A Matrix Model for Heterotic Spin(32)/Z₂ and Type I String Theory.

Morten Krogh

Department of Physics, Jadwin Hall
Princeton University
Princeton, NJ 08544, USA
krogh@princeton.edu

We consider Heterotic string theories in the DLCQ. We derive that the matrix model of the Spin(32)/Z₂ Heterotic theory is the theory living on N D-strings in type I wound on a circle with no Spin(32)/Z₂ Wilson line on the circle. This is an O(N) gauge theory. We rederive the matrix model for the E₈ × E₈ Heterotic string theory, explicitly taking care of the Wilson line around the lightlike circle. The result is the same theory as for Spin(32)/Z₂ except that now there is a Wilson line on the circle. We also see that the integer N labeling the sector of the O(N) matrix model is not just the momentum around the lightlike circle, but a shifted momentum depending on the Wilson line. We discuss the aspect of level matching, GSO projections and why, from the point of view of matrix theory the E₈ × E₈ theory, and not the Spin(32)/Z₂, develops an 11’th dimension for strong coupling. Furthermore a matrix theory for type I is derived. This is again the O(N) theory living on the D-strings of type I. For small type I coupling the system is 0+1 dimensional quantum mechanics.
1. Introduction

A matrix model for the $E_8 \times E_8$ Heterotic string theory has been developed over the past year \[1,2,3,4,5,6,7,8\]. The model can be derived by following Seiberg’s and Sen’s prescription \[9,10\] for M-theory on $S^1/Z_2$. The result is that Heterotic $E_8 \times E_8$ in the sector $N$ of DLCQ is described by the decoupled theory of $N$ D-st rings in type I wound on a circle with a Wilson line on the circle breaking $SO(32)$ to $SO(16) \times SO(16)$. It is important to note that this matrix theory is a description of the $E_8 \times E_8$ Heterotic string with a Wilson line on the lightlike circle, which breaks $E_8 \times E_8$ down to $SO(16) \times SO(16)$. Later we will see that the integer $N$ is not the pure momentum around the lightlike circle but a shifted momentum depending on winding and $E_8 \times E_8$ charges.

In this paper we will follow Seiberg’s prescription for the Spin$(32)/Z_2$ Heterotic string theory in the DLCQ and derive a matrix description of it. This time we do not take a Wilson line on the lightlike circle. The resulting matrix model is again the theory of $N$ D-strings in type I wound on a circle but this time without a Wilson line. Since type I and Heterotic $SO(32)$ are dual in ten dimensions this also gives a matrix model for the type I string theory.

The organization of the paper is as follows. In section 2 we derive the matrix model for the $SO(32)$ Heterotic string. In section 3 we rederive along similar lines the well known result for the $E_8 \times E_8$ Heterotic string. In the process we will get an understanding of the integer $N$ labeling the rank of the matrices. In section 4 we will discuss various aspects of the two Heterotic matrix models, such as level matching, GSO projections and why the $E_8 \times E_8$ theory and not the $SO(32)$ develops an extra dimension for strong coupling. In section 5 we will use the type I - Heterotic $SO(32)$ duality to derive a matrix model for type I. It turns out that type I perturbation theory is described by a 0+1 dimensional quantum mechanics matrix model. We will briefly describe an intuitive picture for understanding type I string theory.

We were informed that a different approach to a matrix model of type I and Heterotic Spin$(32)/Z_2$ string theory is being taken in a forthcoming paper \[11\].

2. The Spin$(32)/Z_2$ Heterotic string

Let us consider the $SO(32)$ Heterotic string with string scale $M$ and coupling $\lambda$. We compactify it on a lightlike circle of radius $R$ with no Wilson line and consider the sector with momentum $N$. We really take this to mean that the theory is compactified on an almost lightlike circle as explained in \[10\]. By a boost and a rescaling of the mass scale
this takes us to a spatial compactification on a circle of length \( R_s \) and string scale \( m_s \). We send \( R_s \to 0 \) keeping
\[
m^2_s R_s = M^2 R
\] (2.1)
The momentum around the circle is \( \frac{N}{\pi} \). The resulting theory in the limit is our answer. We will now apply various well established string dualities to obtain a simple description of the answer.

