Mass Ejection in Failed Supernovae: Variation with Stellar Progenitor

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ABSTRACT
We study the ejection of mass during stellar core-collapse when the stalled shock does not revive and a black hole forms. Neutrino emission during the protoneutron star phase causes a decrease in the gravitational mass of the core, resulting in an outward going sound pulse that steepens into a shock as it travels out through the star. We explore the properties of this mass ejection mechanism over a range of stellar progenitors using spherically-symmetric, time-dependent hydrodynamic simulations that treat neutrino mass loss parametrically and follow the shock propagation over the entire star. We find that all types of stellar progenitor can eject mass through this mechanism. The ejected mass is a decreasing function of the surface gravity of the star, ranging from several $M_\odot$ for red supergiants to $\sim 0.1M_\odot$ for blue supergiants and $\sim 10^{-3}M_\odot$ for Wolf-Rayet stars. We find that the final shock energy at the surface is a decreasing function of the core-compactness, and is $\lesssim 10^{47} - 10^{48}$ erg in all cases. In progenitors with a sufficiently large envelope, high core-compactness, or a combination of both, the sound pulse fails to unbind mass. Successful mass ejection is accompanied by significant fallback accretion that can last from hours to years. We predict the properties of shock breakout and thermal plateau emission produced by the ejection of the outer envelope of blue supergiant and Wolf-Rayet progenitors in otherwise failed supernovae.

Key words: gravitation – hydrodynamics – neutrinos – shock waves – stars: black holes – supernovae: general

1 INTRODUCTION
Understanding the connection between progenitor stellar properties and remnant properties after stellar core-collapse has been a longstanding quest in theoretical astrophysics. This includes the goal of explaining the observed neutron star and black hole mass functions (e.g., Özel et al. 2010; Kochanek 2014). The origin and properties of stellar-mass black holes have received renewed attention, and key new empirical constraints, after the recent detection of gravitational waves from binary black hole mergers by Advanced LIGO (Abbott et al. 2016a,b, 2017).

Our current understanding of stellar core-collapse indicates that black holes can form in a failed explosion or by fallback accretion onto a neutron star in a successful core-collapse supernova (e.g., O’Connor & Ott 2011 and references therein). This can occur up to the onset of pair instability in the stellar core at initial stellar masses $\gtrsim 150M_\odot$ (e.g., Kasen et al. 2011 and references therein). If the pair instability mechanism succeeds, no remnant is left behind. For high enough masses ($\gtrsim 250M_\odot$), however, black holes can form promptly since explosive nuclear burning is unable to reverse the collapse (Fryer et al. 2001).

Direct observational evidence for black hole formation was reported recently (Adams et al. 2017) as part of a survey looking for disappearing massive stars (Kochanek et al. 2008; Gerke et al. 2013). A weak optical transient reaching $\sim 10^3L_\odot$ and lasting for about a year was observed from a red supergiant progenitor, with the flux subsequently decaying by a factor of 6 below the pre-outburst level. At late-times, the bolometric luminosity decays as $t^{-4/3}$, which has been attributed to fallback accretion after the collapse to a black hole. The detection of such a transient implies that a fraction $\sim 10\%$ of all core-collapse supernovae result in failures (Adams et al. 2017). A significant fraction of

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failed supernovae can help explain the absence of massive \((\sim 20 - 25M_\odot)\) red supergiant progenitors associated with Type IIp supernovae and the apparent mass gap between neutron star masses and stellar mass black hole masses \((Kochanek\text{[2014]})\).

The full landscape of observational signatures of black hole formation is not yet well understood, however. Different scenarios are predicted based on the importance of angular momentum. A failed supernova in a star with sufficient rotation should form a black hole accretion disk, from which transients ranging from long gamma-ray bursts to fusion-powered explosions can be generated \cite{Bodenheimer1983, Woosley1993, MacFadyen1999} (see also \cite{Kashiyama2015}).

If rotation is unimportant, a transient associated with black hole formation in a failed supernova can still be generated due to the loss of gravitational mass to neutrinos after collapse. Below the onset of pair instability, black hole formation due to the loss of gravitational mass to neutrinos after black hole formation in a failed supernova can still be generated \cite{Woosley1983, Woosley1993, MacFadyen1999, MacFadyen2001, Ertl2016}. In addition, our understanding of this mass ejection effect explicitly with time-dependent numerical simulations for the case of red supergiant progenitors. The ejecta gives rise to a transient lasting \(\sim 1\) yr, with the main source of power for the light curve being hydrogen recombination. The predicted properties of these transients are broadly similar to the event discovered by \cite{Adams2017}. In addition to long time-scale recombination-powered emission, \cite{Piro2013} showed that shock breakout from these events should be detectable, and \cite{Lovegrove2017} recently computed the shock breakout signal with radiative transfer simulations.

While the neutrino mass loss mechanism can operate in any failed supernova progenitor, its predictions have only been explored for red supergiants. In particular, Wolf-Rayet stars are also strong candidates for black hole formation given the higher compactness of their cores \cite[e.g.,][]{OConnor2013}. This decrease in mass generates an outgoing sound pulse that can steepen into a shock and eject the outer layers of the star \cite{Nadyozhin1980}. \cite{Lovegrove2013} demonstrated this mass ejection effect explicitly with time-dependent numerical simulations for the case of red supergiant progenitors. The ejecta gives rise to a transient lasting \(\sim 1\) yr, with the main source of power for the light curve being hydrogen recombination. The predicted properties of these transients are broadly similar to the event discovered by \cite{Adams2017}. In addition to long time-scale recombination-powered emission, \cite{Piro2013} showed that shock breakout from these events should be detectable, and \cite{Lovegrove2017} recently computed the shock breakout signal with radiative transfer simulations.

We will explore the physics of the neutrino mass loss mechanism for a range of stellar progenitors that cover different values of the core compactness and envelope compactness, including blue supergiants and Wolf-Rayet stars. We will also provide an analytic derivation of the maximum kinetic energy imparted to the outflow from progenitor properties, finding favorable agreement with the results of our simulations (a more detailed analytic description of the relevant physics is given in \cite{Conglin2017}). Finally, we compute the properties of the fallback accretion resulting from these models and estimate the observational signatures of the weak explosions for different progenitors.

Our study makes a number of approximations in order to decrease the computational cost and to efficiently explore parameter space. The main simplification is the parametric treatment of the loss of gravitational mass by the inner core (following LW13). Also, we neglect radiation diffusion, which can alter the dynamics of the ejecta at late times. Nevertheless, many aspects of the physics, including the propagation of the pressure wave inside the star and the transition to an outgoing shock, should be robust and are unlikely to change with further improvements in the modeling of the inner core and radiation transport.

The paper is structured as follows. Section \(2\) discusses our numerical methods, including the progenitors chosen, the setup of our hydrodynamic simulations, the approximations used to model neutrino mass loss, and the parameter range covered by our models. Section \(3\) contains a derivation of the maximum kinetic energy imparted to the outgoing shock, and an estimate of the maximum mass ejected. Section \(4\) describes our numerical results, showing first an overview of fiducial progenitors, analyzing the energetics of mass ejection, surveying the ejecta properties for various progenitors, discussing failed models, and analyzing the properties of fallback accretion. Section \(5\) discusses observational implications, and Section \(6\) summarizes our results. Appendix \(A\) shows tests of our numerical implementation, and Appendix \(B\) contains a derivation of our semianalytic approximation to the fallback accretion rate at small radii.

2 METHODS

2.1 Progenitor Models

We consider non-rotating presupernova stellar models computed with the MESA stellar evolution code version 6794 \cite{Paxton2011, Paxton2013, Paxton2015}, \cite{?}. \(7\). We generate two sets of models, one with solar metallicity and another with \(Z = 10^{-2}Z_\odot\). Each set has main sequence masses in the range \(12 - 100M_\odot\). Models are generated using the same parameters as in \cite{Fuller2015}, particularly their choice of overshoot parameters and the use of the ‘Dutch’ wind model \cite{deJager1988, Nugis2000, Vink2001}. In addition, we set \(\text{Z}_{\text{base}}\) (for opacities) equal to the initial stellar metallicity, turn on the velocity for the entire evolution, and employ the simpler \(\alpha\)-chain \text{aprox21} nuclear network for advanced burning stages, which significantly shortens execution times. The contribution of radiation to the photospheric pressure is set to its default value for massive stars \(\text{P}_{\text{extra}} = -1\). The inlist files used to generate all of our progenitors are publicly available \(1\).

Table \(1\) shows a sample of models from our complete set, which covers a representative range in mass, radius, and envelope binding energies. The discussion will focus on these progenitors, for conciseness. Models are labeled first by the type of star \(\text{R}: \text{RSG}, \text{Y}: \text{YSG}, \text{B}: \text{BSG}, \text{W}: \text{WR}\) and then by their initial Zero-Age Main Sequence (ZAMS) mass and metallicity, e.g. \(\text{R}12z00\) is the \(12M_\odot\) ZAMS with solar metallicity. Each set has main sequence masses in the range \(12 - 100M_\odot\). Models are generated using the same parameters as in \cite{Fuller2015}, particularly their choice of overshoot parameters and the use of the ‘Dutch’ wind model \cite{deJager1988, Nugis2000, Vink2001}. In addition, we set \(\text{Z}_{\text{base}}\) (for opacities) equal to the initial stellar metallicity, turn on the velocity for the entire evolution, and employ the simpler \(\alpha\)-chain \text{aprox21} nuclear network for advanced burning stages, which significantly shortens execution times. The contribution of radiation to the photospheric pressure is set to its default value for massive stars \(\text{P}_{\text{extra}} = -1\). The inlist files used to generate all of our progenitors are publicly available \(1\).

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Also shown in Table \(2\) is the compactness parameter of the stellar core, which has been shown to correlate well with the onset of BH formation \cite{OConnor2013, Ugliano2012, Sukhbold2014, Horinnchi2014}.

\(^1\) https://bitbucket.org/rafernan/bhsn_mesa_progenitors
where according to Pejcha & Thompson (2015). We compute this parameter in the range $\xi_{\text{physics}}$ indicate that the critical value for BH formation lies $M = 2M_\odot$, in solar units $M_\odot$. Core-collapse simulations that include more physics indicate that the critical value for BH formation lies in the range $\xi_{2.5} \approx 0.2 - 0.4$, with more compact stars forming BHs. The presupernova models in our sample cover this range and higher values of $\xi_{2.5}$, with the exception of the $12M_\odot$ solar metallicity model, which is evolved for comparison.

To quantify the degree of binding of the stellar envelope, we also compute a global compactness at the time of core-collapse (Table 1). We use $\xi_{\text{env}}$ as a proxy for the surface gravity of the star. Note that the escape speed from the stellar surface is given by $v_{\text{esc}} \approx 600\xi_{\text{env}}^{1/2}$ km s$^{-1}$.

