Near Horizon Geometry of Rotating Black Holes in Five Dimensions

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Abstract

We interpret the general rotating black holes in five dimensions as rotating black strings in six dimensions. In the near horizon limit the geometry is locally $AdS_3 \times S_3$, as in the nonrotating case. However, the global structure couples the $AdS_3$ and the $S_3$, giving angular velocity to the $S_3$. The asymptotic geometry is exploited to count the microstates and recover the precise value of the Bekenstein-Hawking entropy, with rotation taken properly into account. We discuss the perturbation spectrum of the rotating black hole, and its relation to the underlying conformal field theory.

1 Introduction

In the last few years there has been substantial progress in the quantum description of black holes in string theory. (For reviews of this development see e.g. [1, 2].) However, string theory has not yet offered decisive progress on the notorious questions concerning the information flow in the black hole spacetime, and the physical nature of its singularity. The reason is that, in critical string theory, the internal structure of black holes is represented by world-volume field theories that are decoupled from gravity, and in this description the role of spacetime geometry is unclear. However, it has recently been proposed that the world-volume description is in fact equivalent to string theory in the curved background of the black hole [3, 4, 5]. The defining examples of this correspondance employ geometries that are products of spaces with constant curvature, with one factor being the Anti-deSitter spacetime (AdS). In these cases the world-volume theory is a conformal field theory (CFT), and the proposal
can be justified in explicit computations. Even so, it is not yet clear whether these novel dualities will shed light on the black hole problems alluded to above, but this possibility is an important motivation for their further development.

A significant application of the correspondence between conformal field theory and the geometry of AdS spaces is the counting of microstates \[6, 7\] of the three dimensional black hole of Banados, Teitelboim, and Zanelli (BTZ) \[8, 9\]. The new microscopic derivation of the black hole entropy follows from little more than the fact that the BTZ geometry is asymptotically \(AdS_3\), lending a surprising robustness to the result. This feature of the computation appears to be an important advance over the earlier work of Carlip \[10\]. Additionally, it is notable that the near horizon geometry of the D1-D5 bound state is of the form BTZ \(\times S^3\) \[11\], allowing application of the new method to this case \[6\]. This has lead to a close connection with previous work on D-brane black holes \[12, 13\]. The near horizon geometry of three orthogonally intersecting M5-branes in \(M\)-theory is similarly BTZ \(\times S^2\), and this gives a simple relation to previous work on four dimensional black holes \[14\].

The purpose of the present paper is to investigate this construction in the context of rotating brane configurations. Our starting point is the most general class of black holes in five dimensions \[15\]. We find the exact form of the corresponding six-dimensional black strings, and then take the decoupling limit. The resulting near horizon geometry describes near extremal black holes and is again of the factorized form BTZ \(\times S^3\). Interestingly, this is explicit only in a coordinate system that is rotating at the same rate as the black hole. The parameters of the geometry gives the central charge of the effective conformal field theory, as well as the levels of the black hole states. A microscopic entropy can be inferred from these, and the result agrees precisely with the area law. The result takes into account the precise dependence of the two angular momenta. Black hole entropy is discussed in sec. 2.

A central result of the CFT/AdS correspondence is the relation between the spectrum of perturbations in the \(AdS_3 \times S_3\) geometry, and the elementary excitations in the worldvolume theory of the \(D1 - D5\) system \[12, 13, 16\]. The rotating black hole background is also locally \(AdS_3 \times S_3\) so the spectrum is identical in the two cases, but in the black hole background it is natural to discuss the perturbations in terms of greybody factors. In this way the interpretation of perturbations depends on the boundary conditions, and thus on global issues. Specifically, when rotation is included, the coordinates on the sphere \(S_3\) depend on the \(AdS_3\) coordinates; and this “twisting” of the sphere affects the greybody factors in a universal manner. The perturbations of the black hole are discussed in sec. 3, and the relation between the \(D1 - D5\) system and the black hole is the topic of the concluding remarks in sec. 4.

The BTZ background is locally \(AdS_3\) and so it exhibits an \(SL(2, \mathbb{R})_L \times SL(2, \mathbb{R})_R\) symmetry. A preferred set of coordinates can be defined that makes the factorized form of the symmetry manifest. These coordinates provide a spacetime interpretation of the effective string world-sheet. In this concrete realization the temporal and spatial world-sheet coordinates are naturally associated with the outer and inner horizons, respectively. This connection, discussed in sec. 3, may have universal significance.

There is a generalization of the present work to the case of four dimensional rotating black holes and their representations as rotating strings in five dimensions.
This will be discussed in a separate note [17].

The paper is organized in two parts. In the first part, sec. 2, we focus on the black hole entropy: successive subsections give the metric of the general rotating black string in six dimensions, find the decoupling limit and calculate the entropy. In subsec. 2.4 we give the translation between macroscopic black hole parameters and the microscopic quantum numbers. In the second part, sec. 3, we discuss black hole perturbations. The emphasis is on the effects of rotation, but we also make general remarks. We exhibit the local symmetries and recover the near-horizon wave equation for a minimally coupled scalar field in the rotating background. Finally, in sec. 4, we conclude with a discussion of the relation between the conformal field theory induced by the near horizon geometry of the $D1-D5$ system and that of the black hole.

2 Black Hole Entropy

In this section we present the general form of the rotating black string in six dimensions. We then take the decoupling limit and calculate the microscopic entropy of rotating black holes in five dimensions, following Strominger [6].

