"Tropical" Real Business Cycles?
A Bayesian Exploration

Andrés Fernández
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Andrés Fernández†

Abstract

Can frictionless small open economy models driven solely by technology shocks account for business cycles in developing countries? We don't find evidence of it. We build a DSGE model that jointly includes a variety of real perturbations in addition to technology shocks, such as procyclical fiscal policies; terms of trade fluctuations; and perturbations to the foreign interest rate coupled with financial frictions and estimate it using Bayesian methods on high and low frequency data from a developing -and "tropical"- country, Colombia. We find interest rate shocks to be crucial and that financial frictions play a central role as propagating mechanisms of transitory technology shocks. These two driving forces alone can account well for the observed properties of the Colombian business cycle. Other structural shocks such as terms of trade fluctuations and level shifts in the technology process do not appear to be relevant in the past decade and a half, but their importance increases when a longer span of data is considered.

JEL classification: E32, F41, F47, C11

Key Words: Business cycles; developing economies; dynamic stochastic general equilibrium models; small open economy models; Bayesian estimation.

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¿Ciclos de Negocios Reales en Economías “Tropicales”? Una Exploración Bayesiana*

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Resumen

¿Pueden los modelos de economías pequeñas y abiertas, desprovistos de fricciones reales, y con sólo choques tecnológicos, explicar los ciclos económicos en países en desarrollo? No encontramos evidencia que así se compruebe. Construimos un modelo dinámico y estocástico de equilibrio general (DSGE, por sus siglas en inglés) que incluye, además de choques tecnológicos, perturbaciones reales tales como políticas fiscales procíclicas, fluctuaciones en los términos de intercambio, perturbaciones en el tipo de interés externo junto con fricciones financieras. Estimamos el modelo usando métodos Bayesanos con datos de alta y baja frecuencia de un país en desarrollo —y “tropical”—: Colombia. Encontramos que los choques en el tipo de interés son los más importantes y que las fricciones financieras juegan un papel fundamental como mecanismos de propagación de choques tecnológicos transitorios. Con sólo estas dos fuerzas es posible reproducir las propiedades del ciclo económico colombiano. Otros choques estructurales, tales como las fluctuaciones en los términos de intercambio y los cambios de nivel en el proceso de la tecnología, no parecen haber sido relevantes en la última década y media, pero su importancia aumenta cuando se estudian datos correspondientes a períodos de tiempo más largos.

Clasificación JEL: E32, F41, F47, C11

Palabras clave: Ciclos económicos; economías en desarrollo, modelos dinámicos, estocásticos de equilibrio general (DSGE); modelos de economía pequeña y abierta; estimación Bayesiana.

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1 Introduction

Understanding business cycle regularities in developing countries is a crucial step in the process of designing appropriate stabilization policies and sound macroeconomic management in these countries. A first step toward this understanding must take into account the differences on the business cycles properties in developing countries relative their developed counterparts. As will be shown below, observed business cycles in emerging countries are more volatile relative to their developed counterparts; their trade balance-to-output ratio is countercyclical; and consumption is more volatile than output's at business-cycle frequencies. Explaining these contrasts between emerging and industrialized economies is at the top of the research agenda in small-open-economy macroeconomics (Uribe, 2007).

What are the main driving forces of business cycles in developing countries? To what extent are they responsible for the differences in business cycles properties between developed and developing countries? More specifically, can technology shocks alone, in the spirit of the real business cycle literature, account for these differences? By addressing these questions, the goal of this paper is to contribute to the understanding of business cycles in developing countries.

To do so we use the following approach. First, we make a brief survey of the literature on business cycles in developing countries. As will be documented below, the use of frictionless small open economy models, driven solely by technology shocks to account for business cycles in developing countries has been controversial. On one strand of the literature, some authors have claimed that to properly account for the business cycle in these economies one can rely exclusively on pure technology forces in the form of transitory deviations in the total factor productivity process (e.g. Kydland and Zarága, 2002) or permanent shifts of it (e.g. Aguiar and Gopinath, 2007). Others have stressed as key driving forces the interaction between technology shocks and other real driving forces such as terms of trade (e.g. Mendoza, 1995), or interest rates in world capital markets coupled with financial frictions (e.g. Neumeyer and Perri, 2005).

Second, we use data from Colombia, a developing -and "tropical"- economy that has not yet been analyzed by the literature surveyed above. Using both high frequency-quarterly and low frequency-yearly data, we document the similarities and differences of the Colombian business cycle relative to those observed in an average developing economy. Based upon these stylized facts about the Colombian business cycle, the third element of our approach is to build a dynamic stochastic general equilibrium (DSGE) model that can account for them. Motivated by the observation that, to date, there has been little empirical analysis of the role played by individual shocks, within a multiple-shock setting, in driving business-cycle movements in aggregate variables from developing countries, a central element in our DSGE model is the inclusion of real driving forces other than technology shocks. Based on the literature surveyed in the next section, we include separately three structural driving forces to
the standard neoclassical framework: (i) shocks to the interest rate in world capital markets coupled with financial frictions; (ii) terms of trade fluctuations; and (iii) a procyclical government spending process. While each one of the alternative driving forces has been independently stressed by different strands of the literature on emerging market business cycles, to our knowledge, this is the first time where they will be jointly considered as alternative driving forces to technology shocks. The role of each driving force is empirically quantified by estimating the parameters of the exogenous shocks processes, along with a few other crucial parameters, within a Bayesian-likelihood-based framework, using Colombian macroeconomic data. Thus, we take the model as provider of a complete statistical characterization of the data in the form of a likelihood function. The performance of the model in accounting for the Colombian business cycle is then assessed.

We obtain several results of interest. The data is informative, particularly in terms of the size of the structural shocks impacting the economy. Shocks to the interest rate in world capital markets are a key driving forces of the Colombian business cycle. Transitory technology shocks appear to be relevant as well, to a large extent because financial frictions amplify their macroeconomic effects in the economy. These two driving forces alone can account well for the observed properties of the Colombian business cycle, notably the smooth consumption process, the volatile investment and the strong countercyclicality of the trade balance-to-GDP ratio, and are almost entirely responsible for the sharp downturn experienced in the late 1990s. Other structural shocks such as terms of trade fluctuations and level shifts in the technology process do not appear to be relevant in the past decade and a half, but their importance increases when a longer span of data is considered. Demand shocks, in the form of government consumption innovations account only for a trivial role of the variance of the macroeconomic aggregates but they appear to be relevant for the out-of-sample forecasting fit of the model.

The paper is divided into six sections, including this introduction. The second section presents a brief review of the theoretical literature on business cycles in developing countries and describes the main aspects of the Colombian business cycle. The third section lays out the model. The fourth section describes the Bayesian estimation. The fifth section presents the results. Concluding remarks are given in the sixth section. An appendix summarizes the data sources.

2 Business Cycles in Developing Countries

2.1 A Brief Literature Review

As mentioned above, business cycles in developing countries are different from the ones observed in developed countries. Using the dataset by Aguiar and Gopinath (2007) for a sample of thirteen developed and thirteen developing countries, Table 1 presents the main second moments for these two groups of countries. Comparing the upper and middle panels in Table 1, three dimensions in which these differences manifest are: (i) observed business cycles in emerging countries are more volatile; (ii)
the trade balance-to-output ratio is more countercyclical in emerging countries than in developed countries; and, (iii) consumption appears to be more volatile than output at business-cycle frequencies. These stylized facts, among others, have been widely documented in Mendoza (1995), Agenor et al. (2000), Rand and Tarp (2002), Neumayer and Perri (2005), Aguiar and Gopinath (2007) and Garcia-Cicco et al. (2009).

Despite these important differences a brief review of the literature on general equilibrium emerging markets business cycles models does not show a consensus on the best approach to account for them. One strand of the literature has tried to explain business cycles in developing economies within a neoclassical growth framework augmented by real driving forces in addition to technology shocks. Mendoza (1995) expands a real business cycle model to account for tradable/non-tradable goods in which the terms of trade are an additional driving force. Since emerging countries typically specialize in exports of few primary commodities for which they are small players in the world markets for the goods they export or import, it follows that the terms of trade can be regarded as an exogenous source of aggregate fluctuations. Mendoza (1995) finds they account for 45 to 60 percent of the observed variability of GDP.

The argument of stronger real shocks has also been extended to financial markets. The idea is that developing economies exhibit low levels of aggregate savings forcing them to rely heavily on foreign investment, via capital inflows. Uribe and Yue (2006) explore the significant correlation between the business cycles in emerging markets and the interest rate that these countries face in international financial markets. They find that one third of business cycles in emerging economies is explained by disturbances in external financial variables (e.g. the foreign interest rate and the spread). Moreover, they find evidence of a further increase in the volatility of domestic variables because of the presence of feedback from domestic variables to country spreads. Similarly, Neumayer and Perri (2005) find that eliminating country risk lowers Argentine output volatility by 27%. Another explanation for some of the stylized facts of the business cycles in developing economies explores the role of macroeconomic policies in amplifying the cycle (i.e. procyclical policies) as documented by Agenor et al. (2000) and Kaminsky et al. (2004).

