Un-particle Effective Action

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Abstract
We study un-particle dynamics in the framework of standard quantum field theory. We obtain the Feynman propagator by supplementing standard quantum field theory definitions with integration over the mass spectrum. Then we use this information to construct effective actions for scalar, gauge vector and gravitational un-particles.

1 Introduction
In a seminal paper [1] Banks and Zaks investigated the unusual properties of matter with non-trivial scale invariance in the infra-red regime. Contrary to the intuitive notion of scale invariance as a property of massless particles only, this new kind of stuff has no definite mass at all. For this reason, this presently unknown, scale invariant, sector of the elementary particle spectrum has been dubbed as the un-particle sector [2,3] .

In a recent couple of papers [2,3] it has been proposed that below some critical energy scale $\Lambda_U$ the standard model particles can interact with un-particles. The forthcoming start of LHC activity has focused the interest of the high energy physics community on possible experimental signature of un-particles events at the $TeV$ energy scale [4–8] [9–12] [22,23]. Astroparticle and cosmological unparticle effects have been considered as well [13–16], [17–21].

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On the theoretical side, interesting connections between un-particles and non-standard Kaluza-Klein dynamics in extra-dimensions, and AdS/CFT duality, have been pointed out in [24–27]. Still in the background is the dynamics of unparticle sector itself.

In this letter we would like to consider un-particle dynamics in the familiar language of standard (effective) quantum field theory in four dimensions. This piece of information can be useful in view of studying gravitational effects [28,29] beyond the weak field approximation.

In Sect.2 we use path integral techniques to recover Euclidean propagator, static potential and effective action for scalar and Abelian gauge vector unparticle fields. In Sect.3 we provide an alternative evaluation of the static potential through the expectation value of the Hamiltonian between source physical states. In Sect.4 we extend the results of Sect.2 to the non-Abelian case and to gravity; the final result is the effective action for the un-graviton.

2 Effective Action

We start from the analogy between un-particles and continuous mass spectrum objects [30] and implement this relation in the Euclidean functional integral.

\[
Z_U(J) \equiv \frac{A_{d_U}}{2(\Lambda_U^{2d_U})} \int_0^\infty dm^2 (m^2)^{d_U-2} Z(J)
\] (1)

\[
A_{d_U} = \frac{16\pi^{5/2}}{(2\pi)^{2d_U}} \frac{\Gamma(d_U + 1/2)}{\Gamma(d_U - 1) \Gamma(2d_U)}
\] (2)

where \(d_U\) is a non integral scale dimension of the un-particle field. This parameter is a distinctive feature of un-particle physics which looks like a sort of fractal extension of ordinary particle physics. As fractal geometry allows to approximate non-integer dimension surfaces, un-particle physics provides an effective description of the quantum dynamics of a non-integer number of non-separable quanta. As a reasonable working hypothesis, it is generally assumed \(1 < d_U \leq 2\), even if, on a general ground, higher values of \(d_U\) cannot be dismissed.

\(Z(J)\) is the generating functional for a “particle” quantum field theory. Formula (1) establishes the connection between particle and un-particle quantum field theory. The corresponding Green functions are related in the same way.
\[ G_U(x-y) = \left[ \frac{\delta^2 Z_U(J)}{\delta J(x) \delta J(y)} \right]_{J=0} = \frac{A_{d_U}}{2(\Lambda_U^2)^{d_U-1}} \int_0^\infty dm^2 \left( m^2 \right)^{d_U-2} G(x-y;m^2) \]  

(3)

As a check of our approach let us compute the static potential generated by the exchange of scalar un-particles. Un-particles give rise to long-range forces, thus it is interesting to determine how they modify gravitational and Coulombic interactions [31].

