Estimating CO₂ emissions using a fractional grey Bernoulli model with time power term

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Abstract
Global warming caused by CO₂ emissions will directly harm the health and quality of life of people. Accurate prediction of CO₂ emissions is highly important for policy-makers to formulate scientific and reasonable low-carbon environmental protection policies. To accurately predict the CO₂ emissions of the world’s major economies, this paper proposes a new fractional grey Bernoulli model (FGBM(1,1,$t^{r/\xi}$)). First, this paper introduces the modeling mechanism and characteristics of the FGBM(1,1,$t^{r/\xi}$) model. The new model can be transformed into other grey prediction models through parameter adjustment, so the new model exhibits high adaptability. Second, this paper employs four carbon emission datasets to establish a grey prediction model, calculates model parameters with three optimization algorithms, adopts two evaluation criteria to evaluate the accuracy of the model results, and selects the optimization algorithm and model results that yield the highest model accuracy, which verifies that the FGBM(1,1,$t^{r/\xi}$) model is more feasible and effective than the other six grey models. Finally, this paper applies the FGBM(1,1,$t^{r/\xi}$) model to predict the CO₂ emissions of the USA, India, Asia Pacific, and the world over the next 5 years. The forecast results reveal that from 2020 to 2024, the CO₂ emissions of India, the Asia Pacific region, and the world will gradually rise, but that in USA will slowly decline over the next 5 years.

Keywords CO₂ emissions · Grey Bernoulli model · Grey wolf optimizer · Particle swarm optimizer · Quantum genetic algorithm · Forecasting

Introduction
Global warming caused by the greenhouse effect is one of the factors that seriously threatens human survival and development, and the increase in CO₂ emissions is considered the main cause of the greenhouse effect. According to the Global Climate 2015–2019 report released by the World Meteorological Organization, the growth rate of CO₂ in the atmosphere from 2015 to 2019 was 18% higher than that during the previous 5 years, and the average temperature was 0.2 °C higher than that during the previous 5 years. Hence, this period encompasses the hottest 5 years on record. British Petroleum (BP) World Energy Statistics (2020) indicated that in 2019, the global CO₂ emissions reached 34,169 million tons. Among the various countries, the USA and India rank second and third worldwide,
respectively, in terms of their CO₂ emissions, and the total CO₂ emissions of these two countries account for 21.8% of the world emissions. The Asia Pacific region accounts for 50.5% of the global CO₂ emissions. Upon entering the twenty-first century, the growth of global CO₂ emissions is soaring. Thus, from 2009 to 2019, the CO₂ emissions in the USA basically remained at approximately 5000 MT, revealing a trend of continuous fluctuation. The CO₂ emissions in India grew the fastest, by 55.4%. The growth rate of the CO₂ emissions in the Asia Pacific region reached 30.4%, which is also one of the fastest growing economies in the world. From the perspective of global CO₂ emissions, although the growth rate is not high, with an average annual growth rate of 1.1%, CO₂ emissions remain on the rise. To achieve the target of the Paris Agreement, more than 20 countries put forward carbon neutrality goals at the 2020 Climate Ambition Summit, and more than 40 countries established new commitments to independently improve national contributions. Therefore, accurate prediction of future CO₂ emission data of the USA, India, Asia Pacific region, and the world can help policy-makers formulate more scientific and reasonable environmental policies to truly achieve the goal of carbon emission reduction.

The existing models for CO₂ emission forecast can be classified into three categories. The first category includes nonlinear intelligent models, such as the least squares support vector machine (Sun and Liu 2016), extreme learning machine (Sun and Sun 2017), generalized regression neural network (Heydari et al. 2019); and improved chicken swarm optimization (ICSO-SVM) model using the ICSO algorithm to optimize support vector machine parameters (Wen and Cao 2020). The second category involves the statistical analysis model, which studies the quantitative relationship between CO₂ emissions and influencing factors and applies the relationship equation to predict CO₂ emissions. These approaches include trend analysis (Köne and Büke 2010), logistic equations (Meng and Niu 2011), improved Gaussian process regression (Fang et al. 2018), panel quantile regression (Zhu et al. 2018), log-average decomposition index (Xu et al. 2019), and comprehensive methods combining multiple regression analysis, input–output techniques, and structural decomposition analysis (Xia et al. 2019). The last category is the grey prediction model, which was first proposed by Deng (1982). The grey prediction model was established founded on a small amount of incomplete information to describe the development trend of objects more accurately. Compared to machine learning–based and statistical prediction methods founded on large data samples, the grey prediction model can realize the simulation and prediction of small data samples. The classical grey model GM(1,1) and its extended grey prediction model are widely used in the fields of energy, environment, and social management (Tsai 2016; Liu et al. 2020; Liu et al. 2021a; Wang et al. 2020b; Liu et al. 2021b).

There are two main kinds of grey single-variable forecasting models: the first-order grey differential model GM(1,1) and the grey Bernoulli model GBM(1,1). Deng first proposed the GBM(1,1) model in 1985; namely, a power exponent was introduced into the differential Bernoulli equation. When the exponent is equal to 2, the model is also referred to as the grey Verhulst model. Chen et al. (2008) proposed the nonlinear grey Bernoulli model NGBM(1,1) for the first time. Compared to the general GM(1,1) model, the NGBM(1,1) model can better reflect the nonlinear growth trend of data series. Chen et al. (2010) proposed a new grey prediction model NGBM(1,1), by optimizing the background value and power index simultaneously. Subsequently, many scholars improved the NGBM(1,1) model from different perspectives; e.g., Pao et al. (2012) proposed an iterative method to optimize the parameters of the NGBM(1,1) model. Wang (2017) optimized the background coefficient and initial conditions and considered the weighted method in analysis. Guo et al. (2016) proposed a new model by combining the self-memory principle of a dynamic system with the NGBM(1,1) model. Ma et al. (2019) constructed the NGBM(1, n) model by combining the GMC(1, n) model and Bernoulli equation. Wu et al. (2019b) extended the first-order accumulation operation and established the FANGBM(1,1) model based on fractional accumulation. Liu and Xie (2019) proposed the establishment of the WBGM(1,1) model by combining the fitting performance of the NGBM(1,1) model with the Weibull cumulative distribution. Şahin (2020) proposed the OFANGBM(1,1) model based on the integral mean value theorem. Wu et al. (2020a) established the NGBM(1,1, k, c) model by combining the NGM(1,1, k, c) model with the NGBM(1,1) model. Jiang and Wu (2021) constructed a nonlinear grey Bernoulli model based on the fractional order reverse accumulation, namely, the FANGBM(1,1) model. At present, a simpler and more convenient conformable fractional cumulative grey model (CFGFM) was proposed (Ma et al., 2020). On this basis, Xie et al. (2020) proposed the conformable fractional grey model in opposite direction (CFGOM). Zheng et al. (2021) proposed the conformable fractional nonhomogeneous Bernoulli model (CFNHBGM (1, 1, K)).

The grey prediction model is widely implemented in CO₂ emission prediction. Lin et al. (2011) applied the grey model to predict CO₂ emissions in Taiwan. Pao et al. (2012) used the NGBM model to predict the CO₂ emissions and real GDP growth of China. Lotfall et al. (2013) forecast CO₂ emissions based on a grey model and an autoregressive integrated moving average (ARIMA)
model. The prediction accuracy of these two methods was compared according to the root mean square error (RMSE), mean absolute error (MAE), and mean absolute percentage error (MAPE). Gao et al. (2015) established a new discrete fractional order cumulation model, i.e., FAGM(1, 1, D), to predict CO₂ emissions. Hamzacebi and Karakurt (2015) used the grey prediction model to predict the energy-related CO₂ emissions in Turkey. Yuan et al. (2017) established a linear programming model reflecting the relationship between the economic development and CO₂ emissions in China and employed the GM(1,1) model to predict the parameters of the planning model. Wang and Ye (2017) developed a nonlinear multivariable grey model to discuss the relationship between economic growth and CO₂ emissions. Xu et al. (2019) combined an adaptive grey model with the buffer rolling method to predict the greenhouse gas emissions in China from 2017 to 2025. Wang and Li (2019) adopted the nonequidistant grey Verhulst model to analyze the relationship between CO₂ emissions and economic growth. Wu et al. (2020b) implemented a conformable fractional nonhomogeneous grey model to predict the CO₂ emissions of BRICS countries. Chiu et al. (2020) proposed a multivariate grey prediction model using neural networks based on feature selection and residual correction to predict China’s carbon emissions. Based on the grey Verhulst model, Duan and Luo (2020) introduced an extrapolation method to optimize the background value and predicted the CO₂ emissions for three coal resources in China. Wang et al. (2020a) established the metabolic nonlinear grey model (MNGM)-ARIMA method, established a new MNGM-BPNN combination model based on the MNGM model and back-propagation (BP) neural network and analyzed the CO₂ emissions in China, the USA, and India with these two methods. Zhou et al. (2021) proposed a method to process original sequence data by means of an average weakening buffer operator based on the grey rolling mechanism of the new information priority principle and predicted the trend of CO₂ emissions in China. Xie et al. (2021) established a new continuous conformable fractional nonlinear grey Bernoulli model to forecast CO₂ emissions from fuel combustion in China.

