Need for the intensity-dependent pion-nucleon coupling in multipion production processes

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Abstract

We give reasons in support of the use of an effective intensity-dependent pion-nucleon coupling Hamiltonian for describing the properties of the pion multiplicity distribution and the corresponding factorial moments within the thermal-density matrix approach. We explain the appearance of the negative-binomial (NB) distribution for pions and the well-known empirical relation of Wróblewski. Our model Hamiltonian is written as a linear combination of the generators of the $SU(1,1)$ group. We find the generating function for the pion multiplicity distribution at finite temperature $T$ and discuss the properties of the second-order factorial moment. Also, we show that an intensity-dependent pion-nucleon coupling generates the squeezed states of the pion field. At $T = 0$, these squeezed states become an inherent property of the NB distribution.

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1 Introduction

Recently a considerable amount of experimental information has been accumulated on multiplicity distributions of charged particles produced in \( pp \) and \( p\bar{p} \) collisions in the center-of-mass energy range from 10\( GeV \) to 1800\( GeV \). The Koba-Nielsen-Olesen (KNO) scaling [1], which was previously observed in the ISR c.m.energy range from 11 to 63 \( GeV \) [2], was shown to be violated in the regime of several hundred \( GeV \) [3]. The violation of the KNO scaling is characterized by an enhancement of high-multiplicity events leading to a broadening of the multiplicity distribution with energy.

The shape of the multiplicity distribution may be described either by its \( C \) moments, \( C_q = \langle n^q \rangle / \langle n \rangle^q \), or by its central moments (higher-order dispersions), \( D_q = \langle (n - \langle n \rangle)^q \rangle^{1/q}, q = 2, 3, \ldots \). The exact KNO scaling implies that all \( C_q \) moments are energy independent and that all central moments \( D_q \) satisfy a generalized Wróblewski relation [4,5] as \( \langle n \rangle \rightarrow \infty \):

\[
D_q = A_q \langle n \rangle - B_q, \tag{1}
\]

with the energy-independent coefficients \( A_q \) and \( B_q \). For the \( pp \) and \( p\bar{p} \) inelastic data below 100\( GeV \), the coefficients \( A_q \) and \( B_q \) are approximately equal within errors. This fact implies that the elementary Poisson distribution resulting from the independent emission of particles is ruled out.

The total multiplicity distribution \( P_n \) of charged particles for a wide range of energies (22 – 900\( GeV \)) is found to be well described by a negative-binomial (NB) distribution [3,6] that belongs to a large class of compound Poisson distributions [7]. The NB distribution is a two-step process [8] with two free parameters: the average number of charged particles \( \langle n \rangle \) and the parameter \( k \) that affects the shape (width)
of the distribution. The parameter $k$ is related to the dispersion $D = D_2$ by the relation

$$\left(\frac{D}{\langle n \rangle}\right)^2 = \frac{1}{k} + \frac{1}{\langle n \rangle}.$$  \hspace{1cm} (2)

The observed broadening of the normalized multiplicity distribution with increasing energy implies a decrease of the parameter $k$ with energy. The KNO scaling requires constant $k$.

Although the NB distribution gives information on the structure of correlation functions in multiparticle production, the question still remains whether its clan-structure interpretation [8] is simply a new parametrization of the data or has a deeper physical insight [9]. Measurements of multiplicity distributions in $p\bar{p}$ collisions at $TeV$ energies [10] have recently shown that their shape is clearly different from that of the NB distribution. The distributions display the so-called medium-multiplicity ”shoulder” [11,12]. A satisfactory explanation of this effect is still lacking [12].

In this paper we propose to study another approach to multiplicity distributions based on a pion-field thermal-density operator given in terms of an effective intensity-dependent pion-nucleon coupling Hamiltonian with $SU(1,1)$ dynamical symmetry. We assume that the system of produced hadronic matter (pions) is in thermal equilibrium at the temperature $T$ immediately after the collision.

