Transverse Momentum Distribution in Heavy Ion Collision using q-Weibull Formalism

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We have implemented the Tsallis q-statistics in the Weibull model of particle production known as the q-Weibull distribution to describe the transverse-momentum ($p_T$) distribution of the charged hadrons at mid-rapidity measured at RHIC and LHC energies. The model describes the data remarkably well for the entire $p_T$ range measured in nucleus-nucleus and nucleon-nucleon collisions. The proposed distribution is based on the non-extensive Tsallis q-statistics which replaces the usual thermal equilibrium assumption of hydrodynamical models. The parameters of the distribution can be related to the various aspects of complex dynamics associated with such collision process.

I. INTRODUCTION

The study of relativistic heavy ion collisions provides an excellent means to study the existence of new form of matter at extreme conditions of energy density and temperature \cite{1, 2}. The bulk properties of the system created in such collisions can be studied via the transverse momentum ($p_T$) distribution of the produced charged particles which reflect the conditions during the kinetic freeze out together with the integrated effects of the collective expansion from very initial stages. The bulk properties of the medium created in such collisions is usually studied through the statistical approach. One of the well known approaches to the particle production is governed by hydrodynamics inspired models based on Boltzman-Gibbs (BG) statistics. The models assumes the occurrence of a local thermal equilibrium and hence has a common fixed temperature $T$ associated with the hadronizing system. The transverse momentum ($p_T$) distributions were explained by the Boltzman-Gibbs Blast Wave (BGBW) hydrodynamical model fits to the data to obtain a set of bulk parameters at freeze out such as temperature $T$ and the radial flow velocity ($\beta$) \cite{3, 4}. The successful application of such hydrodynamical models in limited $p_T$ range also provided evidence of collective motion in the continuously expanding system created in heavy ion collisions. Furthermore, the measurement of large elliptic anisotropy in momentum in non-central collisions points towards a nearly thermalized and strongly interacting system. The equilibrium description based on ideal hydrodynamical models where BG statistics can be applied are generally limited to low $p_T$. It is not always a good idea to select or constrain the $p_T$ regime so that the model fitting procedure works.

However, hadronizing systems experience hard QCD scattering processes during initial stages, intrinsic initial state fluctuations due to formation of initial Color Glass Condensate (CGC) formation, fluctuations in temperature, initial energy density \textit{etc.} which can be interpreted as dynamical non-equilibrium effects. These non-equilibrium processes may not be washed away by multi-particle interactions in the initial QGP (Quark Gluon Plasma) state or in the later hadronic phase and can manifest more dominantly in spectral shape at intermediate or high $p_T$ \cite{5, 6}.

Recent studies on the generation of azimuthal anisotropy and $v_2$ mass splitting using multi-phase transport models suggest the dominant role of anisotropic parton escape mechanism to describe the measured azimuthal anisotropy in small system and heavy ion collisions. The escape mechanism shows that the most of the $v_2$ is developed during the parton cascade stage and the origin of mass splitting is attributed to hadron re-scatterings \cite{7, 8}. This clearly shows that evolution of the system based on hydrodynamical scenario is not imperative. In the past, Tsallis non-extensive statistics have been used extensively to understand the particle production and to describe the evolution of $p_T$ spectra in $pp$ and heavy ion collisions at various energies \cite{9, 10}. The $q$ parameter in the Tsallis statistics characterize the deviation from the assumed conditions of local thermal equilibrium in systems created in such collisions. In previous studies, Tsallis distribution have been incorporated into the Blast Wave model (TBW) and have been used successfully to describe the spectra at RHIC energies \cite{11}. The distribution was also applied to describe the data in $pp$ collisions at
LHC energies to observe the onset of radial flow in smaller systems.\[^12\] However, the observation of finite radial flow in smaller systems hintings towards collectivity in such systems is still under investigation and it is highly debatable whether hydrodynamics can be applied in smaller systems.

