Effect of the payload mass on forces acting from the overhead crane drives during movement in the mode of suppressing uncontrolled oscillations

Mikhail S Korytov¹, Vitaly S Shcherbakov¹ and Vladimir V Titenko²
¹Siberian State Automobile and Highway University, Mira Ave., 5, Omsk, Russia
²Omsk State Technical University, Mira Ave, 11, Omsk, 644050, Russia

Abstract. Using two-point Hermite splines, the reference spatial trajectory of a payload on a non-rigid pendulum suspension of an overhead crane was set as a combination of plane controlled oscillations in two mutually perpendicular movement directions of the bridge and the trolley. Under the absent uncontrolled payload oscillations, payload, suspension point coordinates and the rope deviation angles were calculated. Using the example of a constant spatial trajectory of the payload bypassing a single obstacle in the absence of uncontrolled oscillations, the payload mass effect on the forces acting from the side of the bridge drives and the trolley was investigated. The time dependences for horizontal components of the drive side forces were determined using an overhead crane simulation model developed in the Simscape Multibody Matlab application. Payload of various masses was repeatedly moved along the reference constant trajectory by rigidly setting the bridge and trolley movement trajectories. By virtual sensors, the forces acting from the drives on the bridge and trolley were evaluated. The payload mass effect on the maximum values of drive forces is established to be linear. The results can be used in the design and development of intelligent mechatronic control systems for overhead and gantry cranes.

Keywords: payload mass, Hermite spline, payload oscillations, reference trajectory, pendulum suspension, overhead crane

1. Introduction
Overhead cranes (OC) [1] represent mechatronic systems [2] of non-linear dynamics [3], in which the movement of the bridge and the payload trolley [4] cause swinging of the payload [5], thus producing a negative impact on the productivity and safety of the entire work process [6]. An increase in the payload movement velocities by hoisting cranes with a flexible rope suspension leads to an increase in the deviation angles of the payload rope suspension, large dynamic loads and more complex problems in controlling such systems. This is the reason why, though being one of the oldest problems [7], the study of the payload swing phenomenon [8] remains to be an urgent research task [9, 10].

An extensive number of research works [11] on the dynamics of OC [12] and individual elements of their oscillation system [13] have been recently published, including those devoted to the search for effective methods of manual [14] and automatic control [15] of OC mobile parts [16, 17]. For the purpose of limiting uncontrolled oscillations of the payload on a flexible suspension, the following approaches are used: shaping regulation [15, 18], fuzzy logic control [19, 20], fuzzy compensation [21], including adaptive one [22], the neural networks mathematical apparatus [23], geometric analysis...
[24], the inverse dynamics method [25] and others [26, 27]. The latter methods, which allow the problem to be solved in an explicit analytical form [27, 28], have the advantage of being more accurate, simple and fast. The work [28] proposed to use two-point Hermite splines for setting the reference trajectory of the payload on a non-rigid pendulum suspension in a separate spatial plane. In this case, uncontrolled components of the payload oscillations were absent, while the payload and payload suspension point coordinates, as well as the deviation angles of the payload rope from the gravitational vertical, were calculated using analytical expressions. The as-derived form of Hermite splines is suitable for setting the values of a certain number of derivatives at the start and end points of the payload trajectory. The error of this method appears to be negligible at relatively low movement velocities of the OC moving parts, which is typical of the vast majority of real industrial facilities [28]. This makes it possible to apply the superposition of movements and to represent the spatial movements of the dynamic OC system as a set of fully controlled payload oscillations in the bridge and trolley axes.

When designing automatic control systems for OC and drives for their moving parts on the basis of the proposed approach (synthesis and subsequent implementation of the reference movement trajectory of the payload suspension point on the trolley depending on the reference payload movement trajectory), the question of the values for the forces applied to the OC moving parts from the side of their motion drives is of great practical importance. The relatively small values of the required forces and the smooth nature of their change provide the opportunity for the physical implementation of the proposed approach using existing designs of drives and their electric drive controllers based on frequency regulation [12].

This paper sets out to investigate the effect of the payload mass on the forces acting from OC drives during movement in a fully controlled oscillation mode.

