Nonlinear optics of photonic hyper-crystals: Quantum hyper-computing

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Photonic hyper-crystals combine the most interesting features of hyperbolic metamaterials and photonic crystals. Since the dispersion law of extraordinary photons in hyperbolic metamaterials does not exhibit the usual diffraction limit, photonic hyper-crystals exhibit light localization on deep subwavelength scales, leading to considerable enhancement of nonlinear photon-photon interaction. Therefore, similar to their conventional photonic crystal counterparts, nonlinear photonic hyper-crystals appear to be very promising in classical and quantum optical computing applications. Quantum mechanics of photonic hyper-crystals may be formulated in such a way that one of the spatial coordinates would play a role of effective time in a 2+1 dimensional “optical spacetime” describing light propagation in the hyper-crystal. Mapping the conventional quantum computing onto nonlinear optics of photonic hyper-crystals results in a quantum “hyper-computing” scheme, which may considerably accelerate computation time.
1. Introduction

Finding an efficient way to make individual photons interact with each other remains a major challenge in optical quantum computing [1]. A considerable recent progress in this area of research has been achieved by using photonic crystals [2]. A photonic crystal-based nanocavity design enables an ultra-small mode volume, so that considerable nonlinearities may be observed at a single-photon level. This recent success motivates us to look at the nonlinear optics of photonic hyper-crystals [3], which combine the most interesting features of hyperbolic metamaterials and photonic crystals. Photonic hyper-crystals are formed by periodic modulation of hyperbolic metamaterial properties on a scale \( L \), which is much smaller than the free space light wavelength \( \lambda \). Since the dispersion law of extraordinary photons in hyperbolic metamaterials does not exhibit the conventional diffraction limit, such modulation would lead to Bragg scattering of extraordinary photons and formation of photonic band structure no matter how small \( L \) is [4]. Therefore, photonic hyper-crystals exhibit light localization on deep subwavelength scales, which far exceeds the demonstrated mode volume reduction in photonic crystals. It was suggested that such strong field localization would lead to considerable enhancement of nonlinear optical effects in photonic hyper-crystals [3]. In this paper we will demonstrate that the level of nonlinearities achievable in photonic hyper-crystals at single photon levels indeed looks very promising for quantum computing applications.

Our goal is to develop insights into quantum nonlinear optics of photonic hyper-crystals and to suggest promising optical qubit geometries, which would make use of the strong nonlinearities mentioned above. Since nonlinear optics of hyperbolic metamaterials finds natural interpretation in terms of analog gravity in an effective 2+1 dimensional “optical spacetime” describing light propagation in the metamaterial [5], it is not surprising that a similar language is useful in describing the nonlinear optics of
photonic hyper-crystals. Moreover, periodic modulation of hyperbolic metamaterial properties, which is necessary for hyper-crystal formation, may involve creation of hyperbolic metamaterial interfaces with conventional dielectrics. As demonstrated in [6], such interfaces often behave similar to various horizons in Minkowski space-time. Very strong and tightly localized optical field divergences near the apparent horizons at the hyperbolic metamaterial resonator boundaries ensure strong nonlinear photon-photon interaction in such geometry. We should also mention that recent studies (see for example [7]) demonstrated that the issue of losses in hyperbolic metamaterials may be managed by using gain media as a dielectric component of the metamaterial. Thus, both hyperbolic metamaterials and photonic hyper-crystals may be made lossless (and even active) in a narrow frequency range. Therefore, the use of hyperbolic materials in quantum computing applications is justifiable.

This paper is organized as follows. In Section 2 we introduce the photonic hyper-crystal geometry (Fig.1) of particular interest to quantum computing applications, which is made of periodic arrangement of hyperbolic resonators [8]. Linear optical properties of such a photonic hyper-crystal will be described in terms of an effective optical space-time inside an individual resonator. Mutual coupling of these resonators and formation of Bloch waves will be considered as a function of height of the effective potential walls at the resonator boundaries (which look like effective horizons). Section 3 will address nonlinear optical interaction of single photons in such a hyperbolic metamaterial resonator array. It will be demonstrated that single photon qubits may be encoded via the orthogonal $k_{xy}$ directions in the individual resonators, as illustrated in Fig.1. It appears that presence or absence of individual photons in a resonator may considerably alter light reflection and transmission at the resonator boundaries, and therefore change nonlinear optical interaction of single photon qubits. Section 4 will address quantum mechanical properties of photonic hyper-crystals and their potential use in quantum computing applications. Since one of the spatial coordinates plays the
role of effective time in this geometry, the periodic resonator boundaries play the role of a computer “clock”. Such a quantum “hyper-computing” scheme, which maps the computer clock cycle onto the periodic modulation of the photonic hypercrystal, may considerably accelerate computation time, leading to potential advantages over the conventional quantum computing schemes. The paper will be concluded by a brief summary of obtained results.

