Hall and ion-slip effects on MHD mixed convection flow in a vertical microchannel with asymmetric wall heating

Basant K. Jha | Peter B. Malgwi

Abstract
In this article, a theoretical analysis is carried out on hydromagnetic mixed convection flow of viscous incompressible and electrically conducting fluid in a vertical microchannel in the existence of Hall current and ion-slip effects. The walls of the microchannels are preheated and the flow is prompted by the combination of buoyancy forces and pressure gradient. The governing dimensional equations are obtained and reduced to dimensionless form using suitable transformations. The flow equations are solved analytically, and their solutions are obtained in dimensionless form. Some interesting effects of the relevant flow parameters on the flow formation and volume flow rates, in both primary and secondary flow directions, are showed. The results show that, in the presence of Hall current and pressure gradient, the ion-slip current supports flow formation along the primary flow direction, while destabilizing it along the secondary direction. In addition, it is also highlighted that, in the presence of pressure gradient and buoyancy forces, the ion-slip current increases the volume flow rate in both primary and secondary flow directions, whereas the Hall current destabilizes the volume flow rate along the primary direction.

KEYWORDS
hall current, ion-slip current, microchannel, mixed convection

1 | INTRODUCTION

Hydromagnetic flow in the microgeometry has found a wide range of engineering applications, such as in microelectrochemical cell transport, microheat exchanger, microchip cooling amongst others. Recently, highly incorporated systems, such as microelectromechanical systems (MEMS) and nanoelectromechanical system (NEMS) technologies, are gaining enormous attention owing to its vast application in heating or cooling microreactor devices. Due to its cost efficiency, reliability, and simplicity, studies relating to these types of flows remain active over the years. Some articles on this subject includes: Yang et al\(^1\) offered an analysis on the effects of velocity slip and temperature jump on viscous incompressible fluid through a microchannel with heat and mass transfer characteristics. In another work, Jha et al\(^2\) offered the precise solution for hydromagnetic flow due to buoyancy forces in a vertical microporous channel under different microchannel
heating situations. Later on, Weng and Chen\textsuperscript{3} also investigated the influence of wall curvature on viscous fluid though an annular microchannel with isothermal wall heating conditions. Other relevant articles on MHD flow through different geometric situations includes: Soundalgekar and Takhar,\textsuperscript{4} Sheikholeslami et al,\textsuperscript{5} Cai and Liu,\textsuperscript{6} Shojaeian and Shojaeian,\textsuperscript{7} and Weng and Chen.\textsuperscript{8}

It is well known and observed that subjecting a partially ionized gas with low density to a transversely applied magnetic field gives rise to many electromagnetic features like Hall current, ion-slip current and so on (see Cramer and Pai\textsuperscript{9} and Sato\textsuperscript{10}). Such features have significant influence on different flow characteristics like heat transfer and mass transfer. The Hall effect is relevant in MHD generator, MHD accelerator, underground energy storage system, Hall effect sensors, and also in several astrophysical flows. Owing to the strong impact of the magnetic field, the diffusion velocity of ions cannot be neglected giving rise to ion-slip effect. In all aforementioned studies, however, the combined influence of Hall current and ion-slip current was neglected in the magnetohydrodynamic flow equations in order to simplify the equations. As stated by Cowling,\textsuperscript{11} it is important to mention that when the magnetic field becomes strong, the combine effects of Hall and ion-slip current becomes significant on magnetic force term and therefore needs to be incorporated in the flow equations. Application of such flows is evident in many sensor devices in detecting the presence of magnetic field. Mittal et al\textsuperscript{12} studied the impact of Hall current and ion-slip effects on hydromagnetic entrance flow of viscous fluid in a rectangular channel. Soundalgekar et al\textsuperscript{13} investigated the hydromagnetic flow and heat transfer of conducting fluid due to sudden motion of one of the channel wall by incorporating the influence of Hall current and ion-slip effects. Takhar and Jha\textsuperscript{14} analyzed the effects of Hall current and ion-slip effects on hydromagnetic flow of conducting fluid through a channel. Ram et al\textsuperscript{15} investigated on the effect of Hall current and ion slip on hydromagnetic free convection of rotating fluid in a channel with oscillating wall temperatures. Ghosh et al\textsuperscript{16} offered the analytic solution for MHD flow in a revolving system under the influence of Hall current and an inclined magnetic field. Zeeshan et al\textsuperscript{17} analyzed the influence of Hall and ion slip current on hydromagnetic peristaltic motion of conducting fluid in a porous channel. In another work, Bhatti et al\textsuperscript{18} presented an analysis on influence of Hall current on peristaltic motion of conducting fluid through a channel with compliant wall characteristics and heat transfer effects. Rosa\textsuperscript{19} investigated the impact of Hall and ion slip currents on nonuniform gas flows. Other related articles on the above subject includes: Jha et al,\textsuperscript{20} Jha and Malgwi,\textsuperscript{21-23} Noreen et al,\textsuperscript{24} Han et al,\textsuperscript{25} Qureshi et al,\textsuperscript{26} and Jha et al.\textsuperscript{27}

