Posted Price versus Auction Mechanisms in Freight Transportation Marketplaces

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We consider a freight platform that serves as an intermediary between shippers and carriers in a truckload transportation network. The platform’s objective is to design a mechanism that determines prices for shippers and payments to carriers, as well as how carriers are matched to loads to be transported, to maximize its long-run average profit. We analyze three types of carrier-side mechanisms commonly used by freight platforms: posted price, auction, and a hybrid mechanism where carriers can either book loads at posted prices or submit their bids. The proposed mechanisms are constructed using a fluid approximation model to incorporate carrier interactions in the freight network. We show that the auction mechanism has higher expected profits than the hybrid mechanism, which in turn has higher profits than the posted price mechanism. Thus, the hybrid mechanism achieves a trade-off between platform profit and carrier waiting time. We prove tight bounds between these mechanisms for varying market sizes. The findings are validated through a numerical simulation using industry data from the U.S. freight market.

Key words: online platforms, revenue management, mechanism design, freight transportation

1. Introduction

The trucking industry transports 72% of freight in the U.S. and generates a gross revenue of about $800 billion annually [ATA 2021]. However, this large industry is characterized by high market fragmentation. There are over 900,000 for-hire carriers in the U.S., of which 90% operate 6 or fewer trucks [ATA 2021]. Traditionally, the market relies on brokers and freight forwarders to connect shippers and carriers using phones or emails, a process that is time consuming and labor intensive. In recent years, digital freight marketplaces have grown rapidly. These digital platforms allow shippers and carriers to list and book loads through smartphone apps or websites in an automated process that is significantly more efficient than traditional brokers.

A central question faced by operators of digital freight platforms is how to design mechanisms for participants in their marketplaces. Unlike other types of transportation marketplaces such as ridesharing, where matching decisions are made by the platform operator in a centralized manner, most freight platforms allow carriers to browse any open loads and choose which loads they want
to transport. The decentralized matching scheme is a result of demand and supply heterogeneity in freight transportation, as carriers have their specific preferences for load features including cargo type, trailer type, length of haul, etc. A variety of market mechanisms are currently being used in practice to match loads to carriers: Some freight platforms set posted prices for loads; others use auctions that allow carriers to bid their own prices for loads; still others apply hybrid (or dual-channel) mechanisms that allow carriers to choose between posted price and bidding (e.g. Convoy 2018, Uber Freight 2020). For instance, Figure 1 illustrates the carrier app of a freight marketplace that uses such a hybrid mechanism. The platform displays information of available loads to carriers, such as a load’s origin and destination, distance, weight, pickup and drop-off time, as well as a posted price set by the platform (i.e., the expected payment to carriers). If a carrier wants to book a load, they can either book it instantly at the displayed price or submit a bid in order to receive possibly higher payments. However, booking a load through bidding requires the carrier to wait for the platform’s assignment decision, during which the carrier may be outbid by others.

![Figure 1](image)

Figure 1   An example of freight brokerage apps for carriers (source: Uber freight).

*Note: The left screen shows a list of open loads available to carriers for booking. The right screen shows a load being selected that displays two booking options: instant booking at the posted price (bottom right of the screen) or bidding (bottom left of the screen).*

The goal of this paper is to analyze different mechanisms used by freight marketplaces and understand their implications on market efficiency and platform profitability. To formulate the
mechanism design problem, we consider a stochastic model for a two-sided freight marketplace. The market operator observes the movement of loads and carriers in an interconnected freight transportation network. We assume that the behavior of carriers is characterized by a random utility model, and carriers’ private information is their true opportunity costs to transport a given load. By analyzing the decisions of carriers, we consider the three types of mechanisms mentioned earlier: posted price, auction, and hybrid (dual-channel) mechanisms.

1.1. Contributions
The main contributions of this paper are summarized as follows.

- **Freight marketplace model:** We formulate a two-sided freight marketplace as a discrete-time infinite-horizon Markov decision process (MDP). The model is defined over a transportation network where shipper demand and carrier supply arrive dynamically. We assume that all loads are available for one period and unmatched loads are expired. Meanwhile, carriers may stay in the marketplace for multiple periods. A fluid approximation model of the MDP is formulated by replacing random variables with their means. We show that the fluid approximation model can be reformulated as a convex optimization problem. In addition, the optimal objective value of the fluid model serves as an upper bound benchmark of any stable, incentive compatible, and individually rational dynamic mechanisms.

- **Mechanism design and theoretical analysis:** Using the fluid model, we construct several mechanisms and compare their relative performance in terms of long-run average profit. We consider a static posted price with fixed rates on each edge of the network. We also propose a uniform-price auction mechanism based on a decomposition of the network. Furthermore, we consider a hybrid mechanism that allows the carriers to self-select between the auction and the static posted price policy. We show that the expected profit of the auction mechanism is always higher than that of the hybrid mechanism, which in turn is higher than that of the static posted price mechanism. All three mechanisms are asymptotically optimal as the supply and demand both scale up in the system. We provide tight bounds between the posted price mechanism and the auction mechanism as a function of the scaling factor.

- **Numerical experiment:** To evaluate the numerical performance of the proposed mechanisms, we conduct a simulation using real data obtained from the U.S. freight market. We consider a freight network consisting of the lower 48 states, where each state is treated as a node in the network. We simulated the performance of the three proposed mechanisms under different load volume assumptions. In general, the auction mechanism has 4-5% lower carrier costs than the posted price mechanism, whereas the hybrid mechanism has around 2-3% lower costs. In particular, under the hybrid (dual-channel) mechanism, over 90% of carriers choose the posted price option, and this percentage is even higher as the load volume of the platform increases. Both hybrid and auction mechanisms have higher market efficiency, represented by fewer unmatched loads.
1.2. Literature Review

Auction mechanism design. The seminal result of Myerson (1981) characterized the optimal revenue maximizing mechanism for single-parameter mechanism design problems. However, the optimal revenue maximizing mechanism for general multi-item or multi-period problems can be very complex and may not be practically appealing. A stream of literature has studied the performance of simple-to-implement mechanisms such as sequential posted-price mechanisms or first/second-price auctions (Edelman et al. 2007; Chawla et al. 2010; Düttling et al. 2016; Jin et al. 2020; Balseiro et al. 2019); most of these papers are based on applications such as e-commerce and online advertising. Several papers have compared posted price and auction mechanisms (Wang 1993; Vulcano et al. 2002; Einav et al. 2018). We take a similar approach and consider simple mechanisms in this paper. Our contribution is that we consider auction design in a multi-location network setting where the number of bidders in each location is endogenous and depends on previous decisions from other locations.

There is a stream of literature considering hybrid or dual-channel mechanisms that offer both posted price and auction at the same time. Etzion et al. (2006) studied a problem of optimally selecting and designing auction and posted price mechanisms in parallel. Caldentey and Vulcano (2007) extended the work of Etzion et al. (2006) and considered dual channel problems with two variants, i.e., the external posted-price channel or the combined posted-price and auction channels. Osadchiy and Vulcano (2010), Board and Skrzypacz (2016) considered selling goods to strategic buyers using a hybrid mechanism that consists of a dynamic pricing mechanism during the season and an auction for end-of-season clearing sales. Cohen et al. (2022) considered designing dual-channel mechanisms for online advertising. Although the idea of hybrid mechanism is not new, our paper differs from this stream of literature by studying hybrid mechanisms in a network setting.

Pricing in two-sided platforms. Our work is also related to the stream of works on two-sided platforms, especially those on ride sharing. Several recent papers have examined the dynamic matching and pricing problem of two-sided platforms (e.g., Banerjee et al. 2016; Cachon et al. 2017; Özkan and Ward 2020; Chen and Hu 2020; Varma et al. 2020; Banerjee et al. 2021; Feng et al. 2021; Hu and Zhou 2021; Balseiro et al. 2021; Aveklouris et al. 2021). In view of this literature, our paper makes some modeling assumptions that reflect distinct features of freight platforms. First, the majority of papers consider centralized matching control, which is common practice in ridesharing; in other words, it is the platform operator who makes matching decisions for participants in the platform. In contrast, most freight platforms use decentralized matching schemes where carriers are free to choose any available loads, so we assume a setting with decentralized control in this paper. Second, the travel time of a long-haul trip in freight transportation is much longer (up to several days) than a ridesharing trip, so we assume an open network where some carriers leave the
platform at the end of each period and new carriers may arrive, rather than using a closed network assumed in some of the ridesharing papers. Third, our paper studies auction mechanisms, which is commonly used in freight transportation but is less used in ridesharing.

Closer to our work are the ridesharing papers that study pricing with spatial considerations. Bimpikis et al. (2019) consider the spatial transition of a ridesharing network and characterize the value of spatial price discrimination. The authors show that a fixed-commission rate pricing policy could result in significant profit loss if demand is not “balanced” across locations. Guda and Subramanian (2019) consider a two-location two-period setting and show the benefits of using surge pricing when the supply is strategic. Afèche et al. (2018) examine the impact of platforms’ demand-side admission control and supply-side repositioning control on system performance in a two-location network. Relatedly, Besbes et al. (2021) study the optimal spatial pricing strategy in a static equilibrium model, and examine how the platform should respond to short-term supply demand imbalances given that the supply units are strategic.

Freight marketplaces. Figliozzi et al. (2003) proposed a model for freight marketplaces using auction mechanisms and applied Vickery auctions in a simulation study. In a following work, Figliozzi et al. (2005) compared three different sequential auctions numerically. However, the two papers above assume that auctions are run by shippers rather than some freight brokerage platform. Caplice (2007) gave a survey of auctions in truckload transportation marketplaces. Topaloglu and Powell (2007) proposed an optimization model for a carrier’s fleet by integrating shipper pricing and load assignment decisions. Cao et al. (2022) considered a dynamic pricing problem for freight marketplaces that is closely related to this paper. The authors proposed a static pricing policy by setting a fixed price for each lead time. There are several key differences between Cao et al. (2022) and this paper. Their work focused on posted price mechanisms, whereas our paper focuses on comparing posted price mechanisms with auction mechanisms. Moreover, Cao et al. (2022) considered individual lanes and thus did not consider a network setting, whereas our paper explicitly models the interactions of carriers in the freight network.

2. Model
We formulate the marketplace design problem for a freight transportation platform that serves both shippers (the demand side) and carriers (the supply side). On this platform, shippers post information about goods that need to transported, commonly referred to as loads in the freight industry. Carriers book loads on the platform and transport them. Our objective is to design a market mechanism that specifies how the platform should set prices for shippers and determine allocation and payment rules for carriers in order to maximize the expected profits of the platform.

The model is defined in a transportation network represented by a graph $\mathcal{G}(\mathcal{N}, \mathcal{E})$. The set of nodes $\mathcal{N} := \{1, \ldots, n\}$ represents geographical locations, and the set of arcs $\mathcal{E} = \{(i, j) : i, j \in \mathcal{N}\}$
contains all the origin-destination (O-D) pairs, also known as lanes in truckload transportation. We allow self-loops in the network, i.e., \((i, i) \in \mathcal{E}\) for all \(i \in \mathcal{N}\), which represent local lanes. Let \(\delta^+(j) := \{k \in \mathcal{N} : (j, k) \in \mathcal{E}\}\) be the outbound nodes from node \(j\) and \(\delta^-(j) := \{i \in \mathcal{N} : (i, j) \in \mathcal{E}\}\) be the inbound nodes to node \(j\). For simplicity, we assume all loads require exactly one period to transport on any O-D pair. It is possible to generalize our model when travel times of some O-D pairs are more than one period using the approach in [Godfrey and Powell (2002)], but such a generalization will add additional complexity to the model. The numerical experiments in Section 7 will relax the travel time assumption and demonstrate the effectiveness of our proposed mechanisms under a more general setting in which the travel times of certain O-D pairs need multiple time periods.

We model the dynamics in the marketplace as a discrete-time, infinite horizon, average cost Markov decision process (MDP). The objective is to maximize its long-run average profit. The system state of the MDP consists of both the state of loads and the state of carriers. On the carrier side, let \(S_{ijt}\) denote the set of carriers available to deliver loads in lane \((i, j)\) at the beginning of period \(t\), and let \(S_{ijt} := |S_{ijt}|\) be its cardinality. On the demand side, let \(D_{ijt}\) denote the number of loads in the marketplace that need to be transported from node \(i\) to node \(j\) at the beginning of period \(t\), where \(D_{ijt}\) follows a Poisson distribution with rate \(d_{ijt}\). (The Poisson distribution assumption can be replaced with other distributions without loss of generality.) The vector \((S_t, D_t)\) is the system state of the MDP in period \(t\), where \(S_t = (S_{ijt}, \forall (i, j) \in \mathcal{E})\) and \(D_t = (D_{ijt}, \forall (i, j) \in \mathcal{E})\).

In each period of the MDP, the following sequence of events occur on the shipper side of the marketplace: (1) upon observing the current system state \((S_t, D_t)\), the platform determines shipper spot prices for the next period \((t + 1)\); (2) with the announced shipper prices, shippers submit load demand to the platform, which determines \(D_{t+1}\); (3) all the loads submitted by shippers during period \(t\) are processed and will be released to the carriers for booking at the beginning of the next period \(t+1\).

Meanwhile, the following sequence of events occur on the carrier side of the marketplace: (1) at the beginning of period \(t\), upon observing the current system state \((S_t, D_t)\), the platform sets prices and/or auction parameters; (2) during the period, carriers browse the freight brokerage app to decide which load to book or bid, or leave the app without any booking; (3) bookings through posted pricing are confirmed immediately, and bookings through auctions are resolved at the end of the period; (4) after the end of each period, carriers deliver loads, and the platform pays a penalty cost \(b_{ij}\) for each unmatched load on lane \((i, j)\); (5) carriers who delivered loads on \((i, j)\) remain in the marketplace with probably \(q_{ij}\) and become available again at the beginning of the next period at their new locations.
Remark 1. To further explain the penalty term $b_{ij}$, note that when a freight platform is not able to find a carrier for a load in practice, a few possible outcomes may occur. First, the shipper may cancel the load and use another brokerage platform. Because our model accounts for revenues from shippers when they tender loads to the platform, if the load is canceled, then the shipper will be refunded and the penalty term can be set equal to the shipper price. The penalty term can also be adjusted to include any additional compensation that the platform pays to the shipper. Second, the platform may choose to hire a third-party logistics company to transport the load. In this case, the penalty term represents the cost of hiring the third-party company. Finally, it is also possible that the shipper may agree with the platform to reschedule the load for a future date, in which case the penalty term can be set as the current shipper rate and the shipper revenue will be accounted for in the future period to reflect the changes in future shipper rates.

2.1. The Platform’s Mechanism
In this section, we introduce the platform’s mechanism $(M_r, M_p)$, where the shipper-side mechanism $M_r$ sets the prices charged to the shippers, and the carrier-side mechanism $M_p$ specifies payment and load allocation rules for the carriers.

Mechanisms for the Shipper Side
Given the system state $(S_t, D_t)$, the shipper-side mechanism $M_r$ specifies a pricing vector $r_t = (r_{ijt} : (i, j) \in E)$, where $r_{ijt}$ is the spot price for each load that needs to be shipped from node $i$ to node $j$ in period $t$. In response to the pricing vector $r_t$, shippers choose to buy transportation services from the marketplace and submit load information to the platform. The loads that need to be transported from node $i$ to node $j$ are submitted to the platform in period $t$ according to a Poisson distribution with demand rate $d_{ij}(r_{ijt})$. In other words, the demand $D_{ijt+1}$ for the next period is drawn from a Poisson distribution with mean $d_{ij}(r_{ijt})$. (Recall that all loads submitted by shippers during period $t$ will be released to carriers at the beginning of period $t+1$.) Assuming the demand function $d_{ij}(r_{ijt})$ is strictly decreasing, we denote by $r_{ij}(d_{ijt})$ the inverse demand function given demand rate $d_{ijt}$ in period $t$. Finally, we make the standard assumption that the revenue function $r_{ij}(d_{ijt})d_{ijt}$ is concave in $d_{ijt}$. The concavity assumption on the revenue function is commonly assumed in the literature, stemming from the economic principle of diminishing marginal returns.

Remark 2. In addition to spot rates, it is common for freight platforms to offer contract rates to shippers. A truckload contract covers all loads offered by a shipper during a specific period (e.g., a year) at fixed rate formulas. We remark that our shipper model defined above holds for this general setting, where the demand $d_{ijt}$ is interpreted as the sum of contract and spot demands. Specifically, we define the overall demand as $d_{ijt}(r_{ijt}) := d_{ijt}^{\text{contract}} + d_{ijt}^{\text{spot}}(r_{ijt})$, where $d_{ijt}^{\text{contract}}$ is assumed to be exogenous and independent of the spot price $r_{ijt}$. 
Mechanism for the Carrier Side

Carrier-side mechanisms are more complex than shipper-side mechanisms. To formally describe the carrier-side mechanism, we need to introduce some notations to define carriers' utility functions. Consider a carrier $s \in S_{ijt}$. If the carrier $s$ transports a load in lane $(i,j)$ in period $t$ and receives a payment amount $p_{ijt}^s$, his net utility $U_{ijt}^s$ is determined by a random utility model. More specifically, we assume that

$$U_{ijt}^s := \beta p_{ijt}^s - \alpha_{ij} + \epsilon_{ijt}^s, \quad \forall (i,j) \in E,$$

where $\beta > 0$ denotes the price sensitivity parameter, $\alpha_{ij} > 0$ represents the average cost for carriers to transport a load from node $i$ to node $j$, and $\epsilon_{ijt}^s$ represents the idiosyncratic error terms of the carrier $s$, which are assumed to be independent and identically distributed (i.i.d.) random variables. If the carrier $s$ chooses to not book any load from the platform, he may choose any outside option (i.e., the null alternative) resulting in a utility of $U_{i0t}^s := 0$. The definition of this random utility model implies that the opportunity cost of the carrier $s$ for transporting a load in lane $(i,j)$ in period $t$ with the platform is

$$C_{ijt}^s := (\alpha_{ij} - \epsilon_{ijt}^s)/\beta, \quad \forall (i,j) \in E,$$

where Eq (2) is obtained by letting $U_{ijt}^s = U_{i0t}^s = 0$. We assume that $C_{ijt}^s$ has a non-negative lower bound. Let $F_{ij}(\cdot)$ and $f_{ij}(\cdot)$ be the cumulative distribution function and the probability density function of a carrier’s true opportunity cost in lane $(i,j)$, respectively. We further assume that the inverse function of $F_{ij}(\cdot)$, denoted as $F^{-1}_{ij}(\cdot)$, is well defined, and $F^{-1}_{ij}(x_1/x_2)x_1$ is convex and differentiable for all $x_1, x_2 \geq 0$. Finally, let $\psi_{ij}(\cdot)$ be a virtual cost function defined as

$$\psi_{ij}(C_{ijt}^s) := C_{ijt}^s + \frac{F_{ij}(C_{ijt}^s)}{f_{ij}(C_{ijt}^s)}.$$

We assume that $\psi_{ij}$ satisfies the standard regularity condition that it is strictly increasing and its inverse function $\psi^{-1}_{ij}$ exists.

Remark 3. When modeling carrier side mechanisms, we assume that the platform’s policy only affects carriers’ utilities in the current period, but not in future periods. This assumption is reflected by the highly competitive nature of freight marketplaces. Unlike other transportation marketplaces (e.g. ridesharing, delivery) with high market concentration, the truckload freight markets are very fragmented. For example, in the U.S. truckload freight market, there are thousands of brokerage companies; even the largest freight platforms among these have only a few percentage points of market share. If a carrier hauls a load with one platform in one period, it is quite possible that they book loads through another digital platform or through a human broker in the next period. Therefore, a carrier’s utility in future periods is mainly decided by the overall market condition, rather than the decision made by a single platform.
We now define a carrier-side mechanism \( M_p = (A, P) \). By the revelation principle, we focus on direct mechanisms exclusively. In period \( t \), when carriers arrive to the platform, they submit their bids (representing their opportunity costs) to the platform. Given a vector of bids \( c_t \) submitted by carriers, an allocation rule \( A \) determines the probability \( A_{ijt}(c_t) \in [0,1] \) that the carrier \( s \) is allocated a load from origin node \( i \) to destination node \( j \) in period \( t \), and a payment rule \( P \) determines the expected payment \( P_{ijt}(c_t) \) made to the carrier \( s \) for the service provided.

