Characterization of an ArF excimer laser beam from measurements of the Wigner distribution function

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Abstract. An ArF excimer laser beam at a wavelength of 193 nm has been characterized by a quantitative determination of the Wigner distribution function. The setup, comprising a spherical lens, a rotating cylindrical lens and a moveable ultraviolet-sensitive CCD detector, enabled the mapping of the entire four-dimensional phase space within less than 20 min. Experiments yielded complete information about second-order moment-based parameters, spatial coherence, wavefront, beam profiles, as well as beam propagation and local distributions of radiant intensities.

Contents

1. Introduction 2
2. Theory 2
3. Experimental 3
4. Results 4
5. Conclusion 5
Acknowldgments 6
Appendix 6
References 9

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1. Introduction

A couple of laser applications in scientific research as well as industrial production require a more complete beam characterization than can be obtained using standard techniques such as, for example, caustic measurements [1], beam profiling [2] or wavefront sensing [3]–[5]. In particular for the case of partially coherent sources, taking into consideration the second-order correlations, i.e. the mutual intensity [6] of the stochastic wave field, is mandatory for reliable beam propagation or for the design of complex optical systems. The optimization of homogeneous mask illumination, required in 193 nm lithography for instance, requires a locally resolved angular distribution of the beam in order to achieve reliable results during the development stage [7]. Direct measurement of the two-point correlations by, for example Young’s experiment, however, is highly time consuming, and the resulting interference patterns are difficult to evaluate. In contrast, it has been shown that the Wigner distribution [8]–[12], being a two-dimensional (2D) Fourier transform of the mutual intensity \( J \) as well as the 4D Fourier transform of \( J \) itself [13], can be reconstructed from intensity profiles of the beam. However, multi-mode lasers relevant to industrial applications, in particular excimer lasers emitting in the ultraviolet (UV) spectral range, have not yet been investigated, although a comprehensive beam characterization is highly desirable for accurately modeling these partially coherent sources.

Here, after a brief review of the Wigner distribution, we present a setup with a minimum number of optical standard elements and an automated protocol for measurement, followed by a discussion of the results from first measurements of the 4D Wigner distribution at an ArF excimer laser.

2. Theory

In the following, only scalar, quasi-monochromatic and paraxial light beams are considered. Moreover, we average the time-dependent irradiance over the exposure time of the detector and treat the field as stationary. Then, the Wigner distribution \( h \) can be written in terms of the mutual intensity \( J \) as a 2D Fourier transform of the latter according to [14]

\[
h(x, u) = \left( \frac{k}{2\pi} \right)^2 \int \int J(x - s/2, x + s/2) e^{iux} ds^2,
\]

where \( x = (x, y) \) and \( s = (s_x, s_y) \) are 2D spatial and \( u = (u, v) \) angular coordinates in a plane perpendicular to the direction of beam propagation and \( k \) is the mean wave number of light. As \( J \) is Hermitian, \( h \) is real, although it may become negative in some regions. However, its marginal distributions with respect to \( x \) and \( u \) are always nonnegative and yield the irradiance (near-field) \( I(x) \) and the radiant intensity (far-field) \( I_{FF}(u) \), respectively [14].

The propagation of the Wigner distribution \( h \) and its 4D Fourier transform \( \tilde{h} \) through static and lossless paraxial systems, denoted by a \( 4 \times 4 \) optical ray propagation \( ABCD \) matrix \( S \) from an input (i) to an output (o) plane, reads [8, 14, 15]

\[
\tilde{h}_i(Dx - Bu, -Cx + Au) = h_o(x, u),
\]

\[
\tilde{h}_i(A^T w + C^T t, B^T w + D^T t) = \tilde{h}_o(w, t),
\]

where \( (w, t) \) are the Fourier-space variables corresponding to \( (x, u) \).
Considering a set \( \{ p \} \) of parameters and a set of irradiance profiles \( I_{\{ p \}}(x, y) \) recorded at positions that are connected to an arbitrary reference plane via the corresponding ray transformation matrices \( S_{\{ p \}} \), one obtains, according to the marginal property of \( h \) and well-known Fourier relations,

\[
\int \int h_{\{ p \}}(x, u) \, d^2u = I_{\{ p \}}(x) \quad \xrightarrow{\text{FT}} \quad \tilde{I}_{\{ p \}}(w) = \tilde{h}_{\{ p \}}(w, 0),
\]

and from (3) and (4),

\[
\tilde{h}_{\text{ref}}(A_{\{ p \}}^T w, B_{\{ p \}}^T w) = \tilde{I}_{\{ p \}}(w).
\]

