Spherical beamforming for spherical array with impedance surface

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Abstract. Spherical microphone array beamforming has been a popular research topic for recent years. Due to their isotropic beam in three dimensional spaces as well as a certain frequency range, the arrays are widely used in many applications such as sound field recording, acoustic beamforming, and noise source localisation. The body of a spherical array is usually considered perfectly rigid. A sound field captured by the sensors on spherical array can be decomposed into a series of spherical harmonics. In noise source localisation, the amplitude density of sound sources is estimated and illustrated by mean of colour maps. In this work, a rigid spherical array covered by fibrous materials is studied via numerical simulation and the performance of the spherical beamforming is discussed.

1. Introduction

A spherical microphone array is a versatile device that is mainly used in noise source localisation and beamforming. The entire analysis of spherical array with rigid surface is shown in following most-cited articles authored by Rafaely [1, 2]. The working bandwidth of the array is limited by the number of sensors of the array and the size of the spherical body. This leads the array to a narrow working bandwidth that is regarded as a major disadvantage of this technology. There are some attempts to extend the bandwidth of the array. Meyer and Elko suggested that the bandwidth of the array can be extended if the sensors are more directive at high frequency and the best performance can be obtained when the beam width of the sensor is close to the beam width of eigenbeam [3]. Epain and Daniel attempt to push the frequency limit of a spherical array by making a conical waveguide then placing an omnidirectional sensor at the end [4]. That makes the sensors become more directive and the working bandwidth of the array was extended as a consequence. The extension of the spherical beamforming was studied further. Tontiwattanakul suggests that a beamforming technique based on singular value decomposition namely SVD beamforming has capability to extend the bandwidth of the array. It was verified by both numerical study and experiment that arrays with waveguides have slightly better performance [5].

The scattering sound field on the surface of a sphere changes depending on the boundary condition. In microphone array application, sound hard boundary condition is usually assumed. However, there are some studies about concerns on the scattering sound field on a sphere with impedance surface. Treeby et al. studied sound field scattered by a sphere with the impedance surface [7]. The total sound field is derived based on spherical harmonics decomposition. Treeby also considered a sphere with an impedance surface as a human head with hair. The effect of the impedance surface to interaural time
difference (ITD) and interaural level difference (ILD) in comparison to one with rigid surface was discussed in [8].

In this work, a spherical array with impedance surface is studied. Spherical beamforming is employed to a rigid spherical array with or without impedance surface. The performance of the two arrays is evaluated and discussed.

2. Sound field on the surface of a rigid sphere with or without impedance surface

A sound field on the surface of a spherical object can be expressed in terms of spherical harmonics. When a given plane wave with single frequency $\omega$ impinges on the surface of a sphere of radius $r_0$ located at the center of a coordinate system, the total sound field at any position in three-dimensional space denoted by $x$ can be computed by a summation of the incident plane wave $p^{\text{inc}}(x, \omega)$ and the corresponding scattered field $p^{\text{sc}}(x, \omega)$

$$p^{\text{tot}}(x, \omega) = p^{\text{inc}}(x, \omega) + p^{\text{sc}}(x, \omega)$$

The incident sound field can be represented by a series of spherical harmonics. It is also known as Jacobi-Anger expansion [9] given by

$$p^{\text{inc}}(x, \omega) = 4\pi p_0 \sum_{\nu=0}^{\infty} \sum_{\mu=-\nu}^{\nu} (-i)^\nu j_\nu(\kappa x) Y_\nu^\mu(\hat{x}) Y_\nu^\mu(\hat{y})^\star,$$

where $\kappa = \omega / c_0$ denotes the wave number, $c_0$ denotes the speed of sound in air, $i$ denotes the imaginary unit, $\hat{x} = x / x$ denotes a unitary vector indicating the direction such that the pressure on the sphere is evaluated at position $x$ with magnitude $|x| = x$, $\hat{y}$ denotes a unitary vectors indicating the direction of arrival of the incident wave, $j_\nu(\cdot)$ denotes a spherical Bessel function of the first kind of degree $\nu$, and $Y_\nu^\mu(\hat{x})$ and $Y_\nu^\mu(\hat{y})$ denotes a spherical harmonic of order $\nu$ and degree $\mu$ and is evaluated in the directions $\hat{x}$ and $\hat{y}$, respectively.

