On numerical method for modeling oil filtration problems in piecewise-inhomogeneous porous medium

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Abstract. Oil and gas fields in natural conditions are rarely of homogeneous nature. Given the chaotic nature of changes in rock mass permeability within a single layer, its significant parts (areas) can be considered uniform in permeability. If significant in sizes homogeneous zones (regions) are distinguished within the formation, the parameters of such a macro-inhomogeneous formation significantly affect the characteristics of the filtration flows. The area of zonal inhomogeneity consists of the parts of different permeability. Within the same zone, the permeability is on average the same, but differs from the permeability of adjacent zones.

1. Introduction

The issues of filtration and well flow rate have been studied most widely for plane-parallel flows in homogeneous and piecewise-homogeneous formations of constant thickness in the presence of various boundaries of the recharge area, fluid discharge and change in the homogeneity of the formation (layer). The calculation of such flows was performed on the assumption that Forchheimer's postulate of vertical pressure constancy is fulfilled. For piecewise-homogeneous reservoirs, the fluid head is represented by a harmonic function, which made it possible to use the theory of functions of a complex variable, reducing the problem of studying flows in a reservoir to finding an analytical function in terms of flow - the complex flow potential. Within the framework of this theory, all works can be separated into three main groups:

1) when the boundary of homogeneous zones, discharge lines and feeding boundaries are straight;
2) when the boundaries are represented by circles;
3) when the boundaries differ from circles and straight lines:

There are known works in which boundary-value problems of conjugation are solved by the method of integral equations: in the theory of filtration, these are studies.

Filtration in heterogeneous fractured-porous media is considered in the context of the Barenblatt-Zheltov model. It is believed that the movement of fluid in fractures is described by the nonlinear two-term Forchheimer's law in the first form in the form allowed with respect to the filtration rate, and the movement of fluid in porous rock blocks is described by one of the most general nonlinear laws - the DHC law. The main filtration equation is obtained in the considered case and the problem under study
is formulated. To solve it, it is proposed to use either the method of reduction to an integro-differential equation, or the Galerkin method with finite elements. Various options for its application are considered. Finally have been performed numerical calculations.

The paper presents a numerical simulation of two-phase filtration problems in fractured-porous media using a double porosity sample with a highly inhomogeneous permeability coefficient. A system of equations is given for the case of two-phase filtration without taking into account capillary and gravitational forces, which is a coupled system of equations for pressure and saturation in a porous medium with a system of cracks. Various options for setting the functions of the flow between the porous medium and cracks are considered. Numerical implementation for approximating velocity and pressure is based on the finite element method. To discretize the saturation equation by means of the method of introducing artificial diffusion, the classical Galerkin method with upstream approximation is used. The results of computational methods for a model problem using different flow functions are presented [1-3].

In the paper we considered the mathematical model and computational experiments to resolve the problems of bending of three-layer plates of complex configuration. The mathematical model and computational algorithm proposed in the paper are developed according to the Hamilton-Ostrogradky's variation principle, the Bubnov-Galerkin methods and the Rvachev R-functions. The numerical results are presented in tabular and graphical form. Numerical studies of the results obtained are considered using the R-function method in combination with the Bubnov-Galerkin variational method to resolve the problem of calculating the vibration of three-layer plates of complex configuration [4].

For the Analysis of Functioning of Technological Process of Filtering of a Suspension the adequate mathematical model is developed, numerical algorithm and the computing experiments on the computer are carried out. The results of computing experiments are illustrated as the diagrams [5].

The theory of mathematical models of fluid filtration in an anisotropic inhomogeneous porous medium based on the theory of generalized analytical functions and generalized potential is presented. Solved in the final form and numerically based on the method of discrete singularities three-dimensional and two-dimensional boundary-value problems of filtration of a homogeneous liquid and problems of the evolution of the interface of liquids of various physical properties (viscosity, density), which are of interest for the practice of developing oil-bearing (water-bearing) soil layers of complex geological structure and monitoring of groundwater pollution in such layers [6].

In this paper the Cauchy problem for nonlinear systems is considered. The conditions of existence of the solutions on time for the problem Cauchy are given. Furthermore the properties of the finite velocity of a propagation and localization of the disturbance, an asymptotic of self-similar solutions will be defined. The results of numerical solutions will be carried out and on the basis of calculations some necessary statements will be given [7].

This article [8] presents some of the results of many years of scientific research on the creation of a new method of non-invasive measurement of blood glucose (determination without obtaining blood) by studying the biophysical parameters of biologically active points. The authors proposed and substantiated the concept of creating models for an automated system for non-invasive measurement of blood glucose levels by electrical resistance of the skin at informative biologically active points. A functional diagram is proposed, technical means of implementation are outlined, and a methodology for creating an automated system for non-invasive blood glucose measurement is described in detail.

In the design and construction of tunnels, dams and other hydraulic engineering structures modeling of the groundwater flow is necessary. Deep bed filtration of suspension in a porous medium is considered. Forward flow of suspension periodically changes to back flow of clean water and vice versa. Partial lifting of retained particles at the moments of flow direction changes is modeled. Numerical analysis of suspended and retained particles concentrations is carried out. It is shown that at periodic change of flow detection the change of concentrations of suspended and retained particles rapidly reaches a periodic mode [9].

