On the Nature of X(4260)

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Abstract

A careful analysis is made in studying X(4260) property by fitting $J/\psi\pi^+\pi^-$ and $\pi^+\pi^-$ invariant mass spectrum. For the latter final state theorem for $\pi\pi$, $K\bar{K}$ couple channel system is used. As a consequence the ratio $g^2_{XJ/\psi\sigma}/g^2_{XJ/\psi\text{f}_0(980)}$ is estimated. Meanwhile we get a sizable coupling between X and $\omega\chi_c$. Our analysis suggests that the X(4260) resonance is a conventional $c\bar{c}$ state renormalized by $\omega\chi_c$ continuum and the wave function renormalization constant is estimated to be $Z_X \approx 0.8$. The renormalization effect is important in reducing the leptonic decay width of X(4260) and to escape from the existing experimental constraint.

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1 Introduction

A resonant state X(4260) (previously called Y(4260)) has been found by BABAR collaboration in initial-state radiation (ISR) process in 2005 [1], in $J/\psi\pi^+\pi^-$ final state. The mass and width are given as $M = 4259\pm 8(\text{stat.})^{+2}_{-6}(\text{sys.})\text{MeV}$ and $\Gamma = 88\pm 23(\text{stat.})^{+6}_{-4}(\text{sys.})\text{MeV}$. The branching ratio is determined to be

$$\Gamma_{e^+e^-} \times Br(X \rightarrow \pi^+\pi^-J/\psi) = 5.5 \pm 1.0^{+0.8}_{-0.7} \text{eV}. \quad (1)$$

Also the X(4260) state has been confirmed by CLEO [2] and BELLE [3] experiments, and CLEO also has seen it in $\pi^0\pi^0J/\psi$ and $K^+K^-J/\psi$ processes. BELLE collaboration also gave the $\pi^+\pi^-$ invariant mass spectrum. More recently, BABAR has restudied the $J/\psi\pi^+\pi^-$ channel and obtained upgraded results. [4]

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In theory aspects, the existence of X(4260) is very interesting, because it is generally thought there is not enough unassigned vector states in charmonium spectrum, also the masses (including the recently reported Y(4360), X(4630)/Y(4660) states) are inconsistent with naive quark model predictions. [5] The only such $1^{--}$ states expected up to 4.4 GeV are 1S, 2S, 1D, 3S, 2D and 4S, and they seem to be well established. [6] The situation is depicted in Figure 1. It tends to believe that the discrepancy between naive quark model prediction and the observed spectrum is ascribed at least partly to the existence of many open charm thresholds, and the latter will distort the spectrum. The situation is depicted in Figure 2. However the experimental

| X(4260) | $3^3D_1(4.52)$ | $\Lambda^+\Lambda^-$ |
|---------|----------------|---------------------|
| $\Psi(4415)$ | $4^3S_1(4.45)$ |                     |
| X(4360) |                 |                     |
| X(4260) | $2^3D_1(4.19)$ |                     |
| $\Psi(4160)$ | $3^3S_1(4.10)$ |                     |
| $\Psi(4040)$ |                 |                     |
| $\Psi(3770)$ |                 |                     |
| $\Psi(2S)$ | $2^3S_1(3.68)$ |                     |

$\Psi(1S)$ $1^{--}$ particle  $1^3S_1(3.1)$ quark model threshold

Figure 1: X(4260) and nearby resonances from naive quark model calculation [5].

| $J/\Psi K K(4086)$ | $\chi_{c0}\rho(4185)$ | $X(4260)$ | $\chi_{c1}\omega(4293)$ |
|---------------------|-----------------------|---------|-----------------------|
| $D^{*-}D^{*-}(4020)$ | $D^{*-}D^{*-}(4224)$ | $D_1(4289)$ | $\Lambda,\bar{\Lambda}(4572)$ |

Figure 2: X(4260) and its nearby thresholds.
searches seem to indicate a rather negative result. For example, a candidate final state for $X(4260)$ decay is $D \bar{D}^*$, However, BABAR collaboration could not find $X(4260)$ in $\bar{D} D^*, D \bar{D}^*, D^* D^*$ [7]. BELLE collaboration measured the process $e^+ e^- \rightarrow D^0 D^{*-} \pi^+$, and there was no evidence for $X(4260)$ [8]. Rather extensive searches in open charm final states in $e^+ e^-$ annihilation at energies between 3.7GeV and 5.0GeV are made, but only upper limits on $X(4260)$ decays are given [9]. Furthermore BES collaboration did not find the signal of $X(4260)$ by scanning $R$ value as well [10].

Because of above reasons many theoretical papers have been produced devoted to the investigation on $X(4260)$, and it is generally believed that the existence of $X(4260)$ signals a degree of freedom beyond conventional $\bar{c}c$ state. In the literature, many proposals have been made, e.g., charmonium [11], $\chi_{c0}\rho$ molecule [12], $\omega \chi_{c1}$ molecule [13], $c\bar{c}g$ hybrid state [14, 15], $\Lambda_c \bar{\Lambda}$ bayronium [16], $D_1 \bar{D}$ or $D_0 \bar{D}_0^*$ molecule [17, 18], etc.

In this article, we also devote to the study of $X(4260)$ issue, inspired by the recent BABAR experiment [4]. Through a careful analysis to experimental data available, we find that the $X(4260)$ has a sizable coupling to $\omega \chi_{c0}$ channel, but not to other (nearby) channels. The paper is organized as the following: Section 1 is the introduction, in section 2 we review on theoretical tools we use in this paper, special emphasis is made on the final state interactions between pions. In section 3 we give a detailed description to our numerical fit program. In section 4 we extract the $X$ coupling to $J/\psi \sigma$ and $J/\psi f_0(980)$ and determine the pole location of $X(4260)$ on Riemann sheets. In section 5 we estimate the wave function renormalization constant of $X(4260)$ induced by the interaction between $X, \omega$ and $\chi_{c0}$. Finally in section 6 we draw conclusions based on the studies of this paper.

