A Quick Look at Renormalons

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We present a sketchy review of renormalon-based phenomenology. In particular, the leading, \(1/Q\) corrections to various observables, KLN cancellations for power-suppressed corrections and the fixation of operator matrix elements are highlighted.

Renormalons as a pure theoretical construct are known since 1977 [1]. The first attempts to develop a renormalon-based phenomenology are more recent [2,3]. Nowadays it is a fast developing field and a concise review is obviously beyond the scope of the present contribution. Instead, this paper is a compromise between an original paper and a mini-review. Namely we try to put the topics discussed in original papers with the participation of the authors into a more general framework.

1. THEORETICAL ASPECTS.

1.1. Renormalon basics.

The image of renormalons is invariably produced by the renormalon chain which is an insertion of \(n\), where \(n\) is large, vacuum-polarization bubbles into a photon (gluon) line. Denote, furthermore, by \(k\), the 4-momentum flowing through the dressed line and by \(Q\) a large external parameter (like total energy in \(e^+ e^-\)annihilation). If we expand in \(\alpha(Q^2)\) then in the \(n\)-th order one readily obtains for the coefficient \(a_n\) in front of \(\alpha(Q^2)\) the following estimates in terms of the first coefficient of the \(\beta\)-function \(b_0\):

\[
(a_n)_{IR} \sim \int d^4k \left( b_0 \ln Q^2 / k^2 \right)^n \sim n! b_0^{2-n} \quad (1)
\]

in case of \(k^2 \ll Q^2\) and

\[
(a_n)_{UV} \sim \int d^2k \left( b_0 \ln Q^2 / k^2 \right)^n \sim n! (-b_0)^n \quad (2)
\]

in case of \(k^2 \gg Q^2\). The behaviour (1) is process independent provided there is a single soft gauge-boson line \(k^2 \ll Q^2\) and \(Q\) is a euclidean momentum. In this way renormalons indicate the asymptotical nature of perturbative expansions and hence bring an uncertainty to perturbative calculations. Estimating this uncertainty in a standard way one gets :

\[
\delta_{IR} \sim \exp(-2b_0/\alpha_s(Q^2)) \sim (\Lambda_{QCD}/Q)^4, \quad (3)
\]

\[
\delta_{UV} \sim \exp(-b_0/\alpha_s(Q^2)) \sim (\Lambda_{QCD}/Q)^2
\]

where \(\alpha_s(Q^2) \sim (b_0 \ln Q^2 / \Lambda_{QCD}^2)^{-1}\).

Thus, renormalons, although arising within a purely perturbative framework, realize the idea of dimensional transmutation. Also, renormalons indicate presence of non-perturbative power corrections of the same order in \((\Lambda_{QCD}/Q)\) which are now needed to render the theory uniquely determined despite the uncertainties of perturbative expansions. Since renormalons always introduce two different mass scales, that is, \(k^2 \gg Q^2\) or \(k^2 \ll Q^2\) it is natural to invoke operator product expansions to evaluate their contribution. In case of infrared renormalons it is the standard OPE, when applicable. In particular, the series (1) above, for \(n \gg 1\) can be considered as a perturbative contribution to the matrix element of \(\langle 0 | \alpha_s(G_{\mu \nu}^a)^2 | 0 \rangle\) [2,3]:

\[
\frac{\langle 0 | \alpha_s(G_{\mu \nu}^a)^2 | 0 \rangle_{ren}}{24\pi Q^4} = \sum_{n \text{ large}} \frac{3\alpha_s(Q^2)n^{n+1}b_0^n}{2^{n+1}\pi^2} n! \quad (4)
\]

The non-perturbative counterpart was in fact introduced first via QCD sum rules [2]. In case of UV renormalons one can utilize [3] a reverse
OPE which is an expansion in $Q^2/k^2$. The use of an OPE allows us to formulate the renormalon contribution in terms of the running coupling, without direct use of the renormalon chains. The use of the OPE brings also a challenge to theory \[5,6\]. Namely, it turns out that a single renormalon chain does not dominate in fact over two and more chains. Thus, there is no closed set of graphs producing the same \[6\].

In short, the renormalons are a simple and systematic way to parametrize the IR contributions to various observables.

1.2. Limitations of renormalons.

At the one-loop level renormalons are not a unique and even not necessarily the simplest way to probe IR regions perturbatively. Another possibility is an introduction of finite gluon mass \(\lambda\). The gluon mass was tried as a fit parameter about 15 years ago \[7\]. In particular there is an infrared-sensitive perturbative gluon condensate \[8\]:

\[
\langle 0 | \alpha_s(Q^2)(G_{\mu\nu}^a)^2 | 0 \rangle = - \frac{3\alpha_s}{\pi^2} \lambda^4 \ln \lambda^2
\]

which is a substitution for the renormalon contribution \[6\] in case of massless gluons. In recent times, the use of a finite mass \(\lambda \neq 0\) has become very common. In what follows we shall not always distinguish between one-loop calculations with finite \(\lambda\) and a single renormalon chain, labeling generically both techniques as renormalons.