2.1 The Classical Background

Consider the most general rotating black holes in $N = 4$ or $N = 8$ supergravity in five dimensions [15]. The generating solution for these configurations is given in terms its mass $M$, 2 angular momenta $J_{L,R}$, and 3 independent $U(1)$ charges $Q_i$. It is convenient to represent these physical parameters in the parametric form:

$$M = \frac{m}{2} \sum_{i=0}^{2} \cosh 2\delta_i , \quad (1)$$

$$Q_i = m \sinh 2\delta_i \quad ; \quad i = 0, 1, 2 , \quad (2)$$

$$J_{L,R} = m (l_1 \mp l_2) \left( \prod_{i=0}^{2} \cosh \delta_i \pm \prod_{i=0}^{2} \sinh \delta_i \right) . \quad (3)$$

We work in Planck units where the gravitational coupling constant in five dimensions is $G_5 = \frac{\pi}{4}$. The relation to conventional string units is given below, in sec. 2.4.

For the present purpose it is essential that the five dimensional black holes can be interpreted as rotating black strings in six dimensions. The six-dimensional geometry can be determined from the five dimensional form of the metric and the matter fields, given in [15]. In Einstein frame the result is $^1$:

$$ds_6^2 = \frac{1}{\sqrt{H_1 H_2}} \left[ -(1 - \frac{2mf_D}{r^2})d\tilde{t}^2 + d\tilde{y}^2 + H_1 H_2 f_D^{-1} \frac{r^4}{(r^2 + l_1^2)(r^2 + l_2^2)} - 2mr^2 dr^2 \right]$$

$^1$This corrects the result reported in [18]. Also, there is a typo in the expression for the gauge field $A^{(1)}_\phi$ given in [15]. The correct formula, used to derive the result given here, can be obtained from the $A^{(1)}_\psi$ given there, using the symmetry given below in eq. 6.
\[- \frac{4m f_D}{r^2} \cosh \delta_1 \cosh \delta_2 (l_2 \cos^2 \theta d\psi + l_1 \sin^2 \theta d\phi) d\tilde{t} \]
\[- \frac{4m f_D}{r^2} \sinh \delta_1 \sinh \delta_2 (l_1 \cos^2 \theta d\psi + l_2 \sin^2 \theta d\phi) d\tilde{y} \]
\[+ \left( (1 + \frac{l_2^2}{r^2}) H_1 H_2 r^2 + (l_1^2 - l_2^2) \cos^2 \theta \left( \frac{2m f_D}{r^2} \right)^2 \sinh^2 \delta_1 \sinh^2 \delta_2 \right) \cos^2 \theta d\psi^2 \]
\[+ \left( (1 + \frac{l_1^2}{r^2}) H_1 H_2 r^2 + (l_2^2 - l_1^2) \sin^2 \theta \left( \frac{2m f_D}{r^2} \right)^2 \sinh^2 \delta_1 \sinh^2 \delta_2 \right) \sin^2 \theta d\phi^2 \]
\[+ \frac{2m f_D}{r^2} (l_2 \cos^2 \theta d\psi + l_1 \sin^2 \theta d\phi)^2 + H_1 H_2 r^2 f_D^{-1} d\theta^2 \] , \quad (4)

where:
\[H_i = 1 + \frac{2m f_D \sinh^2 \delta_i}{r^2} ; \quad i = 1, 2 \] , \quad (5)
\[f_D^{-1} = 1 + \frac{l_1^2 \cos^2 \theta}{r^2} + \frac{l_2^2 \sin^2 \theta}{r^2} \] , \quad (6)

and:
\[d\tilde{t} = \cosh \delta_0 dt - \sinh \delta_0 dy \] , \quad (7)
\[d\tilde{y} = \cosh \delta_0 dy - \sinh \delta_0 dt \] . \quad (8)

Although the six dimensional rotating black string metric is somewhat involved it is much simpler than the five dimensional black hole that it was derived from. Note that the angular parts are constrained by the symmetry under the simultaneous interchanges:
\[l_1 \leftrightarrow l_2 \quad , \quad \phi \leftrightarrow \psi \quad , \quad \theta \leftrightarrow \frac{\pi}{2} - \theta \] . \quad (9)

This symmetry expresses an automorphism of the algebra of rotations \( SO(4) \approx SU(2)_R \times SU(2)_L \) that corresponds geometrically to a reflection.

The black string metric eq. 1 depends on the parameter \( \delta_0 \) through the boosted differentials \( d\tilde{t} \) and \( d\tilde{y} \) only, thus signifying a momentum along the string. This property is by no means manifest in the five dimensional form of the metric given in [15]; and so it serves as an important check on the algebra that we recover the boost invariance in our solution.

The two charges \( Q_1 \) and \( Q_2 \) appear symmetrically in the black string metric, due to duality of the effective six-dimensional theory. They can be interpreted as the charges of a fundamental string (FS) wrapped around the \( y \)-direction and a NS5-brane wrapped around both the \( y \)-direction and the additional four compact dimensions. The matter fields that are needed for this interpretation were given in [15]. Since duality transformations act on the matter fields, but not on the Einstein metric, we can equally well interpret the metric as the field created by any brane configuration that is dual to the pair NS5-FS. Specifically the \( U(1) \) charges can be interpreted as D1- and D5-brane charges. The advantage of this interpretation is that, unlike the NS5-brane, the D-branes are accessible to a weakly coupled microscopic description. For this reason the D-brane interpretation of the solution is in fact mandatory for the decoupling limit taken below.
2.2 The Near Horizon Geometry

