LEOS-assisted Inter-GEOS Communication via Distributed-storage Coding

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Abstract

We consider a space communication network consisting of geosynchronous earth orbit satellites (GEOSs) and low earth orbit satellites (LEOSs). In case of no direct communication link between two GEOSs, the data exchange between them is through relay by the LEOSs. In particular, the source GEOS sends coded data to multiple LEOSs based on the distributed storage framework. The destination GEOS then retrieves certain amount of data from each LEOS for data reconstruction. For the GEOS-LEOS downlink, a regenerating-code-based transmission scheme is optimized to guarantee data reconstructability, where the transmission power allocation to the LEOSs is proposed to minimize the total transmission energy. We also consider the power allocation to minimize the total transmission time given the total transmission energy. For the LEOS-GEOS uplink, a flexible partial-downloading coding transmission scheme is proposed to guarantee data reconstructability, where the joint uploaded-data size and power allocations are proposed to minimize the total transmission energy or the total transmission time. Extensive simulation results are presented to evaluate the proposed algorithms.

Index Terms

Inter-GEOS communication, relay, distributed storage, regenerating codes, resource allocation, outer approximation (OA) method.

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I. INTRODUCTION

Since the first artificial satellite was launched in the 1950s, satellite communications have experienced rapid development with the explosive growth in the number of Earth-orbiting satellites. Satellites have played a significant and important role in both civil and military applications such as seamless communications, remote sensing and global reconnaissances [1]–[3]. As modern satellites become more and more powerful in acquiring, processing and storing large amount of data, and in the meantime, they become more and more interconnected, efficient data exchange among these satellites becomes a pressing challenge.

Satellites can be classified according to their orbital altitudes as low earth orbit (LEO), medium earth orbit (MEO) and geostationary earth orbit (GEO) systems. For data transmission to the ground stations (GSs), MEO/LEO satellites (MEOSs/LEOSs) have lower propagation attenuation and power consumption compared with the GEO satellites (GEOSs) [4]. However, MEOSs/LEOSs cannot provide consistent coverage due to the fast movements of footprints and limited illuminating region on the earth; while GEOSs on the geo-synchronous orbit can provide consistent and reliable coverages, and thus can serve as data centers capable of storing, processing and distributing data. Therefore, it is easier to establish a reliable communication link between the MEOSs/LEOSs and the GEOSs than do it between the MEOSs/LEOSs and the GSs. Resource allocation over the GEOS-GS link has been extensively studied [5]–[10]. In this paper, we consider the following scenario: one GEOS stores or collects some important data but does not cover the target GSs due to geographical limitations, and there is no direct link to other GEOSs that can cover the target GSs. In such a scenario, we consider the data transmission between the two GEOSs assisted by relaying through MEOSs/LEOSs.

On the other hand, distributed storage techniques have received significant recent interest. The basic idea is to distribute data to different nodes through regeneration coding such that the whole data can be recovered by downloading data from a subset of these nodes [11], [12]. In distributed storage, systems can achieve higher data reconstruction reliability due to the redundancy [13]–[15]. With the developments of low-cost high-volume storage devices and miniaturized payloads, installing distributed space storage and transponder capability on low-orbiting satellites becomes feasible. These satellites can then form a distributed storage system in space to assist the communications between GEOSs.

In this paper, we propose efficient transmission schemes for such inter-GEOSs communications.
assisted by LEOSs using distributed-storage coding, where the direct link between the two GEOs does not exist. The source GEO sends coded data to a group of LEOSs, which are then retrieved by the destination GEO for data reconstruction. The transmission system consists of two types of data-links: GEO-LEOS downlink and LEOS-GEO uplink. For the GEO-LEOS downlink, a regenerating-code-based [11], [16] transmission scheme is proposed to guarantee the data reconstructability, and transmission power allocation to different LEOSs is optimized to minimize the total transmission energy or the total transmission time. For the LEOS-GEO uplink, a flexible partial-downloading transmission scheme is proposed to guarantee the data reconstructability, and a joint uploaded-data size and power allocation is proposed to minimize the total transmission energy or transmission time. Finally, extensive simulation results are presented to evaluate the proposed algorithms.

The remainder of this paper is organized as follows. In Section II, system models are presented and problem formulations are given. In Sections III and IV, algorithms are developed to solve the resource allocation problems for the GEO-LEOS downlink and the LEOS-GEO uplink, respectively. In Section V, simulation results are presented. Finally, conclusions are drawn in Section VI.

II. SYSTEM DESCRIPTIONS AND PROBLEM FORMULATION

A. System Model

We consider a system consisting of 2 GEOs and $N$ circularly orbited LEOSs denoted as $\mathcal{N} = \{1, 2, \ldots, N\}$, as shown in Fig. 1. The GEOs serve as data centers capable of storing,
processing and distributing data. However, there is no direct link between the two GEOSs. Instead, the LEOSs serve as distributed space storage and transponder nodes, which can establish reliable communication links with the GEOSs when entering their coverage areas.

As shown in Fig. 1, let \( H_G \) denote the altitude of the GEOSs, \( H_{L,n} \) denote the altitude of LEOS \( n \) with velocity \( v_n \) for \( n \in \mathcal{N} \) and \( R_E \) denote the radius of the earth. For simplicity, on a two-dimensional plane we use polar coordinates to represent the satellite locations, i.e., \((R_G, 0)\) for GEOS 1, \((R_G, \pi)\) for GEOS 2 and \((R_{L,n}, \varphi_n(t))\) for LEOS \( n \) at time \( t \), where \( \varphi_n(t) \) denotes the instantaneous rotation angle of LEOS \( n \) and is measured by the angle between LEOS \( n \) and GEOS 1/2. Then the instantaneous distance between LEOS \( n \) and GEOS 1/2 is given by

\[
d_n(t) = \sqrt{R_G^2 + R_{L,n}^2 - 2R_GR_{L,n} \cos(\varphi_n(t))}.
\] (1)

We assume that GEOS 2 needs to obtain a source data of \( M \) files \( s = [s_1 \ s_2 \ \cdots \ s_M]^T \) stored on GEOS 1 with the assistance of the \( N \) LEOSs, where the superscript \( T \) denotes the transpose operator and each file is a symbol packet consisting of \( u \) bits. Then the data transmission is composed of two stages: the GEOS-LEOS downlink and the LEOS-GEOS uplink. We employ an \((M, N, K, D, \alpha, \beta)\) regenerating coding scheme [11], [16] for the first stage and a flexible downloading scheme [13], [14] for the second stage.

