Baseline study for higher moments of net–charge distributions at RHIC energies

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Lattice QCD models predict the presence of a critical point in the QCD phase diagram where the first order phase transition between the hadron gas and Quark-Gluon Plasma ceases to exist. Higher moments of conserved quantities, such as net–charge, net–baryon number and net–strangeness, are proposed to be sensitive probes for locating the critical point. The moments of net–charge distributions have been studied as a function of centrality for Au+Au collisions at \( \sqrt{s_{NN}} = 7.7 \) to 200 GeV using three event generators, viz., UrQMD, HIJING, and THERMINATOR-2. The effect of centrality selection, resonance production, as well as contributions from particle species to the net–charge moments and their products have been studied. It is observed that mean of the net–charge distributions are dominated by net–protons, whereas standard deviation, skewness and kurtosis closely follow net–pion distributions. These results, along with the predictions from Hadron Resonance Gas (HRG) model, are presented.

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I. INTRODUCTION

Statistical QCD (Quantum Chromodynamics) calculations predict that at high temperature and/or energy density, a system of strongly interacting particles, consisting of quarks and gluons, is formed where the particles would interact fairly weakly due to asymptotic freedom. Such a phase consisting of (almost) free quarks and gluons is termed as the Quark Gluon Plasma (QGP). Experimentally the QGP can be produced by colliding heavy nuclei at ultra-relativistic energies, thus creating a region of enormous energy density. Understanding the nature of phase transition from normal hadronic matter to QGP has been a topic of tremendous interest over last decades, both in terms of theoretical studies and large scale experiments. The Relativistic Heavy-Ion Collider (RHIC) at Brookhaven National Laboratory has been the major facility for the search and study of QGP and to explore the QCD phase transition. Theoretical models, based on lattice QCD, reveal that at vanishing \( \mu_B \), the transition from QGP to hadron gas is a simple crossover \([1]\), whereas at large \( \mu_B \), the phase transition is of first order \([1-8]\). Therefore, one expects the existence of a critical point at the end of the first order transition. Locating the QCD critical point has been one of the major thrusts of the physics program at RHIC. The collider is capable of accelerating and colliding beams from top energy at \( \sqrt{s_{NN}} = 200 \) GeV, down to as low as \( \sqrt{s_{NN}} = 7.7 \) GeV. Taking advantage of this, a beam energy scan program has been undertaken to exploit wide region of phase diagram and to search for the possible location of the QCD critical point \([6,12]\).

One of the most plausible signatures of the critical point has been predicted to be the large event-by-event fluctuations of thermodynamic quantities measured in high energy heavy–ion collisions \([14]\). This is because of the fact that the thermodynamic susceptibilities \((\chi)\) and the correlation length \((\xi)\) of the system diverge at the critical point \([1,2,15]\). Various QCD inspired models predict that higher moments of conserved quantities, such as distributions of net–charge, net–baryon and the net–strangeness are associated with the higher order thermodynamic susceptibilities and exhibit strong dependence on the correlation strength. This makes the moments, such as, mean \((M)\), standard deviation \((\sigma)\), skewness \((S)\) and kurtosis \((\kappa)\), etc., sensitive probes of the location of the critical point. These moments are related to the correlation length by \( \sigma^2 \sim \xi^2 \), \( S \sim \xi^{4.5} \) and \( \kappa \sim \xi^7 \) \([6,15]\). In order to cancel the volume term in the susceptibilities, different combinations of the moments are constructed, such as, \( S\sigma = \chi^{(3)}/\chi^{(2)} \) and \( \kappa\sigma^2 = \chi^{(4)}/\chi^{(2)} \), which can be used to have a direct comparison of experimental results to lattice calculations. Thus at the critical point, one would expect large non-monotonic behavior in these products. Recent lattice QCD model estimations \([16]\) have proposed the extraction of freeze-out parameters of the collision from the analysis of higher moments of net–charge distribution.

