Nonperturbative approach to quench dynamics. II. Universal electric current of the nonequilibrium Kondo model

Adrian B. Culver* and Natan Andrei†

Center for Materials Theory, Department of Physics and Astronomy, Rutgers University, Piscataway, NJ 08854

(Dated: December 5, 2019)

In the previous paper, we found a series expression for the average electric current following a quench in the nonequilibrium Kondo model driven by a bias voltage. Here, we evaluate the steady state current in the regimes of strong and weak coupling. We obtain the standard leading order results in the usual weak antiferromagnetic regime, and we also find a new universal regime of strong ferromagnetic coupling with Kondo temperature $T_K = D e^{-\frac{\Delta}{K}}$. In this regime, the differential conductance $dI/dV$ reaches the unitarity limit $2e^2/h$ asymptotically at large voltage or temperature.

I. INTRODUCTION

The Kondo model, in which a localized spin interacts via spin flips with one or more reservoirs of electrons, has long been a source of new ideas in theoretical physics and a testing ground for new methods. Starting in the late 1990s, experimenters realized the model in quantum dot systems: a small number of electrons are confined to a nanoscale region, a single unpaired electron acts as the localized spin, and two attached leads serve as reservoirs (see [1–4], for example). These systems can be precisely controlled in ways that solid state systems cannot, allowing for the exploration of nonequilibrium quantities such as the electric current through the dot driven by a voltage drop across the leads.

The universal antiferromagnetic regime of the Kondo model out of equilibrium has been studied theoretically by a variety of approaches, including Keldysh perturbation theory [5–7], flow equations [8], the real-time renormalization group [9], and the variational principle [10]; the Kondo regime has also been studied in the Anderson model using perturbation theory [11], Fermi liquid theory [12], integrability [13], the Scattering Bethe Ansatz [14], Dynamical Mean Field Theory [15], quantum Monte Carlo [16], and matrix product states [17], among other methods. A more complete list of theoretical works on this subject can be found in the references in [10]. In contrast, the strong ferromagnetic regime that we explore later (and prove the universality of) has received little attention.

With universality in mind, we consider the two lead Kondo model in the flat bandwidth limit:

\[ H = -i \int_{-L/2}^{L/2} dx \sum_{\gamma=1}^{2} \frac{d}{dx} \psi_{\gamma}(x) \left( \frac{d}{dx} \psi_{\gamma}(x) \right) + \sum_{\gamma,\gamma'=1,2} \frac{1}{2} J_{\gamma\gamma'}(0) \sigma_{\alpha\alpha'} \psi_{\gamma}(0) \cdot \mathbf{S}. \]  

In the previous paper [18], we introduced a new method for solving quench problems and used it to solve exactly for the time-dependent many body wavefunction $e^{-iHt}|\Psi\rangle$, where the initial state $|\Psi\rangle$ had each lead filled with a Fermi sea (with the bias voltage $V$ appearing as the difference of chemical potentials). In the thermodynamic limit, we found a series expression for the average electric current, which could be expanded either in powers of $J$ for small $J$ or in powers of $1/J$ for large $|J|$. We argued that the series reaches a steady state limit:

\[ I_{\text{steady state}}(T_1, T_2, V) = \lim_{t \to \infty} I(T_1, \mu_1; T_2, \mu_2; t), \]  

where $T_1, \mu_1$ and $T_2, \mu_2$ are the temperatures and chemical potentials of the leads (we set $\mu_1 = 0$ and $\mu_2 = -V$).

In this paper, we evaluate this series in more detail in the steady state limit, exploring both the usual universal regime of weak antiferromagnetic coupling and a new universal regime of strong ferromagnetic coupling.

In each regime, we allow the external parameters $T_1, T_2$, and $V$ to be arbitrary in order to investigate the scaling properties of the steady state current using the Callan-Symanzik equation. We find the standard scaling at leading order for weak antiferromagnetic coupling, and a new $T_K$ for strong ferromagnetic coupling. We then consider two commonly studied special cases – the linear response conductance $G(T)$ and the nonequilibrium zero temperature conductance $G(V)$:

\[ G(T) \equiv \left. \frac{\partial}{\partial V} I_{\text{steady state}}(T_1 = T, T_2 = T, V) \right|_{V=0}, \]  

\[ G(V) \equiv \left. \frac{\partial}{\partial V} I_{\text{steady state}}(T_1 = 0, T_2 = 0, V) \right|_{V=0}. \]

Our results for these quantities again agree with standard results at leading order in the antiferromagnetic regime. In the ferromagnetic regime, we find that $G(T)$ and $G(V)$ both approach the unitarity limit $G_0 \equiv 2e^2/h$ asymptotically for $T \gg T_K$ or $V \gg T_K$. We also discuss the thermoelectric current $I(T_1, T_2, V = 0)$, finding that it vanishes up to and including the equivalent of the third loop order contribution in the antiferromagnetic regime.

The paper is organized as follows. In Sec. II, we set up the calculation of the steady state current, building

* adrianculver@physics.rutgers.edu
† natan@physics.rutgers.edu
II. STEADY STATE LIMIT OF THE CURRENT

From our results in [18], we can write the steady state current in the following series form:

\[ I_{\text{steady state}}(T_1, T_2, V) = \frac{1}{2\pi} Re \left\{ \sum_{n=1}^{\infty} \frac{1}{(i\pi)^{n-1}} \times \sum_{\sigma \in \text{Sym}(n)} W^{(\sigma)}(J) \phi^{(\sigma)}(T_1, T_2, V) \right\} . \]  

The \( W^{(\sigma)}(J) \) terms are spin sums that can be evaluated fairly quickly – see Table I in [18]. The \( \phi^{(\sigma)} \) terms are given by:

\[
\phi^{(\sigma)}(T_1, T_2, V) = \frac{1}{i} \int dx_1 \ldots dx_{n-1} \left[ \prod_{j=1}^{n-1} \left( \frac{e^{i(D-\frac{1}{2})y_j^{(\sigma)}}}{y_j^{(\sigma)}} - \frac{\pi T_1 e^{-i\frac{1}{2}V y_j^{(\sigma)}}}{2 \sinh(\pi T_1 y_j^{(\sigma)})} - \frac{\pi T_2 e^{i\frac{1}{2}V y_j^{(\sigma)}}}{2 \sinh(\pi T_2 y_j^{(\sigma)})} \right) \right] \times \left[ \frac{\pi T_2 e^{i\frac{1}{2}V y_n^{(\sigma)}}}{\sinh(\pi T_2 y_n^{(\sigma)})} - \frac{\pi T_1 e^{-i\frac{1}{2}V y_n^{(\sigma)}}}{\sinh(\pi T_1 y_n^{(\sigma)})} \right],
\]

where \( D \) is the bandwidth (which is first introduced in the calculation as a lower cutoff on the Fermi seas in the initial state \( |\Psi\rangle \)), and:

\[ y_j^{(\sigma)} = \sum_{m \neq j}^{n-1} x_m - \sum_{m = \sigma^{-1}(j)}^{n-1} x_m. \]

Eq. (5) is obtained by setting \( \mu_1 = 0, \mu_2 = -V \) in Eq. (3.33) of [18] and taking the long time limit (which has the effect of deleting the Heaviside function). The linear combinations in (6) (or rather, the dimensionless versions \( v_j \) that appear in Eq. (8a) below) are listed in Table I in Appendix A.

It is to be expected that for large bandwidth, the steady state integrals \( \phi^{(\sigma)}(T_1, T_2, V) \) include powers of \( \ln D/T_1, \ln D/T_2, \) and \( \ln D/V \). These logarithmic divergences – together with the coupling constant dependence contained in the spin sums \( W^{(\sigma)}(J) \) – encode the scaling properties and the emergence of the strong ferromagnetic coupling and weak ferromagnetic coupling, which are non-universal.

This works as long as \( V \) is non-zero; after presenting this formulation, we show that the case of \( V = 0 \) can be treated similarly. In this way, the full parameter space of \( (T_1, T_2, V) \) is covered.

For non-zero bias, we rescale to dimensionless integration variables \( u_j = \frac{1}{2} V x_j \) to find:

\[
\phi^{(\sigma)}(T_1, T_2, V) = V \int_0^{\infty} du_1 \ldots du_{n-1} h \left( \frac{T_1}{V}, \frac{T_2}{V}, u_1^{(\sigma)} \right) \times \prod_{j=1}^{n-1} f \left( \frac{u_j^{(\sigma)}}{v_1^{(\sigma)}}, v_j^{(\sigma)} \right),
\]

where we have defined:

\[ u_j^{(\sigma)} = \sum_{m \neq j}^{n-1} u_m - \sum_{m = \sigma^{-1}(j)}^{n-1} u_m, \]  

\[ f(s_1, s_2, v) = v \left( \frac{\pi s_1 e^{-iv}}{\sinh(2\pi s_1 v)} + \frac{\pi s_2 e^{iv}}{\sinh(2\pi s_2 v)} \right), \]  

\[ h(s_1, s_2, v) = \frac{1}{i} \left( \frac{\pi s_2 e^{iv}}{\sinh(2\pi s_2 v)} - \frac{\pi s_1 e^{-iv}}{\sinh(2\pi s_1 v)} \right). \]

We have explicitly calculated the asymptotic forms of these integrals in the large bandwidth regime for all permutations \( \sigma \) that we need in order to find the current up to and including the \( J^5 \) or \( 1/J^5 \) terms. We find that the rapidly oscillating phases in the integrals defined by (7) generate logarithmic divergences – powers of \( \ln D/V \) with coefficients that depend on the ratios \( T_1/V \) and \( T_2/V \).
In some cases, there are also linear divergences, but they cancel in the final answer for the current at this order.

