Black hole quantum tunnelling and black hole entropy correction

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Parikh-Wilczek tunnelling framework, which treats Hawking radiation as a tunnelling process, is investigated again. As the first order correction, the log-corrected entropy-area relation naturally emerges in the tunnelling picture if we consider the emission of a spherical shell. The second order correction of the emission rate for the Schwarzschild black hole is calculated too. In this level, the result is still in agreement with the unitary theory, however, the entropy of the black hole will contain three parts: the usual Bekenstein-Hawking entropy, the logarithmic term and the inverse area term. In our results the coefficient of the logarithmic term is $-1$. Apart from a coefficient, Our correction to the black hole entropy is consistent with that of loop quantum gravity.

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I. INTRODUCTION

In 2000, Parikh and Wilczek proposed an approach to calculate the emission rate at which particles tunnel across the event horizon [1]. They treat Hawking radiation as a tunnelling process, and the WKB method is used [2, 3]. In this way a corrected spectrum, which is accurate to the first order approximation, is given. Their result is considered to be in agreement with an underlying unitary theory. Following this method, a lot of static or stationary rotating black holes are studied [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31]. The same result, that is, Hawking radiation is no longer pure thermal, unitary theory is satisfied and information is conserved, is obtained. But in all of these literature, the entropy of the black hole only contains the Bekenstein-Hawking entropy. Will the emission process still consist with the unitary theory if the quantum correction of the entropy is taken into account? In present, about the quantum correction, there are different results corresponding to different models and methods [32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43].

The general formulation of the black hole entropy is [44, 45]

$$S_q = \frac{A_H}{4l_p^2} + \alpha \ln \frac{A_H}{4l_p^2} + O\left(\frac{l_p^2}{A_H}\right) + \text{const.}, \quad (1)$$

where $\alpha$ is a model-dependent (dimensionless) parameter. In the case of Loop Quantum Gravity $\alpha$ is a negative coefficient whose exact value was once an object of debate (see e.g. [37]) but has since been rigorously fixed at $\alpha = -1/2$. In String Theory the sign of $\alpha$ depends on the number of field species appearing in the low energy approximation [36]. Therefore, a very interesting work is to introduce the log-corrected entropy-area relation in the
tunnelling framework. Moreover, if the emission rate is calculated to the second order approximation, will the entropy contain the inverse area term as given in equation (1)? In this paper, we first show that in the tunnelling picture, a logarithm correction term occurs in the expression of the black hole entropy if we take the emission particle as a spherical surface wave (spherical shell). Therefore, we expect that Parikh-Wilczek tunnelling framework, in fact, is in agreement with the unitary theory to the first order log-correction of the black hole entropy. Then, we verify that, if we calculate the emission rate to the second order approximation with Parikh-Wilczek tunnelling framework and suppose the emission process to be still in agreement with the unitary theory, the entropy of the black hole will contain three parts: the usual Bekenstein-Hawking entropy, the logarithmic term and the inverse area term. Finally, we give two comments to the Parikh-Wilczek framework and our calculation.

II. BLACK HOLE TUNNELLING AND THE FIRST ORDER CORRECTION TO THE BLACK HOLE ENTROPY

As mentioned above, Parikh and Wilczek applied the WKB approximation to calculate the emission rate of a tunnelling particle (S-shell). We start with a brief review of the WKB method and the barrier penetration. For a massless particle (massless shell), because of the infinite blueshift near the horizon, the characteristic wavelength of any wavepacket of the S-wave (see [1, 2, 3]) is always arbitrarily small there, so that the geometrical optics limit becomes an especially reliable approximation. The geometrical optics limit allows us to obtain rigorous results directly in the language of particles. That is, the WKB method and the expression of the emission rate are the same as that of a classical massive particle. So, in the following discussion we only study the tunneling process of a massive particle (massive shell).

