Supersymmetry, flavor physics and dark matter

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ABSTRACT

In this talk, I first discuss $b \to d$, $b \to s$ and $s \to d$ transitions including time-dependent CP asymmetries in $B_d \to \phi K_s$ and $\text{Re} (\epsilon'/\epsilon_K)$. Then I show that the decay $B_s \to \mu^+\mu^-$ is useful to distinguish various SUSY breaking mechanisms. I will also describe some possible connections between B physics and cosmology: (i) B physics and electroweak baryogenesis within SUSY models, and (ii) the correlation between the neutralino dark matter scattering and $B(B_s \to \mu^+\mu^-)$. In particular, we point out that the current upper bound on $B(B_s \to \mu^+\mu^-)$ form CDF and D0 collaborations can constrain the spin-independent neutralino scattering cross section more strongly than the CDMS bound.

1. Introduction

In the Standard Model (SM), flavor mixing and CP violation in the quark sector have the common origin, namely the CKM mixing matrix. This is dictated by local gauge invariance and renormalizability of the SM with 3 families. This paradigm is well tested by many different observables in K and B meson systems. All the data (except for possible anomalies in the time dependent CP asymmetries in $B_d \to \phi K_s$ and $B_d \to \eta' K_s$ decays, and the baryon number asymmetry in the universe) can be accommodated by the CKM picture, and we have consistent understanding of flavor mixing and CP violation within the SM. Despite this great success of SM, there are many reasons why we consider the SM merely as a low energy effective theory of some fundamental theory. In particular, quadratic divergence in the SM Higgs mass seems to call for new physics beyond the SM around $\sim O(1)$ TeV. SUSY models with $R$–parity conservation are well motivated new physics scenarios due to gauge coupling unification and the presence of dark matter candidates. In SUSY models, the flavor and CP structures of the soft SUSY breaking terms have rich structures, and there could be large deviations in some processes involving B and K mesons, without any conflict with the current status of CKM phenomenology.

In this talk, I will give a few such examples, in which we can have large deviations from the SM predictions, even if the CKM triangle in the SUSY models has the same shape as in the SM. More specifically, we will discuss the branching ratio of $B \to X_d\gamma$ and CP asymmetry therein, CP asymmetries in $B \to X_s\gamma$ and $B_d \to \phi K_s$, $B_s - \overline{B}_s$ mixing (both the modulus and the phase), and $B_s \to \mu^+\mu^-$ as well as $\epsilon'/\epsilon_K$. The future experiments at B factories should study these processes in greater detail, thus testing the CKM paradigm within the SM and exploring the flavor and CP structures of SUSY models.

In phenomenological study of SUSY models, it is crucial to include the soft SUSY breaking terms. However, we do not understand the nature of SUSY breaking in our world, and thus we do not know the flavor and CP structures of soft SUSY breaking
terms. This makes it difficult to study flavor physics and CP violation within SUSY models, and most results are admittedly model dependent. In the following, we take two different approaches: (i) we use the mass insertion approximation (MIA) assuming gluino-squark loop contributions are dominant, or (ii) we work in specific SUSY breaking scenarios which are theoretically well motivated. Even if our current strategies are not perfect, our analysis method could be used in other cases, and we don’t expect that we lose generic features by such strategies. Eventually we will want to measure all the soft SUSY breaking parameters. It would not be easy to get informations on flavor and CP violating soft terms from LHC/NLC alone, and the low energy processes involving K, B mesons and \( \mu, \tau \) leptons will give invaluable informations on flavor and/or CP violating soft SUSY breaking parameters, when combined with the informations on the SUSY particle mass spectra and flavor diagonal couplings measured at LHC and NLC.

The plan of my talk is the following. In Section 2 and Section 3, I will discuss \( b \to d \) and \( b \to s \) transitions within MIA, including \( B \to X_d \gamma \) and CP asymmetry therein, CP asymmetries in \( B \to X_s \gamma \) and \( B_d \to \phi K_S \), and \( B_s - \bar{B}_s \) mixing. In Section 4, I discuss some correlation between \( S_{\phi K} \) and \( \epsilon' / \epsilon_K \) within the RR dominance scenario: namely the \( s_R \to d_L \) transition induced by the \( b_R \to s_R \) transition. In Section 5, I’ll discuss the \( B_s \to \mu^+ \mu^- \) as a useful probe of SUSY breaking mechanisms. In Section 6, I will discuss possible interplay between B physics and cosmology with two examples: (i) B physics and electroweak baryogenesis (EWBGEN) within SUSY models, and (ii) the correlation between \( B_s \to \mu^+ \mu^- \) and the neutralino dark matter (DM) scattering cross section. Then I conclude in Section 7.

2. \( b \to d \) transition: \( B_d - \bar{B}_d \) mixing and CP asymmetry in \( B \to X_d \gamma \)

In general SUSY models, squark mass matrices are not diagonal in the basis where quark masses are diagonal. Therefore the \( \tilde{g} - q_i - \tilde{q}_j \) vertex can change the (s)quark flavor, leading to dangerous flavor changing neutral current (FCNC) processes at one loop level with strong interaction strength. Various low energy data such as \( K^0 - \bar{K}^0 \) and \( B^0_{d(s)} - \bar{B}^0_{d(s)} \) mixings, Re \((\epsilon / \epsilon_K)\) and \( B \to X_d(s) \gamma \) etc. will put strong constraints on such flavor changing \( \tilde{g} - q_i - \tilde{q}_j \) vertex. In the limit of degenerate squark masses, FCNC amplitude vanishes. Therefore the almost degenerate squark masses may be the good starting point to study gluino-mediated FCNC within general SUSY models, and the so-called mass insertion approximation (MIA) is convenient in this case. In this section, we consider \( b \to d \) transition due to gluino mediation within MIA, relegating the \( b \to s \) and \( s \to d \) transitions to the following sections.

Observations of large CP violation in \( B \to J/\psi K_S \) at B factories confirm the SM prediction, and begin to put a strong constraint on new physics contributions to \( B^0 - \bar{B}^0 \) mixing and \( B \to J/\psi K_S \), when combined with

\[
\sin 2\beta_{\psi K} = (0.731 \pm 0.056)
\]

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\[
\Delta m_{B_d} = (0.502 \pm 0.007) \text{ ps}^{-1}, \quad \text{Br}(B \to X_d \gamma) < 1 \times 10^{-5}.
\]
Here the \( B_d \rightarrow X_d\gamma \) branching ratio constraint was extracted from the recent experimental upper limit on the \( B \rightarrow \rho \gamma \) branching ratio \( B(B \rightarrow \rho \gamma) < 2.3 \times 10^{-6} \). Since the decay \( B \rightarrow J/\psi K_S \) is dominated by the tree level SM process \( b \rightarrow c\bar{c}s \), we expect the new physics contribution may affect significantly the \( B_d \rightarrow X_d\gamma \) branching ratio only and not the decay \( B \rightarrow J/\psi K_S \). However, in the presence of new physics contributions to \( B^0 - \bar{B}^0 \) mixing, the same new physics will generically affect the \( B \rightarrow X_d\gamma \) process \( \delta_3 \), which is also loop suppressed in the SM. In the following, we consider \( B^0 - \bar{B}^0 \) mixing, \( B \rightarrow J/\psi K_S \) and \( B_d \rightarrow X_d\gamma \) assuming that the main SUSY contribution is from gluino-squark loops in addition to the usual SM contribution.

