An improved characteristics length in strouhal number for internal flow induced acoustics in corrugated pipe

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Abstract. Corrugated pipes often experience self-sustained oscillations which leads to high intensity noise, known as “whistling” during critical flow conditions. In order to determine the peak whistling Strouhal number ($S_{r_{p-w}}$) and whistling amplitude, a two dimensional (2D) numerical simulation method using Fluent software is proposed in this paper. The flow pattern of the fluid passing through the corrugated pipe and the effect of pipe geometries on the $S_{r_{p-w}}$ and whistling amplitude are investigated. A ratio of $\frac{l}{w+r_{up}}$ is found to be a good indicator of the consistency of $S_{r_{p-w}}$ where the $l$ is the plateau length, $w$ is the width of the corrugation, and $r_{up}$ is the upstream radius of the corrugation. Furthermore, optimal characteristic length is also assessed among three different characteristic lengths including $w + r_{up}$, $w$ and $\frac{l^2}{w+r_{up}}$. The $\frac{l^2}{w+r_{up}}$ is found to be the optimal as the characteristic length. This finding is significant as it can allow us to avoid critical flow condition to minimize and prevent the whistling phenomenon more accurately and effectively by using the proposed equations.

1. Introduction

There has been significance amount of study being done on the vortex induced vibration of pipe as this phenomenon caused the fatigue damage to the pipe [6] compare to the internal flow-induced acoustics of the corrugated pipe. Corrugated pipes are flexible pipes that are specially designed with corrugations to facilitate fluid flow between sea installations and surface equipment on drilling platforms. The main purpose of corrugations is to provide local stiffness and rigidity to prevent collapse of the pipe during bending while maintaining global flexibility. Nevertheless, corrugated pipes often experience self-sustained oscillations that leads to high intensity noise, known as “whistling” or “singing” during critical flow conditions [8], [9].

A special feature that can be observed in the flow of a corrugated pipe is known as shear layer which divides the high-speed flow region from the low speed flow region next to the pipe wall. This is shown in figure 1[1]. The formation of shear layer is normally noticed in a turbulent flow where the viscous sublayer adjacent to the wall does not have sufficient kinetic energy to travel against an adverse pressure gradient. When this occurs, there will be a back flow along the pipe wall in the opposite direction to the main flow. Eventually, a separation of the boundary layer (shear layer) from the pipe wall is resulted at a cavity where there is abrupt pipe widening. This phenomenon also can be observed on the pipe with multiple side branches [7], [10].
According to NakiboĞLu et al. [2], self-sustained oscillations is caused by interaction of shear layer instability with longitudinal local acoustic standing waves as defined in a feedback loop as shown in figure 2 [1]. The shear layers that are produced at the upstream of cavities are the source of sound due to its unsteadiness. The acoustic sound waves in turn impose acoustic pressure which then triggers the rolling up of the shear layer into vortices. These vortices detach periodically to form vortex shedding. According to Popescu et al. [1], a lock-in phenomena will occur where there is a resonance between longitudinal standing acoustic waves and vortex shedding when the frequency of vortex shedding coincides with the acoustic natural frequency of the pipe. The minimum fluid velocity for a lock-in to occur is known as onset velocity.

As shown in figure 3[3], the pipe’s inner diameter ($D_p$) is changing along the pipe length ($L_p$) in both systems periodically. In corrugated pipe, each cavity is in slit shape. Its width and depth are denoted as $W$ and $H$ respectively. The radiuses of cavities are named as $r_{up}$ and $r_{down}$ for upstream and downstream edges, respectively. Besides, the wave length of the variation of pipe diameter is known as pitch ($P_t$). Last but not least, the plateau ($l$) is referring to the part with fixed diameter between two consecutive cavities.
An important dimensionless parameter that is normally used to characterise the oscillating flow is known as Strouhal number, $Sr$.

$$Sr = \frac{fL_c}{U}$$  \hspace{1cm} (1)

where

- $f$ = oscillation frequency, Hz
- $L_c$ = characteristic length, m
- $U$ = average flow velocity in the pipe, m/s

According to Nakiboğlu et al. [2], as the average flow velocity increases, the frequency of whistling shows only slight increase in a particular resonant mode. As a result, the Strouhal number actually decreases at each mode and hence a range of Strouhal numbers will be noticed. The greatest value of Strouhal number in a specific mode is known as critical Strouhal number ($Sr_{cr}$). After that, the rise of average flow velocity increases the sound amplitude until it reaches a maximum value. The Strouhal number at peak sound amplitude at a particular acoustic mode is known as peak whistling Strouhal number ($Sr_{p-w}$). Nakiboğlu et al. [3] explained that $Sr_{p-w}$ number is obtained through a plotting of global gradient of consecutive modes.

