Extending Geostationary Orbit Missions for Lunar Observations

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Abstract. This work investigates an alternative strategy to exploit future communications satellite generations including a final stage of lunar observations. For that, we explore impulsive transfers between geostationary orbits and lunar gravitational capture orbits in a full 4-body dynamical model with the Sun, Earth, Moon and spacecraft. Criteria to seek natural transfer orbits between the geostationary orbit and the vicinity of the Moon are defined considering escape properties of trajectories of the Circular Restricted Three Body Problem (CR3BP) as a guide. Namely, we select initial conditions of the 4-body model with energies that favors Earth-Moon transfers that remain around the Moon for a long time. As a case of study, we selected the current Brazilian geostationary satellite Star One C4. After a broad analysis of initial conditions and their transport behavior, we select potential transfers that reaches a near vicinity of the GEO orbit with an sufficiently small inclination with respect to the terrestrial equator. Time evolution of candidate solutions are analyzed and Δv budget and propellant mass are computed. As well as some current proposals for space debris mitigation, our strategy requires that additional mass of propellant besides onboard propulsion systems to perform final maneuvers have to be foreseen in the design of future generations of these communication satellites.

1. Introduction

Nowadays space mission design should include the implementation of debris mitigation solutions to preserve the space environment and the sustainability of the project. NASA reports approximately 7,400 metric tons for more than 17,000 objects cataloged in Earth orbit [11]. Specifically, a guideline proposed by NASA determines some procedures that a satellite must perform at the end of his mission to limit the risk of explosions and collisions after its life span. These practices involve the depletion of the sources of energy and fuel, the prediction of probabilities of collisions with other debris, and the removal of the spacecraft from Low Earth Orbits (LEO), Medium Earth Orbits (MEO) or Geostationary Orbits (GEO).

Colombo et al. [4] analyzed the effectiveness of some disposal strategies for Libration Point Orbits (LPO) missions. As possible options, they considered a reentry maneuver in the Earth’s atmosphere; lunar impact, both directly and after a weak capture, or injection into a heliocentric graveyard orbit. Besides these options, a transfer to a graveyard orbit, located between 100 and 300 km above the geostationary orbit [12], is also considered usually. These rules and policies
have had an impact reducing the generation of new debris. But graveyard orbits should be exploited only as short term solutions because the number of objects in these orbits cannot grow indefinitely, in order to avoid the increase of the probability of collisions between objects and the production of clouds of new fragments, some of that could collide with other satellites, resulting in an exponential increase of the production of fragments [9]. About 400 communication satellites are currently operating in geostationary orbit, while approximately 300 others are out of service and another considerable part in graveyards orbits. However, recent studies show that these graveyards orbits will certainly lead to atmospheric reentry of such debris, so they should be implemented in a controlled manner to avoid disasters if they fall in large urban centres [1, 4]. So, alternative strategies for space debris mitigation are required.

In this context, we propose an extension of missions with initial stage of geostationary orbit, including a transfer to a lunar osculating orbit. With that, we intend to extend the goal of this kind of mission allowing lunar inspection and observation as a final stage. So, transfer solutions are computed in a 4-body mathematical model including Sun, Earth, Moon and spacecraft, taking the escape properties of trajectories of the Circular Restricted Three Body Problem (CR3BP) [20, 10, 6] into account. Indeed, a scattering region around the Moon is explored performing backward integration of the equations of motion of the full four-body dynamical model, examining different transport processes between regions of the phase space in order to establish transfer routes. Sets of initial conditions of lunar osculating orbits are considered, but only initial conditions in a suitable energy range are evolved. We determine which trajectories are orbits that transit from the lunar region and reach the Earth vicinity with slope in relation to the terrestrial equator less than or equal to 5 degrees. This constraint is considered to favor cheaper costs to reach the geostationary orbit that presents zero inclination with respect with the Earth equatorial plane. Then, we compute the ∆v-budget required to perform a Hohmann transfer maneuver from the trajectories selected in the previous step to a geostationary orbit. Finally, we evaluate of the propellant mass for the accomplishment of the maneuvers. Given that, on-board propulsion systems required for maneuvers can be foreseen. This investigation extend the applications of future generations of communication satellites.

2. Dynamical Model
Two dynamical models are considered in this study: the full N-Body Model (with $N = 4$), accounting for the gravitational attraction of the point-mass Sun, Moon, Earth and spacecraft, and the Spatial Circular Restricted Three-Body Problem (CR3BP). In both cases, the numerical integration is performed by means of RADAU integrator of orders 15 [13, 7].

2.1. The N-Body Model
This problem deals with the movement of $N$ bodies subject to mutual gravitational forces. At first, $N$ bodies are considered as point particles with mass $m_j$ different from zero, where $j = 1, \ldots, N$. By the moment being, we consider four bodies, namely, the Sun ($j = 1$), the Earth ($j = 2$), the Moon ($j = 3$), and the Spacecraft ($j = 4$). Subsequently, the bodies can be considered as spherical masses with their respective mean radius to analyze possible collisions as the distance from one body to the other is smaller than the sum of their mean radii.

