Hydrogen like classification for light nonstrange mesons

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Abstract

The recent experimental results on the spectrum of highly excited light nonstrange mesons are known to reveal a high degree of degeneracy among different groups of states. We revise some suggestions about the nature of the phenomenon and put the relevant ideas into the final shape. The full group of approximate mass degeneracies is argued to be \( SU(2)_f \times I \times O(4) \), where \( I \) is the degeneracy of isosinglets and isotriplets and \( O(4) \) is the degeneracy group of the relativistic hydrogen atom. We discuss the dynamical origin and consequences of considered symmetry with a special emphasis on distinctions of this symmetry from the so-called chiral symmetry restoration scenario.

1 Introduction

The discovery of approximate symmetries in the hadron spectrum played an important role in establishing the structure of hadrons and of underlying strong interactions. The observation of many new resonances in recent years raised a renewed interest in the spectral degeneracies. Broadly speaking, the problem can be framed as follows: If a set of hadrons reveals a clear-cut clustering near certain values of mass, what symmetry is responsible for the observed pattern of approximate mass degeneracy and what are the physical reasons for this symmetry? Needless to say, the correct answer to this question can help considerably in unveiling the underlying universal physics to the first approximation, the next step would be the understanding of the sign and magnitude of fine splittings inside degenerate multiplets, but those phenomena are usually more involved and strongly channel-dependent (e.g., the masses of resonances can be moved seriously by the threshold effects).

A remarkable recent example of such a clustering is provided by the spectrum of unflavored mesons, see [1] for a review. The effect is certainly seen for the well confirmed states from the Particle Data Group (PDG) [2]. A clear-cut cluster structure of the spectrum of light nonstrange mesons was

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convincingly confirmed by the Crystal Barrel experiment on $\bar{p}p$ annihilation in flight [3,4] which ran at the antiproton storage ring LEAR at CERN. In brief, all observed unflavored mesons above the chiral symmetry breaking (CSB) scale in QCD, approximately 1 GeV, cluster into fairly narrow mass ranges with the “centers of gravity” situated near 1340, 1700, 2000, and 2260 MeV. The corresponding spectrum is populated mainly by the radial and orbital excitations of some ground states whose masses lie below the CSB scale. The close values of masses in these “towers” of resonances imply that the states inside each cluster are related by some $\mathcal{X}$-symmetry of unknown nature [1].

The purpose of this paper is to develop a possible candidate for the $\mathcal{X}$-symmetry. We will argue that the full symmetry governing the approximate mass degeneracies in the light nonstrange mesons seems to be $\mathcal{X} = SU(2)_f \times I \times O(4)$, where $I$ means the degeneracy of isosinglets and isotriplets emerging due to the Zweig rule and $O(4)$ is the hydrogen like degeneracy of energy levels.

An immediate question which likely rises here is how the suggested $O(4)$ symmetry is related to QCD, i.e. how it can be understood from the first principles? We do not know a convincing answer to this question. We remind, however, that numerous phenomena and symmetries emerging in the solid state physics originate from Quantum Electrodynamics, on the other hand they are not seen on the level of QED Lagrangian and hardly ever can be derived from the underlying fundamental theory. Even in such a simple system as the classical hydrogen atom, the $SO(4)$ symmetry of energy levels appears, which hardly can be envisaged starting from the QED Lagrangian because it is a dynamical symmetry reflecting the internal structure of the system, it seems to have nothing to do with the approximate symmetries of the QED Lagrangian. Needless to say that QCD is much more complex theory and in the hadron world we can encounter manifestations of similar dynamical symmetries, thus it is not excluded that searching for the complete explanation of observed spectral symmetries in hadrons having at hand the QCD Lagrangian only, one is staying on a false way.

