Supernova neutrinos: difference of $\nu_{\mu} - \nu_{\tau}$ fluxes and conversion effects

Evgeny Kh. Akhmedov$^{a,1}$, Cecilia Lunardini$^{b,2}$ and Alexei Yu. Smirnov$^{c,d,3}$

$^a$ Centro de Física das Interacções Fundamentais (CFIF) Departamento de Física, Instituto Superior Técnico Av. Rovisco Pais, P-1049-001 Lisboa, Portugal
$^b$ Institute for Advanced Study, Einstein drive, 08540 Princeton, New Jersey, USA
$^c$ The Abdus Salam ICTP, Strada Costiera 11, 34100 Trieste, Italy
$^d$ Institute for Nuclear Research, RAS, Moscow 123182, Russia.

Abstract

The formalism of flavor conversion of supernova neutrinos is generalized to include possible differences in the fluxes of the muon and tau neutrinos produced in the star. In this case the radiatively induced difference of the $\nu_{\mu}$ and $\nu_{\tau}$ potentials in matter becomes important. The $\nu_{\mu}$ and $\nu_{\tau}$ flux differences can manifest themselves in the effects of the Earth matter on the observed $\nu_e$ ($\bar{\nu}_e$) signal if: (i) the neutrino mass hierarchy is normal (inverted); (ii) the solution of the solar neutrino problem is in the LMA region; (iii) the mixing $U_{e3}$ is relatively large: $|U_{e3}| \gtrsim 10^{-3}$. We find that for differences in the $\nu_{\mu} - \nu_{\tau}$ ($\bar{\nu}_{\mu} - \bar{\nu}_{\tau}$) average energies and/or integrated luminosities $\lesssim 20\%$, the relative deviation of the observed $\nu_e$ ($\bar{\nu}_e$) energy spectrum at $E \gtrsim 50$ MeV from that in the case of the equal fluxes can reach $\sim 20-30\%$ ($\sim 10-15\%$) for neutrinos crossing the Earth. It could be detected in future if large detectors sensitive to the $\nu_e$ ($\bar{\nu}_e$) energy spectrum become available. The study of this effect would allow one to test the predictions of the $\nu_{\mu}$, $\nu_{\tau}$, $\bar{\nu}_{\mu}$, $\bar{\nu}_{\tau}$ fluxes from supernova models and therefore give an important insight into the properties of matter at extreme conditions. It should be taken into account in the reconstruction of the neutrino mass spectrum and mixing matrix from the supernova neutrino observations. We show that even for unequal $\nu_{\mu}$ and $\nu_{\tau}$ fluxes, effects of leptonic CP violation can not be studied in the supernova neutrino experiments.

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1 Introduction

The study of supernova neutrinos is of great interest to both astrophysics and particle physics. As far as the astrophysics is concerned, the neutrinos provide precious information about the core gravitational collapse, the mechanism of supernova explosion and the properties of matter at extreme conditions of large densities and high temperatures. From the particle physics point of view, supernova neutrinos allow one to probe the neutrino mixings and mass spectrum. Indeed, the properties of the neutrino fluxes are modified by flavor conversion effects inside the star (see [1]-[19] as an incomplete list of relevant works) and in the matter of the Earth (see [20] for an early suggestion and [21, 22, 23] for recent studies).

Until now, the effects of neutrino oscillations and conversion have been considered in the assumption that the fluxes of the muon and tau neutrinos produced in the star are identical. Indeed, while the diffusion of $\nu_e$ in the star differs from that of $\nu_\mu$ due to the effect of the charged current processes, the non-electron flavors, $\nu_\mu$ and $\nu_\tau$, and their antineutrinos are expected to undergo similar interaction effects, dominated by neutral current scatterings. Therefore, the non-electron neutrinos have similar fluxes:

$$F^0_\mu \approx F^0_\overline{\mu} \approx F^0_\tau \approx F^0_\overline{\tau}.$$  

In contrast to this, the fluxes of the $\nu_e$, $\overline{\nu_e}$ and $\nu_\mu$ are expected to be rather different, with the hierarchy of the average energies $\langle E_e \rangle < \langle \overline{E_e} \rangle < \langle E_\mu \rangle$. In calculations of the neutrino transport in the star $\nu_\mu$, $\nu_\tau$, $\overline{\nu}_\mu$ and $\overline{\nu}_\tau$ are considered equivalent and treated effectively as a single species, $\nu_x$.

Several effects can break the $\nu_\mu$–$\nu_\tau$ and the $\nu_\mu$–$\overline{\nu}_\mu$ (or $\nu_\tau$–$\overline{\nu}_\tau$) equivalence, giving rise to small differences between the fluxes of these species. Once effects of weak magnetism are considered, the $\nu_\mu$ and $\overline{\nu}_\mu$ interaction cross section in the matter of the star are different; this can lead to a difference between the average energies and luminosities of the $\nu_\mu$ and $\overline{\nu}_\mu$ fluxes as large as $\sim 10\%$ [24]. The production of muons in the core of the neutron star results in differences in the transport of $\nu_\mu$ and $\nu_\tau$ inside the star, whose effect on the neutrino fluxes exiting the star has not been fully explored yet [22].

The possibility of studying these phenomena through the observation of a difference in the $\nu_\mu$ and $\nu_\tau$ fluxes makes this difference a quantity of particular interest. In addition, if an appreciable $\nu_\mu$–$\nu_\tau$ difference exists, its effects on the neutrino conversion should be taken into account while analyzing the supernova neutrino signals in order to determine the neutrino oscillation parameters.

In this paper we present a generalization of the formalism of neutrino conversion in the star and in the Earth to the case of unequal $\nu_\mu$ and $\nu_\tau$ fluxes. We address the question of whether the $\nu_\mu$–$\nu_\tau$ flux differences could be probed via flavor conversion effects and

\footnote{Another potential source of differences between the $\nu_\mu$ and $\nu_\tau$ fluxes are possible effects of new physics, such as lepton number violating processes or violations of lepton universality. Such effects could, in principle, influence the whole dynamics of neutrino transport in supernovae, and their detailed study goes beyond the scope of the present paper.}
what the effect of these differences would be on the study of the neutrino mixing and mass spectrum with supernova neutrinos.

The paper is organized as follows. After giving some generalities in sec. 2, the effects of $\nu_\mu - \nu_\tau$ flux differences on neutrino conversion in the star are discussed in sec. 3. In sec. 4 we study the Earth matter effects and comment on possible signatures of the $\nu_\mu - \nu_\tau$ flux differences. Discussion and conclusions follow in sec. 5.

2 Supernova neutrinos and different $\nu_\mu$ and $\nu_\tau$ fluxes

2.1 General features

At a given time $t$ from the core collapse the original flux of the neutrinos of a given flavor, $\nu_\alpha$, can be described by a “pinched” Fermi-Dirac spectrum,

$$F^0_\alpha(E, T_\alpha, \eta_\alpha, L_\alpha, D) = \frac{L_\alpha}{4\pi D^2 T^4_\alpha F_3(\eta_\alpha)} \frac{E^2}{e^{E/T_\alpha - \eta_\alpha} + 1},$$

(2)

where $D$ is the distance to the supernova (typically $D \sim 10$ kpc for a galactic supernova), $E$ is the energy of the neutrinos, $L_\alpha$ is the luminosity of the flavor $\nu_\alpha$, and $T_\alpha$ represents the effective temperature of the $\nu_\alpha$ gas inside the neutrinosphere. The typical values of the average neutrino energies produced by the supernova simulations are

$$\langle E_\alpha \rangle = 10 - 12 \text{ MeV}, \quad \langle E_\bar{\alpha} \rangle = 12 - 18 \text{ MeV}, \quad \langle E_x \rangle = 18 - 27 \text{ MeV},$$

(3)

and the integrated luminosities of all neutrino species are expected to be approximately equal, within a factor of two or so [26, 27]. The pinching parameter $\eta_\alpha$ takes the values $\eta_\alpha \sim 0 - 2$ for $\nu_\mu$ and $\nu_\tau$; larger values, $\eta_\alpha \sim 0 - 5$, could be realized for the electron flavor [28, 27]. The normalization factor $F_3(\eta_\alpha)$ is given by

$$F_3(\eta_\alpha) \equiv -6\text{Li}_4(-e^{\eta_\alpha}),$$

(4)

where the polylogarithm function $\text{Li}_n(z)$ is defined as

$$\text{Li}_n(z) \equiv \sum_{k=1}^{\infty} \frac{z^k}{k^n}.$$  

(5)

In the absence of pinching, $\eta_\alpha = 0$, one gets $F_3(0) = 7\pi^4/120 \simeq 5.68$. The average energy $\langle E_\alpha \rangle$ of the spectrum depends on both $T_\alpha$ and $\eta_\alpha$:

$$\langle E_\alpha \rangle = \sigma(\eta_\alpha) T_\alpha,$$

(6)

$$\sigma(\eta_\alpha) \equiv 3 \frac{\text{Li}_4(-e^{\eta_\alpha})}{\text{Li}_3(-e^{\eta_\alpha})}.$$  

(7)
Eqs. (5) and (7) give $\sigma(0) = 3.15$; larger values of $\sigma$ are obtained for larger (positive) $\eta_\alpha$, e.g. $\sigma(2) \simeq 3.6$.

The luminosity, temperature and pinching of the neutrino flux vary with the time $t$; if these variations occur over time scales larger than the duration of the burst, $\tau \sim 10$ s, the integrated $\nu_\alpha$ flux is still given by the expression (2) with $\eta_\alpha \simeq 0$ and the total energy $E_\alpha \sim L_\alpha \tau$ in place of the luminosity $L_\alpha$.

