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Methods of Multidimensional Continuum Mechanics for Economy

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Abstract. A fundamentally new approach is proposed to the development of models of large arrays of dynamically changing data in the economy, based on the construction of a multidimensional feature space and the transition to a continual description of the dynamics of the movement of data clusters. For the motion of clusters of data, the laws of conservation of multidimensional continua are formulated, which are analogs of the laws of conservation of mass, momentum, and angular momentum in classical mechanics. Models of rigid and deformable multidimensional continua are developed. Examples of application of these models for the analysis of economic data on sales in large supermarkets are given.

1. Introduction
Methods of nonlinear intellectual data analysis based on the search and formalization of inner patterns in the data and their dynamics are frequently used to simulate the processes in economics [1-7]. The aim of this work is to elaborate a model for economic big volume economic data dynamics for which the search of inner patterns of data is made using the multidimensional deformable continuum mechanics methods. The model of non-deformable rigid continuum was considered in the work [8,9].

2. Multidimensional Continuum and Conservation laws for Economy
Let there be a set of buyers and the information about their purchases quantity. Let’s introduce the Euclidean space $E^n$, where the coordinates $x$ mean a total number of good $i$ bought or planned for purchase by one consumer for time $t$, with $i=1,\ldots,n$, and $x' \in \mathbb{R}$ [10]. Fixing the coordinates’ values $x'=X'_i$ of the same buyer for time point $t=0$ and keeping these coordinates for this buyer for arbitrary $t>0$ we obtain the buyer’s motion law $x=x(X',t)$ in $E^n$, which defines the Lagrangian-Euler description of buyers’ motion. Using the agglomerative hierarchical clustering [9], we divide the set of buyers at $t=0$ in $E^n$ into groups - i.e. clusters- consisting of buyers with a similar buying behavior. We assign to each cluster the bounding area $V$ in $E^n$. For an arbitrary $t>0$ the area $V(t)$ consisting of one and the same points, i.e. buyers, generally speaking, changes, in other words, deforms.

Let’s pass from the discrete description of points to the continuum with the introduction of the density function $\rho(x',t) = \frac{dM}{dV}$ which is the ratio of buyers’ quantity $dM$ in the local area to the volume of this area $dV$. Then $M = \int_V \rho dV$ is the quantity of buyers in cluster $V$. Let’s further consider the case when $M=\text{const}$ for all $t>0$, this relation can be considered as the mass $M$ conservation law. Let’s introduce axiomatically the analogs of the momentum and angular momentum conservation laws [10] for the cluster $V$. In the economic sense these axioms can be stated as follows: 1) the change of total vector of the cluster purchase frequency $\int_V \rho v dV$, where $v = \partial \mathbf{x}(X',t) / \partial t$ is determined by the total force vector
\( f \) of interaction with other clusters or the external sources, 2) the change of momentum tensor of cluster purchase frequency \( \int_{V} \rho \mathbf{x} \times \mathbf{v} dV \) is determined by the total moment of external forces:

\[
\frac{d}{dt} \int_{V} \rho \mathbf{v} dV = \int_{V} \rho \mathbf{f}_{m} dV + \int_{\Sigma} \mathbf{t}_{\Sigma} d\Sigma , \quad (1)
\]

\[
\frac{d}{dt} \int_{V} \rho \mathbf{x} \times \mathbf{v} dV = \int_{V} \rho \mathbf{x} \times \mathbf{f}_{m} dV + \int_{\Sigma} \mathbf{x} \times \mathbf{t}_{\Sigma} d\Sigma , \quad (2)
\]

where \( \mathbf{x} \times \mathbf{v} \) is the vector product in \( E_{n} \) [11], \( \mathbf{f}_{m} = df / dM \) is the vector of external mass force density that changes the local group purchase frequency, \( \mathbf{t}_{\Sigma} = df / d\Sigma \) is the total vector of surface forces.

