Stellar Helium Burning in Other Universes:
A Solution to the Triple Alpha Fine-Tuning Problem

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Abstract
Motivated by the possible existence of other universes, with different values for the fundamental constants, this paper considers stellar models in universes where $^8\text{Be}$ is stable. Many previous authors have noted that stars in our universe would have difficulty producing carbon and other heavy elements in the absence of the well-known $^{12}\text{C}$ resonance at 7.6 MeV. This resonance is necessary because $^8\text{Be}$ is unstable in our universe, so that carbon must be produced via the triple alpha reaction to achieve the requisite abundance. Although a moderate change in the energy of the resonance (200 – 300 keV) will indeed affect carbon production, an even smaller change in the binding energy of beryllium ($\sim 100$ keV) would allow $^8\text{Be}$ to be stable. A stable isotope with $A = 8$ would obviate the need for the triple alpha process in general, and the $^{12}\text{C}$ resonance in particular, for carbon production. This paper explores the possibility that $^8\text{Be}$ can be stable in other universes. Simple nuclear considerations indicate that bound states can be realized, with binding energy $\sim 0.1 – 1$ MeV, if the fundamental constants vary by a $\sim$ few $–$ 10 percent. In such cases, $^8\text{Be}$ can be synthesized through helium burning, and $^{12}\text{C}$ can be produced later through nuclear burning of beryllium. This paper focuses on stellar models that burn helium into beryllium; once the universe in question has a supply of stable beryllium, carbon production can take place during subsequent evolution in the same star or in later stellar generations. Using both a semi-analytic stellar structure model as well as a state-of-the-art stellar evolution code, we find that viable stellar configurations that produce beryllium exist over a wide range of parameter space. Finally, we demonstrate that carbon can be produced during later evolutionary stages.

Keywords: Fine-tuning; Multiverse; Stellar Nucleosynthesis; Triple Alpha
1. Introduction

Although carbon is a crucial element for the existence of life in our universe, its production in stars is not straightforward. Carbon must be produced via the triple alpha reaction (see below), and the delicate nature of this process has been often used as an example of the so-called fine-tuning of our universe for life \[1, 2, 3, 4, 5\]. This paper offers an alternate solution to this apparent fine-tuning problem. If the fundamental constants of nature are sufficiently different, so that the triple alpha reaction is compromised, then a large fraction of parameter space allows for stable \(^8\text{Be}\) nuclei, which obviates the need for the triple alpha reaction. In such universes, helium burning leads to (stable) beryllium, and subsequent beryllium burning can then produce carbon, oxygen, and other large nuclei. As long as stable \(^8\text{Be}\) exists, all of these nuclei can be synthesized through non-resonant two-body reactions, with no need for the triple alpha process or any particular resonance.

The necessity of the triple reaction for carbon production \[6, 7, 8\] can be summarized as follows. The key issue is that our universe supports no stable nuclei with atomic mass number \(A = 8\). For a universe starting with a composition of primarily hydrogen and helium, the \(^8\text{Be}\) nucleus is the natural stepping stone towards carbon; unfortunately, it decays back into its constituent alpha particles with a half-life of only about \(\sim 10^{-16}\) sec. As a result, the synthesis of carbon depends on the triple alpha process

\[
3\alpha \rightarrow ^{12}\text{C} + \gamma ,
\]

where three helium nuclei combine to make carbon. This reaction, in turn, relies on the temporary formation of \(^8\text{Be}\) nuclei \[9\]. In spite of the instability of \(^8\text{Be}\), a small and transient population of these nuclei builds up and allows for the fusion of a third alpha particle. In order for the reaction to take place fast enough, however, the final step \((^{8}\text{Be} + \alpha \rightarrow ^{12}\text{C})\) must take place in a resonant manner. The existence of this resonance was famously predicted by Hoyle \[10\] and the corresponding energy level in carbon was subsequently measured in the laboratory (e.g., see the review of \[11\]).

Stellar nucleosynthesis calculations \[6, 7, 8\] indicate that the triple alpha process is necessary to produce the observed abundance of carbon in our universe. Subsequent work has shown that the energy level of the resonance, which occurs at 7.644 MeV, cannot vary by a large increment without
compromising carbon production. As one example, calculations of helium burning in 20 $M_\odot$ stars have been carried out [12] using different values for the location of the resonance — but continuing to require the triple alpha reaction. These results show that a 60 keV increase in the crucial energy level in the $^{12}$C nucleus does not significantly alter carbon production in stellar interiors. In contrast, however, an increase of 277 keV (or more) in the energy level leads to different nuclear evolution and much less carbon is synthesized (most of the carbon is burned into oxygen). More recent calculations [13, 14, 15, 16] reach similar conclusions, although the amount of carbon produced depends on both the mass of the star and the stellar evolution code (e.g., compare results from [12], [15], and [16]; see also [17] for further discussion of nuclear structure and [18] for a more comprehensive review).

The above considerations indicate that the resonant energy for $^{12}$C can only change by an increment of order 100 keV without compromising carbon production via the triple alpha process. We note that some carbon can be produced without the triple alpha resonance, but the abundance would be much lower than that of our universe; since we do not know the minimum carbon abundance that is necessary for life, it is not known if such low carbon universes would be habitable. In addition, the triple alpha process is only necessary because $^8$Be is unstable, and its binding energy is higher than two separate alpha particles by a difference of about 92 keV. The similarity of these two energy scales suggests the following solution to the triple alpha fine-tuning problem. If the $^{12}$C resonance changes enough to alter carbon production via the triple alpha process, then $^8$Be will often be stable, so that the triple alpha reaction is no longer necessary. Of course, the changes in nuclear structure must result in the binding energy of $^8$Be being larger (more bound), rather than lower, so that only “half” of the parameter space would be viable.

The scenario considered here implicitly assumes that big bang nucleosynthesis proceeds in essentially the same manner as in our universe, with comparable light element abundances. The exact abundances are expected to vary, of course, as we assume here that nuclear physics changes enough to allow for stable $^8$Be. In our universe, $^7$Li is stable and can be more easily produced than heavier isotopes, but the abundance is only $\sim 10^{-10}$. The abundance of stable $^8$Be in this alternate scenario is expected to be even lower. We thus assume that the universe emerges from its early epochs with a composition dominated by hydrogen and helium, and relatively little mass
in heavier isotopes.

For universes in this regime of parameter space, the key requirement for carbon production is that helium burning can take place readily to form $^8\text{Be}$. As long as the universe under consideration can produce $^8\text{Be}$, and it retains an appreciable supply of $^4\text{He}$, carbon can be produced in subsequent stellar generations through the reaction $^4\text{He} + ^8\text{Be} \rightarrow ^{12}\text{C}$. This reaction is a natural channel to produce carbon — alpha particles are energetically favorable states, so that isotopes produced by adding together alpha particles would naively be the easiest to make. Note that in our universe the most common isotopes (after $^1\text{H}$) are $^4\text{He}$, $^{16}\text{O}$, $^{12}\text{C}$, and $^{20}\text{Ne}$, in decreasing order of abundance. In addition, the isotopes $^{28}\text{Si}$, $^{24}\text{Mg}$, and $^{32}\text{S}$ are also among the top ten most common. All of the small alpha-particle nuclei are thus well-represented with the exception of $^8\text{Be}$.

An underlying assumption of this paper is that the fundamental constants of nature can take on varying values. For completeness, we note that this possibility arises in two separate but related contexts. Within our universe, the constants of nature could vary with time, although observations limit such variations to be quite small [19, 20, 21]. On a larger scale, the constants of nature could vary from region to region within the vast complex of universes known as the multiverse [22, 23]. This latter possibility is often invoked as a partial explanation for why the constants have their observed values. Within the larger ensemble, each constituent universe samples the values of its constants from some underlying distribution. Our local region of space-time — our universe — thus has one particular realization of these parameters. Finally, it is useful to consider different values for the constants of nature, independent of the multiverse, in order to understand the sensitivity of stellar structure to the relevant input parameters.

The number and choice of parameters needed to specify a given universe is relatively large, but no general consensus exists regarding the relevant parameter space [3, 24, 25]. For example, Table 1 of Ref. [25] includes 37 parameters, whereas Ref. [22] includes only 6. The range of allowed variations in the constants also remains undetermined. In previous work, we have considered how stars are influenced by variations in the fine structure constant and the gravitational constant, as well as nuclear parameters determined by the strong and weak forces [26, 27, 28]. The first two quantities are specified
by the dimensionless parameters

\[
\alpha = \frac{e^2}{\hbar c} \quad \text{and} \quad \alpha_G = \frac{G m_p^2}{\hbar c},
\]

where \( m_p \) is the proton mass. For the realization of these constants found in our universe, these parameters have the values \( \alpha \approx 1/137 \) and \( \alpha_G \approx 5.91 \times 10^{-39} \). This past work (see also [29]) shows that the fundamental constants can vary by several orders of magnitude and still allow for functioning stars and habitable planets. In this work, we keep the \((\alpha, \alpha_G)\) fixed and explore the implications of different nuclear structures such as stable \(^8\text{Be}\).

The solution to the triple alpha fine-tuning problem considered here requires two components. First, relatively small changes to the fundamental constants must allow for \(^8\text{Be}\) nuclei that are stable and hence have lower binding energy than two alpha particles. Second, stars in such universes must be able to burn helium into stable beryllium, and then later burn the beryllium into carbon (and heavier elements). This paper considers nuclear models to address the first issue and stellar structure models to address the second. For purposes of showing that stable bound states of \(^8\text{Be}\) exist, the nuclear potential is the key quantity and its depth is specified by the strong coupling constant. Here we show that relatively small increases in the strong force lead to a stable bound state. As long as a stable state exists, the behavior of \(^8\text{Be}\) in stellar interiors is determined by composite parameters \( C \) [26, 27, 29] that specify the rates and yields for a given nuclear reaction (see Section 3). Note that the allowed nuclear structures, and hence the values of \( C \), are complicated functions of the more fundamental parameters appearing in the Standard Model of particle physics (compare Refs. [3, 4, 12, 14, 19, 26, 30]). Using a semi-analytic stellar structure model [26], Section 3 shows that a large fraction of parameter space allows for successful helium burning into beryllium, provided that \(^8\text{Be}\) is a stable isotope. These results are then verified using the state-of-the-art stellar structure code MESA [31, 32] in Section 4. Although the entire parameter space of possible nuclear reactions is too large to fully explore in this work, we demonstrate that stars can successfully burn beryllium into carbon in later stages of evolution (a full exploration of this parameter space is left for future work). The paper concludes in Section 5 with a summary of our results and a discussion of their implications.
2. Nuclear Considerations for Beryllium Bound States

This section considers models for nuclear structure in order to elucidate the requirements for making stable $^8$Be nuclei. Given the difficulties inherent in making a priori models for nuclear structure, along with the large parameter space available for alternate universes, we consider several approximate approaches. First, we review previous calculations using lattice chiral effective field theory [14]. These results indicate that shifts in the fundamental parameters at the level of $\sim 2 - 3\%$ are large enough to allow for stable $^8$Be (Section 2.1). Next, for comparison, we use simple approaches where the $^8$Be nucleus is considered as a bound state of two alpha particles [33]. Under this assumption, we model beryllium bound states using a square well potential (Section 2.2) and a generalization of Bohr theory (Section 2.3). For both of these approximations, the $^8$Be nucleus can have a stable configuration with binding energy $\sim 1$ MeV lower than separate alpha particles, provided that the strong coupling constant is larger by a few percent. For completeness, we also present an order of magnitude estimate using the semiempirical mass formula, which is based on the liquid drop model (Section 2.4). This latter argument also indicates that an increase in the strong coupling constant of a few percent would be sufficient to provide a stable $^8$Be nucleus. Although approximate, all four of these approaches suggest that moderate changes in the fundamental constants would allow for a stable $^8$Be isotope.