2 Theoretical Discussions on $X \rightarrow J/\psi \pi\pi$ Process

2.1 Effective Lagrangian Describing $X \rightarrow J/\psi \pi \pi$ Interactions

Assuming that the $X(4260)$ state being a $J^{PC} = 1^{--}$ chiral singlet, we write down the following effective lagrangian describing $X \rightarrow J/\psi \pi \pi$ process, which is accurate in the leading order in the expansion in terms of $\pi$ momentum in the center of mass frame of $\pi \pi$ system,

$$\mathcal{L}_{\gamma X} = g_0 X_{\mu \nu} F^{\mu \nu},$$
$$\mathcal{L}_{X\psi \pi \pi} = h_1 X_{\mu \nu} \psi^{\mu \nu} < u_\alpha u^\alpha > + h_2 X_{\mu \nu} \psi^{\mu \nu} < \chi_+ > + h_3 X_{\mu \alpha} \psi^{\mu \beta} < u_\beta u^\alpha >,$$

(2)

where we have used the anti-symmetric representation ($X_{\mu \nu}, \psi^{\mu \nu}$) to describe the $1^{--}$ state $X(4260)$ and $J/\psi$, respectively, and $F_{\mu \nu}$ denotes the photon field strength. Notice that in the
present notation, one has,
\[ \Gamma_{e^+e^-} = \frac{4\alpha}{3} \frac{g_0^2}{M_X}, \]  
(3)
when neglecting electron mass. Also notice that, up to \( O(p_\pi^2) \) level, in Eq. (2) there exist only three independent interaction terms with coefficients \( h_1, h_2 \) and \( h_3 \). Further, \( u_\mu = i(u^+\partial_\mu u - u\partial_\mu u^+) \) and

\[ u = \exp\{i \frac{\Phi}{\sqrt{2}F_\pi} \} \]  
(4)
is a parametrization of the pseudo-goldstone octet:

\[ \Phi = \begin{pmatrix} 1/\sqrt{2}\pi^0 + 1/\sqrt{6}\eta_8 & \pi^+ & K^+ \\ -1/\sqrt{2}\pi^0 + 1/\sqrt{6}\eta_8 & \pi^- & K^- \\ \pi^- & K^- & -2/\sqrt{6}\eta_8 \end{pmatrix}. \]  
(5)
The chiral symmetry breaking term with coefficient \( h_2 \) in Eq. (2) reads,

\[ \chi_+ = u^+\chi u^+ + u\chi^+u, \quad \chi = 2B_0 \text{diag}(m_u, m_d, m_s). \]  
(6)
Parameters \( F_\pi \) and \( B_0 \) can be fixed phenomenologically: \( F_\pi \approx 92.4 \text{MeV} \) and \( <\bar{\psi}\psi|0> = -F^2B_0[1 + O(m_\eta)] \). The Eq. (2) can also be rewritten in an explicit form,

\[ \mathcal{L}_1 = \frac{4h_1}{F^2_\pi} X\mu\nu F^{\mu\nu}(\partial_\rho\pi^+\partial^\rho\pi^- + \frac{1}{2}\partial_\rho\pi^0\partial^\rho\pi^0 + \partial_\rho K^+\partial^\rho K^- + \partial_\rho K^0\partial^\rho K^0 + \frac{1}{2}\partial_\rho\eta\partial^\rho\eta) \]  
(7)
\[ \mathcal{L}_2 = -\frac{4h_2}{F^2_\pi} X\mu\nu F^{\mu\nu}(m_\pi^2\pi^+\pi^- + \frac{1}{2}m_\pi^2\pi^0\pi^0 + m_K^2 K^+K^- + m_K^2 K^0K^0 + \frac{2}{3}m_K^2 - \frac{1}{6}m_\eta^2) \eta\eta \]  
(8)
\[ \mathcal{L}_3 = \frac{4h_3}{F^2_\pi} X\mu\nu F^{\mu\nu}(\frac{1}{2}\partial_\beta\pi^+\partial^\beta\pi^- + \frac{1}{2}\partial_\beta\pi^0\partial^\beta\pi^0 + \frac{1}{2}\partial_\beta K^+\partial^\beta K^- + \frac{1}{2}\partial_\beta K^0\partial^\beta K^0 + \frac{1}{2}\partial_\beta\eta\partial^\beta\eta). \]  
(9)

It is noticed that, according to Eq. (2), only \( s \)-wave and \( d \)-wave of the \( \pi\pi \) system are involved (more discussions will be given later). Also Eq. (2) does not take into account of possible strong final state interactions between two pions in the decay product. More discussion on this topic will be left in section 2.3.

### 2.2 Kinematics and tree level amplitudes

Denote the momentums of \( e^-, e^+, X(4260), J/\psi, \pi^+ \) and \( \pi^- \) as \( q_1, q_2, q, q_0, q^+ \) and \( q^- \) respectively, as seen from figure 3. The polarization of \( J/\psi \) is represented as \( e^\lambda \), and define
Figure 3: A depiction of kinematics.

\( k_\pm = q^+ \pm q^- \), then one has the following relations,

\[
\begin{align*}
    s &\equiv k_+^2, \\
k_-^2 &= -s \rho^2 = 4m_\pi^2 - s, \quad (\rho = \sqrt{1 - \frac{4m_\pi^2}{s}}) \\
    q_0^2 &= M_{J/\psi}^2, \quad k_+ \cdot k_- = 0, \\
k_+ \cdot q_0 &= \frac{1}{2}(q^2 - M_{J/\psi}^2 - s). 
\end{align*}
\] (10)

The amplitude of \( X \to J/\psi \pi^+ \pi^- \) process at tree level is

\[
\begin{align*}
    iA_{\text{tree}} &= \frac{i4e q_0}{M_{J/\psi} F^2(q^2) D_X(q^2)} \bar{u}(q_1, s) \gamma_\lambda u(q_2, s') \{ [4h_1 \frac{1}{2}(s - 2m_\pi^2) + 4h_2 m_\pi^2](q^0 \cdot q^\lambda - q \cdot q^0 \epsilon^\lambda) \\
    &+ \frac{1}{2}h_3[-q_0^0 q \cdot \epsilon_\psi(k_+^\lambda k^\alpha_+ - k_-^\lambda k_-^\alpha) + \epsilon_\psi^\alpha q_0^0 q_\beta(k_+^\alpha k_+^\beta - k_-^\alpha k_-^\beta) \\
    &- q^{0\lambda} q_\alpha \epsilon_\psi q_\beta(k_+^\alpha k_+^\beta - k_-^\alpha k_-^\beta) + q^0 \cdot q \epsilon_\psi \epsilon_\lambda (k_+^\lambda k_+^\alpha - k_-^\lambda k_-^\alpha) \} \}, 
\end{align*}
\] (11)

where \( D_X(q^2) \) is the denominator of the \( X(4260) \) propagator. In the simplest form, it can be parameterized as \( D_X(q^2) = M_X^2 - q^2 - iM_X \Gamma_X \).

