New Horizons in Gravity:
Dark Energy and Condensate Stars

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Abstract.
Black holes are an apparently unavoidable prediction of classical General Relativity, at least if matter obeys the strong energy condition \( \rho + 3p \geq 0 \). However quantum vacuum fluctuations generally violate this condition, as does the eq. of state of cosmological dark energy \( \rho = -p > 0 \). When quantum effects are considered, black holes lead to a number of thermodynamic paradoxes associated with the Hawking temperature and assumption of black hole entropy, which are briefly reviewed. It is argued that the largest quantum effects arise from the conformal scalar degrees of freedom generated by the trace anomaly of the stress-energy tensor in curved space. At event horizons these can have macroscopically large backreaction effects on the geometry, potentially removing the classical event horizon of black hole and cosmological spacetimes, replacing them with a quantum phase boundary layer, where the effective value of the gravitational vacuum energy density can change. In the effective theory including the quantum effects of the anomaly, the cosmological term \( \Lambda \) becomes a dynamical condensate, whose value depends upon boundary conditions at the horizon. By taking a positive value in the interior of a fully collapsed star, the effective cosmological term removes any singularity, replacing it with a smooth dark energy de Sitter interior. The resulting gravitational vacuum condensate star (or gravastar) configuration resolves all black hole paradoxes, and provides a testable alternative to black holes as the final quantum mechanical end state of complete gravitational collapse. The observed \( \Lambda_{\text{eff}} \) dark energy of our universe likewise may be a macroscopic finite size effect whose value depends not on Planck scale or other microphysics but on the cosmological Hubble horizon scale itself.

1. Introduction: Classical Black Holes

Just a year after the publication of the field equations of General Relativity (GR), K. Schwarzschild found a simple, static, spherically symmetric solution of those equations, with the line element [1],

\[
ds^2 = -f(r)\, dt^2 + \frac{dr^2}{h(r)} + r^2 \left( d\theta^2 + \sin^2 \theta \, d\phi^2 \right),
\]

(1)

where in this case

\[
f(r) = h(r) = 1 - \frac{r_s}{r} = 1 - \frac{2GM}{rc^2}.
\]

(2)

This Schwarzschild solution to the vacuum Einstein’s equations, with vanishing Ricci tensor \( R^a_b = 0 \) and stress tensor \( T^a_b = 0 \) for all \( r > 0 \) describes an isolated, non-rotating object of total mass \( M \).
If the Schwarzschild solution (1)-(2) is taken seriously for \( r < r_M = 2GM/rc^2 \), the singularity at \( r = 0 \) is present in Einstein’s theory for any mass \( M > 0 \), including the macroscopic mass of a collapsed star with the mass of the sun, \( M_\odot \sim 2 \times 10^{33} \) gm. or even that of supermassive objects with masses \( 10^6 \) to \( 10^9 M_\odot \). The collapse of such enormous quantities of matter to a single mathematical point at \( r = 0 \) certainly presents a challenge to the imagination, and one that it seems Einstein himself sought arguments to avoid [2]. The situation is scarcely more acceptable if the singularity is removed only by the intervention of quantum effects at the extremely tiny Planck length \( L_{Pl} = (G\hbar/c^3)^{1/2} = 1.616 \times 10^{-33} \) cm.

On the other hand, unlike the central singularity of the Schwarzschild metric at \( r = 0 \), and despite the coordinate singularity at \( r = r_M \), local scalar invariant quantities that can be constructed from the contractions of the Riemann curvature tensor remain finite as \( r \rightarrow r_M \). For example the fully contracted quadratic Riemann invariant

\[
R_{abcd}R^{abcd} = \frac{12r_M^2}{r^n},
\]

which diverges at \( r = 0 \) remains finite at \( r = r_M \). Also, although the time for an infalling particle to reach the horizon is infinite for any observer remaining fixed outside the horizon, the *proper time* measured by the particle itself during its infall remains finite as \( r \rightarrow r_M \) [3, 4]. Since the line element (1) is non-singular for \( 0 < r < r_M \), the most straightforward possibility would seem to be to assume that this non-singular vacuum interior can be matched smoothly to the non-singular exterior Schwarzschild solution. This matching was achieved by the coordinate transformations and analytic continuation of the Schwarzschild solution found by Kruskal and Szekeres [5]. In the theses new \((T, X)\) coordinates, the Schwarzschild line element (1)-(2) becomes

\[
ds^2 = \frac{4r^3}{r} e^{r/2r_M} (-dT^2 + dX^2) + r^2 (d\theta^2 + \sin^2 \theta d\phi^2),
\]

where \( r \) is to be regarded as the function of the Kruskal-Szekeres \((T, X)\) defined implicitly by

\[
\left( \frac{r}{r_M} - 1 \right) e^{r/2r_M} = X^2 - T^2.
\]

Thus the (singular) transformation to Kruskal-Szekeres coordinates \((T, X)\) has removed the coordinate singularity of the Schwarzschild line element (1)-(2).

