Modified Newtonian Dynamics and its Implications

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Milgrom has proposed that the appearance of discrepancies between the Newtonian dynamical mass and the directly observable mass in astronomical systems could be due to a breakdown of Newtonian dynamics in the limit of low accelerations rather than the presence of unseen matter. Milgrom’s hypothesis, modified Newtonian dynamics or MOND, has been remarkably successful in explaining systematic properties of spiral and elliptical galaxies and predicting in detail the observed rotation curves of spiral galaxies with only one additional parameter—a critical acceleration which is on the order of the cosmologically interesting value of $cH_0$. Here I review the empirical successes of this idea and discuss its possible extension to cosmology and structure formation.

1. Introduction

Modified Newtonian dynamics (MOND) is an ad hoc modification of Newton’s law of gravity or inertia proposed by Milgrom (1983) as an alternative to cosmic dark matter. The motivation for this and other such proposals is obvious: So long as the only evidence for dark matter is its global gravitational effect, then its presumed existence is not independent of the assumed form of the law of gravity or inertia on astronomical scales. In other words, either the universe contains large quantities of unseen matter, or gravity (or the response of particles to gravity) is not generally the same as it appears to be in the solar system.

The phenomenological foundations for MOND really come down to two observational facts about spiral galaxies: 1.) The rotation curves of spiral galaxies are asymptotically flat, and 2.) There is a well-defined relationship between the rotation velocity in spiral galaxies and the luminosity—the Tully-Fisher (TF) law (Tully & Fisher 1977). This latter implies a mass-velocity relationship of the form $M \propto V^\alpha$ where $\alpha$ is in the neighborhood of 4.

If one wants to modify gravity in some way to explain flat rotation curves or the existence of a mass-rotation velocity relation for spiral galaxies, an obvious first choice would be to propose that gravitational attraction becomes more like $1/r$ beyond some length scale which is comparable to the scale of galaxies. So the modified law of attraction about a point mass $M$ would read

$$F = \frac{GM}{r^2} f(r/r_o)$$

where $r_o$ is a new constant of length with dimensions of a few kpc, and $f(x)$ is a function with the asymptotic behavior: $f(x) = 1$ where $x << 1$ and $f(x) = x$ where $x >> 1$. Equating the centripetal to the gravitational acceleration in the limit $r >> r_o$ would lead to a mass–asymptotic rotation velocity relation of the form $v^2 = GM/r_o$. This is true of any modification attached to a length scale. Milgrom realized that this was incompatible with the observed TF law unless, of course, the mass-to-light ratio ($M/L$) of the stellar population varies systematically with galaxy mass in a very dramatic fashion. Such a drastic variation in $M/L (\propto M^{-2})$, is absolutely inconsistent with everything we think
we know about stellar populations. Moreover, any modification attached to a length scale would imply that larger galaxies should exhibit a larger discrepancy. Anyone who has considered galaxy rotation curves knows that this is totally inconsistent with the observations. There are very small, usually low surface brightness (LSB) galaxies with large discrepancies, and very large high surface brightness (HSB) spiral galaxies with very small discrepancies.

This is shown in the first figure. At the left is a log-log plot of the dynamical $M/L_K$ vs. the radius at the last measured point of the rotation curve for a uniform sample of spiral galaxies in the Ursa Major cluster (Tully et al. 1996, Verheijen and Sancisi, 2001). The dynamical M/L is calculated simply using the Newtonian formula for the mass $v^2r/G$ (assuming a spherical mass distribution) where $r$ is the radial extent of the rotation curve. Population synthesis studies suggest that $M/L_K$ should be about one, so anything much above one indicates a discrepancy– a dark matter problem. It is evident that there is not much of a correlation of M/L with size. On the other hand, the Newtonian M/L plotted against centripetal acceleration $(v^2/r)$ at the last measured point (right figure) looks rather different. There does appear to be a correlation in the sense that $M/L \propto 1/a$ for $a < 10^{-8}$ cm/s$^2$. Any modification of gravity attached to a length scale cannot explain such observations.