On a more orthodox strand, some authors claim to properly account for the business cycle in developing economies by relying exclusively on pure technology forces in the line of the real business cycle school of thought. Kydland and Zarraga (1997, 2002) argue that nominal factors do not seem to be able to account for any significant fraction of the business cycles in Latin American countries, in general. They argue that, in the case of Argentina, the predictions of a standard neoclassical growth model conforms rather well with the observations during the Argentinean 'lost decade' years. More recently, Aguiar and Gopinath (2004, 2007a), claim that accounting for possible regime switches giving rise to changes in the long-run growth trend in these economies is enough to account for the business cycle stylized facts. Their underlying premise is that emerging markets are characterized by frequent regime switches motivated mainly by dramatic reversals in economic policy. Which leads them to conclude that "shocks to trend growth
are the primary source of fluctuations in these [emerging] markets as opposed to transitory fluctuations around a trend”. Thus, the higher volatility of consumption can be explained as agents, seeking to smooth their consumption levels, observe changes in the permanent component of the trend. Aguiar and Gopinath’s conclusion is driven by an estimated volatility of the technological growth process in the Mexican economy four to five times higher than the volatility of the transitory technology shock. In another paper, Aguiar and Gopinath (2008) find this result to be robust under the presence of stochastic interest rate shocks.

The idea that developing countries’ business cycles are, by and large, driven by trend shifts has not gone without criticism. On one hand, Garcia-Cicco et.al. (2009) have argued that in order to properly estimate the parameters of the stochastic trend, long time series are needed. Accordingly, they estimate the Aguiar and Gopinath model on a yearly dataset for Argentina covering over a century of aggregate data and find that the model performs poorly when trying to mimic some of the main moments in the Argentinian macroeconomic data, in particular the higher volatility of consumption relative to output, and the trade balance autocorrelation function. They show how another model that does not rely on growth shocks, but includes other structural shocks instead can overcome these empirical shortcomings. On the other hand, Chang and Fernandez (2010) have shown that a model with foreign interest rate shocks coupled with financial frictions as key amplifying mechanism outperforms the Aguiar and Gopinath model driven solely by transient and permanent technology shocks, if a ranking is made according to the models’ marginal likelihood.

2.2 The Colombian Business Cycles

The lower panel of Table 1 presents the second moments in the main Colombian quarterly macroeconomic aggregates for the period 1994:1 to 2008:4. Colombian data is characterized by some of the main stylized facts from the sample of developing economies highlighted in the middle panel of Table 1. There is a higher macroeconomic volatility measured by the variance of output and the trade balance share is significantly more countercyclical, even when compared to the average developing country. The latter is almost entirely driven by the properties of the time series for investment which exhibit a volatility, relative to output’s, that is also superior to the one in developing countries. There is, however, no evidence of a high volatility of Colombian aggregate consumption. In fact, the standard deviation of consumption appears even lower than the one observed for the average developed country. Importantly, when computing second moments from Colombian data we exclude durable (and semi-durable) goods consumption from aggregate consumption and include it on investment as it is standard in business cycles analysis (see Cooley and Prescott, 1996). It should be noted, however, that the low volatility of consumption with respect to output does not dependent on this transformation. If aggregate consumption is measured including consumption of durables and semidurable goods (as reported by DANE) the standard deviation of consumption growth increases only to 1:04, which is still lower than output’s volatility. It is not specified in Aguiar and Gopinath (2007) whether they also remove durable goods consumption from the aggregate consumption data they report.
The last three rows in Table 1 present additional data on three potential driving forces of the Colombian business cycle that will be included in the theoretical model presented in the next section: (i) $gR$, a proxy for the growth in the gross risky interest rate that countries similar to Colombia have faced in international capital markets, computed adding the real interest rate on U.S. T-Bills and the average EMBI+ spreads for Latin American economies; (ii) $gToT$, a proxy for the growth in the terms of trade faced by Colombian consumers and firms; and (iii) the growth in the level of public consumption\(^2\). Three key stylized facts emerge from the analysis of the second moments of these three variables. First, the proxy for the interest rate is countercyclical and leads the cycle, the same pattern that Neumeyer and Perri (2005) documented for a pool of emerging economies. Second, the terms of trade are highly volatile and procyclical, with a correlation of 0.33 between the terms of trade index and Colombian GDP, which is close to the value found by Mendoza (1995) for a pool of developing countries (0.39). Last, while government expenditure is procyclical, its correlation with output growth (0.17) is lower when compared to studies that have looked at other developing countries as Kaminsky et al. (2004).

To summarize business cycles in Colombia, within the last decade and a half, are characterized by (i) a moderately high variance of output; (ii) a trade balance share of income strongly countercyclical; (iii) a significantly volatile level of investment; (iv) a smooth aggregate consumption path; (v) a leading and countercyclical interest rate in world capital markets; (vi) volatile and procyclical terms of trade; and (vii) a moderately procyclical government expenditure. The following sections will build and estimate a business cycle model of the Colombian economy and its performance will be assessed along these dimensions, among others.

3 A Business Cycle Model for a Small, Open, and "Tropical" Economy.

The model presented here is built following the canonical real business cycle model of a small open and centralized economy first developed by Mendoza (1991). A decentralized version of this model was extended by Chang and Fernandez (2010) by introducing permanent shocks to technology, as discussed by Aguiar and Gopinath (2007) and foreign interest rate shocks that interact with financial imperfections, as discussed by Neumeyer and Perri (2005) and Uribe and Yue (2006). In what follows we modify the model by Chang and Fernandez (2010) in two dimensions: first, we allow for the presence of domestically produced and foreign consumption and investment goods; second, we include the presence of a procyclical government expenditure process.

3.1 Firms and Technology

Time is discrete and indexed by $t = 0; 1; 2; ...$ The domestic good is produced by a representative firm in each period with a Cobb-Douglas technology given by

\(^2\) See the Data Appendix for more details.
where \( Y_t \) denotes output, \( K_t \) capital available in period \( t \), \( h_t \) labor input. We use upper case letters to denote variables that trend in equilibrium, and lower case letters to denote variables that do not. The exogenous variables at and \( \Gamma_t \) represent productivity processes to be specified later.

The firm hires labor for which pays a wage \( W_t \) per worker and rents capital in competitive markets at a rental rate \( u_t \). It faces a friction in the technology for transferring resources to its workers: in order hire workers, the firm needs to set aside a fraction \( \theta \) of the wage bill, \( \theta W_t h_t \), at the beginning of each period.

Thus, because it is assumed that production becomes available at the end of each period, the firm has to borrow \( \theta W_t h_t \) in international markets for which it has to pay an interest rate of equilibrium at the end of the last period, \( R_{t-1} \). There are no frictions in the market for capital. When output becomes available firms use the resources to honor the remaining debts to workers, \( (1-\theta) W_t h_t \),

and to the financial system \( \theta W_t h_t R_{t-1} \), and pay for rented capital capital \( u_t K_t \).

Given \( W_t, u_t \) and \( R_{t-1} \), the firm's problem is to choose labor and capital in order to maximize profits, \( \Pi_t \), given by

\[
\Pi_t = Y_t - W_t h_t - u_t K_t - (R_{t-1} - 1) \theta W_t h_t
\]

subject to the technology available given by 1. The firm's two profit maximizing conditions are then given by

\[
u_t = a_t (1 - \alpha) K_t^{-\alpha} (\Gamma_t h_t)^\alpha
\]

\[
W_t [1 + \theta (R_{t-1} - 1)] = a_t \alpha K_t^{1-\alpha} (\Gamma_t h_t)^{\alpha - 1} \Gamma_t
\]

where the latter implies that the marginal product of labor equals the wage rate inclusive of financing costs. This assumption, first introduced in the emerging markets business cycles literature by Neumeyer and Perri (2005) allows for a direct supply effect of changes in real interest rates.

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3 The only exceptions will be the spread, \( S_p \), and the world and domestic gross interest rates, \( R_p \), to be defined later, which do not trend in equilibrium.
3.2 Households

Households own the capital and labor stock available in the economy. At the beginning of each period a representative household supplies labor and rents its capital to the firms in competitive markets. At the end of the period, the household receives the salary and rents resources for the two inputs and makes consumption and investment decisions. These decisions are made according to the household’s preferences given by the Greenwood, Hercowitz and Huffman - GHH (1988) form:

$$E \sum_{t=0}^{\infty} \beta^t \left( C_t - \tau \Gamma_{t-1} h_0 \right)^{1-\sigma}$$

where $\beta$ is a discount factor between zero and one, $C_t$ denotes consumption and $E(\cdot)$ the expectation operator. As discussed by Neumeyer and Perri (2005) and others, GHHH preferences have been shown to help reproducing some emerging economies’ business cycles facts by allowing the labor supply to be independent of consumption levels. We follow Aguiar and Gopinath (2007) in including $\Pi_i - \pi_i$ in the period utility function to allow for balanced growth.

The resources used for gross investment cover the net increase in the capital stock, the depreciated capital and the costs incurred by adjusting capital as follows:

$$I_t = K_{t+1} - K_t + \delta K_t + \phi \left( \frac{K_{t+1}}{K_t} - \mu \right)^2$$

where the last term is a quadratic capital adjustment cost function that is standard in business cycles models in order to avoid excessive volatility of investment.

