Williamson nanofluid flow over a stretching sheet with varied wall thickness and slip effects

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Abstract. This study investigates the effects of slip parameter and velocity power index parameter along with wall thickness on the magnetohydrodynamic (MHD) boundary layer flow of a Williamson nanofluid through a stretching sheet in porous medium. The governing partial differential equations are transformed into nonlinear ordinary differential equations (ODEs) using the relevant similarity variables. These nonlinear ODEs are solved numerically using the Runge-Kutta Fehlberg in MAPLE software. The effects of the pertinent parameters on the velocity, temperature and nanoparticle volume fraction profiles are presented graphically. The impact of the physical parameters on the skin friction coefficient, the local Nusselt number and the local Sherwood number are computed and analyzed. The velocity profile increases when the velocity slip parameter increases. The temperature slip and nanoparticle slip parameters reduce the temperature and the nanoparticle volume fraction profiles respectively. The temperature and the nanoparticle volume fraction profiles significantly increase due to the increase in the velocity power index. An opposite behaviour is observed on different values of the wall thickness parameter when the power index is less than one compared to greater than one.

1. Introduction

Williamson fluid is featured as a shear thinning non-Newtonian fluid proposed by Williamson [1]. According to him, this fluid is significantly important to distinguish the plastic flow from the viscous flow. A variation apparent viscosity is considered in the momentum governing equation which represents the plastic character of the dispersion. There are many applications on this non-Newtonian fluid especially in the behaviour of pseudoplastic fluid which is widely applied in industrial applications. It is equally important in the biological engineering for example to measure the mass and heat transfer through the vessels in blood and hemodialysis [2]. Hence, many researchers [3-5] investigate this fluid flow respected to various conditions.

Velocity slip occurs when one phase flows faster than the extra phase in multiphase flow. Thermal slip is the difference temperature flows along a surface with certain temperature [6], while concentration slip is the disturbance to the present concentration due to new solute concentration. The slip velocity often occurs in inhomogeneous fluids, especially emulsions, slurries, foams and gels [7-10]. Prasannakumara et al. [11] had investigated the effects of nonlinear thermal radiation on Williamson nanofluid slip flow over stretching sheet in a porous medium. They imposed the Runge-Kutta Fehlberg to solve the non-dimensional equations with shooting methods. The results portrayed the increasing
value of the velocity slip parameter reduced the momentum boundary layer thickness. Both the temperature and nanoparticle volume fraction profiles decreased with an increase in the thermal slip parameter. However, the increase of the nanoparticle slip parameter led to the increase of the nanoparticles volume fraction profile.

Most of the literature mentioned above are restricted to the stretching sheet. The study on the MHD flow over an exponentially stretching sheet are done by Zaman et al. [12] and Khalili et al. [13]. However, there are many conditions require variable thickness of the surface. Reddy et al. [14] studied MHD flow and heat transfer characteristics of Williamson nanofluid over a stretching sheet with variable thickness and variable thermal conductivity. They concluded that the velocity profile decreases for the increase in wall thickness parameter when the velocity power index parameter is less than one and reverse trend is observed for the power index parameter is greater than one.

Shagaiya et al. [15] have presented on boundary layer flow of nanofluid over nonlinear stretched surface with variable thickness in the existence of electric field. They solved the problems using the Keller box method. The skin friction, the rate of heat and mass transfer were reduced with the increase of the wall thickness. Then, MHD flow and heat transfer in a nanofluid through a slender elastic sheet with variable thickness and variable fluid properties were analyzed by Prasad et al. [16]. The study shows there is a decrease in the velocity, temperature and concentration profiles with the increase in the wall thickness parameter.

This paper extends the work done by Reddy et al. [14] by adding slips boundary conditions and the velocity power index parameter along with the variable thickness. Hence, the effects of the slip parameters and the velocity power index parameter on the MHD boundary layer flow of Williamson nanofluid over a stretching sheet with the variable thickness are investigated.

2. Mathematical formulation

This section considers a steady MHD two-dimensional laminar flow of Williamson fluid over a stretching plate with variable wall thickness by considering slip parameters and velocity power index parameter in porous medium.