For \( Z_\phi(J) \) we consider

\[ Z_\phi(J) = \int [D\phi] \exp \left[ -\int d^4x \left( \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + m^2 \phi^2 + J \phi \right) \right] \]  

(4)

where the appropriate normalization factor is understood in the functional integration measure. The gaussian integration leads to the known result

\[ Z_\phi(J) = \exp \left[ -\frac{1}{2} \int d^4x \int d^4y J(x) \frac{1}{-\partial^2 + m^2} \delta(x-y) \right] \]  

(5)

On the other hand, \( Z_\phi(J) \) represents the average value of the exponential interaction energy between the field and the source:

\[ Z_\phi(J) = \langle \exp \left[ -\int d^4x J \phi \right] \rangle \]  

(6)

For “infinitely heavy” sources interaction energy reduces to average of the static potential defined as

\[ Z_\phi[V(\vec{x})] = \langle \exp \left[ -\frac{1}{8\pi} \int_0^T dt V[\vec{x}] \right] \rangle \]  

(7)

Comparison between (5) and (7) gives

\[ V_U(\vec{x}) = 4\pi \int d^3y G_U(\vec{x}-\vec{y}) j(\vec{y}) \]  

(8)

For a (static) point-like source, located in the origin, \( j(\vec{y}) = \kappa \delta(\vec{y}) \), and Eq.(8) gives

\[ V_U(\vec{x}) = 4\pi \kappa G_U(\vec{x}) = 4\pi \kappa \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p} \cdot \vec{x}} G_U(\vec{p}^2) \]  

(9)
¿From the definition (3) we get

\[
G_U (p^2) = - \frac{A_{d_U}}{2 (\Lambda_U^2)^{d_U-1}} \int_0^\infty dm^2 \left( m^2 \right)^{d_U-2} \int_0^\infty ds e^{-s(p^2+m^2)} \\
= - \frac{A_{d_U}}{2\pi (\Lambda_U^2)^{d_U-1}} \Gamma (d_U - 1) \Gamma (2 - d_U) (p^2)^{d_U-2}
\] (10)

By using the relation between Euler gamma functions:

\[
z \Gamma (z) = \Gamma (1 + z), \quad \Gamma (z) \Gamma (-z) = -\frac{\pi}{z \sin (\pi z)}
\] (11)

we recover the Euclidean form of the un-particle propagator, originally found in [3]

\[
G_U (p^2) = \frac{A_{d_U}}{2 (\Lambda_U^2)^{d_U-1}} \frac{(p^2)^{d_U-2}}{\sin (\pi d_U)}
\] (12)

It is straightforward to guess the form of the corresponding effective action:

\[
S_\phi = \frac{\sin (\pi d_U)}{2A_{d_U}} \int d^4x \int d^4s \, s^{d_U-2} \partial_\mu \phi \left[ e^{-s(-\partial^2/\Lambda_U^2)} \right] \partial^\mu \phi
\] (13)

As the exponent \(1 - d_U\) is a real number, a more explicit form of (13) can be obtained by introducing a Schwinger parametrization for higher order D’Alembertian operator

\[
S_\phi = \frac{\sin (\pi d_U)}{2A_{d_U} \Gamma (d_U - 1)} \int d^4x \int_0^\infty ds \, s^{d_U-2} \partial_\mu \phi \left[ e^{-s(-\partial^2/\Lambda_U^2)} \right] \partial^\mu \phi
\] (14)

Furthermore, Eq.(9) allows to determine the corresponding static potential.

\[
V_U (\vec{x}) = 4\pi \kappa \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q} \cdot \vec{x}} \frac{A_{d_U}}{2 (\Lambda_U^2)^{d_U-1}} \frac{(q^2)^{d_U-2}}{\sin (\pi d_U)} \\
= 4\pi \kappa \frac{A_{d_U}}{\Lambda_U^{2d_U-2} \sin (\pi d_U)} \frac{1}{\pi^{3/2} 2^{5-d_U}} \frac{\Gamma (d_U - 1/2)}{\Gamma (2 - d_U)} \frac{1}{|\vec{x}|^{2d_U-1}}
\] (15)
A slightly simpler form can be obtained by inserting the explicit form $A_{d_U}$. Thus, we get

$$V_U(\vec{x}) = \frac{\kappa \Gamma(\frac{d_U}{2} + 1/2) \Gamma(\frac{d_U}{2} - 1/2)}{\Lambda_U^{2d_U-2} (2\pi)^{2d_U-1} \Gamma(2d_U)} \frac{1}{|\vec{x}|^{2d_U-1}}$$

An alternative way to compute the static potential will be discussed in the next section.

