The rubric as an assessment strategy in the mathematical argumentation process

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Abstract. The article shares the proposal of an analytical rubric as a strategy for the assessment and monitoring of learning outcomes in students who develop an argumentative plot from the mathematics field, to solve any problem situation in daily life. The study was based on the theory of mathematical argumentation proposed by Duval and the contributions of León and Calderón, as well as the dimensions presented to us by the logical frameworks in the design of analytical rubrics. The research was developed under the social critical paradigm through the design of pedagogical action research, and the focus group technique was used for the collection of information composed by five professors from the department of basic sciences. As a result, a collective rubric that, in addition to generating processes of self-assessment and self-training in teachers, evidences a decrease in the existent subjectivity of the evaluation processes, thus strengthening its objectivity.

1. Introduction

Mathematical argumentation is one of the fundamental processes in the formation of quantitative reasoning, since it includes the comprehensive and representative dimensions from the use of mathematical language for the resolution of any problem situation. In this sense, [1] has defined these competencies such as interpreting-representing, formulating-executing, and arguing, which, the latter, develops when the student is able to interpret and understand a problem, can also represent and model it in order to take a position that allows them to argue or counter-argument to solve the various problems not always from the context of mathematics.

Based on the foregoing, it is necessary to use formative and evaluative strategies that allow visualizing their development, so training cannot focus only on the management of mechanical exercises, it must go beyond the procedural; it is required of a disciplinary appropriation and reflection that permits the implementation of methodologies fostering scenarios for meaningful learning in the mathematical field, an aspect that is generally not simple, since the assessment process in basic sciences such as mathematics is quite complex [2], that is why the teacher's need or great challenge to rethink discipline from the different epistemological, pedagogical and didactic aspects that strengthen the curriculum and teaching practice in the face of the teaching type processes and learning mathematics that must circulate in the academic scene.

In this way, from the field of educational mathematics, strategies that can develop mathematical argumentation have been proposed, such is the case of the problem solving proposed and applied in...
different investigations by [3-5], where it is shown that the handling of problem situations allows not only to see the application they have in everyday life, but also strengthens the development of competencies since at the time the student faces a problem must interpret himself, understand the statement to be able to model it and in this way get to a solution, visualizing in this process the structure of an argumentative plot (premise - middle term - conclusion), where it is necessary to analyze the production of arguments that affect the type of argumentation present during the solution to the proposed problem situation.

Thus, an argumentative plot [6] constructed for the justification of a problem situation of everyday life, implies that the student assumes a semantic and theoretical epistemic stance typical of the mathematical know how, which allows him to elaborate valid arguments [7], which are analyzed and accepted according to [6] from two dimensions, one of a functional type and the other of a structural type. The functional type dimension allows to analyze the acceptability of the arguments under the criteria of strength and relevance, and the structural type dimension, analyze the premises, middle term and conclusions that are generated in the argumentative plot.

That is why, in the problem-solving strategy and the development of an argumentative plot under the dimensions indicated above, which seek to promote the development of skills in the mathematics field according to current needs. It is proposed in this study to the monitoring of student learning results, an analytical rubric as an assessment strategy for this process, which, according to [8] the use of practices, helps teachers to define excellence and the instructional plan, aligns the objectives of the curriculum and the evaluation proposal, in which, the evaluation is understood not as a process segmented in time, that is, as summative nature, but as a continuous process, of a formative nature, that requires constant feedback.

2. Materials and methods

The research was based on a critical social paradigm, with a qualitative approach and a pedagogical action research design [9] assumed as an emerging design in the field of educational action research [10]. From this logic, in [9] it is proposed that the emphasis of this pedagogical I-A prototype is placed on the teacher's pedagogical practice. There are no pretensions to influence the social change of the immediate context and much less, the radical transformation of the political and social structures of the outline, that is, the generalization of self-assessment processes to investigate the teaching of science, the curriculum and, therefore, the evaluation processes.

Based on a focus group containing five professors from the basic, social and humanities department sciences of the Universidad Simón Bolívar, Colombia, and based on a research process around the field of mathematical argumentation, the present analytical rubric was proposed as a strategy for evaluation of the argumentative process. This rubric was designed according to the dimensions of the argumentative process: functional, structural, epistemological and logical [3] using an Excel matrix and the minimum elements that make up the analytical rubric: categories, dimensions, sub-categories and performance levels.