It might be worth emphasizing, however, that nowadays the finite gluon mass is used mostly not as a fit parameter but rather as a probe of infrared region. Namely, non-analytical in \(\lambda^2\) terms come exclusively from infrared gluons. The power of \(\lambda\) characterizes then the strength of the IR sensitive contributions. Generically, the translation of one-loop calculations with finite gluon mass \(\lambda\) and with IR renormalons looks as follows \[6\]:

\[
\alpha a_0 \ln \lambda^2 + \alpha a_1 \frac{\sqrt{\lambda^2}}{Q} + \alpha a_2 \frac{\lambda^2 \ln \lambda^2}{Q^2} + \ldots \rightarrow \]

\[b_0 \ln \Lambda^2_{QCD} + b_1 \frac{\Lambda_{QCD}}{Q} + b_2 \frac{\Lambda_{QCD}}{Q^2} + \ldots\]

where we keep only infrared sensitive contributions and \(a_i, b_i\) are coefficients.

Among the limitations of the renormalon technique let us mention the following points:

(i) renormalons respect the symmetries of the Lagrangian and cannot, for example, produce a non-vanishing quark condensate \(\langle \bar{q}q \rangle \neq 0\).
(ii) renormalons are “target-blind”, e.g., \(\langle p | G^2 | p \rangle \text{renorm} = \langle 0 | G^2 | 0 \rangle \text{renorm} \).
(iii) renormalons give no direct indication of confinement, say, of a string configuration.

An interesting problem is brought out by renormalons \[10\] in supersymmetric gluodynamics. To render the theory supersymmetric one adds to gluons an equal number of gluinos \(\lambda^a\). The gluinos affect the value of \(b_0\) in Eq. \[6\] but this seems to be the only change. On the other hand, one might argue that \((G_{\mu\nu}^a)^2\) now vanishes. Indeed the vacuum expectation value of the Lagrangian is zero \[11\]:

\[
\langle 0 | - \frac{1}{4} (G_{\mu\nu}^a)^2 + \lambda^2 \{D\lambda^a | 0 \rangle_{SUSY} | 0 \rangle_{SUSY} = 0 \]

since it is an F-component of a superfield. Since \(D\lambda^a = 0\) by virtue of equation of motion, one is inclined to think that \(\langle 0 | \lambda^a \{D\lambda^a | 0 \rangle_{SUSY} \rangle \neq 0\), which is not vanishing in the renormalon approximation and cancels the gluon condensate induced by renormalons. The reason is that in SUSY gluodynamics the gluino wave function renormalization is related to that of the gauge constant while in ordinary QCD it is gauge dependent and in this sense arbitrary. This might be an indication that the dynamics of supersymmetric gauge theories is in fact very different from QCD.

In conclusion, renormalons provide us with a systematic, although incomplete, way to guess at non-perturbative physics in QCD. Theoretically, there are important questions yet to be answered.

2. PHENOMENOLOGY. GENERAL.

2.1. Could-be phenomenology.

The main use of renormalons is in cases when there is no OPE. However, one can try to gauge possible renormalon-based phenomenologies to the case when an OPE is valid. As is mentioned above, evaluating, for example, the T-product of
two electromagnetic currents at large Euclidean momenta \( Q^2 \) one finds that perturbation theory is unreliable in the infrared as far as terms of order \( Q^{-4} \) are concerned. Then the polarization operator \( \Pi(Q^2) \) could be represented as

\[
Q^2 \frac{d\Pi(Q^2)}{dQ^2} = (\text{parton model}) \cdot (1 + a_1\alpha_s(Q^2) + a_2\alpha_s(Q^2)^2 + \ldots + a_{\text{ren}}C\frac{\Lambda_{\text{QCD}}^2}{Q^2})
\]

where \( C \) is a constant related to the procedure of defining the uncertainty associated with an asymptotical expansion and \( a_{\text{ren}} \) varies with the choice of the external current, i.e. is channel dependent. Fitting the data in various channel with a single unknown constant \( C \) one could try to develop a phenomenology similar to QCD sum rules.

This kind of phenomenology, however, would run into apparent difficulties. Indeed, the \( 1/Q^4 \) contribution in is a tiny piece on the background of the first terms in a \( \alpha_s(Q^2) \) expansion. In particular any redefinition of the coupling would resuffle the whole series and the \( 1/Q^4 \) piece could well depend on such a redefinition. Thus, it is inconsistent, generally speaking, to keep the renormalon contributions without keeping many orders in \( \alpha_s(Q^2) \). The phenomenology is painstaking and its principal features are outlined in Ref. for the implementation on the lattice of this approach see for the implementation on the lattice of this approach see.

