Detectability of bi-gravity with graviton oscillations using gravitational wave observations

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The gravitational waveforms in the ghost-free bi-gravity theory exhibit deviations from those in general relativity. The main difference is caused by graviton oscillations in the bi-gravity theory. We investigate the prospects for the detection of the corrections to gravitational waveforms from coalescing compact binaries due to graviton oscillations and for constraining bi-gravity parameters with the gravitational wave observations. We consider the bi-gravity model discussed by the De Felice-Nakamura-Tanaka subset of the bi-gravity model, and the phenomenological model in which the bi-gravity parameters are treated as independent variables. In both models, the bi-gravity waveform shows strong amplitude modulation, and there can be a characteristic frequency of the largest peak of the amplitude, which depends on the bi-gravity parameters. We show that there is a detectable region of the bi-gravity parameters for the advanced ground-based laser interferometers, such as Advanced LIGO, Advanced Virgo, and KAGRA. This region corresponds to the effective graviton mass of \( \mu \gtrsim 10^{-17} \text{cm}^{-1} \) for \( c \sim 1 \gtrsim 10^{-19} \) in the phenomenological model, while \( \mu \gtrsim 10^{-16.5} \text{cm}^{-1} \) and \( \kappa \xi \gtrsim 10^{0.5} \) in the De Felice-Nakamura-Tanaka subset of the bi-gravity model, respectively, where \( c \) is the propagation speed of the massive graviton and \( \kappa \xi \) corresponds to the corrections to the gravitational constant in general relativity. These regions are not excluded by existing solar system tests. We also show that, in the case of 1.4M\(_{\odot}-1.4M_{\odot}\) binaries at the distance of 200 Mpc, \( \log \mu^2 \) is determined with an accuracy of \( \mathcal{O}(0.1)\% \) at the 1\( \sigma \) level for a fiducial model with \( \mu^2 = 10^{-33} \text{cm}^{-2} \) in the case of the phenomenological model.

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I. INTRODUCTION

The second-generation laser interferometers such as Advanced LIGO [1], Advanced Virgo [2], and KAGRA [3, 4], will be in full operation within a few years. These detectors are sensitive to gravitational waves (GWs) in the frequency band between 10 Hz and \(~1000\) Hz. The inspiral of a coalescing compact binary (CCB) system is one of the most promising sources for these detectors. These detectors will be able to see CCB systems, composed of NSs and/or stellar-mass BHs, within 200-1000 Mpc. GW observations of the inspiral signals from CCB systems can be a powerful tool to probe strong-field, dynamical aspects of gravity theories [5]. One of the science targets of these projects is to test the correctness of general relativity (GR) through comparison of observed gravitational waveforms with the prediction.

Cosmological observations of distant Type Ia supernovae have discovered the late-time accelerated expansion of the Universe [6, 7]. Observations of the Type Ia supernovae, the cosmic microwave background anisotropies, and the large scale structure of galaxies consistently suggest the current cosmic acceleration. However, the origin of this late-time cosmic acceleration is still unknown, and it is one of the biggest unsolved problems in cosmology. It may suggest the existence of dark energy. But it may also suggest a sign of breakdown of GR on cosmological scales, and motivates many researchers to study modified gravity (MG) models as cosmological models (see e.g., 8 for a review).

As an alternative model to GR, we focus on the first example of the ghost-free bi-gravity model [9], which is constructed based on the fully nonlinear massive gravity theory [10–12] (see e.g., 13 and 14 for a review). In the ghost-free massive gravity model, it is difficult to construct spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) solutions [15–16], while in the ghost-free bi-gravity model spatially flat FLRW solutions exist [17]. The bi-gravity model with a small mass is interesting phenomenologically. However, such models do not remain to have a healthy background cosmological solution at a high energy. Therefore, it would be required to embed the model into a more fundamental theory which is valid even at higher energies. The first attempt was made in Ref. 20, in which the bi-gravity model is shown to be embedded in the Dvali-Gabadadze-Porrati 2-brane model.
model \[21\], at least, at low energies.

In the bi-gravity theory GWs propagate differently from those in GR. So, direct GW observations will be a powerful probe of the bi-gravity theory. In the ghost-free bi-gravity theory, physical and hidden modes of GWs are both excited. These two gravitons interfere with each other like neutrinos during their propagation, which is called the graviton oscillation and the observed GWs exhibit deviations from GR \[22\]. While there are previous studies on the modified propagation of gravitational waves due to the finite mass of the graviton (see e.g., Refs. \[23\]–\[25\]), those studies were based on the linearized Fierz-Pauli theory \[26\] and did not care about the appearance of a ghost mode at the non-linear level. Once we care about the ghost appearance, we need to consider the ghost-free massive gravity, but the simplest model does not have a suitable FLRW background solution, as we mentioned earlier. In the case of bi-gravity, the situation is very different since we have two gravitons. Furthermore, the linear theory is not sufficient to discuss the solar system constraint, and the generation and propagation of GWs in this model. Owing to the Vainshtein mechanism, the ghost-free bi-gravity model can give almost the same prediction as GR at least in the weak field case. However, the gravitational waveforms differ from those in GR, because of the graviton oscillation effect. De Felice, Nakamura and Tanaka \[22\] (hereafter, DFNT) have pointed out that the interesting parameter range of graviton mass exists, where large deviations from the GR case are produced in GW signals, while it can not be excluded by the solar-system tests. So, one can use gravitational waveforms to identify the effect of modified gravity.

To evaluate the parameter estimation accuracy, the Fisher matrix has often been used \[27\]–\[28\]. Many works \[23\]–\[25\], \[29\]–\[30\] have been done to study the possibility to test the modified propagation of GWs due to the graviton mass by using the Fisher matrix. Bayesian hypothesis testing is also useful for model selection in the GW data analysis \[31\]. Recently, Vallisneri \[32\] has introduced a simple method to test modified gravity within the framework of the Bayesian hypothesis testing. In this method, one can compute the odds-ratio from the fitting factor between the general relativistic and modified gravity. The remainder of this paper is organized as follows. In Sec. \[\text{II}\] we review the ghost-free bi-gravity model, and the derivation of the modified waveforms. In Sec. \[\text{III}\] we briefly review the Vallisneri’s formulas to evaluate the detectability of the bi-gravity model and the Fisher matrix to evaluate the measurement accuracy of the bi-gravity parameters. In Sec. \[\text{IV}\] we show the detectable region of the bi-gravity model on the model parameter space. We discuss the physical explanation on how the detectable range is determined, and the correspondence of the detectable range with the fitting factor between the GR and bi-gravity waveforms. We also evaluate the measurement accuracy of the bi-gravity parameters. Section \[\text{V}\] is devoted to summary and conclusions.