First we perform a T-duality on the circle. This is simple since there is no Wilson line. The momentum number \( N \) turns into fundamental string winding number \( N \). The T-dual theory has coupling \( \lambda' \), string scale \( m'_s \) and radius \( R'_s \) given by
\[
m'_s = m_s
\]
\[
R'_s = \frac{1}{m^2_s R_s}
\]
\[
\lambda' = \frac{\lambda}{m^2_s R_s}
\] (2.2)
Next we employ the ten dimensional type I - SO(32) Heterotic duality which turn fundamental Heterotic strings into D-strings of type I. We thus obtain a type I theory with \( N \) D-strings wound on a circle. The coupling \( \lambda'' \), string scale \( m''_s \) and radius \( R''_s \) are
\[
m''_s = \frac{m'_s}{\sqrt{\lambda'}} = \frac{m^3_s}{r^2} R^2_s \lambda^{-\frac{1}{2}}
\]
\[
R''_s = R'_s = \frac{1}{m^2_s R_s} = \frac{1}{M^2 R}
\]
\[
\lambda'' = \frac{1}{\lambda'} = \frac{m^2_s R_s}{\lambda}
\] (2.3)
There is still no Wilson line on the circle. We see that in the limit \( R_s \to 0 \) with \( m^2_s R_s \) fixed
\[
\lambda'' \to 0
\]
\[
m''_s \to \infty
\] (2.4)
This is exactly the limit in which the theory on the D-strings decouple from the bulk and is described by a SYM theory with \( (0,8) \) supersymmetry. This theory is well known to be an O(N) gauge theory with the following field content
1. 8 scalars \( X^i \) \( i = 1, \ldots, 8 \) in the symmetric part of the \( N \otimes N \) of O(N) and in the vector representation of the transverse spin(8).
2. 8 rightmoving fermions \( \Theta_+ \) in the symmetric part of \( N \otimes N \) of O(N) and in the spinor \( 8_c \) of spin(8).
3. The gauge field $A_\mu$ in the adjoint of O(N).
4. 8 leftmoving fermions $\lambda_-$ in the adjoint of O(N) and 8 of spin(8).
5. Leftmoving fermionic fields, $\chi$, in the fundamental of O(N) and fundamental of the global SO(32).

The $\chi$ come from 1-9 strings. The rest come from 1-1 strings. The gauge coupling is

$$\frac{1}{g^2} = \frac{\lambda^2}{M^4 R^2}$$ (2.5)

The action is standard and can be found in [4].

3. Heterotic $E_8 \times E_8$ String theory

In this section we will derive the well known result for the matrix model of the $E_8 \times E_8$ Heterotic string in the DLCQ. We will follow a similar route to the SO(32) case. We take the theory on a lightlike circle of radius $R$ with mass scale $M$ and coupling $\lambda$. This time we put a Wilson line along the lightlike circle. We take the one which breaks $E_8 \times E_8$ down to $SO(16) \times SO(16)$. We put momentum $\frac{N}{\pi}$ along the circle. By a boost and rescaling we get to a spatial compactification with radius $R_s$ and mass scale $m_s$. Again we take $R_s \to 0$ with

$$m_s^2 R_s = M^2 R.$$ (3.1)

The spatial circle now has the Wilson line.

We do not think of the $E_8 \times E_8$ theory as M-theory on $S^1/Z_2$ as is usually done. The problem with this approach is that with the present knowledge it is impossible to treat the Wilson line rigorously. By shrinking the spatial circle M-theory on $S^1/Z_2$ turns into type IA theory. However type IA requires the specification of the positions of the 8-branes. Obviously these positions are determined by the Wilson line on the shrinking spatial circle, but the exact rules for this are not known yet and one is forced to the reasonable guess that the 8-branes are split evenly between the 2 orientifold planes.

Instead of regarding $E_8 \times E_8$ Heterotic theory as M-theory on $S^1/Z_2$ we will use string dualities to obtain the result. First we apply a T-duality into the SO(32) Heterotic string on a circle with a Wilson line breaking SO(32) to $SO(16) \times SO(16)$. This theory has coupling $\lambda'$, string scale $m'_s$ and radius $R'_s$ given by [12,13]

$$m'_s = m_s,$$

$$R'_s = \frac{1}{2m_s^2 R_s},$$

$$\lambda' = \frac{\lambda}{\sqrt{2m_s R_s}}.$$ (3.2)

1 Pure numbers and factors of $\pi$ are mostly ignored in this paper
Note the extra factor of 2 compared to usual $R \rightarrow \frac{1}{m_s^2 R}$ T-duality.