### 2.2 Numerical Hydrodynamics

We model stellar collapse by solving the time-dependent hydrodynamic equations in spherical symmetry using FLASH3 (Fryxell et al. 2000; Dubey et al. 2009),

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial r}(\rho v_r) = 0,$$  

$$\frac{\partial v_r}{\partial t} + \frac{\partial}{\partial r}(\rho v_r v_r) = 0,$$  

$$\frac{\partial p}{\partial t} + \frac{\partial}{\partial r}(p v_r) = 0,$$

where $p$, $v_r$, $\rho$, and $M(r, t)$ are the fluid density, radial velocity, specific internal energy, pressure, and enclosed mass at radius $r$, respectively, and $D/Dt \equiv \partial/\partial t + v_r \partial/\partial r$. The system of equations (3)-(5) is closed with the Helmholz equation of state (Timmes & Swesty 2000) and solved with the split Piecewise Parabolic Method (Colella & Woodward 1984; Fryxell et al. 1989). We do not include any weak interactions or nuclear burning, since these processes do not influence the dynamics (other than via neutrino mass loss in the protonneutron star, which is parameterized; see (2)).

The computational domain extends from a minimum radius near the edge of the iron core, $R_{\text{min}} = 2 \times 10^8$ cm, to a maximum radius $R_{\text{max}} = 2 \times 10^{10}$ cm, well outside the radius of the largest red supergiant (RSG) in our sample. Smaller outer boundary radii are taken for smaller stars. The grid has logarithmic spacing, with 256 cells per decade in radius, or equivalently a fractional cell size $\Delta r/r \approx 0.9\%$. Outflow boundary conditions are used at both ends of the computational domain. All primitive variables are copied from the active cell adjacent to the boundary into the ghost cells. The radial velocity has an additional $r^{-2}$ dependence so that the mass flux is constant in the ghost cells. No mass is allowed to enter the domain.

We implement a remapping procedure to move the inner boundary outward at specific times in the simulation. The ongoing pressure wave forms within a collapsing medium, and most of the material inside this wave falls toward the center at supersonic speeds. The minimum time step in the simulation is almost always set by the Courant condition at the inner boundary, where material is undergoing supersonic

| Model  | $M_{\text{suns}}$ ($M_\odot$) | $Z$ ($Z_\odot$) | $M_{\text{cc}}$ ($M_\odot$) | $R_{\text{cc}}$ ($10^{11}$ cm) | $L_{\text{cc}}$ ($10^5 L_\odot$) | $T_{\text{eff}}$ (10$^3$ K) | Type | $\xi_{2.5}$ | $\xi_{\text{env}}$ |
|--------|-----------------|----------------|-----------------|-----------------|-----------------|-----------------|-------|-------------|-------------|
| R12z00 | 12              | 1              | 10.0            | 1110            | 770             | 0.9             | 3     | RSG        | 0.15        | 0.009       |
| R15z00*| 15              | 1              | 10.8            | 1060            | 740             | 1.3             | 3     | RSG        | 0.24        | 0.010       |
| Y22z00 | 22              | 1              | 11.1            | 690             | 480             | 2.9             | 5     | YSG        | 0.54        | 0.016       |
| B25z00*| 25              | 1              | 11.7            | 96              | 70              | 3.8             | 15    | BSG        | 0.33        | 0.12        |
| W26z00 | 26              | 1              | 11.9            | 1.1             | 0.8             | 4.0             | 140   | WR         | 0.21        | 10.8        |
| W40z00*| 40              | 1              | 10.3            | 0.38            | 0.3             | 5.7             | 260   | WR         | 0.37        | 27.1        |
| W50z00 | 50              | 1              | 9.2             | 0.42            | 0.3             | 3.4             | 215   | WR         | 0.55        | 21.9        |
| Y25z-2 | 25 $10^{-2}$    | 23.0           | 940             | 650             | 3.4             | 4.6             | YSG   | 0.25       | 0.024       |
| B30z-2 | 30              | 16.0           | 145             | 100             | 5.9             | 13.3            | BSG   | 0.34       | 0.11        |
| B80z-2 | 80              | 55.2           | 70              | 50              | 28              | 28.4            | BSG   | 0.97       | 0.79        |

$\xi_{\text{cc}} = \frac{M_{\text{cc}} / M_\odot}{(R_{\text{cc}} / R_\odot)}$.

We have also computed the parameters $M_4$ and $\mu_4$ as defined by Ertl et al. (2016), which can in principle serve as a more stringent predictor of the threshold for black hole formation. However, using the threshold curves in Ertl et al. (2016) would predict that all of our models – including the $12M_\odot$ solar metallicity model – should form BHs, even after computing these two parameters when the central density of stellar models reaches $5 \times 10^{10}$ g cm$^{-3}$ as in their progenitors.
infall. In order to speed up calculations, we remove the inner-most decade in radius from the computational domain once this region has achieved supersonic infall in its entirety and therefore loses causal contact with the rest of the computational domain (following an approach similar to Hammer et al. [2010]). The procedure is first carried out at \( t = 100 \) s and repeated after every decade in time, unless conditions make it infeasible (e.g. if a reverse shock modifies the velocity field). Since the grid is logarithmic, the minimum time step increases by at least a factor ten each time the procedure is repeated. This allows us to follow the evolution of the shock all the way to the surface of the largest RSGs. More details on this procedure are provided in Appendix A.

The enclosed mass \( M(r, t) \) used to compute the gravitational acceleration in equation (4) is the sum of the gravitational mass inside the inner boundary \( M_c(t) \) and the mass in the computational domain interior to the radius \( r \),

\[
M(r, t) = M_c(t) + 4\pi \int_{R_{\text{min}}}^{r} x^2 \, dx \rho(x, t).
\] (6)

The mass flowing through the inner boundary is added to the baryonic mass \( M_B(t) \) inside \( R_{\text{min}} \),

\[
\dot{M}_B = 4\pi R_{\text{min}}^2 \rho(R_{\text{min}}, t) \max[-v_r(R_{\text{min}}, t), 0].
\] (7)

Equation (7) is integrated using the mass flux obtained from the Riemann solver at the inner boundary, maintaining overall mass conservation close to machine precision (see Appendix A). The gravitational mass \( M_c \) is related to the baryonic mass \( M_B \) through neutrino mass loss, which we discuss in the next subsection.

The specific position of inner boundary can have a \( \sim 10\% \) effect on the shock energy. Appendix A presents test models exploring the impact of this choice, as well as other checks on our numerical implementation.

### 2.3 Evolution of the Inner Core

Properly modeling the loss of gravitational mass by the protoneutron star prior to black hole formation would require neutrino radiation hydrodynamic simulations in general relativity (as in, e.g., O’Connor & Ott [2011]). At present there is no equation of state that smoothly connects the high-density regime required to model the supernova core with the very low density regime needed to follow the shock beyond the stellar surface, thus an approach that employs multiple simulation codes would be needed for self-consistent calculations.

Instead, we choose to parameterize the evolution of the inner supernova core by using approximations similar to those introduced in LW13. By default, we use a simple exponential neutrino cooling model, which is a slight variant of the models considered in LW13. We assume that

\[
\dot{M}_G = \dot{M}_B - \frac{\dot{d}BE_c}{\tau_c} M_G e^{-t/\tau_c},
\] (8)

where \( \tau_c \) is a fiducial neutrino cooling timescale, and

\[
\dot{d}BE_c \simeq 0.084 \left( \frac{M_G}{M_\odot} \right)^2 M_\odot.
\] (9)

is the gravitational binding energy of a cold neutron star (Lattimer & Yahil [1989] Prakash et al. [1997] Lattimer & Prakash [2001]), obtained as a numerical fit to the relation

\[
\dot{d}BE_c = M_B - \dot{M}_G
\] (10)
in cold neutron star models constructed with various equations of state.

For comparison, we also include the full loss model of LW13, which accounts for the thermal energy stored in the protoneutron star,

\[
\dot{M}_G = \dot{M}_B - (1 - \epsilon) \frac{\dot{d}BE_c}{\tau_c} M_B + \dot{M}_\text{th},
\] (11)

where the thermal mass-energy evolution evolves according to

\[
\dot{M}_\text{th} = -\frac{M_{\text{th}}}{\tau_c} + \epsilon \frac{\dot{d}BE_c}{\tau_c} M_B.
\] (12)

with \( \epsilon < 0.5 \) a thermalization factor. In equation (11), a fraction \((1 - \epsilon)\) of the binding energy of accreted matter is immediately radiated away in neutrinos, while the remaining fraction is temporarily stored as thermal energy in the protoneutron star (equation 12). The derivative of \( \dot{d}BE_c \) is obtained from equation (10):

\[
\frac{\dot{d}BE_c}{\dot{M}_B} = \frac{2\dot{BE}_e - 2\dot{BE}_c}{M_G + 2\dot{BE}_c}.
\] (13)

Finally, we also include the maximum loss model of LW13

\[
\dot{M}_G = \dot{M}_B - \frac{\dot{BE}_e(M_{\text{th}})}{\tau_c} e^{-t/\tau_c},
\] (14)

which is identical to the default exponential model except that the binding energy in equation (10) is evaluated at the maximum mass of a cold neutron star \( M_{\text{tov}} \) (corresponding to the Tolman-Oppenheimer-Volkov limit, or TOV for short) supported by the equation of state of nuclear matter.

In the exponential and maximum loss models, neutrino cooling stops when the gravitational mass of the star reaches \( M_{\text{tov}} \). In the full loss model, cooling stops when \( M_G = M_{\text{th}} \) reaches \( M_{\text{tov}} \). Beyond this point, we impose \( \dot{M}_G = \dot{M}_B \).

While more sophisticated models of protoneutron star cooling find a nearly power-law evolution of the neutrino luminosity with time instead of an exponential (e.g., Pons et al. [1999] Roberts [2012]), we employ the simpler parameterizations of LW13 for continuity. In all of our models, we use \( \tau_c = 3 \) s, which is a characteristic timescale over which the neutrino luminosity decreases by a factor of \( \sim 3 \) in more detailed calculations. A more thorough exploration of the effect of this input physics on mass ejection should employ a neutrino transport code.

### 2.4 Initial Condition and Models Evolved

For a given presupernova model, the stellar profile data is mapped into FLASH using interpolation at cell centers. To minimize transients, the mapping uses pressure, density, and mass fractions as independent quantities, recovering the remaining thermodynamic variables using the Helmholtz EOS. The radial velocity is mapped independently, and the remaining thermodynamic variables using the Helmholtz EOS.

