Measurement of the Branching Fractions of the Rare Decays 
\[ B^0 \rightarrow D_s^{(*)} + \pi^-, \quad B^0 \rightarrow D_s^{(*)} + \rho^-, \quad \text{and} \quad B^0 \rightarrow D_s^{(*)} + K^{(*)} \]

B. Aubert, M. Bona, Y. Karyotakis, J. P. Lees, V. Poirer, E. Prencipe, X. Prudent, V. Tisserand, J. Garra Tico, E. Grauges, L. Lopez, A. Falano, M. Pappagallo, G. Eigen, B. Stugu, L. Sun, G. S. Abrams, M. Battaglia, D. N. Brown, J. Button-Shafer, R. N. Cahn, R. G. Jacobsen, J. A. Kadyk, L. T. Kerth, Yu. G. Kolomensky, K. Kulkarni, G. L. Lynch, I. L. Ospienkov, M. T. Ronan, A. Suzuki, K. Tackmann, T. Tanabe, W. A. Wenzel, C. M. Hawkes, N. Soni, A. T. Watson, H. Koch, T. Schroeder, D. Walker, D. J. Asgeirsson, T. Cuhadar-Donszelmann, B. G. Fulsom, C. Hearty, T. S. Mattison, J. A. McKenna, M. Barrett, M. Saleem, L. Teodorescu, V. E. Blinov, A. D. Bukin, A. R. Buzzaev, V. P. Druzhinin, V. B. Golubev, A. P. Omuchin, S. I. Serednyakov, Yu. I. Skovpen, E. P. Solodov, K. Yu. Tsytsarev, M. Bondioli, S. Curry, I. Eschrich, D. Kirkby, A. J. Lankford, P. Lund, M. Mandelkern, E. C. Martin, D. P. Stoker, S. Abachi, C. Buchanan, J. W. Gay, F. Liu, O. Long, B. C. Shen, G. M. Vitug, Z. Yasin, L. Zhang, V. Sharma, C. Campagnari, T. M. Hong, D. Kovalskyi, M. A. Mazur, J. D. Richman, T. W. Beck, A. M. Eisner, C. J. Flacco, C. A. Heusch, J. Kroese, W. S. Lockman, T. Schalk, B. A. Schumm, A. Seiden, L. Wang, M. G. Wilson, L. O. Winstrum, C. H. Cheng, D. A. Doll, B. Echenard, F. Fang, D. G. Hitlin, I. Narsky, T. Pietenko, F. C. Porter, R. Andreassen, G. Mancinelli, B. T. Meadows, K. Mishra, M. D. Sokoloff, F. Blanc, P. C. Bloom, W. T. Ford, A. Gaz, J. F. Hirschauder, A. Kreisell, M. Nagel, U. Nauenberg, A. Olivas, J. G. Smith, K. A. Ulmer, S. R. Wagner, R. Ayad, A. M. Gabareen, A. Soffer, W. H. Toki, R. J. Wilson, D. D. Altenburg, E. Feltesse, A. Hauke, H. Jasper, M. Kurbach, J. Merkel, A. Petzold, B. Spaan, K. Wackser, V. Klose, M. Kobl, H. M. Lacker, W. F. Mader, R. Nogowski, K. R. Schubert, R. Schwierz, J. E. Sundermann, A. Volk, D. Bernard, G. R. Bonneau, E. Latour, Ch. Thiebaux, M. Verderi, J. P. Clark, W. Gradl, S. Player, J. E. Watson, M. Andreotti, D. Bettoni, C. Bozzi, R. Calabrese, A. Cecchi, G. Cibinetto, P. Franchini, E. Lupp, M. Negrini, A. Petrella, L. Piemontese, V. Santoro, F. Anulli, R. Baldini-Ferroli, A. Carlin, R. de Sangro, G. Finocchiaro, S. Pacetti, P. Patitteri, I. M. Peruzzi, M. Piccolo, M. Rama, A. Zallo, A. Buzzo, R. Contri, M. Lo Vetere, M. M. Macri, M. R. Monge, S. Passaggio, C. Patrignani, E. Robutti, A. Santroni, S. Tosi, K. S. Chaisangvanthum, M. Morii, R. S. Dubitzky, J. Marks, S. Schenk, U. Uwer, D. J. Bard, P. D. Dauncey, A. N. Bashir, W. Panduro Vazquez, M. Tibbetts, P. K. Behera, X. Chai, M. J. Charles, U. Mallik, J. Cochran, H. B. Crawford, C. Dong, T. Meyer, S. Prell, E. I. Rosenberg, A. E. Rubin, Y. Y. Gao, A. V. Gritsan, Z. J. Guo, C. K. Lai, A. G. Denig, M. Fritsch, G. Schott, A. Arnaud, J. Béguilé, A. D’Orazio, M. Davier, J. Firmino da Costa, G. Grosdidier, A. Höcker, V. Lepeltier, F. Le Didierber, P. M. Lutz, S. Pruvot, P. Roudeau, M. H. Schune, J. Serrano, V. Sordini, A. Stocchi, W. F. Wang, G. Wormser, D. J. Lange, D. M. Wright, I. Bingham, J. P. Burke, C. A. Chavez, J. R. Ry, E. Gabathuler, R. Gamet, D. H. Hutcroft, D. J. Payne, C. Touramanis, A. J. Bevan, K. A. George, F. Di Lodovico, R. Sacco, M. Sigamani, G. Cowan, H. U. Fleacher, D. A. Hopkins, R. S. Paramasivam, F. Salvatore, A. W. C. Wren, D. N. Brown, C. L. Davis, K. E. Alwyn, N. R. Barlow, R. J. Barlow, R. J. Parmer, R. J. M. Chia, C. L. Edgar, G. D. Lafferty, T. J. West, J. I. Yi, J. Anderson, C. Chen, A. Jawahery, D. A. Roberts, G. Simi, J. M. Tugule, C. Dallapiccola, S. S. Hertzbach, X. Li, E. Salvati, S. Saremi, R. Cowan, D. Dujmic, P. H. Fisher, K. Koeneke, G. Sciolla, M. Spitznagel, F. Taylor, R. K. Yamamoto, M. Zhao, E. M. McLeadhin, P. M. Patel, S. H. Robertson, A. Lazzaro, V. Lombardo, F. Palombo, J. M. Bauer, L. Cremaldi, V. Eschenburg, R. Godang, K. Kroeger, D. A. Sanders, D. J. Summers, H. W. Zhao, S. Brunet, D. Côté, M. Simard, P. Taras, F. B. Viaud, H. Nicholson, G. De Nardo, L. Lista, D. Monorchio, C. Sciaccia, M. A. Baak, G. Raven, H. L. Snoek, C. P. Jessop, K. J. Knoepfel, J. M. LoSecco, G. Benelli, L. A. Corwin, K. Honscheid, H. Kagan, R. Kass, P. J. Morris, J. J. Regensburger, S. J. Sekula, Q. K. Wong, N. L. Blount, J. Brau, R. Frey, O. Ionkina, J. A. Kolb, M. Lu, R. Rahmat, N. B. Sinev, D. Strom, J. Strube, E. Torrence, G. Castelli, N. Gagliardi, M. Margoni, M. Morandin, M. Morozzi, R. Morotto, F. Simonetto, R. Stroili, C. Voci, P. Del Amo Sanchez, B. H. Haim,
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We report the measurement of the branching fractions of the rare decays $B^0 \rightarrow D^{(*)+}_s \pi^-$, $B^0 \rightarrow D^{(*)+}_s K^-$, and $B^0 \rightarrow D^{(*)+}_s K^{(*)+}$ in a sample of $381 \times 10^6$ $\Upsilon(4S)$ decays into $B\bar{B}$ pairs collected with the BABAR detector at the PEP-II asymmetric-energy $e^+e^-$ storage ring. We present evidence for the decay $B^0 \rightarrow D^{(*)+}_s K^{(*)+}$ and the vector-vector decays $B^0 \rightarrow D^{(*)+}_s \rho^-$ and $B^0 \rightarrow D^{(*)+}_s K^{(*)+}$, as well as the first measurement of the vector meson polarization in these decays. We also determine the ratios of the CKM-suppressed to CKM-favored amplitudes $r(D^{(*)+}_s \pi)$ and $r(D^{(*)+}_s \rho)$ in decays $B^0 \rightarrow D^{(*)+}_s \pi^+$ and $B^0 \rightarrow D^{(*)+}_s \rho^+$, and comment on the prospects for measuring the $CP$ observable $\sin(2\beta + \gamma)$.

