Phase Fluctuations and Pseudogap Properties: Influence of Nonmagnetic Impurities

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Abstract

The presence of nonmagnetic impurities in a 2D “bad metal” depresses the superconducting Berezinskii-Kosterlitz-Thouless transition temperature, while leaving the pairing energy scale unchanged. Thus the region of the pseudogap non-superconducting phase, where the modulus of the order parameter is non-zero but its phase is random, and which opens at the pairing temperature is substantially bigger than for the clean system. This supports the premise that fluctuations in the phase of the order parameter can in principle describe the pseudogap phenomena in high-$T_c$ materials over a rather wide range of temperatures and carrier densities. The temperature dependence of the bare superfluid density is also discussed.

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1 Introduction

The differences between the BCS scenario of superconductivity and superconductivity in high-$T_c$ materials are well accepted as experimental facts, although there is no theoretical consensus about their origin. One of the most convincing manifestations is the pseudogap, or a depletion of the single particle spectral weight around the Fermi level (see for example, \cite{1}). Another transparent manifestation is the temperature and carrier density dependencies of the superfluid density in high-$T_c$ superconductors (HTSC) \cite{2, 3, 4} which do not fit the canonical BCS behaviour. In particular the value of the zero temperature superfluid density is substantially less than the total density of doped carriers \cite{5}. Currently there are many possible explanations for the unusual properties of HTSC. One of these is based on the nearly antiferromagnetic Fermi liquid model \cite{6}. Another explanation, proposed by Anderson, relies on the separation of spin

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and charge degrees of freedom. One more approach, which we will follow in this paper, relates the observed anomalies to precursor superconducting fluctuations. Different authors argue that alternative types of superconducting fluctuations are responsible for the pseudogap, e.g. [7], and the scenario based on the fluctuations of the order parameter phase suggested by Emery and Kivelson [8]. The latter scenario we believe to be more relevant due to low superfluid density and practically 2D character of the conductivity in HTSC mentioned above. A microscopic 2D model which elaborates the abovementioned scenario [8] has been studied in the papers [9, 10]. The results obtained show that the condensate phase fluctuations indeed lead to features which are experimentally observed in HTSC both in the normal and superconducting states [1]. It is obvious, however, that the present treatment of the phase fluctuations is incomplete due both to the oversimplified character of the model and the absence of an explanation for the more recent advanced experiments [2, 3, 4] on the temperature and doping dependencies of the superfluid density. It is well known, however, that the theoretical study of HTSC faces a lot of computational difficulties due to, for example, a non-conventional order parameter symmetry, complex frequency-momentum dependence of the effective quasiparticle attraction, general form of the quasiparticle dispersion law, etc. Therefore, in order to obtain analytical results, we have to date only considered nonretarded $s$-wave pairing in the absence of impurities. (The attempts to consider the retardation effects were done in [11].)

Nevertheless, a discussion of the effect of impurities seems to be crucial for a realistic model of the HTSC. Indeed, it is known, that the itinerant holes in HTSC are created by doping which in turn introduces a considerable disorder into the system, for instance, from the random Coulomb fields of chaotically distributed charged impurities (doped ions) [12]. Thus one the purposes of the present paper is to study the model [9, 10] but in the presence of nonmagnetic impurities.

In the theory of “common metals” the Fermi energy $\epsilon_F$ and the mean transport quasiparticle time $\tau_{tr}$ are independent quantities which are always assumed to satisfy the criterion $\epsilon_F\tau_{tr} \gg 1$. In HTSC which are “bad metals” [8] both $\epsilon_F$ and $\tau_{tr}$ are dependent on the doping and the abovementioned criterion may fail [12]. As an illustration we refer to the remarkable linear dependence of the normal state resistivity [1] which implies indeed that $\epsilon_F\tau_{tr}$ may be $\sim 1$. It has been shown [2] that for strongly disordered metallic system superconductivity is absent if the scattering-to-pairing ratio exceeds a critical value and the existence of superconductivity in a finite range of doping if this ratio is not exceeded. We shall not study this case but rather consider here the more usual (and in some sense simple) situation, originally studied in the papers of Anderson [13] and Abrikosov-Gor’kov (AG) [14] (see also [9]), when the superconducting order is preexisting and the criterion $\epsilon_F\tau_{tr} \gg 1$ is satisfied.