The initial condition for equations (7), (8), (11) and (12) is \( M_G = M_B \) and \( M_{\text{th}} = \dot{BE}_c \), with \( M_B \) equal to the mass enclosed by \( r = R_{\text{min}} \) in the progenitor star. In all of our models, we take \( \epsilon = 0.1, \tau_c = 3.0 \) s, and \( M_{\text{tov}} = 2.5 M_\odot \). The latter is the maximum value allowed by causality constraints (e.g., Lattimer & Prakash [2016]).
For numerical reasons, the region outside the star is filled with a constant density ambient medium in hydrostatic equilibrium. The ambient density is chosen to be \( \rho_{amb} = 10^{-18}, 10^{-16}, 5 \times 10^{-13} \) g cm\(^{-3}\) for RSGs/YSGs, BSGs, and WRs, respectively, with the exception of the 50\(M_\odot\) WR for which the ambient density is \( 10^{-12} \) g cm\(^{-3}\). These values are such that the mass in the ambient medium is much smaller than the characteristic ejecta mass over the distances considered, thus avoiding artificially slowing down the ejecta. We note, however, that observed WR winds have densities above these values at radii \( \lesssim 10R_\odot \). If such winds are also present at core collapse, it would modify the shock breakout properties estimated in [3].

In order to implement these low ambient medium densities, the Helmholz EOS is extended below its lower density limit of \( 10^{-10} \) g cm\(^{-3}\) by assuming an ideal gas of electrons instead of its standard table for electrons and positrons. The floor of temperature in the simulation is set to the lower limit of the Helmholz table, \( T_b = 10^4 \) K, at which point hydrogen is still fully ionized. Simulations are stopped when the temperature inside the shock approaches this floor value.

The list of hydrodynamic models evolved is shown in Table 2. Our default neutrino mass loss scheme is the exponential model (equation 5), which we use in all our progenitors. Model names using this prescription have ‘\( e\)’ appended to their names. We adopt three baseline stellar models for more detailed study: the 15\(M_\odot\) solar metallicity RSG (R15z00), the 25\(M_\odot\) solar metallicity BSG (B25z00) and the 40\(M_\odot\) solar metallicity WR (W40z00). These three stellar models are evolved at twice our baseline spatial resolution to test convergence in mass ejection (\( \Delta r/r = 0.45\% \)), and the corresponding model names have ‘HR’ appended to them. We also repeat the three baseline progenitors using the full loss model (equation 11) and maximum loss model (equation 14). These model names have ‘\( f\)’ and ‘\( m\)’ appended to them, respectively.

The maximum simulation time depends on the structure of the progenitor. The RSG model is evolved up to \( 10^7 \) s, shortly after the shock breaks out of the stellar surface. The stopping time is set by the moment when the fluid behind the shock approaches the floor of temperature in the Helmholz EOS (\( 10^4 \) K), at which point simulations are no longer reliable (in particular, the internal energy starts to grow and total energy is not conserved). The BSG and WR models are evolved to \( 10^6 \) s and \( 10^4 \) s, respectively. The criterion for stopping here is to achieve nearly constant kinetic and total energies in the ejected shell while at the same time not sweeping up so much mass in the ambient medium that the shell starts to slow down.

3 ENERGY BUDGET FOR THE OUTGOING SOUND PULSE AND SHOCK

The change in gravitational mass \( \delta M_G \) due to neutrino emission occurs over a finite timescale

\[
\tau_c = \min\{\tau_c, \tau_{tov}\},
\]

where \( \tau_c \) is the neutrino cooling time in the protoneutron star and \( \tau_{tov} \) is the time to collapse to a BH. Changes in the gravitational acceleration act on the different stellar layers.

Table 2. List of hydrodynamic models evolved and summary of results. Columns from left to right show model name, spatial resolution, type of neutrino mass loss (\( exp: \) exponential, \( full: \) full loss, \( max: \) maximum loss), \( 2.3\), cooling time, time to reach TOV\( mass, \) total gravitational mass lost, ejecta mass, total ejecta energy, maximum kinetic energy, radius at which \( \tau_c = \min\{\tau_c, \tau_{tov}\} \) in the progenitor, analytic energy estimate (equation 20), and analytic ejecta mass estimate (equation 21). No ejecta mass or energy is assigned to failed models except for \( Y25z-2 \) in the progenitor, analytic energy estimate (equation 20), and analytic ejecta mass estimate (equation 21).

| Model          | \( \Delta r/r \) (%) | \( \nu\)-loss | \( \tau_c \) (s) | \( \tau_{tov} \) (s) | \( \delta M_G \) (\( M_\odot \)) | \( M_{ej} \) (\( M_\odot \)) | \( E_{ej} \) (\( 10^{47} \) erg) | \( E_{k,\max} \) (\( 10^{47} \) erg) | \( r_c \) (\( 10^6 \) cm) | \( \Delta E(r_c) \) (\( 10^{47} \) erg) | \( \Delta M \) (\( M_\odot \)) |
|----------------|----------------------|---------------|-----------------|---------------------|-----------------------------|-----------------------------|-----------------------------|-------------------------------|-----------------------------|--------------------------------|-----------------------------|
| R15z00_e       | 0.9                  | exp           | 3               | 6.0                 | 3.0                        | 4.2                         | 1.5                         | 4.7                           | 1.5                         | 9.9                            | 4.8                         |
| B25z00_e       | 3.1                  | 0.24          | 4.9E-2          | 1.5                 | 4.5                         | 1.7                         | 12                          | 0.18                          |                             |                               |                             |
| W40z00_e       | 2.6                  | 0.22          | 5.0E-4          | 0.23                | 3.5                         | 1.5                         | 11                          | 8E-3                         |                             |                               |                             |
| R15z00_eHR     | 0.45                 | exp           | 3               | 6.0                 | 3.0                        | 4.2                         | 1.9                         | 4.5                           | 1.5                         | 9.9                            | 4.8                         |
| B25z00_eHR     | 3.1                  | 0.24          | 4.9E-2          | 1.6                 | 4.4                         | 1.7                         | 12                          | 0.18                          |                             |                               |                             |
| W40z00_eHR     | 2.6                  | 0.22          | 5.0E-4          | 0.25                | 3.4                         | 1.5                         | 11                          | 8E-3                         |                             |                               |                             |
| R12z00_e       | 0.9                  | exp           | 3               | 21.0                | 5.5                        | 1.8                         | 3.9                         | 1.4                           | 5.9                         | 5.6                            |                             |
| Y22z00_e       | 1.1                  | 0.12          | ...             | ...                 | 0.4                         | 0.8                         | 4.9                          | 1.2                           |                             |                               |                             |
| Y25z-2_e       | 5.3                  | 0.30          | 2.5             | -1.0                | 8.1                         | 1.5                         | 19                          | 11                            |                             |                               |                             |
| B30z-2_e       | 4.0                  | 0.30          | 0.2             | 1.4                 | 10                          | 1.6                         | 19                          | 1.1                           |                             |                               |                             |
| B80z-2_e       | 0.2                  | 0.03          | ...             | ...                 | 0.03                        | 0.23                        | 1.1                          | 0.03                          |                             |                               |                             |
| W25z00_e       | 6.9                  | 0.30          | 8.1E-3          | 2.6                 | 10                          | 1.5                         | 20                          | 0.03                          |                             |                               |                             |
| W50z00_e       | 1.2                  | 0.13          | 5.7E-5          | 0.02                | 0.63                        | 0.9                         | 5.0                          | 5E-3                          |                             |                               |                             |
| R15z00_f       | 0.9                  | full          | 3               | 8.0                 | 4.7                        | 8.8                         | 12                          | 1.5                           | 25                           | 4.9                            |                             |
| B25z00_f       | 4.2                  | 0.43          | 0.11            | 9.1                 | 18                          | 1.7                         | 40                          | 0.30                          |                             |                               |                             |
| W40z00_f       | 3.6                  | 0.42          | 4.9E-3          | 3.0                 | 17                          | 1.7                         | 35                          | 0.03                          |                             |                               |                             |
| B80z-2_f       | 0.4                  | 0.04          | ...             | ...                 | 0.05                        | 0.42                        | 1.3                          | 0.04                          |                             |                               |                             |
| R15z00_m       | 0.9                  | max           | 3               | 8.4                 | 4.8                        | 13                          | 17                          | 1.5                           | 27                           | 4.9                            |                             |
| B25z00_m       | 3.7                  | 0.37          | 9.5E-2          | 7.0                 | 15                          | 1.7                         | 30                          | 0.27                          |                             |                               |                             |
| W40z00_m       | 3.0                  | 0.33          | 2.6E-3          | 1.5                 | 11                          | 1.7                         | 22                          | 0.02                          |                             |                               |                             |
on the local free-fall time
\[ t_{ff}(r) = \left( \frac{r^3}{GM(r)} \right)^{1/2}, \tag{16} \]
which generally is an increasing function of radius. Regions in the collapsing star for which \( t_{ff} \ll \tau_\nu \) fall onto the black hole without experiencing any significant change in their gravitational acceleration; conversely, regions that satisfy \( t_{ff} \gg \tau_\nu \) respond instantaneously to the change in gravity (Figure 1). The transition between these two regimes lies at a radius \( r_c \) such that
\[ t_{ff}(r_c) = \tau_\nu. \tag{17} \]
For a wide range of stellar masses, this radius has a characteristic value \( r_c \sim 10^9 \) cm for a neutrino cooling timescale of \( \tau_\nu \sim 3 \) s (c.f. Table 2).

Once neutrinos change the gravitational mass by \( \delta M_G \), the inward gravitational force on each mass shell has decreased and thus there is a net outward acceleration due to the excess pressure gradient, with magnitude
\[ a = \frac{G\delta M_G}{r^2}. \tag{18} \]
The outward acceleration produces a net outward velocity over the free-fall time
\[ v = a t_{ff} = \sqrt{\frac{GM(r)}{r} \frac{\delta M_G}{M(r)}}. \tag{19} \]
Note that this velocity is everywhere much less than the local escape speed since \( \delta M_G \ll M(r) \).

The total energy imparted to a given mass shell by the net outward pressure force is given by
\[ \Delta E(r) \simeq \frac{1}{2} M_{shell} v^2 \simeq \frac{G\delta M_G H}{2r} \]
\[ \simeq 1 \times 10^{49} \left( \frac{\alpha}{0.4} \right) \left( \frac{H/\nu}{0.4} \right) \left( \frac{\delta M_G}{0.3 M_\odot} \right) \left( \frac{2 \times 10^9 \text{cm}}{r} \right) \text{erg} \tag{20} \]
where \( M_{shell} = 4\pi r^2 \rho H = (H/\nu) dM(r)/d\ln r \) and \( \alpha \simeq d\ln M(r)/d\ln r \), with \( H \) the pressure scale height. For both super-giant and compact progenitors with \( \gtrsim 25 M_\odot \) we have \( H/\nu \sim 0.3 - 0.5 \) and \( \alpha \sim 0.4 - 0.7 \) at \( r \sim r_c \sim 2 \times 10^9 \) cm.

