Cosmic Rays

II. Evidence for a magnetic rotator Wolf Rayet star origin

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Abstract. Based on 1) a conjecture about the mean free path for particle scattering in perpendicular shock geometries, and 2) a model for Wolf Rayet star winds, we argue that explosions of Wolf Rayet stars can lead through diffusive particle acceleration to particle energies up to $3 \times 10^9 \text{ GeV}$. As a test we first demonstrate that the magnetic fields implied by this argument are compatible with the dynamo limit applied to the convective interior of the main sequence stars which are predecessors of Wolf Rayet stars; second we show that this implies the same strength of the magnetic fields as is suggested by the magnetically driven wind theory. Third, we use data from radio supernovae to check the spectrum, luminosity and time dependence which suggest that the magnetic fields again have to be as high as suggested by the magnetically driven wind theory. This constitutes evidence for the magnetic field strengths required to accelerate particles to $3 \times 10^9 \text{ GeV}$. Fourth, we demonstrate that within our picture the nonthermal radio emission from OB and Wolf Rayet stars can reproduce the proper radio spectra, time variability and radio luminosities. Fifth, the comparison of Wolf Rayet stars and radio supernovae suggests that electron injection into the acceleration process is a step function of efficiency with the shock speed as the independent parameter; the critical speed appears to be that speed at which the thermal downstream electrons become relativistic. We make detailed predictions on the temporal and spectral behaviour of the nonthermal radio emission of OB and Wolf Rayet stars that will allow further checks on our model.

Key words: Plasma Physics – Shockwaves – Supernovae – Wolf-Rayet stars – Cosmic Rays

1. Introduction

The origin of cosmic rays above particle energies of about $5 \times 10^8 \text{ GeV}$ is still not understood. The main proposal that has been introduced by Jokipii and Morfill (1987) argues that there is a terminal shock of the galactic wind which accelerates particles much like the terminal shock of the solar wind, and that then these particles diffuse against the wind "down to us". In this picture it is unclear how the intensity of the cosmic ray flux at the matching point of $5 \times 10^8 \text{ GeV}$ can fit to the lower energy particles which are believed to arise from direct explosions into the interstellar medium (Lagage and Cesarsky 1983) and from explosions of massive stars into their stellar wind cavity (Völk and Biermann 1988, Silberberg et al. 1990). In the latter theory one readily reaches particle energies right up to the knee of the cosmic ray spectrum at $5 \times 10^8 \text{ GeV}$.

Motivated by recent developments in magnetic rotator theory, summarized by Cassinelli (1991, 1993), that account for the strong winds of Wolf Rayet stars, but which require much stronger magnetic fields than had been assumed by Völk and Biermann (1988), we argue in this paper that explosions of Wolf Rayet stars and similar massive stars with strong winds may well account for the dominant cosmic ray component above particle energies of about $10^9 \text{ GeV}$ all the way to about $3 \times 10^9 \text{ GeV}$.

At energies above this presumably the extragalactic component takes over (Biermann 1992, Rachen and Biermann 1992, 1993, paper UHE CR I, Rachen 1992, Rachen et al. 1993, paper UHE CR II).

Here we propose to present the case in several steps: First, we demonstrate that the dynamo mechanism applied to the interior of massive stars - which later get exposed by mass loss - readily can account for magnetic field of substantial magnitude. Then we argue how this leads to the high particle energies; we discuss the spectrum of the cosmic rays between $10^8 \text{ GeV}$ and $3 \times 10^9 \text{ GeV}$ in paper CR I (Biermann 1993); the consequences for the chemical abundances and spectrum of the cosmic rays have been checked against air shower data by Stanev et al. (1993, paper CR IV).

Second, we check that the notion that massive stars have strong magnetic fields in their winds, as suggested by Hartmann and Cassinelli (1981) and Cassinelli (1982) and implied by our model for the origin of energetic Cosmic Rays (papers CR I and CR IV), is consistent with other properties of WR stars, and by implication, also radio supernovae and OB stars. As a test we derive the properties of the expected nonthermal radio emission and show that this is in agreement with the available data; further radio observations may provide an additional detailed test for our concept.

The paper is structured as follows: In section 2 we briefly summarize the basic properties of a fast stellar wind with a strong Parker type magnetic field. In sections 3 to 6 we derive the spectrum of energetic particles accelerated in shocks that traverse such winds, fully analogous to paper CR I. In section 7 we use the dynamo limits to argue that these high magnetic fields are indeed plausible. In section 8 we derive the implica-
tions for the momentum of winds, and show that these winds are quite likely driven by the magnetic field. In section 9 we derive the expected nonthermal radio emission, and also discuss optical thickness effects. In sections 10 and 11 we use data on radiosupernovae to derive numerical estimates for the magnetic field strength in Wolf Rayet star winds independently. In section 12 we then discuss the nonthermal radio emission from Wolf Rayet stars, and in section 13 from OB stars. We finally describe in section 14 further possible tests using detailed radio monitoring of Wolf Rayet and OB stars, and conclude with a summary in section 15.

2. Wolf Rayet star winds

The winds of Wolf Rayet stars are well established by optical, infrared and radio observations. They show a typical mass loss rate of about $10^{-5} \, \text{M}_\odot \, \text{year}^{-1}$, a typical wind velocity of 0.01$c$ and thermal radio emission of several mJy at 5 GHz at a typical distance of 1 kpc. These numbers lead to a reference density in the wind of $7.3 \times 10^6 \, \text{cm}^{-3}$ at a radius of $10^{14} \, \text{cm}$ at 5 GHz. This implies a radius where optical thickness unity for free-free emission - the thermal radio emission - of 1.4 $10^{14}$ cm.

We assume that these winds have a strong magnetic field in the configuration of a Parker spiral with a magnetic field that varies as $1/r$ near the equator and is predominantly azimuthal there and as $1/r^2$ near the pole where it is mostly radial:

$$B_r = B_o \left(\frac{r_o}{r}\right)^2$$

(1)

$$B_\phi = B_o \left(\frac{r_o}{r}\right) \sin \theta$$

(2)

where $\theta$ is the colatitude, with

$$r_o = V_W / \Omega_s$$

(3)

where $V_W$ and $\Omega_s$ are the wind velocity and the angular rotation rate of the star itself. Obviously, if $r_o < r_s$ then we have to replace $r_o$ with the stellar radius $r_s$. The magnetic field inside $r_o$ if $r_o > r_s$ is predominantly radial and decreases with $1/r^2$.

The Alfvén velocity with respect to the generally stronger tangential magnetic field is then given by

$$v_{A\phi} = 2.1 \times 10^8 \, \text{cm} \, \text{sec}^{-1} \, B_{0.5} \left(\frac{M_{-5}}{V_{W,-2}}\right)^{-1/2},$$

(4)

which is independent of radius in the region where the wind velocity is constant. Here the mass loss $M$ is in units of $10^{-5} \, \text{M}_\odot \, \text{year}^{-1}$, and the wind velocity in units of 0.01$c$. From paper CR I we also use the estimate that the magnetic field is 3 Gauss at the reference radius of $10^{14}$ cm, and so we use this value as reference, since it is the consequences of this argument which we propose to test. It is interesting to note that this Alfvén velocity is rather close to the wind velocity, and we will argue below that there is a physical reason for this.

We propose to interpret the nonthermal radio emission both from normal Wolf Rayet stars as well as from supernova explosions into such winds as being caused by particle acceleration and synchrotron emission; hence we have to consider the properties of shockwaves in Wolf Rayet stars. It follows that a shockwave propagating through such a wind encounters for most of its area a magnetic field which is perpendicular to the shock direction, a configuration in which classical Fermi acceleration is known not to work (Drury 1983).

3. The Cosmic Ray particle energies

In the following four sections we reformulate the theory of particle acceleration for perpendicular shocks introduced by Biermann (1993, paper CR I) and apply it in detail to shocks in winds of massive single stars.

