Fundamental Frequency Optimization of Variable Angle Tow Laminates

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Abstract

Variable stiffness composite laminates can improve the structural behaviour of conventional laminates, by allowing new design possibilities. This work intends to explore that larger design space to maximize the fundamental frequency of square and rectangular variable angle tow plates. With that in mind, an optimization framework was implemented that encompasses a finite element model built using the commercial software ABAQUS through a Python script. In addition to it, a Matlab routine was also developed with the aim of locating the manufacturing defects of the respective plate. It is assumed that the plate is symmetric, produced using the automated fiber placement (AFP) manufacturing process with embedded gaps and with a linear fiber angle variation. Both fully clamped (CCCC) and fully simply supported (SSSS) boundary conditions are imposed. The optimal results are presented considering non steered plates and steered plates with and without gaps.

Keywords: Variable stiffness laminate, variable angle tow, fundamental frequency, optimization, automated fiber placement, finite elements

1. Introduction

Nowadays, the use of composites materials is increasing in a wide range of sectors such as the aerospace, automotive, naval and others. This has served as the main motivation to the development of innovative and cost effective composite manufacturing techniques which allow the production of composite laminates that can meet certain requirements in a more effective way. In particular, the development of the automatic fiber placement (AFP) technique opens the possibility of manufacturing variable stiffness composite laminates (VSCL), which, as their name indicates, allow the stiffness to vary spatially. VSCL include, for instance, those made with variable fiber spacing (VFS) and those in which the fibers are deposited following curvilinear paths, denominated as variable-angle tow (VAT) laminates. They allow new design possibilities by enlarging the design space when compared to conventional composite laminates. Hence, several authors have and are currently studying them with the objective of improving composite laminates characteristics. For instance, the studies presented in references [1, 2] show the influence of these kind of laminates in the buckling behaviour of laminated panels.

Focusing now on the study of vibrations, although most studies are performed on composite laminated plates with constant stiffness, some authors also considered the use of VAT laminates. Most of these studies aim to study the influence of the curvature of the fibers on the fundamental frequency achieved by a plate. Akhavan and Ribeiro [3], for example, studied the natural frequencies and mode shapes of VAT with a linear curvilinear fiber path. They used a p-version finite element and a third-order shear deformation theory (TSDT). In addition, a curvature constraint was also imposed. It was verified that the use of curvilinear fibers can be advantageous to adjust frequencies and mode shapes. With that in mind, a few studies with VAT laminated plates were performed with the objective of maximizing the fundamental frequency. For instance, Honda et al. [4] considered splines to represent the fiber paths of curvilinear fibers. Another example is the study from Farsadi et al [5] which included plates with different aspect ratios, boundary conditions and skew angles. The fundamental frequency was also studied and optimized together with other composite characteristics. For example, Honda et al. [6] developed a multi-objective approach in order to maximize both fundamental frequencies and in-plane strengths of VAT plates, while Pereira et al [7] performed a multi-objective optimization considering the fundamental frequency and the first mode spe-
pecific damping capacity (SDC).

The previous studies mentioned assumed that the VAT laminated plate is ideal, which means that the models used to obtain the natural frequencies and mode shapes did not take into account the manufacturing process, in particular, the AFP and the respective manufacturing induced imperfections. As a result of that, some models were developed in order to take defects into consideration and better predict the structural behaviour of VAT laminates. Blom [8] developed a methodology where each ply of the laminate is modeled including gaps and overlaps generated by tow drops. Afterwards, a 2-D mesh is generated on top of each ply and the centroid position of each element is calculated. Next, the centroids of all the elements in all the plies are checked, whether they are or are not located in the tow drop region. If so and in case it is a gap, then at that position resin properties are assigned to the respective element stack. In case it is an overlap, the thickness assigned to the element stack is twice the thickness. It is important to emphasize that a tow drop defect is only identified if the respective element centroid is located over it. In consequence, a very refined mesh is required to have a good precision which is associated to a higher computational cost. A similar strategy was developed by Fayazbakhsh [9, 10]. However, each element properties are assigned in accordance with its volume fraction of gaps and overlaps using micro-mechanics. In this way, it is not required such a refined mesh and computational cost is saved. Marouene et al. [11] study considered this last methodology to take into account the effect of both gaps and overlaps into the finite element model built using ABAQUS. A multi-optimization was performed with the goal of maximizing the in-plane stiffness and the buckling load. The numerical results obtained were then validated with experimental studies and a good accordance between them was found.

Regarding experimental studies with respect to the measurement of the fundamental frequency of VAT laminated plates, the author refers to some publications. In Antunes et. al study [12] experimental modal analysis were performed on a rectangular plate considering various boundary conditions: one edge clamped and the remaining edges free (CFFF), two opposite edges clamped and the other two free (CFCCF). Moreover, modal damping ratios were also identified in addiction to both natural frequencies and mode shapes. In this article, the theoretical model is based on two p-version type finite element models, one based on the Classical Plate Theory (CPT) and another based on the First-order Shear Deformation Theory (FSDT). Similar results were obtained using the two models, which can be justified by the use of a very thin plate. According to the authors, the discrepancies found between the experimental and numerical were due to the occurrence of gaps and overlaps in the plate, which the FEM model did not take into account.

