Multifractal Analysis for the Dynamical Heterogeneity in Strongly Correlated Many-Body Systems

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Abstract. By calculating the non-equilibrium parameter of the probability distribution function and the singularity spectrum of multifractal we have quantified the dynamical heterogeneity in strongly correlated many-body systems.

INTRODUCTION

We have numerically studied the dynamical heterogeneity in strongly correlated many-body systems, for example, the critical spin state[1], the supercooled liquid near the glass transition[2] and the turbulence[3]. These systems are scale-invariant and multifractal. In this paper we briefly review how we quantify the dynamical heterogeneity and discuss in detail the turbulence simulation data recently obtained.

In order to quantify the dynamical heterogeneity we have two measures. One is the non-equilibrium parameter $q$ of the Rényi-Tsallis distribution function. The other is the singularity spectrum $f(\alpha)$. These two measures are closely related and important ingredients of the multifractal analysis.

In the numerical study of the critical spin state[1] we have found that the $q$-parameter represents the degree of non-equilibrium. We have actually quantified the deviation of the $q$-parameter from the equilibrium value according to the spatio-temporal scale of the observation.

In the numerical study of the the supercooled liquid near the glass transition[2] we have found that the broken-bond distribution which reflects the nature of the cooperatively rearranging region is well described by the singularity spectrum $f(\alpha)$ of multifractal. The width of the spectrum becomes broader as approaching the glass transition. Such a broadening is similar to that observed in the numerical study of the Anderson-localization transition.

TURBULENCE SIMULATION

In the numerical study of the turbulence[3] we have adopted the lattice Boltzmann method. Our numerical turbulence on $200^3$ cubic lattice points is sustained by a random forcing and the Reynolds number is about 500.

mean-field analysis

Turbulence is one of the typical phenomena with multiscale motions. Each phenomena of a scale strongly couples with all the other scales of turbulent motion. In order to analyze such a system a scale-dependent entropy, the so-called $\varepsilon$-entropy $h(\varepsilon)$, works well. For example, the time series of the velocity field in turbulence leads to a non-trivial scaling relation, $h(\varepsilon) \propto \varepsilon^{-3}$, expected from Kolmogorov’s scaling. Here $\varepsilon$ is the scale of the observation. The existing experimental data are consistent with this scaling. By our numerical experiment simulating the Navier-Stokes equation we have shown the consistency of the scaling. In contrast to the evaluation of the energy spectrum, which is usually employed for testing Kolmogorov’s scaling and determined by the two-
point correlation function, Kolmogorov’s scaling is easily observed in the ε-entropy, since it is determined by the mean of the exit time from the observation window and fluctuations are averaged out. Such a unifractal scaling is a mean-field description and fluctuations can be taken into account in a multifractal analysis as shown in the following. In the above mentioned mean-field description the velocity difference in temporal and spatial directions have the same fractal scaling exponent. Thus we have confirmed Kolmogorov’s scaling and Taylor’s hypothesis at the same time.

fluctuation analysis

In order to analyze the fluctuations we use two measures, the non-equilibrium parameter $q$ of the Rényi-Tsallis distribution function and the singularity spectrum $f(\alpha)$.

The probability distribution function (PDF) for the velocity or vorticity field is non-Gaussian. The non-Gaussianity is observed at small spatio-temporal scale comparable to that for the coherent vortex in the PDF for the velocity or vorticity difference between two space-time points. By using the wavelet denoising we have clarified that the non-Gaussianity or dynamical heterogeneity results from the existence of the coherent vortex which is a strongly-correlated non-equilibrium region.

The strength of the vortices is shown in Fig. 1 to be intermittent. The spatial distribution of the coherent vortices is quantified by the singularity spectrum of multifractal $f(\alpha)$ as shown in Fig. 2.

The remainder of this subsection is devoted to some speculations. As seen in our previous study[2] the width of the singularity spectrum depends on the degree of intermittency so that we expect some Reynolds-number dependence of the spectrum. It has been discussed by many authors and should be clarified in future systematic study. In our present study the density of the vortex is dilute. Thus the singularity spectrum in Fig. 2 describes the spatial distribution of relatively free vortices. As the Reynolds number is increased the density increases. In this case the interaction among vortices becomes important and the correlation length of the vorticity fluctuation becomes large. While we can observe the vortex only as an individual elementary excitation in our numerical experiment, some collective excitation is expected to dominate at higher Reynolds number. In the limit of divergently large Reynolds number the correlation length becomes divergently large so that we can expect full scale-invariance. We can find a resemblance to the case of the scaling theory in polymers where an ideal scaling relation is realized for dense solutions where polymers are strongly entangled. In the limit of high Reynolds number each boxes counting the coherent vortex for calculating $f(\alpha)$ will be filled by almost equal number of vortices so that intermittency will disappear and unifractal Kolmogorov’s scaling will prevail.

summary for turbulence simulation

Numerical simulation data, in real space and time, for a forced turbulence on the basis of the lattice Boltzmann method have been analyzed by unifractal and multifractal schemes.

Our new findings are summarized into two points. First in the unifractal analysis using the exit-time statistics we have verified Kolmogorov’s scaling and Taylor’s hypothesis at the same time. Second in the analysis using the Rényi-Tsallis PDF and the wavelet denoising we have clarified that the coherent vortices sustain the power-law velocity correlation in the non-equilibrium state.

Finally in the multifractal analysis it is clarified that the intermittent distribution of the coherent vortices in space-time is described as a multifractal.

REFERENCES

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