To clarify the point let us consider a simple example. The spin $J$ and mass $m$ are known to be two independent Casimir invariants of the Poincaré group. Hence, if there is a functional dependence between $J$ and $m$, some higher symmetry takes place. Consider now a classical object of size $r$ rotating at constant velocity, its angular momentum is $J \sim mr$. On the other hand, $n$-dimensional object of constant density has mass $m \sim r^n$, i.e. $r \sim m^{1/n}$. Thus,

$$J \sim m^{1+1/n}. \quad (1)$$
Quantum theory tells us that properties of any quantum system approach to its classical ones if the quantum numbers defining the stationary states of this system are large enough [5]. For this reason the highly excited hadrons are inevitably quasiclassical objects, i.e. the classical arguments can be applied for them as the first approximation. The functional dependence \( m(J) \) is an inherent feature of the Regge theory, experimentally the Regge trajectories are approximately linear on \((J, m^2)\) plane, at least for highly excited hadrons. Consequently, to the extent that the Regge trajectories are linear, the excited hadrons can be viewed quasiclassically as one-dimensional objects of constant density according to Eq. (1). Thus, one arrives at a nice agreement of very general arguments and the real-life phenomenology. What must be emphasized here is that relation (1) is dynamical, we need not any particular Lagrangian to obtain it — the role of interactions is to create the rotating system, the ensuing dynamical dependence (1) is then independent of a concrete kind of underlying interactions. Similarly, the local strong interactions described by the QCD Lagrangian create hadrons which are extended objects, hence, one may expect that some dynamical symmetries come into play.

In summary, the standpoint of the present paper is that the observed spectral degeneracies have somewhat dynamical origin, hence, in order to advance in understanding the spectral degeneracies one should think in terms of internal structure of hadrons rather than analyze dynamics and general properties of QCD. The spirit of our work has something in common with that of Ref. [6] where the spectrum-generating algebra approach was used to deduce the \( SO(4) \) dynamical symmetry from the string-like properties of mesons.

The paper is organized as follows. In Sect. 2 we provide some general arguments justifying the approach we will use. Sect. 3 is devoted to construction of \( O(4) \) classification for mesons. In Sect. 4 the proposed scheme is discussed and compared with some other approaches. We conclude in Sect. 5.

## 2 Preliminary remarks

The isospin invariance \( SU(2)_I \) does not need comments, it is the generally known vector part of the spontaneously broken chiral \( SU(2)_L \times SU(2)_R \) symmetry of the QCD Lagrangian in the limit of vanishing current quark masses. The symmetry \( I \) appears as a consequence of the suppression of transitions between quarks of different flavors, the so-called Zweig (or OZI) rule. The Zweig rule is well understood in the \( 1/N_c \) expansion since it becomes exact at \( N_c = \infty \), i.e., in the planar limit of QCD [7,8]. Usually, the large-\( N_c \) limit works fairly well in the phenomenology, there are sizeable violations of the
OZI-rule only for a relatively small number of states, typically in the scalar sector, reflecting a specific nature of those states which results in a considerable mixture of strange and nonstrange components. It should be noted that the $I$-symmetry is of dynamical origin as it is not present in the QCD Lagrangian. The symmetry $O(4)$ appears to be also dynamical, this novel symmetry will be the subject of our discussions in what follows.

At present there are different ideas (not yet proved rigorously) on the excited light mesons which happened to be quite successful in a global description of the spectroscopic data. In fact, the assumption of $O(4)$-symmetry is likely the only self-consistent way for unification of those ideas. First of all, various arguments and observations indicate that the spin-orbital and spin-spin correlations are strongly suppressed in the excited unflavored hadrons [9–15]. This suggests that, neglecting a possible fine splitting due to such correlations and other non-leading effects, the pattern of mass degeneracies of mesons built from the conventional spinor quarks is the same as that of mesons made of scalar quarks. Since the light mesons are ultrarelativistic systems the use of the potential models is difficult to justify, one should rather solve the Bethe-Salpeter equation for two scalar particles interacting through massless bosons. The corresponding solutions reveal the $SO(4)$-degeneracy, this result goes back to Wick and Cutkosky [16, 17]. The group $SO(4)$ is known to be the dynamical degeneracy group of the nonrelativistic hydrogen (H) atom [18, 19].