Scholars have greatly promoted the optimization and application of grey models. However, although the existing studies have optimized the grey model considering the structure or parameters, each optimization method only improves the model performance to a certain extent, and the accuracy remains insufficient. In addition, most models only apply one optimization algorithm to determine the optimal parameters, and research on the application of multiple optimization algorithms is rare. Therefore, to better predict carbon emissions, based on the optimization of existing models, namely, NGBM(1,1) and FAGM(1,1,r²) (Wu et al. 2019a), this paper proposes a new grey prediction model FGBM(1,1,r²) and considers a variety of optimization algorithms to determine the optimal structural parameters of the model. The main contributions of this paper are as follows:

1. Based on the advantages of the grey Bernoulli prediction model NGBM(1,1) and FAGM(1,1, r²), a new model, i.e., FGBM(1,1, r²), is proposed. The new model can be transformed into other grey prediction models by changing its parameters.

2. In the FGBM(1,1, r²) model, application of the trapezoidal integral method to obtain an approximate solution yields errors, but this paper provides an analytical solution of the time response function in the FGBM(1,1, r²) model, and the result is more accurate.

3. Grey wolf optimization (GWO), particle swarm optimization (PSO), and quantum genetic algorithm (QGA) are applied to solve the parameters of the FGBM(1,1, r²) model.

4. The FGBM(1,1, r²) model is used to predict the CO₂ emissions of four countries and regions over the next 5 years.

This paper is organized as follows: the second section first introduces the basic theory of the NGBM(1,1) and FGBM(1,1, r²) models. Second, a new grey prediction model FGBM(1,1, r²) is constructed, which mainly includes the modeling mechanism, model characteristics, and solution method. Then, three common optimization algorithms (GWO, PSO, and QGA) are employed to solve the parameters of the FGBM(1,1, r²) model, and two widely adopted error measures are introduced. The third section considers the CO₂ emission data of four economies to verify the feasibility and effectiveness of the FGBM(1,1, r²) model. The fourth section applies the FGBM(1,1, r²) model to predict the CO₂ emissions of the USA, India, Asia Pacific region and the world over the next five years. The fifth section contains the conclusion of this paper.

Prerequisite knowledge

Grey Bernoulli model NGBM(1,1) and FAGM(1,1, r²)

This section mainly introduces the grey Bernoulli model NGBM(1,1) and the FAGM(1,1, r²) model as follows:

**Definition 1**: Given a nonnegative sequence $X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\}$, $X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)\}$ is referred to as the first-order generating sequence of $X^{(0)}$, where
\[ x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), \, k = 1, 2, \ldots, n. \] Based on Chen et al. (2008), it can be found that the expression of NGBM(1,1) is:

\[ \frac{dx^{(1)}(t)}{dt} + a x^{(1)}(t) = b(x^{(1)}(t))^\gamma \]  

(1)

The above is a nonlinear equation, and the exponent exponent \( \gamma \neq 1 \).

The parameters \( a, b \) of the NGBM(1,1) model are calculated as follows:

\[ (a, b)^T = (B^T B)^{-1} B^T Y \]  

(2)

\[ B = \begin{pmatrix} -z^{(1)}(2) & \cdots & -z^{(1)}(2)^T \\ -z^{(1)}(3) & \cdots & -z^{(1)}(3)^T \\ \vdots & \ddots & \vdots \\ -z^{(1)}(n) & \cdots & -z^{(1)}(n)^T \end{pmatrix}, \quad Y = \begin{pmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{pmatrix} \]

where \( Z^{(1)} = (z^{(1)}(2), z^{(1)}(3), \ldots, z^{(1)}(n)) \), \( z^{(1)}(k) = 0.5x^{(1)}(k-1) + 0.5x^{(1)}(k), \, k = 2, 3, \ldots, n \), and \( n \) is the sample number of the modeling sequence.

The time response function of the NGBM(1,1) model is as follows:

\[ \dot{x}^{(1)}(k) = \left\{ (x^{(0)}(1))^{1-\gamma} - \frac{b}{a} \right\} e^{-\frac{a}{b}(k-1)} + \frac{b}{a}, \, k = 2, 3, \ldots, n \]  

(3)

The predicted values of the model are as follows:

\[ \hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1), \, k = 2, 3, \ldots, n \]  

(4)

Definition 2: Given a nonnegative sequence \( X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\} \, r \in R^+ \), and its \( r \)-th order accumulation sequence is \( X^{(r)} = \{x^{(r)}(1), x^{(r)}(2), \ldots, x^{(r)}(n)\} \) (Wu et al., 2013). Denoted as matrix \( A' \), the r-AGO matrix that satisfies \( X^{(r)} = A'X^{(0)} \) is:

\[ A' = \begin{pmatrix} r & 0 & 0 & \cdots & 0 \\ \frac{r}{1} & r & 0 & \cdots & 0 \\ \frac{r}{2} & \frac{r}{1} & r & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{r}{n-1} & \frac{r}{n-2} & \frac{r}{n-3} & \cdots & r \end{pmatrix} \]  

(5)

with \( \frac{r}{i} = \frac{(r+1)\cdot(r+2)\cdot\ldots\cdot(r+i)}{i!} \). Denoted as matrix \( D' \), the r-IAGO matrix that satisfies \( X^{(0)} = D'X^{(r)} \) is:

\[ D' = \begin{pmatrix} -r & 0 & 0 & \cdots & 0 \\ -r & -r & 0 & \cdots & 0 \\ -r & -r & -r & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -r & -r & -r & \cdots & -r \end{pmatrix} \]  

(6)

Matrices \( A' \) and \( D' \) satisfy \( A'D' = I_n \).

The whitening differential equation of the FAGM(1,1,\( r^* \)) model is:

\[ \frac{dx^{(r^*)}(t)}{dt} + ax^{(r^*)}(t) = bt^\alpha + c, \, r > 0, \, \alpha \geq 0 \]  

(7)

Applying integration to Eq. (7) in the time interval \([k-1, k]\), we obtain:

\[ \int_{k-1}^{k} \frac{dx^{(r^*)}(t)}{dt} dt + a \int_{k-1}^{k} x^{(r^*)}(t) dt = b \int_{k-1}^{k} t^\alpha dt + c \int_{k-1}^{k} dt \]  

(8)

Based on the trapezoidal equation and \( x^{(r^*)}(k) = 0.5x^{(r^*)}(k-1) + 0.5x^{(r^*)}(k), \, k = 2, 3, \ldots, n \), Equation (8) is written as:

\[ x^{(r^*)}(k) - x^{(r^*)}(k-1) + a x^{(r^*)}(k) = b \frac{k^{1+\alpha} - (k-1)^{1+\alpha}}{1+\alpha} + c \]  

(9)

Based on Eq. (9), unknown parameters \( a, b, c \) of FAGM(1,1,\( r^* \)) satisfy:

\[ (\hat{a}, \hat{b}, \hat{c})^T = (B^T B)^{-1} B^T Y \]  

