CAN A CHANGING $\alpha$ EXPLAIN THE SUPERNOVAE RESULTS?

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ABSTRACT

We show that the supernovae results, which imply that there is evidence for an accelerating universe, may be closely related to the recent discovery of redshift dependence in the fine-structure constant $\alpha$. The link is a class of varying speed-of-light (VSL) theories that contain cosmological solutions that are similar to quintessence. During the radiation-dominated epoch, the cosmological constant $\Lambda$ is prevented from dominating the universe by the usual VSL mechanism. In the matter-dominated epoch, the varying-$c$ effects switch off, allowing $\Lambda$ to eventually surface and lead to an accelerating universe. By the time this happens, the residual variations in $c$ imply a changing $\alpha$ at a rate that is in agreement with observations.

Subject headings: cosmology: observations — cosmology: theory — early universe

1. INTRODUCTION

Two puzzling observations are a challenge to cosmologists. The Supernovae Cosmology Project and the High-$z$ Supernova Search (Perlmutter et al. 1997; Garnavich et al. 1998; Schmidt 1998; Riess et al. 1998) have extended the reach of the Hubble diagram to high redshift and provided new evidence that the expansion of the universe is accelerating. This may imply that there exists a significant positive cosmological constant, $\Lambda$. In separate work, the spacings between quasar (QSO) absorption lines were examined in Keck I data at medium redshifts, $z \sim 1$ (Webb et al. 1999), and compared with those in the laboratory (see also Drinkwater et al. 1998, Damour & Dyson 1996, Shylakher 1976, and Barrow 1987). These observations are sensitive to time variations in the value of the fine-structure constant $\alpha = e^2/(\hbar c)$ (where $e$ is the electron charge, $\hbar$ is Planck’s constant, and $c$ is the speed of light), at a rate 1 million times slower than the expansion rate of the universe. Evidence was found for a small variation in the value of $\alpha$ at redshifts of $\sim 1$. This could be produced by intrinsic time variation or by some unidentified line-blending effect. In this Letter, we assume that the variation is intrinsic and show that there may be a link between the observations of cosmological acceleration and varying $\alpha$.

If $\Lambda > 0$, then cosmology faces a very serious fine-tuning problem, and this has been the motivation for extensive theoretical work. There is no theoretical motivation for a value of $\Lambda$ of currently observable magnitude; a value $10^{120}$ times smaller than the “natural” Planck scale of density is needed if $\Lambda$ becomes important near the present time. Such a small non-zero value of $\Lambda$ is “unnatural” in the sense that making it zero reduces the symmetry of spacetime. A tentative solution is the rolling scalar field exhibiting very long transients. Here we introduce another explanation.

There are a variety of possible physical expressions of a changing $\alpha$. Bekenstein (1982) proposed a varying-$e$ theory. An alternative is the varying speed-of-light (VSL) theory (Moffat 1993; Albrecht & Magueijo 1999; Barrow 1999) in which varying $\alpha$ is expressed as a variation of the speed of light. The choice between these two types of theories transcends experimentation and merely reflects theoretical convenience in the choice of units (Barrow & Magueijo 1998). The simplest cosmology following from the VSL is known to contain an attractor in which $\Lambda$ and matter remain at fixed density ratios throughout the life of the universe (Barrow & Magueijo 1999). Such an attractor solves the fine-tuning problem that is forced on us by the supernovae results. Hence, there is range of possibilities for the observed changing $\alpha$ to be related to the observed acceleration of the universe. In this Letter, we propose a model that leads to good quantitative agreement, given experimental errors, between the observations of acceleration and varying $\alpha$. In §2, we examine the construction of the Hubble diagram in VSL theories and the interpretation of varying-$\alpha$ experiments. Then, in §3, we present an example of a VSL model that can jointly explain the supernovae results and the Webb et al. (1999) varying-$\alpha$ results. We conclude with a discussion of some further aspects of the model proposed, to be investigated elsewhere.

