Transcritical Hopf bifurcation and breathing of limit cycles in sequential tunnelling of superlattices

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Abstract. Within a discrete drift model, we study the evolution of the self-sustained current oscillation (SSCO) solutions (limit cycles) in sequential tunnelling of superlattices under dc bias. We propose two possible modes: one is co-existence of both fixed point and limit cycle solutions, exchanging stabilities at the bifurcation point, termed the transcritical Hopf bifurcation and the other is that, at high doping densities and inside SSCO regime, the breathing motion of the limit cycles, in which the amplitude and frequency of SSCOs oscillate as a function of an applied dc bias, in contrast with a square-root dependence expected in a conventional Hopf bifurcation.

Following the discovery of self-sustained current oscillations (SSCOs) [1]–[3] in superlattices (SLs), a large number of experimental and theoretical studies have focused on different aspects of the SSCOs. Experimentally, it has been found that SSCOs can be induced by illuminating laser light [1], changing the doping concentration [2], applying an external magnetic field parallel to SL layers, as well as varying the sample temperature [3]. Furthermore, the response of an SSCO to an ac bias varies from frequency-locking [4], quasi-periodicity to chaos [5]. On the theoretical side, it is understood that SSCOs are accompanied by the motion of electric-field domain (EFD) boundaries [6]. A discrete drift model capable of describing both the formation of stationary EFDs and SSCOs emerged after many numerical calculations and tedious analysis [7]. Many results have been explained by the concept of EFDs and by the motion of charge monopole or dipole [8], as summarized in recent reviews [9]. Numerical solutions [10]–[14]

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also show possible aperiodic oscillations, either quasi-periodic or chaotic under an external ac bias. Recently, it has been shown [15] that SSCOs can also be understood as the manifestation of one-dimensional attractors—limit cycles. The power of the limit cycle concept lies in its simplicity and universality as it was demonstrated in the explanation of the frequency-locking of SSCOs [14].

It is well known [16] that, depending on control parameters, a nonlinear system can evolve into different types of attractors: zero-dimensional attractors of fixed points or stationary state solutions (SSSs), one-dimensional attractors of limit cycles and strange attractors (chaos). As control parameters are varied, changes may occur in the qualitative structure of the solutions for the dynamical systems. These changes are called bifurcations. Bifurcation between a fixed point and a limit cycle solution is an important subject. One of them is Hopf bifurcation [16] at which a fixed point disappears (appears) while a limit cycle appears (disappears). One of the features of a Hopf bifurcation is that the size of the limit cycle increases as the square root of the deviation of the control parameter from its bifurcation point. A Hopf bifurcation is called supercritical if a branch of stable limit cycles bifurcates from a stable fixed point which loses its stability at the bifurcation point. If a branch of unstable limit cycles bifurcates from a branch of unstable fixed points which will become stable at the bifurcation, it is called subcritical. As the SL is a good non-linear system, bifurcations related to both SSCOs and the static EFD formation in SLs were investigated. Supercritical and subcritical Hopf bifurcations and homoclinic connection were found [17]–[19].

In this paper, the key results are two possible modes in the evolution of limit cycle solutions within a widely employed discrete drift model in SLs. One is the transcritical Hopf bifurcation reminiscent of the transcritical bifurcation between two fixed-point solutions. Both fixed-point and limit-cycle solutions exchange their stabilities at the bifurcation point. The other is a breathing motion of limit cycles at high doping levels, leading to an oscillation behaviour in the amplitude and frequency of the current as a function of an applied dc bias, in contrast with a square-root dependence of the amplitude expected in a conventional Hopf bifurcation.

