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Educational Investments in a Dual Economy

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Abstract:

This paper presents a simple two-period, dual economy model in which migration options may affect the informal financing of educational investments. When credit contracts are universally available and perfectly enforceable, spatially varied returns to human capital have no effect on educational investment patterns. But when financial markets are incomplete and informal mechanisms subject to imperfect contract enforcement must fill the breach, spatial inequality in infrastructure or other attributes that affect the returns to education create spatial differentiation in educational lending and consequently, in educational attainment. Although migration options can increase the returns to education, they can also choke off the informal finance on which poorer rural households depend for long-term, lumpy investments like children’s education.

JEL classification: R23; O15
Keywords: Poverty traps; Informal finance; Education; Rural-Urban migration
INTRODUCTION

The positive relationship between education and expected future income is well established (Schultz 1988, Strauss et al. 1995, Barro and Sala-i-Martin 1995, Psacharopoulos 1985). Yet, despite clear evidence of strong returns to education, many communities exhibit low rates of educational attainment, especially in rural areas of the developing world (Singh 1992, Psacharopoulos 1985). One reason for the apparent underinvestment in children's education is imperfect financial markets that ration poorer households out of the formal market for long-term loans. As Loury (1981) showed, when formal financial markets fail, the logical consequence is not only underinvestment in education but also, derivatively, the propagation of poverty from one generation to the next. Credit market failures, coupled with costly education, limit the poor’s ability to purchase optimal levels of education. The relationship between education and income is thus reversed, generating a poverty trap whereby the poor attain low levels of education due to financial constraints and consequently can expect meager future earnings due to educational deficiencies.

Why, however, don't informal financial markets spring up to fill the educational financing gap when formal markets fail? Elaborate informal credit and insurance mechanisms exist between households, providing finance not available through formal financial institutions (Udry 1993, Townsend 1994, Besley 1995, Morduch 1995). Given the high apparent returns to education and widespread anecdotal evidence of informal financing of others' education, one naturally wonders why informal financial transactions do not resolve the educational investment problem in rural areas of developing countries.

This paper offers an answer to that puzzle. We show that in the presence of financial market imperfections associated with imperfect credit contract enforcement,
spatial variation in the returns to education can induce migration decisions that rationally choke off the informal financing of education in relatively disadvantaged areas. When financial markets are complete and perfect, spatially varied returns to human capital have no effect on educational investment patterns, and they are then Pareto optimal. But when formal financial markets are incomplete and credit contracts must be self-enforcing, spatial inequality in infrastructure and other attributes that increase the returns to education create spatial differentials in educational lending and, consequently, greater geographic and wealth-based variation in educational attainment than would otherwise occur.

The important innovation of this paper is to link the literature on spatially varied productivity and migration with that on informal finance. The extensive literature on migration emphasizes how spatially varied infrastructure, law enforcement, access to lucrative markets and other attributes creates a gradient across space in real returns to education (Banerjee et al. 1998, Stark 1984, Todaro 1997, Williamson 1988). Migration to areas with greater community endowments is an especially attractive option for educated persons living in relatively disadvantaged rural areas with few opportunities for skilled employment (Barnum and Sabot 1975, Schultz 1988). One of the most consistent findings in this literature is of the positive relationship between educational attainment and rural-urban migration (Todaro, 1997).

On the other hand, the literature on informal finance identifies the close-knit associations of traditional communities as the ‘social capital’ that allows for the provision of financial services in informal settings (Stiglitz 1991, Besley et al. 1993). Lenders can access relatively cheap information on potential borrowers due to highly personalized intra-community relationships. They can also assure repayment by the credible threat of
social sanctions: a borrower who is visibly able to pay but neglects his loan commitment will signal dishonesty, thereby eroding his stock of social capital within the community.

Contract enforcement, however, becomes more difficult the farther the contracting parties are from each other. Tracking down debtors becomes costly and as their interaction with the community is diminished, the threat of social sanctions loses some of its power. Prospective rural lenders would thus take borrowers’ migration options into account when deciding whether to extend an educational loan and, if so, for how much and to whom. Put differently, informal financial market equilibria depend on migration incentives. As a consequence, as the spatial differential in the returns to human capital grows, it may choke off informal financing of education in rural areas as lenders increasingly expect borrowers to migrate, making them greater risks for default. In this paper, we develop a theoretical model that demonstrates this explanation for the apparent underinvestment in rural education.

The rest of the paper is structured as follows: Section 2 builds the general structure of a simple two-period, dual economy model that parsimoniously captures the essence of the problem. In section 3, we explore the implications of the model for patterns of educational investment and migration and examine the inefficiencies resulting from credit conditions that deviate from the first best world. Section 4 discusses the policy implications of our findings and concludes.

2 THE MODEL

Consider a two-period dual economy. In period one, the adult household head makes educational investment decisions in the children in the community (no one invests in children outside their own community). Then, in period two, the (now grown) children
make residential/work decisions conditional on the human capital they accumulated in period one.

The economy consists of two locations: A rural area with weak productive infrastructure that represents a more traditional mode of production and an urban area which represents settings enjoying better communications, power, transport and public services that underpin modern industrial and service economies. As such, returns to education are higher in the urban area. We treat the differences in productive infrastructure across locations as exogenous and assume that human capital productivity is increasing in infrastructure. This spatial variation in the returns to education generates incentives to migrate and geographic variation in private education investment, especially in the absence of perfect credit contract enforcement.\textsuperscript{i}

Assume there are $j = 1, ..., N$ households in the rural village, each with one adult decision maker and one child. Each adult decision maker is endowed with wealth $w_j$ and each child with a random assignment of some innate ability $\alpha_j$, where $\alpha \in [0,1]$. Given knowledge of the distribution of abilities across all children in the village, in period one the adults choose (non-cooperatively) how to split their wealth between educating their own children, investing in the education of other children in the village at a given net interest rate $r$, or holding it in the form of a composite, alternative asset that pays marginally less than $r$.\textsuperscript{ii} At the outset of period two, each now-adult child makes a decision as to where to live and work.

