Dirty Paper Coded Rate-Splitting for Non-Orthogonal Unicast and Multicast Transmission with Partial CSIT

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Abstract—A Non-Orthogonal Unicast and Multicast (NOUM) transmission system allows a multicast stream intended to all receivers to be jointly transmitted with unicast streams in the same time-frequency resource blocks. While the capacity of the two-user multi-antenna NOUM with perfect Channel State Information at the Transmitter (CSIT) is known and achieved by Dirty Paper Coding (DPC)-assisted NOUM with Superposition Coding (SC), the capacity and the capacity-achieving strategy of the multi-antenna NOUM with partial CSIT remain unknown. In this work, we focus on the partial CSIT setting and make two major contributions. First, we show that linearly precoded Rate-Splitting (RS), relying on splitting unicast messages into common and private parts, encoding the common parts together with the multicast message and linearly precoding at the transmitter, can achieve larger rate regions than DPC-assisted NOUM with partial CSIT. Second, we study Dirty Paper Coded Rate-Splitting (DPCRS), that marries RS and DPC. We show that the rate region of DPCRS-assisted NOUM is enlarged beyond that of conventional DPC-assisted NOUM and that of linearly precoded RS-assisted NOUM with partial CSIT.

I. INTRODUCTION

Non-Orthogonal Unicast and Multicast (NOUM) transmission, also known as layered division multiplexing (LDM), has been gaining increasing attentions recently. It is considered to be a promising solution to support a mixture of unicast and multicast services for future wireless communication networks. Different from conventional approaches where unicast and multicast services are carried out in orthogonal resource blocks, NOUM allows each user to receive a dedicated unicast message and a multicast message simultaneously based on Superposition Coding (SC) at the transmitter and Successive Interference Cancellation (SIC) at the receivers.

There are two conventional approaches studied in the literature of NOUM for multi-antenna Broadcast Channels (BC). The first approach is the practical Multi-User Linear Precoding (MU–LP)-assisted NOUM [2, 11] where the encoded multicast and unicast streams are linearly precoded and superimposed at the transmitter, each user decodes and removes the multicast stream with the assistance of one layer SIC before decoding its intended unicast stream. The other approach is the non-linear Dirty Paper Coding (DPC)-assisted NOUM relying on DPC to encode unicast messages and SC to encode multicast messages. It is also known as “multi-antenna BC with a common message” in [3–7]. DPC-assisted NOUM has been shown in [7] to achieve the capacity region of two-user multi-antenna NOUM with perfect Channel State Information at the Transmitter (CSIT). However, when the transmitter only has access to partial Channel State Information (CSI), the capacity and the capacity-achieving strategy of multi-antenna NOUM remain an open problem.

Even in the conventional unicast-only multi-antenna BC with partial CSIT, the capacity and capacity-achieving scheme are still unknown. Interestingly, we have shown in our recent work [8] that Dirty Paper Coded Rate-Splitting (DPCRS), that relies on Rate-Splitting (RS) to split user messages into common and private parts, and DPC to encode the private parts, enlarges the rate region of conventional DPC in Multiple-Input Single-Output (MISO) BC with partial CSIT. Moreover, linearly precoded RS, which has been widely studied in multi-antenna networks [9–13], is able to achieve larger rate region than DPC in multi-antenna BC with partial CSIT. The application of linearly precoded RS in multi-antenna NOUM has also been recently studied in [14]. By splitting the unicast messages of users into common and private parts, jointly encoding the multicast message and the common parts of the private messages into a super-common stream, linearly precoding super-common stream and private streams, RS-assisted NOUM has been shown to achieve higher spectral and energy efficiencies than the conventional MU–LP-assisted or Non-Orthogonal Multiple Access (NOMA)-assisted strategies thanks to its robustness and flexibility to manage interference [14].