In this work, steered laminated plates are optimized with the objective of maximizing its fundamental frequency. Moreover, an optimization framework is developed which includes the use of the Defect Layer Method in order to take into consideration manufacturing defects, namely gaps, associated to the AFP process.

2. Variable Angle Tow Laminate

In the AFP process, a band of fibers, denominated course, is composed by individual units denominated tows. The courses are deposited following a reference fiber path which can be curvilinear. Different curvilinear fiber paths have been studied by various authors. In this work, the fiberpath presented next is the one considered to solve the optimization problem.

2.1. Fiber Path Definition

The reference course fiber path considered was first introduced in [13] in 1993. In this formulation, the fiber angle (θ), measured with respect to the x* axis, varies linearly along the axis x*. The respective formulation is presented in equation 1 for a plate with a length in x* equal to a. The variable T0, represents the value of θ in the middle of the plate, while the variable T1 represents the value of θ at both edges in x* of the plate and φ represents the orientation of x* with the respect of the global axis x. For some orientation φ, both middle and edge fiber angles are written as < T0, T1 >. The respective centerline fiber trajectory is presented in equation 2 [14]. The fiber path correspondent to < 60°, 15° > is presented in figure 1. In this work φ is considered as zero.

\[
\theta(x^*) = \phi + (T_1 - T_0)2\frac{|x^*|}{a} + T_0 \quad (1)
\]

\[
f(x) = \begin{cases} 
\frac{1}{2} \left( \frac{\theta(x^*)}{T_1 - T_0} \right) \left[ \ln(\cos(T_0)) + \ln(\cos\left(\frac{T_0 + \frac{2(T_1 - T_0)}{a} x^*}{a}\right)) \right], & \text{if } -\frac{a}{2} \leq x^* \leq 0 \\
\frac{1}{2} \left( \frac{\theta(x^*)}{T_1 - T_0} \right) \left[ \ln(\cos(T_0)) + \ln(\cos\left(\frac{T_0 + \frac{2(T_1 - T_0)}{a} x^*}{a}\right)) \right], & \text{if } 0 \leq x^* \leq \frac{a}{2} 
\end{cases} \quad (2)
\]

2.2. Induced defects

The fiber paths of the adjacent courses are obtained by shifting the centerline fiber path in the y* direction, which leads to the formation of manufacturing defects within the laminate. These defects can be either gaps or overlaps and depend on the way the tows are cut. If the entire course is cut or
restarted when there is an intersection with the adjacent course, it leads to the formation of large gap or overlap areas. So, in order to solve this problem and minimize the defect area, the tow cut and restart capability of the AFP machine is used. It allows to individually control each tow, to cut and restart its deposition and leads to a reduced defect area. In this work, an one sided cut strategy is employed, which means that the towels are only cut on one side of each course. It is also used the zero percent coverage parameter in the tow drop strategy, which implies that the cut is performed as soon as one edge of a tow intersects the boundary of the adjacent course, also known as complete gap strategy. It is assumed that the AFP machine deposits 32 tows, each one with a width of 3.175mm, as it is represented in figure 1(b).

2.3. Manufacturing Constraints
The maximum curvature constraint or inverse, minimum curvature radius, depends on the AFP machine and limits the maximum value of curvature of the reference fiber path of each course. Typical curvature constraints are presented in table 1. This constraint limits the design space by limiting the fiber angle distribution that each lamina possesses. This constraint is dependent on the AFP machine and must be evaluated before the production stage in order to ensure that the composite can be manufactured. For that reason, it is most often used as an optimization constraint when VAT laminates are optimized to maximize or minimize determined characteristics.

The curvature equation was analytically obtained using the fiber reference path presented in equation 1 and also the definition of curvature of a function with a a single variable presented in equation 3.

\[
K = \frac{\frac{d^2 f(x)}{dx^2}}{(1 + (\frac{d f(x)}{dx})^2)^{3/2}} \tag{3}\]

After performing some substitutions it is possible to deduce equation 4. This equation was also considered in references [3] and [16].

\[
K = 2\frac{T_1 - T_0}{a} \cos\left(\frac{(T_1 - T_0)^2|x^*|}{a} + T_0\right) \tag{4}\]

