CAMRI Loss: Improving Recall of a Specific Class without Sacrificing Accuracy

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Abstract—In real-world applications of multi-class classification models, misclassification in an important class (e.g., stop sign) can be significantly more harmful than in other classes (e.g., speed limit). In this paper, we propose a loss function that can improve the recall of an important class while maintaining the same level of accuracy as the case using cross-entropy loss. For our purpose, we need to make the separation of the important class better than the other classes. However, existing methods that give a class-sensitive penalty for cross-entropy loss do not improve the separation. On the other hand, the method that gives a margin to the angle between the feature vectors and the weight vectors of the last fully connected layer corresponding to each feature can improve the separation. Therefore, we propose a loss function that can improve the separation of the important class by setting the margin only for the important class, called Class-sensitive Additive Angular Margin Loss (CAMRI Loss). CAMRI loss is expected to reduce the variance of angles between features and weights of the important class relative to other classes due to the margin around the important class in the feature space by adding a penalty to the angle. In addition, concentrating the penalty only on the important classes hardly sacrifices the separation of the other classes. Experiments on CIFAR-10, GTSRB, and AwA2 showed that the proposed method could improve up to 9% recall improvement on cross-entropy loss without sacrificing accuracy.

Index Terms—Machine Learning, Deep Learning, Multi-class Classification, Loss Function, Cost-Sensitive Learning, ArcFace

I. INTRODUCTION

In recent years, the generalization accuracy of classification prediction using Deep Convolutional Neural Networks (DCNNs) has dramatically improved, and DCNNs are expected to be applied to various fields such as automated driving and medical diagnosis support. However, if the model misclassifies in the real world, it may cause extremely serious accidents.

We consider the multi-class classification problem of identifying road signs in automatic driving. If the model misclassifies a "Stop" sign as other signs, the car will enter the intersection without pausing, likely leading to a traffic accident. By contrast, misclassifying a "No Parking" sign as a "No Parking Daytime" sign is relatively unlikely to lead to accidents or other harm. Therefore, the low recall of classes such as "Stop" and "No Entry," which lead to actual harm from misclassification, is a particularly important problem.

Thus, in order to use the DCNN multi-class classification model in the real world, there are cases where we want to increase the recall of a particularly important class. At the same time, the accuracy should not deteriorate at the cost of improving the recall of a particular class. Therefore, it is necessary to find a way to improve the recall of an important class without compromising the accuracy of entire classes.

A. Related Works

Threshold tuning [1] is considered one of the naive methods for the problem. However, this method improves the recall of a particular class but sacrifices the recall of other classes. In other words, since it only adjusts the trade-off ratio between accuracy and recall of a particular class, it is not possible to both improve recall and maintain accuracy.

There are several approaches with cost-sensitive learning [2], [3]. Cost-sensitive learning is a method of predicting classification with a loss function that imposes a relatively large cost (i.e., misclassification cost) on unacceptable errors. Several methods [4]–[7] consider the importance of the classes that need to improve recall as a penalty and use this penalty as a loss function that weights cross-entropy loss. However, as shown in our experimental results below, these methods do not always sufficiently improve the recall of the important class. Even if the recall is improved, the accuracy may be impaired. The cause of these is discussed in detail in a later section.

For our goal, we need to increase the separation between the important class and other classes while maintaining the separation between unimportant classes. In order to achieve such separation, the model should be trained to generate features so that (1) the feature vectors of the important class are well isolated and localized from the other classes, and (2) the feature vectors of each class are well separated from each other. In recent years, various loss functions have been developed to achieve this [8]–[13]. These methods consider classification by cosine similarity between the representative
vector of each class and the feature vector. Here, the vectors are obtained by projecting onto the unit hypersphere [14], [15]. With this setup, the margin between classes in the feature space is expected to increase. Then, the cosine term is manipulated with some constant factor so that the separation between classes is improved, which is called margin. Liu et al. [8], [9] proposed a method to introduce an angular margin loss between the class corresponding to the feature vector and other classes to promote the expansion of inter-class variance. Liang et al. [10] and Wang et al. [11], [12] proposed an additive angular margin loss (ArcFace) that can be interpreted geometrically, and this ArcFace has reported good performance in face recognition tasks. We remark that these angular-based loss functions are designed to improve separations between classes, whereas they are not designed to improve the recall of a specific important class without sacrificing the overall accuracy.

