CLOSE PAIRS AS PROXIES FOR GALAXY CLUSTER MERGERS

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ABSTRACT

Galaxy cluster merger statistics are an important component in understanding the formation of large-scale structure. Cluster mergers are also potential sources of systematic error in the mass calibration of upcoming cluster surveys. Unfortunately, it is difficult to study merger properties and evolution directly because the identification of cluster mergers in observations is problematic. We use large N-body simulations to study the statistical properties of massive halo mergers, specifically investigating the utility of close halo pairs as proxies for mergers. We examine the relationship between pairs and mergers for a wide range of merger timescales, halo masses, and redshifts (0 < z < 1). We also quantify the utility of pairs in measuring merger bias. While pairs at very small separations will reliably merge, these constitute a small fraction of the total merger population. Thus, pairs do not provide a reliable direct proxy to the total merger population. We do find an intriguing universality in the relation between close pairs and mergers, which in principle could allow for an estimate of the statistical merger rate from the pair fraction within a scaled separation, but including the effects of redshift space distortions strongly degrades this relation. We find similar behavior for galaxy-mass halos, making our results applicable to field galaxy mergers at high redshift. We investigate how the halo merger rate can be statistically described by the halo mass function via the merger kernel (coagulation), finding an interesting environmental dependence of merging: halos within the mass resolution of our simulations merge less efficiently in overdense environments. Specifically, halo pairs with separations less than a few $h^{-1}$ Mpc are more likely to merge in underdense environments; at larger separations, pairs are more likely to merge in overdense environments.

Subject headings: cosmology: theory — dark matter — galaxies: clusters: general — methods: numerical

Online material: color figures

1. INTRODUCTION

Galaxy clusters are of great interest in cosmology, as they are the largest and most recently formed structures in the cosmological hierarchy. The clustering and number density evolution of clusters are sensitive to both the growth function and the expansion history of the universe. Clusters contain a representative sample of baryons and dark matter and are thus also fascinating laboratories in which to study the influence of baryonic physics on the formation of large-scale structure, especially the shape, gravitational potential, and substructure of dark matter halos. Statistical measures such as the mass function, the rate of structure growth, and the clustering of large structures are fundamental predictions of cosmological models. Cluster observations are therefore expected to provide some of the most important constraints on fundamental cosmology and astrophysics (see, e.g., Borgani 2006 for a recent review).

Cluster formation histories are frequently punctuated by large jumps in mass from major mergers (e.g., Cohn & White 2005). These mergers are one of the primary mechanisms for the buildup of mass in clusters and superclusters. In the standard paradigm (Kaiser 1984; Efstathiou et al. 1988; Cole & Kaiser 1989; Mo & White 1996; Sheth & Tormen 1999), observable properties such as the degree of spatial clustering depend only on the cluster mass. However, recent theoretical studies indicate that many cluster observables, such as spatial clustering, concentration, galaxy velocity dispersion, gas distribution and its attendant observables such as X-ray emissions, and Sunyaev-Zel’dovich decrement, depend on the cluster’s formation time, mass accretion history, and large-scale environment (collectively referred to as “assembly bias”; Wechsler et al. 2002, 2006; Zhao et al. 2003; Sheth & Tormen 2004; Gao et al. 2005; Croton et al. 2007; Harker et al. 2006; Wetzel et al. 2007; Gao & White 2007; Wang et al. 2007; Jing et al. 2007; Hahn et al. 2007; Macciò et al. 2007; Sandvik et al. 2007). In addition, there is a dependence on recent merger history (“merger bias”; Scannapieco & Thacker 2003; Rowley et al. 2004; Furlanetto & Kamionkowski 2006, hereafter FK06; Wetzel et al. 2007; Poole et al. 2007; Jeltema et al. 2008; Hartley et al. 2008). And, recent studies have claimed observational detection of assembly bias, although with mixed results (Berlind et al. 2006; Yang et al. 2006). Theory can predict cluster properties as a function of their mass, which is dominated by dark matter and thus cannot be directly measured. Since all methods of observing clusters are sensitive to the effects of assembly/merger bias, it is necessary to develop a more detailed understanding of the mechanisms of structure formation. Specifying a correlation function and mass function, and the evolution of these quantities, may not be sufficient to connect theory to observation. Understanding cluster merger properties is therefore crucial for using these objects as probes of cosmology.

We focus primarily on galaxy cluster mergers. Wetzel et al. (2007) used a large simulation volume to probe high-mass halos with good statistics. They found that halos of mass $M > 5 \times 10^{13} h^{-1} M_{\odot}$ that have undergone a recent (within 1 Gyr or less) major merger or large mass gain exhibit an enhancement in their spatial clustering of up to $\sim 10\%$–$20\%$ on scales of $5$–$25 h^{-1}$ Mpc compared with the entire halo population at the same mass. They noted that this merger bias persists for the redshift range $0 < z < 1$, and that the bias increases with larger merger mass

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discuss the prospects for studying merger bias using close cluster pairs. We conclude in § 7.

