Comparison of linear and quadratic bi-response semiparametric regression models using spline truncated

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Abstract. Semiparametric regression is a regression model consisting of two components, namely the parametric and the nonparametric component. The parametric component is a component with curves are known and the nonparametric component is a component with unknown shape of the curve. Bi-response semiparametric regression is regression analysis which has two response variables with the combination of parametric and nonparametric curves. The purpose of this research is to compare the bi-response semiparametric regression model using spline truncated with component parametric linear and quadratic. In this research, the method that used for the estimation parameter is WLS (Weighted Least Square) and the selection of optimal knot points is done by looking at the minimum GCV (Generalized Cross-Validation) value. WLS is minimizing partial derivative based on a parameter in the model. Each the linear bi-response semiparametric regression and quadratic bi-response semiparametric regression using spline truncated has two parameters that are parameter for parametric and nonparametric components. The difference between the two models contained in the parametric component of the predictor variable and parameter matrix of parametric component \( \hat{Y} = X\hat{\beta} + T\hat{\alpha} \) is estimator linear bi-response semiparametric regression model and \( \hat{Y} = X\hat{\theta} + T\hat{\phi} \) is estimator quadratic bi-response semiparametric regression model using spline truncated.

1. Introduction

One of the statistical methods used for estimating the relation between predictor variables and response variables is regression analysis. Regression analysis is described through a function called a regression curve [1]. Estimation of the regression curve can be done through three approaches that are parametric approaches, nonparametric approach, and semiparametric approach. The parametric approach is used if the regression curve between the predictor variable and response variable assumed by following a certain pattern, for example linear, quadratic, and others. While for the nonparametric approach is used when the pattern regression between predictor variables and the response variable is unknown or not assumed a certain pattern [2]. The semiparametric approach is a combination of parametric and nonparametric approaches, which has a regression curve of partly are known and partly unknown patterns [3]. The most estimation method for nonparametric and semiparametric regression used on research is spline truncated, the previous research is [4], [5], [6], and [7]. Some advantages for estimation with the spline truncated method is spline truncated are good in interpretation statistics and visual and also have the ability to cope with changing behavior in data patterns [2]. Many researchers have been done research about estimator spline truncated for nonparametric or semiparametric
approach. Spline truncated estimator with a nonparametric approach can use one or more predictor variables as well as the truncated spline estimator for the semiparametric approach.

The research about semiparametric regression approach with pattern of parametric regression curve linear has done a lot using spline truncated as the method for estimating nonparametric and semiparametric approach [8], [9]. While the research about semiparametric with pattern of parametric regression curve quadratic is no much done. In modeling, some cases cannot be solved only with one response variable in regression analysis because there is a correlation between one and other response, so the model can be solved with the multiresponse regression model, that is a regression model with more than one response variable and one or more predictor variables. Hence, this research will be estimated spline truncated parameter for bi-response using a semiparametric regression approach, where the parametric component will be approached with two patterns. The first is a parametric component with a linear pattern approach and the second is a parametric component with quadratic approach and then will be compared the parameter estimation of linear and quadratic bi-response semiparametric regression using spline truncated.

2. Theoretical Review
In this section, we will review some of the theories used in this research.

2.1. Semiparametric Regression
Semiparametric regression model is more flexible than the linear regression model because of the presence of parametric and nonparametric components, this will accommodate the relationship between the response variable and the predictors that are linear and nonlinear nature [11]. Data given is predictor variable for the parametric component and is a predictor variable for the nonparametric component is assumed to follow the following semiparametric regression model:

\[ y_i = f(x_i) + g(t_i) + \varepsilon_i \]  

where \( y_i \) is response variable for response \( j^{th} \) in \( i^{th} \) observation, \( f(x_i) \) is a parametric component, \( g(t_i) \) is a nonparametric component and \( \varepsilon_i \) is a random error with zero mean and variance \( \sigma^2 \). Equation (1) can rewrite to matrix model:

\[ Y = X\beta + T\alpha + \varepsilon \]  

with details

\[
\begin{pmatrix}
  y_1 \\
  y_2 \\
  \vdots \\
  y_n \\
\end{pmatrix} = 
\begin{pmatrix}
  1 & x_{11} & \cdots & x_{11} \\
  1 & x_{22} & \cdots & x_{22} \\
  \vdots & \vdots & \ddots & \vdots \\
  1 & x_{nn} & \cdots & x_{nn} \\
\end{pmatrix}
\begin{pmatrix}
  \beta_0 \\
  \beta_1 \\
  \vdots \\
  \beta_p \\
\end{pmatrix} +
\begin{pmatrix}
  g(t_1) \\
  g(t_2) \\
  \vdots \\
  g(t_n) \\
\end{pmatrix} +
\begin{pmatrix}
  \varepsilon_1 \\
  \varepsilon_2 \\
  \vdots \\
  \varepsilon_n \\
\end{pmatrix}
\]