PACS numbers: 12.15.Hh, 11.30.Er, 13.25.Hw
I. INTRODUCTION

The Cabibbo-Kobayashi-Maskawa (CKM) quark flavor-mixing matrix \[^1\] provides an elegant explanation of the origin of CP violation within the Standard Model. CP violation manifests itself as a non-zero area of the unitarity triangle \[^2\]. While it is sufficient to measure one of the angles to demonstrate the existence of CP violation, the unitarity triangle needs to be over-constrained by experimental measurements in order to demonstrate that the CKM mechanism is the correct explanation of this phenomenon. Precision measurements of the sides and angles of the unitarity triangle are the focus of the physics program at the B Factories. While several theoretically clean measurements of the angle \(\beta\) exist \[^3\], constraining the other two angles \(\alpha\) and \(\gamma\) is significantly more challenging. A theoretically clean measurement of \(\sin(2\beta + \gamma)\) can be obtained from the study of the time evolution for \(B^0 \to D^{(*)+}\pi^-\) \[^4\] and \(B^0 \to D^{(*)-}\rho^+\) decays, which are available in large samples at the B factories, and for the corresponding CKM-suppressed modes \(B^0 \to D^{(*)+}\pi^-\) and \(B^0 \to D^{(*)+}\rho^-\). Measurements of CP asymmetries in decays \(B^0 \to D^{(*)+}\pi^\pm\) and \(B^0 \to D^{(*)-}\rho^\pm\) decays have recently been published \[^4\]–\[^7\].

The interpretation of CP asymmetries in \(B^0 \to D^{(*)+}\pi^\pm\) decays as a measurement of \(\sin(2\beta + \gamma)\) requires knowledge of the ratios of the decay amplitudes,

\[
    r(D^{(*)}\pi) = \frac{A(B^0 \to D^{(*)+}\pi^-)}{A(B^0 \to D^{(*)-}\pi^+)}.
\]

However, direct measurements of the doubly Cabibbo suppressed branching fractions \(\mathcal{B}(B^0 \to D^{(*)+}\pi^-)\) and \(\mathcal{B}(B^0 \to D^{(*)+}\rho^-)\) are not possible with the currently available data samples due to the presence of the copious background from \(B^0 \to D^{(*)+}\pi^-, D^{(*)+}\rho^-\). On the other hand, assuming SU(3) flavor symmetry, \(r(D^{(*)}\pi)\) can be related to the branching fraction of the decay \(B^0 \to D_s^{(*)+}\pi^-\) \[^3\] :

\[
    r(D^{(*)}\pi) = \tan\theta_c \frac{f_{D^{(*)}}}{f_{D_s^{(*)}}} \sqrt{\frac{\mathcal{B}(B^0 \to D_s^{(*)+}\pi^-)}{\mathcal{B}(B^0 \to D_s^{(*)-}\pi^+)}} ,
\]

where \(\theta_c\) is the Cabibbo angle, and \(f_{D^{(*)}}/f_{D_s^{(*)}}\) is the ratio of \(D^{(*)}\) and \(D_s^{(*)}\) meson decay constants \[^8\]–\[^10\]. Other SU(3)-breaking effects are believed to affect \(r(D^{(*)}\pi)\) by (10-15)% \[^11\].

The dominant Feynman diagrams for the decays \(B^0 \to D^{(*)+}\pi^+(\rho^+), B^0 \to D^{(*)+}\pi^-(\rho^-), B^0 \to D_s^{(*)+}\pi^-(\rho^-),\) and \(B^0 \to D_s^{(*)-}K^{(*)+}\) are shown in Fig. 1. Since \(B^0 \to D_s^{(*)+}\pi^-\) has four different quark flavors in the final state, a single amplitude contributes to the decay (Fig. 1a). On the other hand, two diagrams contribute to \(B^0 \to D^{(*)-}\pi^+\) and \(B^0 \to D^{(*)-}\pi^-\) : tree amplitudes (Fig. 1b) and color-suppressed direct W-exchange amplitudes (Fig. 1c). The latter are assumed to be negligibly small in Eq. 2. The decays \(B^0 \to D_s^{(*)+}K^{(*)}\) (Fig. 1d) probe the size of the W-exchange amplitudes relative to the dominant processes \(B^0 \to D^{(*)-}\pi^+\).

\[\text{FIG. 1: Dominant Feynman diagrams for (a) CKM-favored decays } B^0 \to D^{(*)+}\pi^+(\rho^+), \text{(b) doubly CKM-suppressed decays } B^0 \to D^{(*)+}\pi^- (\rho^-), \text{ and (c) the SU(3) flavor symmetry related decays } B^0 \to D_s^{(*)+}\pi^- (\rho^-); \text{ (d) the color-suppressed W-exchange contributions to } B^0 \to D^{(*)-}\pi^+ (\rho^+), \text{ (e) } B^0 \to D^{(*)-}\pi^- (\rho^-), \text{ and (f) decay } B^0 \to D_s^{(*)-}K^{(*)+}.\]

The rate of \(B^0 \to D_s^{(*)-}K^{(*)+}\) decays could be enhanced by final state rescattering \[^12\], in addition to the W-exchange amplitude. Such long-distance effects could also affect the vector meson polarization in \(B^0 \to D_s^{(*)-}K^{(*)+}\) decays. The angular distribution in vector-vector decays \(B^0 \to D_s^+ V (V = \rho, K^+)\) is given by

\[
    \frac{d^2\Gamma}{d\cos\theta_{D_s} d\cos\theta_V} \propto \left[ (1 - f_L)(1 + \cos^2\theta_{D_s})\sin^2\theta_V + 4f_L\sin^2\theta_{D_s}\cos^2\theta_V \right],
\]

where \(\theta_{D_s}\) and \(\theta_V\) are the helicity angles of \(D_s^{(*)}\) and the vector meson \(V\), respectively, \(f_L = |A_0|^2/|\Sigma|A_0^2\) is the longitudinal polarization fraction, and \(A_{\lambda=-1,0,+1}\) are the
helicity amplitudes. These distributions are integrated over the angle between the decay planes of \(D_s^+\) and \(V\).

For amplitudes dominated by the short-range (electroweak) currents, \(f_L\) is predicted to be near unity \([13]\), with corrections of order \(O(m^2_V/m^2_W)\), where \(m_V\) is the mass of the vector meson produced by the weak current, and \(m_B\) is the mass of the \(B\) meson. Thus, the measurement of \(f_L\) can constrain the size of the long-distance contributions in \(B^0 \to D_s^{(*)-} K^+\) decays \([12]\).

The branching fractions \(B(B^0 \to D_s^{(*)+} \pi^-)\) and \(B(B^0 \to D_s^{(*)-} K^+)\) have been measured previously by the \(\bar{B}\bar{A}\bar{B}\) Collaboration \([14]\). In this paper we present the first evidence for the decays \(B^0 \to D_s^{(*)+} \rho^-\) and \(B^0 \to D_s^{(*)-} K^+\), and a limit on the rate of \(B^0 \to D_s^{(*)+} \rho^-\). We also update the measurements of the branching fractions \(B(B^0 \to D_s^{(*)+} \pi^-)\) and \(B(B^0 \to D_s^{(*)-} K^+)\) with improved precision, using a 65% larger dataset.

II. DATA SAMPLE AND THE DETECTOR

We use a sample of \(381 \times 10^6 \ U(4S)\) decays into \(B\bar{B}\) pairs collected with the \(\bar{B}\bar{A}\bar{B}\) detector at the PEP-II asymmetric-energy \(e^+ e^-\) collider \([15]\). A detailed description of the \(\bar{B}\bar{A}\bar{B}\) detector is available elsewhere \([16]\). The components of the detector crucial to this analysis are summarized below.

Charged particle tracking is provided by a five-layer silicon vertex tracker (SVT) and a 40-layer drift chamber (DCH). For charged-particle identification, ionization energy loss \((dE/dx)\) in the DCH and SVT, and Cherenkov radiation detected in a ring-imaging device (DIRC) are used. Photons and neutral pions are identified and measured using an electromagnetic calorimeter (EMC), which comprises 6580 thallium-doped CsI crystals. These systems are mounted inside a 1.5-Tesla solenoidal superconducting magnet. We use the GEANT4 \([17]\) software to simulate interactions of particles traversing the \(\bar{B}\bar{A}\bar{B}\) detector, taking into account the varying detector conditions and beam backgrounds.