2. Basics of MOND

Milgrom’s insightful deduction was that the only viable sort of modification is one in which a deviation from Newton’s law appears at low acceleration. (it should be recalled that data such as that shown in Fig. 1 did not exist at the time of Milgrom’s initial papers). Viewed as a modification of gravity, his suggestion was that the actual gravitational acceleration $\mathbf{g}$ is related to the Newtonian gravitational acceleration $\mathbf{g}_n$ as

$$\mathbf{g}_\mu(|g|/a_o) = \mathbf{g}_n$$  

(2)
where $a_o$ is a new physical parameter with units of acceleration and $\mu(x)$ is a function which is unspecified but must have the asymptotic form $\mu(x) = x$ when $x << 1$ and $\mu(x) = 1$ where $x >> 1$.

The immediate consequence of this is that, in the limit of low accelerations, $g = \sqrt{g_o a_o}$. For a point mass $M$, if we set $g$ equal to the centripetal acceleration $v^2/r$, this gives

$$v^4 = GMa_o$$

in the low acceleration regime. So all rotation curves are asymptotically flat and there is a mass-luminosity relation of the form $M \propto v^4$. These are aspects that are built into MOND so they cannot rightly be called predictions. However, in the context of MOND, the aspect of an asymptotically flat rotation curve is absolute. MOND leaves rather little room for maneuver; the idea is in principle falsifiable, or at least it is far more fragile than the dark matter hypothesis. Unambiguous examples of rotation curves (of isolated galaxies) which decline in a Keplerian fashion at a large distance from the visible object would falsify the idea. In effect, a rotational velocity which is constant with radius is Kepler’s law in the limit of low accelerations.

In addition, the mass-rotation velocity relation and implied Tully-Fisher relation is absolute. The TF relation should be the same for different classes of galaxies and the logarithmic slope (at least of the MASS-velocity relation) must be 4— not 3.8 or 4.2— but 4.0. Moreover, it must be the case that the relation is essentially one between the total baryonic mass of a galaxy and the asymptotic flat rotational velocity— not the peak rotation velocity but the velocity at large distance. This is the most immediate and most obvious prediction (see McGaugh & de Blok 1998b and McGaugh et al. 2000 for a discussion of these points).

Converting the M-V relation to the observed luminosity-velocity relation we find

$$\log(L) = 4\log(V) - \log(Ga_0 < M/L>).$$

The near-infrared TF relation for Verheijen’s UMa sample is shown in Figure 2 (Sanders & Verheijen 1998) where the velocity is that of the flat part of the rotation curve. The scatter about the least-square fit line of slope 3.9 ± 0.2 is consistent with observational uncertainties (i.e., no intrinsic scatter). Given the mean $M/L$ in a particular band ($\approx 1$ in the K’ band), this observed TF relation (eq. 4) tells us that $a_o$ must be on the order of $10^{-8}$ cm/s$^2$. It was immediately noticed by Milgrom that $a_o \approx cH_o$ to within a factor of 5 or 6. This cosmic coincidence is quite interesting and suggests that MOND, if it is right, may reflect the effect of cosmology on local particle dynamics.

3. Implications

There are several other immediate consequences of modified dynamics— all of which were explored by Milgrom in his original papers— which do fall in the category of predictions.

1. There exist a critical value of the surface density

$$\Sigma_c \approx a_o/G.$$  (5)

If a system, such as a spiral galaxy has a surface density of matter greater than $\Sigma_c$, that means that the internal accelerations are greater than $a_o$, so the system is in the Newtonian regime. In systems with $\Sigma \geq \Sigma_c$ (HSB galaxies) there should be a small discrepancy between the visible and classical Newtonian dynamical mass within the optical disk. In the parlance of rotation curve observers, a HSB galaxy should be well-represented by the “maximum disk” solution (Sancisi, this volume). But in LSB galaxies ($\Sigma << \Sigma_c$)
there is a low internal acceleration, so the discrepancy between the visible and dynamical mass would be large. These objects should be far from maximum disk. In effect, Milgrom predicted, before the actual discovery of LSB galaxies, that there would be a serious discrepancy between the observable and dynamical mass within the luminous disk of such systems—should they exist. They do exist, and this prediction has been verified—as is evident from the work of McGaugh & de Blok (1998a,b).

