Inhomogeneous Spatial Distribution of Electrons in a Compressed Gas Bubble of Submicron Size

S E Kuratov¹, D S Shidlovski¹, S I Blinnikov¹
¹ Dukhov Research Institute of Automatics (VNIIA), Sushchevskaya str. 22, Moscow, 127055, Russia
E-mail: ser.evg.kuratov@gmail.com, dmitry.shidlovski@yandex.ru

Abstract. The inhomogeneous spatial distribution of the macroscopic amount of electrons (N < 10⁸) in a spherical potential well is considered. It is shown that the electron distribution has two spatial scales. The first scale is the Fermi length. By effect, we mean the existence of the second scale, which is related with the radius of the potential well and much larger than the first scale. Two systems are considered: a system of free electrons in a spherical well and electrons in a compressed gas bubble. Based on the analysis of the distribution function of a system of free semiclassical electrons, we demonstrate the cause of the appearance of the spatial heterogeneity. This result is confirmed by a numerical summation of exact solutions of the wave functions of electrons in an infinite potential well. It is shown that there is an analogous effect for the spatial distribution of electrons in compressed hydrogen gas bubble of submicron size (<0.1 μm). This system can be realized in the thermonuclear experiments. Numerical simulation of the electron distribution was carried out by the DFT method. The limiting factors of the effect are considered. The aim of problem considered in the article deals with the analysis of possible physical processes occurring in the thermonuclear experiments.

Keywords: spatial distribution, hydrogen gas bubble, DFT method.

1. Introduction
We analyze the inhomogeneous spatial distribution of electrons in a strongly compressed gas bubble (H₂, D₂) of submicron size. The characteristic values of the thermodynamic quantities of the compressed gas are the following: \( \rho_{\text{gas}} \sim (10-30) \, \text{g} / \text{cm}^3, \text{Ne} \sim 10^{30-31} \, 1 / \text{m}^3, \text{E}_f \sim (20-100) \, \text{eV}, \text{T}_i \sim \text{T}_e \sim (0.1-1) \, \text{eV}. \) All electrons are ionized and degenerated, and the ions are the classical nonideal gas.

This article demonstrates that there is an inhomogeneous spatial distribution of electrons under certain conditions. The characteristic heterogeneity scale is of the same order of the well bubble radius magnitude. The origin of this heterogeneity is the influence of quantum shell effects affecting the electron gas.

It is known, that the density of the electron distribution has the oscillating behavior because of the presence of shell corrections in a spherically symmetric potential. For example, in an atom the electron density distribution has an oscillatory character. The deviation from the average value of the distribution density can reach up to 20% [1]. In the atom, the number of electrons is relatively small (N < 10³).
In the present article it will be shown that the heterogeneity of the distribution is also present for a macroscopic number of electrons \((N < 10^9)\) in a spherical potential well.

Section 1 of the article is devoted to the theoretical analysis of the distribution of a system of free semiclassical electrons in a spherical potential well. This analysis shows the cause of the appearance of the heterogeneity in the system. It also demonstrates that the spatial scale of the heterogeneity is of the same order of the well radius and can be greater than the distance between the particles by several orders.

Section 2 presents the effective computational technique that allows to calculate the electron density by numerical summation of exact solutions for the wave functions of free electrons in a potential well. The calculations of various systems with the electron number of up to 1 billion particles are presented in this article. These results confirm the existence of the effect.

Section 3 presents the results of DFT calculations of the electron distribution in a compressed gas bubble. We used the jellium model for hydrogen clusters (the number of atoms from 4000 to 100,000). In the conclusion we consider the limiting factors of the effect.

2. A system of free semiclassical electrons in an infinite spherical well. The theoretical analysis

It is important to analyze the distribution of free semiclassical electrons. All electrons are degenerated in this system and their Fermi energy is greater than the thermal and Coulomb energies. Also, electrons have significant orbital angular momentum and therefore they have semiclassical behavior. So this task setting is close to the real situation. The solution of the problem gives us the possibility to obtain analytical dependences of the electron distribution on the system parameters. It is important for the interpretation of the numerical results.

The semiclassical approach was widely used for the analysis of metal clusters [2], for the calculation of the nuclei energy spectrum [3], for the calculation of the electron concentration oscillations in the atom.

