TMD-Factorization inHadron-Hadron Collision

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Proof of transverse-momentum-dependent(TMD) factorization for hadron-hadron collision is given in this paper. We focus on processes without detected soft final hadrons or detected final hadrons that are collinear to initial hadrons. This contradicts the widely accepted viewpoint that TMD-factorization does not hold in such processes even in the generalized sense. The key point is that singular points of the type \( l^+ = 0 \) can be absorbed into Wilson lines of soft gluons, where \( l \) is collinear to the plus direction. Thus one should subtract such singular points from the collinear region of \( l \). After such subtraction, one can make use of Ward identity to absorb effects of scalar-polarized collinear gluons into Wilson lines.

I. INTRODUCTION

Transverse-momentum-dependent(TMD) factorization in hadron-hadron collisions with large transverse momentum back-to-back hadrons produced in the final states is a much non-trivial issue. This is because that the Ward-identity cancelation is prevented by the singular points of the type \( l^+ = 0 \), where \( l \) is collinear to the plus direction. It was first noted in that singular points of such type can cause the process-dependence(diagram-dependence) in Wilson-line structures of TMD parton distribution functions(PDFs) and TMD fragmentation functions(FFs). According to calculations in a model theory, authors in found that the process-dependent Wilson-lines can cause the break down of normal proofs of standard factorization in the inclusive production of two high-transverse-momentum hadrons in hadron-hadron collisions. It was later found in that TMD-factorization in production of back-to-back hadrons in hadron-hadron collisions fails even one allows Wilson-lines in TMD-functions to be process dependent. In addition to the anomalous color caused by the process dependent Wilson lines, the nontrivial dependence of cross sections on transverse momenta of gluons exchanged between spectators and active partons is also identified in as constituting a breakdown of TMD-factorization.

Unlike the case in QCD process, TMD-factorization theorems have been derived rigorously in perturbative QCD(pQCD) for a number of electromagnetic processes including Drell-Yan (DY) process, semi-inclusive deep inelastic scattering (SIDIS) process and the production of back-to-back hadrons in \( e^+e^- \) annihilation process. Even for such processes, one should first deform the integral paths of gluon momenta so that the Grammer-Yennie approximation and the Ward-identity cancelation works. The deformations are different for various processes. For SIDIS and \( e^+e^- \) annihilation the deformations should be consistent with exchanges of gluons between spectators and final particles. While for DY the deformations should be consistent with those between spectators and initial particles. This makes the Wilson lines in SIDIS and \( e^+e^- \) annihilation to be future-pointing and those in DY to be past pointing. Such differences can cause a sign flip for the Sivers function in DY as compared to SIDIS. Thus TMD-functions can be process dependent even for such processes, although the process dependence can be absorbed into future pointing or past pointing Wilson lines. Relations between these functions in SIDIS, \( e^+e^- \) and DY are further studied in. Other issues about the Wilson lines in these processes include, for example, transverse Wilson lines at infinity, rapidity divergences (see, for example, Refs.) and cusp structures of Wilson lines (see, for example, Refs.).

The deformation of integral path is closely related to the issue of Glauber gluons, which are space-like soft gluons with the momenta \( q^\mu \) locate in the region: \( q^0 \ll |\vec{q}| \sim \Lambda_{QCD} \). In the coupling between a soft gluon with momentum \( q \) and a particle collinear-to-plus with momentum \( p \) and mass \( m \), one make the Grammer-Yennie approximation:

\[
A_s^\ell(q) \rightarrow A_{s}^-(q), \quad (p + q)^2 \rightarrow p^2 - m^2 + 2p^+ q^-
\]  

so that the soft gluon \( q \) behave as a scalar polarized gluon in such coupling. One can then make use of Ward identity to absorb this gluon into a Wilson line along the plus direction. Gluons in Glauber region, however, break such approximation. For example, one may consider the case that \( q^\mu \) locate in the region: \( (q^+, q^-) \sim \Lambda_{QCD}(\Lambda_{QCD}/p^+, \Lambda_{QCD}/p^+, 1) \). In this case, we have:

\[
p^2 - m^2 + 2p^+ q^- \sim q^2 - 2\vec{p}_\perp \cdot \vec{q}_\perp \sim \Lambda_{QCD}^2
\]  

Thus the Grammer-Yennie approximation does not work in this case. To make the Grammer-Yennie approximation work, one should first deform the integral paths out of the Glauber region (see, for example, Refs.)
Deformations of the integral paths depend on whether interactions between soft gluons and collinear particles occur before or after the hard collision. There are not universal deformation of integral paths to avoid Glauber in hadron-hadron collisions as both initial and final states interactions exist in such process. It is well known that standard factorization does not hold for hadron-hadron collision with detected hadrons in the target momentum region.\cite{15,16} For inclusive hadron-hadron process, like the Drell-Yan process, final poles in Glauber region cancel out. Thus one can deform the integral paths to be consistent with those between spectators and initial particles so that factorization holds for such processes.\cite{15,17}

The issue of Glauber gluons is also related to collinear factorizations, in which transverse momenta of partons are integrated over in the definitions of PDFs and FFs. In this case, factorization is saved by cancelations of poles of final states interactions once integral over transverse momenta is performed.\cite{11,12} For hadron-hadron collisions with large transverse momentum back-to-back hadrons, however, the cross section is not sufficiently inclusive in transverse momenta. Thus the cancellation does not simply work in such processes.

In this paper, we present an approach to show that the similar cancellation does occur for the hadron-hadron collisions with large transverse momentum back-to-back hadrons produced in the final state. The physical picture of such proof is clear. Couplings between soft gluons and final jets produced in the hard-subprocess can be described by the Grammer-Yennie approximation as such couplings always occurs after the hard scattering. After this approximation, soft gluons couple to eikonal lines along the directions of final jets instead of couple to final jets direct. We can thus factorize the detected jets from the scattering matrix-elements. This is represented by the formula:

\[ M = \prod_{i=1}^{2} M_{\text{det}}^{i} M_{\text{inc}} \]

where \( M \) represents a diagram that contribute to the process, \( M_{\text{det}}^{i} \) represent the collinear subgraph of the \( i \)-th jets. \( M_{\text{inc}} \) includes the hard subgraph, soft subgraph, collinear subgraph of other jets and Wilson lines along the directions of the detected jets. Coherence between \( M_{\text{det}}^{i} \) and \( M_{\text{inc}} \) can only be caused by coherence between their spin structures, color structures and total momenta. Hadrons are color singlets, thus the color charge of \( M_{\text{det}}^{i} \) should be the same as that of its conjugation. One should perform the summation over all possible final particles with definite total momentum, color charge and spin in \( M_{\text{inc}} M_{\text{inc}}^{*} \). The action of QCD is colorless and Poincaré invariant. Thus such states form the invariant sub-space of QCD. After such summation, couplings between soft gluons and collinear particles after the hard collision cancel out in \( M_{\text{inc}} M_{\text{inc}}^{*} \). We can then deform the integral paths to avoid Glauber region in \( M_{\text{inc}} M_{\text{inc}}^{*} \). The Grammer-Yennie works in \( M_{\text{inc}} M_{\text{inc}}^{*} \) after this deformation.

We then absorb soft gluons into Wilson lines along the directions of collinear particles. We subtract contributions of these Wilson lines from couplings between gluons and spectators. After such subtraction, We can take the collinear approximation in the coupling between active partons and such gluons. Singular points of the type \( t^{+} = 0 \) do not give leading order contribution after this subtraction, where \( t^{+} \) represents the momenta of gluons that are is collinear-toplus. We can then make use of Ward identity to absorb collinear gluons into past pointing or future pointing Wilson lines.

The paper is organized as follows. We first consider a simple example in Sec.II in which one gluon is exchanged between spectators and active partons or spectators. According to explicit calculations, we show the cancelation of pinch singular points in Glauber region in this example. We then prove such cancelation in the frame of effective theory in Sec.III Wilson lines of soft gluons and collinear gluons are brought in in Sec.IV. The effective vertex of hard subprocess is also construct in this section. In Sec.V we consider the same process as in Sec.III to show that the nontrivial transverse momenta dependence of cross section can be described by the effective hard vertex brought in Sec.IV. We finish the proof factorization theorem in Sec.VI. Some discussions are given in Sec.VII.

### II. CANCELLATION OF PINCH SINGULARITIES IN GLAUBER REGION AT ORDER \( \alpha_s \)

In this section, we show the cancelation of leading pinch singular surfaces(LPSS) in Glauber region in a simile example. There is one gluon exchanged between spectators and active partons or spectators in the graphs considered in this section. We work in Feynman gauge in this section.

The process we considered in this paper can be written as:

\[ A(p_1) + B(p_2) \rightarrow H_3(k_3) + H_4(k_4) + X \]  

(4)

where \( A \) and \( B \) represent (polarized or unpolarized) initial hadrons with momenta \( p_1 \) and \( p_2 \), \( H_3 \) and \( H_4 \) represent detected back-to-back (polarized or unpolarized) hadrons with momenta \( k_3 \) and \( k_4 \), \( X \) represents any other states.
We work in the center of mass frame of initial hadrons. \( p_1 \) and \( p_2 \) are taken as nearly collinear to plus and minus directions respectively. That is:

\[
p_1 = (p_1^+, M_1^2/2p_1^+, 0) \simeq (p_1^+, 0, 0)
\]

\[
p_2 = (M_2^2/2p_2^-, p_2^-, 0) \simeq (0, p_2^-, 0)
\]

where \( M_1 \) and \( M_2 \) denote the masses of initial hadrons, which are of order \( \Lambda_{\text{QCD}} \). We bring in the notation:

\[
n_1^\mu = \frac{1}{\sqrt{2}}(1, 0, 0, 1), \quad n_2^\mu = \frac{1}{\sqrt{2}}(1, 0, 0, -1)
\]

(7)

\[
\bar{n}_i^\mu = \sqrt{2}(1, 0, 0, 0) - n_i^\mu
\]

(8)

and have:

\[
p_i^\mu = \bar{n}_i \cdot p_i n_i^\mu (1 + O(M_i/Q))
\]

(9)

for \( i = 1, 2 \). We assume that \( k_3 \) and \( k_4 \) nearly collinear to the direction \( n_3^\mu \) and \( n_4^\mu \) respectively, where:

\[
n_3^\mu = \frac{1}{\sqrt{2}}(1, \sin(\theta_3) \cos(\phi_3), \sin(\theta_3) \sin(\phi_3), \cos(\theta_3))
\]

(10)

\[
n_4^\mu = \frac{1}{\sqrt{2}}(1, \sin(\theta_4) \cos(\phi_4), \sin(\theta_4) \sin(\phi_4), \cos(\theta_4))
\]

(11)

with \( 1 - \cos(\theta_3) \sim 1 + \cos(\theta_3) \sim 1, 1 - \cos(\theta_4) \sim 1 + \cos(\theta_4) \sim 1, \pi - (\theta_3 + \theta_4) \sim 0 \) and \( \pi - |\phi_3 - \phi_4| \sim 0 \). We then have:

\[
k_i^\mu = \bar{n}_i \cdot k_i n_i^\mu (1 + O(M_i/Q))
\]

(12)

where

\[
\bar{n}_i^\mu = \sqrt{2}(1, 0, 0, 0) - n_i^\mu
\]

(13)

for \( i = 3, 4 \). We also bring in the notation:

\[
q^\mu = k_3^\mu + k_4^\mu, \quad Q = \sqrt{q^2}
\]

(14)

Then the hard energy scale of the process is set by \( Q \).

We consider the diagrams with one gluon exchanged between remnants of \( p_1 \) and other particles. These are diagrams shown in Fig. 1 and their conjugations.

