Phonon effect on two coupled quantum dots at finite temperature

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The quantum oscillations of population in an asymmetric double quantum dots system coupled to a phonon bath are investigated theoretically. It is shown how the environmental temperature has effect on the system.

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The asymmetric double semiconductor quantum dots system (DQD) is referred as quantum dot molecule due to its similar properties to natural molecule. By using self-assembled dot growth technology[1] we can fabricate these molecule-like dots, in which the confined electrons can transfer between each quantum dot through tunneling effect. Up to now, such system has been studied extensively. Under the influence of an external oscillatory (optical pulse or voltage) driving field, one electron can be excited from the valence to the conduction band in one dot, which can in turn tunnel to the second dot[2][3]. Recent progress in semiconductor nanotechnology indicate that these features have advantages for the application in many quantum devices, such as QD lasers[4][5], QD diodes[6] as well as quantum computing processes[7][8]. However, QDs are embedded in the surrounding solid matrix, thus the effect of electron-phonon interaction during the tunneling is not negligible[9]. Therefore, the environmental temperature will have significant influence on the desired QD devices. In this letter, we analyze how the optically driven asymmetrical DQD device depends on the environmental temperature.

The asymmetrical DQD consists of two dots (the left and the right one) with different geometries and has the ground state |0⟩ (the system without excitation), first excited state |1⟩ (a pair of electron and hole bound in the left dot) and second excited state |2⟩ (one hole in the left dot and one electron in the right dot). Any other states are effectively decoupled from these three states because the valence band levels of two dots become far off-resonance and the hole cannot tunnel to the right dot consequently. Such model can be shown in Fig.1. Considering that the single electron is unavoidably scattered by phonons while tunneling between two dots, the Hamiltonian is given by

\[
H = \sum_{j=0}^{2} \varepsilon_j |j⟩⟨j| + \Omega(e^{i\omega_c t}|0⟩⟨1| + e^{-i\omega_c t}|1⟩⟨0|) + T_c (|1⟩⟨2| + |2⟩⟨1|)
+ \sum_{k=0} \omega_k b^+_k b_k + \frac{1}{2} (|1⟩⟨1| - |2⟩⟨2|) \sum_{k} g_k (b^+_k + b_k),
\]

where \(\varepsilon_j\) is the energy of state \(|j⟩\), \(T_c\) is the electron-tunneling matrix element, \(\omega_c\) is the frequency of the applied field, \(\Omega(t) = ⟨0|\mu E(t)|1⟩\), where \(\mu\)is the electric dipole moment, describes the coupling to the radiation field of the excitonic transition, \(E(t)\) is the optical pulse amplitude. \(b^+_k (b_k)\) and \(\omega_k\) are the creation (annihilation) operator and energy for \(k\)th phonon mode, respectively, \(g_k\)is the coupling constant determined by the crystal material.

**FIG. 1:** Level configuration of a double QD system. A pulsed laser excites one electron from the valence band that can tunnel to other dot with the application of the voltage. We assume that the hole cannot tunnel in the time scale we are considering here.
Applying a canonical transformation with the generator\ref{9}\ref{10}\ref{11}

$$S = (|1\rangle\langle 1| - |2\rangle\langle 2|) \sum_k \frac{g_k}{2\omega_k} (b_k^+ - b_k),$$

we have

$$H' = e^SHe^{-S} = H_0' + H_I',$$

where

$$H_0' = \varepsilon_0|0\rangle\langle 0| + (\varepsilon_1 - \Delta)|1\rangle\langle 1| + (\varepsilon_2 - \Delta)|2\rangle\langle 2| + \sum_k \omega_k b_k^+ b_k,$$

and

$$H_I' = \Omega e^{i\omega t}|0\rangle\langle 1|e^{-A} + e^{-i\omega t}|1\rangle\langle 0|e^{A} + T_e(|1\rangle\langle 2|e^{2A} + |2\rangle\langle 1|e^{-2A}),$$

where

$$\Delta = \sum_k \frac{g_k^2}{4\omega_k}, A = \sum_k \frac{g_k}{2\omega_k} (b_k^+ - b_k).$$