B. Our Contribution

The intuitive approach to improve the recall of the important class is to optimize the cross-entropy loss by giving a larger weight to the loss for the important class. However, as we will show in later experiments, these approaches do not attain our goal even with intensive weight parameter tuning. Our contribution is the following two.

- We propose a loss function, Class-sensitive additive Angular MaRglIn Loss (CAMRI Loss). CAMRI loss adds a margin penalty to the angle between the feature vector and the weight vector corresponding to the important class of the last fully connected (FC) layer only when the feature is labeled with the important class. As a result, it is expected to improve the separability of the features labeled with the important class from other classes in the feature space due to the margin.

- We have empirically shown that CAMRI loss improves the recall of the important class without sacrificing accuracy. We conducted experiments using three datasets: CIFAR-10 [16], German Traffic Sign Recognition Benchmark (GTSRB) [17], and Animals with Attributes 2 (AwA2) [18]. In 8 out of 9 ways, CAMRI loss achieved a higher value of recall improvement than the existing method while maintaining accuracy.

This paper is organized as follows: In Section II, we formulate the multi-class classification and related loss functions. In Section III, we analyze related loss functions based on their contours in the feature space. Then we propose a method that can improve the separation of the important class by setting the margin only for the important class. In Section IV, we evaluate the effectiveness of the proposed method with multiple datasets. In Section V, we present the conclusion.

II. PRELIMINARIES

A. Multi-class Classification

Let \( K \) be the number of classes, \( \mathcal{X} \) be the input space, and \( \mathcal{Y} = \{1, \cdots, K\} \) be the output space. We train a CNN model to represent feature vector \( z_n \in \mathbb{R}^D \) with dimensionality \( D \), using a pair of input images and teacher labels \( \{x_n, t_n\}_{n=1}^N \in \mathcal{D} \) for training. The one hot representation of \( t_n \) is given by \( y_{n,k} \). \( z_n \) is given to the FC layer with weights \( W \in \mathbb{R}^{D \times K} \) and bias \( b \in \mathbb{R}^K \). And the fully-connected layer outputs \( a_n = W^T z_n + b \). The softmax function transforms \( a_n \) into a probability vector \( h_n \), whose elements are interpreted as the predicted probabilities corresponding to each class. The cross-entropy loss is often used for classifier training, which takes \( h_n \) and \( y_n \) as input. By updating the model parameters to minimize the loss function, a multi-class classification model \( f : \mathcal{X} \rightarrow \mathcal{Y} \) can be obtained. We also represent the \( i\)th element in vector \( a \) as \( a_i \) and the \((i, j)\) element in matrix \( A \) as \( A_{i,j} \).

B. Related Loss Functions

In this section, we introduce several types of loss functions related to our research. Let class \( k \) be an important class.

1) Cross-Entropy Loss: First, we formulate the commonly used cross-entropy loss. When the number of samples is \( N \), and the number of classes is \( K \), the cross-entropy loss of multi-class classification is given by

\[
\mathcal{L}_{ce} = - \frac{1}{N} \sum_{n=1}^{N} \sum_{k=1}^{K} \{y_{n,k} \log (h_{n,k})\}.
\]

2) Weighted Cross-Entropy Loss: Second, we introduce Weighted Cross-Entropy loss (WCE) as the method that imposes class-specific penalties on cross-entropy loss. Let \( w_k \in \mathbb{R}^+ \) be the weight parameter of class \( k \). By penalizing the loss with \( w_k \) for each class, WCE [4]–[6] is given by

\[
\mathcal{L}_{wc} = - \frac{1}{N} \sum_{n=1}^{N} \sum_{k=1}^{K} \{w_k y_{n,k} \log (h_{n,k})\}.
\]

For our goal, we set the \( n\)th element in \( w \) to a value greater than 1 and set the other elements to 1.

