The Crystal Ball (CB) collaboration at BNL has recently presented results regarding a study of the $\pi^- A \rightarrow \pi^0 \pi^0 A'$ reaction on $H, D, C, Al$ and $Cu$, using a nearly $4\pi$ detector. Similar results, but for the $\pi^+ A \rightarrow \pi^+ \pi^+ A'$ reaction on $^2H, ^{12}C, ^{40}Ca$, and $^{208}Pb$, have been published earlier by the CHAOS collaboration at TRIUMF. In this Brief Report a comparison of the results of the two measurements is made, which shows that the CHAOS and CB data share relevant common features. In particular, the increase in strength as a function of $A$ seen in the near-threshold $\pi^+ \pi^-$ invariant mass spectra reported by the CHAOS group, is also seen in the $\pi^0 \pi^0$ CB data, when the results from the two groups are compared in a way which accounts for the different acceptances of the two experiments.

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The phenomenon of nuclear medium modification of $\pi \pi$ properties has been the focus of a growing number of recent studies. A selection of the most recent theoretical articles can be found in Ref. [1]. The authors therein have reanalysed the modifications caused by nuclear matter at finite density for the $\pi \pi$ interaction in the $I = J = 0$ channel (the $\sigma$-meson channel), and found a dramatic reshaping of the $\pi \pi$ interaction strength function at around the $2m_{\pi}$ threshold. The reshaping results in a strong enhancement of the strength function around $2m_{\pi}$, which is determined by the combined effect of collective pionic modes and partial restoration of chiral symmetry. The first effect is explained in terms of standard $P-$wave coupling of pions to particle – hole and $\Delta$ – hole correlated states. The effect of partial restoration of chiral symmetry in nuclear matter relates to the modification of the basic $(\pi \pi)_{I=J=0}$ interaction: in the scalar-isoscalar channel, pion pairs strongly interact

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with the nucleons giving rise to the elusive $\sigma$-meson, even at densities below the nuclear saturation density. Such a picture of $\pi\pi$ dynamics in nuclear matter is supported by the experimental results of the CHAOS collaboration. These results, however, seem in part to be contradicted by the recently published data from the CB collaboration. The purpose of this Brief Report is to make a more careful comparison which accounts for the different acceptance of the two experiments. Such a comparison reveals that in fact the main results of the experiments are in good agreement with each other.

The TRIUMF pion production program measured the $\pi^+ A \rightarrow \pi^+ \pi^\pm A'$ reaction on $^2H$, $^{12}C$, $^{40}Ca$, and $^{208}Pb$ at $p_{\pi^+} = 399$ MeV/c, using the CHAOS spectrometer \[2\]. CHAOS consists of a dipole magnet producing a vertical field and four rings of cylindrical wire chambers for the tracking of charged particles. The outermost wire chamber is surrounded by a segmented telescope for particle identification. Particles from a pion production reaction ($\pi, 2\pi$) are measured in the horizontal plane (360°), and $\pm 7^\circ$ out of this plane for a solid angle of $\sim 10\%$ of $4\pi$ sr. The CB collaboration reported results from a study of the $\pi^- A \rightarrow \pi^0\pi^0 A'$ reaction on $H, ^2H, C, Al$ and $Cu$ at $p_{\pi^-} = 408$ MeV/c, using a NaI(Tl) crystal ball which covered 93% of $4\pi$ sr \[3\]. The detector is a multiphoton spectrometer, thus capable of detecting $\pi^0$'s in the final state. Charged particles are vetoed by a plastic scintillator barrel which surrounds the target. Table 1 reports the quantities which are relevant to the further discussion. Since the CB detector has a larger solid angle coverage, the CB data could in principle be analysed and reduced to the CHAOS solid angle. In this manner, the results of the two measurements could be directly compared. This approach, unfortunately, is impractical because of the low statistics of the CB data. In the case of $H$ (Fig. 2 of Ref. \[3\]), the low-energy yield at $\sim 284$ MeV nearly equals the standard deviation thus implying that the bin content is $\sim 1$. About the same amount is accumulated in the $\sim 293$ MeV bin. As well, the deuterium invariant mass ($M_{\pi\pi}$) spectrum (Fig. 3) displays the same features. The comparison will therefore be addressed to the bulk of the ($\pi, 2\pi$) results.

Analyses of the CHAOS work focused on studies of the $A$-dependence of the reaction at threshold \[4\], the reaction mechanism\[5\], the properties of the $\pi\pi$ system in vacuum\[6, 7\] and in nuclear matter \[8, 9\]. Some of the previously published CHAOS results most relevant to this Brief Report are summarised in the following.