2. THE VSL HUBBLE DIAGRAM

The Hubble diagram is a plot of luminosity distance against redshift. The purpose is to map the expansion factor $a(t)$, where $a$ is the comoving proper time. Redshifts provide a measurement of $a$ at the time of emission. If the objects under observation are “standard candles” (as Type Ia supernovae are assumed to be), their apparent brightness gives their (luminosity) distance, which, if we know $c$, tells us their age. By looking at progressively more distant objects, we can therefore map the curve $a(t)$.

We now examine how this construction is affected by a changing $c$. In Albrecht & Magueijo (1999), we show that $E \propto c^2$ for photons in free flight. We also show that quantum mechanics remains unaffected by a changing $c$ if $\hbar \propto c$ (in the sense that quantum numbers are adiabatic invariants). Then all relativistic energies scale like $c^2$. If, for nonrelativistic systems $\hbar \propto 1/c$, the Rydberg energy $E_R = m_e e^4/(2\hbar^2)$ also scales like $c^2$. Hence, all absorption lines, ignoring the fine structure, scale like $c^2$. When we compare lines from near and far systems, we
should therefore see no effects due to a varying $c$; the redshift $z$ is still $1 + z = a_0 a^o a$, where $o$ and $e$ represent epochs of observation and emission, respectively.

In order to examine luminosity distances, we need to reassess the concept of standard candles. For simplicity, let us treat them as blackbodies. Then their temperature scales as $T \propto c^2$ (Albrecht & Magueijo 1999), their energy density scales as $\rho \propto T^{4/3} (hc)^3 \propto c^2$, and their emission power scales as $P = \rho c \propto c$, implying that standard candles are brighter in the early universe if $c < 0$. However, the power emitted by these candles, in free flight, scales like $c^2$, its speed like $c$, and therefore its energy flux like $c^2$. The received flux, as a function of $c$, therefore scales like

$$P_r = \frac{P_c c^2}{4\pi r^2 c} \propto c,$$

where $r$ is the conformal distance to the emitting object, and the subscripts $r$ and $e$ represent received and emitted, respectively. In an expanding universe, we therefore still have

$$P = \frac{P_0}{4\pi r^2 a_0} \left( \frac{a}{a_0} \right)^2,$$

where $P_0$ is the emitting power of standard candles today. Notice that the above argument is still valid if the candles are not black bodies; it depends only on the scaling properties of emitted and received power.

We can now set up the Hubble diagram. Consider the Taylor expansion

$$a(t) = a_0 \left[ 1 + H_0 (t - t_o) - \frac{1}{2} q_0 H_0^2 (t - t_o)^2 + \ldots \right],$$

where $H_0 = \dot{a}_0 a_0$ is the Hubble constant and $q_0 = -\ddot{a}_0 a_0 / \dot{a}_0^2$ is the deceleration parameter. Hence, up to second order, $z = H_0 (t - t_o) + (1 + q_0/2) H_0^2 (t - t_o)^2$, or

$$t_o - t = \frac{1}{H_0} \left[ z - (1 + q_0/2) z^2 + \ldots \right].$$

From equation (2), we find that the luminosity distance $d_L$ is

$$d_L = \left( \frac{P_0}{4\pi P_0} \right)^{1/2} \propto a_0^{-3/2} a = a_0 r (1 + z).$$

The conformal distance to the emitting object is given by $r = \int_0^t c(t) dt/a(t)$. From equation (3), we find that

$$r = c_o \left[ (t_0 - t) + \frac{1 - n}{2} H_0 (t_0 - t)^2 + \ldots \right].$$

where we have assumed that locally $c = c_o a'^{\infty}$ (i.e., $c = c_o[1 + n H_0 (t - t_0) + \ldots]$). Substituting equation (4), we finally have

$$d_L = \frac{c_0}{H_0} \left\{ z + \frac{1}{2} (1 - (q_0 + n)) z^2 + \ldots \right\}.$$

