Cross-flow and torsion coupled dynamic response of submerged floating tunnel under the action of ocean current

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Abstract. Submerged floating tunnel (SFT) is a kind of long distance flexible sea structure. The dynamic behavior of SFT is bound to be affected by the action of ocean current. According to the typical cross-section of the structure, the two-dimensional plane coupled vibration equations are derived based on Lagrange equations to analyze the cross-flow and torsion coupled dynamic response of SFT in ocean current environment. Meantime, the fourth-order Runge-Kutta method is used to solve the vibration equations. Finally, the effect of some key parameters, such as the height-width ratio, eccentric throw and the current velocity are investigated. The results show that the eccentric throw and the current velocity have obvious effects on the coupled structure vibration. With the increase of the eccentric throw and current velocity, the amplitude of the coupled vibration increases. Moreover, it can guarantee the relatively small vibration amplitude in cross-flow direction and torsion, when the height-width ratio is between 0.5 and 0.6.

1. Introduction

Submerged floating tunnel (SFT), also named Archimedes Bridge, is a kind of innovative traffic structure, efficiently connecting the shores while being entirely immersed. It generally consists of tunnel tube suspended in water, anchor cables fixing displacement of tunnel, deep water foundations and revetments connecting quayside. Compared with long span cable-stayed or suspension bridges, the SFT can alleviate the influence of bad weather, protect environment landscape, reduce slope degree of shorten length of tunnel, save engineering investment and have development potential. Thus, it is considered as the most competitive sea-crossing structure for long straits and lakes in the 21st century [1].

Because of these advantages, so many studies are being aggressively promoted in a number of countries. The first serious studies for SFT were undertaken for Italy’s Messina Strait crossing in 1970. A decagon cross-section form of SFT was designed for railway and highway transport [2]. Nearly a decade, Norway’s Hogsfjord project presented a circular cross section with double line highway for the Hogsfjord crossing [3]. In 2000, some scholars from China and Italy conducted a technical feasibility study on SFT for crossing Jintang Strait, which proposed an octagon shape cross section structure [4]. Besides, Japan and America also investigated on SFTs according to their own actual situations in conjunction with civil engineering and ocean engineering technology [5]. While the concept of the SFT was proposed has passed a long time, this technology has not been realized due to safety concerns.

It is well known that various dynamical loads are applied to the SFT during its lifetime, such as wave force, ocean current force and traffic load. The structural parameters of an SFT and its reaction
to the dynamic load are the vital points in the SFT design, which will directly affect the stability and safety of the structure. Luo et al. [6] adopted large eddy simulation to calculate the flow field around five different section forms of SFTs. It showed that the ear shaped and elliptical section forms have great stability and less lift and drag force. Jin et al. [7] investigated the seismic behaviors of a SFT with rectangular cross section at different depth of seawater. Hong et al. [8], basing the theoretical and experimental research, thought that the buoyancy-weight ratio is the most important parameter to dominate the dynamic response of SFT. Moreover, some relevant researches about inclined angle of the cable and cable interval also have been investigated to improve the structural dynamic behavior.

As for an asymmetric cross section structure, the shear center does not coincided with the center of gravity. When the current flow passes this bluff body, it will induce the cross-flow and torsion coupling vibration [9]. In the process of structural construction, concrete vibrating insufficiency, structure bar arrangement difference and artificial errors will increase the inhomogeneity of the structure material and carry the deviation between the shear center and the gravity center of the section in SFT. Because the SFT is operated underwater all year round, it is obviously that the flow induced vibration will occur during the SFT operation.

Therefore, in order to analyze the cross-flow and torsion coupled vibration of SFT in ocean current environment, a two-dimensional plane model of the SFT is derived in the study based on Lagrange equations. The fourth-order Runge-Kutta method is used to solve the vibration equations. Meanwhile, the effect of some key parameters, such as the height-width ratio, eccentric throw and the current velocity are investigated.

2. Mathematical Model

2.1 Model and Assumptions

Fig.1 shows a typical SFT structure. To consider the main factors, some further assumptions the theoretical analytical model are given as follows: (1)The current can be regard as a uniform constant flow. Meantime, the influence of structural vibration in flow field is not considered; (2)The structure is assumed as two-dimensional plane vibration model. Only the cross-flow and torsion coupled vibration is discussed and ignore the effect of the in-line vibration; (3)The SFT is regard as the rigid body and elastically supported by the anchor cable. According to that, the two-dimensional plane model, shown as Fig.2, is established to analyze the cross-flow and torsion coupled vibration of SFT in ocean current environment.

