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Fractional optimal control dynamics of coronavirus model with Mittag–Leffler law

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ARTICLE INFO

Keywords:
Coronavirus
Fractional optimal control
Mittag–Leffler function
Global stability
Uniqueness of solution

ABSTRACT

Most countries around the world are battling to limit the spread of severe acute respiratory syndrome-coronavirus 2 (SARS-CoV-2). As the world strives to get an effective medication to control the disease, appropriate control measures for now remains one of the effective measures to reduce the spread of the disease. In this study, a fractional optimal control model is formulated in Atangana-Baleanu-Caputo derivative sense. The reproduction number and steady state of disease free of the Coronavirus model are examined and found to be globally stable. The existence and uniqueness of solution of the fractional Coronavirus model is established by using the Banach fixed point theorem approach. Three controls are considered in the model and Pontryagins Maximum Principle is used to establish the necessary conditions for optimal control solution. The numerical solution suggests that the best strategy is found to be the utilization of all three controls at the same time.

1. Introduction

Severe Acute Respiratory Syndrome Coronavirus 2 (SARS-CoV-2) since December 2019 has been recognized as one of the causes of respiratory illness. In view of that, understanding the changes of a novel and escalating disease outbreak is important but challenging (Helmy et al., 2020; Sanche et al., 2020). The virus is hypothesized to have originated from a seafood market in Huanan, Wuhan and therefore has zoonotic origin (Coronavirus, 2020). It was established that 55% of the infected cases within Wuhan in the early days of infection was associated with the Seafood Market (Li et al., 2020; Luo et al., 2020; WHO, 2020a; Zhou et al., 2020).

Recent, genetic sequence of the human coronavirus (COVID-19) and that of bat coronavirus show similarity of 96%. Other zoonotic human coronavirus which occurred in 2012 is the Middle East Respiratory Syndrome coronavirus (MERS-CoV). Most common symptoms of this disease includes the following: fever, cough and difficulty in breathing (Ellerin, 2020; Gralinski and Menachery, 2020). It is important to state that some people may exhibit non-respiratory symptoms like vomiting, nausea and possibly diarrhea. The ongoing COVID-19 outbreak has surprised the world due to the fast nature of infection. It has been revealed that by January 23, a total number of 571 confirmed cases has been made known in China (Centers, 2019; Munster et al., 2020; WHO, 2019). A declaration was made by World Health Organization (WHO) in January 30 that the outbreak of the disease is a Global Public Health Emergency of International Concern WHO (2020a). According to the World health Organization, the number of people affected by the novel coronavirus has surpassed 32 million (Chen et al., 2020a; Weekly, 2020; WHO, 2020a; 2020b).

Numerous factors have attributed to the severe challenges faced by scientist to understand the etiology of COVID-19 epidemic. Firstly, the actual origin of the disease is not well understood except being linked in association with some wild animals including bat. Secondly, there is scientific evidence which indicates that typically the incubation period is within 2 to 14 days, However, some people may virtually show no symptoms and could infect others (Ivorra and Ramos, 2020; Rothe et al., 2020; Yang and Wang, 2020). Additionally, some vaccines are being tried and proven positive towards the spread of the disease and also some drugs are helping in the management of COVID-19 pandemic (Yang and Wang, 2020).

Eventually, the control of the disease is heavily relied on by control measures including isolation, quarantine and social distancing. Mathematical models have been observed as a powerful tool for providing good qualitative information about etiology parameters of diseases...
which guide health practitioners to take appropriate decisions (Bonyah et al., 2019b). Since the out-break of the COVID-19 epidemic several mathematical models have been formulated and analyzed with the sole aim of providing some useful information about the disease. In Wu et al. (2020) the authors introduced (SEIR) model and used available data to estimate the reproductive number. Authors in Read et al. (2020) based (2018) the authors introduced (SEIR) model and used available data to obtain a reproductive number of a ratio 3:1. Authors in Yang and Wang (2019) formulated a mathematical model incorporated environmental compartment as a source of COVID-19 infection and observed that the disease would remain endemic in the long run. Optimal control strategies may provide a good scientific platform in controlling the epidemics in the communities (Bonyah et al., 2019b).

Fractional calculus is noticed to be effective in analyzing complex phenomena including disease models (Bonyah et al., 2019a; Khan and Atangana, 2020; Saad, Gómez-Aguilar, Almadiy, 2020). Fractional calculus has been identified by several researchers to possess the ability to include memory effect (Baleanu et al., 2018; Bonyah et al., 2018). This memory relies on the past and present data in order to give future prediction. This unique characteristics make it superior to that of the integer derivatives. In very recent times, several studies have been formulated in fractional derivatives. One of the most effective and reliable operators is the Atangana-Baleanu-Caputo (ABC) (Bonyah et al., 2019a; Khan and Atangana, 2020; Saad, Gómez-Aguilar, Almadiy, 2020). This operator is hinged on the generalized Mittag-Leffler function characterized with non-singular and non-local kernel. Another fascinating property of this operator is the ability to crossover from one operator to one another during predictions (Baleanu et al., 2018). Authors in Sweilam et al. (2019) made use of nonstandard numerical method to obtain qualitative information about tuberculosis with diabetes and obtained accurate result with the nonstandard numerical scheme. In Sweilam et al. (2020) two nonstandard numerical scheme were employed to study the dynamics of cancer treatment and observed that the nonstandard two-step Lagrange interpolation (NS2LIM) had better accuracy.

The aim of this study is to make use of the high accuracy of nonstandard numerical scheme based on two-step Lagrange interpolation to obtain the best control strategy in managing COVID-19 pandemic and controlling the spread of COVID-19 using Mittag-Leffler function To the best of knowledge, coronavirus has been investigated using fractional optimal in the context of Mittag-Leffler function.

