Abstract

We construct the type IIA nonsupersymmetric meta-stable brane configuration consisting of \((2k + 1)\) NS5-branes and D4-branes where the electric gauge theory superpotential has an order \((2k+2)\) polynomial for the bifundamentals. We find a rich pattern of nonsupersymmetric meta-stable states as well as the supersymmetric stable ones. By adding the orientifold 4-plane to this brane configuration, we also describe the intersecting brane configuration of type IIA string theory corresponding to the meta-stable nonsupersymmetric vacua of corresponding gauge theory.
1 Introduction

The dynamical supersymmetry breaking in meta-stable vacua [1, 2] occurs in the $\mathcal{N} = 1$ gauge theory with massive fundamental flavors. The mass term for the quarks in the electric superpotential has led to the fact that some of the F-term equations of magnetic superpotential cannot be satisfied and the supersymmetry is broken. The meta-stable brane constructions of type IIA string theory have been studied in [3, 4, 5].

In [6, 7], other kind of the type IIA non-supersymmetric meta-stable brane configuration was constructed by considering the additional quartic term for the quarks in the electric superpotential besides the mass term. This extra deformation in the supersymmetric gauge theory corresponds to the rotation of D6-branes along the (45)-(89) directions in type IIA string theory. Since the extra quartic term gives rise to the fact that all the F-term equations are satisfied, only supersymmetric ground states are present classically. The non-supersymmetric ground states arise only after the gravitational attraction of NS5-brane is considered.

By adding the orientifold 6-plane to this brane configuration [6], the meta-stable non-supersymmetric brane configuration corresponding to the supersymmetric gauge theory with symmetric flavor as well as fundamental flavors was found [8]. For the antisymmetric flavor plus fundamental flavors case, the corresponding meta-stable brane configuration was also described in [9]. Moreover, the meta-stable brane configuration consisting of three NS5-branes, D4-branes and anti-D4-branes was constructed in [10] by deforming the theory described in [11].

As suggested in [6], what happens when “multiple” NS-branes are rotated relatively? One expects, in the gauge theory side, that the higher order term for the bifundamentals besides the mass term appears in the superpotential. Since there exist multiple outer NS-branes, a rich pattern of non-supersymmetric meta-stable states is possible because the flavor D4-branes can suspend between these two multiple NS-branes in many different way. We’ll elaborate the meta-stable vacuum structure both in the gauge theory side and the type IIA string theory side.

First, we construct the meta-stable brane configuration by rotating and displacing these multiple NS-branes relatively, taking the Seiberg dual, and splitting, reconnecting or displacing the flavor and color D4-branes. When these multiple NS-branes are coincident, then the behavior for the meta-stable brane construction looks similar to the single NS-brane case [10]. However, when they are not coincident with each other, many different pattern for the meta-stable brane construction occurs. Secondly, we describe the meta-stable brane configuration by adding an orientifold 4-plane to this brane configuration and following the previous
method done in unitary gauge group, analyze the vacuum structure for the different gauge theory with matters, and present the different aspects in the presence of orientifold 4-plane.

In section 2, we review the type IIA brane configuration corresponding to the $\mathcal{N} = 1$ $SU(N_c) \times SU(N'_c)$ gauge theory with the bifundamentals and deform this theory by adding both the mass term and the higher order term for the bifundamentals. Then we construct the dual $\mathcal{N} = 1$ $SU(\tilde{N}_c) \times SU(N'_c)$ gauge theory with corresponding dual matter as well as gauge singlet. We describe the nonsupersymmetric meta-stable minimum and present the corresponding intersecting brane configuration of type IIA string theory.

In section 3, we review the type IIA brane configuration corresponding to the $\mathcal{N} = 1$ $Sp(N_c) \times SO(2N'_c)$ gauge theory with a bifundamental and deform this theory by adding the mass term and the higher order term for the bifundamental. Then we describe the dual $\mathcal{N} = 1$ $Sp(\tilde{N}_c) \times SO(2N'_c)$ gauge theory with corresponding dual matters. We construct the nonsupersymmetric meta-stable minimum and present the corresponding intersecting brane configuration of type IIA string theory.

In section 4, we make some comments for the future directions.

2 Meta-stable brane configuration with $(2k+1)$ NS-branes

2.1 Electric theory

The type IIA brane configuration for an $\mathcal{N} = 1$ supersymmetric gauge theory with gauge group $SU(N_c) \times SU(N'_c)$ and a bifundamental $X$ in the representation $(N_c, N'_c)$ and its conjugate field $\tilde{X}$ in the representation $(\overline{N_c}, \overline{N'_c})$ can be constructed as follows \cite{12 13}: the middle NS5-brane(012345), the left $k$ $NS5'_L$-branes(012389), the right $k$ $NS5'_R$-branes(012389), $N_c$- and $N'_c$-color D4-branes(01236) where the number of $NS5'_L,R$-branes is a positive integer $k \geq 1$.

The $N_c$-color D4-branes are suspending between the middle NS5-brane whose $x^6$ coordinate is $x^6 = 0$ and the $NS5'_R$-branes while the $N'_c$-color D4-branes are suspended between the $NS5'_L$-branes and the middle NS5-brane. We consider the arbitrary numbers of color D4-branes with the constraint $N'_c \geq N_c$.

Let us deform this theory which has vanishing superpotential. Displacing the two kinds of $NS5'_L,R$-branes relative each other in the $v$ direction corresponds to turning on a quadratic superpotential for the bifundamentals $X$ and $\tilde{X}$. Rotating these $NS5'_L,R$-branes in the $(v, w)$ plane corresponds to turning on a quartic($k = 1$) or higher order($k \geq 2$) superpotential for the bifundamentals $X$ and $\tilde{X}$ \cite{12 13}. Then the deformed electric superpotential, by changing $\overline{X}$ 1 We introduce two complex coordinates $v \equiv x^4 + ix^5$ and $w \equiv x^8 + ix^9$ \cite{15}.
the position of the left $k$ NS5$_L'$-branes only, can be written as \[12\, 14\, 16\]

$$W_{elec} = -\frac{\alpha}{2} \text{tr}(X\bar{X})^{k+1} + \text{tr} mX\bar{X}, \quad \alpha = \frac{\tan \theta}{\Lambda}, \quad m = \frac{v_{NS5-\theta}}{2\pi \ell_s^2}. \quad (2.1)$$

The $k$ NS5$_L'$-branes are moving to the $+v$ direction together with $N'_c$ D4-branes and are rotating by an angle $-\theta$ in $(w, v)$-plane. Then the $v$ coordinate of NS5$_L'$-branes is denoted by $v = +v_{NS5-\theta}$ and we denote the NS5$_L'$-branes by the NS5-\theta-branes after deformation. Definitely, the number of multiple NS-branes $k$ should be less than $N'_c$ because we'll see, in the magnetic brane configuration, that the $N'_c$-flavor D4-branes are suspended between two $k$ NS-branes. We draw the electric brane configuration in Figure 1.

![Figure 1: The $\mathcal{N} = 1$ supersymmetric electric brane configuration for the gauge group $SU(N_c) \times SU(N'_c)$ and the bifundamentals $X$ and $\bar{X}$ with nonvanishing mass and higher order terms for the bifundamentals. Note that there are multiple outer NS-branes. A “rotation” of $k$ NS5$_L'$-branes in $(w, v)$-plane, which become NS5-\theta-branes, corresponds to a higher order term for the bifundamentals while a “displacement” of $k$ NS5$_L'$-branes in $+v$ direction corresponds to a mass term for the bifundamentals.]

The solution for the supersymmetric vacua can be written as $X\bar{X} = \left[\frac{2m}{\alpha(k+1)}\right]^\frac{1}{2}$ through the F-term conditions. This breaks the gauge group $SU(N_c) \times SU(N'_c)$ to $SU(p_0), SU(N'_c - N_c + p_0)$, and $U(p_1) \times U(p_2) \times \cdots \times U(p_k)$ with $\sum_{i=0}^k p_i = N_c$. If $p_0$ is the number of eigenvalues which are zero with $X = 0 = \bar{X}$. When the middle NS5-brane moves to $+w$ direction, then the three kinds of NS-branes intersect in three points in $(v, w)$-plane. Then we consider that $p_0$ D4-branes are connecting between the middle NS5-brane and the NS5$_R'$-branes. One can decompose $N_c$ D4-branes as $p_0$ plus $(N_c - p_0)$ D4-branes. The latter can be reconnected with the same number of D4-branes among $N'_c$ D4-branes. Then the remaining
$(N'_c - N_c + p_0)$ D4-branes are connecting between the $NS5_{-\theta}$-branes and the middle NS5-brane. Then $(N_c - p_0)$ D4-branes are suspended between $k$ $NS5_{-\theta}$-branes and $NS5'_R$-branes. By separating these NS-branes transversely, each $p_i$ D4-branes (where $i = 1, 2, \cdots, k$) are connecting between the $i$-th $NS5_{-\theta,i}$-brane and the $i$-th $NS5'_R,i$-brane directly.

2.2 Magnetic theory

Let us apply the Seiberg dual to the $SU(N_c)$ factor only and the middle NS5-brane and the right $NS5'_R$-branes are interchanged each other \cite{15}. Then the number of color $\tilde{N}_c$ is given by $\tilde{N}_c = N'_c - N_c$ connecting between the $NS5'_R$-branes and the NS5-brane. One can easily check that the linking numbers of three kinds of NS-branes are preserved during this dual process. For example, the linking number of $NS5'_R$-brane in an electric theory is given by $l_e = -\frac{N_c}{k}$ while the one in magnetic theory is given by $l_m = \frac{\tilde{N}_c}{k} - \frac{N'_c}{k}$. By introducing $N'_c$ D4-branes and $N'_c$ anti-D4-branes between $NS5'_R$-branes and NS5-brane, reconnecting the former with the $N'_c$ D4-branes (that are connecting between the $NS5_{-\theta}$-branes and the $NS5'_R$-branes) and moving those combined D4-branes to $+v$-direction, one gets the final Figure 2 (with $l = 0$) and there exist $(N'_c - \tilde{N}_c)$ anti-D4-branes between $NS5'_R$-branes and NS5-brane when the distance between the multiple NS-branes is large \cite{10}.

