The Influence of the Presence of Viscosity on Evolution of Perturbations in the System of Colliding Plates

Denis Ilnitsky$^{1,2}$, Kirill Gorodnichev$^1$, Alexey Serezhkin$^1$, Sergey Kuratov$^1$, Nail Inogamov$^{1,2}$

$^1$ Dukhov Research Institute of Automatics (VNIIA), 22, ul. Sushchevskaya, Moscow 127055, Russia
$^2$ L.D. Landau Institute for Theoretical Physics, 1a, ul. Akademika Semenova, Chernogolovka, Moscow Region, 142432, Russia
E-mail: cyrgo85@gmail.com

Abstract. The results of research on the effect of the presence of viscosity on evolution of perturbations in the system of colliding plates are presented in this work. The range of amplitudes of the initial perturbations wavelengths, at which the effect of viscosity may be neglected, are determined. Comparison of analytical and numerical results are shown.

1. Introduction

The interaction of planar shock waves with various perturbation fields in a medium has been studied in the last half century in numerous theoretical and experimental works (see, e.g. [1–12]). In particular, in the linear approximation, the interaction of a shock wave with acoustic waves was considered in [2, 11], with an isotropic turbulent vorticity fields in [8], and with the perturbation of the density in [9, 12]. In all this works hydrodynamic approach has been used for the research. It is known, that the presence of viscosity provides a stabilized effect on the evolution of perturbations [13–15].

The interaction of the shock wave with the anisotropic density perturbation field was considered in the linear approximation in [16]. The evolution of perturbation field in the system of colliding plates, one of which has initial density perturbation field, was investigated by means of the hydrodynamic approach. Here, in this work, the results of consideration of analogous problem with account for the presence of viscosity are presented. The range of amplitudes of the perturbation wavelengths, when viscosity plays a significant role on perturbation evolution, are obtained.
2. Definition of the problem

We consider the collision of two semi-infinite plates one of which is at rest (target)(see Fig. 1). A spatially anisotropic density perturbation field initially exist in the incident plate (impactor). Collision results in the appearance of shock waves diverging in both plates from the contact discontinuity (see Fig. 1). It is assumed that the velocity of the impactor is quite high (about 5 km/s). For this reason, the elastoplastic properties of the colliding plates can be neglected and the interaction of the shock wave with the density perturbation field in the impactor can be described by the Navier-Stokes equation system

\[ \rho \left( \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \nabla) \mathbf{v} \right) = -\nabla P + \eta \Delta \mathbf{v} + \left( \nu + \frac{\eta}{3} \right) \text{grad div} \mathbf{v}, \]
\[ \frac{\partial \rho}{\partial t} + \text{div} \rho \mathbf{v} = 0, \]
\[ \frac{\partial}{\partial t} \left( \frac{\rho \mathbf{v}^2}{2} + \rho \varepsilon \right) = -\text{div} \left[ \rho \mathbf{v} \left( \frac{\mathbf{v}^2}{2} + \varepsilon + \frac{P}{\rho} \right) - (\nu \sigma') \right], \]

where

\[ \sigma'_{ik} = \eta \left( \frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i} \right) - 2 \frac{\eta}{3} \delta_{ik} \frac{\partial v_l}{\partial x_l}. \]

In following calculations we set the second viscosity coefficient to zero \( \nu = 0 \). The system of Eqs.(1)-(3) should be supplemented by the Rankine-Hugoniot conditions at the shock front [17]:

\[ v_{1n} - v_{2n} = \sqrt{(P_2 - P_1) \left( \frac{1}{\rho_1} - \frac{1}{\rho_2} \right)}, \]
\[ j^2 = \rho_1 \rho_2 P_2 - \rho_1 P_1, \]
\[ \varepsilon_1 - \varepsilon_2 = \frac{1}{2} \left( \frac{1}{\rho_2} - \frac{1}{\rho_1} \right) (P_1 + P_2), \]

where \( v_{1n} (v_{2n}) \) and \( v_{1n} (v_{2n}) \) are the tangential (normal) components of the velocity ahead of and behind the front, respectively; \( j = v_1 \rho_1 = v_2 \rho_2 \) is the mass flux; and \( \varepsilon_1 \) and \( \varepsilon_2 \) are specific internal energies on both sides of the shock wave.
The equation of state of the medium is used in the form of the Mie-Grüneisen equation [18]:

$$P = \frac{\rho_0 \omega_0^2}{n} \left( \left( \frac{\rho}{\rho_0} \right)^n - 1 \right) + \Gamma \rho \zeta,$$

(6)

where $\rho_0$ is the density of the uncompressed ("cold") medium, $c_0$ is the speed of sound in this medium; $\rho$ is the current density; $\Gamma$ is the Grüneisen coefficient, and $n$ is a constant; and $\zeta$ is the specific thermal energy.

