Experimental and finite element studies on free vibration of cylindrical skew panels

Chikkol V Srinivasa¹, Yalaburgi J Suresh² and Wooday P Prema Kumar³,⁴

Abstract

The present paper deals with experimental and finite element studies carried out on free vibration of isotropic and laminated composite cylindrical skew panels. Natural frequencies were determined using CQUAD8 finite element of MSC Nastran, and comparison was made between the experimental values and the finite element solution. Effects of the skew angle and aspect ratio on the natural frequencies of isotropic cylindrical skew panels were studied. The effects of skew angle, aspect ratio, fiber orientation angle, and laminate stacking sequence (keeping total number of layers in the laminate constant) on the natural frequencies of antisymmetric laminated composite cylindrical skew panels were also studied. The experimental values of the first, second, and third natural frequencies are in good agreement with the finite element solution for both isotropic and laminated composite cylindrical panels. The first, second, and third natural frequencies are found to decrease with the increase in the aspect ratio for both isotropic and laminated composite cylindrical panels.

Keywords: Cylindrical skew panel; Panel angle; Skew angle; Fiber orientation angle; Antisymmetric laminate; Finite element analysis; Natural frequency

Introduction

Cylindrical panels (regular and skewed) have found a wide array of application in the aircraft and spaceship industry. The exact solutions to free vibration problems of cylindrical skew panels are mathematically involved and, in many cases, not available. Hence, most of the available solutions are based on approximate methods, the most commonly used method being the finite element analysis.

Some of the studies made on regular cylindrical panels are those of Selmane and Lakis (1997) who investigated the dynamic and static behavior of thin, open cylindrical shells freely supported along their curved edges with different boundary conditions at the straight edges. Bardell et al. (1997, 1998) studied the free vibration problem of deep cylindrical panels using the h-p version of the finite element method and then extended the method to thin, isotropic conical panels. Singh (1999) studied the free vibration of doubly curved deep sandwich shell panels. He compared the results obtained using the Rayleigh-Ritz method with those obtained using I-DEAS for isotropic circular cylindrical and spherical shell panels. Lee and Han (2001) studied the free vibration of isotropic plates and shells using a nine-node degenerated shell element. In this study, the first-order shear deformation theory has been used. Zhao et al. (2003) studied the vibration behavior of composite cylindrical panels using the mesh-free approach. A refined theory for thick spherical shell has been presented by Voyiadjis and Woelke (2004). This theory not only incorporates the effect of transverse shear deformation, but also takes into account the initial curvature as well as the radial stress. Hossain et al. (2004) presented an improved finite element model for the linear analysis of anisotropic and laminated doubly curved moderately thick composite shell panels. The model has been developed by considering the first-order shear deformation theory. Rectangular-type open circular cylindrical composite shells supported on various combinations of corner and mid-edge points of the panel were investigated by Singh and Shen (2005). Liu et al. (2006) have implemented the element-free Galerkin method for static and free vibration analysis of general shell structures. The formulation has been verified through numerical examples of spatial shell structures. Pradyumna and Bandyopadhyay (2008) investigated the free vibration...
behavior of functionally graded curved panels using a higher-order finite element formulation.

Kandasamy and Singh (2006) carried out a numerical investigation of free vibration of skewed open cylindrical isotropic shells. First-order shear deformation and rotary inertia were included in the formulation. Thin and moderately thick shells were considered. Haldar (2008) used a triangular shallow shell element for free vibration study of laminated composite skewed cylindrical shell panels. Gulshan Taj and Chakrabarti (2013) studied the dynamic response of a functionally graded skew shell using a $C_0$ finite element formulation. Numerical results were presented for cylindrical, spherical, and hyper shells for different boundary conditions and skew angles.

Experimental studies of free vibration and experimental validation of finite element solutions for laminated composite cylindrical skew panels are very scarce in the literature. The present work attempts to address this aspect at least in a partial manner. The effects of varying skew angle, aspect ratio (ratio of panel length to curved width), fiber orientation angle, and laminate stacking sequence on the first, second, and third natural frequencies of cylindrical skew panels are experimentally investigated. The experimental results are compared with those of the finite element solution.

