Models of Supersymmetric
\( U(2) \times U(1) \) Flavor Symmetry

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We use a \( U(2) \times U(1) \) horizontal symmetry in order to construct supersymmetric models where the flavor structure of both quarks and leptons is induced naturally. The supersymmetric flavor changing neutral currents problem is solved by the degeneracy between sfermions induced by the \( U(2) \) symmetry. The additional \( U(1) \) enables the generation of mass ratios that cannot be generated by \( U(2) \) alone. The resulting phenomenology differs from that of models with either abelian or \( U(2) \times GUT \) symmetries. Our models give rise to interesting neutrino spectra, which can incorporate the Super-Kamiokande results regarding atmospheric neutrinos.
1 Introduction

Approximate horizontal symmetries, $H$, can naturally explain the observed flavor structure of fermions. With abelian symmetries [1]-[5] all mass ratios and mixing angles are explained in a straightforward way. On the other hand, with a $U(2)$ symmetry [6] it is quite difficult to explain the large $m_t/m_b$ ratio and the different hierarchies in the down and up sectors. In order to overcome these problems, the $U(2)$ symmetry is often combined with a Grand Unified Theory (GUT) and a very specific choice of flavon representations [7]-[12].

Within Supersymmetry (SUSY), the horizontal symmetries should also suppress new contributions to Flavor Changing Neutral Currents (FCNC). In models with a $U(2)$ symmetry and generations in $2+1$ representations (as in other models of non-abelian horizontal symmetry [13]-[17]), the SUSY FCNC problem is automatically solved by degeneracy between the first two sfermion generations. On the other hand, with Abelian symmetries, the simple alignment mechanism [18] does not give strong enough suppression of the FCNC. In order to solve this problem, one is usually led to rather specific $H$-charge assignments that yield a very precise alignment.

We construct models with a $U(2) \times U(1)$ symmetry that combine the advantages of the two frameworks. The $U(2)$ symmetry solves the SUSY FCNC problem without alignment and the $U(1)$ symmetry accounts for the various mass ratios without invoking a specific GUT structure. The resulting phenomenology is different from that of either framework.

We assume that at some high-energy scale the symmetry $H \equiv U(2) \times U(1) = SU(2) \times U(1)_1 \times U(1)_2$ is realized. The symmetry is broken in the following hierarchical way:

$$U(2) \times U(1) \rightarrow U(1)' \rightarrow 0.$$  

(1)

This is done by giving Vacuum Expectation Values (VEVs) to flavon fields. We quantify all the breaking parameters as powers of a small parameter $\lambda$ which we take to be of $O(0.2)$:

$$\epsilon \sim \lambda, \quad \epsilon' \sim \lambda^2.$$  

(2)

The three generations are in $2+1$ representations. In difference with previous models of $U(2)$, we allow the different SM representations to carry different charges under $U(1)_1 \times U(1)_2$. On one hand, the model is not compatible with an underlying GUT, and loses some of the predictive power found in the GUT scenario. On the other hand, with a small number of flavon fields, we are able to reproduce the mass matrices of the quarks and the leptons and the CKM matrix elements, without invoking any additional different mechanisms or symmetries in order to solve specific problems. Within our framework the large ratio $m_t/m_b$ can be explained without imposing large $\tan \beta$. Also, the $\mu$ term can naturally be of order of the SUSY breaking scale, and does not need to be put by hand.

Our model reproduces the following high-energy scale mass ratios:

$$(m_u; m_c; m_t) \rightarrow (\lambda^7; \lambda^4; 1),$$  

(3)

$$(m_d; m_s; m_b) \rightarrow (\lambda^7; \lambda^5; \lambda^3),$$  

(4)

$$(m_e; m_\mu; m_\tau) \rightarrow (\lambda^8; \lambda^5; \lambda^3).$$  

(5)
The CKM elements are of the experimentally measured order of magnitude, while the Kobayashi-Maskawa phase, $\delta_{KM}$, can receive any experimentally allowed value, and is not restricted to be of $O(1)$. The charges of the matter fields are chosen in such a way that the $U(1)'$ symmetry acts only on the first generation.

