Constraining Hořava-Lifshitz gravity from neutrino speed experiments

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We constrain Hořava-Lifshitz gravity using the results of OPERA and ICARUS neutrino speed experiments, which show that neutrinos are luminal particles, examining the fermion propagation in the earth’s gravitational field. In particular, investigating the Dirac equation in the spherical solutions of the theory, we find that the neutrinos feel an effective metric with respect to which they might propagate superluminally. Therefore, demanding not to have superluminal or subluminal motion we constrain the parameters of the theory. Although the excluded parameter regions are very narrow, we find that the detailed balance case lies in the excluded region.

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I. INTRODUCTION

In September 2011 the OPERA collaboration announced the astonishing result that muon-neutrinos created in CERN CNGS beam in Geneva were detected in Sasso Laboratory in central Italy, faster than the time light would need to cover the same distance in vacuum \( \nu_\mu \). In particular, it was reported that \( \nu_\mu \) neutrinos arrive earlier than expected from luminal speed by a time interval

\[ \delta t = 57.8 \pm 7.8 \text{(stat.)}^{+8.3}_{-5.9} \text{(syst.)} \text{ ns}, \]  

(1)

corresponding to a superluminal propagation of an amount

\[ (v - c)/c = (2.48 \pm 0.28 \text{(stat.)} \pm 0.30 \text{(sys)}) \times 10^{-5}, \]  

(2)

where \( v \) is the neutrino velocity and \( c \) the light speed, a result that is in agreement with earlier 1\( \sigma \) MINOS announcements \[2\]. However, in February 2012, the OPERA collaboration announced that the “measured” superluminality was a result of a loose fibre optic cable. Indeed, on March 2012 the ICARUS collaboration, also using the CNGS neutrino beam, measured that the time of flight difference between the speed of light and the arriving neutrinos was \[2\]

\[ \delta t = 0.3 \pm 4.9 \text{(stat.)}^{+9.0}_{-9.0} \text{(syst.)} \text{ ns}, \]  

(3)

which is compatible with the simultaneous arrival of all events at a luminal speed.

Although the possibility of systematic errors was the reasonable explanation straightaway from the beginning, the OPERA announcement attracted the interest of theorists, who tried to explain it following many different paths: in \[3\] proposing simple models of Lorentz violation by hand, in \[4\] imposing a mass-dependent Lorentz violation, in \[6\] using models of energy-dependent velocities, in \[6\] with a Fermi-point splitting, in \[7\] using the CNGS neutrino beam, measured that the time interval

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In the present work, inspired by the works claiming that neutrino supeluminality is a local effect around earth \[31\] \[33\] and \[34\], the authors proposed the interesting idea of a local effective neutrino supelumination, without Lorentz violation, due to a coupling with a new spin-2 field or a scalar respectively. Finally, we have to mention that straightaway from the beginning there were works claiming that the superluminal interpretation was not correct: in \[35\] \[37\] it was argued that the superluminal neutrinos would decay through a number of channels, while in \[38\] the author put into question the convention for synchronization of clocks in non-inertial frames and in \[39\] \[41\] various other possible systematic errors were discussed.

In the present work, inspired by the works claiming that neutrino supeluminality is a local effect around earth \[31\] \[33\] and \[34\], we investigate the neutrino propagation in the effective background metric of earth’s gravitational field, in the context of Hořava-Lifshitz gravity. Since superluminality may be the case in a small region of the parameters of the theory, we use this result in order to constrain the parameters of Hořava-Lifshitz in order to be consistent with OPERA and ICARUS collaboration non-supeluminal results.

II. SPHERICAL SOLUTIONS IN HORÁVA-LIFSHITZ GRAVITY

Let us briefly review the spherical solutions of simple Hořava-Lifshitz gravity. The dynamical variables are the lapse and shift functions, \( N \) and \( N_i \), respectively, and
the spatial metric \( g_{ij} \) (roman letters indicate spatial indices). In terms of these fields the full metric is written as

\[
ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt),
\]

and the (anisotropic) scaling transformation of the coordinates reads: \( t \to t^2 \) and \( x^i \to lx^i \). As it is known, the action of the theory can be decomposed as

\[
S = \int dt d^3 x \sqrt{g} N \{ \mathcal{L}_0 + \mathcal{L}_1 \}
\]

