On Pruning Adversarially Robust Neural Networks

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Abstract
In safety-critical but computationally resource-constrained applications, deep learning faces two key challenges: lack of robustness against adversarial attacks and large neural network size (often millions of parameters). While the research community has extensively explored the use of robust training and network pruning independently to address one of these challenges, we show that integrating existing pruning techniques with multiple types of robust training techniques, including verifiably robust training, leads to poor robust accuracy even though such techniques can preserve high regular accuracy. We further demonstrate that making pruning techniques aware of the robust learning objective can lead to a large improvement in performance. We realize this insight by formulating the pruning objective as an empirical risk minimization problem which is then solved using SGD. We demonstrate the success of the proposed pruning technique across CIFAR-10, SVHN, and ImageNet dataset with four different robust training techniques: iterative adversarial training, randomized smoothing, MixTrain, and CROWN-IBP. Specifically, at 99% connection pruning ratio, we achieve gains up to 3.2, 10.0, and 17.8 percentage points in robust accuracy under state-of-the-art adversarial attacks for ImageNet, CIFAR-10, and SVHN dataset, respectively. Our code and compressed networks are publicly available\(^1\).

1. Introduction
How can we train deep neural networks that are robust against adversarial examples while minimizing the size of the neural network? In safety-critical and resource-constrained environments, both robustness and compactness are simultaneously necessary. However, existing work is limited in its ability to answer this question since it has largely addressed these challenges in isolation. For example, neural network pruning is an efficient approach to minimize the size of the neural network. In parallel, robust training can significantly improve the adversarial robustness of neural networks. However, improving adversarial robustness has been shown to require even larger networks (Madry et al., 2018; Zhang et al., 2019). Thus it is even more critical to ask whether network pruning techniques can reduce the size of the network while preserving robustness?

A gold standard for network pruning has been the approach of Han et al. (Han et al., 2015), which prunes connections that have the lowest weight magnitude (LWM) under the assumption that they are the least useful. Indeed, in benign settings (absence of adversarial examples), LWM is able to achieve very high compression ratios without sacrificing benign accuracy. In contrast, robust training is a harder learning objective (Madry et al., 2018; Wang et al., 2018a; Cohen et al., 2019) since it aims to optimize for both robust and benign accuracy. In such a challenging learning scenario, we question whether we should be using pruning heuristics that are agnostic to the training objective? Be-

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\(^1\)https://github.com/inspire-group/compactness-robustness
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Figure 2. Given a pre-trained network, comparison of which connections are preserved by each pruning technique for a VGG4 network and CIFAR-10 dataset.

Besides, the stronger dependence of robust training on the size of the network can further limit the success of this heuristic. We confirm the limited robustness achieved by pruning based on LWM heuristic across four different robust training techniques.

In contrast to employing a single pruning heuristic across different robust training objectives, we argue that a better approach is to make the pruning technique aware of the objective itself. We achieve so by formulating the pruning step, i.e., deciding which connections to prune, as an empirical risk minimization problem which can be solved efficiently using stochastic gradient descent (SGD). Given a pre-trained network, we optimize the importance score (Ramanujan et al., 2019) for each connection in the pruning step while keeping the fine-tuning step intact. Connections with the least importance score are later pruned away. For better performance, we initialize the optimization problem with the solution obtained from the LWM pruning.

We demonstrate that fine-tuning the networks pruned with the proposed technique can achieve much higher robust accuracy compared to LWM. Fig. 1 shows these results for adversarial training with a weaker adversary (ε=2) and a stronger adversary (ε=8) on the CIFAR-10 dataset. With increasing pruning ratios, the gap between the robust accuracy achieved with both techniques further increases. In Fig. 2, we compare the connections pruned away by both techniques. We observe that the proposed technique often prunes the highest magnitude connections in favor of connections with smaller magnitude. This difference further depends on the robust training objective.

Recently, (Ramanujan et al., 2019) demonstrated that there exist hidden sub-networks with high benign accuracy within randomly initialized networks. We extend this observation to robust training, where we uncover highly robust (both empirical and verifiable) sub-networks within non-robust networks. Even more, within empirically robust networks, which have no verifiable robustness, we were able to find sub-networks with high verified robust accuracy, even higher than achievable by previous state-of-the-art methods. (Carlini et al., 2019).

**Key contributions:** In this work, we study the interplay of network pruning and robust training. Our key contributions are as follows:

- To improve the robustness of compressed networks, we argue to make pruning techniques aware of robust training objectives. We achieve it by formulating the pruning process as an empirical risk minimization problem, which is then solved with SGD. We demonstrate that this approach is highly successful in achieving a highly compressed yet robust network.

- We evaluate the proposed approach across four robust training objectives, namely iterative adversarial training, randomized smoothing, MixTrain, and CROWN-IBP on CIFAR-10, SVHN, and ImageNet dataset with multiple network architectures. Notably, at 99% connection pruning ratio, we achieve gains up to 3.2, 10.0, and 17.8 percentage points in robust accuracy under state-of-the-art adversarial attacks for ImageNet, CIFAR-10, and SVHN dataset, respectively.

- We also demonstrate the existence of highly robust sub-networks within non-robust or weakly robust networks. In particular, within empirically robust networks that have no verifiable robustness, we were able to find sub-networks with state-of-the-art verified robust accuracy.