In this paper, we consider three types of carrier-side mechanism \( M_p \): posted price mechanisms, auction mechanisms, and hybrid mechanisms that combine pricing and auction. In a posted price mechanism, for each load in the marketplace, the platform sets a price that is offered to carriers for transporting the load. Since carriers have i.i.d. opportunity costs, we shall drop the carrier index \( s \) when we refer to an arbitrary carrier in \( S_{ijt} \). Given a posted price vector \( p_t = (p_{ijt} : (i,j) \in \mathcal{E}) \), the carriers’ utilities defined in Eq (1) imply that a carrier in \( S_{ijt} \) at period \( t \) will choose to transport a load in lane \((i,j)\) with probability \( x_{ijt} \), where

\[
x_{ijt} = F_{ij}(p_{ijt}),
\]

and the probability that a carrier chooses to not book any load and leave the marketplace is given by

\[
w_{ijt} = 1 - x_{ijt}.
\]

It then follows that \( p_{ijt} \) can be expressed as a function of \( x_{ijt} \) as follows:

\[
p_{ijt} = F_{ij}^{-1}(x_{ijt}).
\]  

Alternatively, the platform may use an auction mechanism. We next use a simple example to illustrate how an auction may work in the context of truckload platforms, while deferring the general setting to Section 5. Consider an origin-destination pair \((i,j) \in \mathcal{E}\). Given the observed system state \((S_t, D_t)\) in period \( t \) and the set of bids \( c_t \) that the carriers have reported, the platform sorts the carriers’ bids from the smallest to the largest such that \( c^{[1]}_{ijt} \leq c^{[2]}_{ijt} \leq \cdots \leq c^{[S_{ijt}]}_{ijt} \) and uses the following allocation and payment rules. For each carrier \( s \in S_{ijt} \), \( A_{ijt}(c_t) = 1 \) and \( P_{ijt}(c_t) = c_{ijt}^s \) if \( c_{ijt}^s \leq c^{[\min(S_{ijt},D_{ijt})]}_{ijt} \). Otherwise, \( A_{ijt}(c_t) = 0 \) and \( P_{ijt}(c_t) = 0 \) if \( c_{ijt}^s > c^{[\min(S_{ijt},D_{ijt})]}_{ijt} \). In other words, when the number of available carriers \( S_{ijt} \) is no less than the number of loads \( D_{ijt} \) that need to be transported, the platform allocates the \( D_{ijt} \) loads to the first \( D_{ijt} \) lowest bids with probability one, and the payment amount is equal to each carrier’s reported opportunity cost. When there is not enough carriers, i.e., \( S_{ijt} < D_{ijt} \), each carrier \( s \in S_{ijt} \) is allocated a load with probability one and receives a payment amount equal to their bid \( c_{ijt}^s \). Of course, there are numerous other auction formats that the platform may use, including a hybrid of posted price and auctions that are used by several platforms in practice.
2.2. Dynamic Program Formulation

In this section, we provide a dynamic program formulation for the platform’s optimization problem. At period $t$, given the state $(S_t, D_t)$, a mechanism $\pi$ maps the state to a pair of shipper/carrier mechanisms $(\mathcal{M}_r, \mathcal{M}_p)$. Given the carrier-side mechanism $\mathcal{M}_p$, each available carrier either chooses to book a load or leaves the marketplace without any booking. We consider a full truck load assumption where each carrier can transport only one load at a time. For each $(i, j) \in \mathcal{E}$, let $Y_{ijt}$ be a random variable that denotes the number of carriers who have transported a load in lane $(i, j)$ at period $t$, and let $V_{ijt}$ be the number of carriers in $S_{ijt}$ who leave the marketplace in period $t$. It then follows that for each period $t$, we have

$$Y_{ijt} + V_{ijt} = S_{ijt}, \quad \forall (i, j) \in \mathcal{E}. \quad (5)$$

For a carrier who has just completed a load shipment from node $i$ to node $j$ in period $t$, we assume that the carrier will remain in the marketplace and choose to deliver a load from node $j$ to node $k$ in the next period with probability $q_{jk} \in (0, 1)$, and will leave the system with probability $1 - \sum_{k \in \delta^+ (i)} q_{ik}$. Let $Z_{ijt}$ be a random variable that denotes the number of carriers who decide to stay in the marketplace to deliver a load from node $i$ to node $j$ after completing a load shipment at period $t$. Note that $Z_{it} = (Z_{ijt}, \forall j \in \delta^+ (i))$ follows a multinomial distribution with parameters $(\sum_{k \in \delta^- (i)} Y_{kit}, q_i)$, where $q_i = (q_{ij}, \forall j \in \delta^+ (i))$. Then, the dynamics of how the number of carriers evolves over time is characterized as follows:

$$S_{ijt+1} = Z_{ijt} + \Lambda_{ijt+1}, \quad \forall (i, j) \in \mathcal{E}, \quad (6)$$

where $\Lambda_{ijt}$ is a Poisson random variable with rate $\lambda_{ij} > 0$, which denotes the number of new carriers who exogenously arrive to deliver loads from node $i$ to node $j$ in period $t$.

For carriers who have completed a load shipment in period $t$, the platform makes payments to them. Let $P_{ijt}$ denote the total payment to all the carriers who have transported a load from node $i$ to node $j$ in period $t$. It is then clear from the definitions that $E \left[ \sum_{n=1}^{S_{ijt}} A_{ijt}^n (C_t) | S_t, D_t \right] = E[Y_{ijt} | S_t, D_t]$ and $E \left[ \sum_{n=1}^{S_{ijt}} P_{ijt}^n (C_t) | S_t, D_t \right] = E[P_{ijt} | S_t, D_t]$ for each lane $(i, j) \in \mathcal{E}$. If there are not enough carriers to transport all the loads (i.e., $Y_{ijt} < D_{ijt}$), we assume that the excess demand $(D_{ijt} - Y_{ijt})$ incurs a unit penalty cost $b_{ij}$. Therefore, the total cost that the platform has made during period $t$ is equal to $\sum_{(i,j) \in \mathcal{E}} P_{ijt} + b_{ij} (D_{ijt} - Y_{ijt})$.

Next we consider the shipper side dynamics. Recall that a shipper-side mechanism $\mathcal{M}_r$ specifies the prices $r_t$ charged to the shippers, which determine the number of loads $D_{ijt+1}$ that need to be transported from node $i$ to node $j$ in period $t + 1$. We assume that $D_{ijt+1}$ follows a Poisson distribution with rate $d_{ij}(r_{ijt})$. The revenue that the platform collects from the shippers is $r_{ijt} D_{ijt+1}$.
with mean \( r_{ij}d_{ij}(r_{ij}) \). It is worth pointing out that, although these loads are transported in period \( t+1 \), we account them for the revenue in period \( t \). Therefore, the platform’s expected profit in period \( t \) is given by:

\[
G_t^\pi(S_t, D_t) := E \left[ \sum_{(i,j) \in E} r_{ij}d_{ij}(r_{ij}) - b_{ij}(D_{ijt} - Y_{ijt}) - P_{ijt} \left| S_t, D_t \right. \right].
\] (7)

Define \( \gamma^\pi \) as the long-run average profit per period under a stationary policy \( \pi \):

\[
\gamma^\pi := \lim_{T \to \infty} \frac{1}{T} E \left[ \sum_{t=1}^{T} G_t^\pi(S_t, D_t) \right].
\]

The existence of \( \gamma^\pi \) is implied by Proposition 1 that we detail below in Section 2.3. Let \( \gamma^* \) denote the optimal long-run average profit per period, and let \( h(S_t, D_t) \) denote the differential cost for the state \((S_t, D_t)\). Then the optimality equation is given by

\[
\gamma^* + h(S_t, D_t) = \max_{\pi \in \Pi} \left[ E[G_t^\pi(S_t, D_t) + E[h(Z_t + \Lambda_{t+1}, D_{t+1})] \right| S_t, D_t], \quad \forall(S_t, D_t)
\] (MDP)

where \( \Lambda_t = (\Lambda_{ijt}, \forall(i,j) \in E) \), and \( Z_t = (Z_{ijt}, \forall(i,j) \in E) \).

2.3. Stability, Incentive Compatibility (IC) and Individual Rationality (IR).

In this subsection, we introduce some properties of the platform’s mechanism \((M_r, M_p)\) introduced above. Let \( \pi \in \Pi \) be a stationary policy that maps the system state \((S_t, D_t)\) to a mechanism \((M_r, M_p)\). Proposition 1 below shows that the Markov chain induced by any stationary policy \( \pi \) is stable. The proof uses the Foster-Lyapunov theorem \([Foster1953]\) with a Lyapunov function on the total number of carriers in the system, and we refer the readers to the appendix for the complete proof.

**Proposition 1.** There exists a stationary distribution of the Markov chain induced by the platform’s policy \( \pi \) and the system is stable (i.e., positive recurrent).

By the revelation principle \([Myerson1981]\), we will focus on direct mechanisms that satisfy the Bayesian incentive compatibility (IC) and individual rationality (IR) properties. We next briefly discuss these properties and the conditions under which IC and IR constraints are satisfied.

Recall that \( S_{ijt} \) is the total number of available carriers to deliver loads in lane \((i,j)\) at the beginning of period \( t \) in the marketplace. Consider a carrier \( s \in S_{ijt} \) who chooses to deliver a load from node \( i \) to node \( j \) at the beginning of period \( t \). Let \( c_{ijt}^s \) denote the bid for a load in lane \((i,j)\) submitted by the carrier \( s \) to the platform. In other words, \( c_{ijt}^s \) represents the opportunity cost reported by the carrier \( s \) for transporting a load from node \( i \) to node \( j \) in period \( t \). Let \( C_{ijt}^s = (C_{ijt}^1, ..., C_{ijt}^{s-1}, C_{ijt}^{s+1}, ..., C_{ijt}^{S_{ijt}}) \) be a vector that represents the opportunity costs of all the available
carriers in lane \((i, j)\) at period \(t\) other than \(s\), where \(C^s_{ijt}\) is defined in (2). Let \(g^s(\cdot)\) denote the joint probability density function of \(C^s_{ijt}\). Finally, let \((c^s_{ijt}, C^-_{ijt}) = (C^1_{ijt}, \ldots, C^{s-1}_{ijt}, c^s_{ijt}, C^{s+1}_{ijt}, \ldots, C^S_{ijt})\).

Assume that all the carriers other than \(s\) report their true opportunity costs \(C^-_{ijt}\) when submitting the bids. Then the carrier \(s\) who submitted bid \(c^s_{ijt}\) will be allocated a load to be transported from node \(i\) to node \(j\) in period \(t\) with probability

\[
a^s_{ijt}(c^s_{ijt}) := \int A^s_{ijt}(c^s_{ijt}, C^-_{ijt})g^s(C^-_{ijt})dC^-_{ijt},
\]

and the expected payment to the carrier \(s\) for transporting a load from node \(i\) to node \(j\) is given by

\[
p^s_{ijt}(c^s_{ijt}) := \int P^s_{ijt}(c^s_{ijt}, C^-_{ijt})g^s(C^-_{ijt})dC^-_{ijt}.
\]

Let \(C^s_{ijt}\) denote the true opportunity cost of the carrier \(s\). Then the expected net utility of the carrier \(s\) when he submits a bid \(c^s_{ijt}\) is given by

\[
u^s_{ijt}(c^s_{ijt}) := p^s_{ijt}(c^s_{ijt}) - a^s_{ijt}(c^s_{ijt})C^s_{ijt}.
\]

A carrier-side mechanism \(M_p\) is *incentive compatible* if it is a Bayesian Nash equilibrium for each carrier to report their true opportunity costs. That is, for each carrier \(s\), we have

\[
u^s_{ijt}(C^s_{ijt}) = p^s_{ijt}(C^s_{ijt}) - a^s_{ijt}(C^s_{ijt})C^s_{ijt} \geq p^s_{ijt}(c^s_{ijt}) - a^s_{ijt}(c^s_{ijt})c^s_{ijt}, \quad \forall c^s_{ijt}.
\]

We say a carrier-side mechanism \(M_p\) is *individual rational* if each carrier’s expected net utility is non-negative when they report their true opportunity costs, i.e., \(u^s_{ijt}(C^s_{ijt}) \geq 0\) for each carrier \(s\).

Given a platform mechanism \((M_r, M_p)\) that is IC and IR, the following revenue equivalence principle characterizes the carrier’s expected payment by the allocation rule.

**Proposition 2.** Under a given state \((S, D)\) with the platform’s mechanism \(\pi(S, D) = (M_r, M_p)\) that is IC and IR, the expected payment to a carrier \(s \in S_{ij}\) is

\[
E[p^s_{ijt}(C^s_{ijt})|S, D] = E[a^s_{ijt}(C^s_{ijt})\psi_{ijt}(C^s_{ijt})|S, D],
\]

where \(C^s_{ij}\) denotes the true opportunity cost of the carrier \(s\), and the virtual cost function \(\psi_{ijt}(\cdot)\) is defined in Eq (3).
3. Fluid Approximation and Profit Benchmark

The dynamic programming formulation [MDP] is intractable, since the number of nodes may be large in practice and the state space grows at least exponentially with the number of nodes. This motivates us to consider a fluid approximation of the [MDP] where the random shipper demands and carrier arrivals are replaced with their respective mean values, and we consider the system under the stationary distribution. In this section, we first provide the formulation of the fluid model. Then, we show that the fluid model can be transformed into a convex optimization problem and hence can be solved efficiently. Finally, we show that the optimal objective value of the fluid optimization problem serves as an upper bound for the long-run average profit for the [MDP] under any stationary mechanism. This upper bound is useful in that it can be used as a benchmark to establish performance guarantees of any given mechanism. As we shall show later in Section 4, a simple static posted price mechanism for the [MDP] based on the solution to the fluid optimization problem is asymptotically optimal.

3.1. The Fluid Model

In the fluid model, the random shipper demands $D_{ijt}$ and random carrier arrivals $\Lambda_{ijt}$ in each period $t$ are replaced with their mean values $d_{ij}$ and $\lambda_{ij}$, respectively, for all origin-destination pair $(i,j) \in \mathcal{E}$. We consider the fluid system in a steady state. Let $x_{ij}$ denote the fraction of carriers who choose to transport a load in lane $(i,j)$ under the fluid system, and let $w_{ij}$ be the fraction of carriers who choose to not book any load in lane $(i,j)$ and leave the marketplace. Suppose for now that a posted price mechanism is used on the carrier side. It then follows from Eq (4) that the payment $p_{ij}$ offered to carriers for transporting a load in lane $(i,j)$ is given by

$$p_{ij} = F_{ij}^{-1}(x_{ij}), \quad \forall (i,j) \in \mathcal{E}.$$  

It is worth pointing out that the fraction of carriers who choose loads in lane $(i,j)$ may not be equal to the number of carriers who actually ship loads, since the actual shipment depends on both the carriers’ choices and the shipper demand. In view of this distinction, let $y_{ij}$ be the fraction of carriers who end up hauling loads in lane $(i,j)$, and let $v_{ij}$ be the fraction of carriers who do not deliver any load in lane $(i,j)$ and leave the system. Let $\bar{\lambda}_{ij}$ denote the total available carriers delivering loads in lane $(i,j)$, which includes both the new arrivals and the existing carriers in the marketplace who just finished a shipment and decided to deliver a load in lane $(i,j)$.

We define the fluid approximation model as

$$\max \sum_{(i,j) \in \mathcal{E}} r_{ij}(d_{ij})d_{ij} - F_{ij}^{-1}(x_{ij})\bar{\lambda}_{ij}y_{ij} - b_{ij}(d_{ij} - \bar{\lambda}_{ij}y_{ij})$$  

s.t. $\sum_{k \in \delta^{-}(i)} y_{ki}\bar{\lambda}_{ki}q_{ij} + \lambda_{ij} = \bar{\lambda}_{ij}(y_{ij} + v_{ij}), \quad \forall (i,j) \in \mathcal{E}$.
\[ x_{ij} + w_{ij} = 1, \quad \forall (i, j) \in \mathcal{E}, \tag{9c} \]
\[ y_{ij} + v_{ij} = 1, \quad \forall (i, j) \in \mathcal{E}, \tag{9d} \]
\[ \bar{\lambda}_{ij}y_{ij} \leq d_{ij}, \quad \forall (i, j) \in \mathcal{E}, \tag{9e} \]
\[ y_{ij} \leq x_{ij}, \quad \forall (i, j) \in \mathcal{E}, \tag{9f} \]
\[ x_{ij}, y_{ij}, w_{ij}, v_{ij}, \bar{\lambda}_{ij}, d_{ij} \geq 0, \quad \forall (i, j) \in \mathcal{E}. \tag{9g} \]

In the above fluid model, the objective function (9a) maximizes the platform’s per period profit, where the first term represents the revenues received from shippers, the second term represents payments made to carriers, and the last term represents the penalty costs incurred from unsatisfied demand. Constraint (9b) represents the flow balance equations, where the left-hand side represents the total inflow rate of carriers who decide to deliver loads in lane \((i, j)\), and the right-hand side represents the outflow rate of carriers in lane \((i, j)\). Constraints (9c) and (9d) follow from the definition of probability vectors \(x, w, y,\) and \(v\). Constraint (9e) states that the flow of loaded carriers cannot exceed the number of loads that are available for each O-D pair. Constraint (9f) requires that \(y_{ij}\) cannot exceed \(x_{ij}\) since the actual shipment is constrained by the carriers’ supply.

### 3.2. Convex Reformulation

In this section, we show that the fluid model presented in Section 3.1 can be reformulated as a convex optimization problem and hence can be solved efficiently. To that end, we first introduce some notation. For each O-D pair \((i, j)\) \(\in \mathcal{E}\), define \(\bar{x}_{ij} := \bar{\lambda}_{ij}x_{ij}, \bar{w}_{ij} := \bar{\lambda}_{ij}w_{ij}, \bar{y}_{ij} := \bar{\lambda}_{ij}y_{ij},\) and \(\bar{v}_{ij} := \bar{\lambda}_{ij}v_{ij}\). By the definition of these new variables, it is clear that

\[ p_{ij} = F^{-1}_{ij}(x_{ij}) = F^{-1}_{ij}\left(\frac{\bar{x}_{ij}}{\bar{\lambda}_{ij}}\right), \quad \forall (i, j) \in \mathcal{E}. \]

By substituting the new variables into formulation (9), we have

\[
\begin{align*}
\text{max} & \quad \sum_{(i,j) \in \mathcal{E}} r_{ij}(d_{ij} - F^{-1}_{ij}\left(\frac{\bar{x}_{ij}}{\bar{\lambda}_{ij}}\right) - b_{ij}(d_{ij} - \bar{y}_{ij})) \\
\text{s.t.} & \quad \sum_{k \in \delta^{-}(i)} q_{kj}\bar{y}_{ki} + \lambda_{ij} = \bar{\lambda}_{ij}, \quad \forall (i, j) \in \mathcal{E}, \tag{10b} \\
& \quad \bar{y}_{ij} + \bar{v}_{ij} = \bar{x}_{ij} + \bar{w}_{ij}, \quad \forall (i, j) \in \mathcal{E}, \tag{10c} \\
& \quad \bar{y}_{ij} \leq d_{ij}, \quad \forall (i, j) \in \mathcal{E}, \tag{10d} \\
& \quad \bar{y}_{ij} \leq \bar{x}_{ij}, \quad \forall (i, j) \in \mathcal{E}, \tag{10e} \\
& \quad \bar{y}_{ij} + \bar{v}_{ij} = \bar{\lambda}_{ij}, \quad \forall (i, j) \in \mathcal{E}, \tag{10f} \\
& \quad \bar{x}_{ij}, \bar{y}_{ij}, \bar{w}_{ij}, \bar{v}_{ij}, \bar{\lambda}_{ij}, d_{ij} \geq 0, \quad \forall (i, j) \in \mathcal{E}. \tag{10g} 
\end{align*}
\]

The interpretations of the constraints in the new formulation (10) are straightforward and similar to those in the original formulation (9). Next, we show that the new formulation (10) and the
original fluid model (9) are equivalent. First, it is easy to check that for any feasible solution to the original formulation (9), there exists a corresponding feasible solution to the new formulation (10) with the same objective value by the definition of variables \( x, w, \tilde{y}, \tilde{v}, \) and \( \tilde{\lambda} \). On the other hand, for any given feasible solution \((d, \tilde{x}, \tilde{w}, \tilde{y}, \tilde{v}, \tilde{\lambda})\) to the new formulation, we can set variables \( y_{ij} = \tilde{y}_{ij}/\tilde{\lambda}_{ij}, x_{ij} = \tilde{x}_{ij}/\tilde{\lambda}_{ij}, w_{ij} = \tilde{w}_{ij}/\tilde{\lambda}_{ij}, \) and \( v_{ij} = \tilde{v}_{ij}/\tilde{\lambda}_{ij} \) for each \((i,j) \in \mathcal{E}\) without changing \( d \). It is easy to check that the newly defined variables are feasible to (9), and the new formulation can be reduced to the original formulation.

Let \((d^*, \tilde{x}^*, \tilde{w}^*, \tilde{y}^*, \tilde{v}^*, \tilde{\lambda}^*)\) be an optimal solution to (10). We note that the optimal solution satisfies \( \tilde{x}^* = \tilde{y}^* \). To see this, suppose that there exists an arc \((i,j) \in \mathcal{E}\) such that \( \tilde{y}_{ij} > \tilde{x}_{ij}^* \). This implies that \( \tilde{w}_{ij}^* < \tilde{v}_{ij}^* \). In this case, we can construct a new solution \((d', \tilde{x}', \tilde{w}', \tilde{y}', \tilde{v}', \tilde{\lambda}')\), where \( \tilde{x}_{ij}' := \tilde{y}_{ij}' \) and \( \tilde{w}_{ij}' := \tilde{w}_{ij}' + (\tilde{x}_{ij}' - \tilde{y}_{ij}') \), and the rest of the variables have the same value as that in \((d^*, \tilde{x}^*, \tilde{w}^*, \tilde{y}^*, \tilde{v}^*, \tilde{\lambda}^*)\). It is straightforward to check that \((d', \tilde{x}', \tilde{w}', \tilde{y}', \tilde{v}', \tilde{\lambda}')\) is feasible to (10) and achieves a strictly larger objective value. This leads to a contradiction with the optimality of \((d^*, \tilde{x}^*, \tilde{w}^*, \tilde{y}^*, \tilde{v}^*, \tilde{\lambda}^*)\), and hence we must have \( \tilde{x}^* = \tilde{y}^* \). In view of this, formulation (10) can be simplified as follows:

\[
\text{FA:} \quad \max \sum_{(i,j) \in \mathcal{E}} r_{ij}(d_{ij})d_{ij} - F^{-1}_{ij}\left(\frac{\bar{y}_{ij}}{\bar{\lambda}_{ij}}\right) \bar{y}_{ij} - b_{ij}(d_{ij} - \bar{y}_{ij})
\]

s.t.
\[
\begin{align*}
\sum_{k \in \delta^-(i)} q_{ij} \bar{y}_{ki} + \lambda_{ij} &= \bar{\lambda}_{ij}, & \forall (i,j) \in \mathcal{E}, \\
\bar{y}_{ij} &\leq d_{ij}, & \forall (i,j) \in \mathcal{E}, \\
\bar{y}_{ij} &\leq \bar{\lambda}_{ij}, & \forall (i,j) \in \mathcal{E}, \\
\bar{y}_{ij}, \bar{\lambda}_{ij}, d_{ij} &\geq 0, & \forall (i,j) \in \mathcal{E}.
\end{align*}
\]

Under the standard concavity assumption of the revenue function \( r_{ij}(d_{ij})d_{ij} \) and our assumption that \( F^{-1}_{ij}\left(\frac{\bar{y}_{ij}}{\bar{\lambda}_{ij}}\right) \bar{y}_{ij} \) is a convex function by the regularity condition of \( \psi_{ij} \), it is straightforward to see that the FA is a convex optimization problem.