On the other hand the success of the QCD sum rules is based on a simplifying assumption that the non-perturbative terms matching the renormalon ambiguity are in fact large. It is natural to accept this approach in other applications of renormalons when we have no OPE, as well.

### 2.2. Renormalons and power corrections.

Recent considerations of renormalons have brought to light various power corrections. Common to all the examples which we list below is that they go beyond higher twist effects indicated by the standard OPE.

(i) In case of total cross section, a new type of correction appears due to UV renormalons:

\[
\frac{\sigma_{\text{tot}}(\gamma^* \rightarrow X)}{\sigma_{\text{tot,parton}}(\gamma^* \rightarrow X)} = 1 + a_1\alpha_s + \ldots + c_{\text{UV}}\frac{\Lambda_{\text{QCD}}^2}{Q^2}.
\]

Combined with the idea of enhancement these terms could solve certain problems with QCD sum rules and provide a link to NJL models.

We will concentrate on IR renormalons and in this review only note in passing that already the consideration of the UV induced \( \Lambda_{\text{QCD}}^2/Q^2 \) terms revealed the problem that overshadows all the applications of renormalons. Namely, in the absence of OPE it is much more difficult to relate different channels. In particular, the \( \Lambda_{\text{QCD}}^2/Q^2 \) corrections are welcome on phenomenological grounds in the \( \pi \)-meson channel but not in the \( \rho \)-meson channel. It is not known whether UV renormalons produce such a pattern of \( 1/Q^2 \) corrections.

(ii) Infrared renormalons induce \( 1/Q \) corrections to many observables. The first indications to these corrections were found in the cross section of the Drell-Yan process:

\[
h_1 + h_2 \rightarrow (\mu^+\mu^-) + X.
\]

Shape variables, like the thrust \( T \), also receive \( 1/Q \) corrections. In the language of a finite gluon mass:

\[
1 - T \sim \lambda/Q.
\]

In all the cases these corrections are due to soft gluons with 3-momenta of order \( \lambda \). In the easiest way they can be visualized on the example of a heavy quark mass. The infrared correction to a heavy mass \( M_H \) due to the Coulomb-like field is of order:

\[
\frac{dM_H}{M_H} d^3r \sim \frac{1}{8\pi M_H} \int |E|^2 \sim \alpha_s \frac{\lambda}{M_H}.
\]

where \( E \) is the electric field and by the infrared sensitive piece of the mass one can understand the difference in mass renormalization in cases \( \lambda = 0 \) and \( \lambda \neq 0 \). This contribution is well defined then.

(iii) Renormalons may bring new predictions also in cases when the power corrections could be treated within the standard OPE, like deep-inelastic scattering or inclusive decays of heavy particles. The reason is that in terms of the standard procedures the renormalon calculus unifies...
evaluation of the coefficient functions and of the corresponding matrix elements. As a result new relations may arise. The simplest relation of this kind has been in fact already mentioned. Namely, there is no dependence on the target. We shall discuss further examples in the next section.

Thus, we conclude this section with a remark that at least potentially renormalons may provide us with a new dimension in studies of power corrections. These corrections, in turn, may be important, for example, for the extraction of numerical values of $\alpha_s$ from measurements of event shape variables. For an initial attempt see [21].

3. RENORMALON ”ZEROS”.

3.1. Heavy quark decays.

Renormalon-based predictions naturally fall into two categories, namely, when one gets either a vanishing or a nonvanishing contribution. If we get a zero in a particular calculation, then it is natural to look for a kind of more general explanation, like a symmetry. This indeed turns out to be true, at least for the examples known so far.

As a first example consider inclusive leptonic decays of heavy particles [19]. Confining ourselves to one-loop radiative corrections and keeping $\lambda \neq 0$ we can, generally speaking, parametrize the infrared sensitivity in terms of the coefficients $a_i$ (see also Eq (6)):

$$\Gamma_0^\text{tot} = \Gamma_0^\text{tot}(1 + a_0 \ln \lambda^2 + a_1 \sqrt{\lambda^2} + ... )$$  \hspace{1cm} (13)

where $\Gamma_0^\text{tot}$ is the partonic width, with inclusion of corrections of order $\alpha$. The results of a straightforward calculation are [3]

$$a_0 = a_1 = a_2 = 0.$$  \hspace{1cm} (14)

Now, these zeros have different status in fact. The vanishing of $a_0$ is the well-known Bloch-Nordsieck cancellation. The vanishing of $a_1$ was claimed first [4] on the basis of the OPE for heavy quarks decays (for a review and further references see [5]). This cancellation holds provided that the bare width is proportional to the fifth power of a short-distance mass $M_{sh,d}$ instead of the physical, or pole mass $M_{pole}$:

$$M_{sh,d} \approx M_{pole}(1 - \frac{\alpha_s}{2} \lambda)$$  \hspace{1cm} (15)

where we keep only the infrared sensitive contribution. The physical meaning of this procedure is simple. Indeed, the total decay width is sensitive to the instantaneous energy release. The Coulomb field, on the other hand, is ”shaken off” as a result of a fast decay and the Coulomb correction to the mass (12) does not affect the total width. More elaborate calculations confirm this intuition.

