Extracting Nuclear Symmetry Energies at High Densities from Observations of Neutron Stars and Gravitational Waves

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Abstract. By numerically inverting the Tolman-Oppenheimer-Volkov (TOV) equation using an explicitly isospin-dependent parametric Equation of State (EOS) of dense neutron-rich nucleonic matter, a restricted EOS parameter space is established using observational constraints on the radius, maximum mass, tidal polarizability and causality condition of neutron stars (NSs). The constraining band obtained for the pressure as a function of energy (baryon) density is in good agreement with that extracted recently by the LIGO+Virgo Collaborations from their improved analyses of the NS tidal polarizability in GW170817. Rather robust upper and lower boundaries on nuclear symmetry energies are extracted from the observational constraints up to about twice the saturation density $\rho_0$ of nuclear matter. More quantitatively, the symmetry energy at $2\rho_0$ is constrained to $E_{\text{sym}}(2\rho_0) = 46.9 \pm 10.1$ MeV excluding many existing theoretical predictions scattered between $E_{\text{sym}}(2\rho_0) =15$ and 100 MeV. Moreover, by studying variations of the causality surface where the speed of sound equals that of light at central densities of the most massive neutron stars within the restricted EOS parameter space, the absolutely maximum mass of neutron stars is found to be $2.40 \, M_\odot$ approximately independent of the EOSs used. This limiting mass is consistent with findings of several recent analyses and numerical general relativity simulations about the maximum mass of the possible supermassive remnant produced in the immediate aftermath of GW170817.

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1 Introduction

To understand the nature and constrain the Equation of State (EOS) of dense neutron-rich matter in neutron stars (NSs) are a major goal shared by both nuclear physics and astrophysics \cite{1,2,3,4,5,6,7}. The nucleon specific energy \( E(\rho, \delta) \) in nucleonic matter of density \( \rho \) and isospin asymmetry \( \delta \equiv (n_p - n_n)/\rho \) (where \( n_n \) and \( n_p \) are densities of neutrons and protons, respectively) is a basic input for calculating the EOS of NS matter. Based on very extensive studies within essentially all existing nuclear many-body theories, the \( E(\rho, \delta) \) can be well approximated with the so-called empirical parabolic law, see, e.g., ref. \cite{8}.

\begin{equation}
E(\rho, \delta) = E_0(\rho) + E_{\text{sym}}(\rho) \cdot \delta^2 + O(\delta^4).
\end{equation}

The \( E_0(\rho) \) is the energy in symmetric nuclear matter (SNM) of equal numbers of neutrons and protons while the \( E_{\text{sym}}(\rho) \) is generally called the symmetry energy of isospin asymmetric nuclear matter (ANM).

While much progress has been made over the last few decades in obtaining both a theoretical understanding and observational/experimental constraints of the \( E_0(\rho) \) as well as \( E_{\text{sym}}(\rho) \) around but mostly below the saturation density \( \rho_0 \) of nuclear matter \cite{9,10,11,12,13,14,15,16,17}, even the trend of \( E_{\text{sym}}(\rho) \) at supra-saturation densities is presently controversial. Since it is such an important quantity for investigating many issues in both astrophysics and nuclear physics, essentially all available nuclear many-body theories and interactions have been used to calculate the \( E_{\text{sym}}(\rho) \). While many calculations based on well-known theories predict an increasing \( E_{\text{sym}}(\rho) \) with density, a large number of equally well-respected theories predict that the \( E_{\text{sym}}(\rho) \) first increases with density passing \( \rho_0 \), then stays approximately a constant (or decreases above certain densities) depending on the isospin-dependence of the short-range tensor or three-body nuclear force used in the theories. Therefore, the \( E_{\text{sym}}(\rho) \) at high densities has been broadly recognized as the most uncertain part of the EOS of dense neutron-rich nucleonic matter \cite{15}. For example, over 520 nuclear energy density functionals available by 2014 have been used to predict the \( E_{\text{sym}}(\rho) \), see, e.g., refs. \cite{18,19,20}. Shown in the left window of Fig. \textsuperscript{11} are 60 selected representatives from 6 classes of phenomenological models and/or energy density functional theories including the Relativistic Mean Field (RMF) using 3 different kinds of coupling schemes, Relativistic Hartree-Fock...
The density dependences of nuclear symmetry energy predicted by various kinds of nuclear many-body theories using different interactions, energy density functionals and/or techniques (made by amending a compilation in ref. [21]) in comparison with the constraining boundaries (magenta dot-dashed lines) extracted in this work from studying properties of neutron stars.

Shown in the right window of Fig. 1 are 11 examples [22,23,24,25,26,27,28,29]. They are from the Brueckner Hartree Fock (BHF), Dirac-Brueckner Hartree Fock (DBHF), Chiral Effective Field Theory (Chiral EFT) and the Variational Many Body (VMB) theory using different interactions and/or high-momentum cut-offs. Their predictions also spread broadly at supra-saturation densities. In fact, by design, some of these microscopic theories are valid only at low-energies/densities. When they are extrapolated to high densities, their predictions may not converge and often depend on the high-momentum cut-off used in the theories. A useful measure of the predicted spread of high-density symmetry energies is the value of symmetry energy at twice the saturation density \( E_{\text{sym}}(2\rho_0) \). Information about the EOS and symmetry energy around this density is most relevant for determining the radii of NSs [30] and heavy-ion reactions with radioactive beams of about 400 MeV/nucleon [12]. The examples shown in Fig. 1 have \( E_{\text{sym}}(2\rho_0) \) values scatter between approximately 15 to 100 MeV [31]. The magenta dot-dashed lines are the boundaries of \( E_{\text{sym}}(\rho) \) we extracted in this work from studying properties of NSs as we shall explain in detail in the following. Clearly, the extracted constraint on symmetry energy can already exclude many of the predictions while it is still quite loose at densities above 2\( \rho_0 \).

The proton fraction \( x_p(\rho) \) in NSs at \( \beta \)-equilibrium is uniquely determined by the \( E_{\text{sym}}(\rho) \). Consequently, the composition, critical nucleon density \( \rho_c \) (where \( x_p(\rho_c) = 1/9 \)) above which the fast cooling by neutrino emissions through the direct URCA process can occur, and the crust-core transition density in NSs all depend sensitively on the \( E_{\text{sym}}(\rho) \). It is well known that the radii of NSs are most sensitive to the pressure around 1 - 2\( \rho_0 \) where the symmetry energy makes a significant contribution to the pressure [30]. Moreover, the frequencies and damping times of various oscillations, quadrupole deformations of isolated NSs and the tidal polarizability in NS mergers also depend on the \( E_{\text{sym}}(\rho) \) [31]. Furthermore, there is a degeneracy between the EOS of super-dense neutron-rich matter and the strong-field gravity in understanding both properties of super-massive NSs and the minimum mass to form black holes. Thus, further testing Einstein’s General Relativity (GR) against alternative theories of super-strong gravity also requires reliable knowledge about the EOS of dense neutron stars.
neutron-rich matter, especially its $E_{\text{sym}}(\rho)$ term at suprasaturation densities $^{32,33}$.

The fundamental origin of uncertainties of the high-density $E_{\text{sym}}(\rho)$ is the largely unknown and relatively weak isospin-dependence (i.e., the difference between neutron-proton interactions in the isosinglet and isotriplet channels, while neutron-neutron, neutron-proton and proton-proton interactions are the same in the isotriplet channel due to charge independence) of nuclear forces especially at short distances in the dense neutron-rich medium $^{35}$. Determining the high-density $E_{\text{sym}}(\rho)$ using nuclear reactions induced by high-energy rare isotope beams has been identified as a major science thrust in both the 2015 American and 2017 European nuclear physics long range plans for the next decade $^{36,37}$. Unlike the small isospin effects in laboratory experiments for finite nuclei of normally small neutron/proton ratios, NSs are the natural testing ground of fundamental interactions at extremely high densities and/or isospin asymmetries in cold matter.

