Sensitivity analysis of elastoplastic structures and application to optimal specimen design

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This paper is concerned with the shape improvement of novel specimens for biaxial experiments in terms of optimal stress states, characterized by the stress triaxiality. For this, gradient based optimization strategies are utilized and the needed sensitivities are obtained using a variational approach. Considering elastoplastic material behavior, the deformation history as well as its sensitivity has to be taken into account. In a first step, the material parameters have to be identified for the given material (AlCuMgSi). With the identified parameters, a chosen specimen is numerically analyzed and optimized with the aim to achieve a preferably homogeneous stress triaxiality distribution in the relevant part of the specimen.

1 Introduction

The stress triaxiality $\eta$ is an essential value to characterize the stress state. It is defined as

$$ \eta = \frac{\sigma_{\text{m}}}{\sigma_{\text{eq}}} = \frac{I_1(\sigma)}{3 \sqrt{3} J_2(\sigma)}, $$

(1)

with the mean stress $\sigma_{\text{m}}$, the von Mises equivalent stress $\sigma_{\text{eq}}$ and the invariants of the (deviatoric) stress tensor $I_1$ and $J_2$. It classifies the stress state to be tensile if $\eta > 0$, compressive if $\eta < 0$ or pure shear if $\eta \approx 0$. Gerke et al. \cite{3} developed new biaxial specimens that widen the range of stress triaxiality in one specimen at different loading scenarios. With these specimens it is possible to capture different micro-mechanical damage mechanisms that arise at different stress triaxiality states with only one specimen. In order to identify material parameters for special damage models it is crucial to obtain preferably homogeneous stress triaxiality states.

In this paper, the aim is to further optimize the shape of the chosen XO2-specimen to achieve a homogeneous stress triaxiality distribution in the cross section of the specimen. In a first step, the material parameters for the underlying elastoplastic material model have to be identified, before in a second step the shape optimization can be performed.

2 Response and sensitivity analysis

In this section, the material model, solution strategies to compute the structural response and response sensitivities are briefly outlined. The mechanical model is based on a multiplicative split of the deformation gradient ($F = F^e F^p$) and can be found in the relevant literature, e.g. \cite{2,5,6}. The strain energy function reads

$$ W = \frac{1}{2} \kappa \left[ \frac{1}{2} \left( J^e - 1 \right) - \ln J^e \right] + \frac{1}{2} \mu \left[ \text{tr} B^e - 3 \right], \quad J^e = \det F^e, \quad \bar{B}^e = J^{e-\frac{2}{3}} F^e F^e, $$

(2)

and stresses are obtained using a Return-Mapping-Algorithm, c.f. \cite{2,6}. The nonlinear weak form of equilibrium in the actual configuration $\mathcal{M}$ is solved utilizing Newton’s scheme within a Finite Element framework and reads

$$ r(u; v) = \int_{\mathcal{M}} \sigma(u; h) : \text{grad } v \, dv - \int_{\mathcal{M}} \bar{b}_t \cdot v \, dv - \int_{\partial \mathcal{M}} \bar{t}_t \cdot v \, da. $$

(3)

The set of history variables that has to be saved in each load step and integration point is $h = \{ B^e, \alpha \}$.

Response sensitivities are obtained variationally at continuous level. As Eq. (3) has to hold for any design change $\delta s$, its total variation has to vanish, c.f. \cite{1}

$$ \delta r = \delta_s r + \delta_s B + \delta_h \delta_n = 0 \quad \Rightarrow \quad K \delta u + P \delta s + H \delta n = 0. $$

(4)

The third summand corresponds to the deformation history that has to be taken into account for all variations, c.f. \cite{4,7}. From Eq. (4), the sensitivity matrix $S$ can be derived, that is the total derivative of the structural response w.r.t. design changes

$$ \delta u = -K^{-1} [P \delta s + H \delta h] = -K^{-1} [P + H G_n] \delta s, \quad \text{with} \quad \delta n = \left[ \frac{\partial h_n}{\partial u}, \frac{\partial h_n}{\partial s} \right] \delta s = G_n \delta s. $$

(5)

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3 Results

In a first step, the method mentioned above has been used to fit the material parameters of the constitutive model by means of a standard tensile test. For the sake of brevity, only the obtained results are shown. The parameters have been identified to

\[ E = 61.92 \text{ GPa}, \quad \nu = 0.3, \quad \sigma_0 = 313.3 \text{ MPa}, \quad \sigma_\infty = 119.2 \text{ MPa}, \quad d = 10.72, \quad H = 548.7 \text{ MPa}. \]

Fig. 1: XO2-specimen

Fig. 2: Geometry constraints

Fig. 3: Stress triaxiality at cross section

In a second step, the optimization task is to optimize the shape of the so-called XO2-specimen, see Fig. (1). Design variables are the inner radius \( R_1 \), the outer radius \( R_2 \), the radius in thickness direction \( R_3 \) and the penetration depth \( d \) \((s = [R_1 = 3.003 \text{ mm}, \quad R_2 = 3.003 \text{ mm}, \quad R_3 = 1.500 \text{ mm}, \quad d = 1.000 \text{ mm}])\). With the aim to achieve a preferably homogeneous distribution of stress triaxiality in the cross section of the specimen, the optimization problem to solve reads

\[
\text{max} \ J(s) = ||\hat{\eta}(s)||_2^2, \quad \text{s.t.} \quad s^2 \leq s^i \leq s^u, \quad c^eq = 0, \quad c^in \leq 0. \tag{6}
\]

Here, \( \hat{\eta} \) is the vector of stress triaxiality at the cross section. During the optimization process the cross section has to stay constant at 12 mm\(^2\) and \( R_3 \) must be larger than the penetration depth \( d \) to avoid sharp edges. Thus the constraints reads

\[
c^eq = t_{qs} \cdot l_{qs} - 12 \text{ mm}^2 = 0, \quad c^in = d - R_3 \leq 0. \tag{7}
\]

After 28 iterations the solution is found and the objective function could be increased from \( J^{\text{init}} = 18.292 \) to \( J^{\text{opt}} = 19.314 \). The equality and inequality constraints are fulfilled at the end of the optimization process

\[
c^eq = 3.2925 \times 10^{-10}, \quad c^in = -8.9378 \times 10^{-4}. \tag{8}
\]

The stress triaxiality distribution of the initial and optimized shape is shown in Fig. (3), the optimized design parameters are

\[ R_1 = 1.889 \text{ mm}, \quad R_2 = 1.570 \text{ mm}, \quad R_3 = 1.272 \text{ mm}, \quad d = 1.721 \text{ mm}. \]

4 Summary and Conclusion

An efficient strategy to compute elastoplastic response sensitivities at continuous level utilizing variational principles has been outlined. Simultaneous computation of mechanical response and response sensitivities can be calculated within a Finite Element framework. The obtained gradient information has been used to solve two different inverse problems, namely a parameter identification and a shape optimization. In the cross section of the shape optimized specimen, we can observe an improvement of the homogeneity of the stress triaxiality distribution compared to the initial shape.

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