Profile of mathematical reasoning ability of 8th grade students seen from communicational ability, basic skills, connection, and logical thinking

Sumarsih¹, Budiyono¹ and D Indriati¹
¹Department of Mathematics Education, Universitas Sebelas Maret, Surakarta, Indonesia

Email: smarsih74@yahoo.com

Abstract. This research aims to understand the students’ weaknesses in mathematical reasoning ability in junior secondary school. A set of multiple choice tests were used to measure this ability involving components mathematical communication, basic skills, connection, and logical thinking. A total of 259 respondents were determined by stratified cluster random sampling. Data were analyzed using one-way Anova test with $F_{obs} = 109.5760$ and $F = 3.0000$. The results show that students’ ability from schools with high National Exam in mathematics category was the best and followed by medium and low category. Mathematical connection is the most difficult component performed by students. In addition, most students also have difficulty in expressing ideas and developing logical arguments.

1. Introduction

Reasoning is needed in solving mathematical questions [1]. It is a basic skill of mathematics and is necessary for a number of purposes – to understand mathematical concepts, to use mathematical ideas and procedures flexibility, and to recontruct [2]. Kaur and Lam describe mathematical reasoning as components consisting of communication, basic skills, connection, and logical thinking [3]. It is clear that the “reasoning of mathematics” need to be fully integrated into classrooms, schools, and districts around the country to produce citizens and employees adequately prepared for the 21st century [4]. Based on the above-mentioned research, it can be seen that reasoning skills needs to be applied in learning mathematics in the classroom so that students are able to become proficient communicators, creators, critical thinker, and collaborators.

Based on interviews with some Junior Secondary School students, it is found that the students’ process of reasoning was not yet developed well. The implication is that the students’ critical thinking ability and communication skills were not well-developed. There are some examples showing the lack of students’ mathematical reasoning. First, the students had studied the basic concepts of numerical operations and they had been able to solve arithmetic operations in positive integers without problem. However, in the reality, the students faced difficulties in determining the arithmetic operation results on negative integers and fractions. Related to these, their answers were various and incorrect. For example, in solving $3x = 12$, the students answered correctly in a little amount of time. However, in solving basic questions like $5x = 1$ which should be answered as $x = \frac{1}{5}$, some students answered $x = 1$, $x = -5$, or $x = -4$. Therefore, the students’ ability to make simple examples into complex ones was not yet well-developed [5].
Second, in learning mathematics, the students were experiencing difficulties in writing a statement into mathematical sentence related to operations of addition, subtraction, multiplication, and division. Problem solving-type questions are full of statements and/or sentences, so that in order to solve it, students are expected to rewrite it into mathematical sentences correctly. A hindering factor, which was factor coming from within the students, was found which causes them to have trouble in expressing their ideas [6]. Students in general have troubles in expressing opinion with their own words, mostly because they are afraid to be wrong. The other hindering factors vary: laziness, afraid to be laughed about, afraid to be named as “the smart one”, afraid of teachers, and afraid of teachers’ wrath.

The other difficulties relate to two-dimensional figure discourse matter. Two-dimensional figures have two main elements: sides and corners. Figures like trapezoids, kites, parallelograms, and rhombi can be divided into several two-dimensional figures, which may take forms of squares, rectangles, and triangles. Students were having troubles in determining the side length, even more if stated in variables. The low ability of the students in finding facts from pictures was caused by the students’ inability to free themselves from their mindset. Students often thought about an elaborate solution, while in fact the solution itself is merely a simple one [7].

Based on the description above, it can be concluded that the students’ low ability of mathematical reasoning was indicated by the ability of students in making simple examples into complex ones, which was not yet developed well, the students’ difficulties in expressing their ideas in their own words because of fear of doing wrong, and their low ability in finding facts from pictures because the students were hindered by their own faraway thoughts.

Tajudin and Chinnappan said that: “Further research should be conducted to generate higher level of clarity about the roles of reasoning in mathematics learning. This study indicates that reasoning skills are important at least for the Low-Achievement group” [8]. Ayele argued that to improve students' reasoning skills, teachers should value their creativity in the classroom as well as help and appreciate students' creative work and ideas [9]. Tonels developed Wind Phoenix game with problem solving process through creation of conjectures. Tonels concluded that, with role assumption in the game, the players form logical problem uttering process – puzzles [10].