The H-like $SO(4)$ degeneracy implies the dependence of discrete spectrum on a single ”principal” quantum number $n$,

$$n = l + n_r + 1,$$

where $l$ is the angular momentum and $n_r$ labels the ”radial” excitations. On the other hand, it has been observed recently [1, 14] that the dependence of the meson mass $M$ on $l$ and $n_r$ indeed enters in the combination $l + n_r$, namely, to a rather high accuracy, the whole spectrum of excited unflavored meson resonances can be fitted by the linear relation [1, 20]

$$M^2 = a(l + n_r) + b,$$

with $a \approx 1.1 \text{ GeV}^2$ and $b \approx 0.7 \text{ GeV}^2$. It is interesting to note that the linear dependence of $M^2$ on $l + n_r$ holds in certain quasiclassical strings [21] (see also [22] for the discussions based on the QCD sum rules) and, by construction, in some AdS/QCD models [23, 24], while it cannot be obtained within the existing potential models [25], namely the semirelativistic potential models with linearly rising potential yield typically $M^2 \sim l + cn_r$ with $c \neq 1$. Thus, although we use the nonrelativistic basis, our framework will
not be completely equivalent to old potential models. Introducing the quark
spin in the additive way as in the usual quantum mechanics, one obtains the
physical mesons with the spin \( J = l, l \pm 1 \), which possess the masses dictated
by Eq. (3). The outlined dynamical mechanism seems to be responsible for
the emergence of an approximate degeneracy among resonances of different
spin value. The assumption of suppression of the spin-orbital and spin-spin
correlations inside excited mesons is crucial in this kind of reasoning, other-
wise the angular momentum of \( \bar{q}q \) pair and the intrinsic quark spin cannot be
separated in the relativistic systems under consideration, hence, the formulas
like Eq. (3) may not be written.

All these arguments are quite standard, nevertheless they do not save us
from a certain uneasiness caused by the fact that we are trying to describe
the ultrarelativistic systems by means of the unobservable nonrelativistic
terms. It would be desirable to understand deeper why the nonrelativistic
basis may be useful. For instance, consider a strong decay \( A \to B + C \), where
\( A, B, \) and \( C \) are some mesons. Experimentally one is able to determine the
relative angular momentum \( L \) of the hadron pair \( B \) and \( C \). Intuitively, it
is easy to imagine the following picture: Quark and antiquark inside the
hadron \( A \) have the relative momentum \( l \), then the strong gluon field inside \( A \)
creates from the vacuum a quark-antiquark pair, the whole system rearranges
into two colorless hadrons \( B \) and \( C \) which, in turn, conserve the relative
angular momentum, \( L = l \), if \( l_B = l_C = 0 \), say if \( B \) and \( C \) are pions. In
reality, however, we should confess honestly that we do not know and cannot
imagine the internal structure of meson \( A \). But it is natural to conjecture that
the observable \( L \) reflects somehow this structure. A relevant example is the
observation of excited light mesons with identical quantum numbers and very
close masses, which are related to two different values of \( L \). It is reasonable
to assume that their internal structure is different, an additional argument is
that these two kinds of almost degenerate mesons always have different full
width — this is natural as long as two different quantum systems generically
have different lifetimes. Thus, introducing \( l \) and identifying \( l = L \) (plus
fixing the orientation of intrinsic quark spin) we may expect that thereby
we do an unambiguous mapping of observable \( L \) onto the internal structure
of observed meson, moreover, to a certain extent we may expect that this
mapping is universal for all mesons, this permits then to establish some
relations between mesons, such as relations between masses. It should be
added also that the angular momentum \( l \) and the total quark-antiquark spin
\( s \) can be well defined through the observable P- and C-parities [14] (see their
definitions (4) below). Thus, classifications in terms of unobservable \( l \) can
definitely make sense, in this regard it should be reminded that the standard
\( SU(3)_f \) classifications of hadrons are also based on unobservables, which are
the quarks. The existence of noticeable mass splittings inside \( l \)-multiplets should not be regarded as some kind of drawback of the proposed scheme since they could encode an important physics (say, the spin interactions) like the mass splittings inside the \( SU(3)_f \)-multiplets.

Another argument in favour of our approach is that even essentially relativistic models for light hadron spectrum can possess the property that the states in their spectrum are classified as in nonrelativistic potential models. An example of such models is given in [26] where the mesons are described by a hadron string with massless spinor quarks at its ends. In addition, it is easy to see that in the case of breaking of classical string, a part of its angular momentum is converted into the relative angular momentum of “splinters” and if these ”splinters” are spinless (as it usually happens in real life) this conversion is complete due to the momentum conservation, i.e. one has \( l = L \) just as expected. As long as string models for hadrons are known to be well motivated by QCD, our discussions above are also well motivated.