Figure 2 shows \( \Delta E(r) \) as a function of the initial free-fall time at a given radius for our baseline stellar progenitors. The energy input on the shock is largest at small radii, where the induced outwards pressure force is largest. Note, however, that the vertical dotted line in Figure 2 is where the free-fall time is equal to \( \tau_\nu \sim 3 \) s. Interior to this radius, the energy injection is strongly suppressed relative to equation (20) because the mass is incorporated into the neutron star prior to most of the neutrino binding energy being radiated away. Thus the maximum energy input into the star by the change in gravitational mass is given by equation (20), evaluated at \( r \sim r_c \). Coughlin et al. (2017) present a more detailed derivation of the sound wave excitation, propagation, and energetics induced by neutrino mass loss. Up to factors of order unity (e.g., \( \sim H/\nu \)), their results are consistent with Figure 2 and our derivation here.

The maximum amount of mass that can be ejected by the shock can be estimated by considering the net energy (internal plus gravitational) of the outermost stellar layers (Figure 3). If the total energy of the shock is bounded by \( \Delta E(r_c) \), then the maximum amount of mass \( \Delta M \) than can be ejected by the shock is the outermost mass shell with a...
net energy equal to \( \Delta E(r_e) \),
\[
\Delta E(r_e) = \int_{M_{cc} - \Delta M}^{M_{cc}} (-\epsilon_{tot}) \, dM,
\]
where \( \epsilon_{tot} \) is the total specific energy of stellar material (generally negative). Figure 2 shows that \( \Delta M \) should be a decreasing function of the degree of gravitational binding at the surface of the star, which can be quantified by the surface gravity or the envelope compactness \( \xi_{env} \).

4 NUMERICAL RESULTS

4.1 Overview of Sound Pulse & Shock Evolution

In the absence of a change in the gravitational mass due to neutrino emission, the star simply collapses from the inside out and the velocity is everywhere negative, with the possible exception of the regions outside the stellar surface (see Appendix A for numerical tests involving pure collapse).

When neutrino mass loss operates, a pressure wave is driven due to the change in gravitational acceleration. Figure 3 shows snapshots of the radial velocity, Mach number, density, and temperature in the three fiducial progenitors.

The outgoing wave forms as a sound pulse in the vicinity of \( r = r_e \) over a timescale \( \tau_e = \min(\tau_{cc}, \tau_{cov}) \) (c.f. [3]). Initially, this pulse is subsonic, but as it propagates out, its leading edge gradually steepens into a shock. Three features are noticeable from the velocity and Mach number snapshots in Figure 3. First, once the Mach number exceeds a value \( \sim 0.1 \), the outgoing pressure wave is bound by leading and trailing edges that can be clearly defined. Second, while the leading edge eventually becomes supersonic, its Mach number is only slightly larger than unity while inside the star. Finally, note also that the entire portion of the star outside the wave acquires positive velocity, as implied by equation (18).

Upon reaching the stellar surface, the Mach number at the leading edge of the shock increases, as does the distance between leading and trailing edges. This behavior is expected from the fact that the density and sound speed in the star decrease very steeply with distance from the stellar surface, leading to acceleration of the shock (e.g., Matzner & McKee 1999).

The formation of the pressure wave is similar in all stellar progenitors, hence the evolution at times \( t \lesssim 10 \) s is qualitatively and even quantitatively similar in all cases. This is expected given that the interior structure near \( r_e \) is similar in different models at the onset of core-collapse. Noticeable differences appear once the wave propagates into the stellar envelope. In the RSG case, the shock velocity reaches several 100 km s\(^{-1}\) throughout the stellar interior, with no significant increase upon breakout from the stellar surface. In the BSG model, upon shock breakout the velocity of most of the mass jumps to a few 1000 km s\(^{-1}\), while in the WR case the velocity can reach a few 10\(^4\) km s\(^{-1}\). The general trend is therefore larger shock breakout velocities for increasing envelope compactness \( \xi_{env} \).

Note, however, that despite having converged in mass ejection with resolution, our fiducial models are not fully resolving the outermost layers of the star and therefore do not fully capture shock acceleration during breakout. The gas pressure scale heights at the photosphere in the fiducial RSG, BSG, and WR progenitors are \( H_{\text{phot}}/R_{cc} = 0.01, 0.02, \) and 0.002, respectively, while our highest resolution models have \( \Delta r/r \simeq 0.005 \), thus barely resolving RSG and BSG photospheres and under-resolving WR surfaces. This likely accounts for the behavior of the final ejecta energies in Table 2 which increase by \( \sim 10\% - 30\% \) when doubling the resolution in the fiducial models.

The radial velocities of the leading and trailing edges of the wave are shown in the lower row of Figure 5. The trailing edge \( v_{tr} \), defined as the point at which the velocity changes sign, propagates at a speed very close to the local sound speed. The leading edge \( v_{sh} \) moves at a speed slightly faster than the trailing edge, with the speed difference increasing in magnitude as the leading edge travels into the low density stellar envelope. The speed of the leading edge eventually exceeds the local escape speed, either deep inside the star (RSG and BSG) or very near the surface as it accelerates (WR).

Given the relative weakness of the outgoing shock, the jump in density and temperature is small while the outgoing wave is inside the star. In particular, a shock develops only in regions where the temperature is lower than \( 10^9 \) K, hence explosive nuclear burning is not expected.

For the two RSG cases studied, we obtain shock velocities of the order of a few 100 km s\(^{-1}\) and ejecta masses of the order of a few solar masses, in reasonable agreement with the results of LW13.

4.2 Energetics

The evolution of the energies contained within the pressure wave are shown in Figure 5 for the three fiducial progenitors.
These are calculated by integrating over radii between the leading and trailing edges. In all cases, the kinetic energy $E_k$ initially increases, reaching a maximum value that is close to (but smaller than) the analytical estimate $\Delta E(r)$ evaluated at the radius $r_c$ where $t_{dd} \approx \tau_w$.

The weakness of the shock while deep inside the star is expected given that its energy is much smaller than the local thermal energy $\epsilon$ (Tian et al. 2001). In hydrostatic equilibrium, the latter is close to the gravitational binding energy. For BSG and WR progenitors, the characteristic binding energy is $\sim 10^{51}$ erg for most of the stellar interior, whereas in RSGs the more weakly bound H envelope (Figure 3) provides conditions for a shock to develop deeper in the star.

The existing theory of shock propagation and breakout in stellar interiors (Sedov 1959; Sakurai 1960; Chevalier 1976, 1982; Nadyozhin 1985; Chevalier & Soker 1989; Kazhdan & Murzina 1992; Matzner & McKee 1999) assumes a strong shock in which gravity is negligible, which is not applicable in this problem except when the shock is very close to the stellar surface.

The internal and gravitational energies of the pressure wave, $E_i$ and $E_g$ respectively, are initially much higher than the kinetic energy. As the wave propagates out, $E_i$ and $E_g$ decrease in magnitude. The detailed interplay between internal, gravitational, and kinetic energy after the maximum in $E_k$ depends on the structure of the progenitor.

For the RSG model (R15z00.eHR), the wave acquires positive total energy (black line in Fig. 5) upon reaching the base of the hydrogen envelope at time $t \sim 10^5$ s. This position coincides with a steep radial drop in the binding energy of the outermost layers of the star (Figure 3). By this time the leading edge has steepened into a shock with Mach number $\sim 1.5$. The internal energy stabilizes thereafter, and the gravitational energy reaches a minimum, increasing af-
Mass Ejection in Failed Supernovae

Figure 5. Evolution of the energy, mass, and velocity in the outgoing pressure wave for models R15z00_eHR (left column), B25z00_eHR (middle column), and W40z00_eHR (right column). Tracking of the leading and trailing edge of the wave, \( r_{sh} \) and \( r_{tr} \) respectively (1047), begins once the maximum Mach number exceeds 0.03, for clarity. The vertical dashed line denotes the time at which the leading edge of the pressure wave reaches the stellar radius \( R_{c, s} \) (i.e., shock breakout). Top row: Kinetic energy \( E_k \), internal energy \( E_i \), gravitational energy \( E_g \), and total energy \( E_{tot} \) in the wave. The horizontal dotted line shows the analytic estimate of the maximum kinetic energy of the shock (equation 15) evaluated at a radius \( r_c \), where \( t_f(r_c) = \min(t_{cc}, t_{fbr}) \). Middle row: Mass contained in the outgoing pressure wave (solid black). The red dashed curve shows the total mass with positive velocity, excluding ambient medium. Bottom row: Radial velocity of the leading and trailing edges of the wave, \( v_{rad} \) and \( v_{tr} \), respectively. Also shown are the local sound speed at the trailing edge \( c_s(r_{tr}) \), and the local escape speed at the leading edge \( v_{esc}(r_c) \). The edge velocities are smoothed with a Savitsky-Golay filter to suppress noise from numerical differencing.

Laterward as the shock sweeps up mass. Before reaching the stellar surface, the internal and gravitational energies are higher than the kinetic energy. After shock breakout, the kinetic energy quickly increases to the point at which it matches \( E_i \) and \( E_g \). Since the fluid behind the shock reaches the floor of temperature in the EOS at this point (104 K), the subsequent numerical evolution is unreliable and we do not show it. But assuming that a fraction of the remaining internal energy is used up in adiabatically expanding the shell, we infer that the asymptotic energy of the shell should remain within a factor of a few of the value quoted in Table 2 (\( \sim 2 \times 10^{47} \) erg, measured at \( t = 10^7 \) s).

For both the BSG and WR models (B25z00_eHR and W40z00_eHR, respectively), the internal and gravitational energies decrease almost continuously from the time the wave forms. The kinetic and total energies become roughly constant once \( |E_g| \ll E_k \). The key difference between the BSG and WR models is that in the former the shock acquires positive total energy inside the star, while in the latter it does so only at several stellar radii from the surface.

The position where the entire outgoing wave becomes unbound correlates with the amount of mass in the final unbound shell (shown in the middle row of Fig. 5). For the RSG model, the shock becomes unbound deep inside the star, and sweeps up significant mass in the H envelope. For the BSG and WR models, the mass in the shell decreases continuously until positive energy is achieved, at which point the shell mass stabilizes. The decrease in the mass in the shock is caused by fallback of the innermost layers of the shell that do not become unbound from the star.

Note also that the entire portion of the star outside \( r = r_c \) initially acquires positive velocity (Figure 5). This occurs because the material in this region feels an instantaneous decrease in the acceleration of gravity (equation 15), but it is slower to respond because the free-fall time is long compared to the formation and propagation of the shock through the inner layers. While the amount of mass involved is substantial (Figure 5), most of this material ends up falling back toward the center since it remains gravitationally bound. Coughlin et al. (2017) predict that a second shock can in some cases emerge at the stellar surface due to this outward motion of the stellar envelope, particularly in more compact progenitors. Our simulations do not fully resolve the layers near the stellar surface to capture this effect (14).

