Specific modes of vibratory technological machines: mathematical models, peculiarities of interaction of system elements

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Abstract. The methodological basis of constructing mathematical models of vibratory technological machines is developed in the article. An approach is proposed that makes it possible to introduce a vibration table in a specific mode that provides conditions for the dynamic damping of oscillations for the zone of placement of a vibration exciter while providing specified vibration parameters in the working zone of the vibration table. The aim of the work is to develop methods of mathematical modeling, oriented to technological processes with long cycles. The technologies of structural mathematical modeling are used with structural schemes, transfer functions and amplitude-frequency characteristics. The concept of the work is to test the possibilities of combining the conditions for reducing loads with working components of a vibration exciter while simultaneously maintaining sufficiently wide limits in variating the parameters of the vibrational field.

1. Introduction
Vibrational technological processes and corresponding equipment and machines are widely used in various industries [1 ÷ 3]. The interaction of the elements of vibratory systems is very diverse and essentially depends on the structure of vibrational fields of vibratory machines [4]. Vibrational inertial excitation is widely used in various technological processes, but such approaches have certain drawbacks [5].

The proposed article develops an approach to the construction of vibrational technological complexes, taking into account the possibilities of separating the functions of vibration tables, which are related both to the implementation of the technological process and to the excitation of vibrations, in which the ideas of energy transfer in modes of dynamic damping of oscillations are used.

2. Description of the process unit. Statement of the research task
Let us consider a vibrating table, which consists of a mobile working unit in the form of a rigid weightless rod (a rod with a low moment of inertia). At the ends of the rod, concentrated masses \( m_1 \) and \( m_2 \) are located at points \( B \) and \( C \), respectively. These masses are supported by elastic elements with stiffnesses \( k_1 \) and \( k_2 \). The working unit is essentially a solid body of mass \( M \) and a moment of inertia \( J \) with respect to the center of gravity of point \( O \). The distance to the center of gravity of point \( O \) is \( l_1 \) and \( l_2 \) (Figure 1) [6].
3. Construction of mathematical model

If the longitudinal oscillations along the line of location of characteristic points A, B, C, C₁ and D are neglected, then after the formulation of the expressions for the kinetic and potential energies, let us obtain the equations of motion of the system in coordinates $\bar{y}_1$ and $\bar{y}_2$:

$$\bar{y}_1 \left[ (Ma^2 + Jc^2) p^2 + k_1 \right] + \bar{y}_2 \cdot (Mab - Jc^2) p^2 = 0; \quad (1)$$

$$\bar{y}_1 \cdot (Mb^2 + Jc^2) p^2 + k_2 \bar{y}_2 \cdot (Mab - Jc^2) p^2 = Q. \quad (2)$$

The structural scheme of the system with a force perturbation with respect to the coordinate is shown in Figure 2, where $p = j\omega$ is a complex variable, the symbol “−” means the Laplace image of variables [7, 8].

Figure 2. Structural scheme of the vibration table in coordinates $\bar{y}_1$ and $\bar{y}_2$.

Transfer functions of the system are:

$$W_1(p) = \frac{\bar{y}_1}{Q_2} = \frac{(Jc^2 - Mab)p^2}{A(p)}; \quad (3) \quad W_2(p) = \frac{\bar{y}_2}{Q_2} = \frac{(Ma^2 + Jc^2)p^2 + k_1}{A(p)}; \quad (4)$$

where

$$A(p) = \left[ (Ma^2 + Jc^2)p^2 + k_1 \right] \left[ (Mb^2 + Jc^2)p^2 + k_2 \right] - (Jc^2 - Mab)^2 p^4 \quad (5)$$

− is a characteristic frequency equation.

The relationship between the motions along the $y_1$ and $y_2$ coordinates can also be interpreted as “lever linkages”, which is manifested in the forms of oscillations that are created by the external harmonic effect and are preserved when the frequency of external oscillations changes. The ratio of coordinates $\frac{\bar{y}_1}{\bar{y}_2}$ in the operator form can be obtained from the structural scheme in Figure 2 at input action $Q$, applied to element $m_2$, and can be defined as a transfer function of interpartial bonds:

$$W_{12}(p) = \frac{\bar{y}_1}{\bar{y}_2} = \frac{(Jc^2 - Mab)p^2}{(Ma + (Jc^2 - Mab)p^2 + k_1)}, \quad (6)$$
Substituting \( p = j\omega \) into (6), one can obtain:

\[
\frac{\gamma_1}{\gamma_2} = \frac{-\left( Jc^2 - Mab \right) \cdot \omega^2}{- Ma + (Jc^2 - Mab) \cdot \omega^3 + k_1},
\]  

(7)

from which it can be found that partial frequency \( n \) of the system and the frequency of the dynamic damping of oscillations \( \omega_{\text{din}} \) are connected by the relation:

\[
n^2 = \omega_{\text{din}}^2 = \frac{k_1}{Ma + (Jc^2 - Mab)}. 
\]

(8)

The graph of dependence \( \frac{\gamma_1}{\gamma_2}(\omega) \) is shown in Figure 3 within the range of the frequency of the external action from 0 to \( \omega_{\text{din}} \) and has a negative value; after transition \( \omega = \omega_{\text{din}} \), the ratio becomes positive and tends at \( \omega \to \infty \) to the limit defined by expression:

\[
W_{12}(p) = \frac{\gamma_1}{\gamma_2} = \frac{Jc^2 - Mab}{Ma + (Jc^2 - Mab)} < 1.
\]

(9)

**Figure 3.** The graph of dependence \( \frac{\gamma_1}{\gamma_2} \) on frequency \( \omega \) as long as condition (7) is fulfilled.

