Single Transverse-Spin Asymmetry in $pp^\uparrow \to \pi X$ and $ep^\uparrow \to \pi X$

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Abstract: Cross section formulas for the single spin asymmetry in $p^\uparrow p \to \pi(\ell_T)X$ and $e^\uparrow p \to \pi(\ell_T)X$ are derived and its characteristic features are discussed.

In this report we discuss the single transverse-spin asymmetry for the pion production with large transverse momentum $A_N$ in $pp^\uparrow \to \pi(\ell)X$ and $p^\uparrow e \to \pi(\ell)X$ relevant for RHIC-SPIN, HERMES and COMPASS experiments. According to the QCD factorization theorem, the polarized cross section for $pp^\uparrow \to \pi X$ consists of three twist-3 contributions:

$$
(A) \quad G_a(x_1, x_2) \otimes q_b(x') \otimes D_c(z) \otimes \hat{\sigma}_{ab\to c},

(B) \quad \delta q_a(x) \otimes E_b(x'_1, x'_2) \otimes D_c(z) \otimes \hat{\sigma}'_{ab\to c},

(C) \quad \delta q_a(x) \otimes q_b(x') \otimes \hat{E}_c(z_1, z_2) \otimes \hat{\sigma}''_{ab\to c},
$$

where the functions $G_a(x_1, x_2)$, $E_b(x'_1, x'_2)$ and $\hat{E}_c(z_1, z_2)$ are the twist-3 quantities representing, respectively, the transversely polarized distribution, the unpolarized distribution, and the fragmentation function for the pion. $\delta q_a(x)$ is the transversity distribution in $p^\uparrow$. $a$, $b$ and $c$ stand for the parton’s species, sum over which is implied. $\delta q_a$, $E_b$ and $\hat{E}_c$ are chiral-odd. Corresponding to the above (A) and (C), the polarized cross section for $e^\uparrow p \to \pi X$ (final electron is not detected) receives two twist-3 contributions:

$$
(A') \quad G_a(x_1, x_2) \otimes D_a(z) \otimes \hat{\sigma}_{ea\to a},

(C') \quad \delta q_a(x) \otimes \hat{E}_a(z_1, z_2) \otimes \hat{\sigma}'_{ea\to a}.
$$

The (A) and (B) contributions for $pp^\uparrow \to \pi X$ have been analyzed in [1] and [2], respectively, and it has been shown that (A) gives rise to large $A_N$ at large $x_F$ as observed in E704, and (B) is negligible in all kinematic region. Here we extend the study to the (C) term (see also [3]) at RHIC energy and also the asymmetry in $ep$ collision.

The transversely polarized twist-3 distributions $G_F(x_1, x_2)$ relevant to the (A) term is given in [2]. Likewise the twist-3 fragmentation function for the pion (with
momentum $\ell$) is defined as the lightcone correlation function as $w^2 = 0, \ell \cdot w = 1$

\[
\frac{1}{N_c} \sum_x \int \frac{d \lambda}{2\pi} \int \frac{d \mu}{2\pi} e^{-i \phi_\lambda} e^{-i \mu (\frac{x}{2} - \frac{1}{2})} \langle 0 | \psi_1(0) | \pi X \rangle \langle \pi X | g F^{\alpha \beta}(\mu w) w_\beta \bar{\psi}_j(\lambda w) | 0 \rangle \nonumber
\]

\[
= \frac{M_N}{2z_2} (\gamma_5 \gamma_\nu)_{ij} \epsilon^{\nu \alpha \beta \lambda} w_\beta \ell_\lambda \hat{E}_F(z_1, z_2) + \ldots .
\]

Note that we use the nucleon mass $M_N$ to normalize the twist-3 pion fragmentation function. There is another twist-3 fragmentation function which is obtained from (1) by shifting the gluon-field strength from the left to the right of the cut. The defined function $\hat{E}_{FR}(z_1, z_2)$ is connected to $\hat{E}_F(z_1, z_2)$ by the relation $\hat{E}_F(z_1, z_2) = \hat{E}_{FR}(z_2, z_1)$, which follows from hermiticity and time reversal invariance. Unlike the twist-3 distributions, the twist-3 fragmentation function does not have definite symmetry property.

