The Non-mechanistic Character of Quantum Computation

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Abstract

The higher than classical efficiency exhibited by some quantum algorithms is here ascribed to their non-mechanistic character, which becomes evident by joining the notions of entanglement and quantum measurement. Measurement analogically sets a (partial) constraint on the output of the computation of a hard-to-reverse function. This constraint goes back in time along the reversible computation process, computing the reverse function, which yields quantum efficiency. The evolution, comprising wave function collapse (here a revamped notion), is non-mechanistic as it is driven by both an initial condition and a final constraint. It seems that the more the output is constrained by measurement, the higher can be the efficiency. Setting a complete constraint, by means of a special Zeno effect, yields (speculatively) NP-complete=P.

I. INTRODUCTION

The problem of understanding why quantum computation is more efficient than classical computation has recently received a systematic attention: a reason fundamental enough might be leveraged for broadening the spectrum of efficient quantum algorithms. Ekert and Jozsa (1998) have demonstrated that quantum entanglement is essentially involved in providing the efficiency (see also Kitaev, 1997), but until now quantum measurement has been left in the background. We will show that, by explicitly considering it, quantum computation reveals a non-mechanistic or teleological character as far as it is “intelligently” driven by both initial and final conditions.
Thus, mechanism (everything driven by initial conditions, randomness ≡ ignorance of initial conditions, blindness to final conditions), a central dogma of classical science, would be scientifically disproved by quantum computation efficiency (first shown by Deutsch, 1985).

This will be shown by working on Simon’s algorithm (Simon, 1994), which is summarized in the following. Given a function $f : B^n \rightarrow B^n$, with $B = \{0, 1\}$, 2-to-1 with periodicity $r$ (e.g. fig. 1a), the problem is finding $r$ in poly (n) steps. We have to assume that, given a value $x$ of $x$, the computation of $f(x)$ requires poly(n) steps, whereas given a value $f$ of $f(x)$, the computation of $x$ and $x + r$ such that $\overline{f} = f(x) = f(\overline{x} + r)$ requires exp(n) steps: the function must be hard to “reverse” with classical computation (“invert” is avoided since the function has no inverse).

![Fig. 1](image)

Let $a$ (b) be the register containing $x$ (y), $H_a$ be the Hadamard transform on register $a$, $N = 2^n$. Simon’s algorithm (fig. 1b) is implemented through the following action:

a) prepare: $|\varphi(t_0)\rangle = |0\rangle_a |0\rangle_b$;

b) perform $H_a$: $|\varphi(t_1)\rangle = \frac{1}{\sqrt{N}} \sum_x |x\rangle_a |0\rangle_b$;

c) for each $x$, compute $f(x)$, put result in $b$: $|\varphi(t_2)\rangle = \frac{1}{\sqrt{N}} \sum_x |x\rangle_a |f(x)\rangle_b$;

d) measure $f(x)$, obtaining say $\overline{f} : |\beta(t_3)\rangle = \frac{1}{\sqrt{2}} (|\overline{x}\rangle_a + |\overline{x} + r\rangle_a) |\overline{f}\rangle_b$ ($|\beta(t_3)\rangle$ is not a function of the former states only, this is emphasized by changing notation from $\varphi$ to $\beta$);

step (d) is unnecessary but makes understanding easier;

e) perform $H_a$: $|\beta(t_4)\rangle = \frac{1}{\sqrt{2}} \sum_z (-1)^{\overline{x}z} [1 + (-1)^{r\cdot z}] |z\rangle_a |\overline{f}\rangle_b$; the sign $\cdot$ denotes the module 2 internal product of two binary numbers (seen as row matrices);

f) measure $z$ (time $t_5$): $r \cdot z$ must be 0 for registered $z$; see the form of $|\beta(t_4)\rangle$;

1 This work is influenced by the idea (due to Finkelstein, 1996) that there are only actions, initial and final ones.
g) by repeating the overall process a number of times poly(n) on average, a number of constraints $r \cdot z^{(i)} = 0$ sufficient to identify $r$ is gathered.