We see that besides the direct effects of the VSL on the expansion rate of the universe, it also induces an effective acceleration in the Hubble diagram as an "optical illusion" (we are assuming that $c$ decreases in time: $n < 0$). This is easy to understand. We have seen that the VSL introduces no intrinsic effects in the redshifting spectral line or in the dimming of standard candles with distance and expansion. The only effect that the VSL induces on the construction of the Hubble diagram is that for the same redshift (i.e., the same distance into the past), objects are farther away from us because light traveled faster in the past. But an excess luminosity distance, for the same redshift, is precisely the hallmark of cosmological acceleration. However, we need to consider the other experimental input to our work: the Webb et al. (1999) results. By measuring the fine structure in absorption systems at redshifts $z \sim O(1)$, we can also map the curve $c(t)$. Since $c = c_o [1 + n H_0 (t - t_o) + \ldots]$, we have $c = c_o (1 + n z + \ldots)$, and so to first order, $\alpha = \alpha_0 (1 + 2 n z + \ldots)$. However, the results presented in Webb et al. (1999) show that $n$ is at most of order $10^{-5}$. This means that the direct effects of varying $c$ permitted by the QSO absorption system observations are far too small to explain the observed acceleration. We need to look at a fully self-consistent generalization of general relativity containing the scope for varying $c$.

3. THE MODEL

We start with some general properties of the dynamics of $c$. Drawing inspiration from dilaton theories (like Brans-Dicke gravity), we take $\psi = \log (c/c_0)$ as the dynamical field associated with $c$. Indeed, powers of $c$ appear in all coupling constants, which in turn can be written as $e^\phi$, where $\phi$ is the dilaton. Another theory using a similar dynamical variable is the changing-$\alpha$ theory of Bekenstein (1982), which uses $\log \alpha$.

We then endow $\psi$ with a dynamics similar to the dilaton. The left-hand side for the $\psi$ equation should therefore be $\Box \psi$ (in the preferred Lorentz frame, to be identified with the cosmological frame). This structure ensures that the propagation equation for $\psi$ is second order and hyperbolic; i.e., propagation is causal. Since the VSL breaks Lorentz invariance, other expressions would be possible, but then the field $\psi$ would propagate noncausally. An example is $(g_{\mu \nu} + u u) (\nabla \psi) (\nabla \psi)$, where $u^n$ is the tangent vector of the local preferred frame.

On the other hand, one need not choose (as in Brans-Dicke theories) the source term to be $\rho - 3 p c^2$, where $\rho$ and $p$ are the energy density and pressure of matter, respectively. Without the requirement of Lorentz invariance, other expressions are possible, and using them does not conflict with local causality. If $T^{\mu \nu}$ is the stress-energy tensor, we can choose as a source term $T^{\mu \nu} (g_{\mu \nu} + u u)$; i.e., changes in $c$ are driven by the matter pressure. We find that this choice is the one that gives interesting effects.

For a homogeneous field in an expanding universe, we therefore have $\dot{\psi} + 3(\dot{a} / a) \psi = 4\pi G \omega p c^2$, where $p$ is the energy density of the field.
total pressure of the matter fields and \( \omega \) is a coupling constant (distinct from the Brans-Dicke coupling constant). The full self-consistent system of equations in a matter-plus-radiation universe containing a cosmological constant stress is therefore

\[
\ddot{\psi} + \frac{3}{a} \dot{\psi} = 4\pi G \omega \frac{\rho_\gamma}{3}, \tag{8}
\]

\[
\dot{\rho}_\gamma + \frac{4}{a} \rho_\gamma = -2\rho_\gamma \dot{\psi}, \tag{9}
\]

\[
\dot{\rho}_m + \frac{3}{a} \rho_m = 0, \tag{10}
\]

\[
\frac{a^2}{3} = \frac{8\pi G}{3} (\rho_m + \rho_\gamma + \rho_\Lambda), \tag{11}
\]

\[
(\frac{a}{a_0})^2 = \frac{8\pi G}{3} (\rho_m + \rho_\gamma + \rho_\Lambda), \tag{12}
\]

where subscripts \( \gamma \) and \( m \) denote radiation and matter, respectively. We have assumed that the sink term in equation (10) is reflected in a source term in equation (9) (and not in eq. [11]). This is due to the fact that this term is only significant very early on, when even massive particles behave like radiation.

