Magnetic field instability driven by anomalously magnetic moments of massive fermions and electroweak interaction with background matter

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Abstract

It is shown that the electric current of massive fermions along the external magnetic field can be excited in the case when particles possess anomalously magnetic moments and electroweakly interact with background matter. This current is calculated on the basis of the exact solution of the Dirac equation in the external fields. It is shown that the magnetic field becomes unstable if this current is taken into account in the Maxwell equations. Considering a particular case of a degenerate electron gas, which can be found in a neutron star, it is revealed that the seed magnetic field can be significantly enhanced. The application of the results to astrophysics is also discussed.

The problem of the magnetic field instability is important, e.g., in the context of the existence of strong astrophysical magnetic fields \cite{1}. Besides the conventional magneto-hydrodynamics mechanisms for the generation of astrophysical magnetic fields, recently the approaches based on the elementary particle physics were proposed. These approaches mainly rely on the chiral magnetic effect (CME) \cite{2}, which consists in the appearance of the anomalous current of massless charged particles along the external magnetic field $\mathbf{B}$,

\begin{equation}
\mathbf{J}_{\text{CME}} = \frac{e^2}{4\pi^2} (\mu_R - \mu_L) \mathbf{B},
\end{equation}

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where $e$ is the particles charge and $\mu_{R,L}$ are the chemical potentials of right and left chiral fermions.

If $J_{\text{CME}}$ is accounted for in the Maxwell equations, a seed magnetic field appears to be unstable and can experience a significant enhancement. The applications of CME for the generation of astrophysical and cosmological magnetic fields are reviewed in Ref. [3].

However, the existence of CME in astrophysical media is questionable. As found in Refs. [2,4], $J_{\text{CME}}$ can be non-vanishing only if the mass of charged particles, forming the current, is exactly equal to zero, i.e. the chiral symmetry is restored. For the case of electrons the restoration of the chiral symmetry is unlikely at reasonable densities which can be found in present universe [5]. The chiral symmetry can be unbroken in quark matter owing to the strong interaction effects [6]. The magnetic fields generation in quark matter, which can exist in some compact stars, was discussed in Refs. [7,8]. Nevertheless this kind of situation looks quite exotic.

Therefore the issue of the existence of an electric current $J \sim B$ for massive particles, which can lead to the magnetic field instability, is quite important for the development of astrophysical magnetic fields models. One of the example of such a current in electroweak matter was proposed in Ref. [9]. However, the model developed in Ref. [9] implies the inhomogeneity of background matter. This fact imposes the restriction on the scale of the magnetic field generated.

In the present work, we discuss another scenario for the magnetic field instability. It involves the consideration of the electroweak interaction of massive fermions with background matter along with nonzero anomalous magnetic moments of these fermions. Note that the electroweak interaction implies the generic parity violation which can provide the magnetic field instability.

This work is organized as follows. First, we discuss the Dirac equation for a massive electron with a nonzero anomalous magnetic moment, electroweakly interacting with background matter under the influence of an external magnetic field. Using the previously obtained solution of this Dirac equation, we calculate the electric current of these electrons along the magnetic field direction. This current turns out to be nonzero. Then we consider a particular situation of a strongly degenerate electron gas, which can be found inside a neutron star (NS). Finally we apply our results for the description of the amplification of the magnetic field in NS and briefly discuss the implication of our findings to explain the electromagnetic radiation of compact stars.

Let us consider a fermion (an electron) with the mass $m$ and the anomalous magnetic moment $\mu$. This electron is taken to interact electroweakly with nonmoving and unpolarized background matter, consisting of neutrons and protons, under the influence of the external magnetic field along the $z$-axis, $B = Be_z$. Accounting for the forward scattering off background fermions in the Fermi approximation, the Dirac equation for the electron has the form,

$$\left[\gamma_{\mu}P^\mu - m - \mu B \Sigma_3 - \gamma^0 (V_R P_R + V_L P_L)\right] \psi = 0, \quad (2)$$

where $P^\mu = i\partial^\mu + eA^\mu$, $A^\mu = (0, 0, Bx, 0)$ is the vector potential, $e > 0$ is the absolute value of the elementary charge, $P_{R,L} = (1 \pm \gamma^5)/2$ are the chiral projectors, $\gamma^\mu = (\gamma^0, \gamma)$,
\[ \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3, \] and \[ \Sigma_3 = \gamma^0\gamma^3\gamma^5 \] are the Dirac matrices. The effective potentials of the electroweak interaction \( V_{R,L} \) have the form \[ 10 \],

\[ V_R = -\frac{G_F}{\sqrt{2}} [n_n - n_p (1 - 4\xi)] 2\xi, \quad V_L = -\frac{G_F}{\sqrt{2}} [n_n - n_p (1 - 4\xi)] (2\xi - 1), \] (3)

where \( n_{n,p} \) are the number densities of neutrons and protons, \( G_F = 1.17 \times 10^{-5} \text{GeV}^{-2} \) is the Fermi constant, and \( \xi = \sin^2\theta_W \approx 0.23 \) is the Weinberg parameter.

