Towards an explanation of transverse single-spin asymmetries in proton-proton collisions: the role of fragmentation in collinear factorization

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We study the transverse single-spin asymmetry for single-hadron production in proton-proton collisions within the framework of collinear twist-3 factorization in Quantum Chromodynamics. By taking into account the contribution due to parton fragmentation we obtain a very good description of all high transverse-momentum data for neutral and charged pion production from the Relativistic Heavy Ion Collider. Our study may provide the crucial step towards a final solution to the long-standing problem of what causes transverse single-spin asymmetries in hadronic collisions within Quantum Chromodynamics. We show for the first time that it is possible to simultaneously describe spin/azimuthal asymmetries in proton-proton collisions, semi-inclusive deep-inelastic scattering, and electron-positron annihilation by using collinear twist-3 factorization in the first process along with transverse momentum dependent functions extracted from the latter two reactions.

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Introduction The field of transverse single-spin asymmetries (SSAs) in hard semi-inclusive processes began some four decades ago with the observation of the large transverse polarization (up to about 30 %) of neutral A-hyperons in the process $p Be \rightarrow \Lambda \pi^+ X$ at FermiLab [1]. People noticed early on that the naïve collinear parton model cannot generate such large effects [2]. It was then pointed out that SSAs for single-particle production in hadronic collisions are genuine twist-3 observables for which, in particular, collinear 3-parton correlations have to be taken into account in order to have a proper description within Quantum Chromodynamics (QCD) [3]. This formalism later on was worked out in more detail and also successfully applied to SSAs in processes like hadron production in proton-proton collisions, $p p \rightarrow hX$ — see, e.g., Refs. [4–12]. Here we focus on SSAs in such reactions, which were extensively investigated in fixed target and in collider experiments.

Let us now look at the generic structure of the spin-dependent cross section for $A(p, \vec{S}_L) + B(P') \rightarrow C(P_C) + X$, where the 4-momenta and polarizations of the incoming protons $A$, $B$ and outgoing hadron $C$ are specified. In twist-3 collinear QCD factorization one has

$\frac{d^2\sigma}{d^2x} (x, \vec{p}) = H \otimes f_{a/A}(t) \otimes f_{b/B}(t) \otimes D_{c/C}(t) + H' \otimes f_{a/A}(t) \otimes f_{b/B}(t) \otimes D_{c/C}(t) + H'' \otimes f_{a/A}(t) \otimes f_{b/B}(t) \otimes D_{c/C}(t),$ \hspace{1cm} (1)

with $f_{a/A}(t)$ $(f_{b/B}(t))$ indicating the distribution function associated with parton $a$ ($b$) in proton $A$ ($B$), while $D_{c/C}(t)$ represents the fragmentation function associated with hadron $C$ in parton $c$. The twist of the functions is denoted by $t$. The hard factors corresponding to each term are given by $H$, $H'$, and $H''$, and the symbol $\otimes$ represents convolutions in the appropriate momentum fractions. In Eq. (1) a sum over partonic channels and parton flavors in each channel is understood.

The first term in (1) has already been studied in quite some detail in the literature [4–12]. It contains both quark-gluon-quark correlations and tri-gluon correlations in the polarized proton, where for the former one needs to distinguish between contributions from so-called soft gluon poles (SGPs) and soft fermion poles (SFPs). The second term in (1) arising from twist-3 effects in the unpolarized proton, was shown to be small [13]. Only recently a complete analytical result was obtained for the third term in (1), which describes the twist-3 contribution due to parton fragmentation [14].

For quite some time many in the community believed that the first term in (1) dominates the transverse SSA in $p^0 p \rightarrow hX$ (typically denoted by $A_N$) for the production of light hadrons (see, e.g., Refs. [4–12]), where the SGP contribution is generally considered the most important part. Note that the SGP contribution to $A_N$ is determined by the Qiu-Sterman function $T_F$ [4, 12, 13], which can be related to the transverse-momentum dependent (TMD) Sivers parton density $f_{1T}^{q\perp}$ [13, 14]. For a given parton flavor $q$, these entities satisfy [13, 15]

$T_F^q(x, \vec{p}) = - \int d^2\vec{p}_\perp \frac{p_\perp^2}{M} f_{1T}^q(x, \vec{p}_\perp^2)|_{\text{SIDIS}},$ \hspace{1cm} (2)

where $M$ is the nucleon mass. Because of the relation in (2), one can extract $T_F$ from data on either $A_N$ or on the Sivers transverse SSA in semi-inclusive deep-inelastic scattering (SIDIS) $A_S^{\text{SIDIS}}$. It therefore came as a major surprise when an attempt failed to simultaneously explain both $A_N$ and $A_S^{\text{SIDIS}}$ [13]. The striking result pointed out in Ref. [13] was that the two extractions for $T_F$ differ in sign. This “sign-mismatch” puzzle could not be resolved by more flexible parameterizations of $f_{1T}^{q\perp}$ [18]. Also tri-gluon correlations are unlikely to fix...
this issue \[12\], while SFPs may play some role \[9\].

