Effects of Emerging Bipolar Magnetic Regions in Mean-field Dynamo Model of Solar Cycles 23 and 24

V. V. Pipin, A. G. Kosovichev, and V. E. Tomin

1 Institute of Solar-Terrestrial Physics, Russian Academy of Sciences, Irkutsk, 664033, Russia
2 New Jersey Institute of Technology, Newark, NJ 07102, USA; alexander.g.kosovichev@njit.edu

Received 2022 October 16; revised 2022 December 4; accepted 2022 December 22; published 2023 May 18

Abstract

We model the physical parameters of Solar Cycles 23 and 24 using a nonlinear dynamical mean-field dynamo model that includes the formation and evolution of bipolar magnetic regions (BMRs). The Parker-type dynamo model consists of a complete MHD system in the mean-field formulation: the 3D magnetic induction equation, and 2D momentum and energy equations in the anelastic approximation. The initialization of BMRs is modeled in the framework of Parker’s magnetic buoyancy instability. It defines the depths of BMR injections, which are typically located at the edge of the global dynamo waves. The distribution with longitude and latitude and the size of the initial BMR perturbations are taken from the NOAA database of active regions. The modeling results are compared with various observed characteristics of the solar cycles. Only the BMR perturbations located in the upper half of the convection zone lead to magnetic active regions on the solar surface. While the BMRs initialized in the lower part of the convection zone do not emerge on the surface, they still affect the global dynamo process. Our results show that BMRs can play a substantial role in the dynamo processes and affect the strength of the solar cycles. However, the data driven model shows that the BMR’s effect alone cannot explain the weak Cycle 24. This weak cycle and the prolonged preceding minimum of magnetic activity were probably caused by a decrease of the turbulent helicity in the bulk of the convection zone during the decaying phase of Cycle 23.

Unified Astronomy Thesaurus concepts: The Sun (1693); Solar physics (1476); Solar magnetic fields (1503)

Supporting material: animation

1. Introduction

The basic scenario for the hydromagnetic solar dynamo involves a cyclic mutual transformation of the toroidal and poloidal magnetic fields by means of differential rotation and cyclonic convective motions characterized by kinetic helicity (Parker 1955b). The key idea of Parker suggests that the poloidal magnetic field of the Sun is generated from rising loops of the toroidal magnetic field in the deep convection zone, which are twisted around the radial direction by turbulent cyclonic motions (“the α-effect”). The resulting poloidal fields of the loops coalesce into a large-scale poloidal magnetic field of a new solar cycle. The subsequent stretching of the poloidal field by the differential rotation produces the toroidal field (“the Omega-effect”). The whole 22 yr cyclic process of the magnetic field generation and transformation represents dynamo waves, forming the magnetic butterfly diagram and reversing the magnetic polarity of the Sun’s global magnetic field. Occasional strands of toroidal flux tubes emerge on the surface owing to the magnetic buoyancy instability (Parker 1955a, 1979) in the form of east–west-oriented bipolar magnetic regions (BMRs).

On the other hand, this scenario stems from the turbulent dynamo theory and mean-field dynamo models (Moffatt 1978; Parker 1979; Radler 1980), which are the basis of our current understanding of the nature of magnetic activity in astrophysical objects (Brandenburg & Subramanian 2005; Shakurov & Subramanian 2021). Global convective MHD simulations prove the validity of the basic principles of the dynamo theory (Schrinner et al. 2011; Schrinner et al. 2011; Guerrero et al. 2016; Warnecke et al. 2018, 2021). Naturally, the mean-field solar dynamo models consider the dynamo process as distributed over the convection zone and “shaped” into the butterfly diagram of emerged BMRs in the subsurface rotational shear layer (Brandenburg 2005; Pipin & Kosovichev 2011b).

On the other hand, the phenomenological scenario of Babcock–Leighton (hereafter BL; Babcock 1961; Leighton 1969), based on observations of magnetic field evolution on the solar surface, grew into a popular flux transport model of the solar cycles (see reviews by Charbonneau et al. 2011; Brun et al. 2014; Dikpati 2016). In this scenario, the poloidal field is generated as a result of the latitudinal tilt of BMRs emerging on the solar surface. This field is transported by the surface meridional circulation and turbulent diffusion to the polar regions, where it sinks in the interior and is amplified by the differential rotation, creating a new toroidal field, which produces emerging BMRs. The meridional circulation speed controls the solar cycle duration. For these dynamo models, the surface magnetic activity, which is often considered in the form of an empirical “source term” in the induction equation, is crucial.

In terms of the mean-field theory, the empirical Babcock–Leighton source term calculated from the magnetic flux of BMRs observed on the solar surface can be considered as a near-surface α-effect (Stix 1974). From this point of view, the primary differences between the Parker and Babcock–Leighton scenarios are in the distribution of the α-effect in the convection zone and the role of the Parker’s turbulent magnetic diffusion and meridional circulation in the magnetic flux transport. While the Parker scenario assumes that the α-effect is
distributed in the convection zone and turbulent diffusion plays a major role in the formation of migrating dynamo waves and considers the formation of BMRs as a secondary effect, in the Babcock–Leighton scenario the BMRs play a key role, providing the surface $\alpha$-effect and magnetic flux transported by the meridional circulation to the polar regions.

Theoretical models based on these scenarios have been successful in explaining some observed properties of the solar cycles and magnetic field evolution. The Babcock–Leighton flux transport models explain the magnetic flux emergence and transport observed on the solar surface, as well as the evolution of the polar magnetic field (Dikpati et al. 2016). The recently developed self-consistent Parker-type mean-field model (Pipin 2018) explains the global magnetic structure and evolution (Obridko et al. 2021), as well as the dynamical processes, such as the solar migrating zonal flows, so-called torsional oscillations (Kosovichev & Pipin 2019) and variations of the meridional circulation (Getling et al. 2021), observed by helioseismology. The model also explained the extended solar cycle phenomenon (Pipin & Kosovichev 2019). This model determines the 3D evolution of the large-scale vector magnetic field, coupled with the equations describing large-scale flows and heat transport in the axisymmetric 2D approximation. Thus, unlike in the Babcock–Leighton models, the differential rotation and meridional circulation are not input parameters—they are calculated together with the magnetic field evolution. This allowed comparing the model results with the helioseismic measurements.

Despite the recent advances, both types of dynamo models do not provide a clear understanding of the physical mechanisms causing variations of the solar amplitude and, thus, cannot provide robust solar cycle predictions. In the Babcock–Leighton models, the primary idea is that the solar cycle strength is governed by spatial and temporal variations of the BMR’s tilt, which affects the amount of magnetic flux transported to the polar regions (e.g., Dikpati 2016). On the other hand, Parker-type models attempt to explain the solar cycle amplitudes by long- and short-term variations of the distributed $\alpha$-effect and associated nonlinear processes in the deep convection zone (e.g., Pipin & Kosovichev 2020).