Consider a system consisting of \( N \) quantum wells under a bias voltage of \( U \) at two ends, the averaged bias on each period of SL is \( U/N \). A current \( I_i \) passes through the \( i \)th barrier under a given bias \( V_i \). This current may depend on other parameters, such as doping \( N_D \), and the electric charge in the \( i \)th well, \( n_i \). The dynamics of the system is governed by the discrete Poisson equations [6, 15]

\[
k(V_i - V_{i-1}) = n_i - N_D, \quad i = 1, 2, \ldots, N \tag{1}
\]

and the current continuity equations

\[
J = k \frac{\partial V_i}{\partial t} + I_i, \quad i = 0, 1, 2, \ldots, N, \tag{2}
\]

where \( k \) depends on the SL structure and its dielectric constant. In equation (1), the same doping level in all wells is assumed. It can be shown [14] that all SSSs are stable if \( I_i \) is a function of \( V_i \) only. On the other hand, a SSS may be unstable [6, 7] if one chooses \( I_i = n_i v(V_i) \), where \( v \) is a phenomenological drift velocity which is, for simplicity, assumed to be a function of \( V_i \) only. The constraint equation for \( V_i \) is

\[
\sum_{i=0}^{N} V_i = U. \tag{3}
\]
To close the equations, a suitable boundary condition is required. It is reasonable to assume a constant \( n_0, n_0 = \delta N_D \), with \( \delta \) as a model parameter, if the carrier density in the emitter is much higher than that in wells, and its change is negligible.

Equations (1)–(3), together with \( I_i = n_i v(V_i) \), constitute the discrete drift model. A diffusion term can be added to this model for describing the diffusion current. However, the simple model can simulate the physics of SSCOs in SLs very well and the diffusion term can be safely neglected due to the rapid decrease in the diffusion coefficient [20]. Moreover, since the drift velocity is chosen as a phenomenological parameter in our considerations, the diffusion current can be included in the formalism. Previous studies [6, 15] have shown that this model is capable of describing SSCOs with a negative differential drift velocity. In the numerical calculation, the velocity \( v(V) \) as the sum of two Lorentzian functions, \( v(V) = 0.0081/[(V/E - 1)^2 + 0.01] + 0.36/[(V/E - 2.35)^2 + 0.18] \), is assumed to fit the negative differential resistance behaviour of SLs. This \( v \) has two peaks at \( V = E \) and \( V = 2.35E \). The region from \( V = E \) to \( V = 1.3E \) exhibits negative differential velocity. Thus, \( E \) can be used as a natural unit of bias, and \( 1/v(E) \) as that of the time (the lattice constant is set to be 1). The unit of current is \( kEv(E) \).

For \( N = 30 \) and \( \delta = 1.001 \), the above set of equations can be numerically solved by the fourth-order Runge–Kutta method to show the SSCO solutions. Therefore, the phase diagram of SSCOs in \( (U/N) - N_D \) plane with all other parameters unchanged can be obtained. Figure 1 shows the phase diagram (grey-scale map where the darkness is proportional to the oscillation amplitude) of the SSCOs in \( (U/N) - N_D \) plane in which \( N_D \) is in units of \( kE \). Of course, the diagram depends on the values of \( \delta, N \) and a function of \( v(V) \). The number of branches in phase diagram is only

**Figure 1.** The phase diagram (grey-scale map) in \( U/N - N_D \) plane, where \( U/N \) is in units of \( E \) and \( N_D \) is in units of \( kE \). The system inside the grey-coloured area will have an SSCO solution while it has a static current–voltage characteristic outside this area.
10, which is not equal to the number of the SL wells. We attribute it to the special choice of the model parameters. Especially, the choice of the function $v(V)$ in this formalism can affect this detail. In general, we have also noticed inconsistencies in other theoretical calculations [18] and also in some experiments due to the imperfections of samples.

We are interested in the bifurcation scenarios between the SSSs (fixed points) and the SSCOs (limit cycles) as $U/N$ varies while $N_D$ is fixed or vice versa. To plot the bifurcation diagram, we require a scalar measure of the state vector consisting of the bias on each potential barrier in phase space [16]. We have noticed that the oscillation amplitude is zero at fixed points while it is nonzero in the SSCO regime. Thus, it is natural to choose the current oscillation amplitude as the scalar measure. Hence the bifurcation diagram shows the variation of oscillation amplitudes with control parameters, such as $U/N$ or $N_D$.