2. Linear optical properties of hyperbolic resonator-based photonic hyper-crystals

Let us consider a photonic hyper-crystal geometry, which is based on hyperbolic material resonators separated by thin nonlinear dielectric layers, as shown in Fig.1. The hyperbolic resonators may be made of such artificial hyperbolic metamaterial as porous alumina (Al₂O₃) filled with silver nanowires, which exhibits low loss type I hyperbolic behavior ($\varepsilon_z<0$, $\varepsilon_x=\varepsilon_y>0$) in the visible range [9]. We should also note that pure Al₂O₃ itself exhibits low loss (Q>5) type I hyperbolic behavior in the 19.0-20.4 μm and 23-25 μm ranges [10]. The use of natural hyperbolic materials would greatly facilitate technical challenges involved in fabrication of the photonic hyper-crystal structures.

While more technically challenging, quantum optics in the LWIR range has been made possible due to recent introduction of the LWIR single photon detectors [11], so that consideration of natural hyperbolic materials would also appear to be justified. While propagation losses present a considerable challenge in the hyperbolic metamaterial design, Ni et al. [7] demonstrated that losses in a silver-based hyperbolic metamaterial may be compensated in the visible (~700 nm) frequency range by gain media, such as a dielectric polymer doped with Rh800 dye. When the dye is saturated, the silver-gain medium metamaterial structure becomes almost loss-free. One potential solution in the present case may consist in implanting the porous alumina with a suitable dye operating around 660 nm wavelength.
The hyperbolic resonators are assumed to be separated by thin nonlinear dielectric layers, such as lithium niobate as shown in Fig.1. Alternatively, if operation in the LWIR range is desirable, such nonlinear material as ZnSe (which is highly transparent in the LWIR range) may be used. A grating may be “buried” inside the nonlinear dielectric layer (as shown in Fig.1(b)) to enable controlled single photon-based qubit coupling. Another potentially interesting choice of the nonlinear layer would be to engineer a photonic hyper-crystal cavity and make use of the electromagnetically induced transparency (EIT) effects in either atomic impurities or quantum dots, as it is typically done in the conventional photonic crystal cavity geometries [2]. Kerr-like nonlinearities are enhanced by many orders of magnitude in such configurations. However, for the sake of simplicity let us initially assume that lithium niobate layers are used.

Let us determine the photon eigenfunctions in such a nonlinear photonic hyper-crystal geometry. In order to simplify our analysis, let us assume initially that \( \varepsilon_{xy} = \varepsilon_1 \) is positive and constant everywhere inside the hyper-crystal. Such an assumption is justifiable if similar to [9] the photonic hyper-crystal structure is operated at 660 nm, where the porous alumina/silver nanowire samples were measured to have \( \varepsilon_x = \varepsilon_y = \varepsilon_1 = 5.28 \) and \( \varepsilon_z = -4.8 \). It appears that \( \varepsilon_1^{1/2} = 2.3 \) indeed nearly matches the refractive index of lithium niobate [12]. Thus, at 660 nm wavelength \( \varepsilon_z = \varepsilon_2 \) will exhibit periodic oscillations inside the hyper-crystal as a function of \( z \) with a period \( L << \lambda \), while \( \varepsilon_1 \) may be assumed to be approximately constant. Far from the interfaces, inside the hyperbolic resonators \( \varepsilon_2 = -4.8 = \text{const} \), while near the LiNbO\(_3\) interfaces \( \varepsilon(z) \) experiences fast transition to its +5.28 value within the thin LiNbO\(_3\) layers.

