A quad-stable piezoelectric energy harvester for enhancing energy harvesting from rotational motion: Theoretical model and experiments

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Abstract. Recently, different nonlinear energy harvesters were designed to harvest energy from various rotational motions, which aim to provide power to self-powered wireless sensors. In this paper, a quad-stable piezoelectric energy harvester, comprising of a piezoelectric beam and magnets, is designed to enhance energy harvesting efficiency in rotational motion. In addition, a theoretical model is derived to describe the dynamic performance and the output voltage of the harvester. Furthermore, experimental results demonstrate that the presented harvester in rotational motion exhibits a wider working frequency range compared with that of the bi-stable one. Most importantly, for the rotational speed range of 60 - 420 rpm (1 - 7 HZ), the inter-well oscillation for the harvester can be achieved and corresponding peak voltage is higher than 10 V, which can be used to power wireless sensors. Overall, the quad-stable piezoelectric energy harvester is experimentally verified to be suitable for the low-frequency rotational environment.

1. Introduction

Recently, with the development of Internet of Things (IoT) via sensor networks, the issue of providing power for them has received a great research interest. Energy harvesting from ambition vibrations has become a promising method to achieve self-powered wireless sensors for health structure monitoring [1]. The major mechanisms for vibration energy harvesting are based on piezoelectric [2,3], electrostatic [4-6], electromagnetic [7,8] and triboelectric effects [9,10]. Vibration-based energy harvesters using piezoelectric material have widely explored owning to the advantages of high energy density, simple structure and compact size [11]. Initially, the conventional piezoelectric energy harvesters were designed as linear resonators, however, the vital limitation is that it can only work optimally near their resonance frequencies. It is known that environmental vibration is time-varying in a wide frequency range. In order to obtain high efficiency broadband vibration energy harvesting, researchers proposed different kinds of broadband energy harvesting methods including frequency up-conversion methods [12], nonlinearities [13-18] or multi-degree configurations [19,20], etc.

The nonlinearity of the harvester caused by the magnetic force of magneto-elastic configurations can lead to nonlinear mono-stable [3, 21], bi-stable [22-27], tri-stable [28-33] and quad-stable characteristics [34,35], etc. In case of the bi-stable piezoelectric energy harvester (BPEH), Cottone \textit{et al.} [13] numerically and experimentally demonstrated its advantage with respect to the linear one under random excitation. Stanton \textit{et al.} [22,23] theoretically modelled and experimentally explored the BPEH via a pair of repulsive magnets configuration, and validated the enhanced capabilities. Erturk \textit{et al.} [24,25] and Litak \textit{et al.} [26] experimentally validated the enhancement of energy harvesting for the BPEH under
harmonic and random excitations, respectively. Late on, the tri-stable piezoelectric energy harvesters (TPEH) have attracted more and more attention because of the excellent performance. Zhou et al. [28] proposed a magnetically coupled TPEH, then they experimentally validated its superior performance compared with the bi-stable one under the same excitation. Additionally, Leng et al [30] and Zhu et al [31] numerically and experimentally investigated the performance of the TPEH under several random excitation levels via the Gaussian noise. To further improve the energy harvesting performance, Zhou et al. [34,35] focused on adding external fixed magnets to reduce the potential barriers, thus, the quad-stable characteristics were obtained. They experimentally verified it could realize snap-through easier under random excitation.

Compared with the linear motion, in civil and industrial environments the rotational motion is more widely existing. In recent years, the piezoelectric energy harvester in rotational motion, such as machine [36-40], vehicle wheel [41-43] and wind turbine [44], has become a popular research topic. Khameneifar et al. [38,39] proposed and tested a linear piezoelectric energy harvester utilizing the periodic excitation, which results from the weight component of the tip mass when the hub is in rotational motion, to generate continuous oscillations. Zhang et al. [41,42] used the BPEH to attach on the wheel rim to convert rotational energy and base vibration energy into electrical energy to power the tire pressure monitoring system (TPMS). Febbo et al. [44] designed a cantilever beam-type system for energy harvesting in low-frequency rotational motion, and they experimentally validated the output power range of 26–105 μW over a wide excitation frequency range.