2.1. Lattice Chiral Effective Field Theory

This section reviews nuclear models resulting from Lattice Chiral Effective Field Theory [14] and uses the results to explore the possibility of finding bound states for $^8$Be. The key quantity of interest is the energy difference between a bound state of $^8$Be and two separate alpha particles. This energy difference can be written in the form

$$\Delta E_b = E_8 - 2E_4,$$

where $E_8$ and $E_4$ denote the binding energies of $^8$Be and $^4$He. In our universe, $\Delta E_b \approx +92$ keV, whereas we require $\Delta E_b < 0$ for the synthesis of $^8$Be to be energetically favored. The required change is thus of order 100 keV.

The existing calculations for lattice Effective Field Theory treatments of nuclear structure [14] consider possible changes in binding energy due to variations in the pion mass $M_\pi$ and the fine structure constant $\alpha$. The
changes in binding energies for the nuclei of interest can be written in the form

$$\delta E_j = \frac{\partial E_j}{\partial M_\pi} \delta M_\pi \bigg|_0 + \frac{\partial E_j}{\partial \alpha} \delta \alpha,$$

where the subscripts on the energies label the nuclear species and where the partial derivatives are to be evaluated at the values realized within our universe. This expression represents the leading order correction and is limited to small changes in the binding energies. In this context, the total binding energy of $^8\text{Be}$ (or twice that of $^4\text{He}$) is of order 56 MeV, whereas we are interested in changes of order 0.1 – 1 MeV, so that $(\delta E_j)/E_j \ll 1$ is satisfied.

More specifically, this derivation is based on reference [14], which holds for relative changes as large as $\sim 10\%$. In this scheme, the pion mass is simply related to the light quark masses $M_\pi^2 \propto m_u + m_d$ [14], so that the first term in equation (4) specifies the dependence on $(m_u,m_d)$. The second term arises from variations in the fine structure constant $\alpha$.

In order for variations in the constants of nature to allow for stable $^8\text{Be}$ nuclei, we require that the binding energy of beryllium change by an amount that is not equal to twice the binding energy of helium, i.e., we require $\delta E_8 \neq 2(\delta E_4)$. This condition can be realized as long as the derivatives from equation (4) do not have specific values, i.e., we required $\partial E_8/\partial M_\pi \neq 2 \partial E_4/\partial M_\pi$ and/or $\partial E_8/\partial \alpha \neq 2 \partial E_4/\partial \alpha$. Reference [14] carries out a calculation of these partial derivatives using an Auxiliary Field Quantum Monte Carlo (AFQMC) scheme. In the calculation, the derivatives appearing in equation (4) are expanded into separate terms corresponding to variations in nuclear parameters appearing in the AFQMC action (strength of the pion exchange potential, the coefficients of the isospin terms, etc.). Significantly, for the two nuclear species of interest, the individual partial derivatives (and hence their sums) always satisfy the requirement that $\partial E_8/\partial \xi \neq 2\partial E_4/\partial \xi$, where $\xi$ represents any of the aforementioned parameters. As a result, changes in the nuclear parameters (and $\alpha$) can lead to variations in the binding energy of $^8\text{Be}$.

The energy difference $\Delta E_b$ can be made either larger or smaller through variations in the fundamental constants. As a result, only about half of the allowed variations are expected to lead to nuclear states with lower binding energies. In addition, a bound state for $^8\text{Be}$ requires that the binding energy be lower by at least $(\Delta E)/E \sim 100 \text{ keV}/(56 \text{ MeV}) \sim 0.0018$. This level of variation in the binding energy of $^8\text{Be}$ corresponds to changes in the light quark masses of $2 - 3\%$ and/or changes of $\sim 2.5\%$ in the fine structure
2.2. Square Well Potential

Here we assume that the alpha particles are the basic constituents, and use a square well potential to model the bound state of the two particle system (see [20] for an analogous treatment of diprotons). The reduced mass is given by $m_R = m_1 m_2 / (m_1 + m_2) = m_\alpha / 2 \approx 2 m_N$, where $m_N$ is the nucleon mass (assumed to be the same for protons and neutrons). The system dynamics is then given by the Schrödinger equation in its usual form. Let $E$ be the energy of the two particle system, which can be bound or unbound, but does not include the binding energy of the alpha particles themselves. Here we assume that the potential can be described by a three dimensional square well of depth $V_0$ and width $b$ and limit the discussion to spherically symmetric states ($\ell = 0$). For bound states, the energy $E < 0$, so we define $\epsilon \equiv -E = |E|$, as well as the ancillary quantities

$$\omega^2 \equiv \frac{2m_R(V_0 - \epsilon)}{\hbar^2} \quad \text{and} \quad k^2 \equiv \frac{2m_R \epsilon}{\hbar^2}. \quad (5)$$

The energy levels are given by the quantum condition

$$\omega \cos \omega b \sin \omega b = -k. \quad (6)$$

For convenience we define

$$z^2 = k^2 b^2 \quad \text{and} \quad \lambda^2 = \frac{2m_R V_0 b^2}{\hbar^2}, \quad (7)$$

so that the quantum condition reads

$$(\lambda^2 - z^2)^{1/2} \cos(\lambda^2 - z^2)^{1/2} = -z \sin(\lambda^2 - z^2)^{1/2}. \quad (8)$$

We are interested in the case where $z^2 \ll \lambda^2$. In the limit $z \to 0$, we obtain the solution $\lambda_0 = \pi/2$. This value represents the critical case, so that no bound states exist for $\lambda < \lambda_0 = \pi/2$. In other words, in order for a bound state to exist, we require $\lambda > \pi/2$, or

$$V_0 b^2 > \frac{\pi^2 \hbar^2}{8m_R}. \quad (9)$$

The right hand side has numerical value $\sim 25.5$ MeV fm$^2$. 

constant [14].
For small (but nonzero) values of $z$, we get
\[ \lambda \cos \lambda = -z \sin \lambda + \mathcal{O}(z^2). \] (10)

In this regime, $\lambda$ will be near (but not equal to) $\pi/2$, so we write $\lambda = \pi/2 + \theta$, where $|\theta| \ll 1$, so that $\cos \lambda = -\sin \theta$ and $\sin \lambda = \cos \theta$. Now the quantum condition takes the form
\[ \left( \frac{\pi}{2} + \theta \right) \sin \theta = z \cos \theta, \] (11)
which implies that $z \approx (\pi/2)\theta$, where we have used the fact that $\theta^2 \ll 1$.

The energy level becomes
\[ \epsilon = \frac{\pi^2 \hbar^2}{8m_Rb^2} \left[ \left( \frac{2m_RV_0b^2}{\hbar^2} \right)^{1/2} - \frac{\pi}{2} \right]^2. \] (12)

Now we want to use this result to estimate how much change is required for $^{8}\text{Be}$ to have a bound state. In our universe, with no bound state, the effective depth $V_0$ of the potential well is close to the boundary given by $\lambda = \pi/2$. In an alternate universe, suppose that the potential well has depth
\[ V = fV_0, \] (13)
where $f > 1$ is a dimensionless factor. For the sake of definiteness, we set $b = 1$ fm. Inserting numerical values, the energy of the bound state is given by
\[ \epsilon = 62.7 \text{ MeV} \left( \sqrt{f} - 1 \right)^2. \] (14)

Suppose, for example, we require the bound state to have binding energy $\epsilon = 1$ MeV. The required factor $f \approx 1.27$. If the binding energy is only 0.1 MeV, the required factor $f \approx 1.081$. As a result, working within the square well approximation, the depth of the potential well $V_0$ must change by of order a few percent to allow for a bound state of $^{8}\text{Be}$.

Of course, the direction of the change matters. The deeper value of the potential well arises from an increase in the strength of the strong force. A decrease in the strength of the strong force would lead to the nucleus being

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\(^1\)Since $f = 1$ gives $\epsilon = 0$ in this expression, we are considering the actual binding energy of $-92$ keV for $^{8}\text{Be}$ in our universe to be negligible relative to 62.7 MeV.
even less bound. In addition, we note that if the strong force changes, then
the binding energy of the alpha particles will also change. In this approxi-
mation, we are assuming that the alpha particles are much more bound than
the $^8\text{Be}$ nucleus. If the strong force increases in strength, then the alpha
particles will be more tightly bound and will act as independent entities to
a greater extent. However, it is important that the binding energy of the
$^8\text{Be}$ nucleus does not change at exactly the same rate as that of two alpha
particles (the previous section addresses this issue). Finally, we note that
the potential for the strong force is often modeled with a Yukawa form,

$$V_Y = -\frac{g^2}{r} \exp[-\beta r], \quad (15)$$

where $\beta$ is an inverse length scale that sets the range of the strong force. In
this context, we find that

$$\frac{\Delta g}{g} \propto \frac{\Delta V_0}{V_0} \quad (16)$$

so that the required change in the strong force coupling strength is also of
order a few percent.

2.3. Bohr Model

Here we consider the two alpha particles that make up the $^8\text{Be}$ nucleus
to be in orbit about their common center of mass and held together by the
strong force. Using the form of the Yukawa potential from equation (15),
force balance implies

$$m_R v^2 = \frac{g^2}{r} \exp[-\beta r] (1 + \beta r). \quad (17)$$

The total energy has the form

$$E = \frac{g^2}{2r} \exp[-\beta r] (\beta r - 1). \quad (18)$$

Using the Bohr quantization condition $m_R v r = n \hbar$, the left hand side of
equation (17) can be written

$$m_R v^2 = \frac{n^2 \hbar^2}{m_R r^2}. \quad (19)$$
In our universe, no bound state exists, and the energy $E$ is close to zero. This condition leads us to work in terms of the quantity

$$\frac{1}{\beta_r} = 1 + \eta,$$  \hspace{1cm} (20)

where this expression serves as the definition of $\eta$. Since $\eta$ is small, we can replace the exponential term with its value in the limit $\eta \to 0$ so that $\exp[\beta r] = e$. The energy becomes

$$E = -\frac{g^2 \beta}{2e} \eta,$$  \hspace{1cm} (21)

and the remaining equations can be combined to take the form

$$\frac{\hbar \beta^2}{m_R} (1 + \eta)^2 = \frac{g^2 \beta}{e} (2 + \eta).$$  \hspace{1cm} (22)

Next we define the parameter

$$\Lambda \equiv \frac{g^2 m_R}{e \beta \hbar^2},$$  \hspace{1cm} (23)

so that $\eta$ is given by the solution to

$$1 + 2\eta + \eta^2 = \Lambda(2 + \eta).$$  \hspace{1cm} (24)

In our universe, the energy is essentially zero, so that $\eta \approx 0$ and $\Lambda \approx 1/2$. In general, the parameter $\eta$ is given by

$$\eta = \Lambda/2 - 1 + \left[\frac{\Lambda^2}{4} + \Lambda\right]^{1/2},$$  \hspace{1cm} (25)

where we take the positive root of the quadratic. Given that $\Lambda = 1/2$ in our universe, the energy scale in equation (21) can be written

$$\frac{g^2 \beta}{2e} = \frac{\beta^2 \hbar^2}{4m_R} \approx 5.1 \text{ MeV},$$  \hspace{1cm} (26)

where we have used $\beta^{-1} = 1 \text{ fm}$.