Follow the helicity amplitude decomposition method, [19] choose the basis of tensors

\[
\begin{align*}
    \tilde{t}^{(0)} &= 1, \\
    \tilde{t}^{(1)} &= k_-^\mu, 
\end{align*}
\] (12) (13)
\[ \bar{A}^{(2)} = k_\mu k_\nu - \frac{1}{3} k_\mu g_\mu - \frac{k_\mu k_\nu}{k^2} \],

it is then easy to recognize that terms proportional to \( h_1 \) and \( h_2 \) in Eq. (11) only contribute to the \( s \)-wave whereas the term proportional to \( h_3 \) contributes both to \( s \)-wave and \( d \)-wave. The overall \( s \)-wave amplitude reads,

\[
\begin{align*}
\bar{A}^{tree}_s &= \frac{i 4 e g_0}{M_{J/\psi} F^2 \pi^2 D_X(q^2)} \bar{v}(q_1, s) \gamma_\lambda u(q_2, s') \epsilon_\psi \{ [4 h_1 (s - 2m_\pi^2) + 4 h_2 m_\pi^2] (q^0 \cdot q)^\lambda - q^0 q^0 \lambda ) \\
&+ \frac{1}{2} h_3 [- \frac{1}{3} \rho^2(s) q_0^2 q^0 s + \left( 1 - \frac{1}{3} \rho^2(s) \right) k_\lambda^s q^0 \cdot k_+ + \frac{1}{3} \rho^2(s) q^0 \cdot q \lambda^0 q^0 s \\
&- \left( 1 - \frac{1}{3} \rho^2(s) \right) g^\lambda_\theta k_+ \cdot q k_+ \cdot q^0 - \frac{1}{3} \rho^2(s) q_0^0 q^0 + \left( 1 - \frac{1}{3} \rho^2(s) \right) k_\lambda^q q^0 \cdot q k_+ \\
&+ \frac{1}{3} \rho^2(s) q_0 \cdot q \lambda^0 q^0 s - \left( 1 - \frac{1}{3} \rho^2(s) \right) k_\lambda^s k_+ q^0 \cdot q^0 ] \},
\end{align*}
\]

whereas the \( d \)-wave part is

\[
\begin{align*}
\bar{A}^{tree}_d &= \frac{i 4 e g_0}{M_{J/\psi} F^2 \pi^2 D_X(q^2)} \bar{v}(q_1, s) \gamma_\lambda u(q_2, s') \epsilon_\psi \\
&= \int \frac{d \cos \theta}{4} \sum_{s, s'} |B_d|^2 \left( - \frac{q^2}{135} \rho^4 \left[ M_{J/\psi}^6 (s - 4q^2) + M_{J/\psi}^4 (6q^4 - q^2 s + s^2) + M_{J/\psi}^2 (4q^6 + q^4 s - 34q^2 s^2 + 9s^3) \\
+ M_{\psi}^2 + (q^2 - s)^2 (q^4 + 3q^2 s + 6s^2) \right] \right).
\end{align*}
\]

Since the \( d \)-wave contribution is proportional to the 4-th power of the kinematic factor, it is highly suppressed comparing with the \( s \)-wave contribution. It will be shown later through numerical studies that the \( d \)-wave contribution is roughly less than 1% of the total decay rate.

### 2.3 Final state interactions

The tree level amplitude as described in section 2.2 is not sufficient to describe the \( X \to J/\psi \pi \pi \) decay process, since the \( \pi \pi \) final state is mainly in the \( IJ=00 \) system and undergoes strong
final state interactions (FSI). To include FSI we propose the following decay amplitude:

\[ \mathcal{A}_1 = A_1^{tree} \alpha_1(s) T_{11}(s) + A_2^{tree} \alpha_2(s) T_{21}(s) , \]
\[ \mathcal{A}_2 = A_1^{tree} \alpha_1(s) T_{12}(s) + A_2^{tree} \alpha_2(s) T_{22}(s) , \]

(18)

where subscripts 1,2 denote \( \pi\pi \) and \( K\bar{K} \) final states, respectively. Especially \( T_{11}, T_{12} \) and \( T_{22} \) represent \( \pi\pi \to \pi\pi, \pi\pi \to K\bar{K}, K\bar{K} \to K\bar{K} \) scattering amplitudes, respectively. Functions \( \alpha_i \) are mild polynomials which place the role to offset the ‘left hand’ cuts on the complex \( s \) plane in amplitude \( T \) that would not appear in function \( \mathcal{A} \). Expressions in Eq. (18) are remarkable in the sense that the unitarity relations that the decay amplitudes \( \mathcal{A}_i \) – as an analytic function of \( s \) – have to obey:

\[ \text{Im} \mathcal{A}_1 = A_1^* \rho_1 T_{11} + A_2^* \rho_2 T_{21} , \]
\[ \text{Im} \mathcal{A}_2 = A_1^* \rho_1 T_{12} + A_2^* \rho_2 T_{22} \]

(19)

are automatically satisfied. Noticeably in \( \alpha_1(s) \) an additional pole term is added:

\[ \alpha_1(s) = \frac{c_0^{(1)}}{s - s_A} + c_1^{(1)} + c_2^{(1)} s + \cdots , \]

(20)

where \( s_A \) represents the Adler zero of \( T_{11} \). The role of the pole term is to cancel the Adler zero hidden in \( T_{11} \) but not welcome in \( \mathcal{A} \). [21] An advantage of the pole term in Eq. (20) is that, by appropriately choosing coefficient \( c_0^{(1)} \) as \( \lim_{s \to s_A} \frac{c_0^{(1)}}{s - s_A} T_{11}(s) = 1 \), it guarantees

\[ \mathcal{A}_1 = A_1^{tree} + O(s^2) \]

(21)

in the small \( s \) region, which allows the standard chiral power counting to work here.

2.4 The \( \pi\pi \) scattering \( T \) matrix

We choose three different \( \pi\pi \) scattering \( T \) matrices to perform three different fits to the \( X(4260) \) spectrum and the invariant mass spectrum of the \( \pi\pi \) final state: couple channel Padé approximation (named as fit I, parameters listed in Table 1), [20] K-matrix unitarization (named as fit II, \( K_3 \) parametrization of Ref. [21], using parameters refitted in Ref. [22]), and the PKU representation (fit III). [23] For more details about these \( T \) matrices, we refer to the original literature. Choosing three different \( T \) matrices is not redundant. The PKU representation gives a reliable description to the low energy physics related to the \( \sigma \) meson, but lacks of information on full couple channel dynamics and hence physics related to \( f_0(980) \). The other two methods, on the other side, are not accurate when discussing the \( \sigma \) meson but have full respects to couple
|         | \(L_1\) | \(L_2\) | \(L_3\) | \(L_4\) | \(L_5\) | \(2L_6+L_8\) |
|---------|---------|---------|---------|---------|---------|-------------|
|         | 10.32   | 13.30   | -38.03  | 3.770   | 17.15   | 10.39+4.96  |

Table 1: Low energy constants from couple channel Padé amplitudes of fit I. Here these parameters are refitted and are slightly different from Ref. [20]. The unit is \(10^{-4}\).

|         | Padé         | K-matrix     | PKU          |
|---------|--------------|--------------|--------------|
| sheet–II| 0.459 – 0.229i | 0.549 – 0.230i | 0.463 – 0.247i |
| sheet–II| 0.989 – 0.013i | 0.999 – 0.021i | 0.980 – 0.019i |
| sheet–III| -            | 0.705 – 0.327i | -            |
| sheet–III| -            | 0.977 – 0.060i | -            |

Table 2: Poles’ location on the \(\sqrt{s}\)–plane, in units of GeV.

channel unitarity and hence may provide a better description to the physics related to \(f_0(980)\). Pole positions of the three different \(T\) matrices are listed in Table 2. Regarding to the Adler zero discussed in section 2.3, In this work, we use \(s_A\) and \(c_0^{(1)}\) as calculated from each \(T\) matrix (listed in Table 3), instead of using the \(\chi\)PT \(O(p_\pi^2)\) value to fix the parameters \(c_0^{(1)}(= 16\pi F^2_\pi)\) and \(s_A(= m^2_{\pi}/2)\) in the \(IJ=0,0\) channel.\(^2\)

|         | Padé         | K-matrix     | PKU          |
|---------|--------------|--------------|--------------|
| Alder zero \(s_0\) | 0.00954 – 0.00015i | 0.01880   | 0.00383 |
| \(c_0^{(1)}\) | 0.32956 – 0.00110i | 0.14132 | 0.36088 |

Table 3: The Adler zero location on \(s\)–plane and \(c_0^{(1)}\) (in units of GeV\(^2\)).