Equation (5) corresponds to the projection slice theorem of tomography [16] and states that Fourier transforming a \((u, v)\)-projection of the propagated Wigner distribution \( h_{\{ p \}} \) gives a 2D central slice through the Fourier transformed distribution \( \tilde{h}_{\text{ref}} \) at the reference plane. The linear maps \( S_{\{ p \}} \) determine the angular orientation of the slices that cover the complete reciprocal phase space, provided the parameter set \( \{ p \} \) is chosen properly. In particular, for an experimental arrangement as described below, the variation of the detector position \( z \) and the orientation \( \alpha \) of the cylinder lens together with a discrete translation parameter \( l_d \) are sufficient in order to map, in principle, the complete phase space (see the appendix).

3. Experimental

Figure 1 shows the experimental setup for the measurement of the 4D Wigner distribution. It consists of a 2'' fused silica plano-convex lens with \( f_s = 168 \) mm focal length, a 50 × 60 mm² fused silica cylindrical lens with \( f_c = 347 \) mm at a distance of \( s = 65 \) mm and a 12 bit digital UV-sensitive CCD camera with 2/3'' chip and 1280 × 1024 pixel resolution. The camera is mounted on a PC-driven linear stage and can be moved within a \( z \)-interval of \([34 \, \text{mm}, 184 \, \text{mm}]\) behind the cylindrical lens. Orientation of the latter within the angular range of \([0, \pi]\) is provided by a PC-controlled rotation stage. However, as a consequence of the limited \( z \)-interval the phase
space mapping according to equation (5) with the above parameter set remains incomplete. Therefore an additional motor-driven four-mirror delay line, which can be inserted in front of the spherical lens, was provided to sample missing phase space areas in a second turn. Numerical tests as well as the experiments showed that almost a complete mapping can be achieved with a delay of \( t_d = 800\, \text{mm} \) (see the appendix).

Typically \( n = 20–40 \) \( z \)-positions and \( 10–20 \) evenly spaced angular positions are sampled within a sequence. To enforce an approximate isotropy of the slice distribution, the \( z \)-positions are selected in such a way as to obtain a constant angular spacing \( \Delta \varphi = \pi / n \) between two steps for the \((w', 0, t', 0)\) plane in reciprocal phase space. Sampling was performed fully automated by a PC program with a macro language, and all records were stored in permanent memory for subsequent data processing. The measurement time for 750 profiles was approximately 20 min with an average of 2 frames per record.

Our evaluation starts with the determination of the second-order beam matrix [1] at the reference plane from the centroids and second-order spatial moments \((x^2), (y^2)\) and \((xy)\) of each profile, yielding important beam parameters such as \( M^2 \), beam divergence and waist [1]. For reconstruction, each 2D record is Fourier transformed and subsequently mapped into the 4D reciprocal phase space according to equation (5) and using the nearest-neighbor method for interpolation onto a regular 4D grid. In a final step, \( h \) is obtained from a 4D inverse Fourier transform. The last stage consists of calculating the desired beam characteristics, for example the irradiance \( I(x) \) and the local angular distribution \( \beta(x, u) \),

\[
\beta(x, u) = \lambda^2 P^{-1} h_0 \left( S_{z=0} (x, u)^T \right) \cdot \left( \frac{u}{\sqrt{1 - u^2}} \right),
\]

in arbitrary \( z \)-planes, where \( P \) is the total beam power or pulse energy and \( \lambda \) the mean wavelength, respectively. When integrated over \( u \), \( \beta \) yields the local wavefront gradient but, in view of (6), it is clear that \( \beta \) does not point into the positive half-space whenever \( h < 0 \). However, for highly incoherent sources such as excimer laser beams, it can be shown that \( h \) approaches the classical brightness [6] and thus stays positive in any region containing a considerable amount of power. In those cases, \( P \cdot \beta / \lambda^2 \, d^2x \, d^2u \) gives the power of a light ray emerging from \( x \) into the \((u, \sqrt{1 - u^2})\)-direction. Coherence-related quantities, such as mutual intensity \( J \) and global degree of coherence \( K \) of the beam, can be obtained via [17]

\[
J(x, s) = \int \int h(x, u) \, e^{-ikus} \, d^2u \quad \text{and} \quad K = \frac{\lambda^2}{P^2} \int \int |h(x, u)|^2 \, d^2x \, d^2u.
\]