Given that households can also consume goods produced abroad and that these goods are imperfect substitutes with domestically produced goods, consumption will be defined by an aggregator function:

$$C_t = \left[ \gamma_C^{\frac{1}{\nu_C}} \left( C_t^F \right)^{\frac{\nu_C-1}{\nu_C}} + (1 - \gamma_C)^{\frac{1}{\nu_C}} \left( C_t^H \right)^{\frac{\nu_C-1}{\nu_C}} \right]^{\frac{1}{\nu_C-1}}, \quad \gamma_C \in (0, 1), \nu_C > 0$$

where $C_t^F$ and $C_t^H$ are the consumption levels of foreign and domestic goods, $\gamma_C$ is the share of consumption of foreign goods in total consumption, and $\nu_C$ is the elasticity of substitution between home and foreign goods. Total real expenditure on consumption can be written as follows:

$$p_t^C C_t = p_t^H C_t^H + p_t^F C_t^F$$
where \( p_t^C \) is the aggregate price level of consumption; \( p_t^H \) and \( p_t^F \) are, respectively, the price levels of home and foreign goods. Clearly, only two of these prices are independent, so we choose to express every price in terms of the foreign goods, noting that \( p_t^H / p_t^F \equiv \text{tot}_t \) therefore the terms of trade of this economy, which we assume to be an exogenous process. Given predetermined levels of aggregate consumption, and relative prices, the household's intratemporal problem is to maximize 6 subject to 7; with associated optimality conditions:

\[
C_t^H = C_t (1 - \gamma_C) (p_t^H)^{-\nu_C} \\
C_t^F = C_t \gamma_C (p_t^F)^{-\nu_C}
\]  

(8)  

(9)  

and \( p_t^{HC} = p_t^H / p_t^C \), \( p_t^{FC} = p_t^F / p_t^C \), are relative prices that can be shown, after some algebra, to be determined by the terms of trade, as follows:

\[
p_t^{HC} = \left[ \gamma_C \text{tot}_t^{\nu_C-1} + (1 - \gamma_C) \right]^{\nu_C-1} \\
p_t^{FC} = \left[ \gamma_C + (1 - \gamma_C) \text{tot}_t^{1-\nu_C} \right]^{\nu_C-1} 
\]  

(10)  

(11)  

Households can also invest in home goods or foreign investment goods. Thus, gross investment will also be defined by an aggregator function:

\[
I_t = \left[ \gamma_I^{1/\nu_I} \left( I_t^F \right)^{\nu_I-1} + (1 - \gamma_I)^{1/\nu_I} \left( I_t^H \right)^{\nu_I-1} \right]^{\nu_I}, \quad \gamma_I \in (0, 1), \quad \nu_I > 0 
\]

where \( I_t^F \) and \( I_t^H \) are the investment levels of foreign and domestic goods, \( \gamma_I \) is the share of investment in foreign goods in total investment, and \( \nu_I \) is the elasticity of substitution between home and foreign investment goods. Total real investment can be written as follows:

\[
p_t^I I_t = p_t^H I_t^H + p_t^F I_t^F 
\]

(12)  

It is thus straightforward to see that the optimality conditions for investment will be similar to the ones for consumption:

\[
I_t^H = I_t (1 - \gamma_I) (p_t^H)^{-\nu_I} \\
I_t^F = I_t \gamma_I (p_t^F)^{-\nu_I}
\]  

(13)  

(14)
\[ p_{t}^{HI} = \left[ \gamma_{t} \text{tot}_{t}^{\nu_{t}-1} + (1 - \gamma_{t}) \right]^{\frac{1}{\nu_{t}}} \]  
(15)

\[ p_{t}^{FI} = \left[ \gamma_{t} + (1 - \gamma_{t}) \text{tot}_{t}^{1-\nu_{t}} \right]^{\frac{1}{1-\nu_{t}}} \]  
(16)

Having specified the intratemporal problem of the household, we are ready to specify the household's sequential budget. Recalling that the representative agent has access to a world capital market for noncontingent debt, her budget constraint is, therefore,

\[ p_{t}^{HC} W_{t} h_{t} + p_{t}^{HC} u_{t} K_{t} + p_{t}^{FC} q_{t} D_{t+1} = C_{t} + p_{t}^{FC} I_{t} + p_{t}^{FC} D_{t} + p_{t}^{HC} T_{t} \]  
(17)

where the first two terms in the LHS are factor receipts in period \( t \) in terms of consumption goods: In addition, \( q_{t} \) is the price at which the household can sell a promise to a unit of goods to be delivered at \( t + 1 \); while \( D_{t+1} \) is the number of such promises issued. The first three terms in the RHS describe expenditures in period \( t \), given by consumption, investment, and debt payments; where

\[ p_{t}^{FC} = p_{t}^{FI} / p_{t}^{FC} \]  
(18)

and the last term is given by lump sum taxes paid to the government.

The household chooses consumption, labor, next period debt, and capital to maximize her utility function (4) subject to the sequential budget constraint (17), the capital law of motion (5) and a no-Ponzi condition of the form

\[ \lim_{j \to -\infty} \frac{E_{t} D_{t+j}}{(1 + r)^j} \leq 0 \]  
(19)

Letting \( t \) denote the Lagrange multiplier associated with the sequential budget constraint, the first order conditions of the household's maximization problem are (17), (5), (19) holding with equality, and

\[ \lambda_{t} = (C_{t} - \tau \Gamma_{t-1} h_{t}^{\nu_{t}})^{-\sigma} \]  
(20)

\[ \tau \Gamma_{t-1} \omega h_{t}^{\nu_{t}-1} = p_{t}^{HC} W_{t} \]  
(21)

\[ \lambda_{t} p_{t}^{FC} q_{t} \Gamma_{t-1}^{-\sigma} = \beta \Gamma_{t}^{-\sigma} E_{t+1} \lambda_{t+1} p_{t+1}^{FC} \]  
(22)
\[
\begin{align*}
&= p_t^{IC} \lambda_t \left[ 1 + \phi \left( \frac{K_{t+1}}{K_t} - \mu \right) \right] \Gamma_t^{\sigma} \\
&= \beta \Gamma_t^{\sigma} E_t \lambda_{t+1} \left[ p_t^{HC}_{t+1} + p_t^{IC}_{t+1} (1 - \delta) + \frac{p_t^{IC}_{t+1} \phi}{2} \left( \frac{K_{t+2}}{K_{t+1}} \right) - \mu^2 \right]
\end{align*}
\]

3.3 Government

The government in this economy simply sets taxes equal to an exogenous level of government expenditure in each period:

\[ T_t = GOV_t \]  \hspace{1cm} (24)

Finally, note that, in equilibrium, the trade balance-to-output ratio will be determined as follows:

\[ TBY_t = \frac{Y_t - C_t - I_t - GOV_t}{Y_t} \]  \hspace{1cm} (25)

3.4 Interest Rates and Country Risk

We close the model by providing a simple theory for \( R_t \), the interest rate faced by emerging economies, following Neumeyer and Perri (2005) and Chang and Fernandez (2010). First, the price of the household’s debt is assumed to be given by a debt-elastic interest rate function,

\[ \frac{1}{q_t} = R_t + \psi \left[ \exp \left( \frac{D_t + 1}{\Gamma_t} - d \right) - 1 \right] \]  \hspace{1cm} (26)

where \( R_t \) is the specific rate at which international investors are willing to lend to the small, open, and tropical economy. Formally, this interest rate is defined as follows

\[ R_t = S_t R_t^* \]  \hspace{1cm} (27)

where \( R_t^* \) is the world interest rate for risky asset and \( S_t \) is the country specific spread over that rate, both of which will be assumed to be a stochastic processes to be defined next.

3.5 Driving Forces

There will be five sources of uncertainty in this economy. First, the transitory technology process is assumed to follow an AR(1) process in logs:
\[ \log a_t = \rho_a \log a_{t-1} + \varepsilon_t^a \]  

(28)

where \(|\rho_a| < 1\); and \(\varepsilon_t^a\) is an i.i.d. shock with mean zero and variance \(\sigma_a^2\).

Second, \(\Gamma_t\) is a term allowing for labor augmenting productivity growth. Following Aguiar and Gopinath (2007), we allow it to grow at a stochastic growth rate, \(g_c\). Formally,

\[ \Gamma_t = g_t \Gamma_{t-1} \]  

(29)

where

\[ \ln \left( \frac{g_{t+1}}{\mu} \right) = \rho_g \ln \left( \frac{g_t}{\mu} \right) + \varepsilon_t^g \]  

(30)

\(|\rho_a| < 1, \varepsilon_t^g\) is an i.i.d. process with mean zero and variance \(\sigma_g^2\), and \(\mu\) represents the mean value of labor productivity growth. A positive realization of \(\varepsilon_t^g\) implies that the growth of labor productivity is temporarily above its long run mean.

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Such a shock, however, is incorporated in \(\Gamma_t\) and, hence, results in a permanent productivity improvement.

Third, deviations of the world interest rate for risky asset, \(R_t^*\), from its long-run level are assumed to follow an AR(1) process

\[ \ln \left( \frac{R_t^*}{R^*} \right) = \rho_r \ln \left( \frac{R_{t-1}^*}{R^*} \right) + \varepsilon_t^r \]  

(31)

where \(|\rho_r| < 1\) and \(\varepsilon_t^r\) is an i.i.d. innovation with mean zero and variance \(\sigma_r^2\). Following Chang and Fernandez (2010) we allow for both permanent and transitory shocks to affect the country specific spread. To implement this idea, we assume that deviations of the country spread from its long-run level are a functions of deviations in the total factor productivity (Solow residual):

\[ \log \left( \frac{S_t}{S} \right) = -\eta E_t \log \left( \frac{sol_{t+1}}{sol} \right) \]  

(32)

where \(sol_t\) is the Solow residual, defined as \(sol_t = a_t g_t^\alpha\) and \(sol = \mu^\alpha\). Fourth, the terms of trade are assumed to evolve according to a simple AR(1) process in logs:

\[ \log tot_t = \rho_{tot} \log tot_{t-1} + \varepsilon_t^{tot} \]  

(33)
where \(|\rho_{tot}| < 1\); and \(\varepsilon^{tot}_{t}\) is an i.i.d. shock with mean zero and variance \(\sigma^{2}_{tot}\).