2. Background and related work

2.1. Robust training

Robust training is one of the primary defenses against adversarial examples (Biggio et al., 2013; Goodfellow et al., 2015; Carlini & Wagner, 2017; Madry et al., 2018; Athalye et al., 2018b). It aims to minimize the expected adversarial loss by re-formalizing the network training as the following min-max optimization problem.

\[
\min_{\delta} \mathbb{E}_{(x,y) \sim D} [\max_{\delta \in \Omega} L(\theta, x + \delta, y)]
\]  

(1)

δ denotes the adversarial perturbations bounded within the robustness region (e.g., \( ||\delta||_p \leq \epsilon \)) while \( \theta, x, y \) refers to network parameters and input data, respectively. Given that finding an exact solution for the inner-maximization problem is NP-hard, there exist two key approaches to find an approximate solution.

**Adversarial training.** It under-approximates the solution for inner-maximization with iterative adversarial attacks (Madry et al., 2018; Zhang et al., 2019; Carmon et al., 2019; Wang & Zhang, 2019). Such attacks, for example, projected gradient descent (PGD) attack (Madry et al., 2018), repeatedly search for the worse-case perturbation (δ) within the given region (Ω). Adversarial training further trains the network by minimizing the loss over adversarial examples.
We use the following metrics to capture the performance of Weng et al., 2018; Mirman et al., 2018; Gowal et al., 2018b): \( \text{Vra} \) corresponds to verified robust accuracy (vra) with projected gradient descent based attacks.

**Verifiable robust training.** It aims to obtain a sound over-approximation to the worst-case loss \( L_{\text{ver}}(\theta, x, y, \Omega) \) for the inner maximization in Eq. 1. Such an approach provides provably robust guarantees against adversarial examples. The key insight behind most verification methods (Wang et al., 2015; 2016; Li et al., 2017; Frankle & Carbin, 2019; Lecuyer et al., 2019) also aims to provide certified robustness under the Gaussian noise. To increase robustness against Gaussian noise, it is desirable to train the network with Gaussian noise, \( \mathcal{N}(0, \sigma^2) \).

Step-1: **Pre-training:** It minimizes the pre-training loss objective \( L_{\text{pt}} \) over training data.

\[
\theta_{\text{pretrain}} = \arg\min_{\theta} \mathbb{E}_{(x,y) \sim D} [L_{\text{pt}}(\theta, x, y)]
\]

Step-2: **Pruning the pre-trained network:** In general, most pruning techniques innovate over this step. Least Weight-Magnitude (LWM) based pruning prunes away connections that have the lowest magnitude, assuming that they are the least useful in making predictions. In particular, it generates a pruning mask \( \hat{m} \), where \( \hat{m} = 1(\|\theta_{\text{pretrain}}\| > \theta_{\text{pretrain}}|k]) \), which \( \|\theta_{\text{pretrain}}\| \) refers to \( k \)th percentile of the absolute value of pre-trained weights. Note that the pruning ratio is \( \frac{N}{N} \), which \( N = |\theta_{\text{pretrain}}| \) i.e., the number of parameters in the neural network.

Step-3: Fine-tuning the pruned network.

\[
\theta_{\text{finetune}} = \arg\min_{\theta} \mathbb{E}_{(x,y) \sim D} [L_f(\theta \odot \hat{m}, x, y)]
\]

We will refer the solution \( \theta_{\text{finetune}} \) as the compressed network obtained from this network pruning pipeline. Note that both pruning and fine-tuning steps can be alternatively repeated to perform multi-step pruning (Han et al., 2015), which incurs an extra computational cost than single-step pruning. In addition to this compression pipeline, network pruning can be performed with training, i.e, run-time pruning (Lin et al., 2017; Bellec et al., 2018) or before training (Frankle & Carbin, 2019; Lee et al., 2019; Wang et al., 2020). We focus on pruning after training, in particular, LWM based pruning, since it still outperforms multiple other techniques (Table 1 in Lee et al., 2019) and is a long-standing gold standard for pruning techniques.

Some very recent works have also studied network pruning with robust training (Gui et al., 2019; Sehwag et al., 2019; Ye et al., 2019; Wijayanto et al., 2019a). Sehwag et al., 2019 and Gui et al., 2019 largely demonstrate that empirical adversarial robustness can be achieved with network pruning, in particular with pruning based on least weight magnitudes (LWM). We demonstrate that the performance of LWM based pruning is highly limited with robust training and propose a new pruning technique to significantly improve it. Ye et al. 2019 proposes an alternating direction method of multipliers (ADMM) based technique to achieve robustness with network pruning. In contrast, our approach is to formulate pruning as an empirical risk minimization problem which can be solved with any efficient solver, such as SGD. In addition, our experimental setup is significantly more extensive than earlier works. In comparison to previous work, we experiment with (1) three different verifiable robust training methods (2) Large scale ImageNet dataset (3) state-of-the-art adversarial training techniques (Carmon et al., 2019; Lecuyer et al., 2019)}.
et al., 2019) (4) comparisons to pruning before training approaches. Some of these works (Gui et al., 2019; Wijayanto et al., 2019) also focused on other aspects of compactness that are complementary to ours, such as quantization of weights. Another related line of research aims to use network pruning itself to improve robustness against adversarial examples (Wang et al., 2018d; Feinman et al., 2017; Guo et al., 2018; Dhillon et al., 2018; Rakin et al., 2019).