To compare with the results obtained earlier for the square well model, let us assume that we want the bound state energy to be 1 MeV (instead of zero). We thus need $\eta \approx 0.2$, and hence (from equation (24)) $\Lambda \approx 0.65$. As a result, $(\Delta \Lambda)/\Lambda = 0.30$ and $(\Delta g)/g = 0.15$. In this case, a 15 percent change in the strong coupling constant is necessary to obtain a 1 MeV bound state. If we only require the bound state energy to be 0.1 MeV, then the required values are correspondingly smaller, $(\Delta \Lambda)/\Lambda = 0.03$ and $(\Delta g)/g = 0.015$. 

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2.4. Semiempirical Mass Formula

In this section we derive an order of magnitude estimate for the change in the strong force necessary to make $^8$Be stable against decay to alpha particles. This treatment uses the semiempirical mass formula (SEMF) and is highly approximate, as the SEMF does not work well for small nuclei (especially $^4$He). The binding energy $E$ from the semiempirical mass formula can be written in the form

$$E = a_V A - a_S A^{2/3} - a_C Z^2 A^{-1/3} + a_P A^{-1/2}, \quad (27)$$

where $A$ is number of nucleons and where we have neglected the asymmetry term since it vanishes for the nuclei of interest [34, 35]. We want to compare the binding energy for $^8$Be ($A = 8$) with that for two separate $^4$He nuclei. In our universe, $E(A = 8) \approx 2E(A = 4)$, although the SEMF, as written, does not reflect this finding. Two helium nuclei have more binding energy than a single beryllium nucleus by a small increment $\delta$. The volume term (given by $a_V$) is linear in $A$ and is the same for both states. The Coulomb term (given by $a_C$) is small and can be neglected to the order of interest here. The condition in our universe can thus be written

$$4 \left(2^{1/3} - 1\right) a_S - (1 - 8^{-1/2}) a_P = -\delta. \quad (28)$$

If we want to increase the difference in binding energy from essentially zero (as in our universe) to 1 MeV (the benchmark value used in previous sections), then the first term must increase by 1 MeV. The coefficient $a_S \approx 18$ MeV (e.g., [35]), so the first term has a value of $\sim 18.7$ MeV. The required change in the coefficient is thus $(\Delta a_S)/a_S = 1/18.7 = 0.053$. The value of $a_S$ is determined by the strong force and is proportion to $g^2$. As a result, the required $(\Delta g)/g \approx 0.027$. Once again, a few percent increase in the coupling constant for the strong force is sufficient to allow $^8$Be to be produced from alpha particles as an exothermic reaction with yield $\sim 1$ MeV.

3. Semi-Analytic Stellar Structure Models

Stellar structure and evolution is governed by four coupled differential equations, which describe hydrostatic equilibrium, conservation of mass, heat transport, and energy generation [6, 7, 8, 36, 37]. These equations must be augmented by specification of the equation of state, the stellar opacity, and the nuclear reaction rates. Following previous work [26, 27], this section
develops a polytropic stellar structure model. Although approximate, the model is flexible, and provides solutions over a range of parameter space where the constants of nature vary by many orders of magnitude.

It is useful to define a fundamental scale $M_0$ for stellar masses, i.e.,

$$M_0 \equiv \alpha_G^{-3/2} m_p = \left( \frac{\hbar c}{G} \right)^{3/2} m_p^{-2} \approx 3.7 \times 10^{33} \, \text{g} \approx 1.85 \, M_\odot ,$$

where $\alpha_G$, $\hbar$, and $c$ are the fine-structure constant, Planck constant, and speed of light, respectively, and $m_p$ is the proton mass. The numerical values correspond to standard values of the constants.

Stellar masses are comparable to this benchmark scale, in our universe and others. The mass scale $M_0$ is also comparable to the Chandrasekhar mass.

### 3.1. Hydrostatic Equilibrium Structures

For this model, we use a polytropic equation of state of the form $P = K \rho^\Gamma$, where $\Gamma = 1 + 1/n$. The polytropic index $n$ is expected to be slowly varying over the range of stellar masses. Low mass stars remain convective over much of their lifetimes and have polytropic index $n = 3/2$. On the other hand, high mass stars have substantial radiation pressure in their interiors and the index $n \to 3$. Use of a polytropic equation of state allows us to replace the force balance and mass conservation equations with the Lane-Emden equation

$$\frac{d}{d\xi} \left( \xi^2 \frac{df}{d\xi} \right) + \xi^2 f^n = 0 ,$$

where we use the standard definitions

$$\xi \equiv \frac{r}{R}, \quad \rho = \rho_c f^n, \quad \text{and} \quad R^2 = \frac{K \Gamma}{(\Gamma - 1) 4\pi G \rho_c^{2-\Gamma}}.$$  

For a given index $n$, equation (30) specifies the density profile for given values of the constants $\rho_c$ and $R$. The corresponding pressure profile is then specified through the polytropic equation of state. For stars with the properties required for nuclear fusion, the stellar material obeys the ideal gas law so that the temperature is given by $T = P/(\mathcal{R} \rho)$, with $\mathcal{R} = k/\langle m \rangle$. Integration of equation (30) outwards, subject to the boundary conditions $f = 1$ and $df/d\xi = 0$ at $\xi = 0$, then determines the outer boundary of the star. Specifically, the stellar radius is given by $R_* = R \xi_*$, where the parameter $\xi_*$ is defined to be the value of the dimensionless variable where $f(\xi_*) = 0$. 

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For given values of the constants $\rho_c$ and $R$, and the index $n$, the physical structure of the star is specified. However, for a given stellar mass $M_*$, these parameters are not independent, but rather are related through the integral constraint

$$M_* = 4\pi R^3 \rho_c \int_0^{\xi_*} \xi^2 f^n(\xi) d\xi \equiv 4\pi R^3 \rho_c \mu_0. \tag{32}$$

The second equality defines the dimensionless quantity $\mu_0$, which is a function of the polytropic index $n$ and is of order unity.

### 3.2. Nuclear Reactions

Thermonuclear fusion primarily depends on three physical variables: the temperature $T$, the Gamow energy $E_G$, and the nuclear fusion factor $S(E)$, where the latter quantity sets the reaction cross section. Here we want to determine how the nuclear ignition temperature depends on the other variables of the problem. The Gamow energy can be written in the form

$$E_G = \left(\pi \alpha Z_1 Z_2\right)^2 \frac{2m_1 m_2}{m_1 + m_2} c^2 = \left(\pi \alpha Z_1 Z_2\right)^2 2m_R c^2, \tag{33}$$

where $m_j$ are the masses of the nuclei and $Z_j$ are the charges. The second equality defines the reduced mass $m_R$. For reactions involving two protons, $E_G = 493$ keV, whereas for two helium nuclei, $E_G = 31.6$ MeV. The Gamow energy specifies the degree of Coulomb barrier penetration, and is determined by $\alpha$ (the strength of the electromagnetic force) and the masses of the reacting particles. The strong and weak nuclear forces determine the reaction cross sections. In this setting, the interaction cross sections $\sigma(E)$ are related to the nuclear fusion factor $S(E)$ according to

$$\sigma(E) = S(E) \exp \left[ -\left(\frac{E_G}{E}\right)^{1/2} \right], \tag{34}$$

where $E$ is the energy of the interacting particles. The gas at the center of the star obeys the ideal gas law and its constituent particles have a distribution of energy determined by the temperature. In ordinary stars, under most circumstances, the reaction cross section depends on energy according to $\sigma \propto 1/E$. As a result, the nuclear fusion factor $S(E)$ is a slowly varying function of energy. This form for the cross section arises when the interacting nuclei can be described by non-relativistic quantum mechanics. In this regime, $\sigma$
is proportional to the square of the de Brogile wavelength, so that \( \sigma \sim \lambda^2 \sim (h/p)^2 \sim h^2/(2mE) \).

The energies of the interacting nuclei have a thermal distribution, so that the weighted cross section has the form

\[
\langle \sigma v \rangle = 8 \left[ \frac{1}{\pi m_R} \right]^{1/2} \left[ \frac{1}{kT} \right]^{3/2} \int_0^\infty \sigma(E) \exp\left[ -E/kT \right] E dE. \tag{35}
\]

With the thermal factor (from equation [35]) and the coulomb repulsion factor (from equation [34]), the nuclear reaction rate is controlled by the composite exponential factor \( \exp[-\Phi] \), where the function \( \Phi \) includes two contributions, i.e., \( \Phi = E/(kT) + (E_G/E)^{1/2} \). The function \( \Phi \) has a minimum value at a characteristic energy given by \( E_0 = E_G^{1/3}(kT/2)^{2/3} \). The integral in equation (35) has most of its support for energies \( E \approx E_0 \), where the function \( \Phi \) itself takes the value \( \Phi_0 = 3(E_G/4kT)^{1/3} \).

We can approximate the integral of equation (35) using Laplace’s method, so that the corresponding reaction rate \( R_{12} \) for two nuclear can be written in the form

\[
R_{12} = n_1 n_2 \frac{8}{\sqrt{3\pi\alpha Z_1 Z_2 m_R c(N_S)!}} S(E_0) \Theta^2 \exp[-3\Theta], \tag{36}
\]

where we have defined

\[
\Theta \equiv \left( \frac{E_G}{4kT} \right)^{1/3}. \tag{37}
\]

In this expression, the reacting nuclei have number densities \( n_1 \) and \( n_2 \), and the parameter \( N_S \) counts the number of identical particles involved in the reaction.

3.3. Stellar Luminosity and Energy Transport

If we define \( \varepsilon(r) \) to be the power generated per unit volume, the luminosity of the star is determined through the equation

\[
\frac{dL}{dr} = 4\pi r^2 \varepsilon(r). \tag{38}
\]

The luminosity density \( \varepsilon \) can be written in terms of the nuclear reaction rates so that it takes the form

\[
\varepsilon(r) = C \rho^2 \Theta^2 \exp[-3\Theta], \tag{39}
\]
where $\Theta$ is defined in equation (37) and where we define a nuclear burning parameter

$$C \equiv \frac{\langle \Delta E \rangle R_{12}}{\rho^2 \Theta^2} \exp[3\Theta] = \frac{8\langle \Delta E \rangle S(E_0)}{\sqrt{3\pi} \alpha m_1 m_2 Z_1 Z_2 m_R c (N_s !)},$$

(40)

where $\langle \Delta E \rangle$ is the mean energy generated per nuclear reaction. For proton-proton fusion in our universe, under typical conditions, the nuclear burning parameter $C \approx 2 \times 10^4$ cm$^5$ s$^{-3}$ g$^{-1}$. For deuterium fusion in our universe, for which there is no bottleneck due to the weak interaction, the nuclear constant is much larger, $C \approx 2.3 \times 10^{21}$ cm$^5$ s$^{-3}$ g$^{-1}$ [29].