To determine the electron concentration, we use the Green's function representation \(G(r'',r',e)\) for electrons using the semiclassical approximation [3].

\[
G(r'',r',e) = G_0 - \frac{1}{(2\pi \hbar^2)^{1/2}} \sum_\alpha \left\{ p_\alpha D^{1/2} \exp \left( \frac{i}{\hbar} S_\alpha(r'',r',e) + iv - \frac{i\pi}{4} \right)\right\}_a
\]

\(S_\alpha\)- classical action integral

\[
S_\alpha = \int_{r'}^{r''} p_\alpha dl_\alpha
\]

\[
G_0(r'',r',e) = -\frac{m}{2\pi \hbar^2 |r'' - r'|} \exp \left( \frac{i}{\hbar} |r'' - r'| p(r) \right)
\]

\[
r = \frac{r' + r''}{2}, \quad \mathbf{p} = \frac{dS_\alpha(r',r'',e)}{dr'}, \quad \mathbf{p}'' = \frac{dS_\alpha(r',r'',e)}{dr''}
\]

\[
D = \text{det}
\begin{bmatrix}
\frac{dp'_r}{dz'_r} & \frac{dp'_r}{dz'_r} & \frac{dp'_r}{dz'_r} \\
\frac{dp'_z}{dz'_r} & \frac{dp'_z}{dz'_r} & \frac{dp'_z}{dz'_r} \\
\frac{dp'_z}{dz'_z} & \frac{dp'_z}{dz'_z} & \frac{dp'_z}{dz'_z}
\end{bmatrix}
\]

\[
\frac{dt'_r}{dz'_r} = \frac{dt'_z}{dz'_z} = 0
\]
\[ t_\alpha (r', r'') = \frac{dS_\alpha (r', r'', e)}{de} = \int_{r'}^{r''} \frac{1}{|F|} dl_\alpha \]  \hspace{1cm} (6)

In (1) the summation is extended over all real classical trajectories \( \alpha \) connecting \( r' \) and \( r'' \), \( t_\alpha \) is the time of the motion along the trajectory \( \alpha \).

The electron concentration is determined from the following expression

\[ n(r) = -\frac{2}{\pi} \int_{-\infty}^{\varepsilon_F} \text{Im} G(r, r, e) de \]  \hspace{1cm} (7)

In our case, to calculate the electron concentration (integral in (7)), it is necessary to take into account a variety of non-periodic orbits having coincident initial and final coordinates. The examples of such trajectories are shown in Figure 1.

![Diagram](image)

**Figure 1.** The non-periodic trajectories of the electron.

To determine the cause of the appearance of the spatial heterogeneity we estimate the expected effect by the order of the magnitude.

In our case

\[ S_\alpha \sim p R_0 L_\alpha \left( \frac{r'}{R_0}, \frac{r''}{R_0} \right), \]  \hspace{1cm} (8)

where \( L_\alpha \) is the length of the trajectory \( \alpha \). So

\[ D \sim \frac{m^2}{R_0 p} F \left( \frac{r'}{R_0}, \frac{r''}{R_0} \right). \]  \hspace{1cm} (9)

The expression for Green function has the following form:

\[ G(r'', r', e) \sim \frac{1}{h^2} p \left( \frac{m^2}{R_0 p} \right)^{1/2} F \left( \frac{r'}{R_0}, \frac{r''}{R_0} \right)^{1/2} \exp \left( \frac{i}{h} p R_0 L_\alpha \left( \frac{r'}{R_0}, \frac{r''}{R_0} \right) \right) \]  \hspace{1cm} (10)

\[ \Delta n(r) \sim \int_{-\infty}^{\varepsilon_F} \text{Im} G(r, r, e) de \]  \hspace{1cm} (11)

For the integral calculation, we use the large value (\( \sim 10^3 \)) of the exponent in (10). We obtain the following:
$$\Delta n(r) \sim Im \left[ -i \left( \frac{F(r', r''; R_0)}{L_{\alpha} r_0} \frac{r''}{R_0} \right)^{1/2} \frac{1}{n} \right]^{1/2} \exp \left( \frac{i}{\hbar} p_R L_{\alpha} \left( \frac{r'}{R_0} \frac{r''}{R_0} \right) \right)$$

(12)

$$\Delta n(r) \sim \left( \frac{1}{R_0^2 n} \right)^{1/2} \sin \left( \frac{1}{R_0} p_R L_{\alpha} \left( \frac{r'}{R_0} \frac{r''}{R_0} \right) \right)$$

