Precise frequency measurements of the $D$-lines and fine-structure interval in K

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Abstract

We use a diode laser locked to a Rb transition as the frequency reference along with a scanning Michelson interferometer to make precise measurements on the $D_1$ and $D_2$ lines of potassium. The Rb reference frequency is known with sub-MHz accuracy. We obtain the following values for the energy levels: 12 985.169 60(13)(15) cm$^{-1}$ for the $4P_{1/2}$ state ($D_1$ line); 13 042.875 95(14)(15) cm$^{-1}$ for the $4P_{3/2}$ state ($D_2$ line); and 57.706 35(19)(15) cm$^{-1}$ for the $4P_{3/2} - 4P_{1/2}$ fine-structure interval. The leading source of systematic error cancels in the determination of the interval. The errors represent about an order of magnitude improvement over tabulated values.
Semiconductor diode lasers bring several advantages to the field of laser spectroscopy, the principal ones being tunability and narrow spectral width \[1\]. By placing a single-mode diode in an external cavity and using optical feedback from an angle-tuned grating \[2\], it can be made to operate at single frequency (single longitudinal mode) with linewidth of order 1 MHz and tunability over several nm. In addition, techniques such as saturated-absorption spectroscopy using counter-propagating pump and probe beams can help eliminate the first-order Doppler effect and allow narrow hyperfine transitions within a given atomic line to be resolved. It is thus possible to get an absolute frequency calibration of the laser by locking to such a transition. Recently, we have shown that a diode laser locked to an atomic transition in Rb can be used along with a scanning Michelson interferometer as a precision wavemeter to determine the unknown frequency of a tunable laser \[3\].

In this Letter, we apply this technique to determine the absolute frequencies of the \(D\) lines in potassium. The frequency of the reference laser is known with sub-MHz accuracy, and the unknown laser is tuned to the one of the \(D\)-line transitions in K with \(\sim\)MHz accuracy, therefore the frequencies are also determined with \(\sim\)MHz accuracy. More importantly, we can determine the fine-structure interval in the \(4P\) state with this accuracy. Knowledge of fine-structure intervals is important for several reasons, e.g. in the study of atomic collisions, astrophysical processes, and relativistic calculations of atomic energy levels. Alkali atoms are particularly important because of their widespread use in ultra-cold collision studies, photoassociation spectroscopy, atomic tests of parity violation, and more recently in Bose-Einstein condensation \[4\]. The species we have studied is K in which there has been renewed interest recently because laser cooling can be used to produce a quantum degenerate Fermi gas \[5\]. But the technique is more general and applicable to other alkali atoms such as Li or Cs, alkali-like ions, and indeed any system where precise knowledge of energy levels is important. We achieve an accuracy of 0.00015 cm\(^{-1}\), which is one to two orders of magnitude better than the typical accuracy of the energy-level tables \[6\].

The interferometer used for the ratio measurement has been described extensively in a previous publication \[3\]. For consistency with the terminology there, we will call the two
lasers as “reference” and “unknown”, respectively. The basic idea is to obtain the ratio of the two laser wavelengths using a scanning Michelson interferometer where both lasers traverse essentially the same path. As the interferometer is scanned, the interference pattern on the detectors goes alternately through bright and dark fringes. Since both lasers traverse the same path, the ratio of the number of fringes counted after scanning through a certain distance is the ratio of the two wavelengths. The ratio obtained is a wavelength ratio in air, however, the wavelength ratio in vacuum (or equivalent frequency ratio) is easily calculated by making a small correction for the dispersion of air \[7\] between the reference wavelength and the unknown.

The reference laser is a diode laser system built around a commercial single-mode diode. The output is first collimated using an aspheric lens to give an elliptic beam of 5.8 mm × 1.8 mm \(1/e^2\) diameter. The laser is then frequency stabilized in a standard external-cavity design (Littrow configuration) using optical feedback from an 1800 lines/mm diffraction grating mounted on a piezoelectric transducer \[2\]. Using a combination of temperature and current control, the laser is tuned close to the 780 nm \(D_2\) line in atomic Rb. A part of the output beam is tapped for Doppler-free saturated-absorption spectroscopy in a Rb vapor cell. The injection current into the laser diode is modulated slightly to obtain an error signal and the laser is locked to the \(5S_{1/2}, F = 2 \rightarrow 5P_{3/2}, F' = (2, 3)\) crossover resonance. The frequency of this resonance has been measured previously to be 384 227 981.877(6) MHz \[8\]. The elliptic beam from the laser is directly fed into the Michelson interferometer. The large Rayleigh ranges (\(\sim 34\) m and \(\sim 3\) m in the two directions) ensure that the beam remains collimated over the length of the interferometer and diffraction effects are not significant.