Schrödinger’s equation for the motion of a particle in a centrally symmetric field is

\[ \Delta \psi + \frac{2m}{(\hbar^2)}(E - U(r))\psi = 0. \]  

(2)

Let us consider the following radial equation:

\[ \frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{dR}{dr} \right) - \frac{l(l+1)}{r^2} R + \frac{2m}{\hbar^2} (E - U(r)) R = 0. \]  

(3)

By the substitution

\[ R(r) = X(r)/r \]  

(4)

equation (3) is brought to the form

\[ \frac{d^2 X}{dr^2} + \frac{2m}{\hbar^2} (E - U(r)) - \frac{l(l+1)}{r^2} X = 0. \]  

(5)

For S-wave, \( l = 0 \), the equation of \( X(r) \) is

\[ \frac{d^2 X}{dr^2} + \frac{2m}{\hbar^2} (E - U(r)) X = 0. \]  

(6)

Note that, in the Parikh-Wilczek framework, to calculate the self-gravitation reliably the tunnelling particle is considered as a spherical shell (S-wave). In this way, when it emits from the black hole the matter-gravity system transits from one spherical state to another. So, the de-Broglie wave function of the emission spherical shell should be

\[ \psi(r) = X(r)/r. \]  

(7)
That is, the WKB wave function of a particle can be written as

$$\psi(r) = X(r)/r = \frac{1}{r} \exp\left[\frac{i S(r)}{\hbar}\right],$$

where

$$S(r) = S_0(r) + \left(\frac{\hbar}{i}\right) S_1(r) + \left(\frac{\hbar}{i}\right)^2 S_2(r) + \cdots.$$  

Substituting \( S \) into Schrödinger Equation (6) yields

$$S_0 = \pm \int r p_r \, dr,$$

$$2S_0' S_1' + S_0'' = 0,$$

$$2S_0' S_2' + (S_1')^2 + S_1'' = 0,$$

where we use a prime to denote differentiation with respect to \( r \).

To evaluate the probability of a particle passing through the barrier, we divide the whole region of motion of the particle by two tunnelling points \( a \) and \( b \) into three parts: ingoing and reflecting region I, barrier region II and the outgoing region III. The particle moves as a free particle in region I and III, but region II is classically inaccessible.

In region I, we take the WKB wave function as follows \([46]\)

$$X_I(r) = \frac{2}{\sqrt{v}} \sin \left[ \frac{1}{\hbar} \int_a^r p_r \, dr + \frac{\pi}{4} \right]$$

$$= \frac{1}{\sqrt{v}} \{ \exp \left[ \frac{i}{\hbar} \int_a^r p_r \, dr + \frac{i\pi}{4} \right] - \exp \left[ -\frac{i}{\hbar} \int_r^a p_r \, dr - \frac{i\pi}{4} \right] \},$$

where \( v \) is the velocity of the tunnelling particle. In region II, the WKB wave function is a linear combination of real exponentials. Considering the connexion between the oscillating and the exponential solutions at \( r = a \), the WKB wave function in region II can be written as

$$X_{II}(r) = \frac{1}{\sqrt{v}} \exp \left[ -\frac{1}{\hbar} \int_a^b p_r \, dr \right] \exp \left[ -\frac{1}{\hbar} \int_r^b p_r \, dr \right].$$

And the WKB wave function in region III is

$$X_{III}(r) = -\frac{1}{\sqrt{v}} \exp \left[ -\frac{1}{\hbar} \int_a^b p_r \, dr \right] \exp \left[ \frac{i}{\hbar} \int_r^b p_r \, dr + \frac{i\pi}{4} \right].$$

The probability of barrier penetration is

$$\Gamma_p = \frac{j_{out}}{j_{in}} = \frac{v|\psi_{out}|^2}{v|\psi_{in}|^2} = \frac{v(X_{out}(b)/b)^2}{v(X_{in}(a)/a)^2} = \frac{a^2}{b^2} \exp \left[ -\frac{2\text{Im}S_0}{\hbar} \right].$$

Let’s now calculate the phase space factor corresponding to the black hole tunnelling. For Schwarzschild black hole, the line element in Painlevé coordinates is

$$ds^2 = -c^2(1 - \frac{2MG}{c^2r})dt^2 + 2c\sqrt{\frac{2MG}{c^2r}}dt \, dr + dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2).$$
and the radial null geodesics are
\[ \dot{r} = \frac{dr}{dt} = \pm c \left( 1 - \sqrt{\frac{2MG}{c^2 r}} \right). \] (18)

with the upper(lower) sign in Eq. (18) corresponding to outgoing(ingoing) geodesics, under the implicit assumption that \( t \) increases towards the future[47].