Importantly this specification differs from Mendoza (1995) in that we don’t allow for domestic productivity and terms of trade to be correlated.

Finally, following Canova (2007), the government expenditure process is assumed to be a function of its own past and lagged deviations in the level of output. Formally,

\[
\ln \left( \frac{GOV_{t+1}}{GOV_t} \right) = \rho_{gov} \ln \left( \frac{GOV_{t}}{GOV} \right) + \rho_{GY} \ln \left( \frac{Y_t}{Y} \right) + \varepsilon^{gov}_{t+1}
\]

where \(|\rho_{gov}| < 1\); and \(\varepsilon^{gov}_{t}\) is an i.i.d. shock with mean zero and variance \(\sigma^{2}_{gov}\), and \(\rho_{GY} \in \mathbb{R}\) is intended to capture the degree of procyclicality of public expenditure documented for developing economies.

### 3.6 Competitive Equilibrium

A competitive equilibrium path for this economy is a set of stationary processes along a balanced growth path for twelve allocations,

\[
\{Y_t, K_t, D_t, C_t, C^H_t, C^F_t, I_t, I^H_t, I^F_t, h_t, TBY_t, T_t\}_{t=0}^\infty
\]

and ten relative prices,

\[
\{R_t, q_t, \lambda_t, W_t, u_t, p^H_t, p^F_t, p^C_t, p^H_t, p^C_t\}_{t=0}^\infty
\]

satisfying the three optimality conditions for firms, (1)-(2)-(3); the fifteen intratemporal and intertemporal optimality conditions for the household (5)-(8)-(9)-(10)-(11)-(15)-(13)-(14)-(16)-(17)-(18)-(20)-(21)-(22)-(23); the government balanced budget rule (24); the trade balance-to-output definition (25); the country specific interest rate and spread processes (26)-(27); given the initial conditions for \(K_0\) and \(D_0\); \(\Gamma_{ij}\) and the stochastic processes fat; \(\{a_t, \Gamma_t, g_t, R^t, tot, GOV_t, sol_t\}_{t=0}^\infty\).

### 4 Estimation

We follow a Bayesian estimation strategy that has been increasingly used in the estimation of dynamic stochastic general equilibrium models\(^4\). The following sections briefly describe the estimation technique.

\(^4\) See An and Schorfhheide (2007) for an excellent survey of the theory and applications on DSGE models. For a textbook explanation see also DeJong and Dave (2007).
4.1 Bayesian Estimation Framework

We normalize the variables that trend in equilibrium by dividing them by the (lagged) trend level, \( \Gamma_{t-1} \). Following Schmidt-Grohe and Uribe (2004), the stationary dynamic system of equations is log-linearized and written in the canonical state-space form:

\[
x_{1,t+1} = M(\Theta) x_{1t} + v_{t+1}
\]
\[
x_{2,t} = C(\Theta) x_{1,t}
\]

where \( \{x_1, x_2\} \) are, respectively, state and control variable vectors; \( v_{t+1} \) is a vector of structural perturbations; and the matrices \( M(\theta) \) and \( C(\theta) \) are a function of the vector of structural parameters, \( \theta \). This system can be compactly written as a law of motion equation:

\[
\Psi_{t+1} = \Phi(\Theta) \Psi_t + Bv_{t+1}
\]

(36)

On the other hand, having observed a time series data on a vector \( X_t \), it can be expressed as a non-invertible linear combination of the state variables in a measurement equation:

\[
X_t = \Gamma \Psi_t + \epsilon_t
\]

(37)

where \( \Gamma \) is a conformable matrix that maps the observable time series of the observable elements \( \chi_t \) to their theoretical counterparts in \( \Psi_t \), while \( \epsilon_t \) are exogenous i.i.d. measurement errors. Equations (36) and (37) are the starting point for a time invariant Kalman filter with which one can recursively construct the likelihood function over the \( T \) data points of \( \chi_t \):

\[
L(X|\Theta) = \prod_{t=1}^{T} L(X_t|\Theta)
\]

(38)

From a Bayesian perspective, the observation of \( X \) is taken as given and inferences regarding \( \theta \) center on statements regarding probabilities associated with alternative specifications on \( \theta \) conditional on \( X \). By satisfying the likelihood principle, the Bayesian approach uses all the information from the data to make the probability statements on \( \theta \). Bayes Theorem is used to update our beliefs about \( \theta \). Formally:

\[
p(\Theta|X) \propto p(\Theta) L(X|\Theta)
\]

(39)
where \( p(0) \) is the prior distribution. The posterior distribution then allows us to make probability statements regarding the unknown parameters in our model.

As mentioned in the Introduction, we use quarterly data from Colombia between 1994:1 to 2008:4 with four macroeconomic aggregates: gross domestic product \( (Y) \), consumption \( (C) \), investment \( (I) \), and the trade balance-to-GDP \( (TBY) \). While the first three are observed in log-differences, the latter is observed in first differences. Hence, the observation of \( X \) is:

\[
X = \{ \Delta \ln Y_t, \Delta \ln C_t, \Delta \ln I_t, \Delta TBY_t \}_{t=1994:1}^{2008:4}
\]

and the system of measurement equations (37) is

\[
\begin{align*}
\Delta \ln Y_t &= \ln \mu + (\widehat{g}_t - \widehat{g}_{t-1}) + \widehat{g}_{t-1} + \epsilon^Y_t \\
\Delta \ln C_t &= \ln \mu + (\widehat{c}_t - \widehat{c}_{t-1}) + \widehat{g}_{t-1} + \epsilon^C_t \\
\Delta \ln I_t &= \ln \mu + (\widehat{i}_t - \widehat{i}_{t-1}) + \widehat{g}_{t-1} + \epsilon^I_t \\
\Delta TBY_t &= \widehat{b}_{by_t} - \widehat{b}_{by_{t-1}} + \epsilon^{TBY}_t
\end{align*}
\]

where \( \epsilon^N_t \) is distributed i.i.d. measurement error with mean zero and variance

\[
\sigma^2_N, N = Y, C, I, TBY.
\]

In order to report posterior statistics we need to be able to make random draws from the posterior distribution for which we will make use of advances in Monte Carlo Markov Chain (MCMC) theory to get dependent draws from the posterior distribution, \( p(\theta|\lambda) \). We follow, for the most part, the Random Walk Metropolis algorithm presented in An and Schorfheide (2007) to generate draws from the posterior distribution \( p(\theta|\lambda) \).

The algorithm constructs a Gaussian approximation around the posterior mode, which we first find via a numerical optimization of \( \ln \mathcal{L}(X|\theta) + \ln p(\theta) \), and use a scaled version of the inverse of the Hessian computed at the posterior mode to efficiently explore the posterior distribution in the neighborhood of the mode. It proved useful to repeat the maximization algorithm using random starting values for the parameters drawn from their prior support in order to gauge the possible presence of many modes in the posterior distribution\(^6\). Once this step is completed, the algorithm is used to make 150,000 draws from the posterior distribution of each case. The initial 50,000 draws are burned.

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\(^6\) See Data Appendix for more details.

\(^6\) The MATLAB codes that solve all the model’s extensions as well as the ones that carry out the estimation are available upon request.
Once \( p(\theta|\lambda) \) is approximated, point estimates as well as confidence intervals of the parameters can be obtained from the generated draws, in addition to functions of these parameters. Given that one of our goals is to assess the relative role of each driving force, two functions we will be interested in are structural variance decompositions and impulse response functions.

### 4.2 Benchmark Calibration and Priors

We choose to calibrate some of the deep parameters in the model while we estimate the rest. The choice of which parameters to estimate or calibrate is guided by the objectives of our investigation which is the study of the sources of fluctuations. For that reason we mainly estimate the parameters of exogenous driving forces along with other key parameters in determining business cycles.

Formally, let \( \Theta = [\Theta_1, \Theta_2]^\prime \), where \( \theta_j \) is the vector of parameters that we calibrate:

\[
\Theta_1 = [\sigma, \omega, \mu, \psi, \delta, d]^\prime
\]

(42)

The calibrated parameters are given in Table 2 and take conventional values. The coefficient of relative risk aversion is set at 2, and \( \omega \) is set so as to imply a labor supply elasticity of 1:6. The labor’s share of income, \( \omega \), is set to be 68\%\(^7\). We calibrate the long-run productivity growth, \( \mu \), equal to 1:0077 following consistent with a mean yearly GDP growth rate of 3:1 percent in the dataset.

As it is common in the literature on small open economy models, we set the parameter \( \psi \); determining the interest rate elasticity to debt, to a minimum value that guarantees the equilibrium solution to be stationary (Schmitt-Grohe and Uribe, 2003). The quarterly depreciation rate is assumed to be 20 percent so as to get an investment to GDP ratio close to 0:3, as it is observed in Colombian data. We calibrate \( d \), the debt-to-GDP ratio, to 0:23, the average of external debt as fraction of output in Colombia reported by Avella (2003). The steady state values of some of the variables in the model are also set according to long-run means in the data. We calibrate the government expenditure-to-GDP ratio to 0:19, and the annualized gross risky interest rate to 1:0816. We assume that there is no spread in the steady state, \( \delta = I \), and that \( \tau \) is endogenously determined so as to match a third of the time spent working in the long run, \( h = 1/3 \). Under this parameterization, the discount factor is pinned down in steady state to be \( \beta = 0.9976 \).

