$D = 4$ supergravity dynamically coupled to superstring in a superfield Lagrangian approach

Igor A. Bandos$^*$ and José M. Isidro$^*$

$^*$Departamento de Física Teórica and IFIC (CSIC–UVEG), Universidad de Valencia, 46100-Burjassot (Valencia), Spain. E-mail: bandos@ific.uv.es

Instituto de Física Corpuscular (CSIC–UVEG), Apartado de Correos 22085, Valencia 46071, Spain. E-mail: jmisidro@ific.uv.es and

†Institute for Theoretical Physics, NSC KIPT, UA61108, Kharkov, Ukraine. E-mail: bandos@kipt.kharkov.ua

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We elaborate a full superfield description of the interacting system of dynamical $D = 4, N = 1$ supergravity and dynamical superstring. As far as minimal formulation of the simple supergravity is used, such a system should contain as well the tensor (real linear) multiplet which describes the dilaton and the two-superform gauge field whose pull-back provides the Wess–Zumino term for the superstring. The superfield action is given by the sum of the Wess–Zumino action for $D = 4, N = 1$ superfield supergravity, the superfield action for the tensor multiplet in curved superspace and the Green–Schwarz superstring action. The latter includes the coupling to the tensor multiplet both in the Nambu–Goto and in the Wess–Zumino terms. We derive superfield equations of motion including, besides the superfield supergravity equations with the source, the source-full superfield equations for the linear multiplet. The superstring equations keep the same form as for the superstring in supergravity and 2-superform background. The analysis of gauge symmetries shows that the superfield description of the interacting system is gauge equivalent to the dynamical system described by the sum of the spacetime, component action for supergravity interacting with tensor multiplet and of the purely bosonic string action.

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Introduction

Recently, there has been a new interest in the superfield description of supergravity [1, 2, 3, 4]. It is motivated, in particular, by the search for a superfield formulation of 10-dimensional supergravity incorporating the superstring corrections (see [5, 6, 7] for early studies and [8, 9] for discussions). Supergravity was known to appear at the point-like limit of the superstring which corresponds to $\alpha' \to 0$ limit, i.e. to zeroth order in the decomposition in the Regge slope parameter $\alpha'$. Already at the first order in $\alpha'$, the string corrections modify the supergravity equations of motion. On the other hand, the known superfield formulations of $D = 10$ supergravity provide its on–shell description; it is given by the on–shell constraints on superspace torsion, which imply the dynamical equations for the physical fields. There are just the equations which correspond to the $\alpha' \to 0$ limit of superstring. Thus the incorporation of the $\alpha'$ corrections requires the modification of the standard supergravity equations: a search for a possibility to replace the on-shell constraints by a set of off-shell constraints, or, at least, by a set of ‘on any shell’ constraints including some parameters which specify the right–hand side of the supergravity equations and which can be chosen to describe the superstring corrections to such equations.

Basically the same problem appears when one searches for a superfield description of a superbrane interacting with higher dimensional supergravity. The superbrane is defined as a brane moving in superspace. It is well known [10, 11] that the requirement of a smooth flat superspace limit for the superbrane in curved superspace (which implies that the superbrane action in a curved superspace background should possess the same number of gauge symmetries, including fermionic $\kappa$-symmetries) results in the standard on–shell supergravity constraints. However, as was said above, such constraints imply the ‘free’ supergravity equations of motion without any superbrane source. On the other hand, as is clear from the purely bosonic limit (gravity interacting with a bosonic brane), the brane should provide a source in the Einstein equation [12]. So one could get the (mistaken) impression that supersymmetry forbids interaction with an extended object, at least at the classical level. Certainly this is not the case. The resolution of such a paradox and a search for a consistent (quasi)classical description of the supergravity–superbrane interacting systems is of interest in its own, as well as in relation with its possible applications to the study of quantum gauge theories in the language of classical supergravity–superbrane models along the line of the AdS/CFT correspondence [13, 14, 15].

The study of the complete Lagrangian superfield description of the supergravity–superbrane interacting system, when it is possible, e.g. in $D = 4, N = 1$ curved superspace, might provide new insights into the search for a modification of higher-dimensional ($D = 10, 11$) supergravity constraints in such a way that they would produce dynamical equations with sources, including singular sources from superbranes and nonsingular sources describing the stringy corrections.
Such a study has been carried out in [17] for the interacting system of dynamical $D = 4, N = 1$ supergravity and a massless superparticle source. Here we elaborate the superfield description of the next more complex system which includes, besides dynamical $D = 4, N = 1$ supergravity, the dynamical superstring. It has some specific features in comparison with the system already studied in [17]. First, the source is provided by a supersymmetric extended object. Second, as far as the minimal formulation of $D = 4, N = 1$ supergravity is considered, one finds (see [18]) that the Wess–Zumino term of the superstring describes a coupling to an additional, dynamical tensor (or real linear) multiplet [19, 20] which can be used to formulate the nontrivial 2-superform gauge theory in superspace [21]. Moreover, superstring $\kappa$–symmetry requires the identification of the tensor multiplet with a dilaton superfield and its coupling to the kinetic, Nambu–Goto term of the superstring action. Thus the superfield action for the interacting system which we will study in this paper includes, in addition to the Wess–Zumino action for supergravity [22] and the $D = 4, N = 1$ Green–Schwarz superstring action [23], also a superfield action [24] for the tensor multiplet [19] in a curved superspace of minimal supergravity [18]. It has the form (see the main text for the notations)

$$S = \int d^8 Z \, s \, d\eta (E_M^A) \, \left(1 + s \frac{\Phi}{2} \right) + \frac{1}{2 \pi \alpha'} S_{sstr}, \quad (1)$$

$$S_{sstr} = \frac{1}{2} \int_{W^2} d^2 \xi \, e^{\frac{\Phi}{2}} \sqrt{\det(\tilde{E}_m^a \tilde{E}_n^b)} - \int_{W^2} \tilde{B}_2, \quad (2)$$

where $s$ is the coupling constant for the tensor multiplet, $1/(2 \pi \alpha')$ is the superstring tension, which we set equal to unity in the main text of the present article, $\xi^m = (\tau, \sigma)$ are local coordinates of the string worldsheet $W^2$, $E_m^a := \partial_m \tilde{Z}^A(\xi) E_M^A(\tilde{Z}(\xi))$, the supervielbein $E_M^A(\tilde{Z})$ and 2–superform $B_2 := \frac{i}{2} dZ^M \wedge dZ^N B_{(M} N(\tilde{Z})$ are subject to the constraints given in Sec. I, and, finally, the superfield $e^{\frac{\Phi}{2}}$ satisfies the defining constraints of a tensor multiplet in curved superspace,

$$(\mathcal{D} \mathcal{D} - R) e^{\frac{\Phi}{2}} = 0, \quad (\mathcal{D} \mathcal{D} - R) e^{\frac{\Phi}{2}} = 0. \quad (3)$$

The main result of this paper is the complete set of superfield equations of motion for the interacting dynamical system [11], including the superfield supergravity equations and the dynamical equations for the tensor multiplet [19] with the superstring source [14]. This can provide some insight into a search for source-full superfield equations for more complicated interacting systems in $D = 10,11$ superspaces. These superfield equations can also be used to search for superfield solutions of the superfield supergravity equations [45]; this might open completely new possibilities.

On the other hand, a recent study of supergravity–superbrane interactions indicates the gauge equivalence of the superfield description of the dynamical supergravity interacting with a superbrane source with a simpler system which is described by the sum of a standard (component) supergravity action and of the action of purely bosonic brane (a purely bosonic limit of the associated superbrane). Until now this had been checked by a straightforward study of the $D = 4$ supergravity–superparticle interacting system [17]. The present investigation shows the same result for the supergravity—superstring system and, thus, provides an explicit check of such a gauge equivalence for the dynamical system including an extended object as a source. This gauge equivalence also promises to be useful to search for new solitonic solutions of superfield supergravity, with nontrivial fermionic fields. (Very few such solutions are known; an example is the pp–wave solution of [32].)

This paper is organized as follows. Sec. I is devoted to the Green–Schwarz superstring in curved $D = 4, N = 1$ superspace. The superstring action [23] involves, in addition to (the pull–backs of) the supervielbein, the scalar superfield $\Phi$ and the two–superform $B_2$, neither of which is involved in the superfield description of minimal $D = 4, N = 1$ supergravity. The flat superspace limit of the superstring action is reviewed in Sec. IA. In Sec. IB we show (in a way close to [18]) that the requirement of preservation of the superstring $\kappa$–symmetry in the minimal curved $D = 4, N = 1$ superspace results in constraints for the field strengths $H_3 = dB_2$ and find that these constraints express $H_3$ in terms of the dilaton superfield $\Phi$. Furthermore, a study of the Bianchi identities $dH_3 \equiv 0$ in a way close to the one of Ref. [18, 21] (Sec. IC) concludes that the superfield $e^{\Phi/2}$ obeys the defining constraints of the tensor multiplet, Eqs. [3].

Sec. II collects the necessary information about the superfield supergravity action [22, 37] and their variations [17, 22]. Sec. III describes the admissible variations of the tensor multiplet (dilaton superfield $\Phi$) and of the $B_2$ superform. In Sec. VI we present the complete superfield action for the supergravity—tensor multiplet—superstring interacting system and derive the superfield equations of motion by its variation (Secs. IVA,B,C). [Superfield equations which follow from the sum of the superfield action of the minimal supergravity and the tensor multiplet, without including the superstring action, were considered in [21, 48] in connection with the $D = 4 \ N = 1$ limit/compactification of the heterotic superstring]. The superfield generalizations of the Einstein and Rarita–Schwinger equations with sources are presented in Sec. VD and those of the Kalb–Ramond equations for tensor gauge fields in Sec. VE.