In this paper, the method used is first discussed. The effect of pipe geometries including cavity height ($H$), downstream edge radius ($r_{down}$), upstream edge radius ($r_{up}$), cavity width ($w$) and plateau ($l$) on the $Sr_{p-w}$ and whistling amplitude are then investigated. In Section 3, the optimal characteristic length is identified among $w + r_{up}$, $w$ and $\frac{l^2}{w+r_{up}}$. In the last section, conclusions are drawn on the findings from the simulation work.

2. Numerical Methodology

2.1 Method Used and Benchmarking

2.1.1 Method Used. Before a simulation is conducted by using Fluent, it is necessary to sketch and mesh the 2D model of single cavity corrugate pipe using Gambit 2.3 modelling software. The 2D model is then saved and imported into Fluent for simulation work. Large Eddy Simulation (LES) turbulence model is used as it is very suitable for aero-acoustic simulations. It resolves all eddies with scales larger than grid scale but eliminates small scales of the solution. This is good enough as large eddies possess most energy and interact more strongly with the flow. In the following section, all the results are obtained by using an air velocity of 13.61 m/s unless otherwise specified.

There are two important considerations in order to read off the $Sr_{p-w}$ from the graph obtained from Fluent. Firstly, when the lock-on frequency is changed, a system will go through a minimum of energy according Popescu et al. [1]. Therefore, the peak should only be read off after the first global minimum amplitude. Secondly, the $Sr_{p-w}$ should only be referred to the first global peak after the first global minimum amplitude.
In the following sections, the characteristic length used to conduct the simulation is the summation of cavity width, $w$ and upstream edge radius, $r_{up}$, $(w + r_{up})$. Nevertheless, 3 different types of characteristic lengths including $w + r_{up}$, $w$ and $\frac{\ell^2}{w+r_{up}}$ are used in section 3 to determine the optimal characteristic length. In order to prevent confusion, $Sr_{p-w}$ are represented in $Sr_{w+r_{up}}$, $Sr_{w}$ and $Sr_{\frac{\ell^2}{w+r_{up}}}$ respectively, according to the characteristic length used to calculate the Strouhal number.

2.1.2 Benchmarking. In order to ensure the simulation results obtained from the Fluent software are reliable, several benchmarkings have been made by comparing the simulation results with the experimental work of Nakiboğlu et al. [3]. The simulation results are shown in the figure 4, 5 and 6 as sound pressure level vs Strouhal Number for sample Geo 2, Geo 4, and Geo 5 respectively and further compile as shown in table 1 with the comparison of experimental results of $Sr_{p-w}$ by Nakiboğlu et al. [3] and the simulation results of $Sr_{p-w}$.

| Sample | $W$ (mm) | $H$ (mm) | $r_{up}$ (mm) | $r_{down}$ (mm) | $l$ (mm) | Experimental $Sr_{p-w}$ | Simulation $Sr_{p-w}$ |
|--------|----------|----------|---------------|-----------------|---------|------------------------|----------------------|
| Geo 2  | 4        | 4        | 2             | 2               | 4       | 0.36                   | 0.34                 |
| Geo 4  | 4        | 4        | 2             | 2               | 0       | 0.38                   | 0.35                 |
| Geo 5  | 4        | 4        | 2             | 2               | 8       | 0.40                   | 0.40                 |

Table 1. Comparison of Experimental Results and Simulation Results of $Sr_{p-w}$.