Our computational method is apparently limited to non-gauge particles, as gauge invariance forbids the presence of a mass term in the classical action. This is not strictly correct as it is known from the early work by Stuckelberg [32] that vector particles can be massive without spoiling gauge invariance provided compensating fields are properly introduced [33,34]. In modern language the extra degree of freedom to be added into the Proca-Maxwell Lagrangian is the Goldstone boson $\theta(x)$. The Stuckelberg approach has been subsequently superseded by the Higgs mechanism, as the proper way to provide gauge vector bosons mass, but it still represents a viable alternative in all those cases where a symmetry breaking potential is not available, e.g. in the framework of relativistic extended objects [35–39].

The Proca-Maxwell gauge invariant Lagrangian can be written as

$$L = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m^2}{2} \left( A_\mu - \frac{1}{e} \partial_\mu \theta \right)^2$$

where, $\theta$ is the Stuckelberg compensator which under gauge transformation shifts as the Goldstone boson, that is

$$\theta \longrightarrow \theta + e\lambda$$

The generating functional for the gauge field $A$ can be obtained by integrating $\theta$ in the path integral

$$Z_A(J) = \int [DA] [D\theta] \exp\left( -\int d^4x \, L \right)$$

$$= \int [DA] \exp \left[ -\int d^4x \left( -\frac{1}{4} F_{\mu\nu} \left( 1 - \frac{m^2}{\partial^2} \right) F^{\mu\nu} + J^\mu A_\mu \right) \right]$$

Notice that, despite the presence of a mass term, the generating functional is gauge invariant as the kinetic term is formulated through the field strength only and the source $J$ is assumed to be divergence-free. Thus, we get
\[ Z_U (J) = \frac{A_{dU}}{2 (\Lambda_U^2)^{dU-1}} \int_0^\infty dm^2 \left( m^2 \right)^{dU-2} \int [DA] \times \]
\[ \exp \left[ -\int d^4 x \left( -\frac{1}{4} F_{\mu\nu} \left( 1 - \frac{m^2}{\partial^2} \right) F^{\mu\nu} + J^\mu A_\mu \right) \right] \] (20)

Integration over the mass spectrum gives the un-photon Green function

\[ G^{\mu\nu}_U (x) = \left( \delta^{\mu\nu} - \frac{\partial^\mu \partial^\nu}{\partial^2} \right) \int \frac{d^4 p}{(2\pi)^4} \frac{1}{2\pi^2} G_U \left( p^2 \right) \] (21)

It is again straightforward to build an effective action leading to the propagator (21):

\[ S_A = -\frac{\sin \left( \pi dU \right)}{4 A_{dU}} \int d^4 x \ F_{\mu\nu} \left( \frac{-\partial^2}{\Lambda_U^2} \right)^{1-dU} F^{\mu\nu} \] (22)

### 3 Alternative computation of the static potential

As already mentioned, in this section we discuss an alternative derivation of the static potential (16), which is distinguished by particular attention to gauge invariance. To do this, we shall compute the expectation value of the energy operator \( H \) in the physical state \( |\Phi\rangle \) describing the sources, which we will denote by \( \langle H \rangle_\Phi \). Our starting point is the Lagrangian density [30]:

\[ \mathcal{L} = \sum_{k=1}^N \left[ -\frac{1}{4e_k^2} F^{k\mu\nu} F_{\mu\nu}^k + \frac{m_k^2}{2e_k^2} \left( A^k_\mu - \partial_\mu \varphi^k \right)^2 \right], \] (23)

where \( m_k \) is the mass for the \( N \) scalar fields.

