In this paper, we study the orbital Feshbach resonance of ultracold two-electron $^{173}\text{Yb}$ Fermi gas in a one-dimensional optical lattice. We consider a two-band model under the assumptions of a local-density approximation for the optical lattice potential and a mean-field approximation for the intraband Cooper pairings. We get three crucially important properties for the system, the particle condensation in the momentum space, and the superfluid-LFL phase transition corresponding to the temperature. We explain how to realize and manipulate these properties.

Keywords: optical cooling and trapping of atoms, Fermi gases, optical lattices

(Some figures may appear in colour only in the online journal)
imaging and independent control of external confinement and lattice depth [12]. There are several interesting studies related to the ultracold atoms in the optical lattice which is a defect free set up that allows independent control of various parameters of the system [13–15]. Here, the high controllability over the optical lattice permits one to use the internal atomic degrees of freedom for extension of a one-dimensional (1D) optical lattice into a synthetically yielded two-dimensional lattice [16–20].

In this paper, we consider two nuclear spin states denoted by |↑⟩ (m, F = 1/2) and |↓⟩ (m, F = −1/2) for OFR, e.g., the two nuclear spin states in 171Yb (I = 5/2) with m, F = −5/2, ..., 5/2. As shown in figure 1, the presence of magnetic field causes the two-body internal states, i.e., the open channel |o⟩ = (|g ↓ : e e⟩ − |e ↑ : g g⟩)/√2 and the closed channel |e⟩ = (|g ↑ : e e⟩ − |e ↓ : g g⟩)/√2, introducing the coupling between the orbit and spin degree of freedom [4, 21–23]. We consider a two-band model under the assumptions of a local-density approximation for the optical lattice potential and a mean-field approximation for the intraband Cooper pairings. We find that OFR detuning gradually decrease the critical tunneling energy for superfluid-normal phase transition at zero temperature and there is a narrow superfluid phase region between the two normal phases for a large OFR detuning. We also find that the critical superfluid-normal transition temperature may be reduced even to zero for increasing the tunneling energy of the 1D optical lattice.

Rest of the paper is organized as follows. In section 2, we give the Hamiltonian of OFR in an optical lattice and deduce the thermodynamic potential in momentum space by obtaining the self-consistent equations. In section 3, we discuss obtained results of OFR by tuning different controlling parameters of the system respectively. Finally, section 4 is devoted to the essential conclusions of the work.

2. Model and dynamics

We consider a Fermi gas in 1D optical lattice potential (V(x) = V₀ cos²k,Lx) generated by a standing wave field at

the ‘magic’ wavelength λL = 759 nm, to ensure the lattice dispersions are the same for all states in two channels, as shown in figure 1. The potential strength V₀ is proportional to the laser intensity and corresponds to the maximum depth of the lattice potential. The Hamiltonian describing Fermi gases across an OFR is exactly analogous to that of two-band s-wave superconductors with contact interactions in an optical lattice [8, 11, 12, 21] and we can get the orbital Feshbach two-band Hubbard Hamiltonian by using the single-particle Wannier basis as

\[ \mathcal{H} = -t \sum_{j, \sigma} (c_{j, \sigma}^\dagger c_{j+1, \sigma} + h.c.) - \sum_{j, \gamma, \gamma'} V_{\gamma, \gamma'} b_{j, \gamma}^\dagger b_{j, \gamma'}, \]

where

\[ t = -\int d\omega \omega^a(x - x_j) \left[ -\hbar^2 \nabla^2 \over 2m + V(x) \right] \times \omega^a(x - x_{j+1}) \]

is the tunneling energy between neighboring sites on the same channel, \( \omega^a(x - x_j) \) is the Wannier function, \( x_j \) is the position of the site \( j \) and \( c_{j, \gamma}^\dagger, c_{j, \gamma} \) are the creation (annihilation) operators of a fermion in spin state \( \sigma \) on the \( j \)th lattice site. The subscript \( \sigma (= \uparrow, \downarrow) \) represents the pseudospin projections and \( \gamma(\gamma') = (1, 2) \) note the particles being in the open (closed) channels, respectively. The operator \( b_{j, \gamma}^\dagger = c_{j, \gamma}^\dagger c_{j+1, \gamma} \) means creating pairs of spin up and spin down on the site \( j \) in the open or closed channel. The short-ranged interactions of two-body (\( V_{\gamma, \gamma'} \)) include the intraband interactions with equal amplitudes (\( V_{11} = V_{22} \)) and the interband interactions (\( V_{12} = V_{21} \)), respectively, which are different for different species of atoms. For the sample 171Yb, these interactions are of attractive, i.e., \( V_{\gamma, \gamma'} > 0 \) for both channels.