The dual gauge group is given by $SU(\tilde{N}_c = N'_c - N_c) \times SU(N'_c)$ and the matter contents are the field $Y$ in the representation $(\tilde{N}_c, N'_c)$, its complex conjugate field $\tilde{Y}$ in the representation $(N'_c, \tilde{N}_c)$ and the gauge singlet $M \equiv X \tilde{X}$ in the representation $(1, N'_c^2 - 1) \oplus (1, 1)$. Then the dual magnetic superpotential, by adding the mass term and the higher order term for the bifundamentals $X$ and $\tilde{X}$ from an electric theory \cite{24} to the cubic superpotential between the dual magnetic matters, is given by

$$W_{dual} = \frac{1}{\Lambda} \text{tr} M Y \tilde{Y} - \frac{\alpha}{2} \text{tr} M^{k+1} + \text{tr} MM. \quad (2.2)$$

Note that in the first term, the trace means that the $\tilde{N}_c$ indices of $Y$ and $\tilde{Y}$ are contracted each other. The two $N'_c$ indices of $M$ are contracted with those of $Y$ and $\tilde{Y}$ respectively. The trace in the second and third terms runs over the $N'_c$ indices \footnote{As pointed out in \cite{10}, the conditions $b_{SU(N'_c)}^{mag} < 0$ and $b_{SU(N_c)}^{mag} > 0$ imply that $N'_c < \frac{3}{2} N_c$. Then the range for the $N'_c$ can be written as $N_c < N'_c < \frac{3}{2} N_c$. At the scale $\Lambda_1$, the $SU(N_c)$ theory is strongly coupled and the Seiberg duality occurs. The coefficients of beta function $b_{SU(N'_c)}^{mag}$ becomes negative and $b_{SU(N_c)}^{mag}$ becomes positive. Then at energy scale lower than Landau pole $\Lambda_1$, the theory is weakly coupled. Then under the constraint, $A_2 << (\frac{\Lambda_1}{\mu})^{\frac{3}{2}} \Lambda_1 << \Lambda_1$ where $b \equiv \frac{b_{SU(N'_c)}^{mag} - b_{SU(N_c)}^{mag}}{b_{SU(N'_c)}^{mag}}$, one can ignore the contribution from the gauge coupling of $SU(N'_c)^{mag}$ at the supersymmetry breaking scale. Then one can use the magnetic superpotential safely. Then a further generalization looks straightforward once the tuning of the parameters is chosen in this way.}. In order to obtain the
Figure 2: The $\mathcal{N} = 1$ supersymmetric magnetic brane configuration corresponding to Figure 1 with a misalignment between D4-branes when the gravitational potential of the NS5-brane is ignored. The $N'_c$ flavor D4-branes connecting between $NS5_{-\theta}$-branes and $NS5'_R$-branes are splitting into $(N'_c - l)$ and $l$ D4-branes. The intersection between $NS5_{-\theta}$-branes and $(N'_c - l)$ D4-branes is given by $(v, w) = (0, +v_{NS5_{-\theta}} \cot \theta)$ while the one between $NS5_{-\theta}$-branes and $l$ D4-branes is given by $(v, w) = (+v_{NS5_{-\theta}}, 0)$.

supersymmetric vacua, one computes the F-term equations for the superpotential (2.2):

\[
MY = 0, \quad \tilde{Y}M = 0,
\]

\[
-\frac{1}{\Lambda} \tilde{Y} \tilde{Y} = m - \frac{\alpha(k+1)}{2} M^k.
\]

(2.3)

Let us first consider all the $NS5_{-\theta}$-branes and $NS5'_R$-branes are coincident with each other. Later, we’ll split them in transverse direction.

• Coincident $NS5_{-\theta}$-branes and $NS5'_R$-branes

Since the eigenvalues for the meson field $M$ are either 0 or \( \left[ \frac{2m}{\alpha(k+1)} \right]^\frac{1}{k} \), through F-term equations, one takes $N'_c \times N'_c$ matrix $M$ with $l$’s eigenvalues 0 and $(N'_c - l)$’s eigenvalues \( \left[ \frac{2m}{\alpha(k+1)} \right]^\frac{1}{k} \) as follows:

\[
M = \begin{pmatrix}
0 & 0 \\
0 & \left[ \frac{2m}{\alpha(k+1)} \right]^\frac{1}{k} \mathbf{1}_{N'_c - l}
\end{pmatrix}
\]

(2.4)

where $l = 1, 2, \cdots, N'_c$. In the brane configuration of Figure 2, the $l$ of the $N'_c$-flavor D4-branes are connected with $l$ of $\tilde{N}_c$-color D4-branes and the resulting $l$ D4-branes stretch from
the $NS5_\theta$-branes to the NS5-brane directly and the intersection point between the $l$ D4-branes and the NS5-brane is given by $(v, w) = (+v_{NS5_\theta}, 0)$. This corresponds to exactly the $l$'s eigenvalues 0 of $M$ in (2.4). Now the remaining $(N'_c - l)$-flavor D4-branes between the $NS5_\theta$-branes and the NS5'-brane correspond to the remaining eigenvalues of $M$ in (2.4), i.e., $\left[\frac{2m}{\alpha(k+1)}\right]^\frac{1}{k+1} 1_{N'_c-l}$. The intersection point between the $(N'_c - l)$ D4-branes and the NS5'-brane is given by $(v, w) = (0, +v_{NS5_\theta} \cot \theta)$ from trigonometric geometry [6, 10].

By substituting (2.4) into the last equation of (2.3), one obtains the following expectation value

$$Y\tilde{Y} = \left( \begin{array}{cc} -m\Lambda 1_l & 0 \\ 0 & 0 \end{array} \right). \quad (2.5)$$

In the $l$-th vacuum the gauge symmetry is broken to $SU(\tilde{N}_c - l)$ and the supersymmetric vacuum drawn in Figure 2 with $l = 0$ has $Y\tilde{Y} = 0$ and the gauge group $SU(\tilde{N}_c)$ is unbroken. If we replace $k$ $NS5_\theta$-branes with $N_f$ D6-branes and $k$ NS5'-brane with a single NS5'-brane with $l = \tilde{N}_c$, then the meta-stable brane configuration of Figure 2 reduces to the one in [3, 4, 5, 18, 19].

On the other hand, the theory has many nonsupersymmetric meta-stable ground states due to the fact that there exists an attractive gravitational interaction between the flavor D4-branes and the NS5-brane from the DBI action [11]. When we rescale the meson field as $M = h\Lambda \Phi$, then the Kahler potential for $\Phi$ is canonical and the magnetic quarks $Y$ and $\tilde{Y}$ are canonical near the origin of field space. Then the magnetic superpotential (2.2) can be rewritten in terms of $\Phi, Y$ and $\tilde{Y}$

$$W_{dual} = h\Phi Y\tilde{Y} + \frac{\mu \phi}{2} h^{k+1} \text{tr} \Phi^{k+1} - h\mu^2 \text{tr} \Phi$$

with the new couplings $\mu^2 = -m\Lambda$ and $\mu_\phi = -\alpha \Lambda^{k+1}$.

Now one splits the $(N'_c - l) \times (N'_c - l)$ block at the lower right corner of $M$ and $Y\tilde{Y}$ into blocks of size $n$ and $(N'_c - l - n)$ and then (2.4) and (2.5) are rewritten as follows [7]:

$$h\Phi = \left( \begin{array}{ccc} 1_l & 0 & 0 \\ 0 & h\Phi_n & 0 \\ 0 & 0 & \left[\frac{2m}{\mu_0(k+1)}\right]^\frac{1}{k+1} 1_{N'_c-l-n} \end{array} \right), \quad Y\tilde{Y} = \left( \begin{array}{ccc} \mu^2 1_l & 0 & 0 \\ 0 & \varphi \tilde{\varphi} & 0 \\ 0 & 0 & 0 1_{N'_c-l-n} \end{array} \right). \quad (2.7)$$

Here $\varphi$ and $\tilde{\varphi}$ are $n \times (\tilde{N}_c - l)$ matrices and correspond to $n$-flavors of fundamentals of the gauge group $SU(\tilde{N}_c - l)$ which is unbroken. One can move $n$ D4-branes, from $(N'_c - l)$ flavor D4-branes stretched between the $NS5_\theta$-branes and the NS5'-brane at $w = +v_{NS5_\theta} \cot \theta$, to the local minimum of the potential and the end points of these $n$ D4-branes are at a nonzero
In the brane configuration from Figure 3, \( \varphi \) and \( \bar{\varphi} \) correspond to fundamental strings connecting between the \( n \)-flavor D4-branes and \((\tilde{N}_c - l)\)-color D4-branes. Moreover, the \( h\Phi_n \) and \( \varphi\bar{\varphi} \) are \( n \times n \) matrices. The supersymmetric ground state corresponds to the vacuum expectation values by \( h\Phi_n = \left[ \frac{2\mu^2}{\mu \phi (k+1)} \right]^{\frac{1}{2}} 1_n \) and \( \varphi\bar{\varphi} = 0 \).

The full one loop potential for \( \Phi_n, \varphi, \) and \( \bar{\varphi} \) from (2.6) and (2.7) including the one loop result [1] takes the form

\[
\frac{V}{\| h \|^2} = |\Phi_n \varphi|^2 + |\Phi_n \bar{\varphi}|^2 + \left| \varphi \bar{\varphi} - \mu^2 1_n + \frac{(k+1)h^k}{2} \mu \Phi_n^k \right|^2 + b\| h \mu \|^2 \text{tr} \Phi_n^i \Phi_n, \tag{2.8}
\]

where the positive numerical constant \( b \) is given by \( b = \frac{(\ln 4 - 1)}{8\pi^2} (N'_{c} - N_c) \). Differentiating this potential (2.8) with respect to \( \Phi_n^i \) and putting \( \varphi = 0 = \bar{\varphi} \), one obtains

\[
\left[ -\mu^2 1_n + \frac{(k+1)h^k}{2} \mu \Phi_n^k \right] \frac{k(k+1)}{2} (h^*)^k \mu^* (\Phi_n^i)^{k-1} + b\| h \mu \|^2 \Phi_n = 0. \tag{2.9}
\]

Sine the higher order term superpotential plays the role of small perturbation and we assume that \( \mu^* \mu < \mu \ll \Lambda_m \) [7], the second term of above will be negligible and one gets

\[
\frac{k(k+1)}{2} \mu^2 (h^*)^k \mu^* (\Phi_n^i)^{k-1} \simeq b\| h \mu \|^2 \Phi_n. \tag{2.10}
\]
In order to see the vacuum structure, we consider the particular case where all the parameters are real including the $\Phi_n$ with $k > 2^3$. The general case is, in principle, straightforward. Then one arrives at the following form from \( (2.10) \)

$$
\begin{align*}
  h\Phi_n &\approx \left[ \frac{2b}{k(k+1)\mu_\phi} \right]^{\frac{1}{k-2}} 1_n, \\
  M_n &\approx \left[ \frac{\widetilde{N}_c}{\alpha k(k+1)\Lambda^3} \right]^{\frac{1}{k-2}} 1_n.
\end{align*}
$$

(2.11)

Then the vacuum energy $V$ is given by $V \approx nh^2\mu^4$ and expanding around this solution, one obtains the eigenvalues for mass matrix for $\varphi$ and $\widetilde{\varphi}$: $m_\pm^2 \approx \left[ \frac{2b}{k(k+1)\mu_\phi} \right]^{\frac{2}{k-2}} \pm |h\mu|^2$. For the positive value for these eigenvalues one should have $\left[ \frac{2b}{k(k+1)\mu_\phi} \right]^{\frac{2}{k-2}} > h\mu$ which leads to the quarks $\varphi$ and $\widetilde{\varphi}$ are massive and then the vacuum (2.11) is locally stable. Also note that $|h\Phi_n| << \Lambda_m$.