In our study of neutrino conversions inside the star we use the following matter density profile of the star:

$$\rho(r) = 10^{13} C \left( \frac{10 \text{ km}}{r} \right)^3 \text{ g \cdot cm}^{-3},$$

with $C \simeq 1 - 15$. For $\rho \sim 1 - 10^6 \text{ g \cdot cm}^{-3}$ expression (8) provides a good description of the matter distribution at any time during the neutrino burst emission and propagation through the star [29, 3, 5, 30, 28]. For $\rho \lesssim 1 \text{ g \cdot cm}^{-3}$ the exact shape of the profile depends on the details of evolution of the star, its chemical composition, rotation, etc. The regions at $\rho \gtrsim 10^6 \text{ g \cdot cm}^{-3}$ are reached by the shock-wave propagation during the neutrino burst emission, therefore in these regions the neutrinos cross the shock front, which is characterized by large moving density gradients [31].

### 2.2 $\nu_\mu$ and $\nu_\tau$ spectral difference

In the absence of specific predictions for the $\nu_\mu - \nu_\tau$ flux differences, which could result from a number of physical effects, we consider for these two species the Fermi-Dirac spectra, as in eq. (2), with different temperatures, luminosities and pinching parameters. Then the ratio of fluxes equals:

$$\frac{F_\tau^0}{F_\mu^0} = \left( \frac{L_\tau}{L_\mu} \right) \left( \frac{T_\mu}{T_\tau} \right)^4 \frac{F_3(\eta_\mu)}{F_3(\eta_\tau)} \exp\left[ E/T_\mu - \eta_\mu \right] + 1 \exp\left[ E/T_\tau - \eta_\tau \right] + 1.$$

We consider the relative difference

$$\Delta_F \equiv \frac{F_\tau^0}{F_\mu^0} - 1$$

and adopt a general parameterization of $\Delta_F$ in terms of the relative differences of the temperatures, luminosities and pinching parameters: $\epsilon_T \equiv (T_\tau - T_\mu)/T_\mu$, $\epsilon_L \equiv (L_\tau - L_\mu)/L_\mu$ and $\epsilon_\eta \equiv (\eta_\tau - \eta_\mu)/\eta_\mu$. The presence of charged muons in the star may result in a difference in the pinching parameters with no appreciable difference in the luminosities and temperatures, while different temperatures or luminosities could appear from the effects of lepton number non-conservation or violation of lepton universality.
For $|\epsilon_T|, |\epsilon_L|, |\epsilon_\eta| \ll 1$ the difference $\Delta_F$ can be expanded as

$$\Delta_F \simeq \epsilon_L + \epsilon_T \left[ -4 + \frac{\exp[E/T_\mu - \eta_\mu]}{\exp[E/T_\mu - \eta_\mu] + 1} \left( \frac{E}{T_\mu} \right) \right] + \epsilon_\eta \eta_\mu \left[ \frac{\text{Li}_3(-e^{\eta_\mu})}{\text{Li}_4(-e^{\eta_\mu})} + \frac{\exp[E/T_\mu - \eta_\mu]}{\exp[E/T_\mu - \eta_\mu] + 1} \right],$$

for any value of the energy $E$. It is also useful to consider the high-energy limit, $E \gg T_\mu \eta_\mu$, of eqs. (10) and (11):

$$\frac{F^0_\tau}{F^0_\mu} (E \gg T_\mu \eta_\mu) \sim \left( \frac{L_\tau}{L_\mu} \right) \left( \frac{T_\mu}{T_\tau} \right)^4 \frac{F^\tau_3(\eta_\mu)}{F^\tau_3(\eta_\tau)} e^{\eta_\mu - \eta_\tau} e^E (1/T_\mu - 1/T_\tau),$$

and

$$\Delta_F (E \gg T_\mu \eta_\mu) \sim \epsilon_L + \epsilon_T \left[ -4 + \left( \frac{E}{T_\mu} \right) \right] + \epsilon_\eta \eta_\mu \left[ -\frac{\text{Li}_3(-e^{\eta_\mu})}{\text{Li}_4(-e^{\eta_\mu})} + 1 \right].$$

Let us comment on the contributions of $\epsilon_T$, $\epsilon_L$ and $\epsilon_\eta$ to $\Delta_F$.

1) The effect of different luminosities, $\epsilon_L \neq 0$, gives an energy-independent term in $\Delta_F$, as can be seen in eqs. (10) and (11). If there are no differences in the temperatures and in the pinching factors one merely has $\Delta_F = \epsilon_L$.

2) If the $\nu_\mu$ and $\nu_\tau$ temperatures are different, $\epsilon_T \neq 0$, a critical energy $E^\text{corr}_C$ exists at which the $\nu_\mu$ and $\nu_\tau$ fluxes are equal, $\Delta_F = 0$. If $\nu_\tau$ has a harder spectrum than $\nu_\mu$ ($\epsilon_T > 0$) we have $\Delta_F > 0$ at $E > E^\text{corr}_C$ and $\Delta_F < 0$ for $E < E^\text{corr}_C$. For a given $\epsilon_\eta$, the critical energy $E^\text{corr}_C$ increases with the increase of $\epsilon_T$ and decreases with the increase of $\epsilon_L$. If $\epsilon_T < 0$ the opposite situation is realized: $\Delta_F > 0$ at $E \lesssim E^\text{corr}_C$ and $\Delta_F < 0$ at $E \gtrsim E^\text{corr}_C$. The critical energy $E^\text{corr}_C$ increases with the increase of $\epsilon_T$ (the decrease in absolute value) and with the increase of $\epsilon_L$. Above the energy $E^\text{corr}_C$ the relative difference $\Delta_F$ of the fluxes increases linearly with $E$ in absolute value, as follows from eq. (13). Numerically, taking $T_\mu = 7$ MeV, $\epsilon_T = 0.1$ and $\epsilon_L = \epsilon_\eta = 0$, we find $E^\text{corr}_C \simeq 30$ MeV and eq. (11) gives $\Delta_F \simeq 0.46$ ($\Delta_F \simeq 0.49$) at $E = 60$ MeV and $\Delta_F \simeq -0.35$ ($\Delta_F \simeq -0.29$) at $E = 5$ MeV. In general, for $\epsilon_T \neq 0$ the contribution of the originally non-electron neutrinos to the observed $\nu_\tau$ spectrum is dominated by the flavor with the softer spectrum in the low energy region, while at high energies the flavor with harder spectrum dominates.

3) The contribution due to different pinching parameters, $\epsilon_\eta \neq 0$, increases with energy and turns from negative to positive at $E \sim \langle E_\mu \rangle \sim 18 - 27$ MeV. At larger energies the $\epsilon_\eta$ contribution asymptotically approaches a constant value, as one can see from eqs. (12) and (13). This value is small since the ratio $\text{Li}_3(-e^{\eta_\mu})/\text{Li}_4(-e^{\eta_\mu})$ is close to unity and therefore it undergoes partial cancellation in eq. (13). Taking $\eta_\mu = 1$, $\eta_\tau = 2$ and $\epsilon_L = \epsilon_T = 0$ one gets $\Delta_F \simeq 0.14$ at $E \gtrsim 40$ MeV and $\Delta_F \simeq -0.46$ at $E = 5$ MeV.

The description given here applies to the difference of the $\bar{\nu}_\mu$ and $\bar{\nu}_\tau$ fluxes, $F^0_\bar{\mu}$ and $F^0_\bar{\tau}$, as well. The relative difference $\Delta_F \equiv F^0_\bar{\tau}/F^0_\bar{\mu} - 1$ can be described in terms of the ratios $\bar{\epsilon}_T \equiv (T_\tau - T_\bar{\mu})/T_\bar{\mu}$, $\bar{\epsilon}_L \equiv (L_\tau - L_\bar{\mu})/L_\bar{\mu}$, and $\bar{\epsilon}_\eta \equiv (\eta_\tau - \eta_\bar{\mu})/\eta_\bar{\mu}$ analogously to eq. (11).
3 Conversion effects in the star

3.1 Neutrino mass and mixing schemes

Let us consider a system of three neutrinos, \((\nu_1, \nu_2, \nu_3)\), with masses \((m_1, m_2, m_3)\), related to the flavor eigenstates, \((\nu_e, \nu_\mu, \nu_\tau)\), by the mixing matrix \(U\): \(\nu_\alpha = \sum_i U_{\alpha i} \nu_i\). The indices \(\alpha\) and \(i\) refer to the flavor and mass eigenstates respectively.

We adopt the following parameterization of the mixing matrix (see e.g. [32]):

\[
U = V^{23} I^\delta V^{13} V^{12},
\]

where

\[
I^\delta \equiv \text{diag}(1, 1, e^{i\delta}),
\]

and

\[
V^{12} = \begin{pmatrix}
  c_{12} & s_{12} & 0 \\
 -s_{12} & c_{12} & 0 \\
  0 & 0 & 1
\end{pmatrix},
V^{13} = \begin{pmatrix}
  c_{13} & 0 & s_{13} \\
  0 & 1 & 0 \\
 -s_{13} & 0 & c_{13}
\end{pmatrix},
V^{23} = \begin{pmatrix}
  1 & 0 & 0 \\
  0 & c_{23} & s_{23} \\
  0 & -s_{23} & c_{23}
\end{pmatrix},
\]

(16)

with \(s_{ij} \equiv \sin \theta_{ij}\) and \(c_{ij} \equiv \cos \theta_{ij}\).

We will consider 3\(\nu\)-schemes which explain the atmospheric and the solar neutrino data. The parameters describing the atmospheric neutrino oscillations are:

\[
m_3^2 - m_2^2 \equiv \Delta m_{32}^2 = \pm (1.5 - 4) \cdot 10^{-3} \text{eV}^2, \quad \tan^2 \theta_{23} = 0.48 - 2.1.
\]

The two possibilities, \(\Delta m_{32}^2 > 0\) and \(\Delta m_{32}^2 < 0\), are referred to as normal and inverted mass hierarchies respectively.

The solar neutrino data are explained either by vacuum oscillations (VO solution), or by one of the MSW solutions (LMA, SMA or LOW) with the oscillation parameters

\[
m_2^2 - m_1^2 \equiv \Delta m_{21}^2, \quad \tan^2 \theta_{12},
\]

(see, e.g., numerical values from the analyses [34]-[37]).