(13)

To analyze the heterogeneity scale of the spatial distribution of the electron we calculate the electric field potential

$$\varphi(r) = 4\pi e \int_0^r \Delta n(r') r'^2 dr$$

(14)

To calculate the electric field potential we use the stationary phase method. There are two possible cases. In the first case the function $L_{\alpha}$ has no stationary points on the interval of integration, and so the potential has the obvious oscillating behavior. So the spatial distribution of the electron has one heterogeneity scale. This scale is the Fermi length $\frac{1}{\hbar} p_F r$. In the second case, the function $L_{\alpha}$ has stationary points. So the distribution of the potential has the second spatial scale relating with trajectory having a minimal length. Such trajectory is presented in figure 2. The value of the second spatial scale is order of $R_0/2$

![Figure 2](image-url)

Figure 2. Some possible trajectories of electrons. The green trajectory has a minimal length.

For the electric field potential we have the following estimation:

$$\varphi(r) \sim \frac{e}{\varepsilon_0 R_0} \left( R_0 n_0^{1/3} \right)$$

(15)

3. Numerical calculation of spatial distribution of free electrons in the infinite spherical potential well

To simulate small plasma bubbles with the electron number up to $N=10^8$ we consider a very simple model. This model consists of the spherical potential well with infinite walls and free electrons. All electrons do not interact with each other. The distribution of electron density and potential is very similar to those in small metal clusters [4], where the calculations were carried out for the jellium...
model (SJBM). But calculations for metal clusters using jellium model were performed earlier only for the number of electrons up to 6000 [5]. The model with an infinite potential well is very crude and can misinterpret the behavior electrons, so these two models must be compared for a large number of electrons.

If one considers a large number of \( N \) electrons in a cubic box, then their distribution quickly goes to the semiclassical regime and turns out to be uniform. In contrast, in a spherically symmetric potential in many situations, the density of the electron distribution shows an oscillating behavior along the radius. This can be seen in the solutions by Hartree-Fock, Hartree-Fock-Dirac methods, and in Thomas-Fermi approximation [1,2] and in DFT [4]. Such oscillating behavior is observed, not only for electrons, but also for other particles, and for Coulomb as well as for other interaction potentials between particles (Yukawa potential, hard core, etc.) [6]. In such a situation it is interesting to explore how the quasi-classical regime will proceed with growth of \( N \), how will oscillations behave in the radial distribution of electrons, and accordingly, what orders of electric field may be expected for a local violation of the electrical neutrality of the plasma.

In this section we investigate the distribution of \( N \) electrons in the ground state in a spherically symmetric potential well with impermeable (i.e. infinitely high walls). This problem allows a complete analytical solution (see, for example, books by Fluegge and Messiah [7, 8]). The transition to a large number of \( N \geq 10^8 \) electrons in this problem allows one to trace in detail the oscillating behavior of the electron density in the ground state, to see the difference from the quasi-classical Thomas-Fermi solution and to estimate the magnitude of the electric field, which can arise during adiabatic compression of a cavity containing a degenerate plasma with a similarly large number of electrons. Adiabaticity will be understood here in the spirit of P. Paradoxov [9].

In the subsection 3.1, the starting point of the problem is the statement of the problem in a spherically symmetric potential well, and a long-known exact solution is written down for a well with infinitely high walls. Examples of the oscillating behavior of the electron density are given in the section 3.2. The section 3.3 illustrates the possible distribution of the electrical potential fields under different assumptions about the concentration of a homogeneous distribution ions, partially compensating the field of electrons.

### 3.1. Solution for the wave function of an electron in a spherical well with impermeable walls

Here we reproduce the notation from Fluegge's book [7]: Schrödinger equation in the problem of determining the energy levels of a particle in a spherically symmetric potential. A box with infinite walls and a radius \( R \) admits separation of variables, and the solution can be written in the form

\[
u(r, \theta, \varphi) = \frac{1}{r} \chi_{n,r,l}(r) Y_{l,m}(\theta, \varphi), \tag{16}\]

where \( Y_{l,m}(\theta, \varphi) \) is a spherical function. Radial wave function \( \chi \) satisfies the equation

\[
\chi^{''}_{n,r,l} + \left[ k^2 - \frac{l(l+1)}{r^2} \right] \chi_{n,r,l} = 0, \quad k^2 = \frac{2mE}{\hbar^2}.
\tag{17}\]

The function \( \chi(r) \) corresponds to different energy levels \( E \) in the dependence from the orbital angular momentum \( l \) and the radial quantum number \( n_r \), since \( k \) will be fixed by the boundary conditions. This equation has a solution \( \chi_{n_r,l}(r) = C_{n_r,l} r j_l(\alpha_{n_r,l} r/R) \), where \( j_l(x) \) is called a spherical Bessel function of the order \( l \), \( x_{n_r,l} \) are roots of the Bessel function \( j_{l+1/2}(x) \), \( C_{n_r,l} \) is a normalization constant.

