Production of light antinuclei in heavy ion collisions

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Abstract

The antideuteron and antihelium-3 production rates at high-energy heavy ion collisions are calculated in the framework of fusion mechanism. It is supposed, that \( \bar{p}, \bar{n}, \bar{d}, \bar{^{3}He} \) participating in the fusion are moving in the mean field of other fireball constituents. It is demonstrated that at high energies, where many pions are present in the fireball, the number of produced \( \bar{d} \) and \( \bar{^{3}He} \) is determined by the balance between created and desintegrated (mainly in collisions with pions) \( \bar{d} \) and \( \bar{^{3}He} \). The explicit formulae for coalescence parameters are presented and compared with the data.

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1 Introduction

In recent experiments on heavy ion collisions the production of light antinuclei — antideuterons and antihelium-3 — was measured [1],[2],[3],[4]. Once may expect, that this process proceeds at the intermediate stage of the evolution of the fireball, created in heavy ion collisions. Because of their small binding energy antinuclei are formed at the stage, when the hadrons are already formed, the density of hadronic matter is of order of normal nuclear density, and the particle collisions are still important. We will call this stage as the "dense gas" stage of fireball evolution. It is important to have a theoretical description of antinuclei production in heavy ions collision, because the dense gas stage is evidently after the stage, where quarks and gluons transform into hadrons, but on the other hand this stage precedes the last stage of the fireball evolution (sometimes called the thermal freeze-out), when hadronic spectra are formed. The theoretical calculations of antinuclei production and their comparison with experiment can shed some light on the dense gas stage of the fireball evolution and allows one to do one step back from the final stage, about which we have direct information from experiment. Particularely, as it will be shown, the volume of the
fireball and the hadron densities at the dense gas stage can be estimated from the data on antinuclei production. It is a common belief, that the antinuclei production proceeds as a fusion process:

$$\bar{p} + \bar{n} \rightarrow \bar{d}$$  \hspace{1cm} (1)

in case of antideuterons and

$$\bar{p} + \bar{p} + \bar{n} \rightarrow \bar{3He}$$  \hspace{1cm} (2)

in case of $\bar{3He}$. According to the dominant coalescence mechanism (1,2) it is convenient to characterize $\bar{d}$ and $\bar{3He}$ production by the coalescence parameters

$$B_2 = E_{\bar{d}} \frac{d^3N_{\bar{d}}}{d^3p_{\bar{d}}} \left( E_{\bar{p}} \frac{d^3N_{\bar{p}}}{d^3p_{\bar{p}}} \right)^{-2}, \quad p_{\bar{p}} = p_{\bar{d}}/2 ,$$  \hspace{1cm} (3)

$$B_3 = E_{\bar{3He}} \frac{d^3N_{\bar{3He}}}{d^3p_{\bar{3He}}} \left( E_{\bar{p}} \frac{d^3N_{\bar{p}}}{d^3p_{\bar{p}}} \right)^{-3}, \quad p_{\bar{p}} = p_{\bar{3He}}/3 ,$$  \hspace{1cm} (4)

where one assumes $d^3N_{\bar{n}}/d^3p_{\bar{n}} = d^3N_{\bar{p}}/d^3p_{\bar{p}}$. In what follows we consider only the central heavy ions collisions.

The basic ideas of our approach are the following. We assume, that the coalescence mechanism (1,2) is the dominant one in the production of heavy nuclei. (For $\bar{d}$ production it will be shown by direct calculation, that the contribution of competing process $\bar{p} + \bar{n} \rightarrow \bar{d} + \pi$ is small, see Appendix B.) The fusion reactions (1,2) cannot proceed if all particles are on mass shell. However, in the fireball at the dense gas of its evolution, $\bar{p}$, $\bar{n}$, $\bar{d}$, $\bar{3He}$ are not on mass shell, since they interact with surrounding matter. One may consider their movement as a propagation in the mean external complex field caused by the matter. The interaction with this field leads to appearance of the mass shifts and widths of all particles propagating in the medium (or width broadening for unstable ones), analogous to refraction and attenuation indeces in the case of photon propagation. Another important ingredient of our approach is the balance of antinucleous production and desintegration rates. This balance is achieved because of large density of pions in the fireball and high rate of $\pi + (\bar{d}$ or $\bar{3He}$) collisions leading to antinucleous desintegration. The balance does not imply a statistical equilibrium, but rather a stationary process. The statistical or thermal equilibrium are not assumed in the calculation.

In most of previous investigations the production of deuterons or light nuclei was considered, but not the production of antinuclei. These processes have some common features, but also some differences.

The production of light nuclei in heavy ion collisions has been studied for many years (for an early review see [5]). The first calculation of the deuteron production rate was performed in [6] within simple model of optical potential in nuclei, resposible for the deuteron formation. In [7] it was proposed that the proton and neutron bind together if their relative momentum is less than some $p_0$, a phenomenological parameter to be determined from experiments. Then $B_2 = \frac{2}{E_p} \cdot \frac{2}{3} \cdot \frac{4\pi}{3} p_0^3$ is proportional to the propability to find the proton and neutron inside the sphere of radius $p_0$ in momentum space. A thermodynamical approach to the deuteron formation was applied in [8], where the deuterons were assumed to be in thermodynamical equilibrium with other nucleons in the fireball.
In [9] the sudden approximation of quantum mechanics was first applied to the light nuclei formation. It implies short transition from the high-density stage $|i\rangle$ consisting of the protons and neutrons only, to the low-density stage $|f\rangle$, consisting of the deuterons as well. The amplitude of the deuteron formation is given by the overlap of the wave functions $\langle f|i \rangle$. Under simplifying assumptions that these states are free moving particles, uniformly distributed in a box of volume $V$ and neglecting the deuteron size, one obtains the coalescence parameter:

$$B_2 = \frac{2}{E_p} \cdot \frac{3}{4} \cdot \frac{(2\pi)^3}{V}$$

where $3/4$ is the ratio of the spin weights, the factor $2/E_p$ comes from invariant definition of the phase space element $d^3p/E$.

Most of investigations of the light cluster formation in heavy ion collisions represent extensions of the sudden approximation result for variously prepared initial state (source). Among them are different source geometries [10], [11], expanding systems [12], intranuclear cascade model [13], diagrammatic approach [14], relativistic quantum molecular dynamics model [15], Boltzmann-Uehling-Uhlenbeck transport model [16]. In most cases the results are based on computer simulations and analytical expressions are unavailable.

A novel phenomenon is observed if one tries to go beyond the sudden approximation. In [17] the deuteron formation was considered in the time-dependent perturbation theory. If the interaction, responsible for the deuteron formation, is switched on during some finite time $\tau$, the coalescence parameter gets multiplied on additional factor $\sim (\epsilon \tau)^{1/2}$, where $\epsilon$ is the deuteron binding energy. Exact value of $\tau$ is not determined; as argued in [17], it should be of order of colliding nuclei radius, so $(\epsilon \tau)^{1/2} \sim 1$.

In all mentioned above approaches the interaction of the deuterons with the fireball environment was not accounted. As it will be shown below, this interaction, especially with pions, plays an essential role. As a consequence the coalescence parameter acquires the factor $(\epsilon/\Gamma)^{1/2}$, where $\Gamma$ is the deuteron in-medium width, which is calculated here. According to our calculation $(\epsilon/\Gamma)^{1/2} \sim 1/10$.

Another transport model of the deuteron formation in nuclear collisions is available in literature [18, 19]. It was assumed, that the deuteron formation and desintegration goes mainly via triple nucleon reaction $NNN \leftrightarrow dN$. At low (nonrelativistic) energies the reaction rates were calculated using impulse (Born) approximation with the help of Faddeev equation. Again, the thermodynamical equilibrium was assumed in the calculations. This approach also does not account the deuteron desintegration by the pions.

