Obtaining method of high contributing whole body principal component mode by separated measurements

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Abstract
In this study, we developed a method for obtaining high contributing whole body vibration behavior to the vehicle interior noise at the running condition by modified operational TPA method (OTPA with PC model). The original OTPA with PC model requires measuring all reference and response points simultaneously. However, if the number of the point is so much, applying this method becomes hard. Hence, the proposed method was made to increase the applicability of this method by realizing to obtain the high contributing whole body vibration behavior without the simultaneous measurement of all points. In the method, several operational tests are repeatedly carried out in each measurement group and high contributing vibration behavior of each group to the response point (interior noise) was obtained as the high contributing partial PC mode. Subsequently, the high contributing whole body PC mode was obtained by integrating the partial PC modes. For obtaining it, relationship between each partial PC and the response point and original phase and amplitude compensation method were utilized. As the result, the high contributing whole body PC mode could be obtained well with much more less measurement points by comparing with the original OTPA with PC model and the applicability of the method was increased.

Keywords : Principal component mode, Contribution, Vibration mode, Operational transfer path analysis, Interior noise

1. Introduction

For carrying out effective countermeasure to vehicle interior noise, finding out the contribution of each sound or vibration source and part to the interior noise and measuring the part intensively is essential. Transfer path analysis (TPA) has been proposed to obtain the contribution quantitatively and several methods were developed until now (Plunt, 1998; Noumura, 2006; Brandl, 2008; Lohrmann, 2008; Klerk, D., 2010; Putner, 2012). Operational TPA (OTPA) is one of the methods recently developed and the method calculates the contribution using only the sound and vibration signals at the operational condition (Noumura, 2006; Lohrmann, 2008; Klerk, D., 2010; Putner, 2012). Recently, modified OTPA method (OTPA with principal component model) has also been proposed for obtaining principal component (PC) contribution to the vehicle interior noise or vibration to grasp the important vibration behavior of the target structure for the effective countermeasure (Yoshida, 2016; Yoshida, 2017). However, if we apply the method to obtain high contributing vibration behavior in detail of a large structure such as a vehicle body, a lot of reference points have to be set at around the body on the contrary to the original OTPA. This requires the huge number of simultaneously measurements of the reference and response point signals. Hence, if we do not have adequate numbers of acceleration sensors and measurement system, this method cannot be applied. On the other hand, experimental mode analysis is one of the method for obtaining vibration mode and the natural frequency of the large structure with small experimental set up. However, all vibration modes of the structure are not excited in general by the frequency characteristic of the input force and the input point. Therefore, finding out which vibration mode of the target
structure gives large influence on the interior noise is important to carry out effective countermeasure.

In this study, we then considered a separating measurement and analytical method for the OTPA with PC model to obtain the integrated whole body vibration behavior having large influence on the interior noise to increase the applicability of the method. In addition, we also utilized the method to find out high contributing whole body vibration mode in several modes for carrying out effective countermeasure of the interior noise.

2. Calculation of PC contribution

2.1. Reference and PC contribution calculation

In the original OTPA, the contribution of each reference point to the response point is obtained by multiplying each reference signal with the transfer function. The transfer function in this method is calculated using principal component regression method. The calculation procedure is shown in Fig. 1 (Noumura, 2006).

![Calculation image of transfer function of original OTPA using principal component regression method.](image)

Fig. 1 Calculation image of transfer function of original OTPA using principal component regression method. Left solid box part indicates principal component analysis. Reference signals (A<sub>m</sub>) are transposed to principal components (T) by the principal component analysis using singular value decomposition. Right dotted box part indicates regression analysis. The influence of each principal component (T) on the response signal (A<sub>out</sub>) is obtained in this part.

The calculation procedure of OTPA is described as follows.

Firstly, PC analysis is applied to the reference signal matrix [A<sub>m</sub>] by singular value decomposition (SVD) to remove correlation among reference signals as shown in Eqs. (1) and (2). The calculated uncorrelated signals are PC [T].

\[
[A_m] = [U] [S] [V]^T
\]  
(1)

\[
[T] = [A_m] [V] = [U] [S]
\]  
(2)

The reference signal matrix [A<sub>m</sub>] is obtained by applying FFT repeatedly to the simultaneously measured vibration signals at the reference points. The (i, j)-th element in the reference matrix [A<sub>m</sub>]<sub>i,j</sub> is the data at the j-th reference point in the i-th FFT. Matrices [U], [S], and [V] are obtained as the result of SVD. Here, [V] is the coefficient matrix to transpose the reference matrix [A<sub>m</sub>] to the principal component matrix [T]. The (i,k)-th element in [T] is the k-th principal component in the i-th FFT.