(10)

\[ B = \begin{pmatrix} -z^{(r)}(2) \\ -z^{(r)}(3) \\ \vdots \\ -z^{(r)}(n) \end{pmatrix} \]  

with \( z^{(r)}(i) = \frac{n^{1+\alpha} - (i-1)^{1+\alpha}}{1+\alpha} \). Denoted as matrix \( D'' \), the r-IAGO matrix that satisfies \( X^{(0)} = D''X^{(r^*)} \) is:

\[ D'' = \begin{pmatrix} x^{(r^*)}(2) - x^{(r^*)}(1) \\ x^{(r^*)}(3) - x^{(r^*)}(2) \\ \vdots \\ x^{(r^*)}(n) - x^{(r^*)}(n-1) \end{pmatrix} \]  

(11)

The time response function of the FAGM(1,1,\( r^* \)) model is:

\[ \dot{x}^{(r^*)}(t) = e^{r^* t} \left\{ \left( x^{(0)}(1) + \frac{r^*}{1+\alpha} \right) e^{\alpha t} \right\} + \frac{r^*}{1+\alpha} \]  

(12)

Moreover, the restored value of \( \hat{x}^{(0)}(k), \, k = 2, 3, \ldots, n \) is given as: \( \hat{X}^{(0)} = D'' \hat{X}^{(r^*)} \).
Description of the FGBM(1,1,\(t^\alpha\))

The expression of the FGBM(1,1,\(t^\alpha\)) model is as follows:

\[
\frac{d^\alpha x^{(\alpha)}(t)}{dt^\alpha} + ax^{(\alpha)}(t) = (bt^\alpha + c)(x^{(\alpha)}(t))^{\xi}
\]  

(13)

\[
x^{(\alpha)}(k) = \{\left[x^{(0)}(1)^{1-\xi} - \frac{c'}{\alpha'} \right]e^{-\alpha'(k-1)} + \frac{c'}{\alpha'} + b'e^{-\alpha'(k-1)} \int_1^k \tau^n e^{\alpha'(\tau-1)} d\tau\}^{\frac{1}{\alpha'}} , k = 1, 2, 3 \ldots, n
\]

(14)

The reduction value of \(\tilde{x}^{(\alpha)}(k)\) is \(\tilde{x}^{(0)}(k)\),

\[
\tilde{x}^{(0)}(k) = D^\alpha \tilde{x}^{(\alpha)}(k), k = 1, 2, 3 \ldots, n
\]

(15)

Proof, Multiplying both sides of Eq. (13) by \(x^{(\alpha)}(t)^{-\xi}\). Let 
\(y^{(\alpha)} = [x^{(\alpha)}(t)]^{-\xi}\), one can obtain:

\[
\frac{dy^{(\alpha)}}{dt} + a(1-\xi)y^{(\alpha)}(t) = (bt^\alpha + c)(1-\xi)
\]

(16)

Let the left side of Eq. (13) be 0, \(a(1-\xi) = d', b(1-\xi) = b', c(1-\xi) = c'\), we can get:

\[
C(t) = \int (b't^\alpha + c)e^{d't} dt = C(1) + \int_1^t (b't^\alpha + c)e^{d't} dt = C(1) + b' \int_1^t e^{d't} dt + c'(e^{d't} - e^{d'})
\]

(20)

Parameters estimation of the FGBM(1,1,\(t^\alpha\))

By integrating on \([k-1, k]\) both side of Eq. (16) at the same time, the following conclusion can be obtained:

\[
y^{(\alpha)}(k) - y^{(\alpha)}(k-1) + a' \int_{k-1}^k y^{(\alpha)}(t) dt = \frac{k^{\alpha+1} - (k-1)^{\alpha+1}}{\alpha+1} b' + c'
\]

(25)

According to the integral median theorem, we can get:

\[
\int_{k-1}^k y^{(\alpha)}(t) dt = \lambda y^{(\alpha)}(k) + (1-\lambda)y^{(\alpha)}(k-1)
\]

(26)

where \(0 \leq \lambda \leq 1\).

The results are as follows:

\[
y^{(\alpha)}(k) - y^{(\alpha)}(k-1) + a'[\lambda y^{(\alpha)}(k) + (1-\lambda)y^{(\alpha)}(k-1)] = \frac{k^{\alpha+1} - (k-1)^{\alpha+1}}{\alpha+1} b' + c'
\]

(27)

According to the commonly used method to solve the parameters of grey prediction model, the least square criterion of FGBM (1,1,\(t^\alpha\)) model can be described as the following unconstrained optimization problems:
The solution of this optimization problem is:

\[
\min_{\alpha', \beta', \gamma'} \sum_{k=2}^{n} \left( y^{(r)}(k) - y^{(r)}(k-1) + \alpha' \left( \lambda y^{(r)}(k) + (1 - \lambda) y^{(r)}(k-1) \right) - \frac{k^{\alpha+1} - (k-1)^{\alpha+1}}{\alpha + 1} b' - c' \right)^2
\]

(28)

The expression of the FGBM(1,1,\(t^r\)) model indicates that the FGBM(1,1,\(t^r\)) model can be transformed into other grey prediction models when different values are assigned to parameters \(\xi\) and \(r\), such as GM(1,1) (Deng 1982), FGM(1,1) (Wu et al. 2013), FAGM(1,1,\(t^r\)) (Wu et al. 2019a), NGM(1,1,k,c) (Cui et al. 2009), GM(1,1,\(t^2\)) (Qian et al. 2012), GM(1,1,\(t^r\)) (Qian et al. 2012), NGBM(1,1) (Chen et al. 2008), FANGBM(1,1) (Wu et al. 2019a), and NGBM(1,1,k,c) (Wu et al. 2020a). The relationships between FGBM (1,1,\(t^r\)) with other grey models are shown in the Fig. 2.

To minimize the model error, we must determine the optimal values of parameters \(r\), \(\lambda\), \(\alpha\), \(\xi\) and adopt the MAPE as the main criterion. The optimization problems when solving the optimal parameters are as follows:

Properties of the FGBM(1,1,\(t^r\)) model

The expression of the FGBM(1,1,\(t^r\)) model indicates that the FGBM(1,1,\(t^r\)) model can be transformed into other grey prediction models when different values are assigned to parameters \(\xi\) and \(r\), such as GM(1,1) (Deng 1982), FGM(1,1) (Wu et al. 2013), FAGM(1,1,\(t^r\)) (Wu et al. 2019a), NGM(1,1,k,c) (Cui et al. 2009), GM(1,1,\(t^2\)) (Qian et al. 2012), GM(1,1,\(t^r\)) (Qian et al. 2012), NGBM(1,1) (Chen et al. 2008), FANGBM(1,1) (Wu et al. 2019a), and NGBM(1,1,k,c) (Wu et al. 2020a). The relationships between FGBM (1,1,\(t^r\)) with other grey models are shown in the Fig. 2.

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Fig. 2 Relationships between FGBM (1,1) with other grey models

\[
\min_{r,\alpha, \xi} f(r, \lambda, \alpha, \xi) = \frac{1}{n-1} \sum_{i=2}^{n} \left| \frac{x^{(0)}(t) - \tilde{x}^{(0)}(t)}{x^{(0)}(t)} \right| \times 100% \quad (31)
\]

\[
\begin{align*}
\begin{bmatrix}
0 & \leq & t & \leq & 1, 0 & \leq & a & \leq & 4.0 & \leq & \xi & \leq & 3, \xi \neq 1 \\
[0. \hat{b} \cdot \hat{c}] & = & (\hat{b}^T \hat{B})^{-1} \hat{b}^T Y
\end{bmatrix}
\end{align*}
\]

\[
x^{(1)} = \left\{ \begin{array}{ll}
\lambda & \text{if } \alpha = 1 \\
(\lambda \alpha) & \text{if } \alpha = 2
\end{array} \right.
\]

\[
\begin{align*}
\tilde{y}^{(0)}(t) &= [(\tilde{y}^{(0)}(1))^{1-\xi} - \frac{\tilde{v}_a}{\tilde{v}_b} e^{-\tilde{v}_b(t-\tilde{v}_a)} + \frac{\tilde{v}_c}{\tilde{v}_b} e^{-\tilde{v}_b(t-\tilde{v}_c)} \frac{\tilde{v}_b^{(0)}(1)}{\tilde{v}_b}] \\
\tilde{y}^{(0)}(t) &= ((\tilde{y}^{(0)}(\alpha))^{1-\xi} \cdot t = 1, 2, 3, \ldots, n)
\end{align*}
\]

The above optimization problems are essentially nonlinear programming problems with equality constraints, which can be solved with intelligent optimization algorithms or heuristic algorithms. In this paper, GWO, PSO, and QGA are adopted to solve the parameters. The reason for choosing these three algorithms is: GWO algorithm has the characteristics of simple structure, has few parameters to be adjusted, and is easy to implement. Among them, there are adaptive convergence factors and information feedback mechanism, which can achieve a balance between local optimization and global search. Therefore, GWO algorithm has good performance in solving the problem accuracy and convergence speed. The advantage of PSO algorithm is that it has memory ability. In the implementation of intelligent search, it can combine the individual and global optimal location to realize location. It has a very fast speed of approaching the optimal solution and can effectively optimize the parameters of the system. The population coding method used by QGA greatly enriches the diversity of the population, at the same time, the quantum revolving gate is used to update the population and evolve based on the information of the current optimal individual. It has the characteristics of strong adaptability, fast convergence and suitable for global search. The parameters of the algorithm are shown in Table 1.

