The optimal calculation and machining of the spiral bevel gear pinion based on the same cutter head

Jing DENG*, Chuang JIANG*, Xiaozhong DENG*, Mingzhu ZHANG* and Shaowu NIE*

* Henan Collaborative Innovation Center of Machinery Equipment Advanced Manufacturing,
Henan University of Science and Technology, Luoyang, Henan Province,
48 Xiyuan Rd, Luoyang 471003, China
E-mail: dxz01@163.com

Received: 24 May 2019; Revised: 24 September 2019; Accepted: 16 March 2020

Abstract
In this paper, the method of machining and correcting the two sides of tooth surface of spiral bevel gear pinion with the same cutter head and machine tool is studied. In this method, the cutting procedure and tool path are planned, and the calculation of motion parameters is completed. According to the characteristic that NC machine tool can adjust cutter location in real time, the tooth length and curvature are corrected, and the calculation model of tooth surface deviation is constructed. Five-cutter is used as the target tooth surface to optimize the cutting parameters. An example is introduced to carry out the analysis and gear cutting experiments, which verifies the feasibility of the proposed method.

Keywords: The same cutter, Spiral bevel gear, The curvature correction, Tooth surface comparison, The optimization of cutting parameters

1. Introduction

Spiral bevel gears are widely applied in all kinds of mechanical transmission systems for their upgrade performance in meshing and transmission. At present, there are two main machining methods: Five-cutter cutting method and completed cutting method. Corresponding to the face milling technology, five-cutter cutting method requires five steps to complete gear and pinion rough machining and finish machining, which namely are gear rough and finish cut, pinion rough cut, pinion convex and concave surfaces finish cut. The completed cutting method, corresponding to the face hobbing technology, is the machining of gear pairs are adopted double-surface method and do not need to be divided into rough cutting and finish cutting, gear and pinion need only one cutting to be completed. The Five-cutter started earlier, and relevant scholars have done a lot of studies. Litvin et al., (1991,1994) proposed local Synthesis method to realize the local control of tooth surface. Based on the overall optimization design of the local synthesis method, Fang et al., (2000) comprehensively analyzed the contact area and transmission error curve formed by the gear pair at different positions on the gear surface on the basis of pre-controlling the second-order contact parameters of the gear surface reference meshing point. Wang et al., (2006) realized the control of fourth-order transmission error curve by using the method of hobbing ratio correction. In CNC machine tool, the curvature of tooth length direction is corrected by the control of cutter location to reduce the mounting error sensitivity. Based on the independently developed NC machine tool, the tilt method of spiral bevel gear pinion is applied. Cao et al., (2008) used the active design method of gear surface to control independently the gear surface impression and the geometric transmission error, and achieved the whole gear surface control of the gear surface contact performance. Tang et al., (2009)studied the design of point engagement tooth surface of spiral bevel gear based on predetermined engagement characteristics, strictly limiting the contact trace and the parameters below the third order and the tooth surface structure nearby. Simon et al.,(2009) proposed to optimize the 1-5 order Roll ratio coefficient, used a cutter with bicircular profile and optimized cutter parameters and machining parameters to reduce mounting error sensitivity and improve load distribution. Based on the conjugate tooth surface, Fan et al.,(2010) realized higher order tooth flank form
error correction by controlling the high order motion parameters of the gear in machining. Jiang et al., (2016) achieved the transformation of the motion axis, and then verified the correctness of the calculation by experiments. Mu et al.,(2017) obtained the tooth surface of pinion through the preset seventh-order transmission error curve, and proposed a method to correct the curvature of the tooth surface by correcting some cutting parameters. Nie et al., (2018) established the mapping relationship between the free-controlled tooth surface deviation of pinion and the cutting parameters of machine tools by means of the surface of the second order.

However, due to the large number of procedures, low machining efficiency and difficulty in ensuring accuracy (mainly reflected in the fact that the consistency of gear pair backlash cannot be guaranteed in batch production) of Five-cutter, more and more scholars have carried out research on the Completed cutting method. Wang et al. (1995) deduced the machining parameters of spiral bevel gear by two sides cutting method based on Gleason data. loaded tooth contact analysis. Fan Q et al.,(2006) computerized modeling of Gleason equal-height spiral bevel gear and studied the meshing simulation. Based on the mechanism of face hobbing, Zhang et al., (2007) studied the movement and location relationship between the cutter and the work piece, and established the mathematical model of face hobbing technology. M. Vimercati et al., (2007) established the mathematical model of tilting method of Gleason equal-height spiral bevel gear, and studied the gear modeling, tooth surface contact analysis and stress calculation. Shi et al., (2007) established a unified mathematical model for spiral bevel gear, which is suitable for both bevel gear face milling and bevel gear face hobbing. NISHINO et al., (2009) described the generation of tooth surface mathematically based on the movement of machine tool in end hobbing, and carried out the tooth contact analysis, Stadtfeld et al. (2015) described the two sides cutting method of face hobbing. On this basis, Zhang et al., (2016) put forward the method of setting three reference points, and combined ease-off technology and cutter profile correction technology, the theoretical system of cutting spiral bevel gear by duplex spread blade was established.