We start from the first diagram in Fig. 1. The part of this diagram that depends on the momentum \( l \) can be written as:

\[
M_{1a;pb}^{i_1,i_3}(l) = (-i) \left( \gamma^\mu \eta^a \frac{k_1^\mu + \gamma l}{(k_1^\mu + l)^2 + i\epsilon} \right)_{i_1j_1} \Bigg|_{i_2j_2} \\
\gamma^\nu \left( \frac{k_3^\nu - \gamma l}{(k_3^\nu - l)^2 + i\epsilon} \right)_{i_3j_3}
\]

(15)

where \( \Gamma_H \) denotes the effective vertex of couplings between partons and hadrons, \( i_1 \) and \( i_3 \) denote the Dirac indices of the spinor \( \bar{u}(k_1) \) and \( \bar{u}(k_3) \), \( j_1 \) and \( j_3 \) denote the Dirac indices of the vertex \( \Gamma_H \) (there may be more Dirac indices in \( \Gamma_H \), we do not display them explicitly), \( \rho \) and \( \beta \) represents the Lorentz and color indices of the hard gluon, the factor \(-i\) is that relative to the diagram without gluons. \( l \) is collinear-to-plus or soft (including Glauber) at leading order according to infrared power counting (\[33\]).
We first consider the case that \( l \) locate in the soft region. In this region, we can write \( M_{1a;\rho b}(l) \) at leading order as:

\[
\frac{(-i)t^a}{(k_1^1 + l)^2 + i\epsilon} \Gamma^{i_1j_3}_{H'}(p, k_1 - l, k_1^1 + l) \frac{1 - \cos(\theta_3)}{2(l^2 + i\epsilon)} \]

\[
\left( t^a \frac{2n_3 \cdot k_3}{(k_3 - l)^2 + i\epsilon} \gamma_\rho t^b \frac{k_1^1 \gamma_-}{(k_1 - l)^2 + i\epsilon} \right)_{i_3j_3} \tag{16}
\]

Poles of the internal lines collinear-to-plus, the internal line collinear to \( n^\mu \) and soft internal line read \( l^- \gtrsim M_2^2/p_1^+ \), \( n_3 \cdot l \gtrsim M_3^2/n_3 \cdot k_3 \) and \( l^0 \sim E \) respectively. We have assumed that transverse components of \( k_3 \) are large, thus \( n_3 \cdot l \) do not pinch in Glauber region. We can deform the integral path of \( n_3 \cdot l \) to lower half plane to avoid the Glauber region and take Grammer-Yennie approximation in the internal line \( k_3^1 - l^1 \):

\[
\frac{1}{(k_3 - l)^2 + i\epsilon} \approx \frac{1}{-2n_3 \cdot k_3 n_3 \cdot l + i\epsilon} \tag{17}
\]

After this approximation, we deform the integral path of \( n_3 \cdot l \) back to real axis and define the quantity:

\[
\tilde{M}_{1a;\rho b}^{i_3j_3} \equiv (-i)t^a \frac{2k_1^1}{(k_1^1 + l)^2 + i\epsilon} \Gamma^{i_1j_3}_{H'}(p, k_1 - l, k_1^1 + l) \frac{1 - \cos(\theta_3)}{2(l^2 + i\epsilon)} \]

\[
\left( t^a \frac{1}{-n_3 \cdot l + i\epsilon} \gamma_\rho t^b \frac{k_1^1 \gamma_-}{(k_1 - l)^2 + i\epsilon} \right)_{i_3j_3} \tag{18}
\]

We then make the decomposition:

\[
M_{1a;\rho b}(l) = \tilde{M}_{1a;\rho b}(l) + M_{1a;\rho b}(l) - \tilde{M}_{1a;\rho b}(l) = \tilde{M}_{1a;\rho b}(l) + \hat{M}_{1a;\rho b}(l) \tag{19}
\]

We see that the \( \hat{M}_{1a;\rho b}(l) \) part is power suppressed in the soft region.
We now consider contributions of the momentum region:

\[ |\vec{t}_\perp| \sim M_1, \quad |l^-| \ll |\vec{t}_\perp|, \quad |l^+| \lesssim |\vec{t}_\perp| \]

(20)

The \( \hat{M}_{1 a; \rho b}(l) \) part part is power suppressed in this region and can be neglected. We consider the summation of the first diagram in Fig. 1 and its conjugation. The parts of these two diagrams that depend on \( \vec{t}_\perp \) can be written as:

\[
\sigma_{1 a; \rho b; \nu}^{i_1 i_2 i_3; j_1 j_2 j_3} (\vec{t}_\perp) = \int \frac{dl^+}{2\pi} \frac{dl^-}{2\pi} (-i) \Gamma_H^{i_1 i_2 i_3 ; j_1 j_2 j_3} (p_1, k_1, k'_1) \left( t^\alpha k_1^+ \gamma^- k_1^2 \gamma^\rho \right)_{j_3 j_1 j_2; i_3 i_1 i_2} \\
+ \int \frac{dl^+}{2\pi} \frac{dl^-}{2\pi} (i) t^\alpha \frac{2k_1^+}{(k_1^2 + l^2)^2 + i\epsilon} \Gamma_H^{i_1 i_2 i_3; j_1 j_2 j_3} (p, k_1 - l, k'_1 + l) \frac{1 - \cos(\theta_3)}{2(l^2 + i\epsilon)} \left( t^\alpha \frac{1}{n_3 \cdot l + i\epsilon} \gamma^\rho \gamma^- \frac{k_1^+}{(k_1 - l)^2 - i\epsilon} \right)_{j_3 j_1 j_2; i_3 i_1 i_2} \\
\]  

(21)

We integrate out \( l^- \) by take the residue of the singular point:

\[
l^- = \left( \frac{k_1^2 + \vec{t}_\perp^2}{2(k_1^2 + l^2)} \right) - k_1^- - i\epsilon \simeq \left( \frac{\vec{t}_\perp^2}{2k_1^2} \right) - k_1^- - i\epsilon
\]

(22)

and its conjugation. Initial hadrons are stable states, thus \( k'_1 + l \) and \( k_1 - l \) can not be both on-shell. We then integrate out \( l^+ \) by take the the residue of the singular point:

\[
l^+ \simeq \frac{n_3 \cdot \vec{t}_\perp}{n_3 - i\epsilon}
\]

(23)

and its conjugation. We do not take the residue of the singular point \( l^2 = 0 \) as such singular point do not locate in the region considered here. We then have:

\[
\sigma_{1 a; \rho b}^{i_1 i_2 i_3; j_1 j_2 j_3} (\vec{t}_\perp) \simeq (-i) \Gamma_H^{i_1 i_2 i_3; j_1 j_2 j_3} (p_1, k_1, k'_1) \left( k_1^+ \gamma^- k_1^2 \gamma^\rho t^\nu \right)_{j_3 j_1 j_2; i_3 i_1 i_2} \\
t^\alpha \Gamma_H^{i_1 i_2 i_3; j_1 j_2 j_3} (p, k_1 - l, k'_1 + l) \frac{1}{l^2} \left( t^\alpha \gamma^\rho \gamma^- \frac{k_1^+}{(k_1 - l)^2} \right)_{j_3 j_1 j_2; i_3 i_1 i_2} \\
+ (i) t^\alpha \Gamma_H^{i_1 i_2 i_3; j_1 j_2 j_3} (p, k_1 - l, k'_1 + l) \frac{1}{l^2} \left( t^\alpha \gamma^\rho \gamma^- \frac{k_1^+}{(k_1 - l)^2} \right)_{j_3 j_1 j_2; i_3 i_1 i_2} \\
\]  

\[
= (-i) C_f \frac{k_1^+}{l^2} \Gamma_H^{i_1 i_2 i_3; j_1 j_2 j_3} (p_1, k_1, k'_1) \left( \gamma^\rho \gamma^- \right)_{j_3 j_1 j_2; i_3 i_1 i_2} \\
\left( \Gamma_H^{i_1 i_2 i_3; j_1 j_2 j_3} (p, k_1 - l, k'_1 + l) - \Gamma_H^{i_1 i_2 i_3; j_1 j_2 j_3} (p, k_1 - l, k'_1 + l) \right)
\]

(24)

where \( l^+ \) and \( l^- \) are determined by (23) and (22). \( C_f = \frac{N_c^2 - 1}{2N_c} \) We have made use of the colorlessness of \( \Gamma_H \). \( k'_1 + l \) and \( k_1 - l \) can not be both on-shell as the initial hadron \( p_1 \) is stable particle. We thus drop the \( i\epsilon \) terms in \( (k_1 - l)^2 \pm i\epsilon \).
We drop such terms in $k_1^2 \pm i\epsilon$ according to the same reason. We also drop such terms in $l^2 \pm i\epsilon$ as $l$ can not be on-shell in the region we considered here.

$\Gamma_H$ is Hermitian in both momenta space and spinor space, we thus have:

$$\Gamma_H^{i_1 i_3}(p, k_1 - l, k_1' + l) \Gamma_H^{i_1 i_3}(p_1, k_1, k_1') = \Gamma_H^{i_1 i_3}(p_1, k_1, k_1') \Gamma_H^{i_1 i_3}(p, k_1 - l, k_1' + l)$$

(25)

We then have:

$$\sigma_{1a;pb}^{i_1 i_2 i_3} (l_\perp) \simeq 0$$

(26)

in the region considered here. Thus we can deform the integral path of $l^-$ to the lower half plane with radius of order

$$\min \{ |l_\perp|, \frac{|l_\perp|^2}{|l^+|} \}$$

(27)

in the summation of first diagram of Fig.1 and its conjugation.

For the second and third diagrams of Fig.1 we have the same conclusion. The fourth diagram of Fig.1 is the same as that in Drell-Yan process. Thus we can also deform the integral path of $l^-$ to the lower half plane with radius of order

$$\min \{ |l_\perp|, \frac{|l_\perp|^2}{|l^+|} \}$$

(25)

so that the Grammer-Yennie approximation works in the coupling between $l$ and remnants of the initial hadron $p_1$.

We now consider the integration of $M_{i_1 i_3}^{i_1 i_3}(l)$ over $l$. We deform the integral path of $l^-$ to the lower half plane with radius of order

$$\min \{ |l_\perp|, \frac{|l_\perp|^2}{|l^+|} \}$$

(25)

Residues of the singular points confronted in such deformation do not contribute to the whole process at leading order in $\Lambda_{QCD}/Q$ and can be dropped. We get the integral:

$$\int \frac{dl^+}{2\pi} \int \frac{d^2l_\perp}{(2\pi)^2} \int_{C(l^+, l_\perp)} \frac{dl^-}{2\pi} M_{i_1 i_3}^{i_1 i_3}(l)$$

(28)

where $C(l^+, l_\perp)$ represents the integral path of $l^-$ depending on $l^+$ and $l_\perp$. In the soft region of $l$, we can make the approximation:

$$M_{i_1 i_3}^{i_1 i_3}(l) \simeq S_{1a;pb}^{i_1 i_3}(l)$$

$$\equiv C_f t^b \left( \begin{array}{c} \gamma^+ \\ \gamma^- \end{array} \right) \frac{\Gamma_H^{i_1 i_3} \Gamma_H^{i_1 i_3}}{2(l^2 + i\epsilon)}$$

$$\left( \frac{k_1^+ - \gamma^+}{k_1^+ - k_1^+ l^+ + i\epsilon} \gamma^- \right)^{i_1 i_3}$$

(29)

We then make the decomposition:

$$M_{i_1 i_3}^{i_1 i_3}(l) = S_{1a;pb}^{i_1 i_3}(l) + \Gamma_H^{i_1 i_3} \left( \begin{array}{c} k_1^+ - l^+ \\ \gamma^- \end{array} \right)^{i_1 i_3}$$

(30)

The $S_{1a;pb}^{i_1 i_3}$ part can be represented by the diagram in Fig.2, although the integral path of $l^-$ is not along the real axis at this step, where the double-lines with solid box denote eikonal lines along the directions of the collinear particles.

For the $C_{1a;pb}^{i_1 i_3}(l)$ part in (30), contributions of soft region are power suppressed. We can thus take the approximation:

$$\int \frac{dl^+}{2\pi} \int \frac{d^2l_\perp}{(2\pi)^2} \int_{C(l^+, l_\perp)} \frac{dl^-}{2\pi} C_{1a;pb}^{i_1 i_3}(l)$$

$$\simeq \int \frac{dl^+}{2\pi} \int \frac{d^2l_\perp}{(2\pi)^2} \int_{C(l^+, l_\perp)} \frac{dl^-}{2\pi} C_f t^b$$

$$\left\{ \begin{array}{c} \gamma^+ \\ \gamma^- \end{array} \right)^{i_1 i_3} \frac{\Gamma_H^{i_1 i_3}}{2(l^2 + i\epsilon)}$$

$$\left( \frac{k_1^+ - l^+}{k_1^+} \right)^{i_1 i_3}$$

$$\left( \begin{array}{c} k_1^+ - l^+ \\ \gamma^- \end{array} \right)^{i_1 i_3}$$

(31)
We notice that:

\[
\begin{align*}
\left\{ \gamma^+(k_1^+ + l^-)\gamma^- \right\}_{i_1j_1} & \Gamma_{i_1j_3}^j \left( \frac{(k_1 - l^-)\gamma^-}{(k_1 - l^-)^2 + i\epsilon \gamma^\rho} \right)_{i_3j_3} \\
- \left( \gamma^+ \frac{\gamma^-}{2l^- + i\epsilon} \right)_{i_1j_1} & \Gamma_{i_1j_3}^j \left( \frac{k_1^+ \gamma^-}{k_1^2 - 2k_1^+ l^- + i\epsilon \gamma^\rho} \right)_{i_3j_3} \right\} \frac{1}{l^2 + i\epsilon} \\
\sim O(\Lambda_{QCD}/Q)
\end{align*}
\]

where \( C(l_\perp) = C(0, l_\perp) \) denote the integral path of \( l^- \) with radius of order \( |l_\perp| \). We then have:

\[
\begin{align*}
\int \frac{dl^+}{2\pi} \int \frac{d^2l_\perp}{(2\pi)^2} \int \frac{dl^-}{2\pi} C_{1\alpha;\beta}(l) \\
\int \frac{dl^+}{2\pi} \int \frac{d^2l_\perp}{(2\pi)^2} \int \frac{dl^-}{2\pi} C_{1\alpha;\beta}(l) \\
\left\{ \gamma^+(k_1^+ + l^-)\gamma^- \right\}_{i_1j_1} & \Gamma_{i_1j_3}^j \left( \frac{(k_1 - l^-)\gamma^-}{(k_1 - l^-)^2 + i\epsilon \gamma^\rho} \right)_{i_3j_3} \\
- \left( \gamma^+ \frac{\gamma^-}{2l^- + i\epsilon} \right)_{i_1j_1} & \Gamma_{i_1j_3}^j \left( \frac{k_1^+ \gamma^-}{k_1^2 - 2k_1^+ l^- + i\epsilon \gamma^\rho} \right)_{i_3j_3} \right\} P \left( \frac{1}{-l^+} \right) \frac{1}{l^2 + i\epsilon} \\
\end{align*}
\]

where \( P \) denote the principal value.
For the second and third diagrams of Fig.[1] we can repeat the similar calculations. The difference is that coupling between soft gluon and $k_2$ is absorbed into past pointing Wilson line in the $S_{1a;pb}^{i_1i_3}(l)$. While such couplings are absorbed into future pointing Wilson lines in $S_{1a;pb}^{i_1i_3}(l)$ and $S_{1b;pb}^{i_1i_3}(l)$. This do not affect the $C_{1a;pb}^{i_1i_3}(l)$, $C_{1b;pb}^{i_1i_3}(l)$ and $C_{1c;pb}^{i_1i_3}(l)$ parts as there the singular point $l^+=0$ do not contribute to these parts.