3. Optimization
The optimization problem that this work intends to solve is defined in equation 5. The goal is to maximize the fundamental frequency, treated here as \(f_1\), of a laminated plate. It is assumed that the laminate is symmetric and that the total number of plies \((N_p)\) is equal to eight. The problem was solved considering different cases where the vector of design variables \((\textbf{x})\) depends on whether non-steered or steered fibers (equation 1) were considered. In the cases where only non-steered fibers were considered, the design variables correspond to the orientation of each layer (equations 6 and 7) and, therefore, the number of design variables is four (one per ply for a half laminate). In the cases where steered fibers were considered, the design variables correspond to the angles \(T_0\) and \(T_1\) of each layer (equations 6 and 7). Consequently, there is a total of eight design variables (two per ply for a half laminate). The value of each design variable is between \(-90^\circ\) and \(90^\circ\), except when gaps were considered. In those cases the domain of design variables is allowed to vary between \(-89^\circ\) and \(89^\circ\), due to singularities of the Matlab subroutine used. In some of the steered cases, the maximum curvature constraint, displayed in equation 4, was imposed in order to guarantee that the laminate can be manufactured using an AFP machine. The problem was solved considering the fully clamped and fully simply supported boundary condition.

\[
\begin{align*}
\text{maximize} & \quad f_1(\textbf{x}) \\
\text{by varying} & \quad \textbf{x} \in \mathbb{R} \\
\text{subject to} & \quad K_{n_i}(\textbf{x}) \leq K_{max}, \\
& \quad n = 1, 2, \ldots, N_p, \quad \text{if steered} \\
& \quad \textbf{x} = [x^{(1)} \ldots x^{(i)} \ldots x^{(N_p/2)}]^T \\
\end{align*} \tag{5}\]

where:

\[
K = 2\frac{T_1 - T_0}{a} \cos\left(\frac{(T_1 - T_0)^2|x^*|}{a} + T_0\right) \tag{6}\]

\[
\begin{array}{|c|c|c|}
\hline
\text{tow} & \text{typical minimum} & \text{typical maximum} \\
& \text{width} & \text{turning radius} & \text{curvature} \\
& (mm) & (mm) & (m^{-1}) \\
\hline
3.175 & 635 & 1.57 \\
6.35 & 1778 & 0.56 \\
12.7 & 8890 & 0.1125 \\
\hline
\end{array} \tag{7}\]

Figure 1: Fiber path with \(< 60^\circ, 15^\circ \) >.
\[
x^{(i)} = \begin{cases} [T^{(i)}_0], & \text{if non-steered} \\
[T^{(i)}_0, T^{(i)}_1], & \text{if steered}
\end{cases}
\]

The notation used to distinguish the different cases is orientation distribution - boundary condition - unconstrained/constrained. The orientation distribution notation can be NS, if only non-steered plies are considered or LS (linearly steered), if the fiber path orientation varies linearly across \(x^*\) (equation 1). The boundary conditions fully clamped (CCCC) and fully simply supported (SSSS) are treated here as C and S, respectively. The unconstrained and constrained cases will be denominated UN or CON, respectively. In the constrained cases, it was considered a maximum curvature constraint with two different values: 1.57 \(m^{-1}\) [8], treated here as constraint A, and 3.28 \(m^{-1}\) [14], denominated here as constraint B. For instance, LS-C-CON-A is a plate with a linear fiber angle variation, fully clamped and with a maximum curvature constraint of 1.57 \(m^{-1}\).

The plates considered are symmetric with a total of eight plies, each one with a thickness of 0.159 \(mm\). The material properties considered are displayed in table 2. The length in the \(x^*\) axis, \(a\), is kept constant and equal to 0.5 \(m\) throughout all the plates. A square plate and two rectangular plates with aspect ratios \((AR = a/b)\) equal to 0.5 and 2.0 are considered.

4. Implementation

The developed framework can be divided into two main components: the Matlab and the Finite Element Model (Python/ABAQUS) environments. The optimization process starts in the Matlab where the optimizer, genetic algorithm, is implemented and where each generation’s population is created. After that, the population information is passed to a Python script where the geometry of the plate, property assignment, boundary and loading conditions are defined before running the structural analysis using ABAQUS. The input data generated depends if the case considered only non-steered, steered without imperfections and steered plates with imperfections. Following the structural analysis, its results are then again passed to the Matlab environment where they are post-processed and the result fitness is evaluated by the genetic algorithm optimizer. If there is convergence, the optimization stops, otherwise another population of the next generation is created and the optimization process continues.

4.1. Finite Element Model

The geometry creation, element property assignment as well as the structural analysis is defined in Python scripts and performed by ABAQUS CAE.

It is the orientation of the fibers and material properties assignment that differs and so the input data generated by Matlab. In the non-steered cases all the elements of a ply have the same orientation, so the orientation is defined layer by layer and therefore Matlab only passes each layer orientation to the Python Script. In the steered cases, the element orientation in a ply is calculated in accordance with equation 1, where the fiber orientation varies linearly along the \(x^*\). As a consequence, the orientation can no longer be defined layer by layer and has to be defined element by element in each layer which leads to an increase of computational cost. Matlab generates each ply \(T_0\) and \(T_1\) for half laminate (due to its symmetry) which are read by the Python script. Then, each element orientation in a ply is assigned by substituting in equation 1 each element centroid position in \(x^*\) and its respective ply \(T_0\) and \(T_1\). If defects are being considered in the steered case, each element property is dependent on its resin percentage. So, not only the orientation is defined element by element in each layer, but also the material properties. For that reason, the steered cases considering defects are the ones that require a higher computational cost.