The gear cutting efficiency of Completed cutting method is very high because the gear and pinion are machined on both sides. However, because the same set of cutting parameters is used on the double teeth surfaces, the calculation is very complicated and the meshing performance cannot always reach the ideal state. In addition, the method puts forward extremely high requirements on machine tools and cutters, which greatly increases the cost of making teeth.

With the development of numerical control technology, there are more ways to machine spiral bevel gear. In this paper, the same cutter and two sets of parameters will be used to cut the two tooth surfaces of spiral bevel gear pinion on the same machine tool. This method does not need to be repeatedly loaded, and the adjustment calculation is simple. It can get rid of the dependence on high-end machine tools and cutting tools, thus reducing the production cost.

2. Mathematical model of milling tooth machining
2.1 The description of cutting motion

The Completed cutting method is used to machine the spiral bevel gear pinion. The same machine tool, the same cutter and the same set of cutting parameters are used to machine two sides of the gear tooth at the same time. Different from the Completed cutting method, this paper uses the same machine tool, cutter and two sets of cutting parameters to machine the convex and concave sides of gear tooth respectively.

In machining, first of all, the cutting parameters of the convex side are selected. The generating gear rotates counter-clockwise and the workpiece rotates clockwise. The cutter gradually develops from the toe of the convex side to the tip until it withdraws from meshing. The machine tool switches to the concave cutting parameters and adjusts the
relative position of the cutter and the workpiece automatically. On the contrary, the rotating direction of the generating gear and the workpiece is opposite to that when the convex side is machined. The cutter gradually develops from the tip of the concave side to the toe until the concave side withdraws from meshing. Then the workpiece is indexed and the process is repeated to realize the complete machining of spiral bevel gear pinion. The sketch diagram of the movement track of the gear cutting is shown in Fig. 1.

2.2 The calculation of movement parameters

The model of cutter head is built firstly as Fig. 2. \( S_p \) is the cutting tool coordinate system, \( r_o \) is the cutting tool radius, \( W_t \) is the tool tip distance, \( u_p, \theta_p \) are the surface coordinates.

![The cutting tool model](image)

The cutting edge formulation in the cutting tool coordinate system \( S_p \) is written as

\[
r_p(u_p, \theta_p) = \begin{bmatrix} (r_o \pm u_p \sin \alpha_p) \cos \theta_p \\ (r_o \pm u_p \sin \alpha_p) \sin \theta_p \\ -u_p \cos \alpha_p \end{bmatrix}
\]

(1)

Where the upper symbol in \( \pm \) represents the outer cutting blade, the lower presents the inner cutting blade.

According to the position and motion relationship between the cutter and the workpiece in actual machining, the generating method for machining coordinate system is established as shown in Fig. 3.

![Mathematical Model of Spiral Bevel Gear Machining](image)
In the figure, M-point is the workhead rotation, the center $S_p$ is the cutter head coordinate system, $S_s$ is the coordinate system of virtual generating gear, $S_m$ is machine coordinate system, $S_i$ is pinion coordinate system, $L_p$ is the distance from design intersection point to workhead rotation center, $H$ is horizontal cutter location and $V$ is vertical cutter location, $\gamma_i$ is workpiece mounting angle, $O_iO_s = X_s$ is workhead setting correction, $O_iO_m = X_i$ is sliding base setting correction, $E_i$ is workhead offset.

Let the cutter tooth profile equation and normal vector be $r_i(u_p, \theta_p)$ and $n_i(u_p, \theta_p)$ respectively, and according to the position relationship between cutter and workpiece, the equation of tooth surface and normal vector can be obtained as follows:

$$
\begin{align*}
    r_i(u_p, \theta_p) &= M_{mo} M_{mp} r_i(u_p, \theta_p) \\
    n_i(u_p, \theta_p) &= L_{mi} L_{mp} n_p(u_p, \theta_p)
\end{align*}
$$

In the formula, $(u_p, \theta_p)$ is tool surface coordinate, $M_{im}, M_{mp}$ is coordinate transformation matrix, and $L_{mi}, L_{mp}$ is obtained by removing the last row and column from $M_{im}, M_{mp}$.