2. It is well-known since the work of Ostriker & Peebles (1973), that rotationally supported Newtonian systems tend to be unstable to global non-axisymmetric modes which lead to bar formation and rapid heating of the system. In the context of MOND, these systems would be those with \( \Sigma > \Sigma_c \), so this would suggest that \( \Sigma_c \) should appear as an upper limit on the surface density of rotationally supported systems. This critical surface density is 0.2 g/cm\(^2\) or 860 M\(_\odot\)/pc\(^2\). A more appropriate value of the mean surface density within an effective radius would be \( \Sigma_c/2\pi \) or 140 M\(_\odot\)/pc\(^2\), and, taking \( M/L_b \approx 2 \), this would correspond to a surface brightness of about 22 mag/arc sec\(^2\).

Figure 2. The near-infrared Tully-Fisher relation of Ursa Major spirals (Sanders & Verheijen 1998). The rotation velocity is the asymptotically constant value. The line is a least-square fit to the data and has a slope of 3.9 ± 0.2.

There is such an observed upper limit on the mean surface brightness of spiral galaxies and this is known as Freeman’s law (Freeman 1970, Allen & Shu 1979). The point is that the existence of such a preferred surface density becomes understandable in the context of MOND.

3. Spiral galaxies with a mean surface density near this limit – HSB galaxies– would be, within the optical disk, in the Newtonian regime. So one would expect that the rotation curve would decline in a near Keplerian fashion to the asymptotic constant value. In LSB galaxies, with mean surface density below \( \Sigma_c \), the prediction is that rotation curves would
rise to the final asymptotic flat value. So there should be a general difference in rotation curve shapes between LSB and HSB galaxies. In Fig. 3 I show the rotation curves of two galaxies, a LSB and HSB, where we see exactly this trend. This general effect in observed rotation curves was first noted by Casertano & van Gorkom (1991).

4. With Newtonian dynamics, pressure-supported systems which are nearly isothermal have infinite extent. But in the context of MOND it is straightforward to demonstrate that such isothermal systems are finite with the density at large radii falling roughly like $1/r^4$ (Milgrom 1984). The equation of hydrostatic equilibrium for an isotropic, isothermal system reads

$$\sigma_r^2 \frac{d\rho}{dr} = -\rho g$$

where, in the limit of low accelerations $g = \sqrt{G\alpha_0/r}$. Here $\sigma_r$ is the radial velocity dispersion and $\rho$ is the mass density. It then follows immediately that, in this MOND
Thus there exists a mass-velocity dispersion relation of the form
\[
\left( \frac{M}{10^{11} M_\odot} \right) \approx \left( \frac{\sigma_r}{100 \, \text{km/s}} \right)^4
\]
which is similar to the observed Faber-Jackson relation (luminosity-velocity dispersion relation) for elliptical galaxies (Faber & Jackson 1976). This means that a MOND near-isothermal sphere with a velocity dispersion of 100 km/s to 300 km/s will always have a galactic mass. This is not true of Newtonian pressure-supported objects. Because of the appearance of an additional dimensional constant, \( a_o \), in the structure equation (eq. 5), MOND systems are much more constrained than their Newtonian counterparts.

But with respect to actual pressure supported systems, an even stronger statement can be made. Any isolated system which is nearly isothermal will be a MOND object. That is because a Newtonian isothermal system (with large internal accelerations) is an object of infinite size and will always extend to the region of low accelerations \(< a_o \). At that point \( r_e^2 = GM/a_o \), MOND intervenes and the system will be truncated. This means that the internal acceleration of any isolated isothermal system \( (\sigma_r^2/r_e) \) is expected to be on the order of or less than \( a_o \) and that the mean surface density within \( r_e \) will typically be \( \Sigma_e \) or less (there are low-density solutions for MOND isothermal spheres, \( \rho \ll a_o^2/G\sigma_r^2 \), with internal accelerations less than \( a_o \)). It has been known for some time that elliptical galaxies do have a characteristic surface brightness (Fish 1964). But the above arguments imply that the same should be true of any pressure supported, near-isothermal system, from globular clusters to clusters of galaxies. Moreover, the same \( M - \sigma \) relation (eq. 6) should apply to all such systems, albeit with considerable scatter due to deviations from a strictly isotropic, isothermal velocity field (Sanders 2000). Such deviations will also result in a dispersion of mean internal accelerations about the fiducial value of \( a_o \).