The angle \(\theta_{13}\) is bounded to be small from the results of the CHOOZ and Palo Verde experiments [38, 39]:

\[
\sin^2 \theta_{13} \lesssim 0.02,
\]

(19)

and the CP-violating phase \(\delta\) is still completely undetermined.
3.2 Matter effects: $V_{\tau\mu}$ potential; the level crossing pattern

Due to the very wide range of matter densities inside a supernova, $\rho \sim 0 - 10^{13}$ g · cm$^{-3}$, the conditions for three MSW resonances (level crossings) are met. The crossing of $\nu_\mu$ and $\nu_\tau$ levels, the $\mu\tau$ resonance, is driven by the difference of the $\nu_\mu$ and $\nu_\tau$ potentials which appears at one loop level due to different masses of the muon and tau leptons [40]:

$$V_{\tau\mu} \equiv V_\tau - V_\mu \simeq \pm \frac{3}{2\pi^2} G_F^2 m_\tau^2 \left[ (n_p + n_n) \ln \left( \frac{M_W}{m_\tau} \right) - n_p - 2 \frac{2}{3} n_n \right],$$

(20)

where $G_F$ is the Fermi constant, $m_\tau$ and $M_W$ are the masses of the tau lepton and of the W boson respectively; $n_p$ and $n_n$ denote the number densities of protons and neutrons in the medium. In eq. (20) the + (−) sign refers to neutrinos (antineutrinos). The difference $V_{\tau\mu}$ is much smaller than the $\nu_e - \nu_\mu / \nu_\tau$ effective potential $V_e = \sqrt{2} G_F n_e$, where $n_e$ is the electron number density of the medium. One finds

$$V_{\tau\mu} \simeq 5 \cdot 10^{-5} V_e.$$  

(21)

The conversion induced by the potential (20) is governed by the “atmospheric” mixing and mass splitting, $\Delta m^2_{32}$ and $\theta_{23}$, eq. (17). Depending on the precise value of $\theta_{32}$ the level crossing occurs in the range of densities $\rho \sim 0 - \rho_{\mu\tau}$, where $\rho_{\mu\tau}$ is determined by the condition

$$V_{\tau\mu}(r) \sim \frac{\Delta m^2_{32}}{2E}.$$  

(22)

Numerically, one gets $\rho_{\mu\tau} \sim 10^7 - 10^8$ g · cm$^{-3}$. In fact, as we will see, the actual position of the level crossing is not important.

The probability $P_{23}$ of the transition between the eigenstates of the Hamiltonian in matter, $\nu_{3m}$ and $\nu_{2m}$, can be calculated (following, e.g., [18]) according to

$$P_{23} = \frac{e^\chi \cos^2 \theta_{23} - 1}{e^\chi - 1},$$  

(23)

$$\chi \equiv -2\pi \frac{\Delta m^2_{32}}{2E} \left[ \frac{1}{V_{\tau\mu}(r)} \frac{dV_{\tau\mu}(r)}{dr} \right]^{-1}_{r=r_p},$$  

(24)

where $r_p$ is the distance from the center of the star to the region defined by eq. (22).

With the profile (8) and the parameters (17), eqs. (20), (24) and (22) give $\chi = 500 - 900$, corresponding to a very good adiabaticity: $P_{23} = 0$. The adiabaticity is good also for steeper profiles. We have checked that the adiabaticity condition is satisfied even for the large density gradients which could be produced by the shock wave propagation.

The two other resonances, driven by the $\nu_e - \nu_\mu / \nu_\tau$ effective potential, $V_e$, occur at much smaller densities, $\rho \lesssim 10^4$ g · cm$^{-3}$. For these values of the density the potential $V_{\tau\mu}$
is negligibly small and it is convenient to consider the evolution of the system in the rotated basis of states:

\[ \nu'_\alpha \equiv I^{\dagger} V^{23 \dagger} \nu_\alpha. \]  

The second (H-) resonance, associated to \( \Delta m_{23}^2 \) and \( \theta_{13} \), occurs at \( \rho = \rho_H \sim 10^3 - 10^4 \text{ g} \cdot \text{cm}^{-3} \). The resonance is in the neutrino (antineutrino) channel if the mass hierarchy is normal (inverted). We denote as \( P_H \) the probability of the transition between the eigenstates of the Hamiltonian in this resonance (the hopping probability). As shown in refs. [11, 22], \( P_H \) increases with the decrease of \( \sin^2 \theta_{13} \), varying from perfectly adiabatic conversion (\( P_H = 0 \)) at \( \sin^2 \theta_{13} \gtrsim 10^{-3} \) to strong adiabaticity breaking (\( P_H = 1 \)) at \( \sin^2 \theta_{13} \lesssim 10^{-5} \).

The third level crossing (L resonance) takes place at a lower density, \( \rho = \rho_L \lesssim 10 \text{ g} \cdot \text{cm}^{-3} \). It is determined by the “solar” parameters \( \Delta m_{21}^2 \) and \( \theta_{12} \), eq. (18), and occurs in the neutrino channel (see sec. 3.1). We denote the hopping probability associated to this resonance as \( P_L \). For oscillation parameters in the LMA region the conversion is adiabatic, \( P_L = 0 \), while partial breaking of adiabaticity occurs for other solutions of the solar neutrino problem [11, 22].

The level crossing scheme for the normal mass hierarchy with \( \theta_{23} < \pi/4 \) and a large solar mixing angle \( \theta_{12} \) is given in fig. 1. The dynamics of neutrino conversions inside the star can be considered as occurring in two steps:

1. propagation between the neutrinosphere and the layer with an intermediate density \( \rho_{\text{int}} \) such that \( \rho_H < \rho_{\text{int}} \ll \rho_{\mu \tau} \), (“inner region”) and
2. propagation from the layer with \( \rho \sim \rho_{\text{int}} \) to the surface of the star (“outer region”).

Let us consider the evolution in the inner region. At the neutrinosphere the potentials \( V_e \) and \( V_{\mu \tau} \) dominate over the kinetic energy differences \( \Delta m_{ij}^2/2E \) and the mixing is strongly suppressed, i.e. the matter eigenstates coincide with the flavor ones. At the densities \( \rho \sim \rho_{\text{int}} \), the potential \( V_{\mu \tau} \) is negligible and the matter eigenstates coincide with the states of the rotated basis [23]. Using the level crossing scheme of fig. 1 and taking into account that the adiabaticity is satisfied in whole range of densities \( \rho_H < \rho_{\text{int}} \ll \rho_{\mu \tau} \), we find that in the inner region the following transitions take place:

\[
\begin{align*}
\nu_e &\rightarrow \nu_e, & \nu_\mu &\rightarrow \nu'_\mu, & \nu_\tau &\rightarrow \nu'_\tau, \\
\bar{\nu}_e &\rightarrow \bar{\nu}_e, & \bar{\nu}_\mu &\rightarrow \bar{\nu}'_\mu, & \bar{\nu}_\tau &\rightarrow \bar{\nu}'_\tau,
\end{align*}
\]

provided that the \( \mu \tau \) level crossing occurs in the antineutrino channel. Therefore in the rotated basis \( (\nu_e, \nu'_\mu, \nu'_\tau) \) the fluxes of the states (before they enter the H resonance region) are

\[ F(\nu_e) = F^0_e, \quad F(\nu'_\mu) = F^0_\mu, \quad F(\nu'_\tau) = F^0_\tau. \]

Actually, the observables do not depend on the position of the \( \mu \tau \) resonance: what matters is the flux of the state that crosses the \( \nu_e \) level at the H resonance.
Notice that the role of the potential $V_{\tau\mu}$ is reduced to the suppression of the flavor mixing at high densities; the adiabatic change of this potential allows one to relate the fluxes of the states of the rotated basis to those of the originally produced flavor states.

The evolution of the neutrino states in the outer region is determined by the level crossings at the $H$ and $L$ resonances. If the hierarchy is inverted, then, as shown in fig. 2, the $\mu\tau$ and $L$ resonances are in the neutrino channel, while the $H$ resonance is in the antineutrino channel. In the inner region the conversions $\nu_\tau \to \nu_\mu'$ and $\nu_\mu \to \nu_\tau'$ take place, while the $\nu_e \leftrightarrow \nu_\mu'$ conversion occurs in the $L$ resonance. Antineutrinos first undergo the $\bar{\nu}_\mu \to \bar{\nu}_\mu'$ and $\bar{\nu}_\tau \to \bar{\nu}_\tau'$ conversions in the inner region. At lower densities, the $\nu_e \leftrightarrow \bar{\nu}_\mu'$ / $\bar{\nu}_\tau'$ transitions take place in the $H$ resonance and in the $L$ resonance for large $\theta_{12}$.

### 3.3 Conversion probabilities

As discussed in [11], if $F^0_\mu = F^0_\tau \equiv F^0_x$ the effect of the conversion on the flux $F_e$ of electron neutrinos observed at the Earth can be expressed in terms of a single permutation parameter, $p$, which represents the $\nu_e$ survival probability:

$$F_e = pF^0_e + (1 - p)F^0_x.$$  \hfill (27)

We shall generalize now this result to the case $F^0_\mu \neq F^0_\tau$.

Let us show that the differences between the $\nu_\mu$ and $\nu_\tau$ ($\bar{\nu}_\mu$ and $\bar{\nu}_\tau$) fluxes can manifest themselves only in the following cases:

(i) normal hierarchy, neutrino channel,

(ii) inverted hierarchy, antineutrino channel,

both of them with non-maximal adiabaticity breaking in the $H$ resonance: $P_H < 1$. Indeed, as can be seen from figs. 1 and 2, in the remaining cases (inverted hierarchy, neutrino channel and normal hierarchy, antineutrino channel) neutrinos of one of the non-electron flavors are completely converted into the third mass eigenstate, for which the electron component is small (equal to $\sin^2 \theta_{13}$). Therefore this flavor does not contribute significantly to the observed $\nu_e$ (or $\bar{\nu}_e$) flux, making the study of the $\nu_\mu - \nu_\tau$ inequivalence impossible. The same conclusion is obtained in the schemes (i) and (ii) for maximal adiabaticity breaking in the $H$ resonance, $P_H = 1$, since this corresponds to the complete transition of one of the non-electron flavors to the third mass eigenstate.

