New estimates of Australia’s centenarian population

Wilson, T1* and Terblanche, W2

Abstract

Introduction
The population of Australia at the very highest ages is growing rapidly, like that of many countries. But official population estimates at these ages are of lower quality than those at younger ages, a problem shared by many countries which base their population estimates on census counts. This has implications for many uses of the data, especially rates for which the estimates provide denominators.

Objective
The aims of this paper are to (1) present new population estimates of Australia’s centenarian population (those aged 100 years and above) for 1981 to 2016 which are better quality than official statistics, and (2) illustrate the utility of such estimates as rate denominators by calculating centenarian death rates.

Methods
Population estimates at the highest ages were prepared using a combination of the Extinct Cohort method and a modified Survivor Ratio method. The key modifications of the latter involve projecting and smoothing Survivor Ratios within an iterative set of calculations. Death rates were calculated as standard occurrence/exposure rates. Input data of deaths and official Estimated Resident Populations were obtained from the Australian Bureau of Statistics.

Results
We show that Australia’s centenarian population grew from about 500 in 1981 to just over 3,900 by 2016, equivalent to an annual average growth rate of 5.9%. Centenarian death rates for the 1981-2016 period remained roughly steady, averaging 0.44 for females and 0.51 for males.

Conclusion
Our modified approach adds a degree of stability to the Survivor Ratio method and yields high-quality population estimates and death rates at advanced ages. It could easily be implemented by national statistical offices.

Keywords
Centenarian; Extinct Cohort; Projected Survivor Ratio; projection; smoothing; Australia
The aims of this paper are to (1) present new population estimates of Australia’s centenarian population for the period 1981 to 2016, and (2) illustrate the utility of such estimates as rate denominators by calculating centenarian death rates. The new population estimates are based on recently released deaths data, and updated mid-2016 ERPs for ages 85+. The study also implements a modified version of the standard Survivor Ratio method which includes projections of Survivor Ratios to 2016 and the application of smoothing to the age profile of these Survivor Ratios. We label this the Projected Survivor Ratio method to distinguish it from the standard implementation of the Survivor Ratio method which does not involve projection of the Survivor Ratios.

The paper continues in the following section by describing the data sources used, and the Extinct Cohort and Projected Survivor Ratio methods which were employed to create the new population estimates. Then the paper presents the new centenarian estimates and death rates, and compares them with estimates from other sources, specifically those of the HMD and the ABS. We explain why our estimates differ from these other estimates. As part of the conclusions we emphasise the strengths of the Projected Survivor Ratio method for creating population estimates at the highest ages, and note some minor limitations.

Data and methods

Data

Death counts by sex, single years of age and calendar year were obtained from the ABS from custom data tables (for 1971-2014) and from the ABS online data extraction tool ABS.Stat, http://stat.data.abs.gov.au/ (for 2015 and 2016). In the custom data tables deaths were supplied up to age 109 in single year age groups, with the final age group being 110+; the data from ABS.Stat was available in single years up to age 104, with 105+ being the final age group. Deaths for 2015 and 2016 in the 105+ category were distributed to ages 105, \ldots, 109 and 110+ according to the average distribution over the previous 10 years. Terblanche and Wilson [18] found no obvious problems with Australian deaths data; earlier evaluations by Jdanov and colleagues and Kannisto also classified Australian deaths data as of acceptable quality [22,23].

Because we wished to create mid-year population estimates, deaths were distributed from calendar to financial years, where financial years in Australia span 1st July in one year to 30th June in the next. Deaths were then converted from period-age squares in the Lexis diagram to period-cohort parallelograms [24]. Throughout the data conversion process an equal distribution of deaths within the original calendar year period-age squares was assumed. This was judged to be an approximate, but reasonable, assumption, and one which effectively smooths the data a little.

Mid-2016 Estimated Resident Populations (ERPs) by sex for ages 85+ were obtained from the ABS.Stat online data tool. These estimates are based on 2016 Census counts with adjustments for estimated net undercount, residents temporarily overseas at the time of the census (who are not legally required to complete the census), and the demographic change which occurred during the short period between the 30th June reference date and census night on 9th August.
Population estimation methods

Population estimates at the highest ages by sex and single years were created from the Extinct Cohort method and a modified version of the Survivor Ratio method which we label the Projected Survivor Ratio method. Our estimates were produced for mid-year each year (to be consistent with official ERPs) for the period 1981 to 2016.