**Error metric**

This section considers two error metrics widely used for prediction models to test the effectiveness and applicability of the grey prediction model, as indicated in Table 2.

**Validation of FGBM(1,1,\textit{t}^\alpha)**

In this section, four cases are presented to check the accuracy of the FGBM(1,1,\textit{t}^\alpha), and the forecasting results are compared to those obtained with other grey prediction models, such as NGM(1,1), SIGM, GMP, FGM, NGBM(1,1,k,c), and FAGM(1,1,\textit{t}^\alpha). In Sections 3.1 to 3.4, the raw data in Table 1 and the above seven grey models are used to simulate and

| Table 1 The parameters setting for the three algorithms |
|----------------------------------|-----------------|-----------------|
| Initial population/particle number | Max iterations | Other parameters |
| GWO | 30 | 300 | Default |
| PSO | 30 | 300 | Vmax = 0.6; wMax = 0.9; wMin = 0.2; c1 = 2; c2 = 2 |
| QGA | 30 | 300 | Default |
Table 2: Error metrics of prediction model

| Name                  | Abbreviation | Formulation                                                                 |
|----------------------|--------------|-----------------------------------------------------------------------------|
| Mean absolute        | MAPE         | $\frac{1}{n} \sum_{k=2}^{n} \left| x^{0}(k) - \hat{x}^{0}(k) \right| \times 100$         |
| percentage error     | MAPE         | $\frac{1}{n} \sum_{k=2}^{n} \left| x^{0}(k) - \hat{x}^{0}(k) \right|$     |

predict the CO₂ emissions of the USA, India, Asia Pacific region, and the world. The original time series data from 2009 to 2016 are used to build FGBM(1,1,\(r^n\)), NMG(1,1), SIGM, GMP, FGM, NGBM(1,1,k,c), and FAGM(1,1,\(r^n\)), and the original time series data from 2017 to 2019 are used to check the accuracy of the above grey prediction models.

CO₂ emissions of the USA

According to Statistics Review of World Energy 2020, in 2019, the CO₂ emissions in India reached 2480.35 million tons, accounting for 7.3% of the global CO₂ emissions, ranking third in the world. From 2010 to 2019, the CO₂ emissions in India increased, at an annual growth rate of 5.48%. With the rapid growth of the Indian economy and population, its CO₂ emissions also increased notably, which is estimated to account for 11% of the global CO₂ emissions by 2030. In the face of the call of the international community to promote carbon emission reduction, the Indian government has promised to reduce greenhouse gas emissions by 33–35% from 2015 to 2030. Therefore, this section chooses India as an example to test the accuracy of the FGBM(1,1,\(r^n\)) model. The parameters and MAPE values of the FGBM(1,1,\(r^n\)) model calculated with the three optimization algorithms are listed in Table 6. The MAPE and test_MAPE values for GWO are 0.5090% and 0.7130%, respectively, which are the smallest among the three optimization algorithms, and the performance is the best. Therefore, GWO is selected here. Figure 6 shows the number of iterations and the relationship between MAPE and the parameters. The fitting and prediction results for each model are shown in Fig. 7 and Table 7, and the error measurement results for each model are shown in Fig. 8 and Table 8. The prediction value of the FGBM(1,1,\(r^n\)) model is the closest to the actual value. Based on the prediction and fitting values, the two error measurement indexes of the FGBM(1,1,\(r^n\)) model are the best. This further demonstrates that the FGBM(1,1,\(r^n\)) model is more accurate than the other grey models in predicting the CO₂ emissions of India.

CO₂ emissions of the Asia Pacific region

According to Statistics Review of World Energy 2020, in 2019, the CO₂ emissions of the Asia Pacific region amounted to 17,269.46 million tons, accounting for 50.54% of the world’s CO₂ emissions. From 2010 to 2019, the CO₂ emissions in the Asia Pacific region indicated a rising trend, at an annual average growth rate of 2.60%. This section selects the Asia Pacific region as an example to test the accuracy of the FGBM(1,1,\(r^n\)) model. The parameters and MAPE values of the FGBM(1,1,\(r^n\)) model calculated with the three optimization algorithms are shown in Table 9. The MAPE and test_MAPE values obtained with PSO are 0.1076% and 0.6854%,
respectively, which are the smallest and the best among the three optimization algorithms, so the PSO algorithm is selected here. Figure 9 shows the relationship between the number of iterations, MAPE, and parameters. The fitting and prediction results for each model are shown in Fig. 10 and Table 10, and the error measurement results for each model are shown in Fig. 11 and Table 11. The predicted value of the FGBM(1,1,$\tau^o$) model is the closest to the actual value. According to the prediction and fitting values, the two error metrics of the FGBM(1,1,$\tau^o$) model are the best. This also indicates that the FGBM(1,1,$\tau^o$) model is more accurate than the other grey prediction models in predicting the CO$_2$ emissions in the Asia Pacific region.

**Total CO$_2$ emissions of the world**

According to Statistics Review of World Energy 2020, in 2019, the global CO$_2$ emissions reached 34,169.00 million tons, which has increased greatly over the past few decades. From 2010 to 2019, the global CO$_2$ emissions increased by...
1.102% annually, exhibiting a slow rising trend. However, economic recovery after the COVID-19 pandemic will facilitate a global rebound in CO₂ emissions. Therefore, this section adopts the world as an example to test the accuracy of the FGBM(1,1,\(r^n\)) model. The parameters and MAPE values of the FGBM(1,1,\(r^n\)) model calculated with the three optimization algorithms are summarized in Table 12. The MAPE and test_MAPE values obtained with QGA are 0.168% and 2.352%, respectively, which are the smallest and the best among the three optimization algorithms. Therefore, QGA is selected here. Figure 12 shows the relationship between the number of iterations, MAPE, and parameters.