3. Our approach to network pruning with robust training

A central question in making robust networks compact is to decide which connections to prune? In this work, we argue that an effective pruning approach needs to account for the robust training objective itself. In other words, the robust training objective should guide the selection of connections that get pruned away. We note that this is in contrast with current heuristic-based pruning techniques, such as least weight magnitude (LWM) based pruning.

In LWM based pruning, the connections that have the lowest weight magnitude are pruned away, with the assumption that those connections are the least useful. While this heuristic has demonstrated significant success in benign training settings (in the absence of adversaries), its effectiveness in robust training settings (in the presence of an adversary) is significantly limited. We argue that a better approach would be to perform an architecture search for a neural network with the desired pruning ratio that has the least drop in robust accuracy compared with the pre-trained network. Its performance can then be further improved with fine-tuning.

Pruning as an empirical risk minimization problem (ERM) with adversarial loss objectives. Even in the pruning step itself, we explicitly aim to reduce the degradation of robustness. We achieve this by integrating the robust training objective itself, which we will discuss next. In contrast, architecture search has demonstrated significant success in benign training settings (in the presence of adversaries), its effectiveness in robust training settings is significantly limited. We argue that a better approach would be to perform an architecture search for a neural network with the desired pruning ratio that has the least drop in robust accuracy compared with the pre-trained network. Its performance can then be further improved with fine-tuning.

Scaled-initialization. We observe that the performance of the proposed pruning approach depends heavily on the initialization of importance scores. At high pruning ratios, which we study in this work, we observe slow and poor convergence of SGD with random initialization (He et al., 2015; Glorot & Bengio, 2010) of importance scores. Compared to random initialization, we found that the following scaled-initialization of the importance scores in each layer $i$ ($s_{i}^{(0)}$) can lead to large improvements:

$$s_{i}^{(0)} = \left( \frac{1}{\max(\|\theta_{\text{pretrain},i}\|)} \sqrt{\frac{6}{\text{fan-in}_i}} \right) \times \theta_{\text{pretrain},i}$$

where $\theta_{\text{pretrain},i}$ is the weight corresponding to the $i$th layer in the pre-trained network and fan-in is the product of receptive field size and the number of input channels. We first normalize the weights to keep them in $[0, 1]$ range by diving with the maximum magnitude weight. Then we use the scaling factor to scale the variance which helps in avoiding the gradient explosion and vanishing problem (He et al., 2015). Note that with the scaled-initialization, the optimization problem in the pruning step effectively starts from the same solution as obtained from the least weight magnitude based pruning.

We compare the performance of scaled-initialization with two-widely used random initialization techniques, namely kaiming-init (He et al., 2015) and xavier-init (Glorot & Bengio, 2010). Our results (Table 1) show that random initialization with Gaussian or uniform distribution largely performs even worse than training from scratch, our first baseline which we will discuss next. In contrast, architecture search with our scaled-initialization can significantly improve era of the compressed networks at all pruning ratios.

Comparison with Ramanujan et al. 2019. While our approach to solving the optimization problem in the pruning step is inspired by Ramanujan et al. 2019, we note that the goals of the two works have several significant differences. Their work aims to find sub-networks with high benign accuracy, hidden in a randomly initialized network, without the use of fine-tuning. In contrast, (1) we focus on multiple types of robust training objectives, including verifiably robust training, (2) we employ pre-trained networks.
in our pruning approach, as opposed to randomly initialized networks, and (3) we argue for further fine-tuning of pruned networks resulted from the optimization problem to further boost performance. We further employ an additional scaled-initialization mechanism which is the key driver of the success of our pruning technique. In contrast to their work which searches for sub-networks close to 50% pruning ratio, our goal is to find highly compressed networks (up to 99% pruning ratio).

### 3.1. Comparison with baselines

In this section, we compare the performance of proposed pruning approach with other baselines. Our results are presented in Table 1 for adversarial training with VGG16 network, CIFAR-10 dataset, and $l_\infty$ perturbation ($\epsilon$) of 8/255.

**Robustly training compact networks from scratch.** When the underlying objective is to achieve a compressed and robust network, a natural question is why not perform robust training on a compact network from scratch? However, we observe that it achieves poor robust accuracy. For example, at 99% pruning ratio, the compressed network has only 24.6% \textit{era} which is 29.1 percentage points lower than the non-compressed network. We also observe that the performance gap between sparse networks and non-sparse networks is much larger for robust training than it is for benign training. In other words, \textit{sparcity hurts performance more in the presence of robust training objectives}. We present detailed analysis in Appendix C.1.

**Pruning robustly trained networks via lowest-weight magnitude heuristic.** As our second baseline, we select the well-established least weight magnitude (LWM) based pruning method. We integrate the compression pipeline with robust training by updating the loss objective to a robust loss ($L_{adv}$ or $L_{ver}$) in both pre-training and fine-tuning stages. As reported in Table 1, LWM is able to partially improve the robustness of compressed networks compared to training from scratch. At 99% pruning ratio, it improves the \textit{era} of compressed networks obtained with pruning techniques. For our proposed technique, we also compare the performance of random initialization with the scaled-initialization. We use CIFAR-10 dataset and VGG16 network in this experiment.