Figure 1 shows the schematic diagram of the boundary layer with added porous medium. The sheet is drawn through the fluid medium over the origin where the slit is located. The sheet is stretched exponentially with velocity \( U_w = U_0 (x + b)^m \) where \( U_0 \) and \( b \) are the dimensional constant and \( m \) is the velocity power index. The stretching surface is placed along the \( x \)-axis parallel to the direction of the sheet motion while the \( y \)-axis is perpendicular to the \( x \)-axis. \( T_w \) is the temperature of the fluid at the wall, \( T_a \) is the ambient temperature, \( C_w \) is the nanoparticle volume fraction at the wall, \( C_a \) is the ambient nanoparticle fraction and \( B \) is the transverse magnetic field. The thickness varies as \( y = A(x + b)^{(1-m)/2} \) where \( A \) is very small constant which means the sheet is not flat and sufficiently thin. The sheet becomes a flat sheet when \( m = 1 \).

Cauchy stress tensor \( S \) of Williamson fluid model is defined as \( S = -pI + \tau \) which \( p \) is the pressure, \( I \) is an identity vector and \( \tau = \left( \mu_e + (\mu_0 - \mu_e) / 1 - \Gamma \gamma \right) A_1 \) is an extra stress tensor where \( \mu_0 \) is a limiting viscosity when shear rate at zero, \( \mu_e \) is a limiting viscosity when shear rate at infinite, \( \Gamma > 0 \) is a time constant, \( A_1 \) is the first Rivlin-Erickson tensor and \( \gamma \) defines as \( \gamma = \sqrt{0.5 \pi} \) and \( \pi = \text{trace} \left( A_1^2 \right) \).

Reddy et al. [14] had stated that \( \mu_e = 0 \), \( \Gamma \gamma < 1 \) and then \( \tau \) can be classified as \( \tau = \left( \mu_0 / \left( 1 - \Gamma \gamma \right) \right) A_1 \).

The result is formed as \( \tau = \mu_0 (1 - \Gamma \gamma) A_1 \) when using the binomial expansion.
The formulation of governing equations and boundary conditions are written as [11],[14] and [17]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + \sqrt{2\nu k} \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho} u - \nu \frac{\partial u}{\partial y}
\]  

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{(\rho c_p)_f} \frac{\partial}{\partial y} \left( \kappa \frac{\partial T}{\partial y} \right) - \frac{1}{(\rho c_p)_f} \frac{\partial q_r}{\partial y} + \frac{(\rho c_p)_p}{(\rho c_p)_f} \left[ D_B \frac{\partial C}{\partial y} \left( \frac{\partial T}{\partial y} \right)^2 + \frac{D_T}{T_p} \frac{\partial T}{\partial y} \right]
\]  

where \(u\) and \(v\) are the velocity component in the \(x\) and \(y\) directions, \(\nu\) is the kinematic viscosity, \(\Gamma\) is the material parameter of Williamson fluid and \(\sigma\) is the electrical conductivity. The magnetic field is defined as \(B = B_0(x+b)^{n-1}\) [18]. The symbol \(\rho\) is the density, \(k\) is the permeability of the porous medium, \(T\) is the fluid temperature, \(c_p\) is the specific heat at constant pressure and \((\rho c_p)_f\) is the heat capacity of the nanofluid. Next, \(\kappa\) is the temperature dependent thermal conductivity, \(q_r\) is the radiative heat flux, \((\rho c_p)_p\) is the effective heat capacity of the nanoparticle, \(D_B\) is the Brownian diffusion coefficient, \(C\) is the nanoparticle volume fraction, \(D_T\) is the thermophoretic diffusion coefficient.
The governing equations in (1) to (4) are subjected to the boundary conditions as

\[ u = U_w(x) + L_1 \frac{\partial u}{\partial y} = U_0(x + b)m + L_1 \frac{\partial u}{\partial y}, \quad v = 0, \quad T = T_w + L_2 \frac{\partial T}{\partial y}, \]

\[ C = C_w + L_3 \frac{\partial C}{\partial y} \quad \text{at} \quad y = (x + b)^{1-m} \frac{1-m}{2}, \]

\[ u = 0, \quad T = T_\infty, \quad C = C_\infty \quad \text{at} \quad y \to \infty \]