\[
\mathcal{L}_0 = \frac{2}{\kappa^2} (K_{ij} R^{ij} - \lambda R^2) + \frac{\kappa^2 \mu^2 (\Delta R - 3 \Lambda^2)}{8(3\lambda - 1)}
\]

\[
\mathcal{L}_1 = \frac{\kappa^2}{2w^2} C_{ij} C^{ij} - \frac{\kappa^2 \mu}{2w^2} \sqrt{g} R_{ij} \nabla_j R_k + \frac{\kappa^2 \mu^2}{8} R_{ij} R^{ij} - \frac{\kappa^2 \mu^2 (1 - 4\lambda)}{32(3\lambda - 1)} R^2,
\]

where \( K_{ij} = (g_{ij} - \nabla_i N_j - \nabla_j N_i)/2N \) is the extrinsic curvature and \( C^{ij} = \epsilon^{ijkl} \nabla_k (R^l_j - R^l_j/4)/\sqrt{g} \) the Cotton tensor, and the covariant derivatives are defined with respect to the spatial metric \( g_{ij} \). \( \epsilon^{ijk} \) is the totally antisymmetric unit tensor, \( \kappa, w, \mu \) and \( \Lambda \) are constants (we have already performed the usual analytic continuation of the parameters \( \mu \) and \( w \) and thus \( \Lambda \) is positive), and \( \lambda \) is the dimensionless constant that determines the flow between IR and UV. We mention here that the peculiar second term in (10), which allows for a quantum inheritance principle \([42, 43]\), since the \((D + 1)\)-dimensional theory acquires the renormalization properties of the \( D \)-dimensional one. Finally, it is straightforward to see that for the light speed, the gravitational Newton’s constant\(^1\) and the effective cosmological constant we obtain:

\[
c = \frac{\kappa^2 \mu}{4} \sqrt{\frac{\Lambda}{3\lambda - 1}}, \quad G = \frac{\kappa^2}{32\pi c}, \quad \Lambda_{eff} = \frac{3\kappa^2 \mu^2 \Lambda^2}{16(3\lambda - 1)}.
\]

As one observes, the light speed flows too, however one can still set it to 1, and consider photons to propagate with this speed always, which will be the reference speed in Hořava-Lifshitz gravity.

Under different assumptions there are many spherical solutions in the gravitational scenario at hand \([47, 50]\), which extract the extra terms comparing to General Relativity. For the purpose of this work we desire to remain in a general but still simple level. Thus, we should go beyond the detailed balance condition, which proves to lead to theoretical and observational problems \([51–54]\), but still keeping the structure of the theory simple.

\( ^1 \) Note that in theories with Lorentz invariance breaking the “gravitational” Newton’s constant, that is the one that is read from the action, does not coincide with the “cosmological” Newton’s constant, that is the one that is read from the Friedmann equations \([46]\), but this is irrelevant for the purposes of this work where we focus on non-cosmological scales.

Therefore, it is adequate to deform action (4) as \([47, 48]\)

\[
S = \int dt d^3 x \sqrt{g} N \{ \mathcal{L}_0 + (1 - \epsilon^2) \mathcal{L}_1 \}
\]

with \( \epsilon \) a parameter.

Seeking for static, spherically symmetric solutions with the metric ansatz

\[
ds^2 = -N(r)^2 dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2 \theta d\phi^2),
\]

and setting \( \lambda = 1 \), as expected for earth scales, one obtains:

\[
N(r)^2 = f(r) = 1 + \frac{\Lambda r^2}{1 - \epsilon^2} - \sqrt{\alpha^2 (1 - \epsilon^2) \sqrt{\Lambda r} + \epsilon^2 \Lambda r^4}.
\]

In this expression the integration constant \( \alpha \) can be expressed in terms of the total mass of the spherical object and the gravitational Newton’s constant \([47, 48]\).

