Nucleon parton distributions in the large $N_c$ limit

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Abstract

We review recent progress in calculating the twist–2 parton distribution functions (PDF’s) at a low normalization point in the large–$N_c$ limit, where the nucleon can be described as a soliton of the effective chiral lagrangian. This field–theoretic approach preserves all general requirements on the PDF’s (positivity, normalization etc.). In particular, it allows to calculate the polarized and unpolarized antiquark distributions at the low normalization point.

In this talk we review recent attempts to calculate the leading–twist parton distributions of the nucleon at a low normalization point in the large–$N_c$ limit [1, 2, 3]. The results presented here have been obtained in collaboration with D.I. Diakonov, V.Yu. Petrov, P.V. Pobylitsa, and M.V. Polyakov.

Introduction. The parton distribution functions (PDF’s) of the nucleon are an essential ingredient in the description of deep–inelastic scattering and a variety of other hard processes. The scale dependence of the PDF’s in the asymptotic region is determined by the renormalization group equation of perturbative QCD. A complete description of the experiments requires non-perturbative information in the form of the values of the parton distributions at some initial normalization point. Several sets of input distributions have been determined by parametrizing data from a number of different experiments at large $q^2$ [4, 5, 6, 7]. All modern parametrizations involve both antiquarks and gluons at the low scale.

When attempting to compute the parton distributions at a low normalization point from first principles one must deal with non-perturbative effects such as the dynamical breaking of chiral symmetry and confinement. While a satisfactory theory of confinement is still lacking, the dynamical breaking of chiral symmetry in QCD is well understood theoretically:

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the resulting effective theory at low energies is known, and a dynamical picture of chiral symmetry breaking is provided by the instanton vacuum\(^1\)\(^2\)\(^3\), which has recently received direct support from lattice simulations\(^4\). A description of the nucleon based on the dynamical breaking of chiral symmetry is possible in the large–\(N_c\) limit, where QCD becomes equivalent to an effective theory of mesons, and the nucleon emerges as a soliton of the pion field\(^5\)\(^6\)\(^7\). This picture of the nucleon is known to give a good account of hadronic observables such as the nucleon mass, magnetic moments, form factors etc.\(^8\). It is thus natural to describe also the parton distribution at a low normalization point in this approach. An important advantage of this description of the nucleon is its field–theoretic character, which is essential for the parton distributions to satisfy general requirements such as positivity, proper normalization etc.

Effective chiral theory. In the large–\(N_c\) limit, the consequences of the dynamical breaking of chiral symmetry can be encoded in an effective lagrangian for the pion field, valid at low energies. It can be expressed as a functional integral over quark fields (\(SU(2)\) flavor) in the background pion field, \(\pi(x)\)\(^9\)\(^10\)\(^11\)\(^12\):

\[
\exp\left(iS_{\text{eff}}[\pi(x)]\right) = \int D\psi D\bar{\psi} \exp\left[i\int d^4x \bar{\psi}(i\partial - MU^\gamma_5)\psi\right],
\]

\(U^\gamma_5(x) = \exp\left[i\pi^a(x)\gamma^a\gamma_5\right].\)

Here, \(M\) is the dynamical mass (generally speaking, it is momentum–dependent), which is due to the spontaneous breaking of chiral symmetry. Eq.\(^1\)\(^1\)\(^3\) describes the minimal chirally invariant interaction of quarks with Goldstone bosons. In the long–wavelength limit, expanding in derivatives of the pion field, the effective action Eq.\(^1\)\(^3\) reproduces the Gasser–Leutwyler lagrangian with correct coefficients, including the Wess–Zumino term. The effective theory defined by Eq.\(^1\)\(^3\) is valid for momenta up to an UV cutoff, which is the scale at which the dynamical quark mass drops to zero. For simplicity we shall take in the discussion here the quark mass to be momentum–independent and assume divergent quantities to be made finite by applying some regularization scheme; below we shall see why, and under which conditions, this approximation is justified when computing (anti–) quark DF’s.