| Pruning ratio | 0% | 90% | 95% | 99% |
|---------------|----|-----|-----|-----|
| Scratch       | 45.6 | 42.3 | 24.6 | 24.6 |
| SNIP (Lee et al., 2019) | 43.2 | 40.2 | 27.2 | 27.2 |
| LWM (Han et al., 2015) | 48.8 | 45.2 | 34.3 | 34.3 |
| Xavier-normal | 45.2 | 42.3 | 36.8 | 36.8 |
| Xavier-uniform | 45.0 | 42.4 | 36.5 | 36.5 |
| Kaiming-normal | 44.9 | 42.1 | 36.5 | 36.5 |
| Kaiming-uniform | 44.8 | 42.5 | 36.4 | 36.4 |
| Scaled-initialization | 49.5 | 48.7 | 41.7 | 41.7 |

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**Fine-tuning**

Figure 3. Comparison of our proposed pruning technique in the end-to-end compression pipeline. Both techniques use the identical pre-trained network. It shows that connections preserved by proposed technique are itself able to achieve high \textit{era}, thus leads to significant gains for the compressed network after fine-tuning. to 34.3% but this is still 19.4 percentage points lower than a non-compressed network. We observe similar gaps for varying adversarial strength in adversarial training. These results are discussed in detail in Appendix C.2.

**Pruning before robust training with SNIP.** Similar to SNIP (Lee et al., 2019), we prune connections before training based on saliency calculated over one batch with adversarial loss (Eq. 2). We found its performance comparable to robust training from scratch where it performs better at 99% pruning ratio but worse at other two pruning ratios.

We present the performance of the proposed pruning approach, in comparison to LWM, across the whole compression pipeline in Fig. 3. Both methods start with the same pre-trained VGG16 network which once pruned to 99% achieve 10% \textit{era} on validation set of CIFAR-10. However, starting with this solution, over 20 epochs, our proposed pruning approach is able to find a different set of connections obtaining high robustness even without fine-tuning. Once fine-tuned, the final network achieves 7.4 percentage points higher \textit{era} compared to the compressed network obtained from LWM.

### 4. Experimental setup

We focus on three datasets, namely CIFAR-10, SVHN, and ImageNet. For each dataset, we pre-train the networks with a learning rate of 0.1. We perform 100 training epochs for CIFAR-10, SVHN and 90 epochs for ImageNet. In the pruning step, we perform 20 epochs for CIFAR, SVHN and 90 epochs for ImageNet. We experiment with VGG16 (Simonyan & Zisserman, 2015), Wide-ResNet-28-4 (Zagoruyko & Komodakis, 2016), CNN-small, and CNN-large (Wong & Kolter, 2018) network architectures. For adversarial training and randomized smoothing, we prune with a learning rate of 0.1. We find this learning rate too high for pruning verifiable robust networks trained with
MixTrain and CROWN-IBP, where we select a learning rate of 0.001. Similar to previous works (Liu et al., 2018), we select the learning rate of the fine-tuning step 10× smaller than the pruning step. The \( l_\infty \) perturbation budget for adversarial training is 8/255 for CIFAR-10, SVHN and 4/255 for ImageNet. For verifiable robust training, we choose an \( l_\infty \) perturbation budget of 2/255 in all experiments. These design choices are consistent with previous work (Carmon et al., 2019; Wang et al., 2018a; Zhang et al., 2020). For randomized smoothing we choose an \( l_2 \) budget of 110/255 which gives an upper bound on robustness against an \( l_\infty \) budget of 2/255 for CIFAR-10 and SVHN dataset. For all experiments, we employ cosine learning rate decay. We present a detailed version of our experimental setup in appendix A.

5. Experimental results

Table 2 presents the experimental results on CIFAR and SVHN datasets across three pruning ratios, two network architectures, and four different robust training objectives. The key characteristics of the proposed pruning approach from these results are synthesized below:

**Improved performance across datasets, architectures, and robust training objectives.** Across most experiments in Table 2, the proposed pruning approach is able to achieve a significant improvement in robust accuracy with a mean and maximum improvement of 5.0 and 34.1 percentage points, respectively. Specifically, it achieves a mean improvement in robust accuracy by 5.4, 3.9, 2.0, 8.8 percentage points for adversarial training, randomized smoothing, MixTrain, and CROWN-IBP approach, respectively.

**Higher gains with an increase in pruning ratio.** At 99% pruning ratio, not only is our approach never worse than the baseline but it also achieves the highest gains in robust accuracy. For example, for VGG16 network with CIFAR-10 dataset at 99% pruning ratio, our approach is able to achieve 7.4 and 10.3 percentage points higher robust accuracy with adversarial training and randomized smoothing, respectively. These improvements are larger than the gains obtained at the other two smaller pruning ratios.

For training with MixTrain and CROWN-IBP, we observe that in some cases, the networks fail to converge beyond 95% pruning ratio for SVHN dataset (19.6%), but our pruning approach is more likely to converge. For example, our approach achieves 53.7% vra-m while the LWM based pruned...
network fails to converge at 95% pruning ratio for MixTrain on CNN-large network and SVHN dataset.