2.2. Bi-response Semiparametric Spline Truncated Regression
Bi-response regression analysis is one of the analysis are used to estimate a functional relationship between two response variables with the predictor variable. The relation between the response variable and the predictor variable follows the regression model so that the equation is obtained:

\[ Y_{ij} = \beta_{0j} + \beta_{1j}X_i + \sum_{s=1}^{k} \gamma_{js}T_n + \sum_{p=1}^{p} \alpha_{jp}(t_n - \text{K}_{n})^m + \varepsilon_{ij} \]
where \( i = 1, 2, \ldots, n \) is the subject of observation for every response variable, \( r = 1, 2, \ldots, R \) is a number of predictor variables for parametric component, \( p = 1, 2, \ldots, P \) number of predictor variables for nonparametric component and \( j = 1, 2 \) is the number of response variables.

### 2.3. Weighted Least Square

In parametric regression analysis or semiparametric regression analysis, there are two assumptions that must be fulfilled the variance of the random error in the model is assumed to be homogeneous and variance-covariance error matrix are known [12]. The method that can be used to fulfill the first assumption is WLS (Weighted Least Square). WLS method is used to the estimated parameter by minimizing the number of squares error between observation and the model.

\[
\min_{\beta, \alpha} \left[ R(\beta, \alpha) \right] = \min_{\beta, \alpha} \left( y - X\beta - T\alpha \right)^T W (y - X\beta - T\alpha) 
\]

(5)

### 2.4. Generalized Cross-Validation

The shape of the spline truncated estimator in bi-response semiparametric regression is very influenced by the knot point. Spline truncated estimator optimum was obtained from the value of knots with the minimum GCV value [4] and [13]. The GCV method is generally defined as follows:

\[
GCV(k) = \frac{n^{-1} \sum_{i=1}^{n} \left( y_i - \eta(t_i) \right)^2}{n^{-1} \text{tr} \left( I - A(k) \right)} 
\]

(6)

### 3. Result and Discussion

In this section, we give results and discussion about the comparison estimation parameter of linear and quadratic bi-response semiparametric regression models using spline truncated.

#### 3.1. Estimation parameters of bi-response semiparametric regression models using spline truncated with linear parametric component

Suppose given paired data \( (x_i, t_{i1}, t_{i2}, \ldots, t_{iR}, y_{ij}), j=1,2 \ ; i=1,2,\ldots,n \), with \( x_i \) is predictor variable component parametric, \( t_{i1}, t_{i2}, \ldots, t_{iR} \) is predictor variable for a nonparametric component. Based on equation (4) bi-response semiparametric regression models using spline truncated with component parametric is linear and \( m = 1 \) the equation is obtained:

\[
Y_{i1} = \beta_{01} + \beta_{11}X_i + \sum_{r=1}^{R} \left[ \gamma_{r1}t_{ri} + \alpha_{11r} \left( t_{ri} - K_{11} \right)_s + \cdots + \alpha_{1Pr} \left( t_{ri} - K_{1Pr} \right)_s \right] + \varepsilon_{1i} \\
= \beta_{01} + \beta_{11}X_i + \gamma_{1r1}t_{ri} + \alpha_{111} \left( t_{ri} - K_{11} \right)_s + \cdots + \alpha_{111} \left( t_{ri} - K_{11} \right)_s + \cdots + \alpha_{1Pr} \left( t_{ri} - K_{1Pr} \right)_s + \varepsilon_{1i} \\
+ \cdots + \gamma_{1Rr1}t_{ri} + \alpha_{11R} \left( t_{ri} - K_{11} \right)_s + \cdots + \alpha_{1Pr} \left( t_{ri} - K_{1Pr} \right)_s + \varepsilon_{1i} \\
(7)
\]

