Relativistic heat conduction and thermoelectric properties of nonuniform plasmas

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Abstract

Relativistic heat transport in electron-two-temperature plasmas with density gradients has been investigated. The Legendre expansion analysis of relativistically modified kinetic equations shows that strong inhibition of heat flux appears in relativistic temperature regimes, suppressing the classical Spitzer-Härm conduction. The Seebeck coefficient, the Wiedemann-Franz law, and the thermoelectric figure of merit are derived in the relativistic regimes.

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The relativistic effects of hot electrons in laboratory plasmas have attracted much interest in the past few decades, particularly, in the context of the current drive mode in tokamaks and the electron cyclotron heating in various confinement devices. In the theoretical arena, Braams and Karney (BK) first presented the relativistic Fokker-Planck equations with the extended Rosenbluth potentials, and applied the equations to derivation of the relativistic electrical conductivity. Using the BK collision integrals, Shoucri and Shkarofsky developed a numerical code to survey the relativistic effects on electron cyclotron wave, fast wave, and lower hybrid current drive mode in tokamaks. Making use of the Chapman-Enskog expansion, Mohanty and Baral derived relativistic transport coefficients including magnetic field effects.

Rapid heating of plasma often leads to a bi-Maxwellian electron distribution, consisting of a bulk and high-energy tail, while maintaining relatively cold ions. In fact, the appearances of two-temperature spectrum for electrons have been observed in some experiments with high-intensity lasers. The well-pronounced tail and its velocity moment, which determine the transport properties, are in the relativistic temperature regime and, therefore, important to the fundamental study of relativistic electron transport.

In this Brief Communication, the relativistic transport theory presented in the previous paper is expanded, aiming at the numerical simulation of high-temperature ignition plasmas, and celestial plasmas. We focus on a problem relevant to heat flux inhibition due to relativistic effects of electrons within the framework of the relativistically corrected Spitzer-Härm (SH) formula for electron-two-temperature plasmas with density gradients. The formula is fully consistent with the current-neutral condition, so that one can readily couple the transport coefficients with fluid codes.

The relativistic thermal conductivity is derived below, along the manner developed by BK. Begin with the Legendre expansion for electron distribution function, viz., \( f(r, p, t) \approx f_0(r, p, t) + (p/p) \cdot f_1(r, p, t) + \text{higher order terms} \) for the small parameter which is related to a characteristic field strength. Introducing the relation of \( p_x = p \cos \phi \), a relativistically extended kinetic equation is averaged over the solid angle \( \Omega \), i.e., \( \langle \cdots \rangle = \int \cdots d(\Omega/4\pi) \). After the manipulations, we obtain the first order equation in the form of

\[
\frac{\partial f_{1x}}{\partial t} + v \frac{\partial f_0}{\partial x} - \frac{eE_x}{m_0c} \frac{\partial f_0}{\partial \mu} = \left( \frac{\delta f_{1x}}{\delta t} \right)_c,
\]

where \( v = c\mu/\Gamma, \mu = p/m_0c, \) and \( \Gamma = \sqrt{1 + \mu^2} \). Using the transfer cross section of the rela-
tivistic Mott scattering $\sigma^t_{ei}$, the collision term of Eq. (1) can be approximated by $(\delta f_{1x}/\delta t)_c \simeq -n_i\nu_{ei}\sigma^t_{ei}f_{1x} \equiv -\nu_{ei}f_{1x}$, where $n_i$ is the number density of ions, $\nu_{ei} = (n_i Y_{ei}/e^3)(\Gamma/\mu^3)$ is the electron-ion collision frequency, $Y_{ei} = 4\pi[\bar{Z}e^2/(4\pi\epsilon_0 m_0)]^2\ln\Lambda$, and $m_0$, $\bar{Z}$, and $\ln\Lambda$ are the electron rest mass, the averaged charge number, and the Coulomb logarithm, respectively.

