A model for WZ Sge with “standard” values of $\alpha$

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ABSTRACT
We present a model for the dwarf nova WZ Sge which does not require assuming unusually and unexplained low values of the viscosity $\alpha$ – parameter during exceptionally long quiescent states of this system. We propose that the inner parts of the accretion disc are disrupted by either a magnetic field or evaporation, so that the disc is stable (or very close to being stable) in quiescence, as the mass transfer rate is very low and the disc can sit on the cool, lower branch of the thermal equilibrium curve. Outbursts are triggered by an enhanced mass transfer, which brings the disc into the unstable regime of the standard dwarf nova disc instability model. The resulting eruptions are strongly affected by the irradiation of the secondary star. Our model reproduces very well the recurrence time and the characteristics of the light curve in outburst.

Key words: accretion, accretion discs – novae, cataclysmic variables – stars: individual: WZ Sge

1 INTRODUCTION

Dwarf novae (DN) are cataclysmic variables, which, at usually irregular intervals, undergo eruptions in which the brightness increases by 2 to 7 magnitudes. It now well established that dwarf nova eruptions have their origin in a local, thermal and viscous instability due to an abrupt change in opacities at densities and temperatures at which hydrogen is partially ionized. In the disc instability model (DIM) of dwarf nova outbursts one assumes that the $\alpha$ viscosity parameter is higher in the high state than in quiescence and on obtains in this way a global disc instability which gives a rather good description of the U Gem type dwarf nova properties. Current models of dwarf novae imply values of the $\alpha$ parameter of the order of 0.01 in the low, quiescent state, and about 4 – 10 times larger during outbursts.

The DIM in its standard version cannot however describe outbursts of Z Cam and SU UMa type of dwarf novae. In the ‘standard’ DIM one assumes that the mass transfer from the secondary star is constant, that the only instability operating in the disc is the thermal–viscous one, and one neglects illumination of the disc and of the secondary by the radiation emitted by the accretion on to the white dwarf during outbursts. It is clear that if the Z Cam and SU UMa type of dwarf nova eruptions are due to the same type of local instability as the one that operates in U Gem type dwarf nova systems at least one of the standard assumptions must be dropped.

In the case of SU UMa type dwarf novae it has been proposed by Osaki (see e.g. 1995) that their superoutbursts are due to a tidal-thermal instability (TTI). In this model, successive U Gem type outbursts (‘normal’ outbursts) lead to an accumulation of matter in the disc and an increase of its outer radius. When the disc’s outer rim enters the ‘tidal radius’ in which the 3:1 resonance operates, the tidal couple is assumed to increase the accretion rate and to trigger a superoutburst in which matter accumulated during the supercycle is dropped on to the white dwarf. The TTI model encounters several difficulties. One is that it cannot apply (see e.g Smak 1996) to the 1985 superoutburst (Mason et al. 1988) of U Gem since the tidal instability cannot operate in this system. Another one is connected with the superoutbursts of WZ Sge. The TTI model can describe this system which shows only superoutbursts separated by very long quiescent intervals, only if one assumes a very low value of $\alpha \lesssim 10^{-4}$ (Smak 1993; Osaki 1995). The reason for such a low $\alpha$ in this particular system is however unexplained so that it suggests that the TTI model might not be the correct description of the WZ Sge behaviour.

Lasota et al. (1995) noticed that if the inner disc in WZ Sge was missing because of the presence of a magnetosphere (Livio & Pringle 1992) or because of evaporation (Meyer & Meyer–Hofmeister 1994) the accretion disc would be marginally stable and outbursts could be triggered by an enhanced mass transfer (EMT) from the secondary. The long recurrence time would then be the timescale of fluctuations of the mass transfer and the $\alpha$ parameter would have its ‘usual’ value. The idea that superoutbursts in general could be due to EMT has been proposed by Smak (see e.g. Smak 1996).

Warner et al. (1996) assert that in the case of a truncated inner disc one can obtain long recurrence times in the ‘standard’ DIM with a ‘standard’ value of the $\alpha$ parameter. As we shall show below, the Warner et al. (1996) model is in fact, as far as recurrence time is concerned, practically identical to the one proposed by Lasota et al. (1995). The ‘standard’ DIM with ‘standard’ values of $\alpha$ suffers however from one insurmountable difficulty: the mass contained in
the quiescent accretion disc for the parameters describing WZ Sge and for \( \alpha \sim 0.01 \) is less than \( 10^{23} \) g whereas during the superoutburst of this dwarf nova more than \( 10^{24} \) g has been accreted by the central white dwarf (Smak 1993). In fact this difficulty is the main reason for invoking very low values of the viscosity parameter.