As we are mainly concerned with demonstrating how migration induced by spatial differences in the returns to education leads to rural underinvestment in education by crippling informal finance mechanisms, we make some strong assumptions. Following Banerjee and Newman (1998), we assume that once an individual migrates,
they free themselves of their obligations to non-kin in their original, rural community. This assumed distinction between kin and non-kin derives from an observed, qualitative difference between taking advantage of distance and relative anonymity to default on informal loans provided by non-kin community members and the breaking of ties or responsibility to family. In a comprehensive survey of the relevant literature, Remple and Lobdell (1978) find that a substantial majority of urban remittances go to the household of the migrant with village elders being the only non-kin that receive a significant share of remittances. We incorporate this distinction into our model by allowing households to derive material or non-material (i.e., altruistic) benefits from their child’s income regardless of whether the child migrates.

One way non-family community members can assure returns to their investment is by tracking down emigrants in urban areas and demanding repayment or reciprocity, such as using their home as a base for developing their own ties in the urban area. While emigrants might default on their loan commitment, it is more difficult for them to completely escape traditional norms that call for hospitality and the provision of food and shelter to natal community members who request it. In this way, emigrants can act as ‘beachheads’ for the rural community, establishing a foundation that facilitates greater rural-urban interaction. By utilizing emigrants for this purpose, community lenders can recoup some of their otherwise lost investment.

But while lenders can tap into the benefits emigrants provide to recover part of their loans, the ‘beachhead’ effect alone does not alter a potential lender’s loan decision ex ante because community norms generally require the emigrant to oblige any natal community member who requires assistance in the city, not just those who have extended him credit in the past. So long as emigrants cannot exclude any community members
from assistance, then each potential lender in the rural community has an incentive to free
ride on the ‘beachhead’ opportunity sponsored by some other lender due to the non-
exclusivity of the service being offered. In the interests of simplicity, we therefore
assume away beachhead effects in our model, as they do not affect the qualitative results.

2.1 The Child’s Problem

We follow the standard solution technique of backward recursion, solving the
child’s period two migration decision first, then solving the adults’ first period
educational investment decision conditional on the child’s subsequent best response. Let
$E_{ji}$ denote the educational attainment of child $j$ resulting from an investment by
household $i$. Then let $h_j = \left( \sum_{i=1}^{N} E_{ji} \right) \alpha_j$ be the level of human capital of child $j$, where

$$\sum_{i=1}^{N} E_{ji}$$

represents the total level of education attained by child $j$ by summing up the
contribution of all households in the community to his education. Thus we allow for a
child to have any portion of his education financed by other households. The labor
productivity of a child with human capital $h_j$ is then given by the strictly concave,
monotone and twice differentiable function $\rho(h_j)$. An individual whose productivity is
$\rho(h)$ in the village has an increased productivity level $\lambda \rho(h)$ in the city, where $\lambda > 1$ and
reflects the higher returns to human capital in urban areas.

In the event that their parent’s wealth is insufficient to cover their optimal level of
education, children may have to seek educational loans in period one from other
households. In the absence of credit markets with perfect, exogenous contract
enforcement, children can renege on these loans in period two. For the sake of simplicity
in the model, we assume that the child tries to renege on any loans received from other
households if and only if he migrates to the city. This can provoke retribution, however, which, following Banerjee and Newman (1998), we model as maximal punishment from the village which serves to hold the migrants productive capacity to 0.\textsuperscript{iv} We denote as $1 - \pi$ the probability of catching a reneging child. Educated children will rationally migrate and renege on their educational loan contracts when there is significant spatial variation in the returns to education $\lambda$, the costs of migration $c$ are low and enforcement of loan contracts is weak (i.e., $\pi$ is high).

Suppose a child with human capital $h_j$ stays in the village. His net earnings will then be $\rho(h_j) - (1 + r)P_E \sum_{i \neq j} E_{ji}$ where $r$ is the net interest rate and $P_E$ is the cost of a unit of education.\textsuperscript{v} Should the child decide to migrate, his expected gross earnings will be $\pi \lambda \rho(h_i)$ and he incurs a migration cost, $c$. The migration cost $c$ incorporates both the financial costs of relocation as well as the social costs that result from a loss of social relationships that may be intrinsically as well as instrumentally important. The child’s second period choice is thus quite simple:

$$\text{Max } (\rho(h_j) - (1 + r)P_E \sum_{i \neq j} E_{ji}, \pi \lambda \rho(h_i) - c) \quad (1)$$

Adults make educational investments in children fully knowing this calculus of migration in which children will subsequently engage.

\textbf{2.2 Adult’s Problem.}

All the adults in the village can observe each child’s innate ability by the time they need to make educational investments.\textsuperscript{vi} In deciding how to allocate resources between educating their child and investing in the education of other children, an adult considers
the returns to each investment option, taking into consideration the possibility that children leave the area and subsequently renege on their loan contracts.

Let $E_j \equiv [E_{ij},...,E_{Nj}]$ be the vector of educational units provided to each child $i = [1,...,N]$ by household $j$, and $E^j \equiv [E_{j1},...,E_{jN}]$ be the vector of all educational units received by child $j$ from each household $i = [1,...,N]$. Note that the first subscript indexes the child and the second the household. The adult household head’s first period decision problem can then be characterized by

$$\max_j w_j - \sum_{i=1}^N E_{ij}P_E + \delta(1+r)P_E \sum_{i=j}^N E_{ij} + \delta \beta Y_j$$

$$\delta, \beta \in (0,1) \quad (2)$$

subject to:

$$Y_j = \max (\rho(h_j) - (1+r)P_E \sum_{j \neq i}^N E_{ij}, \lambda \rho(h_j) - c) \quad (3)$$

$$P_E E_j \leq w_j \quad (4)$$

$$E_{ij}[\rho(h_j) - (1+r)P_E (E_i - E_{ij}) - \pi \lambda \rho(h_j) + c] \geq 0 \quad \forall i \neq j \quad (5)$$

where $\delta$ is a discount factor reflecting current valuation of lagged repayments and of the child’s future income. Note that a household’s expenditure on the education of its own child indirectly affects its well being via the function $\delta \beta Y$. The household’s utility increases in its child’s future productivity given by equation (3). The function $\delta \beta Y$ flexibly accounts for parental investments in their children’s education due to any combination of material and nonmaterial (e.g., altruistic, status) purposes. $\beta < 1$ assures that parents do not receive more pleasure than their children from a certain amount of child’s income, and varying $\beta$ changes the valuation households have for their children’s future earnings. Equation (4) is just a budget constraint.