**Help increase in generalization for some cases.** Interestingly, we observe that our pruning approach can obtain robust accuracy even higher than pre-trained networks. For the SVHN dataset and WRN-28-4 network, we observe an increase by 2.7 and 0.1 percentage points for adversarial training and randomized smoothing, respectively at 90% pruning ratio. For verifiable training with CROWN-IBP, we observe improvement in $vra-t$ from 0.9-1.5 percentage points for all networks pruned at 90% ratio. Similar improvements are also observed for CNN-large with MixTrain. Note that the improvement mostly happens for WRN-28-4 and CNN-large architectures, where both networks achieve better robust accuracy than their counterparts. This suggests that there still exists potential room for improving the generalization of these models with robust training. We present additional results in Appendix C.4

**Performance on ImageNet dataset.** To assess the performance of pruning techniques on large-scale datasets, we experiment with the ImageNet dataset. Table 3 summarizes our results. Similar to smaller-scale datasets, our approach also outperforms LWM based pruning for the ImageNet dataset. In particular, at 99% pruning ratio, our approach improves the top-1 $era$ by 4.59 percentage points, and the top-5 $era$ by 6.75 percentage points.

Table 3. Pruning results for ImageNet dataset on ResNet50 network trained with adversarial training for $\epsilon=4/255$.

| Pruning ratio | 0% | 95% | 99% |
|---------------|----|-----|-----|
|               | top-1 | top-5 | top-1 | top-5 | top-1 | top-5 |
| LWM           | 19.6 | 43.3 | 9.8 | 24.4 |
| **Our work**  | **21.4** | **46.6** | **13.0** | **31.2** |
| $\Delta$      | +1.8 | +3.3 | +3.2 | +6.8 |

**6. Further insights**

In this section, we present further insights into our pruning technique. First, we investigate the impact of using different robust training objectives in pre-training and fine-tuning and uncover multiple interesting findings. Second, we probe the source of performance improvements achieved by our proposed approach. We do this by visualizing the distributions of weight parameters pruned by both our proposed approach and LWM based pruning technique.

**6.1. Imbalanced training objectives: Hidden robust sub-networks within non-robust networks.**

We have already demonstrated that the success of the proposed pruning approach largely stems from finding a highly robust yet small sub-network within a pre-trained robust network (Fig. 3). What if the pre-trained network is trained with a different objective than pruning? To answer this question, we prune a pre-trained network with three different objectives, namely benign training, adversarial training, and randomized smoothing. These results are presented in Table 4 where the pruning ratio for each sub-network is 50% with VGG16 network and CIFAR-10 dataset.

Our results show that there exist highly robust sub-networks even within non-robust networks. For example, we were able to find a sub-network with 43.5% $era$ when the pre-trained network was trained with benign training and had 0% $era$. As a reference, the pre-trained network with adversarial training has 51.9% $era$. With randomized smoothing, we found that a sub-network within an adversarial trained network has 63.6% $vra-s$, which is even higher than the pre-trained network with randomized smoothing objective. Again the $vra-s$ for this pre-trained network with adversarial training is equal to a random guess. Under similar setup, we find a sub-network with 61.3% $vra-s$ within an adversarially trained networks on SVHN dataset. In comparison, under similar setup, a pre-trained network with state-of-the-art randomized smoothing approach (Carmon et al., 2019), could only achieve 60.1% $vra-s$.

Table 4. Performance of sub-networks within pre-trained networks. Given a pre-trained network, we search for a sub-network optimized for an individual metric, i.e, test accuracy, $era$ (at $\epsilon=8/255$), or $vra-s$ (at $\epsilon=2/255$). Even within completely non-robust networks, we can find sub-networks with high robust accuracy. We use CIFAR-10 dataset and VGG16 network for this experiment.

| Pre-training objective | Benign accuracy | $era$ | $vra-s$ |
|------------------------|----------------|------|--------|
| **Benign training**    | Pre-trained    | 95.0 | 0.0    | 10.0   |
|                        | Sub-nets       | 95.0 | 43.5   | 53.0   |
| **Adversarial training**| Pre-trained   | 82.7 | 51.9   | 10.0   |
|                        | Sub-nets       | 94.1 | 51.4   | 63.6   |
| **Randomized smoothing**| Pre-trained   | 83.6 | 31.9   | 61.1   |
|                        | Sub-nets       | 93.7 | 48.8   | 60.7   |

**6.2. Visualization of pruned weights**

We visualize the distributions of weights in pruned networks from our proposed approach and the LWM based pruning. Fig. 4 shows our results for each of the four training objectives. For each network, we present the visualization of only one layer in Fig. 4 and for all layers in Appendix D.

Recall that for each of the learning objectives, the SGD in the pruning step starts from the same solution obtained from the least weight-magnitude based pruning due to our scaled-initialization. However, with each epoch, we observe that SGD pruned certain connections with large magnitude...
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We vary the number of samples used from ten to all training images in the dataset for solving the ERM in the pruning step for CIFAR-10 dataset. This phenomenon is particularly visible for adversarial training and randomized smoothing where a significant number of connections with very small magnitude are also pruned away by the solver. This phenomenon also exists for MixTrain and CROWN-IBP but the fraction of such connections is very small and thus not clearly visible in the visualizations. One reason behind this could be that both of these learning objectives are biased towards learning connections with smaller magnitudes (Gowal et al., 2018a).

7. Ablation studies

We now provide further ablation studies for the proposed pruning approach. It is evident from Table 2 that our approach can achieve much higher robust accuracy than the least weight-magnitude heuristic. However, it is not obvious what is the source of these improvements. We shed insight into this question via the following ablation studies. We will focus on adversarial training in particular and measure the performance with empirical robust accuracy.

How much data is needed for supervision in pruning?