III. EVENT SELECTION AND ANALYSIS

The selection of events of interest proceeds in two steps. First, we preselect events with at least three reconstructed charged-particle tracks and a total measured energy greater than 4.5 GeV, as determined using all charged particles and neutral particles with energy above 30 MeV. In order to reject \(e^+ e^- \to qq\) \((q = u, d, s, c)\) continuum background, the ratio of the second to zeroth order Fox-Wolfram moments \([13]\) must be less than 0.5.

Candidates for \(D_s^+\) mesons are reconstructed in the \(D_s^+ \to \phi \pi^+, K^0_s K^+\) and \(\bar{K}^{*0} K^+\) final states, with \(\phi \to K^+ K^-\), \(K^0_s \to \pi^0 \pi^+\), and \(\bar{K}^{*0} \to K^- \pi^+\). The \(K^0_s\) candidates are reconstructed from two oppositely-charged tracks, and their momentum is required to make an angle \(|\theta_{\text{flight}}| < 11^\circ\) with the line connecting their vertex and the \(e^+ e^-\) interaction point. All other tracks are required to originate from the \(e^+ e^-\) interaction region, loosely defined by \(|d_0| < 1.5\) cm and \(|z_0| < 10\) cm, where \(d_0\) and \(z_0\) are the distances of closest approach to the primary \(e^+ e^-\) vertex in the directions perpendicular and parallel to the beams, respectively. In order to reject background from \(D^0 \to K^{0*} \pi^+\) or \(\bar{K}^{*0} \pi^-\), the \(K^+\) candidate in the reconstruction of \(D_s^+ \to K_0^{*0} K^+\) or \(\bar{K}^{*0} K^-\) is required to satisfy positive kaon identification criteria, which have an efficiency of 85% and a 5% pion misidentification probability. The same selection is used to identify kaon daughters of the \(B^0\) and \(K^{*+}\) mesons in decays \(B^0 \to D_s^{(*)-} K^{(*)+}\).

The selection is based on the ratios of likelihoods for kaon, pion, and proton identification in the SVT, DCH, and DIRC. The detector likelihoods are calibrated over a wide range of momenta using particles identified kinematically in clean decay chains, such as \(D^+ \to D^0 \pi^+, D^0 \to K^- \pi^+,\) and \(\Lambda \to p \pi^-\). In all other cases, kaons are not positively identified, but instead candidates passing a likelihood-based pion selection are rejected. The selection efficiency of this “pion veto” is 95% for the kaons and 20% for the pions. Pion daughters of \(B^0\) and \(\rho^-\) mesons in the decays \(B^0 \to D_s^{(*)+} \pi^-\) and \(B^0 \to D_s^{(*)+} \rho^-\) are required to be positively identified. Decay products of \(\phi, \bar{K}^{*0}, D_s^+,\) and \(B^0\) candidates are constrained to originate from a single vertex.

We reconstruct \(\rho^+ \to \pi^+ \pi^0\) candidates by combining a well-identified charged pion with a \(\pi^0 \to \gamma \gamma\) candidate. The \(K^{*+}\) candidates are reconstructed through the decays \(K^{*+} \to K^+ \pi^0\) and \(K^{*+} \to K_0^{*0} \pi^+\). The neutral pion candidates are reconstructed from a pair of photons each with a minimum energy of 30 MeV. The invariant mass of the photon pair is required to be within \(\pm 25\) MeV/c\(^2\) of the nominal value \([20]\). The selected candidates are constrained to the nominal \(\pi^0\) mass before forming the \(\rho^+\) or \(K^{*+}\) candidates. We require that the invariant mass of the two pions forming the \(\rho^-\) candidate be within \(\pm 320\) MeV/c\(^2\) of the nominal value \([20]\), and the invariant mass of the \(K^+ \pi^0\) and \(K_0^{*0} \pi^+\) pairs be within \(\pm 75\) MeV/c\(^2\) of the nominal \(K^*\) mass \([20]\). \(K_0^{*0} \pi^+\) pairs are constrained to a common geometric vertex.

We reconstruct \(D_s^{(*)}\) candidates in the mode \(D_s^{(*)} \to D_s^+ \gamma\) by combining \(D_s^+\) and photon candidates. Photon candidates are required to be consistent with an electromagnetic shower in the EMC, and to have an energy greater than 100 MeV in the laboratory frame. When forming a \(D_s^+\), the \(D_s^+\) candidate is required to have an invariant mass within \(10\) MeV/c\(^2\) of the nominal value \([20]\). For \(B^0 \to D_s^+ \rho^-\) and \(B^0 \to D_s^+ K^{*+}\) modes, we apply a “\(\pi^0\) veto” by rejecting photons that in combination with any other photon in the event form an invariant mass that falls within \(125 < m_{\pi^0\pi^-} < 145\) MeV/c\(^2\).

The efficiency of the initial preselection discussed above varies between 14\% \((B^0 \to D_s^+ \rho^-)\), \(D_s^+ \to \bar{K}^{*0} K^+\) and \(48\% \((B^0 \to D_s^+ \pi^-)\). After the preselection, we identify signal \(B\) decay candidates using a likelihood ratio \(R_L = \mathcal{L}_{\text{sig}}/(\mathcal{L}_{\text{sig}} + \mathcal{L}_{\text{bkg}})\),
where $L_{\text{sig}} = \prod P_{\text{sig}}(x_k)$ is the multivariate likelihood for the signal hypothesis and $L_{\text{bkg}} = \prod P_{\text{bkg}}(x_k)$ is the likelihood for the background hypothesis. Here $x_k$ represents one of the discriminating variables described below, which are computed for each event. The likelihoods for the signal and background hypotheses are computed as a product of the probability density functions (PDFs) $P_{\text{sig}}(x_k)$ and $P_{\text{bkg}}(x_k)$, respectively, for the following selection variables: invariant masses of the $\phi$, $K^0\bar{s}$, $\rho^-$, $K^{++}$, and $K^0$ candidates; $\chi^2$ confidence level of the vertex fit for the $B^0$ and $D^+_s$ mesons; the helicity angles of the $\phi$, $K^0\bar{s}$, $\rho^-$, $K^{++}$, and $D^+_s$ meson decays; the mass difference $\Delta m(D^+_s) = m(D^+_s) - m(D^+_s)$; the polar angle $\theta_B$ of the $B$ candidate momentum vector with respect to the beam axis in the $e^+e^-$ center-of-mass (e.m.) frame; the angle $\theta_B$ between the thrust axis of the $B$ candidate and the thrust axis of all other particles in the event in the e.m. frame; the event topology variable $F$, and the kinematical variable $\Delta E$, described below. Correlations among these variables are small.

The helicity angle $\theta_H$ is defined as the angle between one of the decay products of a vector meson and the flight direction of its parent particle in the meson’s rest frame. Polarization of the vector mesons in the signal decays causes the cosines of their helicity angles to be distributed as $\cos^2 \theta_H (\phi, K^0\bar{s}, \rho^-$, and $K^{++})$ or $1 - \cos^2 \theta_H$ ($D^+_s$), while the random background combinations tend to produce a more uniform distribution in $\cos \theta_H$, with a peak in the forward direction (which corresponds to a low-energy $\pi^0$ for $\rho^-$ and $K^{++}$ candidates. We do not include the helicity angles for $D^+_s$, $\rho^-$, and $K^{++}$ mesons in the likelihood ratio $R_L$ for the vector-vector $B^0 \rightarrow D^+_s\rho^-$ and $B^0 \rightarrow D^+_sK^{++}$ modes, since the polarizations of the vector mesons in these decays are not known a priori. Instead, the helicity angles are used in the multi-dimensional likelihood fit to determine the polarizations, as discussed below.

The variables $\cos \theta_B$, $\cos \theta_T$, and $F$ discriminate between spherically-symmetric $B\bar{B}$ events and jet-like continuum background using event topology. The polar angle $\theta_B$ is distributed as $\sin^2 \theta_B$ for real $B$ decays, while being nearly flat in $\cos \theta_B$ for the continuum. $B\bar{B}$ pairs form a nearly uniform $|\cos \theta_B|$ distribution, while the $|\cos \theta_T|$ distribution for continuum events peaks at 1. A linear (Fisher) discriminant $F$ is derived from the values of sphericity and thrust for the event, and the two Legendre moments $L_0$ and $L_2$ of the energy flow around the $B$-candidate thrust axis [19].

