The effect of a light radion on the triviality bound on higgs mass.

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Abstract

In this paper we study how the triviality bound on higgs mass in the context of the SM is modified by a light stabilized radion of the Goldberger-Wise variety. Our approach is inherently perturbative. Including the radion contribution to \( \beta(\lambda) \) and \( \beta(g_t) \) to one loop we evolve the higgs self coupling \( \lambda \) from the cut off \( \Lambda(=\langle \phi \rangle) \) down to the EW scale \( \mu_0 = v \). The triviality bound is obtained by requiring that \( \lambda(\Lambda) = \sqrt{4\pi} \) which is the perturbative limit. We also study the effect of small changes in the UVBC on the triviality bound both in the presence and absence of a light radion.
1. Introduction

In spite of a lot of searches the higgs particle of Standard Model (SM) has not been discovered so far. LEP2 searches based on the process $e^+ e^- \rightarrow Zh$ have produced the lower bound $m_h > 113.2$ GeV for the SM higgs boson [1]. The higgs particle also contributes to EW radiative corrections and this has been used to derive an upper bound $m_h < 212$ GeV at 95% C.L. [2]. Both these limits are of course valid only in the SM. There also exist several theoretical constraints on $m_h$ like the triviality bound and vacuum stability bound. It is well known that the renormalized $\phi^4$ theory in $3 + 1$ dimension cannot contain an interaction term $\lambda \phi^4$. In other words the $\phi^4$ theory in $3 + 1$ dimension must be trivial (non interacting) if it is to be valid at all scales [3]. This means that in perturbation theory the running coupling $\lambda(\mu)$ diverges at some finite value of $\mu$. A similar effect occurs in the SM, modified to some extent by gauge and Yukawa coupling of top quark [4]. Therefore if the scalar sector of the SM is to be non trivial then the SM must be valid only upto some finite scale $\Lambda$.

Assuming that the SM is an effective theory valid below some cut off scale $\Lambda$, then the running coupling $\lambda(\mu)$ must diverge at $\Lambda$ or above it. This condition gives us a $\Lambda$ dependent upper bound on $m_h$. The triviality bound on $m_h$ is usually derived assuming pure SM interactions. However, in some new physics models one or more light dynamical fields could be present besides the SM particles. If these new light fields couple to the higgs boson with appreciable strength then they could contribute to $\beta(\lambda)$ and thereby generate important modifications to the triviality bound on $m_h$. In this paper we shall consider the modifications to the triviality bound on $m_h$ that arises from a light stabilized radion in the Randall-Sundrum(RS1) model [5]. The
radion will be assumed to be stabilized by the Goldberger-Wise mechanism [6] which is known to produce a light stabilized radion. We shall compare the SM predictions on triviality bound with those of the radion+SM system for cut off scales between 500 GeV and 10 TeV. In our calculations we shall set \( \Lambda = \langle \phi \rangle \) and assume that the KK modes of the graviton are heavier than \( \Lambda \). We shall further assume that the heavy KK modes of the graviton decouple from the light physics at the Electro-Weak (EW) scale. Under this condition the effect of the KK modes on light physics could be represented by a series of higher dimensional gauge invariant operators made out of SM fields. The operators will be suppressed by suitable inverse powers of \( \Lambda \). However, it has been shown that the effect of such higher dimensional operators does not change the triviality bound on \( m_h \) significantly from the SM prediction [7].

On the other hand we shall find that a light radion drastically changes the triviality bound from its SM prediction for \( \langle \phi \rangle \) less than about 1 TeV. In this paper we shall also study the effect of small changes in the UVBC on the triviality bound both in the SM and radion+SM system.

2. Determining the triviality bound for radion + SM system

In order to determine the triviality bound for the radion+SM system corresponding to some physical cut-off \( \Lambda \) by perturbative calculations we need the beta function for the higgs self coupling \( \lambda \) for the same. The SM contribution to \( \beta(\lambda) \) in one loop is well known and is given by [8].

\[
\beta_{SM}(\lambda) = \frac{1}{8\pi^2} \left[ 9\lambda^2 + \lambda \left( 6g_t^2 - \frac{9}{2}g_2^2 - \frac{3}{2}g_1^2 \right) - 6g_t^4 + \frac{3}{16}(g_2^4 + \frac{1}{2}(g_2^2 + g_1^2)^2) \right] \tag{1}
\]

where \( g_t \) stands for the Yukawa coupling of the top quark.
The radion contribution to $\beta(\lambda)$ to one loop can be shown to be given by [9]

$$\beta_{\text{rad}}(\lambda) = \frac{1}{8\pi^2} \left[ \frac{402\lambda^2 v^2}{\langle \phi \rangle^2} + \frac{144\lambda^2 v^4}{\langle \phi \rangle^4} + \frac{5\lambda m_{\phi}^2}{\langle \phi \rangle^2} \right]$$  \hspace{1cm} (2)

The beta function of $\lambda$ for the radion + SM is therefore given by,

$$\beta(\lambda) = \frac{d\lambda}{d\mu} = \frac{1}{8\pi^2} \left[ 9\lambda^2 + \frac{402\lambda^2 v^2}{\langle \phi \rangle^2} + \frac{144\lambda^2 v^4}{\langle \phi \rangle^4} + \frac{5\lambda m_{\phi}^2}{\langle \phi \rangle^2} + \lambda(6g_y^2 - \frac{9}{2}g_2^2 - \frac{3}{2}g_1^2) \right]$$

$$+ \frac{1}{8\pi^2} \left[ -6g_y^4 + \frac{3}{16}(g_2^4 + \frac{1}{2}(g_2^2 + g_1^2)^2) \right]$$ \hspace{1cm} (3)