In Fig. 1 (a), we show the allowed parameter space in the \( (\text{Re}(\delta_{13})_{LL}, \text{Im}(\delta_{13})_{LL}) \) plane for different values of the KM angle \( \gamma \) with different color codes. The region leading to too large a branching ratio for \( B_d \rightarrow X_d\gamma \) is covered by parallel lines. Note that \( B \rightarrow X_d\gamma \) plays an important role here. And the region where the dilepton charge asymmetry \( A_{\text{ll}} \) (see Ref. \[5\] for the definition) falls out of the data \( A_{\text{exp}} = (0.2 \pm 1.4) \% \) within 1\( \sigma \) range is already excluded by the \( B \rightarrow X_d\gamma \) branching ratio constraint \[ Fig. 1 (b) \]. Note that the KM angle \( \gamma \) should be in the range between \( \sim -60^\circ \) and \( \sim +60^\circ \), and \( A_{\text{ll}} \) can have the opposite sign compared to the SM prediction, even if the KM angle is the same as its SM value \( \gamma_{\text{SM}} \approx 55^\circ \) due to the SUSY contributions to \( B^0 - \bar{B}^0 \) mixing. In Fig. 1 (c), we show the direct CP asymmetry in \( B_d \rightarrow X_d\gamma \) as a function of the KM angles \( \gamma \) for the \( LL \) insertion case. The direct CP asymmetry is predicted to be between \( \sim -15\% \) and \( \sim +20\% \). In the \( LL \) mixing case, the SM gives the dominant contribution to \( B_d \rightarrow X_d\gamma \), but the KM angle can be different from the SM case, because SUSY contributions to the \( B^0 - \bar{B}^0 \) mixing can be significant so that the preferred value of \( \gamma \) can change from the SM KM fitting. ( This is the same in the rare kaon decays and the results obtained in
Ref. [8,9] apply without modifications. Therefore, it is possible to have large deviations in the $B_d \to X_d\gamma$ branching ratio and the direct CP violation thereof.

For the LR mixing, the $B(B_d \to X_d\gamma)$ puts an even stronger constraint compared to the LL insertion case [Fig. 2 (a)], whereas the $A_{lt}$ does not put any new constraint [Fig. 2 (b)]. In particular, the KM angle $\gamma$ can not be too much different from the SM value in the LR mixing case, once the $B(B_d \to X_d\gamma)$ constraint is included. Only $30^\circ \lesssim \gamma \lesssim 80^\circ$ is compatible with all the data from the $B$ system, even if we do not consider the $\epsilon_K$ constraint. The resulting parameter space is significantly reduced compared to the LL insertion case. In Fig. 2 (b), we show the predictions for $A_{lt}$ as a function of the KM angle $\gamma$ for the LR insertion only. Note that the $B \to X_d\gamma$ constraint rules out almost all the parameter space region, and the resulting $A_{lt}$ is essentially the same as for the SM case. In Fig. 2 (c), we find that there could be substantial deviation in the CP asymmetry in $B_d \to X_d\gamma$ from the SM predictions, even if the $\Delta m_B$ and $\sin 2\beta$ is the same as the SM predictions as well as the data. For the LL insertion, such a large deviation is possible, since the KM angle $\gamma$ can be substantially different from the SM value. On the other hand, for the LR mixing, the large deviation comes from the complex $(\delta^d_{13})_{LR}$ even if the KM angle is set to the same value as in the SM. The size of $(\delta^d_{13})_{LR}$ is too small to affect the $B^0 - \overline{B^0}$ mixing, but is still large enough to affect $B \to X_d\gamma$. Our model independent study indicates that the current data on the $\Delta m_B$, $\sin 2\beta$ and $A_{lt}$ do still allow a possibility for large deviations in $B \to X_d\gamma$, both in the branching ratio and the direct CP asymmetry thereof. These observables are indispensable to test the KM paradigm for CP violation completely and get ideas on possible new physics with new flavor/CP violation in $b \to d$ transition.

Summarizing this section, we considered the gluino-mediated SUSY contributions to $B^0 - \overline{B^0}$ mixing, $B \to J/\psi K_S$ and $B \to X_d\gamma$ in the mass insertion approximation. We find that the LL mixing parameter can be as large as $|\langle \delta^d_{13}\rangle_{LL}| \lesssim 2 \times 10^{-1}$, but the LR mixing is strongly constrained by the $B \to X_d\gamma$ branching ratio: $|\langle \delta^d_{13}\rangle_{LR}| \lesssim 10^{-2}$. The implications for the direct CP asymmetry in $B \to X_d\gamma$ are also discussed, where substantial deviations
from the SM predictions are still possible both in the \( LL \) and \( LR \) insertion cases even if \( \gamma \simeq \gamma_{SM} \). Our analysis demonstrates that all the observables, \( A_{ll} \), the branching ratio of \( B \to X_d \gamma \) and the direct CP violation thereof are very important, since they could provide informations on new flavor and CP violation from \( (\delta_{13}^d)_{LL,LR} \) (or any other new physics scenarios with new flavor/CP violations). These will provide strong constraints on SUSY flavor models that attempt to solve hierarchies in the Yukawa couplings and SUSY flavor problems using some flavor symmetry groups \[10\]. Also they are indispensable in order that we can ultimately test the KM paradigm for CP violation in the SM, since one can have very different branching ratio and CP asymmetry for \( B \to X_d \gamma \) for the SM values of the CKM matrix elements, if there is a new physics beyond the SM with new sources of flavor and CP violations.

3. \( b \to s \) transition: \( B_d \to \phi K_S \) and \( B_s - \overline{B_s} \) mixing

\( B \to \phi K \) is a powerful testing ground for new physics. Because it is loop suppressed in the standard model (SM), this decay is very sensitive to possible new physics contributions to \( b \to sss \), a feature not shared by other charmless \( B \) decays. Within the SM, it is dominated by the QCD penguin diagrams with a top quark in the loop. Therefore the time dependent CP asymmetries are essentially the same as those in \( B \to J/\psi K_S \): \( \sin 2\beta_{\phi K} \simeq \sin 2\beta_{\psi K} + O(\lambda^2) \)[14].

Recently both BaBar and Belle reported the branching ratio and CP asymmetries in the \( B_d \to \phi K_S \) decay:

\[
A_{\phi K}(t) \equiv \frac{\Gamma(B_0^{\text{phys}}(t) \to \phi K_S) - \Gamma(B_0^{\text{phys}}(t) \to \phi K_S)}{\Gamma(B_0^{\text{phys}}(t) \to \phi K_S) + \Gamma(B_0^{\text{phys}}(t) \to \phi K_S)}
= -C_{\phi K} \cos(\Delta m_B t) + S_{\phi K} \sin(\Delta m_B t),
\]

(2)

where \( C_{\phi K} \) and \( S_{\phi K} \) are given by

\[
C_{\phi K} = \frac{1 - |\lambda_{\phi K}|^2}{1 + |\lambda_{\phi K}|^2}, \quad \text{and} \quad S_{\phi K} = \frac{2 \text{Im}\lambda_{\phi K}}{1 + |\lambda_{\phi K}|^2},
\]

(3)

with

\[
\lambda_{\phi K} \equiv e^{-2i(\beta + \theta_d)} \frac{A(B^0 \to \phi K_S)}{A(B^0 \to \phi K_S)},
\]

(4)

and the angle \( \theta_d \) represents any new physics contributions to the \( B_d - \overline{B_d} \) mixing angle. The current world average is [12]

\[
\sin 2\beta_{\phi K} = S_{\phi K} = (0.34 \pm 0.20),
\]

which is about 2 \( \sigma \) lower than the SM prediction: \( \sin 2\beta_{J/\psi K_S} = (0.731 \pm 0.056) \). The direct CP asymmetry in \( B_d \to \phi K_S \) is also measured, and is consistent with zero [13]: \( C_{\phi K_S} = (-0.04 \pm 0.17) \).