The Hamiltonian in the interaction picture is given by

$$H'' = e^{iH_0't}H_I' e^{-iH_0't} = \Omega [e^{-i(\delta - \Delta)t}|0\rangle\langle 1|X(t) + e^{i(\delta - \Delta)t}|1\rangle\langle 0|X(t)^+]
+ T_e [e^{-i\omega_1zt}|2\rangle\langle 1|X^2(t) + e^{i\omega_1zt}|1\rangle\langle 2|X^2(t)^+],$$

where $\omega_{ij} = \varepsilon_i - \varepsilon_j$, $\delta = \omega_{10} - \omega_e$ is the detuning of the applied field to the the left dot, and

$$X(t) = \exp[-\sum_k g_k \frac{b_k^+}{2\omega_k} (e^{i\omega_k t} - b_k e^{-i\omega_k t})],$$

and

$$X(t)^+ = X(t).$$

In what follows, we assume that relaxing time of the environment (phonon fields) is so short that the excitons do not have time to exchange the energy and information with the environment before the environment returns to its equilibrium state. The excitons interact weakly with the environment so that the equilibrium thermal properties of the environment are preserved. Therefore it is reasonable to replace the operators $X(t)$, $X(t)^+$, $X(t)^+$ and $X^2(t)^+$ with their expectation values over the phonon number states which are determined by a thermal average and write the Hamiltonian as

$$H_{eff} = \Omega [e^{-i(\delta - \Delta)t}|0\rangle\langle 1|e^{-i(\delta - \Delta)t}|1\rangle\langle 0|e^{-(N_{ph} + \frac{1}{2})\lambda}
+ T_e [e^{-i\omega_1zt}|2\rangle\langle 1|e^{i\omega_1zt}|1\rangle\langle 2|e^{-2(N_{ph} + \frac{1}{2})\lambda}]$$

where $\lambda = \sum_k (g_k/2\omega_k)^2$ is the Huang-Rhys factor which corresponds to the electron-phonon interactions\ref{12}. As a result of quantum lattice fluctuations, the exciton-phonon interaction affects our quantum system even at zero temperature. Here we have made an assumption that all the phonons have the same frequency, i.e., $\omega_k = \omega_0$, and write the phonon populations as $N_{ph} = \frac{1}{\omega_0 + \frac{1}{2}i}$\ref{13}\ref{14}.

Now we proceed to solve the equation of motion for the single electron state vector $|\phi(t)\rangle$, i.e.,

$$i\frac{d}{dt} |\phi(t)\rangle = H_{eff} |\phi(t)\rangle.$$  

In general, $|\phi(t)\rangle$ is a linear combination of $|0\rangle$, $|1\rangle$ and $|2\rangle$,

$$|\phi(t)\rangle = c_0(t)|0\rangle + c_1(t)|1\rangle + c_2(t)|2\rangle.$$  

Suppose the initial state of the system is in its ground state $|0\rangle$, i.e., $|0\rangle = 1$ and $|1\rangle = |2\rangle = 0$. We have

$$i\frac{d}{dt} c_0(t) = e^{-(N_{ph} + \frac{1}{2})\lambda} c_1(t)e^{-i(\delta - \Delta)t},$$

\[\text{Equation (13a)}\]
On the other hand, the environment temperature will also affect the beat pattern, which is caused by the electron-phonon coupling. With the rise of the temperature, the beat pattern will decay and the population of the state \(2\) will decrease as well. As the temperature reaches about 50\(\omega_0\), the beat pattern almost disappears. So the environment temperature should be considered in the practical applications.

In conclusion, we have investigated the effect of the environment temperature on an asymmetrical double quantum dot driven by an optical pulse. The Rabi oscillation frequency \(\Omega\) and the electron-tunneling matrix element \(T_e\) are renormalized by the factor of \(\exp[-(N_{ph} + \frac{1}{2})\lambda]\) and \(\exp[-2(N_{ph} + \frac{1}{2})\lambda]\). With the rise of the environment temperature, the population of state \(2\) declines dramatically and the beat pattern, which is caused by the phonon effect, decays.
FIG. 3: Average occupation of state $|0\rangle$ (solid line), state $|1\rangle$ (dash dot line) and state $|2\rangle$ (dot line) as a function of temperature in the case $\delta = \Delta$ and $\omega_{12} = 0$, for the parameters $\Omega = 0.2\omega_c$, $T_c = 0.1\omega_c$ and $\lambda = 0.01$.

FIG. 4: The time evolution of the population in state $|2\rangle$, for the parameters $\delta = \omega_{12} = 0$, $\Omega = 0.2\omega_c$, $T_c = 0.1\omega_c$, $\lambda = 0.01$ and $\Delta = 0.05\omega_c$. (a) is the result for $T = 0$, (b) for $T = 10\omega_0$, (c) for $T = 30\omega_0$ and (d) for $T = 50\omega_0$.

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