\(^7\) Note that in the models with financial frictions, \( \alpha \) is not exactly equal to labor share in the Financial Frictions model but it is rather calibrated as \( \alpha = \text{LaborShare} \cdot [1 + (R - I)]^{-\eta} \).

Thus, it will have an entire distribution determined by the posterior distribution of \( \theta \).
The vector $\theta_2$ gathers the other twenty two parameters we estimate:

$$
\Theta_2 = \begin{bmatrix}
\rho_a, \rho_g, \rho_r, \rho_{gov}, \rho_{tot}, \sigma_a, \sigma_g, \sigma_r, \sigma_{gov}, \sigma_{tot}, \\
\phi, \sigma_Y, \sigma_C, \sigma_I, \sigma_{TBY}, \bar{\theta}, \gamma_C, \gamma_I, \eta, v_C, v_I, \rho_{GY}
\end{bmatrix}^t
$$

(43)

Our prior beliefs over the estimated parameters are described in Table 3 and follow a rather agnostic approach as rather diffuse priors are assumed. All the priors over the AR(1) coefficients in the five stochastic processes are assumed to be distributed with a Beta distribution with mean 0.72 and a large standard deviation of 16 percent. The priors over the standard deviation of both the structural shocks and the data measurement errors are assumed to be distributed with a Gamma distribution with mean 2 percent and a standard deviation of 1 percent. The capital adjustment cost parameter is assumed to be distributed with a Beta distribution with mean 6 and a standard deviation of 346 percent.

Previous studies provide little statistical information on the size of the elasticity of the spread to the country’s fundamentals, $\eta$, and the fraction of the wage bill held as working capital, $\theta$. We use a Gamma prior with mean of 1.0 and a standard deviation of 50 percent for $\eta$, close to the value calibrated by Neumeyer and Perri (2005) to match the volatility of the interest rate faced by Argentina’s residents in international capital markets. As for $\theta$; we decided to specify a very diffuse prior, with the only restriction that it must lie between zero and one. For this purpose we used a Beta function with mean 0.5; and a considerable standard deviation of 22.4 percent.

The weights of importables in the consumption and investment aggregator functions are assumed to be distributed with a Beta function with mean 0.2 and a 10 percent standard deviation. This is motivated by the fact that imports are between 15-25 percent of total GDP in Colombia. The elasticity of substitution in the aggregator of both functions is chosen to be a Gamma distribution with mean 1.0 and a large standard deviation of 50 percent. Finally, the parameter governing the degree of countercyclicality in government expenditure is chosen to be normally distributed with mean 0.0 and a standard deviation of 100 percent.

### 5 Results

This section presents the results of the paper. First the posterior distribution of the estimated parameters is reported, together with functions of these parameters, variance decompositions and impulse response functions. Second, the performance of the estimated model in matching some of the main stylized facts of the Colombian business cycle is assessed as well as its out-of-sample forecasting performance. Finally, a robustness analysis is conducted by using a much longer and yearly dataset spanning from 1925 to 2008.
5.1 Posterior distributions

Table 4 reports the posterior distributions for the twenty two parameters estimated in $\theta$. The table reports for each parameter both the posterior mode and mean together with the 90 percent confidence interval. In addition, a plot of prior and posterior distribution is also presented Figure 1. Finally, impulse response functions and variance decompositions of the main macroeconomic aggregates are computed from the prior distributions and are presented, respectively, in Figure 2 and Table 5. A series of findings emerge from these results.

First, the data appears to be informative for most of the parameters as the posterior distributions significantly differ from the diffuse prior distributions, particularly for the parameters governing the standard deviations of the shocks, the degree of financial frictions, and the persistence of the shocks.

Second, the results clearly favor innovations in the transitory technology process and the interest rate faces in world markets as the most important driving forces of the Colombian business cycle. The forecast error variance decomposition results assign to technology shocks the 74 percent of the variance in output; 43 percent in consumption; 60 percent in investment; and 19 percent in the trade balance-to-GDP ratio. The share of the variability associated to interest rate shocks is most important for the trade balance-to-GDP ratio (76 percent); investment (37 percent); consumption (20 percent); and output (17 percent). From Figure 2, the impulse response of output, measured as deviations from its steady state, following an estimated one standard deviation shock to the transitory technology process peaks near 3 percent; while that associated to a positive interest rate shock makes output fall near 2 percent an its effects are more persistent through time.

Third, and perhaps surprisingly, the other three driving forces play a minor role in accounting for the Colombian business cycles. The estimated posterior mode ratio of the volatilities in the two technology processes is $\frac{\sigma_a}{\sigma_g} = 0.72 = 0.36 = 2:0$, which is clearly at odds with Aguiar and Gopinath (2007)’s finding for Mexico where they obtain a ratio $0:48 = 2:81 = 0:2$. Furthermore, using Aguiar and Gopinath (2007)’s measure for the random walk component of the Solow residual, a nonlinear function of the relevance of trend shocks relative to transitory shocks and defined as follows:

$$RWC = \frac{\alpha^2 \sigma_g^2 / (1 - \rho_g)^2}{[2/(1 + \rho_g)^2] \sigma_a^2 + [\alpha^2 \sigma_g^2 / (1 - \rho_g)]}$$

the mode of the RWC is found to be 0:77, close to two thirds the value estimated for Mexico in Aguiar and Gopinath (0:96). Consequently, the role played by growth shocks in accounting for the variance of the main macroeconomic aggregates is less than 7 percent, except for consumption (26 percent). Likewise, the share of government expenditure and terms of trade perturbations in accounting for the macro volatility is lower than 2 percent for any of the four time series, except for the share of terms...
of trade in accounting for consumption variability (11 percent). Finally, the impulse response functions for output after an estimated one standard deviation shock to any of these three structural shocks is either small and non persistent (0:2 following a growth shock) or non-statistically significant.

Fourth, while the posterior estimate for $\eta$ was high, the one for $\theta$ was close to zero, implying that the degree of financial frictions is important but mainly through the effects that transitory technology shocks have on the spread. The role of this financial friction in propagating transitory technology shocks is of crucial importance. This is evident from the last row of impulse response functions presented in Figure 2 where we plot the counterfactual case setting $\eta = 0$.

It is immediate to see that more than half of the response in output and the other variables is reduced when we artificially set the elasticity of the spread to expected movements in the country fundamentals to zero.

Fifth, the size of the sum of the standard deviation in the measurement errors is rather small when compared to the size of the estimated structural shock’s signaling that misspecification is not a serious problem and that the model successfully accounts for most part of the variability exhibited in the observables.

Sixth, the (little) information that appears to be in the data validates a small shares of importables in total consumption and investment and a low elasticity between home and foreign goods. Last, the data also shows evidence of a procyclical government expenditure.

5.2 Model Performance

The performance of the estimated model in matching some of the main stylized facts of the Colombian business cycle is assessed here by running two separate experiments. First, the model-based second moments of the main macroeconomic aggregates are computed and compared to those computed from the Colombian data. Second, a historical decomposition of the structural shocks is performed by using the smoothing properties of the Kalman filter and their accuracy in replicating the sharp business cycle observed in the late 1990s is assessed.

5.2.1 Selected Second Moments

Table 6 presents the unconditional second moments derived from the estimated model. The model-based moments were computed using the posterior modes for the estimated parameters. Thus, it should be noted that the comparison between the theoretical and sample second moments of the main four macroeconomic aggregates is clearly a stringent test on the model given that the estimation was not designed to match these moments in particular, unlike other methods like GMM. And it is clearly an even more stringent test for the comparison of the second moments in the main driving forces given that these were not even observed in the estimation.
The model achieves, nonetheless, a moderately good fit along most of the important dimensions highlighted in the second section. Indeed, the model successfully replicates the smooth consumption process, the volatile investment and the strong countercyclicality of the trade balance-to-GDP ratio, largely explained by investment variability. In terms of the driving forces, the model also matches closely the leading and countercyclical properties exhibited by the real interest rate. As for the terms of trade, while the model partially replicates the procyclicality observed in this variable it misses in matching its large volatility.

And the model fails completely by grossly overstating the procyclicality of the government expenditure.

5.2.2 Historical Decomposition

The second experiment by which the performance of the estimated model is assessed starts by computing a historical decomposition of the structural shocks using the smoothing properties of the Kalman filter. Following Hamilton (1994) and DeJong and Dave (2007) we use the state space representation (36) together with the observable equation (37) to construct an estimate of the state vector of variables along with innovations to these variables using the information contained in the entire sample:

\[ \{ x_{1t|T}, u_{i,t} \}_{i=1994:4}^{T=2008:4} \]

where the latter can be thought of as a measure of the structural shocks. Next, we use a subset of these structural shocks to simulate the evolution of the main four Colombian macroeconomic aggregates. In particular we are interested in the accuracy of the model in replicating the sharp business cycle observed in the Colombian economy in the late 1990s where a sustained period of growth that started in 1994 was followed by a sharp reversal in 1998 and particularly in 1999.

The time series of the smoothed driving forces together with their innovations are plotted in Figure 3. It is immediate to see that a sharp volatility characterizes the years 1996 to 2000. Positive transitory technology shocks characterize the early years (1996-1997), while a reversal of this trend along with a sharp increase in the smoothed interest rate process characterized the following years (1998-1999).