We show next that the calculation for $V = 0$ reduces to the same dimensionless integral that appears in Eq. (7) with a different oscillating phase and different functions $f$ and $h$. If we assume $T_1 > 0$ with $T_2$ and $V$ arbitrary (in particular this allows $V = 0$), then rescaling to $u_j = \pi T_1 x_j$ yields:

$$\varphi^{(v)}(T_1, T_2, V) = T_1 \int_0^\infty du_1 \ldots du_{n-1} h\left(\frac{T_2}{T_1}, \frac{V}{T_1}, \varepsilon_n^{(v)}\right) \times \prod_{j=1}^{n-1} \frac{e^{i\frac{\partial}{\partial \varepsilon_j^{(v)}}} - f\left(\frac{T_1}{T_2}, \frac{V}{T_1}, \varepsilon_j^{(v)}\right)}{v_j^{(v)}},$$

(9)

with $f$ and $h$ defined in this case by $f(s_1, s_2, v) = \frac{1}{2} \ln \left(\frac{\sinh v - \sinh(s_2 v)}{\sinh v - \sinh(s_1 v)}\right)$ and $h(s_1, s_2, v) = \frac{1}{2} \frac{g_1(s_2 v) - g_2(s_1 v)}{\sinh v - \sinh v}$. In this formulation, the logarithmic divergences are powers of $\ln D/T_1$, with coefficients that depend on $T_2/T_1$ and $V/T_2$.

While the linear response conductance $G(T)$ can in principle be obtained as a special case, it is more straightforward to calculate it directly. We again find the same dimensionless integral that appears in Eq. (7) with a different oscillating phase and different functions $f$ and $h:

$$\left.\frac{\partial}{\partial V}\right|_{V=0} \varphi^{(v)}(T_1 = T, T_2 = T, V) = \int_0^\infty du_1 \ldots du_{n-1} \times h\left(v_n^{(v)}\right) \prod_{j=1}^{n-1} \frac{e^{i\lambda u_j^{(v)}} - f\left(v_j^{(v)}\right)}{v_j^{(v)}},$$

(10)

where $f$ and $h$ are given in this case by $f(v) = h(v) = v/\sinh v$.

Evidently, all cases reduce to the study of the large $\lambda$ behavior of the following general form:

$$\int_0^\infty du_1 \ldots du_{n-1} h\left(v_n^{(v)}\right) \prod_{j=1}^{n-1} \frac{e^{i\lambda u_j^{(v)}} - f\left(v_j^{(v)}\right)}{v_j^{(v)}},$$

(11)

where $f$ and $h$ take various forms (and may depend on ratios such as $T_1/V$). Appendix A presents our asymptotic results for the general form given in (11), with $f$ and $h$ unspecified so that all of the above cases can be considered at once. The simplest non-trivial example is the permutation $\sigma = (2, 1)$, for which we have $v_1^{(v)} = -v_2^{(v)} = u_1$ and the following asymptotic result:

$$\int_0^\infty du_1 \frac{e^{i\lambda u_1} - f(u_1)}{u_1} h(-u_1) \xrightarrow{\lambda \to \infty} -h(0) \ln \lambda$$

$$-h(0) \left(\gamma - i\frac{\pi}{2}\right) + \int_0^\infty du \ln u \frac{d}{du} \left[f(u)h(-u)\right],$$

(12)

where $\gamma$ is the Euler constant (not to be confused with the anomalous dimension $\gamma(g)$ that we discuss later). In the steady state current in the regime of small $J$, the $\ln \lambda$ divergence here will be appear multiplied by $J^3$ – thus, it is the equivalent of the one loop divergence that appears in a Keldysh calculation.

Notice that the constant ($\lambda$-independent) term in (12) is a more complicated functional of $f$ and $h$ than the log term. This is the beginning of a pattern that persists to higher orders. For example, in the case of $\sigma = (2, 3, 1)$ that is explicitly written out in Appendix A, there is a $\ln^2 \lambda$ term that depends only on $h(0)$, a $\ln \lambda$ term involving both $h(0)$ and the same single variable integral over $f$ and $h$ that appears in (12), and then a $\lambda$-independent constant that depends on the same quantities already encountered in $\ln^2 \lambda$ and $\ln \lambda$ and also on a double integral involving $f$ and $h$. These terms then appear in the small $J$ current multiplied by $J^4$ (two loops). This pattern of asymptotic expansion is the mechanism underlying the scaling we find in the following two sections.

### III. ANTI-FERROMAGNETIC REGIME – UNIVERSALITY

We evaluate our current series in the regime of weak antiferromagnetic coupling. We first review what scaling properties are expected on general grounds, then present the results of our calculations. For easier comparison with the literature, we refer to $g \equiv \rho J = \frac{\pi}{\gamma}J$ from now on.

It is expected that, when all other scales in the problem are much smaller than the bandwidth, the current becomes a universal function $I_{\text{universal}}(T_1/T_K, T_2/T_K, V/T_K)$, where the Kondo temperature $T_K = D e^{\frac{-\rho \pi}{\gamma} + \frac{1}{2} \ln \rho}$ is a dynamically generated scale. The “scaling limit” consists of taking $D \to \infty$ and $g \to 0^+$ with $T_K$ fixed; the resulting $I_{\text{universal}}$ is then the same as that which would be obtained from taking the low energy limit of a calculation done with a more realistic Hamiltonian, e.g. three dimensional with a more complicated band structure.

Let us first consider $V$ as the independent variable, with $T_1$ and $T_2$ related to $V$ in fixed ratios. Universal scaling should then manifest itself in a pattern of logarithmic divergences as $D/V$ is sent to infinity. To be precise, the perturbative renormalizability of the Kondo model constrains the answer to the following form at large bandwidth: $I_{\text{steady}}(T_1, T_2, V) \to I_{\text{scaling}}(T_1, T_2, V) = \sum_{n=2, m<n} a_{mn} g^m \ln^n D$, where the coefficients $a_{mn}$ depend only on the ratios $T_1/V$ and $T_2/V$. (This is shown in a very general setting by Delamotte in [19]. We have assumed that the current starts at order $g^2$, as is confirmed by calculation.) This scaling form then should satisfy the Callan-Symanzik equation $\left(\frac{D}{g^2} \frac{\partial}{\partial D} + (\beta(g) \frac{\partial}{\partial g} + \gamma(g))\right) I_{\text{scaling}} = 0$, which is a differential form of the statement that all UV divergences can be absorbed by using a run-
ning coupling constant and rescaling the current operator. The solution to the Callan-Symanzik equation takes the form

\[ I_{\text{scaling}}(T_1, T_2, V) = \int f_{\text{universal}}(T_1/T_K, T_2/T_K, V/T_K) e^{-\int_0^\infty dg' \frac{\gamma(g')}{\beta(g')}} \]

and the anomalous dimension \( \gamma(g) \) should start at the same order or higher in \( g \) as the beta function \( \beta(g) \) so that the \( g \)-dependent scale factor goes to unity in the scaling limit. (Such a scale factor has been seen before in the Kondo problem; see Ref. [20].)

These general expectations are largely met by our series. Up to and including the equivalent of three loops (which is \( g^5 \) in this case), the current at large bandwidth is a scaling form that satisfies the Callan-Symanzik equation with \( \beta(g) \) and \( \gamma(g) \) that are independent of the ratios \( T_1/V \) and \( T_2/V \). The leading order of the beta function \( \beta(g) = -2g^2 \), and the corresponding leading order expression \( T_K = De^{-\frac{g}{V}} \), are in exact agreement with the standard answer [21]. The tree level constant \( (g^2) \) and the one loop constant \( (g^3) \) are in exact agreement with calculations of \( G(T) \) and \( G(V) \) in the literature. The only surprise is that the first correction to beta, and hence \( T_K \), differs by a constant from the expected form; that is, we obtain \( \beta(g) = -2g^2 + 16g^3 \), and hence \( T_K = De^{-\frac{g}{V}} + 4 \ln g \).

The effect of this on physical quantities is to introduce small corrections to the leading order behavior which is set by the tree level answer and the one loop beta function.