The research by Poon & Leung shows that students in schools with higher academic achievements had higher scores in geometry test. Besides, strong correlations are found between the students’ achievement in geometry and their fundamental logical reasoning ability. This finding offers a reason for reflecting present geometry curriculum in junior secondary schools in Hong Kong [11]. Based on the research of Ayele, Tonels, and Poon & Leung, it can be concluded that mathematical reasoning can be developed by appreciating students’ creative works and ideas through games, or by reflecting curriculum which implements learning process that emphasizes on students gaining their own experience. Asyari et al [22] studied Improving critical thinking skills through the integration of problem based learning and group investigation.

This research analyzed more deeply about the profile of students' mathematical reasoning ability in terms of mathematical communication, basic mathematical skills, mathematical connections, and logical think. The components of mathematical reasoning ability in this study are part of "4C", which is included in the critical thinking and communication skills component. This research further experiment with Creative Problem Solving and Group Investigation learning model through Mind Mapping technique. As the result of this research, the teachers and/or educators are expected to identify the characteristics of students’ ability and to determine which components of mathematical reasoning is weak, in order to find an alternative way to improve it.

2. Theoretical basis

Ball and Bass suggested that mathematical reasoning assumes mathematical communication. Communication is an integral part of the process of reasoning [2]. According to Kaur and Lam, communication refers to the ability to use mathematical language to express mathematical ideas and arguments precisely, concisely and logically, to use model with mathematics, and use tools
strategically. The tools are table, chart, diagram, or list [3]. Ball and Bass explained that reasoning is a basic skill of mathematics and is necessary for a number of purposes—to understand mathematical concepts, to use mathematical ideas and procedures flexibility, and to reconstruct [2]. As stated by Kaur and Lam in their quotations about syllabus document of the Ministry of Education of Singapore, mathematical reasoning refers to the ability to analyse mathematical situations and construct logical arguments [3].

Russell stated that mathematical reasoning is essentially about the development, justification and use of mathematical generalizations. The generalizations create an interconnected web of mathematical knowledge—conceptual understanding. Seeing mathematics as a web of interrelated ideas is both a result of an emphasis on mathematical reasoning and a foundation for reasoning further [12]. In the words Kaur and Lam explained that connections refer to the ability to see and make linkage among mathematical ideas, between mathematics and other subjects, and between mathematics and everyday life [3]. Logic is the study of the methods and principles used to distinguish correct from incorrect reasoning. Reasoning is a special kind of thinking in which inference take place, in which conclusions are drawn from premises [13]. It can be concluded that mathematical reasoning is further described into four components, which are communication, basic skills, connection, and the way of mathematical logical thinking.

Higher Ability Selection Test (HAST) is a test package written by the Australian Council Educational Research (ACER), consisting of mathematical reasoning, reading comprehension, abstract reasoning, and written expression [14]. The test are used for selection, or to better differentiate student intakes, and to distinguish between stronger and weaker students. The mathematical reasoning test developed by ACER includes tests to understand, interpret, and analyze mathematical information at the junior level as well as math and science at the middle and senior level. Test participants should apply logical and strategic thinking to work through questions in the exam. Tasks are presented as numbers, texts, diagrams, graphs, and tables including number, measurement, space, time, logical relationships and problems, which must be solved to obtain mathematical components. Two important processes in reasoning are connections and further changes to form an argument or to solve a problem [2]. The process of reasoning consists of deductive reasoning and inductive reasoning [15]. Deduction is defined as the reasoning process from general to specific, whereas induction is defined as the process of reasoning from specific to general. Deductive reasoning includes: ponens mode, tollens mode, and syllogism [13]. Premises, in the argument the propositions upon which inference is bussed, the propositions that are claimed to provide grounds or reason for the conclusion. Conclusion in any argument, the proposition to which the other propositions in the argument are claimed to give support, or for which they are given as reasons. Conclusions with ponens mode, tollens mode, and syllogism can be distinguished as seen in Table 1.

| Table 1. Deductive conclusion |
|--------------------------------|
| **Premise I** | **Ponens mode** | **Tollens mode** | **Syllogism** |
| a→b | a→b | a→b |
| a | ~b | b → c |
| b | ~a | a → c |

Inductive reasoning includes: analogies, generalizations, and causal relationships. Analogy is a process of reasoning that departs from two or more specific events that have similarities with each other. The greater the number of respects in which the entity in the conclusion is similar to the entities in the premise, the more probable in that the conclusion. Generalization is a process of reasoning that departs from a number of facts or the particular phenomenon observed, then drawing general conclusions about the observed phenomenon. Causal relationships are reasoning derived from
interconnected phenomena. Descriptive causal laws asserting a necessary connection between event of two kinds, of which one is the cause and other the effect [13].