It is evident that the $({N'_c} - l - n)$ flavor D4-branes between the $NS5_{-\theta}$-branes and the $NS5'_{R}$-branes are related to the corresponding eigenvalues of $h\Phi$ (2.7), i.e., $\left[ \frac{2b}{k(k+1)\mu_\phi} \right]^{\frac{1}{k-2}} 1_{N'_c - l - n}$ and the intersection point between the $({N'_c} - l - n)$ D4-branes and the $NS5'_{R}$-branes is also given by $(v, w) = (0, +v_{NS5_{-\theta}} \cot \theta)$. Moreover, the remnant $n$ flavor D4-branes between the $NS5'_{R}$-branes and the $NS5_{-\theta}$-branes are related to the corresponding eigenvalues (2.11) of $h\Phi_n$.

- Separated $NS5_{-\theta}$-branes and $NS5'_{R}$-branes

Let us displace the $k$ $NS5_{-\theta}$-branes and $NS5'_{R}$-branes given in Figure 2 in the $v$ direction respectively to two different points denoted by $v_{NS5_{-\theta},j}$ and $v_{NS5'_{R},j}$ where $j = 1, 2, \cdots, k$. The color and flavor D4-branes attached to them are displaced as well. The number of color D4-branes stretched between $j$-th $NS5'_{R,j}$-brane and the NS5-brane is denoted by $\widetilde{N}_c,j$ while the number of flavor D4-branes stretched between the $j$-th $NS5'_{R,j}$-brane and the $j$-th $NS5'_{R,j}$-brane is denoted by $N'_{c,j}$. Many possible supersymmetric configurations, la-

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3For $k = 2$ case, one can express the expectation value from (2.9) exactly: $h\Phi_n = \frac{2}{\mu_\phi} (\mu_\phi - b) \mu_\phi \mu_\phi 1_n$.

4The stability of the SUSY breaking vacua requires $\left[ \frac{2b}{k(k+1)\mu_\phi} \right]^{\frac{2}{k-2}} > h\mu$ and this implies that $h < \frac{\mu_\phi}{\mu_\phi} << 1$. As one flows to IR, the dimensionless coupling $\mu_\phi E^{k-2}$ (the mass dimension of $\mu_\phi$ is equal to $2 - k$) at the energy scale $E$ becomes smaller and smaller when $k > 2$. At the SUSY breaking scale $E = \sqrt{h}\mu$, this dimensionless coupling becomes $h^{\frac{1}{k-2}} \mu_\phi(h\mu)^{k-2}$. When $k > 2$, the first factor $h^{\frac{1}{k-2}}$ is large. Then the second factor should behave as $\mu_\phi(h\mu)^{k-2} << 1$ for the higher order deformation to be small. This is compatible with stability condition. However, the masses of $\varphi$ and $\widetilde{\varphi}$ become large when $k > 2$ and are integrated out. Let us emphasize here that the magnetic theory can stay the IR free region under the condition given in the footnote 2. That is, $\sqrt{h}\mu << \frac{1}{\mu_\phi} << \left( \frac{\Lambda_1}{\Lambda_1} \right)^b \Lambda_1 << \Lambda_1$. On the other hand, in the quartic case($k = 1$) where the quartic deformation becomes relevant operator in the magnetic description, the qualitative difference arises since the first factor $h^{\frac{1}{k-2}} << 1$, as long as $\frac{\mu_\phi}{\mu_\phi}$ is not too large then the above requirement is valid. In other words, the masses of $\varphi$ and $\widetilde{\varphi}$ remain small enough compared to the SUSY breaking scale.
beled by sets of nonnegative integers \((\tilde{N}_{c,1}, \tilde{N}_{c,2}, \cdots, \tilde{N}_{c,k})\) and sets of nonnegative integers \((N'_{c,1}, N'_{c,2}, \cdots, N'_{c,k})\) with the initial conditions on the flavor- and color- D4-branes where \(l = 0\)

\[
\sum_{j=1}^{k} \tilde{N}_{c,j} (\equiv N'_{c,j} - N_{c,j}) = \tilde{N}_c, \quad \sum_{j=1}^{k} N'_{c,j} = N'_c \tag{2.12}
\]

can arise when there is no splitting of color and flavor D4-branes. When all the \(NS5'_{R,j}\)-branes and \(NS5_{-\theta,j}\)-branes \((j = 1, 2, \cdots, k)\) are distinct, the low energy physics corresponds to \(k\) decoupled supersymmetric gauge theories with gauge groups

\[
\prod_{j=1}^{k} SU(\tilde{N}_{c,j}) \times SU(N'_{c,j}).
\]

As we approach the origin of parameter space, \(v_{NS5'_{R,j}} = 0\) and \(v_{NS5_{-\theta,j}} = v_{NS5_{-\theta}}\) for all \(j\), the full \(SU(\tilde{N}_c) \times SU(N'_c)\) gauge group is restored.

Let us focus on \(j\)-th \(NS5_{-\theta,j}\)-brane, \(j\)-th \(NS5'_{R,j}\)-brane, \(N'_{c,j}\) flavor D4-branes, \(\tilde{N}_{c,j}\) color D4-branes and the NS5-brane. Then the submeson field \(M_j\) on this particular brane realization is defined as \(X_j \tilde{X}_j\) and is in the representation \((1, N'_{c,j})^2 - 1) \oplus (1, 1)\) under the gauge group \(SU(\tilde{N}_{c,j}) \times SU(N'_{c,j})\). Note that a bifundamental \(X_j\) in an electric theory is in the representation \((N_{c,j}, \tilde{N}'_{c,j})\) and its conjugate field \(\tilde{X}_j\) is in the representation \((\tilde{N}'_{c,j}, N'_{c,j})\). Generic \((v_{NS5_{-\theta,j}} - v_{NS5'_{R,j}})\), which is equal to \(2\pi \ell_s^2 m_j\), is related to a polynomial superpotential for \(M_j\). One can consider all the mass terms for the bifundamental for all \(j\). More explicitly, the expression \([m_j 1_{N'_{c,j}} - \frac{\alpha(k+1)}{2} M_j^k]\) (where \(j = 1, 2, \cdots, k\)) is nothing but the derivative of superpotential with respect to the \(M_j\) for \(Y_j \tilde{Y}_j = 0\), from the F-term equations \((2.13)\), and has \(k\) distinct minima \(\left[\frac{2m_j}{\alpha(k+1)}\right]^{\frac{k}{2}}\) plus zero with some multiplicities. Note that the F-term equations imply that there exist “zero” eigenvalues. Here \(Y_j\) and \(\tilde{Y}_j\) are dual fields corresponding to \(X_j\) and \(\tilde{X}_j\).

One can write the derivative of superpotential, in terms of its eigenvalues \(x\), with respect to the full meson \(M\) for \(Y \tilde{Y} = 0\) as

\[
W'_{\mathrm{dual}}(x) = \sum_{i=0}^{N'_c} s_i x^{N'_c-i} \equiv - \frac{\alpha(k+1)}{2} \prod_{j=1}^{k} x^{l_j} \left( x - \left[ \frac{2m_j}{\alpha(k+1)} \right]^{\frac{k}{2}} \right)^{-N'_{c,j}-l_j} \tag{2.13}
\]

where we put the sum of multiplicities of zero eigenvalues as nonzero \(l\)

\[
\sum_{j=1}^{k} l_j = l, \quad 0 \leq l_j \leq \tilde{N}_{c,j}. \tag{2.14}
\]
The order of the polynomial (2.13) is $N'_c$. The integers $(N'_{c,1}, N'_{c,2}, \cdots, N'_{c,k})$ are the number of the nonzero plus zero eigenvalues of the meson $M$ residing in the different minima. Thus the set of 

$$(N'_{c,1}, N'_{c,2}, \cdots, N'_{c,k}), \quad (l_1, l_2, \cdots, l_k), \quad \text{and} \quad (m_1, m_2, \cdots, m_k)$$

determines the expectation value of $M$ completely. In the brane realization of Figure 4 (for the time being, the further splitting of $n_j$ flavor D4-branes is ignored), the first two characterize how to split the flavor D4-branes and the last shows the relative distance between the $NS5_{-\theta,j}$-brane and the $NS5'_{R,j}$-brane along $\nu$ direction.

Figure 4: The nonsupersymmetric meta-brane configuration corresponding to Figure 2 when the $NS5_{-\theta}$-branes and the $NS5'_{R}$-branes are separated. We only draw $(N'_{c,j}-l_j-n_j)$-flavor D4-branes, $n_j$-flavor D4-branes, connecting between $j$-th $NS5_{-\theta,j}$-brane and $j$-th $NS5'_{R,j}$-brane, $l_j$ D4-branes connecting between $j$-th $NS5_{-\theta,j}$-brane and the NS5-brane, and $(\tilde{N}_{c,j}-l_j)$-color D4-branes connecting between $j$-th $NS5'_{R,j}$-brane and the NS5-brane. Other $(k-1)$ possible brane configurations we do not draw are assumed.