We have ignored curvature terms because in the quasi-flat attractor as \( \Omega_\Lambda = 0.5 \).

\[
\rho_\Lambda = \frac{A}{a^n} - \frac{n}{n + 2}, \tag{13}
\]

with \( A \) constant, from which it follows that

\[
\frac{\rho}{\rho_\Lambda} = \frac{2}{\delta} \left[ \left( 1 + \frac{\delta \rho_\gamma}{2 \rho_\Lambda} \right) \frac{a_0}{a} - 1 \right]. \tag{14}
\]

We see that, asymptotically, \( \rho/\rho_\Lambda \) grows to infinity if \( \delta > 0 \) (the flat \( \rho_\Lambda = 0 \) attractor of Barrow & Magueijo 1999). However, the growth is very slow even if \( \delta \) is not very small. Our theory displays very long transients and a very slow convergence to its attractor, a property similar to quintessence models (Zlatev et al. 1999). It is therefore possible to achieve \( \rho_\Lambda/\rho \sim 10^{-12} \) at the end of the radiation epoch, with \( \delta \) chosen to be of order 0.1.

Now, why is the change in \( c \) of the right order of magnitude to explain the results of Webb et al. (1999)? With a solution of the form \( c = c_0 a^{n(t)} \), we find that

\[
n(t) \approx \frac{\omega \rho_\gamma}{3(\rho_m + 2\rho_\Lambda)}, \tag{15}
\]

is valid in the matter-dominated era, regardless of the details of the radiation to matter transition. With \( \omega \approx -4 \), we therefore
have

$$n(t_0) \approx \frac{4.23 \times 10^{-5}}{3 \, h^2(1 + \Omega_\Lambda)}$$ (16)

of the right order of magnitude. The order of magnitude of the index $n \sim 10^{-5}$, observed by Webb et al. (1999), is therefore fixed by the ratio of the radiation and the matter energy densities today.

4. DISCUSSION

In this Letter, we propose a theory relating the supernovae results to the observations by Webb et al. (1999). The theory that we have proposed is one example within a class whose members exhibit similar behavior. In these theories, the gravitational effect of the pressure drives changes in $c$, and these convert the energy density in $\Lambda$ into radiation. Thus, $\rho_\Lambda$ is prevented from dominating the universe during the radiation epoch. As the universe cools down, massive particles eventually become the source of pressureless matter and create a matter-dominated epoch. In the matter-dominated epoch, the variation in $c$ comes to a halt, with residual effects at $z \approx 1–5$ at the level observed by Webb et al. As the $c$ variation is switched off, the $\Lambda$ stress resurfaces and dominates the universe for a few expansion times in the matter-dominated era, in agreement with the supernovae results.

In a forthcoming publication, we shall address other aspects of this theory, which are beyond the scope of this Letter. We will mention nucleosynthesis, the location in time of a quantum epoch, and perturbations around the homogeneous solution discussed here (see Barrow & O’Toole 1999). Nucleosynthesis, in particular, may provide significant constraints on this class of models. However, we expect a variation in $\alpha$ to require variations in other couplings if some unification exists. Nucleosynthesis involves many competing effects, with contributions from weak, strong, electromagnetic, and gravitational interactions, and we do not know how to incorporate all the effects self-consistently. Studies of the effects of varying constants coupled by Kaluza-Klein extra dimensions have been made by Kolb et al. (1986) and Barrow (1987). The most detailed study to date was conducted by Campbell & Olive (1995).

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