Among all conserved quantities, experimentally it is advantageous to study net–charge distributions. The fluctuations of net–charge include effects from net–baryon and net–strangeness. Net-charges take into account all the charged particles produced in the collisions, and thus can be directly compared with theoretical calculations \([17,19]\). Analytical calculations \([20]\) also suggest that the net–charge higher moment analysis may be less affected by acceptance effects. To understand various physics processes which contribute to the moments of net–charge distributions, three different models have been used, such as, a QCD based model (HIJING \([21]\)), a trans-
port model (UrQMD [22]) and a thermal model, THERMINATOR-2 [23]. The results of these event generators are compared to those of the Hadron Resonance Gas (HRG) [21] model predictions. Because of the absence of any critical phenomenon in these models, they set a baseline for the measurements at the Relativistic Heavy-Ion Collider (RHIC). Rest of the article is organized as follows. Various moments of the net–charge distributions are introduced in Section II. Importance of centrality selection and centrality bin width correction are presented in Section III. In section IV, contributions of different particle species on the net–charge moments have been discussed, and in section V, effect of resonance decays are studied. In section VI, the products of moments from different event generators are presented and compared to that of the HRG model. Finally, we conclude with a summary.

II. MOMENTS OF NET–CHARGE DISTRIBUTIONS

Experimentally, both the positive \((N_+\)) and negative \((N_-\)) charged particles are measured in a finite acceptance on an event-by-event basis. Net–charge \(\langle N \rangle\) is defined as the difference between the number positive and negative charged particles, \(N = N_+ - N_-\). For a given distribution of \(N\), the first four moments can be expressed as:

\[
\begin{align*}
\text{Mean} & : \quad M = \langle N \rangle, \\
\text{variance} & : \quad \sigma^2 = \langle (\delta N)^2 \rangle, \\
\text{skewness} & : \quad S = \frac{\langle (\delta N)^3 \rangle}{\sigma^3}, \\
\text{and, kurtosis} & : \quad \kappa = \frac{\langle (\delta N)^4 \rangle}{\sigma^4} - 3, \quad (1)
\end{align*}
\]

where \(\delta N = N - \langle N \rangle\). The mean, variance, skewness and kurtosis are measures of the most probable value, width, asymmetry and the peakedness of the distributions, respectively.

The net–charge distributions in a heavy-ion collision are sensitive to the collision centrality, which is normally expressed in terms of impact parameter \((b)\) or the number of participating nucleons \((N_{\text{part}})\). A given centrality class corresponds to certain percentage of total geometrical cross section of the collision, and represented by \((N_{\text{part}})\).

In this study, UrQMD event generator is used to study the beam energy and collision centrality dependence of net–charge distributions for \(\text{Au+Au}\) collisions. Charged particles within a pseudo-rapidity range of |\(\eta| < 0.5\) and transverse momentum range of \(0.2 < p_T < 2.0\) GeV/c are considered. Figure 1(a) shows the net–charge distributions for top central (0-5% of total cross section) collisions for four center-of-mass energies. It has been observed that the mean of the distributions is close to zero for high energy collisions and shifts towards positive values for lower energies. The distributions are seen to be wider for higher energy collisions compared to those of the lower energies. Figure 1(b) shows net–charge distributions for three different centralities (0-5%, 30-40% and 60-70% of total cross section) at \(\sqrt{s_{NN}}=39\) GeV.

III. CENTRALITY BIN WIDTH EFFECT ON HIGHER MOMENTS

A given centrality class is a collection of events having a range of impact parameters or \(N_{\text{part}}\), thus comprising of events with different charged particle multiplicities. This results in additional fluctuations in the number of produced particles within each centrality class. Below we discuss the effect of finite bin
FIG. 2: (Color online). Centrality dependence of (a) mean, (b) standard deviation, (c) skewness, and (d) kurtosis, obtained for Au+Au collisions at $\sqrt{s_{NN}}=39$ GeV. The results using three centrality bins (2.5%, 5% and 10%), without and with the centrality bin width corrections are shown in upper and lower panels, respectively.