Then, in Sec. VA, by studying the gauge symmetries and using known results about fixing the Wess–Zumino gauge (see [17] and refs. therein) we show that the superfield description of the supergravity—tensor multiplet—superstring interacting system is gauge equivalent to a supergravity—tensor multiplet—bosonic string dynamical system described by the sum of the component (space-time) action for supergravity interacting with tensor multiplet [24] and the action for the purely bosonic string, the purely bosonic limit of the Green–Schwarz superstring.
In Sec. VB we check this gauge equivalence at the level of equations of motion by proving that the dynamical equations of motion which follow from the complete superfield description, when considered in the special ‘fermionic unitary gauge’ \( \hat{Z}^M(\xi) \equiv (\hat{x}^\mu(\xi), \hat{\theta}^\alpha(\xi)) = (\hat{x}^\mu(\xi), 0) \) have the same properties as the equations derived from the gauge fixed action (the action in the ‘fermionic unitary gauge’). Namely we show that all the dynamical equations for fermions become sourceless in this gauge. Our conclusions are collected in Sec. VI.

I. GREEN–SCHWARZ SUPERSTRING IN A \( D = 4, N = 1 \) SUPERGRAVITY BACKGROUND

The Green–Schwarz superstring spans a two-dimensional worldsheet \( \mathcal{W}^2 \) in superspace \( \Sigma^{(4|4)} \),

\[
\mathcal{W}^2 \subset \Sigma^{(4|4)}, \quad Z^M(\xi^m) = \hat{Z}^M(\xi^m).
\]

In \( \mathcal{W}^2 \) \( Z^M = (x^\mu, \theta^\alpha) \) are coordinates of \( D = 4, N = 1 \) superspace \( (\mu = 0, 1, 2, 3, \alpha = 1, 2, 3, 4) \), \( \xi^m = (\tau, \sigma) \) are local worldsheet coordinates \( (m = 0, 1) \) and \( \hat{Z}^M(\xi) = (\hat{x}^\mu(\xi), \hat{\theta}^\alpha(\xi)) \) are supercoordinate functions that define the surface \( \mathcal{W}^2 \) in \( \Sigma^{(4|4)} \). One can also say that \( \hat{Z}^M(\xi) \) are defined by the map

\[
\hat{\phi} : \mathcal{W}^2 \to \Sigma^{(4|4)}, \quad \xi^m \mapsto \hat{Z}^M(\xi) = (\hat{x}^\mu(\xi), \hat{\theta}^\alpha(\xi))
\]

of the coordinate chart \( \mathcal{W}^2 \) into \( \Sigma^{(4|4)} \).

The \( D = 4, N = 1 \) version of the Green–Schwarz superstring action reads

\[
S_{str} = \int_{\mathcal{W}^2} \hat{L}_2 = \int_{\mathcal{W}^2} \left[ \frac{1}{4} \epsilon^{\hat{A}} \ast \hat{E}^\hat{A} \wedge \hat{E}^b \eta_{ab} - \hat{B}_2 \right]
\]

\[
= \int d^2 \xi \sqrt{|g|} - \int_{\mathcal{W}^2} \hat{B}_2,
\]

\[(1.3) \quad g = \text{det}(g_{mn}), \quad g_{mn} = \hat{E}_m^a \hat{E}_n^a.
\]

It involves the pull–backs of the forms on superspace to \( \mathcal{W}^2 \),

\[
\hat{E}^a \equiv \hat{\phi}^!(E^a) = d\hat{Z}^M(\xi) E_M^a(\hat{Z}) \equiv d\xi^m \hat{E}_m^a.
\]

for the bosonic supervielbein form \( E^a \) on \( \Sigma^{(4|4)} \),

\[
E^a = \left( E^a, E^2 \right) = (E^a, \hat{E}_a) ;
\]

\[
E^a = dZ^M E_M^a(Z),
\]

\[
E^2 = dZ^M E_M^a(Z) \leftrightarrow \begin{cases} E^a = dZ^M E_M^a(Z) ; \\ \hat{E}^a = dZ^M \hat{E}_M^a(Z). \end{cases}
\]

In Eqs. 1.6–1.8 \( a = 0, 1, 2, 3 \) is a tangent space vector index, \( \alpha = 1, 2, 3, 4 \) is a Majorana spinor index and \( \alpha = 1, 2, \beta = 1, 2 \) are Weyl spinor indices.

The action 1.3 involves also the pull–back \( \hat{\phi} = \hat{\phi}^!(\Phi) = \Phi(\hat{Z}) \) of a dilaton superfield \( \Phi(Z) \) and the pull–back

\[
\hat{B}_2 \equiv \hat{\phi}^!(B_2) = B_2(\hat{Z}(\xi)) = \frac{1}{2} \hat{E}^B \wedge \hat{E}^A B_{AB}(\hat{Z}(\xi))
\]

\[
\equiv \frac{1}{2} \epsilon^{\hat{A}} \wedge d\xi^a \hat{B}_{am}(\xi),
\]

\[
\hat{B}_{am}(\xi) = \partial_m \xi^a \hat{Z}^N B_{NAM}(\hat{Z})
\]

of a two–form on \( \Sigma^{(4|4)} \),

\[
B_2 = \frac{1}{2} E^B \wedge E^A B_{AB}(Z).
\]

The worldsheet Hodge star operator *

\[
* \hat{E}^a = \frac{1}{2} d^2 \xi \sqrt{|g|} \epsilon^{a \dot{b}} \hat{E}_{\dot{b}m} g_{nm},
\]

\[
\Rightarrow * \hat{E}^a \wedge \hat{E}^b = d^2 \xi \sqrt{|g|} g^{mn} \hat{E}_{\dot{b}m} \hat{E}_{\dot{a}n},
\]

(1.11) can be defined using the induced worldsheet metric 1.4.

Then

\[
\frac{1}{4} * \hat{E}^a \wedge \hat{E}^a = \frac{1}{2} d^2 \xi \sqrt{|g|},
\]

(1.12)

\[
\delta(* \hat{E}^a \wedge \hat{E}^a) = 2 * \hat{E}^a \wedge \delta \hat{E}^a.
\]

(1.13)

Substituting 1.12 in 1.3 one arrives at the more familiar form of the Green–Schwarz superstring action 2.

A. Superstring \( \kappa \)-symmetry in flat superspace

In flat superspace one may consider a vanishing dilaton superfield, \( \Phi(Z) = 0 \) (although this is not obligatory in \( D = 4, N = 1 \) superspace, where \( \Phi(z) \) is described by a separate multiplet, see below) and use the standard expression for the supervielbein

\[
\hat{E}^a = dX^\mu \delta^a_\mu - id\theta^a \sigma^a_{\alpha \dot{\alpha}} \hat{\theta}^\dot{\alpha} + i\theta^a \sigma^a_{\alpha \dot{\alpha}} d\hat{\theta}^\dot{\alpha},
\]

(1.14)

\[
E^a = d\theta^\dot{\beta} \delta^a_\dot{\beta} \equiv d\theta^a,
\]

\[
\hat{E}^a = d\theta^\dot{\beta} \delta^a_\dot{\beta} \equiv \hat{d}\theta^a,
\]

(1.15)

where \( \dot{\alpha}, \dot{\beta} = 1, 2, 3, 4 \) shall be treated as a Majorana spinor index, \( \alpha = \alpha \); \( \dot{\beta} = \dot{\beta} \equiv (\theta^\dot{\beta}, \hat{\theta}_\dot{\beta}) \) (i.e. \( \theta^\dot{\beta} \equiv \theta^a \delta^a_\dot{\beta} + \hat{\theta}_\dot{\beta} \delta^\dot{\alpha} \)). The ‘vacuum value’ of the two–form \( \hat{B}_2 \) has to be chosen as

\[
\hat{f} \text{lat superspace }, \quad \Phi(Z) = 0 : \quad B_2 = -\frac{1}{2} dX^\mu \delta^a_\mu \wedge [d\theta^a \sigma_{a\dot{a}\alpha} \hat{\theta}^\dot{\alpha} - \theta^a \sigma_{a\dot{a}\alpha} d\hat{\theta}^\dot{\alpha}].
\]

(1.16)

Then the pull–back \( \hat{B}_2 = \hat{\phi}^!(B_2) \) of the two–form \( B_2 \) produces the standard form of the Wess–Zumino term of the \( D = 4 \) Green–Schwarz superstring action 23. Due to its presence the action possesses a local fermionic symmetry, the seminal \( \kappa \)-symmetry 12, 23. Its transformation rules can be formulated as follows

\[
i_\kappa \hat{E}^a := \delta_\kappa \hat{Z}^M \hat{E}_M^a(\hat{Z}) = 0;
\]

\[
i_\kappa \hat{E}^a \sigma_{a\dot{a}\alpha} (* \hat{E}^a - \hat{E}^a) = 0.
\]