The Anderson theorem [13] states that in 3D the BCS critical temperature is unchanged in the presence of nonmagnetic impurities. However, as discussed in [8], the BCS critical temperature in 2D is the temperature $T^*$ at which the pseudogap opens, while the superconducting transition temperature transition is the temperature $T_{BKT}$ of the Berezinskii-Kosterlitz-Thouless (BKT) transition. In contrast to the former, the latter is defined by a bare superfluid density (given by the delocalized carriers) which is dependent on (see below) the concentration of impurities. Thus in 2D case the superconducting transition temperature, $T_{BKT}$ decreases with

\footnote{Of course, the contribution from the phase fluctuations need not be the only or even the major contribution.}
increasing impurity concentration.

Thus, in the model under consideration, the relative size of the pseudogap phase, \((T^* - T_{\text{BKT}})/T^*\) is larger in the presence of impurities than in the clean limit. Therefore it can be observed over a wider range of densities. The second result obtained is that the value of the zero temperature superfluid density is less than the total density of carriers (dopants), so that the presence of impurities may contribute into this diminishing and in its turn explain the experimental results. Finally we attempt to interpret qualitatively the recent experiments on the temperature dependence of the superfluid density within our scenario.

A brief overview of the paper follows: In Sec. 2 we present the model and derive the main equations. In Sec. 3 we compare the results obtained for clean and dirty limits. In particular we compare the values of \(T_{\text{BKT}}\), the relative sizes of the pseudogap region and the values of the bare superfluid density at \(T = 0\) and \(T\) close to \(T_\rho\). In Sec. 4 an attempt is made to give an explanation for the experimental results.

\section{Model and main equations}

Our starting point is a continuum version of the two-dimensional attractive Hubbard model defined by the Hamiltonian:

\[
H = \int d^2r \left[ \psi_\sigma^\dagger(x) \left( -\nabla^2 - \frac{\Sigma^2}{2m} - \mu \right) \psi_\sigma(x) - V \psi_\uparrow^\dagger(x) \psi_\downarrow^\dagger(x) \psi_\downarrow(x) \psi_\uparrow(x) \right] + U_{\text{imp}}(r) \psi_\sigma^\dagger(x) \psi_\sigma(x),
\]

where \(x = r, \tau\) denotes the space and imaginary time variables, \(\psi_\sigma(x)\) is a fermion field with spin \(\sigma = \uparrow, \downarrow\), \(m\) is the effective fermion mass, \(\mu\) is the chemical potential, \(V\) is an effective local attraction constant, and \(U_{\text{imp}}(r)\) is the static potential of randomly distributed impurities; we take \(\hbar = k_B = 1\). The model with the Hamiltonian is equivalent to a model with an auxiliary BCS-like pairing field which is given in terms of the Nambu variables as

\[
H = \int d^2r \left\{ \left[ \tau_3 \left( -\nabla^2 - \frac{\Sigma^2}{2m} - \mu \right) \right] \right\} \Psi(x) + \frac{|\Phi(x)|^2}{V},
\]

where \(\tau_\pm = (\tau_1 \pm i\tau_2)/2\), \(\tau_3\) are the Pauli matrices, and \(\Phi(x) = V \Psi^\dagger(x) \tau_\pm \Psi(x) = V \psi_\downarrow(x) \psi_\uparrow(x)\) is the complex order field. Then the partition function can be presented as a functional integral over Fermi fields (Nambu spinors) and the auxiliary fields \(\Phi, \Phi^*\).