In this work, we first study the performance of DPC and linearly precoded RS in multi-antenna NOUM with partial CSIT. We show that linearly precoded RS is able to achieve larger rate regions than DPC-assisted NOUM. Motivated by the performance benefits of the linearly precoded RS-assisted NOUM as well as the non-linear DPCRS frameworks in the unicast-only transmission, we further propose a novel DPCRS-assisted NOUM transmission strategy relying on splitting unicast messages into common and private parts, encoding the private parts by DPC and encoding the common parts together with the multicast message at the transmitter. We show that...
such DPCRS-assisted NOUM achieves larger rate regions than conventional DPC-assisted NOUM with partial CSIT.

II. SYSTEM MODEL

Consider a single-cell downlink transmission, which consists of one multi-antenna Base Station (BS) equipped with \( N_t \) antennas simultaneously serving \( K \) single-antenna users, indexed by \( \mathcal{K} = \{1, \ldots, K\} \). Hybrid unicast and multicast services are provided in the system. In each scheduled time frame, the BS delivers one multicast message \( W_0 \) to all users and \( K \) dedicated unicast messages \( W_k, k \in \mathcal{K} \) to the corresponding users. The \( K+1 \) messages are encoded into the stream vector \( s \) and linearly precoded by the precoding matrix \( P \). The resulting transmit signal is \( x = Ps \), which is subject to the transmit power constraint \( \mathbb{E}\{|x|^2\} \leq P_t \). Under the assumption that \( \mathbb{E}\{ss^H\} = 1 \), we obtain that \( \text{tr}(PP^H) = P_t \).

The signal received by user-\( k \) is
\[
y_k = h_k^H x + n_k, \forall k \in \mathcal{K},
\]
where \( h_k \in \mathbb{C}^{N_t} \) is the channel between the BS and user-\( k \), \( n_k \sim \mathcal{CN}(0,1) \) is the Additive White Gaussian Noise (AWGN). Hence, the transmit Signal-to-Noise Ratio (SNR) is equal to \( P_t \).

A. Partial Channel State Information

In this work, we assume the CSI of each user is perfectly known at users (i.e., perfect CSIR) and partially known at the BS (i.e., partial CSIT). The actual CSI known at all users is denoted by \( \hat{H} = [\hat{h}_1, \ldots, \hat{h}_K] \) and the partial instantaneous channel estimate at the BS is denoted by \( \hat{\mathbf{H}} = [\hat{h}_1, \ldots, \hat{h}_K] \).

For a given estimate, the CSI estimation error is denoted by \( \tilde{\mathbf{H}} = [\tilde{h}_1, \ldots, \tilde{h}_K] \). We have the following relationship:
\[
\hat{H} = \hat{\mathbf{H}} + \tilde{\mathbf{H}},
\]

The joint distribution of \( \{\mathbf{H}, \hat{\mathbf{H}}\} \) is assumed to be stationary and ergodic \cite{10}. Though \( \mathbf{H} \) over the entire transmission is unknown at the BS, the conditional density \( \int_{\hat{\mathbf{H}}} \hat{\mathbf{H}}|\mathbf{H}(\hat{\mathbf{H}}, \tilde{\mathbf{H}}) \) is assumed to be known at the BS. Each element of the \( \tilde{\mathbf{H}} \) of \( \hat{\mathbf{H}} \) for user-\( k \) is characterized by an independent and identically distributed (i.i.d.) zero-mean complex Gaussian distribution variable with \( \mathbb{E}\{\tilde{h}_k^2\} = \hat{h}_k^2 \). The variance of the error \( \sigma_{\tilde{h}_k}^2 \) is considered to scale exponentially with SNR as \( \sigma_{\tilde{h}_k}^2 \sim O(P^{-\alpha}) \), \( \alpha \in [0, \infty) \) is the CSIT quality scaling factor \cite{10, 15-18}. \( \alpha = 0 \) and \( \alpha = \infty \) stands for partial CSIT with finite precision and perfect CSIT, respectively.