Following our earlier procedure [42], integrating out the \( \varphi \)-fields induces an effective theory for the \( A^k_\mu \)-fields. Once this is done, we arrive at the following effective Lagrangian density:

\[ \mathcal{L} = \sum_{k=1}^N \frac{1}{e_k^2} \left[ -\frac{1}{4} F_{\mu\nu}^k \left( 1 + \frac{m_k^2}{\Delta} \right) F^{k\mu\nu} \right], \] (24)

Having characterized the theory under study, we can now compute the interaction energy for a single mode in Eq. (24). To this end, we shall first examine the Hamiltonian framework for this theory. The canonical momenta
\[ \Pi^\mu = - \left( 1 + \frac{m^2_k}{\Delta} \right) F^{0\mu}, \] which results in the usual primary constraint, \( \Pi_0 = 0, \) and \( \Pi^i = \left( 1 + \frac{m^2_k}{\Delta} \right) F^{i0}. \) The canonical Hamiltonian is then

\[ H_C = \int d^3x \left\{ -\frac{1}{2} \Pi^i \left( 1 + \frac{m^2_k}{\Delta} \right)^{-1} \Pi_i + \Pi^i \partial_i A_0 + \frac{1}{4} F_{ij} \left( 1 + \frac{m^2_k}{\Delta} \right) F^{ij} \right\}. \quad (25) \]

Time conservation of the primary constraint \( \Pi_0 \) leads to the secondary Gauss-law constraint \( \Gamma_1 (x) \equiv \partial_i \Pi^i = 0. \) The preservation of \( \Gamma_1 \) for all times does not give rise to any further constraints. The theory is thus seen to possess only two constraints, which are first class, therefore the theory described by (24) is a gauge-invariant one. The extended Hamiltonian that generates translations in time then reads

\[ H = H_C + \int d^3x \left( c_0 (x) \Pi_0 (x) + c_1 (x) \Gamma_1 (x) \right), \]

where \( c_0 (x) \) and \( c_1 (x) \) are the Lagrange multiplier fields. Moreover, it is straightforward to see that \( \dot{A}_0 (x) = [A_0 (x), H] = c_0 (x), \) which is an arbitrary function. Since \( \Pi^0 = 0 \) always, neither \( A^0 \) nor \( \Pi^0 \) are of interest in describing the system and may be discarded from the theory. Then, the Hamiltonian takes the form

\[ H = \int d^3x \left\{ -\frac{1}{2} \Pi^i \left( 1 + \frac{m^2_k}{\Delta} \right)^{-1} \Pi_i + \frac{1}{4} F_{ij} \left( 1 + \frac{m^2_k}{\Delta} \right) F^{ij} + c(x) \partial_i \Pi^i \right\}, \quad (26) \]

where \( c(x) = c_1 (x) - A_0 (x). \)

The quantization of the theory requires the removal of nonphysical variables, which is done by imposing a gauge condition such that the full set of constraints becomes second class. A convenient choice is found to be [43]

\[ \Gamma_2 (x) \equiv \int_{\xi x} dz^\nu A_\nu (z) \equiv \int_0^1 d\lambda x^i A_i (\lambda x) = 0, \quad (27) \]

where \( \lambda (0 \leq \lambda \leq 1) \) is the parameter describing the spacelike straight path \( x^i = \xi^i + \lambda (x - \xi)^i, \) and \( \xi \) is a fixed point (reference point). There is no essential loss of generality if we restrict our considerations to \( \xi^i = 0. \) In this case, the only non-vanishing equal-time Dirac bracket is

\[ \{ A_i (x), \Pi^j (y) \}^* = \delta^i_j \delta^{(3)} (x - y) - \partial_i^* \int_0^1 d\lambda x^j \delta^{(3)} (\lambda x - y). \quad (28) \]