From Newton’s Law of Universal Gravitation it is possible to describe the equations of motion in the space written in the geocentric reference system $\{0, X, Y, Z\}$. Defining the position vector of the $j$th body by

$$
\mathbf{r}_j = X_j \mathbf{I} + Y_j \mathbf{J} + Z_j \mathbf{K},
$$

(1)

The total gravitational acceleration on a body of mass $m_j$ is given by the sum of the
gravitational accelerations of all other bodies \((N - 1)\) [10, 6], namely,

\[
\ddot{r}_j = -\sum_{j=1}^{N} \sum_{k=1,k\neq j}^{N} \frac{\mu_k}{r_{kj}^3} (r_k - r_j),
\]

(2)

where, \(\mu_k = Gm_k\) with \(G = 6.674 \times 10^{-11} \text{N.m}^2/\text{Kg}^2\) (Universal Gravitational Constant).

Therefore, the gravitational acceleration acting on the Spacecraft by the Sun, Earth and Moon is given by

\[
\ddot{X}_4 = -\sum_{k=1}^{N-1} \frac{\mu_k}{r_{k4}^3} (X_k - X_4),
\]

(3)

\[
\ddot{Y}_4 = -\sum_{k=1}^{N-1} \frac{\mu_k}{r_{k4}^3} (Y_k - Y_4),
\]

(4)

\[
\ddot{Z}_4 = -\sum_{k=1}^{N-1} \frac{\mu_k}{r_{k4}^3} (Z_k - Z_4),
\]

(5)

where

- \((X_4, Y_4, Z_4, \dot{X}_4, \dot{Y}_4, \dot{Z}_4)\) is the state vector of the Spacecraft (body 4) of mass \(m_4\) in the equatorial reference system centered at the Solar System barycenter, evaluated at a given instant of time.
- \((X_k, Y_k, Z_k, \dot{X}_k, \dot{Y}_k, \dot{Z}_k)\) is the state vector of the body \(k\) of mass \(m_k\) in the equatorial reference system centered at the Solar System barycenter, evaluated at a given instant of time.

### 2.2. The Circular Restricted Three-Body Problem

The CR3BP describes the behavior of a particle with negligible mass moving in the gravitational field of two primaries of masses \(m_1\) and \(m_2\), each revolving around their common center of mass on circular orbits. In this work, \(m_1\) is the Earth and \(m_2\) the Moon [10, 2].

The dimensionless variables are chosen to set the sum of the masses of the primaries, the distance between them and the modulus of the angular velocity of the rotating frame equal to 1. In the actual Earth-Moon system, the mutual distance equals 384,400 km (Earth-Moon distance), the unit of the time is 27.32 days, while the dimensionless mass of the Earth+Moon barycenter is \(\mu = \frac{m_2}{m_1 + m_2} = 0.012150582\) with \(m_1 > m_2\). The dimensionless masses of the primaries are given by \(\mu_1 = 1 - \mu\) and \(\mu_2 = \mu\), the most massive body is located at \((-\mu_2, 0, 0)\), the second one at \((\mu_1, 0, 0)\).

Thus, after some algebraic manipulations equations of motion represented in the synodical reference system can be written as

\[
\ddot{x} - 2\eta \dot{y} - \eta^2 x = -\frac{\mu_1}{r_1^3} (x + \mu_2) - \frac{\mu_2}{r_2^3} (x - \mu_1),
\]

(6)

\[
\ddot{y} + 2\eta \dot{x} - \eta^2 y = -\left(\frac{\mu_1}{r_1^3} + \frac{\mu_2}{r_2^3}\right) y,
\]

(7)

\[
\ddot{z} = -\left(\frac{\mu_1}{r_1^3} + \frac{\mu_2}{r_2^3}\right) z,
\]

(8)
where \(\eta\) is the mean motion, \(r_1 = \sqrt{(x + \mu_2)^2 + y^2 + z^2}\) and \(r_2 = \sqrt{(x - \mu_1)^2 + y^2 + z^2}\) are the distances between the particle \(P\) to the two primaries \(P_1\) and \(P_2\), respectively. This system of equations admits a first integral (the Jacobi integral) given by

\[
\eta^2 (x^2 + y^2) + 2 \left( \frac{\mu_1}{r_1} - \frac{\mu_2}{r_2} \right) - (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) = C_J, \tag{9}
\]

