The Decays of $B^+ \rightarrow \bar{D}^0 + D_{sJ}^+(2S)$ and $B^+ \rightarrow \bar{D}^0 + D_{sJ}^+(1D)$

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ABSTRACT

We analyzed the decays of $B^+ \rightarrow \bar{D}^0 + D_{sJ}^+(2S)$ and $B^+ \rightarrow \bar{D}^0 + D_{sJ}^+(1D)$ by naive factorization method and model dependent calculation based on the Bethe-Salpeter method, the branching ratios are $Br(\bar{D}^0 D_{sJ}^+(2S)) = (0.72 \pm 0.12)\%$ and $Br(\bar{D}^0 D_{sJ}^+(1D)) = (0.027 \pm 0.007)\%$. The branching ratio of decay $B^+ \rightarrow \bar{D}^0 D_{sJ}^+(2S) \rightarrow \bar{D}^0 D^0 K^+$ consist with the data of Belle Collaboration, so we conclude that the new state $D_{sJ}^+(2700)$ is the first excited state $D_{sJ}^+(2S)$. 
BABAR Collaboration reported the observation of a new charmed meson called $D_{sJ}^+(2690)$ with a mass $2688 \pm 4 \pm 3$ MeV and with a broad width $\Gamma = 112 \pm 7 \pm 36$ MeV [1]. Belle Collaboration also reported a charmed state $D_{sJ}^+(2700)$, $M = 2715 \pm 11^{+11}_{-14}$ MeV with width $\Gamma = 115 \pm 20^{+36}_{-32}$ MeV [2], later modified to $M = 2708 \pm 9^{+11}_{-10}$ MeV and $\Gamma = 108 \pm 23^{+36}_{-31}$ MeV [3]. Most authors believe that these two states should be one state, and the most possible candidate is the $2S$ or the $S - D (2^3S_1 - 1^3D_1)$ mixing $c\bar{s} 1^-$ state [4, 5, 6, 7, 8, 9, 10, 11].

Recently, we have analyzed the decays of the $D_{sJ}^+(2S)$ and $D_{sJ}^+(1D)$, we concluded that one can not distinguish them from their decays, because they have the similar decay channels and the corresponding decay widths are comparable [12]. In this paper, we give the calculations of the decays $B^+ \to \bar{D}^0 + D_{sJ}^+(2S)$ and $B^+ \to \bar{D}^0 + D_{sJ}^+(1D)$, we find that we can separate them, and according to the current experimental data of Belle Collaboration, we conclude that the new state $D_{sJ}^+(2700)$ is the first excited state $D_{sJ}^+(2S)$. These channels have also been considered in literatures, for example, Close et al [4, 23] give branching ratios and possible mixing of 2S and 1D; Colangelo et al [7] assume the new state is the 2S state, and extract the decay constant.

The decay amplitude for $B^+ \to \bar{D}^0 + D_{sJ}^+$ can be described in the naive factorization approach:

$$T = \frac{G_F}{\sqrt{2}} V_{cs} V_{cb} a_1 \langle D_{sJ}^+ | J_\mu | 0 \rangle \langle \bar{D}^0 | J_\mu | B^+ \rangle,$$

(1)

where the CKM matrix element $V_{cs} = 0.97334$ and $V_{cb} = 0.0415$ [13], since we focus on the difference between $D_{sJ}^+(2S)$ and $D_{sJ}^+(1D)$, not on the careful study, we have chosen $a_1 = c_1 + \frac{1}{3} c_2 = 1$, where $c_1$ and $c_2$ are the Wilson coefficients [14]. We also delete other higher order contributions like the contributions from penguin operators. And

$$\langle D_{sJ}^+ | J_\mu | 0 \rangle = i F_V M D_{sJ}^+ \epsilon_\mu^\lambda,$$

(2)

$F_V$ and $\epsilon_\mu^\lambda$ are the decay constant and polarization vector of the meson $D_{sJ}^+$, respectively.

