An extended Einstein-Podolsky-Rosen thought-experiment

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We study a generalization of the original Einstein-Podolsky-Rosen thought experiment. It is essentially a delayed choice experiment applied to entangled particles. The basic idea is: given two observers sharing position-momentum entangled photons, one party chooses whether she measures position or momentum of her photons after the particles leave the source. The other party should infer her action by checking for the absence or presence of characteristic interference patterns after subjecting his particles to certain optical pre-processing. An occurrence of apparent signaling is attributed to the difficulty in treating single photons simultaneously at a quantum mechanical and quantum electrodynamic level, as required by the experiment, and points to the need for a careful study of an aspect of the foundations of quantum mechanics and electrodynamics.

I. INTRODUCTION

Quantum information has opened up a new and exciting era in recent times both in fundamental and applied physics. Its nonclassical resources of quantum superposition and entanglement are at the heart of powerful future applications in communication [1,2], computation [3] and cryptographic key distribution [4], among others. And yet, the fundamentally very important question whether the correlated measurements on entangled systems imply a nonlocal transfer of information remains somehow unclear. Einstein, Podolsky and Rosen (EPR) thought that they did, which was the basis of their claim of quantum mechanical incompleteness [5]. Quantum nonlocal correlations have been confirmed in experiments since the mid-1980’s performed on both systems entangled in spin and continuous variables (Refs. [6] and references therein).

Bell’s celebrated theorem [7] tells us only that any realistic model of quantum mechanics should be nonlocal. Informed opinions diverge between on the one hand the view that quantum nonlocality implies no information transfer, but only a change in the mutual knowledge of the two nonlocal systems, to the acknowledgement on the other hand of a tension between quantum theory and special relativity [8]. The tension stems from the possibility that the nonlocal correlations might imply a superluminal transfer of information. A majority of physicists in the field, it would seem, accept the scenario of a spacelike but causal enforcement of correlation, as for example in quantum dense coding [9]. This understanding is echoed in statements of “a deep mystery” [10], and “peaceful coexistence” [11] between quantum nonlocality and special relativity.

In the present article, we propose a generalization of the original EPR Gedanken experiment to facilitate a clearer analysis of the information transfer question, and discuss its implications for the foundations of quantum mechanics/electrodynamics.

II. THOUGHT EXPERIMENT

The proposed thought experiment, a modification of the original Einstein-Podolsky-Rosen experiment [5], involves source S of light consisting of entangled photon-pairs, analyzed by two observers, Alice and Bob, spatially separated by distance $D$, and equidistant from S. Alice is equipped with a device to measure momentum or position of the photons. Bob is equipped with a Young’s double-slit interferometer with the slits separated along the $y$-axis. The optics in front of the interferometer ensures that only a narrow bundle of monochromatic rays making angle $\theta$ to the $x$-axis is interferometrically analyzed, the rest being deflected away (Figure 1). It consists of two convex lenses of identical focal length $f$ and aperture radius $R$, in tandem such that their foci coincide. A reflecting shield, surrounding lens 1, allows only light incident on lens 1 to pass into Bob’s apparatus. A diaphragm with a small hole of radius $h$

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and centered on the lens axis is placed at the mutual focal plane of the lenses. This system is oriented such that the principal axis of the lenses makes an angle $\theta$ with $x$-axis.

By geometrical optics, rays parallel to $x$-axis incident on lens 1 fall at a distance $\eta = f \theta$ from the hole on the diaphragm and are blocked. Plane waves falling on lens 1 at angles larger than $h/f$ do not converge at the hole are absorbed by the diaphragm. Provided $h/f < \lambda_0/s$, where $s$ is the separation between the two slits on Bob’s interferometer and $\lambda_0$ the wavelength of the light incident on the double-slit, the angular tolerance will not greatly affect the visibility of the fringes on Bob’s screen. Furthermore, the diffraction effect at the lens edges can be ignored provided $R \gg \lambda_0$. Therefore, given $hs/f < \lambda_0 < R$, the direction filter thus built up is effective to good approximation. Except for the practical requirement of keeping the integration time low, the hole size $h$ can be made arbitrarily small. A spectral filter behind lens 2 restricts the light to a narrow bandwidth about wavelength $\lambda_0$.

