Electric charge quantization in 331 Models with exotic charges

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Abstract

The extensions of the Standard Model based on the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ gauge group are known as 331 Models. Different properties such as the fermion assignment and the electric charges of the exotic spectrum, that defines a particular 331 model, are fixed by a $\beta$ parameter. In this article we study the electric charge quantization in two versions of the 331 models, set by the conditions $\beta = 1/(3\sqrt{3})$ and $\beta = 0$. In these frameworks, arise exotic particles, for instance, new leptons and gauge bosons with a fractional electric charge. Additionally, depending on the version, quarks with non-standard fractional electric charges or even neutral appear. Considering the definition of electric charge operator as a linear combination of the group generators that annihilates the vacuum, classical constraints from the invariance of the lagrangian, and gauge and mixed gauge-gravitational anomalies cancellation, the quantization of the electric charge can be verified in both versions.

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I. INTRODUCTION

The experimental observation where the electric charge of the particles appears only in quantized units is known as the electric charge quantization. The first efforts to explain this phenomenon included ideas related to higher dimensions [1], [2], magnetic monopoles [3], and grand unified theories [4]. However, these approaches have not yet been experimentally verified. In this work, we follow the perspective introduced by [5], [6], [7] and [8]. This approach considers two main conditions, first, classical constraints imposed by the $U(1)_X$ gauge group invariance of the Yukawa lagrangian and second, the quantum restrictions arising from anomalies cancellation. In that sense, the above approach for electric charge quantization will be applied for two versions of the 331 model, set by the conditions $\beta = 1/(3\sqrt{3})$ [9] and $\beta = 0$ [10]. In these versions, new leptons with charge $\pm 2/3\, e$ ($\pm 1/2\, e$), extra quarks with charges $+1/3\, e$, $0$ ($\pm 1/6\, e$), exotic gauge and scalar bosons with charges $\pm 1/3\, e$ and $\pm 2/3\, e$ ($\pm 1/2\, e$) arise, in addition to a new neutral boson $Z'$.

Also, particles with fractional electric charges have been proposed by other theoretical models [11, 12] and both ATLAS and CMS collaborations have already performed searches of new heavy lepton-like particles with non-standard electric charges [13, 14]. Experimentally, these kinds of particles may be misidentified or unobserved since charged particle identification algorithms generally assume that particles have charges of $\pm 1\, e$ [15]. The new proposal maintains the special features of the 331 models, such as the relation between the number of fermionic families and the number of colors in QCD. Reference [8] considers the quantization of the electric charge in a similar fashion but for the minimal 331 model, that is, for $\beta = -\sqrt{3}$. In this work, the authors argue that for models with $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ symmetry, the quantization of the electric charge is verified when they take into account the three families of fermions, considering or not the neutrino masses, in contrast to the SM. On the other hand, reference [16] explains the charge quantization using the general form of the electromagnetic currents under the parity invariance for 331 models with $\beta = -\sqrt{3}$ and $\beta = -1/\sqrt{3}$. Finally, the objective of this study is to verify if the proposed 331 versions, satisfy the quantization of the electric charge.
II. THE MODELS

The general relation for the electric charge operator \( Q \) in a 331 model is given by

\[
Q = \alpha \ T_3 + \beta \ T_8 + \gamma \ X
\]

where \( T_3 \) and \( T_8 \) are the diagonal generators of SU(3)_L built as \( T_i = \frac{\lambda_i}{2} \) from the Gell-Mann matrices \( \lambda_i \), with \( i = 1, \ldots, 8 \), and \( X \) is the charge of \( U(1)_X \). We consider \( \alpha = 1 \) in order to properly set the \( W \) boson electric charge in the model, as was described in [16]. In addition, the value of the \( \beta \) parameter fixes the fermion assignment, the electric charges of the exotic spectrum and it is used to classify different 331 models [17]. We can write:

\[
Q = \text{diag} \left( \pm \frac{1}{2} \left( 1 + \beta \sqrt{3} \right), \frac{1}{2} \left( \mp 1 \pm \beta \sqrt{3} \right), \mp \frac{1}{3} \beta \sqrt{3} \right) + \gamma X I_{3 \times 3}
\]

where the upper sign corresponds to the fundamental representation of the Gell-Mann matrices and the lower sign to the conjugate representation. The Scalar sector,

\[
\eta \sim (1,3,X_\eta), \quad \rho \sim (1,3,X_\rho), \quad \chi \sim (1,3,X_\chi)
\]

which develop vacuum expectation value (vev) as:

\[
\langle \eta^0 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_\eta \\ 0 \\ 0 \end{pmatrix}, \quad \langle \rho^0 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_\rho \\ 0 \end{pmatrix},
\]

\[
\langle \chi^0 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ v_\chi \end{pmatrix}
\]

With the requirement that the charge operator must annihilate the vacuum, we obtain the following general relations:

\[
\gamma = -\frac{1}{2} \left( 1 + \beta \sqrt{3} \right) \frac{1}{X_\eta} = -\frac{1}{2} \left( -1 + \beta \sqrt{3} \right) \frac{1}{X_\rho} = \frac{\beta \sqrt{3}}{3} \frac{1}{X_\chi}
\]

(5)

Thus, for the particular choice of \( \beta = 1/ (3\sqrt{3}) \), we obtain:

\[
\gamma = \frac{1}{9X_\chi}, \quad X_\eta = -5X_\chi, \quad X_\rho = 4X_\chi
\]

(6)
and for $\beta = 0$:

$$\gamma = -\frac{1}{2X_\eta}, \quad X_\rho = -X_\eta, \quad X_\chi = 0. \quad (7)$$

It is straightforward to verify that the scalar fields fulfill the relation

$$X_\rho + X_\eta + X_\chi = 0 \quad (8)$$

In order to cancel anomalies associated with $SU(3)_L$ gauge group, the leptons and the quark families must be assigned in different $SU(3)_L$ representations. So, for $\beta = 1/(3\sqrt{3})$, the leptons and the third quark family are assigned in triplets, while the first two quark families in anti-triplets.

The leptonic sector includes:

$$\psi_{iL} = (\nu_i, e_i^-, E_i^T)_{L} \sim (1, 3, X_{\ell_i}), \quad e_{iR} \sim (1, 1, X_{e_i}), \quad E_{iR} \sim (1, 1, X_{E_i}) \quad (9)$$

where $i = 1, 2, 3$.

The two first quark families form $SU(3)_L$ anti-triplets

$$Q_{aL} = (d_a, -u_a, D_a)^T_L \sim (3, 3^{*}, X_{Q_a}),$$

$$u_{aR} \sim (3, 1, X_u), \quad d_{aR} \sim (3, 1, X_d),$$

$$D_{aR} \sim (3, 1, X_D), \quad a = 1, 2 \quad (10)$$

and the third family is assigned to $SU(3)_L$ triplet:

$$Q_{3L} = (t, b, T)^T_L \sim (3, 3, X_{Q_3}), \quad t_R \sim (3, 1, X_t),$$

$$b_R \sim (3, 1, X_b), \quad T_R \sim (3, 1, X_T) \quad (11)$$

Furthermore, the Yukawa Lagrangian for quarks:

$$\mathcal{L}_{Y}^{\text{quarks}} = f_u^u \overline{Q_{aL}} \rho^* u_{bR} + f_d^d \overline{Q_{aL}} \eta^* d_{bR}$$

$$+ f_u^D \overline{Q_{aL}} \chi^* D_{bR} + f_3^b \overline{Q_{3L}} \rho b_R$$

$$+ f_t^T \overline{Q_{3L}} \eta t_R + f^T \overline{Q_{3L}} \chi T_R + \text{h.c.} \quad (12)$$

The Yukawa Lagrangian for leptons:

$$\mathcal{L}_{Y}^{\text{leptons}} = F_{ij}^e \overline{\psi_{iL}} \rho e_{jR} + F_{ij}^E \overline{\psi_{iL}} \chi E_{jR} + \text{h.c.} \quad (13)$$
On the other hand, for $\beta = 0$ we have leptons and the third quark family in anti-triplets, while the first two quark families in triplets. In this case, the leptonic sector of the model is

$$
\psi_{iL} = \left(e_i^-, -\nu_i, E_i\right)_L^T \sim (1, 3^*, X_i),
$$
$$
e_{iR} \sim (1, 1, X_{e_i}), \quad E_{iR} \sim (1, 1, X_{E_i})
$$

(14)

where $i = 1, 2, 3$.