2. SIMULATIONS

Our study is conducted using two high-resolution $N$-body simulations of cold dark matter in a flat $Λ$CDM cosmology with parameter values $Ω_m = 0.3$, $Ω_b = 0.046$, $h = 0.7$, $n = 1$, and $σ_8 = 0.9$. Our simulations employ the HOT code (Warren & Salmon 1993) in a $(1.1 h^{-1} $Gpc)$^3$ and a $(2.2 h^{-1} $Gpc)$^3$ volume with periodic boundary conditions, using a Plummer softening length of $35 h^{-1} $kpc. Gaussian initial conditions were randomly generated for the 1024$^3$ particles of mass $10^7 h^{-1} M_\odot$ (smaller simulation) and $8 \times 10^{11} h^{-1} M_\odot$ (larger simulation) at an initial redshift of $z = 34$. Simulation outputs were stored in intervals of 1 Gyr between redshifts $z \approx 1$ and 0, with the last interval of each simulation being shorter: 0.6 Gyr (smaller simulation) and 0.5 Gyr (larger simulation). All time intervals cited below represent the total time elapsed between two simulation outputs. Thus, a merger timescale of $\Delta t = 1$ Gyr indicates that two separate halos have merged within 1 Gyr, an upper limit to the actual time to merger.

We generate a halo catalog for each output using the friends-of-friends (FoF) algorithm (Davis et al. 1985) with a linking length of $b = 0.15$ of the mean interparticle spacing. These groups correspond to a density threshold of $\sim 3/(2\pi b^3)$ and enclose primarily virialized material. In this work we quote FoF masses, which are about 20% smaller than “virial” masses (corresponding roughly to FoF masses with $b = 0.2$, which are more commonly found in the literature; see White [2001] for more details). Since we are examining mergers, we use a smaller linking length to decrease contamination that arises from close neighboring halos being bridged by a narrow string of particles (although using a larger linking length of $b = 0.2$ changes our results by only a few percent; see Fig. 4, discussed below). The halo catalogs of the smaller simulation include all halos of mass greater than $M \approx 5 \times 10^{12} h^{-1} M_\odot$ (>50 particles), although in our study we are concerned primarily with halos of mass $M > 5 \times 10^{13} h^{-1} M_\odot$, of which there are around 75,000 $h^{-3} $Gpc$^{-3}$ at $z = 0$. Our larger ($2.2 h^{-1} $Gpc)$^3$ but less resolved simulation catalogs include all halos of mass greater than $M \approx 4 \times 10^{13} h^{-1} M_\odot$ (>50 particles). We do not consider substructure within host halos.

Merger trees were constructed from the set of halo catalogs by specifying a parent-child relationship, where a “parent” is any halo that contributed mass to a halo at a later time, i.e., a “child.” We define a parent contributing more than half of its mass as a “progenitor.” Under this restriction, a progenitor can never have more than one child. We define a “merger” as a child having more than one progenitor. In cases where we select on a child that had more than two progenitors, we apply the two-body approximation by considering only the two progenitors that contributed the most mass to the child. Using the two most massive progenitors, as opposed to the two most mass-contributing progenitors, changes the progenitor identification in less than 1% of all mergers that we consider. In addition to this progenitor merger tree, a list of all contributing parents (not just progenitors) is stored for each child. This flexible storage of parent data allows us to study two-body mergers, mergers with more than two progenitors, or mass accretion including all parents. However, when considering short timescales ($\leq 1$ Gyr), the two-body criterion is a good approximation (see Wetzel et al. 2007 for more details).

All errors cited are $1 \sigma$ errors derived from dividing the simulations into eight octants and computing the dispersion of the quantity of interest within each octant. Since we probe scales
much smaller than the octants, we treat them as uncorrelated volumes.

3. CLOSE PAIRS AS PREDICTORS OF MERGERS

To examine whether close pairs of galaxy clusters form reliable predictors of mergers, we first extend the work of Wetzel et al. (2007) on halo mergers, using the same simulation and halo catalogs (described in §2). We identify child halos of mass $M > 5 \times 10^{13} \, h^{-1} \, M_{\odot}$ that are products of recent mergers with progenitor masses $M_1$ and $M_2$, where $M_2/M_1 > 0.2$ (major mergers). We explore the distribution of progenitor separations in Figure 1, which shows the number density of progenitors as a function of binned progenitor separation. For time intervals of 1 Gyr, there is a characteristic comoving progenitor separation ($\sim 1.5 \, h^{-1} \, \text{Mpc}$) that does not evolve significantly with redshift. Without the influence of gravitational attraction, halos with typical velocities of $\sim 1000 \, \text{km s}^{-1}$ will travel 1 Mpc within 1 Gyr. Furthermore, the number density of mergers evolves only weakly with redshift. These factors suggest that close pairs might be a reasonable proxy for mergers.

These results represent a postdiction, where we know in advance which halos will merge. We now investigate whether close pairs of objects can reliably be used to predict mergers. The following subsections investigate how frequently pairs will merge as a function of pair mass, (comoving) separation, redshift, and merger time interval. We consider a given pair to have merged if both halos are progenitors of the same child at a later time step. Since observations of galaxy clusters are generally limited by a threshold luminosity, we identify pairs of halos whose individual halo masses are above a given mass cut. We first examine the pair-merger hypothesis (that close pairs are merger proxies) for the best-case scenario in which halo positions and masses are known with complete accuracy (as in our simulations). This will provide a firm upper limit to the utility of pairs in predicting mergers. Then, in §4 we consider observational complexities such as scatter in the halo mass, redshift space distortions, and redshift space errors, which significantly degrade the signal.

