Density dependent synthetic gauge fields using periodically modulated interactions

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We show that density-dependent synthetic gauge fields may be engineered by combining periodically modulated interactions and Raman-assisted hopping in spin-dependent optical lattices. These fields lead to a density-dependent shift of the momentum distribution and may induce superfluid-to-Mott insulator transitions. We show that the interplay between the created gauge field and the broken sublattice symmetry results in an intriguing behavior at vanishing interactions, characterized by the appearance of a fractional Mott insulator. The coupling between density and phase leads as well to anomalous correlations in the superfluid regime.

The emulsion of synthetic electromagnetism in cold neutral gases has attracted a major interest [11, 12]. Artificial electric and magnetic fields have been induced using lasers [3–5]. Moreover, these setups may be extended to generate non-Abelian fields, and in particular spin-orbit coupling [6–13]. Synthetic fields may be generated as well in optical lattices, and recent experiments have created artificial staggered [14–16] and uniform [17, 18] magnetic fields. These fields are however static, as they are not influenced by the atoms.

The dynamical feedback between matter and gauge fields plays, however, an important role in various areas of physics, ranging from condensed-matter [19] to quantum chromodynamics [20], and its realization in cold lattice gases is attracting a growing attention [21]. Schemes have been recently proposed for multi-component lattice gases, such that the low-energy description of these systems is that of relevant quantum field theories [22–31]. The back-action of the atoms on the value of a synthetic gauge field is expected to lead to interesting physics, including statistically-induced phase transitions and anyons in 1D lattices [32], and chiral solitons in Bose-Einstein condensates [33].

Periodically modulated optical lattices open interesting possibilities for the engineering of lattice gases [16–18, 34–40]. In particular, periodic lattice shaking results in a modified hopping rate [34, 35], which has been employed to drive the superfluid (SF) to Mott insulator (MI) transition [36], to simulate frustrated classical magnetism [38], and to create tunable gauge potentials [16]. Interestingly, a periodically modulated magnetic field may be employed in the vicinity of a Feshbach resonance to induce periodically modulated interactions, which result in a non-linear hopping rate that depends on the atom number difference between neighboring sites [41–43].

In this Letter, we show that combining periodic interactions and Raman-assisted hopping may induce a density-dependent gauge field in 1D lattices. The created field results in a density-dependent shift of the momentum distribution that may be probed in time-of-flight (TOF) experiments. Moreover, contrary to the Peierls phase induced in shaken lattices [16], the created field cannot be gaugeed out, and hence affects significantly the ground-state properties of the lattice gas, leading to gauge-induced SF to MI transitions, the emergence of MI at vanishing interaction, and anomalous correlations.

Periodically-modulated interactions—We consider lattice bosons with periodically modulated short-range interactions. Considering a large-enough gap between the first two Bloch bands, we may restrict the description of the system to a single band Bose-Hubbard Hamiltonian:

\[ \hat{H}(t) = -J \sum_{\langle ij \rangle} \hat{b}^\dagger_i \hat{b}_j + \frac{U_0 + U_1(t)}{2} \sum_i \hat{n}_i (\hat{n}_i - 1), \]

where \( \hat{b}_i \) is the bosonic annihilation operator at site \( i \), \( \hat{n}_i = \hat{b}^\dagger_i \hat{b}_i \), \( J > 0 \) is the hopping rate, \( \langle \cdot \rangle \) denotes nearest neighbors, \( U_1(t) = U_1(t + T) \), and \( \int_0^T dt \ U_1(t') = 0 \). Assuming \( \omega = 2\pi/T \gg J/h, U_0/h \) [44], we integrate the modulation to obtain an effective Hamiltonian [41, 42]:

\[ \hat{H}_{\text{eff}} = - \sum_{\langle ij \rangle} \hat{b}^\dagger_i J_{\text{eff}}(\hat{n}_i - \hat{n}_j) \hat{b}_j + \frac{U_0}{2} \sum_i \hat{n}_i (\hat{n}_i - 1), \]

where \( J_{\text{eff}}(\Delta \hat{n}) = \frac{1}{\pi} \int_0^T dt \ e^{iV(t)\Delta \hat{n}} \), and \( \frac{d}{dt} V(t) = U_1(t) \). Hence, periodic interactions result in a hopping rate that depends on the atom number difference between neighboring sites. For \( U_1(t) = \hat{U}_1 \sin(\omega t) \), \( V(t) = \hat{U}_1 [1 - \cos(\omega t)] \), and \( J_{\text{eff}}(x) = J e^{i\Omega x} J_0(\Omega x) \), with \( \Omega = \hat{U}_1/\hbar \) and \( J_0 \) the Bessel function of first kind [44].