Due to the symmetry between the first two generations, a degeneracy between the corresponding sfermions is produced. The degeneracy can be made to be very strong, but here we choose it to be of $O(\epsilon^2)$. This degeneracy is strong enough to solve the SUSY FCNC problem, while mild enough to still allow approximate CP - a description of all CP violating phenomena with small CP violating phases [19, 20]. Since we do not use the alignment mechanism, we do not necessarily reproduce some of its generic features. In particular, $\Delta m_D$ is not close to the experimental bound.

While in our model there is room for approximate CP as a solution to the SUSY CP problem, we do not treat the CP violating phases explicitly. Within this framework there is also the possibility of relaxing the SUSY CP problem by increasing the degeneracy between the first two sfermion generations.

Our models allow various interesting structures of the neutrino sector. In light of the recent results announced by Super-Kamiokande [21] which, in the three generations framework, imply

$$\Delta m_{23}^2 \sim 5 \times 10^{-3} \, eV^2, \quad \sin^2 2\theta_{23} \sim 1,$$

we present two models for the structure of the lepton sector:

- lepton-model I: hierarchy \( m_{\nu_e} \ll m_{\nu_\mu} < m_{\nu_\tau} \),
- lepton-model II: quasi-degeneracy \( m_{\nu_e} \ll |m_{\nu_\mu}| \simeq |m_{\nu_\tau}| \).

The structure of the paper is as follows. In section 2 we present the quark sector of the model. Here we introduce all the flavon fields used in both the quark and the lepton sectors. We study the implications of this model to FCNC processes. In section 3 we present two extensions of the model that describe the lepton sector. Our conclusions are summarized in section 4.

## 2 The quark sector

The superfields of the quark and Higgs sectors of the Supersymmetric Standard Model (SSM) carry $H$-charges, shown in table 1. There, $Q_i$ are the quark doublets, $\bar{d}_i$ and $\bar{u}_i$ are the down and up quark singlets, and $\phi_u$ and $\phi_d$ are the Higgs doublet fields. The electroweak symmetry is spontaneously broken by the VEVs of $\phi_d$ and $\phi_u$, and we assume that

$$\tan \beta \equiv \frac{\langle \phi_u \rangle}{\langle \phi_d \rangle} \sim \frac{1}{\lambda}.$$  \hspace{1cm} (7)

In addition we have standard model singlet superfields: two $U(2)$ doublets and two $U(2)$ singlets. Their $H$-charges are shown in table 3. The horizontal symmetry is broken when
Table 1: $H$ charges of the Higgs and Quark superfields.

| Field     | $SU(2)$ | $(U(1)_1, U(1)_2)$ |
|-----------|---------|-------------------|
| $\phi_u$ | 1       | (0,0)             |
| $\phi_d$ | 1       | (0,-2)            |
| $(Q_1)$   | 2       | (1,4)             |
| $(Q_2)$   | 1       | (0,0)             |
| $(Q_3)$   | 1       | (0,0)             |
| $(\bar{u}_1)$ | 2   | (3,0)             |
| $(\bar{u}_2)$ | 1   | (0,0)             |
| $(\bar{u}_3)$ | 1   | (0,0)             |
| $(\bar{d}_1)$ | 2   | (3,0)             |
| $(\bar{d}_2)$ | 1   | (0,0)             |
| $(\bar{d}_3)$ | 1   | (0,6)             |

Table 2: $H$ charges of the SM-singlet superfields.

| Field | $SU(2)$ | $(U(1)_1, U(1)_2)$ |
|-------|---------|-------------------|
| $\phi_1 = \begin{pmatrix} \phi_{11} \\ \phi_{12} \end{pmatrix}$ | 2 | (-1,-1) |
| $\phi_2 = \begin{pmatrix} \phi_{21} \\ \phi_{22} \end{pmatrix}$ | 2 | (-1,-2) |
| $\chi_1$ | 1 | (0,-2) |
| $\chi_2$ | 1 | (-2,0) |

some of the SM-singlet fields assume VEVs. We take for the VEVs:

$$\frac{1}{M} \begin{pmatrix} \langle \phi_{11} \rangle \\ \langle \phi_{12} \rangle \end{pmatrix} = \begin{pmatrix} \epsilon' \\ \epsilon \end{pmatrix} \sim \begin{pmatrix} \lambda^2 \\ \lambda \end{pmatrix}$$ (8)

$$\frac{1}{M} \begin{pmatrix} \langle \phi_{21} \rangle \\ \langle \phi_{22} \rangle \end{pmatrix} \sim \begin{pmatrix} 0 \\ \epsilon \end{pmatrix} \sim \begin{pmatrix} 0 \\ \lambda \end{pmatrix}$$ (9)

$$\frac{1}{M} \langle \chi_1 \rangle \sim \epsilon \sim \lambda$$ (10)

$$\frac{1}{M} \langle \chi_2 \rangle \sim \epsilon' \sim \lambda^2$$ (11)

where $M$ is a scale in which the information about this breaking is communicated to the SSM. The symmetries of the model allow the choice $\langle \phi_{21} \rangle = 0$ and $\langle \phi_{22} \rangle$, $\langle \chi_1 \rangle$, $\langle \chi_2 \rangle$ real. The additional fields will, in general, receive complex VEVs.

The VEVs and charges allow us to estimate the quark mass matrices $M_f$ and the squark mass-squared matrices $M_{f^2}$. All the terms allowed by SUSY and $U(2) \times U(1)$ are assumed
to appear with coefficients of $O(1)$. When a parameter appears explicitly, it is assumed to be the $O(1)$ coefficient of the corresponding term. We write the effective matrices derived after rotations needed to bring the kinetic terms into their canonical form \[2, 3, 4\]. We get:

$$M^d \sim \langle \phi_d \rangle \begin{pmatrix} \epsilon^3 & \epsilon^2 \epsilon & \epsilon \epsilon^5 \\ \epsilon^2 \epsilon & \epsilon^2 & \epsilon^4 \\ \epsilon^2 \epsilon^3 & \epsilon^4 \epsilon^2 \\ \epsilon^2 \epsilon^3 & \epsilon^4 \epsilon^2 & \epsilon^2 \end{pmatrix},$$  \tag{12}

$$M^u \sim \langle \phi_u \rangle \begin{pmatrix} \epsilon^3 & \epsilon^2 \epsilon^2 & \epsilon \epsilon^3 \\ \epsilon^2 \epsilon^2 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 \epsilon^2 & \epsilon \epsilon^2 & 1 \\ \epsilon^2 \epsilon^2 & \epsilon \epsilon^2 & 1 \end{pmatrix},$$  \tag{13}

$$\tilde{M}^q_{LL} \sim \tilde{m}^2 \begin{pmatrix} a + \epsilon^2 & \epsilon \epsilon & \epsilon \epsilon^4 \\ \epsilon \epsilon & a + \epsilon^2 & \epsilon^2 \\ \epsilon \epsilon^4 & \epsilon^2 & 1 \end{pmatrix},$$  \tag{14}

$$\tilde{M}^q_{RR} \sim \tilde{m}^2 \begin{pmatrix} a + \epsilon^2 & \epsilon \epsilon & \epsilon \epsilon^2 \\ \epsilon \epsilon & a + \epsilon^2 & \epsilon^2 \\ \epsilon \epsilon^4 & \epsilon^2 & 1 \end{pmatrix},$$  \tag{15}

$$\tilde{M}^q_{LR} \sim \tilde{m} M^q.$$  \tag{17}

Note that the ratio $m_t/m_b$ is explained by the horizontal symmetries, as is the difference in hierarchies between the up and the down sectors. This is in contrast to models where the structure of the mass matrices is dictated by $U(2)$ alone.

We can also estimate the size of the bilinear $\mu$ and $B$ terms:

$$\mu \sim \tilde{m} \epsilon,$$  \tag{18}

$$m^2_{12} \sim \tilde{m}^2 \epsilon.$$  \tag{19}

Thus the horizontal symmetry solves the $\mu$-problem in the way suggested in \[22\].