However, in contrast to the usual method, the modulus-phase parametrization \(\Phi(x) = \rho(x) \exp[i\theta(x)]\) is necessary for the 2D model at finite temperatures (see and references therein). To be consistent with this replacement one should also introduce the spin-charge variables for the Nambu spinors

\[
\Psi(x) = \exp[i\tau_3 \theta(x)/2] \Upsilon(x)
\]
with \( \Upsilon(x) \) the field operator for neutral fermions.

From the Hamiltonian \((\ref{eq:hamiltonian})\), following [9], one can derive an effective one which is the Hamiltonian of the classical XY-model

\[
H_{XY} = \frac{1}{2} J(\mu, T, \rho) \int d^2 r |\nabla \theta(x)|^2
\]

where

\[
J(\mu, T, \rho) = \frac{T}{16m\pi^2} \sum_{n=-\infty}^{\infty} \int d^2 k \text{tr} \{ \tau_3 \langle G(i\omega_n, k) \rangle \}
\]

\[
+ \frac{T}{32m^2\pi^2} \sum_{n=-\infty}^{\infty} \int d^2 k k^2 \text{tr} \{ \langle G(i\omega_n, k) \rangle \langle G(i\omega_n, k) \rangle \}
\]

is the bare (i.e. unrenormalized by the phase fluctuations, but including pair breaking thermal fluctuations) superfluid stiffness. Here

\[
\langle G(i\omega_n, k) \rangle = \frac{(i\omega_n \hat{I} - \tau_1 \rho) \eta_n + \tau_3 \xi(k)}{(\omega_n^2 + \rho^2) \eta_n^2 + \xi^2(k)}
\]

with

\[
\eta_n = 1 + \frac{1}{2\tau_r \sqrt{\omega_n^2 + \rho^2}}, \quad \xi(k) = \frac{k^2}{2m} - \mu, \quad \omega_n = \pi(2n + 1)T
\]

is the AG [15] Green’s function of neutral fermions averaged over a random distribution of impurities and written in the Nambu representation [17, 18]. In writing (5) we assumed that \( \langle G(i\omega_n, k)G(i\omega_n, k) \rangle \simeq \langle G(i\omega_n, k) \rangle \langle G(i\omega_n, k) \rangle \). This approximation, as shown by AG [15], does not change the final result for \( J \). Note also that the Green’s function (6) is valid only when \( \epsilon_F \tau_r \gg 1 \) which demands the presence of a well developed Fermi surface, which in turn implies that \( \mu \simeq \epsilon_F \). Thus one cannot use the expression (6) in the so called Bose limit with \( \mu < 0 \) [9]. On the other hand, Fermi surface can be formed even in the bad metals when Ioffe-Regel-Mott criterion proves to be fulfilled [12].

Substituting (6) into (5), and using the inequalities \( \mu \gg T, \rho \) to extend the limits of integration to infinity, one arrives at

\[
J = \frac{\mu}{4\pi} + \frac{T\mu}{4\pi} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} dx \left( \frac{1}{x^2 + (\omega_n^2 + \rho^2) \eta_n^2} - \frac{2\omega_n^2 \eta_n^2}{[x^2 + (\omega_n^2 + \rho^2) \eta_n^2]^2} \right).
\]

(8)

Eq. (8) is formally divergent and demands special care due to the fact that one has to perform the integration over \( x \) before the summation [15]. Finally one can formally cancel the divergence [15] to obtain

\[
J = \frac{\mu \rho^2 T}{4} \sum_{n=-\infty}^{\infty} \frac{1}{(\omega_n^2 + \rho^2) \left[ \sqrt{\omega_n^2 + \rho^2} + \frac{1}{2\tau_r} \right]}.
\]

(9)
The temperature of the BKT transition for the XY-model Hamiltonian \( H \) is determined by the equation
\[
T_{\text{BKT}} = \frac{\pi}{2} J(\mu, T_{\text{BKT}}, \rho(\mu, T_{\text{BKT}})).
\]
(10)

The self-consistent calculation of \( T_{\text{BKT}} \) as a function of the carrier density \( n_f = m \epsilon_F / \pi \) requires additional equations for \( \rho \) and \( \mu \), which together with (10) form a complete set [9].

