Prediction of peak response values of structures with and without TMD subjected to random pedestrian flows

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Abstract. In civil engineering and architecture, the availability of high strength materials and advanced calculation techniques enables the construction of slender footbridges, generally highly sensitive to human-induced excitation. Due to the inherent random character of the human-induced walking load, variability on the pedestrian characteristics must be considered in the response simulation. To assess the vibration serviceability of the footbridge, the statistics of the stochastic dynamic response are evaluated by considering the instantaneous peak responses in a time range. Therefore, a large number of time windows are needed to calculate the mean value and standard deviation of the instantaneous peak values. An alternative method to evaluate the statistics is based on the standard deviation of the response and a characteristic frequency as proposed in wind engineering applications. In this paper, the accuracy of this method is evaluated for human-induced vibrations. The methods are first compared for a group of pedestrians crossing a lightly damped footbridge. Small differences of the instantaneous peak value were found by the method using second order statistics. Afterwards, a TMD tuned to reduce the peak acceleration to a comfort value, was added to the structure. The comparison between both methods is made and the accuracy is verified. It is found that the TMD parameters are tuned sufficiently and good agreements between the two methods are found for the estimation of the instantaneous peak response for a strongly damped structure.

1. Introduction
The recently constructed footbridges in Charleroi (2014) and Avelgem (2015) are illustrative for the trend to design slender footbridges with large spans, enabled by the use of high strength materials and advanced calculation techniques. Slender structures are often characterised by a low stiffness-mass ratio and low natural frequency, making these structures sensitive to human-induced excitation. The typically low modal damping further increases this sensitivity, potentially leading to problems of vibration serviceability [1, 2].

A response prediction often is performed to evaluate the dynamic performance of a footbridge in design stage. Simplified procedures are proposed by guidelines (e.g. Sétara [3], HiVoSS [4]) in which assumptions and simplifications are made for the description of both the dynamic behaviour of the construction and the walking load. The evaluation consists of calculating an acceleration response level which must be measured against comfort criteria for a considered
pedestrian density. A more detailed evaluation method is proposed in [5] where for the
description of the human-induced walking load a more realistic model is used and for the dynamic
behaviour of the footbridge the contribution of multiple modes is considered.

If the calculated vibration levels exceed the required comfort criterion, a Tuned Mass Damper
(TMD) can be added to the structure as a passive vibration control device. The TMD serves as
an energy absorber and is characterised by its mass, stiffness and damping parameter. Optimal
values of these TMD parameters can be derived depending on the type of response (displacement,
acceleration...), the type of loading and the required level of vibration reduction and need to be
tuned to the modal parameters of the main structure.

The parameters of the TMD are often chosen assuming harmonic loading following Den
Hartog [6], disregarding the specific character of the human-induced loads. To account for the
inherent variability of the pedestrian traffic, a frequency approach is often suggested in literature.
Statistical models of the human-induced walking load were derived both in laboratory [7] and
free field [8, 9]. A power spectral density approach of the load is used to describe the variabilities
of the walking load [7, 10]. Based on the spectral density of the load, simple expressions have
been derived to obtain the magnitude of the acceleration response for resonant and non-resonant
walking excitation to give an indication whether a TMD is recommended or not [10]. In [11],
based on the estimation of the spectral load model, a prediction of the acceleration response of
the structure under random pedestrian traffic is made. This approach was followed by Tubino
et al. [12] to tune the parameters of a TMD. The objective was to maximise a scalar measure for
the effectiveness of the TMD considering the standard deviation of the response of the structure
with and without TMD.

In this paper, a detailed simulation of the pedestrian traffic is performed to obtain a realistic
time history of the modal load of a group of pedestrians crossing the footbridge. Based on a
statistical load model, the spectral density function of the load is estimated and used to optimise
the parameters of the TMD. The objective is to minimise the mass of the TMD, taken as
a measure of the cost of the device, while satisfying vibration serviceability requirements. For
the TMD design, variability in the pedestrian walking load is considered.