The paper is organized as follows. In Sec.2 we present the basic ideas of our model. In Sec.3 we discuss the shape of multiplicity distribution, the correlations, and the Wróblewski relation. Finally, in Sec. 4 we conclude with a few remarks on the squeezing properties of the model in connection with the possible extension to include the two-pion coupling in the effective pion-nucleon Hamiltonian.
2 Thermal-density operator for the pion field

At present accelerator energies the number of secondary particles (mostly pions) produced in hadron-hadron collisions is large enough, so that the statistical approach to particle production becomes reasonable. Most of the properties of pions produced in high-energy hadron-hadron collisions can be expressed simply in terms of a pion-field density operator. We neglect difficulties associated with isospin and, for simplicity, consider the production of single-mode pions. As a consequence of this restriction we are only able to calculate multiplicity distributions and multiplicity correlations of pions. The energy dependence of the multiplicity distribution will reveal the scaling properties of the collision dynamics, and the deviation of the distribution from a Poisson distribution will reveal correlations between the produced pions.

We expect that in high-energy collisions most of the pions are produced in the central region $|y| < Y$, where $Y = \ln(s/m^2)$ is the relative rapidity of the colliding particles. In this region the energy-momentum conservation has a minor effect if the transverse momenta of the pions are limited by the dynamics.

The density operator $\hat{\rho}_{0T}$ for a free pion field in a heat bath of temperature $T$ is

$$\hat{\rho}_{0T} = \frac{1}{Z} e^{-\beta \hat{H}_0}, \quad \beta = \frac{1}{k_B T},$$

(3)

where

$$\hat{H}_0 = \omega (a^\dagger a + \lambda),$$

(4)

$$\ln Z = -\beta \lambda \omega - \ln(1 - e^{-\beta \omega}).$$

The quantity $\lambda \omega$ in $\hat{H}_0$ denotes the vacuum energy of the free pion system. The "zero-point energy" corresponds to $\lambda = \frac{1}{2}$. In the limit as $T \rightarrow 0$, the density
operator $\hat{\rho}_{0T}$ reduces to $\hat{\rho}_0 = |0\rangle\langle 0 |$ and represents the density operator for the pion-field vacuum state.

The mean number of thermal (chaotic) pions is

$$\bar{n}_T = \frac{1}{e^{\beta\omega} - 1}. \quad (5)$$

Owing to the interaction with the nucleon field the density operator $\hat{\rho}_{0T}$ is transformed by means of the unitary $S$-matrix operator into

$$\hat{\rho}_T = \hat{S}\hat{\rho}_{0T}\hat{S}^\dagger \quad (6)$$

$$= \frac{1}{Z} e^{-\beta \hat{H}},$$

where

$$\hat{H} = \hat{S}\hat{H}_0\hat{S}^\dagger. \quad (7)$$

We regard $\hat{H}$ as an effective Hamiltonian describing the pion-nucleon system. At $T = 0$, the $\hat{\rho}_T$ becomes $\hat{\rho} = \hat{S} |0\rangle\langle 0 | \hat{S}^\dagger$. The knowledge of $\hat{\rho}_T$ gives the possibility of finding the pion-multiplicity distribution $P_T(n)$ at finite temperature $T$ by calculating

$$P_T(n) = \langle n | \hat{\rho}_T | n \rangle, \quad (8)$$

where $|n\rangle = (n!)^{-1/2}a^n |0\rangle$ are the pion number states.