In this work, using an explicitly isospin-dependent parametric EOS in terms of the $E(\rho, \delta)$ of neutron-rich nucleonic matter with its two components $E_0(\rho)$ and $E_{\text{sym}}(\rho)$ both parameterized as functions of density up to $(\rho/\rho_0)^3$ terms respecting available constraints around $\rho_0$, we invert the TOV equation to investigate how the observational constraints on the radii, maximum mass and tidal polarizability of NSs as well as the causality requirement all together may constrain the high-density $E_{\text{sym}}(\rho)$. Rather strong upper and lower bounds on the $E_{\text{sym}}(\rho)$ at supersaturation densities are obtained. In particular, the symmetry energy at $2\rho_0$ is constrained to $E_{\text{sym}}(2\rho_0) = 46.9 \pm 10.1$ MeV excluding many of the existing predictions as shown in Fig. 1. Simultaneously, the total pressure $P(\epsilon)$ as a function of energy density $\epsilon$ is constrained into a band consistent with that extracted recently by the LIGO+Virgo Collaborations from their improved analyses of the NS tidal polarizability in GW170817 $^{38}$. Moreover, by studying variations of causality surface within the restricted high-density EOS parameter space, the absolutely maximum mass of neutron stars is found to be 2.40 $M_\odot$ almost independent of the EOSs used. The limiting mass for NSs is consistent with recent findings from studying the electromagnetic signals and numerical GR simulations of GW170817 $^{39,40,41,42,43,44}$.

The paper is organized as follows. In the next section, we first outline the approach of numerically inverting the TOV equation using the parametric EOS first used in our previous work $^{45}$. We then study in sect. 3 causality constraints on the maximum mass of NSs and the EOS parameter space. In sect. 4 observational limits on parameters characterizing the high-density EOS are examined. These limits are then transformed into constraints on the pressure $P(\epsilon)$ at $\beta$ equilibrium in NSs and the symmetry energy $E_{\text{sym}}(\rho)$ in sect. 5 and sect. 6, respectively. A summary is given at the end.

2 Explicitly isospin-dependent parametric EOS for high-density neutron star matter

It is well known that the mass (M)-radius (R) relationship of neutron stars can be obtained by solving the TOV equation

$$\frac{dP}{dr} = -\frac{G(m(r) + 4\pi r^3 P/c^2)(\epsilon + P/c^2)}{r(r - 2GM(r)/c^2)}, \quad (2)$$

$$\frac{dm(r)}{dr} = 4\pi r^2$$

for a given input EOS $P(\epsilon)$ where $G$ is the gravitation constant, $c$ is the light speed and $m(r)$ is the gravitational mass enclosed within a radius $r$. Since the main purpose of our work is to constrain the EOS of neutron-rich matter and extract the underlying nuclear symmetry energy $E_{\text{sym}}(\rho)$ using NS observations, we shall invert numerically the TOV equation for a given single or set of NS observables using a parametric EOS with explicit isospin dependence. Detailed discussions of this approach are presented in our previous work $^{35}$. For completeness and ease of our following discussions, we summarize here the main features and justifications of our approach.

The usual representations of the high-density EOS of NS matter widely used in the literature are based on parameterizing the adiabatic index $\Gamma(P)$ defined by $\Gamma(P) = \frac{\epsilon}{P} \frac{dP}{d\epsilon}$ with piecewise analytical functions in each of n density/pressure domains $^{46}$, e.g., the piecewise polynomials from using constant $\Gamma$ values in the n domains, or from constructing $\log(\Gamma(P))$ as a polynomial of $\log(P)$, for a recent review, see, e.g., ref. $^{47}$. While these types of parametric EOSs are indeed sufficient for solving the TOV equation and have been found very useful for many purposes, they carry no non-degenerate information about the internal composition of dense matter and are unable to reveal clearly the underlying nuclear symmetry energy. To extract directly information about the high-density nuclear symmetry energy from inverting the TOV equation using observational constraints, one thus has to parameterize the EOS at a more basic level using the isospin degree of freedom explicitly according to Eq. (1).

The energy density $\epsilon(\rho, \delta)$ in neutron star matter is

$$\epsilon(\rho, \delta) = \epsilon_N(\rho, \delta) + \epsilon_I(\rho, \delta), \quad (4)$$

where $\epsilon_N(\rho, \delta)$ and $\epsilon_I(\rho, \delta)$ are the energy density of nucleons and leptons, respectively. The $\epsilon_N(\rho, \delta)$ is determined by the nucleon specific energy $E(\rho, \delta)$ and average mass $M_N$ via

$$\epsilon_N(\rho, \delta) = \rho E(\rho, \delta) + \rho M_N. \quad (5)$$

The pressure in neutron star matter is then

$$P(\rho, \delta) = \rho^2 \frac{d\epsilon(\rho, \delta)}{d\rho} = \rho^2 \left[E_0(\rho) + E_{\text{sym}}(\rho) \delta^2 + \frac{1}{2} \delta(1 - \delta) \rho E_{\text{sym}}(\rho)\right]. \quad (6)$$
The pressure $P(\rho, \delta)$ and energy density $\epsilon(\rho, \delta)$ become functions of density only once the isospin asymmetry profile $\delta(\rho)$ is determined from the $\beta$-equilibrium condition $\mu_n - \mu_p = \mu_n - \mu_\mu \approx 4\delta\rho\rho_{\text{sym}}(\rho)$ and the charge neutrality condition $\rho_p = \rho_e + \rho_\mu$. The chemical potential for a particle $i$ can be calculated from $\mu_i = \partial\epsilon(\rho, \delta)/\partial\rho_i$. The lepton energy density $\epsilon_l(\rho, \delta)$ is calculated from the noninteracting Fermi gas model as normally done in the literature [43, 58].

To connect self-consistently the core EOS described above with the NV EOS [10] for the inner crust and the BPS EOS [58] for the outer crust, the core-crust transition density and the pressure $P_t$ there were calculated by examining the incompressibility of NS matter for any given set of EOS parameters [58]. In addition, to ensure the thermodynamical stability of NSs, we require the transition pressure to stay positive. This condition limits the low-density behavior of nuclear symmetry energy and is useful to restrict the EOS parameter space.

We note that the dimensionless tidal deformability $\Lambda$ is related to the Love number $k_2$, neutron star mass $M$ and radius $R$ via $\Lambda = \frac{4\pi k_2 (R/M)^3}{M}$. The $k_2$ is determined by the EOS through a differential equation coupled to the TOV equation [41, 52]. It was found earlier that the tidal polarizability is sensitive to the high-density behavior of nuclear symmetry energy but not much to the saturation properties of nuclear matter [53, 54].

To infer the high-density nuclear symmetry energy from NS observables, it is necessary to start from directly parameterizing the $E_0(\rho)$ and $E_{\text{sym}}(\rho)$ in Eq. (1), separately. Since the density profile of isospin asymmetry $\delta(\rho)$ in charge neutral NS matter at $\beta$ equilibrium is completely determined by the $E_{\text{sym}}(\rho)$, the corresponding $P(\epsilon)$ can then be calculated consistently once the $E_0(\rho)$ and $E_{\text{sym}}(\rho)$ are known. We parameterize them as

$$E_0(\rho) = \frac{K_0}{2} (\rho - \rho_0)^2 + \frac{J_0}{6} (\rho - \rho_0)^3, \quad (7)$$

$$E_{\text{sym}}(\rho) = E_{\text{sym}}(\rho_0) + L(\rho - \rho_0) + \frac{K_{\text{sym}}}{2} (\rho - \rho_0)^2 + \frac{J_{\text{sym}}}{6} (\rho - \rho_0)^3. \quad (8)$$

It is worth emphasizing that the above parameterizations naturally approach asymptotically their Taylor expansions when $\rho \rightarrow \rho_0$. While leaving the high-density parameters $J_0$, $K_{\text{sym}}$ and $J_{\text{sym}}$ as free parameters to be determined by NS observables, we fix the $E_0(\rho_0)$, $K_0$, $E_{\text{sym}}(\rho_0)$ and $L$ at their currently known most probable values at saturation density. For instance, extensive studies over the last four decades have determined the most probable incompressibility of symmetric nuclear matter as $K_0 = 230 \pm 20$ MeV [55, 56], while surveys of 53 analyses done over the last two decades have found the most probable magnitude $E_{\text{sym}}(\rho_0) = 31.7 \pm 3.2$ MeV and slope $L = 58.7 \pm 28.1$ MeV of symmetry energy at $\rho_0$ [43, 57]. There are also theoretical and experimental efforts to determine the high-density EOS parameters. However, they are still very uncertain [58, 59]. Nevertheless, the currently known ranges of $-400 \leq K_{\text{sym}} \leq 100$ MeV, $-200 \leq J_{\text{sym}} \leq 800$ MeV, and $-800 \leq J_0 \leq 400$ MeV provide at least a starting point in our efforts to narrow them down using NS observables. Once these parameters are constrained with
NS observables, the \(E_0(\rho), \sym(\rho)\) and subsequently the \(P(\epsilon)\) can all be reconstructed. This prior information albeit inaccurate about the ranges of the high-density EOS parameters is useful compared to directly parameterizing the \(P(\epsilon)\) for which there is even less accurate prior information available.

