Distortion Contribution Analysis with the Best Linear Approximation

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Abstract—A Distortion Contribution Analysis (DCA) obtains the distortion at the output of an analog electronic circuit as a sum of distortion contributions of its sub-circuits. Similar to a noise analysis, a DCA helps a designer to pinpoint the actual source of the distortion. Classically, the DCA uses the Volterra theory to model the circuit and its sub-circuits. This DCA has been proven useful for small circuits or heavily simplified examples. In more complex circuits however, the amount of contributions increases quickly, making the interpretation of the results difficult. In this paper, the Best Linear Approximation (BLA) is used to perform the DCA instead. The BLA represents the behaviour of a sub-circuit as a linear circuit with the unmodelled distortion represented by a noise source. Combining the BLA with a classical noise analysis yields a DCA which is simple to understand, yet capable to handle complex excitation signals and complex strongly non-linear circuits.

Index Terms—Non-linear distortion, Distortion Contribution Analysis, Best Linear Approximation

The decrease of supply voltages in aggressively scaled technologies results in non-linear distortion to become one of the main limiting factors for the dynamic range of analog electronic circuits. Still, the distortion is often taken into account at later stages of the design only by using a single- or two-tone test. The total harmonic distortion or intermodulation distortion is then intended to describe the non-linearity of the circuitry. These numbers give an indication of the total distortion without providing in-depth insight into its origin.

The aim of the Distortion Contribution Analysis (DCA) is to split the total distortion into contributions of each sub-circuit [1]. Comparing the different contributions allows the designer to pinpoint the dominant sources of non-linear distortion and thereby effectively reduce the total distortion [2]. Note that, taken from a bird’s eye view, the DCA closely resembles a noise analysis. The difference is that it is now applied to non-linear distortion sources.

The first DCA methods were based on Volterra theory [1], [3], [4]. The method has been illustrated on small circuits and has been extensively used in both the analysis and the design of electronic circuits [1]. However, the original Volterra-based DCA had some severe limitations:

- Only weakly non-linear circuits could be analysed. In strongly non-linear circuits, the Volterra series obtained around the DC operating point fail to converge, which limits the use of the DCA.
- Only smaller circuits could be analysed. The number of Volterra distortion contributions rises quickly for larger circuits. A simple Miller op-amp, for example, yields over 700 contributions [1], making interpretation of these results more difficult, if not impossible.
- Only the distortion under single-tone or two-tone excitation signals had been considered. Exciting a circuit with practical complex modulated excitation signals, however, has a big influence on the non-linear behaviour of the circuit and hence on the distortion (in Appendix A, we demonstrate this difference on an example) [5].

Many of the limitations of the original Volterra-based DCA have been overcome in more recent years. The phasor method and its variations simplify the obtained distortion expressions such that more complex circuits can be intuitively analysed [4], [6]–[8]. Alternatively, state-space approaches were introduced to circumvent the complexity explosion of the Volterra distortion contributions for larger circuits [9], [10]. Furthermore, an extension to strongly non-linear circuits has been proposed in [11]. Unfortunately, all these techniques mainly capture the distortion generated by single-tone or two-tone excitations, and often require circuit-specific assumptions which prevent the general applicability of these techniques.

More recently, the Best Linear Approximation (BLA) has been used to perform a DCA on analog electronic circuits. The idea was originally proposed in [12] and has been applied to several examples in the past [13]–[15]. In the BLA framework, the behaviour of a non-linear system is approximated in least-squares sense by a linear system. As a consequence, the distortion introduced by the system can be represented by an additive noise source. Combining the BLA analysis with a classic noise analysis yields a DCA which solves some of the drawbacks of the classic Volterra-based implementations, at the cost of an increased simulation time. The main benefits of the method are:

- Linear models are used to describe the dynamic behaviour of the sub-circuits, while the distortion in the circuit is represented by noise-like sources. The concept of linear dynamic systems and noise are familiar to all designers.
- The analysis also applies to modulated signals, which leads to an accurate and realistic representation of the non-linear distortion generated by the circuit in real operation.

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The BLA method does not require simplified device models or accessibility to internal nodes of the device models.

The validity of the BLA is not restricted to weakly non-linear circuits. Strongly non-linear power amplifiers and hard saturation can still be modelled with the BLA. However, in this paper, we rule out strongly non-linear circuits designed for frequency translation, like mixers, phase-locked loops ...

All previous implementations of the BLA-based DCA use a simplified representation of the circuit, ignoring possible correlation of the distortion introduced by different stages on one hand and input-output impedances of the circuit on the other hand [12]–[14], [16]. In this paper, we link the BLA-based DCA to the theoretical framework of the BLA [17]–[19] (Section I and II). A first contribution is to correctly take the correlation between different distortion sources present in the circuit into account. Secondly, the BLA-based DCA is extended to the use of S-parameters to represent the sub-circuits (Section III). This extension takes reverse gain and terminal impedances of the sub-circuits into account, which enables a BLA-based DCA at the transistor level. The introduction of S-parameters moves the sub-circuit representation from Single-Input Single-Output (SISO) sub-blocks to Multiple-Input Multiple-Output (MIMO) sub-blocks, which complicates the identification of the BLA. Section IV details the simulations required to estimate the BLA of the MIMO sub-circuits correctly. Finally, the BLA-based DCA is applied to a two-stage Miller op-amp, a Doherty power amplifier and a Gm-C biquad to show the benefits and general applicability of the method (Section V).

I. THE SINGLE-INPUT SINGLE-OUTPUT
BEST LINEAR APPROXIMATION

Instead of working with deterministic input signals, such as a sine wave or a two-tone, the BLA framework considers noise excitation signals with a fixed Power Spectral Density (PSD) and Probability Density Function (PDF). Examples are filtered white Gaussian noise and telecommunication signals with a

fixed PSD and PDF, the corresponding BLA of the system is then defined as:

\[ G_{U \rightarrow Y}^{BLA}(j\omega) = \frac{S_{up}(j\omega)}{S_{ur}(j\omega)} = \frac{\mathbb{F}\{\mathbb{E}\{y(t) r(t-\tau)\}\}}{\mathbb{F}\{\mathbb{E}\{u(t) r(t-\tau)\}\}} \]  

where \( S_{ur} \) and \( S_{up} \) are the cross-power spectrum between the reference signal \( r(t) \) and the output \( y(t) \) and input \( u(t) \) respectively [17], [18]. \( \mathbb{F}\{x(t)\} \) represents the Fourier transform of \( x(t) \) and the expected value operator \( \mathbb{E}\{\bullet\} \) is taken with respect to the random reference signal \( r(t) \).

