Instability in Casson nanofluids for Darcy-Brinkman model

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Abstract. The convective instability of Casson nanofluids saturating porous medium is investigated analytically and numerically. Darcy-Brinkman law is utilized to frame the momentum equation for the system. The governing equations are simplified using normal mode analysis and linear stability theory. The present study finds the expressions of thermal Rayleigh number for various boundaries and discusses the convective instability under various effects with the help of Wolfram Mathematica software. Out of all the boundaries, the instability of the layer is found to be least for both rigid boundaries. It is established that the convection in the fluid gets delayed due to porosity effects while non-Newtonian property and nano scale effects contribute significantly in making the layer unstable.

Keywords: Nanoparticles; Non-Newtonian fluid; Casson model; Brownian motion; Thermophoresis; Darcy Brinkman model.

Nomenclature

| Symbol | Description |
|--------|-------------|
| a      | wave number |
| αₐ     | thermal diffusivity of fluid |
| αₚ     | specific heat capacity |
| d      | thickness of the layer |
| Cₐ     | diffusion coefficient due to Brownian motion |
| Cₜ     | diffusion coefficient due to thermophoresis |
| eₒ     | deformation rate |
| g      | gravitational acceleration |
| k      | thermal conductivity of nanofluid |
| kₒ     | overall thermal conductivity |
| K      | Permeability |
| aₓ     | x-component of wave number |
| aᵧ     | y-component of wave number |
| p      | pressure |
| s      | rate of growth |
| t      | time |

Greek symbols

| Symbol | Description |
|--------|-------------|
| β      | Casson parameter |
| βₒ     | thermal volumetric coefficient |
| γ      | conductivity variation parameter |
| ε      | porosity |
| μ      | effective viscosity parameter |
| μₜ     | dynamic viscosity |
| τ      | stress tensor |
| ρₜ     | nanoparticle density |
| ρₒ     | fluid density at temperature Tₒ |
| (ρc)ₒ | effective heat capacity |
| σ      | thermal capacity |
| φ      | Relative volume fraction |
| φₒ     | Relative volume fraction |

Non-dimensional parameters

| Symbol | Description |
|--------|-------------|
| Dₐ     | Darcian parameter |
T  temperature
T_0  temperature at top layer
T_1  temperature at bottom layer
\( q_D \)  Darcian velocity \((q_D = (u, v, w) )\)
Y  yield stress for Casson fluid
(x,y,z)  cartesian co-ordinate system
z  an axis of coordinate system

Superscripts
^  perturbed variable
*  non-dimensional variable

1. Introduction

Nowadays, there has been an increasing interest in nanofluids, which are a combination of ordinary fluid and a small number of suspended nanoparticles; the resulting fluid was coined as ‘Nanofluid’ by Choi [1]. Nanofluids transfer heat more efficiently than the conventional fluids and are widely utilized in a variety of applications in diverse fields. Buongiorno [2] made a significant contribution to nanofluid modeling by pointing out that the base fluid velocity and the relative velocity contribute in absolute velocity of nanoparticles. The model was used by many researchers [3-6] to study the convective heat transfer in nanofluids and was further modified by Nield and Kuznetsov [7] to consider the solutal effects on nanofluid layer. The analytical investigations were made to establish the destabilizing influence of solute and nanoparticles on the system. Gupta et al. [8] considered the instability in a binary nanofluid layer subjected to magnetic field. Contrary to solutal and nano scale effects, magnetic field was observed to delay the instability of the layer. To consider the influence of density and conductivity of particles, steady state solution for particle volume fraction was assumed to be invariable [9] and the problem was studied numerically for various nanoparticles in water based nanofluid. Nield and Kuznetsov [10] considered the Brinkman instability in nanofluids for all three boundaries. Further it was established that rotation and magnetic field make the binary nanofluid layer more stable while porosity effects speed up the initiate the convection currents in the fluid layer [11,12]. Dhananjay et al. [13] considered the thermal instability in an internally heated porous layer and porosity effects were found to contribute in the stability while internal heat source and nanoparticles destabilize the system.