We have solved the exact instantaneous Bethe-Salpeter equations [15] (or the Salpeter equations [16]) for 1$^-$ states, the general form for the relativistic Salpeter wave function of vector 1$^-$ state can be written as [17, 18]:

$$\varphi_{1^-}(q_\perp) = q_\perp \cdot \epsilon_\perp \left[ f_1(q_\perp) + \frac{P}{M} f_2(q_\perp) + \frac{q_\perp}{M} f_3(q_\perp) + \frac{P q_\perp}{M^2} f_4(q_\perp) \right] + M \varphi_{1^-} f_5(q_\perp)$$

$$+ g_\perp \left[ f_6(q_\perp) + \frac{P q_\perp}{M} f_7(q_\perp) + \frac{1}{M} \left( P g_\perp q_\perp - P q_\perp \cdot \epsilon_\perp \right) f_8(q_\perp) \right],$$

(3)
where the $P$, $q$ and $\epsilon^\perp_\lambda$ are the momentum, relative inner momentum and polarization vector of the vector meson, respectively; $f_i(q_\perp)$ is the function of $-q^2$, and we have used the notation $q_\perp^\mu \equiv q^\mu - (P \cdot q/M^2)P^\mu$ (which is $(0, \vec{q})$ in the center of mass system).

The 8 wave functions $f_i$ are not independent, there are only 4 independent wave functions [18], and we have the relations

$$
\begin{align*}
    f_1(q_\perp) &= \frac{q_\perp^2 f_3(q_\perp) + M^2 f_5(q_\perp)}{M(m_1 + m_2)q_\perp^2}(m_1 m_2 - \omega_1 \omega_2 + q_\perp^2), \\
    f_2(q_\perp) &= \frac{-q_\perp^2 f_4(q_\perp) + M^2 f_6(q_\perp)}{M(\omega_1 + \omega_2)q_\perp^2}(m_1 \omega_2 - m_2 \omega_1), \\
    f_3(q_\perp) &= f_5(q_\perp) M(-m_1 m_2 + \omega_1 \omega_2 + q_\perp^2), \\
    f_4(q_\perp) &= f_6(q_\perp) M(m_1 \omega_2 - m_2 \omega_1).
\end{align*}
$$

In our method, the $S-D$ mixing automatically exist in the wave function of $1^-$ state, because we give the whole wave function Eq. (3) which is $J^P = 1^-$, but some of the wave function are $S$ wave, some are $D$ wave, for example, see Figure 1-3, we show wave functions. One can see that, for $1S$ and $2S$, the wave function $f_5$ and $f_6$ are dominate, and they are $S$ wave, but there is little $D$ wave mixing in these two states which come from the terms $f_3$ and $f_4$. But for the third state, we labeled as $1D$ state, the terms $f_3$ and $f_4$ are dominate, but these two terms are not pure $D$ wave, they will give contribute as a $S$ wave [17]. So we conclude that the $1S$ and $2S$ states are $S$ wave dominate states, mixed with a little $D$ wave (come from the terms $f_3$ and $f_4$), while the $1D$ state is a $D$ wave dominate state ($f_3$ and $f_4$), mixed with a valuable part of $S$ wave (still come from the terms $f_3$ and $f_4$).

For $c\bar{s}$ vector $1^-$ state, our mass prediction for the first radial excited $2S$ state is $2673$ MeV, and for $1D$, our result is $2718$ MeV [18], so there are two states around 2700 MeV.

In Ref. [18], we only give the leading order calculation for decay constant, which is $F_V = 4\sqrt{N_c} \int \frac{d\vec{q}}{(2\pi)^3} f_5(q)$, the whole equation should be

$$
F_V = 4\sqrt{N_c} \int \frac{d\vec{q}}{(2\pi)^3} f_5(q) - \frac{q^2 f_3}{3M^2},
$$

and our results of the decay constants for vector $c\bar{s}$ system are:

$$
\begin{align*}
    F_V(1S) &= 353 \pm 21 \text{ MeV}, \\
    F_V(2S) &= 295 \pm 13 \text{ MeV}, \\
    F_V(1D) &= 57.1 \pm 5.1 \text{ MeV}.
\end{align*}
$$

Where the uncertainties are given by varying all the input parameters simultaneously within ±5% of the central values in our model, and we will calculate all the uncertainties this way in this letter. The
Figure 1: wave functions of $D^*_s(1S)$.