### 3.3. FA Gives an Upper Bound of the Optimal Profit

In this section, we show that the optimal objective value of the FA provides an upper bound of the long-run average profit for the MDP under any stationary mechanism. For revenue management problems in large networks where exact solutions are intractable, one common approach is to use fluid approximation to get deterministic optimization problems. Gallego and Van Ryzin (1994, 1997) introduced a fluid approximation method for finite-horizon dynamic pricing problems and proposed static pricing policies that are asymptotically optimal. Similar approaches have been applied in many subsequent works (e.g., Cooper 2002, Maglaras and Meissner 2006, Liu and Van Ryzin 2008, Chen et al. 2019), and the fluid approximation model gives an upper bound of
the optimal dynamic pricing mechanism. A new contribution of our analysis is to show that the fluid approximation model is an upper bound for not only pricing but also general freight market mechanisms (including auctions) in an infinite horizon setting.

**Theorem 1.** The optimal value of the fluid problem $FA$ is an upper bound for the long-run average profit of the system under any stationary policy $\pi \in \Pi$.

It is worth noting that although the fluid model ($FA$) is constructed under the posted price mechanism, Theorem 1 holds not only for posted mechanisms, but also for any direct mechanism that is IC and IR. In view of Theorem 1, the optimal objective value of $FA$ provides an upper bound for the long-run average profit for the $MDP$ under any platform mechanism. As a result, this upper bound can serve as a benchmark to evaluate the performance of any platform mechanism. To avoid the trivial case, we assume in the subsequent sections that the optimal objective value of $FA$ is positive, because otherwise it means the platform cannot make an operating profit and thus cannot survive in the long run. The proof of Theorem 1 proceeds in two steps. We first show that the constraints in $FA$ are necessary for any mechanism under which the system is stable. Then we show that the optimal value of the $FA$ is an upper bound of the long-run average profit of any platform mechanism ($M_r, M_p$). The full proof is included in the appendix.

4. Posted Price Mechanisms

In this section, we study a posted price mechanism in which the platform sets prices $r_t = (r_{ijt} : (i, j) \in E)$ to shippers and payments $p_t = (p_{ijt} : (i, j) \in E)$ to carriers for transporting a load in period $t$. Among all the posted price mechanisms, we consider a static pricing mechanism, where the platform offers a fixed price to the carriers and charges a fixed price to the shippers. We show that with a proper choice of the fixed prices, the static posted price mechanism is asymptotically optimal under an asymptotic scaling regime. The static pricing mechanism is easy to implement in practice, and our results provide further theoretical support for its effectiveness. The static posted price mechanism will also be used as a baseline for the analysis of other mechanisms in the subsequent sections.

We consider the following asymptotic regime. Consider a sequence of problem instances $\{MDP^\theta\}$ with scaling parameter $\theta \in \{1, 2, \ldots\}$. In the instance $MDP^\theta$, the arrival rates of shipper demands and carriers are equal to $\theta d$ and $\theta \lambda$, respectively. In other words, the scaling factor $\theta$ can be considered as a measure of the system size. Let $(d^*, y^*, \lambda^*)$ be an optimal solution to the $FA$ and let $\gamma_{FA}$ denote the optimal objective value of the $FA$. Recall in Section 3.2 we showed that $x^* = y^*$ in the optimal solution to the fluid problem (10). As it shall become clear later, it is sometimes more convenient to use $\bar{x}^*$ (and correspondingly, $x^*$) in our analysis, and therefore we will differentiate
between $\bar{x}^*$ and $\bar{y}^*$ (correspondingly, $x^*$ and $y^*$) by using their respective notations (even though they have the same value under the optimal fluid solution). Finally, let $(r^*, p^*)$ respectively be the prices charged to the shippers and the payments paid to the carriers by the platform corresponding to the optimal fluid solution $(d^*, \bar{y}^*, \bar{x}^*)$, where $r^*$ is determined via the inverse shipper demand function, and $p^*$ is given by $p^*_{ij} = F^{-1} (x^*_{ij})$ for each $(i, j) \in E$.

Our proposed static posted price mechanism, denoted as $SP$, applies the prices $(r^*, p^*)$ obtained from the optimal solution to the $FA$ in all system states. Given a system state $(S_t, D_t)$ and the posted price vector $p^* = (p^*_{ij} : (i, j) \in E)$, carriers choose to book a load among the available remaining loads only if their choice maximizes their utilities (i.e., booking a load results in a higher utility than their outside option), and otherwise they would leave the marketplace without booking any load. Therefore, the posted price mechanism $SP$ is IC and IR.

For a problem instance with scaling factor $\theta$, let $\gamma^{SP}(\theta)$ denote the long-run average profit under the proposed $SP$ mechanism. The optimal solution to $FA(\theta)$ is $(\theta d^*, \theta \bar{y}^*, \theta \bar{x}^*)$, and the optimal objective value of $FA(\theta)$, denoted as $\gamma^{FA}(\theta)$, is equal to $\theta \gamma^{FA}$. The following theorem establishes the asymptotic optimality of our proposed static posted price mechanism $SP$.

**Theorem 2.** The static posted price mechanism $(r^*, p^*)$ is asymptotically optimal. More specifically, we have

$$\gamma^{FA}(\theta) - \gamma^{SP}(\theta) \leq O(\sqrt{\theta}),$$

and therefore $\gamma^{SP}(\theta)/\gamma^{FA}(\theta) = 1 - O(1/\sqrt{\theta}) \to 1$ as the scaling factor $\theta$ approaches infinity.

The proof of Theorem 2 can be found in the appendix.

5. **Auction Mechanisms**

Auction mechanisms are widely used in the freight industry to match shippers and carriers [Figliozzi et al. 2005, Caplice 2007]. Digital freight platforms have also been adopting auction mechanisms, especially on the carrier side (Convoy 2018, Uber Freight 2020). Unlike posted price mechanisms where carriers cannot negotiate prices with the platform, auction mechanisms allow the platform to collect information from carriers before it decides how to allocate loads and set payment amounts.

In this section, our goal is to study auction design for the carrier-side of the freight platform. (Meanwhile, we assume that the shipper side uses the same static price $r^*$ as in Section 4.) The carrier-side auction design is nontrivial because of interactions among carriers in the transportation network across multiple periods. It is well known in the mechanism design literature that multi-item auctions are notoriously difficult to analyze. To tackle this challenge, we consider an approach that decomposes the freight network and applies auctions to each lane and each period separately.
This would greatly simplify the implementation of the carrier-side auctions and is aligned with the auction mechanisms used by freight platform in practice.

Our proposed method below will use a sealed-bid uniform price auction with a reserve price for each lane and period. Specifically, for each lane \((i,j)\) in period \(t\), loads are allocated to the lowest \(D_{ij}^t\) bids below the reserve price, where \(D_{ij}^t\) is the demand on the lane in this period. Each winning carrier pays the lower of the \((D_{ij}^t + 1)^{th}\) lowest bid and the reserve price, with ties broken randomly. This auction format is simple to implement and intuitive. In practice, freight platforms typically use discriminatory auctions where carriers’ payments are equal to their own bids rather than uniform price auctions. However, by the revenue equivalence principle, the two auction mechanisms are essentially equivalent, because given the same reserve price, they generate the same allocation outcomes and expected carrier payments (Proposition 2).

We now formally define our uniform price auction mechanism. At the beginning of time period \(t\), each carrier \(s \in S_{ij}^t\) submits a bid \(C_{ij}^s\) for loads in lane \((i,j)\). After receiving bids from the carriers, the platform makes allocation decisions using the following optimization problem and a reserve price \(\xi_{ij}\):

\[
\mathcal{J}_{ij}(C_{ijt}) := \min \sum_{s \in S_{ijt}} C_{ij}^s A_{ij}^s(C_{ijt}) + \xi_{ij} Y_{ij}^0(C_{ijt})
\]

\[
s.t. \ 0 \leq A_{ij}^s(C_{ijt}) \leq 1, \ \forall s \in S_{ijt},
\]

\[
\sum_{s \in S_{ijt}} A_{ij}^s(C_{ijt}) + Y_{ij}^0(C_{ijt}) = D_{ij}^t,
\]

\[
Y_{ij}^0(C_{ijt}) \geq 0.
\]

A few remarks are in order. Due to the network structure and the multi-period dynamics, in the optimal auction, the allocation of loads at a given lane may depend on bids on other lanes. However, in Eq (12), the allocation decisions \(A_{ijt}\) are determined separately on each lane and only depend on the bidding information \(C_{ijt}\) submitted by the carriers in \(S_{ijt}\). The variable \(Y_{ij}^0\) in the objective function (12) represents the number of “dummy” bidders who bid at the reserve price \(\xi_{ij}\). This ensures that carriers whose submitted bids are higher than the reserve price will not receive any load allocation. The first constraint requires that each carrier can be allocated at most one load. Notice that Eq (12) is an assignment problem, so there always exists an integral optimal solution.

After the allocation decisions are set, the payments to carriers are determined as follows. Let \(A_{ij}^*(C_{ijt})\) denote the allocation of loads to carriers in \(S_{ijt}\) in the optimal solution to \(\mathcal{J}_{ij}(C_{ijt})\). Notice that such an optimal solution always exists because the objective function (12) is bounded from above and there always exists a feasible solution \((A_{ijt}(C_{ijt}) = 0, Y_{ij}^0(C_{ijt}) = D_{ijt})\) to the
above optimization problem. With the allocation rule $A_{ijt}^*(C_{ijt})$, the payment to a carrier $s$ in $S_{ijt}$, denoted as $P_{ijt}^s(C_{ijt})$, is given by the following payment rule:

$$P_{ijt}^s(C_{ijt}) = C_{ijt}^s A_{ijt}^s(C_{ijt}) + J_{ij}(C_{ijt}^{-s}) - J_{ij}(C_{ijt}), \quad \forall s \in S_{ijt}, \forall (i, j) \in \mathcal{E}. \quad (13)$$

The following lemma shows that the uniform price auction satisfies the desired incentive compatibility (IC) and individual rationality (IR) properties. The proof of Lemma 1 is relegated to the appendix.

**Lemma 1.** The uniform price auction mechanism is IC and IR.

To fully characterize the uniform price auction, we still need to specify the value of the reserve prices. Introducing reserve prices may improve the platform’s expected profit, since a reserve price imposes an upper bound on the payment to the carriers. For our multi-period multi-location marketplace model, however, a carefully chosen reserve price must also take into account how the allocation decisions in one lane will affect the states of other lanes in the future.

We propose an auction mechanism below denoted as $\text{AUC}$. In $\text{AUC}$, the reserve price $\xi_{ij}^*$ for each origin-destination pair $(i, j) \in \mathcal{E}$ is defined as:

$$\xi_{ij}^* = \max\{\psi_{ij}^{-1}(b_{ij}), p_{ij}^*\} \quad (14)$$

where $\psi_{ij}(\cdot)$ is the virtual cost function defined in Eq (3).

To facilitate the analysis of the auction mechanism $\text{AUC}$, we also consider an alternative auction mechanism below, denoted as $\text{AUC-P}$, which simply uses the optimal fluid carrier-side prices $p^*$ as the reserve price. Compared with the static posted price mechanism proposed in Section 4 which offers a fixed payment price $p^*$ to the carriers, one would expect that the expected payment to the carriers under the auction mechanism $\text{AUC-P}$ would be lower than that under $\text{SP}$, because the payment to each individual carrier under $\text{AUC-P}$ will never exceed the reserve price $p^*$.

### 5.1. Analysis of Auction Mechanisms

We now establish the relationship between the auction mechanisms $\text{AUC}$ and $\text{AUC-P}$ defined earlier and the posted price mechanism $\text{SP}$.

**Theorem 3.** The objective values of the static posted price mechanism and the $\text{AUC-P}$ mechanism satisfy

$$\gamma^\text{SP} \leq \gamma^\text{AUC-P}. \quad (15)$$

In addition, the objective values of $\text{AUC-P}$ and $\text{AUC}$ satisfy

$$\gamma^\text{AUC-P} \leq \gamma^\text{AUC}. \quad (16)$$
The first part of Theorem 3 shows that AUC-P achieves a higher long-run average profit than the static posted price mechanism SP, and the second part shows that AUC has a higher long-run average profit than AUC-P. Together with the asymptotic optimality of SP (Theorem 2), this result implies that both AUC-P and AUC are also asymptotically optimal.

We provide a high-level proof outline for Theorem 3 here, with the detailed proof relegated to the appendix. We first consider Eq (15), which shows that the AUC-P mechanism outperforms the SP mechanism. Recall that the AUC-P mechanism uses the optimal fluid price $p^*$ as the reserve price, which is also the posted price offered in the SP mechanism. Therefore, the payment for a carrier delivering a load from node $i$ to node $j$ under the AUC-P mechanism is never higher than the reserve price $p^*_{ij}$. In addition, we show in the proof of Eq (15) that SP and AUC-P have the same stationary distribution of $Y_{ij}$, which is the number of loads shipped from node $i$ to node $j$ in steady state. As a result, the expected payment made to the carriers under AUC-P is lower than that under SP, while the expected penalty costs under these two mechanisms are the same in view of the same distribution of $Y_{ij}$ in steady state. It then follows that AUC-P achieves a higher long-run average profit than SP since both mechanisms receive the same shipper-side revenue.

In the second part of the proof, we analyze the performance of the auction mechanisms AUC and AUC-P to establish Eq (16). The complex interactions among carriers in the network across time periods and locations make it difficult to analyze the performance of these auction mechanisms. This motivates us to decompose the long-run expected profit under both mechanisms by lanes $(i,j) \in E$, and use a coupling technique to show that the expected profit of AUC dominates that of AUC-P. A key step in the analysis is the following lemma, which shows that the number of available carriers under the AUC mechanism, $S^\text{AUC}_{ij}$, is always stochastically larger than the number of available carriers under the AUC-P mechanism, $S^\text{AUC-P}_{ij}$, for each lane $(i,j) \in E$.

**Lemma 2.** For each lane $(i,j) \in E$ and time period $t$, under the stationary distributions of the AUC-P and AUC mechanisms, $S^\text{AUC-P}_{ij}$ is stochastically dominated by $S^\text{AUC}_{ij}$.

Next, we bound the gap between posted price and auction mechanisms in the asymptotic regime defined in Section 4, in which the arrival rates of loads and carriers are scaled by the same factor $\theta \in \{1, 2, ...\}$. Theorem 2 implies that the gap between the best posted price mechanism and the optimal mechanism is no greater than $O(\sqrt{\theta})$. We show that this bound is tight.

**Theorem 4.** There exists a problem instance such that

$$\gamma^{\text{AUC}}(\theta) - \gamma^{\text{SP}}(\theta) \geq \Omega(\sqrt{\theta}),$$

and therefore $\gamma^{\text{SP}}(\theta)/\gamma^{\text{AUC}}(\theta) = 1 - \Omega(1/\sqrt{\theta})$. Moreover, the gap between the optimal posted price mechanism and the optimal auction mechanism is also lower bounded by $\Omega(\sqrt{\theta})$. 
The result above shows that although both posted price and auction mechanisms are asymptotically optimal as the scaling factor $\theta$ approaches infinity, auction mechanisms can be especially beneficial in markets with low demand and few carriers.

6. Hybrid Mechanisms

The results in the previous section suggest that auction mechanisms can lead to higher long-run average profits for the platform than posted price mechanisms. However, using auctions comes at the expense of longer waiting time for carriers to receive load booking confirmations, as every carrier must wait until the end of a period to learn the result of the auction; in contrast, in the posted price mechanism, carriers can confirm a load booking instantly without waiting for other carriers’ bids.

To counter the downside of auctions, we consider a hybrid mechanism in which carriers can either book a load instantly by accepting a posted price offered by the platform or by bidding in an auction for the load. Such hybrid mechanisms have become increasingly popular in digital freight marketplaces (e.g., Convoy 2018, Uber Freight 2020), as shown earlier in the example of Figure 1, which serve as an attractive strategy for platforms to balance the trade-off between profit and carrier waiting time.

We will analyze a specific hybrid mechanism that combines the two mechanisms we proposed previously, i.e., the static posted price mechanism $SP$ and the auction mechanism $AUC$. One challenge for analyzing hybrid mechanisms is that, although both $SP$ and $AUC$ are IC and IR, the hybrid mechanism that is a simple combination of these two mechanisms may not be IC in general. To see this, consider a carrier whose true opportunity cost is lower than the posted price. If the carrier reports his opportunity cost truthfully, it will be assigned a load instantly under the $SP$ mechanism with a payment equal to the posted price. However, if it turns out that the payout from the auction (which is determined at the end of this period) is higher than the posted price, the carrier may be better off by reporting untruthfully in order to join the auction.

We propose a hybrid mechanism $HYB$ as follows. Similar to the $SP$ and $AUC$ mechanisms, the $HYB$ mechanism sets the same shipper-side price $r^*$ as in Section 4. On the carrier side, consider a carrier who arrives in the marketplace to deliver a load from node $i$ to node $j$ in period $t$. If the submitted bid is less than or equal to the posted price $p^*_{ij}$, then this carrier is assigned a load immediately and the platform guarantees that the payment that carrier would receive is at least $p^*_{ij}$, with the exact payment amount to be determined at a later time. On the other hand, if the submitted bid is higher than $p^*_{ij}$, then this carrier will wait to join an auction with the result to be determined at a later time. An auction will be conducted among all the available carriers if the total number of carriers whose submitted bid does not exceed $p^*_{ij}$ is no more than the demand $D_{ijt}$,
and the format of the auction is same as the uniform price auction with reserve price $\xi^*_{ij}$ defined in Section 5. More specifically, if the number of carriers who have confirmed a load allocation under SP is smaller than $D_{ijt}$, then the AUC auction will be conducted at the end of the period. If all the loads have already been booked (under SP) upon a carrier’s arrival and the bid of this newly arrived carrier is no more than $p^*_{ij}$, the platform makes payment $p^*_{ij}$ to those carriers who have received a load allocation (under SP) and all future carriers will leave the marketplace without receiving any load allocation. Finally, if all the loads have already been booked (under SP) and there is no additional carrier with submitted bid smaller than or equal to $p^*_{ij}$ until the end of the period, then the platform conducts the AUC auction and makes payment to those carriers who have confirmed a load allocation (under SP) according to the AUC payment rule.

We now formally define the carrier-side allocation rule and the payment rule under the hybrid mechanism HYB. Let $X_{ijt}^{SP}(C_{ijt})$ denote the number of carriers who would choose to deliver a load from node $i$ to node $j$ in period $t$ under SP with posted price $p^*_{ij}$ when the opportunity cost vector submitted by the carriers is $C_{ijt}$. The allocation rule of the HYB mechanism is defined as

$$A_{ijt}^{HYB}(C_{ijt}) = \begin{cases} A_{ijt}^{SP}(C_{ijt}), & \text{if } X_{ijt}^{SP}(C_{ijt}) > D_{ijt}, \\ A_{ijt}^{AUC}(C_{ijt}), & \text{otherwise} \end{cases}$$

(17)

where $A_{ijt}^{SP}(C_{ijt})$ and $A_{ijt}^{AUC}(C_{ijt})$ represent the allocations under the SP and AUC mechanisms defined in Sections 4 and 5 respectively. The payment rule of HYB is defined as

$$P_{ijt}^{HYB}(C_{ijt}) = \begin{cases} p^*_{ij}A_{ijt}^{HYB}(C_{ijt}), & \text{if } X_{ijt}^{SP}(C_{ijt}) > D_{ijt}, \\ p_{ij}^* - J_{ij}(C_{ijt}) - J_{ij}(C_{ijt}^*) - A_{ijt}^{HYB}(C_{ijt}), & \text{otherwise} \end{cases}$$

(18)

where $J_{ij}(\cdot)$ represents the optimal objective value of the allocation problem in AUC as defined in Eq (12).

A few remarks are in order. First, we notice that when $X_{ijt}^{SP}(C_{ijt}) > D_{ijt}$, all the loads are allocated under SP and the payment to each carrier is $P_{ijt}^{HYB}(C_{ijt}) = p^*_{ij}$. Otherwise, the payment under the HYB mechanism is given by the payment under the AUC mechanism, $P_{ijt}^{HYB}(C_{ijt}) = P_{ijt}^{AUC}(C_{ijt})$. It is easy to see that $P_{ijt}^{AUC}(C_{ijt}) = C_{ijt}^{[D_{ijt}+1]}$, where $C_{ijt}^{[D_{ijt}+1]}$ is the $(D_{ijt} + 1)$th lowest opportunity cost in the bid vector $C_{ijt}$. Since in this case we have $X_{ijt}^{SP}(C_{ijt}) \leq D_{ijt}$, it then implies that $C_{ijt}^{[D_{ijt}+1]} \geq p^*_{ij}$. Therefore, the payment to a carrier who receives a load allocation under HYB is at least the posted price $p^*_{ij}$, i.e., $P_{ijt}^{HYB}(C_{ijt}) \geq p^*_{ij}$ when $A_{ijt}^{HYB}(C_{ijt}) = 1$. Intuitively, this ensures that a carrier whose true opportunity cost is no more than $p^*_{ij}$ does not have the incentive to bid untruthfully, since he will be able to receive a higher payment based on the auction outcome in case the carrier supply is insufficient. As shown in Lemma 3 below, the hybrid mechanism HYB is IC and IR.
Lemma 3. The HYB mechanism is IC and IR.

We next present our main result in this section. We show that the auction mechanism AUC outperforms the hybrid mechanism HYB in terms of maximizing the long-run average profit, and HYB in turn outperforms the static posted price mechanism SP. Together with Theorem 2, this immediately implies the asymptotic optimality of HYB.