The patterns of optimal investment that result are intuitive. Households will invest in their own child as long as the increase in their well being resulting from a
marginal gain in their child’s productivity exceeds the opportunity cost of investing in another child from the community. An adult will only invest in a child from another household within the community if that child will repay his loan. This creates an incentive compatibility constraint (ICC), reflected in equation (5), such that all children receiving educational loans will be educated only to the point that they have an incentive to migrate to the city and subsequently default on the loan. As we will show, the incentive compatible level of education depends fundamentally on the spatial variation in returns to education, $\lambda$, the cost of migration, $c$, and the enforcement of loan contracts, as reflected in the probability that one can successfully renege on contracts by moving, $\pi$. The ICC for the optimization problem reflects the fact that if household $j$ does not provide any funding for the education of child $i \neq j$, then it is indifferent to child $j$’s decision to migrate. Wealthy households may want their own children to migrate after they are educated, but if they have invested in others’ children’s education, they will not want those children to leave.

3 ANALYSIS

We now analyze the factors that affect the educational outcomes of children and the educational investment decisions taken by adults. Specifically, we investigate how various educational financing schemes affect the optimal education levels in a dual economy setting and how rural educational investments vary in response to changes in the model’s parameters.

To establish a basis for comparison, we first analyze the case in which children only receive educational funding from their own parents and characterize the conditions for migration and the optimal levels of education in each sector. We then allow children
to receive informal loans from other households. We show that the presence of an informal credit market weakly increases the educational attainment of all children but its efficiency is decreasing in the rate of out-migration. Finally, we consider the case of a first-best world, where children can borrow on their future productivity from a formal credit market to finance their education. We make comparisons to show how informal credit markets can break down in the presence of migratory pressures and lead to underinvestment in education.

3.1 Household-Funded Education

In this first scenario, children’s education can only be funded by their own household.

3.1.1 The Child’s Decision

We begin by studying the child’s problem. Suppose that child $j$ receives all of his education from his own household $j$. Then, from (1) we know that he will migrate if his total level of human capital $h_j$ implies

$$\lambda \rho(h_j) - \rho(h_j) \geq c$$

Let $h(\lambda, c)$ denote the level of $h_j$ that solves equation (6) with equality. This is the threshold level of human capital necessary to migrate. Given that $\rho(\cdot)$ is strictly concave and monotonically increasing, we can apply the inverse-function theorem to establish:

$$\frac{\partial (\tilde{h})}{\partial (\lambda)} < 0$$

and

$$\frac{\partial (\tilde{h})}{\partial (c)} > 0$$

Condition (7) says that as the urban/rural infrastructure ratio increases, the human capital threshold level decreases thus more people are likely to migrate. Both within and across nations, actual migration patterns are overwhelmingly toward higher productivity regions. Since, as we show, this leads to an unraveling of informal credit for education in
remote areas, spatial infrastructure differences can lead to educational poverty traps.

Condition (8) simply indicates that as the cost of migration increases, the level of human capital required to migrate also increases. This wedge creates some modest, but bounded, spatial differences in incentives to invest in education.

Furthermore, since \( h = E \alpha \), the threshold level of education needed to induce migration \( E(\alpha)\alpha = h \), is decreasing in natural ability:

\[
\frac{\partial (E(\alpha))}{\partial (\alpha)} < 0
\]

(9)

Thus, everything else equal, high potential individuals are more likely to attain the threshold level and migrate, as reflected in “the brain drain” literature (Galor and Moav 1999, Masson 2001, Stark 1984, 1999).

3.1.2 Household Head’s Decision

We now analyze the adult or household head’s first period problem. We first characterize the conditions under which an adult will spend all of her wealth on the education of her own child.

Recalling that, for the moment, we only permit the parent to pay for her child’s education, suppose child \( j \) migrates. Then, it must be the case that:

\[
P_{E} E_{j} \leq w_{j}
\]

(10)

and

\[
\delta\beta\lambda\rho'(\alpha_{j} E_{jj})\alpha_{j} \geq \delta(1 + r)P_{E} \quad \text{for} \quad E_{jj} \geq E_{j}
\]

(11)

That is, that the adult household head must at least have the level of wealth needed to educate her child such that, given his innate ability, his human capital is above the threshold level and, that for some level of education \( E_{jj} \) above the threshold, the marginal benefit accruing to the household is larger than the opportunity cost. Let \( \overline{w}_{j} = P_{E} E_{j} \) denote the level of wealth a household requires in order
to be able to provide its child with the threshold level of education. In addition, let $\hat{E}_j$ be such that (11) holds with equality. $\hat{E}_j$ represents the optimal level of education that a household $j$ faced with an opportunity cost of $\delta(1+r)P_k$ would provide to its child conditional on the child migrating.