(1.17)
Indeed, as \((\hat{E}^a - \hat{E}^a) = d\xi^a (\delta^m_a + \sqrt{|g|} \epsilon_{nk} \delta^{km}) \hat{E}_m^a\) and, in flat superspace, \(E^a\) has the form of Eq. (1.14), the solution of Eqs. (1.17) with respect to \(\delta_\mu X^\mu\) and \(\delta_\alpha \theta^\alpha\) gives the \(D = 4\) version of the Green–Schwarz expression for the superstring \(\kappa\)-symmetry [23],

\[
\text{flat superspace , } \Phi(Z) = 0: \\
\delta_\mu X^\mu = i \delta_\mu \theta_\alpha \sigma^\alpha_{\alpha\beta} \hat{\theta}^\beta + c.c. , \\
\delta_\alpha \theta^\alpha = i \delta_\alpha \theta_\alpha \sigma^\alpha_{\alpha\beta} \hat{\theta}^\beta = \frac{1}{8} \left( \frac{1}{2} \hat{E}_a \hat{\theta}^\alpha \right) \delta \hat{E}^a + \\
\delta_\alpha \hat{\theta}^\alpha = \frac{1}{8} \left( \frac{1}{2} \hat{E}_a \hat{\theta}^\alpha \right) \delta \hat{E}^a + \frac{1}{8} \hat{E}_a \hat{\theta}^\alpha \delta \Phi - \delta \hat{B}_a , \\
(1.18)
\]

as the only dynamical variables in a curved superspace \(\text{background}\) are the supercoordinate functions \(\hat{Z}^M(\xi)\), one only considers in this case

\[
\delta_\hat{Z} S_{\text{str}} = \int_{W^2} \left[ \frac{1}{2} \hat{E}_a \hat{\theta}^\alpha \delta \hat{E}^a + \\
\frac{1}{8} \hat{E}_a \hat{\theta}^\alpha \delta \Phi - \delta \hat{B}_a \right] ,
(1.19)
\]

and the variations \(\delta_\hat{Z}\) of the pull–backs of differential forms are given by Lie derivatives,

\[
\delta_\hat{Z} \hat{E}^a = \delta_\hat{Z} E^a(\hat{Z}) := E^a(\hat{Z} + \delta \hat{Z}) - E^a(\hat{Z}) = \\
= i \delta_\hat{Z} (d \hat{E}^a) + d(i \delta_\hat{Z} \hat{E}^a) = \\
= i \delta_\hat{Z} T^a + D(i \delta_\hat{Z} \hat{E}^a) + \hat{E}^b \varepsilon_{\hat{Z}}^b w^a , \\
(1.21)
\]

where

\[
i \delta_\hat{Z} E^a(\hat{Z}) := \delta \hat{Z}^M E^a_M(\hat{Z}) , \\
i \delta_\hat{Z} w^{ab} := \delta \hat{Z}^M w^{ab}_M(\hat{Z}) , \\
i \delta_\hat{Z} T^a := \hat{E}^C i \delta_\hat{Z} \hat{E}^B T^a_{BC} , \quad \text{etc.}, \\
(1.25)
\]

the torsion \(T^a\) and the covariant exterior derivative \(D\) are defined below (Eqs. (1.20)–(1.22)) and \(w^{ab} = dZ^M w^M = -w^{ab}\) is the spin connection.

Now it is clear that \(\kappa\)-symmetry is not present in a general curved superspace. As we are interested in the superstring interaction with supergravity, we have to impose first the constraints on the torsion of curved superspace

\[
T^a := dE^a = dE^a - E^b \wedge w^b a = \\
\equiv \frac{1}{2} E^B \wedge E^C T^B_C a , \\
(1.26)
\]

\[
T^a := dE^a = dE^a - E^b \wedge w^b a = \\
\equiv \frac{1}{2} E^B \wedge E^C T^B_C , \\
(1.27)
\]

\[
T^a := dE^a - \bar{E}^b \wedge w^b a = \\
\equiv \frac{1}{2} E^B \wedge E^C T^B_C , \\
(1.28)
\]

and on the Riemann curvature two–form

\[
R^{ab} := dw^{ab} - w^{ac} \wedge w_c^b = \frac{1}{4} E^C \wedge E^D R_{DC}^{ab} ,
(1.29)
\]

The \textit{minimal} off–shell formulation of \(D = 4\), \(N = 1\) supergravity is described by the set of constraints (see [37] and refs. therein)

\[
T_{\alpha\beta} = -2 \sigma_\alpha \sigma_\beta , \\
T_{\alpha\beta} = 0 = T_\alpha \delta^\alpha = 0 , \quad T_{ab} = 0 ,
(1.30)
\]

\[
R_{\alpha\beta} = 0 .
(1.31)
\]

These constraints and their consequences (derived from the Bianchi identities) can be collected in the following expressions for the torsion two–forms [22, 37]

\[
T^a = -2 \sigma_\alpha \sigma_\beta \wedge \sigma_\gamma \wedge \sigma_\delta G^{\alpha\beta} , \\
T^a = \frac{1}{8} E^C \wedge E^D \sigma_\alpha \sigma_\beta \sigma_\gamma \wedge \sigma_\delta G^{\alpha\beta} - \\
- \frac{1}{8} E^C \wedge E^D \sigma_\alpha \sigma_\beta \sigma_\gamma \wedge \sigma_\delta G^{\alpha\beta} = \\
\equiv \frac{1}{4} E^C \wedge E^D \sigma_\alpha \sigma_\beta \sigma_\gamma \wedge \sigma_\delta G^{\alpha\beta} - \\
- \frac{1}{8} E^C \wedge E^D \sigma_\alpha \sigma_\beta \sigma_\gamma \wedge \sigma_\delta G^{\alpha\beta} = \\
\equiv \frac{1}{4} E^C \wedge E^D \sigma_\alpha \sigma_\beta \sigma_\gamma \wedge \sigma_\delta G^{\alpha\beta} , \\
(1.32)
\]

and for the superspace Riemann curvature 2–form

\[
R^{\alpha\beta} := dw^{\alpha\beta} - w^{\alpha c} \wedge w_c^{\beta} = \\
\equiv \frac{1}{8} R_{\alpha\beta} \sigma^\gamma \wedge \sigma^\delta \sigma_\alpha \sigma_\beta R_{\gamma\delta} - \\
\equiv \frac{1}{8} E^C \wedge E^D \sigma_\alpha \sigma_\beta \sigma_\gamma \wedge \sigma_\delta G^{\alpha\beta} - \\
- \frac{1}{8} E^C \wedge E^D \sigma_\alpha \sigma_\beta \sigma_\gamma \wedge \sigma_\delta G^{\alpha\beta} = \\
\equiv \frac{1}{4} E^C \wedge E^D R_{DC}^{\alpha\beta} ,
(1.33)
\]
and the main superfields of the minimal supergravity by symmetrization (antisymmetrization) with unit weight, here and below the brackets (square brackets) denote \( \int \) where we denote \( \int \) for the \( D \) action as \( \Phi \). Moreover, the study of Bianchi identities brings also the equations which, on first sight, seem to be relations between the dilaton superfield and the chiral main superfield of minimal supergravity

\[
R = e^{-\frac{z}{2}} D \nabla e^{\frac{z}{2}}, \quad R = e^{-\frac{z}{2}} D \nabla e^{\frac{z}{2}}.
\]

However, one notes that Eqs. (1.47) can be written as
\[
(DD - R)e^{\frac{z}{2}} = 0, \quad (DD - R)e^{\frac{z}{2}} = 0,
\]

and they just imply that the \( D = 4, N = 1 \) dilaton superfield describes the real linear multiplet. The fact that a two–form in \( D = 4, N = 1 \) superspace is described by real linear multiplet (tensor multiplet) is known from the study in [21].

Thus we conclude that the complete superfield action for the \( D = 4, N = 1 \) interacting system including the superstring should involve, in addition to the superstring action (1.3) and the Wess–Zumino action for the minimal supergravity multiplet, \( \int d^8 Z E \) (see Eq. (1.2) below) also an action for the tensor multiplet, described by the real superfield \( e^{\frac{z}{2}} \) which obeys the constraints (1.43). The kinetic term of the latter action should involve, in particular, the kinetic term for the two–index antisymmetric tensor gauge field (or Kalb–Ramond field [35], first introduced in \( D = 4 \) under the name [39]) which interacts naturally with a string. Such a kinetic term can be written as
\[
\int d^8 Z E \frac{\Phi}{2} e^{\frac{z}{2}}.
\]

Note that the first proposal for the tensor multiplet action was different, \( \int d^8 Z E \Phi \equiv \int d^8 Z E (e^{\frac{z}{2}})^2 \) (see [21] and refs. therein). The tensor multiplet with the action (1.49) was referred to as ‘improved tensor multiplet’ [24]. Its distinguishing property is invariance under the Weyl transformations acting also on the dilaton superfield, \( E^a \rightarrow e^{\Lambda} E^a \), \( E^a \rightarrow e^{\Lambda/2} E^a \), \( \Phi(z) \rightarrow \Phi(Z) - 4 \Lambda \), when \( \Lambda \) is given by a sum of chiral and antichiral superfields, \( \Lambda = i (\phi - \bar{\phi}) \). The fact that the superstring action (1.3) also possesses such a symmetry makes the improved tensor multiplet action preferable for a description of the tensor multiplet–superstring interacting system.

