Effects of Electron Precipitation on E-Region Instabilities: Theoretical Analysis

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Abstract During periods of strong geomagnetic activity, intense currents and electric fields originating in the magnetosphere inundate the high-latitude E-region ionosphere. These strong electric fields drive plasma instabilities, including the Farley-Buneman instability (FBI). These instabilities give rise to small-scale plasma turbulence that modifies the large-scale ionospheric conductance that, in turn, affects the evolution of the magnetosphere-ionosphere-thermosphere system. Also, during geomagnetic storms, high-energy precipitating electrons, \( \geq 5 \text{ keV} \), frequently penetrate down to the same regions where the intense currents and electric fields exist. This research examines the effects of precipitating electrons on the generation of the FBI and shows that, under many common conditions, it can easily suppress the FBI in a predictable manner. We demonstrate that the plasma pressure of superthermal electrons may significantly exceed the regular plasma pressure of the cold ionospheric plasma. This effect will increase the FBI threshold and suppress the instability in auroral regions. However, our detailed theoretical analysis shows that the effect of the superthermal precipitating electrons on the FBI threshold is much stronger than the pressure effect. The energy dependence of the electron-N \(_2\) collision frequency can greatly enhance the effect of this additional pressure, further suppressing the FBI, even at a moderate precipitation level. Therefore, we expect that precipitation will exert an additional significant feedback on the magnetosphere by preventing the elevated conductivity caused by FBI driven turbulence. Both the turbulence-enhanced conductivities and this suppression should be taken into account in global modeling of the magnetosphere-ionosphere coupling.

Plain Language Summary From time to time, elevated solar activity disturbs the near-Earth environment, creating within the Earth's magnetosphere tangible geomagnetic storms. During these events, strong electric fields often penetrate down to the lower high-latitude Earth's ionosphere where intense electric currents close. These currents provide strong magnetosphere-ionosphere coupling. At altitudes between 100 and 120 km, the penetrated electric fields are often intense enough to generate plasma instability which, in turn, causes turbulence, heats electrons, and affects the large-scale ionospheric conductance. This conductance is an important element of a global electric circle surrounding our planet. During the storm time, it is always possible that highly energetic electrons from the distant magnetospheric regions precipitate into the Earth's atmosphere, leading to the spectacular effect of Aurora Borealis. This theoretical research predicts that if these energetic electrons penetrate down to the lower-ionosphere regions at the time of strong electric field able to generate plasma instability then these electrons will suppress this instability within the auroral regions, thus affecting the global ionospheric conductance. Proper understanding of these effects is important for accurate predictive modeling of the near-Earth space plasmas, especially for the timely predictions of potentially devastating effects of solar activity on the human life and technology.

1. Introduction

During periods of intense geomagnetic activity, strong DC electric fields, \( \mathbf{E}_0 \), perpendicular to the geomagnetic field, \( \mathbf{B} \), penetrate from the Earth's magnetosphere into the high-latitude ionosphere where they dissipate energy, form electrojets, and drive plasma instabilities in the E-region ionosphere, at altitudes roughly between 90 and 130 km. In the global picture of magnetosphere-ionosphere coupling, this is the region where most of the field-aligned magnetospheric currents close. E-region instabilities generate plasma density irregularities, typically within the wavelength range from tens of centimeters to tens of meters, coupled with wave-like electrostatic field fluctuations. The density irregularities have been routinely detected as strong coherent radar echoes (e.g., Bahcivan et al., 2005, 2006; Forsythe & Makarevitch, 2015; Hysell et al., 2008). The electrostatic field fluctuations have been detected by rocket flights through the lower ionosphere (Fukao et al., 1998; Pfaff, et al., 1984, 1987, 1992, 1997; Rose et al., 1992). The E-region instabilities include the Farley-Buneman
(Buneman, 1963; Farley, 1963), gradient drift (Hoh, 1963; Maeda et al., 1963), and thermal instabilities (e.g., Dimant & Oppenheim, 2004; Dimant & Sudan, 1997; Makarevich, 2020; Oppenheim et al., 2020, and references therein). The strongest E-region instability, the Farley-Buneman instability (FBI), is excited when the relative speed between the average electron and ion streams exceeds the local ion-acoustic speed. At high latitudes, this usually occurs when $|E_0| \gtrsim 20$ mV/m. This and much stronger fields are not uncommon in the subauroral, auroral, and polar cap areas, especially during geospace storms and substorms. Driven by magnetoospheric activities, small-scale E-region instabilities heat electrons (e.g., Foster & Erickson, 2000; Milikh & Dimant, 2003; Oppenheim & Dimant, 2013, and references therein) and it is expected that this will strongly affect ionospheric conductivities, which in turn exert large-scale feedback on the magnetosphere (Dimant & Oppenheim, 2011; Merkin et al., 2005; Wilthberger et al., 2017).

High-latitude regions are characterized by strong electron precipitation that gives rise to such spectacular phenomena as Aurora Borealis. The electron energy distribution in the aurora displays many different forms that are usually described by qualitative criteria developed by Newell et al. (2009). The majority of these electron energy distributions are classified as diffuse, monoenergetic, or broadband. In Section 2.2, we provide a detailed description of different kinds of aurora.

In this paper, we focus on auroral regions where intense electron precipitation may overlap with strong driving DC fields. We study the distribution function modifications caused by precipitating electrons and the effect of these modifications on instability development. Using a physics-based model of electron precipitation from the SuperThermal Electron Transport (STET) model, we study how this precipitation affects the E-region instability criterion.

We show that precipitating electrons of sufficiently high energies, $E_0 \gtrsim 5$ keV, can easily penetrate down to the E-region. Applying the kinetic STET code, we have analyzed the distribution function expected in the E-region and the effects of these energetic electron distributions on the FBI onset criterion. This analysis shows that the plasma pressure of superthermal electrons may be comparable to, or even significantly exceed, the regular plasma pressure of the cold ionospheric plasma. It is expected to increase the FBI threshold and suppress the instability in auroral regions. However, our detailed theoretical analysis shows that the effect of the superthermal precipitating electrons is much stronger than the effect of the additional pressure. An unexpected finding of our analysis is the discovery that the energy dependence of the electron-N$_i$ collision frequency can greatly enhance the effect of this additional pressure, further suppressing the FBI, even at a moderate precipitation level.

The paper is organized as follows. In Section 2, we discuss in general the E-region instabilities (Section 2.1), as well as electron precipitation and the kinetic tool to treat the latter (Section 2.2). In Section 3, we present the results of our kinetic simulations using STET. Section 4 is the central section of the paper, where we present our principal theoretical analysis. In Section 4.1, we present the general linear theory of the FBI. This theory is based on a hybrid approach: an approximate (but rigorous) kinetic theory for electrons, combined with the fluid model of ions. In Section 4.2, using the results of our kinetic simulations described in Section 3, we give simple estimates of the modified instability threshold in the presence of superthermal electrons. In Section 5, we give the conclusions and the final discussion. In order to look at the predicted effect using a simpler but less accurate analysis, in Appendix A we apply a three-fluid model, two electron fluids and one ion fluid, instead of the full kinetic model presented below. In two other Appendices, we give some details of analytic approximations of the collision cross-section and distribution function.

2. Background

2.1. Outline of the FBI Onset

Here we briefly describe the FBI linear theory relevant to our topic. All E-region instabilities occur within the lower altitude range of the ionosphere where electrons are strongly magnetized, $\omega_{ce} \gg \nu_{ce}$, while ions are fully or partially unmagnetized due to their frequent collisions with neutral atmospheric molecules, $\omega_{ni} \lesssim \nu_{ni}$, where $\omega_{ce}$ and $\omega_{ni}$ are the electron and ion cyclotron frequencies; $\nu_{ce}$ and $\nu_{ni}$ are the electron-neutral ($e-n$) and ion-neutral ($i-n$) mean collision frequencies, respectively (for simplicity, we assume only single-species ions). The mean collision frequencies are altitude-dependent parameters averaged over the entire particle distributions. To avoid a confusion, we note that in the kinetic description of electrons (Section 4) we will use the same notation for the
velocity-dependent $e$-$n$ collision frequency, $\nu_{en}(V)$. At high latitudes, the above conditions usually hold at E-region altitudes between 90 and 120 km.

All E-region instabilities excite low-frequency plasma-density compression/rarefaction waves. Though such waves are usually acoustic-like, no long-lived ion acoustic waves can exist in the highly dissipative E-region ionosphere. Long-lived plasma waves persist there only due to an external DC electric field, $\vec{E}_0 \perp \vec{B}$. These plasma waves are quasineutral, where weak charge separation gives rise to coupled electrostatic field oscillations. The frequencies of these excited waves, $\omega_i$, are typically lower than the characteristic collision frequencies, $\omega \lesssim \nu_{in} \ll \nu_{en}$, while the typical wavelengths are larger than the $i$-$n$ collisional mean free path. The corresponding wave vectors, $\vec{k}$, are largely perpendicular to $\vec{B}$. In the perpendicular to $\vec{B}$ plane, depending on the driving-field magnitude, $E_y$, the wavevectors are mostly directed within a cone of a few tens of degrees wide around the electron $E_0 \times \vec{B}$-drift velocity, $\vec{V}_0 = E_0 \times \vec{B} / B^2$, where $B = |\vec{B}|$

If the particle velocity distributions are close to Maxwellian then, for sufficiently long-wavelength waves (see below), the linear stage of the FBI and other E-region instabilities can be reasonably well described by a closed set of five-moment fluid-model equations that includes the continuity equation, the momentum equation, and the energy-balance equation (e.g., Dimant & Oppenheim, 2004):

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \vec{V}_i) = 0,$$  \hspace{1cm} (1a)

$$m_i \frac{D \vec{V}_i}{Dt} = q_i(\vec{E} + \vec{V}_i \times \vec{B}) - \frac{\nabla P_i}{n} - m_i \nu_{in} \vec{V}_i,$$  \hspace{1cm} (1b)

$$n_i^{3/2} \frac{D}{Dt} \left( \frac{T_i}{n_i^{3/2}} \right) = \frac{2}{3} M_{ii} v_{in} V_i^2 - \delta_{ii} V_i (T_i - T_0).$$  \hspace{1cm} (1c)

Here the subscript $s = e$, $i$ characterizes a specific plasma fluid; $D_i / Dt \equiv \partial_i / \partial t + \vec{V}_i \cdot \nabla_i$; $\vec{V}_i$, $m_i$, $q_i$, and $T_i$ are the $s$-particle mean flow velocities, particle masses, charges ($q_e = +1$, $q_i = -e$), and temperatures (in energy units); $V_i = |\vec{V}_i|$: $m_i$ and $T_i$ are the neutral mass and temperature, respectively; $P_i \approx n_i T_i$ is the $s$-fluid pressure; $M_{ii} = m_i m_j (m_i + m_j)$ is the effective mass of the two colliding particles ($s$ and $n$); and $\delta_{ii}$ is the average fraction of energy lost by the particle of the $s$-species during one $s$-$n$ collision; here $\delta_{ii} \approx 1$ and $\delta_{en} \approx (2 - 4) \times 10^{-3}$ (Gurevich, 1978). Equation 1 implies the frame of reference attached to the mean neutral flow.

In Equation 1, we assume a single ion ($i$) species because the two major E-region ion components, $O^+_i$ and NO$^+_i$, have fairly close masses ($m_i \approx 30 m_p$), where $m_p$ is the proton mass) and comparable collision frequencies; the same can be assumed for neutrals ($n$). Note that this simplified set of moment equations misses a few factors, for example, ionization-recombination, anisotropic pressure, viscosity, and heat conductivity, which play little to no role in E-region waves. This is due to the physical conditions of the E-region instabilities. In particular, the characteristic time of instability development (typically, below one second) is usually much shorter that the typical times of the E-region ionization-recombination balance (often minutes or more). Also, the characteristic wavelengths of the Farley-Buneman linearly unstable waves are longer than the collisional mean free paths of ions and electrons, making the other neglected factors almost negligible.

For the E-region processes, the two sets of the moment equations for plasma particles are usually closed through the quasineutrality condition, $n_i \approx n_e = n$. This condition presumes sufficiently long-wavelength waves, compared to the Debye length. This eliminates the need for Poisson’s equation and allows one to unambiguously determine the turbulent total electric field, $\delta \vec{E} = \vec{E} - \vec{E}_0 = -\nabla \Phi$, where $\Phi$ is the corresponding electrostatic potential.