In the following, we assume that the \( \tilde{b}_A - \tilde{s}_B \) (with \( A, B = L \) or \( R \)) mixing has a new CP violating phase, and study its effects on \( S_{\phi K}, B \to X_s \gamma \), the direct CP asymmetry...
Figure 3: The allowed region in the plane of (a) the \( \text{Re}\ \delta_{LL}, \text{Im}\ \delta_{LL} \), (b) \( S_{\phi K} \) and \( A^{b\rightarrow s\gamma}_{\text{CP}} \), (c) \( S_{\phi K} \) and \( \sin 2\beta \) for the case of a single \( LL \) insertion, with \( m_\tilde{g} = \tilde{m} = 400 \text{ GeV} \). The dotted boxes show the current 1\( \sigma \) experimental bounds, and the hatched regions correspond to \( B(B_d \rightarrow \phi K^0) > 1.6 \times 10^{-5} \).

We calculate the Wilson coefficients of the operators for \( \Delta B = 1 \) effective Hamiltonian at the scale \( \mu \sim \tilde{m} \sim m_W \). Then we evolve the Wilson coefficients to \( \mu \sim m_b \) using the appropriate renormalization group (RG) equations, and calculate the amplitude for \( B \rightarrow \phi K \) using the BBNS approach \cite{18} for estimating the hadronic matrix elements. The details of the effective Hamiltonian and the Wilson coefficients can be found in Ref. \cite{15,16}.

In the numerical analysis presented here, we fix the SUSY parameters to be \( m_\tilde{g} = \tilde{m} = 400 \text{ GeV} \). In each of the mass insertion scenarios to be discussed, we vary the mass insertions over the range \( |\delta_{AB}^{d_{23}}| \leq 1 \) to fully map the parameter space. We then impose two important experimental constraints. First, we demand that the predicted branching ratio for inclusive \( B \rightarrow X_s \gamma \) fall within the range \( 2.0 \times 10^{-4} < B(B \rightarrow X_s \gamma) < 4.5 \times 10^{-4} \), which is rather generous in order to allow for various theoretical uncertainties. Second, we impose the current lower limit on \( \Delta M_s > 14.9 \text{ ps}^{-1} \).

A new CP-violating phase in \( (\delta_{AB}^{d})_{23} \) will also generate CP violation in \( B \rightarrow X_s \gamma \). The current world average of the direct CP asymmetry \( A^{b\rightarrow s\gamma}_{\text{CP}} \) is \( (0.5 \pm 3.6)\% \), which is now quite constraining (see also the discussion in Sec. 5.1 and Fig. 6). Within the SM, the predicted CP asymmetry is less than \( \sim 0.5\% \), and a larger asymmetry would be a clear indication of new physics \cite{19}. Where relevant, we will show our predictions for \( A^{b\rightarrow s\gamma}_{\text{CP}} \).

We begin by considering the case of a single \( LL \) mass insertion: \( (\delta_{LL}^{d})_{23} \). The results are shown in Fig. 3 (a)–(c). We get similar results for a single \( RR \) insertion (see Refs. \cite{15,16} for more details). Scanning over the parameter space consistent with \( B \rightarrow X_s \gamma \) and \( \Delta M_s \) constraints (Fig. 3 (a)), we find that \( S_{\phi K} > 0.5 \) for \( m_\tilde{g} = \tilde{m} = 400 \text{ GeV} \) and for any value of \( |(\delta_{LL}^{d})_{23}| \leq 1 \), the lowest values being achieved only for very large \( \Delta M_s \) (Fig. 3 (b) and (c)). If we lower the gluino mass down to 250 GeV, \( S_{\phi K} \) can shift down to \( \sim 0.05 \), but only in a small corner of parameter space. Similar results hold for a single \( RR \) insertion. Thus we conclude that the effects of the \( LL \) and \( RR \) insertions on \( B \rightarrow X_s \gamma \) and \( B \rightarrow \phi K \) are
not very dramatic, although it can marginally accommodate the current world average of $S_{\phi K}$. Especially it is not likely to generate a negative $S_{\phi K}$, unless gluino and squarks are relatively light. Nonetheless, their effects on $B_d - \bar{B}_d$ mixing could be very large, providing a clear signature for LL or RR mass insertions (Fig. 3 (c)).

Next we consider the case of a single LR insertion. Scanning over the parameter space and imposing the constraints from $B \to X_s \gamma$ and $\Delta M_s$, we find $|\langle \delta_{23}^{LR} \rangle| \lesssim 10^{-2}$. This is, however, large enough to significantly affect $B_d \to \phi K_S$, both its branching ratio and CP asymmetries, through the contribution to the chromomagnetic dipole moment operator. In Fig.s 4 (a) and (b), we show the allowed region in the complex ($\delta_{23}^{LR}$) plane with the contours of $S_{\phi K}$, and the correlation between $S_{\phi K}$ and $C_{\phi K}$. Since the LR insertion can have a large effect on the CP-averaged branching ratio for $B \to \phi K$, we further impose that $B_d \to \phi K < 1.6 \times 10^{-5}$ (which is twice the experimental value) in order to include theoretical uncertainties in the BBNS approach related to hadronic physics. We can see that the $B \to \phi K$ branching ratio constrains $|\delta_{23}^{LR}|$ just as much as $B \to X_s \gamma$. Also we can get a large negative $S_{\phi K}$, but only if $C_{\phi K}$ is also negative. The correlation between $S_{\phi K}$ and the direct CP asymmetry in $B \to X_s \gamma (\equiv A_{CP}^{b \to s\gamma})$ is shown in Fig. 4 (c). We find $A_{CP}^{b \to s\gamma}$ becomes positive for a negative $S_{\phi K}$, while a negative $A_{CP}^{b \to s\gamma}$ implies that $S_{\phi K} > 0.6$. The present world average on $A_{CP}^{b \to s\gamma}$ gives additional constraint on the LR model, and the resulting $S_{\phi K}$ is consistent with the data. Finally, the deviation of $B_s - \bar{B}_s$ mixing from the SM prediction is very small for $|\langle \delta_{23} \rangle| \lesssim 10^{-2}$. Thus we conclude that a single LR insertion can accommodate large deviation in $S_{\phi K}$ from the SM rather easily with $\tilde{m} = m_{\tilde{g}} = 400$ GeV. This scenario can be tested by measuring a positive direct CP asymmetry in $B \to X_s \gamma$ and $B_d - \bar{B}_d$ mixing consistent with the SM.

