Quantum Thermal Effect of Dirac Particles in a Non-uniformly Rectilinearly Accelerating Black Hole with Electronic Charge, Magnetic Charge and Cosmological Constant

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ABSTRACT

The Hawking radiation of Dirac particles in an arbitrarily rectilinearly accelerating Kinnersley black hole with electro-magnetic charge and cosmological constant is investigated by using method of the generalized tortoise coordinate transformation. Both the location and the temperature of the event horizon depend on the time and the polar angle. The Hawking thermal radiation spectrum of Dirac particles is also derived.

Key words: Hawking effect, Dirac equation, non-stationary black hole, generalized tortoise coordinate transformation

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I. INTRODUCTION

An important subject of black hole physics is to reveal the thermal properties of various black holes \[1\]. Last decades has witnessed much progress on investigating the thermal properties of scalar fields or Dirac particles in the stationary axisymmetry black holes \[2,3\]. In the studying of the Hawking evaporation of the non-stationary black holes, a method of the generalized tortoise coordinate transformation (GTCT) suggested by Zhao and Dai \[5\] has been applied to investigate the Hawking thermal radiation of scalar particles in some non-uniformly accelerating black holes \[6\] and in the non-uniformly accelerating Kerr black hole \[7\].

However, it is very difficult to investigate the quantum thermal effect of Dirac particles in the non-stationary black hole. The difficulty lies in the non-separability of the Chandrasekhar-Dirac equation \[8\] in the most general space-times. The Hawking radiation of Dirac particles in some non-static black holes has so far been studied in \[9\].

In this paper, we deal with the Hawking effect of Dirac particles in a non-spherically symmetric and non-stationary Kinnersley black hole with electronic charge, magnetic charge and cosmological constant \[10\]. By making use of the GTCT method, we obtain the equation which determines the event horizon of the Kinnersley black hole. The event horizon equation derived by the limiting form of Dirac equation near the event horizon is exactly the same as those given by the null hypersurface which is not spherically symmetric \[6\]. Then we turn to the second order form of the Dirac equation. With the aid of a GTCT, we adjust the temperature parameter in order that each component of Dirac spinors satisfies a simple wave equation after being taken limits approaching the event horizon.

We show that both the shape and the Hawking temperature of the event horizon of Kinnersley black hole depend on not only the time, but also on the angle. The location and the temperature coincide with those obtained by investigating the Hawking effect of Klein-Gordon particles in the accelerating Kinnersley black hole \[6\].

II. DIRAC EQUATION

The metric of a non-uniformly rectilinearly accelerating Kinnersley black hole with electric charge \(Q\), magnetic charge \(P\) and cosmological constant \(\Lambda\) is given in the advanced Eddington-Finkelstein coordinate system by \[10\].
\[ ds^2 = 2dv(Gdv - dr - r^2 f d\theta) - r^2(d\theta^2 + \sin^2 \theta d\varphi^2) \]  

where \( 2G = 1 - \frac{2M}{r} + \frac{Q^2 + P^2}{r^2} - 4a \cos \theta \frac{Q^2 + P^2}{r^2} - 2ar \cos \theta - r^2 f^2 - \frac{A}{2}r^4, \ f = -a \sin \theta. \) In the above, the parameter \( a = a(v) \) is the magnitude of acceleration, the mass \( M(v) \) and the charges \( Q(v), P(v) \) of the hole are functions of time \( v. \)

We choose a complex null-tetrad \( \{ l, n, m, \overline{m} \} \) such that \( l \cdot n = -m \cdot \overline{m} = 1. \) Thus the covariant one-forms can be written as

\[ l = dv, \quad n = Gdv - dr - r^2 f d\theta, \quad m = \frac{i}{\sqrt{2}} (d\theta + i \sin \theta d\varphi), \quad \overline{m} = \frac{-i}{\sqrt{2}} (d\theta - i \sin \theta d\varphi). \]  

and their corresponding directional derivatives are

\[ D = -\frac{\partial}{\partial r}, \quad \Delta = \frac{\partial}{\partial r} + G \frac{\partial}{\partial r}, \]

