Chromatic Dispersion Compensation Using Filter Bank Based Complex-Valued All-Pass Filter

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Abstract—A long-haul transmission of 100 Gb/s without optical chromatic-dispersion (CD) compensation provides a range of benefits regarding cost effectiveness, power budget, and nonlinearity tolerance. The channel memory is largely dominated by CD in this case with an intersymbol-interference spread of more than 100 symbol durations. In this paper, we propose CD equalization technique based on nonmaximally decimated discrete Fourier transform (NMDFT) filter bank (FB) with non-trivial prototype filter and complex-valued infinite impulse response (IIR) all-pass filter per sub-band. The design of the sub-band IIR all-pass filter is based on minimizing the mean square error (MSE) in group delay and phase cost functions in an optimization framework. Necessary conditions are derived and incorporated in a multi-step and multi-band optimization framework to ensure the stability of the resulting IIR filter. It is shown that the complexity of the proposed method grows logarithmically with the channel memory, therefore, larger CD values can be tolerated with this approach.

I. INTRODUCTION

The performance of fiber optic links in long haul, metro and enterprise networks became limited by chromatic dispersion causing a short optical pulse to broaden as it travels along the fiber leading to intersymbol interference (ISI). To mitigate the effects of dispersion, optical systems include some form of CD compensation either optically or digitally. Digital equalization is attractive because it is less expensive, more flexible and more robust to varying channel conditions compared to the optical equalization.

CD has an almost time-invariant transfer function and only affects the phase of the input signal, i.e., it exhibits an all-pass behavior. In [2], G. Goldfarb and G. Li suggested dispersion compensation (DC) using time domain equalizer (TDE) technique based on infinite impulse response (IIR) all-pass filter. Their methodology requires Hilbert transformer and time reversal operation to design real-coefficients IIR filter separately for real and imaginary part of the transmitted signals. This technique was improved in [3] in which an optimization framework was presented to design a stable complex-valued IIR all-pass filter. The computational complexity of this scheme is less compared with [2] as it does not require the use of Hilbert transformer. However TDE schemes are not feasible for very long optical link as their complexity increases linearly with the channel memory.

Frequency domain equalizers (FDEs) based on fast Fourier transform (FFT) have become the most appealing scheme for CD compensation due to the low computational complexity for large dispersion and the wide applicability for different fiber distances [5]. In state-of-the-art FDE CD compensation design, the size of the FFT to realize fast linear convolution is governed by the specification of the maximum channel memory length with two-fold oversampling and 50% block overlap [4]. Nevertheless, the overlap-save method can be regarded not just as fast convolution method but also as a FB structure with trivial prototype filters and the equalization is done per sub-band, opening the way for more sophisticated sub-band processing [6]. Based on this observation, we propose CD equalization technique based on NMDFT FB with non-trivial prototype filter and complex-valued IIR all-pass filter per sub-band.

The main contribution of the paper is the presentation of an optimization framework to design a stable sub-band complex-valued IIR all-pass filter for CD equalization. Root-raised-cosine (RRC) filter is chosen as a non-trivial prototype filter and optimal filter length and roll-off to achieve certain bit-error-rate (BER) are also investigated. It is shown that the complexity of our approach increases logarithmically with the channel memory. The paper is organized as follows. Section II introduces the channel transfer function for CD. Section III describes the equalizer design based on an IIR filtering and its complexity analysis. FB based sub-band equalizer design is presented in Section IV and necessary condition for stability and filter order of individual sub-bands are derived in Section V. Two design criteria presented in an optimization framework [3] are modified in Section VI for a sub-band equalizer design. Section VII explains an optimization framework to find the sub-band IIR filter coefficients. The complexity analysis of the proposed scheme is derived in Section VIII. Simulation results comparing IIR and FB based IIR filtering are presented in Section IX. Conclusions are given in Section X.