At time $t_1$, part of the light (denoted A) from S is analyzed by Alice while its EPR counterpart (denoted B) is incident upon Bob’s lens 1. Bob completes his observation of the interference pattern on his screen at $t_2 \equiv t_1 + \delta t$. By prior agreement, Alice measures either the transverse position and longitudinal momentum ($\hat{p}_x, \hat{p}_y, \hat{p}_z$), while Bob will observe the resulting interference pattern, if any, on his screen.

### A. Alice measures momentum

The source is prepared so that just before Alice’s measurement at time $t_1$ the pure state of the entangled EPR pairs is described by the wavefunction:

$$
\Psi(x_a, x_b) = \int \int \int e^{i(2\pi i/h)(x_a - x_b + x_0)p} dp_x dp_y dp_z,
$$

where $x \equiv (x, y, z)$ is position, the subscripts $a$ and $b$ refer to the particles in ensembles A and B, respectively, and $p \equiv (p_x, p_y, p_z)$ is momentum. Here integrals with unspecified limits are taken between $-\infty$ and $\infty$. Eq. (1) can be rewritten:

$$
\Psi(x_a, x_b) = \int \int \int \pi_a(x_a) \pi_b(x_b) dp_x dp_y dp_z.
$$

If Alice observes ($\hat{p}_x, \hat{p}_y, \hat{p}_z$), she produces a mixed state wherein each photon in A has collapsed to a momentum eigenstate $\pi_a(x_a)$, which is a plane wave with the eigenvalue $p = (p_x, p_y, p_z)$. Simultaneously, its twin photon in B is left in the eigenstate $\pi_b(x_b)$ with momentum eigenvalue $-p$. The actual value of $(p_x, p_y, p_z)$ is in general different for different photons in A, giving rise to a mixed state in B, too.

Bob’s optics ensures that only plane wavefronts normal to the lenses’ axis incident upon lens 1 and whose frequency falls within the allowed narrowband about $\lambda_0$ have non-vanishing amplitude to fall on the double slit diaphragm. In other words, any particle detection at Bob’s screen could have come only from a narrowband of wavefronts propagating with an orientation $\theta$ and incident on lens 1. Therefore, all “hits” on Bob’s screen are coincidences corresponding to narrow spectral and angular ranges in A measurements. Hence, provided the ensembles A and B are sufficiently large, a Young’s double slit interference pattern forms on Bob’s screen in the single counts.

### B. Alice measures position

Eq. (1) can be expanded in the position-momentum (mixed) bases as:

$$
\Psi(x_a, x_b) = \int \int \int \xi_a(x_a) \xi_b(x_b) dy dz dp_x,
$$

where $\xi_a(x_a) = \delta(y_a - y)\delta(z_a - z)e^{i(2\pi i/h)(x_a p_x)}$, which represents a monochromatic photon wave ray in A moving with momentum $p' = (p_x, 0, 0)$ being transversely localized in the $yz$ plane at $(y_a, z_a) = (y, z)$. And we find:

$$
\xi_b(x_b) = \hbar \delta(y - y_b + y_0)\delta(z - z_b + z_0)e^{i(2\pi i/h)(x_0 - x_b)p_x},
$$

which represents a monochromatic photon wave ray in B moving with momentum $-p'$ being transversely localized at $(y_b, z_b) = (y + y_0, z + z_0)$. 

2
If Alice measures $(\hat{y}, \hat{z}, \hat{p}_x)$ on A, she produces a mixed state wherein each photon has collapsed to some eigenfunction $\xi_\alpha(x_\alpha)$, which can be visualized as a horizontal momentum ray with transverse localization. Simultaneously, each twin photon in B, which is left in the state $\xi_\alpha(x_\alpha)$ and eigenvalues $(y + y_0, z + z_0)$. Where $(y + y_0, z + z_0)$ falls outside the aperture of lens 1, the rays are reflected off Bob’s apparatus by the enclosure. Where the rays fall on his aperture, being horizontal, they converge to a point at distance $\eta$ from the hole, to be blocked by the diaphragm. Hence Bob will not observe any photon on his screen.

The preceding experiment suggests that Alice can transmit a classical binary signal by choosing to measure $(\hat{y}, \hat{z}, \hat{p}_x)$ or $(\hat{p}_x, \hat{p}_y, \hat{p}_z)$ on A. In the following section, we present specific and general arguments that might be expected to render such a signaling system unfeasible. The above experiment is similar to the delayed choice experiment [12], except that it involves an entangled pair rather than a single photon, with the effect of Alice’s choice on Bob’s, rather than her own, photon being considered.