The relevant energy levels of \(^{39}\)K are shown in Fig. 1. The ground state has two hyperfine levels with \(F = 1\) and \(2\). The \(D_1\) line is the \(4S_{1/2} \leftrightarrow 4P_{1/2}\) transition at 770.1 nm. The \(4P_{1/2}\) state again has two hyperfine levels with \(F = 1\) and \(2\). The \(D_2\) line is the \(4S_{1/2} \leftrightarrow 4P_{3/2}\) transition at 767.7 nm. The upper state has 4 excited levels, with \(F = 0, 1, 2,\) and \(3\). The measurements on the \(D\) lines of K are done using a single-frequency tunable Ti-sapphire laser (Coherent 899-21). The laser can be tuned from 700–800 nm and is frequency stabilized
to an ovenized reference cavity that gives it a linewidth of 500 kHz. A part of the output is split off for saturated-absorption spectroscopy in a K vapor cell. The cell is heated to about 70°C to get sufficient vapor. The saturated-absorption spectra on the $D_1$ and $D_2$ lines for transitions starting from the $F = 2$ ground level are also shown in Fig. 1. The two hyperfine transitions in the $D_1$ line are well resolved, as seen in the figure. This is because the hyperfine interval in the $4P_{1/2}$ state is about 60 MHz. On the other hand, all the hyperfine levels in the $4P_{3/2}$ lie within a range of 30 MHz, and the individual transitions in the $D_2$ line are not resolved. However, the peak in the spectrum corresponds to the $F = 2 \rightarrow F' = (2, 3)$ crossover resonance, and this is what we use for the measurements. For transitions starting from the $F = 1$ ground level, the peak corresponds to the $F = 1 \rightarrow F' = (1, 2)$ crossover resonance.

To get several independent measurements of the energy levels, we tune the Ti-sapphire laser to different hyperfine transitions of the $D_1$ and $D_2$ lines in K. In each case, the wavelength ratio is measured about 600 times. The values are statistically analyzed to obtain the mean ratio and the $1\sigma$ (statistical) error in the mean. The mean wavelength ratios obtained from four such measurements (two for each line) are listed in Table I. We extract the hyperfine-free energy level of the state by making two corrections to the measured ratio. First, we convert the wavelength ratio in air to a frequency ratio using the refractive index of air from Edlén’s formula $n = 1.000 275 163$ at 780.0 nm (reference), $n = 1.000 275 231$ at 770.1 nm ($D_1$), and $n = 1.000 275 255$ at 767.7 nm ($D_2$). Then we remove the hyperfine frequency shifts shown in Fig. 1, which are known to sub-MHz accuracy [9]. The extracted values of the energy levels are also listed in Table I.

The main source of statistical error is that the frequency counter counts zero-crossings and does not count fractional fringes. The total number of fringes counted depends on the fringe rate (or cart speed) coupled with the 1 s integration time. We use about 20 cm of cart travel per measurement, which results in a single-shot statistical error of about 5 parts in $10^7$ in each data set [10]. The error of less than 2 parts in $10^8$ in Table I comes after averaging over $\sim 600$ individual measurements. We also plot a histogram of the individual
points to make sure that the distribution around the mean is Gaussian. This guarantees that there is no significant statistical bias in the data.

There are several potential sources of systematic error, the main two being variation in the lock point of the lasers and non-parallelism of the two laser beams in the interferometer. We have checked for the first error by locking the unknown laser to different hyperfine transitions. The transitions are about 500 MHz apart and the saturated-absorption spectrum in each case is quite different. Therefore, we expect the definition of the lock point also to be quite different. However, as seen from Table I, within the errors quoted the different ratios yield consistent values for the hyperfine-free energy level. This implies that there is no significant variation in the lock point of the lasers.