3) Categorical Real-World-Weight Cross-Entropy Loss: Ho et al. [6] introduced Real-World-Weight Cross-Entropy (RWWCE), which penalizes each class’s false negatives and false positives independently with different costs. Multi-class classification extension of RWWCE, termed Categorical RWWCE (CRWWCE), is defined by

\[
\mathcal{L}_{rw} = - \frac{1}{N} \sum_{n=1}^{N} \sum_{k=1}^{K} \left[ c_{n,k} y_{n,k} \log (h_{n,k}) \right] \\
+ \sum_{k' \neq k} C_{k',k} y_{n,k} \log (1 - h_{n,k})
\]

Here \( c_{n,k} \in \mathbb{R}^+ \) is the weight vector whose elements are weights to penalize the false negative cases for each class. \( C_{k',k} \in \mathbb{R}^{K \times K} \) is a square matrix with zero diagonal. The \((k, l)\) element in \( C_{k',k} \) represents the weight parameter for penalizing false positives when a sample of class \( l \) is misclassified to class \( k \). For our goal, we set the \( n\)th element in \( c_{n,k} \) to a value greater than 1, and set the other elements to 1. We also set elements in the \( k \)th column of \( C_{k',k} \) and the \( k \)th row in \( C_{k',k} \), except for the \((k, k)\) element, to values greater than 1, diagonal elements to 0, and the other elements to 1.
4) Wasserstein Loss: Wasserstein loss [7] penalize misclassification by Wasserstein distance between \(y_n\) and \(h_n\) using a distance matrix \(C \in \mathbb{R}^{K \times K}_+\) that defines the penalty for misclassification. Wasserstein loss is defined by

\[
\mathcal{L}_{\text{ws}} = \frac{1}{N} \sum_{n=1}^{N} \left\{ \inf_{T \in \Pi(h_n, y_n)} \langle T, C \rangle - \lambda H(T) \right\}, \tag{4}
\]

where

\[
H(T) = - \sum_{k,k' \in Y} T_{k,k'} \left( \log T_{k,k'} - 1 \right), \tag{5}
\]

\[
\Pi(h, y) = \{ T \in \mathbb{R}^{K \times K}_+ \mid T1 = h, T^T1 = y \}. \tag{6}
\]

The transport matrix \(T\) in (4) is approximated by the Sinkhorn-Knopp algorithm [19].

For our goal, we set elements in the \(k\) column and the \(k\) row in \(C\), except for the \((k, k)\) element, to values greater than 1, diagonal elements to 0, and the other elements to 1.

5) ArcFace: Finally, we describe the additive angular margin loss (ArcFace) [13] as a method that explicitly penalizes the angle between features and weights. ArcFace is defined by

\[
\mathcal{L}_{\text{ArcFace}} = \frac{1}{N} \sum_{n=1}^{N} \log \left( \sum_{k \neq t_n} \exp(s \cos(\theta_{t_n} + m)) \right) \tag{7}
\]

where \(m \in \mathbb{R}\) is margin penalty and \(\theta_{t_n}\) is given by

\[
\theta_{t_n} = \arccos \left( \frac{W_{t_n}^T z_n'}{\|W_{t_n}\| \|z_n'\|} \right), \tag{8}
\]

where \(W_{t_n}'\) and \(z_n'\) are normalized respectively so that its \(L^2\)-norm is 1. Equation (8) means that \(\theta_{t_n}\) is the radian angle between the weight vector \(W_{t_n}'\) of the last FC layer and the feature vector \(z_n'\). ArcFace trains the model by adding an angular margin \(m\) to \(\theta_{t_n}\). At test time, no margin is added. The effect of adding the margin leads to an improvement in the separability between classes, which will be explained in detail in Section III-D.

III. Analysis of Class-Sensitive Separation

Let class \(\kappa\) be an important class, such as a stop sign, and let other classes, such as a no parking sign, be less important. When the misclassification of samples belonging to class \(\kappa\) is significantly harmful compared to those in the other classes, we need to consider improving recall of class \(\kappa\) without reducing overall accuracy. As we already discussed, threshold tuning does not help to attain this goal. As we discuss in this section later, to achieve this goal, we need to find feature representation such that feature vectors of the important class are well separated from feature vectors in the other classes. In this section, we investigate the property of existing loss functions in the feature space and how the separation in the feature space is induced when we penalize the important class using the existing loss functions.