The ratio $R_L$ has a maximum at $R_L = 1$ for signal events, and at $R_L = 0$ for background originating from continuum events. It also discriminates well against $B$ decays without a real $D^+_s$ meson in the final state. The Monte Carlo (MC) simulated distributions of the $R_L$ variable for signal and background events, in $B^0 \rightarrow D^+_s\pi^-$ decays, are shown in Fig. 2.

Finally, two other variables $m_{\text{ES}}$ and $\Delta E$ take advantage of the unique kinematic properties of the $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\bar{B}$ decays. The beam energy spread is significantly smaller than the energy resolution of the reconstructed $B$ mesons, and at the same time larger than the momentum resolution. The momentum of the signal candidates is included in the beam-energy-substituted mass $m_{\text{ES}} = \sqrt{(s/2 \cdot p_B)^2 + (E_1 - p_B)^2}$, where $\sqrt{s}$ is the total c.m. energy, $(E_1, p_1)$ is the four-momentum of the initial $e^+e^-$ system, and $p_B$ is the $B^0$ candidate momentum, both measured in the laboratory frame. The second variable is $\Delta E = E_{B^0} - \sqrt{s}/2$, where $E_{B^0}$ is the $B^0$ candidate energy in the c.m. frame. For signal events, the $m_{\text{ES}}$ distribution is nearly Gaussian and centered at the beam energy, while the $\Delta E$ distribution has a maximum near zero with a resolution of $(17-25)$ MeV. We include $\Delta E$ in the definition of the likelihood ratio $R_L$: $m_{\text{ES}}$ is used as a discriminating variable in the maximum likelihood fit described below.

We parameterize the signal and background PDFs using large samples of simulated events. We select $B^0 \rightarrow D^{(*)+}\pi^-$ and $B^0 \rightarrow D^{(*)-}K^+$ candidates that satisfy $R_L > 0.85$, and accept $B^0 \rightarrow D^{(*)+}\rho^-$ and $B^0 \rightarrow D^{(*)-}K^{++}$ candidates with $R_L > 0.96$. We measure the relative efficiencies $\varepsilon_{R_L}$ of the $R_L$ selections in copious data samples of decays $B^0 \rightarrow D^-\pi^+, D^-\rho^- (D^- \rightarrow K^+\pi^-, K^0\pi^-)$ and $B^+ \rightarrow D^{0}\pi^+, D^{0}\rho^+ (D^{0} \rightarrow K^-\pi^+, \bar{K}^{0}\pi^-)$ in which the kinematics is similar to those of our signal events, and find that they are consistent with Monte Carlo estimates of $\varepsilon_{R_L} \approx 70\%(40\%)$ for $B^0 \rightarrow D^{(*)+}\pi^-$ and $B^0 \rightarrow D^{(*)-}K^+$ ($B^0 \rightarrow D^{(*)+}\rho^-$ and $B^0 \rightarrow D^{(*)-}K^{++}$) modes. The fraction of continuum background events passing the selection varies between 2% and 15%, depending on the mode.

Less than 30% of the selected events in the $B^0 \rightarrow D^{(*)+}\pi^-$, $B^0 \rightarrow D^{(*)-}K^+$, $B^0 \rightarrow D^{(*)+}\rho^-$, and $B^0 \rightarrow D^{(*)-}K^{++}$ channels ($< 2\%$ in $B^0 \rightarrow D^+\pi^-$ and $B^0 \rightarrow D^-K^+$) contain two or more candidates that satisfy the criteria listed above. In such events we select a single $B^0$ candidate based on the event $\chi^2$ formed from $\Delta E$, $m(D_s)$ and (where appropriate) $\Delta m(D^{(*)}_s)$, $m_{\rho}$,
After the $R_L$ requirement is applied, we define the region of interest using the beam-energy-substituted mass $m_{ES}$ and the mass of the $D^{\pm}$ candidate $m(D_s)$. We require $5.2 < m_{ES} < 5.3$ GeV/$c^2$ and $|m(D_s) - m(D_s)_{PDG}| < 50$ MeV/$c^2$ for $B^0 \rightarrow D_s^{*\pm}\pi^0$, $B^0 \rightarrow D_s^{*0}\rho^-$, and $B^0 \rightarrow D_s^{*0}K^{*+}$ modes, where $m(D_s)_{PDG}$ is the world average $D_s$ mass [21]. The invariant mass $m(D_s)$ has a resolution of (5-6) MeV/$c^2$, depending on the $D_s^\pm$ decay mode. The selection is significantly broader than the region populated by the signal events, and allows us to constrain backgrounds in the signal region. For $B^0 \rightarrow D_s^{*\pm}\pi^-$, $B^0 \rightarrow D_s^{*0}\rho^-$, and $B^0 \rightarrow D_s^{*0}K^{*+}$, we require $|m(D_s) - m(D_s)_{PDG}| < 10$ MeV/$c^2$.

Five classes of background events contribute to the fit region. First is the combinatorial background, in which a true or fake $D_s^{(*)}$ candidate is combined with a randomly-selected light meson. Second, B meson decays such as $B^0 \rightarrow D_s^{(*)}\pi^-$ or $\bar{B}^0 \rightarrow D_s^{(*)}\rho^-$ with $D_s^{*\pm} \rightarrow K\pi^+$ or $K^0\pi^+$ can constitute a background for the $B^0 \rightarrow D_s^{*\pm}\pi^-$ and $B^0 \rightarrow D_s^{*0}\rho^-$ modes if the pion in the $D$ decay is misidentified as a kaon (reflection background). The reflection background has nearly the same $m_{ES}$ distribution as the signal but different distributions in $\Delta E$ and $m(D_s)$. The corresponding backgrounds for the $B^0 \rightarrow D_s^{-}K^{(*)+}$ mode ($B^0 \rightarrow D_s^{-}K^{(*)+}$) are negligible. Third, rare charmless $B$ decays into the same final state, such as $B^0 \rightarrow K\pi^0 K^+h$ (where $h = \pi, \rho, K$, or $K^*$), have the same $m_{ES}$ and $\Delta E$ distributions as the $B^0 \rightarrow D_s h$ signal, but are nearly flat in $m(D_s)$. The charmless background is significant in $B^0 \rightarrow D_s^+\pi^-$, $\bar{B}^0 \rightarrow D_s^-\rho^-$, and $B^0 \rightarrow D_s^{-}K^{(*)+}$ decays, but is effectively rejected by the $|\Delta m(D_s^*)|$ variable for the modes with $D_s^{*\pm}$.

Finally, two classes of background events have nearly the same distribution as the signal events in both $m(D_s)$ and $m_{ES}$. For $B^0 \rightarrow D_s^{(*)}\pi^-$ modes we take into account the potential contributions from the non-resonant decays $B^0 \rightarrow D_s^{(*)}\pi^-$ (which have recently been observed [21]), and color-suppressed $B^0 \rightarrow D_s^{(*)}\pi^0$ (unobserved so far). Analogous non-resonant modes $B^0 \rightarrow D_s^{(*)}\pi^-\pi^0$ require the additional popping of a color-matched $q\bar{q}$ pair. They are expected to be small compared to $B^0 \rightarrow D_s^{(*)}\rho^-$ [21] and are ignored. Finally, crossfeed background from misidentification of $\bar{B}^0 \rightarrow D_s^{(*)}\pi^+$ events as $B^0 \rightarrow D_s^{(*)}\pi^+$ signal, and vice versa, needs to be taken into account.

For each mode of interest, we perform an unbinned extended maximum-likelihood (ML) fit to separate the signal events from the backgrounds and extract the signal branching fractions. For $B^0 \rightarrow D_s^+\pi^-$, $B^0 \rightarrow D_s^+\rho^-$, $B^0 \rightarrow D_s^-K^+$, and $B^0 \rightarrow D_s^-K^+$, we perform a two-dimensional fit to the $m_{ES}$ and $m(D_s)$ distributions. For $B^0 \rightarrow D_s^+\pi^-$ and $B^0 \rightarrow D_s^-K^+$ decays, we fit the one-dimensional $m_{ES}$ distribution. In vector-vector modes $B^0 \rightarrow D_s^{(*)}\rho^-$ and $B^0 \rightarrow D_s^{(*)}K^+$, we constrain both the branching fractions of the signal modes and the polarization of the vector mesons by performing a three-dimensional fit to the distribution of $m_{ES}$, and the two helicity angles of the $D_s^{(*)}$ and the two $\rho^-$, and $K^+$ mesons.

