THE RELATION BETWEEN GAS DENSITY AND VELOCITY POWER SPECTRA IN GALAXY CLUSTERS: QUALITATIVE TREATMENT AND COSMOLOGICAL SIMULATIONS

I. Zhuravleva1,2, E. M. Churazov3,4, A. A. Schekochihin5,6, E. T. Lau7,8, D. Nagai7,8,9, M. Gasparet10, S. W. Allen1,2,10, K. Nelson9, and I. J. Parrish11

1 Kavli Institute for Particle Astrophysics and Cosmology, Stanford University, 452 Lomita Mall, Stanford, CA 94305, USA; zhur@stanford.edu
2 Department of Physics, Stanford University, 382 Via Pueblo Mall, Stanford, CA 94305-4060, USA
3 Max Planck Institute for Astrophysics, Karl-Schwarzschild-Strasse 1, D-85741 Garching, Germany
4 Space Research Institute (IKI), Profsoyuznaya 84/32, Moscow 117997, Russia
5 The Rudolf Peierls Centre for Theoretical Physics, University of Oxford, Oxford OX1 3NP, UK
6 Merton College, Oxford OX1 4JD, UK
7 Department of Physics, Yale University, New Haven, CT 06520, USA
8 Yale Center for Astronomy and Astrophysics, Yale University, New Haven, CT 06520, USA
9 Department of Astronomy, Yale University, New Haven, CT 06520, USA
10 SLAC National Accelerator Laboratory, 2575 Sand Hill Road, Menlo Park, CA 94025, USA
11 Canadian Institute for Theoretical Astrophysics, University of Toronto, 60 St. George Street, Toronto, ON M5S 3H8, Canada

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ABSTRACT

We address the problem of evaluating the power spectrum of the velocity field of the intracluster medium using only information on the plasma density fluctuations, which can be measured today by Chandra and XMM-Newton observatories. We argue that for relaxed clusters there is a linear relation between the rms density and velocity fluctuations across a range of scales, from the largest ones, where motions are dominated by buoyancy, down to small, turbulent scales: \( \frac{\delta \rho_k}{\rho} = \eta_1 (V_{ic,k}/c_s)^2 \), where \( \delta \rho_k/\rho \) is the spectral amplitude of the density perturbations at wavenumber \( k \), \( V_{ic,k}^2 = V_{w,k}^2/3 \) is the mean square component of the velocity field, \( c_s \) is the sound speed, and \( \eta_1 \) is a dimensionless constant of the order of unity. Using cosmological simulations of relaxed galaxy clusters, we calibrate this relation and find \( \eta_1 \approx 1 \pm 0.3 \). We argue that this value is set at large scales by buoyancy physics, while at small scales the density and velocity power spectra are proportional because the former is a passive scalar advected by the latter. This opens an interesting possibility to use gas density power spectra as a proxy for the velocity power spectra in relaxed clusters across a wide range of scales.

Key words: galaxies: clusters: intracluster medium – hydrodynamics – methods: analytical – methods: numerical – plasmas – turbulence

Online-only material: color figures

1. INTRODUCTION

Spectacular data accumulated by X-ray observatories on the nearest X-ray brightest clusters of galaxies allow us to probe inhomogeneities in the intracluster medium (ICM) over a broad range of spatial scales. These clusters typically show ~5%–10% density fluctuations on scales from a few tens to a few hundred kiloparsecs (Churazov et al. 2012; Sanders & Fabian 2012; Schuecker et al. 2004). At the same time, the dynamics of the ICM remain largely unknown. For relaxed clusters, numerical simulations predict predominantly subsonic motions of gas on scales from ~megaparsecs down to a few tens of kiloparsecs with approximate Kolmogorov power spectra (PS) of the velocity field (e.g., Norman & Bryan 1999; Dolag et al. 2005; Nagai et al. 2007b; Iapichino et al. 2011; Vazza et al. 2011).