An important point which is often left unstated is that this mathematical procedure of analytic continuation through the null hypersurface of an event horizon at \( r = r_M \) actually involves a *physical assumption*, namely that the stress-energy tensor \( T^a_b \) is vanishing there. Even in the purely classical theory of General Relativity, the hyperbolic character of Einstein’s equations allows generically for stress-energy sources and hence metric discontinuities on the horizon which would violate this assumption. Additional physical information is necessary to determine what happens at the event horizon, and the correct matching of interior to exterior geometry depends on this physics, which may or may not be consistent with (complex) analytic continuation of coordinates around the singularity at \( r = r_M \).

The static Schwarzschild solution of an isolated uncharged mass was generalized to include electric charge by Reissner & Nordström [6], and more interestingly for astrophysically realistic collapsed stars, to include rotation and angular momentum by Kerr [7]. The complete analytic extensions of the Reissner-Nordström and Kerr solutions were found as well [8]. The global properties of these analytic extensions are more complicated and arguably even more unphysical than in the Schwarzschild case. For slowly rotating black holes with angular momentum \( J < GM^2/c \), there are an *infinite* number of black hole interior and asymptotically flat exterior
regions, singularities, and closed timelike curves in the interior region(s), which violate causality on macroscopic distance scales [9]. Again these apparently unphysical features appear in GR only if the mathematical hypothesis of complex analytic extension and continuation through real coordinate singularities is assumed. Based on the Principle of Equivalence between gravitational and inertial mass, Einstein’s theory possesses general coordinate invariance under all regular and real transformations of coordinates. It is the appending to classical General Relativity of the much stronger mathematical hypothesis of complex analytic continuation through singular coordinate transformations that leads to the global aspects of the Schwarzschild solution which may be unrealized in Nature. This analytic continuation is generally invalid if there are stress-tensor sources encountered at or before the breakdown of coordinates.

In the general black hole solution characterized by mass $M$, angular momentum $J$, and electric charge $Q$, one can define a quantity called the irreducible mass $M_{irr}$ by the relation [10]

$$M^2 = \left( M_{irr} + \frac{Q^2}{4GM_{irr}} \right)^2 + \frac{e^2 J^2}{4G^2 M^2_{irr}}, \quad (6)$$

The differential form of (6) is [11, 12]

$$dE = dMc^2 = \frac{c^2}{8\pi G} \kappa dA + \Omega dJ + \Phi dQ \quad (7)$$

which is the Smarr formula for a Kerr-Newman rotating, electrically charged black hole, in which

$$\kappa = \frac{1}{M} \left[ \frac{c^4}{4G} - 4\pi^2 \frac{G^4}{c^4 A^2} (Q^4 + 4e^2 J^2) \right], \quad (8a)$$

$$\Omega = \frac{4\pi J}{MA}, \quad (8b)$$

$$\Phi = \frac{Q}{M} \left[ \frac{c^2}{2G} + \frac{2\pi Q^2}{Ac^2} \right], \quad (8c)$$

$$A = \frac{16\pi G^2}{c^4} M^2_{irr} = \frac{4\pi G}{c^4} \left[ 2GM^2 - Q^2 + 2\sqrt{G^2 M^4 - GM^2 Q^2 - e^2 J^2} \right], \quad (8d)$$

are the horizon surface gravity, angular velocity, electrostatic potential and area respectively. All dimensionful constants have been retained to emphasize that (7)-(8) are formulae derived from classical GR in which no $\hbar$ whatsoever appears. Notice also that the coefficient of $dA$ in (7)

$$\left( \frac{\partial E}{\partial A} \right)_{J,Q} = \frac{c^2 \kappa}{8\pi G} \quad (9)$$

has both the form and dimensions of a surface tension.

2. Quantum Black Holes and Thermodynamic Paradoxes

It has been shown that in any classical process the irreducible mass $M_{irr}$ in (6), and therefore from (8d) the geometric area $A$ of the horizon can never decrease [10, 13]. Since matter falling into a black hole would take its entropy with it, leading to an apparent violation of the Second Law of Thermodynamics, but classically $A$ always increases, Bekenstein proposed that the black hole should itself be assigned an entropy proportional to $A$ [14]. Since $A$ does not have the units of entropy, it is necessary to divide the area by another quantity with units of length squared before multiplying by Boltzmann’s constant $k_B$, to obtain an entropy. However, classical GR (without a cosmological term) contains no such quantity, $G/c^2$ being simply a conversion factor.
between mass and distance. Hence Bekenstein found it necessary for purely dimensional reasons to introduce Planck’s constant $\hbar$ and the Planck length $L_{Pl} = (\hbar G/c^3)^{1/2} = 1.616 \times 10^{-33}$ cm into the discussion, proposing that the entropy of a black hole should be

$$S_{BH} = \gamma k_B A L_{Pl}^2 = \frac{\gamma}{hc} \frac{16\pi G k_B}{M_{irr}} M^2,$$

