Delamination analysis of multilayered viscoelastic beam exposed to chemical degradation

V Rizov\(^1\)

\(^1\)Department of Technical Mechanics, University of Architecture, Civil Engineering and Geodesy, 1046 - Sofia, Bulgaria

E-mail: v_rizov_fhe@uacg.bg

Abstract. Delamination of multilayered viscoelastic non-homogeneous beam structure subjected to chemical degradation is analysed in this paper. The creep behaviour is treated by using a linear viscoelastic model. The influence of chemical degradation is taken into account by decreasing the elasticity modulus and the coefficient of viscosity with time. The time-dependent delamination fracture under chemical degradation is studied by analysing the strain energy release rate. The strain energy is used to obtain the strain energy release rate that accounts for the creep and for the chemical degradation. The solution is verified by applying the compliance method with considering of the chemical degradation.

1. Introduction

Multilayered non-homogeneous structural materials are made of adhesively bonded layers of different non-homogeneous materials (for example, layers of fibre reinforced polymer composites). Very modern non-homogeneous materials are functionally graded materials [1, 2, 3, 4, 5]. These novel materials have a wide application in current engineering mainly due to their specific behaviour involving high strength-to-weight and stiffness-to-weight ratios [6, 7, 8, 9, 10, 11]. Therefore, multilayered non-homogeneous materials are used in advanced load-bearing structures where low weight is of special significance [12, 13, 14, 15, 16, 17].

It should be noted, however, that multilayered materials are employed with caution because of their poor delamination fracture behaviour [18, 19, 20, 21, 22, 23, 24, 25]. The delamination cracks drastically reduce the load-bearing capacity of the multilayered structure. Thus, the safety and reliability of multilayered non-homogeneous constructions depend largely on their delamination fracture behaviour.

The goal of the paper is to analyse the delamination fracture of a multilayered non-homogeneous viscoelastic beam configuration that is exposed to influence of aggressive chemical agent. It should be mentioned that such analysis is important since multilayered beam structures frequently are in contact with various chemical substances in their life-time. It should also be mentioned that previous publications do not consider the influence of chemical degradation on time-dependent delamination of non-homogeneous viscoelastic beam structural components [26, 27, 28]. Therefore, a time-dependent analytical expression of the strain energy release rate that accounts for the chemical degradation of the viscoelastic multilayered non-homogeneous material in a delaminated beam is presented in the paper.
2. Influence of chemical degradation on delamination of multilayered viscoelastic beam

The delamination crack in the multilayered non-homogeneous viscoelastic beam shown in figure 1 is under consideration. The crack has length, \( a \). The lower and the upper crack arms have thickness, \( h_1 \) and \( h_2 \), respectively. The beam has width, \( b \), and thickness, \( h \). The beam is with length, \( l \).

![Figure 1. The multilayered non-homogeneous viscoelastic beam configuration.](image)

The right-hand end of the beam is clamped. Two vertical forces, \( F_1 \) and \( F_2 \), are applied at the free end of the beam (figure 1). The creep behaviour of the \( i \)-th beam layer is treated by a linear viscoelastic model consisting of a spring in a series with a dashpot (damper) (figure 2). For this viscoelastic model, the stress-strain-time relationship under constant stress is written as

\[
\varepsilon = \frac{\sigma_i}{E_i} + \frac{\sigma_i f}{\eta_i} \quad (1)
\]

where \( \varepsilon \) is strain, \( t \) is time, \( \sigma_i \), \( E_i \) and \( \eta_i \) are the stress, the elasticity modulus and the coefficient of viscosity in the \( i \)-th layer, respectively. The layers are smoothly non-homogeneous along the beam width. Therefore, the elasticity modulus of the spring and the coefficient of viscosity of the dashpot in the \( i \)-th layer are distributed continuously along the width. The distributions are written as

\[
E_i = \frac{E_{i_L}}{1 + p_i \frac{b + 2y}{b}} \quad (2a)
\]

\[
\eta_i = \frac{\eta_{i_L}}{1 + q_i \frac{b + 2y}{b}} \quad (2b)
\]

for

\[-b/2 \leq y \leq 0\]
and

\[ E_i = \frac{E_{L_i}}{1 + p_{L_i} \frac{b - 2y}{b}} \quad (3a) \]

\[ \eta_i = \frac{\eta_{L_i}}{1 + q_{L_i} \frac{b - 2y}{b}} \quad (3b) \]

for

\[ 0 \leq y \leq b/2. \]

In formulae (2) and (3), \( E_{L_i} \) and \( \eta_{L_i} \) are, respectively, the values of \( E_i \) and \( \eta_i \) at the lateral surfaces of the beam, \( p_{L_i} \) and \( q_{L_i} \) are material properties that control the non-homogeneity in the width direction, \( y \) is the horizontal central axis of the beam cross-section. According to formulae (2) and (3), \( E_i \) and \( \eta_i \) are distributed symmetrically with respect to \( y = 0 \).