If one wishes to explain superoutbursts of WZ Sge without using extremely low values of \( \alpha \) one cannot avoid adding mass prior to or during the eruption. As discussed by Smak (1996) there is evidence of an increased mass transfer rate during normal outbursts and superoutbursts. In the EMT scenario proposed by Smak (1996) the superoutburst begins with a major enhancement of the mass transfer caused by irradiation during the preceding normal outburst. In the case of WZ Sge which shows only superoutbursts such a trigger does not exist.

In the present article we show that if, as proposed by Lasota et al. (1995), the quiescent accretion disc in WZ Sge is marginally stable, a slight enhancement of mass transfer triggers a ‘normal’ outburst which, due to subsequent irradiation of the secondary, becomes a superoutburst since a substantial amount of matter is added to the disc during the active phase of the cycle.

## 2 WZ SGE IN QUIESCENCE

The orbital period of WZ Sge is 81 min, close to the minimum period of cataclysmic variables; the system is thus very compact, and the accretion disc is quite small, with an outer radius of 1.1 \( 10^{10} \) cm during quiescence (Smak 1993). Here and in what follows we shall use Smak’s (1993) model of WZ Sge according to which the primary and secondary masses are 0.45 and 0.06 \( M_\odot \) respectively. From the luminosity of the hot spot, Smak (1993) determined a mass transfer rate in quiescence \( \dot{M}_q \approx 2 \times 10^{-5} \) g s\(^{-1}\).

WZ Sge has been detected by Einstein (Eracleous et al. 1991), EXOSAT (Mukai & Shiokawa 1993) and ROSAT (van Teeseling et al. 1996); its X-ray luminosity did not significantly vary in the interval from 4 months to more than 7 years after the 1978 outburst, and stayed at the level of \( 3 \times 10^{30} \) erg s\(^{-1}\). If interpreted in terms of accretion rate on to the white dwarf surface, this luminosity corresponds to an accretion rate \( \dot{M}_\text{acc} \) on to the white dwarf surface of:

\[
\dot{M}_\text{acc} = 5.0 \times 10^{13} \eta_X^{-1} r_9^{-1} \left( \frac{M_1}{0.45 M_\odot} \right)^{-1} \left( \frac{L_X}{3 \times 10^{30}} \right) \text{gs}^{-1} \tag{1}
\]

where \( M_1 \) is the primary mass, \( r_9 \) the radius in units of \( 10^9 \) cm, \( L_X \) the X-ray luminosity and \( \eta_X \leq 1 \) the efficiency conversion of gravitational energy into X-ray photons. This would imply that the disc is not very far from steady state, since the mass accretion rate did not vary significantly from soon after the outburst to now, i.e. during an interval which is comparable to the recurrence time.

Moreover, according to the DIM, in the non-equilibrium disc \( \dot{M}_\text{acc} \) has to be low enough so that the inner regions of the disc are on the cool, stable branch of the \( \Sigma - T_{\text{eff}} \) curve, which implies:

\[
\dot{M} < \dot{M}_B = 1.43 \times 10^{13} M_1^{-0.87} r_9^{2.60} \text{gs}^{-1} \tag{2}
\]

(Ludwig et al. 1994). Since the accretion disc has to be in this state very early after outburst one concludes, comparing Eqs. (1) and (2), that in the DIM framework the quiescent viscosity cannot be very small, i.e. that \( \alpha \) cannot be very small.

Consistency between X-ray observations and Eq. (2) requires the radius of the inner edge of the disc \( r_{\text{in}} \) to be larger than:

\[
r_{\text{in}} > 1.43 \times 10^9 \eta_X^{-0.62} \left( \frac{M_1}{0.45 M_\odot} \right)^{0.08} \text{cm} \tag{3}
\]

This is larger than the white dwarf radius, which is easily understood if the inner parts of the disc are disrupted by either a magnetic field (see e.g. Livio & Pringle 1992), or by evaporation (Meyer & Meyer-Hofmeister 1994). At such radii however, the disc is very close to a globally stable configuration (Lasota et al. 1995; Warner et al. 1996). In the following, we assume \( r_{\text{in}} = 4 \times 10^9 \) cm, which ensures that the total disc luminosity is less than the luminosity of the hot spot, in agreement with observations.