II. THE NOTION OF NON-MECHANISTIC COMPUTATION

In order to show the non-mechanistic character of Simon’s algorithm, it is convenient to circumscribe its central part from $t_1$ to $t_3$ where “efficiency” is achieved: the algorithm leading and trailing edges involve neither entanglement nor quantum efficiency. To facilitate exposition, let us say that $|\beta(t_3)\rangle$ already contains the readable period $r$: readable by means of the algorithm trailing edge, in a polynomial number of repetitions of the whole process; or readable for short by ignoring polynomial differences of efficiency.

With reference to steps (c) and (d), we should note that $t_2 < t_3$. $t_2 < t_3$ would allow for $t_2 = t_3$, making the quantum state two-valued ($|\varphi(t_2)\rangle$ and $|\beta(t_3)\rangle$) at the same time), a possibility that should be discarded. It is convenient to introduce the notation $|\beta\rangle_a = \frac{1}{\sqrt{2}}(|\varphi\rangle_a + |\varphi + r\rangle_a)$, thus $|\beta(t_3)\rangle = |\beta\rangle_a |\overline{f}\rangle_b$. Besides producing the random outcome $\overline{f}$ in register $b$, collapse makes an intelligent choice in register $a$, by selecting the superposition $|\beta\rangle_a$ which contains the “readable” period $r$, thus leading toward problem solution. The essential point is that $|\beta\rangle_a$ is a non-redundant function of both the initial condition $|\varphi(t_2)\rangle$ and the measurement outcome $\overline{f} = f(t_3)$, namely a final condition occurring at time $t_3 > t_2$ and not univocally determined by the initial condition since wave function collapse is in between:

$$|\beta\rangle_a = \sqrt{\frac{N}{2}} \langle f(t_3)|_b |\varphi(t_2)\rangle .$$  \hspace{1cm} (1)$$

According to equation (1), the mechanistic notion that everything is determined by initial conditions (with randomness $\equiv$ ignorance of initial conditions), holding in classical computation, is violated: $|\beta\rangle_a$ is clearly determined by both initial and final conditions.

We will show how non-mechanism is leveraged in Simon’s algorithm from two different perspectives:
(i) Things might be clearer by back-dating the outcome of collapse at time $t_{1+}$, after step (b) and before (c). The final actions of (in reverse order) registering $\overline{f}$, measuring register $b$, and computing $f(x)$, change $|\varphi(t_1)\rangle$ into $|\beta(t_{1+})\rangle = \frac{1}{\sqrt{2}} (|\overline{x}\rangle_a + |\overline{x} + r\rangle_a ) |0\rangle_b$, where the arguments $\overline{x}$ and $\overline{x} + r$ are such that their function (computed afterward) will be $\overline{f}$; these final actions have therefore reversed the computation of the direct function, by running it back in time (starting from $t_3$ and $\overline{f}$), thus taking the same time and achieving higher than classical efficiency.

In equivalent terms, it can be said that the measurement outcome goes back in time yielding efficiency; it is therefore essential that the computation process is reversible (Bennett, 1979; Fredkin and Toffoli, 1982). We should note that, according to the current model, this backward propagation is confined within the unobserved, reversible life of the quantum system, namely between the initial measurement (required to prepare the system) and the final measurement, without any possibility of carrying information back in time in the classical world.

(ii) The following yields another perspective. The final action of measuring $f(x)$ both creates the output constraint $f(x) = f(x + r) = \overline{f}$ and selects the superposition of the two arguments $\overline{x}$ and $\overline{x} + r$ which satisfy it. Introducing and satisfying this constraint is an essential step to solve the problem; doing both things at once, analogically, yields quantum computation efficiency. On the contrary, of course no independent constraint can be set on the output of classical, mechanistic computation, which is the deterministic propagation of the input of a reversible Boolean network.

The teleological character of quantum computation is clear: we are dealing with a time evolution (comprising collapse) satisfying (i.e. driven by) both an input condition and an output constraint — as implied by eq. (1). We should note that putting a constraint on the

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2 According to von Neumann, collapse is atemporal as it can be back-dated any time during the unobserved life of the quantum system between initial and final measurement.
output of a hard to reverse function is a general way of creating an NP problem.