\[
Y_{i2} = \beta_{02} + \beta_{12}X_i + \sum_{r=1}^{R} \left[ \gamma_{2r1}t_{ri} + \alpha_{21r} \left( t_{ri} - K_{21} \right)_s + \cdots + \alpha_{2Pr} \left( t_{ri} - K_{2Pr} \right)_s \right] + \varepsilon_{2i} \\
= \beta_{02} + \beta_{12}X_i + \gamma_{2r1}t_{ri} + \alpha_{211} \left( t_{ri} - K_{21} \right)_s + \cdots + \alpha_{211} \left( t_{ri} - K_{21} \right)_s + \cdots + \alpha_{2Pr} \left( t_{ri} - K_{2Pr} \right)_s + \varepsilon_{2i} \\
+ \cdots + \gamma_{2Rr1}t_{ri} + \alpha_{21R} \left( t_{ri} - K_{21} \right)_s + \cdots + \alpha_{2Pr} \left( t_{ri} - K_{2Pr} \right)_s + \varepsilon_{2i} \\
(8)
\]

Equation (7) and (8) can rewrite in the matrix below:
\[
\begin{bmatrix}
Y_{1j} \\
Y_{2j}
\end{bmatrix}
= \begin{bmatrix}
X_1 \beta_1 \\
X_2 \beta_2
\end{bmatrix}
+ \begin{bmatrix}
T_1 \alpha_1 \\
T_2 \alpha_2
\end{bmatrix}
+ \begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2
\end{bmatrix}
\]  

(9)

where

\[
Y_{ji} = \begin{pmatrix}
(y_{11}, y_{12}, \ldots, y_{1n})^T \\
(y_{21}, y_{22}, \ldots, y_{2n})^T
\end{pmatrix}
\]

\[
\beta = \begin{bmatrix}
\beta_{01} & \beta_{02} \\
\beta_{11} & \beta_{12}
\end{bmatrix}
\]

\[
\alpha = \begin{bmatrix}
\gamma_{11}, \alpha_{111}, \ldots, \alpha_{1p1}, \gamma_{12}, \alpha_{11R}, \ldots, \alpha_{1pR} \\
\gamma_{21}, \alpha_{211}, \ldots, \alpha_{2p1}, \gamma_{22}, \alpha_{21R}, \ldots, \alpha_{2pR}
\end{bmatrix}^T
\]

\[
\varepsilon = \begin{bmatrix}
\varepsilon_{11}, \varepsilon_{12}, \ldots, \varepsilon_{1n} \\
\varepsilon_{21}, \varepsilon_{22}, \ldots, \varepsilon_{2n}
\end{bmatrix}^T
\]

\[
X_j = \begin{bmatrix}
1 & 1 & \ldots & 1 \\
1 & x_1 & 2 & \ldots & x_n
\end{bmatrix}
\]

\[
T_j = \begin{bmatrix}
1 & \left(t_{11} - K_{j11}\right) & \ldots & \left(t_{11} - K_{j11}\right) & \ldots & \left(t_{11} - K_{j1R}\right) & \ldots & \left(t_{11} - K_{j1R}\right) \\
1 & \left(t_{12} - K_{j11}\right) & \ldots & \left(t_{12} - K_{j11}\right) & \ldots & \left(t_{12} - K_{j1R}\right) & \ldots & \left(t_{12} - K_{j1R}\right) \\
\vdots & \vdots & \ldots & \vdots & \ldots & \vdots & \ldots & \vdots \\
1 & \left(t_{1n} - K_{j11}\right) & \ldots & \left(t_{1n} - K_{j11}\right) & \ldots & \left(t_{1n} - K_{j1R}\right) & \ldots & \left(t_{1n} - K_{j1R}\right)
\end{bmatrix}
\]

\[
X_j \text{ is a matrix consisting of predictor variables for parametric components and } T_j \text{ is a matrix consisting of predictor variables for nonparametric components. Equation (9) can rewrite follow this equation:}
\]

\[
Y = X\beta + Ta + \varepsilon
\]  

(10)

Parameter estimation of \(\beta\) and \(\alpha\) in equation (10) can be obtained from Weighted Least Square method with weighting matrix \(W\), based on equation (5):

\[
L = (y - X\beta - Ta)^T W(y - X\beta - Ta)
= y^T yW - 2y^T X^T Wy - 2a^T T^T Wx\beta + 2\alpha^T T^T WX\beta + \beta^T X^T WX\beta + \alpha^T T^T WTa
\]  

(11)

Based on Weighted Least Square concept, the partial derivative of equation (11) with respect to parameter \(\beta\) and \(\alpha\):

\[
\frac{\partial L}{\partial \beta} = -2X^T Wy + 2X^T WTa + 2X^T WX\beta
\]
\[
\frac{\partial L}{\partial \alpha} - 2 \mathbf{T}^T \mathbf{W} \mathbf{y} + 2 \mathbf{T}^T \mathbf{W} \mathbf{X} \hat{\beta} + 2 \mathbf{T}^T \mathbf{W} \mathbf{T} \alpha
\]