II. CHANNEL MODEL

The low-pass equivalent model of CD channel of a single mode fiber of length $L_{\text{fiber}}$ can be written as

$$H_{\text{CD}}(\Omega) = \exp \left( -j \cdot \frac{\lambda_0^2}{4\pi c} \cdot D \cdot L_{\text{fiber}} \cdot \Omega^2 \right), \hspace{1cm} (1)$$

where $\Omega$, $\lambda_0$, $D$ and $c$ are baseband radial frequency, operating wavelength, fiber dispersion parameter and speed of light, respectively. If the signal is passed through an ideal low-pass filter of bandwidth $B$ Hz and then sample it at a sampling frequency of $B$ Hz, then the equivalent model in discrete domain can be written as

$$H_{\text{CD}}(\omega) = \exp(-j \cdot \alpha \cdot \omega^2), \hspace{1cm} (2)$$
where \( \alpha = \lambda_0^2 \cdot B^2 \cdot D \cdot L_{\text{fiber}} / (4\pi c) \) and \( \omega \in [-\pi, \pi] \). It is clear from (2) that the CD channel has an all-pass characteristic, i.e., it only changes the phase of the input signal.

III. EQUALIZER DESIGN

The transfer function of an ideal equalizer to compensate CD channel is obtained by taking the inverse of (2)

\[
G_{\text{Ideal}}(\omega) = \exp(+j \cdot \alpha \cdot \omega^2) .
\]

Since the CD channel exhibits an all-pass behavior, therefore, the natural choice to equalize it by the cascade of \( N_{\text{IR}} \) first order IIR all-pass sections of the form

\[
G_{\text{IR}}(z) = \prod_{i=1}^{N_{\text{IR}}} \frac{-a_i^* + z^{-1}}{1 - \alpha_i z^{-1}} = \prod_{i=1}^{N_{\text{IR}}} \frac{-\rho_i e^{-j\theta_i} + z^{-1}}{1 - \rho_i e^{j\theta_i} z^{-1}} ,
\]

where \( \rho_i \) and \( \theta_i \) are the radius and angle of the \( i^{th} \) pole location in the complex \( z \)-plane. Each first order section of complex valued IIR all-pass filter can be efficiently realized by four real multiplications as shown in Fig. 1. It was shown in [3] that the minimum filter order required to approximate (3) using (4) is given as

\[
N_{\text{IR}} = \left( \frac{\lambda_0^2}{2c} \right) \cdot D \cdot B^2 \cdot L_{\text{fiber}}
\]

It can be seen from above relation that the \( N_{\text{IR}} \) increases linearly with the fiber length which is not feasible to equalize very long optical links. In order to extend this framework to equalize long optical channels, we present FB based complex-valued all-pass equalizer design.

IV. FILTER BANK BASED COMPLEX VALUED SUB-BAND ALL-PASS EQUALIZER

Fig. 2 illustrates the general framework of a filter bank. The analysis and the synthesis filters of the \( k \)-th sub-band are \( H_k(z) \) and \( F_k(z) \), respectively. The number of sub-bands is \( M \) and the rate changing factor is denoted as \( L \). In general \( L \leq M \), but in this paper, we will consider the case of non-maximally decimated FB, i.e. \( L < M \), more specifically, we choose \( L = \frac{M}{2} \), even though our approach can be generalized for other values of \( L \).

For a uniform complex modulated FB, the transfer functions of \( H_k(z) \) and \( F_k(z) \) are obtained by complex modulating two low-pass linear phase prototype filters \( H(z) \) and \( F(z) \) of length \( P \), respectively, i.e. \( H_k(z) = z^{-(P-1)/2} H \left( ze^{j\frac{2\pi k}{M}} \right) \), and \( F_k(z) = z^{-(P-1)/2} F \left( ze^{j\frac{2\pi k}{M}} \right) \) \( \forall k = 0, \ldots, M - 1 \). Both analysis and synthesis FBs (AFBs and SFBs) can be efficiently implemented by first applying polyphase decomposition [8] to \( H(z) = \sum_{m=0}^{M-1} z^{-m} G_m(z^M) \) and to \( F(z) = \sum_{m=0}^{M-1} z^{-m} \tilde{G}_m(z^M) \). Then, some important identities for the multirate processing are used [10] and finally the complex modulation by means of DFT and inverse DFT (IDFT) of size \( M \) is applied. The polyphase components \( G_m(z) \) and \( \tilde{G}_m(z) \), \( m = 0, \ldots, M - 1 \) of \( H(z) \) and \( F(z) \), respectively, are static filters with small number of taps \( K \). This leads to the efficient analysis and synthesis FBs structures shown in Figs. 3 and 4, respectively.