As for the vanishing of the coefficient $a_2$ there are no obvious general reasons for it. Moreover, within the OPE one can show [6] that the quadratic corrections are generally related to matrix elements of operators $O_{1,2}$:

$$O_1 = \frac{1}{M_H^2} Q \sigma_{\mu \nu} G_{\mu \nu} Q, \quad O_2 = \frac{1}{M_H^2} Q D^2 Q/(17)$$

where $Q$ is the (operator of) the field of the heavy particle, $G_{\mu \nu}$ is the gluonic field strength tensor (with color indices suppressed), and $D$ is the covariant derivative. It is worth emphasizing that the use of OPE does not assume that the matrix elements of the operators (16) over a free particle state are normalized to zero. Moreover, the infrared-sensitive part of the matrix elements are uniquely determined and are not subject to redefinitions. It just happens that in the renormalon approximation the matrix elements of (16) vanish. This is an example of what we mean in point (iii) of the preceding subsection and we shall return to discuss it in more detail below.

3.2. KLN-vacuum.

In case of heavy quark decays reviewed above one expects the vanishing of the leading $1/Q$ corrections based on the OPE. In case of the Drell-Yan process (10) there is no OPE and one could expect appearance of $1/Q$ corrections. However, a straightforward calculation demonstrated [7] that terms linear in $\lambda$ in fact cancel in one loop. In more detail one evaluates moments $M_n$ from the cross section:

$$\int d\tau \tau^{n-1} \frac{d\sigma(Q^2, \tau)}{dQ^2} = M_n(1 + \alpha_s a_1 \sqrt{\lambda^2} + ...)$$  \hspace{1cm} (17)

where $Q$ is the invariant mass of the lepton pair produced, $\tau = Q^2 / s$ and $\sqrt{s}$ is the invariant mass of the $q\bar{q}$ from the initial hadrons $h_{1,2}$. The result
is \( a_1 = 0 \) provided \( n \) is not very large:
\[
n \cdot \frac{\Lambda_{QCD}}{\sqrt{s}} \ll 1.
\]  
(18)

As argued in [23] the reason for this cancellation is again general and it is a manifestation of the inclusive nature of the moments (17). If one considers, on the other hand very large \( n \) (see (18)) then the integral is practically saturated by an exclusive channel. Moreover, the cancellation of the linear terms in (at least \( U(1) \)) gauge theories appears to be the same general phenomenon as the Bloch-Nordsieck cancellation.

One starts with the Kinoshita-Lee-Nauenberg theorem [24] as the most general statement on infrared cancellations. Moreover, one can argue [25] that the KLN summation over initial and final states eliminates not only the \( \ln \lambda \) terms as is emphasized in the original papers but linear terms as well:
\[
\sum_{i,f} |S_{i \rightarrow f}|^2 \sim 0 \cdot \ln \lambda^2 + 0 \cdot \sqrt{\lambda^2}.
\]  
(19)

Here \( S_{i \rightarrow f} \) are elements of the \( S \)-matrix and relation [13] holds in each order of the perturbative expansion. The rationale behind [13] is simple: the KLN summation cancels the singular, \( 1/\omega \) terms on the level of the amplitudes which implies elimination of both \( 1/\omega^2 \) and \( 1/\omega \) terms in \( \sum |S_{i \rightarrow f}|^2 \).

Note that to visualize the cancellations due to the summation over the degenerate initial states one may think in terms of a ”KLN-vacuum” which is populated by soft gluons. To account for these particles in the initial state the original KLN summation invokes both connected and disconnected graphs. To prove Eqs. (19), (21) on the technical side it is crucial that instead of summing over disconnected graphs one can systematically add to ordinary Feynman graphs those with propagators of soft particles changed into their complex conjugates [23]:
\[
\left( \frac{-i}{k^2 + i\epsilon} \right) \rightarrow \left( \frac{-i}{k^2 + i\epsilon} \right)^*.
\]  
(20)

Adding graphs with the modified propagator (20) is equivalent to using the KLN vacuum and is simple technically. It seems also plausible that the KLN vacuum could be reduced to a finite-temperature vacuum but this analogy has not been elaborated so far.