Casson model fits well to various non-Newtonian fluids [14]. The properties of blood coincide with that of Casson fluid in moderate shear rate flows according to Blair and Spanner [15], thus it is reasonable to use Casson model for blood flow. Casson's equation's success was investigated by Scott Blair [16]. The dual nature solution for the suggested model with thermal radiation impacts on steady as well as unsteady flow of Casson fluid was discovered by Hamid et al. [17]. Natural instability in a partially heated porous medium was studied by Aneja et al. [18] for Casson fluid using penalty finite element approach. Recently, Casson model has been used by researchers for various nanofluid flow problems to study the impact of nanoparticles on blood flow. Boundary layer MHD Casson nanofluid flow with variations in conductivity and viscosity was investigated by Gbadeyan et al. [19]. The convection currents in Casson nanofluids with internal heating effects were considered by Gupta et al. [20]. Recently, Gupta et al. [21] considered binary instability of Casson nanofluids analytically and numerically.

Unlike the previous works, the present paper studies the phenomenon of Darcy-Brinkman convection in Casson nanofluids. The equations based on conservation laws are simplified using normal mode technique [22] and linear stability theory is employed. Top layer of configuration is considered to have more nanoparticle volume fraction as compared to lower which assures the non-occurrence of
oscillatory motions [7, 10]. The stationary convective instability is illustrated through graphs to depict the impact of nanofluid, non-Newtonian and porosity parameters on the Rayleigh number. It is established that Darcy number and porosity have stabilizing impact while Casson parameter, nanoparticle Rayleigh number and Lewis number show destabilizing influence. Also, diffusivity ratio and particle density increment do not show any impact on the instability of the system.

2. Conservation equations for the system

A fluid layer is considered (refer; Figure 1). The temperature and nanoparticle concentration are \( T_1 \) and \( \phi_1 \) at lower layer \( z=0 \) while \( T_0 \) and \( \phi_0 \) at upper layer \( z=d \) (where \( T_1 > T_0 \) and \( \phi_1 > \phi_0 \)).

\[
\begin{align*}
  \rho \frac{\partial T}{\partial t} - \nabla \cdot (k \nabla T) + \rho C_p (T - T_0) &= 0, \\
  \rho \frac{\partial \phi}{\partial t} - \nabla \cdot (D \nabla \phi) &= 0.
\end{align*}
\]

Equations (1) and (3) give

\[
\begin{align*}
  \rho \frac{\partial T}{\partial t} - \nabla \cdot (k \nabla T) + \rho C_p (T - T_0) &= 0, \\
  \rho \frac{\partial \phi}{\partial t} - \nabla \cdot (D \nabla \phi) &= 0.
\end{align*}
\]

At lower layer \( (z = 0) \):

\[
w = 0, \quad \frac{\partial w}{\partial z} + l_1 \frac{\partial^2 w}{\partial z^2} = 0, \quad T = T_1, \quad \phi = \phi_1,
\]

and at upper layer \( (z = d) \):

\[
w = 0, \quad \frac{\partial w}{\partial z} - l_2 \frac{\partial^2 w}{\partial z^2} = 0, \quad T = T_0, \quad \phi = \phi_0.
\]
where the parameters $l_1$ and $l_2$ approach to infinity for free and zero for rigid boundaries.

Now we non-dimensionlize the variables as

\[
(x', y', z') = \left( \frac{x}{l_1}, \frac{y}{l_2}, \frac{z}{l_2} \right), \quad \rho' = \frac{\rho}{\rho_0}, \quad \mu' = \frac{\mu}{\mu_0},
\]

\[
\phi' = \frac{\phi}{(\rho_0 c_0)} / (T' - T_0), \quad \rho' = \frac{\rho}{\rho_0} / (\rho_0 c_0),
\]

where $a_0 = k_0 / (\rho c_0), \quad \sigma = (\rho c_0) / (\rho c_0)$. (8)

Using Eqs. (8), Eqs. (2)-(7) (after dropping the asterisks) become

\[
\nabla q = 0,
\]

\[
\frac{\partial^2}{\partial t^2} + q_0 \nabla^2 = \nabla q_0 - R_1 e_z + R_2 T e_z - R_3 \phi e_z,
\]