It must be stressed that we apply our approach only to the process of antideuteron (or antihelium) formation in high-energy central heavy ion collisions. In this case, prior to antinuclei formation, the hadrons experience at least few collisions with each other and the concept of the hot dense gas seems reasonable to apply. Although the approach is valid for the deuteron (or helium) formation processes, the results should be used with care: a part of the outgoing deuterons may be just some fragments of primary colliding nuclei. Is is rather difficult to separate this part from the deuterons, synthesized from the hot hadronic gas, at least some experimental cuts should be applied. We will not consider this point here, restricting ourselves only by antinuclei processes.

The material of the paper is presented as follows. In Section 2 the effective low energy Lagrangian for fusion processes (1,2) is constructed. The value of effective coupling constant
is found by two ways: 1) by considering the elastic scattering amplitude $\bar{p} + \bar{n} \rightarrow \bar{p} + \bar{n}$ (in case of $\bar{d}$ production); 2) by consideration of $\bar{d}$ or $^3\overline{He}$ polarization operators, on the base of the field theory. With the help of effective Lagrangian the cross sections of fusion reactions are calculated. In Section 3 the transport equation for antinucleous production and propagation in the fireball is formulated. The formulae for the widths and mass shifts of particles, moving in the medium are presented and the formation rates of $\bar{d}$ and $^3\overline{He}$ are calculated in terms of effective $\bar{p}, \bar{n}, \bar{d}, ^3\overline{He}$ widths in the medium. In Section 4 the balance conditions for $\bar{d}$ and $^3\overline{He}$ production are formulated and the explicit formulae for coalescence parameters (3), (4) are presented. Section 5 is devoted to comparison with experimental data. The model of pion and nucleon densities at the dense gas stage of the fireball evolution is formulated. With the help of this model the values of the widths were calculated and it is demonstrated, that all assumptions, used in the calculation are fulfilled. The coalescence parameters are calculated numerically for experimental conditions and compared with the data. Section 6 presents our conclusion. The details of the calculation are given in the Appendices.

2 Effective low energy Lagrangians and fusion cross sections.

At first let us consider the antideuteron production. Evidently, $\sigma_{\bar{p}\bar{n}\rightarrow \bar{d}} = \sigma_{p+n\rightarrow d}$. The low energy effective interaction Lagrangian has the form

$$L = g \psi_p \psi_n \varphi_d^+,$$

where $\psi_p, \psi_n$ and $\varphi_d$ are nonrelativistic $p, n$ and $d$ wave functions and spin effects are neglected. Let us temporarily suppose, that $pn\bar{d}$ interaction is point-like, i.e. neglect the nuclear force radius $r_0$. Following Landau [20], calculate the forward $p + n \rightarrow p + n$ elastic scattering amplitude. In this approximation it corresponds to Fig.1 diagram. According to standard rules the relativistic amplitude is given by:

$$M = \frac{-g^2}{s - m_d^2} \approx \frac{-g^2}{4m(E + \varepsilon)}$$

where $s = (p_p + p_n)^2$, $m$ is nucleon mass, $\varepsilon = m_p + m_n - m_d = 2.225 \text{MeV}$ is the deuteron binding energy, $E$ is the c.m. energy, $\sqrt{s} = m_p + m_n + E$.

On the other hand, the nonrelativistic $s$-wave amplitude $f$ of elastic scattering according to Bethe-Peierls [21] theory is equal

$$f_0 = \frac{1}{k(\cot \delta_0 - i)} \approx \frac{i}{k - i\alpha},$$
where $\alpha = \sqrt{m\varepsilon}$, $k$ is the proton (or neutron) momentum in c.m. frame. At the deuteron pole, $s \to m_d^2$, $k \to i\alpha$, the amplitudes (6) and (7) must be equal (keeping in mind the different normalizations $M = 8\pi\sqrt{s}f$). This requirement determines the value of the effective coupling constant [20]:

$$g^2 = 128\pi m\sqrt{m\varepsilon}$$

(8)

It must be stressed, that $g^2 \to 0$ at $\varepsilon \to 0$, which means that the fusion rate goes to zero at $\varepsilon \to 0$. Physically this is natural: the probability of production of large size antideuteron in heavy ion collision decreases with increasing of its size. It is an easy task to account for the finite value of nuclear force radius $r_0$, see [22] and Appendix A. As a result, the forward scattering amplitude $f_0$ at the pole $k = i\alpha$ gets an additional factor $(1 - \alpha r_0)^{-1} \approx 1.67$. With account of this correction $g^2$ becomes equal:

$$g^2 = 128\pi m\sqrt{m\varepsilon} (1 - \alpha r_0)^{-1}$$

(9)

The invariant cross section for the fusion process $pn \to d$ is found by common rules:

$$E_d \frac{d^3\sigma_{pn \to d}}{d^3p_d} = \frac{3}{4} \cdot \frac{\pi g^2}{4\sqrt{(p_n p_p)^2 - m^4}} \delta^4(p_p + p_n - p_d),$$

(10)

where $E_p, E_n$ and $E_d$ are $p, n$ and $d$ total energies, $3/4$ is the ratio of the spin weights. In fact, the cross-section (10) is valid only for low (nonrelativistic) cm energies. At high energies excited states are produced (for instance $N\Delta$ resonance) which may decay into the antideuteron. Here we do not consider these rather complicated (in medium) processes, assuming they are small enough.

Let us now derive the expression for the effective coupling constant $g^2$ (in the limit $r_0 \to 0$) by another method – by consideration of the deuteron Green function and polarization operator in field theory. This approach, of course, gives the same result as above in case of deuteron, but it will be useful in consideration of $^3He$ production, where the scattering theory is rather complicated.

For nonrenormalized deuteron Green function $D(p^2)$ we have the Schwinger-Dyson equation:

$$[p^2 - m_{d,0}^2 - \Pi(p^2)] D(p^2) = 1,$$

(11)

where $m_{d,0}^2$ is deuteron bare mass. After mass renormalization we get

$$\{p^2 - m_{d,0}^2 - \Pi(m_{d}^2) - [\Pi(p^2) - \Pi(m_{d}^2)]\} D(p^2) = 1,$$

$$m_{d,0}^2 + \Pi(m_d^2) = m_d^2,$$

(12)

where $m_d$ is the physical deuteron mass. Then we perform the Green function renormalization $D(p^2) = Z_2 D_{ren}(p^2)$ so that

$$D_{ren}(p^2)_{p^2 \to m_d^2} \to \frac{1}{p^2 - m_d^2}$$

(13)

The main assumption in this approach, used in derivation of the Bethe-Peierls equation (7), is: vertex corrections and corrections to nucleon propagators are neglected, $d$ is assumed to be the bound state of $p$ and $n$. The polarization operator is contributed only by the diagram
of Fig. 2a) and we can write $\Pi = g_0^2 \bar{\Pi}$, where $g_0^2$ is the nonrenormalized coupling constant, related to renormalized one $g^2$ as:

$$g_0^2 = Z_2^{-1} g^2.$$  

(14)

Representing $\Pi$ in (12) by dispersion relation we get

$$Z_2 \left\{ (p^2 - m_d^2) + \frac{g^2}{Z_2} \frac{1}{\pi} \int ds \frac{\text{Im} \bar{\Pi}(s)}{(s - m_d^2)(s - p^2)} \right\} D_{ren}(p^2) = 1$$  

(15)

The condition of (13) results in:

$$Z_2 + \frac{g^2}{\pi} \int ds \frac{\text{Im} \bar{\Pi}(s)}{(s - m_d^2)^2} = 1$$  

(16)

If deuteron is the bound state of proton and neutron, then the contribution of bare deuteron to Schwinger-Dyson equation vanishes and $Z_2 = 0$. Putting $Z_2 = 0$ in (16) we get the equation for $g^2$:

$$\frac{g^2}{\pi} \int ds \frac{\text{Im} \bar{\Pi}(s)}{(s - m_d^2)^2} = 1$$  

(17)

$\text{Im} \bar{\Pi}(s)$ can be easily calculated for Fig. 2a) diagram (nonrelativistic approximation can be used). As a result the same expression for $g^2$ (8) as in the first method is obtained.