When the modulus of the order parameter \( \rho(x) \) is treated in the mean field approximation, the equation for \( \rho \) takes the following form [9]
\[
\frac{2\rho}{V} = \sum_{n=-\infty}^{\infty} \int \frac{d^2k}{(2\pi)^2} \text{tr}[\tau_1 \langle G(i\omega_n, k) \rangle],
\]
(11)
which formally coincides with the gap equation of the BCS theory. This coincidence allows one to use the Anderson theorem [13] which states the dependence of \( \rho(T) \) is the same as that for the clean superconductor and not affected by the presence of nonmagnetic impurities. It is important to recall that this theorem is, of course, valid only for the \( s \)-wave pairing and small disorder.

There is, however, both physical and mathematical differences between the gap in the BCS theory and \( \rho \) [9, 10]. In particular the temperature \( T_{\rho} \) which is estimated by the condition \( \rho = 0 \) is not related to the temperature of the superconducting transition, but is interpreted as the temperature of pseudogap opening \( T^* \) (see details in [9]). The main point, which we would like only to stress here, is that due to the Anderson theorem [13] the value of \( T_{\rho} \) does not depend on the presence of impurities, while the temperature \( T_{\text{BKT}} \), as we will show, is lowered.

The chemical potential \( \mu \) is defined by the number equation
\[
\sum_{n=-\infty}^{\infty} \int \frac{d^2k}{(2\pi)^2} \text{tr}[\tau_3 \langle G(i\omega_n, k) \rangle] = n_f.
\]
(12)
Since we are interested in the high carrier density region the solution of (12) is \( \mu \simeq \epsilon_F \), so that in Eqs.(9) - (11) one can replace \( \mu \) by \( \epsilon_F \).

Having the temperatures \( T_{\rho} \) and \( T_{\text{BKT}} \) as functions of the carrier density one can build the phase diagram of the model [9] which consists of three regions. The first one is the superconducting (here BKT) phase with \( \rho \neq 0 \) at \( T < T_{\text{BKT}} \). In this region there is algebraic order, or a power law decay of the \( \langle \Phi^* \Phi \rangle \) correlations. The second region corresponds to the pseudogap phase (\( T_{\text{BKT}} < T < T_{\rho} \)). In this phase \( \rho \) is still non-zero but the correlations mentioned above decay exponentially. The third is the normal (Fermi-liquid) phase at \( T > T_{\rho} \) where \( \rho = 0 \). Note that \( \langle \Phi(x) \rangle = 0 \) everywhere. While the given phase diagram was derived for the idealized 2D model, there are indications that even for as complicated layered systems as HTSC the value of the critical temperature for them may be well estimated using \( T_{\text{BKT}} \) [19, 20] even though the transition undoubtedly belongs to the 3D XY class. It was also pointed out in [19] that a nonzero gap in the one-particle excitation spectrum can persists even without long-range order.
3 The comparison of clean and dirty limits

3.1 Clean limit

The transport time $\tau_{tr}$ is infinite in the clean limit, so that

$$J(\epsilon_F, T, \rho(\epsilon_F, T)) = \frac{\epsilon_F \rho^2 T}{4} \sum_{n=-\infty}^{\infty} \frac{1}{(\omega_n^2 + \rho^2)^{3/2}}.$$  \hfill (13)