Note that, the time convention $e^{-i\omega t}$ is adopted throughout this paper. Therefore, the scattered field $p^{\text{sc}}(x, \omega)$ can be represented by means of the spherical harmonic expansion, given by

$$p^{\text{sc}}(x, \omega) = \sum_{\nu=0}^{\infty} \sum_{\mu=-\nu}^{\nu} C_{\nu\mu}(\kappa x) h^{(1)}_\nu(\kappa x) Y_\nu^\mu(\hat{x}),$$

where $h^{(1)}_\nu(\cdot)$ denotes Hankel function of the first kind. When the total pressure on the sphere is considered, Fourier coefficient $C_{\nu\mu}$ can be determined depending on the boundary condition. In case of the sphere with rigid surface, $C_{\nu\mu}$ can be obtained by applying Neumann boundary condition given as the following

$$\frac{\partial}{\partial \hat{x}^\nu} p^{\text{sc}}(x, \omega) = 0.$$
For the sphere with uniform impedance surface with the specific impedance \( Z_{s}(\omega) \), the boundary condition is

\[
\frac{\partial}{\partial x} p_{\text{tot}}^e(x, \omega) + \frac{i \rho c_0}{Z_{s}(\omega)} p_{\text{tot}}^e(x, \omega) = 0.
\] (5)

By combining \( C_{\mu} \) with equations (2) and (1), the total sound field on the surface of the sphere is given by

\[
p_{\text{tot}}^e(r_0, \omega) = \sum_{\nu=0}^{\infty} \sum_{\mu=0}^{\nu} Y_{\nu}^\mu(\hat{x}) Y_{\nu}^\mu(\hat{y}),
\] (6)

where \( r_\nu \) is called radial function and it is given by

\[
r_\nu = 4\pi(-i)^\nu \left( j_\nu(kr_0) - \frac{j_\nu'(kr_0)}{i h_{\nu}^{(1)}(kr_0)} \right),
\] (7)

for the rigid sphere and

\[
r_\nu = 4\pi(-i)^\nu \left( j_\nu(kr_0) - \frac{j_\nu'(kr_0) + i \rho c_0 j_\nu(kr_0) / Z_{s}(\omega) h_{\nu}^{(1)}(kr_0)}{i h_{\nu}^{(1)}(kr_0) + i \rho c_0 h_{\nu}^{(1)}(kr_0) / Z_{s}(\omega)} \right),
\] (8)

for the impedance surface.

Equations (7) and (8) suggest that sound field at any point on the spheres can be computed. The derivation of the expression in equations (7) and (8) is shown in many works by previous authors, for examples, in [2, 7, 9].

Note that when a hard surface is covered by fibrous material, some of the acoustic power convert into active and reactive power. The surface is now considered as an impedance surface associated with the specific acoustic impedance (or complex impedance) as a function of frequency. A model that is widely used to predict the specific impedance \( Z_{s}(\omega) \) of fibrous materials is the one proposed by Delany and Bazley [10] and given by

\[
Z_{s}(\omega) = \rho_0 c_0 \left( 1 + 0.571 \frac{\rho_0 f^{-0.754}}{\sigma} + i 0.087 \frac{\rho_0 f^{-0.732}}{\sigma} \right),
\] (9)

where \( f \) is the frequency of the incident wave and \( \sigma \) is the flow resistivity of the fibrous material [10]. For a given apparent density \( \rho_{av} \) and the mean fibre diameter \( d_{av} \), then the flow resistance can be computed by the following formula [11]

\[
\sigma = \frac{3.18 \times 10^{-9} \times \rho_{av}^{1.53}}{d_{av}^2}.
\] (10)

The relations in equations (9) and (10) are empirical formula obtained by measurement of a large number of samples. The measurement data was then curve-fitted. Sometimes the specific impedance is referred to in the normalised version denoted by \( \zeta \) such that \( Z_{s}(\omega) = \rho_0 c_0 \zeta \) and is given by
\[ \zeta = \gamma + i\xi, \]  
(11)

where \( \gamma \) and \( \xi \) denote resistance and reactive components of the specific impedance, respectively.

3. Directivity of a sensor mounted on the sphere with or without covering materials

In this section, the directivity of the sensor as it is mounted on the surface of the rigid sphere is studied. The sphere of interest has its radius of 0.070 m. There is an omnidirectional microphone, as a sensor, flush mounted on the surface of the sphere in the direction \( \theta = \pi / 2 \) and \( \phi = 0 \), where \( \theta \) and \( \phi \) denote elevation angle and azimuth angle, respectively. The elevation angle starts from the north pole of the sphere and the azimuth angle starts from the x-axis of the Cartesian coordinate. The numerical study is performed in three cases as follows: firstly, the rigid sphere without covering of material; secondly, the rigid sphere with covering of lightweight glass wool with \( \rho_{av} = 12 \text{ kg/m}^3 \) and \( d_{av} = 10 \mu \text{m} \); and thirdly, the rigid sphere with covering of felt with density of 190 kg/m\(^3\) and thickness of 10.3 mm.

