Study on remodeling approach quality of dynamic objects with variable structure

S L Blyumin¹, A V Galkin², P V Saraev² and A S Sysoev¹

¹ Department of Applied Mathematics, Lipetsk State Technical University, Moskovskaya str., 30 RU-398055 Lipetsk, Russia
² Faculty of Automation and Computer Science, Lipetsk State Technical University, Moskovskaya str., 30 RU-398055 Lipetsk, Russia

E-mail: sabl@lipetsk.ru, avgalkin82@mail.ru, psaraev@yandex.ru, anton_sysoyev@mail.ru

Abstract. The paper introduces the procedure of constructing new models based on already existing ones. This process is called Mathematical Remodeling. The inertial torque transformer (ITT) is considered as an object which model is under consideration. The functioning process of ITT varies depending on certain conditions. The initial model of ITT, built on the basis of physical laws underlaying its functioning, is a composite system of nonlinear non-stationary differential equations. Neurostructural models were selected as a class of models, to which the initial model was transformed. It was shown in many studies, that this class of models has a high approximation capabilities. The construction of neurostructural models was carried out on the data obtained by a numerical approximate solution of the original composite systems of differential equations showing ITT workflow. Due to the fact, that the approximate solution could be obtained by different methods, with different integration steps and different sets of initial parameters values, the paper conducted a study on the quality of obtained models, their sustainability (in the sense of changing model parameters with changing the input data).

1. Introduction

The digitalization of production processes, design and other activities involves not only a digital representation of characteristics of objects or processes underlaying an object or a process, their relations, retrospective information about their functioning, but also models describing their functioning. It is the applying adequate models, that makes it possible to predict the development of events, to choose optimal designs of objects and their regimes. Considering a complex technological process, one can face the fact, that it consists of a number of stages, which laws are very different, and consequently, their models are different. Similar situation could be seen in case of technical objects, which modes under different conditions vary greatly. In such cases, one has to deal with a “zoo” of models from different classes, which have to be investigated by different methods, optimized by applying different optimization methods and controlled by synthesizing different control algorithms. To standardize and formalize approaches to the processing of interrelated models of complex processes and objects, heterogeneous models can be unified (transformed to a single class). In fact, the model of one class is converted to the model of another class. The questions of how a new model from a unified class can be used in further studies, whether it has sufficient sustainability, if different data are used for its construction, become relevant.
The idea of the presented study is to construct a new model from a unified class on the basis of the existing model of a complex technical object workflow, to analyze the obtained model for adequacy and sustainability.

2. Remodeling of Dynamic Objects with Variable Structures

By Mathematical Remodeling we mean the construction of an object model (the functioning of the object) on the basis of the already existing model. The reasons for Remodeling could be either:

(i) the simplification of a model for further use of object to be controled (control algorithms cannot be applied to existing models);
(ii) the simplification of the calculations with the model in mind;
(iii) the unification of different kinds of models (reduction to the models of a single class) for the use of unified computational algorithms, etc.

We distinct this feature from the others by knowing, that when constructing a model, we use information not about the object itself or about its functioning, but we use a model, which is already obtained in some way — an imitative or mathematical model.

According to literature review, Mathematical Remodeling approach was not systematically investigated, although there are some studies on it. For example, in the study [1] it is considered to replace PID controller based on fuzzy logic by a neural network controller in order to reduce the computational complexity. By its ideas Mathematical Remodeling is similar to the construction of surrogate models. The motivation presented in the study [2] is to replace an accurate physical model (more precisely, a mathematical model based on a deep understanding of the physical processes occurring in the simulated system), the calculation of which requires considerable time, with a less accurate, but quickly computable surrogate model. Similarly, the study [3] proposes using the transition to a surrogate model which makes it possible to solve optimization problems more efficiently. There used a method of Response Surface Methodology, which is also intended for the construction of approximations for the set of calculations at different points, using the ideas of design of experiment. Application of surrogate models for solutions of inverse problems was investigated in the study [4]. Studies [5, 6] have investigated the construction of ensembles of surrogate models. Options for choosing the quality of construction of surrogate models and applying probabilistic approach (Monte Carlo method) to their construction were considered in the study [7].