The structure of the paper is as follows. First the case study of the footbridge is introduced.
Second, the derivation of the walking load model is discussed and a method to predict the
peak acceleration based on the second order statistics of the response is given. Afterwards,
the proposed method is tested in an assessment of the structure’s vibration serviceability for
an assumed pedestrian density on the lightly damped structure. The paper continues with
formulating the choice of the TMD parameters as an optimisation problem to tune the TMD
and the methods to predict the peak acceleration are verified for the strongly damped structure.

2. The reference footbridge
The modal parameters of the reference footbridge have been selected based on data from a large
number of case studies [13]. The considered reference structure is a single span bridge with
length of 50 m and a width of 3 m. Its first mode is a bending mode with a sinusoidal shape
and a natural frequency of 2.00 Hz. A modal damping of 0.4% is assumed typically used for
the design of steel footbridges [3, 4]. The modal mass of the first mode $m_1$ is 25 ton. In the
response calculation, only the contribution of the first mode is considered.

3. Methodology
This section discusses the methodology adopted in this paper. The single pedestrian walking
force is discussed first followed by a description of the crowd-induced walking force model.
Afterwards, a method to calculate the mean instantaneous peak response is given.
3.1. Single pedestrian

The load of a single pedestrian $k$ can be described by the single-step model of Li et al. [14]. The vertical force due to a single footstep is calculated as:

$$\frac{F_1(t)}{G} = \sum_{n=1}^{5} A_n(f_s) \sin\left(\frac{\pi n}{T_c} t\right) \quad \text{for} \quad 0 \leq t \leq T_c$$

with $F_1(t)$ the force [N] of one single footstep at time $t$, $G$ the weight of the single pedestrian [N], $A_n$ the Fourier coefficient of the $n^{th}$ harmonic component of the generalised load model, $f_s$ the step frequency [Hz] and $T_c$ the time duration of contact between foot and ground. Following Li, the contact time is calculated as $T_c = T_s / 0.76$ with $T_s = 1/f_s$. A step-by-step walking force is obtained by considering a continuous sequence of forces induced by left and right foot, accounting for the contact time. In Figure 1, the step-by-step model is illustrated.

![Figure 1](image.png)

**Figure 1.** Single-step force time history for $f_s = 2$ Hz following Li et al. [14]. Step-by-step model with left foot (–) and right foot (––).

The modal projection of the force $\phi^T F_1(t)$ is given in Figure 2(a) for both a single pedestrian walking at the natural frequency of the footbridge and one walking at a random step frequency crossing the reference footbridge. $\phi \in \mathbb{R}^{n_{DOF} \times n_{DOF}}$ is the matrix containing the mass-normalised mode shapes of the structure and $F_1(t) \in \mathbb{R}^{n_{DOF}}$ the force of the single pedestrian. The walking speed of the pedestrian is in both cases assumed to be the same (1.5 m/s). The corresponding acceleration signals at midspan of the bridge are given in Figure 2(b).
Figure 2. Single pedestrian with step frequency equals to natural frequency of the structure \( \left( \frac{fs}{f_1} = 1 \right) \) and with arbitrary step frequency \( \left( \frac{fs}{f_1} = 0.74 \right) \) with \( T_{\text{cross}} \) the time needed by the pedestrian to cross the bridge and \( T_{\text{resp}} \) the total time taken into account for the response calculation: time history of (a) modal projection of the force, (b) corresponding acceleration response.

3.2. Crowd-induced force

The following method is proposed to upscale the load model for a single pedestrian to a load model for the crowd. In the first step, a large number of single pedestrians is created. Each pedestrian is characterised by his trajectory (straight line on the considered footbridge going from left to right), step frequency \( fs \), weight \( G \) and walking speed \( vs \). The lateral positions of the different pedestrians are not considered here because of the symmetry of the sinusoidal mode shape. The position on the footbridge at each time step is calculated based on given walking speed.