The relatively low energy resolution (~130–150 eV) of current X-ray CCD-type detectors precludes accurate measurements of gas velocities (e.g., Zhuravleva et al. 2013; Tamura et al. 2014). The gain in resolution can be achieved in cool cores of clusters by using grating spectrometers. Such observations provide mostly velocity upper limits approximately a few hundred kilometers per second (e.g., Werner et al. 2009; Sanders & Fabian 2013; Churazov et al. 2004). One can also use Faraday rotation measurements to probe the ICM turbulence indirectly (e.g., Vogt & Enßlin 2003).

The future Japanese–U.S. X-ray observatory Astro-H (launch in 2015; Takahashi et al. 2010) should provide high-resolution (~4–7 eV) X-ray spectra, allowing one for the first time to measure gas velocities directly. However, it will not be trivial to extract the PS of the velocity field (see methods developed in Zhuravleva et al. 2012). The full power of the methods can only be used once Athena+12 is operational.

In the meantime, are there ways to probe the velocity PS with existing and near-term data? In this Letter, we argue that, for subsonic motions in relaxed clusters, there is a linear relation between the PS of density fluctuations derived from X-ray images and the velocity PS. Using analytical descriptions of a passive scalar advected by fluid motions in stratified medium, we show that the linearity holds from large scales where motions are dominated by buoyancy down to small, turbulent scales with the same coefficient of proportionality. In turbulent regimes, linear dependence was found in simulations of massive cluster with solenoidal forcing in Gaspari & Churazov (2013). It is interesting that similar situations arise in the context of solar wind, Earth atmosphere, and the ISM (see, e.g., Armstrong et al. 1995).

Churazov et al. (2012) list the following contributions to measured density variations in clusters: (1) perturbations of the gravitational potential, (2) deviations from the oversimplified model

\[ \text{http://atena2.irap.omp.eu/} \]
profiles, (3) entropy fluctuations, (4) pressure variations associated with gas motions and sound waves, (5) metallicity variations, and (6) the presence of non-thermal and spatially variable components. Cosmological simulations of relaxed clusters (Section 3), which include effects (1)–(4), illustrate that predicted linearity holds approximately in the case of “natural” driving. In the companion paper (Gaspari et al. 2014, hereafter G14), high-resolution simulations in a static gravitational potential with solenoidal forcing of turbulence are used to investigate items (3) and (4) and the role of isotropic thermal conduction.

2. VELOCITY FIELD AND DENSITY FLUCTUATIONS

Let us consider slow, subsonic gas motions in a cluster potential. Different regimes can be distinguished: (1) a large-scale limit, where the dynamics are governed by buoyancy and (2) a turbulent regime at small scales, where the eddy turnover time is shorter than the characteristic buoyancy timescale. Below, we argue that there is a linear relation with the same constant of proportionality between the amplitudes of the gas density and velocity fluctuations in both regimes.

2.1. Buoyancy-dominated Regime (Large Scales)

Assuming that the ICM can be described by standard hydrodynamics or magnetohydrodynamics (MHD), the entropy scales height,13 and the last equality follows (Section 3), which include effects (1)–(4), illustrate that predicted linearity holds approximately in the case of “natural” driving. In the companion paper (Gaspari et al. 2014, hereafter G14), high-resolution simulations in a static gravitational potential with solenoidal forcing of turbulence are used to investigate items (3) and (4) and the role of isotropic thermal conduction.

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In reality, perturbations are not infinitesimal, and the question is to what extent the linear relationship between density and velocity survives in the strongly nonlinear regime when the advection term in Equation (2) is not negligible. The argument that this relationship does survive depends somewhat nontrivially on the strength of the ICM turbulence at the outer (energy-injection) scale. The key parameter is the Froude number, $Fr = \frac{V_{\text{rms}}}{L_\perp N} = \frac{Ma \sqrt{TH_p H_p}}{L_\perp}$, (5) the ratio of the nonlinear decorrelation and linear Brunt–Väisälä frequencies at the outer scale ($V_{\text{rms}}$ is the rms velocity of the turbulent motions, $Ma = V_{\text{rms}}/c_s$ is the Mach number, and $L_\perp$ is the outer scale perpendicular to the radial direction).