**Methods**

**Test specimens and experimental setup**

**Test specimens**

In the present work, isotropic cylindrical skew panels cut from aluminum 7075-T6 tubes were used. The material was supplied by Rio-Tinto Alcon, Canada. The material properties of the isotropic skew panels are $E = 71.7$ GPa, $\mu = 0.33$, and $\rho = 2800$ kg/m$^3$, and these were supplied by the manufacturer. The laminated composite cylindrical skew panels were prepared using unidirectional glass fiber, Epoxy-556 resin, and hardener (HY951) supplied by Ciba-Geigy India Ltd (Mumbai, India). The specimens were prepared using a mandrel of 600-mm length and 76.2-mm diameter. The surface of the cylindrical mandrel was thoroughly cleaned using acetone to remove any dust, dirt, or rust. Then, a layer of thin releasing film was placed over the surface of the mandrel before wrapping the layers of prepreg around it. The laminate was fabricated using hand lay-up technique. After fabrication, the entire surface was covered with a thin layer of releasing film. At a time, one cylindrical panel of 500-mm length and 76.2-mm inner diameter was cast and was later cut into required specimen lengths. The fiber weight percentage is 50:50. The test specimens were prepared in accordance with relevant American Society for Testing and Materials (ASTM) provisions. For laminated glass/epoxy composite skew panel, the material constants $E_1$ and $E_2$ were evaluated experimentally using the Instron Universal Testing Machine as per ASTM Standard D3039/D3039M (ASTM 2006). The average of three experimental determinations was adopted. For the determination of Poisson’s ratio $\nu_{12}$, two strain gages were bonded to the specimen: one in the direction of the loading and the other at right angles to it. The strains were measured in longitudinal and transverse directions using a strain indicator. The ratio of transverse to longitudinal

![Figure 1 The experimental setup.](http://www.advancedstructeng.com/content/6/1/1)
strain gives Poisson's ratio within the elastic range. The average of three experimental determinations was adopted. The shear modulus $G_{12}$ was computed using the standard expression available in Jones (1975). The adopted material properties are $E_1 = 38.07$ GPa, $E_2 = 8.1$ GPa, $G_{12} = 3.05$ GPa, $\nu_{12} = 0.22$, and $\rho = 2200$ kg/m$^3$. In the present study, the skew angle was varied from 0° to 45° and the panel angle was maintained constant at 60°. The panel lengths considered were 100, 150, and 200 mm.

**Experimental setup**

The experimental setup is shown in Figure 1. First, the test specimen was placed in the fixture shown in Figure 1 with two opposite edges fully clamped and the remaining two edges completely free. The piezoelectric accelerometer was directly mounted on the test specimen at the geometric center using an adhesive. The accelerometer was then connected to signal-conditioning unit (fast Fourier transform analyzer), where the signal goes through the charge amplifier and an analog-to-digital converter. The panel was excited at a selected point by means of impact hammer. The impact hammer was struck at the selected point five times, and the average value of the frequency response function (FRF) was input to the computer through a USB port. Sufficient precautions were taken for ensuring that the strike of the impact hammer was normal to the surface of the panel. The pulse lab software accompanying the equipment was used for recording the signals directly in the memory of the computer. The signal was then read and processed to extract different features including frequencies. The frequencies were measured by moving the cursor to the peaks of the FRF. For each specimen, five experimental trials were made, each trial furnishing values of first, second, third, and other higher frequencies. The adopted values of first, second, and third natural frequencies were each the average of five values obtained in five different trials.

**Finite element solution**

A finite element analysis was made for obtaining the first three natural frequencies using MSC Nastran software (MSC Software Corporation, Newport Beach, CA, USA). CQUAD8 (eight-node isoparametric curved shell element) was preferred over the CQUAD4 element in the analysis.