The next step is to relate the KLN sum, which extends over initial and final states, into a summation over the final states alone. It is well known that as far as the most singular terms are concerned it is indeed possible, and the KLN sum so to say folds into twice the Bloch-Nordsieck sum over the final states:
\[
\sum_{i,f} |S_{i \rightarrow f}|^2 \rightarrow_{soft} 2 \cdot \sum_f |S_{i \rightarrow f}|^2
\]  
(21)

where we have indicated that this is true for soft but not collinear gluons. The new development is to show that Eq. (21) holds for linear terms as well. The proof [23] utilizes the Low theorem and is made explicit for the Drell-Yan process. However, the reasoning appears general enough to apply to other processes as well.

One may wonder also how far the use of the KLN vacuum (or, equivalently, of the propagator (20)) extends infrared cancellations. The general answer [25] is that the cancellations continue until one reaches the condensates terms. In particular, in case of the gauge theories the use of the propagator (20) doubles the effect of the perturbative gluon condensate (5). Very recently this function of the modified propagator (20) was emphasized in Ref. [26].

To summarize: at the one-loop level, the linear terms cancel from inclusive cross sections the same way as logarithmic terms do. The basic step in the proof is the use of the KLN vacuum populated with soft particles or, equivalently, addition of graphs with the modified propagator (20). In case of \( U(1) \) gauge theories the cancellation holds for higher loops as well.

### 3.3. Vanishing matrix elements.

A specific feature of the renormalon calculus is that the power corrections get universally expressed in terms of \( \Lambda_{QCD} \) or \( \lambda \) and are not dependent on the target. On the other hand, if the same observable can be treated within OPE the power corrections are routinely related to matrix elements of various operators. Thus, renormalons fix the matrix elements. Whether this fixation
provides satisfactory results, is a different issue which has not been addressed systematically, to our knowledge. Thus, we confine ourselves to a few casual remarks.

As we have already mentioned, in case of heavy quarks, renormalons imply the suppression \( \lambda^2 \ln \lambda^2 \) of the matrix elements of the operators (16):

\[
\langle \text{free particle}| Q_{\mu
u} Q_{\mu
u} | \text{free particle} \rangle = O(\lambda^3) \tag{22}
\]

while on dimensional grounds one would expect terms of order \( \lambda^2 \ln \lambda^2 \).

Technically, the vanishing of the leading terms is due to simple dynamical features of gauge interactions. In particular one observes (27) that the matrix element of the operator of the kinetic energy, \( D^2 \), immediately reduces to a matrix element of a local operator which is nothing else but the vacuum expectation value of the vector potential squared:

\[
\langle \text{free particle}| \bar{Q} D^2 Q | \text{free particle} \rangle \sim C \cdot \langle \mathbf{A}^2 \rangle \tag{23}
\]

It is only natural then that the constant \( C \) turns to be zero because of gauge invariance. As for the matrix element of the magnetic energy, \( \bar{Q} \sigma_{\mu\nu} G_{\mu\nu} Q \), its vanishing is due to the fact that transverse gluons do not interact with a charged particle at rest. While the the matrix elements in point, (22), were calculated directly only at the one-loop level the reason for thier suppression remains true in higher orders as well (27).

It is difficult to comment on the significance of (22). On one hand, the theory of heavy quark decays (for a review see (21)) assumes that the matrix elements in point are determined by the atom-like structure of hadrons and tacitly assumes that for free quarks they are zero. The latter is not obvious (especially in case of confinement). One may say then that this is supported by renormalons (see (23)). On the other hand the very idea that the matrix elements can be target-independent looks very foreign to the whole OPE approach to heavy hadrons decays. It appears more reasonable to apply renormalons only to free particle decays.

In case of deep inelastic scattering one can evaluate power corrections to moments of structure functions. To be specific, consider (24) the first moment of \( F_3(x) \), \( \int dx F_3(x) \), relevant to the Llewellyn-Smith-Gross sum rule. Then the the leading twist contribution and the first power correction are determined by the matrix elements of the following operator (28):

\[
O_{\mu\nu} = \frac{2i}{q^2} \epsilon_{\mu a \beta} q_\alpha (\bar{q} \gamma_\beta q + \frac{4g}{3q^2} q \tilde{G} \gamma_5 \gamma_\alpha q) \tag{24}
\]

Applying the renormalon idea means that one evaluates the power correction in terms of the matrix element of the leading-twist operator. In terms of the IR parameters entering the Feynman graphs this matrix element is a function of the gluon mass \( \Lambda \), quark mass \( m \) and of the quark virtuality \( p^2 - m^2 = \epsilon^2 \) (27). In more detail:

\[
\langle \bar{q} \tilde{G} \gamma_\alpha \gamma_\beta q \rangle = f(p^2, m^2, \lambda^2) \frac{C_F}{2\pi} \frac{4\alpha_s}{3} (\bar{q} q)(25)
\]

where

\[
f(\lambda^2, m^2, \epsilon^2) = \int_0^1 dy X(y) \ln X(y);
\]

\[
X = \epsilon^2 y (y - 1) + m^2 y^2 + \lambda^2 (1 - y). \tag{26}
\]

As one would expect the \( \lambda^2 \ln \lambda^2 \) term disappears if \( m^2 \gg \lambda^2 \) for the same reason as above (see Eq. (24)) and is taken over by the quark mass (for \( \epsilon = 0 \)) as an infrared parameter. On the other hand, the \( \lambda^2 \ln \lambda^2 \) term does represent the power correction if other infrared sensitive parameters are set to be zero. In fact much more detailed calculations, representing the whole \( x \)-dependence of the quadratic power correction are available in this case, or an equivalent thereof (29)(30).