Figures 2(a)–(d) present the bifurcation diagrams, the oscillation amplitude varying with $U/N$, for different fixed values of doping levels. As shown in figure 2(a), at the left bifurcation point (denoted as A, where $U/N = 1.04E$) the oscillation amplitude increases with a square-root dependence on the averaged voltage $U/N$, shown by the dashed line, a typical behaviour of the supercritical Hopf bifurcation type [17, 18]. A stable limit cycle branch bifurcates from a stable fixed point which loses its stability. Besides, we have observed that there are two interesting modes different from the usual Hopf types and proposed two new explanations for them. One of them being that the amplitude (also the frequency, see figure 3 below) of the SSCOs shows an oscillation behaviour as a function of $U/N$, is shown in figures 2(b) and (c). For very high $N_D$, some alternative oscillation windows appear in figure 2(d). The inset in figure 2(d) enlarges two peaks of oscillation windows. These oscillation windows have also been reported [3, 21] and observed experimentally [3, 21]. The bifurcation scenario is different from the conventional Hopf bifurcation whose amplitude shows the expected square-root dependence of control parameter. On the basis of the limit-cycle picture, we propose that such bifurcation scenario can be attributed to a breathing motion of limit cycle, leading to an oscillation behaviour of the current amplitude.

Figures 3(a)–(d) depict the intrinsic frequency $f_0$ of the SSCOs varying with the bias $U/N$ for different $N_D$. The parameters are chosen the same as in figure 2. From figure 3, the frequencies decrease with $U/N$ and show similar oscillation behaviours and oscillation windows as $N_D$ increases. It indicates that the breathing motion of limit cycle leads to such oscillation behaviour of the frequency at high doping levels. Comparison of figures 3 and 2 shows that the frequency decreases as the amplitude increases in the same voltage interval. This implies that the larger the limit cycle, the longer the period of completing one round. This can be qualitatively expected since the velocity on the limit cycle does not change much.

Another interesting phenomenon can also be observed from figures 2 and 3. At the right bifurcation point where the SSCOs disappear (we denote the point B in figure 2(a), at which $U/N = 1.17E$), the oscillation amplitude of the current reduces sharply to zero. A hysteresis loop near this regime where the oscillation vanishes can be observed, as shown in figures 2(e) and 3(e). Previous studies [18, 19] attributed the phenomenon to a subcritical Hopf bifurcation or a homoclinic connection. For a homoclinic connection, the oscillations vanish with a fixed amplitude and zero frequency without any bistability. For a subcritical Hopf bifurcation, the oscillations vanish with a finite amplitude and a finite frequency with a hysteresis loop. For both explanations, there are only fixed-point solutions but no limit-cycle solutions outside the SSCO regime. Here, we provide another possible scenario for the sudden jump in oscillation amplitude. If an unstable limit cycle co-exists with a stable fixed point outside the SSCO regime, the stable limit-cycle branch becomes unstable at the bifurcation point so that the system jumps into a stable
Figure 2. Bifurcation diagrams—amplitude of SSCOs varies with the averaged dc bias $U/N$ for the different doping levels $N_D = 0.05kE$ (a), $0.09kE$ (b), $0.11kE$ (c) and $0.13kE$ (d). In (a), the dashed line shows the square-root dependence from the left bifurcation point A, where $U/N = 1.04kE$. The dash-dotted lines 1 and 2 denote qualitatively the possible evolution trends of the unstable limit cycle with the parameter $U/N$. Point B at which $U/N = 1.17kE$ is the right bifurcation point, where the SSCOs vanish. The inset in (d) enlarges two peaks of oscillation windows. (e) An enlargement scale around the right bifurcation point B in (a) and a hysteresis loop appears when the dc bias is swept up and down. The dashed lines are used as a guide to the eye.
Figure 3. Frequencies of SSCOs vary with $U/N$ for different doping levels $N_D = 0.05kE$ (a), $0.09kE$ (b), $0.11kE$ (c) and $0.13kE$ (d). (e) Enlargement scale around the right bifurcation point in (a) and the hysteresis loop. The dashed lines are used as a guide to the eye.