To operate the FB based all-pass equalizer, sub-band IIR filter is inserted between the analysis and synthesis FB. The complexity of the overall structure is reduced since each frequency band is operating at a lower rate of \( M/2 \). Compared with the FD equalizer [7], the choice of the \( M \) is independent of the channel memory in this case, which can be advantageous from the hardware implementation perspective.

V. COMPLEX VALUED SUB-BAND ALL-PASS EQUALIZER DESIGN

The transfer function of the \( k \)-th sub-band IIR all-pass filter has the form

\[
G_{\text{IR}}^{(k)}(z) = \prod_{i=1}^{N_{\text{IR}}^{(k)}} \frac{-\rho_i^{(k)} e^{-j\theta_i^{(k)}} + z^{-1}}{1 - \rho_i^{(k)} e^{j\theta_i^{(k)}} z^{-1}} , \quad k = 0, \ldots, M - 1 .
\]

The design strategy for each sub-band all-pass equalizer will be same as described in [3]. But we consider it important to derive the general phase response and group delay for the sub-band. Based on the group delay behavior, we will describe the stability criteria and the optimum number of stages for every sub-band.
where \( \alpha \) is given by band after the polyphase filtering and downsampling operation. The phase response and group delay for the sub-band \( k \) are given for the sub-band \( k = M/2 \). Therefore, constant \( \beta' \) is added to the ideal group delay \( \tau_{\text{Ideal}}(\omega') \) for all the sub-bands to make non-negative desired group delay function, i.e.,

\[
\tau_{\text{Desired}}(\omega') = \tau_{\text{Ideal}}(\omega') + \beta',
\]

where

\[
\beta' = \left[ -\tau_{\text{Ideal}}(\omega' = 0) \right] = \left[ 2 \cdot \alpha' \cdot \frac{M}{2} \cdot \pi \right].
\]

The desired phase takes the form

\[
\phi_{\text{Desired}}(\omega') = \alpha' \cdot (\omega' + k' \cdot \pi) - \beta' \cdot (\omega' + k' \cdot \pi) + \phi_{0}^{(k)}.
\]

The phase response and group delay of the IIR all-pass filter in (6) are given by [10]

\[
\phi_{\text{IIR}}^{(k)}(\omega') = \sum_{i=1}^{N_{\text{IIR}}^{(k)}} \left[ -\omega' - 2 \cdot \arctan \left( \frac{\rho_{i}^{(k)} \cdot \sin (\omega' - \theta_{i})}{1 - \rho_{i}^{(k)} \cdot \cos (\omega' - \theta_{i})} \right) \right],
\]

\[
\tau_{\text{IIR}}^{(k)}(\omega') = \sum_{i=1}^{N_{\text{IIR}}^{(k)}} \frac{1 - \rho_{i}^{(k)} \cdot \cos (\omega' - \theta_{i})}{1 + \rho_{i}^{(k)} \cdot \cos (\omega' - \theta_{i})}.
\]

A negative group delay in (12) implies an unstable all-pass filter [9] and the minimum value of an ideal group delay in (10) is given for the sub-band \( k = M/2 \). Therefore, constant \( \beta' \) is added to the ideal group delay \( \tau_{\text{Ideal}}(\omega') \) for all the sub-bands to make non-negative desired group delay function, i.e.,

\[
\tau_{\text{Desired}}(\omega') = \tau_{\text{Ideal}}(\omega') + \beta',
\]

where

\[
\beta' = \left[ -\tau_{\text{Ideal}}(\omega' = 0) \right] = \left[ 2 \cdot \alpha' \cdot \frac{M}{2} \cdot \pi \right].
\]

The desired phase takes the form

\[
\phi_{\text{Desired}}(\omega') = \alpha' \cdot (\omega' + k' \cdot \pi) - \beta' \cdot (\omega' + k' \cdot \pi) + \phi_{0}^{(k)}.
\]

Here \( \phi_{0}^{(k)} \) is a constant of integration and it will be calculated in the optimization framework.