To evaluate the role of BMRs in these models, Pipin (2022) developed a 3D mean-field model, which includes the mechanism of magnetic flux emergence due to the magnetic buoyancy instability resulting in the formation of BMRs on the surface. It was found that the generation of the large-scale poloidal field via the emergence and evolution of the solar BMRs can profoundly affect the global dynamo process. Thus, this new type of mean-field dynamo modeling includes the basic features of both the Parker and Babcock–Leighton scenarios.

In this paper, we study the effects of BMRs on the solar dynamo using the mean-field dynamo model of Pipin & Kosovichev (2020) and the BMR formulation of Pipin (2022). Our aim is to investigate the global parameters of the mean-field dynamo for these solar cycles and evaluate the role of BMRs in the cycle properties. To model the surface magnetic activity, we adopt the data of solar active regions from the NOAA Space Weather Prediction Center for Solar Cycles 23 and 24. In particular, we study the evolution of the polar magnetic field and investigate whether the modeled BMR’s activity can explain the weak Cycle 24.

## 2. Model Formulation

The model is formulated within the mean-field MHD framework of Krause & Rädler (1980). The basic details of the model are given earlier by Pipin & Kosovichev (2019, hereafter PK19) and Pipin (2022, hereafter P22).

The magnetic field evolution is governed by the mean-field induction equation:

$$\partial_{t}\langle B \rangle = \nabla \times \left( \mathcal{E} + \langle U \rangle \times \langle B \rangle \right),$$

where $\mathcal{E} = (u \times b)$ is the mean electromotive force; $u$ and $b$ are the turbulent fluctuating velocity and magnetic field, respectively; and $\langle U \rangle$ and $\langle B \rangle$ are the mean velocity and magnetic field, respectively. We assume that the averaging is done over the ensemble of turbulent flows and magnetic fields. We decompose the induction vector $\langle B \rangle$ into the sum of the axisymmetric and nonaxisymmetric parts,

$$\langle B \rangle = \langle B \rangle + \langle B \rangle,$$

where $\langle B \rangle$ and $\langle B \rangle$ are the axisymmetric and nonaxisymmetric components of the large-scale magnetic field. The mean electromotive force describes the turbulent effects on the mean magnetic field evolution. It consists of two parts:

$$\mathcal{E}_{i} = \mathcal{E}_{i}^{(A)} + \mathcal{E}_{i}^{(BMR)},$$

where $\mathcal{E}_{i}^{(A)}$ is calculated analytically using the double-scale approximation of Roberts & Soward (1975; see, e.g., Kitchatinov et al. 1994; Pipin 2008). The phenomenological part $\mathcal{E}_{i}^{(BMR)}$ describes formations of the surface BMRs from the large-scale toroidal magnetic field.

The expression for $\mathcal{E}_{i}^{(A)}$ reads as follows:

$$\mathcal{E}_{i}^{(A)} = (\alpha_{ij} + \gamma_{ij})\langle B \rangle_{ij} - \eta_{ik}\nabla_{j}\langle B \rangle_{k},$$

where $\alpha_{ij}$ describes the turbulent generation of the magnetic field by helical motions (the $\alpha$-effect), $\gamma_{ij}$ describes the turbulent pumping, and $\eta_{ik}$ is the eddy magnetic diffusivity tensor. The $\alpha$-effect tensor includes the small-scale magnetic helicity density contribution, i.e., the pseudo-scalar $\langle \chi \rangle = \langle a \cdot b \rangle$ (where $\mathbf{a}$ and $\mathbf{b}$ are the fluctuating vector potential and magnetic field, respectively),

$$\alpha_{ij} = C_{\alpha} \psi_{\alpha}(\beta) \beta_{ij}^{(H)} + \alpha_{ij}^{(M)} \psi_{\alpha}(\beta) \langle \chi \rangle \tau_{c} / 4\pi n_{c}^{2},$$

where the expressions of the kinetic helicity tensor $\beta_{ij}^{(H)}$ and the magnetic helicity tensor $\beta_{ij}^{(M)}$ are given by Pipin (2018). The radial profiles of $\beta_{ij}^{(H)}$ and $\beta_{ij}^{(M)}$ depend on the mean density stratification, the profile of the convective rms velocity $u_{\alpha}$, and the Coriolis number $\Omega = 2\Omega_{c}\tau_{c}$, where $\Omega_{c}$ is the angular velocity of the star and $\tau_{c}$ is the convective turnover time. The magnetic quenching function $\psi_{\alpha}(\beta)$ depends on the parameter $\beta = |\mathbf{B}| / \sqrt{4\pi n_{c}^{2}}$. Note that in the presence of the $\mathbf{B}$-field the $\alpha$-effect tensor becomes nonaxisymmetric. It is caused by the $\psi_{\alpha}(\beta)$ quenching and the magnetic helicity effects.

The magnetic helicity evolution follows the global conservation law for the total magnetic helicity, $\langle \chi \rangle^{(tot)} = \langle \chi \rangle + \langle A \rangle \cdot \langle B \rangle$ (see Hubbard & Brandenburg 2012; Pipin
et al. 2013; Brandenburg 2018):

$$\left( \frac{\partial}{\partial t} + \langle U \rangle \cdot \nabla \right) \langle \chi \rangle^{\text{tot}} = - \frac{\langle \chi \rangle}{R_m \tau_c} - 2\eta \langle B \rangle \cdot \langle J \rangle - \nabla \cdot F^x,$$

where we use $2\eta \langle b \cdot j \rangle = \frac{\langle \chi \rangle}{R_m \tau_c}$ (Kleeorin & Rogachevskii 1999). In addition, we introduce the diffusive flux of the small-scale magnetic helicity density, $F^x = -\eta^c \nabla \langle \chi \rangle$, and $R_m$ is the magnetic Reynolds number; we employ $R_m = 10^6$. Following the results of Mitra et al. (2010), we put $\eta^c = \frac{1}{10} \eta$. Further details about the turbulent dynamo effects can be found in P22 and in the above-cited papers. In what follows, we discard the advection of the total helicity by meridional circulation. As a result, the amplitude of the polar magnetic field in the mean-field model decreases in comparison to the standard case (see P22). One purpose of this tuning is to get a stronger impact of the surface BMRs on the deep dynamo in the new model.