For a given pedestrian density, the arrival rate is computed and the arrival times of the individual pedestrians are sampled from a Poisson distribution. In Figure 3, the spatial distribution of the single step forces of the random walking pedestrians is visualised at an arbitrary time point \( t_i \) of the simulation during an interval \( [t_i, t_i + T_{c,k}] \) with \( T_{c,k} \) the contact time of the \( k \)-th pedestrian.
Figure 3. Spatial distribution of single step forces of randomly distributed pedestrians with different characteristics at arbitrary time $t_i$ of the simulation during the interval $[t_i, t_i + T_{c,k}]$ with $T_{c,k}$ the contact time of the $k$-th pedestrian.

The modal projection of the total load induced by the crowd $\phi^T F(t)$ is calculated as the superposition of the modal load signals of the individual pedestrians in time. Afterwards, the acceleration response is calculated. Alternatively, the total acceleration response could also be found directly by superposition of the acceleration responses of the single pedestrians. A sufficiently large time span here must be considered to account for the free decay in the response of the structure when a pedestrian leaves the footbridge.

3.3. Peak acceleration evaluation

The presented method allows for a detailed description of the walking force due to a group of pedestrians. The assessment of the vibration comfort consists of testing a relevant statistic of the response value to a comfort level. Here, the mean value and standard deviation of the instantaneous peak acceleration in a predefined time range are evaluated. Two methods to predict this statistics are compared.

(I) The time history of the acceleration response is subdivided in relevant time windows with length $T$ and time shift $\Delta$ in between to exclude correlation. The instantaneous peak values are found as the maximum absolute acceleration in each time window. The mean value and standard deviation of the instantaneous maxima is calculated afterwards.

(II) Davenport in [15] derives an expression to find the distribution of the maximum instantaneous value in a predefined range $T$ of a stationary random function based on the Vanmarcke distribution [16] which can be used for Gaussian distributed random signals. In [17], a cumulative probability distribution of the maxima of a random function is found. Then, the estimation of the mean value and standard deviation of the instantaneous maxima of the acceleration is given as follows [15, 17, 11]:

\[
\mu_{\ddot{u}_{\text{max}}} = \left( \sqrt{2 \ln(2\nu_e T)} + \frac{0.5772}{\sqrt{2 \ln(2\nu_e T)}} \right) \sigma_{\ddot{u}}
\]

\[
\sigma_{\ddot{u}_{\text{max}}} = \left( \frac{1.2}{\sqrt{2 \ln(2\nu_e T)}} - \frac{5.4}{13 + (2 \ln(2\nu_e T))^{3/2}} \right) \sigma_{\ddot{u}}
\]

with $\nu_e$ the modified expected frequency and $\sigma_{\ddot{u}}$ the standard deviation of the accelerations as found from the acceleration response or based on the classical random vibrations.
theorem by integration of the acceleration spectral density function calculated from the input power spectral density of the modal load.

The modified expected frequency \( \nu_e \) can be derived from following empirical expression [17]:

\[
\nu_e = (1.63q^{0.45} - 0.38)\nu
\]

with \( \nu \) the expected frequency of the output signal in Hz calculated as \( \nu = \sqrt{\frac{\lambda_2}{\lambda_0}} \) with \( \lambda_i = \int_0^{\infty} f^4 G_{uu}(f)df \) and \( q \) a spectral bandwidth parameter. \( \nu \) thus can be interpreted as the frequency at which the most of the energy is concentrated. The spectral bandwidth parameter \( q \) can be written as:

\[
q = \sqrt{1 - \frac{\lambda_1^2}{\lambda_0 \lambda_2}}
\]

For lowly damped structures, the parameter \( q \) can be approximated as follows [18]:

\[
q \approx 2\sqrt{\frac{\xi_j}{\pi}}
\]

with \( \xi_j \) the modal damping for mode \( j \) of the structure. Valid values of \( q \) for Equation (4) are in the range \( 0.1 \leq q \leq 1 \) as determined in [17].