B. Filter Order

The filter order for the \( k \)-th sub-band is selected using the argument of an area under the desired group delay curve. It states that the area under the group delay function of an all-pass filter should match that of the desired group delay, i.e.,

\[
\sum_{i=1}^{N_{\text{IIR}}^{(k)}} \int_{-\pi}^{\pi} \tau_{\text{IIR}}^{(k)}(\omega') \, d\omega' = \int_{-\pi}^{\pi} \tau_{\text{Desired}}(\omega') \cdot d\omega'.
\]

The area under the group delay function of an all-pass filter is given by [3]

\[
\sum_{i=1}^{N_{\text{IIR}}^{(k)}} \int_{-\pi}^{\pi} \tau_{\text{IIR}}^{(k)}(\omega') \, d\omega' = 2\pi N_{\text{IIR}}^{(k)}.
\]

Also the integral of the desired group delay is given by

\[
\int_{-\pi}^{\pi} \tau_{\text{Desired}}(\omega') \cdot d\omega' = (2\pi) \left[ -2\pi \cdot \alpha' \cdot k' + \beta' \right].
\]

Comparing the above two equations yields \( N_{\text{IIR}}^{(k)} \),

\[
N_{\text{IIR}}^{(k)} = -2(2\pi) \cdot \alpha' \cdot k' + \beta'.
\]
VI. DESIGN CRITERIA FOR THE SUB-BAND ALL-PASS FILTERS

The objective is to design a sub-band IIR all-pass equalizer whose phase response matches the desired phase response of (15). Therefore, product of a sub-band all-pass equalizer and the sub-band CD channel has to be ideally

\[ G_{\text{IR}}^{(k)}(\omega) \cdot H_{\text{CD}}^{(k)}(\omega) = e^{-j(\phi_{\text{IR}}^{(k)}(\omega) + \delta' \cdot (\omega' - k' \cdot \pi))} \cdot \]

It can be shown that the above expression is equivalent to

\[ e^{-j(\phi_{\text{desired}}^{(k)}(\omega') - \phi_{\text{IR}}^{(k)}(\omega'))} = 1, \]

We will define the phase error transfer function \( \Upsilon^{(k)}(\omega) \) as

\[ \Upsilon^{(k)}(\omega') = e^{-j(\phi_{\text{desired}}^{(k)}(\omega') - \phi_{\text{IR}}^{(k)}(\omega'))} - 1, \]

and the weighted mean square error (MSE) of the transfer function containing the phase information (trans. phase) is defined as

\[ \text{MSE}_{\text{trans. phase}}^{(k)}(\omega') = \int_{-\pi}^{\pi} W(\omega') \cdot |\Upsilon^{(k)}(\omega')|^2 \cdot d\omega'. \tag{20} \]

The coefficients of a sub-band IIR all-pass equalizer are found by minimizing the cost function

\[ \Psi_{\text{trans. phase}}^{(k)} = \min_{\rho_i^{(k)}, \theta_i^{(k)}} \text{MSE}_{\text{trans. phase}}^{(k)} \quad \text{s.t.} \quad |\rho_i^{(k)}| < 1, \quad i = 1, 2, \ldots, N_{\text{IR}}^{(k)}. \tag{21} \]