We also studied the RL dominance scenario, and the generic feature is similar to the LR insertion case except that (i) the $B \to X_s \gamma$ branching ratio gives a different constraint from the LR insertion case, since the SM contribution does not interfere with the RL contribution, and (ii) direct CP asymmetry in $B \to X_s \gamma$ is zero unless there is additional RR insertion. See Refs. [15,16] for further detail.

Now let us provide possible motivation for values of $|\langle \delta_{23}^{LR,RL} \rangle| \lesssim 10^{-2}$ that could shift $S_{\phi K}$ from the SM value rather easily. In particular, at large $\tan \beta$ it is possible to have
double mass insertions which give sizable contributions to \((\delta_{LR,RL}^d)_{23}\). First a \((\delta_{LL}^d)_{23}\) or \((\delta_{RR}^d)_{23}\) \(\sim 10^{-2}\) is generated. The former can be obtained from renormalization group running even if its initial value is negligible at the high scale. The latter may be implicit in SUSY GUT models with large mixing in the neutrino sector \([20]\). Alternatively, in models in which the SUSY flavor problem is resolved by an alignment mechanism using spontaneously broken flavor symmetries, or by decoupling, the resulting \(LL\) or \(RR\) mixings in the 23 sector could easily be of order \(\lambda^2\) \([10,21]\). However as discussed above, this size of the \(LL\) and/or \(RR\) insertions cannot explain the measured CP asymmetry in \(B_d \to \phi K_S\) unless the squarks and gluinos are rather light. But at large \(\tan \beta\), the \(LL\) and \(RR\) insertions can induce the \(RL\) and \(LR\) insertions needed for \(S_{\phi K}\) through a double mass insertion \([8,9]\):

\[
(\delta_{LR,RL}^d)_{23}^{\text{ind}} = (\delta_{LL,RR}^d)_{23} \times \frac{m_b(A_b - \mu \tan \beta)}{\tilde{m}^2}.
\]

One can achieve \((\delta_{LR,RL}^d)_{23}^{\text{ind}} \sim 10^{-2}\) if \(\mu \tan \beta \sim 10^4\) GeV, which could be natural if \(\tan \beta\) is large (for which \(A_b\) becomes irrelevant). Note that in this scenario both the \(LL(RR)\) and \(LR(RL)\) insertions would have the same CP violating phase, since the phase of \(\mu\) here is constrained by electron and down-quark electric dipole moments. Lastly, one can also construct string-motivated \(D\)-brane scenarios in which \(LR\) or \(RL\) insertions are \(\sim 10^{-2}\) \([16]\).

Summarizing this section, we considered several classes of potentially important SUSY contributions to \(B \to \phi K_S\) in order to see if a significant deviation in its time-dependent CP asymmetry \(S_{\phi K}\) could arise from SUSY effects. The Higgs-mediated FCNC effects are small. The models based on the gluino-mediated \(LL\) and \(RR\) insertions give a rather small deviation in \(S_{\phi K}\) from the SM prediction, unless the squarks and gluinos are relatively light. On the other hand, the gluino-mediated contribution with \(LR\) and/or \(RL\) insertions can lead to sizable deviation in \(S_{\phi Ks}\), as long as \(|(\delta_{LR,RL}^d)_{23}| \sim 10^{-3} - 10^{-2}\). As a byproduct, we found that nonleptonic \(B\) decays such as \(B \to \phi K\) begin to constrain \(|(\delta_{LR,RL}^d)_{23}|\) as strongly as \(B \to X_{s\gamma}\). Besides producing no measurable deviation in \(B^0 - \bar{B}^0\) mixing, the \(RL\) and \(LR\) operators generate definite correlations among \(S_{\phi K}, C_{\phi K}\) and \(A_{CP}^{b \to s\gamma}\), and our prediction for \(S_{\phi K}\) can be easily tested by measuring these other observables. Finally, we also point out that the \(|(\delta_{LR,RL}^d)_{23}| \lesssim 10^{-2}\) can be naturally obtained in SUSY flavor models with double mass insertion at large \(\tan \beta\), and in string-motivated models \([16]\).

4. \(s_R \to d_L\) transition induced by \(b_R \to s_R\) transition: \(\text{Re } (\epsilon'/\epsilon_K)\) vs. \(S_{\phi K}\)

Assuming that the current low value of \(S_{\phi K}\) is a signal of new physics, we need a new CP violating phase in \(b \to s\) transition. An attractive possibility for such new physics beyond the SM occurs in supersymmetric grand unified theories (SUSY GUT’s) scenarios with seesaw mechanism for neutrino masses and mixings. In such scenarios, the large atmospheric neutrino oscillation can be related with a large \(b \to s\) transition through down type squark and gluino loop effects. This flavor changing effect is parametrized by a mixing parameter \((\delta_{23}^d)_{RR}\) with a CP phase \(\sim O(1)\). For low \(\tan \beta\), the single \(RR\) insertion can lead to some deviation in \(S_{\phi K}\), if gluinos and squarks are relatively light. For large \(\tan \beta\) case, the double mass insertion can lead to effective \(RL\) insertion of \(10^{-2}\),
leading to a significant deviation in $S_{\phi K}$ from the SM prediction. In Fig. 5 (a), we show the Feynman diagram for $b \rightarrow sg$ involving a CP violating $(\delta_{23}^d)_{RR}$.

The new CP violating phase in the $RR$ insertion can affect the strange quark chromo-electric dipole moment (CEDM) through triple mass insertions, if there is an $LL$ insertion between third and second generation down squarks. [Fig. 5 (b)] [22,23,24]. Since the $LL$ insertion is generically present in the minimal supergravity (mSUGRA) case, the strange quark CEDM puts a strong constraint on the possible deviation of $S_{\phi K}$ from the SM prediction. However, the substantial theoretical uncertainties occurring when one relates the quark CEDM’s with the hadronic EDM’s suggest that it would be preferable to have some other observable at disposal in addition to the strange quark CEDM in order to constrain $S_{\phi K}$.

In this section, we point out that the phase in the $(\delta_{23}^d)_{RR}$ mixing parameter that would affect $S_{\phi K}$ can also contribute to direct CP violation within the neutral kaon system, namely $\text{Re} (\epsilon' / \epsilon_K)$ through triple mass insertion. The Feynman diagram for $\text{Re} (\epsilon' / \epsilon_K)$ with triple mass insertion [Fig. 5 (c)] is very similar to the Feynman diagram for the strange quark CEDM [Fig. 5 (b)]. Needless to say, making use of the observable $\text{Re} (\epsilon' / \epsilon_K)$ to constrain some SUSY soft breaking parameters also entails theoretical uncertainties mainly ascribed to our ignorance in the evaluation of the relevant hadronic matrix elements. Our discussion shows that, even taking into account such a huge degree of uncertainty, $\text{Re} (\epsilon' / \epsilon_K)$ still constitutes a precious tool in constraining the interesting flavor changing mass insertion parameter $(\delta_{32}^d)_{RR}$ and, in any case, it plays at least a complementary role to the strange quark CEDM in performing such a task.