In paper CR I we have introduced the conjecture that the scattering of energetic particles in a perpendicular configuration is diffusive and that the scattering coefficient $\kappa_{r,2}$ is composed of the radial scale of the region and the velocity difference across the shock, and is independent of energy: Generalizing now for arbitrary wind speed and arbitrary shock strength we obtain for the thickness of the shocked layer

$$\Delta r/r = \frac{U_2}{U_1} \frac{U_1}{V_W + U_1}.$$  

(5)

This reduction of the thickness of the layer for a finite wind velocity is due to the fact that the material which is snowplowed together is not all gas between zero radius and the current radius $r$, but between zero radius and $r(1 - V_W/(V_W + U_1))$, since the gas keeps moving while the shock moves out towards $r$. This then leads to

$$\kappa_{r,1} = \frac{1}{3} U_1 \frac{1}{r} \left(1 - \frac{U_2}{U_1}\right) / (1 + V_W/U_1)$$

(6)

and

$$\kappa_{r,2} = \frac{1}{3} U_2 \frac{1}{r} \left(1 - \frac{U_2}{U_1}\right) / (1 + V_W/U_1).$$

(7)

This then results in the sum of the residence times on the two sides of the shock front, of:

$$\frac{4\kappa_{r,1}}{U_2 c} + \frac{4\kappa_{r,2}}{U_2 c} = \frac{8}{3} \frac{r}{c} \left(1 - \frac{U_2}{U_1}\right) / (1 + V_W/U_1).$$

(8)

The maximum particle energy corrected for the wind velocity then is given by a spatial limit

$$E_{max} = Z e r B_1 / (1 + V_W / U_1).$$

(9)

We emphasize that in the Parker spiral regime of the wind the product of the magnetic field strength and the radius is a constant, i.e. does not vary with radius, and so we can use any radius well outside $r_o$ as reference. Now in order that we can reach particle energies of $3 \times 10^9$ GeV we then require (see paper CR I and eq. 9) that the product of $B$ and $r$ is $10^{16}$ Gauss cm for protons, or $3 \times 10^{14}$ Gauss cm for iron nuclei. We will use in the following the assumption that only iron nuclei (an independent observational check using air shower data on the chemical abundances of high energy cosmic rays is done in paper CR IV) are the highest energy particles, and so use $B = 3$ Gauss at $r = 10^{14}$ cm as reference.
4. Particle Drifts

Consider particles which are either upstream of the shock, or downstream; as long as the gyrocenter is upstream we will consider the particle to be there, i.e. upstream, and similarly downstream. Following paper CR I we assume that drifts are due to both the radial gradient of the magnetic field and the increased curvature due to the turbulence. This increases the $\theta$-drift to

$$V_{d,\theta} = f_d c r_3 / r$$

from the lower level based on pure gradient drift, where

$$f_d = 1 + \frac{1}{r U_1} \frac{r U_1}{U_2} (1 - U_2 / U_1).$$

(11)

It is easily verified that $f_d = 1$ for strong shocks and negligible wind velocity. We take here the curvature length scales to be the same on both sides of the shock, since the curvature is induced by the thickness of the shock region.

The energy gain associated with the $\theta$-drift is given by the product of the drift velocity, the residence time, and the electric field. Upstream and downstream together this energy gain is given by

$$\frac{\Delta E}{E} = \frac{4 U_1}{3 c} f_d (1 + \frac{U_2}{U_1}).$$

(12)

5. The energy gain of particles

Let us consider then one full cycle of a particle remaining near the shock and cycling back and forth from upstream to downstream and back. The energy gain just due to the Lorentz transformations in one cycle can then be written as

$$\frac{\Delta E}{E} = \frac{4 U_1}{3 c} (1 - \frac{U_2}{U_1}).$$

(13)

Adding the energy gain due to drifts we obtain

$$\frac{\Delta E}{E} = \frac{4 U_1}{3 c} (1 - \frac{U_2}{U_1}) x$$

(14)

where

$$x = 1 + 3 \frac{\kappa_{rr,1}}{r U_1} f_d (1 + \frac{U_2}{U_1}) / (1 - \frac{U_2}{U_1}).$$

(15)

which is $9/4$ for negligible wind speeds and a strong shock when $U_1 / U_2 = 4$; on the other hand, for $V_W / U_1 = 1$ we have $x = 2.042$ and for the limiting case of large wind speeds compared to shock speeds $x = 1.833$.

6. Expansion and injection history

Consider how long it takes a particle to reach a certain energy:

$$\frac{dt}{dE} = \left(8 \frac{\kappa_{rr,1}}{U_1 c}\right) \left(\frac{4 U_1}{3 c} (1 - \frac{U_2}{U_1}) x E\right).$$

(16)

Here we have used that $\kappa_{rr,1} / U_1 = \kappa_{rr,2} / U_2$.

Since we have

$$r = U_1 (1 + \frac{V_W}{U_1}) t$$

(17)

this leads to

$$\frac{dt}{t} = \frac{dE}{E} \frac{3 U_1}{U_1 - U_2} \frac{2 \kappa_{rr,1}}{x r U_1} \left(1 + \frac{V_W}{U_1}\right)$$

(18)

and so to a dependence of

$$t(E) = t_0 \left(\frac{E}{E_0}\right)^{\beta}$$

(19)

with

$$\beta = \frac{3 U_1}{U_1 - U_2} \frac{2 \kappa_{rr,1}}{x r U_1} \left(1 + \frac{V_W}{U_1}\right).$$

(20)

Particles that were injected some time $t_0$ ago were injected at a different rate, say, proportional to $r^b$. Also, in $d$-dimensional space, particles have $r^d$ more space available to them than when they were injected. This then leads to a combined correction factor for the abundance of

$$\left(\frac{E}{E_0}\right)^{-(b+d)\beta}.$$  

(21)

The combined effect is a spectral change by

$$\frac{3 U_1}{U_1 - U_2} \frac{2}{x} \left(\frac{d + b}{x} \frac{\kappa_{rr,1}}{r U_1} \left(1 + \frac{V_W}{U_1}\right)\right).$$

(22)

Hence the total spectral difference, as compared with the planeparallel case, is given by

$$\frac{3 U_1}{U_1 - U_2} \left(\frac{\kappa_{rr,1}}{x} - 1\right) + \frac{2}{x} \left(b + d\right) \frac{\kappa_{rr,1}}{r U_1} \left(1 + \frac{V_W}{U_1}\right).$$

(23)

Here the sign convention is such (see paper CR I) that for this expression positive the spectrum is steeper. For a wind we have $b + d = 1$. We note that

$$\kappa_{rr,1} (1 + \frac{V_W}{U_1}) / r U_1$$

(24)

is now independent of the wind speed, and the only effect of the wind which remains is through $x$. For the sequence of $V_W / U_1 = 0, 1.0, and >> 1$ we thus obtain particle spectral index differences, in addition to the index of $7/3$, of 0.0, 0.136, 0.303, corresponding to Synchrotron emission spectral index of an electron population with the same spectrum,
of 0.667, 0.735, 0.818. We will use the first two cases as examples below, and since the work of Owocki et al. (1988) suggests that typical shocks in winds have a velocity in the wind frame similar to the wind velocity in the observers frame itself, which implies that, in the simplified picture here, only spectral indices for the synchrotron emission between 0.667 and 0.735 are relevant, with an extreme range of spectral indices up to 0.818 for strong shocks. Obviously, for weaker shocks with \( U_1/U_2 < 4 \) the spectrum can be steeper, e.g. for \( U_1/U_2 = 3.5 \) we obtain an optically thin spectral index for the synchrotron emission of 0.734 for \( V_W \ll U_1 \) and 0.815 for \( V_W/U_1 = 1 \).

### 7. The strength of the magnetic field

First we propose to demonstrate that high magnetic fields can be generated inside massive stars so that later, when these interiors get exposed to become the surface of Wolf Rayet stars these high magnetic fields can drive the wind. The dynamo mechanism acts in the turbulent zone in the interior of rotating massive stars and, given a seed field, increases the magnetic field strength up to the limit where the Coriolis force equals the magnetic stresses (Ruzmaikin et al. 1988, Gilbert and Childress 1990, Gilbert 1991). The strength of the magnetic field can be estimated from the condition, that the magnetic torque is limited by the Coriolis forces over the size of the convective region. This condition can be written as

\[
B = (\Omega \rho_c R_{cc} v_1 f_1)^{1/2},
\]

(25)

where \( \Omega \) is the rotation rate of the star, \( \rho_c \) is the average density of the convection zone, \( R_{cc} \) is the radius of the convective core, \( v_1 \) is the characteristic turbulent velocity, and \( f_1 \) is a correction factor of order unity in order to allow for a) structural variations ignored here, and also for b) the fact that the dominant length scale is likely to be smaller than the radius of the convective region. The turbulent velocity is estimated from the condition that turbulent convection transports all the luminosity \( L \):

\[
L = 4\pi R_{cc}^2 \rho_c v_1^3 f_2,
\]

(26)

where \( f_2 \) is also a correction factor of order unity to allow for structural variations; we use the average density and the radius of the entire convective region.

Models of Langer (1992, priv.comm.) provide the input together with data from the textbook of Cox & Giuli (1968):

\[
R = 9.0 R_\odot \left( \frac{M}{40 M_\odot} \right)^{0.512},
\]

(27)

\[
M_{cc}/M = 0.628 \left( \frac{M}{40 M_\odot} \right)^{0.466},
\]

(28)

\[
R_{cc}/R = 0.384 \left( \frac{M}{40 M_\odot} \right)^{0.377},
\]

(29)

where \( R \) and \( M \) are the stellar radius and mass, respectively. This leads to a magnetic field of

\[
B = 1.9 \times 10^6 \alpha^{1/4} f_1^{1/2} f_2^{-1/6} \left( \frac{M}{40 M_\odot} \right)^{-0.058} \text{ Gauss},
\]

(30)

where \( \alpha \) is the fraction of critical rotation at the surface, assumed to be solid body rotation. The same argument for Wolf Rayet stars (using data of Langer 1989, with Helium mass fraction \( Y = 1.0 \)) gives

\[
B = 2.3 \times 10^7 \alpha^{1/4} f_1^{1/2} f_2^{-1/6} \left( \frac{M_W}{5 M_\odot} \right)^{0.035} \text{ Gauss},
\]

(31)

Hence this readily produces magnetic fields, dependent on the rotation rate of the star, of up to \( 2 \times 10^8 \) Gauss for O stars, and about 10 times more for Wolf Rayet stars. In both cases the induced magnetic field is nearly independent of stellar mass. Critical rotation here means that the convective core has to rotate at an angular velocity which would correspond at the surface to critical rotation there; but in fact, as we will see below, it is not required that the actual surface rotates this fast.