But in this paper we consider the tunneling of the massive particle. That is, the outgoing particle is a massive shell (de Broglie s-wave). The massive quanta doesn’t follow radial-lightlike geodesics (18). Similar to Ref. [19], we treat the massive particle as a de Broglie wave and obtain the expression of \( \dot{r} \). Namely,
\[ \dot{r} = v_p = \frac{1}{2} v_g = \frac{1}{2} \left( \frac{g_{00}}{2 g_{01}} \right) = \frac{1}{2} \frac{c^2 r^2 - 2MGr}{\sqrt{2MGc^2}}. \] (19)

Note that to calculate the emission rate correctly, we should take into account the self-gravitation of the tunneling particle with energy \( \omega \). That is, we should replace \( M \) with \( M - \omega \) in (17) and (19) to describe the motion of the particle correctly[1, 2, 3].

The canonical momentum \( p_r \) and the imaginary part of the action \( \text{Im} S_0 \) can be easily obtained. Namely,
\[ p_r = \int_0^{r_t} dp_r' = \int dH = -i \pi \frac{\hbar}{l_p^2} r, \] (20)
\[ \text{Im} S_0 = \int_{r_i}^{r_t} p_r dr = -\frac{1}{2} \hbar \left[ A_f \frac{4l_p^2}{4l_p^2} - A_i \frac{4l_p^2}{4l_p^2} \right]. \] (21)

The probability of barrier penetration is
\[ \Gamma_p = \frac{r_i^2}{r_f^2} \exp \left[ -2 \frac{\text{Im} S_0}{\hbar} \right] = \exp \left[ \left( A_f \frac{4l_p^2}{4l_p^2} - \ln A_f \frac{4l_p^2}{4l_p^2} \right) - \left( A_i \frac{4l_p^2}{4l_p^2} - \ln A_i \frac{4l_p^2}{4l_p^2} \right) \right], \] (22)

where \( l_p^2 = \frac{\hbar G}{c^3} \). In this paper, we investigate the transition of the matter-gravity system from one spherical state to another at the same energy. This transition corresponds to the production and barrier penetration of the massive spherical shell (or massless shell). That is, this process contains two stages. The first stage is the production of the spherical shell from the vacuum fluctuation near the event horizon. The second stage is the barrier penetration. The rate of transition from the initial spherical state to the final spherical state is therefore
\[ \Gamma(i \to f) = \Gamma_v \cdot \Gamma_p = \Gamma_v \cdot \exp \left[ \left( A_f \frac{4l_p^2}{4l_p^2} - \ln A_f \frac{4l_p^2}{4l_p^2} \right) - \left( A_i \frac{4l_p^2}{4l_p^2} - \ln A_i \frac{4l_p^2}{4l_p^2} \right) \right]. \] (23)

Let’s Compare (23) with the unitary result in Quantum Mechanics, \( \Gamma(i \to f) = | M_{fi} |^2 \cdot \text{(phase space factor)} \), which is given in Ref. [1]. \( | M_{fi} |^2 \) is the probability amplitude of the process, in this case it is related to the production rate of the particle in the vacuum fluctuation near the event horizon. Thus, we obtain
\[ \text{phase space factor} = \exp \left[ \left( A_f \frac{4l_p^2}{4l_p^2} - \ln A_f \frac{4l_p^2}{4l_p^2} \right) - \left( A_i \frac{4l_p^2}{4l_p^2} - \ln A_i \frac{4l_p^2}{4l_p^2} \right) \right]. \] (24)

If we bear in mind that
\[ \text{phase space factor} = \frac{N_f}{N_i} = e^{S_f - S_i} = e^{S_f - S_i}, \] (25)
we naturally get the expression of the black hole entropy to the first order correction
\[ S_q = \frac{A_H}{4l_p^2} - \ln \frac{A_H}{4l_p^2}. \] (26)
III. SECOND ORDER CORRECTION TO THE BLACK HOLE ENTROPY