Although the linear and nonlinear BPEHs in rotational motion were investigated in previous studies, the quad-stable piezoelectric energy harvester (QPEH) with advantageous performance due to lower potential barrier has not been explored. In this paper, the QPEH in rotational motion is presented. The paper is organized as follows: in section 2, the proposed QPEH in rotational motion is briefly illustrated. In section 3, the related theoretical model is presented, and then dimensionless parametrization for the governing equations is carried out. In section 4, the corresponding experiments are conducted by making a comparison between the QPEH and the BPEH. Lastly, the conclusions are addressed.

2. Design and operating principle

As the figure 1(a) illustrated, the BPEH consists of a cantilever beam partially covered by piezoelectric patch attached on both sides with a tip magnet and an external fixed magnet. Adjusting the gap distance \( d \) between the surface of magnet A and magnet C, the BPEH will achieve two stable equilibrium positions when the elastic restoring force of the piezoelectric beam is balanced by the repulsive force. The snap-through motion between the stable equilibrium positions could improve energy harvesting performance. The potential energy curves of the BPEH (blue line) is shown in figure 1(c). In order to achieve a large-amplitude snap-through motion, the distance between the two potential

![Figure 1](image-url)
wells should be increased, meanwhile, the corresponding potential barriers will increase simultaneously. To overcome this issue, the QPEH with four potential wells is proposed. Based on the configuration of the BPEH in figure 1(a), two fixed magnets are added and then adjust the configuration parameters $d_1$, $h_1$ and $h_2$, the QPEH with four stable equilibrium positions will be obtained as shown in figure 1(b). It should be noted that the similar configuration was presented in Refs. [34,35].

The corresponding potential energy curves of the proposed QPEH are shown in figure 1(c), where $x = \pm x_1$ and $x = \pm x_2$ are the two inner and two outer stable equilibrium positions, respectively. $x_q = 0$ represents the middle unstable equilibrium position, and $x = \pm x_2$ are the two outer unstable equilibrium positions. From figure 1(c), the potential barriers of the QPEH is lower than that of the BPEH, which means that even in low-level excitation the QPEH can achieve the inter-well oscillation easier over the BPEH.

3. Mathematical modelling

The configuration of the QPEH and related advantages are illustrated in last section. In this section, we mainly focus on the modelling of the QPEH in rotational motion. The schematic diagram of the QPEH in rotational motion is shown in figure 2(a). $r$ denotes the radius of the rotating substrate. When the rotating substrate is driven by the rotational motion, the QPEH can also be driven. Importantly, the weigh component of the tip mass resulting from the rotation will produce a harmonic excitation force to the harvester. Via the piezoelectric effect, the vibration energy will be converted into electricity across the electrical load $R$, which can be used to power low-powered sensors networks.

The theoretical models of the bi-stable, tri-stable and quad-stable energy harvesters under the base excitation were presented in the Refs. [23,28,34,35]. Based on these, for the QPEH in rotational motion, a theoretical model that considers the effect of the rotational motion is presented. As shown in figure 2(b), $XYZ$ is the fixed coordinate system, and $X'Y'Z'$ represents the rotational coordinate system. Additionally, $\theta = \int_0^t \omega dt$ is the time-varying angular displacement of the rotating hub, where $\omega = \dot{\theta}$ is its rotational speed and driven by the input rotational motion. We mainly concern the deformation of the piezoelectric beam in the transverse direction, which is time-varying due to the rotational motion. Thus, the theoretical model should be established in a rotational coordinate system as shown in figure 2(b).

