Unification and fermion mass structure.

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Abstract

Grand Unified Theories predict relationships between the GUT-scale quark and lepton masses. Using new data in the context of the MSSM, we update the values and uncertainties of the masses and mixing angles for the three generations at the GUT scale. We also update fits to hierarchical patterns in the GUT-scale Yukawa matrices. The new data shows not all the classic GUT-scale mass relationships remain in quantitative agreement at small to moderate tan β. However, at large tan β, these discrepancies can be eliminated by finite, tan β-enhanced, radiative, threshold corrections if the gluino mass has the opposite sign to the wino mass.

Explaining the origin of fermion masses and mixings remains one of the most important goals in our attempts to go beyond the Standard Model. In this, one very promising possibility is that there is an underlying stage of unification relating the couplings responsible for the fermion masses. However we are hindered by the fact that the measured masses and mixings do not directly give the structure of the underlying Lagrangian both because the data is insufficient unambiguously to reconstruct the full fermion mass matrices and because radiative corrections can obscure the underlying structure. In this letter we will address both these points in the context of the MSSM.

We first present an analysis of the measured mass and mixing angles continued to the GUT scale. The analysis updates previous work, using the precise measurements of fermion masses and mixing angles from the b-factories and the updated top-quark mass from CDF and D0. The resulting data at the GUT scale allows us to look for underlying patterns which may suggest a unified origin. We also explore the sensitivity of these patterns to tan β-enhanced, radiative threshold corrections.

We next proceed to extract the underlying Yukawa coupling matrices for the quarks and leptons. There are two difficulties in this. The first is that the data cannot, without some assumptions, determine all elements of these matrices. The second is that the Yukawa coupling matrices are basis dependent. We choose to work in a basis in which the mass matrices are hierarchical in structure with the off-diagonal elements small relative to the appropriate combinations of on-diagonal matrix elements. This is the basis we think is most likely to display the structure of the underlying theory, for example that of a spontaneously broken family symmetry in which the hierarchical structure is ordered by the (small) order parameter breaking the symmetry. With this structure to leading order the observed masses and mixing angles determine the mass matrix elements on and above the diagonal, and our analysis determines these entries, again allowing for significant tan β enhanced radiative corrections. The resulting form of the mass matrices provides the “data” for developing models of fermion masses such as those based on a broken family symmetry.

The data set used is summarized in Table I. Since the fit of reference [4] (RRRV) to the Yukawa texture was done, the measurement of the Standard-Model parameters has improved considerably. We highlight a few of the changes in the data since 2000: The top-quark mass has gone from $M_t = 174.3 \pm 5$ GeV to $M_t = 170.9 \pm 1.9$ GeV. In 2000 the Particle Data Book reported $m_b(m_b) = 4.2 \pm 0.2$ GeV which has improved to $m_b(m_b) = 4.2 \pm 0.07$ GeV today. In addition each higher order QCD correction pushes down the value of $m_b(M_Z)$ at the scale of the Z bosons mass. In 1998 $m_b(M_Z) = 3.0 \pm 0.2$ GeV and today it is $m_b(M_Z) = 2.87 \pm 0.06$ GeV. The most significant shift in the data relevant to the RRRV fit is a downward revision to the strange-quark mass at the scale $\mu_L = 2$ GeV from $m_s(\mu_L) \approx 120 \pm 50$ MeV to
today's value $m_s(\mu_L) = 103 \pm 20$ MeV. We also know the CKM unitarity triangle parameters better today than six years ago. For example, in 2000 the Particle Data book reported $\sin 2\beta = 0.79 \pm 0.4$ [5] which is improved to $\sin 2\beta = 0.69 \pm 0.032$ in 2006 [1]. The sin $2\beta$ value is about 1.2 $\sigma$ off from a global fit to all the CKM data [8], our fits generally lock onto the global-fit data and exhibit a 1 $\sigma$ tension for sin $2\beta$. Together, the improved CKM matrix observations add stronger constraints to the textures compared to data from several years ago.