In the radius vector from the center of the workhead rotation to the center of the cutter head is

$$
\begin{align*}
    r_{(mo)} &= r_{(mo)} + r_{(o_o)} + r_{(o_o)}
\end{align*}
$$

In the formula

$$
\begin{align*}
    r_{(mo)} &= [-L_p \cos \gamma_i, 0, -L_p \sin \gamma_i] \\
    r_{(o_o)} &= [-X_p \cos \gamma_i, 0, -X_p - X_p \sin \gamma_i] \\
    r_{(o_o)} &= [H, V + E_k, 0]
\end{align*}
$$

If the unit vectors of $S_m$ in three directions are set to be $i, j, k$ respectively in machining, the motion expressions of each axis are as follows

$$
\begin{align*}
    X &= r_{(mo)} \cdot i \\
    Y &= r_{(mo)} \cdot j \\
    Z &= r_{(mo)} \cdot k \\
    A &= i_p (q_s - q_E) \\
    B &= \gamma_i
\end{align*}
$$

In this formula, $i_p$ is the roll ratio of the workpiece to generating gear, $q_s$ and $q_E$ are the initial revolving angle and final revolving angle of generating gear respectively.

### 3. The curvature correction of tooth surface

Tooth surface curvature includes tooth length curvature and tooth depth curvature, the two types of curvature corrections will be discussed separately in this section. The spiral bevel gear is machined by the Five-cutter, and the convex side and concave side of the pinion are machined separately. The radius of the cutter head of the inner cutter is often larger than that of the outer cutter, which is convenient to obtain the ideal contact area. However, when using the same cutter head to cut the two sides of the spiral bevel gear pinion, the contact area is relatively short, and the contact area of the tooth surface is difficult to meet the design requirements, especially for the spiral bevel gear with a transmission ratio of 1. Therefore, it is necessary to correct the curvature of the tooth length direction.

Generally speaking, in the process of machining spiral bevel gear, the radial cutter location is fixed, and different radial cutter locations correspond to different spiral angles. Using NC technology to cut spiral bevel gear, the interpolation motion of two linear axes replaces the cradle, which can realize flexible real-time control of the radial cutter location. It can not only adjust the spiral angle, but also correct the tooth length curvature.
In this paper, the curvature corresponding to the Five-cutter is taken as the objective curvature when correcting the tooth length curvature. Firstly, the spiral angle of each point on the tooth line is calculated, and then the cutter location of each point is calculated.

### 3.1 The calculation of spiral angle

When spiral bevel gears are machined by generating method, the coordinate of curved surface \( u_p = 0 \) is selected and transformed into a tooth line formed by a spatial curve on the generating gear, and a pitch line corresponds to the curve on the gear. Therefore, the two curves have the same properties. In this paper, the spiral angle of the pitch line of the gear will be calculated by the tooth line of the generating gear.

The sketch of spiral angle calculation is established as shown in Fig.4. \( S_0 \) is the generating gear coordinate system. A point \( M \) is selected randomly on the tooth line of generating gear, and its corresponding spiral angle is \( \beta_s \). \( S_1 \) and \( q_i \) are radial and angular cutter locations in the Five-cutter.

According to the established equation of the tooth surface of the generating gear, the unit radius vector of any point on the tooth line of the generating gear is

\[
r_s = r_p(u_p = 0, \theta_p) / r_p(u_p = 0, \theta_p)
\]  

(6)

The unit radius vector corresponding to the tangent vector is

\[
t_s = \frac{dr_p(u_p = 0, \theta_p)}{d\theta_p} = \frac{dr_p(u_p = 0, \theta_p)}{d\theta_p}
\]  

(7)

According to the characteristics of triangle and the definition of helix angle, there is the following relationship

\[
\cos \beta_s = r_s \cdot t_s
\]  

(8)

In the formula, \( \beta_s \) is the spiral angle corresponding to any point of the tooth surface pitch line. When \( \theta_p \) is the angle 0 corresponding to the midpoint of the pitch line, \( \beta_{s'} \) is the spiral angle corresponding to the midpoint of the tooth surface.

### 3.2 The calculation of cutter location

The curvature of tooth length can be corrected by real-time adjustment of cutter location. The sketch of cutter location adjustment is established as shown in Fig. 5.
In the figure, the solid line corresponds to the track of the Five-cutter and the dashed line corresponds to the new cutting method, which is called the New Method in this paper. \( l_1 \) and \( l_2 \) are the tracks of cutter location in Five-cutter and new method, \( A_1, B_1 \) and \( A_2, B_2 \) are the instantaneous centers of cutter head in two methods respectively, \( \Delta q_1 \) and \( \Delta q_2 \) are the D-values of angular cutter location in two methods corresponding to tooth line point, \( \beta_s \) is the spiral angle corresponding to any point \( M \) of the tooth line.