We expect a T-duality to take momentum around the circle into fundamental string winding. However that is not completely correct in this case. The Heterotic $E_8 \times E_8$ theory on a circle is described by a lattice of signature (17,1). States in the theory are labelled by points in the lattice. A point in the lattice can be specified by the momentum around the circle, $N$, winding, $m$, and a lattice vector, $P$, in the weight lattice of $E_8 \times E_8$. Furthermore let $A$ be the vector in $\mathbb{R}^{16}$ specifying the Wilson line on the circle. Here $\mathbb{R}^{16}$ is to be thought of as the vector space spanned by the $E_8 \times E_8$ lattice. The result of T-duality is that the winding number, $\tilde{N}$, in the SO(32) string theory is

$$\tilde{N} = 2(N - m - A \cdot P) \quad (3.3)$$

The factor of two is related to the extra factor of two in (3.2). $\tilde{N}$ is always an integer because $A \cdot P$ is either integer or half-integer. $A \cdot P$ is integer and $\tilde{N}$ is even if $P$ is a weight such that the state is in a true representation (not spinor) of either both SO(16)'s or none of them. Otherwise $A \cdot P$ is half-integer and $\tilde{N}$ is odd. For instance the adjoint of $E_8 \times E_8$ splits under $SO(16) \times SO(16)$ into $(120,1) + (1,120) + (128,1) + (1,128)$. The two first terms correspond to states with even $\tilde{N}$ and the last two to states with odd $\tilde{N}$.

Now we again employ Heterotic SO(32)- type I duality to get the type I theory with $\tilde{N}$ D-strings wound on a circle of radius $R''_s$, string scale, $m''_s$, and coupling, $\lambda''$ given by

$$m''_s = \frac{m'_s}{\sqrt{\lambda'}} = 2^{\frac{1}{2}} m_s^2 R_s^2 \lambda^{-\frac{1}{4}}$$

$$R''_s = R'_s = \frac{1}{2m_s^2 R_s} = \frac{1}{2M^2 R}$$

$$\lambda'' = \frac{1}{\lambda'} = \sqrt{2m_s R_s}$$

There is still a Wilson line on the circle breaking the type I SO(32) to $SO(16) \times SO(16)$. The limit $R_s \rightarrow 0$ works as before

$$\lambda'' \rightarrow 0$$

$$m''_s \rightarrow \infty$$

and the theory on the $\tilde{N}$ D-strings decouple. We get almost the same theory as in section 2. The field content is the same and the gauge group is $O(\tilde{N})$. The difference is the Wilson line which now means that half of the 32 fermions have opposite boundary conditions on going around the circle. The fermions, $\chi$, transform in the $(\tilde{N},32)$ of $O(\tilde{N}) \times SO(32)$. There is a dynamical $O(\tilde{N})$ gauge field, $A_\mu$. The SO(32) holonomy is fixed and it multiplies half of the fermions, $\chi$, by -1 on going once around the circle.
In going through these dualities we have recovered the well-known result for this matrix model. However we now understand better the role of the integer $\tilde{N}$ in $O(\tilde{N})$. It is not equal to the momentum around the lightlike circle but a generalization of it given by eq. (3.3). This also explains the observation in [1], that $E_8 \times E_8$ gauge transformations relate sectors with different values of $\tilde{N}$. In Seiberg’s prescription states with negative momentum around the small spatial circle decouple. The equivalent criterion for decoupling is here $\tilde{N} < 0$. Especially we see that states with negative momentum $N$ can have $\tilde{N} > 0$ if $m$ and $P$ are chosen properly. This result about the change of interpretation of the integer labeling a sector have also been noted by [14] in the case of M-theory on a torus.