Let us turn now to the study of $^3\text{He}$ production. The effective nonrelativistic Lagrangian has the form

$$L = G_0 \psi_p \psi_p \psi_n \psi^n_{He}$$  

(18)

The derivation, performed above in the second approach, can be repeated and eq. (17) with substitution $g^2 \rightarrow G^2$, $m_d \rightarrow m_{He}$ is found. The $\text{Im} \bar{\Pi}(s)$ is described now by the diagram of Fig. 2b). There is, however, the essential difference in calculation of the Fig. 2b) diagram contribution to (17) in comparison with that of Fig. 2a): the integral in (17) is linearly ultraviolet divergent in nonrelativistic approximation. (This divergence still persists in relativistic calculations.) This circumstance physically corresponds to the well known fact, that in nuclear physics the 3-body problem with $\delta$-function potential cannot be correctly formulated: it is necessary to have an additional information about the interaction at small distances. For our purposes it is sufficient to have a rather crude estimation of $G^2$. So, we put ultraviolet cut-off in the integral in (17). The calculation gives:

$$G^2 = 36\sqrt{3} (4\pi)^3 \frac{m}{\Lambda}$$  

(19)

In numerical calculations $\Lambda = 300 \text{ MeV}$ will be taken.
3 Transport equation. Calculation of collisions integral.

As mentioned in the Introduction, we suppose, that antideuterons and antihelium-3 are formed at the dense gas stage of the fireball evolution, which follows after "chemical freeze-out" stage [23, 24]. Let us assume, that particle propagations at this stage can be described by means of classic transport (kinetic) equations.

For definiteness, consider first the \( \bar{d} \)-production. We will use the notation \( q_i(x, p) \), \( i = \bar{p}, \bar{n}, \bar{d}, \pi, \ldots \) for the double densities in coordinate and momentum spaces and \( n_i(x) = \int q_i(x, p) \, d^3p \) for the coordinate densities. (\( q_i(x, p) \) are Lorentz invariant.) Let us choose the c.m. frame of colliding ions. The transport equation for \( \bar{d} \) has the form:

\[
\frac{m_d}{E_d} \frac{\partial q_{\bar{d}}(p_{\bar{d}}, x)}{\partial x_\mu} u_\mu^d = \frac{\partial q_{\bar{d}}}{\partial t} + \mathbf{v}_{\bar{d}} \nabla q_{\bar{d}} = \int d^3p_{\bar{p}} \int d^3p_{\bar{n}} \, q_{\bar{p}}(p_{\bar{p}}) q_{\bar{n}}(p_{\bar{n}}) \omega_{\bar{p}\bar{n}\rightarrow \bar{d}} - q_{\bar{d}}(p_{\bar{d}}) \sum_i \int d^3p_i \, q_i(p_i) \omega_{\bar{d}i\rightarrow X} \tag{20}
\]

where \( u_\mu^d = (1, \mathbf{v}_{\bar{d}})/\sqrt{1 - v_{\bar{d}}^2} \) is antideuteron 4-velocity, \( \omega_{\bar{p}\bar{n}\rightarrow \bar{d}} \) is the fusion reaction probability, proportional to differential cross section

\[
\omega_{\bar{p}\bar{n}\rightarrow \bar{d}} = \frac{\sqrt{(p_{\bar{p}}p_{\bar{n}})^2 - m_{\bar{d}}^4}}{E_{\bar{p}}E_{\bar{n}}} \frac{d^3\sigma_{\bar{p}\bar{n}\rightarrow \bar{d}}}{d^3p_{\bar{d}}} \tag{21}
\]

and similarly for the desintegration process \( \bar{d}i \rightarrow X \) due to collisions of \( \bar{d} \) with \( i \)-th constituent of the fireball (\( i = \pi, K, p, n, \) etc.). The terms, where \( \bar{d} \) appears in the momentum interval \( p_{\bar{d}}, p_{\bar{d}} + \Delta p_{\bar{d}} \) as a result of elastic collisions are neglected. The cross section \( \sigma_{\bar{p}\bar{n}\rightarrow \bar{d}} \) is defined by (10), the cross sections of \( \bar{d} \) collisions with fireball constituents (\( \sigma_{\pi\bar{d}}, \ldots \)) can be found from the experimental data. Necessary applicability conditions of (20) is that particles wave lengths \( \lambda_i = \frac{p_i}{m} \) should be much less, than the mean distances \( d \) between fireball constituents, \( \lambda_i \ll d \).

At the dense gas stage of the fireball evolution all particles inside the fireball should be considered as moving in the mean field of other fireball constituents. As a consequence, the masses are shifted in comparison with their vacuum values. Similarly, due to interaction with medium constituents, the widths \( \Gamma_i \) appear (or width broadening, if the particle has its proper widths). The mass shift \( \Delta m(E) \) and width \( \Gamma(E) \) are expressed in terms of the forward scattering amplitude \( f(E) \) of the particle on \( i \)-th medium constituents (see[25],[26] and references therein)

\[
\Delta m(E) = -2\pi \frac{n_i}{m} Re f_i(E) \quad \Gamma(E) = 4\pi \frac{n_i}{m} Im f_i(E) = \frac{n_i p_i}{m} \sigma_i(E), \tag{22}
\]

where \( E, p \) and \( m \) are particle energy, momentum and mass, \( n_i \) is the density of \( i \)-th constituent in medium. Eq's.(22) take place in the system, where the constituents are at rest. In the case of moving constituents the corresponding Lorentz boost must be performed. (By
definition $\Delta m$ and $\Gamma$ are Lorentz invariant, for details see [27]). The relations (22) must be summed over all fireball constituents.

The applicability conditions of eq. (22) are the following [25, 26]:

i). $\lambda \ll d$

ii). $|f| \ll d$

iii). The main part of the scattering proceeds at small angles, $\theta \ll 1$.

The conditions i) and iii) are well satisfied in the cases under consideration. The condition ii) is fulfilled not quite well and some corrections should be taken into account (see below).

Therefore, $\bar{p}, \bar{n}$ and $\bar{d}$ can be considered as Breit-Wigner resonances with varying masses distributed according to Breit-Wigner formula. In the process of the fireball expansion these Breit-Wigner resonances smoothly evolve to their stable counterparts. So, we substitute (10) in the first term in the right-hand side of eq. (20) and integrate over the masses $m'$ of Breit-Wigner resonances. Using the notation $I/E_{\bar{d}}$ for this term we get:

$$\frac{I}{E_{\bar{d}}} = \int \frac{\Gamma_{\bar{p}}/2\pi}{(m_{\bar{p}} - m_{\bar{n}})^2 + \Gamma_{\bar{p}}^2/4} \frac{\Gamma_{\bar{n}}/2\pi}{(m_{\bar{n}} - m_{\bar{n}}')^2 + \Gamma_{\bar{n}}^2/4} \frac{\bar{\Gamma}_{\bar{d}}/2\pi}{(m_{\bar{d}}' - m_{\bar{d}})^2 + \bar{\Gamma}_{\bar{d}}^2/4}$$

$$\times \frac{3\pi}{16} \frac{g^2}{E_{\bar{d}}'} \int \frac{d^3p_{\bar{p}}}{E_{\bar{p}}'} \frac{d^3p_{\bar{n}}}{E_{\bar{n}}'} \frac{q_{\bar{p}}(p_{\bar{p}}) q_{\bar{n}}(p_{\bar{n}})}{E_{\bar{p}}'} \delta^3(p_{\bar{p}} + p_{\bar{n}} - p_{\bar{d}}) \delta(E_{\bar{p}}' + E_{\bar{n}}' - E_{\bar{d}}')$$