B. Conventional Dirty Paper Coding-Assisted NOUM

Conventional DPC-assisted NOUM relying on DPC to encode unicast messages and SC to encode multicast messages has been studied in \cite{6, 7} for two-user NOUM with perfect CSIT. In this work, we study its performance in the partial CSIT setting.

At the transmitter, the unicast messages \( W_k, k \in \mathcal{K} \) are encoded using DPC based on certain encoding order \( \pi \), where \( \pi \triangleq [\pi(1), \ldots, \pi(K)] \) is a permutation of \( \{1, \ldots, K\} \) such that the message \( W_{\pi(i)} \) is encoded before \( W_{\pi(j)} \) if \( i < j \). The BS encodes the unicast messages \( W_\pi(1), \ldots, W_\pi(K) \) into a set of symbol streams \( s_{\pi(1)}, \ldots, s_{\pi(K)} \) and precodes the streams by \( p_{\pi(1)}, \ldots, p_{\pi(K)} \) based on DPC, where \( p_{\pi(k)} \in \mathbb{C}^{N_t} \) is the precoder for \( s_{\pi(k)} \). The multicast message \( W_0 \) is encoded into the multicast stream \( s_0 \) by treating the interference from all unicast streams as noise. The instantaneous rate at user-\( \pi(k) \) is
\[
R_{\pi(k)} = \log_2 \left( 1 + \frac{|\hat{h}_{\pi(k)}^H p_{\pi(k)}|^2}{\sum_{j \neq \pi(k)} |\hat{h}_{\pi(k)}^H p_{\pi(k)}|^2 + \sum_{\pi(j) \setminus \pi(k)} |\hat{h}_{\pi(k)}^H p_{\pi(k)}|^2} \right).
\]

Once the common message \( \hat{W}_0 \) is decoded, it is then removed from the received signal by SIC. Assuming perfect SIC, the rate at user-\( \pi(k) \) after removing \( \hat{W}_0 \) is
\[
R_{\pi(k)} = \log_2 \left( 1 + \frac{|\hat{h}_{\pi(k)}^H p_{\pi(k)}|^2}{\sum_{i < \pi(k)} |\hat{h}_{\pi(k)}^H p_{\pi(k)}|^2 + \sum_{j > k} |\hat{h}_{\pi(k)}^H p_{\pi(k)}|^2} \right).
\]

Since the BS does not know the exact channel \( \mathbf{H} \), precoder design based on instantaneous rate may be overestimated and unachievable at each user. Therefore, a more robust approach is to design precoders according to the Ergodic Rate (ER), which characterizes the long-term rate performance of each stream over all possible joint fading states \( \{\mathbf{H}, \hat{\mathbf{H}}\} \).

The ERs of decoding the unicast stream \( s_{\pi(k)} \) at user-\( \pi(k) \) is
\[
R_{\pi(k)} = \min \left\{ \log_2 \left( 1 + \frac{|\hat{h}_{\pi(k)}^H p_{\pi(k)}|^2}{\sum_{i < \pi(k)} |\hat{h}_{\pi(k)}^H p_{\pi(k)}|^2 + \sum_{j > k} |\hat{h}_{\pi(k)}^H p_{\pi(k)}|^2} \right) \bigg| k \in \mathcal{K} \right\}.
\]

C. Proposed Dirty Paper Coded Rate-Splitting-assisted NOUM

In this work, we aim at exploring larger rate regions of multi-antenna NOUM with partial CSIT by marrying the benefits of DPC and RS. The proposed strategies, as illustrated in Fig. 1 of \cite{6} are respectively specified in the following.
Denote the stream vector and precoding matrix as \( \pi \). The instantaneous rate to decode the private stream \( c, \pi \) is defined in the same way as the right-hand side of (6), i.e., \( R_{c,\pi}(k) \). The common parts \( \hat{W}_{c\pi}(k) \) are successfully decoded by all users, we also have \( R_{c\pi}(k) = \min \{ R_{c\pi}(k) \mid k \in K \} \). The ER of decoding the unicast stream \( \hat{W}_{c\pi}(k) \) at user-\( \pi \) (k) using 1-DPCRS is

\[
R_{\pi}(k,tot) = C_{\pi} + R_{\pi}(k) .
\]