The turbulent pumping, which is expressed by the antisymmetric tensor $\gamma_{ij}$, is important for reproducing the solar-like evolution of the dynamo-generated magnetic field (Warnecke et al. 2014, 2021). The formulation of $\gamma_{ij}$ for the solar-type mean-field dynamo model was discussed by Pipin (2018). We define it as follows:

$$\gamma_{ij} = \gamma_{ij}^{(\rho)} + \frac{\alpha_{\text{MLT}} u_c}{\gamma} \mathcal{H}(\beta) \tilde{\varepsilon}_{n,ij},$$

where $\mathcal{N}^{(\rho)} = \nabla \log \bar{\rho}$, $\alpha_{\text{MLT}} = 1.9$ is the mixing-length theory parameter, $\gamma$ is the adiabatic exponent, and $u_c$ is the rms convective velocity. In Equation (7), the first term takes into account the mean drift of the large-scale field due to the gradient of the mean density, and the second term describes the magnetic buoyancy effect. Function $\mathcal{H}(\beta)$ takes into account the effect of magnetic tension. For small values of $\beta$, $\mathcal{H}(\beta) \sim \beta^2$, and it saturates as $\beta^{-2}$ for $\beta \gg 1$ (see P22 for details).

We assume that the large-scale flow is axisymmetric. It is decomposed into a sum of the meridional circulation and differential rotation, $\bar{U} = \bar{U}^m + r \sin \theta \Omega(r, \theta) \hat{\phi}$, where $r$ is the radial coordinate, $\theta$ is the polar angle, $\hat{\phi}$ is the unit vector in the azimuthal direction, and $\Omega(r, \theta)$ is the angular velocity profile. The angular momentum conservation and the equation for the azimuthal component of large-scale vorticity, $\omega = (\nabla \times \bar{U}^m)_\phi$, determine the differential rotation and meridional circulation:

$$\frac{\partial}{\partial t} \bar{\rho} r^2 \sin^2 \theta \Omega = - \nabla \cdot (r \sin \theta \mathcal{P}(\bar{L}_\theta + r \sin \theta \Omega \bar{U}^m)) + \nabla \cdot \left( r \sin \theta \frac{(B \cdot \bar{B}_\phi)}{4\pi} \right),$$

where $\mathcal{P}$ is the turbulent stress tensor, $\tilde{T}$ is the turbulent magnetic helicity flux, and $\mathcal{F}^x$ is the anisotropic convective flux (see PK19). The last two terms in Equation (12) take into account the convective energy gain and sink caused by the generation and dissipation of large-scale magnetic and flow fields. The reference profiles of mean thermodynamic parameters, such as entropy, density, and temperature, are determined from the stellar interior model MESA (Paxton et al. 2011, 2013). The radial profile of the typical convective turnover time, $\tau_c$, is determined by the MESA code as well. We assume that $\tau_c$ does not depend on the magnetic field and global flows. The convective rms velocity is determined from the mixing-length approximation,

$$u_c = \frac{\ell_{\text{c}}}{2} \sqrt{- \frac{g}{2 c_p} \frac{\partial \sigma}{\partial r}},$$

where $\ell_{\text{c}} = \alpha_{\text{MLT}} H_\rho$ is the mixing length, $\alpha_{\text{MLT}} = 1.9$ is the mixing-length parameter, and $H_\rho$ is the pressure scale height. Equation (13) determines the reference profiles for the eddy heat conductivity, $\chi_T$, eddy viscosity, $\nu_T$, and eddy diffusivity, $\eta_T$, as follows:

$$\chi_T = \frac{\ell_T^2}{6} \sqrt{- \frac{g}{2 c_p} \frac{\partial \sigma}{\partial r}},$$

$$\nu_T = \nu_T \chi_T,$n

$$\eta_T = \eta_T \nu_T.$$
MESA code, $\tau_c$ increases sharply toward the bottom of the convection zone. This may cause an inversion of the $\lambda$-effect near the bottom of the convection zone. This inversion results in a reverse secondary meridional circulation cell (with clockwise circulation in the northern hemisphere) near the bottom of the convection zone (Pipin & Kosovichev 2018). To get the model with one circulation cell, we smooth the sharp variation of $\tau_c$ toward the bottom of the convection zone following the ansatz of Kitchatinov & Nepomnyashchikh (2017). It has been shown before that the dynamo solutions in our models with one and two meridional circulation cells are similar (Pipin & Kosovichev 2019).

For the convective overshoot region, we assume that the intensity of turbulent mixing decays exponentially with depth from the bottom of the convection zone. We assume that the bottom of the overshoot region rotates as a solid body at the rate of $\Omega_0/2\pi = 430$ nHz. The axisymmetric reference model has several free parameters. We choose them to fit the reference model into solar observations. Here we employ a standard choice to relate eddy heat conductivity to an eddy viscosity $\nu_T = 3/4$. The full dynamo cycle period of about 20 yr is reproduced if $P_{\nu_T} = 11$. The critical threshold of the dimensionless $\alpha$-effect parameter is $C_\alpha = 0.04$. Figure 1 illustrates distributions of the angular velocity, meridional circulation, $\alpha$-effect, and eddy diffusivity calculated for the nonmagnetic reference model. The amplitude of the meridional circulation on the surface is about 13 m s$^{-1}$. We list the critical parameters in Table 1.

![Image](image_url)
penetration of the toroidal dynamo wave toward the surface and supports the equatorward propagation of the toroidal magnetic field due to the Parker—Yoshimura rule (Yoshimura 1975). This can impact the BMR production in the near-surface layers. Noteworthily, the solar observations indicate that the surface toroidal field magnitude is around 1 G (Vidotto et al. 2018). The poloidal magnetic field is potential outside the dynamo domain. For the numerical solution, we use a spectral expansion in terms of the spherical harmonics and employ the FORTRAN version of the SHTNS library of Schaeffer (2013).

2.2. Formation of Bipolar Magnetic Regions

Following ideas of Parker (1955a, 1971), the emergence of the BMRs is modeled by using the mean electromotive force representing the magnetic buoyancy and twisting effects acting on unstable parts of the axisymmetric magnetic field as follows (P22):

\[ E_{i}^{\text{BMR}} = \alpha_{i} \delta_{i} \langle B \rangle_{i} + V_{bi} \left( \nabla \times \langle B \rangle_{i} \right)_{i}, \]

where the first term describes the \( \alpha \)-effect caused by the BMR’s tilt and the second term models the magnetic buoyancy instability. The magnetic buoyancy velocity, \( V_{bi} \), includes the turbulent and mean-field buoyancy effects (Kitchatinov & Rüdiger 1992; Kitchatinov & Pipin 1993; Rüdiger & Brandenburg 1995):

\[ V_{bi} = \frac{\Omega MLT \gamma}{\gamma} \mathcal{H}(\beta_{m}) \xi_{i}(t, r) \]

where function \( \mathcal{H}(\beta) \) describes magnetic tension, \( \beta = \left| B \right| \sqrt{4\pi \mu_{0}} \), and subscript “\( m \)” marks unstable points. Function \( \xi \) defines the location and formation of the unstable part of the magnetic field,

