Active and Reactive Power Coordinated Optimal Dispatch in Active Distribution Network Considering Spatial-temporal Correlation of Wind Power

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Abstract. The high penetration of renewable energy increases the difficulty of optimal dispatch and brings challenges to the safe and economic operation of the distribution network. This paper proposed an active and reactive power coordinated optimal dispatch model, and the spatial-temporal correlation of wind power output is considered. Firstly, the copula function and non-parametric estimation methods are adopted to model the spatial-temporal correlation of wind power. Secondly, the parameters in the copula fitting function are estimated based on the maximum likelihood estimation method. Besides, the active power and reactive power outputs of various flexible resources in the active distribution network (ADN) are coordinated. The proposed model can significantly reduce the network loss of the ADN, improve efficiency, and optimize the power purchase plan. A case study on a modified IEEE 33-bus system verified the effectiveness of the proposed method.

1. Introduction

With the decrease of fossil energy and the continuous growth of power demand, the existing energy structure is changing, which leads to the high penetration of renewable energy sources (RES) in the distribution network (DN). The existence of distributed generations (DGs) can change the power flow distribution, which brings challenges to the safe and economic operation of DN. Therefore, it is necessary to study optimal dispatch in ADN with high penetration of RES.

In [1], a hierarchical model predictive control strategy is proposed to improve the performance of traditional active economic dispatch with large-scale wind power access. Reference [2] proposed a DN and microgrid coordinated scheduling framework. Under this framework, a multi-time scale active power dispatching model is established to coordinate the output of micro-source and energy storage, which ensures the stability of the power system and improves the economic performance of the DN. In [3], an active power dispatching model of ADN based on hierarchical control theory is proposed, and the adaptive particle swarm optimization algorithm is adopted to solve the problem. In the aspect of reactive power optimal dispatch, Reference [4] proposed a dynamic reactive power optimization model (RPOM), which can coordinate the action of DGs and capacitor banks, ensure the voltage level and reduce the system network loss. In [5], a two-stage distributed algorithm is proposed. This algorithm can deal with the privacy problem in the open electricity market. Reference [6] proposed a distributed RPOM based on Wasserstein distance. This model is data-driven and can effectively deal with the randomness of RES.

The literature mentioned above either studies the active power optimal dispatch or reactive power optimal dispatch in the DN, but does not combine them. The reactance-resistance ratio in the DN is
higher than the transmission network, which indicates that the active and reactive power coupling is stronger. A single consideration of active or reactive power optimization cannot guarantee the accuracy of the results, nor can it give full play to the regulation ability of flexible resources in the ADN. Also, the wind power models used in these studies are relatively rough and do not model the complex correlation between wind farms.

References [7], [8] modeled the spatial correlation of wind power output and applied it to the economic dispatch of DN to improve the conservative type of robust optimization problem. In [9], [10], the temporal correlation model of wind power is proposed based on copula theory and applied to power system planning. However, the above literature does not study the spatial correlation and temporal correlation at the same time. As a result, the correlation between wind power output has not been fully revealed. Therefore, it is necessary to model the spatial-temporal correlation of wind power output in the economic dispatch of ADN.

The main contributions of this paper are as follows:

1) The spatial-temporal correlation of output between DGs is modeled by copula and non-parametric estimation methods. The consideration of spatial-temporal correlation can modify the existing predicted values and makes the economic dispatching model in ADN more accurate.

2) The proposed optimal dispatch model can improve the regulation ability of the ADN and fully utilize the regulation ability of various regulation resources. Through the coordinated optimization of active and reactive power, the proposed model not only ensures the stability of the system voltage level but also reduces the system operation costs.