4. Aspects of the Heterotic theories

In this section we will discuss some aspects of the theories of the previous sections. Let us first discuss level matching. Because of S-duality a D-string behaves like a fundamental string. The D-strings in question here are wound on a circle, and we are describing them in static gauge. This corresponds to a fundamental string wound on a circle. Let us review the level matching condition for a wound fundamental string. Let the string have winding number, $n$, and momentum number, $m$. The oscillator number on the rightmoving side is called $M$ and on the leftmoving side $\bar{M}$. These include possible zero point energies. Let the circle have radius $R$ and the string mass be $m_s$. Now the right- and leftmoving Virasoro generators are

$$L_0 = \frac{1}{4} \left( \frac{m}{R} - nm_s^2 R \right)^2 + Mm_s^2$$

$$\bar{L}_0 = \frac{1}{4} \left( \frac{m}{R} + nm_s^2 R \right)^2 + \bar{M}m_s^2$$

The level matching condition is now $L_0 = \bar{L}_0$. This can also be written

$$M - \bar{M} = mn$$

The energy of such a state is $4L_0$. The situation we are interested in is where the winding string (the D-string) has a tension that goes to infinity and $R$ is fixed. Here the main contribution to the energy comes from the winding. We are interested in the total energy minus the winding energy. This difference is the DLCQ $\frac{m^2}{2R}$. This energy is easily calculated to be

$$\text{Energy} - \text{winding energy} = \frac{\bar{M} + M}{nR}$$

We see from eq. (4.2), that in static gauge, the level matching condition for a single wound D-string is that $M - \bar{M}$ is an integer. For an $n$ times wound string this difference should be divisible by $n$. 

5
Let us first look at the $\text{Spin}(32)/\mathbb{Z}_2$ Heterotic string in the sector $N = 1$. The matrix model for this one is the D-string wound on a circle with no SO(32) Wilson line. There are two sectors to consider corresponding to the O(1) holonomy around the circle. In one sector the fermions $\chi$ are periodic, in the other they are antiperiodic. Let us calculate the zero point energies. The rightmovers are easy. Here the bosons and fermions contribute equally but opposite so the zero point energy is zero. This means that $M$ is a non negative integer. On the non-supersymmetric leftmoving side there are 8 bosonic fields $X^i$. There are also 32 fermions, $\chi$. We remember that the zero point energy is $\frac{1}{24}$ for a periodic boson, $\frac{1}{48}$ for a periodic fermion and $\frac{1}{48}$ for an antiperiodic fermion. Let us first calculate the zero point energy in the periodic sector. It is easily seen to be 1. This means that $\bar{M}$ is an integer, which is at least 1. We see that from this sector we do not get any massless states. In the antiperiodic sector we get a zero point energy of -1. This means that $\bar{M}$ is at least -1. Furthermore it follows from eq.(4.2) that $\bar{M}$ is an integer. In other words we have to excite an even number of $\chi$ oscillators. This also follows from O(1) gauge invariance. In the antiperiodic sector we have massless states. We can excite either one $X$ oscillator or two $\chi$ oscillators. These states are the usual states of Heterotic SO(32) string theory, the gravitons and gluons. Furthermore there is also a state with $\bar{M} = -1$. This state has negative energy according to eq.(4.3). It is easy to dualize back to the original SO(32) Heterotic string theory to see what it corresponds to. It is a wound string on the lightlike circle.

Let us now go to $N > 1$ As discussed in [15], [16] and [17] the matrices can configure themselves into long strings. Suppose we have a long D-string with winding number $N$. Level matching now requires the difference $M - \bar{M}$ to be divisible by $N$. States with $M - \bar{M} = 0$ have energies that go as $\frac{1}{N}$ as can be seen from eq.(4.3). States with other values of $M - \bar{M}$ have energies of order one with respect to $N$. This is good, if we hope to recover the full ten dimensional theory for $N \to \infty$, since this shows that wound strings on the lightlike circle decouples in the large $N$ limit. The negative energy state is also abandoned since -1 is not divisible by $N$. So for long strings the low energy states correspond to particle states in ten dimensional SO(32) Heterotic string theory and there are no negative energy states. Similar remarks apply to the $E_8 \times E_8$ case.

Another aspect to discuss is the GSO projection. For the SO(32) case there is no problem. The GSO projection for $N=1$ is simply given by the element $-1 \in O(1)$. This element multiplies all $\chi$ by -1 and leaves the other fields invariant. This is exactly how the GSO projection works for the SO(32) Heterotic string. For larger $N$ the GSO projection is similarly imposed by gauge invariance.