The paper develops a mathematical model of the process of purification of water from oil in a porous medium. Based on this model, the ability to clean a filtration device with a granulated coupling medium that recovers the filtration medium in a fine-grained sedimentation mode without disassembling or replacing the filtration element has been studied. In the course of theoretical
research, a physical model of the process of water purification of oil-containing water in a granular fusion environment has been developed. Based on these models, the factors determining the effectiveness of cleaning are determined. After the implementation of the experimental plan for the first time, a mathematical model of the water treatment process containing the oil in the layer in the volume of the granulated medium was developed on the basis of the regression equation. This model allows the calculation of design and experimental parameters with granulated filter elements [10].

In applied mathematics, when solving a problem on a computer, a technological chain is implemented: the object of research - a mathematical model - an algorithm (numerical methods) - a program on a computer - a computational experiment - analysis (or comparison with experimental and other data). The object of mathematical technology is the computational part of this chain: a mathematical model is a numerical algorithm-program-calculation on a computer. The use of this technology makes it possible to analyze and predict non-stationary oil recovery processes in reservoir conditions and to control them.

2. Mathematical model

Mathematical model of oil filtration problem in a piecewise inhomogeneous porous medium is reduced to the integration of a partial differential equation of parabolic type

\[ \beta \mu \frac{\partial P}{\partial t} = \frac{\partial}{\partial x} \left( k(x,y) \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial y} \left( k(x,y) \frac{\partial P}{\partial y} \right) - Q \]  \hspace{1cm} (1)

The boundary conditions for the integration of equation (1) are:

\[ P(x,y) = P_H(x,y), \quad t = 0, \]  \hspace{1cm} (2)

\[ -\frac{k(x,y)}{\mu} \frac{\partial P}{\partial n} = \alpha (P_A - P), \quad (x,y) \in \Gamma, \]  \hspace{1cm} (3)

\[ \int_{s_i} \frac{k(x,y)}{\mu} \frac{\partial P}{\partial n} ds = -q_i(t), \quad (x,y) \in s_i, \quad i = 1, n_q, \quad Q = \sum_{i=1}^{n_q} q_i \delta. \]  \hspace{1cm} (4)

Here: \( P \) is the pressure in the reservoir; \( P_H \) - the initial reservoir pressure; \( \mu \) - oil dynamic viscosity; \( k \)-permeability of the reservoir; \( \beta \) - coefficient of elastic capacity of the reservoir; \( q_i \) - the flow rate of the \( i \)-th well; \( \delta = \delta(x - \xi_i)(y - \xi_j) \) - Dirac delta function.

To numerically solve problem (1) - (4), the following dimensionless variables are introduced:

\[ P^* = P / P_0, \quad x^* = x / L; \quad y^* = y / L; \quad k^* = k / k_0; \quad \tau = \frac{k_0 L^2}{\beta \mu}; \quad Q^* = \frac{Q \mu}{\pi k_0 P_0 h_0}. \]

Here \( P_0, k_0 \) - are some characteristic values of pressure and permeability of the formation, respectively.

Later, for simplicity, the sign "*" in the equations is omitted. Then, taking this into account, problem (1) - (4) in dimensionless variables is rewritten as follows:

\[ \frac{\partial P}{\partial \tau} = \frac{\partial}{\partial x} \left( k(x,y) \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial y} \left( k(x,y) \frac{\partial P}{\partial y} \right) - Q, \]  \hspace{1cm} (5)

\[ P(x,y) = P_H(x,y), \quad t = 0, \quad (x,y) \in G, \]  \hspace{1cm} (6)

\[ -\frac{k(x,y)}{\mu} \frac{\partial P}{\partial n} = \alpha (P_A - P), \quad (x,y) \in \Gamma, \]  \hspace{1cm} (7)
$$\int_{s_i}^{s_i} k(x, y) \frac{\partial P}{\partial n} ds = -q_i(t), \quad (x, y) \in s_i, \quad i = 1, n_q.$$ (8)

3. Numerical simulation

For the numerical solution of problem (5) - (8) by the differential sweep method, the filtration region $G$ with the outer boundary $\Gamma$ is covered by the grid region $\Omega_{0\gamma} = \left\{ (x = i\Delta h, y = j\Delta h, \tau = r\Delta \tau); \quad i = 1, M; \quad j = 1, N; \quad k = 0, N_r; \quad \Delta \tau = \frac{1}{N_r} \right\}.$

To obtain a differential-difference problem, the algorithmic concept of an implicit scheme of variable directions (longitudinal-transverse scheme) is used. The transition from the $r$-th time layer to the layer $r+1$ takes place in two stages with a step $0.5\Delta \tau$. The result is a successive solution of two systems of differential-difference equations.

$$\frac{d}{dx} \left( k_i(x) \frac{dp_i^{r+0.5}(x)}{dx} \right) - \frac{1}{0.5 \Delta \tau} p_i^{r+0.5}(x) = -\Lambda_i \left[ k_i(x) p_i^r(x) \right],$$ (9)