Wave perturbations of the particle temperatures are crucial for the aforementioned thermal instabilities, but for the pure FBI the temperatures $T_i$ can be assumed constant, $T_i \approx T_0$ (the isothermal regime) or obeying the adiabatic regime, $T_i \propto n_i^{3/2}$ ($P_i \propto n_i^{5/2}$); the latter is derived from Equation 1c if the right-hand side (RHS) equals zero. Assuming any of these regimes, the fluid-model description of the pure FBI no longer requires Equation 1c.

The electron inertia in the corresponding left-hand side (LHS) of Equation 1b never plays a role for low-frequency E-region processes (Dimant & Oppenheim, 2011). For inertialess electrons, Equation 1b yields a simple explicit expression for the electron flow velocity in terms of $E$ and $\nabla P_e$.
\[ \mathbf{V}_e \approx -\frac{1}{m_e} \begin{bmatrix} v_{ce}/\omega_{ce}^2 & 1/\omega_{ce} & 0 \\ -1/\omega_{ce} & v_{en}/\omega_{en}^2 & 0 \\ 0 & 0 & 1/\nu_{en} \end{bmatrix} \times \left[ e\mathbf{E} + \frac{\mathbf{V}_e}{n} \right]. \]  

(2)

where we used the aforementioned condition of \( \omega_{en} \gg \nu_{en} \). The 3-D vector combination \( \mathbf{A} \) on the far right of Equation 2 implies a right-handed Cartesian coordinate system \( [A_x, A_y, A_z] \) with the z-axis directed along \( \mathbf{B} \). In the preceding \( 3 \times 3 \) matrix \( [B_{ij}] \), the two equal diagonal elements, \( v_{ce}/\omega_{ce}^2 \), correspond to the electron Pedersen mobility, the remaining diagonal element, \( 1/\nu_{en} \), corresponds to the parallel to \( \mathbf{B} \) mobility, whereas the two non-zero non-diagonal elements, \( \pm 1/\omega_{en} \), describe the Hall mobility of the strongly magnetized electrons. Equation 2 applies to both the zero-order background electron flow velocity, \( \mathbf{V}_e \), and wave perturbations, \( \delta \mathbf{V}_e \). For ions, however, the particle inertia in the LHS of Equation 1b is crucial for driving the FBI. As a result, the expression for the background ion flow velocity, \( \mathbf{V}_i \), is analogous to Equation 2, while the corresponding wave perturbations, \( \delta \mathbf{V}_i \), are described in a more complex way (see, e.g., Dimant & Oppenheim, 2011).

In the E-region ionosphere, strongly magnetized electrons move against the neutral atmosphere with approximately \( \mathbf{E}_0 \times \mathbf{B} \)-drift velocity, \( \mathbf{V}_e \approx \mathbf{V}_0 \), while unmagnetized ions are almost attached to the neutral atmosphere. The background charged-particle temperatures are usually larger than the neutral temperature, in part due to the ohmic heating by the driving DC field, \( \mathbf{E}_0 \). The background parameters determine the phase velocity of the linearly generated waves, the linear growth rate, and the threshold-field amplitude for exciting the FBI, \( E_{thr} \). The minimum threshold field is usually reached for sufficiently long-wavelength waves, compared to the ion-neutral (\( i-n \)) collision mean free path, and for the wavevectors \( \mathbf{k} \) parallel to \( \mathbf{B} \). Near the optimum wavevector direction in the perpendicular to \( \mathbf{B} \) plane, with small but finite \( k_z \ll k_x \), the FBI threshold-field amplitude can be written in the form given, for example, by Dimant and Milikh (2003); Milikh and Dimant (2003):

\[ E_{thr} = (1 + \psi) \left( \frac{1 + \kappa_i^2}{1 - \kappa_i^2} \right)^{1/2} E_1, \]

(3)

where \( \kappa_i = \omega_i/\nu_i \) are the magnetization parameters for the \( s \)-species and

\[ \psi = \frac{1}{\kappa_i \kappa_s} \left( 1 + \frac{k_i^2 \omega_{ce}^2}{k_s^2 \nu_{en}^2} \right). \]

(4)

Equation 3 implies \( \kappa_i < 1 \), since above the ion magnetization boundary, \( \kappa_i = 1 \), at high latitudes located around 120 km of altitude, the pure FBI cannot be excited, see Dimant and Oppenheim (2004). In the RHS of Equation 3, the smallest FBI threshold field, \( E_1 \), corresponding to altitudes with concurrently small \( \psi \) and \( \kappa_i^2 \) (Dimant & Oppenheim, 2004, Figure 5) is given by

\[ E_1 = C_i B = 20 \left( \frac{T_e + T_i}{600 \, \text{K}} \right)^{1/2} \left( \frac{B}{5 \times 10^4 \, \text{nT}} \right) \text{mV/m}, \]

(5)

where \( C_i = [(T_e + T_i)/m_i]^{1/2} \) is the isothermal ion-acoustic speed. For the adiabatic regime of instability generation, one must replace \( T_i \) with \( (5/3)T_e \).

Equations 1–5 hold for the particle velocity distributions that are reasonably close to Maxwellian. However, during strong diffuse or discrete aurora, the electron distribution function changes dramatically: in addition to the nearly Maxwellian cold-temperature thermal bulk with energies well below 0.1 eV, a significant superthermal tail develops within the eV-to-keV energy range, as we discuss in more detail in the following sections. With significant non-Maxwellian contributions to the velocity distribution, approximate fluid-model Equations 1 and 2 lose their validity, as we demonstrate below in Section 4. Processes with non-Maxwellian velocity distributions require a kinetic description.
For the kinetic treatment, of significant importance is the fact that after an e-n collision most electrons change their momentum at a much higher rate than they lose their energy. This means that e-n collisions effectively scatter electrons by large angles in velocity space with only small relative changes in their kinetic energies. As a result, the electron distribution function remains almost isotropic, \( f_e(V) \approx F_0(V) \) (here \( V \equiv |V_e| \)), but \( F_0(V) \) may deviate significantly from the Maxwellian velocity distribution. Due to this effective isotropization, when calculating integral scalar quantities like the local electron density or pressure, instead of the general 3-D velocity integration one can use a much simpler 1-D speed integration, \( \int f_e(V) dV \approx 4\pi \int_0^\infty \cdots \int F_0(V) V^2 dV. \)

In particular, the total electron density, pressure, and temperature, used in fluid-model Equation 1, become:

\[
\begin{align*}
    n_e & \approx 4\pi \int_0^\infty F_0(V) V^2 dV, \\
    p_e & = n_e T_{tot} \approx \frac{4\pi m_e}{3} \int_0^\infty F_0(V) V^4 dV, \\
    T_{tot} & \approx \frac{4\pi m_e n_e}{3} \int_0^\infty F_0(V) V^4 dV \approx \frac{m_e}{3} \int_0^\infty F_0(V) V^2 dV.
\end{align*}
\]

It is important that the dominant omnidirectional part of the electron distribution function, \( F_0(V) \), includes both the thermal bulk and superthermal tail. As we demonstrate below, the relative addition of superthermal particles to the total electron density, \( n_s \), is usually small and can be neglected, \( n_s \approx n_{el} \) while the total temperature, \( T_{tot} \), due to the additional multiplier \( V^2 \) in the integrand of Equation 6c, can exceed the electron bulk temperature, \( T_{el} \), dramatically. Note that in the high-latitude nighttime E-region ionosphere a significant fraction of the thermal bulk plasma may originate from the electron precipitation followed by ionizing collisions of the precipitated energetic electrons. However, this happens only after multiple collisions, which cause electrons that have already cooled down to become Maxwellian within the cold-temperature thermal bulk. This cold plasma is redistributed by drifts between different locations and can survive without the local precipitation source for a sufficiently long time.

A naive viewpoint suggests using the modified temperature given by Equation 6c to determine the modified FBI threshold by replacing in Equation 5 the regular electron temperature \( T_0 \) with \( T_{tot} \). However, the rigorous kinetic approach of Section 4 below, as well as a simplified three-fluid approach, presented in Appendix A, show that due to the velocity dependence of the e-n collision frequency of electrons the simple modified-temperature approach dramatically understimates the suppression of the instability. Nonetheless, the simple form of Equation 5 can still be used, at least within the E-region core at altitudes 100–110 km, if one replaces the electron temperature with a more complicated integral expression involving the energy-dependent e-n collision frequency, see Equation 43 below. We discuss this in detail in the end of section 4.1.

### 2.2. Electron Precipitation and STET Code

In an aurora, the electron distribution function is classified using different characterization criteria, such as diffuse, monoenergetic, or broadband aurorae. For details, see Newell et al. (2009, 2010), and McIntosh and Anderson (2014).

The diffuse aurora is primarily caused by wave-particle interactions of high energy electrons, \( \mathcal{E}_e \equiv m_e V_e^2/2 > 1 \) keV, within the plasma sheet (Thorne et al., 2010). Note that since in this paper we treat kinetically only electrons, we will drop subscripts \( e \) from any kinetic characteristics (like \( \mathcal{E}_e = \mathcal{E} \) and \( V_e = V \)).

To calculate the aurora electron distribution function, we used SuperThermal Electron Transport (STET) code developed by Khazanov et al. (2016) with the following inputs to this model. The neutral thermospheric densities and temperatures were given by MSIS-90 (Hedin, 1991). The electron profile in the ionosphere was calculated based on the IRI model (Bilitza et al., 2017) and extended into the magnetosphere under the assumption that the electron thermal density distribution in the magnetosphere is proportional to the geomagnetic field as \( n_e \propto B^{1/2} \). Cross-sections for elastic collisions, state-specific excitation, and ionization were taken from Solomon et al. (1988).
There are different settings available for the application of the STET model for studying the diffuse aurora. The first setting involves imposing a spectrum of primary precipitating electrons with energies above 500–600 eV at an altitude of 800 km and keeping the spectra unchanged (Khazanov et al., 2016). This setting implicitly assumes no MI-coupling processes for the energy range of the imposed precipitation and implies the usage of experimental energy fluxes as boundary conditions. Khazanov et al. (2016) introduced a modification of this boundary condition setting to account for the role of multiple reflections (backscatters) of degraded primary electrons traveling between two magnetically conjugate hemispheres. The latest setting is adapted in the studies that are presented below.

The STET setting in the region of monoenergetic aurora is similar, but assumes the existence of an electrostatic acceleration region located at altitudes of (1.5–2)R_E (Marklund et al., 2011), where R_E is the Earth’s radius. Specifically, STET code was set up to run from the northern or southern hemispheres from 90 km to 2R_E with the multiple reflection (backscatter) of the electrons whose electrostatic energies are smaller than the potential drop of the acceleration region (Khazanov et al., 2021).

In the analysis presented below, in order to describe the primary magnetosphere-driven electron precipitation in the region of diffuse aurora, we used only the Maxwellian electron distribution function (EDF) input in the energy range of 600 eV to 30 keV,

$$\Phi(E) = C E e^{-E/E_0}, \quad (7)$$

where $$\Phi = 2E f_s(\vec{V}, s, t)/m^2$$ is the superthermal electron flux (Khazanov, 2011), $$E_0$$ is the characteristic energy of plasmasheet electrons, and C is the normalization constant for the selection of the integrated energy flux driven by magnetospheric processes. In the region of monoenergetic aurora, we selected the Gaussian distribution, as in Banks et al. (1974),

$$\Phi(E) = A \exp \left[ -(E - E_0)^2/(2\sigma^2) \right], \quad \sigma = 0.1E_0. \quad (8)$$

Here, $$E_0$$ is also the characteristic energy of monoenergetic accelerated electrons and A is the normalization constant for the selection of the integrated energy flux as defined above, but only for the electrostatic acceleration region.

Superthermal electrons (SE) make an additional contribution to the total electron energy density and pressure. In accord with Equation 6, the density and pressure of the SE population were found using

$$n_{SE} = 4\pi \int_{E_{min}}^{E_{max}} \frac{\Phi_E}{V(E)} dE, \quad (9a)$$

$$P_{SE} = \frac{4\pi}{3} \int_{E_{min}}^{E_{max}} \frac{E \Phi_E}{V(E)} dE, \quad (9b)$$

where $$V(E) = (2E/m_e)^{1/2}$$. The minimum and maximum superthermal energies used in our simulations were $$E_{min} = 1 \text{ eV}$$ and $$E_{max} = 30 \text{ keV}$$, respectively. These values of $$n_{SE}$$ and $$P_{SE}$$ are calculated below for different types of precipitated electron spectra of Equations 7 and 8, modeling the diffuse and monoenergetic auroras, respectively.