Referring to the opinions of Ball & Bass, Russell, Kaur & Lam, and Copi as well as the adaptation of ACER, the ability of mathematical reasoning is the ability or skills in the form of test results about mathematical communication, basic math skills, mathematical connection, and logical thinking. Indicators of mathematical reasoning ability as measured in this research are stated in Table 2.

Table 2. Indicators of mathematical reasoning ability

| Reasoning Component | Indicators                                                                 |
|---------------------|---------------------------------------------------------------------------|
| Mathematical        |                                                                            |
| communication       | Using mathematical expressions to express mathematical ideas.             |
|                     | Preparing arguments appropriately, concisely, and logically.              |
|                     | Determining images/sketch                                                 |
| Mathematical        |                                                                            |
| basic skills        | Understanding the concept and interpret the data                          |
|                     | Using flexible mathematical ideas and procedures.                         |
|                     | Building a logical argument.                                              |
| Mathematical        |                                                                            |
| connection          | The ability to build inter-discourse material or interdisciplinary connection.|
|                     | Stating relations                                                         |
|                     | Making generalizations                                                    |
| Logical thinking    |                                                                            |
|                     | Using analogies, implications, and syllogism                               |
|                     | Using reasoning in solving problems                                       |

3. Methods
The design of this research is mix-methods research with quantitative and qualitative descriptive approach. This research was conducted on Junior Secondary School students in Sragen, Indonesia. The samples were taken by stratified cluster random sampling [16]. Sampling was done by dividing 49 state SMPs into three categories namely high, medium, and low. Categorization was determined based on the result of National Exam in mathematics of Junior Secondary School in 2016 with mean of 44.35, standard deviation of 18.27, and α = 0.5. The categorization and the number of samples are shown in Table 3.

Table 3. The number of research samples

| No | Category | Number of SMPs | Number of Sample |
|----|----------|----------------|-----------------|
| 1  | High     | 5              | 96              |
| 2  | Medium   | 41             | 96              |
| 3  | Low      | 3              | 67              |
|    | Total    | 49             | 259             |

Data were obtained by interview and test methods. Interview method was used to know students' difficulties in understanding mathematics related to the process of reasoning. A set of mathematical reasoning ability tests consist of four components and are outlined in 11 indicators. Distribution of questions item can be seen in Table 4.

Table 4. Distribution of question item

| Components                   | Indicator | Tryout item | Test item |
|------------------------------|-----------|-------------|-----------|
| Mathematical communication   | 3         | 6           | 3         |
| Mathematical basic skills    | 3         | 10          | 7         |
Content validation was conducted through expert judgment by three experts, namely two lecturers on psychology major and a counseling tutor. Furthermore, tryouts of the mathematical reasoning test instrument consisted of 40 items of question, conducted on 60 8th grade students. A total of 25 question items were selected, having fulfilled power distinct criteria of $D \geq 0.30$ and difficulty level of $0.3 < P < 0.7$ [17] and an index of reliability of 0.84 was obtained.

4. Result and discussion

4.1. Requirements test of one-way anova with unbalanced cells

A set of mathematical reasoning ability tests was conducted to students of schools with National Examination result in three categories namely high, medium, and low. These three categories of data have met the requirements of one-way Anova test with unbalanced cells, namely normality and homogeneity [18] as shown in Table 5 and Table 6.

| Requirements | NE Results Category |
|--------------|---------------------|
| $n_j$        | High    | Medium | Low  |
| $T_j = \Sigma X$ | 96      | 96     | 67   |
| Mean         | 75.2083 | 62.7083| 43.6418 |
| Sd Dev       | 13.9124 | 13.4930| 12.4702 |
| Var ($S_j^2$) | 193.5561| 182.0614| 155.5061 |
| $L_{Max}$    | 0.0675  | 0.0759 | 0.1021 |
| $L_{Table}$  | 0.0904  | 0.0904 | 0.1082 |

Table 6. Requirements homogeneity of variance analysis

| $S_p^2$ | 179.480741 |
| $b_{obs}$ | 0.99635599 |
| $b_k(\alpha; n_1, n_2, n_3)$ | 0.97643359 |

Based on Table 5, the normality test using the Lilliefors method shows that $L_{max} < L_{table}$. This means that each group comes from a normally distributed population. Based on Table 6, the homogeneity test using Bartlett method, it is obtained that $b_{obs} > b_k(0.05; 96, 96, 67)$. This means that all three populations have the same variance.

