Induced Angular Momentum in (2+1)-Dimensional Spinor Electrodynamics in Curved Space. *

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Abstract

Effects due to fermion-vacuum polarization by an external static magnetic field are considered in a two-dimensional noncompact curved space with a nontrivial topology. An expression for the vacuum angular momentum is obtained. Like the vacuum fermion number, it proves to be dependent on the global characteristics of the field and space.

INTRODUCTION

Vacuum quantum effects in strong external fields have long been the subject of intensive investigations (see, for example, [1, 2]). It was found that fermion vacua can possess unusual properties and can be characterized by nontrivial quantum numbers. In particular, the fermion number can be nonzero and even fractional [3, 4]. Fractional fermion numbers are realized in systems of various spatial dimensions. This effect may explain a number of phenomena, both in solid-state physics and in elementary-particle physics (for an

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overview, see [5]). Of special importance was the discovery of the fractional fermion number in (2+1)-dimensional spacetime, because it further resulted in the development of gauge-field models whose action functionals involve Chern-Simons topological term and in the formulation of some fruitful concepts such as anyons and fractional spin statistics, which made it possible to explain satisfactory some aspects of high-temperature superconductivity and fractional quantum Hall effect (see [6-9] and references cited in these studies).

In our opinion, however, analysis of the properties of the fermion vacuum has yet to be completed. First, it is necessary to extend this analysis to quantum numbers other than the fermion one. Second, it is worth considering effects of the geometry of embedding space on these quantum numbers. The only attempts at advancements along these two lines were made in [10-12], where the vacuum angular momentum induced by an external magnetic field was found in a two-dimensional flat (Euclidean) space, and in [6, 7], where the vacuum fermion number induced by an external magnetic field was determined in a two-dimensional curved (Riemann) space.

In this study, we find the vacuum angular momentum induced by an external axisymmetric magnetic field in a two-dimensional axisymmetric non-compact Riemann space with both trivial and nontrivial topologies.

FERMION VACUUM IN STATIC EXTERNAL FIELDS

The two-dimensional Dirac Hamiltonian in the case considered has the form

$$H = -i \alpha^\mu(x) \left[ \partial_\mu + \frac{i}{2} \omega_\mu(x) - i V_\mu(x) \right] + \beta m,$$

where

$$\alpha^\mu(x) \alpha^\nu(x) = g^{\mu\nu}(x) + is \beta \varepsilon^{\mu\nu}(x), \quad \mu, \nu = 1, 2,$$

$g^{\mu\nu}$ and $\varepsilon^{\mu\nu}$ are the metric tensor and the totally antisymmetric tensor of a surface, the values $s = \pm 1$ mark two inequivalent irreducible representations of the Clifford algebra in 2+1-dimensional space-time, $V_\mu$ is the $U(1)$-bundle connection and $\omega_\mu$ is the spin connection. The total magnetic flux (in the units of $2\pi$) through the surface and the total integral curvature (in the units
of $2\pi$) of the surface can be presented in the form

$$\Phi = \frac{1}{2\pi} \int d^2x \sqrt{g} \varepsilon^{\mu\nu} \partial_\mu V_\nu = \Phi^{(+)} - \Phi^{(-)},$$

$$\Phi_K = \frac{1}{2\pi s\beta} \int d^2x \sqrt{g} \varepsilon^{\mu\nu} \partial_\mu \omega_\nu = \Phi^{(+)}_K - \Phi^{(-)}_K,$$

(3)

where

$$\Phi^{(\pm)} = \lim_{\ln r \to \pm \infty} \frac{1}{2\pi} \int_0^{2\pi} d\theta V_\theta(r, \theta), \quad \Phi^{(\pm)}_K = \lim_{\ln r \to \pm \infty} \frac{1}{2\pi s\beta} \int_0^{2\pi} d\theta \omega_\theta(r, \theta),$$

(4)

