Sudakov effects in central-forward dijet production in high energy factorization

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Abstract

We discuss central-forward dijet production at LHC energies within the framework of high energy factorization. In our study, we profit from the recent progress on consistent merging of Sudakov resummation with small-$x$ effects, which allows us to compute two different gluon distributions which depend on longitudinal momentum, transverse momentum and the hard scale of the process: one for the quark channel and one for the gluon channel. The small-$x$ resummation is included by means of the BK equation supplemented with a kinematic constraint and subleading corrections. We test the new gluon distributions against existing CMS data for transverse momentum spectra in forward-central dijet production. We obtain results which are largely consistent with our earlier predictions based on model implementation of Sudakov form factors. In addition, we study dijet azimuthal decorrelations for the forward-central jets, which are known to be sensitive to the modeling of soft radiation.

1 Introduction

Processes with jets remain one of the most important tools used to study Quantum Chromodynamics (QCD) at hadron colliders, in particular at the LHC [1,2] and future Electron Ion Collider (EIC) [3–6]. Amongst them, production of dijets proves particularly useful to address various questions concerning QCD dynamics. When both jets are produced in the central rapidity region, the energy fractions of the incoming partons are comparable and sizable. Theoretical predictions for such configuration can be safely calculated in the framework of collinear factorization. However, when one of the jets moves in the forward direction, $y_{\text{jet}} \gg 0$, one of the incoming hadrons is probed at relatively low momentum fraction $x$, and that leads to the appearance of large logarithms $\ln x$, which have to be resummed. The optimal description of this process is achieved within the hybrid factorization [7–10], where the matrix elements are evaluated with one of the incoming partons being off-shell. The momentum distribution of that parton obeys the BFKL equation [11–14], which depends not only on the longitudinal part of the momentum, but also on its transverse component. We will from now on refer to these as transverse momentum dependent parton distributions (TMDs). In addition, when both jets move forward, the value of $x$ is even smaller and one starts being sensitive to saturation effects [15–16]. The corresponding evolution equation becomes nonlinear [17–21], as density of gluons at low $x$ is very high.

While the small-$x$ effects can be taken into account by using one of the phenomenologically successful TMDs, there is another class of effects relevant for forward jet production which should
also be accounted for, namely the resummation of Sudakov logarithms. They are important as
the hard scale provided by jet transverse momentum opens phase space for logarithmically
enhanced soft and collinear emissions \[22, 26].

As demonstrated in Refs. \[27, 31\], small-\(x\) and Sudakov resummations can be performed
simultaneously in \(b_\perp\) space and can then be cast into transverse momentum dependent gluon
distributions. Such TMDs have already been used in phenomenological calculations of di-hadron
correlations at EIC \[6\] and in proton-nucleus collisions at RHIC \[32\]. In both cases, the Gołąb-
Biernat-Ćwiok model \[33\] was employed to account for small-\(x\) effects.

In the present work, we focus on Sudakov effects in the process of central-forward dijet
production in proton-proton collisions. We perform our calculations in the framework of high
energy factorization (HEF) factorization \[7, 15, 34, 35\], where the cross section is calculated as a
convolution of a hard sub-process \[36, 37\] and nonperturbative parton densities, which take into
account longitudinal and transverse degrees of freedom. At low \(x\), gluons dominate over quarks,
hence we consider only gluon TMDs.

For the central-forward configuration of the final-state jets, one of the longitudinal fractions
of the hadron momenta is much smaller than the other, \(x_B \ll x_A\). This follows from simple
kinematic relations

\[
x_A = \frac{1}{\sqrt{s}} \left( |p_{1\perp}| e^{\nu_1} + |p_{2\perp}| e^{\nu_2} \right), \quad x_B = \frac{1}{\sqrt{s}} \left( |p_{1\perp}| e^{-\nu_1} + |p_{2\perp}| e^{-\nu_2} \right),
\]

where \(\sqrt{s}\) is the center-of-mass energy of the proton-proton collision, while \(p_{1\perp}\) and \(y_i\) are the
transverse momenta (Euclidean two-vectors) and rapidities of the produced jets. The formula for the \textit{hybrid} high energy factorization reads \[8, 10\]

\[
d\sigma_{AA→j_1+j_2+X} = \int dx_A \int dx_B \int d^2k_{B\perp} \frac{1}{\pi} \sum_{a,c,d} f_{g/A} (x_A, \mu) \mathcal{F}_{g^*/B} (x_B, k_{B\perp}, \mu) \ d\hat{\sigma}_{g^*→c+d} (x_A, x_B, k_{B\perp}, \mu),
\]