Interestingly, non-mechanism brings us to re-examine the notion of causality in the quantum framework. Causality should go forward in time during the direct computation of \( f \) and backward in time during reverse computation, in the same time-interval. Let us see that there is no contradiction. In fact both the forward propagation \(|\varphi(t)\rangle\) and the backward propagation \(|\beta(t)\rangle\) (\(\varphi \) and \(\beta\) stand for forward and backward in time) are, up to an undefined overall phase \(\delta\) (Castagnoli, 1995):

\[
|\varphi(t)\rangle = |\psi(t)\rangle_+ - |\psi(t)\rangle_- , \\
|\beta(t)\rangle = e^{i\delta} (|\psi(t)\rangle_+ + |\psi(t)\rangle_-),
\]

where \(|\psi(t)\rangle_+\) is the retarded wave, associated with forward-in-time causality, and \(|\psi(t)\rangle_-\) is the advanced wave, associated with backward-in-time causality, both undergoing the same transformations of \(|\varphi(t)\rangle\). Without going into detail, let us give such two waves in the central part of Simon’s algorithm:

\[
|\psi(t_1)\rangle_\pm = |\psi(t_1+)\rangle_\pm = \pm \frac{1}{2} \left[ \frac{1}{\sqrt{N}} \sum_x |x\rangle_a \pm \frac{1}{\sqrt{2}} e^{i\delta} (|f\rangle_a + |r\rangle_a) \right] |0\rangle_b,
\]

\[
|\psi(t_2)\rangle_\pm = |\psi(t_3)\rangle_\pm = \pm \frac{1}{2} \left[ \frac{1}{\sqrt{N}} \sum_x |x\rangle_a |f(x)\rangle_b \pm \frac{1}{\sqrt{2}} e^{i\delta} (|f\rangle_a + |r\rangle_a) |f\rangle_b \right].
\]

The upper (lower) signs apply to the retarded (advanced) wave. The time-symmetry of a reversible process imposes a gauge symmetry on \(\delta\), it must be a random variable with uniform distribution in \([0, 2\pi]\): this makes the two waves mathematically indistinguishable, either wave can thus be associated with either direction of causality. In conclusion, both directions of causality coexist in either \(|\varphi(t)\rangle\) or \(|\beta(t)\rangle\), and there is no privileged direction at all.

Parenthetically, we should note that each wave, representing a single direction of causality, is an incomplete description (Castagnoli, 1995) — typical of the method of random phases (Finkelstein 1996). Whereas \(|\varphi(t)\rangle\) and \(|\beta(t)\rangle\) are pure quantum states: \(\delta\) either disappears or becomes an irrelevant overall phase.

\[^3\] \(|\beta(t)\rangle\) undergoes the same transformations of \(|\varphi(t)\rangle\), but with a different initial (or final) condition: while \(|\varphi(t)\rangle\) starts from the preparation, \(|\beta(t)\rangle\) (deterministically) ends into the measurement outcome \(|\beta(t_3)\rangle\), or it goes back in time starting from that outcome.
Of course, advanced-retarded wave indistinguishability does not affect the asymmetry between preparation and measurement outcome (thus also between $|\varphi(t)\rangle$ and $|\beta(t)\rangle$), which comes from our capability of controlling the state of the quantum object (see also Vaidman, 1998; Aharonov et al., 1964).

While the preparation is under complete control, obtained by correcting a previous, random measurement outcome, the final measurement outcome is not under complete control in any of the current algorithms. Partial control (yielding the “intelligent choice”) is what provides quantum efficiency in some NP problems. We will show in Section III that total control of the measurement outcome would yield in principle NP-complete = P.

Eventually, let us show that skipping step (d) is indifferent; we will follow a shortcut. Whether step (d) has been performed or skipped is indistinguishable to the observer of register $a$ at time $t_5$ (otherwise there could be superluminal communication between space-separated regions $a$ and $b$). Therefore it is equivalent to keep step (d) there even if it is not performed. This time, measurement of an entangled state is involved, therefore we should not think that the result $z^{(i)}$ (Section I) is already “written” in the state before measurement and that measurement serves to “read” this result. Measurement creates the result, by constraining the output of $f$ computation.

III. AN ALTERNATIVE QUANTUM COMPUTATION PARADIGM

We shall review a speculative but plausible algorithm which puts under complete control a measurement outcome, yielding NP-complete = P (Castagnoli, Sept. 1998). Let $f$ be a general Boolean function (fig. 2) with constraints both on part of the input (which make the function hard to reverse) and on the output; conventionally, the output constraint is 1. The problem is whether there is an assignment of the unconstrained part of the input $x_1, x_2, \ldots$ satisfying all input and output constraints. This is a version of the well-known NP-complete SAT problem.