2. Formulation of the problem

This research considers the main part of the OC workflow, during which the payload is moving along a horizontal plane on a payload rope of a constant length. The \( x(t) \) and \( y(t) \) trajectories of the payload movement in the horizontal plane are given. These trajectories are specified using Hermite splines. In addition, on the basis of the reference payload trajectory, the \( x(t) \) and \( y(t) \) corresponding trajectories of the suspension point, providing the payload movement in the absence of uncontrolled oscillations, are uniquely calculated and set. The movement is carried out from the state of rest and the absence of movement to another similar state characterised by new horizontal coordinates. The payload spatial trajectory, having the shape of an arc to bypass a single obstacle, is defined as a combination of flat trajectories in two mutually perpendicular directions of the bridge and the trolley movement. The research task is to investigate the effect of the \( m \) transported payload mass on the \( F_x \) and \( F_y \) forces acting on the bridge and the trolley, respectively, from the side of the drives during the movement. In this case, the reference spatial trajectory of the payload movement remains fixed.

3. Theoretical part

In order to synthesise the movement trajectory of the suspension point (located on a trolley) in each of two mutually perpendicular planes of the OC bridge and trolley movement separately, flat loading diagrams were applied (see figure 1).
Figure 1. Loading diagrams of the payload oscillations during movement of the suspension point in the movement planes of the overhead crane bridge (a) and the payload trolley (b).

Figure 2. Spatial loading diagram of the payload movement by the overhead crane with a rope suspension.

At angles of rope deviation from the gravitational vertical of less than 5° (the actual operating conditions of most industrial OCs), the spatial payload movement (see figure 2) can be quite accurately described as a superposition of plane oscillations of the payload in two mutually perpendicular planes [12, 28]. These are the plane of the bridge movement (along the $X$ axis, the $\theta_x$ angle, see figure 1(a)) and the plane of the trolley movement (along the $Y$ axis, the $\theta_y$ angle, see figure 1(b)).
The parameters of the pendulum dynamic system are indicated by the following symbols: \( L \) is the length of the payload rope (from the suspension point on the trolley to the payload mass centre), \( m \); \( q \) is the deviation angle of the OC lifting rope from the vertical in a separate plane (identical to the \( \theta \), angle in the bridge movement plane or the \( \theta_y \) angle in the trolley movement plane), rad; \( m \) is the payload mass, kg; \( g \) is the gravity acceleration, \( \text{m/s}^2 \); \( t \) is the current time of payload movement, s; \( T \) is the specified final time of payload movement, s; \( b=B/mL^2 \) is the oscillation suppression coefficient in a separate plane (identical to the \( b_x \) oscillation suppression coefficient in the bridge plane or the \( b_y \) oscillation suppression coefficient of the trolley movement plane), \( \text{s}^{-1} \); \( B \) is the coefficient of the payload rope drag torque to the angular rotation relative to the gravitational vertical in a separate plane, kg·\( \text{m}^2/\text{s} \); \( x \) is the linear horizontal coordinate of the suspension point displacement along the \( X \) axis, m; \( y \) is the linear horizontal coordinate of the suspension point displacement along the \( Y \) axis, m; \( x \) is the linear horizontal coordinate of the payload centre point displacement along the \( X \) axis, m; \( y \) is the linear horizontal coordinate of the payload centre point displacement along the \( Y \) axis, m.

In addition, for a separate elementary stage of the system movement described by the two-point Hermite spline, the following notations were accepted: \( x_{T} \) is the linear horizontal coordinate of the payload displacement along the \( X \) axis at the \( T \) final time, m; \( y_{T} \) is the linear horizontal coordinate of the payload displacement along the \( Y \) axis at the \( T \) final time, m. In order to simplify the analytical expressions without losing their informativeness, at the initial moment of time of a single elementary movement stage, the payload displacement along the \( X \) and \( Y \) axes is assumed to be equal to zero.

The forces from the side of the OC drives, acting on the bridge and trolley, are indicated as \( F_x \) and \( F_y \), respectively. The masses of the bridge and the trolley are \( M_x \) and \( M_y \), respectively. Top points above all variable parameters present their time derivatives.