- **GEOS-LEOS downlink:** When the \( N \) LEOSs enter the coverage area of GEOS 1, each LEOS receives \( \alpha \) linearly coded files of the data from GEOS 1. In particular, the received files of LEOS \( n \) is given by

\[
\mathbf{m}^{(n)} = [m_1^{(n)} \ m_2^{(n)} \ \cdots \ m_{\alpha}^{(n)}]^T
\]

\[
= [h_1^{(n)} \ h_2^{(n)} \ \cdots \ h_{\alpha}^{(n)}]^T s
\]

\(
\triangleq (\mathbf{H}^{(n)})^T s,
\) (2)

where \( \mathbf{h}_i^{(n)} = [h_{i1}^{(n)} \ h_{i2}^{(n)} \ \cdots \ h_{iM}^{(n)}]^T \), \( 1 \leq i \leq \alpha \), and \( \mathbf{H}^{(n)} \), \( n \in \mathcal{N} \), are the encoding matrices of sizes \( M \times \alpha \) known to both GEOSs. Note that the \((M, N, K, D, \alpha, \beta)\) code is such that the original data can be reconstructed by downloading \( \alpha \) files each from any \( K \) LEOSs (termed as \((\alpha, K)\)-reconstructability); moreover, if the stored files on one LEOS get lost, the lost files can be regenerated by downloading \( \beta \) files each from any other \( D \) LEOSs. Parameters \( \alpha \) and \( \gamma \triangleq D\beta \) are called the storage capacity and the repair bandwidth of the code, respectively. The code parameters should satisfy

\[
K \leq D \leq N - 1, \quad M \leq \sum_{i=0}^{K-1} \min\{\alpha, (D - i)\beta\}.
\] (3)
And two types of optimal operating conditions are usually of interest. The condition for the minimum storage regeneration (MSR) point is given by
\[
\alpha = \frac{M}{K}, \quad \gamma = D\beta = \frac{MD}{(D - K + 1)K}; \quad (4)
\]
and the condition for the minimum bandwidth regeneration (MBR) point is given by
\[
\alpha = \frac{2MD}{2KD - K^2 + K}, \quad \gamma = D\beta = \frac{2MD}{2KD - K^2 + K}. \quad (5)
\]

• LEOS-GEOS uplink: When the LEOSs enter the coverage area of GEOS 2, LEOS \( n \) transmits \( \mu_n \leq \alpha \) coded files to GEOS 2 in the form of \( (A^{(n)})^T m^{(n)} \), where \( A^{(n)} \) is an \( \alpha \times \mu_n \) matrix known by both GEOS 1 and GEOS 2. Under the \( \mu \)-reconstructability [14], the source data \( s \) can be reconstructed at GEOS 2 if and only if the following two conditions are met:
\[
\text{rank}\{[H^{(n)}A^{(n)}]_{n \in \mathcal{N}}^T\} = M, \quad (6a)
\]
\[
\text{and } \sum_{n \in \mathcal{N}} \mu_n \geq M, \quad (6b)
\]
where \( \text{rank}\{\cdot\} \) denotes the rank of a matrix and \( [H^{(n)}A^{(n)}, n \in \mathcal{N}] \) denotes the matrix obtained by horizontally stacking matrices \( H^{(n)}A^{(n)} \) for all \( n \in \mathcal{N} \).

Note that we assume that the matrices \( \{H^{(n)}, A^{(n)}, n \in \mathcal{N}\} \) are predesigned and satisfy condition (6a).

B. Communication Link Model

For each GEOS, the multibeam transmitter [17], [18] is employed such that it can use the same frequency band to transmit data to multiple LEOSs simultaneously, and the multi-channel receiver is assumed such that they can receive data from different bands simultaneously. For the LEOSs, they use a common frequency band to receive data from GEOS 1 and use different bands to transmit data to GEOS 2.

For both GEOS-LEOS and LEOS-GEOS links, we assume that the data transmission duration is \( T \) and \( t_s \) denotes the starting time that GEOS 1/2 begins to transmit/receive data. Thus the transmission time interval is \( [t_s, t_s + T] \). For \( n \in \mathcal{N} \), define \( t_{s,n} \) and \( t_{e,n} \) as the starting and ending times for transmission with LEOS \( n \), respectively. Then the transmission time interval between LEOS \( n \) and GEOS 1/2 is denoted as
\[
T_n(t_s) = [t_{s,n}, t_{e,n}] \subseteq [t_s, t_s + T], \quad (7)
\]
with \( \bigcup_{n=1}^{N} T_n(t_s) = [t_s, t_s + T] \).
1) GEOS-LEOS Downlink: Let \( P_n(t), t \in [t_s, t_s + T] \), denote the power allocated to transmit data to LEOS \( n \) with \( P_n(t) \geq 0 \) for any \( t \in T_n(t_s) \) and \( P_n(t) = 0 \) otherwise. Assume that the beam of GEOS 1 to each LEOS can always be tracked in the coverage area and the receive antennas of different LEOSs can always point to GEOS 1 during the movements. Then according to [19], the channel gain of the link from GEOS 1 to LEOS \( n \) can be written as

\[
g_n(t) = \frac{L_n(f)}{d_n^2(t)},
\]

where \( g_n(t) \) denotes the received signal-to-noise ratio (SNR) at LEOS \( n \) and time \( t \) is given by

\[
\Gamma_n(t) = \frac{P_n(t)g_n(t)}{N_0W} = \frac{P_n(t)L_n(f)}{d_n^2(t)},
\]

where \( N_0 \) denotes the noise power spectral density, \( W \) denotes the assigned channel bandwidth and \( L_n(f) \) is given by (1) and \( A_n \) (measured in dB) denotes the attenuation coefficient of signal propagation. Thus the received signal-to-noise ratio (SNR) at LEOS \( n \) and time \( t \) is given by

\[
\Gamma_n(t) = \frac{P_n(t)g_n(t)}{N_0W} = \frac{P_n(t)L_n(f)}{d_n^2(t)}, \tag{8}
\]

2) LEOS-GEOS Uplink: Suppose that the transmission bandwidths of the \( N \) LEOSs are the same and again denoted by \( W \). As we have assumed that each LEOS transmits data via a different frequency band, let \( f_n \) denote the carrier frequency of LEOS \( n \). Define the transmission power of LEOS \( n \) at time \( t \) as \( P_n(t) \) with \( P_n(t) \geq 0 \) for any \( t \in T_n(t_s) \) and \( P_n(t) = 0 \) otherwise. Similarly to (8), the received SNR of GEOS 2 from LEOS \( n \) at time \( t \) is given by

\[
\Gamma_n(t) = \frac{P_n(t)L_n(f_n)}{d_n^2(t)}. \tag{11}
\]

To guarantee the \( \mu \)-reconstructability over the LEOS-GEOS uplink during the link time interval \([t_s, t_s + T]\), the downloaded data from LEOS \( n \) should satisfy

\[
\int_{T_n(t_s)} R(\Gamma_n(\tau))d\tau \geq u\mu_n, \tag{12}
\]

where \( R(\cdot) \) is defined in (9) and \( \sum_{n \in N} \mu_n \geq M \) according to (6b).
C. Problem Formulations

We now formulate problems to be solved for both the GEOS-LEOS downlink and the LEOS-GEOS uplink.

1) Resource Allocation for GEOS-LEOS Downlink under Distributed-storage Coding: Since each LEOS downloads at least $\alpha$ coded files from GEOS 1, the objective is to minimize the total transmission energy consumption over $[t_s, t_s + T]$ for the GEOS-LEOS downlink under the constraint that each LEOS can receive at least $\alpha$ files. We formulate the following power allocation problem:

$$
\min_{\{P_n(t)\}_{n \in \mathcal{N}}} \sum_{n \in \mathcal{N}} \int_{T_n(t_s)} P_n(\tau) d\tau
$$

subject to:

$$
0 \leq P_n(t) \leq P_{\text{max}}, \forall t \in T_n(t_s), \forall n \in \mathcal{N},
$$

where $P_{\text{max}}$ is the maximum transmission power budget of each beam of GEOS 1 [13].

Another problem of interest is to minimize the transmission time given the maximum total transmission energy of GEOS 1. It is equivalent to minimizing the total GEOS-LEOS downlink time period $T$ given the starting time $t_s$ and maximum transmission energy $E_{\text{max}}$ and beam transmission power $P_{\text{max}}$. Then we can formulate the transmission time minimization problem as follows,

$$
\min_{\{P_n(t)\}_{n \in \mathcal{N},T}} \int_{[t_s, t_s + T]} R(\Gamma_n(\tau)) d\tau \geq \alpha u,
$$

subject to:

$$
0 \leq P_n(t) \leq P_{\text{max}}, \forall t \in [t_{s,n}, t_s + T], \forall n \in \mathcal{N},
$$

$$
\sum_{n \in \mathcal{N}} \int_{t_{s,n}}^{t_s + T} P_n(\tau) d\tau \leq E_{\text{max}},
$$

where $t_{s,n}$ denotes the link starting time of LEOS $n$.