The second source of systematic error, namely that the two beams have a small angle between them, is more serious. Any misalignment would cause a systematic increase in the measured ratio given by $1/\cos \theta$, where $\theta$ is the angle between the beams. We have tried to minimize this error in two ways. The first method is as follows. When the beams from the two arms of the interferometer are combined on the beamsplitter, two output beams are produced. Of these, the one on the opposite side of the beamsplitter from the input beam has near-perfect contrast and is detected for the measurement. The unused output beam (the one on the same side of the beamsplitter as the input beam) propagates towards the other laser, and can therefore be used as a tracer for checking the parallelism of the lasers. Using this method, we ensure that the reference and unknown beams are parallel over a distance of about 3 m [1]. The second method to check for parallelism is to look for a minimum in the measured ratio as the angle of the unknown beam is varied. This works because the measured value is always larger than the correct value, whether $\theta$ is positive or negative, and becomes minimum when $\theta = 0$. We find this method to be more reliable and it quickly guarantees us that the beams are perfectly aligned.

It is difficult to get a quantitative estimate of the systematic error due to this non-parallelism. However, it is important to note that the error cancels in the determination of the fine-structure interval. This is because the measurements on the $D_1$ and $D_2$ lines are
done without changing the alignment in the interferometer. The direction of the output of
the Ti:sapphire laser does not change when its wavelength is changed, therefore any mis-
alignment angle remains the same during all the measurements. If there is a misalignment,
the measured frequencies on both the $D_1$ and the $D_2$ lines will be systematically higher
but by the same amount, so that when we take their difference to obtain the fine-structure
interval, the error will cancel.

As a further check on possible systematic errors in our measured frequencies, we have
repeated the measurement scheme using a different reference laser. This set of measurements
was done using a reference diode laser locked to a transition in the $D_1$ line of Rb at 795 nm.
The diode laser system is identical to the first one, except that after stabilization it is locked
to the $5S_{1/2}, F = 3 \rightarrow 5P_{1/2}, F' = 3$ transition in $^{85}$Rb. The frequency of this transition
is 377 106 271.6(4) MHz \[12\]. Since this frequency is also known with sub-MHz accuracy,
we do not expect any decrease in the precision of the measurements. However, because the
laser is different, it requires completely new alignment of the interferometer, which gives us
a good check on systematic errors arising from any misalignment angle between the beams.
More importantly, it gives us a check on the reliability of converting wavelength ratios in air
to vacuum using Edlen’s formula because the reference wavelength has changed from 780
nm to 795 nm.

The results of the second set of measurements are listed in Table II. The values for the
$D_1$ and $D_2$ lines of K are consistent with the earlier results in Table I, within the quoted
errors. Therefore, we have strong reason to believe that our systematic errors are less than
the statistical errors. Combining the results from Tables I and II, we obtain the following
average values

\[
4P_{1/2} : 12\,985.169\,60(13)(15) \text{ cm}^{-1},
\]
\[
4P_{3/2} : 13\,042.875\,95(14)(15) \text{ cm}^{-1},
\]

and a precise value for the fine-structure interval

\[
4P_{3/2} - 4P_{1/2} : 57.706\,35(19)(15) \text{ cm}^{-1}.
\]
The errors are statistical and systematic, respectively.

The values we obtain can be compared to the values listed in the K energy-level tables released by the National Institute of Standards and Technology [6]: 12 985.170 cm\(^{-1}\) for the 4\(P_{1/2}\) state; 13 042.876 cm\(^{-1}\) for the 4\(P_{3/2}\) state, and 57.706 cm\(^{-1}\) for the fine-structure interval. Our results are consistent with these values but have significantly higher accuracy, thus demonstrating the power of our technique to improve the accuracy of the existing energy-level tables.

In conclusion, we have demonstrated that a diode laser stabilized on a Rb transition can be used as an absolute frequency reference to make precise measurements of the energy levels in atoms. The reference laser and a laser tuned to the atomic transition of interest are fed into a scanning Michelson interferometer to obtain their frequency ratio very precisely. We have used this technique to make frequency measurements on the \(D_1\) and \(D_2\) lines of K, and obtain improvement of about an order of magnitude over existing values. The leading source of systematic error is non-parallelism between the two laser beams in the interferometer. While we have several checks to minimize this error, it cancels to first order in the determination of the fine-structure interval. The technique should prove useful in other atomic systems and particularly alkali atoms, where transitions are easily accessible with tunable diode lasers and knowledge of fine-structure intervals is important. Since atomic energy levels are generally known to about 0.01 cm\(^{-1}\) accuracy, our precision wavemeter [3] has the potential to improve these values by up to two orders of magnitude.