2. Preliminaries

This section consists of relevant and necessary definitions (Atangana and Baleanu, 2016; Bonyah, 2020; Sweilam et al., 2019) needed in this study.

Definition 2.1. The Liouville-Caputo (LC)definition of fractional derivative of order $\phi$ is

$$c^\phi \mathcal{D}_{0^+}^\phi r(t) = \frac{1}{\Gamma(1-\phi)} \int_0^t (t-\rho)^{1-\phi} \phi \int_0^\rho (\rho-\tau)^{\phi-1} \rho d\tau d\rho, \quad \phi \in (0,1].$$

Definition 2.2. The Atangana-Baleanu definition in LC sense is

$$\Alpha^\phi \mathcal{D}_{0^+}^\phi r(t) = \frac{\Psi_\phi(1-\phi)}{1-\phi} \int_0^t \left[ r(t) - \frac{\phi}{1-\phi} \right]^{\phi-1} \rho d\rho,$$

where $\Psi_\phi$ is the normalization function and the corresponding integral of ABC is

$$\begin{align*}
\Alpha^\phi \mathcal{D}_{0^+}^\phi r(t) &= \frac{1}{\Psi_\phi(1-\phi)} \int_0^t (t-\rho)^{1-\phi} \phi \int_0^\rho (\rho-\tau)^{\phi-1} \rho d\tau d\rho.
\end{align*}$$

They also found the initial function and ordinary integral for $\phi = 0$ and $\phi = 1$ respectively. Further, the arrived to the following result by computing the Laplace transform:

$$\begin{align*}
\Omega \{ \Alpha^\phi \mathcal{D}_{0^+}^\phi r(t) \} (p) &= \frac{\Psi_\phi(p^\phi \int_0^p (\rho-\tau)^{\phi-1} \rho d\tau)}{(1-\phi)} \left( p^\phi + \frac{1}{p} \right).
\end{align*}$$

Theorem 2.1. ABC satisfies the following results for $r(t) \in C[b_1, b_2]$:

$$\| \Alpha^\phi \mathcal{D}_{0^+}^\phi r(t) \| < \frac{\Psi_\phi(1-\phi)}{1-\phi} \| r(t) \|, \quad \text{where} \quad \| r(t) \| = \max_{b_1 \leq t \leq b_2} |r(t)|,$$

and Lipschits condition:

$$\| \Alpha^\phi \mathcal{D}_{0^+}^\phi r_1(t) - \Alpha^\phi \mathcal{D}_{0^+}^\phi r_2(t) \| \leq \| r_1(t) - r_2(t) \|.$$

3. Mathematical model formulation

This section presents Coronavirus model as a modified version of Khan and Atangana (2020) and Chen et al. (2020b) in which the total host mammal population $S_m$ is partitioned into Susceptible $S_m$. Latent mammal $I_m$. Infected mammal $L_m$. Recovered mammal $R_m$. Thus, the total host population is given by $N_m = S_m + L_m + I_m + R_m$. The human total population is partitioned into Susceptible human $S_h$. Latent human $I_h$. Infected human $I_h$ and Recovered human $R_h$. The recruitment of rate of mammal and human are $\Lambda_m$ and $\Lambda_h$ respectively. The natural mortality rate for mammal is denoted by $\mu_m$ and natural mortality of human is $\mu_h$. The effective contact rate that can result in an infection between Susceptible mammals and Infected mammals is $\beta$ and the effective contact rate that can bring about infection between susceptible human through infected mammals and infected human is denoted by $\beta_2$, $\beta_3$ respectively. The rate at which recovered human loses immunity to join susceptible class is $\gamma$. The recovery rate of human and mammal is $r_h$ and $r_m$ respectively. The rate at which human and mammal move into infected class is given by $\theta_h$ and $\theta_m$ while human disease induced mortality is $\omega$. It is assumed that there is no loss of immunity in mammals. The following nonlinear differential equation represents the interactions among the various compartments.

$$\begin{align*}
\frac{dS_m}{dt} &= \Lambda_m - \beta S_m I_m - \mu_m S_m, \\
\frac{dL_m}{dt} &= \beta S_m I_m - (\mu_m + \theta_m) L_m, \\
\frac{dR_m}{dt} &= \tau_m L_m - \mu_m R_m, \\
\frac{dS_h}{dt} &= \Lambda_h - \beta_2 S_h L_h - \beta_3 S_h I_h + \gamma R_h - \mu_h S_h, \\
\frac{dI_h}{dt} &= \beta_2 S_h L_h + \beta_3 S_h I_h - (\mu_h + \theta_h) I_h, \\
\frac{dR_h}{dt} &= \tau_h I_h - (\mu_h + \omega) R_h.
\end{align*}$$

with the following initial conditions $S_m(0), L_m(0), R_m(0), S_h(0), I_h(0), R_h(0),$ and Lipschits condition:

$$\| \Alpha^\phi \mathcal{D}_{0^+}^\phi r_1(t) - \Alpha^\phi \mathcal{D}_{0^+}^\phi r_2(t) \| \leq \| r_1(t) - r_2(t) \|.$$
$R_0^R = R_0(0)$, $S_n^0 = S_n(0), L_n^0 = L_n(0), P_n^0 = P_n(0), R_n^0 = R_n(0)$.