As an illustration of the high-density behaviors of the \(\sym(\rho)\) to be explored by varying the \(K_{\text{sym}}\) and \(J_{\text{sym}}\) parameters, shown in Fig. 2 are the symmetry energy \(\sym(\rho)\) and the resulting isospin asymmetry profile \(\delta(\rho)\) as functions of the reduced density \(\rho/\rho_0\) by varying only one parameter each time while fixing all others. In the left window, we used \(K_{\text{sym}} = -400, -300, -200, -100, 0,\) and 100 MeV, and in the right window \(J_{\text{sym}} = -200, 0, 200, 400, 600,\) and 800 MeV, respectively. As their names indicate, the slope \(L\), curvature \(K_{\text{sym}}\) and skewness \(J_{\text{sym}}\) of symmetry energy play different roles and in order become increasingly more important at higher densities. Clearly, very diverse density dependences of the \(\sym(\rho)\) spanning the whole range of model predictions shown in Fig. 1 are sampled. The resulting \(\delta(\rho)\) at \(\beta\) equilibrium shown in the lower part of Fig. 2 varies from values for very neutron-poor matter with stiff symmetry energies to pure neutron matter when the \(\sym(\rho)\) becomes zero or even negative.

Lindblom recently reminded us several basic guiding principles that all parameterized EOSs should follow [47]. These include (1) any EOS should be representable by an appropriate parametric representation to any level of desired accuracy, (2) each representation should satisfy the basic laws of physics including the thermodynamic stability and causality, 3) the representation should be efficient. While varying the three high-density EOS parameters within our approach outlined above, necessary physics measures are taken to ensure that our EOS representation meets these requirements. For example, we actually explored the parameter space to locate the crust-core transition boundary by checking signs for the thermodynamical instability in uniform matter starts to occur. As mentioned earlier, we also require the crust-core transition pressure remains positive. Thus the entire EOS from the surface to the core is ensured to be thermodynamically stable. As we have shown earlier, the combined variations of the three high-density parameters are flexible enough for us to mimic essentially all kinds of high-density EOSs predicted by various many-body theories. Of course, we limited ourselves to the EOSs for \(n\rho\mu\) matter in the minimum NS model without phase transitions. As to the causality, we actually use it to limit the allowed parameter space, thus the high-density EOS. As we shall discuss next, reaching the causal limit at central densities of the most massive neutron stars is used to determine the absolutely maximum mass of NSs.

3 Causality limits on the absolutely maximum mass and central density of neutron stars as well as the high-density symmetry energy parameters

The speed of sound is defined by \(c_s^2 = dP/d\epsilon\). The thermodynamical stability and causality condition require that \(0 \leq c_s^2 \leq c^2\). The latter limits naturally the EOS parameter space and determines the absolutely maximum mass of NSs. To use the causal limit for our purposes, we search for combinations of the three high-density EOS parameters leading to \(c_s^2 = c^2\) right at the central density of the most massive NS supported by a given EOS from solving the TOV equation. We refer the surface where the above conditions are met in the 3D \((J_0 - \sym - K_{\text{sym}})\) EOS parameter space as the causality surface in our following discussions.

3.1 Causality contour in the parameter plane of high-density nuclear symmetry energy

First, let us examine the contour of the \(J_0\) parameter in the plane of \(\sym\) versus \(K_{\text{sym}}\) where the causal limit has been reached at the central density of the most massive NS configuration supported by the individual EOS specified by the three high-density EOS parameters in the upper window of Fig. 3. The contour of the corresponding maximum mass is shown in the lower window. We begin from the left side. The lower left corner in the \(\sym - K_{\text{sym}}\) plane is where the nuclear symmetry energy is super-soft with...
both $J_{\text{sym}}$ and $K_{\text{sym}}$ being negative. The maximum mass of NSs in this area is less than 1.4 $M_\odot$. To support NS sequences with a maximum mass as low as $M_{\text{max}} = 0.8 M_\odot$ would require a $J_0$ as high as 400 MeV to make the total pressure strong enough. It clearly demonstrates the complementary contributions of the symmetry energy and the SNM EOS to the total pressure necessary to support stable NSs. Overall, in the left region where the maximum masses are low, the variation of the $J_0$ is more obvious, meaning that maximum masses of these lighter NSs are more sensitive to the variation of the EOS. On the contrary, in the big red region on the right in the lower window the maximum mass is almost a constant of about 2.4 $M_\odot$. Interestingly, the corresponding region in the upper window covers values of $J_0$ ranging from -350 to 400 MeV. Combining the information from both windows, it is seen that there is a big parameter space where the $M_{\text{max}} = 2.4 M_\odot$, namely this limiting value of the maximum mass is almost independent of the EOSs used. We refer it as the absolutely maximum mass (i.e., the maximum of the maximum masses using any EOS) of NSs. It will be further studied next by examining several other features of the causality surface.

The solid line in the lower window of Fig. 3 is the boundary set by the currently observed maximum mass of $M_{\text{max}} = 2.01 M_\odot$. The region to its left is excluded by this observational constraint. Observations of more massive NSs certainly will further limit the EOS parameter space. It is thus interesting to note that the mass of the neutron star PSR J0348+0432 was recently reported to be $2.27^{+0.15}_{-0.13} M_\odot$ [60]. If confirmed, this will raise the minimum maximum mass of NSs, subsequently further the allowed EOS parameter space.

The same information about the causality surface and the associated physics may be more clearly visualized by presenting them in another way. Shown in the left window of Fig. 4 is the maximum mass of NSs on the causality surface as a function of $J_{\text{sym}}$ and $K_{\text{sym}}$ in 3D. For comparisons, a plane with the current minimum maximum mass $M_{\text{max}} = 2.01 M_\odot$ from the confirmed mass of PSR J0348+0432 is also shown. The space below this lower limit is excluded. Thus, the interaction curve of this plane with the causality surface sets a boundary in the $J_{\text{sym}}$ versus $K_{\text{sym}}$ plane. This boundary is the same as the solid line in the lower window of Fig. 3. As we shall discuss later, this boundary sets the lower limit of nuclear symmetry energy. Again, it is clearly seen that the causality surface sets an absolutely upper limit for the mass of NSs at $M_{\text{max}} = 2.4 M_\odot$ almost independent of the EOSs used. This finding has some interesting implications. Given the model-independent nature of inverting the TOV for necessary EOSs to reproduce an observable and the general requirement of causality, the predicted absolutely maximum mass of 2.4 $M_\odot$ is rather general. While the allowed maximum mass varies between 2.01-2.4 $M_\odot$ depending on the high-density behavior of symmetry energy as indicated by the variation of the causality surface (blue) with $J_{\text{sym}}$ and $K_{\text{sym}}$.