In what follows we discuss the conversion probabilities for the scenarios (i) and (ii); results for the remaining cases can be obtained by a simple generalization.

Let us first consider the conversion of neutrinos in the scheme with normal hierarchy, $\Delta m^2_{32} > 0$. As we discussed in sec. 3.2, the neutrinos arrive at the region of the $H$ resonance
as incoherent states $\nu_e, \nu_\mu', \nu_\tau'$ with the fluxes $F^0_e, F^0_\mu, F^0_\tau$. Let us consider the further evolution of these states. As a result of the adiabatic conversions in the star and/or decoherence due to the spread of the wavepackets, the neutrinos leave the star as mass eigenstates. The fluxes $\vec{F} \equiv (F_1, F_2, F_3)$ of these states can be expressed in terms of the original flavor fluxes $\vec{F}^0 \equiv (F^0_e, F^0_\mu, F^0_\tau)$ according to

$$\vec{F} = \mathcal{P} \vec{F}^0,$$

(28)

where $\mathcal{P}$ is the matrix of the conversion probabilities inside the star in the outer region, $\mathcal{P}_{i\alpha} \equiv P(\nu_{\alpha} \to \nu_{\beta})$. In the approximation of factorized dynamics, (factorization of the transition probabilities in H- and L- resonances) the matrix $\mathcal{P}$ is

$$\mathcal{P} = \begin{pmatrix}
P_H P_L & 1 - P_L & (1 - P_H) P_L \\
(1 - P_H) P_L & P_L & (1 - P_H) (1 - P_L) \\
(1 - P_H) & 0 & P_H
\end{pmatrix},$$

(29)

as can be readily derived from the level crossing scheme, fig. 1. Then the fluxes of neutrinos of definite flavors observed at the Earth, $\vec{F} \equiv (F_e, F_\mu, F_\tau)$, are obtained by the projection

$$\vec{F} = S \vec{F}^0,$$

(30)

where the matrix $S$ is defined as

$$S_{\alpha i} \equiv |U_{\alpha i}|^2.$$

(31)

Combining eqs. (28)-(31) one finds the relation between the original fluxes and the observed flavor fluxes:

$$\vec{F} = Q \vec{F}^0,$$

(32)

$$Q \equiv S \mathcal{P}.$$

(33)

Each entry $Q_{i\alpha}$ of the matrix $Q$ represents the total $\nu_{\alpha} \to \nu_{\beta}$ conversion probability, i.e. the probability that a neutrino produced as $\nu_{\alpha}$ in the star is observed as $\nu_{\beta}$ in the detector. From eqs. (14), (16), (31), (33) and (29), neglecting terms of order $\sin^2 \theta_{13}$, we get

$$Q = Q^{(0)} + 2 \tilde{J} Q^{(1)},$$

(34)

where

$$\tilde{J} \equiv s_{12} c_{12} s_{23} c_{23} s_{13} \cos \delta,$$

(35)

and

$$Q^{(0)} = \begin{pmatrix}
P_H f_L & 0 & 0 \\
-s_{23}^2 P_H (1 - f_L) + c_{23}^2 (1 - P_H) & c_{23}^2 f_L & c_{23}^2 (1 - P_H) (1 - f_L) + s_{23}^2 P_H \\
-s_{23}^2 P_H (1 - f_L) + c_{23}^2 (1 - P_H) & s_{23}^2 f_L & s_{23}^2 (1 - P_H) (1 - f_L) + c_{23}^2 P_H
\end{pmatrix},$$

(36)

$$Q^{(1)} = (1 - 2P_L) \begin{pmatrix}
0 & 0 & 0 \\
-P_H & 1 & -(1 - P_H) \\
P_H & -1 & (1 - P_H)
\end{pmatrix}.$$

(37)
In eqs. (36)

\[ f_L \equiv P_L + (1 - 2P_L) \sin^2 \theta_{12} . \]  

(38)

As can be seen from eq. (37), the observed \( \nu_e \) flux does not depend on the CP-violating phase \( \delta \) and, moreover, the terms proportional to \( \tilde{J} \) cancel once the sum \( F_\mu + F_\tau \) is considered. Therefore, since the separate detection of \( \nu_\mu \) and \( \nu_\tau \) is impossible, the study of the CP-violating phase \( \delta \) with supernova neutrinos seems to be not feasible. This result has a general validity and is discussed in more detail in sec. 3.4.

We consider now the antineutrino channel in the case of the inverted hierarchy, \( \Delta m_{32}^2 < 0 \) (fig. 2). Similarly to eqs. (32)-(33), the fluxes \( \vec{F} \equiv (F_e, F_\mu, F_\tau) \) of the antineutrinos at the detector are related to the original fluxes \( \vec{F}^0 \equiv (F^0_e, F^0_\mu, F^0_\tau) \) by

\[ \vec{F} = \vec{Q} \vec{F}^0 , \]  

(39)

\[ \vec{Q} \equiv S \vec{\mathcal{P}} . \]  

(40)

The matrix \( \vec{\mathcal{P}} \) of the \( \bar{\nu}_\alpha \to \bar{\nu}_i \) conversion probabilities is given by

\[ \vec{\mathcal{P}} = \begin{pmatrix} P_H (1 - P_L) & P_L & (1 - P_H)(1 - P_L) \\ P_H P_L & (1 - P_L) & (1 - P_H) P_L \\ (1 - P_H) & 0 & P_H \end{pmatrix} , \]  

(41)

where \( \tilde{P}_L \) represents the hopping probability associated to the L resonance in the antineutrino channel [11]. A form analogous to eq. (34) is obtained for the \( \tilde{Q} \) matrix, \( \vec{Q} = \vec{Q}^{(0)} + 2\tilde{J}\vec{Q}^{(1)} \), where the terms \( \vec{Q}^{(0)} \) and \( \vec{Q}^{(1)} \) are

\[ \vec{Q}^{(0)} = \begin{pmatrix} P_H \tilde{f}_L & (1 - \tilde{f}_L) & (1 - P_H) \tilde{f}_L \\ c_{23}^2 P_H (1 - \tilde{f}_L) + s_{23}^2 (1 - P_H) & c_{23}^2 (1 - P_H)(1 - \tilde{f}_L) + s_{23}^2 P_H \\ s_{23}^2 P_H (1 - \tilde{f}_L) + c_{23}^2 (1 - P_H) & s_{23}^2 (1 - P_H)(1 - \tilde{f}_L) + c_{23}^2 P_H \end{pmatrix} , \]  

(42)

\[ \vec{Q}^{(1)} = -(1 - 2\tilde{P}_L) \begin{pmatrix} 0 & 0 & 0 \\ -P_H & 1 & -(1 - P_H) \\ P_H & -1 & (1 - P_H) \end{pmatrix} . \]  

(43)

Here we defined

\[ \tilde{f}_L \equiv \tilde{P}_L + (1 - 2\tilde{P}_L) \cos^2 \theta_{12} . \]  

(44)

One can see that the results [11]-[13] can be obtained from those in eqs. (24)-(37) by the replacement

\[ P_L \to 1 - \bar{P}_L , \]  

(45)

or, equivalently,

\[ f_L \to \tilde{f}_L . \]  

(46)
3.4 Conversion effects and CP violation

As follows from eqs. (37) and (43), the elements of the second and the third lines of the matrices $Q^{(1)}$ and $\bar{Q}^{(1)}$ have opposite signs and therefore the sums $F_\mu + F_\tau$ and $F_{\bar{\mu}} + F_{\bar{\tau}}$ of the observed non-electron fluxes do not depend on $\delta$. Let us show that this result holds in the general case, without any assumption about the dynamics of neutrino conversion.

Consider the evolution of the neutrino states at densities $\rho \lesssim \rho_{\text{int}}$, where the potential $V_{\tau\mu}$ can be neglected and the conversion can be described in terms of the rotated states (25). The fluxes of these states coincide with the original $\nu_\alpha$ fluxes as a result of the adiabatic conversion in the $\mu\tau$ resonance. From the definition (25) it follows that the evolution of the rotated states $\nu'_\alpha$ and therefore the matrix $\mathcal{P}$, eq. (29), do not depend on the CP-violating phase $\delta$, irrespective of whether the approximation of factorization of transitions in different resonances is used. Terms containing $\delta$ may appear in the conversion probabilities only due to the projection (30) of mass states to the flavor basis by the matrix $S$, eq. (31). We can write the sum of the $\nu_\mu$ and $\nu_\tau$ fluxes as

$$F_\mu + F_\tau = \sum_{i\alpha} (S_{\mu i} + S_{\tau i}) P_{i\alpha} F^0_\alpha .$$

(47)

As a consequence of unitarity we find that for the $\delta$-dependent parts $S^{(\delta)}_{\alpha i}$ of the matrix $S$ the following relation holds:

$$S^{(\delta)}_{ei} + S^{(\delta)}_{\mu i} + S^{(\delta)}_{\tau i} = 0 ;$$

(48)

furthermore, in the parameterization (14) we have:

$$S^{(\delta)}_{ei} = 0 .$$

(49)

The combination of eqs. (48) and (49) yields

$$S^{(\delta)}_{\mu i} + S^{(\delta)}_{\tau i} = 0 .$$

(50)

This means that the sum (47) of the $\nu_\mu$ and $\nu_\tau$ fluxes is independent of $\delta$.