A. Setting

To visualize the properties of existing loss functions in detail, we consider a three-class classification problem and train CNN where the dimension of the feature vector is set to two. Fig. 1 represents the contour plots of loss values in the two-dimension feature space. Each loss function corresponding to these figures is cross-entropy loss (top left), WCE (bottom left), ArcFace without margin (top right), and ArcFace with margin (bottom right). We give a detailed explanation of Fig. 1 when using the cross-entropy loss as an example (Fig. 1, top left). Since it is a three-class classification, there are three corresponding weight vectors in the last FC layer, \(W_1, W_2,\) and \(W_3\). Fig. 1 (top left) represents a contour plot of the cross-entropy loss in the two-dimensional feature space when the ground truth label is 1. In Fig. 1, the weight vector \(W_1\) corresponding to class 1, which we call the ground truth weight vector, is shown in the red arrow. The weight vectors \(W_2\) and \(W_3\) corresponding to class 2 and 3 are shown in black arrows. The blue area in the direction of \(W_1\) indicates that the loss values are low (since the ground truth label is 1), and the yellow area in the opposite direction indicates high loss values.

The last FC layer evaluates the inner product of the weight

\[1\text{This is consistent with the CAMRI loss defined later.}\]
vector $W_1$ and the feature vector $z$ as
\[ W_1^Tz = \|W_1\|\|z\| \cos(\theta_1), \tag{9} \]
where $\theta_1$ is the angle between $W_1$ and $z$. With this representation, the cross-entropy loss is given by
\[ L_{ce} = -\frac{1}{N} \sum_{n=1}^{N} \log \left( \frac{\exp \left( \|W_{t_n}\|\|z_n\| \cos(\theta_{t_n}) \right)}{\sum_{k=1}^{K} \exp \left( \|W_k\|\|z_n\| \cos(\theta_k) \right)} \right). \tag{10} \]

Equation (10) can be minimized by minimizing $\theta_{t_n}$, which is realized by decreasing the angle between the features and the ground truth weight vector.

Looking at the contour plot of cross-entropy loss (Fig. 1, top left) again, the region where the loss has low values spreads in the direction of $W_1$. If we could sharpen the angle of the valley-like landscape represented by blue in the contour plot, we can expect that the separation between classes in the feature space is increased, yielding a higher recall.

**B. Comparison between with and without weighting penalty for cross-entropy loss**

Next, we compare cross-entropy loss (Fig. 1, top left) and WCE (Fig. 1, bottom left), which is a cross-entropy loss penalizing the important class. Because the contour plots of the loss value obtained using CRWWCE and Wasserstein loss have similar shapes to WCE, we omit discussion about them here. Even with penalization to the important class, we can see that the angle discussed earlier is not sharpened. Instead, the counter is shifted toward the direction of $W_1$. Since WCE simply increases the loss values for the important class by multiplying a constant, it does not necessarily reduce the angle between features of the important class and the corresponding weight vector. Therefore, the cost-sensitive learning approach, including WCE, CRWWCE, and Wasserstein loss, would not work well for our purpose.

**C. Comparison of cross-entropy loss and L2-constrained softmax loss**

We now turn our attention to Fig. 1 (top right). L2-constrained softmax loss function [15] explicitly treats the angle between features and weights as a loss. In the training process, $W_i$ and $z$ are normalized so that $\|W_i\| = 1$ and $\|z\| = 1$. With this, (10) can be transformed to
\[ L_{L2} = -\frac{1}{N} \sum_{n=1}^{N} \log \left( \frac{\exp \left( s \cos(\theta_{t_n}) \right)}{\sum_{k=1}^{K} \exp \left( s \cos(\theta_k) \right)} \right), \tag{11} \]
where $s$ is the inverse of the temperature parameter. Comparing Fig.1 (top right) and Fig.1 (top left), we can see that the shape of the contour is different, while the angular sharpness of the valley-like landscape is the same. Thus, this does not improve the separation between classes, either.

