Breakpoint Region in the IV-characteristics of Intrinsic Josephson Junctions

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Abstract. We study theoretically the IV-characteristics of intrinsic Josephson junctions in HTSC. We solve numerically a set of differential equations for N intrinsic Josephson junctions and investigate the nonlinear dynamics of the system. The charging effect is taken into account. We demonstrate that the breakpoint region in the current-voltage characteristics naturally follows from the solution of the system of the dynamical equations for the phase difference. In the breakpoint region the plasma mode is a stationary solution of the system and this fact might be used in some applications, particularly, in high frequency devices such as THz oscillators and mixers.

1. Introduction

A system of intrinsic Josephson junctions (IJJ) in layered superconductors is a perspective object for superconducting electronics and its phase dynamics has attracted a great interest.[1, 2] In Refs.[3, 4, 5] we demonstrated the breakpoint in the current voltage characteristics (IVC) of IJJ in the framework of the capacitively coupled Josephson junctions model with diffusion current (CCJJ+DC model).[6] The breakpoint current as a function of the coupling and dissipation parameters for the stacks with different number of junctions was studied. We showed that the coupling between junctions change crucially the dependence of the return current on dissipation parameter for single junction. Particularly, it leads to the appearance of a plateau on the dissipation dependence of the breakpoint current and oscillation of the breakpoint current as a function of dissipation parameter. Using the idea that a parametric resonance is approached at the breakpoint and longitudinal plasma wave is created, we modeled the coupling and dissipation dependence of the breakpoint current and obtained a good agreement with the results of the numerical simulation.

Different couplings between junctions determine a variety of current-voltage characteristics (IVC) observed in HTSC (see Ref.[7] and reference there). Other two theoretical models are widely used to describe IJJ: capacitively coupled Josephson junctions (CCJJ) model [8, 9, 10, 11] and charge imbalance (CIB) model [12, 13]. In CCJJ model a non-vanishing generalized scalar potential appears due to the breaking of charge neutrality, but in CIB model it is related to the quasiparticle charge imbalance as well. Probably both effects exist in HTSC because the
thickness of superconducting layers is smaller than the Debye length and the characteristic length of disequilibrium relaxation.

In this paper we demonstrate that the breakpoint phenomenon is not specific for CCJJ+DC model only, but it exists in the framework of CCJJ and CIB models as well.

2. Results and Discussion

We develop here an idea that a parametric resonance is approached at the breakpoint and a longitudinal plasma mode is excited by Josephson oscillations. [3, 4, 5]

In the CCJJ model the dynamics of the gauge-invariant phase difference $\varphi_{l,l+1}$ between superconducting layers $l$ and $l+1$ is described by the equation

$$\frac{d^2 \varphi_{l,l+1}}{d\tau^2} = (1 - \alpha \nabla^{(2)})(J - \sin(\varphi_{l,l+1})) - \beta \frac{d\varphi_{l,l+1}}{d\tau}$$

(1)

where $\alpha$ is the coupling parameter, $J$ is the external dc current normalized to the Josephson critical current $J_c$, respectively. The derivative operator $\nabla^{(2)}$ is defined as $\nabla^{(2)} f_l = f_{l+1} + f_{l-1} - 2f_l$. To obtain the dimensionless form of the equation we have used $\tau = \omega_p t$, $\omega_p^2 = 2eJ_c/hC$ and $\beta = 1/\omega_p RC$, where $R$ is the resistance and $C$ is the capacity of the junctions.

The equation for the Fourier component of the difference of phase differences $\delta_l = \varphi_{l+1,l} - \varphi_{l,l-1}$ between neighbor junctions is

$$\dot{\delta}_k + \beta(k)\delta_k + \cos(\Omega(k)\tau)\delta_k = 0,$$

(2)

where $\tau = \omega_p(k)t$, $\omega_p(k) = \omega_p C$, $\beta(k) = \beta/C$, $\Omega(k) = \Omega/C$ and $C = \sqrt{1 + 2\alpha(1 - \cos(k))}$. This equation shows a parametric resonance with changing its parameters $\beta(k)$ and $\Omega(k)$.

