Renormalization group parameter evolution of the minimal supersymmetric standard model with R-parity violation

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A comparison of spectra obtained using the 1–loop MSSM and 2–loop \( R_p \)-MSSM renormalization group equations is presented. Influence of higher loop corrections and \( R \)-parity violating terms is discussed. Some numerical constraints on the \( R \)-parity violating parameters are also given.

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I. INTRODUCTION

The recent observations of neutrino oscillations and reported, although not confirmed, discovery of neutrinoless double beta decay give strong motivation for studying some aspects of physics beyond the Standard Model (SM). One of the most popular approach to such a physics is the incorporation of supersymmetry, which leads, in the simplest case, to the Minimal Supersymmetric Standard Model (MSSM). This framework is very attractive for theorists since it is the simplest supersymmetric extension of SM means introducing a superpartner for each particle and adding a second Higgs singlet, respectively. From the definition follows that or-

where \( Y \)'s are 3x3 Yukawa matrices. \( L \) and \( Q \) are the \( SU(2) \) left-handed doublets while \( E, U \) and \( D \) denote the right-handed lepton, up-quark and down-quark \( SU(2) \) singlets, respectively. \( H_1 \) and \( H_2 \) mean two Higgs doublets. We have introduced color indices \( i, j, k = 1, 2, 3 \) and the \( SU(2) \) spinor indices \( a, b, c = 1, 2 \).

Since supersymmetry is not observed, we need to introduce some mechanism to break it. The simplest possibility is the so-called soft breaking given by the Lagrangian

\[
\mathcal{L}^{soft} = \epsilon_{ab}[(\mathbf{A}E)_{ij}Q^a_i h^b_1 \bar{E}_j + (\mathbf{A}D)_{ij}Q^a_i h^b_1 \bar{D}_j] + (\mathbf{A}U)_{ij}Q^a_i h^b_2 \bar{D}_j + B_1 h^a_1 h^b_2, \tag{2}
\]

where lowercase letters stand for scalar components of respective chiral superfields, and 3x3 matrices \( \mathbf{A} \) as well as \( B_1 \) are the soft breaking coupling constants. We introduce also a scalar mass term of the form

\[
\mathcal{L}^{mass} = m^2_{h_1} h^1_1 h^1_1 + m^2_{h_2} h^2_1 h^2_2 + q^i m^2_{q_i} q + t^l m^2_{t_l} l + u^i m^2_{u_i} u + d^i m^2_{d_i} d + e^i m^2_{e_i} e. \tag{3}
\]

Looking at it, it is clearly visible, that \( W^{MSSM} \) preserves a discrete symmetry called \( R \)-parity and defined as

\[
R_p = (-1)^{3B+L+2S},
\]

where \( B, L, \) and \( S \) are the baryon, lepton and spin numbers, respectively. From the definition follows that ordinary particles have \( R_p = +1 \) whereas supersymmetric (SU SY) particles have \( R_p = -1 \). The consequence of \( R_p \) conservation is a stable lightest SUSY particle, which can be a serious candidate for cold dark matter, and a stable proton.

However, nothing motivates theoretically \( R_p \) conservation, especially that in order to avoid rapid proton decay, at least the lepton number parity \( L_p = (-1)^{L+2S} \) or the baryon number parity \( B_p = (-1)^{B+2S} \) must be conserved, but not necessarily both. It follows that we can add new terms to the superpotential, which violate separately the lepton and baryon number:

\[
W^{R_p} = \epsilon_{ab} \left[ \frac{1}{2} (\mathbf{A}E^a)_{ij} L^a_i L^b_j \bar{E}_k + (\mathbf{A}D^a)_{ij} L^a_i Q^b_k \bar{D}_k \right] + \frac{1}{2} \epsilon_{xyz} (\mathbf{A}U^a)_{jk} t^x_i D^y_j \bar{D}^z_k + \epsilon_{abK} L^a_i H^b_2. \tag{4}
\]
The full superpotential becomes now

\[ W = W_{\text{MSSM}} + W_{\mathcal{R}_p} \]

and defines a new model, often called $\mathcal{R}_p$–MSSM. It introduces nine new Yukawa coupling constants but, as mentioned above, proton stability requires to reject either lepton number or baryon number violating terms. If one wants to apply this model to the description of the neutralino double beta decay

\[ A(Z, N) \rightarrow A(Z + 2, N - 2) + 2e^- , \]

in which the lepton number is violated by two units, the matrices $A_U$, have to be set equal to zero. In this paper, we will not do that in order to stay completely general and to be able to count contributions coming from all sources.