First, let us consider the macroscopic Maxwell equations inside the hyperbolic resonators far from the interfaces. The uniaxial symmetry of this medium reduces the ordinary and the extraordinary waves to respectively the TE (\( \vec{E} \perp \hat{z} \)) and TM (\( \vec{B} \perp \hat{z} \))
polarized modes. Let us introduce an extraordinary (TM) photon wave function as $\varphi = E_z$.

The macroscopic Maxwell equations can be written as

$$\frac{\omega^2}{c^2} \vec{D}_{\omega} = \vec{\nabla} \times \vec{\nabla} \times \vec{E}_{\omega} \quad \text{and} \quad \vec{D}_{\omega} = \vec{\varepsilon}_{\omega} \vec{E}_{\omega},$$

which results in the following wave equation for the $\varphi_{\omega}$ field:

$$-\frac{1}{\varepsilon_1} \frac{\partial^2 \varphi_{\omega}}{\partial z^2} + \frac{1}{\varepsilon_2} \left( \frac{\partial^2 \varphi_{\omega}}{\partial x^2} + \frac{\partial^2 \varphi_{\omega}}{\partial y^2} \right) = \frac{\omega_0^2}{c^2} \varphi_{\omega} \quad (2)$$

Equation (2) is similar to the 3D Klein-Gordon equation describing a massive field propagating in a flat 2+1 dimensional Minkowski spacetime [13], in which the spatial $z$ coordinate behaves as a timelike variable. The opposite signs of $\varepsilon_1$ and $\varepsilon_2$ lead to two important consequences. The dispersion law of the extraordinary waves in such a uniaxial material

$$\frac{k_x^2 + k_y^2}{\varepsilon_2} + \frac{k_z^2}{\varepsilon_1} = \frac{\omega^2}{c^2} \quad (3)$$

describes a hyperboloid in the phase space. As a result, the absolute value of the k-vector is not limited, and the volume of phase space between two such hyperboloids (corresponding to different values of frequency) is infinite. This divergence leads to a formally infinite (in the continuous medium limit) density of photonic states in the hyperbolic frequency bands of the metamaterial [13]. The single photon $|0\rangle$ and $|1\rangle$ qubit states in the photonic hypercrystal geometry shown in Fig.1(a,b) may be encoded via the $k_{xy}$ component of the photon k vector, as indicated by the red and yellow arrows.

The choice of qubit encoding via the propagation direction is quite common in optical quantum computing geometries [1]. The buried grating, which can be switched on and
off via the nonlinear optical interactions of photons, mixes the photon $|0\rangle$ and $|1\rangle$ states, thus enabling optical quantum computations.

Let us now consider the field behaviour near the periodic $\text{Al}_2\text{O}_3/\text{LiNbO}_3$ interfaces. Taking into account the translational symmetry of the system in $x$ and $y$ directions, we can still use the in-plane wave vector $(k_x, k_y)$ as good quantum numbers, so that the propagating waves can be expressed as

$$E_{\omega}(\vec{r}) = E(z) \exp(ik_x x + ik_y y)$$
$$D_{\omega}(\vec{r}) = D(z) \exp(ik_x x + ik_y y)$$
$$B_{\omega}(\vec{r}) = B(z) \exp(ik_x x + ik_y y)$$

Because of the $z$ dependence of $\varepsilon_z$, it is now more convenient to introduce the wavefunction $\psi(\vec{r})$ as the $z$-component of the electric displacement field of the TM wave:

$$\psi(\vec{r}) = D_z(\vec{r}) = \varepsilon_z(z) E_z(\vec{r}) = -\frac{c}{\omega} k_z B$$

so that for the wave equation we obtain

$$-\frac{\partial^2 \psi}{\partial z^2} + \frac{\varepsilon_1}{\varepsilon_z(z)} \psi = \varepsilon_1 \frac{\omega^2}{c^2} \psi$$

(6)