The hopping may hence acquire a density-dependent Peierls phase but it may be gaugeed out by defining new bosonic operators \( \hat{B}_j \equiv e^{-i\Delta \hat{n}_j} \hat{b}_j \). Hence, the complex hopping does not affect the ground-state phase diagram [47]. The Peierls phase does distort however the momentum distribution, which becomes density dependent, but the stochastic
character of the atom number difference between neighboring sites leads to a broadening of the quasi-momentum distribution [44] rather than to an overall shift such as that observed in shaken lattices [14].

**AB model.**—We introduce in the following a set-up that creates a density-dependent Peierls phase that both cannot be gauged out and leads to a net drift of the quasi-momentum distribution. We consider a tilted 1D spin-dependent lattice (see Fig. 1, in which atoms in state |1⟩ (|2⟩) are confined in the sublattice A (B). A first pair of Raman lasers induces Raman-assisted hopping between an A site and the B site to its right, whereas a second pair leads to hopping between an A site and the B site to its left [48]. We consider that within a period T, for 0 < t < T/2 the Raman assisted coupling AB (BA) is on (off) and vice versa for T/2 < t < T. The Hamiltonian of the system is:

$$\hat{H}^{AB} = - \sum_j \left[ J_{AB}(t) \hat{b}_j^\dagger \hat{b}_{j+1} + J_{BA}(t) \hat{b}_{j+1}^\dagger \hat{b}_j + h.c. \right] + \frac{U_A}{2} \sum_j \hat{n}_{2j} - 1 + \frac{U_B}{2} \sum_j \hat{n}_{2j+1} - 1.$$

where $J_{AB} = J$ and $J_{BA} = 0$ for 0 < t < T/2, $J_{AB} = 0$ and $J_{BA} = J$ for T/2 < t < T, and even (odd) site index corresponds to the A (B) sublattice. The interaction of components |1⟩ can be independently modulated from those of |2⟩, such that $U_A = U_{10} + U_{11}(t)$, with $U_{10}(t) = U_{10}(t + T)$ and $\int_0^T dt' U_{11}(t') = 0$, whereas $U_B$ is constant (we consider for simplicity $U_{10} = U_B \equiv U$ [49]). Integrating over the fast modulation we obtain the effective Hamiltonian [50]:

$$\hat{H}_\text{eff}^{AB} = - \sum_j \left[ \hat{b}_j^\dagger \hat{b}_j \hat{J}_{AB}(\hat{n}_{2j}) \hat{b}_{j+1} + \hat{J}_{BA}(\hat{n}_{2j}) \hat{b}_{j+1} + h.c. \right] + \frac{U}{2} \sum_j \hat{n}_{2j} - 1 + \frac{U}{2} \sum_j \hat{n}_{2j+1} - 1,$$

with $\hat{J}_{AB}(\hat{n}_{2j}) = \frac{i}{\sqrt{2}} \int_0^{T/2} dt e^{i \frac{t}{2} \hat{n}_{2j}/\hbar}$, $\hat{J}_{BA}(\hat{n}_{2j}) = \frac{i}{\sqrt{2}} \int_0^{T/2} dt e^{-i \frac{t}{2} \hat{n}_{2j}/\hbar}$, and $V(t) = \int_0^t dt' U_{11}(t') dt'$. For $U_{11}(t) = \tilde{U}_{11} \sin(\omega_A t)$ for 0 < t < T/2 (with $\omega_A = 4\pi / T$), and $U_{11}(t) = -\tilde{U}_{11} \sin(\omega_A t)$ for T/2 < t < T (see Fig. 1(b)), $\hat{J}_{BA}(\hat{n}_{2j}) = \frac{i}{\sqrt{2}} J_{0}(\omega_A \hat{n}_{2j}) e^{i \theta_{AB} \hat{n}_{2j}}$, whereas $\hat{J}_{AB}(\hat{n}_{2j}) = \hat{J}_{BA}(\hat{n}_{2j})^\dagger$, with $\omega_A = \tilde{U}_{11} / \hbar \omega_A$. For more general forms of $U_{11}(t)$ [44], arg[$\hat{J}_{BA}$] = $\phi_{BA} \hat{n}_{2j}$ and arg[$\hat{J}_{BA}$] = $\phi_{BA} \hat{n}_{2j}$. The created Peierls phase cannot be gauged out if $\phi = \phi_{BA} - \phi_{BA} \neq 0$, crucially altering the ground-state properties.