Extinct Cohort method

The Extinct Cohort method is applied to cohorts which are deemed to have completely died out. Starting at the age and time at which there are no survivors, it consists of summing cohort deaths back in time and to younger ages [16]. The cumulative death counts yield the cohort populations at each age. Deaths data are assumed to be complete and accurate, and net international migration is assumed to be negligible. For the application of the Extinct Cohort method to Australian data, male cohorts were judged to have become extinct at age 109 because the average number of deaths at that age over the last 10 years is <0.5 deaths. Female cohorts were assumed to have become extinct at age 111 because our deaths data only extends to age 110+ and the average number of deaths at age 110+ over the last 10 years is very small (2 deaths per year).

The standard Survivor Ratio method

The Survivor Ratio method is applied to male and female cohorts which still have some surviving members at the latest date for which population estimates are required. The method involves estimating the remaining cohort population at the latest date, and then summing cohort deaths back in time and to younger ages (as with the Extinct Cohort method). Estimating the remaining cohort size at the latest date is the more complex part and involves the use of Survivor Ratios.

A Survivor Ratio is defined as a cohort population at time \( t \) divided by its size at time \( t - n \):

\[ R = \frac{P_t}{P_{t-n}} \]  

(1)

where \( R \) denotes the Survivor Ratio, \( P \) population, \( t \) a point in time, and \( t - n \) a point in time \( n \) years prior to \( t \). In many applications \( n \) is equal to 5 years. Figure 1 illustrates a Survivor Ratio in a Lexis diagram for the oldest non-extinct cohort, which in this example is aged 108 at time \( t \) (shown by the dotted pattern). The population at time \( t \) can be determined from an expression which includes the number of cohort deaths between \( t - n \) and \( t \) and the Survivor Ratio \( [25,26] \):

\[ P_t = D \frac{R}{1 - R} \]  

(2)

where \( D \) refers to cohort deaths. Thus the population at the latest date can be found given data on cohort deaths and an assumed Survivor Ratio for that cohort. The actual Survivor Ratio is unobserved, but in standard applications is approximated as the average Survivor Ratio of \( m \) cohorts born just prior the cohort in question, where \( m \) is often 5 (shown by the Survivor Ratios shaded dark grey in Figure 1).

However, to allow for falling mortality over time (and therefore increasing Survivor Ratios), a correction factor, \( c \), is usually applied to equation 2 [18]. If Survivor Ratios are increasing then this factor will be greater than 1.0. The modified equation is:

\[ P_t = D \frac{R}{(1 - R)^c} \]  

(3)

The correction factor \( c \) is calculated as the value which constrains single year of age population estimates at time \( t \) to the official sex-specific 90+ population estimate for that year. Several studies have confirmed that such a constraint to the official 90+, or sometimes 85+, population estimate produces high quality estimates by sex and single years of age [14, 18].

The Survivor Ratio method is applied in descending age order from the highest non-zero age group (e.g. age 108 for Australian males). It yields the population at that age for time \( t \), and then cumulative cohort death counts are added to that population estimate to give population estimates for the cohort at younger ages and earlier years. Once population estimates for this cohort have been estimated, Survivor Ratios can be calculated (which are then used in the calculation of average Survivor Ratios for the slightly younger cohort born in the following year). And then the process is repeated at younger ages and cohorts.

The Survivor Ratio method is applied to produce population estimates at the highest ages for all countries in the Human Mortality Database [18, 26]. It is also used by the UK Office for National Statistics to prepare official population estimates by sex and single years of at the highest ages [27].

The Projected Survivor Ratio method

For the present study we created a modified version of the Survivor Ratio method in which the Survivor Ratios are projected and smoothed. Doing so eliminates noise and yields Survivor Ratios closer to the true underlying values and for the required point in time. This is especially important for the Survivor Ratio method because of the sequential nature of the calculations: Survivor Ratios calculated for older cohorts affect the values of those calculated later for younger cohorts.