2) \textit{M-DPCRS:} M-DPCRS is an extension of 1-DPCRS by embracing the generalized RS framework proposed in [8]. The idea is to split the unicast message of each user into more different parts and encode into multiple layers of common streams, each is intended to one subset of users. The multicast stream is still encoded with some common parts of unicast messages into the super-common stream to be decoded by all users. Due to page limitation, the system model of M-DPCRS is not specified here. It can be easily traced out from M-DPCRS in Section II.C of [8] for MISO BC and 1-DPCRS in Fig. 1(a) for multi-antenna NOUM. Fig. 1(b) illustrates one example of the proposed M-DPCRS for NOUM when \( K = 3 \).

\section{III. Problem Formulation and Optimization Framework}

In this section, we formulate the weighted average sum rate maximization problem and specify the corresponding optimization framework to solve the problem.

\subsection{A. Weighted Average Sum Rate Maximization Problem}

We study the precoder optimization problem at the transmitter with the aim of maximizing the Weighted Ergodic Sum Rate (WESR) of unicast messages subject to the Quality of Service (QoS) rate constraints of multicast and unicast messages. The WESR is defined as \( \sum_{k \in K} u_k R_{\pi}(k,tot) \), where \( u_k \) is the weight for user-\( k \). We further define the Average Rate (AR) of decoding the stream \( s_i, i \in \{ c, k \} \) at user-\( k \) as

\[
R_{i,k}^{\text{WE-SR}}(\hat{W}_{c\pi}(k)) = E_{(H,H)} \left\{ R_{i,k}^{\text{1-DPCRS}}(\hat{W}_{c\pi}(k)) \right\} ,
\]

where \( R_{i,k}^{\text{1-DPCRS}}(\hat{W}_{c\pi}(k)) = R_{i,k}^{\text{1-DPCRS}}(\hat{W}_{c\pi}(k)) \) is simplified to \( R_{i,k}^{\text{1-DPCRS}}(\hat{W}_{c\pi}(k)) \) when \( i = k \). Following [8], the WESR maximization problem is decomposed into Weighted Average Sum Rate (WASR) maximization problems to be solved for all possible channel estimates and DPC encoding orders. For a given weight vector \( u = [u_1, \ldots, u_K] \) and a fixed
DPC encoding order $\pi$, the WASR problem for 1-DPCRS-assisted NOUM is

$$\max_{\hat{e}, \mathbf{P}} \sum_{k \in \mathcal{K}} u_{\pi(k)} (\hat{C}_{\pi(k)} + R_{1,\pi(k)}^{\text{DPCRS}}(\hat{\mathbf{H}}))$$

(11a)

subject to

$$\hat{C}_0 + \sum_{k' \in \mathcal{K}} \hat{C}_{k'}' \leq R_{i,k}^{\text{DPCRS}}(\hat{\mathbf{H}}), \forall k \in \mathcal{K}$$

(11b)

$$\hat{C}_{\pi(k)} + R_{1,\pi(k)}^{\text{DPCRS}}(\hat{\mathbf{H}}) \geq R_{\text{th}}^{\pi(k)}, \forall k \in \mathcal{K}$$

(11c)

$$\hat{C}_0 \geq R_{\text{th}}^{0}$$

(11d)

$$\text{tr}(\mathbf{P} \mathbf{P}^H) \leq P_t$$

(11e)

$$\hat{c} \geq 0$$

(11f)

The rate vector $\hat{c} = [\hat{C}_0, \hat{C}_1, \ldots, \hat{C}_K]$ for 1-DPCRS-assisted NOUM contains the rates allocated to the multicast message $W_0$ as well as the common parts of the unicast messages $W_{p,1}, \ldots, W_{p,K}$ for each $\mathbf{H}$. It is required to be jointly optimized with the precoders so as to maximize the WASR. $R_{\text{th}}^{\pi(k)}$ is the QoS rate constraint of the unicast message $W_{\pi(k)}$ and $R_{\text{th}}^{0}$ is the QoS rate constraint of $W_0$.