Since the eigenvalues for the submeson field $M_j$ are either 0 or $\left[\frac{2m_j}{\alpha(k+1)}\right]^\frac{1}{\varepsilon}$, one takes $N'_{c,j} \times N'_{c,j}$ matrix $M_j$ with $l_j$’s eigenvalues 0 and $(N'_{c,j}-l_j)$’s eigenvalues $\left[\frac{2m_j}{\alpha(k+1)}\right]^\frac{1}{\varepsilon}$ as follows:

$$M = \left[\frac{2}{\alpha(k+1)}\right]^\frac{1}{\varepsilon} \begin{pmatrix} 0 & l_t & 0 & \cdots & 0 \\ 0 & m^\frac{1}{\varepsilon} & 1_{N'_{c,1}-l_1} & 0 & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ 0 & 0 & 0 & 0 & m^\frac{1}{\varepsilon} & 1_{N'_{c,k}-l_k} \end{pmatrix} = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ \cdot & \cdot & \cdot \\ 0 & 0 & M_k \end{pmatrix} \quad (2.15)$$
with (2.14) and (2.12). In the last equation (2.15) we redistributed the zeros of $0_l$ along the diagonal direction into each submeson $M_j$ appropriately. This structure is evident from the brane configuration. In the brane configuration of Figure 4, the $l_j$ of the $N'_{c,j}$-flavor D4-branes are connected with $l_j$ of $\tilde{N}_{c,j}$-color D4-branes and the resulting $l_j$ D4-branes stretch from the $\text{NS}5_{-\theta,j}$-brane to the $\text{NS}5$-brane directly and the intersection point between the $l_j$ D4-branes and the $\text{NS}$-brane is $(v, w) = (+v_{\text{NS5}_{-\theta,j}}, 0)$. This corresponds to exactly the $l_j$’s eigenvalues 0 of $M_j$ in (2.15). Now the remaining $(N'_{c,j} - l_j)$-flavor D4-branes between the $\text{NS}5_{-\theta,j}$-brane and the $\text{NS}5'_{R,j}$-brane correspond to the remaining eigenvalues of $M_j$ in (2.15), i.e., $\left[ \frac{2m_l}{\alpha(k+1)} \right]^{1/2} 1_{N'_{c,j} - l_j}$. The intersection point between the $(N'_{c,j} - l_j)$ D4-branes and the $\text{NS}5'_{R,j}$-brane is given by $(v, w) = (0, +v_{\text{NS5}_{-\theta,j}} \cot \theta)$ from the geometry.

By substituting the above $M$ (2.15) into the F-term equation, one also obtains

$$Y \tilde{Y} = \begin{pmatrix} -m_1 A_1 1_l & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -m_k A_k 1_k & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{pmatrix} \begin{pmatrix} Y_1 \tilde{Y}_1 & 0 & 0 \\ 0 & Y_2 \tilde{Y}_2 & 0 \\ \vdots & \ddots & \ddots \\ 0 & 0 & Y_k \tilde{Y}_k \end{pmatrix} = \begin{pmatrix} Y_j \tilde{Y}_j & 0 & 0 \\ 0 & Y_2 \tilde{Y}_2 & 0 \\ \vdots & \ddots & \ddots \\ 0 & 0 & Y_k \tilde{Y}_k \end{pmatrix}.$$  (2.16)

In the last equation, we redistributed the zeros of $0_{1_{N'_{c,j} - l_j}}$ into each $Y_j \tilde{Y}_j$. In the $l_j$-th vacuum the gauge symmetry is broken to $SU(N_{c,j} - l_j)$ and the supersymmetric vacuum drawn in Figure 4 with $l_j = 0$ has $Y_j \tilde{Y}_j = 0$ and the gauge group $SU(N_{c,j})$ is unbroken.

So far, the ground states are supersymmetric. Moreover, the theory has many nonsupersymmetric meta-stable ground states. One can deform the Figure 3 by displacing the multiple NS-branes along $v$ direction. Then the $n$ curved flavor D4-branes attached to them as well as other D4-branes are displaced also as $k$ different $n_j$’s connecting between $\text{NS}5_{-\theta,j}$-brane and $\text{NS}5'_{R,j}$-brane. For the IR free region, $N_{c,j} < N'_{c,j} < \frac{3}{2} N_{c,j} \prod$, the magnetic theory is the effective low energy description of the asymptotically free electric gauge theory. When we rescale the submeson field as $M_j = h \Lambda \Phi_j$, then the Kahler potential for $\Phi_j$ is canonical and the magnetic quarks $Y_j$ and $\tilde{Y}_j$ are canonical near the origin of field space. Then the magnetic superpotential (2.2) can be rewritten in terms of $\Phi_j$ and $Y_j \tilde{Y}_j$

$$W_{\text{dual}} = \sum_{j=1}^{k} \left[ h \Phi_j Y_j \tilde{Y}_j + \frac{\mu_\phi}{2} h^{k+1} \text{tr} \Phi_j^{k+1} - h \text{tr} \mu_j^2 \Phi_j \right].$$  (2.17)

with $\mu_j^2 = -m_j \Lambda_j$ and $\mu_\phi = -\alpha \Lambda^{k+1}$ as before. Remember that both $h \Phi_j$ and $Y_j \tilde{Y}_j$ are $N'_{c,j} \times N'_{c,j}$ matrices and the $\tilde{N}_{c,j}$ indices are contracted in the product $Y_j \tilde{Y}_j$ in the cubic term of (2.17). One splits the $(N'_{c,j} - l_j) \times (N'_{c,j} - l_j)$ block at the lower right corner of $h \Phi_j$ (2.15).
and $Y_j \tilde{Y}_j$ (2.16) into blocks of size $n_j$ and $(N'_{c,j} - l_j - n_j)$ for all $j$ as follows [7]:

$$h \Phi = \left[ \begin{array}{c} \frac{2}{\mu_\phi(k+1)} \end{array} \right]^\dagger \begin{pmatrix} 0_{l+n} & \mu_\phi^2 & 0 & \cdots & 0 \\ 0 & 0_{N'_{c,1} - l_1 - n_1} & \cdots & \cdots & 0 \\ \vdots & \vdots & \ddots & \cdots & \vdots \\ 0 & 0 & 0 & \mu_\phi^2 & 0_{N'_{c,k} - l_k - n_k} \end{pmatrix} + \text{diag}(0_{l}, h \Phi_{n_1}, \ldots, h \Phi_{n_k}, 0_{1_{N'_{c,l}-n}})$$

(2.18)

and

$$Y \tilde{Y} = \begin{pmatrix} \mu_\phi^2 & 0 & 0 & \cdots & 0 \\ 0 & \ddots & \cdots & \cdots & 0 \\ \vdots & \vdots & \ddots & \cdots & \vdots \\ 0 & 0 & \cdots & \mu_\phi^2 & 0 \\ 0 & 0 & 0 & 0 & 0_{N'_{c,k}-l_k-n_k} \end{pmatrix} + \text{diag}(0_{l}, \varphi_{n_1} \tilde{\varphi}_{n_1}, \ldots, \varphi_{n_k} \tilde{\varphi}_{n_k}, 0_{1_{N'_{c,l}-n}}).$$

(2.19)

Here $\varphi_{n_j}$ and $\tilde{\varphi}_{n_j}$ are $n_j \times (N'_{c,j} - l_j)$ matrices and correspond to $n_j$-flavors of fundamentals of the gauge group $SU(\tilde{N}_{c,j} - l_j)$ which is unbroken. Let us denote the sum of $n_j$ by

$$\sum_{j=1}^{k} n_j = n, \quad 0 \leq n_j \leq N'_{c,j} - l_j$$

which is the number of curved D4-branes in Figure 3 before deformation we are considering.

In the brane configuration from Figure 4, they correspond to fundamental strings connecting between the $n_j$-flavor D4-branes and $(\tilde{N}_{c,j} - l_j)$-color D4-branes. Moreover, the $\Phi_{n_j}$ and $\varphi_{n_j} \tilde{\varphi}_{n_j}$ are $n_j \times n_j$ matrices. The supersymmetric ground state corresponds to the vacuum expectation values by $h \Phi_{n_j} = \left[ \frac{2\mu_\phi^2}{\mu_\phi(k+1)} \right]^\dagger 1_{n_j}$ and $\varphi_{n_j} \tilde{\varphi}_{n_j} = 0$. Note that the $\Phi_j$ contains $\Phi_{n_j}$ as a submatrix.

The full one loop potential for $\Phi_{n_j}$, $\varphi_{n_j}$, and $\tilde{\varphi}_{n_j}$ from (2.17), (2.18) and (2.19) takes the form

$$V/|h|^2 = \sum_{j=1}^{k} \left[ |\Phi_{n_j} \varphi_{n_j}|^2 + |\Phi_{n_j} \tilde{\varphi}_{n_j}|^2 + |\varphi_{n_j} \tilde{\varphi}_{n_j} - \mu_\phi^2 1_{n_j} + \frac{(k+1)h^k}{2} \mu_\phi \Phi_{n_j}^k|^2 + b_j |h \mu_j|^2 \text{tr} \Phi_{n_j}^\dagger \Phi_{n_j} \right],$$

where $b_j = \frac{(n-4)(n-1)}{8\pi^2} (N'_{c,j} - N_{c,j})$ for all $j$. The first term is the absolute valued square of derivative of the superpotential (2.17) with respect to the field $\tilde{\varphi}_{n_j}$, the second term to the field $\varphi_{n_j}$, and the third term to the field $\Phi_{n_j}$. The last term is due to the one loop potential. Differentiating this potential with respect to $\Phi_{n_j}^\dagger$ and setting $\varphi_{n_j} = 0 = \tilde{\varphi}_{n_j}$, one obtains

$$[ -\mu_\phi^2 1_{n_j} + \frac{(k+1)h^k}{2} \mu_\phi \Phi_{n_j}^k ] \frac{k(k+1)}{2} (h^*)^k \mu_\phi^*(\Phi_{n_j}^\dagger)^{k-1} + b_j |h \mu_j|^2 \Phi_{n_j} = 0.$$
Since the higher order term superpotential plays the role of small perturbation and we assume that $\mu_\phi^\pm \ll \mu_j \ll \Lambda_{m,j}$, the second term of above will be negligible and one gets

$$\sum_{j} \frac{k(k+1)}{2} \mu_j^2 (h^*)^k \mu_\phi^\pm (\Phi_n^\pm)^{k-1} \simeq b_j |h\mu_j|^2 \Phi_n^j.$$ 

In order to see the structure of this solution, we consider the particular case where all the parameters are real including the $\Phi_n^j$ with $k > 2$. The general case is, in principle, straightforward. Then one arrives at the following form

$$h\Phi_n^j \simeq \left[ \frac{2b_j}{k(k+1)\mu_\phi} \right]^{\frac{1}{k-2}} 1_{n_j} \quad \text{or} \quad M_n^j \simeq \left[ \frac{\tilde{N}_{c,j}}{\alpha k(k+1)\Lambda_\phi^2} \right]^{\frac{1}{k-2}} 1_{n_j}. \quad (2.20)$$

Of course, the expression $\sum_{j=1}^k \text{tr}(h\Phi_n^j)^{k-2}$, when $b_j = \frac{\mu_j}{k}$ and $n_j = \frac{n_j}{k}$, reduces to $\frac{1}{k} \text{tr}(h\Phi_n^j)^{k-2}$ with (2.11). Then the vacuum energy $V$ is given by $V \simeq \sum_{j=1}^k n_j h^2 \mu_j^2$ and expanding around this solution, one obtains the eigenvalues for mass matrix for $\varphi_n^j$ and $\varphi_n^j$; $m_{\perp}^2 \simeq \left[ \frac{2b_j}{k(k+1)\mu_\phi} \right]^{\frac{1}{k-2}} \pm |h\mu_j|^2$. For the positive value for these, one should have $\left[ \frac{2b_j}{k(k+1)\mu_\phi} \right]^{\frac{1}{k-2}} > h\mu_j$ which leads to the quarks $\varphi_n^j$ and $\varphi_n^j$ are massive and then the vacuum (2.20) is locally stable.