At the next step one has to account for the anomalous dimension of the operator governing the \( Q^{-2} \) correction (see Eq. (24)). In the Minkowskii-space approach the effect of the anomalous dimension corresponds to emission of soft gluons by energetic gluons. This has not been considered so far and it is not clear that \( \lambda \neq 0 \) can be consistently kept at this stage.

Summarizing this section, the vanishing of certain power corrections revealed so far through the use of renormalons can be understood each time within a broader theoretical framework. The development of the corresponding framework was sometimes initiated by renormalons and its completion by including non-abelian theories still represents a challenge.
4. RENORMALONS AND EVENT SHAPES.

Renormalon and renormalon-related techniques have turned out to be instrumental in providing a theoretical basis for the existence of $\Lambda_{QCD}/Q$ corrections in shape variables in $e^+e^-$ annihilation. The phenomenology of these terms is of special interest since they represent, on one hand, leading power corrections and, on the other hand, there does not exist an alternative more general framework to treat these corrections. There are experimental fits to one more general framework to treat these corrections. There are experimental fits to $1/Q$ corrections and a careful experimental study of $1/Q$ has been made in [31,15] and a careful experimental study of $1/Q$ has been made in [32].

The very existence of the $1/Q$ corrections has been demonstrated by various techniques. Let us mention finite gluon mass [33], single renormalon chain [15,33], dispersive approach to the running coupling [29]. It also can be seen from simple estimates. Consider, for example, thrust $T$:

$$T = \max_n \sum_i |p_i \cdot n| \sum_i |p_i|$$ (27)

where $p$ are the momenta of the particles produced while $n$ is a unit vector. Perturbatively $T \neq 1$ arises because of the emission of gluons from quarks. Consider then a contribution to $T$ due to a soft gluon emission:

$$\langle 1-T \rangle_{soft} \sim \int_0^{\Lambda_{QCD}} \frac{d\omega}{\omega} \omega \alpha_s(\Lambda_{QCD}) \sim \frac{\Lambda_{QCD}}{Q}$$ (28)

where the first factor in the integrand comes from the definition of the thrust, $d\omega/\omega$ is the standard factor of emission of a soft gluon, and the running coupling $\alpha_s(\Lambda_{QCD})$ is of order unity. Note that, unlike the inclusive Drell-Yan cross section, evaluation of the thrust assumes that the momenta of final particles are resolved on the infrared sensitive scale and there is no reason, therefore, to expect cancellation of these terms.

Once the existence of $1/Q$ is established, the effort to create phenomenology shifts to deriving relations among various observables and such relations were claimed in all the approaches mentioned above. In particular, in the one-renormalon approximation [3] one gets for the standard shape variables:

$$\frac{1}{2} (1 - T)_{1/Q} = \frac{1}{3\pi} (C)_{1/Q} = \frac{2}{\pi} \left( \frac{\sigma}{\sigma_T} \right)_{1/Q} = \frac{1}{\pi} (E \sin^2 \delta)_{1/Q} = U$$ (29)

where $Q$ is now the total c.m. energy, the subscript $1/Q$ means that only linear power corrections are kept and $U$ is a universal factor:

$$U = \frac{C_F}{\pi Q} \int_0^{\Lambda_{QCD}} \frac{dk_\perp^2}{k_\perp^2} \alpha_s(k_\perp^2) \sim \frac{\Lambda_{QCD}}{Q}. \quad (30)$$

Moreover, according to the rules of the renormalon calculus only contribution of the Landau pole in $\alpha_s(k_\perp^2)$, parametrized in a certain way, is retained in (30). As a result $U \sim \Lambda_{QCD}/Q$ indeed. Similar, although not identical, relations have been obtained within other approaches. The earliest derivation [18] used the finite gluon mass technique. Comparisons with existing data, in general, look favourable [5,15,32].

Having said this, we have to make numerous reservations as to the status of relations of the type (30). The point is that there are uncertainties in derivations which can be removed only at a price of further assumptions. In different approaches these uncertainties arise in different ways but reflect the same difficulty: Namely, perturbative calculations are reliable when the coupling is small. Now we are trying to relate infrared contributions to various observables. This is possible only if a certain extrapolation procedure is accepted and any procedure of this kind is speculative.