In conclusion we find that we rather easily generate magnetic fields at levels deep inside the star beyond the local virial limit at the surface of the star, which gives of the order of a few times \( 10^4 \) Gauss nearly independent of the rotation rate (Maheswaran and Cassinelli 1988, 1992). Rotationally induced circulations can carry these magnetic fields to the surface of the stars on a time scale much shorter than the main sequence life time. In this transport the magnetic field is weakened by flux conservation and so surface fields in the range \( 10^3 \) to \( 10^4 \) Gauss are quite plausible, and are below the local virial theorem limit. Such values are all we require in the following. These estimates are valid for massive stars and extend clearly to below the mass range where we have Wolf Rayet stars as an important final phase of evolution. We thus expect many of the arguments in the following to hold generally for all massive single stars with extended winds, whether slow or fast, whether red or blue supergiant preceding the supernova explosion.

There is a further consequence for the generation of magnetic fields in white dwarfs: The most massive stars that do not become supernovae, but white dwarfs, are sufficiently massive to contain also convective cores which get exposed when the white dwarf is formed. Thus there ought to be a correlation between massive white dwarfs and the detection of strong magnetic fields; this is consistent with the observational data (Liebert 1992, priv.comm., Schmidt et al. 1992).

### 8. Magnetically driven winds

In the standard version of the fast magnetic rotator theory (Hartmann and MacGregor 1982, Cassinelli 1993) it is assumed that the magnetic field is radial close to the stellar surface and then starts bending at the critical point where the radial Alfvén velocity, the local corotation velocity and the wind velocity all coincide. This produces a long lever arm for the loss of angular momentum, and is the essence of the criticism of Nerney and Suess (1987) against the model. The spindown of fast magnetic rotator Wolf-Rayet stars is a topic addressed by Poe, Friend and Cassinelli (1989), where it is again assumed that the field is radial close to the stellar surface.
However, the magnetically driven winds do not require the magnetic field to be radial near the surface. In a convection zone near the surface, it is plausible to assume that the magnetic field is nearly isotropic in its turbulent character below the region where the wind gets started, and radial in the wind zone near to the star, leaving the radial magnetic field lines dominant for as long as the flow velocity is below the radial Alfvén speed.

However, in a star, where the outer layers are radiative, it is not clear at all that the magnetic field is initially radial. Even the slightest differential rotation will tend to make the magnetic field which originates in the central convective region to be strongly tangential, and even the acceleration into a wind does not obviously overturn this tendency completely. In fact, the recent model calculations of Wolf Rayet star winds by Kato and Iben (1992) demonstrate that the critical point of the wind where the wind becomes supersonic is already inside the photosphere, and so it is reasonable to suppose that the other critical point where the wind speed exceeds the radial Alfvén speed may also be inside the star, if there is such a point at all - the radial flow velocity may be faster than the radial Alfvén speed throughout.

In the following we propose to discuss such a wind, generalizing from Parker (1958) and Weber and Davis (1967). As in Weber and Davis we limit ourselves here to the equatorial region. We use magneto-hydrodynamics and Maxwells equations and thus have for the angular momentum transport $L_J$:

$$L_J = r v_{\phi} - \left( \frac{B_r}{4\pi \rho v_r} \right) r B_\phi = \text{const}. \quad (32)$$

Here the components of the magnetic field are $B_\phi$ and $B_r$, and the components of the wind velocity are $v_\phi$ and $v_r$, while $\rho$ is the density and the index s refers to the stellar surface.

Introducing for the magnetic flux

$$F_B = r^2 B_r = r^2 B_{rs} = \text{const}, \quad (33)$$

with flux freezing

$$r (v_r B_\phi - v_\phi B_r) = - \Omega F_B = \text{const}, \quad (34)$$

and mass flux

$$M = 4\pi \rho r^2 v_r = 4\pi \rho_s r^2 v_{rs} = \text{const}, \quad (35)$$

the tangential velocity can be written as

$$v_\phi = \frac{L_J}{r} \left( 1 - \frac{F_B^2}{M v_r} \right) / \left( 1 - \frac{F_B^2}{M r^2 v_r} \right). \quad (36)$$

The angular momentum loss can be written as

$$L_J = \epsilon \Omega v_\phi^2. \quad (37)$$

If there is no Alfvén critical point outside the star, then obviously

$$\epsilon < 1. \quad (38)$$

We note that the term $F_B^2/(M v_r r^2)$ appearing in the expression for the tangential velocity can be rewritten with the surface radial Alfvén Mach number

$$M_{Ars} = \frac{v_{rs}}{v_{Ars}}, \quad (39)$$

where

$$v_{Ars} = \frac{B_r}{(4\pi \rho)^{1/2}}, \quad (40)$$

as

$$\frac{F_B^2}{M} \frac{1}{v_r} = \frac{1}{M_{Ars}^2} \frac{v_{rs}}{v_r} \frac{r_s^2}{r^2} = \frac{1}{M_{Ars}^2}. \quad (41)$$

Similarly we have the relationship

$$\frac{F_B^2}{M} \frac{\Omega}{L_J v_r} = \frac{1}{M_{Ars}^2} \frac{r^2}{r_s^2} \epsilon = \frac{1}{M_{Ars}^2} \frac{v_{rs}}{v_r} \epsilon. \quad (42)$$

Thus the tangential velocity can be written as

$$v_\phi = \frac{L_J}{r} \left( 1 - \frac{v_{rs}}{M_{Ars} \epsilon v_r} \right) / \left( 1 - \frac{1}{M_{Ars}^2} \right). \quad (43)$$

We can reasonably assume that the radial velocity is steadily increasing with radius. The tangential velocity should neither be negative nor exceed the rotational velocity of the star itself, and so we derive the conditions

$$M_{Ars} > 1, \quad (44)$$

and

$$M_{Ars}^{-2} < \epsilon < 1 \quad (45)$$

for the conditions envisaged here, that there is no Alfvén critical point outside the star. These conditions translate into

$$\frac{1}{M_{Ars}^2} \frac{v_{rs}}{v_r} \epsilon < 1, \quad (46)$$

and

$$\frac{1}{M_{Ars}^2} < 1. \quad (47)$$

It follows, e.g., that $M_{Ars} \sim r$ asymptotically.

We define $U_\epsilon$ and $U_M$ and obtain

$$0 < U_\epsilon = 1 - \frac{\epsilon v_s^2}{r^2} < 1, \quad (48)$$

and

$$0 < U_M = 1 - \frac{1}{M_{Ars}^2} < 1. \quad (49)$$

The tangential magnetic field can then be written as
\[ B_0 = -\frac{F_B \Omega}{r v_r} (1 - \frac{\epsilon r^2}{v_r^2})/(1 - \frac{1}{M_{Ar}^2}), \]  

(50)

and is thus also without change of sign outside the star. It is easy to verify that no magnetic flux is transported to infinity (see Kato and Iben (1992)).

The radial momentum equation is

\[ \frac{v_r}{r} \frac{d}{dr} (v_r) (1 - \frac{\epsilon r^2}{v_r^2} ) = \]

\[ 2 \frac{v_r^2}{r} - \frac{G M}{r^2} + \frac{F_{rad} \sigma T N}{m_{\mu} c^3} + \]

\[ \frac{v_r^2}{r} - \frac{1}{8 \pi \rho r^2} \frac{d}{dr} (\varphi B_r)^2. \]

(51)

In this equation the radiation force (flux \( F_{rad} \)) on both lines and continuum is given with the correction factor \( N \) over the Thompson cross-section, also including the effect due to the chemical element composition being different from pure hydrogen; this factor \( N \) depends on the physical state of the gas. The adiabatic speed of sound is \( c_s \). We here proceed to evaluate the last term with the expressions already derived and discuss it in the context of the momentum equation.

The gradient term of \((r B_0)^2 \) in the momentum equation can then be rewritten as (on the right hand side of the momentum equation)

\[ \frac{v_r}{r} \frac{d}{dr} (v_r) \frac{1}{M_{Ar}^2} \left( \frac{r_s \Omega}{v_{rs}} \right)^2 \left( \frac{v_{rs}}{v_r} \right)^3 \frac{U_r^2}{U_{Ar}^2} + \]

\[ - 2 \frac{r}{M_{Ar}^2} r_s \Omega^2 \left( \frac{r_s}{r} \right)^3 \left( \frac{v_{rs}}{v_r} \right)^4 \frac{U_r}{U_{Ar}^3} \left( \epsilon - \frac{1}{M_{Ar}^2} \frac{v_{rs}}{v_r} \right). \]

(52)

A second term on the right hand side of the momentum equation is the centrifugal force, which can be written as

\[ r_s \Omega^2 \left( \frac{r_s}{r} \right)^3 \frac{1}{U_{Ar}^2} \left( \epsilon - \frac{1}{M_{Ar}^2} \frac{v_{rs}}{v_r} \right)^2. \]

(53)

The last term in brackets never goes through zero outside the star because of the conditions we have set above for \( \epsilon \) and \( M_{Ar} \). Comparing now the centrifugal term with the second term from the gradient of the tangential magnetic field, we note that asymptotically these two terms differ by the factor

\[ \frac{1}{2} \left( \epsilon M_{Ar}^2 \right) \frac{v_{rs}}{v_{rs}}, \]

(54)

where \( \epsilon M_{Ar}^2 > 1 \) and \( v_r/v_{rs} \) likely to exceed a value of 2 at large radii \( r \); hence the centrifugal term, which provides some acceleration, is likely to dominate over the other term at large \( r \). Here we have ignored all the terms of order unity that approach unity with both the radius \( r \) and the radial velocity \( v_r \) becoming large. Even close to the star the centrifugal force may dominate.