The composite mass of the two NSs in GW170817 is 2.74 $M_\odot$ [61]. The fate of the remnant in GW170817 is not observationally determined because of the limited sensitivities of the current gravitational wave detectors [61]. Thus the nature of the central remnant of GW170817 remains an open question. While it is generally proposed that the large composite mass of GW170817 would lead to a shortly lived hyper-massive NS or directly produce a black hole, there is no clear evidence to support or rule out a long-lived NS as the merger remnant, see, e.g. refs. [39,62] for detailed discussions. The causality surface clearly forbids the formation of a permanent NS as massive as 2.74 $M_\odot$. Indeed, several recent analyses of GW170817 data and numerical general relativity simula-

Fig. 4. (color online) The mass (left) and radius (right) of the most massive neutron stars on the causality surface as functions of $J_{\text{sym}}$ and $K_{\text{sym}}$, respectively.
However, less is known about effects of the high-density symmetry energy \( E_{\text{sym}}(\rho) \) and its slope \( L(\rho) \) at saturation density \( \rho_0 \) on the radii of NSs have been studied extensively in the literature, see, e.g. ref. [64, 65] for earlier examples and ref. [67] for a recent review. However, less is known about effects of the high-density symmetry energy on the radii of NSs. It is thus interesting to study how the radius of the most massive NS on the causality surface evolves with \( K_{\text{sym}} \) and \( J_{\text{sym}} \). Indeed, as shown in the right window of Fig. 4 these parameters characterizing the high-density symmetry energy have significant effects on the radii of the most massive NSs. It is seen that for \( K_{\text{sym}} \) higher than about \(-100\) MeV regardless of the \( J_{\text{sym}} \) values the radius \( R_{\text{max}} \) stays at a constant of about 12 km. As the \( K_{\text{sym}} \) decreases especially with negative \( J_{\text{sym}} \) values, the symmetry energy becomes softer, the NS matter at \( \beta \) equilibrium becomes more neutron rich. Both the mass and radius become smaller. In fact, the maximum mass drops below 2.01 \( M_\odot \) as shown in the left window of Fig. 4. While it is in a region of parameter space already excluded due to its inability to support NSs with the maximum mass of 2.01 \( M_\odot \), it is worth noting that at the far corner of very negative \( K_{\text{sym}} \) and \( J_{\text{sym}} \) values where the symmetry energy becomes zero or negative, the very low-mass NSs containing almost pure neutron matter can be stable with large radii indicated by the raising \( R_{\text{max}} \).

### 3.2 Mass-radius correlation on the causality surface

To this end, it is necessary and important to compare our \( M_{\text{max}} \) versus \( R_{\text{max}} \) relationship on the causality surface with the causality constraints on the M-R relationship widely used in the literature. An excellent review on this topic can be found in sect. II of ref. [68]. The most frequently used causality constraint is the one derived from the finding by Lindblom that the redshift has a maximum almost independent of the only parameter in his EOS for NS matter \[69\]. For comparisons, it is important to first recall briefly how his EOS was constructed to respect the causal limit. In Lindblom’s work and several subsequent studies confirming his results (for a complete list see ref. [68]), they constructed EOSs that all have \( v_\text{cs}^2 = c^2 \) above a fiducial transition-density \( \rho_f \), namely, the EOS has the simple form \( P(\epsilon) = \epsilon - \epsilon_f + P_f(\epsilon_f) \), for \( \epsilon \geq \epsilon_f(\rho_f) \). Below the \( \rho_f \) considered to be in the envelopes of NSs, either some empirical or “realistic” nuclear EOSs, such as the BPS EOS, or a pure neutron matter EOS were used. The \( \rho_f \) was taken as a model parameter. The redshift was shown to only weakly depend on the value of \( \rho_f \). For \( \rho_f \geq 3\times 10^{14} \text{g cm}^{-3} \) (note that \( \rho_0 = 2.7 \times 10^{14} \text{g cm}^{-3} \) corresponding to a baryon density of 0.16 fm\(^3\), the redshift was found to be

\[
z = \frac{1}{\sqrt{1 - 2GM/Rc^2}} - 1 \leq 0.863,
\]

leading to a limit on the mass-radius relationship

\[
M \leq R/2.83.
\]

The constraint of Eq. (10) has been widely used in the literature as a very general upper causal limit for the stiffness of all EOSs of NS matter. We notice that it was also emphasized by Lindblom that because the EOS reached the causal (stiffness) limit already at \( \rho_f \) which can be quite low, the maximum mass supported can be rather high. Quantitatively, for the \( \rho_f \) changing between 1 to 5 times \( 10^{14} \text{g cm}^{-3} \), the maximum mass \( M_{\text{max}} \) varies between 6.71-3.02 \( M_\odot \), far above the absolutely maximum mass we discussed above, while the redshift \( z \) varies only in a very small range between 0.883-0.858 \[69\], respectively.

Instead of allowing \( v_\text{cs}^2 = c^2 \) in all regions at densities higher than the \( \rho_f \) in all NSs regardless of their masses, on our causality surfaces the condition \( v_\text{cs}^2 = c^2 \) is satisfied only at the central density in the most massive NS configuration in the M-R sequence calculated for a given EOS. All other NSs in the sequence still have \( v_\text{cs} \) less than \( c \) in all density regions reached. Thus, the maximum masses \( M_{\text{max}} \) on our causality surfaces are expected to be below the ones inferred from Lindblom's causality constraint of Eq. (10). This is demonstrated numerically in Fig. 5 by comparing the dashed line for \( M_{\text{max}} = R_{\text{max}}/2.83 \) from Eq. (10) with our results (scattered symbols). The minimum maximum mass of \( M_{\text{max}} = 2.01 \ M_\odot \) is also shown as a reference. The values of \( M_{\text{max}} \) and \( R_{\text{max}} \) are taken from Fig. 4 at the same \( K_{\text{sym}} \) and \( J_{\text{sym}} \) coordinates on the lattice grids. For example, for \( K_{\text{sym}} = 100 \text{ MeV with } J_{\text{sym}} = -200, -100, 0, ..., 800 \text{ MeV}, \) both the \( M_{\text{max}} \) and \( R_{\text{max}} \)
are almost constants, the resulting $M_{\text{max}} - R_{\text{max}}$ correlation, shown as red squares near the right frame, is almost a constant of $M_{\text{max}} = 2.4 M_\odot$ independent of the symmetry energy parameters. This is because near the absolutely maximum mass, the pressure is dominated by the EOS of SNM instead of the symmetry energy. As the $K_{\text{sym}}$ and $J_{\text{sym}}$ decrease to more negative values, making the symmetry energy softer, the $M_{\text{max}}$ decreases slower than the radius $R_{\text{max}}$.

Comparing our results with the M-R correlation of Eq. (10), it is seen that near the limit of $M_{\text{max}} = 2.4 M_\odot$ and $R_{\text{max}} = 11.5 \text{ km}$, our $M_{\text{max}}$ values are about 10% below the Eq. (10) prediction. As the Eq. (10) was derived by assuming the condition $v_s^2 = c^2$ is reached already at densities as low as slightly above $\rho_0$, while we only allow the condition $v_s^2 = c^2$ to be reached at the central densities of the most massive NSs, our smaller maximum masses are easily understandable. Moreover, it is important to emphasize that the mass and radius from Lindblom’s redshift limit are not for the most massive configuration that his EOS can support. In fact, as we mentioned earlier, his EOS can support much more massive NSs as heavy as 6.71 $M_\odot$. We also notice that the $R_{\text{max}}$ from our study is between 10.5 and 11.5 km above the minimum maximum mass of $M_{\text{max}} = 2.01 M_\odot$. This sets a lower limit for the radii of most massive NSs, which is consistent with existing observations as we shall discuss later.

It is also interesting to compare with another scaling found in several earlier studies, see, e.g., refs. [70][71],

$$M_{\text{max}} \approx R_{\text{max}}/4.51.$$  \hspace{1cm} (11)

It was obtained using similarly constructed EOSs as in deriving the scaling of Eq. (10) assuming $v_s^2 = c^2$ above the fiducial transition-density $\rho_f$. Although this scaling is below the minimum maximum mass of $M_{\text{max}} = 2.01 M_\odot$, it is useful for understanding the main physics determining the causality constraint on the mass-radius relation. First, it is important to note that the Eq. (11) is from actually estimating the $M_{\text{max}}$ and $R_{\text{max}}$ for the most massive NSs supported by the EOS used instead of using the approximate invariance of the redshift in deriving the Eq. (10) for all NSs. The Eq. (11) was found by using EOSs giving an incompressibility for SNM less than 150 MeV and assuming $\rho_f \leq 5 \rho_0$ [72]. It is seen in Fig. 5 that this scaling seems to provide a lower bound for our results in the low mass/radius region. Near the limit of $M_{\text{max}} = 2.4 M_\odot$ and $R_{\text{max}} = 11.5 \text{ km}$, our $M_{\text{max}}$ values are about 25% above the Eq. (11) prediction. The difference between the two scalings signifies the model dependence in estimating the causality constraint on the mass-radius relation. Overall, while our numerically calculated $M_{\text{max}} - R_{\text{max}}$ relation on the causality surface is about 10% to 25% different from the estimated scalings, they all share the same linear trend especially near the absolutely maximum mass of $M_{\text{max}} = 2.4 M_\odot$ which is found here to be approximately EOS independent.