(10)

with $\gamma$ a constant of order unity [14]. He showed that if such an entropy were assigned to a black hole, so that it is added to the entropy of matter, $S_{tot} = S_m + S_{BH}$, then this total generalized entropy would plausibly always increase. In fact, this is not difficult at all, and the generalized second law $\Delta S_{tot} \geq 0$ is usually satisfied by a very wide margin, simply because the Planck length $L_{Pl}$ is so tiny, and the macroscopic area of a black hole measured in Planck units is so enormous. Hence even the small increase of mass and area caused by dropping into the black hole a modest amount of matter and concomitant loss of matter entropy $\Delta S_m < 0$ is easily overwhelmed by a great increase in $S_{BH}$, $\Delta S_{BH} \gg |\Delta S_m|$, guaranteeing that the generalized total entropy increases: $\Delta S_{tot} > 0$.

Soon after Bekenstein’s proposal Hawking argued that black holes would also emit radiation at a characteristic temperature [15]

$$T_H = \frac{\hbar \kappa}{2\pi c k_B J=Q=0} = \frac{\hbar c^3}{8\pi G k_B M},$$

(11)

where the first equality is general, and the second equality applies only for a Schwarzschild black hole with $J = Q = 0$. With the temperature inversely proportional to its mass assigned to a black hole by this formula, if we interpret (7) as the first law of thermodynamics in the form

$$dE = dM c^2 = T_H dS_{BH} + \Omega dJ + \Phi dQ,$$

(12)

then the coefficient $\gamma$ in (10) is fixed to be $1/4$. Notice that this involves simply multiplying $\kappa$ and dividing $A$ in the classical Smarr formula (7) by $\hbar$.

Since the temperature $T_H < 0.1 \mu K$ for a solar mass black hole is so exceedingly small, the prospects for testing the Hawking prediction experimentally are correspondingly remote. Nevertheless the formula (12) is simple and appealing, and it has been generally accepted since soon after Hawking’s paper appeared. However, simultaneously and from the very beginning, a number of problems with this thermodynamic interpretation made their appearance as well.

Firstly, the entropy (10) is non-extensive both in not being proportional to the volume but to the area, and in being entirely independent of the number of particle species. This points to a basic problem in trying to account for the entropy by a microcanonical counting of microstates according to Boltzmann’s formula,

$$S = k_B \ln W(E).$$

(13)

Normally one would expect the number of distinct microscopic states $W(E)$ at a given total energy $E$, and hence the entropy, to depend on the number of distinct particle species.

Secondly, tracing the thermal quanta at asymptotic temperature $T_H$ backwards indicates that at late times they must have originated very close to the event horizon at $r = r_{H}$ with local energy

$$\hbar \omega_{loc} \sim \frac{k_B T_H}{f^\frac{1}{2}(r)} = \frac{k_B T_H}{\sqrt{1 - \frac{r_{H}}{r}}} \to \infty,$$

(14)

which is arbitrarily large, and in particular exceeds the Planck energy, at which the semi-classical approximation of a fixed classical background geometry would be expected to fail.
Thirdly, as pointed out by Hawking himself [16], a temperature inversely proportional to the moment $M = E/c^2$ implies that the heat capacity of a Schwarzschild black hole,

$$\frac{dE}{dT_H} = -\frac{8\pi G k_B M^2}{\hbar c} = -\frac{M c^2}{T_H} < 0 \tag{15}$$

is negative, whereas in statistical mechanics the heat capacity of any system in stable equilibrium is related to the energy fluctuations about its mean value $\langle E \rangle$ by (at constant volume)

$$c_v = \left( \frac{d\langle E \rangle}{dT} \right)_v = \frac{1}{k_B T^2} \left\langle (E - \langle E \rangle)^2 \right\rangle > 0, \tag{16}$$

and is necessarily positive [17]. A negative heat capacity can be formally obtained in the microcanonical treatment of certain non-relativistic gravitational systems such as globular clusters with very long relaxation times that have not yet reached a true equilibrium state [18, 19]. However whereas the relaxation time scale for a globular cluster due to two-body stellar encounters can be much larger than the dynamical time scale $\sqrt{R^3/GM}$ (with $R$ a typical dimension of the cluster and $M$ its mass), statistical fluctuations in the Hawking thermal flux occur on the dynamical time scale $2GM/c^3$ itself, which is very short ($\sim 10^{-5}$ sec for a solar mass black hole). This is indeed the time scale for an unstable mode around the equilibrium state to develop [20]. It is therefore by no means clear that equilibrium thermodynamics can be applied to black holes at all.