The total stellar luminosity $L_*$ is determined by integrating over the star,

$$L_* = C 4\pi R^3 \rho_c^2 \int_0^{\xi_*} f^{2n} \xi^2 \Theta^2 \exp[-3\Theta] d\xi \equiv C 4\pi R^3 \rho_c^2 I(\Theta_c).$$

(41)

In this expression, the second equality defines the function $I(\Theta_c)$, where we also define $\Theta_c = \Theta(\xi = 0) = (E_G/4kT_c)^{1/3}$. Note that the temperature is given by $T = T_c f(\xi)$ so that $\Theta = \Theta_c f^{-1/3}(\xi)$. As a result, for a given value of the polytropic index $n$ (which determines the form of $f(\xi)$ through the Lane-Emden equation), this integral is specified up to the constant $\Theta_c$.

At this stage of the derivation, the definition of equation (31), the mass integral constraint (32), and the luminosity integral (41) provide us with three equations for four unknowns: the radial scale $R$, the central density $\rho_c$, the total luminosity $L_*$, and the coefficient $K$ in the equation of state. The fourth equation of stellar structure is thus required to finish the calculation. Here we are primarily interested in larger stars where energy is transported by radiation, so that the energy transport equation takes the form

$$T^3 \frac{dT}{dr} = -\frac{3\rho \kappa}{4ac} \frac{L(r)}{4\pi r^2},$$

(42)

where $\kappa$ is the opacity (cross section per unit mass for photons interacting with stellar material). Next we make the following simplification. The opacity $\kappa$ in stellar interiors generally follows Kramer’s law so that $\kappa \sim \rho T^{-7/2}$ [8 37]. For polytropic equations of state, we find that the product $\kappa\rho \sim \rho^{2-n/2}$. For the index $n = 7/4$, the product $\kappa\rho$ is thus exactly constant. For other values of the index $n$, the quantity $\kappa\rho$ will be slowly varying. As a result, we can make the approximation $\kappa\rho = \kappa_0 \rho_c = \text{constant}$ for purposes
of solving the energy transport equation (42). Equation (41) then simplifies to the form

\[ L_* \int_0^{\xi_*} \frac{\ell(\xi)}{\xi^2} d\xi = aT_c^4 \frac{4\pi c}{3\rho_c \kappa_0} R, \]  

(43)

where we have defined a dimensionless luminosity density \( \ell(\xi) \equiv L(\xi)/L_* \) (see the integral in equation [41] for the full definition of \( \ell(\xi) \)). For purposes of solving equation (43), we can assume that the integrand of equation (41) is sharply peaked toward the center of the star. The temperature in the stellar core can be modeled as an exponentially decaying function of position so that \( T \sim \exp[-\beta \xi] \). With these approximations (see [26] for further detail) the expression for \( \ell(\xi) \) becomes

\[ \ell(\xi) = \frac{1}{2} \int_0^{x_{\text{end}}} x^2 e^{-x} dx \quad \text{where} \quad x_{\text{end}} = \beta \Theta_c \xi. \]  

(44)

and the luminosity (from equation [43]) can be written in the form

\[ L_* = aT_c^4 \frac{4\pi c}{3\rho_c \kappa_0} \frac{R}{\beta \Theta_c}. \]  

(45)

Note that the parameter \( \beta \) is defined implicitly.

3.4. Stellar Structure Solutions

The results of the previous section provide us with four equations and four unknowns. These equations can be solved to obtain the following expression for the central temperature, written here in terms of the variable \( \Theta_c \),

\[ I(\Theta_c)\Theta_c^{-8} = \frac{2^{12}\pi^5}{45} \frac{1}{\beta \kappa_0 c E^3 \hbar^2 c^2} \left( \frac{M_*}{\mu_0} \right)^4 \left( \frac{G\langle m \rangle}{n+1} \right)^7. \]  

(46)

Note that the right hand side of the equation is dimensionless. The quantities \( \mu_0 \) and \( \beta \) are dimensionless measures of the mass and luminosity integrals over the star; these quantities, as well as the index \( n \), are of order unity. The mass of the star is \( M_* \), and the remaining parameters depend on the fundamental constants.

For typical values of the parameters in our universe, the right hand side of this equation is approximately \( 10^{-9} \) for hydrogen burning stars. For helium burning, the Gamov energy is larger by a factor of 64, and the nuclear constant is smaller, perhaps by an order of magnitude. [Keep in mind that
we are working in the counterfactual case where $^8$Be is stable, so that helium burning can proceed via two-body reactions.] The right hand side of the equation for helium burning is thus $\sim 10^{-16}$.

With the central temperature $T_c$ (equivalently, $\Theta_c$) specified by the solution to equation (46), we can solve for the remaining stellar parameters. The stellar radius is given by

$$R_* = \frac{GM_*\langle m \rangle}{kT_c} \frac{\xi_*}{(n+1)\mu_0},$$  

(47)

the luminosity has the form

$$L_* = \frac{16\pi^4}{15} \frac{1}{\hbar^2 c^2 \beta \kappa_0 \Theta_c} \left( \frac{M_*}{\mu_0} \right)^3 \left[ \frac{G\langle m \rangle}{n+1} \right]^4.$$

(48)

Finally, the photospheric temperature $T_*$ is determined by the usual outer boundary condition so that

$$T_* = \left[ \frac{L_*}{4\pi R_*^2 \sigma_{sb}} \right]^{1/4},$$

(49)

where $\sigma_{sb}$ is the Stefan-Boltzmann constant.

### 3.5. Minimum and Maximum Stellar Mass

The semi-analytic stellar structure model (developed above) can be used to explore the range of possible stellar masses that can burn helium in universes with stable $^8$Be. The minimum mass of a star is determined by the onset of degeneracy pressure. For a given stellar mass, degeneracy pressure enforces a maximum temperature that can be attained in the stellar core. If this maximum temperature is lower than the value required for nuclear fusion, the star cannot ignite. Previous work [26, 37] has shown that the maximum temperature that can be realized in a stellar core is given by

$$kT_{\text{max}} = (4\pi)^{-2/3} \frac{5}{36} \frac{m_e}{\hbar^2} G^2 M_*^{4/3} \langle m \rangle^{8/3}.$$  

(50)

Setting this value of the central temperature equal to the minimum temperature $T_{\text{nuc}}$ required for nuclear burning, we obtain the minimum stellar mass

$$M_{\text{min}} = 6(3\pi)^{1/2} \left( \frac{4}{5} \right)^{3/4} \left( \frac{m_p}{\langle m \rangle} \right)^2 \left( \frac{kT_{\text{nuc}}}{m_e c^2} \right)^{3/4} M_0.$$  

(51)
Note that this minimum stellar mass is given by a dimensionless expression times the fundamental stellar mass scale $M_0$ defined in equation (29).

By using the minimum mass from equation (51) to specify the mass in equation (46), we can eliminate the mass dependence and solve for the minimum value of the nuclear ignition temperature $T_{\text{nuc}}$. The resulting temperature is given in terms of $\Theta_c$, which is given by the solution to the following equation

$$\Theta_c I(\Theta_c) = \left( \frac{2^{23} \pi^{7/4}}{5^{11}} \right) \left( \frac{\hbar^3}{c^2} \right) \left( \frac{1}{\beta \mu_0^4} \right) \left( \frac{1}{\langle m \rangle m_e^3} \right) \left( \frac{G \kappa_0 \mathcal{C}}{\kappa_0 C} \right).$$

The parameters on the right hand side of the equation have been grouped to include numbers, constants that set units, dimensionless parameters of the polytropic solution, particle masses, and parameters that depend on the fundamental forces. The function of $\Theta_c$ on the left hand side of equation (52) is a decreasing function of $\Theta_c$ over the range of interest [26]. For larger values of the nuclear parameter $\mathcal{C}$, the solution to equation (52) thus occurs at larger values of $\Theta_c$ and hence lower central temperatures. This feature implies that the lower mass limit for helium burning can be below that for hydrogen burning. The value of $\mathcal{C}$ for helium burning into stable beryllium (Section 3.6) is expected to be comparable to that for deuterium burning in our universe, with a comparable lower mass limit ($M_{\text{min}} \sim 0.015 M_\odot$ for deuterium).

For hydrogen-burning stars with zero metallicity, we can take $\langle m \rangle = m_p$, so that the right hand side of equation (52) becomes $RHS \approx 4 \times 10^{-6}$, where we have used the value of $\mathcal{C}$ appropriate for hydrogen fusion in our universe. The function on the left-hand side of equation (52) has a maximum value of $\sim 0.05$ [26, 27]. In order for working stellar structure solutions to exist, the right-hand side of the equation must be less than this maximum. In our universe, this constraint is satisfied by a factor of $\sim 10^4$, and this lee-way allows (hydrogen-burning) stars to exist over a range of possible masses ($M_* \sim 0.1 - 100 M_\odot$). As outlined below, the nuclear parameter $\mathcal{C}$ for helium burning (into stable beryllium) is a factor of $\sim 10^{16}$ larger than that for hydrogen burning in our universe. For helium-burning stars and a pure helium composition, we can take $\langle m \rangle = 4 m_p$ and the $RHS \sim 10^{-22}$. Because of the larger value of the nuclear parameter $\mathcal{C}$, stars burning helium into beryllium can exist over a wider range of parameter space than those burning hydrogen.

Note that this result is not unexpected — in our universe, the stellar mass
range for burning deuterium (which has a larger value of $C$) is wider than that for hydrogen. In general, this constraint can be written in the form

$$\left(\frac{G}{G_0}\right) \left(\frac{\alpha}{\alpha_0}\right)^{-2} \left(\frac{C}{C_0}\right)^{-1} < 1.2 \times 10^4,$$

where $C_0 = 2 \times 10^4$ (in cgs units).

An estimate for the maximum stellar mass can also be obtained. In massive stars, the central pressure has contribution from gas pressure (through the ideal gas law) and from radiation pressure. If the radiation pressure dominates over the gas pressure, then the star does not have a stable hydrostatic state available to it. In the limit where the radiation pressure is large, the star has the same energy (self-gravity and thermal) for any radial size and thus cannot be stable [37]. If we let $f_g$ be the fraction of the central pressure provided by gas, the maximum stellar mass has the form

$$M_{\ast\text{max}} = \left(\frac{36}{\pi}\right)^{1/2} \left[\frac{3}{a} \frac{(1-f_g)}{f_g^4}\right]^{1/2} G^{-3/2} \left(\frac{k}{\langle m\rangle}\right)^2 = \left(\frac{36\sqrt{10}}{\pi^{3/2}}\right) \left(\frac{m_p}{\langle m\rangle}\right)^2 M_0,$$

where the second equality assumes that $f_g = 1/2$. This value is often used, although the stability boundary is not a perfectly sharp function of $f_g$. With this choice, the above expression becomes $M_{\ast\text{max}} \approx 20 (m_p/\langle m\rangle)^2 M_0$. Since massive stars are highly ionized, $\langle m\rangle \approx 0.6 m_p$ under standard conditions, and hence $M_{\ast\text{max}} \approx 56 M_0 \approx 100 M_\odot$ for our universe. Note that this limit is nearly identical to the constraint derived from the Eddington luminosity [26]. Notice also that this upper limit on stellar mass is independent of the nuclear burning parameter $C$ (to leading order), so that alternate universes are expected to have the same cutoff (for fixed values of $G$ and $m_p$).