Table 1: Relevant quantities in the $CB$ and $CHAOS$ measurements. The notations $T_\pi$ and $M_{\pi\pi}$ indicate the single pion kinetic energy and the $\pi\pi$ invariant mass, respectively.

| Detector | Magnetic field [T] | Solid angle [%] | $T_\pi$ threshold [MeV] | $M_{\pi\pi}$ resolution [MeV] | Reaction channel $|\pi \rightarrow \pi\pi|$ |
|----------|---------------------|----------------|-------------------------|-------------------------------|---------------------------------|
| $CB$     | 0.0                 | 93%            | $\sim 0$               | $3.3 - 7.1$                  | $\pi^- \rightarrow \pi^0\pi^0$ |
| $CHAOS$  | 0.5                 | 10%            | $\sim 11$              | $1.1 - 2.1$                  | $\pi^+ \rightarrow \pi^+\pi^\pm$ |
The $\pi \to \pi\pi$ reaction in nuclei is a quasifree process, which involves a single nucleon and proceeds via $\pi N \to \pi\pi N$ \cite{5}.

Near the $2m_\pi$ threshold, the $\pi^+\pi^-$ interacting system predominantly couples in $S$-wave ($\sim 95\%$), while $P$-wave coupling is negligible \cite{4, 9}. Together with the arguments presented in Ref.\cite{10}, this permits the quantum numbers $I = J = 0$ to be assigned to the $\pi^+\pi^-$ system. The same quantum numbers are readily reached by the pure isospin $0\pi^0\pi^0$ system in the $CB$ measurement. Therefore, the $\pi^+\pi^-$ system can be directly compared to the $\pi^0\pi^0$ system.

The most striking physics result from the CHAOS data is the remarkable $A$-dependence observed in the $\pi^+\pi^-$ invariant mass ($M_{\pi\pi}$) distributions at around the $2m_\pi$ threshold \cite{9}. Unfortunately the data also exhibit a peak in this same region which is an artifact of the limited out-of-plane acceptance of the spectrometer. The $M_{\pi\pi}$ resolution (1.1-2.1 MeV) does not exceed the $M_{\pi\pi}$ binning (8.7 MeV), so resolution does not affect the shape of the $M_{\pi\pi}$ distributions. The physics is in the observed $A$-dependence of the near threshold $M_{\pi\pi}$ distributions, not in the peak itself. Furthermore, the peak is also observed in the simultaneously measured isospin 2 $\pi^+\pi^+$ invariant mass distributions, which exhibit no remarkable $A$-dependence, and appears in the phase-space simulations. This has been consistently explained in each of the CHAOS articles.

The observable $C^A_{\pi\pi}$ was defined in Ref.\cite{8} precisely in order to disentangle the acceptance issue from the physics observed in the $A$-dependence, and therefore focus exclusively on medium effects. $C^A_{\pi\pi}$ is the composite ratio $\frac{M_{\pi\pi}^A}{\sigma_T^A}/\frac{M_{\pi\pi}^N}{\sigma_T^N}$, where $\sigma_T^A$ ($\sigma_T^N$) is the measured total cross section of the $(\pi, 2\pi)$ process in nuclei (nucleon). This observable was shown to yield the net effect of nuclear matter on the $\pi\pi$ interacting system, regardless of the reaction mechanism used to produce the pion pair, and to be nearly unrelated to the CHAOS acceptace. These traits and the points discussed above ensure that the composite ratio $C^A_{\pi\pi}$ should be nearly the same for the CHAOS and the $CB$ measurements, despite the different solid angles subtended by the two experiments.

Before using the acceptance independent $C^A_{\pi\pi}$ to make a meaningful comparison of the $CB$ \cite{3} and the CHAOS \cite{4 – 9} results, some comments on the $CB$ results are needed. The experimental data for the $2\pi^0$ invariant mass distributions (Figs. 2 and 3 of Ref. \cite{3}) are in general characterised by low statistics. Only a few $(\pi, 2\pi)$ events were accumulated in the low-energy yield for $H$ and $^2H$. This is very unfortunate, since these poor statistics occur exactly in the most interesting region of the invariant mass distribution, where $C^A_{\pi\pi}$ departs from a flat behaviour \cite{8}.