2. Spatial-temporal Correlation Modelling of Distributed Generation

Copula function maps the margins of a multi-dimensional random variable (RV) to their joint distribution function. Thus, copula contains the correlation between RVs, so that the correlation of DGs in ADN can be described by copula. One of the bases on which copula is widely used in statistics is Sklar's theorem. According to Sklar's theorem, if $C$ is a copula and $F_i(x_i), i = 1, 2, \cdots, N$ are marginal distribution functions, then the function $F$ defined by (1) is a joint distribution function with margins $F_i(x_i)$.

$$F(x_1, x_2, \cdots, x_N) = C(F_1(x_1), F_2(x_2), \cdots, F_N(x_N))$$  (1)

It is assumed that there are $N$ DGs in the ADN, and the dispatching cycle can be divided into $T$ periods. Since the wind power output at each moment is uncertain, we have $N \times T$ RVs:

$$X_{11}, X_{12}, \cdots, X_{iT}, X_{N1}, X_{N2}, \cdots, X_{NT}$$  (2)

To describe the correlation between different DGs in DN, the marginal distribution function of each DG is obtained by nonparametric estimation based on the historical data of DGs at each time. Secondly, the joint distribution function of DGs is fitted according to Sklar's theorem, and the parameters of $C(\cdot)$ can be determined by maximum likelihood estimation.

$$F(x_{i1}, x_{i2}, \cdots, x_{iT}, \cdots, x_{N1}, \cdots, x_{NT}) = C(F_{i1}(x_{i1}), \cdots, F_{iT}(x_{iT}), \cdots, F_{N1}(x_{N1}), \cdots, F_{NT}(x_{NT}))$$  (3)

Since all marginal distribution function of DGs is included in the calculation, the joint distribution function includes not only the temporal correlation of the same DG but also the spatial-temporal correlation of different DG outputs. The proposed method can accurately capture the spatial-temporal correlation between DG outputs and has lower computational complexity than the general algorithm. This is because, copula allows us to first study the distribution of a single random variable, and then obtain the joint distribution function according to Sklar's theorem.

3. Active and Reactive Power Coordinated Optimal Dispatch Model

3.1. Objective
This paper focuses on the safe and economic operation of ADN. We take the safe operation range of electrical equipment as hard constraints to ensure the safe operation of the ADN. The optimization objective of the proposed model is consistent with the general economic dispatch problem. We minimize the operating cost of the system by optimizing the output of various active-reactive power regulating resources in the ADN. The objective function is as follows.

$$\min C = \sum_T \left( \mu_{\text{Grid},i} P_{\text{Grid},i} + \lambda_i \sum_{ij \in \psi_{\text{down}}} (I_{ij}^{\text{down}})^2 + \sum_{ij \in \psi_{\text{con}}} \mu_{\text{MT},i} P_{\text{MT},i,j} + \sum_{ij \in \psi_{\text{ES}}} \mu_{\text{ES},i} (P_{\text{ch},i,j} + P_{\text{dis},i,j}) \right)$$  \hspace{1cm} (4)

Where $C$ is the total operation cost, $\lambda_i$ and $\mu_{\text{Grid},i}$ are the unit network loss cost and electricity price at t time, $P_{\text{Grid},i}$ is the exchange power between the ADN and the main network, $I_{ij}$ is the current amplitude of $ij$ branch, $r_{ij}$ is the resistance of the $ij$ branch, $\mu_{\text{MT},i}$ and $\mu_{\text{ES},i}$ are the operating costs of micro gas turbines (MGT) and energy storage (ES), respectively, $P_{\text{MT},i,j}$ is the active power output of MGT $i$ at time $t$, $P_{\text{ch},i,j}$ and $P_{\text{dis},i,j}$ are the charge and discharge power of ES, respectively.

3.2. Constraints

ADN needs to provide reliable and high-quality power for users. To ensure the operation performance of the ADN, it is necessary to limit the power flow. At the same time, the regulating resources connected to the network must also be within the normal operating range. When modeling, these limits are described as constraints. In this paper, constraints are divided into three categories: power flow constraints, slow regulating resource operation constraints, and fast regulating resource operation constraints.

3.2.1 Power Flow Constraints

In this paper, the DistFlow branch model is adopted and relaxed to a second-order cone (SOC) constraint. Masoud Farivar and Steven H. Low proved the accuracy of relaxation. The SOC constraint of the power flow equation is relaxed as shown in (5)-(8).