It was chosen a mesh with a total of 2500, 5000, 1250, S4R mesh elements for the square plate and plates with \(AR = 0.5\) and \(AR = 2.0\), respectively. This choice was made after a mesh convergence study with the objective of simultaneously assuring a good compromise between the model convergence and the computational cost necessary to run a structural analysis.

4.2. Matlab environment

The main script starts by generating a population. In case the problem is constrained, this population is generated having to satisfy the optimization constraints which are defined in a different script. After this, each individual is checked by a script that saves the optimization history. That history has saved all the individuals already analysed and their respective fitness evaluation. So, in case a certain individual has already been evaluated, then there is no need to run the structural analysis and this step can be skipped. This has a great importance, because it enables to save computational time. If the individual has not yet been evaluated, the respective input data needed to run the Python Script and ABAQUS CAE is created.

The creation of input data depends on whether the plate considered is non-steered or steered, and if the defects are or are not being considered. It is important to note that only symmetrical composites were considered in all cases, so the input data is only referred to half of the laminate. When defects are considered, namely gaps, they are modeled in a
5 Results

This section is dedicated to present and discuss the optimization results which aim to maximize the fundamental frequency.

5.1 Non steered plates

This section has the goal of presenting the optimal results of plates that only considers non steered plies. The first natural frequency is presented in Table 5 as well as the optimal orientation of each ply ($T_i^{*}$). The computational cost is represented by the number of generations (Gen.) required to perform the optimization.

Focusing first on the square plates, it is observable in Table 5 that the fully clamped supported case achieves a higher first natural frequency (51.601 Hz) than the one obtained in the simply supported case (31.223 Hz). That shows the influence that boundary conditions can have on the natural frequencies of a structure. It is also noticeable that the optimal orientation of each ply is different in both cases. On the one hand, in the NS-C, the optimal orientation alternates between approximately $-90^\circ$ and nearly $0^\circ$. On the other hand, in the NS-S, the exterior ply and its respective symmetric one have an orientation of approximately $45^\circ$, while all the interior ones have an orientation near $-45^\circ$. That again proves that the optimal orientation of each ply is dependent on the imposed boundary conditions. Comparing now both cases, but focusing on the last two columns of Table 5, it is possible to verify that both required the same number of Gen. to perform the optimization. That implies that the computational cost is very similar for both cases. That is something that was expected, because they share the same objective function and design variables being the only difference the imposed boundary conditions.

The non steered rectangular optimal results are also presented in Table 5 that is with aspect ratios different than one ($a/b \neq 1$). When the aspect ratio is equal to 0.5, the optimal orientations are near zero degrees for all the plies and for both boundary conditions. A similar behaviour is observed for an aspect ratio of 2, where all plies have near $90^\circ$ or $-90^\circ$ also for both boundary conditions. So, for non steered plies, these two cases are equivalent, because all plies are oriented parallel to the smaller side of the plate for both boundary conditions considered. Regarding the computational cost, it is visible that all the non steered cases have a similar number of generations due to the fact that all of them have the same number of design variables. The small discrepancy is explained by the randomness associated to the genetic algorithm used.

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Table 2: Carbon epoxy Cytec® G40-800/5276-1 properties [9].

| Property | $E_1$ (GPa) | $E_2$ (GPa) | $G_{12}$ (GPa) | $\nu_{12}$ | $\rho$ (kg/m$^3$) |
|----------|-------------|-------------|----------------|------------|------------------|
| G40-800/5276-1 | 143 | 9.1 | 4.8 | 0.3 | 1650 |
| resin | 3.7 | 3.7 | 1.4 | 0.3 | 1310 |

Matlab subroutine in the same way as in Fayabazh et al.[10]. Afterwards, each element resin percentage is calculated and passed to the Python Script where the equivalent element properties are calculated [10].

Following the input data creation, the next steps include the generation of the plate geometry, element equivalent properties and orientation, the definition of the boundary and loading conditions and the structural analysis. These steps are performed by the Python script which uses ABAQUS to run the structural analysis. After the structural analysis is finished, the output data, in this case, the plate natural frequency, is passed to the Matlab script that defines the objective function in order to evaluate its fitness. Finally, after all the individuals of a generation are evaluated, the optimizer checks if the convergence has yet been achieved. If it has, the optimization process is terminated. Otherwise, the optimizer generates the next population using genetic operators such as selection, crossover and mutation and the optimization process continues. A population size of 80, a function tolerance of $10^{-5}$, a maximum number of stall generations equal to 25 and a constraint tolerance of $10^{-6}$ was considered.