Based on the same cutter head for machining of spiral bevel gear pinion, the cutter head radius \( r_d \) is a known quantity, and set the radius vector to be \( \mathbf{r} = \mathbf{OM} \). The D-value of cutter head radius between the two methods is \( \Delta r = |A_1A_2| = |B_1B_2| \). In \( \Delta O_1B_2M \), the radial cutter location \( S_r \) is

\[
S_r = \left( R_1^2 + r_d^2 - 2R_1r_d \cos(90° - \beta_s) \right)^{1/2}
\]  

(9)

According to the geometric relationship shown in the figure, the angular cutter location \( q_s \) is

\[
\cos \Delta q = \frac{1}{2S_rS_s} (S_s^2 + S_r^2 - \sqrt{\Delta r})
\]

(10)

\[
q_s = q_1 + \Delta q
\]

(11)

According to the formulas (8) and (10), the radial cutter location and angular cutter location at any point of the tooth line can be obtained.

### 3.3 The curvature correction of tooth depth

The curvature correction of tooth length can be achieved by real-time control of tool location. Relatively speaking, the most common method for the curvature correction of tooth depth is to adjust workhead offset. Therefore, the change of tool location and roll ratio can only be adjusted proportionally. For the specific correction principle and adjustment process, please refer to reference (Beijing Gear Factory, 1974). Setting the workhead offset change is \( \Delta E_b \), the corresponding change of tool location and roll ratio is

\[
\Delta S_i = \Delta E_b \cdot \sin q_i
\]

(12)

\[
\Delta i_{\text{sp}} = \frac{i_{\text{sp}} \cdot \tan \beta_s \cdot \Delta E_b}{E_m}
\]

(13)

In the formula, the spiral angle and cone distance of the middle point of the gear are \( \beta_s \) and \( E_m \) respectively.

For the curvature correction of tooth depth, the following section of the optimization of cutting parameters is also based on this principle to achieve the full tooth surface correction.
3.4 The calculation of tooth surface deviation

By comparing the Five-cutter with the New Method, the error between the two tooth surfaces can be analyzed by quantitative analysis, which provides a basis for further optimal calculation of tooth surfaces.

When the New Method and Five-cutter are used, the cutting parameters and the cutter head parameters are different, and the corresponding tooth surfaces will inevitably be in different spatial locations. In order to make the two tooth surfaces coincide at the midpoint of the tooth surface, it is necessary to rotate the tooth surface machined by the New Method and establish the location relationship between the two tooth surfaces as shown in Fig. 6.

![Fig. 6 Location Relationship between Two Tooth Surfaces](image)

In the figure, \( M_r(x_r, y_r, z_r) \) and \( M_s(x_s, y_s, z_s) \) are the midpoints of the tooth surface by the New Method and by the Five-cutter respectively. \( \delta \) is the angle that need to be rotated by the New Method, there is the following relationship

\[
r_e = M_e r_s
\]

In the formula,

\[
M_e = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \Delta \delta & -\sin \Delta \delta & 0 \\
0 & \sin \Delta \delta & \cos \Delta \delta & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} (15)
\]

Then

\[
\Delta \delta = \arcsin(y_e \cos(\arctan(y_e/z_e))/z_e) - \arctan(y_e/z_e) (16)
\]

After the tooth surface is rotated by \( \Delta \delta \) the grid point coordinates after rotation can be obtained according to the matrix coordinate transformation. The location relationship between the New Method and the Five-cutter of tooth surface is shown in Fig. 7. At this time, the two tooth surfaces coincide at the midpoint \( P \) of the tooth surface. When calculating the tooth surface points of the New Method, the grid partition of the tooth surface is \( m \times n \), and any grid point \( p_i(x_i(u_i, \phi_i), y_i(u_i, \phi_i), z_i(u_i, \phi_i)) \) is taken on the tooth surface by the New Method after rotation. Make a perpendicular line to the Five-cutter’s tooth surface through this point and intersect with the Five-cutter’s tooth surface at the point \( P_2(x_2(u_2, \phi_2), y_2(u_2, \phi_2), z_2(u_2, \phi_2)) \). The straight line \( P_1P_2 \) coincides with the normal line \( n_{w_2} \) of the Five-cutter’s tooth surface at point \( P_2 \). Set the coordinate of normal \( n_{w_2} \) as \((x_{w_2}(u_2, \phi_2), y_{w_2}(u_2, \phi_2), z_{w_2}(u_2, \phi_2)) \) and define \( \Delta \varepsilon = |P_1P_2| \) as the deviation between the two tooth surfaces.
There is the following relationship

\[
\begin{align*}
    x_1(u_1, \varphi_1) + \Delta e \cdot x_{2}(u_2, \varphi_2) &= x_1(u_1, \varphi_1) \\
    y_1(u_1, \varphi_1) + \Delta e \cdot y_{2}(u_2, \varphi_2) &= y_1(u_1, \varphi_1) \\
    z_1(u_1, \varphi_1) + \Delta e \cdot z_{2}(u_2, \varphi_2) &= z_1(u_1, \varphi_1)
\end{align*}
\tag{17}
\]

In the formula, \( u_1 \) and \( \varphi_1 \) are known quantities, and \( u_2, \varphi_2 \) and \( \Delta e \) are unknown quantities. The above formula is a non-linear equation group with three variables and three equations, so the tooth surface deviation corresponding to each grid point of the tooth surface can be solved by the New Method.