For each $B$ decay, we simultaneously fit distributions in the three $D_s^{(*)}$ decay modes, constraining the signal branching fractions to a common value. The likelihood function contains the contributions of the signal and the five background components discussed above. The function to be maximized is

$$
\mathcal{L} = \exp \left(-\sum_{k,m} n_{km}\right) \prod_{i=1}^{N_{\text{cand}}} \left( \sum_{j} n_{jm} P_{jm}(\zeta_i) \right)
$$

where $n_{jm}$ is the number of events for each event type $j$ (signal and all background modes) in each $D_s$ decay mode $m$, and $P_{jm}(\zeta_i)$ is the probability density function of the variables $\zeta_i = (m_{ES}, m(D_s), \cos \theta_{D\pi}, \cos \theta_{D\rho})$ for the $i$th event. The likelihood product is computed over all candidates $N_{\text{cand}}$ in the region of interest. We parameterize the event yields as

$$
n_{jm} = N_{B\bar{B}}B_{j}B_{D_m} \epsilon_m ,
$$

where $m$ stands for $D_s^{(*)}\pi^+$, $D_s^{(*)}\rho^-$, and $D_s^{(*)}K^+$, $N_{B\bar{B}} = 381 \times 10^6$, $B_j$ is the $B$ decay branching fraction, $B_{D_m}^m$ is the branching fraction of the $m$-th $D_s^{(*)}$ mode, and $\epsilon_m$ is the reconstruction efficiency.

The branching fractions of the channels contributing to the reflection background are fixed in the fit to the current world average values [20] and the branching fractions of the crossfeed backgrounds are determined by iterating the fits over each $B$ decay mode. The branching fractions of the non-resonant backgrounds are fixed to the values recently measured by BABAR [21]. In the case of $B^0 \rightarrow D_s^{(*)}\pi^+$, which can contribute to $B^0 \rightarrow D_s^{(*)}\pi^0K^+$, we estimate the branching fraction by

$$
B(B^0 \rightarrow D_s^{(*)}\pi^0K^+ \pi^+) \approx \frac{B(B^0 \rightarrow D_s^{(*)}\pi^+ \pi^+)}{B(B^0 \rightarrow D_s^{(*)}\pi^+)}. 
$$

This scaling assumes that the dominant mechanism for producing both $D_s^{(*)}\pi^+\pi^+$ and $D_s^{(*)}\pi^0K^+$ final states is a sub-threshold production of a charmed $D^{*0}$ meson, which subsequently decays into $D_s^{(*)}K^+$, as indicated by the invariant mass spectrum of $D_s^{(*)}K^+$ [21]. Since the decay $B^0 \rightarrow D_s^{(*)}\pi^0$ is color-suppressed compared to $B^0 \rightarrow D_s^{(*)}\pi^+$, we estimate the color suppression factor from the $B^0 \rightarrow D_s^{(*)}\pi^+$ decays. Direct production of the color-suppressed $D_s^{(*)}\pi^0K^+$ final state (without the intermediate $D^{*0}$) results in a smaller branching fraction estimate. We assign a 100% systematic uncertainty to $B(B^0 \rightarrow D_s^{(*)}\pi^0K^+)$. 

$m_{K_s}, m_{\pi^0}$ and $m_{K_s}$, and their uncertainties.
The expected yields of the dominant $B$-decay backgrounds are listed in Table III.

The PDFs and efficiencies for the signal, reflection, and crossfeed backgrounds are determined independently for each $D_s^+$ decay mode using Monte Carlo samples. The signal contribution is modeled as a Gaussian ($B^0 \to D_s^+ \pi^-$ and $B^0 \to D_s^+ K^+$) or a “Crystal Ball” function [22] in $m_{ES}$ and a double Gaussian in $m(D_s)$. The combinatorial background is described in $m_{ES}$ by a threshold function [23], $dN/dx \propto x \sqrt{1 - 2x^2/s} \exp[-\xi (1 - 2x^2/s)]$, characterized by the shape parameter $\xi$. This shape parameter, common to all $D_s^+$ modes, is allowed to vary in the fit. In $m(D_s)$, the combinatorial background is well described by a combination of a first-order polynomial (fake $D_s^+$ candidates) and a Gaussian with (5-6) MeV/$c^2$ resolution (true $D_s^+$ candidates). The charmless background is parameterized by the signal Gaussian shape in $m_{ES}$ and a first order polynomial in $m(D_s)$.

Ideally, the distribution of the helicity angles in the vector-vector decays is given by Eq. (3). The helicity angle $\theta_D$ is defined as the angle between the direction of the photon in $D_s^+ \to D_s^0 \gamma$ and the direction of the $B$ in the rest frame of the $D_s^+$ candidate. The helicity angle $\theta_V$ is similarly defined by the direction of the charged daughter particle in the decays $\rho^+ \to \pi^+\pi^0$, $K^{*+} \to K^+\pi^0$, and $K^{*+} \to K^0\pi^+$. Since the momenta of the decay products in the laboratory frame depend on the helicity angles, acceptance and efficiency effects modify the ideal angular distribution. We determine the PDFs of the signal events using the Monte Carlo simulation, and measure the angular distribution of the combinatorial background in the data region $m_{ES} < 5.27$ GeV/$c^2$.

For $B^0 \to D_s^+ \pi^-, B^0 \to D_s^+ \rho^-$, and $B^0 \to D_s^+ K^+$, the fit constrains 14 free parameters: the shape parameter of the combinatorial background $\xi$ (1 parameter for all $D_s^+$ modes), the slopes of the combinatorial and charmless backgrounds in $m(D_s)$ (3 parameters), the fractions of true $D_s^+$ candidates in combinatorial background (3), the numbers of combinatorial background events (3), the numbers of charmless events (3), and the branching fraction of the signal mode (1). In the $B^0 \to D_s^0 K^{*0}$ mode (6 individual sub-modes, spanning 3 $D_s^+$ channels and 2 $K^{*0}$ channels), 26 free parameters are constrained. For the $B^0 \to D_s^+ \pi^-, B^0 \to D_s^+ K^+$ decays, 5 free parameters are determined by the fit: $\xi$ (1 parameter for all $D_s^+$ modes), the number of combinatorial background events (3), and the branching fraction of the signal mode (1). For $B^0 \to D_s^+ \rho^-$ and $B^0 \to D_s^+ K^{*+}$ fits, we add one more free parameter to the fit: the longitudinal polarization fraction $f_L$ (see Eq. (3)). The total number of free parameters is 6 in $B^0 \to D_s^{*+}\rho^-$ and 9 in $B^0 \to D_s^{*+}K^{*+}$.

The results of the fits are shown in Figs. 3-5 and summarized in Table IV.
found to be negligible for the 1- and 2-dimensional ML fits ($B^0 \to D_s^{(*)}\pi^-, B^0 \to D_s^{(*)}K^-, B^0 \to D_s^- K^{*-}$ modes). On the other hand, we find that in the vector-vector modes ($B^0 \to D_s^+ \rho^-$ and $B^0 \to D_s^- K^{*-}$ decays), the 3-dimensional ML fits underestimate the true values of the signal branching fraction and the fraction of the longitudinal polarization. We correct for the biases of $\Delta B = (-0.37 \pm 0.03) \times 10^{-5}$ and $\Delta f_L = (-5.3 \pm 0.6)\%$ ($B^0 \to D_s^+ \rho^-$) and $\Delta B = (-0.14 \pm 0.04) \times 10^{-5}$ and $\Delta f_L = (-5.5 \pm 0.8)\%$ ($B^0 \to D_s^- K^{*-}$). We assign a conservative uncertainty of 50% of the bias to this correction.

For the longitudinal polarization fractions $f_L$ in the vector-vector modes, the systematic errors are dominated by the uncertainties in the shapes of the signal and background PDFs and the fit bias. The systematic uncertainties for each mode are summarized in Table IV.