\[ \xi_{i}^{(\pm)}(r, t) = C_{i} \tanh \left( \frac{1}{\tau_{0}} \right) \exp \left( -m_{i} \sin^{2} \left( \frac{\theta - \theta_{m}}{2} \right) \right) \]

\[ + \sin \left( \frac{\theta - \theta_{m}}{2} \right) \right) \psi(r, r_{m}^{(\pm)}), |t| < \tau_{0} \]

\[ = 0, t > \tau_{0} \]

where \( \psi \) is a kink-type function of radius,

\[ \psi(r, r_{m}^{(\pm)}) = \frac{1}{4} \left( 1 + \text{erf} \left( \frac{100 (r - r_{m}^{(\pm)})}{R} \right) \right) \]

\[ \times \left( 1 - \text{erf} \left( \frac{100 (r - r_{m}^{(\pm)} + 0.1R)}{R} \right) \right) \]

where \( r_{m} \) and \( \theta_{m} \) are the radius and latitude of the toroidal magnetic field strength extrema in the convection zone, respectively. The reader can find further details in P22 and the above-cited papers. Then, the instability may act both near the bottom and near the top of the convection zone. We handle these situations separately using separate functions: \( \xi^{(\pm)}_{i}(r, t) \) for the low half \( (r_{m}^{(\pm)} < 0.86R) \) and \( \xi^{(\mp)}_{i}(r, t) \) for the upper half \( (r_{m}^{(\mp)} > 0.86R) \) of the convection zone.

Compared to P22, we modify the time evolution of the instability from simple exponent to \( C_{0} \tanh \left( \frac{1}{m_{i}} \right) \) and calculate the unstable points in the whole convection zone. This is similar to our earlier paper (Pipin & Kosovichev 2015). In addition, this formulation allows for more flexible assimilation of the observational data of solar active regions (see Section 2.3). For consistency with the results of P22, we use the parameter \( C_{3} = 180 \). The other parameters are the same as in P22, i.e., the emergence time, \( \tau_{0} = 5 \) days, and the BMR growth rate, \( \tau_{0} = 1 \) day; and with \( m_{1} = 100 \), we get a size of BMRs of about 10 heliographic degrees. The perturbations are randomly initiated in time and longitude in each hemisphere independently.

The radial and latitudinal positions of the unstable points are computed using the instability parameter

\[ I_{3} = -\frac{\partial}{\partial r} \log \left( \frac{B}{\mathcal{P}} \right), \]

where \( B \) is the strength of the axisymmetric toroidal magnetic field and \( \mathcal{P} \) is the density profile. The power-law index \( \xi = 1 \) corresponds to Parker’s instability condition. For a better match with the solar observations, we use \( \xi < 1.2 \). Typically, \( I_{3} > 0 \) at the upper edge of the dynamo wave (see Figure 3 in P22). This condition defines the depth of the unstable perturbations. Therefore, the instabilities are initiated at the points of the \( |B| I_{3}(r, \theta) \) maxima where \( I_{3}(r_{m}, \theta_{m}) > 0 \).

The \( \alpha \)-effect of the BMRs is Equation (18) is given as follows:

\[ \alpha_{i} = C_{\alpha} \beta(1 + \xi_{i} \cos \theta) \psi_{i}(\beta) \psi(\pi, \pi) \]

where the amplitude of the \( \alpha \)-effect is determined by the local magnetic buoyancy velocity. Parameter \( \xi_{i} \) controls random fluctuations of the BMR’s \( \alpha \)-effect. In the current formulation, the \( \alpha \)-effect of the BMRs is readily linked to the BMR’s tilt (Stix 1974). Parameter \( C_{\alpha} \beta \) defines the mean tilt (see P22). The latitudinal dependence of this relationship is governed by the factor \( \cos \theta \); see Equation (24). In addition, we use a step-like function of Equation (21) to define the radial extent of the BMR perturbations.

We model the randomness of the tilt using the parameter \( \xi_{1} \). Similarly to the work of Rempel (2005) and Pipin (2022), the \( \xi_{1} \) evolution follows the Ornstein–Uhlenbeck process,

\[ \xi_{1} = \frac{\xi_{1}}{\tau_{1}}(\xi_{1} - \xi_{2}), \]

\[ \xi_{2} = \frac{\xi_{2}}{\tau_{2}}(\xi_{2} - \xi_{2}), \]

\[ \xi_{2} = \frac{\xi_{2}}{\tau_{2}}(\xi_{2} - \frac{2\xi_{2}}{\tau_{2}}). \]

Here \( g \) is a Gaussian random number. It is renewed at every time step, \( \tau_{1} \). The \( \tau_{2} \) is the relaxation time of \( \xi_{2} \). The parameters \( \xi_{1,2,3} \) are introduced to smooth variations of \( \xi_{0} \). Similarly to the above-cited papers, we choose the parameters of the Gaussian process as follows: \( g = 0, \sigma(g) = 1, \) and \( \tau_{2} = 2 \) months. Parameters \( \xi_{1,2,3} \) vary independently in the northern and southern hemispheres.

2.3. Data-driven BMR Model

In the data-driven models, we compute the radial position of the unstable point using the maximum of product \( |B| I_{3}(r, \theta) \),
the condition $I_{\beta}(r_m, \theta_m) > 0$, and $r_m > 0.86 R$. For the lower half of the convection zone, we use function $\xi_{\beta}^{(-)}$. The latitudinal and longitudinal coordinates $\theta_m$ and $\phi_m$ in function $\xi_{\beta}$ are taken from the active region database. Similarly to the theoretical model, the BMR’s size is controlled by the parameter $m_{lb}$, which is taken in the form

$$m_{lb} = \frac{2}{\sqrt{S_a/10^6}},$$

where $S_a$ is the maximum observed area (in millionths of the hemisphere) of the bipolar active regions. We exclude all regions with $S_a < 500$. The solar observations show a wide range of variations in the BMR’s emergence time and growth rate. In addition, there are periods of simultaneous emergence of several BMRs in one hemisphere. To avoid the overlaps, we reformat the emergence initiation time as follows. First, for such cases, we shift the emergence of subsequent BMRs by two time steps after the end of the previous BMR emergence. Second, we define the minimal emergence time $\delta t_{\text{min}} = 2$ days and assume that these BMRs have smaller $\tau_{\alpha}$ or a higher growth rate. Specifically, we define

$$\tau_{\alpha} = \frac{1}{2} \frac{\delta t_a}{\delta t_{\text{min}}} \tau_0,$$

where $\delta t_a$ is the total emergence time of the BMRs and $\tau_0$ is the growth rate.