The effective action Eq.\(^1\)\(^3\) has been derived from the instanton vacuum, which provides a natural mechanism of dynamical chiral symmetry breaking and enables one to express the parameters intrinsic in Eq.\(^1\)\(^3\) — the dynamical mass, \(M\), and the ultraviolet cutoff — in terms of the QCD scale parameter, \(\Lambda_{\text{QCD}}\)\(^1\)\(^4\). The cutoff is given by the average instanton size,

\[
\bar{\rho}^{-1} \simeq 600 \text{ MeV},
\]

which is the order of the scale at which the coupling constant is fixed in the instanton vacuum. Since the effective theory is not valid beyond \(\bar{\rho}^{-1}\), when applying it to compute PDF’s we are certainly dealing with distributions normalized at a scale not higher than 600 MeV.

An important point to emphasize is that, when working with the effective action, Eq.\(^1\)\(^3\), it is implied that the ratio of the dynamical quark mass to the ultraviolet cutoff is parametrically small, \(M\bar{\rho} \ll 1\). In the instanton vacuum this is guaranteed by the fact that \(M^2\) is

\(^1\)For a recent review of the instanton vacuum see refs.\(^8\)\(^9\).
proportional to the instanton density, so that \((M\bar{\rho})^2\) is proportional to the packing fraction of the instanton medium,

\[
(M\bar{\rho})^2 \propto \left(\frac{\bar{\rho}}{R}\right)^4,
\]
which is a small parameter. However, this requirement is really of a more general nature — otherwise degrees of freedom other than those included in Eq. (1) would play an essential role in the effective dynamics, and the concept of an effective action would not be very meaningful. One may interpret the inverse ultraviolet cutoff, \(\bar{\rho}\), as the size of the “constituent” quark described by the effective theory. The smallness of \(M\bar{\rho}\) implies that the constituent quark is nearly pointlike — its size is small compared to its typical wavelength in the nucleon, which is of order \(M^{-1}\). This fact is essential for the interpretation of the distribution functions computed with the effective chiral theory. For \(M\bar{\rho} \to 0\) one may identify them with the QCD quark and antiquark distributions. When trying to be more accurate, keeping terms of order \((M\bar{\rho})^2\), one begins to resolve the structure of the “constituent” quark, and the distribution functions computed in the effective theory should be regarded as distributions of composite objects which have a substructure in terms of QCD partons.

To describe the gluon distribution one needs a microscopic picture of the non-perturbative gluon degrees of freedom. Such a picture is provided by the instanton vacuum, which on one hand allows to derive the effective chiral action, Eq. (1), by explicit integration over gluon degrees of freedom, on the other hand makes it possible to directly evaluate matrix elements of gluon operators, using the effective operator method developed in ref. [19]. One finds that the twist–2 gluon distribution is of order \((\bar{\rho}/R)^4\), which is consistent with the above statement that the structure of the constituent quark starts to appear at level \((M\bar{\rho})^2\). To compute the twist–2 gluon distribution one needs to construct both the effective action and the effective operators at order \((M\bar{\rho})^2\).

The nucleon as a chiral soliton. In the effective theory defined by Eq. (1) the nucleon is in the large–\(N_c\) limit described by a static classical pion field (“soliton”) [13]. In the nucleon rest frame it is of “hedgehog” form,

\[
U_c^{\gamma_5}(x) = \exp \left[im^a\tau^a P(r)\gamma_5\right], \quad r = |x|, \quad n = \frac{x}{r},
\]
where \(P(r)\) is called the profile function, satisfying \(P(0) = -\pi, P(r) \to 0\) for \(r \to \infty\), which is determined by minimizing the energy of the static pion field. Quarks are described by single-particle wave functions, which are the solutions of the Dirac equation in the external

\[\text{The dominant contribution to the twist–2 quark and antiquark DF’s comes from quarks with momenta of order } M \ll \bar{\rho}^{-1}, \text{ see below.}\]

\[\text{Calculations of the twist–2 gluon distribution in the instanton vacuum using a different approach have recently been performed by Kochelev [20]. However, the contributions taken into account there do not represent the full answer to order } (\bar{\rho}/R)^4; \text{ see refs. [21, 22] for a discussion.}\]
pion field,

\[ H \Phi_n = E_n \Phi_n, \quad H = -i \gamma^0 \gamma^k \partial_k + M \gamma^0 U^\gamma_5. \]  