All the parameters (coupling constants and masses) of $\mathcal{R}_p$–MSSM running parameters in the sense of renormalization group equations (RGE) below appropriate thresholds, where SUSY particles start to contribute, are set to 1 TeV and are dynamically modified.

One wants to apply this model to the description of the neutrinoless double beta decay, and defines a new model, often called $\mathcal{R}_p$–MSSM. It in-

The trilinear soft couplings at $m_{\text{GUT}} = 500$ GeV. Next, all the quantities are evolved down to $m_{\text{GUT}}$. The numbers used in simulation serve as an example. Our investigation showed that the same conclusionscan be drawn for a wide range of $m_0$ and $A_0$.

\[ A_i = A_0 Y_i, \]

with $A_0 = 500$. Next, all the quantities are evolved down to $m_{\text{GUT}}$. The numbers used in simulation serve as an example. Our investigation showed that the same conclusions can be drawn for a wide range of $m_0$ and $A_0$.

| TABLE I: Mass matrices of supersymmetric particles (third family) and values of the coupling constants, calculated using 1–loop MSSM and 2–loop $\mathcal{R}_p$–MSSM renormalization group equations. All supersymmetric masses are unified (500 GeV) at the GUT scale $10^{16}$ GeV. Other MSSM parameters were $\tan \beta = 5$ and positive sign of $\mu$. |
|---|---|---|---|
| 1–loop GUT | 2–loop GUT | 1–loop $m_{\text{GUT}}$ | 2–loop $m_{\text{GUT}}$ |
| MSSM | $\mathcal{R}_p$–MSSM | MSSM | $\mathcal{R}_p$–MSSM |
| $\tan \beta_1$ | 0.0396 | 0.0398 | 0.0169 | 0.0169 |
| $\alpha_2$ | 0.0386 | 0.0402 | 0.0338 | 0.0338 |
| $\alpha_3$ | 0.0387 | 0.0396 | 0.1177 | 0.1177 |
| $Y_\tau$ | 0.0356 | 0.0371 | 0.0527 | 0.0527 |
| $Y_\nu$ | 0.0337 | 0.0343 | 0.0937 | 0.0937 |
| $Y_\mu$ | 0.6356 | 0.6388 | 0.9598 | 0.9598 |
| $m_{\nu_{\mu \tau}}$ | 500 | 500 | 168 | 182 |
| $m_{\nu_{\mu \tau}}$ | 500 | 500 | -1131 | -1170 |
| $m_{\nu_{\mu \tau}}$ | 500 | 500 | -853 | -861 |
| $\mu$ | 500 | 500 | 761 | 805 |
| $B\mu$ | 500 | 500 | -247 | -248 |
| $m_{\nu_{\mu \tau}}$ | 500 | 500 | 210 GeV | 210 GeV |
| $m_{\nu_{\mu \tau}}$ | 500 | 500 | 415 GeV | 414 GeV |
| $m_{\nu_{\mu \tau}}$ | 500 | 500 | 1290 GeV | 1308 GeV |
| $|m_{\nu_{\mu \tau}}|$ | 500 | 500 | 604 GeV | 598 GeV |
| $|m_{\nu_{\mu \tau}}|$ | 500 | 500 | -738 GeV | -782 GeV |
| $m_{\nu_{\mu \tau}}$ | 500 | 500 | 1282 GeV | 1284 GeV |
| $m_{\nu_{\mu \tau}}$ | 500 | 500 | 1281 GeV | 1284 GeV |
| $m_{\nu_{\mu \tau}}$ | 500 | 500 | 1237 GeV | 1249 GeV |
| $m_{\nu_{\mu \tau}}$ | 500 | 500 | 1234 GeV | 1247 GeV |
| $m_{\nu_{\mu \tau}}$ | 500 | 500 | 612 GeV | 607 GeV |
| $m_{\nu_{\mu \tau}}$ | 500 | 500 | 612 GeV | 607 GeV |
| $m_{\nu_{\mu \tau}}$ | 500 | 500 | 612 GeV | 606 GeV |
| $m_{\nu_{\mu \tau}}$ | 500 | 500 | 536 GeV | 532 GeV |
| $m_{\nu_{\mu \tau}}$ | 500 | 500 | 536 GeV | 532 GeV |
| $m_{\nu_{\mu \tau}}$ | 500 | 500 | 534 GeV | 531 GeV |