In the case of a strong shock propagating through a
power-law density medium, two effects compete: the sweeping up of mass, which slows the shock down, and the pressure gradient behind the shock, which speeds it up (Sedov 1959; Herant & Woosley 1991). Which of these dominates depends on the radial steepness of the density profile. For the problem at hand, the shock is not strong enough for gravity to be unimportant, hence we may also need to consider the conversion of internal and kinetic energy into gravitational potential energy as another source of deceleration. Except near the stellar surface, the leading edge of the shock is constantly decelerating as it propagates out in all three fiducial progenitors (Figure 5). This occurs even as the shock sweeps up mass (RSG) or loses mass to fallback (WR). We thus conclude that the low energy of the shock makes the effect of gravity the dominant factor determining the speed of propagation of the shock inside the star. It would be valuable to understand this regime of shock propagation in more detail analytically.

4.3 Dependence on Neutrino Radiation Model

Table 2 also shows models obtained by evolving the fiducial progenitors with the full loss and maximum loss prescription for the inner core evolution ([2.3]). These models serve to quantify the overall uncertainties in our results due to the approximate treatment of the protoneutron star evolution.

In all cases, models lose more gravitational mass than their counterparts that employ the exponential prescription. While the maximum loss prescription results in a larger decrease in the gravitational mass than the full loss model for the RSG progenitor, the opposite result is found for the BSG and WR stars. Differences arise in part from the times needed to reach the TOV limit and in part by the rate at which mass is radiated away.

Table 2 shows that the maximum kinetic energy of the outgoing sound pulse scales approximately as the square of the gravitational mass lost, as expected from equation [20]. The total energy of the ejecta has an even steeper scaling with $\delta M_G$.

The exponential model is the most conservative of the three prescriptions in terms of the amount of gravitational mass lost. The main uncertainty in this model is the maximum mass of a cold, non-rotating neutron star $M_{nov}$, which is constrained from below at $2M_\odot$ by measured neutron star masses (Antoniadis et al. 2013) and from $\sim 2.5M_\odot$ above by causality (e.g., Lattimer & Prakash 2016). The value adopted is at the upper range of allowed values. Alternative choices should lead to factors of less than two difference in the gravitational mass lost (LW13).

4.4 Global Ejecta Properties

The total ejecta energies and masses for all models that employ the exponential neutrino loss prescription are shown in Figure 4. Results are shown as a function of both core compactness $\xi_{c,5}$ and envelope compactness $\xi_{env}$.

The ejecta energy is a monotonically decreasing function of the core compactness, and does not appear to be very sensitive to the envelope compactness. This result can be attributed to two physical effects. First, the energy available to power the shock $\Delta E(r_c)$ depends on the gravitational mass lost to neutrinos, $\delta M_G$, which depends on the neutrino cooling time. Progenitors with a high core compactness reach the TOV limit earlier, and hence do not radiate as much energy. This is shown explicitly in Figure 3, where increasing the core compactness above 0.4 results in a nearly linear decrease in the gravitational mass radiated in neutrinos. Below $\xi_{c,5} = 0.4$, the mass loss to neutrinos saturates at about 0.3$M_\odot$. This is a property of the chosen exponential prescription (equation [3]), which yields $\delta M_G \propto Bc(\Delta E_{nov})[1 - e^{-\frac{1}{\xi_{env}}}]$ unless $\tau_{nov} \ll \tau_{core}$.

Note that while a shorter neutrino cooling time in principle also decreases $\tau_{core}$, the maximum energy $\Delta E(r_c)$ does not increase (Table 2) because of the decrease in $\delta M_G$ and because the maximum energy released tends to flatten out for $\tau_t < 1 s$ at constant $\Delta M_G$ (Figure 2).

The second effect that suppresses the shock energy for high compactness is the larger gravitational energy at the pressure wave formation radius $r_c$. Thus more kinetic and internal energies are spent climbing out of the gravitational potential. Indeed, the only model for which the shock fails to break through the stellar surface (B80z-l-e) has the highest corecompactness of the set and the smallest value of $r_c$.

The final shock energies do not appear to depend sensitively on the envelope compactness. In fact, with the exception of the WR progenitors with higher core-compactness, the energies of successful shocks are not much smaller than the values predicted by $\Delta E(r_c)$.

In contrast, the ejecta mass displays the opposite dependence on progenitor structure. A clear hierarchy is evident in Figure 6: stars that are able to eject mass (with positive energies) do so in amounts that correlate negatively with increasing envelope compactness, in a manner consistent with the ordering shown in Figure 3. In the case of WRs, the ejected mass and energy are strongly correlated (Figure 6). The clear dependence of the ejected mass on core compactness for these progenitors is likely a consequence of the dependence of the energy on core compactness.

A correlation between the total ejected energies and (unbound) mass was predicted by Tan et al. (2001). While we do observe this correlation for WRs and RSGs, this is not the case for BSGs (Figure 6).

4.5 Failed Shocks

Out of our model sample, four cases failed to eject unbound matter by the end of the simulated time. Two factors can lead to failure: a high core compactness, and/or a large envelope mass.

The first case worth noting is model Y25z-2_e, which has about the same core compactness as the fiducial RSG but with a larger envelope due to its lower metallicity (envelope compactness is larger by a factor of 2.5). While the model ejects $2.5M_\odot$ of material, by the time the simulation ends (and the floor of temperature in the EOS is reached) the material is gravitationally bound (net energy $\sim 10^{52}$ erg).

Figure 4 shows the energies and ejected masses of this model in comparison with the fiducial RSG (R15z00_eHRR), also shown in Figure 5. The early evolution of the two models is similar, with the outgoing shock becoming unbound inside the star in both cases. As the shock sweeps up mass in the envelope, however, the deeper gravitational potential in the YSG keeps the gravitational energy in the shock high and
as the kinetic energy decreases, the shock becomes bound again before breakout.

The negative energy at breakout does not mean complete failure, as model W40z00, eHR shows (Figure 4). Nevertheless, the final ejected mass can be significantly lower than that contained in the shock at the time of breakout. Unfortunately, our simulations cannot reach the required times to determine definitely what happens since the temperature floor in the Helmholtz EOS (10^4 K) is reached shortly after breakout and the results become unreliable thereafter.

The other case worth noting is that of models B80z-2, e and B80z-2, f, which correspond to a low-metallicity blue supergiant with a very large core-compactness (ξ_{2.5} = 0.97) and a large envelope (total mass 55 M⊙ at core-collapse). The large core-compactness results in a very short time to reach the TOV mass and a small amount of gravitational mass lost to neutrinos (δM_G = 0.03 M⊙ for B80z-2, e). The resulting sound pulse has a very low energy (E_{k,max} ≈ 3 × 10^{45} erg for B80z-2, e), which is spent mostly climbing out of the large gravitational potential well. The failure of the model is robust to changes in the evolution of the inner core: employing the full loss prescription, which increases the gravitational mass lost by 30% (model B80z-2, f) produces the same qualitative outcome.

Figure 8 shows the evolution of the stellar surface in model B80z-2, e – quantified as the position of the photospheric density – as the shock reaches it. The low shock energy results in an expansion of the star by about ∼ 25% in radius over a period of a few days, followed by a steep infall. Such a star would likely show a modest decrease in effective temperature due to this expansion of the photosphere, just prior to disappearing.

The last unsuccessful case is model Y22z00, e, which has a relatively high core compactness (ξ_{2.5} = 0.54) but an envelope mass similar to that of the successful RSGs (the en-

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**Figure 6.** Global ejecta properties for models that employ the exponential neutrino loss prescription (Table 2), with red circles, blue squares, and black triangles denoting RSGs, BSGs, and WRs, respectively. Left: Total ejecta energy (top) and mass (bottom) as a function of core compactness (equation (a)). Middle: Total ejecta energy (top) and mass (bottom) as a function of envelope compactness (equation (b)). Top right: total ejecta energy versus ejecta mass. Bottom right: gravitational mass lost as a function of core compactness. Purple triangles denote YSGs, and vertical dashed lines correspond to failed models.

**Figure 7.** Evolution of the energies and shock mass in models R15z00, eHR (thin lines) and Y25z-2, e (thick lines), which differ primarily in their envelope compactnesses (ξ_{env} = 0.01 and 0.024, respectively, c.f. Table 1). Top: Kinetic (red), gravitational (blue), and total (black) energies in the outgoing pressure wave. Bottom: mass in the outgoing wave. The YSG ejecta is gravitationally bound by the time the simulation ends and the temperature floor is reached.
velocele compactness is higher by 60% relative to the fiducial RSG). The high core compactness also results in a short time to reach the TOV mass and thus a small amount of gravitational mass lost ($\delta M_C = 0.12 M_\odot$). The shock energy is therefore low. Nevertheless, the model ejects a small amount of mass $\sim 10^{-1} M_\odot$ with marginally negative energy ($\sim 10^{45}$ erg). Since the dynamics at late times after shock breakout for extended stars is not reliable in our simulations, we do not report the ejected mass and energy in Table 2 in contrast to model Y25z-2,e, for which mass ejection is unambiguous (though gravitationally bound). Simulations with a different EOS and/or numerical method will be required in order to better understand these marginal RSG/YSG cases, but it is clear that if there is any unbound material, it will have significantly less mass and energy than the majority of our other progenitors.

4.6 Fallback Accretion

In all of our progenitors, most of the star collapses onto the black hole. The resulting fallback accretion rate – assuming no rotation – can extend from hours to years, depending on the progenitor.

Given that our simulations move the inner boundary to increasingly larger radii in order to save computing time (22), we need to carry out an extra step in order to obtain the fallback accretion rate close to the BH. Fortunately, the accretion rate at small radii depends mostly on time, and the problem is such that an excellent semi-analytic approximation can be obtained (Appendix B).

Since the radial position of the trailing edge of the shock $r_{\text{tr}}$ is defined as the innermost point with zero velocity, we only need to compute the infall from rest of a given mass shell reached by this trailing edge in order to obtain the fallback accretion rate. We provide a detailed derivation of this calculation in Appendix B. The accretion time for the shell is $t_{\text{acc}}(r, t) = t + t_{\text{fall}}(r, r_{\text{tr}}[t])$.

\begin{equation}
\dot{M}(r, t_{\text{acc}}[r, t]) \simeq f_{\text{fall}} \frac{16}{3} r_{\text{tr}}(t)^2 \rho(r_{\text{tr}}[t], t) \sqrt{\frac{2G M(r_{\text{tr}}[t], t)}{r_{\text{tr}}(t)}}.
\end{equation}

where the use of the free-fall speed is a good approximation at late times in the infall ($r \ll r_{\text{tr}}(t)$). The fudge factor $f_{\text{fall}}$ is added to account for the early part of the infall, in which gas pressure effects are important.

Figure 8 shows a quantitative test of equations (22)-(23). We employ a version of model W40z00_e for which the inner boundary is kept constant at $r = 2000$ km for the entire simulation, and compare the accretion rate measured from the simulation with the semianalytic approximation using $f_{\text{fall}} = 1/2$. Agreement is excellent at times $t \gg 10$ s, given that the condition $r \ll r_{\text{tr}}(t)$ is well satisfied. Also, at times $t \lesssim 10$ s the position of the trailing edge of the shock (sound pulse at the time) is not so well defined.