VI. RESULTS

We estimate the significance of a non-zero signal yield by computing $S = \sqrt{-2 \log(L_0/L_{\text{max}})}$, where $L_{\text{max}}$ is the maximum likelihood value, and $L_0$ is the likelihood for a fit in which the signal contribution is set to zero. Including systematic uncertainties and assuming Gaussian-distributed errors, we obtain signal observation significances of $3.9$ ($B^0 \to D_s^+ \rho^-$), $4.6$ ($B^0 \to D_s^- K^{*-}$), and $3.1$ ($B^0 \to D_s^+ K^{*-}$) standard deviations, providing the first evidence for these decays. We test that $S$ measures the probability for the background events to fluctuate to the observed number of signal events with a large ensemble of simulated experiments. For each such experiment, we generate a set of pure background events according to the PDFs and sample composition observed in our dataset. We then fit each simulated event and measure the signal and background yields and, for the vector-vector modes, the polarization fraction $f_L$. By counting the fraction of such pseudo-experiments in which the signal yields are at least as large as the yield observed in the real dataset, we confirm that $S^2$ follows closely the $\chi^2$ distribution with one degree of freedom.

The branching fraction and polarization results are collected in Table III. Since we do not observe a significant event yield in $B^0 \to D_s^+ \rho^-$, we set a 90% confidence-level Bayesian upper limit assuming a constant prior for $B(B^0 \to D_s^+ \rho^-) > 0$.

VII. CONCLUSIONS

We report the following improved measurements of the branching fractions for the rare decays $B^0 \to D_s^{(*)}\pi^-$ and $B^0 \to D_s^{(*)}K^+$, and the first measurements of the branching fractions for the decays $B^0 \to D_s^{(*)}\rho^-$ and $B^0 \to D_s^{(*)}K^{*-}$, as well as the measurements of the longitudinal polarization fractions $f_L$ in vector-vector final states $B^0 \to D_s^{(*)}\rho^-$ and $B^0 \to D_s^{(*)}K^{*-}$:

\[
\begin{align*}
B(B^0 \to D_s^{(*)}\pi^-) &= [2.5 \pm 0.4 \pm 0.2] \times 10^{-5} \\
B(B^0 \to D_s^{(*)}\pi^+) &= [2.6 \pm 0.7 \pm 0.3] \times 10^{-5} \\
B(B^0 \to D_s^{(*)}\pi^+) &= [1.1 \pm 0.9 \pm 0.3] \times 10^{-5} \\
B(B^0 \to D_s^{(*)}\rho^-) &< 2.4 \times 10^{-5} \text{ (90% C.L.)} \\
B(B^0 \to D_s^{(*)}\rho^+) &= [4.4 \pm 1.3 \pm 0.8] \times 10^{-5} \\
f_L(B^0 \to D_s^{(*)}\rho^-) &= 0.86^{+0.26}_{-0.15} \\
f_L(B^0 \to D_s^{(*)}\rho^+) &= [2.9 \pm 0.4 \pm 0.3] \times 10^{-5} \\
f_L(B^0 \to D_s^- K^+) &= [2.4 \pm 0.4 \pm 0.2] \times 10^{-5} \\
f_L(B^0 \to D_s^- K^{*-}) &= [3.6^{+1.0}_{-0.9} \pm 0.4] \times 10^{-5} \\
f_L(B^0 \to D_s^{(*)}K^{*-}) &= [3.0^{+1.4}_{-1.3} \pm 0.3] \times 10^{-5} \\
f_L(B^0 \to D_s^{(*)}K^{*-}) &= 0.96^{+0.38}_{-0.31} \pm 0.08 ,
\end{align*}
\]

where the first quoted uncertainty is statistical, and the second is systematic.

The branching fractions for $B^0 \to D_s^{(*)}K^{(*)}$ are small compared to the dominant decays $B^0 \to D_s^{(*)}\pi^+$ and $B^0 \to D_s^{(*)}\rho^+$, implying insignificant contributions from the color-suppressed $W$-exchange diagrams. The ratios $B(B^0 \to D_s^- K^+)/B(B^0 \to D_s^+ K^*)$ and $B(B^0 \to D_s^+ K^+)/B(B^0 \to D_s^- K^{*-})$ are consistent with the expectation of unity \cite{12}. The predictions for the branching fractions of $B^0 \to D_s^{(*)}\pi^-$ and $B^0 \to D_s^{(*)}\rho^-$ decays are based on the factorization assumption \cite{22} and depend on the estimates of the hadronic form factors. The observed pattern $B(B^0 \to D_s^+ \rho^-) < B(B^0 \to D_s^- \pi^-) \approx B(B^0 \to D_s^+ \pi^-) < B(B^0 \to D_s^+ \rho^-)$ appears to be most consistent with the form factors computed in \cite{20}. The polarizations of the vector mesons in $B^0 \to D_s^{(*)}\rho^-$ and $B^0 \to D_s^{(*)}K^{*-}$ are consistent with expectations \cite{12,13}.

Assuming the SU(3) relation, Eq. \ref{eq:3}, and the recent value of $f_{D_s^{(*)}}^{\gamma}/f_{D_s^{(*)}}$ from an unquenched Lattice QCD calculation \cite{9}, we determine the ratios of the CKM-suppressed to CKM-favored decay amplitudes in decays $B^0 \to D_s^{(*)}\pi^\pm$ and $B^0 \to D_s^{(*)}\rho^\mp$:

\[
\begin{align*}
&\begin{array}{lcl}
&\begin{array}{l}
&\begin{array}{l}
&r(D\pi) = [1.75 \pm 0.14 \text{ (stat)} \pm 0.09 \text{ (syst)} \pm 0.10 \text{ (th)}]\% \\
r(D^+\pi) = [1.81^{+0.17}_{-0.14} \text{ (stat)} \pm 0.12 \text{ (syst)} \pm 0.10 \text{ (th)}]\% \\
r(D\rho) = [0.71^{+0.20}_{-0.26} \text{ (stat)} \pm 0.11 \text{ (syst)} \pm 0.04 \text{ (th)}]\% \\
r(D^+\rho) = [1.50^{+0.22}_{-0.21} \text{ (stat)} \pm 0.16 \text{ (syst)} \pm 0.08 \text{ (th)}]\% \\
&\end{array}
\end{array}
\end{array}
\end{align*}
\]
VIII. ACKNOWLEDGMENTS

We are grateful for the extraordinary contributions of our PEP-II colleagues in achieving the excellent luminosity and machine conditions that have made this work possible. The success of this project also relies critically on the expertise and dedication of the computing organizations that support BABAR. The collaborating institutions wish to thank SLAC for its support and the kind hospitality extended to them. This work is supported by the US Department of Energy and National Science Foundation, the Natural Sciences and Engineering Research Council (Canada), the Commissariat à l’Energie Atomique and Institut National de Physique Nucléaire et de Physique des Particules (France), the Bundesministerium für Bildung und Forschung and Deutsche Forschungsgemeinschaft (Germany), the Istituto Nazionale di Fisica Nucleare (Italy), the Foundation for Fundamental Research on Matter (The Netherlands), the Research Council of Norway, the Ministry of Education and Science of the Russian Federation, Ministerio de Educación y Ciencia (Spain), and the Science and Technology Facilities Council (United Kingdom). Individuals have received support from the Marie-Curie IEF program (European Union) and the A. P. Sloan Foundation.

