Complex mathematical model of the underwater robot operation

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Abstract. The article is devoted to a complex mathematical model of the functioning of an underwater robot that allows solving the problem of operational correction of a service task directly during its execution. The successful usage of UAV largely depends on the quality of preparation and programming of the UAV mission, including the current factors of the water environment. Since UAV works in an aquatic environment, the modeling of its functioning should be carried out taking into account both the environment itself and the dynamics of changes in its properties. The developed mathematical model was implemented in the form of a computer model and confirmed its operability within the declared ranges of initial conditions and initial data. The proposed complex mathematical model can be used for planning of assignments given available energy on board the UAV and allows to evaluate the energy for the operation of the on-board equipment after performing the UAV maneuver in the operating zone and the necessary reserves for the ascent of the unit, its detection and receiving aboard the carrier.

1. Introduction
An absolute trend in the development of modern means of studying the Ocean is the increasingly widespread use of autonomous underwater vehicles (UAV) and unmanned ships [1]. As noted in [2], UAV developing most dynamically. In the future, the creation of small autonomous vehicles with energy-intensive and renewable power sources will allow the implementation of automated networks for both oceanographic measurements and for lighting the underwater situation in all areas of the World Ocean [3, 4].

The successful usage of UAV largely depends on the quality of preparation and programming of the UAV mission, including the current factors of the water environment. One of the possible ways to improve the quality of planning the UAV mission is to develop a special software module that uses a complex model of the functioning of the device in the aquatic environment.

Such a complex model should use well-known, reliable, proven and simple discrete models that allow you to plan the optimal performance of the service task, taking into account the dynamically changing parameters of the water environment.

The proposed approach to the development of a complex model will allow in the future to create a computing core for an on-board intelligent control system that allows you to quickly adjust the service task directly during its execution.
2. Composition and general characteristics of the complex model of the underwater robot operation

The complex model is intended for simulating of the UAV mission and consists of the following discrete models:

- UAV operational model;
- UAV structural and functional models;
- UAV energy model;
- the aquatic environment model.

The operational model describes the sequence of events and changes in the UAV during a typical mission. Under the typical mission, the following sequence of actions implemented by the UAV is accepted: diving from the water surface to a given point in the water space (operating zone), performing special underwater technical works (SUTR), ascent to the water surface. SUTR in the framework of this review means the regular use of UAV on-board equipment in the operating zone.

The UAV structural and functional model describes the element composition of the underwater robot and the functional relationships between them at various stages of the mission. The UAV energy model describes the structure and dynamics of energy consumption by the elements of the underwater robot during the mission. The water environment model describes the dynamics of the interaction of the UAV with the water environment in the operating zone. The parameters of the water environment that significantly affect the progress and results of the performance of the mission may differ significantly depending on the geographical coordinates of the operating area, depth, season and time of day.

3. UAV operational model

In general, the UAV mission can be described by the following sequence of actions:

- UAV approach to the SUTR facility;
- the actual implementation of the SUTR;
- UAV escape from the SUTR facility.

It should be noted that the UAV, in turn, is an element of the deep-sea technical complex that provide the possibility of its use for its intended purpose. Most modern UAV, due to their technical characteristics, cannot be used completely independently. As a rule, the UAV requires a carrier of significantly larger displacement than the device itself. Within the framework of the proposed operational model, we will consider a surface ship as a support vessel and at the same time as an UAV carrier. Thus, the performance of the UAV mission, taking into account the need to use the carrier vessel, can be described as a sequence of the following major stages:

- launching the UAV from the carrier;
- UAV withdrawal from the carrier;
- UAV dive from the water surface to the SUTR depth;
- UAV approach to the SUTR facility;
- the actual execution of the SUTR using on-board equipment;
- UAV escape from the SUTR facility;
- UAV ascent to the water surface;
- UAV receiving on board the carrier.
The use of UAV for SUTR performing is limited to the operating area. Under the operating area of the space refers to the area, a limited lateral surface of a vertical cylinder, the height of which is determined by the maximum immersion depth of AUV, and the radius is restricted by technical means of ship-carrier-detection surfaced after the performance of mission. When programming a mission, the UAV is not allowed to leave the operating zone, and in the case of approaching its border, it is necessary to provide for the execution of a maneuver to keep the device inside the operating zone. In the future, perhaps a more accurate description of the geometrical shape of the operating zone, limiting the space environment, which may provide a sustainable hydroacoustic communication AUV with the ship carrier. In this case, the shape of the operating area will be significantly different from the cylinder, and the requirement for the shape and size of the waiting area of the UAV in the drift will remain.

4. Structural and functional model of UAV
The elemental composition and the connections between the elements of the UAV are shown in figure 1, where the solid lines show the energy connections between the elements, and the dotted lines show the information connections.

![Figure 1. Structural and functional model of UAV.](image)

The movement of the UAV in the water environment is provided by the propulsion and steering complex (PSC) and the buoyancy control system. The calculation of the trajectory parameters, navigation and issuing of control commands is carried out by the autopilot according to the data of the hydroacoustic station (GAS) and other sensors of the control system. The on-board equipment is activated at the point where the SUTR is executed. After completing the mission and surfacing, a radio beacon and a flashing optical beacon are activated.