---

**Figure 2** The geometry of the cylindrical skew panel with dimensions.

**Figure 3** Global and local coordinate systems for cylindrical skew panels used in finite element analysis.
as it yields more accurate results as revealed by the investigation reported by Srinivasa et al. (2013). The geometry of cylindrical skew panel with dimensions is shown in Figure 2. Figure 3 indicates cylindrical skew panel with global and local coordinate systems; \( u \) and \( v \) are the displacement components in \( x \) and \( y \) directions, respectively. Since \( u \) and \( v \) are inclined to the skew edges, the displacement boundary conditions cannot be applied directly. In order to overcome this, a local coordinate system \((x', y')\) normal and tangential to the skew edges is chosen, and the software performs the required transformation. Several options exist in the software in respect of real eigenvalue extraction, and the Lanczos method was used in the present study as it combines the best features of the other methods and computes accurate eigenvalues and eigenvectors.

| Aspect ratio \((a/b)\) | Mode number | Non-dimensional frequency coefficient \(K_f\) |
|----------------------|------------|---------------------------------|
|                      |            | 0° | 15° | 30° | 45° |
|                      | Mode number | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 |
|                      | Experimental | FEM | Experimental | FEM | Experimental | FEM | Experimental | FEM | Experimental | FEM | Experimental | FEM | Experimental | FEM |
| 2.50                 | 1           | 0.074 (1.00) | 0.075 | 0.088 (0.91) | 0.090 | 0.097 (1.10) | 0.100 | 0.112 (1.33) | 0.117 |
| 3.75                 | 1           | 0.039 (1.12) | 0.040 | 0.049 (1.25) | 0.050 | 0.053 (1.25) | 0.054 | 0.059 (0.88) | 0.061 |
| 5.00                 | 1           | 0.026 (1.35) | 0.026 | 0.034 (1.40) | 0.034 | 0.035 (1.45) | 0.036 | 0.037 (1.25) | 0.039 |

The number in parentheses represents the standard deviation in terms of percentage.

Figure 4 Variation of \(K_f\) with aspect ratio \((a/b)\) for isotropic cylindrical skew panels.
Results and discussion

The results of the present work are presented in tables and figures in the form of non-dimensional frequency coefficient $K_f$ defined as $K_f = \omega R \sqrt{\frac{\rho}{E}}$ for isotropic panels and $K_f = \omega R \sqrt{\frac{\rho}{E t^2}}$ for laminated composite cylindrical skew panels (Srinivasa et al. 2013). The standard deviation is stated in terms of percentage within parentheses in the tables. It is also indicated in the relevant figures.

Isotropic cylindrical skew panels

The thickness of the isotropic cylindrical skew panel was taken as 2.0 mm. The panel lengths used were 100, 150, and 200 mm. The skew angle was varied from 0° to 45°, and the panel angle was kept constant at 60°. Table 1 and Figure 4 show the variation of non-dimensional frequency coefficient $K_f$ for isotropic cylindrical skew panels. The experimental values of the non-dimensional frequency coefficient $K_f$ are in very good agreement with the finite element solution. The non-dimensional frequency coefficient $K_f$ increases monotonically with an increase in the skew angle and decreases as the aspect ratio increases. The first three mode shapes obtained by finite element analysis are shown in Figure 5 for an aspect ratio of 2.50.

Laminated composite cylindrical skew panels

The total thickness of the laminate was maintained constant at 2 mm, the number of layers being 20. The panel lengths considered were 100, 150, and 200 mm, and the skew angle was varied from 0° to 45°. Tables 2, 3, 4, 5 and Figures 6, 7, 8, 9 show the variation of the non-dimensional frequency coefficient $K_f$ with aspect ratio and laminate stacking sequence for various values of the skew angle. Four laminate stacking sequences, viz. antisymmetric angle-ply [+0°/−0°/.../−0°], antisymmetric angle-ply [+45°/−45°/.../−45°], antisymmetric angle-ply [+90°/−90°/.../−90°], and antisymmetric cross-ply [0°/90°/.../90°] were considered. The non-dimensional frequency coefficient $K_f$ for the first, second, and third

---

| Skew Angle (°) | Mode Shapes (Panel Angle(θ)=60°, Aspect Ratio(a/b)=2.50) |  |  |  |
|----------------|-------------------------------------------------------------|---|---|---|
| 0°             | ![Mode Shape 1](image1) $f_1=49.59291$ Hz                  | ![Mode Shape 2](image2) $f_2=77.83797$ Hz                  | ![Mode Shape 3](image3) $f_3=121.9359$ Hz                  |
| 15°            | ![Mode Shape 4](image4) $f_4=1877.098$ Hz                  | ![Mode Shape 5](image5) $f_5=2530.766$ Hz                  | ![Mode Shape 6](image6) $f_6=4367.868$ Hz                  |
| 30°            | ![Mode Shape 7](image7) $f_7=2086.000$ Hz                  | ![Mode Shape 8](image8) $f_8=2753.501$ Hz                  | ![Mode Shape 9](image9) $f_9=4885.359$ Hz                  |
| 45°            | ![Mode Shape 10](image10) $f_{10}=2440.812$ Hz             | ![Mode Shape 11](image11) $f_{11}=3187.958$ Hz             | ![Mode Shape 12](image12) $f_{12}=5672.148$ Hz             |