Figure 4. Sound Pressure Level as a Function of Strouhal Number of Geo 2 Corrugated Pipe
Figure 5. Sound Pressure Level as a Function of Strouhal Number of Geo 4 Corrugated Pipe

Figure 6. Sound Pressure Level as a Function of Strouhal Number of Geo 5 Corrugated Pipe

2.2 Effect of Different Pipe Geometries on $S_{r_{w+w_{ap}}}$ and Whistling Amplitude

2.2.1 Effect of Cavity Depth, $H$. From the results obtained from simulation, it is found that the change of cavity depth, $H$ from 4mm to 10mm has only minor effect on the $S_{r_{w+w_{ap}}}$, which is in the range of 0.30 to 0.34. In addition, it has also insignificant effect on the whistling amplitude which only varies between 100dB to 102.5dB.

| $w$ (mm) | $H$ (mm) | $r_{ap}$ (mm) | $r_{down}$ (mm) | $l$ (mm) | Pitch (mm) | $l$ | $S_{r_{w+w_{ap}}}$ |
|--------|--------|--------------|-----------------|--------|-----------|-----|-----------------|
| 4      | 4      | 2            | 2               | 4      | 12        | 2/3 | 100.0           | 0.34 |
| 4      | 6      | 2            | 2               | 4      | 12        | 2/3 | 102.5           | 0.34 |
| 4      | 8      | 2            | 2               | 4      | 12        | 2/3 | 102.5           | 0.30 |
| 4      | 10     | 2            | 2               | 4      | 12        | 2/3 | 102.5           | 0.30 |
This can be inferred from the vortex velocity profile in which all the four types of geometries have similar clockwise vortex velocity magnitude around 1.5m/s. Besides, these four geometries also showed approximately same thickness of boundary layer.

2.2.2 Effect of Downstream Edge Radius, $r_{down}$. The change of downstream edge radius, $r_{down}$ from 2mm to 10mm has varied the $S_{r_w+r_{up}}$ within the range of 0.11 to 0.24, which is very minor. On the other hand, the change of $r_{down}$ has also insignificant effect on the whistling amplitude. The largest difference of whistling amplitude is only 5.7dB. The results is agreed with the work of Nakiboğlu et al. [3] who explained that this is because the shear layer thickness has become less concentrated as the downstream edge is approached. This can also be explained by the small changes of clockwise vortex velocity magnitude from 1.43m/s to 1.52m/s as $r_{down}$ is varied.

| Table 3. $\frac{l}{w+r_{up}}$ ratio, whistling amplitude, at Different Downstream Edge Radius. |
|---|---|---|---|---|---|---|---|
| $w$ (mm) | $H$ (mm) | $r_{up}$ (mm) | $r_{down}$ (mm) | $l$ (mm) | Pitch (mm) | $\frac{l}{w+r_{up}}$ | $S_{r_w+r_{up}}$ |
| 4 | 4 | 2 | 2 | 2 | 10 | 1/3 | 103.8 | 0.15 |
| 4 | 4 | 2 | 2 | 2 | 12 | 1/3 | 101.3 | 0.20 |
| 4 | 6 | 2 | 6 | 2 | 14 | 1/3 | 107.0 | 0.24 |
| 4 | 8 | 2 | 8 | 2 | 16 | 1/3 | 102.0 | 0.16 |
| 4 | 10 | 2 | 10 | 2 | 18 | 1/3 | 102.0 | 0.11 |

2.2.3 Effect of Upstream Edge Radius, $r_{up}$. From table 4, it is observed that as the upstream edge radius, $r_{up}$ is increased from 2mm to 8mm, the $S_{r_w+r_{up}}$ is increased from 0.15 to 0.62. When $r_{up}$ is further increased to 10mm, the $S_{r_w+r_{up}}$ is dropped to 0.42. Nothing can be concluded so far in terms of the effect of $r_{up}$ to $S_{r_w+r_{up}}$. On the other hand, it is observed from the velocity profile that the boundary layer becomes thinner and the clockwise vortex becomes weaker as the $r_{up}$ is increased. This can therefore explain why the whistling amplitude is decreased as the $r_{up}$ is increased until a saturation is observed at $r_{up}$ of 8mm. There is a reduction of 11.3dB when $r_{up}$ is increased from 2mm to 8mm. The results are also in agreement with [3] who discovered that rounding of $r_{up}$ is able to reduce the initial sound absorption and increase the whistling amplitude by a factor of 3 to 5.