In this talk, I’ll illustrate the main point in a simple way, relegating the detailed analysis including theoretical uncertainties to Ref. [25]. If the $(\delta_{23}^d)_{RR}$ mixing is the dominant new physics contribution to $B_d \rightarrow \phi K_S$, we find the following from (5) and (9) in the previous sections:

\[
\text{Re}(\epsilon'/\epsilon_K) : C_{8g}^{\text{SUSY}} (\Delta S = 1) \propto f_1(x) (\delta_{13}^d)_{LL} (\delta_{33}^d)_{LR} (\delta_{32}^d)_{RR},
\]

\[
S_{\phi K} : C_{8g}^{\text{SUSY}} (\Delta B = 1) \propto f_2(x) (\delta_{32}^d)_{RR} + f_3(x) \frac{m_{\tilde{g}}}{m_b} (\delta_{33}^d)_{RL} (\delta_{23}^d)_{RR},
\]

where $f_{i=1,2,3}(x)$ are the loop functions obtained in the previous sections. Now, if the
SUSY contribution saturates \( \text{Re} (\epsilon'/\epsilon_K) \), then it is well known that one has to satisfy
\[
|\text{Re} (\delta_{13}^{d})_{LL}(\delta_{33}^{d})_{LR}(\delta_{32}^{d})_{RR}| \lesssim 10^{-5}
\]
with an \( O(1) \) phase [2]. Since the RG evolution generates \( (\delta_{13}^{d})_{LL} \sim \lambda^3 \) within mSUGRA scenario, we can derive the following upper bound:
\[
|\text{Re} (\delta_{33}^{d})_{LR}(\delta_{32}^{d})_{RR}| \lesssim 10^{-2}. \tag{7}
\]

Note that this combination enters the calculation of \( S_{\phi K} \) and \( B \to X_s \gamma \) through \( C_{b(y(\gamma))}(\Delta B = 1) \) along with \( (\delta_{32}^{d})_{RR} \). For a small \( \mu \tan \beta \) (corresponding to a small \( (\delta_{33}^{d})_{RL} \), one can have larger \( (\delta_{32}^{d})_{RR} \), which is constrained by the lower bound on \( \Delta M_s \) and the \( B \to X_s \gamma \) branching ratio. For a large \( \mu \tan \beta \) (corresponding to a large \( (\delta_{33}^{d})_{RL} \), \( (\delta_{32}^{d})_{RR} \) should be smaller in order to satisfy [7], which would be less constrained by \( \Delta M_s \) and the \( B(B \to X_s \gamma) \). In either case, we can expect that the deviation in \( S_{\phi K} \) cannot be that large for such \( (\delta_{32}^{d})_{RR} \) satisfying the \( \text{Re} (\epsilon'/\epsilon_K) \) constraint, [7].

In Figs. 6 (a) and (b), we show the plots for \( S_{\phi K} \) and \( \text{Re} (\epsilon'/\epsilon_K) \) for \( \mu \tan \beta = 1 \) and 5 TeV, respectively, with \( \tilde{m} = m_{\tilde{g}} = 500 \) GeV. The thick vertical error bar shows the current data on \( S_{\phi K} \), and the two dashed vertical lines delimit the experimental value of \( \text{Re} (\epsilon'/\epsilon_K) \) [3],
\[
\text{Re} (\epsilon'/\epsilon_K) = (16.7 \pm 2.6) \times 10^{-4}. \tag{8}
\]
The full black box shows our estimation of \( S_{\phi K} \) and \( \text{Re} (\epsilon'/\epsilon_K) \) within the SM. Its width and height are the uncertainties in \( \text{Re} (\epsilon'/\epsilon_K) \) and \( S_{\phi K} \), respectively. Note that the \( \text{Re} (\epsilon'/\epsilon_K) \) data gives a strong constraint on the possible value of \( S_{\phi K} \) even if there are large hadronic uncertainties. Note that the constraint from \( \text{Re} (\epsilon'/\epsilon_K) \) is comparable to that from the strange quark CEDM. In particular the positive (negative) \( S_{\phi K} \) is correlated with the positive (negative) \( \text{Re} (\epsilon'/\epsilon_K) \) within minimal SUGRA boundary conditions. In particular, the old Belle data with the negative \( S_{\phi K} \) implies a negative \( \text{Re} (\epsilon'/\epsilon_K) \) in the \( RR \) dominance scenario such as SUSY GUT models with right-handed neutrinos, which is clearly excluded by the data \( \text{Re} (\epsilon'/\epsilon_K) = (16.7 \pm 2.6) \times 10^{-4} \). If the old Belle data were still valid, then the \( RR \) dominance scenario should have been discarded. Our results provide a meaningful correlation between \( S_{\phi K} \) and \( \text{Re} (\epsilon'/\epsilon_K) \) despite of large hadronic uncertainties in both quantities. This is independent of the strange quark CEDM constraint, and probably has less theoretical uncertainties.

If we considered more general flavor structures in the scalar masses at \( M_s \), then our results will be changed accordingly. The Wilson coefficients for \( C_{b(y)}'s \) for both \( \Delta B(S) = 1 \) have to include other mass insertion parameters such as \( (\delta_{23}^{d})_{LR} \), \( (\delta_{12}^{s})_{LL} \), \( (\delta_{23}^{d})_{LL} \), etc., which were neglected in Secs. II and III because they are small within the mSUGRA scenarios. Still we should make it sure that the new flavor physics that affects \( S_{\phi K} \) does not contribute to \( \text{Re}(\epsilon'/\epsilon_K) \) too much, and this could make a strong constraint on new sources of flavor and CP violation despite of theoretical uncertainties on \( \text{Re}(\epsilon'/\epsilon_K) \).

In summary, we showed that if the \( RR \) \( b \to s \) transition is large with \( O(1) \) phase, it can affect not only \( S_{\phi K} \) through the double mass insertion and the strange quark CEDM through triple mass insertion, it affects also the \( \text{Re} (\epsilon'/\epsilon_K) \). The correlation between the
Figure 6: $S_{\phi K}$ vs. Re ($\epsilon'/\epsilon_K$) for (a) $\mu \tan \beta = 1$ TeV and (b) $\mu \tan \beta = 5$ TeV, with $\tilde{m} = m_{\tilde{g}} = 500$ GeV. Experimental bounds on Re ($\epsilon'/\epsilon_K$) and $S_{\phi K}$ are depicted by the vertical dashed lines and the thick vertical error bar, respectively. The SM predictions of them are marked by the black box, whose extent indicates their uncertainties. The black curve does not include hadronic uncertainties, and the gray region includes them. The respective uncertainties in Re ($\epsilon'/\epsilon$) and $S_{\phi K}$ are shown by the horizontal and vertical error bars at some selected points.

The Higgs sector of the MSSM is not Type II but Type III two-Higgs doublet model due to the presence of the soft SUSY breaking terms. Therefore, there are loop induced nonholomorphic trilinear couplings, and this term can induce new FCNC involving neutral Higgs bosons [26]. In the large $\tan \beta$ region, this effect on the $b \to s$-Higgs couplings can be enhanced by $\tan^2 \beta$, and could dominate the $B_s \to \mu^+\mu^-$ process within SUSY models in the large $\tan \beta$ region. Since its branching ratio within the SM is very small ($(3.7 \pm 1.2) \times 10^{-9}$), this decay mode could be a sensitive probe of SUSY models in the large $\tan \beta$ region. In Refs. [27], we studied the correlations between $B_s \to \mu^+\mu^-$ branching ratio, the muon $(g-2)$, and other observables in the $B$ system, imposing the direct search limits on Higgs and SUSY particle masses, and $B \to X_s\gamma$ branching ratio and assuming that $(g-2)^{\text{SUSY}} > 0$ (namely $\mu > 0$). In this section, I report the main results of Refs. [27]. (The correlation between $(g-2)_\mu$ and $B_s \to \mu^+\mu^-$ was first noticed in Ref. [28] within the minimal supergravity scenario.)