$r$ and $\theta$ are the polar coordinates and $g = \det g_{\mu\nu}$; in eqs. (3) and (4) we use the fact that a rotationally-symmetric noncompact Riemann surface has in general the topology of a cylinder. Spinor wave function on such a surface is subject to the condition

$$\psi(r, \theta + 2\pi) = e^{i2\pi \Upsilon} \psi(r, \theta).$$

(5)

The Dirac Hamiltonian $H$ (1) in the gauge

$$V_r = 0, \quad \partial_\theta V_\theta = 0$$

(6)

commutes with the operator

$$M = -ix^\mu \varepsilon_{\mu\nu} \partial_\nu - \Upsilon + \frac{1}{2} s\beta.$$

(7)

Hence a set of functions $\{\psi\}$ can be defined as the complete set of solutions to the equations

$$H\psi(x) = E\psi(x), \quad M\psi(x) = j\psi(x).$$

(8)

If these functions satisfy the condition (5), then the eigenvalues of the operator $M$ (7) are half-integer,

$$j = n + \frac{1}{2}s, \quad n \in \mathbb{Z},$$

(9)

where $\mathbb{Z}$ is the set of integer numbers. Thus eq. (8) describes the angular momentum operator — the generator of rotations in spinor electrodynamics on a rotationally-symmetric surface in a rotationally-symmetric external magnetic field.
Passing from the rotationally-symmetric gauge \((6)\) to an arbitrary one, we get

\[
H \to e^{i\Lambda(x)}He^{-i\Lambda(x)}, \quad M \to e^{i\Lambda(x)}Me^{-i\Lambda(x)},
\]

where the gauge function \(\Lambda(x)\) on a noncompact surface with the topology of a cylinder satisfies the condition

\[
\Lambda(r, \theta + 2\pi) = \Lambda(r, \theta) + 2\pi \Upsilon_\Lambda.
\]

Taking into account

\[
V_\mu(x) \to V_\mu(x) + \partial_\mu \Lambda(x), \quad \psi(x) \to e^{i\Lambda(x)}\psi(x),
\]

one can find

\[
\Phi^{(\pm)} \to \Phi^{(\pm)} + \Upsilon_\Lambda, \quad \Upsilon \to \Upsilon + \Upsilon_\Lambda,
\]

while \(\Phi\) and \(\Phi^{(\pm)} - \Upsilon\) remain gauge invariant.

Let us note that in a two-dimensional space, in distinction to a three-dimensional one, the rotation group is abelian and the group-theoretical arguments restricting the eigenvalues of the angular momentum operator to half-integer (and integer for bosons) numbers are lacking. In the case of simply connected surfaces (topologically equivalent to a plane) we have \(\Upsilon = 0\) and the condition of single-valuedness of spinor wave functions, which is invariant under regular \((\Upsilon_\Lambda = 0)\) gauge transformations, ensures that the eigenvalues of the angular momentum operator are half-integer (and integer for bosons). In the case of punctured surfaces (topologically equivalent to a cylinder), at the same time with eq. \((7)\), it is possible to define the angular momentum operator alternatively (see, for example, \([8]\))

\[
M' = M - \Phi^{(-)} + \Upsilon,
\]

the eigenvalues of \(M'\) \((14)\) on spinor functions satisfying eq. \((5)\) are not half-integer,

\[
j' = n + \frac{1}{2}s - \Phi^{(-)} + \Upsilon, \quad n \in \mathbb{Z}.
\]

The problem of choice between two possible definitions of the angular momentum operator (eq. \((7)\) or \((14)\)) is beyond the scope of current investigation, since to solve this problem one has to take into account the dynamics
of the vector field $V_\mu$. Let us only note here that in the case of a punctured plane ($\Phi^K_+ = \Phi^K_- = 0$) and the regular (i.e. without the $\delta$-function singularity) part of the magnetic field strength being equal to zero the option (7) corresponds to the Maxwell dynamics, and the option (14) corresponds to the Chern-Simons dynamics [9, 10].