Finally, a cold quantum system generally has a low entropy since thermal excitations are suppressed at zero temperature. However in the limit $M \to \infty$ or $\hbar \to 0$, for which (11) gives $T_H \to 0$, (10) gives $S_{BH} \to \infty$, i.e. an infinitely large entropy at absolute zero temperature!

It is instructive to evaluate $S_{BH}$ for typical astrophysical black holes. Taking as our unit of mass the mass of the sun, $M_\odot \simeq 2 \times 10^{33}$ gm., we have

$$S_{BH} \simeq 1.050 \times 10^{77} k_B \left( \frac{M}{M_\odot} \right)^2. \tag{17}$$

This is truly an enormous entropy. For comparison, we may estimate the entropy of the sun as it is, a hydrogen burning main sequence star, whose entropy is given to good accuracy by the entropy of a non-relativistic perfect fluid. This is of the order $Nk_B$ where $N$ is the number of nucleons in the sun $N \sim M_\odot/m_N \sim 10^{57}$, times a logarithmic function of the density and temperature profile which may be estimated to be of the order of 20 for the sun. Hence the entropy of the sun is roughly

$$S_\odot \sim 2 \times 10^{58} k_B, \tag{18}$$

or nearly 19 orders of magnitude smaller than (17). Since (17) makes no reference to how a black hole is formed, we may in principle imagine forming one from the sun adiabatically by an arbitrarily slow contraction. At every time in this adiabatic process the entropy of the sun would remain (18). At the instant that the event horizon is reached this entropy would have to jump discontinuously somehow to (17). From Boltzmann’s formula (13) this means that the number of microstates of a black hole must jump by $\exp(10^{19})$ at that instant at which the event horizon is reached, a truly staggering proposition. The loss of the information represented by this enormous jump in entropy is one form of the “information paradox,” which is so serious that it led Hawking to suggest that quantum mechanics itself must be altered [21]. This is all the more strange when one considers that according to classical GR by a change to Kruskal-Szekeres coordinates (4) nothing at all is supposed to happen at the event horizon.

For all of these reasons the thermodynamic interpretation of (12) remains problematic. On the other hand, if the inserted $\hbar$ in (12) is simply cancelled and one returns to the differential
Smarr relation (7), derived from classical GR, these difficulties immediately vanish. One would only be left to explain in what sense the surface gravity $\kappa$ is a surface tension.

If collapse to a black hole does not produce an equilibrium state, because of its negative heat capacity and rapidly growing unstable negative mode, the question then naturally arises of what is the final equilibrium state of complete gravitational collapse? In condensed matter physics at high density and low temperatures quantum effects always play an important role, and lead generally to a phase transition in which a macroscopic condensate is formed. In the proposal of a quantum phase transition producing a gravitational Bose-Einstein condensate (GBEC) in the interior of a fully collapsed object a physical surface and surface tension replaces the mathematical horizon of classical black hole, and removes all of the thermodynamic paradoxes of quantum black holes [22].

3. Quantum Effects at Horizons: The Role of the Trace Anomaly

The analytic continuation of the static Schwarzschild geometry to the black hole interior by Kruskal coordinates (4) assumes the complete absence of sources to Einstein’s eqs. in the vicinity of the horizon. However, it has long been known that at least in certain states quantum stresses can be quite important there. In the natural “vacuum” state empty of all particles at infinity and corresponding to the static Schwarzschild time $t$, called the Boulware state $\vert B \rangle$ [23], the expectation value of the renormalized stress tensor [24]

$$\langle B \vert T^a_b \vert B \rangle_R \rightarrow -\frac{\pi^2}{90} \frac{\hbar c}{(4\pi r_M)^4} \left( 1 - \frac{r_M}{r} \right)^{-2} \text{diag} (-3, 1, 1, 1),$$

becomes arbitrarily large as $r \rightarrow r_M$. Being a dimension four operator, the $(1 - \frac{r_M}{r})^{-2}$ behavior of $\langle B \vert T^a_b \vert B \rangle_R$ is a kinematic consequence of the blueshift (14). Clearly, if such a state were even close to being realized in practice, its stress-energy would act as a significant physical source for the semi-classical Einstein’s equations,

$$R^a_b - \frac{R}{2} \delta^a_b + \Lambda \delta^a_b = 8\pi G \langle T^a_b \rangle_R,$$

and necessarily influence the background spacetime (1). The assumption that $T^a_b = 0$ on the horizon upon which the analytic continuation of coordinates in (4) critically depends would have to be re-evaluated, or discarded entirely, leading potentially to a very different interior solution.

Let us dispel at this point any worry that large quantum effects on the horizon violate coordinate invariance or the Equivalence Principle. It is only some classical notions about locality that are violated by such effects and in the same way that quantum mechanics itself usually violates them. The quantum effective action in a background field, be it electromagnetic or gravitational, is generally a non-local functional of the background. It is this non-locality which makes particle creation in electromagnetic fields or in curved spacetimes possible. For this same reason the stress tensor (19) derived from the one-loop quantum effective action depends on boundary conditions in the full spacetime, not simply local quantities such as the Riemann curvature (3), which are small on the horizon. Because of quantum mechanics matter behaves as waves and it is these virtual matter waves in the macroscopic quantum state, non-local and coherent on the horizon scale that lead to (19). In a different (Unruh) state it is exactly the same non-local wavelike nature of matter that leads to the Hawking particle creation process.