Fig. 3 of Ref.\cite{3} shows the experimental $2\pi^0$ invariant mass distributions for the studied nuclei corrected for the $CB$ acceptance. The $CB$ data (in arbitrary units) are compared to the model calculation of Vicente-Vacas \cite{11}, dashed line, and Rapp \cite{12}, full line. For $H$
(and $^2H$, see Ref. [1]), the curves are able to reproduce the shape of the $M_{\pi\pi}$ experimental distribution. In the case of $C$, the widening of the distribution toward high energies reflects the Fermi motion of the struck nucleon, $\pi^- p[A-1] \rightarrow \pi^0\pi^0 n[A-1]$. For $Al$ and $Cu$, these features seem contradicted by the shrinking of the distributions. The $M_{\pi\pi}$ high-energy tail appears depleted rather than further expanded even though the models, which include the Fermi motion, predict a non-negligible yield 20-30 MeV above the CB data.

The $2\pi^0$ invariant masses are given in arbitrary units in Ref. [3], which precludes comparison to an absolute prediction. Normalizing simulations (Fig. 2) and model predictions (Fig. 3) to data can lead to ambiguous interpretations. For instance, the $Cu$-target data in Fig. 3 are poorly described by the curves of Vicente-Vacas (dashed line) and Rapp (full line), thus any analysis is inconclusive. If the curves were normalised to the $M_{\pi\pi}$ high-energy tail, then the data would have explicitly displayed an accumulation of strength at threshold. On the other hand, if the curves were normalized to low-energy invariant masses, then the comparison would have disclosed a rather strong depletion of the $M_{\pi\pi}$ high-energy yield. The case for the hydrogen target is different: the theoretical predictions seem to be normalized to the experimental data. The calculation by Vicente-Vacas agrees almost perfectly with the data, which reflects the detailed microscopic description of the elementary $\pi N \rightarrow \pi\pi N$ reaction. The model of Rapp, which misses some important diagrams, overestimates the $M_{\pi\pi}$ yields (from 2 to 3 times) in the low-energy interval. The same quantitative conclusion for the hydrogen target is also reached using the CHAOS data [3].

Finally, the results of CHAOS are compared to the results from the CB Collaboration. The authors of Ref. [3] repeatedly make the point that they do not observe the peak in the near threshold invariant mass that is observed by CHAOS. As noted above, this is merely an expected consequence of the different acceptances of the two detectors.

The results shown in Fig. 4 of Ref. [3] lead to the conclusion that the nuclear medium as represented by nuclei $D$, $C$, $Al$ and $Cu$ changes the $\pi - \pi$ interaction in such a way that the $\pi^0\pi^0$ invariant mass distribution from complex targets is closer to the phase-space distribution than is the same distribution from $H$. These broad features can also be found in the CHAOS data: the phase-space simulations for the $\pi A \rightarrow \pi\pi N[A-1]$ reaction reported in Fig. 9 of Ref. [3] (shaded regions) should just be compared to the $\pi^+\pi^-$ invariant mass distributions (diamonds). The higher A $\pi^+\pi^-$ results clearly match the phase space calculations better than the lower A $\pi^+\pi^-$ results do. These findings, however, do not disclose the nature of the threshold enhancement of the $\pi^+\pi^-$ invariant mass distributions. As pointed out by the CHAOS results, comparison of $M_{\pi^+\pi^-}^A$ with $M_{\pi^+\pi^+}^A$ provides useful insight into $\pi\pi$ dynamics in nuclear matter. Fig. 9 of Ref. [3] shows that the $M_{\pi^+\pi^+}^A$ distributions are accounted for well by the $\pi A \rightarrow \pi\pi N[A-1]$ phase-space from $^2H$ to $^{12}C$ to $^{40}Ca$ and to $^{208}Pb$, and the maximum invariant mass increases with the increase of the nucleon Fermi momentum (i.e., $A$), as one would expect. In contrast, the $M_{\pi^+\pi^-}^A$ strength near threshold is negligible for $^2H$, and strongly increases as $A$ increases. This enhancement is attributed to medium
Figure 1: The composite ratios $C_A^{\pi\pi}$ as a function of the $\pi\pi$ invariant mass for the $^{12}$C target. Full diamonds, the $C_C^{\pi^0\pi^0}$ distribution deduced from the data of Ref. [3] as explained in the text; open diamonds, the CHAOS $C_C^{\pi^+\pi^-}$ distribution taken from [9].

modifications in the $(\pi\pi)_{I=J=0}$ interacting system [8].