In the existing work, we present a theoretical analysis on the combine influence of Hall current and ion-slip effects on magnetohydrodynamic flow of conducting viscous fluid in a microchannel with mixed convection current. The article is an extension of the work of Weng and Chen\textsuperscript{8} by incorporating the influence of Hall current and ion-slip effects in the existence of mixed convection current. The flow equations are obtained in dimensionless form and thereafter solved analytically subject to appropriate conditions. Influence of different flow parameters on fluid velocity, volume flow rate and skin friction are displayed graphically and thereafter discussed. The novelty in this work is that an exact solution responsible for the flow of viscous incompressible and electrically conducting fluid confined between two vertical microchannel walls is obtained in the presence of Hall and ion-slip current. The flow within the microchannel is caused by the combination of free convection and forced convection (due to pressure gradients). Results obtained in this analysis are significant in providing the standards for validating the accuracies of some numerical or empirical methods. Furthermore, the results obtained in this work can be relevant in fluid mechanics and heat transfer applications.

2 MATHEMATICAL ANALYSIS

Flow of conducting viscous fluid confined in an insulated vertical microchannel walls is investigated. The microchannel walls are anticipated to be with temperatures $T_1$ at the right wall located at $y' = 0$ and $T_2$ at the left wall located at $y' = b$ with $T_1 > T_2$. We assume the $x'$–axis is the direction of flow taken vertically upwards and the $y'$–axis perpendicular to it as displayed in Figure 1. A constant magnetic field ($H_0$) and of the form $\vec{H} = (0, H_0, 0)$ is applied perpendicular to the direction of flow. By anticipating a significantly small value of the magnetic Reynold number, the effect of induced magnetic field was neglected. Also, the flow is anticipated to be hydrodynamically and thermally settled such that all parameters depend on $y'$ only. Initially, flow is taken along the $x'$–axis, however, considering the influence of Hall current and ion-slip, another flow is generated along secondary direction $z'$. As stated earlier, the flow is induced by the combine effect of buoyancy forces and pressure gradient applied. The governing equations describing the above physical phenomena is given below under the usual Boussinessq’s approximation following Weng and Chen,\textsuperscript{9} Jha et al,\textsuperscript{28} and Chen and Weng\textsuperscript{29}:
The slip conditions are:

\[ u' = \frac{2 - f_v}{f_v} \lambda \frac{du'}{dy'} \mid y' = 0, \quad w' = \frac{2 - f_v}{f_v} \lambda \frac{dw'}{dy'} \mid y' = 0, \]

\[ u' = \frac{2 - f_v}{f_v} \lambda \frac{du'}{dy'} \mid y' = b, \quad w' = \frac{2 - f_v}{f_v} \lambda \frac{dw'}{dy'} \mid y' = b, \]

The slip conditions are:

\[ T' = T_2 + \frac{2 - f_i}{f_i} \frac{2 \gamma}{\gamma + 1 \Pr} \lambda \frac{dT'}{dy'} \mid y' = 0, \]

\[ T' = T_1 - \frac{2 - f_i}{f_i} \frac{2 \gamma}{\gamma + 1 \Pr} \lambda \frac{dT'}{dy'} \mid y' = b. \]

