Model of diffuse reflection of optical radiation from the surface of biological tissue, implemented on the basis of two-dimensional fractional Brownian motion process

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Abstract. The numerical model of the diffuse reflection of Gaussian beam from the surface of biological tissue is introduced. The two-dimensional fractional Brownian motion (fBm) with the Hurst index $H$ and the scale parameter $\sigma$ was used for the simulations of the tissue surface relief. For the surfaces described by fixed $\sigma = 0.1$ and $H = 0.55$, $H = 0.803$ (corresponds to the surface of a banana fruit), $H = 0.9$, the angular distributions of the reflected radiation intensity were calculated using a Kirchhoff integral approach. The resulting distributions considerably differ from each other. Therefore, the introduced model can be used for the solution of the inverse problem of finding the fBm parameters of tissue surfaces employing the experimentally measured distribution of the reflected radiation intensity.

1. Introduction
The surfaces of biological tissues represent complex spatial structures. Its conformation is conditioned by many external and internal factors. The external impacts can be chemical interactions, e.g. the action of the aceton on the surface of the epidermis causes an increase of the intercellular distance [1] leading to a considerable change of the morphology of its outer layer; mechanical action, e.g. the surface of the artery wall under the effect of regular stretching and compressing leads to an increase of its relief roughness [2]. The internal factors can be both the natural evolution of tissues and the development of pathologies, e.g. the surface of a healthy calcaneus is rougher compared to that affected by the osteoporosis [3]. The relief of the tissue surface serves as a source of information about its physiological state. As follows, the study of the tissue surface relief can be used for the medical diagnostics.

The contact profilometry is a conventional method used for the investigation of the surface relief. This technique is based on measurements of the degree of deformation of the metal needle that touches the surface. However, it is not suitable for the profiling of soft biological tissues. In biology the optical coherence tomography and the scanning electron microscopy (SEM) are usually employed [1, 4]. The first method is based on measurements of the difference between optical paths of the reference beam and the beam that interacts with tissue. In the case of SEM, the spatial diagram of the electrons scattered from the tissue surface is measured. This approach cannot be used for the ex-vivo studying because experiments should be performed in vacuum conditions.
Another, potentially more accessible method for investigation of surface topography is the analysis of the optical radiation reflected from the tissue surface. Since the surface of biological tissue is not absolutely mirror-like, the reflection will be diffuse. The properties of diffusely reflected radiation are determined by the angular function of the intensity distribution or the indicatrix of the diffuse reflection [5]. In the papers [5, 6] it was theoretically demonstrated that the surface relief of a biological tissue affects the indicatrix of reflection. With the increase of the surface roughness degree the intensity of the radiation reflected at large angles to the direction of the incident beam increases. Thus, the analysis of diffusely reflected radiation makes it possible to draw conclusions concerning the structure of the surface relief of a biological tissue.

The indicatrix of diffuse reflection is obtained on the basis of mathematical modeling of the interaction of an electromagnetic wave with a surface. For this purpose, a Kirchhoff integral approach is widely used [7, 8, 9].

In order to find the reflected field through the Kirchhoff integral the functionally defined surface model \( z = Z(x, y) \) in 3-dimensional space is necessary. In the papers [5, 6] the implementation of the stationary (statistical properties do not depend on the coordinates) Gaussian process was used for the description of biological tissues. However, in such case the structural features of real biological tissues are not taken into account. Firstly, the mean level of the surface relief depends on coordinates and the surface structure does not correspond to the stationary property [10]. Secondly, the location of the cell on the tissue surface depends on the location of the neighbor cells because of its common origin. Considering this fact the process has to be non-Markov, i.e. the value of the process in the point \((x_0, y_0)\) depends on the process implementation at \( x < x_0 \) and \( y < y_0 \). In order to satisfy such properties, it was proposed to use the fractional Brownian motion (fBm) [3, 10] that is a non-stationary, non-Markov Gaussian process with fractional qualities [11]. The last property of fBm takes into consideration that in fact all biological objects, from the molecular to ecosystem structure, are fractal structures [12].

fBm is a Gaussian process with the Gaussian distribution of the increments

\[ \Delta Z = Z(x + \Delta x, y + \Delta y) - Z(x, y) \]

that have zero expectation and the dispersion in accordance with the following law:

\[ D = \sigma^2 \left( \sqrt{\Delta x^2 + \Delta y^2} \right)^{2H} \]  \hspace{1cm} (1)

Here \( \sigma \) is the scale parameter, \( H \) is the quantity called the Hurst index \((0 < H < 1)\).

The two-dimensional fBm is essentially a combination of two equivalent one-dimensional fBms [13]: for any fixed coordinate \(\hat{y}\), the process \(Z(x, \hat{y})\) is a one-dimensional fBm with the Hurst index \(H\) and the scale parameter \(\sigma\). The same is valid for any fixed coordinate \(\hat{x}\). Moreover, for any \(\hat{x}\) and \(\hat{y}\): \(Z(\hat{x}, y)\) and \(Z(x, \hat{y})\) are independent. As a result, all properties of the two-dimensional fBm follow from the properties of the one-dimensional fBm.