For the Heterotic $E_8 \times E_8$ string there are two GSO projections. One for each set of 16 $\chi$. The element $-1 \in O(1)$ only takes care of one of these. It multiplies all 32 $\chi$
by -1. We still need another one. As noted in [8] this would be solved by level matching for odd $\tilde{N}$. This is because the rightmovers always have an integer number of excitations. Therefore there must be an even number of excitations of the antiperiodic leftmoving fermions. For a long string composed of an even number $\tilde{N}$ of D-strings we need a GSO projection that distinguishes the two sets of fermions. They are both periodic on going all the way around the long string. However a physical state still has to be invariant under worldsheet translations around the circle once. One set of fermions are antiperiodic under this translation. There has to be an even number of excitations of these. This is the origin of the GSO projection for even $\tilde{N}$. A discussion of this point can be found in [18]. If there had been no SO(32) Wilson line the level matching condition eq.(4.2) would have been enough to make the state invariant under one translation around the circle, because the $\tilde{N}$ units of momentum around the long string are evenly distributed with one unit per single circle. With the SO(32) Wilson line this is not the case automatically and we need the extra condition that an even number of the sets of fermions, which are antiperiodic on going once around the circle, are excited. Together with the gauge invariance mentioned above this implies the full GSO condition.

Let us now discuss how we can see from the matrix model that the $E_8 \times E_8$ theory develops an 11'th dimension at strong coupling, whereas the SO(32) theory does not. Both matrix models have 8 fields X, which correspond to the transverse dimensions. Then there is time and the lightlike circle. For the SO(32) case there should be no more dimensions. For the $E_8 \times E_8$ we expect an interval whose length grows with the string coupling, $\lambda$. It is clear that this extra dimension comes from the O(N) gauge field, $A_\mu$. A Wilson line around the circle corresponds to position in the 11'th direction. With a O(N) Wilson line the charged fields in the 1+1 dimensional theory change boundary conditions around the circle. This will change the frequency of the associated oscillators and hence the zero point energy. This zero point energy acts as an induced potential for the Wilson line. The difference between the SO(32) and $E_8 \times E_8$ case is that for the former the potential locks the Wilson line at a special value, but for the latter the potential is flat.

Let us calculate some zero point energies to verify this picture. The rightmovers are easy. Here the bosons and fermions contribute equally but opposite so the zero point energy is zero. We can apply oscillator raising operators to increase the energy. The increase depends on the periodicity of the oscillator which is determined by the Wilson line. On the non-supersymmetric leftmoving side there are 8 bosonic fields $X^i$ in the symmetric part of $N \otimes N$ and 8 fermions $\lambda_-$ in the adjoint of O(N). They cancel each other except for 8N bosonic fields. There are also 32 fermions, $\chi$, in the N of O(N). We remember that the zero point energy (in units of $\frac{1}{R_\sigma}$ for a wound string) is $\frac{1}{24}$ for a periodic boson, $\frac{1}{24}$ for a periodic fermion and $\frac{1}{48}$ for an antiperiodic fermion.
Let us first calculate the zero point energy with no Wilson line, $A_\mu = 0$. For the $E_8 \times E_8$ case there are $8N$ periodic bosons, $16N$ periodic fermions and $16N$ antiperiodic fermions giving a total zero point energy of zero. For the $SO(32)$ case we have $8N$ periodic bosons and $32N$ periodic fermions giving a total zero point energy of $N$.

Let us now put the Wilson line $-1 \in O(N)$ around the circle. This changes the periodicity of the fermions. For the $E_8 \times E_8$ case we still get zero. For the $SO(32)$ case we now get $-N$. A more generic Wilson line will just give a result in between these two extreme Wilson lines. This is exactly what we expected. For the $E_8 \times E_8$ case it is always zero. For the $SO(32)$ case it is smallest when the Wilson line is locked at $-1$. This state has negative energy and is the string wound on the lightlike circle as discussed above. It disappears when working with long strings instead. To get massless states we have to raise some leftmoving oscillators. We can apply one $X$ oscillator or two $\chi$ oscillators. The last states are the gluons in the adjoint of $SO(32)$.

There is a more pictorial way of understanding all this, namely the T-dual version. Here the Wilson lines turn into positions of 0-branes on the interval in type IIA. This system has been studied by [19,20]. It was shown, by considering zero point energies for instance, that 8-branes repel 0-branes. For the $E_8 \times E_8$ case the 8-branes are split evenly among the two orientifold planes so the 0-branes feel no force. In the $SO(32)$ case all 8-branes are gathered in one end (the rightmost) of the interval. The 0-branes will then be repelled to the other end (leftmost end). The excited $\chi$ oscillators correspond to strings stretched from the 0-branes to the 8-branes in the other end. The excited $X$ oscillators correspond to strings winding once on the interval, starting from a 0-brane and ending on a 0-brane. It is clear from this picture that two $\chi$ oscillators have the same energy as one $X$ oscillator.