4. Rotation curve analysis

Perhaps the most remarkable phenomenological success of MOND is in predicting the form of rotation curves from the observed distribution of detectable matter—stars and gas (Begeman et al. 1991, McGaugh & de Blok 1998, Sanders & Verheijen 1998). The procedure followed can be outlined as follows:

1. One assumes that light traces mass, i.e., \( M/L = \text{constant} \). There are color gradients in spiral galaxies so this cannot be generally true— or at least one must decide which color band is the best tracer of the mass distribution. The general opinion is that the near-infrared emission of spiral galaxies is the optimal tracer of the underlying stellar mass distribution, since the old population of low mass stars contribute to this emission and the near-infrared is less affected by dust obscuration. So where available, near infrared surface photometry is to be preferred.

2. In determining the distribution of detectable matter one must include the observed neutral hydrogen with an appropriate correction for the contribution of primordial helium. The gas can make a dominant contribution to the total mass surface density in some (generally low luminosity) galaxies.

3. Given the observed distribution of mass, \( g_n \), the Newtonian gravitational force, is calculated via the classical Poisson equation. Here it is usually assumed that the stellar and gaseous disks are razor thin. It may also be necessary to add a spheroidal bulge if the light distribution indicates the presence of such a component.
4. Given the radial distribution of the Newtonian force, the true gravitational force, $g$, is calculated from the MOND formula with $a_0$ fixed. Then the mass of the stellar disk is adjusted until the best fit to the observed rotation curve is achieved. This gives M/L of the disk as the single free parameter of the fit (unless a bulge is present).

In comparing to the observed rotation curve one assumes that the motion of the gas is co-planer rotation about the center of the given galaxy. This is certainly not always the case because there are well-known distortions to the velocity field in spiral galaxies caused by bars and warping of the gas layer. In a fully 2-dimensional velocity field these distortions can often be modeled, but the optimal rotation curves are those in which there is no evidence for the presence of significant deviations from co-planer circular motion. In general it should be remembered that not all observed rotation curves are perfect tracers of the radial distribution of force. A perfect theory will not fit all rotation curves because of these possible problems (the same is true of a specified dark matter halo). The point is that with MOND, usually, there is one free parameter per galaxy and that is the mass or M/L of the stellar disk.

I am only going to show two examples of MOND fits to rotation curves, and these are the two galaxies already shown in Fig. 3. The dotted and dashed curves are the Newtonian rotation curves of the stellar and gaseous disks respectively, and the solid curve is the MOND rotation curve with $a_0 = 1.2 \times 10^{-8}$ cm/s$^2$. We see that, not only does MOND predict the general trend for LSB and HSB galaxies, but it also predicts the observed rotation curves in detail from the observed distribution of matter. This procedure has been carried out for about 100 rotation curves and in only about 10 cases is the predicted rotation curve significantly different from the observed curve. For these objects there is usually an obvious problem with the observed curve or its use as a tracer of the radial force distribution.

I have noted that the only free parameter in these fits is the mass-to-light ratio of the visible disk, so one may well ask if the inferred values are reasonable. Here it is useful to consider again the Verheijen UMa sample because all galaxies are at the same distance and there is $K'$-band (near infrared) surface photometry of the entire sample. The sample also contains both HSB and LSB galaxies. Fig. 5 shows the M/L in the B-band required by the MOND fits plotted against B-V color (top) and the same for the $K'$-band (bottom). We see that in the $K'$-band M/L $\approx 1$ with a 30% scatter. In other words, if one were to assume a $K'$-band M/L of one at the outset, most rotation curves would be quite precisely predicted from the observed light and gas distribution with no free parameters. In the B-band, on the other hand, the MOND M/L does appear to be a function of color in the sense that redder objects have larger M/L values. This is exactly what is expected from population synthesis models as is shown by the solid lines in both panels (Bell & de Jong 2000). This is quite interesting because there is nothing built into MOND which would require that redder galaxies should have a higher $M/L_b$; this simply follows from the rotation curve fits.