$$P_{ij,t} = \sum_{k \in \psi_{\text{down}}} P_{kj,t} - \sum_{l \in \psi_{\text{up}}} (P_{lj,t} - I_{lj,t}^2 r_{lj})$$ \hspace{1cm} (5)

$$Q_{ij,t} = \sum_{k \in \psi_{\text{down}}} Q_{kj,t} - \sum_{l \in \psi_{\text{up}}} (Q_{lj,t} - I_{lj,t} x_{lj})$$ \hspace{1cm} (6)

$$V_{ij,t}^2 = V_{ij,t}^2 - 2 (P_{ij,t} r_{ij} + Q_{ij,t} x_{ij}) + (r_{ij}^2 + x_{ij}^2) I_{ij,t}^2$$ \hspace{1cm} (7)

$$I_{ij,t}^2 \geq \frac{P_{ij,t}^2 + Q_{ij,t}^2}{V_{ij,t}^2}$$ \hspace{1cm} (8)

$$V_{ij,t}^\text{min} \leq V_{ij,t} \leq V_{ij,t}^\text{max}$$ \hspace{1cm} (9)

$$I_{ij,t} \leq I_{ij,t}^\text{max}$$ \hspace{1cm} (10)

$$\Delta P_{\text{grid}} \leq P_{\text{grid},t} - P_{\text{grid},t-1} \leq \Delta P_{\text{grid}}^\text{max}$$ \hspace{1cm} (11)

$$Q_{\text{grid}}^\text{min} \leq Q_{\text{grid},t} \leq Q_{\text{grid}}^\text{max}$$ \hspace{1cm} (12)

Where $\psi_{\text{down}}, \psi_{\text{up}}$ denote the downstream/upstream bus set of bus $j$, respectively, $P_{\text{ij},t}, Q_{\text{ij},t}$ are the injected active/reactive power of bus $j$, respectively, $P_{\text{ij},t}, Q_{\text{ij},t}, I_{\text{ij},t}$ are the active power, reactive power,
and current amplitude of bus $i$ to $j$, respectively, $V_{ij}, V_{ji}$ are the voltage amplitudes of bus $i,j$, $x_{ij}$ is the reactance of branch $ij$, $V_{ij}^{\min}, V_{ij}^{\max}$, $I_{ij}^{\max}$ are the lower/upper limit of bus voltage amplitude and the upper limit of branch current, $P_{\text{grid},i}, Q_{\text{grid},i}$ are the active and reactive exchange power between the ADN and the main network, $\Delta P_{\text{grid}}^{\min}, \Delta P_{\text{grid}}^{\max}$, $\Delta Q_{\text{grid}}^{\max}$ are the lower and upper limit of the ramping rate of the transmission network, $Q_{\text{grid}}^{\min}, Q_{\text{grid}}^{\max}$ are the lower and upper limit of reactive power of gateway bus, respectively.

Equation (9)-(10) indicate that the amplitude of bus voltage and branch current should be limited to a certain range, to ensure the quality of voltage and current. Equation (11)-(12) limit the exchange power of the gateway bus to reduce the influence of the power fluctuation of the ADN on the upstream network.

3.2.2 Slow Regulating Resources Operation Constraints

Slow regulating resources refers to the equipment that cannot be adjusted quickly many times due to the limitations of the manufacturing process and operation requirements of the device itself. These limitations of running states are expressed as hard constraints. These constraints ensure the normal operation of the equipment and are beneficial to prolong the service life of the equipment. The slow regulating equipment in the ADN mainly includes on-load tap changer (OLTC), capacitor banks, and interruptible load. The constraint of OLTC is non-convex and nonlinear, so it is difficult to solve the model and the accuracy cannot be guaranteed. Wenchuan Wu accurately relaxes the constraints into linear constraints. For the specific form of OLTC constraints, please refer to [11].

$$Q_{\text{CB},t} = N_{\text{CB},t} \times Q_{\text{CB}}^{\text{Step}}$$ (13)

$$N_{\text{CB},t} \leq N_{\text{CB}}^{\text{max}}, N_{\text{CB},t} \in \text{int}$$ (14)

$$B_{\text{CB},t} \in \{0,1\}$$ (15)

$$\sum_{t=1}^{T} B_{\text{CB},j,t} = B_{\text{CB},j}^{\text{lim}}$$ (16)

$$B_{\text{CB},j,t} \times 1 \times Q_{\text{CB}}^{\text{Step}} \leq |Q_{\text{CB},j,t+1} - Q_{\text{CB},j,t}| \leq B_{\text{CB},j,t} \times N_{\text{CB}}^{\text{max}} \times Q_{\text{CB},t}^{\text{Step}}$$ (17)

