Characterization of quark gluon plasma as seen through the energy loss of heavy quarks

Ashik Ikbal Sheikh and Zubayer Ahammed
Variable Energy Cyclotron Centre, HBNI, 1/AF Bidhan Nagar, Kolkata - 700 064, India

The shear viscosity to entropy density ratio ($\eta/s$) of quark gluon plasma produced in ultra-relativistic heavy-ion collisions has been studied from the energy loss of heavy quarks in QGP medium. We have also studied the bulk viscosity to entropy density ratio ($\zeta/s$) and fluidity measure ($F$) of the medium using the obtained $\eta/s$ values. In addition to that, we have estimated the heavy quark bound state potential ($V$) inside this medium. Our finding of $\eta/s$ agrees well with the results obtained by Lattice QCD (LQCD) and functional renormalization group technique.

PACS numbers: 13.85.Hd, 25.75-q

I. INTRODUCTION

The ultra-relativistic heavy ion collisions in the Relativistic Heavy Ion Collider (RHIC) at BNL and Large Hadron Collider (LHC) at CERN aim to create a hot and dense deconfined state of QCD matter called Quark Gluon Plasma (QGP)\cite{1,2}. The main emphasis of these present day’s heavy ion programs is to characterize the QGP or more precisely to determine its transport coefficients\cite{3}. Immediately after the creation, the QGP will be cooled by expansion due to the large internal pressure and will revert to the hadronic phase. The existence of these two phases (deconfined QGP phase and hadronic phase) has been confirmed by recent LQCD calculations\cite{4,5} and experimental observations\cite{6,7,8,9,10}. During the transition from the deconfined QGP phase to the hadronic phase, the system may encounter the critical point in the QCD phase diagram. The characterization of the medium at critical point is one of the most challenging problem in heavy ion collisions at the relativistic energies. The LQCD calculations indicate that the transition occurs at the critical temperature around $T_c \sim 155$ MeV\cite{12} at zero baryonic chemical potential.

There are many probes in order to characterize the deconfined QGP phase. One of the efficient probe is the heavy quarks which are mostly produced from the fusion of partons in early stage of the collisions. No heavy quarks are produced at the latter stage if the temperature of the system is less than the mass of heavy quark pair and none in the hadronic matters. Hence, the total number of heavy quarks becomes frozen at the very beginning in the history of the collisions which enables them to play a crucial role to characterize the QGP. Immediately after the production of heavy quarks, they will propagate through QGP medium and start losing energy via elastic collisions\cite{13,14,15,16,17,18} and bremsstrahlung gluon radiations\cite{17,19}. There exists a transport parameter $\hat{q}$ which governs these collisional and radiative energy losses of the propagating heavy quarks\cite{28,29,30,31,32,33,34,35,36,37,38}. Generally, this $\hat{q}$ is sensitive to the interaction of the probes with the medium and has been used to calculate the shear viscosity to entropy density ratio ($\eta/s$)\cite{28,31}. If a heavy quark anti-quark($Q\bar{Q}$) bound state is present in this medium, then it’s binding potential will be affected due to the temperature $T$ and $\eta/s$ of this medium\cite{32,33,34}. The effects of the medium are encoded in a temperature dependent $Q\bar{Q}$ bound state potential\cite{35,36,37}. Hence, the estimation of the transport coefficients of QGP by using heavy quarks are of immense interest of research\cite{28,31,35,36,37,38,39}.

In this article, we will revisit the estimation of the various transport coefficients of the QGP. We have estimated $\eta/s$ from the $\hat{q}$ corresponding to the energy losses of heavy quarks in QGP. Along with that, we have also estimated bulk viscosity to entropy density ratio($\zeta/s$) and fluidity measure($F$) of QGP. It is interesting to note that, we found $\eta/s$ a satisfactorily good agreement with the calculations of LQCD and functional renormalization group technique. In addition to that, $QQ$ bound state potential has been reported to understand the effect of the QGP medium we have characterized.

This article is organised as follows: In the next section, we briefly discuss the energy loss of heavy quarks and transport parameter $\hat{q}$ associated to the energy loss. Here we consider the collisional energy loss of heavy quarks by Peigne and Pashier (PP) formalism\cite{10} and the radiative energy loss by Abir, Jamil, Mustafa and Srivastava (AJMS) formalism\cite{20}. In sec. III we discuss some properties of QGP, mainly the $\eta/s$, $\zeta/s$, $F$ and $QQ$ bound state potential. The summary and conclusion are in sec. IV.

