Deviation of resulting load of forestry vehicle from the normal to the bearing surface

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Abstract. The purpose of the article is to develop and analyze relationships for calculating the ratio of the normal pressure of forestry vehicle on the bearing surface and the shear stress caused by the vehicle’s slipping. The study bases on provisions of off-the-road locomotion theory and soil mechanics. The study uses methods of mathematical analysis and applied mathematics for calculations and numeric data processing. The calculations were performed using Maple 2017 software. Calculations show that the ratio of shear stress caused by slipping of the vehicle to the normal pressure on strong forest soil is an average of 17%, reaching 22%; on bearing soil, the ratio averages 22%, reaching 25%; on a weaker forest soil, the ratio is on average 24%, reaching 29% when cutting of the soil layers occurs. It is established that the deviation of the resulting vehicle load from the normal to the bearing surface, at which the cut of the soil layers begins, is 12 - 16 °. The deviation of the resulting load caused by the vehicle slip results in a decrease in the bearing capacity of forest soil by 8.5-10.5%.

1. Introduction

Under influence of a forestry machine mover, several types of deformation arise in the soil: 1) compression of the soil under the influence of the mover normal pressure [1-3], 2) a shift in the soil layers caused by shear stress when the mover is indented into the soil [1], and 3) a shift in the soil caused by the mover slipping [1]. Almost all mathematical models of the interaction of forestry vehicles with the surface, proposed earlier, take into account compression and the shift when the mover is indented into the soil [4, 5]. It was also found that tangential stress along the contact patch increases resulting load on the bearing surface and reduces bearing capacity of the soil, while increasing the thickness of its deformable layer in the direction of the resulting vehicle’s impact [4, 5]. As a result, there is an increase in the total deformation of the soil. Unfortunately, to date, issues of calculating rutting taking into account the ratio of normal and tangential loads have not been fully developed; relations are known that allow qualitative taking into account the influence of an increasing resulting load, but its value is calculated using mostly graphoanalytically [5, 6], which complicates implementation of modelling interaction of vehicles with forest soils.

The aim of the study is to develop and analyze dependencies to calculate the ratio of the normal pressure of the mover of a forest machine and the shear stress caused by its slipping. The study uses methods of mathematical analysis and applied mathematics for calculations and numeric data processing. The calculations were performed using Maple 2017 software.
2. Results and Discussion

The shear stress distributed over the contact patch is determined by the formula [6]:

\[ \tau_s = \left( \frac{1}{\tau_m} + \frac{t_s}{Gj} \right)^{-1}, \]  

(1)

where, \( t_s \) is the grousers spacing, \( j \) is the soil shear strain, \( \tau_m \) is the maximum soil shear resistance, \( G \) is the soil shear modulus.

The maximum soil shear resistance will be found by the formula [6]:

\[ \tau_m = q \tan \varphi + C \xi, \]  

(2)

where \( q \) is the average normal pressure along the contact patch, \( C, \varphi \) are inner cohesion and the angle of internal friction of the soil, respectively, \( \xi \) is a parameter that takes into account the decrease in shear resistance when cutting the soil occurs [6]:

\[ \xi = \left(1 - \frac{j}{t_s}\right)^{\text{Heaviside}} \left(\frac{j_0 - j}{t_s}\right), \]  

(3)

where, \( \text{Heaviside} (...) \) is the Heaviside function, \( j_0 \) is the shear deformation at which the soil cut occurs [6]:

\[ j_0 = \frac{\tau_m t_s}{G} \left(\frac{G}{C}\right)^{0.5} - 1, \]  

(4)

where, \( \tau_{m0} \) is the shear resistance without taking into account a possible cut of the soil [6,7]:

\[ \tau_{m0} = q \tan \varphi + C. \]  

(5)

The shear strain is usually determined taking into account horizontal coordinate \( x \), counted from the beginning of the contact patch along its length [4]:

\[ j = Sx, \]  

(6)

where, \( S \) is the slip ratio.

