Penetration of hot electrons through a cold disordered wire.

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We study a penetration of an electron with high energy \( E \gg T \) through strongly disordered wire of length \( L \gg a \) (\( a \) being the localization length). Such an electron can loose, but not gain the energy, when hopping from one localized state to another. We have found a distribution function for the transmission coefficient \( T \). The typical \( T \) remains exponentially small in \( L/a \), but with the decrement, reduced compared to the case of direct elastic tunnelling: \( \ln T \approx 0.237 \cdot 2L/a \). The distribution function has a relatively strong tail in the domain of anomalously high \( T \); the average \( T \propto (a/L)^2 \) is controlled by rare configurations of disorder, corresponding to this tail.

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The electronic transport in disordered one-dimensional systems was extensively studied in the past 50 years \cite{1,2}. In the non-interacting system all the states are localized \( \xi \), so that the transmission coefficient \( T \) of a finite system exponentially decays with \( L \). \( T \) varies from sample to sample, since it depends on the configuration of disorder. For strongly disordered chains the distribution of \( s \equiv -(1/\alpha) \ln T \) is a narrow gaussian:

\[
F_0(s) \propto \exp\left\{-(s-1)^2/(\Delta s)^2\right\}, \quad \Delta s = B\alpha^{-1/2}, \quad (1)
\]

being the principal large parameter of the theory (the measure of the localization strength), \( B \sim 1 \) being the model-dependent factor \cite{4}. Thus, the conductivity of a noninteracting system is zero at \( L \to \infty \).

The interactions (e.g., with phonons) lead to a finite equilibrium conductivity \( \sigma \) of the hopping type (see \cite{2}). At some \( L \approx L_0(T) \) the exponential \( L \)-dependence of the conductance \( G \) is changed to \( G = \sigma/L \), with \( \sigma \), exponentially dependent on the temperature \( T \) of the system. The specifics of strongly disordered 1d systems was properly taken into account in \cite{6,7}; it was shown that the conductivity in the variable range hopping regime is controlled by rare fluctuations of the density of states at the fermi level – the “breaks”; as a result

\[
\sigma \approx \exp\{-T_0/2T\}, \quad T_0 = 1/ga, \quad (3)
\]

where \( g \) is the average density of states. The result \cite{5} does not obey the Mott law \( \sigma \propto \exp\{-c(T_0/T)^{1/(d+1)}\} \), valid in dimensions \( d \geq 2 \) \cite{2,9}, where electrons can easily circumvent the breaks. The transport in the strong field \( \xi \) was studied in \cite{8,10,11}. Here the current \( I \propto \exp\{-8T_0/\xi E a\} \) strongly depends on \( \xi \), not on \( T \). It is insensitive to the breaks, but, on the other hand, the distribution of electrons is far from equilibrium.

In the present paper we study a different situation, where the current through the system arises due to a small group of strongly nonequilibrium high-energy particles, so that the occupation numbers of most electronic states remain essentially in equilibrium.

A disordered wire of length \( L \) is in equilibrium with two metallic leads (reservoirs) \( R \) at temperature \( T \) and chemical potential \( \epsilon_F \) (see Fig.1). In the left reservoir, however, a small amount of nonequilibrium particles with energies \( E \gg T \) \((E \) is measured with respect to \( \epsilon_F \)) is injected, so that the current \( I_{L} \) reaches the left end of the wire.

![FIG. 1: The setup. A disordered wire of length \( L \) is in a good contact with two reservoirs, the entire system is in equilibrium.](image)

Hot electrons, injected into the wire, weakly interact with the thermal bath, their energy is not conserved. However, as long as \( E \gg T \), only the processes in which the energy is transferred from electron to the bath, not vice versa, are allowed. This is only true for not very long wires \( L < L_{Mott}(T) \), where \( L_{Mott}(T) \sim a(T_0/T)^{1/2} \) is the length of the typical Mott hop at given \( T \). Under this condition the equilibration does not have chance to occur before the electrons escape from the wire. In this letter we also do not take into account correlation effects (like Coulomb gap) due to electron-electron interactions.

Obviously, only the localized states with the energies \( \varepsilon_i \) in the interval between the Fermi energy and the initial energy \( E \) of the injected electron are relevant for our problem. We enumerate them according to their energies: \( 0 < \varepsilon_1 < \varepsilon_2 < \ldots < \varepsilon_n < E \). These “quasiresonant” levels play an important role in the transport physics, as the electron, travelling through the chain with initial energy \( E \), can make intermediate stops only at these sites.
Indeed, all the sites with \( \epsilon_i < 0 \) are occupied, while the sites with \( \epsilon_i > E \) cannot be reached, as no energy can be absorbed. The spatial positions of quasiresonances are \( \ell_i = Lx_i \), the independent random variables \( x_i \) are homogeneously distributed in an interval \( 0 < x_i < 1 \). The number \( n \) of quasiresonances is itself a random variable, described by the poissonian distribution \( p(n, N) = \frac{N^n e^{-N}}{n!} \), where \( N = LEg \) is the average number of quasiresonances. Thus, each wire is characterized by “the configuration” \( C = \{ n, \{ x_1, \ldots, x_n \} \} \) and the average of any \( C \)-dependent quantity \( A(C) \) over the ensemble of wires is \( \overline{A} = \sum_{n=0}^{\infty} p(n, N) \prod_{i=1}^{n} \int_0^1 dx_i A(C) \). In particular, the distribution function \( F_N(s) = \delta(s - s(C)) \). In this letter we focus on the most interesting case \( N \gg 1 \), when the poissonian distribution is sharp and one can simply average over \( x_i \) at fixed \( n \approx N \).