ANGULAR MOMENTUM IN $(2+1)$-DIMENSIONAL SPINOR ELECTRODYNAMICS

In the secondly quantized theory the operator of the dynamical quantity corresponding to $M$ (7) is defined in the conventional way

$$\hat{J} = \frac{1}{2} \int d^2x \sqrt{|g|} [\Psi^+(x), M \Psi(x)]_-=$$

$$= \sum_E \sum_j e^{-tE^2} (a_{E,j}^+ a_{E,j} - b_{E,j}^+ b_{E,j}) - \frac{1}{2} \sum_E \sum_j e^{-tE^2} \text{sgn}(E),$$

where

$$\text{sgn}(u) = \begin{cases} 1, & u > 0 \\ -1, & u < 0 \end{cases},$$

$a_{E,j}^+$ and $a_{E,j}$ ($b_{E,j}^+$ and $b_{E,j}$) are the fermion (antifermion) creation and annihilation operators satisfying the anticommutation relations, the symbol $\sum_E$ implies summation over the discrete and the integration (with a definite measure) over the continuous parts of the energy spectrum and the regularization factor $\exp(-tE^2)$ ($t > 0$) is inserted to tame the divergence at $|E| \to \infty$.

The operator of the dynamical quantity corresponding to $M'$ (14) is defined as

$$\hat{J}' = \hat{J} - (\Phi^(-) - \Upsilon) \hat{N},$$

where $\hat{N}$ is the fermion number operator given by eq. (16) with the unity operator $\mathbb{I}$ substituted for $M$. 

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The c-number piece of the angular momentum operator in the secondly quantized theory $J$ (10) can be presented in the following way

$$\langle \hat{J} \rangle = -\frac{1}{2\pi} \int_{-\infty}^{\infty} du \sum_{E} \sum_{j} jE(E^2 + u^2)^{-1} e^{-tE^2} =$$

$$= -\frac{1}{2\pi} \int_{-\infty}^{\infty} du \int dx \sqrt{\sigma} \langle x | \frac{MH e^{-tH^2}}{H^2 + u^2} | x \rangle . \quad (18)$$

Using the relation

$$[M, \beta]_\pm = 0$$

and the trace identity

$$\frac{i}{2} g^{-1} \partial_{\mu} g^1 \langle x | \alpha^\mu \frac{M\beta H_0}{H_0^2 + m^2 + u^2} e^{-tH_0^2} | x \rangle =$$

$$= \text{tr} \langle x | M\beta e^{-tH_0^2} | x \rangle - (m^2 + u^2) \text{tr} \langle x | \frac{M\beta e^{-tH_0^2}}{H_0^2 + m^2 + u^2} | x \rangle , \quad (19)$$

where $H_0 = H|_{m=0}$, we get

$$\langle \hat{J} \rangle = -\frac{1}{2} \text{sgn}(m) e^{-tm^2} \int d^2 x \sqrt{\sigma} \text{tr} \langle x | \beta M e^{-tH_0^2} | x \rangle +$$

$$+ \frac{im}{4\pi} e^{-tm^2} \int_{-\infty}^{\infty} \frac{du}{m^2 + u^2} \int_\sigma \text{tr} \langle x | \frac{M\beta H_0 e^{-tH_0^2}}{H_0^2 + m^2 + u^2} | x \rangle \varepsilon_{\mu\nu}(x) dx^\nu , \quad (20)$$

where $\sigma$ is the closed contour conditionally bounding a noncompact two-dimensional space (surface) at infinity. We recall that for the surfaces with nontrivial topology the contour $\sigma$ can consist of several disjoint components: for example, if for the infinite plane (trivial topology) the contour $\sigma$ is a circle of infinite radius (one-component boundary at infinity), then for the cylinder of infinite length and finite radius (non-trivial topology) the contour $\sigma$ consists of two circles of finite radius (two-component boundary at infinity).