From the mass matrices we can estimate the mixing angles in the CKM matrix. We find:

$$|V_{us}| \sim \lambda, \quad |V_{ub}| \sim \lambda^4, \quad |V_{cb}| \sim \lambda^2, \quad |V_{td}| \sim \lambda^3.$$  \tag{20}

In order to compare quark-squark-gaugino mixing with the experimental bounds presented in \[23\] we use the formula given in \[18\]:

$$\delta_{MN}^f \sim (V_M^f \tilde{M}_{MN}^f V_N^f) / \tilde{m}^2$$  \tag{21}
where \( \{M, N\} = \{L, R\} \), and \( V^f_M \) are the diagonalizing matrices of \( M^f \). The dimensionless \( \delta^q_{MN} \) matrices have the simple meaning of squark mass-squared matrices (normalized to the average squark mass-squared \( \tilde{m}^2 \)) in the basis where gluino couplings are diagonal and quark mass matrices are diagonal. The comparison is summarized in table 3. There, the phenomenological bounds scale like \( (\tilde{m}/1\ TeV)^2 \), and the CP violating phases are assumed to be of \( O(1) \). We learn the following points from table 3:

| Process | Bound | Model |
|---------|-------|-------|
| \( \text{Re}(\delta_{12}^d)_{LL}^2 \) | \( \Delta m_K \) | \( \lambda^3 \) |
| \( \text{Re}(\delta_{12}^d)_{LL}(\delta_{12}^d)_{RR} \) | \( \Delta m_K \) | \( \lambda^6 - \lambda^7 \) |
| \( \text{Re}(\delta_{12}^d)_{RR}^2 \) | \( \Delta m_K \) | \( \lambda^3 \) |
| \( \text{Re}(\delta_{13}^d)_{LL}^2 \) | \( \Delta m_B \) | \( \lambda^2 \) |
| \( \text{Re}(\delta_{13}^d)_{RR}^2 \) | \( \Delta m_B \) | \( \lambda^6 \) |
| \( \text{Re}(\delta_{12}^d)_{LL}^2 \) | \( \Delta m_D \) | \( \lambda^2 \) |
| \( \text{Re}(\delta_{12}^u)_{LL}^2 \) | \( \Delta m_D \) | \( \lambda^6 \) |
| \( \text{Im}(\delta_{12}^d)_{LL}^2 \) | \( \epsilon_K \) | \( \lambda^6 \) |
| \( \text{Im}(\delta_{12}^d)_{RR}^2 \) | \( \epsilon_K \) | \( \lambda^9 - \lambda^{10} \) |

Table 3: Squark mass parameters: model predictions vs. phenomenological bounds.

- \( \Delta m_K \) receives SUSY contributions comparable to the SM ones.
- Contrary to \( U(2) \times GUT \) symmetry models [10], SUSY contributions to \( \Delta m_B \) are negligible compared to the SM ones.
- Contrary to abelian horizontal symmetry models [2, 18], \( \Delta m_D \) is not expected to be at the experimental limit, but rather \( 1-2 \) orders of magnitude smaller.
- In order not to exceed the measured value of \( \epsilon_K \), the CP violating phase contributing to \( \text{Im}(\delta_{12}^d)_{LL}(\delta_{12}^d)_{RR} \) should be small.

3 The lepton sector

Various anomalies in neutrino experiments provide further input to flavor models [24]-[33]. In the following we show how either an hierarchical spectrum or quasi-degenerate neutrinos consistent with the recent measurements of atmospheric neutrinos [24] are produced naturally in an extension of our model to the lepton sector.
3.1 Model I: Hierarchy

The $H$ charges of the lepton superfields in our model I are given in table 4. There, $L_i$ are the lepton doublets, $\bar{l}_i$ and $\bar{s}_i$ are the charged lepton and neutrino singlets. We get:

| Field | $SU(2)$ | $(U(1)_1, U(1)_2)$ |
|-------|---------|---------------------|
| ($L_1$) | 2       | (1,0) |
| $L_2$ | 2       | (0,0) |
| $L_3$ | 1       | (0,6) |
| ($\bar{l}_1$) | 2       | (3,6) |
| ($\bar{l}_2$) | 1       | (1,2) |
| $\bar{s}_1$ | 1       | (0,3) |
| $\bar{s}_2$ | 2       | (0) |
| $\bar{s}_3$ | 1       | (0) |