3.1. Pair Mergers at z = 0

We select halos at $z = 0.04$ and consider mergers to $z = 0$ ($\Delta t = 0.6 \, \text{Gyr}$). Figure 2 (top) shows the number density of pairs as a function of pair separation, $n_p(r)$, along with the number densities of pairs that merged (solid lines), $n_m(r)$, and did not merge (dashed lines) within the time interval. The number density of pairs terminates at small separations because of halo exclusion, i.e., massive halos have finite radii. The upper set of curves indicates halos of mass $M > 5 \times 10^{13} \, h^{-1} \, M_{\odot}$, and the lower curves have $M > 10^{14} \, h^{-1} \, M_{\odot}$; the results are qualitatively similar. There is a limited range of separations larger than the halo exclusion limit in which the majority of pairs merge. Also plotted (middle) are the same number densities, but as an integrated function of separation, demonstrating the relationship for pairs within the given separation $r$. In 0.6 Gyr at $z = 0$, massive halo pairs will only merge with certainty for separations $\lesssim 2 \, h^{-1} \, \text{Mpc}$.

Figure 2 (bottom) also shows the fraction of pairs that merge within the given separation, $n_m(<r)/n_p(<r)$ (falling curves). This fraction is the “efficiency” of the close-pair method in identifying merger candidates, since it shows the likelihood that a pair within the given separation will merge. It can also be thought of as a measure of contamination of the candidate sample, because its
difference from 1 identifies the fraction of pairs that do not merge. The rising curve shows the fraction of mergers over the total number density of mergers, \( n_m(<r)/n_m,\text{tot} \). This can be interpreted as the merger “completeness,” as it shows the fraction of the total number of mergers found by identifying pairs within the given separation. Figure 2 confirms the intuitive result that because of greater accelerations, larger halos are able to merge from larger separations, which holds at all redshifts and time intervals, as will be shown below.

If close pairs were ideal predictors of mergers, the rising and falling curves of Figure 2 (bottom) would be steep functions, crossing near a fraction (y-value) of 1 and demonstrating a clear dichotomy between those pairs that merge and those that do not. However, because of dynamical effects, halos do not simply accelerate toward one another from rest; they merge with neighbors from a broad distribution of separations. Indeed, the effects of the local environment can hinder the merging of close pairs that would ordinarily occur via nonlinear two-body gravitational interaction (see § 5 for more details).

The intersection of the efficiency and completeness curves represents an easily identifiable pair separation that compromises between maximizing completeness and minimizing contamination. At this intersection the fraction of pairs that merge, \( n_m(<r)/n_p(<r) \), equals the fraction of total mergers, \( n_m(<r)/n_m,\text{tot} \), although the relevant populations are different. If this intersection occurs at a fraction close to 1, most pairs within the separation will merge, and those pairs will represent the majority of all mergers that occur in the time interval. However, if this fraction is low, pairs within the separation are not a representative indicator of mergers. Since the intersection of these two curves occurs at a fraction of ~0.5 for massive pair mergers within \( \Delta t = 0.6 \) Gyr at \( z = 0 \), we conclude that pairs at these redshifts and masses cannot be used to reliably predict mergers.

3.2. Redshift Dependence of Pair Mergers

We next examine whether the pair-merger hypothesis fares better at high redshift. We consider a longer merger timescale of \( \Delta t = 1 \) Gyr for massive halo pairs at \( z \approx 1 \), 0.6, and 0.1. Figure 3 (top) shows the pair merger fractions as a function of (comoving) separations. The pair merger efficiency (falling curves) increases significantly with redshift, while the merger completeness (rising curves) remains approximately invariant. For a given merger timescale, mergers come from pairs of essentially the same (comoving) separation, regardless of redshift (see Fig. 1). However, since gravitational interactions are governed by physical (not comoving) separations, Figure 3 (bottom) shows the same merger fractions as a function of physical separation. The merger efficiency trend is reversed: pairs within a given physical separation are more likely to merge at low redshift, and mergers come from pairs of much larger physical separations at low redshift. The intersection of the efficiency and completeness curves occurs at a higher fraction at high redshift than at low redshift (75%/vs. 60%), indicating that, for a fixed merger timescale, massive halo pairs provide a slightly better proxy for merger rates at higher redshift.

3.3. Mass and Linking Length Dependence of Pair Mergers

While it appears that cluster-mass halo pairs might provide a reasonable proxy for mergers at \( z \approx 1 \), we next examine whether these results are robust as a function of halo mass. Figure 4 (top left) shows the pair merger fractions for halo masses down to \( M > 5 \times 10^{12} h^{-1} M_\odot \) at \( z \approx 1 \), where the pair separation is shown as a multiple of the minimum mass cut virial radius \( r_{500} \). Figure 4 (top right) shows the same but for higher halo masses and a shorter time interval at \( z \approx 0 \). While pairs of higher mass halos merge from larger physical pair separations, when scaled by the halo virial radius we find that the pair merger efficiency and completeness exhibit a nearly universal relation as a function of halo mass. This implies that any of our results in § 3 for a given mass can be approximately scaled to those of another mass.

This universal merger relation also implies that the total merger fraction can be estimated by noting that the following relation is satisfied at the crossing point of the curves: 

\[
\frac{[n_m(<r)/n_p(<r)]}{n_m(<r)/n_m,\text{tot}} = n_m,\text{tot}/n_p(<r) = 1.0.
\]

For example, in Figure 4 (middle) we find that \( n_m,\text{tot} = n_p(<r) \) when evaluated at a scaled pair separation of \( S_r/v_{\text{vir}} \). One can thus estimate the merger rate by counting pairs interior to \( S_r/v_{\text{vir}} \). This is a potentially powerful result, since it is approximately invariant for all mass scales.\(^4\)

\(^4\) We thank the referee for pointing out this universal behavior.

\[\text{Fig. 3.—Fraction of pairs separated by less than } r \text{ that merged (falling curves), indicating efficiency, and the ratio of identified mergers to all mergers above the mass cut within } \Delta t = 1 \text{ Gyr (rising curves), indicating completeness, for various redshifts. The total number density of such mergers is } 458 h^{-3} \text{ Gpc}^{-3} (z = 0.97 - 0.75), 950 h^{-3} \text{ Gpc}^{-3} (z = 0.58 - 0.44), \text{ and } 1464 h^{-3} \text{ Gpc}^{-3} (z = 0.12 - 0.04). \text{ Note the approximate invariance of the total merger fraction (rising curves) with redshift. Bottom: Same, but as a function of physical separation. While pairs of a given comoving separation merge more frequently at high redshift, this trend reverses when we consider physical separations. [See the electronic edition of the Journal for a color version of this figure.]}\]
space distortions will significantly undermine the utility of these results (see § 4).