The first term in the gradient of the tangential magnetic field is more interesting, however. This term becomes a summand to the various terms multiplying \( (v_r \frac{d}{dr} v_r) \) and has there, on the left hand side, a minus sign and is to be compared with unity in the case that we are already at supersonic speeds, as argued by Kato and Iben (1992).

We define an Alfvén Mach number with respect to the total magnetic field with

\[ M_A^3 = \frac{v_r^2 4 \pi \rho}{B_0^2 + B_r^2} \]

(55)

Generally we have obviously

\[ M_A < M_{Ar} \]

(56)

With this generalized Alfvén Mach number we can rewrite the entire factor to \( (v_r \frac{d}{dr} v_r) \) in a simple form

\[ 1 - \frac{1}{M_A^2} - \frac{M_{Ar}^2}{M_A^2 - 1}/(M_A^2 - 1), \]

(57)

where \( M_a \) is the sonic Mach number for the radial flow velocity. Critical points appear whenever this expression goes through zero or a singularity. This expression is easily seen to be equivalent to eq.(6) of Hartmann and MacGregor (1982); they, however, assumed that the magnetic field is initially radial and neglected radiative forces. This shows that we have the critical point of Weber and Davis (1967) again for \( \frac{M_{Ar}}{M_A} \ll 1 \), when and if \( M_{Ar} = 1 \), but we have in our case another critical point at \( M_A = 1 \), which is the fast magnetosonic point (see Hartmann and MacGregor 1982). This critical point may never appear for certain choices of the parameters. In order to make a realistic judgement on this question, we would have to combine the model of Kato and Iben (1992) with a realistic treatment of the magnetic field inside the star, so that we can derive the range of possible properties of the magnetic field near the surface of the star.

However, there are a few general conclusions one can draw already: If the magnetic field inside the star begins as a dominantly tangential field, then it is by no means clear whether there is any point at which we have \( M_{Ar} < 1 \), either inside or outside the star; going outwards the unwinding of the magnetic field is intimately coupled to the initially weak radial flow, and so \( M_{Ar} > 1 \) may hold throughout. In that case the term derived from the radial gradient of the tangential magnetic field goes through unity only when the generalized Alfvén Mach number goes through unity. If, as we argued, \( M_{Ar} > 1 \) possible throughout, then, again going outwards with radius, we start with a low generalized Alfvén Mach numbers \( M_A \), either supersonic or subsonic flow, the entire expression is negative, matching the negative gravitational force on the right hand side. Far out, both sonic and generalized Alfvén Machnumbers are large, and so the expression is positive. It follows that the expression has to go through zero. The condition \( M_A = 1 \) here is the fast magnetosonic point, and the radial velocity there corresponds to the Michel-velocity. Or, in other words, the radial velocity is equal to the total Alfvén velocity.

We know from observations that the winds in OB and Wolf Rayet stars are very strongly supersonic; using our model we deduce that the radial velocity is weakly super-Alfvénic with respect to the tangential (dominant) magnetic field component, and so we have far outside the star a configuration where \( M_r >> 1, M_{Ar} >> 1 \) and \( M_A \geq 1 \), but this latter condition may not necessarily be satisfied by much. This means that the
expression is near to unity and somewhat smaller than unity. This entails the condition that the right hand side has to be positive, which is readily interpreted as possibly arising from line radiation, because that term is the only one which has the same radial dependence as the gravitational force. Comparing then the effect of line driving (Lucy and Solomon 1970, Castor et al. 1975) with and without such a magnetic field, the net effect is an amplification in the sense that for $M_A^2 \gg 1$ the velocity gradient is asymptotically increased by

$$1/(1 - \frac{1}{M_A^2}).$$  

(58)

We thus argue that there is a magnetic field configuration where the field is mostly tangential already inside the star, and remains mostly tangential outside the star, where the initial acceleration of the wind is done by the opacity mechanism discussed by Kato and Iben (1992); outside the star we have a line-driven wind, but the amplification by the effect of the tangential magnetic field really produces the large momentum in the wind.

To see the properties of the equations better, we simplify by dropping all right hand terms in the differential equation except for the gravitational and the radiative force, and introduce characteristic length and velocity scales

$$r_s = \frac{LN}{L_{edd}} \left(\frac{GM}{v_{rs}^2}\right)\frac{v_{vs}}{v_{Ar s}} \frac{4}{3} \left(\frac{v_{vs}}{r_s \Omega}\right)^{4/3},$$

(59)

and

$$v_s = v_{rs} \left(\frac{v_{Ar s}}{v_{rs}}\right)^{2/3} \left(\frac{r_s \Omega}{v_{rs}}\right)^{1/3},$$

(60)

which is easily recognized as the Michel velocity (Hartmann and MacGregor 1982). The Eddington luminosity is given by

$$L_{edd} = \frac{4\pi GM \eta_p c}{\sigma_T}.$$  

(61)

In the approximation that $U_e = U_M \simeq 1$ and $r \Omega \gg v_r$ the Michel velocity corresponds to the overall Alfvén velocity. The differential wind equation then reads in dimensionless length $x$ and velocity $y$

$$\left(1 - \frac{1}{y^3}\right) \frac{dy^2}{dx(1/x)} = -f_{rad},$$  

(62)

with

$$f_{rad} = 1 - \frac{L_{edd}}{NL}.$$  

(63)

Clearly we require then that the radiative force dominates, by however little, to obtain the correct sign of the right hand side, thus $f_{rad} > 0$. The analytic solution yields then two extreme possibilities, either

$$v_{r \infty} = v_c \left(\frac{N L f_{rad}}{L_{edd}}\right)^{1/2},$$  

(64)

or

$$v_{r \infty} = \sqrt{2} \left(\frac{v_{Ar s}}{v_{rs}}\right)^{1/3} \left(\frac{r_s \Omega}{v_{rs}}\right)^{1/3} v_s,$$  

(65)

where we have used the definition

$$v_c = \left(\frac{2GM}{r_s}\right)^{1/2}$$  

(66)

for the surface escape velocity from the star. Obviously, we require that

$$2^{3/2} \left(\frac{v_{Ar s}}{v_{rs}}\right) \left(\frac{r_s \Omega}{v_{rs}}\right) > 1,$$  

(67)

in order to have a final velocity above the Michel velocity. We note that

$$\frac{\Omega r_s}{v_{rs}} = \frac{B_{rs}}{B_{vs}} \frac{U_M s}{U_{cs}},$$  

(68)

and thus the condition is indeed well fulfilled for a magnetic field configuration which is highly tangential at the stellar surface. By the same condition, the initial surface velocity is below the Michel velocity (except for the factor $2^{3/2}$ which is of order unity). This is necessary for the flow to have a transition through the fast magnetosonic point. This sharpens the approximations introduced above.

We have thus in this approximation two possible extreme solutions: First, for small magnetic field, the Alfvén velocity drops out and the line driving is the regulating agency; second, for large magnetic field, the terminal wind velocity is not far from the tangential Alfvén speed, and the line driving effect is small, except in that it provides the source of momentum to be amplified. This latter picture is the concept we propose to explain the momentum in the winds of Wolf Rayet stars. This is different from the solutions of Hartmann and MacGregor (1982) in the sense, that in their wind solutions the main acceleration of the wind is between the slow magnetosonic point and the Alfvén point, both of which are not outside the star in the case which we discuss; our solutions are similar, however, in the sense that both in their case as here, the terminal velocity of the wind in the case of interest is not far from the total Alfvén velocity. In this picture the angular momentum of the star lost refers to a characteristic level inside the star and so the criticism of Neary and Sues (1987) is counteracted, the angular momentum loss of the star through mass loss is minimized.

Using the observed wind velocities then gives an estimate for the Alfvén velocity, and thus implies - assuming our wind driving theory to be correct - that the magnetic field is quite high, and of the order of 3 Gauss at the fiducial radius of $10^{14}$ cm. Or, to turn the argument around, the magnetic field strengths implied by our cosmic ray arguments provide a possible explanation for the origin of the momentum of Wolf Rayet star winds. On the surface of the star the magnetic field strength implied is of order a few thousand Gauss, quite easily within the limits implied by the surface virial theorem. Also the surface of the star may not rotate as fast, but the inside could still rotate at an angular velocity corresponding to near critical at the surface. We emphasize that here a proposed wind driving theory provides an independent argument for the magnetic field strength on the surface of Wolf Rayet stars.
9. The nonthermal radio emission

In this section we derive the basic expressions for the luminosity, spectrum, and time dependence of the nonthermal radio emission from single shocks in the winds of massive stars. In the subsequent sections we then use these expressions to discuss radiosupernovae, young radio supernova remnants in starburst galaxies, Wolf Rayet stars, and OB stars.