where \(\mathcal{F}_{g^*/B}\) is the so-called \textit{unintegrated gluon density} or \textit{transverse momentum dependent gluon
distribution} (see \[38–41\] for more details on different gluon distributions), \(f_{g/A}\) are the collinear
PDFs and \(d\hat{\sigma}_{g^*→c+d}\) is built out of the off-shell gauge-invariant matrix elements. The elements \(a, c, d\) run over the gluon and all the quarks that can contribute to the inclusive dijet production.
Notice that both \(f_{g/A}\) and \(\mathcal{F}_{g^*/B}\) depend on the hard scale \(\mu\), and the latter depends also on the
transverse momentum of the incoming gluon, whose value is linked to the final-state kinematics
by the relation

\[
|k_{\perp}| = |p_{1\perp} + p_{2\perp}| = |p_{1\perp}|^2 + |p_{2\perp}|^2 + 2|p_{1\perp}||p_{2\perp}| \cos \Delta \phi,
\]

where \(\Delta \phi\) is the azimuthal distance between the jets. The hard scale dependence in the TMD
is necessary to properly account for large Sudakov logarithms that appear predominantly in the
back-to-back region, where \(k_{\perp}\) is small, but \(\mu\) remains large for relatively hard jets. As shown
in Ref. \[42\], incorporating the hard scale dependence in the TMD is essential to successfully
describe shapes of dijet spectra.

It is important to mention that, as discussed in Ref. \[40\], the high energy factorization
formula \[2\] is valid only when \(Q_s \ll |k_{\perp}| \ll |p_{1\perp}|, |p_{2\perp}|\), which corresponds to collisions of
relatively dilute hadrons. The process of central-forward dijet production in \(p−p\) collision,
which is the focus of our study, corresponds exactly to that situation. For processes which
involve dense targets, like for example forward-forward dijet production in \(p−A\) collisions,
Eq. \[2\] has to be replaced by a more general factorization formula with multiple transverse
momentum dependent gluon distributions \[39, 40, 43, 44\].
2 Dipole gluon with Sudakov form factor

The Sudakov effects are most conveniently included in position space. The resulting gluon TMD, which incorporates both small-$x$ and soft-collinear resummation, can be then transformed to momentum space as follows \[32\]

\[
\mathcal{F}_{g^* / B}^{ab \rightarrow cd}(x, q_\perp, \mu) = \frac{N_c S_\perp}{2\pi\alpha_s} \int_0^\infty \frac{b_\perp db_\perp}{2\pi} J_0(q_\perp b_\perp) e^{-S^{ab \rightarrow cd}_{\text{Sud}}(\mu, b_\perp)} \nabla_{b_\perp}^2 S(x, b_\perp). \tag{4}
\]

(Notice the difference in the prefactor w.r.t. to Ref. \[32\], which comes from the fact that \(\mathcal{F}_{g^* / B} = \pi \mathcal{F}_{g g}^{(s)}\).) The Sudakov factors come from the resummation of soft-collinear gluon radiation and they depend on the partonic channel. Hence, the gluon with the Sudakov acquires this dependence and, consequently, a single dipole gluon is replaced with a set of gluons \(\{\mathcal{F}_{g^* / B}^{ab \rightarrow cd}\}\).

In practice, the two channels that dominate in the central-forward productions are: \(qg\) and \(gg\). Hence, we will need to determine two gluon TMDs: \(\mathcal{F}_{g^* / B}^{qq \rightarrow gg}\) and \(\mathcal{F}_{g^* / B}^{gg \rightarrow gg}\).

By taking the Fourier transform of Eq. (4), we can express the gluon with Sudakov resummation by the gluon without the Sudakov, all in momentum space

\[
\mathcal{F}_{g^* / B}^{ab \rightarrow cd}(x, k_\perp, \mu) = \int dk_\perp \int dk_\perp' b_\perp b_\perp' J_0(b_\perp k_\perp) J_0(b_\perp k_\perp') \mathcal{F}_{g^* / B}(x, k_\perp) e^{-S^{ab \rightarrow cd}_{\text{Sud}}(\mu, b_\perp)}. \tag{5}
\]

