Gravitational Waves from Primordial Black Holes and New Weak Scale Phenomena

H. Davoudiasl and P.P. Giardino

Submitted to Physical Letters B

May 2017

Physics Department/High Energy Theory

Brookhaven National Laboratory

U.S. Department of Energy
USDOE Office of Science (SC),
High Energy Physics (HEP) (SC-25)

Notice: This manuscript has been co-authored by employees of Brookhaven Science Associates, LLC under Contract No. DE-SC0012704 with the U.S. Department of Energy. The publisher by accepting the manuscript for publication acknowledges that the United States Government retains a non-exclusive, paid-up, irrevocable, world-wide license to publish or reproduce the published form of this manuscript, or allow others to do so, for United States Government purposes.
DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or any third party’s use or the results of such use of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof or its contractors or subcontractors. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.
Gravitational Waves from Primordial Black Holes and New Weak Scale Phenomena

Hooman Davoudiasl * and Pier Paolo Giardino †

Department of Physics, Brookhaven National Laboratory, Upton, NY 11973, USA

We entertain the possibility that primordial black holes of mass $\sim (10^{26} - 10^{29})$ g, with Schwarzschild radii of $O(\text{cm})$, constitute $\sim 10\%$ or more of cosmic dark matter, as allowed by various constraints. These black holes would typically originate from cosmological eras corresponding to temperatures $O(100 - 1000)$ GeV, and may be associated with first order phase transitions in the visible or hidden sectors. In case these small primordial black holes get captured in orbits around neutron stars or astrophysical black holes in our galactic neighborhood, gravitational waves from the resulting “D&G” binaries could be detectable at Advanced LIGO or Advanced Virgo for hours or more, possibly over distances of $O(100)$ Mpc encompassing the Local Supercluster of galaxies. The proposed Einstein Telescope would further expand the reach for these signals. A positive signal could be further corroborated by the discovery of new particles in the $O(100 - 1000)$ GeV mass range, and potentially also the detection of long wavelength gravitational waves originating from the first order phase transition era.

Keywords: Primordial Black Holes; Gravitational Waves; advanced LIGO/VIRGO; Phase Transition

The presence of cosmic dark matter (DM) is firmly established by various cosmological and astronomical observations [1]. However, all existing evidence for DM is from its gravitational effects. While it is widely believed that DM has non-gravitational interactions that governed its production in the early Universe, all attempts to uncover those interactions have been unsuccessful. This situation motivates one to entertain the possibility that DM is of a purely gravitational nature. In particular, if DM is composed of primordial black holes (PBHs) [2–5], formed via gravitational collapse of primordial matter around over density perturbations in the early Universe, it may only manifest itself through its gravitational effects.

The above PBH scenario removes the need to postulate new particles and interactions associated with DM, which is often invoked as strong motivation to search for physics beyond the Standard Model. However, this intriguing possibility is quite constrained by various observations [6–9] over most of the viable parameter space. However, some parts of the parameter space allow for PBHs to be a significant component of DM. In fact, allowing for deviations from a monochromatic spectrum, which is expected to be the case in realistic scenarios [8], some narrow ranges of parameters could possibly allow for all DM to be composed of PBHs.

The primordial nature of the DM black holes implies an interesting correspondence between the masses of PBHs and the era in which they were produced. That is, since PBHs are assumed to be formed by the collapse of matter and energy over a Hubble volume, the mass $M_{\text{PBH}}$ of a PBH is a measure of the horizon size, and hence the temperature of the Universe at the time of the PBH formation.