Let us present these results in more detail. We begin by writing the scaling form that we find for the current. We will write the series in a triangular structure [19] in which the \( n \)th column contains the \( g^{n+1} \) terms, while the \( n \)th row contains terms of the form \( g^{n+j} \ln^{j-1} \frac{D}{V} \) \((j \geq 1)\). The entries in the first row are called the “leading logarithms,” the second row the “sub-leading logarithms,” and so on. For large bandwidth, we find:

\[
I(T_1, T_2, V) = \frac{3\pi}{4} \left\{ g^2 + 4g^3 \ln \frac{D}{2V} + 12g^4 \ln^2 \frac{D}{2V} + 32g^5 \ln^3 \frac{D}{2V} \right. \\
\left. + C_1 \left( \frac{T_1}{V}, \frac{T_2}{V} \right) g^3 + 6C_1 \left( \frac{T_1}{V}, \frac{T_2}{V} \right) g^4 \ln \frac{D}{2V} + \left[ 24C_1 \left( \frac{T_1}{V}, \frac{T_2}{V} \right) - 32 \right] g^5 \ln^2 \frac{D}{2V} \\
\left. + C_2 \left( \frac{T_1}{V}, \frac{T_2}{V} \right) g^4 + \left( 16C_1 \left( \frac{T_1}{V}, \frac{T_2}{V} \right) - 8C_2 \left( \frac{T_1}{V}, \frac{T_2}{V} \right) \right) g^5 \ln \frac{D}{2V} \right. \\
\left. + C_3 \left( \frac{T_1}{V}, \frac{T_2}{V} \right) g^5 + O(g^6) \right\},
\]

where \( C_1 \) and \( C_2 \) are given by:

\[
C_1 \left( \frac{T_1}{V}, \frac{T_2}{V} \right) = 4 \text{ Re} \left\{ \gamma - \int_0^\infty du \ln u \frac{\partial}{\partial u} \left[ f \left( \frac{T_1}{V}, \frac{T_2}{V}, u \right) h \left( \frac{T_1}{V}, \frac{T_2}{V}, -u \right) \right] \right\} \\
C_2 \left( \frac{T_1}{V}, \frac{T_2}{V} \right) = \text{ Re} \left\{ \frac{6\gamma C_1 \left( \frac{T_1}{V}, \frac{T_2}{V} \right) - 12\gamma^2 + 7}{12} \pi^2 - 4 \int_0^\infty \ln^2 u \frac{\partial}{\partial u} \left[ f \left( \frac{T_1}{V}, \frac{T_2}{V}, u \right) h \left( \frac{T_1}{V}, \frac{T_2}{V}, -u \right) \right] \\
+ 8 \int_0^\infty du_1 du_2 \ln u_1 \frac{\partial}{\partial u_1} \frac{\partial}{\partial u_2} \left[ f \left( \frac{T_1}{V}, \frac{T_2}{V}, u_1 \right) f \left( \frac{T_1}{V}, \frac{T_2}{V}, u_2 \right) h \left( \frac{T_1}{V}, \frac{T_2}{V}, -u_1 - u_2 \right) \right] \\
+ 8 \int_0^\infty du_1 du_2 \frac{1}{u_2} \ln \frac{u_1 + u_2}{u_1} \frac{\partial}{\partial u_1} \left[ f \left( \frac{T_1}{V}, \frac{T_2}{V}, u_1 + u_2 \right) f \left( \frac{T_1}{V}, \frac{T_2}{V}, -u_1 \right) h \left( \frac{T_1}{V}, \frac{T_2}{V}, u_2 \right) \right] \right\}.
\]

We omit the explicit form of \( C_3 \) (a sum of integrals over \( f \) and \( h \), including triple integrals), as it is lengthy.

As discussed in more detail by Delamotte, this triangular structure makes clear the operation of perturbative renormalizability. (Delamotte does not consider anomalous scaling \( \gamma(g) \), but this is a simple modification.) One can see that the leading logs are built from pure numbers, the sub-leading logs include pure numbers and the constant \( C_1 \), and so on. We emphasize that we do not require the answer to take this scaling form; we find it as the result of a detailed calculation.

The scaling form Eq. (13) satisfies the Callan-Symanzik equation with:

\[
\beta(g) = -2g^2 + 16g^3 + O(g^4),
\]

\[
\gamma(g) = -32g^2 + (64 + 3\pi^2 - 2\beta_4)g^3 + O(g^4),
\]
where the constant $\beta_4$ would be determined by the next order ($g^6$, or the equivalent of four loops). As expected on general grounds, $\beta(g)$ and $\gamma(g)$ are found to depend only on the coupling constant $g$; the terms $C_1$ and $C_2$ (which contain all dependence on the ratios $T_1/V$ and $T_2/V$) drop out of the scaling equation entirely.

This parameterization, with $V$ as the independent variable, suffices as long as $V$ is non-zero. For the case of strictly zero voltage, we use the alternate parameterization, with $T_1$ as the independent variable, and obtain the exact same $\beta(g)$ and $\gamma(g)$. Thus, we have covered the full parameter space ($T_1, T_2, V$), confirming that the current becomes in the scaling limit a function of $T_1/T_K$, $T_2/T_K$, and $V/T_K$ only. We also note that the same $\beta(g)$ and $\gamma(g)$ are again obtained if we consider the linear response conductance $G(T)$ directly.

We now turn to the two special cases mentioned earlier, $G(T)$ and $G(V)$ (Eqs. (3a) and (3b)). Restoring the dimensionful factor $G_0 = 2e^2/h = (1/\pi$ in the natural units we have been using) to the conductance, we find:

\[
G(T) = \frac{3\pi^2 G_0}{4} \left\{ g^2 + 4g^3 \ln \frac{D}{T} + 12g^4 \ln^2 \frac{D}{T} + 32g^5 \ln^3 \frac{D}{T} - 4 \ln \frac{2\pi e^{1+\gamma}}{T} g^3 - 24 \ln \frac{2\pi e^{1+\gamma}}{T} g^4 \ln \frac{D}{T} - 32 \left( \ln \frac{2\pi e^{1+\gamma}}{T} + 1 \right) g^5 \ln^2 \frac{D}{T} - 7.75g^4 \right. + 138.90g^5 \ln \frac{D}{T} + 9.01g^5 + O(g^6) \right\}, \tag{16a}
\]

\[
G(V) = \frac{3\pi^2 G_0}{4} \left\{ g^2 + 4g^3 \ln \frac{D}{V} + 12g^4 \ln^2 \frac{D}{V} + 32g^5 \ln^3 \frac{D}{V} - 4 \ln \frac{2\pi e^{1+\gamma}}{V} g^3 - 24 \ln \frac{2\pi e^{1+\gamma}}{V} g^4 \ln \frac{D}{V} - 32g^5 \ln^2 \frac{D}{V} - (24 + 17\pi^2) g^5 \ln \frac{D}{V} + 2 \left[ \pi^2 - 32 + 48 \ln 2 - 24\zeta(3) \right] g^5 + O(g^6) \right\}, \tag{16b}
\]

where $\zeta$ is the Riemann zeta function. Using the Callan-Symanzik equation to take the scaling limit, we find the following results in the high energy regime ($T \gg T_K$ or $V \gg T_K$):

\[
G(T) = \frac{3\pi^2 G_0}{16 \ln^2 \frac{D}{T_K}} \left[ 1 + 8 \ln \frac{\ln \frac{T}{T_K}}{\ln \frac{D}{T_K}} + \alpha_1^{(T)} \ln \frac{\ln \frac{T}{T_K}}{\ln \frac{D}{T_K}} + \alpha_2^{(T)} \ln \frac{\ln \frac{T}{T_K}}{\ln \frac{D}{T_K}} + O \left( \ln^2 \frac{\ln \frac{T}{T_K}}{\ln \frac{D}{T_K}} \right) \right], \tag{17a}
\]

\[
G(V) = \frac{3\pi^2 G_0}{16 \ln^2 \frac{D}{V_K}} \left[ 1 + 8 \ln \frac{\ln \frac{V}{V_K}}{\ln \frac{D}{V_K}} + \alpha_1^{(V)} \ln \frac{\ln \frac{V}{V_K}}{\ln \frac{D}{V_K}} + \alpha_2^{(V)} \ln \frac{\ln \frac{V}{V_K}}{\ln \frac{D}{V_K}} + O \left( \ln^2 \frac{\ln \frac{V}{V_K}}{\ln \frac{D}{V_K}} \right) \right], \tag{17b}
\]

where the $\alpha_j^{(T)}, \alpha_j^{(V)}$ constants are:

\[
\alpha_1^{(T)} = 8 \left( 1 + \ln 2 \right) - 2 \ln \frac{2\pi}{e^{1+\gamma}}, \tag{18a}
\]
\[
\alpha_1^{(V)} = 8 \left( 1 + \ln 2 \right), \tag{18b}
\]
\[
\alpha_2^{(T)} = 4 \left( 2 + 3 \ln 2 \right) + 3 \ln \frac{2\pi}{e^{1+\gamma}}, \tag{18c}
\]
\[
\alpha_2^{(V)} = 4 \left( 2 + 3 \ln 2 \right). \tag{18d}
\]

Note that the individual values of $\alpha_1^{(T)}$ and $\alpha_1^{(V)}$ can be changed by rescaling $T_K$ by an overall constant prefactor. In this high energy regime, one can define $T_K^{(T)}$ as the rescaling that sets $\alpha_1^{(T)}$ to zero, with a similar definition for $T_K^{(V)}$; then the ratio $T_K^{(T)}/T_K^{(V)} = \exp \left( \left( \alpha_1^{(T)} - \alpha_1^{(V)} \right) /2 \right) = \frac{e^{\gamma - \gamma_1}}{2\pi}$ is independent of rescaling.

Let us compare these results with the literature. The leading order results for $G(T)$ and $G(V)$ are well-known [5]; they are exactly the resummation of the leading logarithms (i.e. the first row of Eq. (16a) or of Eq. (16b)), or equivalently, the RG improvement of the tree level result $\frac{3\pi^2 G_0}{4} g^2$ via the standard one loop beta function.