We sometimes hear that it is not so surprising that MOND fits rotation curves because that is what it was designed to do. This is certainly not correct. MOND was designed to produce asymptotically flat rotation curves with a given mass-velocity relation (or TF law). It was not designed to fit the details of all rotation curves with a single adjustable parameter (even of galaxies which are gas-dominated with no adjustable parameter), and it was certainly not designed to provide a reasonable dependence of fitted M/L on color. Indeed, there are a couple of well-observed spiral galaxies which are problematic for MOND, and which could, in principle, falsify the idea. One of these is NGC 2841—a large spiral galaxy with a Hubble distance of about 9 Mpc (Begeman et al. 1991). In fact, the rotation curve of the galaxy cannot be fit using MOND if the distance is
Figure 4. Inferred mass-to-light ratios for the UMa spirals (Sanders & Verheijen) in the B-band (top) and the K’-band (bottom) plotted against B-V colors (McGaugh, private communication). The solid lines show predictions from populations synthesis models by Bell and de Jong (2001).

only 9 Mpc. MOND prefers a distance of 19 Mpc as we see in Fig. 5 (the scaling of the centripetal acceleration depends upon the distance). If the distance to this galaxy is really less than about 14 Mpc it is quite problematic for MOND. Now it turns out that a Cepheid distance to this galaxy has just been determined (Macri et al. 2001), and this is 14.1 ± 1.5 Mpc. Given that the distance could easily be as large as 15.6 Mpc, this galaxy now would seem to present no problem for MOND.
Figure 5. MOND fits to NGC 2841 at various distances. The Hubble law distance is 9.3 Mpc (h=0.75), but MOND prefers a distance of 19.3 Mpc. The Cepheid distance is $14.1 \pm 1.5$. The MOND rotation curve at the Cepheid distance $+1\sigma$ (15.6 Mpc) is acceptable, particularly considering the complication of the large warp in the outer regions.

The success of MOND in accounting for galaxy rotation curves with only one free parameter, the M/L of the visible disk which usually assumes quite reasonable values, is remarkable. Whether MOND is correct or not, the success of this simple algorithm implies that galaxy rotation curves are entirely determined by the distribution of visible matter. If you believe in dark matter, then you somehow must explain this phenomenology. How can the distribution of dark matter be so intimately connected with the distribution of visible matter?
Figure 6. The line-of-sight velocity dispersion vs. characteristic radius for pressure-supported astronomical systems. The star-shaped points are globular clusters (Trager et al. 1993), the points are massive molecular clouds in the Galaxy (Solomon et al. 1987), the crosses are massive elliptical galaxies (Jorgensen et al. 1995a,b), and the squares are X-ray emitting clusters of galaxies (White, Jones & Forman 1997). The solid line shows the relation $\sigma^2/r = a_o$ and the dashed lines a factor of 5 variation about this relation.

5. Pressure-supported systems

Fig. 6 is a log-log plot of the velocity dispersion versus size for pressure-supported, nearly isothermal astronomical systems. At the bottom of the plot are globular clusters (star-shaped) and giant molecular clouds (points) in the Galaxy. The group of points in the middle are ellipticals (crosses) and at the top are X-ray emitting clusters of galaxies (squares). The triangles are the dwarf spheroidal systems surrounding the Milky Way and the dashes are compact dwarf ellipticals. The plotted parameters have not been massaged at all but are taken directly from the relevant observational papers. The measure of size is not homogeneous– for ellipticals and globular clusters it is the well-known effective radius, for the X-ray clusters it is an X-ray intensity isophotal radius, and for the molecular clouds it is a isophotal radius of CO emission. The velocity dispersion refers to the central velocity dispersion for ellipticals and globulars; for the clusters it is the thermal velocity dispersion of the hot gas; for the molecular clouds it is just the typical line width of the CO emission. A velocity-dispersion– size correlation has been previously claimed
for individual classes of objects—most notably, the molecular clouds and the clusters of galaxies.