One of the emphasized tasks in applying Remodeling approach is to unify models of a various classes. This approach is valid when modeling objects with variable structure. In this case the object model has the structure of

$$F(x) = \begin{cases} f_1(x), \quad g(x) \in G_1, \\ f_2(x), \quad g(x) \in G_2, \\ \vdots \\ f_n(x), \quad g(x) \in G_n. \end{cases}$$

That means, the specific form depends on whether the conditions of $g(x)$ are fulfilled. The field of $G = \bigcup_{i=1}^n G_i$, where for any values of $i$ and $j$, $G_i \cap G_j = \emptyset$ must determine all possible values of $g(x)$ for $x$ from range of definition $F(x)$. Lets denote with $X_i$ the set of values of $x$ for which $g(x) \in G_i$. Then, the problem of Remodeling an object with variable structure is as follows: for the initial model $F(x)$, it is necessary to select a model from the remodeling class $\tilde{F}(x, \theta)$, which is the closest to the initial value for all values of $x \in \bigcup_{i=1}^n X_i$. All parameters $\theta$ determine objects of the remodeling class.

The approach to Remodeling process is as follows:
Figure 1. Mathematical Remodeling Concept: possible models and their remodeling classes.

- The first step is to select several input data sets of $x$ (one of the variants suitable for a polynomial remodeling class is the methods for design of experiment).
- At the second step the selected inputs are transferred to the initial model to generate input-output values $(x_i, F(x_i))$.
- At the third step with a certain set of input-output values of $(x_i, F(x_i))$ in mind, and the chosen structure of the remodeling class $\tilde{F}(x, \theta)$, the parameters $\theta$ are defined in a way to minimize deviations of the constructed model from the initial $\|F(x) - \tilde{F}(x, \theta)\|$. 

This approach is universal in case of static models. If time occurs among the set of inputs $x$, carrying out remodeling according to the actions described above, can lead to a large error characterized by the magnitude $\|F(x) - \tilde{F}(x, \theta)\|$, due to the fact that dynamics of processes are not taken into account in any way. To take the dynamics into account, the following modification of the remodeling steps can be applied: when forming input-output values, select the time $t$ from the inputs $x$ and add the values of the original function to the previous time points as inputs. In this case, the data set for constructing the model from the remodeling class will look like $\{(x_i, t_i, F(x_i, t_i-k)), F(x_i, t_i)\}$. To preserve the constant structure of the input data, Remodeling requires a constant value of $k$, the number of previous times, in which the value of the original function is used to form input-output values.

It should be noted, that Remodeling Concept could use different types of models as remodeling classes. They could be classified by their properties and physical laws underling described processes and systems. A graphical representation of Mathematical Remodeling concept is shown on the figure 1.

Artificial neural networks [8] are recognized now as an effective structures applied in Mathematical Modeling and consequently in Mathematical Remodeling. Neuro-fuzzy networks and complex-valued neural networks are also popular. It has to be noted that there are many researches devoted to approximation technical systems using neural networks. But the remodeling approach (in case of using artificial neural networks as a basis) is used to approximate complicated systems with different stages.
Conducting Remodeling for the transformation of existing models has a number of drawback, such as additional loss of accuracy, deviation from physical understanding of the simulated process and additional computational costs in the process of model transformation. Therefore, before starting the process, it is necessary to know the purpose of its implementation and to determine the efficiency criteria. These criteria could be:

- the accuracy which is delivered by the approximation of the initial model with a model from a remodeling class. This deviation is determined as a norm of the difference between these two models $\|f(x) - \hat{f}(x, \theta)\|$;
- the influence of changes in data sets used in constructing a new model from the remodeling class on the approximation accuracy and model parameters. The evaluation of these criteria can be carried out by checking the sustainability $\|f(x, \theta_1) - \hat{f}(x, \theta_2)\| \leq M_1\|x^{(1)} - x^{(2)}\|$ and $\|\theta_1 - \theta_2\| \leq M_2\|x^{(1)} - x^{(2)}\|$, where $\theta_1$ and $\theta_2$ are parameter sets identified based on the input data sets $x^{(1)}$ and $x^{(2)}$ respectively, $M_1$ and $M_2$ are constant values;
- computational costs for Remodeling process. A time complexity function of an approximation algorithm could be used for the estimation of these costs.

3. Example of Remodeling of Dynamic Objects with Variable Structure.

Description of the Initial Model

An example of a dynamic object with variable structure is an inertial torque transformer (ITT) [9]. ITT is an automatic continuous variable transmission. The work of ITT is cyclical. Each cycle consists of four stages. Each of the stages is described by a system of nonlinear differential equations.