**D. Comparison of angular-based loss with and without margin**

ArcFace introduces additive constant penalty $m$ to $\theta_{t_n}$ of L2-constrained softmax loss function, which is called additive angular margin. ArcFace is defined by
\[ L_{ArcFace} = -\frac{1}{N} \sum_{n=1}^{N} \log \frac{\exp \left( s \cos(\theta_{t_n} + m) \right)}{\sum_{k\neq t_n} \exp \left( s \cos(\theta_k) \right)}. \tag{12} \]

Comparing the case without (Fig. 1, top right) and with (Fig. 1, bottom right) margin penalty, we can see that the angular sharpness of the valley-like landscape is sharpened. Since angular margin corresponds to geodesic distance margin in a hypersphere [13], ArcFace would improve the separation by making the feature distribution on the hypersphere compact, called intra-class compactness. Therefore, fine-tuning of the margin penalty to all classes equally is expected to improve the generalization ability of the classifier.

**E. Proposal of CAMRI Loss**

Considering the discussion in the last section, we expect that recall improvement of a specific important class without sacrificing overall accuracy is attained by concentrating feature vectors along with the ground truth weight vector and separating feature vectors from non-ground truth weight vectors. For this purpose, we propose Class-sensitive additive Angular MaRgIn Loss (CAMRI Loss), which adds a margin only to the important class. Let $\kappa$ be an important class. The margin vector is defined using one-hot vector as $m = \mu[0, \ldots, 1, \ldots, 0]^T$, where only the $k$th element is $\mu \geq 0$ and the other elements are 0. Using this, CAMRI loss is defined by
\[ L_{CAMRI} = \frac{1}{N} \sum_{n=1}^{N} \log \frac{\exp \left( s \cos(\theta_{t_n} + m_{t_n}) \right)}{\sum_{k\neq t_n} \exp \left( s \cos(\theta_k) \right)}, \tag{13} \]
where $\theta_{t_n}$ is defined by (8) and $s$ is the inverse of the temperature parameter. For each training sample, if the label $t_n$ is $\kappa$, the margin $\mu$ is added, otherwise no margin is added. We note that the margin is applied only in the training process.

Since CAMRI loss adds the angular margin only to the important class, the intra-class compactness of the important class is expected to increase compared to the other classes.

**IV. Experiments**

**A. Experiment Settings**

1) Datasets: We used three datasets, CIFAR-10 [20], GTSRB [17], and AwA2 [18], for the multi-class classification problem. CIFAR-10 is a 10 classes dataset with a balanced number of data per class and has 50,000 training images and 10,000 test images of size $32 \times 32 \times 3$. GTSRB is a color image dataset of German road signs. It has 43 classes with an imbalanced number of data per class and contains
We resized the image size to 64 × 64 × 3 because the image sizes vary from 15 × 15 × 3 to 250 × 250 × 3. AwA2 has 50 classes with an imbalanced number of data per class and contains 37,322 animal images. We resized the image size to 64 × 64 × 3 and divided the dataset into 26,157 training images and 11,165 test images. In all datasets, we normalized all pixels so that pixel values are within the range of [0, 1].

2) CNN Setups: We used Tensorflow [21] to implement loss functions and a deep CNN model. The CNN model consists of a feature extractor and a linear classifier. The feature extractor contains convolution layers, batch normalization, max-pooling, dropout layers, and a global average pooling layer. The linear classifier contains two FC layers. ReLU function activates the FC and convolution layers other than the last FC layer. We set the batch size to 64 and the number of epochs to 300. We used Adam [22] with the 0.001 learning rate for optimization.

3) Parameter Settings: The ranges of weight parameters and margin parameters for each loss function were set in the following manners by preliminary experiments so that the recall of the important class becomes high. In the following, let \( \kappa \) be the index of an important class.

**WCE.** The weight vector \( \mathbf{w} \) of WCE is a \( K \) dimensional vector. Its \( \kappa \)th element was set to 4, 8, 12, \cdots , 40 and the other elements in \( \mathbf{w} \) were set to 1.