Equation (13)-(17) are the mathematical model of capacitor banks (CB). $Q_{\text{CB},t}^{\text{Step}}, Q_{\text{CB},t}$ are the step compensation power and the actual compensation power of CB, $N_{\text{CB},t}, N_{\text{CB}}^{\text{max}}$ are the operation number and the maximum number of compensation groups, $B_{\text{CB},j,t}$ is the action indication variable of the CB, $B_{\text{CB},j}^{\text{lim}}$ is the maximum number of actions of the CB in a scheduling cycle.

3.2.3 Fast Regulating Resources Operation Constraints

Fast regulating resources include ES, static var compensator (SVC), and DGs. These devices are continuously adjustable, and the mathematical model is shown below.

$$E_{\text{ES},t+1} = E_{\text{ES},t} + \eta_{\text{charge}} \times I_{\text{charge}}^{\text{charge}} \times \Delta t - \frac{P_{\text{discharge}}}{\eta_{\text{discharge}}} \times \Delta t, t = 1, 2, \cdots, T - 1$$ (18)

$$0 \leq P_{\text{charge}} \leq P_{\text{charge}}^{\max} \times B_{\text{charge}}^{\text{charge}}$$ (19)

$$0 \leq P_{\text{discharge}} \leq P_{\text{discharge}}^{\max} \times B_{\text{discharge}}^{\text{discharge}}$$ (20)

$$B_{\text{charge}}^{\text{charge}}, B_{\text{discharge}}^{\text{discharge}} \in \{0,1\}$$ (21)

$$B_{\text{charge}}^{\text{charge}} + B_{\text{discharge}}^{\text{discharge}} = 1$$ (22)
Equation (18)-(22) are the constraints of the ES system. \( E_{ES,j} \) is the actual amount of energy stored, \( P_{\text{charge}}^{i,j}, P_{\text{discharge}}^{i,j}, \eta_{\text{charge}}, \eta_{\text{discharge}} \) are the charge/discharge power and efficiency of ES. \( \Delta t \) is the time interval corresponding to the scheduling cycle. \( P_{\text{charge}}^{\text{max}}, P_{\text{discharge}}^{\text{max}} \) are the maximum charging and discharging power of ES. \( B_{\text{charge}}^{i,j}, B_{\text{discharge}}^{i,j} \) are the indicative variables of the charge and discharge status of ES, respectively.

\[
Q_{\text{SVC},i}^{\text{min}} \leq Q_{\text{SVC},i,t} \leq Q_{\text{SVC},i}^{\text{max}} \quad (23)
\]

\[
0 \leq P_{\text{DG},i,t} \leq P_{\text{DG},i,t}^{\text{pre}} \quad (24)
\]

\[
(P_{\text{DG},i,t})^2 + (Q_{\text{DG},i,t})^2 \leq (S_{\text{DG},i})^2 \quad (25)
\]

Equation (23) is the constraint of SVC. \( Q_{\text{SVC},i,t} \) is the optimal output of SVC at bus \( i \) time \( t \), \( Q_{\text{SVC},i}^{\text{min}}, Q_{\text{SVC},i}^{\text{max}} \) are the upper and lower limits of SVC. Equation (24)-(25) are the constraints of DGs. \( P_{\text{DG},i,t}, P_{\text{DG},i,t}^{\text{pre}} \) are the active power output and predicted power of DGs, \( Q_{\text{DG},i,t}, S_{\text{DG},i} \) are the reactive power output and maximum apparent power of DGs grid-connected inverter.

4. Case Study

The proposed method is tested on a modified IEEE 33-bus system as shown in Figure 1. The parameters of each piece of equipment can be found in [12]. The model is solved by CPLEX12.10 in MATLAB2018b on a PC with Intel(R) Core(TM)i5-6500 3.20Ghz, 8G RAM.

![Figure 1. Modified IEEE-33 bus distribution system.](image)

To verify the effectiveness and superiority of the proposed method, the optimization results of four different schemes are compared and analyzed in this paper.