4. Response prediction

The methodology outlined in Section 3 is now applied to predict the peak acceleration in a relevant time period due to pedestrian traffic for the reference footbridge. The example is given considering a pedestrian traffic with a density \( d \) of 0.5 pers./m². It is known that for sparse pedestrian densities (i.e. \( d \leq 1 \) pers./m²), the distribution of step frequencies can be approximated by a Gaussian distribution around the natural frequency of the footbridge with a standard deviation of 0.175 Hz (\( f_s \sim N(f_1,0.175) \)) assuming worst case conditions [5]. The walking speed of all pedestrians is 1.5 m/s. Only the contribution from the first mode is taken into account for the response prediction. The results do not depend on the randomly chosen lateral positions of the pedestrians as only the contribution of the first (symmetric) mode is accounted for in the calculation. A continuous pedestrian flow was simulated to obtain a realistic and representative signal. Both the acceleration response of the structure (at midpoint) and the modal force induced by the flow are retained for comparison and prediction of the peak response according to the two methods.

In Figure 4(a) the time history of the modal force of the simulation is given for the considered pedestrian density. The modal force signal was processed by applying a high-pass filter to exclude static components. The amplitudes of the shape look different for the filtered signal due to the detrend effect (see zoom window). At the The corresponding acceleration signal at midspan of the bridge is given in Figure 4(b). The time vector is a function of the time needed by a pedestrian to cross the bridge \( T_{\text{cross}} \). It was also checked that the effect of the filtering does not influence the time history of the acceleration response.

From Figure 4(b) it follows that rather high acceleration levels up to 6 m/s² are predicted. Instead of retaining the maximum absolute acceleration response, the second order statistics of the peak acceleration are derived according to the methods proposed in Section 3.3. Therefore, multiple time periods are considered. In Figure 4(b), the different time windows with size \( T \) as well as the separation \( \Delta \) which has been considered to obtain independent samples of the peak values are given. From each window the peak acceleration is retained. The mean value and standard deviation of the instantaneous peak value is calculated afterwards. The standard error of the mean estimation \( SE_{\mu} = \sigma_{umax}/\sqrt{N} \) is also given with \( N \) the number of intervals.
Figure 4. Pedestrian traffic of 0.5 pers./m²: (a) Modal projection of the time history of force $F(t)$, including static load contribution (light) and high-pass filtered signal (dark), (b) Corresponding acceleration response at midspan of the bridge with indication of time period $T$.

For the peak estimation according to method (II), the spectral density function of the modal load is estimated using the Matlab [19] function `pwelch`. Figure 5 shows the spectral density function of the modal load. Here, the filtered signal of the modal load must be used to exclude the static components. The spectral density function of the acceleration response is found by multiplying the spectral density function of the modal load from Figure 5 by the acceleration transfer function. Linear interpolated values are obtained for a different discretisation of the frequency vector. The standard deviation of the acceleration response is subsequently used to estimate the mean value $\mu_{\ddot{u}_{\text{max}}}$ and the standard deviation $\sigma_{\ddot{u}_{\text{max}}}$ of the peak acceleration according to Equation (2) and (3) respectively. This value is compared to the respective values, obtained directly from the acceleration signal by evaluating the peak values in the different time windows.

Figure 5. Power spectral density of the modal load $G_{FF}$.