How can an electron get from left reservoir to the right one? Besides the obvious possibility of the direct elastic tunnelling (Fig. 2a), there are also numerous “inelastic staircases” (Fig. 2b,c), in which an electron makes intermediate stops at certain quasiresonant states, while the excess energy at each hop is transferred to the thermostat. Each staircase \( S \) is characterized by the choice of a subset of \( K (0 \leq K \leq n) \) quasiresonances, with \( \epsilon_{k_1} < \epsilon_{k_2} < \cdots < \epsilon_{k_K} \) and \( x_{k_1} > x_{k_2} > \cdots > x_{k_K} \).

Each staircase contributes to the transmission:

\[
\mathcal{T}(C) = \sum_S \mathcal{T}(S|C) \propto e^{-\alpha s(C)}, \quad \mathcal{T}(S|C) \propto e^{-\alpha s(S|C)},
\]

where the summation runs over all the staircases, possible for given configuration \( C \). Under the condition (2) the sum in (3) is dominated by only one – the optimal – staircase \( S_{\text{opt}}(C) \) that corresponds to minimal \( s(S|C) \), so that \( s(C) \approx s(S_{\text{opt}}(C)) = \min_S s(S|C) \). In a typical situation the longest hop in the optimal staircase is the last one, then goes the last but one, etc. Therefore the value of \( s(S|C) \) is controlled by few last hops in \( S \), while the multitude of short hops in the upper part of the staircase are of only secondary importance. The most natural assumption about the structure of the optimal staircase would be the scaling hypothesis: the distribution function \( P_k(\ell_k) \) for random variables \( \ell_k = x_{ik}/x_{ik-1} \) does not depend on \( k \). Such a simple self-similar structure was, however, not observed in our numerical experiments: \( P_k \) manifestly depended on \( k \).

The explicit expression for \( s(S|C) \) can be found with the help of the stationary master equation for the quasiresonant levels populations \( f_i \):

\[
P_{L \rightarrow i} I_{\text{in}} + \sum_{j>i} f_j P_{j \rightarrow i} - f_i P_{i \rightarrow \text{out}} = 0, \quad (5)
\]

The first term in (5) is the incoming flux of particles from the left reservoir; the second term describes the particles, coming to the level \( i \) from all other levels with higher energies (hence the condition \( j > i \)); finally, the third term takes into account all possible escapes from the \( i \)-th level: \( P_{i \rightarrow \text{out}} = P_{i \rightarrow L} + P_{i \rightarrow R} + \sum_{j<i} P_{i \rightarrow j} \). The system’s transmittance is

\[
T \equiv I_R/I_{\text{in}} = P_{L \rightarrow R} + \sum_i P_{i \rightarrow R} (f_i/I_{\text{in}}), \quad (6)
\]

The rate of transitions between \( j \rightarrow i \) is \( p_{j \rightarrow i} = \frac{P_{i \rightarrow j}}{\Delta s(j \rightarrow i)} e^{-\alpha |x_i - x_j|} \). Matrix elements of the electron-thermostat interaction, entering \( P_{j \rightarrow i} \), are smooth power-law functions of the energy transfer \( \epsilon_j - \epsilon_i \). It means, that if we are interested only in the exponential dependencies, we do not have to take these matrix elements into account. Thus, in the exponential approximation we can write

\[
P_{i \rightarrow j} \propto \theta(i - j) e^{-\alpha |x_i - x_j|}, \quad P_{i \rightarrow \text{out}} \propto e^{-\alpha \chi_i}, \quad (7)
\]

\[
\chi_i(C) \equiv \min \left\{ x_i, 1 - x_i, \min_{j<i} \{|x_i - x_j|\} \right\}, \quad (8)
\]

being the distance from the \( i \)-th quasiresonance to its “natural descendant” – a closest neighbor with lower energy, or to one of the two reservoirs. The solution of the system of equations (5) can be written in a recurrent form:

\[
f_i = (P_{L \rightarrow i}/P_{i \rightarrow \text{out}}) I_{\text{in}} + \sum_{j>i} f_j (P_{j \rightarrow i}/P_{i \rightarrow \text{out}}), \quad (9)
\]

which allows for finding \( f_i \) provided all \( f_j \) with \( j > i \) are already found. According to the exponential approximation, justified by the large parameter \( T \), any sum, occurring in (9) or in (6), is dominated by a single term with the smallest negative exponent. As a result, the normalized probability for the electron to make a hop \( i \rightarrow j \) is \( p(i \rightarrow j) = P_{i \rightarrow j}/P_{i \rightarrow \text{out}} = \exp(-\alpha \Delta s(i \rightarrow j)) \), where \( \Delta s(i \rightarrow j) = |x_i - x_j| - \chi_i \). Then, having in mind the following figure:

FIG. 2: (color online) Possible scenarios for electron passing from the left to the right reservoir. “Natural hops” (occurring with the probability close to unity) are shown by thick green arrows, the non-natural hops (that happen with exponentially small probability) are shown by thin red arrows. (a): Direct elastic tunnelling; (b,c): Inelastic staircases with intermediate stops at quasiresonances.
follows. Clearly, to minimize corresponding hops we will call "natural hops" in what a staircase, where all the hops only for some rare "fortunate configurations".

ever, that such a "natural staircase" is possible to find many of them, as possible) are natural. We will see, how-
ination of all staircases, possible for a given configuration, is close to unity. Then, if this upper level is close enough to resonance is the natural descendant of the $i$-th quasiresonance.

than that $s = m \ln 2 - \ln (1/s)$ can be obtained with the help of the Fourier transformation:

$$P_m(\phi) = \int \frac{dq}{2\pi} e^{-iq\phi} \left( \frac{2^{iq+1} - 1}{iq + 1} \right)^m,$$

where $b \sim 1$ is a universal constant. Although this asymptotics describes only a small fraction of the configurations, it turns out to be sufficient for finding the average transmission coefficient:

$$\mathcal{T} \approx \int_0^1 b s \exp\{ -\alpha s \} ds = b \alpha^{-2} \propto L^{-2}. \quad (11)$$

The hopping motion of hot particles, accompanied by the emission of energy, earlier was studied in the context of recombination of photo-excited electron-hole pairs [12]. For that end it was sufficient to take into account only
the natural hops since a particle was assumed to be created in the bulk, far from the ends of the sample, so that it could not escape to a nearby lead (see Fig.4). Under these conditions the evolution of particle density profile, averaged over the ensemble of samples could be described by a peculiar diffusion, in which the distribution of hops lengths is rescaled by a fixed parameter \( q > 1 \) after each hop. If one would extend the same approach to the setup where the particle is initially placed near one of the absorbing leads, then one easily finds for the probability to reach the opposite lead (and thus the average transmission coefficient) \( \overline{T} \propto L^{-\beta} \), where the exponent \( \beta \) (as well as the parameter \( q \)) is universal. Note that this result is in agreement with (11). However, as we have already mentioned, \( \overline{T} \) is controlled by very rare anomalous samples with high transparency, while in a typical sample the natural path would lead to the nearby left lead (see Fig.4). Therefore, to find the probability to reach the right lead in a typical sample one should take into account the non-natural hops, that were ignored in (13). The “diffusional approach” (13), being an adequate instrument for finding \( \overline{T} \), is useless for the determination of the distribution of \( T \).

The distribution \( F_N(s) \) for general \( s \) and \( N \) can only be found numerically. We generated an ensemble of \( \sim 10^6 \) random configurations \( C \) and calculated corresponding \( s(C) \) with the help of the recurrent formula (13). The results of our Monte-Carlo simulations are summarized in Fig.5 The distribution functions \( F_N(s) \) are wide: the dispersion of \( s \) is of order of \( \overline{s} \) for all \( N \). For \( N \gtrsim 300 \) practically \( F_N(s) \approx F_\infty(s) \), and \( F_\infty(s) \) indeed shows a linear low-s asymptotics with \( b \approx 29 \).

The average value \( \overline{s}(N) \) monotonically decreases with \( N \), tending to a finite limit \( \overline{s}(\infty) \approx 0.237 \) at \( N \to \infty \). The convergence is, however, extremely slow: for \( N = 1000 \) the relative difference is still of order of 2%. The convergence can be improved dramatically if one introduces an asymptotic correction according to the empiric formula

\[
\overline{s}(N) \approx 0.237 + 0.598 \ln N/N, \quad \text{for } N \gg 1. \tag{12}
\]

The deviation of experimental \( \overline{s}(N) \) from the asymptotics (12) is less than 2% already for \( N \sim 30 \). The \( N \)-dependent corrections are controlled by many short hops in the beginning of the optimal staircase. The scaling hypothesis, mentioned above, would lead to \( \overline{s}(N) - \overline{s}(\infty) \propto 1/N \), which is inconsistent with our numerical data. This is another argument against the scaling. A fit of our numerical data for the \( N \)-dependence of an average number \( \overline{K} \) of hops in the optimal staircase gives

\[
\overline{K}(N) \approx 0.39(\ln N)^2 + 2.4, \quad \text{for } N \gg 1 \tag{13}
\]

which is perfectly consistent with the result (12) and, again, inconsistent with the scaling hypothesis, (the latter would give \( \overline{K}(N) \propto \ln N \)). The elucidation of the structure of the initial part of the optimal staircase and the origin of the empirical laws (12,13) still remains a challenge.

In conclusion, we have found the distribution function of the transmissi coefficient for the inelastic penetration of a cold disordered wire by a hot electron. The applications of these results to specific physical effects will be presented in a long paper to follow.

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