New Sum Rules for Nucleon Tensor Charges

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Abstract

Two new sum rules for the quark tensor charges of the nucleon are proposed, based
on a relation connecting the quark transversity distributions to the quark helicity
distributions and the quark model spin distributions, and on the sum rules for the
quark helicity distributions. The two sum rules are useful for an estimate of the
values of the quark tensor charges $\delta U$ and $\delta D$ from the measured quantities of $\Gamma^p$,
$\Gamma^n$, $g_A/g_V$ and $\Delta S$, and two model correction factors with limited uncertainties. We
predict a small value for the sum of the quark tensor charges compared to most other
predictions, in analogy to the unexpectedly small quark helicity sum which gave rise
to the proton “spin puzzle”.

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Historically, parton sum rules have played important roles in the understanding of the quark-gluon structure of the nucleons. The confirmation of the Gross-Llewellyn Smith (GLS) [1], Gottfried [2], and Adler [3] sum rules in early deep inelastic scattering (DIS) experiments on unpolarized structure functions were important for identifying the quantum numbers of partons with those of quarks. The recent refined measurements of the proton and neutron structure functions revealed the violation of the Gottfried sum rule and indicated an excess of $d\bar{d}$ quark pairs over $u\bar{u}$ quark pairs in the proton sea [4]. The observation of the violation of the Gourdin-Ellis-Jaffe sum rule [5] in polarized DIS experiments [6, 7, 8, 9] gave rise to the proton “spin crisis” or “spin puzzle” and triggered a vast number of theoretical and experimental investigations on the spin content of the nucleons. There has also been significant progress in the theory of the QCD corrections to the various parton sum rules; e.g., the generalized Crewther relation connects the observables in $e^+e^-$ annihilation and the Bjorken [10] and GLS sum rules in DIS, providing a precision test of the standard model with no scale or scheme ambiguities [11].

All of the above mentioned parton sum rules are related to the quark momentum distributions $q(x)$ and helicity distributions $\Delta q(x)$, two of the fundamental distributions which characterize the state of quarks in the nucleon at leading twist. The above two quark distributions are related to the vector quark current $\bar{q}\gamma^\mu q$ and the axial quark current $\bar{q}\gamma^\mu\gamma^5 q$ respectively. There is another fundamental distribution, the quark transversity distribution $\delta q(x)$ which is related to the matrix elements of the tensor quark current $\bar{q}\sigma^{\mu\nu}i\gamma^5 q$ [12]. Unfortunately, there is still no suggestion of a basic parton sum rule in analogy to the Bjorken sum rule for the quark transversity distributions. However, it has been recently shown that there is a relation [13] which connects the quark transversity distributions to the quark helicity distributions $\Delta q(x)$ and the quark model spin distributions:

$$\Delta q_{QM}(x) + \Delta q(x) = 2\delta q(x), \quad (1)$$

where $\Delta q_{QM}(x)$ is the quark spin distributions as defined in the quark model or in
the nucleon rest frame \[14, 15, 16, 17\]. One can use this relation to measure the quark model spin distributions once the quark helicity distributions and the quark transversity distributions are both measured. In this paper we will show that one can connect the quark tensor charges to the measured quantities \(g_A/g_V\), \(\Gamma_p\), \(\Gamma_n\) and several quantities with limited uncertainties by combining the relation eq. (1) with the parton sum rules for the quark helicity distributions.