In some circumstances the internal structure of a black hole is described accurately by a field theory that couples weakly to the surrounding space. A precise definition of the decoupling limit is given by taking \[ 3 \]:

\[
l_s \to 0 \quad ; \quad r, m, l_{1,2} \to 0 \quad ; \quad \delta_{1,2} \to \infty ,
\]

where the string length \( l_s = \sqrt{\alpha'} \), so that:

\[
rl_s^{-2} \quad ; \quad ml_s^{-4} \quad ; \quad l_{1,2}l_s^{-2} \quad ; \quad Q_{1,2}l_s^{-2} = ml_s^{-2} \sinh 2\delta_{1,2} ,
\]

remain fixed. The decoupling limit is a near horizon approximation, because \( r \to 0 \). Moreover, the “dilute gas” conditions \( \delta_{1,2} \to \infty \) imply that the black hole is necessarily near extremal \[ 10 \].

The metric simplifies dramatically in the limit specified by eq. \[ 10 \]. Note, however, that the function \( f_D \) does not simplify in this limit, and other features due to rotation are similarly retained.

The near horizon geometry is:

\[
\begin{align*}
ds_6^2 &= \frac{r^2}{f_D\lambda^2}[-(1 - 2m f_D^2)d\bar{t}^2 + d\bar{y}^2] + \frac{\lambda^2 r^2}{(r^2 + l_1^2)(r^2 + l_2^2)} dr^2 - \\
&\quad - 2(l_2 \cos^2 \theta d\psi + l_1 \sin^2 \theta d\phi)d\bar{t} - 2(l_1 \cos^2 \theta d\psi + l_2 \sin^2 \theta d\phi)d\bar{y} + \\
&\quad + \lambda^2(\theta^2 + \sin^2 \theta d\phi^2 + \cos^2 \theta d\psi^2) ,
\end{align*}
\]

where we defined the characteristic length scale \( \lambda \equiv (Q_1 Q_2)^{1/4} \). Introducing the shift in the angular variables:

\[
\begin{align*}
d\bar{\psi} &= d\psi - \lambda^{-2}(l_2 d\bar{t} + l_1 d\bar{y}) , \\
d\bar{\phi} &= d\phi - \lambda^{-2}(l_1 d\bar{t} + l_2 d\bar{y}) ,
\end{align*}
\]

the metric becomes:

\[
\begin{align*}
ds_6^2 &= -\frac{(r^2 + l_1^2)(r^2 + l_2^2) - 2mr^2}{\lambda^2 r^2} dr^2 + \frac{r^2}{\lambda^2} (d\bar{y} - \frac{l_1 l_2}{r^2} d\bar{t})^2 + \\
&\quad + \frac{\lambda^2 r^2}{(r^2 + l_1^2)(r^2 + l_2^2) - 2mr^2} dr^2 + \lambda^2[\theta^2 + \sin^2 \theta d\phi^2 + \cos^2 \theta d\psi^2] .
\end{align*}
\]

In this form it is apparent that the geometry is a direct product of two three dimensional spaces. The angular space is a sphere \( S_3 \) with radius \( \lambda \), and the geometry with coordinates \( (\bar{t}, \bar{y}, r) \) is a BTZ black hole in an effective 2 + 1 dimensional theory with cosmological constant \( \Lambda = -\lambda^2 \).

Indeed, the metric can be written in the standard BTZ form \[ 8 \] \[ 9 \]:

\[
\begin{align*}
ds_6^2 &= -N^2 dt_{\text{BTZ}}^2 + N^{-2} dr_{\text{BTZ}}^2 + r_{\text{BTZ}}^2 (d\phi_{\text{BTZ}} - N_\phi dt_{\text{BTZ}})^2 + \lambda^2 d\bar{\Omega}_3^2 ,
\end{align*}
\]

\[
\begin{align*}
N^2 &= \frac{r_{\text{BTZ}}^2}{\lambda^2} - M_3 + \frac{16G_3 J_3}{r_{\text{BTZ}}^2} ,
\end{align*}
\]

\[
\begin{align*}
N_\phi &= \frac{4G_3 J_3}{r_{\text{BTZ}}^2} ,
\end{align*}
\]

5
where:

\[ t_{\text{BTZ}} \equiv \frac{t\lambda}{R_y}, \]  
(19)

\[ \phi_{\text{BTZ}} \equiv \frac{y}{R_y}, \]  
(20)

\[ r_{\text{BTZ}}^2 \equiv \frac{R_y^2}{\lambda^2} \left[ r^2 + (2m - l_1^2 - l_2^2) \sinh^2 \delta_0 + 2l_1l_2 \sinh^2 \delta_0 \cosh^2 \delta_0 \right], \]  
(21)

and the effective three dimensional mass \( M_3 \) and angular momentum \( J_3 \) are:

\[ M_3 = \frac{R_y^2}{\lambda^4} \left[ (2m - l_1^2 - l_2^2) \cosh 2\delta_0 + 2l_1l_2 \sinh 2\delta_0 \right], \]  
(22)

\[ 8G_3J_3 = \frac{R_y^2}{\lambda^3} \left[ (2m - l_1^2 - l_2^2) \sinh 2\delta_0 + 2l_1l_2 \cosh 2\delta_0 \right]. \]  
(23)

We denoted the radius of the compact dimension by \( R_y \).