If there were but a single “vacuum” state of matter in a Schwarzschild background that showed evidence of large quantum effects near the horizon, it might be possible to argue that the single Boulware state $\vert B \rangle$ should be excluded as pathological. In fact, just the opposite is true. By considering the most important non-local terms in the quantum effective action, related to the conformal anomaly, it is possible to show that most states behave as in (19) [25, 26].
That the important quantum effects near horizons are associated with the conformal or trace anomaly is related to the fact that due to the kinematic blueshift (14) becoming greater than all other mass scales as the horizon is approached, all quantum fields, massless or not, become essentially conformal in the horizon region. Quantitatively, it has recently become possible to calculate these effects from the form of the quantum effective action due to the anomaly,

\[ S_{\text{anom}} = \frac{b'}{2} \int d^4x \sqrt{-g} \left\{ -\left( \Box \varphi \right)^2 + 2 \left( R^{ab} - \frac{R}{3} g^{ab} \right) \nabla_a \varphi \nabla_b \varphi + \left( E - \frac{2}{3} \Box R \right) \varphi \right\} + b \int d^4x \sqrt{-g} \left\{ -\left( \Box \varphi \right) \left( \Box \psi \right) + 2 \left( R^{ab} - \frac{R}{3} g^{ab} \right) \nabla_a \varphi \nabla_b \psi + \frac{1}{2} C_{abcd} C^{abcd} \varphi + \frac{1}{2} \left( E - \frac{2}{3} \Box R \right) \psi \right\} \tag{21} \]

in terms of curvature tensors and two new scalar field degrees of freedom \( \varphi \) and \( \psi \) [25]. The form of this effective action is determined by general principles of covariance and the form of the trace anomaly, with only the coefficients \( b \) and \( b' \) dependent upon the number and spin of the underlying quantum fields [27]. The action (21) is clearly a spacetime scalar and therefore in that sense it is certainly consistent with the Equivalence Principle.

If (21) is varied with respect to the scalar fields \( \varphi \) and \( \psi \), one obtains their (linear) eqs. of motion. Solving these linear differential eqs. formally in terms of the Green’s functions of the relevant differential operator and inserting this formal solution for \( \varphi \) and \( \psi \) back into (21) allows one to rewrite the effective action in its original non-local but still covariant form. This non-local effective action of the trace anomaly in four dimensions is the analog of the two-dimensional anomaly effective action [28], which can also be put into a local form with the addition of a new local scalar degree of freedom [29].

The eqs. for the scalar fields \( \varphi(r) \) and \( \psi(r) \) derived from (21) can also be solved in the static spherically symmetric Schwarzschild background, and the results substituted into the stress tensor also obtained from (21) by varying with respect to the metric. This stress tensor incorporating the one-loop quantum effects of the anomaly generally exhibits divergent behavior at the horizon, just as in (19), for a wide variety of states [25]. This verifies explicitly the origin of such divergent behavior as the one-loop non-local quantum effective action of the anomaly, and the generic behavior of the divergences of the stress tensor near \( r = r_\bullet \) due to quantum effects. Let us remark also that it is by no means necessary for the state to have truly divergent behavior on the horizon as in (19) in order to have large quantum effects there. Any state in which \( G \langle T^a_{\mu} \rangle_R \) becomes of order of the (small) classical curvature (23) near the horizon is already enough to produce significant backreaction effects on the geometry. To see this note that Einstein’s equations (20) in the general static spherically symmetric metric (1) imply

\[ \frac{1}{r} \frac{d}{dr} \left( \frac{h}{f} \right) = -8\pi G \frac{\rho + p}{f}, \tag{22} \]

while the Riemann curvature tensor has the component

\[ R^{tr}_{tr} = \frac{h}{4} \left( f'' f^2 - 2 f' f'' - f' \frac{f''}{f} \right) \tag{23} \]

(where primes denote differentiation with respect to \( r \)). In a pure vacuum, \( h(r) = f(r) \) and (23) becomes \(-f''/2\) and hence remains finite. But from (22) if \( \rho + p > 0 \) in the horizon region where \( f \) or \( h \) vanishes, in general \( h \neq f \) and the cancellation of the singularities in (23) will not occur. In that case even the Riemann curvature invariant can become large. The horizon region defined by one or more metric functions \( h \) or \( f \) approaching zero is extremely sensitive to matter source perturbations, and even “small” quantum vacuum polarization effects can have relatively large effects on the spacetime geometry in this region. The anomaly scalar fields \( \varphi(r) \) or \( \psi(r) \)
behaving like $\ln f(r) \to \infty$ shows that non-local quantum effects can easily produce such large perturbations near the horizon.

