ALFVÉN WAVE–DRIVEN SUPERNova EXPLOSION
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ABSTRACT
We investigate the role of Alfvén waves in a core-collapse supernova (SN) explosion. We assume that Alfvén waves are generated by convections inside a proto–neutron star (PNS) and emitted from its surface. These waves then propagate outward, dissipate via nonlinear processes, and heat up matter around a stalled prompt shock. To quantitatively assess the importance of this process for the revival of the stalled shock, we perform one-dimensional time-dependent hydrodynamical simulations, taking into account the heating via the dissipation of Alfvén waves that propagate radially outward along open flux tubes. We show that shock revival occurs if the surface field strength is larger than \(\sim 2 \times 10^{15} \) G and if the amplitude of the velocity fluctuation at the PNS surface is larger than \(\sim 20\% \) of the local sound speed. Interestingly, the Alfvén wave mechanism is self-regulating in the sense that the explosion energy is not very sensitive to the surface field strength or initial amplitude of Alfvén waves, as long as they are larger than the threshold values given above.

Subject headings: MHD — supernovae: general — waves

Online material: color figure

1. INTRODUCTION
The most promising scenario for collapse-driven supernovae (SNe) is currently supposed to be delayed explosion by neutrino heating (Kotake et al. 2006; Janka et al. 2007, and references therein), in which the prompt shock, generated by the core bounce and stalled by neutrino emission and dissociations of nuclei, is heated up and revived by neutrinos coming out of the proto–neutron star (PNS), leading eventually to a SN explosion. Under spherical symmetry, however, no successful explosion has been obtained so far, even though up-to-date microphysics, such as equations of state and weak interaction rates, have been fully incorporated (Janka et al. 2007). Various other effects have also been explored over the years. The implications of stellar rotation and different sorts of hydrodynamical instabilities have been extensively studied. (Kotake et al. 2006; see also Marek & Janka 2007; Mezzacappa et al. 2007 for very recent progress.)

Magnetic fields are drawing much renewed attention from researchers these days, although the history of research is long. After some pioneering papers (LeBlanc & Wilson 1970; Bisnovatyi-Kogan et al. 1976; Meier et al. 1976; Symbalisty 1984), the subject had been forgotten for a while, because it was realized that very strong magnetic fields are required to affect the supernova dynamics, and such strong fields were supposed to be unrealistic at that time. The situation changed with the observational evidence of strongly magnetized neutron stars or magnetars (Thompson & Duncan 1996), and progress in the theoretical understanding of magnetorotational instability or MRI (Balbus & Hawley 1991).

The possible importance of MRI in the generation of magnetic field in core-collapse SNe was first pointed out by Akiyama et al. (2003). Many subsequent papers (Yamada & Sawai 2004; Thompson et al. 2005; Moiseenko et al. 2006; Kotake et al. 2006; Burrows et al. 2007; Wheeler & Akiyama 2007) have investigated the possible roles of magnetic fields in supernova dynamics, especially kinematical aspects such as magnetic pressure and torque induced by rapid rotations in supernova cores. Thompson et al. (2005), on the other hand, considered MHD turbulence that is possibly induced by MRI as a source of viscosity to tap free energy stored in differential rotation. Although the rapid rotation of the stellar core is a prerequisite in these studies, it may not be so easy to obtain in the presence of magnetic fields, according to recent stellar evolution models, because the transfer of angular momentum is efficient and the core may rotate rather slowly just prior to the gravitational collapse (Heger et al. 2005).

More recently, yet another supernova mechanism was put forward: sound waves generated by PNS oscillations of a mainly \(g\)-mode nature, which are probably induced by turbulence caused by the standing accretion shock instability (SASI), may heat up matter through nonlinear dissipations, revive the stalled shock wave, and produce explosions at very late times (Burrows et al. 2006). Since this acoustic mechanism consists of several steps, whose efficiencies appear to be not very high (Yoshida et al. 2007; Marek & Janka 2007), the viability of the mechanism is still controversial, and further explorations from various points of view are needed.

At present, the supernova mechanism is still elusive in spite of these extensive efforts. The above-mentioned research trend naturally leads us to the exploration of still another type of waves that may also contribute to the supernova explosion: Alfvén waves. If the supernova core is magnetized, the oscillation of the PNS will emit not only sound waves but also Alfvén waves. It is also possible that Alfvén waves are excited by convection in the PNS, which has been demonstrated to exist after core bounce (Keil et al. 1996). This is analogous to what happens in the Sun (e.g., Suzuki & Inutsuka 2005, 2006, hereafter SI05, SI06). A

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fraction of these waves propagate outward and dissipate later through nonlinear processes, which then will heat up matter and may lead to the revival of the stalled shock. Such a scenario as a supernova mechanism has not been studied on a quantitative basis so far, although it has been discussed in a qualitative manner (e.g., Wheeler et al. 2000; Woosley & Janka 2005; Burrows et al. 2006) and in the context of nucleosynthesis in $\nu$-driven winds (Suzuki & Nagataki 2005). It should be emphasized that it is the dissipations of fluctuating components of magnetic fields associated with Alfvén waves that heat up matter and revive the shock wave, whereas in most previous papers, torques and pressures exerted by the total magnetic fields were the key players. Since the main aim in this paper is not to construct a realistic model but to elucidate the characteristics of the Alfvén wave heating in a quantitative manner, we employ one-dimensional simplified but dynamical simulations.