Introducing the dimensionless quantities:

\[ y = \frac{y'}{b}, M^2 = \frac{\sigma u^2 H_0^2 b^2}{\rho^2}, \quad (u, w) = \frac{(u', w')}{v}, \quad \text{Pr} = \frac{v}{\alpha}, \quad \theta = \frac{T' - T_0}{T_1 - T_0}, \]

\[ \text{Re} = \frac{U b}{v}, \quad \text{Gr} = \frac{g \beta b^3 (T_1 - T_0)}{v^2}, A_1 = \frac{1}{\rho} \frac{dp}{dx'}, \quad A_2 = \frac{1}{\rho} \frac{dp}{dz'}, \]

Equations (1) to (3) reduces to:

\[ \frac{d^2 u'}{dy'^2} + \frac{M^2}{(\alpha_c^2 + \beta_h^2)} (\alpha_c u' + \beta_h w') - \frac{1}{\rho} \frac{dp}{dx'} = 0, \]

\[ \frac{d^2 w'}{dy'^2} + \frac{M^2}{(\alpha_c^2 + \beta_h^2)} (\beta_h u' - \alpha_c w') - \frac{1}{\rho} \frac{dp}{dz'} = 0, \]

\[ \frac{d^2 T'}{dy'^2} = 0. \]
\[
\frac{d^2w}{dy^2} + \frac{M^2}{(\alpha_\epsilon^2 + \beta_t^2)}(\alpha_\epsilon u - \beta_t w) - A_2 = 0. \tag{7}
\]

\[
\frac{d^2\theta}{dy^2} = 0. \tag{8}
\]

With slip conditions:

\[
u = \beta_v Kn \frac{du}{dy} \bigg|_{y=0}, \quad w = \beta_v Kn \frac{dw}{dy} \bigg|_{y=0},
\]

\[
u = -\beta_v Kn \frac{du}{dy} \bigg|_{y=1}, \quad w = -\beta_v Kn \frac{dw}{dy} \bigg|_{y=1}. \tag{9}
\]

\[
\theta = \xi + \beta_v Kn ln \frac{d\theta}{dy} \bigg|_{y=0}, \quad \theta = 1 - \beta_v Kn ln \frac{d\theta}{dy} \bigg|_{y=1}.
\]

Where,

\[
\beta_v = \frac{2 - f_v}{f_v}, \quad \beta_t = \frac{2 - f_t}{f_t} + \frac{2\gamma_s}{\gamma_s + 1} \frac{1}{Pr}, \quad Kn = \frac{\lambda}{b}, \quad ln = \frac{\beta_t}{\beta_v}, \quad \xi = \frac{T_2 - T_0}{T_1 - T_0}. \tag{10}
\]

All quantities used and their units are defined in the nomenclature above.

Introducing \( F = u + iw \) and \( A = A_1 + iA_2 \), Equations (6)-(7) and slip conditions (9) becomes:

\[
\frac{d^2F}{dy^2} + \frac{Gr}{Re} \theta - M_1^2 F - A = 0, \tag{11}
\]

Where \( M_1^2 = \frac{M^2}{\alpha_\epsilon + \beta_t} \) and \( \alpha_\epsilon = 1 + \beta_t \beta_t \).

\[
F(0) = \beta_v Kn \frac{dF}{dy} \bigg|_{y=0}, \quad F(1) = -\beta_v Kn \frac{dF}{dy} \bigg|_{y=1}, \tag{12}
\]

\[
\theta(0) = \xi + \beta_v Kn ln \frac{d\theta}{dy} \bigg|_{y=0}, \quad \theta(1) = 1 - \beta_v Kn ln \frac{d\theta}{dy} \bigg|_{y=1}.
\]

### 2.1 Solution procedure

Having reduced the dimensional governing equations to their respective dimensionless forms by employing the dimensionless quantities and variables, an analytic solution to Equations (8) and (11) with slip conditions (12) are obtained using the method of undetermined coefficient and presented as follows:

\[
\theta(y) = C_1 + C_2 y, \tag{13}
\]

\[
F(y) = C_3 \cosh(M_1 y) + C_4 \sinh(M_1 y) + \frac{Gr}{Re} \left( \frac{C_1}{M_1^2} + \frac{C_2}{M_1^2} \right) - \frac{A}{M_1^2}. \tag{14}
\]