We first consider the determination of the fundamental mass parameters at the GUT scale in order simply to compare to GUT predictions. The starting point for the light-quark masses at low scale is given by the $\chi^2$ fit to the data of Table [1]

$$m_u(\mu_L) = 2.7 \pm 0.5 \text{ MeV} \quad m_d(\mu_L) = 5.3 \pm 0.5 \text{ MeV} \quad m_s(\mu_L) = 103 \pm 12 \text{ MeV}.$$  \hspace{1cm} (1)

Using these as input we determine the values of the mass parameters at the GUT scale for various choices of $\tan \beta$ but not including possible $\tan \beta$ enhanced threshold corrections. We do this using numerical solutions to the RG equations. The one-loop and two-loop RG equations for the gauge couplings and the Yukawa couplings in the Standard Model and in the MSSM that we use in this study come from a number of sources [6] [9] [10] [11]. The results are given in the first five columns of Table [3]. These can readily be compared to expectations in various Grand Unified models. The classic prediction of $SU(5)$ with third generation down-

| Low-Energy Parameter                  | Value(Uncertainty in last digit(s)) | Notes and Reference |
|---------------------------------------|-------------------------------------|---------------------|
| $m_u(\mu_L)/m_d(\mu_L)$              | 0.45(15)                           | PDB Estimation [1]   |
| $m_u(\mu_L)/m_t(\mu_L)$              | 19.5(1.5)                          | PDB Estimation [1]   |
| $m_u(\mu_L) + m_d(\mu_L)$            | $[8.8(3.0), 7.6(1.6)]$ MeV         | PDB, Quark Masses, pg 15 [11]. (Non-lattice, Lattice) |
| $Q = \sqrt{\frac{m^2_u - (m_c + m_s)^2}{m_s(\mu_L)^2}}$ | 22.8(4)                           | Martemyanov and Sopov [2] |
| $m_u(\mu_L)$                         | 3(1) MeV                           | PDB, Quark Masses, pg 15 [1]. Non-lattice, lattice |
| $m_d(\mu_L)$                         | 6.0(1.5) MeV                       | PDB, Quark Masses, pg 15 [1]. Non-lattice, lattice |
| $m_s(\mu_L)$                         | 6.0(1.5) MeV                       | PDB, Quark Masses, pg 16 [1]. Non-lattice, lattice |
| $m_t(\mu_L)$                         | 1.24(09) GeV                       | PDB, Quark Masses, pg 15 [1]. Non-lattice, lattice |
| $m_b(\mu_b)$                         | 4.20(07) GeV                       | CDF & D0 [3] Pole Mass |
| $M_t$                                 | 170.9 (1.9) GeV                     | 3% uncertainty from neglecting $Y''$ thresholds. |
| $(M_t, M_{\mu}, M_{\tau})$           | $[0.511(15), 105.6(3.1), 1777(53)]$ MeV | PDB Ch 11 Eq. 11.25 [1] |
| $A$ Wolfenstein parameter             | 0.818(17)                          | PDB Ch 11 Eq. 11.25 [1] |
| $\overline{\tau}$ Wolfenstein parameter | 0.221(64)                        | PDB Ch 11 Eq. 11.25 [1] |
| $\lambda$ Wolfenstein parameter      | 0.227(10)                          | PDB Ch 11 Eq. 11.25 [1] |
| $\overline{\tau}$ Wolfenstein parameter | 0.340(45)                        | PDB Ch 11 Eq. 11.25 [1] |
| $|V_{CKM}|$                           | $\begin{pmatrix} 0.97383(24) & 0.2272(10) & 0.00396(99) \\ 0.2271(10) & 0.97296(24) & 0.04222(80) \\ 0.00814(64) & 0.04161(78) & 0.999100(34) \end{pmatrix}$ | PDB Ch 11 Eq. 11.26 [1] |
| $\sin 2\beta$ from CKM               | 0.687(32)                          | PDB Ch 11 Eq. 11.19 [1] |
| Jarlskog Invariant                   | $3.08(18) \times 10^{-5}$         | PDB Ch 11 Eq. 11.26 [1] |
| $v_{Higgs}(M_Z)$                     | 246.221(20) GeV                    | Uncertainty expanded. [1] |
| ($\alpha^{-1}_{EM}(M_Z)$, $\alpha_{s}(M_Z)$, $\sin^2\theta_W(M_Z)$) | (127.904(19), 0.1216(17), 0.23122(15)) | PDB Sec 10.6 [1] |