Electrons suffer massive losses due to Synchrotron radiation and so they can achieve only energies up to that point where the acceleration and loss time become equal; this is a strong function of latitude since the magnetic field varies rather strongly with latitude. The shock transfers a fraction $\eta$ of the bulk flow energy into relativistic electrons. The emissivity of an electron population with the spectrum

$$N(\gamma) \ d\gamma = C \gamma^{-p} \ d\gamma$$

is given by (Rybicki and Lightman 1979) in cgs units for $p = 2.47$

$$\epsilon_\nu = 1.05 \times 10^{-17} \ C \ B^{1.735} \ \nu^{-0.735} \ \text{erg sec}^{-1} \ \text{cm}^{-3} \ \text{Hz}^{-1}. \quad (70)$$

and for $p = 7/3$

$$\epsilon_\nu = 3.90 \times 10^{-18} \ C \ B^{5/3} \ \nu^{-2/3} \ \text{erg sec}^{-1} \ \text{cm}^{-3} \ \text{Hz}^{-1}. \quad (71)$$

Similarly the absorption coefficient for synchrotron self absorption is ($p = 2.47$)

$$\kappa_\nu, \text{syn} = 3.56 \times 10^{12} \ C \ B^{2.235} \ \nu^{-3.235} \ \text{cm}^{-1}, \quad (72)$$

and for $p = 7/3$

$$\kappa_\nu, \text{syn} = 1.23 \times 10^{12} \ C \ B^{13/6} \ \nu^{-19/6} \ \text{cm}^{-1}. \quad (73)$$

In all these expressions we have averaged the aspect angle. We use here at first the case of a wind speed equal to the shock velocity and the limit of a strong shock in a gas of adiabatic index 5/3, which leads to an optically thin synchrotron spectrum of $-0.735$. We will use the limit of large shock speeds below for the discussion of the radiosupernovae, where the radio spectral index is $-2/3$. The free-free opacity is given by

$$\kappa_\nu, \text{ff} = 0.21 T_e^{-1.35} \ \nu^{-2.1} \ n_e^2 \ \text{cm}^{-1}, \quad (74)$$

in an approximation originally derived by Altenhoff et al. (1960) and widely disseminated by Mezger and Henderson (1967). Here $T_e$ and $n_e$ are the electron temperature and density, and the approximation has been used that the ion and electron density are equal, and that the effective charge of the ions is $Z = 1$. We note that for cosmic abundances we have the following approximations (Schmutzler 1987) assuming full ionization:

$$\rho = 1.3621 m_H n, \quad n_e = 1.181 n, \quad n_i = 1.086 n, \quad (75)$$

where $n$ is the total Hydrogen density (particles of mass $m_H$ per cc), and $n_i$ the ion density. With these approximations for full ionization the free-free opacity would increase a factor of two over the approximation by Altenhoff et al. (1960), which, however, is compensated by the incomplete ionization (see, again, the calculations by Schmutzler 1987) in the photon ionized regions (here, winds) near massive stars. In the following we will use the approximation by Altenhoff et al. replacing $n_i = n$.

The synchrotron luminosity in the optically thin limit is then given by an integration over the emitting volume, which we take in the context of our simplified picture to be a spherical shell of thickness $r/4$, for $U_1/V_W = 1$ decreased to $r/8$.

The radio emission only arises from that part of the shell where the shock velocity is larger than the local Alfvén velocity and where the synchrotron loss time is longer than the local acceleration time. These conditions can lead to a restricted range in latitude for the emission. The luminosity is then given by the integral over the latitude dependent emissivity. Here we have to normalize the cosmic ray electron density to the shock energy density, using our induced latitude dependence. In paper CR I we demonstrated that depending on the orientation of the magnetic field the $\theta$-dependence is either peaked toward the poles, when the drift is toward the poles, or peaking near the equator, when the drift is toward the equator. Clearly, the second case produces very much stronger radio emission, since the magnetic field is also strongest near the equator, and so we will concentrate on this case and discuss the opposite case briefly at the end. Using a lower bound for the electron spectrum much below the rest mass energy and an upper bound much above that value we have for the constant $C$ then the expression

$$C(\mu) = C_0 (1 - \mu^2)^{3/2} \quad (76)$$

with

$$\mu_* > \mu > 0 \quad (77)$$

where $\mu_*$ refers to that latitude where the latitude dependent acceleration breaks down:

$$(1 - \mu_*^2)^{1/2} = \frac{3}{4} \frac{U_1}{c} \ll 1, \quad (78)$$

from our argument about the knee energy (see section 8 of paper CR I) for protons and other nuclei. At the equator the energy density of the electrons can be written as

$$C_0 = (p - 2) \eta \rho U_1^2/(m_e c^2) \quad (79)$$

where we use just the relativistic part of the distribution function and incorporate the uncertainty on the existence and strength of any subrelativistic part of the electron distribution function in the factor $\eta$. The integration over latitudes leads to an integral correction factor of

$$\frac{1}{2} B \left( p + \frac{11}{4} \left( \frac{1}{2} \right) \right),$$
where $B(z_1, z_2)$ is the Beta-function, and $p$ the powerlaw index of the electron distribution function. For the two powerlaw indices used here, $p = 7/3$ and $p = 2.470$, this integral is both very close to 0.50 in value. We also have

$$\rho = \frac{\dot{M}}{4\pi r^2 V_W}. \quad (80)$$

Assuming that all latitudes contribute up to $\mu_*$ assumed to be close to unity, the final expression for the luminosity is then

$$L_\nu(nth) = 4.0 \times 10^{24} \text{ erg/sec}/\text{Hz}$$

$$\eta_{-1} \left( \frac{\dot{M}_5}{V_{W,-2}} \right) U_{1,-2}^2 B_{0.5}^{1.735} r_{14}^{-0.735} \nu_{9.7}^{-0.735} \nu_{9.7}. \quad (81)$$

Here the mass loss is in units of $10^{-5} \text{ M}_\odot/\text{yr}$, the unperturbed magnetic field strength at the reference radius of $10^{14} \text{ cm}$ in units of $3 \text{ G}$, the wind and the shock velocity in units of $0.01 \text{ c}$, and as reference frequency we use $5 \text{ GHz}$. For the powerlaw index of $p = 7/3$ this luminosity is

$$L_\nu(nth) = 8.1 \times 10^{24} \text{ erg/sec}/\text{Hz}$$

$$\eta_{-1} \left( \frac{\dot{M}_5}{V_{W,-2}} \right) U_{1,-2}^2 B_{0.5}^{5/3} r_{14}^{-2/3} \nu_{9.7}^{2/3}. \quad (82)$$

Here we note that this emission might become optically thick both to free-free absorption in the lower temperature region outside the shock, or to to Synchrotron self absorption inside the shocked region. In the approximation, that we use the equatorial region in the slab model (i.e., direct central axis radial integration), we obtain for the critical radius $r_{1,ff}$, where the free-free absorption has optical thickness unity:

$$r_{1,ff} = 1.35 \times 10^{14} \left( \frac{\dot{M}_5}{V_{W,-2}} \right)^{2/3} \nu_{9.7}^{-0.70} \text{ cm}. \quad (83)$$

We have used here the analytical approximation for the free-free opacity introduced above and an assumed temperature of $T_e = 2 \times 10^8 \text{ K}$. Similarly, for synchrotron self absorption the critical radius $r_{1,syn}$ is given for $p = 2.470$ by:

$$r_{1,syn} = 3.18 \times 10^{14} \text{ cm}$$

$$\eta_{-1}^{0.309} \left( \frac{\dot{M}_5}{V_{W,-2}} \right)^{0.309} U_{1,-2}^{0.618} B_{0.5}^{0.691} \nu_{9.7}^{1/5}. \quad (84)$$

and for $p = 7/3$:

$$r_{1,syn} = 3.99 \times 10^{14} \text{ cm}$$

$$\eta_{-1}^{0.316} \left( \frac{\dot{M}_5}{V_{W,-2}} \right)^{0.316} U_{1,-2}^{0.632} B_{0.5}^{0.684} \nu_{9.7}^{1/3}. \quad (85)$$

The maximum synchrotron luminosity is given by the dominant absorption process; which one is stronger is given by comparing the relevant radii; in the numerical example synchrotron self absorption happens to be stronger. Free-free absorption is dominant for $p = 2.470$ if

$$\frac{\dot{M}_5}{V_{W,-2}} > 11.0 \eta_{-1}^{0.864} U_{1,-2}^{1.728} B_{0.5}^{1.932} \nu_{9.7}^{-0.839}, \quad (86)$$

and for $p = 7/3$ if

$$\frac{\dot{M}_5}{V_{W,-2}} > 22.0 \eta_{-1}^{0.901} U_{1,-2}^{1.802} B_{0.5}^{1.951} \nu_{9.7}^{-0.856}. \quad (87)$$

We note that the parameter $\eta$ might well be quite low.