![Figure 1. The specific impedance of lightweight glass wool predicted by Delany and Bazley’s formula (left) and curve-fitted values of the specific impedance of felt with density of 190 kg/m\(^3\) and 10.3 mm thick obtained from a measurement (right) ](image-url)

In the first case, the specific acoustic impedance is calculated using Delany and Bazley’s formula as given by equation (9). It should be noted that Delany and Bazley’s formula provide accurate prediction when \( 0.01 < \rho_o f / \sigma < 1.0 \) or \( 10 < f < 1,200 \text{ Hz} \), approximately. Therefore, a big assumption is then made at this state such that Delany and Bazley’s formula is used to predict the specific impedance of the glass wool up to 9,000 Hz. By considering the plot of the specific impedance of the glass wool as shown in figure 3, both resistance and reactive parts of the specific impedance converge at high frequencies. This suggests that the specific impedance at high frequency can be considered as a constant complex number.

In case of the sphere covered by felt, the tested data as reported in \[8\] is used in this study. The specific impedance is measured using impedance tube method according to ISO 10534-2. The data was curve-fitted by 6-th order polynomial in the range between 250 Hz to 9,000 Hz with 95% confidence level. Figure 3 shows the tested data overlaid with the interpolated data. The curve-fitted functions are given by
\[
\gamma = 7.473 \cdot 10^{-22} f^6 - 2.31 \cdot 10^{-17} f^5 + 2.806 \cdot 10^{-13} f^4 - 1.693 \cdot 10^{-9} f^3 + 5.272 \cdot 10^{-6} f^2 - 0.0079 f + 5.361,
\]
and
\[
\xi = 2.882 \cdot 10^{-21} f^6 - 8.915 \cdot 10^{-17} f^5 + 1.085 \cdot 10^{-12} f^4 - 6.578 \cdot 10^{-9} f^3 + 2.074 \cdot 10^{-5} f^2 - 0.0319 f + 19.9.
\]

The total pressure picked up by the sensor is then calculated. In order to study the directivity of the sensor, it is assumed that there are plane waves with unitary amplitude impinging on the spheres from all directions on the azimuth plane (\(\theta = 0\)). The corresponding total pressure field can be computed by using equations (7) and (8). It should be noted that the order of the spherical harmonics must be properly high in order to guarantee the convergence of the series. Treeby et al. discussed in [7] that 30 orders of spherical harmonics are enough to obtain converging result. Nevertheless, 50 orders of spherical harmonics are used in the calculation of this study.

**Figure 2.** Directivity pattern of a sensor flush mounted on a rigid sphere (top half of the left and the right plot), on a rigid sphere with glass wool (bottom half of the left plot), on a rigid sphere with felt (bottom half of the right plot).

**Figure 3.** Normalised directivity pattern of a sensor flush mounted on a rigid sphere (top half of the left and the right plot), on a rigid sphere with glass wool (bottom half of the left plot), on a rigid sphere with felt (bottom half of the right plot).
A polar plot of the magnitude of total pressure at all azimuth angles is now referred to as directivity pattern. Figure 2 shows comparisons of directivity when the sensor mounted on the rigid sphere with or without covering material. It can be seen that the magnitude of total field of the sphere with impedance surface is smaller in comparison to that of the rigid sphere. The total pressure on the circumference is then normalised by the maximum value and is shown in figure 3 in order to provide better illustration for the directiveness. The sensor seems to be slightly more directive when there is covering material.

4. Discrete spherical array beamforming

Let a spherical array of radius $r_0$ have $M$ sensors flush mounted on the surface and the sensors are populated in a nearly uniform scheme. The coordinates of the nearly uniform points on the sphere can be found in [12]. Let plane waves arriving to the array from $S$ directions and $S > M$. Such sound sources distribute uniformly as well as the sensors. The sound pressure picked up by $m$-th sensor can be regarded as a contribution of the sound sources impinging to the sphere from all $S$ directions. This approach is sometimes referred to as plane waves superposition [5, 13] given by

$$p(x, \omega) = \int_{\Omega} H(x, \hat{y}, \omega)a(\hat{y}, \omega)d\Omega(\hat{y}),$$