Now, replacing the $\frac{d}{dt}$ in the system (7) with $\frac{\partial}{\partial \xi}$, we obtain the following system

\[
\begin{align*}
\frac{\partial}{\partial \xi} S_n &= \Lambda_m - \beta_n S_n I_n - \mu_n S_n, \\
\frac{\partial}{\partial \xi} L_n &= \beta_n S_n I_n - (\tau + \mu_n) I_n, \\
\frac{\partial}{\partial \xi} P_n &= \theta_n S_n I_n - (\tau_n + \mu_n) P_n, \\
\frac{\partial}{\partial \xi} R_n &= \tau_n I_n - \mu_n R_n.
\end{align*}
\] (8)

4. Equilibria and positivity invariant region

Here, we will study the positivity invariant region and steady states of the model (8).

Lemma 1. The closed set

\[
\gamma = \left\{ (S_n, L_n, I_n, R_n) \in \mathbb{R}^4_+ : S_n + L_n + I_n + R_n \leq \frac{\Lambda_m}{\mu_n} \right\}
\]

and

\[
\gamma_1 = \left\{ (S_n, L_n, I_n, R_n) \in \mathbb{R}^4_+ : S_n + L_n + I_n + R_n \leq \frac{\Lambda_m}{\mu_n} \right\}
\]

are positively invariant for the Coronavirus model (8) in $\mathbb{R}^4_+\xi$.

Proof 4.1. From the system, the total population of mammals and human respectively are

\[
\begin{align*}
N_n(t) &= S_n(t) + L_n(t) + R_n(t), \\
N_n(t) &= S_n(t) + L_n(t) + I_n(t) + R_n(t).
\end{align*}
\] (9)

Hence, from Corona dynamical model (8) we can write

\[
\begin{align*}
\frac{\partial}{\partial \xi} N_n &= \Lambda_m - \mu_n N_n(t), \\
\frac{\partial}{\partial \xi} N_n &= \Lambda_m - \mu_n N_n(t).
\end{align*}
\] (10)

Now, after utilizing the Laplace transform and simplifying we have

\[
\begin{align*}
\frac{\partial}{\partial \xi} N_n(t) &\leq \left( \frac{\phi}{\phi(1 - \phi)\mu_n} N_n(0) + \frac{(1 - \phi)\Lambda_m}{\phi(1 - \phi)\mu_n} \right) E_{\phi,1}( - \beta_n \phi^{\rho}) \\
+ \frac{\phi \Lambda_m}{\phi(1 - \phi)\mu_n} E_{\phi,1}( - \beta_n \phi^{\rho}) \\
\frac{\partial}{\partial \xi} N_n(t) &\leq \left( \frac{\phi}{\phi(1 - \phi)\mu_n} N_n(0) + \frac{(1 - \phi)\Lambda_m}{\phi(1 - \phi)\mu_n} \right) E_{\phi,1}( - \beta_n \phi^{\rho}) \\
+ \frac{\phi \Lambda_m}{\phi(1 - \phi)\mu_n} E_{\phi,1}( - \beta_n \phi^{\rho}).
\end{align*}
\] (11)

4.1. Steady states

The system (8) has disease free equilibrium (DFE) denoted by $P_0$ and is given by

\[
P_0 = \left( \frac{\Lambda_m}{\mu_n}, 0, 0, 0, \frac{\Lambda_m}{\mu_n}, 0, 0, 0 \right).
\] (15)

The matrices $F$ and $V$ for the next generation method (Ballanu et al., 2018), we have

\[
F = \begin{pmatrix}
0 & \frac{\beta_n \Lambda_m}{\mu_n} & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
-\theta_n & \tau_n + \mu_n & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\] (16)

Now, the basic reproduction number $R_0 = \rho(FV^{-1})$ is $R_0 = R_1 + R_2$, where $R_1 = \frac{\beta_n \Lambda_m}{\mu_n \gamma}$ and $R_2 = \frac{\beta_n \Lambda_m}{\mu_n \gamma}$ represent the individual reproduction of mammals and human respectively.

Now, the endemic equilibrium (EE) is represented by $P^* = (S_n^*, L_n^*, P_n^*, S_n^*, L_n^*, I_n^*, R_n^*)$ and is given by

\[
\begin{align*}
S_n^* &= \frac{\Lambda_m}{\mu_n \gamma} \\
L_n^* &= \frac{\tau_n + \mu_n \gamma}{\theta_n} \\
P_n^* &= \frac{\mu_n \gamma}{\beta_n \gamma (R_1 - 1)} \\
R_n^* &= \frac{\tau_n \gamma}{\mu_n \gamma} \\
S_n^* &= \frac{(\theta_n + \mu_n)(\omega + \mu_n + \tau_n)}{\theta_n (\beta_n \gamma + \theta_n \gamma)} \\
L_n^* &= \frac{(\omega + \mu_n + \tau_n)}{\theta_n} \\
P_n^* &= \frac{(\omega + \mu_n + \tau_n)}{\theta_n}
\end{align*}
\] (18)

4.2. Stability

Before moving towards the local stability we state a theorem

Theorem 4.1. Let $\phi < \frac{1}{\Lambda_m}$ and $\gamma < \frac{\phi}{\mu_n}$, where $\phi_1, \phi_2 \in \mathbb{R}$ such that $\langle \phi_1, \phi_2 \rangle = 1$, then $P_0$ is locally asymptotically stable (LAS) if for all roots, $|\arg(\phi)| < \frac{\phi}{\mu_n}$ for characteristic polynomial det(diag($\phi^\gamma \phi^\gamma \phi^\gamma \phi^\gamma \phi^\gamma \phi^\gamma \phi^\gamma \phi^\gamma$) $- J_P(t)) = 0$, where $\phi$ is for eigenvalues.

Theorem 4.2. If $R_1 < 1$ and $R_2 < 1$, then $P_0$ is locally asymptotically stable (LAS).