(6)

The spectrum of the one-particle Hamiltonian, \( H \), contains a discrete bound–state level, which must be occupied by \( N_c \) quarks to have a state of unit baryon number, as well as the positive and negative energy Dirac continuum, distorted by the presence of the pion field. The nucleon mass is given by the minimum of the bound–state energy and the aggregate energy of the negative Dirac continuum, the energy of the free Dirac continuum subtracted \([13]\). Nucleon states of definite spin-isospin and 3-momentum are obtained by quantizing the rotational and translational zero modes of the soliton, \( i.e. \), applying a flavor rotation and a shift of the center to the soliton field, Eq.(6), and integrating over the corresponding collective coordinates with appropriate wave functions \([13, 16]\).

Quark– and antiquark distributions from the effective chiral theory. In order to compute the twist–2 quark and antiquark distribution functions in the effective chiral theory one may start either from their “parton model” definition as numbers of particles carrying a given fraction of the nucleon momentum in the infinite–momentum frame, or from the “field–theoretic” definition as forward matrix elements of certain light–ray operators in the nucleon. Both ways lead to identical expressions for the quark DF’s in the chiral soliton model. This remarkable fact should be attributed to the field–theoretic nature of this description of the nucleon, and to the fact that the main hypothesis of the Feynman parton model — that transverse momenta do not grow with \( q^2 \) \([23]\) — is satisfied within this model. In the first formulation one considers a nucleon with large 3–momentum,

\[ P_N = \frac{M_N v}{\sqrt{1 - v^2}}, \quad v \to 1, \]  

(7)

and defines the quark and antiquark DF’s as

\[
\begin{align*}
D_{i,f}(x) &= \int \frac{d^3 k}{(2\pi)^3} 2\pi \delta \left( x - \frac{k^3}{P_N} \right) \langle N_\psi | a_{i,f}(k) a_{i,f}(k) \rangle \langle a_{i,f}(k) a_{i,f}(k) | N_\psi \rangle, \\
\bar{D}_{i,f}(x) &= \int \frac{d^3 k}{(2\pi)^3} 2\pi \delta \left( x - \frac{k^3}{P_N} \right) \langle N_\psi | b_{i,f}(k) b_{i,f}(k) \rangle \langle b_{i,f}(k) b_{i,f}(k) | N_\psi \rangle,
\end{align*}
\]

(8)

where \( |N_\psi\rangle \) denotes the state with the fast–moving nucleon and \( a_{i,f}, a_{i,f}^\dagger \) and \( b_{i,f}, b_{i,f}^\dagger \) are the creation/annihilation operators for quarks (antiquarks), characterized by a polarization quantum number \( i \) and flavor \( f \). For large \( N_c \) the nucleon consists of \( O(N_c) \) quarks and antiquarks, so we can hope to describe the distribution functions for values of \( x \) of order

\[ x \sim \frac{1}{N_c}. \]  

(9)

In the large \( N_c \) limit the nucleon matrix element Eq.(8) reduces to a sum of matrix elements between quark single–particle states in the mean pion field (saddle point approximation), which is given in this case by the hedgehog field, Eq.(5), boosted to velocity \( v \), see \([2]\) for
details. Taking the limit \( v \to 1 \) one arrives at the following expressions for the twist–2 quark and antiquark distributions of the nucleon in the large–\( N_c \) limit:

\[
\begin{align*}
D_{i,f}(x) = & \sum_{\text{occ.}} \int \frac{d^3 k}{(2\pi)^3} \phi_{i,f}^\dagger(k)(1 + \gamma^0 \gamma^3) \gamma^0 \Gamma_i \delta(k^3 + E_n \mp xM_N) \phi_{i,f}(k), \\
\bar{D}_{i,f}(x) = & \sum_{\text{non–occ.}} \int \frac{d^3 k}{(2\pi)^3} \phi_{i,f}^\dagger(k)(1 + \gamma^0 \gamma^3) \gamma^0 \Gamma_i \delta(k^3 + E_n \mp xM_N) \phi_{i,f}(k),
\end{align*}
\]

where, for example,

\[
\Gamma_i = \gamma^0 \frac{1 \pm \gamma_5}{2}
\]

for quarks polarized along or against the direction of the nucleon velocity. Here the sums run over all occupied or non-occupied quark levels in the hedgehog pion field, that is, the bound–state level and the negative Dirac continuum or the positive Dirac continuum, respectively. It is understood in Eq.(10) that one subtacts the corresponding sum over eigenstates of the free Hamiltonian, where \( \pi(x) = 0 \).