(23)

where $E_{\bar{p}}' = \sqrt{m_{\bar{p}}^2 + p_{\bar{p}}^2}$ etc. Assume, that $\Gamma_{\bar{p},\bar{n}}$ is much smaller than $m$, and the variation of $q_{\bar{p},\bar{n}}(p)$ on the interval $\Gamma$ is also small enough, $\Gamma dq/dp \ll q$. (In fact, $\Gamma_{\bar{p}} = \Gamma_{\bar{n}} \approx 200$ MeV, see below and $\Delta m \approx 30$ MeV.) Then the distributions $q_{\bar{p}}(p_{\bar{p}}) = q_{\bar{n}}(p_{\bar{n}})$ can be taken out from the integral sign at the values $p_{\bar{p}} = p_{\bar{n}} = p_{\bar{d}}/2$. With a good accuracy we can put $\Gamma_{\bar{p}} = \Gamma_{\bar{n}} \equiv \Gamma$. But $\bar{\Gamma}_{\bar{d}}$ in (23) generally is not equal to the antideuteron in-medium width $\Gamma_{\bar{d}} \approx 2\Gamma$. The reason is that $\pi(n)$ system with $\bar{d}$ quantum numbers at high excitations does not necessarily evolve to $\bar{d}$ in the process of the fireball expansion, but can decay in other ways. One may expect $\bar{\Gamma}_{\bar{d}} < \Gamma_{\bar{d}}$. We keep the ratio $a = \bar{\Gamma}_{\bar{d}}/\Gamma_{\bar{d}}$ as a free parameter in the calculation. As can be seen below, the results weakly depend in this ratio. The calculation of the integral $I$ is straightforward and leads to

$$I = \frac{3\pi^2}{8} g^2 \sqrt{\frac{\Gamma(1 + a)}{2m}} q_{\bar{p}}^2(p_{\bar{p}})$$

(24)

The contributions of direct processes $\bar{p} + \bar{p} \rightarrow \bar{d} + \pi^-, \bar{n} + \bar{n} \rightarrow \bar{d} + \pi^+ \bar{p} + \bar{n} \rightarrow \bar{d} + \pi^0$ are small, they comprise not more than 20% altogether and may be neglected within the accuracy of our calculation, see Appendix B.

We restrict ourselves to consideration of low and intermediate transverse momentum $p_{t,\perp} \lesssim 1$ GeV. At higher $p_{t,\perp}$ many phenomena go into the play: radial and elliptic flow, steep decreasing of spectrum with increasing of $p_{t,\perp}$ etc. The consideration of these effects requires more refined treatment of the problem.

Turn now to the calculation of the second term in the r.h.s. of the transport equation (20), corresponding to $\bar{d}$ desintegration rate. The probability $\omega_{\bar{d} \rightarrow X}$ is proportional to the
total cross section $\sigma_{\bar{d}i \to X}$ in the same way as (21). So one may write the disintegration term in (20) as:

$$\sum_i \int d^3p_i q_i(p_i) \omega_{\bar{d}i \to X} = \frac{m_d}{E_{\bar{d}}} \Gamma_{\bar{d}}$$

(25)

where $\Gamma_{\bar{d}}$ has an obvious interpretation of the antideuteron width. It is equal to:

$$\Gamma_{\bar{d}} = \frac{1}{m_d} \sum_i \int \frac{d^3p_i}{E_i} q_i(p_i) \sqrt{(p_{\bar{d}}p_i)^2 - m_d^2 m_i^2} \sigma_{\bar{d}i \to X}$$

(26)

It is Lorentz invariant generalization of the width (22) and depends on the momentum $p_{\bar{d}}$ of the antideuteron, moving in medium. The process of elastic $\pi\bar{d}$ scattering was neglected in transport equation.

It must be mentioned, that eq. (25) corresponds to gaseous approximation, when the screening corrections are not accounted. In fact, the pion densities in the fireball at SPS or RHIC energies are such, that the account of screening corrections is necessary. Such calculation, based on Glauber theory, is presented in Appendix C. As a result, it was found that the Glauber correction at SPS or RHIC conditions reduces $\Gamma_{\pi}$ approximately by 30%.

### 4 The balance condition. The formula for coalescence parameter for $\bar{d}$. 

Suppose, that the rate of antideuteron collisions with other constituents of the fireball resulting in antideuteron desintegration is much larger, than the rate of the fireball expansion. This happens in case of heavy nucleonic collisions at high energies, when the fireball size at the dense gas stage is large, because of large number of produced pions per nucleon. The large relative pionic density in the fireball leads to high desintegration rate of antideuterons in collisions with pions. (The collisions with nucleons are less essential in $\bar{d}$ desintegration.) In this case one may expect a balance: the antideuteron production rate (first term in the r.h.s. of (20)) is equal to its desintegration rate (second term in r.h.s. of (20)). The balance condition determines $\bar{d}$-density:

$$q_{\bar{d}}(p_{\bar{d}}) = \frac{I}{\Gamma_{\bar{d}} m_d} = \frac{3\pi^2 g^2}{32 m} \sqrt{\frac{1 + a}{\Gamma m}} q^2(p_{\bar{d}})$$

(27)

The momentum distribution $d^3N_{\bar{d}}/d^3p_{\bar{d}}$ entering in (3) is obtained from (25) by integration over the fireball volume

$$\frac{d^3N_{\bar{d}}(p_{\bar{d}})}{d^3p_{\bar{d}}} = \int d^3x q_{\bar{d}}(p_{\bar{d}}, x)$$

(28)

From equations (3),(27),(28) and (9) we find the coalescence parameter

$$B_{2}^{th} = \frac{24\pi^3}{E_{\bar{d}}} \times 1.67 \sqrt{\frac{\varepsilon(1 + a)}{2\Gamma}} \frac{\int d^3x q_{\bar{d}}^2(p_{\bar{d}}, x)}{[\int d^3x q_{\bar{d}}(p_{\bar{d}}, x)]^2}$$

(29)

Since the $x$-dependence of $q_{\bar{d}}(p_{\bar{d}}, x)$ is not known, we replace (29) by:

$$B_{2}^{th} = \frac{24\pi^3}{E_{\bar{d}}} \times 1.67 \sqrt{\frac{\varepsilon(1 + a)}{2\Gamma}} \frac{2}{V} \frac{n_{\bar{d}}^2}{(n_{\bar{d}})^2}$$

(30)
where $V$ is the fireball volume, $\bar{n}_p$ and $\bar{n}_p^2$ are the mean and mean square $\bar{p}$ densities in the fireball. (The coordinate dependence of $\sqrt{\Gamma}$ is neglected). $B_2^{th}$ is Lorentz invariant, as it should be. The volume $V$ in (30) can be understood as a mean value of the fireball volume at the stage, where on the one hand, the hadrons are already formed, i.e. the mean distances between them are larger than the confinement radius $R_c \sim 1/m_\rho \sim (1/4)\text{ fm}$, but on the other hand, hadron interactions are still essential and the mean distances between the antinucleons are of order or larger than the deuteron size, so $\bar{d}$ could be formed in $\bar{p}\bar{n}$ collisions. The antinucleon distributions $n_\bar{p}(r), n_\bar{n}(r)$ inside the fireball are essentially nonuniform: at the stage of the fireball evolution, preceding the dense gas stage, antinucleons strongly annihilated in the internal part of the fireball and in much less extent in its external layer of the thickness of order $\bar{p}(\bar{n})$ annihilation length $l_{ann}$. For this reason $\bar{n}_p^2/\bar{n}_p^2$ may be essentially larger than 1. For the same reason the antinucleons and antideuterons from the backside of the fireball (with respect to the observer) are absorbed in the fireball and cannot reach detector, see Fig. 3. Therefore, only one half of the fireball volume contributes to the number of registered $\bar{p}, \bar{n}$ and $\bar{d}$. The corresponding factor approximately equal to 2 is accounted in (29).

The width $\Gamma$ can be calculated in one or another model of fireball evolution. However, since it enters in the expression for the coalescence parameter $B_2^{th}$ (30) as $\sqrt{\Gamma}$, it influences $B_2^{th}$ not significantly. The same remark refers to not quite certain parameter $a$. Therefore, the comparison with the data allows one to find the most essential parameter $V(\bar{n}_\rho)^2/\bar{n}_p^2$, which would make it possible to check various models of the fireball evolution.