We now characterize the set of children who will not migrate, i.e. for whom $w_j \leq \bar{w}_j$. In such a situation, the adult head will continue to spend on her child so long as $E_{ij}$ satisfies

$$\delta \beta \rho'(\alpha_j E_{ij})\alpha_j \geq \delta(1+r)P_k \quad \text{where} \quad E_{ij} < \bar{E}_j$$

(12)

This condition assures that at the level of education that exhausts the household’s wealth, the marginal benefit to the household from an increase in the child’s education in the rural area is greater than the opportunity cost of investing in the children of other households. Let $\bar{E}_j$ solve (12) with equality. $\bar{E}_j$ represents the optimal level of education that a household $j$ faced with an opportunity cost of $\delta(1+r)P_k$ would provide to its child given that the child will not attain the level of education needed to migrate (that is $w_j \leq \bar{w}_j$).

It is now a simple task to classify the set of all children who will migrate in an environment where a child could only look to his household to finance his education. Given the set of all community-specific parameters $\lambda$ and $c$, whether a child migrates or not depends entirely on his innate ability and the level of his household’s wealth. Intra-community variation in migration and education patterns thus arise due to cross-sectional variation in initial endowments. Recognize that due to the strict concavity of $\rho(.)$ the LHS of equation (12) is decreasing in $E$. Then, since from equation (7) we
know that the threshold level of education, $\bar{E}$, is decreasing in $\alpha$, at low levels of innate ability $\alpha$, $\bar{E}$ gets larger than the optimal level of household provided education in the urban area $\hat{E}$. Let $\alpha^M$ be such that for all children $j$ that satisfy

$$\alpha_j \leq \alpha^M, \quad \hat{E} \text{ is such that } \hat{E}_j \leq \bar{E}_j,$$

or similarly, that $\bar{E}(\alpha^M) = \hat{E}_j$. Therefore, children with $\alpha_j \leq \alpha^M$ will never migrate despite their household wealth.

Figure 1 graphs the combination of educational levels and innate abilities that jointly determine a child’s educational attainment and subsequent locational choice in the second period conditional on $\lambda$ and $c$. For clarity, we call “potential” education the level of education that may be obtained if the household invested all its wealth in education, and distinguish it from “effective” education, the education level actually achieved by the child. In the $(\alpha, E)$ space, potential education, or initial wealth endowment, is represented by an “o” sign, while effective education by a “+” sign.

Suppose first that migration is not an option, and consider the $O\bar{E}$ schedule obtained from equation (12). It represents the maximum educational level an adult will “invest” in his child if the child stays home. In this case, all wealth endowments below the $O\bar{E}$ schedule correspond to cases where the adult will invest all his wealth in education (the marginal product remains higher than the opportunity cost). In other words, effective education is equal to potential education. On the other hand, if the initial endowment is above the $O\bar{E}$ schedule, then only part of the household’s wealth is invested in education, and effective education is just equal to $\bar{E}$. (the “+” lies below the corresponding “o”, indicating readiness to lend for other children’s education).

The $O\hat{E}$ schedule, implied by equation (11), has a similar interpretation, but for children who migrate. As returns to education are higher in the city, this schedule strictly
dominates the former. Both schedules represent the upper limits of effective education. Which schedule is relevant depends on the child’s location. Those children whose effective education lies above the $\bar{E}E$ curve - which corresponds to equation (6) - will migrate, the other ones stay home. This allows for the identification of the critical level of ability $\alpha^M$ where $O\hat{E}$ and $\bar{E}E$ intersect. The shaded area represents all cases where the entire household’s wealth is invested in the child’s education. As such, for these wealth-rationed households, the effective education “+”and the potential education “o” coincide.

3.1.3 The effect of spatial variation in productivity on education levels

Suppose the urban sector underwent a period of heavy investment in its infrastructure, resulting in a relative increase in urban labor productivity. Per equation (11), an increase in $\lambda$ raises the marginal benefit of human capital thus resulting in higher $\hat{E}$ for all levels of $\alpha$. Consequently, the threshold level of $\bar{h}$ drops and, for any given $\alpha$, so does $\bar{E}$ and therefore $\overline{w}$. Since $\bar{E}\alpha^M = \hat{E}$, a decrease in $\bar{h}$ and an increase in $\hat{E}$ implies a decrease in $\alpha^M$. To summarize, we have that $\forall \alpha > 0$

$$\frac{\partial (\hat{E})/\partial (\lambda)}{\partial (\bar{E})/\partial (\lambda)} > 0 \quad (13) \quad \text{and} \quad \frac{\partial (\hat{E})/\partial (\lambda)}{\partial (\bar{E})/\partial (\lambda)} < 0 \quad (14)$$

Figure 2 graphically depicts the effect of increasing urban productivity on educational outcomes and migration rates. As per equations (13) and (11), we see that an increase in $\lambda$ rotates $O\hat{E}$ counter clockwise and shifts $\bar{E}E$ down causing a decrease in $\alpha^M$ from $\alpha^M_0$ to $\alpha^M_1$. The result is an increase in migration in so far as there exists at least one household $j$ such that:

$$w_j < \bar{E}_0\alpha_j, \quad w_j > \bar{E}_i\alpha_j \quad \text{and} \quad w_j \leq \hat{E}_i \quad (15)$$

Condition (15) corresponds to the shaded area in Figure 2. It represents all those
children for whom the increase in $\lambda$ lifted wealth and/or ability constraints enough to make migration attractive. Note that even though their level of education remains the same, they now migrate and thus earn higher wages for any given level of human capital. There will, however, still be those children whose ability and/or household wealth endowment is too low to migrate. Living in a poor household can result in a large, discontinuous reduction in the optimal education that a child receives. The extent of this disparity increases with increasing spatial disparity in productivity. To summarize, as $\lambda$ increases, the differences between rural and urban optimal household sponsored education (for any given level of $\alpha$) increases. That is, $\partial(\hat{E} - \bar{E})/\partial(\lambda) > 0$.