We now turn to the problem of obtaining the interaction energy between point-like sources in the model under consideration. As mentioned above, we will work out the expectation value of the energy operator \( H \) in the physical state \( |\Phi\rangle. \) Now we recall that the physical states \( |\Phi\rangle \) are gauge-invariant [44]. In
that case we consider the stringy gauge-invariant state

$$\Phi \equiv |\bar{\Psi}(y) \Psi(y')\rangle = \bar{\psi}(y) \exp \left( i q \int_{y'}^y dz' A_i(z) \right) \psi(y') |0\rangle, \quad (29)$$

where $|0\rangle$ is the physical vacuum state and the line integral appearing in the above expression is along a space-like path starting at $y'$ and ending $y$, on a fixed time slice. It is worth noting here that the strings between fermions have been introduced in order to have a gauge-invariant function $|\Phi\rangle$. In other terms, each of these states represents a fermion-antifermion pair surrounded by a cloud of gauge fields sufficient to maintain gauge invariance.

Next, from our above Hamiltonian analysis, we note that

$$\Pi_i(x) |\bar{\Psi}(y) \Psi(y')\rangle = \bar{\Psi}(y) \Psi(y') \Pi_i(x) |0\rangle + q \int_y^{y'} dz_i \delta^{(3)}(z-x) |\Phi\rangle. \quad (30)$$

Having made this observation and since the fermions are taken to be infinitely massive (static) we can substitute $\Delta$ by $-\nabla^2$ in Eq. (26). In such a case $\langle H \rangle_\Phi$ reduces to

$$\langle H \rangle_\Phi = \langle H \rangle_0 + V, \quad (31)$$

where $\langle H \rangle_0 = \langle 0 | H | 0 \rangle$. The $V$ term is given by:

$$V = -\frac{q^2}{2} \int d^3x \int_y^{y'} dz'_i \delta^{(3)}(x-z') \frac{1}{\nabla_x^2 - m_k^2} \nabla_x^2 \int_y^{y'} dz^i \delta^{(3)}(x-z), \quad (32)$$

where the integrals over $z^i$ and $z'_i$ are zero except on the contour of integration. This term may look peculiar, but it is just the familiar Yukawa interaction plus self-energy terms. In effect, expression (32) can also be written as

$$V = \frac{q^2}{2} \int_y^{y'} dz'_i \partial^i \int_y^{y'} dz^i \partial^i G(z',z), \quad (33)$$

where $G$ is the Green function

$$G(z',z) = \frac{1}{4\pi} \frac{-e^{m_k |z'-z|}}{|z'-z|}. \quad (34)$$

Employing Eq. (34) and remembering that the integrals over $z^i$ and $z'_i$ are zero except on the contour of integration, expression (33) reduces to the familiar Yukawa interaction after subtracting the self-energy terms. Therefore the potential for two opposite charges located at $y$ and $y'$ is given by

$$V = -\frac{q^2}{4\pi} \frac{e^{-m_k L}}{L}. \quad (35)$$
where \( L \equiv |y - y'| \). However, from (24) we must sum over all the modes in (34), that is,

\[
V = -\frac{q^2}{4\pi} \sum_{k=1}^{N} \frac{1}{E_k^2} \frac{e^{-m_k L}}{L}.
\]  

In effect, as was explained in Ref. [30], in the limit \( k \to \infty \) the sum is substituted by an integral as follows

\[
V = \left( -\frac{q^2}{4\pi} \right) \frac{1}{L} A_{dU} \int_0^\infty t^{d_U - 2} e^{-\sqrt{tL}} dt,
\]  

where \( t = m_k^2 \). Here \( t^{d_U - 2} \) is the spectral density, and \( A_{dU} \) is a normalization factor which is given by expression (2). A direct computation on the \( t \)-variable yields

\[
V = \left( -\frac{q^2}{4\pi} \right) A_{dU} \frac{2}{L^{2d_U - 1}} \Gamma (2d_U - 2).
\]  