The predicted NS maximum masses using various EOSs in the literature still spread out in a large range. There are indeed models predicting maximum masses higher than 2.4 $M_\odot$ but below the scaling of Eq. (10), see, e.g. examples given in refs. [72][77][73]. However, we emphasize here again that the absolutely maximum mass of $M_{\text{max}}=2.4 M_\odot$ found here is based on a minimum NS model. It does not rely on any prior knowledge about the EOS but the physics requirement that the causal limit is reached only at central densities of the most massive NSs supported by the EOSs considered. The absolutely maximum mass is the maximum of the maximum masses supported by all EOSs while the causality is respected. It is therefore theoretically EOS independent. This is very different from comparing the predicted maximum masses using various EOSs with an estimated universal M-R relationship from general causality considerations under some assumptions about the EOS in NSs. Thus, those EOSs predicting the maximum masses higher than 2.4 $M_\odot$ but still below the estimated M-R causality line are not guaranteed to be actually causal as the sound speed $v_s$ depends on the specific EOS used in the model.

3.3 Causality limit on the maximum density reachable in neutron stars and effects of high-density nuclear symmetry energy

Besides the discussions above, one important question also needs to be answered. What are the ultimate baryon and energy densities of observable cold matter? Lattimer and Prakash pointed out that the largest observed mass of NSs sets the lowest upper limit on the maximum energy density [74]. Theoretically, it is interesting to known not only how high but also where and under what conditions the highest densities are reached in NSs. Answers to these questions have several profound implications [74]. Within our minimum model for NSs, the central density reached...
However, it increases quickly as the instant of 5\( \rho \) of the most massive NSs on the causality surface versus (color online) The masses of the most massive neutron stars the most massive NSs on the causality surface provides the text for details. In the most massive NSs on the causality surface provides useful hints to answer these questions.

Shown in Fig. 6 is the central baryon density \( \rho_{\text{max}} \) of the most massive NSs on the causality surface versus \( K_{\text{sym}} \) and \( J_{\text{sym}} \). It is seen that the \( \rho_{\text{max}} \) is about a constant of 5\( \rho_0 \) in the lower left corner where the \( K_{\text{sym}} \) and \( J_{\text{sym}} \) are both positive, leading to stiff symmetry energies. However, it increases quickly as the \( K_{\text{sym}} \) and \( J_{\text{sym}} \) moving toward their lower limits near the upper right corner where the symmetry energy becomes super-soft, supporting only light and very neutron-rich NSs. Since the radius decreases much faster than the mass as the symmetry energy changes from being stiff to super-soft and the density scales with \( M/R^3 \), the \( \rho_{\text{max}} \) thus varies very dramatically with \( K_{\text{sym}} \) and \( J_{\text{sym}} \) as they become negative, towards super-soft symmetry energies. The \( \rho_{\text{max}} \) reaches as high as 15\( \rho_0 \). However the peak is in the region excluded by the minimum maximum mass of 2.01 M\(_{\odot} \), as illustrated in Fig. 4 already. Consequently, the observed most massive NS limits the \( \rho_{\text{max}} \) to the range of 5-9 \( \rho_0 \) depending on the high-density symmetry energy. Of course, as for all minimum models for NSs, there is a maximum range beyond which new physics ingredients, such as non-nucleonic degrees of freedom, various new phases and possible boson condensations, have to be added. The \( \rho_{\text{max}} \) is an indicator when such new physics should be incorporated. The rather high \( \rho_{\text{max}} \) revealed in Fig. 6 reminds us that our results obtained within the minimum model have to be interpreted with some caveats.

To see the maximum energy density reached in most massive NSs and test the conjecture that the Tolman VII analytical solution of Einstein’s field equations mark the upper limit to the energy density inside a star [74], shown in Fig. 7 are the masses versus central energy densities for the most massive NSs on the causality surface. The labels are the same as we used in Fig. 5. For a comparison, as in ref. [74], the lower limit using the earlier reported redshift of \( z=0.35 \) from the x-ray bursting source XTE J1814-338 [75] is also shown. It was shown that all analytic solutions of Einstein’s field equations for fixed compactness obey the scaling \( \epsilon_{\text{cent}} \propto 1/M_{\text{max}}^2 \) and the Tolman VII solution giving \( \epsilon_{\text{cent}} \cdot M_{\text{max}}^2 = 15.3 \times 10^{15} \text{ M}_\odot \text{ g cm}^{-3} \) is the upper limit [74]. Our results for NSs above the minimum maximum mass of 2.01 M\(_{\odot} \) approximately follow this scaling. Indeed, our results fall between the two limits considered. It is useful to know that for \( M_{\text{max}} \geq 2.01 \text{ M}_\odot \), the maximum energy density is between about 1100 to 1700 MeV/fm\(^{-3} \). For the absolutely maximum mass NS with 2.4 M\(_{\odot} \), the central energy density is about 1200 MeV/fm\(^{-3} \) independent of the symmetry energy parameters. As the \( M_{\text{max}} \) decreases to significantly below 2.01 M\(_{\odot} \), effects of the symmetry energy parameters become larger mainly through their influences on the radii as we explained earlier.

In summary of this section, neutron stars’ absolutely maximum mass is found to be 2.4 M\(_{\odot} \), from investigating the causality surface. It is approximately independent of the EOSs used. Depending on the high-density behavior of nuclear symmetry energy, the \( M_{\text{max}} \) ranges between 2.01-2.4 M\(_{\odot} \). The causality surface together with the mass of PSR J0348+0432 set a lower boundary in the plane of \( K_{\text{sym}} \) versus \( J_{\text{sym}} \), limiting the high-density behavior of nuclear symmetry energy. They also limit the radii and maximum density reachable in NSs. These causality limits will be respected and used in our following analyses.

4 Observational restrictions on the parameter space of high-density nuclear EOS and symmetry energy

Since the pioneering works of Lindblom [76] as well as Lat-timer and Prakash et al. [31], extensive studies by many people have confirmed that important constraints on the EOS can be obtained with even a single radius measurement, if it is accurate enough, and that the quality of the constraint is not very sensitive to the mass [32]. While it is necessary to have a series of mass and radius measurements to accurately constrain completely the dense matter EOS [70]. Significant efforts and much progress have been made in measuring/calculating the radii of NSs using various probes/models while some issues/uncertainties remain to be resolved/reduced. In this section, using widely accepted results of mass and radius measurements available in the literature, we examine how they may restrict the 3D EOS parameter space in \( K_{\text{sym}}, J_{\text{sym}} \) and \( J_0 \). Moreover, we shall obtain crosslines of surfaces representing constant radii and the minimum maximum mass of NSs as well as the causal limit. These crosslines will be used in the next section to set boundaries for the high-density EOS and symmetry energy.