PBH masses that could potentially originate from first order phase transitions [10] at temperatures $T \sim O(100 - 1000)$ GeV could offer an interesting window into experimentally accessible particle physics. This range of $T$ can be associated with extensions of the electroweak sector and the Higgs potential and may also lead to long wavelength primordial gravitational waves, which may have been within the reach of future space-based observatories [11]. Those extensions may play a role in electroweak baryogenesis and also provide new possibilities for microscopic DM candidates, with PBHs comprising a subdominant, but potentially significant, DM population. The above range of $T$ roughly corresponds to [10]

$$10^{20} \text{ g} \lesssim M_{\text{PBH}} \lesssim 10^{29} \text{ g}.$$  \hspace{2cm} (1)

Current bounds, including the recent micro-lensing searches from observations of the Andromeda galaxy by the Subaru Hyper Suprime-Cam [9], still allow about 5–10% of DM to be comprised of PBHs, for masses in the above range. With the assumption of a distribution for $M_{\text{PBH}}$, it might be possible that a somewhat larger fraction of DM is made up of PBHs over the range (1). The Schwarzschild radius $R_{\text{Sch}}$ of a black hole scales linearly with its mass $M_{\text{BH}}$ and in the range (1) above, the corresponding Schwarzschild radii are $R_{\text{Sch}} \sim 0.01 - 10$ cm.

In this work, we will consider values of $M_{\text{PBH}}$ in the range (1) and explore the possibility that a neutron star (NS) or an astrophysical black hole (ABH) in our galactic neighborhood may have captured a PBH of such masses in an orbit around them. As we will show, the gravitational wave signals from these “David & Goliath (D&G)” binary systems can be detectable at Advanced

*email: hooman@bnl.gov
†email: pgiardino@bnl.gov

1 Note that, in our version of the confrontation, Goliath fares far better than in the original story.
LIGO (aLIGO) or Advanced Virgo (AdV), at their design sensitivity up to distances of $\sim 10$ Mpc, covering the Local Supercluster of galaxies. The envisioned future Einstein Telescope could possibly extend the reach for the parameters considered here to $\mathcal{O}(50)$ Mpc, going beyond the Local Supercluster.

We point out that there can be two possible formations mechanisms for D&G binaries: (a) through radiative capture; see for example Refs. [12–14] and (b) through adiabatic contraction and dynamical friction. The first possibility has been examined extensively in the context of solar mass black holes and could in principle be applicable here. We suggest the second possibility based on proposed constraints for PBHs, where one estimates the likelihood that a PBH be captured during star formation and later end up within the compact remnants, such as a white dwarf, destroying it [6]. It seems plausible that one may also use this process to form D&G binaries that will later coalesce and yield our signal. We will focus on the first mechanism (a), however possibility (b) may also result in viable candidates. Hence, our estimate for the rate of D&G inspiral events could be considered conservative in this sense. Without a more dedicated analysis - which is outside the scope of this work - it may not be possible to determine which of the (a) or (b) options yield the dominant rate and what a reliable estimate of that rate would be.

As we will discuss in the appendix, formation of D&G binaries via radiative capture is likely a rare occurrence and our estimated rate might be $\sim 10^{-4}$ per year or less. However, our proposed signals could be detected using the existing aLIGO/AdV facilities and do not require dedicated new experiments. In light of the above, and given the major impact of a potential PBH discovery on our understanding of the Universe, an examination of our proposal appears worth while, even if PBHs constitute only a subdominant contribution to DM.\(^2\)

Let us begin with some general information about the astrophysical objects of interest. The known NS and ABH populations have masses $M_{\text{NS}} \sim 1 - 2 M_\odot$ and $M_{\text{ABH}} \gtrsim 10 M_\odot$, where $M_\odot \approx 2 \times 10^{33}$ g is the solar mass. For concreteness, in what follows we will choose

$$M_{\text{NS}} = 1.5 M_\odot \quad \text{and} \quad M_{\text{ABH}} = 10 M_\odot,$$  \hspace{1cm} (2)

as our reference values, however recent gravitational wave observations [17] suggest that values of $M_{\text{ABH}} \sim 30 M_\odot$ are not necessarily uncommon. We note that the nearest known NS and ABH are at distances $d_{\text{NS}} \sim 0.3$ kpc and $d_{\text{ABH}} \sim 1$ kpc, respectively. These objects are known due to optical observations. In principle, there could be isolated compact stellar objects that do not emit detectable optical signals and may be closer to the Solar System. In any event, we will use

$$d \gtrsim 5 \text{ kpc}$$  \hspace{1cm} (3)

as a reasonable lower bound on the distance to potential binaries of interest in our work.