We vary the number of samples used from ten to all training images in the dataset for solving the ERM in the pruning step for CIFAR-10 dataset at 99% pruning ratio. Though a small number of images does not help much, the transition happens around 10% of the training data (5k images on CIFAR-10) after which an increasing amount of data helps in significantly improving the era. We present these results in Appendix B.1.

Number of training epochs for pruning. We vary the number of epochs used to solve the ERM problem for the pruning step from one to a hundred. We observe that even a small number of pruning epochs, such as five, are sufficient to achieve large gains in era and the gains start diminishing as we increase the number of epochs. These detailed results are presented in Appendix B.2.

Multi-step pruning. Due to simplicity and reduced computational overhead, our results so far have leveraged a single-step pruning strategy, for both our proposed pruning approach and LWM. Now we compare the performances of proposed technique (single-step) with multi-step pruning using LWM, and we summarize the top-1 and top-5 era obtained by each pruning strategy in Table 5. Though multi-step pruning can increase the performance of LWM, our approach still outperforms it by a large extent. For example, at 95% pruning ratio, multi-step pruning increases the era by 0.3 percentage points but it is still 1.5 percentage points lower than our proposed approach. Note that the performance of our proposed techniques can also be further increased with a multi-step approach, which however will incur additional computational overhead.

8. Conclusion

In this work, we study the interplay between neural network pruning and robust training objectives. We argue for integrating the robust training objective in pruning technique by formulating pruning as an optimization problem and demonstrate the gains across different datasets, network architectures, and robust training techniques. In future work, we plan to explore more ways to make the pruning process more efficient and further increase robust accuracy.
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A. Experimental Setup

In this work, we focus on CIFAR-10, SVHN, and ImageNet dataset. For each dataset, we train the networks with SGD optimizer with a learning rate of 0.1, and cosine decay. For CIFAR-10, SVHN, and ImageNet dataset, we use 100, 100, and 90 epochs respectively. We experiment with VGG16, WideResnet-28-4, CNN-small, and CNN-large (Wong & Kolter, 2018) network architectures. Since both MixTrain and CROWN-IBP methods only work with small scale networks (without batch-normalization), we use only CNN-large and CNN-small for them. We split the training set into a 90/10 ratio for training and validation for tuning the hyperparameters. Once hyperparameters are fixed, we use all training images to report the final results.

Adversarial training: We use the state-of-the-art iterative adversarial training setup (based on PGD) with $l_\infty$ adversarial perturbations on CIFAR-10 and SVHN dataset. The maximum perturbation budget, the number of steps, and perturbations at each step are selected as 8, 10, and 2 respectively. In particular, for CIFAR-10, we follow the robust semi-supervised training approach from Carmon et al. 2019, where it used 500k additional pseudo-labeled images from the TinyImages dataset. For ImageNet, we train using the free adversarial training approach with 4 replays and perturbation budget of 4 (Shafahi et al., 2019). We evaluate the robustness of trained networks against a stronger attack, where we use 50 iterations for the PGD attack with the same maximum perturbation budget and step size.

Provable robust training: We evaluate our pruning strategy under three different provable robust training settings.

- MixTrain: We use the best training setup reported in Wang et al. 2018a for both CIFAR-10 and SVHN. In specific, we use sampling number $k'$ as 5 and 1 for CNN-small and CNN-large. We select $\alpha = 0.8$ to balance between regular loss and verifiable robust loss. The trained networks are evaluated with symbolic interval analysis (Wang et al., 2018c) to match the results in Wang et al. 2018a.

- CROWN-IBP: We follow the standard setting in Zhang et al. 2020 for CROWN-IBP. We set the $\epsilon$ scheduling length to be 60 epochs (gradually increase training $\epsilon$ from 0 to the target one), during which we gradually decrease the portion of verifiable robust loss obtained by CROWN-IBP while increasing the portion obtained by IBP for each training batch. For the rest of the epochs after the scheduling epochs, only IBP contributes to the verifiable robust loss. We use IBP to evaluate the trained networks.

- Randomized smoothing: We train the network using the stability training for CIFAR-10 and SVHN dataset (similar to Carmon et al. 2019). We calculated the certified robust with $N_0 = 100$, $N = 10^4$, noise variance ($\sigma=0.25$), and $\alpha = 10^{-3}$. 

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Table 6. All neural network architectures, with their number of parameters, used in this work.

| Name         | Architecture                       | Parameters |
|--------------|------------------------------------|------------|
| VGG4         | conv 64 → conv 64 → conv 128 → conv 128 → fc 256 → fc 256 → fc 10 | 0.46m      |
| VGG16        | conv 64 → conv 64 → conv 128 → conv 128 → conv 256 → conv 256 → conv 512 | 15.30m     |
|              | → conv 512 → conv 512 → conv 512 → conv 512 → fc 256 → fc 256 → fc 10 |            |
| CNN-small    | conv 16 → conv 32 → fc 100 → fc 10 | 0.21m      |
| CNN-large    | conv 32 → conv 32 → conv 64 → conv 64 → fc 512 → fc 512 → fc 512 | 2.46m      |
| WideResNet-28-4 | Proposed architecture from Zagoruyko & Komodakis 2016 | 6.11m      |
| ResNet50     | Proposed architecture form He et al. 2016 | 25.50m     |

Pruning and fine-tuning: Except for learning rate and the number of epochs, pruning and fine-tuning have similar training parameters as pre-training. We choose the number of epochs as 20 in all experiments (if not specified). Similar to pre-training, for pruning we choose learning of 0.1 with cosine decay. Often when this learning rate is too high (in particular for MixTrain and CROWN-IBP), we report results with the learning of 0.001 for the pruning step. Fine-tuning is done with a learning rate of 0.01 and cosine decay. To make sure that the algorithm does not largely prune fully connected layers that have most parameters, we constrain it to prune each layer by equal ratio. For multi-step pruning, we follow 50 iterations of pruning and fine-tuning, with a decreasing pruning ratio in each iteration.