4. The optimization of machining parameters and example analysis
4.1 Parameter optimization

Through the real-time adjustment of cutter, the curvature correction of tooth length is completed. Due to the difference of the cutting parameters and the cutter head parameters between the New Method and the Five-cutter, there is still a deviation between the two tooth surfaces. When the cutter head parameters are fixed, the deviation between the two tooth surfaces can be reduced and the cutting performance of the New Method can be improved by correcting other cutting parameters besides the cutter location.

According to the definition of tooth surface deviation, the deviation at any point of tooth surface satisfies the following relationship:

\[
\Delta e(u_i, \varphi_i, \Delta \pi_i) = (x_i^2 - x_i^2)\pi_{i\text{a}} + (y_i^2 - y_i^2)\pi_{i\text{b}} + (z_i^2 - z_i^2)\pi_{i\text{c}}
\tag{18}
\]

In

\[
\Delta \pi_i = (X_1, E_1, X_2, \gamma_1, 1, p)
\tag{19}
\]

In the formula, the superscript \( i = 1 \) represents the new cutting method, the superscript \( i = 2 \) represents the Five-cutter of cutting tooth surface, the subscript \( k = 1, 2, \cdots, N \) represents the sequence number of grid points, and the subscript \( N \) represents the total number of grid points.

Taking \( u_i, \varphi_i, \Delta \pi_i \) as the design variable and the sum of the errors of each grid point on the tooth surface as the objective function, an optimization and correction model of cutting parameters can be constructed as

\[
\begin{align*}
    \min f &= \sum_{i=1}^{N} |\Delta e_i(u_i, \varphi_i, \Delta \pi_i)|^2 \\
    \text{s.t.} \quad \min(\Delta \pi_i) &\leq \pi_i \leq \max(\pi_i) \quad (k = 1, 2, \cdots, N)
\end{align*}
\tag{20}
\]

In the formula, \( \min(\Delta \pi_i) \) and \( \max(\Delta \pi_i) \) are the minimum and maximum allowable trimming values for every parameter.
Based on the optimization and correction model, the cutting parameters are optimized and corrected by Sequential Quadratic Programming (SQP), and the optimal calculation process is established as shown in Fig. 8.

![Flow Chart of Motion Coefficient Correction Calculation](image)

**Fig. 8** Flow Chart of Motion Coefficient Correction Calculation

### 4.2 Example analysis

Based on the above theory, an example is introduced to analyze the process of the curvature correction of tooth length and optimize cutting parameters of spiral bevel gear. Because it is more difficult to adjust the contact area of gear pair with transmission ratio of 1, this paper takes 17/17 gear pair as an example to analyze, and its basic geometric parameters are shown in Tab. 1.

| Name                                | Gear       | Pinion    |
|-------------------------------------|------------|-----------|
| Number of teeth                     | 17         | 17        |
| Direction                           | Right      | Left      |
| Axial angle/(°)                     | 90         | 90        |
| Normal modulus of reference point/(mm) | 8.034     | 8.034     |
| O.D./(mm)                           | 146.24     | 146.24    |
| Whole depth/(mm)                    | 15.17      | 15.17     |
| Addendum/(mm)                       | 6.83       | 6.83      |
| Dedendum/(mm)                       | 8.34       | 8.34      |
| Spiral angle of reference point/(°) | 35         | 35        |
| Pitch angle/(°)                     | 45         | 45        |
| Face angle/(°)                      | 49.93      | 49.93     |
| Root angle/(°)                      | 40.06      | 40.06     |
| Outer cone distance/(mm)            | 96.58      | 96.58     |
| Tooth width/(mm)                    | 35         | 35        |
Based on the same cutter head, the spiral bevel gear pinion is cut, and the concave side adopts the same data as the Five-cutter. The diameter of convex cutter head is 2 mm smaller than that of concave cutter head. The other parameters are calculated separately. The machining parameters and cutter parameters of the two methods are shown in Tab. 2, and the cutter location changes of the New Method are shown in Fig. 9.