Figure 5 Mode shapes for isotropic cylindrical skew panels (a/b = 2.50).
natural frequencies decreases as the aspect ratio increases for all skew angles and laminate stacking sequences. For the laminate stacking sequence [+0°/−0°/.../−0°], the non-dimensional frequency coefficient $K_f$ for the first, second, and third natural frequencies increases monotonically with an increase in the skew angle for all values of aspect ratio. For the laminate stacking sequence [+45°/−45°/.../−45°], the non-dimensional frequency coefficient $K_f$ for the first natural frequency increases monotonically with an increase in the skew angle for all values of aspect ratio. $K_f$ for the second and third natural frequencies for this laminate stacking sequence does not increase monotonically. In respect of other laminate stacking sequences, $K_f$ varies as shown in the tables. Other parameters being the same, antisymmetric angle-ply [+0°/−0°/.../−0°] yields the lowest value for the first, second, and third natural frequencies and highest value for antisymmetric angle-ply [+90°/−90°/.../−90°] in majority of cases. The natural frequencies depend upon the contribution made by extensional stiffness, coupling stiffness, and bending stiffness terms in addition to the boundary conditions and panel geometry among others. The first three mode shapes for different skew angles obtained by finite element analysis are shown in Figure 10.

Table 2 Non-dimensional frequency coefficient for laminated cylindrical skew panel ($\alpha = 0^\circ$)

| Laminate stacking sequence | Mode number | Non-dimensional frequency coefficient ($K_f$) |
|----------------------------|-------------|------------------------------------------|
|                            |             | Aspect ratio ($a/b$) | 2.50 | 3.75 | 5.00 |
|                            |             | Experimental | FEM | Experimental | FEM | Experimental | FEM |
| Angle-ply [+0°/−0°/.../−0°] | 1           | 0.050 (1.45) | 0.050 | 0.028 (1.56) | 0.029 | 0.016 (1.75) | 0.017 |
|                            | 2           | 0.059 (1.35) | 0.059 | 0.029 (1.77) | 0.030 | 0.021 (1.66) | 0.021 |
|                            | 3           | 0.112 (1.55) | 0.113 | 0.063 (1.68) | 0.064 | 0.043 (1.56) | 0.044 |
| Angle-ply [+45°/−45°/.../−45°] | 1         | 0.062 (1.67) | 0.063 | 0.034 (2.12) | 0.035 | 0.019 (1.45) | 0.020 |
|                            | 2           | 0.076 (1.54) | 0.076 | 0.051 (2.34) | 0.052 | 0.037 (2.04) | 0.037 |
|                            | 3           | 0.154 (1.77) | 0.156 | 0.091 (1.57) | 0.092 | 0.052 (1.88) | 0.053 |
| Angle-ply [+90°/−90°/.../−90°] | 1         | 0.068 (1.78) | 0.069 | 0.039 (1.89) | 0.039 | 0.025 (1.99) | 0.026 |
|                            | 2           | 0.087 (1.34) | 0.088 | 0.050 (1.56) | 0.051 | 0.031 (1.65) | 0.032 |
|                            | 3           | 0.165 (1.22) | 0.166 | 0.091 (1.79) | 0.092 | 0.057 (1.79) | 0.059 |
| Cross-ply [0°/90°/.../90°]  | 1           | 0.060 (1.34) | 0.061 | 0.034 (1.11) | 0.035 | 0.023 (0.98) | 0.024 |
|                            | 2           | 0.078 (1.45) | 0.079 | 0.043 (1.45) | 0.043 | 0.026 (1.34) | 0.026 |
|                            | 3           | 0.142 (1.54) | 0.143 | 0.079 (1.89) | 0.080 | 0.051 (1.45) | 0.052 |

The number in parentheses represents the standard deviation in terms of percentage.