(12)

where $a(\hat{y}, \omega)$ is the amplitude density of plane wave, and $\Omega(\hat{y})$ is a solid angle corresponding to the direction $\hat{y}$. It should be noted that when the sound sources $a(\hat{y}, \omega)$ are distributed uniformly then the solid angle is equal. $H(x, \hat{y}, \omega)$ denotes acoustic transfer function. In case of rigid sphere, it is given by

$$H(x, \hat{y}, \omega) = \sum_{l=0}^{\infty} \sum_{\mu} b_l(\kappa x) Y_l^\mu(\hat{x})Y_l^\mu(\hat{y})^*, \quad |x| = x = r_0,$$

(13)

When the sound sources are distributed in nearly uniform configuration, equation (12) can be rewritten in a matrix equation as

$$p = Ha,$$

(14)

where

$$p = \begin{bmatrix} p(x_1, \omega) & p(x_2, \omega) & \cdots & p(x_M, \omega) \end{bmatrix}^T,$$

$$a = \begin{bmatrix} a(\hat{y}_1, \omega)\Delta\Omega(\hat{y}_1) & a(\hat{y}_2, \omega)\Delta\Omega(\hat{y}_2) & \cdots & a(\hat{y}_L, \omega)\Delta\Omega(\hat{y}_L) \end{bmatrix}^T,$$

and

$$H = \begin{bmatrix} H(x_1, \hat{y}_1, \omega) & H(x_1, \hat{y}_2, \omega) & \cdots & H(x_1, \hat{y}_L, \omega) \\ H(x_2, \hat{y}_1, \omega) & H(x_2, \hat{y}_2, \omega) & \cdots & H(x_2, \hat{y}_L, \omega) \\ \vdots & \vdots & \ddots & \vdots \\ H(x_M, \hat{y}_1, \omega) & H(x_M, \hat{y}_2, \omega) & \cdots & H(x_M, \hat{y}_L, \omega) \end{bmatrix}.$$
where \( m \) and \( l \) denote the index number of the sensor and that of the direction of arrival of the impinging plane wave, respectively, and \( \Delta \Omega(\hat{y}_l) \) denotes a solid angle corresponding to \( l \)-th sound source. Position \( \mathbf{x}_m \) denotes the position of the \( m \)-th sensor.

The elements of matrix \( \mathbf{H} \) can be computed using equation (13) and can be rewritten by

\[
H(\mathbf{x}_m, \hat{y}_l, \omega) = \sum_{r=0}^{\infty} b_r(kr_0, r_0) \sum_{\mu=0}^{\infty} Y_{\mu}^\mu(\hat{x}_m) Y_{\mu}^\mu(\hat{y}_l)^*.
\]  

Note that matrix \( \mathbf{H} \) can be rewritten in terms of spherical harmonic matrices \( \mathbf{Y}_x \) and \( \mathbf{Y}_y \), and the corresponding radial weighting matrix \( \mathbf{B} \) as

\[
\mathbf{H} = \mathbf{Y}_x \mathbf{B} \mathbf{Y}_y^H.
\]  

The working bandwidth of a spherical array is limited by size of the array and number of sensors. The array tends to suffer the so-called spatial aliasing when the size of the array is comparable to the wavelength of an impinging wave, in other words, when \( kr_0 > N \) where \( k = \omega / c_0 \) is the wave number of the sound field impinging on the array, \( \omega \) is the angular velocity, \( c_0 \) is the speed of sound, \( r_0 \) is the radius of the array, and \( N \) is the order of spherical harmonic. \( N \) is given by the relation \( N = \sqrt{(M - 1)} \), where \( M \) is the number of sensors distributed uniformly around the sphere \[2\]. In order to avoid spatial aliasing, it is important to note that matrix \( \mathbf{H} \) must be order limited according to the criterion \( N = \sqrt{(M - 1)} \) \[2\].

**Figure 4.** 36 sensors distributed on the surface of the sphere in nearly uniform scheme, where the numbers denote microphone indices.

By substituting equation (16) into (14), thus

\[
\mathbf{p} = \mathbf{Y}_x \mathbf{B} \mathbf{Y}_y^H \mathbf{a}.
\]  

and the amplitude density of the plane waves can be computed by the following relation
\[ \mathbf{a}_{sp} = \mathbf{Y}_y \mathbf{B}' \mathbf{Y}_s'' \mathbf{p}. \]  

(18)

where \((\cdot)'\) and \((\cdot)''\) denote Moore-Penrose pseudoinverse and Hermitian transpose of matrix, respectively. Equation (18) is now referred to as spherical beamforming and \(\mathbf{a}_{sp}\) is the output of the beamformer.

