Observational Semantics of the Prolog
Resolution Box Model

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Abstract. This paper specifies an observational semantics and gives an original presentation of the Byrd box model. The approach accounts for the semantics of Prolog tracers independently of a particular Prolog implementation.

Prolog traces are, in general, considered as rather obscure and difficult to use. The proposed formal presentation of its trace constitutes a simple and pedagogical approach for teaching Prolog or for implementing Prolog tracers. It is a form of declarative specification for the tracers.

The trace model introduced here is only one example to illustrate general problems relating to tracers and observing processes. Observing processes know, from observed processes, only their traces. The issue is then to be able to reconstitute, by the sole analysis of the trace, part of the behaviour of the observed process, and if possible, without any loss of information.

As a matter of fact, our approach highlights qualities of the Prolog resolution box model which made its success, but also its insufficiencies.

1 Introduction

This paper presents a Prolog trace model, often called Byrd box model, in an original way, based on the concept of Observational Semantics (OS). This semantics was introduced in [1] in order to account for the semantics of tracers independently of the semantics of the traced process.

The objective of this paper is to illustrate the merits of an observational semantics with a simple but non trivial example. The result is an original semantics of the Prolog trace as usually implemented, but without either taking into account any particular implementation or describing the totality of the resolution process. Such a semantics also constitutes a form of formal specification of Prolog tracers and makes it possible to easily understand some of their essential properties.

The “box model” was introduced for the first time by Lawrence Byrd in 1980 [2] to help users of the “new” Prolog language⁴ to master the operational

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⁴ it then refers to the implementations of Edinburgh [3] and of Marseille [4]
reading of program executions. Indeed, since the very beginning, users have been complaining about how difficult it is to understand control mechanisms related to the non-determinism of the solutions. Even if thereafter other models were adopted with more complex strategies\(^5\), the four “ports” introduced by Byrd (Call, Exit, Redo and Fail), associated to the four corners of a box and easy to handle in a kind of algebra of Russian headstocks, remained famous and are more or less in all the traces of the still existing Prolog systems.

The Byrd box model fascinates by his apparent simplicity. Often quoted but seldomly well explained, it remains the object of sporadic but regular publications since 1980, for example [5] (1984), [6] (1993), [7] (2000), [8] (2003). Yet, it remains difficult to explain, its various definitions are either too abstract or drowned in a complete formalization of Prolog operational semantics.

In this article, we propose a formal description of a variant of the initial model of Byrd. The originality of this description lies in the fact that it is formally complete, although it contains the ingredients of the original model and it refers as little as possible to the Prolog mechanisms of clause choice and unification.

After an introduction to the traces and the observational semantics (Sections 2 and 3) which outlines the context of this study, we present an observational semantics which specifies the box model (Section 4) and the trace extraction (Section 5). Finally, we give a faithful reconstruction model (Section 6), which establishes a possible reading guideline for the trace, based on the OS. This paper is based on the full report [9] (same title and authors, in French). More details on the motivations can be found in [1] and [10].

2 Introducing traces

We are interested in the observation of dynamic processes starting from the traces which they produce.

One can always consider that between an observer and an observed phenomenon there is an object that we call trace. The trace is the recognized print left by a process and it is thus “readable” by other processes. The observed phenomenon will be regarded here as a closed process, its data and functions are not visible from the outside. External processes can only know its trace. This trace is called the actual trace. It is a sequence of trace events defined on some state. The actual trace is thus a kind of continuous data flow produced by some process.

A trace can also be interpreted as the evolution of an “abstract” state which contains all what one can or wants to know from the process. The sequence of the abstract states can be viewed as a more abstract but meaningful trace. Such a trace is called the full virtual trace.

\(^5\) the Byrd model is limited to the standard strategy of visiting and building a tree.
Definition 1 (Full virtual trace).

A full virtual trace is defined on a set of full virtual states $\mathcal{S}$ and a finite set of event type $\mathcal{R}$. It is an unbounded sequence of trace events of the form $e_t : (t, a_t, S_{t+1})$ where:

- $e_t$: is a unique identifier of the event.
- $t$: is the chrono, a time stamp of the event. It is an integer, incremented by 1 at each event.
- $S_{t+1} = p_{1,t+1}, ..., p_{n,t+1}$ is the reached state of $\mathcal{S}$ represented by the new values of parameters $p_1, ..., p_n$.
- $a_t$: an identifier characterizing the kind of actions performed during the transition from state $S_t$ to state $S_{t+1}$, also called event type.

