How to Use Weak Decays in Analyses of Data on Nucleon Spin Structure Functions

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May 19, 1994

ABSTRACT

The use of weak decays to determine proton spin structure is examined in view of possible violations of the Bjorken and Gottfried Sum rules, flavor symmetry breaking and flavor asymmetry in the sea. The use of the neutron decay is found to be unaffected by all these. A method for including these effects in analyses of hyperon decays shows that a flavor-asymmetric sea produced by SU(3) symmetry breaking has only a small effect on results for the total spin carried by quarks. However the strange quark contribution cannot be reliably obtained from charged lepton scattering and weak decay data alone, and requires additional model-dependent input relating nucleon and hyperon wave functions.
Recent analyses of data on nucleon spin structure [1, 2] confirm the originally surprising conclusion [3] of zero quark spin contribution to the proton spin but continue to use semileptonic weak decay data to determine the fractional contributions of the \( u \), \( d \) and \( s \) - flavored current quarks and antiquarks respectively to the spin of the proton conventionally denote by \( \Delta u(p) \), \( \Delta d(p) \) and \( \Delta s(p) \). However, the weak decays measure flavor-changing transition matrix elements which have been noted[4] to depend upon the wave functions of both the initial and final states. They therefore give information about both nucleon and hyperon spin structures which cannot be disentangled without some hyperon model. Serious difficulties and paradoxes arise in all attempts fit the observed hyperon magnetic moments and weak decays with models for hyperon spin structure[5]. In particular the values for the ratio \( \Delta s(\Sigma)/\Delta s(\Lambda) \) determined by fitting magnetic moment and semileptonic decay data are not simply inconsistent. They differ by a factor of \( 8 \pm 2 \) and it is very difficult to find any simple correction which can fix such a large factor. Until these difficulties are resolved all values quoted for \( \Delta s(p) \) based on using hyperon data with assumptions about \( \Delta s(\Sigma) \) and \( \Delta s(\Lambda) \) should be viewed with suspicion. With this caveat, we attempt to do a bit better than the conventional treatments [2, 3] which assume very simple relations between nucleon and hyperon wave functions; e.g. that the \( \Sigma^- \) wave function is the mirror of the neutron wave function under \( u \leftrightarrow s \) flavor exchange. The possibility of measuring \( \Delta u(p) - \Delta d(p) - \Delta s(p) \) directly by elastic neutrino scattering is now under consideration[2]. We do not consider this further here.

More general relations between weak decays and baryon spin structure [4] were obtained from the current algebra of the electroweak standard model. The initial and final states are assumed to dominate the sum over intermediate states in the evaluation of current commutators, but mirror symmetry is not assumed.

\[
\frac{G_A}{G_V}(n \rightarrow p) \approx \frac{\langle p | \Delta u - \Delta d | p \rangle - \langle n | \Delta u - \Delta d | n \rangle}{2} \tag{1a}
\]

\[
\frac{G_A}{G_V}(\Sigma^- \rightarrow n) \approx \frac{\langle n | \Delta u - \Delta s | n \rangle - \langle \Sigma^- | \Delta u - \Delta s | \Sigma^- \rangle}{2} \tag{1b}
\]

These results are exact in the mirror symmetry limit where the two terms on the right hand side are equal and give information about nucleon spin structure.

The isospin symmetry relevant for neutron decay is valid to the approximation needed for proton spin structure, and completely immune to effects producing violations of the Bjorken and Gottfried sum rules. The Bjorken sum rule relates deep inelastic scattering data to the neutron decay and depends upon QCD to obtain the relevant information from the deep inelastic data. Its failure would reflect on QCD, not on the relations between neutron decay and proton spin structure. The Gottfried sum rule assumes an isospin symmetric sea. Its failure would indicate a charge asymmetry in the sea produced presumably by charge exchange interactions between valence and sea quarks; e.g. by pion or \( \rho \) exchange, and in any case
by strong interactions which conserve isospin. Thus the neutron and proton remain isospin mirrors under $u \leftrightarrow d$ even with flavor asymmetric seas. Substituting this mirror symmetry into eq.(1a) gives:

$$N \equiv \frac{G_A(n \to p)}{G_V} = F + D = \Delta u(p) - \Delta d(p) = \Delta d(n) - \Delta u(n) = 1.2573 \pm 0.0028$$

where we have denoted this quantity by $N$ to simplify further equations, introduced the $F$ and $D$ parameters used in conventional treatments [2,3] and substituted the experimental results[6].