Figure 2: wave functions of $D^*_s(2S)$. 
center values of $S$ waves are little smaller than the estimates in Ref. [18], where $F_V(1S) = 375 \pm 24$ MeV and $F_V(2S) = 312 \pm 17$ MeV, but the value of $D$ wave decay constant is much smaller than the predict in Ref. [18] where we did not shown it, this is because we should not ignore the contribution from the term of $f_3$ when consider a $D$ wave state. Our estimate of $F_V(1S) = 353 \pm 23$ MeV is close to the newer result $F_{Ds^*} = 268 \sim 290$ MeV by Choi [19]. Our estimate of $F_V(2S) = 312 \pm 17$ MeV is a little larger than the estimate $F_V(2S) = 243 \pm 41$ MeV in Ref. [7], where they extracted it from the decay $B^+ \to \bar{D}^0 + D_{sJ}^+(2700)$ assuming the new state $D_{sJ}^+(2700)$ is the first radial excited state $D_{sJ}^+(2S)$.

According to the Mandelstam formalism [20], at the leading order, the transition amplitude for $B^+ \to \bar{D}^0$ can be written as [21]:

$$\langle \bar{D}^0|J^\mu|B^+ \rangle = \int \frac{d\vec{q}}{(2\pi)^3} Tr \left[ \bar{\varphi}_{D^0}^+(\vec{q}) \left( \frac{m_u}{m_c + m_u} \vec{r} \right) \frac{P}{M} \varphi_{B^+}^+(\vec{q}) \gamma^\mu(1 - \gamma_5) \right],$$  

(5)

where $\vec{r}$ is the recoil three dimensional momentum of the final state $\bar{D}^0$ meson, $\varphi^{++}$ is called the positive energy wave function, and $\bar{\varphi}_{D^0}^{++} = \gamma_0(\varphi_{D^0}^{++})^+ \gamma_0$.

The wave function forms for pseudoscalar $B^+$ and $\bar{D}^0$ are similar, for example, the wave function for $B^+$ can be written as [22]

$$\varphi_{B^+}^{++}(\vec{q}) = \frac{M_{B^+}}{2} \left( \varphi_1(\vec{q}) + \varphi_2(\vec{q}) \frac{m_u + m_b}{\omega_u + \omega_b} \right)$$
× \left[ \frac{\omega_u + \omega_b + \gamma_0 - \vec{q}(m_u - m_b)}{m_u + m_b} + \frac{\vec{q} \gamma_0 (\omega_u + \omega_b)}{(m_u \omega_u + m_b \omega_b)} \right] \gamma_5 , \quad (6)

where \( \omega_u = \sqrt{m_u^2 + \vec{q}^2} \) and \( \omega_b = \sqrt{m_b^2 + \vec{q}^2} \); \( \varphi_1(\vec{q}) \), \( \varphi_2(\vec{q}) \) are the radial part wave functions, and their numerical values can be obtained by solving the full Salpter equation of \( 0^- \) state [22].

The decay width is:

\[
\Gamma = \frac{1}{8 \pi} \frac{1}{M_B^2} |T|^2
\]

\[
= \frac{1}{8 \pi} \frac{1 |\vec{p}_{f2}| G_F^2 V_{cs}^2 V_{bc}^2}{M_B^2} F_V^2 M_{f2}^2 X, \quad (7)
\]

where \( \vec{p}_{f2} \) and \( M_{f2} \) are the three dimensional momentum and mass of the final new state \( D_s^{*+} \). \( M_{f2}^2 \) come from the definition of decay constant in Eq.(2), but the square of polarization vector and transition amplitude of Eq.(5) are symbolized as \( X \).