The beam energy dependence of mean, standard deviation, skewness and kurtosis of net–charge distributions are presented in Fig. 3 for central (0-5% centrality bin) Au+Au collisions. The statistical errors are estimated using the Delta theorem [26]. With the increase of center-of-mass energy from 7.7 to 200 GeV, the mean of the distributions decrease, whereas standard deviation has a considerable increase. The skewness decreases with increase of a given centrality class on the moments of distributions and prescribe a method in order to correct for this.

Figure 2 shows the centrality dependence of mean, sigma, skewness and kurtosis of net–charge distributions for Au+Au collisions at $\sqrt{s_{NN}}=39$ GeV, obtained using UrQMD. Centrality classes were chosen using three different centrality bin widths, viz., 2.5%, 5% and 10% of cross section. From Fig. 2(a), it is observed that mean and sigma of the net–charge distributions are close to each other for all three centrality classes, whereas deviations are observed for skewness and kurtosis. This deviation is the result of choosing centrality class with a large bin.

The effect due to the finite centrality bin can be reduced by choosing smaller bins. In some cases, because of practical reasons such as resolution of centrality determination, statistics of available events, it is not possible to choose fine bins. In this case, a centrality bin width weighting method [25] can be used to minimize the effect. For this, net–charge distributions are constructed for finer bins in centrality within a centrality class. For moments of the distributions are obtained for each fine bin and weighted to get the final moments, according to:

$$X = \frac{\sum_i n_i X_i}{\sum_i n_i},$$

where $X$ represents a given moment, the index $i$ runs over each fine centrality bin, $n_i$ is the number of events in the $i^{th}$ bin, and $\sum_i n_i$ is the total number of events in the given centrality class. Figure 2(b) shows the moments of net–charge distributions in each centrality class after recalculating with appropriate weighting using centrality bin width method. After this correction, no centrality bin width dependence is observed in the three centrality classes. Thus the correction method does an appropriate job in correcting for the finite centrality bin width. For the rest of the article, results will be presented after taking into account correction for finite centrality bin width.

The beam energy dependence of mean, standard deviation, skewness and kurtosis of net–charge distributions are presented in Fig. 3 for central (top 5% of total cross section) Au+Au collisions. The statistical errors are estimated using the Delta theorem [26].
FIG. 4: Centrality dependence of (a) mean, (b) standard deviation, (c) skewness, and (d) kurtosis for net–charge, net–pion, net–kaon and net–proton distributions in case of Au+Au collisions at √sNN=39 GeV.

IV. EFFECT OF PARTICLE SPECIES ON HIGHER MOMENTS

Although we mainly focus on moments of inclusive charged particles, it is important to obtain the effect of each particle species which on the total net–charge distributions. These species mostly comprise of π⁺, π⁻, K⁺, K⁻, p, and ¯p, so the effect of net–pion, net–kaon and net–proton distributions on the net–charge distributions needs to be studied. At the generator level, the knowledge of the identity of each particle makes it possible to perform this study.

The centrality dependence of the contributions from different particle species to the moments of net–charge distributions have been studied using UrQMD for Au+Au collisions at √sNN=39 GeV. Inclusive charged particles as well as identified particles are selected in the same pseudo-rapidity (|η| < 0.5) window and same transverse momentum ranges (0.2 < pT < 2.0 GeV/c). Figure 4 shows the mean, standard deviation, skewness and kurtosis of net–charge distributions as a function of centrality. It is observed that the effect of particle species on the net–charge distributions are significant, and different species affect the moments in a different manner. The mean of the net–charge distributions are dominated by the effect of net–protons. The mean of the net–pion distributions, on the other hand, are seen to decrease going from peripheral to central collisions, whereas the trend is opposite for the other three cases. The mean for net–pions shift to negative values for central collisions. The trend for the mean of net–kaons is similar to those of net–charges. The widths of net–charge distributions are close to those of the net–pions. The widths of net–kaons and net–protons are close to each other, but the values are smaller than those of net–charges. The skewness of the net–charge distributions are close to those of the net–pions, whereas net–kaon values are not so far. On the other hand, the skewness for net–proton distributions are much larger compared to net–charges and has a significant centrality dependence. This may be because of the difference in the number of protons and anti-protons produced in different centrality classes at this en-