The first-order moment of $P_T(n)$ gives the average multiplicity

$$\langle n \rangle = \sum n P_T(n) \quad (9)$$

and higher-order moments of $P_T(n)$ give information on dynamical fluctuations from $\langle n \rangle$ and also on multipion correlations. All higher-order moments can be obtained from the pion-generating function

$$G_T(z) = \sum z^n P_T(n) = Tr\{\hat{\rho}_T z^N\} \quad (10)$$
by a suitable differentiation over $z$, where $\hat{N} = a^\dagger a$. Thus the normalized factorial moments $F_q$ are

$$F_q = \frac{\langle n(n-1)\ldots(n-q+1) \rangle}{\langle n \rangle^q} = \langle n \rangle^{-q} \frac{d^q G_T(1)}{dz^q},$$

(11)

and the normalized cumulant moments $K_q$ are

$$K_q = \langle n \rangle^{-q} \frac{d^q \ln G_T(1)}{dz^q}.$$

(12)

These moments are related to each other by the formula

$$F_q = \sum_{l=0}^{q-1} \binom{q}{l} K_{q-l} F_l.$$

(13)

For the Poisson distribution, all the normalized factorial moments are identically equal to 1 and all cumulants vanish for $q > 1$.

As a measure of dynamical fluctuations we shall mostly consider the $q = 2$ moments. They are directly related to the dispersion $D$ of the multiplicity distribution $P_T(n)$:

$$F_2 - 1 = K_2 = \left( \frac{D}{\langle n \rangle} \right)^2 - \frac{1}{\langle n \rangle}.$$

(14)

The quantity $F_2$ is also known as the two-particle correlation function $g^{(2)} = F_2$. Its numerical values are usually used to characterize the shape of the multiplicity distribution as well as to indicate the tendency of particles to bunch. The $g^{(2)}$ can take the following values:
\begin{align*}
g^{(2)} &= 1 \ (\text{Poisson}) \nonumber \\
\langle 1 \ (\text{sub-Poisson/antibunching}) \rangle &= 1 \ (\text{super-Poisson/bunching}) \nonumber \\
= 2 \ (\text{chaotic}) \nonumber \\
\langle 2 \ (\text{enhanced bunching}) \rangle &= 2 \ (\text{chaotic}) \nonumber
\end{align*}

The validity of the Wróblewski relation means that $g^{(2)}$ takes a value between 1 and 2.

The standard pion-nucleon interaction Hamiltonian is linear in the pion field operators. In our simplified model it corresponds to the Hamiltonian

\[ \hat{H} = \omega (a^\dagger a + \frac{1}{2}) + g (a^\dagger + a), \quad (16) \]

where $\omega$ and $g$ are real and positive parameters. This Hamiltonian can be written in the form (7) using the following $S$-matrix operator:

\[ \hat{S} = D^\dagger \left( \frac{g}{\omega} \right) = \text{exp} \left\{ \frac{g}{\omega} (a - a^\dagger) \right\} \quad (17) \]

and $\lambda = \frac{1}{2} - \left( \frac{g}{\omega} \right)^2$. The density operator $\hat{\rho}_T$ corresponding to (16) describes displaced thermal states, i.e., the superposition of coherent and thermal states of pions.

The generating function $G_T(z)$ is [13]

\[ G_T(z) = \frac{1}{1 + (1 - z)\bar{n}_T} \text{exp} \left\{ -\bar{n}_C \frac{1 - z}{1 + (1 - z)\bar{n}_T} \right\}, \quad (18) \]

where $\bar{n}_C = (g/\omega)^2$ denotes the average number of coherent pions. The normalized factorial moments $F_q$ are given in terms of Laguerre polynomials:

\[ F_q = q! (1 + \gamma)^{-q} L_q(-\gamma), \quad (19) \]
where $\gamma = \bar{n}_C/\bar{n}_T$. We note that the average number of pions is

$$\langle n \rangle = \bar{n}_T + \bar{n}_C \quad (20)$$

and the two-pion correlation function is

$$g^{(2)} = 2 - \left( \frac{\gamma}{1+\gamma} \right)^2. \quad (21)$$

According to the relation (14), the square of the dispersion $D$, in the limit $\langle n \rangle \to \infty$ and $\gamma$ fixed, is

$$\left( \frac{D}{\langle n \rangle} \right)^2 \to 1 - \left( \frac{\gamma}{1+\gamma} \right)^2. \quad (22)$$

It is always smaller than one as long as $\gamma \neq 0$.