Recently, the Weibull model was used to describe successfully the particle production in pp(\(p\bar{p}\)) and e\(^+\)e\(^-\) collisions for a broad range of energies.\[^13\] \[^14\] A Weibull-Glauber approach was also developed to understand the charged particle multiplicity and transverse energy distribution in heavy ion collisions.\[^15\] This is natural to assume as the individual nucleon-nucleon collisions inside the nucleus-nucleus collision has an inherent pQCD parton cascade evolution and fragmentation. The Weibull distribution is usually used to describe natural processes where fragmentation and sequential branching is one of the major ingredients of the dynamical evolution.\[^16\] \[^17\]

The aim of the letter is to propose a generalized distribution within the framework of non-equilibrium q-statistics which would describe the \(p_T\) distribution successfully for both small systems (pp and pA) and the system created in heavy ion collisions for all ranges of measured \(p_T\).

The successful description of Weibull model describing particle production can be utilized to incorporate the Tsallis q-statistics to Weibull distribution to obtain a more generalized q-Weibull distribution. The q-Weibull distribution has been previously applied to many complex system in different areas of interest.\[^18\] \[^19\] In this letter, we use q-Weibull distribution to describe the \(p_T\) distribution of charged hadrons emitted in pp, pA and AA systems for broad range of energies. The distribution provides a generalized statistical framework where both small and large systems can be compared. Apart from describing the spectral evolution, the physical interpretation of the model parameters will provide quantitative insight into the dynamical processes in such collisions and help in predicting the spectra for future studies.

### II. THE Q-WIEBULL MODEL

The most common way to incorporate the q-statistic to the Weibull model is to replace the exponential factors by their equivalent q-exponential.\[^18\]

The two parameter Weibull distribution is given by the following expression

\[
P(x; \lambda, k) = \frac{k}{\lambda} \left(\frac{x}{\lambda}\right)^{k-1} e^{-\left(\frac{x}{\lambda}\right)^k} \quad (1)
\]

where \(\lambda\) and \(k\) are the free parameters. The q-Weibull distribution is given by \[^18\]

\[
P_q(x; q, \lambda, k) = \frac{k}{\lambda} \left(\frac{x}{\lambda}\right)^{k-1} e^{-\left(\frac{x}{\lambda}\right)^k} \quad (2)
\]

where

\[
e_q^{-\left(\frac{x}{\lambda}\right)^k} = (1 - (1 - q)^k)^{(\frac{1}{1-q})} \quad (3)
\]

The value of the non-extensive \(q\) parameter is related to the deviation of the system from thermal equilibrium. In general, \(q > 1\) value is attributed to the presence of intrinsic fluctuations in the system and depends upon the observable being measured.

### III. \(p_T\) SPECTRUM AND Q-WIEBULL PARAMETERS

It is worthwhile to investigate whether one can describe the \(p_T\) distribution of charged hadrons in pp, pA and AA (heavy ion) collisions for a broad range of available energies using the q-Weibull approach. This would allow the q-Weibull function the most generalized parametrization to explain the \(p_T\) distribution.

In previous studies,\[^20\] \[^21\] Tsallis distribution was successful in explaining the \(p_T\) distribution of pp collisions for the entire \(p_T\) range for most of the beam energies studied. One expects that the q-Weibull function should describe the \(p_T\) spectrum for charged particles in pp collisions as it incorporates both the Weibull model of particle production and the q-statistics which has been very successful so far. The \(p_T\) distribution of charged hadrons were fitted with q-Weibull function in \(p\bar{p}\) and pp collisions as measured by UA1 experiment\[^22\] and ALICE experiment at various energies, respectively. This is depicted in Figure 1. The values of the extracted parameters are shown in Table 1 and Table 2. It will be interesting to see whether one can extend the functional description to pA and heavy ion collisions where the system created has higher degree of complexity and richer features of particle production mechanism. Figure 2 shows the description for ppBb collisions at 5.02 TeV\[^24\] and PbPb collisions at 2.76 TeV\[^22\] and AuAu\[^26\] collisions at 200 GeV for var-
ious centrality classes are shown in Figure 3 and Figure 4 respectively.