This coincides with the well known phenomenon that as urban centers in developing nations develop at a faster pace than their rural counterparts, the socio-economic disparity between urban elites and rural elites grows, reflected here in terms of increasing optimal levels of education in urban areas only. This does not mean that all, or even most of the urban dwellers achieve this level of education. Indeed, increased relative productivity in the urban area increases the rate of migration by loosening the lower boundaries on the ability and wealth constraints. This means that the urban area begins to attract relatively more skill-poor individuals as well as individuals coming from low wealth households. It is therefore safe to conjecture that such a dynamic not only increases the urban-rural polarization but also results in increasing within-urban inequalities as well. We leave that topic for future extensions of the model.

3.2 Informal Credit Market

Thus far we have restricted our attention to the case in which a child’s education is financed solely by its own household. While this is interesting in its own right, we are most interested in understanding the relationship between spatial variation in the returns
to human capital and the financing of educational investments. We now move on to analyze a household’s decision to invest in other children.

3.2.1 The Supply of Community Funded Education

We first characterize the conditions under which a household will supply educational loans for investment purposes and a child will demand such loans. We assume that the return on investment is independent of the child so long as the child remains in the village. The investor is assured of \((1 + r)\) from non-migrants\(^\text{vii}\). The child is similarly indifferent as to who in the community provides the loans. These assumptions allow us to focus on a representative household whose adult can supply investment loans to a community fund and whose child can apply for educational loans from the same fund. To this end, let \(E^{-j} = \sum_{i \neq j} E_{ji}\) denote the sum total of educational loans that child \(j\) receives from the community. Recall from the ICC that for a community household to invest in a child \(j\) it must be the case that for \(E^{-j} > 0\)

\[
\pi \lambda \rho(\alpha_j(E_{ji} + E^{-j})) - \rho(\alpha_j(E_{ji} + E^{-j})) \leq c - (1 + r)PE^{-j} \tag{16}
\]

This condition assures the lending household(s) that the recipient child will not migrate and thus renege on his loan. The contract is designed so that at the incentive compatible levels of educational investment, the expected net gain from migration is less than the net gain resulting from the opportunity to default on the loan. Let \(E^{-j}\) solve (16) with equality. \(E^{-j}\) then represents the maximum amount of education child \(j\) will be eligible to receive from the community. This is critical to the hypothesis we advance\(^\text{viii}\). Allowing for educational loans shifts the migration threshold since migration effectively generates windfall earnings in the form of loan non-repayments. The incentive compatible threshold level shifts relative to the household financed threshold in response to the size
of loan taken (and thus the magnitude of gains from reneging) as well as the probability of getting caught. The lower the risk of being caught and the larger the loan, the lower will be the threshold as lenders adjust for the increased attractiveness of migration.

To provide a clear picture of the determinants of \( \bar{E}^{-j} \), we graph condition (16) in figure 3 and run some comparative statistics to analyze how shifts in the model’s parameters affect it. Recall that the RHS of (16) captures the net cost of migration. When \( E^{-j} = 0 \), the net cost is simply the parameter \( c \). However as \( E^{-j} \) increases, the net cost decreases at the rate \( (1+r)P_E \) (as migrating now provides the added benefit of freeing the individual from his debt burden). Thus the net cost curve intersects the vertical axis at \( c \) and slopes downward thereafter. The LHS of (16) crosses the vertical axis below \( c \).

Since both functions are strictly concave, and thus their difference is also strictly concave, then for any given \( E_{ji} \), \( (\pi\lambda-1)\rho(\alpha_j(E_{ji} + E^{-j}) \) is increasing in \( E^{-j} \). \( \bar{E}^{-j} \) is the value where the productivity gains equal the net costs.

Recall that \( \pi \) denotes the probability that a reneging child escapes attempts by the community to punish him for breaking the agreement to honor his loans. For large \( \pi \), migrants find it relatively easy to avoid punishment. A rural household with surplus investable resources will rationally seek to protect itself from potentially bad investments. As figure 4 shows, because an increase in \( \pi \) shifts up the expected gain of migrating, it lowers \( \bar{E}^{-j} \), reducing the supply of informal educational loans \( P_k\bar{E}^{-j} \).

A similar graphical representation explains the decrease in \( \bar{E}^{-j} \) resulting from an increase in \( \lambda \). Again, this is merely the rational response of adults protecting their investments in the face of an increased incentive to migrate. On the other hand, in a community with strong networks systems and in which personal welfare is inextricably
linked to social status, the resulting increase in the cost of moving is likely to relax the constraint of loan provision arising from the fear of losing investments to migration. These results highlight the centerpiece of our hypothesis: *For a child $j$ tapping into community educational loans, as the expected benefit of migration increases (captured by either an increase in $\pi$ and/or $\lambda$, or a decrease in $c$), the maximum amount of loans the community is willing to provide, $E^{-j}$, decreases. That is, $\partial(E^{-j})/\partial(\lambda) < 0$, $\partial(E^{-j})/\partial(\pi) < 0$, $\partial(E^{-j})/\partial(c) > 0$.*

The return on educational investment is given by $(1 + r)$. One would expect that increases in investment returns arising from an increase in the going interest rate would increase $P_{E}E^{-j}$, the amount of loans the community is willing to give for each $\alpha$. However, in this case, an increase in the return to investment signifies a larger debt burden per level of education for the child and thus increases his incentive to migrate and renege. *As such, increases in the net return on educational investment $r$ result in a decrease of $E^{-j}$, the maximum level of education the community is willing to invest in any child $j$. That is, $\partial(E^{-j})/\partial(r) < 0$.*

Figure 5 shows this result. An increase in $r$ represents a steeper slope on the net cost to migration curve which then intersects the expected net gain to migration curve at a lower $E^{-j}$.