For the fourth diagram of Fig.[1] $l$ can not locate in collinear region at leading order. We deform the integral path $l^-$ to lower half plane and that of $l^+$ to upper half plane with radius of order $|l_\perp|$ to avoid the Glauber region. There are not $C_{1d;pb}^{i_1i_3}(l)$ term at leading order.

We display the $S_{1a;pb}^{i_1i_3}(l)$, $S_{1b;pb}^{i_1i_3}(l)$, $S_{1c;pb}^{i_1i_3}(l)$ and $S_{1d;pb}^{i_1i_3}(l)$ terms in Fig.[3], where the double-lines with solid box denote eikonal lines along the directions of the collinear particles.

For the $C_{1a;pb}^{i_1i_3}(l)$, $C_{1b;pb}^{i_1i_3}(l)$ and $C_{1c;pb}^{i_1i_3}(l)$ terms, there are not contributions of the singular points $l^+=0$. We then apply the Ward identity to the summation of these three terms. The result is the difference between the two diagrams in Fig.[4], where the double-lines with solid circle represent eikonal lines along the minus direction.

We then deform the integral path of $l^-$ to the real axis. Contributions of poles confronted in such deformation cancel out according to the similar calculations we performed to deform the integral path initially. Fig.[3] and Fig.[4] take the form we demanded. It is, however, not easy to extend such calculations to higher order. Instead of explicit calculations, we consider this problem in the frame of effective theory in next section.

III. CANCELLATION OF PINCH SINGULARITIES IN GLAUBER REGION IN EFFECTIVE THEORY

In this section, we prove that one can deform the integral path so that the Grammer-Yennie approximation works in couplings between soft gluons and collinear particles in processes considered in this paper. We denote the momentum of a soft gluon as $q^\mu$, which is defined as flow into collinear particles. If the soft gluon $q$ couple to particles collinear
to one initial hadron, then we can deform the integral path of \( n \cdot q \) to upper half plane so that the Grammer-Yennie approximation works, where \( n^\mu \) denotes the direction of the collinear particles. If the soft gluon \( q \) couple to particles collinear to other directions, then we can deform the integral path of \( n \cdot q \) to lower half plane so that the Grammer-Yennie approximation works.

The hard subprocess between partons is nearly local in coordinate space with uncertainty of order \( 1/Q \). We denote the space time region in which the hard collision occur as \( H(x, 1/Q) \), where \( x \) is the center of the hard subprocess. After or before the hard collision, there are interactions between collinear and soft particles which are responsible for the formation of final hadrons or the production of initial active partons. The space-time uncertainty of such interactions are of order \( 1/\Lambda_{\text{QCD}} \). We denote the space-time region of such interactions as \( S(x, 1/\Lambda_{\text{QCD}}) \). We also denote the remaining space-time region as \( C(x) \). Different jets and soft hadrons separate from each other in the region \( C(x) \). We can neglect interactions between different jets and soft hadrons in the region \( C(x) \) as they are color singlets and separate from each other far enough.

As in \[43\], We fist consider the classical configuration of quark and gluon fields in the region \( S(x, 1/\Lambda_{\text{QCD}}) \). Different jets separate from each other in this region although they exchange soft gluons. We denote the region which the jet collinear to \( n^\mu \) locate in as \( y_n \). We also denote the region in which there are not collinear jets as \( y_s \). \((y_n \subset S(x, 1/\Lambda_{\text{QCD}}), y_s \subset S(x, 1/\Lambda_{\text{QCD}}))\). We define the soft fields according to the manner:

\[
(D_{x\mu} G_s^{\mu\nu})^a(y_n) = g \bar{\psi}_s(x) \gamma^\nu t^a \psi_s(y_n), \quad A_s^\mu(y_s) \equiv A^\mu(y_s) \tag{34}
\]

and

\[
D_s \psi_s(y_n) = 0, \quad \psi_s(y_s) \equiv \psi(y_s) \tag{35}
\]

where

\[
D_s^\mu \equiv \partial^\mu - ig A_s^\mu, \quad G_s^{\mu\nu} \equiv \frac{i}{g} [D_s^\mu, D_s^\nu] \tag{36}
\]

The equation should be solved perturbatively in the region \( y_n \), that is, soft fields in the region \( y_n \) is defined according to perturbation theory. We then define the collinear fields:

\[
\psi_n(y_n) \equiv \psi(y_n) - \psi_s(y_n) \tag{37}
\]

\[
\psi_n(y_m) = \psi_n(y_s) = 0(m^\mu \neq n^\mu) \tag{38}
\]

\[
A_s^\mu(y_n) \equiv A^\mu(y_n) - A_s^\mu(y_n) \tag{39}
\]

\[
A_s^\mu(y_m) = A_s^\mu(y_m) = 0(m^\mu \neq n^\mu) \tag{40}
\]
We then write the classical Lagrangian density of QCD as:

\[ \mathcal{L} = \sum_{n} \mathcal{L}_{n} + \mathcal{L}_{s} \]

\[ = \sum_{n} i \bar{\psi}_{n}(\not\! \! \partial - ig A_{n} - ig A_{s}) \psi_{n} + \frac{1}{2g^{2}} \sum_{n} \text{tr}_{c}\{[\partial^{\mu} - ig A_{n}^{\mu} - ig A_{s}^{\mu} , \partial^{\nu} - ig A_{n}^{\nu} - ig A_{s}^{\nu}]^{2} - [\partial^{\mu} - ig A_{n}^{\mu} , \partial^{\nu} - ig A_{n}^{\nu}]^{2} \} + i \bar{\psi}_{n}(\not\! \! \partial - ig A_{s}) \psi_{n} + \frac{1}{2g^{2}} \text{tr}_{c}\{[\partial^{\mu} - ig A_{s}^{\mu} , \partial^{\nu} - ig A_{s}^{\nu}]^{2} \} \]  
(41)

where we have dropped the coupling between soft fermion fields and collinear fields as they are power suppressed [33, 34].

We decompose the region \( S(x, 1/\Lambda_{QCD}) \) according to the manner:

\[ S(x, 1/\Lambda_{QCD}) = S_{s}(x, 1/\Lambda_{QCD}) + \sum_{n_{1} \neq n_{2}^{s}, n_{1}^{v} \neq n_{2}^{v}} S_{n_{1}, n_{2}}(x, 1/\Lambda_{QCD}) + S_{n_{1}, n_{2}}(x, 1/\Lambda_{QCD}) \]  
(42)

where \( n_{1}^{s} \) and \( n_{2}^{v} \) are defended in [7]. \( S_{n_{1}, n_{2}}(x, 1/\Lambda_{QCD}) \) denotes the region which the jet collinear to \( n_{1}^{s} \) locate in, \( S_{s}(x, 1/\Lambda_{QCD}) \) denotes the region in which there are not collinear jets.

There are not jets collinear to \( n_{1}^{s} \neq n_{1}^{0}, n_{1}^{s} \neq n_{2}^{0} \) before the hard collision. We can thus deform the integral path [33, 34] so that the Grammer-Yennie approximation works in the coupling between soft fields and collinear fields in the region \( S_{n}(x) \) \( (n_{1}^{s} \neq n_{1}^{0}, n_{1}^{s} \neq n_{2}^{0}) \). We redefine the effective fields [33, 46]:

\[ \psi_{n}^{(0)}(y_{n}) = Y_{n}^{1} \psi_{n}(y_{n}), \quad A_{n}^{(0)\mu}(y_{n}) = Y_{n}^{1} A_{n}^{\mu}(y_{n}) Y_{n}(y_{n}) \]  
(43)

where \( n_{1}^{s} \neq n_{1}^{0}, n_{1}^{s} \neq n_{2}^{0} \) and

\[ Y_{n}(y_{n}) = (P \exp(ig \int_{0}^{\infty} ds n \cdot A_{s}(y_{n} + sn)))^{\dagger} \]  
(44)

The Wilson line travel from \( y_{n} \) to \( \infty \) according to the deformation of the integral path of soft gluons. [33, 34] Coupling between the classical fides \( \psi_{n}^{(0)}(A_{n}^{(0)\mu}) \) and the classical fields \( A_{s} \) do not contribute to the process at leading order in \( \Lambda_{QCD}/Q \). We can then write the effective classical Lagrangian density in the region \( S(x, 1/\Lambda_{QCD}) \) as:

\[ \mathcal{L}_{L} = \sum_{n_{1} \neq n_{2}^{s}, n_{1}^{v} \neq n_{2}^{v}} \mathcal{L}_{n}^{(0)} + \mathcal{L}_{n_{1}} + \mathcal{L}_{n_{2}} + \mathcal{L}_{s} \]  
(45)

\[ \mathcal{L}_{n}^{(0)} = i \bar{\psi}_{n}^{(0)}(\not\! \! \partial - ig A_{n}^{(0)}) \psi_{n}^{(0)} + \frac{1}{2g^{2}} \text{tr}_{c}\{[\partial^{\mu} - ig A_{n}^{(0)\mu} , \partial^{\nu} - ig A_{n}^{(0)\nu}]^{2} \} \]  
(46)

\[ \mathcal{L}_{n_{1}} = i \bar{\psi}_{n_{1}}(\not\! \! \partial - ig A_{n_{1}} - ig A_{s}) \psi_{n_{1}} + \frac{1}{2g^{2}} \text{tr}_{c}\{[\partial^{\mu} - ig A_{n_{1}}^{\mu} - ig A_{s}^{\mu} , \partial^{\nu} - ig A_{n_{1}}^{\nu} - ig A_{s}^{\nu}]^{2} \} \]  
(47)

\[ \mathcal{L}_{n_{2}} = i \bar{\psi}_{n_{2}}(\not\! \! \partial - ig A_{n_{2}} - ig A_{s}) \psi_{n_{2}} + \frac{1}{2g^{2}} \text{tr}_{c}\{[\partial^{\mu} - ig A_{n_{2}}^{\mu} - ig A_{s}^{\mu} , \partial^{\nu} - ig A_{n_{2}}^{\nu} - ig A_{s}^{\nu}]^{2} \} \]  
(48)
\[ \mathcal{L}_s = i\bar{\psi}_s (\partial - ig A_s) \psi_s + \frac{1}{2g^2} tr_c \left\{ [\partial^\mu - ig A^\mu_s, \partial^\nu - ig A^\nu_s]^2 \right\} \]  

(49)

where the derivative \( \partial_n \) is defined as:

\[ \partial_n^\mu \psi_n^{(0)} = \partial^\mu \psi_n^{(0)} , \quad \partial_n^\mu A_n^{(0)} = \partial^\mu A_n^{(0)} \]  

(50)

\[ \partial_n^\mu A_n^\nu \equiv \bar{n}^\mu n \cdot \partial A_n^\nu \]  

(51)

We can then quantize the Lagrangian density \( \mathcal{L}_\Lambda \) by quantize the fields \( \psi_n^{(0)}, A_n^{(0)}(n^\mu \neq n_1^\mu, n^\mu \neq n_2^\mu) \), \( \psi_s, A_s, \psi_n, \) and \( A_n(i = 1, 2) \) independently. While dealing with couplings between soft particles and collinear particles \( (n^\mu \neq n_1^\mu, n^\mu \neq n_2^\mu) \) in the process considered in this paper, such quantization scheme gives the same result as that in QCD at leading order in \( \Lambda_{QCD}/Q \). These fields should be quantized as satisfy the boundary condition provided by classical configurations of effective fields in the region \( C(x) \) and the conditions:

\[ \psi_n(y_m) = \psi_n(y_s) = 0(m^\mu \neq n_1^\mu) \]  

(52)

\[ A_n^\mu(y_m) = A_n^\mu(y_m) = 0(m^\mu \neq n_1^\mu) \]  

(53)

for \( i = 1, 2 \) and

\[ \psi_n^{(0)}(y_m) = \psi_n^{(0)}(y_s) = 0(m^\mu \neq n_1^\mu) \]  

(54)

\[ A_n^{(0)}(y_m) = A_n^{(0)}(y_m) = 0(m^\mu \neq n_1^\mu) \]  

(55)

for \( n^\mu \neq n_1^\mu, n^\mu \neq n_2^\mu \).