4.3. Comparison study

The finite element model was validated by comparing its results with the linear natural frequencies obtained in the study performed by Akbarzadeh et al. [17]. The plate properties considered in the second study are presented in Table 3, where square plates (a=b=1m) are considered. The comparison on results is presented in Table 4. It is worth mentioning that the results were compared considering a defect free plate and the same one with complete gap manufacturing defects. It is also important to note that Akbarzadeh et al. [17] considered a plate with a constant curvature fiber path, which can be the cause of the slight difference in the results displayed in table 4.

The finite element model was validated by comparing its results with the linear natural frequencies obtained in the study performed by Akbarzadeh et al. [17].
5.2. Defect free steered optimization results

The optimal results for the ideal square steered plates are presented in table 6 and visible in figure 2. The optimization was performed considering unconstrained and constrained cases with different maximum curvature constraints (\(K_{\text{max}} = 1.57m^{-1}\) (A) and \(K_{\text{max}} = 3.28m^{-1}\) (B)) in order to assess the curvature effect on the final solution. All these cases where optimized for fully clamped and fully simply supported boundary conditions. Table 6 displays the optimal \(T_0\) and \(T_1\) for the first four plies (because the composite laminate is symmetric) and the respective first natural frequency for the cases considered. It is also visible the number of generations (Gen.) required to perform the optimization.

The optimization results considering the fully clamped boundary condition case are presented in the first three columns. The unconstrained optimization (no maximum curvature considered), the constrained case A \((K_{\text{max}} = 1.57m^{-1})\) and the constrained case B \((K_{\text{max}} = 3.28m^{-1})\) are presented in the first, second and third columns, respectively. An observation that can be made is that the unconstrained case (LS-C-UN) is the one that achieves the highest first natural frequency \((59.471Hz)\) which represents an increase of 15.25% with respect to the square NS-C case (table 5). However, six plies of this case have a maximum curvature \((K_{\text{max}} = 6.42)\) for ply 1 and 8, \(K_{\text{max}} = 6.42\) for ply 2 and 7, \(K_{\text{max}} = 6.42\) for ply 3 and 6) higher than the \(K_{\text{max}}\) values considered here. Consequently, this laminate cannot be manufactured by an AFP machine. In spite of that, the result of the unconstrained case is useful not only to show where the optimal solution is, but also as, with the development of new manufacturing techniques that enable to produce plates with higher fiber curvatures, it could become a feasible design in the close future.

Focusing now in both constrained cases, LS-C-CON-A \((K_{\text{max}} = 1.57m^{-1})\) and LS-C-CON-B \((K_{\text{max}} = 3.28m^{-1})\), table 6 shows that LS-C-CON-B is the case with the highest first natural frequency \((56.814Hz)\) between the two. That constitutes an increase of 10.10% with respect to the square NS-C case (table 5), while the LS-C-CON-A increase is only 4.58%. LS-C-CON-B is also the constrained case that allows a higher maximum curvature, which, again, reinforces the influence of this parameter in the frequency obtained. It is also worth noting that for both cases, \(K_{\text{max}}\) is an active constraint for the first two layers and their respective symmetric ones. Therefore, it can be argued that lowering the maximum curvature allowed tends for the frequency to decrease. Regarding plies four and five of LS-C-CON-B, their maximum curvature approaches zero similarly to the unconstrained case. That is also the case for plies three and six in the LS-C-CON-A case.

The optimization results considering the fully simply supported boundary condition case are presented in the last three columns of table 6. The unconstrained optimization (no maximum curvature considered), the constrained case A \((K_{\text{max}} = 1.57m^{-1})\) and the constrained case B \((K_{\text{max}} = 3.28m^{-1})\) are presented in the fourth, fifth and sixth columns, respectively. The unconstrained optimal solution (LS-S-UN) has a first natural frequency equal to 31.229Hz which is very similar to the one achieved by both constrained cases. The same phenomena is verified with respect to the second and third natural frequencies. So, it is no surprise, that all the cases have similar \(T_0\) and \(T_1\) combinations and, consequently, also similar maximum fiber curvatures \((K_{\text{max}}^*)\) in each ply. Regarding the \(K_{\text{max}}^*\) in each ply, it is important to note that its value in all cases and in all plies is near zero, which implies that the maximum curvature is not an active constraint. It also means that the unconstrained case is possible to manufacture using the AFP machine (because the \(K_{\text{max}}^*\) is lower than both \(K_{\text{max}}\) considered here) and that the first maximum natural frequency achieved is near the one obtained considering only rectilinear fibers. This last affirmation can be proved by comparing the first natural frequency values obtained in table 6 with the one considering only non steered fibers in table 5 for a fully simply

### Table 3: Prepreg and resin properties [17].

| Property | \(E_1\) (GPa) | \(E_2\) (GPa) | \(G_{12}\) (GPa) | \(G_{13}\) (GPa) | \(G_{23}\) (GPa) | \(\nu_{12}\) | \(\rho\) (kg/m³) |
|----------|---------------|---------------|-----------------|-----------------|-----------------|-------------|--------------|
| Prepreg  | 143           | 9.1           | 4.82            | 4.9             | 4.9             | 0.3         | 1500         |
| Resin    | 3.72          | 3.72          | 1.43            | 1.43            | 1.43            | 0.3         | 1100         |