With the minimum value for \( L \) based on \( \frac{\partial L}{\partial (\beta, \alpha)} = 0 \) as follows:

\[
\hat{\beta} = \left( \mathbf{X}^T \mathbf{W} \mathbf{X} \right)^{-1} \left( \mathbf{X}^T \mathbf{W} \mathbf{y} - \mathbf{X}^T \mathbf{W} \mathbf{T} \hat{\alpha} \right) \tag{12}
\]

\[
\hat{\alpha} = \left( \mathbf{T}^T \mathbf{W} \mathbf{T} \right)^{-1} \left( \mathbf{T}^T \mathbf{W} \mathbf{y} - \mathbf{T}^T \mathbf{W} \mathbf{X} \hat{\beta} \right) \tag{13}
\]

Estimation based on the partial derivative is obtained not a mutually free estimator, that is estimator \( \hat{\beta} \) in equation (12) still have estimator \( \hat{\alpha} \), likewise for equation (13). Hence, to obtained the mutually free estimator can be used to a substitution method. Substitution method for estimator \( \hat{\beta} \) obtained:

\[
\hat{\beta} = \mathbf{M} \left( \mathbf{X}^T \mathbf{W} \mathbf{X} \right)^{-1} \left\{ \mathbf{X}^T - \mathbf{X}^T \mathbf{W} \left( \mathbf{T}^T \mathbf{W} \mathbf{T} \right)^{-1} \mathbf{T}^T \right\} \mathbf{W} \mathbf{y}
= \mathbf{A}(\mathbf{K}) \mathbf{y}
\]

where

\[
\mathbf{M} = \left( \mathbf{I} - \left( \mathbf{X}^T \mathbf{W} \mathbf{X} \right)^{-1} \mathbf{X}^T \mathbf{W} \mathbf{T} \left( \mathbf{T}^T \mathbf{W} \mathbf{T} \right)^{-1} \mathbf{T}^T \mathbf{W} \mathbf{X} \right)^{-1}
\]

\[
\mathbf{A}(\mathbf{K}) = \mathbf{M} \left( \mathbf{X}^T \mathbf{W} \mathbf{X} \right)^{-1} \left\{ \mathbf{X}^T - \mathbf{X}^T \mathbf{W} \left( \mathbf{T}^T \mathbf{W} \mathbf{T} \right)^{-1} \mathbf{T}^T \right\} \mathbf{W}
\]

parameters estimation for \( \hat{\alpha} \) are obtained from substitute equation (12) to equation (13), obtained as follow:

\[
\hat{\alpha} = \mathbf{N} \left( \mathbf{T}^T \mathbf{W} \mathbf{T} \right)^{-1} \left\{ \mathbf{T}^T - \mathbf{T}^T \mathbf{W} \mathbf{X} \left( \mathbf{X}^T \mathbf{W} \mathbf{X} \right)^{-1} \mathbf{X}^T \right\} \mathbf{W} \mathbf{y}
= \mathbf{B}(\mathbf{K}) \mathbf{y}
\]

where

\[
\mathbf{N} = \left( \mathbf{I} - \left( \mathbf{T}^T \mathbf{W} \mathbf{T} \right)^{-1} \mathbf{T}^T \mathbf{W} \mathbf{X} \left( \mathbf{X}^T \mathbf{W} \mathbf{X} \right)^{-1} \mathbf{X}^T \mathbf{W} \mathbf{T} \right)^{-1}
\]

\[
\mathbf{B}(\mathbf{K}) = \mathbf{N} \left( \mathbf{T}^T \mathbf{W} \mathbf{T} \right)^{-1} \left\{ \mathbf{T}^T - \mathbf{T}^T \mathbf{W} \mathbf{X} \left( \mathbf{X}^T \mathbf{W} \mathbf{X} \right)^{-1} \mathbf{X}^T \right\} \mathbf{W}
\]