If $Fr \ll 1$, the turbulence will tend to a stratified, anisotropic regime, in which $k_\perp \ll k \approx k_s$, and the gravity-wave frequency $\omega = N k_\perp/k \ll N$ stays comparable to the nonlinear decorrelation rate $k_\perp V_s$ (a type of “critically balanced” state; see Lindborg 2006; Nazarenko & Schekochihin 2011). In this regime, buoyancy remains important scale by scale ($k$ by $k$) as the energy cascades to smaller scales (larger $k$), the relationship (Equation (4)) is approximately satisfied, and the typical velocity of the motions is dominated by the perpendicular velocity: $V \sim V_\perp \sim (k_\perp/c_s) V_r$ (by incompressibility). Therefore, we deduce, at each $k$,

$$\left(\frac{\delta s}{\rho}\right)^2 \sim \frac{\delta s}{s} = \frac{V_r}{c_s} \left(\frac{V_r}{c_s}\right)^2,$$

(6) where $\eta$ is a scale-independent dimensional constant of the order of unity. Here, $\delta s/k$ and $V_r$ are some suitably defined fluctuation amplitudes at scale $k^{-1}$ (see Section 3.2), rather than Fourier components they are related to the three-dimensional (3D) spectrum $E_k$ by $|V_r|^2 \sim k E_k$.

For this kind of turbulence, it is possible to show (see Lindborg 2006; Nazarenko & Schekochihin 2011, and references therein) that the energy spectrum of the perpendicular motions (and, therefore, of the density perturbations) is Kolmogorov in the perpendicular direction, $E_k \sim \varepsilon^{2/3} k^{-5/3} k_\perp$, where $\varepsilon$ is the energy flux and much steeper in the radial direction, $E_k \sim N^2 k^{-3} k_\perp$ (Dewan 1997; Billant & Chomaz 2001). However, as the turbulent cascade proceeds to smaller scales, turbulence becomes less anisotropic, eventually reaching isotropy ($k_{\perp} \sim k_r$, $V_\perp \sim V_r$) at the so-called Ozmidov scale, $k \sim k_0 \sim N^3/2\varepsilon^{1/2}$, where the perpendicular and radial spectra meet and the “local” Froude number is $V_\perp k_0/k \sim 1$ (Ozmidov 1992; for the latest numerical results, see Augier et al. 2012). Beyond this scale ($k > k_0$), the stratified cascade turns into the usual isotropic Kolmogorov cascade, in which $k_{\perp} \sim k_r$, $V_\perp \sim V_r$, $E_k \sim k^{-5/3}$, and the nonlinear decorrelation rate becomes dominant compared to the linear gravity-wave frequency $\omega \sim N$ — and the relation (Equation (6)) continues to be satisfied, but for reasons unrelated to the buoyancy physics considered in Section 2.2.

If $Fr \sim 1$ at the outer scale, the turbulence can be assumed isotropic, with $k_{\perp} \sim k_r \sim k$ and $V_\perp \sim V_r \sim V_{\text{rms}}$ already at $k L_\perp \sim 1$; therefore, the relation (Equation (6)) is satisfied
at the outer scale. Again, it will continue to be satisfied at $kL > 1$, as we are about to explain. This is probably the more common situation: in relaxed clusters (see Table 1), $kL > 1$, as we are about to explain. This is probably the

Note. Mach number $Ma = V_{rms}/c_s$ is calculated as the rms of the velocity in the central 500 kpc after subtracting the mean velocity in this region.

### 2.2. Turbulent Regime (Small Scales)

Consider now the limit of small scales, $k > k_0$. At these scales (the “inertial range”), the nonlinear decorrelation (eddy turnover) rate becomes increasingly (with $k$) greater than the Brunt–Väisälä frequency. This means that the right-hand side of Equation (2) can be neglected, so the entropy fluctuations are a passive scalar field. Since $\delta \rho / \rho = -(1/\gamma) \delta s / s$, so are the density fluctuations.