After eliminating the noise component having very low level, multiple regression analysis is applied between the remained (signal) PCs [T] and the response signal [A<sub>out</sub>]<sub>i</sub> to obtain the influence [B] of each principal component to the response signal as shown in Eqs. (3) and (4).

\[
[A_{out}] = [T] [B]
\]  
(3)

\[
[B] = ([T]^T [T])^{-1} [T]^T [A_{out}]
\]  
(4)

The (i,m)-th element in the response matrix [A<sub>out</sub>] is the data at the m-th response point in the i-th FFT. The (k, m)-th element in the transfer matrix [B] is the transpose coefficient from the k-th principal component to the m-th response point.
Transfer function from reference signal to response signal \([H]\) is calculated by multiplying the coefficient \([V]\), that connects reference signal to principal component, and the regression coefficient \([B]\), that connects principal component and response signal as shown in Eq. (5)

\[
[H] = [V][T][A_{out}]
\]  

(5)

The \((j, m)\)-th element in the transfer matrix \([H]\) is the transfer function from the \(j\)-th reference point to the \(m\)-th response point.

Finally, the reference point contribution and principal component contribution to the response point are calculated as shown in Eqs. (6) and (7), respectively.

\[
[A_{cont}] = [A_{in}]H
\]  

(6)

\[
[T_{cont}] = [T][B]
\]  

(7)

Eq. (6) shows the contribution of reference point. By multiplying each reference point signal \([A_{in}]\) with the transfer function \([H]\), reference point contribution \([A_{cont}]\) can be obtained. Eq. (7) shows the principal component contribution. Here, the PC contribution \([T_{cont}]\) is obtained by multiplying the calculated principal component signal \([T]\) in Eq. (2) with the transpose coefficient \([B]\) in Eq. (4). This is the outline for obtaining contributions by OTPA.

In this study, we focused on the PC contribution as shown in Eq. (7) to obtain important vibration behavior for the reduction of the response signal (Yoshida, 2016; Yoshida, 2017).

2.2. High Contributing PC Mode

Here, we explain the procedure for extracting high contributing vibration behavior (PC mode) of the target structure to the response point using OTPA with PC model.

The PC matrix \([T]\) is calculated by the PC analysis which eliminates the correlation among reference signals as shown in Eqs. (1) and (2). In addition, reference signals are also re-generated by multiplying PC matrix \([T]\) with the inverse (transpose) matrix of unitary matrix \([V]\) as shown in Eq. (8).

\[
[A_{in}] = [T][V]^{-1} = [T][V]^T
\]  

(8)

Here, the relationship between the reference signals and the PCs can be developed as follows (Eq. (9)). Noting that the reference number is reduced to two for simple explanation.

\[
\begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22} \\
\vdots & \vdots \\
a_{n1} & a_{n2}
\end{bmatrix}
= \begin{bmatrix}
t_{11}v_{11} & t_{12}v_{12} & t_{11}v_{11} & t_{12}v_{12} \\
t_{21}v_{11} & t_{22}v_{12} & t_{21}v_{11} & t_{22}v_{12} \\
\vdots & \vdots & \vdots & \vdots \\
t_{n1}v_{11} & t_{n2}v_{12} & t_{n1}v_{11} & t_{n2}v_{12}
\end{bmatrix}
\]  

(9)

In Eq. (9), the left solid box in the right part is the PC1 element in the reference signal 1 and the right solid box is the element in the reference signal 2. Left and right dotted boxes indicate the PC2 elements in the reference signal 1 and 2, respectively. This means that the reference signal can be expressed by the superposition of PC element. In addition, each PC has orthogonality (no correlation) with the other PC by the PC analysis. From these background, PC mode is considered to have similar characteristics of the vibration mode excited at the operational condition and several previous studies focused the high contributing PC mode for extracting high contributing vibration mode to the vehicle interior noise (Yoshida, 2016; Yoshida, 2017).

However, all reference point acceleration signals have to be measured simultaneously with the response point signal to obtain the mode by using the amplitude and phase relationship among the reference signals. Hence, when we apply the method to the large structure such as the vehicle body, a few hundreds of sensors and measurement systems are necessary and this may increase the difficulty to apply this method. Then, we considered a new method for increasing the applicability of the OTPA with PC model by separating the measurement and integrating the analytical.
result to obtain the high contributing PC mode of the whole vehicle body structure to the vehicle interior noise.