Following [1]-[3], we analyze the asymmetries focusing on the soft-gluon pole contributions with $G_F(x, x)$ and $\hat{E}_F(z, z)$. In the large $x_F$ region, i.e. production of pion in the forward direction of the polarized nucleon, the main contribution comes from the large-$x$ and large-$z$ region of distribution and fragmentation functions, respectively. Since $G_F$ and $\hat{E}_F$ behaves as $G_F(x, x) \sim (1 - x)^3$ and $\hat{E}_F(z, z) \sim (1 - z)^{\beta'}$ with $\beta, \beta' > 0$, $|d/dx G_F(x, x)| \gg |G_F(x, x)|, |d/dz \hat{E}_F(z, z)| \gg |\hat{E}_F(z, z)|$ at large $x$ and $z$. In particular, the valence component of $G_F$ and $\hat{E}_F$ dominate in this region. We thus keep only the valence quark contribution for the derivative of these soft-gluon pole functions (“valence-quark soft-gluon approximation”) for the $pp$ collision. For the $ep$ case, we include all the soft-gluon pole contribution.

In general $A_N$ is a function of $S = (P + P')^2 \simeq 2P \cdot P', T = (P - \ell)^2 \simeq -2P \cdot \ell$ and $U = (P' - \ell)^2 \simeq -2P' \cdot \ell$ where $P, P'$ and $\ell$ are the momenta of $p^1$, unpolarized $p$ (or $e$), and the pion respectively. In the following we use $S, x_F = \frac{2q_i}{\sqrt{S}} = \frac{T-U}{S}$ and $x_T = \frac{2q_T}{\sqrt{S}}$ as independent variables. The polarized cross section for the $(C)$ term reads

\[
E_N d^3 \Delta \sigma (S) = \frac{2\pi M_N \alpha_s^2}{S} \epsilon^{\lambda \mu \nu} \sum_a \int_{x_{\min}}^{1} \frac{dz}{z^2} \int_{x_{\min}}^{1} \frac{dx}{x} \frac{1}{xS + U/z} \int_{0}^{1} \frac{dx'}{x'} \nonumber
\]

\[
\times \delta \left( x' + \frac{xT/z}{xS + U/z} \right) \nonumber
\]

\[
\left\{ \sum_b \delta q^a(x) q^b(x') \left[ -z_1^2 \frac{\partial}{\partial z_1} \hat{E}_F^a(z_1, z) \right]_{z_1 = z} \left( -\frac{2p_\alpha}{T} \hat{\sigma}_{ab}^{I} + \frac{2p_\alpha'}{U} \hat{\sigma}_{ab}^{II} \right) + \sum_b \delta q^a(x) q^b(x') \left[ -z^2 \frac{d}{dz} \hat{E}_F^a(z, z) \right] \frac{xp_\alpha + xp'_\alpha}{xT + xU} \left( \hat{\sigma}_{ab}^{I} + \hat{\sigma}_{ab}^{II} \right) \right. \nonumber
\]

\[
\left. + \sum_b \delta q^a(x) q^b(x') \left[ -z \frac{d}{dz} \hat{E}_F^a(z, z) \right] \frac{xp_\alpha + xp'_\alpha}{xT + xU} \left( \hat{\sigma}_{ab}^{I} + \hat{\sigma}_{ab}^{II} \right) \right. \nonumber
\]

\[
\left. + \sum_b \delta q^a(x) q^b(x') \left[ -z \frac{d}{dz} \hat{E}_F^a(z, z) \right] \frac{xp_\alpha + xp'_\alpha}{xT + xU} \left( \hat{\sigma}_{ab}^{I} + \hat{\sigma}_{ab}^{II} \right) \right. \nonumber
\]

\[
\left. + \sum_b \delta q^a(x) q^b(x') \left[ -z \frac{d}{dz} \hat{E}_F^a(z, z) \right] \frac{xp_\alpha + xp'_\alpha}{xT + xU} \left( \hat{\sigma}_{ab}^{I} + \hat{\sigma}_{ab}^{II} \right) \right. \nonumber
\]

\[
\left. + \sum_b \delta q^a(x) q^b(x') \left[ -z \frac{d}{dz} \hat{E}_F^a(z, z) \right] \frac{xp_\alpha + xp'_\alpha}{xT + xU} \left( \hat{\sigma}_{ab}^{I} + \hat{\sigma}_{ab}^{II} \right) \right. \nonumber
\]

\[
\left. + \sum_b \delta q^a(x) q^b(x') \left[ -z \frac{d}{dz} \hat{E}_F^a(z, z) \right] \frac{xp_\alpha + xp'_\alpha}{xT + xU} \left( \hat{\sigma}_{ab}^{I} + \hat{\sigma}_{ab}^{II} \right) \right. \nonumber
\]