A full virtual trace is denoted $T_v = < S_0, v^*_t >$, $t \geq 0$, where $S_0$ is an initial state and $v^*_t$ a possibly empty sequence (if $t = 0$) of trace events or a sequence of length $t + 1$ of the form $e_t, e_{t-1}, ..., e_0$ (if $t \geq 0$).

Example 1. In the Section 4 the box model is introduced with its full virtual trace. The main “virtual objects” described in the parameters consists of a tree whose nodes are labeled by predications and clauses. Such a tree allows to follow the evolution of the proofs during Prolog execution. Starting from a tree reduced to one root node labeled by a unique predication goal and the clauses likely used to solve it, the full virtual trace is the sequence of trees until all possible proof trees have been obtained, including failed partial trees. We do not give more detail here, since the explicit representation of a single state may be quite large and is illustrated in the Section 4.

The full virtual trace may be viewed as an interpretation of the actual trace and the actual trace can be viewed as “extracted” from the virtual one.

Definition 2 (Actual Trace, Trace Schema, Interpretation Schema).

An actual trace is defined on a set of attribute states $\mathcal{A}$. It is an unbounded sequence of trace events of the form $e_t : (t, A_t)$. $e_t$ and $t$ are like in the previous definition and $A_t$ is a tuple of attribute values.

An actual trace is denoted $T_w = < Q_0, w^*_t >$, with the same conventions as for the full virtual trace, where $Q_0$ is a subset of the parameters of $S_0$, the full virtual state.

Each actual trace event is derived from the “transition” $< S_t, S_{t+1} >$ by a function $E$, called extraction function, and such that $A_t = E(S_t, S_{t+1})$.

If $\forall t, A_t = (a_t, S_{t+1})$, the actual trace is a full virtual trace.

Each full virtual trace state can be partly reconstructed from the actual trace by a function $C$, called rebuilding function, and such that $Q_t = C(w^*_t, Q_0)$, where $Q_t$ is a subset of $S_t$. The state domain of $C$, denoted $Q$, is $S$ restricted to the parameters of $Q$, and is called the actual state domain$^6$.

$^6$ $\mathcal{A}$ and $\mathcal{Q}$ are different domains (except if the actual trace is the full virtual trace) and should not be confused.
The description of the extraction function $E$ is called a trace schema. The description of the rebuilding function $C$ is called a trace interpretation schema. By definition, a rebuilding function always exists, when there may be no function of extraction.

Example 2. In the Section 5 we give a short example of actual trace, looking as follows (here the event identifier and the chrono are the same).

chrono attributes:
1 1 1 Call goal
2 2 2 Call p(X)
3 2 2 Exit p(a)
4 3 2 Call eq(a,b)
5 3 2 Fail eq(a,b)
6 2 2 Redo p(a)
...

It will be shown that it describes the evolution of a tree labeled by predicates. However the labels corresponding to clauses are not described.

If the actual trace would be limited to as illustrated below, the actual trace would just describe the evolution of a single tree without labels.

chrono attributes:
1 1 Call
2 2 Call
3 2 Exit
4 3 Call
5 3 Fail
6 2 Redo
...

There are thus two questions one has to consider. The first is related to the rebuilding function (the actual trace interpretation), i.e. how does one interpret the actual trace as (a part of) the full virtual trace; or what is described by the actual trace. The second concerns the existence of the extraction function. The answer to the first question is given by the trace interpretation schema (description of the rebuilding function) which should be given with a trace. The second relates to the existence of a trace model, also called observational semantics.

3 Introducing the Observational Semantics

We present a concrete representation of the observational semantics, the trace schema and the trace interpretation schema. Illustrative examples will be found in the forthcoming sections.
An Observational Semantics (OS) is a model of the full virtual trace production. It describes a transition function between full virtual states of $S$ such that every transition gives rise to a unique trace event (virtual and actual).

In the case of a single observed process, the OS may be considered as a (likely partial) abstract model of the process. If several processes are to be considered with the same actual traces, the OS is thus an abstraction of the semantics of several processes.