The accuracy of the structural shocks in replicating the sharp Colombian business cycle in the late 1990s is assessed in Figure 4. Only shocks to transitory technology and to the interest rate processes are considered. In order to gauge the relevance of financial frictions and interest rate shocks during this episode, the panels in the left column report the simulation using only transitory technology shocks and shutting down the degree of financial frictions, \( \eta = \theta = 0 \); while the panels to the right include interest rate shocks and set the value of \( \eta \) and \( \theta \) equal to their posterior modes.
The results of this experiment are quite surprising. The simulation incorporating solely technology shocks and no financial frictions that propagate these shocks (left panels) misses virtually all the distinctive properties of the Colombian cycle in this period. While the simulation produces only a very moderate fall in GDP, it does not exhibit any fall in consumption nor investment and even counterfactually produces a fall in the trade balance-to-GDP ratio. On the contrary, the simulation that includes both interest rate shocks and financial frictions remarkably matches the evolution of the Colombian macroeconomic time series. In particular, the sharp reversal in the trade balance and the downfall in investment are properly recovered. This corroborates what was mentioned above regarding (i) the relevance of interest rate shocks in accounting for the Colombian business cycle; and (ii) the central role played by financial frictions as propagating mechanism of other real driving forces (i.e. transitory technology shocks).

5.3 Bayesian Model Comparison and Forecasting Performance

When conducting Bayesian estimation of DSGE models, researchers often are interested in the out of sample forecasting performance of the model (see An and Schorfheide, 2007). This is done by computing the marginal likelihood which is done next. Rewriting (39) exactly, the Bayes Theorem implies that posterior beliefs about $\theta$; must respect:

$$p(\Theta|X) = \frac{L(X|\Theta)p(\Theta)}{p(X)}$$

where $p(X)$ is the model's marginal likelihood, defined as:

$$p(X) = \int L(X|\Theta)p(\Theta)d\Theta$$

Following An and Schorfheide (2007) the log-marginal likelihood can be rewritten as

$$\ln p(X) = \sum_{t=1}^{T} \ln p(x_t|X^{t-1})$$

$$= \sum_{t=1}^{T} \ln \left[ \int p(x_t|X^{t-1}, \Theta) p(\Theta|X^{t-1}) d\Theta \right]$$

thereby implying that marginal data densities capture the relative one-step-ahead predictive performance of the model.

The upper panel in Table 7 reports the log-marginal likelihood for the estimated model along with the likelihood and posterior values evaluated at the posterior mode. In order to gauge the forecasting performance of the various structural shocks, we conducted two separate experiments. First, we estimated the model adding only two
structural shocks, one of which was always transitory technology shocks, yielding four possible combinations. Second, we estimated the model removing only one shock at a time, with the exception of transitory technology shocks, again yielding four possible combinations. The results in terms of likelihood, posterior and marginal likelihood for the first and second experiments are reported in the middle and lower panels of Table 7. While the full model does better than most of the restricted models, interestingly, the out-of-sample performance of government shocks appears to be relevant. In that sense, while government expenditure shocks do not appear to contribute much to the in-sample fit of the model, they appear to be relevant for the out-of-sample fit of it.

5.4  A Longer Dataset, Colombia 1925-2008.

Garcia-Cicco et.al. (2009) have recently argued that a more accurate estimation of the relative weights of the growth component in developing countries' business cycles should be done using dataset that span over many years. Following this work, we estimate the model on a yearly dataset covering the period 1925-2008.

The upper panel of Table 8 summarizes the main aspects of this dataset using the same second moments used for the quarterly dataset. While some of the stylized facts remain valid, particularly the strongly countercyclicality of the a trade balance share of income, two noticeable characteristics emerge.

First, there is a sharp increase in the volatility of virtually all variables, particularly in investment, the terms of trade and government expenditure. Second, consumption exhibits now a higher volatility than output.

We estimate the model using this longer dataset and run a similar analysis as before. Table 9 reports posterior modes and compares them with the estimates using the shorter dataset; and Table 10 presents the results of the variance decomposition. Several results stand out. First, the role of growth shocks becomes significantly more relevant now. The ratio $\sigma_g/\sigma_y$ falls from 2:0 to 0:2 and the random walk component increases from 0:77 to 4:19. As a consequence of this almost half (46 percent) of output’s variance is explained by growth shocks, although the share of these shocks in the variance of the other main aggregates is not higher than 19 percent. Second, the role of terms of trade shocks is now much more important, particularly when accounting for the variance of investment (48 percent) and the trade balance share (64 percent). Third, interest rate shocks continue to be relevant, notably in explaining the variance of consumption (81 percent) and their share in output variance remains close to the levels estimated in the quarterly sample (17 percent). Fourth, the model successfully accounts for the new stylized facts as can be seen from the lower panel in Table 8. In particular, the higher volatility of investment and government expenditure are matched together with the relative higher standard deviation of consumption. The model, nonetheless does not generate a countercyclical trade balance share.

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8 Importantly, due to data availability, in these dataset it was impossible to exclude durable (and semi-durable) goods consumption from aggregate consumption and include it on investment as was done before.
6 Concluding Remarks

There exists a consensus regarding the differences in the business cycle patterns in developing and developed economies. Where a consensus does not seem to be emerging is on the key driving forces that can account for these differences.

While some studies argue that a standard RBC-type model, driven only by transitory and/or permanent shocks to the technology process, is enough to properly model business cycles in developing economies, others present conflicting evidence based on dataset covering longer periods or stress the role of other real driving forces.

We contribute to this debate by exploring the business cycle properties of Colombia, a developing -and "tropical"- economy. Our approach is more ambitious in the sense that not only we test for role of technology shocks but we also incorporate other potential real impulses. Motivated by the observation that, to date, there has been little empirical analysis of the role played by individual shocks, within a multiple-shock setting, in driving business-cycle movements in aggregate variables from developing countries, we build a DSGE model including a menu of real driving forces in addition to technology shocks including shocks to the interest rate in world capital markets coupled with financial frictions; terms of trade fluctuations; and a procyclical government spending process. The role of each driving force is empirically quantified by estimating the parameters of the exogenous shocks processes, along with a few other crucial parameters, within a Bayesian framework, using Colombian macroeconomic data.

We find interest rate shocks to be crucial in accounting for the Colombian business cycle while financial frictions play a central role as propagating mechanism for other real driving forces, in particular transitory technology shocks.

These two driving forces alone can account well for the observed properties of the Colombian business cycle such as the smooth consumption process, the volatile investment and the strong countercyclicality of the trade balance-to-GDP ratio.

They both are entirely responsible for the sharp economic downturn experienced in the late 1990s. Other structural shocks such as terms of trade fluctuations and level shifts in the technology process do not appear to be relevant in the past decade and a half, but their importance increases when a longer span of data is considered. Demand shocks, in the form of government consumption innovations account only for a trivial role of the variance of the macroeconomic aggregates but they appear to be relevant for the out-of-sample forecasting fit of the model.

We are thus skeptic as to whether business cycles in developing economies can be modeled with a standard RBC model augmented solely by technology shocks and hope that our findings help stimulate more research into more elaborated models of the business cycles observed in developing economies.
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### Table 1. Business Cycles Moments

| Variable | sd(X) | sd(X) / sd(gY) | Corr(X_t, X_{t+1}) | Corr(X_t, gY_{t-0}) | Corr(X_t, dTBY_i) |
|----------|-------|----------------|----------------------|----------------------|-------------------|
|          |       |                |                      | s = 1                | s = 0             |
|          |       |                |                      | s = -1               |                   |
| Developed Countries | | | | | |
| gY       | 0.97  | 1.00           | 0.10                 | 0.10                 | 0.10              |
| gC       | 0.87  | 0.91           | 0.07                 | 0.17                 | 0.43              |
| gI       | 3.41  | 3.50           | 0.06                 | 0.17                 | 0.46              |
| dTBY     | 0.98  | 1.07           | -0.15                | -0.12                | 0.06              |
| Developing Countries | | | | | |
| gY       | 0.87  | 1.00           | 0.23                 | 0.23                 | 0.23              |
| gC       | 2.82  | 1.62           | 0.11                 | 0.21                 | 0.53              |
| gI       | 7.14  | 4.13           | 0.11                 | 0.34                 | 0.51              |
| dTBY     | 2.49  | 1.56           | 0.11                 | -0.24                | -0.27             |
| Colombia | | | | | |
| gY       | 1.22  | 1.00           | 0.20                 | 0.20                 | 0.20              |
| gC       | 0.83  | 0.67           | 0.33                 | 0.36                 | 0.65              |
| gI       | 6.05  | 4.94           | 0.25                 | 0.29                 | 0.81              |
| dTBY     | 1.05  | 0.86           | 0.30                 | -0.43                | -0.53             |
| gR*      | 0.46  | 0.37           | 0.05                 | 0.00                 | 0.03              |
| gToT     | 5.29  | 4.32           | 0.12                 | 0.01                 | 0.33              |
| gG       | 2.29  | 1.87           | 0.30                 | 0.07                 | 0.17              |