### 3.2.2 The Demand for Community Funded Education

As a result of the fixed rate of repayment $(1 + r)$ that an investor receives per unit of education financed if the child remains in the rural area, the investor may be willing to invest in a child beyond the level that optimizes the child’s productivity. The child, however, will reject all loans whose repayment cost is greater than the resulting increase.
in productivity. Recall that $\tilde{E}_j$ was child $j$’s optimal level of education if he stayed in the rural area and only received education from his household. Now, however, we seek to find the child’s demand for community provided educational loans. Given an optimal level of education provided by his own household, $E_{ji}^*$, child $j$ will accept any level of community funded educational units $E^{-j}$ that satisfies

$$\rho'(\alpha_j(E_{ji}^* + E^{-j}))\alpha_j \geq (1 + r)P_E$$

(17)

Let $\tilde{E}^{-j}$ solve (17) with equality. $\tilde{E}^{-j}$ then denotes child $j$’s optimal demand for community funded education. If community willingness to supply educational loans to child $j$ is greater than the child’s demand for education, i.e. $E^{-j} \geq \tilde{E}^{-j}$, then child $j$’s total educational attainment will be $E_{ji}^* + \tilde{E}^{-j}$ and will not be constrained by the contractual demands of the informal credit market structure. If on the other hand $E^{-j} < \tilde{E}^{-j}$, then the child will receive a total education attainment of $E_{ji}^* + \tilde{E}^{-j}$.

Note that from equations (12) and (17) we have that

$$\delta \rho'(\alpha_j \tilde{E}_j)\alpha_j = \delta(1 + r)P_E$$

(18)

$$\text{and } \rho'(\alpha_j(E_{ji}^* + \tilde{E}^{-j}))\alpha_j = (1 + r)P_E$$

(19)

By the strict concavity of $\rho(\cdot)$, it follows that

$$\tilde{E}_j < E_{ji}^* + \tilde{E}^{-j}$$

(20)

Thus, for any child $j$ for whom $\alpha_j < 0$ and who does not migrate, $\tilde{E}^{-j} > 0$ and they demand a positive level of community funded education. This is true because while a child absorbs the full return from increased productivity resulting from more education, the ensuing indirect increase in the household’s utility is discounted by $\beta$. Furthermore, the household head must factor in the opportunity cost of foregone investment.
Whether a child’s level of community funded education is constrained by the amount of education the community is willing to finance or by the level of education they demand depends on the child’s endowment of ability and his household’s wealth. More specifically, any child who demands an education level that would make migration a rational second-period decision will be constrained by the supply of community educational loans available to him. Unlike the household-only funding scenario in which any child with $\alpha \leq \alpha^m$ would never rationally migrate, any child with $\alpha > 0$ will migrate under informal community level provision if they receive sufficient loans because migration now has the added benefit of offering an opportunity to renege on one’s debt burden. That opportunity becomes more inviting as the probability of getting caught $(1 - \pi)$ decreases, and the marginal cost of education $(1 + r)P_e$ increases. $\bar{E}^{-j}$, the maximum loan amount available for a child $j$ endowed with $\alpha_j$, is thus decreasing in $\alpha$, though even with $\alpha_j = 1$, a child $j$ will always have access to a positive supply of community loans. Meanwhile, a child’s demand for education, and thus for educational loans if his household has insufficient wealth to pay for his schooling, is monotonically increasing in $\alpha$. Figure 6 depicts the demand and supply of community education as a function of $\alpha$.

The bold sections of the demand and supply schedules represent the actual community funding $E^{-j}$ that child $j$ will receive. All those children with $\alpha \leq \alpha^*$ will receive their optimal level of education while children with $\alpha > \alpha^*$ will be constrained by the amount of loan the community is willing to finance. The striking implication is that in a world of imperfectly enforceable credit contracts and migration options, children are implicitly punished for being born intelligent. High innate ability increases the benefits to migration, inducing rational investors to reduce their loan supply as a defense against...
prospective default. This perverse result yields an important, testable hypothesis for future work: are educational loans increasing in innate ability, as would be the case under perfect credit markets (as the next subsection demonstrates) or under altruistic lending (wherein lenders give in response to child demand), or are they decreasing in a child’s innate ability, as predicted by this model?

We built our model of the informal credit market without considering the actual availability of community resources to meet their willingness to invest in the education of each child \( j \), \((\tilde{E}^{-j})\). We modeled a perfectly endowed community financier who is always capable of meeting the demand \((\tilde{E}^{-j})\) subject only to the incentive compatibility constraint. Whether this condition is met depends on the distribution and aggregate level of wealth across households and the distribution of abilities across children in the community. A community poorly endowed with wealth but richly endowed with intelligence is likely to have many loan worthy children demanding loans who nonetheless come up short due to the low level of investible surplus in the community. This simple model captures the key elements of our story: that informal financing weakly dominates the household-only provision of education under any distribution of \( \alpha \) and \( w \), and that spatial disparities in returns to human capital reduce the available supply of educational loans.

### 3.3 The First-Best World

The benefits of informal financing are also sharply limited by financial market imperfections. We now demonstrate this by briefly describing the first-best counterfactual in order to formalize the inefficiencies that result from an imperfect credit market and to show how spatial differences in infrastructure affect those inefficiencies.

While we have shown that informal credit is better than no credit, clearly the
incentive compatibility constraint on the provision of informal loans results in inefficiencies. We investigate these inefficiencies by comparison with a complete, competitive credit market with perfect contract enforcement. In such a world, children would have to repay loans irrespective of their second period locational choice. Therefore, child $j$ migrates only if

$$\rho(\alpha_j(E_{jj}^* + E^{-j})) - (1 + r)P_E E^{-j} < \lambda \rho(\alpha_j(E_{jj}^* + E^{-j})) - (1 + r)P_E E^{-j} + c \quad (21)$$

Since the child has to repay the same amount in both sectors, receiving loans does not change the threshold level and child $j$ will migrate if $E_{jj}^* + E^{-j} > E_j$. Thus migration thresholds are endogenous to lending patterns only in the presence of imperfect financial markets due to contract enforcement problems. Whether the child actually migrates depends on whether there exists $E^{-j}$ such that

$$E_{jj}^* + E^{-j} > E_j \quad \text{and} \quad \lambda \rho'(\alpha_j(E_{jj}^* + E^{-j}))\alpha_j \geq (1 + r)P_E \quad (22)$$