Near $T_\rho$ one can obtain from (13)

$$J(\epsilon_F, T \rightarrow T_\rho^-, \rho \rightarrow 0) = \frac{7\zeta(3)}{16\pi^3} \frac{\rho^2}{T_\rho^2} \epsilon_F,$$  \hfill (14)

where $\zeta(x)$ is the zeta function. This expression must coincide with the result from [9] which was derived using the opposite order for the summation and integration. Inserting the well-known dependence of $\rho(T)$ (see, for example [21])

$$\rho^2(T \rightarrow T_\rho^-) = \frac{8\pi^2}{7\zeta(3)} T_\rho^2 \left(1 - \frac{T}{T_\rho}\right)$$  \hfill (15)

and then substituting (14) into (10) one obtains the following asymptotic expression for the BKT temperature in the clean limit for high carrier densities [22, 9, 23]

$$T_{\text{BKT}} = T_\rho \left(1 - \frac{4T_\rho}{\epsilon_F}\right), \quad T_{\text{BKT}} \lesssim T_\rho.$$  \hfill (16)

In the high density limit one can also use the equation

$$T_\rho = \frac{\gamma}{\pi} \sqrt{2|\varepsilon_b| \epsilon_F},$$  \hfill (17)

where $\gamma \simeq 1.781$ and $\varepsilon_b$ is the energy of the two-particle bound state in vacuum which is a more convenient parameter than the four-fermion constant $V$ [24, 3].

It is obvious from (13) and (17) that the pseudogap region shrinks rapidly for high carrier densities [4] and one may ask (see, for example, [23]) whether this scenario can explain the pseudogap anomalies which are observed over a wide range of temperatures and carrier densities, since in the clean limit the relative size of the pseudogap region $(T_\rho - T_{\text{BKT}})/T_\rho$ is, for instance, less than 1/2 when the dimensionless ratio $\epsilon_F/|\varepsilon_b| \lesssim 128\gamma^2/\pi^2 \approx 41$. A crude estimate for the dimensionless ratio for optimally doped cuprates gives $\epsilon_F/|\varepsilon_b| \sim 3 \cdot 10^2 - 10^3$ [26] which indicates that in the clean superconductor the pseudogap region produced by the phase fluctuations is too small. Of course, all these estimations are qualitative due to the simplicity of the model.

The value of the bare superfluid density, $n_s(T)$, is straightforwardly expressed via the bare phase stiffness, $n_s(T) = 4mJ(T)$. In particular, it follows from (13) that $n_s(T = 0) = n_f$. This is not surprising since $n_s(T = 0)$ must be equal to the full density $n_f$ for any superfluid ground state in a translationary invariant system [27] and the clean system is translationary invariant.

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2 In 2D for s-wave pairing the high density limit is in fact equivalent the weak coupling BCS limit.
We note, however, as stated above that in HTSC \( n_s(T = 0) \ll n_f \) \[3\]. Substituting (15) into (13) one obtains for \( T \) close to \( T_{\rho} \) the bare superfluid density as
\[ n_s(T \to T_{\rho}^-) = 2n_f(1 - T/T_{\rho}). \]
This behaviour of the bare superfluid density is formally the same as the behaviour of the full superfluid density in the BCS theory. Nevertheless it is important to remember that the full superfluid density in the present model undergoes the Nelson-Kosterlitz jump at \( T_{\text{BKT}} \) and is zero for \( T > T_{\text{BKT}} \). We note that one can probe experimentally both the bare superfluid density in high-frequency measurements \[4\] and the full superfluid density in low-frequency measurements \[3\].

3.2 Dirty limit

In the dirty limit the quasiparticle transport time \( \tau_{tr} \) is small (\( \tau_{tr} \ll \rho^{-1}(T = 0) \)) so that one can neglect the radical in the bracket of (9) \[15\]. The remaining series is easily summed and one obtains for the bare superfluid stiffness
\[ J(\epsilon_F, T, \rho(\epsilon_F, T), \tau_{tr}) = \frac{\epsilon_F \tau_{tr} \rho}{4} \tanh\left(\frac{\rho}{2T}\right). \]
(18)