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\frac{\partial^2}{\partial t^2} + q_0 \nabla^2 = \nabla q_0 - R_1 e_z + R_2 T e_z - R_3 \phi e_z,
\]

\[
\frac{\partial^2}{\partial t^2} + q_0 \nabla^2 = \nabla q_0 - R_1 e_z + R_2 T e_z - R_3 \phi e_z.
\]

And boundary conditions for unit depth of layer are

\[
w = 0, T = 1, \phi = 0 \quad \text{at} \quad z = 0,
\]

\[
w = 0, T = 0, \phi = 1 \quad \text{at} \quad z = 1.
\]

3. Steady state solution and perturbation equations

At $t = 0$, let velocity of the fluid is zero and variables vary along horizontal axis. Thus Eqs. (9)-(12) give

\[
T(z) = 1 - z, \quad \phi(z) = z.
\]

Now we add perturbations to initial solutions and write

\[
q_0 = \bar{q}_0 + \hat{q}_0, \quad p = \bar{p} + \hat{p}, \quad \phi = \bar{\phi} + \hat{\phi}, T = \bar{T} + \hat{T}.
\]

Equations (9)-(12) with the help of Eqs. (15 and 16), give us a set of perturbed differential equations as

\[
\nabla \cdot \bar{q}_0 = 0,
\]

\[
\frac{\partial^2}{\partial t^2} + q_0 \nabla^2 = \nabla q_0 - R_1 e_z + R_2 T e_z - R_3 \phi e_z.
\]

Now on Eq. (18) together with Eq. (17) and using the relation $\nabla \cdot \nabla = \nabla^2$, we get

\[
\frac{\partial^2}{\partial t^2} + q_0 \nabla^2 = \nabla q_0 - R_1 e_z + R_2 T e_z - R_3 \phi e_z.
\]

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ and $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ (Laplacian operators).
To use normal mode technique on Eqs. (17)-(21), perturbed variables are written as [refer; 22]

\[
\{\hat{u}, \hat{T}, \phi\} = \{W, T, \phi\}(z) \exp(ia_x x + ia_y y + st).
\]  

(22)

Putting \(s = 0\) for stationary convection and using Eq.(22) in Eqs. (17)-(21); we get

\[
\left(\left(1+1/\beta\right)D_T\left(D^2 - a^2\right) - \left(D^2 - a^2\right)\right)W - a^2 R_T T + a^2 R_T \phi = 0,
\]

(23)

\[
W + \left(D^2 - a^2 + \frac{N}{L_r} (D - 2a^2 J) \frac{L_r}{L} \right)T - \frac{N}{L} D \phi = 0,
\]

(24)

\[
-\frac{1}{e} W + \frac{N}{L_r} (D^2 - a^2) T + \frac{1}{L_r} (D^2 - a^2) \phi = 0,
\]

(25)

with \(D = \frac{d}{dz}\). \(a = (a_x^2 + a_y^2)^{1/2}\).

The boundary conditions (given by Eq. (14)) reduce to

\[
W = T = \phi = 0,
\]

(26)

and

\[
D^2 W = 0 \text{ for free boundary},
\]

(27)

\[
DW = 0 \text{ for rigid boundary}.
\]

(28)

Further Eqs. (23)-(25) are solved to find the expressions of Darcy-Rayleigh number for boundary conditions given by Eqs. (26)-(28).

4. Expressions of Rayleigh number

4.1 Both-free case

The trial solutions satisfying Eqs. (26) and (27) are

\(W, T, \phi = (L, M, N) \sin \pi z\),

(29)

The eigen value equation using Eq. (29) in Eqs. (23)-(25) and eliminating of unknowns L,M and N, gives

\[
R_\infty = \frac{2D_T (1+1/\beta) J^2 + 2J^2 - 2a^2 J L_r}{2a^2 J} - R_T N_{\infty},
\]

(30)

where \(J = \pi^2 + a^2\).

4.2 Rigid-free case

The trial solutions satisfying Eqs. (26)-(28) are

\(W = L z^2 (1-z)(3-2z), T = M(z-z^2), \phi = N(z-z^2)\).