The transition function will be described by a finite set of rules $R$ (one rule by event type in the full virtual trace). It will be represented by a named fraction consisting of four components:

- A rule identifier (rule name).
- A numerator with conditions on current values of the parameters identifying the subset of states to which the rule applies and with some intermediate computation.
- A denominator describing the computations of the new values of the parameters (invariant parameters are not described).
- External conditions (between braces) or properties relating parameters to elements not described by the parameters, but influencing the choice of the rules or the new values of the parameters.

| Name | Conditions characterizing the current state | Computations of the new parameters | \{External Conditions\} |
|------|------------------------------------------|-----------------------------------|-------------------------|

To describe the OS, two kinds of functions will be used: those related to the described objects and their evolution in the virtual trace and those related to events or objects not described in this trace, but likely to occur in the observed process. The functions of the first category are known as “auxiliary”, those of second kind “external”. They relate to parameters not taken into account in the virtual trace. Finally one will also distinguish the functions exclusively used for computation of the attributes during the extraction of the trace, named “auxiliary extraction functions” and those exclusively used for the rebuilding named “auxiliary rebuilding functions”.

The extraction function $E$ will be described by the same kind of rules whose denominator will contain only the produced trace event (attributes only). There is only one produced trace event per rule in the OS. Therefore the extraction function consists of as many components as there are rules in $R$ and is denoted $E_r$ for each rule $r$, i.e. $E = \{E_r | r \in R\}$.

Each rule of the trace schema has the following form.

| Name | Computation of the attributes | \{External Conditions\} |
|------|--------------------------------|-------------------------|

The rebuilding function may take several actual trace events as arguments. In the case of the box model two successive actual trace events and two attributes are sufficient to build a new actual state and to characterize the corresponding transition in the SO. In this case, it is possible to describe the rebuilding function as a family of local rebuilding functions indexed by the same set $R$, $C = \{C_r | r \in R\}$. To describe it, the same kind of rule presentation is used.
In this case, the actual trace is considered as an “external” information and therefore is given in the braces. The numerator of the rule contains the additional conditions which, together with the attributes of the trace events, are used to identify the corresponding applied rule of the OS.

The denominator contains the rebuilding computations (computation of the parameters of the actual state, starting from the given trace events and the previous actual state). The correspondence between the trace interpretation schema and the local rebuilding function is detailed in [9].

Notice that the three sets of rules (OS, trace schema and trace interpretation schema) are pairwise bijective.

The questions advocated at the end of the previous section concern the relationships between the observational semantics and the actual trace and its interpretation. Hence the notion of “faithfulness”.

The faithfulness concerns actual states likely restricted to a subset of parameters. One notes $S/Q$ the restriction of a full state $S$ to the parameters $Q$. $Q$ will be called current actual state and $S/Q$ the virtual state restricted to the parameters of $Q$.

**Definition 3 (Faithful Trace Interpretation).**

Given an actual state domain $Q$, restriction of $S$ to a subset of its parameters, a trace interpretation schema $C$, an OS defined on $S$ by a finite set of transitions $R$ and a trace schema $E$,

$C$ is a faithful trace interpretation w.r.t the OS and $E$, if for all actual trace $T_w = \langle Q_0, w^*_t >$, $t \geq 0$:

$$\forall i \in [0..t-1], C(w^*_i, Q_0) = S_i/Q \land \exists r \in R, \text{ such that } A_i = E_r(S_i, S_{i+1}) \ (A_i \text{ attributes of } w_i).$$

Faithfulness is a kind of total correctness property of an actual trace and the part of the virtual state evolution it represents, with regards to the trace model defined by the observational semantics and the extraction function. It includes indeed some partial correctness statement wrt SO and $E$ by the fact that every actual trace can be interpreted as (part of) a virtual trace produced by the SO and $E$. It includes also a kind of completeness statement (completeness of the observational semantics) since it also states that there is no other actual traces that the one produced by the SO and $E$. In short, faithfulness expresses the commutativity of the extraction/rebuilding schemata.

### 4 An Observational Semantics of the Byrd Box Model

In his articles [2, 11], Byrd illustrates his model using two schemata: a box with its famous four “ports” and “and/or-trees”, an already very widespread structure at that time, which combines representations of proof-tree and search-tree. It
uses neither the concept of partial proof-tree, nor that of search-tree (SLD-tree), still little known, the seminal Clark’s report [12] having hardly just appeared.