In Eq. (1), the effects of magnetic fields are ignored. This approximation is valid for $\nu_{ei} \gg \omega_c$, where $\omega_c = |eB|/(\Gamma m_0)$ is the electron cyclotron frequency. The validity condition gives the allowable parameter range of the magnetic field strength of

$$|B| \ll 1.7 \times 10^7 \frac{\Gamma^2}{(\Gamma^2 - 1)^{3/2}} \left( \frac{\bar{Z}^2 n_i}{10^{27} \text{cm}^{-3}} \right) \left( \frac{\ln\Lambda}{10} \right) \text{ G.} \quad (2)$$

In highly compressed targets irradiating by a relativistic laser pulse [11], the dense plasma parameters are typically $(\Gamma - 1) \sim 10^{-1}$, $n_i \sim 10^{26} \text{ cm}^{-3}$, $\bar{Z} \approx 3.5$ (carbonized deuterium-tritium), and $\ln\Lambda \approx 5$. For such parameters, the right-hand side (RHS) of Eq. (2) reads about 130 MG. Around the tenuous coronae, the laser pulse drives relativistic currents, and induces the self-magnetic fields of magnitude $B \sim m_0\omega_{pe}c/e \sim 10^2$ MG. Recent numerical simulation indicates that at the surface, an intense magnetic field of $B \leq 280$ MG prevents hot electrons from penetrating into the higher density region [12]. The electrons near the channel envelope where the magnetic field is strongest, as well as lower energy electrons, tend to be magnetically trapped [13], since their Larmor radii are comparable to or even less than the channel radius. This stopping effect seems to be subject to the Alfvén current limit [14], which is irrelevant to the limit of energy flux. For a self-focusing electron beam, a fraction of energetic electrons, running along the channel axis, cannot be trapped [13], and generate the relatively small magnetic fields of $B < 10^2$ MG in the denser plasma, as was shown in Ref. [12]. The penetrating electrons have a largely anisotropic momentum distribution, which cannot be treated by the diffusion approximation employed here. In a highly compressed region, however, the transported electrons are expected to be thermalized via dissipative processes [14, 15], and the beam type transport may become diffusive, further decaying the magnetic fields. Although, diffusive transport plays a significant role in heating the final compressed fuel, the details are still not well understood. Hence, here we investigate the fundamental transport properties in the parameter regions of the highly compressed ignitor plasma, where magnetic field effects can be fairly neglected, as far as Eq. (2) is fulfilled. I also mention that the density gradient of ablative plasma is likely to be steep in
the higher density regions, so that nonuniform effects are taken into account here.

For a quasisteady condition of $\partial f_{1x}/\partial t \simeq 0$ in Eq. (11), i.e., omitting the electron inertia, the anisotropic component of electron distribution function is given by

$$f_{1x}(x, \mu) \simeq -\frac{e^4}{n_iY_{ei}} \left( \frac{\mu^4}{1 + \mu^2} \frac{\partial f_0}{\partial x} - \frac{eE_x}{m_0c^2} \frac{\mu^3}{\sqrt{1 + \mu^2}} \frac{\partial f_0}{\partial \mu} \right).$$

(3)

Heat flux of relativistic electrons can be defined by $q_x \equiv m_0c^2 \int_0^\infty (\Gamma - 1) v_x \mu^2 f d\Omega d\mu$. Integrating over solid angle, yields $q_x = \frac{4}{3} \pi m_0 c^3 \int_0^\infty f_{1x} \mu^3 (\Gamma - 1)/\Gamma d\mu$. Making use of Eq. (3), this may be written as

$$q_x = -\frac{4\pi m_0 c^7}{3n_i Y_{ei}} \left( \int_0^\infty \frac{\mu^7}{1 + \mu^2} \frac{\partial f_0}{\partial x} d\mu - \frac{eE_x}{m_0 c^2} \int_0^\infty \frac{\mu^6}{\sqrt{1 + \mu^2}} \frac{\partial f_0}{\partial \mu} d\mu \right).$$

(4)

The longitudinal electric field $E_x$ in Eq. (11) can be determined by the current-neutral condition $j_x \equiv -e \int_0^\infty v_x \mu^2 f d\Omega d\mu = -\frac{4}{3} \pi ec \int_0^\infty f_{1x} \mu^3 /\Gamma d\mu \simeq 0$. That is,

$$\frac{eE_x}{m_0 c^2} = \frac{\int_0^\infty \frac{\mu^7}{(1 + \mu^2)^2} \frac{\partial f_0}{\partial x} d\mu}{\int_0^\infty \frac{\mu^6}{1 + \mu^2} \frac{\partial f_0}{\partial \mu} d\mu}.$$  