4. **Specific aspects of the dynamic properties of the system**

Amplitude-frequency characteristics of the system are shown in Figure 4: more detailed data are given in [6]. Figure 4 shows graphs \( y_1(\omega) \) and \( y_2(\omega) \) upon excitation with respect to the \( y_2 \) coordinate and condition \( R = Jc^2 - Mab > 0 \).
Figure 4. Graphs of the dependence of vibration amplitude of the vibration table on the frequency of external action: curve 1 is dependence $y_1(\omega)$; curve 2 is dependence $y_2(\omega)$.

For $\omega \to \infty$ and $Q_2 = m_0 r \omega^2$, the graph of dependence $y_1(\omega)$ tends to the limit:

$$y_1(\omega) \to N_1 = \left( Jc^2 - Mab \right) m_0 \cdot r.$$ \hspace{2cm} (10)

The ratio (10) to the expression (11) corresponds to the limit:

$$\frac{y_1(\omega \to \infty)}{y_2(\omega \to \infty)} = \frac{Jc^2 - Mab}{Ma + Jc^2 - Mab},$$ \hspace{2cm} (12)

as shown in Figure 3.

Characteristic of dynamic interactions is the relationship:

$$W_{12}(p) = \frac{y_1}{y_2} = \frac{\left( Jc^2 - Mab \right) p^2}{Ma p^2 + (Jc^2 - Mab) p^2 + k_1}.$$ \hspace{2cm} (13)

An important point in the estimation of dynamic properties is the presence of the condition:

$$Jc^2 - Mab < 0; Jc^2 - Mab = 0; Jc^2 - Mab > 0,$$ \hspace{2cm} (14)

which leads to various forms of manifestation of the effects of dynamic damping of oscillations, the frequency is determined as:

$$\omega^2_{\text{din}} = \frac{k_1}{Ma - (Jc^2 - Mab)}.$$ \hspace{2cm} (15)

The shapes of dependency $\frac{y_1}{y_2}(\omega)$ curves depend on the variation of the frequencies of dynamic damping $\omega_{\text{din}}$ and natural oscillations $\omega_1$ and $\omega_2$; the frequencies, at which $\frac{y_1}{y_2} = 1$ and $\frac{y_1}{y_2} = -1$ will be observed, will be different; in addition, the limiting values will also be different as the frequency of perturbation $\omega \to \infty$ increases [9, 10].

To construct a mathematical model, the following parameters are used: $M = 150 \text{ kg}$; $a = 0.1 \text{ m}$; $b = 0.9 \text{ m}$; $J = 286 \text{ kg.m}^2$; $Ma = 15 \text{ kg.m}$; $Mb = 135 \text{ kg.m}$; $R = 4 \text{ kg.m}^2$; $l_1 + l_2 = 4 \text{ m}$.

In Figure 5, a diagram is constructed that allows selecting the parameters of the vibration table (for example, $a$ and $b$, or mass-and-inertia parameters $J$ and $M$), which provide necessary ratios of the oscillation amplitudes with respect to coordinates $y_1$ and $y_2$. 
5. Discussion of results and conclusion

With an appropriate adjustment, the partial frequencies of the systems are close to their natural frequencies, but they do not go beyond them. For the partial frequency $n_2$, small values of the amplitude of the oscillations are observed, i.e. the dynamic damping mode is ensured at the exciter. Depending on the ratio of parameters $a$, $b$ and $J$, the frequencies at which the working member functions as levers of the first and second kinds change: for $R > 0$, the working member behaves like a first-order lever in the resonant region, and like a second-order lever in the inter-resonance region; at $R < 0$, the working member behaves like a first-order lever in the inter-resonance region, and like a second-kind lever in the resonance region. With increasing stiffnesses of elastic supports $k_1$ and $k_2$, the natural frequencies and partial frequencies increase according to the quadratic dependence. When adjusting the vibration table, the values of the upper limits of the partial and natural frequencies of the working parts depend on stiffness $k_1$, and the lower limits of the partial and natural frequencies depend on stiffness $k_2$. When inertial parameter $R$ is changed, the values of the natural frequencies and partial frequencies decrease to the smaller side.

The above data of computational modeling reflect the parameters of the technological vibration complex for strength tests of technical objects at one of the enterprises of the region.

The accuracy of calculations for estimating the oscillation frequencies of the system in the stationary mode ($\approx 30$ Hz) and the oscillation amplitudes is ensured within 5–7%, which corresponds to the technical regulations of the complex operation. Guidance materials and recommendations have been transferred to the enterprise for use in practical tasks, and there are acts of their implementation.

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