3. Results

At follows, the result obtained from the construction of the analytical rubrics for the evaluation of the learning results of a mathematical argumentation process developed by the students at the time of solving a problem situation is shown. In Table 1, Table 2 and Table 3, the components of the rubric for the evaluation of the first theoretical category of the mathematical argumentation process can be observed, which refer to the semantic epistemic value [11].

On the other hand, in Table 4, Table 5 and Table 6, the components of the rubric for the evaluation of the second theoretical category of the argumentative process can be observed, which refer to the theoretical epistemic value [12].

The rubric is organized by the categories of analysis and the elements of the argumentative process. All this, depending on the dimensions: functional, structural, logical and epistemological presented by the theoretical framework in the study of the process of mathematical argumentation.
Table 1. Analytical rubric with semantic epistemic value: Premises.

| Argumentative process elements | Semantic epistemic value (functional dimension) and operational statute (structural dimension) |
|--------------------------------|---------------------------------------------------------------------------------------------|
| Argument premises             |                                                                                            |
| Level 1: Low                  | Level 2: Medium                                                                            |
| (0.0 to 2.5)                  | (2.6 to 3.9)                                                                               |
| Level 3: High                 |                                                                                             |
| (4.0 to 5.0)                  |                                                                                             |
| The synthetic and semantic content of the premises demonstrate the lack of understanding of the statement. | The synthetic and semantic content of the premises are valid but not relevant to the mathematical context of the problem situation. | The synthetic and semantic content of the premises are valid and relevant to the mathematical context of the problem situation. |

Table 2. Analytical rubric with semantic epistemic value: Middle term.

| Argumentative process elements | Semantic epistemic value (Functional dimension) and Operational Statute (Structural dimension) |
|--------------------------------|---------------------------------------------------------------------------------------------|
| Argument premises             |                                                                                            |
| Level 1: Low                  | Level 2: Medium                                                                            |
| (0.0 to 2.5)                  | (2.6 to 3.9)                                                                               |
| Level 3: High                 |                                                                                             |
| (4.0 to 5.0)                  |                                                                                             |
| It does not use relevant processes from the mathematics field. | Uses relevant processes from the mathematics field but with some synthetic and / or semantic errors. | Uses relevant processes from the mathematics field without synthetic and / or semantic errors. |

Table 3. Analytical rubric with semantic epistemic value: Conclusions.

| Argumentative process elements | Semantic epistemic value (functional dimension) and operational statute (structural dimension) |
|--------------------------------|---------------------------------------------------------------------------------------------|
| Argument conclusions          |                                                                                            |
| Level 1: Low                  | Level 2: Medium                                                                            |
| (0.0 to 2.5)                  | (2.6 to 3.9)                                                                               |
| Level 3: High                 |                                                                                             |
| (4.0 to 5.0)                  |                                                                                             |
| It does not make a conclusion of the process performed. It does not validate the problem. | It partially concludes the problem but does not validate it. | It concludes the problem, validates it and is pertinent with the developed argumentative plot. |

Table 4. Analytical rubric with theoretical epistemic value: Premises.

| Argumentative process elements | Theoretical epistemic value (functional dimension) and operational statute (structural dimension) |
|--------------------------------|--------------------------------------------------------------------------------------------------|
| Argument premises             |                                                                                            |
| Level 1: Low                  | Level 2: Medium                                                                            |
| (0.0 to 2.5)                  | (2.6 to 3.9)                                                                               |
| Level 3: High                 |                                                                                             |
| (4.0 to 5.0)                  |                                                                                             |
| It does not represent the problem situation in mathematical language. It does not define variables. | Defines some variables for the representation in mathematical language of the statement premises. | Partially defines the variables. | Uses appropriate mathematical signs for the representation in mathematical language of the problem situation. Defines the variables correctly. |

Table 5. Analytical rubric with theoretical epistemic value: Middle term.

| Argumentative process elements | Theoretical epistemic value (functional dimension) and operational statute (structural dimension) |
|--------------------------------|--------------------------------------------------------------------------------------------------|
| Argument premises             |                                                                                            |
| Level 1: Low                  | Level 2: Medium                                                                            |
| (0.0 to 2.5)                  | (2.6 to 3.9)                                                                               |
| Level 3: High                 |                                                                                             |
| (4.0 to 5.0)                  |                                                                                             |
| It does not use valid processes from the theoretical corpus of mathematics according to the statement. | Uses valid processes from the theoretical corpus of mathematics according to the statement but still presents some epistemological errors. | Uses valid processes from the theoretical corpus of mathematics according to the statement and the premises proposed. |
Table 6. Analytical rubric with theoretical epistemic value. Argument conclusions.