So our result are:

\[
\Gamma(B^+ \to \bar{D}^0 D_s^{*+}(2S)) = F_V^2(2S) \times (3.50 \pm 0.38) \times 10^{-14} \text{ GeV}, \quad (8)
\]

\[
\Gamma(B^+ \to \bar{D}^0 D_s^{*+}(1D)) = F_V^2(1D) \times (3.25 \pm 0.30) \times 10^{-14} \text{ GeV}. \quad (9)
\]

If we ignore the mass difference between \( D_s^{*+}(2S) \) and \( D_s^{*+}(1D) \), and use 2700 MeV as input, the results become

\[
\Gamma(B^+ \to \bar{D}^0 D_s^{*+}(2S)) = F_V^2(2S) \times (3.36 \pm 0.25) \times 10^{-14} \text{ GeV}, \quad (10)
\]

\[
\Gamma(B^+ \to \bar{D}^0 D_s^{*+}(1D)) = F_V^2(1D) \times (3.36 \pm 0.25) \times 10^{-14} \text{ GeV}. \quad (11)
\]

So the difference between this two channel mainly come from the difference of decay constants. Then our predictions of branching ratios are:

\[
Br(B^+ \to \bar{D}^0 D_s^{*+}(2S)) = (0.72 \pm 0.12)\%, \quad (12)
\]

\[
Br(B^+ \to \bar{D}^0 D_s^{*+}(1D)) = (0.027 \pm 0.007)\%. \quad (13)
\]

In Ref. [12], we have calculated the main decay channels of \( D_{sJ}^+(2S) \) and \( D_{sJ}^+(1D) \), and we have the following estimates:

\[
Br(D_{sJ}^+(2S) \to D^0 K^+) = 0.20 \pm 0.03
\]
and

\[ Br(D_{sJ}^+(1D) \to D^0 K^+) = 0.32 \pm 0.04, \]

so we obtain:

\[ Br(B^+ \to \bar{D}^0 D_{sJ}^{+(2S)}) \times Br(D_{sJ}^{+(2S)} \to D^0 K^+) = (1.4 \pm 0.5) \times 10^{-3} \]  \hspace{1cm} (14)

and

\[ Br(B^+ \to \bar{D}^0 D_{sJ}^{+(1D)}) \times Br(D_{sJ}^{+(1D)} \to D^0 K^+) = (0.9 \pm 0.3) \times 10^{-4}. \]  \hspace{1cm} (15)

Our estimate of \( Br(D_{sJ}^{+(2S)} \to D^0 K^+) \) \( \approx 0.20 \) is larger than than the estimate 0.11 in Ref. [4] and the estimate 0.05 in Ref. [5], if we use their value as input, we will obtain a smaller value of \( Br(B^+ \to \bar{D}^0 D_{sJ}^{+(2S)}) \times Br(D_{sJ}^{+(2S)} \to D^0 K^+) \). But our estimate of \( Br(D_{sJ}^{+(1D)} \to D^0 K^+) \) \( \approx 0.32 \) consist with the value 0.34 in Ref. [5]. One can also see that, the final branching ratios depend strongly on the decay constants, but at this time few papers have calculated the values of decay constants for \( D_{sJ}^{+(2S)} \) and \( D_{sJ}^{+(1D)} \). In Ref. [7], they assumed that the new state \( D_{sJ}^{+(2700)} \) is \( D_{sJ}^{+(2S)} \), and they extracted decay constant of \( D_{sJ}^{+(2S)} \): \( F_{D_{sJ}^{+(2S)}} = 243 \pm 41 \) MeV.