We note that in comparison with CCJJ+DC model, where $\beta(k) = \beta C$, the effective dissipation parameter $\beta(k)$ in this model is less than $\beta$. We may conclude that with increase in $\alpha$ it is more favorable for the system to have the plasma waves. We solved the Eq. (2) numerically and result is presented in Fig. 1(left), where the resonance region for the stack of 10 IJJ at $\alpha = 1$, $\beta = 0.2$ and $\gamma = 0$ is shown. For $\Omega(k)$ and $\beta(k)$ inside of the resonance regions the solution of the equation increases in time. It means that we observe a parametric resonance in this region. Outside of this region, starting from any initial condition, the solutions attenuate in time to zero.

![Figure 1](image)

Figure 1. Left- Phase diagram in $\Omega(k) - \beta(k)$ space for the stack of 10 IJJ at $\alpha = 1$, $\beta = 0.2$ and $\gamma = 0$; Right - One loop current voltage characteristics at different $\alpha$. 

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Figure 2. The total branch structure in the IVC of 10 IJJ for different values of nonequilibrium parameter \( \eta \).

In the CIB model the dynamics of the gauge-invariant phase difference \( \varphi_{l,l+1} \) between superconducting layers \( l \) and \( l + 1 \) is described by the equation

\[
\frac{d^2\varphi_{l,l+1}}{dt^2} = \left( 1 - \alpha \nabla^{(2)} \right) \left( J - \sin(\varphi_{l,l+1}) - \psi_l + \psi_{l+1} - \beta \frac{d\varphi_{l,l+1}}{dt} \right) - \frac{\dot{\psi}_l - \dot{\psi}_{l+1}}{\beta} \tag{3}
\]

and kinetic equations

\[
\begin{align*}
\dot{\psi}_0 &= \eta \gamma_d (b J - \beta \dot{\varphi}_{0,1} + \psi_1 - \psi_0) - \psi_0 \\
\dot{\psi}_l &= \eta \beta (\dot{\varphi}_{l-1,l} - \dot{\varphi}_{l,l+1}) + \nabla^{(2)} \dot{\psi}_l - \psi_l \\
\dot{\psi}_n &= \eta \gamma_d (3 \dot{\varphi}_{n,n+1} - b J + \psi_{n-1} - \psi_n) - \psi_n
\end{align*} \tag{4}
\]

In these equations \( J \) is the external current in \( J_c \) units, parameter \( b \) determines the injection of quasiparticles from boundaries, \( \psi_l \) - the charge imbalance potential on superconducting layers \[12], \( \alpha \) - the coupling parameter, \( \beta \) is related with the McCumber parameter \( \beta_c \) as \( \beta = 1/\sqrt{\beta_c} \), \( \eta \) - the nonequilibrium parameter, \( \zeta = \omega_p \tau_{qp} \), \( \omega_p \) - the plasma frequency, \( \tau_{qp} \) - the charge imbalance relaxation time, \( \gamma_d = s/s_0 = s/s_N \) and \( s_0, s_N \) are the thickness of the first and last \( S \)-layers, respectively. The Laplacian on the boundaries is defined as \( \nabla^{(2)} f_{0,1} = f_{1,2} - (1 + \gamma) f_{0,1} \), \( \nabla^{(2)} f_{N-1,N} = f_{N-2,N-1} - (1 + \gamma) f_{N-1,N} \).

To obtain the voltage we use the generalized Josephson relation

\[
\frac{V_{l,l+1}}{\beta J_c R} = (1 - \alpha \nabla^{(2)})^{-1}(\dot{\varphi}_{l,l+1} + \frac{\psi_l - \psi_{l+1}}{\beta}) \tag{5}
\]

We have solved numerically this system of equations using fourth order Rounge-Kuta method in the presence of very small noise with maximum \( 10^{-10} \). We show the effect of \( \alpha \) on the breakpoint current in the outermost branch in Fig. 1(right), where one loop results for different \( \alpha \) at \( \eta = 0.3, \beta = 0.2, \zeta = 100 \) and \( \gamma = 0 \) are presented. As we can see, the increase in \( \alpha \) leads to the increase in the breakpoint current value. The width of the breakpoint region (the distance between the breakpoint and transition to another branch) is growing with increase in coupling parameter at chosen values of the other parameters.