| Table 4. $\frac{l}{w+r_{up}}$ ratio, whistling amplitude, $S_{r_w+r_{up}}$ at Different Upstream Edge Radius. |
|---|---|---|---|---|---|---|---|
| $w$ (mm) | $H$ (mm) | $r_{up}$ (mm) | $r_{down}$ (mm) | $l$ (mm) | Pitch (mm) | $\frac{l}{w+r_{up}}$ | $S_{r_w+r_{up}}$ |
| 4 | 4 | 2 | 2 | 2 | 10 | 1/3 | 103.8 | 0.15 |
| 4 | 4 | 2 | 2 | 2 | 12 | 1/4 | 97.0 | 0.37 |
| 4 | 6 | 2 | 6 | 2 | 14 | 1/5 | 95.0 | 0.57 |
| 4 | 8 | 2 | 8 | 2 | 16 | 1/6 | 92.5 | 0.62 |
| 4 | 10 | 2 | 10 | 2 | 18 | 1/7 | 93.0 | 0.42 |

2.2.4 Effect of Cavity Width, $w$. When the width of the cavity, $W$ is changed, a similar observation is noted as to that of the change of $r_{up}$. There is no a clear relationship between $w$ and $S_{r_w+r_{up}}$ because the $S_{r_w+r_{up}}$ fluctuates as the $w$ is changed. On the other hand, although thicker boundary layer and
stronger clockwise vortex are observed, the whistling amplitude is reduced significantly as the \( w \) is increased. As seen in table 5, the whistling amplitude is reduced from 107.5dB to 83dB when \( w \) is increased from 2mm to 12mm. A closer investigation to the vortex profile reveals that a secondary counter clockwise vortex is formed at the upstream of the primary clockwise vortex when \( w \) is 6mm and 8mm. Besides, a tertiary clockwise vortex is also formed at the upstream of the secondary counterclockwise vortex when \( W \) is 10mm and 12mm. It is inferred that these secondary and tertiary vortices are contributing to the reduction of whistling amplitude through a “cancelling” effect to the primary vortex.

Table 5. \( \frac{l}{w+r_{up}} \) ratio, whistling amplitude, \( S_{r_{w+r_{up}}} \) at Different Cavity Widths.

| \( w \) (mm) | \( H \) (mm) | \( r_{up} \) (mm) | \( r_{down} \) (mm) | \( l \) (mm) | Pitch (mm) | \( \frac{l}{w+r_{up}} \) (dB) | \( S_{r_{w+r_{up}}} \) |
|---|---|---|---|---|---|---|---|
| 2 | 4 | 2 | 2 | 2 | 8 | 1/2 | 107.5 | 0.30 |
| 4 | 4 | 2 | 2 | 2 | 10 | 1/3 | 103.8 | 0.15 |
| 6 | 4 | 2 | 2 | 2 | 12 | 1/4 | 92.5 | 0.31 |
| 8 | 4 | 2 | 2 | 2 | 14 | 1/5 | 89.0 | 0.58 |
| 10 | 4 | 2 | 2 | 2 | 16 | 1/6 | 84.0 | 0.57 |
| 12 | 4 | 2 | 2 | 2 | 18 | 1/7 | 83.0 | 0.44 |

2.2.5 Effect of Plateau, \( l \). From table 6, it is found that the whistling amplitude does not show an explicit trend as plateau, \( l \) is changed. According to Nakiboğlu and Hirschberg [4], there is a shift of peaks as \( l \) is varied. In other words, a secondary peak will form and grow at certain plateau length to replace the first peak which diminishes in the process until it disappears. Furthermore, an important finding is observed from table 6. It is noted that the \( S_{r_{w+r_{up}}} \) fluctuates when \( l \) is varied from 0mm to 6mm. Nevertheless, the \( S_{r_{w+r_{up}}} \) becomes more constant as \( l \) is changed from 6mm to 10mm.

The value of 6mm is exactly the same as the summation of \( w \) and \( r_{up} \), \( (w + r_{up}) \). Therefore, it is inferred that there is a relationship between the \( l \) and \( (w + r_{up}) \). In order to investigate this relationship, the ratio of \( \frac{l}{w+r_{up}} \) is computed for every geometry from table 2 to table 5 and tabulated in figure 7.