Our approach is very different from the so-called chiral symmetry restoration (CSR) scenario, which is claimed to be completely relativistic and QCD-based explanation of many observed spectral degeneracies [15]. First of all, the CSR explains degeneracies among states of equal spin, e.g., the parity doubling, while the observed degeneracy is much broader [4]. A detailed comparison of our scheme with the CSR one is presented in Sect. 4, here we would give the following general remark. The CSR scenario treats the observed degeneracies as a completely quantum effect, i.e., it does not have a well understood classical limit, this point has been already criticized in [14] from the point of view of linearity of Regge trajectories. The hadrons are bound states of quarks, therefore they are described by some theory of bound states and it is quite difficult to imagine that such a theory does not have the quasiclassical limit. The quantum effects are decisive in phenomena like boundary effects (e.g., the Casimir effect) or quantum tunneling, but in bound states, they commonly result in fine splittings of energy levels which are the next-to-leading effects. Our standpoint is that the theory of bound quarks does have the classical limit, we try to guess the dynamical symmetry in this limit and use it as a starting point for further analysis.

In what follows, we proceed to explanation of observed degeneracies in light nonstrange mesons on the base of nonrelativistic basis, finally it will turn out that the scheme can be reformulated in terms of observable hadron spin. The detailed phenomenological analysis based on Eq. (3) was carried out in [1, 20] and we will not repeat it here, of our concern will be the group-theoretical aspects and their physical sense.
3 Construction of $O(4)$ classification

The light nonstrange mesons are characterized by the quantum numbers $I^G(J^{PC})$, with the $P$, $C$, $G$ parities defined as

$$P = (-1)^{l+1}, \quad C = (-1)^{l+s}, \quad G = (-1)^{l+s+l},$$

where $s$ is the total quark-antiquark spin. The $G$-parity is not of interest for us since it is just a combination of the $C$-parity and isospin. Changing the angular momentum $l$ by one unit we change immediately the $P$ and $C$ parities. Define the pure and mixed $P$ and $C$ transformations as

$$P : |\Delta l| = 1, \quad l + s = \text{const},$$

$$C : |\Delta s| = 1, \quad |\Delta l| = 0,$$

$$PC : |\Delta l| = 1, \quad |\Delta s| = 0.$$  \hspace{1cm} (5)\hspace{1cm} (6)\hspace{1cm} (7)

The change of $l$ can be compensated by that of $n_r$ such that the sum $l + n_r$ remains constant, the meson mass then is not affected due to Eq. (3). The $C$-transformation preserves the meson mass by virtue of the assumed quark spin orientation independence of the hadron masses. Thus, there is a possibility to relate, in some sense, the $P$ and $C$ invariances of the QCD Lagrangian to the same invariances of the resonance spectrum.

Supplementing the $P$ and $C$ transformations defined in Eqs. (5)-(7) by the $I$-transformation discussed above (the mass-conserving transitions from isosinglet channels to the isotriplet ones and *vice versa*) we obtain the complete set of transformations relating different states within a degenerate cluster. For instance, consider the first cluster of unflavored mesons. It is populated by the well-established states from the PDG [2], the fine splitting does not exceed 10% of meson mass except for the $h_1(1170)$-meson, the fine splitting is known to reduce progressively in the higher clusters [4]. We can “walk” along the whole tower of states, e.g., in the following way,

\[ a_2(1320) \xrightarrow{I} f_2(1270) \xrightarrow{CI} b_1(1235) \xrightarrow{I} h_1(1170) \xrightarrow{C} f_1(1285) \xrightarrow{I} a_1(1260) \xrightarrow{PC} \]

\[ \rho(1450) \xrightarrow{I} \omega(1420) \xrightarrow{CI} \pi(1300) \xrightarrow{P} a_0(1450) \xrightarrow{I} f_0(1370) \xrightarrow{P} \eta(1295). \]  \hspace{1cm} (8)

Similarly, one is able to go over the resonances in the higher towers, those clusters contain more mesons including some missing states.