II. ENERGY LOSS AND TRANSPORT PARAMETER

In relativistic heavy ion collisions, the initially produced energetic heavy quarks will have to go through
multiple scatterings and induced gluon bremsstrahlung radiations as it propagates through the medium. The calculation of the collisional energy loss per unit length \( -dE/dx \) is reported by several authors\[13–16\]. One of the reliable calculation is made by Peigne and Pashier\[16\]. Another important and dominant process of energy loss from a fast energetic parton in QGP is due to the gluon radiations. This radiative energy loss has also been calculated in the past (see Ref.\[17–26\]). The energy loss by gluon emission has been calculated based on generalized dead cone approach in the Ref.\[26\]. We calculate these energy loss over the QGP life time and finally average over the temperature evolution. The initial condition used for the hydrodynamic medium evolution as in Ref.\[27\]. We take initial time \( \tau_0 = 0.3 \) fm and freeze-out time \( \tau_f = 6 \) fm. In the context of these energy loss, the transport parameter \( \hat{q} \) can be defined as the square of average momentum transfer between the probe and bath particles per unit length. If a highly energetic parton travels a distance \( d \) in QGP so that it loses its energy per unit length \( -dE/dx \), then the \( \hat{q} \) takes the form\[30\]:

\[
\hat{q} = \frac{1}{\alpha_s} \left( \frac{dE}{dx} \right) ,
\]

where \( \alpha_s \) is the strong coupling constant.

\[\text{FIG. 1: The energy loss per unit length, } -dE/dx, \text{ of a charm quark inside QGP medium as a function of its energy } E, \text{ obtained using PP formalism}\[16\] (for Collisional loss (Coll.)) and AJMS formalism\[26\] (for Radiative loss (Rad.)).\]

\[\text{FIG. 2: Variation of transport parameter } \hat{q} \text{ with temperature } T \text{ of the QGP medium. We choose } d = 5 \text{ fm.}\]

\[\text{FIG. 3: Variation of transport parameter } \hat{q} \text{ with temperature } T/T_c \text{ when the heavy quarks undergo both collisional and radiative processes. We compare } \eta/s \text{ as the function of } T/T_c.\]

\[\text{III. PROPERTIES OF QGP}\]

\[\text{A. Shear viscosity to entropy density ratio } (\eta/s)\]

Viscosity measures the resistance of a fluid deformed either by tensile stress or shear stress. The less viscosity means greater fluidity. In order to characterize the QGP medium, amongst many, \( \eta/s \) is one of the important quantity. It is an important dimensionless measure of how imperfect or dissipative the QGP is. In this work, we evaluate \( \eta/s \) by calculating the transport parameter \( \hat{q} \) from the energy loss of heavy quarks in QGP. A heavy quark with certain momentum while propagating through QGP encounters the QGP bath particles and hence the momentum exchange occurs with the bath particles. We can define \( \hat{q} \) by square of this average momentum exchange per unit length. So, the momentum of the energetic heavy quarks is shared by the low momentum (on an average) bath particles which causes minimization of momentum (or velocity) gradient in the system. Therefore, it is related to the shear viscous coefficients of the system which drive the system towards a depleted velocity gradient.

With the definition of \( \hat{q} \), the expression of \( \eta/s \) reads as\[28\]:

\[
\eta/s \approx 1.25 T^3 \hat{q}/4 ,
\]

where \( T \) is the temperature of the QGP medium.

Ads/CFT calculations\[45\] predicts a lower bound of \( \eta/s \), which is \( \frac{\eta}{s} \geq \frac{1}{4\pi} \). In Fig.\[3\] we display \( \eta/s \) as a function of \( T/T_c \) when the heavy quarks undergo both collisional and radiative processes. We compare \( \eta/s \)
with the results obtained by Mazumder et. al.\cite{39} from charm quark’s energy loss, LQCD calculations\cite{40–42} and calculations from functional renormalization group technique\cite{43}. Our finding of $\eta/s$ is in a good agreement with the LQCD calculations\cite{40–42} and calculations from functional renormalization group technique\cite{43}.