The method was operationalised in Microsoft Excel in an iterative calculation scheme which involved the following steps.

(a) Survivor Ratios by sex and single years of age from 85 upwards were projected to 2016 using linear extrapolation of 1996-2015 Survivor Ratios (Figure 2). Age in this instance refers to cohort age at time \( t \) (shown at the top-right of the Survivor Ratio in Figure 1 with the dotted pattern).

(b) Projected 2016 Survivor Ratios were smoothed across age at the very highest ages. From experimentation we discovered that ratios of Survivor Ratios at adjacent ages change approximately linearly with age, and made use of this relationship to smooth the data. First, ratios of Survivor Ratios at age \( a+1 \) to those at age \( a \) were calculated at the highest ages, i.e.

\[ r_{(a+1)/a} = \frac{R_{a+1}}{R_a} \]  

(4)

where \( r \) is the ratio of adjacent age-specific Survivor Ratios. A straight line was fitted to these \( r \) ratios over ages 90/89
Notes: The Survivor Ratio (R) shown with the dotted pattern is used to estimate the population aged 108 at time $t$, and is not known initially. In the standard application of the Survivor Ratio method it is approximated as the average of Survivor Ratios ($R$) for several (often five) adjacent older cohorts, which are depicted by the dark grey shading.

to 100/99 and the values extrapolated to higher ages (Figure 3). The ratios were then converted back to Survivor Ratios, and for ages 101 and above the modelled Survivor Ratios were used in place of the original values.

(c) Projected Survivor Ratios from step (b) were input into equation 3 to calculate mid-2016 populations. The correction factors were estimated so that the mid-2016 population estimates by single years of age summed to the official mid-2016 male and female 85+ ERPs published by the ABS. The correction factors needed to achieve consistency with the 85+ ERPs were 0.989 for females and 1.011 for males.

(d) Population estimates for each cohort were then calculated by adding deaths back down the cohort from the mid-2016 populations.

(e) Survivor Ratios were then updated based on the new population estimates.

(f) The calculations from (a) to (e) were then performed iteratively by Excel until convergence was achieved.

Results

Centenarian estimates and death rates

Figure 4 illustrates the growth in the number of centenarians in Australia over the last three-and-a-half decades. Their numbers increased from 499 in 1981 to 3,924 in 2016, which equates to an average annual rate of 5.9%, considerably greater than the 1.4% for the Australian population as a whole. Over the 35 year period to 2016 female centenarians increased in number from 411 to 3,223 (nearly an eight-fold increase), while the population of male centenarians rose from just 88 to 701 (an eight-fold increase, albeit from a lower base). Even over the last decade centenarian numbers grew from 2,110 to 3,924, representing an 86% increase. Female centenarians increased in number by 81% over the decade (from 1,782 to 3,223) while the male population grew by 113% (329 to 701).

On average over the last 35 years, male and female centenarian numbers increased at a similar rate. However, from the late 1990s onwards male centenarian numbers increased faster than those of females, growing at an average annual rate of 8.1% from 1998 to 2016 compared to 5.7% for females. This is due to greater improvements in male survival between ages 65 and 100 [3, 12]. A change in the prevalence of smoking among males relative to females has been cited as one reason for the different rates of mortality improvements for males and females [28-30]. This represents a reversal of the trend of faster female centenarian growth previously. As a result, the ratio of female to male centenarians decreased from 7.2 in 1998 to 4.6 in 2016.

Figure 5 shows population pyramids for the Australian centenarian population in 1981 and 2016, and the approximate mid-point of 1999. The graph illustrates significant increases in numbers at single years of age for both males and females over the study period. From 1981 to 1999 male centenarian numbers doubled and female centenarian numbers increased 3-fold. From 1999 to 2016 male and female centenarian numbers increased 4-fold and 2.6-fold respectively. Despite the huge increase in numbers, the age distribution of male and female centenarians has hardly changed so that the mean age of centenarians has remained relatively stable at 101.1 years for males and 101.3 for females.