3 Numerical Studies and Discussions

3.1 Introduction to the fit program and nearby cut singularities

The X(4260) may have other decay channels besides the \(J/\psi\pi\pi(\bar{K}K)\) one. It may also couple to a channel that it does not decay for kinematical reason. The nearby thresholds are \(\chi_{c0} \omega (4197\text{MeV}), D^+_s \bar{D}^0_s \bar{D}^0_s (4224\text{MeV}), D^- D^{-}_s (2420) (4291\text{MeV}), \chi_{c1} \omega (4293\text{MeV}), \text{etc.} \) It

\(^2\)Allowing these two parameters to vary does not make any significant changes on other physical output in the numerical fit discussed in section 3.
is possible that X(4260) couples to these channels. To be specific we attempt to write down the denominator of the propagator of X(4260) as,

\[
D_X(q^2) = M_X^2 - q^2 - i\sqrt{q^2}(g_1 k_1 + g_2 k_2 + \Gamma(q^2) + \Gamma_0),
\]

(22)

where \( k_1 \) (\( k_2 \)) is the c.m. momentum of channel 1 (2) that lies below (above) the peak position around 4260MeV. For example, channel 1 may denote \( D^*_s + D^*_s \) or \( \omega \chi_{c0} \), channel 2 may denote \( D^- D^{+}_1(2420) \) or \( \chi_{c1} \omega \). The latter two are indistinguishable in our study since they are both s-wave decays and the thresholds are very close to each other. \( \Gamma(q^2) \) is calculable (see for example Eq. (17)) which includes both \( J/\psi \pi \pi \) and \( J/\psi \bar{K} K \) decays, \( \Gamma_0 \) is a constant and simulates the decay width into other (un-observed) channels.

In our program we fit the X(4260) line shape in the region \( 4.15 \text{GeV} < \sqrt{q^2} < 4.47 \text{GeV} \), using data from Ref. [3] (17 data points) and Ref. [4] (17 data points) (see Figure 4a). If one tries to fit a larger \( q^2 \), one encounters the problem that \( s \) becomes too large and hence invalidates the parametrization of low energy \( T \) matrices.\(^3\) For \( \pi \pi \) invariant mass spectrum we use the data given in figure 4b of Ref. [3], which corresponds to \( m_{\pi^+\pi^-J/\psi} \in [4.2, 4.4]\text{GeV} \) (17 data points), and the data \( \in [4.15, 4.45]\text{GeV} \) (41 data points) from Ref. [4]. In Ref. [3], a MC study of efficiency correction at \( q^2 = 4.26\text{GeV} \) is also given. Through a numerical test we however find that the efficiency curve is well reproduced by the \( \pi \pi \) two body phase space up to a normalization constant, hence in our fit we simply use the two body phase space instead of the efficiency corrected one. We also assume the BABAR data of Ref. [4] have a similar behavior. There is no \( \bar{K} K \) invariant mass spectrum available. But one has the integrated \( J/\psi K \bar{K} \) events and this contributes another data point, \( \sigma(K^+K^-J/\psi) = 9^{+9}_{-5} \pm 1 \text{ pb} \).\(^2\) Summing up there are totally 93 data points to be used.

Parameters needed in our fit are the following: Firstly, Eq. (2) describing tree level X(4260) \( J/\psi \pi \pi (K \bar{K}) \) interactions and the \( \gamma - X \) transition provides 4 parameters. Secondly, it is found that taking \( \alpha_{1,2}(s) \) to be linear polynomials (except the Adler zero term) is already good enough for data fitting, hence the two \( \alpha_i(s) \) contribute another 4 parameters.\(^4\) Thirdly, through rather extensive numerical tests in different environment, we find that the coupling constant \( g_2 \) in Eq. (22) is always vanishingly small, therefore we often freeze parameter \( g_2 \) in the fit. Furthermore, in the absence of experimental evidence on X(4260) decay into other channels we often assume \( \Gamma_0 \) being vanishing.\(^5\) So we often switch off these two parameters unless stated otherwise. Hence

\(^3\)For \( \sqrt{q^2} = 4.47\text{GeV} \), it corresponds to a value \( \sqrt{s} \approx 1.17\text{GeV} \) which is within the range that \( T \) can provide a reasonable description.

\(^4\)In PKU parametrization there is no \( \alpha_2 \) so there are only two parameters here, only tree level amplitude of \( X \rightarrow J/\psi K \bar{K} \) are used.

\(^5\)Adding \( \Gamma_0 \) will actually not lead to any important change in physical conclusions in this paper.
Eq. (22) contains another 2 parameters. Finally, there are two normalization factors $N_1$ and $N_2$ for Belle and Babar $\pi^+\pi^-$ invariant mass spectrum. So summing up there are totally 12 (10 for PKU representation) parameters.

To begin with, we assume channel 1 represents $D_s^*\bar{D}_s^*$ and channel 2 represents $DD_1(2420)$ (and/or $\omega \chi_{c1}$). In this case it is noticed that the $X \to D_s^*\bar{D}_s^*$ decay is $p$ wave, hence in Eq. (22) $k_1$ should be replaced by $k_3$. It is found that all three fits give a fair description to the data. The parameters are determined and listed in Table 4 (w/o $\Gamma_0$) and Table 5 (with $\Gamma_0$). In the fit we find that, as already mentioned before, the coupling constant $g_2$ is vanishingly small. It suggests that the coupling of X(4260) to $DD_1(2420)$ and $\omega \chi_{c1}$ is negligible. On the other side, the

| Fit I (Padé) | Fit II (K-matrix) | Fit III (PKU) |
|--------------|-------------------|----------------|
| $\chi^2_{d.o.f}$ | $\frac{125.3}{93-13}$ | $\frac{127.8}{93-13}$ | $\frac{150.2}{93-11}$ |
| $g_0$(MeV) | $2.582 \pm 0.105$ | $2.616 \pm 0.090$ | $2.944 \pm 0.218$ |
| $g_1$ | $0.76 \times 10^{-8} \pm 0.00005$ | $0.54 \times 10^{-3} \pm 0.0003$ | $0.17 \times 10^{-6} \pm 0.0005$ |
| $g_2$ | $0.69 \times 10^{-7} \pm 0.00001$ | $0.43 \times 10^{-3} \pm 0.0003$ | $0.57 \times 10^{-7} \pm 0.0001$ |
| $M_Y$(GeV) | $4.261 \pm 0.005$ | $4.261 \pm 0.005$ | $4.269 \pm 0.007$ |