Byrd fustigates nevertheless the implementors who, at backtrack (which is displayed in the trace by an event with port Redo), force to return directly to the selected choice point, and do not express explicitly in the trace all the steps back. Byrd estimates that this is likely to lose the user and that it is preferable to demolish step by step what was explicitly made at the time of the successive uses of the clauses in order to solve goals.

Even if one wants to remain as close as possible to this model, we will however not follow this point of view, and will currently adopt that of the implementors, more widespread, and which seems quite as easy to understand, considering that the whole matter is formalized there. Indeed, the box model obliges to follow the calls of clauses through a system embedded boxes. It is thus easy to understand that, as each box has a unique identifier, the access to a choice-point, deeply located in a large box stacking, can be done as clearly by jumping directly to the deepest box rather than by descending carefully the staircase resulting from stacking, or by following strictly the opposite way. One will thus avoid to explicitly detail the manner of reaching the box by backtracking.

Even if we do not describe exactly the model initially defined by Byrd, we estimate that we keep its historically essential elements, namely the building-visit of tree and the boxes in which clauses, or a subset of them, are stored. The approach formalized here will be refered as the simplified box model.

The stacking of the boxes and its evolution will thus be described by a building-visit of tree in which each node corresponds to a box. The visit strategy corresponds to standard Prolog strategy (ISO Prolog [13]), that is to say a top-down left-to-right building-visit. Each new node, or box, is associated with a number which is incremented by 1 at each time a node is created.

Each node is labelled with a predication and a packet of clauses. Each box is thus the root of a subtree which is spread as a “treemap”, thus resulting in a kind of algebra of embedded boxes.

We use the vocabulary of ISO Prolog [13].

**Full virtual trace parameters**

A full state has 9 parameters:

\{T, u, n, num, pred, claus, first, ct, flr\}.

1. **T**: T is a tree labelled by creation numbers, predications and subsets of clauses of the program \(P\). It is described here by its functions of building-visit-rebuilding (see below) and its labels. No specific data representation is requested here. However we use in the examples a notation “à la Dewey”.

Each node is represented by a sequence of integers and for example denoted \(\epsilon, 1, 11, 12, 112, \ldots\). \(\epsilon\) is the empty word, 1, 11 are direct successors and 11, 12 are “brothers”. The lexicographic ordering is as follows: \(u, v, w\) are words, \(u < uv (v \neq \epsilon)\), and \(uiw < ujw\) if \(i < j\).

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\[\] According to the ISO Prolog glossary, a predication is a term whose outermost functor is a predicate.
2. \( u \in T \): \( u \) is the current node in \( T \) (visited box).
3. \( n \in \mathcal{N} \): \( n \) is a positive integer, number of the last created node in \( T \).
4. \( \text{num} : T \to \mathcal{N} \). Abbrev.: \( \text{nu} \). \( \text{nu}(v) \) is the creation number (positive integer) labelling node \( v \) in \( T \).
5. \( \text{pred} : T \to \mathcal{H} \). Abbrev.: \( \text{pd} \). \( \text{pd}(v) \) is the predication labelling the node \( v \) in \( T \). It is a term of \( \mathcal{H} \) (non ground Herbrand base).
6. \( \text{claus} : T \to 2^P \). Abbrev.: \( \text{cl} \). \( \text{cl}(v) \) is the list of clauses in \( P \) (same order as in \( P \)) whose heads use predication \( \text{pd}(v) \) associated with node \( v \) in \( T \). \([]\) is the empty list. Depending on the clauses in \( \text{cl}(v) \), one gets several models of trace. We will assume that only useful clauses are in \( \text{cl}(v) \) (clauses whose head is unifiable with \( \text{pd}(v) \)). If \( \text{cl}(v) \) is empty, there is no way to solve the corresponding predication \( \text{pd}(v) \) and the node is thus in "failure". This list is created externally when a predication is called (see \( \text{claus}_{\text{pred init}} \) in external functions) and updated each time the node is revisited (see \( \text{update}_{\text{claus}_{\text{and pred}}} \) in auxiliary functions).
7. \( \text{first} : T \to \mathbb{B} \). Abbrev.: \( \text{fst} \). \( \text{fst}(v) \) is true iff \( v \) is a not yet visited node in \( T \) (it is a leaf of \( T \)).
8. \( \text{ct} \in \mathbb{B} : \text{ct} = \text{true} \) iff all nodes in the tree \( T \) have been completely visited (the current node is \( \epsilon \) again after completion of a building-visit).
9. \( \text{flr} \in \mathbb{B} : \text{flr} = \text{true} \) if (and not iff) the current subtree is failed. \( \text{flr} = \text{false} \) otherwise (which does not mean that the subtree is successful).