Neutrino propagation in an asymmetric matter can give rise to matter-induced T violation effects in neutrino oscillations even in the absence of the fundamental CP and T violation (i.e. when $\delta = 0$). In [41] it was pointed out that such effects cannot be observed in supernova neutrino experiments as a consequence of the assumption $F^0_\mu = F^0_\tau$ and experimental indistinguishability of $\nu_\mu$ and $\nu_\tau$. We note here that this result holds true even if the condition $F^0_\mu = F^0_\tau$ is relaxed. This is a consequence of the fact that the effects of matter-induced T violation (as well as those of the fundamental T violation) drop out from the sum (47), irrespective of whether the original $\nu_\mu$ and $\nu_\tau$ fluxes coincide.
3.5 Expected spectra

Let us discuss the observed $\nu_e$ signal for normal mass hierarchy (the case (i) of sec. 3.3).

From eqs. (32)-(37) one finds that the detected flux of electron neutrinos, $F_e$, is

$$F_e = F_0^0 Q_{ee} + F_0^0 Q_{e\mu} + F_0^0 Q_{e\tau} ,$$

$$= F_0^0 P_H f_L + F_0^0 (1 - f_L) + F_0^0 (1 - P_H) f_L ,$$

(51)

These expressions reduce to the equal fluxes form, eq. (27), if one of the following possibilities is realized:

1) the adiabaticity is maximally violated in the H resonance, $P_H = 1$ (as discussed in sec. 3.3),
2) $f_L = 0$, or
3) $f_L = 1$.

As can be seen from eq. (38), the last two possibilities are realized only if the angle $\theta_{12}$ is small, which is disfavored by the solar neutrino data (see e.g. [34]-[37]). In the following we focus on large solar mixing angles and therefore neglect cases (2) and (3).

Expression (51) can be conveniently rewritten as

$$F_e = F_0^0 P_H f_L + F_0^0 (1 - P_H f_L) + (F_0^0 - F_0^0) (1 - P_H) f_L ,$$

(53)

where the third term is entirely due to the difference of the $\nu_\mu$ and $\nu_\tau$ fluxes and vanishes if either condition (1) or (2) is fulfilled. From eq. (53) one finds the relative deviation, $R$, of the observed $\nu_e$ flux from the one in the case of equal fluxes:

$$R = \frac{(1 - P_H) f_L}{(F_e^0/F_\mu^0) P_H f_L + (1 - P_H) f_L} ,$$

(54)

where $\Delta F$ was given in eqs. (10) and (11). As follows from eq. (54), the sign of $R$ is determined by that of $\Delta F$. Therefore $R$ vanishes at the critical energy $E_C^{corr}$ and, if $\epsilon_T > 0$ ($\epsilon_T < 0$), we have $R < 0$ ($R > 0$) for $E < E_C^{corr}$ and $R > 0$ ($R < 0$) for $E > E_C^{corr}$. As discussed in sec. 2.2, if $\epsilon_T = \epsilon_\eta = 0$ no change of sign occurs and the quantity $R$ is determined by the difference in the luminosities only.

It is useful to introduce another critical energy $E_C^{(2)}$ defined as the energy at which the difference $F_0^0 - F_0^0$ changes its sign [14]. At $E \ll E_C^{(2)}$ the electron neutrino flux dominates over the $\nu_\mu$ and $\nu_\tau$ fluxes; in contrast, for $E \gg E_C^{(2)}$ the latter fluxes – having larger average energy – are dominant. Numerically, taking $T_e = 3.5$ MeV, $T_\mu = 7$ MeV, $L_e = L_\mu$ and $\eta_e = \eta_\mu = 0$ one finds $E_C^{(2)} \approx 20$ MeV.

Taking $P_L = 0$ (which is the case for solar neutrino parameters in the LMA region), from eq. (54) we find that

$$0 \leq \frac{R}{\Delta F} \leq \sin^2 \theta_{12} ,$$

(55)
between two events to be significantly larger than $t_D$. Notice that the condition $P_H = 0$ corresponds to no contribution of the original $\nu_e$ flux to the observed $\nu_e$ signal: $Q_{ee} = 0$ (see eq. 10). Thus, the observed $\nu_e$ flux is given by a mixture of the original $\nu_\mu$ and $\nu_\tau$ fluxes. The effect is suppressed, $R \simeq 0$, for $P_H \sim 1$ or at low energies, $E \ll E_C^{(2)}$, where the ratio $F_0^e/F_0^\mu$ is large. Numerically, taking $P_L = P_H = 0$, $T_\mu = 7$ MeV, $\epsilon_T = 0.1$, $\eta_\mu = \eta_\tau = \epsilon_L = 0$ and $\tan^2 \theta_{12} \simeq 0.33$ one gets $R \simeq -0.1$ at $E = 5$ MeV and $R \simeq 0.12$ at $E = 60$ MeV.

In general for $|\epsilon_T|, |\epsilon_L| < 0.2$ and $|\epsilon_\tau| \lesssim 1$ the relative deviation $R$ can be as large as $\sim 40 - 50\%$ (in absolute value) at high energies, $E \gg E_C^{(2)}$, and $\sim 15\%$ in the low energy limit, $E \ll E_C^{(2)}$.

For antineutrino conversion with inverted mass hierarchy the effects of the difference of the $\bar{\nu}_\mu$ and $\bar{\nu}_\tau$ fluxes are described by a straightforward generalization of eqs. (51) - (54). The results are similar to those presented here.

Let us consider the possibility of observation of the difference of the $\nu_\mu$ and $\nu_\tau$ fluxes. It is easy to see that, despite relatively large values of $R$ found here, the effect of the $\nu_\mu - \nu_\tau$ inequivalence will be very difficult to identify. Indeed, the corresponding distortion of the observed $\nu_e$ spectrum, which for $\epsilon_T \neq 0$ consists in a broadening of the energy distribution, can be mimicked by other effects and therefore does not represent a unique signature of the unequal $\nu_\mu$ and $\nu_\tau$ fluxes.

In particular, taking, e.g., $\epsilon_L = \epsilon_\eta = 0$, we find that the distorted $\nu_e$ spectrum is very accurately reproduced by the equal fluxes case with a temperature

$$T_x \approx (1 - f_L)T_\mu + (1 - P_H)f_LT_\tau$$

(56)

of the $\nu_x$ spectrum. In eq. (13) the $\nu_\mu$ and $\nu_\tau$ temperatures are weighted by the same coefficients that multiply the corresponding fluxes in the expression of the observed $\nu_e$ flux, eq. (12). If $\epsilon_T > 0$ ($\epsilon_T < 0$), we have $T_\mu < T_x < T_\tau$ ($T_\tau < T_x < T_\mu$) and the differences $|T_x - T_\mu|$ and $|T_x - T_\tau|$ are well within the uncertainty of the predicted values of $T_\mu$ and $T_\tau$. A numerical example is given in fig. 3 which shows the energy spectrum of events at the SNO detector from the $\nu_e + d \to e^- + p + p$ detection process with equal and different $\nu_\mu$-$\nu_\tau$ fluxes. We considered thermal time-integrated spectra as in eq. (2) with no pinching, equal integrated luminosities $E_\mu = E_\tau = 5 \cdot 10^{52}$ ergs and a distance $D = 8.5$ kpc from the supernova. LMA oscillation parameters ($\tan^2 \theta_{12} = 0.33$) were assumed, together with normal mass hierarchy and $P_H = 0$. As follows from the figure, the $\nu_e$ spectrum obtained

$^2$Events from the $\bar{\nu}_e + d \to e^+ + n + n$ reaction can in principle be distinguished from those from $\nu_e + d \to e^- + p + p$ due to the detection of at least one neutron by capture on deuterium, with characteristic time $t_{nd} \sim 5$ ms, in coincidence with the charged lepton. The coincidence can be established with high efficiency ($\sim 75-80\%$) due to the relatively small event rates of the two processes at SNO ($\sim 100-200$ events for a supernova at $D = 10$ kpc distance, see e.g. 14), which guarantees the average time interval between two events to be significantly larger than $t_{nd}$. 

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with different temperatures, \( T_{\mu} = 7 \text{ MeV} \) and \( T_{\tau} = 8.4 \text{ MeV} \), is well reproduced by that with the common temperature \( T_x = 7.3 \text{ MeV} \), according to eq. (54).

Global fits of simulated SNO and SuperKamiokande data \([14]\), performed under the assumption of equal temperatures of all the non-electron neutrino species and exact equipartition of the total energy \( E_B \), allow to determine these parameters with precision better than \( \sim 10\% \). Considering that the data set is dominated by the high statistics in SuperKamiokande, this result should be interpreted as a determination of the effective temperature and integrated luminosity of the non-electron antineutrinos, \( T_x \) and \( E_x \).

It can be checked that, if the \( \nu_\mu \) and \( \nu_\tau \) spectra have different pinching factors and luminosity difference of \( \sim 10-20\% \), the equal fluxes case accounts very well for the observed \( \nu_e \) spectrum once effective values of \( T_x \) and \( E_x \) are chosen within their uncertainty intervals, as they could be measured by SuperKamiokande.

In principle, the possibility exists that the differences between the \( \nu_\mu \) and \( \nu_\tau \) (or \( \bar{\nu}_\mu \) and \( \bar{\nu}_\tau \)) fluxes will be probed at future large detectors (UNO \([13]\), HyperKamiokande \([16]\), etc.) once all the neutrino oscillation parameters are known to a sufficient accuracy.

4 Earth matter effects

4.1 Unequal fluxes and conversion in the Earth

If the neutrino burst crosses the Earth before detection, the fluxes of the various neutrino flavors in the detector are affected by the regeneration in the matter of the Earth.