Condition (22) assures that child $j$’s sum total education is greater than the threshold for migration and that the amount $E^{-j}$ of his education that was formally financed provides net benefits to him at the margin. Let $E^{-j}$ equate (22). $E^{-j}$ is the unconstrained optimal level of education for child $j$. In a first best world, all rural children for whom $E_{jj}^* + E^{-j} > E_j$ will migrate and borrow the amount needed to fund $E^{-j}$ units of education. If $E_{jj}^* + E^{-j} \leq E_j$, then child $j$ would never migrate and thus reverts to making his borrowing decision conditional on staying in the rural area. This corresponds to him borrowing the amount $E^{-j}$ that equates (17) and thus, the first best optimal level of rural education is exactly equal to the optimal demand for community funded education $E^{-j}$. It follows that all children $j$ who don’t migrate and satisfy $E^{-j} \leq \ddot{E}^{-j}$ receive their first-best level of credit financed education even under a regime of informal credit markets.
For those children $j$ such that $\tilde{E}^{-j} > \tilde{E}^{-j}$ however, the introduction of formal credit markets will allow them to take the amount of loans that will fund their optimal level of education. As they were previously subject to the incentive compatibility constraint necessary in an informal financing regime, a release of this constraint means that all such children will now migrate.

Improving loan contract enforcement in the rural area also improves the educational outcomes of children who subsequently migrate. Using equations (11) and (22) and conducting the same exercise that led to equation (20), it follows that

$$\hat{E}_j < E_{jj}^* + \tilde{E}^{-j}$$

(23)

And by a similar argument to the proof in footnote 13, we find that: *Any child $j$ that migrates will demand a positive level of education loans from the credit market and will attain a level of education greater than his optimal level of household financed education $\hat{E}_j$. That is, for all $j$ such that $E_{jj}^* + \tilde{E}^{-j} > \tilde{E}_j$, $\tilde{E}^{-j} > 0$ and $E_{jj}^* + \tilde{E}^{-j} > \hat{E}_j$.*

Moreover, there also exist children who can migrate (and thus achieve their full potential) only if they have access to a formal credit market. From a rearranging of equation (22) we get

$$E_{jj}^* + \tilde{E}^{-j} = \rho^{-1}\left(\frac{(1+r)P_e}{\alpha_j \lambda}\right) \frac{1}{\alpha_j}$$

(24)

Since the RHS of (24) is a constant for each $j$, this implies a one to one tradeoff between $E_{jj}^*$ and $\tilde{E}^{-j}$. Whereas, child $j$ (should he migrate) will always attain a total level of education equal to $E_{jj}^* + \tilde{E}^{-j}$ in the first-best world, how much he actually borrows to finance his education will depend on the amount $E_{jj}^*$ that his household contributes to his
education. However, though this will affect the gross income that a child earns, it will not affect the level of education he will attain. Indeed, should a child $j$ come from a household of zero wealth, he will simply borrow enough to finance 

$$\rho^{-1}(1 + r)P_E \frac{1}{\alpha_j}$$

units of education. It follows that the only condition child $j$ has to satisfy to migrate is that $\hat{E}^{-j} > \bar{E}_j$. This depends solely on his innate ability $\alpha_j$. Household wealth is no longer a factor in a child’s decision to migrate. In addition, introduction of perfect credit markets also lowers the threshold level of innate ability needed to migrate. To see this, recall that $\alpha^M$ is such that $\bar{E}(\alpha^M) = \hat{E}_j$. From equation (24) it follows that

$$\bar{E}(\alpha^M) < E^*_\beta + \hat{E}^{-j} \quad (25)$$

Denote the level of $\alpha$ that equates (26) as $\alpha^{FB}$. Since $\bar{E}(\alpha)$ is strictly decreasing in $\alpha$, it follows that $\alpha^{FB} < \alpha^M$. To summarize, the provision of a formal market for credit increases the rate of migration by making the migration decision independent of household wealth and lowering the threshold level of innate ability needed to migrate.

Figure 7 graphs the preceding results. The schedules $0\hat{E}$ and $0\bar{E}$ obtained from equations (22) and (17) represent the optimal education levels for a child who migrates and one who remains in the rural area respectively. The analytical expressions of these curves are almost identical to $0\hat{E}$ and $0\bar{E}$ respectively from figure 1, but for the absence of parameter $\beta$. As the migration conditions remain unchanged, educational attainment for all children is represented by the bold segments of the two curves

In summary, perfect credit markets provide Pareto optimal education financing.
They increase the optimal level of education and of available educational credit for all but the lowest ability children who would remain in the rural village regardless. Perfectly enforceable credit contracts also yield an optimal spatial distribution of individuals. Absent first-best credit markets, informal credit provides a means for certain rural children to overcome the constraints of household financing and receive a better education. But informal credit fails to meet the needs of higher ability children from low wealth households, creating important inefficiencies and inequities.

4 DISCUSSION AND POLICY IMPLICATIONS

The central results of our model highlight three key points. First, we demonstrate the consequences of spatial inequality in productive infrastructure for rural education investment in an environment of imperfect credit markets. Spatially varied returns to education tighten the incentive compatibility constraints inherent to informal credit markets and thereby limit the usefulness of informal educational loans. Second, our model underscores the crucial importance of the presence of credit contract enforcement mechanisms for the optimal investment in and attainment of education. Perfectly enforceable education loans afford children the opportunity to realize their full potential and to break free of poverty traps caused by low household wealth endowments. This does not carry over fully to imperfectly enforceable informal financing.