## 3 THE OUTBURST

As mentioned in the introduction two “standard \( \alpha \)” mechanisms have been proposed for triggering outbursts in WZ Sge. In one of them, the disc is supposed to be stable on the cool branch in quiescence, in which case outbursts have to be triggered by an increase of the mass transfer from the donor star, causing the disc to become thermally unstable (Lasota et al. 1995); in the other one, the disc is supposed to be marginally unstable and to undergo a ‘standard’ DIM outburst. (Warner et al. 1996). It must however be noted that, because the disc must remain on the cool, stable branch during quiescence, its mass cannot be more than the integral of the maximum surface density \( \Sigma \), i.e. \( M_d < M_{\text{max}} \approx 6 \times 10^{21} \alpha^{-0.8} \text{g} \) (Smak 1993). If the mass transfer rate from the secondary remains constant, the recurrence time can be estimated as the time it takes to increase \( M_d \) to this maximum value, i.e.

\[
t_{\text{rec}} \sim \frac{M_{\text{max}}}{M_d - M_{\text{acc}}} \sim 4 \left( \frac{\alpha}{10^{-2}} \right)^{-0.8} \left( 1 - \frac{M_{\text{acc}}}{M_d} \right)^{-1} \text{yr} \tag{4}
\]

If one requires that \( \alpha \) is not extremely small, \( M_{\text{acc}} \) must be very close to \( M_d \) (within 10\% ) during the whole quiescent phase in order to get very long recurrence times. It is therefore most likely that an outburst would be triggered by a small fluctuations of \( M_d \) that is expected to occur within the long recurrence time of WZ Sge. Since in the Lasota et al. (1995) model the disc is marginally stable, both models are in fact identical as far as the recurrence time is concerned.

In the standard DIM, the duration of the outburst is at most the time it takes to empty the disc, i.e.

\[
t_{\text{outb}} = \frac{M_{\text{max}}}{M_{\text{acc}} - M_d} \sim 3 \left( \frac{\alpha}{10^{-2}} \right)^{-0.8} \left( 1 - \frac{M_d}{M_{\text{acc}}} \right)^{-1} \text{days} \tag{5}
\]

where we took \( M_{\text{acc}} = 10^{18} \) g s\(^{-1}\) (Smak 1993). The long duration of WZ Sge outbursts (about one month) implies, if
\( \alpha \) is not very small, that is during outburst, the mass transfer rate had increased by two orders of magnitude, and was of the same order as the accretion rate. This would very naturally result from the illumination of the secondary. Effects of irradiation of the secondary were clearly observed during the outburst of SS Cyg (Hessman et al. 1984) and an increase of \( \dot{M}_t \) by factors \( \approx 2 \) has been observed in DNs such as Z Cha and U Gem (Smak 1995). Illumination effects are expected to be even more important in WZ Sge, in which the quiescent \( \dot{M}_t \) is particularly low, so that the secondary surface temperature is expected to be low, and which has the shortest orbital period, so that the X-ray flux heating the secondary is large. For example, Smak (1993) obtains for the effective temperature of the secondary during the outburst \( T_{\text{eff},2} \approx 17000 \) K.

In the simplest model for illumination, the mass transfer rate is proportional to \( \exp(-\Delta r/H) \), where \( \Delta r \) is the distance between the secondary photosphere and the Lagrangian point \( L_1 \), and \( H \) the atmospheric scale height, proportional to the secondary surface temperature. In the case of WZ Sge, \( \Delta r/H \) would typically be \( \gtrsim 1 \) in quiescence, and less than unity during outburst, leading to large variations of the mass transfer rate. The response of the secondary to illumination has been discussed by e.g. Osaki (1985) and Hameury et al. (1986), but the presence of screening effects, of flows from the secondary’s poles to \( L_1 \), and the dependence on the emitted spectrum are very complex and make it difficult to describe; the exponential dependence quoted above is certainly not a very good approximation. As a preliminary step, we assume here that

\[
\dot{M}_t = \gamma \dot{M}_{\text{acc}}
\]  

with \( \gamma < 1 \); this is similar to the approach of Augustein et al. (1993) in the context of soft X-ray transients.

The expected outcome of the model is that during quiescence, the disc stays on the cool stable branch. A fluctuation of the mass transfer rate from the secondary triggers the viscous/thermal instability, most probably at the outer edge of the disc. A heat front then propagates towards the compact object. Once it reaches the inner edge of the disc, \( \dot{M}_t \) increases up to a value \( \gamma \dot{M}_{\text{acc}} \), and then, since the viscous time of the disc is short as compared to the total duration of the outburst, the disc would be close to steady state, with a mass transfer rate equal to \( \gamma \) times the mass accretion rate, while the disc introduces a delay equal to the viscous time \( t_{\text{visc}} \). This naturally produces an exponential behaviour, with a decay time equal to \( t_{\text{visc}} / \ln \gamma \). Eventually, \( \dot{M}_t \) becomes less than the critical value below which the hot, stable solution in the \( \Sigma-T_{\text{eff}} \) diagram does not exist any longer; then a cooling wave starts from the outer edge of the disc, and brings it into quiescence in a short time scale.