The $(N'_{c,j} - l_j - n_j)$ flavor D4-branes between the $NS5_{-\theta,j}$-brane and the $NS5_{R,j}'$-brane are related to the corresponding eigenvalues of $h\Phi$ (2.18), i.e., $\left[ \frac{2\mu_j^2}{\mu_\phi(k+1)} \right]^{\frac{1}{k-2}} 1_{N'_{c,j} - l_j - n_j}$ and the intersection point between the $(N'_{c,j} - l_j - n_j)$ D4-branes and the $NS5_{R,j}'$-brane is also given by $(v, w) = (0, +v_{NS5_{-\theta,j}} \cot \theta)$. Moreover, the remnant $n_j$ flavor D4-branes between the $NS5_{R,j}'$-brane and the $NS5_{-\theta,j}$-brane are related to the corresponding eigenvalues (2.20) of $h\Phi_n^j$ and corresponding nonzero $w$ coordinate of these curved flavor D4-branes can be determined.

Therefore, the meta-stable states, for fixed $k$ which is related to the order of the superpotential polynomial and $\theta$ which is a deformation parameter by rotation angle of $NS5_{-\theta,j}$-brane, are classified by the number of various D4-branes and the positions of multiple NS-branes:

$$\left( N_{c,j}, N'_{c,j}, l_j, n_j \right) \quad \text{and} \quad \left( v_{NS5_{-\theta,j}}, v_{NS5_{R,j}'} \right).$$

That is, the former is given by $i)$ $N_{c,j}$ which appears in the number of dual color D4-branes through $\tilde{N}_{c,j}$ and in $h\Phi_n^j$ (2.20) through $b_j$, $ii)$ $N'_{c,j}$ which is encoded also in the number of dual color D4-branes through $\tilde{N}_{c,j}$, in “straight” flavor D4-branes, in the multiplicities of expectation value $h\Phi$ (2.18), and in $h\Phi_n^j$ through $b_j$, $iii)$ $l_j$ which characterizes the number of splitting D4-branes and the multiplicities of expectation value $h\Phi$, and $iv)$ $n_j$ which is the number of “curved” flavor D4-branes and determines the multiplicities of the expectation value for $h\Phi_n^j$ and the multiplicities of expectation value $h\Phi$. The latter is given by $i)$ $v_{NS5_{-\theta,j}}$
which provides the locations of both \( l_j \) D4-branes and straight flavor D4-branes through \( \mu_j \) (2.18), and \( \mu_j \) which determines the locations of both dual color D4-branes and straight flavor D4-branes through \( \mu_j \).

3 Meta-stable brane configuration with \((4k+3)\) NS-branes plus O4-plane

3.1 Electric theory

The type IIA brane configuration [20] corresponding to \( \mathcal{N} = 1 \) supersymmetric gauge theory with gauge group \( Sp(N_c) \times SO(2N'_c) \) and a bifundamental \( X \) in the representation \((2N_c, 2N'_c)\) can be described by a middle NS5-brane, the left \((2k+1)\) NS5\(_R\) -branes and the right \((2k+1)\) NS5\(_L\) -branes, \( 2N_c \) - and \( 2N'_c \) -color D4-branes, and an O4-plane(01236). The O4-plane acts as \((x_4, x_5, x_7, x_8, x_9) \rightarrow (-x_4, -x_5, -x_7, -x_8, -x_9)\) as usual [21, 22]. The number of NS5\(_{L,R}\) -branes is a nonnegative integer \( k \geq 0 \). The \( 2N_c \) D4-branes and O4\(^+\)-plane are suspended between the middle NS5-brane whose \( x^6 \) coordinate is \( x^6 = 0 \) and NS5\(_R\) -branes while the \( 2N'_c \) D4-branes and O4\(^+\)-plane are suspended between the NS5\(_L\) -branes and the middle NS5-brane. We take the arbitrary numbers of color D4-branes with the constraint \( N'_c \geq N_c + 2 \).

Let us deform this gauge theory. Displacing the two kinds of NS5\(_{L,R}\) -branes relative each other in the \( \pm v \) direction corresponds to turning on a quadratic mass-deformed superpotential for the bifundamental \( X \) while rotating them in the \((v, w)\) plane corresponds to turning on a quartic \( k = 0 \) or higher order \( k \geq 1 \) superpotential for the bifundamental \( X \). The deformed electric superpotential, by changing the left \((2k+1)\) NS5\(_L\) -branes, is as follows:

\[
W_{elec} = -\frac{\alpha}{2} \text{tr}(XX)^{2k+2} + \text{tr} m XX, \quad \alpha = \frac{\tan \theta}{\Lambda}, \quad m = \frac{v_{NS5_{L,R}}}{2\pi \ell_s^2}.
\]  

Each \((k + \frac{1}{2})\) NS5\(_L\) -branes are moving to the \( \pm v \) directions respectively together with \( N'_c \) D4-branes and are rotating by an angle \( -\theta \) in \((w, v)\)-plane. Then the \( v \) coordinate of NS5\(_L\) -branes is denoted by \( v = \pm v_{NS5_{L,R}} \). The order of this superpotential (3.1) can be obtained by replacing \( k \) in (2.1) with \((2k+1)\). We present the electric brane configuration in Figure 5.

The solution for the supersymmetric vacua can be written as \( XX = \left[ \frac{m}{\alpha(k+1)} \right]^{\frac{1}{2k+2}} \) through the F-term condition. This breaks the gauge group \( Sp(N_c) \times SO(2N'_c) \) to \( Sp(p_0) \), \( SO(2N'_c - 2N_c + 2p_0) \), and \( U(2p_1) \times U(2p_2) \times \cdots \times U(2p_k) \) with \( 2\sum_{i=0}^{k} p_i = 2N_c \) [18]. The \( 2p_0 \) is the number of eigenvalues which are zero with \( X = 0 \). When the middle NS5-brane moves to \( \pm w \) direction, then the three kinds of NS-branes intersect in three points in \((v, w)\)-plane(and their mirrors). Then we consider that \( 2p_0 \) D4-branes are connecting between the middle NS5-brane.
Figure 5: The $\mathcal{N} = 1$ supersymmetric electric brane configuration for the gauge group $Sp(N_c) \times SO(2N'_c)$ and a bifundamental $X$ with nonvanishing mass and higher order terms for the bifundamental. There are two deformations by rotation and displacement of upper and lower $(k + \frac{1}{2})$ NS5$_L$-branes. Note that there exist multiple outer NS-branes.

and the zero-th NS5$_{R,0}$-brane. One can decompose $2N_c$ D4-branes as $2p_0$ plus $2(N_c - p_0)$ D4-branes. The latter can be reconnected with the same number of D4-branes among $2N'_c$ D4-branes. Then the remaining $2(N'_c - N_c + p_0)$ D4-branes are connecting between the zero-th NS5$_{\theta,0}$-brane and the middle NS5-brane. Then $2(N_c - p_0)$ D4-branes are suspended between $2k$ NS5$_{-\theta}$-branes and NS5$_{R}$-branes. By separating these NS-branes transversely, each $p_i$ D4-branes(where $i = 1, 2, \cdots, k$) are connecting between the $i$-th NS5$_{-\theta,i}$-brane and the $i$-th NS5$_{R,i}$-brane directly(and their mirrors).

3.2 Magnetic theory

In order to apply the Seiberg dual to the $Sp(N_c)$ factor, the NS5-brane is moved to the right all the way past the NS5$_{R}$-branes. Let us introduce $2N'_c$ D4-branes and $2N'_c$ anti-D4-branes between NS5$_{R}$-branes and NS5-brane, recombine the former with the $2N'_c$ D4-branes that are connecting between the NS5$_{-\theta}$-branes and the NS5$_{R}$-branes. By moving those combined D4-branes to $\pm v$-direction, one gets the final Figure 6(with $l = 0$).

The gauge group is given by $Sp(\tilde{N}_c = N'_c - N_c - 2) \times SO(2N'_c)$ and the matter contents are the field $Y$ in the representation $(2\tilde{N}_c, 2N'_c)$ under the dual gauge group and the gauge-singlet $M(\equiv XX)$ is in the representation $(1, N'_c(2N'_c - 1))$ under the dual gauge group.
Figure 6: The $\mathcal{N} = 1$ supersymmetric magnetic brane configuration corresponding to Figure 5 with a misalignment between D4-branes if we ignore the gravitational potential of NS5-brane. The upper $N_c'$-flavor D4-branes connecting between the upper half NS5$_{θ}$-brane and NS5$'_R$-brane are splitting into $(N_c' - l)$ and $l$ D4-branes. The location of intersection between the upper NS5$_{θ}$-branes and the upper $(N_c' - l)$ D4-branes is given by $(v, w) = (0, v_{NS5_{θ}} \cot \theta)$ while the one between the upper NS5$_{θ}$-brane and the upper $l$ D4-branes is given by $(v, w) = (v_{NS5_{θ}}, 0)$.