The flavor-SU(3) symmetry relevant for $\Sigma^-\to n$ decay is broken. We now correct the results of ref. [2] for the most obvious symmetry breaking mechanism, the higher mass of the strange quark which reduces the production of strange quark pairs by gluons in the sea and therefore destroys the mirror symmetry under $u \leftrightarrow s$. We assume that the sea contribution to $\Delta s$ is equally suppressed in both states and therefore that $\Delta u - \Delta s$ is the same in both hadron seas, rather than being mirror images with opposite signs, and that flavor polarization effects analogous to those in the $u - d$ sector can be neglected. A symmetry-conserving strangeness exchange between valence and sea; e.g. $K$ or $K^*$ exchange would give a hyperon wave function with nonstrange valence quarks and a sea containing an extra strange quark in addition to $s\bar{s}$ pairs. Both strangeness exchange and the introduction of an extra strange quark in the sea are expected to be suppressed in comparison with the $\pi$ or $\rho$ exchange and the extra $u$ or $d$ quark in the sea which could arise in the nonstrange sector and give rise to violation of the Gottfried sum rule.

A flavor-asymmetric sea which is the same for the neutron and $\Sigma^-$ can be treated[7] by decomposing the quantities $\Delta u(p)$, $\Delta d(p)$ and $\Delta s(p)$ into valence and sea contributions denoted by the superscripts $v$ and $s$ respectively. Since the neutron contains no valence strange quarks, we can write

$$\Delta s^v(n) = 0; \quad \Delta u^s(\Sigma^-) = \Delta u^s(n) = (1 + \epsilon)\Delta s(n)$$

where $\epsilon$ describes the flavor asymmetry of the sea in the $u - s$ sector and has the same value for the neutron and the $\Sigma^-$. Substituting this result into eq. (1b) gives

$$\Sigma \equiv \frac{G_A(\Sigma^- \to n)}{G_V} = \Delta u^v(n) - \Delta s^v(n) = \Delta s^v(\Sigma^-) - \Delta u^v(\Sigma^-)$$

where we have used the shortened notation $\Sigma$. Substituting eqs. (3) and the experimental
value [6] into (15) gives

\[ \Sigma = F - D = \Delta u(n) - \Delta s(n) - \{ \Delta u(n) - \Delta s(n) \} = \]

\[ = \Delta u(n) - (1 + \epsilon)\Delta s(n) = \Delta d(p) - (1 + \epsilon)\Delta s(p) = -0.340 \pm 0.017. \quad (5) \]

We can compare our treatment with the conventional SU(3) analyses [2, 3] of weak decays by noting that Eqs. (2) and (5) give \( F = 0.459 \pm 0.009, \) \( D = 0.799 \pm 0.009 \) and

\[ \frac{2\Sigma + N}{\sqrt{3}} = \frac{3F - D}{\sqrt{3}} = \]

\[ = \frac{[\Delta u(p) + \Delta d(p) - 2(1 + \epsilon) \cdot \Delta s(p)]}{\sqrt{3}} = 0.356 \pm 0.020 \quad (6a) \]

\[ F/D = 0.574 \pm 0.013 \quad (6b) \]

These numerical results are essentially indistinguishable from those obtained in the standard treatment [2]; namely, 0.34±0.020 for the expression (6a) which transforms in the SU(3) symmetry limit like the isoscalar component of an octet and \( F/D = 0.58 \pm 0.02. \) This seems strange, since we have only considered the \( \Sigma^- \rightarrow n \) decay and not used the additional input available from the experimental data for the other two weak decays. We show below that errors on the other decays are too large to have any impact on the \( F \) and \( D \) values determined by fitting all the data.