The only strong deviation from this nearly universal mass relation occurs for merger completeness of $5 \times 10^{12} \, h^{-1} M_\odot$ halos (Fig. 4, top left), which results in an intersection of merger efficiency and completeness at a lower fraction (10%-15% decrease) for lower mass halos, a trend which we find does not depend strongly on redshift or merger timescale. The use of pairs as proxies for merger rates becomes increasingly unreliable as we approach massive galaxy mergers at high redshift, where galaxies are found primarily in distinct host halos.

These results augment those of Berrier et al. (2006), who found that the evolution of close galaxy pairs cannot be used to measure the host halo merger rate. Specifically, they found that the host halo merger rate evolves as $(1+z)^7$, while the number of close galaxy pairs evolves little with redshift. This arises because, at low redshift, the merger rate of host halos is low, but there are multiple galaxy pairs merging within massive host halos. At high redshift, the host halo merger rate is high, but the number density of halos massive enough to host more than one bright galaxy is low. Our results imply that even when considering major mergers of massive galaxies at high redshift, where each halo hosts a single bright galaxy, the close pair population will not reliably trace the merger population.

We have also compared two different linking lengths to explore the dependence of our results on the FoF process. In Figure 4 (bottom) we show the results using a linking length of $b = 0.20$. Since changing the linking length changes the mass of a given halo, we have scaled the mass threshold for $b = 0.20$ to match the number densities of the $b = 0.15$ sample, thereby probing the same halo population. We have also scaled the halo separations by $r_{\text{vir}} = r_{500}$. In the case of $b = 0.20$, the intersection of the efficiency and completeness curves is shifted outward by $\sim r_{\text{vir}}$, which is not surprising, since a halo found using $b = 0.20$ is expected to be ~30% larger than one found using $b = 0.15$. From the figure we see that changing the linking length results in a change of only a few percent in the merger fraction at the intersection of the efficiency and completeness curves. Thus, changing the linking length does not qualitatively alter any of our results. The weak dependence on the linking length suggests that for these mergers, the FoF procedure does not give rise to significant artificial bridging of nearby halos, and our results would not differ significantly from a similar analysis performed using a spherical overdensity halo finder (Lacey & Cole 1994).

3.4. Merger Timescale Dependence of Pair Mergers

Pair merging is also strongly dependent on the choice of merger timescale. Figure 5 shows that the merger efficiency (falling curves) increases with timescale; i.e., pairs are more likely to merge
4. SCATTER IN MASS, REDSHIFT SPACE DISTORTIONS, AND REDSHIFT ERRORS

The situation becomes more complicated when we consider redshift space distortions, redshift space errors, and scatter in the estimated mass of the halos. To identify the impact of imprecise determination of dark matter halo masses, we have recomputed several of the statistics of §3 after introducing an rms scatter of 0.2M_{\text{cut}}, where M_{\text{cut}} is the threshold for detecting the pairs in the mock observation. The scatter in the mass causes some halos to fall out of the sample and others to enter it, resulting in a different population of halos near the threshold. Because the mass function is steep, many more low-mass halos enter the sample than high-mass halos leave it. Thus, there are both more pairs to consider as merger candidates and more actual mergers between members of the observed sample. For example, if M_{\text{cut}} = 5 \times 10^{13} h^{-1} M_{\odot}, the number of pairs increases more than the number of mergers, resulting in a decrease in n_{\text{mergers}}/n_{\text{clusters}} of ~5% for pairs separated by 2 h^{-1} Mpc < r < 6 h^{-1} Mpc over 0.58 < z < 0.97 (\Delta t = 2 Gyr). The effect is similar for other thresholds, redshifts, and intervals. Scatter in the mass has a more pronounced impact at high values of M_{\text{cut}} where the mass function is steeper.

In contrast, redshift space distortions from Doppler shift due to the halo peculiar velocities v_p are catastrophic for the pair-merger hypothesis. When pairs are identified in redshift space, virtually none of them merge. In redshift space, the line-of-sight component is shifted relative to its real position by an amount

\[ \Delta \chi = \int_{z}^{z+t_{\text{Halo}}/c} \frac{cdz}{H(z)}. \]

For typical halo velocities in our simulation, this shift amounts to a few h^{-1} Mpc. Figure 6 illustrates that this distortion makes many close pairs appear to be highly separated and pairs that are highly separated appear close. At redshift space separations less than 5 h^{-1} Mpc, fewer than 5% of merger candidates actually merge in \Delta t = 0.6 Gyr at z = 0. Even at z = 0.58, where redshift space distortions are less severe, only ~10% of candidates merge over \Delta t = 1 Gyr. The situation improves modestly for longer time intervals, with ~60% of pairs merging over \Delta t = 5.6 Gyr, but this falls far short of the fraction in real space.

