Control system for nonlinear dynamic processes in a buck converter

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Abstract. The paper deals with a control system for nonlinear dynamic processes in a buck converter based on the target-oriented control and belonging to the class of discrete automatic control systems. The main task of the proposed system is to stabilize the desired dynamic mode, which provides a minimum amplitude of output voltage fluctuation. The author proposes a hardware-software calculation method of the fixed point coordinates in the desired periodic mode, when the average values of state variables are determined by hardware using low-frequency filters, and the fluctuation amplitude of the output physical quantity in the desired dynamic mode is estimated on the basis of analytical expressions. Computer simulation of the proposed system is performed and its efficiency is shown. The proposed approach to estimating the coordinates of the fixed point of the desired mode in real time can be applied to a wide class of DC/DC converters.

1. Introduction
Pulse-width DC/DC converters are widely used in modern industry because they provide efficient power conversion [1]. At the same time, they need to be further improved in order to improve their technical characteristics. The main indicator of the quality of the output DC voltage is the fluctuation amplitude, the value of which is determined by the technical specification for the design of the converter. The drift of a number of converter parameters, such as input voltage, load resistance, and reference action is known to have a significant effect on the character of output physical value fluctuation. In this case, as a result of bifurcations, periodic, quasi-periodic and chaotic fluctuation with a large amplitude may occur, which is associated with the loss of stability of the desired dynamic mode or the influence of interference when the system operates in the area of multistability, when both the desired 1-cycle and undesired modes are stable [2].

Pulse-width DC/DC converters are known to be closed-loop automatic control systems (ACS) with feedback on the required physical value (output voltage or current). They belong to the class of nonlinear dynamical systems, which in some cases requires the application of the theory of bifurcations and chaos at the design stage (the so-called bifurcation approach) in order to exclude undesired dynamic modes with a high fluctuation amplitude of the output physical quantity. The main task in this case is to ensure the stability of the desired dynamic mode (1-cycle), when the frequency of output voltage fluctuations coincides with the frequency of pulse-width modulation. As a rule, the minimum amplitude of output voltage fluctuation is observed in this mode. Undesired periodic modes usually have the cycle ratio of $m$, i.e. the fluctuation frequency is $m$ times less than the pulsed-width modulation (PWM) frequency [2].
The most optimal approach to design is structural-algorithmic synthesis, associated with the construction of specific control systems with the function of controlling nonlinear dynamic processes, operating on the basis of specialized control algorithms [3-7]. The most well-known methods for controlling dynamic processes of nonlinear systems include: the time-delay feedback control (TDFC), the method of linearization of the Poincare map, and the target-oriented control (TOC). These methods were originally developed by specialists in the field of cybernetic physics, so the number of publications devoted to their practical application in technical systems is significantly lower than the number of theoretical publications.

One of the most promising methods is the target-oriented control [5], which was first used by the author to control nonlinear dynamic processes of DC/DC converters. As suggested in [5], this method allows you to eliminate undesired dynamic modes in a wider range of system parameters than other methods, but for the control system to work, you need to know the coordinates of the fixed point of the desired mode, the determination of which is the most important scientific problem when using this method.

The point mapping method is known to be used for the analysis of nonlinear dynamical systems [8]. In this case, the mathematical description of the dynamic system is restricted a stroboscopic map of the form

\[ X_k = \Psi(X_{k-1}) , \]

where \( X_{k-1} \) is the vector of state variables at the end of the \( k-1 \) clock interval, \( X_k \) is the vector of state variables at the end of the \( k \)-th clock interval. The fixed point of a stroboscopic map is the point that the display \( \Psi \) maps to itself. So, in the case of the desired dynamic mode (1-cycle), when the frequency of fluctuation at the output of the converter is equal to the PWM frequency, the fixed point is described by the equation

\[ X^* = \Psi(X^*) . \]

The essence of the point mapping method is explained in figure 1.

![Figure 1. Explanation of the point mapping method.](image)

Here \( \gamma_1 \) is the 1-cycle trajectory, \( \gamma_2 \) is the 2-cycle trajectory. In this case, a plane is drawn in the phase space so that the trajectory intersects it at a non-zero angle. The fixed point of the map is the point where the trajectory of this plane intersects. In the case of a 2-cycle (orange trajectory), when the fluctuation frequency is half the PWM frequency, two points are displayed on the specified plane. The desired 1-cycle can be settled by introduction of additional control actions \( u_k \) (green trajectory).

As mentioned earlier, in order to implement the TOC in the conditions of changing system parameters, it is necessary to constantly evaluate the coordinates of the fixed point of the 1-cycle that needs to be stabilized. In [5], to determine the coordinates of the fixed point, the author proposed using pre-trained neural networks, which imposes certain requirements on the computing resources of the control microcontroller. In addition, it introduces a certain static error in the operation of the control...
system. Work [7] is devoted to obtaining analytical expressions for calculating the fixed point of the desired mode. To do this, the system’s variable parameters (input voltage, load resistance) are measured, and then the coordinates of the fixed point are calculated based on analytical expressions that are quite complex and can be difficult to use in real time.