In the following we give then the maximum luminosities calculated by using the proper optical depth for a central axis approximation in a slab geometry for the radiative transfer at the equator; this gives an additional factor of about 2/3 (see below). However, for the total emission we do integrate properly over all latitudes, while for the absorption we approximate by using the equatorial values. This is a fair approximation, since both emission and absorption decrease towards the poles, but the absorption decreases even faster. The exact numerical value in our approximation is used here.

In the case that free-free absorption dominates, the maximum luminosity is then given by $p = 2.470$

$$L_\nu(nth) = 1.8 \times 10^{24} \text{ erg sec}^{-1} \text{ Hz}^{-1}$$

$$\eta_{-1} \left( \frac{\dot{M}_5}{V_{W,-2}} \right)^{0.510} U_{1,-2}^2 B_{0.5}^{1.735} \nu_{9.7}^{0.221}. \quad (88)$$

and for $p = 7/3$ by

$$L_\nu(nth) = 3.8 \times 10^{24} \text{ erg sec}^{-1} \text{ Hz}^{-1}$$

$$\eta_{-1} \left( \frac{\dot{M}_5}{V_{W,-2}} \right)^{0.556} U_{1,-2}^2 B_{0.5}^{1.667} \nu_{9.7}^{0.200}. \quad (89)$$

In the case that Synchrotron self absorption dominates, these maximum luminosities are given by $p = 2.470$

$$L_\nu(nth) = 1.2 \times 10^{24} \text{ erg sec}^{-1} \text{ Hz}^{-1}$$

$$\eta_{-1}^{0.773} \left( \frac{\dot{M}_5}{V_{W,-2}} \right)^{0.773} U_{1,-2}^{1.546} B_{0.5}^{1.227}. \quad (90)$$

and for $p = 7/3$ by

$$L_\nu(nth) = 2.2 \times 10^{24} \text{ erg sec}^{-1} \text{ Hz}^{-1}$$

$$\eta_{-1}^{0.789} \left( \frac{\dot{M}_5}{V_{W,-2}} \right)^{0.789} U_{1,-2}^{1.579} B_{0.5}^{1.211}. \quad (91)$$

Here we do not consider mixed cases.

Obviously, the ratio of mass loss rate and wind velocity can be different by many orders of magnitude among predecessor supernova stars, and their numerical values will have to be argued on the basis of observations.

It is useful to also calculate the thermal radio emission, using the same standard parameters, then adjust our parameters to a realistic range, and then compare the nonthermal luminosities. The thermal emission can be properly integrated, allowing for the sphericity of the wind structure (Biermann et al. 1990) to give:
\[ L_{\nu}(th) = 4\pi^2 B_\nu(T) r_{1, ff}^2 \Gamma \left( \frac{1}{3} \right) \]

(92)

With our standard parameters this luminosity is at the frequency of maximum emission (i.e. what we observe in a steady wind)

\[ L_{\nu}(th) = 3.0 \times 10^{17} \left( \frac{M_{-5}}{V_{W,-2}} \right)^{4/3} \nu_{0.7}^{0.66} \text{erg/sec/Hz}, \]

(93)

weakly dependent on electron temperature.

In the following we make a number of consistency checks on the basic notions used above:

First, we note that shocks can only exist when the shock velocity is larger than the Alfvén velocity. Since in our wind driving theory developed above the wind velocity is just a little larger than the Alfvén velocity itself, this implies that the shock velocity in the frame of the flow has to be at least the same velocity as the wind, and so - within our analytical approximations - we have the condition that always

\[ U_1 \geq V_W, \]

(94)

where the limit of the equality corresponds to a spectral index for the Synchrotron emission of \(-0.735\) and in the limit of large shock velocity to \(-2/3\). Here we have to note that the Alfvén velocity is colatitude \(\theta\) dependent and is proportional to \(\sin \theta\).

Thus, even for lower shock speeds, there is a latitude range where a shock can be formed, but then the luminosity is very much reduced.

Second, we have to check that the Synchrotron loss time is larger than the acceleration time, because otherwise we would not have any electrons at the appropriate relativistic energies. This condition can be rewritten as

\[ \nu_{0.7} B_{0.5}^3 \geq U_{1,-2}^2 r_{14}. \]

(95)

This is the condition at the equator, and the condition gets weaker at higher latitudes. This makes it obvious, that for all reasonable ranges of the parameters acceleration can succeed to the required electron energies. It also shows that at higher frequencies or smaller radii the condition would fail. At smaller radii the emission is usually optically thick (see above), and higher frequencies are as yet difficult to observe at the required sensitivity. The implied high frequency cutoff in the observed synchrotron spectra would, however, be an important clue.

Third, we have to check whether the implication that the shock speeds are typically similar to the wind speeds, is supported by data on Wolf Rayet stars, and their theoretical understanding. The wind calculations with shocks suggest that the typical shock velocities are indeed of order the wind speed itself or somewhat higher (Owocki et al. 1988), and so we expect a range in radio spectral indices of \(-0.667\) to \(-0.735\) or steeper if the shocks are not strong, i.e. if \(U_1/U_2 < 4\). We note that the actual value of the nonthermal luminosity is changed only moderately in this range of spectral indices.

Fourth, we have to discuss optical thickness effects in more detail: A shock travels from the region inside of where free-free absorption dominates through this region to the outside. The emission then is first weak and has a steep spectrum due to the strong frequency dependence of the free-free absorption and the exponential cutoff induced, then approaches a peak in emission with a spectral index approaching \(-0.67\) to near about \(-0.735\) or steeper as discussed above, and then becomes weaker with the optically thin spectrum. At the location where our ray encounters the shock, the temperature of the gas increases drastically, and so the differential optical depth for free-free absorption goes to zero. Hence we have the simple case that we have emission inside the shock and absorption outside.

We limit ourselves to the central axis as a first approximation, and also neglect the spatial variation of the emission itself (see above). This is equivalent to pure screen-like absorption, however with a screen which extends from the shock to the outside and so is variable. The radio luminosity is given by (using free-free absorption)

\[ L_{\nu}(nth) = e^{-\tau_{\nu}} L_{\nu}(nth, no abs). \]

We have the spectral index \(-\alpha_{\text{thin}}\) in the optically thin regime, and the time dependence of the spectral index given by

\[ \alpha(\nu) = -\alpha_{\text{thin}} (1 - (t^*/t)^3). \]

(97)

(97)

The spectral index is positive and very steep at first and then goes through zero to become slowly negative approaching asymptotically the optically thin spectral index. The nonthermal luminosity considered as a function of time sharply rises at first, then peaks at optical depth \(\alpha_{\text{thin}}/3\), obviously strongly dependent on frequency, and finally drops off as \(1/t^{\alpha_{\text{thin}}}\):

\[ \frac{d\ln L_{\nu}(nth)}{d\ln t} = -\alpha_{\text{thin}} (1 - \frac{10}{7} (t^*/t)^3). \]

(98)

Here we see that the time dependence of the flux density at a given frequency and the time dependence of the spectral index are closely related. The time of luminosity maximum depends on frequency as

\[ t_{\text{max L}} \sim \nu^{-0.7}. \]

(99)

from the frequency dependence of the optical depth

\[ \tau_{\nu} \sim t^{-3} \nu^{-2.1}. \]

(100)

These relationships can be used to check on the importance of free-free absorption in our approximations.

Synchrotron self absorption is due to internal absorption inside the shell, and so again in the central axis approximation we have

\[ L_{\nu}(nth) = \frac{1 - e^{-\tau_{\nu}}}{\tau_{\nu}} L_{\nu}(nth, no abs). \]

(101)

This then results in the time dependence for the spectral index of

\[ \alpha(\nu) = \frac{5}{2} + \frac{5 + 2\alpha_{\text{thin}}}{2} \tau_{\nu} - \frac{5 + 2\alpha_{\text{thin}}}{1 - e^{-\tau_{\nu}}} \]

(102)

with
\[ \tau_{\nu} \sim t^{-(5+2\alpha)/2} \nu^{-(5+2\alpha)/2}. \]  
(103)

The time dependence of the double logarithmic derivative of luminosity on time is exactly the same

\[ \frac{d\ln L_\nu(nth)}{d\ln t} = \alpha(\nu). \]  
(104)

It follows that the time of maximum depends on frequency as

\[ t_{\text{max}} \sim \nu^{-1}. \]  
(105)

This is a characteristic feature for synchrotron self absorption in the approximation used (and well known from radioquasars) and differs from the case considered above, for free-free absorption.

Thus, the true maximum of the luminosity is given by an optical depth less than unity, which gives a correction factor to the luminosities introduced above (eqs. 88 through 91) of about 2/3; we have corrected the luminosities introduced there for this factor.