Compared with problem (19) in [8] for 1-DPCRS-assisted MISO BC, the main difference of problem (11) comes from constraints (11b) and (11c) due to the additional multicast message $W_0$ to be transmitted for all users. The WASR problem of DPC-assisted NOUM is formulated by turning off $\hat{C}_1, \ldots, \hat{C}_K$ in (11). The problem of M-DPCRS-assisted NOUM can be formulated if readers understand problem (20) in [8] for M-DPCRS-assisted MISO BC and problem (11) for 1-DPCRS-assisted NOUM.

**B. Optimization Framework**

The formulated problem (11) is stochastic and non-convex. To solve the problem, we extend the optimization framework proposed in [10]. Specifically, we first transform the original stochastic problem into a deterministic form by using the Sample Average Approximation (SAA) approach. The approximated deterministic problem is further transformed into an equivalent Weighted Minimum Mean Error (WMMSE) problem, which is then solved by using Alternating Optimization (AO) algorithm. Each step of the optimization framework is further explained in the following.

SAA is first adopted to approximate the stochastic AR (10) into the corresponding deterministic expression. Assuming that the conditional density $f_{\mathbf{H} \mid \mathbf{H}}(\mathbf{H} \mid \hat{\mathbf{H}})$ is known at the BS. For a given $\hat{\mathbf{H}}$, the BS generates a sample of $M$ user channels, indexed by $M = \{1, \ldots, M\}$ as

$$\mathbb{H}^{(M)}(\hat{\mathbf{H}}) \triangleq \{ \mathbf{H}^{(m)} = \hat{\mathbf{H}} + \mathbf{h}^{(m)} \mid \hat{\mathbf{H}}, m \in M \}$$

(12)

With the introduced channel sample in (12) and the strong Law of Large Number (LLN), the ARs $R_{i,k}^{\text{DPCRS}}(\hat{\mathbf{H}})$ specified in equation (10) is equivalent to

$$R_{i,k}^{\text{DPCRS}}(\hat{\mathbf{H}}) = \lim_{M \to \infty} \frac{1}{M} \sum_{m=1}^{M} R_{i,k}^{\text{DPCRS}}(\mathbf{H}^{(m)}, \hat{\mathbf{H}})$$

(13)

Denote $\hat{R}_{i,k}^{\text{DPCRS}(M)}(\hat{\mathbf{H}}) \triangleq \frac{1}{M} \sum_{m=1}^{M} R_{i,k}^{\text{DPCRS}}(\mathbf{H}^{(m)}, \hat{\mathbf{H}})$ as the sampled AR with sample size $M$, problem (11) is transformed into its deterministic form, which is given by

$$\max_{\hat{e}, \mathbf{P}} \sum_{k \in \mathcal{K}} u_{\pi(k)} (\hat{C}_{\pi(k)} + \hat{R}_{i,k}^{\text{DPCRS}(M)}(\hat{\mathbf{H}}))$$

(14a)

subject to

$$\hat{C}_0 + \sum_{k' \in \mathcal{K}} \hat{C}_{k'}' \leq \hat{R}_{i,k}^{\text{DPCRS}(M)}(\hat{\mathbf{H}}), \forall k \in \mathcal{K}$$

(14b)

$$\hat{C}_{\pi(k)} + \hat{R}_{i,k}^{\text{DPCRS}(M)}(\hat{\mathbf{H}}) \geq R_{\text{th}}^{\pi(k)}, \forall k \in \mathcal{K}$$

(14c)

$$\hat{C}_0 \geq R_{\text{th}}^{0}$$

(14d)

$$\text{tr}(\mathbf{P} \mathbf{P}^H) \leq P_t$$

(14e)

$$\hat{c} \geq 0$$

(14f)

where the precoder $\mathbf{P}$ and the common stream allocation vector $\hat{c}$ are designed over all the $M$ channel samples.