On the other hand, by keeping the coupling constant $g$ fixed and letting the temperature approach either zero or infinity (variable $\gamma$), we find that $g^{(2)} = 1$ for coherent production and $g^{(2)} = 2$ for chaotic production. In this case, the Wróblewski relation (1) with a constant slope $A$ cannot be obtained. To remedy this, we require that $\gamma$ is an energy-independent parameter [14]. This requirement means that the square of the coupling constant $g^2$

$$g^2 = \omega^2 \frac{\gamma}{1+\gamma} \langle n \rangle \quad (23)$$

increases linearly with $\langle n \rangle$ or with $T$, which has the same effect. In this way, the Wróblewski relation can be satisfied for all energies. In analogy with quantum optics [15], we shall call the coupling of the type (23) intensity dependent.

We propose to study the following form of the effective pion-nucleon intensity-dependent coupling Hamiltonian:

$$\hat{H} = \epsilon (\hat{N} + \lambda) + \kappa (a \sqrt{\hat{N} + 2\lambda - 1} + h.c.), \quad (24)$$

where $\epsilon^2 = \omega^2 + 4\kappa^2$ and $\lambda = \langle 0 \mid \hat{H} \mid 0 \rangle / \epsilon$ is related to the vacuum energy of the system. In the limit $\lambda \to \infty$ and $\kappa \to 0$, so that $2\kappa^2\lambda = g$ is finite, the interaction part of the Hamiltonian $\hat{H}$ reduces to the standard pion-nucleon Hamiltonian (16).
We observe that the operators

\begin{align*}
K_0 &= \hat{N} + \lambda, \\
K_- &= a\sqrt{\hat{N} + 2\lambda - 1}, \\
K_+ &= \sqrt{\hat{N} + 2\lambda - 1}a^\dagger
\end{align*}

form the standard Holstein-Primakoff [16] realizations of the $su(1,1)$ Lie algebra, the Casimir operator of which is

\[ \hat{C} = K_0^2 - \frac{1}{2}[K_+K_- + K_-K_+] = \lambda(\lambda - 1)\hat{I}. \]  

The Hamiltonian $\hat{H}$ is thus a linear combination of the generators of the $SU(1,1)$ group:

\[ \hat{H} = \epsilon K_0 + \kappa(K_+ + K_-). \]  

The corresponding S-matrix that diagonalizes this Hamiltonian is

\[ \hat{S}(\theta) = e^{-\theta(K_+ - K_-)}, \]  

with

\[ th \theta = \frac{2\kappa}{\epsilon}. \]

The initial-state vector for the pion field, $\hat{S}(\theta) \mid 0 \rangle \equiv \mid \theta \rangle$, is [17]

\[ \mid \theta \rangle = (1 - th^2\theta)^\lambda \sum_n (-th\theta)^n \left( \frac{\Gamma(n + 2\lambda)}{n!\Gamma(2\lambda)} \right)^{1/2} \mid n \rangle. \]

We note that the matrix element squared $\mid \langle n \mid \theta \rangle \mid^2$ can be written in the form of the NB distribution

\[ p_n^{NB}(\theta) \equiv \mid \langle n \mid \theta \rangle \mid^2 = \frac{\Gamma(n + 2\lambda)}{n!\Gamma(2\lambda)} \left( \frac{\bar{n}(\theta)}{\bar{n}(\theta) + 2\lambda} \right)^n \left( \frac{2\lambda}{\bar{n}(\theta) + 2\lambda} \right)^{2\lambda}, \]

where $\bar{n}(\theta) = 2\lambda sh^2(\theta)$. 