FIG. 5: Centrality dependence of (a) Sσ and (b) κσ² for net–charge, net–pion, net–kaon, and net–proton distributions in case of Au+Au collisions at √sNN=39 GeV.

FIG. 6: Beam energy dependence of (a) Sσ and (b) κσ² for net–charge, net–pion, net–kaon, net–proton distributions in case of central Au+Au collisions.
ergy. The kurtosis of net-charge distributions are close to those of net-pions, and smaller than those of the for net-kaons and net-protons.

As the products of moments (such as $S\sigma$ and $\kappa\sigma^2$) can directly be compared to lattice calculations, we have studied their centrality dependence in Fig. 5 as a function of centrality for Au+Au collisions at $\sqrt{S_{NN}}=39$ GeV. It is observed that the $S\sigma$ values do not show centrality dependence at this energy. $S\sigma$ for net-charges are close to those of the net-kaons, whereas $S\sigma$ for net-pions are close to zero. These values for net-protons are much larger compared to net-charges. The $\kappa\sigma^2$ values for net-charges are close to those of the net-pions, and show strong centrality dependence. For net-kaons and net-protons, these values remain close to unity.

The beam energy dependence of $S\sigma$ and $\kappa\sigma^2$ for top central Au+Au collisions, obtained using UrQMD, has been presented in Fig. 6 where the upper and lower panels show the results for $S\sigma$ and $\kappa\sigma^2$, respectively. It is observed that $S\sigma$ for net-pions are close to zero at all energies, whereas a decreasing trend with increase of energy is observed for all other cases. For net-charges, $S\sigma$ values get close to zero at higher energies. The energy dependence of net-proton is quite prominent with much larger $S\sigma$ values. The values of $\kappa\sigma^2$ for net-charge are close to those of net-pions, and show increasing trend with increase of beam energy. For net-proton and net-kaon distributions, $\kappa\sigma^2$ are close to unity at all energies.

V. RESONANCE EFFECTS ON HIGHER MOMENTS

The production of charged particles and net-charge distributions in proton-proton and heavy-ion collisions are affected by resonance decays. This can be studied by an event generator where one can track each of the particles in order to know its origin and history of the decay. THERMINATOR-2 offers such possibility for Au+Au collisions at $\sqrt{S_{NN}}=200$ GeV, where decays of resonances, such as $\Xi$, $\Delta^{++}$, $\rho$, $\phi$, and $\omega$, and their anti-particles, can be turned on and off. Net-charge distributions have been made with resonance decays turned on and off, and the effects on the products of moments are studied. Figure 7 shows the results for $S\sigma$ and $\kappa\sigma^2$ as a function centrality. $S\sigma$ shows no centrality dependence, and the values with resonance decay turned on are closer to zero compared to those without the decay. Values of $\kappa\sigma^2$ without resonance decay are higher compared to those with the decay at all centralities. The reason for this could be the presence of double charged baryons, like $\Delta^{++}$, which may affect the net-charge distributions, and enhance the higher order moments.