The extremal limit of the six dimensional string is given by:

\[ m, l_{1,2} \to 0 ; \delta_0 \to \infty, \]  
(24)

with fixed \( Q_0 = m \sinh 2\delta_0 \). In this limit one of the angular momenta \( J_R \to 0 \), but the other \( J_L \) remains finite.

It has previously been found that, in the near horizon region, the extremal rotating black hole preserves 1/4 of the maximal supersymmetry, twice the amount that is preserved in the bulk [20]. Now this result follows from the much simpler analysis of supersymmetry in the context of BTZ black holes [21]. Additionally, it follows that the near-horizon region exhibits superconformal invariance [22], except for a global obstruction that can be removed by decompactification of the 6th dimension.

### 2.3 Counting States

In this section we count the microscopic states of the black hole, following [6].

The effective gravitational coupling in three dimensions \( G_3 \) can be related to the gravitational coupling in five dimensions \( G_5 \) by comparing two different dimensional reductions from six dimensions, as in [3, 14]. It is:

\[ \frac{1}{G_3} = \frac{1}{G_5} \frac{A_3}{2\pi R_y}, \]  
(25)

where \( A_3 = 2\pi^2\lambda^3 \) is the area of the \( S_3 \). This result is independent of the rotational parameters because the effective cosmological constant depends only on the charges of the branes.

The isometry group of the asymptotic \( AdS_3 \) induces a conformal field theory on the boundary at the conformal infinity of the BTZ black hole. Its central charge is given in terms of the cosmological constant [23]:

\[ c = \frac{3\lambda}{2G_3} = 6 \frac{Q_1 Q_2}{R_y} \frac{\pi}{4G_5}. \]  
(26)
Note that the central charge is also independent of angular momentum. This suggests that the rotating black holes can be interpreted as states in the same conformal field theory that describes the nonrotating black holes.

The relation between the symmetry generators of the induced conformal symmetry, and the effective mass and angular momentum are:

\[ M_3 = \frac{8G_3}{\lambda}(L_0 + \bar{L}_0) \]
\[ J_3 = L_0 - \bar{L}_0 \]

where the eigenvalues of the operators \( L_0 \) and \( \bar{L}_0 \) are the conformal dimensions \( h_L \) and \( h_R \), respectively. Then Cardy’s formula for the statistical entropy [24]:

\[ S = 2\pi \left( \sqrt{\frac{ch_L}{6}} + \sqrt{\frac{ch_R}{6}} \right) \]

(27)

(28)

(29)

(30)

(31)

The general formula for the macroscopic entropy of rotating black holes in five dimensions is [15]:

\[ S \equiv \frac{A_5}{4G_5} = \frac{\pi}{4G_5} 2\pi m \left[ \prod_{i=0}^{2} \cosh \delta_i + \prod_{i=0}^{2} \sinh \delta_i \right] \sqrt{2m - (l_1 - l_2)^2} + \left( \prod_{i=0}^{2} \cosh \delta_i - \prod_{i=0}^{2} \sinh \delta_i \right) \sqrt{2m - (l_1 + l_2)^2} \]

(32)

In the limit \( \delta_{1,2} \gg 1 \) this becomes:

\[ S = \frac{\pi}{4G_5} \sqrt{Q_1 Q_2} \left[ \sqrt{2m - (l_1 - l_2)^2} e^{\delta_0} + \sqrt{2m - (l_1 + l_2)^2} e^{-\delta_0} \right] \]

(33)

Thus the microscopic entropy eq. [31] precisely reproduces the macroscopic entropy eq. [32] in the decoupling limit eq. [10] where the microscopic calculation applies. The range of parameters that are considered here is as general as the previous D-brane results [25].

Cardy’s formula eq. [28] can be derived from unitarity and modular invariance of the boundary conformal field theory. In the present context unitarity cannot be taken for granted; superconformal WZW-models with the noncompact target space \( SL(2, \mathbb{R}) \) are in fact nonunitary. Thus the justification of the calculation ultimately rests on the existence of an underlying unitary framework, such as the one realized in the full string theory. Despite these caveats, we find it impressive that the method accurately reproduces an entropy with the complexity apparent in eq. [33].
2.4 Quantization Conditions

It is instructive to rewrite some of the formulae in microscopic units. Then:

\[ G_5 = \frac{\pi}{4} \frac{(\alpha')^3 g^2}{R_1 R_2 R_3 R_4 R_y}, \quad (34) \]

where the \( R_i \) are the radii of the compact dimensions and the type IIB string coupling \( g \) is normalized so that \( g \to 1/g \) under S-duality. Our convention hitherto was \( G_5 = \frac{\pi}{4} \), except where \( G_5 \) is written explicitly. The quantization conditions on the D-brane charges are [26],

\[ Q_1 = n_1 g \frac{\alpha'}{R_1 R_2 R_3 R_4}, \quad (35) \]
\[ Q_2 = n_2 g \alpha', \quad (36) \]

where \( n_{1,2} \) are the number of D1- and D5-branes, respectively. It immediately follows from eq. [26] that \( c = 6n_1 n_2 \), as expected [27].