As can be observed from the figures, the q-Weibull fits provide an excellent description of the data for different systems and for broad range of energies in the measured $p_T$ range. As q-Weibull appears to be the most generalized q-distribution to explain the features of $p_T$ distribution, it seems important to study the evolution of its parameters with beam energy, type of colliding systems and centrality classes considered. This would allow to attribute some dynamics associated with the collisions processes to the parameters and thus one can have better understanding of the complex phenomena and have some quantitative estimates for future studies.

Figure 5 shows the centrality dependence of the $q$ parameter extracted in $PbPb$ collisions at 2.76 TeV. The non-extensive parameter $q$ which quantifies the deviation from local equilibrium goes up as we move from peripheral to central collisions if we consider the entire $p_T$ regime for the fit. The behavior is on expected lines as the initial pQCD hard scattering processes followed by fragmentation and hadronization dominates the particle production mechanism in hadronic interactions and supports the non-equilibrium scenario in such collisions. The processes becomes more 'harder' in more central collisions and the relative contributions from initial state fluctuations is also more in central collisions. Hence, we observe an increasing trend of deviation from equilibrium with centrality. However, the $q$ values shows a slight decreasing trend or remains more or less constant around 1.0 if we constrain the $p_T$ limit of the fit to lower values (up to 2 GeV/c). This is in agreement with the hydrodynamical expectation where one sees a local equilibrium to be attained as the low $p_T$ hadrons emanate from a nearly thermally equilibrated system. The $q$ parameter in $pp$ collisions is consistently greater than 1.0 and one can see larger $q$ values at highest energies as shown in Table 1. This supports a similar mechanism of particle production via initial hard-scatterings across all energies concerned with an increase of relative contributions of hard or semi hard processes with energy.

Figure 6 shows the behavior of $\lambda$ parameter for different centrality in $PbPb$ and $AuAu$ collisions. The $\lambda$ value shows an increase from peripheral to central collisions for both integrated $p_T$ regime and the low $p_T$ regime. This shows that the $\lambda$ parameter is related to the mean $p_T$ of the distribution. This can also be related to a common radial velocity (as per transverse expansion scenario in hydrodynamic evolution) in the low $p_T$ regime which goes up with centrality.

The $\lambda$ parameter in $pp$ collisions is more or less constant across the beam energies considered if one restricts the fit to a common value (16 GeV/c in this case) which is expected.

The $k$ parameter shows a decrease as we move from central to peripheral collisions in both $PbPb$ and $AuAu$ collisions as shown in Figure 3. However, the trend is reversed in the low $p_T$ regime. This indicates that $k$ parameter is related to the dynamics of particle production and its value increases with onset of hard QCD scatterings, initial fluctuations and other processes leading to non-equilibrium conditions.

To understand the physical implications of the parameters better, one requires a systematic study of q-Weibull model fits from ISR to LHC energies in case of hadronic collisions and from SPS to LHC energies for heavy ion collisions. The fits to identified particle spectra will allow deeper understanding of the parameters and characterize the system evolution based the species (baryon/meson) and mass of the hadrons. The successful description of the spectra and the detailed interpretation of the model parameters will certainly be useful in providing qualitative estimates for future studies.

| $\sqrt{s}$ (TeV) | $k$ | $\lambda$ | $q$ |
|------------------|-----|---------|-----|
| 0.9              | 1.078 ± 0.023 | 0.1629 ± 0.007 | 1.133 ± 0.004 |
| 2.76             | 1.09 ± 0.029  | 0.1711 ± 0.008 | 1.154 ± 0.005 |
| 7                | 1.063 ± 0.03  | 0.1663 ± 0.008 | 1.1565 ± 0.005 |
| 13               | 1.010 ± 0.024 | 0.157 ± 0.006  | 1.151 ± 0.005 |

TABLE I. The parameters $k$, $\lambda$ and $q$ obtained from the fits of $p_T$ distributions using q-Weibull function in $pp$ collisions ($|\eta| < 0.8$) for different energies as measured by ALICE experiment at LHC. The fits are constrained to an upper limit of $p_T$ for uniform comparison of parameters.