4. Results

Figure 2 (left) shows the irradiance distribution of an ArF excimer laser (Lambda Physik Novaline A2010) 0.5 m behind the laser exit. 608 profiles were recorded for the measurement of the Wigner distribution. The large number of modes contributing to the emission results in a large \( M^2 \) of 153 and 42 in the direction of the vertical and horizontal axes, respectively, as well as a low global coherence of \( K = 4.6 \times 10^{-4} \). Correspondingly, the degree of coherence \( |\Gamma(x_1, x_2)/\sqrt{T(x_1)T(x_2)}| \) decreases quite rapidly with increasing mutual distance as shown in the right-hand diagram of figure 2. For that reason no negative values of the Wigner distribution appear within the beam support, as can be discovered from the projected Wigner distributions (figure 3). Negative values outside the beam support are reconstruction artifacts from the limited number of measurements.

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Figure 2. Experimental irradiance distribution 0.5 m behind the laser exit (left) and calculated mutual coherence (right) averaged over the indicated central part of an ArF excimer laser beam (Lambda Physik Novaline A2010) at 2 Hz and 10 mJ pulse$^{-1}$.

Figure 3. Wigner distribution $h_x(x,u)$ and $h_y(y,v)$ of the ArF excimer laser projected on the horizontal (left) and vertical (right) coordinate planes (cf text).

Furthermore, the low beam coherence is revealed in a considerable spread of the local propagation direction, which can be seen in figure 4, where the local brightness at a plane 1.5 m from the laser exit is displayed as a polar plot for nine positions within the beam profile. Taking $(x, z_{ref}), (u\sqrt{1-u^2})$ as the ray coordinates with the associated weight function $h(x, u)$, this information may serve as an input for ray tracing in order to design or optimize optical systems under partially coherent illumination.

5. Conclusion

An industrial excimer laser has been investigated using a setup for the measurement of the 4D Wigner distribution as a tool for the characterization of partially coherent light beams.
Figure 4. Angular distribution $\beta(x, u)$ of the ArF laser beam for nine selected lateral positions within the beam profile, calculated 1.5 m behind the laser exit.

Full automation enables measurement periods of the order of only 20 min, depending on the complexity of the test beam, reducing the stability demands on the source. Thus, together with the comprehensive amount of information yielded, the method may find increasing use in scientific applications or even in production environments. Apart from excimer laser applications in lithography, where the design of illumination optics and beam delivery may be optimized, some partially coherent sources such as self-amplified spontaneous emission-type free electron lasers or high-power diodes and solid-state lasers may benefit from quantitative determination of the beam coherence. In the latter case, the modal decomposition of the mutual intensity may, for instance, yield valuable information for the improvement of resonator design or efficient fiber coupling.

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Appendix

To show that the set of parameters $\{z, \alpha, l_d\}$ is sufficient for mapping the reciprocal phase space, consider first the corresponding $4 \times 4 S_{\{z, \alpha, l_d\}}$-transformation matrix through the measurement system:

$$
S_{\{z, \alpha, l_d\}} = S_z R_a S_{f_s} R_a^T S_z S_{f_c} S_{l_d} R_a S_{f_s} S_{f_c} S_{l_d} R_a T_{\{z, l\}} R_a^T.
$$

(A.1)

In (A.1) $S_z$, $S_s$ and $S_{l_d}$ denote the free space translations with distances $z$, $s$ and $l_d$, respectively, $S_{f_s}$ and $S_{f_c}$ represent the focusing action of the spherical and the horizontally aligned cylindrical lenses with focal lengths $f_s$ and $f_c$, whereas $R_a$ describes the rotation of the
latter with respect to the detector reference frame:

\[
S_z = \begin{pmatrix} I & B_z \\ 0 & I \end{pmatrix}, \quad S_x = \begin{pmatrix} I & B_x \\ 0 & I \end{pmatrix}, \quad S_{l_d} = \begin{pmatrix} I & B_{l_d} \\ 0 & I \end{pmatrix},
\]

\[
S_{f_s} = \begin{pmatrix} I & 0 \\ C_{f_s} & I \end{pmatrix}, \quad S_{f_c} = \begin{pmatrix} I & 0 \\ C_{f_c} & I \end{pmatrix}, \quad R_\alpha = \begin{pmatrix} R & 0 \\ 0 & R \end{pmatrix},
\]