**Quasi-momentum distribution.**—Since the Peierls phase depends on the occupation of A sites, and not on the population difference between neighboring sites, this results in a net drift of the quasi-momentum distribution in the SF regime. As in recent experiments on shaken lattices [16], this shift may be probed in TOF (details about experimental detection are discussed below). Fig. 2(a) shows the quasi-momentum distribution as a function of the average density $\langle \hat{n} \rangle$ for an homogeneous system with $\Omega_{AB} = \pi / 4$ and $U = 0.2 J$. However, in contrast to shaken lattice experiments, the momentum shift is density dependent. This dependence results in a non-trivial behavior of the quasi-momentum distribution in the presence of an external harmonic confinement, which may be accounted for by an additional term $V_T \sum_j (j - L/2)^2 \hat{n}_j$ in the Hamiltonian [4]. As shown in Fig. 2(b), for larger $V_T$ the quasi-momentum distribution shifts due to growing central density, and broadens due to the inhomogeneous density distribution $\langle \hat{n}_j \rangle$.

**Ground-state phase diagram.**—The non-gaugeable density-dependent Peierls phase and the associated broken AB symmetry are crucial for the ground-state physics of the AB model (see Fig. 3 in which $\mu$ is the chemical potential). MI phases at half-integer filling are induced by the AB asymmetry, opening immediately at any finite $J$. For $\langle \hat{n} \rangle = 1/2$ at $J/U \ll 1$ we may project on the manifold with 0 or 1 particle per site. Performing Jordan-Wigner transformation into fermionic operators $\hat{b}_j \rightarrow (-1)^{\sum_{i < j} \hat{n}_i} \hat{c}_j$, we obtain up to $O(J^2 / U)$ the effective Hamiltonian $\hat{H}_{JW} = \hat{H}_0 + \hat{H}_2$, with $\hat{H}_0 = -J \sum_j (\hat{c}_j^\dagger \hat{c}_{j+1} + h.c.)$, $\hat{H}_2 = -\sum_j \left[ \hat{c}_j \hat{c}_{j+1} + \frac{\Delta^2}{2} \hat{n}_{2j+1} \hat{n}_{2j} + h.c. \right]$. For $\Delta^2 = 0$ the perturbative corrections result in nearest neighbor interactions and staggered correlated hopping. The latter becomes immediately relevant for free hard-core particles, and hence any AB-dependent $\Gamma$ opens a (band insulator) gapped phase at half-filling for $U \rightarrow \infty$. A similar reasoning applies for higher half-integer fillings $\hat{n} + 1/2$, by considering hard-core particles on top of a pseudo-vacuum with $\hat{n}$ particles per site. Note that the Mott boundaries depend on $\Phi$ and hence varying $\Phi$ at constant $J/U$ results in gauge-induced phase transitions (Fig. 3(b)), similar as the statistical transitions of Ref. [52]. In particular, for $\Phi \rightarrow \pi$ one observes a strong enhancement of the MI gaps. Half-integer and integer MI may be revealed by the appearance of density plateaus in the presence of a harmonic trap [51].

**Vanishing on-site interaction.**—The effect of the density-
ploying bosonization \cite{54}: correlations in the SF regime. This is best understood by employing the rotated fields \(\tilde{\rho} = \rho_0 - i \nabla \phi(x) + \rho_0 \sum_{\rho \neq 0} e^{i2\rho(\eta_0 + \phi(x))}, \rho_0\) the average density, and \(x_j\) the position of site \(j\). The fields \(\phi(x)\) and \(\phi(x)\) characterize the density and phase, respectively, whereas \(\eta\) is for a global gaugeable phase shift. The density-dependent Peierls phase results in a coupling between phase and density, \(\sim \partial_x \phi \partial_x \eta\), absent when \(\Phi = 0\). By introducing the rotated fields \(\tilde{\phi} = \phi \cos(\gamma) + \phi \sin(\gamma)\) and \(\tilde{\phi} = -\phi \sin(\gamma) + \phi \cos(\gamma)\) we diagonalize the Hamiltonian into the Luttinger liquid form \(\cite{54}\):

\[
\hat{H}_L = \frac{u}{2\pi} \int dx \left[ K \partial_x^2 \tilde{\phi} + K^{-1} \partial_x^2 \tilde{\theta} \right],
\]