5. Type I Matrix Model

Since type I and $SO(32)$ Heterotic string theory are dual in ten dimensions we also have a matrix model for type I. Let us start with type I with string scale $m_I$ and coupling $\lambda_I$ on a lightlike circle of radius $R$. This is dual to $SO(32)$ Heterotic theory with radius $R$, coupling $\lambda = \frac{1}{\lambda_I}$ and string scale $M = m_I \lambda_I^{-\frac{1}{2}}$. The matrix model for type I is thus again the theory of $N$ D-strings wound on a circle with no Wilson line. The parameters of the type I theory where the D-strings live are given by substitution in eq.(2.3). Like before

$$\lambda'' \to 0$$

$$m_s'' \to \infty$$

$$R_s'' = \frac{\lambda_I}{m_I^2 R}$$

(5.1)
The gauge coupling is given from eq. (2.5),
\[ \frac{1}{g^2} = \frac{1}{m^2 I R^2}. \] (5.2)

The matrix model is thus a 1+1 dimensional O(N) theory on a circle of radius \( \frac{\lambda}{m^2 R}. \) Especially momentum around the circle has energy \( \frac{m^2 R}{\lambda I}. \) This means, naively at least, that perturbative type I theory is reproduced by the dimensional reduction of this model to 0+1 dimensions. This is a quantum mechanics system!

In the T-dual version the restriction to quantum mechanics means that we do not consider strings that go from one end of the interval to the other except for those needed to fulfill level matching for a long string. We remember there were two ways of doing this, either exciting one \( X^i \) oscillator or two \( \chi \) oscillators. In the type IA picture exciting an \( X^i \) oscillator means exciting a string stretched around the interval coming back to the 0-branes. The \( \chi \) oscillator corresponds to a string stretched from a 0-brane to an 8-brane in the other end. We need two of these. It is astonishing how much this resembles type I perturbation theory. The first kind of 0-brane corresponds to a closed string. The second kind to an open string with Chan-Paton factors. From this picture it is clear that a closed string can open up and become an open string. The fact that we need to excite oscillators mean that the theory is not quite the naive dimensional reduction. One has to keep track of other oscillators too. Hopefully it is still tractable.

Let us have a look at the sector N=1. Here we get the states \( (8_v + 8_c) \otimes 8_v \) from the “closed string sector” and \( 8_v + 8_c \) in the adjoint of SO(32) from the open string sector (we discarded the negative energy state which is absent for a long string anyway). These are the massless states in type I. We see that we do not get any massive states in type I. This is not an immediate contradiction since massive states are unstable for finite coupling and are not asymptotic states. Exciting more oscillators we get states with energies that go like \( \frac{1}{\lambda I}. \) These correspond to D-strings of type I. It would be interesting to see from this picture of type I why fundamental strings can end on D-strings.

6. Conclusions

We have seen how the two Heterotic strings have a matrix model description as a 1+1 dimensional O(N) gauge theory. This was derived for very special Wilson lines. It would be very interesting to understand the general case. It might be related to the recent discoveries by Connes, Douglas, Hull and Schwarz [21,22]. We saw how the integer N of the matrix model was not just momentum around the lightlike circle but a generalized
momentum eq. (3.3). For general Wilson lines it is hard to get an integer out of a formula containing the Wilson line. It would be interesting to figure out what matrix models with non-integer N means, if they exist.

We also saw how the type I theory is described by a 1+1 dimensional O(N) gauge theory. For small coupling it reduces to a quantum mechanical problem. There is however a problem in that we have to keep an excited oscillator. This means a naive dimensional reduction to 0+1 dimensions is too simple. We have to keep a finite number of excited oscillators. It is still a quantum mechanical system however. It would be very interesting to do a scattering calculation and compare to traditional type I results.