As an experiment, we have also evaluated equations (22)-(23) using the initial density profile of the stellar progenitor. Figure 9 shows that the resulting accretion rate drops steeply shortly after the time of shock breakout in this model ($\sim 100$ s). This demonstrates that the enhanced accretion measured at late times comes from material in the outgoing shell which is not gravitationally unbound and thus falls back to the BH (accounting for the decreasing mass in Figure 5).

Figure 10 shows the result of applying equations (22)-(23) with marginally negative energy ($\sim 10^{45}$ erg). Since the dynamics at late times after shock breakout for extended stars is not reliable in our simulations, we do not report the ejected mass and energy in Table 2 in contrast to model Y25z-2,e, for which mass ejection is unambiguous (though gravitationally bound). Simulations with a different EOS and/or numerical method will be required in order to better understand these marginal RSG/YSG cases, but it is clear that if there is any unbound material, it will have significantly less mass and energy than the majority of our other progenitors.

\begin{equation}
\text{Figure 8.} \text{ Evolution of the stellar surface of model B80z-2,e, which fails to eject any mass. The stellar radius is quantified by the position of the initial photospheric density } \rho_{\text{ph}} \simeq 2.5 \times 10^{-10} \text{ g cm}^{-3}. \text{ The small increase in the stellar radius is due to the weak sound pulse/shock approaching the surface. This would likely manifest itself as a small decrease in the stellar effective temperature prior to its disappearing.}
\end{equation}

\begin{equation}
\text{Figure 9.} \text{ Accretion rate – assuming no rotation – as a function of time in a version of model W40z00_e for which the inner boundary is kept constant at } r = 2000 \text{ km. The black curve shows the accretion rate measured in the simulation, while the red dashed curve shows the analytic approximation to the accretion rate at small radii in equations (22)-(23) using the density at the trailing edge of the shock and } f_{\text{fall}} = 1/2. \text{ For comparison, the blue curve shows the result of applying the same equations to the initial density profile of the stellar progenitor. Accretion onto the newly formed black hole extends much later in time in the full simulations due to the marginally bound ejecta generated by neutrino-induced mass loss.}
\end{equation}
to all the models that employ the exponential neutrino loss scheme. Except for the failing model B80z-2.e, all progenitors lead to sustained accretion from hours to years. In the case of RSGs, this accretion has a shallow time dependence, since the surface of the star has not yet fallen in by the time the simulation stops (the collapse of the initial density profile for model R15z00.eHR is shown in Figure 10a, for reference). This suggests that fallback accretion will last for many years for these extended progenitors. The fallback accretion declines more rapidly in time for BSGs and WRs, following approximately a $t^{-5/3}$ time dependence at very late times once fallback from the rear of the ejecta reaches the BH.

More realistically, not all of the star will fall radially onto the black hole. The specific angular momentum required to circularize just outside of the innermost stable circular orbit is (e.g., Margalit et al. 2015)

$$j_{\text{isco}} \simeq \left(4 \times 10^{16} - 10^{17}\right) M_{\odot} \text{ cm}^2 \text{s}^{-1},$$  \hspace{1cm} (24)

where $M_{\text{cc,10}} = M_{\odot}/(10 M_{\odot})$, and the numeric range accounts for the black hole spin. Massive stars are rapid rotators, with surface rotational velocities $v_{\text{rot}} \sim 100 \text{ km s}^{-1}$ on the main sequence (e.g., Fukuda 1982). The specific angular momentum of material at the stellar surface in a presupernova star is

$$j_{\text{surt}} = 10^{18} v_{\text{rot},7} R_{\text{cc,11}} \text{ cm}^2 \text{s}^{-1},$$  \hspace{1cm} (25)

where $v_{\text{rot},7} = v_{\text{rot}}/\left(10^7 \text{ cm s}^{-1}\right)$ and the presupernova radius is $R_{\text{cc,11}} = R_{\odot}/(10^{11} \text{ cm})$. The corresponding circularization radius of this surface material would be

$$r_{\text{circ}} \simeq 7 \times 10^8 v_{\text{rot},7}^2 R_{\text{cc,11}} M_{\odot}^{-1/2} \text{ cm}.$$  \hspace{1cm} (26)

Rotation rates for WR stars are difficult to measure (Crowther 2007; St-Louis et al. 2009), although they are predicted to be in the range $10 - 100 \text{ km s}^{-1}$ (Meynet & Maeder 2003) with slow rotation more likely due to the intense mass loss. Except for the case of very slowly-rotating stars with $v_{\text{rot}} < 10 \text{ km s}^{-1}$ and a non-spinning black hole, the formation of a fallback disk out of material in the outer layers is almost guaranteed. The larger radii of BSG progenitors and the small amount of mass ejected makes disk formation even more likely.

In the case of RSGs, ejection of the entire hydrogen envelope means that the last material to fall back is located at the base of this envelope. For our fiducial progenitor, this radius is located at $r_{\text{base}} \sim 4 R_{\odot} \simeq 3 \times 10^{13} \text{ cm}$. Assuming uniform rotation in the envelope, the rotational velocity at this radius is $v_{\text{rot},7} \sim (r_{\text{base}}/R_{\odot}) \sim 4 \times 10^{-3}$, which would bring $j_{\text{surt}}$ (equation 25) below $j_{\text{isco}}$ (equation 24) and an accretion disk may not form. If, on the other hand, the specific angular momentum is constant with radius in the H envelope, disk formation is likely to occur.

The incidence of these late-time disks in failed supernovae has been considered previously (Quataert & Kasen 2012; Woosley & Heger 2012) and estimates of the accretion lifetimes extend to thousands of years (Perna et al. 2014). This fallback accretion might power long time-scale high energy transients. In addition, Kashiwaya & Quataert (2015) predicted a UV/optical transient lasting $\sim 10$ days assuming an outflow from the radiatively inefficient fallback disk that circularizes at small radii.

The temporal dependence of the decay in the bolometric luminosity ($\sim t^{-4/3}$) measured by Adams et al. (2017) for their failed supernova candidate suggests the existence of such a fallback disk, as this temporal slope is expected for a super-Eddington slim disk model (e.g., Perna et al. 2014). We note, however, that the fallback rates in RSGs are likely to be super-Eddington for many years (Fig. 10). In this regime, the luminosity is unlikely to decline much as the accretion rate does, and may in fact be roughly constant until the fallback accretion rate is below Eddington. It is thus somewhat puzzling that the luminosity of the Adams et al. (2017) transient decays at a rate of order the expected fallback accretion rate on year timescales. Future work should address the formation, long-term evolution, and emission of the fallback accretion with multi-dimensional time-dependent simulations.

5 OBSERVATIONAL IMPLICATIONS

In this section we estimate the observational manifestation of the weak explosions calculated in the previous sections. We focus on the most robust predictions, which are associated with the spherically symmetric shock breakout and

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recombination powered emission. Fallback accretion might in some cases produce a separate transient if a significant amount of mass becomes rotationally supported at late times. In future work it would be interesting to quantitatively apply the fallback accretion rates found here to such models (e.g., [Quataert & Kasen 2012] [Woosley & Heger 2012] [Kashiyama & Quataert 2015]).

5.1 Shock Breakout

The emergence of a successful shock from the stellar surface is accompanied by a brief burst of radiation once photons trapped at the leading edge of the shock diffuse out ([Colgate 1974] [Falk 1978]). Radiation starts escaping once the shock reaches an optical depth $\tau_{bo} = c/v_k$ from the stellar surface (e.g., [Sapir et al. 2011]). The duration of this signal is the longest of the diffusion time over the characteristic width of a radiation-dominated shock $\delta \tau = \tau_{bo}$ ([Weaver 1976])

$$t_{diff,bo} \approx \tau_{bo} \frac{(R_{cc} - R_{bo})}{c}$$

(27)

and the light-crossing time over the stellar surface ([Ensman & Burrows 1992])

$$t_{lc} \approx \frac{R_{cc}}{c}$$

(28)

with $R_{bo}$ the radius where the optical depth is $\tau_{bo}$. The breakout luminosity is simply the radiation energy within the transition region $E_{rad,bo}$ divided by the breakout time (e.g., [Piro 2013])

$$L_{bo} \approx \frac{E_{rad,bo}}{max\{t_{diff},t_{lc}\}}$$

(29)

Table 8 shows estimates for the bolometric shock breakout luminosity and timescales for the baseline model set. Given that we are not fully resolving the layers near the photosphere ([41]), we estimate the shock breakout velocity $v_{bo}$ with the formulae in ([Waxman & Katz 2016]) using the ejecta energy and mass from Table 2 and the progenitor radius $R_{cc}$. The resulting values are in good agreement with the value measured in the simulation for the RSG and within a factor of two for the BSG. In the case of the WR, for which we have the poorest resolution at the photosphere, the analytic estimate is a factor $\sim 4$ larger than the velocity in the simulation.

Given the breakout velocity, the resulting optical depth $\tau_{bo} = c/v_k$ is used to measure the radial distance from the surface of the star at which breakout occurs. This yields the diffusion time and allows measuring the radiation energy $E_{bo}$ contained in the transition region for use in equation (29). For the WR progenitor, this procedure results in only one cell in the simulation contributing to the radiation energy, which we consider unreliable, so for this progenitor we use instead the analytic estimate from ([Waxman & Katz 2016]). The breakout temperature is estimated assuming black body radiation $L_{bo} = 4\pi R_{cc}^2 \sigma T_{bo}^4$.

Shock breakout in the RSG can reach peak luminosities $\sim 2 \times 10^{40} \erg \, \s^{-1}$ ($10^6 - 10^7 L_\odot$) and last for a few days. These values agree favorably with the estimates from ([Piro 2013] and [Lovegrove et al. 2017]) given the mass and energy of the ejecta in our models.

Shock breakout in the BSG model produces a luminosity $\sim 10^{42} \erg \, \s^{-1}$ over a timescale of $\sim 3$ h, presumably in the UV given $T_{bo} \sim 7 \times 10^4 \K$. Our models do not include radiation diffusion and hence the detailed values for the temperature might change when more physics is included. Nonetheless, a transient with this brightness and timescale can be a promising target for future wide-field, very short-cadence surveys (e.g., [Sako et al. 2016]).

The WR model reaches a similar bolometric breakout luminosity as the BSG ($\sim 10^{42} \erg \, \s^{-1}$ or $10^8 L_\odot$) over $\sim 1$ s. For this model the light crossing time sets the light curve timescale. Given the estimated temperature, the emission should come out in the UV to soft X-rays, although detailed calculations of radiation mediated shocks are needed to make firm predictions. Note that these estimates assume that the circumstellar medium is a vacuum. In reality, the star will be surrounded by material ejected by the powerful stellar winds that occurred earlier in its life. The properties of shock breakout in a dense wind can be different, including acceleration of particles to high energies in collisionless shocks (e.g., [Katz et al. 2012]).