The quantum numbers \( p, \epsilon, \) and \( j_{R,L} \) for momentum, energy, and angular momenta, respectively, are introduced through:

\[ Q_0 = m \sinh 2\delta_0 = \frac{p}{R_y} \frac{4G_5}{\pi}, \quad (37) \]
\[ E = m \cosh 2\delta_0 = \frac{\epsilon}{R_y} \frac{4G_5}{\pi}, \quad (38) \]
\[ J_{R,L} = j_{R,L} \frac{4G_5}{\pi}. \quad (39) \]

Then the conformal weights can be written as:

\[ h_{L,R} = \frac{\lambda M_3 \pm 8G_3 J_3}{16G_3} = \frac{\pi R_y}{16G_5} [2m - (l_1 \mp l_2)^2] e^{\pm 2\delta_0} = \frac{1}{2} (\epsilon \pm p) - \frac{1}{n_1 n_2} j_{L,R}^2. \quad (40) \]

In this form it is manifest that the effective levels agree precisely with the results previously derived using D-branes [28, 25].

The quantization conditions on the angular momenta derived from the periodicity conditions on the angles are \( j_{R,L} = \frac{1}{2}(j_\phi \pm j_\psi) \), where \( j_\phi \) and \( j_\psi \) are quantized as integers [28, 29]. It follows from:

\[ h_L - h_R = p - \frac{1}{n_1 n_2} j_\phi j_\psi, \quad (41) \]

that the natural spacing of the conformal weights is in multiples of \( 1/n_1 n_2 \). Periodicity around the \( y \)-direction would imply that the momentum quantum number \( p \) is integral, but in fact \( p \) is also quantized in multiples of \( 1/n_1 n_2 \). Although this fractionation is suggested by the classical geometry [30, 31, 32], it is most convincingly seen using D-branes [33]. In this way the presence of angular momentum makes a qualitatively important effect more apparent.
The effective level:

\[ N_{L,R} \equiv \frac{c}{6} h_{L,R} = n_1 n_2 \frac{\epsilon \pm \rho}{2} - j_{L,R}^2 , \tag{42} \]

is designed to take into account the fractionation. The \( N_{L,R} \) can be written in the duality invariant form:

\[ N_{L,R} = 2m^3 \left( \prod_i \cosh \delta_i \pm \prod_i \sinh \delta_i \right)^2 \left( \frac{\pi}{4G_5} \right)^2 - j_{L,R}^2 . \tag{43} \]

This generalization of the effective level may account for the black hole entropy eq. 32 arbitrarily far from extremality [34, 35, 29].

3 Black Hole Perturbations

The isometry group \( SO(2, 2) \simeq SL(2, \mathbb{R})_L \times SL(2, \mathbb{R})_R \) of \( AdS_3 \) can be exploited in several ways. The computation of the entropy relies on the fact that the BTZ black hole is asymptotically \( AdS_3 \), so that a conformal field theory is induced at the boundary at infinity. However, the BTZ geometry is in fact locally \( AdS_3 \). This has important consequences for the spectrum of black hole perturbations, and for the dynamics encoded in the greybody factors. In the present section we discuss these issues with emphasis on the effects of rotation.

3.1 The Local \( AdS_3 \times S_3 \)

We first make the local \( AdS_3 \) manifest. The near horizon metric eq. 12 defines a quadratic form that can be diagonalized and written as:

\[ ds_6^2 = -\lambda^{-2} \frac{r_+^2 - r_-^2}{r_+ - r_-} (r_+ d\tilde{t} - r_- d\tilde{y})^2 + \lambda^{-2} \frac{r_+^2 - r_-^2}{r_+^2 - r_-^2} (r_+ d\tilde{y} - r_- d\tilde{t})^2 + \frac{\lambda^2 r_+^2}{(r_+^2 - r_-^2)(r_+^2 - r_-^2)} dr^2 + \lambda^2 d\tilde{\Omega}_3^2 , \tag{44} \]

where the loci \( r_\pm \) of the outer and inner horizons are:

\[ r_\pm = \frac{1}{2} \left[ \sqrt{2m} - (l_1 - l_2)^2 \pm \sqrt{2m} - (l_1 + l_2)^2 \right] . \tag{45} \]

The eigenvectors of the metric are parametrized by the dimensionless differentials:

\[ d\tau = \lambda^{-2} (r_+ d\tilde{t} - r_- d\tilde{y}) \quad ; \quad d\sigma = \lambda^{-2} (r_+ d\tilde{y} - r_- d\tilde{t}) , \tag{46} \]

and the dimensionless radial variable \( \rho \) is introduced through:

\[ \frac{1}{2} \cosh 2\rho \equiv \frac{r_+^2 - \frac{1}{2}(r_+^2 + r_-^2)}{r_+^2 - r_-^2} \equiv x \quad ; \quad r \geq r_+ , \tag{47} \]
with the coordinate $x$ defined for later use. Then the metric becomes:

$$ds_6^2 = \lambda^2 [- \sinh^2 \rho \ d\tau^2 + \cosh^2 \rho \ d\sigma^2 + d\rho^2 + d\tilde{\Omega}_3^2].$$

(48)

In this form it is manifest that the geometry is locally $AdS_3 \times S_3$ outside the horizon. The result also holds inside the horizon, as can be shown using alternative definitions of the radial coordinate $\rho$ that apply in different patches. Since the local geometry is just $AdS_3 \times S_3$, the origin of nontrivial causal structure is of purely global nature [36].

The rotation similarly does not affect the local structure, but the shifts of the angular coordinates eqs. [13–14] tie the $S_3$ to the $AdS_3$. We explore their precise effect in more detail in the following.

### 3.2 Black Hole Perturbations

An important consequence of the local $AdS_3 \times S_3$ form of the metric is that the spectrum of black hole perturbations is organized into multiplets of the superconformal algebra. This allows a complete classification of all perturbations, as carried out in [16]. The spectrum of perturbations follows from local properties of $AdS_3$, and so it is identical for the entire class of black holes considered here.