Diagrams with disconnected hard parts do not contribute to the process at leading order in \( \Lambda_{QCD}/Q \). We bring in an effective hard vertex \( \Pi(x) \) to describe the effects of hard sub-process. According to the gauge invariance, we have:

\[ \Pi(x) = \Pi(Y_n \psi_n^{(0)}(x + n \cdot y_n \bar{n}), Y_n A_n^{(0)} Y_n^\dagger(x + n \cdot y_n \bar{n}), \ldots, \]

\[ (\psi_n + \psi_s)(x + n_1 \cdot y_{n_1} \bar{n_1}), (A_{n_1} + A_s)(x + n_1 \cdot y_m \bar{n_1}), \]

\[ (\psi_{n_2} + \psi_s)(x + n_2 \cdot y_{n_2} \bar{n_2}), (A_{n_2} + A_s)(x + n_2 \cdot y_m \bar{n_2}) \]  

(56)

where we have dropped the \( \psi_s(x + y_s) \) and \( A_s(x + y_s) \) terms as couplings between soft particles and modes with \( |k^2| \geq Q^2 \) are suppressed as power of \( \Lambda_{QCD}/Q \). \( \Pi(x) \) is the hard vertex, we have:

\[ y_n^0 \sim y_s^0 \sim 1/Q \]  

(57)

To get \( \Pi(x) \), one should integrate out degrees of freedom with \( |k^2| \geq Q^2 \). We assume that this can be performed without breaking the gauge invariance.

According to our quantization scheme, time evolution of the effective fields are:

\[ Y_n \psi_n^{(0)}(x + y_n) = e^{i H Ax^0} Y_n \psi_n^{(0)}(\vec{x} + y_n) e^{-i H Ax^0} \]  

(58)

\[ Y_n A_n^{(0)} Y_n^\dagger(x + y_n^\prime) = e^{i H Ax^0} Y_n A_n^{(0)} Y_n^\dagger(x + y_n^\prime) e^{-i H Ax^0} \]  

(59)

\[ (\psi_{n_1} + \psi_s)(x + y_{n_1}) = e^{i H Ax^0} (\psi_{n_1} + \psi_s)(\vec{x} + y_{n_1}) e^{-i H Ax^0} \]  

(60)

\[ (A_{n_1} + A_s)(x + y_{n_1}^\prime) = e^{i H Ax^0} (A_{n_1} + A_s)(\vec{x} + y_{n_1}^\prime) e^{-i H Ax^0} \]  

(61)

\[ (\psi_{n_2} + \psi_s)(x + y_{n_2}) = e^{i H Ax^0} (\psi_{n_2} + \psi_s)(\vec{x} + y_{n_2}) e^{-i H Ax^0} \]  

(62)
\[(A_{n_2} + A_x)(x + y'_{n_2}) = e^{iH Ax^0} (A_{n_2} + A_x)(\vec{x} + y'_{n_2})e^{-iH Ax^0}\]  
\[(63)\]

where \(H_A\) is the Hamiltonian corresponding to \(\mathcal{L}_A\). We then have:

\[\Pi(x) = e^{iH Ax^0} \Pi(\vec{x})e^{-iH Ax^0}\]  
\[(64)\]

We then write the transition amplitude of the process in the frame of the effective theory:

\[\lim_{T \to \infty} T \langle H_3 H_4 X | e^{-iH_A(T - x^0)} \Pi(\vec{x}) e^{-iH_A(x^0 + T)} | p_1 p_2 \rangle_T - T\]  
\[(65)\]

where the subscripts \(T\) and \(-T\) denote the states at the time \(t = T\) and \(t = -T\) in the Schrödinger picture. We take the modular square of such amplitude and make the summation over all possible states \(X\) and the point at which the hard collision occurs, this gives:

\[\lim_{T \to \infty} \sum_X \int d^4x_2 \int d^4x_1\]
\[\langle p_1 p_2 | e^{iH_A(x_2^0 + T)} e^{i\sum_{\mu, \nu \neq n_1^r, n_\nu \neq n_2^r} H_n(\vec{x_2}^0 + T)} \Pi(\vec{x}_2) e^{iH_A(T - x_2^0)} | H_3 H_4 X \rangle_T\]
\[T \langle H_3 H_4 X | e^{-iH_A(T - x_1^0)} \Pi(\vec{x}_1) e^{-iH_A(x_1^0 + T)} | p_1 p_2 \rangle_T - T\]  
\[(66)\]

We write \(H_A\) as:

\[H_A = \sum_{n_\mu \neq n_1^r, n_\nu \neq n_2^r} H_n^{(0)} + H_{n_1} + H_{n_2} + H_s \equiv \sum_{n_\mu \neq n_1^r, n_\nu \neq n_2^r} H_n^{(0)} + H_{cs}\]  
\[(67)\]

where \(H_n^{(0)} (n_\mu \neq n_1^r, n_\mu \neq n_2^r), H_{n_1}, H_{n_2}\) and \(H_s\) represent the Hamiltonian corresponding to \(\mathcal{L}_n^{(0)} (n_\mu \neq n_1^r, n_\mu \neq n_2^r), \mathcal{L}_{n_1}, \mathcal{L}_{n_2}\) and \(\mathcal{L}_s\) respectively. We have \([H_n^{(0)}, H_{cs}] = 0\) and write \[(66)\] as:

\[\lim_{T \to \infty} \sum_X \int d^4x_2 \int d^4x_1\]
\[-T \langle p_1 p_2 | e^{iH_{cs}(x_2^0 + T)} e^{i\sum_{n_\mu \neq n_1^r, n_\nu \neq n_2^r} H_n^{(0)}(x_2^0 + T)} \Pi(\vec{x}_2)\]
\[e^{i\sum_{n_\mu \neq n_1^r, n_\nu \neq n_2^r} H_n^{(0)}(T - x_2^0)} e^{iH_A(T - x_2^0)} | H_3 H_4 X \rangle_T\]
\[T \langle H_3 H_4 X | e^{-iH_{cs}(T - x_1^0)} e^{-i\sum_{n_\mu \neq n_1^r, n_\nu \neq n_2^r} H_n^{(0)}(T - x_1^0)} \Pi(\vec{x}_1)\]
\[e^{-i\sum_{n_\mu \neq n_1^r, n_\nu \neq n_2^r} H_n^{(0)}(x_1^0 + T)} e^{-iH_{cs}(x_1^0 + T)} | p_1 p_2 \rangle_T - T\]  
\[(68)\]

The states \(|H_3 H_4 X\rangle_T\) form an invariant subspace of \(H_{cs}\) as \(H_3\) and \(H_4\) decouple from \(H_{cs}\). We have:

\[e^{iH_{cs}(T - x_2^0)} |H_3 H_4 X\rangle_T = \langle H_3 H_4 X | e^{-iH_{cs}(T - x_2^0)}\]
\[= e^{-iH_{cs}(x_1^0)} |H_3 H_4 X\rangle_T \quad T \langle H_3 H_4 X | e^{iH_{cs}x_2^0}\]
\[= e^{-iH_{cs}(x_1^0 + T)} |H_3 H_4 X\rangle_T \quad T \langle H_3 H_4 X | e^{iH_{cs}(x_2^0 + T)}\]  
\[(69)\]

We can then write \[(68)\] as:

\[\lim_{T \to \infty} \sum_X \int d^4x_2 \int d^4x_1\]
\[-T \langle p_1 p_2 | e^{iH_{cs}(x_2^0 + T)} e^{i\sum_{n_\mu \neq n_1^r, n_\nu \neq n_2^r} H_n^{(0)}(x_2^0 + T)} \Pi(\vec{x}_2)\]
\[e^{i\sum_{n_\mu \neq n_1^r, n_\nu \neq n_2^r} H_n^{(0)}(T - x_2^0)} e^{-iH_{cs}(x_2^0 + T)} | H_3 H_4 X \rangle_T\]
\[T \langle H_3 H_4 X | e^{iH_{cs}(x_1^0 + T)} e^{-i\sum_{n_\mu \neq n_1^r, n_\nu \neq n_2^r} H_n^{(0)}(T - x_1^0)} \Pi(\vec{x}_1)\]
\[e^{-i\sum_{n_\mu \neq n_1^r, n_\nu \neq n_2^r} H_n^{(0)}(x_1^0 + T)} e^{-iH_{cs}(x_1^0 + T)} | p_1 p_2 \rangle_T - T\]  
\[(70)\]

According to \[(57)\], we see that contributions of coupling between remnants of a initial hadron and other particles (soft or collinear to the initial hadron) after the hard collision cancel out at leading order.
We notice that:

\[ e^{-iH_{cs}(x_2^0 - x_1^0)} = e^{-i(H_{cs})_0 x_2^0} (T e^{-i \int_{q_1^0}^{q_2^0} dy y^0 (H_{cs})_i(y)} ) e^{i(H_{cs})_0 x_2^0} \]

\[ e^{-i(H_{cs})_0 x_2^0} \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{x_1^0}^{x_2^0} dy_1^0 \cdots \int_{x_1^0}^{x_2^0} dy_n^0 \]

\[ T[ (H_{cs})_I(y_0^0) \cdots (H_{cs})_I(y_n^0) ] e^{i(H_{cs})_0 x_2^0} \]

(71)

for arbitrary \( x_1 \) and \( x_2 \), where \( (H_{cs})_0 \) represents the free part of \( H_{cs} \), \( (H_{cs})_I \) represents the interaction part of \( H_{cs} \) in the interaction picture. Thus the \( \delta \)-function of momenta conversation at a vertex of coupling between remnants of a initial hadron and other particles should be substituted with the function:

\[ (2\pi)^4 \delta(4)(q_i) \rightarrow \frac{i(2\pi)^3 \delta(3)(\vec{q}_i)}{q_i^0 + i\epsilon} e^{-i q_i^0 x^0} \]

(72)

in (70). where \( x^0 = x_1^0 \) or \( x^0 = x_1^0 \) in (70), \( q_i^0 \) denotes the total momenta flowing into the vertex. We consider the coupling between remnants of the initial hadron that collinear-to-plus and other particles. We make the substitution:

\[ \frac{i(2\pi)^3 \delta(3)(\vec{q}_i)}{q_i^0 + i\epsilon} \rightarrow \frac{\sqrt{2} i(2\pi)^3 \delta(3)(\vec{q}_i)}{q_i^0 + i\epsilon} \]

(73)

We then take the approximation:

\[ e^{-i q_i^0 x^0} \simeq e^{-i q_i^0 x^0} \]

as we can drop the component \( q_i^- \) in the \( \delta \)-function of momenta conversation of the whole process. For the \( \delta \)-function \( \delta(q_i^0) \), we have:

\[ \delta(q_i^0) = \sqrt{2} \delta(q_i^0 + q_i^-) \]

(75)

We can then take \( q_i^- \) as independent variable and integrate out \( \delta(q_i^0) \) to determine \( q_i^+ \). In the propagators of particles collinear-to-plus with momenta \( k \), we can drop the small parts of \( k^+ \). This is equivalent to take the approximation:

\[ \delta(q_i^0) \simeq \sqrt{2} \delta(q_i^+) \]

(76)

According to these analyses, we have:

\[ \frac{i(2\pi)^3 \delta(3)(\vec{q}_i)}{q_i^0 + i\epsilon} e^{-i q_i^0 x^0} \simeq \frac{2i(2\pi)^3 \delta(q_i^+)}{q_i^0 + i\epsilon} (2\pi)^2 \delta(2)(\vec{q}_i^\perp) e^{-i \frac{1}{2} q_i^+ x^0} \]

(77)

We notice that minus momenta of lines at the vertex are independent of each other in this case. Singular point locating in the Glauber region can only be produced by the \( \frac{1}{q_i^- + i\epsilon} \) term. If there is one Glauber gluon flow into the vertex, the momentum of which we denote as \( q_i^G \). Then the other end of the \( q_i^G \) should be collinear particles with large minus momenta if we work in the Feynman gauge. Thus we can deform the integral path of \( q_i^{G-} \) to the upper half plane with radius of order \( \min\{|q_i^{G-}|, |q_i^{G-}|^2/|q_i^{G+}|\} \) so that the Gramma-Yennie approximation works. We have proved our claim at the beginning of this section.

IV. WILSON LINES OF COLLINEAR AND SOFT GLUONS

In this section, we bring in Wilson lines of soft and scalar polarized collinear gluons. We will show that effects of soft gluons and scalar-polarized collinear gluons can be absorbed into these Wilson lines.