To solve the above Hamiltonian, it is convenient to work in the momentum space. For this goal, the equation (1) can be transformed to an expression in momentum space by using Fourier transformation. We introduce an intraband order parameter \( \Delta_\gamma = \Delta_\gamma = -V_{\gamma, \gamma'} \langle b_{j, \gamma}^\dagger \rangle \) for each channel with \( \langle b_{j, \gamma}^\dagger \rangle \) being the thermal average under the mean-field pairing approximation, and the resulting Hamiltonian now reads

\[ \mathcal{H} = \sum_{k, \sigma, \gamma} \xi_k c_{k, \sigma}^\dagger c_{k, \sigma} + \Delta_\gamma^\sigma \sum_{k, \gamma} c_{k, \gamma}^\dagger c_{k, -\gamma} + \Delta^k \sum_{k, \gamma} c_{k, \gamma}^\dagger c_{-k, -\gamma} + \Delta_\gamma \sum_{k, \gamma} c_{-k, \gamma}^\dagger c_{k, \gamma} + U_{\gamma, \gamma'} \Delta_\gamma^\sigma \Delta^\gamma_{\gamma'} \]

The operators \( c_{k, \sigma}^\dagger \) create a single-particle with the spin \( \sigma \) in channel \( \gamma \), momentum \( k \) and dispersion \( \xi_k = \xi_k - \mu_\gamma \), where \( \xi_k = h^2 k^2/2m \). Similarly, the effective chemical potentials are given by \( \mu_1 = \mu + 2\gamma \cos ka - \delta/2 \) with \( \delta \) being the detuning of OFR and \( \alpha = \lambda_0/2 \) being the lattice constant. Matrix \( U \) is inverse of the interaction strength matrix \( V \). The Hamiltonian in equation (3) can be diagonalized by using Bogoliubov transformation and
minimizing the ground state thermodynamic potential

\[ \Omega = \sum_{k, \sigma, \gamma} \xi_{E_k} - E_k - 2T \ln(1 + e^{-E_k/T}) \]

\[ + \frac{V_1 |\Delta_2|^2 + V_2 |\Delta|^2 - V_1 \Delta_1^* \Delta_2 - V_1 \Delta_1 \Delta_2^*}{|V|}, \]

in the grand canonical ensemble. The self-consistent parameters are given by

\[ \Delta_1 = \sum_k V_1 \frac{\Delta_1}{2E_{k1}} \tanh \left( \frac{E_{k1}}{2T} \right) \]

\[ + V_2 \frac{\Delta_2}{2E_{k2}} \tanh \left( \frac{E_{k2}}{2T} \right), \]

\[ \Delta_2 = \sum_k V_2 \frac{\Delta_1}{2E_{k1}} \tanh \left( \frac{E_{k1}}{2T} \right) \]

\[ + V_2 \frac{\Delta_2}{2E_{k2}} \tanh \left( \frac{E_{k2}}{2T} \right), \]

and

\[ n = \sum_{k, \gamma} \left[ 1 - \frac{\xi_{E_k}}{E_k} \tanh \left( \frac{E_k}{2T} \right) \right]. \]

Here \( \Delta_1 (\Delta_2) \) is the order parameter of open (closed) channel describing the minimum energy to break up a Cooper pair. Similarly, \( E_k = \sqrt{\xi_{E_k}^2 + |\Delta|^2} \) is the energy of the quasiparticle excitations in the \( \gamma \) channel, \( T \) represents temperature of the Fermi gas, and \( n \) is the density of atoms. By solving the self-consistent equations, we find that there are two solutions, i.e., an in-phase solution for both \( \Delta_2 \) and \( \Delta_1 \) having the same signs and an out-of-phase solution for both \( \Delta_2 \) and \( \Delta_1 \) having different signs. However, the in-phase solution is trivial, which has nothing to do with the BCS–BEC crossover associated with the OFR [24]. We will focus on the discussion of the out-of-phase solution of the two pairs potentials, which is, in fact, an excited-state in the landscape of the thermodynamic potential. The excited-state (out-of-phase) solution is predominantly characterized by the OFR in a \(^{173}\text{Yb} \) Fermi gas [8].