### Table 4 Fitting and prediction results of CO₂ emissions of USA

| Year | data | NGM | SIGM | GMP | FGM | NGBM(1,1,k,c) | FAGM(1,1,\(r^n\)) | FGBM |
|------|------|-----|------|-----|-----|--------------|----------------|-----|
| 2009 | 5289.14 | 5289.14 | 5289.14 | 5289.14 | 5289.14 | 5289.14 | 5289.14 |
| 2010 | 5485.72 | 5453.79 | 5458.8 | 5453.91 | 5397.34 | 5453.79 | 5381.41 | 5485.72 |
| 2011 | 5336.44 | 5322.85 | 5324.18 | 5318.94 | 5340.07 | 5322.85 | 5347.20 | 5375.56 |
| 2012 | 5089.97 | 5239.79 | 5239.46 | 5283.4 | 5239.79 | 5285.63 | 5284.68 |
| 2013 | 5249.6 | 5187.09 | 5186.15 | 5189.98 | 5227.34 | 5187.09 | 5227.87 | 5222.82 |
| 2014 | 5254.57 | 5153.67 | 5152.6 | 5157.66 | 5171.88 | 5153.67 | 5179.98 | 5177.19 |
| 2015 | 5141.41 | 5132.46 | 5131.49 | 5134.68 | 5117 | 5132.46 | 5141.41 | 5141.41 |
| 2016 | 5042.43 | 5119.01 | 5118.21 | 5116.87 | 5062.7 | 5119.01 | 5110.48 | 5112.17 |
| 2017 | 4983.87 | 5110.47 | 5109.85 | 5101.92 | 5008.98 | 5110.47 | 5085.63 | 5087.57 |
| 2018 | 5116.79 | 5105.06 | 5104.59 | 5088.55 | 4955.83 | 5105.06 | 5065.69 | 5066.42 |
| 2019 | 4964.69 | 5101.62 | 5101.28 | 5076.05 | 4903.25 | 5101.62 | 5049.79 | 5047.92 |

### Table 5 Error metrics of CO₂ emissions of USA

| Fitting | NGM | SIGM | GMP | FGM | NGBM(1,1,k,c) | FAGM(1,1,\(r^n\)) | FGBM |
|---------|-----|------|-----|-----|--------------|----------------|-----|
| MAPE(%) | 1.2262 | 1.2146 | 1.2037 | 1.1934 | 1.2262 | 1.3043 | **1.1320** |
| MAE | 63.4644 | 62.8228 | 62.315 | 62.1516 | 63.4644 | 67.8710 | **58.2463** |
| Prediction | NGM | SIGM | GMP | FGM | NGBM(1,1,k,c) | FAGM(1,1,\(r^n\)) | FGBM |
| MAPE(%) | 1.8425 | 1.8391 | 1.7212 | 1.629 | 1.8425 | 1.5848 | 1.5805 |
| MAE | 91.7527 | 91.5873 | 85.8817 | 82.5036 | 91.7527 | 79.3186 | 79.0989 |

### Table 6 Parameters and MAPEs of the FGBM (1,1,\(r^n\)) model based on different optimization algorithms(Case 2)

| Algorithm | \(r\) (Parameter 1) | \(\lambda\) (Parameter 2) | \(\alpha\) (Parameter 3) | \(\xi\) (Parameter 4) | MAPE(%) | test_MAPE(%) |
|-----------|-----------------|-----------------|-----------------|-----------------|--------|-------------|
| GWO       | 0.0005 | 0.4788 | 0.4393 | 0.0067 | 0.5090 | 0.7130          |
| PSO       | 1.0000 | 0.5173 | 3.3792 | 0.0000 | 0.5893 | 0.9856          |
| QGA       | 0.0045 | 0.5024 | 2.2159 | 0.0001 | 0.5126 | 7.7080          |
The fitting and prediction results for each model are shown in Fig. 13 and Table 13, and the error measurement results for each model are shown in Fig. 14 and Table 14. The predicted value of the FGBM(1,1,\(t^n\)) model is the closest to the actual value. Based on the prediction and fitting values, the two error metrics of the FGBM(1,1,\(t^n\)) model are the best. This further verifies that the FGBM(1,1,\(t^n\)) model is more accurate than the other grey prediction models in predicting the global CO\textsubscript{2} emissions.

The above results reveal that although FGBM(1,1,\(t^n\)) achieves the best effect in predicting the global CO\textsubscript{2} emissions among the many models, due to the sudden increase in the original data of CO\textsubscript{2} emissions in 2016, the fitting effect of the model is good, but the prediction error is large. The predicted global CO\textsubscript{2} emissions over the next 5 years may be far from the real value. To resolve this problem, global carbon emission data can be divided into two parts, namely, Organisation for Economic Co-operation and Development
Table 7 Fitting results and prediction results of CO₂ emissions of India

| Year | data | NGM | SIGM | GMP | FGM | NGBM(1,1,k,c) | FAGM(1,1,\(t/U_1\)) | FGBM |
|------|------|-----|------|-----|-----|---------------|----------------------|------|
| 2009 | 1596.24 | 1596.24 | 1596.24 | 1596.24 | 1596.24 | 1596.24 | 1596.24 | 1596.24 |
| 2010 | 1660.65 | 1649.02 | 1649.03 | 1649.8 | 1660.65 | 1648.09 | 1654.19 | 1656.16 |
| 2011 | 1735.15 | 1749.11 | 1749.45 | 1751.62 | 1737.49 | 1751.26 | 1745.24 |
| 2012 | 1848.13 | 1849.32 | 1849.41 | 1848.95 | 1846.66 | 1847.28 | 1845.79 |
| 2013 | 1929.35 | 1949.64 | 1949.65 | 1948.07 | 1956.97 | 1944.88 | 1949.24 |
| 2014 | 2083.54 | 2050.07 | 2050.17 | 2047.3 | 2060.50 | 2045.31 | 2051.17 |
| 2015 | 2149.38 | 2150.63 | 2150.95 | 2154.88 | 2155.62 | 2149.30 | 2149.38 |
| 2016 | 2242.89 | 2251.29 | 2251.99 | 2243.4 | 2242.89 | 2257.37 | 2242.89 |
| 2017 | 2352.08 | 2352.09 | 2353.26 | 2391.72 | 2323.4 | 2352.08 | 2369.91 | 2331.37 |
| 2018 | 2453.00 | 2453.01 | 2454.75 | 2498.25 | 2398.17 | 2453.00 | 2487.30 | 2414.90 |
| 2019 | 2554.03 | 2554.04 | 2556.47 | 2605.12 | 2468.07 | 2554.03 | 2609.87 | 2493.72 |

Fig. 8 Error metrics of CO₂ emissions of India

Table 8 Error metrics of CO₂ emissions of India

|                | Fitting | NGM | SIGM | GMP | FGM | NGBM(1,1,k,c) | FAGM(1,1,\(t/U_1\)) | FGBM |
|----------------|---------|-----|------|-----|-----|---------------|----------------------|------|
| MAPE(%)        | 0.6654  | 0.6656 | 0.6685 | 0.5555 | 0.5429 | 0.6646 | 0.5090 |
| MAE            | 12.8783 | 12.8825 | 12.9674 | 10.8937 | 10.4717 | 13.1051 | 9.8836 |
| Prediction     | NGM | SIGM | GMP | FGM | NGBM(1,1,k,c) | FAGM(1,1,\(t/U_1\)) | FGBM |
| MAPE(%)        | 1.3154 | 1.3158 | 1.3889 | 1.0248 | 0.9953 | 2.7872 | 0.7130 |
| MAE            | 32.1452 | 32.1552 | 33.9366 | 25.1379 | 24.3432 | 68.1367 | 17.5085 |

Table 9 Parameters and MAPEs of the FGBM (1,1,\(t/U_1\)) model based on different optimization algorithms (Case 3)

| Algorithm | \(r_1\) (Parameter 1) | \(a_1\) (Parameter 2) | \(a_2\) (Parameter 3) | \(z\) (Parameter 4) | MAPE(%) | test_MAPE(%) |
|-----------|-----------------------|-----------------------|-----------------------|---------------------|---------|--------------|
| GWO       | 0.2262                | 0.5336                | 3.8539                | 0.1932              | 0.1095  | 0.8011       |
| PSO       | 0.2731                | 0.5320                | 3.6715                | 0.0000              | 0.1076  | 0.6854       |
| QGA       | 0.2187                | 0.5337                | 3.9843                | 0.2037              | 0.1100  | 1.2388       |
Fig. 9 Iterations, MAPE, and parameters of the three optimization algorithms