Figure 6 shows a histogram of the difference between a pair’s separation in redshift space (dashed line), as would be seen in an observation, and its true separation in configuration space (solid lines) at z = 0.04. Merger candidates identified in real and redshift space are widely disjointed sets. The three sets of curves...
errors that result in a shift of more than a few 
are not correlated with the high-density regions. Redshift space
Pair counts, even in redshift space, provide a rough proxy to the
group-mass halos and when the galaxy masses are comparable.
side a host halo are influenced by dynamical friction, especially in
satellite galaxy and the central galaxy, and merger dynamics in-
2006). Inside a halo, galaxy mergers are predominantly between a
sample of merger candidates. The reason for the asymmetry
between the pair population that left the sample (solid lines, right side) and the population that entered the sample and replaced it (dashed lines, left side) is that pairs are most often found in overdense regions toward which many other halos are streaming.
The impact is that the pair population identified in redshift space
is larger and has a broad distribution of physical separations; virtually none of the “merger candidates” we identified actually merge. This result is robust for all redshifts, merger intervals, and halo masses.

To a degree, this result also applies to the relationship between
galaxy pairs and galaxy mergers. Observing galaxy pairs to study
the merger rates of singly occupied host halos at high redshift will
be similarly affected by redshift space distortions, as will using
galaxy pairs to deduce the merger rates of multiply occupied host halos at lower redshift. However, the impact of redshift space dis-
tortions on using galaxy pairs to deduce the galaxy-galaxy merger rates within a single host halo may be less severe (Berrier et al. 2006). Inside a halo, galaxy mergers are predominantly between a
satellite galaxy and the central galaxy, and merger dynamics inside a host halo are influenced by dynamical friction, especially in
group-mass halos and when the galaxy masses are comparable. Pair counts, even in redshift space, provide a rough proxy to the
halo mass and will therefore correlate well with the merger rate
within the halo.

The impact of redshift space errors is similar to that of redshift
space distortions, except that, unlike with velocities, the errors are not correlated with the high-density regions. Redshift space
errors that result in a shift of more than a few \( h^{-1} \) Mpc in the
apparent position have an impact similar to that of redshift space distortions.

5. THE MERGER KERNEL AND THE DENSITY
DEPENDENCE OF MERGERS

Since spatial information is a weak probe of halo merger sta-
tistics, we now turn to statistically describing the merger rate via
the halo mass function. For any population of objects that are
built up by binary mergers of smaller constituents, the rate at
which the number of objects of a given mass changes can be
described by the Smoluchowski coagulation equation, which has
been applied to the evolution of dark matter halos (Silk & White
1978; Cavaliere et al. 1992; Sheth & Pitman 1997; Benson et al.
2005). The abundance of halos at a given mass is increased through
the creation of such halos by mergers of smaller halos and decreased as these halos merge into more massive ones. The
rate of change of the number density of halos of a given mass
can then be determined by knowing the number density of halos
at all masses, i.e., the mass function, and the proper merger kernel
to relate the mass function to a merger rate. Historically, it has
been difficult to study the coagulation of cluster-mass halos through
simulation because these events are rare. However, with our large
simulation volume the merger kernel can be computed in a statisti-
cally significant way.

We define the merger kernel \( Q(m_1, m_2, z) \) as in FK06, via the relation
\[
Q(m_1, m_2, z) = \frac{n_{\text{merg}}(m_1, m_2, z)}{n(m_1, z)n(m_2, z)\Delta t},
\]
where \( n_{\text{merg}}(m_1, m_2, z) \) is the number density of mergers between parents of mass \( m_1 \) and \( m_2 \) in a time \( \Delta t \) and \( n(m, z) \) is the halo mass function. The quantity \( Q \) can be interpreted as the efficiency of merging for a pair of objects, such that the rate of merger is a product of this efficiency with the densities of available parents.

Note that in previous contexts the term “efficiency” refers to the
product of this efficiency with the densities of available parents.
The time interval \( \Delta t \) should be sufficiently short that there is no significant
evolution of the halo mass function. Satisfying this requirement
drives down the number of mergers in a fixed volume, making
statistically significant measurements difficult. However, with our
large simulation volume we measure \( Q(m_1, m_2, z = 0) \) on the in-
terval \( 0 < z < 0.04 \) (\( \Delta t = 0.6 \) Gyr) by classifying the two pro-
genitors that contributed the most mass to each halo at \( z = 0 \)
into 10 logarithmically spaced mass bins in the range \( 10^{13} < h^{-1} M_{\odot} < 10^{15} \). The results are shown in Figure 7.

We find that \( Q \) follows the simple functional form
\[
Q(m_1, m_2, z = 0) = A \left( \frac{m_1 + m_2}{h^{-1} M_{\odot}} \right)^{\alpha}.
\]

The best-fit values for \( A \) and \( B \), found using a linear least-squares
fit to the data, are presented in Table 1 (bottom row). This func-
tional form satisfies the formal requirement that the merger kernel
be symmetric in its two arguments. For comparison, Benson et al.
(2005) analytically found that \( Q \propto (m_1 + m_2) \) when \( P(k) \propto k^n \)
with \( n = 0 \), which is approximately true in the translinear regime of
cluster-mass halos.
TABLE 1

| $\tilde{\delta}_i$ | $V$ (h$^{-1}$ Gpc)$^3$ | $A$ (h$^{-1}$ kpc)$^3$ Gyr$^{-1}$ | $B$ | $\chi^2_{\text{red}}$ |
|-------------------|-----------------|-----------------|-----|----------------|
| –0.56 ........    | 0.47            | (0.121 ± 1.99) × 10$^{-3}$ | 0.88 | 1.00          |
| –0.21 ........    | 0.33            | (0.086 ± 1.30) × 10$^{-3}$ | 0.88 | 1.55          |
| 0.20 .............| 0.27            | (0.073 ± 0.98) × 10$^{-4}$ | 0.88 | 1.46          |
| 1.33………………….| 0.21            | (0.066 ± 7.10) × 10$^{-3}$ | 0.88 | 1.41          |
| 0.0,………………….| 1.28            | (0.088 ± 2.23) × 10$^{-2}$ | 0.88 ± 0.008 | 3.09 |