In this wave equation the periodic $\varepsilon_1/\varepsilon_z$ ratio acts as a periodic effective potential. Solutions of eq.(6) may be found as Bloch waves

$$\psi(z) = \sum_{m=0}^{\infty} \psi_m \exp(i(k_z + \frac{2\pi m}{L})z)$$

(7)

where $k_z$ is defined within the first Brillouin zone $-\pi/L < k_z < \pi/L$. Strong Bragg scattering is observed near the Brillouin zone boundaries at $k_z \sim \pi/L >> \pi/\lambda$, leading to
Let us consider this wave function behavior near the Al$_2$O$_3$/LiNbO$_3$ interfaces. Let us assume that the thickness of a very thin transition layer between Al$_2$O$_3$ and LiNbO$_3$ is very small but finite ($\delta<<\lambda$), so that similar to [6], the following transition behaviour may be assumed near one of these interfaces located at $z=0$:

$$\varepsilon_2(z) = \varepsilon_1 \varepsilon_2 \frac{(1-\exp(z/\delta))}{(\varepsilon_1 - \varepsilon_2 \exp(z/\delta))} + \frac{i\Gamma}{(1-(\varepsilon_2/\varepsilon_1)\exp(z/\delta))},$$

where $\Gamma = 0.24$ equals the imaginary part of $\varepsilon_2$ of the wire array metamaterial at $\lambda=660$ nm [9] (a justification for such a transition layer to exist at the Al$_2$O$_3$/LiNbO$_3$ interface and extension of this model to the nonlinear optical properties of the interface will be done in Section 3). The corresponding effective potential $V=\varepsilon_1/\varepsilon_2$ experienced by extraordinary photons inside the photonic hyper-crystal is shown in Fig.2(a). Note that the potential step at the Al$_2$O$_3$/LiNbO$_3$ interface diverges in the limit of perfect loss compensation $\Gamma\to 0$ (see also Fig.3(a)). Substituting Eq.(8) into Eq.(6) and assuming $\Gamma<<1$, the wave equation near the Al$_2$O$_3$/LiNbO$_3$ interface may be re-written as

$$\left(u^2 + u\right)\frac{\partial^2 \psi}{\partial u^2} - \frac{Au + B}{u - 1} \psi = 0,$$

where $u=\exp(z/\delta)$,

$$A = \left(k^2 - \frac{\varepsilon_1 \omega^2}{c^2}\right)\delta^2,$$

$$B = \left(\frac{\varepsilon_1 \omega^2}{c^2} - \frac{\varepsilon_1}{\varepsilon_2} k^2\right)\delta^2,$$

and $k$ is the wave number. Note that $A<0$ if the dielectric may support a propagating wave with wave number $k$, and $A>0$ otherwise. As described in detail in [6], the general
solution of Eq. (9) in the limit $\Gamma \ll 1$ may be expressed in terms of the hypergeometric function $\,_{2}F_{1}(a,b,c,u)$ [14]:

$$
\psi(u) = u^{i\sqrt{B}}\,_{2}F_{1}^{*}(\sqrt{A} - i\sqrt{B}, \sqrt{A} - i\sqrt{B}, 1 - 2i\sqrt{B}, u) + ru^{-i\sqrt{B}}\,_{2}F_{1}^{*}(\sqrt{A} + i\sqrt{B}, \sqrt{A} + i\sqrt{B}, 1 + 2i\sqrt{B}, u)
$$

(12)

where the reflection coefficient $r$ is defined as

$$
r = \frac{\Gamma(\sqrt{A} + i\sqrt{B})\Gamma(1 + \sqrt{A} + i\sqrt{B})}{\Gamma(\sqrt{A} - i\sqrt{B})\Gamma(1 + \sqrt{A} - i\sqrt{B})} \frac{\Gamma(1 + 2i\sqrt{B})}{\Gamma(1 + 2i\sqrt{B})} \exp(-2\pi\sqrt{B})
$$

(13)

An example of calculated electric and magnetic field intensities in the $\Gamma \ll 1$ limit plotted near the interface in the case of A=30 and B=40 is shown in Fig.3(b). The electric field of extraordinary waves is strongly enhanced near the interface. For a wire array hyperbolic medium this field divergence may be explained via the well-known lightning rod effect at the tips of the silver nanowires.