Putting it all together, we obtain the instability function for the data-driven modeling, $\xi_{\beta}^{(a)}$,

$$\xi_{\beta}^{(a)}(r, t) = C_{\beta} \tanh \left( \frac{1}{\tau_a} \right) \exp \left( - \frac{2}{\sqrt{S_a/10^6}} \sin^2 \left( \phi - \phi_m \right) \right) + \sin^2 \left( \frac{\theta - \theta_m}{2} \right) \psi(r, x_{\beta}^{(+)}), \quad t < \delta t_a$$

$$= 0, \quad t > \delta t_a.$$  

(26)

The parameters $S_a$, $\theta_m$, and $\phi_m$ are taken from the NOAA database of solar active regions (https://www.swpc.noaa.gov/).

### 3. Results

#### 3.1. Formation of Bipolar Magnetic Regions and Their Parameters

Table 2 summarizes the key parameters of the model runs. Model T0 is a base axisymmetric dynamo model without BMRs. It reproduces basic observed properties of the solar cycles, such as the magnetic butterfly diagram, polar field reversals, migrating zonal flows (torsional oscillations), variations of the meridional circulation, and the extended solar cycle phenomenon (Pipin & Kosovichev 2019, 2020). Models T1 and T2 include the BMR’s initiation driven by the magnetic buoyancy instability with initial perturbations with the radius according to the instability criterion and randomly in longitude and latitude. The perturbations are initiated in the whole convection zone in model T1 and only in the upper half of the convection zone in model T2. Models S0–S2 are data-driven models. Like in models T1 and T2, the BMR’s sources are distributed with the radius according to the magnetic buoyancy criterion, but in the upper half of the convection zone the latitudinal and longitudinal distributions correspond to the location of the solar active regions observed during Solar Cycles 23 and 24. The BMR Injection Table 2 summarizes the key parameters of the model runs.

| Model | BMR Injection | $\alpha_\beta$ | $C_{\alpha}$ | $t$ (yr) | Period (yr) |
|-------|---------------|---------------|--------------|---------|------------|
| T0    | 0             | 0             | 0.045        | ...     | 10.4       |
| T1    | $\xi_{\beta}^{(-)}$, $\xi_{\beta}^{(+)}$ | $\xi_{\alpha}$ | 0.045 | $\geq 0$ | 10.6, 10.8, 10.5 |
| T2    | 0, $\xi_{\beta}^{(+)}$ | $\xi_{\alpha}$ | 0.04 | $\geq 0$ | 11.2, 11.3, 11.1 |
| S0    | $\xi_{\beta}^{(-)}$, $\xi_{\beta}^{(+)}$ | $\xi_{\alpha}$ | 0.045, 0.035, 0.044 | $\geq 0$, $\geq 5$, $\geq 11$ | 11.2, 11.6 |
| S1    | $\xi_{\beta}^{(-)}$, $\xi_{\beta}^{(+)}$ | $\xi_{\alpha} > 0.95R$ | 0.045, 0.035, 0.044 | $\geq 0$, $\geq 5$, $\geq 11$ | 11.4, 11.6 |
| S2    | 0, $\xi_{\beta}^{(+)}$ | $\xi_{\alpha}$ | 0.045, 0.035, 0.044 | $\geq 0$, $\geq 5$, $\geq 11$ | 11.4, 12 |

Note. T0 is the axisymmetric base model without BMRs. T1 and T2 are models with the random initialization of BMRs. S0–S2 are data-driven models with the initialization corresponding to Solar Cycles 23 and 24 in the upper half of the convection zone. The second column shows the implementation of the BMR’s perturbations in the lower half (the first function) and the top half (the second function) of the convection zone (see Equations (20) and (26)). The third column shows whether the models employ the BMR’s tilt (see Equations (24) and (25)). The fourth column shows the parameters of the global mean-field $\alpha$-effects (see the text). The fifth column shows the time intervals for the corresponding $C_{\alpha}$ values (after the start of Solar Cycle 23 for models S0–S2). The sixth column shows the duration of the activity cycles (half dynamo periods of the magnetic cycles).
Figure 2. Formation of two magnetic regions in the southern hemisphere during the growth phase of Cycle 23 in model S0. The left column shows the field lines of the nonaxisymmetric magnetic field, and time is shown in days. The right column shows the surface radial magnetic field. The full animation of the magnetic field evolution is available online. The animation illustrates the formation of BMRs in the southern hemisphere. The BMRs start as magnetic bubbles, which eventually appear at the surface. In addition, we see the effect of the large-scale flow and the eddy magnetic diffusivity on the evolution of the BMR’s remnants.

(An animation of this figure is available.)

Figure 3. Model S0. (a) Distribution of the BMR’s area and the magnetic flux. (b) The BMR’s tilt. Blue circles show BMRs from the northern hemisphere, red circles indicate those from the southern hemisphere, the solid line shows the mean tilt, and the dashed lines show the tilt variance.

Figure 2 illustrates the formation of BMRs simulated in model S0. The BMRs start as magnetic bubbles that eventually appear at the surface. In addition, we see the corresponding poleward magnetic flux transport events that are formed from remnants of the BMR’s evolution due to the effects of differential rotation, meridional circulation, and magnetic eddy diffusivity.

Next, we look at distributions of the BMR’s size, tilt angles, and magnetic fluxes. We consider snapshots of the synoptic maps of the nonaxisymmetric radial magnetic field and calculate the continuous area of the magnetic regions using the threshold of $10^{20}$ Mx in the pixel. We find the linear relation between the BMR’s area and flux; see Figure 3(a)). This agrees with observational results (Nagovitsyn & Pevtsov 2021). The distribution of the tilt angle shown in Figure 3(b) is also in close agreement with observations (Nagovitsyn et al. 2021), including the nonlinear behavior at latitudes $>25^\circ$. 
Figure 4 shows results for the probability distributions of the BMR’s flux magnitude, $\Phi$, and the power of the magnetic flux occupying the area $S_a$, $\Phi \cdot S_a$. These parameters show the power-law probability distributions, similarly to the observational analyses of Parnell et al. (2009), Muñoz-Jaramillo et al. (2015), and Nagovitsyn et al. (2021). However, the power-law indexes of in models are higher than in the observations, e.g., Parnell et al. (2009) found $P \propto (\Phi \cdot S_a)^{-1.8}$. Our model reproduces the power law but with a steeper index $\approx -3.1$.

3.2. The Modeled Magnetic Field Evolution

Figure 5 shows the time–latitude diagram of the surface radial magnetic field and the toroidal magnetic field in the subsurface shear layer for model S0. In addition, we show the evolution of the BMR’s injection locations in radius. The butterfly diagrams are similar to the results of P22. The radial locations of the BMR’s injections mark the propagation of the dynamo wave from the bottom of the convection zone toward the surface (Kosovichev & Pipin 2019; Pipin & Kosovichev 2019). Figure 6 illustrates this propagation in a series of magnetic field meridional snapshots.