The optimization problem in (21) is non-convex and non-linear so we can only solve it by non-linear optimization techniques. But usually such solvers require good initial guess of the solution and it may get stuck into the local minima if the initial solution is far from the global minima. To overcome this problem, we first minimize the mean square error in the group delay \( \text{MSE}_{\text{GD}}^{(k)} \) metric by using the group delay cost function \( \Psi_{\text{GD}}^{(k)} \), i.e.,

\[ \Psi_{\text{GD}}^{(k)} = \min_{\rho_i^{(k)}, \theta_i^{(k)}} \text{MSE}_{\text{GD}}^{(k)} \quad \text{s.t.} \quad |\rho_i^{(k)}| < 1, \quad i = 1, 2, \ldots, N_{\text{IR}}^{(k)}. \tag{22} \]

where

\[ \text{MSE}_{\text{GD}}^{(k)} = \int_{-\pi}^{\pi} W(\omega') \cdot |\tau_{\text{desired}}^{(k)}(\omega') - \sum_{i=1}^{N_{\text{IR}}^{(k)}} \tau_{\text{IR}}^{(k)}(\omega')|^2 \cdot d\omega'. \tag{23} \]

Although (22) is also non-linear but the initial solution can easily be found by the Abel-Smith algorithm. The initial solution provided by the Abel-Smith algorithm is used to find the sub-optimal solution to (22). This sub-optimal result then provides a good starting solution to solve optimization problem (21). Moreover, the choice of the appropriate weighting function \( W(\omega') \) in (20) and (23) will be discussed in Section IX.

VII. OPTIMIZATION FRAMEWORK

The flow chart of FB based IIR all-pass sub-band optimization framework for solving (21) is shown in Fig. 5. The initial estimate of the radii and angles are found by the Abel-Smith algorithm which is then refined by a non-linear solver to find the solution of (23). The solution of (23) is used as an initial starting guess to find the final optimal solution of (21). Any gradient based non-linear solver can be used to solve optimization problem (21) and (23) respectively. In the following, we will provide details of all the steps in our optimization framework.

A. Abel-Smith Algorithm

In [9], J.S. Abel and J.O. Smith described a method to extract filter coefficients for an all-pass design with an arbitrary group delay. Their design procedure is as follows:

1) Divide \( \tau_{\text{Desired}}^{(k)}(\omega') \) into 2\( \pi \)-area frequency bands.
2) Fit a first-order (complex) all-pass section \( G^{(k)}_{\text{IR}_i}(\omega') \) to each band as described below.
3) Cascade the first-order sections to form the all-pass filter,

\[ G_{\text{IR}}^{(k)}(\omega') = \prod_{i=1}^{N_{\text{IR}}^{(k)}} G_{\text{IR}_i}^{(k)}(\omega'). \]

The pole frequency is taken to be the band midpoint,

\[ \theta_i^{(k)} = \frac{\omega_{i-1} + \omega_i}{2}. \tag{24} \]

The expression for the pole radius is derived as,

\[ \rho_i^{(k)} = \mu_i^{(k)} - \sqrt{\mu_i^{(k)}}^2 - 1, \tag{25} \]

where

\[ \mu_i^{(k)} = \frac{1 - \zeta \cdot \cos(\Delta_i^{(k)})}{1 - \zeta}, \quad \Delta_i^{(k)} = \frac{\omega_i - \omega_{i-1}}{2}, \]

and \( \zeta \) is taken from the interval \[ 0.75 \quad 0.85 \] as proposed in [9].

B. Non-linear Optimization, \( \Psi_{\text{GD}}^{(k)} \)

The solution provided by the Abel-Smith algorithm is improved by finding a solution to (22) using a non-linear solver.
C. Non-linear Optimization, $\Psi_{\phi_0}^{(k)}$

Till this point, we have only estimates of the radii and angles. In order to get the initial phase correction term $\phi_0$, the following unconstrained optimization problem is solved

$$\Psi_{\phi_0}^{(k)} = \min_{\phi_0} \text{MSE}_{\text{trans. phase}}^{(k)}.$$  \hspace{1cm} (26)

D. Non-linear Optimization, $\Psi_{\text{trans. phase}}^{(k)}$

The final step of our framework solves (21) using a non-linear optimization solver with initial estimate of radii, angles and phase rotation provided by the first three steps of the optimization framework.