The relevant 1– and 2–loop equations for MSSM can be found in \[5\] with $\mathcal{R}_p$–MSSM corrections in \[11\]. Our procedure of running the parameters consists of few steps. We take into account mass thresholds where SUSY particles start to contribute \[12\] and use the SM 2–loop renormalization group equations (RGE) below appropriate thresholds and 2–loop $\mathcal{R}_p$–MSSM RGE above it. Initially all the thresholds are set to 1 TeV and are dynamically modified during the running of mass parameters. After evolving the dimensionless couplings from the electroweak scale $m_Z$ up to $m_{\text{GUT}} \sim 10^{16}$ GeV, we unify the masses of gauginos, sfermions and squarks to be $m_0 = 500$ GeV. The trilinear soft couplings at $m_{\text{GUT}}$ are set according to formula

\[ A_i = A_0 Y_i, \]
results, all these couplings should obey

\[(\Lambda)_{ii} \leq 0.015, \quad (i = 1, 2, 3), \quad (8)\]

otherwise high vacuum expectation values are generated from the down quark mass evolution. What is more, values of other parameters nearly did not change for various \(\Lambda\)'s. From (8) we can make use of the following relation

\[\Lambda_{D1} \leq \xi \left( \frac{10^{24}y}{T_{\beta\beta\text{exp}}} \right)^{-1/4}, \quad (9)\]

Using the \(\xi\) parameter for \(^{76}\text{Ge}\) isotope for the unification mass 500 GeV, and inserting the experimentally known half-life given by the Heidelberg–Moscow collaboration \(^{4}\) \(T_{1/2}^{\beta\beta\text{HMexp}} = (0.8 - 18.3) \times 10^{25}y\), we obtain

\[\Lambda_{D1} \leq 0.275, \quad (10)\]

which is much weaker than our constraint from (8). If one uses limit (8) the half-life to be of the order of \(\sim 1.3 \times 10^{30}y\) which is rather an overestimation. On the other hand, the newest data from the IGEX Collaboration \(^{14}\) sets the lower bound on the half-life in germanium to be \(T_{1/2}^{\beta\beta\text{IGEXexp}} > 1.57 \times 10^{25}y\) without bounds from above.

Values for various parameters in both 1-loop MSSM and 2-loop \(R_p\)-MSSM cases are given in Tab.\(^{1}\) The \(SU(3)_c \times SU(2)_L \times U(1)_Y\) dimensionless coupling constants \(g_i\) are represented here in the usual way by \(\alpha_i = g_i^2 / 4\pi\). Whenever the parameter has a matrix form, it has been diagonalized and the entry \(\{33\}\) is listed in the table. Masses are given respectively for gauginos, left-handed Higgs doublets, left-handed squark doublets, right-handed up- and down-type squark singlets, left-handed slepton doublets, and right-handed slepton singlets, with respect to the \(SU(2)\) group.

In general, as expected, all parameters are changed by at most few percent. One should keep in mind that the new results include two corrections: the 2-loop terms and the terms connected with \(R\)-parity violation. Our investigation shows, that the differences are mainly driven by the 2-loop corrections, whereas the \(R\)-parity terms change the results on the third decimal place.

This situation can be understood from the evolution of \(R_p\)-violating couplings. In Fig.\(^{11}\) an example of running of \(\Lambda\) coupling constants is shown. It is essential, that their values decrease with increasing energy. As a consequence the contributions coming from \(\Lambda\)'s can be safely neglected in actual calculations. It is, however, a surprising result, since normally one would expect to see exotic processes at high energies. Fig.\(^{11}\) clearly shows, that to observe a lepton number violating process, like the neutrinoless double beta decay, one should search for

\[\begin{align*}
\Lambda_{u1} & \quad \Lambda_{d1} \\
\Lambda_{u2} & \quad \Lambda_{d2} \\
\Lambda_{u3} & \quad \Lambda_{d3}
\end{align*}\]
it in low temperatures rather than in accelerators.

IV. FINAL REMARKS

In conclusion, we have presented the spectrum of supersymmetric particles and coupling constants using the 1–loop MSSM and 2–loop $R_p$–MSSM renormalization group equations. Inclusion of the $R$-parity violating terms does not change the results significantly. On the other hand we confirmed the well known rule that the 2–loop corrections should be used in all cases, where quantitative rather than qualitative features are important.

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