### 3.6. Helium Burning into Beryllium

We now consider stellar structure models for universes where beryllium has a stable $A = 8$ isotope. In this scenario, after hydrogen burning has run its course, the helium that has built up in the stellar core can burn directly into $^8\text{Be}$. We thus assume that the reaction

$$^4\text{He} + ^4\text{He} \rightarrow ^8\text{Be} + \gamma$$

is viable. In order to explore the properties of stars fueled by this reaction, we use the stellar model developed in previous sections.
As a start, we set the structure parameters $\alpha$ and $\alpha_G$ to have the standard values in our universe. For varying choices of the nuclear parameters, we can then compare the properties of these helium burning stars to those in our universe. The composition of the star must also be specified. Here we consider the stellar core to be composed entirely of helium, i.e., we assume that hydrogen burning in the core has proceeded to completion.

Because the reaction (55) does not occur in an exothermic manner in our universe, the reaction rate cannot be measured, but we still need to specify the nuclear parameter $C$. Keep in mind that this quantity includes both the energy per reaction $\Delta E$ and the nuclear fusion factor $S(E_0)$ (see equation [40]). Since the reaction (55) takes places through the strong force, we expect the reaction rate to be relatively large. More specifically, the proton-proton chain for hydrogen burning (in our universe) requires the weak interaction, as two protons must be converted into neutrons to make helium, and $C \sim 10^4$ in cgs units. For comparison, for deuterium burning reactions the weak force does not come into play, and $C \sim 10^{22}$ in the same units. Although the energy per reaction $\Delta E$ for helium burning (here into beryllium) is likely to be smaller than that for helium production in our universe, the difference is likely to be less than a factor of ten; the lack of a weak interaction bottleneck should more than compensate. We thus expect the nuclear parameter $C$ for $2\alpha \rightarrow ^8\text{Be}$ to be closer to that for deuterium burning ($C \sim 10^{22}$ cgs) than for hydrogen burning ($C \sim 10^4$ cgs). Given the uncertainty in this parameter, we explore a range of values, from $C = 10^{12}$ to $10^{24}$ cm$^5$ sec$^{-3}$ g$^{-1}$.

The H-R diagram for these helium burning stars is shown in Figure 1, where these results are obtained using the semi-analytic stellar structure model. The locations of the helium burning main-sequence are shown for four choices of the nuclear parameter $C$ (where the spacing of the values is even in the logarithm). For each value of $C$, the range of allowed stellar masses is different (see Section 3.5). Note that these helium burning stars occupy approximately the same region of the H-R diagram as stars in our universe. In Figure 1, the thin black curve shows the hydrogen burning main-sequence for stars operating with only a single proton-proton nuclear reaction chain (calculated using the semi-analytic model). As another comparison, the thick black curve shows the helium burning main-sequence for stars in our universe (calculated numerically as described in Section 4). This curve varies with time and corresponds to the condition that the carbon abundance in the stellar core has reached 2%. In our universe where the triple alpha process operates, helium burning is suppressed by the need for three-body reactions,
Figure 1: H-R diagram for stars that can burn helium into beryllium. The colored curves show the main-sequence for helium burning in alternate universes where $^8$Be is stable, for four choices of the nuclear burning parameter, from left to right: $C$ (in cgs units) = $10^{12}$ (cyan), $10^{16}$ (blue), $10^{20}$ (green), and $10^{24}$ (red). For comparison, the thick black curve shows the helium burning main-sequence for stars in our universe where the triple alpha process is active. The thin black curve shows the main sequence for hydrogen burning stars with a single $p$-$p$ nuclear reaction chain and the structure of an $n = 3/2$ polytrope.
Figure 2: Luminosity versus stellar mass relation for stars that burn helium into beryllium. The curves show the luminosities for helium burning stars in alternate universes where $^8$Be is stable, for four choices of the nuclear burning parameter, from top to bottom: $C$ (in cgs units) = $10^{12}$ (cyan), $10^{16}$ (blue), $10^{20}$ (green), and $10^{24}$ (red). For comparison, the upper black line shows the relation for hydrogen burning stars using the simplified stellar model (see text).

but is enhanced due to the Hoyle resonance. Compared to the alternate scenario explored here, helium burning stars in our universe have somewhat higher central temperatures and luminosities. Nonetheless, the luminosities are roughly comparable to those of stars in our universe.

The stellar luminosity is plotted versus stellar mass in Figure 2. Note that the allowed mass range for helium burning stars extends down to much lower masses than for the hydrogen burning stars in our universe. This extension is a direct result of the larger value of $C$. With higher $C$, nuclear reactions can take place at lower temperatures, which can be achieved with smaller stellar masses. This effect is modest — the variation in stellar mass range is much smaller than the variation in $C$ — because of the extreme sensitivity
Figure 3: Central temperature versus stellar mass relation for stars that burn helium into beryllium. The colored curves show the temperatures for helium burning stars in alternate universes where $^8$Be is stable, for four choices of the nuclear burning parameter, from top to bottom: $C$ (in cgs units) = $10^{12}$ (cyan), $10^{16}$ (blue), $10^{20}$ (green), and $10^{24}$ (red). For comparison, the black curves shows the central temperature as a function of mass for stars burning hydrogen (through the p-p chain).

of nuclear reactions to temperature. Deuterium burning in our universe also proceeds via the strong force, with a large value of $C$, and deuterium burning extends down to $M_* \approx 0.015 M_\odot$ (small compared to the minimum mass of about $0.1 M_\odot$ for hydrogen burning). Except for the differences in the allowed mass range, the mass-luminosity relation is similar for all values of $C$ and for hydrogen burning stars. The variation in the luminosity with $C$ for a given mass is much smaller than the variation in luminosity with mass.

For completeness, the central temperatures of the helium burning stars are shown in Figure 3. Again, the range of stellar masses extends to lower values, but the range of central temperatures is roughly comparable to those of hydrogen burning stars in our universe, from a few million to a few hundred
million Kelvin. The slope of the central temperature curve is less steep for helium burning stars. This difference arises due to the greater temperature sensitivity — a small change in central temperature $T_c$ leads to an enormous change in the nuclear reaction rates. As a result, the range of central temperatures for a given value of the nuclear parameter $C$ is smaller for helium burning stars (compared to hydrogen burning stars in our universe).

Taken together, Figures 1–3 show that stars burning helium into stable beryllium have properties that are roughly comparable to ordinary hydrogen burning stars in our universe.

4. Numerical Stellar Evolution Models

The previous section demonstrates that stars in other universes can successfully burn helium into beryllium, provided that that latter nucleus has a stable state. These results were obtained using an extremely simple stellar model in order to understand the process at the most basic level. Since the regime of operation for these stars is quite different from those of ordinary stars in our universe, this section pursues the issue further by considering stellar evolution models using the state-of-the-art computational package MESA [31, 32]. The semi-analytic model is robust because of its simplicity. However, the model uses only a single nuclear reaction (at a given time), considers only stellar structure rather than time evolution, and is hard-wired with a polytropic equation of state. In contrast, the MESA package can evolve the stars from before their pre-main-sequence phase, onto the hydrogen burning main-sequence, through helium burning, and beyond. The code includes contributions to the equation of state from degeneracy pressure, radiation pressure, and gas pressure; it also includes both convective and radiative energy transport.

4.1. Modifications to the MESA Numerical Package

In order to use MESA to study $^8$Be production in other universes, we had to modify two principal components of the code. First, we needed to include a stable $^8$Be isotope. In our universe, the binding energy of $^8$Be, denoted here as $B_8$, is given by

$$B_8 = [4m_p + 4m_n - M(^8\text{Be})]c^2,$$

where $M(^8\text{Be})$ the mass of $^8$Be, $m_p$ is the mass of a proton, and $m_n$ is the mass of a neutron. With the standard values, the binding energy is $B_8 \simeq 56.5 \text{ MeV}$.
For comparison, the binding energy of $^4\text{He}$ is

$$B_4 = [2m_p + 2m_n - M(^4\text{He})]c^2 \simeq 28.296 \text{ MeV}. \quad (57)$$

As a result, the binding energy of two $^4\text{He}$ nuclei is larger than that of a single $^8\text{Be}$ nucleus by $\sim 100 \text{ keV}$. Within the isotope list of MESA, we changed the mass of $^8\text{Be}$ so that its binding energy increases to $B_8 \simeq 56.8 \text{ MeV}$. We preserved the mass of $^4\text{He}$ so that a single $^8\text{Be}$ nucleus has a larger binding energy than two $^4\text{He}$ nuclei by $\sim 200 \text{ keV}$. With the change in binding energies, $^8\text{Be}$ no longer decays into two $^4\text{He}$ nuclei and therefore subsists in appreciable amounts in stellar interiors.

The second component that we modified in MESA is the averaged product of cross section and speed, denoted $\langle \sigma v \rangle$, for the new reactions. Because $^8\text{Be}$ is unstable in our universe and decays on a time scale much shorter than stellar evolution time scales, previous treatments included neither the nuclide nor any reactions associated with that nuclide. Instead, a small equilibrium abundance of unstable $^8\text{Be}$ is built up in the core, and some fraction immediately is converted into $^{12}\text{C}$, so that the nuclear properties of $^8\text{Be}$ effectively cancel out (for further detail, see [6, 8, 7]). In the alternate universes under consideration here, the isotope $^8\text{Be}$ is stable, so that we no longer need the triple alpha ($3\alpha$) channel to synthesize $^{12}\text{C}$ from $^4\text{He}$. As a result, we broke apart the $3\alpha$ reaction into two separate reactions, where the first one synthesizes beryllium,

$$^4\text{He} + ^4\text{He} \leftrightarrow ^8\text{Be} + \gamma, \quad (58)$$

and the second reaction converts beryllium into carbon,

$$^4\text{He} + ^8\text{Be} \leftrightarrow ^{12}\text{C} + \gamma. \quad (59)$$

Unlike the case of the triple alpha reaction in our universe, the second reaction (59) can take place at any later time, and even in a different star (in a different stellar generation). We thus turn our attention to the reaction in equation (58), which we denote as $\alpha(\alpha, \gamma) ^8\text{Be}$. We provide the motivation for our choice of $\langle \sigma v \rangle$ for $\alpha(\alpha, \gamma) ^8\text{Be}$ by considering the $3\alpha$ reaction. The cross section $\langle \sigma v \rangle$ for $3\alpha$ is the sum of two constituent parts, a resonant component and a nonresonant component. The resonance channel is not relevant for our studies, so we focus on the nonresonant channel. Reference
[39] calculates an analytic approximation for $\langle \sigma v \rangle$ in the nonresonant channel of $\alpha(\alpha, \gamma) {}^8\text{Be}$, denoted here as $\langle \alpha\alpha \rangle^\ast$, with the following form:

$$
\langle \alpha\alpha \rangle^\ast = 6.914 \times 10^{-15}\ \text{cm}^3/\text{s} \ T_9^{-2/3} \ \exp(-13.489 T_9^{-1/3}) \ (60)
$$

$$
\times (1 + 0.031 T_9^{1/3} + 8.009 T_9^{2/3} + 1.732 T_9 + 49.883 T_9^{4/3} + 27.426 T_9^{5/3})
$$

where $T_9$ is the temperature in units of $10^9\text{K}$. Using equations (35) and (40), we can calculate the $S(E_0)$ factor from equation (60), which leads to the result

$$
S(E_0) \approx 6.760 \times 10^{-31} \text{erg cm}^2 \approx 422 \text{ keV barn}.
$$