3. Propagation of the perturbations in the impactor

Let the initial density perturbation field in the impactor be described by the expression

$$\delta \rho_1 = A_\delta \rho_1 \exp(ik_{0x}x + ik_{0y}y).$$

The axis of anisotropy for such perturbation is directed along the vector $k_0$ (Fig. 1). The shock wave moves from the right to the perturbation at the velocity $(d_1 - v_1)$ in the the negative direction of the $x$ axis $(d_1 - v_1 < 0)$. The angle between the vector $k_0$ and 0$x$ axis is $\theta_0$ $(\tan \theta_0 = k_{0y}/k_{0x})$. At the interaction of the density perturbation field with the shock wave, acoustic and entropy-vortex waves are formed behind the shock front [17].

Perturbations of the hydrodynamic values are the functions of spatial coordinates and time $f(x, y, t)$. Taking into account the expression for the initial perturbation field we will find the solution for acoustic waves in the form $f(x, y, t) = A_f^{(s)} \exp(ik_{0y}y + ik_{2x}x - i\omega_2t)$, and for entropy-vortex waves - $f(x, y, t) = A_f^{(ev)} \exp(ik_{0y}y + i\kappa_{2x}x - i\Omega_2t)$. From linearized system of the equations (1) – (3) we obtain:

for acoustic waves

$$\left( \omega_2 - v_{2x}k_{2x} + \frac{in}{\rho_2}k^2 + \frac{in}{3\rho_2}k^2 k_{2x}^2 \right) \delta v_{2x} = \frac{k_{2x}}{\rho_2} \delta P_2 - \frac{in}{3\rho_2} k_{2x} k_{0y} \delta v_{2y},$$

(7)

$$\left( \omega_2 - v_{2x}k_{2x} + \frac{in}{\rho_2}k^2 + \frac{in}{3\rho_2}k^2 k_{2y} \right) \delta v_{2y} = \frac{k_{0y}}{\rho_2} \delta P_2 - \frac{in}{3\rho_2} k_{2x} k_{0y} \delta v_{2x},$$

(8)

$$(\omega_2 - v_{2x}k_{2x}) \delta P_2 = \rho_2 c_2^2 \left( k_{2x} \delta v_{2x}^{(s)} + k_{0y} \delta v_{2y} \right),$$

(9)

for entropy-vortex waves

$$(\Omega_2 - v_{2x}\kappa_{2x}) \delta v_{2x}^{(ev)} = -\frac{in}{\rho_2} \kappa^2 \delta v_{2x}^{(ev)} - \frac{in}{3\rho_2} \left( \kappa_{2x}^2 \delta v_{2x}^{(ev)} + k_{0y} \kappa_{2x} \delta v_{2y}^{(ev)} \right),$$

(10)

$$(\Omega_2 - v_{2x}\kappa_{2x}) \delta v_{2y}^{(ev)} = -\frac{in}{\rho_2} \kappa^2 \delta v_{2y}^{(ev)} - \frac{in}{3\rho_2} \left( k_{0y} \delta v_{2x}^{(ev)} + k_{0y} \kappa_{2x} \delta v_{2x}^{(ev)} \right),$$

(11)

$$\kappa_{2x} \delta v_{2x}^{(ev)} + k_{0y} \delta v_{2y}^{(ev)} = 0,$$

(12)

where $k_2 = \sqrt{k_{2x}^2 + k_{0y}^2}$, $\kappa_2 = \sqrt{\kappa_{2x}^2 + k_{0y}^2}$.

From (4) - (5) it results

$$k_{0x}(v_1 - d_1) = \omega_2 - k_{2x}d_1 = \Omega_2 - \kappa_{2x}d_1.$$  

(13)

It follows from Eqs. (7) – (9) and (13) that the $x$ - projection of the wave vector of acoustic waves behind the shock front $k_{2x}$ is given by the expression $k_{2x}$.
Figure 2: The dependence of imaginary part to real part ratio \( \text{Im} (k^2x) / \text{Re} (k^2x) \) for the \( x \)-projection of the wave vector of acoustic waves behind the shock front on the first coefficient of viscosity \( \eta \).

Figure 3: The dependence of imaginary part to real part ratio \( \text{Im} (k^2x) / \text{Re} (k^2x) \) for the \( x \)-projection of the wave vector of entropy-vortex waves behind the shock front on the first coefficient of viscosity \( \eta \).

\[
d_1 (k_{0x} - k_{2x}) - k_{0x}v_1 + k_{2x}v_{2x} =
\]
\[
= -4i\eta k_{0y}^2 + 3d_1k_{0x}\rho_2 - 3d_1k_{2x}\rho_2 - 3k_{0x}\rho_2 v_1 + 3k_{2x}\rho_2 v_2.
\]

From Eqs. (10) – (12) and (13) we receive the expression for \( x \)-projection of the wave vector of entropy-vortex waves behind the shock front \( \kappa_{2x} \):

\[
\kappa_{2x} = -\frac{i}{2\eta} \left[ d_1\rho_2 + \rho_2v_2 - \sqrt{(d_1\rho_2 + \rho_2v_2)^2 - 4i\eta (i\eta k_{0y}^2 + (v_1 + d_1) k_{0x}\rho_2)} \right].
\]

The frequencies for both types of waves are obtained from (13).

