A POINTLESS MODEL FOR THE CONTINUUM AS THE FOUNDATION FOR QUANTUM GRAVITY

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SUMMARY In this paper, we outline a new approach to quantum gravity; describing states for a bounded region of spacetime as eigenstates for two classes of physically plausible gedanken experiments. We end up with two complementary descriptions in which the point set continuum disappears.

The first replaces the continuum of events with a handlebody decomposition of loop space. We conjecture that techniques from algebraic topology will allow us to extend state sum models on spacetime to loop space.

The second picture replaces the continuum with a nondistributive lattice; the classical limit seems more tractable in this picture.
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We want to implement the recent suggestion [1] that the continuum picture disappears in highly curved regions in quantum general relativity.

A body of recent work in relativity has shown us that only a finite dimensional Hilbert space of information can pass from any bounded region in spacetime to its exterior [2]. The description of the geometry of a region as a metric on an open subset of a manifold contains more information than external observers can ever detect. Such information can play no role in a quantum theory of the observable geometry of the region.

Here we will outline an approach to the construction of a quantum theory of the geometry of a bounded region R in spacetime, using only the results of external experiments probing the geometry of the region. We assume the area of the spatial boundary of the R is known, and that the exterior of R is simple, causal, flat, topologically trivial, and can be treated classically. This will produce a relational geometry, since the probes would communicate different information to different external observers, and a pointless geometry in that the finite information observable would not distinguish an infinite point set.

We need to include the back reaction of the probes on the spacetime geometry of R, which would focus the trajectories of the probes and redshift the information, and cause the information limitation mentioned above. Too large of a probe or too many probes would cause an event horizon to form and therefore not transmit information to the outside. Including the back reaction is in the spirit of quantum mechanics, since we cannot ignore the effect of the probe on the system, i.e. on the spacetime geometry of R.

We shall discuss two different kinds of probes, external, and internal, which will give us two complementary descriptions of R. Measuring one type would disturb the result of the other, much as the position and momentum descriptions of a particle are complementary.

EXTERNAL PROBES. THE TELESCOPE-MICROSCOPE HYPOTHESIS

In the first type of probing experiment, we imagine sending a probe along some path through the region of spacetime. The probe is thought of as including gyroscopes and a clock. This probe would measure the holonomy of the connection on the spacetime along the path. In a classical spacetime, assuming that measurements did not disturb the spacetime geometry, we could imagine measuring holonomy along each causal path, from which the metric could be recovered.

Quantum mechanically, however, we cannot think of a probe as following a definite path, and the back reaction compresses the infinite number of
holonomies of the causal paths to a finite dimensional hilbert space of information.

At this point we propose:

**TELESCOPE-MICROSCOPE HYPOTHESIS** all the observable information from causal path holonomies is contained in the families of null geodesics from events in the causal past of R to events in its causal future.

The motivation for this suggestion is the thought that an electromagnetic wave through R samples all causal paths at once as in a path integral. Classically, a light pulse would travel along null geodesics only because of Fermat’s principle. So we are proposing that the information which survives is exactly the apparent position of all the classical images of past events seen through R.

In making this hypothesis, we reason by analogy with the results of astronomical observation of distant objects through highly curved regions such as galaxies We are motivated in what follows by the mathematical theory of gravitational lensing [3].

To restate, we replace the probes through all possible causal paths with an optical gravitational lensing experiment, and conjecture that the probabilities of the different multiple images exhaust the transmissible information.

This hypothesis seems unnatural for a large flat region of spacetime, but if we think of a region small enough that quantum gravity is relevant, it would be full of geometrical fluctuations of near black hole strength at near Planck scale dimensions, so a pulse of light passing through the region would have infinitely many multiple images as viewed by any observer in the future of R.

The case of a large flat region could be included in this picture by thinking of the wave optical corrections to a lensing experiment as given by the average of all the multiple images in the quantum fluctuations of the geometry on the region, reproducing a path integral version of quantum mechanics. This implements the suggestion that quantum general relativity automatically imposes quantum mechanics on a particle [4]. Recovering standard quantum mechanical propagators from this picture will be a stern test for this proposal.

**MORSE THEORY ON LOOP SPACE**

Now let us consider the images of a wavefront generated at a single event in the causal past of R, as seen by an observer in the future of R.

Since the elapsed time is a morse function on the space of paths between two events, and since that space is homotopy equivalent to the based loop space on R, $\Omega(R)$, the multiple images seen by each observer can be understood as critical points of a morse function on $\Omega(R)$. This is equivalent to a handlebody decomposition of the loop space [5].

(If we combine the images all possible future observers could see, we obtain a decomposition of the free path space of R. This is a (contractible) complex of a
slightly more general type, since singularities of a higher order would generically occur. Nevertheless, a type of cellular complex would result.)

In this description, the events or localized subregions of $R$ have disappeared, to be replaced by a geometry on a cellular complex.

Next we need a theory that attaches probability amplitudes to finite handlebody approximations to $\Omega(R)$, and to the apparent sizes of the handles to external observers (evidenced by the apparent distances between images and the angular region surrounding $R$ in which they are observable.)

This problem is similar to the constructions of TQFTs and combinatorial state sums for quantum gravity in 3 and 4 dimensions [6,7]; because in homotopy theory $\Omega(R)$ looks like a 3-manifold, while the differential graded algebra of complexes on it can be obtained from that of $R$ by the cobar construction, and takes the form of a limit of multiple copies of the DG algebra on $R$ [8].

If we replace $R$ by a superposition of quantum amplitudes on handlebodies, how could the classical picture of a spacetime region be recovered?

**INTERNAL PROBES**

Now let us imagine probing $R$ by sending probes inside it which set off pulses of light at some time. If $R$ were a classical spacetime region, we could reconstruct its spacetime geometry by noticing when and from what angle different external observers saw the same pulse, combining the past light cones of each observer along its worldline into regions of Minkowski space, and using the correlations for different observers to glue together the patches of Minkowski space into a manifold. (In order to directly observe the distance of an event, we should think of our observer as binocular, and use parallax).

Different metrics on $R$ would cause different identifications of the apparent past patches, so the set of apparent positions for one metric would not coincide with those for another.

If we take it that “position” in $R$ means apparent position as seen by all observers, then the set of apparent positions in $R$ for a quantum state of its geometry would not be described as subsets of a point set. As we have argued in another paper [9], it would be a non-distributive lattice, where the number of regions we could see would be limited by the dimension of the Hilbert space of transmissible information, or the boundary area of $R$ in Planck units.

In situations that were essentially classical, the nondistributivity of the lattice would only appear at negligible scales, and the lattice would be fine enough to approximate a continuous point set.

**SUMMARY**

Our proposal reduces to the following steps:

1. Lift the BC model [7] for quantum gravity to $\Omega(R)$ using the cobar construction,
2. Find the form of the transformation from the external probe basis, as described by handlebody decompositions of $\Omega(R)$ to the internal probe basis, described by non-distributive lattices of apparent regions.

3. Investigate the form of the quantum theory in the internal probe basis, and verify the classical limit.

This is conceptually similar to ordinary quantum mechanics, in which dynamics are worked out in the momentum basis, then transformed to the position basis to obtain the usual propagators, differential equations etc.

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