(5)

For the isotropic component $f_0(x, \mu)$, I employ the superposition of the two-temperature populations of electrons,

$$f_0(x, \mu) = \frac{1}{4\pi} \sum_j n_{e,j}(x) \alpha_j(x) \frac{\alpha_j(x)}{K_2[\alpha_j(x)]} \exp \left[ -\alpha_j(x) \sqrt{1 + \mu^2} \right],$$

(6)

where $K_\nu(\alpha_j)$ is the modified Bessel function of index $\nu$ with its argument of $\alpha_j(x) \equiv m_0c^2/T_j(x)$, and $j = c, h$ indicate the cold and hot components, respectively. The normalization is given by $n_e(x) = \bar{Z} n_i(x) = \sum_j n_{e,j}(x) = 4\pi \int_0^\infty f_0(x, \mu) \mu^2 d\mu$.

In a steep temperature gradient plasma, depending on the collisional mean-free path, $\lambda$, the transport properties may not be locally defined. In this sense, local transport theory is valid only for the case of $\lambda \ll |L_T|$, where $L_T = T/(\partial T/\partial x)$ is the characteristic length of the temperature gradient. Concerning the relation of $|\partial T/\partial x| \sim c|E_x|$ derived from Eq. (5), the parameter range involving the electric field can be estimated as $|E_x| \ll E_c \sim 10^{12}(100 \text{ keV}/T)(\bar{Z}^2 n_i/10^{27} \text{ cm}^{-3})(\text{ln} \Lambda/10) \text{ V/m}$. In the case of ignitor physics, the relativistic electron transport establishes the temperature gradient in the high-density plasma. Assuming the spatial gradient of $\Delta T/\Delta x \sim -100 \text{ keV}/100 \mu\text{m}$, the electric field strength can be estimated as $|E_x| \sim 10^9 \text{ V/m} [\text{see also Eq. (11) below}]$. For $T \leq 10^2 \text{ keV}$,
$Z^2 n_i \sim 10^{27}$ cm$^{-3}$, and $\ln \Lambda \approx 5$, we read $|E_x| < 10^{-2} E_c$. For the case of $|E_x| > (0.01-0.1) E_c$, one may solve kinetic transport equations to determine the full self-consistent spectral distribution, instead of using Eq. (1).

Substituting Eq. (6) into Eqs. (4) and (5), we obtain the relativistic heat flux of $q_x \equiv -\kappa_{rel}(\partial T_h/\partial x)$ for the temperature gradient of hot electrons, and may decompose the coefficient as $\kappa_{rel} = f_{rel}\kappa_{SH}$. Here, $\kappa_{SH}(T_h) = 256(2\pi)^{1/2}\epsilon_0 T_h^{5/2}/(Ze^4 m_0^{1/2}\ln \Lambda)$ is the familiar nonrelativistic SH heat conductivity of the Lorentz plasmas [17], and the factor $f_{ref}$ corresponds to the relativistically corrected flux limiter which can be expressed as

$$f_{rel} = \frac{(2\pi)^{1/2}}{384}\alpha_h^{7/2}\{C_{1,c} + \theta C_{1,h} + \epsilon [C_{2,c}\Theta_1(\alpha_c) + C_{2,h}\Theta_1(\alpha_h)]\}, \quad (7)$$

where the abbreviations are

$$\epsilon = \frac{\theta C_{2,c}[\Theta_1(\alpha_c) + C_{3,c}\Theta_2(\alpha_c)] + C_{2,h}[\Theta_1(\alpha_h) + C_{3,h}\Theta_2(\alpha_h)]}{\alpha_c C_{2,c}\Theta_2(\alpha_c) + \alpha_h C_{2,h}\Theta_2(\alpha_h)}, \quad (8)$$

$$C_{1,j} = -\frac{C_{2,j}[C_{3,j}\Theta_1(\alpha_j) - \Theta_3(\alpha_j)]}{\alpha_j}, \quad (9a)$$

$$C_{2,j} = \frac{n_{e,j}\alpha_j}{n_e K_2(\alpha_j)}, \quad (9b)$$

$$C_{3,j} = 3 - \delta_j + \frac{\alpha_j K_1(\alpha_j)}{K_2(\alpha_j)}, \quad (9c)$$