2) Resource Allocation for LEOS-GEOS Uplink under $\mu$-Reconstructability: For the data transmission and reconstruction from the $N$ LEOSs to GEOS 2 over $[t_s, t_s + T]$, we formulate the following joint uploaded-data size and power allocation problem:

$$
\min_{\{P_n(t)\}_{n \in \mathcal{N} \cdot \mathbf{\mu}}} \sum_{n \in \mathcal{N}} \int_{T_n(t_s)} P_n(\tau) d\tau
$$

subject to:

$$
\sum_{n \in \mathcal{N}} \mu_n = M,
$$

$$
\mu_n \in \{0, 1, \ldots, \alpha\},
$$

$$
0 \leq P_n(t) \leq P_{\text{max}}, \forall t \in T_n(t_s), \forall n \in \mathcal{N},
$$

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where \( P_{\text{max}} \) is the maximum transmission power budget of each LEOS and \( \mu_n \) files are downloaded among the \( \alpha \) files of LEOS \( n \).

Similarly to (14), we can also formulate the transmission time minimization problem for the LEOS-GEOS uplink as follows,

\[
\min_{\{P_n(t)\}_{n \in \mathbb{N}}, \mu, T} T
\]

\[
\text{s.t. } \sum_{n \in \mathbb{N}} \mu_n = M, \mu_n \in \{0, 1, \ldots, \alpha\},
\]

\[
\int_{t_n}^{t_n+T} R(\Gamma_n(\tau))d\tau \geq \mu_n u,
\]

\[
0 \leq P_n(t) \leq P_{\text{max}}, \forall t \in [t_n, t_n + T], \forall n \in \mathbb{N},
\]

\[
\sum_{n \in \mathbb{N}} \int_{t_n}^{t_n+T} P_n(\tau)d\tau \leq E_{\text{max}},
\]

where \( E_{\text{max}} \) denotes the transmission energy budget of the whole LEOS network.

### III. GEOS-LEOS Downlink Resource Allocation

#### A. Solution to Problem (13)

Since there is no coupled constraint on \( \{P_n(t)\}_{n \in \mathbb{N}} \), problem (13) can be decoupled as \( N \) power minimization sub-problems where the \( n \)th sub-problem is given by

\[
\min_{P_n(t)} \int_{T_n(t_n)} P_n(\tau)d\tau
\]

\[
\text{s.t. } \int_{T_n(t_n)} \log_2 \left( 1 + \frac{P_n(\tau)L_n(f)}{d_n^2(\tau)} \right) d\tau \geq \frac{u\alpha}{W},
\]

\[
0 \leq P_n(t) \leq P_{\text{max}}, \forall t \in T_n(t_n).
\]

It is easy to see that problem (17) is convex. As [20] has shown that the Lagrangian coefficients of the linear power constraints in a power optimization problem can be ignored, we can directly define the following Lagrangian function

\[
L(P_n(t), \lambda) = \int_{T_n(t_n)} P_n(\tau)d\tau - \lambda \left( \int_{T_n(t_n)} \log_2 \left( 1 + \frac{P_n(\tau)L_n(f)}{d_n^2(\tau)} \right) d\tau - \frac{u\alpha}{W} \right), \forall t \in T_n(t_n),
\]

where \( \lambda \geq 0 \) and \( 0 \leq P_n(t) \leq P_{\text{max}} \). The Karush-Kuhn-Tucker (KKT) conditions can then be written as

\[
1 + \frac{P_n(t) \cdot L_n(f)}{d_n^2(t)} = \frac{\lambda L_n(f)}{d_n^2(t) \ln 2}, \forall t \in T_n(t_n),
\]

\[
\text{and } \int_{T_n(t_n)} \log_2 \left( 1 + \frac{P_n(\tau)L_n(f)}{d_n^2(\tau)} \right) d\tau = \frac{u\alpha}{W}.
\]
Taking into account $0 \leq P_n(t) \leq P_{\text{max}}$ and solving for $P_n(t)$ from (19a), we obtain
\[
P_n(t) = \left[ \frac{\lambda}{\ln 2} - \frac{d_n^2(t)}{L_n(f)} \right]^+, \quad \forall t \in T_n(t_s),
\]
where $[x]^+ = \max\{x, 0\}$, $[x]_a = \min\{x, a\}$ and $\lambda$ is chosen to meet (19b). To obtain the solution to $P_n(t)$, the time-domain constrained waterfiling algorithm is presented in Algorithm [1].

### B. Solution to Problem (14)

We first prove by contradiction that the minimum transmission time is achieved by the optimal power allocation $\{P_n(t)\}_{n \in \mathcal{N}}$ that minimizes the total transmission energy over the given transmission interval. Assume that the optimal solution pair to problem (14) is $\{(P_n^{\text{Opt}}(t))_{n \in \mathcal{N}}, T^{\text{Opt}}(t)\}$ and the transmission power minimizing the total transmission energy over $T^{\text{Opt}}(t)$ is $\{P_n(t), T^{\text{Opt}}(t)\}_{n \in \mathcal{N}}$.

If $P_n^{\text{Opt}}(t) \neq P_n(t, T^{\text{Opt}})$ for certain $n \in \mathcal{N}$, then
\[
\sum_{n \in \mathcal{N}} \int_{t_{s,n}}^{t_{s,n} + T} P_n(\tau, T^{\text{Opt}}) d\tau < \sum_{n \in \mathcal{N}} \int_{t_{s,n}}^{t_{s,n} + T} P_n^{\text{Opt}}(\tau) d\tau.
\]

Note that given any feasible $T$, we have that
\[
\sum_{n \in \mathcal{N}} \int_{t_{s,n}}^{t_{s,n} + T} P_n(\tau, T) d\tau = \min_{\{P_n(t)\}_{n \in \mathcal{N}}} \sum_{n \in \mathcal{N}} \int_{t_{s,n}}^{t_{s,n} + T} P_n(\tau) d\tau
\]
\[
\int_{t_{s,n}}^{t_{s,n} + T} \log_2 \left( 1 + \frac{P_n(\tau)L_n(f)}{d_n^2(\tau)} \right) d\tau \geq \frac{W}{W},
\]
\[
0 \leq P_n(t) \leq P_{\text{max}}, \quad \forall n \in \mathcal{N}.
\]

It’s easy to see that $\sum_{n \in \mathcal{N}} \int_{t_{s,n}}^{t_{s,n} + T} P_n(\tau, T) d\tau$ is decreasing with $T$. Then there should exist a smaller feasible $T_s$ such that $T_s < T^{\text{Opt}}(t)$ and
\[
\sum_{n \in \mathcal{N}} \int_{t_{s,n}}^{t_{s,n} + T_s} P_n(\tau, T_s) d\tau = \sum_{n \in \mathcal{N}} \int_{t_{s,n}}^{t_{s,n} + T^{\text{Opt}}(\tau)} P_n^{\text{Opt}}(\tau) d\tau
\]
hold. This contradicts the assumption that $T^{\text{Opt}}(t)$ is the optimal solution. Therefore, based on the solving process of (20), problem (14) can be simplified as
\[
\min_{T} \quad T \quad \text{s.t.} \quad \sum_{n \in \mathcal{N}} \int_{t_{s,n}}^{t_{s,n} + T} P_n(\tau, T) d\tau \leq E_{\text{max}},
\]
where
\[
P_n(t, T) = \left[ \frac{\lambda_n}{\ln 2} - \frac{d_n^2(t)}{L_n(f)} \right]^+, \quad \forall t \in [t_{s,n}, t_{s,n} + T], \quad \forall n \in \mathcal{N},
\]
and $\lambda_n$ is chosen such that $\int_{t_{s,n}}^{t_{s,n} + T} \log_2 \left( 1 + \frac{P_n(\tau)L_n(f)}{d_n^2(\tau)} \right) d\tau = \frac{W}{W}.$