VI. COMPARISON OF MODEL PREDICTIONS

The net-charge distributions are sensitive to the particle production mechanisms. HIJING treats the heavy-ion collisions as a superposition of nucleon-nucleon collisions. It is well suited to study the effects of jets and mini-jets on produced particles. UrQMD is a hadronic transport model including strings. It has been used successfully to describe the stopping power and hadronic re-scattering along with various hadronic resonances. The Lund string model is used for the particle production both in the HIJING and UrQMD models. On the other hand, THERMINATOR-2 gives a good description for the thermal model of particle production. The moments and their products for the net-charge distributions are analyzed for the three event generators for Au+Au collisions at $\sqrt{S_{NN}}=200$ GeV. Figure 8 shows the mean, standard deviation, skewness and kurtosis as a function of centrality. Although the trends of all the distributions are similar, there are some differences in terms of magnitudes. The $M$ and $\sigma$ increase from peripheral to central collisions for all three models, whereas $S$ and $\kappa$ values decrease with increase in collision centrality. The values of $M$ are lower in THERMINATOR-2 as compared to HIJING and UrQMD, but the standard deviations are similar in all three cases. The centrality dependence
of $S$ and $\kappa$ for THERMINATOR-2 are also weaker as compared to the other two cases.

The centrality evolution of the higher moments can be better understood by invoking the Central Limit Theorem (CLT) \cite{28,29}, which gives the dependence of the moments on the number of participating nucleons:

$$M \propto \langle N_{\text{part}} \rangle,$$

$$\sigma \propto \sqrt{\langle N_{\text{part}} \rangle},$$

$$S \propto \frac{1}{\sqrt{\langle N_{\text{part}} \rangle}} ,$$

and $\kappa \propto \frac{1}{\langle N_{\text{part}} \rangle}. \quad (3)$

The centrality evolution of the moments, as shown by different lines in Fig. 8, follow the trend of the CLT for all the three event generators.

Combinations of the moments, such as $M/\sigma^2$, $S\sigma$ and $\kappa\sigma^2$, have been constructed for central (0-5% of cross section) Au+Au collisions at $\sqrt{s_{NN}}=200$ GeV using the three event generators. These are shown in Fig. 9 along with the predictions from the HRG model calculations. It is observed that $M/\sigma^2$ and $S\sigma$ decreases with increasing colliding energy in all cases. The results from the three event generators are close together and HRG gives higher values. The values of $\kappa\sigma^2$ from HRG model is seen remain constant close to 2, whereas variations are seen for the event generators. The consideration of double charged baryons in HRG model is probably responsible for $\kappa\sigma^2$ to be close to 2. HIJING and UrQMD values are close to unity at low energy, after which they steadily increase as a function of energy. THERMINATOR-2 is available only at $\sqrt{s_{NN}}= 200$ GeV, and the value is close to unity. At this energy, the models with thermal equilibrium (HRG and THERMINATOR-2) have produced lower values of $\kappa\sigma^2$ compared to HIJING and UrQMD.

VII. SUMMARY

Locating the QCD critical point is one of the major tasks in our understanding of the QCD phase diagram. Higher moments of net-charge distributions are proposed to provide one of the most sensitive probes towards the search for the critical point. The
moments and their products have been studied at RHIC energies for Au+Au collisions within a range of $\sqrt{s_{NN}} = 7.7$ to 200 GeV using UrQMD, HIJING and THERMINATOR-2 event generators. The net-charge distributions are seen to be sensitive to the width of centrality class, and a centrality bin width correction is needed in order to take into account inherent fluctuations within the centrality class. The first four moments, viz., mean, sigma, skewness and kurtosis have been determined for all the energies at regular centrality intervals. For a given collision energy, the mean and sigma of the net-charge distributions increase in going from peripheral to central collisions, whereas the skewness and kurtosis have decreasing trends. For top central (0-5%) collisions, whereas the skewness and kurtosis have increasing trend, and kurtosis values show negligible beam energy dependence. Net-charge distributions inherently contain the contributions from net-pion, net-proton and net-kaon. The effect of these particle species on the final net-charge moments have been studied using UrQMD. It is observed that mean of the net-charge is dominated by net-protons whereas standard deviation, skewness and kurtosis have been determined for all the energies and THERMINATOR-2 event generators. The net–charge distributions are seen to be sensitive to the QCD critical point.

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