Towards “Weak” Information Theory:
Weak-Joint Typicality Decoding Using Support Vector
Machines May Lead to Improved Error Exponents*

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Abstract

In this paper, the authors report a way to use concepts from statistical learning to gain an advantage in terms of error exponents while communicating over a discrete memoryless channel. The study utilizes the simulation capability of the scientific computing package MATLAB to show that the proposed decoding method performs better than the traditional method of joint typicality decoding. The advantage is secured by modifying the traditional specification of what constitutes a decoding error. This is justified by the paradigm, also used in the program of ‘utilizing’ noisy feedback, that one ought not to declare a condition as an error if some further processing can extract useful information from it.

Part I
Theoretical Foundations

Noisy feedback has been found useful in recent information theory literature. In the same spirit, we investigate the potential use of discarded “erroneous” conditions in this paper. In this part we provide some theoretical underpinnings of our computer investigations. The investigations relate to the concept of error exponents. These functions capture the performance of a communication system in a subtle and important way. Along with channel capacity, the error exponent is an important metric of communication system performance.

*This project was commenced in early 2020. It has been partly presented in initial form in the first author’s Anabhayin series of self-published books.

1See, for example, the papers of Prof. Anant Sahai (UC Berkeley) and Prof. Sekhar Tatikonda (Yale).
1 Brief Outline

Consider Theorem 7.7.1 of Cover and Thomas, page 200 [1]. In the proof of this theorem, the authors lay out a few points. The sixth point details how joint typicality decoding is to be used in the proof. Motivated by ‘weak values’ in quantum measurement theory and ‘strong learning’ and ‘weak learning’ in the book on machine learning by Kearns and Vazirani [2], we propose to ‘weaken’ this decoding step. Specifically, in weak-joint typicality decoding, the receiver declares that the index $\hat{W}$ was sent if the following conditions are satisfied:

1. $(X^n(\hat{W}), Y^n)$ are jointly typical
2. There may or may not be other indices $W' \sim \hat{W}$ such that $(X^n(W'), Y^n)$ belongs to the jointly typical set

This is a sort of fuzzy decoding set and gives us the idea that neural networks could ‘learn’ in this setting. If no such $\hat{W}$ exists, satisfying condition 1. above, then an error is declared and the receiver outputs a dummy index such as zero in this case.

Suppose we decode to $\hat{W}$ since $(X^n(\hat{W}), Y^n)$ is a jointly typical pair. We also find that $\hat{W}_k, k = 1, 2, ..., L$ are such that $(X^n(\hat{W}_k), Y^n)$ are jointly typical. So we have $L + 1$ decodings, potentially. We can look for clusters in this data via unsupervised learning, and possibly decode to a point chosen at random from the largest cluster [4]. Following this set up, we can finally evaluate the probability of error so obtained.

In the next section we will provide the details of the transmission scheme which will be computerized in the next part.

2 Details of the Transmission Scheme

Consider a communication system, the transmission scheme over which consists of $M$ messages, belonging to the set $\{W_1, W_2, W_3, ..., W_M\}$. The encoder maps these $M$ messages into input sequences for the channel. These input sequences, or codewords, belong to the set $\{X^n(W_i)\}_{i=1}^M$. Each member of this latter set is of length $n$ and there are $M$ members. One of these codewords is transmitted per cycle. Due to channel noise, it is received at the receiver in the form $Y^n$.

Now suppose that the transmitted sequence is unknown, and we only have access to $Y^n$. The question is how do we decode the message hidden in $Y^n$. In particular, suppose that under joint typicality decoding, two distinct transmitted sequences $X^n(W_1)$ and $X^n(W_6)$ are jointly typical with $Y^n$. This raises a quandry for the decoder. Suppose there is a ‘difference function’ on sequence space, $d(\cdot, \cdot)$ which yields a new sequence which is the difference between the two input sequences. For example, it might be based on binary Hamming distance. Let $Z^n_i = d(X^n(W_i), Y^n)$ for $i = 1, 6$. Whereas the codeword is deterministic [3], the received sequence is random due to the noise and so the formed $Z$-sequences are also random. We can then use unsupervised learning to form $k$ clusters out of these $Z$-sequences. We then search for and find the largest among the $k$ clusters. Next we find its mean and denote it by $m$. Finally, we decode to that $Z$-sequence which is closest to $m$.

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2 See k-means clustering from Haykin’s book, *Neural Networks and Learning Machines*, page 242, section 5.5 [3].
3 However, if the codebook is also chosen randomly as in the proof of Theorem 7.7.1 [1], then the codeword is no longer deterministic. Furthermore, we may draw the message also at random using a uniform distribution over the message set.
In the next part, we will perform MATLAB simulations of communication over a discrete channel and perform decoding as outlined above, as well as the more standard joint typicality decoding.

Part II
Computer Experimentation

In this part, just as laid out earlier in this paper, we relax the error-event definition in joint typicality decoding over a noisy channel and invoke Support Vector Machines (SVMs) to help us decode in the presence of several ‘matches’ between the received word and codewords from the codebook. We can also extend this to source coding and rate distortion theory in future work.

3 Study of Error Exponents

In Figure 1 we plot the formula

\[ -\frac{\ln(\text{Pe})}{\text{blocklength}} \]

for various blocklengths, for the discrete memoryless binary symmetric channel BSC(0.05). This is done for the case of joint typicality decoding. The x-axis is the block length simulated and the y-axis is the error exponent.

Figure 1: Joint typicality decoding error exponent versus block length.

\[ ^{4}\text{Thanks are due to Prof. Rajesh, CSE, DSU for suggesting the use of SVMs instead of k-means clustering due to the vector nature of the codewords.} \]
Next, in Figure 2 we plot the same formula $[1]$, but for weak-joint typicality decoding, and for the same channel. We can see that the exponent is higher for certain rates. The x-axis is the block length simulated and the y-axis is the error exponent.

Figure 2: Weak-joint typicality decoding error exponent versus block length. The y-axis peak value is nearly double that of the y-axis peak value in Figure 1.

4 Results

In this section we present a figure that summarizes the performance of the new decoding method. As Figure 3 shows, there is a definite advantage in using weak-joint typicality decoding over regular joint typicality decoding. The x-axis is the probability of a ‘1’ symbol in the codebook and the y-axis is the difference between the (maximum, simulation-obtained, value of the) two types of decoding-exponents, with a block length of up to 600 symbols considered for codeword length. The graph always remains below zero, demonstrating the greater magnitude of the (maximum) weak-joint typicality decoding exponent. The discrete channel used was the BSC(0.4).

Recall that the rate is nothing but the number of bits carried by a codeword divided by the length of the codeword.
5 Discussion

While the differences are not glaring with and without weak-joint typicality decoding, we have obtained a modification of Shannon's strategy that performs better in terms of error exponents, under certain conditions. It remains to do an analytical study. Additionally, it was suggested recently that our strategy might be related to list decoding [4]. However, a preliminary reading of [5] indicates that there might be some differences between these two types of decoding.

To summarize, in this paper we have discussed a potentially new way of using machine learning to enhance communication system performance as captured by error exponents. There are limitations to our work, both in theory and simulation. Theory can be improved by considering other machine learning algorithms and the simulations can be more exhaustive. However, this is a preliminary study of new frontiers in information theory and statistical learning. It has several practical implications for those implementing data communication systems in the twenty-first century.

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