Let us first consider these effects in the scheme with normal mass hierarchy. According to a simple generalization of eqs. (32) and (33), the fluxes \( \vec{F}^D \equiv (F^D_e, F^D_\mu, F^D_\tau) \) of the neutrino flavor states in the detector can be expressed as

\[
\vec{F}^D = Q^\oplus \vec{F}^0 , \quad Q^\oplus \equiv S^\oplus \mathcal{P} , \tag{57}
\]

where the matrix \( \mathcal{P} \) was given in eq. (29). Each entry \( S^\oplus_{\alpha i} \) of the matrix \( S^\oplus \) represents the probability that a neutrino arriving at the Earth as \( \nu_i \) interacts, after crossing the Earth, as a \( \nu_\alpha \) in the detector: \( S^\oplus_{\alpha i} = P(\nu_i \to \nu_\alpha) \). From eqs. (29) and (57) the observed \( \nu_e \) flux equals:

\[
F^D_e = F^0_e P_H f^\oplus_L + F^0_\mu (1 - f^\oplus_L) + F^0_\tau (1 - P_H) f^\oplus_L , \tag{58}
\]

where

\[
f^\oplus_L \equiv P_L + (1 - 2P_L) S^\oplus_{e2} . \tag{59}
\]

In eqs. (58)-(59) terms of order \( \sin^2 \theta_{13} \) were neglected \( ^3 \).

\[ ^3 \text{It can be checked } [11] \text{ that the contributions of these terms to the difference } (60) \text{ are smaller than } 10^{-3} , \text{ and therefore negligible.} \]
Combining eqs. (38), (52) and (58)-(59) we obtain the deviation due to the Earth matter effects of the observed $\nu_e$ flux, $F_{e}^{D}$, from that predicted for conversion in the star only:

$$F_{e}^{D} - F_{e} = (1 - 2P_L) f_{\text{reg}} \left[ P_H F_{e}^0 - F_{\mu}^0 + (1 - P_H) F_{\tau}^0 \right] ,$$

(60)

where the regeneration factor,

$$f_{\text{reg}} \equiv S_{e2}^0 - \sin^2 \theta_{12} ,$$

(61)

accounts for the matter effects inside the Earth. It depends on the solar neutrino parameters, the neutrino energy, and the direction of the neutrino trajectory inside the Earth.

In what follows we consider $\Delta m^2_{21}$ and $\theta_{12}$ in the LMA region. (The Earth matter effect is small for the other solutions of the solar neutrino problem; see e.g. the discussion in ref. [22]). For the LMA oscillation parameters the conversion in the L resonance is adiabatic, thus we set $P_L = 0$ from now on.

As can be seen from eq. (60), the contribution of $F_{e}^0$ to the difference $F_{e}^{D} - F_{e}$ is small for large violation of adiabaticity in the H resonance and vanishes for $P_H = 1$, according to the discussion in sec. 3.3. In this case the difference of the $\nu_\mu$ and $\nu_\tau$ fluxes clearly has no observable effects. Conversely, in the case of good adiabaticity, $P_H = 0$, the contribution of the $\nu_e$ flux, $F_{e}^0$, is absent and the whole Earth effect is due to the difference $F_{e}^0 - F_{\mu}^0$.

It is useful to split the expression (60) into two terms according to their dependence on $P_H$:

$$F_{e}^{D} - F_{e} = \Delta F_{e}^{(2)} + \Delta F_{e}^{\text{corr}} ,$$

(62)

where

$$\Delta F_{e}^{(2)} = f_{\text{reg}} P_H (F_{e}^0 - F_{\tau}^0) , \quad \Delta F_{e}^{\text{corr}} = f_{\text{reg}} (F_{\tau}^0 - F_{\mu}^0) .$$

(63)

Let us comment on the quantities $\Delta F_{e}^{(2)}$ and $\Delta F_{e}^{\text{corr}}$.

• $\Delta F_{e}^{(2)}$ describes the whole Earth matter effects if the $\nu_\mu$-$\nu_\tau$ fluxes are equal. Being proportional to the difference $F_{e}^0 - F_{\tau}^0$, it changes the sign at the critical energy $E_C^{(2)}$, discussed in sec. 3.3. If the regeneration factor $f_{\text{reg}}$ is positive (as it is for conversion in the mantle of the Earth only [11, 22]) we have $\Delta F_{e}^{(2)} > 0$ for $E < E_C^{(2)}$ and $\Delta F_{e}^{(2)} < 0$ for $E > E_C^{(2)}$. The quantity $\Delta F_{e}^{(2)}$ is also proportional to $P_H$ and vanishes if $P_H = 0$, according to the fact that if the H resonance is completely adiabatic the flux $F_{e}^0$ does not undergo the L resonance. Conversely, $\Delta F_{e}^{(2)}$ is maximal if $P_H = 1$.

• $\Delta F_{e}^{\text{corr}}$ is proportional to the difference of the $\nu_\mu$ and $\nu_\tau$ fluxes. As discussed in sec. 2.2 $\Delta F_{e}^{\text{corr}}$ changes sign at $E = E_C^{\text{corr}}$. If $f_{\text{reg}}$ is positive and $\epsilon_T > 0$ we have $\Delta F_{e}^{\text{corr}} < 0$ for $E < E_C^{\text{corr}}$ and $\Delta F_{e}^{\text{corr}} > 0$ for $E > E_C^{\text{corr}}$. The opposite signs are realized for $\epsilon_T < 0$, while there is no change of sign (see sec. 2.2) if the $\nu_\mu$ and $\nu_\tau$ fluxes differ only in the luminosity.
The relative size of $\Delta F^{(2)}_e$ and $\Delta F^{\text{corr}}_e$ depends on $P_H$. In particular, in the high energy limit, $E \gg E^{(2)}_e$, where $F^0_e/F^0_\mu \ll 1$ and the regeneration factor is large \[22\], the condition $|\Delta F^{\text{corr}}_e/\Delta F^{(2)}_e| \gtrsim 1$ requires

$$P_H \lesssim P^\text{eq}_H \tag{64}$$

$$P^\text{eq}_H \equiv \left| \frac{\Delta F}{1 + \Delta F} \right|, \tag{65}$$

as follows from eqs. \[63\] and \[10\]. Taking $E = 60 \text{ MeV}$, $T_\mu = 7 \text{ MeV}$ $\epsilon_L = \epsilon_\eta = 0$ and $\epsilon_T = 0.1$ one gets $P^\text{eq}_H \simeq 0.33$. For $P_H \gtrsim P^\text{eq}_H$ (i.e. $P_H \sim 1$) the Earth matter effect is dominated by the $\Delta F^{(2)}_e$ term, while if $P_H \ll P^\text{eq}_H$ the larger contribution to the effect is given by $\Delta F^{\text{corr}}_e$, which, as we have discussed, is entirely due to the differences in the $\nu_\mu - \nu_\tau$ fluxes.

For $P_H \gtrsim P^\text{eq}_H$, the effect of the $\nu_\mu - \nu_\tau$ flux difference is very difficult to identify due to the uncertainties in the values of the various parameters: for $|\epsilon_T|, |\epsilon_T| \lesssim 0.2$ the flux $F^D_e$ observed in the detector is very accurately reproduced by the equal $\nu_\mu - \nu_\tau$ fluxes case with values of the parameters within their uncertainties. In contrast to this, if $P_H \ll P^\text{eq}_H$ the term $\Delta F^{\text{corr}}_e$ due to the unequal fluxes dominates, and in the limit $P_H = 0$ it represents the only Earth matter effect. The absence of competing effects offers the possibility of probing the differences in the $\nu_\mu - \nu_\tau$ fluxes and therefore makes this case of particular interest. This is true especially if $P_H$ is known to be very small (i.e. $\sin^2 \theta_{13} > 10^{-3}$) from independent measurements provided, e.g., by neutrino factories or superbeams \[17\]. A possibility is also related to the comparison of charged current and neutral current events from the neutronization burst at SNO \[14\]. If the value of $P_H$ is unknown, the possibility that the Earth effect is due to $F^0_\mu \neq F^0_\tau$ with $P_H = 0$ could be distinguished, in principle, from that of equal fluxes with a non-zero $P_H$ (eq. \[63\]) at least when the two effects have different signs at high energies, i.e. for $\epsilon_T > 0$ and/or $\epsilon_L > 0$.

The description of the Earth matter effects on the conversion of antineutrinos with inverted mass hierarchy is analogous to that presented above. Similarly to eqs. \[52\]-\[63\], the difference of the observed $\bar{\nu}_e$ fluxes with and without Earth effects is given by

$$F^D_{\bar{\nu}} - F_{\bar{\nu}} = \Delta F^{(2)}_{\bar{\nu}} + \Delta F^{\text{corr}}_{\bar{\nu}}, \tag{66}$$

$$\Delta F^{(2)}_{\bar{\nu}} = \bar{f}_{\text{reg}} P_H \left( F^0_{\bar{\nu}} - F^0_\tau \right), \quad \Delta F^{\text{corr}}_{\bar{\nu}} = \bar{f}_{\text{reg}} (F^0_\tau - F^0_\mu), \tag{67}$$

where

$$\bar{f}_{\text{reg}} \equiv S^\oplus_{1\bar{e}} - \cos^2 \theta_{12}, \tag{68}$$

and $S^\oplus_{1\bar{e}}$ is the $\bar{\nu}_1 \rightarrow \bar{\nu}_e$ conversion probability in the matter of the Earth. The results obtained above for neutrinos can be immediately extended to antineutrinos. However, compared to
the neutrino channel, the regeneration factor $f_{\text{reg}}$ is smaller in the antineutrino case, thus giving a smaller effect (see e.g. \[\text{22}\]).

It should be noted that, although the potential $V_{\tau\mu}$ is induced by radiative corrections and therefore is much smaller than the potential $V_e$, it plays a very important role in the evolution of neutrino states in the star provided that the original fluxes of $\nu_\mu$ and $\nu_\tau$ are different. If one had disregarded it, one would have had an additional suppression factor $\cos 2\theta_{23} \ll 1$ in the expressions for $\Delta F_{e}^{\text{corr}}$ and $\Delta F_{\bar{e}}^{\text{corr}}$ and so much smaller Earth matter effects.