Actually in the study of \( D = 4, N = 1 \) limit/compactification of the heterotic string [15, 25], [27, 28, 29], it was argued that such a limit rather provided the minimal supergravity–tensor multiplet action [15, 25]. We are not addressing this problem here but rather considering the \( D = 4, N = 1 \) interacting system (including the \( D = 4, N = 1 \) superstring) as a relatively simple model for the (quasi)classical description of a more complicated, higher dimensional (\( D = 10, 11 \)) supergravity–superbrane interacting system [in particular, the \( D = 10, N = 1 \) supergravity–super Yang–Mills–heterotic string interacting system described by
D. Superstring $\kappa$–symmetry in supergravity and
tensor multiplet background

With the constraints (1.32), (1.33), (1.34) and (1.43) the variation (1.42) of the superstring action (1.3) becomes

$$\delta_2 S_{sttsr} = -\frac{1}{4} \int_W [D(e^\Phi * \hat{E}_a) - \frac{1}{4} e^\Phi * \hat{E}_b \wedge \hat{E}^b \nabla_a \hat{\Phi} + i \delta_2 E^a -
+ i \int_W e^\Phi (\hat{E}_a - * \hat{E}_a) \wedge \sigma_{\lambdaa} \hat{E}^{\alpha} + c.c. +
+ \frac{i}{2} \int_W e^\Phi (\hat{E}_b - * \hat{E}_b) \wedge \hat{E}^b \nabla_a \hat{\Phi} i \delta_2 E^a + c.c.] (1.50)$$

Eq. (1.50) makes evident the presence of the local fermionic $\kappa$–symmetry defined by Eq. (1.17), but now with curved space supervielbein,

$$i_\kappa \hat{E}^a = 0, \quad i_\kappa \hat{E}^a \sigma_{\lambdaa} (\hat{E}^a - \hat{E}^a) = 0. \quad (1.51)$$

The solution of Eqs. (1.50) (cf. Eq. (1.13) and above) provides us with the explicit form of the $\kappa$–symmetry transformations,

$$\delta_\kappa \hat{Z}^M (\xi) = \delta_\kappa^a (\xi) - \sqrt{g} \epsilon_{a b k} g^{k m} \hat{E}^a_m \hat{\sigma}^a \sigma_a^M (\hat{Z}) + c.c., \quad (1.52)$$

where $g^{m n}(\xi)$ is the matrix inverse to the induced metric $g_{m n}(\xi) = E^a_m (\hat{Z}) E^b_n (\hat{Z}) \epsilon_{a b}$

$$= \partial_m \hat{Z}^a (\xi) \epsilon_{a b} \hat{Z}^b (\xi) E^a (\hat{Z}) E^b (\hat{Z}). \quad (1.53)$$

The standard flat superspace Green-Schwarz $\kappa$–
symmetry transformations (1.13) can be derived from (1.52) by substitution of the flat superspace expressions for the (inverse) supervielbein coefficients $E^a_M(\hat{Z})$ and for $E^a_M(\hat{Z})$ in (1.53).

II. $D = 4 \ N = 1$ SUPERFIELD SUPERGRAVITY

ACTION

The action of $D = 4 \ N = 1$ supergravity is given by the invariant supervolume of $D = 4, N = 1$ superspace

$$S_{SG} = \int d^4 x d^2 \theta \ sdet(E^A_M) = \int d^8 Z \ E, \quad (2.1)$$

where $E := sdet(E^A_M)$ is Berezinian (superdeterminant) of the supervielbein $E^A_M(\hat{Z})$, Eq. (1.7), and $E^A_M(\hat{Z})$ are assumed to be subject to the constraints (1.30), (1.31).

A. Admissible variations of supervielbein

As the supervielbein is considered to be restricted by the constraints, its variation cannot be treated as independent. To find admissible variation one can, following [22], denote the general variation of the supervielbein and spin connections by

$$\delta E^A_M (\hat{Z}) = E^B_M K^A_B (\delta), \quad \delta u^a_B (\delta) = E^a_B u^b_B (\delta), \quad (2.2)$$

and obtain the equations to be satisfied by $K^A_B (\delta), u^b_B (\delta)$ from the requirement that the constraints (1.30), (1.31) are preserved under (2.2).

Quite complicated but straightforward calculations result in the following expression (2.1) for admissible variations of the supervielbein

$$\delta E^a = E^a (\Lambda (\delta) + \tilde{\Lambda} (\delta)) - \frac{1}{4} E^a \hat{\sigma}^a \sigma_a (\hat{D}^a, \hat{D}_a) \delta H^a + \frac{i}{4} E^a \hat{D}^a \delta H^a - i E^a \hat{D}_a \delta H^a, \quad (2.3)$$

$$\delta E^a = E^a \Xi^a (\delta) + E^a \Lambda (\delta) + \frac{1}{8} \hat{E}^a R \sigma_{\lambdaa} \sigma_a H^a. \quad (2.4)$$

In Eqs. (2.3), (2.4), $\Lambda (\delta), \tilde{\Lambda} (\delta)$ are given by

$$\Lambda (\delta) = \frac{1}{2} \hat{\sigma}^a (\hat{D}_a, \hat{D}_a) H^a + \hat{D}^a \delta H^a + \frac{1}{2} \hat{G}_a \delta H^a + \frac{1}{2} \hat{H}^a \delta H^a + 2 \hat{D}^a \hat{D}_a \delta \hat{H}^a + \hat{D}^a \hat{D}_a \delta \hat{H}^a \quad (2.5)$$

$$\tilde{\Lambda} (\delta) = \frac{1}{2} \hat{\sigma}^a (\hat{D}_a, \hat{D}_a) \delta H^a + \hat{D}^a \delta H^a + \frac{1}{2} \hat{G}_a \delta H^a + \frac{1}{2} \hat{H}^a \delta H^a + \hat{D}^a \hat{D}_a \delta \hat{H}^a + \hat{D}^a \hat{D}_a \delta \hat{H}^a \quad (2.6)$$

the explicit expression for $\Xi^a (\delta)$ in (2.4) as well as the expression for the basic variations of the spin connection, $\hat{\sigma}^a (\delta)$ in Eq. (2.4), will not be needed below (they can be found in [22]).

Note that the variations representing the manifest
gauge symmetries of supergravity are factored out from the above expressions. These are the superspace local
Lorentz transformations and the variational version of the superspace general coordinate transformations (see [22]).

For the free supergravity action (2.1) the nontrivial dynamical equations of motion should follow from the
variations (2.3), (2.4) with (2.3), (2.4) only. The variation of the superdeterminant $E = sdet(E^A_M)$ under (2.3),
(2.4), has the form (see [22])

$$\delta E = E \left[ - \frac{1}{12} \hat{\sigma}^a (\hat{D}_a, \hat{D}_a) H^a + \frac{1}{6} \hat{G}_a \delta H^a + 2 (\hat{D}^a \hat{D}_a \delta \hat{H}^a + \hat{D}^a \hat{D}_a \delta \hat{H}^a) \right]. \quad (2.7)$$

In the light of the identity

$$\int d^8 Z E (D_A \xi^A (\hat{L}) - (\hat{L}) \equiv 0, \quad (2.8)$$

all the terms with derivatives can be omitted in (2.7) when one considers the variation of the action (2.1). The
first equation in (2.8) uses the minimal supergravity constraints which imply \( T_{BA}^\alpha (−1)^A = 0 \). Hence,

\[
\delta S_{SG} = \int d^8 Z \delta E = \int d^8 Z E \left[ \frac{1}{16} \frac{d}{d \delta \bar{U}} \right] \delta H^a - 2 R \delta \bar{U} - 2 \bar{R} \delta U \]

(2.9)

and one arrives at the following superfield equations of motion for ‘free’, simple \( D = 4 \), \( N = 1 \) supergravity:

\[
\frac{\delta S_{SG}}{\delta H^a} = 0 \implies G_a = 0 , \quad (2.10) \\
\frac{\delta S_{SG}}{\delta \delta \bar{U}} = 0 \implies R = 0 , \quad (2.11) \\
\frac{\delta S_{SG}}{\delta \delta U} = 0 \implies \bar{R} = 0 . \quad (2.12)
\]

III. TENSOR MULTIPLE IN CURVED SUPERSPACE

A. The ‘improved’ action for tensor multiplet

As was argued in Sec. II D (see also [18, 25, 27]), the most suitable action for the description of a tensor multiplet interacting with a superstring is provided by the Weyl invariant de Wit–Roˇtiplet interacting with a superstring is provided by the

\[
\delta H^a = \frac{1}{4} \left( \bar{\sigma} [a] \sigma [b] \right) \beta^a \bar{\sigma} \delta H^b , \quad (3.3)
\]

\[
\delta \bar{R} = - \frac{i}{4} \left( \bar{\sigma} [a] \sigma [b] \right) \delta^a \bar{\sigma} \delta \bar{R} , \quad (3.4)
\]

\[
\delta R = - \frac{i}{4} \left( \bar{\sigma} [a] \sigma [b] \right) \delta^a \bar{\sigma} \delta R . \quad (3.5)
\]

As the components of the superfield strength of the two–form \( B_2 \) are expressed through the dilaton superfield, it should not be a surprise that the preservation of

\[
\delta e^\frac{\hat{h} \Phi}{2} = \frac{i}{4} \bar{D}_\alpha (\bar{D} \hat{D} - \tilde{R}) \delta \nu^a - \frac{i}{4} \bar{D}_\alpha (\bar{D} \hat{D} - \tilde{R}) \delta \nu^a . \quad (3.6)
\]

where \( (\Lambda(\delta) + \tilde{\Lambda}(\delta)) \) is defined in Eq. (2.40).

IV. INTERACTING ACTION AND SUPERFIELD EQUATIONS OF MOTION

Now that we have found all the necessary basic variations, we may turn to varying the coupled action

\[
S = \int d^8 Z sdet(E_{\alpha}^A) \left( 1 + s \frac{\hat{h} \Phi}{2} \right) + S_{sstr} , \quad (4.1)
\]

\[
S_{sstr} = \int \left[ \frac{1}{4} \ast \hat{E}_a \wedge \hat{E}^a + \hat{B}_2 \right] , \quad (4.2)
\]

to derive the equations of motion.