We are interested in signals from an Extreme Mass Ratio Inspiral [22]. Here we comment on a possible formation mechanism for such a binary, option (a) mentioned before, by emission of gravitational radiation during the initial D&G encounter [12–14]. One finds that the resulting binary orbits initially have $\mathcal{O}(1)$ eccentricities $e$. The orbits get circularized as the binary evolves, however for very hierarchical mass ratios the rate at which the eccentricity decreases $de/dt \propto -M_{\text{PBH}}/M_{\text{ABH}}$ [23] is slow and the eccentricity may still be sizable at the final merger. Hence, the circular orbit approximation may not be very accurate for the systems we focus on. One of the main consequences of having $e \sim 1$ is that the gravitational radiation emitted by the binary is not dominated by quadrupolar $n = 2$ harmonic and has significant components from higher harmonics [24, 25]. These effects do not, by and large, change the orders of magnitude for our estimates.

In order to estimate the proposed signal strengths, we will need to make sure that parameters of the orbits we examine can yield valid results. In this regard, we need to know the last stable orbit (LSO) for our systems. According to the results in Ref. [26], for a test particle going around a black hole of mass $M$ in an orbit with eccentricity $e$, the radius of the LSO is given by

$$r_{\text{LSO}} = \frac{G_N (6 + 2e) M}{c^2 (1 + e)},$$  \hspace{1cm} (4)

where $G_N = 6.67 \times 10^{-8}$ cm$^3$g$^{-1}$s$^{-2}$ is Newton’s constant and $c = 3.00 \times 10^{10}$ cm/s is the speed of light. For a circular orbit with $e = 0$ we get the familiar result $r_{\text{LSO}} = 3 R_{\text{Sch}}$ and for $e = 1$ we find $r_{\text{LSO}} = 2 R_{\text{Sch}}$. Hence, as long as we choose $r > 3 R_{\text{Sch}}$, we can assume stable orbits in our analysis. For simplicity, we will use $r_{\text{LSO}} = 3 R_{\text{Sch}}$ for both the NS and ABH cases. The results of Ref. [27] suggest that this would also be a good estimate for the NS case.

Gravitational waves cause oscillations in the local metric as they travel through spacetime. These oscillations give rise to strain, i.e. variations in physical length scales, the size of whose amplitude we denote by $h$. Measurement of strain is the basis of gravitational wave detection. In the following, non-relativistic speeds and orbits large compared to radii of the compact stellar objects are assumed. The simple formalism that we will use suffices to get reasonable order of magnitude estimates. See e.g.

\(\text{\textcopyright 2023 American Physical Society.}}$
Ref. [28] for an accessible presentation and Ref. [29] for a detailed exposition to the relevant subjects.

For a binary system, with component masses \( M_1 \) and \( M_2 \), in a circular orbit of size \( r \) at a distance of \( d \) from the observer, we have

\[
h = \frac{4G_N^2 M_1 M_2}{c^4 r d}.
\]

(5)

The frequency of the corresponding gravitational waves are then given by

\[
f = \frac{1}{\pi} \left[ \frac{G_N (M_1 + M_2)}{r^3} \right]^{1/2}.
\]

(6)

The radiation of gravitational waves by the binary system causes the decay of its orbital radius \( r \) to a smaller radius \( r_f \) after a time [23]

\[
\Delta t_f (r) = \frac{5 c^5}{256 G_N} \left[ \frac{r^4 - r_f^4}{M_1 M_2 (M_1 + M_2)} \right].
\]

(7)

Of particular interest is the time \( \Delta t_{LSO} \), which we obtain from Eq. (7), required for the system to evolve to the LSO at \( r_f = r_{LSO} \).