For a higher order check of $G(T)$ and $G(V)$, we com-
pare to the real-time renormalization group calculation of Pletyukhov and Schoeller (PS) [9]. While these authors calculated the full conductance curves numerically, we are concerned for the moment with comparing to the analytical expressions they find for the first two terms ($g^2_{\alpha}$ and $g^3_{\alpha}$) of $G(T)$ and $G(V)$ as power series in the running coupling $g_R$. Re-expressing their answers in terms of bare quantities, we note that the $D$-independent $g^3$ terms of our series (the distinctive number $ln^{2}\pi g$ for $G(T)$ and zero for $G(V)$) are in exact agreement with PS. In turn we have exact agreement for the ratio $T_K^{(C)}/T_K^{(V)}$. Our scaling differs from theirs at higher order, seeing as they find the conventional expression $\beta(g) = -2 g^2 + 2 g^3 + O(g^4)$. Conventional scaling would have been obtained in our calculation had an additional contribution $3 \pi^2 G_0 \left( g^4 \ln \frac{D}{T} - g^3 \ln^2 \frac{D}{T} \right)$ been present in $G(T)$ (or the same term in $G(V)$ with $V$ replacing $T$), but extensive checks have not detected any such contribution. The first terms in the final answers Eq. (17a) and Eq. (17b) that affects are the double log terms $\ln \ln \frac{T}{K}$ and $\ln \ln \frac{V}{K}$; with the conventional beta function, the coefficient 8 would instead be 1. (Note that the coefficients of the leading terms, $1/\ln^2(T/T_K)$ and $1/\ln^2(V/T_K)$, are unaffected.)

We therefore conclude that our approach yields the correct leading behavior in the high energy regime ($T \gg T_K$ or $V \gg T_K$), with the leading correction apparently differing by an overall constant from the conventional cutoff scheme. We have confirmed this surprising result by doing the calculation in several equivalent ways which are described in Appendix B; more checks are also reported in Appendix F of the previous paper.

We also find that the thermoelectric current $I(T_1, T_2, V = 0)$ vanishes up to and including the third loop order term ($g^3$). This follows from a detailed cancellation of many terms, and we do not know if there is any non-vanishing contribution from higher orders in $g$.

IV. FERROMAGNETIC REGIME – UNIVERSALITY

Our approach also reveals a new universal regime of the Kondo model: strong ferromagnetic coupling ($g < 0$, $|g| \gg 1$). We note that there are several proposed mesoscopic realizations [22–24] of the weak ferromagnetic model; it may be possible to realize strong ferromagnetism by modifying these proposals to use the charge Kondo effect [25]. We find that the strong ferromagnetic model generates a new Kondo temperature given at leading order by $T_K = D e^{4\tilde{C}_1 g^2}$. A very similar discussion applies in this case as in the antiferromagnetic regime. (Indeed, the quantity $-1/g$, which is small and positive, plays much the same role as a small antiferromagnetic coupling, though the parallel is not exact.) The scaling limit in this regime consists of taking $D \to \infty$ and $g \to -\infty$ with $T_K$ fixed; the resulting universal functions are expected to agree with the low energy results from a more realistic Hamiltonian.

We begin in the same way as in the antiferromagnetic case, by examining the scaling. We again set $V$ to be the independent variable and take $T_1/V$ and $T_2/V$ to be fixed constants. The same integrals appear again; the only change we need to make is to expand the spin sums $W(\sigma)(J)$ about $J = -\infty$ instead of $J = 0$. (We can actually expand the spin sums about $|J| = \infty$ with the same result for either sign; we return to the case of large positive $J$ in Sec. V.) We find the following scaling form at large bandwidth:

$$I(T_1, T_2, V) = \frac{1}{\pi} V \left\{ 1 - \frac{4}{9 \pi^2} \left[ \frac{7}{g^2} - 16 \pi^2 g^3 \ln \frac{D}{2V} + \frac{64}{\pi^4 g^4} \ln^2 \frac{D}{2V} - \frac{2048}{9 \pi^6 g^6} \ln^3 \frac{D}{2V} \right] - C_1 \frac{16}{\pi^2 g^3} + C_1 \frac{128}{\pi^4 g^4} \ln \frac{D}{2V} \right.$$  

$$+ \left( 3 C_2 + 6 \pi \tilde{C}_1 - 22 \pi^2 \right) \frac{16}{g^4} + \left( 32 - 8 C_2 + 16 C_1 \right) \frac{16}{-12 \pi \tilde{C}_1 + 11 \pi^2} \frac{64}{9 \pi^6 g^6} \ln \frac{D}{2V}$$  

$$+ C_4 \frac{1}{g^6} + O \left( \frac{1}{g^8} \right) \right\} \right\}$$  

where $C_1$, $C_2$, $\tilde{C}_1$, and $C_4$ depend on the ratios $T_1/V$ and $T_2/V$; the first two have been defined already in Eq. (14a) and Eq. (14b), $\tilde{C}_1$ is the imaginary part of the same quantity that appears in $C_1$,

$$\tilde{C}_1 \left( \frac{T_1}{V}, \frac{T_2}{V} \right) = 4 Im \left\{ \gamma - \int_0^\infty du \ln u \frac{\partial}{\partial u} \left[ f \left( \frac{T_1}{V}, \frac{T_2}{V}, u \right) h \left( \frac{T_1}{V}, \frac{T_2}{V}, -u \right) \right] \right\},$$

and $C_4$ is given by a lengthy sum of integrals over $f$ and $h$, which we omit. This expansion is valid for either sign.
of $g$, though we focus on the ferromagnetic case $g < 0$ for now.

For $T_1 = T_2$, we find that the Callan-Symanzik equation holds with a non-zero anomalous dimension $\gamma(g)$:

$$\beta(g) = -\frac{8}{3\pi^2} \left[ 1 + \frac{32}{9\pi^2 g} + \frac{\tilde{\beta}_2}{\pi^4 g^2} + O \left( \frac{1}{g^3} \right) \right], \quad (21a)$$

$$\gamma(g) = \frac{256}{27\pi^4 g^3} \left[ 1 + \frac{56}{9\pi^2 g} + \frac{1}{\pi^4} \left( \frac{7\tilde{\beta}_2}{4} - \frac{115}{9\pi^2} + \frac{64}{3\pi^4} \right) \frac{1}{g^2} + O \left( \frac{1}{g^3} \right) \right], \quad (21b)$$

where the constant $\tilde{\beta}_2$ would be determined by the next order ($1/g^6$). The scaling invariant is the Kondo temperature for this regime:[26]

$$T_K \equiv D e^{\frac{3\pi^2}{4} - \frac{4}{\ln |g|}}. \quad (22)$$

Let us emphasize that the non-zero anomalous dimension $\gamma(g)$ for the current operator is necessary in this case to resum even the leading logarithms. Concretely, this means that one would not obtain the correct beta function by compensating a change in coupling constant in the $1/g^2$ term by a change of bandwidth in the $(1/g^3) \ln \frac{D}{T}$ term; the resulting beta function would not be consistent with the next term, $(1/g^4) \ln^2 \frac{D}{T}$. One is forced rescale the whole observable as well, which is equivalent to introducing $\gamma(g)$.

Curiously, the scaling breaks down if the lead temperatures are different ($T_1 \neq T_2$).

For the special cases $G(T)$ and $G(V)$, we obtain:

$$G(T) = G_0 \left\{ 1 - \frac{4}{9\pi^2} \left[ \frac{7}{9\pi^2 g^2} \ln \frac{D}{T} + \frac{16}{\pi^4 g^3} \ln^2 \frac{D}{T} - \frac{2048}{9\pi^6 g^5} \ln^3 \frac{D}{T} \right] + \frac{16}{\pi^2 g^3} \ln \frac{2\pi}{e^{\gamma_T}} - \frac{128}{\pi^4 g^3} \ln^2 \frac{2\pi}{e^{\gamma_T}} \ln \frac{D}{T} + \frac{2048}{9\pi^6 g^6} \left( 3 \ln \frac{2\pi}{e^{\gamma_T}} + 1 \right) \ln^2 \frac{D}{T} - 4.39 \frac{1}{g^4} + 1.61 \frac{1}{g^5} \ln \frac{D}{T} - 0.22 \frac{1}{g^6} + O \left( \frac{1}{g^7} \right) \right\}, \quad (23a)$$

$$G(V) = G_0 \left\{ 1 - \frac{4}{9\pi^2} \left[ \frac{7}{9\pi^2 g^2} \ln \frac{D}{V} + \frac{16}{\pi^4 g^3} \ln^2 \frac{D}{V} - \frac{2048}{9\pi^6 g^5} \ln^3 \frac{D}{V} \right] + \frac{16}{\pi^2 g^3} \ln \frac{2\pi}{e^{\gamma_V}} - \frac{128}{\pi^4 g^3} \ln^2 \frac{2\pi}{e^{\gamma_V}} \ln \frac{D}{V} + \frac{2048}{9\pi^6 g^6} \left( 3 \ln \frac{2\pi}{e^{\gamma_V}} + 1 \right) \ln^2 \frac{D}{V} - 436 \frac{1}{9\pi^2 g^4} + 1.61 \frac{16}{27\pi^6 g^5} \left[ 192 \left( 4 - 6 \ln 2 + 3\zeta(3) \right) - 24\pi^2 \right] \ln \frac{D}{V} + O \left( \frac{1}{g^7} \right) \right\}, \quad (23b)$$