where \(u\) is a velocity, and \(K\) is the Luttinger parameter. The fact that the fields \(\tilde{\phi}\) and \(\tilde{\theta}\) in Eq. (6) are not those describing density and phase fluctuations are crucial for the correlations in the SF regime. Without density-dependent phase \(\gamma = 0\), \(\langle \tilde{\rho}_i \tilde{\rho}_j \rangle \sim |i - j|^{-1/2\alpha}\) and \(\langle \tilde{\eta}_i \tilde{\eta}_j \rangle \sim |i - j|^{2 \alpha + \beta}\), with \(\alpha = \beta = K\) (inset of Fig. 4). On the contrary, for \(\gamma \neq 0\), \(\alpha\) and \(\beta\) become different functions of \(K\) and \(\gamma\), which in turn depend on \(\Omega_{AB}\). Figure 4 shows \(\alpha\) and \(\beta\) as a function of \(\Omega_{AB}\) for the case \(\Phi = 2\Omega_{AB}\) in Eq. (4), confirming the non-trivial behavior of correlations in the Luttinger liquid.

Adiabatic preparation.– We have focused above on the effective model \(\cite{4}\). As for shaken lattices \(\cite{55}\), one may start from the ground-state without modulated interactions, and adiabatically increase \(U_{AB}\). We have studied this preparation.
FIG. 5: (Color online) Quasi-momentum $k_{\text{max}}$ at which the quasi-momentum distribution of the B sublattice is maximal as a function of $\Omega_{AB} \langle \hat{n} \rangle$ for $\omega = 20 J$ and $U = J$. Solid (dashed) lines denote the results obtained from the effective model \(4\) with $\langle \hat{n} \rangle = 3/2$ ($1$). The error bars denote the uncertainty (time average and standard deviation) of $k_{\text{max}}(t)$ for the case of a linear ramp of $U_{A1}$ with a ramp time of $\tau = 200T$ (see text). (Inset) Solid and dash-dotted lines show $k_{\text{max}}(t)$ for $\langle \hat{n} \rangle = 3/2$ with $\Omega_{AB} = 0.4$ and $0.8$, whereas the dotted line indicates the value of $k_{\text{max}}$ for the effective model \(4\). We depict with a dashed line the ramp $U_{A1}(t)$.

by means of time-evolving block decimation (TEBD) \[56\] simulations of the dynamics of Eq. \(3\) when applying a linear ramp $U_{A1}(t) = \frac{t}{\tau} U_{A1}$ for $t < \tau$, and constant afterwards \[44\]. Fig. \(5\) depicts the value $k_{\text{max}}$ at which the momentum distribution is maximal, showing that the evolved momentum distribution is in very good agreement with that of the effective model. Note that the drift $k_{\text{max}}$ is only linear with $\Omega_{AB} \langle \hat{n} \rangle$ for a sufficiently small value of $\Omega_{AB} \langle \hat{n} \rangle$. For larger $\Omega_{AB} \langle \hat{n} \rangle$ it presents a non-trivial density dependence, especially at low $\langle \hat{n} \rangle$, due to number fluctuations.

Detection.— Whereas the density distribution of the effective model corresponds to that measured in the laboratory frame, the measurement of the momentum distribution in TOF presents some features that differ significantly from the shaken lattice case \[16\]. First, since the lattice is not actually shaken, the overall momentum envelope resulting from the Fourier transform of the Wannier functions does not oscillate in time. Second, whereas the momentum distribution of the B sublattice measured in TOF corresponds to that of the effective model, the distribution of the A sublattice just coincides with that of the effective model (and also with that of the sublattice B) when $V(t) = 0$. For intermediate times, the phase appearing in the conversion between both reference frames leads to a broadening, and eventual blurring, of the TOF peaks \[44\].

Outlook.— Periodic interactions combined with Raman-assisted hopping may create a density-dependent Peierls phase that results in non-trivial ground-state properties, characterized by a density-dependent momentum distribution, gauge-induced SF to MI transitions, a MI phase at vanishing hopping, and anomalous correlations in the SF phase. The AB model may be extended to create a density-dependent gauge field in a square lattice, in which each row is an exact copy of the AB lattice as that discussed above, and rows are coupled by direct (not Raman-assisted) hops. Tilting the lattice, leads to a row-dependent $\langle \hat{n} \rangle$, and hence to a different Peierls phase at each row when modulating the interactions. In this way a finite flux may be produced in each plaquette, proportional to the density difference between neighboring rows. As a result, density dependent synthetic magnetic fields may be created, opening interesting possibilities that deserve further investigation.

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More involved modulations of the interactions may lead in general to a Peierls phase which is not linear in the number difference, but in general an odd function. The momentum distribution will hence show a stochastic broadening rather than a net drift, as for the linear case. Moreover, the cubic part of the phase is only relevant when the modulus of the hopping is very small, due to the large argument of the Bessel function. It has hence negligible consequences in the ground-state phase diagram.