- Method 1: Active power optimal dispatch;
- Method 2: Reactive power optimal dispatch;
- Method 3: Active and reactive power coordinated optimal dispatch, regardless of spatial-temporal correlation of wind power;
- Method 4: Active and reactive power coordinated dispatching, considering the spatial-temporal correlation of wind power.

First of all, we make a comparative analysis of method 1, method 2 and method 3, and the results are shown in Table 1.

| Methodology | Electricity cost | MGT Cost | Network loss | Total cost |
|-------------|------------------|----------|--------------|-----------|
| Method 1    | 12104$           | 4312$    | 61.936kW     | 16816$    |
| Method 2    | 17179$           | 0$       | 61.042kW     | 17210$    |
| Method 3    | 12079$           | 4323$    | 42.292kW     | 16423$    |
In this paper, Method 1 means that network operators can utilize ES, MGT to optimize system operation, without considering the regulation role of reactive power compensation devices. Method 2 mainly considers the reactive power compensation capacity of SVC and CB and optimizes their output. Compared with the active power optimal dispatch, the traditional reactive power optimization dispatch has a better effect in reducing the network loss, but the price is to pay more electricity bills. Active power optimal dispatch can provide services for users by using cheap MGT in the ADN. The proposed method (Method 3) in this paper can combine the advantages of the two and achieve the best results. The network loss can be reduced by 31.72%. The comprehensive operating cost is reduced by 4.79% and 2.34% compared with Method 2 and Method 1.

![Network loss comparison of Method 1,2,3](image)

Figure 2 shows the network loss of ADN under the three methods. As can be seen from the figure, the network loss level of method 1 is similar to that of Method 2 and is higher than that of Method 3. In contrast, the network loss level of Method 3 is not only lower but also more balanced during the scheduling cycle. In particular, the network loss during the high load period (17:00-22:00) can be greatly reduced, which greatly improves the operating efficiency of the system.

It is assumed that the forecast output of wind power is accurate the volatility of wind power cannot be ignored. To investigate the impact of wind power output correlation on economic dispatch, this paper employs the method proposed in Section II to capture wind power correlation and uses the scenario method to optimize equipment output and power purchase plan. The results are shown in Table 2.

| Methodology                  | Electricity cost | Network loss | Total cost   |
|------------------------------|------------------|--------------|--------------|
| Method 3                     | 13028$           | 44.375kW     | 17373$       |
| Method 4, t-Copula           | 12094$           | 43.230kW     | 16438$       |
| Method 4, Gaussian-Copula    | 12178$           | 41.139kW     | 16521$       |
| Method 4, Clayton-Copula     | 12221$           | 42.661kW     | 16565$       |

It can be seen from Table 2 that the consideration of wind power correlation can reduce the cost of power purchase, mainly because the consideration of spatial-temporal correlation can reduce the uncertainty of wind power output and reduce the scope of wind power output scene set. Therefore, the dispatching plan can reduce the wind curtailment, and the power purchase plan is more economical. Comparing the optimization results of three kinds of copula functions, we can see that t-copula has the lowest operating cost. This is because the wind power output has the characteristics of the upper thick tail, while the t-copula can accurately describe the thick tail characteristics of the distribution.
Although the form of Gaussian-Copula is simple and widely used, its tail thickness is not enough. Clayton-Copula is not sensitive to the characteristics of the upper thick tail, so the effect is poor.

5. Conclusions
This paper proposed an economic dispatch model of active distribution network considering the spatial-temporal correlation of wind power output. Firstly, the marginal distribution of multi-wind farms is obtained based on historical data and non-parametric estimation method. Secondly, the joint distribution function of wind power output is fitted by copula function, and the parameters are determined by maximum likelihood estimation. In the optimal dispatch, the active and reactive power dynamic optimization is carried out by comprehensively considering various regulation resources in the ADN. The simulation results of a modified IEEE 33-bus system show that the coordinated optimal dispatch of active and reactive power can greatly reduce the level of network loss and improve the operation efficiency of the system compared with a single active and reactive power optimization. Besides, the consideration of the spatial-temporal correlation of wind power output can reduce wind curtailment and optimize the power purchase plan of ADN. Based on the analysis of the optimization results of three kinds of copula functions, it is concluded that t-copula is the most accurate description of the upper thick tail characteristics and spatial-temporal correlation of wind power output.

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