Table 3 Non-dimensional frequency coefficient for laminated cylindrical skew panel ($\alpha = 15^\circ$)

| Laminate stacking sequence | Mode number | Non-dimensional frequency coefficient ($K_f$) |
|----------------------------|-------------|------------------------------------------|
|                            |             | Aspect ratio ($a/b$) | 2.50 | 3.75 | 5.00 |
|                            |             | Experimental | FEM | Experimental | FEM | Experimental | FEM |
| Angle-ply [+0°/−0°/.../−0°] | 1           | 0.053 (1.34) | 0.054 | 0.029 (1.55) | 0.029 | 0.016 (1.34) | 0.017 |
|                            | 2           | 0.061 (1.43) | 0.062 | 0.031 (1.21) | 0.032 | 0.022 (1.01) | 0.023 |
|                            | 3           | 0.118 (1.67) | 0.120 | 0.067 (1.00) | 0.068 | 0.043 (1.31) | 0.044 |
| Angle-ply [+45°/−45°/.../−45°] | 1         | 0.078 (2.45) | 0.079 | 0.036 (2.00) | 0.037 | 0.020 (1.95) | 0.021 |
|                            | 2           | 0.082 (2.11) | 0.083 | 0.049 (2.30) | 0.050 | 0.035 (1.34) | 0.036 |
|                            | 3           | 0.177 (1.89) | 0.180 | 0.093 (1.89) | 0.096 | 0.054 (2.00) | 0.056 |
| Angle-ply [+90°/−90°/.../−90°] | 1         | 0.079 (1.33) | 0.081 | 0.043 (1.66) | 0.044 | 0.029 (1.21) | 0.030 |
|                            | 2           | 0.096 (1.45) | 0.098 | 0.050 (1.65) | 0.052 | 0.030 (1.23) | 0.031 |
|                            | 3           | 0.181 (1.34) | 0.184 | 0.097 (1.55) | 0.100 | 0.063 (2.12) | 0.065 |
| Cross-ply [0°/90°/.../90°]  | 1           | 0.069 (1.65) | 0.070 | 0.039 (1.11) | 0.040 | 0.025 (0.88) | 0.026 |
|                            | 2           | 0.086 (1.45) | 0.088 | 0.043 (1.30) | 0.044 | 0.027 (1.25) | 0.027 |
|                            | 3           | 0.161 (1.77) | 0.164 | 0.086 (1.56) | 0.088 | 0.057 (1.43) | 0.059 |

The number in parentheses represents the standard deviation in terms of percentage.
for an aspect ratio of 2.50 with antisymmetric cross-ply [0°/90°/…/90°] laminate stacking sequence.

Conclusions
The following conclusions are made based on the above study of isotropic and laminated composite cylindrical skew panels:

- The experimental values of the first, second, and third natural frequencies agree well with the finite element solution in the case of both isotropic and laminated composite cylindrical skew panels.
- In the case of isotropic cylindrical skew panels, the first, second, and third natural frequencies increase with an increase in the skew angle of the panel and decrease as the aspect ratio increases.
- The non-dimensional frequency coefficient $K_f$ for first, second, and third natural frequencies decreases as the aspect ratio increases for all skew angles and laminate stacking sequences. For the laminate