In the renormalon language, the problem is that all orders of the perturbative expansion which is an expansion in a small parameter in the UV region, collapse to the same order of magnitude in the IR region. Indeed, since

$$\alpha_s^2(k_\perp^2) \sim \Lambda_{QCD} \frac{d\alpha_s(k_\perp^2)}{d\Lambda_{QCD}}$$ (31)

we have

$$\int_{1/R} \frac{dk_\perp^2}{k_\perp^2} \alpha_s^2(k_\perp^2) \frac{k_\perp^2}{Q} \sim U \sim \frac{\Lambda_{QCD}}{Q}. \quad (32)$$

Thus, one is invited to address the problem in higher orders as well.
There is a hope that the universality relations (30) hold in higher orders as well. Namely, it is known that all the log terms which dominate in perturbative region are universally related to the so called cusp anomalous dimension $\gamma_{\text{cusp}}$. If one retains only these terms in IR as well then the universal factor $U$ in Eq. (30) becomes:

$$ U = \int_0^{-Q^2} \frac{dk_\perp^2}{Q \cdot k_\perp} \gamma_{\text{cusp}}(\alpha_s(k_\perp^2)). $$

(33)

The reservation is that the terms which dominate in UV region do not necessarily dominate upon the continuation into the IR region.

An attractive possibility is to relate the factor $U$ in Eq. (30) to parameters of hadronization models \cite{15}. Indeed, the renormalon technique parametrizes contribution of the region where the running coupling $\alpha_s$ blows up. Since in the perturbative regime the coupling runs with $k_\perp^2$ renormalons, at least intuitively, correspond to introducing intrinsic transverse momentum for hadrons in a quark jet. In the two-jet limit this relation can be made quantitative \cite{13}. Namely, let $\bar{\rho}(z, p_\perp)$ denote the appropriately normalized distribution of hadrons in a jet with longitudinal momentum fraction $z$ and perpendicular component $p_\perp$. Then

$$ U \to \int d^2 p_\perp \rho(p_\perp) \frac{p_\perp}{Q} $$

(34)

where $\rho(p_\perp) \equiv \bar{\rho}(0, p_\perp)$. Numerical value of (34) can be obtained from fits to jet masses within the tube model (for a review see Ref. \cite{34}) which identifies $\rho(p_\perp)$ with the $p_\perp$ distribution of hadrons in a rapidity-$p_\perp$ "tube". Using the experimental data one then gets

$$ Q \cdot U \approx 0.5 GeV, $$

(35)

the value which also fits well the data on the $1/Q$ terms in shape variables.

Thus, Eq. (34) can be considered as an attempt to formulate the enhancement hypothesis (see subsection 2.2) in pure phenomenological terms. Theoretically it would be very attractive to formulate this hypothesis in terms of matrix elements of some operators. Note therefore the attempts to develop a kind of OPE valid for jet physics \cite{33}.

We have spelled out in some detail the difficulties of a phenomenology based on renormalon chains. It is worthwhile to mention that other approaches suffer uncertainties as well. For example, the prediction for the thrust $T$ depends on whether one keeps the gluon mass $\lambda \neq 0$ in the denominator of Eq. (27) or not. The prediction closest to the renormalon chain arises if this kinematical effect is neglected \cite{29}.

In view of the model dependence of the prediction for the $1/Q$ corrections in shape variables, it would be important to list predictions which could distinguish between various models. This has not been done however and we confine ourselves only to a single remark of this kind \cite{13}. Namely, the renormalon-chain predictions outlined above allow easily for an enhancement hypothesis. That is, if two-jet events are observed the $1/Q$ corrections to a heavy jet mass $M_h$ and to the light jet mass $M_l$ could be comparable. The only relation which is expected to hold is

$$ \langle 1 - T \rangle_{1/Q} = \langle M_h^2 / Q^2 \rangle_{1/Q} + \langle M_l^2 / Q^2 \rangle_{1/Q}. $$

(36)

This relation is simply an expression of the fact that the $1/Q$ corrections arise due to soft gluons. On the other hand, the models with a finite gluon mass or the frozen coupling do not allow for such an enhancement.

Data at relatively low energies \cite{31} do indicate

$$ \langle M_h^2 / Q^2 \rangle_{1/Q} \approx \langle M_l^2 / Q^2 \rangle_{1/Q}. $$

(37)

which can be considered as a support to the particular enhancement mechanism described above.

To comprehend the significance of data at higher energies more theoretical work is needed. The point is that the $1/Q$ form of the leading power corrections has been established in the two-jet limit. At high energies, however, the two-jet events themselves are suppressed by a Sudakov form-factor. It is for this reason that the $1/Q$ corrections from the very beginning \cite{17} were claimed for resummed cross sections. To ensure the two-jet dominance one could introduce a corresponding weight factor. In case of the thrust, for example, one can consider \cite{33} the following
average as far as the $1/Q$ terms are concerned:

$$\langle 1 - T \rangle_{1/Q} \rightarrow \langle \exp(-\nu(1 - T)) \rangle_{1/Q} \quad (38)$$

where $\nu$ is a new parameter which is to be large enough to ensure the dominance of the region $(1 - T) \ll 1$.

