Modal identification of cylindrical shell with high damping using reduction method focusing on symmetry of mode shape

Daiki Tajiri¹*, Masaki Ojiro¹, Masami Matsubara² and Shozo Kawamura²

1 Graduate School, Toyohashi University of Technology, 1-1 Hibarigaoka, Tempaku Toyohashi Aichi 441-8580, Japan
2 Department of Mechanical Engineering, Toyohashi University of Technology, 1-1 Hibarigaoka, Tempaku Toyohashi Aichi 441-8580, Japan
*E-mail: d189103@edu.tut.ac.jp

Abstract. The viscoelastic structures such as tires have high damping characteristics, so that resonance peaks do not appear clearly and they are difficult to identify the existence of modes in the high frequency region. Previous studies have proposed methods that make it easier to understand modes. In that method, the number of modes are reduced by using the fact that the basis function of the mode shape of the cylindrical structure can be expressed by a periodic function. However, a large number of measurement points are placed on the structure and data is collected, so the workload of the experiment is large. In this study, we conducted a numerical experiment of the mode reduction method using some measurement point data for the purpose of reducing the workload, and evaluate the influence on the identification accuracy. As a result, it was confirmed that the identification results of the natural frequency and the structural damping coefficient had sufficient accuracy even if some measurement points were used.

1. Introduction

The experimental modal analysis is widely used to understand the vibration characteristics of mechanical structures. This is a method for identifying modal characteristics by curve fitting with the theoretical formula based on the frequency response function (FRF) obtained from vibration tests [1][2]. Recently, highly accurate mode identification is required. This is because the problem of vibration and noise has shifted to the high frequency side including many modes [3], making mode identification difficult. Furthermore, viscoelastic structures such as tires have high damping characteristics, so that resonance peaks do not appear clearly and they are difficult to identify the existence of modes in the high frequency region [4]. Therefore, tire mode identification is generally targeted at the low frequency region [5][6][7].

From such a background, a method of identifying after reducing the number of modes from FRF has been proposed. In the complex mode indicator function (CMIF) method [8], only the one-degree-of-freedom mode component is extracted by performing singular value decomposition on the response obtained from the measurement points installed in the entire machine structure. In this method, the extraction accuracy of the mode component depends on the position and number of measurement points. Therefore, there is a problem that the number of experiment man-hours increases to obtain the necessary identification accuracy. There is also a method (mode expansion reduction method) that reduces the number of modes by focusing on the mode shape of the mechanical structure [9]. This method reduces the FRF by utilizing the fact that the mode shape can be expressed by a periodic function in the circumferential direction when a cylindrical shell such as a tire is used. In previous studies, it has been reported that mode identification in high damping systems and high frequency regions is possible by...
applying the reduction method and the conventional curve fitting method. On the other hand, the experimental load is not reduced because a large number of measurement points are placed on the entire cylindrical shell. In addition, although the mode identification results have been reported, the accuracy of the reduction method has not been fully discussed. Therefore, in this study, we conduct a numerical experiment of the mode reduction method by using some measurement points data for the purpose of reducing the workload, and evaluate the influence on the identification accuracy. About the target structure, we set the cylindrical shell with high damping and used a periodic function as the basis function. Here, we define a high damping system as a system with a structural damping coefficient of 5.0×10⁻² or higher. Besides, in order to improve the identification accuracy, the linear fit method [10][11] is used for modal identification.

2. Circumferential mode reduction method for cylindrical structures

In the previous study [9], the response points were set on multiple virtual rings that arranged in the whole cylindrical shell as shown in figure 1. The response of each ring could be expressed using a periodic function. In this study, only the response of the measurement points set in the range of 0 to 180 ° of the virtual ring is used for the purpose of reducing the workload of the experiment. The reason is that the case of a cylindrical shell, the positions of the antinodes and nodes of the mode shape excited by the position of the input point can be easily grasped. Here, the basic theory of circumferential mode reduction when using the response of the whole measurement point is explained.

![Figure 1. Input points and response points set on the virtual ring](image)

The excitation points \( P \) are arbitrarily arranged on the cylindrical shell. The response points are arranged with \( M \) virtual rings at equal intervals in the axial direction and \( N \) points at equal intervals \((2\pi/N)\) in the circumferential direction of each ring. Therefore, the total of the response points are \( M \times N \) points. The FRF of all response points are represented by the following matrix \( H(\omega) \). \( \omega \) is an angular frequency.

\[
H(\omega) = \begin{bmatrix} H_1(\omega) \\ H_2(\omega) \\ \vdots \\ H_N(\omega) \end{bmatrix}
\]

(1)

\[
H_{m,p}(\omega) = \begin{bmatrix} H_{1,m}(\omega) \\ \vdots \\ H_{N,m}(\omega) \end{bmatrix} \begin{bmatrix} m = 1, \cdots, M \\ p = 1, \cdots, P \end{bmatrix}
\]

(2)

\( H(\omega) \) is \( M \times N \times P \) matrix in equation (1). \( H_{m,p}(\omega) \) is \( N \) matrix between the \( p \)-th input point and the response point on the \( m \)-th virtual ring.