Table 2  Cutter Parameters and Machining Parameters

| Parameters              | The Five-cutter | The New Method |
|-------------------------|-----------------|----------------|
|                         | Convex side     | Concave side   | Convex side     | Concave side   |
| Cutter diameter/(mm)    | 196.99          | 183.36         | 181.36          | 183.36         |
| Pressure angle /(°)     | 22              | 16             | 22              | 16             |
| Radial location/(mm)    | 80.26           | 82.57          | --              | 82.57          |
| Angular location/(°)    | -71.44          | -74.22         | --              | -74.22         |
| Roll ratio              | 1.3805          | 1.4329         | 1.4877          | 1.4329         |
| Vertical offset /(mm)   | -3.32           | 3.087          | 2.94            | 3.087          |
| Machine center to back /(mm) | 1.12       | -1.58          | 0.44            | -1.58          |
| Sliding base /(mm)      | -0.72           | 1.02           | 0.29            | 1.02           |
| Mount angle /(°)        | 40.06           | 40.06          | 40.06           | 40.06          |

Fig. 9  Comparison of the Cutter Location Change

Based on the curvature correction technology of tooth length proposed in this paper, the New Method is adopted to machine the convex side of the pinion. The cutter head is located in the fourth quadrant of the virtual cradle. During generation, the machining area moves from toe to tip. As can be seen from Fig. 9(a), with the rotation of the cradle, the radial cutter location gradually decreases from toe to tip, and the radial cutter location at the toe is larger than that at the Five-cutter, while the radial cutter location at the tip is smaller than that at the Five-cutter. From Fig. 9 (b), it can be seen that the angular cutter location increases gradually during the machining and tends to be gentle near the tip. In the whole process, the angular cutter location of the New Method is always larger than the radial cutter location of the Five-cutter.

According to the theory of tooth surface deviation calculation, the Five-cutter tooth surface and curvature correction tooth surface are calculated respectively. Take the Five-cutter tooth surface as the target tooth surface, and then calculate the deviation between the two tooth surfaces. The result is shown in Fig. 10.
From Fig. 10, it can be seen that there are deviations of tooth length curvature, tooth depth curvature and torsion deviation between the Five-cutter tooth surface and the curvature correction tooth surface. The maximum upper deviation is 0.41 mm, and the maximum lower deviation is 0.15 mm.

According to the theory of optimization and correction of cutting parameters, the vertical offset, machine center to back, sliding base, roll ratio and mount angle of the New Method are corrected. The corrected parameters are shown in Tab. 3, and the corrected tooth surface deviation is shown in Fig. 10.

| Name                  | After correction | corrections |
|-----------------------|------------------|-------------|
| Machine center to back/(mm) | 2.07            | 3.65        |
| Vertical offset/(mm)  | 4.56             | 1.47        |
| Sliding base/(mm)     | 0.24             | 0.24        |
| Roll ratio            | 1.3973           | -0.0356     |
| Mount angle/(°)       | 40.15            | 0.09        |

According to Fig. 11, after optimizing the cutting parameters, the maximum upper deviation is 0.029 mm and the maximum lower deviation is 0.013 mm compared with the Five-cutter. Compared with the results before correction, the tooth surface deviation is effectively reduced.

Based on the above calculation results, the tooth surface contact marks (the concave side of gear and the convex side of pinion) and transmission errors of the Five-cutter and the New Method are calculated respectively, and the results are shown in Fig. 12.
According to Fig. 12, the contact areas of the two methods are both in the form of inner opposite angle. Compared with the Five-cutter, the angle between the contact trace of tooth surface and the root of tooth is too large when the New Method used, and the transmission error amplitude decreases slightly. It should be noted that the result is only valid for this example, and different examples need different calculation and analysis.

Through the whole analysis, it can be seen that the meshing performance of gear pair can be effectively improved by using the same cutter head to machine the tooth surface of spiral bevel gear based on the method proposed in this paper.

5. Experimental verification and discussion

Based on the results of theoretical analysis, the tooth cutting experiment is carried out on the gear pair by adopting the optimized cutting parameters and the curvature correction technology of tooth length. The whole tooth cutting process is carried out on a domestic 4-axis CNC Spiral Bevel Gear Machine. According to the planned tooth cutting procedures, firstly, the convex side of pinion is machined from toe to tip, and the location relationship between the cutter and the workpiece is adjusted to complete the cutting of the concave side. An instantaneous state in the experimental procedures is shown in Fig. 13. After completed, the measurement of the pinion is conducted on Gleason 650 GMS measuring machine, and the five cutter milled surface is used as the reference tooth surface during the measurement. The measurement state and its corresponding results are given in Fig. 14 and Fig. 15.
As illustrated in Fig.15(a) and by contrast with the actual tooth surface and the theoretical tooth surface, the largest error of convex is 0.020mm, which exists in addendum of heel. Besides, the biggest error of concave is 0.027mm, existing in dedendum of outer. Both the pressure angle and the helix angle are all well agreed. As illustrated in Fig.15(b), the index of both convex and concave surfaces reaches grade 6 by using the method proposed in this paper, and thus the machining accuracy can be verified.