The two first quark families form $SU(3)_L$ triplets

$$
Q_{aL} = (u_a, d_a, U_a)_L^T \sim (3, 3, X_{Q_a}),
$$
$$
u_{aR} \sim (3, 1, X_{\nu_a}), \quad d_{aR} \sim (3, 1, X_{d_a}),
$$
$$
U_{aR} \sim (3, 1, X_{U_a}), \quad a = 1, 2
$$

(15)

and the third family is assigned to $SU(3)_L$ anti-triplet:

$$
Q_{3L} = (b, -t, T)_L^T \sim (3, 3^*, X_{Q_3}), \quad t_R \sim (3, 1, X_t),
$$
$$
b_R \sim (3, 1, X_b), \quad T_R \sim (3, 1, X_T)
$$

(16)

Finally, for this version the Yukawa Lagrangian for quarks is:

$$
-L_{Y}^{\text{quarks}} = f_{iu} \overline{Q}_{iL} \eta u_{aR} + f_{id} \overline{Q}_{iL} \rho d_{aR}
$$
$$
+ f_{iu} \overline{Q}_{iL} \chi U_{aR} + f_{ia} \overline{Q}_{3L} \eta^* d_{aR}
$$
$$
+ f_{ia} \overline{Q}_{3L} \rho^* u_{aR} + f_{ia} \overline{Q}_{3L} \chi^* U_{aR} + \text{h.c.}
$$

(17)

with $i = 1, 2; a = 1, 2, 3; u_{aR} = u_R, c_R, t_R; d_{aR} = d_R, s_R, b_R$ and $U_{aR} = U_{1R}, U_{2R}, T_R$. The Yukawa Lagrangian for leptons take the form:

$$
-L_{Y}^{\text{leptons}} = F_{ij} \overline{\psi}_{iL} \rho e_{jR} + F_{ij} \overline{\psi}_{iL} \chi E_{jR} + \text{h.c.}
$$

(18)

Now, in order to obtain the different electric charges of the particles in these models, we will use classical and quantum constraints.

5
III. CONSTRAINTS FROM FAMILIES REPLICA

Since the SM particles and the exotic particles in the models present replicas between families, this allows us to reduce the number of hypercharge variables. For $\beta = 1 / (3\sqrt{3})$:

\begin{align}
X_{Q_1} &= X_{Q_2} \equiv X_Q \\
X_{\ell_1} &= X_{\ell_2} = X_{\ell_3} \equiv X_{\ell} \\
X_{u_1} &= X_{u_2} = X_{t} \equiv X_u \\
X_{d_1} &= X_{d_2} = X_{b} \equiv X_d \\
X_{e_1} &= X_{e_2} = X_{e} \equiv X_e \\
X_{E_1} &= X_{E_2} = X_{E_3} \equiv X_E \\
X_{D_1} &= X_{D_2} \equiv X_D
\end{align}

and for $\beta = 0$, we obtain the same conditions, but instead of (25), we have:

\begin{align}
X_{U_1} &= X_{U_2} = X_T \equiv X_U
\end{align}

IV. CONSTRAINTS FROM $U(1)_X$ INVARIANCE

From the $U(1)_X$ invariance of the Yukawa lagrangian we obtain for $\beta = 1 / (3\sqrt{3})$:

\begin{align}
X_Q &= X_d - X_\eta \\
X_Q &= X_D - X_\chi \\
X_Q &= X_u - X_\rho \\
X_{Q_3} &= X_\eta + X_t \\
X_{Q_3} &= X_\chi + X_T \\
X_{Q_3} &= X_\rho + X_b \\
X_e &= X_\ell - X_\rho \\
X_E &= X_\ell - X_\chi
\end{align}
and for $\beta = 0$

\[
X_Q = X_u + X_\eta \quad (35)
\]
\[
X_Q = X_d + X_\rho \quad (36)
\]
\[
X_Q = X_\chi + X_U \quad (37)
\]
\[
X_{Q3} = X_d - X_\eta \quad (38)
\]
\[
X_{Q3} = X_u - X_\rho \quad (39)
\]
\[
X_{Q3} = X_U - X_\chi \quad (40)
\]

the equations (33) and (34) are still being fulfilled for this case.

V. CONSTRAINTS FROM ANOMALIES CANCELLATION

As it is known, an anomaly is a symmetry which has been conserved in the classical theory but is broken at the quantum level. In the context of quantum field theory involving chiral fermions, it is important to cancel gauge anomalies in order to obtain a renormalizable theory. In the present paper, we are focusing on the cancellation of gauge and mixed gauge-gravitational anomalies. Thus, the quantum restrictions arising from anomalies cancellation imply

\[
[SU(3)_C]^2 U(1)_X \rightarrow A_C = 3 \sum_q X_{qL} - \sum_q X_{qR} = 0
\]
\[
[SU(3)_L]^2 U(1)_X \rightarrow A_L = 3 \sum_q X_{qL} + \sum_\ell X_{\ell L} = 0
\]
\[
[\text{Grav}]^2 U(1)_X \rightarrow A_G = 3 \sum_{\ell,q} [X_{\ell L} + 3X_{qL}] - \sum_{\ell,q} [X_{\ell R} + 3X_{qR}] = 0
\]
\[
[U(1)_X]^3 \rightarrow A_X = 3 \sum_{\ell,q} [X_{3\ell L}^3 + 3X_{3qL}^3] - \sum_{\ell,q} [X_{3\ell R}^3 + 3X_{3qR}^3] = 0
\]
where $q_L$ and $\ell_L$ are the doublets, and $q_R$ and $\ell_R$ are the singlets, for the SM and exotic fields, then:

$$
A_C = 3 \{2X_Q + X_{Q_3}\} \\
- (2X_u + 2X_d + 2X_{D,Ux} + X_\ell + X_b + X_T) = 0
$$

(41)

$$
A_L = 3 \{2X_Q + X_{Q_3}\} + 3X_\ell = 0
$$

(42)

$$
A_G = 3 (3X_\ell) - \{3X_e + 3X_E\} = 0
$$

(43)

From equation (43), we have:

$$
A_G = 9X_\ell - 3X_e - 3X_E = 0
$$

(44)

For $\beta = 1/(3\sqrt{3})$:

Using equations (6), (33) and (34) in (44), we obtain:

$$
X_\ell = -5X_\chi
$$

(45)

For $\beta = 0$:

Replacing equations (7), (33) and (34) in (44), we have

$$
X_\ell = X_\eta
$$

(46)

VI. RESULTS

A. For $\beta = 1/(3\sqrt{3})$:

Subtracting equations (27) and (32) and using (22) and (8):

$$
X_{Q_3} = X_Q - X_\chi
$$

(47)

Replacing the equations (45) and (47) in (42), we obtain:

$$
X_Q = 2X_\chi
$$

(48)
Then

\[ X_{Q_3} = X_X, \quad X_D = 3X_X \]
\[ X_T = 0, \quad X_d = -3X_X, \quad X_u = 6X_X \]
\[ X_e = -9X_X, \quad X_E = -6X_X \]

(49)

In this case the charge operators equation (2) is given by:

\[ Q = \text{diag} \left( \pm \frac{5}{9}, \mp \frac{4}{9}, \mp \frac{1}{9} \right) + \frac{X}{9X_X} I_{3 \times 3} \]

(50)

As it was previously explained, the upper sign corresponds to the fundamental representation (triplet) of the Gell-Mann matrices and the lower sign to the conjugate representation (anti-triplet).