The multiplets predicted by Eq. (3) are drawn in Fig. 1. The states lying on the diagonal line have $n_r = 0$, they form the leading Regge trajectory, with the spin being $J = l$ or $J = l + 1$ in the real situations. It should be emphasized that these resonances do not possess $P$-parity doublets — the
Figure 1: A graphical representation of Eq. (8) with physical values of parameters in GeV^2. The principal quantum number n is defined in Eq. (2). The dots denote the corresponding states (only several low-lying levels are shown). The numbers in brackets display the predicted mean mass in MeV.

states of equal spin and close mass but with the opposite P-parity — as the $P$-transformation (5) for such mesons cannot conserve the spin and mass simultaneously.

Besides spin, a complete extension of Fig. 1 to the real mesons must include the isospin and doubling of both $P$ and $C$ parities. The isospin can be incorporated by a reflection with respect to the axis $M^2$, the values of mass remain intact due to the $I$-invariance. There are two possible ways of $P$-parity doubling, they correspond to $P$ and $PC$ transformations. The former case is depicted in Fig. 2 the states on the leading Regge trajectories have $s = 0$, hence, $J = l$. The latter possibility is displayed in Fig. 3 the resonances belonging to the leading trajectories have then $s = 1, J = l + 1$. As remarked above, the resonances on the leading trajectories are $P$-parity singlets, all other states are $P$-parity doubled. The last step is to superimpose Fig. 3 on Fig. 2 identifying the $M^2$ axes and dashed lines and turn one of figures through angle 90°, in this way we incorporate also the $C$-parity. The horizontal lines of degenerate states in Fig. 2 and Fig. 3 will form then planes, the clusters of degenerate states live on these equidistant and parallel planes. For example, the states in cluster (8) populate the lowest such plane. The resulting three-dimensional picture of meson degeneracies is easy to imagine, although the corresponding figure appears to be beyond author’s artistic abilities.

The states below 1.9 MeV in Fig. 2 and Fig. 3 are taken from the PDG [2], the nonstrange nature of those resonances is usually indicated by their decay channels. Above 1.9 MeV the states are mainly from a review [3], the PDG lists them in section "Other States". A complementary test for the non-strangeness of included states is that they belong to the relevant families of
Figure 2: A hydrogen like classification for the states with $J = l$ and for their $P$-parity doublets. The dashed line denotes symbolically the CSB scale, the given classification is not expected to be reliable below this scale.

Figure 3: The same as in Fig. 2 but for the states having $J = l + 1$ and for their PC-parity doublets.
Regge trajectories [3, 27–29]. The numbers in brackets serve for orientation only as long as often they refer to the traditional names of particles given by the PDG rather than to the actual mass. For instance, the mass of $\rho_5(2350)$ is $2330 \pm 35$ MeV according to the PDG [2], in the $\bar{p}p$ annihilation it is seen with the mass $2300 \pm 45$ MeV [3], the mass of $\omega_5(2250)$ was estimated in the $\bar{p}p$ annihilation as $2250 \pm 70$ MeV [3], thus, it is not excluded that $\rho_5(2350)$ and $\omega_5(2250)$ are exactly degenerate despite so different numbers in brackets, which would mean the exact $I$-symmetry for them. Another example is the $\pi_2(2100)$-meson of the PDG, its mass looks considerably bigger than the averaged value 2000 MeV in the corresponding cluster, however, such an observation may turn out to be misleading since, say, in the $\bar{p}p$ annihilation this resonance was seen in the region $2005 \pm 15$ MeV [3]. The same can be said about the $\eta_2(1870)$-meson, which was observed in the $\bar{p}p$ annihilation at $2030 \pm 16$ MeV [3]. In all other cases any judgements about the fine splittings within the clusters should be also made with caution.

Notably, within the presented classification of light nonstrange mesons there is no place for the states $f_0(600)$, $f_0(980)$, and $a_0(980)$, the nature of which is highly controversial.