Fig. 10 Results of CO$_2$ emissions of Asia Pacific
Table 10 Fitting results and prediction results of CO2 emissions of Asia Pacific

| Year  | data   | NGM   | SIGM  | GMP   | FGM   | NGBM(1,1,k,c) | FAGM(1,1,t) | FGBM   |
|-------|--------|-------|-------|-------|-------|--------------|-------------|--------|
| 2009  | 13,244.47 | 13,244.47 | 13,244.47 | 13,244.47 | 13,244.47 | 13,244.47 | 13,244.47 | 13,244.47 |
| 2010  | 13,993.5 | 14,039.21 | 14,004.53 | 14,037.98 | 13,993.5 | 13,989.08 | 14,044.40 | 13,993.5 |
| 2011  | 14,876.64 | 14,850.3 | 14,843.42 | 14,866.61 | 14,828.75 | 14,862.94 | 14,769.40 | 14,869.13 |
| 2012  | 15,310.55 | 15,338.96 | 15,343.17 | 15,339.22 | 15,339.94 | 15,338.78 | 15,284.81 | 15,394.69 |
| 2013  | 15,666.93 | 15,633.38 | 15,640.89 | 15,619.27 | 15,655.5 | 15,623.05 | 15,626.99 | 15,666.93 |
| 2014  | 15,802.62 | 15,810.76 | 15,818.25 | 15,795.15 | 15,842.54 | 15,802.63 | 15,840.96 | 15,801.7 |
| 2015  | 15,894.14 | 15,917.63 | 15,923.9 | 15,914.68 | 15,939.34 | 15,921.87 | 15,963.44 | 15,898.41 |
| 2016  | 16,022.09 | 15,982.01 | 15,986.85 | 16,003.74 | 15,969.78 | 16,005.85 | 16,022.09 | 16,041.41 |
| 2017  | 16,357.09 | 16,020.8 | 16,024.34 | 16,076.31 | 15,949.83 | 16,069.61 | 16,037.07 | 16,304.71 |
| 2018  | 16,863.32 | 16,044.18 | 16,046.68 | 16,139.97 | 15,890.71 | 16,122.45 | 16,022.88 | 16,755.99 |
| 2019  | 17,269.46 | 16,058.26 | 16,059.99 | 16,198.8 | 15,800.65 | 16,170.25 | 15,989.84 | 17,459.35 |

Fig. 11 Error metrics of CO2 emissions of Asia Pacific

Table 11 Error metrics of CO2 emissions of Asia Pacific

| Fitting/MAE | NGM   | SIGM  | GMP   | FGM   | NGBM(1,1,k,c) | FAGM(1,1,t) | FGBM   |
|-------------|-------|-------|-------|-------|--------------|-------------|--------|
| MAPE(%)     | 0.1933 | 0.1696 | 0.1668 | 0.2072 | 0.1234 | 0.3123 | 0.1076 |
| MAE         | 29.3878 | 26.22 | 25.3164 | 32.305 | 19.1706 | 47.3506 | 16.594 |

| Prediction  | NGM   | SIGM  | GMP   | FGM   | NGBM(1,1,k,c) | FAGM(1,1,t) | FGBM   |
|-------------|-------|-------|-------|-------|--------------|-------------|--------|
| MAPE(%)     | 4.6423 | 4.6268 | 4.0686 | 5.5876 | 4.172 | 4.7834 | 0.6854 |
| MAE         | 788.8784 | 786.2858 | 691.5954 | 949.5596 | 709.1898 | 813.3621 | 116.5323 |

Table 12 Parameters and MAPEs of the FGBM (1,1,t) model based on different optimization algorithms(Case 4)

| Algorithm | r(Parameter 1) | a(Parameter 2) | a1(Parameter 3) | g1(Parameter 4) | MAPE(%) | test_MAPE(%) |
|-----------|----------------|---------------|----------------|----------------|---------|--------------|
| GWO       | 0.0000         | 0.5671        | 4.0000         | 3.0000         | 0.1891  | 2.4410       |
| PSO       | 0.0000         | 0.5671        | 4.0000         | 3.0000         | 0.1891  | 2.4410       |
| QGA       | 0.0624         | 0.5545        | 0.1410         | 0.9082         | 0.1680  | 2.3515       |
(OECD) and non-OECD data. The trend of CO₂ emissions based on OECD and non-OECD data is more stable than that based on global data. Therefore, this paper applies the FGBM(1,1,ε) model to fit and predict the CO₂ emissions based on OECD and non-OECD data, as listed in Tables 15, 16, and 17. Tables 15 and 16 reveal that between the OECD and non-OECD data, the prediction error obtained with QGA is the smallest, so QGA is selected to calculate the model parameters. The global CO₂ emissions are determined by adding the OECD and non-OECD-based CO₂ emissions,
### Table 13 Fitting results and prediction results of CO₂ emissions of total world

| Year data | NGM   | SIGM  | GMP   | FGM   | NGBM(1,1,k,c) | FAGM(1,1,\(t/u_1\)) | FGBM   |
|-----------|-------|-------|-------|-------|---------------|-----------------------|--------|
| 2009      | 29,745.21 | 29,745.21 | 29,745.21 | 29,745.21 | 29,745.21 | 29,745.21 | 29,745.21 |
| 2010      | 31,085.53 | 31,127.57 | 31,087.02 | 31,127.78 | 31,085.54 | 31,097.99 | 31,085.61 |
| 2011      | 31,973.37 | 31,942.26 | 31,938.15 | 31,929.02 | 31,939.21 | 31,927.01 | 31,973.1 |
| 2012      | 32,273.53 | 32,402.62 | 32,395.41 | 32,396.32 | 32,394.97 | 32,392.79 | 32,434.11 |
| 2013      | 32,795.55 | 32,656.09 | 32,666.91 | 32,656.47 | 32,670.83 | 32,656.76 | 32,770.42 |
| 2014      | 32,804.72 | 32,794.40 | 32,814.93 | 32,803.29 | 32,826.66 | 32,800.37 | 32,880.37 |
| 2015      | 32,787.20 | 32,867.92 | 32,879.79 | 32,875.37 | 32,908.30 | 32,880.00 | 32,880.00 |
| 2016      | 32,936.07 | 32,886.28 | 32,884.68 | 32,895.7 | 32,936.28 | 32,936.28 | 32,936.28 |
| 2017      | 33,279.49 | 32,933.55 | 32,879.78 | 32,844.47 | 32,925.14 | 32,981.3 | 32,981.3 |
| 2018      | 34,007.89 | 32,911.08 | 32,826.48 | 32,895.7 | 32,936.28 | 32,936.28 | 32,936.28 |
| 2019      | 34,169.00 | 32,952.52 | 32,666.30 | 32,769.20 | 32,826.52 | 32,981.3 | 32,981.3 |

### Table 14 Error metrics of CO₂ emissions of total world

| Algorithm | r(Parameter 1) | \(\beta\) (Parameter 2) | \(\alpha\) (Parameter 3) | \(\gamma\) (Parameter 4) | MAPE(\%) | test_MAPE(\%) |
|-----------|----------------|-------------------------|-------------------------|-------------------------|----------|---------------|
| GWO       | 0.0337         | 0.7190                  | 1.8236                  | 2.9951                  | 0.1972   | 0.2853        |
| PSO       | 0.0327         | 0.7232                  | 1.3789                  | 3.0000                  | 0.1557   | 1.0823        |
| QGA       | 0.0336         | 0.7198                  | 1.7555                  | 3.0000                  | 0.2851   | 1.0763        |

### Table 15 Parameters and MAPEs of the FGBM (1,1,\(r\)) model based on different optimization algorithms (OECD)

| Algorithm | r(Parameter 1) | \(\beta\) (Parameter 2) | \(\alpha\) (Parameter 3) | \(\gamma\) (Parameter 4) | MAPE(\%) | test_MAPE(\%) |
|-----------|----------------|-------------------------|-------------------------|-------------------------|----------|---------------|
| GWO       | 0.2690         | 0.5229                  | 0.2772                  | 0.1436                  | 0.1489   | 3.1289        |
| PSO       | 0.2147         | 0.5267                  | 4.0000                  | 0.0000                  | 0.1557   | 1.1457        |
| QGA       | 0.8747         | 0.5157                  | 3.9997                  | 0.0691                  | 0.1392   | 1.0637        |

### Table 16 Parameters and MAPEs of the FGBM (1,1,\(r\)) model based on different optimization algorithms (non-OECD)

| Algorithm | r(Parameter 1) | \(\beta\) (Parameter 2) | \(\alpha\) (Parameter 3) | \(\gamma\) (Parameter 4) | MAPE(\%) | test_MAPE(\%) |
|-----------|----------------|-------------------------|-------------------------|-------------------------|----------|---------------|
| GWO       | 0.0337         | 0.7190                  | 1.8236                  | 2.9951                  | 0.1972   | 0.2853        |
| PSO       | 0.0327         | 0.7232                  | 1.3789                  | 3.0000                  | 0.1557   | 1.0823        |
| QGA       | 0.0336         | 0.7198                  | 1.7555                  | 3.0000                  | 0.2851   | 1.0763        |

**Fig. 14** Error metrics of CO₂ emissions of total world
as summarized in Table 17. The fitting and prediction errors of the global CO₂ emissions are listed in Table 18 and are lower than those listed in Table 13. Table 19 provides the OECD and non-OECD based CO₂ emissions over the next 5 years. The global CO₂ emissions over the next 5 years can be obtained by adding the two types of emissions. The fitting and prediction results are more accurate than the direct prediction results of the global CO₂ emissions.