Let us first summarize what we have learned about the radii of NSs before trying to use them to constrain the high-density EOS and symmetry energy. The radii of NSs have been measured using several approaches. For example, the radii of 14 NSs have been extracted from
analyzing thermal emissions from quiescent low-mass X-ray binaries (QLMXBs) and photospheric radius expansion (PRE) bursts [77,78,79,80,81,82]. While there are still some variations and issues in different analyses of the same data, see e.g., ref. [83] for a recent review, the radius $R_{1.4}$ of canonical $1.4$ M$_\odot$ NSs was found generally to be around $10.62 \leq R_{1.4} \leq 12.83$ km [67]. The upper limit of the dimensionless tidal deformation $A_{1.4} \leq 800$ for canonical NSs from the first analysis of GW170817 by the LIGO+Virgo Collaborations [61] has been used in a number of studies to extract the $R_{1.4}$. While the lower limit has somewhat larger variations in different studies, a rather consistent upper limit of about $R_{1.4} \leq 13.7$ km has been found [63-65,66-68,69-70]. An improved analysis of the GW170817 event by the LIGO+Virgo Collaborations has now provided both the upper and lower limits for the tidal polarizability. The $A_{1.4}$ is found to be in the range of $70-580$ [38], leading to a refined constraint on the radius in the range of $10.5-13.3$ km. We notice that an independent reanalysis of the GW170817 data has put the $A_{1.4}$ in larger ranges depending on the mass priors used in their Bayesian analyses. However, a common radius of $8.7 \leq R_{1.4} \leq 14.1$ km across all mass priors was reported [92]. Some explanations for the differences in the two analyses of the GW170817 data are given in refs. [85,92]. It is also very interesting to mention that $R_{1.4} \leq 14.4$ km was obtained earlier from studying the fastest-spinning radio pulsar ever found and confirmed, the PSR-J1748-2446ad spinning at 716 Hz [93].

To this end, we emphasize since it is often overlooked that terrestrial nuclear experiments, especially heavy-ion reactions with radioactive beams at intermediate energies, do provide strong constraints on the EOS of dense neutron-rich matter and subsequently the radii of NSs as reliably as astrophysical observations mentioned above, see, e.g. refs. [11,12] for earlier reviews and ref. [94] for an example of very recent studies. As an earlier example from 2006, using the MDI (Momentum-Dependent Interaction) energy density functional [95,96,97] with its EOS for SNM constrained up to about 4.5$\rho_0$ by nucleon collective flow and kaon production data [1] in relativistic heavy-ion collisions, and its symmetry energy term constrained up to about 1.2$\rho_0$ by isospin diffusion data [98] in intermediate energy heavy-ion collisions then extrapolated to higher densities based on the MDI energy density functional, the radius of canonical NSs was predicted to be $11.5$ km $\leq R_{1.4} \leq 13.6$ km [65]. The same MDI EOS was also used to place bounds on the quadrupole deformation of isolated neutron stars, strain amplitude as well as the frequency and damping time of several modes of NS oscillations which are all potential sources of gravitational waves [99,100,101,102]. Ironically, given all the existing uncertainties in every method used to extract the radii of NSs, the predicted radius range using the MDI EOS partially constrained by the terrestrial nuclear reaction experiments is in very good agreement with the astrophysical observations mentioned above.

Overall, it is very encouraging to see that the constraints on the radius $R_{1.4}$ especially its upper limit from all analyses are rather consistent. This impressive consistency further validates and signifies the multi-messengers approach of constraining the EOS of dense neutron-rich matter using all available probes in both astrophysical observations and terrestrial experiments. In the following, we adopt the range $10.62 \leq R_{1.4} \leq 12.83$ km from analyzing X-rays [67] as an example in constraining the high-density EOS parameters. This range is almost identical to that of $10.42 \leq R_{1.4} \leq 12.80$ km extracted from a very recent analysis of the tidal polarizability in GW170817 within a Bayesian statistical framework incorporating constraints on the EOS from laboratory measurements of nuclei and state-of-the-art chiral effective field theory methods [91].

Since it is generally understood that the tidal polarizability is more sensitive to the EOS because of its dependence on $R^2$, while its current uncertainty range from the single event GW170817 is still relatively larger than those in measuring $R$ using some other approaches, we study separately in the left and right blocks of Fig. 8 how the existing results of $A_{1.4}$ and $R_{1.4}$ together with the mass measurements and causality condition can limit the 3D EOS parameters. For a given observable, we perform an inversion of the TOV equation by looping through the 3D EOS parameters. While a single observable can often restrict the parameter space, large degeneracies in the EOS parameters are expected. In fact, a constant surface of a given observable can only give the required combinations of the EOS parameters to reproduce the observed value. The crosslines of two constant surfaces reduce the degeneracies. To uniquely determine all three parameters would require at least three surfaces to cross at a point. This is consistent with the earlier expectation that the measurements of masses and radii of 2-3 NSs are necessary to completely determine the EOS of NS matter [76].

In the left block of Fig. 8, the magenta surface of a constant $A_{1.4} = 580$ is the upper limit of tidal polarizability from the improved analysis of GW170817 by the LIGO+Virgo Collaborations. It is rather vertical and covers the whole range of both $J_0$ and $J_{\text{sym}}$ considered, but spans a narrow range in $K_{\text{sym}}$. Since the $J_0$ and $J_{\text{sym}}$ are coefficients of the $(\rho/\rho_0 - 1)^2$ term in our parameterizations of the EOS, it is easy to understand that the $A_{1.4}$ is not sensitive to really high-density EOS as the average density reached in $1.4$ M$_\odot$ NSs are considered intermediate that is not high enough for the $J_0$ and $J_{\text{sym}}$ to play really big roles. On the other hand, the $K_{\text{sym}}$ as the coefficient of the $(\rho/\rho_0 - 1)^2$ term in parameterizing the symmetry energy plays a bigger role at the intermediate densities reached in canonical NSs. This means that the upper limit of $A_{1.4}$ is insensitive to the EOS of SNM but can limit the symmetry energy to a narrow region through its constraints on the $K_{\text{sym}}$ parameter. Very low values of $J_0$ and/or $K_{\text{sym}}$ as well as $J_{\text{sym}}$ together make the EOSs too soft to support NSs with masses as high as $1.4$ M$_\odot$. In fact, only regions with large $J_0$ can reach the lower limit of $A_{1.4} = 70$ if the symmetry energy is very soft. To make our presentations clear while still convey the main physics, we use the yellow surface to indicate the integrated parameter space leading to $70 \leq A_{1.4} \leq 415$. 
Fig. 8. (color online) Observational restrictions on the 3 dimensional EOS parameter space in $K_{\text{sym}}$, $J_{\text{sym}}$ and $J_0$. Left: the magenta and yellow surfaces have the tidal polarizability for canonical neutron stars $A_{1.4} = 580$ and $70 \leq A_{1.4} \leq 415$, respectively. On the light-brown surface, the maximum mass of neutron stars is set at $M_{\text{max}} = 2.17 M_\odot$. Right: the green, magenta, yellow and blue surfaces represent $M_{\text{max}} = 2.01 M_\odot$, $R_{1.4} = 12.83$ km, $R_{1.4} = 10.62$ km and the causality surface, respectively.

The causality surface (blue) limits the $J_0$ from the above. Interestingly, however, the $J_0$ may not be able to go as high as the causality surface indicates. As we mentioned earlier, several studies have estimated the maximum mass of the possible super-massive remnant produced in the immediate aftermath of GW170817. For example, using electromagnetic constraints on the remnant imposed by the kilonova observations and the gravitational wave information, Margalit and Metzger [40] found a maximum mass of $M_{\text{max}} \leq 2.17 M_\odot$ with 90% confidence. The constant surface of this mass is shown in light brown. It is seen that the crossline of this surface with the causality surface provides a lower boundary from the right for the $K_{\text{sym}}$-$J_{\text{sym}}$ relation significantly tighter than that along the crossline of causality and the condition $70 \leq A_{1.4} \leq 415$. Moreover, if confirmed, the $M_{\text{max}} \leq 2.17 M_\odot$ together with the minimum maximum mass of $2.01 M_\odot$ shown as the bottom surface in the right block will limit the $J_0$ to the range of about $-200 \pm 25$ MeV.