Table 4: Fit results assuming an X(4260) coupling to $D_s^*\bar{D}_s^*$ and $DD_1$; $\Gamma_0$ not included in the fit.

| Fit I (Padé) | Fit II (K-matrix) | Fit III (PKU) |
|--------------|-------------------|----------------|
| $\chi^2_{d.o.f}$ | $\frac{125.9}{93-14}$ | $\frac{127.3}{93-14}$ | $\frac{149.9}{93-12}$ |
| $g_0$(MeV) | $3.358 \pm 0.199$ | $2.836 \pm 0.085$ | $3.745 \pm 2.088$ |
| $g_1$ | $0.14 \times 10^{-6} \pm 0.001$ | $0.74 \times 10^{-6} \pm 0.0006$ | $0.14 \times 10^{-6} \pm 0.0006$ |
| $g_2$ | $0.18 \times 10^{-8} \pm 0.0002$ | $0.80 \times 10^{-7} \pm 0.0001$ | $0.13 \times 10^{-6} \pm 0.00002$ |
| $M_Y$(GeV) | $4.249 \pm 0.004$ | $4.257 \pm 0.006$ | $4.260 \pm 0.013$ |
| $\Gamma_0$(GeV) | $0.098 \pm 0.010$ | $0.020 \pm 0.018$ | $0.052 \pm 0.077$ |

Table 5: Fit results assuming an X(4260) coupling to $D_s^*\bar{D}_s^*$ and $DD_1$; $\Gamma_0$ included in the fit.

coupling $g_1$ is also negligible, but the reason is different from neglecting $g_2$. Unlike the $s$-wave, the $XD_s^*\bar{D}_s^*$ coupling is $p$ wave, and this contribution in Eq. (22) is severely suppressed.
3.2 Coupling to $\omega \chi_{c0}$ channel

Remember that the $\omega \chi_{c0}$ channel is in $s$-wave, we therefore redo the above analysis by letting channel 1 being $\omega \chi_{c0}$. 6 The result is plotted in Figure 4 and Table 6. Comparing Table 6 with Table 4 (5) we find that including coupling the $\omega \chi_{c0}$ channel significantly improves the fit quality. Meanwhile, the pole position of X(4260) is determined on sheet III of $\sqrt{s}$ plane, i.e., the

| Parameter | fit I (Padé) | fit II (K-matrix) | fit III (PKU) |
|-----------|--------------|-------------------|---------------|
| $g_0$ (MeV) | $33.416 \pm 19.828$ | $34.404 \pm 19.391$ | $34.592 \pm 14.944$ |
| $g_1$ | $0.476 \pm 0.056$ | $0.446 \pm 0.048$ | $0.507 \pm 0.060$ |
| $M_\gamma$ (GeV) | $4.274 \pm 0.007$ | $4.271 \pm 0.005$ | $4.279 \pm 0.007$ |
| $N_1 (pb^{-1})$ | $54.031 \pm 4.958$ | $55.326 \pm 4.647$ | $52.494 \pm 4.858$ |
| $N_2 (pb^{-1})$ | $19.103 \pm 1.775$ | $19.761 \pm 1.771$ | $16.909 \pm 1.672$ |
| $h_1$ | $0.0542 \pm 0.033$ | $0.107 \pm 0.051$ | $0.018 \pm 0.009$ |
| $h_2$ | $-0.554 \pm 0.496$ | $-1.077 \pm 0.520$ | $-0.085 \pm 0.037$ |
| $h_3$ | $-0.004 \pm 0.048$ | $0.0007 \pm 0.025$ | $-0.014 \pm 0.008$ |
| $c_1^{(1)}$ | $0.633 \pm 0.430$ | $0.321 \pm 0.212$ | $5.876 \pm 1.908$ |
| $c_2^{(1)}$ | $-0.271 \pm 0.282$ | $-0.155 \pm 0.110$ | $-3.341 \pm 1.155$ |
| $c_1^{(2)}$ | $-0.038 \pm 0.011$ | $-0.003 \pm 0.014$ | $0$ |
| $c_2^{(2)}$ | $0.029 \pm 0.013$ | $-0.0006 \pm 0.013$ | $0$ |

Table 6: Parameters given by fit to X(4260) data, assuming an X$\omega \chi_{c0}$ coupling $g_1$.

one seen by experiments. The results are listed in Table 7. It is remarkable to see that the (real part of the) pole is very close to (or even below) the $\omega \chi_{c0}$ threshold, and be much lower than the peak position. There is another pole, lying roughly in similar position, on sheet IV. On the other side, from Figure 4a) one finds the BABAR data peak at 4.227GeV. The discrepancy between the peak position and the pole position is due to the complicated analytic structure around the $\omega \chi_{c0}$ threshold. To further test the importance of the $\omega \chi_{c0}$X coupling $g_1$, we also provide the fit result by switching off $g_1$ in Table 8 and in Figure 5. The pole position when eliminating $g_1$ is also looked for and the result is given in Table 9.

At this stage let us firstly summarize what we have obtained: We find the coupling to channel $\omega \chi_{c0}$ very important in influencing the fit quality and the output physical picture related

6For the parameter $\Gamma_0$ and $g_2$ in Eq. (22), it is noticed that including them does not improve the fit quality and does not alter any of the major conclusions given below. Also the fit in this subsection gives vanishingly small values of $\Gamma_0$ and $g_2$. For this reason, we no longer discuss these parameter hereafter.
Figure 4: A fit to the X(4260) invariant mass spectrum by assuming that it couples to $\omega \chi_{c0}$.

to X(4260). When turning on the coupling $g_1$, one gets a large $g_0$ as well, and hence a large $\Gamma_{X \rightarrow e^+ e^-}$. That means the $\text{Br}(X \rightarrow J/\psi \pi \pi)$ is very small. As a result, poles on sheet III and sheet IV are very close to each other numerically and both are very close to the $\omega \chi_{c0}$ threshold at 4197 MeV. The picture may suggest that X(4260) be a $\omega \chi_{c0}$ molecular resonance. If one however switch off the $\omega \chi_{c0}$ channel, one gets a quite different scenario: Coupling $g_0$ becomes very small
as seen from Table 8, which corresponds to $\Gamma_{e^+e^-} \simeq 10$eV. Meanwhile X decays dominantly into $J/\psi\pi\pi$ and the pole locates at around 4250MeV. Confidentially, the $\chi^2$ fit favors the former scenario. Comparing Table 6 with Table 8, it is found that the total $\chi^2$ increases rather rapidly, which corresponds to a significance level for parameter $g_1$ being 3.6$\sigma$ (I), 4.3$\sigma$ (II) and 4.2$\sigma$ (III), respectively. This strongly suggests that X(4260) has a sizable coupling to channel $\omega \chi c_0$.