The parallel lines are not fits but represent fixed internal accelerations. The solid line corresponds to \( \sigma_l^2/r = 10^{-8} \text{ cm}/\text{s}^2 \) and the parallel dashed lines to accelerations 5 times larger or smaller than this particular value. It is clear from this diagram that the internal accelerations in these systems all lie within a factor of a few of \( a_o \). This also implies that the surface densities in these systems are near the MOND surface density \( \Sigma_c \).

So these astronomical objects appear to have a characteristic internal acceleration or a characteristic surface density as MOND predicts. I emphasize that these objects are not only pressure-supported, but they are also nearly isothermal; i.e., there is not a large variation in the line-of-sight velocity dispersion across these objects. Stars are also pressure-supported systems but they would lie far outside the upper left boundary of this plot. However, stars are very far from isothermal.

It has been noted above that, with MOND, such self-gravitating near-isothermal systems would be expected to have internal accelerations comparable to or less than \( a_o \). But it is not at all evident how Newtonian theory can account for the fact these different classes of astronomical objects, covering a large range in size and located in very different environments, all appear to have comparable internal accelerations near the cosmologically interesting value of \( cH_o \).

With MOND, systems that lie below the line, i.e., with low internal accelerations, would be expected to exhibit larger discrepancies. This is particularly true of the dwarf spheroidal systems. Systems above the line (ellipticals) are high surface brightness systems and if interpreted in terms of Newtonian dynamics, would not exhibit much need for dark matter inside an effective radius. This seems to be the case. I just add that the MOND M-\( \sigma \) relation (eq. 7) is very sensitive to variations from strict homology which would be expected to lead to a large scatter in the observed Faber-Jackson law. However, MOND imposes boundary conditions on the inner Newtonian solution which restrict non-homologous objects to lie on a narrow fundamental plane similar to that implied by the traditional virial theorem (Sanders 2000).

Note that clusters of galaxies lie below the \( \sigma_l^2/r = a_o \) line in Fig. 6; thus, these objects would be expected to exhibit significant discrepancies. That this is the case has been known for 70 years (Zwicky 1933), although the subsequent discovery of hot X-ray emitting gas goes some way in alleviating the original discrepancy. For an isothermal sphere of hot gas at temperature \( T \), the Newtonian dynamical mass within radius \( r_o \), calculated from the equation of hydrostatic equilibrium, is

\[
M_n = \frac{r_o kT}{G m} \left( \frac{d \ln(\rho)}{d \ln(r)} \right),
\]

where \( m \) is the mean atomic mass and the logarithmic density gradient is evaluated at \( r_o \). For the X-ray clusters plotted in Fig. 6 this turns out to be typically about a factor of 4 or 5 larger than the observed mass in hot gas and in the stellar content of the galaxies. This rather modest discrepancy viewed in terms of dark matter has led to the so-called baryon catastrophe—enough non-baryonic dark matter in the context of standard CDM cosmology (White et al. 1993).

With MOND, the dynamical mass (eq. 6) is given by

\[
M_m = (G a_o)^{-1} \left( \frac{kT}{m} \right)^{\frac{3}{2}} \left( \frac{d \ln(\rho)}{d \ln(r)} \right)^{\frac{3}{2}},
\]

and the discrepancy, using the same value of \( a_o \) determined from nearby galaxy rotation curves, is on average reduced to about a factor of 2 larger than the observed mass. There
does indeed seem to be a remaining discrepancy. This could be interpreted as a failure, or one could say that MOND predicts that the baryonic mass budget of clusters is not yet complete and that there is more mass to be detected (Sanders 1999). It would have certainly been devastating for MOND had the predicted mass turned out to be typically less than the observed mass in hot gas and stars.