Using Eq. (31) in Eqs. (23)-(25) gives

\[
R_\infty = \frac{2[D_T e(1+1/\beta)(4536 + 432a^2 + 19a^4) + e(216 + 19a^2))(10 + a^2)^2 - 507a^2 (10 + a^2)}{507a^2 e(10 + a^2)} - R_T N_{\infty},
\]

(32)

4.3. Both-rigid case

The trial solutions satisfying Eqs. (26) and (28) are

\(W = L z^2 - 2z^3 + z^4), T = M(z-z^2), \phi = N(z-z^2)\).

(33)

Equations (23)-(25) with Eq. (33) give
5. Approximate solution and results

5.1. Both-free case

When \( \beta \to \infty \) in the Rayleigh number expression in Eq. (30) matches the one given by Nield and Kuznetsov [10]

\[
R_e = \frac{56(D_e(1+\beta)(504+24a^2+a^4)+e(12+a^2)))(10+a^2)^2 - 54a^2L_yR_y(10+a^2)}{54a^2e(10+a^2)} - R_y N_e.
\]  

(34)

When \( \beta \to \infty \) in the Rayleigh number expression in Eq. (30) matches the one given by Nield and Kuznetsov [10]

\[
R_e = \frac{D_e(1+\beta)(504+24a^2+a^4)+e(12+a^2)))(10+a^2)^2 - 54a^2L_yR_y(10+a^2)}{54a^2e(10+a^2)} - R_y N_e.
\]  

(35)

Also Eq. (30) can be written as

\[
R_e + R_y (N_e + \frac{L_y}{e}) = \frac{D_e(\alpha^2 + \alpha^4)(1+\frac{1}{\beta}) + (\pi^2 + \alpha^2)^2}{\alpha^2}.
\]

(36)

When \( D_e = 0 \) then \( R_e + R_y (N_e + \frac{L_y}{e}) = 4\pi^2 \), that is the lowest value attained at \( a = \pi \).

When \( D_e = 1 \) then \( R_e + R_y (N_e + \frac{L_y}{e}) = \frac{27}{4} \pi^2 \), that is the lowest value attained at \( a = \frac{\pi}{\sqrt{2}} \).

Therefore, using the large value of \( D_e \) as unity, critical wave number for both-free boundaries is around 2.22 and critical Rayleigh number is around 657.5.

5.2. Rigid-free case

Equation (32) by putting \( \beta \to \infty \), provides

\[
R_e = \frac{28[D_e(1/1 + \beta)(4536+432a^2+19a^4) + 216+19a^4)](10+a^2)^2 - R_y (N_e + \frac{L_y}{e}).
\]  

(37)

which is same as that of Nield and Kuznetsov [10] for rigid-free boundaries. Also, Eq. (37) can be written as

\[
R_e + R_y (N_e + \frac{L_y}{e}) = \frac{28[D_e(1+1/\beta)(4536+432a^2+19a^4) + 216+19a^4)](10+a^2)}{507a^2}.
\]  

(38)

When \( D_e = 0 \) then \( R_e + R_y (N_e + \frac{L_y}{e}) = 48.01 \), which is minimum value obtained at \( a = 3.27 \).

When \( D_e = 1 \) and \( \beta \to \infty \) then \( R_e + R_y (N_e + \frac{L_y}{e}) = 1139 \), which is minimum value obtained at \( a = 2.67 \).

Hence, the critical wave number for the case is 2.67 and critical Rayleigh number as 1139 when \( D_e \) is as large as unity.

5.3. Both-rigid case

For \( \beta \to \infty \) in Eq. (34), we get expression of Rayleigh number

\[
R_e = \frac{28[D_e(504+24a^2+a^4)+(12+a^2)](10+a^2)^2 - R_y (N_e + \frac{L_y}{e})}{27a^2}.
\]  

(39)

This expression agrees with Nield and Kuznetsov [10] for porous layer only. Eq. (39) can also be written as

\[
R_e + R_y (N_e + \frac{L_y}{e}) = \frac{28[D_e(1+1/\beta)(504+24a^2+a^4)+(12+a^2)](10+a^2)}{27a^2}.
\]  

(40)
When $D_x = 0$ then $R_x + R_x(N_x + \frac{L_x}{\varepsilon}) = 43.92$, which is minimum value obtained $a = 3.31$.