Then the dual magnetic superpotential\footnote{The conditions $b_{Sp(N_c)}^{mag} < 0$ and $b_{Sp(N_c)} < 0$ imply that $N_c' < \frac{3}{2} N_c + \frac{3}{2}$. Then the range for the $N_c'$ can be written as $N_c + 2 < N_c' < \frac{3}{2} N_c + \frac{3}{2}$. Moreover, $b_{SO(2N'_c)} > 0$ and $b_{SO(2N'_c)}^{mag} > 0$. In the low energy description, the $SO(2N'_c)$ is asymptotically free. At the scale Landau pole $A_1$, the $Sp(N_c)$ theory is strongly coupled and the Seiberg duality occurs. The coefficients of beta function $b_{Sp(N_c)}^{mag}$ becomes negative and $b_{SO(2N'_c)}^{mag}$ becomes positive. Then at energy scale lower than $A_1$, the theory is weakly coupled. We require that $SO(2N'_c)^{mag}$ theory is less coupled than the $Sp(N_c)^{mag}$ at the supersymmetry breaking scale $\mu$. This provides a stronger constraint on $A_2$. Then under the constraint, $A_2 << \left( \frac{1}{\lambda} \right)^b A_1 << A_1$ where $b \equiv \frac{b_{Sp(N_c)}^{mag} - b_{SO(2N'_c)}^{mag}}{b_{SO(2N'_c)}}$, one can ignore the contribution from the gauge coupling of $SO(2N'_c)^{mag}$ at the supersymmetry breaking scale and one relies on the one loop computation.} by adding the mass term and higher order term for the bifundamental $X$ (3.1) to the cubic superpotential, is given by

$$W_{dual} = \frac{1}{\Lambda} \text{tr} MYY - \frac{\alpha}{2} \text{tr} M^{2k+2} + \text{tr} mM$$

(3.2)

and the F-term equations are

$$MY = 0, \quad -\frac{1}{\Lambda} YY = m - \alpha (k + 1) M^{2k+1}.$$  

(3.3)

Let us consider all the NS5$_{θ}$-branes and NS5$'_R$-branes are coincident with each other.

- Coincident NS5$_{θ}$-branes and NS5$'_R$-branes
The matrix equation $mM = \alpha(k+1)M^{2(k+1)}$ implies that the eigenvalues for the meson field $M$ are either 0 or $\left[ \frac{m}{\alpha(k+1)} \right]^{1/2k+1}$, and one takes $2N'_c \times 2N'_c$ matrix with $2l'$'s eigenvalues 0 and $2(N'_c - l)$'s eigenvalues $\left[ \frac{m}{\alpha(k+1)} \right]^{1/2k+1}$:

$$M = \begin{pmatrix} 0 & 1_{2l'}^1 \\
0 & \left[ \frac{m}{\alpha(k+1)} \right]^{1/2k+1} 1_{N'_c - l} \otimes i\sigma_2 \end{pmatrix}$$

(3.4)

where $l = 1, 2, \cdots, 2N'_c$. Therefore, in the brane configuration of Figure 6, the $l$ of the upper $N'_c$ flavor D4-branes are connected with $l$ of $\tilde{N}_c$ color D4-branes and the resulting D4-branes stretch from the upper NS5-θ-branes to the NS5-brane directly and the intersection point between the $l$ upper D4-branes and the NS5-brane is given by $(v, w) = (+v_{NS5\_θ}, 0)$. Similarly the mirrors are located at $(v, w) = (-v_{NS5\_θ}, 0)$. This corresponds to exactly the $2l'$'s eigenvalues 0 of $M$ above (3.4). The remaining $(N'_c - l)$ upper flavor D4-branes between the NS5-θ-branes and the NS5'$_R$-brane are related to the corresponding half eigenvalues of $M$ which is equal to $\left[ \frac{m}{\alpha(k+1)} \right]^{1/2k+1} 1_{N'_c - l} \otimes i\sigma_2$. The intersection point between the $(N'_c - l)$ upper D4-branes and the NS5'$_R$-branes is given by $(v, w) = (0, +v_{NS5\_θ} \cot \theta)$ corresponding to half eigenvalues of $M$ from geometry. The mirrors are located at $(v, w) = (0, -v_{NS5\_θ} \cot \theta)$ corresponding to other half eigenvalues of $M$ [10].

Substituting (3.4) into the second equation of (3.3) gives rise to

$$YY = \begin{pmatrix} -m\Lambda & 1_{2l'} \\
0 & 0 & 1_{2N'_c - 2l'} \end{pmatrix}$$

(3.5)

In the $l$-th vacuum the gauge symmetry is broken to $Sp(\tilde{N}_c - l)$ and the supersymmetric vacuum drawn in Figure 6 with $l = 0$ has $Y = 0$ and the gauge group $Sp(\tilde{N}_c)$ is unbroken. The expectation value of $M$ (3.4) in this case is given by $M = \left[ \frac{m}{\alpha(k+1)} \right]^{1/2k+1} 1_{N'_c} \otimes i\sigma_2 = \left[ \frac{m\Lambda \cot \theta}{(k+1)} \right]^{1/2k+1} 1_{N'_c} \otimes i\sigma_2$. If we replace $(2k+1)$ $NS5\_θ$-branes with $2N_f$ D6-branes and $(2k+1)$ $\tilde{NS5}'_R$-branes with a single $NS5'_R$-brane, then the meta-stable brane configuration of Figure 6 reduces to the one in [23, 24, 19].

The theory has many nonsupersymmetric meta-stable ground states if an attractive gravitational interaction between the flavor D4-branes and the NS5-brane from the DBI action is considered [11]. For the IR free region [1, 23], the magnetic theory is the effective low energy description of the asymptotically free electric gauge theory. When we rescale the meson field

\footnote{The mass matrix $m$ is antisymmetric in the indices and is given by $m = \text{diag}(i\sigma_2 m_1, i\sigma_2 m_2, \cdots, i\sigma_2 m_{N'_c})$ due to the antisymmetric matrix $M$. In the matrix equation $mM$, we assumed this property of mass matrix. In (3.4), we use the same notation for the equal mass $m \equiv m_1 = m_2 = \cdots = m_{N'_c}$.}
as \( M = h \Lambda \Phi \), then the Kahler potential for \( \Phi \) is canonical and the magnetic “quarks” are canonical near the origin of field space. Then the magnetic superpotential (3.2) can be written in terms of \( \Phi \)

\[
W_{\text{dual}} = h \Phi YY + \frac{\mu_\phi}{2} h^{2k+2} \text{tr} \Phi^{2k+2} - h \mu^2 \text{tr} \Phi
\]

with \( \mu^2 = -m \Lambda \) and \( \mu_\phi = -\alpha \Lambda^{2k+2} \).

The classical supersymmetric vacua given by (3.4) and (3.5) can be described in terms of new variables. Now one splits the \( 2(N'_c - l) \times 2(N'_c - l) \) block at the lower right corner of \( h \Phi \) and \( YY \) into blocks of size \( 2n \) and \( 2(N'_c - l - n) \) as follows:

\[
h\Phi = \begin{pmatrix}
0 & 1_{2l} \\
1_{2l} & h\Phi_{2n}
\end{pmatrix},
\]

\[
Y^2 = \begin{pmatrix}
\mu^2 1_{2l} & 0 & 0 \\
0 & \varphi \varphi & 0 \\
0 & 0 & 0\end{pmatrix},
\]

where \( \varphi \) is \( 2n \times 2(\tilde{N}_c - l) \) matrix and corresponds to \( 2n \) flavors of fundamentals of the gauge group \( Sp(\tilde{N}_c - l) \) which is unbroken by the nonzero expectation value of \( Y \). One can move \( n \) upper D4-branes, from upper \( (N'_c - l) \) D4-branes stretched between the \( N S 5'_R \)-brane and the upper \( N S 5_{-\theta} \)-brane at \( w = +v_{NS 5_{-\theta}} \cot \theta \), to the local minimum of the potential and the end points of these \( n \) D4-branes are at a nonzero \( w \) as in Figure 7. In the brane configuration in Figure 7, \( \varphi \) corresponds to fundamental strings connecting the \( n \) upper flavor D4-branes and \( (\tilde{N}_c - l) \) color D4-branes(and their mirrors). The \( \Phi_{2n} \) and \( \varphi \varphi \) are \( 2n \times 2n \) matrices. The supersymmetric ground state corresponds to the vacuum expectation values by \( h\Phi_{2n} = \frac{\mu^2}{\mu_\phi(k+1)} \frac{1}{8\pi^2} 1_n \otimes i\sigma_2 \) and \( \varphi = 0 \).

The full one loop potential for \( \Phi_{2n} \) and \( \varphi \) from (3.6) and (3.7) including the one loop result takes the form

\[
\frac{V}{|h|^2} = |\Phi_{2n} \varphi|^2 + |\varphi \varphi - \mu^2 1_{2n} + (k + 1)h^{2k+1} \mu_\phi \Phi_{2n}^{2k+1}|^2 + b|h|^2 \text{tr} \Phi_{2n} \Phi_{2n},
\]

where the positive numerical constant \( b \) is given by \( b = \frac{(\ln 1 - 1)}{8\pi^2} (N'_c - N_c - 2) \). Differentiating this potential with respect to \( \Phi_{2n} \) and putting \( \varphi = 0 \), one obtains

\[
[-\mu^2 1_{2n} + (k + 1)h^{2k+1} \mu_\phi \Phi_{2n}^{2k+1}] (2k + 1)(k + 1)(h^*)^{2k+1} \mu_\phi^* \Phi_{2n}^{2k} + 2b|h|^2 \Phi_{2n} = 0.
\]

Since we assume that \( \mu_\phi^{\frac{1}{k+1}} \ll \mu \ll \Lambda_m \), the second term of above will be negligible and one gets

\[
(2k + 1)(k + 1)h^*(h^*)^{2k+1} \mu_\phi^* \Phi_{2n}^{2k} \simeq 2b|h|^2 \Phi_{2n}.
\]
Figure 7: The nonsupersymmetric meta-brane configuration corresponding to Figure 6 when we consider the gravitational potential of NS5-brane and is obtained by moving \( n \) flavor D4-branes from \( (N'_c - l) \) flavor D4-branes of Figure 6. The nonzero positive \( w \) coordinate for \( n \) curved flavor D4-branes can be determined.

Then one arrives at the following form from (3.9)

\[
h\Phi_{2n} \simeq \left[ \frac{2b}{(2k+1)(k+1)\mu_\phi} \right]^{\frac{1}{2k-1}} 1_n \otimes i\sigma_2, M_{2n} \simeq \left[ \frac{\tilde{N}_c}{\alpha(2k+1)(k+1)\Lambda^2} \right]^{\frac{1}{2k-1}} 1_n \otimes i\sigma_2. (3.10)
\]

Then the vacuum energy \( V \) is given by \( V \simeq 2nh^2\mu^4 \) and expanding around this solution, one obtains the eigenvalues for mass matrix for \( \phi \):

\[
m^2_{\pm} \simeq \left[ \frac{2b}{(2k+1)(k+1)\mu_\phi} \right]^{\frac{1}{2k-1}} \pm |h\mu|^2.
\]

For the positive value for these eigenvalues one should have \( \left[ \frac{2b}{(2k+1)(k+1)\mu_\phi} \right]^{\frac{1}{2k-1}} > h\mu \) which leads to the quarks \( \phi \) are massive and then the vacuum (3.10) is locally stable. Note that \( |h\Phi_{2n}| \ll \Lambda_m \).