Let’s now calculate the tunnelling rate to the second order approximation. In order to get the second order correction of the black hole entropy, we write the WKB wave function to the second order approximation. Namely,

$$X(r) = \exp\left[\frac{iS_0(r)}{\hbar} + S_1(r) + \frac{\hbar}{i}S_2(r)\right],$$

where

$$S_2 = \int_r^a - \frac{(S_1'' + S_1')}{2S_0'} dr. \quad (28)$$

Like the treatment in section II, the wave function in region I can be taken as

$$X_I(r) = \frac{2}{\sqrt{v}} \sin\left[\frac{1}{\hbar}\left(\int_r^a p_r dr - \hbar^2 S_2(r)\right) + \frac{\pi}{4}\right],$$

$$= \frac{1}{i\sqrt{v}} \exp\left[\frac{i}{\hbar}\left(\int_r^a p_r dr - \hbar^2 S_2(r)\right) + \frac{i\pi}{4}\right] - \exp\left[\frac{-i}{\hbar}\left(\int_r^a p_r dr - \hbar^2 S_2(r)\right) - \frac{i\pi}{4}\right]. \quad (29)$$

In this region the expression of $S_2(r)$ is

$$S_2 = \int_r^a - \frac{(S_1'' + S_1')}{2S_0'} dr. \quad (30)$$

In order to reduce to the first order approximation case, the connexion between the oscillating and the exponential solutions at $r = a$ should be

$$\frac{2}{\sqrt{v}} \sin\left[\frac{1}{\hbar}\left(\int_r^a p_r dr - \hbar^2 S_2(r)\right) + \frac{\pi}{4}\right] \equiv \frac{1}{\sqrt{v}} \exp\left[-\frac{1}{\hbar}\left(\int_a^r |p_r| dr - \hbar^2 S_2(r)\right)\right]. \quad (31)$$

On the right hand of the connexion (31), the expression of $S_2(r)$ is

$$S_2 = \int_r^a - \frac{(S_1'' + S_1')}{2S_0'} dr. \quad (32)$$

The connexion at $r = b$ is

$$\frac{1}{\sqrt{v}} \exp\left[\frac{1}{\hbar}\left(\int_b^r p_r dr - \hbar^2 S_2\right)\right] \equiv -\frac{1}{\sqrt{v}} \exp\left[\frac{i}{\hbar}\left(\int_b^r p_r dr - \hbar^2 S_2\right) + \frac{i\pi}{4}\right], \quad (33)$$

and the wave function in region III is

$$X_{III}(r) = -\frac{1}{\sqrt{v}} \exp\left[-\frac{1}{\hbar}(\text{Im}S_0 - \hbar^2 \text{Im}S_2)\right] \exp\left[\frac{i}{\hbar}\left(\int_b^r p_r dr - \hbar^2 S_2\right) + \frac{i\pi}{4}\right], \quad (34)$$

where

$$\text{Im}S_2 = \text{Im} \int_a^b - \frac{(S_1'' + S_1')}{2S_0'} dr. \quad (35)$$

Since

$$\psi(r) = X(r)/r, \quad (36)$$
in region I, the ingoing flux density is
\[ j_{\text{in}} = -\frac{i\hbar}{2m}(\psi_{\text{in}}^* \frac{\partial}{\partial r} \psi_{\text{in}} - \psi_{\text{in}} \frac{\partial}{\partial r} \psi_{\text{in}}^*) = v|\psi_{\text{in}}|^2 = \frac{1}{\sigma^2}. \] (37)
and in region II the outgoing flux density is
\[ j_{\text{out}} = -\frac{i\hbar}{2m}(\psi_{\text{out}}^* \frac{\partial}{\partial r} \psi_{\text{out}} - \psi_{\text{out}} \frac{\partial}{\partial r} \psi_{\text{out}}^*) = v|\psi_{\text{out}}|^2 = \frac{1}{\beta^2} \exp\left[-\frac{2}{\hbar}(\text{Im}S_0 - \hbar^2 \text{Im}S_2)\right]. \] (38)
Therefore,
\[ \Gamma_p = \frac{j_{\text{out}}}{j_{\text{in}}} = \frac{a^2}{\beta^2} \exp\left[-\frac{2}{\hbar}(\text{Im}S_0 - \hbar^2 \text{Im}S_2)\right]. \] (39)
For Schwarzschild black hole tunnelling, in classically inaccessible region, we have
\[ S'_{0} = p_r = -i\pi \frac{\hbar}{l_p}, \quad S''_{0} = -i\pi \frac{\hbar}{l_p^2}, \] (40)
and
\[ S'_1 = -\frac{1}{2} S''_0, \quad S''_1 = \frac{1}{2 r^2}. \] (41)
From (41) we can easily obtain
\[ S'_2 = -\frac{1}{2S'_0}(S''_1 + S''_1) = -\frac{3i}{8\pi} \frac{l_p^2}{\hbar} \cdot \frac{1}{r^3}. \] (42)
So,
\[ S_2 = \int_{r_i}^{r_f} S_2' \, dr = \frac{3i}{4\hbar} \left( \frac{l_p^2}{A_f} - \frac{l_p^2}{A_i} \right). \] (43)
Substituting (21), (43) into (39) and considering \( \Gamma(i \rightarrow f) = | M_{fi} |^2 \cdot \text{(phase space factor)} \), yields
\[ \text{phase space factor} = \exp\left[\left(\frac{A_f}{4l_p^2} - \ln \frac{A_f}{4l_p^2} + \frac{3}{2} \frac{l_p^2}{A_f}\right) - \left(\frac{A_i}{4l_p^2} - \ln \frac{A_i}{4l_p^2} + \frac{3}{2} \frac{l_p^2}{A_i}\right)\right]. \] (44)
Comparing (44) with (25), we get the expression of the black hole entropy to the second order correction
\[ S_q = \frac{A_H}{4l_p^2} - \ln \frac{A_H}{4l_p^2} + \frac{3}{2} \frac{l_p^2}{A_H} + \text{const.}, \] (45)
which is consistent with an unitary theory and is also in agreement with the general formulation of the black hole entropy. The emission rate is
\[ \Gamma(i \rightarrow f) \sim e^{\Delta S_q}. \] (46)