II. GRAVITATIONAL WAVES IN THE BI-GRAVITY MODEL

In this section, we briefly review graviton oscillations in the ghost-free bi-gravity model.

A. Ghost-free bi-gravity theory

We describe the first example of ghost-free bi-gravity model \[\text{II}\]. The action of this model is given as

\[
S = \frac{M_{\odot}^2}{2} \int d^4x \sqrt{-\text{det} g} [\dot{g} R] + \frac{\kappa M_{\odot}^2}{2} \int d^4x \sqrt{-\text{det} \tilde{g} R[\tilde{g}]} - m^2 M_{\odot}^2 \int d^4x \sqrt{-\text{det} g} \sum_{n=0}^{4} c_n V_n (Y_\mu^\nu) + S_m [g],
\]

(1)

where \(Y_\mu^\nu = \sqrt{g_{\mu\nu}} g_{\alpha\beta} Y^\alpha Y^\beta\), and \(V_n\) are elementary symmetric polynomials \[\text{III}\] defined as

\[
V_0 = 1, \quad V_1 = [Y], \quad V_2 = [Y]^2 - [Y^2], \quad V_3 = [Y]^3 - 3[Y][Y^2] + 2[Y^3], \quad V_4 = [Y]^4 - 6[Y]^2[Y^2] + 8[Y][Y^3] + 3[Y^2]^2 - 6[Y^4],
\]

(2)

where the trace of \(Y^n\) is expressed as \([Y^n] = \text{tr}(Y^n) = Y_{a_1}^{a_1} Y_{a_2}^{a_2} \cdots Y_{a_n}^{a_n - 1}\). \(c_n\) are dimensionless constants and the matter action \(S_m [g]\) only couples to the physical metric \(g_{\mu\nu}\). \(g_{\mu\nu}\) is an additional dynamical tensor field,
which we refer to as the hidden metric. $R$ and $\tilde{R}$ denote the scalar curvatures for $g_{\mu \nu}$ and $\tilde{g}_{\mu \nu}$, respectively. $M_G = 1/(8\pi G_N)$ is the Planck mass, $\kappa$ is a constant which expresses the ratio between the two gravitational constants for $\tilde{g}_{\mu \nu}$ and $g_{\mu \nu}$ and graviton mass parameter $m^2$ can be absorbed into the parameters $c_i$. The action consists of the standard Einstein-Hilbert kinetic terms for both $g_{\mu \nu}$ and $\tilde{g}_{\mu \nu}$, and coupling terms between $g_{\mu \nu}$ and $\tilde{g}_{\mu \nu}$. The theory is free from the Boulware-Deser ghost \cite{35} in both $g_{\mu \nu}$ and $\tilde{g}_{\mu \nu}$ sectors \cite{10–12}.

We assume the spatially flat FLRW background \cite{19}

$$ds^2 = a^2(-dt^2 + dx^2), \quad \tilde{ds}^2 = \tilde{a}^2(-c^2dt^2 + dx^2), \quad (3)$$

where the scale factors $a$, $\tilde{a}$, and the propagation speed of the hidden graviton $\tilde{c}$ are functions of the conformal time coordinate $t$. We focus on a healthy branch of background cosmological solutions \cite{22, 36}; in which $\tilde{c} = c$. We also focus on the case of $m^2 \gg \rho_m / M_G^2$, where $\rho_m$ is the matter energy density. In this limit, we can regard $\xi \equiv \tilde{a}/a$ as a constant, $\tilde{\xi}_c$. Now, the usual Friedmann equation for the physical metric $g_{\mu \nu}$ is given as

$$3H^2 \approx \tilde{M}_G^{-2} \rho_m, \quad (4)$$

where $H \equiv \dot{a}/a^2$ is the Hubble parameter and $\tilde{M}_G \equiv M_G^2(1 + \kappa\tilde{\xi}_c^2)$ is the effective gravitational constant.

### B. Propagation of gravitational waves

By using the nonlinear Hamiltonian analysis \cite{12}, we find that there are in general seven propagation degrees of freedom in the ghost-free bi-gravity theory. The seven modes consist of one massive and one massless spin-2 fields. Dominant contributions to GW radiation in the theory are two plus two helicity-2 modes for physical and hidden sectors, both of which are generated in the same way as in GR \cite{13}. Here we consider the double FLRW background solutions and denote the perturbations around them as $\delta g_{ij} = a^2(\tilde{h}x_{ij} + \tilde{h}^x_{ij})$ and $\delta \tilde{g}_{ij} = \tilde{a}^2(\tilde{h}x_{ij} + \tilde{h}^x_{ij})$, where $x_{ij}$ and $x^x_{ij}$ represent the polarization tensors for plus and cross modes.

The physical and hidden gravitational modes mix during their propagation, because of their coupling through the interaction term. The mixing of the gravitational wave modes is interpreted as graviton oscillations in analogy with neutrino oscillations.

Neglecting the effects of cosmic expansion, we have the following propagation equations for gravitational waves \cite{22, 36}.