3. Results of STET Simulation

Before presenting the kinetic simulation results, we note that in the simulations we neglect any collisions between the superthermal particles themselves, compared to their collisions with the thermal bulk particles. Furthermore, in the lower ionosphere the e-n collisions vastly dominate over Coulomb collisions between the charged particles. With neglect of electron-electron collisions, the corresponding kinetic equation becomes linear with respect to the superthermal particle flux $$\Phi_E$$. As a result, given the energy distribution of the precipitated electrons, any superthermal particle-energy-integrated characteristics, such as the density and pressure, will be proportional to the mean energy flux, $$\Phi_E$$. The latter is defined as the total SE energy (in ergs) per unit square (in cm$^2$) per unit time (in s) at a given altitude. We will represent the SE density and pressure defined by Equation 9 as
The characteristic SE density and pressure values, \( A_n \) and \( P \), corresponding to a moderately high energy-flux value of 10 erg cm\(^{-2}\) s\(^{-1}\), depend on the SE velocity distribution and the ionosphere-thermosphere parameters at given ionospheric altitudes. For our simulations, we picked three E-region altitudes: 100, 110, and 120 km, that best characterize the typical altitude range for the FBI generation.

We performed specific kinetic simulations for the following conditions. Bearing in mind both the discrete and diffuse aurorae, we modeled the Maxwellian EDF given by Equation 7 and the Gaussian flux given by Equation 8. To characterize various precipitation conditions, we have chosen different values of \( E_0 \) for each EDF (see below). We believe that these values are most characteristic for each kind of storm-time EDF.

To compare the SE contributions to the total electron density and pressure, we have chosen the following values of the nighttime cold thermal background from the IRI model, as shown in Table 1. The background electron temperature for all three altitudes was taken equal, \( T_e = 370 \) K (corresponding to 0.0318 eV).

Figure 1 shows examples of the corresponding SE velocity distributions. Tables 2–4 show some simulation results for the Maxwellian-input EDF. Tables 5–7 show some simulation results for the Gaussian EDF.

Using the table values along with Equation 10 and comparing the simulation results with the typical background parameters, we see that the contribution of the SE tail, \( n_{SE} \), to the total electron density, \( n_{tot} = n_e + n_{SE} \), is usually small compared to the background density, \( n_e \). For reasonable values of the mean energy flux, \( \Phi_E \lesssim 10 \) erg cm\(^{-2}\) s\(^{-1}\), even for the large values of \( E_0 \), the SE contribution to \( n_{tot} \) can be neglected.

An entirely different situation, however, takes place for the total pressure, \( P_{tot} \), and hence for the total electron temperature, \( T_{tot} \). Only for the Maxwellian EDF with the smallest calculated SE energy \( E_0 = 1 \) keV are the SE contributions comparable to the background values of the electron pressure and temperature; for all other values of \( E_0 \), \( \Phi_E \gtrsim 10 \) erg cm\(^{-2}\) s\(^{-1}\), and both kinds of the EDF, are the SE contributions are much larger than the corresponding background values. To the best of our knowledge, at lower ionospheric altitudes, a possible

\[
n_{SE} = A_n \left( \frac{\Phi_E}{10 \text{ erg cm}^{-2} \text{ s}^{-1}} \right), \quad P_{SE} = A_P \left( \frac{\Phi_E}{10 \text{ erg cm}^{-2} \text{ s}^{-1}} \right). \tag{10}
\]

We believe that these values are most characteristic for each kind of storm-time EDF.

Table 1: Nighttime Cold Thermal Background From the IRI Model

| Altitude, km | Background density, cm\(^{-3}\) | Background pressure, eV cm\(^{-3}\) |
|-------------|-------------------------------|----------------------------------|
| 100         | \(1.67 \times 10^3\)           | 53.1                             |
| 110         | \(2.59 \times 10^3\)           | 82.4                             |
| 120         | \(1.08 \times 10^3\)           | 34.2                             |

Figure 1. Superthermal electron energy distribution function: some results of kinetic (STET) simulations (Maxwellian input); the parameters are shown in the figure.
significant contribution of superthermal particles to the total plasma pressure and to the corresponding effective temperature has not been addressed in the published literature.

4. Effect of Superthermal Electrons on the FBI Threshold: Analytical Treatment

As mentioned, the naive calculation of the modified instability threshold based on replacing the undisturbed electron cold-plasma temperature $T_e$ with the modified temperature $T_{\text{eto}}$ defined by Equation 6 turns out to be incorrect. This becomes clear after implementing a simple three-fluid approach based on two distinct Maxwellian velocity distributions of electrons (the cold thermal bulk and the energetic tail of precipitated electrons) with two different $e-n$ collision frequencies. This three-fluid approach has been implemented in Appendix A.

At E-region altitudes, however, the entire superthermal energy distribution of precipitating, secondary, tertiary, etc., electrons deviates significantly from a Maxwellian distribution. Also, the $e-n$ collision frequency depends smoothly on the electron energy and hence cannot be reduced to only two constant values. All this requires the proper description of the electron behavior to be kinetic. At the same time, for sufficiently long-wavelength and low-frequency waves, as specified below by Equation 22, the ion behavior can be successfully described by the much simpler fluid model.

For the kinetic treatment of electrons, we will mostly follow the approximate kinetic approach developed in Dimant and Sudan (1995a). This approach is based on the assumption that $e-n$ collisions lead to much faster angular scatter of electrons in velocity space than to losses of their energies, as we already mentioned in Section 2.1. For superthermal electrons with energies $E \gtrsim 1$ keV, the two rates are comparable, but the approximate approach of Dimant and Sudan (1995a) is still useful and should lead to reasonably accurate analytic results.

4.1. Kinetic Analysis of the FBI Onset for the General Electron Distribution Function

In this section, we develop a kinetic linear theory of the FBI for a system with general non-Maxwellian electron distributions. Our hybrid theoretical approach combines the fully kinetic description of electrons with the fluid-model description of ions.

As above, we will restrict our treatment to the E-region altitudes where the electrons are highly magnetized, $a_i \gg \nu_{ei}$, while ions are at least partially unmagnetized, $a_i \lesssim \nu_{ie}$. These conditions typically hold within the core of the high-latitude E-region ionosphere between 90 and 120 km. Under these magnetization conditions, electrons are essentially $E_0 \times B$ drift, while ions mostly move with the dominant neutral component. The linear instability onset is described by analyzing small harmonic wave perturbations of the plasma particle motion and the coupled electrostatic potential.

Before proceeding with the fluid-model ion description and kinetic description of electrons, we introduce dimensionless variables and parameters

$$
\eta_{n,k} \equiv \frac{\delta n_{e,k}}{n_0}, \quad \phi_{e,k} \equiv \frac{e \Phi_{e,k}}{T_{e0}}, \quad \beta_T \equiv \frac{T_{ei}}{T_{e0}},
$$

(11)

where $\omega$ is the complex wave frequency and $\vec{k}$ is the wave vector, denoting a given Fourier harmonic $\exp[i(\vec{k} \cdot \vec{r} - \omega t)]$ of small linear perturbations. Here, $T_{ei}$ is an effective electron temperature, while the entire ion population is assumed to be Maxwellian with the constant temperature $T_{\text{eto}}$. For the general non-Maxwellian electron distribution, we will not specify the parameter $T_{\text{eto}}$. We have introduced it here as a convenient normalization constant, but the final expressions will not depend on $T_{\text{eto}}$.

For isothermal ions, the fluid model Equations 1a and 1b yield the following relation between $\phi_{e,k}$ and $\eta_{n,k}$.

| Table 2 |
|----------|
| **Maxwellian Electron Distribution Function, $E_0 = 1$ keV** |
| Altitude, km | $A_n$, cm$^{-3}$ | $A_p$, eV cm$^{-3}$ |
| 100 | 2.06 | 18.6 |
| 110 | 21 | 180 |
| 120 | 82 | 635 |

| Table 3 |
|----------|
| **Maxwellian Electron Distribution Function, $E_0 = 10$ keV** |
| Altitude, km | $A_n$, cm$^{-3}$ | $A_p$, eV cm$^{-3}$ |
| 100 | 10 | 604 |
| 110 | 23 | 1,441 |
| 120 | 37 | 2,195 |
and \( V_T = (T_i/m_i)^{1/2} \) is the ion thermal velocity. Recall that \( \mathbf{V}_0 = \mathbf{E}_0 \times \mathbf{B} / B^2 \) is the \( \mathbf{E}_0 \times \mathbf{B} \)-drift velocity and \( \kappa_i = \omega_i / \nu_{ic} \) is the ion magnetization parameter. The full expression for \( \mathbf{V}_0 \) is of importance only for altitudes above 115 km where \( \kappa_i \geq 1 \). At altitudes well below 115 km, the ions are essentially unmagnetized, \( \kappa_i \ll 1, \nu_{ic} \gg \omega_i \), so that the mean ion-flow speed is negligibly small compared to the mean speed of the highly magnetized electrons \( V_0 = |\mathbf{V}_0| = e \mathbf{E}_0 / (m_i \omega_i) \).

Now we generalize the kinetic description of electrons in Dimant and Sudan (1995a) by assuming arbitrary ion magnetization and, more importantly, by assuming non-Maxwellian velocity distribution of the background electrons.

If we suppose that collisional angular scattering of electrons is much faster than the corresponding energy changes then the electron velocity distribution consists mostly of two different parts,

\[
f_{e}(\mathbf{V}, \mathbf{r}) \approx F_0(\mathbf{V}, \mathbf{r}) + \frac{f_1(\mathbf{V}, \mathbf{r}) \mathbf{V}}{V}, \quad |f_1(\mathbf{V}, \mathbf{r})| \ll F_0(\mathbf{V}, \mathbf{r}),
\]

where \( \mathbf{V} \) is the electron velocity and \( V \) is the corresponding speed. The function \( F_0(\mathbf{V}, \mathbf{r}) \) is the dominant omnidirectional, that is, isotropic (in velocity space) part of \( f_{e}(\mathbf{V}, \mathbf{r}) \), while \( f_1(\mathbf{V}, \mathbf{r}) \) is a small directional part that includes the shift in velocity space due to the Hall and Pedersen electron drifts. The dominant isotropic part \( F_0(\mathbf{V}, \mathbf{r}) \) is responsible for the integral scalar quantities like the total electron density, pressure,

\[
n_e(\mathbf{r}) \approx 4\pi \int_0^{\infty} F_0(\mathbf{V}, \mathbf{r}) V^2 dV, \quad P_e(\mathbf{r}) \approx \frac{4\pi m_e}{3} \int_0^{\infty} F_0(\mathbf{V}, \mathbf{r}) V^4 dV,
\]

and temperature, \( T_{ee}(\mathbf{r}) = P_e(\mathbf{r})/n_e(\mathbf{r}) \), while the small directional part is responsible for various fluxes and currents, like the total particle number flux,

\[
\bar{\Gamma}(\mathbf{r}) \approx \frac{4\pi}{3} \int_0^{\infty} f_1(\mathbf{V}, \mathbf{r}) V^3 dV
\]

and the corresponding energy flux. Equation 14 represents the two highest-order terms of the Legendre polynomial series (Gurevich, 1978; Khazanov, 2011; Shkarofsky et al., 1966). No higher-order angular dependencies of \( f_{e}(\mathbf{V}, \mathbf{r}) \), responsible for the anisotropic pressure, viscosity tensors, etc., are included in approximate Equation 14.

Under conditions of strong isotropization of the electron distribution function, the general kinetic equation reduces to a set of two coupled integro-differential equations (Dimant & Sudan, 1995a, Equations 11 and 12), whose further simplification results in \( f_1(\mathbf{V}, \mathbf{r}) \) explicitly expressed in terms of \( F_0(\mathbf{V}, \mathbf{r}) \) (Dimant & Sudan, 1995a, Equations 13 and 14). This allows one to obtain a closed partial differential equation for \( F_0(\mathbf{V}, \mathbf{r}) \),

\[
\left( \frac{d}{dt} + \mathbf{V}_d \cdot \nabla + \mathbf{\dot{R}} \right) F_0(\mathbf{V}, \mathbf{r}) = 0,
\]

where the total \( \mathbf{E} \times \mathbf{B} \)-drift velocity is given by

\[
\mathbf{V}_d \equiv \mathbf{V}_0 + \frac{e}{m_i \omega_{ic}} \mathbf{b} \times \nabla \Phi,
\]
and the differential operator \( \hat{R} \) is given by

\[
\hat{R} \equiv -\frac{1}{3V^2} \left( \hat{K}_\perp \cdot \frac{V^2\nu_{\text{tr}}(V)}{\omega_{\text{ce}}^2} \hat{K}_\perp + \hat{K}_\parallel \frac{V^2}{\nu_{\text{tr}}(V)} \hat{K}_\parallel \right).
\]

(19)

where \( \partial/\partial z \) is the derivative in the \( \overline{B} \) direction; \( E_\parallel \) and \( E_\perp \) are the parallel and perpendicular to \( \overline{B} \) components of the total electrostatic field, \( \overline{E} = E_\parallel - \Phi \); and \( b = \overline{B}/b \) is the unit vector in the \( \overline{B} \) direction. Equations 17–19 differ from Dimant and Sudan (1995a, Equations 16–23) by some notations and, most importantly, by neglecting here the \( e-e \) collisions and the terms describing the thermal exchange between electrons and neutrals through \( e-n \) collisions. The former is important for sufficiently dense and low-energy particles, while the latter is crucial for the electron thermal instability (ETI) (Dimant & Sudan, 1995b, 1995c, 1997; Oppenheim et al., 2020, and references therein).