3. Operational test in separated measurement
3.1. Employed vehicle model

Simple vehicle body model was made for the operational test as shown in Fig. 2. The length, width and height were 850 x 300 x 300 mm. Total weight was 25 kg including four tires. Thickness of each body panel was 3 mm and the material of the panel was Aluminum. The cavity surrounding by the panel was regarded as the vehicle interior.

Fig. 2 Vehicle body model made for the operational test. Input force was given from under the tire and interior noise was recorded as the response point signal. Vibration acceleration signals at multiple points on the body panel were recorded as the reference signal.

As the input force, four electrical magnetic exciters (Modalshop: K2007E01) were put under the four tires to give the forces. Uncorrelated random noise was given in each tire for 50 s. As the response point signal, that was the target for contribution separation, vehicle interior noise was recorded by a microphone in cabin as shown in Fig. 2. As the reference point, a lot of measurement points had better to be set for obtaining detail vibration behavior of whole body. However, this OTPA with PC model requires simultaneously measurement of all reference and response signals, hence, the analysis may be unrealistic if the target structure is large. Then, we attempted to apply this method through separated measurement in this study for increasing the applicability of the method. Accordingly, the reference point around the body was separated in each panel and the reference point number varies from 16 to 24 in each panel. Thus, the operational test was carried out in total nine times. In each test, the reference point signals of a panel and the identical response signal (interior noise) was recorded simultaneously in the same input condition. As an instance, Fig. 3 shows the averaged SPL of interior noise when the reference points were set on the left side panel where 22 reference points were set.

Fig. 3 Sound pressure level of the measured interior noise at the operational test. Large SPL peak was observed at around 200 Hz as shown by orange arrow. Horizontal and vertical axes show the frequency and the SPL, respectively.

As shown in the figure, the interior noise was found to have large SPL peak at around 200 Hz. Then, this frequency was set as the target frequency for the following analysis to obtain the high contributing whole body vibration behavior.
3.2. Obtaining PC contribution (Left side panel)

As the first analysis, we applied OTPA with PC model to the simultaneously measured vibration acceleration signals (reference signals) on the left side panel and the interior noise (response signal) and obtained the PC contribution. Figure 4 shows the averaged PC contributions of PC1 to PC4 having relatively large influence to the response point in total 22 PCs.

![Fig. 4](image)

Fig. 4 Calculated PC contribution of left side panel by using the measured vibration acceleration and sound pressure signals at the operational test. Horizontal and vertical axes show the frequency and SPL, respectively. Pink, red, green, and blue curves show the PC1, 2, 3, and 4 contributions, respectively. Around the target frequency (200 Hz band), PC1 was found to have the highest contribution and PC2 was the second.

As shown in the result, PC1 of the left side panel was observed to be dominant contribution at around 200 Hz band except for 190 Hz where the level was not so high but PC2 had similar contribution with PC1. Then, the PC1 mode of the target panel, which had relatively larger contribution in all PC contributions around the target frequency band (200 Hz), was obtained as an instance in the following analysis.

3.3. High contributing PC mode (Left side panel)

We calculated the high contributing PC1 mode at 200 Hz, where large SPL peak was found at interior, when the target panel was set on the left side. Figure 5 shows the high contributing PC1 mode calculated by using Eq. (9) at 200 Hz.

![Fig. 5](image)

Fig. 5 High contributing PC1 mode of left side panel at 200 Hz obtained by the analysis to the operational data. In this vibration behavior, the center of the left side panel had large vibration amplitude.

As shown in upper figure, the PC mode was found to have large vibration at the center of the left side panel. However, the PC mode only shows the vibration behavior of the left side panel and considering suitable countermeasure to the whole body using this information is hard. Then, we considered a method to obtain the whole body vibration behavior affecting largely to the vehicle interior noise through the separated measurement.

4. Integrated high contributing PC mode of whole body

4.1 Methodology

In this study, the operational test was carried out separately by setting reference points on each panel and the identical response point. And we also obtained PC contribution and high contributing PC mode at 200 Hz by applying
the OTPA with PC model to the reference and response signals in the other panel as same as the left side panel. Here, we attempted to obtain the high contributing PC mode of the “whole” body by using the previously obtained high contributing “partial” PC mode of each panel. But the reference signals in different panels were not measured simultaneously and the actual phase and amplitude relationship among panels of the whole body are not retained at present. Then, we considered the integrating method to obtain the whole body PC mode affecting largely to the interior noise using the partial PC mode of each panel as follows.