A.1. Strength of adversarial attacks for evaluation

Iterative adversarial training (Madry et al., 2018) has long withstood its performance against attacks of varied strength (Athalye et al., 2018a). It is natural to ask whether our compressed networks bear the same strength. To evaluate it, we measure the robustness of our compressed networks against adversarial attacks with increasing step-size i.e., enabling stronger adversary to search for adversarial examples. We choose the perturbation budget for each step with the $\frac{2\times\epsilon}{\text{steps}}$ rule suggested by Madry et al. 2018. Figure 5 shows the results for networks trained on CIFAR-10 and compressed to 99% i.e., 100$\times$ smaller networks. It shows that gains in adversarial attacks strength saturates after a certain number of attacks steps since the robust accuracy stops decreasing. We use 50 attack steps for all adversarial attacks in our evaluation.

A.2. Network architectures

Table 6 contains the architecture and parameters details of the neural networks used in this work. For WideResNet-28-4 and ResNet-50, we use the original architectures proposed in Zagoruyko & Komodakis 2016 and He et al. 2016, respectively. CNN-large and CNN-small are similar to architectures used in Wong & Kolter 2018. VGG4 and VGG16 are the the variants of original VGG architecture (Simonyan & Zisserman, 2015).

B. Further details on ablation studies

In this section, we discuss the ablation studies for the pruning steps in detail.

B.1. How much data is needed for supervision in pruning?

We vary the number of samples used from ten to all training images in the dataset for solving the ERM in the pruning step for CIFAR-10 dataset at a 99% pruning ratio. Fig. 6 shows there results. Data corresponding to zero samples refers to the least weight-magnitude based heuristic as it is used to initialize the pruning step. As the amount of data (number of samples) used in the pruning step increases, the robustness of the pruned network after fine-tuning also increases. For CIFAR-10, a small number of images doesn’t help much in finding a better pruned network. However, the transition happens around 10% of the training data (5k images for CIFAR-10) after which an increasing amount of data helps in significantly improving the era.

Figure 5. Empirical adversarial accuracy (era) of compressed networks with increasing number of steps in projected gradient descent (PGD) attack. Beyond a certain number of steps, era is largely constant with increase in steps. Results are reported for compressed networks at 99% pruning ratio with CIFAR-10 dataset.
C. Additional experimental Results

In this section, we first study the impact of sparsity in the network in the presence of benign and robust training. Next, we present the limitation of least weight magnitude pruning in the presence of robust training and discuss the choice of this heuristic as a baseline. After that, we study the improvement in the generalization of some networks after the proposed pruning technique. Next, we provide additional visualization on comparison of both techniques across the end-to-end compression pipeline. Finally, we demonstrate the success of the proposed pruning technique with benign training.

C.1. Sparsity hurts more with robust training.

We first study the impact of sparsity in the presence of benign training and adversarial training. Fig. 8 shows these results, where we train multiple networks from scratch with different sparsity ratio and report the fractional decrease in performance compared to the non-sparse network trained from scratch. For each training objective (adversarial training or benign training) and sparsity ratio, we train an individual VGG4 network. These results show that robustness decreases at a faster rate compared to clean accuracy with increasing sparsity. Consider robust training against a stronger adversary (\(\epsilon=8\)), where at 75% sparsity ratio, the \(\text{-era}\) reduced to a fraction of 0.74 of the non-sparse network. The fractional decrease in test accuracy for a similar setup is only 0.92. Even defending against a weaker adversary (\(\epsilon=2\)), robust accuracy is hard to achieve in the presence of sparsity. The fractional decrease in \(\text{era}\) is .79 against this weaker adversary at 75% sparsity level. With the increasing size of the baseline network, such as VGG16, WideResNet-28-4 size, the rate of degradation of robustness with sparsity decreases but it still decays faster than the test accuracy.

This observation is closely related to the previously reported relationship between adversarial training and the size of neural networks (Madry et al., 2018; Xie & Yuille, 2019). In particular, Madry et al. (Madry et al., 2018) demonstrated that increasing the width of the network improves robust accuracy to a large extent. We complement these observations by highlighting that further reducing the number of parameters (before training) reduces the robustness at a much higher rate.

C.2. Combining network pruning with robust training

We can further integrate the network pruning pipeline with robust training by updating the loss objective. For example, to achieve an empirically robust network, we can pretrain and fine-tuning a network with adversarial training (selecting \(L_{pt} = L_f = L_{adv}\)). Similarly, for other robust training mechanisms, we can use their respective loss functions.
Next, we discuss the limited performance of least weight magnitude (LWM) based pruning.