| Laminate stacking sequence | Mode number | Non-dimensional frequency coefficient ($K_f$) |
|---------------------------|-------------|---------------------------------------------|
|                           |              | Aspect ratio (a/b)                           |
|                           |             | 2.50 | 3.75 | 5.00 | Experimental | FEM | Experimental | FEM | Experimental | FEM |
| Angle-ply [+0°/−0°/…/−0°] | 1           | 0.064 (1.66) | 0.066 | 0.030 (1.77) | 0.032 | 0.017 (2.13) | 0.018 |
|                           | 2           | 0.067 (1.45) | 0.069 | 0.038 (1.55) | 0.039 | 0.027 (1.67) | 0.028 |
|                           | 3           | 0.140 (1.43) | 0.144 | 0.077 (1.32) | 0.080 | 0.045 (1.87) | 0.047 |
| Angle-ply [+45°/−45°/…/−45°] | 1         | 0.082 (2.45) | 0.084 | 0.039 (1.89) | 0.040 | 0.023 (2.45) | 0.024 |
|                           | 2           | 0.086 (2.65) | 0.088 | 0.043 (2.22) | 0.044 | 0.031 (2.65) | 0.032 |
|                           | 3           | 0.180 (2.76) | 0.186 | 0.092 (1.85) | 0.096 | 0.061 (2.45) | 0.063 |
| Angle-ply [+90°/−90°/…/−90°] | 1         | 0.094 (1.90) | 0.097 | 0.044 (1.35) | 0.046 | 0.026 (1.45) | 0.027 |
|                           | 2           | 0.104 (1.77) | 0.107 | 0.049 (1.78) | 0.051 | 0.037 (1.76) | 0.038 |
|                           | 3           | 0.197 (1.67) | 0.203 | 0.100 (1.35) | 0.104 | 0.063 (1.98) | 0.066 |
| Cross-ply [0°/90°/…/90°] | 1           | 0.086 (1.76) | 0.089 | 0.040 (2.00) | 0.041 | 0.023 (1.45) | 0.024 |
|                           | 2           | 0.094 (1.56) | 0.097 | 0.047 (2.18) | 0.048 | 0.036 (1.33) | 0.037 |
|                           | 3           | 0.190 (1.23) | 0.196 | 0.096 (1.98) | 0.099 | 0.060 (1.65) | 0.062 |

The number in parentheses represents the standard deviation in terms of percentage.

Table 5 Non-dimensional frequency coefficient for laminated cylindrical skew panel ($\alpha = 45^\circ$)

| Laminate stacking sequence | Mode number | Non-dimensional frequency coefficient ($K_f$) |
|---------------------------|-------------|---------------------------------------------|
|                           |              | Aspect ratio (a/b)                           |
|                           |             | 2.50 | 3.75 | 5.00 | Experimental | FEM | Experimental | FEM | Experimental | FEM |
| Angle-ply [+0°/−0°/…/−0°] | 1           | 0.079 (1.54) | 0.082 | 0.034 (1.87) | 0.036 | 0.019 (2.00) | 0.020 |
|                           | 2           | 0.085 (1.76) | 0.089 | 0.052 (2.10) | 0.054 | 0.037 (2.13) | 0.039 |
|                           | 3           | 0.179 (1.64) | 0.187 | 0.087 (2.11) | 0.091 | 0.049 (2.43) | 0.052 |
| Angle-ply [+45°/−45°/…/−45°] | 1         | 0.089 (2.43) | 0.092 | 0.046 (2.76) | 0.048 | 0.028 (3.01) | 0.030 |
|                           | 2           | 0.103 (2.65) | 0.108 | 0.049 (2.90) | 0.052 | 0.031 (3.21) | 0.032 |
|                           | 3           | 0.201 (2.54) | 0.209 | 0.102 (2.54) | 0.107 | 0.066 (2.98) | 0.069 |
| Angle-ply [+90°/−90°/…/−90°] | 1         | 0.107 (1.43) | 0.112 | 0.043 (1.98) | 0.045 | 0.022 (2.30) | 0.024 |
|                           | 2           | 0.114 (1.54) | 0.118 | 0.063 (1.65) | 0.066 | 0.043 (2.53) | 0.046 |
|                           | 3           | 0.201 (1.70) | 0.210 | 0.101 (1.76) | 0.106 | 0.057 (2.64) | 0.060 |
| Cross-ply [0°/90°/…/90°] | 1           | 0.102 (1.54) | 0.107 | 0.042 (1.32) | 0.044 | 0.021 (1.99) | 0.023 |
|                           | 2           | 0.114 (1.87) | 0.118 | 0.067 (1.43) | 0.071 | 0.047 (2.31) | 0.049 |
|                           | 3           | 0.213 (1.76) | 0.222 | 0.103 (1.87) | 0.108 | 0.058 (1.89) | 0.061 |

The number in parentheses represents the standard deviation in terms of percentage.
Figure 6 Variation of $K_f$ with aspect ratio ($a/b$) for different laminate stacking sequences ($\alpha = 0^\circ$).