Table 4: Model I: $H$ charges of the lepton superfields.

$$M^l \sim \langle \phi_d \rangle \left( \begin{array}{ccc} \epsilon^3 \epsilon & \epsilon^2 \epsilon^2 & \epsilon \epsilon^3 \\ \epsilon^2 \epsilon^2 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 \epsilon^3 & \epsilon^2 & \epsilon^2 \end{array} \right).$$ (22)

The matrices $M^\nu_{RR}$ and $M^\nu_{LR}$ are given in their naive form before the rotations to the canonical form are made:

$$M^\nu_{RR} \sim M_L \left( \begin{array}{ccc} a \epsilon^2 \epsilon & a \epsilon' \epsilon^2 & c \epsilon \epsilon^3 \\ a \epsilon' \epsilon^2 & b \epsilon^2 \epsilon^3 & c \epsilon^3 \\ c \epsilon^3 & \epsilon^3 & \epsilon^3 \end{array} \right),$$ (23)

$$M^\nu_{LR} \sim \langle \phi_u \rangle \left( \begin{array}{ccc} a \epsilon^2 \epsilon & (a+b) \epsilon' \epsilon & c \epsilon \epsilon \\ (a-b) \epsilon' \epsilon & a \epsilon^2 & c \epsilon^2 \\ 0 & d \epsilon & 0 \end{array} \right).$$ (24)

Using the see-saw mechanism, and arranging in the canonical form:

$$M^\nu_{LL} \sim M^\nu_{LR} M^\nu_{RR}^{-1} M^\nu_{LR} M_L \left( \begin{array}{ccc} \epsilon^2 \epsilon^{-1} & \epsilon' & \epsilon' \\ \epsilon' & \epsilon & \epsilon \\ \epsilon' & \epsilon & 1 \end{array} \right).$$ (25)

The hierarchy of the neutrino masses in this model is:

$$m_{\nu_e} \ll m_{\nu_\mu} < m_{\nu_\tau}.$$ (26)
There is no degeneracy between any of the neutrinos. Using as input the new data from Super-Kamiokande \[21\], we find:

\[
\begin{align*}
  m_{\nu_e} &\sim \lambda^3 m_{\nu_r} \sim 5 \times 10^{-4} \text{ eV}, \\
  m_{\nu_\mu} &\sim \lambda m_{\nu_r} \sim 0.01 \text{ eV}, \\
  m_{\nu_\tau} &\sim 0.07 \text{ eV}, \\
  \sin \theta_{12} &\sim \lambda, \\
  \sin \theta_{13} &\sim \lambda, \\
  \sin \theta_{23} &\sim 1.
\end{align*}
\] (27)

We also get

\[
M_L \sim 5 \times 10^{14} \text{ GeV}.
\] (29)

The neutrinos do not contribute significantly to the dark matter. The mass of \(\nu_\mu\) together with the mixing angle \(\sin \theta_{12}\) might point at the large angle matter enhanced solution to the solar neutrino problem (although the mass is a bit too large) \[31, 34\].

For the slepton mass-squared matrices, we get

\[
\tilde{M}_{LL}^2 \sim \tilde{m}^2 \begin{pmatrix}
  a + \epsilon^2 & \epsilon \epsilon' & \epsilon' \epsilon^2 \\
  \epsilon' \epsilon & a + \epsilon^2 & \epsilon^2 \\
  \epsilon^2 & \epsilon' & 1
\end{pmatrix},
\] (30)

\[
\tilde{M}_{RR}^2 \sim \tilde{m}^2 \begin{pmatrix}
  a + \epsilon^2 & \epsilon \epsilon' & \epsilon^2 \epsilon' \\
  \epsilon' \epsilon & a + \epsilon^2 & \epsilon' \epsilon^2 \\
  \epsilon^2 \epsilon' & \epsilon' \epsilon^2 & 1
\end{pmatrix},
\] (31)

\[
\tilde{M}_{RR}^2 \sim \tilde{m}^2 \begin{pmatrix}
  a + \epsilon^2 & \epsilon \epsilon' & \epsilon' \\
  \epsilon' \epsilon & a + \epsilon^2 & \epsilon^2 \\
  \epsilon' \epsilon & \epsilon^2 & 1
\end{pmatrix},
\] (32)

\[
\tilde{M}_{LR}^2 \sim \tilde{m} \tilde{M}^I.
\] (33)