In this paper, we propose a hardware-software method for estimating the coordinates of the fixed point of the desired mode in real time, based on simplifying the computational problem solved by the microcontroller, since a number of calculations are replaced by the results of measurements on a real object, which increases the accuracy of calculating the fixed point of the desired mode. The results presented in this paper are obtained for the first time and can be extended to other types of DC/DC converters.

2. The control system of DC/DC converters with nonlinear dynamics processes control

This section discusses a modified control system for nonlinear dynamic processes based on the target-oriented control. The main task in this case is to implement a hardware-software method for determining the coordinates of the desired fixed point of the 1-cycle in order to reduce the requirements for the microcontroller.

A functional diagram of an ACS based on the buck converter using TOC, described by a system of 2nd-order differential equations, is shown in figure 2. In the figure, the following notation is used: PWC is the pulse-width converter, NDPCS is the nonlinear dynamic processes (NDP) control system, MC is the master clock, RG – ramp generator, R is the the inductor active resistance, L – the inductor inductance, C is the the capacitor capacitance, \( R_{LD} \) is the load resistance, \( VD \) is the power diode, \( VT \) is the power transistor, \( U_{in} \) is the input voltage, \( U_{ref} \) is the voltage reference signal, \( U_{err} \) is the voltage error signal, \( U_{con} \) is the control signal, \( U_{rmp} \) is the ramp generator signal, \( \alpha \) is the proportional gain of the voltage controller, \( \beta \) is the scale factor of the voltage feedback circuit, \( U_{LD} \) is the load voltage, \( I_{LD} \) is the load current, \( x_i \) is the \( i \)-th phase variable, \( U_{mc} \) is the output signal of the master clock generator, \( u_k \) is the correction signal, \( U_p \) – control pulsed voltage.

The control system consists of the main control system, which provides stabilization of the average value of the output voltage and NDPCS, which provides a set dynamic mode, introducing every clock interval the corrective action \( u_k \) in the loop of the general control system.

Figure 2. Buck converter based on the automatic control system with control of nonlinear dynamic processes.

Figure 3 shows the NDPCS function diagram. The following designations are accepted here: \( \beta_i \) is the scale factor of the \( i \)-th phase variable, \( SH_i \) is the sample and hold circuit of the \( i \)-th phase variable, \( SB_i \) is the subtractors, \( PAB \) is the parameters adaptation block, \( FPC \) is the fixed point computer, \( K_i \) is the proportionality coefficients for the \( i \)-th phase variable, \( \Delta K_i \) is the deviation of \( K_i \), \( x_{ik} \) is the \( i \)-th phase variable at the \( k \)-th stroboscopic moment of time, \( u_{ik} \) is the deviation component for the \( i \)-th phase variable, \( x_{iref} \) – \( i \)-th coordinate of fixed point, \( ADD \) is the adder.
The essence of the TOC is as follows. The phase variables of the $x_{ik}$ system are stored using $SH_i$, which are clocked by the $MC$ master clock generator at the beginning of the $k$-th clock interval. The $FPC$ fixed point computer determines the coordinates of the required fixed point of the desired periodic mode (1-cycle) [5].

Then, using $SB_i$ subtractors the deviation of the current operating point from the $i$-th coordinate of the specified fixed point of the stroboscopic map is calculated for each $i$-th phase variable, followed by scaling with the coefficient $K_i$

$$ u_{ik} = K_i \left( x_{i \text{ref}} - x_{ik} \right). $$

To calculate the corrective action on the $k$-th clock interval, the corrective actions for each phase variable are summed, i.e.

$$ u_k = \sum_{i=1}^{n} u_{ik}. $$

It is obvious that when the system reaches the given fixed point $x_{i \text{ref}}$, the correction effect $u_k$ takes the zero value. The optimal values of the $K_i$ coefficients are calculated by $PAB$ using the method [6] in order to ensure the stability of the desired mode.

![Figure 3. Control system for nonlinear dynamic processes.](image)

Thus, it can be concluded that the control system of a pulse voltage converter with the function of controlling nonlinear dynamic processes consists of two parallel subsystems, one of which provides stabilization of the average value of the output voltage and it is the leading subsystem, and the second one is discrete and guided and provides stabilization of the fixed point of the 1-cycle, having a minimal impact on the general control system. However, research shows that such an impact is still possible due to the additive nature of corrective actions, which leads to an additional, although insignificant, static error due to the error in estimating the coordinates of the fixed point of the desired 1-cycle. At the same time, the main system, being the leading one, has a direct by introduction on the guided NDPCS, since it is the general control system that sets the coordinates of the fixed point of the desired mode, which are used when working with NDPCS.

The $FPC$ block calculates the coordinates of the fixed point in the desired dynamic mode. Figure 3 shows that the input of the $FPC$ block is supplied with a vector of phase variables of the system, as well as three changing parameters of the system: input voltage, load current and reference voltage. Based on
this information, the coordinates of the fixed point of the desired 1-cycle $x_i$ are issued on the FPC output. Obviously, the requirements for computing resources in the software implementation of the FPC algorithm should be minimal.