The approximated deterministic problem (14) is still non-convex due to the non-convex approximated rate expressions of the common stream and the private streams. To solve the non-convex problem (14), we further extend the WMMSE algorithm proposed in [10, 19]. User-$\pi(k)$ employs equalizer $g_{\pi(k)}^i$ to decode data stream $s_i$. The Mean Square Error (MSE) of stream $s_i$, $i \in \{c, \pi(k)\}$ at user-$\pi(k)$ is

$$e_i^\pi \triangleq \mathbb{E}[|\hat{s}_i - s_i|^2] = |s_i|^2 T_i^{\pi(k)} - 2R g_{\pi(k)}^{H} s_i^H p_i + 1$$

(15)

where $T_i^{\pi(k)} \triangleq \sum_{j \in \mathcal{K} \cup \{c\}} |h_{\pi(k)}^{H} p_j|^2 + 1$, $i = c, T_i^{\pi(k)} \triangleq \sum_{j \in \mathcal{K}} |h_{\pi(k)}^{H} p_j|^2 + 1$, $i = \pi(k)$.

Define the Weighted MSE (WMMSE) of decoding $s_i$ at user-$\pi(k)$ as $\xi_i^\pi(\mathbf{H}, \hat{\mathbf{H}}) \triangleq w_i^\pi e_i^\pi$, where $w_i^\pi$ is the introduced weight for MSE of user-$\pi(k)$. By taking the equalizers and weights as optimization variables, the Rate–WMMSE relationship for an instantaneous channel realization is established as

$$\xi_i^\pi(\mathbf{H}, \hat{\mathbf{H}}) \triangleq \min_{\mathbf{w}, \mathbf{e}} \sum_{i \in \{c, \pi(k)\}} w_i^\pi e_i^\pi(\mathbf{H}, \hat{\mathbf{H}}) = 1 - R_{i,\pi(k)}^{\text{DPCRS}}(\mathbf{H}, \hat{\mathbf{H}}).$$

By defining $w_i^{\ell(m)} = g_{\pi(k)}^{\ell(m)}$ as the weights and equalizers associated with the $m$th channel realization in $\mathbb{H}^{(M)}$, the relationship for an instantaneous channel realization is then extended to the average Rate–WMMSE relationships over $\mathbb{H}^{(M)}$ as

$$\bar{\xi}_i^\pi(\mathbf{H}, \hat{\mathbf{H}}) \triangleq \frac{1}{M} \sum_{m=1}^{M} \min_{\mathbf{w}, \mathbf{e}} \sum_{i \in \{c, \pi(k)\}} w_i^{\ell(m)} e_i^\pi(\mathbf{H}^{(m)}, \hat{\mathbf{H}})$$

(16)

$$\bar{\xi}_i^\pi(\mathbf{H}, \hat{\mathbf{H}})$$

is simplified to $\bar{\xi}_i^\pi(\mathbf{H}, \hat{\mathbf{H}})$ when $i = \pi(k)$. Based on (16), problem (13) is equivalently transformed into the WMMSE problem, which is given by

$$\min_{\mathbf{P}, \mathbf{P}_k, \mathbf{w}} \sum_{k \in \mathcal{K}} u_{\pi(k)} (\hat{C}_{\pi(k)} + \bar{\xi}_i^\pi(\mathbf{H}, \hat{\mathbf{H}}))$$

(17a)

subject to

$$\hat{X}_0 + \sum_{k' \in \mathcal{K}} \hat{X}_{k'} + 1 \geq \bar{\xi}_c(\mathbf{H}, \hat{\mathbf{H}}), \forall k \in \mathcal{K}$$