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The solution of Eq. (2) has the form \[ 11 \],

\[ \psi = \exp (-iEt + ip_y y + ip_z z) \times (C_1 u_n - 1, iC_2 u_n, C_3 u_n - 1, iC_4 u_n)^T, \]

\[ u_n(\eta) = \left( \frac{eB}{\pi} \right)^{1/4} \exp \left( -\frac{\eta^2}{2} \right) \frac{H_n(\eta)}{\sqrt{2^n n!}}, \quad n = 0, 1, \ldots, \] (4)

where \( -\infty < p_y, z < +\infty \), \( H_n(\eta) \) are the Hermite polynomials, \( \eta = \sqrt{eBx + p_y/\sqrt{eB}} \), and \( C_i \), with \( i = 1, \ldots, 4 \), are the spin coefficients. For the definiteness, we will use below the chiral representation for the Dirac matrices. It is convenient to normalize the wave function \( \psi \) as

\[ \int d^3x \psi^\dagger_p \gamma^5 \psi_p = \delta(p_y - p'_y) \delta(p_z - p'_z) \delta_{nn'}, \] (5)

at any moment of time.

The energy levels \( E \) for \( n > 0 \) have the form \[ 11 \],

\[ E = \bar{V} + \epsilon, \quad \epsilon = \sqrt{p_z^2 + m^2 + 2eBn + (\mu B)^2 + V_5^2 + 2sR^2}, \]

\[ R^2 = (p_z V_5 - \mu Bm)^2 + 2eBn [V_5^2 + (\mu B)^2]. \] (6)

where \( s = \pm 1 \) is the discrete spin quantum number, \( \bar{V} = (V_L + V_R)/2 \), and \( V_5 = (V_L - V_R)/2 \). At \( n = 0 \), the energy spectrum reads

\[ E = \bar{V} + \sqrt{(p_z + V_5)^2 + (m - \mu B)^2}. \] (7)

It should be noted that, at lowest energy level, the electron spin has only one direction since \( C_1 = C_3 = 0 \). In Eqs. (4) and (5), we present the solution only for particles (electrons) rather than for antiparticles (positrons).

Using the exact solution of the Dirac equation, we can calculate the electric current of electrons in this matter. This current has the form \[ 2 \],

\[ J = -e \sum_{n=0}^{\infty} \sum_s \int_{-\infty}^{+\infty} dp_y dp_z \psi \gamma \psi f(E - \chi), \] (8)

where \( f(E) = [\exp(\beta E) + 1]^{-1} \) is the Fermi-Dirac distribution function, \( \beta = 1/T \) is the reciprocal temperature, and \( \chi \) is the chemical potential.
First, we notice that the transverse components of the electric current \( J_{x,y} \sim \bar{\psi}\gamma^{1,2}\psi \) are vanishing because of the orthogonality of Hermite functions with different indexes. The contribution of the lowest energy level with \( n = 0 \) to the electric current along the magnetic field \( J_z \sim \bar{\psi}\gamma^3\psi \) is also vanishing: \( J_z^{(n=0)} = 0 \). This result is valid for arbitrary parameters \( m, \mu, V_5, \) and \( \chi, \) and \( B \).