Based on the Lagrangian equation, the theoretical model of the proposed QPEH in rotational motion [45] is derived and expressed in Eqs. (1) and (2):

$$M_0 \ddot{q}(t) + C_q(t) + (K_c + K_\theta \dot{\theta}^2)q(t) + \chi \ddot{\theta} - \partial_q v(t) + F_m = -\Gamma g \cos \theta$$  
$$C_p \dot{v}(t) + R^{-1}v(t) + \partial_q \dot{q}(t) = 0$$

Figure 2. (a) The schematic diagram of the proposed QPEH in rotational motion; (b) the modelling diagram of the QPEH in rotational coordinate system.
where \( q(t) \) is the tip displacement of the harvester in the transverse direction as shown in figure 2(b). \( v(t) \) is the output voltage across the electrical load \( R_e \). \( M_e \) is the equivalent mass. \( C \) is the mechanical damping coefficient. \( C_p \) is the equivalent capacitance of the PZT. \( \vartheta_p \) is the electromechanical coupling coefficient. Moreover, \( \dot{\vartheta} \) and \( \ddot{\vartheta} \) are the angular velocity and acceleration of the rotational motion, respectively. \( K_e \) is the linear coefficient of the harvester, and \( K_c \) is the coefficient of the coupled term \( \dot{\vartheta}^2 q \). \( \chi \) is the coefficient of the angular acceleration, and \( \Gamma \) is the equivalent excitation from the gravity of tip mass caused by the rotational motion. \( g \) is gravitational acceleration \( (g = 9.8 \text{ m/s}^2) \). \( F_a \) is the nonlinear magnetic force along the tip displacement direction.

Furthermore, the equivalent restoring force \( F_e \) [46], which is composed of the elastic force of the beam and the nonlinear magnetic force \( F_{ae} \), can be expended into a polynomial because of the four stable and three unstable equilibrium positions as shown in figure 1(b). As follows:

\[
F_e = K_e q(t) + F_{ae} = K_e q(t) + K_e q(t)^3 + K_e q(t)^5 + K_e q(t)^7
\]

(3)

where \( K_e \) is the linear coefficient of the harvester, and \( K_1, K_3, \) and \( K_5 \) represent nonlinear coefficients of the restoring force, respectively. Thus, Eq. (3) can be rewritten as an expression comprising of all the equilibrium positions:

\[
F_e = K_e \left( x_1^2 - x_2^2 \right) \left( x_2^2 - x_3^2 \right) \left( x_3^2 - x_4^2 \right) (x - x_4)
\]

(4)

where \( x_4 \) represents the middle unstable equilibrium position, and is equal to zero for the QPEH. Thus, the relationship between the coefficients and the equilibrium positions, can be expressed as:

\[
K_5 = -K_1 \left( x_1^2 + x_2^2 + x_3^2 \right)
\]

(5)

\[
K_7 = K_1 \left( x_1^2 x_2^2 + x_2^2 x_3^2 + x_3^2 x_4^2 \right)
\]

(6)

\[
K_9 = -K_1 x_1^2 x_2^3 x_3^2
\]

(7)

The potential energy function of the QPEH can be written as:

\[
U = \frac{K_e}{2} q(t)^2 + \frac{K_1}{4} q(t)^4 + \frac{K_3}{6} q(t)^6 + \frac{K_5}{8} q(t)^8
\]

(8)

The paper mainly focuses on investigating the performance including the dynamic response and the output voltage of the QPEH under various constant rotational speeds without external base excitation, namely, \( \dot{\vartheta} \) is the constant and \( \theta = \vartheta t \). Therefore, \( \dot{\vartheta} = 0 \) in Eq. (1). The mechanical governing equation Eq. (1) can be rewritten as:

\[
M_e \ddot{q}(t) + Cq(t) + \left( K_e + K_1 \dot{\vartheta}^2 \right) q(t) + K_3 q(t)^3 + K_5 q(t)^5 + K_7 q(t)^7 - \vartheta_p v(t) = -\Gamma g \cos \left( \vartheta \cdot t \right)
\]

(9)