Table 1: Low-energy observables. Masses in lower-case $m$ are $M_S$ running masses. Capital $M$ indicates pole mass. The light quark’s $(u,d,s)$ mass are specified at a scale $\mu_L = 2$ GeV. $V_{CKM}$ are the Standard Model’s best fit values.
quark and charged-lepton masses given by the coupling $B \bar{5}_f.10_f.5_H$ is $m_u(M_X)/m_\tau(M_X) = 1$ \textsuperscript{12}. This ratio is given in Table 2 where it may be seen that the value agrees at a special low tan $\beta$ value but for large tan $\beta$ it is some 25% smaller than the GUT prediction\textsuperscript{2}. A similar relation between the strange quark and the muon is untenable and to describe the masses consistently in $SU(5)$ Georgi and Jarlskog [13] proposed that the second generation masses should come instead from the coupling $C \bar{5}_f.10_f.45_H$ leading instead to the relation $3m_u(M_X)/m_\tau(M_X) = 1$. As may be seen from Table 2 in all cases this ratio is approximately 0.69(8). The prediction of Georgi and Jarlskog for the lightest generation masses follows from the relation $Det(M^d)/Det(M^l) = 1$. This results from the form of their mass matrix which is given by\textsuperscript{3}

$$M^d = \begin{pmatrix} 0 & A' \\ A & C & B \end{pmatrix}, \quad M^l = \begin{pmatrix} 0 & A' \\ A & -3C & B \end{pmatrix} \quad (2)$$

in which there is a (1, 1) texture zero\textsuperscript{4} and the determinant is given by the product of the (3, 3), (1, 2) and (2, 1) elements. If the (1, 2) and (2, 1) elements are also given by $\bar{5}_f.10_f.5_H$ couplings they will be the same in the down-quark and charged-lepton mass matrices giving rise to the equality of the determinants. The form of eq (2) may be arranged by imposing additional continuous or discrete symmetries. One may see from Table 2 that the actual value of the ratio of the determinants is quite far from unity disagreeing with the Georgi Jarlskog relation.

In summary the latest data on fermion masses, while qualitatively in agreement with the simple GUT relations, has significant quantitative discrepancies. However the analysis has not, so far, included the SUSY threshold corrections which substantially affect the GUT mass relations at large tan $\beta$. A catalog of the full SUSY threshold corrections is given in [16]. The particular finite SUSY thresholds discussed in this letter do not decouple as the superpartners become massive. We follow the approximation described in Blazek, Raby, and Pokorski (BRP) for threshold corrections to the CKM elements and down-like mass eigenstates [17]. The finite threshold corrections to $Y^e$ and $Y^u$ and are generally about 3% or smaller

$$\delta Y^u, \delta Y^d \lesssim 0.03 \quad (3)$$

and will be neglected in our study. The logarithmic threshold corrections are approximated by using the Standard-Model RG equations from $M_Z$ to an effective SUSY scale $M_S$.

The finite, tan $\beta$-enhanced $Y^d$ SUSY threshold corrections are dominated by the a sbottom-glino loop, a stop-higgsino loop, and a stop-chargino loop. Integrating out the SUSY particles at a scale $M_S$ leaves the matching condition at that scale for the Standard-Model Yukawa couplings:

$$\delta m_{sch}^u Y^{u SM} = \sin \beta \quad Y^u \quad (4)$$

$$\delta m_{sch}^d Y^{d SM} = \cos \beta \quad U^d \hat{Y}^{d \dagger}_L \Gamma^d \hat{V}^{\dagger}_{CKM} \Gamma^u \hat{V}_{CKM} \quad U^d \quad (5)$$

$$Y^e \quad \hat{Y}^{e \dagger}_L \quad (6)$$

All the parameters on the right-hand side take on their MSSM values in the $\overline{DR}$ scheme. The factor $\delta m_{sch}$ converts the quark running masses from $\overline{MS}$ to $\overline{DR}$ scheme. The $\beta$ corresponds to the ratio of the two Higgs VEVs $v_u/v_d = \tan \beta$. The $U$ matrices decompose the MSSM Yukawa couplings at the scale $M_S$: $Y^u = U^d \hat{Y}^u \Gamma^u$ and $Y^d = U^d \hat{Y}^d \Gamma^d$. The matrices $Y^u$ and $Y^d$ are diagonal and correspond to the mass eigenstates divided by the appropriate VEV at the scale $M_S$. The CKM matrix is given by $V_{CKM} = U^d \hat{Y}^d_U$. The left-hand side involves the Standard-Model Yukawa couplings. The matrices $\Gamma^u$ and $\Gamma^d$ encode the SUSY threshold corrections.

If the squarks are diagonalized in flavor space by the same rotations that diagonalize the quarks, the matrices $\Gamma^u$ and $\Gamma^d$ are diagonal: $\Gamma^d = \text{diag}(\gamma_d, \gamma_d, \gamma_d)$, $\Gamma^u = \text{diag}(\gamma_u, \gamma_u, \gamma_u)$. In general the squarks are

\textsuperscript{1}The $\bar{5}_f, 10_f$ refer to the $SU(5)$ representations making up a family of quarks and leptons while $5_H$ is a five dimensional representation of Higgs scalars.