We mention here that the peculiar second term in \([110]\), which will play the central role in the following discussion, arises in the majority of the corresponding solutions \([47, 50]\). For example, if we take the Kehagias-Sfetsos (KS) model \([50]\) and extend it to the minimal beyond-detailed-balance case (that is taking a general coefficient of the Ricci scalar term in the action (11)) we can obtain the KS spherical solution

\[
N_{KS}(r)^2 = f_{KS}(r) = 1 + qr^2 - \sqrt{q^2 r^3 + 4qMG},
\]

where \( q \) in now a free parameter, negative due to the analytic continuation (in this work we have transformed the parameters \( \mu \) and \( w \) of \([50]\) as \( \mu \to i\mu \) and \( w^2 \to -iw^2 \) \([47]\), and \( M \) is the total mass (note that one could alternatively obtain the above solution keeping the detailed-balance version of KS model, but move slightly away from \( \lambda = 1 \)).

### III. NEUTRINOS MOTION IN EARTH’S GRAVITATIONAL FIELD

Let us now investigate the propagation of fermions, and in particular of neutrinos, in earth’s gravitational field. Considering a massive Dirac field in a curved background \( g_{\mu\nu} \), the equation of motion reads \([55]\):

\[
[\gamma^a \epsilon^a_{\mu} (\partial_\mu + \Gamma_\mu) + \frac{m}{\hbar}] \Psi = 0,
\]

where \( m \) is the fermion mass and \( \hbar \) the Planck’s constant. In this relation \( \gamma^a \) is the inverse of the vierbein tetrad field \( e^a_{\mu} \), defined as \( g_{\mu\nu} = \eta_{ab} e^a_{\mu} e^b_{\nu} \) with \( \eta_{ab} = \text{diag}(-1,1,1,1) \), \( \gamma^a \) are the Dirac matrices (taken in the standard representation \([54]\)) and \( \Gamma_\mu \) is the spin connection given by

\[
\Gamma_\mu = \frac{1}{8} [\gamma^a, \gamma^b] e^a_{\nu} e_{\nu\mu;\lambda},
\]
where the covariant derivative of $e_{b
u}$ is as usual $e_{b
u;\mu} = \partial_{\mu}e_{b\nu} - \Gamma_{\mu\nu}^{\rho}e_{b\rho}$.

Let us investigate the Dirac equation in the earth's background, considering that its gravitational field is given by (9), that is in a vierbein reading as

$$e^\mu_a = \text{diag} \left( \frac{1}{\sqrt{f(r)}}, \frac{1}{r}, \frac{1}{r \sin \theta} \right),$$

(14)

with $f(r)$ given by (10) or (11). Neglecting for simplicity the spin connection $\Gamma^\mu_{\nu\rho}$, which proves to be vary small, the Dirac equation (15) under the geometry (14) reads:

$$ \left( \frac{\gamma^0}{\sqrt{f(r)}} \partial_t + \sqrt{f(r)} \gamma^1 \partial_r + \frac{\gamma^2}{r} \partial_\theta + \frac{\gamma^3}{r \sin \theta} \partial_\phi + \frac{m}{\hbar} \right) \Psi = 0. $$

(15)

From this relation one can immediately see that the neutrinos feel an effective metric, and that their velocity is simply

$$ v(r) = f(r), $$

(16)

and in particular if they propagate in an approximately constant $r$, equal for instance with the earth’s radius $r = R_\oplus$, their speed will be $v = f(R_\oplus) = \text{const}.$

Additionally, we can verify this result by approximately solving the Dirac equation (15) under certain assumptions. In particular, in the standard Dirac matrices representation the fermion wave function is written as

$$ \Psi(t, r, \theta, \phi) = \begin{pmatrix} A(t, r, \theta, \phi) \\ B(t, r, \theta, \phi) \end{pmatrix} \exp \left[ \frac{i}{\hbar} I(t, r, \theta, \phi) \right]. $$

(17)

Without loss of generality, and in order to avoid difficulties of solving Dirac equation in spherical coordinates, and using the spherical symmetry, we can assume that $A$ and $B$ are constants, while $I(t, r, \theta, \phi) = -\omega t + p(r)r + \Theta(\theta, \phi) \frac{\hbar}{\Lambda}$, with $p(r)$ the neutrino momentum. In such a case the two relevant equations read

$$ - \frac{A}{\sqrt{f(r)}} \omega + B \sqrt{f(r)} p(r) + mA = 0 $$

and

$$ - \frac{B}{\sqrt{f(r)}} \omega - A \sqrt{f(r)} p(r) + mB = 0, $$

(18)

and thus the solution condition (the determinant of $A$, $B$ coefficients to be zero) leads to the dispersion relation

$$ \omega^2 = f(r)^2 p(r)^2 + m^2 f(r). $$

(19)

In the massless case we can see that both the group velocity $\partial \omega/\partial p$ and the phase velocity $\omega/p$ are equal to $f$, that is $v(r) = f(r)$.