The spin-dependent structure functions for the proton and the neutron, when expressed in terms of the quark helicity distributions \(\Delta q(x)\), should read

\[
\begin{align*}
g_1^p(x) &= \frac{1}{2}\left[\frac{4}{9}(\Delta u(x) + \Delta \bar{u}(x)) + \frac{1}{9}(\Delta d(x) + \Delta \bar{d}(x)) + \frac{1}{9}(\Delta s(x) + \Delta \bar{s}(x))\right], \\
g_1^n(x) &= \frac{1}{2}\left[\frac{1}{9}(\Delta u(x) + \Delta \bar{u}(x)) + \frac{4}{9}(\Delta d(x) + \Delta \bar{d}(x)) + \frac{1}{9}(\Delta s(x) + \Delta \bar{s}(x))\right].
\end{align*}
\]

The measured Gourdin-Ellis-Jaffe integrals \(\Gamma_p = \int_0^1 dx g_1^p(x)\) and \(\Gamma_n = \int_0^1 dx g_1^n(x)\) from polarized DIS experiments \[1, 2, 3, 4\] have been found to be in conflict with the corresponding sum rules \[5\] based on assumptions of zero strangeness, zero gluon spin contribution, and SU(3) symmetry for the octet baryons. The quark axial charge or the quark helicity related to the axial quark current \(\bar{q}\gamma^\mu\gamma^5 q\) is expressed by \(\Delta Q = \int_0^1 dx [\Delta q(x) + \Delta \bar{q}(x)]\). Combining eqs. (2) and (3) one obtains the sum of the quark axial charges (or quark helicities) for the light flavors

\[
\Delta U + \Delta D = \frac{18}{5}(\Gamma_p + \Gamma_n) - \frac{2}{5}\Delta S,
\]

from which we know that the sum of quark helicities \(\sum \Delta Q\) including the strangeness contribution should be

\[
\sum \Delta Q = \Delta U + \Delta D + \Delta S = \frac{18}{5}(\Gamma_p + \Gamma_n) + \frac{3}{5}\Delta S,
\]

where the quantities \(\Gamma_p\), \(\Gamma_n\), and \(\Delta S\) can be measured independently in different experiments. The Bjorken sum rule defined by

\[
\Gamma_p - \Gamma_n = \frac{1}{6}(\Delta U - \Delta D) = \frac{1}{6} \frac{g_A}{g_V},
\]

(6)
where $g_A/g_V$ is determined from the neutron $\beta$ decay, is a more basic result and has been found to be valid with the observed values of $\Gamma^p$ and $\Gamma^n$ within experimental uncertainties by taking into account QCD radiative corrections \[1, 8, 9\].

The tensor charge, defined as $\delta Q = \int_0^1 dx [\delta q(x) - \delta \bar{q}(x)]$, is chiral-odd due to the charge conjugation properties of the tensor current $\bar{q} \sigma^{\mu\nu} i \gamma^5 q$. Therefore the quark tensor charge $\delta Q$ and the quark axial charge $\Delta Q$ have different chiral parities. The quark helicity distributions, $\Delta q(x)$ and $\Delta \bar{q}(x)$, should be measured for quarks and anti-quarks separately in applying Eq. (1). In order to get the tensor charge for each flavor, we must first isolate eq. (1) for both quarks and anti-quarks of each flavor, and integrate. In practice, one expects the antiquark contributions to be small. For example, the anti-quark contributions to $\Delta Q$ and $\delta Q$ are zero in the meson-baryon fluctuation model \[18\] and in a broken-U(3) version of the chiral quark model \[19\]. There has been an explicit measurement of the helicity distributions for the individual $u$ and $d$ valence and sea quarks by the Spin Muon Collaboration (SMC) \[20\]. The helicity distributions for the $u$ and $d$ anti-quarks are consistent with zero in agreement with the results of the light-cone meson-baryon fluctuation model of intrinsic $q\bar{q}$ pairs.