The contributions of the higher energy levels with \( n > 0 \) to \( J_z \) can be obtained using the expressions for the spin coefficients \( C_i \) also found in Ref. [11],

\[
J_z^{(n>0)} = -\frac{e^2 B}{(2\pi)^2} \sum_{n=1}^{\infty} \sum_{s=\pm1} \int_{-\infty}^{+\infty} \frac{dp_z}{E} \left[ p_z \left( 1 + s \frac{V_5^2}{R^2} \right) - s \frac{\mu B m V_5}{R^2} \right] f(E - \chi). \tag{9}
\]

The first nonzero term in Eq. (9) is proportional to \( \mu B \) and \( V_5 \),

\[
J_z = \mu m V_5 B^2 \frac{e^2}{\pi^2} \sum_{n=1}^{\infty} \int_{0}^{+\infty} \frac{dp}{\epsilon_{\text{eff}}} \left[ \left( 1 - \frac{3p^2}{\epsilon_{\text{eff}}} \right) \left( f' - \frac{f}{\epsilon_{\text{eff}}} \right) + \frac{p^2}{\epsilon_{\text{eff}}} f'' \right], \tag{10}
\]

where \( \epsilon_{\text{eff}} = \sqrt{p^2 + m_{\text{eff}}^2} \) and \( m_{\text{eff}} = \sqrt{m^2 + 2eBn} \). The argument of the distribution function in Eq. (10) is \( \epsilon_{\text{eff}} + V - \chi \).

Let us consider the case of a strongly degenerate electron gas. In this situation, \( f = \theta(\chi - V - \epsilon_{\text{eff}}) \), where \( \theta(z) \) is the Heaviside step function. We can also disregard the positrons contribution to \( J_z \). The direct calculation of the current in Eq. (10) gives

\[
J_z = -2\mu m V_5 B^2 \frac{e^2}{\pi^2 \tilde{\chi}^3} \sum_{n=1}^{\infty} \sqrt{\tilde{\chi}^2 - m_{\text{eff}}^2} \theta(\tilde{\chi} - m_{\text{eff}}), \tag{11}
\]

where \( \tilde{\chi} = \chi - V \).

One can see that \( J_z \) in Eq. (11) is nonzero if \( B < \tilde{B} \), where \( \tilde{B} = (\tilde{\chi}^2 - m^2) / 2e \). If the magnetic field is strong enough and is close to \( \tilde{B} \), then only the first energy level with \( n = 1 \) contributes to \( J_z \), giving one \( J_z = -8\mu m V_5 B^2 \alpha_{\text{em}} \sqrt{\tilde{\chi}^2 - m^2 - 2eB} / \pi \tilde{\chi}^3 \rightarrow 0 \), where \( \alpha_{\text{em}} = e^2 / 4\pi \approx 7.3 \times 10^{-3} \) is the fine structure constant. In the opposite situation, when \( B \ll \tilde{B} \), one gets that \( J_z = -8\alpha_{\text{em}} \mu m V_5 B (\tilde{\chi}^2 - m^2 - 2eB) / 3\pi e \tilde{\chi}^3 \approx -8\alpha_{\text{em}} \mu m V_5 B / 3\pi e \), i.e. the current is proportional to the magnetic field strength.

To study the evolution of the magnetic field in the presence of the current in Eq. (11) we return there to the vector notations,

\[
\mathbf{J} = \Pi \mathbf{B}, \quad \Pi = -8\mu m V_5 B \frac{\alpha_{\text{em}}}{\pi \tilde{\chi}^3} \sum_{n=1}^{N} \sqrt{\tilde{\chi}^2 - m_{\text{eff}}^2}, \tag{12}
\]

where \( N \) is maximal integer, for which \( \tilde{\chi}^2 - m^2 - 2eBN \geq 0 \), and take into account the current in Eq. (12) in the Maxwell equations along with the usual ohmic current \( \mathbf{J} = \sigma_{\text{cond}} \mathbf{E} \), where \( \sigma_{\text{cond}} \) is the matter conductivity and \( \mathbf{E} \) is the electric field.
Considering the magnetohydrodynamic approximation, which reads $\sigma_{\text{cond}} \gg \omega$, where $\omega$ is the typical frequency of the electromagnetic fields variation, we derive the modified Faraday equation for the magnetic field evolution,

$$\frac{\partial \mathbf{B}}{\partial t} = \frac{1}{\sigma_{\text{cond}}} \nabla \times (\Pi \mathbf{B}) + \frac{1}{\sigma_{\text{cond}}} \nabla^2 \mathbf{B},$$

(13)

where we neglect the coordinate dependence of $\sigma_{\text{cond}}$.