### 3. Results and discussions

In this section, we want to examine how do the basic natures of the two-channel system, such as order parameter \( \Delta_n \), density of atoms \( (n_a) \) and momentum distribution \( (n_m) \) of the ultracold Fermi gas, vary depending on the dimensionless parameters \( t/E_F \) (the tunneling energy), \( T/E_F \) (the temperature) and \( \delta/E_F \) (the OFR or related to the magnetic field). That is, for convenience, we use \( E_k = \hbar^2 k^2 / 2m \) (Fermi energy) as a unit of energy to normalize the system. Here \( k_F = \left( 3\pi^2 n \right)^{1/3} \) is referred to as the Fermi wavelength and \( n = 10^{29} m^{-3} \) is the typical density of the Fermi gas. It is worth pointing out that one can achieve an orbital resonance by tuning the Zeeman energy difference \( \delta \) to a threshold

\[ \delta_{\text{th}} = \frac{\hbar^2}{m(a_0 - r_0)^2} = 3.14E_F, \]

where \( m \) is the mass of a single atom, \( a_0 = 960a_0 \), \( a_0 \) is the Bohr radius of the \(^{173}\text{Yb} \) atoms and one can choose \( r_0 = 0 \) for the system [3, 8, 21].

#### 3.1. The crossover of the superfluid to normal state corresponding to the tunneling energy

The order parameter \( \Delta_n \), and density fraction \( n_a/n \) can be tuned by the tunneling energy \( t/E_F \) according to equations (5)–(7). In figure 2, we plot these quantities as a function of the tunneling energy for different values of OFR detuning \( \delta \) in the case of the out-of-phase \( \Delta_1 < 0, \Delta_2 > 0 \) and \( |\Delta_2| > |\Delta_1| \) solutions.

Due to the existence of tunneling, atoms are able to hop between the open channel and closed channel of the neighboring sites. The probabilities of hopping to open and closed channel are different with the variation of the OFR detuning and the tunneling energy. In figures 2(a) and (b), we plot the order parameter \( \Delta_n/E_F \) and the particle density \( n_a/n \) versus the tunneling energy \( t/E_F \) for two channels for different values of \( \delta \), respectively. One can see that, for relative small
tunneling energy $t$ and small $\delta$ the description of the two channels model is important, because in this case two channels have different properties of superfluidity and particle densities. Figure 2(a) shows that for small $t$ and $\delta$ it is easy to form Cooper pairs and the system exhibits the superfluidity. When $t$ is large enough (for any $\delta$), $\Delta_1$ (for $\gamma = 1, 2$) tend to zero. This means Cooper pairs disappear and the system goes to normal phase. This property can be referred to as the superfluid-normal phase crossover corresponding to the tunneling energy. For a deep lattice potential ($V_0/E_F \gg 1$), the hopping between neighboring sites is forbidden and atoms are trapped in separated lattice sites, or say in the harmonic potential [9]. For a nonzero tunneling energy $t$, atoms can hop between neighboring sites. The hopping can prevent from the formation of pairs on all sites and then with $t$ becoming large, the system carries out a crossover from superfluid to normal phase. It is worth noticing that $\delta$ also influence the superfluid-normal crossover. When $\delta$ increases, the superfluid-normal crossover shifts toward a smaller $t$. An interesting phenomenon is that for a small $\delta$ case, at a large $\delta$, for example $\delta = 10E_F$, the order parameters of two channels have an individually increased peak.

Figure 2(b) shows the variation of $n_{\gamma}/n$ (particles in two channels) versus $t/E_F$ (the tunneling energy) for different values of $\delta$. The relation with $\delta$ means that the particle densities can be adjusted by the magnetic field according to the definition of $\delta$. One can see that, in the region $t < 1.5E_F$, the particle density in the open channel is usually larger than that in the closed channel. If the magnetic field intensity (as well as $\delta$) is increased, the particle density in the open channel increases accordingly, even up to all occupation in the open channel when the magnetic field is large enough. There is a critical value, $\delta_{c1}/E_F = 3.14$. Above this value (for large $\delta$), the particle density for the closed channel may rapidly tend to zero (the closed channel becomes empty), then all particles are located in the open channel. However, when $\delta/E_F < 3.14$, the system usually has $n_2 < n_1$, but it normally does not tend to zero. One can see from the figure that all curves vary smoothly, i.e. no sudden change happens. Then no phase transition happens corresponding to the tunneling energy for the system.