In the case of a noncompact surface with the topology of a cylinder we get

$$\langle \hat{J} \rangle = -\frac{1}{2} e^{-tm^2} \left[ \text{sgn}(m) A^{(M)}(t) + S_+^{(M)}(m, t) - S_-^{(M)}(m, t) \right] , \quad (21)$$
where
\[ A^{(M)}(t) = \int d^2 x \sqrt{g} \text{tr} \langle x | \beta Me^{-tH_0^2} | x \rangle \tag{22} \]
and
\[ S^{(M)}_\pm (m, t) = -\frac{sm}{2\pi} \int_{-\infty}^{\infty} \frac{du}{m^2 + u^2} \int_0^{2\pi} d\theta \]
\[ \text{tr} \langle \ln r \to \pm \infty, \theta | r \sqrt{g} \alpha^g \frac{H_0 Me^{-tH_0^2}}{H_0^2 + m^2 + u^2} | \ln r \to \pm \infty, \theta \rangle. \tag{23} \]

We prove the existence of the asymptotical expansion
\[ \langle \hat{J} \rangle = t \to 0^+ \sum_{l=0}^{\infty} C_l t^\frac{l}{2}, \]
and calculate the coefficients corresponding to \( l = 0, 1, 2 \) (details will be published elsewhere). It turns out that the coefficients \( C_0 \) and \( C_1 \) are depending on \( \Phi^+(\pm) \) but independent of \( \Phi^-(\pm) \) and \( \Upsilon \). The latter circumstance allows us to define the renormalized vacuum value as
\[ \langle \hat{J} \rangle_{\text{ren}} = \lim_{t \to 0^+} \left( \langle \hat{J} \rangle - \langle \hat{J} \rangle |_{\Phi^+(\pm) = \Upsilon = 0} \right). \tag{24} \]

It is clear that this definition ensures the normal ordering of the operator product in the non-interacting theory \( \langle \hat{J} \rangle_{\text{ren}} |_{\Phi^+(\pm) = \Upsilon = 0} = 0 \).

Let us present the final form for the renormalized vacuum value of angular momentum on a surface with the topology of a cylinder:
\[ \langle \hat{J} \rangle_{\text{ren}} = -\frac{1}{4} s \text{sgn}(m) \left[ (\Phi^+ - \Upsilon)^2 - (\Phi^- - \Upsilon)^2 \right] - \]
\[ -\frac{1}{2} (\Phi^+ - \Upsilon) \xi_+ \left[ m, \text{fract} \left( \Phi^+ - \Upsilon + \frac{1}{2} \right), \Phi^+_K \right] - \]
\[ -\frac{1}{4} \tau_+ \left[ m, \text{fract} \left( \Phi^+ - \Upsilon + \frac{1}{2} \right), \Phi^+_K \right] + \]
\[ +\frac{1}{2} (\Phi^- - \Upsilon) \xi_- \left[ m, \text{fract} \left( \Phi^- - \Upsilon + \frac{1}{2} \right), \Phi^-_K \right] + \]
\[ +\frac{1}{4} \tau_- \left[ m, \text{fract} \left( \Phi^- - \Upsilon + \frac{1}{2} \right), \Phi^-_K \right], \tag{25} \]
where

\[ \xi_{\pm}(m, v, \Phi_{K}^{(\pm)}) = \begin{cases} 
0, & \Phi_{K}^{(\pm)} \leq 1 \\
s \left\{ \arctg \left[ \cosh(\pi R_{\pm} m) \tanh(\pi v) \right] - \text{sgn}(m) v \right\}, & \Phi_{K}^{(\pm)} > 1 \\
s \text{sgn}(m) \left[ \frac{1}{2} \text{sgn}_{0}(v) - v \right], & \Phi_{K}^{(\pm)} \geq 1 
\end{cases} \]  