If increasing educational attainment in less-favored rural communities, perhaps especially among high ability children, is an objective for policymakers, then our analysis suggests two possible means by which public investment might “crowd in” private educational investment. First, governments and donors might improve rural infrastructure in ways that encourage private business investment that stimulates skilled
employment and thereby raises the expected returns to human capital. This might include programs of rural electrification, improvements in rural communication infrastructure, road improvement and maintenance, and provision of police protection. Improving rural infrastructure reduces incentives to migrate out of rural villages, making informal loan contract enforcement easier and thereby increasing the provision of private, informal finance. Second, governments and donors can invest in credit contract enforcement, perhaps through credit reporting bureaus or improved juridical enforcement of contracts.

Third, and perhaps the most worrying implication of our model, is that increasing spatial inequality in productive infrastructure is accompanied by a weakened ability of community social norms to ensure contract compliance. Without any significant improvements in formal contract enforcement mechanisms, we would expect to see increased incentives for rural-to-urban migration, but with the relatively wealthy increasingly disproportionately able to capitalize on these opportunities as informal finance for education becomes increasingly difficult to obtain in rural villages. More poor children of high innate ability thus become consigned to a sort of low-education poverty trap of the sort first posited by Loury (1981).

It would be instructive to expand on this model to allow for dynamics in order to explore the potential divergence of rural and urban livelihoods and the prospective intergenerational reproduction of poverty. Indeed, as poorer and less skilled individuals tend to remain in the rural area, we would expect that spatial mobility combined with imperfect credit markets would yield over time a rural population with a distribution of innate abilities and wealth that results in a decreased rate of migration and a low steady-state level of rural educational attainment and productivity. The simple two-period model we have developed nonetheless provides a credible answer to the puzzle of
underinvestment of education in rural areas based on the twin empirical regularities of spatially varied returns to human capital and imperfect loan contract enforcement in rural credit markets.
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FIGURES

Figure 1: Educational Outcomes and Migration Level Conditioned on Child Intelligence and Household Wealth

Figure 2: Effect of Increasing Urban Productivity on Educational Outcome and Migration Rate
Figure 3: Determinants of Maximum Level of Community Financed Education

\[ (\pi \lambda - 1) \rho(\alpha_j (E_{jj} + E^{-j})) \]

\[ c - (1 + r)P^e E^{-j} \]

Figure 4: Response of Community Financed Education to Increases in Urban Productivity and Ease of Default.

\[ (\pi \lambda - 1) \rho(\alpha_j (E_{jj} + E^{-j})) \]

\[ c - (1 + r)P^e E^{-j} \]
Figure 5: Response of Community Funded Education to Decreases in Costs of Migration

\[(\pi \lambda - 1) \rho(\alpha_j (E_{ji} + E^{-j}))\]

\[c - (1 + r)P_j E^{-j}\]

Figure 6: Demand and Supply of Community Funded Education

\[\frac{c}{(1 + r)P_j}\]
Figure 7: Effect of Formalizing Credit Markets on Educational Outcomes and Migration Rates
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NOTES

i We focus on the rural economy and use the urban area only as a magnet for migrant laborers from the village. In our framework, it would never be rational for an urban dweller to migrate to the rural area, given the decreased return on their human capital that would result.

ii This composite alternative asset serves just as a benchmark against which educational investments are measured.

iii A household indexed by \( j \) denotes the household in which a child \( j \) under analysis is attached to. A household indexed by \( i \) denotes any other household in the community where \( i \neq j \).

iv By driving a defaulter’s income to zero, no lender would ever fund a migrating child so informal finance flows only to non-migrating children. Our main aim, to show that migration options reduce the loan pool for education, is robust to this simplifying assumption.

v A child does not have to explicitly repay education financed by his parents. This allows for an adult’s decisions on their children’s education to involve additional considerations beyond merely material investment returns.

vi As primary education is often free or subsidized, the need for educational investments arises mainly at the secondary level making this a tenable assumption.

vii We assume the existence of other investment possibilities that in equilibrium establish the opportunity cost of capital, \( r \).

viii Note that the supply of loans \( \tilde{E}^{-j} \) for a child \( j \) is calculated after household \( j \) decides how much to invest in their children’s education \( E^+_{j,j} \), independently from lending or borrowing options. Then, starting from the optimal educational expenses provided by the household, informal (or formal) lending may take place.

ix We know that \( E^{-j} > 0 \) implies that \( E^+_{j,j} < \tilde{E}_j \). Thus, since \( \tilde{E}_j \) is such that \( (\lambda - 1)\rho(\alpha_j \tilde{E}_j) = c \), then for \( E^{-j} = 0 \) and \( \pi \in (0,1) \), it follows that \((\lambda \pi - 1)\rho(\alpha_j (\tilde{E}_j + E^{-j})) < c \).

x One can prove this as follows. Suppose not. Then \( \tilde{E}^{-j} = 0 \). Equation (21) then implies that \( \tilde{E}_j < E^+_{j,j} \). This is a contradiction since given that the optimal level of household funded rural education is \( \tilde{E}_j \), it must be that \( E^+_{j,j} \leq \tilde{E}_j \). It follows that \( \tilde{E}_j > 0 \).

xi This result follows from the definition of \( \tilde{E}_j \) and \( \tilde{E}_j \). \( \tilde{E}_j \) is such that \((\lambda - 1)\rho(\alpha_j \tilde{E}_j) = c \) and \( \tilde{E}^{-j} \) solves \((\lambda \pi - 1)\rho(\alpha_j (\tilde{E}_j + \tilde{E}^{-j})) = c - (1 + r)P_j \tilde{E}^{-j} \). Let \( \alpha_j = 1 \) and \( \pi \in (0,1) \). Suppose \( \tilde{E}^{-j} = 0 \), this implies that \((\lambda - 1)\rho(\alpha_j \tilde{E}_j) = c \), and \((\lambda \pi - 1)\rho(\alpha_j \tilde{E}_j) = c \). This is a contradiction and thus \( \tilde{E}^{-j} > 0 \).

xii The low-density exception are children of households whose wealth and resulting investment choices bring the child nearly to the migration threshold, but a single unit of community-financed education would provide education sufficient to make it worth the child’s while to migrate.