T2K Signals Non-Maximal Atmospheric Neutrino Mixing

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From recent groundbreaking experiments, it is now known that the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing differs significantly from the tribimaximal model in which \( \theta_{13} = 0 \) and \( \theta_{23} = \pi/4 \). Flavor symmetry can require that the departures from these two equations are linearly related. \( T \) and \( A_4 \), which successfully accommodated the pre-T2K PMNS matrix, predict that \( 38.07^\circ \leq \theta_{23} \leq 39.52^\circ \) at 95% C.L.. The best fit values, combining the model predictions with T2K, MINOS, Double Chooz, Daya Bay, and RENO data, are \( \theta_{23} = 38.7^\circ \) and \( \theta_{13} = 8.9^\circ \).

Of the parameters in the standard model of particle theory, we will focus on the mixing matrices for down-type quarks and for neutrinos, named respectively for Cabibbo, Kobayashi, and Maskawa (CKM) \cite{CKM}, and for Pontecorvo, Maki, Nakagawa, and Sakata (PMNS) \cite{PMNS}. Without losing generality, we choose a basis in which the flavor and mass eigenstates coincide for the three up-type quarks and all three charged leptons.

This investigation will consider one of three mixing angles, \( \theta_{13} \) and \( \theta_{23} \) listed in the 2010 Review of Particle Physics \cite{RPP} since these two are, we suggest, both changed by the T2K measurement \cite{11,12}. The values then were:

\[
36.8^\circ \leq \theta_{23} \leq 45.0^\circ, \quad 0.0^\circ \leq \theta_{13} \leq 11.4^\circ \quad (1)
\]

consistent with vanishing \( \theta_{13} \) and maximal \( \theta_{23} \).

The other angles are not considered to be variables in this analysis, although the superior experimental accuracy of the CKM Gell-Mann-Lévy quark mixing angle \( \theta_{12} \),

\[
\theta_{12} = (13.03 \pm 0.06)^\circ, \quad (2)
\]

played an important role in our investigation of flavor symmetry.

To accommodate the new data, we invoke broken binary tetrahedral \( (T') \) flavor symmetry as a promising approach to explaining the mixing angles \( \theta_{13}, \theta_{23} \).

This flavor symmetry was first used in Ref. \cite{13} solely as a symmetry for quarks, because neutrinos were still believed to be massless. After neutrino masses and mixings were discovered \cite{13}, the mixing matrix for neutrinos was measured and found to be very different from the CKM mixing matrix for quarks. A number of theories arose \cite{24,28} to explain this. Eventually a useful approximation to the empirical PMNS mixing was determined to be the tribimaximal (TBM) matrix \cite{29}:

\[
U_{TBM} = \left( \begin{array}{ccc}
\sqrt{2/3} & \sqrt{1/3} & 0 \\
-\sqrt{1/6} & \sqrt{1/3} & -1/\sqrt{2} \\
-\sqrt{1/6} & \sqrt{1/3} & 1/\sqrt{2}
\end{array} \right). \quad (3)
\]

Flavor symmetry based on the Tetrahedral Group, \( A_4 = T' \), was introduced by Ref. \cite{30} to underpin TBM neutrino mixing. Further investigation revealed that this model could not be extended to quarks because a viable CKM matrix could not be obtained \cite{31}. \( A_4 \) is not a subgroup of its double cover \cite{20}, \( T' \), nevertheless from the viewpoint of kronecker products used in model building \cite{14}, \( A_4 \) behaves as if it were a subgroup. This explains why the larger group can act as a successful flavor symmetry for both quarks and leptons.

We shall consider only the projection on the two-dimensional \( \theta_{23} - \theta_{13} \) plane of the three-dimensional \( \theta_{12} - \theta_{23} - \theta_{13} \) space. At leading order, requiring \( \sin \alpha \sim \epsilon_0^4 \) for \( \theta_{13} \) and \( (\pi/4 - \theta_{23}) \), the calculation of the perturbation of this projection from the TBM matrix in Eq. \( \ref{3} \) is independent of the solar neutrino mixing angle \( \theta_{12} \). The relevant perturbation away from Eq. \( \ref{3} \) was explicitly calculated in Ref. \cite{18,19}.