The expression for the electron density for the filled shells:

\[
\rho_e(r) = \frac{1}{4\pi R^3} \sum_{n_r,l} g_e (2l+1) j^2_l \left( \frac{\alpha_{n_r,l} r}{R} \right) \left( j^2_l(x_{n_r,l}) - j_{l-1}(x_{n_r,l}) j_{l+1}(x_{n_r,l}) \right)^{-1}. \tag{18}\]
It immediately shows that if the physical radius of the cavity $R$ is changed for any set of filled states the density varies self-similarly, simply as $f(x)/R^3$, where $f(x)$ is determined by the sum on orbitals in (18).

Potential is computed by the formula

$$V(r) = 4\pi \left[ \frac{1}{R} \int_0^R \rho_c(x)x^2\,dx + \int_r^R \rho_c(x)x\,dx \right].$$

(19)

Here, the charge density is $\rho_c = |e|(\rho_i - \rho_e)$, the elementary charge is $e$, the electron density $\rho_e(r)$ was taken from the calculation using the formula (18), and for ions the simplest approximation was applied $\rho_i(r) = \text{const}(r)$. The constant was chosen either from the electroneutrality condition within the radius $R$, or was chosen so as to minimize the potential difference inside a sphere with radius $R$.

### 3.2. Density distribution for various values of radius $R$

The results for $\rho$ are self-similar when the radius of the cavity $R$ is changed. For illustration, we fix the average value of electron density assuming that inside $R$ we have a deuterium plasma at a density of $5$ g/cm$^3$. The Fermi energy is then $47.9$ eV.

Figure 3 shows the density calculated directly from the formula (18) for $R = 3.958$ nm. Figure 4 shows the density calculated from the formula (18) for $R = 39.85$ nm near the outer border on a large scale.

**Figure 3.** Distribution of the number density of states for $R = 3.958$ nm (dimensionless energy $E = 2 \cdot 10^4$).

**Figure 4.** Distribution of the number density of states for $R = 39.85$ nm (dimensionless energy $E = 2 \cdot 10^6$) at the edge.

### 3.3. Distribution of the potential of electric field

In this section we show the potential distributions $\varphi(r)$.

Figure 5 shows the potential distribution calculated by the formula (19) for $R = 3.958$ nm, homogeneous distribution $\rho_i$ with complete compensation of the electron charge. In this case, the potential difference is large. On such a chart, no oscillations of the potential are seen, which give density oscillations, since density is the second derivative of $\varphi$. But some oscillations become visible if we choose the compensating distribution of the ions $\rho_i$, where the potential difference is minimal - see figure 6. Here $\rho_i$ is increased to compensate for the negative charge in the central zones, since there $\rho_e$ is higher than the mean $\rho_e$ over the entire volume. The value of the excess density, sufficient to minimize the potential difference, in this case, $2.48\%$ of the original density.
From these graphs, we see in one case a potential difference of order $\sim 15\text{V}$, in another $\sim 1.5\text{kV}$. What kind of potential difference can be expected in reality? This is the most important issue of the problem. It is necessary to understand that we have here a strong idealization: in a box with high mirror walls is filled with $N$ electrons that have no potential except the walls to feel. This means, that there are $N$ of classical singly charged ions, which compensated all the charges, and there are no potential differences!

If the ions were distributed strictly uniformly, compensating only full charge of electrons, it would be great potential, as in the figure 5. There is the potential well for ions in the center: there is an excess of electrons, and they pull positive charges toward the center.

**Figure 5.** Distribution of potential for $R = 3.958\text{ nm}$ (dimensionless energy $E = 2 \cdot 10^4$) in case of uniform distribution $\rho_i$ with full compensation of electron charge.

**Figure 6.** Distribution of potential for the same set as in figure 6, but with $\rho_i$ fitted to minimize the difference of potentials.