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The full pion-density operator \( \hat{\rho} \equiv \hat{\rho}(\theta) \) is

\[
\hat{\rho}(\theta) = \sum_n p_T(n) |n, \theta\rangle \langle n, \theta|,
\]

where \( p_T(n) = (\bar{n}_T)^n/(1 + \bar{n}_T)^{n+1} \) represents the Bose-Einstein (geometric) distribution function and \( |n, \theta\rangle = \hat{S}(\theta) |n\rangle \). It is easy to see that the states \( |n, \theta\rangle \) form a complete orthonormal set of eigenvectors of the Hamiltonian \( \hat{H} \), i.e.,

\[
\hat{H} |n, \theta\rangle = \omega(n + \lambda) |n, \theta\rangle.
\]

The calculation of the matrix element \( \langle n | \hat{S}(\theta) | m \rangle = \langle n | m, \theta \rangle \) is much facilitated if we present the operator \( \hat{S}(\theta) \) in the antinormal form

\[
\hat{S}(\theta) = e^{th(\theta)K_-} (ch(\theta))^{2K_0} e^{-th(\theta)K_+}.
\]

For \( n \geq m \), we find

\[
\langle n | m, \theta \rangle = (ch(\theta))^{-2\lambda}(-th(\theta))^{n-m} \left( \frac{\Gamma(n+2\lambda)m!}{\Gamma(m+2\lambda)n!} \right)^{1/2} P_m^{(n-m,2\lambda-1)}(1 - 2th^2(\theta)),
\]

where \( P_m^{(\alpha,\beta)} \) are Jacobi polynomials [18]. A similar expression is obtained for \( n \leq m \):

\[
\langle n | m, \theta \rangle = (ch(\theta))^{-2\lambda} (th(\theta))^{m-n} \left( \frac{\Gamma(m+2\lambda)m!}{\Gamma(n+2\lambda)n!} \right)^{1/2} P_n^{(m-n,2\lambda-1)}(1 - 2th^2(\theta)).
\]

These matrix elements squared are used to find the pion multiplicity distribution

\[
P_T(n) = \sum_m p_T(n) | \langle n | m, \theta \rangle |^2
\]

and its generating function

\[
G_T(z) = \sum_m p_T(m) \langle m, \theta | z^\hat{N} | m, \theta \rangle
= \sum_{m,n} p_T(m) z^n | \langle n | m, \theta \rangle |^2.
\]
3 Pion-generating function and its moments

The average multiplicity $\langle n \rangle$, the dispersion $D$, and all higher-order moments $\langle n^q \rangle$, $q = 1, 2, \ldots$, at the temperature $T$ are obtained by a suitable differentiation over $z$ from the pion-generating function $G_T(z)$. The close analytic form of $G_T(z)$ can be found by observing that

$$\langle m, \theta | z^N | m, \theta \rangle = \langle \theta | z^N | \theta \rangle y^m P_m^{(0, 2\lambda - 1)}(x),$$

(39)

where

$$x = \frac{z + (1 - z)^2 \text{sh}^2(\theta) \text{ch}^2(\theta)}{z - (1 - z)^2 \text{sh}^2(\theta) \text{ch}^2(\theta)},$$

(40)

$$y = \frac{z - (1 - z) \text{sh}^2(\theta)}{1 + (1 - z) \text{sh}^2(\theta)}.$$

It is easy to see that

$$\langle \theta | z^N | \theta \rangle \equiv G_0(z)$$

(41)

$$= \sum_m z^m p_m^{NB}(\theta)$$

$$= [1 + (1 - z) \bar{n}(\theta) \frac{2}{2\lambda}]^{-2\lambda}$$

(42)

is exactly the generating function of the NB distribution with a constant shape parameter $2\lambda$ and the average number of pions equal to $\bar{n}(\theta)$.