As explained above, due to the Anderson theorem, the expressions (15) for \( \rho \) and (17) for \( T_{\rho} \) remain unchanged in the presence of impurities. Again substituting (15) into (18) one obtains
\[ T_{\text{BKT}} = T_{\rho} \left(1 - \frac{14\zeta(3)}{\pi^3} \frac{1}{\epsilon_F \tau_{tr}}\right) , \quad T_{\text{BKT}} \lesssim T_{\rho}. \]
(19)

One can see that the size of the pseudogap region is now controlled by the new phenomenological parameter \( \tau_{tr} \) which is an unknown function of \( \epsilon_F \) for HTSC. The experimental data \[1\] suggest that \( \tau_{tr} \) is almost independent on doping level in the underdoped region.

It is difficult to obtain more than a qualitative estimate using Eq.(19) since in its derivation we have assumed that \( \epsilon_F \tau_{tr} \gg 1 \). In HTSC however, as discussed above (see also [12]), this assumption is not always justified. Bearing in mind that the dirty limit implies that the condition \( \tau_{tr}^{-1} \gg \rho(T = 0) \sim T_{\rho} \) is satisfied, one can easily see that the value of \( T_{\text{BKT}} \) for this case is less than that given by (16) for the clean superconductor. Since impurities are inevitably present in HTSC, phase fluctuations can in fact give rise to a pseudogap region that is of comparable size to that experimentally observed. We note that our arguments are in fact quite similar to that given in [22] for the best observing conditions for the BKT physics in superconducting films. However, in contrast to this paper, the gap opening below \( T_{\rho} \) is particularly emphasized here.

While Eq.(19) was derived under assumption \( T_{\text{BKT}} \lesssim T_{\rho} \) in general case when \( T_{\text{BKT}} \) can be substantially less than \( T_{\rho} \) the self-consistent equation (11) with \( J(\epsilon_F, T_{\text{BKT}}, \rho(\epsilon_F, T_{\text{BKT}})) \) given by (18) must be solved. Recall, however, that to make any quantitative estimates, the more realistic \( d \)-wave model has to be considered and the inequality \( \epsilon_F \tau_{tr} \gg 1 \) should not be assumed \[12\].

The value of the zero temperature superfluid density is now given by \( n_s(T = 0) = \pi n_f \tau_{tr} \rho \ll n_f \) since \( \tau_{tr} \rho \ll 1 \). This does not contradict the results of [27] because the system is not translationary invariant in the presence of impurities \[28\]. Furthermore as one can see, the low value of the superfluid density in HTSC \[3\] may be related to the impurities which are inevitably present in HTSC. Another reason which leads to lowering of the superfluid density is
the presence of lattice which also destroys a continuous translational invariance. We note that as was pointed quantum fluctuations also lead to a decrease in the superfluid density.

4 The temperature dependence of the bare superfluid density

In this section we try to correlate the temperature dependence of the observed in-plane resistivity $\rho_{ab}(T)$ with the recently measured temperature dependence of the bare superfluid density [2].

For $T > T_{\text{BKT}}$ the expression for the bare superfluid density in the dirty limit (18) can be rewritten in terms of the in-plane conductivity, $\sigma = e^2n_f\tau_e/m$, where $e$ is the charge of electron:

$$J(\sigma(\epsilon_F, T), \rho(\epsilon_F, T)) = \frac{\pi \sigma \rho}{4 e^2} \tanh \frac{\rho}{2T}.$$ (20)

The in-plane resistivity, $\rho_{ab} \sim \sigma^{-1}$ in cuprates has been extensively studied [1] and its temperature and concentration dependencies must reflect the pseudogap properties observed in other experiments. One can say that $\rho_{ab}(T)$ is linear above $T^* \simeq T_\rho$ and roughly linear between $T_{\text{BKT}}$ and $T_\rho$ but with a lower slope. Thus in the interval $T_{\text{BKT}} < T < T_\rho$ the resistance can be approximately written as $\rho_{ab}(T) = aT + b$, where $a$ and $b$ are functions of $\epsilon_F$ but not of temperature.