For each channel, the Sudakov factors can be written as

\[
S^{ab \rightarrow cd}_{\text{Sud}}(b_\perp) = \sum_{i=a,b,c,d} S_i^p(b_\perp) + \sum_{i=a,c,d} S_{i,p}(b_\perp), \tag{6}
\]

where \(S_i^p(b_\perp)\) and \(S_{i,p}(b_\perp)\) are the perturbative and non-perturbative contributions. As argued in Ref. \[32\], as small-$x$ gluon TMDs for parton \(b\) may already contain some non-perturbative information at low-$x$, the non-perturbative Sudakov factor associated with that incoming gluon \(b\) should not be included. In addition, according to the derivation in Ref. \[27\], the single logarithmic term in the perturbative part of the Sudakov factor – the so-called \(B\)-term – should also be absent for the incoming small-$x$ gluon. The perturbative Sudakov factors are given by \[32\]

\[
S^{qq \rightarrow gg}_{p}(Q, b_\perp) = \int \frac{Q^2}{\mu^2} \frac{d\mu^2}{\mu^2} \frac{2(C_F + C_A)\alpha_s}{2\pi} \ln \left( \frac{Q^2}{\mu^2} \right) - \left( \frac{3}{2} C_F + C_A \beta_0 \right) \frac{\alpha_s}{\pi}, \tag{7}
\]

\[
S^{gg \rightarrow gg}_{p}(Q, b_\perp) = \int \frac{Q^2}{\mu^2} \frac{d\mu^2}{\mu^2} \frac{4C_A\alpha_s}{2\pi} \ln \left( \frac{Q^2}{\mu^2} \right) - 3C_A \beta_0 \frac{\alpha_s}{\pi}, \tag{8}
\]

where \(\beta_0 = (11 - 2n_f/3)/12\), \(\mu_b = 2e^{-\gamma_E}/b_s\), and \(b_s = b_\perp/\sqrt{1 + b_\perp^2/b_{\max}^2}\). The \(gg \rightarrow qg\) channel is negligible for the kinematics of this study. Following Ref. \[32\], for the non-perturbative Sudakov factor, we employ the parameterization \[45,46\]

\[
S^{gg \rightarrow gg}_{np}(Q, b_\perp) = \left( 2 + \frac{C_A}{2C_F} \right) \frac{g_1}{2} b_\perp^2 + \left( 2 + \frac{C_A}{2C_F} \right) \frac{g_2}{2} \ln \frac{Q}{Q_0} \ln \frac{b_\perp}{b_s}, \tag{9}
\]

\[
S^{gg \rightarrow gg}_{np}(Q, b_\perp) = \frac{3C_A}{2C_F} \frac{g_1}{2} b_\perp^2 + \frac{3C_A}{2C_F} \frac{g_2}{2} \ln \frac{Q}{Q_0} \ln \frac{b_\perp}{b_s}, \tag{10}
\]

with \(g_1 = 0.212\), \(g_2 = 0.84\), and \(Q_0^2 = 2.4\text{GeV}^2\).

As a basis for all calculations presented in this study, we use the nonlinear KS (Kutak-Sapeta) gluon TMD \[17\], which comes from the extension of the BK (Balitsky-Kovchegov) equation \[48\].
Figure 1: KS gluon with various Sudakov form factors.
following the prescription of Ref. [49] to include kinematic constraint on the gluons in the chain, non-singular pieces of the splitting functions, as well as contributions from sea quarks. The parameters of this gluon were set by a fit to $F_2$ data from HERA.

We introduce the Sudakov effects into the KS gluon following the formalism described above. In addition, for reference, we use two methods employed in our earlier studies [42, 50]. Those calculations used the Sudakov form factor, understood as the DGLAP evolution kernel, that has been applied on the top of the gluon TMD, together with constraints such as unitarity. Those methods should therefore be considered as models, in contrast to the proper resummation of Sudakov logarithms considered in this work. Nevertheless, the approaches used in Refs. [42, 50] were phenomenologically successful (see also [51]), and one of the objectives of this study is to check how the predictions of those simplistic models compare with the proper way of including the Sudakov effects into the small-$x$ gluon. The reference models are:

- Model 1: The survival probability model [42], where the Sudakov factor of the form [52]

$$T_s(\mu_F^2, k_{\perp}^2) = \exp \left(- \int_{k_{\perp}^2}^{\mu_F^2} \frac{dk_2^2}{k_2^2} \frac{\alpha_s(k_2^2)}{2\pi} \sum_a \int_0^{1-\Delta} dz' P_{a,a}(z') \right),$$

is imposed at the level of the cross section. This procedure corresponds to performing a DGLAP-type evolution from the scale $\mu_0 \sim |k_{\perp}|$ to $\mu$, decoupled from the small-$x$ evolution.