**Initial state** \( S_0 \):

Due to space limits we will sometimes use \( \text{tu} \) (\( \text{fa} \)) instead of \( \text{true} \) (resp. \( \text{false} \)).

\[
\{ \epsilon \}, \epsilon, 1, \{ (\epsilon, 1) \}, \{ (\epsilon, \text{goal}) \}, \{ (\epsilon, \text{list} \text{and} \text{goal} \text{claus}) \}, \{ (\epsilon, \text{tu}) \}, \text{fa}, \text{fa}
\]

The model is based on a building-visit of partial proof-trees, built then rebuilt after backwards. The nodes are built just before being visited for the first time. Each visit of a node (or box) produces a trace event.

**Auxiliary Functions** (parameters manipulations):

- \( \text{parent} : T \to T \). Abbrev.: \( \text{pt} \). \( \text{pt}(v) \) is the unique direct ancestor of \( v \) in \( T \).
  To simplify the model, it is assumed that \( \text{pt}(\epsilon) = \epsilon \).
- \( \text{leaf} : T \to \mathbb{B} \). Abbrev.: \( \text{lf} \). \( \text{lf}(v) \) is true iff \( v \) is a leaf of \( T \).
- \( \text{may have new brother} : T \to \mathbb{B} \). Abbrev.: \( \text{mhnb} \). \( \text{mhnb}(v) \) is true iff \( \text{pd}(u) \) is not the last predication in the body of the currently used clause (which is the first clause in the box of the parent node of \( v \) in \( T \)). The root (node \( \epsilon \)) has no brother.
- \( \text{create child} : T \to T \). Abbrev.: \( \text{crc} \). \( \text{crc}(v) \) is the new child of \( v \) in \( T \).
- \( \text{create new brother} : T \to T \). Abbrev.: \( \text{crnb} \). \( \text{crnb}(v) \) is the new brother of \( v \) in \( T \). Defined if \( v \) is different from \( \epsilon \).
- \( \text{has a choice point} : T \to \mathbb{B} \). Abbrev.: \( \text{hcp} \). \( \text{hcp}(v) \) is true iff there is a choice point \( w \) in the subtree rooted \( v \) in \( T \) (\( \text{cl}(w) \) has at least one clause).
- \( \text{greatest choice point} : T \to T \). Abbrev.: \( \text{gcp} \). \( \text{w} = \text{gcp}(v) \) is the greatest choice point in the subtree of root \( v \) in \( T \) (i.e. such that \( \text{cl}(w) \) has at least one clause) according to the lexicographic ordering of nodes in \( T \).
The Resolution Box Model

- **fact**: \( T \rightarrow \text{Bool} \). Abbrev.: \( ft \). \( ft(v) \) is true iff the first clause in \( cl(v) \) is a fact.

- **update_number**: \( Fu, T \rightarrow Fu \). \( Fu \) stands respectively for a set of functions and a set of nodes representing a tree. Abbrev.: \( upm \). \( upm(nu, v) \) updates the function \( num \) removing all defining pairs related to removed nodes in \( T \) until node \( v \) (not removed).

- **update_claus_and_pred**: \( F, T, H \rightarrow F \). Abbrev.: \( upcp \). \( upcp(claus, v) \), \( upcp(pred, v) \) (2 arguments) or \( upcp(pred, v, p) \) (3 arguments): updates the functions \( claus \) and \( pred \) removing all pairs related to removed nodes in \( T \) until node \( v \), updating too, if requested by the external function \( pred \_update \), the value of \( pd(v) \) with the pair \((v, p)\) and updating the value of the function \( claus \) at node \( v \), removing the last used clause.

**External functions:**

They correspond to the actions not described in the full virtual trace but with some influence, in particular all the aspects of the resolution related to the unification, which are omitted in this OS.

- **success**: \( T \rightarrow \text{Bool} \). Abbrev.: \( scs \). \( scs(v) \) is true iff \( v \) is a leaf in \( T \) and the current predication has been successfully unified with the head of the clause selected in the current box (box associated with the node \( v \)), or \( v \) is not a leaf in \( T \) and the subtree of root \( v \) is a sub-proof-tree (all leaves are successful).