To avoid a special weighting function one should have developed the theory of $1/Q$ corrections for three-jet events and so on. This has not been done. For a discussion of the effect of intrinsic $k_\perp$ near three-jet configurations see Ref. [35].

Summarizing this section, relations among $1/Q$ terms in various observables are model dependent. It looks plausible at this point that the renormalon-based model will merge with the old-fashioned hadronization models.

REFERENCES

1. G. ’t Hooft, in "The whys of subnuclear physics" Erice 1977, Ed. Zichichi, Plenum, (1979), p. 94;
B. Lautrup, Phys. Lett., B69 (1978) 109.
2. A. Mueller, Nucl. Phys. B250 (1985) 327.
3. V.I. Zakharov, Nucl. Phys. B385 (1992).
4. M.A. Shifman, A.I. Vainshtein, and V.I. Zakharov, Nucl. Phys. B147, (1978) 385, 419.
5. A.I. Vainshtein and V.I. Zakharov, Phys. Rev. Lett. 73 (1994) 1207.
6. N. V. Krasnikov and A. A. Pivovarov, Mod. Phys. Lett. A11, 835 (1996); Yu. L. Dokshitzer and N. G. Uraltsev, hep-ph/9512407;
S. Peris and E. de Rafael, hep-ph/9603359;
S. V. Faleev and P. G. Silvestrov, hep-ph/9610344.
7. G. Parisi and R. Petronzio, Phys. Lett. B94 (1980) 51; J.M. Cornwall, Phys. Rev. D26 (1982) 1453.
8. V.P. Spiridonov and K.G. Chetyrkin, Sov. J. Nucl. Phys. 47 (1988) 522.
9. M. Beneke, V. Braun, and V.I. Zakharov, Phys. Rev. Lett. 73 (1994) 3058.
10. B. Blok, V.I. Zakharov, unpublished.
11. V.I. Zakharov and M.B. Voloshin, JETP Letters, 34 (1981) 485;
H.P. Niles, Phys. Lett. 312 (1982) 455.
12. A. Mueller, Phys. Lett. B308 (1993) 355.
13. G. Martinelli and C. T. Sachrajda, hep-ph/9605336
14. G. Martinelli and C. T. Sachrajda, hep-lat/9607018.
15. R. Akhoury and V.I. Zakharov, Phys. Lett. B357 (1995) 646; Nucl. Phys. B465 (1996) 295.
16. V. I. Zakharov, In Continuous Advances in QCD, Minnesota '95.
17. H. Contopanagos and G. Sterman, Nucl. Phys., B437 (1995) 415.
18. B.R. Webber, Phys. Lett. B339 (1994) 148.
19. I.I. Bigi, M.A. Shifman, N.G. Uraltsev, and A.I. Vainshtein, Phys. Rev. D50 (1994) 2234.
20. M. Beneke and V.M. Braun, Nucl. Phys. B426 (1994) 301.
21. A.I. Vainshtein, hep-ph/9512419.
22. M. Beneke and V.M. Braun, Nucl. Phys. B454 (1995) 253.
23. R. Akhoury and V.I. Zakharov, Phys. Rev. Lett. 76, (1996) 2238.
24. T. Kinoshita, J. Math. Phys. 3 (1962) 3; T.D. Lee, M. Nauenberg, Phys. Rev. 133 (1964) 1549.
25. R. Akhoury, L. Stodolsky, and V.I. Zakharov, hep-ph/9609368.
26. P. Hoyer, hep-ph/9610270.
27. R. Akhoury and V.I. Zakharov, in preparation.
28. E.V. Shuryak and A.I. Vainshtein, Nucl.Phys. B199 (1982) 451.
29. Yu. L. Dokshitzer, G. Marchesini, and B.R. Webber, Nucl. Phys. B469 (1996) 93.
30. E. Stein, M. Meyer-Herrmann, L. Mankiewicz, and A. Schäfer, Phys. Lett. 376 (1996) 177.
31. F. Barreiro, Fortschr. Phys. 34 (1986) 503.
32. K. Hamacher, talk at the conference QCD '96, Montpellier, July (1996).
33. G. Korchemsky and G. Sterman, hep-ph/9505391; Nucl. Phys. B437 (1995) 415.
34. B.R. Webber, Proc. Summer School, Zuoz, Switzerland, (1994), ed. M.P. Locher (PSI, Villingen, 1994).
35. S. D. Ellis, Phys. Lett. B117 (1982) 333.