(A.2a)

with \( I \) being the \( 2 \times 2 \) unit matrix and

\[
B_z = \begin{pmatrix} z & 0 \\ 0 & z \end{pmatrix}, \quad B_x = \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix}, \quad B_{l_d} = \begin{pmatrix} l_{d} & 0 \\ 0 & l_{d} \end{pmatrix},
\]

\[
C_{f_s} = \begin{pmatrix} -f_s^{-1} & 0 \\ 0 & -f_s^{-1} \end{pmatrix}, \quad C_{f_c} = \begin{pmatrix} -f_c^{-1} & 0 \\ 0 & 0 \end{pmatrix},
\]

(A.2b)

\[
R = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix}. \]

Here \( s, f_s, \) and \( f_c \) are regarded as fixed, \( z \) and \( \alpha \) vary continuously within \([z_{\text{min}}, z_{\text{max}}]\) and \([0, \pi]\), respectively, and \( l_{d} \) may assume a couple of discrete values. The composite matrix \( T_{z,l} \) of the nonrotational terms reads

\[
T_{[z,l]} = \begin{pmatrix} A_x & 0 & B_x + l_d A_x & 0 \\ 0 & A_y & 0 & B_y + l_d A_y \\ C_x & 0 & D_x & 0 \\ 0 & C_y & 0 & D_y \end{pmatrix},
\]

(A.3)

with diagonal \( 2 \times 2 \) submatrices and

\[
A_x = 1 - \frac{s}{f_s} - z \left( \frac{1}{f_c} + \frac{1}{f_s} - \frac{s}{f_c f_s} \right), \quad A_y = 1 - \frac{s}{f_s} - z, \quad B_x = s + z \left( 1 - \frac{s}{f_c} \right), \quad B_y = s + z,
\]

(A.4)

\( C_{x,y} \) and \( D_{x,y} \) being unimportant here. The mapping

\[
\begin{pmatrix} w' \\ t' \end{pmatrix} = \begin{pmatrix} R & 0 \\ 0 & R \end{pmatrix} T_{[z,l]}^{T} \begin{pmatrix} R & 0 \\ 0 & R \end{pmatrix}^{T} \begin{pmatrix} w \\ 0 \end{pmatrix} \leftrightarrow \begin{pmatrix} w \\ t \end{pmatrix}
\]

\[
\begin{pmatrix} A_{p,w}^{T} & B_{p,w}^{T} \\ 0 & 0 \end{pmatrix}^{T}
\]

(A.5)

according to equation (5) then reads explicitly

\[
w'_x \cos \alpha - w'_y \sin \alpha = A_x (w_x \cos \alpha - w_y \sin \alpha), \quad w'_y \cos \alpha + w'_y \sin \alpha = A_y (w_y \cos \alpha + w_x \sin \alpha),
\]

\[
t'_x \cos \alpha - t'_y \sin \alpha = (B_x + l_{d} A_x) (w_x \cos \alpha - w_y \sin \alpha), \quad t'_y \cos \alpha + t'_y \sin \alpha = (B_y + l_{d} A_y) (w_y \cos \alpha + w_x \sin \alpha),
\]

(A.6a–d)
Equation (A.6) is a nonlinear set of four equations and, in the following, we solve it in the three nontrivial cases, (a) \( w' \neq 0, t' \neq 0 \), (b) \( w' = 0, t'_x \neq 0 \) and (c) \( t' = 0 \).

(a) Provided \( w', t' \neq 0 \), dividing the third and fourth equations of (A.6) by the first and second ones, respectively, yields

\[
\tan \alpha = \frac{B_y w'_y - A_y (t'_y - l_d w'_y)}{A_y (t'_y - l_d w'_y) - B_y w'_y} = \frac{B_x w'_x - A_x (t'_x - l_d w'_x)}{B_x w'_x - A_x (t'_x - l_d w'_x)}. \tag{A.7}
\]

After multiplying with the common denominator, the second equality in (A.7) becomes

\[
A_y A_y + (B_y B_y + l_d (A_y B_y + A_y B_x) + l_d^2 A_y A_y) q^2 - (B_y A_y + B_y A_x + 2 l_d A_y A_y) q \cos \chi = 0, \tag{A.8}
\]

which depends on (\( w', t' \)) only through

\[
q = |w'|/|t'| \quad \text{and} \quad \frac{w' \times t'}{|w'\times t'|} = \cos \chi.
\]