4. Numerical simulation of the nonuniform electron distribution in a compressed gas bubble of nanometer size

4.1. Description of the jellium model

Numerical simulation of the electron and potential distribution is made using density functional theory (DFT) in the spherical jellium background model (SJBM). In the jellium model ions are represented as a continuous fixed distribution of a positive charge. In this form the spherically symmetric distribution of ions and their potential is possible, that allows to reduce the problem to one-dimensional. Here the jellium model modification is used, which is so called stabilized jellium model [10,11] with a correction, that takes into account an average difference between the jellium and point ions.

When the external potential has spherical symmetry, as in section 3.1, one-electron wavefunction can be decomposed as (16) to a radial $\chi_{n,r,l}(r)$ and a spherical $Y_{lm}(\theta, \varphi)$ wavefunction, if we suppose that the electron distribution also has a spherical symmetry. For the radial wavefunction Kohn-Sham equation [12] has the following form (in atomic units):

$$\left( -\frac{1}{2} \frac{d^2}{dr^2} + V_{KS} + \frac{l(l+1)}{2r^2} \right) \chi_{n,r,l}(r) = E_{n,r,l} \chi_{n,r,l}(r), \quad (20)$$

$$V_{KS} = V_{ion} + V_H + V_{xc} + (\delta\nu)_{WS} = V_{ES} + V_{xc}, \quad (21)$$

where $V_{KS}$ is the effective Kohn-Sham potential, $V_H$ and $V_{ion}$ are electrostatic potentials arising from the electronic density and the ionic jellium, respectively, $V_{xc}$ is the exchange-correlation potential,
$\delta v_{WS}$ is the stabilized jellium correction. In this section all electrostatic potentials are potentials for electrons, so they are multiplied by -1.

Equations for the single electron wavefunction are solved self-consistently using simple iteration method, at this electrons with equal values of quantum numbers $n$ and $l$ have the same radial wavefunctions, so electron density equals

$$n_e(r) = \sum_{n_r,l} N_{n_r,l} \frac{P_{n_r,l}^2(r)}{4\pi r^2}.$$  \hspace{1cm} (22)

Here $N_{n_r,l}$ is a number of electrons in every shell, for fully occupied shells $N_{n_r,l} = 2(2l + 1)$, for partially occupied, if there are more than one, such values used, that their energy eigenvalues $E_{n_r,l}$ will be equal. As a result, the number of the single electron wavefunction, that should be found, is much less than the electron number.

For spherically symmetric computations in the stabilized jellium model an own code was used [13], which was solving one-dimensional Kohn-Sham equations with the exchange-correlation functional PZ (Perdew, Zunger) [14] in the local density approximation (LDA).

In this work only calculations with the homogeneous ion distribution are considered. In the stabilized jellium model it looks like this: the ionic jellium density, which is also the positive charge density equals the average density $\bar{n}_e = N(3/4\pi R^3)$ inside the sphere and 0 outside.

4.2. Calculation results

A series of calculations was carried out using the jellium model for ionized hydrogen bubbles with different density values of homogeneous ionic jellium and radius values. The results are obtained for density $10^{30}$, $10^{31}$ and $10^{32}$ m$^{-3}$, three radius values used for each, parameters are listed in table 1.

Because the ionic jellium density is constant and approximately equal to the average electron density in a bubble, as the electron density oscillations we can use difference between the electron density and the ionic jellium density $\Delta n = n_e - \bar{n}_{ion}$.

To get the electrostatic potential oscillations $\Delta V_{ES}$ we can, for example, extract from $V_{ES}$ average value, which is approximately expressed in terms through $E_F$:

$$\Delta V_{ES} = V_{ES} + V_{xc}(n_{ion}) + \left( E_F(n_{ion}) - E_{n_r,l}^{\text{max}} \right),$$  \hspace{1cm} (23)

since

$$V_{ES} + V_{xc} = V_{KS} \approx -E_F.$$  \hspace{1cm} (24)

Figures 7–9 show the difference between the electron density and the ionic jellium density $\Delta n$ and also the electrostatic potential oscillations, obtained using formula (23), for one radius value at each density.