where \(C_J\) is the so called Jacobi constant [10]. This integral defines a five-dimensional manifolds of the six-dimensional phase space at which trajectories are immersed. This integral of motion also defines the regions of phase space that the particle \(P\) can or cannot access. The boundary between the accessible and forbidden regions are the zero velocity surface defined by the zero velocity condition \((\dot{x}^2 + \dot{y}^2 + \dot{z}^2 = 0)\) in the Eq. (9). Thus, for each value of \(C_J\), accessible regions are defined in which the motion of the particle is possible. The intersection of these surfaces with the \(xy\) plane forms the Zero Velocity Curves [10, 6, 16, 17]. The qualitative study of Zero Velocity Curves is possible using the five values associated with Jacobi Constant at the \(L_i, (i = 1, \ldots, 5)\) equilibrium points, for which the following relationship holds

\[
C_J(L_1) > C_J(L_2) > C_J(L_3) > C_J(L_4) = C_J(L_5). \tag{10}
\]

3. Selected Mission and Constraints

The selected mission for the detailed analysis is Star One C4 (current). The Star One C4 is a Brazilian geostationary communication satellite built by Space Systems/Loral (SS/L) [3]. It is located in the orbital position of 70 degrees west longitude along with Star One C2 and is operated by Embratel Star One, subsidiary of Embratel. The satellite was based on the LS-1300 platform and its life expectancy is 15 years [19, 18].

3.1. Mission scenario

The first stage of our investigation consists on the definition of appropriate sets of initial conditions around the Moon to be backward evolved by natural dynamics in order to seek suitable transfer solutions from the lunar vicinity to the proximity of the Geostationary orbit. So, we generate sets of initial conditions of lunar osculating orbits obtained by varying values of the orbital elements, namely, the semi-major axis \((a)\), eccentricity \((e)\), inclination \((i)\), argument of perigee \((\omega)\) and longitude of ascending node \((\Omega)\). With this, it is expected to produce solutions that resemble periodic orbits and quasi-periodic solutions. More specifically, the orbital elements relative to the Moon are inspected as follows: \(a\) is varied from 1,750 km to 66,750 km with steps of 130 km; \(e\) is varied from 0 to 0.99 with steps of 0.01; \(i\) is varied from 0 deg to 15 deg, in steps of 0.4 deg; \(\omega\) is chosen from 0 to 315 deg, in steps of 45 deg; and \(\Omega\) is chosen from 0 to 315 deg, in steps of 45 deg.

Each selected initial condition is evolved backward by the full 4-body dynamical model (Eq. (2)) from time equals zero to the final time \(t_f\) of \(-1,000\) days (so, searched final states of our numerical analysis must correspond to arrival orbits close to the geostationary orbit, that will be the initial states in real time). However, given the huge amount of possible initial conditions to be explored, we adopted a criterion to decide which initial conditions are of interest, reducing the required processing time. For that, we compute the Jacobi constant associated to the Earth-Moon system of each initial condition and select for time evolution only those with \(C_J\) in the range between \(C_J(L_2)\) and \(C_J(L_1)\). With that, considering the good approximate description provided by the CR3BP, we aim to select trajectories that only can transit from the lunar region through the \(L_1\) neck of the Earth-Moon system. It is important to note that the Jacobi constant value in the full 4-body dynamics varies with time and it is used only as a guide for the choice of suitable initial conditions. By the other side, we remark that for initial
conditions with inclination close to zero deg, the satellite is very close to the Moon, in such a way that the gravitational forces due to the Earth and the Sun can be considered as small perturbations.

Due to the configuration of the initial orbital elements, the initial two-body (2B) energy has a negative value with respect to the Moon (closed orbit). With the time evolution and the perturbation of the third and fourth bodies (the Earth and the Sun), the 2B energy changes its value. When the orbital energy changes to a positive value (open orbit), this value can be assigned as the capture time of the trajectory. The integration is interrupted if the particle collides with the Moon or the Earth before \( t_f \), or if time exceed \( t_f = -1000 \) days. If the particle does not escape in the period of \(-1,000\) days, its trajectory is called prisoner. However, the cases of interest are those at which the Spacecraft escapes from the Moon vicinity to the Earth region, specially, approaching a Geostationary Orbits (GEO).

Propagating all trajectories candidates to \( L_1 \) escape, it will be possible to identify which transit trajectories reach a closest vicinity to the Geostationary Orbit, which is circular and has a radius of approximately 42,164 km and zero deg inclination in relation to the terrestrial equator. So, seeking suitable transfer solutions, trajectories that reach distances to the center of mass of the Earth lower than 300,000 km with an inclination equal or lower than 5 deg in relation to the terrestrial equator orbits were selected.

To illustrate the obtained results, Figs. 1 to 3 show the initial conditions selected according to the qualitative dynamical behavior as a function of the semi-major axis, the eccentricity, and the inclination, keeping both \( \omega = 90 \) deg and \( \Omega = 315 \) deg constant, for \( C_J(L_2) > C > C_J(L_1) \).