The expression for $\beta(\lambda)$ besides depending on radion mass $m_{\phi}$ and radion vev $\langle \phi \rangle$ also depends on the top Yukawa coupling $g_t$ and the EW gauge couplings $g_1$ and $g_2$. We need the beta functions of the later couplings also for the radion+SM system in order to evolve $\lambda$ from the cut off scale $\Lambda$ down to the EW scale. Since the radion is an EW singlet, the beta functions for $g_1$ and $g_2$ in the radion+SM system are given by the same expressions as in the SM. The radion however does couple to the top quark quite strongly and contributes to $\beta(g_t)$. It can be shown that the beta function of $g_t$ for the radion+SM system to one loop order is given by [10]

$$\beta(g_t) = \beta_{\text{SM}}(g_t) + \beta_{\text{rad}}(g_t) = \frac{g_t}{16\pi^2} \left[ \frac{9}{2}g_t^2 - 8g_3^2 - \frac{9}{4}g_2^2 - \frac{17}{12}g_1^2 \right]$$

$$+ \frac{g_t}{16\pi^2\langle \phi \rangle^2} \left[ 4m_{\phi}^2 + \frac{31}{2}g_2^2 v^2 + 9\lambda v^2 \right]$$ \hspace{1cm} (4)

The radion contribution is given by those terms that vanish in the limit $\langle \phi \rangle \rightarrow \infty$. With the above expressions for $\beta(\lambda)$ and $\beta(g_t)$, we are now in a position to evolve $\lambda$ from the cut-off scale $\Lambda$ down to the EW scale. Although the choice of the cut off $\Lambda$ for the SM is somewhat arbitrary, for the radion+SM system in RS1 model a natural cut off is provided by $\langle \phi \rangle$. For a viable solution to the hierarchy problem the radion must be so stabilized that $\langle \phi \rangle$ lies between few hundreds of GeV to a few TeV.
In this paper we shall determine the triviality constraint on $m_h$ for the radion+SM system for cut-offs $\langle \phi \rangle$ lying between 500 GeV and 10 TeV and compare them with the corresponding SM predictions. In perturbation theory the triviality bound can be determined from the assumption that the running coupling $\lambda(\mu)$ blows up at the cut off. So to determine the triviality bound perturbatively we shall work with the UVBC $\lambda(\langle \phi \rangle) = \sqrt{4\pi}$ which is the perturbative limit. Further to study the variation of the triviality bound with changes in the UVBC we shall also determine the triviality bound for a different UVBC namely $\lambda(\langle \phi \rangle) = 2.0$. In Figure 1 we have plotted the triviality bound on $m_h$ for cut offs $\langle \phi \rangle$ lying between 500 GeV and 10 TeV for the UVBC $\lambda(\langle \phi \rangle) = \sqrt{4\pi}$. Figure 2 is a repeat of Figure 1 but for the UVBC $\lambda(\langle \phi \rangle) = 2.0$.

We would like to note the following points:

- The triviality bound on $m_h$ changes quite significantly under the change in UVBC that we have made both in the SM and radion+SM system. But the qualitative nature of the variation of triviality bound with $\langle \phi \rangle$ remains the same for both the SM and radion+SM system.

- The triviality constraint increases monotonically with decreasing cut off in the SM. This fact can be easily understood because with decreasing cut off the running coupling $\lambda(\mu)$ gets smaller and smaller momentum interval to run down from its value at the cut off. On the other hand for the radion+SM, for large enough cut offs the triviality bound increases with decreasing cut off which is expected since the radion decouples from the SM for large enough $\langle \phi \rangle$. However for cut offs smaller than 3 TeV the triviality bound decreases with decreasing cut off. This behaviour is due to the fact that for the radion+SM as the cut off
\langle \phi \rangle$ decreases the radion contribution to $\beta(\lambda)$ increases. As a result for small enough $\langle \phi \rangle$ although the running interval shrinks with decreasing cut off, the beta function becomes much stronger and causes $\lambda(\mu)$ to fall off very fast. The net result therefore is a decreasing triviality bound with decreasing cut off instead of an increasing one.

In this paper we have assumed that the KK modes of the graviton are several times heavier than the compactification scale $\langle \phi \rangle$ and that they decouple from the light SM physics. In the RS1 model even the lightest KK mode of the graviton turns out to be several times heavier than the compactification scale $\langle \phi \rangle$. It is therefore reasonable to assume that the KK graviton modes decouple from the SM.

**Conclusion**

In conclusion in this paper we have studied how a light stabilized radion in the RS1 model affects the triviality bound on higgs mass for varying cut offs between 500 GeV and 10 TeV. We have also studied the effect of changing the UVBC i.e. the value of $\lambda(\langle \phi \rangle)$ on the triviality bound. We find that for $\langle \phi \rangle$ greater than 3 TeV, the triviality bound increases with decreasing cut off for the radion+SM system like in SM. However for $\langle \phi \rangle$ less than 3 TeV the triviality bound decreases with decreasing cut offs in sharp contrast to its SM behaviour where the triviality bound increases monotonically with decreasing cut off.

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Figure 1: Triviality bound on $m_h$ in the radion+SM system corresponding to $\lambda(\langle \phi \rangle) = \sqrt{4 \pi}$. 

$\lambda(\Lambda = \langle \phi \rangle) = 3.5449$

$\mu_0 = 247 \text{ GeV}$

$\ln(\frac{\Lambda}{\mu_0})$
Figure 2: Triviality bound on $m_h$ in the radion+SM system corresponding to $\lambda(\langle \phi \rangle) = 2.0$. 

$\lambda(\Lambda = \langle \phi \rangle) = 2.0$

$\mu_0 = 247$ GeV