For concreteness, we will consider \( f_\star = 150 \) Hz as a typical value where aLIGO/AdV reach for gravitational waves is optimal. Our estimates do not sensitively depend on the exact value of \( f_\star \) near our reference value. Using our reference values in Eq. (2), Eq. (6) yields the radius \( r_\star \) corresponding to \( f_\star \)

\[
r_\star \approx 97 \text{ km (NS)} \quad \text{and} \quad r_\star \approx 182 \text{ km (ABH).}
\]

Note that for the “D&G” binaries of interest here, we have \( M_{PBH} \ll M_\odot \) and hence the frequency \( f_\star \) of the waves is independent of \( M_{PBH} \), to a very good approximation. We see that for the above choice of parameters, \( r_\star \) is well above the radius of the NS, about 10 km, and the implied value of \( r_{LSO} \) from Eq. (4).

The decay time \( \Delta t_{LSO} \) versus \( M_{PBH} \) is plotted in Fig. 1, for \( M_{NS} = 1.5 M_\odot, M_{ABH} = 10 M_\odot \), and \( f_\star = 150 \) Hz. We see that \( 4 \times 10^4 \) s \( \lesssim \Delta t_{LSO} \lesssim 10^7 \) s. We will choose the “observation time”

\[
t_{\text{obs}} = \Delta t_{LSO},
\]

(9)

which we will assume over the parameter space of our analysis.

In Fig. 2, we have plotted the expected size of the strain signal \( h \left| \frac{1}{N} \sqrt{t_{\text{obs}}} \right| \), with \( t_{\text{obs}} = N t_{\text{coh}} \), versus \( M_{PBH} \) for \( M_{NS} = 1.5 M_\odot \) and distance from Earth 5 kpc \( \leq d \leq 50 \text{ Mpc} \). Here, \( t_{\text{coh}} \) is the time scale over which the signal can be coherently observed. The value of \( r_\star \) has been chosen from Eq. (8) corresponding to the NS case. The horizontal dotted, dashed, and dot-dashed lines mark the projected AdV, aLIGO, and the proposed Einstein Telescope (ET) [30] sensitivities at \( f = f_\star \), in

\[1/\sqrt{\text{Hz}}, \text{of approximately } 5 \times 10^{-24}, 4 \times 10^{-24} [31], \text{and } 4 \times 10^{-25} [32], \text{respectively. We have used } t_{\text{coh}} = 2000 \text{ s (see for example Ref. [33]) in obtaining the results in Fig. 2. We see that for most of the range of } M_{PBH} \text{ considered here, the entire Milky Way } (d \lesssim 50 \text{ kpc}) \text{ is within the reach of aLIGO/AdV.}

We note that the rate of the frequency increase for the systems we examine is intrinsically quite slow, and one could also focus the search on O(2000) known “pulsars” in our Galaxy whose optical signals determine their positions in the sky. This feature allows one to account
for signal modulation due to the motion of the observer with respect to the barycenter of the Solar System, which may lead to $t_{\text{obs}} = t_{\text{coh}}$, enhancing the reach for Galactic NS-PBH systems.

The values of $hN^{-1/4} \sqrt{t_{\text{obs}}}$ versus $M_{\text{PBH}}$ are given in Fig.3, for the ABH case is Eqs.(2) and (8), where we have again assumed $t_{\text{coh}} = 2000$ s. Our results in Fig.3 suggest that for $M_{\text{PBH}} \approx 10^{29}$ g, aLIGO/AdV can be sensitive to the gravitational wave signals of a PBH-ABH binary out to distances of $O(10)$ Mpc, while ET can probe $d \lesssim 50$ Mpc, beyond our Local Supercluster.