- Model 2: The model with a hard scale introduced in Ref. [50]. The Sudakov form factor of the same form as in Eq. (11) is imposed on top of the KS gluon in such a way that, after integration of the resulting hard scale dependent gluon TMD, one obtains the same result as by integrating the KS gluon.

In Fig. 1 we show the KS gluons, with and without Sudakov form factors, as functions of the transverse momentum $k_{\perp}$ and the hard scale $\mu$. Three columns correspond to three different $x$ values. The first row shows the original KS gluon, which, as expected, does not depend on the value of $\mu$. In the second row, we show the KS hardscale gluon of Ref. [50] (the other model [42] does not allow one to plot gluon distribution, as it applies Sudakov effects at the cross section level via a reweighting procedure). Here, the dependence on $\mu$ is non-trivial and we see that the gluon develops a maximum in that variable. As shown in the figure, this maximum is rather broad. In the third and the fourth row of Fig. 1 we present our new KS gluon with the Sudakov form factor described in this section. As explained earlier, this gluon exists in two versions, one for the $qg$ and the other for the $gg$ channel. The dependence on $k_{\perp}$ and $\mu$ is qualitatively similar between the new gluons and the naive KS hardscale gluon. In the former case, however, the peak is significantly narrower in $\mu$ as compare to the naive model of Ref. [50]. It is interesting to note that the $qg$ gluon is broader than the $gg$ gluon. This can be understood by comparing the colour factors in the Sudakov functions (7) and (8). The colour factor is bigger for the $gg$ channel, hence, in that case, the Sudakov suppression is stronger along the $\mu$ direction.

We have as well computed linear versions of the KS gluons with the Sudakov, using the KS linear gluon of Ref. [47]. We also used them to calculate differential distributions discussed in the following section. We observed that both sets of gluons (linear and nonlinear) give comparable results for the phenomenological observables. This is consistent with the expectation that saturation plays a limited role in central-forward dijet production in $p\bar{p}$ collisions. Therefore, given that the nonlinear KS gluon comes from a better fit to $F_2$ than its linearized version [47], in the following, we present only the results obtained with the nonlinear gluon density.

The new gluons presented in this section are available publicly from the recent version of the KS package and can be downloaded from \url{http://nz42.ifj.edu.pl/~sapeta/KSgluon-2.0.tar.gz}. 

5
Distributions of the longitudinal momentum fractions, $x_A$, $x_B$, defined in Eq. (1) from calculations with various version of the KS gluon discussed in the article.

3 Differential distributions

We now turn to the discussion of differential distributions in jets’ transverse momenta calculated in the framework described in the preceding sections. We calculated the cross sections using the selection criteria of CMS [53]. The two leading jets were required to satisfy the cuts $p_{T1}, p_{T2} > 35$ GeV and $|y_1| < 2.8, 3.2 < |y_2| < 4.7$. We used the CTEQ18 NLO PDF set [54] and LHAPDF [55] for the collinear PDFs and the KS gluons with and without Sudakov for the gluon TMDs.

Our calculations have been performed and cross checked using two independent Monte Carlo programs [56, 57] implementing the high energy factorization together with the off-shell matrix element calculated following the methods of Refs. [58–60]. We used the average transverse momentum of jets as both the renormalization and factorization hard scales.

We start by showing in Fig. 2 distributions of the longitudinal momentum fractions probed by the central-forward dijet configurations. These results are consistent with the discussion of Section 1, in particular Eq. (1), and provide justification to the use of the hybrid factorization formula (2).

If Fig. 3 we show differential cross sections as function of the momenta of the forward and central jets. We compare central values of various predictions which differ by the gluon TMDs used in the HEF formula (2). The black dotted histograms correspond to the gluon without Sudakov, while the other three histograms use gluons with some form of Sudakov resummation. The main result of this paper is shown as a blue solid line, while the green and the red dashed curves correspond to the naive Sudakov modelling of Refs. [42,50].

We see that the predictions with the Sudakov effects included describe the data better, especially in the region of small $p_{T}$. It is worth noticing that the proper Sudakov used in this study (blue solid line) gives predictions which are fairly close to the models 1 and 2 used in our earlier works [42,51].

If Fig. 4, we show the same distributions of transverse momenta, but, here, we plot only two models (without Sudakov and with Sudakov from Section 2). This time, we show also the theoretical errors, estimated by the usual renormalization and factorization scale variation by the factors $2^{\pm 1}$.
Figure 3: The transverse momentum spectra of the central (left) and the forward (right) jets obtained with the KS gluon, with and without Sudakov effects, computed for the central value of the factorization and renormalization scale, compared to CMS data [53].