Table 6. \( \frac{l}{w+r_{up}} \) ratio, whistling amplitude, \( S_{r_{w+r_{up}}} \) at Different Plateaus.

| \( w \) (mm) | \( H \) (mm) | \( r_{up} \) (mm) | \( r_{down} \) (mm) | \( l \) (mm) | Pitch (mm) | \( \frac{l}{w+r_{up}} \) (dB) | \( S_{r_{w+r_{up}}} \) |
|---|---|---|---|---|---|---|---|
| 4 | 4 | 2 | 2 | 0 | 8 | 0 | 97.0 | 0.35 |
| 4 | 4 | 2 | 2 | 2 | 10 | 1/3 | 103.8 | 0.15 |
| 4 | 4 | 2 | 2 | 3 | 11 | 1/2 | 72.0 | 0.28 |
| 4 | 4 | 2 | 2 | 4 | 12 | 2/3 | 100.0 | 0.34 |
| 4 | 4 | 2 | 2 | 5 | 13 | 5/6 | 99.0 | 0.27 |
| 4 | 4 | 2 | 2 | 6 | 14 | 1 | 83.0 | 0.39 |
| 4 | 4 | 2 | 2 | 7 | 15 | 7/6 | 97.5 | 0.39 |
| 4 | 4 | 2 | 2 | 8 | 16 | 4/3 | 87.5 | 0.40 |
| 4 | 4 | 2 | 2 | 9 | 17 | 3/2 | 96.3 | 0.40 |
| 4 | 4 | 2 | 2 | 10 | 18 | 5/3 | 92.0 | 0.43 |
Figure 7. Graph of $S_r (w + r_{up})$ against $\frac{l}{w + r_{up}}$

From Figure 7, it is observed that when $l < w + r_{up}$, the value of $S_r (w + r_{up})$ is fluctuating. $S_r (w + r_{up})$ is at a maximum value of 0.62 when $\frac{l}{w + r_{up}}$ is 0.1667 and at a minimum value of 0.11 to 0.24 when $\frac{l}{w + r_{up}}$ is 0.3333. There is a smaller fluctuation for $0.3333 < \frac{l}{w + r_{up}} < 1$.

Nevertheless, when $l \geq w + r_{up}$ or $\frac{l}{w + r_{up}} \geq 1$, the value of $S_r (w + r_{up})$ is far more constant in the range of 0.39 to 0.43. A conclusion that can be made at this point is that when $l < w + r_{up}$, $w$ and $r_{up}$ have dominant effect over $l$ in affecting the flow over corrugated pipe, therefore value of $S_r (w + r_{up})$ is fluctuating. However, when $l \geq w + r_{up}$, $l$ is dominant over $w$ and $r_{up}$. The change of $S_r (w + r_{up})$ in this range is insignificant, which means that $l$ does not contribute much to the change of $S_r (w + r_{up})$ as compared to $w$ and $r_{up}$ as discussed earlier.

Although the ratio of $\frac{l}{w + r_{up}}$ is a good indicator of the consistency of $S_r (w + r_{up})$, it cannot be used to infer the value of $S_r (w + r_{up})$. In other words, even though two corrugated geometries have the same ratio of $\frac{l}{w + r_{up}}$, they do not necessarily have the same value of $S_r (w + r_{up})$. This can be seen by comparing Table 6 and 7. Although some of the geometries in two tables have the same ratio of $\frac{l}{w + r_{up}}$, their $S_r (w + r_{up})$ are significantly different.

| Table 7. $\frac{l}{w + r_{up}}$ ratio, whistling amplitude, $S_r (w + r_{up})$ at Different Geometries. |
|-----------------|-------|-------|-------|-------|-------|-------|-------|-------|
| $w$ (mm) | $H$ (mm) | $r_{up}$ (mm) | $r_{down}$ (mm) | $l$ (mm) | Pitch (mm) | $\frac{l}{w + r_{up}}$ | dB | $S_r (w + r_{up})$ |
| 11    | 6     | 6     | 2     | 10    | 42    | 10/17 | 88.8  | 0.67 |
| 11    | 4     | 4     | 2     | 10    | 27    | 2/3   | 98.8  | 0.28 |
| 6     | 4     | 4     | 2     | 10    | 22    | 1     | 97.0  | 0.21 |
| 4.5   | 4     | 3     | 2     | 10    | 19.5  | 4/3   | 104.0 | 0.22 |
| 4.5   | 4     | 1.5   | 2     | 10    | 18    | 5/3   | 94.0  | 0.17 |
It should be noted that Table 7 also shows a similar phenomenon as mentioned earlier, where a consistency of value of $S_{r_w + r_{up}}$ is observed when $l \geq w + r_{up}$ or $\frac{l}{w + r_{up}} \geq 1$.