6. Cosmology and structure growth.

Let me just summarize what I have said so far. MOND not only allows the form of rotation curves to be precisely predicted from the distribution of observable matter, but it also explains certain systematic aspects of the photometry and kinematics of galaxies and clusters: the presence of a preferred surface density in spiral galaxies and ellipticals—the so-called Freeman and Fish laws; the fact that pressure-supported nearly isothermal systems ranging from molecular clouds to clusters of galaxies are characterized by a specific internal acceleration \(a_o\); the existence of a TF relation with small scatter—specifically a correlation between the baryonic mass and the asymptotically flat rotation velocity of the form \(v^4 \propto M\); the Faber-Jackson relation for ellipticals, and with more detailed modeling, the Fundamental Plane; not only the magnitude of the discrepancy in clusters of galaxies but also the fact that mass-velocity dispersion relation which applies to elliptical galaxies (eq. 6) extends to clusters (the mass-temperature relation). And it accomplishes all of this with a single new parameter with units of acceleration—an parameter determined from galaxy rotation curves which is within an order of magnitude of the cosmologically significant value of \(cH_0\). This is why several of us believe that, on an epistemological level, MOND is more successful than dark matter.

But, of course, MOND must fit into a larger picture. One may naturally ask—what are the larger-scale implications of modified dynamics—specifically what are implications for gravitational lensing and does MOND imply a reasonable cosmology and cosmogony? These are questions which require a more basic theory underlying MOND, and this is, at present, the essential weakness of the idea.

Frequently, the absence of a covariant theory is presented as an argument against MOND. But the criterion for judging a scientific hypothesis surely must be its empirical success. The absence of a successful covariant version is simply an aspect of its incompleteness. People don’t reject general relativity because there is not yet a viable theory of quantum gravity. At the same time, it is fair to say that MOND will never be entirely credible to most astronomers and physicists until it makes some contact with more familiar physics.

There have been several attempts to construct a more general theory, most notably by Bekenstein (1987), and while these are very nice ideas, none of these attempts is entirely satisfactory for various reasons (Bekenstein & Sanders 1994, Sanders 1997). A different approach is to consider MOND as modified inertia (Milgrom 1994), perhaps resulting from the interaction of an accelerating particle with vacuum fields (Milgrom, 1999). Here the coincidence between \(a_o\) and \(cH_0\) plays a central role: if inertia results from influence of the vacuum on accelerated motion, then, because a cosmological constant has a non-trivial effect upon the vacuum, we might expect that it also has a non-trivial effect upon inertia. It is beyond my mission to describe these ideas in detail, but I would just like to comment upon the possible shape of a MOND cosmology.

First of all, I take it that the experimental foundations of the standard Big Bang are so well-established, that any underlying theory of MOND should not lead to a radically different cosmology, at least not in the early Universe. Then, to say that MOND is an alternative to dark matter does not mean that every baryon in the Universe must be
Figure 7. The growth of fluctuations with an initial amplitude of $10^{-5}$. The solid lines show the growth of fluctuations on various comoving scales in the context of the simple non-relativistic MOND theory (Sanders 2001) and dotted line is the usual Newtonian growth in the pure baryonic Universe. The vertical dashed line indicates the scale factor at which the cosmological term begins to dominate the expansion in this model universe.