Let us consider the evolution of the magnetic field given by the Chern-Simons wave, corresponding to the maximal negative magnetic helicity, $\mathbf{B}(z,t) = \mathbf{B}(t) \left( e_x \cos kz + e_y \sin kz \right)$, where $k = 1/L$ is the wave number determining the length scale of the magnetic field $L$ and $B(t)$ is the wave amplitude which can depend on time. In this situation we can neglect the coordinate dependence of $\Pi$ in Eq. (13) and the equation for $B$ takes the form,

$$\dot{B} = -\frac{k}{\sigma_{\text{cond}}} (k + \Pi) B.$$

(14)

Since $\Pi$ in Eq. (12) is negative, the magnetic field described by Eq. (14) can be unstable since $\dot{B} > 0$.

We shall apply Eq. (12) to describe the magnetic field amplification in a dense degenerate matter which can be found in NS. In this situation, $n_n = 1.8 \times 10^{38} \text{ cm}^{-3}$ and $n_p \ll n_n$. Using Eq. (3), one gets that $V_5 = G_F n_n / 2\sqrt{2} = 6 \text{ eV}$. The number density of electrons can reach several percent of the nucleon density in NS. We shall take that $n_e = 9 \times 10^{36} \text{ cm}^{-3}$, which gives one $\chi = (3\pi^2 n_e)^{1/3} = 125 \text{ MeV}$ [12]. Thus electrons are ultrarelativistic and we can take that $\tilde{\chi} \approx \chi$. We shall study the magnetic field evolution in NS in the time interval $t_0 < t < t_{\text{max}}$, where $t_0 \sim 10^2 \text{ yr}$ and $t_{\text{max}} \sim 10^6 \text{ yr}$. In this time interval, NS cools down from $T_0 \sim 10^8 \text{ K}$ mainly by the neutrino emission [13]. In this situation, the matter conductivity in Eq. (14) becomes time dependent $\sigma_{\text{cond}}(t) = \sigma_0 (t/t_0)^{1/3}$ [12], where $\sigma_0 = 2.7 \times 10^6 \text{ GeV}$.

We shall discuss the amplification of the seed magnetic field $B_0 = 10^{12} \text{ G}$, which is typical for a young pulsar. In such strong magnetic fields, the anomalous magnetic moment of an electron was found in Refs. [14,15] to depend on the magnetic field strength. We can approximate $\mu$ as

$$\mu = \frac{e}{2m} \frac{\alpha_{\text{em}}}{2\pi} \left( 1 - \frac{B}{B_c} \right),$$

(15)

where $B_c = m^2/e = 4.4 \times 10^{13} \text{ G}$. Note that Eq. (15) accounts for the change of the sign of $\mu$ at $B \approx B_c$ predicted in Ref. [14].

The evolution of the magnetic field for the chosen initial conditions is shown in Fig. 1 for different length scales. One can see that, if the magnetic field is enhanced from $B_0 = 10^{12} \text{ G}$, it reaches the saturated strength $B_{\text{sat}} \approx 1.3 \times 10^{13} \text{ G}$. Thus, both quenching factors in Eqs. (12) and (15) are important. One can see in Fig. 1 that a larger scale magnetic field grows slower. The further enhancement of the magnetic field scale compared to $L = 10^5 \text{ cm}$, shown in Fig. 1(b), is inexpedient since the growths time would significantly exceed $10^6 \text{ yr}$.
At such long evolution times, NS cools down by the photon emission from the stellar surface rather than by the neutrino emission [13].

The energy source, powering the magnetic field growth shown in Fig. 1, can be the kinetic energy of the stellar rotation. To describe the energy transmission from the rotational motion of matter to the magnetic field one should take into account the advection term $\nabla (v \times B)$ in the right hand side of Eq. (13). Here $v$ is the matter velocity.

Moreover one should assume the differential rotation of NS [16]. The NS spin-down because of the magnetic field enhancement can be estimated basing on the conservation of the total energy of a star: $I\Omega^2/2 + B^2V/2 = \text{const}$, where $I$ is the moment of inertia of NS, $\Omega$ is the angular velocity, and $V$ is the NS volume.

Taking the NS radius $R \sim 10$ km and the initial rotation period $P_0 \sim 10^{-3}$ s, we get for $B_{\text{sat}} \approx 1.3 \times 10^{13}$ G, shown in Fig. 1, that the relative change of the period is $(P - P_0)/P_0 \sim 10^{-9}$. Hence only a small fraction of the initial rotational energy is transmitted to the energy of a growing magnetic field.

The obtained results can be used for the explanation of electromagnetic flashes emitted by magnetars within the recently proposed model of a thermoplastic wave [17], which can be excited by small-scale, with $L \sim (10^2 - 10^3)$ cm, fluctuations of the magnetic field having the strength $B \gtrsim 10^{13}$ G [8]. The evolution of fields with such characteristics is shown in Fig. 1.