\textsuperscript{2}We’d like to thank Ilja Dorsner for pointing out that the tan $\beta$ dependence of $m_b/m_\tau(M_X)$ is more flat than in previous studies (e.g. ref. [13]). This change is mostly due to the higher effective SUSY scale $M_S$, the higher value of $\alpha_s(M_Z)$ found in global standard model fits, and smaller top-quark mass $M_t$.

\textsuperscript{3}The remaining mass matrix elements may be non-zero provided they do not contribute significantly to the determinant

\textsuperscript{4}Below we discuss an independent reason for having a (1, 1) texture zero.
not diagonalized by the same rotations as the quarks but provided the relative mixing angles are reasonably small the corrections to flavour conserving masses, which are our primary concern here, will be second order in these mixing angles. We will assume $\Gamma_a$ and $\Gamma^d$ are diagonal in what follows.

Approximations for $\Gamma^u$ and $\Gamma^d$ based on the mass insertion approximation are found in [13,19,20]:

$$\Gamma_t \approx y^t_2 \mu A_t \frac{\tan \beta}{16\pi^2} I_3(m^2_{\tilde{u}_1}, m^2_{\tilde{u}_2}, \mu^2) \sim y^t_2 \frac{\tan \beta \mu A_t}{32\pi^2} \frac{m^2_{\tilde{t}}}{m^2_{\tilde{t}}},$$

$$\Gamma_u \approx -g^2_2 M_2 \mu \frac{\tan \beta}{16\pi^2} I_3(m^2_{\tilde{u}_1}, m^2_{\tilde{u}_2}, m^2_{\tilde{b}}) \sim 0,$$

$$\Gamma_b \approx \frac{8}{3} g_3^2 \frac{\tan \beta}{16\pi^2} M_3 \mu I_3(m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}, M^2) \sim \frac{4}{3} g_3^2 \frac{\tan \beta \mu M_3}{16\pi^2} \frac{m^2_{\tilde{b}}}{m^2_{\tilde{b}}},$$

$$\Gamma_d \approx \frac{8}{3} g_3^2 \frac{\tan \beta}{16\pi^2} M_3 \mu I_3(m^2_{\tilde{d}_1}, m^2_{\tilde{d}_2}, M^2) \sim \frac{4}{3} g_3^2 \frac{\tan \beta \mu M_3}{16\pi^2} \frac{m^2_{\tilde{d}}}{m^2_{\tilde{d}}},$$

where $I_3$ is given by

$$I_3(a^2, b^2, c^2) = \frac{a^2 b^2 \log \frac{b^2}{a^2} + b^2 c^2 \log \frac{c^2}{a^2} + c^2 a^2 \log \frac{a^2}{c^2}}{(a^2 - b^2)(b^2 - c^2)(a^2 - c^2)}.$$  

In these expressions $\tilde{q}$ refers to superpartner of $q$, $\chi^j$ indicate chargino mass eigenstates. $\mu$ is the coefficient to the $H^u H^d$ interaction in the superpotential. $M_1, M_2, M_3$ are the gaugino soft breaking terms. $A_t$ refers to the soft top-quark trilinear coupling. The mass insertion approximation breaks down if there is large mixing between the mass eigenstates of the stop or the sbottom. The right-most expressions in equations (7,9,10) assume the relevant squark mass eigenstates are nearly degenerate and heavier than $M_3$ and $\mu$. These expressions (11)}
eqs 7 - 10 provide an approximate mapping from a supersymmetric spectra to the \( \gamma_t \) parameters through which we parameterize the threshold corrections; however, with the exception of Column A of Table 4 we do not specify a SUSY spectra but directly parameterize the thresholds corrections through \( \gamma_t \).

The separation between \( \gamma_b \) and \( \gamma_d \) is set by the lack of degeneracy of the down-like squarks. If the squark masses for the first two generations are not degenerate, then there will be a corresponding separation between the (1,1) and (2,2) entries of \( \Gamma^d \) and \( \Gamma^u \). If the particle spectra is designed to have a large \( A_t \) and a light stop, \( \gamma_t \) can be enhanced and dominate over \( \gamma_0 \). Because the charm Yukawa coupling is so small, the scharm-higgsino loop is negligible, and \( \gamma_u \) follows from a chargino squark loop and is also generally small with values around 0.02 because of the smaller \( g_2 \) coupling. In our work, we approximate \( \Gamma^{d}_{22} \sim \Gamma^{u}_{11} \sim 0 \). The only substantial correction to the first and second generations is given by \( \gamma_d \).