When $D_x = 1$ and $\beta \rightarrow \infty$ then $R_x + R_x(N_x + \frac{L_x}{\varepsilon}) = 1750$, which is minimum value obtained at $a = 3.12$.

Thus when $D_x$ approaches to unity and in the absence of Casson parameter, the critical wave number is 3.12 and critical Rayleigh number 1750 approximately.

Here, Eqs. (36), (38), (40) reveal that rigid-rigid boundaries give the system most stability.

6. Numerical results using Wolfram Mathematica software

Equations (31), (34), and (37) are dependent on multiple parameters. Let us take fixed values of parameters as follows: $L_x = 100, N_x = 1, N_r = 0.01, R_x = 0.1, D_o = 0.5, \varepsilon = 0.9$ and vary one of the variables to analyze its impact on instability of the system. Figure 2 depicts the relationship between critical Rayleigh number and Casson parameter $\beta$ for various values of $R_x$ with all three boundaries. It is noted that when $R_x$ increases, there is a fall in values of critical Rayleigh number with respect to Casson parameter $\beta$ and hence $R_x$ hastens the instability of the system. Figure 3 plots the graph of critical Rayleigh number versus Casson parameter $\beta$ for various values of $N_x$. One can easily observe that critical Rayleigh number for all three boundaries do not get affected by increment in modified diffusivity ratio $N_x$ and hence doesn’t have any impact on the initiation of instability in the fluid.

There is no significant change in the stability curves for variations in $N_r$ as interpreted in Figure 4 and hence doesn’t have much impact on the onset of instability of the fluid layer. Figure 5 shows the stability curves of critical Rayleigh-number in the presence of Casson parameter $\beta$ for variations in $L_x$. The critical Rayleigh number falls for increasing values of $L_x$ and therefore a destabilizing impact of Lewis number is established for all three boundaries.
Figure 6. Critical $R_c$ versus $\beta$ for $\varepsilon$.

Figure 7. Critical $R_c$ versus $\beta$ for $D_s$.

Figure 6 depicts the impact of porosity $\varepsilon$ on the onset of instability and it is noted that as $\varepsilon$ rises, there is a rise in the value of critical Rayleigh number. So, it is indicating that porosity parameter delays the onset of convection currents in the layer. Figure 7 depicts the curves of critical Rayleigh number as function Casson parameter $\beta$ for variations in Darcy number and it is shown to delay the convection in the layer. The reason is the increment in effective viscosity with the increment in Darcy parameter which retards the fluid flow. In Figs. 2-7, for smaller values of Casson parameter, critical Rayleigh number decreases quickly whereas for increasing values of $\beta$ doesn’t show much variation in its values.

To understand the complete effect of Darcy parameter, let us draw the graphs of $\log D_s$ versus critical Rayleigh number and $\log D_s$ versus critical wave number $\alpha_c$. Here, one can observe that critical Rayleigh number increases when Darcy parameter increases in Figure 8 and hence has a delaying influence on the onset of instability. Figure 9 establishes that there is an increase in the size of convection cell due to presence of Darcy number which postpones the thermal instability of the system.
Impact of Casson parameter $\beta$ on $a_c$ is depicted in Figure 10. It is observed that the values of $a_c$ do not vary much with the variation in Casson parameter $\beta$ and only small values of Casson parameter has a slight impact on $a_c$. Thus, non-Newtonian behaviour largely doesn’t show any impact on critical wave number.

7. Concluding remarks

The convective instability of a non-Newtonian nanofluid layer using Casson model is studied analytically and numerically using Darcy-Brinkman model. Method of normal modes is utilized to solve differential equations to get an eigen value equation for both free, rigid-free and both rigid cases. Effect of Casson parameter on critical Rayleigh number for various nanofluid and porosity parameters are illustrated graphically using the software Mathematica. It is established that Darcy number and porosity delay the convective instability of the system while other nanofluid parameters show a destabilizing effect on the fluid layer. Diffusivity ratio and particle density increment do not show any effect on the system. Non-Newtonian property destabilizes the system significantly for small values of Casson parameter which otherwise doesn’t show much influence on the stability of the fluid. Out of all three boundaries, the layer is found to be most stable for realistic rigid-rigid boundaries.

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