(5)

where \( L_1, L_2, \) and \( L_3 \) are velocity, temperature and nanoparticle fraction slip factors respectively. The temperature dependent thermal conductivity is in the form of \( \kappa = \kappa_0 \left(1 + \omega(T - T_\infty)/(T_w - T_\infty)\right) \) which \( \omega = (pc_p)_p/(pc_f)_f \) is the ratio between the nanoparticles material effective heat capacity and fluid heat capacity. The Rosseland approximation for radiation is denoted as

\[ q_r = \frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y} \]

(6)

where \( \sigma^* \) is the Stefan–Boltzmann constant, \( T^4 \) is written as a linear function of the temperature which is the temperature differences within the flow and \( k^* \) is the mean absorption coefficient. After neglecting higher order terms of the Taylor series expansion of \( T^4 \), we obtain

\[ T^4 = 4T_\infty^4 T - 3T_\infty^4 \]

(7)

Then, using equations (6) and (7), we obtain

\[ \frac{\partial q_r}{\partial y} = -\frac{16\sigma^*}{3k^*} \frac{T^3}{T} \frac{\partial^2 T}{\partial y^2} \]

(8)

The similarity variables being considered are

\[ \eta = \sqrt{\frac{U_0(m+1)}{2\nu}} \left(y(x+b)^{m-1} - A\right), \quad \psi = \sqrt{\frac{2\nu U_0}{m+1}} (x+b)^{m+1} f(\eta), \]

\[ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \]

(9)

where \( \eta \) is the similarity variable, \( A \) is a constant, \( \psi \) is the dimensionless stream, \( f(\eta) \) is the dimensionless stream function, \( \theta(\eta) \) is the temperature function and \( \phi(\eta) \) is the nanoparticle volume fraction function. Substituting (9) into (1) – (4) along with the boundary conditions (5), the partial differential equations are transformed into ordinary differential equations as

\[ f''' + \lambda f f'' + f' - \frac{2m}{m+1} f'' - (M + kp)f' = 0 \]

(10)

\[ \left(1 + \frac{4}{3} R + \varepsilon \theta\right) \theta'' + \varepsilon \theta' + \frac{Nc}{Le} \phi' \theta' + \frac{Nc}{Le Nbt} \theta'^2 = 0 \]

(11)

\[ \phi'' + \frac{Le \ Pr f \phi' + 1}{Nbt} \theta'' = 0 \]

(12)
where \( \lambda = \Gamma \sqrt{U_0^3 (m+1)(x+b)^{m-1}} / \nu \) is the Williamson parameter, \( m \) is the velocity power index parameter, \( M = 2 \sigma B_i^2 / \rho U_0 (m+1) \) is the magnetic parameter, \( kp = 2 \nu (x+b)^{m-1} / kU_0 (m+1) \) is the permeability parameter, \( R = 4 \sigma T_0^3 / \kappa k' \) is the radiation parameter, \( \varepsilon \) is the thermal conductivity parameter, \( Pr = \nu / \alpha \) is the Prandtl number, \( Nc = \rho_p c_p (C_w - C_w) / (\rho c)_f \) is the heat capacity parameter, \( Le = \alpha / D_n \) is the Lewis number and \( Nbt = D_n (T_w - C_w) / D_T (T_w - T_w) \) is the Brownian diffusivity parameter. The transformed boundary conditions are

\[
f'(0) = 1 + (N1)f', \quad f(0) = \alpha \left( \frac{1-m}{m+1} \right) f'(0), \quad \phi(0) = 1 + (N2)\phi', \quad \theta(0) = 1 + (N3)\theta'
\]

\[
f'(\infty) = 0, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0
\]

where \( \alpha \) is the wall thickness parameter, \( N1 \) is the velocity slip parameter, \( N2 \) is the temperature slip parameter and \( N3 \) is the nanoparticle fraction slip parameter.

