GUTs) which can be made finite to all-loop orders, including the soft supersymmetry breaking sector. The constructed finite unified $N = 1$ supersymmetric SU(5) GUTs predicted correctly from the dimensionless sector (Gauge-Yukawa unification), among others, the top quark mass \cite{1,2}. Eventually, the full theories can be made all-loop finite and their predictive power is extended to the Higgs sector and the s-spectrum \cite{3}. For a detailed discussion see \cite{4–6}.

Consider a chiral, anomaly free, $N = 1$ globally supersymmetric gauge theory based on a group G with gauge coupling constant $g$. The superpotential of the theory is given by

$$W = \frac{1}{2} m^{ij} \Phi_i \Phi_j + \frac{1}{6} \frac{C^{ijk}}{C_2(G)} \Phi_i \Phi_j \Phi_k ,$$

(1)

where $m^{ij}$ (the mass terms) and $C^{ijk}$ (the Yukawa couplings) are gauge invariant tensors, and the matter field $\Phi_i$ transforms according to the irreducible representation $R_i$ of the gauge group $G$. All the one-loop $\beta$-functions of the theory vanish if the $\beta$-function of the gauge coupling $\beta_2^{(1)}$, and the anomalous dimensions of the Yukawa couplings $\gamma_i^{(1)}$, vanish, i.e.

$$\sum_i \ell(R_i) = 3C_2(G) , \quad \frac{1}{2} C_{ipq} C^{ipq} = 2\delta_i^j g^2 C_2(R_i) ,$$

(2)
where $\ell(R_i)$ is the Dynkin index of $R_i$, and $C_2(G)$ is the quadratic Casimir invariant of the adjoint representation of $G$. A theorem given in [7–10] then guarantees the vanishing of the $\beta$-functions to all-orders in perturbation theory. This requires that, in addition to the one-loop finiteness conditions (2), the Yukawa couplings are reduced in favour of the gauge coupling.

In the soft breaking sector, the one- and two-loop finiteness for the trilinear terms $h^{ijk}$ can be achieved by [11]

$$h^{ijk} = -MC^{ijk} + \ldots = -M\rho^{ijk}(0) g + O(g^5).$$

(3)

It was also found that the soft supersymmetry breaking (SSB) scalar masses in Gauge-Yukawa and finite unified models satisfy a sum rule [12, 13]

$$\left( \frac{m_i^2 + m_j^2 + m_k^2}{MM^\dagger} \right) = 1 + \frac{g^2}{16\pi^2} \Delta^{(2)} + O(g^4)$$

(4)

for $i, j, k$, where $\Delta^{(2)}$ is the two-loop correction, which vanishes when all the soft scalar masses are the same at the unification point.

2. $SU(5)$ Finite Unified Theories

We will examine here all-loop Finite Unified theories with $SU(5)$ gauge group, where the reduction of couplings has been applied to the third generation of quarks and leptons. An extension to three families, and the generation of quark mixing angles and masses in Finite Unified Theories has been addressed in [14], where several examples are given. These extensions are not considered here. The particle content of the models we will study consists of the following supermultiplets: three ($\overline{5} + 10$), needed for each of the three generations of quarks and leptons, four ($\overline{5} + \overline{5}$) and one $24$ considered as Higgs supermultiplets. When the gauge group of the finite GUT is broken the theory is no longer finite, and we will assume that we are left with the MSSM.

A predictive Gauge-Yukawa unified $SU(5)$ model which is finite to all orders, should have the following properties:

(i) One-loop anomalous dimensions are diagonal, i.e., $\gamma^{(1)}_{ij} \propto \delta_{ij}$.

(ii) Three fermion generations, in the irreducible representations $\overline{5}_i, 10_i$ ($i = 1, 2, 3$), which obviously should not couple to the adjoint $24$.

(iii) The two Higgs doublets of the MSSM should mostly be made out of a pair of Higgs quintet and anti-quintet, which couple to the third generation.

In the following we discuss two versions of the all-order finite model. The model of ref. [1, 2], which will be labeled A, and a slight variation of this model (labeled B), which can also be obtained from the class of the models suggested in ref. [15] with a modification to suppress non-diagonal anomalous dimensions.

The superpotential which describes the two models takes the form [1, 2, 16]

$$W = \sum_{i=1}^{3} \left[ \frac{1}{2} g_{i}^{u} 10, 10_i H_i + g_{i}^{d} 10, \overline{5}_i \overline{\mathcal{P}}_i \right]$$

$$+ g_{23}^{u} 10_2 10_3 H_4 + g_{23}^{d} 10_2 \overline{5}_3 \overline{\mathcal{P}}_4 + g_{32}^{d} 10_3 \overline{5}_2 \overline{\mathcal{P}}_4$$

$$+ \sum_{a=1}^{4} g_{a}^{d} H_a 24 \overline{\mathcal{P}}_a + \frac{g_{a}^{n}}{3} (24)^3,$$

(5)

where $H_a$ and $\overline{\mathcal{P}}_a$ ($a = 1, \ldots, 4$) stand for the Higgs quintets and anti-quintets.
The main difference between model A and model B is that two pairs of Higgs quintets and anti-quintets couple to the 24 in B, so that it is not necessary to mix them with $H_4$ and $\overline{H}_4$ in order to achieve the triplet-doublet splitting after the symmetry breaking of $SU(5)$ [16]. Thus, although the particle content is the same, the solutions to the finiteness equations and the sum rules are different, which will reflect in the phenomenology.