The number of supercentenarians (those aged 110 and
Figure 2: Actual female Survivor Ratios and linear projections, 1996-2016

Note: The highest Survivor Ratio is for age 90, the lowest for age 110.

Figure 3: Ratios of female Survivor Ratios, 2016

Note: Age labels refer to the ages of the two Survivor Ratios in each ratio, e.g. 90/89 refers to the Survivor Ratio at age 90 (cohort age at time $t$) divided by the Survivor Ratio at age 89.
Figure 4: Growth in Australia’s centenarian population by sex, 1981 to 2016

Source: Authors’ calculations incorporating the Projected Survivor Ratio method

Figure 5: Australia’s centenarian population by single years of age and sex in 1981, 1999 and 2016

Source: Authors’ calculations incorporating the Projected Survivor Ratio method
older) in Australia remains too small to assess trends (Figure 5). However, it is possible to examine the growth of semi-supercentenarians (those aged 105 and older) instead. Their numbers increased from 29 in 1981 to 184 in 2016. Male semi-supercentenarians increased from just 5 in 1981 to 21 in 2016, while female numbers increased from 24 to 163. Over the 35 years to 2016, the average annual growth rate of female semi-supercentenarians (5.5%) was higher than that of males (3.9%).

The new centenarian population estimates presented in this paper enable the calculation of robust demographic and socio-economic rates using the new population numbers in the denominator. As an example, Figure 6 shows death rates for Australia’s centenarian population. Importantly, these rates possess numerator-denominator consistency because both numerator and denominator are based on deaths data. Over the whole 1981 to 2016 period females experienced an average rate of 0.44 and males a rate of 0.51, although there is clearly considerable fluctuation due to small numbers. Interestingly, there is no apparent trend of declining death rates over time, though this is consistent with trends in some other countries, such as Sweden [31].

Comparison with other statistics

How do our centenarian population estimates and death rates compare with official Australian statistics [2] and those from the HMD [19]? Figure 7 shows population estimates calculated using the Extinct Cohort and Projected Survivor Ratio methods alongside equivalent estimates published by the ABS and the HMD. For comparison purposes, estimates have also been created using the standard Survivor Ratio method which does not use projected or smoothed Survivor Ratios.

As can be seen, the ABS estimates differ noticeably from all other estimates in most years, with significant differences apparent between 1996 and 2003. The greatest differences occur in 2001 for males where the ABS’s estimated number of centenarians is 171% higher than the Projected Survivor Ratio estimate, and in 2000 for females when the ABS estimate is 22% higher. ABS estimates exceed those of others in most years, though for females they fall below the other estimates for certain periods when numbers drop (in the early 1980s, early 1990s, and in recent years). Such large fluctuations in the centenarian population would only be plausible if there had been proportionately huge fluctuations in birth trends 100 or more years earlier. The problems with the ABS estimates originate with problematic census counts of population at the highest ages.

The HMD centenarian estimates are similar to those of the Projected Survivor Ratio method in most years, with the exception of 2008-2013 when the number of female centenarians in the HMD differs by up to 8% (in 2011). This is the result of different death counts at single years of age from age 100 upwards. The deaths data used by the HMD up to 2009 were available at single years of age to 105 and in total for ages 106+. For the years 2010-2014 total deaths for ages 100+ were used. The total deaths in this open-age interval were disaggregated to single years of age by fitting a logistic function, referred to as the Kannisto model of old-age mortality [26]. This resulted in more deaths at earlier ages (closer to 100) and fewer deaths at higher ages compared to actual deaths provided by the ABS. The resulting Survivor Ratios calculated by the HMD at ages 100 and older were smaller than those of the authors, and hence the estimated population numbers at these ages were smaller. Estimates at ages 85-99 were very similar.

Population estimates from the standard and Projected Survivor Ratio methods are almost indistinguishable in Figure 7. The only perceptible differences occur in the most recent two years where the standard method gives higher populations, but only by a small margin. Recall that both methods constrain preliminary 2016 single year of age populations to 85+ ERPs. The difference is due to the projection and smoothing of Survivor Ratios in the modified method. It gives slightly different Survivor Ratios for 2016, and therefore slightly different preliminary and constrained populations. The largest numerical differences in populations are at ages 100 and 101, which is not surprising given they are the largest single year age groups in the centenarian ages (Figure 5). If you go further back in time, the difference between the two sets of centenarian numbers disappears as the larger numerical differences in cohort populations at ages 100 and 101 in 2016 shift to ages below 100, and both sets of estimates are increasingly based on the Extinct Cohort method alone.