5 The model of dense gas stage of fireball evolution.

We accept the following model for the dense gas stage of fireball evolution [26]. (A related model had been suggested long ago [28, 29]: it may be called Fermi–Pomeranzuk model). Neglect for a moment contributions of all particles except for nucleons and pions. Assume that at dense gas stage any participant – nucleon or pion occupies the volume $v_N$ or $v_\pi$, respectively. Then

$$n_N = \frac{N_N}{V} = \frac{n_N^0}{1 + Q_\pi \beta}, \quad n_\pi = \frac{N_\pi}{V} = \frac{n_\pi^0 Q_\pi}{1 + Q_\pi \beta}$$

(31)

where $n_N^0 = 1/v_N$, $\beta = v_\pi/v_N$, $Q_\pi = N_\pi/N_N$. For numerical estimations we take $\beta = (r_\pi/r_N)^3 = 0.55$, where $r_\pi = 0.66\text{ fm}$ and $r_N = 0.81\text{ fm}$ are pion and nucleon electric radii.
Table 1: Experiments, where antinuclei were observed

| Experiment   | NA44 [1] | STAR [2] | E684 [3] |
|--------------|----------|----------|----------|
| √s/A, GeV    | 17.4     | 130      | 4.8      |
| p_{pt}, GeV  | 0.55     | 0.33     | 0.17     |
| E_{cm}, GeV  | 1.5      | 1.05     | 0.99     |
| # of "wounded" nucleons | 362 [31, 32] | 320 [33] | 350 [34] |
| Q_{π}        | 5.2 [31, 32] | 7 ± 1 [35] | 1.6 [34] |

We take n_{0}^{N} as a parameter varying in the interval from n_{0}^{N} = 0.17 fm^{-3} (normal nuclear density) to 0.30 fm^{-3}.

Check first the applicability of our approach. Choose n_{0}^{N} = 0.24 fm^{-3} and Q_{π} = 5.2. (The latter value was found at NA44 experiment [30] at SPS.) We have n_{N} = 0.062 fm^{-3}, n_{π} = 0.325 fm^{-3}, n = n_{N} + n_{π} = 0.39 fm^{-3} and the mean distances between the fireball constituents is d = 1/n^{1/3} = 1.4 fm. Evidently, the conditions \( \lambda_{π} = 1/p_{π} \ll d \) and \( R_{c} \ll d \) are well satisfied. Check now the balance conditions that the probability of deuteron desintegration exceeds the fireball expansion rate. The former is given by \( 2\Gamma(m/E_{π}) \). In calculation of \( \Gamma \) we assume, that the hadronic spectra at the dense gas stage of fireball evolution differ not too much from the spectra at the final (freeze-out) stage. Since, as was explained above, rather crude estimation of \( \Gamma \) is sufficient for our purposes, we believe, that such approximations is satisfactory. Using (25) and the pion spectrum, presented in [32], \( \Gamma \) was found, \( \Gamma \approx 200 \) MeV (Glauber correction was accounted). The estimation for the fireball expansion is \( w \approx (1/5) \) fm\(^{-1} \). We have: \( 2\Gamma(m/E_{π}) \approx 1.4 \) fm\(^{-1} \gg 0.2 \) fm\(^{-1} \). \( (E_{π} \approx 1.5 \) GeV at NA44 experiment [1]). So, this condition is also fulfilled. Finally, check the condition ii) of Section 3, which is equivalent to the requirement \( Im f \ll d \). Before taking into account the Glauber correction \( Im f \approx 1 \) fm and this condition is not well satisfied. After accounting 30% Glauber correction the situation improves, but not too much. For this reason the values of \( \Gamma \), presented above, have a large (may be 50%) uncertainty. Since \( \sqrt{\Gamma} \) enters (30), the error in \( B_{2}^{th} \) reduces to 25%.

6 Comparison with data on \( \bar{d} \)-production.

The antideuteron production in heavy ion collisions was observed in NA44 experiment at CERN in \( Pb + Pb \) collisions at \( \sqrt{s} = 17.4 A \) GeV [1], STAR experiment at RHIC in \( Au + Au \) collisions at \( \sqrt{s} = 130 A \) GeV [2] and E864 experiment at AGS in \( Au + Pb \) collisions at \( \sqrt{s} = 4.8 A \) GeV [3]. The data of these experiments are presented in Table 1.

Table 2 gives the values of coalescence parameters \( B_{2}^{exp} \), measured in these experiments and relevant centralities. It must be mentioned that \( B_{2}^{exp} \) rather strongly depends on the centrality. For example in NA44 experiment the results for 0–5% centrality are about 1.5 times lower. Also, \( B_{2}^{exp} \) has significant \( p_{pt} \) dependence: such dependence was observed at PHENIX experiment at RHIC (we do not analyse PHENIX data here, since only preliminary results were published up to date [4]).

Table 2 presents also the results of our calculation for the values of \( n_{0}^{N} \), introduced in (31): \( n_{0}^{N} = 0.17, 0.24, 0.30 \) fm\(^{-3} \). In the calculation of mean volume \( V \) at the dense gas
stage the number of "wounded" nucleons and the number of pions were taken from Table 1, the corrections 15%, 20% and 10% were accounted for other particles, except for nucleons and pions, in cases of NA44, STAR and E684 experiments respectively. The antiproton annihilation length was estimated as $\bar{l}_{\bar{p},\text{ann}} \approx 3$ fm in case of NA44 and STAR experiments and $\bar{l}_{\bar{p},\text{ann}} \approx 1.5$ fm in case of E684. In all cases we can put with sufficient accuracy $n^2/\bar{n}^2 \approx 2$ and $a = 1/2$.

The calculated values of the coalescence parameters $B_2^{th}$ are presented in Table 2. It must be mentioned, that in case of E684 experiment the validity conditions of our approach are on the edge of their applicability. So, the theoretical expectations for $B_2$ in this case are valid only by the order of magnitude. In two other cases the agreement of the theory with the experiment is quite satisfactory. Let us remark, that the main uncertainty arises from the effective width $\Gamma$ which, probably, is known with an accuracy of order 50%. Taking in mind all other possible uncertainties, we believe that the accuracy of theoretical predictions for $B_2$ is also about 50%. The inspection of the Table 2 shows, that the total hadronic density (pions+nucleons) at the dense gas stage of fireball expansion, where the antideuterons are formed, is of order $n_N + n_\pi \sim 0.4$ fm$^{-3}$. Much lower or much higher densities would lead to contradiction with experiment.

Using the pion and nucleon densities and spectra chosen above, we can estimate the typical energy density at the dense gas stage: $\epsilon \sim 0.3 - 0.5$ GeV/fm$^3$. As the colliding ions are initially strongly Lorentz-compressed and the produced fireball expands in the longitudinal direction with an almost velocity of light, it takes approximately $R \approx 10$ fm to reach the proposed volume. It is instructive to put these values on the Fig. 27 from [36], where the density of energy/time evolution of the fireball is represented. We see that the dense gas stage begins just after the moment when the total hadronization happened and this is a good check for self-consistency of our assumptions.
7 Production of $^3\overline{He}$.