As described in BRP, the threshold corrections leave \( |V_{us}| \) and \( |V_{ub}/V_{cb}| \) unchanged to a good approximation. Threshold corrections in \( \Gamma^u \) do affect the \( V_{ub} \) and \( V_{cb} \) at the scale \( M_S \) giving

\[
\frac{V_{ub}^{SM} - V_{ub}^{MSSM}}{V_{ub}^{MSSM}} \leq \frac{V_{cb}^{SM} - V_{cb}^{MSSM}}{V_{cb}^{MSSM}} \leq - (\gamma_t - \gamma_u). (12)
\]

The threshold corrections for the down-quark masses are given approximately by

\[
\begin{align*}
m_d &\approx m_d^0 (1 + \gamma_d + \gamma_u)^{-1} \\
m_s &\approx m_s^0 (1 + \gamma_d + \gamma_u)^{-1} \\
m_b &\approx m_b^0 (1 + \gamma_b + \gamma_u)^{-1}
\end{align*}
\]

where the superscript 0 denotes the mass without threshold corrections. Not shown are the nonlinear effects which arise through the RG equations when the bottom Yukawa coupling is changed by threshold effects. These are properly included in our final results obtained by numerically solving the RG equations.

Due to our assumption that the squark masses for the first two generations are degenerate, the combination of the GUT relations given by \((\det M^d/\det M^u) (3 m_s/m_\mu)^2 (m_b/m_\tau) = 1\) is unaffected up to nonlinear effects. Thus we cannot simultaneously fit all three GUT relations through the threshold corrections. A best fit requires the threshold effects given by

\[
\begin{align*}
\gamma_b + \gamma_t &\approx -0.22 \pm 0.02 \\
\gamma_d + \gamma_u &\approx -0.21 \pm 0.02.
\end{align*} (13) (14)
\]

giving the results shown in the penultimate column of Table 2 just consistent with the GUT predictions. The question is whether these threshold effects are of a reasonable magnitude and, if so, what are the implications for the SUSY spectra which determine the \( \gamma_t \)? From eqs 9-10, at \( \tan \beta = 38 \) we have

\[
\frac{\mu \cdot M_3}{m_b^0} \sim -0.5, \quad \frac{m_b^2}{m_d^2} \sim 1.0
\]

The current observation of the muon’s \((g - 2)_\mu\) is 3.4 \(\sigma\) away from the Standard-Model prediction. If SUSY is to explain the observed deviation, one needs \( \tan \beta > 8 \) [22] and \( \mu M_3 > 0 \) [23]. With this sign we must have \( \mu M_3 \) negative and the \( \tilde{d}, \tilde{s} \) squarks only lightly split from the \( \tilde{b} \) squarks. \( M_3 \) negative is characteristic of anomaly mediated SUSY breaking[24] and is discussed in [25][26][20][27]. Although we have deduced \( M_3 < 0 \) from the approximate eqs 9-10, the correlation persists in the near exact expression found in eq(23) of ref [17]. Adjusting to different squark splitting can occur in various schemes[28]. However the squark splitting can readily be adjusted without spoiling the fit because, up to nonlinear effects, the solution only requires the constraints implied by eq (13), so we may make \( \gamma_b > \gamma_d \) and hence make \( m_b^2 < m_d^2 \) by allowing for a small positive value for \( \gamma_t \). In this case \( A_\tau \) must be positive.

It is of interest also to consider the threshold effects in the case that \( \mu M_3 \) is positive. This is illustrated in the last column of Table 2 in which we have reversed the sign of \( \gamma_d \), consistent with positive \( \mu M_3 \), and chosen \( \gamma_b \approx \gamma_d \) as is expected for similar down squark masses. The value of \( \gamma_t \) is chosen to keep the equality between \( m_b \) and \( m_\tau \). One may see that the other GUT relations are not satisfied, being driven further away by the threshold corrections. Reducing the magnitude of \( \gamma_b \) and \( \gamma_d \) reduces the discrepancy somewhat but still limited by the deviation found in the no-threshold case (the fourth column of Table 2).
Table 3: Results of a $\chi^2$ fit of eqs(15,16) to the data in Table 2 in the absence of threshold corrections. We set $a'$ as indicated and set $c' = d' = d = 0$ and $f = f' = 1$ at fixed values.