A. Superstring equations

Clearly, the superstring equations of motion for the interacting system keep the same form as the superstring equations in the superspace background of the superfield supergravity and tensor multiplet,

\[
\ast \hat{E}^a - \hat{E}^a \wedge (\sigma_{a a} \hat{E}^b - \frac{1}{4} \hat{E}^b (\sigma_{a b} \sigma_\beta \hat{\nabla}_\beta \hat{\Phi})) = 0 , \quad (4.3) \\
\text{and c.c.}
\]

\[
\bar{D}(e^\frac{\hat{h} \Phi}{2} \wedge \hat{E}^a \wedge \hat{E}^b \hat{H}_{a b c} - \frac{1}{2} \hat{E}_b \wedge \hat{E}_a \hat{\nabla}_c \hat{\Phi} + \hat{E}^c \wedge \hat{E}^a (\sigma_{a b} \hat{\nabla}_b \hat{\Phi}) - 2 i \hat{E}^a \wedge \hat{E}^b \sigma_{a a} \hat{E}^b = 0 . \quad (4.4)
\]
B. Superfield equations for tensor multiplet

The equations of motion for the tensor multiplet appear as a result of the \( \delta \mathcal{H} \) and \( \delta \bar{\mathcal{H}} \) variations of the dilaton superfield, Eq. (3.7), and the 2-superfield \( B_2 \), Eqs. (3.9)–(3.10). They are

\[
s(D\bar{D} - R) \left( e^{-\frac{1}{2} F} D_a e^{\frac{1}{2}} \right) = - (D\bar{D} - R) D_a K_a^a + 4i \sigma_{\beta\bar{\beta}} (D\bar{D} - R) W^{\beta\bar{\beta}} - \frac{1}{2} (\sigma_a \sigma_b) \alpha^\beta (D \bar{D} - R) D_{\bar{a}} W^{a b} \ , \quad (4.5)
\]

\[
s(D\bar{D} - \bar{R}) \left( e^{-\frac{1}{2} F} D_a e^{\frac{1}{2}} \right) = - (D\bar{D} - \bar{R}) D_a K_a^a + 4i \sigma_{\beta\bar{\beta}} (D\bar{D} - \bar{R}) W^{\beta\bar{\beta}} + \frac{1}{2} (\sigma_a \sigma_b) \alpha^\beta (D \bar{D} - \bar{R}) D_{\bar{a}} W^{a b} \ , \quad (4.6)
\]

where

\[
W^{BA} := \frac{1}{2} \int_{W^2} \frac{1}{E} \bar{E}^B \wedge \bar{E}^A \delta^8(Z - \bar{Z}) \quad (4.7)
\]

are current prepotentials which appear naturally in any variation of the Wess-Zumino term of the superstring action. In the same manner, any variation of the Nambu-Goto terms of the superstring action will be expressed through the current prepotential

\[
K_a^B := \frac{1}{4} \int_{W^2} \frac{e^{\frac{1}{2} F}}{E} \bar{E}_a \wedge \bar{E}^B \delta^8(Z - \bar{Z}) \quad (4.8)
\]

(cf. with the superparticle current prepotentials in [17]).

C. Superfield supergravity equations

Now let us turn to the supergravity equations for the coupled system, which appear as a result of the \( \delta \mathcal{H} \), \( \delta \mathcal{U} \), \( \delta \bar{\mathcal{U}} \) variations (see Sec.II for free supergravity).

The first observation is that, in accordance with (3.7) and (3.8), the variation of the Nambu-Goto terms of the superstring action with respect to supergravity superfields does not contain an input from \( \Lambda(\delta), \bar{\Lambda}(\delta) \), defined in Eq. (2.10), (2.11),

\[
\delta U S_{str} = \int_{W^2} \left[ \frac{1}{2} * \bar{E}_a \wedge \delta_4 \bar{E}^a \wedge e^{\frac{1}{2}} + \frac{1}{4} * \bar{E}_a \wedge \bar{E}^a \delta_4 e^{\frac{1}{2}} \right] = \frac{1}{2} \int_{W^2} * \bar{E}_a \wedge \bar{E}^a \delta_4 (D\bar{D} - \bar{R}) \delta U = -\frac{1}{4} \int_{W^2} * \bar{E}_a \wedge \bar{E}^a e^{\frac{1}{2}} 2(D\bar{D} - \bar{R}) \delta U = 0 \quad (4.9)
\]

Clearly, no such inputs come from the variations of the pull-back \( B_2 \) of the two-form \( B_2 \), as Eqs. (3.9)–(3.10) do not contain \( \Lambda(\delta), \bar{\Lambda}(\delta) \) at all. As the chiral variations \( (D\bar{D} - \bar{R}) \delta U \) c.c. are involved in the variations of the supervielbein only inside the \( \Lambda(\delta), \bar{\Lambda}(\delta) \) combinations, this means that the equations \( \frac{\delta S}{\delta U} = 0 \) and \( \frac{\delta S}{\delta \bar{U}} = 0 \) do not possess an input from the superbrane source (as was also the case with the supergravity—superparticle system, see [17]). These equations can acquire an input from the action of the real linear multiplet, which in general, has the form \( s \int d^8 Z E f(\frac{\delta S}{\delta Z}) \) with an arbitrary function \( f \). However, one can check that an input in the \( \delta U \) variation of the improved kinetic term, \( s \int d^8 Z E \frac{\delta S}{\delta Z} \), also vanishes

\[
\delta U \int d^8 Z E \frac{\delta S}{\delta Z} = 0 \quad (4.10)
\]

Thus, for the coupled action \( [17] \), one finds that the chiral superfield equation \( \frac{\delta S}{\delta U} = 0 \) remains the same as in the case of 'free' supergravity,

\[
\frac{\delta S}{\delta U} = 0 \quad \Rightarrow \quad \bar{R} = 0 \quad (4.11)
\]

\[
\frac{\delta S}{\delta \bar{U}} = 0 \quad \Rightarrow \quad R = 0 \quad (4.12)
\]

Thus, in this case, as in the case of supergravity—massless superparticle \( [17] \), only the vector superfield supergravity equation \( \frac{\delta S}{\delta H} = 0 \) acquires a source term from the superstring. However, in the coupled system under consideration, these equations are more complicated due to the supergravity interaction with the tensor multiplet,

\[
\frac{\delta S}{\delta H^a} = 0 \quad \Rightarrow \quad G_a (1 - s e^{\Phi}) = \mathcal{J}_a + s (5 + 3 \Phi) \delta_{a}^{\beta\bar{\beta}} [D_{\beta} \bar{D}_{\bar{\beta}}] e^{\Phi} + 3s \delta_{a}^{\beta\bar{\beta}} e^{-\frac{1}{2} \bar{D} \bar{\Phi} e^{\bar{F}}} D_{\beta} \bar{D}_{\bar{\beta}} e^{\Phi} \quad (4.13)
\]

The superstring current potential

\[
\mathcal{J}_a = -6 \frac{\delta S_{str}}{\delta H^a} \quad (4.14)
\]

entering the r.h.s of Eq. (4.13), can be expressed through two types of current prepotentials, Eqs. (4.8) and (4.9), as follows

\[
-\frac{1}{6} \mathcal{J}_a = 2i D_{\beta} K^a_{\beta} - 2i \bar{D}_{\bar{\beta}} K^\beta_a - \delta_{a}^{b\beta} [D_{\beta} \bar{D}_{\bar{\beta}}] K^b_a - \frac{1}{2} K^b_a \delta_{a}^{b\beta} [D_{\beta} \bar{D}_{\bar{\beta}}] e^{\Phi} + \frac{i W^{a b} (\eta_{ba} + (\sigma_a \sigma_b) D_{\bar{\beta}} \bar{D}_{\bar{\beta}}) e^{\Phi}}{s} + \frac{i W^{a b} (\eta_{ba} + (\sigma_a \sigma_b) D_{\bar{\beta}} \bar{D}_{\bar{\beta}}) e^{\Phi}}{s} + \frac{i W^{a b} (\eta_{ba} + (\sigma_a \sigma_b) D_{\bar{\beta}} \bar{D}_{\bar{\beta}}) e^{\Phi}}{s} \ . \quad (4.15)
\]

The first line of the r.h.s of Eq. (4.15) has exactly the same form as the expression for the current through the superparticle current potential in the supergravity—superparticle coupled system \( [17] \); the second line contains the trace \( K^a_b \) of the bosonic current potential (4.8) which vanishes in the superparticle case but is nonzero for the superstring; the remaining part of the r.h.s of Eq. (4.15) contains the current prepotentials (4.7) which come from the variation of the superstring Wess-Zumino term.
Note that on the shell of Eqs. (1.11), (1.12) \( R = 0 = \tilde{R} \), the equations for the tensor multiplet (4.15), (4.16) simplify to
\[
\mathcal{D}[se^{-\frac{2}{3}}\mathcal{D}_\alpha e^\frac{2}{3}] + \mathcal{D}_\alpha K^a - 4i\sigma_{b\alpha\beta}W^{b\beta} + \frac{1}{2}(\sigma_{[a}\sigma_{b]}\beta)\mathcal{D}_\beta W^{ab} + \mathcal{D}_\alpha K^a - 4i\sigma_{b\alpha\beta}W^{b\beta} - \frac{i}{4}(\delta_{[a}\sigma_{b]}\beta)\mathcal{D}_\beta W^{ab} = 0. \tag{4.16}
\]

D. Superfield generalization of the Einstein and Rarita–Schwinger equations with sources

In the minimal (off-shell) supergravity the superfield generalization of the Ricci tensor and of the Rarita–Schwinger spin–tensor are expressed through the vector and chiral scalar superfields as follows (see [37] as well as \[ 17 \], \[ 22 \], \[ 37 \], refs. therein) and chiral scalar superfields as follows (see [37] as well as \[ 17 \], \[ 22 \], \[ 37 \], refs. therein and also \[ VA \].

\[ R_{bc}^{\alpha\beta} = \frac{1}{32}(\mathcal{D}^\beta\mathcal{D}^{[\alpha}\mathcal{G}^{\beta\beta]}(\Phi) - \mathcal{D}^{[\beta}\mathcal{D}^{\alpha]\mathcal{G}^{\beta}\beta]}(\Phi))\sigma_{\alpha\alpha}^a\sigma_{b\beta}^b + \frac{1}{32}(\mathcal{D}^{[\alpha}\mathcal{D}^{\beta]}G^{\alpha\beta}(\Phi)\mathcal{D}^{\beta}\mathcal{D}^{[\alpha}\mathcal{G}^{\beta\beta]}(\Phi) + \mathcal{D}^{[\alpha}\mathcal{D}^{\beta]}(\mathcal{J}^{\alpha\beta}\mathcal{G}^{\alpha\beta}(\Phi))/\mathcal{D}^{\alpha}(\Phi). \tag{4.24}
\]

The spacetime Einstein and Rarita–Schwinger equations can be obtained as the leading \( (\theta = 0) \) components of the superfield equations (4.21), (4.23) in the Wess–Zumino gauge (see \[ 17 \], \[ 22 \], \[ 37 \], refs. therein and also \[ VA \].