![Fig.1 SFT with tethers to the bottom](image1)

2.2 Vibration governed equations

In the analytical model of SFT, the shear center and the center of gravity of the section form is represented as $O$ and $G$, respectively. The distance between the shear center and the center of gravity calls the eccentric throw, which represented by $e$. So the kinetic energy and the potential energy of the analytical model can be expressed as:
\[ T = \frac{1}{2} m \dot{y}^2 + \frac{1}{2} J \dot{\theta}^2 - S_y \dot{\theta} \dot{y} \]  
\[ V = \frac{1}{2} k_y y^2 + \frac{1}{2} k_t \theta^2 \]  \hspace{1cm} (1)

where, \( m \) is the mass per unit length; \( J \) is the polar mass moment of inertia of the section; \( S_y \) is the eccentric moment, \( S_y = m \cdot e \). Both \( m \) and \( J \) include the add mass of entrained fluid. Moreover, \( k_y \) and \( k_t \) are the cross-flow direction stiffness and torsional stiffness respectively. When the SFT adopts equal interval to layout the anchor cables, the stiffness can be calculated by the following equations.

\[ k_y = 2 \frac{E_A}{s_l_c} \sin^2 \alpha \]  
\[ k_t = 2 \frac{E_A}{s_l_c} r^2 \sin^2 \alpha \]  \hspace{1cm} (2)

where, \( E_c, A_c, s_c, l_c \) and \( \alpha \) are the elastic modulus, section area, length, cable interval, and inclination angle of the tether, respectively. \( r \) is the distance from the anchor point to the centroid of the cross section. The equations of motion are derived by Lagrange’s equations

\[ \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = Q_i \]  \hspace{1cm} (3)

where \( L \) is the Lagrange function, \( L = T - V \); \( Q_i \) is the generalized force with respect to the generalized coordinate \( q_i \), and obtained from virtual work considerations:

\[ Q_y(t) = \frac{1}{2} \rho_v v^2 D_j \mu_t(\beta) - 2m \zeta_y \omega \dot{y} \]  
\[ Q_t(t) = \frac{1}{2} \rho_v v^2 D_j \mu_r(\beta) - 2J \zeta_t \omega \dot{\theta} \]  \hspace{1cm} (4)

where, \( \rho_v, v \) and \( D \) are the density of the fluid, current velocity and the width of the cross section, respectively. \( \zeta_y \) and \( \zeta_t \) are the damping ratio of the SFT in the cross-flow direction and torsion. In Eqs (6) and (7), the coefficients \( \mu_t(\beta) \) and \( \mu_r(\beta) \) are nonlinear functions of the angle of attack \( \beta \). Generally, these coefficients should be measured from some hydrodynamic experiments. Blevins and Iwan [10] have approximated the coefficients by using the cubic polynomials

\[ \mu_t(\beta) = a_1 \beta + a_3 \beta^3 \]  
\[ \mu_r(\beta) = b_1 \beta + b_3 \beta^3 \]  \hspace{1cm} (5)

where \( a_1, a_3, b_1, b_3 \) are the hydrodynamic coefficient curve fit parameters. And for small angles of attack, it can be approximately expressed as

\[ \beta = \theta - R \frac{\dot{\theta} - \dot{y}}{v} \]  \hspace{1cm} (6)

where \( R \) has been chosen to be one-half cord for torsion of rectangles about their geometric center. Then Eq. (5) can be transformed into following equations by substituting Eq. (4) ~ Eq. (6) into Eq. (3).

\[ m \ddot{y} + 2m \zeta_y \omega \dot{y} - S_y \ddot{\theta} + k_y y = \frac{1}{2} \rho_v v^2 D \mu_t(\beta) \]  \hspace{1cm} (7)

\[ J \ddot{\theta} + 2J \zeta_t \omega \dot{\theta} - S_y \ddot{y} + k_t \theta = \frac{1}{2} \rho_v v^2 D \mu_r(\beta) \]  \hspace{1cm} (8)

From Eqs. (7) and (8), \( S_y \dot{\theta} \) is the inertia force in cross-flow direction caused by structure torsion; \( S_y \ddot{y} \) is the inertia force in torsion direction caused by cross-flow vibration. When the water flows through SFT, the structure suffers from the lift force and torsion respectively. Due to the effect of eccentric throw, the structural vibrations of cross-flow and torsion coupled together. Besides, the lift and torsion coefficients are nonlinear functions of the angle of attack, which make the vibration have nonlinear characteristic.