The laser arrangement is basically the same as that of [D. Jaksch and P. Zoller, New. J. of Phys. 5, 56 (2003)] proposed for the creation of a synthetic (static) magnetic field. However, here we do not demand a spatial dependence of the Rabi frequencies and the AB and BA lasers are switched on and off. The tilting must be sufficiently large to be resolved by the two different Raman pairs. The tilting must be also larger than the Raman-induced hopping rate and the interaction energy. Note also that the tilting should be chosen avoiding photon-assisted resonances [C. Sias et al., Phys. Rev. Lett. 100, 040404 (2008)], which could result in a significant BA hopping even during the AB pulses.

An even more intriguing phase space could emerge from assuming \( U_{AB} \neq U_B \), however this goes beyond the scope of this current article.

The AB model resembles the anyon model of Ref. [32], in which the inter-site hopping depends on the occupation of the left site. The model of Ref. [32] requires twice as many Raman lasers as the maximal occupation per site, and on-site interactions larger than the laser linewidth. The AB model works with only one laser pair, and for interaction shifts smaller than the laser linewidth (for \( \langle \hat{n} \rangle = 5 \) and \( U = 0.2J \), the linewidth required must be larger than \( U/\langle \hat{n} \rangle = J \); for \( J \) of the order of tens of Hz this is a realistic assumption for typical linewidths [13]). The AB model may be recast as an anyon model without Peierls phase by defining \( \hat{a}_{2j} = e^{i\Omega_{AB} \sum_{l<j} \hat{a}_l \hat{b}_{2j}} \), and \( \hat{a}_{2j+1} = e^{i\Omega_{AB} \sum_{l<j} \hat{a}_l \hat{b}_{2j+1}} \), where the \( \hat{a} \) operators fulfill, for \( j' > j \), \( e^{i\Omega_{AB} \hat{a}^\dagger_{2j} \hat{a}_{2j'}} = \hat{a}_{2j'} \hat{a}^\dagger_{2j} \), \( e^{i\Omega_{AB} \hat{a}^\dagger_{2j} \hat{a}_{2j'+1}} = \hat{a}_{2j'+1} \hat{a}^\dagger_{2j}, \hat{a}^\dagger_{2j+1} \hat{a}_{2j'+1} = \hat{a}_{2j'+1} \hat{a}_{2j} \).

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In this supplementary material, we provide additional details on the gauge fixing condition for periodically modulated interactions, the calculation of the scattering length of the AB-model, some aspects of the time of flight (TOF) imaging as well as details on the numerical simulation of real-time evolutions.

A. PERIODICALLY MODULATED INTERACTIONS AND GAUGE FIXING

In the following we discuss the choice of $V(t)$ for periodically modulated interactions, and in particular how to fix the gauge uncertainty. Starting from Eq. (1) of the main text we perform the transformation $|\psi(t)\rangle = \hat{R}(t)|\psi(t)\rangle$, with $\hat{R}(t) = e^{\frac{iVt}{\hbar}\sum_{\langle i,j\rangle} n_i(n_j-1)}$, such that $\frac{d}{dt} V(t) = U_1(t)$ (note that $V(t) = V(t+T)$ since $U_1(t)$ is unbiased). In the transformed frame: $i\hbar \partial_t |\psi(t)\rangle = \hat{H}'(t)|\psi(t)\rangle$, with $\hat{H}' = \hat{R}\hat{H}\hat{R}^\dagger - i\hbar \frac{d}{dt}\hat{R}\hat{R}^\dagger$. Assuming a fast modulation, $\omega = 2\pi T \gg J/\hbar$, $U_0/\hbar$, we integrate the modulation to obtain the effective time-independent Hamiltonian (Eq. (2) of the main text) with an effective density-dependent hopping $J_{\text{eff}}(\Delta \hat{n}) = \int_0^T dt \ e^{iV(t)\Delta \hat{n}}$ in the transformed frame (see Refs. [2, 3] for further details).