Here, the bending vibration of the virtual ring is expressed as a basis function using a periodic function. The bending vibration of the cylindrical shell is excited in the radial direction. Specifically, it is excited
every wave numbers in the circumferential direction and every half wave numbers in the axial direction. Therefore, the wave number \( i \) in the circumferential direction is defined as the circumferential \( i \)-th mode, and the half wave number \( j \) in the axial direction is defined as the \( j \)-th mode in the axial direction. Figure 2 shows an example of the circumferential 0th to 4th modes.

\[ H_n(\omega) = \frac{A_0(\omega)}{2} + \sum_{i=1}^{N/2-1} \left[ A_i(\omega) \cos \left( i \frac{2\pi n}{N} \right) + B_i(\omega) \sin \left( i \frac{2\pi n}{N} \right) \right] + \frac{A_{N/2}(\omega)}{2} \cos(\pi n) \]  

(3)

\[ A_i(\omega) = \frac{2}{N} \sum_{n=1}^{\infty} H_n(\omega) \cos \left( i \frac{2\pi n}{N} \right) \]  

(4)

\[ B_i(\omega) = \frac{2}{N} \sum_{n=1}^{\infty} H_n(\omega) \sin \left( i \frac{2\pi n}{N} \right) \]  

(5)

equations (3) and (4) are the sums of the circumferential direction 0 to \( N/2 \)-th mode components. So the circumferential \( i \)-th mode component can be extracted as following equation.

\[ H_n^{(i)}(\omega) = A_i(\omega) \cos \left( i \frac{2\pi n}{N} \right) + B_i(\omega) \sin \left( i \frac{2\pi n}{N} \right) \]  

(6)

From the above, \( H_{n,p}(\omega) \), which have \( N \) degrees of freedom, is reduced to a matrix with two degrees of freedom for each circumferential mode order as shown in the following equation.

\[ \mathbf{H}_{n,p}^{(i)}(\omega) = \begin{bmatrix} A_i(\omega) \\ B_i(\omega) \end{bmatrix} \]  

(6)

Therefore, the FRF matrix \( \mathbf{H}(\omega) \) that had \( M \times N \times P \) degrees of freedom is reduced to an FRF matrix with \( 2 \times P \times M \) degrees of freedom for each circumferential mode order. At this time, only the circumferential \( i \)-th mode component is extracted, so that modes with different axial orders exist as eigenmodes, and the mode density is greatly reduced.
3. Modal identification using finite element model of cylindrical shell

3.1. Cylindrical shell model and analysis conditions

Figure 3 shows the cylindrical shell model created by commercial software MSC.Patran. Table 1 shows the design parameters. FRF is calculated by a hysteretic damping system with a structural damping coefficient set to \(5.0 \times 10^{-2}\). The cylindrical shell is simply supported at both ends. The input point is the point of \(\theta = 0\) in the second ring (marked \(\bigcirc\) in figure 3), and the input direction is perpendicular to the cylindrical shell surface. The response points are 80 points (8 virtual rings \(\times\) 10 points on the ring) distributed at equal intervals in the range of \(0 \leq \theta \leq \pi\) on the cylindrical shell. This method is effective for measuring the response of cylindrical structures. The reasons are that the positions of the antinodes and nodes of the excited mode shape are determined by the positions of the input points, and the positions can be easily grasped.

Table 1. The design parameters

| Parameters | Values      |
|------------|-------------|
| Young's modulus | 200 [GPa]   |
| Poisson's ratio  | 0.3 [-]    |
| Radius       | 0.5 [m]    |
| Length       | 1.26 [m]   |
| Thickness    | 0.01 [m]   |

Figure 3. Input and response points on virtual ring in the cylindrical shell model

3.2. Effects of circumferential mode reduction and results of modal identification

As an example of numerical analysis, using the driving point FRF in the cylindrical shell model, the circumferential 3rd mode and 5th mode components are extracted based on the circumferential mode reduction method. Figure 4 shows the effect of circumferential mode reduction. Figure 4 (a) shows the original FRF (Original; solid black line) and all mode components calculated in the cylindrical shell model. Figure 4 (b) shows the original FRF and the reduced circumferential mode components of all orders. It can be seen that modes that could not be clearly identified from original FRF were separated by circumferential mode reduction. This effect makes it relatively easy to identify the mode characteristics.