Bearing in mind the pure FBI, we disregard here any thermal-instability effects.

Equation 17 holds for the entire isotropic part of the electron distribution function, \( F_0(V, \overline{r}) \), which includes the spatially homogeneous background distribution, \( f_0(V) \), and all linear wave perturbations, \( f_\omega(V) \). Linearizing this equation for a given wave harmonic, after some algebra we arrive at Equation 38 from Dimant and Sudan (1995a):

\[
\left( i\Delta_{\omega, k} + \hat{D}_{\omega, k} \right) f_{\omega, k}(V) = \left( \hat{B}_{\omega, k} F_0(V) \right) \phi_{\omega, k},
\]

(20)

where various differential operators acting on both \( f_{\omega, k} \) and \( F_0 \) are defined by

\[
\hat{D}_{\omega, k} \equiv -\frac{1}{3V^2} \left( \hat{K}_{\perp}^{(0)} \cdot \frac{V^2\nu_{\text{tr}}(V)}{\omega_{\text{ce}}^2} \hat{K}_{\perp}^{(0)} + \hat{K}_{\parallel}^{(0)} \frac{V^2}{\nu_{\text{tr}}(V)} \hat{K}_{\parallel}^{(0)} \right)
\]

(21a)

\[
\Delta_{\omega, k} \equiv k \cdot \overline{V}_0 - \omega,
\]

(21b)

\[
\hat{B}_{\omega, k} \equiv \frac{T_{\nu, k}}{3e} \left[ \left( \frac{k^2_1\nu_{\text{tr}}(V)}{\omega_{\text{ce}}^2} + \frac{k^2_3}{\nu_{\text{tr}}(V)} \right) V \frac{d}{dV} + 2i \frac{e\overline{E}_0}{m_e\omega_{\text{ce}}^2V^2} \frac{d}{dV} \left( V^2\nu_{\text{tr}}(V) \frac{d}{dV} \right) \right].
\]

(21c)

Equation 20 implies an arbitrary background distribution function \( F_0(V) \) that provides convergence of any integrals, like those in Equation 15. In our case, \( F_0(V) \) includes both the low-energy Maxwellian bulk distribution and the high-energy superthermal tail. We will specify these components later, but now will proceed with arbitrary \( F_0(V) \). Note that the definition \( \hat{B}_{\omega, k} \) includes the normalization constant \( T_{\nu, k} \) in the numerator, while the wave potential \( \phi_{\omega, k} \), defined in Equation 11, contains \( T_{\nu, k} \) in the denominator, so that the RHS of Equation 20 is actually \( T_{\nu, k} \)-independent.

In accord with the above discussion, we drop all terms proportional to \( (\overline{K}_k \cdot \overline{E}_0) \) because these terms describe frictional heating and will eventually lead to the electron thermal instability (ETI). The pure (FBI) is described by the remaining terms, like the first term in the RHS of Equation 21c \( \propto Vd\nu dV \). The procedure of dropping the terms \( \propto (\overline{K}_k \cdot \overline{E}_0) \) is justified in an intermediate wavelength range in which \( kV \gg \delta_{\nu} \nu_{\text{tr}} \), while the conditions imposed by below Equation 22 hold. These concurrent inequalities are compatible due to the fact that \( \delta_{\nu} \ll 1 \), as follows from the explanation below Equation 1c; for more discussion, see Dimant and Sudan (1995a).

Now we obtain the second relation between \( \eta_{\omega, k} \) and \( \phi_{\omega, k} \), analogous to Equation 12. Before proceeding, we specify the main conditions for the vast majority of the FBI-driven waves. These low-frequency and long-wavelength waves usually satisfy

\[
k_1 \ll k_\perp \approx k, \quad \gamma \ll \omega, \quad kV_0 \ll \nu_{\text{tr}}, \quad k\lambda_D \ll 1,
\]

(22)
so that the following inequality holds,

$$|\hat{D}_{\omega_k}| \ll |\Delta_{\omega_k}|.$$  \hspace{1cm} (23)

This symbolic relation means that the operators $\hat{D}_{\omega_k}$ and $\Delta_{\omega_k}$ apply to $f_{\omega_k}(V)$ and the results are compared by the absolute value. The main point of Equation 23 is that one can apply to Equation 20 a formal Taylor expansion with respect to the small ratio $|\hat{D}_{\omega_k}|/|\Delta_{\omega_k}|$. This leads to

$$f_{\omega_k}(V) \approx \left(\frac{\hat{A}_{\omega_k} F_0(V)}{i\Delta_{\omega_k}} + \frac{\hat{B}_{\omega_k} F_0(V)}{\Delta_{\omega_k}^2} + \frac{\hat{B}_{\omega_k} F_0(V)}{\Delta_{\omega_k}}\right) \phi_{\omega_k},$$  \hspace{1cm} (24)

where the order of the two differential operators $\hat{D}_{\omega_k}$ and $\hat{B}_{\omega_k}$ matters. This formal expansion procedure is equivalent to a regular perturbation technique when one initially neglects in Equation 20 the term $\propto \hat{D}_{\omega_k}$ and then finds the first-order correction. Proceeding from $f_{\omega_k}$ to $\eta_{\omega_k}$ through the relation

$$\eta_{\omega_k} \equiv \frac{4\pi \int_0^\infty f_{\omega_k}(V) V^2 dV}{\int_0^\infty F_0(V) V^2 dV} = \frac{\int_0^\infty f_{\omega_k}(V) V^2 dV}{\int_0^\infty F_0(V) V^2 dV},$$

following from Equations 11 and 15, we obtain

$$\eta_{\omega_k} \approx \frac{\langle \hat{B}_{\omega_k} \rangle}{i\Delta_{\omega_k} \langle \hat{A}_{\omega_k} \rangle} \left(1 + \frac{i\langle \hat{D}_{\omega_k} \hat{B}_{\omega_k} \rangle}{\Delta_{\omega_k} \langle \hat{B}_{\omega_k} \rangle}\right) \phi_{\omega_k},$$  \hspace{1cm} (25)

where the speed averaging of any operator or function $\hat{A}$ is defined as

$$\langle \hat{A} \rangle \equiv \frac{4\pi}{n_0} \int_0^\infty (\hat{A} F_0(V)) V^2 dV = \frac{\int_0^\infty (\hat{A} F_0(V)) V^2 dV}{\int_0^\infty F_0(V) V^2 dV}.$$  \hspace{1cm} (26)

Expressing $\phi_{\omega_k}$ again to the first-order accuracy with respect to the small parameter $|\langle \hat{D}_{\omega_k} \hat{B}_{\omega_k} \rangle/|\Delta_{\omega_k}\langle \hat{B}_{\omega_k} \rangle|$ we obtain Equation (52) from Dimant and Sudan (1995a):

$$\phi_{\omega_k} \approx \left(\frac{i\Delta_{\omega_k}}{\langle \hat{B}_{\omega_k} \rangle} + \frac{\langle \hat{D}_{\omega_k} \hat{B}_{\omega_k} \rangle}{\langle \hat{B}_{\omega_k} \rangle^2}\right) \eta_{\omega_k}.$$  \hspace{1cm} (27)

Combining Equations 12 and 27, we obtain the FBI dispersion relation:

$$\frac{\Omega_{\omega_k}}{k^2 V_F^2} \left[\left(1 - k_i^2\right) \Omega_{\omega_k} + i\nu_v(1 + k_i^2)\right] = 1 + \frac{1}{\nu_f} \left(\frac{\langle \hat{D}_{\omega_k} \hat{B}_{\omega_k} \rangle}{\langle \hat{B}_{\omega_k} \rangle^2} + \frac{i\Delta_{\omega_k}}{\langle \hat{B}_{\omega_k} \rangle}\right),$$  \hspace{1cm} (28)

which generalizes Equation (58) from Dimant and Sudan (1995a) for general $F_0(V)$ and arbitrary ion magnetization.

The differential operators $\hat{D}_{\omega_k}$ and $\hat{B}_{\omega_k}$ are defined by Equation 21. The terms proportional to $\overline{E}_0$ are crucial for the ETI, but for the FBI they play no role, so that we can reduce these operators to simpler expressions,

$$\hat{D}_{\omega_k} \Rightarrow \frac{k_i^2 V^2 m_e \psi_e(V)}{3m_e V_F}, \quad \hat{B}_{\omega_k} \Rightarrow - \frac{k_i^2 T_0}{3m_e V_F} V \psi_e(V) \frac{d}{dV},$$  \hspace{1cm} (29)

where

$$\psi_e(V) \equiv \frac{V_{ei}(V) v_{ei}}{\omega_{ei} \omega_{ci}} \left(1 + \frac{k_i^2 \omega_{ci}^2}{k_i^2 V_e^2(V)}\right),$$  \hspace{1cm} (30)

is the kinetic analog of the standard fluid-model parameter $\psi$ defined by Equation 4. The reduced expression for $\hat{D}_{\omega_k}$ fully agrees with Equation (53) from Dimant and Sudan (1995a) after neglecting in that equation the term
\( \zeta \). At the same time, the operator \( \hat{B}_{\omega k} \) can be reduced to Dimant and Sudan (1995a, Equation (54)), for Maxwellian \( F_0(V) \). For general \( F_0(V) \), in accord with Equation 26, we obtain after integration by parts:

\[
\langle \mathbf{B}_{\omega k} \rangle = \frac{4\pi T_0 k^2}{3n_0 m_i V_i} \int_0^\infty F_0(V) \frac{d (V^3 \psi \omega)}{dV} dV.
\]

Under conditions of Equation 22, in both sides of dispersion Equation 28 the imaginary parts dominate. This allows us to easily separate the wave phase-velocity relation, \( \omega_r(k) \), from the wave growth/damping relation, \( \gamma(k) \).

The wave phase-velocity relation is obtained to the zeroth-order accuracy, after neglecting the small real parts, as well as small \( \gamma \) in \( \omega = \omega_r + i\gamma \). This yields:

\[
\omega_r = \frac{k \cdot [\overline{V}_0 + (1 + \kappa_i^2)\overline{\psi} \overline{V}_0]}{1 + (1 + \kappa_i^2)\overline{\psi}},
\]

where the constant parameter

\[
\overline{\psi} = \frac{\beta r V_{i0} (\mathbf{B}_{\omega k})}{k^2 V_i^2} \equiv \frac{4\pi}{3n_0} \int_0^\infty F_0(V) \frac{d (V^3 \psi \omega)}{dV} dV
\]

unlike \( \psi \) (V), generalizes the conventional parameter \( \psi \) for the entire electron population. Equations 13 and 32 yield the real part of \( \Omega_{\omega k} \):

\[
\langle \mathbf{\Omega}_{\omega k} \rangle = \omega_r - \frac{k \cdot \overline{V}_0}{1 + (1 + \kappa_i^2)\overline{\psi}}.
\]

To the first-order accuracy, Equation 28 yields:

\[
\gamma \approx \frac{\overline{\psi} [1 - \kappa_i^2] \Omega_{\omega k}^2 - k^2 \mathcal{C}_i^2]}{(1 + \overline{\psi}) V_i},
\]

where \( \mathcal{C}_i \) is a modified ion-acoustic speed,

\[
\mathcal{C}_i^2 = V_i^2 + \frac{(\mathbf{\hat{D}}_{\omega k} \mathbf{\hat{B}}_{\omega k}) V_i^2}{\beta_r (\mathbf{\hat{B}}_{\omega k})^2}.
\]

Using Equation 31, the second term in the RHS of Equation 36 can be written as

\[
\frac{(\mathbf{\hat{D}}_{\omega k} \mathbf{\hat{B}}_{\omega k}) V_i^2}{\beta_r (\mathbf{\hat{B}}_{\omega k})^2} = \frac{m_e n_0}{4\pi m_i} \int_0^\infty F_0(V) \left[ \frac{d (V^3 \psi \omega)}{dV} \right] V_i \frac{dV}{dV} dV.
\]