### 3. Numerical simulation

#### 3.1 Basic parameters

So far, there is no practice project of the SFT in the world. The parameters of the SFT are taken from some design study cases [6, 11]. The height, width and thickness of the section are 10m, 30m, and 0.5m, respectively. The mass per unit length and the initial eccentric throw of structure are assumed to be \( 1.1 \times 10^5 \text{kg/m} \) and 0.4m. In order to moor the SFT, cables are symmetrically employed at each mooring plans with 100m interval. The elastic modulus, area, length, inclination angle of cable are
1.95×10^5 MPa, 0.0946 m^2, 231 m, and π/3, respectively. Besides, the density and the velocity of ocean current are 1028 kg·m^3 and 6 m/s [12]. Meantime, the dimensionless damping ratios are considered as constant and equivalent to 0.0001. As to the equations of objects motion, the fourth-order Runge-Kutta method on MATLAB is employed for iterative solution.

3.2 Numerical Results and Analysis

Time-history responses of SFT in case 1 are shown in Fig. 3(a). The magnitude of initial disturbance in cross-flow direction is 0.01 m. The maximum amplitudes of cross-flow direction and torsion are 1.003×10^-2 m and 1.363×10^-5 rad. That is to say, the eccentric throw affects the cross-flow direction and torsion vibration. The polar mass moment of inertia is larger than the mass per unit length. So the magnitude of torsion is very small in the hydrostatic pressure environment.

When eccentric throw e equal to 0, the structure subjected the uniform constant flow vibrate independently. Over time the vibration of cross-flow direction and torsion increase gradually and tend to be stable, known from Fig. 3(b). The reason is the lift and torsion coefficients are related to the angle of attack, which is depended by the vibration velocity from Eq. (4). The maximum amplitudes of cross-flow direction and torsion are 1.188 m and 6.193×10^-4 rad. Compared with Case 1, the magnitude of vibration amplitudes has raised a lot.

In this case 3, the effects of eccentric throw and current are both considered in the system. The maximum amplitudes of cross-flow direction and torsion are 1.204 m and 6.193×10^-4 rad. Compared with Case 1, the magnitude of vibration amplitudes has raised a lot.

In this case 3, the effects of eccentric throw and current are both considered in the system. The maximum amplitudes of cross-flow direction and torsion are 1.204 m and 1.011×10^-3 rad. Comparing the results with that of Case 2, the eccentric throw strengthen the amplitudes of structure vibration in cross-flow direction and torsion. The increment ratios of cross-flow direction and torsion reach to 1.3% and 63.2%. The coupled vibration, caused by eccentric throw, extremely amplifies the torsion dynamic response. It will leave some safety problems in the structural vibration and should be avoided.

4. Parametric Study

4.1 Effect of height-width ratio
During SFT cross section design process, height-width ratio is one of critical parameters for non-circular section. Generally, the width will change with the number of traffic lanes setting in SFT [12]. Therefore, we maintain the height of SFT at 10m and change the height-width ratio of section at 1.0. Known from Fig.4, the maximum amplitudes of cross-flow direction and torsion are 0.886m and $4.532 \times 10^{-3}$ rad. Compared with the results of Case 3, the magnitude of cross-flow direction has reduced 26.41%, but the magnitude of torsion has increased 352.2%. The time of structure reach to stable vibration is shorter than that of Case 3, which means that the vibration of SFT strongly coupled with higher height width ratio.

Fig.5 depicts the variation of amplitudes of SFT at different height-width ratio values. Known from Fig.5, the magnitude of cross-flow direction reduces with increasing height-width ratio, but the magnitude of torsion increases gradually with increasing height-width ratio. As well known, cross-flow direction and torsional vibrations have adverse effects on the structure, which can cause fatigue damage to structural members. So it is necessary to choose appropriate height-width ratio to reduce this effects. According to the numerical results, it can guarantee the relatively small vibration amplitude in cross-flow direction and torsion, when the height-width ratio is between 0.5 and 0.6.

### 4.2 Effect of eccentric throw

For considering the effect of eccentric throw, Fig.6 gives the time-history response of SFT at eccentric throw equal to 1.0m. The maximum amplitudes of cross-flow direction and torsion are 1.230m and $2.180 \times 10^{-3}$ rad. Compared with the results of Case 3, the magnitude of cross-flow direction and torsion have increased 2.16% and 352.2% respectively. That is to say, eccentric throw has greater effect on the vibration of torsion.