The mean velocity at any cross section of the microchannel can be obtained by:

\[
\int_0^1 F(y)dy = 1. \tag{15}
\]
Using Equation (14) in Equation (15) we obtain:

\[ A = \frac{1 - \frac{Gr}{Re}k_{15}}{k_{14}}. \]

To obtain the shear stress at the microchannel walls we proceed as follows:

At surface \( y = 0 \),

\[ \tau_0 = \frac{dF}{dy} \bigg|_{y=0} = \tau_{x0} + i\tau_{z0}, \]

where \( \frac{dF}{dy} \bigg|_{y=0} = M_1C_4 + \frac{Gr}{Re} \frac{C_2}{M_1^2}. \) (16)

At surface \( y = 1 \),

\[ \tau_1 = \frac{dF}{dy} \bigg|_{y=1} = \tau_{x1} + i\tau_{z1}, \]

where \( \frac{dF}{dy} \bigg|_{y=1} = \frac{Gr}{Re} \frac{C_2}{M_1^2} + M_1(C_3 \sinh(M_1) + C_4 \cosh(M_1)). \) (17)

With \( \tau_x = \text{real}(\tau) \) and \( \tau_z = \text{Imag}(\tau) \).

To obtain the dimensionless volume flow rate (\( \delta \)) we proceed as follows:

\[ \delta = Q_x + iQ_z = \int_0^1 F(y)dy, \]

\[ = C_3 \frac{\sinh(M_1)}{M_1} + C_4 \frac{\cosh(M_1) - 1}{M_1} + \frac{Gr}{Re} \left( \frac{C_1}{M_1^2} + \frac{C_2}{2M_1^2} \right) - \frac{A}{M_1^2}. \] (19)

Where \( Q_x = \text{Re}(\delta) \) and \( Q_z = \text{Im}(\delta) \).

And \( C_1, \ldots C_4 \) and \( k_1, \ldots k_{23} \) are constants defined in the appendix.

### 2.2 Limiting cases

In the absence of pressure gradient, Hall and ion-slip currents the mathematical model and result presented in this work is in agreement with those of Chen and Weng. Furthermore, results presented in this work also coincide with the work of Weng and Chen in the absence of transverse magnetic field and pressure gradient.

### 3 RESULTS AND DISCUSSION

In this section, we use the expression for different flow features like temperature and velocity obtained in the previous section to prepare a Matlab program in order to visualize the influence of relevant flow parameters. Numerical values for related flow parameters taken are: \( 5 \leq M \leq 10, 0.5 \leq \beta_h \leq 1.5, 0.1 \leq \beta_i \leq 0.5, 0 \leq \frac{Gr}{Re} \leq 100, \) and \( 0.01 \leq \beta_vKn \leq 0.10, \) whereas \( M = 5, \beta_i = 0.25, \beta_h = 0.5, \frac{Gr}{Re} = 100, In = 1.667, \) and \( \beta_vKn = 0.05 \) are taken for fixed values, following Weng and Chen, Avci and Aydin, Jha et al, and Chen and Weng. In each of the figures presented, three different cases for microchannel heating have been considered by utilizing the wall ambient temperature difference ratio \( \xi \): \( \xi = 1 \) represents the physical situation when the two walls are heated symmetrically, \( \xi = 0 \) represents the physical situation when one of the microchannel wall is heated and the other not heated, and \( \xi = -1 \) represents the physical situation when one of the wall is heated and the other is cooled.

Figure 2A,B illustrate the impact of rarefaction parameter (\( \beta_vKn \)) on primary and secondary velocity profiles in a microchannel heated either symmetrically (\( \xi = 1 \)) or asymmetrically (\( \xi = 0, -1 \)) in presence of buoyancy forces and pressure gradient. It is evident from the figure that for \( \xi = 1 \) (symmetric heating), the limiting influence of the microchannel
walls drops with the increase in $\beta_{vKn}$ causing an enhancement in primary velocity about the microchannel walls while it reduces within the central region. In addition, in a region away from the center of the microchannel, a point of intersection is observed where the influence of the rarefaction parameter becomes insignificant on fluid velocity. When the walls are heated asymmetrically ($\xi = 0, -1$), however, it is evident that increase in rarefaction parameter ($\beta_{vKn}$) pushes the momentum boundary layer toward the heated microchannel wall causing an increase in fluid velocity while it reduces about the cold wall. A related phenomenon is observed for fluid velocity along the secondary flow direction.