Figure 4. The effect of circumferential mode reduction
Subsequently, the modal characteristics are identified by using the FRF of the extracted circumferential
3rd. mode component and 5th. mode component shown figure 5 (a)(b) (Extracted FRF; blue solid line).
The FRFs in figure 5 (a)(b) has five resonant peaks. These are the axial 1st to 5th mode from the low
frequency side. An example of a mode shape representing the coupling in the circumferential direction
and the axial direction is also shown in figure 5.
In this study, the modal characteristics are identified using the linear fit method [10][11], which is one
of the curve fitting methods. This method uses simultaneous equations of the real and imaginary parts
of the FRF. In these simultaneous equations, the natural frequency and damping characteristic are
unknown quantities, and they are obtained using the least squares method (LSM).
Table 2 shows the identified natural frequency $f_{np}$, structural damping coefficient $\eta_p$, and their relative
errors with respect to their exact values. The identification results of natural frequency and structural
damping coefficient are close to exact values, and it can be seen that both can be identified within a very
small error range. In addition, it was confirmed that there was no difference in identification accuracy
when using the response of the entire cylindrical shell ($0 \leq \theta \leq 2\pi$). Similarly, the mode characteristics
can be identified with high accuracy for other circumferential modes. Figures 5(a)(b) shows the
reconstructed FRF (Reconstructed FRF; red broken line) using the identified mode characteristics. Since
all the modal characteristics were identified with high accuracy, the blue solid line and the red broken
line coincide.

**Table 2. The exact values and identification values of modal characteristics**

| Axial order | Exact value | Identification value | Relative error [%] |
|-------------|-------------|----------------------|-------------------|
|             | $f_{np}$ [Hz] | $\eta_p$ [-] | $f_{np}$ [Hz] | $\eta_p$ [-] | $f_{np}$ [Hz] | $\eta_p$ [-] |
| (a) Circumferential 3rd mode | | | | | |
| 1 | 350.45 | 5.00 x 10^{-2} | 350.46 | 5.00 x 10^{-2} | 2.59 x 10^{-6} | 4.30 x 10^{-6} |
| 2 | 696.44 | 5.00 x 10^{-2} | 696.44 | 5.00 x 10^{-2} | 2.42 x 10^{-7} | 2.07 x 10^{-5} |
| 3 | 1023.92 | 5.00 x 10^{-2} | 1023.92 | 5.00 x 10^{-2} | 4.35 x 10^{-6} | 3.77 x 10^{-5} |
| 4 | 1268.98 | 5.00 x 10^{-2} | 1268.98 | 5.00 x 10^{-2} | 2.88 x 10^{-6} | 2.91 x 10^{-5} |
| 5 | 1453.26 | 5.00 x 10^{-2} | 1453.26 | 5.00 x 10^{-2} | 2.98 x 10^{-6} | 3.01 x 10^{-5} |
| (b) Circumferential 5th mode | | | | | |
| 1 | 311.60 | 5.00 x 10^{-2} | 311.60 | 5.00 x 10^{-2} | 9.00 x 10^{-7} | 1.98 x 10^{-5} |
| 2 | 500.33 | 5.00 x 10^{-2} | 500.33 | 5.00 x 10^{-2} | 2.54 x 10^{-6} | 3.69 x 10^{-5} |
| 3 | 752.76 | 5.00 x 10^{-2} | 752.76 | 5.00 x 10^{-2} | 5.65 x 10^{-6} | 2.78 x 10^{-5} |
| 4 | 1017.51 | 5.00 x 10^{-2} | 1017.51 | 5.00 x 10^{-2} | 4.09 x 10^{-6} | 3.24 x 10^{-5} |
| 5 | 1271.93 | 5.00 x 10^{-2} | 1271.94 | 5.00 x 10^{-2} | 2.49 x 10^{-6} | 3.63 x 10^{-5} |

**Figure 5. Comparison of the original and the reconstructed FRF**
4. Conclusion
In this study, we conducted a numerical experiment of the mode reduction method using some measurement point data for the purpose of reducing the workload. Then, the influence on the identification accuracy was evaluated. The target structure was a cylindrical shell with high damping, and a periodic function was used as the basis function. Although the circumferential 3rd mode and 5th mode were described as examples, the accuracy of the identification results of the natural frequency and the structural damping coefficient was very high in the circumferential modes of all orders. From the above, it was understood that the circumferential mode reduction method had sufficient accuracy and could reduce the experimental workload even when only the data of some measurement points were used.

5. Future works
In the future, the influence on the identification accuracy will be grasped by numerical analysis considering measurement noise. Then, we plan to conduct experimental verification on cylindrical structures such as automobile tires.

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