Proof 4.2. For the required result, the Jacobian matrix calculated at $P_0$ is
It’s corresponding characteristic equation is given by

\[(\varphi^k + \mu_k)(\varphi^k + \mu_h + \gamma)(\varphi^k + \mu_h)^3 (\varphi^k + \mu_h + \varphi)(\varphi^k + \mu_h + \varphi + \gamma) = 0\]

(20)

where the coefficients \( C_j \) for \( j = 1, 2, 3, 4 \) are given by

\[ C_1 = \omega + \theta_h + \tau_h + 2\mu_h + \theta_h + \tau_h + 2\mu_h, \]
\[ C_2 = (\omega + \theta_h + \tau_h + 2\mu_h)(\theta_h + \tau_h + 2\mu_h) + (\theta_h + \mu_h)(\mu_h + \tau_h)(1 - R_2), \]
\[ C_3 = (\omega + \theta_h + \tau_h + 2\mu_h)(\theta_h + \mu_h)(\mu_h + \tau_h)(1 - R_2), \]
\[ C_4 = (\theta_h + \mu_h)(\omega + \tau_h + \mu_h)(\theta_h + \mu_h)(\mu_h + \tau_h)(1 - R_2)(1 - R_2). \]

The arguments of the roots of \( \varphi^k + \mu_h = 0, \varphi^k + \mu_h + \gamma = 0 \) and \( (\varphi^k + \mu_h + \gamma)^2 = 0 \) are as follows:

\[ |\arg(q_i)| > \frac{\pi}{\theta_h} + k \frac{2\pi}{\varphi}, \quad \frac{\pi}{\theta_h} + k \frac{2\pi}{\varphi} > \frac{\pi}{2\pi}. \]

where \( k = 0, 1, 2, 3, \ldots, \phi - 1 \). Further, if \( R_1, R_2 < 1 \), then the arguments of the remaining roots also satisfy the said condition. Hence, \( P_0 \) is LAS if \( R_1 < 1 \) and \( R_2 < 1 \). For global stability of the coronavirus model (8) at \( P_0 \), we use the Castillo-Chavez et al. (2002) technique. We re-write the model (8):

\[ \chi' = K(\chi, \nu), \]
\[ \chi = M(\chi, \nu), M(\chi, 0) = 0. \]

(22)

where \( \chi(t) = (S_m, R_m, S_h, R_h) \in \mathbb{R}^4 \) and \( \nu(t) = (I_m, I_h, L_h, L_h) \in \mathbb{R}^4 \) represent the infected free and infected classes respectively. The DFE \( P_0 = (\chi^0, \nu^0) = (\chi^0, 0) \) is globally asymptotically stable (GAS) if the model (8) satisfies the following two conditions (C1) and (C2):

\[ \text{(C1) : For } \frac{d\chi(t)}{dt} = K(\chi, 0), \chi^0 \text{ is GAS,} \]
\[ \text{(C2) : For } M(\chi, \nu) = M_0\nu - M_1(\chi, \nu), \quad M_1(\chi, \nu) \geq 0 \text{ for all } (\chi, \nu) \in \mathcal{F}, \]

where \( M_0 = \Sigma\mu \mu^2, M_0(\mu^2, 0) \) is an M-matrix.

**Lemma 4.1.** The DFE \( P_0 = (\chi^0, \nu^0) = (\chi^0, 0) \) of the model (8) is GAS if \( R_0 < 1 \) and the above two conditions (C1), (C2) are satisfied.

**Theorem 4.3.** If \( R_0 < 1 \), then \( P_0 \) is GAS.

**Proof 4.3.** Let \( X = (S_m, R_m, S_h, R_h) \in \mathbb{R}^4 \) and \( \nu(t) = (I_m, I_h, L_h, L_h) \in \mathbb{R}^4 \) with \( P_0 \), we get

\[ \chi' = K(\chi, 0), \]
\[ \chi' = \begin{pmatrix} \Lambda_m - \mu_m S_m^0 \\ 0 \\ \Lambda_h - \mu_h S_h^0 \\ 0 \end{pmatrix}, \]

(23)

(24)

as \( t \to \infty \). \( \chi(t) \to \chi^0(t) \). So \( \chi(t) = \chi^0(t) = (S_m^0, R_m^0, S_h^0, R_h^0) \) is GAS.

Now, to ensure the second condition, let

\[ M_0 = \begin{pmatrix} -\mu_m & \beta_1 S_m^0 & 0 & 0 \\ \beta_1 S_m^0 & \beta_1 S_m^0 & 0 & 0 \\ 0 & \beta_1 S_h^0 & -\mu_h & 0 \\ 0 & 0 & \mu_h & -\tau_h \end{pmatrix}, \]
\[ \chi(t) = \begin{pmatrix} L_m \\ L_h \\ I_m \\ I_h \end{pmatrix} \] and \( M_1(\chi, \nu) = \begin{pmatrix} \beta_1(S_m^0 - S_m)I_m \\ 0 \\ \beta_1(S_h^0 - S_h)I_h \\ 0 \end{pmatrix}. \)

(25)

(26)

So, \( M(\chi, \nu) \) can be written as \( M_0\nu - M_1(\chi, \nu) \). Clearly \( M_1(\chi, \nu) \geq 0 \) as \( S_m^0 \) and \( S_h^0 \) are upper bounds. Also \( M_0 \) is an M-matrix. Hence, both the conditions are satisfied. Thus by lemma (4.1), \( P_0 \) is GAS. Due to the complex nature of the endemic stability analysis this study did not focus on the endemic stability.

