A Mellin Space Program for $W^\pm$ and $Z^0$ Production at NNLO

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We present a program for the evaluation of full unpolarized cross sections for the $W^\pm$ and $Z^0$ production in the narrow width approximation at NNLO in perturbative QCD using Mellin space techniques.

1 Introduction

The Drell-Yan process, originally described in the context of the parton model [1], concerns the production of a lepton pair of large invariant mass in hadron-hadron collisions. With the increase of the centre of mass energy at particle accelerators, the Drell-Yan process led to the discovery of $W^\pm$ and $Z^0$ bosons at UA1 and UA2 experiments [2,3]. Since then the properties of massive vector bosons have been studied in great detail. At present the production of $W^\pm$ and $Z^0$ provides an important benchmark for the LHC and a test of the Standard Model (SM) in a new range of centre of mass energies [4].

As guaranteed by the factorisation theorem [5], one can separate the physics of soft energy scales from the physics at hard energy scales where perturbation theory applies. The higher order QCD corrections to the Drell-Yan process have been calculated up to next-to-next-to-leading order (NNLO), see [6–8] and references therein. The full cross section is obtained as a convolution with the parton distribution functions (PDFs) that encode the non-perturbative information.

In this paper, we present a program for evaluation of the full inclusive cross section for $W^\pm$ and $Z^0$ production in a fast and accurate way using a Mellin space approach. After a brief description of the basic ingredients of the calculation we give formulae for the Mellin transforms. We then present a comparison with the code ZWPROD [7,8] and discuss possible applications and extensions within this framework.

2 Formalism

We consider the inclusive production of a single vector boson $V = W^+, W^-$ or $Z^0$ in hadron-hadron collision with a centre of mass energy $s$ which subsequently decays into a lepton pair of an invariant mass $Q^2$. The decay of the vector boson is treated within the narrow width approximation which replaces the propagator by a delta function such that $Q^2 = M_V^2$. We consider massless quarks. The cross section for this process can be expressed as

$$\sigma^{h_1h_2\to V\to l_1l_2}(s) = x\sigma^{V\to l_1l_2}\sigma^V(x, Q^2), \quad x = Q^2/s,$$

where $\sigma^{V\to l_1l_2}$ represents the kinematically independent part of the Born level subprocess $q\bar{q} \to V \to l_1l_2$ (the point-like cross section) multiplied by the appropriate branching ratio. The exact form of the point like cross section can be found in Ref. [7], formulae (A.10) and (A.11). [9]
The perturbative coefficients are known up to NNLO \cite{7,8}, reads

\[ W^V(x, Q^2) = \sum_{a,b=q,g} C_{a,b}^V \left[ f_a(\mu_f^2) \otimes f_b(\mu_f^2) \otimes \Delta_{ab}(Q^2, \mu_f^2, \mu_r^2) \right](x). \] (2)

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\[ \Delta_{ab}^{\text{NNLO}}(x, Q^2, \mu_f^2, \mu_r^2) = \sum_{n=0}^{k} \frac{\alpha_s^n(\mu_f^2)}{4\pi} \Delta_{ab}^{(k)}(x, Q^2, \mu_f^2, \mu_r^2). \] (3)

The factor \( C_{a,b}^V \) in Eq. (3) contains information about couplings of vector bosons to partons \( a \) and \( b \). For the detailed form of the Eq. (3) we refer the reader to the paper of Hamberg, Matsuura and van Neerven \cite{7} whose notation we follow closely. The convolution sign represents an integral

\[ (f_1 \otimes f_2 \otimes \cdots \otimes f_k)(x) = \int_0^1 dx_1 \int_0^1 dx_2 \cdots \int_0^1 dx_k \delta(x - x_1 x_2 \cdots x_k) f_1(x_1) f_2(x_2) \cdots f(x_k). \] (4)

In principle one can perform the integrals in Eq. (3) directly however, the problem is much better addressed after transforming to Mellin space,

\[ f(N) = \int_0^1 dx x^{-N-1} f(x). \] (5)

This transformation turns the integrals in Eq. (3) into ordinary products such that the structure function reads

\[ W^V(N, Q^2) = \sum_{a,b=q,g} C_{a,b}^V f_a(N, Q^2) f_b(N, Q^2) \Delta_{ab}(N, Q^2) \quad \mu_f = \mu_r = Q^2, \] (6)

and therefore it is possible to evaluate it in a fast and efficient way. The formula for the inverse Mellin transform defines how to recover the original momentum space result,

\[ W^V(x, Q^2) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dN x^{-N} W^V(N, Q^2), \] (7)

where \( c \) represents a point on the real axis such that all poles \( N_i \) in the function \( W(N, Q^2) \) lie to the left from \( c \). Further on, we will refer to functions in Mellin space as \( N \) space functions and functions from momentum space as \( x \) space functions.