Using the relationship involving Gamma functions, that is,

\[
\Gamma (2d_u - 2) = (2\pi)^{-1/2} 2^{2d_u - 5/2} \Gamma (d_u - 1) \Gamma \left( d_u - \frac{1}{2} \right),
\]  

expression (38) then becomes

\[
V = \left( -\frac{q^2}{4\pi} \right) \frac{4}{\pi^{2d_u - 1}} \frac{\Gamma \left( d_u + \frac{1}{2} \right) \Gamma \left( d_u - \frac{1}{2} \right)}{\Gamma (2d_u)} \frac{1}{(L)^{2d_u - 1}}.
\]  

By introducing the scale factor \( l = \frac{1}{2^{2d_u - 2} A_{dU}} \), the corresponding modified Coulomb potential may be written as

\[
V = \left( -\frac{q^2}{4\pi} \right) \frac{1}{L} \left[ 1 + \frac{2}{\pi^{2d_u - 1}} \frac{\Gamma \left( d_u + \frac{1}{2} \right) \Gamma \left( d_u - \frac{1}{2} \right)}{\Gamma (2d_u)} \left( \frac{l}{L} \right)^{2d_u - 2} \right].
\]  

The above potential profile is analogous to the one encountered in [28].

4 Conclusion Remarks

Having obtained (22) it is tempting to extend this result both to Yang-Mills fields and to gravity.
In the first case, the passage from Abelian to non-Abelian un-vector gauge boson start from the massive Yang-Mills non-local action, which can be formally written as
\[ S^A_U = -\frac{1}{4} \int d^4x \, \text{tr} \left[ F_{\mu\nu} \left( 1 + m^2 \left( -D_\mu D^\mu \right)^{-1} \right) F^{\mu\nu} \right] \] (42)

where, the inverse gauge D’ Alembertian operator can be defined through a perturbative series in the coupling constant. Integration over \( m \) leads to the non-Abelian version of (22)

\[ S_{YM}^U = -\frac{\sin (\pi d_U)}{4A_{dU}} \int d^4x \, \text{tr} \left[ F_{\mu\nu} \left( \frac{-\partial^2}{\Lambda_U^2} \right)^{1-d_U} F^{\mu\nu} \right] \] (43)

By choosing a static charge distribution for \( J^\mu \), one recovers the un-particle correction to the Coulomb potential [28]. Colored un-particle dynamics has been perturbatively investigated in [40], where the production cross section for scalar unparticles mediated by a gauge interaction has been computed. More in detail, Eq.(3.1) in [40] is the momentum space version of Eq.(13) with the addition of an infrared cut-off \( m \), but the gauge effective action Eq.(4.1) is simply the ordinary effective action times a factor \( (2 - d_U) \). As a consequence the production cross section at the first order in the coupling constant results to be \( (2 - d_U) \) times the standard result. On the other hand, our effective action (43), is a gauge invariant non-perturbative result taking into account un-particle effects to any order in the coupling constant. To our knowledge, equations (22), (43) are new results.

The un-gravity effective action is little more involved as one starts from linearized gravity. Un-particle modifications to the Newton potential has also been discussed in [28] and possible test in planetary system in [41]. The action for linearized, general coordinate transformation respecting, massive gravity can be written as

\[ S^h = \frac{1}{2} \int d^4x \, h_{\mu\nu} \left( 1 + \frac{m^2}{\partial^2} \right) \Delta^{\mu\nu\alpha\beta} h_{\alpha\beta} \] (44)

where, \( \Delta^{\mu\nu\alpha\beta} \) is the covariant D’Alembertian for rank-two symmetric tensors. Once more, integration over the mass spectrum leads to the un-graviton effective action

\[ S^h_U = \frac{\sin (\pi d_U)}{2A_{dU}} \int d^4x \, h_{\mu\nu} \Delta^{\mu\nu\alpha\beta} \left( \frac{-\partial^2}{\Lambda_U^2} \right)^{1-d_U} h_{\alpha\beta} \] (45)