$$\ddot{h} - \Delta h + m^2 \Gamma_c(h - \tilde{h}) = 0,$$

$$\ddot{\tilde{h}} - \tilde{c}^2 \Delta \tilde{h} + \frac{m^2 \Gamma_c}{\kappa^2 \tilde{\xi}_c^2}(\tilde{h} - h) = 0, \quad (5)$$

where $\xi_c$ and $\Gamma_c$ are constants. Later, $\Gamma_c$ is absorbed into the effective mass for graviton defined as $\mu^2 \equiv (1 + 1/\kappa^2 \tilde{\xi}_c^2)m^2 \Gamma_c$. Since the propagation equations are identical for both polarizations, we have omitted the index $+/\times$. Solving eqs. \ref{5}, we obtain two eigen wave numbers for a given gravitational wave frequency $f$ as

$$k_{1,2}^2 = \frac{(2\pi f)^2 - \mu^2}{2} \left(1 + x \mp \sqrt{1 + 2x - \frac{1}{1 + \kappa^2 \tilde{\xi}_c^2} + x^2}\right), \quad (6)$$

and the corresponding eigenfunctions are given as

$$h_1 = \cos \theta_g h + \sin \theta_g \sqrt{\kappa \xi_c} \tilde{h}, \quad (7)$$

$$h_2 = -\sin \theta_g h + \cos \theta_g \sqrt{\kappa \xi_c} \tilde{h}, \quad (8)$$

with the mixing angle

$$\theta_g = \frac{1}{2} \cot^{-1} \left(\frac{1 + \kappa^2 \tilde{\xi}_c^2}{2\sqrt{\kappa \xi_c} x + \frac{1 - \kappa^2 \tilde{\xi}_c^2}{2\sqrt{\kappa \xi_c}}}\right),$$

and

$$x \equiv \frac{2(2\pi f)^2(\tilde{c} - 1)}{\mu^2}. \quad (9)$$

In the case of the usual Vainshtein mechanism, the Compton wavelength of the graviton should be as large as 300 Mpc or so to pass the solar system constraints. In that case the effect of the graviton mass is hardly detected even if we consider the propagation of GWs over the cosmological distance scale. However, in the bi-gravity model discussed in Ref \cite{22}, thanks to the enhanced Vainshtein mechanism, it is possible to keep the effective graviton mass $\mu$ much larger \cite{22}. When the Vainshtein mechanism \cite{37} works, metric tensor perturbations on both sectors are equally excited inside the Vainshtein radius.

### C. Modified inspiral waveforms due to graviton oscillations

Here we discuss only the inspiral phase of gravitational waves from CCB systems in the ghost-free bi-gravity model. Both $h$ and $\tilde{h}$ are excited exactly as in the case of GR \cite{13}. By using the stationary phase approximation, the observed signal in the frequency-domain is given as\footnote{For simplicity, we assume a signal from a face-on binary system at the zenith.}

$$h(f) = A(f)e^{i\Phi(f)} \left[B_1e^{i\delta\Phi_1(f)} + B_2e^{i\delta\Phi_2(f)}\right], \quad (10)$$

where the amplitude $A(f)$ (up to Newtonian order), the bi-gravity corrections $B_i$ and the phase function $\Phi(f)$ (up to 3.5PN order) and the phase corrections $\delta\Phi_{1,2}$ are...
given as

\[ A(f) = \sqrt{\frac{5\pi}{24}} \frac{M^2}{(8\pi M_G^3)^2 D_L} f^{-7/6}, \]

\[ B_1 = \cos \theta_g (\cos \theta_d + \sqrt{\kappa \xi} \sin \theta_d), \]

\[ B_2 = \sin \theta_g (\sin \theta_d - \sqrt{\kappa \xi} \cos \theta_d), \]

\[ \Phi(f) \equiv 2\pi f t_c - \Phi_c - \pi/4 + \frac{3}{128} y^{-5/3}\left\{1 + \right. \]

\[ + \left. \left( \frac{3715}{756} + \frac{55}{9} \eta \right) \eta^{-2/5} y^{2/3} - 16\pi \eta^{-3/5} y \right. \]

\[ + \left. \left( \frac{15}{80} + \frac{27}{504} \eta \right) \eta^{-4/5} y^{4/3} \right. \]

\[ + \left. \left( \frac{38}{756} - \frac{65}{9} \eta \right) \left[ 1 + \left( \frac{y}{y_{ISCO}} \right) \pi \eta^{-1} y^{5/3} \right] \right\}, \]

\[ \delta \Phi_{1,2} = -\frac{\mu D_L \sqrt{\epsilon - 1}}{2 \sqrt{2x}} \left( 1 + x \pm \sqrt{1 + x^2 + 2x \frac{1 - \kappa \xi^2}{1 + \kappa \xi^2}} \right), \]

where \( y \equiv Mf/(8M_G^3) \), \( M \equiv (m_1 m_2)^{3/5}/(m_1 + m_2)^{1/5} \) is the chirp mass, \( \eta = m_1 m_2/(m_1 + m_2)^2 \) is the symmetric mass ratio, \( t_c \) is the coalescence time and \( \Phi_c \) is the phase at the coalescence, \( \gamma_E = 0.577216 \ldots \) is the Euler constant. \( D_L \) is the luminosity distance to the source. \(^2\)

The first and second terms in Eq. (10) show the contributions of \( h_1 \) and \( h_2 \), respectively. In the above waveform, we can follow the following five parameters as independent parameters for GR, \( \theta_{GR} = \{D_L, m_1, m_2, t_c, \Phi_c \} \). On the other hand, there are 8 independent parameters for the phenomenological bi-gravity model, \( \theta_{MG} = \{ \log \mu^2, \log(\tilde{c} - 1), \kappa \xi^2, \theta_{GR} \} \). In the DFNT subset of the bi-gravity model, \( \log(\tilde{c} - 1) \) is not an independent variable, but it depends on the matter density.

From Eq. (10), we have the formula for the amplitude of the wave in frequency domain.

\[ |h(f)| = A(f) \left( 1 + 2B_1 B_2 (\cos(\Delta \Phi) - 1) \right)^{1/2}, \]

\[ \Delta \Phi \equiv \Phi_1 - \Phi_2. \]

Thus, unless \( B_1 B_2 \) or \( \Delta \Phi \) is zero, amplitude modulation occurs in the bi-gravity waveform which is caused by the interference between two modes. The peak amplitude of the modulated waveform is determined by \( 1 + 2B_1 B_2 (\cos(\Delta \Phi) - 1) \).

Figure 1 shows the frequency-domain gravitational waves \( h(f) \) for different values of the model parameter sets of \((\mu^2, \tilde{c} - 1)\). The curves are plotted for (a) GR (solid (blue)) and for the bi-gravity models with (b) \((\mu^2, \tilde{c} - 1) = (10^{-33} \text{ cm}^{-2}, 10^{-17.8}) \) (dot-dashed (green)), (c) \((10^{-33} \text{ cm}^{-2}, 10^{-18}) \) (long-dashed (red)), and (d) \((10^{-32.8} \text{ cm}^{-2}, 10^{-18.2}) \) (dashed (black)), respectively, at fixed \( \kappa \xi^2 = 100 \). Here we consider BNS at the distance, \( D_L = 200 \text{ Mpc} \). The SNR and the fitting factor between GR waveform and each waveform in this figure become as follows. \( \text{SNR, FF} = (a) (8.7, 1.0), (b) (31, 0.5), \) (c) \((26, 0.47)\), (d) \((21, 0.53)\). Definition of FF is given in Eq. (23).