In [28], the authors proposed to use the Hermite polynomial with the \( m=4 \) order of the highest derivative to specify a separate elementary stage of movement along the reference spatial trajectory of the payload. All derived coordinates of the payload at the initial \((t=0)\) and final \((t=T)\) moments of movement, except the zero derivative at the final moment \((x_{T} \text{ or } y_{T})\), are taken equal to zero. In this case, the analytical expressions for the time dependences of the payload coordinates and their first two derivatives will have the form given below [28]. For the \( x \) payload coordinate:

\[
x_i(t) = s_i t^0 + s_2 t^2 + s_4 t^4 + s_6 t^6 + s_8 t^8 ;
\]

(1)

\[
\dot{x}_i(t) = 9 s_1 t^1 + 8 s_3 t^3 + 7 s_5 t^5 + 6 s_7 t^7 + 5 s_9 t^9 ;
\]

(2)

\[
\ddot{x}_i(t) = 72 s_1 t^2 + 56 s_3 t^4 + 42 s_5 t^6 + 30 s_7 t^8 + 20 s_9 t^{10} ,
\]

(3)

where

\[
s_1 = ((70 \cdot x_{T} ) / T^0) ; s_2 = ((315 \cdot x_{T} ) / T^2) ; s_4 = ((540 \cdot x_{T} ) / T^4) ; s_6 = ((420 \cdot x_{T} ) / T^6) ; s_8 = ((126 \cdot x_{T} ) / T^8).
\]

(4)

For the \( y \) payload coordinate, the expressions are similar to expressions (1) - (4):

\[
y_i(t) = s_i t^0 + s_2 t^2 + s_4 t^4 + s_6 t^6 + s_8 t^8 ;
\]

(5)

\[
\dot{y}_i(t) = 9 s_1 t^1 + 8 s_3 t^3 + 7 s_5 t^5 + 6 s_7 t^7 + 5 s_9 t^9 ;
\]

(6)

\[
\ddot{y}_i(t) = 72 s_1 t^2 + 56 s_3 t^4 + 42 s_5 t^6 + 30 s_7 t^8 + 20 s_9 t^{10} ,
\]

(7)

where

\[
s_1 = ((70 \cdot y_{T} ) / T^0) ; s_2 = ((315 \cdot y_{T} ) / T^2) ; s_4 = ((540 \cdot y_{T} ) / T^4) ; s_6 = ((420 \cdot y_{T} ) / T^6) ; s_8 = ((126 \cdot y_{T} ) / T^8).
\]

(8)

The time dependence of the payload rope deviation angle and the expression of the integration constant for both coordinates are:
\[ q(t) = C_1 e^{-\frac{gt}{b}} - \frac{2t^3}{g} \left( 36 s_1 t^4 + 28 s_2 t^3 + 21 s_3 t^2 + 15 s_4 t + 10 s_5 \right) + \frac{362880 L^7 b^7 s_1}{g^8} + \]
\[ + \frac{5040 L^7 b^6 \left( 36 s_1 t^2 + 8 s_2 t + s_3 \right)}{g^6} + \frac{120 L^7 b^5 \left( 126 s_1 t^4 + 56 s_2 t^3 + 21 s_3 t^2 + 6 s_4 t + s_5 \right)}{g^5} - \]
\[ - \frac{40320 L^7 b^4 \left( s_2 + 9 s_1 t \right)}{g^4} - \frac{720 L^7 b^4 \left( 84 s_1 t^3 + 28 s_2 t^2 + 7 s_3 t + s_4 \right)}{g^5} + \]
\[ + \frac{6 L b t^2 \left( 84 s_1 t^4 + 56 s_2 t^3 + 35 s_3 t^2 + 20 s_4 t + 10 s_5 \right)}{g^2} - \]
\[ - \frac{24 L^7 b^7 \left( 126 s_1 t^4 + 70 s_2 t^3 + 35 s_3 t^2 + 15 s_4 t + 5 s_5 \right)}{g^3} . \]

\[ C_1 = \frac{720 L^7 b^4 s_1}{g^5} - \frac{120 L^7 b^5 s_3}{g^4} - \frac{5040 L^7 b^6 s_2}{g^6} + \frac{40320 L^7 b^7 s_2}{g^7} - \frac{362880 L^7 b^7 s_1}{g^8}. \] (10)

The constant coefficients in expressions (9) and (10) for the X and Y coordinates are determined by expressions (4) and (8), respectively.