Notes. —Where $(m_1 + m_2)$ is the sum of the two progenitor masses. The first four rows give the results for each of the four density subdivisions, while the bottom row gives the best fit for the entire simulation. The average density and total volume of each subdivision are listed at left, and the reduced $\chi^2$ for each subdivision is listed at right.

5.1. Density Dependence of the Merger Kernel

Because the densities of the parent populations have been divided out, it is conceivable that the merger kernel $Q$ is independent of the large-scale density field. Significant dependence on density would indicate that environmental effects other than progenitor densities, such as halo velocity distributions, halo impact parameters, and tidal fields, are at play in determining the merger rate. To investigate the dependence of $Q$ on the large-scale density field we construct a coarse (64$^3$) density grid in our simulation and assign the dark matter particles to the nearest grid point, effectively smoothing the field on a scale of $\sim$17 h$^{-1}$ Mpc. Each halo at $z = 0$ is also assigned to the coarse grid, and these halos are sorted based on their large-scale density environment and then divided into quartiles of density, with each quartile containing approximately the same number of halos. Using the coarse grid, we compute the total volume and overdensity of each quartile in the simulation, and these appear in the first two columns of Table 1. The mean overdensity of a quartile is defined as

$$\langle \delta_i \rangle = \left( \frac{\rho}{\langle \rho \rangle} \right), \quad i = 1, 2, 3, 4,$$

where the angular brackets denote an average of the $\sim$17 h$^{-1}$ Mpc$^3$ cells of the coarse grid. The merger kernel $Q$ is fit for each quartile to the form of equation (3), using the value of $B$ determined from the entire simulation. The results are summarized in Table 1. The best-fit values of $A$ differ by several $\sigma$, and the improvement in the reduced $\chi^2_{\text{red}}$ statistic indicates that each of the individual density fits is a much better fit that a fit to the entire volume. This indicates that there is a clear trend in $Q$ with the large-scale density field: as the density increases, the efficiency of merging for a given system mass decreases. Thus, the environmental effects in dense environments are hindering the merging process in comparison to less dense environments.

Figure 7 shows $Q$ in each of the four density environments plotted individually. The top panel shows $Q$ in the highest and lowest density quartiles, while the bottom panel shows the results from the inner two quartiles. The solid line indicates the best-fit model for the entire box, which is only a good approximation for environments close to the mean density. The shaded region results from allowing $A$ and $B$ to deviate by their $1 \sigma$ values. Figure 7 demonstrates clearly that a large component of the dispersion in the best fit to the entire simulation originates in the density dependence of $Q$.

Figure 7 clearly indicates that merging is less efficient in high-density environments than in low-density environments, but we note that with regard to our fit there is an ambiguity as to whether this implies a lower normalization $A$ or a shallower power law $B$. Given the rarity of these events at the high-mass end and the limited dynamic range that is consequently driving the fit, there is large covariance between the fit values of $A$ and $B$. We have performed the two-parameter fit to equation (3) for each of the density quartiles individually and find that, indeed, $A$ increases slightly with density (in contrast to the one-parameter fit), while $B$ decreases. Unfortunately, the formal errors in the parameters (from inversion of the covariance matrix) become so large that the four regions are statistically indistinguishable. However, the individual fits hint at an interesting possibility: if $B$ decreases and $A$ increases with density, then the merger efficiencies cross over as a function of system mass. Specifically, for lower total system masses $(m_1 + m_2)$, merging becomes more efficient in high-density environments. In our simulation, all four $Q$ curves cross over in the mass range $10^{10}$ h$^{-1}$ M$_\odot < (m_1 + m_2) < 10^{12}$ h$^{-1}$ M$_\odot$, suggesting that galaxy-scale mergers may be more efficient in denser environments. While this evidence is extrapolated from higher mass, the trend is potentially worth further investigation, given that other halo properties, e.g., clustering as a function of concentration/formation time, reverse the trend from $M > M_\ast$ to $M < M_\ast$ (Gao et al. 2005; Wechsler et al. 2006).

We note that the merger kernel $Q$ as computed in this section is not a direct cosmological observable. There is an observational counterpart to the merger kernel: the pair kernel, $Q_p$, computed using the number density of pairs with separations $r < r_{\Delta t}$. The threshold separation is calibrated from simulations to any desired tolerance for completeness or contamination, but the result is insensitive to $r_{\Delta t}$. In all cases, there are many more pairs than actual mergers, the amplitude $A$ of $Q_p$ is larger, and the power law $B$ is shallower, both by several $\sigma$. It is feasible to use pairs at any mass scale as a proxy to compute merger rates.