### B. Bulk viscosity to entropy density ratio ($\zeta/s$)

One another important quantity for QGP characterization is bulk viscosity to entropy density ratio ($\zeta/s$). Bulk viscosity acts as a resistance against the volume expansion of a fluid and it slows down the evolution of the system. A number of theoretical calculations have been done over the years to uncover the temperature dependence of $\zeta/s$ for the QGP.

The bulk viscosity coefficient of QGP phase is parametrized as\cite{44},

$$\zeta/s \approx 15 \frac{\eta}{s} \left( \frac{1}{3} - c_s^2 \right)^2,$$

(3)

Here the values of $\eta/s$ are used from our findings(from Eq(2)) and $c_s$, the velocity of sound is taken from LQCD calculations\cite{3}. In Fig(3) we show $T$ dependence of $\zeta/s$. $\zeta/s$ peaks near $T_s \sim 155$ MeV and decreases after that as $T$ increases. The peak of $\zeta/s$ near $T_s$ is due to the lower value of $c_s$ around $T_s$ and large conformal breaking $\left( \frac{1}{3} - c_s^2 \right)^2$. $\zeta/s$ is insignificant at higher temperature and hardly plays any role in the high temperature regime.

### C. Fluidity measure ($F$)

The measure of fluidity($F$) is based on the propagation of sound wave within the medium having dissipation due to viscosity and under what condition the sound wave propagation has stopped because of dissipation\cite{46}. Two length scales $L_\eta$ and $L_n$ in the medium have been introduced. $L_\eta$ provides a measure for the minimal wavelength of a sound wave to propagate in such a viscous medium and $L_n$ is the de-correlation length of a certain density-density spatial correlator which gives a natural scale of short range order in the system. In the most cases, this de-correlation length $L_n$ is the interparticle distance. Finally, the ratio of these two length scales gives a dimensionless quantity, termed as fluidity measure($F$). The expression of $F$ for a system having zero thermal conductivity and vanishing net charge density is taken from the Ref.\cite{47} as:

$$F \approx \frac{L_\eta}{L_n}.$$

Here the values of $\eta/s$ are used from our findings(\cite{3}) and $c_s$, the velocity of sound is taken from LQCD calculations\cite{3}. In Fig(4) we show $T$ dependence of $\zeta/s$. $\zeta/s$ peaks near $T_s \sim 155$ MeV and decreases after that as $T$ increases. The peak of $\zeta/s$ near $T_s$ is due to the lower value of $c_s$ around $T_s$ and large conformal breaking $\left( \frac{1}{3} - c_s^2 \right)^2$. $\zeta/s$ is insignificant at higher temperature and hardly plays any role in the high temperature regime.

FIG. 4: Temperature ($T$) variation of bulk viscosity to entropy density ratio $\zeta/s$.
\[ F = \frac{\rho^{1/3}}{2\sqrt{2}} \left[ \zeta^2 + 8\xi \eta + 16\eta^2 + 2(P + \epsilon)\beta_0 \xi^2 \left( \frac{\partial P}{\partial \epsilon} \right) + 48\eta^2 \beta_2 (P + \epsilon) \left( \frac{\partial P}{\partial \epsilon} \right) \right]^{1/2} / (P + \epsilon) \left( \frac{\partial P}{\partial \epsilon} \right)^{1/2}. \] (4)

Where \( P, \rho \) and \( \epsilon \) are the pressure, number density and energy density of the system respectively. \( \left( \frac{\partial P}{\partial \epsilon} \right)^{1/2} = c_s \), is the velocity of sound. Using \( P + \epsilon = sT, \beta_2 = \frac{3}{\pi^4}, \eta = \frac{4}{\pi} \) and \( \zeta = 0 \) (with \( s = \) entropy density of the system), we can arrive at

\[ F = \frac{L_\eta}{L_n}. \] (5)

Where,

\[ L_\eta = \frac{1}{2\sqrt{2}} \frac{1}{T} \left[ 16\eta^2 \left( \frac{\eta}{s} \right)^2 + 144 \left( \frac{\eta}{s} \right)^2 \right] \] (6)

and

\[ L_n = \left( \frac{4}{\pi} \right)^{1/3}. \] (7)

Here \( c_s \) and \( s \) are used based on LQCD calculations.\(^4\)

Fig. 4 shows the fluidity measure \( F \) of QGP as a function of \( T \). Near \( T_c \), the value of \( F \) is less compared to other \( T \) which implies that the QGP is more close to the perfect fluidity near \( T_c \). As \( T \) increases, \( F \) increases which suggests that the system is deviating from near perfect fluidity.

\[ \frac{\partial P}{\partial \epsilon} = 0 \] (9)

where \( \eta' \equiv 4\pi \eta/s \).