Figure 3A,B demonstrates the combine effects of Hall current parameter ($\beta_h$) and wall ambient temperature difference ratio ($\xi$) on velocity profiles. Analysis from the figure reveals that in existence of ion-slip current ($\beta_i$) and pressure gradient ($\frac{\text{Gr}}{\text{Re}}$), increase in Hall current parameter destabilizes the flow toward the primary flow direction, yielding an observable decrease in fluid velocity for all wall heating situations ($\xi$). A close observation of the figure also reveals that for $\xi = -1$ depicting the physical situation when one of the microchannel wall is heated and the other wall cooled, a reverse flow is observed close to the wall at $y = 0$. For secondary component on the other hand, it is interesting to mention that increase in $\beta_h$ poses a significant effect on flow formation yielding a pronounced increase in fluid velocity toward the heated right microchannel wall for both $\xi = 0$ and $\xi = -1$ cases. For symmetric heating ($\xi = 1$), however, it could be observed that increase in Hall current parameter ($\beta_h$) leads to increase in fluid velocity within the central region of the microchannel.
This is because when the Hall current ($\beta_h$) rises, the effect of the applied magnetic field decreases thus strengthening the convective current toward the secondary direction.

Figure 4A,B displays the influence of the ion-slip current parameter ($\beta_i$) on flow formation for various microchannel heating ($\xi$). Results from the figure reveals that in presence of mixed convection current ($\frac{Gr}{Re}$) and Hall current parameter ($\beta_h$), ion-slip parameter ($\beta_i$) supports fluid velocity toward the heated right microchannel wall while it decreases toward the left microchannel wall. This phenomenon is observed for asymmetric wall heating ($\xi = 0, -1$). For asymmetric heating, however, it is clear that fluid velocity toward the primary direction rises all through the flow domain with increase in $\beta_i$. Results obtained from Figure 4B shows that irrespective of the wall ambient temperatures, increase in ion-slip current ($\beta_i$) dampens the flow toward the secondary direction causing a reduction in secondary velocity.

Figure 5A,B shows the impact of Hartmann number ($M$) on flow formation for different microchannel heating conditions ($\xi$) in presence of mixed convection current ($\frac{Gr}{Re}$), Hall current ($\beta_h$), and ion-slip ($\beta_i$). As expected, the result indicate that with asymmetric wall heating ($\xi = 0, -1$), fluid velocity along the primary flow is lowered toward the heated right microchannel wall when the Hartmann number ($M$) rises while it enhances the flow toward the left microchannel wall. This result is, however, reversed along the secondary flow direction as fluid velocity is observed to increase toward the right microchannel wall, whereas it decreases toward the left wall when the Hartmann number rises. For symmetric wall

**FIGURE 4** Influence of ion-slip parameter ($\beta_i$) on fluid velocity toward the primary and secondary directions when $\beta_h = 0.5, \beta_iKn = 0.05, ln = 1.667$ and $\frac{Gr}{Re} = 100$

**FIGURE 5** Influence of Hartmann number ($M$) on fluid velocity toward the primary and secondary directions when $\beta_i = 0.25, \beta_h = 0.5, ln = 1.667$ and $\frac{Gr}{Re} = 100$
heating on the contrary, it is clear that a rise in Hartman number causes primary fluid velocity to display a dual character. Thus, a rise in magnetic field yields a decrease in fluid velocity about the center of the microchannel, whereas it increases near the microchannel walls.

Influence of mixed convection parameter \( \left( \frac{Gr}{Re} \right) \) on flow formation is depicted in Figure 6A,B for various values of \( \xi \). From the figure, it is remarkable to observe that with asymmetric wall heating \( (\xi = 0, -1) \), the fluid velocity along the primary flow direction is lowered toward the cold microchannel wall as the mixed convection parameter \( \left( \frac{Gr}{Re} \right) \) rises while it supports the flow toward the heated microchannel wall yielding an enlargement in velocity profile. The figure also reveals that about the central region, fluid velocity is unaffected by the increase in \( \frac{Gr}{Re} \). A similar occurrence is observed for the secondary flow (Figure 6B).