5. Existence and uniqueness

Here, we use the fixed point theory. For this we re-write (8) as:

\[ \begin{cases} \frac{dS_m}{dt} = b(t, r(t)) \\ r(0) = r_0, \quad 0 < t < T < \infty \end{cases} \]

(27)

The vector \( r(t) = (S_m, L_m, R_m, S_h, L_h, I_m, L_h, R_h) \) represents the state variables and \( b \) is a continuous vector function such that:
with initial condition $r_0$. Moreover, the Lipschitz condition is satisfied by $b$ as $b$ is a quadratic vector function i.e. there exists $\mathcal{M}^0 \in \mathbb{R}$ such that:

$$
\| b(t, r_1(t)) - b(t, r_2(t)) \| \leq \mathcal{M}^0 \| r_1(t) - r_2(t) \| .
$$

(28)

The existence and uniqueness of the fractional proposed model (8) will be investigated by the following theorem.

**Theorem 5.1.** The fractional proposed model (8) has unique solution if below condition holds:

$$
(1 - \phi) \mathcal{M}^0 + \frac{\phi}{N(\phi)\Gamma(\phi)} T^\phi \mathcal{M}^0 \leq 1.
$$

(29)

**Proof.** We apply the definition (3) on (27), we get

$$
r(t) = r_0 + \frac{1 - \phi}{N(\phi)} b(t, r(t)) + \frac{\phi}{\mathcal{M}^0 N(\phi)\Gamma(\phi)} \int_0^t (t - \eta)^{\phi - 1} b(\eta, r(\eta)) d\eta.
$$

(30)

Let $K = (0, T)$ and the operator $B : \mathcal{C}(K, \mathbb{R}^3) \to \mathcal{C}(K, \mathbb{R}^3)$ such that

$$
B[r(t)] = r_0 + \frac{1 - \phi}{N(\phi)} b(t, r(t)) + \frac{\phi}{\mathcal{M}^0 N(\phi)\Gamma(\phi)} \int_0^t (t - \eta)^{\phi - 1} b(\eta, r(\eta)) d\eta.
$$

(31)

It gives

$$
r(t) = B[r(t)]
$$

(32)

Let $\| . \|_K$ denotes the supremum norm on $K$. Thus

$$
\| r(t) \|_K = \sup_{t \in K} \| r(t) \|,
$$

(33)

So $\mathcal{C}(K, \mathbb{R}^3)$ with $\| . \|_K$ is a Banach space. Moreover, the following relation holds:

$$
\int_0^t \mathcal{L}(t) r(t) \eta d\eta \leq T \| \mathcal{L}(t) \| \| r(t) \|_K,
$$

(34)

with $r(t) \in \mathcal{C}(K, \mathbb{R}^3)$, $\mathcal{L}(t, \eta) \in \mathcal{C}(K^2, \mathbb{R})$ such that

$$
\| \mathcal{L}(t, \eta) \|_K = \sup_{\tau \in K} \| \mathcal{L}(t, \eta) \|,
$$

(35)

Thus Eq. (31) can be written as:

$$
B[r_1(t)] - B[r_2(t)] \|_K \leq (1 - \phi) \mathcal{M}^0 \| b(t, r_1(t)) - b(t, r_2(t)) \| + \frac{\phi}{\mathcal{M}^0 N(\phi)\Gamma(\phi)} \int_0^t (t - \eta)^{\phi - 1} b(\eta, r_1(\eta) - r_2(\eta)) d\eta.
$$

(36)

$$
\leq \left( \frac{1 - \phi}{N(\phi)\Gamma(\phi)} T^\phi \mathcal{M}^0 \right) \| b(t, r_1(t)) - b(t, r_2(t)) \|,
$$

(37)

$$
\leq \left( \frac{1 - \phi}{N(\phi)\Gamma(\phi)} \mathcal{M}^0 + \frac{\phi}{\mathcal{M}^0 N(\phi)\Gamma(\phi)} T^\phi \mathcal{M}^0 \right) \| r_1(t) - r_2(t) \|.
$$

(38)

Finally, we obtain

$$
\| B[r_1(t)] - B[r_2(t)] \|_K \leq \mathcal{L} \| r_1(t) - r_2(t) \|_K.
$$

(40)

where

$$
\mathcal{L} = \frac{(1 - \phi)\mathcal{M}^0}{N(\phi)\Gamma(\phi)} T^\phi + \frac{\phi}{N(\phi)\Gamma(\phi)} \mathcal{M}^0 T^\phi.
$$

If $\mathcal{L} < 1$, then the operator $B$ is called a contraction. Hence, the fractional system (8) possesses a unique solution.

6. Fractional optimal control problem formulation

Here, we formulate an optimal control problem similar to Sweilam et al. (2019), Bonyah (2020) for the corona model (8) by incorporating three different time dependent control functions $u_i(t)$ for $i = 1, 2, 3$. The control $u_1(t)$ is designed to bring about effective social distancing between infected mammals and susceptible mammals. The control $u_2(t)$ is also preventive control purposely for public education aiming at ensuring social distancing in all human activities. The control $u_3(t)$ which is treatment control is designed to ensure drug efficacy and lies between $0 \leq u_3(t) \leq 1$. These drugs are those ones that have proven to improve immunity and reduce the associated burdens of COVID-19. Therefore, the following system of equation is obtained

$$
\begin{aligned}
&\frac{d\phi}{dt} = \phi \eta S_0 - \beta_1 S_0 I_0 - \mu \phi S_0,
&\frac{\phi}{\mathcal{M}^0} \frac{\phi}{\mathcal{M}^0} R_0 - \beta_1 S_0 I_0 - \mu \phi S_0,
&\phi \eta S_0 - \beta_1 S_0 I_0 - \mu \phi S_0,
&\phi \eta S_0 - \beta_1 S_0 I_0 - \mu \phi S_0,
&\phi \eta S_0 - \beta_1 S_0 I_0 - \mu \phi S_0,
&\phi \eta S_0 - \beta_1 S_0 I_0 - \mu \phi S_0,
&\phi \eta S_0 - \beta_1 S_0 I_0 - \mu \phi S_0.
\end{aligned}
$$