In flat space one can understand the large quantum effects of the anomaly on the light cone (hence a null surface such as an event horizon of a black hole) as the result of the massless poles of the conformal $\varphi$ and $\psi$ scalar propagators, which describe correlated two-particle states of the underlying quantum fields at threshold [30]. Two-particle pairing at the Fermi surface is common in many-body physics, where it is responsible for superfluidity and superconductivity at low temperatures. In these non-relativistic quantum systems as the temperature is lowered or the pressure raised there is a phase transition to a condensate, in which a macroscopically large number of paired fermions or bosons collapse into a single coherent quantum state. In relativistic quantum field theory, anomalies (both axial and conformal) seem to be the only cases where two-particle pairing to form a spin zero bosonic state at the kinematic threshold $k^2 = 0$ of the Dirac sea (for a fermion pair) can occur consistent with Lorentz or general coordinate invariance. In this case, of course, it is the vacuum itself which becomes unstable. The strong effects of these anomalous quantum two-particle correlations near black hole horizons invites us then to consider the possibility that a phase transition in spacetime itself is possible, in which the black hole interior is replaced by a kind of low temperature gravitational vacuum condensate.

4. Gravitational Vacuum Condensate Stars

The region near the horizon $r = r_M$ in which quantum effects can be important is very narrow. The spatial extent of the region can be estimated by asking how close to $r_M$ must $r$ be for the quantum stresses (19) to give curvature corrections through Einstein’s eqs. (20) of the same order as the classical Riemann curvature (23). This gives the estimate

$$|f(r_M \pm \delta r)| = \left| 1 - \frac{r_M}{r_M \pm \delta r} \right| \approx \frac{\delta r}{r_M} \equiv \epsilon \approx \frac{M_{Pl}}{M} \approx 10^{-38} \frac{M_\odot}{M},$$

in solar mass units. Thus $\delta r \approx L_{Pl}$. However because the physical distance is given by the line element (1) this corresponds to a physical distance

$$\ell \approx \frac{\delta r}{f^2} \approx \sqrt{L_{Pl} r_M} \approx 3 \times 10^{-14} \sqrt{\frac{M}{M_\odot}} \text{ cm},$$

of the order of the diameter of an atomic nucleus for $M$ of a few solar masses, which is tiny on astrophysical scales but still much larger than the microscopic Planck length. Well away from this boundary layer $f(r) = (1 - r_M/r) \to 1$, and quantum effects are of order $\epsilon^2$, and hence completely negligible. Thus we expect the general conditions for an effective field theory treatment to hold everywhere except within this thin boundary layer of thickness $\ell$, where the quantum effects are relatively large. Mathematically, this situation is similar to those encountered in hydrodynamic shock waves where flows can be accurately described by the continuum Navier-Stokes eqs. except for a thin boundary layer inside the shock front of order of the mean free path of the molecular constituents of the fluid where the continuum approximation breaks down. A second example from fluid mechanics of a boundary layer is the Prandtl layer of the flow of a fluid around an obstacle, such as a ship’s bow or an airplane’s wing, which is responsible for the full macroscopic drag on the body. In both of these situations one can obtain an accurate description in the continuum fluid description, provided one supplies higher derivative terms usually neglected and/or proper boundary conditions derived from conservation laws across the layer.

Away from the boundary layer on either side of it Einstein’s eqs. (20) apply with negligible quantum effects. However, because the layer is a phase boundary, in which the quantum vacuum itself has changed its character, the parameters of the low energy description in terms of classical
General Relativity need not have the same values on opposite sides of the phase boundary. On general grounds the “latent heat” of the ground state of a system can change in a phase transition, and for the vacuum itself this is the cosmological term $\Lambda$ in Einstein’s eqs. (20). Moreover it has been shown that the fluctuations of the anomaly scalar degrees of freedom in (21) which are responsible for the large quantum effects near the horizon also give rise to the cosmological term becoming dynamical, varying in space and time [29, 31]. A non-zero $\Lambda$ can also be interpreted thermodynamically as a zero entropy density condensate because of the Gibbs’ relation

$$\rho + p = sT + \mu n. \quad (26)$$

Since $\Lambda > 0$ corresponds to a positive vacuum energy density with negative pressure $p = -\rho$, the entropy density $s$ must vanish if there is no conserved number density or associated chemical potential $\mu = 0$. We are familiar with phenomenological low energy effective field theories such as the electroweak theory of the Standard Model where the vacuum energy and effective $\Lambda$ changes due to the Higgs field developing a non-zero condensate $\langle \Phi \rangle \neq 0$ which is a pure quantum state with zero entropy, so that $W = 1$ and $S = 0$ in (13). In the bag model of hadrons there is a vacuum energy, the bag constant, associated with the rapid crossover (not a true phase transition) between the interior of a hadron where quarks and gluons are approximately free and the exterior where they cannot propagate. Note also that in all these cases the vacuum condensate eq. of state $p = -\rho$ with $\rho > 0$ violates the strong energy condition $\rho + 3p \geq 0$ which is used to prove the classical black hole singularity theorems [9].