We call it a ‘transcritical Hopf bifurcation’ reminiscent of the transcritical bifurcation between two fixed-point solutions.

In order to clarify the mechanism, we have investigated the response to an external ac bias $V_{ac} \sin(2 \pi f_{ac} t)$ applied in the static regimes. In the case of figure 2(a), we choose two
positions A’ and B’ just outside the SSCO regime in phase diagram, where at A’ the parameter is $U/N = 1.03E$ and at B’, $U/N = 1.19E$. We have not shown the two points in figure 2(a) since A’ and B’ are very close to A and B, respectively. For the case without external ac bias, there are no SSCO solutions for the kind of parameters corresponding to the two positions A’ and B’. For the case of applying an external ac bias, we argue that, if there is only the stable fixed points at the position, the response frequency should always be equal to the ac driven frequency (the forced oscillation case). However, if an unstable limit cycle co-exists with the fixed points in the static region, applying an ac bias with a large enough amplitude should excite the system into the unstable limit cycle. Thus, we may expect that the ac response of an unstable limit cycle shows the similar structure of devil’s staircases, as discussed in [14].

Figure 4(a) shows the ac response when applying an external ac bias at the position B’. The amplitude of ac bias $V_{ac} = 2.86E$, or the ratio of ac amplitude to dc bias $r = V_{ac}/U = 0.08$. We plot the figure of $f_{ac}/f - f_{ac}/f_{0}$ as in [14], where $f_{ac}$ is the ac frequency and $f$ is the fundamental frequency of the response oscillation when ac bias is driven. The fundamental frequency $f_{0}$ of the SSCO at the bifurcation point B is 0.0122$v(E)$. The ac response shows a devil’s staircase-like structure. Widths of $f = f_{ac}/2$ and $f = f_{ac}/4$ are shown in figure 4(a). The responses in the region of $f_{ac}/f_{0} ∈ [1, 1.3]$ appear to be either aperiodic or high values of $f_{ac}/f$ with very narrow widths and they are not displayed. The staircase-like structure implies a possibility of co-existence of an unstable limit cycle with stable fixed points at the position B’. As a comparison, the similar figure of the ac response at the position A’ has been plotted in figure 4(b). Now $f_{0} = 0.0250v(E)$, which is the fundamental frequency of the self-oscillation at the left bifurcation point A. Figure 4(b) shows the forced oscillation case in which the response frequency $f$ always equals $f_{ac}$. It implies that there are only stable fixed-point solutions at the position A’.

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Figure 5. Bifurcation diagram of the current at time \( mT_{ac} \) (for sufficiently large \( m \)) versus the driven ac amplitude \( r = V_{ac}/U \) with a fixed ac frequency \( f_{ac} = 0.0183v(E) \). The solid lines and dots are ac response at the position B' and the dashed line is the response at the position A'.