4.2. One-way variance analysis test with unbalanced cells

ANOVA test results with unbalanced cells as shown in Table 7. Based on $F_\alpha = 3.0000$ and $F_{obs} = 109.5760 > F_\alpha$, the decision is $H_0$ is rejected [18]. It was concluded that not true that all categories have the same mean. Because of the decision that $H_0$ is rejected, then it conducted the multiple comparison test by the Scheffe’s method. The results are as shown in Table 8.
Table 7. Summary of one-way anova test with unbalanced cells

| Source                          | SS     | Df | MS       | F_{obs} | F_{a} |
|---------------------------------|--------|----|----------|---------|-------|
| Mathematical reasoning ability (A) | 39333.548 | 2  | 19666.774 | 109.5760 | 3.0000 |
| Error (G)                       | 45947.070 | 256 | 179.481 |         |       |
| Total (T)                       | 85280.618 | 258 |          |         |       |

Table 8. Summary of multiple comparison test

| Comparison | F_{obs} | F_{a} |
|------------|---------|-------|
| F_{1-2}    | 41.787213 | 6.0000 |
| F_{2-3}    | 79.925334 |       |
| F_{1-3}    | 219.0759 |       |

The analysis result shows that student’s mathematical reasoning ability in schools with high National Exam (NE) in mathematics category are better than students in schools with medium and low category. As well as student’s mathematical reasoning ability in schools with medium categories are better than low category.

4.3. Qualitative descriptive analysis of student’s mathematical reasoning ability

Descriptive qualitative analysis was carried out based on the percentage of score on each component and indicator of mathematical reasoning ability. Table 9 demonstrates score result on each component of mathematical reasoning ability from schools with low, medium, and high NE results category in 2016.

Table 9. Score result of mathematical reasoning ability components

| Mathematical Reasoning Components | NE Results Category |
|-----------------------------------|--------------------|
|                                   | High   | Medium | Low  |
| a. Mathematical communication     | 82%    | 67%    | 47%  |
| b. Mathematical basic skills      | 71%    | 64%    | 33%  |
| c. Mathematical connection        | 59%    | 31%    | 32%  |
| d. Logical thinking               | 82%    | 72%    | 40%  |

Table 9 illustrates that in the schools that fall into the high NE result category, students have low levels of basic mathematical skills and mathematical connection abilities. Meanwhile, students from schools with medium and low category have low levels of abilities on all components. In addition, Table 9 demonstrates that students have low levels of mathematical connection abilities even though they are from the schools with high NE result category. Connection relate with how people can organize the ideas. In this global era, the ability to analyze an information will determine someone’s success to take advantage of opportunities and win the competition [4]. To overcome this case, mathematics teachers need to apply learning method that can improve connections ability.

Further, the analysis of each indicator from reasoning components in Table 9 are then shown in Table 10, 11, 12, and 13. Table 10 illustrates the score results of the mathematical communication component.
Table 10. Scores results of mathematical communication component

| Component / Indicators                          | NE Results Category |
|------------------------------------------------|---------------------|
| **a. Mathematical communication**              |                     |
| 1) Using the language of mathematics to express mathematical ideas. | 100% 88% 71% |
| 2) Develop an argument with precise, concise, and logical. | 86% 72% 45% |
| 3) Determining picture/sketches                | 60% 43% 27% |

Table 10 illustrates that schools in all NE result categories, students have low levels of determining picture/sketches. Communication is related to how students are able to transform a problem into a symbolic mathematical problem, to reorganize the mathematical system, and to interact and dialogue with existing media. The importance of mathematization as a crucial process in the learning and teaching of mathematics [6]. There is a need for a better understanding of teachers’ learning and the quality of their development of know-how and skills about language in mathematics [20]. Communication is so crucial in the 21st Century that the cooperative and collaborative approaches to learning depicted [19]. Based on interview result with some students, it is found that the cause of the lack of ability to identify pictures is that students have difficulty in analysing information from images into appropriate mathematical models. To overcome this problem, it is necessary for teachers to apply mind mapping learning technique.