It is of interest to investigate the maximum energy consumption, which corresponds to the minimum transmission duration and is characterized by $P_n(t, T) = P_{\text{max}}$ during the transmission.
Algorithm 1  Constrained waterfilling algorithm for computing (20)

**Input:** $T_n(t_s), L_n(f), \{d_n(t) : \forall t \in T_n(t_s)\}, \alpha, u, W, P_{\text{max}}$

**Output:** $\{P_n(t) : \forall t \in T_n(t_s)\}$

1: **Initialization:** Denote $T_{n,1}$ and $T_{n,2}$ as the time intervals such that $P_n(t) = 0$ and $P_n(t) = P_{\text{max}}$, respectively. According to (20), reformulate $P_n(t)$ as

(A1) \[ P_n(t) = \begin{cases} \frac{\lambda}{\ln 2} - \frac{d_n^2(t)}{L_n(f)}, & \forall t \in T_n(t_s) \setminus T_{n,1} \setminus T_{n,2}, \\ P_{\text{max}}, & \forall t \in T_{n,2}, \\ 0, & \forall t \in T_{n,1}. \end{cases} \]

Set $T_{n,1} = T_{n,2} = \emptyset$ and substitute (A1) into (19b) to obtain $\lambda = \frac{\ln 2}{L_n(f)} \exp \left( \frac{u \alpha \ln 2}{|T_n(t_s)| W} \right) \exp \left( \frac{\int_{T_n(t_s)} \ln(d_n(t)^2) dt}{|T_n(t_s)|} \right)$, where $|T_n(t_s)|$ denotes the length of $T_n(t_s)$. Then update $P_n(t)$ for all $t \in T_n(t_s)$ by (A1).

2: **Update:**

while $\exists t$ such that $P_n(t) < 0$ or $P_n(t) > P_{\text{max}}$ do

a. – while $\exists t$ such that $P_n(t) < 0$ do

1) Update $T_{n,1}$ by $T_{n,1} = \{t : P_n(t) \leq 0\}$;

2) Solve $\lambda$ by substituting (A1) into (19b);

3) Update $P_n(t)$ for all $t \in T_n(t_s)$ by (A1);

– end while

b. – if $\exists t$ such that $P_n(t) > P_{\text{max}}$ do

1) Update $T_{n,2}$ by $T_{n,2} = \{t : P_n(t) \geq P_{\text{max}}\}$;

2) Solve $\lambda$ by substituting (A1) into (19b);

3) Update $P_n(t)$ for all $t \in T_n(t_s)$ by (A1);

– end if

end while

3: **return** $\{P_n(t) : \forall t \in T_n(t_s)\}$.

Let $T_{n0}$ denote the minimum transmission duration with respect to LEOS $n$, then $T_{n0}$ can be
obtained by using the bisection method to solve the following equations:

\[
\begin{aligned}
P_n(t, T_{n0}) &= P_{\text{max}}, \quad \forall t \in [t_{s,n}, t_{s} + T_{n0}], \\
\int_{t_{s,n}}^{t_{s} + T_{n0}} \log_2 \left( 1 + \frac{P_n(\tau, T_{n0})L_n(f)}{d_n^2(\tau)} \right) d\tau &= \frac{\alpha u}{W}. 
\end{aligned}
\] (24)

Define \( T_0 \triangleq \max_{n \in \mathcal{N}} T_{n0} \) and \( E_0 \triangleq \sum_{n \in \mathcal{N}} \int_{t_{s,n}}^{t_{s} + T_0} P_n(\tau, T_0) d\tau \), then \( T_0 \) denotes the common minimum transmission time for problem (22) and \( E_0 \) denotes the corresponding maximum transmission energy consumption when \( E_{\text{max}} \geq E_0 \). While when \( E_{\text{max}} < E_0 \), the optimal \( T \) to problem (22) should achieve \( \sum_{n \in \mathcal{N}} \int_{t_{s,n}}^{t_{s} + T} P_n(\tau, T) d\tau = E_{\text{max}} \). Thus problem (22) is equivalent to finding a time interval \( T \) such that the total transmission energy over \( T \) is equal to \( E_{\text{max}} \). Then the optimal solution to problem (22) can be solved by

\[
\begin{aligned}
\sum_{n \in \mathcal{N}} \int_{t_{s,n}}^{t_{s} + T} P_n(\tau, T) d\tau &= E_{\text{max}}, \\
P_n(t, T) &= \left[ \frac{\lambda_n}{\ln 2} - \frac{d_n^2(t)}{L_n(f)} \right]^+, \quad \forall t \in [t_{s,n}, t_{s} + T], \\
\int_{t_{s,n}}^{t_{s} + T} \log_2 \left( 1 + \frac{P_n(\tau, T)L_n(f)}{d_n^2(\tau)} \right) d\tau &= \frac{\alpha u}{W}, \quad \forall n \in \mathcal{N}.
\end{aligned}
\] (25)

As given \( T, P_n(t, T) \) for all \( n \in \mathcal{N} \) can be obtained by Algorithm 1, and thus the corresponding total transmission energy can be obtained. Based on the decreasing property of \( \sum_{n \in \mathcal{N}} \int_{t_{s,n}}^{t_{s} + T} P_n(t, T) dt \) with respect to \( T \) according to (21), we can use the bisection method to obtain \( T \) that satisfies (25). The detailed procedure is presented in Algorithm 2.

IV. LEOS-GEOS UPLINK RESOURCE ALLOCATION

A. Solution to Problem (15)

For notational simplicity, denote \( p_n = \{P_n(t), \forall t \in T_n(t_{s})\} \) and \( P = \{p_n\}_{n \in \mathcal{N}} \). Then we re-write (15) as

\[
\min_{P, \mu} \quad f_E(P) \\
\text{s.t.} \quad \sum_{n \in \mathcal{N}} \mu_n = M, \quad \mu_n \in \{0, 1, \cdots, \alpha\}, \\
f_n(p_n, \mu_n) \leq 0, \quad \forall n \in \mathcal{N}, \\
0 \leq P \leq P_{\text{max}},
\] (26)
Algorithm 2: Bisection-based constrained waterfilling algorithm for solving problem (14)