- **failure**: \( T \rightarrow \text{Bool} \). Abbrev.: \( flr \). \( flr(v) \) is true iff \( v \) is a leaf and no head of any clause of the program can be unified with the current predication (in this case \( cl(v) \) is empty).

- **claus_pred_init**: \( T \rightarrow (\text{pred}, \text{list of clauses}) \). Abbrev.: \( cpini \). \( (c, p) = cpini(v) \) (1) updates in the function \( cl \) the pair \((v, c)\) where \( c \) is the list of clauses defining the predication \( pd(v) \) which thus can be used successfully to try all possible solutions, and (2) updates in the function \( pd \) the pair \((v, p)\) where \( p \) is the predication to be associated to the node \( v \). The elements (clause and predication) computed by \( cpini(v) \) will be respectively denoted \( c_{cpini}(v) \) and \( p_{cpini}(v) \).

- **pred_update**: \( T \rightarrow H \). Abbrev.: \( pud \). \( pud(v) \) is the new value given to the current predication \( pd(v) \) labelling the node \( v \) in \( T \), following a successful unification.

Notice that \( \forall u, flr(u) \Rightarrow flr = \text{true} \) (see the **Tree failed** rule).

The OS is defined by the rules of the Figure 1. Each rule is commented in the following.

- **Leaf reached**: The current node is a leaf and the called predication will be solved by a fact. This node will thus remain a leaf. The choice point is updated (a clause is removed from the box).

- **Lf rcd & go down**: the current node is a leaf but the associated predication is solved with a clause whose head has been successfully unified and whose body is not empty. This node will be expanded. A new node is created of which
Fig. 1. Observational Semantics of Prolog resolution (full virtual trace)

box \( v \) is filled with useful clauses and a calling predication is associated. The choice point is updated.

- **Tree success**: successful exit from the last predication of the body of the current clause. \( \text{pred}(u) \) is updated (it is not necessarily the same one as at the time of the call). Step up with a successful subtree without creation of any new branch.

- **Tree suc & go right**: successful exit from a predication of the body of the current clause with creation a new “sister” (new leaf \( v \), in case of using a clause with more than one predication in the body). The box \( v \) is filled with useful clauses and node \( v \) is labelled with a calling predication.

- **Tree failed**: step up with a failed subtree as long as there is no choice point in the subtree.

- **Backtrack**: backtrack following success or failure, if there is a choice point in the subtree opening for a possible solution (or new solution if the current node is the root). As discussed at the beginning of this section, in this model, one does not repeat all the **Redo**, following all the steps down and back until the choice point, as in the original Byrd’s model.

- **Bkt & gd**: backtrack following success or failure, if there is a choice point in the subtree, opening for a possible solution (or new solution if the current node is the root). As above, but with the creation of a successor as in the Lf rcd & go down rule.
In the initial state $S_0$, only one of the rules Leaf reached or Lf rcd & go down may apply. Whatever is the state, only one rule in $R$ can be applied as long as a complete tree has not been built. No rule applies any more if the built tree is complete and it does not contain choice point.

5 Extraction of the actual trace

Each application of a rule of the OS gives place to the extraction of a trace event whose chrono is incremented of a unit each time. For the extraction one needs an auxiliary function.

Auxiliary extraction function.

- $lpath : T \rightarrow N$. Abbrev.: $lp$.
  Byrd calls it the “depth of recursion”. $lp(v)$ is the number of nodes on the path from the root to the node $v$. It is thus the length of the path from the root to the node $v$ +1. $lp(\epsilon) = 1$.

In the box model, the actual trace has 4 attributes and each event has the form

$$t \ r \ l \ port \ p$$

where

- $t$ is the chrono.
- $r$ is the creation number of the current node $u$ (the node in the current state), $nu(u)$.
- $l$ is the depth in the tree $T$ of the current node, that is to say $lp(u)$.
- $port$ is the action identifier having produced the trace event (Call, Exit, Fail or Redo).
- $p$ is the predication associated with the current node, that is to say $pd(u)$.