5.2. Density Dependence of Close Pair Mergers

The trend in the merger kernel is driven by the fact that the number of mergers grows more slowly with density than the parent mass functions. By studying changes in the relationship between close pairs and mergers in regions of differing local density, we can demonstrate that this is not universally true for all pair separations. We proceed in an analogous manner, smoothing over $\sim$17 h$^{-1}$ Mpc and defining four density quartiles, each containing the same number of mergers. We consider pairs merging between $z = 0.12$ and 0.04 ($\Delta t = 1$ Gyr), although our results are insensitive to the time interval and redshift. Figure 8 shows the results for pairs of halos above $5 \times 10^{12}$ h$^{-1}$ M$_\odot$ (top) and $10^{14}$ h$^{-1}$ M$_\odot$ (bottom). The relationship between close pairs and mergers changes with the large-scale density. Pairs within a given separation are a less complete sample of merger candidates in overdense environments than in underdense regions (rising curves, top and bottom). Objects in overdense regions have higher velocities, which allow mergers to come from a broader range of progenitor separations and can extend the infall time of closer pairs by generating larger angular momenta.

Figure 8 also indicates that, at large separations, pairs are more efficient predictors of mergers in overdense regions than in underdense regions (falling curves) because higher velocities in overdense regions allow pairs from larger separations to merge. However, at small separations ($\leq 2$ h$^{-1}$ Mpc), the relationship between pairs and mergers depends on the pair masses. Here,
show that pairs within this separation constitute the majority of pair mergers in the given timescale. Figure 8 (bottom) shows that high-mass pairs are more likely to merge in overdense regions at all pair separations. However, since mergers between two halos that both have $M > 10^{14} h^{-1} M_\odot$ are extremely rare (as opposed to mergers between a high- and low-mass object), these contribute less to the merger kernel at a given child mass.

6. THE CLUSTERING OF CLOSE PAIRS AND MERGER BIAS

For two halos to merge, they must have been located in close physical proximity at an earlier time. Since closely spaced halos are more likely to be found in overdense regions, recently merged halos may exhibit enhanced spatial clustering. Moreover, if recently merged halos cluster differently from the general population (“merger bias”) and this is unaccounted for, conclusions drawn about halos on the basis of their clustering could be compromised. For example, the use of cluster self-calibration to infer cluster masses (and, in turn, cosmological parameters) requires a precise knowledge of the clustering of clusters as a function of mass (Majumdar & Mohr 2003). A number of earlier studies have looked for such merger bias (Gottlöber et al. 2002; Percival et al. 2003; Scannapieco & Thacker 2003; Wetzel et al. 2007), with mixed results.

Recently, FK06 developed an analytic model to predict the merger bias. Assuming that mergers correspond to closely spaced objects at an earlier time, they compared the clustering of close pairs to that of single objects, thus computing the pair bias as a proxy for merger bias. On scales much larger than the pair separations, they found an enhancement of clustering for pairs of mass $M > M_\ast$, implying that recently merged high-mass halos should exhibit a clustering bias. We now use FK06’s framework to determine whether the clustering of close halo pairs of mass $M \gg M_\ast$ provides an accurate proxy for the clustering of mergers.

Using simulations, Wetzel et al. (2007) found the most prominent merger bias (~20%) for major mergers ($M_2/M_1 > 0.3$) of high-mass halos ($M > 5 \times 10^{13} h^{-1} M_\odot$) at $z = 0$ over $\Delta t = 0.6$ Gyr. We consider a similar mass and temporal regime. To improve our statistics, we also use a box of 8 times the volume previously used (see § 2), which allows us to probe merger effects of more massive halos. We use a shorter timescale of $\Delta t = 0.5$ Gyr at $z = 0$ to define our merger interval to preserve the signal in the comparison population. Contrary to expectation, the merger bias does not increase with mass; it remains a 10%–20% enhancement with similar statistical significance up to halos of mass $M > 4 \times 10^{14} h^{-1} M_\odot$.

In computing the pair bias, FK06 define “close” halo pairs by demanding that the probability of finding three or more halos in a sphere of a given radius is small compared to that of finding two. This is approximately equivalent to the restriction that the probability of finding two or more neighbors, $P(\geq 2)$, within a given separation from a halo is small compared to that of finding one, $P(1)$, which is what we measure. For halos $M > 5 \times 10^{13} h^{-1} M_\odot$ at $z \approx 0$, we find that $P(\geq 2)/P(1) \approx 0.1$ for a (comoving) sphere radius of $4 h^{-1} \text{Mpc}$. This separation restriction yields ~6800 pairs per ($h^{-1}$ Gpc)$^3$ out of ~77,000 halos per ($h^{-1}$ Gpc)$^3$. While this is a sufficient number density of pairs for a robust correlation function measurement in our (2.2 $h^{-1}$ Gpc)$^3$ simulation volume, § 3 demonstrates that close pairs do not reliably predict the merger population.