A scaling theory for such passive fluctuations in the inertial range can be constructed along the lines of the Obukhov–Corrsin theory (Obukhov 1949; Corrsin 1951). The total variance of $\delta \rho / \rho$ is conserved, so there should be a constant flux $\varepsilon_{\rho}$ of it toward smaller scales:

$$\left( \frac{\delta \rho_k}{\rho} \right)^2 \sim \frac{\varepsilon_{\rho} t_k}{c_s^2}, \quad (7)$$

where $\varepsilon_{\rho}$ is the scalar dissipation rate, $t_k$ is the typical “cascade time,” and $c_s^2$ appears in the right-hand side simply to ensure convenient normalization of the flux $\varepsilon_{\rho}$ to energy units. Similarly, for the turbulent motions themselves,

$$V_k^2 \sim \varepsilon_{\rho} t_k, \quad (8)$$

where $\varepsilon_{\rho}$ is the flux of the kinetic energy and $t_k$ is the same cascade time as in Equation (7) because both $\delta \rho / \rho$ and $V$ are mixed by the same velocity field. From Equation (8), $t_k \sim V_k^2 / \varepsilon_{\rho}$ and, therefore, from Equation (7),

$$\left( \frac{\delta \rho_k}{\rho} \right)^2 \sim \frac{\varepsilon_{\rho}}{\varepsilon_{\rho} V_k^2 / c_s^2}. \quad (9)$$

Thus, the spectrum of the passive scalar follows the spectrum of the velocity field in the inertial range. The constant of proportionality, $\varepsilon_{\rho} / \varepsilon_{\rho} V_k$, depends on how much scalar variance, compared to kinetic energy, arrives into the inertial range from the larger, buoyancy-dominated scales. Therefore, the relationship (Equation (9)) should be matched with Equation (6):

$\varepsilon_{\rho} \varepsilon_{\rho} V_k \sim \eta^2$.

Note that these conclusions are independent of precisely what the spectrum of the turbulence is because the argument above was based only on the assumption that the cascade times are the same for the passive scalar and for the turbulence that advects it. This is reassuring because ICM turbulence is certainly not a simple hydrodynamic Kolmogorov turbulence of an inertial fluid. At the very least, it is MHD, as clusters are known to host dynamically significant magnetic fields (see, e.g., Schekochihin & Cowley 2006; Enßlin & Vogt 2006, and references therein).

Since $\beta = 8\pi P / B^2 \sim 10^2 \gg 1$, these fields do not significantly modify buoyancy physics at large scales. In the inertial range, the turbulence may be MHD rather than hydrodynamic, but density fluctuations continue to behave as a passive scalar in Alfvénic MHD and even kinetic turbulence (Schekochihin & Cowley 2007; Schekochihin et al. 2009); therefore, the above argument continues to hold.

Thus, the conclusion from this section is that the relation (Equation (6)) holds with the same proportionality constant both at scales where buoyancy physics matters and at those where it does not and independently of whether Fr is small or of the order of unity (i.e., independently of how vigorous the turbulence is and of whether it is an isotropic Kolmogorov turbulence or an anisotropic stratified one). Below, we verify these arguments using a sample of cosmological hydrodynamic simulations of galaxy clusters.

### 3.3. Simulations and Sample of Galaxy Clusters

We use high-resolution cosmological simulations of galaxy clusters in a flat ΛCDM model with Wilkinson Microwave Anisotropy Probe 5 yr cosmological parameters: $\Omega_m = 1 - \Omega_\Lambda = 0.27$, $\Omega_b = 0.0469$, $h = 0.7$, and $\sigma_8 = 0.82$. The simulations are performed using the adaptive refinement tree N-body+gas-dynamics code (Kravtsov 1999; Kravtsov et al. 2002; Rudd et al. 2008), which is an Eulerian code that uses adaptive refinement in space and time and non-adaptive refinement in mass (Klypin et al. 2001) to achieve the dynamic ranges necessary to resolve the cores of halos formed in self-consistent cosmological simulations. Details of the zoom-in simulations used here can be found in Nelson et al. (2014, Section 3.1).

Our sample includes six relaxed clusters at $z = 0$. Their X-ray morphology exhibits spherical or elliptical symmetry with no filamentary or clumpy substructures within $r_{500}$ (see Nagai et al. 2007). We analyze non-radiative (NR) runs only because these involve physical processes directly relevant to those discussed in Section 2.