Before T2K, the neutrino mixing angles were all empirically consistent with the TBM values. However, as the experimental accuracy has now improved in recent data from T2K \cite{6,11}, MINOS \cite{32,38}, Double Chooz \cite{39,43}, Daya Bay \cite{44,45}, and RENO \cite{46,47}, this situation has changed dramatically, as discussed in the global fits of Refs. \cite{15,50}; of these we shall use Fogli et al. \cite{49}. These five remarkable experiments have provided us with a rich new perspective on mixing angles. From flavor symmetry, it is then possible to predict quantitatively how departures from the TBM values,

\[
\theta_{12} = \tan^{-1} \left( \frac{1}{\sqrt{2}} \right), \quad \theta_{23} = (\pi/4), \quad \theta_{13} = 0, \quad (4)
\]

are related. The model allows one to address this question by relating the perturbations around TBM,

\[
\theta_{ij} = (\theta_{ij})_{TBM} + \epsilon_k, \quad (5)
\]

1 The reader is directed to the references summarized in RPP.

2 This is a \( < 1\% \) approximation for \( \theta_{13} \) and \( (\pi/4 - \theta_{23}) \) since both angles are less than \( \alpha = 12^\circ = 0.2094 \) radians with \( \sin \alpha = 0.2079 \).
FIG. 1. The global analysis of Ref. [49], incorporating SBL, LBL, Solar, and Atmospheric neutrino observations, excludes the red-shaded region at 2σ. The same assessment excludes the orange-shaded region at 1σ. The best fit value for θ_{13} is indicated by the vertical green line at θ_{13} = 8.9°. Extreme values of the linear correlation coefficient, η, are indicated by dashed lines at η = 0.902 and η = 3.29, while our predicted correlation of η = √2 is indicated by the solid dark blue line. The intersection of our correlation prediction and the θ_{13} best fit occurs at θ_{13} = 8.9° and θ_{23} = 38.7°, a close match to the current experimental best fit of θ_{23} = 38.4°. (The color plot is in the online version of the paper.)

(3) A_4 is also capable of producing Eq. (8) with η = √2, though we give preference in this paper to T' for its capacity to explain CKM mixing.

4 It is notable that Eq. (8) with η ≃ √2 appears en passant in Ref. [51]; see also Ref. [52], which implied that η ~ 2. Another, model-independent, correlation was developed in Ref. [53] including the three PMNS mixing angles and the CP-violating phase.
that $(\theta_{13}, \theta_{23})$ are respectively closer to $(8.9^\circ, 38.7^\circ)$ than to $(0.0^\circ, 45.0^\circ)$. Before T2K, $\eta$ was unconstrained, $0 \leq \eta < \infty$. With the current global fit data, we find $0.902 \leq \eta \leq 3.29$.

This is in sharp difference from the previous widespread acceptance of a maximal $\theta_{23} = \pi/4$ which fitted so well with vanishing $\theta_{13} = 0$ in the TBM context.

As the measurement of $\theta_{13}$ sharpens experimentally, so will the prediction for $\theta_{23}$ from Eq. [19], and measurement of the atmospheric neutrino mixing’s departure from maximality will provide an interesting test of the binary tetrahedral flavor symmetry.

Several years ago Super-Kamiokande showed $\theta_{23} > 36.8^\circ$ [54], and current analysis places it at $\theta_{23} \approx 40.7^\circ$ [53]. Once combined in a global fit of $3\nu$ oscillation, Ref. [49] states the best fit of $\theta_{23} = 38.4^\circ$, tantalizingly close to our central value of $\theta_{23} = 38.7^\circ$.

This suggests to us that the $T$ flavor symmetry, introduced in Ref. [13], should now be taken much more seriously. As errors in $\theta_{13}$ and $\theta_{23}$ diminish even further, it will be interesting to see how the prediction of Eq. [9] by $T$ perseveres, as it would inspire further investigation into other mixing angles for quarks and leptons. This, in turn, may show that $T$, first mentioned in physics as an example of an $SU(2)$ subgroup [56], is actually a useful approximate symmetry in the physical application of quark and lepton flavors.

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