Alternatively, one can start from the QCD definitions of the twist–2 quark distribution functions as matrix elements of certain light–ray operator, which act as generating functions of the series of local twist–2 operators \([24, 25]\); in the unpolarized case

\[
\begin{align*}
D_{\text{unpol}, f}(x) &= \frac{1}{4\pi} \int_{-\infty}^{\infty} dz e^{i x p^+ z^-} \langle P| \bar{\psi}_f(0) \gamma^+ \psi_f(z) |P\rangle \big|_{z^+ = 0, z^\perp = 0}, \\
\bar{D}_{\text{unpol}, f}(x) &= -\{x \to -x\},
\end{align*}
\]

and in the longitudinally polarized case

\[
\begin{align*}
D_{\text{pol}, f}(x) &= \frac{1}{4\pi} \int_{-\infty}^{\infty} dz e^{i x p^+ z^-} \langle P,S| \bar{\psi}_f(0) \gamma^+ \gamma_5 \psi_f(z) |P,S\rangle \big|_{z^+ = 0, z^\perp = 0}, \\
\bar{D}_{\text{pol}, f}(x) &= \{x \to -x\}.
\end{align*}
\]

These matrix elements can be evaluated in the nucleon rest frame, expanding the quark fields in Eqs.(12, 13) in the basis of single–particle wave functions, Eq.(6); the result is identical to Eq.(10).

To obtain the unpolarized or polarized (anti–) quark distributions for a nucleon state of definite spin and isospin, one has to apply a flavor rotation to the quark wave functions in the basic expression Eq.(10), \( \Phi_{n,f} \to R_{ff'}(t)\Phi_{n,f'} \), and integrate over slow rotations, projecting on a nucleon state with given spin and isospin \([1]\). At this point characteristic differences between the different isospin combinations of the unpolarized and polarized distributions
appear. The angular velocity of the soliton, $R^\dagger (\partial R/\partial t)$, is of order $1/N_c$ (the soliton moment of inertia is $O(N_c)$), and in the leading order of the $1/N_c$–expansion one may neglect it. The distribution functions which are non-zero at this level are the isosinglet unpolarized and isovector polarized — they are the leading ones in the $1/N_c$–expansion. The isovector unpolarized and isosinglet polarized distributions, on the other hand, are non-zero only after expanding to first order in the angular velocity; they are thus suppressed relative to the former by a factor $1/N_c$. Thus the $1/N_c$–expansion leads to the following classification of quark/antiquark DF’s in “large” and “small” ones:

|       | unpolarized | polarized (longitud.) | polarized (transv.) |
|-------|-------------|-----------------------|---------------------|
| “large” | $u + d$     | $\Delta u - \Delta d$ | $h^u_1 - h^d_1$    |
| “small”| $u - d$     | $\Delta u + \Delta d$ | $h^u_1 + h^d_1$    |

More precisely, bearing in mind that $x \sim 1/N_c$, we can say that the (anti–) quark DF’s in the large $N_c$ limit behave as

$$D^{\text{large}}(x) \sim N_c^2 f(N_c x), \quad D^{\text{small}}(x) \sim N_c f(N_c x),$$

where $f(y)$ is a stable function in the large $N_c$–limit which depends on the particular distribution considered. One may easily convince oneself that this is consistent with the well–known $N_c$–dependence of the lowest moments of the DF’s: the first moments of $u + d$ (baryon number) and of $\Delta u - \Delta d$ (isovector axial coupling, $g_A^{(3)}$) are $O(N_c)$, while those of $u - d$ (isospin) and of $\Delta u + \Delta d$ (isosinglet axial coupling, $g_A^{(0)}$) are $O(N_c^0)$.