### Table 4: Verification of the Defect Layer Method [17] - fundamental frequency \((f_1[Hz])\).

| \(a/h\)     | Layup       | Manufacturing defects | FSĐT (\(c_f = 5/6\)) [17] | Results | Difference (%) |
|-------------|-------------|-----------------------|----------------------------|---------|----------------|
| 200 \([±58°39']_4s\) | Defect-free | 16.4566               | 16.2358                    | 1.34    | 0.02           |
|              | Complete gap| 15.7916               | 15.7874                    |         |                |

### Table 5: Prepreg and resin properties [17].

| Property | \(E_1\) (GPa) | \(E_2\) (GPa) | \(G_{12}\) (GPa) | \(G_{13}\) (GPa) | \(G_{23}\) (GPa) | \(\nu_{12}\) | \(\rho\) (kg/m³) |
|----------|---------------|---------------|-----------------|-----------------|-----------------|-------------|--------------|
| Prepreg  | 143           | 9.1           | 4.82            | 4.9             | 4.9             | 0.3         | 1500         |
| Resin    | 3.72          | 3.72          | 1.43            | 1.43            | 1.43            | 0.3         | 1100         |
supported boundary condition. The respective increase of the first natural frequency caused by the introduction of steered fibers is only 0.02%. So, it can be concluded that the use of non steered fibers is almost as efficient as steered fibers in order to maximize the first natural frequency for this boundary condition.

Focusing now on the plates with \( AR = 0.5 \) (visible in figure 3), columns one and three of table 7, it is observable that the orientation of each ply is near \( 0^\circ \) for the fully clamped boundary condition. Therefore, the optimal solution found is similar to the one obtained considering only rectilinear fibers (table 5). So, it appears that the use of curvilinear fibers is not justifiable in this case. On the contrary, for the simply supported boundary condition, there is a slight improvement of 1.49% on the optimal solution with the introduction of curvilinear fibers. The influence of the curvature of the fibers is especially relevant because the maximum curvature is an active constraint in the first and last two plies (\( K_{max} = 1.57m^{-1} \)). Regarding the plates with \( AR = 2.0 \) (fifth and seventh columns of table 7), the maximum curvature is also an active constraint for the fully simply supported boundary condition. For this case, the orientation in the middle of all plies (\( T_0 \)) is near \( \pm 90^\circ \), which is also the optimal orientation of all plies for the correspondent non steered case (table 5). The orientation at the edge of the plies is clearly restricted by the maximum curvature constraint, that, as aforementioned, is an active constraint in all plies. Analyzing now the case with the fully clamped boundary condition, it is visible that all plies have almost zero curvature and an orientation near \( \pm 90^\circ \). So, the optimal solution obtained is very similar to the one achieved in the non steered case.

5.3. Complete gap steered optimization results

The same optimization was performed, but considering the complete gaps as manufacturing defects. The optimization was only done for the LS-C-CON-A and LS-S-CON-A, meaning that \( K_{max} = 1.57m^{-1} \) for each layer of the laminate. The results are shown in table 8 and figure 2 for square plates. It is observable that the existence of this manufacturing defect leads to optimal plates with a lower highest fundamental frequency when compared with the defect free steered cases. This conclusion, could also be taken using the data of the study done by Akharzadeh et al. [17], whose results where used to validate the finite element model in section 4. This can be justified by a reduction of the elastic properties and stiffness of the plate in the mesh elements where there are pockets of resin. Focusing now on the fully clamped plate, it is visible that the decrease of the fundamental frequency is only about 0.8% when compared with the defect free steered optimal solution. However, it is worth mentioning that the optimal \( T_0 \) and \( T_1 \) combination changed and that the maximum curvature constraint is now an active constraint in the first three plies and their respective symmetric ones (\( K_{max} = 1.57m^{-1} \)). It is also important to note that although there is a slight reduction in the fundamental frequency caused by the existence of gaps, the fundamental frequency achieved is still approximately 3.7% higher than the one obtained in the non steered optimal plate (table 5). The optimal solution of the simply supported case considering this defects is also lower, but near the one where no defects were considered. The observed decrease is around 0.58% which means that the achieved fundamental frequency is now bellow the one obtained in the optimal non steered laminate (table 5). Therefore, it can be said, for the considered manufacturing process, that it is not advantageous to use curvilinear fibers in the fully simply supported boundary condition.