(17b)

$$\hat{X}_c + \bar{\xi}_c(\mathbf{H}, \hat{\mathbf{H}}) \leq 1 - R_{\text{th}}^{0}, \forall k \in \mathcal{K}$$

(17c)

$$\hat{X}_0 \leq -R_{\text{th}}^{0}, \forall k \in \mathcal{K}$$

(17d)

$$\text{tr}(\mathbf{P} \mathbf{P}^H) \leq P_t$$

(17e)

$$\hat{c} \leq 0,$$

(17f)

where $\hat{\mathbf{x}} = [\hat{X}_0, \hat{X}_1, \ldots, \hat{X}_K]$ is the transformation of $\hat{c}$ satisfying $\hat{\mathbf{x}} = -\hat{c}$. w = $\{w_i^{\ell(m)} \mid i \in \{c, \pi(k)\}, k \in \mathcal{K}, m \in M\}$ and
Problem (20) is a standard Quadratically Constrained Quadratic Program (QCQP), which can be solved using interior-point methods [20]. Therefore, we obtain the AO algorithm specified in Algorithm 1. The weights w, equalizers g, precoders and common rate vectors (P, x) are updated iteratively until the WASR of the system WASR[\eta] converges. The convergence proof of Algorithm 1 follows [8, 10], which is not specified here. By using the same method, we could also obtain the formulated problem and the corresponding optimization framework for DPC and M-DPCRS-assisted NOUM.

IV. NUMERICAL RESULTS

In this section, we evaluate the performance of the proposed 1-DPCRS and M-DPCRS strategies for NOUM. CVX toolbox [21] is adopted to tackle problem (20) that requires to be solved by the interior-point method. The exact channel H_k and the channel estimation error \hat{H}_k have i.i.d. complex Gaussian entries drawn from the distributions \mathcal{CN}(0, \sigma_k^2), \mathcal{CN}(0, \sigma_c^2), respectively and \sigma_k^2, \sigma_c^2 are constraints (or constant vectors/matrices) averaged over a sample of \eta user channels, i.e., \tilde{w}_{\pi(k)} = 1/M \sum_{m=1}^{M} w_{\pi(k)}(m). Their corresponding values in each channel instance \theta = \{\theta_{\pi(k)}\} are updated as

\begin{align}
\theta_{\pi(k)} &= \theta_{\pi(k)} + \Delta \theta_{\pi(k)}
\end{align}

Fig. 2 illustrates the two-user ER region comparison of different strategies with partial CSIT for multi-antenna NOUM, averaged over 100 random channel realizations, SNR=20 dB, K = 2, \alpha = 0.6, \sigma_k^2 = 1, R_{th}^0 = 0.5 bit/s/Hz.

\begin{algorithm}
\caption{WMMSE-based AO algorithm}
\begin{algorithmic}[1]
\State Initialize: \eta \leftarrow 0, P, WASR[\eta];
\Repeat
\State \eta \leftarrow \eta + 1;
\State \eta \leftarrow \eta - 1;
\State \eta \leftarrow \eta + 1;
\State \eta \leftarrow \eta - 1;
\State update g and w by \eta^{*}(P^{\eta-1}) and \eta^{*}(P^{\eta-1}) specified in (18) and (19), respectively;
\State update (P, x) by solving (20) using the updated w, g;
\Until \|WASR[\eta] - WASR[\eta-1]\| \leq \epsilon;
\end{algorithmic}
\end{algorithm}

where \eta_{\pi(k)} = \eta_{\pi(k)} + \Delta \eta_{\pi(k)} and it is simplified to \eta_{\pi(k)}(H) when \eta = \pi(k). \Omega_{\pi(k)} = \sum_{j \in \mathcal{K} \cup \{c\}} \eta_{\pi(k)}^j + \sum_{j < k} \eta_{\pi(k)}(H) \Phi_{\pi(k)}(H) if \eta = \pi(k). \Psi_{\pi(k)} = \Psi_{\pi(k)}(H) \Phi_{\pi(k)}(H) if \eta = \pi(k).