In the present context the perturbations are naturally interpreted as test fields that interact with the black hole background. The wave function of the perturbations then gives the greybody factor, expressing the form factor of the Hawking radiation as function of particle quantum numbers, such as energy and spin, and of the black hole parameters $[13, 37, 38, 39, 40, 41]$. The greybody factors provide a semiclassical testing ground for dynamical properties. In the special case of minimally coupled scalars in the $S$-wave they agree precisely with calculations in string theory [45]. The microscopic processes responsible for other modes can be modelled in terms of an effective string theory with dynamics that reproduces the black hole greybody factors qualitatively [37, 38].

It is the local $AdS_3 \times S_3$ geometry of the near horizon that makes the effective string description possible. Moreover, this connection has made it feasible to complete the list of conformal dimensions that was previously known only in part [16].

The most general black hole depends on 6 parameters (given in eq. 3), but only 4 parameters remain in the decoupling limit specified in eq. 10. They can be chosen as $ml_s^{-4}, \delta_0, l_1 l_s^{-2}$, or $M, Q_0, J_{L,R}$, but for the present purpose it is more convenient to choose them as the potentials:

$$\beta^{L,R} = \frac{2\pi \lambda^2 e^{\mp \delta_0}}{\sqrt{2m - (l_1 \mp l_2)^2}} = \frac{2\pi \sqrt{n_1 n_2}}{\sqrt{\frac{1}{2} (\epsilon \pm p) n_1 n_2 - j_{L,R}^2/2}},$$

(49)

$$\beta_{H \Omega}^{L,R} = \frac{2\pi (l_1 \mp l_2)}{\sqrt{2m - (l_1 \mp l_2)^2}} = \frac{2\pi j_{L,R}}{\sqrt{\frac{1}{2} (\epsilon \pm p) n_1 n_2 - j_{L,R}^2}}. $$

(50)

$^2$In the case of minimally coupled scalars the wave function on $AdS_3 \times S_3$ can be matched directly on to the asymptotic Minkowski space, but in general the geometry interpolating between the near horizon region and the asymptotic Minkowski space results in further distortions [42]. See [13, 14] for very recent discussions in a related context.
The $\beta^{L,R}$ are conjugate to the left and right moving energy along the string, respectively; and the $\beta_H \Omega^{L,R}$ are conjugate to the two independent angular momenta $J_{L,R}$.

The rotating background breaks the rotational invariance so the wave function depends nontrivially on the azimuthal quantum numbers $m_{L,R}$. However, the rotational invariance is restored in the shifted coordinates eqs. 13–14:

$$\begin{align*}
\frac{1}{2}(\tilde{\psi} \pm \tilde{\phi}) &= \frac{1}{2}(\psi \pm \phi) - \frac{l_1 \pm l_2}{\lambda^2} e^{\mp \delta_i}(t \pm y).
\end{align*}$$

(51)

Simultaneous translations of the $\psi \pm \phi$ and $t \pm y$ that can be absorbed in translations of $\tilde{\psi} \pm \tilde{\phi}$ must leave the wave function invariant, except for an overall phase. However, the system may transform nontrivially under translations that leave $\frac{1}{2}(\tilde{\psi} \pm \tilde{\phi})$ fixed.

The wave function is written in general as:

$$\Phi \equiv \Phi_0 (r) \chi(\theta) e^{-i\omega R(t+y) - i\omega L(t-y) + im_L(\phi - \psi)} + im_R(\phi + \psi),$$

(52)

where $m_{R,L} \equiv \frac{1}{2}(m_\phi \pm m_\psi)$ and $\omega_{R,L} \equiv \frac{1}{2}(\omega \mp q)$. Note that the coefficient of the second term in eq. 51 is the ratio of eqs. 50 and 49. Translations in $\psi \pm \phi$ and $t \pm y$ that leave $\frac{1}{2}(\tilde{\psi} \pm \tilde{\phi})$ invariant are therefore conjugate to $\beta^{L,R} \omega_{L,R} - m_{L,R} \beta_H \Omega^{L,R}$. Thus the entire dependence of the radial wave function $\Phi_0$ on $m_{L,R}$ and $\beta_H \Omega^{L,R}$ can be taken into account by the shifts:

$$\beta^{L,R} \omega_{L,R} \rightarrow \beta^{L,R} \omega_{L,R} - m_{L,R} \beta_H \Omega^{L,R}.$$

(53)

This rule gives the exact wave functions in the rotating background, when the non-rotating ones are known. It is valid for all fields in the near horizon region, without regard to the details of their couplings.

### 3.3 The Scalar Wave Equation

Let us make this discussion explicit in the case of a minimally coupled scalar field. We use the original coordinates $t, y$ and the radial variable $x$, introduced in eq. 14. Then metric is:

$$\begin{align*}
\text{ds}_6^2 &= \lambda^{-2} \left[ -(x - \frac{1}{2}) (r_+ d\tilde{t} - r_- d\tilde{y})^2 + (x + \frac{1}{2}) (r_+ d\tilde{y} - r_- d\tilde{t})^2 \right] + \\
&\quad + \lambda^2 \left[ \frac{1}{4} dx^2 + d\theta^2 + \cos^2 \theta d\tilde{\psi}^2 + \sin^2 \theta d\tilde{\phi}^2 \right],
\end{align*}$$