We analyze fluctuations in the central $\sim$500 kpc (radius) region. The gas motions are subsonic with $Ma \sim 0.2$–0.4 (Table 1). X-ray images of the NR relaxed clusters do not show any prominent subhalos or clumpy structures within $r_{500}$ (Figure 1, inset). Only cluster CL25 has a small subhalo, which we remove from the analysis. Subhalos/clumps, which are

| ClusterID | $r_{500}$ (kpc) | $M_{500}$ ($10^{14} M_\odot$) | $Ma$ |
|-----------|----------------|-------------------------------|------|
| CL149     | 814.4          | 1.83                          | 0.48 |
| CL21      | 1215.2         | 6.08                          | 0.21 |
| CL223     | 824            | 1.9                           | 0.29 |
| CL25      | 1095.8         | 4.46                          | 0.32 |
| CL27      | 1154.5         | 5.21                          | 0.23 |
| CL42      | 1297.7         | 7.4                           | 0.27 |

14 At least not on the qualitative level. Strictly speaking, one ought to worry about the instabilities caused by anisotropic heat fluxes (see, e.g., Balbus 2000; Kunz 2011; Quataert 2008; Parrish et al. 2010, and references therein).

15 Matters may become more complicated at scales below the collisional mean free path where density fluctuations are subject to collisionless damping (Schekochihin et al. 2009), but such small scales are unlikely to be observable in the near future.
present in 3D data and are not visible in projection, only slightly affect our results, e.g., for cluster CL149, the exclusion of clumps with $\delta \rho / \rho > 1$ removes $\sim 1.8\%$ of the volume and changes the total rms of $\delta \rho / \rho$ by $\sim 10\%$. The changes in $M_a$ are less than $1\%$.

### 3.2. Power Spectra of Density and Velocity Fluctuations

For each cluster in our sample, we calculated the 3D emissivity-weighted electron density as $n_{e,X} = n_e \sqrt{\Lambda(T)}$, where $\Lambda(T)$ is the gas emissivity (Sutherland & Dopita 1993) and the three components of the normalized velocity field are $V_{x,y,z}/c_s$. Projecting the squared density along one of the directions, the X-ray surface brightness (SB), $I_X(x, y, z) \propto \int n_{e,X}^2(x, y, z) dz$, is obtained. We calculate the spherically symmetric radial profile of the SB and approximate it with a $\beta$ model (Figure 1). Dividing the density by the corresponding 3D $\beta$ model and subtracting unity, the 3D density fluctuations $\delta \rho / \rho$ are obtained. The center of each cluster is chosen as the peak of the gas density within the central $\sim 50$ kpc region and confirmed by the visual inspection.

We then use the modified $\Delta-$variance method (Arévalo et al. 2012) to calculate $P_{3D}(k)$, the PS of the 3D density fluctuations, and velocity. These spectra are converted to fluctuation amplitudes $A_k = \sqrt{4\pi P_{3D}(k)k^3}$. Figure 2 shows an example of such amplitudes of the density fluctuations $\delta \rho / \rho$ and of rms velocity component $V_{1,k}$ for cluster CL21. These amplitudes follow each other over a broad range of scales. Their ratio $\eta_1 = (\delta \rho / \rho)/(V_{1,k}/c_s)$ is close to unity with $\sim 10\%$ deviations.

Even though the relationship (Equation (6)) is found to hold over a broad range of scales, the amplitudes at the smallest and the largest scales are affected by several artifacts. At $k \gtrsim 4 \cdot 10^{-2}$ kpc$^{-1}$, the limit of numerical resolution is reached. At the largest scales, the amplitude is (1) sensitive to the underling SB model and (2) affected by uncertainties due to stochastic nature of perturbations. The sensitivity to (1) is estimated by experimenting with different underlying models (e.g., non-spherical $\beta$ models, averaged profiles). In order to evaluate the uncertainties of (2), we experimented with multiple realizations of a Gaussian field that had a PS similar to that of density/velocity fluctuations in simulated clusters. As expected, there are large variations at $k \sim 1/L$, where $L = 1000$ kpc is the size of the box. At $k \gtrsim 3/L \sim 3 \times 10^{-3}$ kpc$^{-1}$, the variations drop down to $\sim 5\%–10\%$.