We have proved in last section that one can deform the integral path to avoid Glauber region in couplings between soft gluons and collinear particles in (66). We can thus take the Grammer-Yennie approximation in these couplings. We make such approximation and write the classical effective Lagrangian density in the region \( S(x, 1/\Lambda_{QCD}) \) as:

\[ \mathcal{L}_{eff} \equiv \sum_{n} \mathcal{L}^{(n)}_{n} + \mathcal{L}_{s} \]

(78)
\[ \mathcal{L}_s^{(0)} = \frac{i}{2g^2} \text{tr} \left\{ \left[ (\bar{\partial}_n^\mu - ig A_1^{(0)\mu}, \bar{\partial}_n^\nu - ig A_2^{(0)\nu})^2 \right] \right\} \]

where the effective fields \( \psi_n^{(0)} \) and \( A_n^{(0)\mu} \) are defined as:

\[ \psi_n^{(0)}(y_n) = Y^\dagger_n \psi(y_n), \quad A_n^{(0)\mu} = Y^\dagger_n A_n^\mu Y_n(y_n) \]

and

\[ Y_n(y_n) = \begin{cases} 
\{ P \exp(ig \int_0^0 ds n \cdot A_s(y_n + sn) \} & \text{for } n^\mu = n_1^\mu \text{ or } n^\mu = n_2^\mu \\
( P \exp(ig \int_0^0 ds n \cdot A_s(y_n + sn) )^\dagger & \text{for } n^\mu \neq n_1^\mu \text{ and } n^\mu \neq n_2^\mu 
\end{cases} \]

The classical Lagrangian density in the region \( S(x, 1/A_{QCD}) \) can then be written as:

\[ \mathcal{L}_{\text{eff}} = \sum_{n^\mu} \mathcal{L}_n^{(0)} + \mathcal{L}_s \]

\[ \mathcal{L}_n^{(0)}(y_n) = \frac{i}{2g^2} \text{tr} \left\{ \left[ (\bar{\partial}_n^\mu - ig A_1^{(0)\mu}, \bar{\partial}_n^\nu - ig A_2^{(0)\nu})^2 \right] \right\} \]

We extract the large momenta components of collinear fields. That is:

\[ \psi_n^{(0)}(x_n) = \sum_{n^\mu p^\nu} \psi_{n,\bar{n}n}^{(0)}(x_n) e^{i \bar{n} \cdot p \cdot x_n} \]

\[ A_n^{(0)\mu}(x_n) = \sum_{n^\mu p^\nu} A_{n,\bar{n}n}^{(0)\mu}(x_n) e^{i \bar{n} \cdot p \cdot x_n} \]

Then the large momenta components become labels on the effective fields. The classical Lagrangian density in the region \( S(x, 1/A_{QCD}) \) can then be written as:

\[ \mathcal{L}_{\text{eff}} = \sum_{n^\mu} \mathcal{L}_n^{(0)} + \mathcal{L}_s \]

\[ \mathcal{L}_s = \frac{i}{2g^2} \text{tr} \left\{ (\bar{\partial}_s^\mu - ig A_1^\mu, \bar{\partial}_s^\nu - ig A_2^\nu)^2 \right\} \]

We then quantize \( \mathcal{L}_{\text{eff}} \) by quantizing the effective fields \( \psi_n^{(0)}(x_n), A_n^{(0)\mu}, \psi_s \) and \( A_s \). Hadrons collinear to \( n^\mu \) move along the direction \( n^\mu \) with uncertainties of order \( 1/A_{QCD} \). Thus boundary conditions in the path integral produced
by the initial or final collinear hadrons are constraints on classical configurations of the fields $\psi_n^{(0)}(x_n)$ and $A_n^{(0)\mu}(x_n)$ at $\vec{n} \cdot x_n \to \pm \infty$. Boundary conditions produced by final soft hadrons are constraints on classical configurations of the fields $\psi_s(x_s)$ and $A_s^{(\mu)}(x_s)$ at $x_s^0 \to \infty$.

We bring in an effective vertex $\Gamma(x)$ to describe the hard-subprocess in the region $H(x, 1/Q)$. We have:

$$\Gamma(x) = \Gamma(Y_n \psi_n^{(0)}(x_n), \ldots, Y_m A_n^{(0)\mu}(x_n) Y_m^+(x))$$

(90)

The residual momenta of the fields $\psi_n^{(0)}(x_n)$ and $A_n^{(0)\mu}(x_n)$ only contribute to the momenta conversation of the whole process. We thus set the coordinates of the fields $\psi_n^{(0)}(x_n)$ and $A_n^{(0)\mu}(x_n)$ as equal to $x$.

We pause here to discuss the modes $A_n^{(0)}$ with $\vec{n} \cdot p = 0$. In usual perturbative calculations, one preform the integral over the whole momentum space. Thus one can not distinguish the modes $A_n^{(0)}, n, p = 0$ and $A_s$ in perturbative calculations. We absorb contributions of the modes $A_n^{(0)}, n, p = 0$ into those of the modes $A_s$. The modes $A_n^{(0)}, n, p = 0$ are not gauge invariant. Thus this should be performed in a given gauge condition, for example, the Feynman gauge. We make the decomposition:

$$A_n^{(0)\mu}(x_n) = n \cdot A_n^{(0)\mu}(x_n) = n \cdot A_n^{(0)\mu}(x_n) = n \cdot A_n^{(0)\mu}(x_n) = n \cdot A_n^{(0)\mu}(x_n) = n \cdot A_n^{(0)\mu}(x_n)$$

(91)

where the subscript $n \perp$ denotes that the vector $(\vec{x})_{n\perp}$ fulfill the condition:

$$(\vec{x})_{n\perp} \cdot \vec{n} = 0$$

(92)

Then contribution of the modes $n \cdot A_n^{(0)\mu}(x_n)$ can be absorbed into Wilson lines of the soft gluons. Contributions of the modes $A_n^{(0)\mu}(x_n) - n \cdot A_n^{(0)\mu}(x_n)$ are power suppressed as the transition probability is free of pinch singular surfaces in Glauker region at leading order in $\Lambda_{QCD}/Q$. We absorb them to collinear modes. It is equivalent to replace:

$$A_n^{(0)\mu}(x_n) = A_n^{(0)\mu}(x_n) = A_n^{(0)\mu}(x_n) = A_n^{(0)\mu}(x_n) = A_n^{(0)\mu}(x_n) = A_n^{(0)\mu}(x_n)$$

(93)

in $\{s\}$. This is what we do in Sec II.

We then consider the evolution of the jets collinear to $n^\mu$ with the coordinates $\vec{n} \cdot x_n$. According to the similar procedure in last section. We see that effects of couplings between particles collinear to $n^\mu(i = 1, 2)$ occur at the points $x_n$, with $\vec{n} \cdot x_n > \vec{n} \cdot x$ cancel out. While effects of couplings between particles collinear to $n^\mu(n^\mu \neq n_1^\mu, n^\mu \neq n_2^\mu)$ occur at the points $x_n$ with $\vec{n} \cdot x_n < \vec{n} \cdot x$ cancel out. Thus energy flow from the jets collinear to $n^\mu(i = 1, 2)$ to the hard vertex and then to the jets collinear to $n^\mu(n^\mu \neq n_1^\mu, n^\mu \neq n_2^\mu)$. We define the momenta of collinear particles that participate in the hard-subprocess as flow from the jets collinear to $n^\mu(i = 1, 2)$ to the hard vertex or flow from the hard vertex to the jets collinear to $n^\mu(n^\mu \neq n_1^\mu, n^\mu \neq n_2^\mu)$. Then we have $\vec{n}_i \cdot p_n \geq 0(i = 1, 2)$ and $\vec{n} \cdot p_n \geq 0(n^\mu \neq n_1^\mu, n^\mu \neq n_2^\mu)$, where $p_n$ and $p_n$ represent particles collinear to $n_1^\mu$ and $n_2^\mu$ that participate in the hard-subprocess. To see this, we consider a particle collinear-to-plus with momentum $l$ that participate in the hard sub-process. We repeat the similar calculations as in last section. At the end of this particle that connect to the jet collinear-to-plus, we get the factor:

$$\frac{(2\pi)^6 \delta(-l^+ + \ldots)}{-l^+} e^{-i \int_{-l^+}^{l^+} (-l^+ + \ldots) x^0}$$

(94)

where the ellipsis denotes terms that is independent of $l$. The denominator of the propagator of $l$ is:

$$l^2 = 2l^+(l^+ - |l^+_\perp|^2 + i\epsilon l^+)$$

(95)

The other end of $l$ is the hard vertex and is independent of $l^-$. We see that singular points of $l^+$ all locate in the upper half plane if $l^+ < 0$. Thus we have $l^+ \geq 0$. For particles collinear to other directions, we can repeat the similar calculations and get the conclusion that $\vec{n}_i \cdot p_n \geq 0(i = 1, 2)$ and $\vec{n} \cdot p_n \geq 0(n^\mu \neq n_1^\mu, n^\mu \neq n_2^\mu)$.

We now consider the effects of scalar polarized collinear gluons in the hard sub-process that collinear to $n^\mu$ as $l_{1\perp}^\mu$. $l_{1\perp}^\mu$ is defined as flow from the jets collinear to $n_1^\mu(i = 1, 2)$ to the hard vertex or from the hard vertex to the jets collinear to $n_1^\mu(n^\mu \neq n_1^\mu, n^\mu \neq n_2^\mu)$. We work in Feynman gauge in this section thus the other end of $l_1^\mu$ should be polarized in the direction $\vec{n}_1^\mu$ at leading order in $\Lambda_{QCD}/Q$. According to our decomposition in (91) and the substitution in (93), contribution of these modes are
power suppressed if \( \vec{n} \cdot l_n^i = 0 \). We thus constrain that \( \vec{n} \cdot l_n^i > 0 \). We can drop the \( i\epsilon \) term in the eikonal propagators. That is to say:

\[
\frac{1}{\vec{n} \cdot l_n^i \pm i\epsilon} = P\left(\frac{1}{\vec{n} \cdot l_n^i}\right)
\]

(96)

\[
\frac{1}{\vec{n} \cdot (l_n^1 + \ldots + l_n^n) \pm i\epsilon} = P\left(\frac{1}{\vec{n} \cdot (l_n^1 + \ldots + l_n^n)}\right)
\]

(97)

where \( P \) denote the principal value. Singular points of the type \( \vec{n} \cdot l_n^i = 0 \) or \( \vec{n} \cdot (l_n^1 + \ldots + l_n^n) = 0 \) do not give contribution to \( \Gamma(x) \) at leading order. We can then apply the Ward identity to \( \Gamma(x) \) to absorb the fields \( \vec{n} \cdot A_m^{(0)} \) into Wilson lines. We bring in the Wilson lines (14–46):

\[
\tilde{W}_{n,x}^{(0)} = \begin{cases} 
\exp\left\{ g \sum_{\vec{n} \cdot p > 0} e^{-i\vec{n} \cdot p \cdot x} \frac{\bar{\gamma} \cdot A^{(0)}_{\bar{\gamma} \cdot p}(x)}{\vec{n} \cdot \bar{\gamma} \cdot p(x)} \right\} & \text{for } n^\mu = n_1^\mu \text{ or } n^\mu = n_2^\mu \\
\exp\left\{ g \sum_{\vec{n} \cdot p < 0} e^{-i\vec{n} \cdot p \cdot x} \frac{\bar{\gamma} \cdot A^{(0)}_{\bar{\gamma} \cdot p}(x)}{\vec{n} \cdot \bar{\gamma} \cdot p(x)} \right\} & \text{for } n^\mu \neq n_1^\mu \text{ and } n^\mu \neq n_2^\mu
\end{cases}
\]

(98)

where the operator \( \vec{n} \cdot P_n \) is defined as:

\[
\vec{n} \cdot P_n \psi^{(0)}_{n,m,p} = \vec{n} \cdot P_n \psi^{(0)}_{n,\vec{n},\vec{n},p}, \quad \vec{n} \cdot P_n A^{(0)\mu}_{n,m,\nu,p} = \vec{n} \cdot P_n A^{(0)\mu}_{n,\vec{n},\nu,p} = 0 (m^\mu \neq n^\mu)
\]

(99)

(100)

We notice that contributions of the modes \( \vec{n} \cdot A^{(0)}_{\bar{\gamma} \cdot p}(x) \) with \( \vec{n} \cdot p = 0 (i = 1, 2) \) and \( \vec{n} \cdot A^{(0)}_{\bar{\gamma} \cdot p}(x)(n^\mu \neq n_1^\mu, n^\mu \neq n_2^\mu) \) with \( \vec{n} \cdot p = 0 \) are power suppressed. We may tentatively relax the constraints \( \vec{n}_i \cdot p > 0 (i = 1, 2) \) and \( \vec{n}_i \cdot p < 0 (m^\mu \neq n_1^\mu, n^\mu \neq n_2^\mu) \) in the Wilson lines to that \( \vec{n}_i \cdot p \geq 0 (i = 1, 2) \) and \( \vec{n}_i \cdot p \leq 0 (m^\mu \neq n_1^\mu, n^\mu \neq n_2^\mu) \). The operators \( \vec{n} \cdot P_n \) are singular if we include the modes \( \vec{n} \cdot A^{(0)}_{\bar{\gamma} \cdot p}(x) \) with \( \vec{n} \cdot p = 0 \). Thus we need some regularization of the operators \( \vec{n} \cdot P_n \). It is convenient to regularize these operators so that singular points of the type \( \vec{n} \cdot p = 0 \) do not affect the form of \( \Gamma(x) \).