4 Discussions

It is interesting to notice that although we have used the nonrelativistic arguments in building our classification, the final scheme turns out to be relativistic as long as formally the spectrum depends on the spin $J$ and the number $n$ enumerating the daughter trajectories, in principle, now one can detach from the nonrelativistic interpretations at all, regarding Fig. 2 and Fig. 3 as classifications for the states generated by the leading Regge trajectories of unnatural, $P = (-1)^{J+1}$, and natural, $P = (-1)^J$, P-parity, respectively. In addition, the proposed classification coincides with the classification of energy levels in the relativistic H-atom, see [30] for references. The latter scheme was used for description of the light nonstrange baryons in 1960s (see, e.g., [31, 32]; numerous references are collected in a review [30]). In essence, the H-like description of the light nonstrange mesons contains only one substantial complication in comparison with the baryons — the resulting picture of mass degeneracies is three-dimensional due to the existence of C-parity. The relativistic $O(4)$ description of the H-atom emerged in 1960s from a remarkable group-theoretical discovery: The full relativistic theory of the H-atom (without account for electron spin) can be formulated as a dynamical group theory based on $O(4, 2)$, the conformal group. The unitary irreducible representations of $O(4, 2)$ are labelled then by $|nJm_\pm\rangle$, where $m$ is
the usual magnetic quantum number, \( n \) is the relativistic principal quantum number, and \( \pm \) refers to the P-parity, which is determined from the parity of the ground state. While the \( O(4) \) symmetry relates only states within a degenerate energy level, the \( O(4,2) \) symmetry relates also different energy levels, in our case the latter relation is given by Eq. (3). One of reduction of \( O(4,2) \) to \( O(4) \) corresponds to the relativistic H-atom, where all states for a given \( n \) are P-parity doublets, except the state \( J = n - 1 \) which is a singlet. The P-parity doubling distinguishes the relativistic \( O(4) \) H-like assignment of energy levels from the nonrelativistic \( SO(4) \) one, the group \( O(4) \) is just the extension of \( SO(4) \) by P-parity. The absence of P-parity partners for the states lying on the principal Regge trajectories is a remarkable feature of the H-based scheme since such partners have never been observed in the mesons.

As was mentioned in Sect. 2, the most known recent explanation of spectral degeneracies among the highly excited states is based on the effective axial and chiral symmetry restoration at high energies, the relevant ideas are summarized in a review [15] (see also [30, 33]). In this regard, it would be instructive to compare in detail our scheme with the CSR one. Resorting to some semiclassical arguments, the latter idea suggests that the highly excited hadrons fall into the multiplets of approximate chiral \( SU(2)_L \times SU(2)_R \) symmetry of QCD extended by P-parity, the resulting parity-chiral group is isomorphic to \( O(4) \), we will call it \( O(4)_{pc} \) in what follows. First of all, the possible physical origins of the H-like \( O(4)_H \) symmetry and that of \( O(4)_{pc} \) are completely different, the former invariance is a dynamical symmetry reflecting the internal space structure of mesons and the centrosymmetric character of interactions between the constituents, while the latter one is a classical symmetry of the QCD Lagrangian. The \( O(4)_H \) symmetry can relate states with different spin, while the \( O(4)_{pc} \) symmetry relates states of equal spin only.

Consider as an example the \( \rho_J \)-mesons. The degenerate states of equal spin value can be obtained with the help of certain combinations of the \( P, C, \) and \( I \) transformations in the way depicted in Fig. 4. The presented diagram provides also all other spin-preserving transformations, they can be trivially performed through the "center" \( \rho_J \) taking into account that double transformation of any kind is unity. For instance, the line \( a_J \xrightarrow{PC} \rho_J \xrightarrow{P} b_J \) gives the C-parity doubling for \( a_J \) and \( b_J \).

The same chain of degeneracies as in Fig. 4 follows from a classification of mesons according to multiplets of \( O(4)_{pc} \) and of axial \( U(1)_A \) [15]. However, there exists a crucial difference between the two classifications: The CSR scenario predicts P-parity doublets for all highly excited states, while the H-like scenario predicts that the states lying on the principal Regge trajectories
Figure 4: A diagram for the spin-preserving transformations with the center \( \rho_J \). The symbols \( J_o \) and \( J_e \) mean that the given transformation can be performed for the odd or, respectively, even values of spin \( J \) only.

are P-parity singlets, all other mesons are P-parity doubled. As mentioned above, experimentally P-parity doublers for the states belonging to the principal meson Regge trajectories have not been observed, the Crystal Barrel experiment confirmed this phenomenological fact [1, 4, 20]. The absence of such P-parity partners is a strong advantage of the H-based scheme over the CSR scenario: Usually the states on the daughter trajectories are less reliable than the resonances on the principal trajectories, hence, the CSR scheme fails completely in the most reliable part of the meson spectrum.