As discussed in the main text, we are interested in probing the effective model by TOF measurements. Note, however, that TOF measurements will monitor the evolution of the quasi-momentum distribution in the laboratory frame, $\rho_L(k,t) = \frac{1}{N_L} \sum_{i,j} e^{-i k (i-j)} \langle \psi(t)|b_i^\dagger b_j|\psi(t)\rangle$, with $N$ the number of particles, and $L$ the number of sites. The single-particle correlation function in the laboratory frame fulfills: $\langle \psi(t)|b_i^\dagger b_j|\psi(t)\rangle = \langle \psi(t)|b_i^\dagger e^{iV(t)0}\langle \hat{n}_i-\hat{n}_j|b_j|\psi(t)\rangle$, and hence for general times $\rho_L(k,t)$ does not coincide with the quasi-momentum distribution of the effective model $\rho_{\text{E}}(k) = \frac{1}{N_L} \sum_{i,j} e^{-i k (i-j)} \langle \psi|b_i^\dagger b_j|\psi\rangle$. We will be interested in stroboscopic measurements at times $t = nT$, with $n = 0, 1, \ldots$, such that $\rho_L(k,nT) = \rho_{\text{E}}(k)$. This condition demands $V(0) = 0$, fixing the gauge uncertainty (we assume this gauge fixing henceforth). Hence measurements at times $t = nT$ allow to probe an effective model, $\hat{H}_{\text{eff}}$ (see Eq. (2) of the main text). In the following we consider for simplicity $U_1(0) = 0$.

Note that if the modulation starts at time $-T < -t_0 < 0$, $U_1(-t_0) = 0$, then the time evolution between $t = -t_0$ and $t = 0$ must be explicitly considered, i.e. the initial condition for the time evolution under the effective model is $|\psi'(0)\rangle = T e^{-i\int_{-t_0}^0 \hat{H}'(t')dt'} |\psi(-t_0)\rangle$, where $T$ denotes time ordering. The discrete time evolution at times $nT$ may be then evaluated, in a very good approximation for $\hbar\omega \gg U_0$, $J$, by evolving with $\hat{H}_{\text{eff}}$ starting with the calculated $|\psi'(0)\rangle$. Note, however, that the measurements will probe a different effective model, due to the different (shifted) form of $U_1(t)$, and hence of $V(t)$, and in turn of $J_{\text{eff}}(\Delta \hat{n})$.

For a sinusoidal modulation $U_1(t) = \tilde{U}_1 \sin(\omega t)$, $V(t) = \frac{\tilde{U}_1}{\omega} [1 - \cos(\omega t)]$, and hence $J_{\text{eff}}(\Delta \hat{n}) = J e^{i\Omega\Delta \hat{n}} J_0(\Omega \Delta \hat{n})$, with $\Omega = \tilde{U}_1/\hbar\omega$ and $J_0$ the Bessel function of first kind. The stroboscopic measurement of $\rho_L(k,nT)$ allows hence to probe $\rho_{\text{E}}(k)$ for an effective model with a complex $J_{\text{eff}}(\Delta \hat{n}) = |J_{\text{eff}}(\Delta \hat{n})| e^{i\phi(\Delta \hat{n})}$, with a quantum Peierls phase $\phi(\hat{n}_i - \hat{n}_j) = \Omega(\hat{n}_i - \hat{n}_j)$, dependent on the population difference between nearest sites.

As mentioned in the main text the appearance of this phase does not result in an overall shift of $\rho_{\text{E}}(k)$. This may be understood by realizing that by construction $J_{\text{eff}}(-\Delta \hat{n}) = J_{\text{eff}}(\Delta \hat{n})^*$, and hence $\phi(-\Delta \hat{n}) = -\phi(\Delta \hat{n})$. For an homogeneous superfluid, the number difference between neighboring sites presents quantum fluctuations.
around a zero mean, and the quantum Peierls phases acquire a stochastic character, varying randomly from bond to bond between positive and negative values. As a result the system experiences an effective decoherence. We illustrate this effect in Fig. S2(a), where we show DMRG results for $\rho_E(k)$. Note that when increasing $\Omega, \rho_E(k)$ broadens, and may even saturate the Brillouin zone, as a consequence of the dephasing.

A net drift of the quasi-momentum distribution may be however achieved in the presence of a density gradient, which may in turn result from a lattice tilting. Although the created phase still depends on population differences, the density gradient $\langle \hat{n}_j - \hat{n}_{j+1} \rangle \neq 0$ leads to a non-zero average Peierls phase. This is illustrated in Fig. S1(b), where we show that a density gradient results in a net drift of the momentum distribution, in addition to the broadening mentioned above.

**B. CREATION OF ARBITRARY DENSITY DEPENDENT PEIERLS PHASES**

In the main text the derivation of the AB-model is described for the case $U_{AB}(0 < t < T/2) = -U_{AB}(T/2 < t < T) = \tilde{U}_{AB}\sin(\omega_{AB}t)$ which leads to the density dependent hopping amplitude and phase $\tilde{J}_{AB}(\tilde{n}_{j2}) = \frac{2}{\sqrt{2}}\tilde{J}_0(\Omega_{AB}\tilde{n}_{j2}) e^{i\Omega_{AB}\tilde{n}_{j2}}$. So here the phase is always strictly coupled to the modulus of the hopping.