III. DISCUSSION

It is generally agreed that classical signal transmission via nonlocality is not possible because of the ‘no-signaling’ condition, according to which Bob’s probability of measurement of any eigenstate is unaffected by Alice’s action [9]. This prompts the question: what is the origin of the nonlocal classical signal in the above thought-experiment?

By way of preliminary response, first we note a disparity forced on us. In the thought-experiment, the photon is analyzed in two distinct ways before and after its incidence on the lens. The wave field before the lens, described by Eqs. (1) through (4), is the unphysical (i.e., unobservable) quantum mechanical probability wave. On the other hand, the wave field after the lens is the measurable quantized radiation field and governed by quantum electrodynamics (QED). The consistency of the experimental prediction is based on the assumption that there is a continuous passage from the incident wave field to that transmitted beyond the lens. However, in fact no rigorous proof exists that this reasonable assumption is justified— a difficulty modern quantum formalism should contend with [13].

One point to be taken into consideration in evaluating the assumption is that QED is genuinely covariant, whereas in quantum mechanics, as is well known, spatial coordinates are operators, but time is merely a parameter. As position and momentum don’t commute, phase space is not well represented in quantum mechanics. On the other hand, in QED, spatial coordinates are also parameters like time, and not operators, which are furnished by the fields. It remains a difficulty how the quantum mechanical operator algebra may be recovered as an “effective” theory in the one-particle non-relativistic limit of QED.

For example, it is not position, but phase, that the quantum electrodynamics momentum operator $\hat{P} \equiv \sum_{k,\alpha} \hbar \hat{N}_{k,\alpha}$ does not commute with. Here $\hat{N}_{k,\alpha}$ is number operator for mode $k$ with polarization $\alpha$. It is possible to derive a relation akin to the familiar Heisenberg uncertainty principle $\Delta p_x \Delta x \geq \hbar$, based on the number-phase non-commutation relation (for all $k, \alpha$) $\{\hat{N}, \hat{\phi}\} = i$, but this is meaningful only as applied to light beams, and not to an individual photon, whose localization it actually seems to prohibit (since the photon number becomes highly indeterminate if phase relations between modes are definite enough to localize the radiation field).

Hence, the lens in Figure 1 ends up acting as an interface between two disparate wave fields, whose consistency is not established in the relevant limit. From this viewpoint, we conclude that the claimed signaling finds its origin in the lack of a seamless union between the applications of the two quantum formalisms and that resolving this disparity is expected to reinstate no-signaling.

Following a different line of thought, one might consider possible quantum information theoretic principles that might be brought to bear upon the experiment. It is an intriguing observation in quantum information that the ingredients that enforce no-signaling, linearity [13], unitarity (as evidenced by no-cloning [14]) and the tensor product character of the Hilbert space of composite quantum systems [17] are essentially non-relativistic. It might be conjectured that there lies a deeper connection between quantum information and the causal structure of spacetime whereby (e.g.,) the entangled light in the experiment somehow decoheres into separable momentum pointer states [18] before reaching Bob’s double-slit, so that he will always find a Young’s double-slit interference pattern irrespective of Alice’s action.

Whether this, or a re-examination of the quantum formalism, suggested earlier, or any other uncovered general principles are permitted by Nature to resolve the problem posed by the gedanken experiment, only suitable practical experiments (e.g., [10][20]) can adjudicate. The gedanken experiment shows us how quantum optics/information can shed light on basic issues in quantum theory.
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FIG. 1. Bob’s equipment: Its double-lens-and-reflector system acts as a direction filter that permits only rays parallel to the principal axis of the lens, which is inclined at angle $\theta$ to the $x$-axis, to fall on the double slit. The spectral filter restricts the light from lens 2 to a narrow bandwidth about $\lambda_0$. The enclosure with a reflecting exterior ensures that the only light entering the experiment is via lens 1. The lenses’ size is large compared to $\lambda_0$ in order to minimize correlation loss produced by diffraction at the lens edges. A $(\hat{p}_x, \hat{p}_y, \hat{p}_z)$-measurement by Alice has a non-vanishing chance of producing an interference pattern on Bob’s screen. But $(\hat{y}, \hat{z}, \hat{p}_x)$ leaves Bob’s particles as rays parallel to the $x$-axis. They converge to points $\eta$ and do not enter the interferometer.