Finally, we have to comment on the sign of the magnetic field. We have used here throughout the assumption that the magnetic field is oriented such that the drifts are towards the equator for the particles considered. Clearly, since we consider stars with a magnetic field driven by turbulent convection in a rotating system, we can expect that there are sign reversals of the magnetic field just as on the Sun. For the other sign of the magnetic field and the other drift direction there is by many powers of ten less nonthermal emission and so it is to be expected that at any given time we should detect at most half of the stars in nonthermal radio emission; occasionally we might even catch a shock travelling through the region in the wind where the sign is reversing itself, because the wind mirrors the time history of the star in terms of magnetic field.

In the following we will first discuss the radio observations of supernovae, then Wolf Rayet stars, and finally OB stars.

10. Radiosupernovae

The adopted value for the magnetic field, however, was derived from the notion that the subsequent supernova shocks accelerate particles to extremely high energies. This argument can be checked with the observations of those supernovae of which the radioemission in the wind was really observed, five sources discussed by Weiler and colleagues in a number of papers (Weiler et al. 1986, 1989, 1990, 1991, Panagia et al. 1986) and the supernova 1987A (Turtle et al. 1987, Jauncey et al. 1988, Staveley-Smith et al. 1992) and related radio sources in starburst galaxies. However, we restrict ourselves to those supernovae for which we can reasonably assume that the predeccessor star was indeed a Wolf Rayet star, and this we will do using statistical arguments in the subsequent section.

In fact, our model can be paraphrased as a numerical version of Chevaliers (1982) model with the parameters fixed: In the screen approximation valid for free-free absorption (see above for details), the time evolution of the nonthermal emission in Chevaliers model can be written as

\[ L_\nu(nth) = K_1 \nu^\alpha \delta \nu^{-K_2 z^\delta} \]  
(106)

with \( \alpha, \beta \) and \( \delta \) to be fitted to the data. For fast strong shocks our model predicts that \( \alpha = \beta = -2/3 \) and \( \delta = -3 \), and for slower shocks that still \( \alpha = \beta \) but larger in number, for \( U_1/V_W = 1, \alpha = \beta = -0.735 \), for instance. This is very close to the detailed fits for the four supernovae listed above (Weiler et al. 1986).

For SN 1986J the data clearly cover a sufficiently large time to test this numerical model in more detail; Weiler et al. (1989) find that this simple model in its screen approximation does not provide a good fit to the early epochs. We suspect similar to Weiler et al., that this lack of a good fit can be traced to the simplification that we consider the external absorption as a simple screen, and disregard the lateral structure. Further possible reasons for a failure to strictly adhere to the simplified model are the following: a) The pre-shock wind may not be smooth in its radial behaviour, there might have been a weaker shock running through earlier which nevertheless can disturb the radial density profile (see the calculations by MacFarlane and Cassinelli 1989). b) Mixing between free-free absorption (outside the shock region) and synchrotron self-absorption (inside the shock region). c) The structure of acceleration in its latitude dependence is considered here only for the acceleration of particles (see paper CR I), but not for absorption and radiative transfer. Given a very detailed multifrequency data set it would be interesting to model the data fully.

In fact, we can use the numerical values for the radii for maximum luminosity derived above to obtain the pre-shock wind density, which is proportional to \( M/V_W \). With the data given by Weiler et al. (1986) this yields for the supernova 1979c

\[ \frac{\dot{M}_5}{V_W^{-2}}(1979c) = 3.0 \times 10^7 U_1^{-3/2}, \]

and for the supernova 1980k

\[ \frac{\dot{M}_5}{V_W^{-2}}(1980k) = 3.4 \times 10^2 U_1^{-3/2}. \]

Clearly, these numbers tell us that the predecessor stars had a slow wind, and thus were probably red supergiants (see Weiler et al. 1986). This then implies for shock speeds of 0.03 \( c \), \( \eta = 0.1 \), and free-free absorption being dominant, that the nonthermal luminosities at 5 GHz expected versus observed are

\[ L_{\text{max}}(1979c) = 3.0 \times 10^{27} \text{versus 2.10}^{27} \text{erg sec}^{-1} \text{Hz}^{-1}, \]

and

\[ L_{\text{max}}(1980K) = 9.0 \times 10^{26} \text{versus 1.10}^{26} \text{erg sec}^{-1} \text{Hz}^{-1}, \]

both for the assumed strength of the magnetic field. Since the luminosity is proportional to the magnetic field strength to the power 5/3, and directly proportional to the electron efficiency parameter \( \eta \), we derive thus a lower limit to the magnetic field, given an upper limit on \( \eta \). Since \( \eta = 0.1 \) is unlikely to be surpassed by much, using the ratio of the luminosities expected/observed of the two cases above yields an estimated
lower limit to the magnetic field strength at our reference radius of

\[ B > 1.5 \eta^{-3/5} U_{-1.5}^{-6/5} \] Gauss.

This implies a strong lower limit to the magnetic field from using \( \eta = 1 \) of 0.4 Gauss. The uncertainty in these estimates is clearly at least a factor of 2. For 1987A, the initial radio luminosity was indeed of order \( 10^{25} \) erg/sec/Hz and so, applying our model with a shock speed of order 0.03 c is consistent with our expectation of then \( 3.4 \times 10^{25} \eta_{-1} \) erg/sec/Hz; we do not wish to discuss 1987A in any detail here.

This demonstrates as argued in section 7, that strong magnetic fields also exist in the winds of massive stars in the red part of the Hertzsprung–Russell diagram, where the winds are slow; note that the predecessor to supernova 1987A was a blue supergiant with a fast wind.

The only unknown parameter in all these predictions is the efficiency of electron acceleration \( \eta \) and the strength of the magnetic field; with \( \eta \) close to 0.1, clearly close to the maximum number reasonable, leads then to the requirement that the magnetic field strength is near to what we assumed, 3 Gauss at \( 10^{14} \) cm, but even higher, if \( \eta \) is very much less than unity. This again confirms independently that indeed the magnetic field has to be as high as argued by Cassinelli (1982, 1991) to drive the winds and as is required to accelerate cosmic rays particles to energies near \( 3 \times 10^9 \) GeV.

For supernova predecessor stars with fast winds like Wolf Rayet and OB stars, it is of interest to ask whether the expected luminosity violates the Compton limit, famous from the study of radioquasars. At the Compton limit the first order inverse Compton X-ray luminosity becomes equal to the synchrotron luminosity, and it has been found from observations that compact radioquasars are close to this limit and indeed have strong X-ray emission. This question can be formulated as a limit to the brightness temperature of the radio source (using here synchrotron self absorption) which then gives the limit

\[ \eta_{-1} \left( \frac{M_{-5}}{W_{-2}} \right) U_{-1.5}^2 B_{0.5}^{1.27} \nu_{-7}^{2.7} \leq 1.5 \times 10^7. \] (107)

For free-free absorption the limit is

\[ \eta_{-1} \left( \frac{M_{-5}}{W_{-2}} \right)^{-0.777} U_{-1.5}^2 B_{0.5}^{5/3} \nu_{-7}^{-0.6} < 0.18. \] (108)

The first of these conditions is almost certainly always fulfilled, while the second one may be so tight as to suggest that inverse Compton X-rays might be observable. This has indeed been checked with modelling successfully the observed X-ray spectra beyond photon energies of 2 keV by Chen and White (1991a) for Orion OB stars. This suggests that the inverse Compton X-ray luminosity ought to be less, usually considerably less than the Synchrotron luminosity. Most of the X-ray emission from hot stars is thought to arise from these same shocks in free-free X-ray emission which we consider for particle acceleration; a modelling of this was done by White and Long (1986) and MacFarlane and Cassinelli (1989).

11. The statistics of Wolf Rayet stars and supernovae

There are a variety of ways to estimate the relative frequency of Wolf Rayet star supernova explosions relative to supernova explosions of lower mass stars (Hidayat 1991, Leitherer 1991, Massey and Armandroff 1991, Shara et al. 1991).

In our Galaxy there are between 300 and 1000 Wolf Rayet stars, which have an average lifetime of about \( 10^7 \) years. This gives an estimated occurrence of Wolf Rayet supernovae of about one every 100 to 300 years. Since the total rate of supernovae in our Galaxy is estimated at about one every 30 years, this means that roughly one in 3 to one in 10 supernovae ought to represent the explosion of a Wolf Rayet star.

The numbers of stars on the main sequence between 8 solar masses and above are 25 solar masses, and between 25 solar masses and the upper end of the main sequence also ought to correspond to the ratio of Wolf Rayet stars and the rest of those stars which explode as supernovae. Using for the simple estimate the Salpeter mass function gives here an estimated ratio of 1 in 5 supernova events which originate from a Wolf Rayet star.

The model for the origin of the high energy population of relativistic cosmic rays proposed in paper CR I and tested successfully in paper CR IV suggests from the energetics a ratio of about 1 in 3, assuming the amount of energy pumped into cosmic rays per supernova to be the same for all kinds. However, here we lump all supernova explosions into stellar winds together, both with fast and slow winds.

