Time variation of the gravitational and fine structure constants in models with extra dimensions

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We derive formulae for the time variation of the gravitational “constant” and of the fine structure “constant” in various models with extra dimensions and analyze their consistency with the observational data.

1 Introduction

Physical theories contain certain parameters characterizing the strength of the interaction which are assumed to be constant and fundamental. Examples of such parameters are the gravitational constant $G$, which characterizes the gravitational interaction, or the fine structure constant $\alpha$ determining the strength of the electromagnetic interaction. However, there are experimental evidences of the temporal variation of some of these constants. In particular, it has been recently reported a positive detection of the variability of the fine structure constant, by comparing quasar absorption lines at $z \simeq 3$ with laboratory spectra. Results on the time variation of the gravitational constant $G$ usually yield experimental bounds $|\dot{G}/G| < 10^{-11} \text{yr}^{-1}$. Finally, more conclusive results, based on the analysis of the Hubble diagram for type Ia supernovae were reported in Ref. [3]. On the other hand, there are a few theoretical schemes which predict such variations of the fundamental constants — see Ref. [2] for a comprehensive review on this subject. Here we study models which appear as a low-energy limit of some “fundamental” theory in $(4 + d)$-dimensions. Our interest in such theories is motivated by the fact that they provide a natural and self-consistent framework for such variations. Previous studies on this subject can be found in [4, 5]. In what follows we study three classes of multidimensional models and derive relationships for the time variation of $\alpha$ and $G$. These relationships will be used for establishing predictions for the time variation of the gravitational constant.

1Talk presented by Yu.A. Kubyshin at the Spanish Relativity Meeting ERE-2002 (Maó, Spain, September 22-24, 2002)
2 Theories with extra dimensions

T. Kaluza and O. Klein in their pioneering papers \[6\] discovered that the zero-mode sector of the Einstein gravity in the five-dimensional space-time \(M^4 \times S^1\) — where \(M^4\) is the Minkowski space-time and \(S^1\) is the circle — is equivalent to the Einstein gravity and Maxwell electrodynamics in \(M^4\). The relation between the parameters of the four-dimensional theory, \(G\) and \(\alpha\), and the constant \(\hat{G}(5)\) of the five-dimensional theory is given by the following reduction formula:

\[
G = \frac{\hat{G}(5)}{2\pi R}, \quad \alpha = \frac{\hat{\alpha}(5)}{2\pi R^3}.
\]

(1)

The construction was later generalized \[7\] to more general spaces \(M^4 \times K(d)\) (see Refs. \[8\] for reviews). In this case the \((4 + d)\)-dimensional Einstein gravity reduces to the gravity and Yang-Mills theory in four dimensions with the gauge group being the isotropy group of the space of extra dimensions. The reduction formulae are

\[
G = \frac{\hat{G}(4+d)}{V(d)} \propto \frac{\hat{\alpha}(4+d)}{R^d}, \quad \alpha = \kappa \frac{\hat{G}(4+d)}{R^{d}V(d)},
\]

(2)

where \(\kappa\) is some numerical factor which depends on the specific model and \(V(d)\) is the volume of the space of extra dimensions. From these relations one easily concludes that \(\dot{G}/G = -d(\dot{R}/R)\), and \(\dot{\alpha}/\alpha = -(d+2)(\dot{R}/R)\). Consequently,

\[
\frac{\dot{\alpha}}{\dot{\alpha}} = \frac{d + 2}{d} \frac{\dot{G}}{G}.
\]

(3)

Let us consider now an Einstein-Yang-Mills theory formulated in a \((4 + d)\)-dimensional space-time of the form \(M(4) \times K(d)\) \[9\] \[10\]. The theory includes gravity and the Yang-Mills field, and its action is given by

\[
S = \int_{M(4) \times K(d)} d^{4+d}x \sqrt{-\hat{g}} \left[ \frac{1}{16\pi \hat{G}(4+d)} R^{(4+d)} + \frac{1}{4\hat{g}^2(4+d)} Tr \hat{F}_{MN} \hat{F}^{MN} \right],
\]

(4)

where, as above, \(\hat{G}(4+d)\) is the multidimensional gravitational constant and \(\hat{g}(4+d)\) is the multidimensional gauge coupling. Both are supposed to be constant in time. The dimensionally reduced theory includes the Einstein gravity, the four-dimensional gauge fields and scalar fields. The explicit form of the dimensionally reduced theory depends on the topology and geometry of the space of extra dimensions and the multidimensional gauge group. The four dimensional couplings are given by

\[
G = \frac{\hat{G}(4+d)}{V(d)} \propto \frac{G(4+d)}{R^d}, \quad \alpha = \frac{\hat{\alpha}(4+d)}{V(d)} \propto \frac{\hat{\alpha}(4+d)}{R^d}.
\]
As a consequence the time variations of $G$ and $\alpha$ are related as follows:

$$\frac{\dot{\alpha}}{\alpha} = \frac{\dot{G}}{G}. \tag{5}$$

Finally, let us consider the Randall-Sundrum (RS) model proposed in Refs. \[11\] (see \[12, 13\] for reviews on the subject). It describes the five-dimensional Einstein gravity with the cosmological constant in the space-time $M^4 \times S^1/Z_2$. Here $S^1/Z_2$ is the orbifold, the space obtained from the circle $S^1 = \{y|0 \leq y < 2\pi R\}$ of radius $R$ by the identification $y \cong (-y)$. There are two 3-branes located at the fixed points $y = 0$ and $y = \pi R$ of the orbifold, one with positive brane tension ($\sigma$) and the other with the negative brane tension ($-\sigma$). The gravity propagates in the five-dimensional bulk, while matter fields are supposed to be localized on the branes. Usually the brane with negative tension is located at $y = \pi R$ and identified with our physical 3-space. The RS model provides an elegant geometrical solution to the hierarchy problem and predicts some physical effects which, in principle, can be observed in future collider experiments. To find the reduction formula for the gravitational constant we transform first to the coordinates which are Galilean on the physical brane; that is, on the brane at $y = \pi R$ \[14\]. In these coordinates the Planck mass is expressed through the fundamental scale $M_{(5)}$ of the five-dimensional theory (the five-dimensional Planck mass) as follows \[13, 14\]