Results of two simulations of the total branch structure for the stack of 10 IJJ at different values of \( \eta \) and \( \zeta \) are shown in Fig. 2. The structure in the IVC is close to the equidistant one and the both IVC demonstrate the breakpoint regions on their outermost branches.
A detailed investigation of the influence of the model parameters on the vortex structure in the CIB model will be presented somewhere else.

In the CIB model we find the equation for the Fourier component of the difference of phase differences $\delta_l = \varphi_{l+1,t} - \varphi_{l,t-1}$ between neighbor junctions

$$\dot{\delta}_k + \cos(\varphi)\delta_k + (\beta(k) - \frac{\eta(k)}{\zeta})\delta_k + (1 + \frac{\eta(k) + 1}{\zeta\beta(k)})\eta(k)\beta(k) \int_0^\tau dt\delta_k e^{\frac{i\pi(k)(t-\tau)}{\sqrt{\eta(k)}}}$$

$$+ \Delta\psi\eta e^{\frac{i\pi(k)}{\sqrt{\eta(k)}}} = 0$$

(6)

where the dimensionless parameters $\psi'$ and $\eta(k)$ are defined as $\psi' = 2(1 - \cos(k))\psi$ and $\eta(k) = 2(1 - \cos(k))\eta$. Other parameters have the same form as in CCJJ model. As we can see, the dynamical equations are different for different modes $k$ even at $\alpha = 0$. To find the resonance regions for parameters of the system, we have solved the equation (6) numerically. The results of calculations are presented in Fig. 3. The decrease in $\Omega(k)$ means the decrease in the voltage, so we may conclude from the Fig. 3(left) that for $\beta(k)$ smaller than some value $\beta_0$, the increase in $\eta(k)$ increases the voltage value of the breakpoint, while for $\beta(k)$ bigger than $\beta_0$, the increase in $\eta$ decreases the breakpoint voltage. Fig. 3(right) demonstrates a decrease of the breakpoint voltage with the increase in $\beta$.

3. References

[1] R. Kleiner, F. Steinmeyer, G. Kunkel and P. Muller, Phys. Rev. Lett. 68, 2394 (1992).
[2] G. Oya, N. Aoyama, A. Irie, S. Kishida, and H. Tokutaka, Jpn. J. Appl. Phys., 31, L829 (1992).
[3] Yu. M. Shukrinov, F. Mahfouzi, Supercond. Sci. Technol., 19, S38-S42 (2007).
[4] Yu. M. Shukrinov, F. Mahfouzi, N. F. Pedersen, Phys. Rev. B 75, 104508 (2007).
[5] Yu. M. Shukrinov, F. Mahfouzi, Phys. Rev. Lett 98, 157001 (2007).
[6] Yu.M.Shukrinov, F.Mahfouzi, P.Seidel. Physica C449, 62 (2006).
[7] Yu.M.Shukrinov and F.Mahfouzi, J.of Phys.:Conf.Series 43, 1143 (2006).
[8] T. Koyama and M. Tachiki, Phys. Rev. B 54, 16183 (1996)
[9] H.Matsumoto, S.Sakamoto, F.Wajima, T. Koyama, M. Machida, Phys. Rev. B 60, 3666 (1999)
[10] M.Machida, T. Koyama, Phys. Rev. B 70, 024523 (2004)
[11] Yu.M.Shukrinov and F.Mahfouzi, Physica C434, 6 (2006).
[12] D. A. Ryndyk, Phys. Rev. Lett. 80, 3376 (1998); D. A. Ryndyk, J. Keller, and C. Helm, J. Phys.: Condens. Matter 14, 815 (2002)
[13] D. A. Ryndyk, J. Keller, Phys. Rev. B 71, 054507 (2005)