At this point one may start to question the dominance of the first term in \[1\]. In fact, data on the transverse SSA in inclusive DIS \[14, 20\] seem to support this point of view, i.e., that the first term in \[1\] is not the main cause of \(A_N\) \[21\]. Therefore, in the present work we study the potential role of the twist-3 fragmentation part of \[1\]. After fixing the SGP contribution to \(A_N\) through the Sivers function extracted from data on \(A_{\text{SIDIS}}^{\text{Siv}}\) \[22, 23\] and the relation in \[24\], we obtain a very good fit to all high transverse-momentum forward-region pion data for \(A_N\) from the Relativistic Heavy Ion Collider (RHIC). As explained below in more detail, our analysis shows for the first time that one can simultaneously describe \(A_N\) using collinear factorization, \(A_{\text{SIDIS}}^{\text{SS}}\), the Collins transverse SSA \(A_{\text{SIDIS}}^{\text{Col}}\) in SIDIS, and \(A_{\text{SIDIS}}^{\text{Coll}}\) that represents a particular azimuthal asymmetry in electron-positron annihilation into two hadrons, \(e^+e^- \to h_1 h_2 X\) \[24\].

**Fragmentation contribution to \(A_N\)**

The fragmentation contribution to the cross section in (1) reads \[14\]

\[
P_h^{\text{frag}}(S_{\perp}) = -2\frac{\alpha_s^2 M_h}{S} \epsilon_{\alpha\beta}(S_{\perp}^2) P_h^\beta \sum_i \sum_{a,b,c} \int_{z_{\min}}^{1} dz \frac{1}{z^3} \times \int_{x_{\min}}^{1} dx' \frac{1}{x' x' S + T + z - x' \hat{x} \hat{u} + h_i'(x) f_i'(x')} \times \left\{ \frac{\hat{H}^{C/c}(z) - z \frac{d\hat{H}^{C/c}(z)}{dz}}{z} S_H^i + \frac{1}{z} \hat{H}^{C/c}(z) S_H^i \right\} + 2z^2 \int_{z}^{\infty} dz_1 \frac{1}{z_1 - z_1} \hat{H}^{C/c,3}(z_1, z_1) \frac{1}{z} S_H^{i} \right\} , \tag{3}
\]

where \(i\) denotes the channel, \(x = -x'(U/z)/(x' S + T + z)\), \(x' = -(T/z)/(U + S)\), \(z_{\min} = -(T + U)/S\), and \(\xi = (1 - z)/z\). Here we used the Mandelstam variable \(S = (P + P')^2\), \(T = (P - P')^2\), and \(U = (P' - P)^2\), which on the partonic level give \(s = x' x' S\), \(\hat{x} = x T/z\), and \(\hat{u} = x' U/z\). Oftentimes one also uses \(x_F = 2P_{hz}/\sqrt{S}\), where \(P_{hz}\) is the longitudinal momentum of the hadron, as well as the pseudo-rapidity \(\eta = -\ln(\tan(\theta/2))\), where \(\theta\) is the scattering angle. The variables \(x_F, \eta\) are further related by \(x_F = 2P_{h\perp} \sinh(\eta)/\sqrt{S}\), where \(P_{h\perp}\) is the transverse momentum of the hadron. The non-perturbative parts in \[3\] are the transversity distribution \(h_1\), the unpolarized parton density \(f_1\), and the three (twist-3) fragmentation functions (FFs) \(\hat{H}, H, \hat{H}^3_{FU}\), with the last one parameterizing the imaginary part of a 3-parton correlator. The definition of those functions and the results for the hard scattering coefficients \(S_i\) can be found in \[14\]. (An alternative notation of the relevant FFs is given in Ref. \[25\], where twist-3 effects in SIDIS were computed.) We note that the so-called derivative term in \[3\], associated with \(dH/dz\), was first obtained in \[26\].