Based on Eqs. (2) and (9), the following non-dimensional equations can be derived:

\[
\ddot{y}(\tau) + 2 \zeta \dot{y}(\tau) + \left( 1 + \gamma \omega^2 \right) y(\tau) + k_3 u^3(\tau) + k_5 u^5(\tau) + k_7 u^7(\tau) - \sigma^2 y(\tau) = F \cos(\omega \tau)
\]

(10)

\[
\dot{\vartheta}(\tau) + \lambda y(\tau) + \dot{u}(\tau) = 0
\]

(11)

where \( \omega_n \) is the nominal short-circuit frequency, and other variables are shown as follows:

\[
\zeta = \frac{C}{\sqrt{M_e K_1}}; \quad \sigma^2 = \frac{\vartheta_p^2}{C_p K_1}; \quad \omega_n = \frac{\sqrt{K_e}}{M_e}; \quad \gamma = \frac{K_1}{M_e}; \quad \tau = t \omega_n; \quad \omega = \frac{\dot{\vartheta}}{\omega_n};
\]

\[
k_3 = \frac{K_3}{K_1 \omega_n^2}; \quad k_5 = \frac{K_5}{K_1 \omega_n^4}; \quad k_7 = \frac{K_7}{K_1 \omega_n^6}; \quad \lambda = \frac{1}{C_p R_e \omega_n}; \quad F = \frac{\Gamma g}{M_e}.
\]

4. Experimental validation

In this section, experiments at various constant rotational speeds are carried out. The experimental setup is shown in figure 3(a). The rotational motion is produced by a servo motor (SGM7J, YASKAWA), which is controlled by a matching servo controller (SGD7S, YASKAWA). The harvesters including the QPEH and the BPEH, and corresponding laser sensors (HG-C1100, Panasonic) are installed on a
rotation plate as shown in figure 3(a). The electrical slip ring (SENRING, CHINA) with a rotor and a stator, is mounted on the rotation axis to address the issue of wire connection in rotational motion. Additionally, the output voltage and the displacement of each harvester are monitored and measured by a digital oscilloscope (DSOX3014T, KEYSIGHT), whose internal resistance is 1 MΩ. In the experiments, all the magnets are NdFeB cylinder magnets. For the QPEH, the three external fixed magnets have the same dimension of D12×1.7 mm³ (D represents the diameter). The tip mass (magnet A) is consisting of two magnets D12×2.7 mm³ and one D12×1.7mm³. The substrate layer is made of stainless steel and has the dimension of 95×12×0.2 mm³. It should be pointed out that the displacement in all experiments are measured at the distance 54 mm from the fixed end of the harvester by the fixed laser sensors on a rotational plate. For the BPEH, the two stable equilibrium positions are illustrated in figure 3(b), and d=14 mm as shown in figure 2(a). The four stable equilibrium positions of the QPEH are shown in figure 3(c), and the related parameters are h₁=20 mm, h₂=20 mm and d=14 mm as the schematic diagram shown in figure 2(b).

Figure 3. (a) The experiment setup; (b) two stable equilibrium positions of the BPEH; (c) four stable equilibrium positions of the QPEH.

The experimental results, obtained from the rotational speeds 60 rpm, 200 rpm, 240 rpm, 410 rpm, 500 rpm and 840 rpm are respectively shown in figure 4 and figure 5. When the rotational speed is 60 rpm, the inter-well oscillation cannot be triggered for the BPEH, however, the chaotic oscillation appears for the QPEH with lower potential barriers as shown in figure 4(a) and 4(b). As observed from the figure 4(c) and 4(d) when the rotational speed is 200 rpm, the BPEH is still performing the intra-well oscillation without any snap-through phenomenon, while the high-energy global inter-well oscillation for the QPEH is achieved. When the rotational speed is increased to 240 rpm, the BPEH begins to experience the high-energy inter-well oscillation, and the peak voltage of the BPEH and the QPEH are 7 V and 10 V respectively as shown in figure 4(e) and 4(f).
Figure 4. Experimental displacements and output voltages of the BPEH (blue line) and the QPEH (red line): (a) and (b) 60 rpm; (c) and (d) 200 rpm; (e) and (f) 240 rpm.