Note: gX and dX denote log differences and linear difference, respectively. Y is output; C is private consumption; I is investment; TBY is trade balance-to-GDP ratio; R* is a proxy for the gross risky interest rate available to emerging economies similar to Colombia; ToT is a proxy of Colombian terms of trade index; and G is the level of public consumption. The source of data for Developed and Developing countries was Aguiar and Gopinath (2007). Colombian data is quarterly from 1994:1 to 2008:4. See appendix for more data sources and details.
Table 2. Calibrated Parameters

| Parameter | Description                  | Calibrated Value |
|-----------|------------------------------|------------------|
| \( \sigma \) | Intertemporal Elasticity of Substitution \(1 / \sigma\) | 2.0              |
| \( \omega \) | Labor Supply Elasticity \( \frac{1}{\omega - 1} \) | 1.6              |
| \( h \cdot \text{Share} \) | Labor Share of Income | 0.68             |
| \( R^* \) | Gross Annual Foreign Interest Rate | 1.0816           |
| \( \mu \)  | Long-run Gross Productivity Growth Rate | 1.0077           |
| \( \psi \) | Debt Elastic Interest Rate Parameter | 0.001            |
| \( \beta \) | Discount Factor | 0.9976           |
| \( S \) | Long-run Gross Country Interest Rate Premium | 1.0             |
| \( \delta \) | Depreciation Rate of Capital | 0.20             |
| \( d \)  | Debt-to-GDP Ratio \((D/Y)\) | 0.100            |
| \( G/Y \) | Government Expenditure Share of Income | 0.19             |
| \( h \) | Labor in steady state | 1/3              |

Note: Note that \( \alpha \) is not exactly equal to labor share \((h \cdot \text{Share})\) but it is rather \( \alpha = h \cdot \text{Share} \left(1 + (R^* - 1)\theta\right) \) and its distribution is a function of the distribution of \( \theta \).
### Table 3. Prior Distributions

| Parameter | Range | Density | Mean | S.D (%) | 90% Conf. Interval |
|-----------|-------|---------|------|---------|-------------------|
| $\rho_S$  | [0,1) | Beta [ 5.0 ; 2.0 ] | 0.72 | 16 | [ 0.42 ; 0.94 ] |
| $\sigma_S$ | $R^+$ | Gamma [ 4.0 ; 0.005 ] | 2.00 | 1.0 | [ 0.70 ; 3.91 ] |
| $\phi$    | $R^+$ | Gamma [ 3.0 ; 2.0 ] | 6.00 | 346 | [ 1.62 ; 12.6 ] |
| $\sigma_X$| $R^+$ | Gamma [ 4.0 ; 0.005 ] | 2.00 | 1.0 | [ 0.70 ; 3.91 ] |
| $\theta$  | [0,1] | Beta [ 2.0 ; 2.0 ] | 0.50 | 22.4 | [ 0.13 ; 0.87 ] |
| $\gamma_J$| [0,1] | Beta [ 3.0 ; 12.0 ] | 0.20 | 10.0 | [ 0.06 ; 0.39 ] |
| $\eta$    | $R^+$ | Gamma [ 4.0 ; 0.25 ] | 1.00 | 50 | [ 0.35 ; 1.95 ] |
| $\nu_J$   | $R^+$ | Gamma [ 4.0 ; 0.25 ] | 1.00 | 50 | [ 0.35 ; 1.95 ] |
| $\rho_{GR}$| $R$  | Normal [ 0.0 ; 1.0 ] | 0.00 | 100 | [ -1.66 ; 1.66 ] |
Note: Estimates obtained using four observables, \( \{g_Y, g_C, g_I, dTBY\} \) from the Colombian quarterly data, 1994:1-2008:4 (see Appendix for data sources). Estimations were done using measurement errors in all four variables. RWC refers to the random walk component, see text for details. Numbers in brackets report the 90 percent confidence intervals from each posterior distribution.

### Table 4. Prior / Posterior Distributions

| Parameter | Prior Distribution | Posterior Distribution | Parameter | Prior Distribution | Posterior Distribution |
|-----------|-------------------|------------------------|-----------|-------------------|------------------------|
| \( \rho_a \) | 0.72 [0.42, 0.94] | 0.97 [0.94, 0.99] | 100\( \sigma_y \) | 2.00 [0.70, 3.91] | 0.27 [0.11, 0.51] |
| \( \rho_g \) | 0.72 [0.42, 0.94] | 0.65 [0.43, 0.96] | 100\( \sigma_c \) | 2.00 [0.70, 3.91] | 0.37 [0.15, 0.67] |
| \( \rho_r \) | 0.72 [0.42, 0.94] | 0.98 [0.83, 0.99] | 100\( \sigma_I \) | 2.00 [0.70, 3.91] | 2.32 [1.96, 3.24] |
| \( \rho_{gov} \) | 0.72 [0.42, 0.94] | 0.78 [0.51, 0.86] | 100\( \sigma_{TBY} \) | 2.00 [0.70, 3.91] | 0.26 [0.09, 0.51] |
| \( \rho_{tot} \) | 0.72 [0.42, 0.94] | 0.86 [0.64, 0.98] | \( \gamma_c \) | 0.20 [0.06, 0.39] | 0.18 [0.02, 0.34] |
| 100\( \sigma_a \) | 2.00 [0.70, 3.91] | 0.72 [0.56, 0.87] | 100\( \sigma_g \) | 2.00 [0.70, 3.91] | 0.36 [0.09, 0.50] |
| 100\( \sigma_g \) | 2.00 [0.70, 3.91] | 0.36 [0.09, 0.50] | 100\( \sigma_r \) | 2.00 [0.70, 3.91] | 0.66 [0.25, 0.81] |
| 100\( \sigma_r \) | 2.00 [0.70, 3.91] | 0.66 [0.25, 0.81] | 100\( \sigma_{gov} \) | 2.00 [0.70, 3.91] | 0.84 [0.30, 1.72] |
| 100\( \sigma_{tot} \) | 2.00 [0.70, 3.91] | 1.64 [1.40, 3.38] | \( \eta \) | 1.00 [0.35, 1.95] | 0.89 [0.41, 1.21] |
| \( \phi \) | 6.00 [1.62, 12.6] | 6.89 [5.77, 9.38] | \( \nu_c \) | 1.00 [0.35, 1.95] | 0.75 [0.23, 1.61] |
| \( \theta \) | 0.50 [0.13, 0.87] | 0.04 [0.00, 0.12] | \( \nu_I \) | 1.00 [0.35, 1.95] | 0.75 [0.21, 0.65] |
| RWC | 2.73 [0.70, 24.59] | 0.77 [0.04, 27.84] | \( \rho_{GY} \) | 0.00 [-1.66, 1.66] | 1.04 [0.66, 1.94] |
Table 5. Forecast Error Variance Decompositions

| Structural Shock | $g_Y$ | $g_C$ | $g_I$ | $dTBY$ |
|------------------|-------|-------|-------|--------|
| $\varepsilon^a$  | 74.2  | 43.1  | 60.4  | 19.3   |
| $\varepsilon^g$  | 6.9   | 26.0  | 2.1   | 1.9    |
| $\varepsilon^f$  | 17.0  | 19.9  | 37.4  | 75.5   |
| $\varepsilon^{gov}$ | 0.0 | 0.0 | 0.0 | 2.0 |
| $\varepsilon^{tot}$ | 1.9 | 11.0 | 0.2 | 1.4 |

Note: $gX$ denotes log-differences, $dX$ denotes first differences. Variance decompositions computed from the estimation using four observables and measurement errors in all variables. Numbers reported using posterior mode estimates. Standard Errors are omitted for brevity but are available upon request. In the variance decomposition computations only the role of the structural shocks was taken into account. A time horizon of 40 quarters was used when computing the variance decomposition.

Table 6. Sample and Model-Based Business Cycles

| Variable | sd(X) | sd(X) / sd($g_Y$) | Corr($X_t$, $g_Y_{t-s}$) | Corr($X_t$, $g_Y_{t-s}$) | Corr ($X_t$, $dTBY_t$) |
|----------|-------|------------------|------------------------|------------------------|------------------------|
|          |       |                  | s=1                    | s=0                    | s=-1                   |
|          |       |                  |                         |                        |                        |
| Colombia |       |                  |                         |                        |                        |
| $g_Y$    | 1.22  | 1.00             | 0.20                   | 0.20                   | 1.00                   | 0.20                   | -0.53                  |
| $g_C$    | 0.83  | 0.67             | 0.33                   | 0.36                   | 0.65                   | 0.34                   | -0.49                  |
| $g_I$    | 6.05  | 4.94             | 0.25                   | 0.29                   | 0.81                   | 0.11                   | -0.84                  |
| $dTBY$   | 1.05  | 0.86             | 0.30                   | -0.43                  | -0.53                  | -0.16                  | 1.00                   |
| $gR^*$   | 0.46  | 0.37             | 0.05                   | 0.00                   | 0.03                   | -0.21                  | -0.11                  |
| $gToT$   | 5.29  | 4.32             | 0.12                   | 0.01                   | 0.33                   | 0.09                   | -0.39                  |
| $gG$     | 2.29  | 1.87             | 0.30                   | 0.07                   | 0.17                   | 0.06                   | 0.06                   |

Model Based Moments

| Variable | sd(X) | sd(X) / sd($g_Y$) | Corr($X_t$, $g_Y_{t-s}$) | Corr ($X_t$, $dTBY_t$) |
|----------|-------|------------------|------------------------|------------------------|
| $g_Y$    | 1.79  | 1.00             | 0.53                   | 0.53                   | 1.00                   | 0.53                   | -0.30                  |
| $g_C$    | 1.26  | 0.71             | 0.55                   | 0.55                   | 0.81                   | 0.48                   | -0.28                  |
| $g_I$    | 5.62  | 3.14             | 0.08                   | 0.12                   | 0.61                   | 0.37                   | -0.59                  |
| $dTBY$   | 1.18  | 0.66             | 0.20                   | -0.31                  | -0.30                  | -0.38                  | 1.00                   |
| $gR^*$   | 0.66  | 0.37             | -0.01                  | 0.01                   | 0.02                   | -0.19                  | 0.80                   |
| $gToT$   | 1.70  | 0.95             | -0.07                  | -0.01                  | 0.13                   | -0.02                  | 0.10                   |
| $gG$     | 5.56  | 3.11             | 0.92                   | 0.74                   | 0.52                   | 0.44                   | -0.51                  |