Hence in the sample of radio supernovae of type II (6 sources) presented by Weiler et al. (1986) there ought to be between none and two events based on Wolf Rayet star explosions; in the sample of radio sources likely also to be very young supernova remnants (28 sources with radio luminosities at 5 GHz) in the starburst galaxy M82 (Kronberg et al. 1985) there ought to be about at least between 3 and 10 sources which originate from a Wolf Rayet star explosion. On the other hand, all of these objects are possibly explosions of stars into former stellar winds (there is evidence that the initial mass function in starburst galaxies is biased in favor of massive stars), and so the proportion of the stars among them that are due to Wolf Rayet star explosions, could be even higher than estimated here. The radio luminosities are very similar for the sources in M82 and the radio supernovae of Weiler et al. (1986); the average luminosity calculated in a variety of ways is always in the range of \( 3 \times 10^{25} \) erg sec\(^{-1}\) Hz\(^{-1}\) and \( 10^{26} \) erg sec\(^{-1}\) Hz\(^{-1}\), fitting our expectation (see above) for \( U_1 \approx 0.03 \) c rather well.

For our arguments on cosmic rays it is not relevant whether the predecessor stars were Wolf Rayet stars or other massive stars with extended winds permeated by strong magnetic fields. But we conclude that a sufficient fraction of the young radio supernova events and remnants are likely to be from Wolf Rayet stars, that we can use the implied properties for Wolf Rayet stars.

12. Wolf Rayet stars

The theoretical luminosities derived can be compared with the observations of Wolf Rayet stars, which yield (Abbott et al. 1986) nonthermal luminosities up to about \( 5 \times 10^{29} \) erg/sec/Hz at 5 GHz and thermal luminosities up to
higher nonthermal emission as well, by a factor of 3.

\[ \eta \approx 10^{-6} \pm 1 \] for the shock speeds

to be normal in Wolf Rayet star winds, and of order \eta \approx 0.1 for supernova explosions. The data thus suggest that the injection of electrons into the diffusive shock acceleration appears to exhibit a step function property at a shock speed near to \( 7 \times 10^3 \) cm sec\(^{-1}\). If true, this might be important also for electron injection in quasars, which also exhibit a dichotomy into radioloud and radioweak objects. In paper CR III (Biermann and Strom, 1993) we will then test whether this concept leads to a successful estimate for the electron/proton ratio of the lower energy cosmic rays.

Now we can go back and ask again, whether free-free absorption or synchrotron self absorption dominates in the various cases considered. Clearly, when \eta, the efficiency for electron injection, is very small, then free-free absorption always dominates, and so the maximum luminosities during a radio variability episode of a Wolf rayet (or OB) star should be frequency dependent. Also, for slow winds in supernova predecessor stars, again free-free absorption will dominate normally (e.g. in the two examples used above). On the other hand, for fast winds of the supernova predecessor stars (as in the case of a Wolf Rayet star exploding), synchrotron self absorption is likely to be stronger than free-free absorption and then the maximum luminosities at different radio frequencies should be independent of frequency. This is a testable prediction, since it relates radio and optical properties of a young supernova.

13. OB stars

The theoretical luminosities derived can be compared with the observations of OB stars, which yield (Bieging et al. 1989) nonthermal luminosities up to about \( 7 \times 10^{19} \) erg/sec/Hz at 5 GHz, and thermal luminosities up to \( 2.5 \times 10^{19} \) erg/sec/Hz. Since the most important parameter, that enters here is the density of the wind, or in terms of wind parameters, the ratio of the mass loss to the wind speed \( M/V_w \), we induce that the most extreme stars have a higher value for \( M/V_w \) by 27.6. This translates into a higher nonthermal emission as well, by a factor of 5.4 to give \( 9.8 \times 10^{24} \) erg/sec/Hz \( \eta_{1/2} U_{1/2}^2 B_1^{7/5} \). This is very much more than the observed nonthermal luminosities.

Since the shock properties are likely to be similar to Wolf Rayet stars, this is again consistent with the idea that the injection efficiency of electrons might be quite low, in conjunction with magnetic field values not too far from what we argued to be valid for Wolf Rayet stars.

We can also compare the detection statistics and observed spectral indices: Since in a variability episode a shock comes through from below through the region of optical thickness near unity, the nonthermal emission increases rapidly to its maximum, when its spectral index is

\[ \alpha_{\nu, \text{max}} = -0.3 \alpha_{\text{thin}}, \] (109)

which is approximately \(-0.2\); Thereafter, as the luminosity more slowly decreases, the spectral index gradually approaches the optically thin index of \(-0.67\) or slightly steeper. Therefore we expect the spectral index distribution of detected sources to be a broad distribution from near \(-0.2\) to near \(-0.7\); this is what has been found (Bieging et al. 1989). Because of the two possible signs of the magnetic field orientation, and the associated drift energy gains for particles, we also expect that at any given time at most half of all sources are detectable with nonthermal emission, even at extreme sensitivity. This is consistent with the observations. We predict a similar behaviour for Wolf Rayet stars.

Here we have to ask how the magnetic fields can penetrate the radiative region from below. This question cannot presently be tackled by simulations on a very large computer yet, but it is likely that the circulations induced by rotation transport magnetic fields to the surface, where even a rather slight differential rotation draws out the magnetic field into a mostly tangential configuration. In this case, clearly the origin of the momentum of the wind can be readily accounted for from line driving (Lucy and Solomon 1970, Castor et al. 1975), and so we suspect that the magnetic driving adds only little (which is the ”other” case discussed above near the end of the wind section, section 8).

We can also compare with the only existing theory to explain the nonthermal radio emission of single massive stars with winds, by White (1985). White’s theory is based on a concept involving a large number of shocks and does not explain the data as already demonstrated by Bieging et al. (1989) in terms of i) radio spectral index, ii) time variability nor iii) of the statistics of detection. Our theory is based on using single shocks and readily provides spectral indices in the entire range that is observed, it explains the time variability and easily accounts for a fair fraction of undetected sources. On the other hand, a second conclusion of Chen and White (1991b) and White and Chen (1992) is likely to be correct also in our
theory, that OB and WR stars might be strong sources in the Gamma ray range from the secondaries resulting from hadronic interactions of the energetic nuclei.

14. An observational check

Consider then the case where we observe the time evolution of the radio emission of massive stars: We measure at maximum luminosity, and at two times later in the optically thin regime the nonthermal luminosity again. Then the ratio of the two luminosities in the optically thin regime yields the zero point of the shock event time. The ratio of the luminosities at maximum and at one of the optically thin epochs yields the time scale of expansion, the scaling for the shock event. From an observation of the free-free emission, say, before the nonthermal outburst, we obtain the density and with that the radial scale, where the optical thickness is unity to free-free absorption. The ratio of this length scale and the explosion timescale gives the sum of the wind speed and the shock speed. Given an estimate for the wind speed from UV data then yields the shock speed. And, finally, given the density, the shock speed and the nonthermal luminosity we can obtain an estimate for the magnetic field.

We note that the shock travelling through the region where we normally have optical depth of order unity to free-free absorption also causes the thermal emission to decrease since the shock increases the temperature drastically. Hence the total radio emission should reflect the increase of the nonthermal emission and subsequent decrease, while at the same time the thermal emission will decrease and subsequently recover. Doing this exercise at several frequencies then yields a control on the entire procedure, and also checks on the spectral index dependence on wind speed. Therefore, an intensive observing campaign at the VLA should enable us to check on our model in some detail. Together with more detailed modelling of the radio emission from radio supernovae such results would then strengthen or weaken the case for the concept we propose here for the acceleration of Cosmic Ray particles in the energy range from $10^4$ GeV to $3 \times 10^9$ GeV.

15. Conclusions

In this paper we wanted to check whether the suggestion from cosmic ray arguments that the magnetic field strengths in the wind of early type stars are quite high, leads to any contradictions. We are able to draw several conclusions:

1) The magnetic fields in massive stars can be generated by a dynamo in the convective interiors. We can only speculate how they penetrate the radiative zones during the main sequence phase. In those massive stars that do not explode as supernovae, but become white dwarfs instead, the strong magnetic fields start already on the surface of the star to be nearly tangential.

2) The strong magnetic fields can put the major amount of momentum into wind and accelerate them to slightly above the local Alfvén velocity; critical to the argument is that the magnetic fields start already on the surface of the star to be nearly tangential.

3) Indeed, with the parameters as suggested we are able to understand the properties of radio supernovae, as regards i) the optically thin radio spectrum, ii) the maximum luminosity, and iii) the time dependence of the emission both during the optically thick and thin phase. We derive a lower limit to the magnetic field strength very close to the value implied by cosmic ray arguments.

4) Using the same arguments for Wolf Rayet stars implies that the electron injection is a step function of the shock velocity, with the step possibly connected to the shock velocity when the thermal electron speed behind the shock reaches relativistic values.

5) Similarly, the nonthermal radio emission of OB stars can be interpreted.

6) In summary, the strength of the magnetic field in Wolf Rayet stars as estimated from fast magnetic rotator theory, finds a variety of supportive arguments, from cosmic rays (papers CR I and Stanev et al. 1993, paper CR IV), from the radio emission of radio supernovae, and the radio emission of Wolf Rayet stars. This demonstrates that the proposal, that cosmic ray particles can be accelerated to extremely high energies in the ultimate explosions of Wolf Rayet stars and other massive stars with extended winds, up to energies of $3 \times 10^{9}$ GeV, is fully consistent with the available data on radio supernova and Wolf Rayet stars.

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