However, before jumping to a quick conclusion we notice that there is a potentially severe problem regarding to the results given in Table 6. The central value of $g_0$ given there, which characterizes the transition between X(4260) and the photon field, or $\Gamma_{X\rightarrow e^+e^-}$ according to Eq. (3), is too large to avoid the upper bound established by an analysis on the $R$ value measured by BES experiment: Ref. [24] gives $\Gamma_{X\rightarrow e^+e^-} < 420$eV at 90\% confidence level. Nevertheless the value of $g_0$ given in Table 6 contains a large error bar as well. It is actually understood well why such a phenomenon occurs. When $g_0$ being large, parameters $h_i$ have to become small to accommodate for the experimental value given in Eq. (1), as can be understood from Eq. (15). This results in a non-important $\Gamma(q^2)$ contribution to the denominator of X(4260) propagator (see Eq. (22)). As a consequence, $\chi^2$ becomes inert with respect to the change of $g_0$ when it is large enough since the effect can be counterbalanced by a change of $h_i$. The situation is clearly depicted in Figure 6 which exhibits an approximate scaling law for parameter $g_0$. This means that we can not reliably determine parameter $g_0$ at all when $g_0$ is large enough, though the value

\[ L = \tilde{g}_1 Y_{\mu\nu} \omega^{\mu\nu} \chi c_0, \]

then if taking $g_1 \simeq 0.5$ according to Table 6 we get an estimation on $\tilde{g}_1 \simeq 7.2$GeV.

| Pole Location | Fit I (Padé) | Fit II (K-matrix) | Fit III (PKU) |
|---------------|--------------|-------------------|---------------|
| sheet-III     | 4236.6-63.4i | 4237.5-58.9i      | 4236.1-68.5i  |
| sheet-IV      | 4237.0-63.0i | 4237.8-58.6i      | 4236.6-68.0i  |

Table 7: The pole location of X(4260). Assuming an $X(\omega \chi c_0)$ coupling $g_1$.  

| Parameters       | fit I (Padé) | fit II (K-matrix) | fit III (PKU) |
|------------------|--------------|-------------------|---------------|
| $\chi^2_{d.o.f}$| 121.6        | 126.2             | 152.9         |
| $g_0$(MeV)       | 2.529 ± 0.087| 2.533 ± 0.095     | 2.790 ± 0.168 |
| $M_Y$(GeV)       | 4.260 ± 0.005| 4.259 ± 0.005     | 4.264 ± 0.005 |

Table 8: Parameters given by fit to X(4260) data with $g_1$ switched off.
of $g_1$ is very stable when varying $g_0$ in Figure 6. It is important to notice from Figure 6 that there exists a large enough space for $g_0$ to maintain a (almost) minimal $\chi^2$ while being easily escappable of the BES $\Gamma_{X\rightarrow e^+e^-}$ bound given in Ref. [24]. As an example, we list the fit result in Table 10 by fixing $g_0$ at 9.62MeV which corresponds to $\Gamma_{X\rightarrow e^+e^-} \sim 211 eV$. The pole location in such a situation is listed in Table 11.
Table 9: The pole location of X(4260) with $g_1$ switched off.

|        | Fit I         | Fit II        | Fit III        |
|--------|---------------|---------------|---------------|
| sheet-II | 4250.2-51.3i | 4253.0-33.0i | 4256.6-49.1i |

Figure 6: The approximate scaling law of parameter $g_0$.

Before ending this section it maybe worthwhile to briefly comment on the $d$-wave contribution. It is controlled by parameter $h_3$ in Eq. (11). According to Table 6, $h_3$ is very small and probably be consistent with 0. The estimate is however very unstable, maybe partly due to the fact that we do not yet take into account $d$ wave final state interactions. [25] It is also worth mentioning that our analysis reveals that there ought to be non-negligible $K\bar{K} \to \pi\pi$ (and vice versa) final state interaction. This can be understood from Table 4 by comparing the $\chi^2$ difference between the PKU representation and the other two approaches.

At last we would like to point out that, we also tested the possibility whether the above fit results be distorted by possible interfering background. To do this we add a background term by
changing the X propagator:

\[
\frac{1}{M_X^2 - q^2 - i\sqrt{q^2}(g_1 k_1 + \Gamma(q^2))} \rightarrow \frac{1}{M_X^2 - q^2 - i\sqrt{q^2}(g_1 k_1 + \Gamma(q^2))} + \frac{c_1 e^{i\phi}}{M_1^2 - q^2 - iM_1 \Gamma_1} + c_2 .
\]  

(23)

Taking fit I for example, in doing this we observe that adding 6 additional parameters (\(c_2\) is complex) only reduces total \(\chi^2\) by 3. Moreover, parameter \(g_1\) only decreases very slightly. The only important change comes from \(g_0\) – its central value is now roughly 4.1MeV. Hence we conclude that the qualitative picture obtained in previous discussions of this section is rather robust.

| \(\chi^2_{d.o.f}\) | fit I (Padé) | fit II (K-matrix) | fit III (PKU) |
|----------------|-------------|-----------------|---------------|
| \(g_0\) (MeV) | 9.615       | 9.615           | 9.615         |
| \(g_1\)       | 0.435±0.048 | 0.412±0.043     | 0.450±0.048   |
| \(M_Y\) (GeV) | 4.273±0.006 | 4.270±0.005     | 4.277±0.007   |
| \(N_1\) (pb\(^{-1}\)) | 54.239±4.954 | 55.511±5.140 | 52.799±4.953 |
| \(N_2\) (pb\(^{-1}\)) | 19.147±1.807 | 19.795±1.935 | 16.951±2.053 |
| \(h_1\)    | 0.187±0.063 | 0.383±0.255     | 0.063±0.065   |
| \(h_2\)    | -2.030±0.798| -3.835±2.126    | -0.296±0.149  |
| \(h_3\)    | 0.010±0.151 | 0.002±0.0773    | -0.049±0.007  |
| \(c_1^{(1)}\) | 0.583±0.477 | 0.313±0.411     | 5.928±7.26   |
| \(c_2^{(1)}\) | -0.250±0.271 | -0.150±0.195 | -3.360±4.354 |
| \(c_1^{(2)}\) | -0.031±0.055 | -0.003±0.022   | 0            |
| \(c_2^{(2)}\) | 0.022±0.056 | -0.0006±0.021  | 0            |

Table 10: Parameters given by fit to X(4260) data, assuming an X\(\omega\)X\(\chi_c\) coupling \(g_1\) and keeping \(g_0\) fixed.
|        | Fit I       | Fit II      | Fit III     |
|--------|-------------|-------------|-------------|
| sheet-III | 4239.1-61.2i | 4239.9-57.2i | 4240.9-66.1i |
| sheet-IV  | 4243.3-56.1i | 4243.2-52.9i | 4246.5-59.0i |

Table 11: The pole location of X(4260). Assuming an $X\omega_{X_c0}$ coupling $g_1$, $g_0$ fixed at 9.62MeV. The results of this table is similar to Table 7.