The system of differential equations describing the first stage:

$$
\begin{align*}
B_1(\psi)\ddot{\phi}_{21} + B_2(\psi)\ddot{\phi}_{22} - B_4(\psi)(\dot{\phi}_{21} - \dot{\phi}_{22})^2 + B_6(\psi)\ddot{\phi}_{22}^2 &= M_D, \\
B_2(\psi)\ddot{\phi}_{21} + B_3\ddot{\phi}_{22} - B_6(\psi)\dot{\phi}_{21} &= 0, \\
J_P\ddot{\phi}_{1} &= -M_c. \\
\end{align*}
$$

The system of differential equations describing the second stage:

$$
\begin{align*}
B_1(\psi)\ddot{\phi}_{21} + B_2(\psi)\ddot{\phi}_{22} - B_4(\psi)(\dot{\phi}_{21} - \dot{\phi}_{22})^2 + B_6(\psi)\ddot{\phi}_{22}^2 &= M_D, \\
B_2(\psi)\ddot{\phi}_{21} + B_3\ddot{\phi}_{22} - B_6(\psi)\dot{\phi}_{21} &= -M_c. \\
\end{align*}
$$

The system of differential equations describing the third stage matches with the system (1).

The system of differential equations describing the fourth stage looks like:

$$
\begin{align*}
B_1(\psi)\ddot{\phi}_{21} + B_4(\psi)\ddot{\phi}_{22}^2 &= M_D, \\
J_P\ddot{\phi}_{1} &= -M_c. \\
\end{align*}
$$

Where $\phi_i$, $\dot{\phi}_i$ are generalized coordinates and generalized speeds of the drive shaft, reactor and driven shaft;

$$
\begin{align*}
B_1(\psi) &= J_{21} + nme^2 + nmed(1 + a)\cos{\psi} + nJ_G(1 + a)^2; \\
B_2(\psi) &= -anJ_G(1 + a) - nmaed\cos{\psi}; \\
B_3 &= J_{22} + nJ_Ga^2; \\
B_4(\psi) &= nmaed(1 + a)\sin{\psi}; \\
B_5 &= B_3 + J_P; \\
B_6(\psi) &= nmaed\sin{\psi}; \\
\psi(t) &= a(\dot{\phi}_{21} - \dot{\phi}_{22});
\end{align*}
$$
$a$ is the internal gear ratio; $n$ is the number of cargo links; $m$ is the mass of the cargo link; $d$ is the distance from the axis of rotation of the load to its center of gravity; $e$ is the distance from the axis of rotation of the ITT to the axis of rotation of the cargo link; $J_{21}$ is reduced moment of momentum of the leading elements; $J_{22}$ is the reduced moment of momentum of the leading part of the reactor; $J$ is the reduced moment of momentum of the driven elements; $J_G$ is the reduced moment of momentum of the cargo link. Initial conditions for the first section $\phi_{21}(0) = \phi_{210}$, $\dot{\phi}_{21}(0) = \dot{\phi}_{210}$, $\phi_{22}(0) = \phi_{220}$, $\dot{\phi}_{22}(0) = \dot{\phi}_{220}$, $\phi_1(0) = \phi_{10}$, $\dot{\phi}_1(0) = \dot{\phi}_{10}$. As initial values for the next sections, the final values of the previous sections are used, which follows from the continuity of the process. Terms of transition from stage to stage:

- from the first to the second stage: reaching the angular velocity of the reactor to the angular velocity of the driven shaft $\dot{\phi}_1(t_1) = \dot{\phi}_{22}(t_1)$;
- from the second to the third stage: the change in the angle of rotation of the satellite in the relative motion by $\pi$ radians $\phi_{21}(t_2) - \phi_{22}(t_2) = \frac{\pi}{a}$;
- from the third stage to the fourth stage: reaching the reactor zero angular velocity $\dot{\phi}_{22}(t_3) = 0$;
- the end of the fourth stage (the end of the cycle): the change in the angle of rotation of the satellite in relative motion on $2\pi$ radians $\phi_{21}(t_4) - \phi_{22}(t_4) = \frac{2\pi}{a}$.

The systems of differential equations (1)–(3) do not have an exact analytic solution. The figure 2 shows the solution obtained by the fourth-order Runge-Kutta method with a variable integration step.

![Figure 2](image-url)  
**Figure 2.** Changing angular velocities of drive shaft, reactor and driven shaft.

In the tasks of optimizing the ITT workflow, it is required to define a set of model parameters values, that ensures a quick transition of workflow into an established one. The solution of optimization problem is complicated by the fact that the model of functioning is changing and does not have an explicit analytical solution. Obtaining an explicit analytical solution will open
new possibilities of using various optimization methods. The development of such a solution can be carried out on the basis of the input-output values generated from the available model.