The calculation of $^3\overline{He}$ production proceeds along the same lines as $\overline{d}$. The only difference is that now the collision integral $I$ – the first term in the r.h.s. of eq. (20) corresponds to the formation of $^3\overline{He}$ in collision of three antinucleons: $\overline{p}_1, \overline{p}_2$ and $\overline{n}$ and the effective coupling constant is given by (19). Instead of Breit-Wigner off-shell mass distributions for antiparticles $\overline{p}_1, \overline{p}_2, \overline{n}$ and $^3\overline{He}$ we now take the Gaussian ones:

$$f_i(m'_i) = \frac{2}{\sqrt{\pi} \Gamma_i} e^{-(m'_i-m_i)^2/(\Gamma/2)^2}$$  \hspace{1cm} (32)

The reason is that the Breit-Wigner distributions do not provide the necessary convergence of collision integral. The collision integral takes the form:

$$I_{^3\overline{He}} = \frac{\pi G^2}{64 E_{^3\overline{He}}} \int \left\{ \prod_{i=1,2,3} dm'_i \right\} \int \frac{d^3p_1}{E'_{p_1}} \frac{d^3p_2}{E'_{p_2}} \frac{d^3p_n}{E'_{p_n}} q_p(p_1) q_p(p_2) q_n(p_n) \times \delta^3(p_1 + p_2 + p_n - p_{^3\overline{He}}) \delta(E'_{p_1} + E'_{p_2} + E'_{p_n} - E'_{^3\overline{He}})$$  \hspace{1cm} (33)

(The distribution of $^3\overline{He}$ was taken as a δ-function, what is equivalent to $a = \tilde{\Gamma}_{^3\overline{He}}/(3\Gamma) \ll 1$). The integral is equal to:

$$I = \frac{\pi^4}{96\sqrt{6}} G^2 \Gamma^2 q^3(p)$$  \hspace{1cm} (34)

Applying the balance condition and (19) we find the coalescence parameter:

$$B_3 = 96\pi^7 \frac{1}{\sqrt{2}} \frac{\Gamma}{\Lambda} \frac{1}{V^2} \frac{n^3}{\overline{n}^3}$$  \hspace{1cm} (35)

The accuracy of calculation of $B_3$ is lower, that in case of $B_2$ since additional uncertainties appear: strong dependence on ultraviolet cut-off Λ etc. So we may pretend only on the estimation of $B_3$, correct up to order of magnitude (in the best case up to factor of 2). The coalescence parameter for $^3\overline{He}$ production in Au + Au collisions at $\sqrt{s} = 130A$ GeV was measured by STAR Collaboration [2]. In this experiment the centrality was up to 18%, $^3\overline{He}$ transverse momentum $1.0 < p_{\perp,^3\overline{He}} < 5.0$ GeV and rapidity $|y| < 0.8$. These limitations corresponds to the average antiproton energy $E_{\bar{p}} \approx 1.4$ GeV. STAR found:

$$B_{^3\overline{He}}^{exp} = (2.1 \pm 0.6 \pm 0.6) \times 10^{-7} \text{ GeV}^4$$  \hspace{1cm} (36)

The theoretical value according to (35) at $n_N^0 = 0.24 \text{ fm}^{-3}$, $\overline{n}^3/\overline{n}^3 = 3$, and $\Lambda = 300$ MeV is:

$$B_{^3\overline{He}} = 3.3 \times 10^{-7} \text{ GeV}^4$$  \hspace{1cm} (37)

The agreement with experiment is good, despite of many theoretical uncertainties. It demonstrates the validity of basic ideas of theoretical approach.
8 Summary and conclusion.

The coalescence parameters for production of antideutrons and antihelium-3 in heavy ion collisions were calculated. The obtained results are based on three assumptions: i) the main mechanism of light antinucleous production is coalescence (fusion) mechanism — eq. (1,2); ii) all particles, participating in fusion process are moving in the mean field of other fireball constituents; iii) the number of produced antinucleous is determined by the balance conditions: the equality of produced and desintegrated — mainly by pions — antinuclei. The production of antinucleous proceeds at the dense gas stage of the fireball evolution, when the hadrons are already formed, but their interaction is still important. Statistical or thermal equilibrium are not supposed at the dense gas stage. In fact, the final results depend on one parameter — the volume of the fireball at this stage (or, equivalently, on the hadron densities.) Good agreement with experimental data for coalescence parameters was obtained for experiments at CERN, RHIC and AGS for the values of $n_N^0$, defined by eq. (31), $n_N^0 \sim 0.17 \div 0.30 \text{ fm}^{-3}$, close to normal nucleous density. Much lower, or much higher values of $n_N^0$ lead the to values of coalescence parameters, incompatible with the data. It must be stressed, that the same values of $n_N^0$ well describe the coalescence parameter for $^3\text{He}$ production, which demonstrates the effectiveness of the method. The values of the fireball volume $V$ are about 2 times larger, than those found in [23] at the so-called chemical freeze-out stage, about 2 times smaller than at thermal freeze-out [35, 37] and in agreement with limitations on the volumes found in [24]. The same method can be applied to the production of deuterons and $^3\text{He}$ nucleous. But in these cases, in order to avoid the background, consisting of $d$ or $^3\text{He}$ nucleous falling apart from colliding nuclei, it is necessary to select the events with rapidity close to zero. One may expect, that for such events the expressions for coalescence parameters, found above are valid. The further investigations of this problem — both theoretical and experimental are very desirable, since they can shed light also on the most interesting stage — the stage of hadron formation.

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Appendix A: Account of the finite nuclear force radius

According to Bethe the general form of $pn$ scattering phase $\delta$ at low energies is

$$k \cot \delta_0 = -\alpha + \frac{1}{2}(\alpha^2 + k^2)r_0$$

(A.1)

Here $k$ is the proton (or neutron) momentum in c.m. frame, $r_0$ is defined as a radius of nuclear forces: at $r > r_0$ the $pn$ potential $V_{pn}(r)$ may be neglected. (A.1) is an expansion in terms of $kr_0^2$, higher order terms, $\sim k^4r_0^3$ are omitted. For the scattering amplitude (7) we
get from (A.1)
\[ f_0 = \frac{1}{k(\cot \delta_0 - i)} = \frac{i}{k - i\alpha} \frac{1}{1 + i(k + i\alpha)r_0/2} \] (A.2)

The value of effective coupling constant is determined by the residue of the amplitude \( f \) at the pole \( k \to i\alpha \). It is easy to see, that in comparison with the case \( r_0 = 0 \), the value of the residue changes by the factor \((1 - \alpha r_0)^{-1}\). Higher order corrections \( \sim k^4 r_0^2 \) are negligible.

There are several different definitions of the nuclear force radius \( r_0 \). On the other hand, all \( r_0 \)-corrections to the \( pnd \)-coupling can be expressed in terms of the normalization constant \( A_S \) of the deuteron radial wave function \( R = A_S e^{-\sqrt{m\varepsilon}r} \) at large \( r \). Indeed, it determines the residue of the amplitude (7) at the pole \( E = -\varepsilon \):
\[ f_0 = -\frac{A^2_S}{m} \frac{1}{E + \varepsilon} \quad (E \to -\varepsilon) \] (A.3)

In case of zero radius of nuclear force \( A_S = (4m\varepsilon)^{1/4} = 0.68 \text{ fm}^{1/2} \). Analysis of experimental data on elastic \( pd \) scattering [38] as well as various potential models give the value around \( A_S = 0.88 \text{ fm}^{1/2} \). Comparison with the amplitude (7) gives the following coupling value:
\[ g^2 = 64\pi m A^2_S \] (A.4)

It exceeds the zero radius result (9) by 1.67 times. It corresponds to \( r_0 = 1.7 \text{ fm} \).

**Appendix B: Estimation of direct process \( NN \to d\pi \)**

The main processes of the deuteron formation in vacuum is \( NN \to d\pi \). In case of exact isospin symmetry the total cross sections of each channel are related as
\[ \sigma_{pp \to d\pi^+} = \sigma_{nn \to d\pi^-} = 2\sigma_{pn \to d\pi^0} \] (B.1)

similarly for antinucleons. The process \( pp \to d\pi^+ \), as well as inverse one, has been accurately measured in many experiments. The cross section is shown in Fig 4. It has a resonance peak near \( \sqrt{s} = 2.17 \text{ GeV} \), the mass of the \( N\Delta \) system and decreases steeply above \( s \sim 2.5 \text{ GeV} \).