The soft SUSY breaking parameters at electroweak scale is determined by RG evolution with the initial condition at the messenger scale $M_{\text{mess}}$ within a given SUSY breaking scenario. The initial conditions depend on SUSY breaking mediation mechanisms: super-
gravity (including scenarios motivated by superstring theories, $M$-theories and $D$-brane models), gauge mediation (GMSB), anomaly mediation (AMSB), gaugino mediation, to name a few. Many of these scenarios predict flavor blind soft terms at the messenger scale, and nontrivial flavor dependence in the soft terms are generated by RG evolution from $M_{\text{mess}}$ to electroweak scale $\mu_{EW}$. Then the dominant contribution to $b \to s$ transition comes from the chargino-stop loop diagram. Therefore, in order to have a large branching ratio for $B_s \to \mu^+ \mu^-$, we need large $\bar{t}_L - \bar{t}_R$ mixing, light chargino and stops, and large $\mu \tan \beta$. If these conditions cannot be met, there would be no chance to observe $B_s \to \mu^+ \mu^-$ in the near future at the Tevatron.

As an example, let us consider GMSB scenarios, which are specified by the following set of parameters: $M$, $N$, $\Lambda$, $\tan \beta$ and $\text{sign} (\mu)$, where $N$ is the number of messenger superfields, $M$ is the messenger scale, and the $\Lambda$ is SUSY breaking scale, $\Lambda \approx \langle F_X \rangle / \langle X \rangle$, where $X$ is a gauge singlet superfield $X$, the vacuum expectation value of which (both in the scalar and the $F$ components) will induce SUSY breaking in the messenger sector. If the messenger scale (where the initial conditions for the renormalization group (RG) running for soft parameters are given) is low such as $10^6$ GeV, the flavor changing amplitude involving the gluino-squark is negligible and only the chargino-upsquark contribution is important in $B \to X_s \gamma$. Also, in the GMSB scenario with low messenger scale, the charged Higgs and stops are heavy and their effects on the $B \to X_s \gamma$ and $B_s \to \mu^+ \mu^-$ are small. Also $A_t$ is small, since it can generated by only RG running. Therefore the stop mixing angle becomes also small. These effects lead to very small branching ratio for $B_s \to \mu^+ \mu^- (\lesssim 10^{-8})$, making this decay unobservable at the Tevatron Run II. On the other hand, the $a_\mu^{\text{SUSY}}$ can be as large as $60 \times 10^{-10}$. If we assume the messenger scale
be as high as the GUT scale, the RG effects become strong and the stops get lighter. Also the $A_t$ parameter becomes larger at the electroweak scale, and so is the stop mixing angle. Therefore the chargino-stop loop contribution can overcompensate the SM and charged Higgs-top contributions to $B \to X_s \gamma$ and this constraint becomes more important compared to the lower messenger scale. Also the $A_t$ parameter becomes larger at the electroweak scale, and so is the stop mixing angle. Therefore the chargino-stop loop contribution can overcompensate the SM and charged Higgs-top contributions to $B \to X_s \gamma$ and this constraint becomes more important compared to the lower messenger scale. Also the $B_{s} \to \mu^{+}\mu^{-}$ branching ratio can be enhanced (upto $2 \times 10^{-8}$ for $\tan \beta = 50$, for example), because stops become lighter and larger $\tilde{t}_L - \tilde{t}_R$ mixing is possible [Fig. 7 (a)]. If the number of messenger field is increased from $N = 1$ to 5, for example, the scalar fermion masses become smaller at the messenger scale, and stops get lighter in general. Therefore the chargino-stop effects in $B \to X_s \gamma$ and $B_{s} \to \mu^{+}\mu^{-}$ get more important than the $N = 1$ case, and the $B_{s} \to \mu^{+}\mu^{-}$ branching ratio can be enhanced upto $2 \times 10^{-7}$ [Fig. 7 (b)]. In short, the overall features in the GMSB scenarios with high messenger scale look alike the mSUGRA with $A_0 = 0$. Especially the branching ratio for the decay $B_{s} \to \mu^{+}\mu^{-}$ can be much more enhanced for large $\tan \beta$ in the GMSB scenario with high messenger scale [Fig. 7 (c)]. Thus, if $a_{\mu}^{\text{SUSY}} > 0$ and the decay $B_{s} \to \mu^{+}\mu^{-}$ is observed at the Tevatron Run II with the branching ratio larger than $2 \times 10^{-8}$, the GMSB scenario with $N = 1$ would be excluded upto $M_{\text{mess}} \sim 10^{10}$ GeV and $\tan \beta \lesssim 50$.

In the AMSB scenario, the hidden sector SUSY breaking is assumed to be mediated to our world only through the auxiliary component of the supergravity multiplet (namely super-conformal anomaly) [29]. In this scenario, the gaugino masses are proportional to the one-loop beta function coefficient for the MSSM gauge groups, whereas the trilinear couplings and scalar masses are related with the anomalous dimensions and their derivatives with respect to the renormalization scale. Since the original AMSB model suffers from the tachyonic slepton problem, we simply add a universal scalar mass $m_0^2$ to the scalar fermion mass parameters of the original AMSB model, and assume that the aforementioned soft parameters make initial conditions at the GUT scale for the RG evolution. Thus, the minimal AMSB model is specified by the following four parameters: $\tan \beta$, $\text{sign}(\mu)$, $m_0$, $M_{\text{aux}}$. We scan these parameters over the following ranges: $20 \text{ TeV} \leq m_{\text{aux}} \leq 100 \text{ TeV}$, $0 \leq m_0 \leq 2 \text{ TeV}$, $1.5 \leq \tan \beta \leq 60$, and $\text{sign}(\mu) > 0$. In the case of the AMSB scenario with $\mu > 0$, the $B \to X_s \gamma$ constraint is even stronger compared to other scenarios, and almost all the parameter space with large $\tan \beta > 30$ is excluded. Also stops are relatively heavy in this scenario mainly due to the universal addition of $m_0^2$. Therefore the branching ratio for $B_{s} \to \mu^{+}\mu^{-}$ is smaller than $4 \times 10^{-9}$, and this process becomes unobservable at the Tevatron Run II. For the detailed discussions on other variations of AMSB scenarios, see Refs. [27].