3. Model of the deformable multidimensional continuum

Let’s introduce the deformable multidimensional continuum model in the \( E_{n} \) space which motion law is a superposition of cluster mass center translational motion \( \mathbf{x}_{0} = \mathbf{x}_{0}'(t) \mathbf{e}_{i} \), and prompt-rotatory movement of the whole cluster as a solid body around the mass center, and the cluster tension or compression along the certain major axes \( O^{'} \mathbf{p}_{j} \) represented by tensor \( \mathbf{S} \):

\[
\mathbf{x} = \mathbf{x}_{0} + \mathbf{S} \cdot \mathbf{Q} \cdot \mathbf{x}' , \quad (3)
\]

where \( \mathbf{x}_{0}' \) is Cartesian (Euler) coordinate of a cluster mass center, \( \mathbf{x}' = \mathbf{X}' \mathbf{e}_{j} \) is a radius-vector of a concerned point in the Lagrangian description, \( \mathbf{e}_{i} \) is a Cartesian basis, \( \mathbf{Q} = Q'^{ij}(t) \mathbf{e}_{i} \otimes \mathbf{e}_{j} \) is an orthogonal rotation tensor, and \( \otimes \) is a tensor multiplication operator. Tension tensor \( \mathbf{S} \) is supposed to be symmetric and positively defined, and there exists real-valued proper basis \( \mathbf{p}_{j} \) in which this tensor has a diagonal form \( \mathbf{S} = S'^{ij} \mathbf{e}_{j} \otimes \mathbf{e}'_{i} = \sum_{i=1}^{n} S_{ij} \mathbf{p}_{j} \otimes \mathbf{p}_{i} \), where \( S'^{ij} \) are components of the symmetric cluster deformations matrix, \( S_{ij} \) are the real positive eigenvalues of tensor \( \mathbf{S} \), \( \mathbf{p}_{j} \) is the proper basis.

In this model we suppose that the only reason for change of distance between points in cluster (i.e. deformation) is the changing ratio between a buyer’s financial reserve \( \hat{e} \) and its initial value \( \hat{e}_{0} \), that’s why the eigenvalues \( S_{ij} \) are supposed to be axiomatically given: \( S_{ij} = S_{j}(\hat{e} - \hat{e}_{0}) \). In the simplest case this relation is considered to be linear \( S_{ij} = 1 + A_{j}(\hat{e} - \hat{e}_{0}) \), where \( A_{j} \) are coefficients. We suppose further that the major axes \( O^{'} \mathbf{p}_{i} \) of tension tensor \( \mathbf{S} \) are fixed relatively to the major axes of cluster inertia \( O^{'} \mathbf{x}' \).

The deformable cluster purchase frequency vector is defined by the expression

\[
\mathbf{v} = \frac{d\mathbf{x}}{dt} = \dot{\mathbf{x}}_{0}(t) + (\mathbf{S} \cdot \mathbf{Q}) \cdot \mathbf{x}'. \quad \text{This formula can be also rewritten in the form}
\]

\[
\mathbf{v} = \mathbf{v}_{0} + \dot{x} \cdot \mathbf{W}, \quad \mathbf{W} = \mathbf{W} + \ln \mathbf{S}' , \quad (4)
\]

where \( \mathbf{v}_{0} = \dot{\mathbf{x}}_{0} \) is a motion velocity of a cluster rotation center, \( \dot{x} = \mathbf{x} - \mathbf{x}_{0} \) is a relative radius-vector, \( \mathbf{W} = \mathbf{Q} \cdot \mathbf{Q}' \) is a skew-symmetric vorticity tensor, and \( \mathbf{S}' \) is the Jaumann derivative [5] of a tension tensor. Formula (4) is a generalization of the \( n \)-dimensional Euler formula for the velocities distribution in the non-deformable multidimensional solid body.

The purchase frequency moments tensor for a deformable cluster can be rewritten taking into account (5) as follows:

\[
\mathbf{\dot{m}} = -\mathbf{\alpha} \otimes \mathbf{\alpha}_{0} + (\mathbf{W} \cdot \mathbf{I}) \cdot \mathbf{\alpha}_{0} + (\ln \mathbf{S}' \cdot \mathbf{I}) \cdot \mathbf{\alpha}_{0} , \quad \text{where}
\]

\[ ^{a}\mathbf{\alpha} = e_{a,ij} e^{i} \otimes \ldots \otimes e^{i} \] is the Levi-Civita tensor, \( e_{a,ij} \) are the Levi-Civita symbols, and \((\cdots)\) is the double scalar multiplication operator [11], \( \mathbf{I} = \int_{V} \rho \mathbf{\ddot{x}} \otimes \mathbf{\ddot{x}} dV \) is a cluster inertia moments tensor.
Plugging in the formula (4) and the expression for \(\frac{n}{2}\mathbf{m}\) into the system (1)-(2), we come to the Cauchy problem for a deformable cluster motion:

\[
\begin{cases}
M \frac{dv_n}{dt} = \overline{f}, \quad \frac{dx_0}{dt} = v_0, \quad Q + Q \cdot W = 0, \\
\frac{d}{dt} (W' \cdot \mathbf{I}) \cdot \mathbf{x} + \frac{d}{dt} (\ln S' \cdot \mathbf{I}) \cdot \mathbf{c} = n \mathbf{\mu}, \\
t = 0: \quad x_0 = x_0^0, \quad v_0 = v_0^0.
\end{cases}
\]  

(5)

where \(\overline{f} = \int \rho f_m dV + \int \sum t_{ij} d\Sigma + \overline{f}_\rho\) is a total vector of forces and \(n \mathbf{\mu} = \int \rho x \times f_m dV + \int x \times t_{ij} d\Sigma + x \times \overline{f}_\rho\) is a moment of the force tensor. These last two expressions include the additional force \(\overline{f}_\rho\) and the moment \(n \mathbf{\mu}\), which are generated by the additional condition of non-negativity of purchase quantity \(v^j = v_0^j + \mathbf{z}' \mathbf{W}' \geq 0\). As \(\overline{f}_\rho\) and \(n \mathbf{\mu}\) depend upon \(\mathbf{W}\) and \(v_0\), the equations in (5) are connected and don’t break up into separated groups. The third group of equations represents the n-dimensional Poisson equations.

4. Example of model application

The elaborated model was applied to analysis of experimental data for a sales market with five price categories of large household appliances via online-shop (n=5) in thirty months. The space \(E_n\) thus was 5-dimensional. We executed clusterization of customers in \(E_n\) space at time \(t=0\) using the agglomerative hierarchical clustering. For a motion analysis of customers’ clusters we applied the model of a multidimensional deformable cluster. When modeling we approximated a cluster with a multidimensional ellipsoid.

The study of ellipsoids’ mass centers motion showed that this motion had a linear form along all the goods’ axes \(x^j\) in the space \(E_n\), when there was no external forces influence as it followed from the model elaborated. Under the external forces influence the mass center motion trajectory changes the slope angle. Figure 1 shows the motion of the \(n\)-dimensional ellipsoid defined by the deformable cluster model and dissected by the plane \((x^3, x^4)\) for time points \(t_2, t_5\) and \(t_{11}\): the light dots designate the points inside the approximating ellipsoid, while the black dots indicate the ones outside this ellipsoid.

![Figure 1 - Motion of the n-dimensional ellipsoid constructed from the model of the deformable cluster at the control points t2, t5 and t11.](image-url)

Taking into account the change of distance between cluster points depending upon the customers financial reserve allows us to raise the precision of the model in comparison with the model of a rigid cluster, where distances between points are considered to be in a whole time period [9]. The model precision is reached however not because of the excessive cluster increase, as the maximum values of
modeled ellipsoid’s semiaxes’ lengths do not exceed the lengths of semiaxes of the ellipsoid calculated by the approximation of data at each time point. The model precision rises with an increase of the points being part of a cluster. The precision reaches its highest value with the usage of the initial data for $Q^j(0)$ and $I^j(0)$ calculated by the methods with averaging.

The deformable cluster method demands more input information than the rigid cluster model [9] – one need to put into the model the data for a customers’ financial reserve and its impact on the change of purchase quantity. In the cases when this information is not available, the rigid cluster model can be more preferable. For a more precise forecast of the data dynamics, the deformable cluster model can be recommended.

5. Conclusions

The application of continuum mechanics methods for the analysis of large arrays of dynamically changing data in the economy represents a fundamentally new direction - both for the mathematical economy and for the mechanics of continuous media itself, since a completely new object that has not been studied before, a multi-dimensional deformable continuum, laws of conservation.

The classical models of mechanics of 3-dimensional continua can hardly be transferred to the modeling of economic processes, since "economic continuous media" obey other laws.

Special models of rigid and deformable multidimensional continua are developed. Examples of application of these models for the analysis of economic data on sales in large supermarkets are given.

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