3. Optimal characteristic Length

Numerous works have been done previously to determine the optimal characteristic length and the sum of the width of cavity and the upstream edge radius, $w + r_{up}$, which is also known as modified gap width, is found to be the best representation of characteristic length. Since the ratio of $\frac{l}{w + r_{up}}$ is found to be a good indicator of the consistency of $S_{r_w + r_{up}}$, it is therefore possible to use it as a characteristic length in calculating the Strouhal number provided it can give a lesser variation of $S_{r_p - w}$ than the use of $w + r_{up}$ as characteristic length.

Nevertheless, $\frac{l}{w + r_{up}}$ is only a ratio but not a length. It cannot be used as a characteristic length unless it has a unit of meter. In order to make it a characteristic length with unit of meter, $\frac{l^2}{w + r_{up}}$ is proposed in this project. This parameter is verified to be a good indicator of the consistency of $S_{r_w + r_{up}}$ as a similar pattern in figure 7 is also observed in figure 8, in which the $S_{r_w + r_{up}}$ is plotted against $\frac{l^2}{w + r_{up}}$.

![Graph of $S_{r_w + r_{up}}$ against $\frac{l^2}{w + r_{up}}$](image)

Figure 8. Graph of $S_{r_w + r_{up}}$ against $\frac{l^2}{w + r_{up}}$

Furthermore, $w$ is also used as a characteristic length in this paper and the results of $S_{r_p - w}$ obtained by using air velocity from 11.61 m/s to 21.61 m/s for three different types of characteristic length are tabulated in Table 8.

| $w$ (mm) | $H$ (mm) | $r_{up}$ (mm) | $r_{down}$ (mm) | $l$ (mm) | Pitch (mm) | Velocity (m/s) | $S_{r_{w + r_{up}}}$ | $S_{r_w}$ | $S_{r} \frac{l^2}{w + r_{up}}$ |
|---------|---------|---------------|-----------------|---------|-----------|---------------|-----------------|---------|-----------------|
| 4       | 4       | 2             | 2               | 4       | 12        | 11.61         | 0.38            | 0.29    | 0.10            |
| 4       | 4       | 2             | 2               | 4       | 12        | 13.61         | 0.34            | 0.25    | 0.05            |
| 4       | 4       | 2             | 2               | 4       | 12        | 15.61         | 0.30            | 0.20    | 0.13            |
| 4       | 4       | 2             | 2               | 4       | 12        | 17.61         | 0.30            | 0.09    | 0.06            |
| 4       | 4       | 2             | 2               | 4       | 12        | 19.61         | 0.43            | 0.17    | 0.08            |
| 4       | 4       | 2             | 2               | 4       | 12        | 21.61         | 0.30            | 0.08    | 0.05            |

Table 8. $S_{r_{w + r_{up}}}$, $S_{r_w}$ and $S_{r} \frac{l^2}{w + r_{up}}$ at Different Geometries.
From Table 8, it is observed that the use of \( w + r_{up} \) as the characteristic length results in variation of Strouhal number of \( 0.3 \leq Sr_{w+r_{up}} \leq 0.43 \). The use of \( w \) as the characteristic length has led to larger variation of Strouhal number in the range of \( 0.09 \leq Sr_w \leq 0.29 \). The smallest variation in Strouhal number is observed when \( \frac{l^2}{w+r_{up}} \) is used as the characteristic length, which is limited to \( 0.05 \leq Sr_{\frac{l^2}{w+r_{up}}} \leq 0.13 \).