Equations 32 and 36 totally agree with Equations 5 and 6 from Dimant and Milikh (2003) after replacing there the standard parameters \( \psi \) and \( \mathcal{C}_i^2 \) with \( \overline{\psi} \) and \( \mathcal{C}_i^2 \), respectively. Notice that if the ion magnetization is sufficiently high, \( \kappa_i > 1 \), then the FBL driving mechanism, described in Equation 35 by the term \( (1 - \kappa_i^2) \Omega_{\omega k}^2 \), becomes stabilizing, as discussed in detail in Dimant and Oppenheim (2004). This happens above the magnetization boundary, \( \kappa_i = 1 \), which at the high-latitude ionosphere is located about 120 km of altitude (e.g., Dimant & Oppenheim, 2004, Figure 5). In this paper, we will restrict our analysis to lower E-region altitudes where \( \kappa_i < 1 \).
We can rewrite the expression for the modified ion-acoustic speed, $\tilde{C}_i$, in a more traditional way as

$$\tilde{C}_i = \left( \frac{T_i + T_{\text{eff}}}{m_i} \right)^{1/2}, \quad T_{\text{eff}} = \frac{4\pi m_e}{9n_0\bar{\psi}^2} \int_0^\infty F_0(V) \frac{d\left(V^3\bar{\psi}^2\right)}{dV} dV.$$

We emphasize that $\tilde{C}_i$ is not the actual ion-acoustic speed because in the highly dissipative lower ionosphere no ion-acoustic wave can survive for a time duration $\gtrsim \nu_{ei}^{-1}$. For ion-acoustic waves, the collisional damping is even more detrimental than the collisionless ion Landau damping at much higher ionospheric altitudes (if there $T_e \sim T_i$). In the highly dissipative E-region ionosphere, the analogs of the ion-acoustic-like waves are precisely the compression/decompression waves driven by the FBI and other plasma instabilities. These waves, however, can survive for a time duration much longer than $\nu_{ei}^{-1}$ only because they are sustained by the external DC electric field, $E_0 \perp B$.

For constant $\nu_{ei}$ (and hence for constant $\nu_e$), the above expressions reduce to the fluid-model FBI wave phase velocity and growth/damping rate relations. Indeed, in this case Equation 33 yields $\bar{\psi} = \psi_e = \bar{\psi}$, so that Equation 32 reduces to the fluid-model phase-velocity relation, see, for example, Equation 5 from Dimant and Milikh (2003), even for arbitrary background electron distribution function $F_0(V)$. For constant $\bar{\psi}_e$, Equation 35 reduces to fluid-model Equation 6 from Dimant and Milikh (2003) for isothermal ions and adiabatic electrons,

$$\gamma = \frac{\bar{\psi}([1 - \kappa_i^2/(\Omega_en_0)] - k^2C_i^2)}{(1 + \bar{\psi})\nu_{ei}}, \quad C_i^2 = \frac{T_i + (5/3)F_{\text{tot}}}{m_i},$$

where $T_{\text{tot}}$ is defined by Equation 6c. In reality, however, the kinetic quantity $\nu_{ei}$ is strongly velocity-dependent, so that the exact form of the omnidirectional function $F_0(V)$ does really matter.

The fact that for constant $\nu_{ei}$ the electron temperature term in $C_i^2$ includes the single-atom adiabaticity coefficient $5/3$ is associated with the fact that we have neglected here the frictional heating and the corresponding collisional cooling of electrons. This approximation works for waves having sufficiently high wave frequencies, $\omega$, $kV_e \gg \delta_{\text{col}}\nu_{ei}$, while still satisfying the low-frequency, long-wavelength conditions imposed by Equation 22. Here $\delta_{\text{col}} \approx (2-4) \times 10^{-3}$ is the mean relative fraction of collisional losses of the electron energy during one $e$-n collision (Dimant & Sudan, 1995a; Gurevich, 1978). In the opposite limit of very low-frequency, long-wavelength waves, $\omega, kV_e \ll \delta_{\text{col}}\nu_{ei}$, the electron thermal behavior is mostly determined by the heating/cooling balance, so that the factor $5/3$ disappears and the destabilizing ETI mechanism for the optimal $\vec{k}$ directions becomes efficient (Dimant & Sudan, 1995b, 1995c). The net result of this change is that the minimum threshold field is reached for longer-wavelength waves than for those prone to the pure FBI excitation, though mostly for altitudes below the 100 km latitude (Dimant & Sudan, 1997).

The linear instability develops if the DC electric field exceeds the threshold field determined by $\gamma = 0$. According to Equations 34 and 35, this yields the threshold parameters

$$\frac{E_{\text{Thr}}}{B} = V_{\text{Thr}} = \frac{(1 + \kappa_i^2) \left[ 1 + (1 + \kappa_i^2)\bar{\psi} \right]}{(\cos \theta - \kappa_i \sin \theta) \sqrt{1 - \kappa_i^2}} \tilde{C}_i,$$

where $\theta$ is the angle between the wavevector $\vec{k}$ and the $E_0 \times B$-drift direction (the “flow” angle).

Crucial for the onset of the FBI is the minimum threshold field at a given location. The driving field and the corresponding $E_0 \times B$-drift speed reach their minimal values at the optimal direction of the wavevector, $\vec{k} \parallel \vec{U}$, corresponding to $\theta = -\arctan \kappa_i$ and $k_\parallel = 0$:

$$\left( \frac{E_{\text{Thr}}}{B} \right)_{\text{min}} = (V_{\text{Thr}})_{\text{min}} = \frac{(1 + \kappa_i^2)}{1 - \kappa_i^2} \left[ 1 + (1 + \kappa_i^2)\bar{\psi} \right] \tilde{C}_i = \sqrt{\frac{1 + \kappa_i^2}{1 - \kappa_i^2} \left[ 1 + (1 + \kappa_i^2)\frac{I_1}{5I_0} \right]} \left( \frac{T_i}{m_i} + \frac{m_eI_0I_2}{m_i} \right)^{1/2},$$

where in the last equality we expressed $\bar{\psi}$ (for $k_\parallel = 0$) and $\tilde{C}_i$ in terms of the following integral parameters:
\[ I_0 = \frac{n_0}{4\pi} = \int_0^\infty F_0 V^2 dV = \frac{\sqrt{2}}{m_i^{3/2}} \int_0^\infty F_0 \sqrt{E} dE, \]  
\[ I_1 = \int_0^\infty F_0 \frac{d}{dV} \left( \frac{V^3 \psi_e}{E} \right) dV = \left( \frac{2}{m_i} \right) \int_0^\infty F_0 \frac{d}{dE} \left( \frac{E^{3/2} \psi_e}{E} \right) dE, \]  
\[ I_2 = \int_0^\infty F_0 \frac{d}{dV} \left( \frac{V^5 \psi_e^2}{E} \right) dV = \left( \frac{2}{m_i} \right) \int_0^\infty F_0 \frac{d}{dE} \left( \frac{E^{5/2} \psi_e^2}{E} \right) dE. \]  

In the two equivalent forms for each \( I_k, k = 0, 1, 2 \), both \( F_0 \) and \( \psi_e \) should be taken as functions of either the electron speed \( V \) or the corresponding kinetic energy, \( E = m_i V^2 / 2 \), depending on the integration variable.

It is conventional to express the distribution function and collision frequencies in terms of the electron kinetic energy, rather than of the electron speed, so that the integral forms in terms of \( E \) are more convenient for specific calculations. The form of Equation 40 in terms of \( I_k \) is convenient because it makes the threshold field totally insensitive to the normalization of \( F_0 \) since, in the relevant fractions, the common coefficients in different \( I_k \) cancel. This allows one to pick an arbitrary (but common for all \( I_k \)) normalization of the distribution function, provided \( F_0 \) includes the entire electron population that consists of the thermal bulk and the superthermal tail.

If both \( \tilde{\psi} = (1 + \kappa_i^2) I_1 / (3 I_0) \) and \( \kappa_i \) are small (this dual condition is usually satisfied at altitudes between 100 and 110 km) then Equation 40 reduces to a simpler relation,

\[ \frac{(E_{Thc})_{min}}{B} = (V_{Thc})_{min} \approx \left( \frac{T_i + m_i I_0 I_2}{m_i I_1} \right)^{1/2}. \]  

In this case, \( (E_{Thc})_{min} \) becomes insensitive to the normalization of the function \( \psi_e \) as well. This allows one to simultaneously replace in all integrals \( I_k \) the energy-dependent function \( \psi_e \) with merely the \( e-n \) collision frequency, \( \nu_{en} \), so that for \( \psi_e \), \( \kappa_i \ll 1 \) we have

\[ \frac{(E_{Thc})_{min}}{B} = (V_{Thc})_{min} \approx \left( \frac{T_i + T_{eff}}{m_i} \right)^{1/2}, \]

\[ T_{eff} = \left( \frac{\int_0^\infty F_0(E) \sqrt{E} dE}{\int_0^\infty F_0(E) dE} \right) \left( \frac{\int_0^\infty F_0(E) dE}{\int_0^\infty F_0(E) dE} \right)^{1/2} \]  

In the constant-\( \nu_{en} \) limit, the effective FB-threshold temperature reduces to

\[ (T_{eff})_{\nu=const} = \frac{10}{9} \frac{\int_0^\infty F_0(E) E^{3/2} dE}{\int_0^\infty F_0(E) \sqrt{E} dE}. \]  

For Maxwellian EDF, \( F_0(E) \propto \exp(-E/T_{ce}) \), this further reduces to \( (5/3) T_{ce} \) in full accord with Equation 39. For the general, non-Maxwellian EDF, for example, for the combined cold bulk electrons and SE, Equation 44 would correspond to merely including the total electron pressure.

The main result of our FBI linear analysis for general \( f_i(V) \approx F_0(V) \) is given by Equations 32, 35, and 40; the following relations just represent various simplifications. It is to be noted, however, that at altitudes closely approaching the magnetization boundary, \( \kappa_i = 1 \), the effect of ion-thermal instability (ITI) driving becomes important (Dimant & Oppenheim, 2004). The ITI driving modifies both the optimum angles of the instability onset and the threshold field values. Furthermore, the ITI driving even extends the unstable range of altitudes by a few kilometers above the magnetization boundary, where the pure FBI mechanism becomes stabilizing. Unlike the ETI mechanism, the ITI mechanism destabilizes waves largely in the same wavelength range as does the FBI mechanism, so that the effect of ITI driving is inseparable from the FBI. We have not included in the present analysis any thermal effects because that would make our theoretical treatment much more complicated. This may be a subject of a future work.
4.2. Specific Calculations for Superthermal Electrons Produced by Electron Precipitation

In order to estimate the contribution of the superthermal energy tail formed by precipitating electrons, we apply the equations derived above to specific calculations of the FBI threshold. We separate the dominant omnidirectional part of the total electron velocity distribution, \( f_e(\mathbf{V}) \approx F_0(\mathbf{E}) \), into two distinct components,

\[
F_0(\mathbf{E}) \approx F_{TB}(\mathbf{E}) + F_{SE}(\mathbf{E}),
\]

namely, the undisturbed thermal bulk described by the Maxwellian distribution,

\[
F_{TB}(\mathbf{E}) = n_{TB} \left( \frac{m_e}{2\pi k_{TB}} \right)^{3/2} \exp \left( -\frac{\mathbf{E}}{k_{TB}} \right),
\]

and the superthermal EDF, \( F_{SE}(\mathbf{E}) \), calculated numerically using the kinetic code STET, as described above in Sections 2.2 and 3. The Maxwellian thermal bulk electron distribution, \( F_{TB} \), is fully determined by the values of the undisturbed temperature, \( T_{TB} \), and density, \( n_{TB} \). We take these values from ionospheric models, as described in Section 3. The superthermal EDF, \( F_{SE}(\mathbf{E}) \), was calculated by STET in the energy range between 1 eV and 30 keV. In the low-energy range below 1 eV, the main contributions into all relevant integrals come almost exclusively from Maxwellian \( F_{TB}(\mathbf{E}) \), while the entire energy range above 1 eV is overwhelmingly dominated by \( F_{SE}(\mathbf{E}) \). This allows us to disregard possible inaccuracies of the EDF within the interface energy range of \( \mathbf{E} \sim 1 \) eV.

For simplicity, we will do our specific calculations for the intermediate E-region altitudes where both conditions \( \psi \ll 1 \) and \( \kappa_i \ll 1 \) hold. Since \( \psi \propto \kappa_i^{-1} \), there is an overlapping altitude range, roughly between 100 and 110 km, where both conditions hold concurrently. In this case, the minimum threshold field is approximately given by Equation 43, where normalizations of both \( F_0(\mathbf{E}) \) and \( v_{\infty}(\mathbf{E}) \) can be ignored, provided they are common across all four integrals in the expression for \( T_{eff} \).