We may explain the properties of the system how does it look like in this case. According to the definition of $E_F$ (Fermi energy), for a non-interacting single-channel Fermi gas at zero temperature, the upper closed channel is completely empty for the energy difference $\delta/2 > E_F$. Now in this two-channel system, for all possible $\delta$ with small $t$, particles are scattered in two channels, this is because of the coupling between two channels. However, for the case $\delta/2 \gg E_F$, the two-channel coupling is not strong enough to overcome the detuning barrier of the Fermi gas. As a consequence, particles in the closed channel vanish and only concentrate in the open channel, which leads to the two-channel pairings collapsing as well ($\Delta_1 = 0$). One may notice that, for a small OFR detuning ($\delta/E_F < 3.14$) (for example, $\delta/E_F = 1$), the densities $n_1$ and $n_2$ behavior oppositely. First of all, we have to emphasize that $n_1 + n_2 = n$, the total particles are conservative, so when $n_1$ increases, $n_2$ must decrease, and vice versa. The variation of $n$ is connected with $\Delta_\gamma$, the chemical potentials $\mu_\gamma$, and so on. These connections carry out $n$ having a nonlinear relation with the tunneling energy $t$, which is especially connected with the formation and variation of Cooper pairs.

### 3.2. A condensation in the momentum space

The existences of the recoil energy $E_L = \hbar^2k^2_L/2m$ in optical lattice and the tunneling effect may influence the momentum distributions of particles. So we can investigate the relation of particle numbers with the relation of momentum, and define the particle density of the momentum distributions of the ultracold Fermi gas as $n_{\gamma,k}$, and $n_{\gamma}/n = \int n_{\gamma,k}dk$ is the same as given in equation (7). In figure 3, we plot the momentum density distributions of two channels with the variation of $k/2k_L$ under the conditions of $t = 0$ and $0.45E_F$. One can see that in the case $t = 0$ (no tunneling energy), both $n_{\gamma,1}$ and $n_{\gamma,2}$ vary continuously and smoothly. However, for $t = 0.45$ the variation of curves are quite different. They show that there is another peak correspondingly. This is a very interesting property. To examine why this peak happens, in the inset we plot a figure of $\xi_{kL}/E_F$ versus $k/2k_L$. One can find that two peaks of $n_{\gamma,k}$ are exactly matching the positions of the minima of $\xi_{kL}$. While $\xi_{kL}$ is physically related with the energy of Fermi gas, it is this relation leads to the peak of the distribution. The first peak is well understood because for most particles of the ultracold Fermi gas their momenta must be very small, so there are a lot of particles locate in a small momentum region. Then for the minimum $\xi_{kL}$ value, there is another peak, the accumulation of particles at this position are referred to as the particle condensation in the momentum space.
3.3. The phase transitions corresponding to the temperature

The characteristic with temperature $T$ of the Fermi gas is another important nature. In figures 4(a) and (b), we plot the order parameters and densities of two channels versus with the temperature $T$. How these parameters vary with the temperature is obvious and intuitionistic. It is not necessary to describe again in more detail. However, we have to emphasize a very important physics meaning of curves, i.e. the phase transitions corresponding to the temperature of the system. Let us see figure 4(a) first, it shows that both channels are in the superfluid states at low temperature. Up to the critical temperature $T_c$, which is dependent on the tunneling energy, two channels of the system undergo a transition from the superfluid states to the normal state ($\Delta_n = 0$), simultaneously. This is the analog of the typical phase transition in condensed matter physics. So this result shows the superfluid-normal phase transition of a two-channel ultracold Fermi gas corresponding to the temperature in the optical lattice.

This phase transition is also reflected in the nature of the particle densities of two channels as shown in figure 4(b). One can see that two-channel densities vary with the increase of the temperature, up to the critical temperature, the densities of two channels undergo a transition which is shown by the variation of the derivative of the densities. The discontinuity of the derivative of the densities expresses a phase transition from an orderly superfluid phase to a disordered normal phase, which corresponds the Landau–Fermi-liquid (LFL). Over the phase transition point, the ‘pairing’ order in the superfluid is broken down. In the superfluid phase ($T < T_c$), the particle density bias of two channels ($n_1 - n_2$) increases with temperature. It illustrates that as the temperature rises, more and more atoms accumulate to the open channel of the lower energy, and meanwhile the particles in the upper closed channel are exhaled. In contrast, for the normal phase ($T > T_c$), the particle density bias of two channels decreases with the temperature.