\[ \delta = \frac{1}{\sqrt{2r}} \left(-r^2 f \frac{\partial}{\partial r} + \frac{i}{\sin \theta} \frac{\partial}{\partial \varphi} + \frac{i}{\sin \theta} \frac{\partial}{\partial \varphi} \right), \]

\[ \overline{\delta} = \frac{1}{\sqrt{2r}} \left(-r^2 f \frac{\partial}{\partial r} + \frac{i}{\sin \theta} \frac{\partial}{\partial \varphi} - \frac{i}{\sin \theta} \frac{\partial}{\partial \varphi} \right). \]  

Inserting for the following relations among the Newman-Penrose [11] spin-coefficients [1]

\[ \epsilon - \rho = -\frac{1}{r}, \quad \bar{\pi} - \alpha = \frac{\cot \theta}{2\sqrt{2r}} - \sqrt{2} f, \]

\[ \mu - \gamma = \frac{G}{r} + \frac{G^*}{2}, \quad \beta - \tau = \frac{\cot \theta}{2\sqrt{2r}} - \frac{f}{\sqrt{2}}, \]  

into the spinor form of the coupled Chandrasekhar-Dirac equation [8], which describes the dynamic behavior of spin-1/2 particles, namely

\[ (D + \epsilon - \rho) F_1 + \overline{\delta + \bar{\pi} - \alpha} F_2 = \frac{i\mu_0}{\sqrt{2}} G_1, \]

\[ (\Delta + \mu - \gamma) F_2 + (\delta + \beta - \tau) F_1 = \frac{i\mu_0}{\sqrt{2}} G_2, \]

\[ (D + \epsilon^* - \rho^*) G_2 - (\bar{\delta} + \bar{\pi} - \alpha^*) G_1 = \frac{i\mu_0}{\sqrt{2}} F_2, \]

\[ (\Delta + \mu^* - \gamma^*) G_1 - (\bar{\delta} + \bar{\beta} - \tau^*) G_2 = \frac{i\mu_0}{\sqrt{2}} F_1, \]

where \( \mu_0 \) is the mass of Dirac particles, one obtains

\[ -D_1 F_1 + \frac{1}{\sqrt{2r}} (L - r^2 f D_2) F_2 = \frac{i\mu_0}{\sqrt{2}} G_1, \]

\[ \left( \frac{\partial}{\partial v} + GD_1 + G^{*r}/2 \right) F_2 + \frac{1}{\sqrt{2r}} \left( L^* - r^2 f D_1 \right) F_1 = \frac{i\mu_0}{\sqrt{2}} G_2, \]

\[ -D_1 G_2 - \frac{1}{\sqrt{2r}} \left( L^* - r^2 f D_2 \right) G_1 = \frac{i\mu_0}{\sqrt{2}} F_2, \]

\[ \left( \frac{\partial}{\partial v} + GD_1 + G^{*r}/2 \right) G_1 - \frac{1}{\sqrt{2r}} (L - r^2 f D_1) G_2 = \frac{i\mu_0}{\sqrt{2}} F_1, \]

\[ \]  

1Here and hereafter, we denote \( G^{*r} = dG/dr \) , etc.
in which we have defined operators
\[ D_n = \frac{\partial}{\partial r} + \frac{n}{r}, \quad L = \frac{\partial}{\partial \theta} + \frac{i}{2} \cot \theta - \frac{i}{\sin \theta} \frac{\partial}{\partial \varphi}, \quad L^\dagger = \frac{\partial}{\partial \theta} + \frac{i}{2} \cot \theta + \frac{i}{\sin \theta} \frac{\partial}{\partial \varphi}. \]