We start by approximating analytically the function \( v_{\infty}(\mathbf{E}) = n_e \sigma_{\infty}(\mathbf{E}) V(\mathbf{E}) \), where \( n_e \) is the neutral density and \( \sigma_{\infty}(\mathbf{E}) \) is the energy-dependent e-n collision momentum transfer cross section. We need to fit \( v_{\infty}(\mathbf{E}) \) by a continuous analytic function of the electron energy, \( \mathbf{E} \), because the expression for the effective temperature, (43), involves the energy derivative of \( v_{\infty}(\mathbf{E}) \).

At the altitudes of interest, the neutral atmosphere consists mostly of the molecular nitrogen and oxygen (\( \sim 80\% \) of \( \mathrm{N}_2 \) and \( \sim 20\% \) of \( \mathrm{O}_2 \)), so that \( \mathrm{N}_2 \) is more abundant. Besides, the e-\( \mathrm{N}_2 \) collisional cross section vastly dominates over the e-\( \mathrm{O}_2 \) collisional cross section (e.g., Solomon et al., 1988). This allows us to neglect the e-\( \mathrm{O}_2 \) collisions and approximate the entire neutral population by the nitrogen molecules. In this approximation, the e-n cross section \( \sigma_{\infty}(\mathbf{E}) \) becomes a universal function of the electron energy \( \mathbf{E} \), see Figure 2. For this paper, we have approximated the data presented in Itikawa (2006, Table 2), with an addition of the top value of \( \mathbf{E} = 10 \) keV from Solomon et al. (1988). The analytic expression is given by \( \sigma_{\infty}(\mathbf{E}) = 10^{-16} \times \Sigma(\mathbf{E}) \), where the piecewise function \( \Sigma(\mathbf{E}) \) is expressed in the polynomial-fractional form \( \Sigma(\mathbf{E}) = \sum a_n \mathbf{E}^n / \sum b_n \mathbf{E}^n \) with \( \mathbf{E} \) expressed in eV. The details of this approximation are given in Appendix B. The universal approximation given by Equation B1 can be effectively employed in many kinetic problems involving collisions of electrons with molecules \( \mathrm{N}_2 \).

The effective temperature \( T_{eff} \) is insensitive to the normalization of the collision frequency \( v_{\infty}(\mathbf{E}) = n_e \sigma_{\infty}(\mathbf{E})(2E/m_e)^{1/2} \), so that we can replace the latter with \( \Sigma(\mathbf{E})/\sqrt{\mathbf{E}} \), and obtain:

\[
T_{eff} = \frac{K_1 K_2}{K_3}
\]
where

\[ K_1 = \int_0^{\infty} F_0(\xi) \sqrt{\xi} d\xi, \]
\[ K_2 = \int_0^{\infty} F_0(\xi) \frac{d}{d\xi} \left( \xi^{5/2} \Sigma(\xi) \right) d\xi, \]
\[ K_3 = \int_0^{\infty} F_0(\xi) \frac{d}{d\xi} \left( \xi^2 \Sigma(\xi) \right) d\xi. \]

and the total distribution function of Equation 45 can be written as

\[ F_0(\xi) = \xi \exp(-31.44\xi) + \eta F_{SE}(\xi), \]

with \( \xi = 3.931 \times 10^{-7}, \eta = 1.616 \times 10^{-19} \), and \( \xi \) in eV. The normalization coefficients \( \xi \) and \( \eta \) provide the distribution function to be measured in \( s^3 m^{-6} \).

For a specific STET simulation, we pick the Maxwellian input with the energy flux \( 10 \) erg cm\(^{-2} \) s\(^{-1} \), the characteristic energy 30 keV, at the 110 km of altitude. The simulated electron distribution function in the superthermal range of energies between 1 eV and 30 keV is reasonably well approximated by a piecewise expression given by Equation C2. Using these analytic fits given by Equations B1 and C2, after all numeric integrations, we obtain

\[ K_1 \approx 1.976 \times 10^{-21}, \quad K_2 \approx 2.676 \times 10^{-19}, \quad K_3 \approx 3.665 \times 10^{-21}. \]

In \( K_1 \) and \( K_3 \), the thermal bulk distribution, \( F_{TB}(\xi) \), vastly dominates the total integrals, whereas in \( K_2 \), due to the higher power of \( \xi \) in the integrand, on the contrary, the superthermal distribution \( F_{SE}(\xi) \) determines essentially the entire integral value.

Equation 48 leads to the effective temperature \( T_{eff} \approx 39.4 \) eV (while the effective electron temperature based on the total electron pressure, see Equation 44, would yield a much smaller value of 0.11 eV). This extremely high value of \( T_{eff} \) increases the regular FBI threshold corresponding to \( T_{eff} = 300 \) K (about 0.026 eV), \( E_{Thr} = 20 [B/(5 \times 10^4 nT)] \) mV/m, by a significant factor (almost 30). This results in the enormous threshold field, \( E_{Thr} \approx [(T_i + T_{eff})/600 K]^{1/2} E_{Thr} \approx 0.55[B/(5 \times 10^4 nT)] V/m \). To excite the FBI under these conditions, the convection DC electric field mapped from magnetosphere down to the E-region altitudes must exceed this field. At the ionosphere altitudes, such huge convection electric fields, that would correspond to the \( \vec{E} \times \vec{B} \)-drift speed as large as almost 11 km/s, have never been reported. This means that for this level of precipitation, the strongly elevated FBI threshold can hardly be reached during extreme geomagnetic storm events, and even during modest ones.

This specific simulation was performed for a relatively strong precipitation with the mean energy flux \( \Phi_e = 10 \) erg cm\(^{-2} \) s\(^{-1} \). As we discussed in Section 3, any superthermal particle-energy-integrated characteristics will be proportional to \( \Phi_e \). Since the contribution of the superthermal energy tail to the integrals \( K_{1,3} \) is negligible, while \( K_2 \) is almost entirely determined by \( F_{TB}(\xi) \), the effective FBI threshold “temperature,” \( T_{eff} \), is in direct proportion to the energy flux, \( T_{eff} \propto \Phi_e \). Thus we can generalize the previous result as \( T_{eff} \approx 39.4(\Phi_e/10 \) erg cm\(^{-2} \) s\(^{-1} \) eV).

Furthermore, according to Figure 1, at a given E-region altitude (e.g., 110 km) the superthermal EDF does not vary significantly in the broad range of plasmasheet electron characteristic energies, \( \xi_0 \), between 5 and 30 keV (at least, for the Maxwellian precipitation input). This allows us to roughly use the approximation of Equation C1 to be a “universal” EDF within the energy domain, say, between 1 eV and a given SE cutoff energy \( \xi_{max} \) with the zero values outside (in the above calculation, \( \xi_{max} = 20\text{keV} \)). This allows us to obtain an explicit analytic expression for the SE-dominated effective temperature.

The idea of this calculation is as follows. Assuming \( \xi_{max} \) to be in the energy domain between 1 and 30 keV, we can separate the major integral \( K_1 \) into two parts: a lower-energy part between 1 eV and 1 keV and the higher-energy remainder. The lower-energy part can be calculated numerically, which is done above, in Equation 49. This calculation yields a specific number. For the remaining integral between 1 keV and \( \xi_{max} \), we can use the large-energy asymptotics of both functions \( F_0(\xi) \) and \( \Sigma(\xi) \).
where \( \beta = 0.025 \), \( A = 634.6 \), \( B = 2.721 \times 10^6 \), \( m = 373 \), \( p = 35.21 \).

For energies above 1 eV these asymptotics are very close to the original functions. Combining the two parts of the integral, after dropping some small and inconsequential terms, we obtain for the total SE contribution into \( K_2 \) a simple algebraic function of \( \beta \max \):

\[
K_2 \approx 5.622 \times 10^{-28} \beta \max^2 + 7.531 \times 10^{-25} \beta \max + 3.085 \times 10^{-21} \ln \beta \max - 4.966 \times 10^{-21}.
\]

(50)

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\]

(50)

In the entire energy domain of 1–30 keV, the integrals \( K_{1,3} \) are vastly dominated by the cold bulk-electron energy distribution with the specific values given by Equation 49, while \( K_2 \) is almost entirely determined by the SE energy distribution and given by Equation 50. As a result, we obtain

\[
T_{\text{eff}}(\text{eV}) \approx 8.35 \times 10^{-2} \beta \max^2 + 0.112 \beta \max + 0.46 \beta \max + 2.43,
\]

(51)

where, unlike the above, we express \( \beta \max \) in keV. In the particular case of \( \beta \max = 20 \) keV, Equation 51 reproduces the specific value of \( T_{\text{eff}}(\text{eV}) \approx 39.4 \) eV obtained above. Figure 3 shows the energy dependence given by Equation 51 and similar for different values of the energy flux, \( \Phi \beta \) (the values of \( T_{\text{eff}} \) are proportional to \( \Phi \beta \)).

In this paper, we have restricted our treatment to the FBI, disregarding other instability drivers, such as the ETI and ITI driving mechanisms. The latter instabilities may reduce the instability threshold. We may include the other instability drivers in our future analysis, but one should hardly expect any drastic changes in the predicted increase of the instability threshold caused by strong electron precipitation.

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In principle, the predicted effect of suppression of the E-region instabilities by strong electron precipitation is verifiable by observations. Unfortunately, there are almost no simultaneous colocated observations of electron
precipitation and E-region irregularities caused by the FBI because such observations and their accurate interpretation represent a certain challenge. Nevertheless, we are aware of at least one work where the authors have reported on observations of colocated optical and radar auroras (Bahcivan et al., 2006). The data presented there indicate suppression of the FBI inside the auroral arc, versus the arc edges where radar aurora still exists. In their conclusion 4, Bahcivan et al. (2006) state: “The radar aurora was often observed at the discrete arc boundary and suppressed inside the arc. Radar aurora could sometimes be seen inside an arc at the very peak of optical intensification.” The authors’ interpretation of the observed suppression is that within the arc the driving electric field might had been dramatically reduced due to the enhanced conductivity, so that the E-field magnitude might have dropped below the regular FBI threshold value. While E-field reduction is a possible mechanism, one must also consider dramatically increased FBI threshold due to precipitating electrons within the arcs. In order to sustain or disprove our theory, more future observations with better statistics and more careful data analysis are needed.

5. Conclusions

During events of strong geomagnetic activity, most of the intense magnetospheric currents close through the high-latitude E-region ionosphere. At E-region altitudes between 100 and 120 km, strong DC electric fields, mapped down from the magnetosphere, can drive plasma instabilities, such as the (FBI) and others. These instabilities give rise to anomalous electron heating and enhanced plasma particle transport, affecting the global ionospheric conductances and, hence, the entire dynamics of the near-Earth's plasma.

During geomagnetic perturbations, strong electron precipitation also happens, resulting in Aurora and enhanced ionization. The regions of strong electric fields that drive E-region instabilities and the auroral regions of strong electron precipitation may overlap in space, so that the two effects might interact. This work shows that the intense electron precipitation can modify significantly the conditions necessary to drive the instability.

We have analyzed theoretically how strong electron precipitation affects the threshold conditions for the FBI driving. To this end, we performed a series of kinetic simulations of superthermal electrons, using the sophisticated kinetic code STET (Khazanov et al., 2016), using various kinds of the input particle fluxes. These simulations predict distributions of superthermal electrons in the energy range between 1 eV and 30 keV. While the total number density of the superthermal electrons (SE) is usually small compared to the total number density of the electron thermal bulk (TB), the total energy of the entire hot SE population is often many times that of the entire cold TB population. This means that the SE total pressure, \( P_{SE} \), exceeds dramatically the regular pressure of the dominant TB population, \( P_{TB} = n_T e^2 \). Since the SE particle density is typically small compared to \( n_T \), then the dominant SE pressure creates an effective plasma temperature in proportion to \( P_{SE} \). The elevated electron temperature increases the particle diffusion and hence the instability threshold, thus suppressing the instability or at least reducing its efficiency. In order to quantify the threshold conditions, a naive viewpoint might suggest to just replace the regular electron temperature with the elevated effective temperature \( T_{e^{SE}/n_T} \) in the conventional threshold conditions for the E-region instabilities.

The actual situation, however, turns out to be more complicated. Even a simple three-fluid model, in which the TB and SE electron populations are treated as two different Maxwellian distribution functions, demonstrates that the FBI threshold field involves the \( e\text{-}n \) collision frequencies whose values differ dramatically for the two electron populations. The fact that the \( e\text{-}n \) collision frequency varies gradually with energy and cannot be reduced to just two distinct values means that a quantitative analysis of growth rates requires kinetic theory.