Thus the effect of SU(3) breaking is simply incorporated in the standard SU(3) analysis which determines the \( F \) and \( D \) parameters from the \( n \rightarrow p \) and \( \Sigma^- \rightarrow n \) decays and uses them to determine two independent linear combinations of \( \Delta u, \Delta d \) and \( \Delta s \) which transform like the isovector and isoscalar components of an SU(3) octet; namely (2) and (6a). One simply replaces \( \Delta s(p) \) by \( (1 + \epsilon)\Delta s(p). \)

We now introduce this SU(3)-breaking correction into the standard analysis of the EMC experiment and weak decays which begins with the relation [3]

\[ \frac{4}{9}\Delta u(p) + \frac{1}{9}\Delta d(p) + \frac{1}{9}\Delta s(p) = E \quad (7) \]

where \( E \) denotes an experimental number obtained from polarized deep inelastic scattering data with various QCD corrections [3] to the naive parton model result.
Solving the three equations (7), (2) and (5) for $\Delta u(p)$, $\Delta d(p)$ and $\Delta s(p)$ gives

$$
\Delta u = \frac{1}{6 + 5\epsilon} \cdot \{9(1 + \epsilon)E + (2 + \epsilon)N + \Sigma\} \quad (8a)
$$

$$
\Delta d = \frac{1}{6 + 5\epsilon} \cdot \{(1 + \epsilon)(9E - 4N) + \Sigma\} \quad (8b)
$$

$$
\Delta s = \frac{1}{6 + 5\epsilon} \cdot \{9E - 4N - 5\Sigma\} = \frac{6}{6 + 5\epsilon} \cdot \Delta s(\epsilon = 0) \quad (8c)
$$

where we have suppressed the argument $(p)$ for all cases referring to the proton spin structure. The total quark spin contribution to the proton spin is then

$$
\Delta q \equiv \Delta u + \Delta d + \Delta s = \frac{9E - 2N - \Sigma}{2} \cdot \left(1 - \frac{\epsilon}{6 + 5\epsilon}\right) + \epsilon \cdot \left(\frac{N + 2\Sigma}{6 + 5\epsilon}\right)
$$

$$
\approx \Delta q(\epsilon = 0) \cdot \left(1 - \frac{\epsilon}{6 + 5\epsilon}\right) + \frac{7\epsilon}{12(6 + 5\epsilon)} \quad (9a)
$$

where we have inserted the approximate experimental values $N \approx (5/4)$ and $\Sigma \approx -(1/3)$ to give an estimate of the effects of SU(3) breaking and

$$
\Delta q(\epsilon = 0) \equiv \frac{9E - 2N - \Sigma}{2} \quad (9b)
$$

is the total contribution of quarks to the proton spin in the SU(3) symmetry limit. The total contribution of the sea quarks is given by

$$
\Delta q^s \equiv (3 + 2\epsilon) \cdot \Delta s = 3\Delta s(\epsilon = 0) \cdot \left(1 - \frac{\epsilon}{6 + 5\epsilon}\right) \quad (9c)
$$

The recent analysis \[2\] assuming SU(3) symmetry gives

$$
\Delta q(\epsilon = 0) = 0.12 \pm 0.17 \quad (10a)
$$
\[ \Delta s(\epsilon = 0) = -0.19 \pm 0.06 \]  

(10b)

from which we can extract the sea and valence contributions as

\[ \Delta q^s(\epsilon = 0) = -0.57 \pm 0.18 \]  

(10c)

\[ \Delta q^v(\epsilon = 0) = 0.69 \pm 0.25 \]  

(10d)

Substituting \( \epsilon = 1 \) into eqs. (8-10) gives the result for the case of an asymmetric sea with \( \Delta s \) reduced by a factor of 2 with respect to \( \Delta d \),

\[ \Delta q(\epsilon = 1) = \Delta u + \Delta d + \Delta s \approx 0.16 \pm 0.15 \]  

(11a)

\[ \Delta s(\epsilon = 1) = (6/11) \cdot \Delta s(\epsilon = 0) = -0.10 \pm 0.03 \]  

(11b)

\[ \Delta q^s(\epsilon = 1) = -0.50 \pm 0.15 \]  

(11c)

\[ \Delta q^v(\epsilon = 1) = 0.66 \pm 0.21 \]  

(11d)

The correction to \( \Delta q \) is very small but \( \Delta s \) is significantly reduced.