Example 1 below presents a program and the extracted trace corresponding to the goal :- goal, (u current node)

\begin{align*}
c1 &: \text{goal:-p(X),eq(X,b).} \\
c2 &: \text{p(a).} \\
c3 &: \text{p(b).} \\
c4 &: \text{eq(X,X).} \\
:- \text{goal.}
\end{align*}

| chrono | nu(u) | lp(u) | port | pd(u) | Virtual State reached |
|--------|-------|-------|------|-------|-----------------------|
| 1      | 1     | 1     | Call | goal  | S2                    |
| 2      | 2     | 2     | Call | p(X)  | S3                    |
| 3      | 2     | 2     | Exit | p(a)  | S4                    |
| 4      | 3     | 2     | Call | eq(a,b) | S5                |
The trace schema is described in figure 2. Note that in these rules, the node $u$ refers to the current virtual state $S_t$.

Leaf reached: $\langle \nu(u) \quad lp(u) \quad \text{Call} \quad pd(u) \rangle > \{\}

Lf rcd & go down: $\langle \nu(u) \quad lp(u) \quad \text{Call} \quad pd(u) \rangle > \{\}$

Tree success: $\langle \nu(u) \quad lp(u) \quad \text{Exit} \quad p \rangle > \{p = pud(u)\}$

Tree suc & go right: $\langle \nu(u) \quad lp(u) \quad \text{Exit} \quad p \rangle > \{p = pud(u)\}$

Tree failed: $\langle \nu(u) \quad lp(u) \quad \text{Fail} \quad pd(u) \rangle > \{\}$

Backtrack: $v \leftarrow gcp(u)$$\langle \nu(v) \quad lp(v) \quad \text{Redo} \quad pd(v) \rangle > \{\}$

Bkt & go down: $v \leftarrow gcp(u)$$\langle \nu(v) \quad lp(v) \quad \text{Redo} \quad pd(v) \rangle > \{\}$

Fig. 2. Trace schema (actual trace extraction function)

6 Actual Trace Interpretation

To only account for the elements specific to the box model (evolution of the tree and predication labels) 4 parameters are sufficient. One will thus take as restricted virtual state the following parameters:

$$Q = \{T, u, \text{num}, \text{pred}\}.$$

Note that one could have added the parameters $ct$ and $flr$. But that does not appear necessary a priori because $ct$ is true (except at the first trace event) iff the first or the second attribute is 1 (in fact they are 1 together); and $flr$ becomes false (failure) for any trace event of port $\text{Fail}$. In particular if we are at the root, we know if we are in failure (event of port $\text{Fail}$ at the root) or in success (event of port $\text{Exit}$ at the root). Then we know if we have a failure tree or a complete proof tree (success).

So the initial state $S_0/Q$ is: (see the complete state at the previous section)

$$\{\{\epsilon\}, \epsilon, \{(\epsilon, 1)\}, \{(\epsilon, \text{goal})\}\}$$
The rebuilding function $C$ of the restricted virtual trace now is described starting from the initial actual state and the actual trace.

**Auxiliary function of rebuilding:**
To rebuild the partial current state, one auxiliary function only is necessary, namely the inverse function of $\text{num}$, noted $\text{node}$.

$\text{node} : \mathcal{N} \rightarrow T$. Abbrev.: $\text{nd}$. Inverse function of $\text{num}$. $v = \text{nd}(n)$ is the node of $T$ whose creation rank is $n$ (such that $\text{nu}(v) = n$). By definition $\text{nd}(\text{nu}(v)) = v$ and $\text{nu}(\text{nd}(n)) = n$.

The interpretation schema is given in figure 3 by the family $\{C_r | r \in R\}$.

**Leaf reached**

$\begin{align*}
\frac{r' = r}{< r \ l \ Call \ p > ; < r' >}
\end{align*}$

**Lf rcd & go down**

$\begin{align*}
r' > r & \quad u' \leftarrow \text{crc}(\text{nd}(r)), \quad T' \leftarrow T \cup \{u'\}, \quad \text{nu}'(u') \leftarrow r', \quad \text{pd}'(u') \leftarrow p' \quad \{< r \ l \ Call \ p > ; < r' p' > \}
\end{align*}$

**Tree success**

$\begin{align*}
r' > r \land u = \epsilon & \quad u' \leftarrow \text{pt}(u), \quad \text{pd}'(u) \leftarrow p \quad \{< r \ l \ Exit \ p > ; < r' > \}
\end{align*}$