To obtain the superfield generalizations of the Rarita–Schwinger equations with sources for the interacting system one substitutes in Eqs. (4.11), (4.13) with \( R = 0 \) the formal solution
\[ G_a = \frac{1}{1 - se^{\frac{2}{3}}} \mathcal{J}_a + \mathcal{G}_a(\Phi) \tag{4.21}
\]
of the superfield equation (4.18). In (4.21) \( \mathcal{G}_a(\Phi) \) denotes the on–shell ‘value’ of the \( \mathcal{G}_a \) superfield in the system of supergravity interacting with a dynamical tensor multiplet, \( i.e. \) in the absence of the superstring,
\[ \mathcal{G}_a(\Phi) := \frac{5 + 3\frac{2}{3}}{1 - se^{\frac{2}{3}}} \sigma_{\alpha\alpha}^a[D_{\beta}, \mathcal{D}_{\beta}]e^{\frac{2}{3}} + \frac{3s}{1 - se^{\frac{2}{3}}} \sigma_{\alpha\alpha}^a e^{\frac{2}{3}} \mathcal{D}_{\beta} e^{\frac{2}{3}} \mathcal{D}_{\beta} e^{\frac{2}{3}}. \tag{4.22}
\]

Thus the superfield generalizations of the Rarita–Schwinger and the Einstein equations in the supergravity–tensor multiplet—superstring interacting system read
\[ \Psi_{\alpha}^a := e^{abcd}T_{bc}^{\alpha} \sigma_{do\alpha} = \frac{5 + 3\frac{2}{3}}{1 - se^{\frac{2}{3}}} \sigma_{\alpha\alpha}^a[D_{\beta}, \mathcal{D}_{\beta}]e^{\frac{2}{3}} + \frac{3s}{1 - se^{\frac{2}{3}}} \sigma_{\alpha\alpha}^a e^{\frac{2}{3}} \mathcal{D}_{\beta} e^{\frac{2}{3}} \mathcal{D}_{\beta} e^{\frac{2}{3}}. \tag{4.23}
\]

E. Superfield generalization of the Kalb–Ramond gauge field equations with source for the interacting system

Taking the vector covariant derivative of the expression \( (4.40) \) for \( H_{abc} \), one finds the off–shell expression for the l.h.s of the 2–superform gauge field equation,
\[ \mathcal{D}^c H_{abc} = \frac{1}{32}(\sigma_{[a}\mathcal{S}_{b]}^\alpha)\mathcal{D}(\mathcal{D}^c\mathcal{D}^c)\mathcal{D} e^{\frac{2}{3}} + c.c. - \frac{1}{2}[\mathcal{D}_a, \mathcal{D}_b]e^{\frac{2}{3}} + \frac{1}{2}G_a\mathcal{D} e^{\frac{2}{3}} + \frac{1}{2}H_{abc}G^c + \frac{5}{32}e^{abcd}\mathcal{D}^c e^{\frac{2}{3}} e^{\frac{2}{3}} - \frac{i}{6}\sigma_{[a}\mathcal{S}_{b]}^\alpha W_{abc\gamma} e^{\frac{2}{3}} + c.c. \tag{4.27}
\]

To derive from (4.27) the (superfield generalization) of the antisymmetric tensor gauge field equations (Kalb–Ramond equations), we shall substitute the expression for \( \mathcal{D}(\mathcal{D}^c\mathcal{D}^c)\mathcal{D} e^{\frac{2}{3}} \) which follows from acting by the spinor covariant derivative \( \mathcal{D}_{\beta} \) on the superfield equations of motion (4.16) for the tensor multiplet and, then, substitute (4.21) for \( G^a \). The equations thus obtained have quite a complicated form. Writing explicitly only the terms
with the maximal number of the spinor covariant derivatives acting on the current prepotentials (see below for a special rôle of such terms) one gets

\[ sD^eH_{abc} = - \frac{1}{32} e^{-\frac{1}{2} (\sigma_4 \tilde{\sigma}_b)} \partial_\alpha D_\alpha \tilde{D}_\beta D_\beta K_a^a + \]

\[ + \frac{i}{8} e^{-\frac{1}{2} (\sigma_4 \tilde{\sigma}_b)} \sigma_c \partial_\alpha D^n \tilde{D}^n W^{ac} - \]

\[ - \frac{1}{64} e^{-\frac{1}{2} (\sigma_4 \tilde{\sigma}_b)} \sigma_c \partial_\alpha D_\beta \tilde{D}^b D^n W^{cd} + \]

\[ + c.c. + \ldots \]  

(4.28)

A further study of the complete form of the superfield equations for the tensor multiplet and for the Kalb–Ramond gauge field entering that multiplet will be the subject of a separate paper.

Below we will show that the knowledge of the general form of the tensor multiplet superfield equations with the source, Eq. (4.1), and of its relation to the gauge field equation (through Eq. (4.28)) already allow one to make interesting conclusions that provide a shortcut in the study of the interacting system.

V. GAUGE EQUIVALENT DESCRIPTION OF THE SUPERFIELD INTERACTING SYSTEM

A. Superdiffeomorphism symmetry gauge fixing

The interacting action (4.1) is manifestly invariant under the local Lorentz symmetry and under superdiffeomorphisms

\[ Z'^M = Z^M + b^M(Z) : \left\{ \begin{array}{l}
\dot{\chi}^\mu = \chi^\mu + b^\mu(x, \theta), \\
\dot{\theta}^\alpha = \theta^\alpha + \epsilon^\alpha(x, \theta)
\end{array} \right. \]  

(5.1)

\[ E'^A(Z') = E^A(Z), \quad \Phi'(Z') = \Phi(Z) \]

\[ w'^{ab}(Z') = w^{ab}(Z), \quad etc., \]  

(5.2)

which act on the superstring variables, coordinate functions \( Z^M = \tilde{Z}^M(\xi) \equiv (\tilde{x}^\mu(\xi), \tilde{\theta}^\alpha(\xi)) \), by the pull–back of the transformations (5.1),

\[ \dot{Z}'^M = \dot{Z}^M + b^M(\dot{Z}) : \left\{ \begin{array}{l}
\dot{\chi}^\mu = \dot{\chi}^\mu + b^\mu(\dot{x}, \dot{\theta}), \\
\dot{\theta}^\alpha = \dot{\theta}^\alpha + \epsilon^\alpha(\dot{x}, \dot{\theta})
\end{array} \right. \]  

(5.3)

The action (4.1) is also invariant under the worldsheet reparametrizations and under the \( \kappa \)-symmetry (1.52), which act on the coordinate functions only.

Thus, omitting the worldvolume reparametrization for simplicity, the complete variation of the superstring coordinate function under the local symmetries of the interacting action (4.1) is given by

\[ \delta \tilde{Z}^M(\xi) = b^M(\tilde{Z}(\xi)) + \delta_\kappa \tilde{Z}^M(\xi), \]  

(5.4)

where \( \delta_\kappa \tilde{Z}^M(\xi) \) is defined in (1.52).

Now we observe (17, 35) that the superdiffeomorphism symmetry can be used to fix the ‘fermionic unitary gauge’

\[ \tilde{g}_\alpha(\xi) = 0 \iff \tilde{Z}^M(\xi) = (\hat{z}^\mu(\xi), 0). \]  

(5.5)

Moreover, in the same manner as in (17) one can show that this gauge can be fixed simultaneously with the Wess–Zumino gauge for supergravity (see (40, 17) and refs. therein),

\[ \theta^\alpha E^\mu_\alpha(x, \theta) = 0, \quad \theta^\alpha (E^\alpha_\beta(x, \theta) - \delta^\alpha_\beta) = 0, \]

\[ \theta^\alpha w^{ab}(x, \theta) = 0, \]  

(5.6)

where, in particular, (22, 35)

\[ E^a_\mu |_{\theta = 0} \propto \epsilon^a_\mu(x), \quad E^a_\alpha |_{\theta = 0} \propto \psi^a_\alpha, \]  

(5.7)

\[ E^\alpha_\beta |_{\theta = 0} = 0, \quad E^\alpha_\beta |_{\theta = 0} = \delta^\alpha_\beta, \]

(5.8)

\[ w^{ab}(x, \theta) \propto \omega^{ab}(x). \]  

(5.9)

This can be understood by observing that, although both gauges, Eqs. (5.6) and (5.5), are fixed with the use of the same superdiffeomorphism symmetry with the parameter \( b^M(Z) = b^M(x, \theta) \), the transformation rules of the supergravity superfields involve only derivatives of \( b^M(x, \theta) \) (characteristic property of the gauge field transformations), while the transformation rules of the coordinate functions \( \tilde{Z}^M(\xi) \) contain the additive contribution of \( b^M(\tilde{Z}(\xi)) = b^M(\dot{x}(\xi), \dot{\theta}(\xi)) \) (characteristic property of the Goldstone field transformations, but for Goldstone fields defined on a surface in superspace, see Sec. VIIB of (17) and (35, 36)).