3. Results and discussion

The numerical values of the skin friction coefficient for the governing equations are obtained at various values of the velocity power index parameter when \( M = \lambda = kp = 0 \) and \( \alpha = 0.5 \). Table 1 shows the comparison of skin friction coefficient, \( f''(0) \) between the results obtained by Khader and Megahed [19], Reddy et al. [14] and present results are in excellent agreement.

| \( m \) | Khader & Megahed (2013) | Reddy et al. (2017) | Present results |
|---|---|---|---|
| 0 | 0.9577 | 0.957644 | 0.957644 |
| 0.5 | 0.9798 | 0.979949 | 0.979949 |
| 1.0 | 1.0000 | 1.000008 | 1.000008 |
| 2.0 | 1.0234 | 1.023420 | 1.023420 |
| 3.0 | 1.0358 | 1.035883 | 1.035883 |
| 5.0 | 1.0486 | 1.048628 | 1.048628 |
| 7.0 | 1.0551 | 1.055062 | 1.055062 |
| 9.0 | 1.0588 | 1.058934 | 1.058934 |
| 10.0 | 1.0603 | 1.060343 | 1.060343 |
Figures 2 and 3 show the effects of three different values of the velocity slip parameter $N_1$ on the velocity and temperature profiles respectively. Figure 2 shows the rises of the velocity slip parameter $N_1$ will increase the velocity profile as well as the fluid velocity within the boundary layer. This is due to the positive value of the gradient of the fluid velocity adjacent to the surface of the sheet, hence reducing the momentum boundary layer thickness. Moreover, an increasing of the velocity slip parameter allows more fluid to slip through the sheet, consequently increases the fluid flow in the boundary layer. However, figure 3 shows the temperature profile drops when the velocity slip parameter rises which leads to the decrement of thermal boundary layer thickness.

Figure 4 indicates the effects of the temperature slip parameter on the temperature profile. The temperature slip reduces the temperature profile hence the temperature boundary layer thickness decreases as the heat transfer from the surface to the fluid decreases.
Figure 5. Nanoparticle volume fraction profile for different values of $N3$ when $Le=5$, $e=0.2$, $m = M = kp = R = Nc = 0.5$, $\lambda = 0.2$, $Pr = Nb = Le = 2$, $N1 = N2 = 0.1$ and $\alpha = 1.5$.

Figure 6. Velocity profile for different values of when $M = 1$, $kp = e = R = 0.2$, $\lambda = 0.2$, $Pr = Nb = Le = 2$, $Nc = \alpha = 0.5$ and $N1 = N2 = N3 = 0.1$.

Figure 7. Temperature profile for different values of $m$ when $M = 1$, $kp = e = R = 0.2$, $\lambda = 0.2$, $Pr = Nb = Le = 2$, $Nc = \alpha = 0.5$ and $N1 = N2 = N3 = 0.1$.

Figure 8. Nanoparticle volume fraction profile for different values of $m$ when $M = 1$, $kp = e = R = 0.2$, $\lambda = 0.2$, $Pr = Nb = Le = 2$, $Nc = \alpha = 0.5$ and $N1 = N2 = N3 = 0.1$.

The results show, the temperature and the nanoparticle volume fraction profiles are significantly increased while the velocity profile is slightly increased due to the increase in the velocity power index $m$ as shown in figures 6-8. Physically, the increment in the velocity power index parameter $m$ enhances the mass and the heat transfer as the plate is stretched exponentially. Consequently, it increases the thermal and nanoparticle boundary layer thicknesses.
Figure 9. Velocity profile for different values of $\alpha$ when $m < 1$, $M = 1$, $\lambda = 0.2$, $\varepsilon = R = 0.1$, $kp = Nc = 0.5$, $Pr = Nbt = 2$, $N1 = N2 = N3 = 0.2$ and $Le = 3$.

Figure 10. Temperature profile for different values of $\alpha$ when $m < 1$, $M = 1$, $\lambda = 0.2$, $\varepsilon = R = 0.1$, $kp = Nc = 0.5$, $Pr = Nbt = 2$, $N1 = N2 = N3 = 0.2$ and $Le = 3$.