Note that our signal will not be mistaken for that of a small planet or asteroid captured around an NS or ABH. This is because our gravitational wave signals are obtained for $r_\ast \sim 100$ km. This should be compared to the much larger radius of the Earth $R_\oplus \sim 6000$ km, whose mass $M_{\oplus} \sim 6 \times 10^{27}$ g is in the $M_{\text{PBH}}$ range of our proposal. In any event, a compact star will tidally destroy a terrestrial scale rocky object, well before reaching an orbit comparable to its size.

In conclusion, we illustrated, as a proof of principle, that if a primordial black hole of mass $\sim 10^{26} - 10^{29}$ g is captured by a neutron star or an astrophysical black hole in our galactic neighborhood, gravitational wave signals of their “D&G” confrontation could be detected by aLIGO/AdV or the proposed Einstein Telescope. Current constraints allow these primordial black holes to constitute a significant fraction of cosmic dark matter. Although the signals we consider might be rare, their discovery could shed light on early Universe phase transitions in the visible and hidden sectors relevant to weak scale phenomena. As such, we may also expect that our signals may be accompanied by discovery of new states $\sim 10 - 100$ GeV and also long wavelength primordial gravitational waves from the phase transition era. Therefore, we believe that searching for these signals in the existing and future data is well motivated.

The observation of gravitational waves by LIGO has opened an exciting new front in the exploration of the Cosmos. We hope that our work would further expand the range of questions that could potentially be examined at this front.

We thank Scott Hughes for very helpful comments and constructive criticism regarding our proposal and Tongyan Lin for useful discussions. This work is supported by the United States Department of Energy under Grant Contract DE-SC0012704.

Appendix

Here, we provide an order of magnitude estimate for the rate of D&G binary signal. As discussed before, the binaries may form either in the process of star formation, via the capture of a PBH by a massive star whose remnant later forms a binary with the PBH, or through radiative capture. Here, we focus on the second possibility, and estimate the rate for an ABH to capture a PBH through gravitational radiation; the realistic rate may potentially be larger. Also, there is some contribution from NS-PBH binaries that could add to the expected signal rate. In any event, given the multitude of contributing factors, the following should be viewed as a rough guide.

Following the discussions in Refs. [13, 14], let $\eta \equiv M_{\text{PBH}}M/ M_{\text{tot}}^2$, where $M$ is the mass of the NS or ABH and $M_{\text{tot}} \equiv M_{\text{PBH}} + M$. The maximum impact parameter $b$ that leads to the formation of the binary, assuming a relative velocity of $w$, is given by

$$b_{\text{max}} = \left( \frac{340\pi}{3} \right)^{1/7} \frac{M_{\text{tot}} \eta^{1/7}}{w^{9/7}} G_N c^{-2}. \quad (10)$$

We will choose $M_{\text{PBH}} \sim 10^{29}$ g, since it offers the farthest reach in our range of PBH masses in (1) as seen from Fig.3, and set $M = M_{\text{ABH}} \sim 10M_\odot$. The results of Ref. [34] suggest that within the inner 100 pc of the Milky Way, one could have a DM content of $\sim 4 \times 10^8 M_\odot$, though this quantity has large uncertainties. Hence, assuming some enhancement of DM density towards smaller radii, we can reasonably assume that the DM mass contained within the central 10 pc of the Galaxy is $\sim 10^8 M_\odot$. The simulations of Ref. [35] also imply that $\sim 10^5$ ABHs of mass $10M_\odot$ could be contained within the same radius. Hence, the contribution of DM (including a sub-dominant PBHs population) and ABHs can be comparable and of order $10^8 M_\odot$. Assuming that the total mass within 10 pc of the center of the Galaxy
is \( \sim \) few \( \times 10^6 M_\odot \), we find that \( w \sim 30 \sim 40 \text{ km/s} \) can be a fair estimate.

For the above set of parameters, one finds the cross section \( \sigma_{\text{ABH}} \sim \pi b_{\text{max}} \sim 10^{-12} \text{ km}^2 \). Assuming that the PBHs are distributed around the value chosen here, we find a PBH number density of \( n_{\text{PBH}} \sim 10^{-34} \text{ km}^{-3} \).