(41)

To define the frac...
\[ J = \int_0^{T_{nu}} \left\{ \mathcal{H}_i(S_u, L_u, R_u, \delta C, L_u, R_u, u_1, u_2, u_3, t) \right\} dt. \tag{45} \]

Thus for FOCP the Hamiltonian (\(\mathcal{H}_i\)) is as follows:

\[ \mathcal{H}_i(r(t), u_1, u_2, u_3, t) = \Theta(r(t), u_1, u_2, u_3, t) + \int r(t, u_1, u_2, u_3, t) dt. \tag{47} \]

In the above equation \(r(t) = (S_m, L_m, I_m, R_m, S_l, I_l, R_l, \delta C)\). Now, the following are essential and sufficient conditions for formulating the FOCP (Bonyah, 2020):

\[ f^T \partial \mathcal{H}_i \delta \delta \delta = \frac{\partial f^T}{\partial c} \frac{\partial \mathcal{H}_i}{\partial u} \delta \delta \delta = \frac{\partial f^T}{\partial \delta} \frac{\partial \mathcal{H}_i}{\partial u} \delta \delta \delta = \frac{\partial f^T}{\partial \Lambda} \frac{\partial \mathcal{H}_i}{\partial u} \delta \delta \delta = \frac{\partial f^T}{\partial \Lambda} \frac{\partial \mathcal{H}_i}{\partial u} \delta \delta \delta = 0. \tag{50} \]

Moreover, with \(\delta_i(T_{nu}) = 0\), for \(i = 1, 2, \ldots, 8\) establishes the Lagrange multiplier equations. The equations (48) and (50) provide the necessary conditions for FOCP in terms of Hamiltonian. The following theorem states:

**Theorem 6.1.** The optimal controls \(u_1', u_2', u_3'\) and the solutions \(S_m, L_m, R_m, S_l, I_l, R_l\) and \(\delta C\) of the corresponding model (41) that minimizes \(J(u_1, u_2, u_3)\) over \(t\). Then there exist adjoint state variables \(\delta_i\) for \(i = 1, 2, \ldots, 8\) satisfying the three results:

1. **Equations of adjoint state variables**

\[ \frac{\partial}{\partial c} \frac{\partial \mathcal{H}_i}{\partial u_1} \delta \delta \delta = \ldots = (1 - u_1)\beta_3 L_u(\delta_{I_u} - \delta_{L_u}) - \mu_u \delta_{I_u} \tag{53} \]

\[ \frac{\partial}{\partial c} \frac{\partial \mathcal{H}_i}{\partial u_2} \delta \delta \delta = 1 + \delta_u(\delta_{I_u} - \delta_{L_u}) - \mu_u \delta_u \tag{54} \]

\[ \frac{\partial}{\partial c} \frac{\partial \mathcal{H}_i}{\partial u_3} \delta \delta \delta = 1 + (1 - u_3)\beta_3 L_u(\delta_{I_u} - \delta_{L_u}) + \tau_u(\delta_{L_u} - \delta_{I_u}) - \mu_u \delta_u \tag{55} \]

\[ \frac{\partial}{\partial c} \frac{\partial \mathcal{H}_i}{\partial u_4} \delta \delta \delta = -\mu_u \delta_u \tag{57} \]

\[ \frac{\partial}{\partial c} \frac{\partial \mathcal{H}_i}{\partial u_5} \delta \delta \delta = (1 - u_2)\beta_3 L_u(\delta_{I_u} - \delta_{L_u}) + \tau_u(\delta_{L_u} - \delta_{I_u}) - \mu_u \delta_u \tag{56} \]

\[ \frac{\partial}{\partial c} \frac{\partial \mathcal{H}_i}{\partial u_6} \delta \delta \delta = \ldots = (1 - u_2)\beta_3 L_u(\delta_{I_u} - \delta_{L_u}) + \tau_u(\delta_{L_u} - \delta_{I_u}) - \mu_u \delta_u \tag{58} \]

\[ \frac{\partial}{\partial c} \frac{\partial \mathcal{H}_i}{\partial u_7} \delta \delta \delta = 1 + \delta_u(\delta_{I_u} - \delta_{L_u}) - \mu_u \delta_u \tag{59} \]

\[ \frac{\partial}{\partial c} \frac{\partial \mathcal{H}_i}{\partial u_8} \delta \delta \delta = (1 - u_1)\beta_3 L_u(\delta_{I_u} - \delta_{L_u}) + u_3 \tau_u(\delta_{L_u} - \delta_{I_u}) - (\mu_u + \omega) \delta_u \tag{60} \]

\[ \frac{\partial}{\partial c} \frac{\partial \mathcal{H}_i}{\partial u_9} \delta \delta \delta = \ldots = \delta_u \tag{61} \]

2. **With transversality conditions**

\[ \delta_i(T_{nu}) = 0, \text{ for } j = 1, 2, \ldots, 8 \tag{62} \]