The ac response of the static regimes by varying the ac amplitude at fixed ac frequency can also be investigated. \( f_{ac} = 0.0183v(E) = 1.5f_B^0 \) is chosen, where \( f_B^0 \) is the fundamental frequency of the self-oscillation at the point B. An external ac bias is applied at A' and B' with the ac amplitude \( r \) ranging from 0 to 0.15, where \( r = V_{ac}/U \). The Poincaré mapping method is employed to detect the response period visually. Denoting the period of ac bias \( T_{ac} = 1/f_{ac} \), we sample the oscillation current at times \( T_m = mT_{ac}, (m = 0, 1, \ldots) \), (after waiting enough time for the transients to have decayed). Figure 5 shows such a bifurcation diagram giving the \( I(mT_{ac}) \) as a function of \( r \). The solid lines and dots give the ac response at B'. Note that the number of branches gives the response oscillation period. For example, at \( r = 0.08 \), there are two branches, giving the oscillation period \( T = 2T_{ac} \), or \( f_{ac}/f = 2 \). There also exist \( 4T_{ac}, 8T_{ac}, \ldots \), periodic-doubling windows and aperiodic windows in figure 5. For a comparison, the ac response at the position A' is also plotted, shown by a dashed line in the same figure. It is obvious that one branch indicates the response frequency always equals to ac frequency. So far, we find that the ac response on the right-hand side of SSCOs regime in the \((U/N)–N_D\) plane (grey area in figure 1) is very different from that on the left-hand side. Based on this difference we propose a possible bifurcation scenario that the stable limit cycle becomes unstable when the control parameter passes the bifurcation point. An unstable limit cycle may co-exist with fixed points on the right-hand side of SSCOs regime in the \((U/N)–N_D\) plane.

For further discussion, firstly, it may be noted that the sharp change in amplitude cannot be observed between the oscillation windows when \( N_D \) is high by zooming in two peaks of oscillation windows, shown in the inset of figure 2(d). It implies that unstable limit cycles do not co-exist in the static windows. Thus, the breathing limit cycle is gradually generated and annihilated at high doping levels. Secondly, the bifurcation scenarios by choosing the doping \( N_D \) as a control parameter with fixed voltage \( U/N \) can be investigated as well. A sudden jump
in amplitude can also be observed, which is consistent with the case of employing $U/N$ as a control parameter. It implies that the right regime outside the SSCO\textsuperscript{s} might correspond to the co-existence of the unstable limit cycle and fixed points while the left regime outside the SSCO\textsuperscript{s} in the phase diagram has only stable fixed point solutions. Finally, in the co-existence regime of the unstable limit cycle with fixed points, we may ask how the size of the unstable limit cycle evolves when the control parameter passes the bifurcation point. In other words, will the unstable limit cycle evolve qualitatively along branch 1 or branch 2 on the right-hand side of point B, as shown in figure 2(a)? By assuming the rule for the ac response of a stable limit cycle\textsuperscript{[14]} to be also applicable for the case of an unstable limit cycle, we can extract the possible intrinsic frequency $f_{0}^{B}$ of the unstable limit cycle at the point B'. From the observations of $f_{ac}/f = 2$ at $f_{ac} = 1.5 f_{0}$ and $f_{ac}/f = 4$ at $f_{ac} = f_{0}$ in figure 4(a), we can deduce $f_{0}^{B'}$ to be approximately $3 f_{0}^{B}/4$, where $f_{0}^{B}$ is the intrinsic frequency at the point B. The smaller $f_{0}$ at B' than that at B implies that the size of the unstable limit cycle may evolve with branch 1, indicating the unstable limit cycle becoming larger as the bias increases. A detailed investigation is worthy of a future work.

In conclusion, we have studied the evolution of the SSCO solution, which corresponds to a limit cycle in phase space, in sequential tunnelling of SLs under dc bias within a widely employed discrete drift model. We propose two possible modes. One of them is called the transcritical Hopf bifurcation. By applying an external ac bias outside the SSCO regime, a devil's staircase-like structure has been observed, suggesting that an unstable limit cycle may co-exist with stable fixed points, exchanging the stabilities at the bifurcation point. At high doping levels and inside the SSCO regime, the limit cycle undergoes a breathing motion, leading to an oscillation behaviour in the amplitude and frequency of SSCO\textsuperscript{s} as a function of dc bias, in contrast with a square-root dependence of the amplitude expected in a conventional Hopf bifurcation.

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