5. Energy model of UAV
The main problem of planning mission is to ensure the energy balance of the UAV, which can be expressed by the ratio:

$$Q_{\text{available}} > Q_{\text{required}},$$

where $Q_{\text{available}}$—available power supply on board of UAV; $Q_{\text{required}}$—the estimated energy reserve on board of UAV, which ensures the mission.

The following design properties of the underwater vehicle itself will also affect the energy consumption during the mission:

- changes in the volume of the UAV under the influence of hydrostatic pressure;
- the hydrodynamic form of UAV;
- distribution of energy consumption between the UAV subsystems in the overall structure of energy consumption during the mission.
It should be noted that the energy consumption of the on-board source of the UAV during the mission is uneven. You can conditionally distinguish three stages of mission in terms of energy consumption:

- dive and arrive at the starting point of mission;
- driving on a given route;
- surfacing and waiting for ascent to the support vessel.

At the first stage, the main energy consumers are the following subsystems of the UAV: PSC, the drives of the buoyancy and trim control system, and the autopilot. At the second stage, in addition to the subsystems listed above, the on-board equipment that provides the performance of mission consumes energy. At the third stage, before surfacing, the main energy consumers are the PSC, the drives of the buoyancy and trim control system, the autopilot, and after surfacing, the equipment is used to quickly detect the UAV and lift it on board the support vessel (radio and sonar beacons, optical flashing beacon). At the same time, after ascent on the surface of water the PSC, the buoyancy and trim control systems, the autopilot may not be used, the UAV is in free drift. The on-board hardware is also disabled. The energy consumption for each of the consumers can be set based on experimental data, and in case of their absence in the form of percentages based on expert estimates. The energy consumption for movement (PSC operation) significantly depends on both the hydrodynamic shape of the UAV and the speed of its movement. In general, the higher the speed of movement, the greater the hydrodynamic resistance and, accordingly, the energy consumption of the PSC. On the other hand, the speed of movement less than the speed of the current will lead to the displacement of the UAV and its withdrawal outside the operating zone. In addition to the flow, the energy consumption of the UAV is significantly affected by the properties of the water, which depend on the hydrological conditions of operating zone.

6. Model of the aquatic environment

Since UAV works in an aquatic environment, the modeling of its functioning should be carried out taking into account both the environment itself and the dynamics of changes in its properties:

- water density, depending on its salinity, temperature and hydrostatic pressure;
- the direction and speed of water flow in the operating zone.

A mathematical model of the medium that allows us to take into account the dynamics of changes in water density as a function of salinity, temperature, and hydrostatic pressure is represented by a set of generally accepted, well-tested dependencies [5]. The density \( \rho \) of seawater as a function of the practical salinity \( S \), temperature \( t \), and hydrostatic pressure \( p \) is determined by the equation:

\[
\rho(S, t, p) = \frac{\rho(S, t, 0)}{1 - p / K(S, t, p)},
\]

where \( K(S, t, p) \) — average modulus of elasticity; 0 — corresponds to the standard atmosphere unit (101 325 Pa). The specific volume \( V \) of seawater is defined as:

\[
V(S, t, p) = \frac{V(S, t, 0)}{1 - p / K(S, t, p)}.
\]

The density is given by the ratio:

\[
\rho(S, t, 0) = \rho_w + \left( b_0 + b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 \right) S + \left( c_0 + c_1 t + c_2 t^2 \right) S^{3/2} + d_0 S^2,
\]
where $b_0 = 8.24493 \cdot 10^{-1}; b_1 = 8.24493 \cdot 10^{-1}$; $b_2 = 7.6438 \cdot 10^{-5}; b_3 = -8.2467 \cdot 10^{-7}; b_4 = 5.3875 \cdot 10^{-9}; c_0 = -5.72466 \cdot 10^{-3}; c_1 = 1.0227 \cdot 10^{-4}; c_2 = -1.6546 \cdot 10^{-6}; d_0 = 4.8314 \cdot 10^{-4}; \rho_w$—density of standard mid-ocean clean water:

$$\rho_w = a_0 + a_1t + a_2t^2 + a_3t^3 + a_4t^4 + a_5t^5,$$

where $a_0 = 999.842594; a_1 = 6.793952 \cdot 10^{-2}; a_2 = -9.095290 \cdot 10^{-3}; a_3 = 1.001685 \cdot 10^{-4}; a_4 = -1.120083 \cdot 10^{-6}; a_5 = 6.536332 \cdot 10^{-9}.$