The upper \( (N'_c - l - n) \) flavor D4-branes between the upper NS5-\( \theta \)-branes and the NS5'\( R \)-branes are related to the corresponding positive eigenvalues of \( h\Phi \) (3.7) which is equal to \( \left[ \frac{\mu^2}{\mu_\phi(k+1)} \right]^{\frac{1}{2k+1}} 1_{(N'_c - l - n)} \otimes i\sigma_2 \). The intersection point between the upper \( (N'_c - l - n) \) D4-branes and the NS5'\( R \)-branes is given by \( (v, w) = (0, +v_{NS5-\theta} \cot \theta) \). The \( n \) upper curved flavor

\[\text{Figure 7: The nonsupersymmetric meta-brane configuration corresponding to Figure 6 when we consider the gravitational potential of NS5-brane and is obtained by moving } n \text{ flavor D4-branes from } (N'_c - l) \text{ flavor D4-branes of Figure 6. The nonzero positive } w \text{ coordinate for } n \text{ curved flavor D4-branes can be determined.}
\]

Then one arrives at the following form from (3.9)

\[h\Phi_{2n} \simeq \left[ \frac{2b}{(2k+1)(k+1)\mu_\phi} \right]^{\frac{1}{2k-1}} 1_n \otimes i\sigma_2, M_{2n} \simeq \left[ \frac{\tilde{N}_c}{\alpha(2k+1)(k+1)\Lambda^2} \right]^{\frac{1}{2k-1}} 1_n \otimes i\sigma_2. (3.10)\]

Then the vacuum energy \( V \) is given by \( V \simeq 2nh^2\mu^4 \) and expanding around this solution, one obtains the eigenvalues for mass matrix for \( \phi \): \( m^2_{\pm} \simeq \left[ \frac{2b}{(2k+1)(k+1)\mu_\phi} \right]^{\frac{1}{2k-1}} \pm |h\mu|^2 \). For the positive value for these eigenvalues one should have \( \left[ \frac{2b}{(2k+1)(k+1)\mu_\phi} \right]^{\frac{1}{2k-1}} > h\mu \) which leads to the quarks \( \phi \) are massive and then the vacuum (3.10) is locally stable. Note that \( |h\Phi_{2n}| \ll \Lambda_m \).

The upper \( (N'_c - l - n) \) flavor D4-branes between the upper NS5-\( \theta \)-branes and the NS5'\( R \)-branes are related to the corresponding positive eigenvalues of \( h\Phi \) (3.7) which is equal to \( \left[ \frac{\mu^2}{\mu_\phi(k+1)} \right]^{\frac{1}{2k+1}} 1_{(N'_c - l - n)} \otimes i\sigma_2 \). The intersection point between the upper \( (N'_c - l - n) \) D4-branes and the NS5'\( R \)-branes is given by \( (v, w) = (0, +v_{NS5-\theta} \cot \theta) \). The \( n \) upper curved flavor

\[\text{The stability of the SUSY breaking vacua implies that } h < \frac{\mu^2}{\mu_\phi(k+1)} \ll 1. \text{ As one flows to IR, the dimensionless coupling } \mu_\phi E^{2k-1} \text{ at the energy scale } E \text{ becomes smaller and smaller when } k > 0. \text{ At the SUSY breaking scale } E = \sqrt{h\mu}, \text{ this dimensionless coupling becomes } h^{\frac{1-2k}{2k-1}} \mu_\phi(h\mu)^{2k-1}. \text{ When } k > 0, \text{ the first factor } h^{\frac{1-2k}{2k-1}} \text{ is large. Then the second factor should behave as } \mu_\phi(h\mu)^{2k-1} \ll 1 \text{ for the higher order deformation to be small. This is compatible with stability condition. However, the mass of } \phi \text{ becomes large when } k > 0 \text{ and are integrated out. On the other hand, in the quartic case } (k = 0), \text{ the qualitative difference arises since the first factor } h^{\frac{1-2k}{2k-1}} \ll 1, \text{ as long as } \frac{\mu_\phi}{h\mu} \text{ is not too large then the above requirement is valid. In other words, the mass of } \phi \text{ remains small enough compared to the SUSY breaking scale. In the higher order case } (k > 0), \text{ the conditions on } \mu_\phi \text{ and } \mu \text{ push the SUSY breaking scale too high for the deformation to stay small enough.}\]
D4-branes between the $\text{NS}5_\theta$-branes and the $\text{NS}5'_R$-branes are related to the corresponding positive eigenvalues (3.10) of $h\Phi_{2n}$.

Let us consider all the $\text{NS}5_\theta$-branes and $\text{NS}5'_R$-branes are separated each other.

- Separated $\text{NS}5_\theta$-branes and $\text{NS}5'_R$-branes

We displace the $k$ upper $\text{NS}5_\theta$-branes and upper $\text{NS}5'_R$-branes, given in Figure 6, in the $+v$ direction respectively to two $k$ different points denoted by $\nu_{\text{NS}5_\theta,j}$ and $\nu_{\text{NS}5'_R,j}$ where $j = 1, 2, \cdots, k$ (and their mirrors in the $-v$ direction). For convenience, let us put the “half” $\text{NS}5_\theta$-brane as the lowest value of $v$ coordinate and one remaining $\text{NS}5'_R$-brane at $v = 0$. Then the $k$ upper $\text{NS}5_\theta$-branes are located above “half” $\text{NS}5_\theta$-brane and $k$ upper $\text{NS}5'_R$-branes are located above a “single” $\text{NS}5'_R$-brane whose $v$ coordinate is zero. The number of color D4-branes stretched between $j$-th upper $\text{NS}5'_R,j$-brane and the NS5-brane is denoted by $\tilde{N}_{c,j}$ while the number of flavor D4-branes stretched between the $j$-th upper $\text{NS}5_\theta,j$-brane and the $j$-th upper $\text{NS}5'_R,j$-brane is denoted by $N'_{c,j}$. Let us denote $\text{NS}5_\theta,j=0$-brane by “half” $\text{NS}5_\theta$-brane and $\text{NS}5'_R,j=0$-brane by a “single” $\text{NS}5'_R$-brane. Then the supersymmetric configurations, labeled by sets of nonnegative integers $(\tilde{N}_{c,0}, \tilde{N}_{c,1}, \tilde{N}_{c,2}, \cdots, \tilde{N}_{c,k})$ and sets of nonnegative integers $(N'_{c,0}, N'_{c,1}, N'_{c,2}, \cdots, N'_{c,k})$ with

\[
\sum_{j=0}^{k} \tilde{N}_{c,j} = \sum_{j=0}^{k} N'_{c,j} = N'_c \quad (3.11)
\]

can arise. When all the upper $\text{NS}5'_R,j$-branes and $\text{NS}5_\theta,j$-branes ($j = 0, 1, 2, \cdots, k$) are distinct, the low energy physics corresponds to $(k + 1)$ decoupled supersymmetric gauge theories with gauge groups

\[
\prod_{j=0}^{k} Sp(\tilde{N}_{c,j}) \times SO(2N'_{c,j}).
\]

As we approach the origin of parameter space, $\nu_{\text{NS}5'_R,j} = 0$ and $\nu_{\text{NS}5_\theta,j} = \nu_{\text{NS}5_\theta}$ for all $j$, the full $Sp(\tilde{N}_c) \times SO(2N'_c)$ gauge group is restored.

Then each submeson field $M_j \equiv X_j X_j$ is in the representation $(1, N'_{c,j}(2N'_{c,j} - 1))$ under the gauge group $Sp(\tilde{N}_{c,j}) \times SO(2N'_{c,j})$. Note that a bifundamental $X_j$ is in the representation $(2N_{c,j}, 2N'_{c,j})$. Generic $(\nu_{\text{NS}5_\theta,j} - \nu_{\text{NS}5'_R,j})$, which is equal to $2\pi l_s m_j$, is related to a polynomial superpotential for $M_j$. The expression $[m_j 1_{N'_{c,j}} - \alpha (k + 1) M_j^{2k+1}]$ (where $j = 0, 1, 2, \cdots, k$) is the derivative of superpotential with respect to the $M_j$ for $Y_j = 0$ and has $k$ distinct minima $\left[ \frac{m_j}{\alpha (k+1)} \right]^{\frac{2k+1}{l_s}}$ plus zero with some multiplicities. Here $Y_j$ are dual fields corresponding to $X_j$.

One can write the derivative of superpotential with respect to the full meson $M$ for $Y = 0$
\[ W'_{dual}(x) = \sum_{i=0}^{N'_c} s_i x^{2(N'_c-i)} \equiv -\alpha(k+1) \prod_{j=0}^{k} x^{2l_j} \left( x^2 - \left[ \frac{m_j}{\alpha(k+1)} \right]\frac{1}{2k+1} \right)^{N'_{c,j} - l_j} \] (3.12)

where we put the sum of multiplicities of zero eigenvalues as \( l \) appeared in Figure 6

\[ \sum_{j=0}^{k} l_j = l, \quad 0 \leq l_j \leq \tilde{N}_{c,j}. \] (3.13)

The order of the polynomial (3.12) is \( 2N'_c \). The integers \( (N'_{c,0}, N'_{c,1}, \ldots, N'_{c,k}) \) are the number of the nonzero- plus zero-eigenvalues of the meson \( M \) residing in the different minima. Thus the set of

\[ (N'_{c,0}, N'_{c,1}, \ldots, N'_{c,k}), \quad (l_0, l_1, \ldots, l_k), \quad \text{and} \quad (m_0, m_1, \ldots, m_k) \]
determines the expectation value of \( M \) completely.