Table 11. Scores result of mathematical basic skills component

| Component / Indicators                        | NE Results Category |
|------------------------------------------------|---------------------|
| **b. Mathematical Basic Skills**              |                     |
| 1) Understanding the concepts and interpret data | 83% 80% 41% |
| 2) Using mathematical ideas and procedures with flexible. | 67% 47% 28% |
| 3) Building a logical argument                 | 47% 65% 27% |

Based on the results shown in Table 11, it can be seen that most students already understand concepts and interpret information. However, all students have not succeeded in applying mathematical ideas and building logical arguments. In this component, teachers need to improve students’ competence in applying mathematical ideas and building logical arguments. This basic skills indicator is really important in mathematics learning [12]. Because to solve every mathematics questions, students must understand the concept mathematically and be able to give a logical argumentation. Based on interview with students, it is found that the cause of the lack of ability in applying mathematical ideas and building logical arguments is that students are not confident to express their ideas. Self-confidence is necessary in mathematics learning. This is in line with Warner and Kaur’s opinion, they stated that without confidence/self-efficacy in one’s ability, students cannot perform to their potential or at their highest standard. It is even possible that learners with lesser abilities, but with confidence, can outperform higher ability students because belief in oneself can be a powerful influence. Bandura refers to situation-specific, self-confidence as self-efficacy, which is the strength of an individual’s belief that they can successfully perform a given activity or task [19].
Table 12. Score result of mathematical connection component

| Component / Indicators | NE Results Category |
|------------------------|---------------------|
|                        | High    | Medium | Low     |
| c. Mathematical connection |        |        |         |
| 1) Connection capabilities between subjects or materials | 59%     | 31%    | 32%     |
| 2) Stating relations | 70%     | 42%    | 29%     |
| 3) Make generalizations | 51%     | 21%    | 42%     |

Table 12 illustrates that schools in all NE result categories, students have low levels on all indicators of the mathematical connection ability component. Based on the interview, the weakness of mathematical connection ability is caused by the low creativity in thinking and organizing the ideas. They understand the lessons when it described, but students still difficulty making connection between subjects or materials. Based on this analysis, teachers are important to develop innovative learning models to produce students who ready within the competitive world of work in the 21st century [1]. Bertoncelli and Lynass state that employers in the 21st century stimulate that employees be critical thinkers, effective collaborators, innovators, and excellent communicators [21]. Hence, there is a challenge for educators to transform how they prepare learners for the inevitable and impending workforce.

Table 13. Scores acquisition of logical thinking component

| Component / Indicators | NE Results Category |
|------------------------|---------------------|
|                        | High    | Medium | Low     |
| d. Logical thinking |        |        |         |
| 1) Using the analogy, the implications, the syllogism | 82%     | 72%    | 40%     |
| 2) Using the reasoning in problem solving | 81%     | 73%    | 39%     |

In the logical thinking component (Table 13), the schools with high category obtained that the highest percentage score followed by medium and low category. The results of the interviews obtained the information that students can solve the problems which were described, but they can not use analogy to solve more complicated problems. Analogy is parallel drawn between two (or more) entities by indicating one or more respects in which they are similar [21]. In fact, if the students can develop analogy and logical thinking then the student will be easily to solve the problems. Based on this description teachers are be advised to improve the frequency in learning to develop analogical and logical thinking.
Figure 1. Map the low of student’s mathematical reasoning ability

Based on illustrations in Table 10, 11, 12, and 13, the low of student’s mathematical reasoning ability from schools with high, medium, and low category of NE results can be mapped out as shown in Figure 1. Based on Figure 1, it is concluded that students in high category have no problem on logical thinking component. Students in medium category have no problem on indicators using the language of mathematics to express mathematical ideas (a₁) and understanding the concepts and interpret data (b₁). Students in low category have low percentage on all indicators.