For the presented case, a time shift of $\Delta = 40$ s and a relevant window $T$ of size 1, 2, 5 and 10 times $T_{\text{cross}}$ is proposed. Table 1 shows that the mean instantaneous peak acceleration is slightly
overestimated by the statistical procedure which relies on the spectral density function (method (II)). Both the absolute values of the mean instantaneous values according to both methods and the mutual relative difference $\Delta_{\text{rel}}$ are given. A positive value of $\Delta_{\text{rel}}$ denotes an overestimation by the second method. The difference becomes smaller for a larger time window for which the relevance increases to evaluate the instantaneous peak response. For the estimation of the standard deviation of the instantaneous peak values, a larger discrepancy is found between both methods. Here, an underestimation of the standard deviation is found for method (II). Since the absolute value of the standard error of the mean can not explain the difference between the two methods, a reason for both the small overestimation of the mean and the underestimation of the standard deviation is that the condition for the applicable range for the bandwidth parameter $q$ is not satisfied. The very narrow acceleration spectrum and the low modal damping result in a small value just below the lower limit. Despite the small overestimation of the mean, it is concluded that the method enables to approximate the mean value of the instantaneous peak response per time window.

**Table 1.** Instantaneous peak acceleration [m/s$^2$] for structure without TMD for different time windows obtained (I) as the statistical moment of the maximum values for different time windows and (II) estimated based on the standard deviation of the acceleration response using Equations (2) and (3).

| $T$  | $[\times T_{\text{cross}}]$ | 1    | 2    | 5    | 10   |
|------|-----------------|------|------|------|------|
| (I) $\mu_{\text{u max}}$ ($SE_{\mu}$) [m/s$^2$] | 3.31 (0.04) | 3.76 (0.05) | 4.37 (0.06) | 4.84 (0.07) |
| (II) $\mu_{\text{u max}}$ [m/s$^2$] | 3.98 | 4.37 | 4.83 | 5.16 |
| $\Delta_{\text{rel},\mu}$ [%] | 20.34 | 16.07 | 10.64 | 6.74 |
| (I) $\sigma_{\text{u max}}$ [m/s$^2$] | 0.95 | 0.93 | 0.86 | 0.74 |
| (II) $\sigma_{\text{u max}}$ [m/s$^2$] | 0.75 | 0.69 | 0.62 | 0.57 |
| $\Delta_{\text{rel},\sigma}$ [%] | $-20.93$ | $-26.34$ | $-27.94$ | $-22.34$ |

**5. Response control**

**5.1. TMD design**

The predicted acceleration levels (Table 1) exceed the minimal level of 1 m/s$^2$ for vibration comfort [3, 4] so the installation of a TMD is recommended. The location of the TMD is at the midpoint of the footbridge where the maximum modal deformation of the first mode occurs. A schematic view of the TMD installation is given in Figure 6.

![Figure 6](image)

**Figure 6.** Schematic view of reference footbridge equipped with TMD at midpoint.

For the design of the TMD, the power spectral density of the acceleration response is recomputed based on the previously derived spectral density function of the modal load and the frequency response function of the structure equipped with TMD. The design of the TMD is formulated as an optimisation problem where the cost function is the total cost of the TMD,
which is assumed to depend linearly on the TMD mass \( m_{\text{TMD}} \) as an indication of the size of the TMD. The decision variables of the problem are the TMD mass \( m_{\text{TMD}} \), stiffness \( k_{\text{TMD}} \) and damping \( c_{\text{TMD}} \). The limit value \( \ddot{u}_{\text{comfort}} \) of 1 m/s\(^2\) is imposed as a design constraint for the instantaneous peak acceleration. The constraint is defined as limiting the probability that the instantaneous peak value exceed the threshold value. Therefore, the relevant statistic is defined by \( \mu_{\dot{u}_{\text{max}}}(T) + n\sigma_{\dot{u}_{\text{max}}}(T) \). The constraint must be satisfied for the considered time window \( T = 10 \times T_{\text{cross}} \) as was proposed in [11] as evaluation interval for a similar structure. An upper and lower limit for the stiffness \( k_{\text{TMD}} \) and damping \( c_{\text{TMD}} \) of the TMD is included to ensure buildability. This leads to the following formulation of the optimisation problem:

\[
\begin{align*}
\text{minimise} & \quad m_{\text{TMD}} \\
\text{subject to} & \quad \mu_{\ddot{u}_{\text{max}}}(T) + n\sigma_{\ddot{u}_{\text{max}}}(T) \leq \dddot{u}_{\text{comfort}} \\
& \quad k_{\text{min}} \leq k_{\text{TMD}} \leq k_{\text{max}} \\
& \quad c_{\text{min}} \leq c_{\text{TMD}} \leq c_{\text{max}}
\end{align*}
\]

with the limit values of the constraints summarised in Table 2 and with \( n = 2 \). The optimisation problem was solved using an interior-point algorithm as implemented in the Matlab function \textit{fmincon}. For the constraint calculation, the exact formula for \( q \) (Equation (5)) now is used because the conditions for using the approximating formula (6) are no longer valid as the total damping has increased due to the TMD effect. \( \mu_{\ddot{u}_{\text{max}}}(T) \) and \( \sigma_{\ddot{u}_{\text{max}}}(T) \) are calculated according to Equations (2) and (3).

**Table 2.** Constraints TMD optimisation problem.

| \( \ddot{u}_{\text{comfort}} \) | 1.00 m/s\(^2\) |
|---|---|
| \( k_{\text{TMD}} \) | \([0 - 10^6]\) N/m |
| \( c_{\text{TMD}} \) | \([0 - 10^5]\) Ns/m |

The solution of the optimisation problem (7) is given in Table 3. The absolute values are given as well as the dimensionless TMD mass ratio \( \mu_{\text{TMD}} = \frac{m_{\text{TMD}}}{m_1} \), frequency ratio \( \rho_{\text{TMD}} = \frac{f_{\text{TMD}}}{f_1} \) and damping ratio \( \xi_{\text{TMD}} = \frac{c_{\text{TMD}}}{2\sqrt{m_{\text{TMD}}k_{\text{TMD}}}} \). The TMD parameters are tuned such that the constraints are satisfied at minimum cost, i.e. with a minimal value of the TMD mass.

**Table 3.** TMD parameters as solution of the optimisation problem (7).

| \( m_{\text{TMD}} \) | 1526 kg |
|---|---|
| \( \mu_{\text{TMD}} \) | 6.10 % |
| \( k_{\text{TMD}} \) | 247639 N/m |
| \( c_{\text{TMD}} \) | 3426 Ns/m |
| \( \rho_{\text{TMD}} \) | 1.01 |
| \( \xi_{\text{TMD}} \) | 8.81 % |

5.2. Verification

To verify the correctness of the followed procedure for the strongly damped structure (equipped with TMD), a new simulation is performed. The previously used modal load signal is retained as it is not influenced by the presence of the TMD as human-structure interaction is not considered. However, a second response calculation is performed to allow comparison with the acceleration...
response as found by summing the response of the single pedestrians on the structure with TMD. The acceleration response for the structure equipped with TMD is given in Figure 7. The instantaneous peak acceleration is evaluated in the two ways (directly and using Equations (2) and (3)). The comparison of the mean $\mu_{u_{\text{max}}}(T)$, standard deviation $\sigma_{u_{\text{max}}}(T)$ and relevant statistic $\mu_{u_{\text{max}}}(T) + 2\sigma_{u_{\text{max}}}(T)$ according to both methods is given in Table 4 with their relative difference. The TMD performs well since it reduces the peak acceleration $\mu_{u_{\text{max}}}(T) + 2\sigma_{u_{\text{max}}}(T)$ to approximately $1\,\text{m/s}^2$ as calculated by the total acceleration signal (method (I)). For the time window $T = 10 \times T_{\text{cross}}$, according to method (II) the value of $u_{\text{comfort}}$ is reached exactly because this method was used in the optimisation routine for tuning the TMD parameters. The advantage of the latter method is the fast response evaluation by calculating the acceleration spectral density function using the transfer function of the structure equipped with TMD. This means that for the considered time window, the design constraint is active. Good agreements are found according to both methods for the estimation of the mean and standard deviation of the instantaneous peak values. In comparison with the lightly damped structure, the relative differences between the methods strongly decreased since the value of $q$ increased to the applicable range. The discrepancy between the mean values of both methods is acceptable according to the standard error of the mean estimation.