By substituting equation (17) into (18), an analysis can be taken at this state

\[ \mathbf{a}_{sp} = \mathbf{Y}_y \mathbf{B}' \mathbf{Y}_s'' \mathbf{Y}_s \mathbf{B}' \mathbf{Y}_y'' \mathbf{a}. \]  

(19)

It should be noted that the multiplication \(\mathbf{Y}_s'' \mathbf{Y}_s\) does not result an identity matrix due to the lag of the completeness of an order limited system. Thus, some modification can be taken by replacing Hermitian transpose by pseudoinverse. Pseudoinverse beamforming is given as below

\[ \mathbf{a}_{\text{inv}} = (\mathbf{Y}_y'')' \mathbf{B}' \mathbf{Y}_s' \mathbf{p}. \]  

(20)

where \(\mathbf{a}_{\text{inv}}\) denotes beamformer output of the pseudoinverse beamformer.

5. Performance of the spherical beamformer

In this work, there are 36 sensors distributed nearly uniformly all around the sphere. Figure 4 depicts the spherical body with sensors arranged in nearly uniform scheme. By using the criterion discussed in the previous section, the rigid spherical array does not suffer from spatial aliasing for the frequency up to 3,900 Hz. The performance of the array above the aliasing limit is discussed in this section.

The total pressure picked up by the sensors of the array in three following cases is computed using equation (6). The output from the pseudoinverse beamformer, which is given by equation (20), is calculated and plotted as shown in figures 5 through 8 in the dynamic range of 20 dB in reduction. Note that the black dots overlaid on the colour map plots indicate the positions of the 36 sensors. By considering the directivity pattern of the beamformer output, it can be seen that large side lobes appear at frequencies above aliasing frequency. However, this phenomena is acceptable in some applications, for example, in noise source localisation.
The performance of the array can be evaluated based on a quantitative index. The directivity index (DI) is widely used as an indicator and is given by

\[
DI(\omega) = 10\log_{10} \left\{ \frac{4\pi |p_{\alpha}|^2}{\int_0^{2\pi} \int_0^\pi |p(\theta,\phi)|^2 \sin\theta d\theta d\phi} \right\},
\]

where \( p_{\alpha} \) denotes magnitude of the beamformer output at direction of the main lobe, \( p(\theta,\phi) \) denotes magnitude of the beamformer output at direction \((\theta,\phi)\). The directivity index of the spherical array with or without impedance surface is computed from 250 to 9,000 Hz and is plotted in figure 9. The decaying of the DI in figure 9 suggests that the array loses its performance at around 3,500 Hz.

In practice, the low frequency limit of the array is usually constrained by the measurement error of the sound field, i.e. numerical error when a sound field is computed for the numerical simulation and the noise of sensor. However, this point is beyond the discussion of this paper.

Figure 5. The directivity pattern beamformer output at 500 Hz of the rigid sphere (top left plot), the rigid sphere with glass wool (top right plot), and the rigid sphere with felt (bottom)
Figure 6. The directivity pattern beamformer output at 3,000 Hz of the rigid sphere (top left), the rigid sphere with glass wool (top right), and the rigid sphere with felt (bottom)

Figure 7. The directivity pattern beamformer output at 5,500 Hz of the rigid sphere (top left), the rigid sphere with glass wool (top right), and the rigid sphere with felt (bottom)
Figure 8. The directivity pattern beamformer output at 8,000 Hz of the rigid sphere (top left), the rigid sphere with glass wool (top right), and the rigid sphere with felt (bottom).

Figure 9. The directivity index (DI) of the beamformer outputs as a function of frequency.
6. Conclusion

This paper discusses the effect of the spherical array with or without impedance surface. Some numerical studies were carried out. It can be concluded that the impedance surface plays an important role to the scattering of sound field on the surface of the sphere. The numerical studies carried out in this work suggest that the impedance surface also affects the performance of the spherical array. The flush mounted sensor on the impedance surface tends to be more directive than one without the impedance surface.

By considering figure 1, it suggests that magnitude of the resistance part of the two materials is not much different as it seems to level off at around 1 but the difference in the reactive part is considerable. It implies that the reactive part of the material might be a factor that obstructs the improvement of the sensor’s directivity.

Figures 5 and 6 illustrate the directivity pattern of the beamformer below the aliasing frequency. It can be seen that the main lobe distinguishably indicates the direction of sound source. Figures 7 and 8 illustrate the performance of the three beamformer outputs. Some large sidelobes appear and become larger at higher frequency. Although, sidelobes seem to be smaller for the case that the array is covered by fibrous material, it is difficult to assess the performance by visualising.

The DI plot in figure 9 suggests that the array with felt seem to have lower performance than the others since its DI is always lower than that of the others. The one with glass wool seems to have slightly better performance (2 dB higher) at high frequencies above 5,000 Hz.

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