Noteworthily, the BMR’s injections from the bottom of the convection zone do not produce the surface BMRs. This may be related to the restricted radial sizes of the BMR’s initiation sources (see Equation (21)). Nevertheless, as discussed later, such magnetic flux injections affect the surface nonaxisymmetric magnetic field. With the described tuning of the $C$ parameter, the BMR’s activity in model S0 satisfactorily fits the time–latitude variations of the near-surface toroidal magnetic field. Yet, near the equator, the BMR’s activity goes outside the modeled toroidal field evolution during the epoch of Cycle 23 minimum. The situation is clarified by Figure 6. During the activity minima, the magnetic buoyancy mechanism initiates BMRs not only from weak remnants of the toroidal magnetic field of the old cycle in the subsurface layers very near the equator but also from the deeper layers on the edge of the new dynamo wave of the next cycle.

Figure 7 shows the time–latitude diagrams for models S1, T0, and T1. Similarly to the results of P22, we find that the model produces the smooth evolution of the surface radial magnetic field if we neglect the BMR’s tilt. The comparison of models S0 and T1 shows that details of the magnetic buoyancy mechanism and latitudinal locations of the BMR’s activity are important for the magnetic cycle parameters (Mackay &
Figure 6. Snapshots of the axisymmetric magnetic field evolution during Cycle 23 in model S0. The BMR formation locations are marked by squares. To show the weak toroidal field near the equator, we use an oversaturated color range. Vector potential streamlines show the poloidal magnetic field; dashed lines reflect the counterclockwise direction of the field lines.

Yeates 2012; Miesch & Dikpati 2014). For comparison, we show the results for the pure axisymmetric model, T0, as well. For the given $C_\alpha$, model T0 has the activity cycle (half of the full dynamo period) of about 10.5 yr, which is shorter than the periods of models T1 and S0, which include the BMR’s activity.

Figure 8 shows the mean absolute magnitude of the surface radial magnetic field, $|\mathbf{B}_r|$, and the ratio of the mean magnitude of the axisymmetric surface field, $\langle \mathbf{B}_r \rangle$, to $\langle \mathbf{B}_r \rangle$. This ratio characterizes the level of the nonaxisymmetry of the surface activity in our models. To compare with observations, we use the synoptic maps of the radial magnetic field from the KPO, SOLIS, and SDO/HMI data archives (Harvey et al. 1980; Scherrer et al. 2012; Bertello et al. 2014). Using the synoptic maps, we calculate the mean of the unsigned radial magnetic field and the same for the axisymmetric radial magnetic field. We find that, in the observations, the value of $\langle \mathbf{B}_r \rangle$ reaches about 20–25 G at the solar maxima, and it was around 15 G in Solar Cycles 23 and 24. Bearing in mind the large-scale character of the magnetic activity in our model, we also compare our results with $\langle \mathbf{B}_r \rangle$ calculated for nonaxisymmetric spherical harmonics of the angular order $m < 11$. We find a satisfactory agreement between models S0 and S2 with the solar observations. Model S0 shows that Solar Cycle 25, started in 2019, can be the same as or a little higher than Cycle 24. The same is likely true in model S2.

However, the basal level of $|\mathbf{B}_r|$ during the solar minima is about 2 smaller times than in the solar observations. This is reflected in the behavior of the nonaxisymmetry parameter $|\mathbf{B}_s|/|\mathbf{B}_r|$ as well. It seems that our model misses some parts of the dynamo process that are essential for the large-scale nonaxisymmetric magnetic field of the Sun. Interestingly, the contribution of the BMR’s activity to the axisymmetric magnetic field increases to the observational level just after the magnetic cycle maximum (see the dashed red curve in Figure 8(a)). Model S2 shows the same behavior.

Figure 9 shows the evolution of the polar magnetic field calculated by averaging the north–south antisymmetric radial field components for latitudes higher than 60° and 70°. We see that the polar field in the solar observations shows only a little change between these two measurements during the epoch of solar minimum around 1996. At the same time, the dynamo model shows significant changes in the latitude range in the polar field definition. Model T0 without BMRs shows the same magnitude of the polar field for the cycle minima in 1996, 2007, and 2018. Model T1, with random initialization of BMRs, shows a slight increase of the polar magnetic field during the minima due to the BMR contribution to the polar field magnitude. The same effect is found for model S0, although the magnitude of the mean-field $\alpha$-effect in this model was decreased in Cycles 23 and 24 to match the observed properties of Cycle 24. The mean latitude of the BMR injections in the data-driven model S0 (NOAA data-driven model) is lower than in model T1 with the random BMR initialization. Model S2 shows the best fit for the solar data reproducing plateaus during the minima of Solar Cycles 23 and 24.

In our simulations, we investigated various possibilities to reproduce the basic parameters of Solar Cycles 23 and 24. Besides the long-term variations of the mean-field parameter $C_\alpha$, we considered $\alpha-\beta$ variations. In such cases, to bring the model into the best agreement with observations, we needed to assume very small values $C_\beta$ (corresponding to the mean BMR’s tilt) during the declining phase of Cycle 23 and the growth phase of Cycle 24. However, the observational results of Tlatov et al. (2013) do not show strong variations of the mean BMR’s tilt in different solar cycles. However, we cannot exclude that Cycles 23 and 24 were affected by the emergence of the so-called “rogue” active regions (Nagy et al. 2017; Kumar et al. 2021). This point should be studied separately.

3.3. Torsional Oscillations and Meridional Circulation

The BMR’s activity contributes substantially to the variations of zonal flows (“torsional oscillations”) and meridional circulation. Figure 10 shows the surface time–latitude diagrams of the zonal acceleration, the Lorentz force from the BMRs, and variations of the meridional circulation for Model S2.

The Lorentz force induced by the BMR’s activity, $\mathbf{F}_L$, is determined by the azimuthal average of the magnetic stress as follows:

$$\mathbf{F}_L = \frac{1}{2r \sin \theta} \nabla \left( r \sin \theta \frac{\mathbf{B} \cdot \mathbf{B}_\phi}{4\pi} \right)$$

Similarly to the results of Pipin & Kosovichev (2019), the dynamo model reveals the extended cycles of the torsional oscillations. During the extended Solar Cycle 23, the zonal
acceleration wave, which started at about 50° latitude around 1997, reached the equatorial regions after about 20 yr, in agreement with the helioseismic observations (Kosovichev & Pipin 2019). The time—latitude pattern of the torsional oscillation results from a complicated force balance. The angular momentum balance includes the dynamo-induced variations of turbulent stresses, inertia forces, and variations of the meridional circulation (Pipin & Kosovichev 2019). Noteworthily, the amplitude of the forces that contribute to the balance is more than an order of magnitude higher than the...
magnitude of the zonal acceleration. Here we see that the BMR’s activity produces the additional azimuthal acceleration of the near-equatorial regions of the Sun.