At $\tan \beta$ near 50 the non-linear effects are large and $b - \tau$ unification requires $\gamma_b + \gamma_\tau \sim -0.1$ to $-0.15$. In this case it is possible to have $t - b - \tau$ unification of the Yukawa couplings. For $\mu > 0, M_3 > 0$, the “Just-so” Split-Higgs solution of references [29, 30, 31, 32] can achieve this while satisfying both $b \rightarrow s \gamma$ and $(g - 2)_\mu$ constraints but only with large $\gamma_b$ and $\gamma_\tau$ and a large cancellation in $\gamma_\beta + \gamma_\tau$. In this case, as in the example given above, the threshold corrections drive the masses further from the mass relations for the first and second generations because $\mu M_3 > 0$. It is possible to have $t - b - \tau$ unification with $\mu M_3 < 0$, satisfying the $b \rightarrow s \gamma$ and $(g - 2)_\mu$ constraints in which the GUT predictions for the first and second generation of quarks is acceptable. Examples include Non-Universal Gaugino Mediation [33] and AMSB; both have some very heavy sparticle masses ($\gtrsim 4$ TeV) [20]. Minimal AMSB with a light sparticle spectra ($\lesssim 1$ TeV), while satisfying $(g - 2)_\mu$ and $b \rightarrow s \gamma$ constraints, requires $\tan \beta$ less than about 30 [23].

We turn now to the second part of our study in which we update previous fits to the Yukawa matrices responsible for quark and lepton masses. As discussed above we choose to work in a basis in which the mass matrices are hierarchical with the off-diagonal elements small relative to the appropriate combinations of on-diagonal matrix elements. This is the basis we think is most likely to display the structure of the underlying theory, for example that of a spontaneously broken family symmetry, in which the hierarchical structure is ordered by the (small) order parameter breaking the symmetry. With this structure to leading order in the ratio of light to heavy quarks the observed masses and mixing angles determine the mass matrix elements on and above the diagonal provided the elements below the diagonal are not anomalously large. This is the case for matrices that are nearly symmetrical or for nearly Hermitian as is the case in models based on an $SO(10)$ GUT.

For convenience we fit to symmetric Yukawa coupling matrices but, as stressed above, this is not a critical assumption as the data is insensitive to the off-diagonal elements below the diagonal and the quality of the fit is not changed if, for example, we use Hermitian forms. We parameterize a set of general, symmetric Yukawa matrices as:

$$Y^u(M_X) = y^u_{33} \left( \begin{array}{ccc} d \epsilon_u^4 & b' \epsilon_u^3 & c' \epsilon_u^2 \\ b' \epsilon_u^3 & f' \epsilon_u^2 & a' \epsilon_u \\ c' \epsilon_u^2 & a' \epsilon_u & 1 \end{array} \right).$$

(15)

$$Y^d(M_X) = y^d_{33} \left( \begin{array}{ccc} d \epsilon_d^4 & b \epsilon_d^3 & c \epsilon_d^2 \\ b \epsilon_d^3 & f \epsilon_d^2 & a \epsilon_d \\ c \epsilon_d^2 & a \epsilon_d & 1 \end{array} \right).$$

(16)

Although not shown, we always choose lepton Yukawa couplings at $M_X$ consistent with the low-energy lepton masses. Notice that the $f$ coefficient and $\epsilon_d$ are redundant (likewise in $Y^u$). We include $f$ to be able to discuss the phase of the $(2,2)$ term. We write all the entries in terms of $\epsilon$ so that our coefficients will be $\mathcal{O}(1)$. We will always select our best $\epsilon$ parameters such that $|f| = 1$.

RRRV noted that all solutions, to leading order in the small expansion parameters, only depend on two
phases $\phi_1$ and $\phi_2$ given by

$$
\phi_1 = (\phi_b - \phi_f)' - (\phi_b - \phi_f) \tag{17}
$$

$$
\phi_2 = (\phi_c - \phi_a) - (\phi_b - \phi_f). \tag{18}
$$

where $\phi_x$ is the phase of parameter $x$. For this reason it is sufficient to consider only $b'$ and $c$ as complex with all other parameters real.