Theorem 5. The objective values of the hybrid mechanism and the (pure) auction mechanism satisfy
\[ \gamma^{HYB} \leq \gamma^{AUC}. \]  
(19)
In addition, the objective values of static posted price mechanism and the hybrid mechanism satisfy
\[ \gamma^{SP} \leq \gamma^{HYB}. \]  
(20)

7. Numerical Studies
In this section, we conduct a case study to provide further insights regarding the performance of different mechanisms. In our numerical studies, we use the freight transportation data (United States Census Bureau 2017) and the national trucking rates from DAT Freight & Analytics (DAT 2022) to calibrate our model parameters. The 2017 federal government data include average delivery miles and volumes (tons) for each state-level O-D pair, and the 2022 national flatbed rates include regional levels rates. The details of data sources and parameter estimation are relegated to the appendix.

In our numerical studies, we relax the travel time assumption in the model so that the travel times of different lanes are heterogeneous and not necessarily equal to one period. Our data set has information about the distance (average miles) for each lane (O-D pair). We assume each period is equal to one day. To obtain the number of transportation periods needed for each lane, we divide the average miles for each lane by 500 miles, which is about the maximum distance that a truckload driver can make under the federal regulation, and round up the value to the nearest integer. Therefore, in our case study, the travel times of different lanes are heterogeneous and not necessarily equal to one period. Next, we calculate the daily demand rate \( d_{ij} \) for each O-D pair \((i,j)\) as follows:
\[ d_{ij} = \frac{\text{delivery volumes in lane } (i,j) \text{ per year}/365}{\text{Container Volume}} \times \text{Market Share}, \]
where the container volume is set to 20 tons and the market share of a platform is set to 1.0% (the market share of Uber Freight is approximately 1.0% in 2021). In our numerical studies, we exclude
lanes with extremely small daily demand rates (i.e., smaller than 0.2). Our final data set includes the daily demand rate, the average miles, and the transportation period for 1728 lanes in 48 states within the United States.

We use simulation to generate independent sample paths, each with a total number of $T = 1000$ time periods. For each sample path, the first $T_0 = 200$ time periods are discarded and the performance of the system under a given mechanism (e.g., average cost) is evaluated against the remaining $T - T_0$ time periods. Furthermore, we set the number of carriers at the beginning of period 1 as $S_{ij1} = \lceil \bar{\lambda}_{ij} \cdot \text{Market Share} \rceil$ for each lane $(i, j) \in E$, where Market Share is the percentage of loads transacted on a focal platform and $\bar{\lambda}_{ij}$ is the optimal solution to the fluid problem $FA$. This helps to reduce the number of iterations to reach the stationary distribution.

### 7.1. Cost Gap Ratio and Booking Channel

Our first set of numerical experiments compare the performance of the static posted price mechanism $SP$, the auction mechanism $AUC$, and the hybrid mechanism $HYB$. As our proposed mechanisms ($SP$, $AUC$, $HYB$) share the same shipper-side mechanism, we mainly focus on the carrier-side mechanisms to compare their performance from a cost minimization perspective.

The first performance metric that we consider is the cost gap ratio. More specifically, the cost gap ratio of a given policy $\pi \in \Pi$ is defined as

$$\frac{\kappa^\pi - \kappa^{FA}}{\kappa^{FA}}.$$ 

In the above cost gap ratio, $\kappa^\pi$ denotes the long-run average cost incurred by the platform under policy $\pi$:

$$\kappa^\pi := \sum_{(i,j) \in E} E[P_{ij}^\pi + b_{ij}(D_{ij} - Y_{ij}^\pi)],$$  

which consists of payments made to the carriers on the platform and penalty costs incurred due to unsatisfied demand (if any).

In addition to the cost gap ratio, we are also interested in the SP ratio, which captures carriers’ booking channel selection behavior. More specifically, the SP ratio under a given policy is defined as the percentage of carriers who confirmed a load booking immediately upon their arrival through posted pricing, among all carriers who delivered a load. Under the posted price mechanism, all carriers can confirm a load instantly and the SP ratio is equal to one. In contrast, under the auction mechanism, all carriers have to wait until the end of a period for load confirmation and the SP ratio is equal to zero. For the hybrid mechanism, the SP ratio is somewhere in between zero and one, which reflects the proportion of carriers who do not need to wait and are able to book a load instantly upon arrival.
Table 1 summarizes the cost gap ratio and the SP ratio under SP, AUC and HYB. We observe that the gap between the simulated long-run average cost under all the three policies and the fluid bound decreases as the platform’s market share (and hence the system size) becomes larger, which is consistent with our findings in Theorems 2-5. Moreover, our numerical results suggest that the SP ratio under HYB increases and becomes closer to one as the market size grows. This implies that most carriers can confirm a booking instantly and do not have to wait for load confirmation under the HYB mechanism. From Table 1, we can clearly see a trade-off between the two competing performance metrics, the cost gap ratio and the SP ratio. In particular, a lower long-run average cost comes at the expense of a higher proportion of carriers who need to wait for load allocations. Compared with SP and AUC, HYB achieves a balanced trade-off between the cost gap ratio and the SP ratio. In view of this, the HYB mechanism can be an attractive alternative for platforms that care about both their cost performance and the carriers’ waiting time experience.

| Market Share | Cost Gap Ratio (%) | SP Ratio (%) |
|--------------|--------------------|--------------|
| 0.1%         | 35.96              | 31.54        |
| 0.5%         | 23.00              | 17.90        |
| 1%           | 18.39              | 13.36        |
| 5%           | 9.41               | 5.25         |

Table 1 Cost Gap Ratio under SP, AUC, and HYB.

To gain further insights about the performance of the three policies, we break down the long-run average cost into two components, the payment made to the carriers in the marketplace and the penalty incurred (or, payment made to third-party companies) due to the excess demand. The cost ratio, payment ratio, and penalty ratio of a given policy \( \pi \in \Pi \) are respectively defined as

\[
\frac{\kappa^\pi}{\kappa^{FA}}, \frac{\sum_{t=T_0+1}^{T} \sum_{(i,j) \in E} P_{ijt}^\pi}{(T-T_0)\kappa^{FA}}, \text{ and } \frac{\kappa^\pi}{\kappa^{FA}}, \sum_{t=T_0+1}^{T} \sum_{(i,j) \in E} P_{ijt}^\pi \frac{\kappa^{FA}}{(T-T_0)\kappa^{FA}}.
\]

Table 2 summarizes the ratios between the total cost and the decomposed cost components relative to the long-run average cost of FA, under the three different policies (SP, AUC, HYB). First, we observe that SP incurs the highest average total cost ratio. Second, we observe that HYB makes the highest average payment to the carriers among the three policies in general. This is intuitive because HYB offers the higher payment of that under SP and AUC by the definition of the payment rule (18). As for the average penalty, SP incurs the highest penalty cost due to unsatisfied demand. Recall that the reserve price \( \xi^* \) under AUC and HYB is higher than the posted price \( p^* \). Then intuitively, both AUC and HYB accept more carriers to transport loads, i.e., \( E[Y_{ijt}^{HYB}] = E[Y_{ijt}^{AUC}] \geq E[Y_{ijt}^{SP}] \), which leads to a lower average penalty cost than that under SP. Notice that the average penalty cost of AUC and HYB are the same, since these two policies share the same stationary distribution in steady state.
Table 2 Cost decomposition under SP, AUC, and HYB.

| Market Share | Cost Ratio | Payment Ratio | Penalty Ratio |
|--------------|------------|---------------|---------------|
|              | SP AUC HYB | SP AUC HYB    | SP AUC HYB    |
| 0.1%         | 1.36 1.32 1.34 | 0.57 0.68 0.71 | 0.79 0.63 0.63 |
| 0.5%         | 1.23 1.18 1.21 | 0.71 0.81 0.85 | 0.52 0.37 0.37 |
| 1%           | 1.18 1.13 1.17 | 0.77 0.86 0.89 | 0.41 0.28 0.28 |
| 5%           | 1.09 1.05 1.08 | 0.88 0.94 0.97 | 0.21 0.11 0.11 |

7.2. Robustness Checks

In this section, we conduct additional numerical experiments to test the robustness of the insights obtained from our earlier results. Table 3 summarizes how the performance of the three different policies (SP, AUC, HYB) change with respect to the penalty cost parameter. In our simulation, we fix the platform’s market share as 0.5%. All the other parameters remain the same as those in Section 7.1 except the penalty cost parameter. We vary the penalty cost parameter such that $b_{ij}/p_{ij} \in \{1.25, 1.50, 1.75, 2.00\}$ for each O-D pair $(i, j) \in E$, where $p_{ij}$ is an estimated shipping cost calculated from the data. From Table 3, we observe that the cost gap ratios of all three policies increase as the penalty cost parameter becomes larger, with SP having the most significant increase compared with the other two policies. Intuitively, the posted price mechanism has the largest amount of excess demand, and hence is most affected by the change in the penalty cost parameter.

Table 3 Impact of penalty cost parameter $b$ on cost gap ratio.

| $b_{ij}/p_{ij}$ | Cost Gap Ratio(%) |
|----------------|------------------|
|                | SP AUC HYB       |
| 1.25           | 4.57 3.85 4.56   |
| 1.50           | 9.73 7.56 9.72   |
| 1.75           | 16.59 12.80 16.30|
| 2.00           | 22.96 17.85 21.37|

In addition to the penalty cost parameter, we have also conducted additional numerical experiments to investigate the impact of the probability that a carrier will stay in the marketplace after completing a load transportation. The market share is fixed at 0.5%, and all the other parameters remain the same as those in Section 7.1 except the staying probability parameter. Note that $q_i := \sum_{j \in \delta^+ (i)} q_{ij}$ represents the probability that a carrier would stay in the marketplace after finishing a load delivery at node $i$. In our numerical studies, we assume that $q_i = q \in \{0.1, 0.2, 0.3, 0.4\}$ for all $i$. It is worth noticing that changing the value of this probability would affect the exogenous inflow rate of the carriers based on our parameter estimation process. Therefore, the results below measure the overall effect of the remaining probability, with the exogenous carriers' arrival rate taken into account. As shown in Table 4, the cost gap ratios of all three mechanisms decrease...
as $q$ increases. Intuitively, a larger staying probability would increase carrier availability in the marketplace, which can help reduce penalty cost due to excess demand, and potentially decrease the payments to the carriers due to more competitive bidding.

![Table 4](image)

| $q$ | SP | AUC | HYB |
|-----|----|-----|-----|
| 0.10 | 23.65 | 18.46 | 22.03 |
| 0.20 | 22.96 | 17.85 | 21.37 |
| 0.30 | 22.24 | 17.20 | 20.73 |
| 0.40 | 21.96 | 16.65 | 20.14 |

Table 4  Impact of staying probability $q$ on cost gap ratio.

8. Concluding Remarks

In this paper, we study a mechanism design problem for freight marketplaces. We consider a freight platform who serves as an intermediary between shippers and carriers in a truckload transportation network and aims to maximize its long-run average profit. We have proposed and analyzed three types of mechanisms: posted price mechanisms, auction mechanisms, and a hybrid of both. First, we show that a static posted price mechanism based on fluid approximation is asymptotically optimal when the shipper demand and the carrier supply are both large. Second, we study an auction mechanism to determine the load allocation and payments to the carriers. Our proposed auction mechanism applies the uniform price auction to a decomposition of the transportation network. We show that this auction mechanism generates higher profits for the platform than the static posted price mechanism. Finally, we study a hybrid mechanism, in which carriers can either book a load instantly by accepting the posted price offered by the platform, or join an auction to seek higher payments. We show that the hybrid mechanism can achieve a trade-off between platform profit and carrier waiting time, and is asymptotically optimal.

There are several possible directions to extend our research. First, our model assumes that the lead time of each load is one period. That is, any load arriving at the beginning of a period will expire at the end of the period and cannot be carried over to the next period. It would be interesting to generalize our model and consider loads with heterogeneous, multi-period lead times. Second, our proposed static posted price mechanism uses a fixed price for each O-D pair. To improve the platform’s profit, considering prices that dynamically change over time in response to the system states can be a good extension. Lastly, it would be more desirable that a platform can choose different types of mechanisms in each lane. The effect of this flexibility may offer more managerial insights in practice.
References

Aféche, P., Liu, Z., and Maglaras, C. (2018). Ride-hailing networks with strategic drivers: The impact of platform control capabilities on performance. Working paper available at SSRN: https://ssrn.com/abstract=3120544.

ATA (2021). Americal Trucking Associations: Economics and industry data. https://www.trucking.org/economics-and-industry-data/ [Online; accessed 9-March-2021].

Aveklouris, A., DeValve, L., and Ward, A. R. (2021). Matching impatient and heterogeneous demand and supply. Working paper available at https://arxiv.org/abs/2102.02710.

Balseiro, S. R., Besbes, O., and Weintraub, G. Y. (2019). Dynamic mechanism design with budget-constrained buyers under limited commitment. Operations Research, 67(3):711–730.

Balseiro, S. R., Brown, D. B., and Chen, C. (2021). Dynamic pricing of relocating resources in large networks. Management Science, 67(7):4075–4094.

Banerjee, S., Freund, D., and Lykouris, T. (2021). Pricing and optimization in shared vehicle systems: An approximation framework. Forthcoming at Operations Research.

Banerjee, S., Johari, R., and Riquelme, C. (2016). Dynamic pricing in ridesharing platforms. ACM SIGecom Exchanges, 15(1):65–70.

Besbes, O., Castro, F., and Lobel, I. (2021). Surge pricing and its spatial supply response. Management Science, 67(3):1350–1367.

Bimpikis, K., Candogan, O., and Saban, D. (2019). Spatial pricing in ride-sharing networks. Operations Research, 67(3):744–769.

Board, S. and Skrzypacz, A. (2016). Revenue management with forward-looking buyers. Journal of Political Economy, 124(4):1046–1087.

Cachon, G. P., Daniels, K. M., and Lobel, R. (2017). The role of surge pricing on a service platform with self-scheduling capacity. Manufacturing & Service Operations Management, 19(3):368–384.

Caldentey, R. and Vulcano, G. (2007). Online auction and list price revenue management. Management Science, 53(5):795–813.

Cao, Y., Kleywegt, A., and Wang, H. (2022). Dynamic pricing for two-sided marketplaces with offer expiration. Available at SSRN 3700227.

Caplice, C. (2007). Electronic markets for truckload transportation. Production and Operations Management, 16(4):423–436.

Chawla, S., Hartline, J. D., Malec, D. L., and Sivan, B. (2010). Multi-parameter mechanism design and sequential posted pricing. In Proceedings of the forty-second ACM symposium on Theory of computing, pages 311–320.

Chen, Y., Farias, V. F., and Trichakis, N. (2019). On the efficacy of static prices for revenue management in the face of strategic customers. Management Science, 65(12):5535–5555.
Chen, Y. and Hu, M. (2020). Pricing and matching with forward-looking buyers and sellers. *Manufacturing & Service Operations Management*, 22(4):717–734.

Cohen, M. C., Désir, A., Korula, N., and Sivan, B. (2022). Best of both worlds ad contracts: Guaranteed allocation and price with programmatic efficiency. *Management Science*.

Convoy (2018). Introducing instant bidding: Helping carriers find the right job at the right price. [Online; accessed 21-January-2021].

Cooper, W. L. (2002). Asymptotic behavior of an allocation policy for revenue management. *Operations Research*, 50(4):720–727.

DAT (2022). National flatbed rates. [Online; accessed 21-february-2022].

Dütting, P., Fischer, F., and Klimm, M. (2016). Revenue gaps for static and dynamic posted pricing of homogeneous goods. *arXiv preprint arXiv:1607.07105*.

Edelman, B., Ostrovsky, M., and Schwarz, M. (2007). Internet advertising and the generalized second-price auction: Selling billions of dollars worth of keywords. *American economic review*, 97(1):242–259.

Einav, L., Farronato, C., Levin, J., and Sundaresan, N. (2018). Auctions versus posted prices in online markets. *Journal of Political Economy*, 126(1):178–215.

Etzion, H., Pinker, E., and Seidmann, A. (2006). Analyzing the simultaneous use of auctions and posted prices for online selling. *Manufacturing & Service Operations Management*, 8(1):68–91.

Feng, G., Kong, G., and Wang, Z. (2021). We are on the way: Analysis of on-demand ride-hailing systems. *Manufacturing & Service Operations Management*, 23(5):1237–1256.

Figliozzi, M. A., Mahmassani, H. S., and Jaillet, P. (2003). Framework for study of carrier strategies in auction-based transportation marketplace. *Transportation Research Record*, 1854(1):162–170.

Figliozzi, M. A., Mahmassani, H. S., and Jaillet, P. (2005). Impacts of auction settings on the performance of truckload transportation marketplaces. *Transportation research record*, 1906(1):89–96.

Foster, F. G. (1953). On the stochastic matrices associated with certain queuing processes. *Annals of Mathematical Statistics*, 24(3):355–360.

Gallego, G. and Van Ryzin, G. (1994). Optimal dynamic pricing of inventories with stochastic demand over finite horizons. *Management science*, 40(8):999–1020.

Gallego, G. and Van Ryzin, G. (1997). A multiproduct dynamic pricing problem and its applications to network yield management. *Operations research*, 45(1):24–41.

Godfrey, G. A. and Powell, W. B. (2002). An adaptive dynamic programming algorithm for dynamic fleet management, ii: Multiperiod travel times. *Transportation Science*, 36(1):40–54.

Guda, H. and Subramanian, U. (2019). Your uber is arriving: Managing on-demand workers through surge pricing, forecast communication, and worker incentives. *Management Science*, 65(5):1995–2014.
Hu, M. and Zhou, Y. (2021). Dynamic type matching. published online at *Manufacturing & Service Operations Management*.

Jasin, S. (2014). Reoptimization and self-adjusting price control for network revenue management. *Operations Research*, 62(5):1168–1178.

Jin, Y., Lu, P., Tang, Z. G., and Xiao, T. (2020). Tight revenue gaps among simple mechanisms. *SIAM Journal on Computing*, 49(5):927–958.

Liu, Q. and Van Ryzin, G. (2008). On the choice-based linear programming model for network revenue management. *Manufacturing & Service Operations Management*, 10(2):288–310.

Maglaras, C. and Meissner, J. (2006). Dynamic pricing strategies for multiproduct revenue management problems. *Manufacturing & Service Operations Management*, 8(2):136–148.

Myerson, R. B. (1981). Optimal auction design. *Mathematics of Operations Research*, 6(1):58–73.

Osadchiy, N. and Vulcano, G. (2010). Selling with binding reservations in the presence of strategic consumers. *Management Science*, 56(12):2173–2190.

¨Ozkan, E. and Ward, A. R. (2020). Dynamic matching for real-time ride sharing. *Stochastic Systems*, 10(1):29–70.

Topaloglu, H. and Powell, W. (2007). Incorporating pricing decisions into the stochastic dynamic fleet management problem. *Transportation Science*, 41(3):281–301.

Uber Freight (2020). Uber freight announces in-app bidding for carriers. [https://www.uber.com/blog/does-uber-freight-allow-bidding/](https://www.uber.com/blog/does-uber-freight-allow-bidding/) [Online; accessed 21-January-2021].

United State Census Bureau (2017). Geographic area series: Shipment characteristics by origin geography by destination geography by commodity by mode: 2017. [https://data.census.gov/cedsci/table?q=cf1700a21&hidePreview=true&tid=CFSAREA2017.CF1700A21/](https://data.census.gov/cedsci/table?q=cf1700a21&hidePreview=true&tid=CFSAREA2017.CF1700A21/) [Online; accessed 21-february-2022].

Varma, S. M., Bumpensanti, P., Maguluri, S. T., and Wang, H. (2020). Dynamic pricing and matching for two-sided queues. In *Abstracts of the 2020 SIGMETRICS/Performance Joint International Conference on Measurement and Modeling of Computer Systems*, pages 105–106.

Vulcano, G., Van Ryzin, G., and Maglaras, C. (2002). Optimal dynamic auctions for revenue management. *Management Science*, 48(11):1388–1407.

Wang, R. (1993). Auctions versus posted-price selling. *The American Economic Review*, 83(4):838–851.
Appendix

Notations

| Symbol | Description |
|--------|-------------|
| $\mathcal{N}$ | The set of nodes (locations) |
| $\mathcal{E}$ | The set of arcs (lanes) |
| $S_{ijt}$ | The set of carriers available to deliver loads in lane $(i,j)$ at period $t$ |
| $\delta^+(j)$ | $\{k \in \mathcal{N} : (j,k) \in \mathcal{E}\}$, the set of outbound nodes from node $j$ |
| $\delta^-(j)$ | $\{i \in \mathcal{N} : (i,j) \in \mathcal{E}\}$, the set of inbound nodes to node $j$ |
| $S_{ijt}$ | Number of available carriers in lane $(i,j)$ at period $t$ |
| $D_{ijt}$ | Number of loads that need to be shipped in lane $(i,j)$ at period $t$ |
| $C_{ijt}$ | True opportunity cost of carrier $s$ transporting a load in lane $(i,j)$ at period $t$ |
| $X_{ijt}$ | Number of carriers who would choose to book loads in lane $(i,j)$ at period $t$ |
| $Y_{ijt}$ | Number of carriers who are awarded loads in lane $(i,j)$ at period $t$ |
| $Z_{ijt}$ | Number of carriers who decide to stay and deliver a load in lane $(i,j)$ after completing a shipment at period $t$ |
| $V_{ijt}$ | Number of carriers in $S_{ijt}$ who leave the marketplace at the end of period $t$ |
| $P_{ijt}$ | Total payment to carriers who transported loads in lane $(i,j)$ at period $t$ |
| $\Lambda_{ijt}$ | Number of exogenous arrival of carriers in lane $(i,j)$ at period $t$ |
| $FA$ | Optimal objective value of FA |
| $b_{ij}$ | Unit penalty cost for unsatisfied demand in lane $(i,j)$ |
| $q_{ij}$ | Probability that a carrier will stay in the marketplace and choose to deliver a load in lane $(i,j)$ |
| $\lambda_{ij}$ | Exogenous arrival rate of carriers in lane $(i,j)$ |
| $d_{ij}$ | Optimal demand rate of loads in lane $(i,j)$ |
| $r_{ij}$ | Optimal spot price for loads that need to be shipped in lane $(i,j)$ |
| $\lambda^r_{ij}$ | Optimal total inflow of carriers in lane $(i,j)$ |
| $x^*_{ij}$ | Optimal probability that a carrier chooses to deliver a load in lane $(i,j)$ |
| $y^*_{ij}$ | Optimal flow of carriers who transported a load in lane $(i,j)$ |
| $v^*_{ij}$ | Optimal flow of leaving carriers in lane $(i,j)$ |
| $\rho_{ijt}$ | Auxiliary notation used in the proofs, see Eq (EC.12) |
| $\mathcal{H}_{ijt}$ | Auxiliary notation used in the proofs, see Eq (EC.13) |
| $\mathcal{K}_{ijt}$ | Auxiliary notation used in the proofs, see Eq (EC.17) |

Table EC.1 Summary of important notations.