$$\Theta_1(\alpha_j) = \left(1 - \frac{1}{\alpha_j} + \frac{2}{\alpha_j^2} + \frac{42}{\alpha_j^3} + \frac{120}{\alpha_j^4} + \frac{120}{\alpha_j^5}\right)\exp(-\alpha_j) + \alpha_j\text{Ei}(-\alpha_j), \quad (10a)$$

$$\Theta_2(\alpha_j) = \left(1 - \frac{1}{\alpha_j} + \frac{2}{\alpha_j^2} - \frac{6}{\alpha_j^3} - \frac{24}{\alpha_j^4} - \frac{24}{\alpha_j^5}\right)\exp(-\alpha_j) + \alpha_j\text{Ei}(-\alpha_j), \quad (10b)$$

$$\Theta_3(\alpha_j) = \left(\frac{48}{\alpha_j^2} + \frac{288}{\alpha_j^3} + \frac{720}{\alpha_j^4} + \frac{720}{\alpha_j^5}\right)\exp(-\alpha_j), \quad (10c)$$

where $\text{Ei}(-\alpha_j)$ is the exponential integral function, and $\delta_j = \partial \ln n_{e,j}/\partial \ln T_j$ and $\theta = \partial \ln T_e/\partial \ln T_h$ reflect the nonuniformity of plasma. Namely, for $\theta \to 0$, the formula describes the energetic transport in the plasma that the cold electron component is isothermal, and for $\delta_j \to 0$ and $-1$, in the plasma that the electron component $j$ is isochoric ($n_{e,j} = \text{const}$) and isobaric ($n_{e,j} T_j = \text{const}$), respectively.
The geometrical constraint of $\nabla n_{e,j} \parallel \nabla T_j$ due to ignoring two-dimensional (2D) effects means that thermoelectric magnetic fields, which can be prominent, for example, in intense laser-plasma interactions [18], are not taken into account at the moment. 2D effects are important, because they prefer to short out electric fields and pinch directional flows by the toroidal magnetic fields. The complexities of magnetic inhibition in heat flux might be effectively considered by introducing a reduction factor $f_B < 1$: Bohm’s $f_B = (1 + \omega_c \tau)^{-1}$ or Braginskii’s $f_B = (1 + \omega_c^2 \tau^2)^{-1}$ [18], where $\tau$ denotes a collision period. That is, one can practically utilize the cross-field conductivity approximated by $\kappa_{\perp rel} \approx f_B f_{rel} \kappa_{SH}$. Note that Eq. (2) reflects the much smaller Hall parameter $\omega_c \tau \ll 1$, such that $f_B \to 1$, and $\kappa_{\perp rel} \simeq \kappa_{\parallel rel} = \kappa_{rel}$.

In the following, more elemental issues are investigated, i.e., relativistically extended longitudinal thermoelectric effects. With regard to the longitudinal thermal diffusion that develops an electrostatic potential, one should note the important relation $\epsilon = L_{Th}[eE_x/(m_0 c^2)]$. For the special case of $n_{e,h}/n_e \to 1$, $C_{2,c} \to 0$, and $\theta \to 1$, namely, the one-temperature model for electrons, the self-consistent electric field Eq. (8) reduces to

$$\epsilon \simeq \frac{1}{\alpha} \left[ \Theta_1(\alpha) + C_3(\alpha, \delta) \right],$$

where $\alpha \equiv \alpha_j$ and $C_3 \equiv C_{3,j}$. In the thermoelectric point of view, the relativistic Seebeck coefficient can be defined by $s \equiv \alpha \epsilon/e$. The temperature dependence of Eq. (11) is shown in Table II for $\delta \equiv \delta_j = 0$ and $-1$. In the nonrelativistic limit of $\alpha \gg 1$, Eq. (11) for $\delta = 0$ asymptotically approaches $\epsilon \to -5/(2\alpha)$ [9]. In the isobaric case of $\delta = -1$, owing to the pressure-balance effects, the field strength reduces to $50-60\%$ of the $\delta = 0$ case. Noted is that in this case the flux limiter $f_{rel}$ does not depend on $\delta$, and the similar property appears again in the following other cases.