Equation (40) defines the nuclear burning parameter $C$, which includes both the nuclear fusion factor $S(E_0)$ and the mean energy per reaction $\Delta E$. For the reaction $\alpha(\alpha, \gamma) {}^8\text{Be}$, the nuclear burning parameter takes the following form:

$$
C^\ast = \frac{8 \langle \Delta E \rangle^\ast S(E_0)}{\sqrt{3\pi\alpha m^2 Z^2 m_\text{RC}}},
$$

(62)

where $\langle \Delta E \rangle^\ast = 2B_4 - B_8$ is the endothermic energy “yield” for $2(^4\text{He}) \rightarrow {}^8\text{Be}$. After substituting the values from our universe into equation (62), we find $C^\ast = 5.69 \times 10^{23}\ \text{cm}^5 \text{s}^{-1} \text{ g}^{-1}$. When calculating the cross section for the triple alpha process in a reaction network, it is common to multiply equation (60) by additional factors in order to fit to experimental data (see Ref. [40, 41]). For our purposes, we do not include the fitting factors, but instead include an overall constant. As a result, our expression for $\langle \sigma v \rangle$ in alternate universes can be written in the form

$$
\langle \alpha\alpha \rangle = \frac{C}{5.69 \times 10^{23}} \left( \frac{\langle \Delta E \rangle^\ast}{\langle \Delta E \rangle} \right) \langle \alpha\alpha \rangle^\ast, \ (63)
$$

where $\Delta E$ (without the superscript) is the positive energy difference $\Delta E = B_8 - 2B_4$ for the universe in question (and where the expression is in cgs units). We incorporated equation (63) into the MESA reaction library and added the reaction (58) to the network. Note that this procedure does not preserve unitarity for the $^8\text{Be}$ compound nucleus. To preserve unitarity, the differential cross sections for all of the reactions that include a compound $^8\text{Be}$ nucleus would also have to be modified. Such a task is difficult and is beyond the scope of this present work (see Ref. [42] for further details about unitarity in nuclear reaction networks).
4.2. Results for Helium Burning into Beryllium

This section presents the results obtained for a range of possible nuclear reaction rates for $^8$Be production. As outlined above, in order to run a version of MESA with the new $^8$Be physics, we added $^8$Be to the list of isotopes and $\alpha(\alpha, \gamma)^8$Be to the reaction list. Specifically, the list of isotopes includes the following: $^1$H, $^3$He, $^4$He, $^8$Be, $^{12}$C, $^{14}$N, $^{16}$O, $^{20}$Ne, and $^{24}$Mg. The resulting reaction list contains 18 reactions, including the reaction for $\alpha(\alpha, \gamma)^8$Be but excluding the $3\alpha$ reaction. With this restricted list of isotopes and reactions, we can study helium burning into beryllium, as well as the production of carbon and oxygen. Note that the inverse reactions must also be included. The production of heavier elements (e.g., iron) is not considered here.

These simulations were performed using the initial value $Z = 10^{-4}$ for the metallicity of the star. This relatively low value of metallicity allows us to more easily interpret the production of heavy nuclei through stellar processes and to compare results with the simple model of the previous section. On the other hand, for technical reasons, the numerical code runs more robustly for $Z \neq 0$. The elemental abundances for the more common isotopes for this choice of metallicity ($Z = 10^{-4}$) are listed in Table 1. To obtain these abundances, we started with the values given in Ref. [43] for the metals that are stable in our universe (those nuclei with atomic number $> 2$ excluding $^8$Be) and scaled them such that they preserved their relative abundances and summed up to 95%. We then added in an ad-hoc $^8$Be mass fraction of 5%. Finally, we multiplied all of the metal mass fractions by $10^{-4}$ and added them to the hydrogen and helium mass fractions to obtain the resulting set of abundances in Table 1.

The other two parameters that must be specified are the stellar mass $M_*$ and the nuclear burning parameter $C$ for the production of $^8$Be. For the sake of definiteness, we consider the range of stellar masses to be similar to that of our universe, i.e., $M_* \sim 0.1 - 100M_\odot$. Due to convergence issues, we sometimes use a somewhat smaller range of masses. Because the cross section for helium burning ($^8$Be production) is unknown, we consider a wide range for the nuclear burning parameter $C = 10^{16} - 10^{24}$ (in cgs units). The upper end of this range is comparable to that appropriate for deuterium burning in our universe, which proceeds rapidly because the reaction only involves the strong force. For comparison, hydrogen burning in our universe is characterized by a much smaller effective value of $C$ (specifically, $C \sim 10^4$ in cgs units for the p-p reaction chain). The smaller value for hydrogen burning
| Isotope | Mass Fraction |
|---------|---------------|
| $^1\text{H}$ | 0.76          |
| $^3\text{He}$ | $3.0 \times 10^{-5}$ |
| $^4\text{He}$ | 0.24          |
| $^8\text{Be}$ | $4.9 \times 10^{-6}$ |
| $^{12}\text{C}$ | $1.7 \times 10^{-5}$ |
| $^{14}\text{N}$ | $4.8 \times 10^{-6}$ |
| $^{16}\text{O}$ | $4.5 \times 10^{-5}$ |
| $^{20}\text{Ne}$ | $1.0 \times 10^{-5}$ |
| $^{24}\text{Mg}$ | $2.0 \times 10^{-5}$ |

Table 1: Table of initial mass fractions for nonzero metallicity. (Note that the values for H and $^4\text{He}$ are slightly smaller than those listed so that the total adds up to unity.)

reflects the fact that the nuclear reaction chain must convert two protons into neutrons (thereby involving the weak force) in order to synthesize helium.

For given choices of metallicity, stellar mass, and nuclear burning parameter, the MESA code evolves the star from an initial state, down its pre-main-sequence track, onto the main-sequence where it burns hydrogen into helium, and then through the helium burning phase. With the reactions considered here, including a stable $^8\text{Be}$ isotope, the stars reach a helium burning main-sequence analogous to that found in the previous sections using the semi-analytic model.

Figure 4 shows resulting Hertzsprung-Russell (H-R) diagram for a range of values for the nuclear parameter $C$. In the figure, the curves represent the zero-age-main-sequence for helium burning (into beryllium). The onset of helium burning is not perfectly sharp in time, so that we need to specify the criterion used to define the helium burning main-sequence. The main-sequences shown in Figure 4 correspond to the epoch when the abundance of $^8\text{Be}$ reaches 2% in the stellar core, where the core is defined to be the inner 10% of the star by mass. Note that these main-sequences are in good

\[2\text{Note that stars take } \sim 0.1 - 0.2\text{ Myr to form [44], at least in our universe, so that the first stages of this evolutionary sequence are not physically realistic. In particular, the pre-main-sequence evolutionary time scale is shorter than the formation time for stars more massive than } M_* \sim 7M_\odot, \text{ so that massive stars do not have a pre-main-sequence phase.}\]
qualitative agreement with those calculated from the semi-analytic model, as shown in Figure 1. Notice also that all stars must get brighter as they burn through their nuclear fuel supply, and that the helium burning phase is much shorter than the hydrogen burning phase. As a result, the location of the main-sequence varies with time, much more than the case of hydrogen burning.

The H-R diagram of Figure 4 also includes the main sequence for helium burning in our universe using the standard $3\alpha$ process (for metallicity $Z = 10^{-4}$). For this $3\alpha$ main-sequence, shown as the solid black curve, the results are plotted at the epoch when the mass fraction of $^{12}\text{C}$ in the core reached 2%. Note that for this regime of low metallicity, the $3\alpha$ reaction has a nuclear burning parameter with an effective value $C^* \approx 6 \times 10^{23}$ in cgs units (from equation 62). As a result, the helium burning main-sequence occupies roughly the same region of the H-R diagram for the $3\alpha$ process in our universe and the beryllium producing process in other universes. In detail, however, stars burning helium in our universe have somewhat lower surface temperatures than the $C \sim 10^{24}$ curve for other universes, i.e., the helium burning main-sequence in our universe falls to the right of those in alternative universes that produce $^8\text{Be}$. One reason for this difference is the composition of the stellar core. In our universe, the stars burn most of the hydrogen into helium before the onset of the triple alpha process. For large values of $C$ in other universes, however, helium burning starts while the core maintains a significant fraction of hydrogen.

The location of the main-sequence found here for the $3\alpha$ process depends sensitively on the stage of evolution when the “main-sequence” is defined. The tracks of massive stars move back and forth as they evolve beyond the (hydrogen-burning) main-sequence. As a result, for a fixed stopping condition in the numerical code, the resulting main-sequence for the $3\alpha$ process suffered from minor numerical artifacts at both low and high stellar masses. The tracks in the H-R diagram displayed complicated non-monotonic behavior, with small oscillations superimposed on the otherwise smooth curve. To address this issue, we employed a smoothing algorithm to obtain the black solid curve shown in Figure 4. The raw data contained points that fell to the right side (lower temperatures) of the black curve. The smoothing algorithm removed those points, essentially yielding an envelope, which thus provides an upper limit to the temperature for the helium burning main-sequence. Notice also that helium burning in our universe occurs much more rapidly than hydrogen burning, so that stars move relatively quickly in the H-R di-
Figure 4: H-R diagram at the start of $^4$He burning for universes with different values of $C$ (values given in cgs units), from left to right: $10^{16}$ (blue), $10^{20}$ (green), and $10^{24}$ (red). Initially, the stars have metallicity $Z = 10^{-4}$. In these simulations, the start of helium burning is defined to occur when the amount of $^8$Be rises to 2% in the core. For comparison, the solid black curve shows the main-sequence for $^4$He burning via the triple alpha process. The start of helium burning for the triple alpha curve occurs when the $^{12}$C abundance reaches 2% in the stellar core.

Figure 5 shows the mass versus luminosity relationships for helium burning stars in other universes, again for starting metallicity $Z = 10^{-4}$. For these stars, the luminosity at a given mass does not vary appreciably with the nuclear burning parameter $C$, which is varied over 8 orders of magnitude in the figure. For any value of $C$, the luminosity of the star must adjust so that its energy generation ultimately supplies enough pressure to hold up the star against its self-gravity. The stellar mass and the opacity (which determines the energy loss rate) is the same for all values of $C$, so the luminosity
Initially, the stars have metallicity $Z = 10^{-4}$.
Figure 6: Central-temperature plotted as a function of stellar mass at the start of $^4$He burning for universes with different values of $\mathcal{C}$ (in cgs units), from top to bottom: $10^{16}$ (blue), $10^{20}$ (green), and $10^{24}$ (red). Initially, the stars have metallicity $Z = 10^{-4}$. For comparison, the central temperature for stars in our universe, operating via the triple alpha reaction, is shown as the solid black curve.

is slowly varying (as a function of $\mathcal{C}$). Moreover, to leading order, all of the curves show the expected power-law scaling $L_* \propto M_*^3$, which is essentially the same as that found using the semi-analytic model (see Figure 2). A more detailed comparison shows that the full stellar evolution code (the MESA results shown in Figure 5) produces a mass-luminosity relation with more curvature than that of the semi-analytic model. This curvature results from the more complicated physics included in the numerical code (see below).