In the $I = J = 0$ channel, both the $CB$ and $CHAOS$ measurements find the $M_A^{\pi\pi}$ yield to be close to zero near the $2m_\pi$ threshold for $A = (1 \text{ or } 2$, while it increases with increasing $A$. The composite ratio $C_A^{\pi^+\pi^-}$ discussed in Refs. [8, 9] can be derived from the $CB$ data by dividing the $M_A^{\pi^0\pi^0}$ by the $M_A^{\pi^0\pi^0}$ yields. $C_A^{\pi^0\pi^0}$ for the $C$-target (the only nucleus in common to the two experiments) is shown in Fig. 1 with full diamonds. As discussed above, $C_C^{\pi^0\pi^0}$ can be directly compared with $C_C^{\pi^+\pi^-}$. Since the $M_A^{\pi^0\pi^0}$ distributions were given in arbitrary units, $C_C^{\pi^0\pi^0}$ is normalized to $C_C^{\pi^+\pi^-}$ in the energy range above 350 MeV, where the two distributions are flat. The large error bars associated with $C_C^{\pi^0\pi^0}$ reflect the poor statistical content of $M_A^{\pi^0\pi^0}$ around threshold. It is worthwhile comparing the $CB$ and $CHAOS$ ratios for $C$ since they convey relevant common features: they both are flat at $\sim 350$ MeV, are peaked at threshold and are comparable in strength. In fact the agreement between the two experiments is remarkably good. The curious depletion of the invariant mass yield for $Al$ and $Cu$ observed in Ref. [3] makes a close comparison between $C$s’ for these two targets of questionable significance. Their $C_C^{\pi^0\pi^0}$ distributions, however, resemble the $C_C^{\pi^+\pi^-}$ one.
In summary, the number of current international workshops devoted to the study of hadronic properties in nuclear matter reveals the increasing importance of this field of nuclear physics. In this realm, the correlated system of two pions in the $I = J = 0$ channel embodies a special role: $(\pi\pi)_{I=J=0}$ is the lighter object carrying the quantum numbers of the QCD vacuum. On the other hand, experimental studies of the behaviour of pion pairs in nuclear matter are fairly recent. The $CB$ measurement examined the $A$-dependence of pion-induced pion production at 408 MeV/c in the $\pi^0\pi^0$ neutral channel. An earlier study undertaken by the $CHAOS$ collaboration reported on the $\pi^+\pi^\pm$ channels at an incident pion momentum of 399 MeV/c. As far as the $(\pi\pi)_{I=J=0}$ interacting system is concerned, the $CB$ results share relevant common features with the $CHAOS$ results. When the two experiments are compared using an observable designed to mitigate acceptance differences, the results agree with each other very well, although the statistical quality of the $CB$ data in the most interesting, near-threshold region is marginal. This agreement may be interpreted as an independent confirmation by the $CB$ measurement of the nuclear matter modifications in the $I = J = 0$ $\pi\pi$ system, first reported by $CHAOS$.

References

[1] D. Davesne, Y. J. Zhang and G. Chanfray, Phys. Rev. C62, 024604 (2000); Z. Aouissat, G. Chanfray, P. Schuck, and J. Wambach, Phys. Rev. C61, 012202 (2000); D. Jido, T. Hatsuda and T. Kunihiro, Phys. Rev. D63, 011901-1 (2000); T. Hatsuda, T. Kunihiro and H. Shimizu, Phys. Rev. Lett. 63, 2840 (1999).

[2] G.R. Smith et al., Nucl. Instr. and Meth. in Phys. Res. A362, 349 (1995). Further details on $CHAOS$ can be found in the references therein quoted.

[3] A. Starostin et al., Phys. Rev. Lett. 85, 5539 (2000).

[4] F. Bonutti et al., Phys. Rev. Lett. 77, 603 (1996).

[5] F. Bonutti et al., Phys. Rev. C55, 2998 (1997).

[6] F. Bonutti et al., Nucl. Phys. A638, 729 (1998).

[7] M. Kermani et al., Phys. Rev. C58, 3419 (1998).

[8] F. Bonutti et al., Phys. Rev. C60, 018201 (1999).

[9] F. Bonutti et al., Nucl. Phys. A677, 213 (2000).

[10] D. Lohse, J.W. Durso, K. Olinde and J. Speth, Phys. Lett. B234, 235 (1990).

[11] M. J. Vicente-Vacas and E. Oset, Phys. Rev. C60, 064621 (1999).
[12] R. Rapp, J. W. Durso, Z. Aouissat G. Chanfray, O. Krehl, P. Schuck J. Speth and J. Wambach, Phys. Rev. C59, R1237 (1999).

[13] Proceedings of the international workshops on \(\sigma\)-Meson and Hadron Physics, KEK Proceedings 2000-4; and Workshop XXVIII on Gross Properties of Nuclei and Nuclear Excitations, Hirschegg (Austria); and workshop on Chiral Fluctuations in Hadronic Matter, September 26-28 2001, Paris (France).