In Fig. I the temperature dependence of the relativistically corrected flux limiter are shown. The ratios of hot/total electron density are chosen for $n_{e,h}/n_e = 0.1 - 1$, fixing the temperature scale length equal, $\theta = 1$. For the electron-two-temperature models of $n_{e,h}/n_e \neq 1 (C_{2,c} \neq 0)$, set the temperature of cold component to $\alpha_c = 10^2 (T_c = 5.11 \text{ keV})$ as an example. The densities are set to be uniform ($\delta_j = 0$), except for the case of $n_{e,h}/n_e = 0.1$ that the nonuniformity ($\delta_j = -1$) is taken into consideration. Actually our major interests are in the relativistic heat flux carried by the high-energy tail electrons of $\Gamma > \Gamma_0 = (\alpha_h - \alpha_c)^{-1} \ln(C_{2,c}/C_{2,h})$, where the spectral population of hot electrons is larger than
that of cold ones. As expected, in the lower energy regions of $\Gamma < \Gamma_0$, energy transport by cold components is dominant. In Fig. such criterion seems to appear as pseudo cut-off in the lower temperature region.

Now one finds that the heat flux is strongly inhibited in the relativistic regime. For example, in the one-temperature model for electrons, the flux limiters are $f_{rel} \approx 0.73$ for $T = 0.1m_0c^2 (\alpha = 10)$ and $f_{rel} \approx 0.37$ for $T = m_0c^2 (\alpha = 1)$, as shown in Fig. (solid curve). This is due to the drift velocity carrying heat asymptotically close to the speed of light. Moreover, it is found that a fall in the hot electron population leads to further decrease of the conductivity, and indeed, the degree of the depletion reflects the abundance of hot electrons. Regarding the electron transport in laser-produced plasmas, typically a flux limiter of order of $10^{-2} - 10^{-1}$ has been empirically employed [16], consistent with the experimental results [19]. In this aspect, the present results imply that the relativistic effects on sparsely populated high-energy tails can also participate in lowering the flux limit. These properties do not largely depend on $\delta_j$ as seen in Fig. For example, in the case of $n_{e,h}/n_e = 0.1$, the difference of the flux limiter between the case of $\delta_j = 0$ (dotted curve) and $-1$ (crosses) is about 10% at most.

Taking the limit of $n_{h,e}/n_e \to 1$, $C_{2,c} \to 0$, and $\theta \to 1$, Eqs. (7)-(10) reduce to

$$f_{rel} \approx \left(\frac{2\pi}{384}\right)^{1/2} \frac{\alpha^{7/2}}{K_2(\alpha)} \left[\Theta_2^2(\alpha) + \Theta_3(\alpha)\right].$$

(12)

This corresponds to the standard relativistic SH heat conductivity having the temperature dependence of $\kappa_{HM}(T) = f_{rel}(T)\kappa_{SH}(T) \propto T^2 - T^{5/2}$ [9], which exhibits the asymptotic property of $\kappa_{SH}(T) \propto T^{5/2}$ ($f_{rel} \to 1$) in the nonrelativistic limit of $\alpha \gg 1$ [17], whereas $\kappa_{DT}(T) = \left[\frac{5}{2}\right]^{1/2} [\kappa_{SH}(T) \propto T^2 (f_{rel} \propto T^{-1/2})$ by Dzhavakhishvili and Tsintsadze in the ultrarelativistic limit of $\alpha \ll 1$ [20]. These characteristics are also shown in Fig. (solid curve), and summarized in Table I.

Here let us take the ratio of the thermal to electrical conductivity. The key relation is known as the Wiedemann-Franz law for metallic states of matters [21]. The ubiquitous nature is derived from a simple assumption of the elastic scattering of conduction electrons. As for fully ionized plasmas, the relativistically extended law can be expressed as

$$\frac{\kappa_{HM}}{\sigma_{BK}} = -\frac{T}{e^2} \frac{\Theta_1^2(\alpha) + \Theta_2(\alpha)\Theta_3(\alpha)}{\Theta_2^2(\alpha)} > 0,$$