Based on equation (10) estimation curve for linear bi-response semiparametric regression models using spline truncated sequence \( m = 1 \) and \( knot = 1 \) are obtained:

\[
\mathbf{Y} = \mathbf{X} \hat{\beta} + \mathbf{T} \hat{\alpha}
= \mathbf{X} \mathbf{A}(\mathbf{K}) \mathbf{y} + \mathbf{T} \mathbf{B}(\mathbf{K}) \mathbf{y}
= \mathbf{C}(\mathbf{K}) \mathbf{y}
\]

where

\[
\mathbf{C}(\mathbf{K}) = \left( \mathbf{M} \left( \mathbf{X}^T \mathbf{W} \mathbf{X} \right)^{-1} \left\{ \mathbf{X}^T - \mathbf{X}^T \mathbf{W} \left( \mathbf{T}^T \mathbf{W} \mathbf{T} \right)^{-1} \mathbf{T}^T \right\} \mathbf{W} \right) + \left( \mathbf{N} \left( \mathbf{T}^T \mathbf{W} \mathbf{T} \right)^{-1} \left\{ \mathbf{T}^T - \mathbf{T}^T \mathbf{W} \mathbf{X} \left( \mathbf{X}^T \mathbf{W} \mathbf{X} \right)^{-1} \mathbf{X}^T \right\} \mathbf{W} \right)
\]
3.2. Estimation parameters of bi-response semiparametric regression models using spline truncated with quadratic parametric component

Estimation with quadratic parametric component is done with the same process when estimating parameter with linear parametric component, response variable, and predictor variable is the same that is for predictor variables in parametric component and predictor variables in nonparametric component. Based on equation (4) bi-response semiparametric regression models using spline truncated with quadratic component parametric and equation is obtained:

\[ Y_{ji} = \theta_{0j} + \theta_{1j}X_i + \theta_{2j}X_i^2 + \sum_{r=1}^{R} h_r(t_{ri}) + \varepsilon_{ji} \]  

(15)

Equation (15) can be written to matrix shape based on equation (8). The difference between parameter estimation bi-response semiparametric regression linear parametric component with bi-response semiparametric regression quadratic component contained in the matrix predictor variable for parametric and nonparametric component obtained the matrix is:

\[ \theta = \begin{bmatrix} \theta_{01}, \theta_{11}, \theta_{21} \\ \theta_{02}, \theta_{12}, \theta_{22} \end{bmatrix}^T \]

\[ X_j = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_1, x_2, \cdots, x_n \\ x_1^2, x_2^2, \cdots, x_n^2 \end{bmatrix}^T \]

Based on equation (15) quadratic bi-response semiparametric regression using spline truncated is following:

\[ Y_{ji} = \theta_{0j} + \theta_{1j}X_i + \theta_{2j}X_i^2 + \sum_{r=1}^{R} \phi_r t_{ri} + \sum_{p=1}^{P} \phi_{jpr} (t_{ri} - K_{jpr}) \varepsilon_{ji} \]  

(16)

Equation (16) can rewrite as a matrix:

\[ Y = X\hat{\theta} + T\hat{\phi} + \varepsilon \]  

(17)

Using Weighted Least Square (WLS) method, and then substitute the partial derivative of parameter \( \hat{\theta} \) and \( \hat{\phi} \), then estimator quadratic bi-response semiparametric regression model using spline truncated are obtained:

\[ Y = X\hat{\theta} + T\hat{\phi} = XD(K)y + TE(K)y \]

(18)

where

\[ F(K) = \left( Q(X^TWX)^{-1}\{X^T - X^TW(T^WT)^{-1}T^T\}W + N(T^WT)^{-1}\{T^T - T^TW\left(X^TWX\right)^{-1}X^T\}W \right) \]

4. Conclusion

Based on the analysis has been given on the section result, then we can be concluded:

1. The curve regression of parameter estimation for linear bi-response semiparametric regression model using spline truncated as following:
\[ Y = X\beta + T\alpha \]
\[ = XA(K)y + TB(K)y \]
\[ = C(K)y \]

where
\[ C(K) = \left( M(X'WX)^{-1} \left\{ X' - X'WT(T'WT)^{-1}T' \right\} W + N(T'WT)^{-1} \left\{ T' - T'WX(X'WX)^{-1}X' \right\} W \right) \]

2. The curve regression of parameter estimation for quadratic bi-response semiparametric regression model using spline truncated as following:

\[ Y = X\hat{\theta} + T\hat{\phi} \]
\[ = XD(K)y + TE(K)y \]
\[ = F(K)y \]

where
\[ F(K) = \left( Q(X'WX)^{-1} \left\{ X' - X'WT(T'WT)^{-1}T' \right\} W + N(T'WT)^{-1} \left\{ T' - T'WX(X'WX)^{-1}X' \right\} W \right) \]

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