In case a PC mode of panel A (PC_A mode) is a part of the global (whole body) vibration mode, the time variance of PC_A value in an operational condition has perfect correlation with the time variance of the global vibration mode. In addition, if the global vibration mode has large influence on the interior noise (reference signal) and has the correlation of them is 0.8, the PC_A mode must has the same correlation (0.8) with the response signal in the operational test as shown in Fig. 6. On the other hand, In case a PC mode of panel B (PC_B mode) is also a part of the same global vibration mode, the time variance of PC_B in the other operational condition has perfect correlation with the global vibration mode. And, the PC_B mode also has the same correlation (0.8) with the response signal in the operational test as same as PC_B because the PC_B mode is a part of the global vibration mode having 0.8 correlation with the response signal as shown in Fig. 6. This means if both PC_A and PC_B modes are part of the identical global vibration mode having large influence on the response signal, the correlation of each partial PC mode to the response signal in each separated operational test becomes similar and high. Then, we considered the associated high contributing partial PC mode for all panels can be found by using the correlation between each partial PC and the response signal in each operational test.

![Image](https://example.com/image.png)

**Fig. 6** Image of related PC of each panel and the response point. As shown in the left three figures, if the same vibration mode is excited in each part, the obtained partial PC mode is considered to have perfect correlation in case the PC mode expresses the part of the vibration mode. And the correlation between the partial PC mode and the response point also becomes the same as the vibration mode.

After obtaining the associated high contributing partial PC number in each panel, we subsequently considered how to compensate the phase and amplitude relationship of the partial PC mode in each panel for obtaining the integrated high contributing whole body PC mode as follows.

To obtain relationship of each partial PC mode, setting common measurement point in every separated measurement and calculating relative phase and amplitude of each reference point in each high contributing partial PC mode by dividing them by the common signal is one of the solutions. However, if the common measurement point is close to the node of the global mode, the calculated relative phase and amplitude becomes inaccurate because the signal to noise ratio of the measured common signal is low. Then, adding several common points is better to obtain stable relative phase and amplitude, however this requires more sensors and decreases maximum number of the measured acceleration signals for making PC mode. Then, we considered another method to obtain accurate high contributing whole body PC mode without setting any additional measurement points.

In this method, the amplitude and phase of high contributing partial PC is calculated if unit response signal (interior noise) is generated by the PC by using the PC transfer function ([B]) in Eq. (7) as shown in Fig. 7. Subsequently, the amplitude and phase of the reference points composing the partial PC in the target panel are also calculated by using the transpose coefficient matrix [V] in Eq. (2). By applying this process to each partial PC mode in each panel, the phase and amplitude compensated partial PC mode is obtained.
Fig. 7  Image of compensation of relative amplitude and phase using unit response signal. In the method, the reference point vibration is calculated using the PC transfer function when the unit response point (interior noise) is generated in each panel.

4.2. Obtaining integrated high contributing PC mode of the whole body

Here, we attempted to obtain the high contributing whole body PC mode at 200 Hz by referring the proposed method. Firstly, the correlation coefficient of each PC and the response signal were calculated in each operational test to find out the associated PC number in each panel. Figure 8 shows the coefficient from PC1 to PC4 having relatively large level in all PCs.

Fig. 8  Correlation coefficient of each calculated PC signal and the response signal through the analysis to the operational data. Pink, red, green, and blue bars show the correlation coefficient between each partial PC1, 2, 3, and 4 with the response signal, respectively. As shown in this figure, PC1 of most panel and PC2 of front panel were found to have largest correlation with the response point generally. Thus, these PCs are considered to express the same global vibration mode.

As shown in this figure, PC1 had largest correlation with the response signal in most panels except for front panel. This indicates that PC1 in most panels and PC2 in the front panel were high contributing partial PC modes and they composed the high contributing whole body PC mode. Secondly, the whole body high contributing PC mode at 200 Hz was made after calculating compensated phase and amplitude of each reference point of each high contributing partial PC mode by using the unit response signal. Figure 9 shows the integrated high contributing whole body PC mode.
Fig. 9 Integrated high contributing whole body PC mode at 200 Hz obtained by using only the operational data. The vibration behavior had large vibration amplitude at the center of the left and right side panel. The amplitude of the left side panel was larger than the right side panel and the phase of them was opposite.

As shown in the figure, the high contributing whole body PC mode was indicated to have large vibration on the center of left and right side panels. But the amplitude of the left side panel was larger than the right side panel and the phase of them was opposite.