\(^2\) The phase shifts are not integer powers of PN expansion parameter \( y \).
FIG. 2. The time-domain gravitational waveform $h(t)$. The coalescence time $t_c$ is set to 0. The parameters and the definitions of the curves are the same as those of Fig. 1.

to $x \approx 1$:

$$f_{\text{peak}} = \frac{1}{2\pi} \left( \frac{\mu^2}{2(c-1)} \right)^{1/2}. \quad (18)$$

The corresponding time at the highest peak is given as

$$\tau_{\text{peak}} \equiv t_c - t_{\text{peak}} = \frac{5}{256} \eta f_{\text{peak}}^{8/3} M^5/3. \quad (19)$$

with the total mass $M_t = m_1 + m_2$.

The value of $f_{\text{peak}}$ and $\tau_{\text{peak}}$ for the parameters in Figs. 1 and 2 are (b) (67 Hz, -6.2 s), (c) (107 Hz, -1.8 s), and (d) (169 Hz, -0.5 s), respectively. We can confirm that these values match the location of the highest peaks in Figs. 1 and 2 well.

These large deviations of the waveform from GR are produced by the mixing of the two gravitons, and they depend on the bi-gravity parameters. Thus, these deviations help us put constraints on the bi-gravity with the GW observations.

The amplitude of the peak is determined by Eq. (16). The phase difference at the highest peak, which occurs at $x \approx 1$, becomes,

$$\Delta \delta \Phi \sim \frac{\sqrt{2\mu\sqrt{c} - 1} D_L}{\sqrt{1 + \kappa \xi_c^2}}. \quad (20)$$

For all sets of the bi-gravity parameters in Fig. 1, $\Delta \delta \Phi$ and $B_1 B_2$ at the peak in Eq. (19) take the same value. Thus, there is no difference in the amplification of the

FIG. 3. The same as Fig. 1 but for different values of $\kappa \xi_c^2$ in the case of $(\mu^2, \tilde{c} - 1) = (10^{-33} \text{ cm}^{-2}, 10^{-18})$. The curves are for (a) GR (solid (blue)) and the bi-gravity model with (b) $\kappa \xi_c^2 = 50$ (dot-dashed (green)), (c) $\kappa \xi_c^2 = 100$ (long-dashed (red)) and (d) $\kappa \xi_c^2 = 1000$ (dashed (black)), respectively. Each curve corresponds to $(\text{SNR}, \text{FF}) = (a) (8.7, 1.0), (b) (19, 0.58), (c) (26, 0.47), (d) (34, 0.41).

FIG. 4. The time-domain gravitational waveform $h(t)$. The parameters are the same as those of Fig. 3.
highest peak caused by the bi-gravity effect. The difference of these peak amplitudes in Fig. 1 is just caused by the difference of $A(f_{\text{peak}})$.

In Figs. 3 and 4 we compare the waveforms with different values of $\kappa\xi_h^2$ in the case of $(\mu^2, \tilde{c} - 1) = (10^{-33} \text{ cm}^{-2}, 10^{-18})$. As can be seen in Eq. (18), $f_{\text{peak}}$ does not depend on $\kappa\xi_h^2$. Thus, the peak frequency does not change at all in Figs. 3 and 4. On the other hand, we find in Figs. 3 and 4 that the deviation of the bi-gravity waveforms is larger for larger $\kappa\xi_h^2$. This can be understood as a consequence of larger value of $|B_1B_2|$ for larger $\kappa\xi_h^2$ in Eq. (16).

### III. ANALYSIS METHODS FOR TESTING MODIFIED GRAVITY THEORY

In this section, we briefly review the methods to test the MG theories. Vallisneri [32] has proposed a model comparison analysis of simple MG, and derived a formula that characterizes the possibility to detect the effects of MG on gravitational waves.

First, we define the noise-weighted inner product of signals $h_A$ and $h_B$ by

$$ (h_A|h_B) \equiv 4\text{Re} \int_{f_{\text{min}}}^{f_{\text{max}}} \frac{h_A(f)h_B(f)}{S_n(f)} df, \quad (21) $$

where $S_n(f)$ is the one-sided noise power spectrum density of a detector.

The limits of integration $f_{\text{min}}$ and $f_{\text{max}}$ are taken to be $f_{\text{min}} = f_{\text{low}}$ and $f_{\text{max}} = f_{\text{ISCO}}$ where $f_{\text{low}}$ is the lower cutoff frequency which is defined for each detector, while $f_{\text{ISCO}}$ is the frequency at the innermost stable circular orbit of the binary. We adopt $f_{\text{ISCO}} = (6^{3/2}\pi M)^{-1}$ as an approximation.

The signal-to-noise ratio for a given signal $h$ is its norm defined as

$$ \text{SNR} = |h| = \sqrt{(h|h)}. \quad (22) $$

We also define the fitting factor (FF) [38] which is used to characterize the deviation of a MG waveform from the GR waveform. The FF is defined as

$$ \text{FF}(\theta_{\text{MG}}) = \max_{\theta_{\text{GR}}} \frac{|h_{\text{GR}}(\theta_{\text{GR}})|h_{\text{MG}}(\theta_{\text{MG}})|}{|h_{\text{GR}}(\theta_{\text{GR}})||h_{\text{MG}}(\theta_{\text{MG}})|}, \quad (23) $$

where $h_{\text{GR}}(\theta_{\text{GR}})$ and $h_{\text{MG}}(\theta_{\text{MG}})$ are the GR and MG waveforms, $\theta_{\text{GR}}$ represents the source parameters in GR, and $\theta_{\text{MG}}$ represents the parameters in the MG theory.

By definition, the maximum of FF is 1, which is realized when the MG waveform coincides with the GR waveform. Thus, $1 - \text{FF}$ measures the strength of the MG corrections that cannot be absorbed by the variation of the GR source parameters.