5.2 Plateau Emission

As the ejecta expands, it converts part of its internal energy into kinetic energy and radiates the rest away. Emission occurs from a photosphere at the recombination front of its dominant species, above which the opacity drops sharply ([Grassberg et al. 1971]). This results in plateau emission, with a luminosity and timescale ([Popov 1993] [Kasen & Woosley 2009] [Kleiser & Kasen 2014])

$$L_{pl} \approx 1.8 \times 10^{30} E_{ej,47}^{5/6} M_{ej,1}^{-1/2} R_{cc,500}^{3/6} \frac{\tau_{bo,4}^{-1/3}}{T_{6000}} \, \erg \, \s^{-1}$$

(30)

$$t_{pl} \approx 220 E_{ej,47}^{1/6} M_{ej,1}^{1/6} R_{cc,500}^{1/6} \tau_{bo,4}^{2/3} \, \text{days}$$

(31)

where $E_{ej,47} = E_{ej}/(10^{47} \erg)$, $M_{ej,1} = M_{ej}/(1 M_\odot)$, $R_{cc,500} = R_{cc}/(500 R_\odot)$, and $\tau_{bo,4}$ is the opacity in units of $0.4 \cm^2 \, \g^{-1}$. Equations (30)-(31) account for the diffusion of radiation in an expanding medium with a receding photosphere, and assume that the ejecta are radiation pressure dominated, which remains true for our models despite the low ejecta energies (although only marginally for the RSG case). The luminosity in equation (30) is a black body at the recombination surface with the recombination temperature $T_{6000} \times 6000 \K$. For hydrogen and oxygen-dominated ejecta, this recombination temperature is approximately $6000 \K$, while for helium-dominated ejecta it increases to $10^5 \K$ (e.g., [Kleiser & Kasen 2014]). The absence of radioactive energy injection results in a sharp drop in the emission once the recombination front reaches the base of the ejecta (e.g., [Kasen & Woosley 2009] [Piro & Nakar 2013]).

Table 3 shows the bolometric luminosities and timescales associated with plateau emission for the three baseline progenitors. A recombination temperature of $6000 \K$ is used in the RSG and WR models, while $10,000 \K$ is used for the BSG model given its higher surface abundance of helium. Also shown is the final velocity of the ejecta, which is assumed to be $v_{exp} = \sqrt{2E_{ej}/M_{ej}}$.  

---

3 We also assume $f_\rho = \kappa_{0,34} = 1$ in their equations.
Table 3. Bolometric emission properties inferred from the fiducial model set. Columns from left to right show: model name, shock breakout luminosity (eq. 29), breakout time $t_{bo} = \min(t_{diff}, t_{bo})$ (eq. 27-28), shock velocity at breakout $v_{bo}$, gas temperature at breakout $T_{bo}$, plateau luminosity (eq. 30), plateau duration (eq. 31), final shock velocity $v_{\text{exp}} = \sqrt{2E_{\text{ej}}/M_{\text{ej}}}$, mass fraction of hydrogen, helium, carbon, and oxygen at the stellar surface. Shock breakout parameters are in part analytic estimates based on Waxman & Katz (2016) (see §5), given that we are not fully resolving the regions close to the photosphere and hence the shock acceleration in those regions.

| Model          | $L_{bo}$ $(L_{\odot})$ | $t_{bo}$ (d) | $v_{bo}$ (km s$^{-1}$) | $T_{bo}$ (K) | $L_{pl}$ $(L_{\odot})$ | $t_{pl}$ (d) | $v_{\text{exp}}$ (km s$^{-1}$) | $X_H$ | $X_{He}$ | $X_C$ | $X_O$ |
|----------------|------------------------|--------------|------------------------|--------------|------------------------|--------------|------------------------|-------|---------|-------|-------|
| R15z00_cHR    | 6E+6                   | 3d           | 70                     | 9E+3         | 6E+5                   | 400          | 70                     | 0.67  | 0.31    | 0     | 0     |
| B25z00_cHR    | 2E+8                   | 3h           | 900                    | 7E+4         | 2E+6                   | 20           | 600                    | 0.41  | 0.57    | 0     | 0     |
| W40z00_cHR    | 3E+8                   | 1s           | 12,000                 | 1E+6         | 5E+4                   | 2            | 2000                   | 0     | 0.18    | 0.49  | 0.30  |

Plateau emission for the RSG model is again consistent with the results of LW13 and the estimates of Piro (2013), with a duration of about 400 days and a luminosity $\sim 2 \times 10^{49}$ erg s$^{-1}$, brighter by a factor $\sim 4$ relative to the progenitor star. The inferred final velocity ($\lesssim 100$ km s$^{-1}$) is very low compared to normal supernovae.

The BSG progenitor has a plateau that can last for about 20 days, reaching a luminosity of $\sim 10^{48}$ erg s$^{-1}$, brighter by a factor $\sim 5$ relative to the stellar progenitor. Such a brightening might be detectable if the star is monitored every few days.

Finally, the WR progenitor has a plateau phase lasting for less than one day, and with a luminosity that is about 10 times fainter than the progenitor. In this case, we expect a spike of radiation following shock breakout, followed by a steep decrease of the luminosity to a plateau a few magnitudes fainter than the progenitor. After a day, the star should disappear.

5.3 Failed shocks

The case of failed shocks might still be interesting observationally. As shown in Figure 8, arrival of the pressure wave to the stellar surface results in an expansion of the star. For this particular model, the radius expands by 25% over a timescale of days. The luminosity is likely to be unchanged due to rapid photon diffusion, so that the increase in surface area will lead to a modest decrease in effective temperature prior to the star disappearing.

6 SUMMARY AND DISCUSSION

We have studied the properties of the ejecta generated by non-rotating massive stars that undergo core-collapse and fail to produce a successful supernova. Neutrino radiation during the protoneutron star phase decreases the mass of the core of the star by $\sim 0.1 - 0.5 M_{\odot}$ over a few seconds. The part of the progenitor exterior to a radius $\sim$ few $10^9$ cm experiences this change as an effectively instantaneous decrease in the mass of the star. These layers of the star are thus over-pressured, resulting in an outward going sound pulse that steepens into a shock as it travels out through the star. We have used time-dependent hydrodynamic simulations that follow the propagation of the outgoing pressure wave through the entire star, using an approximate prescription for the neutrino radiation from the inner protoneutron star. Our analysis extends the earlier work of LW13 by studying this mechanism of mass ejection in failed supernovae for a wide range of stellar progenitors. We also provide a more detailed physical understanding of, and analytic estimates for, the mass ejection process. Our main results are the following:

1. Successful mass ejection due to the loss of gravitational mass to neutrinos can occur in all types of stellar progenitors, not just red supergiants (Figure 5 and Table 2).

2. The explosion energy is a monotonically decreasing function of the core compactness, and the ejected mass is a monotonically decreasing function of the envelope compactness (or equivalently, of the escape speed at the stellar surface; Figure 6).

3. The maximum kinetic energy of the shock is set by the change in the gravitational acceleration over a free-fall time, at a radius where the free-fall time equals the neutrino cooling time (Figures 1 and 2). This is $\sim 10^{47} - 10^{48}$ erg for most progenitors. Propagation through the stellar envelope decreases the kinetic energy from its maximum as the pressure wave (and later shock) moves out in the gravitational potential (Figure 5; hence the analytic estimate (equation 20) is an upper limit on the final ejecta energy. This in turn translates into an upper limit on the ejected mass, which is $\sim 5$, 0.2, and 0.01 $M_{\odot}$ for RSGs, BSGs, and WR stars, respectively (Figure 4).

4. For RSGs, the change in gravitational mass due to neutrino radiation unbinds the hydrogen envelope, which likely has the vast majority of the angular momentum of the progenitor. In this case, it is likely that the resulting black hole will be relatively slowly spinning. For BSGs and WRs, however, the ejected mass and angular momentum are negligible, so that the resulting black hole mass and spin is very close to that implied by the total mass and angular momentum of the pre-collapse progenitor (with the caveat that if the stellar angular momentum implies a black hole dimensionless spin $\gtrsim 1$ this mapping cannot hold). These conclusions are important for interpreting gravitational wave and X-ray binary inferred black hole masses and spins.

5. Stars that have a high core compactness or high envelope compactness relative to the average of its class ($\xi_{\text{env}} \gtrsim 0.5$, $\xi_{\text{env}} \gtrsim 0.02$ for RSGs; $\xi_{\text{env}} \gtrsim 0.5$, $\xi_{\text{env}} \gtrsim 0.5$ for BSGs) fail to eject unbound matter or any matter at all (e.g., Figure 5). While none of our WRs fail, they all eject relatively small
amounts of unbound mass.

6. Successful mass ejection also results in fallback accretion over periods of time ranging from hours to years (Figure 10). Depending on the uncertain angular momentum distribution of the stellar progenitor, this fallback accretion might power a variety of transients, including ultra-long-duration gamma-ray bursts and rapid optical transients (Quataert & Kasen 2012; Woosley & Heger 2012; Kashiwai & Quataert 2015).

7. We estimate the shock breakout and recombination-powered plateau emission for our fiducial RSG, BSG, and WR progenitors (Table 3). These are the most robust observational signatures of failed supernovae. For RSGs our estimates are in good agreement with the previous work of LW13, Piro (2013), & Lovegrove et al. (2017). We find that BSGs have shock breakouts that last for hours with luminosities comparable to those of normal supernovae. The plateau emission is a factor of several brighter than the progenitor star, lasting for several weeks. In the case of WRs, shock breakout is extremely bright but very short-lived and likely at UV to soft X-ray energies. The plateau emission is bolometrically a factor ~ 10 fainter than the progenitor star, and lasts for about a day, after which the star would likely truly disappear. An interesting possibility to explore in future work is the interaction between the weak explosions found here and the pre-collapse stellar wind, particularly for WR stars. This circumstellar interaction might well be brighter than the plateau emission estimated here.

Our predictions can be improved in several ways. The simplest is to update the equation of state to include neutral hydrogen, allowing us to follow shocks from red supergiant expansion. Similarly, inclusion of radiation diffusion with suitable opacities would allow a more accurate evolution of the internal energy of the ejecta after shock breakout. Finally, use of a proper core-collapse supernova code to calculate self-consistently the loss of gravitational mass would remove the uncertainty in the change in gravitational mass of the stellar core $\delta M_G$. The final ejecta energy and ejecta mass are sensitive to the exact value of $\delta M_G$.

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APPENDIX A: NUMERICAL TESTS

A1 Neutrino mass loss and choice of inner radial boundary

We first test that no shock is generated without loss of gravitational binding energy to neutrinos. Figure A1 shows the result of turning off the neutrino mass loss for the collapse of the baseline WR progenitor (M40z00). In contrast to the model in which the gravitational mass decreases, a model with no neutrino mass loss does not develop a shock. Positive velocities outside the rarefaction wave have amplitudes smaller than 10 km s$^{-1}$ at $t = 100$ s. Larger velocities develop outside the star in the low-density ambient medium, but this has no significant effect on the dynamics.