The model shows the north—south asymmetry of the meridional circulation variations in Cycles 23 and 24. This asymmetry is accompanied by trans-equatorial meridional flows of small magnitude during Cycles 23 and 24 (Figure 10(d)). Similar results were found recently by Getting et al. (2021) from helioseismology. In Figure 10(c), we saturate the variations of high magnitude to show the weak variations of the meridional circulation in the near-polar regions. These variations can be interpreted as reverse high-latitude circulation cells. Although the variations at the high latitudes are significantly weaker than the variations in the activity belts, they can affect the surface magnetic flux transport and evolution of the polar magnetic field (e.g., Dikpati et al. 2010).

4. Discussion and Conclusions

We model the physical parameters of Solar Cycles 23 and 24 using the nonlinear 3D mean-field dynamical dynamo model and the observational active region data. Our algorithm for the emergence of BMRs is based on the magnetic buoyancy effect acting on the unstable part of the large-scale magnetic field. The radial positions of the unstable regions are calculated using Parker’s magnetic instability condition. This condition leads to the instability of the toroidal magnetic field at the front edge of the dynamo waves near the bottom of the convection zone and in its upper part, as illustrated in Figure 6. For modeling the BMRs injected from the upper part of the convection zone, we use the NOAA database for coordinates and areas of active regions to specify the locations and sizes of the initial perturbations. In the unstable areas of the lower part of the convection zone, the initial perturbations were distributed randomly in time and longitude.

Our results show that, in most cases, magnetic flux injections from the lower part of the convection zone do not result in the BMR formation on the surface. However, these injections affect the magnitude of the background nonaxisymmetric magnetic field on the surface. Our dynamo models show that the basal level of the nonaxisymmetric magnetic field during the solar minima is about a factor of two smaller than in the solar observations. The possible reason is that our BMR algorithm is not sufficient for deep initialization sources. This affects the onset and decay of the large-scale nonaxisymmetric magnetic activity.

The BMR distributions generated by our dynamo models show that the magnetic flux is directly proportional to BMR’s areas. This result is equally applied to the data-driven models S0, S1, and S2 and to models T1 and T2 with the random distribution of the BMR’s sizes and longitudinal initialization points. The same proportionality was found in observations by Nagovitsyn & Pevtsov (2021) for the sunspot groups areas. The distribution of the BMRs versus the magnetic flux and area shows the inverse power law of index $-3.1$. This qualitatively agrees with the results of Parnell et al. (2009), Muñoz-Jaramillo et al. (2015), and Nagovitsyn et al. (2021). However, they found a less steep power law for the large-scale part of the magnetic field distributions.

In our models, the BMR’s tilt is given theoretically, and it is directly related to the near-surface $\alpha$-effect (Stix 1974; Pipin 2022). The latitudinal profile of the tilt follows the $\cos \theta$ dependence, where $\theta$ is colatitude. We choose the BMR’s tilt to be randomly fluctuating about the mean. The resulting tilt distribution agrees with the solar observations (e.g., Nagovitsyn et al. 2021). In addition, these authors found a tendency for the nonlinear behavior of tilt at latitudes $>25^\circ$. Our model shows similar behavior. It is caused by the $\alpha$-effect modulation due to the large-scale toroidal magnetic field.

In general, the dynamo solution of our model follows the Parker–Yoshimura dynamo-wave law (Pipin 2021). In the weakly nonlinear regime, the dynamo period depends on the $\alpha$-effect magnitude (Noyes et al. 1984). Naturally, the dynamo models also demonstrate the Waldmeier rules (Pipin & Kosovichev 2011). We exploited these properties to model the prolonged minimum of Cycle 23 using the temporal decrease of the turbulent $\alpha$-effect. Generally, we cannot...
exclude other reasons for the unusual behavior of Cycle 23. In particular, Dikpati et al. (2010), using the flux transport model, argued that the long minimum of Cycle 23 could be due to an increase in the latitudinal extent of the meridional circulation cell and a slowdown in the return-flow speed in the deep convection zone.

Pipin & Kosovichev (2019) found that the solar dynamo models show a decrease in the surface poleward flow at high latitudes during the magnetic activity maxima. Here we demonstrate this effect in the data-driven model of Cycles 23 and 24. This result is consistent with the helioseismology analysis of Cycle 24 by Getling et al. (2021). In addition, we find that the hemispheric asymmetry of the sunspot activity during Cycles 23 and 24 results in the north—south asymmetry of the meridional circulation variations. This asymmetry is accompanied by the trans-equatorial meridional flow of a small magnitude.

The variations of the meridional circulation are closely related to the torsional oscillations. These variations show the north—south asymmetry. The model indicates that the BMR’s activity induces additional azimuthal acceleration force. It has the same order of magnitude as the other forcing sources of the torsional oscillations, caused by large-scale dynamo-induced variations of turbulent stresses, inertia forces, and variations of the meridional circulation (see Pipin & Kosovichev 2019).

We conclude that our initial modeling of Solar Cycles 22 and 23, which includes a combination of the global mean-field dynamo and emerging BMRs, shows that the BMR’s activity plays a significant role by affecting the strength and duration of the solar cycles. It was missing in the previous Parker-type dynamo models of the solar cycle. The model qualitatively reproduces the observed north—south asymmetry of these cycles and the variations of the zonal flows (torsional oscillations) and the meridional circulation, including the extended cycle phenomenon, as well as the enhancement of meridional flows converging toward the emerging BMRs and the cross-equatorial meridional flows during the solar maxima. However, our data-driven models show that the BMR’s effect alone cannot explain the prolonged solar minimum between Cycles 23 and 24 and the weak Cycle 24. Instead, we find that the decrease in the Cycle 24 amplitude and the prolonged preceding minimum were probably caused by a decrease of the

Figure 10. (a) The blue-red background color shows variations of the zonal acceleration on the solar surface; green squares show the BMR’s active latitudes. (b) The time—latitude variations of the azimuthal force induced by the BMR’s activity. (c) The time—latitude variations of the meridional velocity, \( \delta U_{\theta} \), in the south—north direction. (d) The amplitude of the cross-equatorial meridional flow in the south—north direction.
turbulent helicity in the bulk of the convection zone during the decaying phase of Cycle 23.

V.P. and V.T. acknowledge the financial support of the Ministry of Science and Higher Education of the Russian Federation (Subsidy No.075-GZ/C3569/278). A.K. acknowledges the partial support of NASA grants NNX14AB70G, 80NSSC20K0602, 80NSSC20K1320, and 80NSSC22M0162.

Data Availability Statements. The data underlying this article are available by request.