This is a question that has been considered by McGaugh (1999, 2000), who applied the widely-used CMBFAST program (Seljak & Zeldarriaga 1996) in the case of a pure baryonic Universe. Before the Boomerang and Maxima results appeared (Hanany et al. 2000, Lang et al. 2001), McGaugh pointed out that a pure baryonic universe, with the dominant constituent of the Universe being in vacuum energy density, would imply that the second peak in the angular power spectrum should be much reduced with respect to the expectations of the concordance ΛCDM cosmology. The reason for this suppression is basically Silk damping (Silk 1968) in a low $\Omega_m$, pure baryonic universe— the shorter wavelength fluctuations are exponentially suppressed by photon diffusion. When the Boomerang results appeared, much of the excitement was generated by the unexpected low amplitude
of the second peak. With $\Omega_{\text{total}} = 1.01$ and $\Omega_m = \Omega_b$ (no CDM or non-baryonic matter of any sort) McGaugh produced a rather nice match to the Boomerang results. A further prediction is that the third peak should be even more reduced. There are indications from the recent more complete analyses of BOOMERANG data (Netterfield et al. 2001) that this may not be the case, but the systematic uncertainties remain large. In addition, the SNIa results on the accelerated expansion of the Universe (Perlmutter et al. 1999) as well as the statistics of gravitational lensing (Falco et al. 1998) seem to exclude a pure baryonic and vacuum energy Universe, although it is unclear that all systematic effects are well-understood. It is also possible that a MOND cosmology may differ from standard Friedmann cosmology in the low-$z$ Universe (note that in some brane-world scenarios late-time cosmology diverges from Friedmann cosmology, e.g., Deffayet 2001).

Of course, if we live in a Universe of only baryons, then how does structure form? After all a primary motivation for non-baryonic cosmic dark matter is the necessity of forming the observed structure in the Universe by the present epoch via gravitational growth of very small density fluctuations. As we all know, non-baryonic dark matter helps because it offers the possibility that fluctuations can begin growing before the epoch of hydrogen recombination. The expectation is that MOND, by providing stronger effective gravity in the limit of low accelerations, might also help.

In the absence of a proper theory, this question can be considered by making several Ansätze in the spirit of the existing bits of the theory:
1. The MOND acceleration parameter $a_o$ is constant with cosmic time. This could be the case if $a_o$ is related to the cosmological term ($a_o \approx c\sqrt{\Lambda}$).

2. MOND is applied in determining the peculiar accelerations– those accelerations which develop around density perturbations– and not to the overall Hubble flow. That is to say, the Hubble flow remains intact. One might imagine that MOND should be applied to the Hubble flow; that is, as soon as the deceleration of the Hubble flow over a finite size region falls below $a_o$, then the dynamics of that region begins to deviate from the standard Friedmann solutions. This would lead to the eventual collapse of any finite size region regardless of its initial density and expansion velocity (Felten 1984, Sanders 1998). With this sort of cosmology the evolution of the early Universe would be as it is in the standard Big Bang (the deceleration on relevant scales is much larger than $a_o$) but the present Universe would look rather different than it actually does.

3. Although the MOND does not affect the Hubble flow, the deceleration or acceleration of the Hubble flow enters as a background field which influences the development of peculiar accelerations in the MOND regime.

It is possible to construct a non-relativistic Lagrangian-based theory which incorporates these three assumptions (Sanders 2001)– this is similar to the 2-field version of the theory of Bekenstein and Milgrom (1984) . Following the same procedure as in Newtonian cosmology, I find a growth equation for small density fluctuations which is non-linear even in the regime where the density fluctuations are small (this is because MOND is fundamentally non-linear). The growth of fluctuations becomes dramatically rapid when the non-linear term dominates as is evident in Fig. 7 which is a plot of the fluctuation amplitude as a function of scale factor in a baryonic-vacuum energy dominated Universe. Fluctuations of smaller wavelength grow to larger amplitude because they enter the MOND regime earlier.

The non-linear term becomes important when the background acceleration vanishes, i.e., when the density in vacuum energy becomes comparable to the matter energy density. Thus in a MOND Universe we might expect structure and massive galaxies to form about when the cosmological constant begins to dominate the expansion. Starting with an initial Harrison-Zeldovitch power spectrum normalized by COBE, the final power spectrum is shown in Fig. 8 where it is compared to the $\Lambda$CDM power spectrum. We see that it is quite similar apart from the baryonic oscillations.

So MOND offers the possibility of overcoming the slow growth of fluctuations in a pure baryonic Universe. It also offers an explanation of why we are observing the Universe at an epoch when $\Lambda$ has only recently emerged as the dominant term in the Friedmann equation. If this scenario remains as an aspect of a fully covariant theory, then the cosmological argument is no longer a unique rationale for non-baryonic dark matter.

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