**CRWWCE.** The weight vector \( \mathbf{w}^{\text{fn}} \) of WCE is a \( K \) dimensional vector, and matrix \( \mathbf{W}^{\text{fp}} \), a \( K \times K \) matrix, of CRWWCE were set as follows. \( \kappa \)th element in \( \mathbf{w}^{\text{fn}} \) was set to 1, 4, 8, 12, \cdots , 40, and the other elements were set to 1. Each element in \( \mathbf{W}^{\text{fp}} \) was set as follows: (1) all elements in the \( \kappa \)th row and \( \kappa \)th column were varied as 1.0, 1.2, \cdots , 4.0, (2) all diagonal elements were set to 0, and (3) the other elements were set to 1.

**Wasserstein loss.** The distance matrix \( \mathbf{C} \) of Wasserstein loss is a \( K \times K \) matrix. Each element in \( \mathbf{C} \) was set as follows: (1) all elements in the \( \kappa \)th row and \( \kappa \)th column were varied as 1.0, 1.2, \cdots , 4.0, (2) all diagonal elements were set to 0, and (3) the other elements were set to 1.

We tested our model with three classes with the worst, second-worst, and median recall among all classes when the model had been trained with cross-entropy loss. With this criterion, \{cat, dog, airplane\}, \{limit-80-end, no-parking-end, limit-80\}, and \{mouse, beaver, moose\} were chosen as the important classes respectively for CIFAR-10, GTSRB, and AwA2. We made ten training trials with all combinations of the parameters described in the previous subsection. With the following criteria, the results are summarized in Table I.

1) Recall and accuracy with the regular cross-entropy loss are used as the baseline.

2) Among penalty parameter settings specified in the previous subsection, the results that maintain equal or better accuracy compared to the baseline are selected. The results of methods that do not achieve equal accuracy in any parameter setting are not shown in the table.

### TABLE I

| Method   | CIFAR-10 | GTSRB | AwA2 |
|----------|----------|-------|-------|
|          | cat      | dog   | airplane | limit-80-end (worst1) | no-passing-end (worst2) | limit-80 (worst1) | mouse | beaver | moose |
| WCE      | 0.802 ± 0.034 | 0.879 ± 0.005 | 0.879 ± 0.005 | 0.879 ± 0.005 | 0.879 ± 0.005 | 0.879 ± 0.005 | 0.651 ± 0.013 | 0.651 ± 0.013 | 0.651 ± 0.013 |
| CRWWCE   | -        | -     | -       | -       | -       | -       | -       | -       | -       |
| CAMRI (proposal) | 0.792 ± 0.027 | 0.850 ± 0.019 | 0.917 ± 0.018 | 0.835 ± 0.033 | 0.960 ± 0.035 | 0.995 ± 0.002 | 0.129 ± 0.046 | 0.238 ± 0.053 | 0.645 ± 0.053 |
| ArcFace  | 0.765 ± 0.040 | 0.842 ± 0.031 | 0.910 ± 0.028 | 0.808 ± 0.038 | 0.941 ± 0.070 | 0.993 ± 0.002 | 0.101 ± 0.061 | 0.187 ± 0.052 | 0.612 ± 0.046 |
| Wasserstein | -        | -     | -       | -       | -       | -       | -       | -       | -       |

The distance matrix \( \mathbf{C} \) of Wasserstein loss is a \( K \times K \) matrix. Each element in \( \mathbf{C} \) was set as follows: (1) all elements in the \( \kappa \)th row and \( \kappa \)th column were varied as 1.0, 1.2, \cdots , 4.0, (2) all diagonal elements were set to 0, and (3) the other elements were set to 1.
3) For each method, the results obtained with the parameter setting that achieves the highest recall are shown.

4) The results of the method which achieved the highest recall are shown in bold letters.

B. Performance Evaluation

The results are summarized in Table I. As we discussed already, improving recall with sacrificing accuracy is extremely easy. For this reason, cells in Table I are left blank when the accuracy is less than the baseline (cross-entropy loss), even if the corresponding recall is improved.