As mentioned above the data favours a texture zero in the (1,1) position. With a symmetric form for the mass matrix for the first two families, this leads to the phenomenologically successful Gatto Sartori Tonin (34) relation

$$V_{us}(M_X) \approx |bd - |b'|e^{i\phi_1}e_a| \approx \left| \sqrt{\frac{m_d}{m_u}}0 - \sqrt{\frac{m_d}{m_c}}0e^{i\phi_1} \right|. \tag{19}
$$

This relation gives an excellent fit to $V_{us}$ with $\phi_1 \approx \pm 90^\circ$, and to preserve it we take $d, d'$ to be zero in our fits. As discussed above, in SU(5) this texture zero leads to the GUT relation Det($M^d$)/Det($M^f$) = 1 which, with threshold corrections, is in good agreement with experiment. In the case that $c$ is small it was shown in RRRV that $\phi_1$ is to a good approximation the CP violating phase $\delta$ in the Wolfenstein parameterization. A non-zero $c$ is necessary to avoid the relation $V_{ub}/V_{cb} = \sqrt{m_u/m_c}$ and with the improvement in the data, it is now necessary to have $c$ larger than was found in RRRV. As a result the contribution to CP violation coming from $\phi_2$ is at least 30%. The sign ambiguity in $\phi_1$ gives rise to an ambiguity in $c$ with the positive sign corresponding to the larger value of $c$ seen in Tables 3 and 4.

Table 4: A $\chi^2$ fit of eqs (13 16) including the SUSY threshold effects parameterized by the specified $\gamma_i$.  

| Parameter | A       | B       | C       | B2      | C2       |
|-----------|---------|---------|---------|---------|---------|
| $\tan \beta$ | 30      | 38      | 38      | 38      | 38      |
| $\gamma_b$  | 0.20    | -0.22   | +0.22   | -0.22   | +0.22   |
| $\gamma_t$  | -0.03   | 0       | -0.44   | 0       | -0.44   |
| $\gamma_d$  | 0.20    | -0.21   | +0.21   | -0.21   | +0.21   |
| $a'$        | 0       | 0       | 0       | -2      | -2      |
| $\epsilon_u$| 0.0495(17) | 0.0483(16) | 0.0483(18) | 0.0485(17) | 0.0485(18) |
| $\epsilon_d$| 0.131(7)  | 0.128(7)  | 0.102(9)  | 0.127(7)  | 0.101(9)  |
| $|b'|$       | 1.04(12) | 1.07(12) | 1.07(11) | 1.05(12) | 1.06(10) |
| $\text{arg}(b')$ | 90(12)$^o$ | 91(12)$^o$ | 93(12)$^o$ | 95(12)$^o$ | 95(12)$^o$ |
| $a$         | 2.17(24) | 2.27(26) | 2.30(42) | 2.03(24) | 1.89(35) |
| $b$         | 1.69(13) | 1.73(13) | 2.21(18) | 1.74(10) | 2.26(20) |
| $|c|$        | 0.80(16) | 0.86(17) | 1.09(33) | 0.81(17) | 1.10(35) |
| $\text{arg}(c)$ | -41(18)$^o$ | -42(19)$^o$ | -41(14)$^o$ | -53(10)$^o$ | -41(12)$^o$ |
| $Y^u_{33}$  | 0.48(2)  | 0.51(2)  | 0.51(2)  | 0.51(2)  | 0.51(2)  |
| $Y^d_{33}$  | 0.15(1)  | 0.34(3)  | 0.34(3)  | 0.34(3)  | 0.34(3)  |
| $Y^c_{33}$  | 0.23(1)  | 0.34(2)  | 0.34(2)  | 0.34(2)  | 0.34(2)  |
| $(m_b/m_{\tau})(M_X)$ | 0.67(4)  | 1.00(4)  | 1.00(4)  | 1.00(4)  | 1.00(4)  |
| $(3m_s/m_{\beta})(M_X)$ | 0.60(3)  | 0.9(1)   | 0.6(1)   | 0.9(1)   | 0.6(1)   |
| $(m_d/3m_{\tau})(M_X)$ | 0.71(7)  | 1.04(8)  | 0.68(6)  | 1.04(8)  | 0.68(6)  |
| $\left| \det Y^{d}(M_X) \right|$ | 0.3(1)   | 0.92(14) | 0.4(1)   | 0.92(14) | 0.4(1)   |
| $\left| \det Y^{c}(M_X) \right|$ | 0.3(1)   | 0.92(14) | 0.4(1)   | 0.92(14) | 0.4(1)   |

$^5$ As shown in ref. [35], it is possible, in a basis with large off-diagonal entries, to have an Hermitian pattern with the (1,1) and (1,3) zero provided one carefully orchestrates cancelations among $Y^u$ and $Y^d$ parameters. We find this approach requires a strange-quark mass near its upper limit.
associated with this ambiguity, allowing for $O(1)$ coefficients $a'$. In all the examples in Table 3, the mass ratios, and Wolfenstein parameters are essentially the same as in Table 2.