One may choose more generally $U_{AB}(0 < t < T/2) = \tilde{U}_{AB}\sin(\omega_{AB}t + \phi_1)$ and $U_{AB}(T/2 < t < T) = \tilde{U}_{AB}\sin(\omega_{AB}t + \phi_2)$. Note that $\phi_1 = 0$, $\phi_2 = \pi$ reproduces the case shown in figure 1 of the main text. The effective tunneling is given by $\tilde{J}_{AB}(\tilde{n}_{j2}) = \frac{2}{\sqrt{2}}\tilde{J}_0(\Omega_{AB}\tilde{n}_{j2}) e^{i\Omega_{AB}\cos(\phi_1)\tilde{n}_{j2}}$ and $\tilde{J}_{BA}(\tilde{n}_{j2}) = \frac{2}{\sqrt{2}}\tilde{J}_0(\Omega_{AB}\tilde{n}_{j2}) e^{i\Omega_{AB}\cos(\phi_2)\tilde{n}_{j2}}$. A unitary gauge transformation $\tilde{B}_{j2} \rightarrow \tilde{B}_{j2} e^{-i(\Phi_{AB} + \Phi_{BA})/2n_{j2}}$ may be used to obtain $\tilde{J}_{AB}(\tilde{n}_{j2}) = \frac{2}{\sqrt{2}}\tilde{J}_0(\Omega_{AB}\tilde{n}_{j2}) e^{i\Phi/2n_{j2}} = \tilde{J}_{BA}(\tilde{n}_{j2})$ in Eq. (4) of the main text. Hence, $\Phi = \Phi_{AB} - \Phi_{BA} = \Omega_{AB}(\cos(\phi_1) - \cos(\phi_2))$ may be changed keeping the hopping modulus unaffected as in Fig. 3(b) of the main text.

**C. THE TWO PARTICLE SCATTERING PROBLEM**

In the following we provide a detailed description of the calculation of the two-particle scattering length for the AB-model as given in Eq.(5) of the main text. A general bosonic two particle state is given by $|\Psi_Q\rangle = \left[ \sum_x \frac{c_{x,x}}{\sqrt{2}} (b_x^\dagger)^2 + \sum_{x,y>x} c_{x,y} b_x^\dagger b_y \right] |0\rangle$, where $|0\rangle$ is the vacuum. Due to the conservation of total momentum in the scattering process one can express the amplitudes as $c_{x,x+r} = C_r e^{iQ(x+\frac{r}{2})}$ for $x$ in one of the $A$ sites and $c_{x,x+r} = D_r e^{iQ(x+\frac{r}{2})}$ for $x \in B$. Here $Q = q_1 + q_2$, the total momentum (below we employ $q = (q_1 - q_2)/2$ as the half relative momentum). The Schrödinger equation $H_{eff}^{AB} |\Psi\rangle = \epsilon |\Psi\rangle$ for the two particle problem leads to the following system of coupled equations for the amplitudes $C_r$ and $D_r$ with $\Gamma = \frac{1}{\sqrt{2}}\tilde{J}_0(\Omega_{AB}) e^{i\Phi/2}$.

$$
(\epsilon - U)C_0 = -\sqrt{2}J |\Gamma| \left( D_1 e^{iQ/2} + C_1 e^{-iQ/2} \right)
$$

$$
(\epsilon - U)D_0 = -\sqrt{2}J |\Gamma| \left( C_1 e^{iQ/2} + D_1 e^{-iQ/2} \right)
$$

$$
\epsilon C_1 = -\sqrt{2}J |\Gamma| \left( C_0 e^{iQ/2} + D_0 e^{-iQ/2} \right) - J/2 \left( C_2 e^{-iQ/2} + D_2 e^{iQ/2} \right)
$$

$$
\epsilon D_1 = -\sqrt{2}J |\Gamma| \left( C_0 e^{-iQ/2} + D_0 e^{iQ/2} \right) - J/2 \left( C_2 e^{iQ/2} + D_2 e^{-iQ/2} \right)
$$

$$
\epsilon C_{r \geq 2} = -J/2 \left( C_{r-1} e^{iQ/2} + C_{r+1} e^{-iQ/2} + D_{r-1} e^{-iQ/2} + D_{r+1} e^{iQ/2} \right)
$$

$$
\epsilon D_{r \geq 2} = -J/2 \left( D_{r-1} e^{iQ/2} + D_{r+1} e^{-iQ/2} + C_{r-1} e^{-iQ/2} + C_{r+1} e^{iQ/2} \right)
$$

