Drag Prediction of NASA Common Research Model Under Stochastic Inflow Conditions

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Abstract. Numerical investigations of NASA Common Research Model under stochastic inflow conditions are analysed in this paper by Non-intrusive Polynomial Chaos (NIPC) method. Reynolds number, Mach number and temperature of inflow conditions are assumed to be independent stochastic variables. It is found that predicted drag mainly depends on Mach number of incoming flow. The contribution of Reynolds number and temperature to the variance of drag is negligible. The mean value of drag shows consistent convergence with grid refinement. The investigation of this paper quantitates the uncertainty induced by stochastic inflow conditions to drag prediction and recognizes the most significant input variable. This will help the validation of numerical methods with experiment.

1. Introduction
To assess computational methods for prediction of industry geometries, AIAA initiated Drag Prediction Workshop (DPW) Series [1-5]. As shown in the report of DPW-V [5], there are significant differences between numerical predictions and wind tunnel tests. One of the possible causes may be that CFD simulations and wind tunnel tests both contain complex uncertainties. From the analysis below, it can be found that inflow conditions in experiments and simulations are not completely consistent. Therefore, the effects of stochastic inflow conditions on drag prediction need to be quantified.

The simulation of DPW-VI [6] model is performed in this paper under stochastic inflow conditions. The geometry and computational grids are introduced, followed by numerical methods presented. The results section shows the statistical characteristics of predicted drag under stochastic inflow conditions.

2. Geometry and Computational Grids
The selected model, presented in Figure 1, is Common Research Model (CRM) [6], which is representative of a modern transonic commercial transport designed to cruise at \( Ma = 0.85 \) and \( Cl = 0.5 \).

The test case is drag prediction under the cruise condition. The deterministic incoming flow conditions are: \( Ma = 0.85 \), \( Re = 5 \times 10^6 \), \( T = 300 K \). A series of three nested unstructured grids are used in computations. The surface mesh of coarse grid is shown in Figure 2, and grid information is presented in Table 1.

The model has been extensively investigated in several wind tunnels recently. However, the experimental data also has obvious uncertainty. Figure 3 shows the comparison of the test results in the NTF wind tunnel and the Ames wind tunnel. The drag under \( Cl = 0.5 \) can be obtained by interpolation. The measured drag is 249.0-249.7 cnts in the NTF wind tunnel and 241.4-243.4 cnts in the Ames wind tunnel.
tunnel. The revised Mach number is between 0.8484 and 0.8502. The Reynolds number is between 4.981 million and 5.005 million. The inflow conditions are not strictly consistent between previous deterministic CFD computations and experiments. So, it is necessary to quantify the drag under non-deterministic flow conditions.

Table 1. Information of three nested grids

|         | Coarse      | Medium     | Fine        |
|---------|-------------|------------|-------------|
| Total cells | 17698918    | 26072640   | 40305331    |
| Tetrahedral | 7275236    | 11010753   | 17013480    |
| Prism   | 10379279    | 14997002   | 23203707    |
| Pyramid | 44403       | 64885      | 88144       |

Figure 1. NASA CRM configuration

Figure 2. Surface mesh of coarse grid

Figure 3. Comparison of experimental data in different wind tunnels

3. Numerical methods
The deterministic simulations in this paper are performed with in-house solver named MFlow [7]. The Polynomial Chaos (PC) approach [8-10] is used to quantify the uncertainty of results. Assume a stochastic input variable $\xi$ that satisfies a determined probability density distribution. For any stochastic output variable $y$ can be expanded in the spectral space formed by the orthogonal basis functions series of the input variable, ie:

$$ y = \sum_{j=0}^{p} \alpha j \phi_j(\xi) $$

(1)

Where $\alpha j$ is the deterministic component, also known as freedom, $\phi_j$ is the basis function of the stochastic variable. The choice of the orthogonal basis function is determined by the probability density function of the stochastic input variable, which satisfies:
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\[ <\psi_j, \psi_j> \overset{\text{def}}{=} \int \psi_j(\xi)\psi_j(\xi)f(\xi)d\xi = \delta_{ij} <\psi_j, \psi_j> \] (2)

Where \( \{\psi_j, j \geq 0\} \) is the sequence of orthogonal polynomials with a weight function of \( f(\xi) \). For a uniformly distributed stochastic variable, the optimal basis function sequence is Legendre polynomials; and for a normally distributed stochastic variable, the optimal basis function sequence is Hermite polynomials. The optimality mentioned here refers to the fact that the statistical information output by the system converges with the increase of the polynomial order and the speed is consistent with the theory. In this paper, NIPC method is used to solve the degree of freedom in the expansion by the regression method. The response value \( \{y_1, y_2, K, y_N\} \) of samples \( \{\xi_1, \xi_2, K, \xi_N\} \) is obtained by deterministic CFD method. The number of samples \( N \) is related to the number of the degree of freedom in PC,

\[ N = n_p (P + 1) = n_p \frac{(n + p)!}{n!p!} \] (3)