In conclusion it is interesting to compare the appearance of the new current along the magnetic field in Eq. (10) with CME [2], which is known to get the contribution only from massless electrons at the zero Landau level in an external magnetic field. Since left electrons move along the magnetic field and right particles move in the opposite direction, the current in Eq. (11) is nonzero until there are different populations of the zero Landau level by left and right particles, i.e. $\mu_R \neq \mu_L$. Electrons at higher Landau levels can move arbitrarily with respect to the magnetic field. Therefore, CME is caused by an asymmetric motion of massless particles along the external magnetic field.

If we consider massive electrons with nonzero anomalous magnetic moments, elec-
troweakly interacting with background matter, then, unlike CME, the motion of such particles at the lowest energy level with \( n = 0 \) is symmetric with respect to the magnetic field, i.e. \( -\infty < p_z < +\infty \) for them (one can see it if we replace \( p_z \to p_z - V_5 \) in Eq. (7)). On the contrary, higher energy levels with \( n > 0 \) in Eq. (6) are not symmetric with respect to the transformation \( p_z \to -p_z \). The reflectional symmetry cannot be restored by any replacement of \( p_z \). Therefore electrons having \( p_z > 0 \) and \( p_z < 0 \) will have different energies and hence different velocities \( v_z = p_z/\mathcal{E} \). Thus \( J_z \sim \langle v_z \rangle \neq 0 \), with only higher energy levels contributing to it. It is interesting to mention that the term in Eq. (9), which violates the reflectional symmetry \( p_z \to -p_z \), is proportional to \( \mu_B m V_5 \). It is this factor which \( J_z \) in Eq. (10) is proportional to. Thus, one can see that the nonzero current \( \mathbf{J} \sim \mathbf{B} \) results from the asymmetric motion of particles along \( \mathbf{B} \). This asymmetry is caused by the simultaneous presence of three factors: nonzero \( m \) and \( \mu \), as well as the electroweak interaction with background matter \( \sim V_5 \).

It should be noted that, in addition to the electroweak interaction between electrons and background fermions, taken into account in Eqs. (2) and (3), and leading to an electric current \( \mathbf{J} \sim \mathbf{B} \) in Eq. (12), electrons also interact electromagnetically with background protons and neutrons. Considering, for definiteness, the electromagnetic interaction between electrons and a homogeneous gas of non-moving protons with a constant density, we find that the following additional term appears in the left hand side of Eq. (2): \( \sim [\cdots + e^2 \gamma^0 f_0] \psi \), where \( f_0 \sim n_p/\omega_p^2 \) is a quantity proportional to the zero component of the proton current and \( \omega_p \) is the plasma frequency in the considered matter.

The strength of the electromagnetic interaction is much higher than that of the electroweak interaction, \( e^2 f_0 \gg G_F n_p \), since, in the degenerate matter, one has \( \omega_p^2 \sim \alpha_{\text{em}} \chi^2 \) and \( e^2 f_0 \sim 10^2 \text{GeV}^{-2} \times n_p \) for \( \chi \sim 10^2 \text{MeV} \) (see above). Nevertheless, the additional contribution of the electromagnetic interaction in Eq. (2) can be eliminated by the gauge transformation \( \psi \to \psi' = \exp(-ie^2 f_0 t)\psi \) in case of matter with constant density, \( f_0 \sim n_p = \text{const.} \). The contribution of the electroweak interaction in Eq. (2), \( \gamma^0 (V_R P_R + V_L P_L) \to \gamma^0 \gamma^5 V_5 \) can not be eliminated by any gauge transformation because of the presence of the matrix \( \gamma^5 \) indicating the parity violation in electroweak interactions. Similarly, it can be shown that the electromagnetic interaction between electrons and neutrons, due to the presence of the magnetic form factor of neutrons \( f_1 \), does not give rise to a current \( \mathbf{J} \sim \mathbf{B} \) in the case of a homogeneous, unpolarized neutron matter with a constant density.

The necessity of the presence of the contribution of a parity violating interaction in the generation of the current \( \mathbf{J} = \Pi \mathbf{B} \) in Eq. (12) in the system of massive fermions follows also from the fact that the parameter \( \Pi \) should be a pseudoscalar. Electromagnetic interaction is known to be parity conserving. That is why it does not contribute to \( \Pi \) in the system in question.

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