Thus it is reasonable to assume that the effective value of $\Lambda$ will change at the quantum phase transition and become non-zero in the interior of the quantum boundary layer at $r \simeq r_M$. Einstein’s eqs. again apply within with this new value of $\Lambda$ describing the interior vacuum condensate state. Just as the vacuum Einstein’s eqs. possess a static, spherically symmetric solution for an isolated mass, namely the Schwarzschild solution (2), they possess a static, spherically symmetric vacuum solution for positive cosmological term, namely the de Sitter solution (1) with [32]

$$f(r) = h(r) = 1 - \frac{r^2}{r_H^2}, \quad \text{with} \quad r_H = \sqrt{\frac{3}{\Lambda}}. \quad (27)$$

Just as the exterior Schwarzschild geometry will attract all classical matter into its horizon at $r = r_M$, the de Sitter geometry will sweep out all classical matter outwards towards its horizon at $r = r_H$. Thus a strictly static solution of Einstein’s eqs. is possible if both the exterior and condensate interior are free of other matter or stress energy sources. As the de Sitter horizon is approached from the interior there are again quantum effects of the anomaly action and stress tensor in generic quantum states that grow like $f^{-2}(r) = (1 - H^2 r^2)^{-2}$ [25, 33]. Hence a globally static solution is possible only if the interior and exterior geometries are matched in their mutual near horizon regions: $r_H \simeq r_M$. This fixes the interior vacuum condensate energy density to

$$\rho_{\text{cond}} = \frac{3c^4H^2}{8\pi G} = \frac{3c^8}{32\pi G^3M^2}, \quad (28)$$

so that the total mass of the interior condensate is

$$\frac{4\pi r_H^3}{3} \frac{\rho_{\text{cond}}}{c^2} = M, \quad (29)$$

the total mass of the Schwarzschild solution in the exterior. This static matching of the interior de Sitter vacuum condensate (27) to the exterior Schwarzschild geometry (2) is what we have called a gravitational vacuum condensate star or gravastar [22, 34].
A key point about matching an interior de Sitter solution to an exterior Schwarzschild solution is that this matching cannot be achieved across a null surface such as an horizon because such a null surface can contain radiation leading to arbitrary discontinuities of the metric functions $f(r)$ and $h(r)$. However, once the null horizon singularity of these functions is regulated by a finite boundary layer of order $\ell$ in (25), then the entire boundary layer is a timelike tube. In ref. [22] this was done by application of the Israel junction conditions, assuming a $p = \rho$ relativistic fluid eq. of state within the boundary layer [35]. A more accurate matching is now possible with the effective action (21) and stress tensor derived from it. The stability question can then be re-examined through the Lagrangian variational principle the effective action provides.

The gravastar proposal for the final equilibrium quantum ground state for complete gravitational collapse eliminates all of the paradoxes associated with black holes. Since the interior is described by the static patch of de Sitter spacetime (27), there is evidently no singularity and no place for information to disappear into. Quantum unitarity is preserved. Since the interior is a pure quantum vacuum state which can be regarded as a macroscopic condensate, it has zero entropy. Any entropy is associated with the fluctuations present in the quantum boundary layer at $r \simeq r_M = r_H$, and may be estimated to be of order [22]

$$S \sim \frac{\rho}{T} \ell A \sim k_B \left( \frac{M}{M_{Pl}} \right)^{\frac{3}{2}} \simeq 10^{57} k_B \left( \frac{M}{M_{\odot}} \right)^{\frac{3}{2}}, \tag{30}$$

which is within an order of magnitude or so of a typical stellar progenitor. The $M^{\frac{3}{2}}$ dependence on mass is consistent with estimates from ordinary thermodynamics for a relativistic star (see e.g. [36]). Hence there is no enormous mismatch of entropies to be explained as for (17), and no information paradox. There is also no Hawking radiation from a gravastar, the black hole event horizon having been replaced with a boundary layer or thin shell of finite thickness (25) and finite energy density. Although this energy density is enormous by terrestrial standards, it is of the same order as that of a neutron star, i.e. nuclear energy density, and far below Planck scale energy densities. Hence there is no trans-Planckian problem and the effective field theory of Einstein’s equations (20) augmented by the quantum effects of the anomaly action (21) should be sufficient to describe the main macroscopic features of gravastars. The final state of a completely cooled gravastar as its temperature approaches absolute zero is a quantum stable ground state of zero entropy similar to a finite sized Bose-Einstein condensate of cold atoms at absolute zero, trapped by its own self-gravity.