The difference between the actual output \( y(t) \) of the non-linear system and the output predicted by the BLA is denoted by \( d(t) \). The distortion term \( d(t) \) is zero mean, uncorrelated with the reference signal \( r(t) \) and behaves like noise [18]. In the frequency domain, the input-output relation at each excited frequency bin \( k \) is written as:

\[ Y(k) = G_{U \rightarrow Y}^{BLA}(j\omega_k) U(k) + D(k) \]  

wherein \( Y(k) \) and \( U(k) \) are the DFT spectra of \( y(t) \) and \( u(t) \) respectively, evaluated at the \( k^{th} \) frequency bin. Equation (2) is the key expression that allows to use the BLA in a DCA. It tells that the output of each sub-circuit can be written as the sum of the output of a signal-dependent linear dynamic circuit and an additive noise source \( D(k) \) representing the distortion. Calculating the frequency response function from each distortion source to the considered output of the total circuit allows one to compute the different distortion contributions.

A. Multisine Excitations

Instead of working with noisy excitation signals directly, Random Phase Multisine (RPM) are commonly used to estimate the BLA for a Gaussian input signal. A RPM is a sum of harmonically related sine waves with a random phase:

\[ r(t) = \sum_{k=1}^{N} A_k \sin(2\pi f_0 t + \phi_k) \]  

where \( f_0 \) is the base frequency of the multisine. \( A_k \) and \( \phi_k \) are the amplitude and phase of the \( k^{th} \) tone in the multisine. If the phases are drawn randomly from a uniform distribution \([0, 2\pi]\), the multisine PDF converges to a Gaussian PDF when a large number of frequencies \( N \) is considered. The amplitude coefficients \( A_k \) in the multisine can be chosen in a deterministic way to set the required PSD. In RF applications, the multisine only excites frequency bins around a centre frequency \( f_c \) between a minimum frequency \( f_{\text{min}} \) and a maximum frequency \( f_{\text{max}} \). \( f_{\text{min}}, f_{\text{max}} \) and \( f_c \) are all set to integer multiples of the base frequency \( f_0 \). In baseband applications, lowpass multisines are used which excite frequencies starting from DC, so \( f_{\text{min}} \) is equal to \( f_0 \) in that case.

To separate even and odd non-linear distortion contributions in a baseband circuit, odd lowpass multisines are commonly used \((A_k = 0 \text{ for even } k)\). An even non-linearity always combines an even number of frequencies in the multisine, so its distortion contributions will fall on the even frequency

\[ \]
We introduce the “robust method” to determine the BLA of a circuit wherein the uncertainty on the estimate can all be used to determine the steady-state response to the multisine excitation. The steady-state spectrum of the reference signal, input signal and output signal of the circuit under excitation by the multisines are applied to the system. In those different-phase multisines, the steady-state response of the circuit is obtained in a two-step procedure. First, the Single-Input Multiple-Output (SIMO) BLA of the reference signal, input signal and output signal of the circuit under excitation by the multisines is determined by averaging over the steady-state response for each multisine. The steady-state spectrum of the reference signal, input signal and output signal of the circuit are separated. Finally, the BLA of the system operating in feedback to the different-phase multisines: 

\[ \text{min} \] 

where

\[ C(\omega) = \sigma_r^2 \] 

is the sample covariance matrix of \( Z \), expressing the uncertainty on the estimate. *H* indicates the Hermitian transpose. Finally, the BLA of the system operating in feedback is determined as:

\[ G_{U \rightarrow Y}^{\text{BLA}}(\omega_k) = \frac{G_{R \rightarrow U}^{\text{BLA}}(\omega_k)}{G_{R \rightarrow Y}^{\text{BLA}}(\omega_k)} \] 

B. Determining the SISO BLA

We introduce the “robust method” to determine the BLA of a circuit as it allows to estimate the BLA and the distortion in the circuit with the highest accuracy [21]. \( M \) different-phase multisines are applied to the system. In those different-phase multisines only the \( \phi_k \) are changed in (3), the amount of tones \( N \) and the amplitude of the tones \( A_k \) is kept the same for each multisine. The steady-state response of the circuit to each of the different-phase multisines is then determined using a large-signal simulation\(^1\). The steady-state spectrum of the reference signal, input signal and output signal of the circuit under excitation by the different-phase multisine is labelled \( R^{(m)}, U^{(m)} \) and \( Y^{(m)} \) respectively.

The BLA of the system is now obtained in a two-step procedure. First, the Single-Input Multiple-Output (SIMO) BLA from the reference signal to the stacked output-input vector \( (Z) \) is determined by averaging over the steady-state response to the different-phase multisines:

\[ Z^{(m)}(j\omega_k) = \begin{bmatrix} Y^{(m)}(j\omega_k) \\ U^{(m)}(j\omega_k) \end{bmatrix} \left( R^{(m)}(j\omega_k) \right)^{-1} \]

\[ Z(j\omega_k) = \frac{1}{M} \sum_{m=1}^{M} Z^{(m)}(j\omega_k) = \begin{bmatrix} G_{R \rightarrow Y}^{\text{BLA}}(j\omega_k) \\ G_{R \rightarrow U}^{\text{BLA}}(j\omega_k) \end{bmatrix} \] 

\[ r^{(m)}_Z(j\omega_k) = Z^{(m)}(j\omega_k) - Z(j\omega_k) \]

\[ C_Z(j\omega_k) = \frac{1}{M(M-1)} \sum_{m=1}^{M} r^{(m)}_Z(j\omega_k) \left( r^{(m)}_Z(j\omega_k) \right)^H \]

wherein \( C_Z \) is the sample covariance matrix of \( Z \), expressing the uncertainty on the estimate. *H* indicates the Hermitian transpose. Finally, the BLA of the system operating in feedback is determined as:

\[ G_{U \rightarrow Y}^{\text{BLA}}(j\omega_k) = \frac{G_{R \rightarrow Y}^{\text{BLA}}(j\omega_k)}{G_{R \rightarrow U}^{\text{BLA}}(j\omega_k)} \] 

Furthermore, the uncertainty on the BLA-estimate can be calculated as:

\[ \sigma_r^2 = \left[ \frac{1}{G_{R \rightarrow U}^{\text{BLA}}(j\omega_k)} \right]^2 \mathbf{V}(j\omega_k) C_Z(j\omega_k) \mathbf{V}^H(j\omega_k) \] 

\[ \mathbf{V}(j\omega_k) = \begin{bmatrix} 1 & -G_{R \rightarrow Y}^{\text{BLA}}(j\omega_k) \end{bmatrix} \]

\(^1\) A Transient simulation. Periodic Steady-State (PSS), Harmonic Balance (HB) or Envelope simulation can all be used to determine the steady-state response to the multisine excitation.

More details about the experiments and algorithm needed to determine the BLA are given in Section IV and in references [17]–[19].

The estimate of the uncertainty on the BLA is used to determine the number of different-phase multisines needed to obtain a sufficiently certain estimate of the BLA. When the uncertainty is too high for the specific application, more different-phase multisines are simulated and added to the set of signals until a sufficiently low uncertainty is obtained. In strongly non-linear circuits, it can take several hundreds of different-phase multisines to obtain a good estimate as the standard deviation only decreases with the square-root of this number.