By substituting
\[ F_1 = \frac{1}{\sqrt{2r}} P_1, \quad F_2 = P_2, \quad G_1 = Q_1, \quad G_2 = \frac{1}{\sqrt{2r}} Q_2, \]
into Eq. (6), they have the form
\[ -D_0 P_1 + (L - r^2 f D_2) P_2 = i\mu_0 r Q_1, \]
\[ r^2 \left( 2 \frac{\partial}{\partial v} + 2GD_1 + G_r \right) P_2 + \left( L^\dagger - r^2 f D_0 \right) P_1 = i\mu_0 r Q_2, \]
\[ -D_0 Q_2 - \left( L^\dagger - r^2 f D_2 \right) Q_1 = i\mu_0 r P_2, \]
\[ r^2 \left( 2 \frac{\partial}{\partial v} + 2GD_1 + G_r \right) Q_1 - (L - r^2 f D_0) Q_2 = i\mu_0 r P_1. \]

III. EVENT HORIZON

An apparent fact is that the Chandrasekhar-Dirac equation (6) could be satisfied by identifying \( Q_1, Q_2 \) with \( P_2^*, -P_1^* \), respectively. So one may deal with a pair of components \( P_1, P_2 \) only. Although Eq. (7) can not be decoupled, to deal with the problem of Hawking radiation, one may concern about the behavior of Eq. (7) near the horizon only. As the space-time we consider at present has a symmetry about \( \varphi \)-axis, we can introduce the generalized tortoise coordinate transformation [5]
\[ r_* = r + \frac{1}{2\kappa} \ln[r - r_H(v, \theta)], \]
\[ v_* = v - v_0, \quad \theta_* = \theta - \theta_0, \]
where \( r_H \) is the location of the event horizon, \( \kappa \) is an adjustable parameter and is unchanged under tortoise transformation. Both parameters \( v_0 \) and \( \theta_0 \) are arbitrary constants. From formula (8), we can deduce some useful relations for the derivatives as follows:
\[ \frac{\partial}{\partial r} = \left[ 1 + \frac{1}{2\kappa(r-r_H)} \right] \frac{\partial}{\partial r_*}, \]
\[ \frac{\partial}{\partial v} = \frac{\partial}{\partial v_*} - \frac{r_H v}{2\kappa(r-r_H)} \frac{\partial}{\partial r_*}, \]
\[ \frac{\partial}{\partial \theta} = \frac{\partial}{\partial \theta_*} - \frac{r_H \phi}{2\kappa(r-r_H)} \frac{\partial}{\partial r_*}. \]
Under the transformation (8), Eq. (7) with regards to \((P_1, P_2)\) can be reduced to the following limiting form near the event horizon:

\[
\frac{\partial}{\partial r^*} P_1 + (r_{H,\theta} + r_{H,f}^2) \frac{\partial}{\partial r^*} P_2 = 0,
\]

\[
-(r_{H,\theta} + r_{H,f}^2) \frac{\partial}{\partial r^*} P_1 + 2r_{H}^2 (G - r_{H,v}) \frac{\partial}{\partial r^*} P_2 = 0,
\]

after being taken limits \(r \to r_H(v_0, \theta_0), v \to v_0\) and \(\theta \to \theta_0\). A similar form holds for \(Q_1, Q_2\).

If the derivatives \(\frac{\partial}{\partial r^*} P_1\) and \(\frac{\partial}{\partial r^*} P_2\) in Eq. (9) do not be equal to zero, the existence condition of non-trial solutions for \(P_1\) and \(P_2\) is that the determinant of Eq. (9) vanishes, which gives the following equation to determine the location of horizon:

\[
2G - 2r_{H,v} + r_{H,f}^2 + 2f r_{H,\theta} + \frac{r_{H,\theta}^2}{r_{H}^2} = 0.
\]

The event horizon equation (10) can be inferred from the null hypersurface condition, \(g^{ij} \partial_i F \partial_j F = 0\), and \(F(v, r, \theta) = 0\), namely \(r = r(v, \theta)\). The location of the event horizon is in accord with that obtained in the case of discussing about the thermal effect of Klein-Gordon particles in the same space-time [3]. It follows that \(r_H\) depends not only on \(v\), but also on \(\theta\). So the location of the event horizon and the shape of the black hole change with time.