The function \(\hat{H}\) is related to the TMD Collins function

\[
\hat{H}^{h/q}(z) = z^2 \int d^2 k_{\perp} \frac{k_{\perp}^2}{2M_h^2} H_1^{h/q}(z, z^2 k_{\perp}^2) . \tag{4}
\]

This relation can be considered the fragmentation counterpart of Eq. \[2\]. Exploiting the universality of the Collins function \[22\], one can simultaneously extract \(H_1^+\) and \(h_1\) from data on \(A_{\text{SIDIS}}^{\text{Siv}}\) \[22, 23\] and data on \(A_{\text{SIDIS}}^{\text{Coll}}\) (see \[33\] and references therein). Below we utilize such information for \(H_1^+\) and \(h_1\) when describing \(A_N\). The FFs in \[2\] are related via \[14\]

\[
H^{h/q}(z) = -2z\hat{H}^{h/q}(z) + 2z^3 \int_z^{\infty} dz_1 \frac{1}{z_1 - z} \hat{H}^{h/q,3}(z, z_1) , \tag{5}
\]

implying that in the collinear twist-3 framework one has two independent FFs. It is important to realize that this is different from the so-called TMD approach for \(A_N\), where only \(H_1^+\) enters the fragmentation piece \[33\].

**Phenomenology of \(A_N\) for pion production**

We consider \(A_N\) for \(p^+ p \to \pi X\) in the forward region of the polarized proton, which has been studied by the STAR \[32, 37\], BRAHMS \[38, 39\] and PHENIX \[40\] collaborations at RHIC. We mainly focus on data taken at \(\sqrt{S} = 200\) GeV for which typically \(P_{h\perp} > 1\) GeV. Throughout we use the GRV98 unpolarized parton distributions \[41\] and the DSS unpolarized FFs \[42\]. Note that the GRV98 parton distributions were also used in Refs. \[22, 23, 32\] for extracting the Sivers function and the transversity, which we take as input in our calculation. The SGP contribution to \(A_N\) is computed by fixing \(T_F\) through Eq. \[2\] with two different inputs for the Sivers function — SV1: \(f_{\text{SV1}}^F\) from Ref. \[22\], obtained from SIDIS data on \(A_{\text{SIDIS}}^{\text{Siv}}\) \[43, 44\], and SV2: \(f_{\text{SV2}}^F\) from Ref. \[22\], “constructed” such that, in the TMD approach, the contribution of the Sivers effect to \(A_N\) is maximized while maintaining a good description of \(A_{\text{SIDIS}}^{\text{Siv}}\). These two inputs for \(f_{\text{SV}}^F\) are mainly distinct by their quite different large-\(x\) behavior. To compute the contribution in \[3\] we take \(h_1\) and \(H_1^+\) (which fixes \(H\) through \[1\] from \[33\]). For favored fragmentation into \(\pi^+\) we make for \(\hat{H}^{+/d}_{FU}\) the ansatz

\[
\frac{\hat{H}^{+/d}_{FU}(z, z_1)}{D^{+/d}(z) D^{+/d}(z_1)} = \frac{N_{\text{fav}}}{2I_{\text{fav}} I_{\text{fav}}} z^{\alpha_{\text{fav}}}(z/z_1)^{-\beta_{\text{fav}}} \times (1 - z)^{\beta_{\text{fav}} - 1} \equiv I_{\text{fav}}(z, z_1) , \tag{6}
\]

with the parameters \(N_{\text{fav}}, \alpha_{\text{fav}}, \beta_{\text{fav}}\) and the unpolarized FF \(D\). Note that the allowed range for \(z\) and \(z_1\) is \([0, 1]\) \[45\] and that our ansatz satisfies the constraint \(\hat{H}_{FU}(z, z) = 0\) \[45, 46\]. With the use of DSS FFs \[42\], the factor \(I_{\text{fav}}\) reads \(I_{\text{fav}} = I_{\text{fav}} \equiv I_{\text{fav}}\).
full analogy to (6), introducing the additional parameters $P_{\text{fixed}}$ through charge conjugation, and the

\[ \int \frac{dz}{z} H_{\pi^+}^z(z) = 0 \]

for the SV1 input. Also shown is $H_{\pi^+/q}$ without the contribution from $\tilde{H}_{FU}^z$ (dashed line).