For \( H > \frac{1}{2} \) the process is positively correlated, and for \( H < \frac{1}{2} \) the correlation is negative. As follows, in the first case the process is persistent, and in the second one it is antipersistent [11]. The larger value of \( H \) means that it is more likely that \( Z \) will keep increasing or decreasing in the next point. The larger Hurst index corresponds to the smoother surface implementation (see Fig. 1).
FBm is widely used for the simulations of natural surfaces [3, 10, 14]. In the paper [3] the fBm was used for the description of the calcaneus surface relief, and in the paper [10] it was used for the investigation of the surface profile of a banana fruit. The artificial profiles were generated basing on the fBm implementation with the geometric properties close to natural ones. It was demonstrated that fBm describes well the real surfaces of biological tissues and it was proposed to use the fBm parameters as the characteristics of the surface relief [10].

However, to the best of our knowledge, there are no numerical models that describe the reflection of optical radiation from tissue surfaces using fBm realizations, which could be used for the development of experimental methods for the research and diagnosis of biological tissue diseases.

In this work, the angular distributions of the intensity of the diffusely reflected Gaussian beam from the surfaces, constructed using the implementation of fBm with different Hurst indices H and fixed σ, were calculated. The simulations were performed using a Kirchhoff integral approach. This model is supposed to use for the processing of the experiment, based on the goniometric measurements of the diffuse reflected light from the tissue surface with a single-mode optical fiber as a radiation source.

2. Model description
The spatial distribution of the electric field amplitude $E^s$ of the electromagnetic wave was calculated using the Kirchhoff integral:

$$E^s = \iiint_E \frac{1}{4\pi} \left[ i k + \frac{1}{r} \right] E \cos \alpha - i \frac{\partial E}{\partial n} \frac{e^{-ikr}}{r} dF$$  \hspace{1cm} (2)

Here $k$ is the wavenumber; $E$, $\frac{\partial E}{\partial n}$ are the field amplitude at the surface and its derivative along the normal vector to the surface respectively; $\alpha$ is the angle between the normal vector and the radius vector $\vec{r}$ from the surface element $dF$ to the point above the surface. The amplitude of the electric field of the incident Gaussian beam propagating along $z$ axis can be expressed as:

$$E(x, y, z) = E_0 \frac{w_0}{w(z)} \exp \left( i \frac{k}{2q(z)} (x^2 + y^2) - ikz \right)$$  \hspace{1cm} (3)

Here $w_0 = 10 \, \mu m$ is the beam waist radius, $E_0 = 1000 \frac{V}{m}$ is the electric field amplitude at the origin, $k$ is the wave number, $w(z) = w_0 \sqrt{1 + \left( \frac{z}{z_r} \right)^2}$, $R(z) = z \left( 1 + \left( \frac{z}{z_r} \right)^2 \right)$, $q(z) = \left( \frac{1}{R(z)} + \frac{k}{\pi w(z)^2} \right)^{-1}$, $z_r = \frac{\pi w_0^2}{\lambda}$, $\lambda = 1 \, \mu m$ is the wavelength. The distance from the waist to the plane $Z = 0$ is $1 \, mm$ (see Fig. 2).

The integration was performed for the surfaces $Z(x, y)$ obtained as the realizations of a two-dimensional fBm with a scale parameter $\sigma = 0.1$ and different Hurst indices. The fBm surface simulation
algorithm was based on the Fourier filtering [12]. For this purpose, the random spectrum that satisfies the fBm spectral density theorem was constructed, and then its inverse Fourier transform was calculated. The resulting Fourier image was the approximation of the fBm implementation.

![Geometry of the model](image)

**Figure 2.** Geometry of the model

Numerical integration was performed over 180 μm × 180 μm surface area, while the incident beam diameter equals approximately 70 μm at the plane Z = 0. The x and y mesh consisted of 256 nodes. In order to obtain the angular distributions of the electric field amplitude, the Kirchhoff integral (2) was calculated for 800 points with y = 0 and z=1 mm, the distance between neighboring nodes along x was 10 μm. In order to obtain the intensity distribution at the screen (z=1 mm), the integral was calculated for 80×80 points located in the xy plane with the distance of 50 μm between the neighboring nodes in each direction.

3. Results

In Fig. 3 the indicatrices of the diffuse reflection of the normally incident Gaussian beam at y=0 are shown for the surfaces with σ=0.1 and H=0.55, H=0.803 (corresponds to the H of the banana fruit surface [10]), H=0.9. Also in Fig. 3 the corresponding distributions of the intensity of the reflected radiation at the 4×4 mm² screen located at the distance of 1 mm from the plane Z=0 and parallel to this plane are shown.
It can be observed that with the decrease of the parameter $H$ the reflection at large angles starts to make larger contribution to the total angular distribution of the intensity of diffuse reflected radiation. The smaller is $H$, the larger is the spot diameter at the screen. The characteristic spot diameters are approximately equal to 0.5 mm, 1 mm and 2.5 mm for the values of $H = 0.9$, $H = 0.803$ and $H = 0.55$ respectively.

4. Conclusion
The obtained results reveal that application of the Kirchhoff integral approach for the simulations provides sufficient sensitivity and can be used for the solution of the inverse problem, i.e. restoration of the surfaces Hurst indices, which characterize its relief, using the reflection indicatrix experimental data. It is highly likely that the experimentally measured angle distributions of diffuse reflected radiation from the different types of biological tissues will differ. On the basis of such differences it will be possible to calculate fBm parameters of real tissues and therefore analyze its relief.

Studies of the diffuse reflection of optical radiation from the surfaces of biological tissues in its different physiological states will make it possible to create devices for the diagnosis of pathologies that affect the surfaces morphology.

5. References
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