3. Method for estimating the fixed point of the desired mode in real time

Let's consider the proposed method for estimating the coordinates of the fixed point of the desired 1-cycle in real time using the hardware-software method. Figure 4 shows time diagrams of the inductor current and the capacitor voltage of the buck converter [1]. Red dots indicate the coordinates of the fixed point that correspond to samples of state variables at clock points in time. It is obvious that the position of the fixed point of the display coincides with the position of a certain point, the coordinates of which are determined by the average values of the inductor current and the voltage on the capacitor, but taking into account their high-frequency fluctuation. Figure 4 shows that the coordinates of the fixed point of map for the buck converter are calculated as

$$I_{fp} = I_{L,avg} - \Delta i_L;$$

$$U_{fp} = U_{c,avg};$$

where $U_{c,avg}, I_{L,avg}$ are the average values of capacitor voltage and inductor current, $\Delta i_L$ is the amplitude of the inductor current fluctuation.

In the proposed method, when estimating the coordinates of the fixed point in real time, the average values of state variables are determined by digitizing the voltages after filtering using analog filters, the input of which is supplied with state variables. In this case, the author used low-pass filters of the 2nd order with a transfer function

$$W(p) = \frac{K}{T_1 p^2 + T_2 p + 1}.$$ 

Also, in the case of using microcontrollers with a DSP core, it is possible to use higher-order digital filtering or any other approach that allows you to determine the average value with high accuracy by averaging over a sufficiently long period of time. The choice of a long period of time for averaging is related to the potential complexity of the emerging dynamic modes.

![Figure 4. To the explanation of the evaluation method of the fixed point periodic mode.](image)

The functional diagram of the proposed FPC is shown in figure 5. Here $F_i$ is the low-pass filter for the phase variable $x_i$, $\beta_i$ is the scale factor of the $i$-th phase variable, as in figure 3, $\Delta x_i$ is the absolute correction for the $i$-th phase variable. In the case being considered $\Delta x_1 = \Delta i_L$, $\Delta x_2 = 0$.

To calculate $\Delta x_1 = \Delta i_L$ the analytical expression that was obtained earlier by the author can be used [7]

$$\Delta i_L = \frac{(U_{in} - U_{c,avg}) (R + R_{ld})}{2LR_{ld} U_{in}} U_{c,avg} a.$$
4. Simulation of an automatic control system with the function of controlling nonlinear dynamic processes based on a buck converter

This section presents the simulation results of a control system based on the proposed fixed point estimation method. Simulation of a buck converter was performed with the following system parameters: the inductance of the inductor $L=0.1\ \text{H}$, the condenser capacity $C=1\ \text{uF}$, the resistance of the inductor active resistance $R=1\ \text{Ohm}$, the load resistance $R_{LD}=50\ \text{Ohm}$, the gain of the proportional controller $\alpha=80$, the feedback gain $\beta=0.01$, the amplitude of the ramp voltage $U_{\text{ramp}}=10\ \text{V}$, the PWM period $a=0.0001\ \text{s}$, $T_1=0.1\ \text{s}$, $T_2=0.1\ \text{s}$.

The simulation results are shown in figure 6 in the form of dynamic mode maps in the space of $U_{\text{ref}}$ and $\alpha$ parameters and time diagrams.

On dynamic mode maps, the symbols $D_i$ mark the areas of different modes, where the symbol $i$ is the multiplicity of the cycle $m$, and the symbol $j$ is the number of the periodic mode with the multiplicity $i$ on the map. For example, $D_{1,1}$ is the area of existence of the desired 1-cycle. The areas $D_{ch,j}$ correspond to chaotic modes of operation of the converter ($m\rightarrow\infty$).

As it can be seen from figure 6(b) the use of NDPCS as part of the control system leads to the disappearance of undesired dynamic modes in the entire selected range of system parameters. At the same time, without using the NDP control (figure 6(a)). The areas of undesired modes are quite large and the map contains both the areas of undesired periodic modes and the areas of chaotic fluctuation (marked in gray in figure 6(a)).

Figure 7 show the time diagrams which correspond to the dynamic mode map in figure 6(b).
Figure 7. $U_{ref}=4.1$ V, $U_{in}=1000$ V.

The time diagrams of the output voltage ($u_c$) are marked in black, and the output signal of the second order filter determining $U_{c,avg}$ is marked in red. At the moment $t=0.1$ s NDPCS starts working and the system switches from the chaotic mode to the 1-cycle with a small pulsation amplitude. The figure shows that before the start of NDPCS, transition processes in the filters $F1$ and $F2$ must end (figure 5).

5. Conclusion
The proposed hardware-software method for estimating the coordinates of the fixed point of the desired mode in real time reduces the requirements for computing resources of the master microcontroller, which in this sense allows us to bring the hardware requirements of this method closer to the hardware requirements of the method with time-delay feedback while maintaining the advantages of TOC. In addition, hardware estimation of the average values of state variables by measuring them on a real object allows you to provide greater compliance with the reality of control actions generated by NDPCS.

The proposed approach to estimating the coordinates of the fixed point of stroboscopic map can be applied to a wide class of transistor DC/DC converters, taking into account the use of specific expressions for calculating the $\Delta x_i$ corrections.

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