The energy of the two scattered particles is given by $\epsilon = -2J\cos(q)\cos(Q/2)$. In order to extract scattering properties we solve this set of equations with the ansatz $C_r = e^{-\alpha r} + ve^{i\alpha r} + \beta \alpha^r$ and $D_r = e^{-i\alpha r} + ve^{i\alpha r} - \beta \alpha^r$ for $r > 1$. The equations for $r > 2$ can be solved by this ansatz if $2\alpha \cos(q)\cos(Q/2) = (-1 + \alpha^2)\sin(Q/2)$. We choose $|\alpha| < 1$ and solve the remaining four equations for $C_0, D_0, v$ and $\beta$. Since the $\alpha$ part decays exponentially fast, we can extract the scattering length $a = -\lim_{q \rightarrow 0} \partial_q \delta$ with $v = e^{2\delta}$ which after some algebra results in Eq.(5) of the main text.
FIG. S2: Quasi-momentum distribution of the A and the B components in the laboratory frame as a function of time for \( \langle \hat{n} \rangle = 3/2 \) and \( \Omega_{AB} = 0.8 \), and same parameters as those of Fig. 5 of the main text.

D. TIME OF FLIGHT IMAGING

As in recent experiments on shaken lattices \cite{11}, the shifted quasi-momentum distribution \( \rho_E(k) \), may be detected in TOF experiments. However, as mentioned in the main text, the relation between the quasi-momentum distribution of the effective model and TOF imaging presents some features that differ significantly from the shaken lattice case. Interestingly, since atoms at sites A and B belong to different species, it is actually possible to visualize the quasi-momentum distribution of atoms in state \( |1\rangle \) and \( |2\rangle \) separately (see Fig. S2). Note that for the B sublattice, \( \langle \psi(t)|\hat{b}_{2i+1}^{\dagger}\hat{b}_{2j+1}\psi(t)\rangle = \langle \psi'(|\hat{b}_{2i+1}^{\dagger}\hat{b}_{2j+1}|\psi') \), and hence the quasi-momentum distribution observed in TOF will be exactly the same as that of the effective model at any time. In contrast, for the A sublattice \( \langle \psi(t)|\hat{b}_{2i}^{\dagger}\hat{b}_{2j}\psi(t)\rangle = \langle \psi'(|\hat{b}_{2i}^{\dagger}\hat{b}_{2j}|\psi') \). As a result, the quasi-momentum distribution of the A sublattice just coincides with that of the effective model (and also with that of the sublattice B) at times \( t = nT \). For intermediate times, the phase appearing in the conversion between both reference frames leads to a broadening, and eventual blurring, of the TOF peaks (Fig. S2). Note that this blurring is in itself a result of the number-dependence of the effective model, being related with the stochastic phase discussed in Sec. A.

E. CORRELATION FUNCTIONS

In Fig. S3 we show two examples of single-particle and density-density correlations without and with density dependent phase and the fit of the power-law behavior including small oscillatory-parts which are due to the broken AB-symmetry and conformal corrections as presented in \cite{8}.

The very same reasoning on anomalous behavior of correlation functions of the main text is indeed true for any model with a coupling between phase and density. In fig. S4 we show for completeness the anomalous behavior of correlation functions of a pure anyon Hubbard model (as described in \cite{2}).

F. DETAILS OF THE NUMERICAL SIMULATION OF REAL TIME EVOLUTIONS

For the dynamical calculations of Fig. 5 of the main text, and Fig. S2 we have used TEBD calculations for 16 sites with up to 300 states, and a maximal site occupation of 4 bosons. As in Ref. \cite{8}, we may simulate rather long evolution times\( (t \sim 400\tau) \) due to the quasi-
adiabatic character of the dynamics. We have carried out our TEBD simulations for time steps $dt = T/400$ and $m = 300$ matrix states, which compare well to simulations with $dt = T/600$ and $m = 400$, showing the convergence of the results. Smaller system sizes, with a correspondingly decreased ramping and evolution time, display very similar behavior and error-bars. The non-adiabaticity of the finite ramping time leads to oscillations in the expectation value of $k_{\text{max}}$ after the ramping procedure. The time-average and standard deviation are shown as points and error-bars in Fig. 5 of the main text and compare very well to the ground-state expectation.

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