Example 1: BLA of a Miller op-amp in feedback

Before we use the BLA in a DCA, let us illustrate how the BLA is used to describe the behaviour of an op-amp placed in a negative feedback configuration as shown in Fig. 2.

The reference signals are lowpass random-odd RPM with \( f_0 = f_{\text{min}} = 0.1kHz \) and \( f_{\text{max}} = 100kHz \) (Figure 3). The amplitude of the multisines is chosen flat as a function of frequency and such that the Root Mean Square (RMS) voltage equals 50mV.

\[ \text{Figure 3. Spectrum of the reference signal.} \]

The input and output voltages obtained with a Harmonic Balance (HB) simulation clearly contain non-linear distortion (Fig. 4), as there is energy appearing at non-excited frequency lines. Even-order distortion (blue) and odd-order distortion (red) are separated by using the odd multisines. The in-band odd non-linear distortion is visible on the detection lines. The BLA obtained with 7 different-phase multisines is shown in Fig. 5. A compression of 0.1dB is observed with respect to the results obtained with an AC simulation.

II. DISTORTION CONTRIBUTION ANALYSIS & BLA

By fixing the input signal class (fixed PSD and fixed PDF) and working with the BLA framework, the non-linear distortion in a circuit can be treated as if it were noise. Combining the BLA with a noise analysis then allows one to determine the dominant
source of non-linear distortion in a system. The basic idea is simple [12], [22], but a rigorous treatment of the concept has not been detailed in literature. The main difference between the noise analysis in a BLA-based DCA and the classic noise analysis is that all distortion sources are correlated. Taking this correlation into account is very important to obtain the correct result for the DCA and is one of the main contributions of this paper.

Consider $N$ SISO non-linear systems embedded in a linear feedback structure as is shown in Fig. 6. The whole system is excited by random-phase multisines $R$ with a specified PSD and PDF. Using (2), the output of the system at the $k$\textsuperscript{th} bin of the DFT can be written as:

$$Y_t(k) = G_{R\rightarrow Y}^{\text{BLA}}(j\omega_k) R(k) + D_t(k)$$ (7)

This expression indicates that the output contains a best linear contribution to the input ($G_{R\rightarrow Y}^{\text{BLA}}$, $R$) and a distortion term $D_t$. The goal of the DCA is to write $D_t$ as the sum of contributions stemming from the $N$ non-linear blocks in the circuit. As explained in the previous section, $D_t(k)$ has noise-like properties which means that only the power of the output distortion, or $\mathbb{E} \{ D_t(k)^2 \}$, can be considered. Although (7) also holds for systems excited by single-tone and two-tone signals, the distortion term $D_t(k)$ for these deterministic signals exhibits different statistical properties compared to noise excitations (Appendix A). At present, there does not exist a method which relates the distortion term obtained with noise excitations to the one obtained with deterministic signals. As a result, the proposed BLA-based DCA is currently only applicable to noise excitation signals.

To determine the distortion contributions separately, first consider the BLAs of the different non-linear systems. All inputs and outputs of the non-linear sub-circuits are gathered frequency by frequency in column vectors $U(k)$ and $Y(k)$:

$$U(k) = \begin{bmatrix} U_{[1]}(k) \\ \vdots \\ U_{[N]}(k) \end{bmatrix} \quad Y(k) = \begin{bmatrix} Y_{[1]}(k) \\ \vdots \\ Y_{[N]}(k) \end{bmatrix}$$ (8)

where $Y_{[n]}(k)$ and $U_{[n]}(k)$ indicate the output and input DFT spectra of the $n$\textsuperscript{th} sub-circuit respectively. The different SISO BLA, as defined in (1), are grouped in a diagonal matrix:

$$G_{U\rightarrow Y}(j\omega_k) = \begin{bmatrix} G_{U\rightarrow Y}^{\text{BLA}}(j\omega_k) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & G_{U\rightarrow Y}^{\text{BLA}}(j\omega_k) \end{bmatrix}$$ (9)

The input-output relation of all non-linear systems can now be written simultaneously as follows:

$$Y(k) = G_{U\rightarrow Y}^{\text{BLA}}(j\omega_k) U(k) + D(k)$$ (10)

where $D \in \mathbb{C}^{N \times 1}$ contains the non-linear distortion introduced by the $N$ sub-systems. From here, the frequency indices $(k)$ and $(j\omega_k)$ will be omitted for notational simplicity. It is shown in Appendix B that the output signal $Y_t$ of the total system can be written as:

$$Y_t = B (I_N + G_{U\rightarrow Y}^{\text{BLA}})^{-1} G_{U\rightarrow Y}^{\text{BLA}} A R + B (I_N + G_{U\rightarrow Y}^{\text{BLA}})^{-1} D$$ (11)

where $B$, $A$ and $M$ are the linear blocks connected to the non-linear sub-circuits as shown in Fig. 6. $I_N$ is the identity matrix of size $N$. The second part of (11) yields the expression for the output distortion as a function of the distortion in the sub-systems. Considering the power of the distortion at $(k)$:

$$\mathbb{E} \{ D_t D_t^H \} = T_{\text{out}} \mathbb{E} \{ D D^H \} T_{\text{out}}^H$$ (12)

with $T_{\text{out}} = B (I_N + G_{U\rightarrow Y}^{\text{BLA}})^{-1}$ a row vector of length $N$ that contains the Frequency Response Function (FRF) from each distortion source to the output. The $n$\textsuperscript{th} element of $T_{\text{out}}$ will be called $t_{[n]}$ from now on. $\mathbb{E} \{ D D^H \} = C_D$ is the covariance matrix of the distortion introduced by the non-linear sub-systems.

A two-step procedure is used to obtain an estimate of $C_D$, which is similar to the way we determined the BLA itself.
in section I [19]. First, the covariance matrix of the stacked input-output vectors $C_Z$ is determined as in equation (5), but now $Y^{(m)}$ and $U^{(m)}$ are replaced by the stacked input and output signals defined in (8). $C_Z$ is multiplied by the number of different-phase multisines $M$, as we are interested in the power of the distortion, rather than in the uncertainty on the BLA-estimate. This $C_Z$ is now a $2N \times 2N$ matrix. To obtain a full-rank estimate of $C_Z$, the response to at least $2N$ different-phase multisines must be simulated. $C_D$ is then calculated starting from $C_Z$ in the following way:

$$C_D = M \begin{bmatrix} I_N & -G^\text{BLA}_{U \rightarrow Y} \end{bmatrix} C_Z \begin{bmatrix} I_N & -G^\text{BLA}_{U \rightarrow Y} \end{bmatrix}^T$$

(13)

The matrix product in (12) can be re-written as:

$$E\{D_i D_j^H\} = \sum_{i=1}^{N} \sum_{j=1}^{N} [C_D]_{i,j} T_i^H T_j^T$$

herein $C_D$ is an Hermitian matrix. The complex conjugate contributions of $[C_D]_{i,j}$ and $[C_D]_{j,i}$ will therefore combine to form a single, real-valued distortion power contribution. The expression for the distortion at the output can now be simplified as follows:

$$E\{D_i D_j^H\} = \sum_{i=1}^{N} [C_D]_{i,i} T_i^H T_i^T + \sum_{i=2}^{N} \sum_{j=1}^{N-1} 2R\{[C_D]_{i,j} T_i^H T_j^T\}$$

(14)

Equation (14) contains all the different distortion contributions: each element of the covariance matrix of the distortion sources is transferred to the output. The total distortion at the output is then the sum of all these contributions. The contributions can be sorted according to their magnitude to determine the dominant distortion contribution.