In summary, we showed that the neutrino’s velocity in the earth’s gravitational field in Hořava-Lifshitz gravity is equal to $v = f(r)$, with $f(r)$ given by (10) or (11) according to the specific solution subclass one uses. Observing the form of $f(r)$ we can clearly see that $v(r)$ may becomes superluminal, that is $v(r) > 1$. Clearly this is not the case in General Relativity spherical solutions, where the examination of the Dirac equation, similarly to the above procedure, leads always to $v < 1$.

Let us now come to the OPERA and ICARUS experiments. Since the neutrino motion takes place approximately on earth’s surface, we deduce that the neutrinos have a constant velocity $v = f(R_\oplus)$. Thus, if we want this not to be superluminal but not subluminal either, at an accuracy of $10^{-7}$ of the ICARUS result, we deduce that the parameter $\epsilon$ in solution subclass (10) must be in the interval $\epsilon \gtrsim 10^{-30}$ (we use relations (11) in order to set $c$ and $G$ to 1 and then we use the values of $\Lambda_{eff}$, $R_\oplus$ and $M_\oplus$ in these units). Similarly, for the Kehagias-Sfetsos solution subclass we can see that the observed neutrino luminality is obtained for $\epsilon \lesssim -10^{-21}$.

IV. DISCUSSION

In the present work we constrained Hořava-Lifshitz gravity using the data form OPERA and ICARUS neutrino speed experiments which show that neutrinos are luminal particles, by examining the fermion propagation in the earth’s gravitational field, considering the gravitational sector to be of Hořava-Lifshitz type. In particular, we used the spherical solutions of the theory going beyond the detailed-balance condition, and in such a background we investigated the Dirac equation. We found that the neutrinos feel an effective metric with respect to which they might propagate superluminally. The reason for such a behavior is that in spherical Hořava-Lifshitz solutions one obtains an extra positive term in the effective metric, and subsequently in the fermion velocity. In general, such a result is expected for Lifshitz-type theories and it plays the role of the “anti-gravity” source that is needed for superluminality, and indeed our own result in the specific case of Hořava-Lifshitz gravity is in agreement with the general qualitative result of [25, 26].

Therefore, if one desires not to have superluminal or subluminal motion, then the parameter $\epsilon$ in [47, 48] formulation must be in the range $\epsilon \geq 10^{-30}$, while the parameter $q$ in KS formulation of [50] must be in the range $q \lesssim -10^{-21}$. Clearly the excluded parameter regions are very narrow, that is Hořava-Lifshitz gravity predicts luminal motion in a large subspace of its parameter space. However, we can clearly see that the detailed balance case (corresponding to $\epsilon = 0$ in [47, 48]) is excluded (KS solution is already beyond the detailed balance), which is in agreement with theoretical works that exclude this case due to instabilities [51, 52].

We close this work with two comments. The first is that if one desires to apply the above analysis in neutrinos coming from galactical distances, then he should take into account that away from the earth’s surface the background metric is not spherical and it is not determined by the earth anymore, but from the sun, the other planets, the other stars etc, resulting to the Friedmann-
Robertson-Walker metric, where the above procedure results to luminal speed for massless neutrinos. This is in agreement with anti-neutrino observations from the SN1987A supernova, which impose the stringent constraint $(v - c)/c < 2 \times 10^{-9}$.

The second point is what version of Hořava-Lifshitz gravity must be used, and which solution subclass. In the present work we desired to provide two examples where superluminality is theoretically possible in Hořava-Lifshitz context, thus we chose a simple version of Hořava-Lifshitz gravity, allowing also from a departure from the detailed-balance condition, as a representative example, despite the fact that more complicated extensions seem to be theoretically more robust. Clearly, one should repeat the above procedure for such modified theories in order to constrain them, however the complication of the scenario does not allow even for an acceptable examination of general spherical solutions.

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