FIG. 6: Variation of heavy quark anti-quark bound state \((\bar{Q}Q)\) potential \( V \) with spatial separation \( L \) between \( Q \) and \( \bar{Q} \) in the bound state at different values of \( T \) and \( \eta' \).

In Fig 6, we depict \( \bar{Q}Q \) bound state potential \( V \) as a function of spatial separation \( L \) between \( Q \) and \( \bar{Q} \) in the bound state for different values of \( \eta/s \) and \( T \) of the QGP medium. At vacuum \((T = 0)\), \( V \) is negative, i.e. the potential is attractive. As the medium comes into the picture, the nature of \( V \) is modified. \( V \) decreases (in magnitude) as the \( \eta/s \) and \( T \) increases, i.e. \( V \) becomes weaker due to the presence of medium. More precisely, the medium screens the potential.

IV. SUMMARY AND CONCLUSION

In summary, we have estimated the transport parameter of heavy quarks propagating in the QGP medium. From the obtained transport parameter, we have estimated \( \eta/s, \zeta/s \) and fluidity measure \( F \) of the QGP medium. Our results of \( \eta/s \) are in good agreement with the LQCD and renormalization group calculations. In addition to that, the effect of this medium on the heavy quark bound state has also been discussed and found significant effects on it of the medium.

V. ACKNOWLEDGEMENT

A.I.S. acknowledges Mahfuzur Rahaman and Golam Sarwar for fruitful discussions.
VI. REFERENCES

[1] I. Arsene et al. (for BRAHMS Collaboration), Nucl. Phys. A 757, 1 (2005); B. B. Back et al. (for PHOBOS Collaboration), Nucl. Phys. A 757, 28 (2005); J. Adams et al. (for STAR Collaboration), Nucl. Phys. A 757, 102 (2005); K. Adcox et al. (for PHENIX Collaboration), Nucl. Phys. A 757, 184 (2005).

[2] B. Muller, J. Schukraft and B. Wyslouch, Ann. Rev. Nucl. Part. Sci. 62, 361 (2012).

[3] B. Muller, Phys. Scripta T158 (2013) 014004.

[4] S. Borsanyi, Z. Fodor, S. D. Katz, S. Krieg, C. Ratti and K. Szabo, J. High Energy Phys. 01, 138 (2012).

[5] A. Bazavov et al. (HotQCD Collaboration), Phys. Rev. D 86, 034509 (2012); A. Bazavov, H.-T. Ding, P. Hegde, O. Kaczmarek, F. Karsch, E. Laermann, Y. Maezawa, S. Mukherjee et al., Phys. Rev. Lett. 113, 072001 (2014).

[6] S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg and K. K. Szabo, Phys. Lett. B730, 99 (2014).

[7] I. Arsene et al. (BRAHMS Collaboration), Nucl. Phys. A 757,1(2005).

[8] B. B. Back et al. (PHOBOS Collaboration), Nucl. Phys. A 757, 28(2005).

[9] K. Adcox et al. (PHENIX Collaboration), Nucl. Phys. A 757, 184(2005).

[10] J. Adams et al. (STAR Collaboration), Nucl. Phys. A 757, 102(2005).

[11] K. Aamodt et al.(ALICE Collaboration), Phys. Rev. Lett.105,(2010)252302, K. Aamodt et al.(ALICE Collaboration), Phys. Rev. Lett.107, (2011)032301.

[12] F. Karsch and E. Laermann, in Quark-Gluon Plasma III, pp. 1-59, R. Hwa (ed.);hep-lat/0305025.

[13] M.H. Thoma and M. Gyulassy, Nucl. Phys. B351 (1991) 491-506.

[14] E. Brateen and M.H. Thoma, Phys. Rev. D 44, R2625 (1991).