**Input:** $N$, $M$, $\alpha$, $u$, $W$, $\{L_n(f), t_{s,n}\}_{n \in \mathbb{N}}$, $t_s$, $P_{\max}$, $E_{\max}$

**Output:** $T$, $\{P_n(t), \forall t \in [t_{s,n}, t_s + T]\}_{n \in \mathbb{N}}$

1: **Initialization:** Set $E = 0$, $T_{\min} = 0$ and assign a large feasible value to $T_{\max}$.
2: **Step 1:** Solve $\{T_{n0}\}_{n \in \mathbb{N}}$ based on (24) via the bisection method since $\int_{t_{s,n}}^{t_{s}+T_{n0}} \log_2 \left(1 + \frac{P_{\max} L_n(f)}{d_n^2(\tau)} \right) \, d\tau$ is increasing with respect to $T_{n0}$;
3: **Step 2:** Set $T_0 = \max_{n \in \mathbb{N}} T_{n0}$ and use Algorithm [1] to calculate $E_0 = \sum_{n \in \mathbb{N}} \int_{t_{s,n}}^{t_{s}+T_0} P_n(\tau, T_0) \, d\tau$, where $P_n(\tau, T)$ is given by (23);
4: **Step 3:** Solve problem (22) according to the relationship between $E_{\max}$ and $E_0$:
   - if $E_{\max} \geq E_0$ do
     Set $T = T_0$ and $P_n(t) = P_n(t, T_0)$ given by (23) for $n \in \mathbb{N}$;
   - else
     Solve $T$ and $\{P_n(t)\}_{n \in \mathbb{N}}$ based on (25) via the bisection method using Algorithm [1]
     - while $E \neq E_{\max}$ do
       1) Update $T = (T_{\min} + T_{\max})/2$ and update $E = \sum_{n \in \mathbb{N}} \int_{t_{s,n}}^{t_{s}+T} P_n(\tau, T) \, d\tau$ via Algorithm [1]
       2) If $E < E_{\max}$ then $T_{\max} = T$; otherwise, $T_{\min} = T$.
     - end while
   - end if
5: **return** $T$, $\{P_n(t), \forall t \in [t_{s,n}, t_s + T]\}_{n \in \mathbb{N}}$.

where

\[ f_E(P) \triangleq \sum_{n \in \mathbb{N}} \int_{T_n(t_s)}^{T_n(t_s+T)} P_n(\tau) \, d\tau, \]

\[ f_n(p_n, \mu_n) \triangleq \frac{u \mu_n}{W} - \int_{T_n(t_s)}^{T_n(t_s+T)} \log_2 \left(1 + \frac{P_n(\tau) L_n(f_n)}{d_n^2(\tau)} \right) \, d\tau. \]  

(27)

Problem (26) is a mixed integer nonlinear program (MINLP), which will be solved using the outer approximation (OA) method [21], [22]. The main idea of the OA method is to repeatedly solve a linear relaxation of the MINLP, known as the master problem, which is obtained by linearizing the nonlinear constraints at the OA-point set. The OA-point set is initialized by the solution to the continuous relaxation of the MINLP and then updated by accumulating all the solutions to the master problem that have been obtained. The feasible region is updated by the
lower and upper bounds which are obtained by the solution to the master problem and the feasible solution to the MINLP, respectively. The main theoretical justification is that the OA method can achieve the same optimal solution to the MINLP if the following two main conditions hold [22]: the variable-dependent nonlinear functions in the problem are convex and twice continuously differentiable, and the feasible set of the problem is a bounded polyhedral set and contains a strictly interior feasible point. For problem (26), it is easy to see that the program is convex and twice continuously differentiable and its feasible set is a bounded polyhedral. As the case at the boundary point can be solved separately, then the program should contain a strictly interior feasible point if it is solvable. Thus problem (26) satisfies the above two conditions and can be solved by the OA method. The procedure for solving (26) based on the OA method is derived in Appendix A and summarized in Algorithm 3.

---

**Algorithm 3** OA algorithm for solving problem (26)

**Input:** $N$, $M$, $\alpha$, $u$, $W$, $\{L_n(f_n)\}_{n \in N}$, $\{T_n(t_n)\}_{n \in N}$.

**Output:** $(P, \mu)$.

1. **Initialization:** Choose a tolerance $\epsilon \geq 0$, set the lower bound $z_L = -\infty$ and the upper bound $z_U = +\infty$ for $f_E(P)$ given in (27) and set $k = 0$ and $S_{P, \mu} = \emptyset$.

2. **Step 1:** Solve NLPR (the continuous relaxation of (26)) formulated in (35) using a convex solver and obtain its optimal solution pair $(P_{\text{min}}^{(\text{NLPR})}, \mu_{\text{min}}^{(\text{NLPR})})$. If $\mu_{\text{min}}^{(\text{NLPR})}$ is an integer vector, then $f_E(P_{\text{min}}^{(\text{NLPR})})$ is the optimal value of (26), thus return $(P_{\text{min}}^{(\text{NLPR})}, \mu_{\text{min}}^{(\text{NLPR})})$ and stop; otherwise, set $S_{P, \mu} = \{(P_{\text{min}}^{(\text{NLPR})}, \mu_{\text{min}}^{(\text{NLPR})})\}$.

3. **Step 2:** OA:

4. while $z_U - z_L > \epsilon$ do

   1) Solve OA-ILP formulated in (37) using an MILP solver and obtain its optimal solution pair $(P_{\text{OA}}^{(k)}, \mu_{\text{OA}}^{(k)})$;

   2) Set $\hat{\mu} := \mu_{\text{OA}}^{(k)}$ and solve the NLP formulated in (36) using a convex solver to obtain its optimal solution $P_{\text{NLP}}^{(k)}$;

   3) Set $z_L = f_E(P_{\text{OA}}^{(k)})$ and $z_U = f_E(P_{\text{NLP}}^{(k)})$;

   4) Update $S_{P, \mu} := S_{P, \mu} \cup \{(P_{\text{NLP}}^{(k)}, \mu_{\text{OA}}^{(k)})\}$ and $k := k + 1$.

5. end while

6. return $(P_{\text{NLP}}^{(k)}, \mu_{\text{OA}}^{(k)})$. 

---
B. Solution to Problem (16)

Note that $P_n(t) = P_{\text{max}}$ characterizes both the minimum transmission duration and the maximum transmission energy consumption. Let $T_0$ and $E_0$ denote the corresponding transmission duration and energy consumption, respectively. Then according to (16), $T_0$ can be obtained by

$$T_0 = \min_{\mu, T} T$$

s.t. $\sum_{n \in \mathcal{N}} \mu_n = M$, $\mu_n \in \{0, 1, \cdots, \alpha\}$,

$$\frac{W}{u} \int_{t_{s,n}}^{t_s+T_0} \log_2 \left(1 + \frac{P_{\text{max}}L_n(f_n)}{d_n^2(\tau)}\right) d\tau \geq \mu_n, \ \forall n \in \mathcal{N},$$

(28)

and then

$$E_0 = \min_{\{P_n(t)\}_{n \in \mathcal{N}}, \mu} \sum_{n \in \mathcal{N}} \int_{t_{s,n}}^{t_s+T_0} P_n(\tau) d\tau,$$

s.t. $\sum_{n \in \mathcal{N}} \mu_n = M$, $\mu_n \in \{0, 1, \cdots, \alpha\}$,

$$\frac{W}{u} \int_{t_{s,n}}^{t_s+T_0} \log_2 \left(1 + \frac{P_n(\tau)L_n(f_n)}{d_n^2(\tau)}\right) d\tau \geq \mu_n, \ \forall n \in \mathcal{N}.$$  

(29)

For problem (28), it can be solved by the bisection method, where $T$ is adjusted to guarantee that the summation $\sum_{n \in \mathcal{N}} \mu_n = M$ via the following equation:

$$f(T) \triangleq \sum_{n \in \mathcal{N}} \left[ \frac{W}{u} \int_{t_{s,n}}^{t_s+T} \log_2 \left(1 + \frac{P_{\text{max}}L_n(f_n)}{d_n^2(\tau)}\right) d\tau \right] = M,$$

(30)

where $\lfloor \cdot \rfloor$ denotes the flooring operation, and in each round of search the lower bound $T_{\text{min}}$ and upper bound $T_{\text{max}}$ of $T$ are updated by $f(T_{\text{min}}) < M$ and $f(T_{\text{max}}) = M$, respectively. The flooring operation is to guarantee that LEOS $n$ for $n \in \mathcal{N}$ should transmit at least integer $\mu_n$ files while the way of setting $T_{\text{min}}$ and $T_{\text{max}}$ is to find the minimal $T_0$ such that at least one LEOS can exactly transmit integer number of files under the maximum transmission power $P_{\text{max}}$, i.e.,

$$\frac{W}{u} \int_{t_{s,n}}^{t_s+T_0} \log_2 \left(1 + \frac{P_{\text{max}}L_n(f_n)}{d_n^2(\tau)}\right) d\tau = \left[ \frac{W}{u} \int_{t_{s,n}}^{t_s+T_0} \log_2 \left(1 + \frac{P_{\text{max}}L_n(f_n)}{d_n^2(\tau)}\right) d\tau \right] = \mu_n, \text{ for certain } n \in \mathcal{N}.$$  

(31)

It’s easy to prove by contradiction that $T_0$ is the optimal solution to problem (28). While for problem (29), it is a convex and twice continuously differentiable MINLP and is equivalent to problem (15), and thus the OA method proposed in Section IV-A can be used to solve it.