Proofs in Section 2

Proof of Proposition 1. We prove Proposition 1 by showing the stability of a modified system whose expected total number of carriers is greater than or equal to that of the original system. We use the Foster’s theorem (Foster 1953) to prove the stability of the new system, and we show that the existence of the stationary distribution of the new system implies the stability of the original
system. Notice that it suffices to consider the stability of the carrier side since the new shipper demand \(D_t\) follows a Poisson distribution with known rate and all the loads are served either by the carriers in the marketplace or the third-party.

Consider a modified model in which a carrier stays on the platform with probability \(\hat{q}\) after completing a load shipment in each period, where \(\hat{q} := \max_{(i,j) \in E} q_{ij}\), regardless of whether the carrier hauls a load. Let \(\hat{S}_t\) be the total number of carriers in the modified system at the beginning of period \(t\). Let \(\hat{\lambda} = \sum_{(i,j) \in E} \lambda_{ij}\) be the total arrival rate of carriers in each period. The number of remaining carriers at the end of period \(t\), \(\hat{Z}_t\), follows a Binomial distribution with parameters \(\hat{q}\) and \(\hat{S}_t\). To explicitly show the dependency of \(\hat{Z}_t\) on the parameter \(\hat{q}\), we write \(\hat{Z}_t(\hat{q}|\hat{S}_t)\) whenever necessary. Then in the modified model, the carriers available at the next period are

\[
\hat{S}_{t+1} = \hat{S}_{t}(\hat{q}|\hat{S}_t) + \hat{\lambda}_{t+1}.
\]

The one-step transition probability from state \(l\) to state \(k\) is given by

\[
\Pr(\hat{S}_{t+1} = k | \hat{S}_t = l) = \sum_{m=0}^{\min\{l,k\}} \binom{l}{m} \hat{q}^m (1 - \hat{q})^{l-m} e^{-\hat{\lambda}} \frac{\hat{\lambda}^{k-m}}{(k-m)!}.
\]

Clearly, \(E[\sum_{i \in N} S_{ij}] \leq E[\hat{S}_t]\) if \(E[\hat{S}_t]\) exists, where \(S_{ij}\) is the number of carriers in lane \((i,j)\) at period \(t\) in the original model. Therefore, to prove Proposition 1, it suffices to show the existence of \(E[\hat{S}_t]\).

We use Foster’s Theorem to show that a stationary distribution exists in the modified system. Let \(Z^+\) denote the set of non-negative integers. Define a Lyapunov function \(\Phi\) as

\[
\Phi(\hat{S}_t) := \hat{S}_t.
\]

By the definition of \(\hat{S}_t\), we assume \(\Phi(\hat{S}_t) \geq 0\) without loss of generality. By Foster’s Theorem, the Markov chain of the modified platform is positive recurrent if the Lyapunov function \(\Phi\) satisfies

\[
\sum_{k=0}^{\infty} \Pr(\hat{S}_{t+1} = k | \hat{S}_t = l) \Phi(k) \leq \Phi(l) - \epsilon, \quad \forall l \notin F,
\]

for some finite set \(F\) and \(\epsilon > 0\).

We first show that \(\sum_{k=0}^{\infty} \Pr(\hat{S}_{t+1} = k | \hat{S}_t = l) \Phi(k) < \infty\) for any state \(l \in Z^+\). We have

\[
\sum_{k=0}^{\infty} \Pr(\hat{S}_{t+1} = k | \hat{S}_t = l) \Phi(k) = \sum_{k=0}^{\infty} \Pr(\hat{S}_{t+1} = k | \hat{S}_t = l) k = \sum_{k=0}^{l} \Pr(\hat{S}_{t+1} = k | \hat{S}_t = l) k + \sum_{k=l+1}^{\infty} \Pr(\hat{S}_{t+1} = k | \hat{S}_t = l) k.
\]
It is clear that $\sum_{k=0}^{l} \Pr(\hat{S}_{t+1} = k|\hat{S}_{t} = l)k$ is bounded. Consider the second term in the above equation:

$$\sum_{k=0}^{\infty} \Pr(\hat{S}_{t+1} = k|\hat{S}_{t} = l)k = \sum_{k=1}^{\infty} \sum_{m=0}^{l} \binom{l}{m} \hat{q}^{m}(1-\hat{q})^{l-m} e^{-\hat{\lambda} \hat{k-m} \hat{\lambda}} \frac{(k-m)!}{(k-m)!} k$$

$$= \sum_{k=m}^{\infty} \sum_{m=0}^{l} \binom{l}{m} \hat{q}^{m}(1-\hat{q})^{l-m} e^{-\hat{\lambda} \hat{k-m} \hat{\lambda}} (k-m+m)$$

$$= \sum_{m=0}^{l} \binom{l}{m} \hat{q}^{m}(1-\hat{q})^{l-m} \sum_{k=m}^{\infty} e^{-\hat{\lambda} \hat{k-m} \hat{\lambda}} (k-m+m).$$

We can get an upper bound of the above inner summation term as follows:

$$\sum_{k=m}^{\infty} e^{-\hat{\lambda} \hat{k-m} \hat{\lambda}} (k-m+m) \leq \sum_{k=m}^{\infty} e^{-\hat{\lambda} \hat{k-m} \hat{\lambda}} (k-m+m)$$

$$= \sum_{k=m}^{\infty} e^{-\hat{\lambda} \hat{k-m} \hat{\lambda}} (k-m) + \sum_{k=m}^{\infty} e^{-\hat{\lambda} \hat{k-m} \hat{\lambda}} m$$

$$\leq \hat{\lambda} + m.$$ 

Then, we have

$$\sum_{k=0}^{\infty} \Pr(\hat{S}_{t+1} = k|\hat{S}_{t} = l)k \leq \sum_{m=0}^{l} \binom{l}{m} \hat{q}^{m}(1-\hat{q})^{l-m}(\hat{\lambda} + m)$$

$$\leq \hat{\lambda} + l\hat{q}(1-\hat{q}).$$

It then immediately follows that $\sum_{k=0}^{\infty} \Pr(\hat{S}_{t+1} = k|\hat{S}_{t} = l)\Phi(k) < \infty$ for all $l \in \mathbb{Z}^+.$

We next show that there exist a finite set $F^*$ and a real positive number $\epsilon^*$ that satisfy the second condition of the Foster’s theorem:

$$\sum_{k=0}^{\infty} \Pr(\hat{S}_{t+1} = k|\hat{S}_{t} = l)\Phi(k) \leq \Phi(l) - \epsilon^*, \quad \forall l \not\in F^*.$$ 

We have

$$\sum_{k=0}^{\infty} \Pr(\hat{S}_{t+1} = k|\hat{S}_{t} = l)\Phi(k) - \Phi(l)$$

$$= \sum_{k=0}^{\infty} \Pr(\hat{S}_{t+1} = k|\hat{S}_{t} = l)k - l$$

$$= \sum_{k=0}^{\infty} \Pr(\hat{S}_{t+1} = k|\hat{S}_{t} = l)(k-l)$$

$$= \sum_{k=0}^{l} \Pr(\hat{S}_{t+1} = k|\hat{S}_{t} = l)(k-l) + \sum_{k=l+1}^{\infty} \Pr(\hat{S}_{t+1} = k|\hat{S}_{t} = l)(k-l).$$
Then, the condition can be expressed as:

\[
\sum_{k=0}^{l} \Pr(\hat{S}_{t+1} = k|\hat{S}_t = l)(k - l) + \sum_{k=l+1}^{\infty} \Pr(\hat{S}_{t+1} = k|\hat{S}_t = l)(k - l) \leq -\epsilon^*, \quad \forall l \notin F^*.
\]

In what follows, we will show the existence of \(\epsilon^*\) and \(F^*\) that satisfy the following two inequalities:

\[
\sum_{k=0}^{l} \Pr(\hat{S}_{t+1} = k|\hat{S}_t = l)(k - l) < -\hat{\lambda} - \epsilon^*, \quad \forall l \notin F^*, \tag{EC.1}
\]

\[
\sum_{k=l+1}^{\infty} \Pr(\hat{S}_{t+1} = k|\hat{S}_t = l)(k - l) \leq \hat{\lambda}, \quad \forall l \notin F^*. \tag{EC.2}
\]

To show (EC.1), we first show

\[
\sum_{k=0}^{l} \Pr(\hat{S}_{t+1} = k|\hat{S}_t = l)k \leq \hat{\lambda} + l\hat{q}(1 - \hat{q}) (*)
\]

for any \(l \in \mathbb{Z}^+\), and then show the existence of \(\epsilon^* > 0\) and \(F^*\) such that

\[
\sum_{k=0}^{l} \Pr(\hat{S}_{t+1} = k|\hat{S}_t = l)l > l\hat{q}(1 - \hat{q}) + 2\hat{\lambda} + \epsilon^*, \quad \forall l \notin F^*. \tag{**}
\]

Combining the above two inequality immediately leads to (EC.1).

We first show (\*). We have

\[
\sum_{k=0}^{l} \Pr(\hat{S}_{t+1} = k|\hat{S}_t = l)k = \sum_{k=0}^{l} \sum_{m=0}^{k} \binom{k}{m} \hat{q}^m (1 - \hat{q})^{k-m} e^{-\hat{\lambda}} \frac{\hat{\lambda}^{k-m}}{(k-m)!} k
\]

\[
= \sum_{k=0}^{l} \sum_{m=0}^{k} \binom{k}{m} \hat{q}^m (1 - \hat{q})^{k-m} e^{-\hat{\lambda}} \frac{\hat{\lambda}^{k-m}}{(k-m)!} (k-m+m)
\]

\[
= \sum_{m=0}^{k} \binom{k}{m} \hat{q}^m (1 - \hat{q})^{k-m} \sum_{k=0}^{l} e^{-\hat{\lambda}} \frac{\hat{\lambda}^{k-m}}{(k-m)!} (k-m+m)
\]

\[
\leq \sum_{m=0}^{k} \binom{k}{m} \hat{q}^m (1 - \hat{q})^{k-m} (\hat{\lambda} + m)
\]

\[
\leq \hat{\lambda} + l\hat{q}(1 - \hat{q}).
\]

We next show (**). Consider any \(k, l \in \mathbb{Z}^+\) such that \(l > k\). By the probability mass function of the Binomial distribution, we have

\[
\Pr(Z_t(\hat{q}|l) = l - k) = \binom{l}{l-k} \hat{q}^{l-k}(1 - \hat{q})^k.
\]

It then follows that

\[
\lim_{l \to \infty} \frac{\Pr(Z_t(\hat{q}|l) = l - k)}{\Pr(Z_t(\hat{q}|l+1) = l + 1 - k)} = \lim_{l \to \infty} \frac{l - k + 1}{(l+1)\hat{q}} = \frac{1}{\hat{q}}.
\]
which implies that \( \lim_{l \to \infty} \Pr(Z_t(\hat{q}|l) = l - k) = 0. \) Then, for any \( \epsilon > 0 \) and \( k \), there exists some \( N \in \mathbb{Z}^+ \) such that
\[
\Pr(Z_t(\hat{q}|l) = l - k) < \epsilon, \quad \forall l > N.
\]
Consider some \( \epsilon^* \) such that \( 0 < \epsilon^* < 1 - \tilde{q} + \tilde{q}^2 \). As \( \hat{\Lambda}_t \) follows a Poisson distribution, there exists a \( \lambda' \in \mathbb{Z}^+ \) such that \( \Pr(\hat{\Lambda}_t > \lambda') < \epsilon^*/2 \). Given \( \epsilon^* > 0 \), consider \( N_k \in \mathbb{Z}^+ \) such that \( \Pr(Z_t(\hat{q}|l) = l - k) < \epsilon^*/(2\lambda') \) for all \( l > N_k \). Let \( N' := \max_{k \leq \lambda'}{N_k} \). We have
\[
\Pr(Z_t(\hat{q}|l) > l - \lambda') = \sum_{k=0}^{\lambda'-1} \Pr(Z_t(\hat{q}|l) = l - k) < \epsilon^*/2, \quad \forall l > N'.
\]
Combining the two inequalities \( \Pr(\hat{\Lambda}_{t+1} > \lambda') < \epsilon^*/2 \) and \( \Pr(Z_t(\hat{q}|l) > l - \lambda') < \epsilon^*/2 \), we have
\[
\Pr(Z_t(\hat{q}|l) > l - \lambda' \text{ or } \hat{\Lambda}_{t+1} > \lambda') < \epsilon^*, \quad \forall l > N'.
\]
Recall that \( \hat{S}_{t+1} = \hat{Z}_t(\hat{q}||\hat{S}_t) + \hat{\Lambda}_{t+1} \). If \( \hat{S}_{t+1} > \hat{S}_t \) and \( \hat{\Lambda}_{t+1} \leq \lambda' \), then we have \( \hat{Z}_t(\hat{q}||\hat{S}_t) > \hat{S}_t - \hat{\Lambda}_{t+1} \geq \hat{S}_t - \lambda' \), which leads to \( \Pr(\hat{S}_{t+1} > \hat{S}_t) \leq \Pr(\hat{Z}_t(\hat{q}||\hat{S}_t) > \hat{S}_t - \lambda' \text{ or } \hat{\Lambda}_{t+1} > \lambda') \). It then implies that
\[
\sum_{k=l+1}^{\infty} \Pr(\hat{S}_{t+1} = k|\hat{S}_t = l) < \epsilon^*, \quad \forall l > N'.
\]
Or equivalently,
\[
\sum_{k=0}^{l} \Pr(\hat{S}_{t+1} = k|\hat{S}_t = l) \geq 1 - \epsilon^*, \quad \forall l > N'.
\]
Let \( N^* := \max\{\lfloor (2\lambda + \epsilon^*)/(1 - \tilde{q} + \tilde{q}^2 - \epsilon^*) \rfloor, N'\} \). We have
\[
\sum_{k=0}^{l} \Pr(\hat{S}_{t+1} = k|\hat{S}_t = l) \geq (1 - \epsilon^*)l > l\tilde{q}(1 - \tilde{q}) + 2\lambda + \epsilon^*, \quad \forall l > N^*.
\]
Let \( F^* := \{l \in \mathbb{Z}^+: l \leq N^*\} \) and recall that \( \sum_{k=0}^{l} \Pr(\hat{S}_{t+1} = k|\hat{S}_t = l)k \leq \hat{\lambda} + l\tilde{q}(1 - \tilde{q}) \) for any \( l \). It then leads to
\[
\sum_{k=0}^{l} \Pr(\hat{S}_{t+1} = k|\hat{S}_t = l)(k - l) < -\hat{\lambda} - \epsilon^*, \quad \forall l \notin F^*.
\]
Finally, it remains to show (EC.2). We have
\[
\sum_{k=l+1}^{\infty} \Pr(\hat{S}_{t+1} = k|\hat{S}_t = l)(k - l) = \sum_{k=l+1}^{\infty} \sum_{m=0}^{l} \binom{l}{m} \tilde{q}^m (1 - \tilde{q})^{l-m} e^{-\hat{\lambda}} \frac{\hat{\lambda}^{k-m}}{(k-m)!}(k-l)
\]
\[
= \sum_{m=0}^{l} \binom{l}{m} \tilde{q}^m (1 - \tilde{q})^{l-m} \sum_{k=l+1}^{\infty} e^{-\hat{\lambda}} \frac{\hat{\lambda}^{k-m}}{(k-m)!}(k-m)l
\]
\[
\leq \sum_{m=0}^{l} \binom{l}{m} \tilde{q}^m (1 - \tilde{q})^{l-m} \sum_{k=l+1}^{\infty} e^{-\hat{\lambda}} \frac{\hat{\lambda}^{k-m}}{(k-m)!}(k-m)
\]
\[
\leq l \tilde{q}^l (1 - \tilde{q})^{l-m} \hat{\lambda} = \hat{\lambda}.
\]
Therefore, we have $\sum_{k=0}^{\infty} \Pr(\hat{S}_{t+1} = k | \hat{S}_t = l)(k - l) < -\epsilon^*$ for all $l \notin F^*$ where $F^* := \{ l \in \mathbb{Z}^+ : l \leq N^* \}$.

The existence of $F^*$ and $\epsilon^*$ that satisfy the two conditions for the Foster’s theorem implies that the modified system is stable. By definition, the expected total number of carriers in the modified system is an upper bound of that in the original system. Therefore, the expected total number of carriers in our original system is bounded, which completes the proof.  

Proof of Proposition 2. Since our analysis does not depend on the specific lane, and with the assumption of symmetric carriers, we will drop the node index $i$, $j$, $t$, and the carrier index $s$ for notation simplicity whenever the context is clear.

Under a state $(S, D)$ of an IC and IR mechanism, consider the net utility $u(C)$ of a carrier with true opportunity costs $C \in \mathbb{R}$. By the IC constraint, we have

$$u(C) = p(C) - a(C)C \geq p(c) - a(c)C = u(c) - a(c)(C - c)$$

for any $c \in \mathbb{R}$. This implies that $u(C)$ is a convex function with gradient $-a(C)$. Recall that $p(C) = u(C) + a(C)C$ and $\psi(C) = C + \frac{F(C)}{f(C)}$. Note that $\lim_{C \to \infty} a(C) = 0$ because the platform should reject the carrier whose opportunity cost is $\infty$. Then, the expected payment to the carrier is

$$\hat{E}[p(C)] = \hat{E}[u(C) + a(C)C]$$

$$= \int_{0}^{\infty} \int_{C}^{\infty} a(\zeta) f(C)d\zeta dC + \int_{0}^{\infty} a(C)C f(C)dC$$

$$= \int_{0}^{\infty} \int_{0}^{\zeta} a(\zeta) f(C)dC d\zeta + \int_{0}^{\infty} a(C)C f(C)dC$$

$$= \int_{0}^{\infty} a(\zeta) F(\zeta)d\zeta + \int_{0}^{\infty} a(C)C f(C)dC$$

$$= \int_{0}^{\infty} a(C) \left(C + \frac{F(C)}{f(C)} \right) f(C)dC$$

$$= \hat{E}[a(C)\psi(C)] ,$$

which completes the proof.  

Proofs in Section 3

Lemma EC.1. Constraints (11a)-(11d) in FA are necessary conditions for any mechanism under which the system is stable.
Proof of Lemma EC.1. We showed that any mechanism has a stationary distribution by Proposition 1. Consider any platform mechanism $\pi \in \Pi$. As the system is assumed to be in a steady state, we have

$$E[S_{t+1}] = E[S_t],$$

or alternatively,

$$E[S_{ij,t+1}] = E[S_{ij,t}], \quad \forall (i,j) \in \mathcal{E},$$

where the expectation is taken with respect to the stationary distribution of the Markov chain induced by a given mechanism. Taking expectation on both sides of Eq (5) and defining $\bar{S}_{ij} := E[S_{ij,t}]$, $\bar{Y}_{jk} := E[Y_{jkt}]$, and $\bar{V}_{ij} := E[V_{ij,t}]$, we have

$$\bar{S}_{ij} = E[Y_{ij,t} + V_{ij,t}] = \bar{Y}_{ij} + \bar{V}_{ij}. \quad (EC.3)$$

Taking expectation on both sides of the system dynamics equation Eq (6), we have

$$\bar{S}_{ij} = E[Z_{ij,t} + \Lambda_{ij,t+1}] = \sum_{k \in \delta^-(i)} q_{ij} E[Y_{ki,t}] + \lambda_{ij} = \sum_{k \in \delta^-(i)} q_{ij} \bar{Y}_{ki} + \lambda_{ij},$$

Notice that $\bar{Y}_{ij}$ represents the long-run average fluid rate of carriers who actually ship a load in lane $(i,j)$, which should be no more than the (fluid) demand rate $d_{ij}$ in lane $(i,j)$: $\bar{Y}_{ij} \leq d_{ij}$. Then, the following constraints are necessary for any mechanism under which the system is stable:

$$\sum_{k \in \delta^-(i)} q_{ij} \bar{Y}_{ki} + \lambda_{ij} = \bar{Y}_{ij} + \bar{V}_{ij}, \quad \forall (i,j) \in \mathcal{E},$$

$$\bar{Y}_{ij} \leq d_{ij}, \quad \forall (i,j) \in \mathcal{E},$$

$$\bar{Y}_{ij}, \bar{V}_{ij} \geq 0, \quad \forall (i,j) \in \mathcal{E}.$$

Therefore, we can conclude that the constraints of FA are necessary conditions for any mechanism under which the system is stable. □

We next analyze the expected total payment made to all the carriers delivering loads in a particular lane. Consider lane $(i,j) \in \mathcal{E}$ under a state $(S,D)$ in the stationary distribution, and we index the available carriers in lane $(i,j)$ as $s \in \{1, \ldots, S_{ij}\}$. Let $P_{ij}$ denote the payment made to all the carriers who have transported a load in lane $(i,j)$, where $P_{ij} := \sum_{s=1}^{S_{ij}} p_{ij}^{s} (C_{ij}^{s})$. 