The “large” DF’s are given by simple sums over quark levels; the isosinglet unpolarized DF by (the sum over flavor indices is understood on the R.H.S)

\[
[u + d](x) = N_c M_N \sum_{\text{occ.}} \int \frac{d^3 k}{(2\pi)^3} \Phi^\dagger_n(k)(1 - \gamma^0 \gamma^3)\delta(k^3 + E_n + xM_N) \Phi_n(k),
\]

\[
[\bar{u} + \bar{d}](x) = -N_c M_N \sum_{\text{non-occ.}} \{x \rightarrow -x\}.
\]

An alternative representation of the DF is obtained by applying in Eq.(12) the anticommutation relation of quark fields before taking the light–cone limit; this leads to a representation of the quark DF’s as sums over non-occupied, and of the antiquark DF as a sum over occupied states:

\[
[\bar{u} + \bar{d}](x) = -N_c M_N \sum_{\text{occ.}} \{x \rightarrow -x\},
\]

where the braces again denote the corresponding expressions appearing in Eq.(15). One may regard Eq.(12) together with Eq.(17) as one universal function, describing the quark DF at $x > 0$ and minus the antiquark distribution at $x < 0$. The isovector–polarized DF is given by\(^4\)

\[
[\Delta u - \Delta d](x) = \frac{1}{3}(2T_3) N_c M_N \sum_{\text{occ.}} \int \frac{d^3 k}{(2\pi)^3} \Phi^\dagger_n(k)(1 + \gamma^0 \gamma^3)\gamma_5\delta(k^3 + E_n + xM_N) \Phi_n(k),
\]

\(^4\)For the corresponding formulas in the case of transverse polarization, see ref.[3].
\[
[\Delta \bar{u} - \Delta \bar{d}] (x) = \frac{1}{3} (2T_3) N_c M_N \sum_{\text{non-occ.}} \{ x \rightarrow -x \},
\]

or, alternatively,

\[
[\Delta \bar{u} - \Delta \bar{d}] (x) = -\frac{1}{3} (2T_3) N_c M_N \sum_{\text{occ.}} \{ x \rightarrow -x \},
\]

where \(2T_3 = \pm 1\) for proton and neutron, respectively. The “small” DF’s in the large-\(N_c\) limit are given by double sums over quark levels divided by the moment of inertia of the soliton; see ref.\([4]\) for details.

From the explicit representations Eqs.\((15–20)\) it can easily be seen that the large-\(N_c\) quark and antiquark DF’s satisfy the following properties:

**Positivity:** The singlet unpolarized quark and antiquark DF’s, Eqs.\((15, 16)\), are explicitly positive (the Dirac matrices in these expressions are a projector and thus positive definite).

**Sum rules:** Integrating Eq.\((15)\) minus the representation Eq.\((17)\) over \(x\) one finds

\[
\int dx [u + d - \bar{u} - \bar{d}] (x) = N_c \left( \text{number of occupied levels in the soliton} - \text{number of occupied levels in the vacuum} \right),
\]

which coincides with the baryon number of the nucleon in the large-\(N_c\) limit. One can also show that the momentum sum rule is satisfied by virtue of the fact that the soliton field is a stationary point of the static energy. Similarly, integrating \(\Delta u - \Delta d + \Delta \bar{u} - \Delta \bar{d}\), Eq.\((15)\) minus Eq.\((20)\), one obtains the expression for the isovector axial coupling, \(g_A^{(3)}\), in the chiral soliton model \([15, 16]\), i.e., the Bjorken sum rule is satisfied within this approach.

For a proof of sum rules for the “small” DF’s \((e.g.\) the isospin sum rule for \(u - d\)), and a discussion of the Gottfried sum we refer to ref.\([1]\).