We should also note that based on Eq.(2), the factor $(-\varepsilon_2/\varepsilon_1)^{1/2}$ plays the role of a scale factor of a 2+1 dimensional effective “optical spacetime”, which “interval” may be introduced as

$$
\,ds^2 = -\,dz^2 + (-\varepsilon_2/\varepsilon_1)\left(dx^2 + dy^2\right)
$$

(14)

Since $\varepsilon_2$ is negative, in the case of hyperbolic metamaterials the concept of “optical spacetime” replaces the “optical space”, which is typically introduced in transformation optics [15]. The scale factor of the effective Minkowski spacetime calculated using Fig.2(a) is plotted in Fig.2(b). If the spacetime analogy is used, each Al$_2$O$_3$/LiNbO$_3$ interface of the photonic hyper-crystal behaves as either big bang or big crunch singularity. Inflation-like behaviour of the effective scale factor is observed near each interface, leading to extremely large field and strong nonlinearities near the interfaces.
3. Nonlinear photon-photon interaction in hyperbolic resonator-based photonic hyper-crystals

Let us now consider how the introduction of photons into the individual hyperbolic resonators alters the effective potential $V = \varepsilon_1 / \varepsilon_2$. The Maxwell-Garnett approximation may be used to evaluate the nonlinear optical effects in a wire array hyperbolic metamaterial structure and in the transition layer of thickness $\delta$ between the metamaterial and the lithium niobate. The diagonal components of the permittivity tensor of the wire array metamaterial may be obtained as [16]:

$$
\varepsilon_1 = \varepsilon_{x,y} = \frac{2\alpha \varepsilon_m \varepsilon_d + (1-\alpha) \varepsilon_d (\varepsilon_d + \varepsilon_m)}{(1-\alpha)(\varepsilon_d + \varepsilon_m) + 2\alpha \varepsilon_d} \approx \frac{1 + \alpha}{1 - \alpha} \varepsilon_d , \text{ and (15)}
$$

$$
\varepsilon_2 = \varepsilon_z = \alpha \varepsilon_m + (1-\alpha) \varepsilon_d \text{ (16)}
$$

where $\alpha$ is the volume fraction of the metallic phase, and $\varepsilon_m < 0$ and $\varepsilon_d > 0$ are the dielectric permittivities of the metal and dielectric component of the metamaterial, respectively (typically, $-\varepsilon_m >> \varepsilon_d$). Assuming the same material parameters as in ref. [9] at 660 nm, the dielectric permittivity of $\text{Al}_2\text{O}_3$ is $\varepsilon_d = 2.4$, while the permittivity of silver is $\varepsilon_m = -21.6+0.8i$, so that $\alpha = 0.3$. Based on Eqs.(15,16), the transition layer of thickness $\delta$ between the metamaterial and the lithium niobate (which was approximated by Eq.(8) in Section 2) may be ascribed to the gradual $\alpha \rightarrow 0$ transition near the metamaterial surface, which is accompanied by surface roughness of the $\text{Al}_2\text{O}_3$/LiNbO$_3$ interface. We will assume that the nonlinear optical effects do not affect $\varepsilon_m$ (since light does not penetrate substantially into silver nanowires), so that only the dielectric permittivities of $\text{Al}_2\text{O}_3$ and LiNbO$_3$ will be influenced by the interfacial electric field divergence (shown in Fig.3(b)) via the nonlinear Kerr effect:

$$
n = n_0 + n_2 I \text{ (17)}
$$
According to [12], the nonlinear refractive index of doped LiNbO$_3$ reaches up to $n_2 = 1.73 \times 10^{-10}$ cm$^2$/W, while much lower values $n_2 \approx 3.3 \times 10^{-16}$ cm$^2$/W of the nonlinear refractive index has been reported in the literature for Al$_2$O$_3$ [17]. Thus, based on the arguments above and similar to Section 2, we may assume that near the interface located at $z=0$

$$n_2(z) = \frac{(n_{2+} + n_{2-})}{2} - \left(\frac{(n_{2+} - n_{2-})}{2}\right) \frac{(1 - \exp(z/\delta))}{(1 + \exp(z/\delta))} \approx n_{2+} \exp(z/\delta)$$

and that the effective potential in the presence of photon field may be estimated as follows:

$$V(z) = \frac{\varepsilon_1}{\varepsilon_2} \approx \frac{\varepsilon_1^{(0)}}{\varepsilon_2^{(0)}} + 2\sqrt{\varepsilon_1^{(0)} n_2(z) I(z)} \approx \frac{\varepsilon_1^{(0)}}{\varepsilon_2^{(0)}} + 2\sqrt{\varepsilon_1^{(0)} n_2(z) I(z)}$$