The SNR and FF of each waveform in Figs. 1 and 2 become as follows. (SNR, FF) = (a) $(8.7, 1.0)$, (b) $(31, 0.50)$, (c) $(26, 0.47)$, (d) $(21, 0.53)$. The same values for Fig. 3 become as follows: (SNR, FF) = (a) $(8.7, 1.0)$, (b) $(19, 0.58)$, (c) $(26, 0.47)$, (d) $(34, 0.41)$.

Now, we explain Vallisneri’s formula which is based on the Bayesian hypothesis testing. The Vallisneri’s formula can be used for estimating the SNR value required for discrimination of gravity models based on FF. This analysis is valid for large SNR signals and Gaussian detector noise.

In this method, the odds-ratio is a key quantity which is interpreted as the odds of MG over GR. The Bayesian odds-ratio for MG over GR is defined as

$$ O = \frac{P(\text{MG}|s)}{P(\text{GR}|s)} = \frac{P(\text{MG})P(s|\text{MG})}{P(\text{GR})P(s|\text{GR})}, \quad (24) $$

where $P(\text{MG}|s)$ and $P(\text{GR}|s)$ are the posterior probabilities of the MG and GR hypotheses for a given data $s$, $P(\text{MG})$ and $P(\text{GR})$ are the prior probabilities of the MG and GR hypotheses, and $P(s|\text{MG})$ and $P(s|\text{GR})$ are the fully marginalized likelihood or evidence of the MG and GR hypotheses. The odds-ratio when the data contain a MG signal is given by $O_{\text{MG}} = P(\text{MG}|s_{\text{MG}})/P(\text{GR}|s_{\text{MG}})$, while the odds-ratio when the data contain a GR signal is given by $O_{\text{GR}} = P(\text{MG}|s_{\text{GR}})/P(\text{GR}|s_{\text{GR}})$, where $s_{\text{MG}}$ is the data which contain the MG signal and $s_{\text{GR}}$ is the data which contain the GR signal. Cornish et al. [39] have shown that in the limit of large SNR and small MG deviations, the logarithm of the odds ratio scales as $\text{SNR}_{\text{res}}^2$ where the residual signal-to-noise ratio, $\text{SNR}_{\text{res}}$, is defined as $\text{SNR}_{\text{res}} \equiv \text{SNR}_f - \text{FF}$. We declare the detection of MG when the odds ratio exceeds a certain threshold $O_{\text{thr}}$. We set the threshold $O_{\text{thr}}$ by requiring a given false alarm probability, $F$, which is the fraction of observation in which $O$ happens to exceed $O_{\text{thr}}$ in the case of GR signal. The efficiency of the detection, $E$, is the fraction of observation in which $O$ exceeds $O_{\text{thr}}$ in the case of MG signal. When one computes $E$ as a function of $F$, $E$ is a simple function of the residual signal-to-noise ratio $\text{SNR}_{\text{res}}$.

The formula is given as [32]

$$ E = 1 - \frac{1}{2} \left( \text{erf}(-\text{SNR}_{\text{res}} + \text{erf}^{-1}(F)) - \text{erf}(-\text{SNR}_{\text{res}} - \text{erf}^{-1}(F)) \right), \quad (25) $$

where $z = \text{erf}^{-1}(F)$ is the solution of $\text{erf}(z) = F$. In this paper, we assume $E = 1/2$ and $F = 10^{-4}$. The solution of (24), with $E = 1/2$ and $F = 10^{-4}$, is denoted as $\text{SNR}_{\text{res}} = \text{SNR}_{\text{eff}}$. The SNR required for confident MG detection is then given as $\text{SNR}_{\text{req}} = \text{SNR}_{\text{res}}/\sqrt{1 - \text{FF}}$. We can find that the SNR required to detect 10% deviations from GR (FF $= 0.9$) is 8.699.

Del Pozzo et al. [33] have shown that the scaling that the logarithm of the odds ratio scales as $\text{SNR}_{\text{res}}^2$ holds in the case of 2 or more MG parameters at the lowest order of $(1 - \text{FF})^2$. Thus, Eq. (25) holds for 2 or more MG parameters.

When the bi-gravity signal is detected, the next question is how accurately the bi-gravity parameters can be measured. To quantify the measurement accuracy of pa-
rameters, we compute the standard Fisher matrix,

$$\Gamma_{ab} \equiv \left( \frac{\partial h}{\partial \theta^a} \right) \left( \frac{\partial h}{\partial \theta^b} \right),$$  \hspace{1cm} (26)

which is an $8 \times 8$ matrix in the present context. For sufficiently strong signal, the measurement accuracy of a parameter $\theta^a$ can be evaluated as

$$\Delta \theta^a \equiv \sqrt{(\langle \theta^a \rangle - \langle \theta^a \rangle)^2} = \sqrt{(\Gamma^{-1})^{aa}}.$$  \hspace{1cm} (27)

**IV. PHENOMENOLOGICAL MODEL**

First, we consider the phenomenological model, in which the bi-gravity parameters $\mu^2$, $\tilde{c} - 1$, and $\kappa \xi^2$ are treated as independent parameters, although $\mu^2$ and $\tilde{c} - 1$ are related with each other in the case of the ghost-free bi-gravity. This case is discussed in the succeeding section.

**A. Detectability of the bi-gravity corrections to the waveforms**

In this section, we evaluate the detectable region of the parameters of the bi-gravity theory with the observation of gravitational waves by an advanced laser interferometer. We consider the three cases of binary with masses, $(1.4M_\odot, 1.4M_\odot)$ ($f_{\text{ISCO}} = 1570$ Hz), $(1.4M_\odot, 10M_\odot)$ ($f_{\text{ISCO}} = 386$ Hz), and $(10M_\odot, 10M_\odot)$ ($f_{\text{ISCO}} = 219$ Hz). In this paper, we consider the face-on binaries which are located at the zenith direction from the detector. We thus do not consider the dependence on the inclination, the source location on the sky, and the polarization angle of the wave.