Due to the limited scope of the paper, the analytical expressions for the first two derivatives of the rope angle obtained by integrating expression (9) are not provided. The expressions relating linear displacement, velocity and acceleration of the suspension point to similar parameters of the payload point centre are as follows (at X and Y coordinates, respectively):

\[ \ddot{x}_i(t) = x_i(t) - L \cdot \dot{q}(t) ; \quad \ddot{\dot{x}}_i(t) = \ddot{x}_i(t) - L \cdot \ddot{q}(t) ; \quad \dddot{x}_i(t) = \dddot{x}_i(t) - L \cdot \dddot{q}(t) ; \] (11)

\[ \ddot{y}_i(t) = y_i(t) - L \cdot \dot{q}(t) ; \quad \ddot{\dot{y}}_i(t) = \ddot{y}_i(t) - L \cdot \ddot{q}(t) ; \quad \dddot{y}_i(t) = \dddot{y}_i(t) - L \cdot \dddot{q}(t). \] (12)

The derivation of the analytical dependences (9) and (10) is based on the well-known differential equation for the oscillations of a pendulum with a movable suspension point in a separate plane [28]. In the case of small angles and taking into account the suppression of oscillations, this linearised differential equation has the form (for example, for the bridge coordinates)

\[ \dddot{q} + \dddot{x}_i / L + b \dddot{q} + q \cdot g \cdot L = 0. \] (13)

By setting the T final time of stage movement and the final coordinate of the payload stage movement \( x_{iT} \) or \( y_{iT} \), the use of analytical expressions (1) - (12) provides for the description of individual elementary movement stages for a pendulum payload system with a moving suspension point along a given spatial trajectory. Herewith, the latter is a combination of several elementary movement steps along the X and Y coordinates, which can either border or overlap each other.

4. Experimental results

As an example, a computational experiment was carried out on a reference payload trajectory presenting a combination of one elementary movement stage along the X axis at T time and two elementary movement stages along the Y axis at T/2 time (see figure 3). The movements along the Y axis in two stages were equal in absolute value and opposite in sign. The end of the first elementary stage of movement along the Y axis at the same time was the beginning of the second elementary stage of movement along the same axis. This trajectory allows the payload to bypass a fairly common form of a single obstacle such as a wall.
The above formulas (1) - (13) do not take into account the magnitude of the forces from the side of the OC drives acting on the bridge and the payload trolley. Rather, these formulas use the principle of kinematic excitation of oscillations as an assumption. Such an approach simplified the differential equation for payload oscillations in the plane (13) and the analytical formulas derived on its basis, as well as allowed analytical expressions to be derived for the necessary movements, velocities and accelerations provided by the payload suspension point when moving the latter in the absence of uncontrolled oscillation mode.

In order to determine the values of the forces acting on the bridge and the trolley from the side of the OC drives, as well as necessary for the exact implementation of the synthesised movement trajectory of the payload suspension point, the OC simulation mathematical model developed in the MATLAB Simscape Multibody system was used (see figure 4).
Figure 4. Simulation mathematical model of an overhead crane in the MATLAB Simscape Multibody designations.

In this model, the reference trajectory of the payload suspension point is simulated. To this end, the bridge and the OC trolley were imparted of reference movements by means of force excitation. The forces acting on the bridge and the OC trolley were set according to the following dependencies:

\[
F_x = \left( (x) - (x) \right) \cdot c_x; \quad F_y = \left( (y) - (y) \right) \cdot c_y,
\]

where \( (x) \) is the required value of the bridge coordinate, \( m \); \( (x) \) is the actual value of the bridge coordinate, \( m \); \( (y) \) is the required value of the trolley coordinates, \( m \); \( (y) \) is the actual value of the trolley coordinates, \( m \); \( c_x \) is the reduced stiffness coefficient of the bridge drive, \( N/m \); \( c_y \) is the reduced stiffness coefficient of the trolley drive, \( N/m \).