Figure 11. Nanoparticle volume fraction profile for different values of $\alpha$ when $m < 1$, $M = 1$, $\lambda = 0.2$, $\varepsilon = R = 0.1$, $kp = Nc = 0.5$, $Pr = Nbt = 2$, $N1 = N2 = N3 = 0.2$ and $Le = 3$.

Figure 12. Velocity profile for different values of $\alpha$ when $m > 1$, $M = 1$, $\lambda = 0.2$, $\varepsilon = R = 0.1$, $kp = Nc = 0.5$, $Pr = Nbt = 2$, $N1 = N2 = N3 = 0.2$ and $Le = 3$.

Figure 13. Temperature profile for different values of $\alpha$ when $m > 1$, $M = 1$, $\lambda = 0.2$, $\varepsilon = R = 0.1$, $kp = Nc = 0.5$, $Pr = Nbt = 2$, $N1 = N2 = N3 = 0.2$ and $Le = 3$.

Figure 14. Nanoparticle volume fraction profile for different values of $\alpha$ when when $m > 1$, $M = 1$, $\lambda = 0.2$, $\varepsilon = R = 0.1$, $kp = Nc = 0.5$, $Pr = Nbt = 2$, $N1 = N2 = N3 = 0.2$ and $Le = 3$. 
Table 2. Numerical values of wall thickness parameter when $\lambda = 0.2$, $M = 1$, $\varepsilon = R = 0.1$, $N1 = N2 = N3 = 0.2$, $kp = Nc = 0.5$, $Pr = Nb t = 2$ and $Le = 3$.

| $m$ | $f''(0)$ | $-\theta'(0)$ | $-\phi'(0)$ |
|-----|----------|---------------|-------------|
| 0.5 | 1.96787  | 0.68733       | 1.22175     |
| 0.6 | 2.09902  | 0.85117       | 1.58483     |
| 1.2 | 2.23789  | 1.01420       | 1.90562     |
| 3.0 | 2.22165  | 0.67019       | 1.20393     |
| 0.6 | 2.04630  | 0.42580       | 0.61767     |
| 1.2 | 1.88669  | 0.20957       | 0.16907     |

Figures 9 through 11 show that the changes of wall thickness parameter $\alpha$ when $m < 1$, while figures 12 through 14 show the cases for value of $m > 1$. Figures 9-11 portray when the wall thickness parameter $\alpha$ increases, the velocity, temperature and nanoparticle volume fraction profiles decrease. Apparently, the decrease in the three profiles lead to the reduction on the boundary layers near the plate respectively. Hence, the rate of heat and mass transfer increase. The graphs in figures 12-14 illustrate the three profiles increase with the increment of wall thickness parameter $\alpha$. These results show an opposite behavior compared to the results when $m < 1$ since as the wall thickness parameter $\alpha$ increases, the slower the fluid flow and the smaller the heat and mass transfer move away from the wall as depicted in table 2. These analysis are quite similar to the results reported by Reddy et al. [9] without the slip parameters and porous medium.

4. Conclusion
This paper studied the effects of the slip parameter and the velocity power index parameter along with wall thickness on the MHD boundary layer flow of a Williamson nanofluid through a stretching sheet in porous medium. In summary, the velocity profile increases when velocity slip parameter increases. An increasing of the velocity slip parameter allows more fluid to slip through the sheet, consequently increases the fluid flow in the boundary layer. The increasing of the temperature slip parameter resulted in the decrease of boundary layer thickness as well as the heat transfer from the surface to the fluid decreases. While, the nanoparticle fraction slip parameter decreases the nanoparticle volume fraction profile which leads to the decrease of the mass transfer from the surface to fluid. Hence, nanoparticle boundary layer thickness decreases. The temperature and the nanoparticle volume fraction profiles are significantly increased while the velocity profile is slightly increased due to the increase in the velocity power index. There are two cases for the wall thickness parameter which are when $m < 1$, the velocity, the temperature and the nanoparticle volume fraction profiles decrease when wall thickness parameter increases with the increase of the skin friction coefficient, the local Nusselt number and the local Sherwood number and when $m > 1$, the effects are vice versa.

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