In this appendix we estimate the contribution of these processes to the deuteron formation rate. It is convenient to use invariant definition of the cross-section in relativistic calculations.

We assume \( q_p(p) = q_n(p) \). The processes with identical particles in initial state \( pp \to d\pi^+ \), \( nn \to d\pi^- \) should be taken with weight \( 1/2 \) to avoid the double counting from the same regions of the phase space. So the total deuteron production rate can be written as:
\[ \frac{m_d}{E_d} u_d^\mu \frac{\partial q_d(p_d)}{\partial x^\mu} = \frac{3}{2} \int d^3p_1 d^3p_2 d^3p_\pi q_p(p_1) q_p(p_2) \omega_{pp \to d\pi^+} = \frac{I_{\text{direct}}}{E_d} \] (B.2)

where \( \omega \) is the reaction probability:
\[ \omega_{pp \to d\pi^+} = \frac{\sqrt{(p_1 p_2)^2 - m^4_p}}{E_1 E_2} \frac{d\sigma_{pp \to d\pi^+}}{d^3p_d d^3p_\pi} \] (B.3)
Figure 4: Total cross section $\sigma_{pp \to d\pi^+}$ in mb versus $\sqrt{s}$ in GeV. The experimental data are taken from [39]. The line represents the fit of [40].

The value $I_{direct}$ should be compared with the collision integral $I$ of the fusion process, obtained in (23,24). Integrating by the momenta $p_\pi$, one obtains:

$$I_{direct} = \frac{3}{2} \int \frac{d^3p_1}{E_1} \frac{d^3p_2}{E_2} q_p(p_1) q_p(p_2) \sqrt{(p_1p_2)^2 - m_p^2} E_d \frac{d^3\sigma_{pp \to d\pi^+}}{d^3p_d} \quad (B.4)$$

We compute this integral in the center of mass frame. Let $(E, k)$ and $(E_d, k_d)$ be the c.m. energies and momenta of the first proton and the deuteron respectively. Then the lab momenta are given by the Lorentz boost, $v$ is the center of mass velocity:

$$p_1 = k + (\gamma - 1) \frac{v(k \cdot v)}{v^2} + \gamma vE, \quad E_1 = \gamma (E + vk)$$

$$p_2 = -k - (\gamma - 1) \frac{v(k \cdot v)}{v^2} + \gamma vE, \quad E_2 = \gamma (E - vk) \quad (B.5)$$

where $\gamma = (1 - v^2)^{-1/2}$. Now in the integral (B.4) we come from the variables $p_1, p_2$ to $v, k$ according to:

$$\frac{d^3p_1}{E_1} \frac{d^3p_2}{E_2} = 8\gamma^4 E d^3v d^3k$$

Notice, that $Ed^3\sigma/d^3p$ is invariant cross-section, so $E_d d^3\sigma/d^3p_d = E_d d^3\sigma/d^3k_d$. Then the integral (B.4) can be written as:

$$I_{direct} = \frac{3}{2} \cdot 16 \int d^3v d^3k q_p(p_1) q_p(p_2) \gamma^4 E^2 k \frac{d^3\sigma_{pp \to d\pi^+}}{d^3k_d} \quad (B.6)$$

And finally, it could be convenient instead of the velocity $v$ to substitute the deuteron cm momentum $k_d$ according to:

$$v = \frac{2w}{1 + w^2}, \quad w = \frac{p_d - k_d}{E_d + E_d} \quad (B.7)$$
The velocity integration measure becomes:

\[ \gamma^4 d^3v = \frac{8}{(1-w^2)^3} d^3w = \left( \frac{E_d + \mathcal{E}_d}{m_d + E_d + \mathbf{p}_d \mathbf{k}_d} \right)^2 d^3k_d \frac{\mathcal{E}_d}{\mathcal{E}_d} \]  

(B.8)

Then the integral (B.6) can be written as follows:

\[ I_{\text{direct}} = \frac{3}{2} \cdot 16 \int d^3k \, d^3k_d \frac{d^3\sigma_{pp \rightarrow d\pi^+}}{d^3k_d} \frac{k \mathcal{E}^2 (E_d + \mathcal{E}_d)^2}{(m_d^2 + E_d \mathcal{E}_d + \mathbf{p}_d \mathbf{k}_d)^2} q_p(p_1) q_p(p_2) \]  

(B.9)

To evaluate it further, one needs to know the antiproton distribution \( q_p(p) \) in the fireball. For the first approximation, we could take \( q_p(p) \approx q_p(p_d/2) \), as we did in (24). This however would be an overestimation of the production rate, since it gets the main contribution from the resonance area (see Fig. 4), but not from the threshold. More careful estimation can be obtained by taking the Boltzmann distribution for the antiprotons:

\[ q_p(p) \sim e^{-E/T} \]  

(B.10)

Then one can perform the integration over the angles and \( k_d \). The result is:

\[ I_{\text{direct}} = q_p^2(p_d/2) \frac{3\pi}{4m_d^2} \int_{(m_d + m_\pi)^2}^{\infty} ds \cdot s(s - 4m_d^2) \sigma_{pp \rightarrow d\pi^+} \]

\[ \times \exp \left( - \frac{E_d(s - m_d^2 - m_\pi^2)}{2Tm_d^2} \right) \frac{\text{sh}(\sqrt{s}p_d k_d/(Tm_d^2))}{\sqrt{s}p_d k_d/(Tm_d^2)} \]  

(B.11)

where \( \sigma_{pp \rightarrow d\pi^+} \) is the total cross section, \( k_d^2 = [s - (m_d - m_\pi)^2][s - (m_d + m_\pi)^2]/(4s) \).

The integral (B.11) reaches maximum for the deuteron at rest in the frame, where the system is described by Boltzmann distribution (B.10). So for numerical estimation we take \( p_d = 0 \) and the temperature \( T \) of order of the antiproton inverse slope parameter \( T = 300 \text{ MeV} \). We integrate (B.11) up to \( \sqrt{s} = 2.5 \text{ GeV} \). Above this value other channels appear, where experimental data are rare. However the total cross section \( pp \rightarrow dX \) at high \( \sqrt{s} \) is negligible, so this will not be an underestimation. The result is

\[ I_{\text{direct}} = 7.5 \text{ GeV}^2 \times q_p^2(p_d/2) \]  

(B.12)

It should be compared with the fusion rate integral (24). According to our estimations \( I = 40 \text{ GeV}^2 \times q_p^2(p_d/2) \). It confirms our assumption that the contribution of direct process of antideuteron formation \( \bar{N}\bar{N} \rightarrow \bar{d}\pi \) is small in the heavy ion collisions.

### Appendix C: Particle width in dense medium

The relation (22) for \( \Gamma_{\bar{p}} \) corresponds to the case of nucleon moving through the medium of pions at the dense gas stage of fireball evolution. It turns out that \( \Gamma_{\bar{p}} \) weakly depends on the pion spectrum details, so with sufficient accuracy one can use the observed pion spectrum which is formed on the freeze-out stage. To obtain numerical results we exploit eq. (26) for pion-antiproton collisions, where the pion distribution is extracted from [32].
It is usually parametrized in terms of transverse mass $m_\pi^\perp = \sqrt{(p_\pi^\perp)^2 + m_\pi^2}$ and rapidity $y = \frac{1}{2} \ln[(E + p^\parallel)/(E - p^\parallel)]$:

$$2\pi E_\pi q_\pi(p) = \frac{d^2 n_\pi}{m_\pi^\perp dm_\pi^\perp dy} = C \exp\left\{-(m_\pi^\perp - m_\pi)/T_\perp - y^2/b^2\right\}, \quad (C.1)$$

where $T_\perp = 110$ MeV, $b = 1.9$ and $C \approx 38.6$ GeV$^{-2}$ is the normalization constant, determined from the requirement $\int q_\pi(p) d^3p = n_\pi$. Performing the calculation we find that the numerical value of $\Gamma_\bar{p}$ is about 290 MeV at the pion density $n_\pi = 0.32$ fm$^{-3}$ and antiproton energy $E_\bar{p} = 1.5$ GeV. The effect of averaging over pion momentum is practically negligible as the obtained value of $\Gamma_\bar{p}$ differs by less than 5% from its value calculated within the assumption that all pions are at rest.