We obtain $\text{SNR}_{\text{res}}$ from Eq. (25) by setting $E = 1/2$ and $F = 10^{-4}$. The detectable region of the bi-gravity correction is the region where $\text{SNR} > \text{SNR}_{\text{res}} = \text{SNR}_{\text{res}}/\sqrt{1-FF}$ is satisfied. Figure 5 shows the detectable region of $(\mu^2, \tilde{c} - 1)$ in the case of $(m_1, m_2) = (1.4M_\odot, 1.4M_\odot)$ and $\kappa \xi^2 = 100$. Curves correspond to the distance to the source $D_L = 200$ Mpc (solid line) and 100 Mpc (dashed line) respectively. The upper-right regions of these lines are the region in which the bi-gravity correction is detectable. The regions shown in Fig. 5 have not been excluded with the solar system experiments yet (see Ref. 22 for the detail.). Thus, this figure shows an interesting possibility to constrain and to detect the bi-gravity correction from CCB.

By comparing the regions in Fig. 5, we find that, the detectable region for $D_L = 100$ Mpc is slightly larger than that for $D_L = 200$ Mpc. The effect of larger SNR for smaller distance turns out not to be very large.

We compare the effect of the masses of the binaries on the detectable region. We consider NSBH with $(m_1, m_2) = (1.4M_\odot, 10M_\odot)$ and BBH with $(m_1, m_2) = (10M_\odot, 10M_\odot)$. We set the distance of these systems so that the SNR in the GR limit is 8.7, which is the value for BNS at 200Mpc. The distance with SNR = 8.7 becomes 416Mpc for NSBH and 980Mpc for BBH. The upper and right-hand-side of the lines in Fig. 5 represents the detectable regions on $(\mu^2, \tilde{c} - 1)$ plane. For simplicity, we do not consider the cosmological redshift effect. We find that, the detectable region in the case of NSBH is slightly smaller than that of BNS. On the other hand, the detectable region is slightly larger for BBH than for BNS.

We also consider the cases with different values of $\kappa \xi^2$. In Fig. 7 we show the detectable region for $\kappa \xi^2 = 50$, 100 and 1000 for BNS at 200Mpc. We find that, the detectable region does not strongly depend on the parameter $\kappa \xi^2$.

**B. Interpretation of the detectable region**

Now, we investigate the origin of the shape of the detectable region in Figs. 5-7. Eq. (18) represents the peak frequency of amplitude of the bi-gravity waveform in the frequency domain as a function of $\tilde{c} - 1$ and $\mu^2$. We
FIG. 6. A plot similar to Fig. 5 but for the waveforms from BNS with $(m_1, m_2) = (1.4 M_\odot, 1.4 M_\odot)$ at 200Mpc (solid), NSBH with $(m_1, m_2) = (1.4 M_\odot, 10 M_\odot)$ at 416Mpc (dashed), and BBH with $(m_1, m_2) = (10 M_\odot, 10 M_\odot)$ at 980Mpc (dot-dashed), respectively. We set $\kappa \xi c^2 = 100$. The detectable region is upper and right-hand side of these curves. SNR of the gravitational waves from these systems in GR limit are 8.7.

We recover the dimension and rewrite Eq. (18) as

$$\tilde{c} - 1 \simeq 1.1 \times 10^{-18} \left( \frac{\mu^2}{10^{-32} \text{cm}^{-2}} \right) \left( \frac{10^3 \text{ Hz}}{f_{\text{max}}} \right)^2.$$  

When the value of $f_{\text{peak}}$ is located within the detector’s sensitivity band, and less than $f_{\text{ISCO}}$, the bi-gravity effects can be detected easily. We take the maximum frequency of the detector’s sensitivity band to be 1000Hz corresponding to the sensitivity curve of advanced LIGO used in this paper. Then, the above equation becomes

$$\tilde{c} - 1 \gtrsim 1.1 \times 10^{-19} \left( \frac{\mu^2}{10^{-32} \text{cm}^{-2}} \right) \left( \frac{10^3 \text{ Hz}}{f_{\text{max}}} \right)^2.$$  

We can see that this equation approximately express the lower boundary of the region for $\mu^2 > 10^{-32} \text{ cm}^{-2}$ in Fig. 5.

As discussed in Sec. II C, the largest effect of bi-gravity model can occur when $x \approx 1$. In such a case, Eq. (15) is rewritten as

$$\tilde{c} - 1 \simeq 1.3 \times 10^{-18} (\Delta \delta \Phi)^2 \left( \frac{10^{-34} \text{cm}^{-2}}{\mu^2} \right) \left( \frac{1000 \text{ Hz}}{f} \right) \left( \frac{\kappa \xi c^2}{100} \right)^{1/2} \times \left( \frac{200 \text{ Mpc}}{D_L} \right)^2.$$  

These two equations can give the lower boundary for $\mu^2$ and $\tilde{c} - 1$. By setting $\Delta \delta \Phi \sim 0.3$, we can see that these two equations represent approximately the lower bound of the detectable region for $\mu^2 \lesssim 10^{-34} \text{cm}^{-2}$ in Fig. 5.

If $\Delta \delta \Phi \neq 0$, the deviation of bi-gravity from GR becomes possible to detect. By setting $(\Delta \delta \Phi) \sim 0.3$, we can see that Eq. (30) roughly represents the lower boundary of the detectable region for $\mu^2 \lesssim 10^{-34} \text{cm}^{-2}$ in Fig. 5.

We can also eliminate $\tilde{c} - 1$ or $\mu^2$ from Eqs. (28) and (30). We obtain

$$\mu^2 \simeq 3.4 \times 10^{-35} (\Delta \delta \Phi) \left( \frac{f}{10 \text{ Hz}} \right) \left( \frac{\kappa \xi c^2}{100} \right)^{1/2} \times \left( \frac{200 \text{ Mpc}}{D_L} \right) \text{ cm}^{-2},$$  

$$\tilde{c} - 1 \simeq 3.9 \times 10^{-20} (\Delta \delta \Phi) \left( \frac{1000 \text{ Hz}}{f} \right) \left( \frac{\kappa \xi c^2}{100} \right)^{1/2} \times \left( \frac{200 \text{ Mpc}}{D_L} \right).$$  

FIG. 7. A plot similar to Fig. 5 but for $\kappa \xi c^2 = 50$ (dashed), 100 (solid), and 1000 (dot-dashed), respectively. The masses are $(m_1, m_2) = (1.4 M_\odot, 1.4 M_\odot)$ and the distance is 200Mpc.
to \( \mu^2 > 10^{-33} \text{ cm}^{-2} \) in Fig. 6.