**Ts & gr**

$\begin{align*}
r' > r \land u \neq \epsilon & \quad u' \leftarrow \text{crnb}(u), \quad T' \leftarrow T \cup \{u'\}, \quad \text{nu}'(u') \leftarrow r', \quad \text{pd}'(u') \leftarrow p, \quad \text{pd}'(u') \leftarrow p' \quad \{< r \ l \ Exit \ p > ; < r' p' > \}
\end{align*}$

**Tree failed**

$\begin{align*}
< r \ l \ Fail \ p >
\end{align*}$

**Backtrack**

$\begin{align*}
\frac{r' = r}{v = \text{nd}(r), \quad T' \leftarrow T - \{y | y > u\} \quad \{< r \ l \ Redo \ p > ; < r' > \}}
\end{align*}$

**Bkt & gd**

$\begin{align*}
r' > r & \quad \frac{v \leftarrow \text{nd}(r), \quad T' \leftarrow T - \{y | y > u\}, \quad u' \leftarrow \text{crc}(v), \quad \text{nu}' \leftarrow \text{upn}(\text{nu}, v) \cup \{(u', r')\}, \quad \text{pd}' \leftarrow \text{upcp}(\text{pd}, v) \cup \{(u', p')\} \quad < r \ l \ Redo \ p > ; < r' p' > \}}{< r' >}
\end{align*}$

**Fig. 3.** Trace Interpretation Schema (simplified box model)

A complete proof of the faithfulness of the interpretation trace schema $C$ wrt the given OS and $E$, can be found [9].

### 7 Discussion and Conclusion

The main point of this paper is the illustration of an original approach to give a semantics to traces. The example of the box model, used here, is primarily anecdotic. But, in fine, the result is undoubtedly a complete formalization, which is also one of the simplest formalizations of this model (simple because restricted to only elements necessary to its comprehension).
Our first observations will relate to the comprehension of the trace given by the rules of figure 3. They give an first interpretation of the trace, provided the actual trace reflects a true execution (for example it is the trace of a tree construction according to some strategy). The faithfulness property guarantees that it can be related with a more complete semantics given by the observational semantics and the associated actual trace extraction function.

This approach also immediately highlights the difficulties of interpretation of such a model. We will retain two of them. At first it will be observed that the trace interpretation requires to analyse two actual trace events (Section 6). This could be avoided if some information on the clause would be displayed in the actual trace event (fact or not fact), making thus the trace simpler to read.

In second remark one will observe a contrario that the trace contains an useless attribute. The depth (attribute 1) finally does not contribute to the comprehension of the trace and overloads it unnecessarily. In fact the depth could contribute to the comprehension of the partial proof tree by combining it with an adequate coding of the nodes. This choice is made for example in the trace of GNU-Prolog [14] where the nodes are coded, not by their order of creation, but by their rank in the tree. The combination of the two attributes then allows a direct location of the current node in the tree $T$. This choice constitutes indeed an improvement of the original trace.

The few articles quoted in the introduction translate the permanent search for improvements of the comprehension of control and also of the unification. So [5] (1984) [15] (1985) propose improvements of Byrd trace with a more reduced number of events, thus bringing a more synthetic vision of the traversed tree, and they also propose new ports concerning the unification and the choice of the clauses. [6] (1993) explicitly introduces an algebra of boxes supported by graphics, but this model, which wants to deal with all the aspects of the resolution, remains rather complex. [7] (2000) proposes a semantics of trace based on a denotational semantics of Prolog. The principal disadvantage is that the comprehension of the trace needs a good comprehension of a complete model of Prolog, synthetic but requiring a certain familiarity with the continuations. In the paper [8] (2003) the approach is similar but it is based directly on the ports whose possible sequences constitute its skeleton. The result is also that the comprehension of the trace needs the assimilation of a relatively complex semantics of Prolog, which is connected more with a semantics based on the “magic sets” than with a direct explanation of the trace.

One may consider that the trace model presented here is not so easy to understand, since some auxiliary and external functions are not formally described. However the auxiliary functions always refer to wellknown data structure manipulation with an usual unambiguous semantics. Some external ones refer to a non described Prolog semantics (successful unification, clauses selection, proof-tree, ...) and are not defined here. It must be clear that the given model is a formal definition of a tracer, not of the observed processor. It means that the given model is not an executable specification and that its semantics depends on the interpretation given to undefined functions. In particular the faithfulness
property does not depend on these interpretations. The interest and potential simplicity of such an approach precisely lies in the fact that there is no need to describe entirely the semantics of the observed processes.

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