### IV. HAWKING TEMPERATURE

To investigate the Hawking radiation of spin-1/2 particles, one may only deal with the behavior of \(P_1, P_2\) components of Dirac equation near the event horizon because one can set

\[
Q_2 = -P_1^* , \quad Q_1 = P_2^*.
\]

A direct calculation gives the second-order form of Dirac equation for the two-component spinor \((P_1, P_2)\)

\(^2\text{Throughout the paper, we make a convention that all coefficients in the front of each derivatives term take values at the event horizon when a GTCT is made and followed by taking limits approaching the event horizon.}\)
Given the GTCT in Eq. (8) and after some tedious calculations, the limiting form of Eqs. (12, 13), when $r$ approaches $r_H(v_0, \theta_0)$, $v$ goes to $v_0$ and $\theta$ goes to $\theta_0$, reads

\[
[r^2(2\frac{\partial}{\partial v} + 2GD_0 + G_r)D_0 + ((\mathcal{L} - r^2fD_1)(\mathcal{L} - r^2fD_0))P_1 \\
+ r^2(-(2\frac{\partial}{\partial v} + 2GD_0 + G_r)(\mathcal{L} - r^2fD_2)) \\
+ (\mathcal{L} - r^2fD_1)(2\frac{\partial}{\partial v} + 2GD_1 + G_r)]P_2 = \mu_0^2r^2P_1,
\]

(12)

and

\[
[r^2D_1(2\frac{\partial}{\partial v} + 2GD_1 + G_r) + ((\mathcal{L} - r^2fD_1)(\mathcal{L} - r^2fD_2))]P_2 \\
+ [D_0, \mathcal{L} - r^2fD_0)]P_0 = \mu_0^2r^2P_2.
\]

(13)

With the aid of the event horizon equation (10), we know that the coefficient $A$ is an infinite limit of $\frac{\partial}{\partial v}$ type. By use of the L' Hôpital rule, we get the following result

\[
A = \lim_{r\to r_H(v_0, \theta_0)} \frac{2r^2(G - r_{H, v}) + r^4f^2 + 2fr_{H, \theta}r_{H, \theta}}{r - r_H} \\
= 2r_H^2G_r + 4r_H(G - r_{H, v}) + 4r_H^3f^2 + 4fr_Hr_{H, \theta} \\
= 2r_H^2G_r + 2r_H^3f^2 - \frac{2r_H^2r_{H, \theta}}{r_H}.
\]

(16)

Now let us select the adjustable parameter $\kappa$ in Eqs. (14, 15) such that

\[
r_H^2 = \frac{A}{2\kappa} + 2r_H^2(2G - r_{H, v}) + 2r_H^4f^2 + 2fr_H^2r_{H, \theta} \\
= \frac{r_H^3G_r + r_H^4f^2 - r_H^2r_{H, \theta}}{\kappa r_H} + 2Gr_H^2 + r_H^4f^2 - r_H^2r_{H, \theta},
\]

(17)
which means the temperature of the horizon is

\[ \kappa = \frac{r^2_H G_r + r^3_H f^2 - r^2_H r_H}{r^2_H (1 - 2G) - r^4_H f^2 + r^3_H} \].

Such a parameter adjustment can make Eqs. (14,15) reduce to

\[
\frac{\partial^2}{\partial r^2} P_1 + 2 \frac{\partial}{\partial r} \frac{\partial}{\partial v} P_1 - 2 \left( f + \frac{r_{H,\theta}}{r_H} \right) \frac{\partial^2}{\partial r \partial \theta} P_1 + \left[ r_H f^2 - G_r \right. \\
\left. - f,\theta - f, \cot \theta_0 - (r_H f + \cot \theta_0) \frac{r_{H,\theta}}{r_H} + \frac{2 r_{H,\theta}}{r_H} - \frac{r_{H,\theta\theta}}{r_H} \right] \frac{\partial}{\partial r^*} P_1 \\
+ 2 \left[ G,\theta + r^2_H f_v - \frac{Gr_{H,\theta}}{r_H} - r^2_H f \left( G_r + \frac{r_{H,v} - 2G}{r_H} \right) \right] \frac{\partial}{\partial r^*} P_2 = 0 ,
\]