In order to receive values of the perturbations amplitudes the system of Eqs. (10) – (12) should be supplemented by the Rankine-Hugoniot conditions written in the linear approximation (4) – (5). Analytical solution of this system of equations leads to a very lengthy expressions, and so we don’t present it in this work. Quantitative results will be shown below.

We investigate the influence of the presence of viscosity on perturbation field development behind the shock front on specific example. We consider the collision of two semi-infinite plates: impactor (iron) and target (aluminum). Definition of the problem is analogous to that described in section 5. The dependence of imaginary part to real part ratio \( \text{Im} (k_{2x}) / \text{Re} (k_{2x}) \) for the \( x \)-projection of the wave vector of acoustic waves behind the shock front on the first coefficient of viscosity \( \eta \) is shown in Fig. 2. Analogous dependence for entropy-vortex waves is presented in Fig. 3. For both metals we use \( \eta \sim 30 \text{ Pa s} \) [13, 14]. From Fig. 2 - 3 it is clear that the effect of viscosity is negligible for perturbation wavelengths more than 20 mkm.

It was shown [20] that the regime of propagation of sound behind the shock front depends on the angle between the axis of anisotropy of the perturbation field and the velocity of the
shock wave. There are three types of qualitatively different regimes of this propagation. In this work we consider only the case when sound propagates from the shock wave towards the contact discontinuity. For this reason we should investigate interaction of this wave with the contact discontinuity.

4. Interaction of perturbations with the contact discontinuity

Conditions of the equality of pressures and normal components of the velocity take place on both sides of the contact discontinuity. From this equations it follows:

\[ k_{2x}v_2 - \omega_2 = k_{2x}^{ref}v_2 - \omega_2^{ref} = k_{3x}^{tr}v_2 - \omega_3^{tr}, \]  

where \( k_{2x}^{ref}, \omega_2^{ref} \) are wave vector and frequency of the acoustic/entropy-vortex wave, reflected from the contact boundary; \( k_{3x}^{tr}, \omega_3^{tr} \) are wave vector and frequency of the acoustic/entropy-vortex wave, transmitted through the contact boundary.

The contact boundary values for the entropy-vortex wave is joining with the homogeneous solution in the target. This homogeneous solution decays exponentially with moving away from the contact discontinuity. Thus, the entropy-vortex perturbations exist only in very narrow region near the contact in the target.

For the wave vectors of reflected and transmitted acoustic waves we obtain:

\[ k_{2x}^{ref} = -k_{2x}, \]
\[ k_{3x}^{tr} = \frac{1}{4\eta\rho_2c_2^2(k_2^2) - \rho_3c_3^2[4\eta k_2^2 - 3i\rho_2(\omega_2 - k_{2x}v_2)]} \times \left[ \rho_3c_3^2k_0^2 \left( 4\eta(k_{2x}^2)^2 - 3i\rho_2(\omega_2 - k_{2x}v_2) \right) - \rho_2c_2^2k_2^2 \left( 4\eta k_0^2 - 3i\rho_3(\omega_2 - k_{2x}v_2) \right) \right]. \]

For the pressure amplitudes of reflected and transmitted acoustic waves we receive:

\[ \delta P_{2}^{ref} = -\frac{k_{2x}}{3\rho_2(\omega_2 - k_{2x}v_2) + 4i\eta(k_{2x}^2 + k_y^2)} - \frac{k_{3x}^{tr}}{3\rho_3(\omega_2 - k_{2x}v_2) + 4i\eta((k_{3x}^{tr})^2 + k_y^2)} \delta P_2, \]
\[ \delta P_{3}^{tr} = \delta P_2 + \delta P_2^{ref}. \]

5. Results of the numerical calculations

The problem of two semi-infinite plates impact is analyzed numerically also. For this task we use hydrocode TIS-2D [19], which is based on the finite-volume method. We consider collision of the iron impactor with initially perturbed density field with homogeneous aluminum target. The initial perturbations amplitude is \( \delta\rho_1/\rho_1 \sim 5\% \).

Two different wavelengths of the initial perturbation field was taken into consideration \( \lambda_x = 100 \, \text{mkm} \) and \( \lambda_x = 50 \, \text{mkm} \). The results of numerical calculations for the case \( \lambda_x = 100 \, \text{mkm} \) at instant 150 ns are presented in Fig. 4 and 5. It is evident that attenuation is negligible and that the numerical data confirms the analytical results.
The results of numerical calculations for the case $\lambda_x = 50$ mkm at instant 75 ns are presented in Fig. 6 and 7. In this case attenuation is significant. The results of theoretical and numerical analysis are in good agreement. For lesser wavelengths attenuation increases and the linear approximation for the analytical consideration is not sufficient.

6. Conclusions

The evolution of the initial density perturbation field in the system of two colliding plates are considered with account for the presence of the viscosity. The parameters of the perturbation field in the compressed region was found analytically. Numerical calculations confirms theoretical results. It is shown, that the presence of viscosity has significant effect at the initial perturbation wavelengths $\lambda_x$ in order and less than 100 mkm.
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