The kinetic theory developed here confirms that the strongly energy-dependent \( e\text{-}n \) collision frequency plays a crucial role in the FBI threshold conditions. The physical reason is that the instability threshold is determined by diffusive losses, where the Pedersen diffusion (i.e., diffusion along the total electrostatic field and perpendicular to the magnetic field) plays the principal role in low-frequency plasma density waves. The Pedersen diffusion coefficient of magnetized electrons is proportional to the \( e\text{-}n \) collision frequency \( \nu_{en} \), so that for the general non-Maxwellian electron velocity distributions the energy dependence of \( \nu_{en} \) cannot be canceled out. Furthermore, specific calculations for realistic conditions shows that the energy dependence of \( \nu_{en} \) results in much more severe suppression of the instability compared to the naïve model of the just pressure-dependent threshold: the effective “temperature” for the FBI threshold may exceed that determined by the modified electron pressure alone by more than order of magnitude.
While there is some observational evidence of the FBI suppression within the optically active arcs of intense electron precipitation, those observations cannot be considered as definite proof of our theory because alternative explanations also exist (Bahcivan et al., 2006). Nevertheless, we believe that our theoretical treatment is based on solid physical foundations, so that its major conclusion of possible dramatic suppression of E-region instabilities by precipitating electrons should be correct.

**Appendix A: Three-Fluid Linear Analysis of the FBI**

In this Appendix, we develop a simplified 3-fluid model approach by assuming two electron fluids and a single ion one. This simplified approach does not rival the rigorous kinetic approach implemented in Section 4 of the main text, but it provides an additional insight into the effect of electron precipitation on the FBI threshold conditions and helps identify the key factors.

The closed set of fluid-model Equation 1 is only valid for the particle velocity distributions reasonably close to Maxwellian, so that in this approach we will use the model of two Maxwellian populations of electrons, \( f_e = f_{TB} + f_{SE} \), where \( TB \) stands for the thermal bulk and \( SE \) stands for the superthermal electron tail. Each Maxwellian population has its own density and temperature: \( f_{TB} = n_{TB}(m_e/2\pi kTB)^{3/2} \exp(-\epsilon/kTB) \) and \( f_{SE} = n_{SE}(m_e/2\pi kSE)^{3/2} \exp(-\epsilon/kSE) \). Under the actual conditions of electron precipitation, the conditions \( T_{SE} \rightarrow T_{TB} \), \( n_{SE} \ll n_{TB} \) usually hold, although for this specific treatment these conditions are of no importance and will not be imposed.

Adding to the two electron fluids an ion fluid and assuming for simplicity the isothermal regime of the pure FBI with constant \( T_{TB}, T_{SE} \), and \( T_i \) but variable densities, \( n_{SE}, n_{TB} \approx n_e \), we will need only two first fluid-model Equations 1a and 1b,

\[
\frac{dn_j}{dt} + \nabla \cdot (n_j \overrightarrow{v}_j) = 0,
\]

\[
m_j \left( \frac{\partial}{\partial t} + \overrightarrow{v}_j \cdot \nabla \right) \overrightarrow{v}_j = q_j (\overrightarrow{E} + \overrightarrow{v}_j \times \overrightarrow{B}) - T_j \frac{\nabla n_j}{n_j} - m_j \nu_j \overrightarrow{v}_j.
\]

where the subscript \( j \) denotes either each of the two electron species, \( j = TB \) and \( j = SE \), or the single ion species, \( j = i \). The kinetic \( e-n \) collision frequency \( \nu_j \) depends strongly on the individual electron velocity. To mimic this in our simplified three-fluid model, we will assign for each electron population its own constant value of \( \nu_j \); the mean thermal bulk value, \( \nu_{TB} \), and the mean superthermal value, \( \nu_{SE} \). These values of \( \nu_e \) may be vastly different.

Bearing in mind the long-wavelength and low-frequency E-region processes, we will close all three sets of fluid equations by the quasineutrality condition, \( n_j = n_{TB} + n_{SE} \), for both the undisturbed plasma background, \( n_i(0) = n_{TB}(0) + n_{SE}(0) \), and linear wave perturbations, \( \delta n_j = \delta n_{TB} + \delta n_{SE} \). In what follows, we will mostly operate with the relative fractions of each background electron population, \( \rho_{TB} = n_{TB}(0)/n_i(0) \) and \( \rho_{SE} = n_{SE}(0)/n_i(0) \), so that \( \rho_{TB} + \rho_{SE} = 1 \).

For the undisturbed background flows, after setting \( \partial/\partial t \to 0 \), \( \nabla \to 0 \), Equation 1b yields

\[
\overrightarrow{V}_{j0} = \left( \frac{q_j E_0}{m_j \nu_j} + \kappa_j^2 \overrightarrow{v}_0 \right) \left( \frac{1}{1 + \kappa_j^2} \right),
\]

where the \( \overrightarrow{E}_0 \times \overrightarrow{B} \)-drift velocity \( \overrightarrow{V}_0 \) and magnetization parameters \( \kappa_j = \omega_j / \nu_j \) were defined in Section 2.1; \( q_j = e \), \( q_{TB} = q_{SE} = -e \), and \( m_{TB} = m_{SE} = m_e \). In spite of the common value of the electron gyrofrequency \( \omega_{ce} \), each Maxwellian electron population, \( TB \) and \( SE \), has different magnetization parameters \( \kappa_j, \kappa_{TB} = \omega_j / \nu_{TB} \neq \kappa_{SE} = \omega_j / \nu_{SE} \).

In what follows, we will assume both electron fluids to be strongly magnetized, \( \kappa_{TB}, \kappa_{SE} \gg 1 \), so that the average background flow of all electrons is close to the \( \overrightarrow{E}_0 \times \overrightarrow{B} \)-drift velocity, \( \overrightarrow{V}_{TB0} \approx \overrightarrow{V}_{SE0} \approx \overrightarrow{V}_0 \). For the following analysis, it is also convenient to introduce the velocity difference between the background electron and ion flows:

\[
\overrightarrow{U} \approx \overrightarrow{V}_0 - \overrightarrow{V}_{i0} = \left( \overrightarrow{V}_0 - \frac{q_j E_0}{m_j \nu_j} \right) \left( \frac{1}{1 + \kappa_j^2} \right).
\]
In all low-frequency E-region processes, electric fields are electrostatic, \( \vec{E} = -\nabla \Phi \), while the magnetic field \( \vec{B} \) remains essentially constant. For linear wave perturbations of all space/time-varying quantities, we will set the standard harmonic-wave ansatz: \( \delta \alpha \propto \exp[i(\vec{k} \cdot \vec{r} - \omega t)] \) with real \( \vec{k} \), but complex frequency, \( \omega = \omega_r + i\gamma \).

Introducing dimensionless variables for each species \( j \):

\[
\eta_j \equiv \frac{\delta n_j}{n_{j0}}, \quad \phi \equiv \frac{\epsilon \delta \Phi}{T_{\omega r}},
\]

we obtain from continuity Equation 1a a simple relation:

\[
\eta_j = \frac{\vec{k} \cdot \delta \vec{V}_j}{\Omega_j},
\]

where \( \Omega_j \equiv \omega - \vec{k} \cdot \vec{V}_{j0} \) is the Doppler-shifted wave frequency in the frame of reference of the \( j \)-species mean flow. The \( j \)-fluid velocity perturbation \( \delta \vec{V}_j \) should be found from momentum-balance Equation 1b. In the dimensionless variables, all \( \delta \vec{V}_j \) become proportional to the normalized linear combinations of the linearized wave electric field with the particle pressure perturbations, \( \alpha \phi + \eta_j \), where

\[
\alpha_i = 1, \quad \alpha_{TB} = -\frac{T_{\omega 0}}{T_{TB}}, \quad \alpha_{SE} = -\frac{T_{\omega 0}}{T_{SE}}.
\]

Then Equation 5 yields \( \eta_j \) in terms of \( \phi \),

\[
\eta_j = \frac{\alpha_i A_j}{1 - A_j} \phi, \quad A_j \equiv \frac{\vec{k} \cdot \delta \vec{V}_j}{(\alpha_i \phi + \eta_j) \Omega_j},
\]

via still undetermined coefficients \( A_j \). In the direction parallel to \( \vec{B} \), Equation 1b yields

\[
\delta \vec{V}_{\beta j} = -i \frac{k_j V_j^2}{v_j} \frac{(\alpha_i \phi + \eta_j)}{v_j (1 - i\Omega_j/v_j)},
\]

while in the perpendicular to \( \vec{B} \) component of \( \delta \vec{V}_j \), we have a more complicated relation,

\[
\delta \vec{V}_{\perp j} = -i \frac{V_j^2}{v_j} \left( \frac{1 - i\Omega_j/v_j}{(1 - i\Omega_j/v_j)^2 + k_j^2} \right) \frac{(\alpha_i \phi + \eta_j)}{(1 - i\Omega_j/v_j)^2 + k_j^2},
\]

where \( \vec{k}_j \) and \( \vec{k}_p \) are the wavevector components in the parallel and perpendicular to \( \vec{B} \) directions, respectively. For the coefficients \( A_j \), these equations yield

\[
A_j = -i \frac{V_j^2}{v_j \Omega_j} \left[ \frac{(1 - i\Omega_j/v_j) k_j^2}{(1 - i\Omega_j/v_j)^2 + k_j^2} + \frac{k_j^2}{1 - i\Omega_j/v_j} \right] .
\]

Using the quasineutrality condition for the wave perturbations, \( \eta_j = \rho_{TB} \alpha_{TB} + \rho_{SE} \alpha_{SE} \), we obtain from Equation 7 and 10 the three-fluid FBI dispersion relation:

\[
D(\omega, \vec{k}) \equiv 1 + \frac{1}{A_j} \left( \frac{\rho_{TB} \alpha_{TB} |A_{TB}|}{1 - A_{TB}} + \frac{\rho_{SE} \alpha_{SE} |A_{SE}|}{1 - A_{SE}} \right) = 0,
\]

where we have used the fact that both \( \alpha_{TB} \) and \( \alpha_{SE} \) are negative, as seen from Equation 6.

General three-fluid FBI dispersion relation, Equation 11, does not have a simple general solution. Fortunately, it can be simplified by taking into account the fact that fluid-model Equation 1 is valid only in the long-wavelength limit in which all wavevectors are much larger than the corresponding ion collisional mean free paths, while the wave frequencies are small compared to the ion-neutral collision frequencies, \( |\omega|, kV_{\parallel}, kV, \ll v_i \). Otherwise, ion Landau damping becomes crucial, requiring the kinetic treatment of ions. It is also important that the minimum values of the FBI threshold field are always reached in the same long-wavelength limit, where we automatically obtain \( |A_j| \ll 1 \). Assuming also \( |\Omega_{TB}/v_{TB}|, |\Omega_{SE}/v_{SE}| \ll |\Omega_j/v_j|, k_{\parallel} \ll k_{\perp}, \) and \( k_j \leq 1 \), we obtain for all \( A_j \) simpler expressions:
\[ A_{TB} \approx -i \frac{k_T^2 V_T^2}{v_{TB} \Omega_T} \left( 1 + \frac{k_T^2}{k_{TB}^2} \right) \left( 1 + \frac{k_T^2}{k_{TB}^2} \right), \]  
\[ A_{SE} \approx -i \frac{k_T^2 V_{SE}^2}{v_{SE} \Omega_{SE} k_{SE}^2} \left( 1 - i \Omega_{SE}/v_{SE} \right), \]  
\[ A_i \approx -i \frac{k_T^2 V_i^2}{v_i \Omega} \left( 1 - i \frac{\Omega}{v_i} \right). \]

Then, to the first-order accuracy with respect to small \(|A_i|\), general three-fluid FBI dispersion Equation A11 reduces to

\[ D(\omega, \kappa) \approx 1 + \frac{\rho_{TB}|\sigma_{TB}|A_{TB}}{A_i} (1 + A_{TB} - A_i) + \frac{\rho_{SE}|\sigma_{SE}|A_{SE}}{A_i} (1 + A_{SE} - A_i) = 0. \]  

This reduced dispersion relation has certain advantages over general Equation A11. First, in the assumed long-wavelength limit, \(|\text{Im}D(\omega, \kappa)\)| turns out to be automatically small compared to \(|\text{Re}D(\omega, \kappa)\|, as well as the growth/damping rate, \(|\gamma|\), becomes small compared to the real wave frequency, \(\omega_i\). This allows one to treat the wave phase-velocity relation derived by the dominant real part of \(D(\omega, \kappa)\), separately from the instability driving derived by the small imaginary part of \(D(\omega, \kappa)\). Second, Equation A13 allows one to expose all instability driving and loss mechanisms as separate linear terms. This is convenient for the general instability analysis, although here we restrict ourselves to the purely isothermal FBI.