In the high energy regime ($T \gg T_K$ or $V \gg T_K$), the running coupling constant is large and negative, and we can use the Callan-Symanzik equation to find the following universal results:

$$G(T) = G_0 \left\{ 1 - \frac{3\pi^2}{16 \ln^2 \frac{T}{T_K}} \left[ 1 + \frac{8 \ln \ln \frac{T}{T_K}}{3 \ln \frac{T}{T_K}} + \frac{\tilde{\alpha}(T)}{3 \ln^2 \frac{T}{T_K}} + \frac{16 \ln^2 \ln \frac{T}{T_K}}{3 \ln^2 \frac{T}{T_K}} + \frac{\tilde{\alpha}(T)}{\ln^2 \frac{T}{T_K}} + O \left( \frac{1}{\ln^2 \frac{T}{T_K}} \right) \right] \right\}, \quad (24a)$$

$$G(V) = G_0 \left\{ 1 - \frac{3\pi^2}{16 \ln^2 \frac{V}{T_K}} \left[ 1 + \frac{8 \ln \ln \frac{V}{T_K}}{3 \ln \frac{V}{T_K}} + \frac{\tilde{\alpha}(V)}{3 \ln^2 \frac{V}{T_K}} + \frac{16 \ln^2 \ln \frac{V}{T_K}}{3 \ln^2 \frac{V}{T_K}} + \frac{\tilde{\alpha}(V)}{\ln^2 \frac{V}{T_K}} + O \left( \frac{1}{\ln^2 \frac{V}{T_K}} \right) \right] \right\}, \quad (24b)$$

where the $\tilde{\alpha}(T), \tilde{\alpha}(V)$ constants are:

$$\tilde{\alpha}(T) = \frac{8}{3} - \frac{8}{3} \ln \frac{3\pi^2}{8} - 2 \ln \frac{2\pi}{e^{\gamma_T}},$$

$$\tilde{\alpha}(V) = \frac{8}{9} - \frac{8}{3} \ln \frac{3\pi^2}{8}, \quad (25b)$$

$$\frac{1}{\tilde{\alpha}_1} = \frac{8}{9} - \frac{8}{3} \ln \frac{3\pi^2}{8}.$$
\[ \bar{\alpha}_2^{(T)} = -8 \left( \frac{4}{9} \ln \frac{27\pi^6}{512} + \ln \frac{2\pi}{e^{1+\gamma}} \right), \]  
\[ \bar{\alpha}_2^{(V)} = -\frac{32}{9} \ln \frac{27\pi^6}{512}. \]  

Notice that the unitarity limit is reached asymptotically at high energy. This is the main novel prediction of our method so far. Ultimately, the unitary conductance traces back to the fact that the bare \( S \)-matrix of the model becomes a single particle phase shift of \( \pi/2 \) in the limit \( |J| \to \infty \); see the “quasiparticle basis” discussed in Sec. II.D of the previous paper.

To see the predicted rise towards unitarity experimentally, one would need a hierarchy of scales \( T_K \ll V \ll E_{\max} \) or \( T_K \ll T \ll E_{\max} \), where \( E_{\max} \) is the lowest energy scale at which the Kondo model is no longer an accurate description of the system. Defining \( T_K^{(T)} \) and \( T_K^{(V)} \) in the same way as in the antiferromagnetic case (see (17b) and below), we find that the universal ratio is the same in this regime: \( T_K^{(T)}/T_K^{(V)} = -\frac{e^{1+\gamma}}{2\pi} \).

\section{V. ON THE NON-UNIVERSAL REGIMES}

The basic picture of scaling in the antiferromagnetic Kondo model is that the theory is effectively strongly coupled at low energies (\( T, V \ll T_K \)), even though the coupling constant that appears in the original Hamiltonian is small \((0 < g \ll 1)\). Loosely speaking, one says that the coupling constant increases as one reduces the measurement scale, reaching infinity at zero energy. It is tempting to suggest, then, that a calculation using the Kondo Hamiltonian with large \( g \) (expanding in powers of \( 1/g \)) would reproduce the low energy regime of the model with small \( g \). In this section, we show that this is not so, both by general arguments and by examining our explicit answers in the large \( g \) regime. Starting from weak coupling and flowing to strong coupling at low energy is not the same as starting the theory at strong coupling.

Our statement does not contradict the many successes of the effective field theory approach to the low energy regime (of the model with small \( g \)), which refers to the leading irrelevant operators around the strong coupling fixed point. Instead, the conclusion is that the effective field theory approach is more sophisticated than the simple idea of taking \( g \) to be large in the original Hamiltonian.

To clarify the point, we must carefully set up the field theoretic version of the renormalization group. For definiteness, we consider a dimensionless observable \( O(D, g, T) \) with temperature \( T \) as the only external scale. Our analysis is not confined to equilibrium, though, and \( T \) can be replaced by any single energy scale (such as a bias voltage). Suppose the observable is calculated as a power series in \( g \), with the leading term being \( g^2 \); then a series expansion in \( g \) must take the form:

\[ O(D, g, T) = g^2 + \sum_{n=3}^{\infty} g^n F_n(D/T), \]  
where \( F_n(D/T) \) are some functions. As discussed in [19], these functions are constrained by the perturbative renormalizability of the model to take a logarithmic form in the \( T \ll D \) regime:

\[ F_n(D/T) = \sum_{m=0}^{n-1} a_{nm} \ln^m \frac{D}{T} + \ldots, \]  

where the \( a_{nm} \) coefficients are pure numbers that depend on the observable being evaluated. The logarithmic terms define the “scaling form” part of the observable:

\[ O_{\text{scaling form}}(D, g, T) = g^2 + \sum_{n=3}^{\infty} \sum_{m=0}^{n-1} a_{nm} g^n \ln^m \frac{D}{T}. \]  

The scaling form satisfies the RG scaling (or Callan-Symanzik) equation:

\[ \left[ D \frac{\partial}{\partial D} + \beta(g) \frac{\partial}{\partial g} + \gamma(g) \right] O_{\text{scaling form}} = 0. \]  

Assuming (as we find for the current) that the leading order of the anomalous dimension term \( \gamma(g) \) starts at the same order or higher than the leading order of the beta function, the solution of the Callan-Symanzik equation then implies that the scaling form can be written as a function of \( T/T_K \) only (where \( T_K \) is the scaling invariant defined by \( (D \frac{\partial}{\partial D} + \beta(g) \frac{\partial}{\partial g}) T_K = 0 \)), up to corrections that vanish as \( g \to 0^+ \):

\[ O_{\text{scaling form}}(D, g, T) = f_{\text{universal}}(T/T_K) \left[ 1 + O(g) \right]. \]  

In the Kondo model, the leading order of the beta function has negative sign. This implies that \( T_K \) can be held fixed while taking the limit \( D \to \infty \) and \( g \to 0^+ \), which means that the function \( f_{\text{universal}}(T/T_K) \) is a universal result for the observable \( O \). In contrast, the scaling invariant cannot be held fixed in the limit \( D \to \infty \) and \( g \to 0^- \) (the ferromagnetic case), and then the function \( f_{\text{universal}}(T/T_K) \) only represents what would happen if the simplified model itself were realized.

Let us focus on the antiferromagnetic \((g > 0)\) case for now. The procedure for calculating the asymptotic behavior of \( f_{\text{universal}}(T/T_K) \) for \( T \gg T_K \) using the first few series coefficients \( a_{nm} \) is well-known. One finds that the solution of the Callan-Symanzik equation is characterized by a running coupling \((g_T = \frac{1}{32\pi} T/T_K \) at the leading approximation) which is found to grow as \( T \) is reduced. As \( T \) approaches \( T_K \) from above, one finds that infinitely many series coefficients are needed; however, non-perturbative techniques confirm that the running coupling keeps growing as \( T \) is reduced. If one ignores momentarily the distinction between the running
coupling and the bare coupling, one can imagine that a series in $1/g$ would provide information about the low temperature behavior of $f_{\text{universal}}(T/T_K)$, much in the same way that a series in $g$ yields the high temperature behavior.

The basic problem with this approach is that if one repeats the same steps with the $1/g$ series – i.e., expand each order of the series for large bandwidth and declare the logarithmic part to be the “scaling form” – one arrives at a scaling form that may not be the same as the one found from the $g$ series. Since the ultimate goal is to take $g \to 0^+$ with $T_K$ fixed, the scaling form of the $g$ series is the correct one. But the parts of this scaling form that describe the small $T/T_K$ behavior of the function $f_{\text{universal}}(T/T_K)$ may appear to be negligible in the $1/g$ series.

A simple example illustrates the point. It is known that the universal conductance curve $G(T)$ reaches unitarity at $T = 0$ with corrections of the form $T^2/T_K^5$. Thus, the scaling form for the conductance must include a contribution of the form $T^2$, seeing as this term becomes $T^2/D^2$ in the $g \to 0^+$ scaling limit (we assume the conventional expression $T_K = D e^{-1/g^r + 1/a_1}$ in this discussion). Since this term vanishes for large bandwidth rather than diverging logarithmically, it is exactly the type of term that is dropped in determining the scaling form of the $1/g$ series. The logarithmically diverging terms, on the other hand, can easily be negligible in the $g \to 0^+$ scaling limit; consider, e.g., the expansion $g + \ln D/T = \frac{1}{g} - \frac{1}{g} \ln \frac{D}{T} + \ldots$ in powers of $1/g$. Thus, no finite number of terms of the $1/g$ series will yield the low temperature behavior, since there is no obvious way to identify which contributions are important in the $g \to 0^+$ scaling limit.