In the experiment, the five-cutter cutting method and the method proposed in this paper are adopted to cut the pinions respectively. The time used in each stage is shown in Tab. 4.

|                      | Roughing | Changing procedure | Concave finishing | Changing procedure | Concave finishing | Total |
|----------------------|----------|--------------------|-------------------|--------------------|-------------------|-------|
| Five-cutter          | 10       | 0.66               | 8                 | 0.66               | 8                 | 27.32 |
| Method in this paper | 10       | 0.66               | 8                 | 0                  | 8                 | 26.66 |

Referring to Tab. 4, compared with the five-cutter cutting method, the improvement of the cutting efficiency of the method proposed in this paper is mainly reflected in the fact that it is not necessary to change the machine tool when finish cutting the two tooth surfaces of the pinion. The total machining time is reduced by 2.4% with the new method. However, the machining accuracy is improved despite the fact that the machining time is almost the same.
The difference between the five-cutter cutting method and the method proposed in this paper on gear accuracy is mainly reflected in the gear pair backlash. Secondary installation exists in the process of gear pair machining with the five-cutter cutting method, and the consistency of the gear pair backlash is not easy to be guaranteed. The same problem does not exist in the approach presented in this paper. In order to compare the influence on the backlash, 20 sets of gear pairs are machined by two methods respectively. Based on the 0.22mm backlash, the number of gear pairs within the corresponding range is counted, and the specific statistical situation is shown in Tab. 5.

| Tolerance of gear pair backlash(mm) | 0~±0.005 | ±0.005~±0.010 | ±0.010~±0.015 | ±0.015~±0.020 |
|-------------------------------------|-----------|---------------|---------------|---------------|
| Number of gear pairs                | Five-cutter Method in this paper | 4 | 5 | 7 | 4 |
|                                     | 17 | 3 | 0 | 0 |

As shown in Tab. 5, there are 4, 5, 7, 4 pairs of gears distributed in the backlash tolerance range of 0~±0.005 mm, ±0.005 mm~±0.010 mm, ±0.010 mm~±0.015 mm, ±0.015 mm~±0.020 mm with the five-cutter cutting method, and the maximum tolerance is 0.04 mm. When machining with the method mentioned in this paper, there are 17 pairs of gears distributed in the backlash tolerance range of 0~±0.005 mm, only 3 pairs of gears distributed in the backlash tolerance range of ±0.005 mm~±0.010 mm, and the maximum tolerance is 0.02 mm. Therefore, the conclusion that the consistency of backlash of the proposed method is better than that of the five-cutter cutting method can be obtained.

To further verify the meshing performance, rolling test is carried out subsequently. Because the cutting parameters of the concave side of pinion are the same as those of the Five-cutter, the meshing area of the convex side is only analyzed here. The final result of meshing mark test of the convex side is shown in Fig. 16.

As can be seen from Fig. 15, in the concave side of the driven gear (ring gear), the contact area is located in the middle of the face width and presents the form of inner opposite angle, which is basically consistent with the theoretical calculation results and verifies the accuracy of the new method proposed in this paper.

The authors anticipate providing a method for machining of spiral bevel gear. However, this approach has some limitations, it is only proved to be applicable to spiral bevel gears(with no offset) at present, but it cannot be proved to be applicable to hypoid gears(with offset), further study is needed in the next step, and the scope of use also needs to be further studied.

6. Conclusion

It is feasible to use the same cutter head to cut and correct the tooth surfaces of spiral bevel gear pinion. According to the location relationship between the cutter and the workpiece, the curvature of tooth length can be corrected by adjusting the cutter location, the tooth surface used by Five-cutter cutting method as the target tooth surface, and the cutting parameters can be optimized by using the method proposed in this paper. On this basis, through numerical example analysis and tooth cutting experiment, it is verified that the two sides of spiral bevel gear pinion can be machined with the same cutter head to achieve a good contact performance, which provides a reference for the efficient cutting of spiral bevel gear pinion.

[DOI: 10.1299/jamdsm.2020jamdsm0045] © 2020 The Japan Society of Mechanical Engineers
Acknowledgments

The authors are grateful to the National Natural Science Foundation Council of China, This project was performed under the Grants No. 51705135 and No.51475141.