The non-degenerate and isolated solutions to $\gamma_i^{(1)} = 0$ for model FUTA, which are the boundary conditions for the Yukawa couplings at the GUT scale, are:

$$
(g_1^2) = \frac{8}{5} g^2 , \quad (g_2^d_i)^2 = \frac{6}{5} g^2 , \quad (g_3^y_i)^2 = \frac{8}{5} g^2,
$$

$$
(g_2^d_i)^2 = (g_3^y_i)^2 = \frac{6}{5} g^2 , \quad (g_2^{y_i} e) = 0 , \quad (g_2^{d_i} d) = 0 , \quad (g_3^{d_i} u) = 0 .
$$

In the dimensionful sector, the sum rule gives us the following boundary conditions at the GUT scale for this model [16–18]:

$$
m_{H_u}^2 + 2 m_{10}^2 = m_{H_d}^2 + m_{\overline{H}_u}^2 + m_{\overline{H}_d}^2 = M^2 ,
$$

and thus we are left with only three free parameters, namely $m_{\overline{H}_u} = m_{\overline{H}_d}$, $m_{10} = m_{10_3}$, and $M$.

For the model FUTB the non-degenerate and isolated solutions to $\gamma_i^{(1)} = 0$ give us:

$$
(g_1^2) = \frac{8}{5} g^2 , \quad (g_2^d_i)^2 = \frac{6}{5} g^2 , \quad (g_3^y_i)^2 = \frac{8}{5} g^2,
$$

$$
(g_2^d_i)^2 = (g_3^y_i)^2 = \frac{3}{5} g^2 , \quad (g_2^{y_i} e) = \frac{4}{5} g^2 , \quad (g_2^{d_i} d) = (g_2^{d_i} u) = \frac{3}{5} g^2 ,
$$

$$
(g_3^{d_i} u) = \frac{15}{7} g^2 , \quad (g_4^l)^2 = (g_4^f)^2 = \frac{1}{2} g^2 , \quad (g_4^l)^2 = 0 , \quad (g_4^f)^2 = 0 ,
$$

and from the sum rule we obtain:

$$
m_{H_u}^2 + 2 m_{10}^2 = M^2 , \quad m_{H_d}^2 - 2 m_{10}^2 = -\frac{M^2}{3} , \quad m_{\overline{H}_u}^2 + 3 m_{10}^2 = \frac{4 M^2}{3} ,
$$

i.e., in this case we have only two free parameters $m_{10} = m_{10_3}$ and $M$.

3. Predictions of the $SU(5)$ models

We confront now the predictions of the four models with the experimental data, starting with the heavy quark masses (see refs. [3,6] for more details). Since the gauge symmetry is spontaneously broken below $M_{GUT}$, the finiteness conditions do not restrict the renormalization properties at low energies, and all it remains are boundary conditions on the gauge and Yukawa couplings (6), the $h = -MC$ relation eq. (3), and the soft scalar-mass sum rule (4) at $M_{GUT}$, as applied in each of the models. Thus we examine the evolution of these parameters according to their RGEs up to two-loops for dimensionless parameters and at one-loop for dimensionful ones with the relevant boundary conditions. Below $M_{GUT}$ their evolution is assumed to be governed by the MSSM. We further assume a unique supersymmetry breaking scale $M_s$ and therefore below that scale the effective theory is just the SM.

As a first step, we compare the predictions of the two models, FUTA and FUTB (both with $\mu > 0$ and $\mu < 0$), with the experimental values of the top and bottom quark masses. We use for the top quark the value for the pole mass [19]

$$
M_{t}^{exp} = (173.1 \pm 1.3)
$$

(10)
where we notice that the theoretical values for $M_t$ may suffer from a correction of $\sim 4\%$ \cite{4}. For the bottom quark mass we use the value at $M_Z$ \cite{20}

$$m_b(M_Z) = 2.83 \pm 0.10 GeV$$  \hspace{1cm} (11)

to avoid errors that come from the further running from the $M_Z$ to the $m_b$ mass, and where we have taken the $\Delta b$ effects into account.

From fig. 1 it is clear that the model FUTB with $\mu < 0$ is the only one where both top and bottom quark masses lie within experimental limits. In this case the value of $\tan \beta$ is found to be $\sim 48$. Thus, we will concentrate now on the results for FUTB, $\mu < 0$. In the case where all the soft scalar masses are universal at the unification scale, there is no region of $M$ below $\mathcal{O}$ (few TeV) in which $m_\tau > m_{\chi^0}$ is satisfied (where $m_\tau$ is the lightest $\tilde{\tau}$ mass, and $m_{\chi^0}$ the lightest neutralino mass). This problem can be solved naturally, thanks to the sum rule (9).