VIII. COMPLEXITY ANALYSIS

The complexity of the IIR equalizer in terms of real multiplications is given by

$$C_{\text{IIR}} = 4 \cdot (N_{\text{IIR}} + 1),$$

where $N_{\text{IIR}}$ is given by (5). If we implement the IIR filter in the sub-band domain then the complexity reduces by a factor $2$ since each sub-band is running at a lower rate. But at the same time, the overall filter order $N_{\text{IIR}}$ increases by an overlapping factor $\kappa$ which is 2 in this case. Therefore, complexity of IIR with the FB is given by

$$C’_{\text{IIR}} = 4 \cdot \frac{2}{M} \cdot (\kappa \cdot N_{\text{IIR}} + M) = \frac{8 \cdot \kappa \cdot N_{\text{IIR}}}{M} + 8.$$  \hspace{1cm} (27)

The total complexity of the NMDFT FB (AFB and SFB) [8] and IIR is given as

$$C_{\text{FB+IIR}} = 4 \log_2 (M) - 6 + 8 \cdot K + \frac{8 \cdot \kappa \cdot N_{\text{IIR}}}{M}.$$  \hspace{1cm} (28)

In (28), the only variable to reduce the complexity is the number of sub-bands. The minimum value of $C_{\text{FB+IIR}}$ is obtained by setting the derivative of (28) with respect to $M$ to zero and solving it to find the optimal value of $M$, i.e.,

$$M_{\text{opt}} = 2 \cdot \kappa \cdot N_{\text{IIR}} \cdot \ln(2).$$  \hspace{1cm} (29)

Substituting $M_{\text{opt}}$ in (28), we will get

$$(C_{\text{FB+IIR}})_{\text{opt}} = 4 \log_2 (2 \cdot \kappa \cdot N_{\text{IIR}} \cdot \ln(2)) - 6 + 8 \cdot K + \frac{4}{\ln(2)},$$

which increases logarithmically with $N_{\text{IIR}}$.

IX. SIMULATIONS

A 14 GBaud QPSK transmission with digital coherent receiver and sampling frequency $B = 28$ Gs/s is used to verify our technique for the CD equalization. The system specifications are $\lambda_0 = 1550$ nm, $D = 16$ ps/nm/km and $L_{\text{fiber}} = 2000$ km. Both transmit and receive pulse shaping filters are assumed to be an ideal low-pass filters. It is assumed that the transmitted QPSK symbols have zero mean and variance $\sigma_n^2$. The noise samples are drawn from an i.i.d. circularly symmetric complex Gaussian random process with zero mean and variance $\sigma_n^2$. The SNR in dB is defined as

$$\text{SNR} = 10 \log_{10} \left( \frac{\sigma_n^2}{\sigma_n^2} \right) \text{ dB}.$$  \hspace{1cm} (31)

The FB based all-pass equalizer design contains the following parameters to be optimized

- appropriate choice and parameters of the prototype filters (AFB and SFB).
- design of a sub-band all-pass filter coefficients based on the optimization framework.

The prototype filter can be designed according to different goals, although here we concentrate our results on a finite length Root Raised Cosine (RRC) filter with a roll-off, $\xi_{\text{RRC}}$ and filter length, $P = K \cdot M$, where $K$ is any positive integer. For the performance analysis, the signal to noise ratio (SNR) at an uncoded bit error ratio (BER) of $10^{-3}$ is chosen as a figure-of-merit. Apart from the CD channel impairments, all the transmitter and receiver imperfections such as phase noise, synchronization, fiber non-linearities are not considered.