The comparison between lepton-slepton-gaugino parameters, defined analogously to the definitions in the quark sector (eq. [21]), and the experimental bounds presented in \[23\], is summarized in table 6. There, the phenomenological bounds for the process \(\mu \rightarrow e\gamma\) scale like \((\tilde{m}/1 \text{ TeV})^2\), while the bound for the Electric Dipole Moment (EDM) of the electron scales like \((\tilde{m}/1 \text{ TeV})\). The \(CP\) violating phases are assumed to be of \(O(1)\). The bounds appear only for processes for which the bound is \(\leq 1\). The following points should be noted in table 6:

- The decay \(\mu \rightarrow e\gamma\), if the slepton masses are close to \(m_Z\), is expected to be close to the experimental limit.

- The EDM can be close to the experimental limit, if the \(CP\) violating phases are large.

### 3.2 Model II: Quasi-degeneracy

The \(H\) charges of the lepton superfields in our model II are given in table 5. We get:

\[
\tilde{M}^I \sim \langle \phi_d \rangle \begin{pmatrix}
  \epsilon^2 \epsilon' & \epsilon' \epsilon & \epsilon' \epsilon^2 \\
  \epsilon' \epsilon^2 & \epsilon' \epsilon & \epsilon^2 \epsilon' \\
  \epsilon^2 \epsilon' & \epsilon' \epsilon & \epsilon'
\end{pmatrix}.
\] (34)
Table 5: Model II: $H$ charges of the lepton superfields.

The matrices $M_{RR}^\nu$ and $M_{LR}^\nu$ have the following structure:

$$M_{RR}^\nu \sim M_L \left( \begin{array}{ccc} a e^4 & a e^3 \epsilon & 0 \\ a e^3 \epsilon & a e^2 \epsilon^2 & b \epsilon \\ 0 & b \epsilon & 0 \end{array} \right),$$  \hspace{1cm} (35)

$$M_{LR}^\nu \sim \langle \phi_u \rangle \left( \begin{array}{ccc} a e^4 & (a + b) e^3 \epsilon & 0 \\ (a - b) e^3 \epsilon & a e^2 \epsilon^2 & c \epsilon \\ 0 & d \epsilon & 0 \end{array} \right).$$  \hspace{1cm} (36)

Using the see-saw mechanism, and arranging in the canonical form, we get:

$$M_{LL}^\nu \sim \frac{\langle \phi_u \rangle^2}{M_L} \left( \begin{array}{ccc} e^4 \epsilon & e^3 \epsilon & e' e^2 \\ e^3 \epsilon & e^2 \epsilon^2 & \epsilon \\ e' e^2 & \epsilon & e^2 \epsilon^3 \end{array} \right).$$  \hspace{1cm} (37)

The hierarchy of the neutrino masses in this model is:

$$m_{\nu_e} \ll |m_{\nu_\mu}| \simeq |m_{\nu_\tau}|.$$

The degeneracy between $m_{\nu_\mu}$ and $m_{\nu_\tau}$ is $O(\lambda^5)$. Analyzing this using the new data from Super-Kamiokande, we find:

$$m_{\nu_e} \sim \lambda^7 m_{\nu_\tau}, \quad m_{\nu_\mu} \simeq m_{\nu_\tau} \sim 3 \text{ eV},$$

$$\sin \theta_{12} \sim \lambda^2, \quad \sin \theta_{13} \lesssim \lambda^3, \quad \sin \theta_{23} \simeq \frac{1}{\sqrt{2}},$$

and

$$M_L \sim 2 \times 10^{12} \text{ GeV}.$$  \hspace{1cm} (41)