Figure 5. Experimental displacements and output voltages of the BPEH (blue line) and the QPEH (red line): (a) and (b) 410 rpm; (c) and (d) 500 rpm; (e) and (f) 840 rpm.
As the rotational speed is increased to 410 rpm, both the BPEH and the QPEH are still exhibiting the high-energy inter-well oscillation, and corresponding peak voltages are dramatically increased to almost 15 V as shown in figure 5(a) and 5(b). It means that in this rotational frequency the weigh component of both two harvesters can overcome their corresponding potential barriers perfectly. However, when the rotational speed is increased to 500 rpm, it is found from the figure 5(c) and 5(d) that the dynamic performance of both two harvesters decreases significantly, and corresponding output voltage also deceases due to the centrifugal stiffness effect. Namely when the rotational speed reaches a high speed, the tip mass (magnet A) of each harvester will produce a non-negligible centrifugal force caused by the rotational motion. Thus, the centrifugal stiffness effect occurs. Furthermore, the phenomenon also be verified while the rotational speed is increased to 840 rpm, especially for the QPEH (red line) as shown in figure 5(e) and 5(f).

5. Conclusions

In summary, the paper explores the QPEH for enhancing energy harvesting performance in rotational motion. The theoretical model of the QPEH is presented, and then dimensionless parametrization for the governing equations is carried out. Based on this model, the dynamic characteristics and the output voltage can be predicted. Experiments are conducted under various constant rotating speeds by making a comparison between the BPEH and the QPEH. Experimental results demonstrate that the presented QPEH is superior to the BPEH in terms of the broadband working frequency range in rotational motion. For the rotational speed range of 60 rpm - 420 rpm (1 HZ - 7HZ), the inter-well oscillation for the QPEH can be achieved and corresponding peak voltage is higher than 10 V, which can be used to power wireless sensors in rotational motion. In the future work, we will focus on the self-tuning harvester based on the centrifugal stiffness effect for optimizing vibration energy harvesting in rotational motion.