| $\sqrt{s}$ (TeV) | $k$ | $\lambda$ | $q$ |
|------------------|-----|---------|-----|
| 0.2              | 0.91 ± 0.062  | 0.13 ± 0.013 | 1.07 ± 0.014 |
| 0.5              | 0.921 ± 0.065 | 0.130 ± 0.015 | 1.079 ± 0.015 |
| 0.9              | 0.906 ± 0.045 | 0.132 ± 0.019 | 1.080 ± 0.013 |

TABLE II. The parameters $k$, $\lambda$ and $q$ obtained from the fits of $p_T$ distributions using q-Weibull function in $p\bar{p}$ collisions ($|\eta| < 2.5$) for different energies as measured by UA1 experiment.
FIG. 1. (Color online) (Upper Panel) $p_T$ distribution of charged hadrons in $pp$ collisions at 0.9 TeV, 2.76 TeV and 7 TeV from pseudo-rapidity region $|\eta| < 0.8$ as measured by ALICE experiment at LHC [23]. (Lower Panel) $p_T$ distribution of charged hadrons in $pPb$ collisions at 5.02 TeV for $|\eta| < 0.3$ as measured by ALICE experiment at LHC [24]. The solid line is the $q$-Weibull fit to the data points. The data points are properly scaled for visibility.

IV. SUMMARY

We have used the Tsallis $q$- statistics in the Weibull model of particle production and applied it for the first time to describe the transverse momentum distribution of charged hadrons in different colliding system for a broad range of energies. The $q$-Weibull function successfully describes the $p_T$ distribution for all ranges of $p_T$ measured. The parameter $q$, which characterizes the degree of deviation from thermal equilibrium in a system decreases systematically from peripheral to the central collisions in heavy ion collisions if the $p_T$ of the particles considered are constrained to lower values (less than 2 GeV/c). This indicates an evolution from a non-equilibrated system in peripheral collisions towards a more thermalized system in central heavy ion collisions. However, the trend is reversed if we consider the all inclusive $p_T$ regime which supports an increase of relative contribution of hard pQCD processes in central collisions. The $\lambda$ parameter can be associated with the mean $p_T$ or collective expansion velocity of hadrons which shows an expected increase with centrality of collisions. The $k$ parameter can be related to system dynamics associated with the collisions types and centrality classes. The quantitative evolution of the $k$ parameter and deeper physical interpretation requires systematic studies of identified particle spectra for a broad range of energies. We have successfully demonstrated that $q$-Weibull is the most generalized statistical model which can be used to describe and predict the $p_T$ distribution of charged particles in different collision systems for broad range of energies.

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FIG. 3. (Color online) $p_T$ distribution of charged hadrons for various centrality classes in PbPb collisions at 2.76 TeV from pseudo-rapidity region $|\eta| < 0.8$ as measured by ALICE experiment at LHC [25]. The solid line is the q-Weibull fit to the data points. The data points are properly scaled for visibility. The upper panel shows the data and fit where the entire range of measured $p_T$ is taken while the lower panel shoes the fit for $p_T < 2.0 GeV/c$

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FIG. 4. (Color online) $p_T$ distribution of charged hadrons for various centrality classes in AuAu collisions at 200 GeV in pseudo-rapidity region $|\eta| < 0.18$ as measured by PHENIX experiment at RHIC [26]. The solid line is the q-Weibull fit to the data points. The data points are properly scaled for visibility. The upper panel shows the data and fit where the entire range of measured $p_T$ is taken while the lower panel shoes the fit fot $p_T < 2.0\text{GeV/c}$.

FIG. 5. (Color online) (Upper Panel) Variation of $q$ as a function of centrality for $p_T < 2.0\text{GeV/c}$. (Lower Panel) Variation of $q$ as a function of centrality for the entire range of $p_T$ measured.

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FIG. 6. (Color online) (Upper Panel) Variation of $\lambda$ as a function of centrality for $p_T < 2.0\text{GeV}/c$. (Lower Panel) Variation of $\lambda$ as a function of centrality for the entire range of $p_T$ measured.

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FIG. 7. (Color online) (Upper Panel) Variation of $k$ as a function of centrality for $p_T < 2.0 \text{ GeV/c}$. (Lower Panel) Variation of $k$ as a function of centrality for the entire range of $p_T$ measured.