We consider a gluon collinear to plus with momentum \( l \) that participate in the hard sub-process. We have:

\[
P\left(\frac{1}{l^+}\right) = \frac{1}{2} \left(\frac{1}{l^+ + i\epsilon} + \frac{1}{l^+ - i\epsilon}\right)
\]

(101)

We repeat the calculations in above paragraphs. At the end of \( l \) that connect to the jet collinear-to-plus, we get the factor:

\[
\frac{2i(2\pi)^3(-l^+ + \ldots)}{-l^+ + \ldots + i\epsilon} (2\pi)^2 \delta^2(-l^+ + \ldots) e^{-i\frac{1}{2\epsilon}(-l^+ + \ldots)x^0}
\]

(102)

where the ellipsis denotes terms that is independent of \( l \). The denominator of the propagator of \( l \) is:

\[
l^2 = 2l^+(l^+ - |l| l^+ + i\epsilon l^+)
\]

(103)

The other end of \( l \) is the hard vertex and is independent of \( l^+ \). We see that if \( l^+ = -i\epsilon \) then all singular points of \( l^- \) locate in the upper half plane. In this case, the singular point \( l^+ = -i\epsilon \) does not affect \( \Gamma(x) \). While the singular point \( l^+ = i\epsilon \) does affect \( \Gamma(x) \). Thus we regularize \( \vec{n}_1 \cdot P_{\vec{n}_1} \) in the Wilson lines by making the substitution:

\[
\frac{1}{\vec{n}_1 \cdot P_{\vec{n}_1}} \to \frac{1}{\vec{n}_1 \cdot P_{\vec{n}_1} + i\epsilon}
\]

(104)

in the Wilson lines. According to the similar analysis, we make the substitution:

\[
\frac{1}{\vec{n}_i \cdot P_{\vec{n}_i}} \to \frac{1}{\vec{n}_i \cdot P_{\vec{n}_i} + i\epsilon} (i = 1, 2)
\]

(105)

\[
\frac{1}{\vec{n} \cdot P_{\vec{n}}} \to \frac{1}{\vec{n} \cdot P_{\vec{n}} - i\epsilon} (n^\mu \neq n_1^\mu, n^\mu \neq n_2^\mu)
\]

(106)
We bring in the notations: denoted as: in the Wilson lines.

Thus \( \Gamma(\bar{n},p) \) can then be written as:

\[
\Gamma(x) = \sum_{n,\bar{n},p,...,m,\bar{m},p'} Y_m(\bar{m} \cdot \bar{n} - igA_n^{(0)\mu}Y^{\dagger}_{m})(x)
\]

We then bring in the effective fields:

\[
\begin{align*}
\hat{\psi}^{(0)}_{n,x}(x_n) &= W^{(0)}_{n,x} \psi^{(0)}_{n}(x_n) \\
(\partial_{\mu} - igA_n^{(0)\mu})(x_n) &= W^{(0)\dagger}_{n,x} (\partial_{\mu} - igA_n^{(0)\mu})W^{(0)\dagger}_{n,x}(x_n)
\end{align*}
\]

\( \Gamma(x) \) can then be written as:

\[
\Gamma(x) = \sum_{n,\bar{n},p,...,m,\bar{m},p'} Y_m(\bar{m} \cdot \bar{n} - igA_n^{(0)\mu}Y^{\dagger}_{m})(x)
\]

where the short distance coefficients in \( \Gamma \) are functions of the large momenta components \( n \cdot p, m \cdot \bar{n} \) denotes the vectors that fulfill the condition \( m \cdot \bar{m} = 0 \). Physical fields in \( \Gamma^\mu \) should connect to different jets at leading order. Thus \( \Gamma^\mu \) is multi-linear with its variables.

Up to this step, we are dealing with the Wilson lines structures in Hadron-Hadron collision. Our conclusion is that effects of soft gluons and scalar polarized collinear gluons can be absorbed into Wilson lines of these gluons. The remain question is whether the non-trivial dependence of cross section on transverse momenta of exchanged gluons in (112) really break the factorization. We will consider this in next section.

V. NONTRIVIAL TRANSVERSE MOMENTA DEPENDENCE OF HADRON-HADRON COLLISIONS

In this section we inspect the nontrivial transverse momenta dependence of hadron-hadron collisions shown in [12]. Our conclusion is that such non-trivial transverse momentum dependence can be described by the effective hard vertex (109).

To compare our results with those in (124), we consider the inclusive production of a back-to-back hadron-photon pair in hadron-hadron collision (see Fig.7):

\[
p_1 + p_2 \rightarrow \gamma(k_3,\lambda_3) + H_4(k_3, S_4) + X
\]

where \( p_1 \) and \( p_2 \) represent initial hadrons, \( \gamma \) represents the detected final photon, \( H_4 \) represents the detected final hadron, \( X \) represents any other particles, \( k_3 \) and \( \lambda_3 \) denote the momentum and helicity of the detected photon, \( k_4 \) and \( S_4 \) denote the momentum and spin of the detected hadron.

\( k_3 \) and \( k_4 \) are constrainers to be not nearly collinear to \( p_1 \) or \( p_2 \). The hard energy scale of this process is set by the quantity:

\[
Q \equiv \sqrt{(k_3 + k_4)^2}
\]

As in (12), we consider the \( gq \rightarrow \gamma q \) channel in this paper. An example of such channel at tree level is shown in Fig.5.

We work in the center of mass frame of initial hadrons. The momenta \( p_1 \) and \( p_2 \) are approximately light-like and denoted as:

\[
p_1 \simeq (p_1^+, 0, 0), \quad p_2 \simeq (0, p_2^- , 0)
\]

We bring in the notations:

\[
n_1^\mu \equiv \frac{1}{\sqrt{2}}(1, 0, 0, 0) = \frac{1}{\sqrt{2}}(1, 0, 0, 0)
\]
FIG. 5. An example of the inclusive production of hadron-photon pair in hadron-hadron collision at tree level

\[ n_2^\mu = \frac{1}{\sqrt{2}} (1, 0, 0, -1) = \frac{1}{\sqrt{2}} (0, 1, 0) \] (114)

\[ \bar{n}_i^\mu \equiv \sqrt{2}(1, 0, 0, 0) - n_i^\mu (i = 1, 2) \] (115)

then we have:

\[ p_1^\mu \simeq \bar{n}_1 \cdot p_1 n_1^\mu, \quad p_2^\mu \simeq \bar{n}_2 \cdot p_2 n_2^\mu \] (116)

The incoming partons are approximately collinear to their parent hadrons, thus \( k_1 \) and \( k_2 \) scale as:

\[ (k_1^+, k_1^-, |\vec{k}_1|) \sim (Q, \Lambda_{QCD}^2/Q, \Lambda_{QCD}), \quad (k_2^+, k_2^-, |\vec{k}_2|) \sim (\Lambda_{QCD}^2/Q, Q, \Lambda_{QCD}) \] (117)

\( k_3 \) and \( k_4 \) are also collinear at the limit \( Q \rightarrow \infty \). They scale as:

\[ (k_3^+, k_3^- |\vec{k}_3|) \sim (Q, Q, Q), \quad (k_4^+, k_4^- |\vec{k}_4|) \sim (Q, Q, Q) \] (118)

We bring in the notation:

\[ n_3^\mu = \frac{1}{\sqrt{2}} (1, \bar{n}_3) \] (119)

\[ n_4^\mu = \frac{1}{\sqrt{2}} (1, \bar{n}_4) \] (120)

where

\[ |\bar{n}_3| = |\bar{n}_4| = 1 \] (121)

\[ n_3 \cdot n_4 = 1 + O(|\vec{q}_\perp|/Q) \] (122)

We have:

\[ k_3^\mu \simeq \bar{n}_3 \cdot k_3 n_3^\mu, \quad k_4^\mu \simeq \bar{n}_4 \cdot k_2 n_4^\mu \] (123)

where

\[ \bar{n}_i^\mu \equiv \sqrt{2}(1, 0, 0, 0) - n_i^\mu \] (124)
where $Q_q$ is the electric-charge of the quark $k_2$. At leading order in $\Lambda_{QCD}/Q$, such scattering amplitude can be reproduced by the effective operator:

\begin{equation}
J^\mu B^\mu(x) = \sum_{n_1^\perp, n_1} \sum_{k_1} \sum_{n_2^\perp, n_2} \sum_{k_2} \bar{\psi}^{(0)}_{n_4, n_4, k_4, x}(\bar{y}_{n_2}^{n_2 \perp} + i g A^{(0)n_2 \perp}_{n_2, n_2, k_2, x})(x)
\end{equation}

where $B^\mu$ denotes the photon field, $n_2^\perp$ denotes the vectors that fulfill the condition $n_2^\perp \cdot \bar{n} = 0$, the derivative $\bar{\partial}_n^\mu$ is defined in [83] and [84].

According to the form of $\Gamma(x)$ in [109], the effective operators that contribute to the process considered here is:

\begin{equation}
\Gamma^\mu B^\mu(x) = \sum_{n_1^\perp, n_1} \sum_{k_1} \sum_{n_2^\perp, n_2} \sum_{k_2} \bar{\psi}^{(0)}_{n_4, n_4, k_4, x}(\bar{y}_{n_2}^{n_2 \perp} + i g A^{(0)n_2 \perp}_{n_2, n_2, k_2, x})(x)
\end{equation}

We then consider the diagrams in Fig[8]. The gluon $k_2$ is collinear gluon, thus we can drop the Wilson lines $Y_{n_i}$ in...
we have:

\[\Gamma^\mu B_\mu(x)_{\text{one gluon}} \equiv \sum_{n_1^\mu, n_1, k_1} \sum_{n_2^\mu, n_2, k_2} \sum_{n_3} \sum_{n_4, k_4} \tilde{\psi}_{n_4, n_4, k_4, x}(ig)(\bar{A}_{n_2, n_2, k_2, x}^{(0)}(ig)A_{n_2, n_2, k_2, x}^{(0)} - (i\tilde{\partial}_{(n_2\cdots)^\mu})_{n_2, n_2, k_2, x}^{(0)}n_2 \cdot k_2)
\]

The term does not contribute to scattering amplitude according to momenta conservation. We make the substitution:

\[A_{n_2, n_2, k_2, x}^{(0)}(ig)A_{n_2, n_2, k_2, x}^{(0)} - (i\tilde{\partial}_{(n_2\cdots)^\mu})_{n_2, n_2, k_2, x}^{(0)}n_2 \cdot k_2)
\]

which do not change the amplitude at leading order according to Ward identity. We bring in the operator:

\[\bar{\Gamma}^\mu B_\mu(x)_{\text{one gluon}} \equiv \sum_{n_1^\mu, n_1, k_1} \sum_{n_2^\mu, n_2, k_2} \sum_{n_3} \sum_{n_4, k_4} \tilde{\psi}_{n_4, n_4, k_4, x}(ig)(\bar{A}_{n_2, n_2, k_2, x}^{(0)}(ig)A_{n_2, n_2, k_2, x}^{(0)} - (i\tilde{\partial}_{(n_2\cdots)^\mu})_{n_2, n_2, k_2, x}^{(0)}n_2 \cdot k_2)
\]

The scattering amplitude of the process can then be written as:

\[iM \equiv \langle k_3, \lambda_3; k_4, S_4|T(\bar{\Gamma}^\mu B_\mu)|_{\text{one gluon}}|p_1 p_2\rangle
\]

where \(T\) denotes the time-ordering operator. This is in accordance with the corresponding result in [12].

### B. Calculations of Hard Sub-process Involving Two Gluons

We now consider the case that there are two gluons connect to the hadron \(p_2\). There is the case that the additional gluon do not connect to hard sub-diagram. That is to say, the additional gluon is exchanged between spectators. This gluon should be soft at leading order in \(\Lambda_{QCD}/Q\). We perform the summation over all possible final cuts of spectators and absorb soft gluons in to the Wilson lines of soft gluons in Sec[V]. Thus we do not show the calculations of such case. This is enough to compare with the results in [12].