We thus can assume that the anti-quark contributions are negligible. We thus obtain, combining eqs. (1) and (8),

$$\delta U - \delta D = \frac{1}{2} \left( (\Delta U - \Delta D) + (\Delta U_{QM} - \Delta D_{QM}) \right),$$

where the first term in the right side satisfies the Bjorken sum rule and the second term satisfies a Bjorken-like sum rule in which one can approximate the quantity $\Delta U_{QM} - \Delta D_{QM}$ by the non-relativistic value $5/3$ for the naive quark model. Therefore we have a Bjorken-like sum rule for the isovector tensor charge

$$\delta U - \delta D = \frac{1}{2} \left( \frac{g_A}{g_V} + \frac{5}{3} c_1 \right),$$

where $g_A/g_V$ might be the value from the neutron $\beta$ decay or $g_A/g_V = 6(\Gamma^p - \Gamma^n)$ from eq. (3) and $c_1$ is an unknown correction factor reflecting the deviation from
the naive quark model value $\Delta U_{QM} - \Delta D_{QM} = 5/3$ and might range from 0.9 to 1. Similarly, combing eqs. (1) and (3), we obtain the second sum rule for the isoscalar tensor charge
\[ \delta U + \delta D = \frac{1}{2}[(\Delta U + \Delta D) + (\Delta U_{QM} + \Delta D_{QM})] = \frac{9}{5}(\Gamma^p + \Gamma^n) - \frac{1}{5}\Delta S + \frac{1}{2}c_2, \tag{9} \]
where $c_2$ is another unknown correction factor reflecting the deviation from the naive quark model value $\Delta U_{QM} + \Delta D_{QM} = 1$ and might range from 0.75 to 1.

From eqs. (8) and (9), we can predict the quark tensor charges $\delta U$ and $\delta D$ by use of the measurable quantities $\Gamma_p, \Gamma_n, g_A/g_V$ and $\Delta S$, and the correction factors $c_1$ and $c_2$ with limited uncertainties. The quantities $\Gamma^p$ and $\Gamma^n$ at several different $Q^2$ have been measured from polarized DIS experiments [6, 7, 8, 9], and $\Delta S$ has also been extracted from analysis of the polarized DIS data and it might range from about -0.01 [18] to -0.13 [21]. The value of $\Delta S$ from those analysis is sensitive to the assumption of SU(3) symmetry. It would be better to measure $\Delta S$ from other independent processes and there have been suggestions for this purpose [22, 23]. Nevertheless, we notice that the predicted values of $\delta U$ and $\delta D$ are not sensitive to $\Delta S$. In case $g_A/g_V = 6(\Gamma_p - \Gamma^n)$ is adopted (we denote case 1), for $\Gamma^p(E143) = 0.127$ and $\Gamma^n(E143) = -0.037$ at $\langle Q^2 \rangle = 3$ GeV$^2$, we have
\[ \delta U = 0.89 \rightarrow 1.01; \]
\[ \delta D = -0.28 \rightarrow -0.39, \tag{10} \]
and for $\Gamma^p$(SMC) = 0.136 and $\Gamma^n$(SMC) = -0.063 at $\langle Q^2 \rangle = 10$ GeV$^2$, we have
\[ \delta U = 0.93 \rightarrow 1.04; \]
\[ \delta D = -0.34 \rightarrow -0.46. \tag{11} \]

Combining the above two constraints and taking into account further the uncertainties (0.05) introduced by the data, we have
\[ \delta U = 0.84 \rightarrow 1.09; \]
\[ \delta D = -0.23 \rightarrow -0.51. \tag{12} \]
In case the value $g_A/g_V = 1.2573$ from neutron $\beta$ decay is adopted (we denote case 2), we obtain
\[
\delta U = 0.94 \rightarrow 1.06; \\
\delta D = -0.36 \rightarrow -0.48
\]
corresponding to eq. (10) and
\[
\delta U = 0.96 \rightarrow 1.07; \\
\delta D = -0.34 \rightarrow -0.46
\]
corresponding to eq. (11). We notice that the difference between eqs. (13) and (14) is much smaller than that between eqs. (10) and (11). This indicates the sensitivity to the quantity $g_A/g_V$ used in the sum rule (8). Combining the constraints (13) and (14) and taking into account also the uncertainties 0.05, we obtain
\[
\delta U = 0.89 \rightarrow 1.11; \\
\delta D = -0.29 \rightarrow -0.53
\]
Further progress in the precision of the data and in the knowledge of the correction factors can further constrain the results. Therefore the predicted $\delta U$ and $\delta D$ are within limited ranges from the two sum rules eqs. (8) and (9).