By the substitution of \( A_x, A_y, B_x \) and \( B_y \) from equation (A.4), one obtains a quadratic equation in \( z \), which, for \( l_d \) fixed, may yield complex solutions for some regions in the \((q, \chi)\)-plane. However, in the limit \(|l_d| \to \infty\) the real and distinct solutions of (A.8) are the zero points \( z_x \) and \( z_y \) of \( A_x \) and \( A_y \):

\[
z_x = \frac{f_c (f_s - s)}{f_c + f_s - s} \quad \text{and} \quad z_y = f_s - s, \tag{A.9}
\]

respectively. Thus, given a domain \( \sum_{q, \chi} = (0, q_{\max}) x [0, \pi) \) in the \((q, \chi)\)-plane, an optical configuration \( \{s, f_s, f_c, z_{\min}, z_{\max}\} \) with \( z_{x,y} \in [z_{\min}, z_{\max}] \) and a distance \( z_{\text{ref}} \) of the reference plane to the spherical lens, there exists a delay distance \( l_{d,\min} \in (-\infty, \infty) \) such that for all \((q, \chi) \in \sum_{q, \chi}\) at least two solutions \( z \) of (A.8): \( \sum_{z} = \{z_{1,2}(l_d = z_{\text{ref}}), z_{1,2}(l_d = z_{\text{ref}} + l_{d,\min})\} \) are real and satisfy \( z \in [z_{\min}, z_{\max}] \).

The subset \( \sigma_1 \subset \Sigma_1 \) that simultaneously solves (A.7) can then be figured out by back-substitution. Once \( z \) is determined, \( \alpha \) follows from the first part of (A.7) and \( w \) from the first and second equations of (A.6).

Figure A.1 at the end of this section shows the result of a numerical evaluation of (A.8) for the experimental setup described in section 3. Only a small part of the phase space remains uncovered. For a larger delay, the complete space could have been reached, but that was not feasible in this case for technical reasons.

(b) If \( w' = 0 \) and \( t'_x \neq 0 \), on the other hand, \( z \) is determined from \( A_x = 0 \), yielding

\[
z = \frac{f_c (f_s - s)}{f_c + f_s - s}, \tag{A.10}
\]

which satisfies equation (A.6a) \( A_y = 0 \) will do equally well for (A.6b) if \( t'_y \neq 0 \). Then the substitution of

\[
w_x = B_x^{-1} t'_x \cos^2 \alpha - B_x^{-1} t'_y \sin \alpha \cos \alpha + (B_y + l_d A_y)^{-1} t'_y \sin \alpha \cos \alpha + (B_y + l_d A_y)^{-1} t'_x \sin^2 \alpha, \]

\[
w_y = (B_y + l_d A_y)^{-1} t'_y \cos^2 \alpha + (B_y + l_d A_y)^{-1} t'_y \sin \alpha \cos \alpha - B_x^{-1} t'_x \sin \alpha \cos \alpha + B_x^{-1} t'_y \sin^2 \alpha, \tag{A.11}
\]
Figure A.1. Camera $z$-position according to (A.8), calculated at $50 \times 50 (q, \chi)$ points between $(0, 0)$ and $(0.01, \pi)$ for $f_c = 347$ mm, $f_s = 168$ mm and $l_d = 0, 800$ mm. Complex roots or $z$-positions outside the camera range of [34 mm, 184 mm] are shown in red.

which follows from (A.6c) and (A.6d), into (A.6b) gives a cubic equation in $\tan \alpha$:

$$t'_y + t'_x \cdot \tan \alpha + t'_y \cdot \tan^2 \alpha + t'_x \cdot \tan^3 \alpha = 0,$$

(A.12)

which is solved by

$$\tan \alpha = -t'_y/t'_x.$$

(A.13)

Finally, back substitution of (A.10) and (A.13) into (A.11) yields $(w_x, w_y)$.

(c) At least, if $t' = 0$, $\alpha$ is determined from

$$\alpha = \arctan \left[ w'_x / w'_y \right]$$

(A.14)

by setting (A.6a) equal to zero. $z$ follows from $B_y + l_d A_y = 0$ to

$$z = -\frac{l_d f_s + s f_k - s l_d}{f_k - l_d},$$

(A.15)

and $w$ is obtained via the linear system

$$w_x \cos \alpha - w_y \sin \alpha = 0,$$

$$w_x \sin \alpha + w_y \cos \alpha = A_y^{-1} \left( w'_x \sin \alpha + w'_y \cos \alpha \right).$$

(A.16)

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