ORCID iDs
V. V. Pipin https://orcid.org/0000-0001-9884-1147
A. G. Kosovichev https://orcid.org/0000-0003-0364-4883

References

Babcock, H. W. 1961, ApJ, 133, 572
Bertello, L., Pevtsov, A. A., Petrie, G. J. D., & Keys, D. 2014, SoPh, 289, 2419
Brandenburg, A. 2005, ApJ, 625, 539
Brandenburg, A. 2018, JPhPh, 84, 735840404
Brandenburg, A., & Subramanian, K. 2005, PhR, 417, 1
Brun, A., Garcia, R., Houdek, G., Nandy, D., & Pinsoneault, M. 2014, SSRv, 196, 3031
Charbonneau, P. 2011, LRSP, 2, 2
Dikpati, M. 2016, AsJPh, 25, 341
Dikpati, M., Suresh, A., & Burkepile, J. 2016, SoPh, 291, 339
Getling, A. V., Kosovichev, A. G., & Zhao, J. 2021, ApJL, 908, L50
Hunter, J. D. 2007, CSE, 9, 90
Kitchatinov, L. L., Pipin, V. V., & Radier, G. 1994, AN, 315, 157
Kitchatinov, L. L., & Radier, G. 1992, A&A, 260, 494
Kleeorin, N., & Rogachevskii, I. 1999, PhRvE, 59, 6724
Kosovichev, A. G., & Pipin, V. V. 2019, ApJ, 871, L20
Krause, F., & Rädler, K.-H. 1980, Mean-Field Magnetohydrodynamics and Dynamo Theory (Berlin: Akademie-Verlag), 271
Kumar, P., Nagy, M., Lemerle, A., Binay Karak, B., & Petrovay, K. 2021, ApJ, 909, 87
Leighton, R. B. 1969, ApJ, 156, 1
Mackay, D., & Yeates, A. 2012, LRSP, 9, 6
Miesch, M. S., & Dikpati, M. 2014, ApJL, 785, L8
Mitra, D., Candelaresi, S., Chatterjee, P., Tavakol, R., & Brandenburg, A. 2010, AN, 331, 130
Nagy, M., Lemerle, A., Labonville, F., Petrovay, K., & Charbonneau, P. 2017, SoPh, 292, 167
Noyes, R. W., Weiss, N. O., & Vaughan, A. H. 1984, ApJ, 287, 769
Obridko, V. N., Pipin, V. V., Sokoloff, D., & Shibalova, A. S. 2021, MNRAS, 504, 4900
Parker, E. N. 1955a, ApJ, 121, 491
Parker, E. N. 1955b, ApJ, 122, 293
Parker, E. N. 1971, ApJ, 163, 279
Parker, E. N. 1979, Cosmical Magnetic Fields: Their Origin and Their Activity (Oxford: Clarendon Press)
Parnell, C. E., DeForest, C. E., Hagenaa, H. J., et al. 2009, ApJ, 698, 75
Paxton, B., Bildsten, L., Dotter, A., et al. 2011, ApJS, 192, 3
Paxton, B., Cantillo, M., Arras, P., et al. 2013, ApJS, 208, 4
Pipin, V. 2018, VVpipin/2DSDPdy v0.1.1, Zenodo, doi:10.5281/zenodo.1413149
Pipin, V. V. 2008, GApFD, 102, 21
Pipin, V. V. 2018, IASTP, 179, 185
Pipin, V. V. 2021, MNRAS, 502, 2565
Pipin, V. V. 2022, MNRAS, 514, 1522
Pipin, V. V., & Kichatinov, L. L. 2000, AREp, 44, 771
Pipin, V. V., & Kosovichev, A. G. 2011a, ApJ, 741, 1
Pipin, V. V., & Kosovichev, A. G. 2011b, ApJL, 727, L45
Pipin, V. V., & Kosovichev, A. G. 2015, ApJ, 813, 134
Pipin, V. V., & Kosovichev, A. G. 2018, ApJ, 854, 67
Pipin, V. V., & Kosovichev, A. G. 2019, ApJ, 887, 215
Pipin, V. V., & Kosovichev, A. G. 2020, ApJ, 900, 26
Pipin, V. V., Sokoloff, D. D., Zhang, H., & Kuzanyan, K. M. 2013, ApJ, 768, 46
Radler, K.-H. 1980, AN, 301, 101
Rempel, M. 2005, ApJ, 631, 1286
Roberts, P. H., & Soward, A. M. 1975, AN, 296, 49
Ruediger, G., & Brandenburg, A. 1995, A&A, 296, 557
Schaeffer, N. 2013, GGG, 14, 751
Scherrer, P. H., Schou, J., Bush, R. I., et al. 2012, SoPh, 275, 207
Schrinner, M. 2011, A&A, 533, A108
Schrinner, M., Pettidemange, L., & Dormy, E. 2011, A&A, 530, A140
Shukurov, A., & Subramanian, K. 2021, Astrophysical Magnetic Fields: From Galaxies to the Early Universe, Cambridge Astrophysics (Cambridge: Cambridge Univ. Press), doi:10.1017/9781139046657
Sisti, M. 1974, A&A, 267, 121
Sullivan, C., & Kaszynski, A. 2019, JSS, 4, 1450
Tlatov, A., Illarionov, E., Sokoloff, D., & Pipin, V. 2013, MNRAS, 432, 2975
Vidotto, A. A., Lehmann, L. T., Jardine, M., & Pevtsov, A. A. 2018, MNRAS, 480, 477
Virtanen, P., Gommers, R., Oliphant, T. E., et al. 2020, NatMe, 17, 261
Warnecke, J., Rheinhardt, M., Tuomisto, S., et al. 2018, A&A, 609, A51
Warnecke, J., Rheinhardt, M., Viviani, M., et al. 2021, ApJL, 919, L13
Yoshimura, H. 1975, ApJ, 201, 740

Moffatt, H. K. 1978, Magnetic Field Generation in Electrically Conducting Fluids (Cambridge: Cambridge Univ. Press)
Muñoz-Jaramillo, A., Senkpeil, R. R., Windmuller, J. C., et al. 2015, ApJ, 800, 48
Papanytyn, Y. A., Ospovoa, A. A., & Pevtsova, A. A. 2021, MNRAS, 501, 2782
Papanytyn, Y. A., & Pevtsova, A. A. 2021, ApJ, 906, 27
Parker, E. N. 1955a, ApJ, 121, 491
Parker, E. N. 1955b, ApJ, 122, 293
Parker, E. N. 1971, ApJ, 163, 279
Parker, E. N. 1979, Cosmical Magnetic Fields: Their Origin and Their Activity (Oxford: Clarendon Press)

The Astrophysical Journal, 949:7 (13pp), 2023 May 10