Variation of various governing parameters with regards the volume flow rate along the primary and secondary flow directions \( (Q_x, Q_z) \) are displayed in Figures 7 and 8 under different heating conditions. Figure 7 depicts the effect of Hartmann number \( (M) \) as well as rarefaction parameter \( (\beta_vKn) \) on volume flow rate \( (Q_x, Q_z) \) in presence of Hall current \( (\beta_h) \), ion-slip current \( (\beta_i) \), and mixed convection \( \left( \frac{Gr}{Re} \right) \). Details from the figure reveals that irrespective of the microchannel heating conditions, the volume flow rate along the primary flow direction could be lowered by simultaneously reducing the strength of the magnetic field \( (M) \) and the rarefaction parameter \( (\beta_vKn) \). The contrast of this phenomenon is found along the secondary direction.

**Figure 6** Influence of mixed convection parameter \( \left( \frac{Gr}{Re} \right) \) on fluid velocity toward the primary and secondary directions when \( \beta_i = 0.25, \beta_h = 0.5, ln = 1.667 \) and \( \beta_vKn = 0.05 \)

**Figure 7** Combined effect of Hartmann number \( (M) \) and rarefaction parameter \( (\beta_vKn) \) on primary and secondary volume flow rates for \( \beta_i = 0.25, \beta_h = 0.5, ln = 1.667, \) and \( \frac{Gr}{Re} = 5 \).
The combined effect of Hall and ion-slip parameters ($\beta_h, \beta_i$) on primary and secondary volume flow rates ($Q_x, Q_z$) is illustrated in Figure 8 under various heating situations ($\zeta$). From the figure, it is remarkable to observe that in the existence of mixed convection current, ion-slip current ($\beta_i$) supports volume flow rate in both primary and secondary flow directions yielding an observable increase in volume flow rate ($Q_x, Q_z$) for both symmetric ($\zeta = 1$) and asymmetric heating situations ($\zeta = 0, -1$).

Table 1 illustrates the effect of pertinent flow parameters like mixed convection parameter ($Gr/Re$), Hall current parameter ($\beta_h$), and ion-slip parameter ($\beta_i$) on shear stress ($\tau$) located at $y = 0$ and $y = 1$ correspondingly when $M = 5$, $\xi = 0$ and $\ln = 1.667$ toward the primary and secondary flow directions.

| $Gr/Re$ | $\beta_i$ | $\beta_h$ | $\tau_{x0}$ | $\tau_{z0}$ | $\tau_{x1}$ | $\tau_{z1}$ |
|---------|-----------|-----------|--------------|--------------|--------------|--------------|
| 0       | 0.1       | 0.5       | 5.1690       | 1.3108       | -5.1690      | -1.3108      |
|         | 1.0       | 4.3490    | 1.8095       | -4.3490      | -1.8095      | -1.0647      |
|         | 1.5       | 3.8835    | 1.9337       | -3.8835      | -1.9337      | -1.0647      |
| 0.3     | 0.5       | 5.1946    | 1.1916       | -5.1946      | -1.1916      | -1.4364      |
|         | 1.0       | 4.5603    | 1.6147       | -4.5603      | -1.6147      | -1.4364      |
|         | 1.5       | 4.2190    | 1.7290       | -4.2190      | -1.7290      | -1.4364      |
| 0.5     | 0.5       | 5.2071    | 1.0871       | -5.2071      | -1.0871      | -1.4364      |
|         | 1.0       | 4.7004    | 1.4346       | -4.7004      | -1.4346      | -1.4364      |
|         | 1.5       | 4.4445    | 1.5227       | -4.4445      | -1.5227      | -1.4364      |
| 100     | 0.1       | 0.5       | 0.7357       | 0.7212       | -9.6022      | -1.9003      |
|         | 1.0       | -0.4888   | 0.8932       | -9.1867      | -2.7259      | -2.7259      |
|         | 1.5       | -1.3343   | 0.9071       | -9.1014      | -2.9602      | -2.9602      |
| 0.3     | 0.5       | 0.6661    | 0.6603       | -9.7230      | -1.7229      | -2.7259      |
|         | 1.0       | -0.3719   | 0.8374       | -9.4926      | -2.3921      | -2.3921      |
|         | 1.5       | -1.0507   | 0.8809       | -9.4887      | -2.5771      | -2.5771      |
| 0.5     | 0.5       | 0.5911    | 0.6060       | -9.8231      | -1.5682      | -1.5682      |
|         | 1.0       | -0.3220   | 0.7689       | -9.7227      | -2.1004      | -2.1004      |
|         | 1.5       | -0.8874   | 0.8160       | -9.7764      | -2.2295      | -2.2295      |
When both buoyancy forces and pressure gradient supports the flow that is, \( \frac{Gr}{Re} > 0 \), a rise in ion-slip (\( \beta_i \)) and Hall current parameter (\( \beta_h \)) leads to reduction in primary as well as secondary skin friction (\( \tau_x \) and \( \tau_z \)) at \( y = 0 \) while it increases at \( y = 1 \).