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[10] The calculated error from missing one fringe over a distance of 20 cm is about 1 part in 10^6 at 780 nm. Our error is smaller because we are measuring a wavelength ratio close to 1 and chances are high that either both counters see a zero crossing or miss it.

[11] Ideally, the unknown beam should enter exactly where the reference beam exits so that both beams traverse exactly the same path, albeit in opposite directions. However, this causes one laser to feed into the other and destabilize it. Therefore we align the beams with a small offset and check for parallelism over a finite distance.

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FIGURES

FIG. 1. Energy levels of $^{39}\text{K}$. The figure shows the relevant energy levels of $^{39}\text{K}$ in the ground $4S$ state and first excited $4P$ state. The various hyperfine levels are labeled with the value of the total angular momentum $F$, and the number on each level is the energy displacement (in MHz) from the unperturbed state. The two insets on the left-hand side are saturated absorption spectra on the $D_1$ and $D_2$ lines for transitions starting from the $F = 2$ ground level.
TABLES

TABLE I. The table lists the measured wavelength ratios and the energy levels in K. The reference laser was locked to the $5S_{1/2}, F = 2 \rightarrow 5P_{3/2}, F' = (2,3)$ crossover resonance in the $D_2$ line of $^{87}\text{Rb}$, corresponding to a frequency of 384 227 981.877(6) MHz. The unknown laser was tuned to various hyperfine transitions of the $D_1$ and $D_2$ lines in $^{39}\text{K}$, as listed. The hyperfine-free line wavenumber in vacuum was extracted by first correcting for the dispersion of air and then removing the hyperfine shifts shown in Fig. 1. The errors are statistical 1$\sigma$ deviations.

| Measured transition | Wavelength ratio | Energy (cm$^{-1}$) |
|---------------------|-----------------|--------------------|
| $D_1$: $F = 1 \rightarrow F' = (1, 2)$ | 1.013 163 848(18) | 12 985.169 64(23) |
| $D_1$: $F = 2 \rightarrow F' = (1, 2)$ | 1.013 162 640(19) | 12 985.169 56(24) |
| $D_2$: $F = 1 \rightarrow F' = (1, 2)$ | 1.017 666 373(19) | 13 042.875 93(24) |
| $D_2$: $F = 2 \rightarrow F' = (2, 3)$ | 1.017 665 211(16) | 13 042.875 93(20) |

TABLE II. Similar to Table I but now the reference laser was locked to the $5S_{1/2}, F = 3 \rightarrow 5P_{1/2}, F' = 3$ hyperfine transition in the $D_1$ line of $^{85}\text{Rb}$, corresponding to a frequency of 377 106 271.6(4) MHz. The unknown laser was tuned to hyperfine transitions of the $D_1$ and $D_2$ lines in $^{39}\text{K}$, as before.

| Measured transition | Wavelength ratio | Energy (cm$^{-1}$) |
|---------------------|-----------------|--------------------|
| $D_1$: $F = 2 \rightarrow F' = (1, 2)$ | 1.032 296 511(16) | 12 985.169 59(20) |
| $D_2$: $F = 2 \rightarrow F' = (2, 3)$ | 1.036 884 083(24) | 13 042.876 03(30) |
\( F = 2 \rightarrow F' \) 

**D\textsubscript{2} line**

**D\textsubscript{1} line**

766.7 nm (D\textsubscript{2} line) 
770.1 nm (D\textsubscript{1} line) 

\( 4 \ ^2\text{P}_{1/2} \)

\( 4 \ ^2\text{S}_{1/2} \)

Fine-structure interval

\[ +173.1 \]
\[ -288.6 \]
\[ +21.6 \]
\[ -36.1 \]
\[ +14.3 \]
\[ -6.7 \]
\[ -16.0 \]
\[ -19.2 \]

Frequency (MHz)