References

Beijing Gear Factory. Spiral Bevel Gear(1974), p. 321, Science Press.
Cao X. M., Fang Z. D. Function-oriented Active Tooth Surface Design of Spiral Bevel Gears and Experimental Tests. Journal of Mechanical Engineering, Vol. 44, No. 7 (2008), pp. 209 -214.
Fan Q. Computerized modeling and simulation of spiral bevel and hypoid gears manufactured by Gleason face hobbing process. Journal of Mechanical Design, Vol. 128 (2006), pp. 1315-132.
Fan Q. Tooth surface error corrections for face-hobbed hypoid gears. Journal of Mechanical Design, Vol. 129 (2010), pp. 31–37.
H.J. Stadtfeld. The basics of Gleason face hobbing. Gleason Works, (2016), pp. 40-46.
Jiang C., Deng J., Deng X. Z., Zhang H., Nie S. W. and Geng L. L.. Study of motional parameters calculation and tooth cutting experiment based on the new type of tilt milling machine, Journal of Advanced Mechanical Design Systems and Manufacturing, Vol. 10, No. 4 (2016), pp.1–19.
Litvin F L. Gear Geometry and Applied Theory(1994), p. 99, Prentice Hall.
Litvin F L, A.G. Wang. Local Synthesis and Tooth Contact Analysis of Face –Milled, Uniform Tooth Height Spiral bevel gear(1991) , p. 52, NASA Lewis Research Center.
M. Vimercati. Mathematical model for tooth surfaces representation of face-hobbed hypoid gears and its application to contact analysis and stress calculation. Mechanism and Machine Theory, Vol. 42 (2007), pp. 668-690.
Mu Y. M. , Fang Z.. An ease-off flank correction method for high contact ratio spiral bevel gear with modified curvature motion, Journal of Advanced Mechanical Design Systems and Manufacturing, Vol. 11, No. 3 (2017), pp.1-12.
NISHINO, Takayuki. Computerized Modeling and Loaded Tooth Contact Analysis of Hypoid Gears Manufactured by Face Hobbing Process, Journal of Advanced Mechanical Design, Systems, and Manufacturing, Vol. 3, No. 3 (2009), pp. 224-235.
Nie S. W., Deng J., Deng X. Z., Li T. X. and Li J. B.. A flank correction method for spiral bevel gear based on mismatch topography adjustment, Journal of Advanced Mechanical Design Systems and Manufacturing, Vol. 12, No. 2 (2018), pp.1–15.
Shih Y P, Fong Z H, Lin C Y. Mathematical model for a universal face hobbing hypoid gear generator. Journal of Mechanical Design, Vol. 129, No. 1 (2007), pp. 38-47.
Tian X. B., Fang Z. D. Overall Optimization on Machine-Tool Settings of Spiral Bevel Gears. Journal of Aerospace Power, Vol. 15, No. 1 (2000), pp. 75-77.
V. Simon. Head-cutter for optimal tooth corrections in spiral bevel gear. Mechanism and Machine Theory, Vol. 44 (2009), pp. 1420-1435.
Wang Z., Liu Q. M., Zhang D. J., et al. The Exact Duplex Helical Method in Generation of Spirial Bevel Gears(I)—The blank design of the spiral bevel gears. Journal of Jilin Forestry University, Vol. 11, No. 2 (1995), pp. 76-79.
Wang Z., Liu Q. M., Li Q. S., et al. The Exact Duplex Helical Method in Generation of the Spiral Bevel Gears(II)—The machine setting of generating the gear of the spiral bevel gears. Journal of Jilin Forestry University, Vol. 11, No. 2 (1995), pp. 80-82.
Wang Z., Liu Q. M., Zhang D. J. The Exact Duplex Helical Generation of the Spiral Bevel Gears(III)—The determination of the calculating point and its curvature of the gear surface of the spiral bevel gears [J]. Journal of Jilin Forestry University, Vol. 11, No. 3 (1995), pp. 138-141.
Wang Z., Liu Q. M., Li Q. S. The Exact Duplex Helical Generation of the Spiral Bevel Gears(IV)—The determination of the calculating point and its curvature of the pinion surface of the spiral bevel gears. Journal of Jilin Forestry University, Vol. 11, No. 3 (1995), pp. 142-145.
Wang Z., Liu Q. M., Sun W. C. The Exact Duplex Helical Generation of the Spiral Bevel Gears(V)—The machine setting generating the pinion of the spiral bevel gears. Journal of Jilin Forestry University, Vol. 11, No. 4 (1995),
Wang P Y, Fong Y H. Fourth-order kinematic synthesis for face-milling spiral bevel gear with modified radial motion correction. Journal of Mechanical Design, Vol. 128 (2006), pp. 457-467.
Zhang Y, Wu Z. Geometry of tooth profile and fillet of face-hobbed spiral bevel gear. Mechanical Engineering, (2007), pp. 181-192.
Zhou K. H., Tang J. Y., Zeng T. CNC Manufacturing Technology of Point Meshing Gear Based on Predetermined Meshing Characteristics. Journal of Mechanical Engineering, Vol. 45, No. 9 (2009), pp. 173-182.
Zhang Y. Research on Methods of Machine-Tool Settings Calculation and Meshing Performance Pre-control for Spiral Bevel and Hypoid Gears Generated by Duplex Helical Method(2016), p.24, Central South University.