It is therefore concluded that \( \frac{l^2}{w+r_{up}} \) is the optimal choice of characteristic length as the use of \( \frac{l^2}{w+r_{up}} \) in the calculation of Strouhal number led to smallest variation. This finding is significant as it can allow a better design of flow condition to minimize and avoid the whistling phenomenon. The equation 1 can be rearranged to obtain the following formula:

\[
U_{cr} = \frac{f_n L_c}{Sr_{p-w}}
\]  

(2)

where
- \( f_n \) = acoustic natural frequency of the pipe, Hz
- \( L_c \) = characteristic length, m
- \( U_{cr} \) = critical flow velocity in the pipe, m/s
- \( Sr_{p-w} \) = Strouhal number

This formula can allow us to predict the critical flow condition in which whistling phenomenon will occur and therefore avoid it. The value of \( Sr_{p-w} \) can be obtained from the data obtained from simulations when a particular value of characteristic length \( \frac{l^2}{w+r_{up}} \) is selected. The acoustic natural frequency of the pipe, \( f_n \), can then be calculated by using Cumming’s formula as proposed in Kristiansen and Wiik [5] as follow:

\[
f_n = 0.89 \frac{nc}{2L'}
\]  

(3)

where
- \( n \) = mode number (1, 2, 3, ….)
- \( c \) = speed of sound, m/s
- \( L' \) = effective pipe length, m

When the critical flow velocity, \( U_{cr} \), has been determined, it can be easily avoided by manipulating the fluid flow velocity. It is also possible to select or design the parameters of the corrugated pipe if the flow velocity cannot be manipulated.

For instance, given a 2m long corrugated pipe and speed of sound of 340.29m/s, the second mode of acoustic natural frequency is calculated to be 151.43Hz by using equation 3. If the \( \frac{l^2}{w+r_{up}} \) is found to be 0.1 from the simulation for the pipe with \( \frac{l^2}{w+r_{up}} \) of 0.005, the \( U_{cr} \) can then be determined to be around 7.57m/s from equation 2. In order to avoid whistling phenomenon, the flow condition should be much larger or smaller than this value. It is important to note that other modes of resonance frequency should also be calculated and avoided accordingly. With the proposed method, the whistling phenomenon can be overcome effectively.

In the cases where flow velocity is constant, equation 2 can also be used. For instance, if a 2m corrugated pipe with \( l = 8\text{mm}, w = 10\text{mm} \) and \( r_{up} = 2.8\text{mm} \) with second mode of acoustic natural frequency of 151.43Hz (calculated using equation 3) is selected for a flow condition of 8m/s, its \( Sr_{\frac{l^2}{w+r_{up}}} \) can be determined with equation 2 to be around 0.0946. This value is then compared with simulation method by using \( \frac{l^2}{w+r_{up}} \) of 0.005. If the values of \( Sr_{\frac{l^2}{w+r_{up}}} \) is different between calculated
value and simulation results, this particular design of corrugated pipe is suitable to be used for the flow condition. However, a different pipe should be re-selected shall the values of $\frac{Sr_{l^2}}{w+r_{up}}$ is similar or same between calculated value and simulation results. With the proposed method, the whistling phenomenon can be overcome effectively.

It should also be noted that the simulation method proposed in this project is able to contribute effectively in the research of flow induced acoustics in corrugated pipe. This is because the conduction of simulation is much simpler than running an experiment. Different models of corrugated pipe can be easily drawn with few minutes and simulation results can be obtained easily with high accuracy. Hence, time can be saved and the progress of research can be accelerated.

Furthermore, the use of simulation method is cheaper than the use of experimental method. This is because simulation method can provide a wide range of flow conditions and information without the need of costly experimental setup provided the initial conditions and boundary conditions are specified correctly. This can also reduce waste of material since different configurations of corrugated pipe are not required.

Last but not least, large amount of space is not required by using simulation method since no equipment are needed. This can save the cost indirectly while not compromising the accuracy of the results.

4. Conclusion

In this paper, the effects of various corrugated pipe geometries on the $Sr_{p-w}$ and whistling amplitude are simulated using Fluent software. The simulation method is first compared with the experimental results from Nakiboğlu et al. [3] to ensure the reliability of the results obtained. The 2D numerical simulation method is found to be accurate in determining the $Sr_{p-w}$ and whistling amplitude.