Average value of the shear stress distributed along the contact patch, we find as the integral [4, 5, 7]:

\[ \tau = \frac{1}{l} \int_0^l \tau_s dx. \]  

(7)

The resulting load on the soil \( p \) and the angle of its deviation from the normal to the bearing surface \( \beta \) will be found by following formulas [1, 4, 5]:

\[ p = \sqrt{q^2 + \tau^2}, \]  

(8)

\[ \beta = \arctan \frac{\tau}{q}. \]  

(9)

Figure 1 shows calculation results for the average tangential stress according to formula (7) for three categories of forest soil with the mover parameters: \( q = 0.05E \) (approximately corresponds to half of the bearing capacity of forest soil \( p_s \)), \( d = 1.333 \text{ m}, l = d / 2 \) (indicative values for a wheeled forestry mover [8]). For strong forest soil (I) \( E = 3 \text{ MPa} \), for a bearing forest soil (II) \( E = 1 \text{ MPa} \), for a
weak forest soil (III) $E = 0.4 \text{ MPa}$ [9-12]. Hereinafter, we will evaluate the physic-mechanical properties of the soil using the formulas [9-12]:

\begin{align}
C &= 10.774 E^{0.7737}, \\
\varphi &= 13.669 E^{0.1819}, \\
G &= 0.244 E^{0.117}. 
\end{align}

**Figure 1.** The average tangential stress along the contact patch.

Figure 2 shows the results of calculating the angle of deviation of the resulting load from the normal to the bearing surface.

**Figure 2.** The angle of deviation of the resulting load from the normal to the bearing surface.
Figure 3 illustrates the ratio of the average tangential stress and the normal pressure along the contact patch.

![Figure 3](image_url)  
**Figure 3.** The ratio of the average tangential stress and the normal pressure along the contact patch.

Calculations show that the percentage of shear stress on the contact patch caused by slipping of the mover to normal pressure on strong forest soil is an average of 17%, reaching 22%; on a bearing forest soil the ratio averages 22%, reaching 25%; on a weak forest soil, the ratio of shear stress and normal pressure is on average 24%, reaching 29% at the beginning of the soil layers cutting.

We express the slip ratio corresponding to the beginning of a decrease in shear resistance due to the occurrence of the soil cut, according to formulas (3) - (6), then:

\[
S = \frac{1}{l} \frac{t_g (q \tan \varphi + C)}{G} \left( \frac{G}{C} \right)^{0.5} - 1. \tag{13}
\]

Note that the slip ratio according to formula (13) on a weak forest soil and on a bearing forest soil corresponds to the loss of vehicle passability, since the shear resistance begins decreasing. According to calculations, on strong forest soils after the beginning of its cut, shear resistance does not change significantly.

For \( S \), limited by formula (13), integral (7) has the form:

\[
\tau = \frac{G}{2t_g (G - C)^2} \left( SlC^2 - CGlS + 2Gq_t g \tan \varphi + 2CGl_t g \right) - \frac{G^2}{Sl(G - C)^3} \left( q^2 \cos^2 \varphi - 2Cq \tan \varphi - q^2 + q^2 \cos^2 \varphi \right) \times \ln \left( 1 + \frac{Sl}{t_g \tan \varphi + Ct_g} \right). \tag{14}
\]

Substituting expression (13) into formula (14), we obtain the shear stress corresponding to the beginning of the cut of the soil layers due to slipping of the mover:

\[
\tau = (q \tan \varphi + C) \zeta, \tag{15}
\]

where, \( \zeta \) is dimensionless parameter determined by the shear properties of the soil.
Basing on formula (9), taking into account (15), (16), we obtain expressions for the angle of deviation of the resulting load from the normal to the bearing surface at the beginning of the soil cut:

$$\beta = \arctg \left( \tan \varphi + \frac{C}{q} \zeta \right).$$  (17)

We note that formulas (15) - (17) are invariant with respect to geometrical parameters of the contact patch, but it should be mentioned that the value of the slip ratio corresponding to the beginning of the cut of the soil depends on the length of the contact patch and the grousers spacing t_g, formula (13).

The results of calculations by formula (16) are presented in figure 4.

**Figure 4.** The parameter $\zeta$ in the equation for calculating the average tangential stress corresponding to the beginning of the cut of soil layers.