where $\varepsilon_1^{(0)}$ and $\varepsilon_2^{(0)}$ are the dielectric tensor components at $I=0$. Eq.(19) makes clear that the main effect of nonlinearities consists in shifting the position of the potential wall with respect to $z=0$. This behaviour is illustrated in Fig. 4(a) for two increasing levels of light intensity assuming 0.4x0.4x0.4 $\mu$m$^3$ hyperbolic resonator dimensions (same as in Fig.2) and either $2.3 \times 10^4$ or $7 \times 10^4$ photons in the resonator. Reducing the resonator dimensions to $\lambda/10$ (or 66 nm on the side) would reduce the required number of photons in the resonator to achieve the same effect to either 100 or 300, respectively. While this light level is relatively far from the single photon limit, Fig.4(b) illustrates that nonlinear photonic hyper-crystals based on such simple nonlinear dielectrics as LiNbO$_3$ or ZnSe may be used in classical optical computing schemes. For example, control light may be used to drastically change transmission of the photonic hyper-crystal via broadening of the tunnelling gaps around the nonlinear dielectric layers. Alternatively, mode coupling properties of a buried grating may be altered between the “on” and “off” states via positioning the potential wall of the $V=\varepsilon_1/\varepsilon_2$ potential barrier either in front or behind the grating. As demonstrated e.g. in recent ref.[18], classical
optical computing remains an attractive intermediate option between the conventional electronic classical computing and the future quantum computing schemes. Therefore, the optical “hyper-computing” scheme described in more detail in Section 4 below, may turn out to be useful even at the classical level.

An obvious way to increase $n_2$ by two to three orders of magnitude compared to LiNbO$_3$, and reach the single photon level is to implement a photonic hyper-crystal cavity design, and make use of the electromagnetically induced transparency (EIT) effects in either atomic impurities or quantum dots, as it is typically done in the conventional photonic crystal cavity geometries [2]. Kerr-like nonlinearities are enhanced by several orders of magnitude in such configurations, so that considerable photon-photon interaction may be observed at a single-photon level. Engineering of such photonic hyper-crystal nanocavities is possible due to formation of photonic band structure in hyperbolic metamaterials at any desired hyper-crystal periodicity $L$, which may be much smaller than the free space wavelength $\lambda$. Examples of the photonic hyper-crystal band structure calculations may be found in refs.[3,4]. Much higher levels of $n_2$ nonlinearity are achieved due considerably smaller mode volumes in such nanocavities. We should also mention a very recent study of Purcell enhancement of parametric luminescence in hyperbolic metamaterials [19], which reported a 1000 fold enhancement of the down-converted emission rate in experimentally realistic nanostructures, which gives yet another example of several orders of magnitude enhancement of nonlinear optical effects in hyperbolic nanostructures. In the next section we analyze the quantum mechanical properties of the nonlinear photonic hyper-crystals.
4. Quantum mechanical properties of photonic hyper-crystals: quantum hyper-computing

Quantum mechanics of light in a hyperbolic metamaterial has been considered in detail in ref.[20]. The transition from classical to quantum optics occurs when the number of photons in any given mode is no longer large. Using the correspondence principle, the wave equation describing extraordinary photons propagating inside the hyperbolic metamaterial (Eq.(2)) may be re-written as follows:

\[ -\frac{1}{\varepsilon_1} \frac{\partial^2}{\partial z^2} + \frac{1}{|\varepsilon_2| \varepsilon_2} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) - \frac{m^*}{\hbar^2} c^2 \frac{\partial^2}{\partial z^2} \varphi_\omega = 0 \]  

(20)

where \( \varphi_\omega \) is now understood as a quantum mechanical photon wave function, and the effective mass \( m^* \) equals

\[ m^* = \frac{\hbar \omega}{c^2} \]  

(21)

In the “non-relativistic” limit in which the kinetic energy (second term) in the parenthesis in Eq.(20) is much smaller than the effective rest energy \( m^* c^2 \), eq.(20) reduces to a standard Schrödinger equation:

\[ i \frac{\hbar c}{\varepsilon_1} \frac{\partial}{\partial z} \varphi_\omega = \hat{H} \varphi_\omega = \pm \left( m^* c^2 - \frac{\hbar^2}{\varepsilon_2 |2m^*|} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \right) \varphi_\omega = \pm \left( m^* c^2 + \frac{\hat{p}_x^2 + \hat{p}_y^2}{\varepsilon_2 |2m^*|} \right) \varphi_\omega \]  