As it was pointed in the text, Eq. (22) implies that the scattering amplitude of the incident particles on the medium constituents is much less then the mean distance between them and the scattering can be treated as the sum of independent processes. However, this condition is not fulfilled quite well in the dense matter. In this case $n\sigma$ has the meaning of the total cross section on all the constituents of fireball (divided by its volume) which is not simply reduced to the sum of individual cross sections. This lack of additivity is well known from the nucleon-deuteron scattering at high energies [41]. Within the accuracy of our approach it is sufficient to take only the first term in density expansion. Consider the scattering amplitude at small angles when the incident nucleon interacts with the target consisting from two pions confined in a cube with side $a$. From the calculation of $\Gamma_\bar{p}$ it follows that the main contribution to the integral for $\Gamma_\bar{p}$ arises from the pions with small momenta, so one can assume that the antiproton momentum is much larger than the pion momentum. Then the antiproton scattering amplitude can be written in the framework of method developed by Glauber [41]:

$$f(\bar{q}) = \frac{k}{2\pi i} \int d^2\bar{\rho} e^{-i\bar{q}\bar{\rho}} \frac{d^3\bar{r}_1 d^3\bar{r}_2}{a^6} [\exp\{2i\delta_1(\bar{\rho} - \bar{r}_1^\perp) + 2i\delta_2(\bar{\rho} - \bar{r}_2^\perp)\} - 1], \quad (C.2)$$

where $\delta_i$ is the phase shift induced by the scattering of antiproton on $i$-th pion and $r_i^\perp$ is the transverse coordinate of $i$-th pion.

Using the relation

$$\exp\{2i\delta(r) - 1\} = \frac{2\pi i}{k} \int f_i(\bar{q}) e^{i\bar{q}\bar{\rho}} \frac{d^2\bar{q}}{(2\pi)^2} \quad (C.3)$$

and identity $e^{(x+y)} - 1 = (e^x - 1) + (e^y - 1) + (e^x - 1)(e^y - 1)$, we get the following equation for the forward scattering amplitude:

$$f(0) = f_1(0) + f_2(0) + \frac{2\pi i}{ka^2} f_1(0) f_2(0). \quad (C.4)$$

To obtain quantitative result we assume that $f_1 = f_2$ and $Re f_i = 0$ (actually, $Re f/Im f \approx 20\%$) and note that $n = 2/a^3$ which gives the first correction to $n\sigma$:

$$n_\pi\sigma \rightarrow n_\pi\sigma \left[1 - \frac{\sigma_{\pi\bar{p}}}{4a^2} \left(\frac{n_\pi}{2}\right)^{2/3}\right] \quad (C.5)$$
The factor in square brackets represents the screening effect and also appears in $\Gamma_{\bar{p}}$. At the pion density $n_\pi = 0.32 \text{ fm}^{-3}$ and $\sigma_{\pi\bar{p}} \approx 30 \text{ fm}^2$ the screening effect reduces the value of $\Gamma_{\bar{p}}$ to 190 MeV. (It is worth to note that the screening correction ($C.5$) to $\Gamma_{\bar{d}}$ is larger than the same one to $\Gamma_{\bar{p}}$ as $\sigma_{\pi\bar{d}} > \sigma_{\pi\bar{p}}$ which is a direct indication that $\Gamma_{\bar{d}} < 2\Gamma_{\bar{p}}$.) Taking different values of the pion density one can find the other values of $\Gamma_{\bar{p}}$ presented in Table 2.

References

[1] I.G.Bearden et al, Phys.Rev.Lett. 85, 2681 (2000).
[2] C.Adler et al, Phys.Rev.Lett. 87, 262301 (2001).
[3] T.A.Armstrong et al, Phys.Rev.Lett. 85, 2685 (2000).
[4] T.Chujo, Nucl.Phys. A715, 151 (2003)
[5] L.R.Csernai, J.I.Kapusta, Phys. Rep. 131, 223 (1986);
[6] S.T.Butler, C.A.Pearson, Phys.Rev.Lett. 7, 69 (1961); Phys.Rev. 129, 836 (1963).
[7] A.Schwarzshild, C.Zupancic, Phys.Rev. 129, 854 (1963)
[8] A.Z.Mekjian, Phys.Rev.Lett. 38, 640 (1977); Phys.Rev. C17, 1051 (1978).
[9] R.Bond, P.J.Johansen, S.E.Koonin, S.Garpman, Phys.Lett. B71, 43 (1977).
[10] H.Sato, K.Yazaki, Phys.Lett. 98B, 153 (1981).
[11] S.Mrowczynsky Phys.Lett. B308, 216 (1993).
[12] C.Dover, U.Heinz, E.Schnedermann, J.Zimanyi, Phys.Rev. C44, 1636 (1991).
[13] M.Gyulassy, K.Frankel, E.A.Remler Nucl.Phys. A402 596, (1983).
[14] R.P.Duperray, K.V.Protasov, A.Yu.Voronin, Eur.Phys.J. A16, 27 (2003).
[15] J.L.Nagle, B.S.Kumar, D.Kuznezev, H.Sorge, R.Matiello, Phys.Rev. C53, 367 (1996).
[16] L.W.Chen, C.M.Ko, B.A.Li, Nucl.Phys. A729, 809 (2003)
[17] J.I.Kapusta, Phys.Rev. C21, 1301 (1980).
[18] P.D.Danielewicz, G.F.Bertsch, Nucl.Phys. A533, 712 (1991).
[19] M.Beyer, W.Schadow, C.Kuhrts, G.Ropke, Phys.Rev. C60, 034004 (1999).
[20] L.D.Landau, Sov.Phys. ZhETF 39, 1856 (1960).
[21] H.A.Bethe and R.E.Penierls, Proc.Roy.Soc. A149, 176 (1935).
[22] H.A.Bethe, Elementary nuclear theory, N.Y., 1947.
[23] P.Braun-Munziger, I.Heppe and J.Stachel, Phys.Lett. B465, 15 (1999).
[24] E.V.Shuryak and G.E.Brown, Nucl.Phys. A717, 322 (2003).
[25] V.L.Eletsky and B.L.Ioffe, Phys.Rev.Lett. 78, 1010 (1997).
[26] V.L.Eletsky, B.L.Ioffe and J.I.Kapusta, Eur.Phys.J. A3, 381 (1998).
[27] V.L.Eletsky and B.L.Ioffe, Phys.Lett. B401, 327 (1997).
[28] E.Fermi, Prog.Theor.Phys. 5, 570 (1950).
[29] I.Pomeranchuk, Doklady Akad.Nauk USSR 78, 889 (1951).
[30] S.V.Afanasiev et al, Phys.Rev. C66, 054902 (2002).
[31] M.Gaždzicki et al, nucl-ex/0403023.
[32] M.van Leewen, Nucl.Phys. A715, 161 (2003).
[33] C.Adler et al, Phys.Rev.Lett. 87, 112303 (2001).
[34] W.Cassing and E.L.Bratkovskaya, Phys.Rep. 308, 65 (1999).
[35] P.Braun-Munziger, D.Magesto, K.Redlich, J.Stachel, Phys.Lett. B518, 41 (2001).
[36] L. McLerran, Surveys in High Energy Physics, 2003 Vol 18 (1-4), p.101
[37] J.Stachel, Nucl.Phys. A654, 119 (1999).
[38] I. Borbely, W. Gruebler, V. Koning et al. Phys. Lett. B160 (1985) 17
[39] F.Shimizu et al, Nucl.Phys. A386 (1982) 571
[40] B.J. VerWest, R.A. Arndt, Phys. Rev. C25 (1982) 1979
[41] R.J. Glauber, Phys.Rev. 100 (1955) 242