Given the differences between the centenarian population estimates calculated for this paper and the official ERPs (Figure 7), it is not surprising to find that centenarian death rates calculated using these two sets of denominators vary significantly. They are compared for the male centenarian population in Figure 8, alongside death rates calculated using data from the HMD. The authors’ estimates using the Extinct Cohort and Projected Survivor Ratio methods and those of the HMD are in broad agreement, notwithstanding some variations in noise. But death rates calculated using ERP denominators clearly tell a very different story – one which is hard to accept given the huge swings in trend and lower average rates. Any other rates based on 100+ ERP denominators are likely to be similarly problematic. And population projections based on such death rates are liable to generate unreliable centenarian populations.

Discussion and Conclusions

This paper has employed the Extinct Cohort method and the Projected Survivor Ratio method to produce updated estimates of Australia’s centenarian population. Using these methods we calculated revised and updated estimates of Australia’s centenarian population from 1981 to 2016, revealing substantial growth from about 500 in 1981 to a little over 3,900 in 2016. Centenarians remain a tiny proportion of the national population for now, but that will change in the coming decades as a result of larger birth cohorts, migration, and continuing improvements in survival in the elderly age groups. Reliable population estimates and projections at the highest ages are important for planning, service delivery, budgeting and policy related to these rapidly growing age groups. Good quality population estimates are also essential for providing reliable denominators for a wide range of demographic measures, such as death rates, and other socio-economic indicators.

Our modification to the standard Survivor Ratio method consists of projecting Survivor Ratios to the latest year in the
Figure 6: Centenarian death rates, 1981-2016

Source: Authors’ calculations incorporating the Projected Survivor Ratio method

Figure 7: Comparison of centenarian estimates for Australia, 1981-2016

Source: Authors’ calculations; ABS; HMD.
Note: SR = Survivor Ratio method; HMD = Human Mortality Database

Figure 8: Comparison of male centenarian death rates for Australia, 1981-2016

Source: Authors’ calculations; ABS; HMD.
Note: SR = Survivor Ratio method; HMD = Human Mortality Database
estimates time series and then smoothing, thereby obtaining Survivor Ratios which should be close to their true underlying values. The smoothing adds more stability to the results, especially at the very highest ages where the Survivor Ratios experience increasing noise with rising age.

In the application for Australia, it was shown that at most ages the trend in Survivor Ratios was near-linear over time (Figure 2), and the ratios of projected 2016 Survivor Ratios at adjacent ages also appeared to be near-linear (Figure 3), permitting simple linear regression methods to be applied. The constraining of population estimates by single years of age to the 85+ ERP involved an adjustment of just over 1% for both males and females. Empirically, the results do not differ much from the application of the standard Survivor Ratio method, but arguments for its use can be made on conceptual grounds. In addition, the method does not have onerous data requirements, is relatively simple to calculate in a spreadsheet, and could be easily implemented by the Australian Bureau of Statistics and other statistical agencies with minimal resources.

Although conceptually attractive, the Projected Survivor Ratio method does possess some minor limitations. If mortality, and thus Survivor Ratio, trends were to change direction then the projected Survivor Ratios for the most recent date would be inaccurate. And, like the standard Survivor Ratio method, if the constraining 85+- or 90+- population estimate was inaccurate then those inaccuracies would be transferred to the single year of age population estimates.

Further research could involve empirical comparisons of the Projected Survivor Ratio method with other variants of the method for Australia, and assess its suitability for other countries. It would also be useful to determine how the Projected Survivor Ratio method without constraining to an independent population estimate compared to that with constraining.

Statement on conflicts of interest

The authors have no conflicts of interest to declare.

Abbreviations

ABS Australian Bureau of Statistics
ERP Estimated Resident Population
HMD Human Mortality Database

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