Furthermore, we impose the conditions of successful radiative electroweak symmetry breaking, $m_\tau^2 > 0$ and $m_\tau > m_{\chi^0}$, plus the constraints coming from B physics, namely the experimental bounds on BR($b \rightarrow s \gamma$) and BR($B_s \rightarrow \mu^+ \mu^-)$ (which we have evaluated with Micromegas \cite{21}). This way, we find a prediction for the lightest Higgs mass and the s-spectra. From the analysis we find that the lightest observable particle (LOSP) is either the stau or the second lightest neutralino, with mass starting around $\sim 500$ GeV. The prediction for the lightest Higgs mass is

$$M_h \sim 121 - 126 GeV$$  \hspace{1cm} (12)

as shown in fig.2, where the red points satisfy the loose CDM constraint $\Omega h^2 < 0.3$. To this value one has to add $\pm 3$ GeV coming from unknown higher order corrections \cite{22}. See \cite{3, 6} for a more detailed analysis.

4. Finite $SU(3)^3$ model

We now examine the possibility of constructing realistic FUTs based on product gauge groups. Consider an $N = 1$ supersymmetric theory, with gauge group $SU(N)_1 \times SU(N)_2 \times \cdots \times SU(N)_k$, with $n_f$ copies of the supersymmetric multiplet $(N, N^*, 1, \ldots, 1) + (1, N, N^*, \ldots, 1) + \cdots + (N^*, 1, 1, \ldots, N)$. The one-loop $\beta$-function coefficient in the renormalization-group equation of each SU($N$) gauge coupling is simply given by

$$b = \left(-\frac{11}{3} + \frac{2}{3} \right) N + n_f \left(\frac{2}{3} + \frac{1}{3} \right) \left(\frac{1}{2} \right) 2N = -3N + n_f N.$$  \hspace{1cm} (13)

This means that $n_f = 3$ is the only solution of eq. (13) that yields $b = 0$. Since $b = 0$ is a necessary condition for a finite field theory, the existence of three families of quarks and leptons is natural in such models, provided the matter content is exactly as given above.
Figure 2. The lightest Higgs mass $M_h$ as function of the unified gaugino mass $M$. The red (dark) points satisfy the loose CDM constraint.

The model of this type with best phenomenology is the $SU(3)^3$ model discussed in ref. [23], where the details of the model are given. It corresponds to the well-known example of $SU(3)_C \times SU(3)_L \times SU(3)_R$ [24–27]. Thus, this $SU(3)^3$ model is finite between the Planck $M_P$ and the unification $M_{GUT}$ scales, then breaks spontaneously down to the MSSM at $M_{GUT}$ [26]. Notice that this model has extra exotic particles above $M_{GUT}$, which are Higgs-like and down-quark like.

With three families, the most general superpotential contains 11 $f$ couplings, and 10 $f'$ couplings, subject to 9 conditions, due to the vanishing of the anomalous dimensions of each superfield. The conditions are the following

$$\sum_{j,k} f_{ijk}(f_{ijk})^* + \frac{2}{3} \sum_{j,k} f'_{ijk}(f'_{ijk})^* = \frac{16}{9} g^2 \delta_{il},$$

(14)

The assumptions below the GUT scale are the same as for the $SU(5)$ models, where now we have the boundary conditions given by eq. (14), the $h = -MC$ relation, and the soft scalar-mass sum rule (4) at $M_{GUT}$.

We will concentrate here on a two-loop finite solution, in which we keep $f'$ non-vanishing and we use it to introduce the lepton masses. This model is finite only up to two-loops since the corresponding solution of eq. (14) is not isolated. In this case we have the following boundary conditions for the Yukawa couplings

$$f^2 = r \left( \frac{16}{9} \right) g^2, \quad f'^2 = (1 - r) \left( \frac{8}{3} \right) g^2.$$  

(15)

As for the boundary conditions for the soft scalars, we have the universal case.

As with the case of the $SU(5)$ models, we first look at the predictions for the top and quark masses, noticing that we have here an extra parameter, namely $r$. We found that there is a small region where both top and bottom quark masses are in the experimental range for the same value of $r$. But if we take some of the exotic particles into account, decoupling below the unification scale, the situation improves [28]. This can be seen in Fig. 1, where we took only one down-like exotic particle decoupling at $10^{14}$, below that the usual MSSM. In this case, for $r \sim 0.5 \sim 0.62$ we have reasonable agreement with experimental data for both top and bottom quark masses, where the red points in the figure are the ones that satisfy the B physics constraints [20].

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Figure 3. The figure shows the values for the top and bottom quark masses for the FUT model $SU(3)^3$, with $\mu < 0$, where the darker horizontal line is the experimental central value, and the lighter ones are the one and two sigma limits. The red points are the ones that satisfy the B physics constraints.

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