\[
\left. + \sum_b \delta q^a(x) q^b(x') \left[ -z \frac{d}{dz} \hat{E}_F^a(z, z) \right] \frac{xp_\alpha + xp'_\alpha}{xT + xU} \left( \hat{\sigma}_{ab}^{I} + \hat{\sigma}_{ab}^{II} \right) \right. \nonumber
\]

\[
\left. + \sum_b \delta q^a(x) q^b(x') \left[ -z \frac{d}{dz} \hat{E}_F^a(z, z) \right] \frac{xp_\alpha + xp'_\alpha}{xT + xU} \left( \hat{\sigma}_{ab}^{I} + \hat{\sigma}_{ab}^{II} \right) \right. \nonumber
\]

\[
\left. + \sum_b \delta q^a(x) q^b(x') \left[ -z \frac{d}{dz} \hat{E}_F^a(z, z) \right] \frac{xp_\alpha + xp'_\alpha}{xT + xU} \left( \hat{\sigma}_{ab}^{I} + \hat{\sigma}_{ab}^{II} \right) \right. \nonumber
\]
\[ +\delta q^a(x)G(x') \left[ -z_1^2 \frac{\partial}{\partial z_1} \hat{E}^a_F(z_1, z) \right]_{z_1=z} \left( \frac{-2p_0}{T} \hat{\sigma}^I_{ag\to a} + \frac{-2p_0}{U} \hat{\sigma}^{II}_{ag\to a} \right) \]

\[ +\delta q^a(x)G(x') \left[ -z_1^2 \frac{d}{dz} \hat{E}^a_F(z, z) \right] \frac{x p_0 + x' p'_a}{|xT + x' U|} \left( \hat{\sigma}^I_{ag\to a} + \hat{\sigma}^{II}_{ag\to a} \right) \}, \quad (2) \]

where the lower limits for the integration variables are \( z_{\text{min}} = -(T + U)/S = \sqrt{x_F^2 + x_T^2} \) and \( x_{\text{min}} = -U/z(S + T/z). \) Using the invariants in the parton level, \( \hat{s} = (xp + x'p')^2 = xx'S, \hat{t} = (xp - \ell/z)^2 = xT/z \) and \( \hat{u} = (x'p' - \ell/z)^2 = x'U/z, \) the partonic hard cross sections read

\[
\hat{\sigma}^I_{qq'\to q} = \frac{\hat{s}u}{18t^2} - \frac{\hat{s}}{54t} \delta_{qq'}, \quad \hat{\sigma}^{II}_{qq'\to q} = \frac{\hat{s}u}{18t^2}, \quad \hat{\sigma}^I_{qq'\to q} = -\frac{\hat{s}u}{18t^2} \]

\[
\hat{\sigma}^I_{qq'\to q} = -\frac{\hat{s}u}{18t^2} + \frac{\hat{s}}{54t} \delta_{qq'}, \quad \hat{\sigma}^{II}_{qq'\to q} = \frac{7\hat{s}u}{18t^2} - \frac{\hat{s}}{54t} \delta_{qq'}, \quad \hat{\sigma}^{II}_{qq'\to q} = -\frac{\hat{s}u}{9t^2}, \]

\[
\hat{\sigma}^I_{qq'\to q} = \frac{\hat{s}u}{9t^2}, \quad \hat{\sigma}^{II}_{qq'\to q} = \frac{7\hat{s}u}{9t^2} + \frac{\hat{s}}{54t} \delta_{qq'}, \quad \hat{\sigma}^{II}_{qq'\to q} = -\frac{\hat{s}u}{9t^2} \quad (3) \]

At large \( x_F (\sim U \gg T), \) \( \sigma^I \) becomes more important because of \( 1/T \) factor in (2).

![Figure 1: \( A_{pp}^N \) at \( \sqrt{S} = 20 \) GeV and \( \ell_T = 1.5 \) GeV.](image)