### 4.2 Observable signals

Let us now discuss the sensitivity of the Earth matter effect to the $\nu_\mu-\nu_\tau$ flux differences. Consider the effect in the neutrino channel for normal mass hierarchy, solar neutrino parameters in the LMA region ($P_L = 0$) and $\sin^2 2\theta_{13} > 10^{-3}$ (i.e. $P_H \simeq 0$). In this case the relative Earth matter effect, $r \equiv (F_D^{e} - F_e)/F_e$, is entirely due to the differences in the $\nu_\mu-\nu_\tau$ fluxes:

$$ r = \frac{\Delta F_{e}^{\text{corr}}}{F_e} ,$$

and, from eqs. (10), (63) and (51) one gets

$$ r = \frac{\Delta F}{1 + s_{12}^2 \Delta F} f_{\text{reg}} .$$

A study of the ratio $r$ is presented in figs. 4-10, in which the cases of different $\nu_\mu$ and $\nu_\tau$ temperatures and luminosities are considered. The effect of different pinching factors, being energy-independent at high energy, $E \gg E_C^{(2)}$ (see sec. 2.1), is equivalent to a difference in the luminosities and therefore will not be discussed here.

Let us first consider the situation in which the difference between the $\nu_\mu$ and $\nu_\tau$ fluxes is mainly due to different temperatures and take $\epsilon_T > 0$ and $\epsilon_L = 0$. The ratio $r$ depends on the neutrino energy via the regeneration factor $f_{\text{reg}}$ and via the difference $\Delta F$, eqs. (10) and (11). The regeneration factor determines the oscillatory behavior of $r$ with the energy, while the dependence of $r$ on $\Delta F$ leads to the change of the sign of $r$ at $E = E_C^{\text{corr}}$. This sign changes from negative at lower energies to positive at higher energies, provided that $f_{\text{reg}} > 0$. Both $f_{\text{reg}}$ and $\Delta F$ increase with energy (see eq. (11)), therefore the largest effect is achieved in the high energy part of the spectrum. It also increases with increasing $\epsilon_T$. These features are illustrated in fig. 4 which shows $r$ as a function of the energy $E$ for different values of $\epsilon_T$ and of the nadir angle $\theta_n$ of the neutrino trajectory in the Earth. The figure has been obtained using a realistic density profile of the Earth [48]. As fig. 4 shows, the oscillatory distortions of the neutrino energy spectrum have larger periods for larger $E$, due to the larger oscillation length in matter, and for larger nadir angle $\theta_n$ (i.e. shorter trajectory inside the Earth). The size of the effect increases with the energy due...
to the linear increase of $\Delta F$ with $E$ (see eq. (13)) and to the larger regeneration factor. Taking into account that $f_{\text{reg}} \approx 0.7$ and that $r/f_{\text{reg}}$ can be as large as $r/f_{\text{reg}} \sim 0.45$ for $E \sim 60 - 70$ MeV and $\epsilon_T \sim 0.1$, we find $r \lesssim 0.3$. For larger temperature difference, $\epsilon_T \sim 0.2$, the effect can reach $\sim 70\%$.

If $\epsilon_T < 0$, $r$ is positive for $E < E_{\text{corr}}$ and negative for $E > E_{\text{corr}}$. Besides this change of the sign, the general features and the size of the effect are similar to those discussed above for positive $\epsilon_T$. This is expected, considering that $r$ is linear in $\epsilon_T$ when the latter is small (see eqs. (11) and (70)). The linear approximation is good for $|\epsilon_T| \lesssim 0.05 - 0.1$.

Let us now consider effect of different luminosities: $\epsilon_T = 0$ and $\epsilon_L \neq 0$. In this case we have $\Delta F = \epsilon_L$ (eq. (11)), and therefore $r \approx \epsilon_L f_{\text{reg}}$, as follows from eq. (70). The only energy dependence comes from the regeneration factor $f_{\text{reg}}$. For the luminosity differences $|\epsilon_L| \lesssim 0.2$ and $f_{\text{reg}} \approx 0.7$ we find $|r| \lesssim 0.15$. This is confirmed by the results shown in the fig. 4.

In general, both the temperatures and the integrated luminosities of the $\nu_\mu$ and $\nu_\tau$ fluxes may differ. The interplay of these two differences can enhance or suppress the Earth matter effect $r$ compared to the cases we have discussed above. Since, as we have shown, the larger contribution to the Earth matter effect comes from the temperature difference, it is illuminating to consider the effect of a small luminosity difference for a given difference $\epsilon_T$ of the temperatures. We focus on the high-energy part of the spectrum because at low energies the regeneration factor $f_{\text{reg}}$ is small.

As can be understood from eqs. (9), (11) and (70), at high energies the relative difference of fluxes $\Delta F$, and therefore the ratio $r$, is larger (in absolute value) when $\epsilon_L$ and $\epsilon_T$ have the same sign.

Conversely, if $\epsilon_L$ and $\epsilon_T$ have opposite signs, the effects of the two differences in eq. (11) tend to cancel each other. This is illustrated in fig. 4, which shows the ratio $r$ as a function of energy for $\epsilon_T = 0.1$ and different values of $\epsilon_L$, with the same parameters as in fig. 4. From the figures it follows that the enhancement or suppression of the Earth matter effect due to the $\nu_\mu - \nu_\tau$ luminosity difference can be as large as $40 - 50\%$, depending on the neutrino energy.

If the mass hierarchy is inverted and $P_H = 0$, the Earth matter effect due to the $\bar{\nu}_\mu - \bar{\nu}_\tau$ flux difference, $\Delta F_{\bar{\nu}_\tau}^{\text{corr}}$, appears in the antineutrino channel without competing effects ($\Delta F_{\bar{\nu}_\bar{e}}^{(2)} = 0$, see sec. 4.1). The expression for the relative Earth matter effect $\bar{r}$ can be obtained from eqs. (33), (12) and (66)-(67):

$$
\bar{r} = \frac{\Delta F}{1 + c_{12}^2 \Delta F} \bar{f}_{\text{reg}} \cdot (71)
$$

As can be seen from eqs. (70) and (71), for equal relative differences of fluxes, $\Delta F = \Delta_f$, the quantity $\bar{r}$ differs from $r$ by small terms of order $\Delta_f^2$ and by the suppression factor
\( f_{\text{reg}} / f_{\text{reg}} < 1 \). For the antineutrino channel plots analogous to those in figs. 4 and 5 are presented in figs. 7 and 8.

As a further illustration, in figs. 9 and 10 we show the binned energy spectra of the events expected in the heavy water tank of SNO for different \( \nu_\mu \) and \( \nu_\tau \) fluxes with and without Earth crossing. In the calculation of the expected numbers of events the detection efficiency and energy resolution have been taken into account following the discussion in ref. \[22\]. In fig. 9 we considered unpinched spectra with \( \epsilon_L = 0 \), \( \epsilon_T = 0.1 \) and the other neutrino parameters the same as in fig. 4. The nadir angle \( \theta_n = 0^\circ \) was taken; the distance \( D \) and the integrated luminosities are the same as in fig. 3. Fig. 10 shows the same as fig. 9 but with \( \Delta m^2_{21} = 3 \cdot 10^{-5} \text{ eV}^2 \) and \( \theta_n = 40^\circ \).

As follows from the figures, the Earth matter effect caused by the difference of \( \nu_\mu \) and \( \nu_\tau \) fluxes leads to a distortion of the observed energy spectrum as a whole or in isolated energy bins, depending on the depth of the neutrino trajectory in the Earth. This distortion amounts to 10 – 20% at most, and, due to the expected limited number of events, is not statistically significant. This deviation could have the significance of 3\( \sigma \) or larger for a supernova at a smaller distance (2-3 kpc) and/or in a larger experiment. For instance the proposed liquid argon detector LANNDD is expected to observe about 3000 \( \nu_e \) events \[49\] – more than 10 times larger statistics than SNO – for the parameters used in figs. 9 and 10. The sensitivity of this detector to the \( \nu_e \) energy spectrum, however, remains to be investigated. Moreover, as discussed in \[22\], establishing the Earth matter effects may require the comparison of the spectra observed by the detectors at different locations, and at present it is not clear if more than one large \( \nu_e \) detector with the necessary characteristics\( ^4 \) will be built in future.

The Earth matter effect in the antineutrino channel, eq. (71), can be detected via the \( \bar{\nu}_e + p \rightarrow e^+ + n \) reaction in Cherenkov or scintillator detectors \[5\]. The distortions of the observed energy spectrum compared to the case of no Earth crossing by neutrinos do not exceed 10%. Establishing this difference with at least 3\( \sigma \) statistical significance would require a comparison of data from two large detectors. We find that, unless the distance to the supernova is very small (\( D = 2 \text{–} 3 \text{ kpc} \)), two detectors of SuperKamiokande (SK) size do not have enough sensitivity. In contrast to this, large water Cherenkov detectors of next generation like UNO \[45\], HyperKamiokande \[46\] or TITAND \[50\] (\( \sim 18, 40 \text{ and } 60 \) times larger than SK respectively) could establish the effect with significance 3\( \sigma \) or larger for distances \( D \) up to \( \sim 10 \text{ kpc} \) at least.

The results for the time-integrated energy spectra presented here can be extended to

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\(^4\) Besides the high statistics, the observation of the Earth matter effect requires \[22\]: (i) separate detection of \( \nu_e \) and \( \nu_\mu + \nu_\mu + \nu_\tau \) signals, (ii) separate detection of neutrinos and antineutrinos, and (iii) the reconstruction of the energy spectrum.

\(^5\) We checked that the contributions of charged current scatterings on nuclei, e.g. \( \nu_e +^{16}O \rightarrow^{16}F + e^- \), to the event rates is smaller than 2 – 4\% and therefore negligible.
the spectra of events collected in any (small) time interval within the duration of the burst. Taking into account that the difference between the fluxes of the original non-electron neutrinos may vary with time, a study of the time dependence of the signal would be of interest to test the effect we have discussed. This is especially true if a specific time dependence of the $\nu_\mu - \nu_\tau$ flux difference is predicted by supernova models.