Figure 7. Acceleration response at midspan of the bridge equipped with TMD with parameters according to Table 3 for pedestrian traffic of 0.5 pers./m$^2$. 
Table 4. Instantaneous peak acceleration \([\text{m/s}^2]\) for structure with TMD for different time windows obtained (I) as the statistical moment of the maximum values for different time windows and (II) estimated based on the standard deviation of the acceleration response using Equations (2) and (3).

| \(T\) \(\times T_{\text{cross}}\) | 1 | 2 | 5 | 10 |
|---|---|---|---|---|
| (I) \(\mu_{\ddot{u}_{\text{max}}} (SE_{\mu})\) [m/s²] | 0.69 (0.01) | 0.75 (0.01) | 0.82 (0.01) | 0.87 (0.01) |
| (II) \(\mu_{\ddot{u}_{\text{max}}}\) [m/s²] | 0.68 | 0.73 | 0.80 | 0.84 |
| \(\Delta_{\text{rel,}\mu}\) [%] | -1.10 | -2.67 | -2.82 | -3.09 |
| (I) \(\sigma_{\ddot{u}_{\text{max}}}\) [m/s²] | 0.11 | 0.10 | 0.09 | 0.08 |
| (II) \(\sigma_{\ddot{u}_{\text{max}}}\) [m/s²] | 0.10 | 0.09 | 0.08 | 0.08 |
| \(\Delta_{\text{rel,}\sigma}\) [%] | -7.80 | -13.13 | -3.16 | -6.33 |
| (I) \(\mu_{\ddot{u}_{\text{max}}} + 2\sigma_{\ddot{u}_{\text{max}}}\) [m/s²] | 0.90 | 0.96 | 0.99 | 1.03 |
| (II) \(\mu_{\ddot{u}_{\text{max}}} + 2\sigma_{\ddot{u}_{\text{max}}}\) [m/s²] | 0.88 | 0.91 | 0.96 | 1.00 |
| \(\Delta_{\text{rel,}\mu + 2\sigma}\) [%] | -2.69 | -4.95 | -2.88 | -3.62 |

6. Conclusions

Due to the inherent random character of the human-induced walking load, variability on the pedestrian characteristics must be considered. Accounting for a variability in the step frequency around the natural frequency of the structure, the simulation of a continuous pedestrian traffic is performed for a chosen pedestrian density. The instantaneous peak values were opted here as the statistic to evaluate the acceleration response.

Two methods to predict the instantaneous peak acceleration are studied. In a first method, the mean value and standard deviation of the peak acceleration is obtained directly from the acceleration signal by evaluating the peak values in the different time windows. The second method estimates the moments of the instantaneous peak acceleration based on second order statistics of the acceleration response.

The comparison of the both methods was first investigated for a lightly damped footbridge. A continuous pedestrian flow was simulated to obtain both the acceleration response at midpoint of the bridge and the modal force time history. A small overestimation of the mean and an underestimation of the standard deviation of the instantaneous peak values was found by the method using second order statistics. In a second application, a TMD was added to the structure to reduce the vibration levels to a predefined comfort level. The parameters of the TMD were tuned such that the instantaneous peak acceleration is reduced to the comfort value. Again, the comparison between both methods is made and their accuracy is verified. It is found that the TMD parameters are tuned sufficiently and good agreements are found between both methods to predict the instantaneous peak values of the strongly damped structure.

In next steps, the method will be further investigated considering the effect of multiple modes and can be extended by accounting for multiple traffic classes. The effect of uncertainties on the modal parameters of the structure will be included in the TMD tuning to obtain a robust design accounting for uncertainties in the modal parameters and by considering variability in the load.

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