Here the neutrinos play an important role in structure formation and contribute a significant part to the hot dark matter. The spectrum, however, does not seem to be compatible with any of the suggested solutions to the solar neutrino problem [31, 34].
The sfermion mass-matrices have the following structure:

\[ \tilde{M}_{\ell\ell}^2 \sim \tilde{m}^2 \begin{pmatrix}
    a + \epsilon^2 & \epsilon' \epsilon & \epsilon^3 \epsilon^2 \\
    \epsilon' \epsilon & a + \epsilon^2 & \epsilon^3 \epsilon^2 \\
    \epsilon^3 \epsilon^2 & \epsilon^2 \epsilon^2 & 1
\end{pmatrix}, \]  

(42)

\[ \tilde{M}_{\ell\ell}^2 \sim \tilde{m}^2 \begin{pmatrix}
    a + \epsilon^2 & \epsilon' \epsilon & \epsilon' \epsilon^4 \\
    \epsilon' \epsilon & a + \epsilon^2 & \epsilon^2 \epsilon^2 \\
    \epsilon^3 \epsilon^2 & \epsilon^2 \epsilon^2 & 1
\end{pmatrix}, \]  

(43)

\[ \tilde{M}_{\ell\ell}^2 \sim \tilde{m}^2 \begin{pmatrix}
    a + \epsilon^2 & \epsilon' \epsilon & \epsilon^3 \epsilon^2 \\
    \epsilon' \epsilon & a + \epsilon^2 & \epsilon^3 \epsilon^2 \\
    \epsilon^3 \epsilon^2 & \epsilon^2 \epsilon^2 & 1
\end{pmatrix}. \]  

(44)

The comparison between lepton-slepton-gaugino parameters and the experimental bounds presented in [23], is summarized in table 6. We point out that

| Process | Bound | Model I | Model II |
|---------|-------|---------|----------|
| \(|(\delta_{12})_{LL}|\) | \(\mu \to e\gamma\) | \(\lambda^2\) | \(\lambda^3\) |
| \(|(\delta_{12})_{RR}|\) | \(\mu \to e\gamma\) | \(\lambda^2\) | \(\lambda^3\) |
| \(|(\delta_{12})_{LR}|\) | \(\mu \to e\gamma\) | \(\lambda^2\) | \(\lambda^3\) |
| \(|Im(\delta_{11})_{LR}|\) | EDM | \(\lambda^8\) | \(\lambda^7\) |

Table 6: Slepton mass parameters: model predictions vs. phenomenological bounds.

- If the CP violating phases are large, the EDM can be close to the experimental limit.

4 Conclusions

Approximate flavor symmetries naturally explain the smallness and hierarchy of the flavor parameters in SUSY models, while suppressing sources for FCNC. Abelian horizontal symmetries explain the mass ratios in a straightforward way, but need to invoke an alignment mechanism through specific \(H\) charge assignments in order to suppress FCNC. Horizontal \(U(2)\) symmetries suppress FCNC with a built-in degeneracy between the first two sfermion generations, but need the framework of \(GUT\) in order to explain various mass ratios.

In this work, we presented a hybrid model of abelian and non-abelian symmetries. The model combines the characteristics of both symmetries in such a way as to produce all the required flavor parameters and suppressions naturally, with no additional ingredient. The \(U(2)\) symmetry allows for a hierarchical breaking \(U(2) \times U(1) \xrightarrow{\epsilon} U(1)\)' \(\epsilon \to 0\) and gives the solution to the SUSY FCNC problem. The additional \(U(1)\) enables generation of various mass ratios and mixing parameters in a simple way. It also allows for a natural solution of the SUSY \(\mu\) problem.
The phenomenology of the hybrid model is different than that of either abelian or non-abelian symmetry models. Unlike in usual non-abelian models, here different SM representations carry different charges so the model is not compatible with GUT. On the other hand, $\Delta m_D$ is not close to the experimental limit, as it is in models with alignment. This framework leaves room for different possible solutions to the SUSY CP problem, including approximate CP.

Different viable neutrino spectra can arise within this framework. We gave two examples both of which are compatible with the recent observations of atmospheric neutrinos by Super-Kamiokande. The first produces a hierarchy of neutrino masses, while the second produces quasi-degenerate neutrinos, that might play a significant role in cosmology.

The model presented here is not unique. It intends to demonstrate how within the hybrid framework a simple model with very few flavon fields can be built, that at the same time agrees with all measured flavor parameters and suggests attractive spectra for the neutrino masses.

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