Thus, our analysis shows that the hypothetic effective restoration of chiral and axial symmetries of the classical QCD Lagrangian in the upper part of the hadron spectrum does not necessary constitute a piece of the broader degeneracy \( \mathcal{X} \) existing in that part of the spectrum (in contrast to the point of view taken in [15]). If the symmetry \( \mathcal{X} \) is of the type advocated in the present paper, the predictions of the CSR scheme are included into \( \mathcal{X} \) partly only. It is quite important to emphasize that the ”not overlapped” with \( \mathcal{X} \) part of the CSR predictions lies completely in the unobserved part of the meson spectrum.

It is interesting to mention that a pattern of P-parity singlets similar to that of \( O(4)_H \) assignment emerged naturally in the geometrical string like and bag like models proposed in [34].

Let us try to figure out qualitatively a possible physical origin of the \( O(4)_H \) symmetry in the meson spectrum. On the intuitive level, it is clear that both in the H-atom and in the mesons one deals with quantum two-body systems interacting via centrosymmetrical forces, the appearance of an universal dynamical symmetry is then quite conceivable. In QCD, the CSB disturbs drastically the low-energy part of the spectrum, for this reason a
manifestation of this universal dynamical symmetry should be naturally expected above the CSB scale. The observation of the same symmetry among the excited light baryons may indicate on their quardiquark structure, in fact, historically the \( O(4)_H \) symmetry was first proposed for baryons on the base of analyses of a rather rich baryon spectrum, which was available already at 1960s, see [30–32] for references. On the other hand, it is not excluded that the \( O(4)_H \) symmetry might be given an interpretation as a ”survived” part of a broken fundamental classical symmetry. Indeed, in the very high energy limit, the QED and QCD Lagrangians possess the conformal invariance \( O(4,2) \) (more generally, the high-energy density of states of any \( d \)-dimensional renormalizable field theory is that of \( O(d,2) \) conformal theory), which is incompatible with the existence of bound states as long as the spectrum of conformal theories is massless or continuum. From the group-theoretical point of view, the \( O(4,2) \) is also incompatible with the existence of a finite number of degenerate states at some energy since the unitary irreducible representations (UIR) of \( O(4,2) \) are infinite-dimensional. However, the group \( O(4,2) \) contains subgroups with finite-dimensional UIR, which already are able to accommodate the discrete spectrum and certain degeneracies in their multiplets. The maximal such subgroup is exactly \( O(4) \). This intriguing relation of \( O(4) \) and \( O(4,2) \) might give a chance to relate the observed spectral degeneracy to the fundamental theory.

5 Conclusions

We have proposed a classification scheme for light nonstrange mesons which explains completely the observed approximate mass degeneracies, only a few of states are missing and we hope they are to be discovered in future experiments. By and large, the accuracy of the mass degeneracies in the proposed multiplets is similar to that of the unitary \( SU(3)_f \) symmetry. The classification looks most naturally in terms of unobservable angular momentum of quark-antiquark pair, but it can be reformulated also in terms of observable hadron spin.

The main message of the present work is that the observed spectrum of light nonstrange mesons is similar to nothing but the discrete spectrum of the classical hydrogen atom. Such ideas appeared about forty years ago in the baryon spectroscopy, so the present analysis may be regarded as a revival of those forgotten ideas in application to mesons.

If this result is correct, a natural question arises as to why the non-relativistic symmetries can work in the excited light hadrons, which represent ultrarelativistic systems? This question reminds the old question why
the nonrelativistic model of constituent quarks works in the domain where naively it should not work? The understanding of the latter problem took a long way, now we know that clue lies somewhere in the fact that at low energies the effective physical degrees of freedom are not those of the QCD Lagrangian, but the exact implementation of this mechanism is still a riddle. It may be that with the nonrelativistic symmetries in the highly excited light hadrons we are also staying at the beginning of a long way...

In conclusion, we have tried to demonstrate that a mere observation of hadron clusters may open the door to a new line of research, where far-reaching results could be obtained. At present, there exists only one experiment which systematically looked for the excited unflavored mesons in a broad energy range, the Crystal Barrel one [3], and its results we have actively used in this work. It would be really nice if experimentalists taught us more about the particle content of the hadron clusters, elevating thereby the clustering from the present somewhat speculative level to a rather unexpected new direction in the particle physics. A tentative program of relevant physical experiments is proposed in [35].

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