The ratio $\eta_1$ averaged over a sample of relaxed clusters is consistent with unity at scales $\sim 30–300$ kpc, with a relatively modest scatter $\lesssim 30\%$ (Figure 3). There are many possible reasons for this scatter. In particular, the presence of individual subhalos, the choice of the cluster center, and the underlying model, effects of adaptive mesh refinement (AMR) resolution, can all contribute to variations at this level.

In order to assess the effects of the AMR resolution on our results, we re-simulated one cluster, varying the maximum refinement levels from six to nine (the default level is eight). In the lowest-resolution runs, both density and velocity fluctuations are suppressed compared to the high-resolution runs at all scales, except for the largest at $\sim 300$ kpc. The amplitudes of density fluctuations in simulations with the refinement levels 8 and 9 globally follow each other at scales $\sim 50–300$ kpc. However, at
some scales, deviations are up to a factor of 1.3. The velocity amplitudes are the same at scales \(\sim 300–100 \text{ kpc}\) and differ by a factor of 1.5 at \(\sim 50 \text{ kpc}\). Despite individual PS varying with the AMR resolution, the ratio in all runs is still close to unity, with the scatter up to 25\% at scales \(\sim 50–300 \text{ kpc}\).

4. CONCLUSIONS

In this Letter, we have addressed the problem of constraining the gas velocity PS in relaxed galaxy clusters using the observed density fluctuations. We argue the following.

1. The rms of density and velocity fluctuations are linearly related across a broad range of scales in both buoyancy-dominated and turbulent regimes.

2. The constant of proportionality between them is set at large scales by gravity-wave physics and remains approximately the same in the nonlinear turbulent regime.

3. Cosmological simulations of relaxed clusters give the proportionality coefficient \(\eta_1 \sim 1 \pm 0.3\).

It is an interesting conclusion that if the energy-injection scales are large enough (e.g., \(\sim 10^2 \text{ kpc}\) for merger-driven turbulence), stratification leads to anisotropy \(\left(V_{\perp} \gg V_r, k_{\perp} \ll k_r\right)\), whereas turbulence driven at small scales (e.g., \(\sim 10 \text{ kpc}\), as in the active galactic nucleus-driven case) will be isotropic—these are the \(F_r \ll 1\) and \(Fr \sim 1\) cases discussed in Section 2.1. Indeed, in cosmological simulations where turbulence is primarily driven by mergers, perpendicular velocities are slightly larger than the radial ones in the central 500 kpc.

Admittedly, our simulations suffer from insufficient dynamic range and lack of relevant physics. For instance, thermal conduction could erase some of the temperature/density fluctuations and break the relation (Equation (6)). This is considered in the companion paper (G14) where a series of hydrodynamic simulations are carried out, with varying Ma and isotropic conductivity. In the presence of tangled magnetic fields and, consequently, in a saturated state of a system that may have been unstable to heat-flux-driven buoyancy instability, the density/velocity linearity may persist, but with the temperature scale height replacing entropy scale height in Section 2.1. Cooling can also produce density fluctuations, which do not trace velocity. However, to the first approximation, the cooling can be ignored since typically \(f_{\text{edd}} \ll f_{\text{cool}}\). In our future work, we plan to consider the simulations with additional physics to clarify the role of these effects.

It should be possible to verify the relation (Equation (6)) using future direct velocity measurements with Astro-H (combined with current observations). Strong deviations from \(\eta \sim 1\) would suggest interesting microphysics or the dominance of other sources of density fluctuations.

In conclusion, we have shown that the analysis of SB fluctuations in X-ray images offers a novel way to estimate the velocity PS in relaxed galaxy clusters. In general, proportionality between the density and velocity amplitudes for subsonic motions is probably a generic feature of small perturbations in stratified atmospheres.

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