From Table I, we can see that CAMRI loss, achieved the improvement of the recall of the important classes in all settings. Also, ArcFace improves the recall of the important classes in all settings except for mouse class of AwA2. Comparing CAMRI loss and ArcFace, CAMRI loss achieves higher recall than ArcFace, which suggests that adding angular margin only to the important class induces better results.

Wasserstein, CRWWCE, and WCE improve the recall by sacrificing the accuracy in many cases. This is because these methods do not sharpen the angle between the distribution of feature vectors and ground-truth weight vectors as described in Section III. Therefore, these methods need to decrease the accuracy to improve the recall.

C. Effects of Margin on Separability

From Table I, we can see that adding the angular margin only to the important class (i.e., CAMRI loss) attains better results than adding the angular margin to all classes equally (i.e., ArcFace). We investigate the reason for this result by visualizing the feature space and observing the effect of class-sensitive angular margin on the separability of features.

For the visualization purpose, we trained a CNN with two-dimension feature space using MNIST [23]. Fig. 2 shows the ground truth weight vectors and the corresponding distribution of normalized feature vectors in the feature space when the loss function is without angular margin (L2-constrained softmax loss), with the equal angular margin (ArcFace), and with the class-sensitive angular margin (CAMRI loss). Here, we set “3” as the important class (shown in red color).

Comparing the case without margin (Fig. 2, left) and with the equal margin for all classes (Fig. 2, right), we can see that there is no improvement in the separability of class “3” (represented by red color). On the other hand, when the margin is added only to “3” (Fig. 2, bottom), the angle between the weight vector for “3” and the adjacent weight vectors are well separated. The features are distributed in the direction of the weight vector for “3”, and the separability of class “3” from other classes is relatively increased. This increase in the separability of the important class is caused by adding the angular margin penalty only to the important class, yielding the improvement of the recall.

D. Reducing Intra-class Angular Variance of Important Class by Class-sensitive Angular Margin

We observed that class-sensitive angular penalization enhances the intra-class compactness of feature vectors in the feature space. For measuring the intra-class compactness, we measured the standard deviations of the angle between the ground truth weight vectors and corresponding feature vectors.

Table II shows the standard deviations of the angles when cat, dog, and airplane are set as the important class (average of ten trials). For each important class, the minimum and the median excluding the important class are also shown, and the smallest value is shown in bold letters. The margin parameter values of CAMRI loss and ArcFace follows the setting of

| class               | CAMRI | ArcFace | WCE (w = 4) | ArcFace (m = 0) | Cross-entropy |
|---------------------|-------|---------|-------------|-----------------|--------------|
| cat (important)     | 0.0695| 0.0384  | 0.0603      | 0.0928          | 0.0842       |
| min excl. cat       | 0.0948| 0.0168  | 0.0540      | 0.0937          | 0.0570       |
| median excl. cat    | 0.1143| 0.0403  | 0.0766      | 1.009           | 0.0806       |
| dog (important)     | 0.0717| 0.0256  | 0.0667      | 1.009           | 0.0815       |
| min excl. dog       | 0.0919| 0.0158  | 0.0602      | 0.0928          | 0.0570       |
| median excl. dog    | 0.1116| 0.0328  | 0.0835      | 1.009           | 0.0806       |
| airplane (important)| 0.0367| 0.0512  | 0.0727      | 0.0512          | 0.0806       |
| min excl. airplane  | 0.0439| 0.0441  | 0.0627      | 0.0441          | 0.0570       |
| median excl. airplane| 0.0558| 0.0555  | 0.0714      | 0.0555          | 0.0815       |

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Fig. 2. Normalized feature vectors of MNIST $z'$ (represented as dots) and weight vectors (represented as solid lines) in the two-dimensional feature space. The vertical and horizontal axes represent the elements $z'_1, z'_2$ of $z'$, respectively. The left shows L2-constrained softmax loss, the right shows ArcFace, and the bottom shows CAMRI loss. In the results, class “3” (shown in red) was set as the important class.
Table I. WCE is measured with a fixed value of $w = 4$. The value of $s$ for Arcface ($m = 0$) is the same as that of CAMRI loss.