The Belle Collaboration have the data [3, 13]

\[ Br(B^+ \to \bar{D}^0 D_{sJ}^{+(2700)}) \times Br(D_{sJ}^{+(2700)} \to D^0 K^+) = (1.13^{+0.26}_{-0.36}) \times 10^{-3}. \]

Because we only calculated the decay widths of six main channels for \( D_{sJ}^{+(2S)} \) (\( D_{sJ}^{+(1D)} \)), and based on the summed width of these six channels, not full width, we give a relative larger branching ratio \( Br(D_{sJ}^{+(2S)} \to D^0 K^+) \) and \( Br(D_{sJ}^{+(1D)} \to D^0 K^+) \), the real branching ratios should be smaller than our estimates, so our estimate of \( B^+ \) decay to \( D_{sJ}^{+(2S)} \) is close to the data, while the estimate of \( B^+ \) decay to \( D_{sJ}^{+(1D)} \) is much smaller than the data, then we can draw a conclusion that the new state \( D_{sJ}^{+(2700)} \) is \( D_{sJ}^{+(2S)} \).

We have another method to estimate the branching ratio, because the mass of \( B^+ \) is much heavier than the mass of \( \bar{D}^0 \), and the mass of \( D_{sJ}^{+(2700)} \) is close to the mass of \( D_{sJ}^{+} \), as a rough estimate, we ignore the mass difference of \( D_{sJ}^{+(2700)} \) and \( D_{sJ}^{+(2112)} \), from Eq.(1) and Eq.(2), then we have

\[ \frac{Br(B^+ \to \bar{D}^0 D_{sJ}^{+(2S)})}{Br(B^+ \to \bar{D}^0 D_{sJ}^{+(1S)})} \approx \frac{F_\chi^2(2S)}{F_\chi^2(1S)}, \]  \hspace{1cm} (16)

and from Particle Data Group [13], the branching ratio of \( B^+ \to \bar{D}^0 + D_{sJ}^{+(1S)} \):

\[ Br(B^+ \to \bar{D}^0 D_{sJ}^{+(1S)}) = (7.8 \pm 1.6) \times 10^{-3}. \]
So we have:

\[
Br(B^+ \rightarrow \bar{D}^0 D_s^{*+}(2S)) \simeq \frac{F^2_0(2S)}{F^2_0(1S)} \times (7.8 \pm 1.6) \times 10^{-3} \simeq (5.4 \pm 1.7) \times 10^{-3},
\]

(17)

\[
Br(B^+ \rightarrow \bar{D}^0 D_s^{*+}(1D)) \simeq \frac{F^2_0(1D)}{F^2_0(1S)} \times (7.8 \pm 1.6) \times 10^{-3} \simeq (2.0 \pm 1.0) \times 10^{-4}.
\]

(18)

This rough estimate results are very close to our calculations (Eq.(12) and Eq.(13)), so we have the same conclusion that \(D_{sJ}^+(2700)\) is \(D_{sJ}^+(2S)\).

In Ref. [12], we estimate the full widths of \(D_{sJ}^+(2S)\) and \(D_{sJ}^+(1D)\) by six main decay channels, the estimated full widths are \(46.4 \pm 6.2\) MeV for \(D_{sJ}^+(2S)\), \(73.0 \pm 10.4\) MeV for \(D_{sJ}^+(1D)\), comparing with experimental data, \(\Gamma = 112 \pm 7 \pm 36\) MeV for \(D_{sJ}^+(2690)\) and \(\Gamma = 108 \pm 23^{+36}_{-31}\) MeV for \(D_{sJ}^+(2700)\), there is still the possible that there are two states around 2700 MeV, one is the \(S\) wave dominate \(D_{sJ}^+(2S)\), the other is \(D\) wave dominate \(D_{sJ}^+(1D)\).

As summary, from the decays \(B^+ \rightarrow \bar{D}^0 + D_{sJ}^+(2S)\) and \(B^+ \rightarrow \bar{D}^0 + D_{sJ}^+(1D)\), we conclude that the new state \(D_{sJ}^+(2700)\) is the first radial excited state \(D_{sJ}^+(2S)\), and there may exist another state around 2700, \(D_{sJ}^+(1D)\), with a mass around 2718 MeV, and a width \(73.0 \pm 10.4\) MeV.

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