(54)

where $(\tilde{t}, \tilde{y})$ and $(\tilde{\psi}, \tilde{\phi})$ are defined in eqs. 8 and eqs. 13–14, respectively. Inserting the ansatz for the wave function (eq. 52) into the Klein-Gordon equation:

$$\begin{align*}
\frac{1}{\sqrt{-g}} g^{\mu \nu} \partial_{\mu} (\sqrt{-g} g^{\mu \nu} \partial_{\nu} \Phi) &= \mu^2,
\end{align*}$$

(55)
we find:
\[
\left[ \frac{\partial}{\partial x} (4x^2 - 1) \frac{\partial}{\partial x} + \frac{1}{x - \frac{1}{2}} \left( \frac{\beta^R \omega_R + \beta^L \omega_L - m_R \beta_H \Omega^R - m_L \beta_H \Omega^L}{2\pi} \right)^2 - \frac{1}{x + \frac{1}{2}} \left( \frac{\beta^R \omega_R - \beta^L \omega_L - m_R \beta_H \Omega^R + m_L \beta_H \Omega^L}{2\pi} \right)^2 \right] \Phi_0 = (\Lambda + \lambda^2 \mu^2) \Phi_0 ,
\]
(56)

after somewhat lengthy transformations. The eigenvalues of the angular Laplacian:
\[
\hat{\Lambda} = -\frac{1}{\sin 2\theta} \frac{\partial}{\partial \theta} \sin 2\theta \frac{\partial}{\partial \theta} - \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \phi^2} - \frac{1}{\cos^2 \theta} \frac{\partial^2}{\partial \psi^2}
\]
(57)

were denoted \( \Lambda \) and take the form \( \Lambda = l(l+2) \) where \( l = 0, 1, \cdots \). The wave equation eq. 56 agrees with the near horizon limit of the general one given in [29], except that there the ansatz for the wave function did not allow dependence on the compact coordinate \( y^3 \). The present generalization gives an even more symmetric result.

It is immediately apparent from the form of the wave equation that its solutions depend on \( m_{L,R} \) and \( \beta_H \Omega^{L,R} \) only through the prescriptions eq. 53. The solution to the near horizon wave equation is a hypergeometric function found in [38, 46, 47] (present notation is used in [29]).

The recent work on black hole greybody factors has focussed on massless fields, but here a mass \( \mu \) is included in the Klein-Gordon equation, eq. 55. This leads to a constant on the right hand side of the radial equation eq. 56 and so the effect of the mass can be absorbed in the conformal dimension. The reason that the mass term does not change the form of the radial equation is that the determinant of the metric is independent of the radial variable. This property is nontrivial: for example, massive fields that couple to the Einstein metric in five dimensions experience a potential induced by the effective dilaton. These fields are therefore more complicated than their six dimensional analogues considered here. It is possible that this observation points towards a simple spacetime description of string states that are massive from the six-dimensional point of view.

In rotating backgrounds there are not in general enough isometries to guarantee that the variables can be separated [4]. It is therefore a surprise that this is possible for massless minimally coupled scalars in the five dimensional black hole geometry [29]. The statements made in the preceding paragraph can be verified even for the general nonextremal metric eq. 4. Thus we find that separation of variables remains true for massive scalars, when the mass is measured by the Einstein metric in six dimensions. These properties are precisely analogous to those of Kerr black holes in four dimensions. This is encouraging for the hope that the rotating black holes in string theory admit conserved Killing-Stackel tensors and Killing-Yano spinors that are analogous to those of the Kerr black hole (for some discussion and references see [48, 49]).

\[ \text{3} \text{An equation that applies in the general nonextremal case and includes the dependence on the compact coordinate} \ y \ \text{can be obtained from the master equation given in [29] without extensive calculations, by exploiting boost invariance in the y dimension.} \]

\[ \text{4} \text{We would like to thank G. Gibbons for reminding us of this fact.} \]
3.4 The local $SL(2, \mathbb{R})_L \times SL(2, \mathbb{R})_R$

The explicit form of the local $SL(2, \mathbb{R})_L \times SL(2, \mathbb{R})_R$ generators were inferred from the wave equation in [29], without realizing the connection to the BTZ black hole. The result agrees with the one found directly in the BTZ geometry [36], except for the modification due to rotation. It can be written:

$$R_{\pm} \equiv R_1 \pm iR_2 = \frac{1}{2}e^{\pm(\tau+\sigma)}[\mp\partial_\rho + (\coth \rho \partial_\tau + \tanh \rho \partial_\sigma)] ,$$

$$R_3 = \frac{1}{2}(\frac{\partial}{\partial \tau} + \frac{\partial}{\partial \sigma}) ,$$

and the $SL(2, \mathbb{R})$ algebra is normalized so:

$$[R_3, R_{\pm}] = \pm R_{\pm} ; \ [R_{\pm}, R_{\pm}] = -2R_3 .$$

The generators $\vec{L}$ of the $SL(2, \mathbb{R})_L$ are found by taking $\sigma \to -\sigma$. They commute with the $\vec{R}$. The effect of rotation is taken into account by evaluating the derivatives at fixed value of $(\tilde{\psi}, \tilde{\phi})$.

The $(\tau, \sigma)$ are the coordinates that are acted on in a simple way by the local $SL(2, \mathbb{R})_L \times SL(2, \mathbb{R})_R$. It is therefore natural to interpret these variables as the spacetime realizations of the effective string world-sheet coordinates. The $\tau$ and the $\sigma$ are proportional to the $t$ and $y$, respectively, for a static string wrapped around the $y$ direction. When the string has a momentum along $y$ the world-sheet coordinates the $(\tau, \sigma)$ are proportional to the boosted coordinates $\tilde{t}$ and $\tilde{y}$. However, in the case of a rotating string there is no obvious geometrical interpretation of the linear relation eq. 46 between $(\tau, \sigma)$ and $(t, y)$.