The valence contributions are seen to be consistent with the value \( (3/4) \) given by naive constituent quark models with parameters adjusted to fit \( G_A \) (n → p) = 1.25. The sea contributions nearly cancel the valence contributions to give a low value for \( \Delta q \). These results are very insensitive to symmetry-breaking corrections. However, the flavor composition of the sea is seen to be not well determined by the data, and the value of \( \Delta s \) is sensitive to model-dependent SU(3) breaking corrections.

These qualitative features are illustrated by a useful general inequality independent of the details of SU(3) symmetry breaking. We assume only that the value of \( \Delta s(\epsilon = 0) \) determined in the symmetry limit has the correct sign and that symmetry breaking reduces its magnitude.

\[ 0 \geq \Delta s \geq \Delta s(\epsilon = 0) \]  

(12)

It is then convenient to rewrite the expression (7) [8] to express the total quark contribution to the proton spin in terms of \( E \) and \( N \) which are directly measured and \( \Delta s \) whose value
satisfies the inequality (12).

\[ \Delta q \equiv \Delta u + \Delta d + \Delta s = \frac{18}{5} \cdot E - \frac{3}{5} \cdot N + \frac{3}{5} \cdot \Delta s. \]  

Then

\[ \Delta q(\epsilon = 0) \leq \Delta q \leq \Delta q(\epsilon = 0) - \frac{3}{5} \cdot \Delta s. \]  

Substituting the results (10) gives

\[ 0.12 \pm 0.17 \leq \Delta q \leq 0.23 \pm 0.17 \]  

Thus the correction to \( \Delta q \) is still less than one standard deviation even in the limit of large SU(3) breaking which reduces \( \Delta s \) to zero.

Further insight into the roles of valence and sea quarks in weak decays is illustrated by comparing the information obtained from these decays with that obtained from deep inelastic scattering. The EMC result (7) weighs quark and antiquark contributions equally and therefore also weighs valence and sea contributions equally. In deep inelastic scattering of a polarized \( W^- \) beam on a nucleon target where one observes inclusively all final states with a given strangeness, the contributions from both the quarks and antiquarks in the nucleon are equally weighted. The results for \( W^- p \) inclusive scattering into all nonstrange final states therefore give the quantity \( \Delta u(p) - \Delta d(p) \). Similarly \( W^- n \) inclusive scattering into all strange final states gives \( \Delta u(n) - \Delta s(n) \).

We now examine whether the weak decays give the same information and in particular whether the quark and antiquark transitions or the valence and sea transitions at the quark level are equally weighted. We can compare the relative weighting of valence and sea contributions obtained from weak decays and from deep inelastic \( W^- \) scattering by noting that the neutron and \( \Sigma^- \) decays are related respectively by time reversal to exclusive neutron and \( \Sigma^- \) production in \( W^- N \) scattering,

\[ W^- + p \to n; \quad W^- + n \to \Sigma^- \]  

A sea transition changes the electric charge of the sea. In inclusive scattering this causes no problem, all final states are considered. But in the exclusive transitions (15) the sea contribution can be reduced by a wave-function overlap factor between initial and final states with seas having different charges.
If the sea is flavor symmetric, the sea contributions to $\Delta u(p) - \Delta d(p)$ or $\Delta u(n) - \Delta s(n)$ exactly cancel. Thus the relative weighting of sea and valence contributions is irrelevant to the determination of these differences and the same results are expected in inclusive transitions and in the exclusive reactions (15).

If the sea is not isospin symmetric, there is a sea contribution to $\Delta u(p) - \Delta d(p)$. But isospin invariance then requires the proton wave function to have a component where the electric charge of the valence quarks is zero and the proton charge is carried by the sea and predicts the same result for $\Delta u(p) - \Delta d(p)$ from neutron decay and deep inelastic scattering. This is confirmed by our result (2).

Similarly, if the sea is not SU(3) symmetric SU(3) symmetry would requires the $\Sigma^-$ wave function to have a component where the strangeness of the valence quarks is zero and the strangeness is carried by the sea. This is not expected; there should be no sea contribution to $\Sigma^-$ decay. Thus we see again that $\Delta u(n) - \Delta s(n)$ should be given by the result (4) in which only valence quarks contribute. This argument for the results (3-11) is more general, since it does not assume that the seas in the neutron and $\Sigma^-$ are the same. The results thus apply even if the sea is not isospin symmetric and the isospin asymmetries in the neutron and $\Sigma^-$ are different. Note that eq. (5) relates only $\Delta u(n)$ and $\Delta s(n)$ which are equal by isospin to $\Delta d(p)$ and $\Delta s(p)$ but makes no assumption about $\Delta u(p)$ and $\Delta d(n)$.