We also allow the $\beta$-parameters $\beta_u^T = \beta_d^T$ of the transversity to vary within the error range given in [32]. All integrations are done using the Gauss-Legendre method with 250 steps. For the SV1 input the result of our 8-parameter fit is shown in Tab. II. Note that the values for $\beta_{uv}^T, \beta_{ud}^T$ are at their lower limits, which we introduce to guarantee a finite integration upon $z_1$ in [33] and a proper behavior of $A_N$ at large $x_F$, respectively. For the SV2 input the values of the fit parameters are similar, with an equally successful fit ($\chi^2$/d.o.f. = 1.10).

The very good description of $A_N$ is also reflected by Fig. I. We emphasize that such a positive outcome is non-trivial if one keeps in mind the constraint in [34] and the need to simultaneously fit data for $A_N^{n\pi}$ and $A_N^{\pi^\pm}$. Results for the FFs $H_{\pi^+/q}$ and $\tilde{H}_{FU}^{\pi^+/q}$ are displayed in Fig. 2. In either case the favored and disfavored FFs have opposite signs. This is like for $H_{\perp}$ where such reversed signs are actually “preferred” by the Schäfer-Teryaev (ST) sum rule $\sum_{h} \sum_{d} \int_0^1 dz \, M_h \tilde{H}^{h/q}(z) = 0$ [47]. Note that the ST sum rule, in combination with [37], implies a constraint on a certain linear combination of $H^{h/q}$ and (an

\begin{table}[h]
\centering
\caption{Fit parameters for SV1 input.}
\begin{tabular}{cc}
\hline
$\chi^2$/d.o.f. & 1.03  \\
$N_{\text{d.o.f.}}$ & 0.0338 \\
$N_{\text{dis}}$ & 0.216  \\
$\alpha_{\text{f}}$ & 0.198  \\
$\alpha_{\text{d}}$ & 0.0 \\
$\beta_{\text{uv}}$ & 0.180  \\
$\beta_{\text{ud}}$ & 0.34  \\
$\beta_u^T$ & 3.34  \\
$\beta_d^T$ & 1.10  \\
\hline
\end{tabular}
\end{table}
integrand of) $\tilde{H}_{F_U}^{\pi/q,3}$. In view of that, reversed signs between favored and disfavored FFs like in Fig. 2 are actually beneficial. Also depicted in Fig. 4 is $H^{\pi/q}$ when $\tilde{H}_{F_U}^{\pi/q,3}$ is switched off. As shown in Fig. 4, in such a scenario, i.e., by turning off the 3-parton FF, one cannot describe the data for $A_N$. According to Fig. 3 the $H$ term (including its derivative) in Eq. 1 contributes only very little to $A_N$. Also the SGP pole term is small, except for the SV2 input at large $x_F$, where its contribution is opposite to the data. Clearly $A_N$ is governed by the $H$-term in Fig. 4. This result can mainly be traced back to the hard scattering coefficients: e.g., for the dominant $gg \to gg$ channel one has $S_H \propto 1/t^l$, but $S_{H} \propto 1/t^2$ in the forward region where $t$ is small. Finally, Fig. 3 shows the $P_{h\perp}$-dependence of $A_N$ for $\sqrt{S} = 500$ GeV. Preliminary data from STAR, extending to almost $P_{h\perp} = 10$ GeV, show that $A_N$ is rather flat. The twist-3 calculation agrees with that trend, and also the magnitude of $A_N$ is in line with the data. Note that the data of Ref. 48 were not included in our fit and that only statistical errors are shown in Fig. 4.

**Conclusions** Collinear twist-3 QCD factorization can be considered the most natural and rigorous approach to the transverse SSA $A_N$ in $p^p \to hX$. However, the sign-mismatch issue of the Sivers effect had put this framework into question. Here we have demonstrated for the first time that, despite the sign-mismatch problem, twist-3 factorization actually can describe high-energy RHIC data for $A_N$ very well if one takes the fragmentation contribution into account. We re-emphasize that this result is non-trivial. Since in the twist-3 approach part of $A_N$ can be fixed by spin/azimuthal asymmetries in SIDIS and in $e^+e^- \to h_1h_2X$, we have shown that at present a simultaneous description of all those observables is possible. We repeat that the fragmentation contribution in twist-3 factorization goes beyond the pure Collins effect. Independent information on the FFs $H^{\pi/q}$, $\tilde{H}_{F_U}^{\pi/q,3}$ from other sources is needed before one can ultimately claim the intriguing data on $A_N$ is fully understood. However, the fact that $\tilde{H}_{F_U}^{\pi/q,3}$ gives a reasonable contribution to (the numerically dominant) $H^{\pi/q}$ (see Fig. 2) allows one to be optimistic in this regard.

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![Graph](image-url)
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