5 Discussion and conclusions

If the solution of the solar neutrino problem is in the LMA region, the Earth matter effects appear in the spectra of $\nu_e$ or $\bar{\nu}_e$ from a supernova or in both, depending on the type of the neutrino mass hierarchy, mixing angle $\theta_{13}$ and possible differences between the $\nu_\mu, \nu_\tau, \bar{\nu}_\mu$ and $\bar{\nu}_\tau$ fluxes. In particular, if (i) $F_\mu^0 = F_\tau^0$ ($F_\bar{\mu}^0 = F_\bar{\tau}^0$), (ii) the hierarchy is normal (inverted), and (iii) the conversion in the H resonance is adiabatic, $P_H \simeq 0$ (i.e. $\sin^2 \theta_{13} > 10^{-3}$), the Earth matter effects exist in the $\bar{\nu}_e$ ($\nu_e$) channel only \cite{1, 22}.

This is no longer true if there are differences between the original $\nu_\mu$ and $\nu_\tau$ or $\bar{\nu}_\mu$ and $\bar{\nu}_\tau$ fluxes. If $F_\mu^0 \neq F_\tau^0$ ($F_\bar{\mu}^0 \neq F_\bar{\tau}^0$) we find that, even for $P_H = 0$, effects of regeneration in the matter of the Earth as large as $\sim 70\%$ ($\sim 20\%$) can take place in the $\nu_e$ ($\bar{\nu}_e$) spectrum for normal (inverted) hierarchy.

Therefore, if $P_H$ is known to be very small from independent measurements (e.g. from neutrino factories or superbeams), the observation of a large Earth effect in the antineutrino channel and of a smaller one in the neutrino channel will testify for normal mass hierarchy and differences between $\nu_\mu$ and $\nu_\tau$ fluxes. Moreover, the character of the $\nu_\mu - \nu_\tau$ difference can be found from the sign of the observed Earth effect: a positive (negative) effect at high energies will correspond to $T_\tau > T_\mu$ or $T_\tau = T_\mu$ with $L_\tau > L_\mu$ ($T_\tau < T_\mu$ or $T_\tau = T_\mu$ with $L_\tau < L_\mu$).

If the same experimental result is observed but $P_H$ is unknown, an ambiguity exists between the two possibilities: (1) $F_\mu^0 \neq F_\tau^0$ with $P_H = 0$, and (2) $F_\mu^0 = F_\tau^0$ with $0 < P_H < 1$. In some circumstances the ambiguity can be resolved by considering the sign of the effect: if the effect is positive the case (1) will be established, with $T_\tau > T_\mu$ or $T_\tau = T_\mu$ with $L_\tau > L_\mu$. Moreover, the information that $P_H \ll P_H^{eq}$ is obtained. If the effect is negative, the case (1) is allowed only if $T_\tau < T_\mu$ or $T_\tau = T_\mu$ with $L_\tau < L_\mu$. Therefore a full discrimination requires the knowledge of the sign of $\epsilon_T$ and $\epsilon_L$.

If a large Earth matter effect is observed in the $\nu_e$ spectrum together with a small one in the $\bar{\nu}_e$ spectrum, the inverted hierarchy and the existence of the $\bar{\nu}_\mu - \bar{\nu}_\tau$ flux differences will be established, with implications analogous to those given here for the neutrino channel.

Let us summarize our main results.

1. The formalism of flavor conversion of supernova neutrinos has been generalized to the case of unequal fluxes of the non-electron neutrinos. A proof was given that, even for different fluxes of $\nu_\mu$ and $\nu_\tau$ neutrinos, the effects of CP violation are not observable in the
supernova neutrino signal since they do not affect the observed $\nu_e$ flux and cancel in the sum of the $\nu_\mu$ and $\nu_\tau$ fluxes in the detector.

2. It was found that possible $\nu_\mu - \nu_\tau$ ($\bar{\nu}_\mu - \bar{\nu}_\tau$) flux differences modify the observed $\nu_e$ ($\bar{\nu}_e$) flux compared to what is predicted for equal fluxes if the mass hierarchy is normal (inverted) and the adiabaticity violation in the H resonance is not maximal, $P_H < 1$.

3. For conversion in the star only, the effect of these differences consists in a distortion (broadening) of the observed $\nu_e$ or $\bar{\nu}_e$ energy spectrum. For relative differences between the temperatures and/or integrated luminosities of the muon and tau neutrino fluxes $\lesssim 20\%$ the relative deviation $R$ of the spectrum can be as large as $R \sim 0.4$ at high energies. However, the spectrum is consistent with the undistorted one within the uncertainties due to the approximate knowledge of the neutrino temperatures and luminosities.

4. If the solution of the solar neutrino problem is in the LMA region and the neutrino trajectory crosses the Earth, oscillatory distortions of the $\nu_e$ or $\bar{\nu}_e$ energy spectra occur. For $P_H \sim 1$ ($\sin^2 \theta_{13} < 10^{-4}$) the effect of the $\nu_\mu - \nu_\tau$ ($\bar{\nu}_\mu - \bar{\nu}_\tau$) flux differences is small and indistinguishable from the equal fluxes case due to the uncertainties in the neutrino temperatures and luminosities.

5. Conversely, if $P_H \simeq 0$ ($\sin^2 \theta_{13} \gtrsim 10^{-3}$) and the hierarchy is normal (inverted), the Earth matter effect in the neutrino (antineutrino) channel is entirely due to the $\nu_\mu - \nu_\tau$ ($\bar{\nu}_\mu - \bar{\nu}_\tau$) flux difference and therefore the existence of the flux difference can, in principle, be established. In the neutrino channel, the relative deviation $r$ of the observed spectrum from the prediction in the case of no Earth crossing can be typically as large as $r \simeq 0.2 - 0.3$ at high energy, $E \approx 70 - 80$ MeV, and can reach $r \sim 0.7$ for the largest considered values of the difference $T_\tau - T_\mu$. This effect could be observable in future if large $\nu_e$ detectors with good energy resolution become available. In the antineutrino channel the effect does not exceed $\sim 10 - 15\%$ and could be tested by the next generation of large Cherenkov detectors.

6. The observation of the Earth matter effect that we considered here would be a signal of the difference between $\nu_\mu$ and $\nu_\tau$ (or $\bar{\nu}_\mu$ and $\bar{\nu}_\tau$) fluxes, with interesting implications for the models of neutrino transport in the star and for the underlying physics. Jointly with other observations, it would give information on the neutrino mass hierarchy and on the mixing angle $\theta_{13}$. The effects of the possible $\nu_\mu - \nu_\tau$ flux difference should be properly taken into account in the interpretations of the results of the supernova neutrino experiments.

7. The potential $V_{\tau \mu}$ is induced by radiative corrections and therefore is much smaller than the potential $V_{\nu_e}$. Nevertheless, it plays an important role in the evolution of the neutrino states in the star provided that the original fluxes of $\nu_\mu$ and $\nu_\tau$ are different. Had one disregarded it, one would have arrived at a much smaller Earth matter effect, suppressed by the factor $\cos 2\theta_{23} \ll 1$. 

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Figure 1: The level crossing scheme of neutrino conversion in the matter of the star. The figure refers to the case of normal mass hierarchy with $\theta_{23} < \pi/4$ and a large solar angle $\theta_{12}$. The semi-plane with negative density, $n_e < 0$, describes the conversion in the antineutrino channel.
Figure 2: The same as in fig. 1 but for the inverted mass hierarchy.
Figure 3: The energy spectrum of the $\nu_e + d \rightarrow e^- + p + p$ events expected at SNO for conversion in the star only with equal (dotted and dashed curves) and different $\nu_\mu$ and $\nu_\tau$ fluxes (solid curve). The dotted curve was obtained considering normal mass hierarchy with $P_H = 0$, $T_\mu = 7$ MeV, $\eta_\mu = 0$, and LMA oscillation parameters ($P_L = 0$ and $\sin^2 2\theta_{12} = 0.75$). The solid curve refers to the same parameters with the difference $\epsilon_T = 0.2$. The dashed curve mimicks the effect of the different $\nu_\mu$-$\nu_\tau$ fluxes with $T_\mu = T_\tau = 7.3$ MeV. A distance $D = 8.5$ kpc from the supernova and integrated luminosity $E_\mu = E_\tau = 5 \cdot 10^{52}$ ergs were taken.
Figure 4: The relative Earth matter effect in the neutrino channel, $r$, as a function of the neutrino energy for different values of the trajectory nadir angle, $\theta_n$, and for a number of values of the relative $\nu_\mu - \nu_\tau$ temperature difference $\epsilon_T$. We considered normal mass hierarchy, $P_H = 0$, $\epsilon_L = 0$, $T_\mu = 7$ MeV, $\sin^2 2\theta_{12} = 0.75$ and $\Delta m^2_{21} = 5 \cdot 10^{-5}$ eV$^2$. 
Figure 5: The same as in fig. 4 but for $\epsilon_T = 0$ and different values of the relative differences of luminosities, $\epsilon_L$. 
Figure 6: The relative Earth matter effect in the neutrino channel, $r$, as a function of the neutrino energy for $\epsilon_T = 0.1$ and different values of $\epsilon_L$. We have taken $\theta_n = 0^\circ$; the values of the other parameters are the same as in fig. 4.
Figure 7: The same as in fig. 6 but for the antineutrino channel and inverted mass hierarchy.
Figure 8: The same as in fig. but for $\bar{\epsilon}_T = 0$ and different values of the relative differences of the integrated luminosities $\bar{\epsilon}_L$. 
Figure 9: The spectrum of events expected at SNO from the $\nu_e + d \rightarrow e^- + p + p$ process with and without Earth crossing. We took $\epsilon_L = 0$ and $\epsilon_T = 0.1$, a distance to the supernova $D = 8.5$ kpc, nadir angle $\theta_n = 0^\circ$ and integrated luminosities $E_\mu = E_\tau = 5 \cdot 10^{52}$ ergs. The other parameters are the same as in fig. 8.
Figure 10: The same as fig. 9 but for $\Delta m_{21}^2 = 3 \cdot 10^{-5} \text{ eV}^2$ and $\theta_n = 40^\circ$. 