Summarizing this section, we showed that there are qualitative differences in correlations among $(g - 2)_{\mu}$, $B \to X_s \gamma$, and $B_{s} \to \mu^{+}\mu^{-}$ in various models for SUSY breaking mediation mechanisms, even if all of them can accommodate the muon $a_{\mu}$: $10 \times 10^{-10} \lesssim a_{\mu}^{\text{SUSY}} \lesssim 40 \times 10^{-10}$. Especially, if the $B_{s} \to \mu^{+}\mu^{-}$ decay is observed at the Tevatron Run II with the branching ratio greater than $2 \times 10^{-8}$, the GMSB with low number of messenger fields $N$ and certain class of AMSB scenarios would be excluded. On the other hand, the minimal supergravity scenario and similar mechanisms derived from string models, GMSB with large messenger scale and the deflected AMSB scenario can
accommodate this observation without difficulty for large \( \tan \beta \). Therefore search for \( B_s \to \mu^+\mu^- \) decay at the Tevatron Run II would provide us with important informations on the SUSY breaking mediation mechanisms, independent of informations from direct search for SUSY particles at high energy colliders. This is remarkable, since \( B_s \to \mu^+\mu^- \) could be an excellent discriminator of SUSY breaking mediations without directly producing SUSY particles at all. Let us stay tuned with updated data analysis on this decay by CDF and D0 collaborations at the Tevatron.

6. Interplay of B physics with cosmology

6.1. B physics and electroweak baryogenesis (EWBGEN) within SUSY models

Let us first discuss an effective SUSY model with minimal flavor violation \[21\]. In this model, the 1st and the 2nd generation squarks are very heavy and almost degenerate, thus evading SUSY flavor/CP problem. And flavor violation comes through CKM matrix, whereas CP violation originates from the \( \mu \) and \( A_t \) phases as well as the KM phase. Therefore the stop-chargino loop have additional source of CP violation in addition to the KM phase in the SM. One-loop electric dipole moment (EDM) constraint is evaded in the effective SUSY model due to the decoupling of the 1st/2nd generation sfermions, but there are poentially large two-loop contribution to electron/neutron EDM’s through Barr-Zee type diagram in the large \( \tan \beta \) region \[30\]. Imposing this two-loop EDM constraint and direct search limits on Higgs and SUSY particles, we make the following observations \[31,32\]:

- No new phase shifts in \( B_d - \overline{B}_d \) and \( B_s - \overline{B}_s \) mixings: Time dependent CP asymmetries in \( B_d \to J/\psi K_S \) still measures the KM angle \( \beta = \phi_1 \)
- \( \Delta M_{B_d} \) can be enhanced upto \( \sim 80\% \) compared to the SM prediction
- Direct CP asymmetry in \( B \to X_s\gamma \) (\( A_{\text{CP}}^{b\to s\gamma} \)) can be as large as \( \pm 15\% \) ( Fig. 8 (a) and (b) ) which is now strongly constrained by the data \( (0.5 \pm 3.6)\% \) \[13\]
- \( R_{\mu\mu} \equiv B(B \to X_s\mu^+\mu^-)/B(B \to X_s\mu^+\mu^-)_{\text{SM}} \) can be as large as 1.8, which is now strongly constrained by the data from B factories \[33\]
- \( \epsilon_K \) can differ from the SM value by \( \sim 40\% \).

Therefore we predict substantial deviations in certain observables in the \( B \) and \( K \) systems in SUSY models with minimal flavor violation and complex \( \mu \) and \( A_t \) parameters. Even if the \( A_t \) phase is set to zero, the predictoins do not change much. Now this model is beginning to be strongly constrained by new data on the direct CP asymmetry in \( B \to X_s\gamma \) and \( R_{\mu\mu} \) from B factories \[33\].

This class of models includes the electroweak baryogenesis (EWBGEN) within the MSSM \[34\] and some of its extensions such as NMSSM or extra \( U(1) \) gauge symmetry, where the chargino and stop sectors are the same as in the MSSM and the \( \mu \) phase plays a
Figure 8: Direct CP asymmetry in $B \to X_s \gamma$ as functions of (a) $B(b \to s\gamma)$ and (b) $m_{\tilde{\chi}}$ (the lighter chargino mass). The parameter space excluded by two loop EDM constraint is denoted by x.

key role to generate baryon number asymmetry. In the EWBGEN scenario within MSSM, Murayama and Pierce argued that there could no large CP violating effects from the $\mu$ phase on $B$ physics, except for the $B_d(s) = \overline{B_d(s)}$ mixing: $1 \leq \Delta M_s/(\Delta M_s)_{SM} \leq 1.30$ [35]. This is mainly because of the strong tension between the light $\tilde{t}_R$ and heavy $\tilde{t}_L$. In the EWBGEN scenario, we need a strong 1st order phase transition, and this requires a light $\tilde{t}_R$. On the other hand, the current LEP bound on the lightest Higgs mass $m_0^H$ for $3 \lesssim \tan \beta \lesssim 6$ (for larger $\tan \beta$, the $\mu$ phase effect drops out) calls for heavy $\tilde{t}_L$ to generate large stop loop corrections to $m_0^H$.

However, the LEP bound on the lightest Higgs mass becomes less problematic in the extensions of MSSM such as NMSSM or MSSM with extra $U(1)$ gauge group, because there are tree level contributions to the Higgs mass. Therefore the tension between the light $\tilde{t}_R$ and the heavy $\tilde{t}_L$ becomes much milder compared to the MSSM, and our predictions on the $B$ system still remain valid in such scenarios.

6.2. Neutralino dark matter scattering and $B_s \to \mu^+\mu^-$

In SUSY models with $R$–parity conservation, the lightest superparticle (LSP) is stable and becomes a good candidate for dark matter of the universe. In particular, the neutralino ($\chi$) LSP is a nice candidate for cold dark matter, and could be detected in the laboratory through (in)elastic scattering with nuclei. There are several direct search experiments going on around the world [36]. A few years ago, the DAMA Collaboration reported a positive signal in the range of $\sigma_{\tilde{\chi}p} = (10^{-6} - 10^{-5})$ pb with $m_{\chi}$ at electroweak scale [37]. However this was not confirmed by other experiments. Recently, the DAMA signal region has been excluded by the CDMS cryogenic DM search experiment [35] in the range of

$$\sigma_{\tilde{\chi}p} = (10^{-6} - 10^{-5})$$ pb,

with the corresponding DM mass depends on galactic halo models. Since the CDMS experiment probes the DM scattering down to $3 \times 10^{-7}$ pb level in certain range of DM mass, it is important to calculate the DM scattering cross section within well defined
and/or motivated SUSY models, in which the cross section can be in the CDMS sensitivity. Anyway the present sensitivity of the ongoing DM scattering experiments is roughly $10^{-6}$ pb, and it is important to identify the parameter space of general MSSM which can be probed by the DM scattering experiments.

In the following we show that there is a strong correlation between $\sigma_{\chi p}$ and $B_s \to \mu^+ \mu^-$ [39]. In the large $\tan \beta$ region of SUSY models, both processes are dominated by neutral Higgs exchange diagram, and the amplitudes for these two processes depend on $\tan \beta$ as

$$M(B_s \to \mu^+ \mu^-) \propto \tan^3 \beta / m_A^2,$$

$$M(\chi^0 p \to \chi^0 p) \propto \tan \beta / m_A^2.$$  \hspace{1cm} (9)

Therefore one can expect some correlation between the two observables in the large $\tan \beta$ limit. Since the current limit on $B(B_s \to \mu^+ \mu^-)$ is already tight enough, this could provide an important constraint on the neutralino DM scattering cross section.