### Table 17 Fitting and prediction results of CO₂ emissions in OECD, non-OECD and total world

| Year | OECD       | FGBM       | Non-OECD | FGBM | Total World | Add_values |
|------|------------|------------|----------|------|-------------|------------|
| 2009 | 12,507.58  | 12,507.58  | 17,237.63| 17,237.63 | 29,745.21  | 29,745.21  |
| 2010 | 12,957.49  | 12,957.46  | 18,128.05| 18,128.05 | 31,085.53  | 31,085.52  |
| 2011 | 12,783.1   | 12,822.77  | 19,190.27| 19,190.27 | 31,773.37  | 32,002.55  |
| 2012 | 12,580.34  | 12,671.53  | 19,693.19| 19,770.25 | 32,273.53  | 32,441.78  |
| 2013 | 12,661.94  | 12,547.74  | 20,133.61| 20,112.97 | 32,795.55  | 32,660.71  |
| 2014 | 12,441.45  | 12,443.81  | 20,363.27| 20,325.81 | 32,804.72  | 32,769.62  |
| 2015 | 12,347.76  | 12,352.68  | 20,439.44| 20,488.73 | 32,787.20  | 32,841.41  |
| 2016 | 12,270.06  | 12,270.02  | 20,666.00| 20,666.10 | 32,936.07  | 32,936.12  |
| Year | OECD       | FGBM       | Non-OECD | FGBM | Total World | Add_values |
| 2017 | 12,300.25  | 12,193.21  | 20,979.24| 20,916.26 | 33,279.49  | 33,109.47  |
| 2018 | 12,372.33  | 12,120.54  | 21,635.56| 21,296.18 | 34,007.89  | 33,416.72  |
| 2019 | 12,011.96  | 12,050.83  | 22,157.05| 21,864.04 | 34,169.00  | 33,914.88  |

### Table 18 Error metrics of CO₂ emissions in total world(Add_values)

| Fitting | MAPE | MAE | Prediction | MAPE | MAE |
|---------|------|-----|------------|------|-----|
| FGBM    | 0.1852 | 60.2361 | FGBM | 0.9976 | 338.4367 |

### Table 19 Predictions for the CO₂ emissions over the next 5 years in OECD and Non-OECD

| Year | 2020 | 2021 | 2022 | 2023 | 2024 |
|------|------|------|------|------|------|
| OECD | 11,983.30 | 11,917.34 | 11,852.53 | 11,788.53 | 11,725.16 |
| Non-OECD | 22,680.69 | 23,810.63 | 25,322.78 | 27,291.05 | 29,794.83 |

### Table 20 Predictions for the CO₂ emissions over the next 5 years

| Year | US | India | Total Asia Pacific | Total World |
|------|----|------|-------------------|-------------|
| 2020 | 5031.49 | 2568.18 | 18,477.36 | 34,663.99 |
| 2021 | 5016.79 | 2638.66 | 19,872.37 | 35,727.97 |
| 2022 | 5003.49 | 2705.52 | 21,707.38 | 37,175.31 |
| 2023 | 4991.37 | 2769.12 | 24,046.64 | 39,079.60 |
| 2024 | 4980.25 | 2829.79 | 26,956.05 | 41,519.99 |

#### Forecasting CO₂ emissions over the next 5 years

In this section, we employ the FGBM(1,1,t<sup>e</sup>) model to predict the CO₂ emissions of the USA, India, Asia Pacific region, and the world over the next 5 years (2020–2024). The prediction results are summarized in Table 20 and Fig. 15. The CO₂ emissions in India, the Asia Pacific region, and the world will gradually increase over the 5 years. In addition, the CO₂ emissions in the USA will slowly decline.

#### Conclusions and policy implications

To better describe the future CO₂ emissions of the USA, India, Asia Pacific region, and the world, a new grey prediction model, i.e., the FGBM(1,1,t<sup>e</sup>) model, is proposed based on the NGMB(1,1) and FAGM(1,1,t<sup>e</sup>) models, and a precise solution of the new model is obtained via the numerical integration method. Moreover, this paper applies three common optimization algorithms to calculate the model parameters. By changing the model parameters, the FGBM(1,1,t<sup>e</sup>) model can be transformed into other models, so the model achieves a strong adaptability. The CO₂ emission fitting and forecasting results in the above four economies indicate that the FGBM(1,1,t<sup>e</sup>) model is more effective and accurate than the existing NGM(1,1), SIGM, GMP, FGM, NGMB(1,1,k,c), and FAGM(1,1,t<sup>e</sup>). Moreover, we employ the FGBM(1,1,t<sup>e</sup>) model to predict the CO₂ emissions of the USA, India, Asia Pacific region, and the world over the next 5 years. The forecast results reveal that from 2020 to 2024, the CO₂ emissions of India, the Asia Pacific region, and the world will gradually rise. The CO₂ emissions of the USA will slowly decline over the next 5 years. Notably, the grey prediction FGBM(1,1,t<sup>e</sup>) model can be applied not only in
the prediction of CO₂ emissions but also in the prediction of other data, with a high adaptability.

In order to meet the requirements for carbon emission reduction in the sustainable development goals, several suggestions are put forward based on the above findings. First, promoting the green and low-carbon transformation of the energy system is the key. We should focus on energy conservation and improve the dual control system of total energy consumption and intensity. We should increase the utilization ratio of renewable energy such as wind power and photovoltaic power generation, and develop hydropower, geothermal energy, marine energy, hydrogen energy, biomass energy, and photothermal power generation according to local conditions. Second, the fiscal and tax support should be strengthened to encourage the research and development of green and low-carbon technologies. Financial funds should be used to support the development of green environmental protection industry and energy efficient utilization, and a number of scientific and technological projects should be arranged in the fields of energy conservation and environmental protection, cleaner production, and clean energy. Third, the recycling of renewable resources should be strengthened to realize a green and low-carbon lifestyle. We should promote the classification, reduction, and resource utilization of domestic waste according to local conditions, accelerate the construction of waste material recovery system, and strengthen the recycling of waste paper, waste plastics, waste tires and other resources.

This study still has some limitations. First, the model has many hyper-parameters, which may lead to over-fitting problems. Second, the model is a univariate model, which may ignore some important influencing factors (e.g., economic development, industrial policy, etc.), so the multivariate grey model can also be applied to predict carbon emissions, which is the optimization direction in the future.

Author contribution All authors contributed to the study conception and design. Material preparation, data collection, and analysis were performed by Huiping Wang and Yi Wang. The first draft of the manuscript was written by Yi Wang. All authors read and approved the final manuscript.

Funding This work was supported by the Shaanxi Province Education Department Philosophy and Social Science Key Institute Base Project (No. 19JZ048), the Social Science Project of Shaanxi (No.2021D062), the Youth Innovation Team of Shaanxi Universities (No. 21JP044), and the Scientific Research Project of China (Xi’an) Institute for Silk Road Research (No. 2019YA08).

Data availability The datasets used or analyzed during the current study are available from the corresponding author on reasonable request.
Declarations

Ethics approval Not applicable.

Consent to participate Not applicable.

Consent to publish Not applicable.

Competing interests The authors declare no competing interests.