3. **Moreover, we state the optimality condition for FOCP as follows**:

\[ \mathcal{H}_i(S_u, L_u, I_u, R_u, \delta C, L_u, R_u, u_1', u_2', u_3', \delta) \]

\[ = \min_{u_1', u_2', u_3'} \mathcal{H}_i(S_u, L_u, I_u, R_u, \delta C, L_u, R_u, u_1', u_2', u_3', \delta) \tag{63} \]

\[ u_1' = \min\left( 1, \max\left( 0, \frac{\beta_3 S_u I_u(\delta_{I_u} - \delta_{L_u})}{L_u} \right) \right) \tag{65} \]

\[ u_2' = \min\left( 1, \max\left( 0, \frac{\beta_3 L_u(\delta_{L_u} - \delta_{I_u})}{L_u} \right) \right) \tag{66} \]

\[ u_3' = \min\left( 1, \max\left( 0, \frac{\tau_u(\delta_{L_u} - \delta_{I_u})}{L_u} \right) \right) \tag{67} \]

**Proof 6.1.** We obtain the Hamiltonian function by using Eq. (47).

\[ \mathcal{H}_i(t) = L_u + I_u + \frac{1}{2} \left( L_u^2 + I_u^2 + \lambda_u \delta_u^2 \right) \tag{68} \]

\[ + \delta_{I_u} \frac{\partial}{\partial c} \frac{\partial \mathcal{H}_i}{\partial u_1} \delta \delta \delta + \delta_{I_u} \frac{\partial}{\partial c} \frac{\partial \mathcal{H}_i}{\partial u_2} \delta \delta \delta + \delta_{I_u} \frac{\partial}{\partial c} \frac{\partial \mathcal{H}_i}{\partial u_3} \delta \delta \delta + \delta_{I_u} \frac{\partial}{\partial c} \frac{\partial \mathcal{H}_i}{\partial u_4} \delta \delta \delta \tag{69} \]

\[ + \delta_{L_u} \frac{\partial}{\partial c} \frac{\partial \mathcal{H}_i}{\partial u_5} \delta \delta \delta + \delta_{L_u} \frac{\partial}{\partial c} \frac{\partial \mathcal{H}_i}{\partial u_6} \delta \delta \delta + \delta_{L_u} \frac{\partial}{\partial c} \frac{\partial \mathcal{H}_i}{\partial u_7} \delta \delta \delta + \delta_{L_u} \frac{\partial}{\partial c} \frac{\partial \mathcal{H}_i}{\partial u_8} \delta \delta \delta \tag{70} \]

constitutes the Hamiltonian. Furthermore, the condition \(\delta_i(T_{nu}) = 0, j = 1, 2, 3, \ldots, 8\) also exists, and the optimal control parameterization in Eq. (65) can be determined from Eq. (50). Making a Substitution of \(u'_i, i = 1, 2, 3 \text{ in } (8), \) we can arrive at the following state system:

\[ \frac{\partial}{\partial c} \frac{\partial \mathcal{H}_i}{\partial u_1} \delta \delta \delta = \ldots = \delta_u \tag{71} \]

7. **Scheme for FOCP**

For a general initial value problem (Sweilam et al., 2019)

\[ \frac{\partial}{\partial c} \frac{\partial \mathcal{H}_i}{\partial u} \delta \delta \delta = \ldots = \delta_u \tag{72} \]

\[ e(0) = e_0 \tag{73} \]

With the help of fundamental theorem of fractional calculus to Eq. (72), we have

\[ e(t) - e(0) = \frac{1 - \phi}{\Gamma(\phi)} r(t, e(t)) + \frac{\phi}{\Gamma(\phi)} \int_0^t r(\eta, e(\eta))(t - \eta)^{\phi-1} d\eta. \tag{74} \]

With normalization function \(R(\phi) = 1 - \phi + \frac{\phi}{\Gamma(\phi)}\) at a \(t_{n+1}\). After discretization, we have
\[ e_{n+1}(t) - e(0) = \frac{\Gamma(\phi)(1 - \phi)}{\Gamma(\phi)(1 - \phi) + \phi} \sum_{m=0}^{\infty} \frac{e}{\Gamma(\phi)(1 - \phi) + \phi} \times \int_{t_n}^{t_{n+1}} r(t_{n+1} - \eta)^{\phi - 1} d\eta. \]  

(75)

Now, approximating \( r(\eta, e(\eta)) \) by two-step Lagrange interpolation (Bonyah, 2020; Solís-Pérez et al., 2018)

\[ r(\eta, e(\eta)) \approx \frac{r(t_m, e_m)(\eta - t_{m-1})}{h} + \frac{r(t_{m-1}, e_{m-1})(\eta - t_m)}{h} \]  

(76)

Now, we get

\[ e_{n+1}(t) - e(0) = \frac{\Gamma(\phi)(1 - \phi)}{\Gamma(\phi)(1 - \phi) + \phi} \sum_{m=0}^{\infty} \frac{1}{\Gamma(\phi)(1 - \phi) + \phi} \times \left\{ h^\phi r(t_m, e_m) \{(n + m^2 + \phi) - (n - m^2 + 2\phi) \} - h^\phi r(t_{m-1}, e_{m-1}) \{(n + 2\phi) - (n - m^2 + 1\phi) \} \right\} \]  

(78)

Fig. 1. Simulation results of the two preventive controls \((u_1, u_2)\) at \( \phi = 0.75 \).
To get high stability, we incorporate a simple modification (Patidar, 2016; Sweilam et al., 2019) such that replacing $h$ (step size) by $\phi(h)$ with $\phi(h) = h + O(h^2); 0 < \phi(h) \leq 1$. This new scheme is a nonstandard one characterized by unconditional stability and details can be established in Sweilam et al. (2020), Patidar (2016) and we obtain the following scheme

$$e_{n+1}(t) - e(0) = \frac{\Gamma(\phi)(\phi)}{\Gamma(\phi)(1 - \phi) + \phi} r(t_n, e_n)$$

$$+ \frac{1}{(1 + \phi)(\Gamma(\phi)(1 - \phi) + \phi)} \sum_{m=0}^{n} \times$$

(80)

The new scheme is therefore utilized in Eq. (80) to obtain numerical solution to the state system in Eq. (71). Further, we make use of the implicit finite difference method in order to derive the solution of the co-state system Eqs. (61) together with the transversality conditions in Eq. (62).