From figures 4(a) and (b), one can see that the phase transition here is actually related to the tunneling energy of the system. So, we are interested in how this relation is. For this purpose, we plot figure 4(c), which is the relation of the critical temperature $T_c$ versus the tunneling energy $t$. It is worth noticing that the critical superfluid-normal transition temperature is reduced with increasing the tunneling energy $t$. Actually for $t \gg E_F$, Fermi atoms do not form the superfluid order at all, even in very low temperature. So the critical temperature tends to zero for a large value of $t$.

3.4. Additional remarks

Up to now, we have actually got all the basic results of the system. In this subsection, we plot figure 5 as additional remarks for detailed influences of OFR detuning on the behavior of the Fermi gas in the optical lattice. Figures 5(a) and (b) show the variation of the order parameters and the particle densities with the OFR detuning $\delta$ under the condition $t = 0.45E_F$. These two figures show explicitly how does the magnetic field control the evolutions of the order parameters and the particle densities. In
Figure 5. (a) The order parameters $\Delta_\gamma$ and (b) The number density $n_\gamma$ versus the OFR detuning $\delta$ for the open (solid line) and closed (dotted–dashed) channels. (c) The momentum density distribution $n_\gamma k$ for different OFR detuning $\delta$. Here, all curves are obtained at zero temperature and $t/E_F = 0.45$.

4. Conclusions

In summary, we have studied OFR of $^{173}$Yb ultracold atoms in a 1D optical lattice. The investigation is based on the excited-state (out-of-phase) solution of a two-channel model. We found three fundamentally important properties: the superfluid-LFL crossover corresponding to the tunneling energy, the particle condensation in momentum space, and the superfluid-LFL phase transition corresponding to the temperature. All these properties can be manipulated by the OFR. For ultracold atoms, it is understood that the distribution corresponding to the momentum must be nonuniform. But a condensation corresponding to the momentum obtained here is unexpected. This condensation is especially evident when the OFR detuning $\delta$ is large as shown in figure 5(c). In this case, the condensation happens mainly in two areas in the open channel: $0 \leq k/2k_L \leq 0.4$ and $0.7 \leq k/2k_L \leq 1.1$. The reason for this momentum condensation relies on the property of the energy dispersion relation. A phase transition in any system is crucially important in physics. The superfluid-LFL phase transition corresponding to the temperature here can also be manipulated by the tunneling energy $t$ and OFR detuning $\delta$. It is found that the critical superfluid-normal transition temperature may be reduced even to zero with increasing values of the tunneling energy $t$. The superfluid phase disappears as the critical superfluid-normal transition temperature is $T_c \approx 0$ for the tunneling energy $t/E_F \geq 1.7$. In general, for a given system, $t$ may not be changed, so the OFR detuning $\delta$ is a very suitable parameter for controlling the critical temperature of the phase transition. The method developed in this paper can be extended to the analysis of other related problems in an optical lattice, such as the chiral edge currents of these topological edge states in synthetic ladders [17, 18, 25–28].

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References

[1] Eagles D M 1969 Phys. Rev. 186 456–63
[2] Leggett A J 1980 Diatomic molecules and Cooper pairs Modern Trends in the Theory of Condensed Matter Proc. 16th Karpacz Winter School of Theoretical Physics 19 February–3 March 1979. Karpacz, Poland ed A Pęksalski and J A Przystawa (Berlin: Springer) (https://doi.org/10.1007/BFb0120123)
[3] Zhang R, Cheng Y, Zhai H and Zhang P 2015 Phys. Rev. Lett. 115 135301
[4] Höfer M, Rieger L, Scazza F, Hofrichter C, Fernandes D R, Parish M M, Levinse J, Bloch I and Fölling S 2015 Phys. Rev. Lett. 115 265302
[5] Chin C, Grimm R, Julienne P and Tiesinga E 2010 Rev. Mod. Phys. 82 1225–86
[6] Kato S, Sugawa S, Shibata K, Yamamoto R and Takahashi Y 2013 Phys. Rev. Lett. 110 173201