From now on, we will refer to the distortion contributions due to the diagonal elements of the distortion covariance matrix as direct distortion contributions ($C_{i,i}$). The contributions due to the off-diagonal elements will be called correlation distortion contributions ($C_{i,j}$).

Example 2: DCA of a non-linearity followed by its inverse

To clarify the interpretation of the different distortion contributions obtained with the BLA-based DCA, we consider the trivial example of a static non-linearity followed by its inverse (Figure 7). The first non-linear block is an exponential function and the second its inverse: a logarithm. The cascade of both blocks results in a perfectly linear system.

![Figure 7. Cascade of two non-linear systems studied in this example.](image)

The system is excited by random-phase multisines with an $f_0 = 1$Hz which excite all frequencies up to 100Hz. The RMS of the multisine was set to 0.5V. The steady-state spectrum of the signal $I$, measured between the two non-linear blocks, is shown in Fig. 8.

![Figure 8. Spectrum of the signal $I$ between the non-linear blocks.](image)

The frequency bins excited by the multisine are shown in black, the remaining frequency lines in magenta. The measured distortion power at the internal signal is shown with the magenta line in the plot. The amount of non-linear distortion at the intermediate signal in this cascade is very high (signal to distortion ratio of 10dB), but all the distortion is completely cancelled out by the second block, so that the input and output signals are exactly the same. We calculate the BLAs of the two non-linear blocks using (4)-(6). $10^4$ different-phase multisines were simulated to obtain an adequately low uncertainty on the BLA-estimates in this strongly non-linear circuit. The obtained BLAs and their $3\sigma$ uncertainty bound are shown in Fig. 9.

![Figure 9. BLAs of the two non-linear blocks in the cascade.](image)

With these BLAs, the covariance matrix of the distortion sources can be calculated using (13). In this simple example, there are two distortion sources, one for each non-linearity. This results in a $2 \times 2$ covariance matrix $C_D$. The elements on the diagonal of $C_D$ describe the power of each of the distortion sources. The off-diagonal elements indicate the correlation between both sources. The values of the distortion covariance matrix are shown in Fig. 10.

![Figure 10. Elements of $C_D$ as a function of frequency.](image)

The three distortion contributions to the output can now be calculated. The FRF from each distortion source to the output can be obtained using (11). Here, we have:

$$T_{[1]}(j\omega_k) = G^\text{BLA}_{I \rightarrow Y}(j\omega_k) \quad T_{[2]}(j\omega_k) = 1$$
With \( C_D \), \( T_{[1]} \) and \( T_{[2]} \), we can calculate the distortion contributions to the output of the circuit. The direct distortion contributions due to the first stage is:
\[
C_{[1]}(j\omega_k) = \left| G_{R \rightarrow Y}(j\omega_k) \right|^2 \left| C_D(j\omega_k) \right|_{1,1}
\]
The direct distortion contributions due to the second stage is:
\[
C_{[2]}(j\omega_k) = \left| C_D(j\omega_k) \right|_{2,2}
\]
The correlation distortion contribution is given by:
\[
C_{[1,2]}(j\omega_k) = G_{Y \rightarrow Y}(j\omega_k) \left| C_D(j\omega_k) \right|_{1,2}
\]

The obtained distortion contributions are shown in Fig. 11. The two direct contributions are equal in amplitude and both positive. The correlation contribution is equal to the sum of the two direct contributions, but opposite in sign. The sum of all contributions (shown in gray on the plot above) therefore lies very close to zero.

With this very simple example we have shown the effectiveness of the BLA-based DCA to predict the distortion contributions of a strongly non-linear circuit under a modulated excitation signal. Additionally, we have shown that it is important to keep the correlation distortion contributions into account to obtain a correct result.

III. BLA-BASED DCA WITH S-PARAMETERS

The previous expressions can be used in a DCA on system-level simulations, where every sub-circuit is represented by a SISO system. In actual electronic circuits however, a port-based representation of the sub-blocks has to be used to represent the terminal impedances and to include the forward and reverse gain of each sub-circuit in the circuit. In the remainder of this paper, S-parameters will be used to represent the behaviour of the different circuit blocks. Similar expressions can be obtained for the \( Y \) and \( Z \) parameters, but this is considered to be outside of the scope of this paper.

The reasoning in this section is very similar to the one detailed in previous section but, instead of working with SISO BLA, each of the sub-circuits is described by a MIMO BLA.

Fig. 12 shows the general circuit under test for the DCA. There are \( N \) non-linear sub-circuits embedded in a package. The whole circuit is excited by different-phase multisines \( R \) and the output of the circuit is terminated in a load impedance.

The steady-state port voltages and currents of the sub-circuits are measured and transformed into waves using the classical expression [23]:
\[
B_i = \frac{V_i - Z_0 I_i}{2\sqrt{Z_0}} \quad A_i = \frac{V_i + Z_0 I_i}{2\sqrt{Z_0}}
\]

The circuit under test will consist of \( N \) non-linear sub-circuits embedded in a linear package. The whole circuit is excited by a reference signal \( R \). The goal of the DCA is split the distortion in the output wave \( B_t \) into its contributions.

where \( V_i \) is the port voltage and \( I_i \) is the port current flowing into the sub-circuit port. \( Z_0 \) is a user-chosen reference impedance. The \( A \) and \( B \) waves at the \( p_n \) ports of the \( n \)th sub-circuit are gathered in vectors, giving:
\[
B_{[n]} = \begin{bmatrix} B_{[n]1} \\ \vdots \\ B_{[n]p_n} \end{bmatrix} \quad A_{[n]} = \begin{bmatrix} A_{[n]1} \\ \vdots \\ A_{[n]p_n} \end{bmatrix}
\]

The relation between \( A_{[n]} \) and \( B_{[n]} \) is given by the MIMO BLAs \( S_{A,[n] \rightarrow B,[n]} \):
\[
B_{[n]} = S_{A,[n] \rightarrow B,[n]} A_{[n]} + D_{[n]}
\]

in which \( D_{[n]} \) is the vector of distortion sources. Determining the MIMO BLAs is more complex than what has been done for the SIMO procedure described in Section I. The algorithm needed is described in the following section. For now, assume the MIMO BLAs to be known. All the different BLAs are gathered in a block diagonal matrix, similarly to what was done in (10):
\[
\begin{bmatrix}
B_{[1]} \\
\vdots \\
B_{[N]}
\end{bmatrix} = \begin{bmatrix} S_{A,[1] \rightarrow B,[1]} & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & S_{A,[N] \rightarrow B,[N]} \end{bmatrix} \begin{bmatrix} A_{[1]} \\
\vdots \\
A_{[N]} \end{bmatrix} + \begin{bmatrix} D_{[1]} \\
\vdots \\
D_{[N]} \end{bmatrix}
\]

(15)

The total number of ports of all the sub-circuits is denoted by \( P \). The vector of distortion sources \( D \in \mathbb{C}^{P \times 1} \) is again noise-like, so the covariance matrix \( C_D = \mathbb{E} \{ DD^H \} \) is used to describe it. Determining \( C_D \) is done in the same way as explained in Section II and equation (13).