As astrophysical objects gravastars would be cold, dark and compact, and therefore mimic classical black holes in almost every respect observationally. In complete isolation it would be virtually impossible to tell a gravastar from a black hole. The accretion of matter onto a gravastar would be similar to that of a black hole except at the very last stage when it interacts with the boundary layer. At the boundary layer, matter can be converted into pure energy and baryon and lepton number is likely also violated by anomalous processes in the Standard Model. Extensions to more realistic solutions with rotation, magnetic fields and taking into account Standard Model matter interactions through the anomaly effective action (21) are possible. Although the release of energy by matter striking the boundary layer can be very large, since the boundary layer thickness (25) is so small, any radiation from a non-rotating gravastar would be extremely redshifted out of $\gamma$-rays or X-rays to much lower energies. Thus the standard arguments for the absence of a surface [37] do not apply [38]. Gravitational waves should be generated at the natural oscillation frequencies of the physical thin shell, and perhaps would provide the cleanest evidence of the surface distinguishing a gravastar from a black hole. Once rotation, particularly rapid rotation is considered, the situation may be quite different, and the efficient conversion of matter into energy could conceivably provide the central powerhouse for Gamma Ray Bursters and some of the most energetic sources observed in the universe.
A New Model for Cosmological Vacuum Energy

The scalar $\varphi$ and $\psi$ fields couple to the metric particularly strongly on the cosmological Hubble horizon scale where they have their largest effect. Thus the fluctuations of the scalar degrees of freedom determined by the anomaly may lead to a phase transition to precisely the conformally invariant phase of gravity described by the fixed point $\Lambda = 0$ [29, 31] in the near vicinity of the horizon. Since the interior condensate phase has the eq. of state $p = -\rho$, exactly that of the cosmological dark energy observations suggest pervade our universe [39], and the gravastar solution automatically fixes the vacuum condensate energy density $\rho_{\text{cond}}$ in terms of its size by (28), an interesting possibility raised by this solution is that the observable universe itself could be the interior of a gravastar. In that case the observed cosmological dark energy of our universe some 72% of the critical energy density $\rho_{\text{crit}}$ defined by the present value of the Hubble parameter $H_0$ would be identified with the condensate energy density

$$\rho_{\text{cond}} \simeq (0.72) \rho_{\text{crit}} = (0.72) \frac{3c^4H_0^2}{8\pi G} \simeq 6.5 \times 10^{-9} \text{ erg/cm}^3.$$ 

If our location is far from the bubble wall, it would be difficult to distinguish the cosmological model of a spherical bubble of vacuum energy condensate from the standard Friedmann-Robertson-Walker (FRW) model, although on the cosmological Hubble horizon scale $H_0^{-1} \approx 4.2$ Gpc and of course globally it would be dramatically different, with a preferred origin and a physical surface. At the horizon the quantum fluctuations of the scalar degrees of freedom described by the anomaly action (21) would be large. In ref. [33] it was pointed out that these anomaly scalars couple to the metric near the horizon and generate fluctuations that are similar to those in inflationary models. Thus it may be that these fluctuations emanating from the horizon are what we observe in the Cosmic Microwave Background (CMB) anisotropies, without any need to introduce an inflaton. Moreover, since the fluctuations of the anomaly scalars lead to a conformally invariant phase of gravity [29, 31], the fluctuations should have a spectrum and statistics consistent with conformal invariance, including in their non-Gaussian features. Thus, one way of testing this cosmological model would be to detect the non-Gaussian bi-spectrum of the CMB with the shape predicted by conformal invariance [40, 41, 42].

The new cosmological model is then a kind of bubble or ‘bag’ of vacuum energy, containing these gravitationally coupled degrees of freedom, which is well described as approximately a de Sitter universe of finite size of the order of the Hubble radius. Like an ordinary bubble of gas, the pressure inside is determined by the size of the bubble and its surface tension, not any UV cutoff, and the interior pressure adjusts itself dynamically to the boundary conditions at the Hubble scale. In the effective field theory of Einstein gravity augmented by the anomaly action (21), the vacuum energy of infinite flat space is identically zero, and the residual small $\rho_{\text{cond}}$ of (31) is a boundary effect of a macroscopic condensate. Then the essential cosmological problem becomes to explain why $\Omega_{\text{cond}} = \rho_{\text{cond}}/\rho_{\text{crit}} \simeq 0.72$ rather than unity, and how the vacuum energy and pressure adjusts itself dynamically at the boundary to maintain $\Omega_{\text{cond}} \approx 1$ over time.

Clearly additional details of the gravastar model incorporating matter and radiation need to be worked out and it then must pass the many successful tests of the usual FRW big bang model before it can be considered a realistic alternative cosmology. The essentially automatic incorporation of dark energy with the approximately correct value and solution of the cosmological dark energy problem would seem to make the model worth pursuing further.

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