The rectangular plate results are visible in table 7 considering \( K_{max} = 1.57m^{-1} \) for both boundary conditions and aspect ratios. As it could be expected, the fundamental frequencies achieved are lower than the ones obtained when defects were not considered. Focusing first on the plates with \( AR = 0.5 \) (visible in figure 3), there is a decrease of 0.31% and 0.74% for the LS-C-CON-A and LS-S-CON-A, respectively, with respect to their equivalent cases where complete gaps are not included (table 7). Regarding the fully clamped cases, it is

| Design | \( T_{01}[^\circ] \) | \( T_{12}[^\circ] \) | \( T_{03}[^\circ] \) | \( T_{31}[^\circ] \) | \( f_1[Hz] \) | Gen. |
|--------|----------------|----------------|----------------|----------------|----------------|-----|
| \( a/b = 1.0 \) | NS-C | -89.855 | 0.556 | -89.467 | 0.538 | 51.601 | 55 |
| | NS-S | 44.989 | -44.998 | -44.998 | -44.946 | 31.223 | 55 |
| \( a/b = 2.0 \) | NS-C | -90.000 | 89.992 | 89.915 | -89.472 | 197.443 | 61 |
| | NS-S | -89.857 | 89.890 | -89.765 | -89.528 | 88.275 | 43 |
| \( a/b = 0.5 \) | NS-C | -0.014 | -0.028 | -0.116 | 0.122 | 49.260 | 39 |
| | NS-S | 1.662 | -1.043 | -0.846 | -2.397 | 22.042 | 44 |
observable that the maximum curvature of each ply tends to zero while the constraint is active in some plies in the simply supported boundary condition, as in the respective defect free cases. It is also worth noting that there is a decrease for the fully clamped boundary condition of approximately 0.31% when compared to the optimal solution considering only non steered plies (table 5). As a result it can be said that it is preferable to use rectilinear fibers for this case. On the contrary, when the BC is simply supported there is an increase near 0.74% when compared to the respective optimal NS case (table 5), even taking into consideration the occurrence of gaps.

A similar phenomenon occurs for plates with $AR = 2.0$, however in these cases the frequencies achieved are much lower. The plate’s maximum curvature tends to zero in all plies for the fully clamped boundary condition and it is near the maximum curvature constraint in some plies for the in the simply supported boundary condition (table 7). A reduction in frequency of 0.73% and 2.21% for LS-C-CON-A and B, respectively, in relation to their optimal equivalent defect free cases (table 7). Since their is a reduction of 0.73% in the LS-C-CON-A with respect to its respective optimal non steered case (table 5), the non steered optimal solution is preferable for this boundary condition just like in the previous case. Focusing now on the simply supported boundary condition, there is a slight decrease of 0.14% with respect to its corresponding non steered optimal solution (table 5), although the maximum curvatures especially in plies 2 and 4, and their respective symmetric ones, are near the maximum curvature constraint. Therefore, it can also be said that it is also preferable to use non steered plies for this boundary condition as opposed to the previously case with an $AR = 0.5$. Since the maximum curvature of the plies does not tend to zero, it is possible to speculate that a better solution could be found if the maximum curvature constraint imposed was higher.

6. Conclusions

In this work, an optimization framework is implemented with the objective of maximizing the first natural frequency of composite laminated plates. With that in mind, the optimization is performed considering both non steered and steered fibers in order to evaluate the effects of having curvilinear fibers in the structural performance. On top of that, the occurrence of tow drop gaps inside the composite is also taken into consideration using the Modified Defect Layer Method with the objective of assessing its repercussions in the fundamental frequency. It was revealed that the fibers’ curvature effect on increasing the fundamental frequency depends not only on the boundary condition considered, but also on the plate’s geometry. For example, in square plates it was verified that the use of steered fibers is advantageous when a fully clamped boundary condition is imposed, while the optimal fiber orientation is rectilinear when a fully simply supported boundary condition is imposed. On the contrary, the advantage of using curvilinear fibers was only verified considering the simply supported boundary condition in rectangular plates. Furthermore, the presented results show that having a fully clamped boundary condition leads to higher frequencies when compared to a fully simply supported one. Moreover, they also revealed that increasing the aspect ratio of the plate leads to higher frequencies. As a result, the plate’s fundamental frequency is highly dependent on the imposed boundary condition and on the plate’s geometry. When tow drop defects, namely, gaps were considered, results show that these rich resin areas lead to a reduction of the fundamental frequency, which appears to be a consequence of a reduction of the material’s equivalent elastic properties. Moreover, the fundamental frequency reduction increases with the increase of the volumetric fraction of these defects. Keeping that in mind, it was also shown that the optimal $< T_0, T_1 >$ is dependent not only the fibers curvature but also on the occurrence of gaps inside the laminated plate.