The other hyperon decays can be treated with SU(3) symmetry breaking by analogy with eqs. (3-5) if we assume that the sea is isoscalar,

$$\Xi \equiv \frac{G_A}{G_V} (\Xi^- \rightarrow \Lambda) = \frac{1}{3} \left[ \Delta u(p) + \Delta d(p) - 2(1 + \epsilon) \cdot \Delta s(p) \right]$$

$$\Lambda \equiv \frac{G_A}{G_V} (\Lambda \rightarrow p) = \frac{1}{3} \left[ 2\Delta u(p) - \Delta d(p) - (1 + \epsilon) \cdot \Delta s(p) \right]$$

where we have again introduced the shortened notation $\Xi$ and $\Lambda$. The experimental values can be compared with the SU(3) predictions using the $F$ and $D$ values obtained from eqs. (6).

$$0.25 \pm 0.05 = \Xi = F - \frac{D}{3} = 0.193 \pm 0.009$$

$$0.718 \pm 0.015 = \Lambda = F + \frac{D}{3} = 0.729 \pm 0.009$$

We also obtain two additional independent values for the linear combination (6a) which
transforms like the isoscalar component of an octet,

\[
\frac{3F - D}{\sqrt{3}} = 2\sqrt{3} \cdot \Lambda - \sqrt{3} \cdot N = 0.310 \pm 0.05
\]

\[
= \sqrt{3} \cdot \Xi = 0.43 \pm 0.09 \quad (18)
\]

The experimental values are in agreement with SU(3), and again the effect of SU(3) breaking on information obtained about proton spin structure is to replace \(\Delta s\) by \((1 + \epsilon)\Delta s\). These results, even with SU(3) symmetry breaking, still satisfy the SU(3) “equal spacing rule” \([9]\).

\[
(1/3)[\Delta u(p) - 2\Delta d(p) - (1 + \epsilon) \cdot \Delta s(p)] = 
\]

\[
= N - \Lambda = \Lambda - \Xi = \Xi - \Sigma = \frac{1}{3} \cdot (N - \Sigma) \quad (19a)
\]

\[
0.439 \pm 0.015 = 0.47 \pm 0.05 = 0.60 \pm 0.05 = 0.532 \pm 0.006 \quad (19b)
\]

These equations show that breaking SU(3) only by suppressing the strange quark contribution in an isoscalar sea leaves the SU(3) relations between the different hyperon decays intact, and affects only the relation between the decay rates and the spin structure of the baryons by introducing the factor \((1 + \epsilon)\) in the coefficient of \(\Delta s\) everywhere.

However, the experimental errors are so large in the \(\Xi\) and \(\Lambda\) decays that using them together with the other data to determine the \(D\) and \(F\) parameters and the isoscalar component \((6a)\) provides a negligible improvement. This is seen most easily in the equal-spacing parameter \((19)\) where the error in the value obtained by using only the neutron and \(\Sigma^-\) decays is much smaller than in values obtained by using other decays. Thus even in unbroken SU(3) it is simpler not to use the \(F\) and \(D\) parametrization, deal directly with the data and use the experimental fact that the error in \(n \to p\) is much smaller than in all other decays. Better experimental values, especially for \(\frac{G_A}{G_V}(\Xi^- \to \Lambda)\) would provide a significant test of SU(3) breaking, as well as providing more insight into the \(n - \Lambda - \Sigma^-\) paradox \([5]\).

We also note that our results obtained by using only the \(\Sigma^- \to n\) data do not assume an isoscalar sea, while the results \((19)\) are valid only for an isoscalar sea. Thus if there is evidence that the sea is not isoscalar it is also safer to use only \(\Sigma^- \to n\) data.

Stimulating and clarifying discussions with Marek Karliner and Jechiel Lichtenstadt are gratefully acknowledged. This research was partially supported by the Basic Research Foundation administered by the Israel Academy of Sciences and Humanities and by grant No. 90-00342 from the United States-Israel Binational Science Foundation (BSF), Jerusalem, Israel.
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