**Lemma EC.2.** Under a state in the stationary distribution of the system induced by the platform’s mechanism $\pi(S,D) = (\mathcal{M}_r, \mathcal{M}_p)$ that is IC and IR, the expected total payment to all the carriers in this period is lower bounded by

$$
E[P_{ij}] \geq F_{ij}^{-1} \left( \frac{E[Y_{ij}]}{E[S_{ij}]} \right) E[Y_{ij}], \quad \forall (i,j) \in \mathcal{E}.
$$

**Proof of Lemma EC.2.** Consider a state $(S,D)$ in the stationary distribution of the system. For notation simplicity, we will use $\hat{E}[]$ to denote the conditional expectation $E[\cdot|S,D]$ in the proof.

By Proposition 2, we have $\hat{E}[\pi^*(C_{ij})] = \hat{E}[a_{ij}^*(C_{ij})\psi_{ij}(C_{ij})]$. Then the expected total payment made by the platform to all carriers in lane $(i,j)$ is given by

$$
\hat{E}[P_{ij}] = \hat{E} \left[ \sum_{s=1}^{S_{ij}} a_{ij}^*(C_{ij}^s)\psi_{ij}(C_{ij}^s) \right]
$$

$$
\geq \hat{E} \left[ \min_{s=1}^{S_{ij}} \zeta_{ij}^s \psi_{ij}(C_{ij}^s), \text{ s.t. } \zeta_{ij}^s \in \Delta_{ij}, \sum_{s=1}^{S_{ij}} \zeta_{ij}^s \geq \sum_{s=1}^{S_{ij}} a_{ij}^*(C_{ij}^s) \right]
$$

$$
\geq \hat{E} \left[ \min_{s=1}^{S_{ij}} \zeta_{ij}^s \psi_{ij}(C_{ij}^s) + \epsilon_{ij} \left( \sum_{s=1}^{S_{ij}} a_{ij}^*(C_{ij}^s) - \sum_{s=1}^{S_{ij}} \zeta_{ij}^s \right), \text{ s.t. } \zeta_{ij}^s \in \Delta_{ij} \right]
$$

where $\Delta_{ij} := \{ \zeta_{ij} \in \mathbb{R}_+: \zeta_{ij} \leq 1 \}$. The last inequality follows from relaxing the constraints $\sum_{s=1}^{S_{ij}} \zeta_{ij}^s \geq \sum_{s=1}^{S_{ij}} a_{ij}^*(C_{ij}^s)$ with Lagrangian multipliers $\epsilon_{ij} \geq 0$ (the value of $\epsilon_{ij}$ will be specified later). It then follows that

$$
\hat{E} \left[ \min_{s=1}^{S_{ij}} \zeta_{ij}^s \psi_{ij}(C_{ij}^s) + \epsilon_{ij} \left( \sum_{s=1}^{S_{ij}} a_{ij}^*(C_{ij}^s) - \sum_{s=1}^{S_{ij}} \zeta_{ij}^s \right), \text{ s.t. } \zeta_{ij}^s \in \Delta_{ij} \right]
$$

$$
= \epsilon_{ij} \hat{E}[Y_{ij}] + S_{ij} \hat{E}[\min \psi_{ij}(C_{ij}) - \epsilon_{ij}], \text{ s.t. } \zeta_{ij} \in \Delta_{ij}
$$

$$
= \epsilon_{ij} \hat{E}[Y_{ij}] + S_{ij} \hat{E}[\psi_{ij}(C_{ij}) - \epsilon_{ij}], \text{ s.t. } \zeta_{ij} \in \Delta_{ij}
$$

$$
= \epsilon_{ij} \hat{E}[Y_{ij}] + S_{ij} \hat{E}[I_{ij}(C_{ij}, \epsilon_{ij})(\psi_{ij}(C_{ij}) - \epsilon_{ij})]
$$

$$
= \epsilon_{ij} \hat{E}[Y_{ij}] - S_{ij} \hat{E}[I_{ij}(C_{ij}, \epsilon_{ij})] + S_{ij} \hat{E}[I_{ij}(C_{ij}, \epsilon_{ij})(\psi_{ij}(C_{ij}) - \epsilon_{ij})].
$$

A few remarks are in order. Eq (EC.4) follows from the assumption of symmetric carriers where $C_{ij}^s$ are independent and identically distributed, and index $s$ is dropped from this equation onwards. Eq (EC.5) follows from the relationship between the allocation probabilities $a_{ij}^*$ and the number
of transported loads $Y_{ij}$ for each O-D pair, where $\hat{E}\left[ \sum_{s=1}^{S_{ij}} a_{ij}^s(C_{ij}^s) \right] = \hat{E}[Y_{ij}]$. Eq (EC.6) holds by the definition of the 0-1 vector $I_{ij}(C_{ij},t_{ij})$, where the value is equal to one if $\psi_{ij}(C_{ij}) - t_{ij}$ is non-positive, and equal to zero otherwise. Note that the function $\hat{E}[I_{ij}(C_{ij},t_{ij})]$ is continuous (since $C_{ij}$ follows a continuous distribution with CDF $F_{ij}$) and increasing in $t_{ij}$. Therefore, for any $Y_{ij}$ such that $\hat{E}[Y_{ij}] \leq S_{ij}$, we can pick some $t_{ij}$ such that $\hat{E}[Y_{ij}] = S_{ij} \hat{E}[I_{ij}(C_{ij},t_{ij})] = S_{ij} F_{ij}(t_{ij})$. Then it follows from Eq (EC.7) that

$$\hat{E}[P_{ij}] \geq S_{ij} \hat{E}[I_{ij}(C_{ij},t_{ij})] \psi_{ij}(C_{ij}). \quad (EC.8)$$

Consider a (dynamic) posted price mechanism with price $\tau_{ij}$. The choice probabilities of carriers under this price vector are $\hat{E}[Y_{ij}]/S_{ij}$. Note that this posted price mechanism has the same ex post allocation rule as a mechanism with allocation function $I_{ij}(C_{ij},t_{ij})$. By the revenue equivalence principle (Proposition 2), any mechanism that results in the same allocation probability must have the same expected payment. Therefore, we have

$$S_{ij} \hat{E}[I_{ij}(C_{ij},t_{ij})] \psi_{ij}(C_{ij}) = S_{ij} \tau_{ij} \hat{E}[Y_{ij}]/S_{ij} = \tau_{ij} \hat{E}[Y_{ij}],$$

where the left-hand side is a lower bound of the expected payment of the mechanism being considered (see Eq (EC.8)) and the right-hand side is the expected payment of the posted price mechanism. Combining the above equation with Eq (EC.8), it then follows that

$$E[P_{ij}] = E[\hat{E}[P_{ij}]] \geq E\left[ S_{ij} \hat{E}[I_{ij}(C_{ij},t_{ij})] \psi_{ij}(C_{ij}) \right] = E[t_{ij} \hat{E}[Y_{ij}]] = E\left[ F_{ij}^{-1} \left( \frac{\hat{E}[Y_{ij}]}{S_{ij}} \right) \hat{E}[Y_{ij}] \right] \geq F_{ij}^{-1} \left( \frac{E[Y_{ij}]}{E[S_{ij}]} \right) E[Y_{ij}],$$

where the last inequality follows from the Jensen’s inequality. □

Proof of Theorem 1. By Proposition 1 and Lemma EC.1, any platform policy under which the system is stable should satisfy the constraints of the FA. The stability of the system implies that the long-run average profit is the same as the expected profit under the stationary distribution. In the remainder of the proof, we will show that the optimal solution to FA gives an upper bound on the expected profit of any policy $\pi \in \Pi$ under the stationary distribution. By the revelation principle, we can restrict our focus to IC and IR mechanisms. Consider the expected payment made by the platform $E[P_{ij}]$ under an IC and IR mechanism $\pi(S,D) = (\mathcal{M}_r, \mathcal{M}_p)$. By Lemma EC.2, we have
\[ E[P_{ij}] \geq F^{-1}\left( \frac{E[Y_{ij}]}{E[S_{ij}]} \right) E[Y_{ij}]. \]

Then, it follows that the expected profit of the platform mechanism \( \pi \) is upper bounded by

\[
\sum_{(i,j) \in E} E[r_{ij}(d_{ij})d_{ij} - b_{ij}(D_{ij} - Y_{ij}) - P_{ij}]
\leq \sum_{(i,j) \in E} r_{ij}(d_{ij})d_{ij} - b_{ij}(d_{ij} - E[Y_{ij}]) - F^{-1}\left( \frac{E[Y_{ij}]}{E[S_{ij}]} \right) E[Y_{ij}]
\leq \sum_{(i,j) \in E} r^*_i(d^*_i)d^*_i - b_{ij}(d^*_i - \bar{y}^*_i) - p_{ij}\bar{y}^*_i.
\]

Therefore, the optimal value of the FA provides an upper bound on the long-run average profit of any platform mechanism, which completes the proof. □

**Proofs in Section 4**

In order to prove Theorem 2, we first present some auxiliary results which will prove useful for the analysis of the system performance under SP.

**Lemma EC.3.** Given a problem instance with scaling factor \( \theta \), we have

\[
E[X_{ij}(\theta)] - E[Y_{ij}(\theta)] \leq O(\sqrt{\theta}), \text{ and } \theta \bar{\lambda}^*_ij - E[S_{ij}(\theta)] \leq O(\sqrt{\theta}), \quad \forall (i,j) \in \mathcal{E},
\]

where \( E[X_{ij}(\theta)] \), \( E[Y_{ij}(\theta)] \), and \( E[S_{ij}(\theta)] \) are respectively the expected number of carriers who choose to ship a load in lane \((i,j)\), the expected number of carriers who actually delivered a load in lane \((i,j)\), and the expected number of available carriers in lane \((i,j)\), in a state of the stationary distribution under the SP mechanism.

**Proof of Lemma EC.3.** We will prove the two bounds separately. We first focus on

\[
E[X_{ij}(\theta)] - E[Y_{ij}(\theta)] \leq O(\sqrt{\theta}), \quad \forall (i,j) \in \mathcal{E}.
\]

Consider a problem instance with scaling parameter \( \theta \). Let \( S_{ij}(\theta) \) and \( D_{ij}(\theta) \) denote the number of carriers and the number of loads in the marketplace in a state of the stationary distribution under the SP mechanism. Let \( X_{ij}(\theta) = (X_{ij}(\theta) : (i,j) \in \mathcal{E}) \) and \( Y_{ij}(\theta) = (Y_{ij}(\theta) : (i,j) \in \mathcal{E}) \), where \( X_{ij}(\theta) \) and \( Y_{ij}(\theta) \) respectively denote the number of carriers who choose to ship a load and who actually shipped a load from node \( i \) to node \( j \) in a state under the stationary distribution. Consider an optimal solution \((\theta d^*, \theta y^*, \bar{\lambda}^*)\) to the FA(\( \theta \)) and the corresponding variables \((d^*, y^*, \bar{\lambda})\) and \((r^*, p^*)\) associated with this solution. We first show

\[ E[S_{ij}(\theta)] \leq \theta \bar{\lambda}^*_ij, \quad \forall (i,j) \in \mathcal{E}. \quad \text{(EC.9)} \]
Given the posted carrier-side prices $p^*$, we have $x_{ij}^* = E[X_{ij}^{SP}(\theta)]/E[S_{ij}^{SP}(\theta)]$. Let $y_{ij} = E[Y_{ij}^{SP}(\theta)]/E[S_{ij}^{SP}(\theta)]$. As $E[Y_{ij}^{SP}(\theta)] \leq E[X_{ij}^{SP}(\theta)]$, we have $y_{ij} \leq x_{ij}^*$. It follows that $E[S_{ij}^{SP}(\theta)] \leq \theta \lambda_{ij}^*$ for each $(i, j) \in \mathcal{E}$. Moreover, we have

$$
\bar{\lambda}_{ij}^* x_{ij}^* \leq d_{ij}^*, \quad \forall (i, j) \in \mathcal{E},
$$

(EC.10)

since $\bar{\lambda}_{ij}^* x_{ij}^* = \bar{x}_{ij}^* \leq d_{ij}^*$.

Now consider the expected number of carriers in excess of shipper demands $E[X_{ij}^{SP}(\theta)] - E[Y_{ij}^{SP}(\theta)]$:

$$
E[X_{ij}^{SP}(\theta)] - E[Y_{ij}^{SP}(\theta)] = E[(X_{ij}^{SP}(\theta) - D_{ij}^{SP}(\theta))^+] = E[(X_{ij}^{SP}(\theta) - \theta d_{ij}^* + \theta d_{ij}^* - D_{ij}^{SP}(\theta))^+]
$$

$$
\leq E[(X_{ij}^{SP}(\theta) - E[X_{ij}^{SP}(\theta)] + \theta d_{ij}^* - D_{ij}^{SP}(\theta))^+]
$$

$$
\leq E[|X_{ij}^{SP}(\theta) - E[X_{ij}^{SP}(\theta)]| + |\theta d_{ij}^* - D_{ij}^{SP}(\theta)|]
$$

$$
\leq \sqrt{E[(X_{ij}^{SP}(\theta) - E[X_{ij}^{SP}(\theta)])^2] + E[(\theta d_{ij}^* - D_{ij}^{SP}(\theta))^2]}
$$

$$
= \sqrt{Var[X_{ij}^{SP}(\theta)] + Var[D_{ij}^{SP}(\theta)]}
$$

$$
= \sqrt{E[S_{ij}^{SP}(\theta)] x_{ij}^*(1 - x_{ij}^*) + \theta d_{ij}^*}
$$

$$
\leq \sqrt{\theta \bar{\lambda}_{ij}^* x_{ij}^*(1 - x_{ij}^*) + \theta d_{ij}^*}.
$$

A few remarks are in order. The first inequality follows from $E[X_{ij}^{SP}(\theta)] = E[S_{ij}^{SP}(\theta)] x_{ij}^*$, and Eq (EC.9) and Eq (EC.10):

$$
E[X_{ij}^{SP}(\theta)] = E[S_{ij}^{SP}(\theta)] x_{ij}^* \leq \theta \bar{\lambda}_{ij}^* x_{ij}^* \leq \theta d_{ij}^*.
$$

The third inequality follows from the triangle inequality, and the fourth inequality holds by the Cauchy-Schwartz inequality. The last equality holds by the variance of random variable that follow the Binomial distribution and the Poisson distribution. Therefore, $E[X_{ij}^{SP}(\theta)] - E[Y_{ij}^{SP}(\theta)] \leq O(\sqrt{\theta})$.

Next, we will show

$$
\theta \bar{\lambda}_{ij}^* - E[S_{ij}^{SP}(\theta)] \leq O(\sqrt{\theta}), \quad \forall (i, j) \in \mathcal{E}.
$$

By Lemma [EC.1] the SP mechanism satisfies

$$
\sum_{k \in \delta^{-}(i)} q_{ij} E[Y_{ki}^{SP}(\theta)] + \theta \lambda_{ij} = E[Y_{ij}^{SP}(\theta)] + E[V_{ij}^{SP}(\theta)] = E[S_{ij}^{SP}(\theta)], \quad \forall (i, j) \in \mathcal{E}.
$$

Let $y_{ij}^{SP}(\theta) = E[Y_{ij}^{SP}(\theta)]/E[S_{ij}^{SP}(\theta)]$ and $\tilde{\lambda}_{ij}^{SP}(\theta) = E[S_{ij}^{SP}(\theta)]/\theta$. Then, we have

$$
\sum_{k \in \delta^{-}(i)} q_{ij} y_{ki}^{SP}(\theta) \tilde{\lambda}_{ij}^{SP}(\theta) + \lambda_{ij} = \tilde{\lambda}_{ij}^{SP}(\theta), \quad \forall (i, j) \in \mathcal{E}.
$$
Consider a deterministic system with a $\theta$-scaled problem instance. Let $S_{ijt}(\theta)$ denote the number of carriers in this deterministic system in lane $(i,j)$ at period $t$. Given any initial state $S_{ij0}(\theta)$, the system dynamics of this deterministic system are given by

$$S_{ijt+1}(\theta) = \sum_{k \in \delta^{-}(i)} q_{ij} x_{ki}^* S_{k1t}(\theta) + \theta \lambda_{ij}, \quad \forall (i,j) \in \mathcal{E}. \quad \text{(EC.11)}$$

Recall that the optimal solution of the FA satisfies the following equations:

$$\tilde{\lambda}_{ij}^* = \sum_{k \in \delta^{-}(i)} q_{ij} x_{ki}^* \lambda_{ki}^* + \lambda_{ij}, \quad \forall (i,j) \in \mathcal{E}.$$

Comparing the above two equations, we observe that with the initial state given as $S_{ij0}(\theta) = \theta \tilde{\lambda}_{ij}^*$, $\lim_{t \to \infty} S_{ijt}(\theta) = \theta \tilde{\lambda}_{ij}^*$ because the initial state does not affect the steady state of the system.

Consider the deterministic system with initial state $S_{ij0}(\theta) = \theta \lambda_{ij}^{SP}(\theta)$. Let $\Delta_{ijt}(\theta) := S_{ijt}(\theta) - \theta \lambda_{ij}^{SP}(\theta)$ and $t_{ij}(\theta) := \sum_{k \in \delta^{-}(i)} q_{ij} (x_{ki}^* - y_{ki}^{SP}(\theta)) \lambda_{ki}^{SP}(\theta)$. Then, by Eq (EC.11), $S_{ij1}(\theta)$ is given by

$$S_{ij1}(\theta) = \sum_{k \in \delta^{-}(i)} q_{ij} x_{ki}^* \lambda_{ki}^{SP}(\theta) + \theta \lambda_{ij}$$

$$= \sum_{k \in \delta^{-}(i)} q_{ij} (x_{ki}^* + y_{ki}^{SP}(\theta) - y_{ki}^{SP}(\theta)) \lambda_{ki}^{SP}(\theta) + \theta \lambda_{ij}$$

$$= \sum_{k \in \delta^{-}(i)} q_{ij} y_{ki}^{SP}(\theta) \lambda_{ki}^{SP}(\theta) + \theta \lambda_{ij} + \sum_{k \in \delta^{-}(i)} q_{ij} (x_{ki}^* - y_{ki}^{SP}(\theta)) \lambda_{ki}^{SP}(\theta)$$

$$= \theta \lambda_{ij}^{SP}(\theta) + t_{ij}(\theta),$$

where the last equation holds by $\sum_{k \in \delta^{-}(i)} q_{ij} y_{ki}^{SP}(\theta) \lambda_{ki}^{SP}(\theta) + \lambda_{ij} = \lambda_{ij}^{SP}(\theta)$. By the system dynamics Eq (EC.11), $S_{ij2}(\theta)$ can be expressed as

$$S_{ij2}(\theta) = \sum_{k \in \delta^{-}(i)} q_{ij} (x_{ki}^* + y_{ki}^{SP}(\theta) - y_{ki}^{SP}(\theta)) S_{k11}(\theta) + \theta \lambda_{ij}$$

$$= \sum_{k \in \delta^{-}(i)} q_{ij} y_{ki}^{SP}(\theta) S_{k11}(\theta) + \theta \lambda_{ij} + \sum_{k \in \delta^{-}(i)} q_{ij} (x_{ki}^* - y_{ki}^{SP}(\theta)) S_{k11}(\theta)$$

$$= \sum_{k \in \delta^{-}(i)} q_{ij} y_{ki}^{SP}(\theta) (r_{ki}^{SP}(\theta) + \Delta_{k11}(\theta)) + \theta \lambda_{ij} + \sum_{k \in \delta^{-}(i)} q_{ij} (x_{ki}^* - y_{ki}^{SP}(\theta)) r_{ki}^{SP}(\theta) + \Delta_{k11}(\theta))$$

$$= \theta \lambda_{ij}^{SP}(\theta) + \sum_{k \in \delta^{-}(i)} q_{ij} y_{ki}^{SP}(\theta) r_{ki}^{SP}(\theta) + \Delta_{k11}(\theta) + \sum_{k \in \delta^{-}(i)} q_{ij} (x_{ki}^* - y_{ki}^{SP}(\theta)) r_{ki}^{SP}(\theta) + \Delta_{k11}(\theta))$$

$$= \theta \lambda_{ij}^{SP}(\theta) + \sum_{k \in \delta^{-}(i)} q_{ij} (x_{ki}^* - y_{ki}^{SP}(\theta)) r_{ki}^{SP}(\theta) + \Delta_{k11}(\theta) + \sum_{k \in \delta^{-}(i)} q_{ij} (x_{ki}^* - y_{ki}^{SP}(\theta)) \lambda_{ki}^{SP}(\theta)$$

$$= \theta \lambda_{ij}^{SP}(\theta) + \sum_{k \in \delta^{-}(i)} q_{ij} (x_{ki}^* - y_{ki}^{SP}(\theta)) \Delta_{k11}(\theta) + t_{ij}(\theta),$$

where the fourth equation holds by $\sum_{k \in \delta^{-}(i)} q_{ij} y_{ki}^{SP}(\theta) \lambda_{ki}^{SP}(\theta) + \lambda_{ij} = \lambda_{ij}^{SP}(\theta)$. From the last equation above, we have

$$S_{ij2}(\theta) - \theta \lambda_{ij}^{SP}(\theta) = \Delta_{ij2}(\theta) = \sum_{k \in \delta^{-}(i)} q_{ij} x_{ki}^* \Delta_{k11}(\theta) + t_{ij}(\theta).$$
It is easy to see that the above relationship between $\Delta_{ij2}(\theta)$ and $\Delta_{k11}(\theta)$ can be generalized to

$$\Delta_{ijt}(\theta) = \sum_{k \in \Delta^*(i)} q_{ij} x^*_{ki} \Delta_{kt-1}(\theta) + \ell_{ij}(\theta), \quad \forall t > 1.$$  