We observe good agreement of our predictions with the CMS data [53], except the tail of the central-jet transverse momentum distribution. One has to remember however that, following Eq. (1), the tails of $p_T$ distributions are sensitive to the region of large $x$, where, in principle, the gluon TMDs are not valid. Indeed, we have seen in our calculation that the KS gluon with the Sudakov can sometimes get negative for larger $x$ values. We interpret that as a sign of going outside of the validity region of the gluon distribution and, hence, in such situations, we set it to zero in the cross section calculation.

In Fig. 5 (left) we compare predictions for the distribution of the azimuthal angle between the two leading jets (aka azimuthal decorrelations). Again, we show results corresponding to calculations with and without the Sudakov. We observe that inclusion of Sudakov effects leads to qualitatively the same modification of $\Delta \phi$ distributions. Namely, the region of large $\Delta \phi$ is depopulated w.r.t. the result without Sudakov, while the opposite happens in the region of smaller $\Delta \phi$.

While qualitatively the predictions from KS gluon + Sudakov from this work look similar to the earlier Sudakov models, quantitatively those cross sections differ to a certain degree, as seen in Fig. 5. In particular, the models 1 and 2, lead to convex functions for the azimuthal decorrelations, while the Sudakov of this study produces a concave curve.

We would also like to mention that the predictions using model 1 were shown to successfully reproduce the shapes of preliminary CMS data for the azimuthal decorrelations [42]. Since, as of today, these data are not published, we refrain from comparing them with the predictions of this work. We would only like to comment that, based on the comparison shown in Fig. 5, we expect the predictions from this study to be largely compatible with the earlier naive models, within theoretical errors.

Finally, in Fig. 5 (right) we show rapidity distributions resulting from the various versions of the KS gluon, for the central and the forward jet. We see marked differences between predictions without and with Sudakov. Interestingly, inclusion of the Sudakov from this work suppresses both the central and the forward jet distribution, and this is largely consistent with the naive model 1. However, model 2 shows enhancement (central jet) or almost no effect (forward jet) in the rapidity differential cross sections.
Figure 4: As Fig. [3] but we only show predictions obtained with the original KS gluon and the predictions with the KS gluon with Sudakov from this work. The bands correspond to varying the renormalization and factorization scale by factors $2^{\pm 1}$.

4 Summary

We discussed Sudakov effects in central-forward dijet production at LHC energies within the framework of high energy factorization. Our study was triggered by recent progress on consistent merging of Sudakov resummation with the small-$x$ effects, which allowed us to compute hard-scale dependent gluon TMDs. As explained in Section 2, we were able to combine the phenomenologically successful KS gluon [47] with the Sudakov factors directly in momentum space.

In our study, we used the Sudakov factors derived within perturbative QCD in Refs. [27–31]. For comparison, we also used simpler Sudakov models employed in our earlier studies [42,50].

We have calculated theoretical predictions for the differential cross sections as functions of $p_\perp$ of the central and the forward jet, as well as azimuthal distance between the jets. The results are largely consistent with our earlier predictions based on simple phenomenological Sudakov models. We also achieved good description of CMS data for $p_\perp$ distributions. Finally, we presented predictions for dijet azimuthal decorrelations.

It is worth emphasising that our framework is relatively simple and all the parametrizations of non-perturbative physics were taken from external analyses. Hence, no additional parameters were introduced in the calculation of the results presented in this work.

Overall, we conclude that the Sudakov resummation has a moderate effect on $p_\perp$ spectra and a fairly sizable effect on the shapes of decorrelations. This is consistent with earlier phenomenological studies [42,51], which showed preference for gluons with Sudakov effects included.

Our future work will concern developing a full set of TMD gluon distributions exhibiting saturation effects and the Sudakov resummation, following the same perturbative calculations we used in the present paper. Such TMDs are necessary to confirm our previous calculations for forward-forward dijets [51] that show interplay of saturation effects and Sudakov effects consistent with the ATLAS data, where, however, the more naive Sudakov model was used.

Furthermore, in the future, we plan to address the dijet production in DIS, and a good understanding of the interplay of Sudakov effects and saturation is needed in order to provide robust predictions for the EIC [3] jet observables. We expect that by starting with central ra-
Figure 5: Differential cross sections as functions of the azimuthal distance between the jets $\Delta \phi$ (left) and jet rapidities (right) obtained with the KS gluon with and without Sudakov effects.

... going to more forward rapidities, one will be able to incrementally see the increasing importance of saturation effects and disentangle them from Sudakov effects.

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