### 3 Implementation

The main ingredients of the calculation are the coefficient functions up to NNLO and the parton distribution functions in Mellin space in terms of a complex variable \( N \). The condition \( N \in \mathbb{C} \) is required for the numerical evaluation of the inversion formula (7). For this we adopted the technique implemented in QCD-PEGASUS \cite{10}. The complex integral (7) is rewritten in terms of an integral over a real variable \( z \)

\[ W(x, Q^2) = \frac{1}{\pi} \int_0^\infty dz \text{Im}[e^{i\phi} x^{-c-z} e^{i\phi} W(N, Q^2)] \quad N = c + z \exp^{i\phi} \in \mathbb{C} \] (8)

and evaluated using Gaussian quadratures. The parameter \( \phi > \pi/2 \) represents the angle with respect to the positive real axis. Since the rightmost pole of the structure function is \( N_{\text{max}} = 1 \), we chose \( c = 1.5 \). These

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2 Several typos appearing in Ref. \cite{7} have been pointed out in \cite{9}
values as well as the maximum value of the integration variable $z$ are flexible and can be modified by the user in the main program if desired. For a more detailed description of the shape of the integration contour we refer to the QCD-PEGASUS manual [10].

The coefficient functions in $N$ space were published previously in Ref. [11], including corrections to the previous literature. The corresponding FORTRAN code is DY.f used together with ANCONT [12]. We performed the Mellin transforms starting from the $x$ space expressions [13] using the harmpol package [14]. The results can be expressed mostly in terms of complex-valued simple harmonic sums [15] and several more complicated ones which we approximated by using the minimax method[3] worked out in detail in [12] previously. The absolute accuracy of our approximation is better than $10^{-9}$ over the whole kinematic range.

At the moment there are two options for the input parton distribution functions in $N$ space. A toy input corresponds to the one used for the 2001/2 benchmark tables [18] and is used for comparisons with ZWPROD [7, 8] assuming no evolution of PDFs. The general form reads

$$ x f_i,\text{toy}(x, \mu_0^2) = nx^a(1-x)^b, \quad i = q, \bar{q}, g, \quad n, a, b \in \mathbb{R}, \quad (9) $$

which is in Mellin space represented by an Euler beta function

$$ f_i,\text{toy}(N, \mu_0^2) = n \beta(a + N, b + 1). \quad (10) $$

The second option for the PDF input is using the FORTRAN code QCD-PEGASUS [10] which can be linked to our program.

![Comparison with ZWPROD](image)

**Fig. 1** Cross section for $W^-$ production up to NNLO in the narrow width approximation using the toy parton distribution functions and a fixed value of the strong coupling constant. Upper part: The full cross section. Lower part: Relative accuracy with respect to the ZWPROD.

### 4 Results And Outlook

There are several programs on the market using the standard momentum space evaluation [19–21] which can provide a cross-check for our $N$ space calculation. We performed comparisons of the full cross sections with a program ZWPROD written by the authors of the original calculation of the NNLO Drell-Yan

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3 We used the MINIMAX routine implemented in Maple

4 Exact expressions were given in [17].
coefficient functions [7,8]. The Fig. 1 shows a comparison for the $W^-$ cross section using toy input for PDFs corresponding to the Eq. (10) with no evolution and a fixed value of the coupling constant $\alpha_s = 0.1$. The relative accuracy is better than $6 \times 10^{-6}$ in the relevant kinematical range $x \in (10^{-4}, 0.8)$. As an intermediate check, we compared the Mellin inversion of $N$ space coefficient functions against the $x$ space expressions using a program of Gehrmann and Remiddi [22] for the numerical evaluation of harmonic polylogarithms. The framework presented here is suitable for a further implementation of those cross sections where $N$ space coefficient functions are also available, like Higgs production and deep inelastic scattering (DIS) [11, 23–26]. The setup is well suited for merging the program with threshold resummation calculations which are typically performed in Mellin space (see e.g. [27]). For the extraction of PDFs from $W^\pm$ and $Z^0$ production it would be desirable to have an access to the rapidity distributions in which case one will need to apply double Mellin transforms of two variables $N_1$ and $N_2$ however, this is a subject to further study. On the side of PDFs we aim for a direct interface to the LHAPDF grids [28]. Recent results [29] on $N$ space input parametrizations also allow for more flexible input PDF parametrisations in QCD-PEGASUS. Further improvements with respect to the speed of the code are foreseen and together with an upgrade on the input PDFs this code can become a tool for PDF fits, where fast and accurate evaluations of cross sections are needed. The current version of the c++ code can be downloaded from http://www-zeuthen.desy.de/~kpetra/sbp.

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