Fig. 2

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The computer register must comprise both the input and output qubits appearing in fig. 2; each unconstrained input is prepared in $\frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$. By using conventional quantum computation, we compute the direct function, reaching in polynomial time an entangled state $|\varphi\rangle$ which is an equally weighted superposition of all tensor products of qubits eigenstates which satisfy the input constraint and $f$ (while the output constraint may not be satisfied). At this time interaction has ceased and all registers’ qubits are independent of each other.

To simplify, say that the network has exactly one solution: the problem is to find it. All $|\varphi\rangle$’s tensor products but one end with $|0\rangle_y$. A $\pi/2$ rotation is now applied to the independent qubit $y$ (in order to bring it from about $|0\rangle_y \langle 0|_y$ to about $|1\rangle_y \langle 1|_y$), while the overall network state is (speculatively) submitted to a continuous measurement, such that it is continuously projected on the constrained Hilbert subspace $\mathcal{H}^c$. By definition, $\mathcal{H}^c$ is the span of all the vectors which satisfy the input constraints and $f$ (note that $|\varphi\rangle$ as all subsequent measurement outcomes belong to $\mathcal{H}^c$). This generates a unitary propagation of the network state which, in one computation step (the $\pi/2$ rotation), leads to a state close to the solution (ending with $|1\rangle_y$). Qubit measurement gives the solution with high probability.

The notion of continuous measurement, as the associated “special” Zeno effect (keeping the propagation inside $\mathcal{H}^c$), is strictly based on a retarded-advanced wave model. Without it, we would remain with the mechanistic notion of frequent measurement in the limit of infinite frequency. This would bring in a different Zeno effect freezing the propagation (no more driven by both initial and final conditions) in its initial state. By the way, this discrepancy between continuous and frequent measurement might lead to ascertaining the existence of an advanced propagation.

IV. DISCUSSION

We have shown that a quantum propagation comprising measurement and wave function collapse (here a revamped notion) is strictly non-mechanistic in character, being driven by both initial and final conditions.
Although non-mechanism justifies the efficiency of quantum computation, it should be noted that it came out in the past in several forms, apparently without being acknowledged. Maybe, because the notion of causality going back and forth in time is often “aborred”, considered with either no consequences (an idle interpretation) or too many (time-travel). Hopefully, this work should convince the reader that both concerns are not justified.

Let us see how the notion of non-mechanism (tacitly) has developed in the past. It has been a progressive development, characterized by an increasing control of the measurement outcome:

(i) In the simplest case of an elementary (un-compound) object, eq. (1) tells that the outcome of measurement is a function of itself (is what it is): the only constraint is that it is one of the outcomes allowed by the state before measurement. This is freedom from the past — namely non-mechanism. Here the measurement outcome can be controlled only in a stochastic way (the probability amplitudes can be prepared).

(ii) In the case of a compound object in an entangled state, non-mechanism also appears in the novel form of correlation between simultaneous eigenvalues of the measurement outcome — like in EPR measurement. This is called non locality of course. However, such a correlation can be described without giving up locality, provided that causality is allowed to go back and forth in time (Bennett, 1995; Castagnoli, 1995).

(iii) A third level of unfolding of non-mechanism appears when it is leveraged to yield quantum computation efficiency. In current algorithms, as already in (ii), part of the outcome is random, part is correlated. This yields a capability of solving in polynomial time some NP problems. We have shown a further level of unfolding of non-mechanism, speculative for the time being. This corresponds to putting the outcome of measurement under a complete control, by means of a special Zeno effect. This would allow to solve the SAT problem in polynomial time, in principle.

We should further note that, through steps (i)-(iii), the teleological character of the evolution, more and more controlled by the final condition, increases.

Non-mechanism appears to be an exclusively quantum feature of evolutions comprising
wave function collapse. Like all exclusively quantum features, on the one hand it is strictly confined between initial and final measurement, on the other hand it yields consequences in the classical world, in fact quantum computation efficiency.

Hopefully, the perspective developed in this work will help in the quest for new efficient computation algorithms. Thanks are due to G. Baget Bozzo, T. Beth, A. Ekert, D. Finkelstein, and V. Vedral for stimulating discussions.

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Fig. 1

(a) (b)

Fig. 2

\[ y = 1 \]