Since the eigenvalues for the submeson field \( M_j \) are either 0 or \( \left[ \frac{m_j}{\alpha(k+1)} \right]\frac{1}{2k+1} \), one takes \( 2N'_{c_j} \times 2N'_{c_j} \) matrix \( M_j \) with \( 2l_j \)'s eigenvalues 0 and \( 2(N'_{c,j} - l_j) \)'s eigenvalues \( \left[ \frac{m_j}{\alpha(k+1)} \right]\frac{1}{2k+1} \) as follows:

\[
\begin{pmatrix}
0 & 1_{2l} & 0 & 0 & \cdots & 0 \\
0 & m_0 \frac{1}{2k+1} & 1_{N'_{c,0} - l_0} \otimes i\sigma_2 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & m_k \frac{1}{2k+1} & 1_{N'_{c,k} - l_k} \otimes i\sigma_2
\end{pmatrix}
\] (3.14)

with (3.13) and (3.11). In the brane configuration of Figure 8 where we ignore the splitting of \( n_j \) flavor D4-branes for the time being, the \( l_j \) of the upper \( N'_{c,j} \)-flavor D4-branes are connected with \( l'_j \) of upper \( \tilde{N}_{c,j} \)-color D4-branes and the resulting \( l_j \) D4-branes stretch from the upper \( NS5_{\theta,j} \)-brane to the NS5-brane directly and the intersection point between the \( l_j \) D4-branes and the NS5-brane is \((v, w) = (v_{NS5_{\theta,j}}, 0)\). This corresponds to exactly the \( l_j \)'s eigenvalues 0 of \( M_j \) in (3.14). Now the remaining upper \( (N'_{c,j} - l_j) \)-flavor D4-branes between the upper \( NS5_{\theta,j} \)-brane and the upper \( NS5'_{R,j} \)-brane correspond to the remaining eigenvalues of \( M_j \) in (3.14), i.e., \( \left[ \frac{m_j}{\alpha(k+1)} \right]\frac{1}{2k+1} 1_{N'_{c,j} - l_j} \). The intersection point between the upper \( (N'_{c,j} - l_j) \)-flavor D4-branes and the upper \( NS5'_{R,j} \)-brane is given by \((v, w) = (0, +v_{NS5_{\theta,j}} \cot \theta)\).

By substituting the above \( M \) (3.14) into the F-term equation, one also obtains

\[
YY = \begin{pmatrix}
-m_1\Lambda_0 1_{2l_0} & 0 & 0 & \cdots & 0 & 0 \\
0 & \cdot & \cdot & \cdots & \cdot & 0 \\
0 & 0 & 0 & \cdots & -m_k\Lambda_k 1_{2l_k} & 0 \\
0 & 0 & 0 & \cdots & 0 & 1_{2N'_c - 2l}
\end{pmatrix}
\] (3.15)
Figure 8: The nonsupersymmetric meta-brane configuration corresponding to Figure 5 when the $NS_{5-\theta}$-branes and the $NS'_{R}$-branes are separated. We only draw $(N'_{c,j} - l_j - n_j)$-flavor D4-branes, $n_j$-flavor D4-branes, connecting between $NS_{5-\theta,j}$-brane and $NS'_{R,j}$-brane, $l_j$ D4-branes connecting between the $NS_{5-\theta,j}$-brane and the NS5-brane, and $(\tilde{N}_{c,j} - l_j)$-color D4-branes connecting between the $NS'_{R,j}$-brane and the NS5-brane. For simplicity, we draw here the case where $v_{NS'_{R,j}}$ goes to zero. In general, the $(2k+1)$ $NS'_{R}$-branes are separated, like as $NS_{5-\theta}$-branes, along $v$ direction.

In the $l_j$-th vacuum the gauge symmetry is broken to $Sp(\tilde{N}_{c,j} - l_j)$ and the supersymmetric vacuum drawn in Figure 8 with $l_j = 0$ has $Y_j = 0$ and the gauge group $Sp(\tilde{N}_{c,j})$ is unbroken. So far, the ground states are supersymmetric. Moreover, the theory has many nonsupersymmetric meta-stable ground states. For the IR free region, $N_{c,j} + 2 < N'_{c,j} < \frac{3}{2}(N_{c,j} + 1)$ [1, 23], the magnetic theory is the effective low energy description of the asymptotically free electric gauge theory. When we rescale the submeson field as $M_j = h\Lambda \Phi_j$, then the Kahler potential for $\Phi_j$ is canonical and the magnetic quarks $Y_j$ are canonical near the origin of field space. Then the magnetic superpotential (3.2) can be rewritten in terms of $\Phi_j$ and $Y_j$

$$W_{\text{dual}} = \sum_{j=0}^{k} \left[ h\Phi_j Y_j Y_j + \frac{\mu_{\phi}}{2} h^{2k+2} \text{tr} \Phi_j^{2k+2} - h \text{tr} \mu_j^2 \Phi_j \right]$$  \hspace{1cm} (3.16)

with $\mu_j^2 = -m_j \Lambda_j$ and $\mu_{\phi} = -\alpha \Lambda^{2k+2}$ as before. In the brane configuration, this is equivalent to deform the Figure 7. Then the $n$ curved flavor D4-branes also are splitted as $k$ different $n_j$’s(and their mirrors). One splits the $2(N'_{c,j} - l_j) \times 2(N'_{c,j} - l_j)$ block at the lower right corner of $h\Phi_j$ (3.14) and $Y_j Y_j$ (3.15) into blocks of size $2n_j$ and $2(N'_{c,j} - l_j - n_j)$ for all $j$ as follows.
The brane configuration from Figure 8, they correspond to fundamental strings connecting

gauge group $Sp$.

Since the higher order term superpotential plays the role of small perturbation and we assume

hence $\mu_n << \Lambda_{m,n} \equiv \Lambda_{m,n}$. The second term of above will be negligible and one gets

$$(2k + 1)(k + 1)\mu_n^2 (h^*)^{2k+1} \phi \Phi_{2n_j}^2 \simeq 2b_j |h\mu_j|^2 \phi \Phi_{2n_j}.$$
In order to see the structure of this solution, we consider the particular case where all the parameters are real. The general case is, in principle, straightforward. Then one arrives at the following form

$$h\Phi_{2n_j} \simeq \left[ \frac{2b_j}{(2k+1)(k+1)\mu_\phi} \right]^{\frac{1}{n_j}} 1_{n_j} \otimes i\sigma_2,$$

or

$$M_{2n_j} \simeq \left[ \frac{\tilde{N}_{c,j}}{\alpha(2k+1)(k+1)\Lambda^3} \right]^{\frac{1}{n_j}} 1_{n_j} \otimes i\sigma_2. \tag{3.19}$$

Then the vacuum energy \( V \) is given by \( V \simeq \sum_{j=0}^{k} 2n_j h^2 \mu_j^4 \) and expanding around this solution, one obtains the eigenvalues for mass matrix for \( \varphi_{2n_j} \): 

\[
m_{\pm}^2 \simeq \left[ \frac{2b_j}{(2k+1)(k+1)\mu_\phi} \right]^{\frac{1}{n_j}} \pm |h\mu_j|^2.
\]

For the positive value for these eigenvalues, one should have 

\[
(2k+1)(k+1)\mu_\phi > h\mu_j
\]

which leads to the quarks \( \varphi_{2n_j} \) are massive and then the vacuum (3.19) is locally stable.

The \( (N'_{c,j} - l_j - n_j) \) flavor D4-branes between the \( NS5_{-\theta,j} \)-brane and the \( NS5'_{R,j} \)-brane are related to the corresponding eigenvalues of \( h\Phi \) (3.17), i.e., 

\[
\sum_{j=0}^{k} 2n_j h^2 \mu_j^4 \]

and the intersection point between the \( (N'_{c,j} - l_j - n_j) \) D4-branes and the \( NS5'_{R,j} \)-brane is also given by \((v, w) = (0, +v_{NS5_{-\theta,j}} \cot \theta)\). Moreover, the remnant \( 2n_j \) flavor D4-branes between the \( NS5'_{R,j} \)-brane and the \( NS5_{-\theta,j} \)-brane are related to the corresponding eigenvalues (3.19) of \( h\Phi_{2n_j} \).

Then, the meta-stable states, for fixed \( k \) and \( \theta \), are classified by the number of various D4-branes and the positions of multiple NS-branes. That is, the former is given by \( i) \ N_{c,j} \) which appears in the number of dual color D4-branes and in \( h\Phi_{2n_j} \) (3.19), \( ii) \ N'_{c,j} \) which is encoded also in the number of dual color D4-branes, in “straight” flavor D4-branes, in the multiplicities of expectation value \( h\Phi \) (3.17), and in \( h\Phi_{2n_j} \), \( iii) \ l_j \) which characterizes the number of splitting D4-branes and the multiplicities of expectation value \( h\Phi \), and \( iv) \ n_j \) which is the number of “curved” flavor D4-branes and determines the multiplicities of the expectation value for \( h\Phi_{2n_j} \) and the multiplicities of expectation value \( h\Phi \). The latter is given by \( i) \ v_{NS5_{-\theta,j}} \) which provides the locations of \( l_j \) D4-branes and stright flavor D4-branes, and \( ii) \ v_{NS5'_{R,j}} \) which determines the locations of dual color D4-branes and straight flavor D4-branes.

By applying the Seiberg dual to the \( SO(2N'_c) \) gauge group, one can construct the magnetic brane configuration. Then the dual gauge group is given by \( Sp(N_c) \times SO(2N'_c - 2N'_c + 4) \) and the matter contents are the field \( Y \) in the representation \( (2N_c, 2\tilde{N}'_c) \) and the gauge-singlet \( M \) is in the representation \( (N_c(2N_c + 1), 1) \) under the dual gauge group [10]. The discussion for the supersymmetric or nonsupersymmetric vacua in previous description can be applied here also without any difficulty. Or if we change the charges of orientifold 4-plane in Figures 5-
8, then the symplectic(orthogonal) gauge group changes to the orthogonal(symplectic) gauge group and the matter contents also change. One can also analyze the (meta-)stable states.

4 Conclusions and outlook

We have found the type IIA nonsupersymmetric meta-stable brane configuration, presented by Figures 3 and 4, consisting of \((2k+1)\) NS-branes and different kinds of D4-branes where the electric gauge theory superpotential \((2.1)\) initially has an order \(2(k+1)\) polynomial for the bifundamentals. There exists a rich pattern of nonsupersymmetric meta-stable states in the presence of \(n_j\)-flavor D4-branes as well as the supersymmetric stable ones, in Figures 2 and 4, without those \(n_j\)-flavor D4-branes. By adding the orientifold 4-plane to this brane configuration, we have also described the intersecting brane configuration of type IIA string theory, presented by Figures 7 and 8, consisting of \((4k+3)\) NS-branes, various different kinds of D4-branes, and an orientifold 4-plane where the electric gauge theory superpotential \((3.1)\) has an order \(4(k+1)\) polynomial for the bifundamentals. The presence of orientifold 4-plane determines the gauge groups, matter contents and the positions of NS-branes and D4-branes.

It would be interesting to see whether the quartic case for the supersymmetric gauge theory with adjoint matter \([25]\) can be generalized to the higher order superpotential by increasing the number of NS5-branes, like as this paper. One can also study, in principle, how the meta-stable brane configurations for higher order superpotential case occur when there exists an orientifold 6-plane. For example, in the brane configurations \([26, 27, 28, 29, 30, 31, 32]\), it is natural to ask what happens if we increase the number of NS5-branes by looking at the supersymmetric magnetic brane configurations and analyzing the brane motion for the flavor and color D4-branes.

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