The scaling form of the $1/g$ series describes a different physical problem: one in which the bare coupling constant is large in magnitude. The sign of the beta function then indicates that the strong ferromagnetic regime is universal and the strong antiferromagnetic regime is non-universal. The quantity $-\frac{1}{g}$ behaves much like $g$ does in the antiferromagnetic case; that is, the $g = -\infty$ point behaves like $g = 0^+$, and $g = 0^-$ behaves like $g = \infty$. Let us state this more definitely. A system with large negative bare coupling $g$ has a running coupling that is also large and negative at high energies; an RG-improved power series in $\frac{1}{g}$ produces accurate results. At low energies, a more powerful technique is needed; neither a series in $\frac{1}{g}$ nor a series in the inverse parameter $g$ gives any information about the low energy behavior (unless one has all terms of the series), because in this case the correct scaling form is the one generated by the $1/g$ series (which can differ from the scaling form generated by the $g$ series).

Our calculation yields the beginning of the RG flow in the strong ferromagnetic regime: starting at the unstable fixed point $g_R = -\infty$, the running coupling constant becomes smaller in magnitude according to $g_R = -\frac{\pi^2}{D} \ln \frac{T}{T_K}$. As $T$ approaches $T_K$ from above, $|g_R|$ becomes too small for our calculation to be valid. We expect that $g_R$ continues to flow to the stable fixed point $g_R = 0^-$ without any other fixed points in between (much like the corresponding antiferromagnetic flow from $g_R = 0^+$ to $g_R = \infty$). The ground state of the system would flow from a triplet at high energy, with entropy $\ln 3$, to a free spin at low energy, with entropy $\ln 2$. We emphasize again that perturbation theory in small, bare, ferromagnetic $g$ provides no information at all about the low energy behavior of a system with strong ferromagnetic $g$ except the extreme point. In other words, the conductance in the universal strong ferromagnetic regime should be zero at $T = V = 0$, but calculating the approach to zero requires another method (such as an analysis of leading irrelevant operators, or NRG).

VI. CONCLUSION AND OUTLOOK

Our calculation has predicted a universal strong ferromagnetic regime in which the conductance approaches unitarity for large voltage or temperature. This regime could be accessible by other non-perturbative methods. We expect that the same basic picture of RG flow will be found if the calculation can be repeated using a conventional cutoff scheme.

It would be interesting to study the rate of entropy production of the NESS using the approach of Melita and Andrei [27].

Another direction would be to adapt either the self-consistent rate equation used in [6] or the Dyson equation used in [28] to the many body wavefunction approach presented here, in order to repeat the calculation of the electric current in the presence of a non-zero magnetic field on the dot (particularly in the strong ferromagnetic regime). Though we set the magnetic field to zero in this paper, the exact wavefunction is given in the previous paper for arbitrary magnetic field.

To take full advantage of the fact that the wavefunction for a fixed number of electrons is exact, it is essential to find a different way of taking the thermodynamic limit of observables other than the approach we took here of expanding in powers of $J$ or $1/J$. We hope that the technology we have introduced for using the new wavefunctions in the thermodynamic limit can eventually reach the advanced state of development found in equilibrium calculations with the Bethe Ansatz.

ACKNOWLEDGMENTS

We are grateful to Chung-Hou Chung, Garry Goldstein, Yashar Komijani, Yigal Meir, Andrew Mitchell, Achim Rosch, and Hubert Saleur for helpful discussions. We have benefited from working on related problems with Huijie Guan, Paata Kakashvili, Christopher Munson, and...
Roshan Tourani. This material is based upon work supported by the National Science Foundation under Grant No. 1410583.

Appendix A: Asymptotic evaluation of integrals

We study the asymptotic behavior as $\lambda \to \infty$ of the general form (11), namely:

$$ R^{(\sigma)}\{f,h\}, \lambda \equiv \int_{\sigma=0}^{\infty} du_1 \ldots du_{n-1} \left[ \prod_{j=1}^{n-1} \frac{e^{i\lambda v_j^{(\sigma)}} - f(v_j^{(\sigma)})}{v_j^{(\sigma)}} \right] h(v_n^{(\sigma)}), $$ (A1)

where $\sigma \in \text{Sym}(n)$ and the $v_j^{(\sigma)}$ variables are linear combinations of the integration variables:

$$ v_j^{(\sigma)} = \sum_{m=j}^{u-1} u_m - \sum_{m=\sigma^{-1}(j)}^{u-1} u_m \quad (1 \leq j \leq n). $$ (A2)

These linear combinations are listed in Table I for all of the eleven permutations $\sigma$ that we need in order to evaluate the current up to and including the $J^5$ or $1/J^5$ term.

| $\sigma \equiv (\sigma_1, \ldots, \sigma_n)$ | $v_1^{(\sigma)}$ | $v_2^{(\sigma)}$ | $v_3^{(\sigma)}$ | $v_4^{(\sigma)}$ |
|----------------------------------------|----------------|----------------|----------------|----------------|
| $(1)$                                  | $1$            | $-$            | $-$            | $-$            |
| $(2, 1)$                                | $u_1$          | $-u_1$         | $-$            | $-$            |
| $(3, 1, 2)$                             | $u_1$          | $u_2$          | $-u_1 - u_2$   | $-$            |
| $(2, 3, 1)$                             | $u_1 + u_2$    | $-u_1$         | $-u_2$         | $-$            |
| $(3, 2, 1)$                             | $u_1 + u_2$    | $0$            | $-u_1 - u_2$   | $-$            |
| $(2, 3, 4, 1)$                          | $u_1 + u_2 + u_3$ | $-u_1$   | $-u_2$         | $-u_3$         |
| $(2, 4, 1, 3)$                          | $u_1 + u_2$    | $-u_1$         | $u_3$          | $-u_2 - u_3$   |
| $(3, 1, 4, 2)$                          | $u_1$          | $u_2 + u_3$    | $-u_1 - u_2$   | $-u_3$         |
| $(3, 4, 1, 2)$                          | $u_1 + u_2$    | $u_2 + u_3$    | $-u_1 - u_2$   | $-u_2 - u_3$   |
| $(4, 1, 2, 3)$                          | $u_1$          | $u_2$          | $u_3$          | $-u_1 - u_2 - u_3$ |
| $(4, 3, 2, 1)$                          | $u_1 + u_2 + u_3$ | $u_2$       | $-u_2$         | $-u_1 - u_2 - u_3$ |

We use brackets to indicate that $R^{(\sigma)}\{f,h\}, \lambda$ is a functional of $f$ and $h$ and a function of the real parameter $\lambda$. As discussed in the main text, $\lambda$ is essentially either $D/V$ or $D/T$, and the functions $f$ and $h$ take various forms depending on which case is being considered.

We have found the asymptotic form as $\lambda \to \infty$ of $R^{(\sigma)}\{f,h\}, \lambda$ for all eleven of the necessary permutations. By leaving $f$ and $h$ unspecified, we can cover all cases discussed in the main text at once.

We will not attempt to characterize exactly what properties of $f$ and $h$ are necessary for our calculations below to be valid. At the very least, we assume that $f$ and $h$ are both analytic in the U(1) integration variables that have poles only along the imaginary axis (but no pole at the origin), that $f(0) = 1$ (otherwise $R^{(\sigma)}\{f,h\}, \lambda$ would be ill-defined due to the denominators), and that $h(v)$ decays like $1/v$ or faster as $v \to \infty$; we also assume that $f'(0) = 0$ and that $h(0)$ is real, although these conditions could easily be relaxed. All of these approximations hold for the particular $f$ and $h$ functions defined in the main text.

Before presenting the full results, we show one more example. We have already given the simplest non-trivial example in (12) in the main text, which is the asymptotic expansion of $R^{(2,1)}\{f,h\}, \lambda$. An example result from the next order ($n = 3$) is:

$$ R^{(2,3,1)}\{f,h\}, \lambda \equiv \int_{0}^{\infty} du_1 du_2 e^{i\lambda(u_1 + u_2)} - f(u_1 + u_2) e^{-i\lambda u_1} - f(-u_1) \frac{h(-u_2)}{u_1} \lambda \to \infty \frac{1}{2h(0)} \ln^2 \lambda + \left[ -h(0) \left( \gamma + \frac{\pi}{2} \right) + \int_{0}^{\infty} du \frac{d}{du} (f(u)h(u)) \right] \ln \lambda - \left( \frac{7\pi^2}{24} + \frac{1}{2} \gamma^2 + \frac{1}{2} \pi \gamma \right) h(0) $$




\[
+ \left(\gamma + i \frac{\pi}{2}\right) \int_0^\infty du \, \ln u \, \frac{d}{du} [f(u)h(-u)] + \frac{1}{2} \int_0^\infty du \, \ln^2 u \, \frac{d}{du} [f(u)h(-u)] \]
\[
- \int_0^\infty du_1 du_2 \, \frac{1}{u_2} \ln \frac{u_1 + u_2}{u_1} \frac{\partial}{\partial u_1} [f(u_1 + u_2)f(-u_1)h(-u_2)],
\]
where \(\gamma\) is the Euler constant. Notice that here and in the simpler example (12), the asymptotic expansion consists of powers of \(\ln \lambda\) with coefficients that are functionals of \(f\) and \(h\); higher powers of \(\ln \lambda\) are multiplied by simpler functionals, and the highest power is \(\ln^{n-1} \lambda\).