IV. CONCLUSION AND COMMENTS

We showed how a log-corrected entropy-area relation can emerge in the tunnelling picture if we consider the emission particle as a spherical shell. We also showed that, if the emission rate is calculated to the second order approximation, the black hole entropy will contain three parts: the usual Bekenstein-Hawking entropy, the logarithmic term and the inverse area term. In our calculation the logarithmic term and the inverse area term is the consequence of requesting
the process to satisfying the unitary theory. Apart from a coefficient, Our correction to the black hole entropy is consistent with that of loop quantum gravity. In the following, we give two comments to the Parikh-Wilczek method and our calculation.

1) In this paper, we only take into account the emission of the massive particle. The motion of a massless particle (S-wave) is very different from that of a massive particle. However, as mentioned in the first paragraph of the section II, a massless shell can be treated in the language of particle. That is, for massless shell we can also apply the WKB method and obtain the same functional form of emission rate as that of the massive particle. So, Eqs. (45) and (46) are also suitable for massless particle’s emission.

2) In the first order approximation, the previous expression of the emission rate can be written in the following explicit form

$$\Gamma \sim \exp (\Delta S_q) = (1 - \frac{\omega}{M})^n \exp (-8\pi GM\omega(1 - \frac{\omega}{2M})).$$

(47)

In Refs. [6] and [13] the authors pointed out that the coefficient of the log-corrected term in the black hole entropy should be positive, otherwise, the probability of emission will diverge when the emission particle’s mass, $\omega$, approaches to $M$. In fact, if we consider the applying condition of the WKB method, the emission particle’s mass, $\omega$, will never approach to $M$, it is far smaller than the black hole mass $M$. Here is our derivation.

The WKB method is established in the conditions:

$$\hbar |S'_0| \ll |S'^2_0|,$$

(48)

and

$$2\hbar |S'_0S'_1| \ll |S'^2_0|. \quad (49)$$

From (48) and (49), above conditions (48) and (49) can be incorporated into an inequality, that is

$$\hbar \frac{|dp_r|}{dr} \ll |p^2_r|. \quad (50)$$

Considering $p_r = -i\pi r$ and $2(M - \omega) \leq r \leq 2M$, (50) becomes

$$2(M - \omega) \gg \sqrt{\frac{\hbar}{\pi}}. \quad (51)$$

That is,

$$M \gg \omega. \quad (52)$$

It means that the Parikh-Wilczek framework is only suitable for the emission of the particle whose energy is far less than the mass of the black hole. Most of the time in the evaporation of the black hole this condition is satisfied, that is, the mass of the emission particle will not approach to the black hole mass $M$. Therefore, the coefficient of the log-corrected term in the black hole entropy is not constrained to be positive. However, in the last stage of evaporation the emission will be very strong, and the mass of the emission particle will be very great, the WKB conditions will not be satisfied, then one would have to resort to other mechanisms to describe the last stage of the evaporation.
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