Other differences are produced by the difference of distance in Eq. (30). For NSBH and BBH, the distance is larger and the lower boundary becomes lower than that of BNS. We can also understand most of the lowest boundary of \( \mu^2 \) and \( \tilde{c} - 1 \) in Fig. 6 from the dependence on the distance of Eqs. (31) and (32). However, the difference between BNS and NSBH of the lowest boundary for \( \mu^2 \) is very small.

In Fig. 7, we see that the difference of \( \kappa \xi^2 \) produces only a small difference in the detectable region. As we saw in Figs. 3 and 4, the amplitude of bi-gravity waveform becomes larger when \( \kappa \xi^2 \) is larger. Thus, SNR of the signal becomes larger. However, from Eqs. (30), (31) and (32), we find that larger \( \kappa \xi^2 \) raises the lower boundary of \( \mu^2 \) and \( \tilde{c} - 1 \). These two effects compensate each other, and the difference of the detectable region becomes very small in Fig. 7. Only the difference we can see is the boundary for \( \mu^2 > 10^{-32} \text{ cm}^{-2} \), for which Eq. (29) determines the boundary. Since Eq. (29) does not depend on \( \kappa \xi^2 \), large SNR for larger \( \kappa \xi^2 \) produces slightly wider detectable region.

Here, we mention the correspondence between Fig. 5 and the contours of the fitting factor between the GR and bi-gravity waveforms, which are plotted in Fig. 8. The FF is computed by maximizing Eq. (23) with respect to \( m_1 \) and \( m_2 \) for each value of \((\mu^2, \tilde{c} - 1)\), at fixed \( \kappa \xi^2 = 100 \). We find that the detectable region of the bi-gravity corrections in Fig. 5 is very similar to the red solid contour of \( \text{FF} = 0.9 \) in Fig. 8. This fact shows that the detectable region in Fig. 5 is almost determined by the value of the fitting factor in this case.

Figure 9 shows the contour of SNR for BNS. By comparing SNR_{req} from Fig. 5 and SNR from Fig. 9, we can obtain the detectable region of Fig. 5 as the region where SNR > SNR_{req} is satisfied.

### C. Constraints on bi-gravity parameters

Next, we evaluate the measurement accuracy of the bi-gravity parameters. We compare the error contour on the \((\mu^2, \tilde{c} - 1)\) plane for the sources at different distances. In order to see the genuine effect of the bi-gravity on the waveform through the different source distance, we renormalize the amplitude of the waveforms so that the signals have the same SNR. In Fig. 10 we show the measurement accuracy in the case of \((\mu^2, \tilde{c} - 1) = (10^{-33} \text{ cm}^{-2}, 10^{-18})\), and for the BNS at 200 Mpc and 100 Mpc, but with SNR renormalized to \( \text{SNR} = 10 \). In this case, the expected accuracy of \( \log \mu^2 \) is \( O(0.1\%) \) at \( 1\sigma \) level. We find that the accuracy is better for the 200 Mpc case. Note that the phase shift, \( \delta \Phi_{1,2} \), in Eq. (15) depends on the distance. For the parameters in Fig. 10 the factor \( 1 + 2B_1 B_2 \cos(\Delta \Phi) - 1 \) is 97.1 for \( D_L = 200 \text{ Mpc} \) and 41.1 for \( D_L = 100 \text{ Mpc} \). Thus, bi-gravity effect is larger for the 200 Mpc case. In Fig. 11 we show the error contour in the case of different parameters of

![FIG. 8. Contour plots of the fitting factor between the GR and bi-gravity waveforms in the \((\mu^2, \tilde{c} - 1)\) parameter space. Here we adopt the model \( \kappa \xi^2 = 100 \). Curves correspond to contours of FF = 0.9 (solid), FF = 0.95 (dashed), and FF = 0.99 (dotted). We assume BNS at \( D_L = 200 \text{ Mpc} \).](image)

![FIG. 9. Contour plots of the SNR of bi-gravity waveforms in the \((\mu^2, \tilde{c} - 1)\) parameter space. The parameters are the same as those of Fig. 5. Curves correspond to contours of SNR = 8.75 (solid), SNR = 10 (dashed), SNR = 18 (dotted), and SNR = 25 (dot-dashed). We assume BNS at \( D_L = 200 \text{ Mpc} \).](image)
$(\mu^2, \tilde{c} - 1) = (10^{-32} \text{ cm}^{-2}, 10^{-19})$. We find the same trend as above: the 1σ error of $\log \mu^2$ is $O(0.1)\%$, and the accuracy is better for the 200 Mpc case.

Finally, we note that the measurement accuracy of the bi-gravity parameters do not strongly depend on the mass of the source. This is because there is no binary’s mass dependence on the amplitude correction factor, $1 + 2B_1B_2(\cos(\Delta \Phi) - 1)$.

V. THE DFNT SUBSET OF THE BI-GRAVITY MODEL

Next, we study the DFNT subset of the bi-gravity model [22], in which the bi-gravity parameters obey the relation

$$\tilde{c} - 1 = 3H_0^2 \frac{\rho_m}{\rho_c} \left( 1 + \kappa c^2 \right), \quad (33)$$

where $H_0$ is the Hubble parameter at the present epoch and $\rho_c$ is the critical density. The value of $\tilde{c} - 1$ is large in the high density region, while it is small in the low density region. We assume GWs are generated in a galaxy where the density is higher than the average density in the intergalactic space. We also assume that GWs experience much lower density during the propagation between galaxies. We neglect the effect of the high density region on the phase corrections $\delta \Phi_1, 2$, and we evaluate the phase corrections by using the background density of the Universe. On the other hand, we assume that the dispersion relations of the modes 1, 2 adiabatically evolve because of the slow evolution of the background. Therefore, by assuming conservation of energy for each mode, we evaluate the amplitude corrections $B_1, 2$ with the average density in the galaxy, $\rho_{\text{gal}}$, where binaries are embedded. Figure 12 shows the gravitational waveforms for the DFNT subset of the bi-gravity model for different values of the average density in the galaxy. Curves in Fig. 12 are for (a) GR (solid (blue)) and for the DFNT subset of the bi-gravity model with $\rho_{\text{gal}} = (b) 10^{5.5} \rho_c$ (dot-dashed (green)), (c) $10^5 \rho_c$ (long-dashed (red)), and (d) $10^4 \rho_c$ (dashed (black)), respectively. We set $(\mu^2, \kappa c^2) = (10^{-32} \text{ cm}^{-2}, 100)$ and $D_L = 200$ Mpc. The gravitational waveforms for the DFNT subset of the bi-gravity model are significantly different from those for the phenomenological bi-gravity model. From Eqs. (18) and (33), we see that $f_{\text{peak}}$ increases as $\mu^2$ increases, $\kappa c^2$ decreases, and $\rho_{\text{gal}}$ decreases, and does not depend on $D_L$. The value of $f_{\text{peak}}$ for the parameters in Fig. 12 are (b) 44 Hz, (c) 78 Hz, and (d) 138 Hz. The SNR and FF of each waveform in Figs 12 become as follows.