For the lepton triplet, we obtain:

\[ Q\psi_L = \left( \text{diag} \left( \frac{5}{9}, -\frac{4}{9}, -\frac{1}{9} \right) - \frac{5}{9} I_{3 \times 3} \right) \psi_L \]
\[ Q\psi_L = \text{diag} \left( 0, -1, -\frac{2}{3} \right) \psi_L \]

(51)

Thus, we find the quantization of the electric charge with the correct electric charges for leptons

\[ Q_{\nu_e,\mu,\tau} = 0, \quad Q_{e,\mu,\tau} = -1, \quad Q_{E,M,T} = -\frac{2}{3} \]

(52)

and for the quark anti-triplet:

\[ QQ = \text{diag} \left( -\frac{1}{3}, \frac{2}{3}, \frac{1}{3} \right) Q \]

(53)

\[ Q_{d,s} = -\frac{1}{3}, \quad Q_{u,c} = \frac{2}{3}, \quad Q_{D_1,D_2} = \frac{1}{3} \]

(54)

for the quark triplet

\[ QQ_3 = \text{diag} \left( \frac{2}{3}, -\frac{1}{3}, 0 \right) Q_3 \]

(55)

\[ Q_b = -\frac{1}{3}, \quad Q_t = \frac{2}{3}, \quad Q_T = 0 \]

(56)
B. For $\beta = 0$

Substracting equations (36) and (38) and using (7):

$$X_{Q_3} = X_Q \quad (57)$$

Replacing the equations (46) and (57) in (42), we obtain:

$$X_Q = -\frac{1}{3} X_\eta \quad (58)$$

Then

$$X_{Q_3} = -1/3 X_\eta, \quad X_{U,T} = -1/3 X_\eta$$

$$X_d = 2/3 X_\eta, \quad X_u = -4/3 X_\eta$$

$$X_e = 2 X_\eta, \quad X_E = X_\eta \quad (59)$$

and

$$Q = \text{diag} \left( \mp 1/2, \mp 1/2, 0 \right) - \frac{X}{2X_\eta} I_{3 \times 3} \quad (60)$$

For the lepton triplet

$$Q\psi_L = \left( \text{diag} \left( -1/2, 1/2, 0 \right) - \frac{1}{2} I_{3 \times 3} \right) \psi_L$$

$$Q\psi_L = \text{diag} \left( -1, 0, -1/2 \right) \psi_L \quad (61)$$

and

$$Q_{\nu_e,\nu_\mu,\nu_\tau} = 0, \quad Q_{e,\mu,\tau} = -1, \quad Q_{E,M,T} = -1/2 \quad (62)$$

For the quarks triplet

$$QQ_3 = \text{diag} \left( -1/3, 2/3, -1/6 \right) Q \quad (63)$$

$$Q_b = -1/3, \quad Q_t = 2/3, \quad Q_T = -1/6 \quad (64)$$

Finally, for the quarks, anti- triplet:

$$QQ = \text{diag} \left( 2/3, -1/3, -1/6 \right) Q_3 \quad (65)$$

$$Q_{u,c} = 2/3, \quad Q_{d,s} = -1/3, \quad Q_{U_1,U_2} = -1/6 \quad (66)$$
In this work, we have considered two versions of the 331 model, with the particular feature of containing extra leptons with fractional electric charges and non-standard electric charges for the new quarks. By considering constraints from the classical and quantum level, we have shown, for both versions $\beta = 1/ (3\sqrt{3})$ and $\beta = 0$, that the quantization of the electric charge can be obtained by using the Yukawa sector and the chiral anomalies cancellation when the three families are taken together and independent of the neutrino, as happens in the others 331 versions. As it can be observed from our procedure, different $\beta$ values produce different constraint equations as a result of imposing the $U(1)_\chi$ invariance of the Yukawa Lagrangian and the anomalies cancellation. This is due to the parameter $\beta$ fixes the fermion representations in the multiplets of the group. In that sense, in our approach, we think that the extension of the charge quantization for an arbitrary $\beta$ is not straightforward. An analysis using, the general form of the electromagnetic currents under parity invariance for arbitrary beta, and the cancellation of chiral anomalies for two specific values of the mentioned parameter, allows to obtain the quantization of the electric charge as was shown in the reference [16].