Taking into account expressions (10) - (12), for a forest soil, the dimensionless parameter as a function of the angle of deviation of the resulting load, determined by the shear properties of the soil, is almost exactly determined by the value of the total strain modulus $E$:

$$\zeta = 0.54E^{-0.17}.  \quad (18)$$

The estimation of the deviation angle at the beginning of the cut of soil layers is illustrated in figure 5 (the calculation was performed at $q = 0.05E$).
Figure 5. The angle of deviation of the resulting load from the normal to the bearing surface at the beginning of the cut of the soil.

As the graph shows, that the deviation of the resulting load from the normal to the bearing surface, at which the cut of the soil layers begins, is 12 - 16 °. This range can be considered as a limit, since with a greater slip ratio, the shear resistance of a weak and bearing forest soil decreases, while the resistance of a strong forest soil changes slightly (figure 1).

Let us evaluate the effect of the deviation of the resulting load from the normal on the bearing capacity of the soil. The bearing capacity is calculated by the formulas [5]:

\[ p_s = p_{s0} \alpha_Z \]

\[ p_{s0} = 0.5K_1K_{p1}N_1b + N_2h + K_3K_{p3}N_3C \]

\[ K_1 = \frac{l}{l + 0.4b}; K_3 = \frac{l + b}{l + 0.5b} \]

\[ K_{p1} = \frac{L}{L + 0.4B}; K_{p3} = \frac{L + 0.5B}{L + B} \]

\[ N_1 = \frac{1 - S^4}{S^4}; N_2 = \frac{1}{S^2}; N_3 = \frac{2(1 + S^2)}{S^3}; S = \tan \left( \frac{\pi}{4} - \frac{\varphi}{2} \right) \]

\[ \alpha_Z = 1 + \frac{H}{\cos \beta} \left( H \cos \beta - h - 0.25H_\varphi \right); H_\varphi = 0.707 \exp \left[ \frac{\pi + 3\varphi}{4} + \frac{3\varphi}{4} \right] b \cos \frac{3\varphi}{4} \cos \frac{3\varphi}{4} \tan \frac{3\varphi}{4} \]

(19)

where, \( p_{s0} \) is the bearing capacity of the soil with unlimited thickness of the deformable layer, \( \alpha_Z \) is the coefficient taking into account the thickness of the deformable layer of the soil, \( K_{p1}, K_{p3} \) are coefficients that take into account the angle of deviation of the resulting load from the normal to the bearing surface, \( K_1, K_3 \) are coefficients that take into account the contact patch geometry, \( N_1, N_2, N_3 \) are coefficients that take into account the internal friction of soil particles, \( S, H_\varphi \) are auxiliary notations, \( H \) is the thickness of the deformable layer of soil, \( \gamma \) is the specific gravity of the soil [9-12]:

\[ H = 0.4714E^{-0.479} \]

\[ \gamma = 0.0084E^{-0.1168} \]

(20)

(21)
The calculation results according to formulas (19) for \( \beta \) according to expression (17) and for \( \beta = 0 \) are presented in figure 6 (calculations were performed at \( h = 0.5H \)).

![Graph showing bearing capacity of forest soil](image_url)

**Figure 6.** Bearing capacity of forest soil (solid line – no deviation of resulting load from the normal to bearing surface; dash – deviation of the resulting load).

Analysis of the calculation results shows that the deviation of the resulting load from the normal to the bearing surface caused by slipping of the mover leads to a decrease in the bearing capacity of the forest soil by 8.5-10.5%.

3. Conclusion

Analysis of results of calculating indicators of interaction of a wheeled mover with forest soil before a start of cutting the soil layers, subsequent loss of the mover trafficability with cutting the bearing surface with the grousers, allowed the following conclusions:

- Calculations show that the percentage of shear stress on the contact spot caused by slipping of the mover to normal pressure on strong soil is an average of 17%, reaching 22%; on a bearing forest soil, the ratio averages 22%, reaching 25%; on a weak forest soil, the ratio of the shear stress and the normal pressure is on average 24%, reaching 29% at the beginning of the soil layers cutting.

- Deviation of the resulting load from the normal to the bearing surface, at which the soil layers cutting begins, is 12 - 16 °. This range can be considered as a limit, since with a greater slip ratio the shear resistance of a weak and bearing forest soil decreases, the resistance of a strong forest soil changes slightly (figure 1).

- The deviation of the resulting load caused by slipping of the mover leads to a decrease in the bearing capacity of the forest soil by 8.5-10.5%.

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