The QoS rate constraint of the multicast stream is \mathcal{R}_{th} = 0.5 bit/s/Hz, SNR = 20 dB. We compare the following eight transmission strategies in the results. “1-DPCRS” and “M-DPCRS” are the strategies we proposed in Section II-C. “DPC” is the strategy described in Section II-B. “Generalized RS”, “1-layer RS”, “SC–SIC”, “SC–SIC per group” and “MU–LP” are the linearly precoded Strategies proposed in [13] for multi-antenna NOUM.

\begin{align}
\begin{cases}
\text{min} & \sum_{k \in \mathcal{K}} u(x_k) + \lambda_{\pi(k)}(\bar{\Phi}_{\pi(k)}(H)) \\
\text{s.t.} & \bar{X}_k + 1 \geq \lambda_{\pi(k)}(\bar{\Phi}_{\pi(k)}(H)), \forall k \in \mathcal{K} \\
& \bar{X}_{\pi(k)} + \lambda_{\pi(k)}(\bar{\Phi}_{\pi(k)}(H)) \leq 1 - R_{th}^k, \forall k \in \mathcal{K} \\
\end{cases}
\end{align}

where \lambda_{\pi(k)}(\bar{\Phi}_{\pi(k)}(H)) \triangleq \lambda_{\pi(k)}(\bar{\Phi}_{\pi(k)}(H)) - \lambda_{\pi(k)}(\bar{\Phi}_{\pi(k)}(H)) if \eta = \pi(k). \Omega_{\pi(k)} = \sum_{j \in \mathcal{K} \cup \{c\}} \eta_{\pi(k)}^j \bar{U}_{\pi(k)}^j + \sum_{j < k} \eta_{\pi(k)}^j \Phi_{\pi(k)}(H) \Phi_{\pi(k)}(H) if \eta = \pi(k). \Psi_{\pi(k)} = \Psi_{\pi(k)}(H) \Phi_{\pi(k)}(H) if \eta = \pi(k).

\begin{align}
\begin{cases}
\text{min} & \sum_{k \in \mathcal{K}} u(x_k) + \lambda_{\pi(k)}(\bar{\Phi}_{\pi(k)}(H)) \\
\text{s.t.} & \bar{X}_k + 1 \geq \lambda_{\pi(k)}(\bar{\Phi}_{\pi(k)}(H)), \forall k \in \mathcal{K} \\
& \bar{X}_{\pi(k)} + \lambda_{\pi(k)}(\bar{\Phi}_{\pi(k)}(H)) \leq 1 - R_{th}^k, \forall k \in \mathcal{K} \\
\end{cases}
\end{align}
represent both M-DPCRS and 1-DPCRS and use the term “RS” to represent both generalized RS and 1-layer RS since M-DPCRS and generalized RS respectively reduces to 1-DPCRS and 1-layer RS. In all subfigures, DPCRS maintains the largest rate region compared with the rate regions of all other strategies. Interesting, we found that the existing linearly precoded RS strategies (generalized RS and 1-layer RS) outperform non-linear DPC especially in the region with strong CSIT inaccuracy.

V. Conclusion

In this work, we propose a novel strategy, namely, Dirty Paper Coded Rate-Splitting (DPCRS) that incorporates RS with DPC to assess the rate region of multi-antenna non-orthogonal unicast and multicast transmission with partial CSIT. By splitting the unicast messages of each user into common and private parts, using DPC to encode the private parts, jointly encoding the multicast message with the common parts of the unicast messages, DPCRS is able to partially decode the interference and partially treat interference as noise, further restrain the interference between multicast and unicast messages as well as the multi-user interference among unicast messages. Numerical results show that linearly precoded RS-assisted NOUM is able to achieve larger rate region than DPC-assisted NOUM but with a much lower hardware and computational complexities. The proposed DPCRS-assisted NOUM outperforms all existing strategies. It is more robust to CSIT inaccuracies, network loads and user deployments.

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