To estimate the above contribution, we introduce a simple model ansatz as \( \hat{E}^a_F(z, z) = K_a D_0(z) \) with a flavor dependent factor \( K_a. \) \( K_a \)’s are determined to be \( K_u = -0.11 \) and \( K_d = -0.19 \) so that (2) approximately gives rise to \( A_{pp}^N \) observed in E704 data at \( \sqrt{S} = 20 \) GeV and \( \ell_T = 1.5 \) GeV. As noted before, \( \hat{E}_F(z_1, z_2) \) does not have definite symmetry property unlike the twist-3 distribution \( G_F(x_1, x_2). \) Nevertheless we assume \( \left[(\partial/\partial z_1)\hat{E}_F(z_1, z)\right]_{z_1=z} = \left(1/2\right)(d/dz)\hat{E}_F(z, z) \). \) We refer the readers to [2] for the adopted distribution and fragmentation functions. The result for \( A_{pp}^N \) from the (C) (chiral-odd) term is shown in Fig. 1 in comparison
with the (A) (chiral-even) contribution. (See [2] for the detail.) Both effects give rise to $A_N^{CA}$ similar to the E704 data. The origin of the growing $A_N$ at large $x_F$ is (i) large partonic cross sections in (3) ($\sim 1/\hat{t}^2$ term) and (ii) the derivative of the soft-gluon pole functions. With the parameters $K_a$ fixed, $A_N^{PP}$ at RHIC energy ($\sqrt{S} = 200$ GeV) is shown in Fig. 2 at $\ell_T = 1.5$ GeV. Both (A) and (C) contributions give slightly smaller $A_N^{CA}$ than the STAR data reported at this conference [4]. Fig. 3 shows the $A_N^{PP}$ as a function of $\ell_T$ at $\sqrt{S} = 200$ GeV and $x_F = 0.6$, indicating quite large $\ell_T$ dependence in both (A) and (C) contributions at $1 < \ell_T < 4$ GeV region.

![Figure 2: $A_N^{PP}$ at $\sqrt{S} = 200$ GeV and $\ell_T = 1.5$ GeV.](image)

We next discuss the asymmetry $A_N^{CP}$ for $p^\uparrow e \to \pi(\ell)X$ where the final electron is not observed. In our $O(\alpha_s^0)$ calculation, the exchanged photon remains highly virtual as far as the observed $\pi$ has a large transverse momentum $\ell_T$ with respect to the $ep$ axis. Therefore experimentally one needs to integrates only over those virtual photon events to compare with our formula.

![Figure 3: $\ell_T$ dependence of $A_N^{PP}$ at $\sqrt{S} = 200$ GeV and $x_F = 0.6$.](image)

Using the twist-3 distribution and fragmentation functions used to describe $pp$ data, we show in Fig. 4 $A_N^{CP}$ corresponding to (A')(chiral-even) and (C')(chiral-odd) contributions. Remarkable feature of Fig. 4 is that in both chiral-even and chiral-odd contributions (i) the sign of $A_N^{CP}$ is opposite to the sign of $A_N^{PP}$ and (ii) the magnitude of $A_N^{CP}$ is much larger than that of $A_N^{PP}$, in particular, at large $x_F$, and it even overshoots one. (In our convention, $x_F > 0$ corresponds to the production of $\pi$ in the forward hemisphere of the initial polarized proton both in $p^\uparrow p$ and $p^\uparrow e$ case.) The origin of these features can be traced back to the color
factor in the dominant diagrams for the twist-3 polarized cross sections in \( ep \) and \( pp \) collisions. Of course, the asymmetry can not exceed one, and thus our model estimate needs to be modified. First, the applied kinematic range of our formula should be reconsidered: Application of the twist-3 cross section at such small \( \ell_T \) may not be justified. Second, our model ansatz of \( G_F^a(x, x) \sim q^a(x) \) and \( E_F^a(z, z) \sim D^a(z) \) is not appropriate at \( x \to 1 \) and \( z \to 1 \), respectively. The derivative of these functions, which is important for the growing \( A_{pp}^N \) at large \( x_F \), eventually leads to divergence of \( A_{pp}^N \) at \( x_F \to 1 \) as \( \sim 1/(1 - x_F) \).

As a possible remedy to this pathology we tried the following. For the (A) (chiral-even) contribution we have a model \( G_F^a(x, x) \sim q^a(x) \sim x \to 1 \beta_a \) where \( \beta_u = 3.027 \) and \( \beta_d = 3.774 \) in the GRV distribution we adopted. Tentatively we shifted \( \beta_u, d \) as \( \beta_a \to \beta_a(x) = \beta_a + x^3 \), which suppresses the divergence of \( A_N \) at \( x_F \to 1 \) but still causes rising behavior of \( A_N \) at large \( x_F \). This avoids overshooting of one in \( A_{pp}^N \) but reduces \( A_{pp}^N \), typically by factor 2. So the twist-3 contribution to \( A_{pp}^N \) shown in Fig. 1 and 2 is further reduced, making the deviation from E704 and STAR data bigger.

To summarize we have studied the \( A_N \) for pion production in \( pp \) and \( ep \) collisions. Although our approach provides a systematic framework for the large \( \ell_T \) production, applicability of the formula to the currently available low \( \ell_T \) data still needs to be tested, in particular, comparison of \( \ell_T \) dependence with data is needed.

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