(13)

for the case of $n_{e,h}/n_e = 1$. Here, $\sigma_{BK}(\alpha) = -\left[(2\pi)^{1/2}/96\right]^{1/2} \Theta_2(\alpha)/K_2(\alpha)\sigma_0 > 0$ [5],
and $\sigma_S(T) = 64(2\pi)^{1/2}e^2T^{3/2}/(Ze^4m_0^{1/2}\ln\Lambda)$ stands for the nonrelativistic Spitzer conductivity. Evidently, the ratio depends on the temperature only, without involving the intrinsic parameters of plasmas. Figure 2 shows the temperature dependence of Eq. (13). In the nonrelativistic limit of $\alpha \gg 1$, it asymptotically approaches $\kappa_{HM}/\sigma_{BK} \rightarrow \kappa_{SH}/\sigma_S = 4T/e^2$ [17]. This value slightly decreases as the temperature increases, to take the minimum value of $(\kappa_{HM}/\sigma_{BK})_{\text{min}} = 3.92T/e^2$ at $\alpha = 19.6$ ($T = 26.1$ keV). As seen in the figure, it increases up to $\kappa_{DT}/\sigma_{BK} = 5T/e^2$ in the ultrarelativistic regime. It may be instructive to mention that the transport equation of the Fermi liquid in metals or condensed plasmas yields $\kappa/\sigma \simeq \pi^2T/(3e^2) \simeq 3.3T/e^2$ [21, 22], which is lower than $(\kappa_{HM}/\sigma_{BK})_{\text{min}}$ in the ordinary plasmas.

The heat conductivity holds the larger power index of temperature. Thus, fast heating of plasma can drive the nonlinear heat-wave accompanied with a well-defined wave front, where an electrostatic field tends to be well developed. This leads to an idea that such a thermally non-equilibrated plasma can be essentially compared to a thermoelectric converter. And, in general, its efficiency can be quantitatively evaluated by invoking a thermoelectric figure of merit. Along the conventional notation used in material physics, we now define the thermoelectric figure of merit by $Z \equiv s^2\sigma_{BK}/\kappa_{HM} = (\alpha e/e)^2(\sigma_{BK}/\kappa_{HM})$ for $n_{e,h}/n_e = 1$. Making use of Eqs. (11) and (13), this multiplied by $T$ can be written in the dimensionless form,

$$ZT = -\frac{[\Theta_1(\alpha) + \Theta_2(\alpha)C_3(\alpha, \delta)]^2}{\Theta_1^4(\alpha) + \Theta_2^2(\alpha)\Theta_3(\alpha)} > 0. \quad (14)$$

Note that Eq. (14) depends on $\delta$, in contrast to Eqs. (12) and (13). For $\delta = 0$ and $-1$, the temperature dependence of Eq. (14) and the coefficient $s$ are also shown in Fig. 2. It is found that for $\alpha \gg 1$, Eq. (14) asymptotically approaches $ZT(\delta = 0) \rightarrow 25/16$ and $ZT(\delta = -1) \rightarrow 9/16$, whereas for $\alpha \ll 1$, $ZT(\delta = 0) \rightarrow 4/5$ and $ZT(\delta = -1) \rightarrow 1/5$. Particularly, in the nonrelativistic plasmas with uniform density, i.e., $\alpha \gg 1$ and $\delta = 0$, one can extract the higher figure of merit $ZT \simeq 0.4-1.3$ as indicated by the arrow in Fig. 2. Notice that the Carnot efficiency can be achieved for $ZT \gg 1$. The dimensionless values of $(\kappa_{HM}/\sigma_{BK})(e^2/T)$, $(se)^2$, and $ZT$ for some $\alpha$ and $\delta$ values are summarized in Table II.