The distortion at the output of the system is defined by considering the BLA from the reference multisine to the output wave \( B_t \):
\[
B_t = G_{R \rightarrow B_t} R + D_t
\]

(16)

The goal of the DCA is to split the power in \( D_t \) as a sum of contributions from \( C_D \). Classical papers on wave-based circuit and noise analysis have dealt with this problem already [24], [25], and describe how to determine a row vector \( T_{out} \) that is used to refer \( C_D \) to the output wave (See appendix C). This results in the following formula:
\[
\mathbb{E} \{ D_t D_t^H \} = T_{out} C_D T_{out}^H
\]

(17)
This expression can be re-written in a similar way as in (14) to obtain a list of direct distortion contributions and correlation distortion contributions.

A. Dealing with the combinatorial explosion

In circuits, each port of each sub-circuit will create several distortion contributions: a single direct contribution and some correlation contributions. The number of contributions can therefore rise quickly, especially in fully differential, complex, circuits. In a circuit with \( P \) ports, there are \( 1/2P(P-1) \) distortion contributions to the output.

If the amount of contributions is too large to be easily tractable and interpretable, the different contributions of a single sub-circuit can be combined into a single contribution by simply summing the contributions of each of the ports of one sub-circuit. The covariances can also be combined in the same way. Combining the contributions of each stage reduces the amount of contributions for \( N \) sub-circuits to \( 1/2N(N-1) \). If this amount of contributions is still too large for an easy interpretation of the results, the contributions of several sub-circuits can be combined into one contribution of a larger sub-circuit. It is a clear advantage that the BLA-based DCA can easily be applied hierarchically as this allows to zoom in selectively on the most contributing parts of the circuit, while leaving the other sub-circuits aggregated at a higher level of abstraction.

IV. ESTIMATING THE MIMO BLA OF SUB-CIRCUITS

In the previous section, it was assumed that the BLA of the sub-circuits was known. Determining the BLA is the most difficult and time-consuming step in the BLA-based DCA, as it requires averaging the large-signal steady-state response of the circuit over many different-phase multisines. For SISO sub-circuits, the estimation steps detailed in equations (4)-(6) can be used. In case of MIMO sub-circuits, extra excitation signals need to be added to the circuit to determine the BLA.

A. MIMO identification in feedback

The MIMO BLA of the \( n^{\text{th}} \) sub-circuit with \( p_n \) inputs and outputs, is defined as an extension of (6):

\[
S_{A[n] \rightarrow B[n]}(j\omega_k) = G_{R \rightarrow B[n]}^{\text{BLA}}(j\omega_k) \left[ G_{R \rightarrow A[n]}^{\text{BLA}}(j\omega_k) \right]^{-1}
\]

where \( G_{R \rightarrow B[n]}^{\text{BLA}} \in \mathbb{C}^{p_n \times nr} \) is the MIMO BLA taken from the \( n_r \) reference signals to the output waves of the \( n^{\text{th}} \) sub-circuit. \( G_{R \rightarrow A[n]}^{\text{BLA}} \in \mathbb{C}^{p_n \times n_r} \) is the MIMO BLA from the reference signals to the input waves of the sub-circuit. Since \( G_{R \rightarrow A[n]}^{\text{BLA}} \) must be invertible, at least \( p_n \) independent reference signals have to be present in the circuit. Most circuits are only excited by one main reference signal, so the required extra reference signals have to be added artificially to the circuit to allow estimation of the MIMO BLA of the sub-circuits. The extra multisines have to be very small in amplitude with respect to the large signal to avoid changing the BLA of the circuit, which is why they are called tickler multisines [16].

If the ticklers are placed on the same frequency grid as the main multisines, their response will be overwhelmed by the circuit response to the main multisines. To avoid this overlap, the frequency grid of the tickler is shifted by a frequency \( \epsilon f_{l,t} \). This technique is called ‘Zippering’ [26]. The zippered tickler multisines are defined by:

\[
r_{\text{tickler}}(t) = \sum_{h=1}^{N} A_h \sin (2\pi (f_{0} + f_{l,t}) t + \phi_h)
\]

where the phases of the tickler multisine (\( \phi_h \)) are drawn randomly from [0, 2\( \pi \]). These tickler multisines will create contributions on frequencies that are always \( f_{l,t} \) away from the spectral lines of the main multisines, so the responses of the main and tickler multisines are easily separated by looking at the correct frequency bins. Because their amplitude is very small, the tickler signals can either be applied one-by-one, while the main multisine remains active, or applied all simultaneously since each of the ticklers can be given an independent \( f_{l,t} \).

To obtain the BLA with the zippered multisines, the SIMO BLA from each reference signal to the stacked output-input vector of the sub-circuit is determined first by simple averaging as in (4). The SIMO BLA from reference signal \( R_r \) to the stacked output-input vector will be denoted \( G_{R \rightarrow Z}^{\text{BLA}} \), while its uncertainty is expressed by the covariance matrix \( C_{G_{R \rightarrow Z}^{\text{BLA}}} \).

The \( G_{R \rightarrow Z}^{\text{BLA}} \) are known at the frequencies of the spectral lines of multisines \( R_r \). All \( G_{R \rightarrow Z[n]}^{\text{BLA}} \) are then linearly interpolated to the spectral lines of the main multisines and gathered in a large matrix:

\[
G_{R \rightarrow Z[n]}^{\text{BLA}} = \begin{bmatrix} G_{R \rightarrow B[1]}^{\text{BLA}} & \cdots & G_{R \rightarrow B[n]}^{\text{BLA}} \\ G_{R \rightarrow A[1]}^{\text{BLA}} & \cdots & G_{R \rightarrow A[n]}^{\text{BLA}} \end{bmatrix} = \begin{bmatrix} G_{R \rightarrow Z[1]}^{\text{BLA}} & \cdots & G_{R \rightarrow Z[n]}^{\text{BLA}} \end{bmatrix}
\]

The BLA is now obtained using expression (18). To obtain the uncertainty on the BLA, the uncertainty on the \( G_{Z} \)-matrix is transformed by the following expression [19]:

\[
C_{\text{vec}(G_{Z})} = TC_{\text{vec}(G_{R \rightarrow Z})} \cdot T^H
\]

where \( T \) is the covariance matrix \( C_{\text{vec}(G_{Z})} \) contains the different covariance matrices of the SIMO BLA on its diagonal:

\[
C_{\text{vec}(G_{Z})} = \begin{bmatrix} C_{G_{R \rightarrow Z[1]}^{\text{BLA}}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & C_{G_{R \rightarrow Z[n]}^{\text{BLA}}} \end{bmatrix}
\]

Determining the MIMO BLA with the method described here can be very time-consuming because many large-signal simulations have to be run. A speed-up can be obtained using advanced non-parametric estimation techniques like the local polynomial method [28]. Alternatively, rational approximations can be estimated to reduce the noisiness of the obtained BLA estimates.