The required values of the bridge and trolley coordinates were set according to formulas (11) and (12). During movement, the actual values of the bridge and trolley coordinates were obtained using virtual sensors of the simulation model.

The time dependences of the forces from the side of the overhead crane drives during movement in the mode of suppressing uncontrolled oscillations are shown in figure 5(a). Large amplitudes of forces correspond to large values of the payload mass.
Figure 5. Temporal dependences of forces from the side of the overhead crane drives in the mode of uncontrolled oscillation suppression (a) and the dependence of the force maximum value on the payload mass value (b): — of the bridge; - - - of the trolley.

In figure 5(b), the effect of the payload mass on the maximum values of the forces acting from the side of the bridge and OC trolley drives, appearing during the movement of the payload in the mode of uncontrolled oscillation suppression along the same reference trajectory is presented (see figure 3).

5. Discussion
The obtained functional dependences for the maximum values of the forces acting from the side of the bridge and OC trolley drives on the value of the payload mass appear to be linear. They can be characterised by both the initial values at zero mass of the payload and proportionality coefficients. The equations of the corresponding lines are of the following form:

\[
(F_x)_{\text{max}} = 1016.45 + m \cdot 0.09731; \quad (F_y)_{\text{max}} = 440.96 + m \cdot 0.20509.
\]

The implementation of the necessary movement for the payload suspension point using the simulation mathematical model demonstrated that the maximum error of the payload trajectory specified by the Hermite polynomial was less than 6 mm in the most unfavourable case. The most unfavourable case is realised under the maximum payload mass of 12,000 kg. All results were obtained both for a fixed reference payload trajectory presented in figure 3 and for a constant movement time of 30 s.

6. Conclusions
1. The description of the in-plane reference trajectory of the payload in the form of two-point Hermite splines with the highest order of derivatives equal to 4 produced analytical expressions of the movements, velocities and accelerations of the payload suspension point necessary to move the payload along a separate plane in the absence of uncontrolled oscillations. In this case, the principle of kinematic excitation of payload oscillations and the corresponding differential equation of motion were used to derive analytical dependencies.
2. By using the principle of superposition, the spatial movement trajectory of the payload was realised in the combination of plane-controlled oscillations in two mutually perpendicular directions of the bridge and the trolley movement. At small values of the deviation angle of the payload rope from the gravitational vertical (less than 5°), no significant increase in the errors of the linear coordinates of the payload is observed resulting from a superposition of oscillations in two mutually perpendicular directions.
3. In this case, the spatial payload movement is carried out along a reference arced curvilinear trajectory in the absence of uncontrolled oscillations with smooth acceleration and deceleration to zero velocities. This allows the payload to bypass a single obstacle.
4. Using a simulation mathematical model developed in the MATLAB Simscape Multibody, the specified movements of the payload suspension point obtained from the derived analytical expressions
are implemented. By applying the developed model, the implementation of the specified movements of the payload suspension point located on the crane trolley demonstrated the following. The maximum error in the realisation of the payload trajectory specified by the Hermite polynomial was less than 6 mm in the most unfavourable case with the mass of the transported payload equal to 12000 kg, i.e., with the maximum value of the payload mass from the considered range. This error can be considered negligible when moving the payload at distances exceeding few meters.

5. The developed simulation model was used to study the effect of transported payload mass on the forces acting on the bridge and the crane trolley from the side of the crane drives during movement. A linear increase in the maximum values of forces from the drive side was revealed against the linear increase in the payload mass. Equations of lines were derived, which approximate the obtained functional dependences of the force maximum values on the payload mass. The maximum value of the force acting on the crane trolley increases with a greater slope coefficient as opposed to the force acting on the bridge.

All calculations were carried for a fixed reference arced payload trajectory with overall dimensions of 10 by 5 m and for a constant time of movement of 30 s. The length of the payload rope was 10 m.

The results may be of interest to researchers involved in the design and development of intelligent mechatronic control systems for bridge and gantry cranes with the function of uncontrolled sway suppression.

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