There is a striking relation between $(\tau, \sigma)$ and the black hole horizons. In the metric eq. 54 we see that $g_{\tau\tau}$ vanishes precisely once, at the outer horizon, and $g_{\sigma\sigma}$ similarly vanishes once, at the inner horizon. Thus, upon embedding into one dimension higher, the two well known zeros of the $g_{tt}$, at the two horizons, have been split symmetrically between the remaining time-like, and the additional space-like coordinate. This structure is reflected in the wave equation eq. 56 by the presence of simple poles at both horizons, with an apparent symmetry between their coefficients. The wave equation implies that the potentials conjugate to $L$- and $R$- moving energy $\beta_{L,R}$, given in eq. 19, are related to the surface accelerations $\kappa_{\pm}$ at the outer and inner horizons, respectively, as [29]:

$$\beta_{L,R} = \frac{2\pi}{\kappa_+} \mp \frac{2\pi}{\kappa_-} .$$

It is reasonable that the effective string theory of black holes treats the effective world-sheet coordinates $(\tau, \sigma)$ symmetrically, but it is not obvious that this should imply a symmetry between the two horizons of the black hole. We suspect that this structure may have important implications.

---

5We change the signature from the Euclidian one used in [29] by taking $R_{\pm}^{\text{here}}(\tau, \sigma) = iR_{\pm}^{\text{here}}(i\tau, i\sigma)$ and $R_3^{\text{here}}(\tau, \sigma) = iR_3^{\text{here}}(i\tau, i\sigma)$. 

13
The coordinates \((\tau, \sigma)\) exhibit nontrivial global properties, due to the periodicity of \(y\). It is natural to take both \(t\) and \(y\) compact. The generators eq. 58 determine the periodicity of \(\sigma\) as \(2\pi i\), and then the relation:

\[
\tau \pm \sigma = \lambda^{-2}(r_+ \mp r_-)e^{\mp \frac{\pi}{\kappa_+}}(t \pm y) = \frac{2\pi}{\beta_{L,R}}(t \pm y) ,
\]

(61)
gives the complex structure of Euclidianized spacetime \(z = (y + it)/2\pi R_y\) in terms of the black hole parameters as:

\[
\tau_{\text{modulus}} = \left( \frac{2\pi}{\kappa_+} + \frac{i 2\pi}{\kappa_-} \right) \frac{i}{2\pi R_y}.
\]

(62)

In this way the two parameters that specify the global structure in spacetime are mapped to a complex modulus of the effective string. The remaining two black hole parameters, associated with rotation, are similarly associated with the inequivalent embeddings of the two \(SU(2)\) algebras into the isometry group of the sphere \(S_3\). Thus the global structure is parametrized by a total of four moduli, both in spacetime and on the effective string world-sheet.

### 4 Discussion

In string theory black holes are described as quantum states that are constructed by exciting a fundamental ground state [27]. A given black hole is described by a projection on the highly excited states that are consistent with the specified left moving (L) and right moving (R) energy and the (L and R) angular momenta. The projected theory forms a legitimate CFT but it is only its finite excitations that can be considered, or else the state belongs to a sector that is better described in terms of different projection. Thus the four parameters that describe the projections act as moduli.

On the spacetime side of the AdS/CFT correspondence the fundamental ground state is identified with the \(AdS_3\) geometry of the \(M = -1\) BTZ black hole [21, 6], and the excitations are general multiparticle states in the Fock space, constructed over the perturbative spectrum of the \(AdS_3 \times S_3\) [3, 12, 13]. The present paper, and the works on greybody factors, considered the actual black hole spacetime, rather than the vacuum geometry. The geometry remains locally \(AdS_3 \times S_3\) in the black hole spacetime, just as in the vacuum, so the induced CFTs have the same spectrum in the two cases. However, they are not equivalent: the boundary conditions satisfied by the wave functions that describe the perturbations are in one-to-one correspondence with the parameters that describe the black hole, and thus with the moduli of the effective string. The “fundamental” CFT is just a specific point in moduli space, albeit one that defines the vanishing of entropy.

It is inherent to this discussion that the black hole microstates are “made out of” excitations of a different background that is not itself a black hole. Thus it is not

\[\footnote{Note, however, that this subtlety does not affect the differentials \((d\tau, d\sigma)\) and the differential operators used in the generators eq. 58.} \]
meaningful to ask where in the black hole spacetime the microstates reside, because the geometry is itself an emerging phenomenon that is only present in the projected theory with finite moduli turned on. This picture therefore resoles the obvious tension between the no hair theorem, and the counting of black hole microstates.

An important aspect of the black hole puzzles is that general relativity applies whenever the curvature is small, and specifically in the horizon region of large black holes. This condition can only be met in the backgrounds described by highly excited states, and hence general relativity should always be compared with the effective string CFT, rather than the fundamental one. Moreover, the effective string CFT reduces to general relativity when curvatures are small, so all is in order.

However, in this framework, information flow must be analyzed in terms of the quasistatic evolution on the moduli space, and the resolution of the black hole singularities requires a geometric interpretation of the CFT that persists in the strong coupling region. It is not presently clear what techniques would allow such studies so we are not yet in a position to answer the central puzzles posed by the black holes.

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