In conclusion, I have derived solutions for the heat conductivity and related thermoelectric coefficients in a relativistic nonuniform plasma. These results indicate that the relativistic
effects on the high-energy tail electrons significantly limit the heat flux. This mechanism might play an additional role of the stopping of relativistic electrons in the context of ignitor physics \[15\], although this work ignores 2D thermoelectric effects such as \(\nabla_\perp n \times \nabla_\parallel T\), which may be important for typical ignitor geometries.
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FIG. 1: Hot electron temperature dependence of the relativistic Spitzer-Härm (SH) heat conductivity normalized by the nonrelativistic one: The flux limiter of hot electrons $f_{rel}(T_h) = \kappa_{rel}/\kappa_{SH}$ is shown for density ratios of $n_{e,h}/n_e = 1$ [Eq. (12): solid curve], 0.5 (dashed curve), 0.2 (dotted-dashed curve), and 0.1 (dotted curve). For the case of $n_{e,h}/n_e \neq 1$, $T_c/m_0c^2 = 0.01$ and $n_{e,j} = \text{const}$ are chosen as examples. For comparison, we plot another case of $n_{e,j} \propto T_j^{-1}$, only for $n_{e,h}/n_e = 0.1$ and $T_c/m_0c^2 = 0.01$ (crosses).
FIG. 2: Electron temperature dependence of the dimensionless thermoelectric figure of merit $ZT$ (solid curves), the Wiedemann-Franz ratio $\kappa_{HM}/\sigma_{BK}$ multiplied by $e^2/T$ (dotted-dashed curve), and the Seebeck coefficient $s$ multiplied by $|e|$ (dotted curves). Note that in the present case $\kappa_{HM}/\sigma_{BK}$ is independent on $\delta$ and takes the minimum values at $T/m_0c^2 \simeq 0.051$. 
TABLE I: Temperature dependence of $f_{rel}$ and $\alpha\epsilon$ for $\delta = 0$ and $-1$.

| $\alpha$ | $T$ (keV) | $\alpha\epsilon|_{\delta=0}$ | $\alpha\epsilon|_{\delta=-1}$ | $f_{rel}^a$ |
|----------|-----------|------------------------------|-------------------------------|------------|
| $\ll 1$  |           | $-2$                         | $-1$                          |            |
| 1        | $5.11 \times 10^2$ | $-2.0178$                    | $-1.0178$                     | 0.36792    |
| 5        | $1.02 \times 10^2$ | $-2.1692$                    | $-1.1692$                     | 0.62668    |
| 10       | 51.1      | $-2.2692$                    | $-1.2692$                     | 0.73598    |
| 20       | 25.5      | $-2.3560$                    | $-1.3560$                     | 0.83044    |
| 100      | 5.11      | $-2.4638$                    | $-1.4638$                     | 0.95529    |
| $\gg 1$  |           | $-2.5$                       | $-1.5$                        | 1          |

$^a f_{rel} = \kappa_{HM}/\kappa_{SH}$, to give $f_{rel} \simeq 1$ [17] and $\propto T^{-0.5}$ [20] for $\alpha \gg 1$ and $\ll 1$, respectively.
TABLE II: Temperature dependence of $(\kappa_{HM}/\sigma_{BK})(e^2/T)$, $(se)^2$, and $ZT$ for $\delta = 0$ and $-1$.

| $\alpha$ | $T$ (keV) | $(\kappa_{HM}/\sigma_{BK})(e^2/T)^a$ | $(se)^2|_{\delta=0}$ | $(se)^2|_{\delta=-1}$ | $ZT|_{\delta=0}$ | $ZT|_{\delta=-1}$ |
|----------|-----------|------------------------------------|---------------------|---------------------|-----------------|-----------------|
| $\ll 1$  | $-$       | 5                                  | 4                   | 1                   | 4/5             | 1/5             |
| 0.5      | $1.02 \times 10^3$ | 4.9032                          | 4.0145             | 1.0072             | 0.81874         | 0.20542         |
| 1        | $5.11 \times 10^2$ | 4.7331                          | 4.0717             | 1.0360             | 0.86026         | 0.21888         |
| 5        | $1.02 \times 10^2$ | 4.0886                          | 4.7053             | 1.3670             | 1.1508          | 0.33434         |
| 10       | 51.1      | 3.9529                            | 5.1495             | 1.6110             | 1.3027          | 0.40754         |
| 20       | 25.5      | 3.9221                            | 5.5509             | 1.8388             | 1.4153          | 0.46883         |
| 100      | 5.11      | 3.9669                            | 6.0704             | 2.1428             | 1.5303          | 0.54170         |
| $\gg 1$  | $-$       | 4                                  | 25/4               | 9/4                | 25/16           | 9/16            |

$^a\sigma_{BK}(T)$ is introduced in Ref. [3].