Taking into account the generating function of Jacobi polynomials [18]

$$\sum_m P_m^{(0, 2\lambda - 1)}(x)y^m = 2^{2\lambda - 1} R^{-1}(1 + y + R)^{1 - 2\lambda},$$

(43)

$$R = \sqrt{1 - 2xy + y^2},$$

we obtain the final form of the pion-multiplicity generating function $G_T(z)$:

$$G_T(z) = G_0(z)(1 + \bar{n}_T)^{-2\lambda - 1} R_T^{-1}(1 + y_T + R_T)^{1 - 2\lambda},$$

(44)

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where now \( y_T = \frac{\bar{n}_T}{1 + \bar{n}_T} y \). Using this generating function the following average number of pions and the multiplicity dispersion at the temperature \( T \) are found:

\[
\langle n \rangle = \bar{n}(\theta) + \bar{n}_T + \frac{1}{\lambda} \bar{n}(\theta) \bar{n}_T,
\]

\[
D^2 = d^2_T + d^2(\theta)[1 + \frac{2\lambda + 3}{\lambda} \bar{n}_T + \frac{4}{\lambda} \bar{n}_T^2],
\]

where

\[
d^2_T = \bar{n}_T^2 + \bar{n}_T,
\]

\[
d^2(\theta) = \frac{1}{2\lambda} \bar{n}^2(\theta) + \bar{n}(\theta).
\]

Combining the two relations in (45), we can rewrite \( D^2 \) in the form of a quadratic function of \( \langle n \rangle \):

\[
D^2 = A_0(T) + A_1(T) \langle n \rangle + A_2(T) \langle n \rangle^2,
\]

with temperature-dependent coefficients \( A_i(T) \), \( i = 0, 1, 2 \). In order to explain the Wroblewski relation, the coefficient \( A_2(T) \),

\[
A_2(T) = 2 - \frac{6\lambda - 3}{2\lambda} (1 + \frac{\bar{n}_T}{2\lambda})^{-1} + \frac{\lambda - 1}{\lambda} (1 + \frac{\bar{n}_T}{2\lambda})^{-2},
\]

should be less than one. If \( A_2(T) \) is fixed, say, by experiment, then it gives a definite relation between the vacuum energy \( \lambda \) of the pion field and the number \( \bar{n}_T \) of the produced thermal pions. Thus, for example, the data [9] on negatively charged particles produced in \( pp \) collisions suggest \( A_2(T) = 1/3 \). This value of the coefficient \( A_2(T) \) restricts the average number of thermal pions to \( \bar{n}_T < 1 \) for all values of \( \lambda \geq 3/2 \).

We notice that at \( T = 0 \) the \( G_T(z) \) becomes the generating function of the NB distribution, \( G_0(z) \), and

\[
\frac{D^2}{\langle n \rangle^2} \bigg|_{T=0} = \frac{d^2(\theta)}{\bar{n}^2(\theta)} = \frac{1}{2\lambda} + \frac{1}{\bar{n}(\theta)},
\]
as it is to be expected from the NB distribution. The shape parameter $2\lambda$ of our NB distribution is given by the vacuum energy of the pion field in the nucleon environment, and has nothing to do with either the number of pion sources (cells) or the number of clans. The Wróblewski relation at $T = 0$,

$$d(\theta) \approx A\bar{n}(\theta) + B, \quad \bar{n}(\theta) \gg 1,$$

has energy-independent coefficients $A = (2\lambda)^{-1/2}$ and $B = (\lambda/2)^{1/2}$. If $\lambda > 1/2$, we have $A < 1$.

For the temperature $T$ going to infinity, we obtain

$$\frac{D^2}{\langle n \rangle^2} \bigg|_{T \to \infty} = 2 - \left(1 + \frac{\bar{n}(\theta)}{\lambda}\right)^2$$

$$= 1 + \hbar^2(2\theta).$$

This result shows that at very high temperature the distribution of pions will become chaotic if $\theta$ is very small.