### 4 | CONCLUSION

Hydromagnetic flow formation of conducting fluid in a vertical microchannel due combined effect of buoyancy forces and pressure gradient has been studied theoretically. Influence of Hall and ion-slip current has been taken into account. Impact of different governing parameters on relevant flow features have been identified and discussed. As stated earlier, results obtained in this analysis are significant in providing the standards for validating the accuracies of some numerical or empirical methods. Furthermore, the results obtained in this work can be relevant in fluid mechanics and heat transfer applications.

The major conclusions derived from this study include:

1. Flow formation in the microchannel is governed by five basic parameters: rarefaction parameter (\( \beta_vKn \)), mixed convection parameter \( \left( \frac{Gr}{Re} \right) \), wall ambient temperature difference ratio(\( \xi \)), fluid wall interaction parameter (\( In \)), Hartmann number (\( M \)), Hall current parameter (\( \beta_h \)), and ion-slip parameter (\( \beta_i \)).

2. In the absence of mixed convection \( \left( \frac{Gr}{Re} \right) \), Hall and ion-slip effects the mathematical model presented in this work is in excellent agreement and those of Weng and Chen.\(^8\)

3. When the ion-slip parameter (\( \beta_i \)) rises, the momentum boundary layer along the primary flow direction thickens close to the heated wall while it becomes thinner about the cold wall for both asymmetric wall heating situations: \( \xi = 0 \) and \( \xi = -1 \).

4. For fixed values of the mixed convection parameter \( \left( \frac{Gr}{Re} \right) \) and ion-slip current parameter (\( \beta_i \)), the Hall current parameter (\( \beta_h \)) strengthens the flow toward the secondary flow while it retards the flow toward the primary direction.

### PEER REVIEW INFORMATION

*Engineering Reports* thanks the anonymous reviewers for their contribution to the peer review of this work.

### CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

### AUTHOR CONTRIBUTIONS

Basant K. Jha contributed to Conceptualization-Lead, Methodology-Lead, Resources-Lead, and Supervision-Lead. Peter B. Malgwi contributed to Conceptualization-Supporting, Data curation-Supporting, Formal analysis-Supporting, Investigation-Lead, Methodology-Supporting, Writing-original draft-Lead, and Writing-review & editing-Equal.

### NOMENCLATURE

- \( b \) channel gap (m)
- \( C_p, C_v \) specific heat at constant pressure and constant volume, respectively \( (Jkg^{-1}K^{-1}) \)
- \( f_v, f_t \) thermal and tangential momentum accommodation coefficient, respectively (dimensionless variables)
- \( g \) acceleration due to gravity \( (ms^{-2}) \)
- \( \frac{Gr}{Re} \) mixed convection parameter (dimensionless)
- \( A_1 \) constant pressure gradient along \( x' \)-axis (dimensionless)
- \( A_2 \) constant pressure gradient along \( z' \)-axis (dimensionless)
- \( H_0 \) Constant magnetic flux density \( (Tesla) \)
- \( Kn \) Knudsen number \( \lambda/b \) (dimensionless variable)
- \( In \) fluid-wall interaction parameter \( \beta_i/\beta_v \) (dimensionless variable)
- \( M \) Hartmann number (dimensionless variable)
- \( k \) thermal conductivity \( (wmt^{-1}K^{-1}) \)
- \( T \) temperature of the fluid \( (K) \)
- \( T_1 \) wall temperature at \( y' = b \) \( (K) \)
$T_2$  wall temperature at $y' = 0$ (K)