References

Chen CI, Chen H, Chen S (2008) Forecasting of foreign exchange rates of Taiwan's major trading partners by novel nonlinear grey Bernoulli model NGBM(1, 1). Commun Nonlinear Sci 13:1194–1204

Chen CI, Hsin PH, Wu CS (2010) Forecasting Taiwan's major stock indices by the Nash nonlinear grey Bernoulli model. Expert Syst Appl 37:7557–7562

Chiu YJ, Hu YC, Jiang P, Xie JC, Ken YW (2020) A multivariate grey prediction model using neural networks with application to carbon dioxide emissions forecasting. Math Probl Eng 2:1–10

Cui J, Dang Y, Liu S (2009) Novel gray forecasting model and its modeling mechanism. Control Decis 24(11):1702–1706

Deng J (1982) Control problems of grey systems. Syst Control Lett 15(5):288–294

Duan HM, Luo XL (2020) Grey optimization Verhulst model and its application in forecasting coal-related CO₂ emissions. Environ Sci Pollut Res 27:43884–43905

Fang DB, Zhang XL, Yu Q, Jin TC, Tian L (2018) A novel method for carbon dioxide emission forecasting based on improved Gaussian processes regression. J Clean Prod 173:143–150

Gao M, Mao S, Yan X, Wen J (2015) Estimation of Chinese CO₂ emissions based on a discrete fractional accumulation grey model. J Grey Syst 27:114–130

Guo XI, Liu SF, Yang YJ, Jin JL (2016) Forecasting China's SO₂ emissions by the nonlinear grey Bernoulli self-memory model. J Grey Syst 28:77–87

Hamzacebi C, Karakurt I (2015) Forecasting the energy-related CO₂ emissions of Turkey using a grey prediction model. Energ Source Part A 37:1023–1031

Heydari A, Garcia DA, Keynia F, Bisegna F (2019) Renewable energies generation and carbon dioxide emission forecasting in microgrids and national grids using GRNN-GWO methodology. Energy Procedia 159:154–159

Köne AÇ, Büke T (2010) Forecasting of CO₂ emissions from fuel combustion using trend analysis. Renew Sustain Energ Rev 14:2906–2915

Jiang JM, Wu WZ (2021) Nonlinear grey Bernoulli Model with fractional-order opposite-direction accumulation and its application. Math Pract Theory 51:48–53

Lin CS, Liou FM, Huang CP (2011) Grey forecasting model for CO₂ emissions: a Taiwan study. Appl Energy 88:3816–3820

Liu C, LAO TF, Wu WZ, Xie WL (2021a) Application of optimized fractional grey model-based variable background value to predict electricity consumption. Fractals 29(02).

Liu C, Wu WZ, Xie WL, Zhang J (2020) Application of a novel fractional grey prediction model with time power term to predict the electricity consumption of India and China. Chaos Soliton Fract 141:110429.

Liu C, Xie WL, Wu WZ, Zhu HG (2021b) Predicting Chinese total retail sales of consumer goods by employing an extended discrete grey polynomial model. Eng Appl Artif Intell 102(3):104261.

Liu X, Xie NM (2019) A nonlinear grey forecasting model with double shape parameters and its application. Appl Math Comput 360:203–212

Lotfalipour MR, Falahi MA, Bastam M (2013) Prediction of CO₂ emissions in iran using grey and arima models. Int J Energy Econ Policy 3:229–237

Ma X, Liu ZB, Wang Y (2019) Application of a novel nonlinear multivariate grey Bernoulli model to predict the tourist income of China. J Comput Appl Math 347:84–94

Ma X, Wu WQ, Zeng B, Wang Y, Wu XX (2020) The conforable fractional grey system model. ISA Trans 96:255–271

Meng M, Niu D (2011) Modeling CO₂ emissions from fossil fuel combustion using the logistic equation. Energy 36:3355–3359

Pao HT, Fu HC, Tseng CL (2012) Forecasting of CO₂ emissions, energy consumption and economic growth in China using an improved grey model. Energy 40:400–409

Qian WY, Dang YG, Liu SF (2012) Grey GM(1,1) model with time power and its application. Syst Eng Theory Pract 32(10):2247–2252

Sun W, Liu MH (2016) Prediction and analysis of the three major industries and residential consumption CO₂ emissions based on least squares support vector machine in China. J Clean Prod 122:144–153

Sun W, Sun JY (2017) Prediction of carbon dioxide emissions based on principal component analysis with regularized extreme learning machine. Environ Eng Res 22:302–311

Şahin U (2020) Projections of Turkey’s electricity generation and installed capacity from total renewable and hydro energy using fractional nonlinear grey Bernoulli model and its reduced forms. Sustain Prod Consump 23:52–62

Tsai SB (2016) Using grey models for forecasting China’s growth trends in renewable energy consumption. Clean Technol Environ Policy 18:563–571

Wang Q, Li SY, Pisarenko Z (2020a) Modeling carbon emission trajectory of China, US and India. J Clean Prod 258:120723.

Wang ZX (2017) A weighted nonlinear grey Bernoulli model for forecasting nonlinear economic time series with small data sets. Econ Comput Econ Cybern Stud Res 51:169–185

Wang ZX, Li Q (2019) Modelling the nonlinear relationship between CO₂ emissions and economic growth using a PSO algorithm-based grey Verhulst model. J Clean Prod 207:214–224

Wang ZX, Wang ZW, Li Q (2020b) Forecasting the industrial solar energy consumption using a novel seasonal GM(1,1) model with dynamic seasonal adjustment factors. Energy 200:117460.

Wang ZX, Ye DJ (2017) Forecasting Chinese carbon emissions from fossil fuel energy consumption using non-linear grey multivariable models. J Clean Prod 142:600–612

Wen L, Cao Y (2020) Influencing factors analysis and forecasting of residential energy-related CO₂ emissions utilizing optimized support vector machine. J Clean Prod 250:119492.

Wu LF, Liu SF, Yao LG, Yan SL, Liu DL (2013) Grey system model with the fractional-order accumulation. Commun Nonlinear Sci Numer Simul 18(7):1775–1785

Wu WQ, Ma X, Zeng B, Lv WY (2020a) A novel Grey Bernoulli model for short-term natural gas consumption forecasting. Appl Math Model 84:393–404

Wu WQ, Ma X, Zeng B, Wang Y, Cai W (2019a) Forecasting short-term renewable energy consumption of China using a novel fractional nonlinear grey Bernoulli model. Renew Energy 140:70–87
Wu WQ, Ma X, Zhang YY, Li WP, Wang Y (2020b) A novel conformable fractional non-homogeneous grey model for forecasting carbon dioxide emissions of BRICS countries. Sci Total Environ 707:135447.
Wu WQ, Ma X, Zhang YY, Wang Y (2019b) Analysis of novel FAGM(1,1,) model to forecast health expenditure of China. Grey Syst: Theory Appl 9:232–250
Xia Y, Wang HJ, Liu WD (2019) The indirect carbon emission from household consumption in China between 1995–2009 and 2010–2030: A decomposition and prediction analysis. Comput Ind Eng 128:264–276
Xie WL, Wu WZ, Liu C, Zhao JJ (2020) Forecasting annual electricity consumption in China by employing a conformable fractional grey model in opposite direction. Energy 202:11768
Xie WL, Wu WZ, Liu C, Zhang T, Dong ZJ (2021) Forecasting fuel combustion-related CO$_2$ emissions by a novel continuous fractional nonlinear grey Bernoulli model with grey wolf optimizer. Environ Sci Pollut Res 28:38128–38144
Xu N, Gong DS, YD Bai J, (2019) Forecasting Chinese greenhouse gas emissions from energy consumption using a novel grey rolling model. Energy 175:218–227
Yuan CQ, Yang YJ, Liu SF, Fang ZG (2017) An investigation into the relationship between China’s economic development and carbon dioxide emissions. Clim Dev 9:66–79
Zheng CL, Wu WZ, Xie WL, Li Q (2021) A MFO-based conformable fractional nonhomogeneous grey Bernoulli model for natural gas production and consumption forecasting. Appl Soft Comput 99:106891.
Zhou WH, Zeng B, Wang JZ, Luo XS, Liu XZ (2021) Forecasting Chinese carbon emissions using a novel grey rolling prediction model. Chaos Soliton Fract 147:110968.
Zhu HM, Xia H, Guo YW, Peng C (2018) The heterogeneous effects of urbanization and income inequality on CO$_2$ emissions in BRICS economies: evidence from panel quantile regression. Environ Sci Pollut Res 25:17176–17193

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