Material parameters for computer analysis of fibre reinforced concrete structures

R Pukl¹, D Lehký², M Lipowczan³ and D Novák⁴

¹ Project manager, Červenka Consulting s.r.o., Prague, Czech Republic
² Assoc. Prof., Institute of Structural Mechanics, Faculty of Civil Engineering, Brno University of Technology, Brno, Czech Republic
³ PhD student, ISM FCE BUT, Brno, Czech Republic
⁴ Professor, ISM FCE BUT, Brno, Czech Republic

E-mail: radomir.pukl@cervenka.cz

Abstract. Methods and software tools for material parameter identification of high-performance cementitious composites are presented. The aim is to provide techniques for the purpose of advanced assessment of their fracture–mechanical properties and subsequent numerical simulations of components/structures made of these materials. The paper describes development of computational and material models utilized for efficient material parameter determination of studied composite. Such a determination is performed with the help of experimental data from four-point bending test used in inverse analysis based on artificial neural networks. It is a part of complex methodology for statistical and reliability analyses of fibre reinforced concrete structures.

1. Introduction

Advanced assessment of fracture–mechanical properties is primarily important for subsequent numerical simulations of components/structures made of fibre reinforced concrete (FRC). The variability of experimental results using specimens made of quasi-brittle materials such as FRC is high due to the natural heterogeneity of this materials. Then an assessment of fracture–mechanical parameters is much more difficult and problematic. To remain at deterministic level is therefore impossible and without virtual statistical approach, simulation and probabilistic result assessment the consequent practical design of quasi-brittle material-based structures can be risky. The aim of the performed research is to deepen knowledge about the complexity of the behaviour of this advanced and progressive composite material especially in relation to its resistance to crack propagation. The obtained knowledge is a prerequisite for efficient design of this composite and the consequent expansion of its applicability for increase of sustainability of constructed elements, structures and buildings.

The research is focused on several topics: First, development of suitable constitutive law for FRC cementitious composites under the framework of nonlinear fracture mechanics software ATENA [1] is of primary importance. The computational model of unnotched beam subjected to four-point bending has been created and verified using data from Dura Technology Sdn Bhd., Malaysia. Sensitivity analysis which shows importance of parameters of computational model have been performed. Finally, material parameters identification will be performed based on the combination of fracture tests, numerical simulation and artificial neural network-based (ANN) inverse analysis [2, 3].
2. Computational model

The nonlinear numerical simulation of the four-point bending beam tests (Figure 1) has been performed using FEM software ATENA [5]. It has been developed by the Červenka Consulting company for realistic simulation of concrete and reinforced concrete structures at deterministic level. ATENA software enables to calculate response of the structural member including material damage and is used here as an advanced failure function.

![Diagram of the unnotched specimen tested in the four-point bending configuration.](image)

The crucial role by the nonlinear numerical analysis plays the constitutive law at material point level. The realistic structural response, damage and failure calculated by the computer model depends strongly on the material model quality, which decides about the exactness and accuracy of the achieved results. Since the fibre reinforced concrete is rather complex heterogeneous material with strongly nonlinear response the “3DNonlinerCementitious2User” material model has been utilized for the realistic computation of this composite. This material model can capture all the important aspect of the FRC material behavior and response under tensile as well as compressive loading.

Concrete in tension is described within the smeared cracks concept by the nonlinear fracture mechanics with crack band approach. The main material model parameters are tensile strength and four parameters C1 to C4 as shown in Figure 2. The form of the function is described by tensile strength and four additional parameters C1 to C4 as shown in Figure 2.

Concrete in compression exhibits strong confinement effect – i.e. increase of the compressive strength under concentrated three-dimensional compressive stress state. This effect is covered by the special plasticity theory with a non-associated plastic flow law in the combined fracture-plastic models in ATENA. The compressive ductility of FRC should be appropriately accounted in the material model as well.
Figure 2. The proposed tensile softening function of the studied composite.

The created computer model of the four-point bending fracture test beam is shown in Figure 3. It consists of 500 four-node iso-parametric finite elements. It is loaded in 120 load steps by prescribed displacement at the marked loading point locations (see Figure 3, top). The nonlinear solution is performed by modified Newton-Raphson iterative method. The resulting crack pattern from the deterministic analysis when reaching the ultimate capacity of the cement matrix is shown in Figure 3, bottom.

Figure 3. FEM model of beam tested in four-point bending configuration (top) and crack pattern when reaching the ultimate capacity of cement matrix (bottom).

3. Material parameter identification and its software implementation
Mechanical fracture parameter values are determined using the results of fracture tests in suitable configurations. In case of FRC, four-point bending (4PB) tests on unnotched prism specimens is widely used. The outcome of each test is a force–deflection diagram ($F$–$d$ diagram), which is subsequently used for mechanical fracture parameter determination.

An artificial intelligence-based inverse procedure developed by Novák and Lehký [2] transforms fracture test response data into the desired mechanical fracture parameters. This approach is based on matching laboratory measurements with the results gained by reproducing the same test numerically.
The ANN is used here as a surrogate model of an unknown inverse function between input mechanical fracture parameters and corresponding response parameters.

The set for training the ANN is prepared numerically via the utilization of an FEM model (Figure 3) which simulates 4PB test with random realizations of material parameters. These are generated with the help of the stratified sampling method and by performing an inverse transformation of the distribution function in order to reflect the probability distribution of the parameter.

The random responses from the computational model and the corresponding random realizations of parameters serve as input–output elements for the ANN training set. After training, the ANN is ready to solve the main task, which is to provide the best material parameters in order for the numerical simulation to achieve the best agreement with the experiment. This is performed by simulating a network using the previously measured responses as an input. This results in a set of identified material parameters. The last step is result verification – the calculation of the computational model using the identified parameters. Comparison with the experiment will show the extent to which the inverse analysis was successful.