![Figure 2. Viscoelastic model.](image)

The multilayered non-homogeneous viscoelastic beam structure is exposed to the influence of aggressive chemical agent which leads to chemical degradation of the material in layers. Thus, \( E_{L_i} \) and \( \eta_{L_i} \) decrease with time on exponential laws [30]:

\[ E_{L_i} = E_{L_i,0} e^{-\beta_{E_i}y - \psi_{E_i}y^\sqrt{r}} \quad (4a) \]

\[ \eta_{L_i} = \eta_{L_i,0} e^{-\beta_{\eta}y - \psi_{\eta}y^\sqrt{r}} \quad (4b) \]

where \( E_{L_i,0} \) and \( \eta_{L_i,0} \) are, respectively, the values of \( E_{L_i} \) and \( \eta_{L_i} \) at the initiation of the chemical degradation, \( \beta_{E_i} \), \( \psi_{E_i} \), \( \beta_{\eta} \) and \( \psi_{\eta} \) are properties that describe the process of decrease of the elasticity modulus and the coefficient of viscosity in the layer with time.

The delamination fracture behaviour is analysed by investigating of the strain energy release rate with considering the chemical degradation of the material. The strain energy release rate, \( G \), that takes into account the chemical degradation is obtained by using the formula

\[ G = \frac{dU}{bda} \quad (5) \]

where \( U \) is the strain energy in the beam structure, \( da \) is an elementary increase of the length of the
delamination. The strain energy is written as
\[ U = U_1 + U_2 + U_3 \]  
(6)
where \( U_1 \), \( U_2 \) and \( U_3 \) are the strain energies in the lower and upper arms of the delamination and in the un-cracked beam portion, \( a \leq x \leq l \), where \( x \) is the longitudinal centroidal axis (figure1). The strain energy in the lower arm of the delamination is found as
\[ U_1 = \sum_{i=1}^{n_1} \int_{V_i} u_{0i} \, dV \]  
(7)
where \( u_{0i} \) is the density of the strain energy, \( V_i \) is the volume of the \( i \)-th layer of the lower arm of the delamination, and \( n_1 \) is the number of layers in this delamination arm.

The density of the strain energy in the \( i \)-th layer is expressed as
\[ u_{0i} = \frac{1}{2} \sigma_i \epsilon \]  
(8)
By substituting (1) in (8), one obtains
\[ u_{0i} = \frac{\sigma_i^2}{2E_i} + \frac{\sigma_i^2 l}{2\eta_i} \]  
(9)
The distribution of strains along the thickness of the lower arm of the delamination is written as
\[ \epsilon = \kappa_1 (z_1 - z_{in}) \]  
(10)
where \( \kappa_1 \), \( z_{in} \) and \( z_1 \) are the curvature, the neutral axis coordinate and the vertical centric axis of the cross-section, respectively. Formula (10) follows from the fact that validity of the Bernoulli’s hypothesis for plane section is assumed since the beam under consideration has a high length to thickness ratio. The curvature and the neutral axis coordinate are determined by using the equations of equilibrium of the lower delamination arm.

The curvature, the neutral axis coordinate and the strain energy in the upper crack arm and in the un-cracked beam portion are found in analogous manner.

It should be noted that solution (5) is time-dependent since the strain energy densities in the layers are functions of the time.

The strain energy release rate that accounts for the chemical degradation of the viscoelastic multilayered beam is found also through the compliance method. According to this method, the strain energy release rate is written as
\[ G = \frac{1}{2b} \left( F_1^2 \frac{dC_{F_1}}{da} + F_2^2 \frac{dC_{F_2}}{da} \right) \]  
(11)
where the beam compliances, \( C_{F_1} \) and \( C_{F_2} \), are expressed as
\[ C_{F_1} = \frac{w_1}{F_1} \]  
(12a)
\[ C_{F_2} = \frac{w_2}{F_2} \]  
(12b)
In formulae (12), \( w_1 \) and \( w_2 \) are the vertical displacements of the application points of the external forces, \( F_1 \) and \( F_2 \), respectively. These vertical displacements are found by applying the integrals of Maxwell-Mohr. The result is

\[
\begin{align*}
    w_1 &= \int_{0}^{a} \kappa_1 x \, dx + \int_{a}^{l} k_1 x \, dx \\
    w_2 &= \int_{0}^{a} \kappa_2 x \, dx + \int_{a}^{l} k_3 x \, dx
\end{align*}
\]  

(13a)  
(13b)

where \( k_2 \) and \( \kappa_3 \) are the curvatures of the upper crack arm and the un-cracked beam portion, respectively.