$$(\text{SNR}, \text{FF}) = (a) (8.7, 1.0), (b) (26, 0.71), (c) (24, 0.72), (d) (19, 0.73).$$

Figure 13 shows the detectable region of $(\mu^2, \kappa c^2)$ for the DFNT subset of the bi-gravity model in the case...
FIG. 12. The frequency-domain gravitational waves \( h(f) \) for DFNT subset of the bi-gravity model for different values of the average density in the galaxies \( \rho_{\text{gal}} \), where GWs are generated. The curves are plotted for (a) GR (solid (blue)) and for the DFNT subset of the bi-gravity model with (b) \( \rho_{\text{gal}} = 10^{5.5} \rho_c \) (dot-dashed (green)), (c) \( 10^{5.5} \rho_c \) (long-dashed (red)), and (d) \( 10^{4.5} \rho_c \) (dashed (black)), respectively, at fixed \( (\mu^2, \kappa \xi^2) = (10^{-32} \text{ cm}^{-2}, 100) \). Here we consider BNS at the distance, \( D_L = 200 \text{ Mpc} \). The SNR and the fitting factor between GR waveform and each waveform in this figure become as follows. \((\text{SNR}, \text{FF}) = (a) (8.7, 1.0), (b) (26, 0.71), (c) (24, 0.72), (d) (19, 0.73)\).

FIG. 13. The detectable region of the bi-gravity corrections to the waveforms for DFNT subset of the bi-gravity model in the case \((m_1, m_2) = (1.4M_\odot, 1.4M_\odot) \) and \( D_L = 200 \text{ Mpc} \). Curves correspond to the average density in the galaxies \( \rho_{\text{gal}} = 10^{5.5} \rho_c \) (dashed), \( 10^{5} \rho_c \) (solid), and \( 10^{4.5} \rho_c \) (dot-dashed) on the detectable region. Figure 14 shows the detectable region for \( D_L = 100 \text{ Mpc} \) and \( 200 \text{ Mpc} \) for BNS at \( 200 \text{ Mpc} \). The detectable region for \( D_L = 100 \text{ Mpc} \) is slightly larger than that for \( D_L = 200 \text{ Mpc} \). This is because of larger SNR for smaller distance.

VI. SUMMARY AND CONCLUSIONS

In this paper, we investigated the detectability of the ghost-free bi-gravity theory with the observation of gravitational waves from inspiraling compact binaries. Graviton oscillations generate deviations of the gravitational waveform from that of GR. These effects can be used to put constraints on the bi-gravity model.

We calculated modified inspiral waveforms and observed the amplitude modulation due to graviton oscillations in the phenomenological model and in the DFNT subset of the bi-gravity model. We found that there is a characteristic frequency for the peak of the amplitude of the inspiral waveforms which is determined by the bi-gravity parameters.

In order to assess the detectability of the deviation of the waveform from GR prediction due to bi-gravity effects, we used the formula derived by Vallisneri which is based on the Bayesian hypothesis testing, and which
FIG. 14. A plot similar to Fig. 13 but for $D_L = 100$ Mpc (dashed) and 200 Mpc (solid), respectively. The masses are $(m_1, m_2) = (1.4 M_\odot, 1.4 M_\odot)$. We set $\rho_{gal} = 10^2 \rho_c$.

uses the fitting factor to compute the Bayesian odds ratio. With this method, we evaluated the detectability of the deviations of the waveforms by an advanced laser interferometer. We found that there is a region of the parameter space of the bi-gravity model where the deviation can be detected. The detectable region corresponds to the effective graviton mass of $\mu^2 \approx 10^{-34}$ cm$^{-2}$, and the propagation speed of the hidden graviton mode of $\tilde{c} - 1 \approx 10^{-19}$ for the phenomenological model, and $\mu^2 \approx 10^{-34}$ cm$^{-2}$ and $\kappa_\xi^2 \approx 10^{9.5}$ for the DFNT subset of the bi-gravity model.

The shape of the detectable region can be easily understood by using the formula which describe the bi-gravity correction to the waveform. The existence of the detectable region is rather robust and is not strongly affected by the source parameters within the region of interest. We thus conclude that GW observations can be powerful probe of graviton oscillations.

In the phenomenological model, we also studied the possibility to constrain the bi-gravity parameters which characterize graviton oscillations by the observations of the GW from binary inspirals. We found that accuracy in determining the effective graviton mass $\log \mu^2$ is $O(0.1\%)$ for the particular model with $(\mu^2, \tilde{c} - 1) = (10^{-33}$ cm$^{-2}$, $10^{-18})$. We also investigated the dependence of the accuracy on binaries’s masses and the distance to the source.

In this paper, we fixed the distance to the source when we calculated the FF. In the real data analysis, it is possible to determine the distance as well as the direction to the source and the inclination angle by using a network of GW detectors. Even in that case, it would be very helpful if electromagnetic follow-up observations could determine the distance by identifying the host galaxy. Also, we have not included the spins of the stars in the binaries. If spin precession effect exits, there will be an amplitude modulation due to spin precession effect. Such modulation will be mixed with the modification caused by the bi-gravity effects, and the waveform will become more complicated. In such a case, the results in this paper may be changed. Since the spin may not be neglected for black holes, it is important to investigate the effects of spin. We plan to investigate it in the future.

If we consider future detectors such as Einstein Telescope [40], eLISA/NGO [41] or DECIGO/BBO [12-44], it will be possible to detect GWs from coalescing binaries at much larger distance, and at different frequency region. We also plan to investigate such cases in the future.

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