The effects of the large $\tan \beta$ threshold corrections are shown in Table 4. The threshold corrections depend on the details of the SUSY spectrum, and we have displayed the effects corresponding to a variety of choices for this spectrum. Column A corresponds to a “standard” SUGRA fit - the benchmark Snowmass Points and Slopes (SPS) spectra 1b of ref. Because the spectra SPS 1b have large stop and sbottom squark mixing angles, the approximations given in eqns. break down, and the value for the correction $\gamma_i$ in Column A need to be calculated with the more complete expressions in BRP. In the column A fit and the next two fits in columns B and C, we set $a'$ and $c'$ to zero. Column B corresponds to the fit given in the penultimate column of Table 2 which agrees very well with the simple GUT predictions. It is characterized by the “anomaly-like” spectrum with $M_3$ negative. Column C examines the $M_3$ positive case while maintaining the GUT prediction for the third generation $m_b = m_t$. It corresponds to the “Just-so” Split-Higgs solution. In the fits A, B and C the value of the parameter $a$ is significantly larger than that found in RRRV. This causes problems for models based on non-Abelian family symmetries, and it is of interest to try to reduce $a$ by allowing $a'$, $b'$ and $c'$ to vary while remaining $O(1)$ parameters. Doing this for the fits B and C leads to the fits B2 and C2 given in Table 4 where it may be seen that the extent to which $a$ can be reduced is quite limited. Adjusting to this is a challenge for the broken family-symmetry models.

Although we have included the finite corrections to match the MSSM theory onto the standard model at an effective SUSY scale $M_S = 500$ GeV, we have not included finite corrections from matching onto a specific GUT model. Precise threshold corrections cannot be rigorously calculated without a specific GUT model. Here we only estimate the order of magnitude of corrections to the mass relations in Table 2 from matching the MSSM values onto a GUT model at the GUT scale. The $\tan \beta$ enhanced corrections in eq. arise from soft SUSY breaking interactions and are suppressed by factors of $M_{SU3}/M_{GU3}$ in the high-scale matching. Allowing for $O(1)$ splitting of the mass ratios of the heavy states, one obtains corrections to $y^3$ (likewise for the lighter generations) of $O\left(\frac{x^2}{(4\pi)^2}\right)$ from the $X$ and $Y$ gauge bosons and $O\left(\frac{y^2}{(4\pi)^2}\right)$ from colored Higgs states. Because we have different Higgs representations for different generations, these threshold correction will be different for correcting the $3m_s/m_t$ relation than the $m_b/m_t$ relation. These factors can be enhanced in the case there are multiple Higgs representation. For an $SU(5)$ SUSY GUT these corrections are of the order of $2\%$. Plank scale suppressed operators can also induce corrections to both the unification scale and may have significant effects on the masses of the lighter generations. In the case that the Yukawa texture is given by a broken family symmetry in terms of an expansion parameter $\epsilon$, one expects model dependent corrections of order $\epsilon$ which may be significant.

In summary, in the light of the significant improvement in the measurement of fermion mass parameters, we have analyzed the possibility that the fermion mass structure results from an underlying supersymmetric GUT at a very high-scale mirroring the unification found for the gauge couplings. Use of the RG equations to continue the mass parameters to the GUT scale shows that, although qualitatively in agreement with the GUT predictions coming from simple Higgs structures, there is a small quantitative discrepancy. We have shown that these discrepancies may be eliminated by finite radiative threshold corrections involving the supersymmetric partners of the Standard-Model states. The required magnitude of these corrections is what is expected at large $\tan \beta$, and the form needed corresponds to a supersymmetric spectrum in which the gluino mass is negative with the opposite sign to the Wino mass. We have also performed a fit to the recent data to extract the underlying Yukawa coupling matrices for the quarks and leptons. This is done in the basis in which the mass matrices are hierarchical in structure with the off-diagonal elements small relative to the appropriate combinations of on-diagonal matrix elements, the basis most likely to be relevant if the fermion mass structure is due to a spontaneously broken family symmetry. We have explored the effect of SUSY threshold corrections for a variety of SUSY spectra. The resulting structure has significant differences from previous fits, and we hope will provide the “data” for developing models of fermion masses such as those based on a broken family symmetry.

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