4. Building a Model from a Remodeling Class and Analysis of Adequacy and Sustainability

As a remodeling structure for models (1)–(3) a neural network is used. Previous studies (cf. [10, 11]) have shown, that the best result of accuracy for the particular task under consideration is provided by a neural network of direct propagation with a hidden layer consisting of 100 neurons and using the sigmoid function as an activation function

\[ y_k = \sigma_1 \left( \sum_{j=1}^{100} w_j \cdot \sigma_2 \left( \sum_{m=1}^{3} w_{jm}x_m \right) \right), \]  

(4)

where \( y_k = \{\dot{\phi}_{21}(t), \dot{\phi}_{22}(t), \phi_1(t)\} \), \( x_m = \{J_{22}, i, t, \dot{\phi}_{21}(t-1), \dot{\phi}_{22}(t-1), \dot{\phi}_1(t-1)\} \), \( \sigma_1(\text{net}) = \text{net} \) and \( \sigma_2(\text{net}) = \frac{1}{1 + \exp(-\text{net})} \) are linear and sigmoid activation function respectively.

The use of the input values at the output from some time ago allows to take into account the dynamics of the approximating model. In this case, the task of Remodeling goes to the problem of parametric identification of the model (4); that means, to the differentiation of weight coefficients \( w_j \) and \( w_{jm} \).

The initial values of the model parameters were chosen as \( \phi_{210} = 0 \text{ sec}^{-1}, \dot{\phi}_{210} = 418 \text{ sec}^{-1}, \phi_{220} = 0 \text{ sec}^{-1}, \dot{\phi}_{220} = 0 \text{ sec}^{-1}, \phi_{10} = 0 \text{ sec}^{-1}, \dot{\phi}_{10} = i \cdot \phi_{210} \text{ sec}^{-1}, J_{21} = 0.2645 \text{ kg-m}^2, n = 5, a = 1.13 \text{ m}, m = 1.205 \text{ kg}, e = 0.099 \text{ m}, J_P = 10.3 \text{ kg-m}^2, d = 0.0188 \text{ m}, M_P = 130 \text{ H-m}, M_C = M_D/i, J_P = 0.00196 \text{ kg-m}^2 \). The following parameters were varied: the reduced moment of momentum of the leading part of the reactor \( J_{22} \) and the gear ratio \( i \). Three different sets of input-output data were generated, which were then used as training samples during the study of the sustainability of models obtained during Remodeling.

For the construction of the first neural network (Model I), 3251 cases of the process were used, for the construction of the second network (Model II) 3147 cases and for the construction of the third network (Model III) 3408 cases of the process we used respectively. As a test sample, the results of the solution of the analytical model obtained by the Runge-Kutta method were used for the parameters: time \( t = \{0.0001, 0.0002, ..., 0.0156\} \), \( i = 0.4 \), \( J_{22} = 0.03 \). A comparison of the results obtained by solving the equations of the model (1)–(3) by the Runge-Kutta method and the results obtained from Models I–III constructed in Remodeling are presented on the figure 3. Analysis of the graphs allows one to judge the similarity of the obtained results.

Further comparison of models to confirm the sustainability of the method was carried out in several stages. At the first stage, the series of weight coefficients of the obtained models were compared. For one output of each model, 605 weight coefficients were obtained. Figures 4–6 show the sweep curves for pairwise deviation of the model coefficients. The proximity of the median values of the range to zero indicates a high sustainability of the coefficients obtained.

At the second stage, there were compared variations of the solutions obtained from Models I–III by the numerical solution of systems of differential equations (1)–(3). The percentage value of the outputs obtained with the help of Models I–III was studied in the limits of the exact solution: for the output \( \phi_{21}(t) \) with possible deviations by 0.5% to the right and left; for the output \( \phi_{22}(t) \) with possible deviations by 15% to the right and left, for the output \( \phi_1(t) \) with possible deviations by 0.5% to the right and left of the exact solution. The results of the comparison are presented in the table 1. According to the results, all models provide similar values for specified limits of impact.
Figure 3. Angular velocities obtained using Runge-Kutta method and Models I–III.

Figure 4. Comparing weights for $\dot{\phi}_{21}(t)$.

