Multi–Layer Structure in the Strongly Coupled 5D Abelian Higgs Model

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We explore the phase diagram of the 5–D anisotropic Abelian Higgs model by Monte Carlo simulations. In particular, we study the transition between the confining phase and the four dimensional layered Higgs phase. We find that, in a certain region of the lattice parameter space, this transition can be first order and that each layer moves into the Higgs phase independently of the others (decoupling of layers). For more information about the layer phase formation see (\cite{1}–\cite{4}) and the talk by S. Nicolis\textsuperscript{5} in the same volume.

1. Formulation of the model

We write down the lattice action of the 5-D Abelian Higgs model in typical notation. Direction \( \hat{5} \) (transverse) will be singled out by couplings (primed) that will differ from the corresponding ones in the remaining four directions.

\[
S = \beta_g \sum_x \sum_{1 \leq \mu < \nu \leq 4} (1 - \cos F_{\mu\nu}(x)) \\
+ \beta'_g \sum_x \sum_{1 \leq \mu \leq 4} (1 - \cos F_{\mu5}(x)) \\
+ \beta_h \sum_x \text{Re}[4\varphi^*(x)\varphi(x) - \sum_{1 \leq \mu \leq 4} \varphi^*(x)U_{\hat{\mu}}(x)\varphi(x + \hat{\mu})] \\
+ \beta'_h \sum_x \text{Re}[\varphi^*(x)\varphi(x) - \varphi^*(x)U_{\hat{5}}(x)\varphi(x + \hat{5})] \\
+ \sum_x [(1 - 2\beta_R - 4\beta_h - \beta'_h)\varphi^*(x)\varphi(x) \\
+ \beta_R(\varphi^*(x)\varphi(x))^2] \quad (1)
\]

The order parameters that we use are the plaquette and the link defined either on 4-dimensional volume or in the transverse direction. We define also the Higgs measure \( R^2 \) on the 5-D volume. Here, we show only the space–like link \( L_S \) and transverse–like link \( L_T \):

\[
L_S \equiv \frac{1}{4N^5} \sum_x \sum_{1 \leq \mu \leq 4} \cos(\chi(x + \hat{\mu}) + A_{\hat{\mu}}(x) - \chi(x)) > (2)
\]

\[
L_T \equiv \frac{1}{N^5} \sum_x \cos(\chi(x + \hat{5}) + A_{\hat{5}}(x) - \chi(x)) > (3)
\]

\( N \) is the linear dimension of a symmetric \( N^5 \) lattice.

2. Monte Carlo Results

and the Confining–Layered Transition

For the simulations we use 5-hit Metropolis algorithm for the updating of both the gauge and Higgs fields. In order to get better behaviour we use a global radial algorithm and an overrelaxation algorithm for the updating of the Higgs field. We simulated the system for \( 4^5, 6^5, \) and \( 8^5 \) lattices. We made use, mainly, of the hysteresis loop technique to establish the phase diagram of the system. For the results shown here we set \( \beta_g = 0.5, \beta'_h = 0.001 \) and \( \beta_R = 0.1 \). Thus the phase diagram has been found in the \( \beta'_g - \beta_h \) subspace. In Figure 1 the phase diagram for \( \beta_R = 0.1 \) is given. S denotes the strong phase, C is the 5-D Coulomb phase, \( H_5 \) is the 5-D Higgs phase and \( H_4 \) is the layer Higgs phase in 4-D. Further analysis for the phase diagrams and for the order of the phase transitions can be found in \cite{6}.
2.1. The Multi–Layer structure

In Figure 2 we show the very different way for the transition of \( L_S \) calculated either on the five dimensional volume and on four layers. In the figure 2a the \( S - H_4 \) transition is presented where the layers show a decoherent behaviour on the phase transition in contrast with the \( S - H_5 \) phase transition Fig. 2b where the transition for \( L_S \) is identical for all layers, so it coincides with the mean values over the 5D–volume. This specific behaviour of hysteresis loops, may actually serve as a “criterion” to characterise the layered phase.

This behaviour can be further confirmed by long runs in the transition region. From the hysteresis loop results found it is to be expected that the \( S - H_4 \) phase transition is of first order. Therefore, we would expect a two peak signal in the order parameter distribution at equilibrium. Nevertheless the situation, shown in Figure 3(a) concerning the distribution for the order parameter \( L_S \) for a value of \( \beta_h \) near the transition region for \( V = 6^5 \) is by no means what one would expect normally. The multipeak structure seems rather strange. It should be noticed that the same occurs for the order parameter \( P_S \) too. However, the study of the same order parameter defined on each space–like volume is more illuminating.

For example in Figure 3(b), we can see the distribution of \( L_S \) values on each layer. We show four out of six distributions of \( L_S \) corresponding to the four space–like layers within the five–dimensional volume. The distributions which are produced appear quite usual and they show that at the same time one layer is in the strong phase (called 2nd in the figure), another has already
Figure 3. Distribution for $L_S$ on the volume (a) and on each layer (only four of the six are shown in the figure)(b) for $6^5$ lattice in the critical region for the $S - H_4$ phase transition.

passed to the broken phase ($\gamma_{cd}$) and others produce a two peak signal result. The result is that the strange picture of the distribution formed in Figure 3(a) is resolved if we analyze the behaviour of the system on each layer as the system undergoes the phase transition. This result is found for all of the volume sizes (e.g. $4^5, 6^5, 8^5$) which we have worked on. Although it is consistent with a first order phase transition it lends support to the view of a *decoherent* behaviour for every four-dimensional volume (layer) in the transition region between five-dimensional strong phase and the four-dimensional layered Higgs phase. This certain behaviour describes a dynamical decoupling of the layers and provides a possible mechanism for localization of the fields on the layers.

3. Conclusions

We have shown, by using Monte–Carlo methods, that a layered Higgs phase actually exists in the phase diagram of the five-dimensional Abelian Higgs model. Furthermore, this layered phase is separated by a phase transition with the five-dimensional strong phase. In the region of parameter space that we explored the phase transition in question is first order and its additional feature is the multi–layer formation. The latter occurs as a consequence of the independent formation of each layer as the phase transition takes place.

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REFERENCES

1. Y.K. Fu and H.B. Nielsen, Nucl. Phys. B 236, 167 (1984); Nucl. Phys. B 236, 127 (1985).
2. C.P. Korthals-Altes, S. Nicolis and J. Prades, Phys. Lett. B 316 339 (1993) [hep-lat/9306017]; A. Hulsebos, C.P. Korthals-Altes and S. Nicolis, Nucl. Phys. B 450 437 (1995) [hep-th/9406004].
3. P. Dimopoulos, K. Farakos, A. Kehagias, G. Koutsoumbas Lattice Evidence for Gauge Field Localization on a Brane [hep-th/0007079].
4. P. Dimopoulos, K. Farakos, G. Koutsoumbas, C.P. Korthals-Altes, S. Nicolis, J. High Energy Phys. 02(005) (2001) [hep-lat/0012028].
5. S. Nicolis, Branes in the 5D (Abelian) Higgs Model, these proceedings.
6. P. Dimopoulos, K. Farakos, S. Nicolis, [hep-lat/0105014].