When neutrino cooling is included, the position of the inner boundary can influence the results because it determines the initial value of $M_G$ and $M_B$, and thus it influences the amount of mass lost to neutrinos, all else being equal. Following LW13, we adopt $R_{\text{min}} = 2 \times 10^8$ cm, which is very near the outer edge of the iron core for most stars. Figure A2 shows the effect of changing the position of this boundary radius for the baseline WR progenitor using the exponential loss model (equation 3). At $t = 60$ s, the amplitude of the shock increases monotonically with increasing inner boundary radius. The maximum shock amplitude at this time lies in the range $580 - 720$ km s$^{-1}$ for $R_{\text{min}} = 500$ km to $4000$ km, respectively, with the corresponding kinetic energies in the range $(2.5 - 3.7) \times 10^{47}$ erg.

For the three smaller values of $R_{\text{min}}$ shown in Figure A2, the evolution of $M_G(t)$ becomes nearly identical after $t = 1$ s, with the time to reach the TOV mass being nearly the same $(2.6 - 2.7)$ s, since accretion inside $r = 2 \times 10^8$ cm occurs faster than outside this radius. A larger initial value of $M_B$ (larger $R_{\text{min}}$) results in a larger amount of mass lost to neutrinos in the exponential loss model (equation 3), and a larger decrease in the gravitational acceleration. The model with $R_{\text{min}} = 4 \times 10^9$ cm starts

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out with a larger baryonic mass, but its rate of increase is similar to that of the model with $R_{\text{min}} = 2 \times 10^8$ cm, hence it reaches the TOV limit at a significantly earlier time (2.1 s). Despite losing less mass to neutrinos than the model with $R_{\text{min}} = 2 \times 10^8$ cm ($0.216M_\odot$ versus $0.222M_\odot$, respectively), the shorter timescale over which the mass changes results in a larger energy release (kinetic energy $3.7 \times 10^{47}$ erg; §3).

We conclude that the position of the inner boundary can introduce an uncertainty of $\sim 10\%$ in the outflow energy. Improving upon this uncertainty requires treating neutrino mass loss with full physics simulations.

A2 Resolution and baryonic mass conservation

To diagnose the degree of convergence of our results with spatial resolution, we compute the same baseline WR model as in the previous subsection using 64, 128, and 256 cells per decade in radius, corresponding to a fractional cell size $\Delta r/r = 3.7\%$, $1.8\%$, and $0.9\%$, respectively (our baseline resolution is the latter). Figure A2 shows the velocity profile around the shock at time $t = 60$ s, with higher resolution resulting in a shock with larger amplitude but narrower width. The kinetic energies of material with positive velocity at $t = 60$ s for the three spatial resolutions from low to high are respectively $2.9$, $3.6$, and $3.4 \times 10^{47}$ erg. The kinetic energy is higher in the model with $\Delta r/r = 1.8\%$ because the mass with positive velocity is larger than in the model with highest resolution, $0.224$ vs $0.212M_\odot$, respectively. This is visible in Figure A2b in a wider shock for $\Delta r/r = 1.8\%$. We therefore conclude that at our base resolution, results are converged to within $10\%$ in shock kinetic energies.

Increasing the spatial resolution also decreases the magnitude of the velocity fluctuations outside the star (visible in Figure A2), and the degree to which the outer stellar boundary moves before being reached by the shock. Since these regions have very low densities, we do not optimize our resolution to minimize these transients, as they do not affect the overall energetics of the shock. The velocity of the forward shock at breakout depends on how the very surface layers of the star are resolved (c.f.4.1); we defer a more detailed study of this process for future work and focus on the global energetics of the shock, which are captured with our current resolution to within $\sim 10\%$.

Finally, the spatial resolution has a very moderate effect on the overall mass conservation in our simulation. Given that we employ the mass flux from the split PPM Riemann solver in FLASH to update the point mass for accretion (eq. [7]), mass conservation should in principle hold to machine precision. Figure A3 shows the quantity

$$
\frac{M_{\text{dom}}(t) + M_{\text{in}}(t)}{M_{\text{dom}}(0) + M_{\text{in}}(0)} - 1,
$$

(A1)

where $M_{\text{dom}}(t)$ is the baryonic mass in the computational domain, for a baseline WR model with no neutrino mass loss and for which the outer boundary has been set to reflecting. Any deviations from zero are thus accumulated errors in the integration of equation [7]. Conservation to near machine precision is indeed maintained up to about $t = 10$ s, after
which the large number of time steps ($10^5$ for the highest resolution) results in oscillations. The spatial resolution does not appear to have a significant effect on the degree to which mass is conserved, since all fluctuations are kept smaller than $10^{-13}$. Since other uncertainties cause much greater changes in our results, we consider this effect to be a negligible source of error.

### A3 Remapping

Following the shock evolution all the way to the surface of a RSG is a non-trivial calculation. At the typical shock velocities obtained at large radii ($\sim 10^7$ cm s$^{-1}$) and for characteristic RSG sizes ($\sim 10^{14}$ cm; Table 2) the required physical evolution time is $\sim 10^7$ s. At our baseline resolution, the Courant time step is approximately 

$$\Delta t_{\text{CFL}} \sim 10^{-3} \left( \frac{10^6 \text{cm s}^{-1}}{v[R_{\text{min}}]} \right) \left( \frac{R_{\text{min}}}{2000 \text{ km}} \right) \left( \frac{\Delta r/r}{1%} \right) \text{s},$$

(A2)

which means that $\sim 10^{10}$ time steps would be needed for the shock to reach the stellar surface if the size of the computational domain was kept fixed.

The problem simplifies due to the straightforward collapse of the inner layers of the star toward the BH, quickly reaching supersonic velocities. This means that these inner layers become causally disconnected from the rest of the star. The acceleration of gravity at any point depends only on the enclosed gravitational mass and not on the detailed mass distribution as long as spherical symmetry is maintained. One can therefore move the position of the inner boundary outward as long as infall is supersonic, allowing longer time steps (equation A2), without affecting the dynamics. A similar approach was used by Hammer et al. (2010) for evolving a successful core-collapse supernova shock over long timescales.

Figure A4 shows the Mach number at different radii in the evolution of the baseline RSG model (M15z00). Within 100 s of evolution, the inflow Mach number at the initial inner boundary exceeds 5, while at a radius ten times larger ($2 \times 10^9$ cm) the flow is also increasingly supersonic. At the times indicated by the vertical dotted lines in Figure A4, the inner decade in radius is removed from the computational domain, and the mass contained in this removed domain is added to both the baryonic and gravitational masses. We choose the time for the first remapping (100 s) so that the TOV mass has already been reached and no neutrino mass loss is occurring. We have checked that the subsequent evolution is identical whether this inner decade in radius is removed or not. The corresponding gains in time step are at least a factor 10 in each case, easily allowing evolution of the shock to the surface of the RSG. No remapping is carried out after $10^4$ s, since strong reverse shocks cause the infall velocity to decrease in magnitude (while still remaining supersonic at the inner boundary).

### APPENDIX B: CALCULATION OF THE ACCRETION RATE

Here we provide a derivation of the semi-analytic expression for the accretion rate in equations (22) - (23). We assume that a given mass shell experiences free-fall from rest from the time at which the trailing edge of the outgoing shell reaches it, thus neglecting pressure forces which decrease the infall velocity from the free-fall value. Stellar rotation is also neglected.

The time it takes a given shell at radius $r_0$ to fall to a
radius $r$ from rest is [Bethe 1990]:

\[
t_{\text{fall}}(r, r_0) = \frac{1}{\sqrt{2GM(r_0)}} \int_{r_0}^{r} \frac{dr}{\sqrt{r'^2 - 1}}
\]

(B1)

\[
= \frac{r^{3/2}}{\sqrt{2GM(r_0)}} \int_{r_0}^{1} dx \sqrt{\frac{x}{1 - x}}
\]

(B2)

\[
\approx \frac{r_0^{3/2}}{\sqrt{2GM(r_0)}} \left[ \frac{\pi}{2} - \arcsin \left( \frac{r}{\sqrt{r_0}} \right) \right]
\]

(B3)

\[
\approx \frac{r_0^{3/2}}{\sqrt{2GM(r_0)}} \left[ \frac{\pi}{2} - \frac{2}{3} \left( \frac{r}{r_0} \right)^{3/2} \right]
\]

(B5)

where free-fall motion from rest has been assumed ($\alpha = 1$ in [Bethe 1990]), and the latter equality is valid for $r \ll r_0$. Note that the mass enclosed by the shell $M(r_0)$ is assumed to remain constant.

Mass conservation in the infalling shell implies

\[
\rho(r) r^2 dr = \rho_0(r_0) r_0^2 dr_0,
\]

(B6)

where $\rho_0$ is the density at the time infall begins. For $r \ll r_0$ and fixed infall time $t_{\text{fall}}$, changes in the initial and final radii are related by

\[
\left( \frac{dr_0}{dr} \right)_t = \frac{4}{3\pi} \left( \frac{r}{r_0} \right)^{1/2},
\]

(B7)

where we have also assumed $\rho_0 r_0^3 \ll M(r_0)$, which is generally true for shells in the outer envelope of the star ($r_0 \gg r_c$).

Substituting into equation (B6) yields a relation between the initial and final densities ([Bethe 1990])

\[
\rho(r) = \frac{4}{3\pi} \left( \frac{r_0}{r} \right)^{3/2} \rho_0(r_0).
\]

(B8)

Assuming that the infalling shell has reached the free-fall velocity by the time it reaches a radius $r$ (an excellent approximation if $r \ll r_0$), the accretion rate is

\[
M(r, r_0) = f_{\text{fall}} 4\pi r^2 \rho(r) \sqrt{\frac{2GM(r_0)}{r}}
\]

(B9)

\[
= f_{\text{fall}} \frac{16}{3} \sqrt{\frac{\rho_0(r_0)}{r_0}} \sqrt{\frac{2GM(r_0)}{r}}.
\]

(B10)

where we have added a fudge factor $f_{\text{fall}}$ to correct the density mapping (equation [B7]) for the effects of gas pressure during the early part of the infall. In practice, we find that setting $f_{\text{fall}} = 1/2$ provides excellent agreement with our simulations (c.f. Figure 6).

Applying equation (B9) to the trailing edge of the outgoing shock, we have $r_0 = r_{\text{in}}(t)$. The time from the beginning of the simulation at which the infalling shell reaches a radius $r$ is

\[
t_{\text{acc}}(t) = t + t_{\text{fall}}(r, r_{\text{in}}[t]),
\]

(B11)

which is the time at which the accretion rate in equation (B9) is valid. The very weak dependence of $t_{\text{fall}}$ on $r$ for $r \ll r_0$ means that the accretion rate is primarily a function of time, not of position.

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4 [Bethe 1990] uses a factor $\alpha$ for the infall velocity to account for the effects of gas pressure.

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