Now, it is tempting to conjecture the exact form of gravity action in the un-particle sector. As the graviton kinetic term comes from the weak field expansion of the Ricci scalar, \( R \), and the effect of integrating over the mass
spectrum amounts to modify the standard action by inserting the operator 
\((-\partial^2 / \Lambda_U^2)^{1-d_U}\), we conclude that

\[
S^h_U = \frac{\sin (\pi d_U)}{32\pi G_N A_{d_U}} \int d^4x \sqrt{g} \left( -\Delta^\mu \partial_\mu \right)^{1-d_U} R
\]

(46)

where, \(\Delta^\mu\) is the generally covariant derivative. To prove that this is the correct effective action reproducing (44) one has to remember that the covariant derivative is metric compatible, thus the covariant D’Alembertian acting on the Ricci, \(R = g^{\mu\nu} R_{\mu\nu}\), scalar is blind to \(g^{\mu\nu}\) and can freely moved in front of the Ricci tensor.

To conclude, we summarize the main results of this paper. We have constructed effective actions for scalar, gauge vector and tensor un-particles by starting from the explicit expression of the Feynman propagator. While the scalar effective was already available in the literature [30], the Abelian/Nonabelian and gravitational cases are new results obtained through a proper implementation of the Stuckelberg compensating method. In alternative to the known procedure, based on conformal scaling arguments, we explicitly evaluated the Green function through standard functional techniques supplemented by a mass spectrum integration. In the case of gauge un-particle we applied the mass integration to a Stuckelberg type action for massive gauge vectors. The gauge invariant effective action (22) has been extended to the Yang-Mills case (43). The same procedure has been applied to tensor un-particles to recover the gravitational effective action (46). All these results are new ones and deserve further investigations.

Finally, we recovered the static potential generated by un-particle exchange in a gauge-invariant path dependent framework, which has been previously introduced to study color confinement in Yang-Mills theories. An important feature of this methodology is that it provides a physically-based alternative to the usual Wilson loop approach.

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References

[1] T. Banks and A. Zaks, Nucl. Phys. B 196, 189 (1982).

[2] H. Georgi, Phys. Rev. Lett. 98, 221601 (2007).
[3] H. Georgi, Phys. Lett. B 650, 275 (2007).

[4] M. J. Strassler, “Why Unparticle Models with Mass Gaps are Examples of Hidden Valleys,” arXiv:0801.0629 [hep-ph].

[5] Y. Liao and J. Y. Liu, Phys. Rev. Lett. 99, 191804 (2007).

[6] Y. Liao, “Impact of Unparticles on Asymptotic Freedom and Unification of Gauge Couplings,” arXiv:0708.3327 [hep-ph].

[7] Y. Liao, Phys. Rev. D 76, 056006 (2007) [arXiv:0705.0837 [hep-ph]].

[8] T. G. Rizzo, JHEP 0710, 044 (2007).

[9] K. Cheung, W. Y. Keung and T. C. Yuan, Phys. Rev. D 76, 055003 (2007).

[10] M. Bander, J. L. Feng, A. Rajaraman and Y. Shirman, Phys. Rev. D 76, 115002 (2007).

[11] K. Cheung, W. Y. Keung and T. C. Yuan, “Collider signatures for unparticle,” arXiv:0710.2230 [hep-ph].

[12] K. Cheung, W. Y. Keung and T. C. Yuan, Phys. Rev. Lett. 99, 051803 (2007).

[13] T. Kikuchi and N. Okada, “Unparticle Dark Matter,” arXiv:0711.1506 [hep-ph].

[14] I. Lewis, “Cosmological and Astrophysical Constraints on Tensor Unparticles,” arXiv:0710.4147 [hep-ph].

[15] J. McDonald, ‘Cosmological Constraints on Unparticles,” arXiv:0709.2350 [hep-ph].