It is more easy to work with tickler current sources, as they can be added to a circuit’s netlist without introducing extra
nodes. Choosing the best nodes to place the tickler multisines and determining the amplitude of the ticklers will usually require some user intervention. When the BLA from two reference signals to the input and output waves of the circuit are too similar, the $G_{R \rightarrow A}^{BLA}$ matrix will be badly conditioned and its inverse will be difficult to compute. To obtain a good conditioning of the $G_{R \rightarrow A}^{BLA}$, it is recommended to connect the tickler sources to nodes that are close to the ports of the circuit under test.

The amplitude of the tickler multisines should be small, but the circuit’s response to the tickler signal should lie above the numerical noise floor of the input and output waves. Setting the correct amplitude of the tickler signals is therefore done by increasing the amplitude of the tickler until its response is clearly visible in the steady-state spectra of all input and output signals of the sub-circuits. In circuits with very small reverse gains, obtaining a good estimate of the reverse gain can be very difficult. For the small reverse gain, the small-signal behaviour of the sub-circuit is used if it cannot be estimated reliably [29].

Example 3: MIMO BLA of a class-C amplifier

To demonstrate how the MIMO BLA of a circuit is estimated, consider the class-C amplifier shown in Fig. 13.

![Class-C amplifier](image)

The class-C amplifier is excited by different-phase RPM $R_1$ that excites 41 frequencies in a band of 40MHz around 1GHz with a RMS value of 0.2V. The transistor in the amplifier is placed in a common-source configuration, so it will be modelled by a two-port. There’s only one reference multisine in the circuit so a second multisine current source $R_2$ is added at the output (shown in blue in the figure). The tickler multisines are shifted 1Hz away from the frequency grid of the main multisine and are given an RMS current of 40µA. The resulting $B_2$ wave, obtained with HB is shown in Fig. 14.

![Spectrum of the $B_2$ wave](image)

In this figure, the frequency bins of the main multisines are shown in black, while its distortion is shown in red. In between the frequency bins of the main multisines, the response to the tickler is visible. The frequency bins of the tickler are indicated with green, while all remaining bins are grey. The amplitude of the tickler was chosen such that the tickler’s response was clearly visible above the numerical noise floor.

The obtained BLAs and their $3\sigma$ uncertainty interval are shown in Fig. 15.

![Estimated MIMO BLAs for the transistor in the amplifier](image)

The dashed lines in Fig. 15 are the small-signal S-parameters. The largest differences are observed on $S_{21}$ and $S_{22}$, which is to be expected for a class-C biased transistor.

B. Weakly non-linear circuits

In weakly non-linear circuits, the BLA doesn’t deviate a lot from the small-signal frequency response of the circuits, so the small-signal behaviour can be used to replace the BLA. When the MIMO identification scheme is avoided, a significant reduction in simulation time for the DCA is obtained. The small-signal S-parameters can be obtained quickly in modern simulators combining several AC or S-parameter simulations.

Combining small-signal and large-signal results is trivial when the response of the circuit to the multisine excitations is obtained with HB. If the response is obtained with a time-domain simulation, frequency warping should be properly taken into account:
- A trapezoidal integration method should always be used to avoid artificial damping of the circuit poles [30].
- A fixed time step $T_s$ should be used to allow calculating the spectrum easily with the DFT.
- The remaining frequency warping introduced by going to the discrete time domain should be taken into account.

When the trapezoidal integration method is used, each frequency bin of the large-signal simulation $k f_0$ is warped to a frequency $f_{\text{warp},k}$ according to the following relationship:

$$f_{\text{warp},k} = \frac{1}{\pi T_s} \tan (\pi k f_0 T_s)$$  \hspace{1cm} (20)$$

It is as if the circuit is working on the frequency grid determined by $f_{\text{warp},k}$. Hence, the small-signal behaviour should preferably be determined on the frequency grid $f_{\text{warp},k}$, or alternatively should be interpolated to the warped frequency grid to minimise any errors.
When the BLA deviates too far from the small-signal S-parameters due to strongly non-linear behaviour of the circuit, one cannot replace the BLA by small-signal S-parameters without introducing errors. It is therefore important to assess the quality of the small-signal behaviour when using it to predict the distortion in the circuit. The perfect assessment could be obtained by comparing the small-signal behaviour to the estimated MIMO BLA of each sub-circuit, but calculating the MIMO BLA beats the purpose of using the small-signal behaviour in the first place. Instead, the BLA from the reference signal to the input and output waves \( G_{BLA}^{\text{BLA}}(\mathbf{Z}_n) \) as defined in (19)) can be compared to the frequency responses obtained with an AC simulation. When the difference between \( G_{BLA}^{\text{BLA}}(\mathbf{Z}_n) \) and the AC result lies significantly above the distortion level, the small-signal behaviour deviates too far from the correct BLA and should not be used in the DCA.

As an example, this test is applied to the class-C amplifier from before. The FRF from the main reference signal to the output wave is calculated in three different ways: First, the BLA from the reference to the output wave is calculated using the SISO techniques described in Section I. Second, the small-signal frequency response from the reference to the output wave was calculated with an AC simulation. Finally, the MIMO BLA of the transistor was used to predict the same frequency response (Appendix D). The three frequency responses are shown in Fig. 16. The frequency response obtained with the AC simulation deviates strongly from the BLA from the reference to the output wave. When the estimated MIMO BLA is used to predict the frequency response, the correct result is obtained.

In this class-C amplifier, it is therefore not possible to use the small-signal S-parameters in the DCA, which is to be expected for such a strongly non-linear circuit. In the examples shown in the next section, two circuits are shown where this small-signal assumption is valid.

V. EXAMPLES

The BLA-based DCA will now be applied to several examples. First, a two-stage Miller op-amp is analysed to show the importance of the correlation between distortion sources in a real examples. With the second example, a Doherty power amplifier, the DCA is shown to work for strongly non-linear circuits as well. The weakly non-linear assumption doesn’t hold in such an amplifier, so the BLA of the sub-circuits is estimated with zippered multisines. The third and last example deals with a considerably larger circuit. The distortion contribution analysis of a a fully differential Gm-C biquad is used to illustrate how the DCA can be applied hierarchically.