In order to test this, we have calculated the time-dependent evolution of a disc initially stable with a low \( \dot{M}_t \), taken to be \( 1.5 \times 10^{15} \) g s\(^{-1} \), which is suddenly increased to \( \max(5 \times 10^{15}, 0.87 \dot{M}_{\text{acc}}) \) g s\(^{-1} \). The enhanced mass transfer rate decays exponentially in 5 days. \( \alpha \) is taken to be 0.01 in the cool branch and 0.1 in the hot one. All other parameters are those quoted here for WZ Sge; the code used is described in Hameury et al. (1996). The disc is assumed to be truncated

![Predicted visual light curve for the outburst of WZ Sge as a result of the presence of a magnetic field, so that:](image)

\[
\text{time (days)}
\]

\[
\text{visual magnitude}
\]

\[
\frac{r_{\text{in}}}{4} = 4 \times 10^9 \left( \frac{\dot{M}_{\text{acc}}}{1.5 \times 10^{15} \text{g s}^{-1}} \right)^{2/7} \text{cm}
\]

The disc behaves exactly as described above; for illustration, the visual magnitude of the disc (i.e. that does not include contributions from the secondary or from the white dwarf) is displayed in Fig. 1. It is seen that the shape of the light curve is in good agreement with observations.

## 4 CONCLUSION

We have shown that the unusually long recurrence time and outburst duration in WZ Sge does not require the viscosity in this system to be much lower than in all other systems; these characteristics would result from (1) a low value of the mass transfer rate, so that the system is marginally stable during quiescence; (2) a truncated disc, which is required in many other systems to account for e.g. the observed optical-UV delay; and (3) a significant illumination effect that increases the mass transfer rate from the secondary by two orders of magnitude. WZ Sge would thus be explained by a combination of the two different model proposed by Osaki (1974, 1985), in a way similar to the proposition of Duschl & Livio (1989); a fluctuation of the mass transfer rate produces a thermal/viscous disc instability that brings the disc into a hot state, leading to an sudden increase of \( \dot{M}_t \), which then slowly decreases until a cooling wave rapidly brings the disc back into its quiescent cool state.

The possible tests of this model are not very different from those proposed by Warner et al. (1996), since the outburst is an outside-in outburst, and since the enhancement of mass transfer that triggers the outburst is not very large, so that the behaviour of the disc outer radius need not be very different from that of a pure disc instability. We do however predict an increase of the hot spot luminosity a few days before the onset of an outburst; we also predict that the occurrence of outburst should be irregular (for example of a shot noise type). Finally, the hot spot should be much brighter during outbursts; this has been observed by Patterson et al. (1975) who inferred an enhancement of mass transfer from the secondary by a factor of 60 to 1000;
they deduced then that the cause of the outburst was a mass transfer instability, although they could not exclude the possibility “that a brightening of the white dwarf or disc could be the event that triggers unstable mass transfer from the secondary”.

REFERENCES

Augusteijn T., Kuulkers E., Shaham J., 1993, A&A, 279, L9
Duschl W. J., Livio M., 1989, A&A, 209, 183
Eracleous M., Halpern J., Patterson J., 1991, ApJ, 290,300
Hameury J.-M., King A. R., Lasota J.-P., 1986, A&A, 162, 71
Hameury J.-M., Huré J.-M., Lasota J.-P., in preparation
Hessman F.V., Robinson E.L., Nather, R.E., Zhang, E.-H., ApJ, 286, 747
Lasota J.-P., Hameury J.-M., Huré J.-M., 1995, A&A, 302, L29
Livio M., Pringle J., 1992, MNRAS, 259, 23p
Ludwig K., Meyer-Hofmeister E., Ritter H., 1993, A&A, 290, 473
Mason, K.O., Cordova, F.A., Watson, M.G., King, A.R., 1988 MNRAS, 232, 779
Meyer F., Meyer-Hofmeister E., 1994, A&A, 288, 175
Mukai K., Shiokawa K., 1993, ApJ, 418, 863
Osaki Y., 1974, PASJ, 26, 429
Osaki Y., 1985, A&A, 144, 369
Osaki Y., 1995, PASJ, 47, 47
Patterson J., McGraw J.T., Coleman L., Africano J.L., 1981, ApJ, 248, 1067
Smak J., 1993, Acta astron., 43, 101
Smak J., 1995, Acta astron., 45, 355
Smak J., 1996, in Cataclysmic Variables and Related Objects, IAU Coll. 158, p. 45
van Teeseling A., Beuermann K., Verbunt F., 1996, A&A, in press
Warner B., Livio M., Tout C. A., 1996, MNRAS, 282, 735

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