## 4 Couplings $g_{X\Psi\sigma}$ and $g_{X\Psi f_0}(980)$

The analytic continuation of amplitude $\mathcal{A}$ – as a function of complex variable $s$ – to different sheets can be obtained:\(^8\)

\[
\mathcal{A}^{II} = \mathcal{A}B^{II} \equiv \mathcal{A}\left(\begin{array}{cc} 1 & -2i\rho_1 T_{11}^I \\
1+2i\rho_1 T_{11}^I & 1+2i\rho_1 T_{11}^I 
\end{array}\right),
\]

\[
\mathcal{A}^{III} = \mathcal{A}B^{III} \equiv \mathcal{A}\left(\begin{array}{cc} \frac{1+2i\rho_2 T_{12}^I}{\det S} & -\frac{2i\rho_1 T_{12}^I}{\det S} \\
-\frac{2i\rho_2 T_{12}^I}{\det S} & \frac{1+2i\rho_1 T_{12}^I}{\det S} 
\end{array}\right),
\]

\[
\mathcal{A}^{IV} = \mathcal{A}B^{IV} \equiv \mathcal{A}\left(\begin{array}{cc} 1 & 0 \\
-2i\rho_2 T_{21}^I & 1+2i\rho_2 T_{22}^I 
\end{array}\right),
\]

where $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$, and $T$ matrix expressions on different sheets are similar,

\[
T^{II} = TB^{II}, \quad T^{III} = TB^{III}, \quad T^{IV} = TB^{IV}.
\]

With these expressions the residues or coupling constants of second and third sheet poles can be extracted. In the vicinity of pole position on sheet II, one has

\[
T^{II}(\pi\pi \rightarrow \pi\pi)(s \rightarrow z_{II}) = \frac{T_{11}(z_{II})}{S_{II}'(z_{II})(s - z_{II})},
\]

\[
\mathcal{A}^{II}(X \rightarrow \Psi\pi\pi)(s \rightarrow z_{II}) = \frac{\mathcal{A}_1(z_{II})}{S_{II}'(z_{II})(s - z_{II})}
\]

and through the definition of coupling constants:

\[
T(\pi\pi \rightarrow \pi\pi)(s \rightarrow z_{R}) \equiv \frac{g_{f\pi\pi}^2}{z_{R} - s},
\]

\[
\mathcal{A}(X \rightarrow \Psi\pi\pi)(s \rightarrow z_{R}) \equiv \frac{g_{X\Psi f f\pi\pi}}{z_{R} - s},
\]

\(^8\)For more details on how to obtain these formulas one is referred to Ref. [26].
one obtains the expressions of $g_{f\pi\pi}$ and $g_{X\psi f}$ for second sheet pole $f$:\footnote{One can instead get expressions for $g_{fKK}^2$ and $g_{X\psi f} \times g_{fKK}$ and deduces the same expression for $g_{X\psi f}$, as it should.}

\begin{equation}
  g_{f\pi\pi}^2 = \frac{T_{11}(z_{III})}{S'_{11}(z_{III})}, \quad g_{X\psi f g_{f\pi\pi}} = \frac{A_1(z_{III})}{S'_{11}(z_{III})}.
\end{equation}

Denote the third sheet pole as $z_{III}$, which is the zero of $\det S$, then

\begin{equation}
  \det S(s \to z_{III}) = (\det S)'(z_{III})(z - z_{III}).
\end{equation}

Coupling constants are hence obtainable, according to Eq. (25) and Eq. (28):

\begin{equation}
  g_{f\pi\pi}^2 = \frac{S_{22}(z_{III})}{2i\rho_1 \det S'(z_{III})}, \quad g_{fKK}^2 = \frac{S_{11}(z_{III})}{2i\rho_2 \det S'(z_{III})}, \quad g_{f\pi\pi} g_{fKK} = \frac{-T_{12}(z_{III})}{\det S'(z_{III})}.
\end{equation}

From Eq. (24) one further gets

\begin{align}
  g_{X\psi f} \times g_{f\pi\pi} &= -\frac{A_1(z_{III})S_{22}(z_{III})}{(\det S)'(z_{III})} + \frac{A_2(z_{III})2i\rho_2(z_{III})T_{12}(z_{III})}{(\det S)'(z_{III})}, \\
  g_{X\psi f} \times g_{fKK} &= \frac{A_1(z_{III})2i\rho_1(z_{III})T_{12}(z_{III})}{(\det S)'(z_{III})} - \frac{A_2(z_{III})S_{11}(z_{III})}{(\det S)'(z_{III})},
\end{align}

or more easily, by making use of Eq. (18),

\begin{equation}
  g_{X\psi f} = A_{1\text{tree}}(z_{III})\alpha_1(z_{III})g_{f\pi\pi} + A_{2\text{tree}}(z_{III})\alpha_2(z_{III})g_{fKK}.
\end{equation}

These expressions enable us to get the same value of $g_{X\psi f}$, for the third sheet pole $f$, as expected.

From the method as mentioned above, we can predict the $X(4260)$-$J/\psi$-$f_0(600)$ coupling and the $X(4260)$-$J/\Psi$-$f_0(980)$ coupling. However, owing to the approximate scaling law discussed in section 3.2, the magnitude of $g_{X\psi f}$ is not physical and only the ratio $g_{X\psi f}/g_{X\psi f_0(980)}$ makes sense in the scaling region. Table 12 provides an example of such calculation. It is seen however that this ratio is not quite stable and is rather sensitive to different fit method. Nevertheless one expects that one can evaluate the order of magnitude of the ratio through Table 12. For example both Fit II and Fit III give a value of $|g_{X\psi f_0}|$ a few times larger than $|g_{X\psi f_0(980)}^2|$, hence the $X(4260) \to J/\psi\pi\pi$ decay mainly goes through the $\sigma$ or $f_0(600)$ intermediate state. This agrees qualitatively with the estimation [4],

\begin{equation}
  \frac{Br(X(4260) \to J/\psi f_0(980), f_0(980) \to \pi^+\pi^-)}{Br(X(4260) \to J/\psi \pi^+\pi^-)} = 0.17 \pm 0.13.
\end{equation}
Table 12: The coupling of X(4260) to \( f_0(980) \) and \( \sigma \) (or \( f_0(600) \)). The last three columns provide the results by taking \( g_0 = 9.62 \).