Where \( n \) is the dimension of the stochastic input variable, \( p \) is the order of the PC, and \( n_p \) is defined as the oversampling rate, set to 2. An overdetermined equation is formed,

\[ \Psi \alpha = Y \] (4)

\( \Psi \) is the response of samples matrix, \( \psi_j(\xi) \). \( \alpha \) can be solved by the least squares method. Then, the output statistics are,

\[ \mu(y) = \alpha_0 \]
\[ \sigma^2(y) = \sum_{j=1}^{p} \alpha_j^2 \langle \psi_j \psi_j \rangle \] (5)

The Sobol’s indices [11] are used in global sensitivity analysis as a tool for ranking the stochastic input variables.

\[ \frac{\alpha_j^2 \langle \psi_j \psi_j \rangle}{\sigma^2} \] (6)

4. Uncertainty Quantification

It is assumed that Mach number, Reynolds number and temperature of incoming flow are independent stochastic variables satisfying Gaussian distributions. Their respective mean values are the deterministic ones, and the standard deviation is 10% of the mean. There is no strict theoretical analysis about the standard deviation, so the setting here is somewhat arbitrary. The uncertainty of the inflow conditions will be further studied.

In Figure 4, it shows mean values and standard deviation of drag with grids refinement. It can be clearly seen that the mean values of drag have an approximately linear variation with grid refinement, which is similar to the grid convergence curve of deterministic calculations.

For skin frictional drag, the difference between the two is small. For total drag and pressure drag, the mean value under stochastic conditions is about 0.8 cnts higher than deterministic calculations. The standard deviation does not change much with grid refinement. The standard deviation of pressure drag is about an order of magnitude larger than that of skin friction drag and is higher than the standard deviation of total drag. The standard deviation of total drag is about 5.4 cnts, which is about 2.6% of the mean value.
For multivariate UQ problems, it is necessary to evaluate the sensitivity of input variables to identify which factors have a large impact on output. This paper uses two methods for sensitivity analysis. One is qualitative correlation analysis. It is obvious from Table 2 that the Mach number has the greatest correlation relation in predicting drag.

Table 2. Correlation between incoming condition and predicted drag on medium grid

|       | Cd | Cdp | Cdσ |
|-------|----|-----|-----|
| Re    | -0.2701 | -0.2456 | -0.0627 |
| Ma    | 0.9421 | 0.9473 | 0.9361 |
| T     | -0.4702 | -0.4807 | 0.5682 |

The second approach is to use Sobol’s indices to quantify the contribution of each input variable to the variance of output, as shown in Table 3. The results are consistent on three nested grids. Mach number is the only variable that plays a decisive role. Reynolds number only has a certain contribution to the variance of skin friction drag. The uncertainty in inflow temperature can be ignored. The conclusions are basically consistent with correlation analysis. Therefore, for better comparison of CFD simulation and the test data under the cruise conditions, it is necessary to pay extra attention to the discrepancy of Mach number of incoming flow.

Table 3. Sobol’s indices for predicted drag

|       | Cd | Cdp | Cdσ |
|-------|----|-----|-----|
| Re    | 0.006 | 0.006 | 0.006 |
| Ma    | 0.994 | 0.994 | 0.994 |
| T     | 0.004 | 0.003 | 0.004 |

The uncertainty in inflow temperature can be ignored. The conclusions are basically consistent with correlation analysis. Therefore, for better comparison of CFD simulation and the test data under the cruise conditions, it is necessary to pay extra attention to the discrepancy of Mach number of incoming flow.
The relationship between predicted drag and Mach number is shown in Figure 5 with all samples considered. Total drag and pressure drag show a certain nonlinear trend with the change of Mach number, which explains the mean values under stochastic conditions are slightly higher than those under the cruise condition.

In order to further study the influence of uncertainty of incoming flow conditions on numerical predictions, the pressure coefficient and skin friction coefficient at every grid points on the wing surface are taken as the output quantities of interest. The standard deviation distribution of the pressure coefficient and friction coefficient on the upper and lower surfaces of the wing are shown in Figure 6, both of those show a relatively high variation near the shock wave on the upper surface of the wing.

The comparison of pressure distribution between PC predicted mean value and deterministic computation at three spanwise stations are presented in Figure 7, together with standard deviation distribution. The difference in pressure distribution is mainly reflected in the vicinity of the shock wave, where the mean distribution is more even.
5. Conclusion
In this paper, the NIPC method is used to predict the drag of CRM under stochastic inflow conditions. The focus is on statistical results of predicted drag, pressure coefficient and skin friction coefficient. The mean values of drag have an approximately linear relationship with grid refinement. The Mach number of incoming flow is identified to be the only parameter that affects the variation of predicted drag. For better comparison of CFD simulation and the test data, it is necessary to pay extra attention to the discrepancy of Mach number of incoming flow.

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