In the minimal supergravity model with $R-$parity conservation, the LSP is binolike neutralino in most parameter space, and the spin-independent dark matter scattering cross section $\sigma_{\chi p}$ turns out to be very small $\lesssim 10^{-8}$ pb, after imposing various constraints from Higgs and SUSY particle masses, $B \to X s \gamma$, etc. [ Fig. 9 (a) ]. The mSUGRA models cannot give a large enough $\sigma_{\chi p}$ in the signal regions of DAMA and CDMS or in the sensitivity region of other experiments down to $\sim 10^{-8}$ pb. However, the usual minimal SUGRA boundary conditions for soft parameters are too much restrictive without theoretical justification, and it is important to study the dark matter scattering in more general supergravity models with nonuniversal soft terms [40]. In such case, one has to be careful not to overproduce flavor changing neutral current processes, which is a subject of this subsection.

As discussed before, the universal soft parameters are too restricted assumption without solid ground within supergravity framework. In order to consider more generic situation within supergravity scenario, let us relax the assumption of universal soft masses as follows:

$$m^2_{H_u} = m^2_0 (1 + \delta_{H_u}), \quad m^2_{H_d} = m^2_0 (1 + \delta_{H_d}),$$

(10)

whereas other scalar masses are still universal. Here $\delta$'s are parameters with $\lesssim O(1)$. By allowing nonuniversality in the Higgs mass parameters, the situation changes, however. For illustration of our main point, let us take the numerical values of $\delta$'s as in Refs. [36,40].

$$(I) \quad \delta_{H_d} = -1, \quad \delta_{H_u} = 1,$$

$$(II) \quad \delta_{H_d} = 0, \quad \delta_{H_u} = 1.$$  \hspace{1cm} (11)

For $\delta_{H_u} = +1$, $\mu$ becomes lower and the Higgsino component in the neutralino LSP increases so that $\sigma_{\chi p}$ is enhanced, as discussed in Ref. [36]. The change of $|\mu|$ also has an impact on the higgs masses because

$$m^2_A = m^2_{H_u} + m^2_{H_d} + 2\mu^2 \simeq m^2_{H_d} + \mu^2 - M_Z^2 / 2$$

at weak scale. For $\delta_{H_d} = -1$, $m_A$ and $m_H$ becomes further lower, and both $\sigma_{\chi p}$ and $B(B_s \to \mu^+ \mu^-)$ are enhanced. These features are shown in Fig.s 8 (b) and (c) for Case (I).
Figure 9: \( \sigma_{\tilde{\chi} p} \) vs. \( B(B_s \to \mu^+\mu^-) \) within (a) mSUGRA with universal Higgs mass parameters for \( \tan \beta = 10, 35 \) and 55 (from the left to the right), in SUGRA with nonuniversal Higgs mass parameters: (b) \( \delta_{H_u} = 1 \) and \( \delta_{H_d} = -1 \) and (c) \( \delta_{H_u} = 1 \) and \( \delta_{H_d} = 0 \). Black dots for \( \Omega_{\chi} h^2 \geq 0.13 \), red dots for \( 0.095 \leq \Omega_{\chi} h^2 \leq 0.13 \) and green dots for \( \Omega_{\chi} h^2 \leq 0.095 \).

Figure 10: \( \sigma_{\tilde{\chi} p} \) vs. the neutralino LSP mass, with the CDMS upper bound. The points which are excluded by \( B(B_s \to \mu^+\mu^-) < 5.7 \times 10^{-7} \) are denoted by red dots.

and (II), respectively. Note that the CDF upper bound \( B(B_s \to \mu^+\mu^-) < 5.8 \times 10^{-7} \) provides a very strong constraint on the neutralino DM scattering cross section \( \sigma_{\tilde{\chi} p} \), and removes the parameter space where the DM scattering is within the reach of the current DM search experiments.

The impact of the \( B(B_s \to \mu^+\mu^-) \) branching ratio constraint becomes more transparent in Fig.s 10 (a) and (b). Here we plot the neutralino DM scattering cross sections as functions of the LSP mass, imposing the CDMS bound as well as the \( B(B_s \to \mu^+\mu^-) \). The red points in the plots are excluded by \( B(B_s \to \mu^+\mu^-) < 5.7 \times 10^{-7} \). For \( \tan \beta = 35 \), both constraints are comparable. However, for \( \tan \beta = 50 \), the \( B(B_s \to \mu^+\mu^-) \) branching ratio puts a much stronger constraint than the direct search by CDMS. This clearly shows the importance of \( B_s \to \mu^+\mu^- \) when we study the neutralino DM scattering.

We also considered nonuniversal gaugino masses, in which case the most important one is the gluino mass parameter via RG running. Therefore we allowed nonuniversality only in the gluino mass parameter, and found that the qualitative feature is similar as in nonuniversal Higgs masses. In particular the current limit on \( B(B_s \to \mu^+\mu^-) \) already
puts a strong constraint on $\sigma_{\chi p}$ in the large $\tan \beta$ region.

In summary, we found that the upper limit on $B(B_s \rightarrow \mu^+\mu^-)$ is an important constraint on SUSY parameter space in the large $\tan \beta$ region, and the DM scattering cross section could be strongly affected by this constraint. This is an example of an interesting interplay between particle physics and cosmology.

7. Conclusion

In this talk, I discussed flavor physics within SUSY models, in particular where we may expect large deviations from the SM predictions, even if the unitarity triangle is the same as the SM case. This includes $B \rightarrow X_d \gamma$, $B \rightarrow X_s \gamma$, $B_d \rightarrow \phi K_S$, $B_s - \overline{B}_s$ mixing, and $B_s \rightarrow \mu^+\mu^-$ as well as $\epsilon'/\epsilon_K$. Also I discussed some interplay between B physics and cosmologically interesting SUSY scenarios. In EWBGEN scenarios within SUSY models, one may expect a large direct CP violation in $B \rightarrow X_s \gamma$, which is now strongly constrained by the data. Dark matter scattering cross section and $B_s \rightarrow \mu^+\mu^-$ exhibit a strong correlation for large $\tan \beta$. In particular, the branching ratio of $B_s \rightarrow \mu^+\mu^-$ can exceed the current CDF limit, when the DM scattering cross section becomes large within the sensitivity of the current DM search experiments: $\sigma_{\chi p} \sim (10^{-6} - 10^{-7})$ pb. This is an example where B physics and cosmology show an interesting interplay, and the upper limit on the branching ratio for $B_s \rightarrow \mu^+\mu^-$ becomes an important constraint on SUSY parameter space in the large $\tan \beta$ region, even stronger than the CDMS bound. In short, it is still possible to have substantial SUSY effects in the $b \to s$ transition without conflict with any other observed phenomena as of now. Therefore these processes should be actively searched for at B factory experiments in the coming years. By doing so, we can verify the CKM paradigm for flavor and CP violation, and better constrain the flavor and CP structures of SUSY models. Or we may encounter some nice surprise from the $b \to s$ transition.

Note: Recently, the D0 collaboration presented a new data [41]:

$$B(B_s \rightarrow \mu^+\mu^-) < 4 \times 10^{-7} \text{ (90\% C.L.)},$$

which is better than the CDF bound we used in the published paper. This D0 data would improve slightly the bounds discussed in Sec. 4 and Sec. 5.2.

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