| Argument process elements | Theoretical epistemic value (functional dimension) and operational statute (structural dimension) | Argument conclusions |
|---------------------------|---------------------------------------------------------------------------------------------|---------------------|
| Level 1: Low (0.0 to 2.5) | Level 2: Medium (2.6 to 3.9)                                                                  | Level 3: High (4.0 to 5.0) |
| It does not make a conclusion of the process developed or does not conclude according to the corpus theory of mathematics. | Partially concludes the problem but still presents epistemological errors. | It concludes the problem, from the theoretical corpus of mathematics and it is pertinent with the developed argumentative plot. |

3.1. Categories of analysis

From the functional point of view, an argument is accepted or rejected based on two fundamental aspects, the strength and relevance of the arguments [6]; In this phase the functional dimension is combined with the epistemological dimension.

In this phase the general category, "mathematical argumentation", the argumentative dimensions "heuristic argumentation and rhetorical argumentation" will be found. Understood the heuristic argumentation as that argumentation given in the field of the disciplinary, which is to say in the field of the mathematical and the rhetorical argumentation like such argument that presents absence of preliminary theoretical corpus from the mathematics field.

The categories in terms of acceptability or rejection of a “strength and relevance” argument and the sub-categories in terms of the functional dimension of a “semantic epistemic value” argument related to the category of relevance of the argument and "theoretical epistemic value" related to the strength category of an argument.

For [3], an argument is strong when it contains a theoretical and epistemological corpus from the disciplinary field where it is argued, in other words, it evidences a content of axioms, theorems, laws, etc., typical of mathematical knowledge. On the other hand, a relevant argument is one that evidences a semantic and syntactic relationship with the content of the statement, that is, that there is an epistemic value of “truth”. Of course, all of the above can only be validated in the field of heuristic argumentation, to be precise, in the field of mathematical argumentation.

Finally, the rhetorical type argument is not of the interest of the mathematical argumentation process, however, it is important to highlight the fact that in some rhetorical type arguments, the students demonstrate logical and reasonable elements but without strength in them. The teacher’s task will design an excellent problem situation that generates and stimulates in the student the development of an argumentative plot from the context of the mathematical and of course from the context of logical semiotics and mathematical language.

3.2. Elements of the argumentative process

In this context, the argumentative process is investigated from the logical and structural dimension of the argumentative plot. For this, the structure of a logical reasoning passage proposed by [6], premises, middle term and conclusion are taken into account.

It is expected that when a student expresses their premises, they demonstrate the sense and meaning in terms of the understanding that the subjects are giving to the problem, the process followed, the student evidences in the "middle term" the use of mathematical signs (semiotics logic-mathematical language) where they should be able to represent and model the problem situation (this is the transition from a daily language to a mathematical language) and the use of processes, methods and procedures typical of mathematics and according to the problem situation raised. Finally, the student must show elements of conclusion where he argues in a strong and relevant way, the solution to the problem posed.

Each structural dimension presents levels of evaluation around the logic posed by the analytical rubric; it is proposed for this rubric to address three levels in each dimension: level 1. Low, Level 2.
Medium and Level 3. High, which, from the logical dimension, describes the criteria to be taken into account when generating an evaluative affirmation.

The weighting for this case, is based on a maximum score of five (5.0), where, each level depending on the logical evaluation criteria, presents an option range to be known: low level from 0.0 to 2.5, average level from 2.6 a 3.9 and high level from 4.0 to 5.0.

With this rubric it is hoped to recognize in a more precise way the aspects to be improved by the students, at the same time, it brings the opportunity of a didactic agreement between the students and the teacher to carry out a process of evaluation of the mathematical argumentation competence and finally, of the competence of quantitative reasoning.

4. Conclusions
The use of the analytical rubric as an assessment strategy in the mathematical argumentation field, allowed reducing the levels of subjectivity on the part of the agents involved in the process of teaching and learning mathematics.

The analytical rubric allowed a more objective and formative follow-up to the development of the students learning results who develop argumentative plots for the resolution of problem situations, not always from the mathematics field.

The use of the rubric as an evaluation strategy strengthens the elements of self-training and self-evaluation in teachers, thus determining emerging elements and learning opportunities in the mathematics field teaching and at a general level in the field of science didactics. The use of the rubric strengthens the formative dimension of the assessment, allows for a more objective monitoring of the process carried out by the students, found in a timely manner, opportunities for improvement and specific aspects in which the student should be reinforced.

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