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References

[1] Yang Z, Zhou S, Zu J and Inman D J 2018 Joule 2 642-697.
[2] Wang J, Zhou S, Zhang Z and Yurchenko D 2019 Energ. Convers. Manage. 181, 645-652.
[3] Fan K, Tan Q, Zhang Y, Liu S, Cai M and Zhu Y 2018 Appl. Phys. Lett. 112 123901.
[4] Suzuki Y 2011 IEEE T. Electr. Electr. 6 101-111.
[5] Tao K, Tang L, Wu J, Lye S W, Chang H and Miao J 2018 J. Microelectromech. S. 27 276-288.
[6] Zhang Y, Wang T, Zhang A, Peng Z, Luo D, Chen R and Wang F 2016 Rev. Sci. Instrum. 87 125001.
[7] Liu H, Hou C, Lin J, Li Y, Shi Q, Chen T and Lee C 2018 Appl. Phys. Lett. 113 203901.
[8] Li Z, Zuo L, Luhrs G, Lin L and Qin Y 2013 IEEE T. Veh. Technol. 62 1065-74.
[9] Meng X, Cheng Q, Jiang X, Fang Z, Chen X, Li S and Wang Z 2018 Nano Energy 51 721-727.
[10] Wang P, Pan L, Wang J, Xu M, Dai G, Zou H and Wang Z 2018 ACS Nano 12 9433-40.
[11] Liu H, Zhong J, Lee C and Lin L 2018 Appl. Phys. Rev. 5 041306.
[12] Li M, Wen Y, Li P, Yang J and Dai X 2011 Sensor Actuat. A-Phys. 166 102-110.
[13] Cottone F, Vocca H and Gammaitoni L 2009 Phys. Rev. Lett. 102 080601.
[14] Daqaq M F, Masana R, Erturk A and Quinn D 2014 Appl. Mech. Rev. 66 040801.
[15] Ferrari M, Ferrari V, Guizzetti M, Andò B, Baglio S and Trigona C 2010 Sensor Actuat. A-Phys. 162 425-431.
[16] Ferrari M, Bau M, Guizzetti M and Ferrari V 2011 Sensor Actuat. A-Phys. 172 287-292.
[17] Zhou S, Cao J, Erturk A and Lin J 2013 Appl. Phys. Lett. 102 173901.
[18] Huang D, Zhou S, Litak G 2019 Commun. Nonlinear Sci. 69 270-286.
[19] Zou H, Zhang W, Li W, Wei K, Gao Q, Peng Z and Meng G 2017 Energ. Convers. Manage. 148 1391-98.
[20] Wang H and Tang L 2017 Mech. Syst. Signal Pr. 86 29-39.
[21] Kumar A, Ali S and Arockiarajan A 2018 Appl. Phys. Lett. 112 233901.
[22] Stanton S C, McGeehe C C and Mann B P 2010 Physica D. 239 640-653.
[23] Stanton S C, Erturk A, Mann B P and Inman D J 2010 J. Appl. Phys. 108 074903.
[24] Erturk A, Hoffmann J and Inman D J 2009 Appl. Phys. Lett. 94 254102.
[25] Erturk A and Inman D J 2011 J. Sound Vib. 330 pp 2339-53.
[26] Litak G, Friswell M I and Adhikari S 2010 Appl. Phys. Lett. 96 214103.
[27] Zheng R, Nakano K, Hu H, Su D and Cartmell M P 2014 J. Sound Vib. 333 2568-87.
[28] Zhou S, Cao J, Inman D J, Lin J, Liu S and Wang Z 2014 Appl. Energ. 133 33-39.
[29] Zhou S and Zuo L 2018 Commun. Nonlinear Sci. 61 271-284.
[30] Leng Y, Tan D, Liu J, Zhang Y and Fan S 2017 J. Sound Vib. 406 146-160.
[31] Zhu P, Ren X, Qin W, Yang Y and Zhou Z 2017 Arch. Appl. Mech. 87 1541-54.
[32] Panyam M and Daqaq M F 2017 J. Sound Vib. 386 336-358.
[33] Kim P, Son D and Seok J 2016 Appl. Phys. Lett. 108 243902.
[34] Zhou Z, Qin W and Zhu P 2016 Sensor Actuat. A-Phys. 243 151-158.
[35] Zhou Z, Qin W, Yang Y and Zhu P 2017 Sensor Actuat. A-Phys. 265 297-304.
[36] Elhadidi M, Helal M, Nassar O, Arafa M and Zeyada Y 2015 SPIE. 9431 94310S.
[37] Xie Z, Xiong J, Zhang D, Wang T, Shao Y and Huang W 2018 Appl. Sci. 8 1418.
[38] Khameneifar F, Moallem M and Arzanpour S 2011 J. Vib. Acoust. 133 011005.
[39] Khameneifar F, Arzanpour S and Moallem M 2013 IEEE-ASME T. Mech. 18 1527-34.
[40] Guan M and Liao W 2016 Energ. Convers. Manage. 111 239-244.
[41] Zhang Y, Zheng R, Kaizuka T, Su D and Nakano K 2015 J. Phys.: Conf. Ser. 660 012126.
[42] Zhang Y, Zheng R, Nakano K and Cartmell M P 2018 Appl. Phys. Lett. 112 143901.
[43] Askari H, Hashemi E, Khajepour A and Khamesee M B, Wang Z 2018 Adv. Mater. Technol. 1800105.
[44] Febbo M, Machado S P, Gatti C D and Ramirez J M 2017 Energ. Convers. Manage. 152 166-175.
[45] Mei X, Zhou S, Yang Z, Kaizuka T and Nakano K 2019 Appl. Phys. Express in press https://doi.org/10.7567/1882-0786/ab0b75
[46] Zhou S, Cao J, Inman D J, Liu S, Wang W and Lin J 2015 Appl. Phys. Lett. 106 093901.