Based on the solutions obtained from (28) and (29), when $E_{\text{max}} \geq E_0$, the minimum transmission time is $T_0$; while when $E_{\text{max}} < E_0$, we can have that the optimal $T$ achieves that
\[
\min_{\{P_n(t)\}_{n \in \mathcal{N}}, \mu} \sum_{n \in \mathcal{N}} \int_{t_{s,n}}^{t_{s,n}+T} P_n(\tau) \, d\tau = E_{\text{max}}, \quad \text{then problem (16) is equivalent to finding a time interval } T \text{ to solve the following equation:}
\]

\[
E_{\text{max}} = \min_{\{P_n(t)\}_{n \in \mathcal{N}}, \mu} \sum_{n \in \mathcal{N}} \int_{t_{s,n}}^{t_{s,n}+T} P_n(\tau) \, d\tau
\]

\[
\text{s.t.} \quad \sum_{n \in \mathcal{N}} \mu_n = M, \quad \mu_n \in \{0, 1, \ldots, \alpha\},
\]

\[
\int_{t_{s,n}}^{t_{s,n}+T} R(\Gamma_n(\tau)) \, d\tau \geq \mu_n u,
\]

\[
0 \leq P_n(t) \leq P_{\text{max}}, \quad \forall t \in [t_{s,n}, t_{s,n}+T], \quad \forall n \in \mathcal{N}.
\]

Note that the optimization on the right-hand side of (32) is equivalent to problem (15) under a given \( T \) while the optimal value can be viewed as a decreasing function of \( T \), thus a combination of the OA method proposed in Section IV-A and the bisection method can be used to solve (32). The procedure for solving problem (16) is summarized in Algorithm 4.

V. NUMERICAL RESULTS

In this section, we perform numerical simulations to evaluate the proposed algorithms. The system parameters are presented in Table I. Assume that there are \( N = 5 \) LEOSs in the space and the number of source files initially stored on GEOS 1 is \( M = 30 \) and the regenerating code at the MSR point is used. The GEOS coverage angle is \( \theta_G = 12^\circ \), which means the GEOS coverage area is from \(-6^\circ\) to \(+6^\circ\) as seen from the sub-satellite point of the GEOS. According to the CoRaSat scenarios defined in [23], [24], Ku band for the GEOS-LEOS downlink and Ka band for the LEOS-GEOS uplink are used, respectively. And the transmit carrier frequencies of the 5 LEOSs used in the LEOS-GEOS uplink are equi-spaced. For each type of data links, suppose that LEOS 5 is the first satellite that enters the GEOS coverage area and set the entering time by \( t = 0 \). The values of the initial angle difference \( \{\phi_{0n}\}_{n \in \mathcal{N}} \) presented in Table I denote the angle differences (in absolute values) at \( t = 0 \) between LEOS \( n \in \mathcal{N} \) and LEOS 5 as seen from the center point of the earth. Then the instantaneous rotation angle \( \varphi_n(t) \) for \( n \in \mathcal{N} \) is given by

\[
\varphi_n(t) = \frac{v_n t}{R_{L,n}} - \phi_{0n} + \varphi_5(0), \quad t \geq 0,
\]

where \( v_n \) denotes the velocity of LEOS \( n \), \( R_{L,n} = H_{L,n} + R_E \) and \( \varphi_5(0) = -41.06^\circ \) which is obtained according to the GEOS-LEOS geometry and value of \( \theta_G \).
Algorithm 4: Bisection-based OA algorithm for solving problem (16)

Input: \( N, M, \alpha, u, W, \{L_n(f_n), t_{s,n}\}_{n \in \mathcal{N}}, t_s, P_{\text{max}}, E_{\text{max}}\)

Output: \( T, \{P_n(t), \forall t \in [t_{s,n}, t_s + T]\}_{n \in \mathcal{N}}, \mu \)

1: **Initialization**: Set \( E = 0, T_{\text{min}} = 0 \) and assign a large feasible value to \( T_{\text{max}} \).

2: **Step 1**: Solve \( T_0 \) based on (28) via the bisection method since \( f(T) \) defined in (30) is non-decreasing with respect to \( T \);

3: **Step 2**: Solve \( E_0 \) based on (29) via Algorithm 3.

4: **Step 3**: Solve problem (16) according to the relationship between \( E_{\text{max}} \) and \( E_0 \):
   - if \( E_{\text{max}} \geq E_0 \)
     - Set \( T = T_0 \) and update \( \{P_n(t), \forall t \in [t_{s,n}, t_s + T]\}_{n \in \mathcal{N}} \) and \( \mu \) according to the solution obtained from (29);
   - else
     - Solve \( T, \{P_n(t)\}_{n \in \mathcal{N}} \) and \( \mu \) based on (32) via the bisection method using Algorithm 3:
       - while \( E \neq E_{\text{max}} \) do
         1) Update \( T = (T_{\text{min}} + T_{\text{max}})/2 \) and update \( E = \sum_{n \in \mathcal{N}} \int_{t_{s,n}}^{t_s+T} P_n(\tau)d\tau \) by solving (32) via Algorithm 3.
         2) If \( E < E_{\text{max}} \) then \( T_{\text{max}} = T \); otherwise, \( T_{\text{min}} = T \).
       - end while
   - end if

5: return \( T, \{P_n(t), \forall t \in [t_{s,n}, t_s + T]\}_{n \in \mathcal{N}}, \mu \).

A. Results for the GEOS-LEOS Downlink

We first compare the transmission power allocation algorithm proposed in Section III-A with the sub-optimal constant power allocation strategy that satisfies the constraint:

\[
\begin{cases}
\int_{T_n(t_s)} R \left( \frac{P_n L_n(f)}{d_n^2(\tau)} \right) d\tau = \alpha u, \\
0 \leq P_n \leq P_{\text{max}} \forall n \in \mathcal{N},
\end{cases}
\]  

(34)

where \( T_n(t_s) \) is defined in (7), \( R(\cdot) \) is defined in (9), \( P_n \) denotes the constant transmission power allocated to transmit data to LEOS \( n \), \( d_n^2(\tau) \) and \( L_n(f) \) are defined in (1) and (8), respectively. Then the bisection method can be used to solve \( P_n \) for \( n \in \mathcal{N} \) based on (34).