We list in Table 1 our predictions of the quark tensor charges $\delta U$ and $\delta D$ and the values of the two sums (8) and (9). There have been a number of calculations of the quark tensor charges $\delta U$ and $\delta D$, and a comparison of our results with several existing predictions [17, 23, 24, 25, 26, 27, 28] is also made in Table 1. From the table we notice the significant difference between the predictions. One interesting feature we notice is that the value of the first sum (i.e., the isovector tensor charge $\delta U - \delta D$) in our work is consistent with most other predictions except the lattice QCD result, whereas the value of the second sum (i.e., the isoscalar tensor charge $\delta U + \delta D$) is small and only consistent with the lattice QCD result [28]. The small $\delta U + \delta D$ in our work seems to be more reasonable in analogy to the unexpected small quark helicity sum $\Delta U + \Delta D$ which gave rise to the “spin puzzle”. It is also supported by a Skyrme model analysis in which $\delta U + \delta D$ is of the order of $1/N_c$ relative to $\delta U - \delta D$ in the
| Name of work                                      | \( \delta U \) | \( \delta D \) | \( \delta U - \delta D \) | \( \delta U + \delta D \) |
|--------------------------------------------------|----------------|----------------|---------------------------|---------------------------|
| Case 1 of this work                              | 0.84 → 1.09    | -0.23 → -0.51  | 1.24 → 1.43               | 0.51 → 0.69               |
| Case 2 of this work                              | 0.89 → 1.11    | -0.29 → -0.53  | 1.38 → 1.46               | 0.51 → 0.69               |
| Light-cone quark model [17]                      |                |                |                           |                           |
| QCD sum rule [24]                                | 1.167          | -0.292         | 1.458                     | 0.875                     |
| Chiral soliton model [25]                        | 1.12           | -0.42          | 1.54                      | 0.70                      |
| Chiral chromodielectric model [26]               | 0.969          | -0.250         | 1.219                     | 0.719                     |
| Spectator model [27]                             | 1.218          | -0.255         | 1.473                     | 0.963                     |
| Lattice QCD [28]                                 | 0.84           | -0.23          | 1.07                      | 0.61                      |
| Non-relativistic limit                           | \( \frac{1}{3} = 1.333 \) | \( -\frac{1}{3} = -0.333 \) | \( \frac{5}{3} = 1.667 \) | 1                          |
| Ultra-relativistic limit                         | \( \frac{2}{3} = 0.667 \) | \( -\frac{1}{3} = -0.167 \) | \( \frac{5}{3} = 0.833 \) | \( \frac{1}{2} = 0.5 \) |

large-\( N_c \), SU(3)-symmetric limit [29]. From another point of view, a small \( \delta U + \delta D \) can be naturally understood within a framework of the SU(6) quark spectator model [16] plus the baryon-meson fluctuation model [18]: the flavor asymmetry between the Melosh-Wigner rotation factors for the \( u \) and \( d \) quarks will cause a reduction of \( \delta U + \delta D \) relative to the flavor symmetric case [17], and a further reduction comes from an additional negative contribution to \( \delta D \) due to the intrinsic \( d\bar{d} \) fluctuations related to the Gottfried sum rule violation. The future experimental measurements of \( \delta U \) and \( \delta D \) can test the above predictions and reveal more information of the quark-gluon structure of the nucleon if the measured values will be out of the predicted ranges.