and

\[
\frac{\partial^2}{\partial r^2} P_2 + 2 \frac{\partial}{\partial r} \frac{\partial}{\partial v} P_2 - 2 \left( f + \frac{r_{H,\theta}}{r_H} \right) \frac{\partial^2}{\partial r \partial \theta} P_2 + \left[ 5 r_H f^2 + G_r \right. \\
\left. - f,\theta - f, \cot \theta_0 + (3 f r_H - \cot \theta_0) \frac{r_{H,\theta}}{r_H} + \frac{2 r_{H,\theta}}{r_H} - \frac{r_{H,\theta\theta}}{r_H} \right] \frac{\partial}{\partial r^*} P_2 + \frac{r_{H,\theta}}{r_H} \frac{\partial}{\partial r^*} P_1 = 0 .
\]

Using Eq. (9), Eqs. (19,20) can be recast into the following standard wave equation near the horizon in an united form

\[
\frac{\partial^2}{\partial r_s^2} \Psi + 2 \frac{\partial}{\partial r_s \partial v_s} \Psi - 2 C_1 \frac{\partial^2}{\partial r_s \partial \theta_s} \Psi + 2 C_2 \frac{\partial}{\partial r_s} \Psi = 0 ,
\]

where \( C_1, C_2 \) will all be regarded as finite real constants,

\[ C_1 = f + \frac{r_{H,\theta}}{r_H} , \]

\[ 2C_2 = -r_H f^2 - G_r - f,\theta - f \cot \theta_0 - (r_H f + \cot \theta_0) \frac{r_{H,\theta}}{r_H} + \frac{2 r_{H,\theta}}{r_H} - \frac{r_{H,\theta\theta}}{r_H} \]

\[ - \frac{r^2_H f + r_{H,\theta}}{(G - r_{H,v}) r_H^3} \left[ G_{v,H} - r_H G_r - r^3_H f_v + r^2_H f \left( G_r r_H + r_{H,v} - 2G \right) \right] \]

for \( \Psi = P_1 \), and

\[ 2C_2 = 3 r_H f^2 - f,\theta - f \cot \theta_0 + G_r + \frac{4G - 2r_{H,v}}{r_H} \]

\[ + (2 f r_H - \cot \theta_0) \frac{r_{H,\theta}}{r_H} + \frac{r^2_{H,\theta}}{r_H^3} - \frac{r_{H,\theta\theta}}{r_H^2} \]

for \( \Psi = P_2 \).
V. THERMAL RADIATION SPECTRUM

Now separating variable as follows

\[ \Psi = R(r_*)\Theta(\theta_*)e^{-i\omega r_* + im\varphi} \]

and substituting this into equation (21), one gets

\[ \Theta' = \lambda \Theta, \]
\[ R'' = 2(i\omega - C_0)R', \]

where \( \lambda \) is a real constant introduced in the separation variables, \( C_0 = C_2 - \lambda C_1 \). The solutions are

\[ \Theta = e^{\lambda \theta_*}, \]
\[ R \sim e^{2(i\omega - C_0)r_*}; R_0. \]

The ingoing wave and the outgoing wave to Eq. (21) are

\[ \Psi_{\text{in}} = e^{-i\omega r_* + im\varphi + \lambda \theta_*}, \]
\[ \Psi_{\text{out}} = e^{-i\omega r_* + im\varphi + \lambda \theta_*}e^{2(i\omega - C_0)r_*}, \quad (r > r_H). \]

Near the event horizon, we have

\[ r_* \sim \frac{1}{2\kappa} \ln(r - r_H). \]

Clearly, the outgoing wave \( \Psi_{\text{out}}(r > r_H) \) is not analytic at the event horizon \( r = r_H \), but can be analytically extended from the outside of the hole into the inside of the hole through the lower complex \( r \)-plane

\[ (r - r_H) \to (r_H - r)e^{-i\pi} \]

to

\[ \tilde{\Psi}_{\text{out}} = e^{-i\omega r_* + im\varphi + \lambda \theta_*}e^{2(i\omega - C_0)r_*}e^{i\pi C_0/\kappa}e^{\pi \omega/\kappa}, \quad (r < r_H). \]

