Introducing Loopedia*

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Loopedia is a new project for a public database of loop integrals. After stating the goals and desired properties of the project, two possible ways to characterize Feynman graphs are explained here: the Adjacency List and the Nickel Index. Furthermore, a first version of the Loopedia website is presented. It allows for automatic graph generation and the conversion from Adjacency List to Nickel Index.

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1. Introduction

Loop integrals play an important role in particle physics — especially for high-precision calculations. For one-loop integrals there are already public tools and repositories such as: LoopTools [1], Golem95 [2], PJFry [3, 4], QCDloop [5], ONELOOP [6], and Hepforge [7]. For two-loop and multi-loop integrals, however, there is no one-stop repository yet.

Loopedia aims to improve this situation. It will be a public database of loop integrals. The infrastructure is provided by the contributors of the project in a way that allows the physics community to upload integrals. Interested readers are invited to play with the existing Loopedia Web interface now and contribute integrals once it is fully online.

Loopedia stores the results for the integral along with the references to the work where it was computed. This will help researchers to find results from the literature.

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2. Characterization of graphs

Loop integrals are closely related to Feynman graphs. This allows us to use graph-theoretical methods for their characterization. We use two possible ways of describing a graph: the Adjacency List and the Nickel Index. Both are briefly explained in this section.

2.1. Adjacency List

To get the Adjacency List for a graph we have to number the vertices with integers. Next, we write down every connection (line) that exists among the vertices. For example, the graph of Fig. 1 would have the following Adjacency List:

\([-1, 0], [0, 1], [0, 2], [-1, 1], [1, 2], [2, -1]\). Each of the square brackets represents a line. For example \([1, 2]\) represents the connection between vertex 1 and vertex 2. The order of the connections is not important. External fields are denoted with \(-1\). In the current version of Loopedia, external fields cannot be numbered with \(-2, -3, -4, \ldots\), only with \(-1\).

The Adjacency List depends on the numbering of the vertices so that there is no obvious way to obtain a canonical (unique) one.

![Fig. 1: Triangle diagram with the Nickel Index: $e_{12} | e_{2} | e$ and an Adjacency List: $[-1, 0], [0, 1], [0, 2], [-1, 1], [1, 2], [2, -1]$.](image)

2.2. Nickel Index

The Nickel Index consists of several parts — one part for each vertex. Each part is closed with a vertical bar | (pipe symbol). Let us consider again the diagram from Fig. 1. It has the Nickel Index $e_{12} | e_{2} | e$. To construct this index, we first have to number the vertices with integers, starting at zero. Here, we have three vertices so that the labels are 0, 1, 2.

The first part of the Index (read the usual way from left to right) reads $e_{12}$ and lists just all connections that vertex number 0 has. Here, $e$ stands
for an external field. So this part tells us that vertex 0 is connected to vertex 1, vertex 2, and an external field.

The second part of the index, corresponding to vertex number 1, reads $e2|$ and lists all new connections of vertex 1 that were not stated in the previous part. In this example, the connection from 0 to 1 was already mentioned.

The third and last part, describing the connections of vertex 2, reads $e|$ and lists the one external connection of this vertex that was not mentioned by the previous parts.

Note that each line of the graph is represented by exactly one character of the index. Also, as each vertex has to be represented by one part, the number of vertical bars in the index is exactly the same as the number of vertices. This way, it may happen that a particular vertex has no new connections that we can mention in its corresponding part of the Nickel Index. Thus, we just write a closing vertical bar. An example of this case is shown in Fig. 2a.

![Graph](image)

(a) Nickel Index: $e12|34|56|e6|e6||$

Adjacency list: $[-1,1],[1,2],[1,4],[2,3],[3,4],[4,6], [3,5],[2,7],[6,-1],[6,5],[5,7],[7,-1]$

(b) Nickel Index: $e111|e|$

Adjacency list: $[-1,1],[1,2],[1,2],[1,2],[2,-1]$

Fig. 2: Further examples

This index depends highly on the way we number the vertices. The crucial part is that if we write down all possible indices (for all numberings) according to the presented rules, there will be exactly one of them which is minimal in a specific sense. This minimal index is called the real Nickel Index and can be used to extract many properties of the graph.

Two further example graphs and their corresponding Adjacency List and Nickel Index are shown in Fig. 2.

A more detailed description of the Nickel Index can be found in the paper of Batkovich et al. [8], where they introduce the GraphState library. It is used by Loopedia to manipulate Nickel Indices.
3. First version of Loopedia

The first version of Loopedia is already online and can be found at this address: http://www.mpp.mpg.de/~papara/loopediatest

At this stage, the user can type in an Adjacency List or a Nickel Index and the website will generate the corresponding graph using Graphviz [9]. If an Adjacency List was given, it will also display the corresponding Nickel Index. See screen-shots in Figs. 3 and 4.

4. Conclusion

The Loopedia project, a public database of loop integrals, was introduced. In order to characterize those integrals two notations were presented: the Adjacency List and the Nickel Index. A first version of the Loopedia website was demonstrated. It allows for automatic graph generation and conversion from Adjacency List to Nickel Index.

The next step is to implement an upload function so that the database can be filled and tested.
Fig. 4: If you select “Adjacency List”, type it in and click on “Click Here!”

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