We may then estimate the capture rate for D&G binaries of interest, near the core of the Milky Way, as \( R \sim \sigma_{\text{ABH}} n_{\text{PBH}} w N_{\text{ABH}} \sim 10^{-8} \text{ yr}^{-1} \). Here, \( N_{\text{ABH}} \sim 10^6 \) is the number of ABHs within the inner \( 10 \text{ pc} \) of the Galaxy. Given our results in Fig.3, we may assume that for the chosen parameters aLIGO/AdV could be sensitive to sources \( \sim 10 \text{ Mpc away} \), which covers most of the Local Supercluster, comprising \( \sim 2000 \) large galaxies. Hence, we may roughly set the expected rate for aLIGO/AdV at \( \sim \) few \( \times 10^{-5} \text{ yr}^{-1} \). This rate could potentially be enhanced if we also include expected signals from NS-PBH mergers, as well as binary formation processes besides radiative capture. Therefore, we can tentatively assume a signal rate \( \lesssim 10^{-4} \text{ yr}^{-1} \).

---

[1] K. A. Olive et al. [Particle Data Group Collaboration], Chin. Phys. C 38, 090001 (2014). doi:10.1088/1674-1137/38/9/090001
[2] B. J. Carr and S. W. Hawking, Mon. Not. Roy. Astron. Soc. 168, 399 (1974).
[3] P. Mezasros, Astron. Astrophys. 37, 225 (1974).
[4] B. J. Carr, Astrophys. J. 201, 1 (1975). doi:10.1086/153853
[5] For a review on the subject see for example M. Y. Khlopov, Res. Astron. Astrophys. 10, 495 (2010). doi:10.1088/1674-4527/10/6/001 [arXiv:0801.0116 [astro-ph]].
[6] F. Capela, M. Pshirkov and P. Tinyakov, Phys. Rev. D 87, no. 2, 023507 (2013) doi:10.1103/PhysRevD.87.023507 [arXiv:1209.6021 [astro-ph.CO]].
[7] K. Griest, A. M. Cieplak and M. J. Lehner, Phys. Rev. Lett. 111, no. 18, 181302 (2013). doi:10.1103/PhysRevLett.111.181302
[8] B. Carr, F. Kuhnel and M. Sandstad, arXiv:1607.06077 [astro-ph.CO].
[9] H. Niikura et al., arXiv:1701.02151 [astro-ph.CO].
[10] K. Jedamzik, Phys. Rev. D 55, 5871 (1997) doi:10.1103/PhysRevD.55.5871 [astro-ph/9605152].
[11] C. Grojean and G. Servant, Phys. Rev. D 75, 043507 (2007) doi:10.1103/PhysRevD.75.043507 [hep-ph/0607107]; P. Schaller, Phys. Rev. Lett. 115, no. 18, 181101 (2015) doi:10.1103/PhysRevLett.115.181101 [arXiv:1504.07263 [hep-ph]].
[12] M. Turner, Astrophys. J. 216, 610 (1977).
[13] R. M. O’Leary, B. Kocsis and A. Loeb, Mon. Not. Roy. Astron. Soc. 395, no. 4, 2127 (2009) doi:10.1111/j.1365-2966.2009.14653.x [arXiv:0807.2638 [astro-ph]].
[14] I. Cholis, E. D. Kovetz, Y. Ali-Hamoud, S. Bird, M. Kamionkowski, J. B. Munoz and A. Raccanelli, arXiv:1606.07437 [astro-ph.HE].
[15] S. Bird, I. Cholis, J. B. Munoz, Y. Ali-Haimoud, M. Kamionkowski, E. D. Kovetz, A. Raccanelli and A. G. Riess, Phys. Rev. Lett. 116, no. 20, 201301 (2016) doi:10.1103/PhysRevLett.116.201301 [arXiv:1603.00846 [astro-ph.CO]].