Figure 7 Variation of $K_f$ with aspect ratio ($a/b$) for different laminate stacking sequences ($\alpha = 15^\circ$).
Figure 8 Variation of $K_f$ with aspect ratio ($a/b$) for different laminate stacking sequences ($\alpha = 30^\circ$).

Figure 9 Variation of $K_f$ with aspect ratio ($a/b$) for different laminate stacking sequences ($\alpha = 45^\circ$).
stacking sequence \([+0^\circ/-0^\circ/\ldots]/-0^\circ\] , the non-dimensional frequency coefficient \(K_f\) for the first, second, and third natural frequencies increases monotonically with an increase in the skew angle for all values of aspect ratio. For the laminate stacking sequence \([+45^\circ/-45^\circ/\ldots]/-45^\circ\] , the non-dimensional frequency coefficient \(K_f\) for the first natural frequency increases monotonically with an increase in the skew angle for all values of aspect ratio. The value of \(K_f\) for the second and third natural frequencies for this laminate stacking sequence does not increase monotonically. In respect of other laminate stacking sequences, \(K_f\) varies as shown in the tables.

- Other parameters being the same, antisymmetric angle-ply \([+0^\circ/-0^\circ/\ldots]/-0^\circ\] yields the lowest value for the first, second, and third natural frequencies and highest value for antisymmetric angle-ply \([+90^\circ/-90^\circ/\ldots]/-90^\circ\] in majority of cases considered in the present study.

Abbreviations
\(a\): Projected width of panel; \(b\): Length of panel; \(E\): Modulus of elasticity of the material of isotropic panel; \(E_1\): Young’s modulus of the lamina in the longitudinal direction; \(E_2\): Young’s modulus of the lamina in the transverse direction; \(G_{12}\): In-plane shear modulus of the lamina; \(K_f\): Non-dimensional frequency coefficient; \(NL\): Number of layers in the laminate; \(R\): Panel radius; \(t\): Panel thickness; \(V_{12}\): Major Poisson’s ratio of the lamina; \(\mu\): Poisson’s ratio of the material of isotropic plate; \(\alpha\): Skew angle of the panel; \(\theta\): Angle subtended by the panel at the center of curvature; \(\alpha\): Fiber orientation angle of the lamina; \(\rho\): Mass density of the material of panel; \(\omega\): Natural angular frequency of panel in radian/second.

Competing interests
The authors declare that they have no competing interests.

Authors’ contributions
CVS carried out the experimental and finite element analysis under the guidance of YJS and WPPK. CVS and WPPK drafted the manuscript. All the three authors read and approved the final manuscript.

Authors’ information
CVS received his M.Tech. in Design Engineering from Visvesvaraya Technological University, Belgaum, Karnataka, India in 2003 by securing a III rank. He is an assistant professor at the Department of Mechanical Engineering, GM Institute of Technology, Davangere, Karnataka, 577006, India. Presently, he is pursuing his Ph.D. at the Research Centre, Department of Mechanical Engineering, JNN College of Engineering, Shivarogga, affiliated to Visvesvaraya Technological University, Karnataka, India. His research interests include stress analysis, finite element method, and experimental material characterization. He is a member of ISTE (India), ISTAM, MRSI. YJS is a professor at the Department of Mechanical Engineering, JNN College of Engineering, Shivarogga, Karnataka, India. He received his Ph.D. from Indian Institute of Technology, Madras, India and has more than 30 years of experience in teaching and research. His current area of research includes design, stress analysis, finite element analysis, and vibration. He has

| Skew Angle (\(\alpha\)) | Mode Shapes (Panel Angle (\(\theta\)) = 60\(^\circ\), Aspect Ratio (\(a/b\)) = 2.50) |
|------------------------|---------------------------------|
| 0\(^\circ\)             | ![Mode Shape 1](image1)          |
|                        | \(f_1 = 960.4996\) Hz           |
| 15\(^\circ\)            | ![Mode Shape 2](image2)          |
|                        | \(f_1 = 1117.419\) Hz           |
| 30\(^\circ\)            | ![Mode Shape 3](image3)          |
|                        | \(f_1 = 1408.250\) Hz           |
| 45\(^\circ\)            | ![Mode Shape 4](image4)          |
|                        | \(f_1 = 1691.422\) Hz           |