Figure 6 shows the central temperatures for helium burning stars as a function of stellar mass, for initial metallicity $Z = 10^{-4}$. For these stars, the central temperatures vary from about $10^7$ K to just under $10^8$ K across the range of stellar masses. Stars with lower values of the nuclear burning parameter $\mathcal{C}$ require higher central temperatures to produce (almost) the same
luminosity (see Figure 6), i.e., the higher central temperature compensates for the lower reaction cross section. As expected, larger stars require higher central temperatures to support their higher mass. These same general trends are indicated by the semi-analytic stellar model (compare with Figure 3). Figure 6 also shows the central temperature during helium burning for stars in our universe operating via the triple alpha process (black solid curve). Note that the central temperatures for these stars are higher by factors of \( \sim 4 - 10 \) compared to stars that produce stable \(^8\)Be. In our universe, stars thus have to reach higher central temperatures for the triple alpha process to operate efficiently. The black curve does not extend down to the lowest masses. Stars in this regime experience a helium flash \([7]\), and this complication makes it difficult to define the helium burning main-sequence in the same manner as for larger stars; nonetheless, helium burning in these stars takes place at a central temperature \( T_c \sim 10^8 \) K \([6, 7]\), which corresponds to an extension of the curve.

Finally, we note that the semi-analytic model and the detailed numerical package of MESA produce results that are in excellent qualitative and good quantitative agreement. For a wide range of values for the nuclear parameter \( C \), which includes the cross sections and yields for helium burning into \(^8\)Be, both approaches produce helium burning main-sequences that occupy the same region of the H-R diagram (compare Figure 1 with Figure 4). Both approaches also predict mass versus luminosity relations of the basic form \( L_\star \propto M_\star^3 \) (compare Figure 2 with Figure 5). Finally, the central temperatures of the stars, as a function of stellar mass, are also similar for the two approaches (compare Figure 3 with Figure 6). The differences arise due to the more complicated physics that is included in the numerical model. Because the semi-analytic model uses a simple polytopic equation of state, the physical structure of the star is decoupled from its thermodynamic processes and the resulting stars have limited forms available. In contrast, the numerical model has many more degrees of freedom. For example, during the evolution of high mass stars \( (M_\star = 30 - 100M_\odot) \), the equation of state contains contributions from both radiation pressure and ordinary gas pressure. Since stars dominated by radiation pressure have the same energy with different radial sizes \([8, 37]\), such stars are close to instability. They pulsate as they evolve, and thus cycle through different radii; as a result, their surface temperatures vary significantly with time and with stellar mass. In addition, such stars are often in a state where the energy transport mechanism alternates between being convective and radiative. This variation,
which is included automatically in the numerical treatment of MESA, would correspond to different choices for the polytropic index $n$ in the semi-analytic model, but $n$ is not allowed to vary. These complications, and others, thus lead to the modest differences found in the results from the semi-analytic and numerical approaches. On the whole, however, it is encouraging that two such widely different treatments lead to essentially the same results.

4.3. Carbon Production

The discussion thus far has focused on the production of $^8\text{Be}$ through the process of helium burning. The semi-analytic model demonstrates that $^8\text{Be}$ is readily produced. The numerical simulations not only confirm this result, but also show that helium burning naturally follows (or partially overlaps with) the main-sequence phase of hydrogen burning. Nonetheless, the overall goal of this work is to demonstrate that alternate universes with stable $^8\text{Be}$ can produce carbon (as well as oxygen and other heavy elements necessary for life) without the triple alpha reaction. Universes of this class will contain $^8\text{Be}$ from helium burning (as shown here) and additional $^4\text{He}$, both from its early epoch of big bang nucleosynthesis and from hydrogen burning in ordinary stars. The required nuclear reaction to produce carbon, $^4\text{He} + ^8\text{Be} \rightarrow ^{12}\text{C}$, can then take place in a variety of settings.

In some stars, a beryllium burning phase can follow the helium burning phase, so that carbon is produced in the same stellar core at a later time. In other cases, however, the star could burn all of its helium into beryllium, so that no alpha particles are left over for carbon production in that particular star. Sufficiently massive stars can continue nucleosynthesis, starting with the reaction $^8\text{Be} + ^8\text{Be} \rightarrow ^{16}\text{O}$, and continuing to $^{56}\text{Fe}$. In such cases, some fraction of the $^8\text{Be}$ will be returned to the interstellar medium through stellar winds and/or supernova explosions, so that later generations of stars will be formed with a mix of $^8\text{Be}$ and $^4\text{He}$. Carbon can then be produced by those later stellar generations. Similarly, oxygen can be produced through the reaction $^4\text{He} + ^{12}\text{C} \rightarrow ^{16}\text{O}$, along with the new possible reaction $^8\text{Be} + ^8\text{Be} \rightarrow ^{16}\text{O}$.

As stellar evolution proceeds, the required networks of nuclear reactions become increasingly complicated [7]. For the case of helium burning (to produce $^8\text{Be}$) considered thus far, we had to introduce only a single additional composite parameter $C$ that incorporates the cross section for the (single) reaction as well as the energetic yield (which is determined by the binding energy). With only one parameter to consider, we could study a range of
its values spanning many orders of magnitude, and could consider a range of possible stellar masses. In order to follow the nuclear reaction chains up the periodic table, however, we would have to introduce additional nuclear parameters for each possible reaction. The allowed parameter space for the resulting nuclear reaction network is enormous and a complete study is beyond the scope of this present paper. Here we expand the models to allow for carbon production (see below), which requires specification of a second nuclear parameter. However, this set of simulations does not include nuclear reactions that produce oxygen, neon, and heavier elements, so the simulations are stopped once the carbon abundance in the core reaches 50%.

We demonstrate the feasibility of carbon production through a representative set of numerical simulations. To start, we allow the beryllium and carbon producing reactions to have different rates. For the helium burning reaction \( ^4\text{He} + ^4\text{He} \rightarrow ^8\text{Be} \) we use the nuclear parameter \( C = 10^{20} \) in cgs units, whereas for the carbon production reaction \( ^6\text{He} + ^8\text{Be} \rightarrow ^{12}\text{C} \) we use the larger value \( C_C = 10^{28} \) in the same units. Note that these two nuclear parameters must have different values – otherwise the stellar core will burn all of the available helium into beryllium before any carbon can be produced. For these particular nuclear parameters, the resulting tracks in the H-R diagram are shown in Figure 7 for two choices of stellar mass, \( M_* = 5 \) and \( 15 \, M_\odot \).

For the sake of definiteness, the stars have metallicity \( Z = 10^{-4} \) at the start of their evolution. Both stars evolve to a configurations where they burn hydrogen, but relatively soon enter into a helium burning phase that produces stable \(^8\text{Be}\) nuclei. Somewhat later, carbon production begins through the burning of beryllium, while helium burning continues. The simulations are stopped after the carbon abundance in the stellar core (which encompasses 10% of the stellar mass) has reached a mass fraction of 50%. Note that the tracks also include the pre-main-sequence phases for both stars. For the \( M_* = 15 \, M_\odot \) star, however, the pre-main-sequence time scale is shorter than the expected formation time, so that star is not expected to be optically visible during that phase.

In the H-R diagram of Figure 7, the dashed curve depicts an effective zero-age helium burning main-sequence, i.e., the locus of points where the stars can first burn helium. This curve is defined as the epoch when the \(^8\text{Be}\) abundance reaches 2% in the stellar core. Note that the onset of hydrogen burning takes place earlier, when the stars are somewhat dimmer but somewhat hotter. The tracks are labeled at the benchmark epoch where the abundances of \(^4\text{He}\) and \(^8\text{Be}\) are both equal to 50% in the stellar core.
At this epoch, helium burning is well-developed and the stars lie above the helium-burning zero-age main-sequence marked by the dashed curve. The production of carbon begins after the stars evolve further off of this main sequence. The end-point of the tracks corresponds to the epoch when the abundance of $^{12}$C reaches 50% in the stellar core.

Unlike stars in our universe, where hydrogen fusion dominates the nuclear reactions for an extended span of time, these stars begin to burn helium into beryllium soon after the ignition of hydrogen. In our universe, after stars exhaust the supply of hydrogen in their cores, they experience significant readjustment in order to burn helium. An extreme change in the stellar configuration is necessary to burn helium through the triple alpha process, which requires a high central temperature (e.g., see [6, 7, 8, 37]). In these stars, however, the required central temperatures are lower (see Figure 6) and the adjustment is more modest. As a result, the stars do not have a clean delineation between their hydrogen burning phase and their helium burning phase. Instead, the two classes of nuclear reactions can take place simultaneously. Similarly, with the values of the nuclear burning parameters used here, the synthesis of carbon takes place while the stars continue to burn helium into beryllium. As a result, during carbon production the stars are still relatively close to the main-sequence (compared to stars in our universe). In general, these stars thus have smoother transitions between their different burning phases.

Another way to illustrate nuclear processing inside stars is to plot the abundances of the elements as a function of time. Figure 8 shows the mass fractions of hydrogen, helium, beryllium, and carbon as a function of time for a star with mass $M_* = 15M_\odot$. The star starts with metallicity $Z = 10^{-4}$ and thus has essentially no beryllium or carbon at $t = 0$. In this star, the central core is hot enough, and the nuclear burning parameter $C$ is large enough, that helium burning (beryllium production) takes place shortly after the star reaches the main-sequence and begins burning hydrogen. The two nuclear processes take place simultaneously, so that the mass fraction of $^8$Be increases while the mass fraction of $^1$H decreases. The mass fraction of $^4$He initially decreases, but then reaches a steady state where helium production from hydrogen burning is approximately balanced by helium burning into beryllium. After $\sim 10$ Myr, the abundance of beryllium becomes comparable to that of hydrogen and the star starts to produce carbon. The simulation is stopped after 11 Myr because the code requires more complex nuclear reactions and hence the specification of more $C$-values.
Figure 7: Tracks in the H-R diagram for stars in universes with stable $^8$Be. The upper blue track corresponds to stellar mass $M_* = 15 \, M_\odot$ and the lower red track corresponds to $M_* = 5 \, M_\odot$. The end points of the tracks correspond to the locations in the diagram where the carbon abundance has reached 50% in the stellar core. The label marks the point along the track where the isotope $^8$Be reaches a mass fraction of 50%. The dashed curve marks the location of the ‘zero-age’ helium burning main-sequence, corresponding to the point where beryllium production begins. These simulations demonstrate that carbon can be produced in other universes without the triple alpha process.
Figure 8: Elemental abundances as a function of time for the evolution of a star with mass $M_*=15\, M_\odot$. The curves show the mass fractions, averaged over the entire star, versus time for a star with initial metallicity $Z=10^{-4}$. The nuclear parameters are $C=10^{20}$ for helium burning and $C_C=10^{28}$ for beryllium burning (in cgs units).