We have shown analytically that for all of the eleven necessary permutations, the asymptotic form of \(R^{(\sigma)}\{\{f, h\}, \lambda\}\) is a sum of logarithmic terms (including a constant term, i.e. \(\ln^0 \lambda\)) and a linear term:

\[
R^{(\sigma)}\{\{f, h\}, \lambda\} \xrightarrow{\lambda \to \infty} z^{(\sigma)}_{\text{linear}}\{\{f, h\}\} \lambda + \sum_{j=0}^{n-1} z^{(\sigma)}_j\{\{f, h\}\} \ln^j \lambda,
\]
(A3)

where \(z^{(\sigma)}_{\text{linear}}\{\{f, h\}\}\) and \(z^{(\sigma)}_j\{\{f, h\}\}\) are complex numbers (functionals of \(f\) and \(h\)). Let us first discuss the coefficient \(z^{(\sigma)}_{\text{linear}}\{\{f, h\}\}\) of the linear term. This coefficient vanishes for all of the eleven permutations except for \((3, 2, 1)\) and \((4, 3, 2, 1)\); for these two permutations, we find:

\[
z^{(3, 2, 1)}_{\text{linear}}\{\{f, h\}\} = -i \pi z^{(4, 3, 2, 1)}_{\text{linear}}\{\{f, h\}\} = -i \int_0^\infty du \, f(u)h(u).
\]
(A4)

In the current, these linear terms cancel at the order we are working to \((J^5\) or \(1/J^5\)), so we can ignore them.

We proceed to the logarithmic terms. It turns out that for all eleven permutations, the coefficients \(z^{(\sigma)}_j\{\{f, h\}\}\) can be expressed entirely in terms of the following three functionals:

\[
\rho_1\{\{f, h\}\} = \left(\gamma + i \frac{\pi}{2}\right) h(0) + \int_0^\infty du \, \ln u \, \frac{d}{du} [f(u)h(-u)],
\]
(A5a)

\[
\rho_2\{\{f, h\}\} = -\left(\gamma - i \frac{\pi}{2}\right)^2 h(0) + 2 \left(\gamma - i \frac{\pi}{2}\right) \int_0^\infty du \, \ln u \, \frac{d}{du} [f(u)h(-u)]
\]
\[
+ \frac{1}{2} \int_0^\infty du \, \ln^2 u \, \frac{d}{du} [f(u)h(-u)] - \int_0^\infty du_1 du_2 \, \frac{1}{u_2} \ln \frac{u_1 + u_2}{u_1} \frac{\partial}{\partial u_1} [f(u_1 + u_2)f(-u_1)h(-u_2)],
\]
(A5b)

\[
\rho_3\{\{f, h\}\} = \left(\gamma - i \frac{\pi}{2}\right)^2 h(0) - 2 \left(\gamma - i \frac{\pi}{2}\right) \int_0^\infty du \, \ln u \, \frac{d}{du} [f(u)h(-u)]
\]
\[
+ \int_0^\infty du_1 du_2 \, \ln u_1 \ln u_2 \frac{\partial}{\partial u_1} \frac{\partial}{\partial u_2} [f(u_1)f(u_2)h(-u_1 - u_2)],
\]
(A5c)

Table II contains our results for the coefficients \(z^{(\sigma)}_j\{\{f, h\}\}\) of the asymptotic expansion. These results completely specify the integrals we need for \(n = 1, 2,\) and 3, while for \(n = 4,\) they provide the complete expansion except for the coefficient \(z^{(\sigma)}_0\{\{f, h\}\}\) of the smallest term (the \(\lambda\)-independent constant); these remaining coefficients can also be
written as lengthy functionals of $f$ and $h$ (including triple integrals), and we list their approximate numerical values in Table III for the two special cases corresponding to $G(T)$ and $I_{\text{steady state}}(T_1 = 0, T_2 = 0, V)$.

Our asymptotic results are in good agreement with Monte Carlo evaluation [29]. An example of this agreement is shown in Fig. 1.

![Fig. 1](image)

**FIG. 1.** (Color online). Sample numerical checks of our asymptotic result for $R^{(3,1,4,2)}[[f, h], \lambda]$. Case 1 is $f(v) = h(v) = v/\sinh v$, which is used in the calculation of $G(T)$; case 2 is $f(v) = \cos v$ and $h(v) = \sin v$, which is used in the calculation of $I_{\text{steady state}}(T_1 = 0, T_2 = 0, V)$ (and hence, $G(V)$). Only the real part of $R^{(3,1,4,2)}[[f, h], \lambda]$ appears in the answer to the order we consider ($J^5$ or $1/J^5$), but the agreement for the imaginary part is similar.

The calculations that produce Table II are lengthy; to illustrate the method used, we derive the asymptotic expansion (12) in the main text. The integral to be studied is:

$$R^{(2,1)}[[f, h], \lambda] = \int_0^\infty du_1 \frac{e^{i\lambda u_1} - f(u_1)}{u_1} h(-u_1). \quad (A6)$$

We would like to separate the $\lambda$-dependent term of (A6), but cannot do so because $e^{i\lambda u_1}/u_1$ by itself diverges too strongly at $u_1 = 0$. We therefore integrate by parts, finding (note that $h$ falls off sufficiently rapidly at infinity so that the boundary contribution is zero):

$$R^{(2,1)}[[f, h], \lambda] = R_1^{(2,1)}[[f, h], \lambda] + R_2^{(2,1)}[[f, h]], \quad (A7)$$

where:

$$R_1^{(2,1)}[[f, h], \lambda] = - \int_0^\infty du_1 \ln u_1 \left( e^{i\lambda u_1} h(-u_1) \right), \quad (A8a)$$

$$R_2^{(2,1)}[[f, h]] = \int_0^\infty du_1 \ln u_1 \left( f(u_1) h(-u_1) \right). \quad (A8b)$$

We evaluate $R^{(2,1)}[[f, h], \lambda]$ for large $\lambda$ using a contour argument based on a example 1 in section 6.6 of Ref [30]. The essential idea is to turn the rapidly oscillating phase into a decaying exponential.
Recall that any poles of $h$ are on the imaginary axis. Write $C$ for the contour that starts at 0 and extends to $i\infty$ going slightly to the right ($Re u_1 > 0$) around each of the poles. This contour $C$ taken in reverse, the original integration contour from 0 to $i\infty$, and a semicircular arc from $i\infty$ to $i\infty$ form a closed contour that contains no poles. Furthermore, it can be verified that the semicircular arc makes no contribution. Therefore, the original contour can be replaced by $C$:

$$R_1^{(2,1)}\{[f,h], \lambda\} = - \int_C du_1 \ln u_1 \frac{d}{du_1} [e^{i\lambda u_1} h(-u_1)].$$  \hfill (A9)

For large $\lambda$, the function $h$ can be replaced by its value at zero; the reason for this is that the difference $h(-u_1) - h(0)$ starts at linear order, which permits integration by parts:

$$- \int_C du_1 \ln u_1 \frac{d}{du_1} [e^{i\lambda u_1} (h(-u_1) - h(0))] = \int_C du_1 \frac{1}{u_1} (h(-u_1) - h(0)) e^{i\lambda u_1}$$

$$= \int_C du_1 \frac{d}{du_1} \left[ \frac{1}{u_1} (h(-u_1) - h(0)) \frac{1}{i\lambda} e^{i\lambda u_1} \right]$$

$$- \int_C du_1 \frac{d}{du_1} \left[ \frac{1}{u_1} (h(-u_1) - h(0)) \frac{1}{i\lambda} e^{i\lambda u_1} \right] (A11)$$

$$= O\left(\frac{1}{\lambda}\right).$$  \hfill (A12)

We have therefore shown:

$$R_1^{(2,1)}\{[f,h], \lambda\} = - \int_C du_1 \ln u_1 \frac{d}{du_1} [e^{i\lambda u_1} h(0)] + O\left(\frac{1}{\lambda}\right).$$ \hfill (A13)

Since there are no longer any poles, we can shift the contour $C$ to be exactly the positive imaginary axis; then the remaining integrals are elementary after the change of variables $s_1 = \lambda u_1$:

$$R_1^{(2,1)}\{[f,h], \lambda\} = - \int_0^\infty du_1 \ln(iu_1) \frac{d}{du_1} [e^{-\lambda u_1} g(0)] + O\left(\frac{1}{\lambda}\right)$$  \hfill (A14a)

$$= g(0) \left(- \ln \lambda - \gamma + i \frac{\pi}{2}\right) + O\left(\frac{1}{\lambda}\right).$$ \hfill (A14b)

Adding this to Eq. (A8b), we obtain the second row of Table II.