[15] A. Meistrenko, A. Pashier, J. Uphoff and C. Greiner, Nucl. Phys. A901 (2013) 51-64.

[16] S. Peigne and A. Peshier, Phys. Rev. D 77, 114017 (2008).

[17] M.G. Mustafa, D. Pal, D.K. Srivastava and M.H Thoma, Phys. Lett. B 428 (1998) 234; Erratum: Phys. Lett. B438, 450 (1998).

[18] M. G. Mustafa, D. Pal and D. K. Srivastava, Phys. Rev. C 57, 889 (1998).

[19] Y. L. Dokshitzer and D. E. Kharzeev, Phys. Lett. B519, 199 (2001).

[20] Y. L. Dokshitzer, V. A. Khoze, and S. I. Troian, J. Phys. G17, 1602 (1991).

[21] S. Wicks, W. Horowitz, M. Djordjevic, and M. Gyulassy, Nucl. Phys. A 784, 426 (2007); Nucl. Phys. A 783, 493 (2007); M. Djordjevic and M. Gyulassy, Nucl. Phys. A 733, 265 (2004).

[22] N. Armesto, M. Cacciari, A. Dainese, C. A. Salgado, and U. A. Wiedemann, Phys. Lett. B673, 362 (2006).

[23] B.Z. Kopeliovich, I.K. Potashnikova, I. Schmidt, Phys. Rev. C 82 (2010) 037901.

[24] W.C. Xiang, H.T. Ding, D.C. Zhou, D. Rohrich, E. Phys. J. A 25 (2005) 75.

[25] I. Vitev, J. Phys. G 35 (2008) 104011.

[26] R. Abir, U. Jamil, M.G Mustafa and D.K Srivastava, Phys. Lett. B 715 (2012) 183.

[27] A.I. Sheikh, Z. Ahammed, P. Shukla and M.G. Mustafa, arXiv:1711.06245.

[28] A. Majumder, B. Muller and X.N. Wang, PRL 99, 192301 (2007).

[29] R. Baier, Y. L. Dokshitzer, A. H. Mueller, S. Peigne, and D. Schiff, Nucl. Phys. B483, 291 (1997); B484, 265 (1997).

[30] R. Baier, Nucl. Phys. A 715, 209 (2003).

[31] R. A. Lacey et al., Phys. Rev. Lett. 103, 142302 (2009).

[32] P. Petreczky and J. Weber, Nuclear Physics A 967 (2017) 592595.

[33] J. Noronha and A. Dumitru, Phys. Rev. D 80, 014007 (2009).

[34] M. Hasan, B. Chatterjee and B.K. Patra, Eur.Phys.J. C77 (2017) no.11, 767.

[35] F. Karsch, M. T. Mehr and H. Satz, Z. Phys. C 37, 617 (1988).

[36] B. K. Patra and D. K. Srivastava Phys. Lett. B 505 113 (2001).

[37] U. Kakade, B. K. Patra, Phys. Rev. C 92, (2015) 024901.

[38] S.K. Das, J. Alam and P. Mohanty, Phys. Rev. C 82, 014908 (2010).

[39] S. Mazumder, T. Bhattacharyya and J. Alam, Phys. Rev. D 89, 014002 (2014).

[40] H. B. Meyer, Phys. Rev. D 76 (2007).

[41] H. B. Meyer, Nucl. Phys. A 830 (2009) 641C.

[42] S. Borsanyi, Z. Fodor, S. Mages, A. Schafer and K. K. Szabo PoS (LATTICE2014) 232.

[43] M. Haidenbauer, B. K. Patra, Eur Phys. J. A 25 (2005) 75.

[44] S. Weinberg, Astrophys. J. 168, 175 (1971).

[45] P. Kovtun, D.T. Son, A.O. Starinets, Phys. Rev. Lett. 94, 111601 (2005); G. Policastro, D.T. Son, A.O. Starinets, Phys. Rev. Lett. 87 081601 (2001); A. Buchel, Phys. Lett. B 609, 392 (2005).

[46] J. Liao and V. Koch, Phys. Rev. C 81, 014902 (2010).

[47] M. Rahman and J. Alam, arXiv:1712.09175.

[48] J. Noronha and A. Dumitru, Phys. Rev. D 80, 014007 (2009).