Note: $gX$ and $dX$ denote log differences and linear difference, respectively. See appendix for data sources. Model-based moments were computed using posterior mode. Confidence intervals are omitted for brevity but are available upon request.
| Models | Log-Likelihood | Log-Posterior | Marginal Log-Likelihood |
|--------|----------------|---------------|------------------------|
| Estimating the Full Model: with 5 Structural Shocks: \{\varepsilon^a, \varepsilon^g, \varepsilon^r, \varepsilon^{gov}, \varepsilon^{tot}\} | Full Model | 702.59 | 723.72 | 650.92 |
| Estimating the Model with Only Two Structural Shocks: \{\varepsilon^a, \varepsilon^x\} | \{\varepsilon^a, \varepsilon^g\} | 669.86 | 680.68 | 642.43 |
| | \{\varepsilon^a, \varepsilon^r\} | 668.73 | 680.01 | 638.08 |
| | \{\varepsilon^a, \varepsilon^{gov}\} | 685.94 | 694.58 | 654.96 |
| | \{\varepsilon^a, \varepsilon^{tot}\} | 672.06 | 687.46 | 645.25 |
| Estimating the Model Removing Only One Shock at a Time | No Interest Rate Shocks \{\varepsilon^a, \varepsilon^g, \varepsilon^{gov}, \varepsilon^{tot}\} | 699.87 | 715.59 | 655.23 |
| | No Terms of Trade Shocks \{\varepsilon^a, \varepsilon^g, \varepsilon^r, \varepsilon^{gov}\} | 701.53 | 716.27 | 651.94 |
| | No Growth Shocks \{\varepsilon^a, \varepsilon^r, \varepsilon^{gov}, \varepsilon^{tot}\} | 702.66 | 721.25 | 657.31 |
| | No Government Expenditure Shocks \{\varepsilon^a, \varepsilon^g, \varepsilon^r, \varepsilon^{tot}\} | 669.42 | 687.35 | 627.28 |

Note: Log-Likelihood levels computed in the posterior mode. Results on marginal data densities are approximated by Geweke's harmonic mean estimator with truncation parameter 0.5.
Table 8. Sample and Model-Based Business Cycles

| Variable | sd(X) | sd(X) / sd(gY) | Corr(Xₜ, Xₜ₋₁) | Corr(Xₜ, gYₜ₋₉) | Corr(Xₜ, dTBYₜ) |
|----------|-------|----------------|----------------|-----------------|----------------|
|          |       |                | s=1 | s=0 | s=-1 |                  |
| gY       | 2.44  | 1.00           | 0.32 | 0.32 | 1.00 | 0.32 -0.28       |
| gC       | 6.12  | 2.51           | -0.23 | 0.11 | 0.55 | 0.00 -0.46       |
| gI       | 17.78 | 7.29           | -0.21 | 0.27 | 0.26 | 0.30 -0.60       |
| dTBY     | 3.72  | 1.53           | -0.16 | -0.25 | -0.28 | -0.15 1.00       |
| gR*      |       |                |      |      |      |                  |
| gToT     | 14.44 | 5.92           | 0.00 | 0.02 | 0.09 | -0.01 -0.06      |
| gG       | 11.28 | 4.63           | -0.09 | 0.15 | 0.20 | 0.01 -0.05       |

Model Based Moments

| Variable | sd(X) | sd(X) / sd(gY) | Corr(Xₜ, Xₜ₋₁) | Corr(Xₜ, gYₜ₋₉) | Corr(Xₜ, dTBYₜ) |
|----------|-------|----------------|----------------|-----------------|----------------|
|          |       |                | s=1 | s=0 | s=-1 |                  |
| gY       | 3.29  | 1.00           | 0.42 | 0.42 | 1.00 | 0.42 0.11       |
| gC       | 5.71  | 1.74           | -0.05 | 0.18 | 0.44 | -0.07 -0.24     |
| gI       | 17.66 | 5.37           | -0.13 | -0.11 | 0.24 | 0.43 -0.48     |
| dTBY     | 3.80  | 1.16           | 0.02 | 0.18 | 0.11 | -0.03 1.00     |
| gR*      | 0.79  | 0.24           | -0.02 | 0.03 | 0.03 | -0.33 -0.01     |
| gToT     | 4.75  | 1.45           | -0.22 | 0.03 | 0.17 | -0.16 0.67     |
| gG       | 14.79 | 4.50           | 0.95 | 0.30 | 0.10 | 0.02 -0.22     |

Note: gX and dX denote log differences and linear difference, respectively. See appendix for data sources. Model-based moments were computed using posterior mode. Confidence intervals are omitted for brevity but are available upon request.
Table 9. Prior / Posterior Distributions. Estimation with Annual Data: 1925-2008

| Parameter | Prior Mode | Posterior Distribution | Parameter | Prior Mode | Posterior Distribution |
|-----------|------------|------------------------|-----------|------------|------------------------|
| $\rho_a$  | 0.72       | 0.97                   | $100\sigma_y$ | 2.00       | 0.27                   |
| $\rho_g$  | 0.72       | 0.65                   | $100\sigma_c$ | 2.00       | 0.37                   |
| $\rho_r$  | 0.72       | 0.98                   | $100\sigma_I$ | 2.00       | 2.32                   |
| $\rho_{gov}$ | 0.72      | 0.78                   | $100\sigma_{TBY}$ | 2.00      | 0.26                   |
| $\rho_{tot}$ | 0.72      | 0.86                   | $\gamma_c$ | 0.20       | 0.18                   |
| $100\sigma_a$ | 2.00     | 0.72                   | $\gamma_I$ | 0.20       | 0.15                   |
| $100\sigma_g$ | 2.00     | 0.36                   | $\eta$    | 1.00       | 0.89                   |
| $100\sigma_r$ | 2.00     | 0.66                   | $\nu_c$   | 1.00       | 0.75                   |
| $100\sigma_{gov}$ | 2.00   | 0.84                   | $\nu_I$   | 1.00       | 0.75                   |
| $100\sigma_{tot}$ | 2.00   | 1.64                   | $\rho_{gy}$ | 0.00       | 1.04                   |
| $\phi$   | 6.00       | 6.89                   | $RWC$     | 2.73       | 0.77                   |
| $\theta$ | 0.50       | 0.04                   |           | 0.80       | 4.19                   |

Note: Estimates obtained using four observables, (gY, gC, gI, dTBY) from the Colombian annual data, 1925-2008 (see Appendix for data sources). Estimations were done using measurement errors in all four variables. RWC refers to the random walk component, see text for details.
Table 10. Forecast Error Variance Decompositions.

| Structural Shock | gY  | gC  | gI  | dTBY |
|------------------|-----|-----|-----|------|
| $\varepsilon^a$  | 26,9| 1,4 | 9,9 | 1,0  |
| $\varepsilon^g$  | 45,7| 15,3| 18,6| 12,3 |
| $\varepsilon^r$  | 16,8| 80,9| 23,1| 22,6 |
| $\varepsilon^{gov}$ | 0,0 | 0,1 | 0,0 | 0,0  |
| $\varepsilon^{tot}$ | 10,6| 2,3 | 48,3| 64,0 |

Note: $\text{gX}$ denotes log-differences, $\text{dX}$ denotes first differences. Variance decompositions computed from the estimation using four observables and measurement errors in all variables. Numbers reported using posterior mode estimates. Standard Errors are omitted for brevity but are available upon request. In the variance decomposition computations only the role of the structural shocks was taken into account. A time horizon of 40 quarters was used when computing the variance decomposition.
Figure 1. Priors and Posterior Distribution Plots

Note: Each plot presents the kernel smoother of prior and posterior distributions.
Figure 2. Estimated Impulse Response Functions

Note: Each column tracks the response of output (Y); consumption (C); investment (I), trade balance-to-GDP ratio (TBY); and employment (h) as deviations from steady states, after an estimated one standard deviation shock to the transitory technology process (first row); the growth process (second row); the interest rate process (third row); the government expenditure process (fourth row); the terms of trade process (fifth row). Red dashed lines depict 90% confidence interval based upon the posterior distribution. The fifth row presents the estimated impulses after a one standard deviation shock to the transitory technology process (blue) and the impulse under the counterfactual experiment $\eta = \theta$. 
Figure 2. Estimated Impulse Response Functions (cont)
Figure 3. Smoothed Driving Forces and Innovations

Note: The first column tracks the smoothed driving force processes and the second column plots the smoothed innovations to these driving forces. Both are computed using the Kalman smoother and red dashed lines depict 90% confidence interval based upon the posterior distributions.
Figure 4. Simulating the Colombian Business Cycle: 1997-2000

Simulation using only transitory technology shocks and no financial frictions

Simulation using only transitory technology shocks and interest rate shocks with financial frictions

Note: The first column tracks the evolution of the main Colombian macro aggregates in logs (except for the trade balance-to-GDP ratio) using the Kalman-smoothed process of the transitory technology process assuming no financial frictions ($\eta = \theta = 0$). The second column tracks the evolution of the same aggregates using the Kalman-smoothed processes of the transitory technology and the interest rate and setting the parameters governing the degree of financial frictions ($h_q$) equal to their posterior mode estimates. The smoothed innovations were obtained using posterior modes. The simulations were computed without the smoothed measurement errors.