In the right block of Fig. 8 the green, magenta and yellow surfaces represent $M_{\text{max}} = 2.01 M_\odot$, $R_{1.4} = 12.83$ km and $R_{1.4} = 10.62$ km, respectively. The allowed regions are indicated by the arrows. First of all, comparing the two magenta surfaces in the left and right blocks, it is seen that the one representing $A_{1.4} = 580$ is more vertically oriented than the one on the right representing $R_{1.4} = 12.83$ km, indicating that a larger region in $K_{\text{sym}}$ can be used to give the same radius compared to the one needed to give the same tidal polarizability. While both surfaces span the whole ranges of $J_0$ and $J_{\text{sym}}$, considered. This confirms the expectation that the tidal polarizability is more sensitive to the symmetry energy than the radius itself and none of them is much affected by the high-density EOS of SNM. It is interesting to see that the causality and $M_{\text{max}} = 2.01 M_\odot$ surfaces together not only limit the range for $J_0$ from the top and bottom, respectively, their crossline also determines the $K_{\text{sym}}$-$J_{\text{sym}}$ relation along the right boundary. It is seen that the lower limit $R_{1.4} = 10.62$ km of the radius is actually outside this boundary. Thus, if

Fig. 9. (color online) Boundaries of the allowed $K_{\text{sym}}$-$J_{\text{sym}}$ plane obtained from the crosslines shown in the right window of Fig. 8.
one believes in the causality and the measured minimum maximum mass of $M_{\text{max}} = 2.01 \, M_\odot$, the reported small radii of NSs less than about 10 km is clearly ruled out at least within the rather general model framework of this work. On the hand, it is seen that the crossline of the $M_{\text{max}} = 2.01 \, M_\odot$ and $R_{1.4} = 12.83 \, \text{km}$ surfaces provide a limit on the $K_{\text{sym}}$-$J_{\text{sym}}$ relation from the left. The results shown in this block signifies the need of improving the accuracy of radius measurements especially regarding the lower limit of $R_{1.4}$.

If we consider the tidal polarizability from GW170817 and the radii from other measurements as independent and assume they are all equally accurate, comparing the results in the two blocks of Fig. [8] it is seen that the radius constraint $R_{1.4} \leq 12.83 \, \text{km}$ provides a slightly tighter constraint on the $K_{\text{sym}}$-$J_{\text{sym}}$ boundary than the condition $A_{1.4} \geq 580$. While the lower limits of both $R_{1.4} = 10.62 \, \text{km}$ and $A_{1.4} \geq 70$ do not provide any additional constraints as they are both outside the crossline between the causality surface and the minimum maximum mass constraint $M_{\text{max}} \geq 2.01 \, M_\odot$. In the following, we thus use the radius $R_{1.4} \leq 12.83 \, \text{km}$ instead of $A_{1.4} \geq 580$ to set the left boundary in the $K_{\text{sym}}$-$J_{\text{sym}}$ plane. This choice is also physically necessary as we are interested in making an unbiased comparison of the pressure extracted independently from our analyses directly using the radius data with that extracted from GW170817 by the LIGO+Virgo Collaborations. Using the $A_{1.4}$ in our following analyses would introduce self-correlations. In addition, to ensure the thermodynamical stability through out the star, we also use the condition that the transition pressure $P_\tau$ at the crust-core transition density is always positive as we discussed earlier. This latter condition has been discussed in detail in our earlier work in ref. [45] where the crust-core transition point was found by explicitly investigating where the incompressibility of NS matter start to become negative.

Under the above conditions, the allowed region in the $K_{\text{sym}}$-$J_{\text{sym}}$ plane is shown in Fig. [9] as the hatched area. Different segments obtained from the crosslines of two different observables and/or physical conditions are labeled, separately. It is seen that the $J_{\text{sym}}$ is still completely unconstrained with respect to its known uncertain range in the literature. This will lead to uncertainties in our extracted symmetry energy work when the $(\rho/\rho_0 - 1)^3$ term becomes significant enough. Nevertheless, since the correlations between the $K_{\text{sym}}$ and $J_{\text{sym}}$ are constrained along the boundaries, the uncertainty due to $J_{\text{sym}}$ is not as big as if it is allowed to vary freely between -200 and 800 MeV. The above observational boundaries will be used to set limits on the high-density EOS and symmetry energy as we shall discuss in detail next.

5 Combined astrophysical constraints on the pressure in neutron star matter

Within the allowed region in the $K_{\text{sym}}$-$J_{\text{sym}}$ plane, the $J_0$ is limited from above by the causality surface and below by the mass constraint $M_{\text{max}} \geq 2.01 \, M_\odot$, respectively. The pressure of NS matter at $\beta$ equilibrium in this allowed 3D space is calculated using the formalism given in sect. [2]. It is shown as a function of energy density in the two windows of Fig. [10]. While the symmetry energy contributes to the pressure and determines the composition of NSs at $\beta$ equilibrium, the total pressure calculated self-consistently is dominated by the SNM contribution at high densities. The upper and lower boundaries of the pressure are thus dominated by the causality and mass constraint $M_{\text{max}} \geq 2.01 \, M_\odot$ through the $J_0$ parameter, while the variation of the spread in pressure is due to the variation of all three high-density EOS parameters.

The inset in the left window is the pressure as a function of baryon density in comparison with the pressures at 90% confidence level extracted by the LIGO+Virgo Collaborations. In our approach, the upper and lower boundaries represent 100% confidence level of finding the pressure to be between them. With this understanding, the overlap of the two results is overwhelming. Since we did not use the tidal polarizability in deriving the constraining band on pressure, there is no self-correlation in the comparison with the LIGO+Virgo result. On the other hand, it is also not surprising since a number of independent analyses have extracted from the tidal polarizability of GW170817 the upper limit on the radius $R_{1.4}$ consistent with the value $R_{1.4} \leq 12.83 \, \text{km}$ we used in deriving the pressure.

It is also useful to compare the extracted pressure band with predictions using EOSs available in the literature in the right window of Fig. [10]. We only selected typical EOSs that can support a maximum mass of NSs higher than 2.01 $M_\odot$. The EOSs are ALF2 of Alford et al. [103] for hybrid (nuclear + quark matter) stars, APR3 and APR4 of Akmal and Pandharipande [104], ENG of Engvik et al. [105], MPA1 of Muther, Prakash and Ainsworth [106], SLy of Douchin and Haensel [107], WWF1 and WWF2 of Wiringa, Fiks and Fabrocini [25], the QMFL40, QMFL60 and QMFL80 within the Quark Mean Field model with $L=40$, 60 and 80 MeV, respectively, from the recent work of Zhu et al. [108]. The individual predictions of the maximum masses are in parentheses following the model names in Fig. [10]. The black symbols indicate the maximum pressure and energy density reached at the maximum mass supported by the EOS of the mode. While the constraining band for the pressure is still quite wide especially at high energy densities, predictions of several of these theoretical EOSs are too stiff to be bounded by the constraining band from our analyses of the astrophysical observations.

6 Combined astrophysical constraints on nuclear symmetry energies at high densities

We now turn to the combined astrophysical constraints on nuclear symmetry energies at high densities. This is achieved by examining the $E_{\text{sym}}(\rho)$ functions using limiting $K_{\text{sym}}$ and $J_{\text{sym}}$ parameters on the constraining boundaries shown in Fig. [9]. The shaded area is allowed by all as-
Fig. 10. (color online) Left: the pressure as a function of energy density (baryon density in the inset) in neutron star matter at β equilibrium extracted in this work from the astrophysical observational constraints (shaded regions). A comparison with the LIGO+Virgo result at 90% confidence level (red boundary) is shown in the inset. Right: Comparisons of the extracted pressure in this work with the predictions of several EOS models predicting maximum masses higher than 2.01 M⊙. The maximum masses predicted are in parentheses following the model names. The black symbols indicate the maximum pressure and energy density reached at the maximum mass predicted in the specified model.

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Left: demonstrations of how each segment of the upper and lower boundaries of the symmetry energy are obtained using the boundaries in the $K_{\text{sym}}-J_{\text{sym}}$ plane shown in Fig. 10. Right: a comparison with the Prakash, Ainsworth and Lattimer parameterizations of nuclear symmetry energy.