Now, substituting $\sigma \sim \rho_{ab}^{-1}(T)$ into Eq.(21), one obtains

$$n_s(T) \sim \frac{\rho}{aT + b} \tanh \frac{\rho}{2T}.$$ (21)

Our estimations based on Eq.(21) are shown in Fig. 1. One can see that, in contrast to the almost linear BCS dependence of $n_s(T)$, we have convex behaviour and the superfluid density becomes zero at $T_\rho$. We stress that the curvature of $n_s(T)$ is the result of both the temperature dependence of $\rho(T)$ and $\sigma(T)$ for $T_{\text{BKT}} < T < T_\rho$. More importantly the slope of the curve $n_s(T)$ at $T_\rho$ for the dirty metal is substantially less than for the clean one. The experiment [2] shows the same curvature for $n_s(T)$ but indicates that the bare superfluid density disappears at a lower temperature, $T_s < T^*$. Since the slope $dn_s(T)/dT$ at $T_\rho$ is very small, as predicted by Eq.(21) and observed experimentally, the non-zero value of $n_s(T)$ between $T_s$ and $T_\rho$ may however simply be too small to be experimentally observed. A definitive answer to this question demands further experiments and theoretical studies. In particular, $\rho$-fluctuations should be taken into account [9, 10].

One can also comment on the experimentally observed change in the curvature of the full superfluid density, $N_s(T)$, with changing carrier density [3] even though $N_s(T)$ cannot be directly linked to the bare superfluid density $n_s(T)$ discussed here. Although the full superfluid density disappears above $T_{\text{BKT}}$, the curvature present in the bare superfluid density $n_s(T)$ seems to be retained as a curvature in the full superfluid density, $N_s(T)$, below $T_{\text{BKT}}$ [3, 4]. For low carrier densities (the underdoped region) the pseudogap region, $T_{\text{BKT}} < T < T_\rho$, is larger and therefore the curvature in $n_s(T)$ is more pronounced. This behaviour seems to be reflected in the full superfluid density, $N_s(T)$ below $T_{\text{BKT}}$ [3]. It is important however to study experimentally and theoretically the concentration dependence of the bare superfluid density, $n_s(T)$ to make a full comparison with the results from [3] for $N_s(T)$.

The experimental data of [3] also show that $N_s(T)$ does not display the Nelson-Kosterlitz jump. This is probably related to the influence of the interlayer coupling, see Refs. in [10].
5 Conclusion

Since in HTSC the pairing scale $T^*$ is different from the superconducting transition temperature the role of non-magnetic impurities is not traditional and they in fact define the superconducting properties of a “bad metal”. In particular the presence of nonmagnetic impurities strongly increases the size of the pseudogap phase originating from the fluctuations of the phase of the order parameter. In addition the behaviour of the superfluid density in the presence of the impurities is closer to that experimentally observed.

Our results are only qualitative since we have considered the model with non-retarded $s$-wave attraction and an isotropic fermion spectrum. However, it is likely that for $d$-wave pairing, the properties obtained will persist. There is, of course, a problem why a strong disorder does not destroy $d$-wave superconductivity when non-magnetic impurities are pair-breaking. As was suggested by Sadovskii [30] even $d$-wave pairing may persist if coupling is strong enough. Further studies are necessary, for example, it is important to explain the concentration dependence of the superfluid slope, $dn_s(T)/dT$ at $T = 0$ [3, 4]. Our results also indicate that it would be interesting to study the BCS-Bose crossover problem in the presence of impurities, especially in $d$-wave case [30].

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Figure 1: The behaviour of $n_s(T)$ in the clean (upper curve) and dirty (lower curve) limits. The value of $n_s(T)$ is normalized by $n_s(T = 0)$ for clean system, $T$ is given in units $T_\rho$. 