The imaginary parts of eikonal propagators are not concerned in [12], as they do not violate maximally generalized factorization TMD-factorization in the one-extra-gluon example. To simplify the calculations, we will also drop such
terms in this paper. We do not distinguish collinear or soft gluons in these calculations as contributions of the singular points $\vec{n}_i \cdot p_{n_i} = 0$ are not concerned in these calculations, where $p_{n_i}$ denotes the momentum of a internal line that is collinear to $n_i^\mu$. We can thus make the substitution:

$$Y_n \hat{\psi}^{(0)}_{n,x} \rightarrow \hat{\psi}_{n,x}$$

$$Y_n (\bar{\partial}_n^\mu - igA_n^{(0)\mu}) Y_n \rightarrow (\bar{\partial}_n^\mu - igA_n^\mu)$$

in (127), where

$$\hat{\psi}_{n,x}(x_n) = W^\dagger_{n,x}(x_n) \psi_{n}(x_n)$$

$$(\bar{\partial}_n^\mu - igA_n^\mu) = W^\dagger_{n,x}(\bar{\partial}_n^\mu - igA_n^\mu)W_{n,x}$$

We now consider the effective operator $\Gamma^\mu B_\mu$ in (127) at the two-gluon level (both gluons connect to $p_2$). We have:

$$\Gamma^\mu B_\mu|_{\text{two} \text{ gluons}}(k_1, k_2, k_3, k_4)$$

$$\equiv - \frac{g^2}{2} g^2(Q_{q\bar{q}}) B^\mu(k_3) \sum_{n_1^\nu} \sum_{n_2^\nu} \sum_{n_3^\nu} \hat{\psi}_{n_3}(k_4)$$

$$\left\{ - \int \frac{d^4l_2}{(2\pi)^4} l_2^{n_2, \perp \nu} P \left( \frac{\vec{n}_2 \cdot A_n^\mu(k_2 - l_2)}{\vec{n}_2 \cdot (k_2 - l_2)} \right) P \left( \frac{\vec{n}_2 \cdot A_n^\mu(l_2)}{\vec{n}_2 \cdot l_2} \right) \right.$$  

$$+ \int \frac{d^4l_2}{(2\pi)^4} l_2^{n_2, \perp \nu} P \left( \frac{\vec{n}_2 \cdot A_n^\mu(k_2 - l_2)}{\vec{n}_2 \cdot (k_2 - l_2)} \right) P \left( \frac{\vec{n}_2 \cdot A_n^\mu(l_2)}{\vec{n}_2 \cdot l_2} \right) \right.$$  

$$- 2 \int \frac{d^4l_2}{(2\pi)^4} A_n^\mu(k_2 - l_2) P \left( \frac{\vec{n}_2 \cdot A_n^\mu(l_2)}{\vec{n}_2 \cdot l_2} \right)$$

$$\left\{ \gamma_\mu (\vec{n}_1 \cdot k_4 \not{n}_4 - \vec{n}_2 \cdot k_2 \not{n}_2) \gamma_\mu t^c \right.$$  

$$\left. - 2 \vec{n}_2 \cdot \vec{n}_2 \not{n}_4 \cdot \not{k}_4 \not{n}_2 \cdot n_4 + i\epsilon \right.$$  

$$+ \gamma_\mu (\vec{n}_1 \cdot k_1 \not{n}_1 + \vec{n}_2 \cdot k_2 \not{n}_2) \gamma_\mu t^c \right\} \hat{\psi}_{n_1}(k_1)$$

where we have turn in to momenta space to simplify the expression, $P$ denotes the principal value. The $l_2^{n_2, \perp \nu}$ term is indeed the same as that in the formula (134) of [12]. One should not be confused with the global coefficient 1/2 in our formula. We do not contract fields in the operator $\Gamma^\mu B_\mu$ with initial or final states at this step. There are two possible manners to contract the two gluon fields in $\Gamma^\mu B_\mu$ and the two initial gluons. We have dropped terms with the color factor $\frac{1}{2} \{t^a, t^b\}$ in above formula as they do not contribute to the amplitude of the process considered here. The scattering amplitude of the process can then be written as:

$$iM \equiv \langle k_3, \lambda_3; k_4, S_4 | T(\Gamma^\mu B_\mu)_{\text{two} \text{ gluons}} | p_1 p_2 \rangle$$

This is in accordance with the corresponding result in [12].

We see that the non-trivial transverse momenta dependence of cross section, which is considered as the symbol of break down of factorization, can be described by the effective operator $\tilde{\Gamma}^\mu$ in (127) or more generally, by the effective operator $\Gamma(x)$ in (109). We will work in the frame of effective theory and finish the proof of factorization theorem in next section.

VI. FACTORIZATION.

In this section, we finish the proof of QCD factorization for processes considered in this paper.
We start from the transition probability:

\[ |M|^2 \equiv \lim_{T \to \infty} \sum_X \int d^4 x_2 \int d^4 x_1 \]

\[ -T \langle p_1 p_2 | e^{iH_{\text{eff}}(x_2^0 + T)} \Gamma (\vec{x}_2) e^{iH_{\text{eff}}(T - x_1^0)} | H_3 H_4 X \rangle_T \]

\[ T \langle H_3 H_4 X | e^{-iH_{\text{eff}}(T - x_1^0)} \Gamma (\vec{x}_1) e^{-iH_{\text{eff}}(x_2^0 + T)} | p_1 p_2 \rangle_{-T} \]

\[ = \lim_{T \to \infty} \sum_X \int \frac{d^4 k}{(2\pi)^4} \int d^4 x e^{-i k \cdot x} \]

\[ -T \langle p_1 p_2 | e^{iH_{\text{eff}}(x_2^0 + T)} \Gamma (\vec{x}_2) e^{iH_{\text{eff}}(T - x_1^0)} | H_3 H_4 X \rangle_T \]

\[ T \langle H_3 H_4 X | e^{-iH_{\text{eff}}(T - x_1^0)} \Gamma (\vec{0}) e^{-iH_{\text{eff}}(x_2^0 + T)} | p_1 p_2 \rangle_{-T} \] (137)

where \( \Gamma (x) \) take the form shown in (109), \( H_{\text{eff}} \) is the Hamiltonian corresponding to \( \mathcal{L}_{\text{eff}} \) in (87). We have made the directions \( n^\mu \) to be slightly space like so that the time ordering and anti-time ordering operators do not affect the Wilson lines.

We next factorize \( \Gamma (x) \) according to the manner:

\[ \Gamma (x) \equiv \sum_{\Gamma_c, \Gamma_s} \Gamma_c (\vec{\psi}_n, \ldots, \vec{\partial}_n m^\perp - i g \vec{A}_m (0) \Gamma_s (Y_n, Y_m) (x) \] (138)

where \( \Gamma_c^\mu \) is multi-linear with \( \vec{\psi}_n (0) \) and \( \vec{A}_m (0) \) as physical collinear fields in \( \Gamma^\nu \) connect to different jets at leading order, Wilson lines that appear in \( \Gamma_s \) depend on type of partons that appear in \( \Gamma_c \). We have dropped possible color indices of \( \Gamma_c \) and \( \Gamma_s \) for simplicity.

The detected hadrons can locate in the back-to-back region, thus Wilson lines of soft gluons do not necessary to cancel out. In this case, there are only two final jets in the hard sub-diagrams at leading order. (50) There are only Wilson lines along the directions of initial hadrons or the two final jets in this case. We write the part of \( |M|^2 \) in (137) that depend on soft gluons as:

\[ \langle 0 | \Gamma_s (x) e^{-iH_s x^0} \Gamma_s (\vec{0}) | 0 \rangle \]

\[ \simeq \langle 0 | \Gamma_s (x_\perp) \Gamma_s (\vec{0}) | 0 \rangle \]

\[ \simeq \langle 0 | 1 \rangle + \langle 0 | \Gamma_s (x_\perp) \Gamma_s (\vec{0}) | 0 \rangle | \text{two back-to-back final jets} \] (139)

where \( H_s \) is the Hamiltonian corresponding to \( \mathcal{L}_s \) in (89). We can then write \( |M|^2 \) in (137) as:

\[ |M|^2 \simeq \lim_{T \to \infty} \sum_X \int \frac{d^4 k}{(2\pi)^4} \int d^4 x e^{-i k \cdot x} \]

\[ -T \langle p_1 p_2 | e^{i \sum_n H_n (0) (x^0 + T)} \Gamma (\vec{x}_2) e^{i \sum_n H_n (0) (T - x_1^0)} | H_3 H_4 X \rangle_T \]

\[ T \langle H_3 H_4 X | e^{-i \sum_n H_n (0) T} \Gamma (\vec{0}) e^{-i \sum_n H_n (0) T} | p_1 p_2 \rangle_{-T} \]

\[ \left( 1 + \langle 0 | \Gamma_s (x_\perp) \Gamma_s (\vec{0}) | 0 \rangle \right) | \text{two back-to-back final jets} \] (140)

where \( H_n (0) \) is the Hamiltonian corresponding to \( \mathcal{L}_n \) in (88).

We then define the annihilation operator:

\[ \tilde{a}_{n, p} = \sqrt{2 E_{p}} \int d^3 \vec{x}_n \bar{u}^s (p) \vec{\psi}_{n, x}^{(0)} (\vec{x}_n) e^{-i \vec{p} \cdot \vec{x}_n} \] (141)

\[ \tilde{a}_{n, p}^\dagger = -\sqrt{2 E_{p}} \int d^3 \vec{x}_n \bar{\psi}_{n, x}^{(0)} (\vec{x}_n) u^s (p) e^{-i \vec{p} \cdot \vec{x}_n} \] (142)

for collinear fermions and anti-fermions respectively. One can verify that \( \tilde{a}_{n, p}^\dagger | 0 \rangle = 0 \). For collinear gluons, we define that:

\[ \tilde{a}_{n, p}^\dagger = \frac{i}{\vec{n} \cdot p} \sqrt{2 E_{p}} \int d^3 \vec{x}_n e^{-i \vec{p} \cdot \vec{x}_n} \vec{n} \cdot \partial \tilde{A}_n (\vec{x}_n) \]

\[ = \frac{i}{\vec{n} \cdot p} \sqrt{2 E_{p}} \int d^3 \vec{x}_n e^{-i \vec{p} \cdot \vec{x}_n} \tilde{e}_n^s (p) \]

\[ \tilde{W}_n (0) \tilde{W}_n (0)^\dagger (\vec{x}_n) \] (143)
One can also verify that \( \hat{a}_{n,p}^\dagger |0\rangle = 0 \). Hadron states can then be expanded according to the parton states \( |\hat{p}_n\rangle \) which are created by the creation operator \( \hat{a}_{n,p} \). That is:

\[
|\hat{p}_n\rangle = \sqrt{2E_{p_n}a_{n,p}^\dagger} |0\rangle
\]

where \( a_{n,p}^\dagger \) denote the conjugation of the operators \([141],[142]\) or \([143]\).

\( \Gamma \) is multi-linear with \( \hat{\psi}_{n,x}^{(0)} \) and \( \hat{A}_{m,x}^{(0)\perp\mu} \), where \( n \neq m \). There can be no more than one parton states that are created by the operator \( \hat{a}_{n,p}^\dagger \) contracted with \( \Gamma^\mu \) for each collinear jets. We denote these active partons as \( |\hat{p}_1\rangle \) and \( |\hat{k}_1\rangle \) and have:

\[
|M|^2 \simeq \frac{1}{D(G)} \lim_{T \to \infty} \sum_X \int \frac{d^4k}{(2\pi)^4} \int d^4x e^{-ik \cdot x} [2E_{p_1}^i tr_c \{ -T \langle p_i | e^{-iH_{n_i}^{(0)}(x^0 + T)} a_{n_i,x,p_i}^\dagger e^{-iH_{n_i}^{(0)}T} |p_i\rangle_T \} \]
\[\times \prod_{i=3}^4 \{ \langle 0 | \hat{a}_{n_i,x,k_i} e^{iH_{n_i}^{(0)}(T-x^0)} |H_iX_{n_i}\rangle_T
\]
\[\times \langle \hat{p}_1^2 \hat{p}_2^2 |\Gamma \langle \hat{x} | e^\sum_{n_i} H_{n_i}^{(0)}(T-x^0) |\ldots \hat{k}_1 \ldots X_n\rangle_T
\]
\[\times T \langle \ldots \hat{k}_1 \ldots X_n | e^{-i\sum_{n_i} H_{n_i}^{(0)}T} \Gamma |\hat{0}\rangle |\hat{p}_1^2 \hat{p}_2^2 \}
\]
\[\left(1 + \langle 0 | \Gamma_{\perp}(\hat{x}_\perp) \Gamma_{\parallel}(\hat{0}) |0\rangle \text{two back-to-back final jets} \right) \]

where \( \frac{1}{D(G)} \) denotes the color factor, \( X_{n_i} \) denotes undetected hadrons that collinear to \( n_i^\mu \), \( \hat{n}_i \) denotes the directions of collinear hadrons that are quite different from those of initial hadrons and detected final hadrons.

We notice that the states \( |\hat{p}_n\rangle \) is invariant under the gauge transformation \( U_n(x_n)(U_n(\infty) = 1) \):

\[
\psi_n^{(0)} \to U_n \psi_n^{(0)}, \quad A_n^{(0)\mu} \to U_n (A_n^{(0)\mu} + \frac{i}{g} \partial^\mu) U_n^\dagger
\]

\[
\psi_s \to \psi_s, \quad A_s^\mu \to A_s^\mu
\]

\( \Gamma(x) \) and \( \Gamma_{\perp}(x) \) is also invariant under such gauge transformation even if we choose different \( U_n \) for different directions \( n^\mu \). Especially, we can choose that \( U_{\hat{n}_i} = 1 \). The matrix-elements between parton states in \([146]\) are invariant under such gauge transformations. We can take the lowest perturbation of the fields \( \hat{n}_i \cdot A_{n_i}^{(0)}(i = 1, \ldots, 4) \) in such
matrix-element as they are scalar polarized. We then have:

$$|M|^2 \simeq \frac{1}{D(G)} \lim_{T \to \infty} \sum_X \int \frac{d^4 k}{(2\pi)^4} \int d^4 x e^{-ik \cdot x}$$

$$\prod_{i=1}^2 \left( 2E_{p_i}^\dagger \text{tr} \left\{ -T \langle p_i | e^{iH_n^{(0)}(x^0 + T)} \hat{a}_{n_i,x \cdot p_i}^\dagger e^{-iH_n^{(0)}T | p_i \rangle_T} \right\} \right)$$

$$\prod_{i=3}^4 \left( 2E_{k_i}^\dagger \text{tr} \left\{ \langle 0 | \hat{a}_{n_i,x \cdot k_i} e^{iH_n^{(0)}(T - x^0)} | H_i X_{n_i} \rangle_T \right\} \right)$$

$$\prod_{i=1}^2 \left( 2E_{p_i}^\dagger \text{tr} \left\{ -T \langle p_i | e^{iH_n^{(0)}(x^0 + T)} \hat{a}_{n_i,x \cdot p_i}^\dagger e^{-iH_n^{(0)}T | p_i \rangle_T} \right\} \right)$$

$$\prod_{i=3}^4 \left( 2E_{k_i}^\dagger \text{tr} \left\{ \langle 0 | \hat{a}_{n_i,x \cdot k_i} e^{iH_n^{(0)}(T - x^0)} | H_i X_{n_i} \rangle_T \right\} \right)$$

where $p_i^1$ is the usual partons produced by the operator $a_{n_i,p_i}^\dagger \hat{a}_{n_i} = a_{n_i,p_i}^\dagger | \tilde{n} \cdot A_n = 0 \rangle$. The condition $\tilde{n} \cdot A_n = 0$ should be treated as the lowest perturbation of the fields $\tilde{n} \cdot A_n$ not the axial gauge.