In a number of astrophysics studies, the PAL (Prakash, Ainsworth and Lattimer) parameterizations:

$$E_{\text{sym}}(\rho)(\text{stiff}) = 12.7(\rho/\rho_0)^{2/3} + 38(\rho/\rho_0)^{2/(1 + \rho/\rho_0)},$$

$$E_{\text{sym}}(\rho)(\text{soft}) = 12.7(\rho/\rho_0)^{2/3} + 19(\rho/\rho_0)^{1/2},$$

are often used as examples of the “stiff” and “soft” nuclear symmetry energies. A recent comparison indicates that one can not clearly distinguish the two PAL parameterizations in the density range of $1 - 3\rho_0$. At higher densities, there are some weak indications that the stiff symmetry energy is preferred. However, in both the analyses of ref. [12] and ref. [35] polytropes knowing nothing about the underlying $E_{\text{sym}}(\rho)$ are used to parameterize directly the pressure in the core. As we have shown in Fig. 10 the pressure extracted from our analyses also agree well with that from the LIGO+Virgo Collaborations. We found that the upper/lower bound on the pressure is mainly determined by the uncertainty of the SNM EOS through the $J_0$ parameter. Since the $E_{\text{sym}}(\rho)$ has much less effects on the pressure itself at supra-saturation densities, it is thus hard to extract any reliable information about the high-density $E_{\text{sym}}(\rho)$ from directly comparing the pressures. In fact, it was shown numerically in Fig. 146 of ref. [12] that the isospin-asymmetric pressure $P_{\text{sym}}$ in NSs at $\beta$ equilibrium dominates only near the saturation density. Above a transition density around $\rho_{\text{transition}} = 1.3\rho_0 - 2.5\rho_0$ depending on the stiffness of the symmetry energy, the total pressure is clearly and increasingly determined by the contribution from symmetric nuclear matter. Thus, there is a better chance to learn something about the $E_{\text{sym}}(\rho)$ at densities below the $\rho_{\text{transition}}$ from directly comparing the pressures inferred from heavy-ion reactions at intermediate energies and gravitational waves [94]. Our explicitly isospin-dependent parameterizations of the EOS at a more basic level through the specific nucleon energies in neutron-rich matter enabled us to extract more accurately and self-consistently the whole $E_{\text{sym}}(\rho)$ underlying the pressure as a function of density. A comparison of the PAL parameters with the $E_{\text{sym}}(\rho)$ constraining band in the right window of Fig. 10 clearly favors PAL’s $E_{\text{sym}}(\rho)(\text{soft})$ parameterization.

The strong power of our extracted constraining band on the $E_{\text{sym}}(\rho)$ in distinguishing existing model predictions is clearly demonstrated in Fig. 14. Although the band is still quite wide above about $2.5\rho_0$, like it or not, a large number of predictions especially those based on phenomenological models or energy density functionals run out the constraining boundaries. Compared to the spread of predicted $E_{\text{sym}}(2\rho_0)$ values between 15 MeV to 100 MeV by various theories shown in Fig. 10 the value of $E_{\text{sym}}(2\rho_0) = 46.9 \pm 10.1$ MeV extracted here represents a significant progress in the field.

It is worth emphasizing that in our current study the most probable values of the empirical saturation properties are used as we mentioned earlier. All of them still have some uncertainties. Since the lower bound we extracted is from the crossline of causality and the maximum mass condition $M_{\text{max}} \geq 2.01M_\odot$ and none of them depends sensitively on the saturation properties, the lower bound is expected to be approximately the same if we
loose the constraints on the saturation properties. On the other hand, since the upper bound is from the conditions of $M_{\text{max}} \geq 2.01 \, M_\odot$ and $R_{1.4} \leq 12.83 \, \text{km}$ and the radius is known to have some dependences on the $L$ parameter, if we lose the constraint on the later, the upper bound is expected to be altered. While we expect this possible modification is small and our bounds are already rather conservative, an investigation of the probability distribution functions of all the $E_{\text{sym}}(\rho)$ parameters within the Bayesian framework is underway. Effects of all uncertainties and their correlations will be studied and reported elsewhere. The upper/lower bound we extracted here from basically only two NS observables represents the most probable bound under the conservative conditions used.

7 Summary and outlook

In summary, the density dependence of nuclear symmetry energy $E_{\text{sym}}(\rho)$ is the most uncertain part of the EOS of neutron-rich nucleonic matter especially at supra-saturation densities. Essentially all available nuclear many-body theories and interactions have been used to predict the $E_{\text{sym}}(\rho)$. However, the predictions diverge widely especially at high densities. Among the difficulties of predicting accurately the $E_{\text{sym}}(\rho)$ are our poor knowledge about the weak isospin-dependence of strong force, the spin-isospin dependence of three-body nuclear forces and the tensor-force induced isospin-dependence of short-range nucleon-nucleon correlations in dense matter besides the challenges of solving accurately nuclear many-body problems. It is well known that the $E_{\text{sym}}(\rho)$ has many important effects on properties of neutron-rich nuclei and nuclear reactions. It also affects some properties of NSs and gravitational waves from sources/events involving NSs.

Using an explicitly isospin-dependent parametric EOS with three parameters characterizing nucleon specific energy in dense neutron-rich matter, for a single given NS observable, such as its mass, radius or tidal polarizability we can find all required combinations of the EOS parameters by inverting numerically the TOV equation. Compared to the widely used isospin-independent polytropes of pressure as a function of energy or baryon density for the core of NSs, our isospin-dependent parameterization of nucleon specific energy is at a more basic level and necessary for extracting the underlying nuclear symmetry energy. Applying our approach using observational constraints on the radii, maximum mass and tidal polarizability of NSs as well as the causality condition all together, we have learned the following new and important physics compared to the existing knowledge in the literature:

- By studying the variation of causality surface where the speed of sound is the same as that of light at central densities of the most massive NSs within the uncertain ranges of high-density EOS parameters, the absolutely maximum mass of NSs is found to be $2.40 \, M_\odot$ approximately independent of the EOSs used. This limiting mass is consistent with the findings of several recent analyses about the maximum mass of the possible super-massive remanent produced in the immediate aftermath of GW170817.

- Boundaries are established for the high-density EOS parameter space by examining the crosslines of the minimum maximum mass $M_{\text{max}} \geq 2.01 \, M_\odot$ and the radius range of $R_{1.4} = 10.62-12.83 \, \text{km}$ for canonical NSs as well as the causality surface. These boundaries lead to constraining bands for both the pressure as a function of energy (baryon) density and the density dependence of nuclear symmetry energy. Our EOS presented using pressure as a function of energy density is in good agreement with that extracted recently by the LIGO+Virgo Collaborations from their improved analyses of the NS tidal polarizability in GW170817.

- The pressure constraining band is also compared with predictions of several typical EOSs available in the literature. Several predictions were found to run out of the constraining band at high densities. The pressure boundaries are mostly determined by the minimum maximum mass and causality conditions with little influences from variations of nuclear symmetry energy, making it difficult to extract any reliable information about the high-density nuclear symmetry energy from studying directly the total pressure of NS matter itself.

- Rather robust upper and lower boundaries for the symmetry energy are extracted up to about $2.5 \rho_0$ while at higher densities the boundaries suffer some uncertainties. The upper bound is obtained from the crosslines of the $R_{1.4} = 12.83 \, \text{km}$ and $M_{\text{max}} = 2.01 \, M_\odot$ surfaces in the 3D EOS parameter space, while the lower one is from the crossline of the causality and $M_{\text{max}} = 2.01 \, M_\odot$ surfaces. Many available predictions for nuclear symmetry energy run out of the extracted boundaries at various densities. The symmetry energy at $2 \rho_0$ is constrained to $E_{\text{sym}}(2 \rho_0) = 46.9 \pm 10.1 \, \text{MeV}$ excluding many of the existing predictions scattered between $E_{\text{sym}}(2 \rho_0) = 15$ and 100 MeV. Thus, the $E_{\text{sym}}(\rho)$ at supra-saturation densities extracted in this work from observations of NSs represent a significant progress in the field compared to the prior knowledge in the literature.

At densities higher than about twice the saturation density of nuclear matter, the symmetry energy is still not well constrained by the astrophysical observables and physics conditions used here. To narrow down the $E_{\text{sym}}(\rho)$ above $2 \rho_0$, independent measurements of other observables and/or improvements of the accuracy of radius measurements are necessary. In addition, ongoing efforts using high-energy heavy-ion reactions at several radioactive beam facilities may also help further constrain nuclear symmetry energies above twice the saturation density. We are hopeful that eventually the multi-messengers approach of combining probes in both astrophysical observations and terrestrial experiments will lead us to a narrow stripe of nuclear symmetry energies at high densities.
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