[16] H. Davoudiasl, Phys. Rev. Lett. 99, 141301 (2007).

[17] P. K. Das, Phys. Rev. D 76 123012 (2007).

[18] G. L. Alberghi, A. Y. Kamenshchik, A. Tronconi, G. P. Vacca and G. Venturi, “Cosmological Unparticle Correlators,” arXiv:0710.4275 [hep-th].

[19] S. Hannestad, G. Raffelt and Y. Y. Y. Wong, Phys. Rev. D 76, 121701 (2007), [arXiv:0708.1404 [hep-ph]].

[20] A. Freitas and D. Wyler, “Astro Unparticle Physics” arXiv:0708.4339 [hep-ph].

[21] S. L. Chen, X. G. He, X. P. Hu and Y. Liao, “Thermal Unparticles: A New Form of Energy Density in the Universe,” arXiv:0710.5129 [hep-ph].

[22] T. Kikuchi and N. Okada, “Unparticle physics and Higgs phenomenology,” arXiv:0707.0893 [hep-ph].

[23] T. Kikuchi, N. Okada and M. Takeuchi, “Unparticle physics at the photon collider” arXiv: 0801.0018 [hep-ph].

[24] M. A. Stephanov, Phys. Rev. D 76, 035008 (2007), [arXiv:0705.3049 [hep-ph]].
[25] D. I. Kazakov and G. S. Vartanov, “Phenomenology of the $1/N_f$ Expansion for Field Theories in Extra Dimensions”, arXiv:0710.4889 [hep-ph].

[26] D. I. Kazakov and G. S. Vartanov, JHEP 0706, 081 (2007).

[27] J. P. Lee, “Unparticles and holography,” arXiv:0710.2797 [hep-ph].

[28] H. Goldberg and P. Nath, “Ungravity and Its Possible Test,” arXiv:0706.3898 [hep-ph].

[29] J. R. Mureika, “Unparticle-Enhanced Black Holes at the LHC” arXiv:0712.1786 [hep-ph].

[30] N. V. Krasnikov, Phys. Lett. B 223, 32 (1989); N. V. Krasnikov, Int. J. Mod. Phys. A 22, 5117 (2007).

[31] N. G. Deshpande, S. D. H. Hsu and J. Jiang, Phys. Lett. B 659, 888 (2008).

[32] E. C. G. Stuckelberg Helv. Phys. Acta 11, 299 (1938).

[33] A. Burnel, Phys. Rev. D 33 (1986) 2981.

[34] A. Burnel, Phys. Rev. D 33, 2985 (1986).

[35] A. Aurilia, A. Smailagic and E. Spallucci, Phys. Rev. D 47, 2536 (1993).

[36] A. Smailagic and E. Spallucci, Phys. Rev. D 61, 067701 (2000).

[37] S. Ansoldi, A. Aurilia, L. Marinatto and E. Spallucci, Prog. Theor. Phys. 103, 1021 (2000).

[38] A. Smailagic and E. Spallucci, Phys. Lett. B 489, 435 (2000).

[39] S. Ansoldi, A. Aurilia and E. Spallucci, Phys. Rev. D 64, 025008 (2001).

[40] G. Cacciapaglia, G. Marandella and J. Terning, JHEP 0801, 070 (2008).

[41] S. Das, S. Mohanty and K. Rao, “Test of unparticle long range forces from perihelion precession of Mercury”, arXiv:0709.2583 [hep-ph].

[42] P. Gaete, Phys. Rev. D 59, 127702 (1999), P. Gaete and I. Schmidt, Phys. Rev. D 76, 027702 (2007), P. Gaete and E. Spallucci, Phys. Rev. D 77, 027702 (2008).

[43] P. Gaete, Z. Phys. C76, 355 (1997).

[44] P. A. M. Dirac, The Principles of Quantum Mechanics (Oxford University Press, Oxford, 1958), 4th ed.; Can. J. Phys. 33, 650 (1955).