In order to analyze the effect of variation of eccentric throw, Fig.7 shows the increment ratios of SFT at different eccentric throw values. Increment ratio is that the change of amplitude caused by
variable takes up the maximum change of amplitude. From Fig.7, the increment ratio of cross-flow direction keeps a steady step to increase, but the increment ratio of torsion tends to flatten with increasing eccentric throw. It means that eccentric throw leads the structure vibrate nonlinearly. With the increase of eccentric throw, the amplitude of structural vibration increases until destroyed.

4.3 Effect of current velocity

Fig.8 Time-history response of SFT at current velocity equal to 12.0m/s

Fig.9 Increment ratios of the SFT at different eccentric throw values

Ocean current velocity is another vital factor affecting structural vibration. Fig.8 gives the time-history response of SFT at current velocity equal to 12m/s. We know that the maximum amplitudes of cross-flow direction and torsion are 2.423m and 2.971×10⁻³ rad. Compared with the results of current velocity equal to 6m/s, the magnitude of cross-flow direction and torsion have increased 101.2% and 197.1% respectively. Meanwhile, the time of structure reach to stable vibration becomes shorter than low current velocity.

Besides, the increment ratios of SFT at different current velocity values are shown in Fig.9. The increment ratio of torsion angle satisfies the law of increasing with the current velocity. The vibration of torsion results into structure rotation in the current environment and bring catastrophe to the SFT. So it should take measures to reduce the torsion response particularly in water flow environment.

5. Conclusion

(1) The magnitude of cross-flow direction reduces with increasing height-width ratio, but the magnitude of torsion increases gradually with increasing height-width ratio. According to the numerical results, it can guarantee the relatively small vibration amplitude in cross-flow direction and torsion direction, when the height-width ratio is between 0.5 and 0.6.

(2) Eccentricity error leads the structure vibrates nonlinearly. With the increase of eccentric throw, the amplitude of structural vibration increases. Meantime, eccentric throw has greater effect on the vibration of torsion.

(3) The current velocity has obvious influence on the coupled vibration. The faster current velocity results into higher dynamic response of torsion. So it should take some measures to reduce the torsion response particularly in the strong water flow environment.

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References

[1] Xiang Y, Yang Y. (2016) Challenge in Design and Construction of Submerged Floating Tunnel and State-of-art. Procedia Eng. 166:53-60.

[2] Faggiano B, Landolfo R, Mazzolani F M. (2001) Design and modelling aspects concerning the submerged floating tunnels: an application to the Messina Strait crossing. In:
Proceedings of the 3rd International Conference on Strait Crossings, Bergen, Norway.

[3] Ahrens D. (1997) Submerged floating tunnels—a concept whose time has arrived. Tunn Undergr Sp Tech. 12(2): 317-336.

[4] Chao C. (2013) Dynamic response analysis and experiment of submerged floating tunnel based on fluid-structure interaction. PhD Thesis, Zhejiang University, September, Hangzhou, China. (In Chinese)

[5] Sakurai S. (1996) Submerged floating tunnel proposed to connect kansai, kobe airports. Tunnelling and Underground Space Technology, 11:511–514.

[6] Luo G., Zhou X., Deng F., et al. (2013) Analysis on Characteristics of Flow Passing Submerged Floating Tunnels of Different Sections. J China Rail Soc. 35(1): 115-120. (In Chinese)

[7] Jin H L, Seo S I, Mun H S. (2016) Seismic behaviors of a floating submerged tunnel with a rectangular cross-section. Ocean Eng. 127:32-47.

[8] Hong Y, Ge F, Lu W. (2016) On the Two Essential Concepts for SFT: Synergetic Buoyancy-weight Ratio and Slack-taut Map. Procedia Eng. 166:221-228.

[9] Desai Y M, Shah A H, Popplewell N. (1990) Galloping Analysis for Two-Degree-of-Freedom Oscillator. J Eng Mech-ASCE. 116(12):2583-2602.

[10] Blevins R D, Iwan W D. (1974) The Galloping Response of a Two-Degree-of-Freedom System. J Appl Mech-T ASME. 41(4):1113.

[11] Xiang Y, Yang Y. (2017) Spatial dynamic response of submerged floating tunnel under impact load. Mar Struct. 53:20-31.

[12] Ding H, Li Q, Jiang S, et al. (2016) Research Progress and Outlook on Critical Characteristics of Submerged Floating Tunnels. Procedia Eng. 166:347-354.