It is essential that both the contributions of the bound–state level as well as the Dirac continuum are taken into account in the representation of the large–\(N_c\) quark and antiquark DF’s, Eqs.\((15–20)\). Restricting oneself to the bound–state level only one would, for example, violate either the the baryon number sum rule or the positivity of the singlet unpolarized antiquark distribution — since the level makes a negative contribution to Eq.\((17)\), depending on whether one writes the antiquark distribution in the form Eq.\((16)\) or Eq.\((17)\). Adding the contribution of the Dirac continuum Eq.\((16)\) and Eq.\((17)\) give identical results, the antiquark distribution is positive, and the baryon number unity.

In the discussion of the large–\(N_c\) DF’s so far we have not explicitly taken into account the UV cutoff, which is an essential ingredient in the effective chiral theory, Eq.\((1)\). In fact, the expressions for the distribution functions as sums over quark levels, Eqs.\((15–20)\) contain UV divergences which are made finite by the UV cutoff. When choosing a regularization method to implement this UV cutoff it is of utmost importance that the regularization do not violate any of the fundamental properties of the DF’s. A crucial requirement is that the regularization should preserve the completeness of the set of quark single–particle wave functions in the soliton field, Eq.\((6)\). An incomplete set of basis functions would mean a
violation of the local anticommutation relation of the quark fields, that is, a violation of causality. For an extensive discussion of the many facets of this we refer to ref. [2]. Let us mention here only that the equivalence of the representations of the (anti–) quark DF’s as sums over non-occupied or occupied states, cf. Eqs. (16, 17), which is instrumental in ensuring both positivity and proper normalization of the isosinglet DF, depends crucially on the completeness of the basis.

A regularization by subtraction, for example, a Pauli–Villars cutoff, meets the above requirement, while a regularization by cutoff (for example, an energy cutoff or the popular proper–time regularization of the fermion determinant) is equivalent to working with an incomplete set of states and thus unacceptable. The Pauli–Villars cutoff is implemented by subtracting from the sums Eqs. (15 – 20) a multiple of the corresponding sums computed with a regulator mass, $M_{PV} \simeq \bar{\rho}^{-1}$.

$$D(x)_{PV} = D(x) - \frac{M^2}{M_{PV}^2} D(x)_{M_{PV}}.$$  

(22)

It can be shown that such regularization preserves all qualitative properties of the DF’s (positivity, sum rules) for which the completeness of states is an essential condition, thus justifying our above discussion of “naive” unregularized expressions [2]. (Note that only UV divergent quantities such as the total isosinglet distribution $u + d + \bar{u} + \bar{d}$ should be regularized; UV finite quantities such as the valence distribution $u + d - \bar{u} - \bar{d}$ should be left unregularized.) In short, the UV cutoff in the effective theory Eq. (1) can be implemented in a way that the field–theoretic character of the description of quark and antiquark DF’s is preserved.

It is important to note that, with a regularization preserving completeness, the actual dependence of the twist–2 DF’s on the cutoff of the effective theory is very weak: the DF’s Eqs. (15 – 20) are only logarithmically divergent for fixed $x$, and so are all their moments, see ref. [2]. The dominant contributions to the DF’s thus come from degrees of freedom with momenta of order $M \ll \bar{\rho}^{-1}$. This fact is crucial for the consistency of the effective theory approach: the quark and antiquark DF’s are determined by genuine dynamics of the effective theory — that is, by degrees of freedom with momenta much smaller than the cutoff — not by the details of the UV regularization, which, at any rate, is known only schematically. In particular, this justifies working with a suitable generic regularization (Pauli–Villars subtraction) rather than with the momentum–dependent quark mass obtained from the instanton vacuum.

One exception to the above is the region of values of $x \leq (M\bar{\rho})^2/N_c$, where the dominant contribution to the (anti–) quark DF’s comes from states with large (positive or negative) energies. In this parametrically small region of $x$ the momentum dependence of the quark mass is essential and, in the case of the singlet unpolarized distribution, Eqs. (15, 17), leads to a vanishing of the distribution at $x = 0$, as has recently been shown in connection with

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[We remind that we are discussing the (anti–) quark DF’s at a low normalization point, whose small–$x$ behavior has no direct relation to that of the DF’s at experimental scales, which is determined essentially by perturbative evolution.]
an investigation of the off-forward PD’s in the large-$N_c$ approach [27]. We note also that, in contrast to the twist–2 DF’s, matrix elements of higher–twist operators generally receive contributions from momenta of order of the cutoff, $\bar{\rho}^{-1}$, when computed in the effective theory. Such matrix elements can be estimated within the effective theory derived from the instanton vacuum, taking into account the full momentum dependence of the constituent quark mass [21, 22].