A. Miller Op-amp

As a first example of a DCA on the circuit level, we consider a two-stage Miller-compensated op-amp designed in a commercial 0.18µm CMOS technology (Fig. 17). The op-amp is placed in an inverting feedback configuration with a gain of 5 and drives a load capacitance of 10pF, resulting in a gain-bandwidth product of 10MHz. The circuit is split into three sub-circuits: the input stage, which has three ports, the current mirror, and the output stage which both have two ports.

The amplifier is excited by lowpass random-odd multisines with a base frequency \( f_0 = 100\text{kHz} \) and \( f_{\text{max}} = 10\text{MHz} \). The RMS voltage of the multisines is set to 0.1V. The steady-state response of the circuit to 50 different-phase multisines is obtained with HB simulation. The obtained spectrum of the output wave \( B_t \) is shown in Fig. 18.

The Miller op-amp can be considered to be weakly non-linear, so the small-signal S-parameters were used to represent each sub-circuit. To test the validity of this small-signal assumption, the BLA from the reference multisine to all waves in the circuit was compared to the result obtained with an AC simulation as explained in Section (IV-B). The largest difference is observed on the frequency response from the reference to the output wave of the second stage and it is shown in Fig. 19. This difference is small enough to allow using the small-signal S-parameters instead of the MIMO BLA to represent each sub-circuit.

The results of the DCA at four different frequencies are shown in Fig. 20. The contributions are combined for each sub-circuit.
Figure 19. The frequency response from input voltage to the output wave of the circuit (-) doesn’t lie far from the corresponding BLA (+), so the small-signal S-parameters can be used to represent the sub-circuits instead of the MIMO BLA.

Figure 20. Distortion contributions to the output wave of the Miller op-amp at four different frequencies.

The even and odd distortion contributions can be split again because odd RPM were used. At 200kHz (top left in figure 20), we obtain only even-order non-linear contributions, since this is an even frequency line as defined by the chosen frequency grid. The input stage seems to generate the most distortion with a direct contribution which is 150% of the total output distortion at that frequency bin, but its contribution is largely cancelled in its correlation with the current mirror, which has a contribution of −120% of the total distortion. This leads to the output stage as the dominant source of distortion at 200kHz. This shows the importance of the correlation between the distortion sources in the circuit.

The bottom series of plots in Fig. 20 shows the results at the high-frequency end of the analysed band. At 9.8MHz, the output stage dominates the even-order contributions. At 9.9MHz, the odd contribution of the output stage (350%) is compensated with a covariance with the input stage (−350%), making the input stage the dominant source of distortion at this particular frequency.

With this example, we have shown that the BLA-based DCA can be used in circuit-level simulations to obtain the distortion contributions. Taking the correlation between distortion sources of different sub-circuits into account is crucial to obtain an accurate representation of the circuit’s distortion.

B. Doherty Power Amplifier

The second example that will be considered is a Doherty power amplifier found in the example library of Keysight’s Advanced Design System (ADS). The amplifier is built with two Freescale MRF8S21100H transistors for a centre frequency of 2.14GHz (Fig. 21). The main transistor is biased in class-AB with a quiescent current of 0.7A. The auxiliary transistor is biased deep in class-C with a quiescent current of 1.1mA.

The amplifier is excited by bandpass multisines that have 41 spectral lines in a band of 10MHz around 2.14GHz. The RMS of the input multisines is 22dBm. The steady-state response of the circuit to the different-phase multisine excitations is obtained by HB simulation. The spectrum of the output wave $B_t$ around the centre frequency is shown in Fig. 22.

This circuit can be considered to be strongly non-linear, especially because of the auxiliary amplifier. To obtain the BLAs of the two transistors in the circuit, tickler multisines are added to the circuit at the output of the total amplifier. The added multisines are current sources which insert an RMS current of 1µA on frequency bins in between the frequencies of the main multisine. 60 different-phase multisines were used to estimate the MIMO BLA and derive the distortion present in the circuit. The total simulation time for this Doherty Power amplifier was 2 hours (approximately 2 minutes for one different-phase multisine). Increasing the number of different-phase multisines decreases the variability of the distortion estimate in Fig. 22 but inevitably increases the computational complexity and cost.

The main distortion contributor is found to be the main transistor (Fig. 23). This can be expected, as the auxiliary...
amplifier only kicks in for limited amounts of time in this Doherty configuration. A similar Doherty amplifier was analysed in [31] with a Volterra-based DCA under two-tone excitation\(^4\). It was concluded there that the auxiliary amplifier only contributes significantly to the distortion for very high amplitudes in the two-tone. With modulated signals, like the multisines used in the BLA-based DCA, the peaks only occur from time to time, so the average contribution of the auxiliary amplifier to the total distortion is low.

The information given by the BLA-based DCA is limited for the Doherty power amplifier, because only the signals around the centre frequency are used here. Designers are also interested in how the low-frequency signals in the bias and supply networks are up-converted in-band through the second-order non-linearities [2]. The current implementation of the DCA doesn’t split the up-converted low-frequency signals from the high-frequency odd-order non-linear distortion appearing in-band. A more advanced BLA-based DCA can be implemented using the higher-order BLA [32]. With the higher-order BLA, one could obtain a similar result to [31], but for modulated signals, instead of two-tones.

### C. Gm-C filter

The final example is a fully differential Gm-C biquad [33] designed in the same commercial 0.18\(\mu\)m CMOS technology as the other examples (Fig. 24). Each Operational Transconductance Amplifier (OTA) in the biquad consists of an input pair and a cascode stage. The common-mode feedback in the OTA is active. The biquad is configured to create a resonant pole pair at 10MHz.

The differential mode of the biquad is excited by full lowpass RPM (all frequency lines are excited). The multisines have \(f_0= f_{\text{min}}=200\text{kHz}\) and \(f_{\text{max}}=100\text{MHz}\). In a resonant system like this, the frequency resolution of the multisines should be chosen to have several lines in the resonance [34]. If, for example, only a single spectral line is placed in a sharp resonance, the PDF of the internal signals will tend to that of a sine wave, instead of the wanted Gaussian PDF. The wanted noise-like properties of the internal signals in the circuit then disappear, which is unwanted if the results are to be valid for Gaussian input signals.

The RMS of the multisines was set to 50mV and the steady-state response of the circuit to 50 different-phase multisines was obtained with HB. The resulting spectrum at the differential output is shown in Fig. 25. The output distortion lies 50dB below the signal level, so the circuit is behaving close to linear. No even-order contributions are present due to the differential nature and perfect symmetry in the simulations of the circuit. Note that the obtained odd-order distortion at the output shows a strong frequency dependence around the resonance. The sub-circuits in this biquad are assumed to be weakly non-linear, so the 4-port S-parameters of each OTA were used in the DCA.