The analytical model of FK06 predicts a significant merger bias in its application to the clustering of massive halos ($M \gg M_\ast$). For such objects, it predicts a correlation function of pairs
halos are approximately the same mass as the child products of the couples. We consider pairs 0.5 Gyr prior to the effect of mass dependence on the halo correlation function, otherwise, the merger bias becomes entangled with the effect that larger halos are more clustered. To adjust for this in the halos, the comparison population should be an ensemble of halos of the same characteristic mass as the child halo, not the parent halos. Otherwise, the merger bias becomes entangled with the effect of the halo mass; more massive halos are more highly clustered. For a given halo mass, this can result in anomalously high pair bias. This is because the halo correlation function is implicitly a function of the halo mass; more massive halos are more highly clustered. When computing the merger bias (or its proxy, the pair bias) the comparison population should be an ensemble of halos of the same characteristic mass as the child halo, not the parent halos. Otherwise, the merger bias becomes entangled with the effect that larger halos are more clustered. To adjust for this in the analytic model we note that $\xi_p(r, 2M) \approx 1.1\xi_c(r, M)$ near $M \sim 10^{14} M_\odot$. On scales $>10 h^{-1}$ Mpc, $\xi(r)$ is less than 1, so this leads to a pair bias of $b_p^2 \approx 3.6$. This pair bias is still significantly larger than the 10%–20% merger bias seen in simulations for halos up to $M > 4 \times 10^{14} h^{-1} M_\odot$. As an alternative to an analytic approach, we next measure the clustering of close pairs in simulations to discover whether pair bias can predict merger bias. We consider pairs 0.5 Gyr prior to $z = 0$. To assign a unique position to a pair of neighboring halos, we impose a “couples” restriction, namely, that each member of a pair is the other’s nearest neighbor. This restriction remains robust for two-body mergers, a good approximation for our time interval. Within the pair separations we consider ($<2 h^{-1}$ Mpc), couples constitute essentially all pairs. We select halo couples where each halo is above $5 \times 10^{13} h^{-1} M_\odot$ at $z = 0.04$ and use their geometric centers to evaluate the couple correlation function. We use couples with separations less than 1.6 $h^{-1}$ Mpc, 80% of which correspond to mergers in our time interval (Fig. 4). To limit the effect of mass dependence on the halo correlation function, we compare this correlation function with that obtained from single halos above $10^{14} h^{-1} M_\odot$ at $z = 0$. While this ignores the effects of mass scatter in mergers, such effects remain small for this short timescale.

Figure 9 shows that with a $(2.2 h^{-1}$ Gpc$)^3$ simulation volume, no statistically significant pair bias is found. Such poor statistics arise because only a small fraction (15%) of total mergers are represented by couples at such close separations. Trying to increase statistics by considering couples at larger separations is undermined by the fact that the fraction of couples that merge is a steeply decreasing function of separation. As already mentioned, one cannot consider longer merger timescales, as this permits a larger fraction of the halo population to have undergone a major merger, thereby washing out the signal in the comparison population. We find similar results when looking at the pair bias for mass cuts from $M > 10^{13}$ to $M > 2 \times 10^{14} h^{-1} M_\odot$, and thus we conclude that pair bias cannot be used to reliably predict merger bias observationally, in simulation, or through current analytic treatment.

7. CONCLUSIONS

Cluster merger statistics may provide insight into the nature of hierarchical structure formation and the mechanisms by which the largest coherent objects in the universe form. We use large-volume, high-resolution N-body simulations to investigate the utility of close spatial pairs of galaxy clusters as proxies for cluster mergers. We characterize merger statistics through the merger kernel and examine the density dependence of merger efficiency. We highlight our conclusions as follows:

1. Close pairs of galaxy clusters at very small separations ($<1$–2 $h^{-1}$ Mpc) can be used to reliably predict mergers. However, since these constitute a small fraction of the total merger population, close pairs are not a reliable proxy for cluster merger rates. We quantify this by measuring the efficiency and completeness of merger candidates identified via close pairs as a function of separation and find that their intersection typically occurs at a low merger fraction (0.5–0.6).

2. We find that close pairs are even poorer proxies for mergers between massive galaxy-size halos. This indicates that galaxy pairs will not provide a reliable proxy for galaxy merger rates at high redshift, where most galaxies reside in distinct halos.

3. We note that the failure of close pairs as proxies for mergers indicates that determination of merger rates from spatial statistics, such as the correlation function, cannot be trusted outside the physical size of a single halo.

4. We examine the mass, redshift, and timescale dependence of pair mergers, finding that the pair-merger hypothesis (that close pairs are proxies for mergers) at a given comoving separation is most accurate at high redshift, high mass, and low merger timescales. In our best scenarios, the intersection of merger efficiency and completeness is at 75%; i.e., 75% of pairs within a given separation merge, and these constitute 75% of all mergers. We also find a nearly universal relation for pair merger efficiency and completeness for different mass halos. This relation begins to break down as we approach massive galaxy-size halos and is compromised by redshift space distortions.

5. Redshift space distortions have a devastating impact on detecting close galaxy cluster pairs in surveys; nearly all of the merger candidates identified in redshift space do not merge. Although they are an extrapolation, we expect these results to be robust for galaxy-size halos at high redshift.

6. We present the first fit from simulation to the merger kernel—a means to describe halo merger rates via the halo mass function (coagulation).
7. The merger kernel exhibits dependence on local ($\sim 17 h^{-1} \text{Mpc}$) density. Specifically, halo merging in our high-mass regime is more efficient in underdense regions.

8. Pairs at large separations ($\geq 3 h^{-1} \text{Mpc}$) are more likely to merge in overdense regions. For pairs at small separations, low-mass halos are more likely to merge in underdense regions, while high-mass halos exhibit no environmental dependence.

9. We sought to use cluster pairs to measure the merger bias by using the pair bias as a proxy for the merger bias. We extended the treatment of previous analytic work to include the fact that mergers result in mass bias; when computing the bias of recently merged halos, the comparison population should be a set of halos of the same mass as the children, instead of the parents. Close spatial pairs that reliably merge are too rare to produce a statistically significant measure of merger bias, even in a $(2.2 h^{-1} \text{Gpc})^3$ simulation volume.

In conclusion, we have shown that close spatial pairs of galaxy clusters are of limited value as a probe of overall cluster merger rates. We have determined the merger kernel for halo coagulation for the first time from simulations, finding that a statistical description of halo mergers is more promising. Further work is needed to extend our parameterization of the merger kernel to lower masses and higher redshifts and to explore whether the environmental dependence of the merger rate persists in these regimes.

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