Results for quark and antiquark distributions. To compute the DF’s one needs to evaluate the sums over quark levels, Eqs. (15 – 20), taking into account the contributions of the bound–state level as well as the polarized Dirac continuum. This can be done either by numerical diagonalization of the hamiltonian [2], or by rewriting the sums over levels in terms of the quark Green function in the background pion field, which allows an approximate evaluation of the continuum contribution using the so-called interpolation formula [1].

Results for the isosinglet–unpolarized and isovector–polarized distributions are shown in Fig.1 and 2, where we compare with the parametrizations of Glück, Reya et al. [6, 7]. In the calculations reported here we have chosen a simple UV regularization by way of

![Graph showing quark and antiquark distributions.](image-url)
Eq.\((22)\) and used the standard parameters for the effective chiral theory (see refs.\([1, 2]\) for details); we have not attempted to make a best fit. Fig.\(1\) shows the isosinglet unpolarized valence and antiquark DF. We note that the antiquark distribution obtained in our approach is of the “valence-like” (\textit{i.e.}, non-singular at small \(x\)) form assumed which was assumed in the fits of ref.\([3]\). Fig.\(2\) shows the isovector polarized total distribution (quarks plus antiquarks) as well as the antiquark distribution, which was set to zero in the fit of ref.\([7]\). That the calculated total distribution is systematically smaller than the fit is due to the fact that the isovector axial coupling, \(g_A^{(3)}\), is underestimated by the large–\(N_c\) limit; the \(1/N_c\)-corrections in this channel are large and have been computed \([10]\). In Figs.\(1\) and \(2\) we compare the calculated DF’s with the parametrizations at face value, without taking into account evolution; the normalization point of the calculated DF’s may actually be lower than that of the parametrizations.

We stress that we are computing the twist–2 parton distributions at a low normalization
point, not the structure functions (cross sections) at low $q^2$, which are, in principle, directly measurable. The latter are affected by higher-twist (power) corrections, which become large at low $q^2$. Our calculated distributions should be used as input for perturbative evolution, starting with a scale below the cutoff, $\bar{\rho}^{-1} \simeq 600 \text{MeV}$. A direct comparison with the data can be performed only after perturbative evolution to sufficiently large $q^2$.

Results of a calculation of the “small” DF’s in the large–$N_c$ limit, the isovector unpolarized and isosinglet polarized, have been reported by Wakamatsu and Kubota [26].

Recently also the off-forward quark DF’s have been computed in the chiral soliton model [27]. These generalized DF’s are defined as nonforward matrix elements of light–ray operators of the type of Eq.(12) (non-zero momentum transfer). It was found that the large–$N_c$ approach predicts a strong dependence of the isosinglet off-forward distribution on the longitudinal momentum transfer.

Summary. To summarize, we have shown that the large–$N_c$ picture of the nucleon as a soliton of the effective chiral theory is able to explain the essential features of the quark and antiquark DF’s at a low normalization point. As we have shown, the success of this approach is essentially due to two circumstances:

- the parametric smallness of the ratio of the dynamical quark mass to the cutoff of the effective theory, $M\bar{\rho}$ (related to the small packing fraction of the instanton medium), which allows to identify the twist–2 QCD (anti–) quark DF’s with the distributions computed in the effective theory;
- the field–theoretic character of this description, which ensures that the DF’s satisfy all general requirements such as positivity, normalization etc.

An important direction for future work is the refinement of this approach to a level that allows one to take into account effects of higher order in $(M\bar{\rho})^2$, i.e., to resolve the structure of the “constituent” quark. This can be done in the framework of the instanton vacuum, which allows to derive the effective chiral action by explicit integration over non-perturbative gluon degrees of freedom. In particular, at order $(M\bar{\rho})^2$ the gluon distribution will start to appear. Also, this will allow to determine the normalization point of the calculated DF’s more accurately.

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