Table 1. Comparing the results of the remodeling model outputs falling into the borderline of the exact solution.

| Output          | $\dot{\phi}_{21}(t)$ | $\dot{\phi}_{22}(t)$ | $\dot{\phi}_{1}(t)$ |
|-----------------|-----------------------|-----------------------|----------------------|
| Deviations to the left and to the right, % | 0.5 | 15 | 0.5 |
| Remodeling accuracy, % | Model I: 100 | 76.9 | 100 |
|                  | Model II: 100 | 71.2 | 100 |
|                  | Model III: 100 | 83.3 | 100 |

Then the deviations of Models I–III were compared with the exact solution by Akaike criterion [12]. The Akaike information criterion is used to compare several models for balances, taking
into account the number of identifiable coefficients and process realizations. The results of the comparison are given in the table 2. There is a slight variation of values of the Akaike criterion for model outputs, which indicates the high sustainability of the obtained remodeling structures.

Table 2. Information values of the Akaike criterion in comparison of Models I–III and the exact solution.

|       | \( \dot{\phi}_{21}(t) \) | \( \dot{\phi}_{22}(t) \) | \( \dot{\phi}_{1}(t) \) |
|-------|----------------|----------------|----------------|
| Model I | 1803.91 | 2353.98 | 528.92 |
| Model II | 1797.17 | 2358.04 | 522.25 |
| Model III | 1791.05 | 2371.89 | 525.69 |

According to a certain quality criteria of Remodeling process, approximation and sustainability quality was evaluated. Deviation rates between constructed models and the initial one and between each other (the figure 2, tables 1 and 2) demonstrate a high quality of the approximation. Deviation rates between obtained parameters of constructed based on different
data sets neurostructural models indicate the sustainability of Remodeling process in this case (figures 4–6).

The obtained during Remodeling process explicit analytical solution for systems of differential equations could be used to solve optimization problems, containing in the quality criteria a function of angular velocities of the drive shaft rotation, reactor and slave link. For example, it could be applied to determine parameters providing a quick output of the inertial transformer in a steady regime. The optimization criterion in this problem is: to find arg min\(_x\) \(f(x)\), where

\[
\begin{align*}
f(x) &= \sum_{i=0}^{1} (\dot{\phi}_{1}(x, i) - \dot{\phi}_{1}''(i))^{2},
\end{align*}
\]

\(\dot{\phi}_{1}\) is a velocity value of the flywheel at the end of the cycle, \(\dot{\phi}_{1}''\) is a velocity value of the flywheel at the beginning of the cycle and \(x\) are parameters to be optimized.

**Conclusion**

Applying Mathematical Remodeling approach to the model of ITT workflow, there were build three models from the remodeling class of models (in particular case — neurostructural models). These models are not structurally different, but different in the sense of their parameters values, as they were built on different data sets obtained by solving the original composite systems of differential equations for different values of the initial parameters. The comparison of these models with the initial one and with each other has shown their good adequacy. In addition, the variation of the parameters of these models is also small. Thus, neural network are powerful remodeling class, that allows transforming original heterogeneous models into a unified form. In particular, this class of models could be applied for modeling dynamic objects, the laws of functioning of which change in time.

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**References**

[1] Taifu Li, Yingying Su, Bingxiang Zhong 2011 *Fuzzy Information and Engineering (ICFIE)* ASC 40 p 714–725
[2] Kuleshov A P, Bernstein A V and Burnaev E V 2010 Proc. of the 3rd Int. Conf. on Inductive Modelling Kyiv Ukraine pp 64–71
[3] Jansson T, Nilsson L and Redhe M 2003 *Structural and Multidisciplinary Optimization* 25(2)
[4] Cnockuylt I, Aernouts J, Deschrijver, D et al. 2013 *Engineering with Computers* 29
[5] Goel T, Haftka R T, Shyy W et al. 2007 *Structural and Multidisciplinary Optimization* 33(3)
[6] Zhao D and Xue D 2011 *Engineering with Computers* 27
[7] Huang C, Rady B and Hami A E 2016 *The International Journal of Advanced Manufacturing Technology* 86(9)
[8] Ossowski S 2002 Sieci neuronowe do przetwarzania informacji (Warsaw: Of. Ed. Pol.) *(In Polish)*
[9] Bazhenov S P 2003 Besstupenchatyi peredachi tyagoviyh i transportnyih mashin [Infinitely variable transmission of traction and transport vehicles] (Lipetsk: Lipetsk State Technical University Press) *(In Russian)*
[10] Galkin A, Sysoev A and Saraev P 2017 Proc. of the International Conference On Industrial Engineering, Applications And Manufacturing (ICEAM)
[11] Saraev P V, Blyumin S L, Galkin A V and Sysoev A S 2018 *Advances in Intelligent Systems and Computing* 679
[12] Akaike H 1974 *IEEE Transactions on Automatic Control* 19