The small-signal assumption was verified by comparing the frequency response from the input of the total circuit to each of the waves in the circuit with the corresponding BLA. The largest difference was observed on the frequency response from the reference to the output waves of OTA4 (shown in Fig. (26)), but this difference is small enough to consider the small-signal assumption to be valid.

The first OTA is found to be the dominant source of distortion in the resonance peak of this circuit (Fig. 27, left). The fourth OTA also introduces a considerable contribution. To find out which part of the OTA is mainly responsible, the first and fourth OTA were split into two parts and the DCA was applied again. With this hierarchical application of the BLA-based DCA, it is found that the first stage of both OTA 1 and 4 are the dominant contribution (Fig. 27 right).

With this final example, we have demonstrated how the BLA-based DCA can be used in larger circuits and how it can be used hierarchically to zoom in on certain sub-circuits to

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\(^4\)We don’t have access to the circuit simulated in [31], nor to their DCA method, so only a qualitative comparison between both methods can be obtained.
determine the actual source of non-linear distortion.

VI. CONCLUSIONS

Combining the BLA with a noise analysis creates a DCA that pin-points the sources of non-linear distortion in a circuit under excitation of complex, modulated signals. The method doesn’t require special models and can be implemented easily with the help of a few post-processing steps on top of a commercial simulator.

The BLA-based DCA works for both weakly and strongly non-linear circuits, returns a single distortion contribution for each sub-circuit, and takes the correlation between the distortion introduced by the sub-circuits into account. The method can be applied hierarchically and is therefore suitable to be applied to large circuits without introducing hundreds of contributions.

The method has been demonstrated on several examples and is shown to be able to provide information about the source of non-linear distortion in a two-stage op-amp, a Doherty power amplifier and a fully differential Gm-C filter.

APPENDIX A

POWER DEPENDENCE OF THE BLA AND THE DISTORTION

In this appendix, we demonstrate how the BLA and non-linear distortion change in function of the power of the input multisines and how both can be different from the results obtained with a single-tone or two-tone excitation signal. Consider the following static non-linearity \( y = \tanh(u) \) which is a saturation function that closely resembles the voltage-to-current transfer function of a bipolar transistor [35] or a MOST differential pair biased in weak inversion [36]. The validity of the harmonic distortion scaling rules in function of the input power is verified by applying a single-tone, two-tone and random-odd RPM (Section I-A) to this weakly nonlinear function (Fig. 28). Even though the different excitations result in a different absolute value for the distortion component, this specific example shows that for static weakly non-linear systems the same scaling rules for 2\(^{nd}\)-order distortion (1 dBW/dBW) and 3\(^{rd}\)-order distortion (2 dBW/dBW) apply until compression occurs.

In a more realistic setting (low-noise amplifier in 90 nm CMOS), large discrepancies in the inter-modulation distortion between two-tones and multisines were documented (Fig. 29) [16]. In the case of the two-tone, a local minimum (the so-called sweet-spot) appears which gives a wrong impression about the actual inter-modulation distortion present in the circuit when complex modulated signals are used.
APPENDIX B

OBTAINING EXPRESSION (11)

In the circuit shown in Fig. 6, the following equations describe the behaviour of the input, output and feedback dynamics:

\[ U = AR - MY \]  \hspace{1cm} (21)
\[ Y_1 = BY \]  \hspace{1cm} (22)

The relation between \( U \) and \( Y \) is given by the BLA:

\[ Y = G_{U \rightarrow Y}^{BLA} U + D \]  \hspace{1cm} (23)

Plugging equation (21) into (23) and solving for \( Y \), we obtain:

\[ Y = (I_N + G_{U \rightarrow Y}^{BLA} M)^{-1} (G_{U \rightarrow Y}^{BLA} AR + D) \]

Using this expression in (22) and grouping the terms in \( R \) and \( D \) yields equation (11).

APPENDIX C

BLA WITH S-PARAMETERS

To calculate the contributions of the distortion sources to the output of the circuit, an algorithm similar to the one described in [25] is used. The different S-matrices of the components in the circuit shown in Fig. 12 are gathered in a matrix \( T \), while the distortion sources are gathered in a vector \( N \):

\[
T = \begin{bmatrix}
\Gamma_{in} & \Gamma_{out} \\
0 & \text{S}_{A \rightarrow B}^{BLA}
\end{bmatrix}
\]

\[
N = \begin{bmatrix}
0 \\
0_{P+2 \times 1} \\
D
\end{bmatrix}
\]

where \( \Gamma_{in} \) and \( \Gamma_{out} \) are the reflection factors presented to the circuit by the reference source and load respectively. \( \text{S}_{A \rightarrow B}^{BLA} \) is the block diagonal matrix of size \( P \times P \) which contains the BLAs of the circuits as defined in equation (15). \( D \) is the vector of distortion sources of length \( P \) defined in the same expression. \( P \) is the S-matrix of the package defined in Fig. 12 of size \( (P+2) \times (P+2) \) for a circuit with 2 external ports.

The interconnection between the different parts of the circuit is represented by the following matrix:

\[
C = \begin{bmatrix}
0_{2 \times 2} & I_{2 \times 2} & 0_{4 \times P} \\
I_{2 \times 2} & 0_{2 \times 2} & 0_{P \times 4} \\
0_{P \times 4} & 0_{P \times P} & I_{P \times P} \\
0_{P \times P} & 0_{P \times P} & 0_{P \times P}
\end{bmatrix}
\]

The incident-waves at all ports generated by the sources in \( N \) is given by the following expression:

\[ A_{all} = (C - T)^{-1} N = W^{-1} N \]  \hspace{1cm} (24)

Since we are only interested in the wave incident to the load, just the second row of \( W^{-1} \) is used. Also, the first \( P + 4 \) elements of \( W^{-1} \) can be ignored, because the first \( P + 4 \) elements of \( N \) are zero. This finally leads to the expression for \( T_{out} \) used in equation (17):

\[ T_{out} = [W^{-1}]_{2,P+5.2P+4} \]  \hspace{1cm} (25)

APPENDIX D

PREDICTING THE BLA FROM REFERENCE TO THE WAVES IN THE CIRCUIT

Predicting the frequency response from the main reference signal to the input and output waves at the ports of the sub-circuits is done using the same matrix \( W^{-1} \) as was used in Appendix C. But now, the \( N \)-vector is set to the following:

\[ N = \begin{bmatrix}
\frac{1 - \Gamma_{S}}{2 \pi Z_0} \\
0_{2P+1 \times 1}
\end{bmatrix}
\]

where \( Z_0 \) is the chosen reference impedance and \( \Gamma_{S} \) is the reflection factor presented by the reference source. All A-waves in the circuit are now predicted by (24). The frequency response from the reference voltage source to the A-waves radiating into the sub-circuits are found at \([A_{all}]_{4P+1.4P+2P+1}\).

The frequency response from the reference voltage source to the B-waves at the ports of the sub-circuits are found at \([A_{all}]_{5.5+P+1}\).

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