(26)

\[ \tau_{\pm}(m, v, \Phi_{K}^{(\pm)}) = \begin{cases} 
0, & \Phi_{K}^{(\pm)} \leq 1 \\
s \left\{ -\frac{2}{\pi} \int_{1/2}^{v} du \arctg \left[ \cosh(\pi R_{\pm} m) \tanh(\pi u) \right] + \right. \\
+ \text{sgn}(m) \left( v^2 - \frac{1}{4} \right) + \frac{1}{\pi} R_{\pm} m \ln \left[ 1 - \frac{\cos^2(\pi v)}{\sin^2(\pi R_{\pm} m)} \right], & \Phi_{K}^{(\pm)} = 1 \\
\text{sgn}(m) \left[ \frac{1}{2} \text{sgn}(v) - v \right]^2, & \Phi_{K}^{(\pm)} \geq 1 
\end{cases} \]  

(27)

and we introduce the following notations

\[ \text{fract}(u) = u - \text{integ}(u), \quad \text{sgn}_{0}(u) = \begin{cases} 
\text{sgn}(u), & u \neq 0 \\
0, & u = 0 
\end{cases} \]

integ\((u)\) is the nearest to \(u\) integer;

\[-\frac{1}{2} < \text{fract}(u) < \frac{1}{2}, \quad \lim_{\varepsilon \to 0} \text{integ}(n + \frac{1}{2} \pm \varepsilon) = n + \frac{1}{2} \pm \frac{1}{2}.\]

Note that the functions \(\xi_{+}\) and \(\xi_{-}\) which determine the boundary contribution to the vacuum value of the fermion number on a surface with the topology of a cylinder have been found earlier in [11] (where they are denoted as \(\xi\) and \(\xi_{0}\) respectively). In the case of the alternative definition of angular momentum (17) we get

\[ <\hat{J}'_{\text{ren}} \equiv <\hat{J}_{\text{ren}} - (\Phi(-\gamma) - \Upsilon) <\hat{N}_{\text{ren}} = -\frac{1}{4}s \text{sgn}(m) \Phi^2 - \\
\quad - \frac{1}{2} \Phi_{\xi_{+}} \left[ m, \text{fract} \left( \Phi^{(+)} - \gamma + \frac{1}{2} \right), \Phi_{K}^{(+)} \right] - \\
\quad - \frac{1}{4} \tau_{+} \left[ m, \text{fract} \left( \Phi^{(+)} - \gamma + \frac{1}{2} \right), \Phi_{K}^{(+)} \right] + \\
\quad + \frac{1}{4} \tau_{-} \left[ m, \text{fract} \left( \Phi^{(-)} - \gamma + \frac{1}{2} \right), \Phi_{K}^{(-)} \right]. \]  

(28)
The latter expression at fixed values of \( \Phi \) depends on \( \Phi(-) - \Upsilon \) (or \( \Phi(+) - \Upsilon \)) periodically with the period equal to 1, which is also peculiar for the renormalized vacuum value of fermion number on a surface with the topology of a cylinder \([6, 7]\)

\[
\langle \hat{N} \rangle_{\text{ren}} = \lim_{t \to 0^+} \langle \hat{N} \rangle =
\begin{align*}
&= -\frac{1}{2}s \text{sgn}(m) \Phi - \frac{1}{2} \xi_+ \left[ m, \text{fract} \left( \Phi^{(+) - \Upsilon + \frac{1}{2}}, \Phi^{(+) K} \right) \right] + \\
&+ \frac{1}{2} \xi_- \left[ m, \text{fract} \left( \Phi^{(-) - \Upsilon + \frac{1}{2}}, \Phi^{(-) K} \right) \right].
\end{align*}
\]

(29)

Let us emphasize that the results obtained are invariant under gauge transformations, including singular ones, \( \Upsilon_A \neq 0 \) \([11]\). The gauge invariance of (25) and (28) is stipulated by the choice of the operators \( M (7) \) and \( M' (14) \) in the capacity of the angular momentum operator in the first-quantized theory: although the operators \( M, M' \) are changed (covariantly, as well as the Hamiltonian \( H \) \([11]\) does, see eq.(10)) under gauge transformations, their eigenvalues, as well as the values of the energy, remain gauge invariant. Expressions (25), (28) and (29) are also invariant under simultaneous substitution \( s \to -s \) and \( m \to -m \), which means that the transition to inequivalent representation can be implemented by changing the sign of the fermion mass.