Recall that we have shown $E[S^\text{SP}_{ij}(\theta)] \leq \theta \bar{\lambda}^*_{ij}$ in the proof of Lemma \ref{lem:EC.3}. Then, we have $\lim_{t \to \infty} \Delta_{ijt}(\theta) = \theta \bar{\lambda}^*_{ij} - \theta \bar{\lambda}^\text{SP}_{ij}(\theta) = \theta \bar{\lambda}^*_{ij} - E[S^\text{SP}_{ij}(\theta)] < \infty$. In addition, $\ell_{ij}(\theta)$ is bounded by $O(\sqrt{\theta})$ since

$$\theta \bar{\lambda}^\text{SP}_{ij}(\theta)(x^*_{ij} - y^\text{SP}_{ij}(\theta)) = E[X^\text{SP}_{ij}(\theta)] - E[Y^\text{SP}_{ij}(\theta)] \leq O(\sqrt{\theta}).$$

Intuitively, $\Delta_{ijt}(\theta)$ can be considered as a linear function of $\ell(\theta)$ with non-negative coefficients, and these coefficients converge to certain real numbers because $\lim_{t \to \infty} \Delta_{ijt}(\theta)$ is bounded. We note that these coefficients depend on $x^*$ and $q$, but independent of the SP mechanism and $\theta$. It then follows that

$$\lim_{t \to \infty} \Delta_{ijt}(\theta) = \theta \bar{\lambda}^*_{ij} - \theta \bar{\lambda}^\text{SP}_{ij}(\theta) = \theta \bar{\lambda}^*_{ij} - E[S^\text{SP}_{ij}(\theta)] \leq O(\sqrt{\theta}),$$

which completes the proof. \hfill \square

In view of Lemma \ref{lem:EC.3}, the expected number of drivers in excess of the shipper demands in steady state under SP, as well as the difference between the expected number of carriers in steady state under SP and that in the fluid system, are bounded above by $O(\sqrt{\theta})$. With Lemma \ref{lem:EC.3}, we are now ready to complete the proof of Theorem \ref{thm:2}

**Proof of Theorem \ref{thm:2}** Given a problem instance with scaling parameter $\theta$, the long-run average profit of the platform under the SP mechanism, $\gamma^\text{SP}(\theta)$, is given by

$$\gamma^\text{SP}(\theta) = E \left[ \sum_{(i,j) \in E} \theta r^*_{ij} d_{ij}(r^*_{ij}) - b_{ij}(D^\text{SP}_{ij}(\theta) - Y^\text{SP}_{ij}(\theta)) - p^*_{ij} Y^\text{SP}_{ij}(\theta) \right]$$

$$= E \left[ \sum_{(i,j) \in E} \theta r^*_{ij} d_{ij}(r^*_{ij}) - b_{ij}(D^\text{SP}_{ij}(\theta) - X^\text{SP}_{ij}(\theta)) - b_{ij}(X^\text{SP}_{ij}(\theta) - D^\text{SP}_{ij}(\theta))^+ - p^*_{ij} Y^\text{SP}_{ij}(\theta) \right]$$

$$\geq E \left[ \sum_{(i,j) \in E} \theta r^*_{ij} d_{ij}(r^*_{ij}) - b_{ij}(D^\text{SP}_{ij}(\theta) - X^\text{SP}_{ij}(\theta)) - b_{ij}(X^\text{SP}_{ij}(\theta) - D^\text{SP}_{ij}(\theta))^+ - p^*_{ij} X^\text{SP}_{ij}(\theta) \right]$$

$$= E \left[ \sum_{(i,j) \in E} \theta r^*_{ij} d_{ij}(r^*_{ij}) - b_{ij} D^\text{SP}_{ij}(\theta) + (b_{ij} - p^*_{ij}) X^\text{SP}_{ij}(\theta) \right] - E \left[ \sum_{(i,j) \in E} b_{ij} (X^\text{SP}_{ij}(\theta) - D^\text{SP}_{ij}(\theta))^+ \right],$$

where the first inequality follows from the definition of $Y^\text{SP}_{ij}(\theta)$. The second term in the last equation is bounded by $O(\sqrt{\theta})$ as shown in Lemma \ref{lem:EC.3}. Consider the first term in the last equation:

$$E \left[ \sum_{(i,j) \in E} \theta r^*_{ij} d_{ij}(r^*_{ij}) - b_{ij} D^\text{SP}_{ij}(\theta) + (b_{ij} - p^*_{ij}) X^\text{SP}_{ij}(\theta) \right]$$
By Lemma [EC.3], $\lambda^s_{ij} - E[S^s_{ij}(\theta)]$ is bounded from above by $O(\sqrt{\theta})$. Therefore, $\gamma^F(\theta) - \gamma^S(\theta)$ is bounded from above by $O(\sqrt{\theta})$. It then follows that

$$\gamma^F(\theta) - \gamma^S(\theta) \leq O(\sqrt{\theta}).$$

This completes the proof.  □

**Proofs in Section 5**

Proof of Lemma [4]. Consider lane $(i,j)$ at time period $t$. Since the uniform price auction mechanism is applied to each lane separately, we will drop the indices $i$, $j$, and $t$ to simplify the notations. Let $(C^{-s}, c^s)$ be the opportunity cost vector submitted by the carriers, where we assume that all carriers other than the carrier $s$ submit their bids truthfully. Now consider the payoff of the carrier $s$ under the uniform price auction mechanism:

$$P^s(C^{-s}, c^s) - C^s A^{ss}(C^{-s}, c^s)$$

$$= c^s A^{ss}(C^{-s}, c^s) + J(C^{-s}) - J(C^{-s}, c^s) - C^s A^{sy}(C^{-s}, c^s)$$

$$= J(C^{-s}) - J(C^{-s}, c^s) + (c^s - C^s) A^{sy}(C^{-s}, c^s)$$

$$= J(C^{-s}) - \sum_{s' \in S \setminus \{s\}} C^{s'} A^{s'y}(C^{-s}, c^s) - \xi^s Y^0(C^{-s}, c^s)$$

$$= J(C^{-s}) - \sum_{s' \in S} C^{s'} A^{s'y}(C^{-s}, c^s) - \xi^s Y^0s(C^{-s}, c^s)$$

$$\leq J(C^{-s}) - J(C)$$

$$= P^s(C) - C^s A^{ss}(C).$$

Therefore, it is optimal for the carrier $s$ to bid truthfully, so the uniform price auction is incentive compatible. In addition, we have $J(C^{-s}) \geq J(C)$. This is because a feasible solution to problem $J(C)$ can be constructed from the optimal solution to $J(C^{-s})$ by adding additional relevant variables associated with the carrier $s$ and restricting the value of these variables equal to zero, and therefore the optimal objective value of $J(C)$ is no more than that of $J(C^{-s})$. As a result, the uniform price auction is also individual rational.  □
Proof of Theorem 3 (Part 1: SP and AUC-P). Consider the allocation problem [12] of AUC-P with reserve price $p^*$. As each carrier only has one destination, it is easy to see that the optimal solution can be obtained by allocating carriers in an increasing order of their true opportunity costs. Let $C_{ij}^{[D_{ij}+1]}$ denote the $(D_{ij}+1)^{th}$ lowest opportunity cost among all the carriers originating from node $i$ to node $j$. If $C_{ij}^{[D_{ij}+1]} \leq p^*_{ij}$, i.e., there are enough carriers with opportunity costs below the reserve price to cover the demand $D_{ij}$, then the payment made by the platform to carriers who get allocated a load is $C_{ij}^{[D_{ij}+1]}$. To see this, consider the uniform price payment rule:

$$P_{ijt}^s(C_{ijt}) = C_{ijt}^s + J_{ij}(C_{ijt}^s) - J_{ij}(C_{ijt}).$$

As the platform allocates carriers in an increasing order of their opportunity costs, excluding the carrier $s$ only affects the carrier with the $(D_{ij}+1)^{th}$ lowest opportunity cost. Therefore, we have $J_{ij}(C_{ijt}^s) = J_{ij}(C_{ijt}) - C_{ijt}^s + C_{ij}^{[D_{ij}+1]}$, and the payment to the carriers who are allocated a load at lane $(i,j)$ in period $t$ is $C_{ij}^{[D_{ij}+1]}$. Otherwise, if there are not enough carriers with opportunity costs no more than $p^*_{ij}$ to cover all the demand, then the platform sets a payment $p^*_{ij}$ by the use of dummy carriers. Therefore, the payment $P_{ijt}^s(C_{ijt})$ to carriers under the AUC-P mechanism does not exceed the reserve price $p^*_{ij}$.

Next, we show that the number of loads shipped from node $i$ to node $j$ in steady state $Y_{ij}$ has the same distribution under the auction mechanism AUC-P and the static posted mechanism SP. Recall that the variable $X_{ijt}$ represents the number of carriers who choose to deliver a load from node $i$ to node $j$ at period $t$, and $Y_{ijt} = \min\{X_{ijt}, D_{ijt}\}$. Under AUC-P, $X_{ijt}$ corresponds to the number of carriers whose opportunity costs are no more than the reserve price $p^*_{ij}$, which is the same as the counterpart under SP with a posted price $p^*_{ij}$. Therefore, these two mechanisms share the same $X_{ijt}$ and $Y_{ijt}$ under a given state and realization of carriers’ opportunity costs.

Since the payment made to the carriers under AUC-P is no more than the reserve price $p^*$ (which is the payment offered in SP) and both SP and AUC-P mechanisms share the same stationary distribution of $Y_{ijt}$, AUC-P allocates the same expected number of carriers with less payment compared to SP. In addition, the expected penalty costs under these two mechanisms are the same in view of the same distribution of $Y_{ijt}$ in steady state. It then follows that AUC-P achieves a higher long-run average profit than SP since both mechanisms receive the same shipper-side revenue. □

To prove the second part of Theorem 3 we establish another lemma below. Consider the platform’s one period objective function Eq (7) for a given state:

$$E \left[ \sum_{(i,j) \in E} r_{ij}^* d_{ij}(r_{ij}^* - b_{ij}(D_{ijt} - Y_{ijt}) - P_{ijt} \mid S_t, D_t) \right].$$
For notation convenience, we define $\rho_{ijt}$ and $H_{ijt}$ for each lane $(i,j) \in \mathcal{E}$ as follows:

$$\rho_{ijt} := r_{ij}^* d_{ij}(r_{ij}^*) - b_{ij} D_{ijt}, \quad \text{(EC.12)}$$

$$H_{ijt} := b_{ij} Y_{ijt} - P_{ijt}. \quad \text{(EC.13)}$$

We define a single-period single-lane mechanism design problem under a given system steady state $(S_t, D_t)$ as follows:

$$\max_{Y_{ijt}, P_{ijt}} \sum_{(i,j) \in \mathcal{E}} E \left[ \rho_{ijt} + H_{ijt} | S_t, D_t \right] \quad \text{(EC.14a)}$$

s.t. $Y_{ijt} \leq S_{ijt}$, \quad $Y_{ijt} \leq D_{ijt}$, \quad $Y_{ijt} \geq 0$. \quad \text{(EC.14b, 14c)}$

In the above formulation, constraint (EC.14b) follows from Eq (5), and constraint (EC.14c) follows from the definition of $Y_{ijt}$. We show that the AUC mechanism achieves the optimal solution to this problem.

**Lemma EC.4.** The long-run expected profit of any IC and IR mechanism is given by

$$\gamma = \sum_{(i,j) \in \mathcal{E}} E \left[ E \left[ \rho_{ijt} + H_{ijt} | S_t, D_t \right] \right],$$

where the outer expectation is taken over the steady state distribution of $(S_t, D_t)$ under the given mechanism. If $\psi_{ij}(p_{ij}^*) \leq b_{ij}$, the AUC mechanism is an optimal solution to single-period problem (EC.14) conditional on any given state $(S_t, D_t)$.

**Proof of Lemma EC.4.** By the law of total expectation, the long-run average profit of any given mechanism is equal to

$$\gamma = \sum_{(i,j) \in \mathcal{E}} E \left[ E \left[ \rho_{ijt} + H_{ijt} | S_t, D_t \right] \right],$$

where the outer expectation is taken over the steady state distribution of $(S_t, D_t)$ under the given mechanism. Following the definition of $\rho_{ijt}$ and $H_{ijt}$, the first part of the lemma is proved.

Next we prove the second part of the lemma regarding the AUC mechanism. Notice that given a system state $(S_t, D_t)$, the term $E\left[ \rho_{ijt} | S_t, D_t \right]$ in the objective function (EC.14a) is a constant. Therefore, it suffices to focus on the term $E \left[ H_{ijt} | S_t, D_t \right]$. For notation simplicity, let $\hat{E} \left[ \cdot \right] := E \left[ \cdot | S_t, D_t \right]$. By the definition of $A_{ijt}$ and the assumption of homogeneous carriers, we have

$$\sum_{s \in S_{ijt}} \hat{E} \left[ A_{ijt}(C_{ijt}) \right] = \sum_{s \in S_{ijt}} \hat{E} \left[ a_{ijt}(C_{ijt}^s) \right] = \hat{E}[Y_{ijt}].$$
Because AUC is IC (Lemma 1), by the definition of $P_{ijt}$ and Proposition 2, we have
\[
\hat{E}[P_{ijt}] = \sum_{s \in S_{ijt}} \hat{E}[p_{ijt}(C_{ijst})] = \sum_{s \in S_{ijt}} \hat{E}[a_{ijt}(C_{ijst})\psi_{ij}(C_{ijst})].
\]

Then, $\hat{E}[H_{ijt}]$ can be written as
\[
\hat{E}[H_{ijt}] = \hat{E}[b_{ij}Y_{ijt} - P_{ijt}] = \sum_{s \in S_{ijt}} b_{ij} \hat{E}[a_{ijt}(C_{ijst})] - \sum_{s \in S_{ijt}} \hat{E}[a_{ijt}(C_{ijst})\psi_{ij}(C_{ijst})] = \sum_{s \in S_{ijt}} \hat{E}[(b_{ij} - \psi_{ij}(C_{ijst}))A_{ijt}^s(C_{ijt})].
\]

By Eq (EC.15) and $Y_{ijt} = \sum_{s \in S_{ijt}} A_{ijt}^s(C_{ijt})$, it is easy to see that the single-period problem (EC.14) can be reformulated as follows:
\[
\max \hat{E} \left[ \rho_{ijt} + \sum_{s \in S_{ijt}} (b_{ij} - \psi_{ij}(C_{ijst}))A_{ijt}^s(C_{ijt}) \right]
\text{s.t. } 0 \leq A_{ijt}^s(C_{ijt}) \leq 1, \quad \forall s \in S_{ijt},
\sum_{s \in S_{ijt}} A_{ijt}^s(C_{ijt}) \leq D_{ijt}.
\]

Denote $\mathcal{K}_{ijt}(C_{ijt})$ as the optimal value of the following problem:
\[
\mathcal{K}_{ijt}(C_{ijt}) := \max \sum_{s \in S_{ijt}} (b_{ij} - \psi_{ij}(C_{ijst}))A_{ijt}^s(C_{ijt})
\text{s.t. } 0 \leq A_{ijt}^s(C_{ijt}) \leq 1, \quad \forall s \in S_{ijt},
\sum_{s \in S_{ijt}} A_{ijt}^s(C_{ijt}) \leq D_{ijt}.
\]

Let $A_{ijt}^{*s}(C_{ijt})$ be the optimal solution to the above allocation problem $\mathcal{K}_{ijt}(C_{ijt})$. Recall that $\xi_{ij}^* = \psi_{ij}^{-1}(b_{ij})$ by Eq (14) when $\psi_{ij}(p_{ij}^*) \leq b_{ij}$. By the monotonicity of $\psi_{ij}$, the optimal solution $A_{ijt}^{*s}(C_{ijt})$ is to allocate loads to carriers whose opportunity costs are less than $\xi_{ij}^*$ in an increasing order of their opportunity costs. This is exactly the same allocation by the AUC mechanism. In other words, the allocation rule of AUC gives the optimal solution to problem $\mathcal{K}_{ijt}(C_{ijt})$, which completes the proof. □

Proof of Lemma 2. To prove the first part of the lemma, a coupling method and the fact that $\xi_{ij}^* \geq p_{ij}^*$ are used. Consider a coupling $(\tilde{S}_{ijt}^{AUC-P}, \tilde{S}_{ijt}^{AUC})$ for all $(i,j) \in \mathcal{E}$ and for all $t$ on the same probability space, where $\tilde{S}_{ijt}^{AUC-P} \sim S_{ijt}^{AUC-P}$ and $\tilde{S}_{ijt}^{AUC} \sim S_{ijt}^{AUC}$. We will show that $\tilde{S}_{ijt}^{AUC-P} \leq \tilde{S}_{ijt}^{AUC}$ for all $(i,j) \in \mathcal{E}$ and for all $t$ by induction.
Base Case. When \( t = 1 \), it is clear that \( \tilde{S}_{ijt}^{\text{AUC-P}} = \tilde{S}_{ijt}^{\text{AUC}} = 0 \) for all \((i,j) \in \mathcal{E}\) as we assume zero carrier in the marketplace at the beginning of period 1.

Induction Step. Suppose \( \tilde{S}_{ijt}^{\text{AUC-P}} \leq \tilde{S}_{ijt}^{\text{AUC}} \) for all \((i,j) \in \mathcal{E}\). Set \( D_{ijt}^{\text{AUC-P}} = D_{ijt}^{\text{AUC}} \) and \( C_{ijt}^{\text{AUC-P}} = (C_{ijt}^{\text{AUC-P}} - \tilde{C}_{ijt}^{\text{DIFF}}) \), where \( \tilde{C}_{ijt}^{\text{DIFF}} \) is the opportunity cost vector for those \((\tilde{S}_{ijt}^{\text{AUC}} - \tilde{S}_{ijt}^{\text{AUC-P}}) \) carriers in the marketplace under AUC but not in the system under AUC-P. Recall that \( X_{ijt}^{\text{AUC-P}} \) is the number of carriers whose opportunity costs are less than \( \xi_{ijt}^* \), where \( \xi_{ijt}^* \geq p_{ijt}^* \). Then, we have \( X_{ijt}^{\text{AUC-P}} \leq X_{ijt}^{\text{AUC}} \).

It then follows that

\[
Y_{ijt}^{\text{AUC-P}} = \min\{X_{ijt}^{\text{AUC-P}}, D_{ijt}^{\text{AUC-P}}\} \leq \min\{X_{ijt}^{\text{AUC}}, D_{ijt}^{\text{AUC}}\} = Y_{ijt}^{\text{AUC}}.
\]

We set \( Z_{ijt}^{\text{AUC-P}} = Z_{ijt}^{\text{AUC}} + \tilde{Z}_{ijt}^{\text{DIFF}} \) and \( \Lambda_{ijt}^{\text{AUC-P}} = \Lambda_{ijt}^{\text{AUC}} + \tilde{Z}_{ijt}^{\text{DIFF}} \), where \( \tilde{Z}_{ijt}^{\text{DIFF}} \) follows the multinomial distribution with parameters \((\sum_{k \in \delta^{-}(i)} Y_{kit}^{\text{AUC}} - Y_{kit}^{\text{AUC-P}}, q_i)\). Then, let

\[
\tilde{S}_{ijt+1}^{\text{AUC-P}} = Z_{ijt}^{\text{AUC-P}} + \Lambda_{ijt+1}^{\text{AUC-P}} + \tilde{Z}_{ijt}^{\text{DIFF}} = \tilde{S}_{ijt+1}^{\text{AUC}} + \tilde{Z}_{ijt}^{\text{DIFF}}, \forall (i,j) \in \mathcal{E}.
\]

As \( \tilde{Z}_{ijt}^{\text{DIFF}} \geq 0 \), we have \( \tilde{S}_{ijt+1}^{\text{AUC-P}} \geq \tilde{S}_{ijt+1}^{\text{AUC}} \), \( \forall (i,j) \in \mathcal{E} \). This completes the proof for the induction step. Therefore, we conclude that \( \tilde{S}_{ijt}^{\text{AUC-P}} \leq \tilde{S}_{ijt}^{\text{AUC}} \) for all \((i,j) \in \mathcal{E} \) and for all \( t \). As \( t \to \infty \), \( \tilde{S}_{ijt}^{\text{AUC-P}} \) and \( \tilde{S}_{ijt}^{\text{AUC}} \) converge to the stationary distribution of AUC-P and AUC, respectively, which implies that the stationary distribution of \( S_{ijt}^{\text{AUC-P}} \) is stochastically smaller than the stationary distribution \( S_{ijt}^{\text{AUC}} \) by the monotone convergence theorem.

\[\square\]

Proof of Theorem \[3\] (Part 2: AUC-P and AUC). By Lemma \[EC.4\], we have

\[
\gamma_{ijt}^{\text{AUC-P}} = \sum_{(i,j) \in \mathcal{E}} \mathbb{E}[\rho_{ijt}^{\text{AUC-P}} + H_{ijt}^{\text{AUC-P}}] \quad \text{and} \quad \gamma_{ijt}^{\text{AUC}} = \sum_{(i,j) \in \mathcal{E}} \mathbb{E}[\rho_{ijt}^{\text{AUC}} + H_{ijt}^{\text{AUC}}].
\]

We first compare \( H_{ijt}^{\text{AUC-P}} \) and \( H_{ijt}^{\text{AUC}} \). Consider a coupling \((\hat{H}_{ijt}^{\text{AUC-P}}, \hat{H}_{ijt}^{\text{AUC}})\) in the same probability space, where \( \hat{H}_{ijt}^{\text{AUC-P}} \sim H_{ijt}^{\text{AUC-P}} \) and \( \hat{H}_{ijt}^{\text{AUC}} \sim H_{ijt}^{\text{AUC}} \). By Lemma \[2\], there exists a coupling \((\tilde{S}_{ijt}^{\text{AUC-P}}, \tilde{S}_{ijt}^{\text{AUC}})\) that satisfies \( \tilde{S}_{ijt}^{\text{AUC-P}} \leq \tilde{S}_{ijt}^{\text{AUC}} \). In this coupling, we let \( D_{ijt}^{\text{AUC-P}} = D_{ijt}^{\text{AUC}} \) and \( C_{ijt}^{\text{AUC-P}} = (C_{ijt}^{\text{AUC-P}}, \tilde{C}_{ijt}^{\text{DIFF}}) \), where \( \tilde{C}_{ijt}^{\text{DIFF}} \) is the opportunity cost vector for those \((\tilde{S}_{ijt}^{\text{AUC}} - \tilde{S}_{ijt}^{\text{AUC-P}}) \) carriers in the marketplace under mechanism AUC but not in the system under AUC-P. Then, we have \( \hat{\mathbb{E}}[\hat{H}_{ijt}^{\text{AUC-P}}] \leq \hat{\mathbb{E}}[\hat{H}_{ijt}^{\text{AUC}}] \). To see this, notice that by the definition of the single-origin subproblem \[EC.17\], we can construct a feasible solution to \( \mathcal{K}_{ijt}(C_{ijt}^{\text{AUC}}) \) from the optimal solution to \( \mathcal{K}_{ijt}(C_{ijt}^{\text{AUC-P}}) \) by adding additional \((\tilde{S}_{ijt}^{\text{AUC}} - \tilde{S}_{ijt}^{\text{AUC-P}}) \) variables and restricting the value of these added variables equal to zero.