2. MODELS

In this section, we describe the scenario we have in mind and explain the nonlinear dissipation processes of Alfvén waves. Based on these, we give the basic equations and approximations employed in detail.

2.1. Basic Picture

Convection around the neutrinosphere that roughly coincides with the surface of a PNS is induced by negative gradients of lepton fraction and entropy about a few tens of milliseconds after core bounce. According to hydrodynamical simulations of PNS convections by Keil et al. (1996), the amplitude of velocity fluctuations near the PNS surface is $\approx 4 \times 10^3$ km s$^{-1}$ on average and becomes as high as $10^4$ km s$^{-1}$. Recent simulations also reported that SASI in accreting flows might excite a comparable order of surface fluctuations (Ohnishi et al. 2006). Nonradial oscillations of PNS are another obvious possible source of Alfvén waves.

These activities in PNS generate various modes of waves with the amplitude of velocity fluctuations mentioned above, which then emanate from its surface and propagate outward. If the magnetic field is sufficiently strong, two types of magnetic waves are important, Alfvén waves and fast waves. The Alfvén waves propagate along field lines and are less subject to dissipation, thanks to their incompressive character. On the other hand, the fast waves propagate almost isotropically and can traverse field lines, although they suffer more damping owing to their compressive nature. Therefore, Alfvén waves can transport energy farther away along magnetic filed lines unless they form closed loops. The fast waves could become important, however, in the equatorial region if the PNS rotates rapidly and the field lines are tightly wound up. In this paper, we assume strong magnetic fields of a realistic model, which should be the next step, but to understand the essential features of Alfvén wave heating in the postbounce core, we make the models as simple as possible, incorporating only minimum ingredients. Since stellar rotation is not an indispensable player in the mechanism considered here, we neglect it, and assume radial magnetic fields as the simplest configuration for open field lines in this paper. The field strength is given by

$$B_r = B_{r,0} \frac{r^2}{r_0^2},$$  \hspace{1cm} (1)

where $B_{r,0}$ is the field strength at the PNS surface, $r = r_0$. We perform one-dimensional (1D) simulations, ignoring the effects of neighboring magnetic fields.

Even if the stellar rotation is fast, our models will still be applicable locally to the polar region, where the centrifugal force is a minimum, although the simple prescription for the expansion factor of flux tubes, $\propto r^2$, adopted in this paper may need further elaboration. In the equatorial region, on the other hand, the validity of the assumptions in this paper depend not only on the rotation period but also on the reconnection efficiency among the field lines frozen into the accreted matter. This issue will be discussed again in § 4.1.

2.2. Dissipation of Alfvén Waves

The effects of the Alfvén waves on accreting matter are two-fold: (1) the heating of matter by the Alfvén wave dissipations and (2) the extra pressure exerted by the Alfvén waves. We take into account only the former in this paper. The Alfvén waves with linear amplitudes are nondissipative due to their incompressive nature. However, nonlinear Alfvén waves suffer various dissipation processes. The nonlinearity, $w$, of Alfvén waves can be defined as

$$w \equiv \frac{\delta v_{\perp}}{v_A} = \frac{\delta B_{\perp}}{B_r},$$  \hspace{1cm} (2)

where $\delta v_{\perp}$ and $\delta B_{\perp}$ are the amplitudes of the velocity and magnetic field, the subscript “$\perp$” denotes the transverse directions with respect to $B_r$, and $v_A = B_r/(4\pi \rho)^{1/2}$ is the Alfvén speed. Here we have used the relation $\delta v_{\perp} = \delta B_{\perp}/(4\pi \rho)^{1/2}$, satisfied by the Alfvén wave (see § 10 of Lamers & Cassinelli 1999). Even though the initial amplitude at the launch from the PNS surface is in the linear regime, $w \ll 1$, it grows thanks to the decrease of $B_r$ (eq. [1]). Eventually, the Alfvén wave becomes nonlinear, $w \sim 1$, and various dissipation processes set in.

The excitation of compressive waves by nonlinear mode conversions is one route of the dissipation. If the Alfvén wave is not strictly circularly polarized, the magnetic pressure, $\delta B_{\perp}/8\pi r$, fluctuates along with $B_r$ and induces longitudinal compressive motions, most of which correspond to slow magnetohydrodynamical (MHD) waves (SI05, SI06). Even if the Alfvén wave is circularly polarized, it is subject to the parametric decay instability, which generates outgoing slow MHD waves and incoming Alfvén waves.
waves (Goldstein 1978; Terasawa et al. 1986). The velocity amplitudes of the slow waves are also amplified as they propagate outward and the density decreases. Eventually, the wave fronts steepen to form shocks and heat up matter.