5 An Estimation of Wave Function Renormalization Constant of X(4260)

Coupling constant \( g_0 \) is related to the two point proper vertex describing \( \gamma - X \) transition. After considering the interaction between X, \( \chi_{c0} \) and \( \omega \), \( g_0 \) gets renormalized,

\[
g^R_0 = Z_X^{1/2} g^B_0 ,
\]

where \( g^B_0 \) is the value of \( g_0 \) at tree level, which may be recognized as the value obtained from simple potential model calculation without considering the continuum mixing, and \( g^R_0 \) is the one measured experimentally in the leptonic decay process. On the other side the wave function renormalization constant can be expressed as

\[
Z_X = \frac{1}{1 - \text{Re} \Sigma(\mu^2)} ,
\]

where \( \Sigma \) is the self-energy, \( \mu \) is the physical (pole) mass. According to the approximation made in previous sections,

\[
\Sigma(q^2) \simeq -i \sqrt{q^2} g_1 k_1 .
\]

More accurately it is recasted in an expression using dispersion relation,

\[
\Sigma(\mu^2) = \frac{1}{\pi} \int_{(M_X + m_\omega)^2}^{\infty} \frac{\text{Im} \Sigma(q^2)}{q^2 - \mu^2} dq^2
\]
from which one further deduces

$$\Sigma'(\mu^2) = \frac{1}{\pi} \int_{(M_\chi + m_\omega)^2}^{\infty} \frac{\text{Im} \Sigma(q'^2)}{(q'^2 - \mu^2)^2} dq'^2.$$  \hspace{1cm} (39)$$

The above integral is divergent in relativistic theory and hence not calculable. However, in the non-relativistic approximation, it becomes calculable for $s$ wave interaction:

$$\text{Im} \Sigma(q^2) \simeq - \sqrt{q^2} g_1 k_1 \rightarrow - \frac{g_1}{2} \sqrt{4 M_\chi m_\omega \sqrt{(q^2 - (M_\chi + m_\omega)^2)},}$$  \hspace{1cm} (40)$$

and bringing it back to Eq. (39) one gets

$$Z_X^{-1} = 1 - \text{Re} \Sigma'(\mu^2) = 1 + \frac{g_1}{2} \text{Re} \left[ \frac{\sqrt{M_\chi m_\omega}}{\sqrt{M_{th}^2 - \mu^2}} \right],$$  \hspace{1cm} (41)$$

where $M_{th}^2 = (M_\chi + m_\omega)^2$. If $X$ is a bound state then

$$Z_X = \frac{1}{1 - \text{Re} \Sigma'(\mu^2)} \simeq 1 + \frac{1}{2 \sqrt{2} M_R \epsilon},$$  \hspace{1cm} (42)$$

where we have let $\mu = M_{th} - \epsilon$ and $\epsilon$ is the binding energy, $m_R = \frac{M_\chi m_\omega}{M_\chi + m_\omega} = 637 \text{MeV}$. If $X$ is a resonance then when $\mu \simeq M_{th} - i \Gamma/2$ one gets instead of Eq. (42) the following formula,

$$Z_X = \frac{1}{1 - \text{Re} \Sigma'(\mu^2)} \simeq 1 + \frac{1}{2 \sqrt{2} \Gamma},$$  \hspace{1cm} (43)$$

If for example taking $g_1 \simeq 0.435$ and the third sheet pole of Fit I from Tables 10, 11, it is estimated $Z_X(4260) \simeq 0.74$ using Eq. (43), and $Z_X \simeq 0.82$ using Eq. (41). The potential model calculation of $g_0$ does not take into account the hadron loop effect and hence only corresponds to a ‘tree level’ calculation. The $\omega \chi_{c0}$ loop correction leads to that the ‘tree level’ value of $\Gamma_{e^+e^-}$ be reduced by a factor $Z_X$.

Finally we comment a few words on X(3872) before ending the present discussion. Considering the first row of Table 3 in Ref. [27] for example. From which one gets $g_1 = 0.0937$, and here $m_R \simeq 966.5 \text{MeV}, \Gamma \simeq 0.4 \text{MeV}$. Bring them back to Eq. (43) one gets $Z_X(3872) \simeq 0.38$, which is rather close to the one obtained in Ref. [28]. In obtaining this number other contributions like $J/\psi \rho$ and $J/\psi \omega$ are not yet considered, hence one concludes that the mixing effects from continuum in X(3872) is stronger than in X(4260). Axiomatic quantum field theory indicates that $0 \leq Z \leq 1$, and $Z = 0$ for a pure composite particle. In this sense, one may conclude that X(4260) is more elementary than X(3872).
6 Conclusions

To conclude, in the analysis made in this paper we find a sizable $X\omega \chi_{c0}$ coupling. Other couplings to nearby thresholds are found vanishingly small with no significance importance. For example, there is no evidence for a $g_{X D_s^* D_s^*}$. This does not support the proposal of $X(4260)$ being a $D_s^* D_s^*$ molecule. We also find no evidence to include an $XDD_1$ coupling. This is a rather negative observation towards the hybrid assignment of $X(4260)$, since the latter indicates a large hybrid coupling to a spin 0 and spin 1 meson. [29] The large coupling to $\chi_{c0} \omega$ is also of spin 0, 1 type, but a $c\bar{c}g$ hybrid should couple much stronger to a charmed meson pair than to a $\bar{c}c$ state plus a light meson, since the latter needs a color rearrangement. Also, the observation made in this paper seems to exclude the $D\bar{D}_1 (\omega \chi_{c1})$ assignment of $X(4260)$. Finally the $\pi\pi$ spectrum is apparently and overwhelmingly in isoscalar $s$ wave, this certainly excludes the proposal that $X$ be a $\chi_{c0}\rho$ molecule.

Of course once the $g_1$ coupling is switched off the $X(4260)$ behaves very differently: It has only a tiny electron decay width but with a dominant $J/\psi \pi\pi$ decay mode. Nevertheless, this picture is disfavored by the $\chi^2$ analysis. After all, the numerical study of this paper suggests that the $X(4260)$ state contains a sizable $\omega \chi_{c0}$ component, and one may attempt to describe $X(4260)$ as an $\omega \chi_{c0}$ molecule, or more appropriately, a molecular resonance. Nevertheless, this seemingly natural conclusion is unlikely for two reasons. Firstly the coupling $g_1$ is still far from being large enough to regard $X(4260)$ as a pure molecule. The second evidence against the molecule interpretation comes from the value of $g_0$ given in Table 6 and Figure 6, where it is seen that the fit prefers a larger $g_0$, even though we can not actually determine $g_0$ reliably. A larger $g_0$ easily leads to a $\Gamma_{e^+e^-}$ of a few hundreds eV, which is hard to understand in a pure molecule picture. It is actually highly plausible that $X(4260)$ contains a large portion of $\bar{c}c$ component. The preference for a large $g_0$, in our understanding, seems to support the $\bar{c}c$ origin of $X(4260)$. In Ref. [30] a screening potential is used to calculate the charmonium spectrum and it is found that the mass of $4^3S_1$ state is around 4273MeV with a $\Gamma_{4^3S_1\rightarrow e^+e^-} \simeq 970$eV. The latter is dangerously large considering the BES bound [24]. Taking into account S–D mixing effect may roughly reduce the above leptonic width by half [30]. Here it is pointed out that the renormalization from $\omega \chi_{c0}$ continuum can further reduce $\Gamma_{e^+e^-}$ of the bare $4^3S_1$ state roughly by a factor 0.8. This is helpful in allowing $X(4260)$ as a candidate $\bar{c}c$ state to escape from the constraint set up by BES experiment [24]. Furthermore, a mixture with $\omega \chi_{c0}$ continuum is certainly helpful in reducing the decay rates into open charm channels as well.
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