$$M_{Pl}^2 = \frac{M_{(5)}^3}{k} \left[ e^{2\pi kR} - 1 \right], \tag{6}$$

where $k$ is a parameter related to the brane tension $\sigma$. It turns out that the fundamental scale and $k$ must be of order of 1 TeV. For the hierarchy problem to be solved the value $kR$ must be approximately $kR \approx 11 - 12$. From Eq. (6) one easily gets that

$$G = \frac{k}{16\pi M_{(5)}^3} \frac{1}{e^{2\pi kR} - 1}. \tag{7}$$

In the original formulation \[11\] all the fields of the Standard Model were localized on the negative tension brane. It is easy to see that in this case the model does not provide any mechanism for the time variation for the fine structure constant $\alpha$. Extended versions of the RS model were also considered in the literature \[15, 16\]. One of the possibilities is to allow gauge and some matter fields to propagate in the bulk. In this case the four-dimensional gauge coupling on the brane is related to the five-dimensional one as follows \[16\]: $g_{(4)} = \tilde{g}_{(5)}/\sqrt{2\pi R}$. Now suppose that the size of the space of extra dimensions varies with time. This leads to the time variation of the gravitational and fine structure parameters given by
\[
\frac{\dot{G}}{G} = -(2\pi kR) \frac{1}{1 - e^{-2\pi kR}} \frac{\dot{R}}{R} \approx -(2\pi kR) \frac{\dot{R}}{R}. \quad (8)
\]

From these two relations we get
\[
\frac{\dot{\alpha}}{\alpha} \approx \frac{1}{2\pi kR G} \frac{\dot{G}}{G}. \quad (9)
\]

To summarize, we have analyzed three classes of models with extra dimensions, namely the “classical” Kaluza-Klein models, the Einstein-Yang-Mills models and the extended version of the RS model with gauge fields propagating in the bulk. We have obtained that for these three cases there is a relationship between the respective rates of variations of \(\alpha\) and \(G\):
\[
\frac{\dot{\alpha}}{\alpha} = \beta(R) \frac{\dot{G}}{G}, \quad (10)
\]

where \(\beta(R)\) depends on the adopted model. The factor \(\beta\) is \(\sim 1\) for the first and second class of models. For the RS-type model, since \(kR \approx 11 - 12\) the parameter \(\beta\) is \(\sim 10^{-2}\). It is important to realize that in all the cases \(\beta(R) > 0\). Consequently, the time variations \(\dot{\alpha}/\alpha\) and \(G/G\) are of the same sign. We emphasize that this feature appears to be quite generic and model independent for theories with extra dimensions, and therefore our result is rather robust.

Let us see some implications of our result. The recent results of [1] yield \(\Delta \alpha/\alpha \equiv (\alpha_z - \alpha_0)/\alpha_0 \sim -10^{-5}\) for \(0.5 \leq z \leq 2\). Assuming, as it is usual, a constant rate and using a typical look-back time \(\Delta t \approx 8 \cdot 10^9\) yr (at \(z = 1\)) one obtains \(\dot{G}/G \sim +10^{-15}\) yr\(^{-1}\) for the first two models and \(\dot{G}/G \sim +10^{-13}\) yr\(^{-1}\) for the RS model. We see that in both cases the predicted time variation for the gravitational constant is positive.

Let us examine now the experimental bounds on the time variation of \(G\). From the best fit to the Hubble diagram of SNIa at 1\(\sigma\) C.L. [3] bounds on \(\dot{G}/G\) for various values of \((\Omega_M, \Omega_\Lambda)\) can be obtained. Here we present a few examples of them for \(z = 0.5\):

\[
\begin{align*}
\Omega_M &= 1.0, \quad \Omega_\Lambda = 0.0 & \quad & -3.0 \cdot 10^{-11} \text{ yr}^{-1} < \dot{G}/G < -0.8 \cdot 10^{-11} \text{ yr}^{-1}, \\
\Omega_M &= 0.3, \quad \Omega_\Lambda = 0.7 & \quad & -0.8 \cdot 10^{-11} \text{ yr}^{-1} < \dot{G}/G < +1.4 \cdot 10^{-11} \text{ yr}^{-1}, \\
\Omega_M &= 0.5, \quad \Omega_\Lambda = 0 & \quad & -2.0 \cdot 10^{-11} \text{ yr}^{-1} < \dot{G}/G < -1.0 \cdot 10^{-12} \text{ yr}^{-1}.
\end{align*}
\]

The comparison of these estimates with our predictions shows that for a wide range of \((\Omega_M, \Omega_\Lambda)\) the models with extra dimensions considered here are at odds with the existing experimental bounds on the time variation of \(G\) and \(\alpha\).

Note, however, that our analysis relies on the results of [1], which have been challenged [2] and need an independent confirmation. Also, we expect that future improvements in the experimental data and/or new experiments will give better bounds on \(\dot{G}/G\) and at least will determine its sign. There exists as well the possibility that the discrepancy between our theoretical predictions and the experimental
bounds quoted above is of a deeper nature and points towards some drawbacks of the multidimensional models considered here or even of the multidimensional approach as such. Of course, questioning the applicability of the multidimensional approach to the description of the fundamental interactions needs further studies.

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