So the relative scattering probability of the outgoing wave at the horizon is easily obtained

\[ \left| \frac{\Psi_{\text{out}}}{\tilde{\Psi}_{\text{out}}} \right|^2 = e^{-2\pi \omega/\kappa}. \]
According to the method suggested by Damour and Ruffini [2] and developed by Sannan [12], the thermal radiation Fermionic spectrum of Dirac particles from the event horizon of the hole is given by

$$\langle N(\omega) \rangle = \frac{1}{e^{\omega/T_H} + 1},$$

(27)

with the Hawking temperature being

$$T_H = \frac{\kappa}{2\pi},$$

whose obvious expression is

$$T_H = \frac{1}{4\pi r_H} \cdot \frac{M r_H - r_H^3 a \cos \theta_0 + (2r_H a \cos \theta_0 - 1)(Q^2 + P^2) - \frac{A}{3} r_H^4 - \frac{r_H^2}{2} a}{M r_H + r_H^3 a \cos \theta_0 + (2r_H a \cos \theta_0 - 1/2)(Q^2 + P^2) - \frac{A}{6} r_H^4 + \frac{r_H^2}{2} a}. $$

(28)

It follows that the temperature depends not only on the time, but also on the angle $\theta$ because it is determined by the surface gravity $\kappa$, a function of $v$ and $\theta$. The temperature is in consistent with that derived from the investigating of the thermal radiation of Klein-Gordon particles [3].

VI. CONCLUSIONS

Equations (10) and (18) give the location and the temperature of event horizon of the hole, which depend not only on the advanced time $v$ but also on the polar angle $\theta$. Eq. (27) shows the thermal radiation spectrum of Dirac particles in an arbitrarily rectilinearly accelerating Kinnersley black hole.

In conclusion, we have studied the Hawking radiation of Dirac particles in an arbitrarily accelerating Kinnersley black hole whose mass and charges change with time. The Chandrasekhar-Dirac equation can not be decoupled in the most general black hole background, however, under the generalized tortoise coordinate transformation, the limiting form of its corresponding second order equation takes the standard form of wave equation near the event horizon, to which separation of variables is possible.

Both the location and the temperature of the event horizon of the accelerating Kinnersley black hole depend on the time and the angle. They are just the same as that obtained in the discussing on thermal radiation of Klein-Gordon particles in the same space-time.

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**APPENDIX: NEWMAN-PENROSE COEFFICIENTS**

The complex null-tetrad \{l, n, m, m\} that satisfies the orthogonal conditions \(l \cdot n = -m \cdot m = 1\) in the Kinnersley black hole is chosen as

\[
\begin{align*}
l &= dv, \\
n &= Gdv - dr - r^2 f d\theta, \\
m &= \frac{-r}{\sqrt{2}} (d\theta + i \sin \theta d\varphi), \\
m &= \frac{-r}{\sqrt{2}} (d\theta - i \sin \theta d\varphi).
\end{align*}
\]  

(A.1)

It is not difficult to determine the twelve Newman-Penrose complex coefficients [1] in the above null-tetrad as follows

\[
\begin{align*}
\bar{\kappa} = \bar{\lambda} = \sigma = \epsilon = 0, \\
\rho &= \frac{1}{r}, \\
\mu &= \frac{G}{r}, \\
\gamma &= -G_r/2, \\
\tau &= -\bar{\pi} = \frac{f}{\sqrt{2}}, \\
\alpha &= -\frac{\cot \theta}{2\sqrt{2r}} + \frac{f}{\sqrt{2r}}, \\
\beta &= \frac{\cot \theta}{2\sqrt{2r}}, \\
\nu &= \frac{1}{\sqrt{2r}} \left[ (2rG - r^2 G_r) f + r^2 f_v + G_\theta \right].
\end{align*}
\]

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