(22)

However, the role of effective Hamiltonian operator in this equation is played by

\[ \hat{H} = i \frac{\hbar c}{\varepsilon_1} \frac{\partial}{\partial z} \]  

(23)

(note that in the non-relativistic quantum mechanics the \( m^* c^2 \) term is usually omitted by re-defining the zero energy). If \( \varepsilon_1 \) and \( \varepsilon_2 \) are allowed to vary, an effective potential energy term would also appear in eq.(22). Thus, Eq.(22) replicates a version of non-
relativistic quantum mechanics in a 2+1 dimensional Minkowski spacetime, in which the spatial \( z \) coordinate plays the role of effective time. However, within the scope of this model, measurements performed in the “past” and in the “future” (defined with respect to an arbitrary “time arrow” along the \( z \) coordinate) must be treated on absolutely equal footing. This model of quantum mechanics is perfectly “time symmetric” and both pre-selection and post selection of the quantum states is allowed (as long as the experimental arrangement is symmetric with respect to \( k_z \rightarrow -k_z \) transformation). Note also that in a more general “relativistic” situation where the effective kinetic energy is no longer much smaller than the rest energy \( m^*c^2 \), the “relativistic” Eq.(20) must be used. Thus, for all practical purposes the quantum mechanics of hyperbolic metamaterials and photonic hyper-crystals looks very similar to conventional quantum mechanics, except that the spatial \( z \) coordinate plays the role of time.

While such a re-definition of quantum mechanics in hyperbolic metamaterials may not seem to be useful at first glance, it has a very interesting consequence. The speed of conventional computations (both classical and quantum) is ultimately limited by the speed of light. It limits, for example, the clock frequency of the computer. Since both classical and quantum computing rely on the clocks to perform a set of programmable steps, one step after another, a given computation requires a time interval given by the number of computational steps \( N \) divided by the clock frequency \( \nu \).

The model of quantum mechanics outlined above seems to be able to map such a conventional (temporal) computation onto a computation performed in a hyperbolic metamaterial using \( N \) spatial steps instead of \( N \) temporal ones. As illustrated in Fig.5, due to such a periodic spatial “clock”, which is used in the mapping, the hyperbolic metamaterial becomes a photonic hyper-crystal. The potential advantage of a photonic hyper-crystal based hyper-computing scheme would be a much faster computation
speed, which is important in time-sensitive applications. However, the potential importance of experimental realization of a quantum hyper-computing scheme would go far beyond simple mapping from $z$ to $t$ in the differential equations describing quantum mechanics of hyperbolic metamaterials. This mapping leads to deep and non-trivial physical questions related to the very foundations of quantum mechanics. For example, conventional non-relativistic quantum mechanics treats spatial and temporal coordinates differently (time is not an operator). This difference appears to be related to the issues of quantum non-locality and post-selection, which are hotly debated in the quantum mechanics literature. A quantum hyper-computer built according to the prescription shown in Fig.5 would interchange $z$ and $t$, and may provide novel experimental insights into these issues.

We should also note that the quantum hyper-computing proposal outlined above have some features in common with the recently suggested “faster-than-light” computing proposal based on immersion of an optical computer into a medium having index of refraction smaller than one [21], thereby trespassing the speed-of-light communication barrier. While both “faster-than-light” and “hyper-computing” proposals rely on the artificial metamaterial media for their accelerated operation, the proposal of Putz and Svozil [21] still relies on the more conventional “temporal” computing and communication scheme.

The large density of optical states in hyperbolic metamaterials implies reduced group velocity compared to the velocity of light in vacuum, which may impede the quantum hyper-computing computation scheme. However, the magnitude of group velocity remains very large in hyperbolic materials. For example, recent detailed calculations performed in [22] indicate group velocities in the hyperbolic frequency domains of realistic metal-dielectric metamaterials of the order of $v_g \sim 0.1c_0 \sim 0.3c_0$, where $c_0$ is the velocity of light in vacuum. Since the photonic hyper-crystal geometry implies
subwavelength dimensions $L$ of the hyperbolic metamaterial resonators (see Fig. 1a), the corresponding information propagation time between the resonators $L/v_g$ would fall into the femtosecond range, which far exceeds the $\sim$GHz clock speed of both classical and quantum state of the art computers.