In addition, the objective value under this feasible solution to \( \mathcal{K}_{ijt}(C_{ijt}^{\text{AUC-P}}) \) is the same as the optimal objective value of \( \mathcal{K}_{ijt}(C_{ijt}^{\text{AUC-P}}) \). As a result, the optimal objective value of \( \mathcal{K}_{ijt}(C_{ijt}^{\text{AUC-P}}) \) is always less than or equal to that of \( \mathcal{K}_{ijt}(C_{ijt}^{\text{AUC}}) \) under this coupling. It then follows that

\[
\hat{\mathbb{E}}[\hat{H}_{ijt}^{\text{AUC-P}}] \leq \hat{\mathbb{E}}[\hat{H}_{ijt}^{\text{AUC}}].
\]
Since $\tilde{H}_{ijt}^{AUC-P} \sim H_{ijt}^{AUC-P}$ and $\tilde{H}_{ijt}^{AUC} \sim H_{ijt}^{AUC}$, we have $E[H_{ijt}^{AUC-P}] \leq E[H_{ijt}^{AUC}]$.

Because the expectation of the $\rho_{ijt}$ term is only affected by the shipper price, which is the same for both mechanisms, we have $E[\rho_{ijt}^{AUC-P}] = E[\rho_{ijt}^{AUC}]$. By Lemma EC.4, it then follows that

$$\gamma^{AUC-P} = \sum_{(i,j) \in E} E[\rho_{ijt}^{AUC-P} + H_{ijt}^{AUC-P}] \leq \sum_{(i,j) \in E} E[\rho_{ijt}^{AUC} + H_{ijt}^{AUC}] = \gamma^{AUC},$$

and this completes the proof of Theorem 3. □

**Proof of Theorem 4.** We prove the theorem using the following problem instance. Suppose the penalty costs and optimal shipper rates are $b_{ij} = r_{ij}^* = 1$. Shipper demand $D_{ijt}$ follows a Poisson distribution with mean $\theta/2$ under the optimal shipper rates. Carriers arrivals $\Lambda_{ijt}$ follow i.i.d. Poisson distributions with mean $\theta$. The carrier costs are distributed i.i.d. uniformly on $[0, 1]$. The probability that carriers stay in the next period $q_{ij} = 0$.

It is easily verified that under these assumptions, the optimal solution to the fluid model is $p_{ij}^* = 1/2$ for all $(i, j) \in E$, the optimal value of the fluid model is $\theta/4$, and the carrier virtual cost function is $\psi_{ijt}(c) = 2c, \forall c \in [0, 1]$. Since $p_{ijt}^* = \psi_{ijt}^{-1}(b_{ij}) = 1/2$, The AUC mechanism defined in Eq (14) is a uniform price auction with reserve price $\xi_{ijt}^* = 1/2$ (note: AUC-P has the same reserve price, so it is identical to AUC). The mechanism design problem is decomposable for each period and for each lane, in which case we know the optimal mechanism is simply to apply the Myerson mechanism on each lane separately. In fact, the Myerson mechanism for this instance is precisely the AUC mechanism.

We complete the proof by using two existing results from the revenue management literature. First, the profit gap between AUC and FA is bounded by $\gamma^{FA}(\theta) - \gamma^{AUC}(\theta) = O(\log \theta)$. In fact, the $O(\log \theta)$ gap with respect to FA can be achieved by applying the heuristic in Jasin (2014) that charges a discriminative and adaptive price for each arriving carrier. The gap between AUC and FA can only be smaller because AUC is the optimal mechanism. Second, the gap between the optimal posted price mechanism (denoted by OPP hereafter), where all carriers on lane $(i, j)$ in one period are offered a fixed price, and the fluid model is at least $\gamma^{FA}(\theta) - \gamma^{OPP}(\theta) = \Omega(\sqrt{\theta})$ (Gallego and Van Ryzin 1994). Together, these two results imply that $\gamma^{AUC}(\theta) - \gamma^{OPP}(\theta) = \Omega(\sqrt{\theta}) - O(\log \theta) = \Omega(\sqrt{\theta})$. Because $\gamma^{SP}(\theta) \leq \gamma^{OPP}(\theta)$, we also have $\gamma^{AUC}(\theta) - \gamma^{SP}(\theta) = \Omega(\sqrt{\theta})$. Finally, we get $\gamma^{SP}(\theta) / \gamma^{AUC}(\theta) = 1 - \Omega(1/\sqrt{\theta})$ by noting that $\gamma^{FA}(\theta)$ is linear in $\theta$. □

**Proofs in Section 6**

**Proof of Lemma 3.** Suppose carriers report their opportunity costs $C_{ijt}$ to the platform. For simplicity, indices $i, j, t$ are dropped. We first show the HYB mechanism is IR. If $X_{SP}(C) > D$, then the payments and the load allocations follow the SP mechanism which is IR. Otherwise, the
payments and the load allocations follow the AUC mechanism, which is IR. Therefore, the HYB mechanism is also IR.

We next show the HYB mechanism is IC. Suppose the bid vector submitted by the carriers in a lane at period \( t \) is \(( \mathbf{C}^{-s}, \tilde{c}^s \)) . We will show the following inequality holds by considering three cases:

\[
P^{s,\text{HYB}}(\mathbf{C}^{-s}, \tilde{c}^s) - C^s A^{s,\text{HYB}}(\mathbf{C}^{-s}, \tilde{c}^s) \leq P^{s,\text{HYB}}(\mathbf{C}) - C^s A^{s,\text{HYB}}(\mathbf{C}).
\]

**Case 1.** If \( A^{s,\text{HYB}}(\mathbf{C}^{-s}, \tilde{c}^s) = 0 \), then we have \( P^{s,\text{HYB}}(\mathbf{C}^{-s}, \tilde{c}^s) = 0 \) because both SP and AUC mechanisms do not make any payment to carriers who do not receive a load allocation. Then, we have

\[
P^{s,\text{HYB}}(\mathbf{C}^{-s}, \tilde{c}^s) - C^s A^{s,\text{HYB}}(\mathbf{C}^{-s}, \tilde{c}^s) = 0 \leq P^{s,\text{HYB}}(\mathbf{C}) - C^s A^{s,\text{HYB}}(\mathbf{C}).
\]

The above inequality holds because the HYB mechanism is IR.

**Case 2.** If \( A^{s,\text{HYB}}(\mathbf{C}^{-s}, \tilde{c}^s) = 1 \) and \( X^{\text{SP}}(\mathbf{C}^{-s}, \tilde{c}^s) > D \), then in this case, we have \( P^{s,\text{HYB}}(\mathbf{C}^{-s}, \tilde{c}^s) = p^* \) and \( A^{s,\text{SP}}(\mathbf{C}^{-s}, \tilde{c}^s) = A^{s,\text{HYB}}(\mathbf{C}^{-s}, \tilde{c}^s) = 1 \). If \( C^s < p^* \), then \( A^{s,\text{SP}}(\mathbf{C}) = 1 \) because the outcome of the SP mechanism is not affected by changing opportunity costs less than \( p^* \). In addition, \( C^s < p^* \) together with \( A^{s,\text{SP}}(\mathbf{C}) = 1 \) imply that \( A^{s,\text{HYB}}(\mathbf{C}) = 1 \) because \( X^{\text{SP}}(\mathbf{C}) = X^{\text{SP}}(\mathbf{C}^{-s}, \tilde{c}^s) > D \). It then results in \( P^{s,\text{HYB}}(\mathbf{C}) \geq p^* \). Otherwise if \( C^s \geq p^* \), then we have

\[
P^{s,\text{HYB}}(\mathbf{C}^{-s}, \tilde{c}^s) - C^s A^{s,\text{HYB}}(\mathbf{C}^{-s}, \tilde{c}^s) = p^* - C^s \leq 0.
\]

It then follows that

\[
P^{s,\text{HYB}}(\mathbf{C}^{-s}, \tilde{c}^s) - C^s A^{s,\text{HYB}}(\mathbf{C}^{-s}, \tilde{c}^s) \leq P^{s,\text{HYB}}(\mathbf{C}) - C^s A^{s,\text{HYB}}(\mathbf{C}).
\]

**Case 3.** If \( A^{s,\text{HYB}}(\mathbf{C}^{-s}, \tilde{c}^s) = 1 \) and \( X^{\text{SP}}(\mathbf{C}^{-s}, \tilde{c}^s) \leq D \), then in this case, we have \( P^{s,\text{HYB}}(\mathbf{C}^{-s}, \tilde{c}^s) = P^{s,\text{AUC}}(\mathbf{C}^{-s}, \tilde{c}^s) \) and \( A^{s,\text{AUC}}(\mathbf{C}^{-s}, \tilde{c}^s) = A^{s,\text{HYB}}(\mathbf{C}^{-s}, \tilde{c}^s) = 1 \). If \( C^s < P^{s,\text{AUC}}(\mathbf{C}^{-s}, \tilde{c}^s) \), then \( P^{s,\text{AUC}}(\mathbf{C}^{-s}, \tilde{c}^s) = P^{s,\text{AUC}}(\mathbf{C}) \) because the payment of the AUC mechanism is not affected by changing an opportunity cost less than the payment. Otherwise if \( C^s \geq P^{s,\text{AUC}}(\mathbf{C}^{-s}, \tilde{c}^s) \), then we have

\[
P^{s,\text{HYB}}(\mathbf{C}^{-s}, \tilde{c}^s) - C^s A^{s,\text{HYB}}(\mathbf{C}^{-s}, \tilde{c}^s) \leq 0.
\]

It then follows that

\[
P^{s,\text{HYB}}(\mathbf{C}^{-s}, \tilde{c}^s) - C^s A^{s,\text{HYB}}(\mathbf{C}^{-s}, \tilde{c}^s) \leq P^{s,\text{HYB}}(\mathbf{C}) - C^s A^{s,\text{HYB}}(\mathbf{C}).
\]

By combining the above three cases, we conclude that the HYB mechanism is IC and IR. □
Proof of Theorem 5. We first show $\gamma_{\text{HYB}} \leq \gamma_{\text{AUC}}$. Let $X_{ijt}^{\text{HYB}}$ denote the number of carriers in lane $(i,j)$ with opportunity cost no more than $\xi_i^* s$ in period $t$ under the HYB mechanism. Notice that this is the same as $X_{ijt}^{\text{AUC}}$. As the HYB and AUC mechanisms share the same distribution of demands, HYB has the same stationary distribution of $X_{ijt}$, $Y_{ijt}$, and $S_{ijt}$ with AUC. Then under the stationary distributions of the SP, AUC-P, AUC, and HYB mechanisms, we have

$$E[X_{ijt}^{\text{SP}}] = E[X_{ijt}^{\text{AUC-P}}] \leq E[X_{ijt}^{\text{AUC}}] = E[X_{ijt}^{\text{HYB}}],$$
$$E[Y_{ijt}^{\text{SP}}] = E[Y_{ijt}^{\text{AUC-P}}] \leq E[Y_{ijt}^{\text{AUC}}] = E[Y_{ijt}^{\text{HYB}}],$$
$$E[S_{ijt}^{\text{SP}}] = E[S_{ijt}^{\text{AUC-P}}] \leq E[S_{ijt}^{\text{AUC}}] = E[S_{ijt}^{\text{HYB}}].$$

(EC.18)

It then follows from Eq (EC.18) that $\gamma_{\text{HYB}} \leq \gamma_{\text{AUC}}$, since both mechanisms share the same stationary distribution of $Y_{ijt}$, whereas the HYB mechanism pays more than the AUC mechanism does because $p_{ijt}^{\text{HYB}}(C_{ijt}) \geq p_{ijt}^{\text{AUC}}(C_{ijt})$ by Eq (18). Intuitively, the AUC achieves a better market efficiency by delaying the allocation decisions until the end of each time period.

To prove Eq (20), we first show

$$E \left[ \rho_{ijt}^{\text{SP}} + \mathcal{H}_{ijt} \right] \leq E \left[ \rho_{ijt}^{\text{HYB}} + \mathcal{H}_{ijt}^{\text{HYB}} \right], \quad \forall (i,j) \in \mathcal{E},$$

where the terms $\rho_{ijt}$ and $\mathcal{H}_{ijt}$ under a given mechanism are defined in Eq (EC.12) and Eq (EC.13), respectively. By Lemma EC.4 we have

$$\sum_{(i,j) \in \mathcal{E}} E \left[ \rho_{ijt}^{\text{SP}} + \mathcal{H}_{ijt}^{\text{SP}} \right] = \sum_{(i,j) \in \mathcal{E}} E \left[ r_{ij}^* d_{ij} (r_{ij}^*) - b_{ij} (D_{ijt} - Y_{ijt}^{\text{SP}}) - P_{ijt}^{\text{SP}} \right] = \gamma_{\text{SP}},$$
$$\sum_{(i,j) \in \mathcal{E}} E \left[ \rho_{ijt}^{\text{HYB}} + \mathcal{H}_{ijt}^{\text{HYB}} \right] = \sum_{(i,j) \in \mathcal{E}} E \left[ r_{ij}^* d_{ij} (r_{ij}^*) - b_{ij} (D_{ijt} - Y_{ijt}^{\text{HYB}}) - P_{ijt}^{\text{HYB}} \right] = \gamma_{\text{HYB}},$$

so if we can prove the above inequality, it immediately implies Eq (20).

We first compare $\mathcal{H}_{ijt}^{\text{SP}}$ and $\mathcal{H}_{ijt}^{\text{HYB}}$. Consider a coupling $(\mathcal{H}_{ijt}^{\text{SP}}, \mathcal{H}_{ijt}^{\text{HYB}})$ in the same probability space, where $\mathcal{H}_{ijt}^{\text{SP}} \sim \mathcal{H}_{ijt}^{\text{SP}}$ and $\mathcal{H}_{ijt}^{\text{HYB}} \sim \mathcal{H}_{ijt}^{\text{HYB}}$. In Lemma 2, we have shown that there exists a coupling $(\tilde{S}_{ijt}^{\text{SP}}, \tilde{S}_{ijt}^{\text{HYB}})$ satisfying $\tilde{S}_{ijt}^{\text{SP}} \leq \tilde{S}_{ijt}^{\text{HYB}}$. As $S_{ijt}^{\text{SP}} \sim S_{ijt}^{\text{SP}}$ and $S_{ijt}^{\text{HYB}} \sim S_{ijt}^{\text{HYB}}$, there also exists a coupling $(\tilde{S}_{ijt}^{\text{SP}}, \tilde{S}_{ijt}^{\text{HYB}})$ that satisfies $\tilde{S}_{ijt}^{\text{SP}} \leq \tilde{S}_{ijt}^{\text{HYB}}$. In this coupling, we set $D_{ijt}^{\text{SP}} = D_{ijt}^{\text{HYB}}$ and $C_{ijt}^{\text{DIFF}} = (C_{ijt}^{\text{SP}}, C_{ijt}^{\text{DIFF}})$, where $C_{ijt}^{\text{DIFF}}$ is the opportunity cost vector for those $(\tilde{S}_{ijt}^{\text{SP}} - \tilde{S}_{ijt}^{\text{HYB}})$ carriers in the marketplace under mechanism HYB but not in the marketplace under mechanism SP. If $X_{ijt}^{\text{SP}}(C_{ijt}^{\text{HYB}}) > D_{ijt}^{\text{HYB}}$, then the HYB operates the same as the SP mechanism. In this case, we have $\tilde{\gamma}_{\text{SP}} \\leq \tilde{\gamma}_{\text{HYB}}$ because $\sum_{s \in \tilde{S}_{ijt}^{\text{SP}}} A_{ijt}^{\text{SP}} (C_{ijt}^{\text{SP}}) \leq \sum_{s \in \tilde{S}_{ijt}^{\text{HYB}}} A_{ijt}^{\text{HYB}} (C_{ijt}^{\text{HYB}}) = D_{ijt}^{\text{HYB}} = D_{ijt}^{\text{HYB}}$ and $\rho_{ijt} \leq b_{ij}$. Otherwise, the HYB operates the same as the AUC mechanism. By the definition of problem (EC.17), we can construct a feasible solution to $\mathcal{K}_{ijt}(C_{ijt}^{\text{HYB}})$ from the optimal solution to $\mathcal{K}_{ijt}(C_{ijt}^{\text{SP}})$ by adding additional $(\tilde{S}_{ijt}^{\text{HYB}} - \tilde{S}_{ijt}^{\text{SP}})$ variables and restricting the value of these added variables equal to zero. In addition, the objective value under this feasible solution to $\mathcal{K}_{ijt}(C_{ijt}^{\text{HYB}})$ is the same as the optimal objective
value of $K_{ijt}(C_{ijt})$. As a result, the optimal objective value of $K_{ijt}(C_{ijt})$ is always less than or equal to that of $K_{ijt}(C_{HYB})$ under this coupling. It then follows that $\hat{E}[H_{SP}^{ijt}] \leq \hat{E}[H_{HYB}^{ijt}]$. Since $H_{SP}^{ijt} \sim H_{SP}^{ijt}$ and $H_{HYB}^{ijt} \sim H_{HYB}^{ijt}$, it then follows that $E[H_{SP}^{ijt}] \leq E[H_{HYB}^{ijt}]$ in both cases.

Because the expectation of $\rho_{ijt}$ is not affected by carrier side mechanisms, we have $E[\rho_{SP}^{ijt}] = E[\rho_{HYB}^{ijt}]$. By Lemma EC.4, it then follows that

$$\gamma_{SP} = \sum_{(i,j) \in E} E[\rho_{SP}^{ijt} + H_{SP}^{ijt}] \leq \sum_{(i,j) \in E} E[\rho_{HYB}^{ijt} + H_{HYB}^{ijt}] = \gamma_{HYB},$$

and this completes the proof. \(\square\)

**Data Set and Model Calibration**

In this section, we provide detailed information about our data set and the parameter estimation procedure to calibrate our model. The freight data used in our numerical case studies come from two sources, Census Bureau and DAT Freight & Analytics. The Census Bureau provides U.S. mode data on the website [https://data.census.gov/cedsci/](https://data.census.gov/cedsci/). By using the keyword CFSAREA2017.CF1700A20 in the search box, we obtain a table “Geographic Area Series: Shipment Characteristics by Origin Geography by Destination Geography: 2017”, which includes information on annual shipment volumes and average mile per shipment per each geographical origin-destination pair. Our second data source is DAT Freight & Analytics, which provides national flatbed rates on their website [https://www.dat.com/industry-trends/trendlines/flatbed/national-rates](https://www.dat.com/industry-trends/trendlines/flatbed/national-rates). In our numerical studies, we accessed the website and retrieved the average outbound flatbed rates of five regions in the U.S. on February 28, 2022: West ($3.10), West South ($2.78), Mid West ($3.46), South East ($3.03), and North East ($2.94).

In our numerical studies, we assume that the true opportunity cost of a carrier follows a uniform distribution, which satisfies the regularity assumption of $\psi$. More specifically, a carrier’s opportunity cost for lane $(i,j)$ is uniformly distributed between $\frac{1}{2}p_{ij}$ and $\frac{3}{2}p_{ij}$, where $p_{ij}$ represents the average freight shipping cost in lane $(i,j)$. To calculate this average shipping cost, we utilize the data by DAT Freight & Analytics, which provides the average regional flatbed rates per mile of five regions within the U.S. We term this average regional flatbed rate as the **normal freight rate**. We assume that 90% of daily demand is satisfied by normal freight rates; while the rest of demand that cannot be served by the carriers in the marketplace is fulfilled by penalty shipping rates (e.g., third-party carriers’ shipping costs) which are assumed two times more expensive than the normal freight rates. Then the average freight shipping in lane $(i,j)$ can be computed as follows:

$$p_{ij} = (\text{Origin } i \text{ rate} + \text{Destination } j \text{ rate})/2 \times \text{Average miles}/(2 \times 0.1 + 0.9),$$

where the origin and destination rates are based on the DAT regional flatbed rate data.
Next, we specify the values of the staying probabilities $q$. We assume that the probability that a carrier will remain in the marketplace after transporting a load is 0.2, and the probability that a staying carrier chooses to deliver a load in lane $(i,j)$ is proportional to the demand rate $d_{ij}$ for each $(i,j) \in \mathcal{E}$. For example, consider two O-D pairs $(i,j)$ and $(i,k)$ originating from node $i$ with demand rates $d_{ij} = 15$ and $d_{ik} = 45$. Then, we set $q_{ij} = 0.05$ and $q_{ik} = 0.15$ where $q_{ij} + q_{ik} = 0.2$. Finally, in our case study, we assume that $x_{ij} = 0.5$ for all $(i,j) \in \mathcal{E}$, and the carriers’ external arrival rate to the marketplace $\lambda_{ij}$ for each lane $(i,j)$ can be obtained by solving the flow balance Equation (11a).