B. Gauge fixed action

Let us discuss what happens with the interacting action (4.1), (12) in the gauge (5.5). After integration over the Grassmann coordinates, the Wess–Zumino supergravity action (5.5) becomes the standard supergravity action with the minimal set of auxiliary fields (11),

\[ S_{SG} = \int d^4 x \tilde{d}_\theta sdet(E^A_M) \propto \]

\[ \propto S_{sg} = \int d^4 x (eR + e^{\mu
u\rho\sigma} \tilde{\psi}_\mu \gamma_5 \gamma_\nu D_\rho \psi_\sigma) \]

\[ + \mathcal{O}(g_a(x), r(x), \bar{r}(x)), \]  

(5.10)

where \( R = R_{\mu\nu} \) is the scalar curvature of spacetime, \( e = det(e^a_\mu(x)) \) and \( \mathcal{O}(g_a(x), r(x), \bar{r}(x)) \) denotes terms with auxiliary fields

\[ G_a |_{\theta = 0} \propto g_a(x), \quad R |_{\theta = 0} \propto r(x), \quad \bar{R} |_{\theta = 0} \propto \bar{r}(x) \]  

(5.11)

which are not essential for the consideration below. The improved tensor multiplet action (1.49) also entering
\( S_{TM} = s \int d^4x \left[ e^{\phi(x)/2} \kappa(x) \right] \), becomes, schematically, \( \text{(5.11)} \)

\[
S_{TM} = \int d^4x \left[ \frac{1}{2e^{\phi(x)/2}} \left( H^\mu_{\nu\rho}H_{\mu\nu\rho} + ie^2 \delta_{\alpha\beta} \right) \right] , \quad (5.12)
\]

where \( H_{\mu\nu\rho} \) denotes the spacetime covariant derivatives and the component fields of the tensor multiplet are defined by

\[
\Phi_\alpha \left|_{\theta=0} \right. = \phi(x), \quad D_\alpha e^\phi \left|_{\theta=0} \right. = \chi_\alpha(x) , \quad (5.13)
\]

and (cf. 1.43)

\[
[D_\alpha,D_\beta]e^\phi \left|_{\theta=0} \right. = \sigma_{\alpha\beta\gamma}e_\mu(x)e^{\mu\rho\sigma}H_{\nu\rho\sigma} , \quad (5.14)
\]

\( H_{\mu\nu\rho}(x) = 3\partial_\mu B_{\nu\rho}(x) \) in \( \text{(5.10)} \). In \( \text{(5.12)} \) we have written explicitly only the kinetic terms; the remaining ones may be extracted from the formulae in [24] and are not essential for what follows.

Finally, the superstring action \( \text{(5.12)} \) in the gauge \( \text{(5.14)} \) reduces to the bosonic string action

\[
S_{\text{str}} \left|_{\theta=0} \right. \propto S_{bstr} = \int_{W^2} \left[ \frac{1}{4}\kappa(x) + \kappa(x) \right] B_2(\hat{x}) \right] \right] , \quad (5.15)
\]

where

\[
B_2(\hat{x}) = \int_{W^2} \left[ \frac{1}{2e^{\phi(x)/2}} \right] \right] \right] \right] , \quad (5.16)
\]

with \( S_{\text{str}} \), \( S_{\text{bstr}} \) and \( S_{\text{int}} \) defined in \( \text{(5.10)} \), \( \text{(5.12)} \) and \( \text{(5.16)} \), respectively.

\section*{C. Supersymmetry of the gauge fixed action}

Note that, although the gauge fixed action \( \text{(5.10)} \) includes the action for the purely bosonic string \( \text{(5.15)} \), it possesses 1/2 of the local supersymmetry characteristic for the supergravity action. Actually, the direct proof of this fact can be found in [34]. Here we will show this in a different way which is based on the observation that the symmetries of the gauge fixed action \( \text{(5.16)} \) can be identified as a subset of the symmetries of the complete (superfield) action \( \text{(5.15)} \) which preserve the gauge \( \text{(5.16)} \) and the Wess–Zumino gauge (see 17) for the supergravity—superparticle interacting system.

Firstly note that in the Wess–Zumino gauge \( \text{(5.16)} \), the index of the superspace Grassmann coordinate is identified with the Lorentz group spinor index. Indeed, due to the second equation in \( \text{(5.10)} \),

\[
\theta^\alpha = (\theta^\alpha, \tilde{\theta}^\alpha) \gamma^\alpha E^\beta_{\alpha\beta}(Z) = \theta^\alpha \tilde{\theta}^\alpha . \quad (5.17)
\]

Clearly, the same is true for the superstring spinorial Grassmann coordinate functions. Their transformation rules can be read off Eq. \( \text{(5.18)} \),

\[
\delta \theta^\alpha = b^M(\hat{x}, \tilde{\theta}) E_M^\alpha(\hat{x}, \tilde{\theta}) + \delta_{\hat{\theta}} \theta^\alpha \equiv 0 \quad (5.18)
\]

where \( \delta_{\hat{\theta}} \theta^\alpha \) (read 11, \hbox{Eqs. 1.52} and 5.17)

\[
\delta_{\hat{\theta}} \theta^\alpha = E_M^\alpha(\hat{x}, \tilde{\theta}) \tilde{\theta}^\alpha \equiv 0 \quad (5.19)
\]

Clearly, the superdiffeomorphism parameter \( b^M(x, \theta) \) in \( \text{(5.18)} \) is to be restricted by the conditions of the preservation of the Wess–Zumino gauge. However (see \( \text{22, 40, 17} \) and refs therein) the parameter \( \epsilon_\alpha(x) = b^M(x, 0) E_M^\alpha(0, 0) \) remains unrestricted and is identified with the parameter of the local supersymmetry of the component (spacetime) formulation of supergravity.

Now, the preservation of the gauge \( \text{(5.19)} \), \( \hat{\theta}^\alpha(\theta) = 0 \), imposes the condition \( \hat{\delta} \theta^\alpha(\hat{x}) \mid_{\theta^\alpha(\hat{x}) = 0} \equiv 0, \) i.e.

\[
\epsilon_\alpha(\hat{x}) = -\delta_{\hat{\theta}} \theta^\alpha(\hat{x}) \mid_{\theta^\alpha(\hat{x}) = 0} = -\tilde{\kappa}_\alpha^\beta(\hat{x})(\tilde{\delta}_\alpha^\beta - \sqrt{|g|} \epsilon_{\alpha\beta} g^{\mu\nu}) e^\mu_\alpha(\hat{x}) \tilde{\theta}^\beta \equiv 0 . \quad (5.20)
\]

on the parameter of the local supersymmetry. This restriction appears only on the string worldsheet and expresses the pull–back \( \epsilon_\alpha(\hat{x}) \) of the supersymmetry parameter \( \epsilon_\alpha(x) \) through a worldsheet parameter \( \tilde{\kappa}_\alpha^\beta(\hat{x}) \) contracted (on both indices) with the expression \( \delta(\hat{x}, \tilde{\theta}) \) of the Green–Schwarz \( \kappa \)-symmetry ‘projector’ and makes only one parameter included in \( \tilde{\kappa}_\alpha^\beta(\hat{x}) \) being involved effectively in the expression.

Thus on the worldsheet \( W^2 \) of a (dynamical) string the 4 parameters of local supersymmetry of the free supergravity are reduced, by the condition of the invariance of the gauge fixed interacting action \( \text{(5.16)} \), to the 2 effective parameters of the \( \kappa \)-symmetry–like transformations, while out of \( W^2 \) these 4 parameters, \( \epsilon_\alpha(x), \tilde{\epsilon}_\alpha(x), \) remain unrestricted. This can be characterized by stating the preservation of the 1/2 of the local supersymmetry of the free supergravity by the gauge fixed interacting action \( \text{(5.16)} \).
particular, this means that in the gauge 5.5, 5.6 neither the Rarita–Schwinger equations, nor the equations for the fermionic field of the tensor multiplet will include a source term from the superstring (although they will be written with covariant derivatives for the spin connections satisfying the sourceful Einstein equations with an input from the superstring energy–momentum tensor).

This is a manifestation of a counterpart of the super–Higgs effect in general relativity interacting with a simplest supersymmetric extended object:

\[ \hat{\theta}^\alpha = 0 \] is only the second derivative of the current potential may produce a term with a nonvanishing leading component (\( \theta^0 \)-independent part),

\[ \mathcal{D}^2_{BC} \mathcal{J}_a \propto \theta^0 \] (5.30)

The spacetime fermionic equations of motion of the interacting system may be obtained as leading (\( \theta = 0 \)) components of the Eqs. (5.23) and (5.29). Ignoring the inputs with a smaller number of derivatives applied to the current potential, one can write the leading components of Eq. (5.29) as

\[ (\Psi^a \theta^a - \Xi^a(\Phi))|_{\theta=0} \propto \mathcal{D}_B \mathcal{J}_a|_{\theta=0} \] (5.31)

where \( \Xi^a(\Phi) \) denotes the tensor multiplet contribution to the gravitino equation, which is given by the first term in the r.h.s. of Eq. (5.29). Then Eq. (5.29) implies that the r.h.s. of Eq. (5.32) vanishes in the gauge (5.5), i.e. that in this gauge Eq. (5.32) reads

\[ \Psi^a|_{\theta=0} = \Xi^a(\Phi)|_{\theta=0} \] (5.32)

and does not contain an explicit input from the superstring.