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Table 6: Optimal defect free steered results - square plate.

| Design | LS-C-UN | LS-C-CON | LS-S-UN | LS-S-CON |
|--------|---------|----------|---------|----------|
|        | A       | B        | A       | B        |
| $< T_0^*, \ T^*_1 \geq 1$ | $< 89.985,$ | $< -74.854,$ | $< -81.806,$ | $< -42.833,$ | $< 42.291,$ | $< 42.055,$ |
|        | $-1.905 >$ | $-43.733 >$ | $-28.432 >$ | $-45.874 >$ | $46.043 >$ | $46.043 >$ |
| $< T_0^*, \ T^*_1 \geq 1$ | $< -89.706,$ | $< 63.344,$ | $< 74.941,$ | $< 47.781,$ | $< -47.148,$ | $< -47.974,$ |
|        | $3.670 >$ | $35.667 >$ | $23.651 >$ | $44.003 >$ | $< 43.819 >$ | $< 43.718 >$ |
| $< T_0^*, \ T^*_1 \geq 1$ | $< -89.995,$ | $< -89.833,$ | $< 87.103,$ | $< 47.922,$ | $< -48.488,$ | $< -49.109,$ |
|        | $3.583 >$ | $-89.236 >$ | $32.428 >$ | $43.881 >$ | $< 43.859 >$ | $< 43.805 >$ |
| $< T_0^*, \ T^*_1 \geq 1$ | $< -5.374,$ | $< 80.938,$ | $< 1.993,$ | $42.698,$ | $-49.943,$ | $-47.316,$ |
|        | $-5.871 >$ | $48.178 >$ | $2.639 >$ | $45.290 >$ | $< 42.988 >$ | $< 43.892 >$ |
| $f^*_1 (Hz)$ | 59.471 | 53.962 | 56.814 | 31.229 | 31.229 | 31.229 |

Table 7: Optimal steered results for rectangular plates.

| Design | LS-C-CON A | LS-S-CON A | LS-C-CON A | LS-S-CON A |
|--------|------------|------------|------------|------------|
|        | Free | Gap | Free | Gap | Free | Gap | Free | Gap |
| $< T_0^*, \ T^*_1 \geq 1$ | $2.70e^{-2},$ | $-0.263 <$ | $0.122, 1.159$ | $< 89.936,$ | $87.401,$ | $< -89.888,$ | $< -84.085,$ |
|        | $0.343 >$ | $1.001$ | $22.606 >$ | $-23.472$ | $89.965 >$ | $87.300 >$ | $-52.872 >$ | $-59.169 >$ |
| $< T_0^*, \ T^*_1 \geq 1$ | $< 0.243,$ | $1.027$ | $< -5.406,$ | $-1.106$ | $< -89.551,$ | $< -84.935,$ | $< 89.970,$ | $< 84.898,$ |
|        | $6.11e^{-2} >$ | $-0.354 <$ | $-27.968 >$ | $-23.636$ | $89.522 >$ | $81.697 >$ | $52.907 >$ | $51.358 >$ |
| $< T_0^*, \ T^*_1 \geq 1$ | $< 1.36,$ | $0.034$ | $< 2.524,$ | $-6.053$ | $< 89.180,$ | $87.905,$ | $< 88.755,$ | $< 86.181,$ |
|        | $-0.542 >$ | $-2.12$ | $-17.068 >$ | $-28.554$ | $88.592 >$ | $86.896$ | $52.156 >$ | $51.154 >$ |
| $< T_0^*, \ T^*_1 \geq 1$ | $2.680 >$ | $-3.972$ | $32.586 >$ | $29.987$ | $88.290 >$ | $83.190 >$ | $52.876 >$ | $65.530 >$ |
| $f^*_1 (Hz)$ | 49.258 | 49.107 | 22.370 | 22.205 | 197.436 | 195.988 | 90.146 | 88.151 |
| $v_G [%]$ | 0 | 3.078 | 0 | 3.77 | 0 | 0.96 | 0 | 1.87 |
| Gen. | 59 | 68 | 66 | 62 | 82 | 84 | 117 | 91 |

Table 8: Optimal steered results with complete gaps - square plate.

| Design | LS-C-CON | LS-S-CON |
|--------|----------|----------|
|        | A       | B        | A       | B        |
| $< T_0^*, \ T^*_1 \geq 1$ | $< 66.049,$ | $< -53.852,$ | $< -41.182,$ | $< 47.580,$ |
|        | $37.649 >$ | $-6.606 >$ | $-46.913 >$ | $47.836 >$ |
| $< T_0^*, \ T^*_1 \geq 1$ | $< -56.187,$ | $< 65.293,$ | $< 44.730,$ | $< -51.157,$ |
|        | $-30.178 >$ | $16.399 >$ | $44.857 >$ | $-41.094 >$ |
| $< T_0^*, \ T^*_1 \geq 1$ | $< -36.489,$ | $< 80.028,$ | $< 52.273,$ | $< -43.176,$ |
|        | $-13.389 >$ | $27.309 >$ | $41.830 >$ | $-45.536 >$ |
| $< T_0^*, \ T^*_1 \geq 1$ | $< 84.566,$ | $< 55.145,$ | $< -39.640,$ | $< 88.990,$ |
|        | $-13.433 >$ | $8.204 >$ | $-39.683 >$ | $33.960 >$ |
| $f^*_1 (Hz)$ | 53.517 | 55.311 | 31.047 | 31.080 |
| $v_G [%]$ | 1.88 | 2.21 | 1.64 | 2.24 |
| Gen. | 122 | 75 | 83 | 88 |
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