The validity of the Wróblewski relation implies that the produced pions have a tendency to bunch. In the standard approach, the bunching of pions is usually attributed to the presence of a quadratic (two-pion) interaction term in the Hamiltonian (16), which is now of the form

$$\hat{H} = \omega(a^\dagger a + \frac{1}{2}) + g(a^\dagger + a) + \frac{1}{2}\kappa(a^2 + a^2).$$

Recently the properties of the density matrix $\hat{\rho}_T$, corresponding to this Hamiltonian, were investigated for the photon field [19]. It was found that $\hat{\rho}_T$ then defined the squeezed coherent thermal states [20].

Our effective intensity-dependent coupling Hamiltonian (24) is, however, highly nonlinear and nonquadratic in the pion field. It is, therefore, to be expected that it also generates squeezing that, in our model, should depend on the value of
the parameter $\lambda$. Namely, we know that for $\lambda \to \infty$ the pion distribution reduces to the superposition of the coherent and thermal states that show no squeezing.

We study the squeezing properties of the pion field in two Hermitian quadrature operators $a_1$ and $a_2$ defined by

$$a = a_1 + i a_2,$$

which satisfy $[a_1, a_2] = i/2$. The corresponding uncertainty relation is $\Delta a_1 \Delta a_2 \geq 1/4$, where variances $\Delta a_{1,2}$ are defined by $(\Delta a_{1,2})^2 = \langle a_{1,2}^2 \rangle - \langle a_{1,2} \rangle^2$. A state of the pion field is considered squeezed if either $\Delta a_1$ or $\Delta a_2$ are smaller than $1/2$. If we define the relative variance with respect to $(\Delta a_{1,2})^2_{coh} = 1/4$ as

$$S_{1,2} = 4(\Delta a_{1,2})^2 - 1,$$

then the squeezing condition becomes

$$-1 \leq S_i < 0, \ i = 1 \text{ or } 2.$$  \hfill (55)

At $T = 0$ our model gives

$$S_1 = 2(\langle a^\dagger a \rangle + \langle a^\dagger a^2 \rangle - 2\langle a^\dagger \rangle^2),$$

$$S_2 = 2(\langle a^\dagger a \rangle - \langle a^\dagger a^2 \rangle),$$

where

$$\langle a^\dagger a \rangle = \bar{n}(\theta),$$

$$\langle a^\dagger \rangle = -\text{th}(\theta) \sum_n p_n^{NB}(\theta) \sqrt{n + 2\lambda},$$

$$\langle a^\dagger a^2 \rangle = \text{th}^2(\theta) \sum_n p_n^{NB}(\theta) \sqrt{(n + 2\lambda)(n + 2\lambda + 1)}.$$  \hfill (57)

$^2$The finite-$T$ case is considered elsewhere.
It is easy to see that for $\lambda \gg \frac{1}{2}$, the relative variance is $S_1 > 0$ and $S_2 \approx 0$.

However, for moderate values of $\lambda > \frac{1}{2}$, and such that

$$\sqrt{(n + 2\lambda)(n + 2\lambda + 1)} \approx (n + 2\lambda) + \frac{1}{2} - \frac{1}{8(n + 2\lambda)} + \cdots,$$

we find squeezing for $S_2 < 0$.  

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4 Conclusions

In this paper we have proposed to study an intensity-dependent pion-nucleon coupling Hamiltonian with $SU(1, 1)$ dynamical symmetry, within a multipion-production model in which the pion field is represented by the thermal-density operator.

We have shown that this Hamiltonian explains in a natural way the appearance of the NB multiplicity distribution for pions and the Wróblewski relation. The shape parameter of the NB distribution is related to the vacuum energy of the pion field in the nucleon environment.

Also, we have shown that, depending on the value of the parameter $\lambda$, an intensity-dependent pion-nucleon coupling is able to generate the squeezed states of the pion field. At $T = 0$, these squeezed states $| \theta \rangle$ then become an inherent property of the NB distribution.

For $T \neq 0$, we have found the explicit analytic form of the pion-multiplicity generating function that may be used for obtaining all higher-order moments of the pion field.

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