In the same manner one finds that the dynamical equation for the fermionic component of the tensor multiplet, given by the leading component of 4.110

\[ s \mathcal{D} \mathcal{D} |e^{-\frac{1}{2}(\mathcal{D}_a \mathcal{D}_a + \mathcal{D}_a \mathcal{D}_a)}|_{\theta=0} = -\mathcal{D} \mathcal{D} \mathcal{D}_a K_{a}^{a} - \frac{1}{2} (\sigma_\alpha \hat{\theta}^\beta) \mathcal{D} \mathcal{D} \mathcal{D}_a W^{|\beta|}_{a b} \] (5.33)
implies, in the light of Eq. (5.20), that the superstring-produced r.h.s. of this equation vanishes in the gauge (5.19):

$$s\tilde{D}[e^{-\frac{D}{2}D_a e^{\frac{D}{2}}}]|_{\theta=0} = 0.$$  (5.34)

As far as the bosonic superfield equations are concerned, they preserve the nontrivial input from the superstring source in the gauge (5.5). Indeed, the leading components of Eqs. (5.23)–(5.27) contain the second derivatives of the current potential and fourth derivatives of the current prepotentials, which remain nonvanishing in accordance with Eqs. (5.30) and (5.20).

The explicit form of the Einstein equation can be derived in a way close to the one used in [17] for the supergravity—superparticle interacting system (although the presence of both Nambu–Goto and Wess–Zumino terms in the superstring action, as well as of the tensor multiplet in the action of the interacting system, makes the Einstein equation of the supergravity—tensor multiplet—superstring system a bit more complicated). As far as the Kalb–Ramond field equations are concerned, taking into account Eqs. (4.24), (4.28), and the conditions of the Wess–Zumino gauge (5.5), (5.6), one finds that, in the gauge (5.5), the leading component of the superfield equation (4.38) becomes

$$sD^e H_{abc}|_{\theta=0} = \frac{1}{16} e^{-\frac{D}{2}} w^{cd} + \ldots.$$  (5.35)

where

$$w^{ba} := \frac{1}{2} \int W^a \frac{1}{e^{\phi}} \delta^e (x - \hat{x}).$$  (5.36)

Hence, in the gauge (5.5), the superstring input on the r.h.s. of the Kalb–Ramond gauge field equation for the supergravity—tensor multiplet—superstring interacting system is nonvanishing and, moreover, coincides with the input of the bosonic string.

Thus we have checked that the spacetime equations of motion for the fermionic fields which follow from the complete superfield action (5.10) of the supergravity—tensor multiplet—superstring interacting system become sourceless in the gauge (5.5). This is true both for the gravitino equations and for the fermionic field of the tensor multiplet. At the same time, the corresponding bosonic equations are clearly sourceful in any gauge. This is a characteristic property of the equations which follow from the gauge fixed action (5.10).

We should note that the gauge fixed fermionic equations are not completely decoupled from the superstring. They are written in terms of the spacetime covariant derivatives with (composed) spin–connections satisfying the Einstein equation with a source. The same is true for the equations derived directly from the gauge fixed action (5.10).

F. On possible application of the gauge equivalence

Note that the gauge fixed description (5.10) of the superfield interacting system (5.16) is complete in the following sense. Along the line of [15] one may check that the gauge fixed action (5.10), (5.12) reproduces the gauge fixed version of all the dynamical equations which might be derived from the complete superfield action, including the fermionic equations for the bosonic string. This is the manifestation of the purely gauge (or Goldstone) nature of the superstring coordinate functions $Z^M(\xi)$ (not to be confused with the supercoordinates $Z^M$; see [30] and [32] for further discussion).

A counterpart of the gauge fixed action (5.10) can be written in any dimension, for any supergravity interacting with any superbrane. Hence, using the above described gauge equivalence one may already proceed with studying the $D = 11$ supergravity interacting with super–M2–branes and super–M5–branes, as well as the $D = 10$ type II supergravity interacting with super–Dp–branes (in spite of the fact the $D = 10, 11$ superfield supergravity actions are not known).

VI. CONCLUSIONS

In this paper we studied the full superfield Lagrangian description of the $D = 4$ interacting system of dynamical supergravity and a superstring described by the sum $S = S_{SG} + S_{TM} + S_{sstr}$ of the superfield supergravity action $S_{SG}$ [22], the Green–Schwarz superstring action $S_{str}$ [23] and a superfield action for the dynamical tensor multiplet $S_{TM}$ [24].

The superfield theoretical system $S = S_{SG} + S_{TM}$ was argued to be related to the low–energy limit of $D = 4$ compactification of the heterotic superstring [25, 18] (see also [28]) and considered in [27, 24]. So, our main interest here has been to analyze the influence of the superstring action $S_{sstr}$ on the dynamics of the interacting system, namely in the superstring–produced source terms both in the superfield and component form of the equations.

We have obtained the complete set of superfield equations with sources provided by the superstring. In the supergravity sector we found that the scalar superfield equation remains the same as for free supergravity, while the vector superfield equation is modified both by the interaction with the tensor multiplet and by the source (current potential) coming from the superstring. The current potential is constructed from the two types of current prepotentials coming from the variation of the Nambu–Goto and the Wess–Zumino terms of the superstring action, respectively. The superfield equations for the tensor multiplet are also modified by inputs from the above mentioned current prepotentials. The equations of motion for the superstring variables are the same as in the background of supergravity interacting with dilaton and super–2–form superfields.

By analyzing the gauge symmetries and taking into
account the properties of the Wess–Zumino gauge (see 17
and refs. therein) we have shown that there exists a
complete gauge equivalent description of the ‘superfield
action for supergravity $S_{sg}$ (without auxiliary fields), the
component action for the tensor multiplet $S_{tm}$ and the
action for a bosonic string $S_{str}$ (which appears as the
purely bosonic ‘limit’ of the supersting action $S_{sstr}$). We
calculated this gauge equivalence by studying the prop-
erties of the gauge fixed version of the equations of mo-
tion derived from the complete superfield action. Despite
the quite complicated form of the superfield generaliza-
tions of the Einstein–Rarita–Schwinger equations, as well as of the Kalb–Ramond equations and the equa-
tions for the fermionic fields of the tensor multiplet, it
turned out quite easy to show that in the above men-
tioned ‘fermionic unitary gauge’ all the fermionic equa-
tions are sourceless (although they include the covariant
derivatives with the spin connections obeying the source-
ful Einstein equations) while the bosonic equations, in-
cluding the Einstein and Kalb–Ramond field equations,
acquired a source from the supersting.

This extends the supergravity–massless superparticle
results of 17 to the case of dynamical interacting sys-
tems including supergravity and an extended supersym-
metric object and strongly supports that the above men-
tioned gauge equivalence is not an artifact of the sim-
pler massless superparticle case, but rather is a general
property of the above mentioned interacting systems. As
the component actions for supergravity are known in all
dimensions, including $D = 10, 11$ (the most interesting
from an M–theoretic perspective), our results allows one
to obtain and to study the complete set of equations for
dynamical supergravity interacting with dynamical super–$p$–brane, at least in its gauge fixed version. This
promises to be a useful tool in a future search for new
solitonic solutions of higher dimensional supergravity in-
cluding the ones with nonvanishing fermionic fields.

An analysis of the superfield equations with sources ob-
tained in this paper, the search for their possible higher
dimensional generalization as well as the investigation of the $D = 10, 11$ supergravity–superbrane interacting
systems with the use of the gauge equivalence of their
complete superfield description with the description by
the sum of spacetime (component) action for supergrav-
ity and the action for bosonic brane will be the subject
of future work.

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sider the $D = 4$ supergravity–superstring interacting system as a relatively simple model for a more complicated $D = 10, 11$ supergravity–superbrane systems. An interesting alternative possibility is to consider the new minimal formulation of superfield supergravity [30], where the supergravity auxiliary fields are provided by a tensor multiplet, interacting with the Green–Schwarz superstring. This, however, is beyond the score of the present paper.

[44] Note that superfield equations of motion for the field theoretical part of our interacting system, i.e. following from the action $S = \int d^8 Z s \det(E_M^A)\left(1 + s \frac{\Phi^2}{2}\right)$, but without the $S_{evt}$ term, were considered in [27, 29].

[45] Note that a toy model of superfield interacting system, the coupled system of $D = 2$ supergravity and a superparticle, was studied in [31]. We thank W. Kummer and A. Nurmagambetov for pointing out this reference.

[46] One should keep in mind that $(\mathcal{D} \mathcal{D} - \bar{R})$ is a chiral projector, i.e. that $D_\alpha (\mathcal{D} \mathcal{D} - \bar{R}) U \equiv 0$ for any superfield $U = U(Z)$.

[47] In the framework of the conformal tensor calculus, to arrive at the component form of the action of the new minimal (off-shell) supergravity one starts from the improved tensor multiplet action in flat superspace, $\int d^8 Z L \ln L$ with $D D L = 0 = \bar{D} D L$, performs the Grassmann integration in it, then one introduces the coupling to the conformal supergravity and makes a gauge choice for the special superconformal transformation. As a result, one arrives at the action $\int d^4 x (\epsilon R + \epsilon^{\mu\nu\rho\sigma} \bar{\psi}_\mu \gamma_\nu \gamma_\rho \psi_\sigma + A_\mu \epsilon^{\mu\rho\sigma} \partial_\rho B_\sigma + \frac{1}{2} (\epsilon^{\mu\nu\rho\sigma} \partial_\nu B_\rho) J)$, which includes both the antisymmetric tensor $B_{\mu\nu}$ and the vector gauge field $A_\mu$ as auxiliary fields (see [24]). In contrast, in our case the interacting action involves the improved tensor multiplet action coupled to minimal supergravity in superfield formulation, $\int d^8 Z E L \ln L$ with $\ln L = \Phi^2 / 2$. In this case the antisymmetric tensor field $B_{\mu\nu}$ is dynamical and its equations of motion contain a source from the superstring.

[48] Note that with an independent variation of the supervielbein, the action would lead to the equation stating the vanishing of the superdeterminant $E$, which contradicts the original assumption about its nondegeneracy.