[16] S. Cleisse and J. Garca-Bellido, arXiv:1603.05234 [astro-ph.CO].
[17] B. P. Abbott et al. [LIGO Scientific and Virgo Collaborations], Phys. Rev. Lett. 116, no. 6, 061102 (2016) doi:10.1103/PhysRevLett.116.061102 [arXiv:1602.03837 [astro-ph.CO]].
[18] T. Nakamura, M. Sasaki, T. Tanaka and K. S. Thorne, Astrophys. J. 487, L139 (1997) doi:10.1086/310886 [astro-ph/9708060].
[19] K. T. Inoue and T. Tanaka, Phys. Rev. Lett. 91, 021101 (2003) doi:10.1103/PhysRevLett.91.021101 [gr-qc/0303058].
[20] M. Kesden and S. Hanasoge, Phys. Rev. Lett. 107, 111101 (2011) doi:10.1103/PhysRevLett.107.111101 [arXiv:1106.0011 [astro-ph.CO]].
[21] K. M. Belotsky et al., Mod. Phys. Lett. A 29, no. 37, 1440005 (2014) doi:10.1142/S0217732314400057 [arXiv:1410.0203 [astro-ph.CO]].
[22] S. A. Hughes, Class. Quant. Grav. 18, 4067 (2001) doi:10.1088/0264-9381/18/19/314 [gr-qc/0008058].
[23] P. C. Peters, Phys. Rev. D 136, B1224 (1964). doi:10.1103/PhysRevD.136.B1224
[24] P. C. Peters and J. Mathews, Phys. Rev. 131, 435 (1963). doi:10.1103/PhysRev.131.435
[25] V. Piero, I. M. Pinto, A. D. Spallicci, E. Laserra and F. Recano, Mon. Not. Roy. Astron. Soc. 358 (2001) doi:10.1046/j.1365-8711.2001.04422.x [gr-qc/0005040].
[26] C. Cutler, D. Kennefick and E. Poisson, Phys. Rev. D 50, 3816 (1994). doi:10.1103/PhysRevD.50.3816
[27] G. Torok, M. Urbanec, K. Adamek and G. Urbancova, Mon. Not. Roy. Astron. Soc. 258, 1538 (2001) doi:10.1046/j.1365-8711.2001.04422.x [gr-qc/0005040].
[28] P. C. Peters, Phys. Rev. Lett. 116, 094013 (2011) doi:10.1103/PhysRevLett.116.094013 [arXiv:1103.00464 [gr-qc]].
[29] Exploring Black Holes: Introduction to General Relativity. Copyright ©2010 Edmund Bertschinger, Edwin F. Taylor, & John Archibald Wheeler (exploringblackholes.com).
[30] Gravitation, Charles W. Misner, Kip S. Thorne, and John Archibald Wheeler, W. H. Freeman and Company, 1973.
[31] M. Punturo et al., Class. Quant. Grav. 27, 084007 (2010). doi:10.1088/0264-9381/27/8/084007
[32] J. Aasi et al. [LIGO Scientific and VIRGO Collaborations], Living Rev. Rel. 19, no. 1 (2016) doi:10.1007/lrr-2016-1 [arXiv:1304.0670 [gr-qc]].
[33] S. Hild et al., Class. Quant. Grav. 28, 094013 (2011) doi:10.1088/0264-9381/28/9/094013 [arXiv:1102.0908 [gr-qc]].
[34] J. Aasi et al. [LIGO and VIRGO Collaborations], Class. Quant. Grav. 31, 085014 (2014) doi:10.1088/0264-9381/31/8/085014 [arXiv:1311.2409 [gr-qc]].
[34] V. Gammaldi, V. Avila-Reese, O. Valenzuela and A. X. Gonzales-Morales, arXiv:1607.02012 [astro-ph.HE].

[35] F. Antonini, Astrophys. J. 794, no. 2, 106 (2014) doi:10.1088/0004-637X/794/2/106 [arXiv:1402.4865 [astro-ph.GA]].