$u'$  velocity along $x'$-direction (ms$^{-2}$)

$w'$  velocity along $z'$-direction (ms$^{-2}$)

$u$  velocity along $x'$-direction (dimensionless)

$w$  velocity along $z'$-direction (dimensionless)

$F$  complex velocity $F = u + iw$ (dimensionless)

$\delta$  volume flow rate $\delta = Q_x + iQ_z$ (dimensionless)

$Q_x$  volume flow rate along the primary flow direction (dimensionless)

$Q_z$  volume flow rate along the secondary flow direction (dimensionless)

GREEK LETTERS

$\beta$  coefficient of thermal expansion (K$^{-1}$)

$\gamma_s$  specific heat ratio $C_p/C_v$

$\sigma$  electrical conductivity of the fluid

$\mu$  dynamic viscosity

$\xi$  wall-ambient temperature difference parameter

$\rho$  density (kgm$^{-3}$)

$\alpha$  thermal diffusivity

$\lambda$  molecular mean free path (m)

$\beta_t$  temperature jump (dimensionless)

$\beta_v$  velocity slip (dimensionless)

$\beta_H$  Hall current parameter ($\omega_e \tau_e$)

$\beta_i$  ion-slip parameter ($\omega_i \tau_i$)

$\nu$  kinematic viscosity (m$^2$s$^{-1}$)

$\theta$  dimensionless temperature (K)

$\tau_{x0}$  skin friction owing to primary flow at $y = 0$ (dimensionless)

$\tau_{z0}$  skin friction owing to secondary flow at $y = 0$ (dimensionless)

$\tau_{x1}$  skin friction owing to primary flow at $y = 1$ (dimensionless)

$\tau_{z1}$  skin friction owing to secondary flow at $y = 1$ (dimensionless)

ORCID

Peter B. Malgwi  https://orcid.org/0000-0001-7772-964X

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How to cite this article: Jha BK, Malgwi PB. Hall and ion-slip effects on MHD mixed convection flow in a vertical microchannel with asymmetric wall heating. Engineering Reports. 2020;2:e12241. https://doi.org/10.1002/eng2.12241

APPENDIX A

\[ C_1 = \xi + \frac{\beta_c KnIn(1-\xi)}{1 + 2\beta_c KnIn}, \quad C_2 = \frac{1-\xi}{1 + 2\beta_c KnIn}, \quad C_3 = Ak_7 + k_8 \frac{Gr}{Re}, \quad C_4 = Ak_5 + k_10 \frac{Gr}{Re}, \quad k_1 = \beta_c KnM_1, \]

\[ k_2 = \beta_c Kn \frac{C_2}{M_1^2} - \frac{C_1}{M_1^2}, \quad k_3 = \frac{1}{M_1^2}, \quad k_4 = \cosh(M_1) + \beta_c KnM_1 \sinh(M_1), \]

\[ k_5 = \sinh(M_1) + \beta_c KnM_1 \cosh(M_1), \quad k_6 = C_1 k_3 + C_2 (\beta_c Kn k_3 + k_3), \quad k_7 = \frac{k_3 k_5 - k_6 k_1}{k_5 + k_1 k_4}, \]

\[ k_8 = \frac{k_3 k_5 - k_6 k_1}{k_5 + k_1 k_4}, \quad k_9 = -\frac{k_3 - k_6 k_4}{k_5 + k_1 k_4}, \quad k_{10} = -\frac{k_4 + k_2 k_4}{k_5 + k_1 k_4}, \quad k_{11} = \frac{C_1}{M_1^2} + \frac{C_2}{2M_1^2}, \quad k_{12} = \frac{\sinh(M_1)}{M_1}, \]

\[ k_{13} = \frac{\cosh(M_1) - 1}{M_1}, \quad k_{14} = k_7 k_{12} + k_9 k_{13} - k_3, \quad k_{15} = k_8 k_{12} + k_{10} k_{13} + k_{11} \]