![Simulation results of the preventive control $u_1$ and treatment control $u_3$ at $\phi = 0.75$.](Fig. 2)
8. Numerical simulations

This section is devoted to a modified version of the numerical scheme Sweilam et al. (2019) which Eq. (80) is used to obtain numerical simulation result of the fractional-order optimal system in Eqs. (41) and (61) with the transversality condition in Eq. (61) using the parameter values and initial conditions given in (Chen et al. (2020b)). For purpose of illustrations the following parameter values were utilized as in (Chen et al. (2020b)) \( \Lambda_m = 600, \Lambda_h = 9000, \beta_1 = 0.009, \mu_m = 0.000474, \theta_m = 0.1, \tau_m = 0.9, \beta_2 = 0.009, \beta_3 = 0.009, \gamma = 0.07, \mu_h = 0.0009, \theta_h = 0.9, \omega = 0.8, \tau_h = 0.5 \).

8.1. Prevention \((u_1)\) and treatment \((u_2)\) only

The control \(u_1\) and \(u_2\) are preventive controls designed to ensure effective social distancing among the infective classes while treatment control is kept zero throughout in this strategy. In Fig. 1(a) there is vast difference between controlled case and without application of control. However, the Latent class increases at the end of the intervention. For Fig. 1(b)–(d) there is a vast significant difference between applied control and without control and the number of cases in each class is controlled at the end of the intervention. The Fig. 1(e) is the control profile for this strategy where treatment control \(u_3\) is kept at zero.
throughout and control $u_1$ is fully utilized (100%) for 118 days and reduced for the rest of the intervention. The control $u_2$ is kept at 23% initially and rise to 40% within 50 days and is finally reduced for the rest of the intervention. It can be inferred that this control is very effective in controlling the spread of the coronavirus.

8.2. Prevention ($u_1$) and treatment ($u_3$) only

The control strategy employed in this regard is the utilization of $u_1$ and control $u_3$ while control $u_2$ is set at zero in the entire intervention. It can be observed in the Fig. 2(a)–(b) that the control strategy is effective in reducing the number of Latent and Infected mammals in contact with susceptible mammals in the communities. However, these number of individuals in these classes increase at the end of the intervention. Fig. 2(c)–(d) indicate that the control strategy has no effect on reducing the number of Latent and Infected human individuals in these classes. The crossover property of the ABC operator enhances the quality level of prediction. Fig. 2(e) is the control profile where control $u_2$ is set at zero while control $u_1$ is kept initially at 100% for 68 days and it gradually reduces towards the end of the intervention. The control $u_3$ is initiated at 100% for 63 days and gradually reduces as the strategy gets closer to the end of the intervention.

Fig. 4. Simulation results of the preventive controls $u_1$, $u_2$ and treatment $u_3$ at $\phi = 0.75$. 
8.3. Preventive ($u_2$) and treatment ($u_3$) only

The social distancing control $u_2$ and treatment control $u_3$ are made use of while $u_1$ is set at zero. It is clearly seen in Fig. 3(a)–(b) that the control strategy designed for this enterprise did not have any positive impact on the dynamics of mammals in spreading the disease among themselves. However, different situation can be observed in Fig. 3(c)–(d) where the control strategy has had a positive effect on controlling the spread of the disease amongst human beings. It can be inferred that the ABC crossover property allows the operator to convert from one operator to another during prediction. Fig. 3(e) is the control profile used for this strategy where control $u_1$ is set at zero but control $u_2$ is initially set at 100% for 18 days and is reduced to 25% and maintained same throughout the intervention period. Control $u_3$ is set at 4% and gradually reduces during the rest of the intervention.

8.4. Prevention, treatment and treatment ($u_1$, $u_2$, $u_3$)

In this strategy, the entire controls $u_1$, $u_2$, $u_3$ strategies are simultaneously in motion with the aim of optimizing the objective function. In Fig. 4(a)–(d) one can observed that there are vast significance difference between the application of control and without control strategy. It can be seen that these controls strategies still become effective after the intervention period of three months. It can be suggested that the ABC operator crossover property has positive influence on the quality of prediction. Fig. 4(e) is the control profile for this enterprise where control $u_1$ is kept at 10% for the initial day and gradually reduced during the rest of the intervention while control $u_2$ is set at full 100% for 18 days and reduced to 27% which is kept same throughout the rest of the intervention. The control $u_2$ is began at 4% initially which gradually reduced for the rest of the intervention.

9. Conclusions

This work had been analyzed in the context of fractional optimal control approach with Mitta-Leffler Law. The basic properties and the steady states of the coronavirus model were examined. The reproduction number of the model was calculated. The existence and uniqueness of solutions were investigated using Banach fixed point theorem. A fractional numerical scheme for the coronavirus model in ABC sense was constructed. The numerical simulation results indicated that the two preventive controls were effective in controlling the spread of the disease. However, it was further established that the combination of all the three controls ensured that the disease was reduced even after the intervention period. Thus, it is suggested that researchers can apply fractional control in ABC sense in obtaining the best strategy in managing complex models.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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