We should mention that since there is no fundamental physical tensor current, the proposed sum rules have then the correction coefficients, i.e., they are not exact.
We have neglected the contributions from anti-quarks, gluons, $Q^2$ dependence due to higher twist effects, and different evolution behaviors between $\Delta Q$ and $\delta Q$ in the above analysis. In principle the corrections due to these sources can be further taken into account from theoretical and experimental progress and they should be topics for later study. We indicate that the contributions due to gluons or sea quarks might be canceled in $\delta U - \delta D$ and $\Delta U_{QM} - \Delta D_{QM}$, in analogy to the situation of $\Delta U - \Delta D$ \cite{17}. Therefore the first sum rule (8) might be more basic than the second one (9), and that is also why we adopted a small uncertainty ($0.9 \rightarrow 1$) for the correction factor $c_1$ compared to $c_2$ with a large uncertainty ($0.75 \rightarrow 1$) due to the possible negative contribution from the sea quarks \cite{18}.

One of the known constraints for the quark transversity distributions is Soffer’s inequality \cite{30}:

$$q(x) + \Delta q(x) \geq 2|\delta q(x)|,$$

which is valid for each flavor, likewise for antiquarks. We need to check whether our predicted values for $\Delta U$ and $\delta D$ satisfy this inequality, if we neglect antiquark contributions as was explained before. At a first sight one may have doubt since $\delta D$ can be -0.5 from Table 1, whereas the measured $\Delta D$ is around -0.35 and the integrated $\int_0^1 [dx]d(x)$ for valence quark is only 1. However, one should take into account the $d$ sea quarks for the first term of (16). From the Gottfried sum rule violation \cite{4} we know that the excess of $d\bar{d}$ over $u\bar{u}$ should be of the order 0.15 and in principle there could be also unlimited numbers of extrinsic sea quarks in the nucleon sea \cite{18}. Thus there is no difficulty to satisfy the Soffer’s inequality for the values of $\delta U$ and $\delta D$ predicted from the two sum rules (8) and (9).

We also list in Table 1 the values of $\delta U$ and $\delta D$ in the non-relativistic and ultra-relativistic limits \cite{15, 17} of the simple three quark light-cone model. The predicted quark tensor charges $\delta U$ and $\delta D$ listed in Table 1 are also presented in Fig. 1. It is interesting to note that the tensor charges still have finite values in the ultra-relativistic limit, compared to the corresponding case of vanishing axial charges \cite{15}.
Figure 1: The predictions of the quark tensor charges $\delta U$ and $\delta D$. The markers are predictions from several models: the light-cone quark model $\circ$ [17], the chiral soliton model $\otimes$ [25], the chiral chromo-electric model $\bigtriangleup$ [26], the spectator model $\diamondsuit$ [27], and lattice QCD $\bullet$ [28]. The solid box represents the range within the non-relativistic and ultra-relativistic limits of the simple three quark light-cone model [15, 17], the dotted box represents the prediction from the QCD sum rule [24], and the dashed box represents the range predicted from the two sum rules Eqs. (8) and (9).
We also notice that the predicted values for $\delta U$, $\delta U - \delta D$, and $\delta U + \delta D$ are within the values between the two limits, whereas the predicted $\delta D$ may have an additional negative contribution beyond the naive quark model. This is similar to the case of the axial charges discussed in Ref. [18]. Unlike most other predictions, the QCD sum rule analysis [24, 31] predicted a shift of $\delta D$ beyond the quark model limits in an opposite direction. Thus any evidence of the measured $\delta D$ beyond the range $-1/6 \rightarrow -1/3$ will be useful to confirm contribution from the intrinsic $d$ sea quarks predicted in Refs. [13, 18] or other new physics.

In summary, we proposed in this paper two new sum rules, based on a known relation connecting the quark transversity distributions to the quark helicity distributions and the quark model spin distributions, and on the sum rules for the quark helicity distributions. Though the two sum rules are simple, they are useful to predict the values of the quark tensor charges $\delta U$ and $\delta D$ from the measured quantities of $\Gamma^p$, $\Gamma^n$, $g_A/g_V$ and $\Delta S$, and two model correction factors with limited uncertainties. We also predicted a small value for the sum of the quark tensor charges compared to most other predictions, and this seems to be reasonable in analogy to the unexpected small quark helicity sum which gave rise to the proton “spin puzzle”.

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