[1] N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).
[2] C. Jarlskog, Adv. Ser. Direct. High Energy Phys. 3, 3 (1989).
[3] BABAR Collaboration, B. Aubert et al., Phys. Rev. Lett. 87, 091801 (2001); Belle Collaboration, K. Abe et al., Phys. Rev. Lett. 87, 091802 (2001).
[4] Charge conjugation is implied throughout this letter, unless explicitly stated.
[5] I. Dunietz, Phys. Lett. B 427, 179 (1998); I. Dunietz and R. G. Sachs, Phys. Rev. D 37, 3186 (1988); D. A. Suprun, C.-W. Chiang, and J. L. Rosner, Phys. Rev. D 65, 054025 (2002).
[6] BABAR Collaboration, B. Aubert et al., Phys. Rev. Lett. 92, 251801 (2004); Phys. Rev. Lett. 92, 251802 (2004).
[7] BABAR Collaboration, B. Aubert et al., Phys. Rev. D 73, 111101 (2006).
[8] D. Becirevic, hep-ph/0310072; C. Aubin et al., Phys. Rev. D 70, 094505 (2004); E. Follana et al., arXiv:0706.1726.
[9] C. Aubin et al., Phys. Rev. Lett. 95, 122002 (2005).
[10] CLEO Collaboration, K. M. Ecklund, et al., arXiv:0712.1175 (2007), submitted to Phys. Rev. Lett.
[11] M. Baak, Ph.D. thesis, report SLAC-R-858 (2007).
[12] S. Mantry, D. Pirjol, I. W. Stewart, Phys. Rev. D 68, 114009 (2003).
[13] A. Ali et al., Z. Phys. C 1, 269 (1979).
[14] BABAR Collaboration, B. Aubert et al., Phys. Rev. Lett. 90, 181803 (2003); Phys. Rev. Lett. 98, 081801 (2007).
[15] PEP-II Conceptual Design Report, SLAC-0418 (1993).
[16] BABAR Collaboration, B. Aubert et al., Nucl. Instrum. Methods Phys. Res., Sect. A 479, 1 (2002).
[17] S. Agostinelli et al., Nucl. Instrum. Methods Phys. Res., Sect. A 506, 250 (2003).
[18] G.C. Fox and S. Wolfram, Phys. Rev. Lett. 41, 1581 (1978).
[19] BABAR Collaboration, B. Aubert et al., Phys. Rev. Lett. 89, 281802 (2002); Phys. Rev. D 70, 032006 (2004).
[20] Particle Data Group, W.-M. Yao et al., Journal of Physics G33, 1 (2006).
[21] BABAR Collaboration, B. Aubert et al., arXiv:0707.1043 (2007), submitted to Phys. Rev. Lett.
[22] M. J. Oreglia, Ph.D Thesis, report SLAC-236 (1980), Appendix D; J. E. Gaiser, Ph.D Thesis, report SLAC-255 (1982), Appendix F; T. Skwarnicki, Ph.D Thesis, report DESY F31-86-02 (1986), Appendix E.
[23] ARGUS Collaboration, H. Albrecht et al., Z. Phys. C 48, 543 (1990).
[24] CLEO Collaboration, J. P. Alexander et al., arXiv:0801.0680 (2008), submitted to Phys. Rev. Lett.
[25] D. Choudhury et al., Phys. Rev. D 45, 217 (1992); C. S. Kim et al., Phys. Rev. D 63, 094506 (2001).
[26] M. Wirbel, B. Stech, and M. Bauer, Z. Phys. C 29, 637 (1985); Z. Phys. C 34, 103 (1987).
| Signal mode | Background mode | $N(D_s^+ \to \phi \pi^+)$ | $N(D_s^+ \to \bar{K}^0 K^+)$ | $N(D_s^+ \to K^0 K^+)$ |
|-------------|-----------------|------------------|------------------|------------------|
| $B^0 \to D_s^+ \pi^-$ | $B^0 \to D_s^+ K^+$ | 1.5 ± 0.2 | 0.4 ± 0.1 | 0.3 ± 0.0 |
| | $B^0 \to D_s^+ \pi^+$ | 17.1 ± 1.3 | 21.1 ± 1.6 | 13.9 ± 1.0 |
| $B^0 \to D_s^+ \pi^-$ | $B^0 \to D_s^+ K^+$ | 0.9 ± 0.2 | 0.3 ± 0.1 | 0.3 ± 0.0 |
| | $B^0 \to D_s^- \rho^+$ | 0.5 ± 0.1 | 3.5 ± 0.7 | 1.8 ± 0.4 |
| | $B^0 \to D_s^- \pi^+ , D_s^- \to D^- \pi^0$ | 0.3 ± 0.1 | 1.2 ± 0.2 | 0.8 ± 0.1 |
| $B^0 \to D_s^+ \rho^-$ | $B^0 \to D_s^+ \rho^-$ | 6.9 ± 2.0 | 1.4 ± 0.4 | 1.6 ± 0.4 |
| | $B^0 \to D_s^+ \pi^-$ | 6.3 ± 1.1 | 1.3 ± 0.2 | 1.6 ± 0.3 |
| | $B^0 \to D_s^+ \pi^0$ | 3.9 ± 0.7 | 1.1 ± 0.2 | 1.0 ± 0.2 |
| | $B^0 \to D^- \rho^+$ | 26.0 ± 4.4 | 35.2 ± 5.9 | 30.1 ± 5.0 |
| | $B^0 \to D^- \rho^+ , D^- \to D^0 \pi$ | 0.3 ± 0.0 | 6.1 ± 3.7 | 8.5 ± 1.4 |
| | $B^0 \to D^- \rho^+ , D^- \to D^- \pi^0$ | 0.9 ± 0.4 | 1.5 ± 0.6 | 2.2 ± 0.5 |
| $B^0 \to D_s^- \rho^-$ | $B^0 \to D^- \rho^-$ | 0.7 ± 0.2 | 1.7 ± 0.4 | 2.6 ± 1.2 |
| | $B^0 \to D^- \rho^+ , D^- \to D^- \pi^0$ | 0.1 ± 0.0 | 0.8 ± 0.1 | 0.8 ± 0.1 |
| $B^0 \to D_s^- \pi^-$ | $B^0 \to D_s^- \pi^-$ | 0.6 ± 0.1 | 0.3 ± 0.0 | 0.2 ± 0.0 |
| | $B^0 \to D_s^- K^+$ | 1.4 ± 0.3 | 0.3 ± 0.1 | 0.2 ± 0.0 |
| | $B^0 \to D^- K^+$ | 0.9 ± 0.3 | 2.2 ± 0.2 | 1.3 ± 0.4 |
| $B^0 \to D_s^0 K^+$ | $B^0 \to D_s^0 K^+$ | 0.9 ± 0.1 | 0.2 ± 0.0 | 0.1 ± 0.0 |
| | $B^0 \to D_s^0 K^+$ | 1.0 ± 0.2 | 0.4 ± 0.1 | 0.3 ± 0.1 |
| | $B^0 \to D_s^0 \pi^-$ | 0.5 ± 0.1 | 0.2 ± 0.0 | 0.2 ± 0.0 |
| $B^0 \to D_s^- K^+, K^+ \to K_s^0 \pi^+$ | $B^0 \to D_s^- K^+$ | 0.4 ± 0.2 | 0.1 ± 0.0 | 0.1 ± 0.0 |
| | $B^0 \to D_s^- K^0$ | 1.9 ± 0.7 | 0.8 ± 0.1 | 0.6 ± 0.1 |
| | $B^0 \to D_s^- K^+ , K^+ \to K^+ \pi^0$ | 2.6 ± 0.4 | 0.9 ± 0.2 | 0.9 ± 0.2 |
| | $B^0 \to D_s^- K^+$ | 1.1 ± 0.5 | 0.3 ± 0.2 | 0.4 ± 0.2 |
| | $B^0 \to D_s^- K^+ \pi^0$ | 0.4 ± 0.4 | 0.1 ± 0.1 | 0.1 ± 0.1 |
| $B^0 \to D_s^- K^+, K^+ \to K_s^0 \pi^+$ | $B^0 \to D_s^- K^+$ | 0.2 ± 0.0 | 0.1 ± 0.0 | 0.0 ± 0.0 |
| | $B^0 \to D_s^- K^+ K^0$ | 0.6 ± 0.4 | 0.3 ± 0.2 | 0.2 ± 0.1 |
| | $B^0 \to D_s^- K^+ \pi^0$ | 0.5 ± 0.1 | 0.1 ± 0.0 | 0.2 ± 0.1 |
| | $B^0 \to D_s^- K^+ \pi^0$ | 0.1 ± 0.1 | 0.0 ± 0.0 | 0.0 ± 0.0 |
TABLE III: Relative systematic uncertainties for the branching fractions of $B^0 \to D_s X$ modes (%).