So far the only matrix model which is 0+1 dimensional is the original model of M-theory in the DLCQ [23]. One can do scattering calculations there but the problem is that there is only supergravity to compare with. It is not clear whether the system in the DLCQ is describable by supergravity and possible disagreements might be because we are outside the regime of validity of supergravity in this special kinematical situation. In the type I model the situation is seemingly better in this respect since we have a string theory to compare with. Unfortunately it might not be better, since type I on a small circle is not perturbative [24]. So it might not be possible to compare with a string calculation. Certainly the arguments in this paper imply that the 1+1 dimensional model describes type I in the DLCQ, where DLCQ is defined as a limit of an almost lightlike circle. For small \( \lambda_I \) the model becomes 0+1 dimensional. However defining the DLCQ as a limit of an almost lightlike circle the theory is Lorentz equivalent to type I on a very small circle. This is not perturbative for small \( \lambda_I \), so we have not derived agreement between perturbative type I and the 0+1 dimensional matrix model described here for finite N. Following the philosophy of matrix theory so far we could hope that for large N all traces of the lightlike compactification disappears and type I with a small coupling would be perturbative. In this case the result would be that perturbative type I is reproduced by a large N quantum mechanics. We saw one hint of the disappearance of the lightlike circle for large N, namely the Heterotic string wound on the lightlike circle was not present for a long D-string in the Matrix model, even though it was present for a single D-string.

Acknowledgments

I wish to thank Ori Ganor for fruitful discussions and Jan Ambjorn for hospitality at the Niels Bohr Institute. This work was supported by The Danish Research Academy.
References

[1] Shamit Kachru and Eva Silverstein, “On Gauge Bosons in the Matrix Model Approach to M Theory,” hep-th/9612162
[2] Nakwoo Kim and Soo-Jong Rey, “M(atrix) Theory on an Orbifold and Twisted Membrane,” hep-th/9701139
[3] Tom Banks and Lubos Motl, “Heterotic Strings from Matrices,” hep-th/9703218
[4] David A. Lowe, “Heterotic Matrix String Theory,” hep-th/9704041
[5] Soo-Jong Rey, “Heterotic M(atrix) Strings and Their Interactions,” hep-th/9704158
[6] Petr Horava, “Matrix Theory and Heterotic Strings on Tori,” hep-th/9705055
[7] Suresh Govindarajan, “Heterotic M(atrix) theory at generic points in Narain moduli space,” hep-th/9707164
[8] Lubos Motl and Leonard Susskind, “Finite N Heterotic Matrix Models and Discrete Light Cone Quantization,” hep-th/9708083
[9] Ashoke Sen, “D0 Branes on T^n and Matrix Theory,” hep-th/9709220
[10] N. Seiberg, “Why is the Matrix Model Correct?,” hep-th/9710009
[11] Soo-Jong Rey and Savdeep Sethi, to appear.
[12] K. Narain, M. Sarmadi and E. Witten, Nucl. Phys. B279 (1987) 369.
[13] P Ginsparg, Phys. Rev. D35 (1987) 648.
[14] N.A. Obers, B. Pioline and E. Rabinovici, “M-theory and U-duality on T^d with Gauge Backgrounds,” hep-th/9712084
[15] Lubos Motl, “Proposals on nonperturbative superstring interactions,” hep-th/9701025
[16] Tom Banks and Nathan Seiberg, “Strings from Matrices,” hep-th/9702187
[17] R. Dijkgraaf, E. Verlinde and H. Verlinde, “Matrix String Theory,” hep-th/9703030
[18] Tom Banks, “Matrix Theory,” hep-th/9710231
[19] Ulf Danielsson, Gabriele Ferretti and Igor R. Klebanov, “Creation of Fundamental strings by crossing D-branes,” hep-th/9705084
[20] Ulf Danielsson and Gabriele Ferretti, “Creation of strings in D-particle Quantum Mechanics,” hep-th/9709171
[21] Alain Connes, Michael R. Douglas and Albert Schwarz, “Noncommutative Geometry and Matrix Theory: Compactification on Tori,” hep-th/9711162
[22] Michael R. Douglas and Chris Hull, “D-branes and the Noncommutative Torus,” hep-th/9711165
[23] T. Banks, W. Fischler, S.H. Shenker and L. Susskind, “M Theory As A Matrix Model: A Conjecture,” hep-th/9610043
[24] Joseph Polchinski and Edward Witten, “Evidence for Heterotic- Type I String Duality,” hep-th/9510169