Subsequently, the vibration mode of the target structure at around 200 Hz was obtained through the experimental mode analysis to verify whether the integrated whole body PC mode actually indicates the global vibration mode of the whole body or not. As the result, two vibration modes were observed at 192 Hz and 201 Hz around 200 Hz as shown in Fig. 10. Figure 10 shows the point inertance at the center of the left side panel and the vibration mode shapes at around 200 Hz obtained by the experimental mode analysis. Through the comparison of the integrated whole body PC mode in Fig. 9 and the vibration modes at around 200 Hz in Fig. 10, the high contributing PC mode (Fig. 9) at 200 Hz was observed to have very similar vibration behavior with the vibration mode at 201 Hz but the mode shape was quite different from the vibration mode at 192 Hz. This shows that this method could extract the high contributing vibration mode in a few modes by using the integrated high contributing PC mode.

Fig. 10 Point inertance at the center of the left side panel and the vibration mode shapes at 192 Hz and 201 Hz obtained by the experimental mode analysis. The vibration mode shape at 192 Hz had the large vibration at the center of the left and right side panels and the vibration had the same phase. On the other hand, the vibration mode shape at 201 Hz also had the large vibration at the center of the left and right side panels but the phase was opposite.

5. Interior noise reduction considering high contributing integrated whole body PC mode

Main purpose of the proposed method in this study is to extract the whole body vibration behavior (vibration mode) affecting largely to the interior noise through the separated measurement at the operational condition. Accordingly, if the method can actually indicate the high contributing whole body PC mode, the interior noise is expected to decrease effectively by applying the countermeasure to the obtained high contributing mode. Then we
attempted to apply countermeasure to verify the proposed method as follows. The high contributing integrated whole body PC mode at 200 Hz was shown to have large vibration at the center of left and side panels along the opposite direction and the PC mode had almost same vibration behavior with the vibration mode at 201 Hz. Hence, to verify whether the extracted vibration mode had actually large influence on the interior noise, we performed a countermeasure considering the mode shape. In this instance, we inserted a steel bar between the center of the left and right panels to constrain the oppositely movement of the side panels as shown in Fig.11 (a).

![Countermeasure Image](image)

(a) Countermeasure  (b) Point inertance at center of left side panel  (c) Interior noise SPL

**Fig. 11** Countermeasure image and the influence on the vibration and interior noise at the experiment. (a) shows the countermeasure image. Steel rod was inserted to constrain the side panel vibration. (b) shows the comparison of point inertance at the left side center point by the impact hammering test before and after the countermeasure. Red and blue curve indicate the inertance before and after countermeasure, respectively. The inertance was found to decrease largely over 150 Hz. (c) shows the comparison of the interior noise SPL at the operational test. Red and blue curves show the SPL before and after countermeasure, respectively. Apparently, the interior noise was found to decrease largely only at the target frequency band of 200 Hz.

After inserting the steel rod, we carried out impact measurement test to obtain point inertance at the left panel center and also performed operational test again at the same condition to measure interior noise before and after countermeasure. Figure 11 (b) and (c) shows the point inertance comparison and the interior SPL comparison by the impact hammering test and the operational test, respectively. As the results, the point inertance at the left side panel was observed to decrease largely at wide frequency band over 150 Hz as shown in Fig. 11 (b), but the interior noise was found to decrease largely only at 200 Hz band (reduction target frequency) as shown in Fig. 11 (c). This reveals that the side panel vibrated at the wide frequency band but the side panel vibration affects largely to vehicle interior noise only at 200 Hz band, hence the interior noise at 200 Hz was mainly decreased by the countermeasure. From these results, these integrated whole body PC modes was clarified to have large influence on the interior noise at 200 Hz actually.

6. Summary

In this study, we developed a method for integrating high contributing PC mode of the whole body to the interior noise using OTPA with PC model through separated measurement. In the method, the high contributing whole body PC mode was obtained by using the following two procedures. The first one is the associated PC selection method at different panel using the correlation of each partial PC with the response signal. The second one is the compensation method for obtaining relative phase and amplitude of each reference point of the high contributing partial PC mode using the unit response signal. The obtained whole body PC mode was confirmed to have the same vibration mode of the whole body, and the PC mode also indicated which vibration mode of the target structure actually had large influence on the interior noise in a few vibration modes.

From these results, this OTPA with PC model can be applied if we cannot prepare large measurement system and sensors adequate for the simultaneously measurement to all reference points by carrying out the separating measurement with the proposed method. And the method could extract high contributing global vibration mode excited at the operational condition and giving large influence on the interior noise for the effective countermeasure.
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