Figure 9 shows the corresponding chemical evolution diagram for a star with mass $M_*=5\, M_\odot$. The star follows a similar time sequence. Hydrogen burning takes place after a brief pre-main-sequence phase, but the onset of helium burning occurs shorter thereafter. The two classes of reactions subsequently power the star for 60 – 70 Myr. During this epoch, the hydrogen abundance decreases and the beryllium abundance increases, while helium reaches a steady-state mass fraction of about 10%. After $\sim 68$ Myr, the central core grows hot enough to ignite carbon, which increases its abundance to make up half of the stellar core a few Myr later. Note that the chemical evolution of the 5 and 15 $M_\odot$ stars are nearly the same except for the longer time scale of the smaller star.

For completeness we note that for sufficiently small changes to the nuclear physics, the triple alpha process should not change, and carbon production
Figure 9: Elemental abundances as a function of time for the evolution of a star with mass $M_* = 5 \, M_\odot$. The curves show the mass fractions, averaged over the entire star, versus time for a star with initial metallicity $Z = 10^{-4}$. The nuclear parameters are $C = 10^{20}$ for helium burning and $C_C = 10^{28}$ for beryllium burning (in cgs units).
would proceed in the same manner as in our universe (provided that the Hoyle resonance is present). It is useful to estimate how much change in the nuclear properties is necessary for the carbon production scenario to be different. In our universe, a population of $^8\text{Be}$ is built up in the stellar core, even though the isotope is unstable. In nuclear statistical equilibrium, this abundance is given by the solution to the nuclear version of the Saha equation \[6\], which implies that the mass fraction $X$ of unstable $^8\text{Be}$ has the form

$$X(^8\text{Be}) = 1.79 \times 10^{-11} \rho X_\alpha^2 T_9^{-3/2} \exp[-1.066/T_9], \quad (64)$$

where the quantity in the exponent is given by $1.066/T_9 = \Delta E_b/kT$. In this expression, $\Delta E_b = B_8 - 2B_4 = +92$ keV, where the plus sign indicates that the $^8\text{Be}$ isotope is unbound. Under typical conditions in the cores of helium burning stars, where $\rho \approx 10^3$ g cm$^{-3}$, $T_9 \approx 0.2$, and $X_\alpha \approx 1$, we find the mass fraction $X(^8\text{Be}) \approx 10^{-9}$. Under these conditions, the decay mode of $^8\text{Be}$ is the same as its production mode, so that the reaction $\alpha + \alpha \leftrightarrow ^8\text{Be}$ reaches equilibrium. As long as $X(^8\text{Be}) \ll 1$, the next step of the nuclear reaction chain proceeds at a rate such that the mass fraction $X(^8\text{Be})$ cancels out \[6\] [7].

In order to get a different scenario for carbon production, we thus need to change the properties of the $^8\text{Be}$ nucleus so that the equilibrium abundance predicted by the Saha equation is large enough that the stellar core can build up its beryllium abundance (instead of maintaining the trace amounts consistent with nuclear statistical equilibrium). For the sake of definiteness, we find the conditions necessary for an abundance of $X(^8\text{Be}) = 0.10$, which implies the constraint

$$\exp[(\Delta E_b^{\text{old}} - \Delta E_b^{\text{new}})/T_9] \sim 10^8 \quad \text{or} \quad -\Delta E_b^{\text{new}} \sim 110\text{keV} \left(\frac{T}{10^8\text{K}}\right). \quad (65)$$

If stars burning helium in other universes had the same central temperature as those in our universe, a binding energy of about 200 keV would be sufficient to lead to a different scenario for carbon production. As shown here, however, the central temperatures can be lower by factors of 3 – 10 (see Figure \[6\]), so that a binding energy of only 20 – 60 keV can be enough to make the equilibrium abundance of $^8\text{Be}$ large enough to allow for new scenarios for carbon nucleosynthesis. Note that this energy is comparable to the energy increment (92 keV) by which $^8\text{Be}$ is unbound in our universe and the increment $O(100$ keV) by which the triple alpha resonance can be moved without compromising carbon production \[12\].
5. Conclusion

This paper argues that the sensitivity of our universe — and others — to the triple alpha reaction for carbon production is more subtle and less confining than previously reported. If nuclear structures are different in other universes, so that the carbon resonance is not present at the proper energy level, then previous work has shown that carbon production through this process can be highly suppressed. If the nuclear structures are different, however, then it remains possible for the isotope $^8\text{Be}$ to be stable in other universes. In such universes with long-lived $^8\text{Be}$, carbon production can proceed without the need for the triple alpha process, or the need for any particular resonance.

More specifically, the isotope $^8\text{Be}$ can be stable if its binding energy is changed by an increment $\sim 100$ keV. For comparison, a larger change in the location of the triple alpha resonance ($\sim 300$ keV) is required to compromise carbon production through the standard reaction network \cite{12}. In order to produce changes to the nuclear binding energies of this order, the fundamental constants must be varied by a few percent (see Section \ref{sec2} and references therein). Moreover, effective field theory calculations \cite{14} show that the binding energies of $^8\text{Be}$ and $^4\text{He}$ do not generally change at the same rate as the fundamental parameters are varied\footnote{More specifically, the derivative of the $^4\text{He}$ binding energy with respect to the fundamental parameters of the theory is not equal to half of the derivative of the $^8\text{Be}$ binding energy with respect to the same parameter. This lack of equality is necessary to allow changes in the binding energy of $^8\text{Be}$ relative to that of two alpha particles}, so that bound states of $^8\text{Be}$ can be realized.

This paper has also shown that, given a stable $^8\text{Be}$ isotope, stars can readily produce beryllium through helium burning. We have addressed this issue using both a semi-analytic model and a state-of-the-art numerical stellar evolution code (MESA). Both approaches are in good agreement and show that the helium burning main-sequence in other universes corresponds to stars with properties (luminosities, surface temperatures, central temperatures) that are roughly similar to those of helium burning stars in our universe. To carry out these calculations, we have to specify the values of the nuclear reaction cross sections and the yields for helium burning into beryllium; in this treatment, these quantities are encapsulated into the nuclear burning parameter $C$, which we allow to vary by many orders of magnitude. In spite
of the large range of possible values for $C$, the helium burning main-sequences are well-defined and vary by less than an order of magnitude in surface temperature and a factor of $\sim 100$ in luminosity (Figures 1 and 4). Similarly, the central temperatures vary by about one order of magnitude for the entire range of $C$ considered here (Figures 3 and 6). The mass versus luminosity relationship varies by even less and (approximately) follows the expected scaling law $L_* \propto M_*^3$, similar to that for hydrogen burning stars in our universe (see Figures 2 and 5). It is significant the simplest possible model that includes the relevant physics (Section 3 and [26]) and one of the most sophisticated stellar evolution codes that is currently available (Section 4 and [31]) both give essentially the same results.

After a star produces $^8$Be via helium burning, carbon production can take place during the subsequent evolution of the same stellar core and/or during subsequent stellar generations. Using a straightforward extension of the nuclear reaction network to include $^4$He + $^8$Be $\rightarrow ^{12}$C, we have used the MESA stellar evolution code to demonstrate that carbon can be produced in stellar cores after helium burning is sufficiently advanced (see Figures 7 – 9). This set of simulations thus indicates that stars can synthesize carbon without the triple alpha process. However, the abundance of carbon produced in an alternate universe, and the abundances of other relevant nuclei such as oxygen, depends on a number of additional properties. In order to follow the chain of nuclear reactions to ever higher atomic numbers, we would need to specify the rates and yields for each reaction involving the new stable isotope $^8$Be, as well as any additional reactions that operate in that universe (see the discussion below). The ratios of the various nuclear burning parameters $C$ (defined in equation [40]) are particularly important. A full assessment of the possible isotopic ratios in alternate universes is beyond the scope of any single paper and is left for future work.

We note that the apparent fine-tuning problem posed by the triple alpha process has (at least) two components. One can ask if the $^{12}$C resonance is necessary to produce the abundances of carbon, oxygen, and other heavy elements observed in our universe. On the other hand, one can consider what nuclear properties are required to produce enough carbon for some type of life to exist. Previous work has shown that nuclear physics cannot be changed greatly without altering the abundances of carbon and oxygen observed in our universe, which utilizes the triple alpha process. In contrast, this paper shows that the triple alpha process is not a necessary ingredient for carbon production in other universes.
The results of this paper do not predict the range of possible isotope ratios. Such a determination must consider the rates (and yields) for all of the relevant nuclear reactions, where these rates can vary from universe to universe. A related complication is that the reaction rates are secondary parameters that are ultimately determined by the underlying fundamental theory. The changes in the reaction rates and other nuclear properties are thus coupled, but the coupling is not known. This paper has focused on non-resonant two-body reactions, and shows that carbon can be produced without the triple alpha resonance. However, resonances are simply manifestations of the energy levels of the interacting nuclei, so that every universe is expected to have some collection of resonances, which will in turn affect the isotopic ratios. The locations of these resonances can favor some nuclear reactions over others (by enhancing particular cross sections) and will help produce a wide range of different isotope abundances across the multiverse.

In the absence of a specification of the possible reaction rates, yields, and resonances, one can explore the parameter space for all of the possible nuclear reactions (e.g., with and without stable $^8$Be) and determine which cases produce favorable isotopic ratios. In addition to the large possible parameter space, such a program is further complicated because stars of varying masses produce different nuclear yields, and the distribution of stellar masses can also vary. Moreover, even if the allowed parameter space for producing given abundances of carbon could be fully delineated, we do not know how much carbon is actually required for life. Finally, the probability of a universe being habitable ultimately depends not only on the range of allowed parameters, but also on the distribution from which the parameters are sampled; unfortunately, this underlying probability distribution also remains unknown. As a result of these complications, this paper represents only one step toward understanding carbon production in other universes (see also [1, 2, 3, 4, 5, 18] and references therein).

Although carbon is (most likely) necessary for a universe to be habitable, it is not sufficient. Universes favorable for the development of life require additional heavy elements, including oxygen, nitrogen, and many others. Although a full treatment of heavy element production for all possible universes is beyond the scope of this paper, we can outline some basic requirements. Hydrogen is a necessary ingredient, so it is important that big bang nucleosynthesis does not process all of the protons into heavier nuclei and that star formation is not overly efficient. The natural endpoint for stellar nucleosynthesis is to produce large quantities of the element with the highest
binding energy per particle, i.e., iron (in our universe) or its analog (in other universes). As a result, nuclear processing in stars cannot be too efficient. In our universe, stars span a range of masses, from those that can barely burn hydrogen up to those that produce iron cores and explode as supernovae. This range of stellar masses results in a wide range of endpoints for nuclear reaction chains and is thus favorable for producing a diverse ensemble of heavy elements. Habitable universes thus reside in a regime with intermediate properties. Star formation must take place readily in order to produce the heavy elements, energy, and planets necessary for life, but cannot be so efficient that no hydrogen is left over for water. Stars must be able to synthesize the full distribution of heavy elements necessary for life, but cannot be so efficient that all nuclei become iron (or whatever nuclide has the highest binding energy in the given universe). This paper generalizes the class of potentially habitable universes to include those without the triple alpha process for carbon production, but a great deal of additional work remains to sort out the full parameter space.

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