Comparing the standard deviation of the important class and the other classes, we can see that CAMRI loss makes the standard deviation of the important class 0.783 times lower than the minimum excluding of the important class on average. In contrast, the other methods do not necessarily make the standard deviation of the important class lowest, which means that the other methods do not improve the intra-class compactness even with penalization.

We remark that our method does not necessarily attain the lowest standard deviation for the important class among other methods. The standard deviation of the important class is made lower than other classes relatively, which is considered to be sufficient to improve the recall of the important class.

E. Effects of Penalty Changes on Recall and Accuracy

For further investigation of the relationship between the penalty given to the important class and recall/accuracy, we evaluated the changes of recall and accuracy with respect to penalty parameters of CAMRI loss and WCE.

Cat, dog, and airplane classes of CIFAR-10 are trained as the important class with changing the penalty parameters. Fig. 3 shows how the recall and the accuracy change when the angular margin $m_κ$ of CAMRI loss and the weight penalty $w_κ$ of WCE change, where $κ$ is the index of the important class. As $w_κ$ increases, WCE improves the recall but sacrifices the accuracy. In contrast, as $m_κ$ increases, CAMRI loss improves the recall, and the accuracy is almost maintained.

The results show that WCE improves the recall by controlling the trade-off of recall and accuracy, while CAMRI improves the recall by acquiring a feature representation having better intra-class compactness without sacrificing the accuracy.

F. Impacts of Improving Recall of the Important Class on the Other Classes

When CAMRI loss improves recall of an important class while maintaining accuracy, it is thought that recall of other classes may be decreased. Therefore, we observed the difference of confusion matrixes with CAMRI loss and WCE.

Fig. 4 (a) shows the confusion matrix when CIFAR-10 is trained with cross-entropy loss. Fig. 4 (b), (c), and (d) show the difference of the confusion matrix when CIFAR-10 is trained with cross-entropy loss and CAMRI loss, where cat, dog, and airplane are set as the important classes, respectively. Both are the average of ten training trials.

When the recall of the important class (say, class A) improves, some misclassified samples as the important class are correctly classified. At the same time, the recall of some other classes (say, class B) decreases. Between class A and class B, there appears to be a trend of decreasing the number of class A samples misclassified as class B. Looking at Fig. 4 (b), the recall of cat (important class) improves, while the recall of bird and dog decreases. At the same time, the number of samples of cat misclassified as bird and cat misclassified as dog decreases, and the recall of bird and dog decreases. Fig. 4 (c) also shows the same tendency. The number of samples of dog (important class) misclassified as bird decreases, and the recall of dog improves while the recall of bird decreases. Similarly, Fig. 4 (d) shows the number of samples of airplane (important class) misclassified as bird decreases, and the recall of airplane improves while the recall of bird decreases.

These results show, firstly, that samples that were misclassified as the important class are correctly classified, which contributes to improving recall of the important class. Secondly, the recall of classes to which samples that contribute to the recall improvement belong tends to decrease. These two factors suggest that the separation between the important class and another class is impacted when CAMRI loss improves the recall of the important class.
V. CONCLUSION

We proposed CAMRI loss, which improves the recall of an important class without sacrificing overall accuracy. First, by analyzing the contour plots of existing loss functions, we found that it is necessary to reduce the angle between the feature vectors and the ground truth weight vectors for improving the separation of different classes in the feature space (intra-class compactness). In order to achieve this, we introduced class-sensitive additive angular margin. Experimental results showed that CAMRI loss improved the recall of the important class without sacrificing the accuracy compared to other methods. We experimentally confirmed that CAMRI loss makes the intra-class compactness of the important class relatively smaller than the other classes.

In this study, the number of the important class is assumed to be one, but the number of the important class is often multiple in the real world. Whether the recall of multiple important classes can be improved with our proposal remains our future work.

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Fig. 4. The left (a) is the baseline confusion matrix of CIFAR-10 obtained by trained with cross-entropy loss. (b), (c), and (d) are the differences between the confusion matrix with cross-entropy loss (i.e., (a)) and that with the CAMRI loss, where cat, dog, and airplane be set to the important class, respectively. Each result is the average of ten trials and corresponds to the result shown in Table I.