5. Conclusions
Based on the results of the research and discussion, it was concluded that the students’s mathematical reasoning ability from schools with high category in mathematics National Examination result is better than medium and low category. The medium is better than the low category. The mathematical connection component is the most difficult component for students in all categories. The weakness of mathematical connection is due to the low creativity of thinking and organizing ability between ideas and concepts in mathematics. In addition to the components connection, in the parts of the mathematical communication component students also have difficulty in pouring ideas and developing logical arguments. This is because, by their lack of confidence in the opinion. This condition needs to be addressed with teachers to apply learning models which can facilitate the students to develop creative thinking and good communication. Looking at the 21st century, schools should prepare their students in order to be able to be critically thinker and be able to organize the ideas. Therefore, there is a challenge for educators to transform how to prepare the students for the inevitable and impending workforce.

Acknowledgments
The authors would like to thank to the referees for their suggestions and this research is part of Sebelas Maret University, Surakarta, Indonesia in the academic year of 2018

References
[1] Zivkovic S 2016 A model of critical thinking as an important attribute for success in the 21st century Procedia–Social and Behavioral Sciences 232 102 (Turkey: Elsevier)
[2] Ball D L and Bass H 2003 Making mathematics reasonable in school (Reston, VA: National Council of Teachers of Mathematics)
[3] Kaur B and Lam T 2012 *Reasoning, Communication and Connections in Mathematics*. (Singapore: World Scientific Publishing Co. Pte. Ltd.)

[4] Roekel D V 2011 *Preparing 21st century students for a global society an educator’s guide to the “Four Cs”* (USA: National Education Association)

[5] Forgues H L, Tian J and Siegler R S 2016 Why is learning fraction and decimal arithmetic so difficult? *Developmental Review ScienceDirect* 38 201

[6] Jupri A and Drijvers P 2016 Student difficulties in mathematizing word problems in algebra *Eurasia Journal of Mathematics, Science & Technology Education* 12 2481

[7] Pantziara M, Gagatsis A and Elia I 2009 Using diagrams as tools for the solution of non-routine mathematical problems Published online: 10 February 2009 *Springer Science + Business Media*

[8] Tajudin N M and Chinnappan M 2016 Relationship between scientific reasoning skills and mathematics achievement among malaysian student *Geografia online™ Malaysian Journal of Society* 46 96

[9] Ayele M 2016 Mathematics teachers’ perceptions on enhancing students’ creativity in mathematics. *IEJME Mathematics Education* 11 3521

[10] Tonels C N 2016 The act of playing and the logical and mathematical reasoning in digital games. *Elsevier B.V. All rights reserved. http://dx.doi.org/10.1016/j.entcom.2016.10.001*

[11] Poon, K. K. & Leung, C. K. (2016) A study of geometric understanding via logical reasoning in Hong Kong *International Journal for Mathematics Teaching and Learning (IJMT).17 3*

[12] Brodie K 2010 *Teaching Mathematical Reasoning in Secondary School Classrooms* (New York: Springer Science+Business Media)

[13] Copi I M, Cohen c and McMahon K 2014 *Person New International Edition Introduction to Logic* (USA: Person Education Limited)

[14] ACER 2017 The Tests Higher Ability Selection from http:// www.acer.org/hast-secondary/the-tests

[15] Solso R L 1991 *Cognitive Psychology* (Boston: Alliyn and Bacon)

[16] Phrasisomphant K 2009 *Sample Size and Sampling Methods* (Vientiane: University of Health Sciences)

[17] Hale C D 2014 *Measuring Learning & Performance: A Primer* (Florida: Charles Dennishale.org)

[18] Heron E 2009 *Analysis of Variance-ANOVA* from https://www.tcd.ie

[19] Warner S and Kaur A 2017 The perceptions of teachers and students on a 21 st century mathematics instructional model *IEJME - Mathematics Education* 12 193

[20] Hajer M 2017 Teachers’ knowledge about language in mathematics professional development courses: from an intended curriculum to a curriculum in action *EURASIA Journal of Mathematics Science and Technology Education* 13 4087

[21] Bertoncelli T, Mayer O and Lynass M 2016 Creativity, learning technique and TRIZ *Elsevier Procedia CIRP 39* 191

[22] Asyari M, Al Muhdhar M, Susilo H and Ibrohim 2016 Improving critical thinking skills through the integration of problem based learning and group investigation *IJLLS* 10 42