Set the maximum transmission power constraint for each beam of GEOS 1 by \( P_{\text{max}} = 40\) W.
TABLE I
SIMULATION PARAMETERS

| Distributed-storage coding parameters | value          |
|--------------------------------------|----------------|
| MSR: \((M, N, K, D, \alpha, \beta)\) | (30, 5, 3, 4, 10, 5) |
| File size \(u\)                      | 20MB           |

| Basic space network parameters       | value          |
|--------------------------------------|----------------|
| GEOS altitude \(H_G\)               | 35786km        |
| Earth radius \(R_E\)                | 6371km         |
| GEOS coverage angle \(\theta_G\)    | 12°            |
| LEOS altitudes \(\{H_{L,n}\}_{n \in \mathcal{N}}\) | [500 700 900 1100 1300]km |
| LEOS velocities \(\{v_n\}_{n \in \mathcal{N}}\) | [7.2 7.3 7.4 7.5 7.6]km/s |
| Initial angle difference \(\{\phi_{0n}\}_{n \in \mathcal{N}}\) | [12° 9° 6° 3° 0°] |
| Path attenuation \(\{A_n\}_{n \in \mathcal{N}}\) | [10 8 6 4 2]dB |

| GEOS-LEOS downlink parameters       | value          |
|--------------------------------------|----------------|
| GEOS carrier \(f\) & bandwidth \(W\) | 19.7GHz & 40MHz |
| AGs \(G_{T,n}\) & \(G_{R,n}\) \((n \in \mathcal{N})\) | 40dB & 10dB |
| Noise power \(N_0\)                 | -126.56dB      |

| LEOS-GEOS uplink parameters         | value          |
|--------------------------------------|----------------|
| LEOS carriers \(\{f_n\}_{n \in \mathcal{N}}\) | 29.5 – 31GHz (5 carriers) |
| LEOS bandwidth \(W\)                | 20MHz          |
| AGs \(G_{T,n}\) & \(G_{R}\) \((n \in \mathcal{N})\) | 20dB & 20dB |
| Noise power \(N_0\)                 | -129.08dB      |

Set the total transmission time period \(T = 600\)s, and for all \(n \in \mathcal{N}\), let \(t_{s,n} = \max\{t_s, t_{0n}\}\) and \(t_{e,n} = t_s + T\), which denote the starting and ending times of GEOS 1 transmitting data to LEOS \(n\), respectively, with \(t_s\) representing the starting transmission time of GEOS 1 for the LEOS network and \(t_{0n}\) representing the enter time of LEOS \(n\) to the coverage area of GEOS 1 and being calculated by (33). Fig. 2(a) only presents the transmission power of GEOS 1 allocated to LEOS 1 for \(t_s = 0\)s and the transmission powers for other LEOSs are nearly the same as that of LEOS 1. It can be seen that although the transmission power obtained by the proposed allocation algorithm (denoted by “Opt”) can be higher at some time points than the constant power allocation (denoted by “Sub-opt”), the nonzero power values are distributed along a shorter continuous time period and are all below the maximum beam power constraint \(P_{\text{max}} = 40\)W. The reason for some zero points at the beginning for the constant power allocation is that the starting link time of LEOS 1 \(t_{s,1}\) is later than \(t_s = 0\)s and LEOS 5 establishes the link first to
GEOS 1, i.e., $t_{s,5} = t_s = 0$s. Moreover, when we calculate the transmission energy of GEOS 1 on each LEOS and present them in Fig. 2(b), it can be seen that the proposed transmission power allocation consumes lower transmission energy for each GEOS-LEOS downlink than that of the constant power allocation strategy. The main cause of the largest energy consumption for LEOS 5 is the smallest channel link gain from GEOS 1 to LEOS 5.

Fig. 2. Optimization results for minimizing transmission energy over the GEOS-LEOS downlink with $T = 600$s. (a) Transmission power v.s. transmission time for starting time $t_s = 0$s. (b) The transmission energy of GEOS 1 for starting time $t_s = 0$s.

Next we compare the transmission power allocation (denoted by “Opt”) with the constant power allocation (denoted by “Sub-opt”) for minimizing the total transmission time. The maxi-
mum beam transmission power and total transmission energy budgets for GEOS 1 are set to be $P_{\text{max}} = 40\text{W}$ and $E_{\text{max}} = 3.7 \times 10^4\text{J}$, respectively. The transmission time for the constant power allocation is obtained by solving problem (14) under the assumption that the transmission power $P_n(t)$ for any $n \in \mathcal{N}$ is constant. Thus the corresponding problem can also be solved by using the same method as that for solving (14). Fig. 3(a) presents a comparison on the transmission power of LEOS 1 at the starting time $t_s = 0\text{s}$. Minimizing the total transmission time is actually equivalent to minimizing the ending transmission time of GEOS 1 since the starting time is assumed to be known. We can see that the proposed transmission power allocation shows earlier ending transmission time. The results of the total transmission time of the two algorithms for the starting time $t_s$ ranging from 0s to 600s are presented in Fig.3(b), from which we can still see that the proposed allocation shows shorter transmission time. And the main cause of the same transmission time at $t_s = 450\text{s}$ for the two algorithms is the utilization of the same energy budget $E_{\text{max}}$ for all the starting time points in the simulation, while $E_{\text{max}}$ is superfluous for GEOS 1 at $t_s = 450\text{s}$ to transmit $N\alpha$ files to the $N$ LEOSs, i.e., the actually consumed maximum transmission energy $E_0 < E_{\text{max}}$ and the common minimum transmission time is $T_0 = \max_{n \in \mathcal{N}} T_{n0}$ with $T_{n0}$ obtained by solving (24). Note that the total transmission time decreases with the starting time since the channel gain of the whole GEOS-LEOS link increases with the starting time.

B. Results for the LEOS-GEOS Uplink

We compare the joint uploaded-data size and power allocation to minimize the total transmission energy with the constant power allocation. In the following simulation results, we use “Opt” to represent the allocations containing transmission power allocation while use “Sub-opt” to represent the allocations using constant power. Similarly to the simulations performed in Section V-A, we set $t_{s,n} = \max\{t_s, t_{0n}\}$ and $t_{e,n} = t_s + T$ for the $N = 5$ LEOSs and let $T = 600\text{s}$. We assume that the LEOSs will transmit totally 30 files directly under the $(\alpha, K)$-reconstructability by $\mu_1 = \mu_2 = \mu_3 = \alpha = 10$; while for the joint uploaded-data size and power allocation, we can obtain the optimal file numbers transmitted by the five LEOSs via optimization for any given $t_s$. For example, when $t_s = 133\text{s}$, the obtained $\mu = [0 5 10 10 5]^T$, which implies that LEOSs 2-5 need to transmit, and for simplicity, Fig. 4(a) only presents the transmission power of LEOS 3 over the time interval $T$ and the transmission powers of other LEOSs that transmit data behave similarly. We can see from the figure that the joint uploaded-
Fig. 3. Optimization results for minimizing total transmission time over the GEOS-LEOS downlink with $P_{\text{max}} = 40\,\text{W}$ and $E_{\text{max}} = 3.7 \times 10^4\,\text{J}$. (a) Transmission power v.s. transmission time for starting time $t_s = 0$. (b) Total transmission time v.s. starting time $t_s$ from 0s to 450s.

data size and power allocation needs a shorter continuous transmission time period. Although it may need higher instantaneous transmission power at certain time, all the higher values are below the maximum power constraint $P_{\text{max}} = 900\,\text{W}$. To analyze the total transmission energy, four transmission strategies are considered: constant transmission power without resource allocation by letting $\mu = [10\,10\,10\,0\,0]^T$, transmission power allocation based on $\mu = [10\,10\,10\,0\,0]^T$, constant transmission power based on $\mu$ obtained by the joint uploaded-data size and power.
allocation, and directly using the joint uploaded-data size and power allocation. The results with respect to $t_s$ from 0s to 600s are presented in Fig. 4(b), where the constant transmission power without resource allocation, which can be viewed as $(\alpha, K)$-reconstructability, achieves the highest total transmission energy since it does not include any optimization while the joint uploaded-data size and power allocation achieves the lowest total transmission energy since it includes downloaded-file size and transmission power allocations.