Figure 10 Mode shapes for laminated composite cylindrical skew panel \((a/b = 2.50,\text{ cross-}90^\circ/90^\circ/\ldots/90^\circ)\).
Acknowledgements
Chikkel V Srinivasa would like to thank the management and principal, Dr. S.G. Hiremath, of GM Institute of Technology, Davangere, Karnataka, India for the kind encouragement and support provided. Yalaburgi J Suresh would like to thank the management of Jawaharlal Nehru College of Engineering, Shiamogga, Karnataka, India for the kind encouragement and support provided. Wooday P Prema Kumar would like to thank the management, principal, Dr. N Ranaprathap Reddy, and head of the Department of Civil Engineering, Dr. Y. Ramalinga Reddy, Reva Institute of Technology and Management, Bangalore, Karnataka, India for the kind encouragement and support provided.

Author details
1Department of Mechanical Engineering, GM Institute of Technology, Karur Village, PB Road, Davangere, Karnataka 577006, India. 2Department of Mechanical Engineering, JNN College of Engineering, Shiamogga, Karnataka 577204, India. 3Department of Civil Engineering, Reva Institute of Technology and Management, Bangalore, Karnataka 560064, India. 4Academic Council, Reva University, Bangalore, Karnataka 560064, India.

Received: 17 September 2013 Accepted: 8 December 2013

Published: 21 Jan 2014

References
ASTM (2006) ASTM D3039/D3039M: standard test method for tensile properties of polymer matrix composite materials. ASTM, Pennsylvania
Bardell NS, Dunson JM, Langley RS (1997) On the free vibration of completely free, open, cylindrically curved, isotropic shell panels. J Sound Vib 207:647–669
Bardell NS, Dunson JM, Langley RS (1998) Free vibration of thin, isotropic, open conical panels. J Sound Vib 217:297–320
Gulshan Taj MNA, Chakrabarti A (2013) Dynamic response of functionally graded skew shell panel. Lat Am J Solids Stru 10:1243–1266
Haldar S (2008) Free vibration of composite skewed cylindrical shell panel by finite element method. J Sound Vib 311:9–19
Hossain SJ, Sinha PK, Sheikh AH (2004) A finite element formulation for the analysis of laminated composite shells. Comput Struct 82:1623–1638
Jones RM (1975) Mechanics of composite materials. McGraw-Hill, New York
Kandasamy S, Singh AV (2006) Free vibration analysis of skewed open circular cylindrical shells. J Sound Vib 290:1100–1118
Lee SJ, Han SE (2001) Free vibration analysis of plates and shells with a nine-node assumed natural degenerated shell element. J Sound Vib 241(4):605–633
Liu L, Chua LP, Ghista DN (2006) Element free Galerkin method for static and dynamic analysis of spatial shell structures. J Sound Vib 295:388–406
Pradumna S, Bandyopadhyay JN (2008) Free vibration analysis of functionally graded curved panels using a higher-order finite element formulation. J Sound Vib 318:176–192
Selmane A, Lakis AA (1997) Dynamic analysis of anisotropic open cylindrical shells. Comput Struct 62:1–12
Singh AV (1999) Free vibration analysis of deep doubly curved sandwich panels. Comput Struct 73:385–394
Singh AV, Shen L (2003) Free vibration of open circular cylindrical composite shells with point supports. ASCE J Aerospace Eng 18:120–128
Srinivasa CV, Suresh YJ, Prema Kumar WP (2013) Finite element studies on free vibration of laminated composite cylindrical skew panels. In: Shin CS (ed) Advances in Mechanical Engineering, , Hindawi, New York, in press
Voyiadjis GV, Woelke P (2004) A refined theory for thick spherical shells. Solids Struct 41:3747–3769
Zhao X, Liew KM, Ng TY (2003) Vibration analysis of laminated composite cylindrical panels via a meshfree approach. Int J solids Struct 40:161–180

Submit your manuscript to a SpringerOpen journal and benefit from:

- Convenient online submission
- Rigorous peer review
- Immediate publication on acceptance
- Open access: articles freely available online
- High visibility within the field
- Retaining the copyright to your article

Submit your next manuscript at https://springeropen.com