| $B^0 \to D_s^- \pi^+$ | $B^0 \to D_s^+ K^+$ | $B^0 \to D_s^+ \rho^-$ | $B^0 \to D_s^- K^{*+}$ | $K^{*+} \to K^0_s \pi^+$ | $K^{*+} \to K^0_s \pi^+$ | $K^{*+} \to K^0_s \pi^+$ |
|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| $N_{B\pi}$          | 1.1                  | 1.1                  | 1.1                  | 1.1                  | 1.1                  | 1.1                  |
| Tracking efficiency  | 1.6                  | 1.3                  | 1.7                  | 0.8                  | 1.7                  | 0.8                  |
| PID efficiency       | 1.0                  | 1.0                  | 1.0                  | 1.0                  | 1.0                  | 1.0                  |
| $\pi^0$ efficiency   | -                    | -                    | 3.0                  | 3.0                  | 3.0                  | 3.0                  |
| $R_L$--cut efficiency | 1.6                  | 1.4                  | 2.8                  | 2.0                  | 2.0                  | 2.0                  |
| MC statistics        | 0.8                  | 0.7                  | 1.8                  | 1.1                  | 1.1                  | 1.1                  |
| $K^0_s$ efficiency   | 0.4                  | 0.3                  | 0.9                  | 0.0                  | 0.0                  | 0.0                  |
| PDF parameters       | 0.8                  | 0.7                  | 2.8                  | 1.4                  | 1.4                  | 1.4                  |
| $\Delta E, m(D_s)$ PDFs | 0.6                  | 1.7                  | 3.2                  | 1.7                  | 1.7                  | 1.7                  |
| $B(D_s^+ \to \phi \pi^-)$ | 7.7                  | 8.3                  | 14.7                 | 7.7                  | 7.7                  | 7.7                  |
| $B(D_s^+ \to K^0_s \pi^+)$ | 2.8                  | 3.1                  | 1.8                  | 3.1                  | 3.1                  | 3.1                  |
| $B(D_s^+ \to K^0_s \pi^+)$ | 1.6                  | 1.4                  | 3.7                  | 0.6                  | 0.6                  | 0.6                  |
| Reflection background | 2.0                  | 0.7                  | 10.2                 | 0.6                  | 0.6                  | 0.6                  |
| Crossfeed background | 0.9                  | 0.4                  | 15.6                 | 2.5                  | 2.5                  | 2.5                  |
| Resonant lineshape   | 1.6                  | 1.7                  | 1.8                  | 4.9                  | 4.9                  | 4.9                  |
| **TOTAL**            | 9.3                  | 9.7                  | 25.0                 | 13.1                 | 13.1                 | 13.1                 |
### TABLE IV: Relative systematic uncertainties for the branching fractions of $B^0 \rightarrow D^*_s X$ modes (%).

| Source                      | $B^0 \rightarrow D_s^{*+}\pi$ | $B^0 \rightarrow D_s^{*+}K^+$ | $B^0 \rightarrow D_s^{*+}\rho$ | $B^0 \rightarrow D_s^{*+}K^{**}$ |
|-----------------------------|---------------------------------|-------------------------------|---------------------------------|----------------------------------|
| $N_{PB}$                    | 1.1                            | 1.1                           | 1.1                             | 1.1                             |
| Tracking efficiency         | 1.6                            | 1.3                           | 1.7                             | 2.0                             |
| PID efficiency              | 1.0                            | 1.0                           | 1.0                             | 1.0                             |
| $\pi^0$ efficiency          | -                              | -                             | 3.0                             | 3.0                             |
| $\gamma$ efficiency         | 1.8                            | 1.8                           | 1.8                             | 1.8                             |
| $\mathcal{B}(D_s^{*+}\rightarrow D_s^+\gamma)$ | 0.7                            | 0.7                           | 0.7                             | 0.7                             |
| $\Delta m(D_s^{*+})$        | 1.0                            | 1.0                           | 1.0                             | 1.0                             |
| $\cos\theta_{D_s^{*+}}$    | 1.0                            | 1.0                           | 1.0                             | 1.0                             |
| $\pi^0$ veto efficiency     | -                              | -                             | 2.0                             | 2.0                             |
| Fit bias                    | -                              | -                             | 4.1                             | 2.3                             |
| $R_L$ cut efficiency        | 1.5                            | 1.7                           | 2.3                             | 1.6                             |
| MC statistics               | 0.8                            | 0.8                           | 1.1                             | 1.6                             |
| $K^0_S$ efficiency          | 0.4                            | 0.0                           | 0.2                             | 0.3                             |
| PDF parameters              | 1.9                            | 2.1                           | 3.0                             | 3.1                             |
| $\Delta E, m(D_s)$ PDFs    | 3.0                            | 0.8                           | 0.7                             | 3.4                             |
| $\mathcal{B}(D_s^+\rightarrow \phi\pi^+)$ | 7.6                            | 8.4                           | 16.0                            | 5.9                             |
| $\mathcal{B}(D_s^+\rightarrow K^{*-}K^+)$ | 2.7                            | 3.8                           | 3.4                             | 4.1                             |
| $\mathcal{B}(D_s^+\rightarrow K^0_S K^+)$ | 1.5                            | 1.3                           | 0.9                             | 1.3                             |
| Reflection background       | 3.4                            | 0.0                           | 2.1                             | 0.0                             |
| Crossfeed background        | 1.9                            | 0.4                           | 0.0                             | 1.6                             |
| Resonant lineshape          | 0.8                            | 0.4                           | 1.6                             | 0.3                             |
| **TOTAL**                   | 10.5                           | 10.2                          | 18.2                            | 10.6                            |

### TABLE V: Absolute systematic uncertainties for the longitudinal polarization fraction (%).

| Source                      | $B^0 \rightarrow D_s^{*+}\rho$ | $B^0 \rightarrow D_s^{*+}K^{**}$ |
|-----------------------------|---------------------------------|----------------------------------|
| Fit bias                    | 2.7                            | 2.8                             |
| $R_L$ cut efficiency        | 0.7                            | 0.2                             |
| MC statistics               | 0.6                            | 0.7                             |
| $K^0_S$ efficiency          | 0.4                            | 0.0                             |
| PDF parameters              | 14.7                           | 7.0                             |
| $\Delta E, m(D_s)$ PDFs    | 0.0                            | 0.5                             |
| $\mathcal{B}(D_s^+\rightarrow \phi\pi^+)$ | 0.8                            | 0.8                             |
| $\mathcal{B}(D_s^+\rightarrow K^{*-}K^+)$ | 0.7                            | 0.2                             |
| $\mathcal{B}(D_s^+\rightarrow K^0_S K^+)$ | 0.5                            | 0.2                             |
| Reflection background       | 0.5                            | 0.0                             |
| Crossfeed background        | 0.0                            | 0.6                             |
| Resonant lineshape          | 0.6                            | 0.1                             |
| **TOTAL**                   | 15.0                           | 7.6                             |
FIG. 3: (a,c,e,g) $m_{ES}$ projection of the fit with $|m(D^+_s) - m(D^+_{s})_{PDG}| < 10$ MeV/$c^2$ and (b,d,f,h) $m(D_s)$ projection with $5.275 < m_{ES} < 5.285$ GeV/$c^2$ for (a,h) $B^0 \rightarrow D^+_s\pi^-$, (c,d) $B^0 \rightarrow D^-_sK^+$, (e,f) $B^0 \rightarrow D^+_s\rho^-$, and (g,h) $B^0 \rightarrow D^-_sK^{*+}$. The black solid curves correspond to the full PDF from the combined fit to all $D^+_s$ decay modes. Individual contributions are shown as solid red (signal), green dashed (combinatorial background), and blue dotted (sum of reflection, charmless, crossfeed, and non-resonant backgrounds) curves.
FIG. 4: $m_{ES}$ projection of the fit for (a) $B^0 \rightarrow D_s^{*-} \pi^-$ and (b) $B^0 \rightarrow D_s^{*-} K^+$. The black solid curves correspond to the full PDF from the combined fit to all $D_s^+$ decay modes. Individual contributions are shown as solid red (signal), green dashed (combinatorial background), and blue dotted (sum of reflection, charmless, and crossfeed backgrounds) curves.

FIG. 5: Projections of the fit on to (a,b) $m_{ES}$, (c) $\cos \theta_\rho$, (d) $\cos \theta_{K^*}$, and (e,f) $\cos \theta_{D_s^*}$ for (a,c,e) $B^0 \rightarrow D_s^{*-} \rho^-$ and (b,d,f) $B^0 \rightarrow D_s^{*-} K^+$. For helicity projections, a selection $5.275 < m_{ES} < 5.285$ GeV/$c^2$ is applied. The black solid curves correspond to the full PDF from the combined fit to all $D_s^+$ decay modes. Individual contributions are shown as solid red (signal), green dashed (combinatorial background), and blue dotted (sum of reflection, charmless, crossfeed, and non-resonant backgrounds) curves.