5. Conclusion

In conclusion, we have demonstrated that photonic hyper-crystals exhibit light localization on deep subwavelength scales, leading to considerable enhancement of nonlinear photon-photon interaction. Like their conventional photonic crystal counterparts, nonlinear photonic hyper-crystals appear to be very promising in quantum computing applications. Since spatial coordinate plays the role of time in this geometry, a quantum “hyper-computing” scheme may be suggested, which considerably accelerates computation time, leading to considerable advantages over the conventional quantum computing schemes in time-sensitive applications. According to their definition, time-sensitive applications are driven by real world demands and are characterized by timing constraints that must be satisfied for successful operation. One example of such a time-sensitive application is cryptanalysis, which is known to be performed faster by quantum computers. In addition, quantum computers are believed to be able to efficiently solve real life “big data” problems which are not practically feasible on classical computers. These applications may benefit from the faster quantum hyper-computing scheme.

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Figure Captions

Figure 1. (a) Schematic geometry of a photonic hyper-crystal based on hyperbolic material resonators separated by thin nonlinear dielectric layers. The single photon $|0\rangle$ and $|1\rangle$ qubit states are encoded via the $k_{xy}$ component of the photon k vector, as indicated by the red and yellow arrows. Effective horizons leading to field divergences occur at hyperbolic/dielectric interfaces, thus enabling nonlinear photon-photon interaction. (b) Side view of the nonlinear dielectric layer having a “buried grating”. The grating, which can be switched on and off mixes the photon $|0\rangle$ and $|1\rangle$ qubit states.

Figure 2. (a) Effective potential $\varepsilon_1/\varepsilon_2$ experienced by extraordinary photons inside the Al$_2$O$_3$/LiNbO$_3$ photonic hypercrystal. In this simulation the hypercrystal parameters are: $L=0.4\mu m$, LiNbO$_3$ layer thickness is $d=0.08\mu m$, the transition layer thickness is $\delta=20nm$, $\Gamma=0.24$. (b) Scale factor $(\varepsilon_2/\varepsilon_1)^{1/2}$ of the effective optical 2+1D Minkowski spacetime experienced by the extraordinary photons inside the hyperbolic metamaterial resonators.

Figure 3. (a) Effective potential $\varepsilon_1/\varepsilon_2$ near the Al$_2$O$_3$/ LiNbO$_3$ interface calculated without loss compensation using $\Gamma=0.24$ magnitude of the imaginary part of $\varepsilon_2$ of the wire array metamaterial [9]. Note that the potential step at the interface diverges in the limit of perfect loss compensation $\Gamma\to0$. (b) Electric (red) and magnetic (blue) field intensities calculated near the interface in the $\Gamma<<1$ limit for the A>0 case (A=30, B=40).

Figure 4. (a) Shifting position of the potential wall with respect to z=0 using control light. These simulations assume 0.4x0.4x0.4 mm$^3$ hyperbolic resonator dimensions (same as in Fig.2) and either $2.3\times10^4$ or $7\times10^4$ photons in the resonator. Reducing the
resonator dimensions to $\lambda/10$ (or 66 nm on the side) would reduce the required number of photons in the resonator to achieve the same effect to approximately 100 and 300, respectively. (b) In a classical optical computing scheme control light may be used to drastically change transmission of the photonic hyper-crystal via broadening of the tunnelling gaps around the nonlinear dielectric layers. Alternatively, mode coupling properties of a buried grating may be altered between the “on” and “off” states via positioning the potential wall of the $V=\varepsilon_1/\varepsilon_2$ potential barrier either in front or behind the grating.

**Figure 5.** Mapping of a conventional (temporal) quantum computation onto a quantum computation performed in a hyperbolic metamaterial using $N$ spatial steps instead of $N$ temporal ones: The top portion of the figure shows schematically the succession of operations performed on a set of several qubits in a conventional (temporal) quantum computer. These operations are performed one at a time in sync with a temporal clock signal. The bottom portion of this figure shows how these operations may be mapped onto operations performed with single-photon qubits in a photonic hyper-crystal. The hyper-crystal periodicity plays the role of the clock signal.
Fig. 1
Fig. 2
Fig. 3
Fig. 4
Fig. 5

conventional (“temporal”) quantum computing:

quantum hyper-computing:

photonic hyper-crystal