The strain energy release rate obtained by (11) is in excellent agreement with that calculated by (5) which prove the correctness of analysis performed.

3. Numerical results

The time-dependent analysis described in section 2 of the paper is applied here in order to evaluate the influence of the chemical degradation of the multilayered viscoelastic material on the delamination fracture behaviour.

![Figure 3](image)

**Figure 3.** The non-dimensional strain energy release rate presented as a function of non-dimensional time (curve 1 – for multilayered inhomogeneous viscoelastic beam exposed to chemical degradation, curve 2 – for multilayered inhomogeneous viscoelastic beam that is not exposed to chemical degradation).

The strain energy release rate is expressed in non-dimensional form by using the formula
\[ G_N = \frac{G}{(E_{L0} b)}. \]

Calculations of the strain energy release rate are carried out for a three-layered cantilever beam exposed to chemical degradation. The thickness of each layer is \( h_d \). The following data are used: \( b = 0.020 \) m, \( h_d = 0.008 \) m, \( h = 0.024 \) m, \( l = 0.250 \) m, \( a = 0.100 \) m, \( F_1 = 2 \) N and \( F_2 = 3 \) N.

The change of the strain energy release rate with time is analysed. For this purpose, calculations of the strain energy release rate are performed at various values of time. The results obtained are illustrated in figure 3 where the non-dimensional strain energy release rate is presented as a function of the non-dimensional time. The curves in figure 3 indicate that the strain energy release rate increases with time. This behaviour is due to the creep and to the chemical degradation of the multilayered viscoelastic material. The change of the strain energy release rate with time is analysed also assuming that chemical degradation does not occur. For this purpose, calculations are carried out after substituting of \( \beta_{Ei} = 0, \psi_{Ei} = 0, \beta_{\eta} = 0 \) and \( \psi_{\eta} = 0 \) in (5). The results obtained are shown in figure 3. It can be observed in figure 3 that the chemical degradation leads to increase of the strain energy release rate. This finding indicates that chemical degradation has to be taken into account in fracture mechanics based structural design of multilayered non-homogeneous viscoelastic beams.

![Figure 4](image-url). Figure 4. The non-dimensional strain energy release rate presented as a function of \( p_{i,j} \) (curve 1 – at \( q_{Li} = 0.5 \), curve 2 – at \( q_{Li} = 1.0 \) and curve 3 – at \( q_{Li} = 2.0 \)).

It is analysed also how the material non-homogeneity along the thickness of layers influences delamination of multilayered viscoelastic beam subjected to chemical degradation. For this purpose, calculations of the strain energy release rate are performed at various values of \( p_{Li} \) and \( q_{Li} \). The influence of material non-homogeneity on the delamination is displayed in figure 4 where the non-
dimensional strain energy release rate is presented as a function of $p_{L_i}$ at three values of $q_{L_i}$. The curves in figure 4 show that the strain energy release rate increases with increasing of $p_{L_i}$ when the beam is exposed to chemical degradation. The increase of material property, $q_{L_i}$, leads also to increase of the strain energy release rate under chemical degradation (figure 4).

4. Conclusions
A time-dependent delamination analysis of a multilayered non-homogeneous viscoelastic beam configuration subjected to chemical degradation is developed. The beam is made of adhesively bonded non-homogeneous viscoelastic layers of different thicknesses and material properties. The beam is exposed to influence of aggressive chemical agent. This leads to chemical degradation of the material in layers of the beam. As a result of the chemical degradation, the elasticity modulus and the coefficient of viscosity decrease with time. The time-dependent delamination fracture behaviour is studied by analysing the strain energy release rate with considering of the chemical degradation. For this purpose, the strain energy is used to derive the strain energy release rate. The strain energy is calculated by taking into account the chemical degradation. In this way, the solution obtained accounts for both the creep behaviour of the non-homogeneous structure and the chemical degradation of the material. The compliance method with considering of the chemical degradation of the multilayered non-homogeneous viscoelastic material is applied to verify the solution. The change of the strain energy release rate with time is analysed. The analysis reveals that the strain energy release rate increases with time (this behaviour is due to the creep and to the chemical degradation). Concerning the influence of material non-homogeneity, the calculations show that the strain energy release rate increases with increasing of $p_{L_i}$ and $q_{L_i}$ under chemical degradation.

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