![Graph](image-url)

**Fig. 4.** Optimization results for minimizing total transmission energy over the LEOS-GEOS uplink with $T = 450s$ and $P_{\text{max}} = 900W$. (a) Transmission power v.s. transmission time for $t_s = 133s$. (b) Total transmission energy v.s. starting time $t_s$ from 0s to 600s.
As both Algorithms 3 and 4 employ the OA method which is an iterative procedure, we show its convergence behavior in Fig. 5, where the values of the objective function in (26) is plotted against the number of iterations when $t_s = 0$ and $t_s = 133$, respectively. The optimal values are obtained by using the constrained waterfilling algorithm according to the obtained optimal $\mu$. From the figure we can see that for both cases 2 or 3 iterations suffice to obtain the optimal solutions.

![Fig. 5. Convergence of the OA method used in Algorithm 3](image)

Next we compare the joint uploaded-data size and power allocation to minimize the total transmission time with the constant power allocation. Set the total transmission energy budget $E_{\text{max}} = 5.8 \times 10^5$J and let the other parameters be the same with the forgoing analyses. We assume that the LEOSs will transmit totally 30 files directly by $\mu_3 = \mu_4 = \mu_5 = \alpha = 10$ for the constant power allocation; while for the joint allocation, the obtained optimal file numbers transmitted by the five LEOSs for $t_s = 0$ are $\mu = [0 \ 5 \ 9 \ 10 \ 6]^T$. The results of the transmission power of LEOS 4 for the two allocations are presented in Fig. 6(a). As minimizing the total transmission time is actually equivalent to minimizing the ending transmission time of the LEOS network since the starting time is assumed to be known, we can see from the figure that the joint allocation just needs a shorter transmission time and all the values of the transmission power are below $P_{\text{max}} = 900$W. To further analyze the total transmission time, two additional allocations are also considered: minimizing time via solving problem (16) given that $\mu = [0 \ 0 \ 10 \ 10 \ 10]^T$
and minimizing time via solving problem (16) given that $P_n(t)$ for all $n \in N$ are constant. Since the subsequent two allocations are the special cases of the joint allocation given by (16), both of them can be easily solved and we omit the details of their solving procedures. The minimum total transmission times of the four allocations with respect to $t_s$ from 0s to 600s are presented in Fig. 6(b), from which we can see that the joint uploaded-data size and power allocation shows the shortest total transmission time while the constant power allocation under $\mu = [0 0 10 10 10]^T$ shows the longest transmission time and the results of the other two allocations line between of the two. This is because the constant power allocation under $\mu = [0 0 10 10 10]^T$ (denoted by “Sub-opt” in Fig. 6(b)) just takes the maximum individual transmission time of the LEOSs as the common minimum transmission time under the energy budget constraint and does not optimize the power allocation over time.

VI. Conclusions

In this paper, we have considered relay-assisted inter-GEOSs communications through LEOSs using distributed-storage coding. We have proposed and optimized the transmission schemes for both the GEOS-LEOS downlink and the LEOS-GEOS uplink. For the GEOS-LEOS downlink, a regenerating-code-based transmission scheme has been optimized to guarantee data reconstructability, and to minimize the total transmission energy or time. For the LEOS-GEOS uplink, we have proposed a flexible partial-downloading coding transmission scheme to guarantee data reconstructability, and have considered the problem of joint uploaded-data size and power allocation to minimize the total transmission energy or time. Extensive simulation results have been presented to show the advantages of the proposed algorithms.

Appendix A

Derivations of the OA Algorithm for Problem (26)

The iterative computation in the OA method is to initialize and improve the bounds of the obtained solutions over feasible region. The lower bounds are obtained by relaxing the original problems while the upper bounds are obtained by finding feasible points. To solve the MINLP formulated in (26), three problems will be considered. The first problem is the continuous
Fig. 6. Optimization results for minimizing total transmission time over the LEOS-GEOS uplink with $P_{\text{max}} = 900\, \text{W}$ and $E_{\text{max}} = 5.8 \times 10^5\, \text{J}$. (a) Transmission power v.s. transmission time for starting time $t_s = 0$. (b) Total transmission time v.s. starting time $t_s$ from 0s to 600s.

The relaxation of the MINLP given by

\[
\text{NLPR}\left\{ \begin{array}{ll}
\min_{\mathcal{P}, \mu} & f_E(\mathcal{P}) \\
\text{s.t.} & \sum_{n \in \mathcal{N}} \mu_n = M, \quad 0 \leq \mu_n \leq \alpha, \quad \forall n \in \mathcal{N}, \\
& f_n(\mathcal{P}_n, \mu_n) \leq 0, \quad \forall n \in \mathcal{N}, \\
& 0 \leq \mathcal{P} \leq P_{\text{max}},
\end{array} \right.
\]

(35)
which can be solved by a convex solver and lead to a lower bound of problem (26).

Letting \( \hat{\mu} \) be a feasible point of problem (26), we can obtain the second problem formulated as

\[
\text{NLP} \left\{ \begin{array}{ll}
\min_{\mathcal{P}} & f_E(\mathcal{P}) \\
\text{s.t.} & f_n(p_n, \hat{\mu}_n) \leq 0, \ \forall n \in \mathcal{N}, \\
& 0 \leq \mathcal{P} \leq P_{\text{max}},
\end{array} \right.
\]

which can be directly solved by a convex solver and is used to determine the upper bound on \( f_E(\mathcal{P}) \) in each iteration.

Initialize the OA-point set \( \mathcal{S}_{\mathcal{P}, \mu} \) by solving (35) and repeatedly solve the relaxed master problem of (26), which is the third problem obtained by replacing the nonlinear constraints by their linear outer approximations at the points of set \( \mathcal{S}_{\mathcal{P}, \mu} \). The corresponding master problem is given by

\[
\text{OA–ILP} \left\{ \begin{array}{ll}
\min_{\mathcal{P}, \mu} & f_E(\mathcal{P}) \\
\text{s.t.} & f_n(\bar{p}_n, \bar{\mu}_n) + \nabla^T f_n(\bar{p}_n, \bar{\mu}_n) \left( \begin{array}{c}
p_n - \bar{p}_n \\ \mu_n - \bar{\mu}_n
\end{array} \right) \leq 0, \ \forall (\bar{p}_n, \bar{\mu}_n) \in \mathcal{S}_{\mathcal{P}, \mu}, \\
& \sum_{n \in \mathcal{N}} \mu_n = M, \ \mu_n \in \{0, 1, \cdots, \alpha\}, \ \forall n \in \mathcal{N},
\end{array} \right.
\]

which is a mixed integer linear program (MILP) and can be readily solved by many efficient branch-and-cut-based linear programming solvers such as CPLEX, LINDO, INTLINPROG, etc. \([25], [26]\). In each iteration, \( \mathcal{S}_{\mathcal{P}, \mu} \) is updated by collecting \((\mathcal{P}_{\text{NLP}}, \mu_{\text{OA}})\), where \( \mathcal{P}_{\text{NLP}} \) is the optimal solution to (36) under \( \hat{\mu} = \mu_{\text{OA}} \) while \( \mu_{\text{OA}} \) is the optimal solution to (37), and the lower bound on \( f_E(\mathcal{P}) \) is updated by the optimal value of (37).

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