For the higher order integrals, the basic strategy is the same: use integration by parts to rewrite the integral in a form that can be separated into a sum of simpler terms, shift integration contours to turn oscillating phases into decaying exponentials, and replace functions by their values at zero via integration by parts. In the case of $\sigma = (4,3,2,1)$, this last step has to be done more carefully due to the linear divergence.

**Appendix B: Further checks**

We summarize a number of checks that confirm the consistency of our results. We have found the large bandwidth asymptotic form of the basic steady state integral $\varphi(T_1 = 0, T_2 = 0, V)$ in an alternate way that agrees with the results of Appendix A and also provides the analytical formula for the bandwidth-independent $g^5$ and $1/g^5$ terms in $G(V)$ in the main text. We have repeated the calculation of $G(T)$ in an alternate cutoff scheme in which the Fermi function smoothly drops to zero at large negative energies, rather than being sharply cut off. We have also repeated the calculation of $G(V)$ and $G(T)$ in the XXZ anisotropic Kondo model, finding the usual scaling of $g_\perp$ and $g_\parallel$ at leading order.

The basic integral that appears in our current series is given by (see Eq. (3.23) and Eq. (3.32) of the previous paper):

$$\varphi^{(\sigma)}(T_1, \mu_1; T_2, \mu_2; t) = \left(\frac{i}{2}\right)^{n-1} \frac{\partial}{\partial t} \int_0^t dx_1 \ldots dx_n \Theta(x_n < \cdots < x_1)$$

$$\times \int_{-D}^D dk_1 \ldots dk_n \left[ \prod_{j=1}^{n-1} (n_1(k_j) + n_2(k_j)) \right] \left[ n_1(k_n) - n_2(k_n) \right] \prod_{\ell=1}^n e^{i(k_n - k_{\ell}) x_\ell},$$ \hfill (B1)
where the Fermi functions of the leads are:

\[ n_s(k) \equiv n(T_\gamma, \mu_\gamma, k) \equiv \frac{1}{e^{(k-\mu_\gamma)/T_\gamma} + 1} \quad (\gamma = 1, 2). \]  

The approach taken in the main text (starting in the previous paper) was to do the momentum integrals via an asymptotic formula for the Fourier transform of the Fermi function with a cutoff, resulting in an integral in which \( t \) could be sent to infinity by deleting a Heaviside function. Using the notation of Appendix A, the result in the special case of zero temperature can be written as:

\[
\frac{1}{V} \lim_{t \to \infty} \varphi^{(\sigma)}(T_1 = 0, \mu_1 = 0; T_2 = 0, \mu_2 = -V; t) = \frac{1}{V} \varphi^{(\sigma)}(T_1 = 0, T_2 = 0, V) = R^{(\sigma)} \left[ \{f, h\}, 2 \frac{D}{V} - 1 \right],
\]

where \( f(v) = \text{sinc} v \) and \( h(v) = \cos v \). The asymptotic expansion of \( R^{(\sigma)} \left[ \{f, h\}, 2 \frac{D}{V} - 1 \right] \) for \( D/V \gg 1 \) can be read off from Table II and the third column of Table III; our task is to calculate \( \frac{1}{V} \varphi^{(\sigma)}(T_1 = 0, T_2 = 0, V) \) in an alternate way as a check.

An alternate approach in this special case is to do the position integrals in Eq. (B1) before the momentum integrals, arriving at the long time limit by means of the Laplace transform. Recall that the long time limit of a function \( F(t) \) is determined by the behavior of is Laplace transform near the origin:

\[
\lim_{t \to \infty} F(t) = \lim_{s \to 0^+} s \tilde{F}(s), \quad \text{where} \quad \tilde{F}(s) = \int_0^\infty dt \, e^{-st} F(t).
\]

Taking the Laplace transform and doing the position integrals, we find:

\[
s^{\tilde{\varphi}^{(\sigma)}}(T_1, \mu_1; T_2, \mu_2; s) = \left( \frac{i}{2} \right)^{n-1} \int_{-D}^D dk_1 \cdots dk_n \left[ \prod_{j=1}^{n-1} (n_1(k_j) + n_2(k_j)) \right] [n_1(k_n) - n_2(k_n)]
\times \prod_{\ell=1}^{n-1} k_{\sigma_1} + \cdots + k_{\sigma_\ell} - k_1 - \cdots - k_\ell + is \quad \text{(B5)}
\]

The point of these manipulations is that if we set \( T_1 = T_2 = 0 \), we obtain a form that is tractable analytically. After some relabellings of coordinates, we obtain:

\[
s^{\tilde{\varphi}^{(\sigma)}}(T_1 = 0, \mu_1 = 0; T_2 = 0, \mu_2 = -V; s) = i^{n-1} \sum_{m=0}^{n-1} \left( \frac{1}{2} \right)^m \binom{n-1}{m} \int_{-V}^0 dk_1 \cdots dk_{m+1-n}
\times \prod_{\ell=1}^{n-1} k_{\sigma_1} + \cdots + k_{\sigma_\ell} - k_1 - \cdots - k_\ell + is \quad \text{(B6)}
\]

where the symmetrized \( S_{k_1 \ldots k_{n-1}} \) acts on the first \( n - 1 \) momenta of any function \( X \) via:

\[
S_{k_1 \ldots k_{n-1}} X(k_1, \ldots, k_n) = \frac{1}{(n-1)!} \sum_{\sigma' \in \text{Sym}(n-1)} X(k_{\sigma'_1}, \ldots, k_{\sigma'_{n-1}}, k_n). \quad \text{(B7)}
\]

By lengthy computer evaluation, these integrals were done analytically for all of the eleven permutations; then the limit \( s \to 0^+ \) was taken and an expansion was done for large \( D/V \). The final results are conveniently written in the following form:

\[
\frac{1}{V} \varphi^{(\sigma)}(T_1 = 0, T_2 = 0, V) = \frac{1}{V} \lim_{s \to 0^+} s^{\tilde{\varphi}^{(\sigma)}}(T_1 = 0, \mu_1 = 0; T_2 = 0, \mu_2 = -V; s)
\]
\[
\approx \frac{D}{V} b^{(\sigma)} + \sum_{n=0}^{3} \sum_{m=0}^{n} \frac{1}{m!} \ln^m \frac{D}{V} \quad \text{for} \quad D \gg V \quad \text{(B9)}
\]

where \( b^{(\sigma)} \) is zero for all eleven permutations except for \( b^{(3,2,1)}_{\text{linear}} = -\frac{i}{2} b^{(4,3,2,1)}_{\text{linear}} = -i\pi/2 \), and where the remaining coefficients are listed in Table IV. These results are in good agreement with Table II and the third column of Table III.
Another check is provided by varying the cutoff scheme that regulates UV divergences of the model. The cutoff scheme we have used amounts to multiplying the Fermi function by a Heaviside function $\Theta(k + D)$ (the cutoff of large positive energies turns out to be unimportant due to the exponential suppression of the Fermi function there). For an alternate cutoff scheme, we replace the Heaviside function by a smoothly decaying function (which is chosen for convenience to have the form of a Fermi function); the resulting Fourier transform for the cutoff Fermi function is:

$$
\int_{-\infty}^{\infty} dk \, n(T, \mu, k) \frac{1}{e^{\frac{1}{T}(D' + k)} + 1} e^{-iky} = \frac{\pi}{i} \frac{e^{iD'y} - e^{-iy}}{\sinh(\pi Ty)},
$$

again with exponentially small corrections $O(e^{-(D' + \mu)/T})$. We write the cutoff as $D'$ as a reminder that, while it plays the same role, it is not identical to the sharp cutoff $D$ except in the case $T = 0$. In this alternate cutoff scheme, we repeated the calculation of the integrals $R^{(\sigma)}[\{f, h\}, \lambda]$ by Monte Carlo integration at several logarithmically-spaced values of $\lambda$. The results indicate that $D'$ and $D$ yield equivalent answers in the large bandwidth regime; we have shown this analytically for some of the integrals using the contour method described in Appendix A.

Still another check is obtained by repeating the calculation allowing anisotropy in the Kondo interaction. As discussed in more detail in Appendix C of the previous paper, the $XXZ$ model has exactly the same wavefunction with a more general form of the $T$-matrix. The same series answer for the current is obtained, with the only change being a modification of the spin sums $W^{(\sigma)}(J)$. The leading log results are:

$$
G(V) = \frac{3\pi^2}{4} G_0 \left[ \frac{2}{3} g_{11}^2 + \frac{1}{3} g_{21}^2 + 4 g_{12}^2 g_{11} \ln \frac{D}{V} + 12 \left( \frac{1}{3} g_{11}^4 + \frac{2}{3} g_{21}^2 g_{11}^2 \right) \ln^2 \frac{D}{V} + 32 \left( \frac{2}{3} g_{12}^4 g_{11} + \frac{1}{3} g_{12}^2 g_{21}^3 \right) \ln^3 \frac{D}{V} + O(g^6) \right],
$$

and the same for $G(T)$ with with $V$ replaced by $T$. The Callan-Symanzik equation is satisfied with the following beta functions at leading order:

$$
\beta_{g_{1\perp}}(g_{1\perp}, g_{21}) = -2g_{1\perp} g_{21} + O(g^3),
$$

$$
\beta_{g_{1\parallel}}(g_{1\parallel}, g_{21}) = -2g_{1\parallel}^2 + O(g^3),
$$

which are standard [21].

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