Limitations of least weight magnitude based heuristic. Though pruning with least weight magnitude based heuristic brings some gains in improving the robust accuracy of the network, there still exists a large room for improvement. For example, at a 99% pruning ratio for a VGG16 network, it still incurs a decrease in \( \epsilon \) by 17.6 percentage points compared to the non-pruned i.e., pre-trained network. We also observe a non-linear drop in performance with increasing adversarial strength in adversarial training. Consider Fig. 9, where we report the performance of the pre-trained networks along with the compressed network (at 99% pruning ratio) from the pruning pipeline for different adversarial perturbation budgets in adversarial training.

Against a weaker adversary, where the pre-trained network is highly robust, weighted-based pruning heuristic struggles to achieve high robustness after compression. At smaller perturbation budgets, this gap increases further with the increase in adversarial strength.

C.3. Why focus on pruning and fine-tuning based compression pipeline

We focus on pruning and fine-tuning approach because it achieves the best results among all three pruning strategies namely pruning before training, run-time pruning, and pruning after training i.e., pruning and fine-tuning. This is because the other approaches are constrained and tend to do pruning in a less flexible or with incomplete information (Lee et al., 2019; Lin et al., 2017). On the other hand, despite the simplicity, pruning and fine-tuning based on least weight magnitude (Han et al., 2015) can itself achieve highly competitive results (Lee et al., 2019). With similar motivation, we integrate this approach with robust training and select it as the baseline. This simplicity also allows us to integrate different training objectives, such as adversarial training and verifiable robust training.

C.4. Increase in generalization with pruning

For verifiable training with CROWN-IBP, we observe improvement in generalization across all experiments ranging from 0.9-1.5 percentage points. Note that both proposed and baseline techniques can improve the generalization. This further highlights how network pruning itself can be used to improve the generalization of verified training approaches. Table 7 summarizes these results for proposed pruning methods where we observe improvement in robust accuracy after pruning at multiple pruning ratios.

Table 7. Verified robust accuracy with CROWN-IBP with the proposed pruning methods for CNN-large network and CIFAR-10 dataset.

| Pruning ratio | 0  | 10 | 30 | 50 | 70 | 80 | 90 |
|---------------|----|----|----|----|----|----|----|
| vra-t         | 45.5 | 46.1 | 46.0 | 45.9 | 45.9 | 46.0 | 46.1 |

C.5. Additional comparisons across end-to-end pruning pipeline

In Fig. 10, we present additional comparisons of LWM and proposed pruning approach across the end-to-end compression pipeline. Though both approaches use the identical pre-trained network, the proposed approach searches for a better pruning architecture in the pruning steps itself. Fine-tuning further improves the performance of these networks. For the WRN-28-4 network on the SVHN dataset, we also observe that the fine-tuning step decreases the performance to some extent for the proposed approach. We hypothesize that this behavior could be due to an imbalance in the learning rate at the end of the pruning step and the start
of the fine-tuning step. With further-hyperparameter tuning, our approach can achieve higher gains for this network. However, for an impartial comparison with baseline, we avoid excessive tuning of hyperparameters for the proposed approach and use a single set of hyperparameters across all networks. The results are reported on a randomly partitioned validation of the CIFAR-10 and SVHN dataset at a 99% pruning ratio.

C.6. Performance with benign training

In this work, we have largely focused on demonstrating the success of the proposed pruning approach with multiple robust training objectives. However, it is natural to ask whether the proposed approach also has the same advantage with benign training i.e., in the absence of an adversary. We compare the performance of LWM and our approach for VGG16 and WRN-28-4 across CIFAR-10 and SVHN dataset in Table 8. Similar to robust training, our approach is also successful with benign training where it outperforms LWM based pruning in all experiments. In particular, even at a 99% pruning ratio, the proposed approach can maintain the accuracy within 1.2 percentage points for the SVHN dataset.

Table 8. Performance of LWM and proposed pruning technique for benign training.

| Architecture | VGG16           | WRN-28-4         |
|--------------|-----------------|------------------|
|              | Pruning ratio   |                  |
| Cifar-10     | 0% | 95% | 99% | 0% | 95% | 99% |
| LWM                  | 95.1±0.1        | 86.1±0.1         | 95.8±0.2         | 94.9±0.2 | 89.2±0.2 | 94.9±0.2 |
| Our work               | 94.6±0.1        | 90.4±0.2         | 95.5±0.2         | 91.2±0.2 |
| SVHN     | 0% | 95% | 99% | 0% | 95% | 99% |
| LWM                  | 95.9±0.1        | 93.6±0.1         | 96.4±0.1         | 91.9±0.1 |
| Our work               | 95.6±0.2        | 95.2±0.1         | 96.3±0.1         | 95.2±0.2 |

D. Visualization of pruned weights

Fig. 11, 12 present the visualization for pruned connection of adversarial training and randomized smoothing for each layer in the VGG16 network on CIFAR-10 dataset. Fig. 13, 14 present similar visualization for MixTrain and CROWN-IBP for CNN-large networks.
Figure 10. Comparison of proposed pruning approach with least weight magnitude (LWM) based pruning at 99% pruning ratio for robust training with iterative adversarial training.
Figure 11. Histogram of weights pruned by the baseline and proposed technique for adversarial training.
Figure 12. Histogram of weights pruned by the baseline and proposed technique for randomized smoothing based training.
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Figure 13. Histogram of weights pruned by the baseline and proposed technique for MixTrain.

Figure 14. Histogram of weights pruned by the baseline and proposed technique for CROWN-IBP.