The value of $K(S, t, p)$ is defined by the expression:

$$K(S, t, p) = K(S, t, 0) + Ap + Bp^2,$$

$$K(S, t, 0) = K_w + (f_0 + f_1t + f_2t^2 + f_3t^3)S + (g_0 + g_1t + g_2t^2)S^{3/2},$$

where $f_0 = 54.6746; f_1 = -0.603459; f_2 = 1.09987 \cdot 10^{-2}; f_3 = -6.1670 \cdot 10^{-5}; g_0 = 7.944 \cdot 10^{-2}; g_1 = 1.6483 \cdot 10^{-2}; g_2 = -5.3009 \cdot 10^{-4};$

$$A = A_w + (i_0 + i_1t + i_2t^2)S + j_0S^{3/2},$$

where $i_0 = 2.2838 \cdot 10^{-3}; i_1 = -1.0981 \cdot 10^{-5}; i_2 = -1.6078 \cdot 10^{-6}; j_0 = 1.91075 \cdot 10^{-4};$

$$B = B_w + (m_0 + m_1t + m_2t^2)S,$$

where $m_0 = -9.9348 \cdot 10^{-7}; m_1 = 2.0816 \cdot 10^{-8}; m_2 = 9.1697 \cdot 10^{-10};$

$$K_w = e_0 + e_1t + e_2t^2 + e_3t^3 + e_4t^4,$$

where $e_0 = 19652.21; e_1 = 148.4206; e_2 = -2.327105; e_3 = 1.360477 \cdot 10^{-2}; e_4 = -5.155288 \cdot 10^{-5};$

$$A_w = h_0 + h_1t + h_2t^2 + h_3t^3,$$

where $h_0 = 3.239908; h_1 = 1.43713 \cdot 10^{-3}; h_2 = 1.16092 \cdot 10^{-4}; h_3 = -5.77905 \cdot 10^{-7};$

$$B_w = k_0 + k_1t + k_2t^2,$$

where $k_0 = 8.50935 \cdot 10^{-5}; k_1 = -6.12293 \cdot 10^{-6}; k_2 = 5.2787 \cdot 10^{-8}.$

To determine the current value of the water temperature depending on the depth and geographical location of the operating zone, the interpolation of the graphs (figure 2) given in [6] is used.

The decrease in the displacement of the UAV associated with the action of hydrostatic pressure, i.e. its crimp when performing a service task depends on the individual design and technological characteristics of a particular device and is taken into account by a constant. In the future, when refining a specific model of the UAV, it is possible to apply the experimentally obtained dependence of the change in the displacement of the device with the depth of immersion. Within the framework of the introduced model, limit the depth of the operating zone to 1000 m, since at present most of the UAV in operation are designed for this limit value. The size of the operating zone is set from the condition of the possibility of visual detection from the UAV carrier that floated to the surface after performing a mission and is in the drift. From [6, 7] we determine the position of the border of the visual detection zone of the UAV that is drifting on the water surface: $De = 2.08 \sqrt{\varphi}$, where $De$ is the range of the visible horizon in miles, and $\varphi$ is the height of the observer’s eyes in meters. For example, by setting the height of the observer’s eyes to 5 m, we get the radius of the operating zone equal to 4.7 miles or 7.4 km.
Figure 2. Types of water temperature changes with depth: EqT—equatorial-tropical; ET—eastern tropical; T—tropical primary; SubT—subtropical; Med—Mediterranean and red sea; APa—Atlantic–Pacific; SubP—subpolar; P—polar.

For a complete description of the dynamics of the water environment in the operating room, we set the current. According to [8], the primary cause of surface currents in the World Ocean is the winds of the lower atmosphere, which create tangential stresses on the ocean surface. The same source notes that the thickness of the water layer affected by surface circulation depends on the stratification of the water column and extends to a depth of up to 300–500 meters in low latitudes and up to thousands of meters in the Arctic and Antarctic. In the proposed model, the flow is defined by cylinders (flow tubes) with diameters $D$ and located at different depths at arbitrary angles and offset from the center of the operating zone. In this case, the condition is met: the value of $D$ significantly exceeds the maximum linear size of the UAV. Since in practice the change in the direction and velocity of the flow changes smoothly with increasing depth, the change in the flow parameters in the model is achieved by moving the flow tube synchronously with the movement of the UAV with a simultaneous change in the flow velocity vector.

In the model, the geometry of the UAV is not taken into account, the body of the device is taken as a material point, and the axis of the tube section passes through this material point. The model allows setting the actual dimensions of specific UAV with a corresponding increase in the diameter of the flow tube. The specific numerical characteristics of the currents (speed and direction at different horizons) depend significantly on the geographical location, time of year, and time of day, and are taken from the corresponding latitudes as deterministic values.

7. Conclusions
The developed mathematical model was implemented in the form of a computer model and confirmed its operability within the declared ranges of initial conditions and initial data. The proposed complex mathematical model can be used for planning of assignments given available energy on board the UAV and allows to evaluate the energy for the operation of the on-board equipment after performing the UAV maneuver in the operating zone and the necessary reserves for the ascent of the unit, its detection and receiving aboard the carrier. With the help of the proposed model, the problem of calculating the program trajectory of the UAV movement can be solved, ensuring its maximum energy efficiency, i.e. minimizing the energy consumption of the UAV system during dive and transition to the area of mission performance and return to the surface. At the same time, the greatest energy reserve is provided and, consequently, the operating time of the on-board equipment of the UAV.
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