Note that the importance of the training sample preparation has been emphasized and tested by Tong and Liu [4], including LHS scheme. In spite of the fact that these authors concluded that the number-theoretic methods appear as the most efficient, LHS scheme also provided very good results. Moreover, our focus to LHS is also determined by the general applicability of this small-sample simulation technique for practical statistical, sensitivity and reliability analyses in many fields of engineering.

The methodology described above for parameter identification combining non-linear simulations with the training of an artificial neural network is relatively time consuming and of high complexity. Therefore, the whole procedure has been implemented in the software FraMePID-3PB [3], successfully used for material parameters identification of plain concrete. Now, the software is being modified and updated for fibre reinforced concrete including constitutive law as described in Section 2. Screenshot of the first version of the software named FRCID-4PB is shown in Figure 4.

![FRCID-4PB software screenshot.](image)

**Figure 4.** FRCID-4PB software – screenshot.
4. Results
The approach has been applied for fibre reinforced concrete produced by DURA Technology company, Malaysia. The presented results show outcomes from a pilot study serving for determination of the model feasibility and obtaining a range and sensitivity of the particular model parameters for the purpose of the consequent identification of FRC material parameters using the neural network technology.

4.1. Deterministic simulation of experiment
A set of five reference experimental load-deflection diagrams obtained from Dura Technology Sdn Bhd's laboratories was used to create and set up a four-point bending test computational model (Figure 5). The manufacturer's recommendations have been used to set the default parameters of 3DNonlinerCementitious2User material model. The four specific parameters $C_1$-$C_4$ of the tensile softening model were set based on experience and engineering estimation. The numerical simulation with the initial set of parameters results in a $F$–$d$ diagram shown in Figure 6. Note that the range of experimental curves is plotted using grey dashed lines.

![Figure 5](image1.png)  
**Figure 5.** Experimental load vs. deflection diagrams from the four-point bending test.

![Figure 6](image2.png)  
**Figure 6.** Load vs. deflection diagram from model simulation with initial set of parameters.

4.2. Sensitivity analysis
A parametric study was performed to understand the effect of the individual material parameters on the beam response in the four-point bending test. The aim was to identify the so-called dominant parameters, to determine the parameters that do not influence the response and thus can be excluded from the inverse analysis, to find out the real ranges of individual parameters and to propose a set of suitable response parameters that will be used as the input of the inverse analysis.

The parametric study was performed gradually for six model parameters – tensile strength, compressive strength and four parameters of the tensile softening model $C_1$-$C_4$. For each parameter, its value was gradually changed, always in four steps (with the exception of compressive strength, where only two steps were considered), the remaining parameters always remained unchanged at their default values. One of the cases always corresponded to the default set of parameters – marked with the abbreviation "ini" in the figures below. The resulting $F$–$d$ diagrams are shown in Figure 7.
Figure 7. Four-point bending test simulation for various values of tensile strength (top left), compressive strength (top right), parameter $C_1$ (middle left), parameter $C_2$ (middle right), parameter $C_3$ (bottom left), and parameter $C_4$ (bottom right).
The following conclusions can be drawn from the parametric study results:

- Tensile strength is one of the dominant parameters. Increasing its value leads to increased resistance across the entire load spectrum.
- The compressive strength, on the other hand, belongs to parameters that have no effect on the resulting response when the sample is loaded in a four-point bending. Therefore, the parameter will not be included in the identification and will be considered at its default value.
- Parameter $C_1$ related to the delayed activation of fibres in the cement matrix is expected to have the greatest influence on the shape of the diagram when the cement matrix capacity is exhausted. Its size controls the "depth" of the first curve drop.
- Parameter $C_2$ does not affect the initial part of the diagram, but affects the amount of deformation when the ultimate bearing capacity of the specimen is reached.
- Parameter $C_3$ is also closely related to the ultimate bearing capacity of the sample, but this time it mainly influences the value of the ultimate force and slope of the second part of the diagram, when it is strengthened due to the joint action of the matrix and the fibres.
- Parameter $C_4$ influences the maximum deformation that will be achieved during the test. At a low value, the numerical test is terminated at a low deflection value.

For the purposes of inverse analysis, we can conclude:

- Due to the clear effect of $C_1$ parameter on the first visible decrease of the $F$–$d$ diagram when reaching the ultimate bearing capacity of the cement matrix it is possible to exclude it from the identification and consider it with a constant value of $C_1 = 0.8$. In case of future need to identify samples with significantly “deeper” first drop in diagram, $C_1$ parameter may be included in the identification.
- Another parameter whose value can be considered constant and can be excluded from identification is parameter $C_4$. Its initial value $C_4 = 0.1$ is large enough to reach the required deformation, which corresponds to the ductility of the material.
- The last parameter that falls out of the identification is the compressive strength. Its effect on the four-point bending response is negligible.
- Identification set contains four parameters of the material model – tensile strength, $C_2$, $C_3$ parameters, and the modulus of elasticity, whose influence on the slope of the initial linear part of the diagram are well known.

5. Conclusion

The paper describes methodology and software tool which can be routinely used for indirect determination of the fracture-mechanical parameters of fibre reinforced concrete based on an artificial neural network. User-friendly software has been developed for application based on the three-point bending testing of notched specimens, and a neural network has been trained and predefined for such a configuration for the purpose of future inverse analysis. Material parameters identification technique is based on the combination of the statistical simulation method. Methodology and software tool have been verified using data from DURA technology company, Malaysia.

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