Under condition of \(|\gamma| \ll \omega_i\), if we also neglect the corresponding first-order small terms in the RHS of Equation A13 and substitute \(\omega \approx \omega_i\) in all highest-order terms, we obtain the equation for the real wave frequency, \(\text{Re}D(\omega, \kappa) \approx D_0(\omega, \kappa)\). The solution of \(D_0(\omega, \kappa) = 0\) for \(\text{Re} \omega = \omega_i\) provides the zeroth-order phase-velocity relations for the linear harmonic waves, \(\omega_0(\kappa)\). In the next step, we add the small imaginary parts and solve for the first-order equation with \(i \gamma\) included in the complex wave frequency. This gives

\[ \gamma \approx -\frac{\text{Im}D(\omega, \kappa)}{\frac{\partial D(\omega, \kappa)}{\partial \omega}} \bigg|_{\omega=\omega_i}. \]  

The zeroth-order relation for the dominant real part of the wave frequency is obtained by neglecting in the RHS of Equation A13 all terms proportional to \(A_j\), except the ratios \(A_{TB}/A_i\) and \(A_{SE}/A_i\). This yields

\[ D_0(\omega_0, \kappa) = 1 + (1 + \kappa_0^2) \left( 1 + \frac{\kappa \cdot \hat{V}}{\Omega_0} \right) \Psi = 0, \]  

where \(\Omega_0\) is the common Doppler-shifted wave frequency for all electrons, \(\Omega_0 = \Omega_{TB} = \Omega_{SE} \approx \omega_0 - \kappa \cdot \hat{V}_0\),

\[ \Psi \equiv \rho_{TB} \psi_{TB} + \rho_{SE} \psi_{SE}, \quad \psi_j \equiv \frac{1}{\kappa_j^2} \left( 1 + \frac{\kappa_j^2 k_j^2}{k_{TB}^2} \right), \]

and \(\hat{U}\) is defined by Equation A3 with the use of the relation \(\Omega_0 = \Omega_0 - \kappa \cdot \hat{U}\). The solution of Equation A15 for \(\Omega_0\) yields

\[ \Omega_0(\kappa) = -\frac{(1 + \kappa_0^2)(\hat{\kappa} \cdot \hat{U})\Psi}{1 + (1 + \kappa_0^2)\Psi}, \]

\[ \Omega(\kappa) = \frac{\kappa \cdot \hat{U}}{1 + (1 + \kappa_0^2)\Psi}, \]

\[ \omega_0(\kappa) = \frac{\kappa \cdot [\hat{V}_0 + (1 + \kappa_0^2)\Psi \hat{V}_0]}{1 + (1 + \kappa_0^2)\Psi}. \]
These expressions provide the phase-velocity relation, \( \mathbf{V}_{\text{ph}} = -\frac{\mathbf{k} \cdot \mathbf{V}_0}{\Omega} \), in various frames of reference. For the low ion magnetization, \( \kappa_i^2 \ll 1, |\mathbf{V}_{\text{ph}}| \ll V_0, \mathbf{U}_i \approx \mathbf{V}_0 \), that usually takes place at E-region altitudes below 115 km, we have much simpler relations:

\[
\Omega_e \approx -\frac{\Psi(k \cdot V_0)}{1 + \Psi}, \quad \Omega_i \approx \frac{k \cdot V_0}{1 + \Psi}.
\]  

(A18)

They become the conventional FBI expressions if the parameter \( \Psi \) defined in Equation A16 is replaced by the single-group parameter \( \psi = \nu_e \), each proportional to \( n_0e^2/(\epsilon_0m_e) \) \(^{1/2} \) is the ion plasma frequency. Equating \( \psi \) and \( -2i \) is the ion plasma frequency. Equating \( \psi \) and \( -2i \), respectively, \( \mathbf{V}_{\text{ph}} \) and \( \mathbf{U}_i \), respectively, \( \mathbf{V}_0 \), that usually takes place at E-region altitudes below 115 km, we have much simpler relations:

\[
\Omega_e \approx -\frac{\Psi(k \cdot V_0)}{1 + \Psi}, \quad \Omega_i \approx \frac{k \cdot V_0}{1 + \Psi}.
\]  

(A18)

They become the conventional FBI expressions if the parameter \( \Psi \) defined in Equation A16 is replaced by the single-group parameter \( \psi = \nu_e \), each proportional to \( n_0e^2/(\epsilon_0m_e) \) \(^{1/2} \) is the ion plasma frequency. Equating \( \psi \) and \( -2i \)

Calculating the FBI growth rate requires more cumbersome algebra. Skipping most of it, we obtain

\[
\gamma = \frac{\Psi}{(1 + \psi^2 \Psi)} \left( 1 - \frac{1 + \kappa_i^2}{\frac{\nu_i}{\kappa_i^2}} \right) \Omega_i^2
\]

(A19)

where \( \Omega_i \) is given by Equation A17 and \( V_{\text{ph}}^2 = T_i/m_i \). Then Equation A14 yields

We will not analyze here expression Equation A19 because the simple three-fluid model cannot provide accurate quantitative description of the FBI in the presence of the electron precipitation. The main point of this analysis was to demonstrate the unexpected complexity of the parameter dependence. The contribution of the partial parameters \( \psi_{TB} \) and \( \psi_{SE} \) into the total parameter \( \Psi \), as described by Equation A16, is natural and well-expected generalization. What was not expected though is the explicit involvement of these partial parameters into the weighting factors for \( T_{TB} \) and \( T_{SE} \) within the FBI diffusion loss term \( \kappa_i^2 V_{\text{ph}}^2 \). If there were no additional weighting factors \( \psi_{TB} \) and \( \psi_{SE} \) then the three-fluid model expression for the growth/damping rate \( \gamma \) would correspond to the naively suggested replacement of the electron temperature with the effective temperature \( T_{TB} \) defined by Equation 6c. However, the additional multipliers \( \psi_{TB}^2 \) and \( \psi_{SE}^2 \), each proportional to \( v_{TB}^2 \) and \( v_{SE}^2 \), respectively, makes the contribution of each electron group into the FBI loss term much less obvious with potentially significant quantitative consequences.

Since the \( e-n \) collision frequency has a gradual dependence on the individual electron velocity, this dependence cannot be accurately reduced to just two different constant values \( \nu_{TB} \) and \( \nu_{SE} \), as we have done in this simple analysis. The only proper and accurate way to quantitatively treat electron collisions for the two-component electron distribution with non-Maxwellian high-energy tail is by employing the rigorous kinetic theory. This is done in section 4.1.

**Appendix B: Analytic Approximation of the Energy-Dependent \( e-N_2 \) Collision Cross-Section**

In this appendix, we approximate analytically the energy-dependent cross-section of electron collisions with \( N_2 \) molecules. For \( \sigma_n(E) \), we use the most up-to-date numerical model compiled by Itikawa (2006), which is in full agreement with the corresponding data published earlier by Solomon et al. (1988, Figure A1a). We approximate the function \( \Sigma \), see Section 4.2, as
\[
\Sigma(\mathcal{E}) = \begin{cases} 
\frac{a_0 + a_1 \mathcal{E} + a_2 \mathcal{E}^2 + a_3 \mathcal{E}^3 + a_4 \mathcal{E}^4 + a_5 \mathcal{E}^5}{b_0 + b_1 \mathcal{E} + b_2 \mathcal{E}^2 + b_3 \mathcal{E}^3 + b_4 \mathcal{E}^4 + b_5 \mathcal{E}^5} & \text{if } \mathcal{E} \leq 7.484, \\
\frac{A_0 + A_1 \mathcal{E} + A_2 \mathcal{E}^2 + 373 \mathcal{E}^3}{B_0 + B_1 \mathcal{E} + B_2 \mathcal{E}^2 + \mathcal{E}^{3.75}} & \text{if } \mathcal{E} > 7.484.
\end{cases}
\]  

(B1)

Here the electron energy \( \mathcal{E} \) is expressed in eV and the numeric parameters \( a_k, b_k, A_k, \) and \( B_k (k, p = 1, 2, 3, \ldots) \) are given by

\[
\begin{align*}
 a_0 &= 1.0, & a_1 &= 29.343 718 19, & a_2 &= -48.032 970 58, \\
 b_0 &= 0.570 827 0397, & b_1 &= 1.773 322 602, & b_2 &= -3.690 608 199, \\
 A_0 &= 40.281 607 78, & A_1 &= -151.764 .479 1, & A_2 &= 13.134 .794 16, \\
 a_1 &= 28.820 983 19, & a_4 &= -7.611 134 714, & a_5 &= 0.755 097 368 6, \\
 b_1 &= 2.357 341 373, & b_4 &= -0.641 715 008 1, & b_5 &= 0.064 .793 543 438, \\
 B_0 &= 4189 .550 .095, & B_1 &= -117 .601 .051 2, & B_2 &= 3517 .072 043 .
\end{align*}
\]  

(B2)

Figure 2 shows that Equation B1 agrees with the published tabulated data almost perfectly. At the interface energy between the two pieces, \( \mathcal{E}_i = 7.484 \) eV, the function \( \Sigma(\mathcal{E}_i) \approx 11.41 \) is continuous but not smooth; the corresponding derivatives on both sides of \( \mathcal{E}_i \) differ by an order of magnitude, \( d\Sigma/d\mathcal{E}|_{\mathcal{E}_i-\Delta} \approx 0.02 \) and \( d\Sigma/d\mathcal{E}|_{\mathcal{E}_i+\Delta} \approx 0.24 \), where \( \Delta \) is an infinitesimal positive number. According to Equation 43, the derivatives of \( \Sigma(\mathcal{E}) \) are involved in the integrations, but both values of \( d\Sigma/d\mathcal{E} \) around \( \mathcal{E} = \mathcal{E}_i \) are so small that the inaccuracy caused by the fitting discontinuity is inconsequential. It is important that the analytical fit described by Equations B1 and B2 describes adequately all major details of \( \sigma_{\text{tot}}(\mathcal{E}) \), including the well-known \( N_2 \) vibrational excitation peak around 2.5 eV.

**Appendix C: Analytic Approximation of the SE Distribution Function (Maxwellian Input \( \Phi_{\mathcal{E}} = 10 \text{ erg cm}^{-2}\text{s}^{-1}, 30 \text{ keV, 110 km} \)**

In this appendix, we approximate analytically the STET-simulated distribution function for the Maxwellian input with the energy flux 10 erg cm\(^{-2}\)s\(^{-1}\), the characteristic energy 30 keV, at the 110 km of altitude. The simulated electron distribution function in the superthermal range of energies between 1 eV and 30 keV is reasonably well approximated by a piecewise expression similar in form to Equation B1.

\[
F_{SE}(\mathcal{E}) \approx \begin{cases} 
\frac{p_0 + p_1 \mathcal{E} + p_2 \mathcal{E}^2 + p_3 \mathcal{E}^3 + p_4 \mathcal{E}^4 + p_5 \mathcal{E}^5}{q_0 + q_1 \mathcal{E} + q_2 \mathcal{E}^2 + q_3 \mathcal{E}^3 + q_4 \mathcal{E}^4 + q_5 \mathcal{E}^5} & \text{if } \mathcal{E} \leq 10, \\
\frac{M_0 + M_1 \mathcal{E} + M_2 \mathcal{E}^2 + M_3 \mathcal{E}^3 + M_4 \mathcal{E}^4}{N_0 + N_1 \mathcal{E} + N_2 \mathcal{E}^2 + N_3 \mathcal{E}^3 + N_4 \mathcal{E}^4} & \text{if } \mathcal{E} > 10.
\end{cases}
\]

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This analytic fitting is shown in Figure C1. This fitting matches the actual simulated data reasonably well, except the low-energy range of 1–2 eV and, to some degree, above $A_\parallel = 3$ keV. The former range plays no role, while the latter may introduce some error, but not very significant.

Data Availability Statement
This theoretical research used no data.

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Figure C1. Analytic fit of superthermal electron energy distribution function (the green curve); the parameters are shown in the figure.

\[
\begin{align*}
M_0 &= 488\,045\,957.3, & M_1 &= 8068\,167.375, & M_2 &= 73\,402.0464, \\
M_3 &= 24.854\,365\,16, & M_4 &= 0.025, & N_0 &= -1059\,681.101, \\
N_1 &= 207\,060.510\,8, & N_2 &= -13\,361.654\,83, & N_3 &= 359.556\,316\,9.
\end{align*}
\]
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