Figures 1 and 2 present the initial conditions of the trajectories that collide, respectively, with the surface of the Moon and with the surface of the Earth, before \( t_f \) is reached. The collisional time in both figures is represented by the color code.

**Figure 1.** Trajectories that collide with the Moon surface before \( t_f \) as a function of initial values of \( a \), \( e \) and \( i \).

**Figure 2.** Trajectories that collide with the Earth surface before \( t_f \) as a function of initial values of \( a \), \( e \) and \( i \).

Figure 3 presents the initial conditions of trajectories that approach geostationary orbits, i.e., reaches a minimum distance to the center of the Earth of 300,000 km with an inclination in relation to the terrestrial equator \( i_{24} \) equal or lower than 5 deg.
Figure 3. Trajectories that approach geostationary orbits, i.e., that reach distances to the center of mass of the Earth lower than 300,000 km with an inclination equal or lower than 5 deg in relation to the terrestrial equator. As in previous figures, solutions are presented as a function of $a$, $e$ and $i$, for $\omega = 90$ deg and $\Omega = 315$ deg fixed and $C_j(L_1) > C_j > C_j(L_2)$.

The solutions shown in the Fig. 3 are candidates for the mitigation maneuver analysis, outlined in the next Section. In this analysis, we compute the $\Delta v$-budget and the amount of propellant needed to perform this disposal maneuver, as well as, a scalar that quantifies the hyperbolic excess velocity with respect to the Moon.

4. Results

In this section, we choose one of the solutions that satisfy the conditions of proximity to the geostationary orbits, shown in Fig. 3. The initial condition of this selected solution is given by $a = 25,150$ km, $e = 0.72$, $i = 1.7$ deg, $\omega = 90$ deg, and $\Omega = 315$ deg.

The propagation of the orbit was performed and for each instant, a Hohmann transfer [8] with three $\Delta v$ was calculated to transfer the satellite from the location of the 4B-dynamics trajectory to a geostationary orbit. Using Tsiolkovsky’s equations [21] the propellant mass required to perform each of the impulses was computed considering a specific impulse ($I_s = 455$ s) referring to liquid oxygen / liquid hydrogen [6].

In addition, to rank solutions based on characteristics, besides the $\Delta v$-budget, the time of flight and the required propellant mass, the half of the hyperbolic excess velocity $C_3$ may also be used [22, 4], defined as a function of the orbital elements of the state of the spacecraft just before impact as:

$$\frac{1}{2} C_3 = \frac{v^2}{2} - \frac{\mu_3}{r} = -\frac{1}{2} \frac{\mu_3}{h^2} (1 - e^2) = -\frac{\mu_3}{2a},$$

where $v$ and $r$ are, respectively, the spacecraft velocity and position, $h$ is the angular momentum and $e$ the eccentricity. The $C_3$ value provides an evaluation of the robustness of the transfer, i.e., the lower the value, the more ballistic the capture at the Moon, and thus the more robust the transfer is in case of contingencies. Thus, a trajectory with low $C_3$ value corresponds to a quasi-captured solution by the Moon. Therefore, this value is useful as another parameter to compare one particular transfer with another.

Figures 4 to 6 present, respectively, the Hohmann transfer time, the propellant mass required, and the $C_3$ value together as a function of the flight time. The color code presents also the $\Delta v$-budget for the Star One C4.
Given these preliminary analyzes and the examination of the obtained results, we are able to establish more suitable criteria for the selection of solutions for transfer possibilities and then quantify required costs and transfer time for this strategy.

5. Conclusion
This paper investigates a mission design strategy that extends the goal and applicability of usual geostationary missions. We propose a transfer from the GEO to an osculating solution around the Moon exploiting the natural dynamics of a full 4-Body Model. This proposal will enable a new scientific return given by this additional stage of the mission.

This contribution is in agreement with another works found in the literature [1, 4]. As expected, the $\Delta v$-budget required for geostationary missions is larger than transfers computed for Libration Point Orbit and Highly Elliptical Orbit missions, since that geostationary orbits are gravitationally captured by the Earth. As claimed by Alessi[1] and Colombo et al. [4], the scientific return of such a mission justify the transfer to the Moon, besides that it also contributes to mitigate the space debris of geostationary missions.

The main drawback of the proposed strategy may be the chaotic nature of the involved trajectories. In the future, we aim to investigate the role of the uncertainties arising from the
orbit determination using Kalman Filter or other estimation method [5, 14, 15]. Possible future works can include a more detailed analysis of the role of the attitude, and orbit prediction.

A possible future investigation corresponds to the direct injection of a satellite into a heliocentric orbit graveyard [1], analyzing the ∆v-budget and the propellant mass required to carry out this strategy. Both possibilities may be explored according to the specific convenience and goals. The actual proposal has the advantage of a further usage of the satellite to explore lunar environment.

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