The change of cavity depth, Hand downstream edge radius, $r_{down}$ are found to have insignificant effect on the $Sr_{p-w}$ and whistling amplitude. It is found that the change of cavity depth, $H$ from 4mm to 10mm only changes $Sr_{w+r_{up}}$ from 0.30 to 0.34. In addition, it has also insignificant effect on the whistling amplitude which only varies between 100dB to 102.5dB. The change of downstream edge radius, $r_{down}$ from 2mm to 10mm has varied the $Sr_{w+r_{up}}$ within the range of 0.11 to 0.24, which is very minor. On the other hand, the change of $r_{down}$ does not affect the whistling amplitude greatly. The largest difference of whistling amplitude is only 5.7dB.

However, the increase of upstream edge radius, $r_{up}$ and cavity width, $w$ lead to a fluctuation of $Sr_{p-w}$ and a reduction of whistling amplitude. Whistling amplitude is dropped from 103.8dB to 93dB when $r_{up}$ is increased from 2mm to 10mm. Besides, whistling amplitude is reduced from 107.5dB to 83dB when $W$ is increased from 2mm to 12mm.

The strongest finding is that when $l < w + r_{up}$, the value of $Sr_{w+r_{up}}$ is fluctuating. $Sr_{w+r_{up}}$ is at a maximum value of 0.62 when $\frac{l}{w+r_{up}}$ is 0.1667 and at a minimum value of 0.11 to 0.24 when $\frac{l}{w+r_{up}}$ is 0.3333. There is a smaller fluctuation for $0.3333 < \frac{l}{w+r_{up}} < 1$.

Nevertheless, when $l \geq w + r_{up}$ or $\frac{l}{w+r_{up}} \geq 1$, the value of $Sr_{w+r_{up}}$ is far more constant in the range of 0.39 to 0.43. It can be concluded that when $l < w + r_{up}$, $w$ and $r_{up}$ have dominant effect over $l$ in affecting the flow over corrugated pipe, therefore value of $Sr_{w+r_{up}}$ is fluctuating. However, when $l \geq w + r_{up}$, $l$ is dominant over $w$ and $r_{up}$. The change of $Sr_{w+r_{up}}$ in this range is insignificant, which means that $l$ does not contribute much to the change of $Sr_{w+r_{up}}$ as compared to $w$ and $r_{up}$ as discussed earlier.

Since the ratio of $\frac{l}{w+r_{up}}$ is found to be a good indicator of the consistency of $Sr_{w+r_{up}}$, it is therefore possible to use it as a characteristic length in calculating the Strouhal number. Nevertheless, $\frac{l}{w+r_{up}}$ is
only a ratio but not a length. It cannot be used as a characteristic length unless it has a unit of meter. In order to make it a characteristic length with unit of meter, \( l^2 \) is proposed in this project.

Lastly, it is found that the use of \( w + r_{up} \) as the characteristic length results in variation of Strouhal number of \( 0.30 \leq S_{w+r_{up}} \leq 0.43 \). The use of \( W \) as the characteristic length has led to larger variation of Strouhal number in the range of \( 0.09 \leq S_W \leq 0.29 \). The smallest variation in Strouhal number is observed when \( \frac{2}{w+r_{up}} \) is used as the characteristic length, which is limited to \( 0.05 \leq S_{\frac{2}{w+r_{up}}} \leq 0.13 \). It is therefore concluded that \( \frac{2}{w+r_{up}} \) is the optimal choice of characteristic length.

This finding is significant as it can allow us to avoid critical flow condition to minimize and prevent the whistling phenomenon more accurately and effectively by using equation 2 and 3. It is also possible to design the pipe parameters if the flow condition cannot be manipulated in certain conditions. With the proposed method, the whistling phenomenon can be overcome effectively.

Lastly, the simulation method proposed in this project is able to contribute effectively in the research of flow induced acoustics in corrugated pipe. This is because the simulation method is able to save cost, time and space compare to experimental method while not compromising the accuracy of the results obtained. In other words, large amount of information can be obtained by using simulation methods which are especially useful for the researchers in understanding the science and arts behind this topic of interest. There have been multiple proposals on the mitigation of the flow-induced acoustics in corrugated pipe through the smoothing the internal surface of the corrugated pipe or by introducing liquid droplet into the flow [11]. However, these method still not feasible for commercial practice.

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