In the case of a simply connected surface we have

\[
\begin{align*}
\Phi(-) &= 0, & \Phi^{(-)} &= 0, & \Upsilon &= 0, \\
\Phi^{(+)} &= \Phi, & \Phi^{(+) K} &= \Phi^{K},
\end{align*}
\]

(30)

and the expressions for the renormalized vacuum values take the form

\[
\langle \hat{N} \rangle_{\text{ren}} = -\frac{1}{2}s \text{sgn}(m) \Phi - \frac{1}{2} \xi_+ \left[ m, \text{fract} \left( \Phi + \frac{1}{2}, \Phi^{K} \right) \right]
\]

(31)

and

\[
\langle \hat{J} \rangle_{\text{ren}} = -\frac{1}{4}s \text{sgn}(m) \Phi^2 - \frac{1}{2} \Phi \xi_+ \left[ m, \text{fract} \left( \Phi + \frac{1}{2}, \Phi^{K} \right) \right] - \\
- \frac{1}{4} \tau_+ \left[ m, \text{fract} \left( \Phi + \frac{1}{2}, \Phi^{K} \right) \right].
\]

(32)
It is instructive to compare the results in two very simple particular cases differing in topology. In the case of a plane we have the vacuum fermion number \(\langle \hat{N}\rangle_{\text{ren}} = -\frac{1}{2} s \text{sgn}(m) \Phi\) \hspace{1cm} (33)

and the vacuum angular momentum \(\langle \hat{J}\rangle_{\text{ren}} = -\frac{1}{4} s \text{sgn}(m) \Phi^2\). \hspace{1cm} (34)

In the case of a punctured plane and \(\Phi = 0\) we have the vacuum fermion number \(\langle \hat{N}\rangle_{\text{ren}} = \frac{1}{2} s \text{sgn}(m) \left\{ \frac{1}{2} \text{sgn}_0 \left[ \text{fract} \left( \Phi(-) - \Upsilon + \frac{1}{2} \right) \right] - \right.\)

\(\left. - \text{fract} \left( \Phi(-) - \Upsilon + \frac{1}{2} \right) \right\} \left(35\right)\)

and the vacuum angular momentum

\(\langle \hat{J}\rangle_{\text{ren}} = \frac{1}{2} s \text{sgn}(m) \left( \Phi(-) - \Upsilon \right) \left\{ \frac{1}{2} \text{sgn}_0 \left[ \text{fract} \left( \Phi(-) - \Upsilon + \frac{1}{2} \right) \right] - \right.\)

\(\left. - \text{fract} \left( \Phi(-) - \Upsilon + \frac{1}{2} \right) \right\} + \frac{1}{4} s \text{sgn}(m) \left\{ \frac{1}{2} \text{sgn} \left[ \text{fract} \left( \Phi(-) - \Upsilon + \frac{1}{2} \right) \right] - \right.\)

\(\left. - \text{fract} \left( \Phi(-) - \Upsilon + \frac{1}{2} \right) \right\}^2 \hspace{1cm} \left(36\right)\)

or

\(\langle \hat{J}'\rangle_{\text{ren}} = \frac{1}{4} s \text{sgn}(m) \left\{ \frac{1}{2} \text{sgn} \left[ \text{fract} \left( \Phi(-) - \Upsilon + \frac{1}{2} \right) \right] - \right.\)

\(\left. - \text{fract} \left( \Phi(-) - \Upsilon + \frac{1}{2} \right) \right\}^2 \hspace{1cm} \left(37\right)\)

In conclusion we note that the vacuum fermion number induced on a noncompact Riemann surface of a more general form — topologically finite
orientable surface (possessing a disconnected multi-component boundary at infinity and handles) — was calculated in [6, 7]. The inducing of the vacuum angular momentum on such a surface will be considered elsewhere.

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