![Figure 7](image-url)  

Figure 7. The difference between the electron density $n_e$ and the jellium density $n_{ion}$ (a), the electrostatic potential oscillations $\Delta V_{ES}$ (b) for density $n_{ion}$ equal to $10^{30}$ m$^{-3}$ and radius 2.5 nm.
In graphs we can see, that, if we do not take into account fluctuations close to the center, double amplitudes of the electron density oscillations are in a good agreement with values, which can be obtained from (13):

\[
\frac{2\Delta n}{n} \sim \frac{2}{(R_0^3 n)^{1/2}} = 2 \left(\frac{4\pi}{3N}\right)^{1/2} \approx \frac{4}{N^{1/2}} .
\] (25)

A comparison of values \(2\Delta n/n\) and \(4/N^{1/2}\) for all cases is presented in table 1. For all values the difference is about 2 times, a value of \(2\Delta n/n\) is taken approximately from graphs.

As can be clearly seen from the table 1, the amplitude of the electrostatic potential oscillations decrease with a number of electrons increase. Also in many cases the electrostatic potential oscillations can be observed, which have a scale of the order of the system size, unlike small oscillations with a scale of the order of the interatomic distance or in other words \(h/p_F\). Especially this is noticeably for the density \(10^{32} \text{ m}^{-3}\).

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{figure8a}
\includegraphics[width=0.4\textwidth]{figure8b}
\caption{The same as the previous figure for density \(10^{31} \text{ m}^{-3}\) and radius 1.25 nm.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{figure9a}
\includegraphics[width=0.4\textwidth]{figure9b}
\caption{The same as the previous figure for density \(10^{32} \text{ m}^{-3}\) and radius 0.5 nm.}
\end{figure}

\begin{table}[h]
\centering
\caption{A comparison of values \(4/N^{1/2}\) and double relative oscillation amplitudes \(2\Delta n/n\) obtained in the numerical simulation (n is the density in m\(^{-3}\), R is the radius in nm, N is the number of electrons).}
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline
\(n, \text{ m}^{-3}\) & \multicolumn{3}{c|}{\(10^{30}\)} & \multicolumn{3}{c|}{\(10^{31}\)} & \multicolumn{3}{c|}{\(10^{32}\)} \\
\hline
\(R, \text{ nm}\) & 1.0 & 2.0 & 2.5 & 0.5 & 1.0 & 1.25 & 0.25 & 0.5 & 0.625 \\
\hline
\(N\) & 4188 & 33510 & 65450 & 5236 & 41888 & 81812 & 6544 & 52360 & 102266 \\
\hline
\(4/N^{1/2}\) & 0.062 & 0.022 & 0.016 & 0.055 & 0.020 & 0.014 & 0.050 & 0.017 & 0.013 \\
\hline
\(2\Delta n/n\) & 0.030 & 0.012 & 0.0085 & 0.022 & 0.010 & 0.0075 & 0.020 & 0.009 & 0.007 \\
\hline
\end{tabular}
\end{table}
5. Conclusion
Analysis of the spatial distribution of electrons in mesoscopic systems is presented in this paper. We consider two mesoscopic systems: a macroscopic number of free quasiclassical electrons in a spherical potential well and a compressed gas bubble.

From analysis of first system we show that the electron distribution has the spatial scale which is of the order of the well radius magnitude. The appearance of this effect is caused by influence of the shell corrections in a spherically symmetric potential. We obtained the analytical estimation for magnitude of the spatial heterogeneity of electron distribution

$$\Delta n(r) \sim \left( \frac{1}{R_0^2 n} \right)^{1/2}$$

The compressed gas bubble of submicron size is more realistic system. This system can be realized in the thermonuclear experiments. Numerical simulation of the electron distribution was carried out by the DFT method. Jellium model used in the study is relevant to describe the realistic physical systems. The numerical calculation demonstrate the existence of the spatial distribution of electrons in compressed hydrogen gas bubble too. Account the spatial distribution of electrons is important for description of the thermonuclear experiments.

A number of factors limit and weaken the effect considered in the article. These factors have different origin.

The first of these is associated with the relaxation of the classical ionic system under the influence of the generated electric field. The motion of classical ions will tend to neutralize the arising charge. The estimates, following from an analysis of the gas-dynamic motion of ions, show that the neutralization time is order of 0.2 ns. Essentially this is the time of the effect's existence.

The second factor is associated with the interaction of almost free electrons with ions and with each other. After the collision the electron forgets about the spherical well. Therefore, the analyzed effect is limited by the bubble radius magnitude. The radius estimated in this way is of order of 0.1 μm.

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