[1] O. Klein, The Atomicity of Electricity as a Quantum Theory Law, Nature 118 (1926) 516.
[2] Arun, M.T., Saha, P. Gravitons in multiply warped scenarios: At 750 GeV and beyond. Pramana - J Phys 88, 93 (2017).
[3] P. A. M. Dirac, Quantised singularities in the electromagnetic field, Proc. Roy. Soc. A133 (1931) 60.
[4] J. C. Pati and A. Salam, Lepton number as the fourth "color", Phys. Rev. D10 (1974) 275; H. Georgi and S. L. Glashow, Unity of All Elementary-Particle Forces, Phys. Rev. Lett. 32 (1974) 438; L. B. Okun, M. B. Voloshin and V. I. Zakharov, Electrical neutrality of atoms and grand unification models, Phys. Lett. B138 (1984) 115.
[5] K. S. Babu and R. N. Mohapatra, Is there a connection between quantization of electric charge and a Majorana neutrino?, Phys. Rev. Lett 63, 938 (1989); K. S. Babu and R. N. Mohapatra, Quantization of electric charge from anomaly constraints and a Majorana neutrino, Phys. Rev
D41, 271 (1990).

[6] R. Foot, G. C. Joshi, H. Lee and R. R. Volkas, Charge quantization in the standard model and some of its extensions, Mod. Phys. Lett. A5, 2721 (1990); R. Foot, New Physics From Electric Charge Quantization?, Mod. Phys. Lett A6, 527-530, (1991); R. Foot et al, Electric-charge quantization, J. Phys. G: Nucl. Part. Phys. 19 361, (1993).

[7] Felice Pisano, A simple solution for the flavor queestion, Mod. Phys. Lett. A11, 32n33, pp. 2639-2647 (1996).

[8] Pires, C. A. de S. and Ravinez, O. P., Electric charge quantization in a chiral bilepton gauge model, Phys. Rev. D 58, 035008, (1998).

[9] E. Ramirez Barreto and D. Romero Abad, Heavy long-lived fractionally charged leptons in novel 3 − 3 − 1 model, arXiv:1907.02613 [hep-ph].

[10] Le Tho Hue and Le Duc Ninh, The simplest 3-3-1 model, Mod. Phys. Lett. A, Vol. 31, No. 10 (2016) 1650062.

[11] Paul Langacker and Gary Steigman, Requiem for a fractionally charged, massive particle, Phys. Rev. D 84, 065040 (2011).

[12] S. Davidson, B. Campbell, and D. Bailey, Limits on particles of small electric charge, Phys. Rev. D 43, 2314 (1991).

[13] ATLAS collaboration, Search for magnetic monopoles and stable particles with high electric charges in 8 TeV pp collisions with the ATLAS detector, Phys. Rev. D 93, 052009 (2016).

[14] ATLAS collaboration, Search for heavy long-lived multi-charged particles in pp collisions at $\sqrt{s} = 8$ TeV using the ATLAS detector, Eur. Phys. J. C (2015) 75: 362.

[15] Tobias Golling, LHC searches for exotic new particles, Progress in Particle and Nuclear Physics, 90:156-200, (2016).

[16] Phung Van Dong and Hoang Ngoc Long, Electric charge quantization in $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ models, Int. J. Mod. Phys. A 21, No. 32, pp. 6677-6692 (2006).

[17] Diaz, Rodolfo A. and Martinez, R. and Ochoa, F., SU(3)(c) x SU(3)(L) x U(1)(X) models for beta arbitrary and families with mirror fermions, Phys.Rev. D72 (2005) 035018 hep-ph/0411263.