Here

$$\begin{align*}
\Lambda_i \left[ k_i(x) P_{ij}^r \right] &= \frac{k_{i,0.5,j} p_{i,j}^{r} - \left( k_{i-0.5,j} + k_{i+0.5,j} \right) p_{i,j}^{r} + k_{i+0.5,j} p_{i+1,j}^{r}}{h_{xj}^2}, \\
\Lambda_i \left[ k_i(y) P_{ij}^{r+0.5} \right] &= \frac{k_{i,j-0.5} p_{i,j}^{r+0.5} - \left( k_{i,j-0.5} + k_{i,j+0.5} \right) p_{i,j}^{r+0.5} + k_{i,j+0.5} p_{i,j+1}^{r+0.5}}{h_{yj}^2}.
\end{align*}$$

The resulting system of differential-difference equations (9) is solved by the differential sweep method along each of the straight lines $x_i$ with initial conditions known at $\tau = \tau_r$, and then along each of the lines $y_j$, where the found values corresponding to the $r+0.5$-th layer are taken as the initial condition.

According to the differential sweep method, the solution of the first equation of the system of differential-difference equations (9) on the $r+0.5$-th time layer is derived by formulas:

$$P_j(x) = \frac{\gamma_{j}(x) u_j(x) - \alpha_{j}(x) w_j(x)}{\alpha_{j}(x) v_j(x) - \beta_{j}(x) u_j(x)}; \quad \frac{dP_j}{dx} = \frac{1}{k_j(x)} \frac{\gamma_{j}(x) v_j(x) - \beta_{j}(x) w_j(x)}{\alpha_{j}(x) v_j(x) - \beta_{j}(x) u_j(x)}.$$ (10)

Here the coefficients of the left-hand $\alpha(x), \beta(x), \gamma(x)$ and right-hand $u(x), v(x), w(x)$ sweep are found as solutions to the following Cauchy problems

$$\begin{align*}
\frac{d u_j(x)}{dx} &= v_j(x), \quad u_j(0) = 1, \\
\frac{d v_j(x)}{dx} &= R_j u_j(x), \quad v_j(0) = 0, \\
\frac{d w_j(x)}{dx} &= F_j u_j(x), \quad w_j(0) = 0;
\end{align*}$$ (11)

Here
The solution of the second differential-difference equation (9) on the \( r+1 \)-th time layer with boundary conditions (6) - (8) is determined similarly.

Numerical integration of the Cauchy problem is carried out by the Runge-Kutta method using the normalization procedure of the sweep coefficients and the coefficients of this method.

In each step, when calculating vector \( \vec{U}_{i+1} \), the normalized vector \( \vec{U} = (u, v, w) \) or \( \vec{U} = (\alpha, \beta, \gamma) \) is substituted into the right-hand part of the system of equations instead of \( \vec{U}_i \). The normalization procedure can be omitted if the chosen method steadily solves the Cauchy problem.

4. Computational experiments and their visualization

Based on the developed mathematical model, the Matlab program developed a software product called "Computer modeling of the filtration process of liquids in a non-homogeneous porous medium." The program includes blocks that include the filtration time of liquids, initial pressure, viscosity of the liquid, length of the layer in \( x \), \( y \) coordinates, layer thickness, well flow rate, permeability coefficients of the fragmented medium. Based on the parameters, that we entered in the program, when you press the button to calculate the indicators, the result will display images in 3D graphics and cut-outs, and write the numerical result in the file result.txt.

![Figure 1. Matlab software product interface window.](image)

In order to conduct computational experiments, the following initial data were considered: reservoir length \( L = 10000 \) m; reservoir thickness \( h = 20 \) m; initial reservoir pressure \( P_n = 300 \) atm; permeability of reservoir \( k = 0.5 \) and 0.05 Darcy; reservoir elastic capacity \( \beta = 2.3 \times 10^{-5} \) cm²/kg; oil viscosity \( \mu = 4 \) cps (centipoise).

Figures 2-5 show the results of calculation in the form of a graphical object.
Figure 2. 3D graph of pressure distribution (lower profile)

Figure 3. 3D graph of pressure distribution (upper profile)

Figure 4. Graph of pressure distribution profiles in section $x$: 1-360, 2-720, 3-1080 days

To ensure computing process control, an information array $I = \{i_{nj}\}$, $i = \overline{1, N}$, $j = \overline{1, M}$ is created.

In this case, the permeability values for each zone have the form:

- $G_1$: $k_1 = 0.05$
- $G_2$: $k_2 = 0.5$
- $G_3$: $k_3 = 0.05$

Figure 5. Graph of pressure distribution profiles in sections: 1-$x$, 2-$y$, 1080 days

Figure 6. Filtration area in piecewise inhomogeneous porous medium

Figure 7. Information array for permeability of porous medium

5. Conclusion

The developed mathematical software tool provides the calculation of basic indices of oil field development and the visualization of results of a computational experiment in a graphical form. The software tool provides numerical calculations in graphical visual form and can be used in design, analysis and forecasting of new oil field development.
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