If we do not consider the back-to-bak region, then we can set that:

$$x \to \tilde{x}_n \equiv (n \cdot x, 0, \tilde{0})$$

in the fields collinear to $n^\mu$. In the back-to-back region, we can set that:

$$x \to \tilde{x}_n \equiv (n \cdot x, 0, \tilde{x}_{n \perp})$$

in the fields collinear to $n^\mu$. We can then write the transition probability as:

$$|M|^2 = |M|_c^2 + |M|^2_R$$

$$|M|_c^2 = \frac{1}{D_c(G)} \lim_{T \to \infty} \sum_X \int \frac{d^4 k}{(2\pi)^4} \int d^4 x e^{-ik \cdot x}$$

$$\prod_{i=1}^2 \left( 2E_{p_i}^\dagger \text{tr} \left\{ -T \langle p_i | e^{iH_n^{(0)}(x^0 + T)} \hat{a}_{n_i,x \cdot p_i}^\dagger e^{-iH_n^{(0)}T | p_i \rangle_T} \right\} \right)$$

$$\prod_{i=3}^4 \left( 2E_{k_i}^\dagger \text{tr} \left\{ \langle 0 | \hat{a}_{n_i,x \cdot k_i} e^{iH_n^{(0)}(T - x^0)} | H_i X_{n_i} \rangle_T \right\} \right)$$

$$\langle p_1^1 p_2^1 | \Gamma_{c}(\tilde{x}) e^{i \sum H_n^{(0)}(T - x^0)} | \ldots k_i^1 \ldots X_{\tilde{n}} \rangle_T$$

$$T \langle \ldots k_i^1 \ldots X_{\tilde{n}} | e^{-i \sum H_n^{(0)} T} \Gamma(\tilde{0}) | p_1^1 p_2^1 \rangle_{\tilde{n} \cdot A_n = 0}$$

(150)

(151)

(152)

(153)
\[ |M|^2 = \frac{1}{D_4(\mathcal{G})} \lim_{T \to \infty} \sum_X \int \frac{d^4k}{(2\pi)^4} \int d^4xe^{-ik \cdot x} \]
\[
\prod_{i=1}^{2} \left( 2E_{p_i} \text{tr}_c \left\{ -T \langle p_i | e^{iH_n^0(x_n^0 + T)\hat{a}_{n1}^\dagger} e^{-iH_n^0|p_i/T \rangle} \right\} \right) 
\]
\[
\prod_{i=3}^{4} \left( 2E_{k_i} \text{tr}_c \left\{ \langle 0|\hat{a}_{n1},\bar{x}_n, k_1^1 e^{iH_n^0(T-x_n^0)}|H_sX_n \rangle \right\} \right) 
\]
\[
T \langle H_sX_n | e^{-iH_n^0T\hat{a}_{n1}^\dagger} | 0 \rangle \right\} 
\]
\[
\langle p_1 | p_2 | \Gamma_1^\dagger(x)e^{\sum l_i^0(x-T-x_n^0)}|k_1^1 \ldots \rangle_T 
\]
\[
T \langle \ldots k_1^1 \ldots | e^{-i\sum l_i^0(T-H_n^0 \Gamma(0) |p_1 |p_2)\hat{a}_{n1}^\dagger} | 0 \rangle \right\} 
\]
\[
tr_c \langle \langle 0|\Gamma_1^\dagger(x_\perp)\Gamma_2(0)|0 \rangle \right\} \| \text{two back-to-back final jets} \right\} 
\]

(154)

where \( |M|^2 \) and \( |M_0|^2 \) represent contributions of large transverse momenta region and back-to-back region respectively, \( D_4(\mathcal{G}) \) and \( D_2(\mathcal{G}) \) represent the color factors.

The formula (152) is our final result. Soft gluons and scalar polarized gluons that collinear to initial and detected final hadrons decouple from the matrix-element between parton states in (152). Such matrix-element can be calculated perturbatively.

### VII. CONCLUSION

We have presented the proof of TMD-factorization in hadron-hadron collisions in this paper. We constrain that the detected hadrons are not collinear to initial hadrons. This is essential in the cancellation of pinch singular surfaces in Glauber region. Our result contradicts the widely accepted viewpoint that TMD-factorization does not hold for such process. We do not claim that calculations in the literatures are incorrect. We simply point out that there are some missing aspects in these calculations.

The Ward identity cancellation of scalar polarized gluons is prevented by singular points of the type \( \bar{n} \cdot l_i^n = 0 \) or \( \bar{n} \cdot (l_1^n + \ldots l_n^n) = 0 \), where \( l_i^n \) represent momenta of scalar polarized gluons that collinear to \( n^\alpha \). Such singular points indeed locate in soft region. The collinear approximation does not hold in this region. One may first deform the integral path of \( \bar{n} \cdot l_i^n \) to collinear region so that the collinear approximation works. However, \( \bar{n} \cdot l_i^n \) is pinched in the soft(non-Glauber) region. Thus one can not simply deform the integral path before contributions of the soft region is subtracted from the whole integral.

We first make some subtraction(like that we do in Sec [II]) so that contributions of the soft region do not affect the remaining part. This can be written as:

\[ M = M_s + M - M_s \simeq M_s + M_c \]

(155)

where \( M \) represents the whole amplitude or the modular square of the whole amplitude, \( M_s \) is achieved by apply some approximation to \( M \) which behaves the same as \( M \) at leading order in soft region, \( M_c \) represents that we take the collinear approximation in the \( M - M_s \) part. The Ward identity cancelation works in \( M_c \) part as singular points of the type \( \bar{n} \cdot l_i^n = 0 \) or \( \bar{n} \cdot (l_1^n + \ldots l_n^n) = 0 \) do not contribute to \( M_c \) at leading order.

If the gluons exchanged between collinear particles in \( M_s \) can be absorbed into Wilson lines along the directions of the collinear particles, then both \( M_c \) and \( M_s \) take the factorized form like that shown in Fig [3] and Fig [4]. To achieve this, one should first prove the cancelation of pinch singular surfaces in Glauber region so that the Grammer-Yennie approximation works in soft region. This is performed in sec [III]. The deformations depend on whether the collinear particles collinear to initial hadrons or other directions. However, this only affect the Wilson lines of soft gluons.

We see that different deformations of the integral path can only affect the \( M_s \) part as \( M_c \) is free of singular points of the type \( \bar{n} \cdot l_i^n = 0 \) or \( \bar{n} \cdot (l_1^n + \ldots l_n^n) = 0 \). For \( M_s \) part we do need different Wilson lines while gluons couple to different jets. These Wilson lines only depend on which jets the gluons couple to. They do not depend the other ends of the gluon propagators. Thus these Wilson lines can be written into a soft factor after we sum over all undetected final states.
The soft factors do not cancel out if there detected hadrons in back-to-back region. There are only four Wilson lines along the directions of initial hadrons or the two detected hadrons in this case. Thus the soft factor only depend on properties of the initial and detected hadrons. We see that the soft factors do not affect the factorization.

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[1] C. J. Bomhof, P. J. Mulders, and F. Pijlman, Phys. Lett. B596, 277 (2004), arXiv:0406099 [hep-ph].
[2] A. Bacchetta, C. Bomhof, P. Mulders, and F. Pijlman, Phys. Rev. D72, 034030 (2005), arXiv:0505268 [hep-ph].
[3] C. J. Bomhof, P. J. Mulders, and F. Pijlman, Eur. Phys. J. C47, 147 (2006), arXiv:0601171 [hep-ph].
[4] C. J. Bomhof and P. J. Mulders, Nucl. Phys. B795, 409 (2008), arXiv:0709.1390 [hep-ph].
[5] J. Collins and J.-W. Qiu, Phys. Rev. D75, 114014 (2007), arXiv:0708.2141 [hep-ph].
[6] W. Vogelsang and F. Yuan, Phys. Rev. D76, 094013 (2007), arXiv:0708.4398 [hep-ph].
[7] J. Collins, (2007), arXiv:0708.4410 [hep-ph].
[8] A. Bacchetta, C. Bomhof, U. DAlesio, P. J. Mulders, and F. Murgia, Phys. Rev. Lett. 99, 212002 (2007), arXiv:0703153 [hep-ph].
[9] C. J. Bomhof, P. J. Mulders, W. Vogelsang, and F. Yuan, Phys. Rev. D75, 074019 (2007), arXiv:0701277 [hep-ph].
[10] D. Boer, P. J. Mulders, and C. Pisan, Phys. Lett. B660, 360 (2008), arXiv:0712.0777 [hep-ph].
[11] T. C. Rogers and P. J. Mulders, Phys. Rev. D81, 034030 (2005), arXiv:0505268 [hep-ph].
[12] C. J. Bomhof, P. J. Mulders, and F. Pijlman, Eur. Phys. J. C47, 147 (2006), arXiv:0601171 [hep-ph].
[13] T. C. Rogers, Phys. Rev. D85, 094006 (2012), arXiv:1204.4251 [hep-ph].
[14] C. J. Bomhof and D. E. Soper, Nucl. Phys. B185, 381 (1981).
[15] C. J. Bomhof and D. E. Soper, Nucl. Phys. B194, 445 (1982).
[16] G. T. Bodwin, Phys. Rev. D31, 2616 (1985).
[17] J. C. Collins, D. E. Soper, and G. Sterman, Phys. Rev. D31, 104 (1985).
[18] J. C. Collins, D. E. Soper, and G. Sterman, Nucl. Phys. B308, 833 (1988).
[19] X. d. Ji, J. p. Ma, and F. Yuan, Phys. Lett. B597, 299 (2004), arXiv:0405085 [hep-ph].
[20] J. C. Collins, F. Hautmann, Phys. Lett. B472, 129 (2000), arXiv:9908467 [hep-ph].
[21] J. C. Collins, Acta. Phys. Polon. B34, 3103 (2003), arXiv:0304122 [hep-ph].
[22] F. Hautmann, Phys. Lett. B655, 26 (2007), arXiv:072196 [hep-ph].
[23] J. Collins, Phys. Lett. B536, 43 (2002), arXiv:0204004 [hep-ph].
[24] D. Boer, P. J. Mulders, and F. Pijlman, Nucl. Phys. B667, 201 (2003), arXiv:0303034 [hep-ph].
[25] J. Collins and F. Hautmann, Acta. Phys. Polon. B34, 3103 (2003), arXiv:0304122 [hep-ph].
[26] J. C. Collins, Nucl. Phys. B597, 299 (2001), arXiv:0107177 [hep-ph].
[27] J. C. Collins, Nucl. Phys. B597, 299 (2001), arXiv:0107177 [hep-ph].
[28] J. C. Collins, Acta. Phys. Polon. B34, 3103 (2003), arXiv:0304122 [hep-ph].
[29] J. C. Collins, Nucl. Phys. B597, 299 (2001), arXiv:0107177 [hep-ph].
[30] J. C. Collins, Nucl. Phys. B597, 299 (2001), arXiv:0107177 [hep-ph].
[31] J. C. Collins, Nucl. Phys. B597, 299 (2001), arXiv:0107177 [hep-ph].
[32] J. C. Collins, Nucl. Phys. B597, 299 (2001), arXiv:0107177 [hep-ph].
[33] J. C. Collins, Nucl. Phys. B597, 299 (2001), arXiv:0107177 [hep-ph].
[34] J. C. Collins, Nucl. Phys. B597, 299 (2001), arXiv:0107177 [hep-ph].
[35] J. C. Collins, Nucl. Phys. B597, 299 (2001), arXiv:0107177 [hep-ph].
[36] J. C. Collins, Nucl. Phys. B597, 299 (2001), arXiv:0107177 [hep-ph].
[37] J. C. Collins, Nucl. Phys. B597, 299 (2001), arXiv:0107177 [hep-ph].
[38] J. C. Collins, Nucl. Phys. B597, 299 (2001), arXiv:0107177 [hep-ph].
[49] J. M. F. Labastida and G. Sterman, Nucl. Phys. B254, 425 (1985).
[50] J. C. Collions, Phys. Rev. D21, 2962 (1980).