We will first investigate the importance of the weighting function, $W(\omega)$ in (20) and (23) on the BER performance. The number of the sub-bands are chosen as $M = 32$, and the parameters of the RRC filter are taken as $\xi_{\text{RRC}} = 0.2$ and $K = 8$ in the simulations. Fig. 6 shows the BER comparison between the FB based sub-band IIR filter design, and the pure IIR filter equalizing the whole Nyquist band [3]. The performance of the sub-band equalizer is worse compared with the pure IIR equalizer when unity weighting, $W(\omega) = 1$ is used in the optimization framework. The unit weighting (or trivial weighting) function is not a good choice for the sub-band equalizer design and it motivates us to use a good weighting function. As the power spectral density (PSD) of the received signal in each sub-band is shaped by the magnitude of the RRC filter. Therefore, we will choose the magnitude of the RRC filter (in frequency domain) as a weighting function with a certain cut-off, $\omega_{\text{cut-off}}$ and roll-off, $\xi_{\omega_{\text{cut-off}}}$. We found out that the good parameters of the weighting function are $\omega_{\text{cut-off}} = 0.60 \pi$ and $\xi_{\omega_{\text{cut-off}}} = 0.1$. Fig. 6 shows the improvement with the RRC weighting function and it slightly outperforms the pure IIR equalization. Although one can also use any other appropriate weighting function but we refrain ourselves to the RRC.

In the second set of simulations, we focus on optimizing the roll-off parameter, $\xi_{\text{RRC}}$ of the prototype filter. The overall complexity of a FB base processing also depends on the prototype filter length and it is reflected in the parameter $K$ in (28) for a fixed $M$. The number of taps of the prototype filter is large for a smaller roll-off and vice versa. Therefore, we will investigate the performance loss in reducing the length of prototype filter for different roll-offs. Table I shows the effect of reducing the length for a small and high roll-off factors.

| $\xi_{\text{RRC}}$ | $P = K \cdot M$ | SNR at $10^{-3}$ BER |
|-------------------|-----------------|----------------------|
| 0.2               | 256             | 10.1                 |
| 0.2               | 128             | 11.2                 |
| 0.9               | 256             | 10.1                 |
| 0.9               | 128             | 10.4                 |
| 0.2               | 64              | 10.2                 |

TABLE I. BER performance for different roll-off factor and filter length of the RRC prototype filter. The number of sub-bands are chosen as $M = 32$ for the simulations.

It can be observed from the table that if $K$ is reduced from the value of 8 to 4, there is a significant loss for the $\xi_{\text{RRC}} = 0.2$
Comparison of BER with SNR for different values of $M$.

One of the major advantages of our methodology compared with the FD equalization methods [6], [7] is the flexibility in selecting the number of sub-bands. The reason is simple, the sub-band processing in our proposed methodology uses an IIR filter with a filter order, $N_{IR}^{(k)}$. In case of the existing FD equalization methods which can also be regarded as a FB structure with a trivial prototype filter [6], sub-band processing is performed using a single-tap FIR filter. One can also use a multi-tap sub-band FIR filter for small values of $M$, but then it has two major drawbacks. Firstly, it will probably require non-trivial prototype filters. Secondly, it was shown in [3] that the number of taps required by the FIR filter is double compared to an IIR equalizer for the same CD channel. Therefore, our proposed technique is a good choice when flexibility is required in selecting $M$. Fig. 7 shows the performance of our technique for different values of $M$. It is pertinent to mention that the computational complexity of our propose method is slightly more compared with the FD methods because of the non-trivial prototype filters requirement in analysis and synthesis FB.

X. CONCLUSIONS

We presented a NMDFT FB based framework with an RRC filter in the AFB and SFB to design a sub-band IIR all-pass filter for the CD compensation. The necessary conditions based on the group delay characteristic are derived to select a minimum filter order and stability of the each sub-band IIR all-pass filter. Very long optical links can be compensated by this method as the complexity increases logarithmically with the fiber length. Simulation results show that the BER performance of the proposed method with appropriate selection of the weighting function in the optimization framework is better compared with pure IIR equalization.

Additionally we show that a compromise between the complexity and performance can be reached by selecting a roll-off factor and filter length of the RRC prototype filter. One of the advantage offered by our scheme over the FDE is the flexibility in selecting $M$ which might be attractive from hardware implementation point of view. It is left as a future work to do a thorough analysis between the complexity and performance comparison between the FD equalization methods and our proposed scheme.

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