If \( B_r \) has a transverse gradient (along the \( \perp \) direction), fast MHD waves that propagate perpendicularly to \( B_r \) are also excited from Alfvén waves (Nakariakov et al. 1997). These fast waves also heat ambient matter by shock formation. Moreover, the transverse inhomogeneity of \( B_r \) leads to phase mixing (Heyvaerts & Priest 1983), which is another channel for the dissipation of Alfvén waves.

The turbulent cascade may also work to dissipate Alfvén waves. In the PNS, the Alfvén speed is not constant along the radial direction, and density decreases. Eventually, the wave fronts steepen to form shocks and heat up matter.

Outward propagating and incoming Alfvén waves will lead to the formation of smaller scale (large wavenumber) structures, mainly in the perpendicular directions (Goldreich & Sridhar 1995). This turbulent cascade to higher wavenumber proceeds up to the dissipation range, where resistivities (for magnetic field fluctuations) and/or viscosities (for velocity fluctuations) become important. Then, the energy that Alfvén waves are carrying is finally transferred to the ambient matter.

As a result of these various dissipation processes, the non-linearity, \( w \), of the outgoing Alfvén waves is saturated at a certain level. According to dynamical simulations of solar (SI05, SI06) and stellar (Suzuki 2007) winds, the saturation level is found to be \( w \approx 0.3 \rightarrow 1 \), which we apply in this paper to the Alfvén waves in the supernova core. It should be noted that saturation with a constant \( w \) implies that \( \delta B\perp \), itself decreases as the Alfvén waves propagate farther outward, since \( B_r \) is declining (see eq. [2]). This means a dissipation of the energy of the Alfvén waves and results in the heating of the matter.

2.3. Formulation

We evaluate the Alfvén wave heating by considering the conservation of wave energy under the WKB, or short wavelength, approximation. This treatment enables us to obtain estimations in a simple manner without solving the fully nonlinear MHD equations. The conservation of wave energy can be expressed as (Jacques 1977)

\[
\frac{\partial \mathcal{E}_w}{\partial t} + \nabla \cdot \mathbf{F}_w - \mathbf{v} \cdot \nabla P_w = -\rho \dot{q}_w, \tag{3}
\]

where \( \rho \) and \( \mathbf{v} \) are the density and velocity of the accretion flow, respectively, \( \mathcal{E}_w \) is the wave energy density, \( \mathbf{F}_w \) is the wave energy flux, \( P_w \) is the wave pressure, and \( \dot{q}_w \) is the dissipation rate of wave energy per unit mass. As for the Alfvén waves, \( \mathcal{E}_w = \rho \langle \delta B^2 \rangle / \langle \mu \rangle / 4\pi \) and \( P_w = \mathcal{E}_w/2 \), where the angle bracket \( \langle \ldots \rangle \) stands for the average over a period of the Alfvén wave. The last term on the left-hand side denotes the rate of work done by the Alfvén waves on the accretion flow, which we neglect in this paper, as stated above. The right-hand side represents the energy deposition by the Alfvén waves to the accreting matter. When \( \dot{q}_w > 0 \), the matter is heated up by wave dissipations; the Alfvén waves travel without dissipation if \( \dot{q}_w = 0 \).

It is convenient for later use to introduce an adiabatic constant, the so-called wave action, \( \mathbf{H}_w \), defined as (Jacques 1977)

\[
\nabla \cdot \mathbf{F}_w - \mathbf{v} \frac{dP_w}{dr} = \frac{v_\perp}{v_\parallel} \nabla \cdot \mathbf{H}_w. \tag{4}
\]

We neglect relativistic corrections because they are minor outside the PNS. It should be noted that the wave action, \( \mathbf{H}_w \), unlike \( F_w \), is conserved in moving media. The specific form of \( \mathbf{H}_w \) is given as (Jacques 1977)

\[
\mathbf{H}_w = \left( \frac{\delta B^2}{4\pi} \right) (v_\perp + v)(v_\perp + v). \tag{5}
\]

In this paper we assume a steady propagations of Alfvén waves and neglect the time derivative, \( \partial / \partial t \), in equation (3). This is valid when the Alfvén transit time is shorter than a typical timescale of the system, which will be discussed in § 4.2.

The initial amplitude of Alfvén waves at the PNS surface is supposed to be an order of convective velocities at the surface of PNS, which are suggested to be a fraction of the sound speed, \( \epsilon \approx 10\% \) of the light speed, \( \epsilon = 0.1 \rightarrow 0.3 \) in our simulations. Because \( c_{s,0} \approx 0.1 \) km s\(^{-1} \), which is comparable to the values obtained in hydrodynamical simulations (Keil et al. 1996). For a sufficiently large background magnetic field at the PNS surface, \( B_{r,0} \approx 5 \times 10^{14} \) G, the initial wave amplitude is small, in the sense that \( w < 0.1 \). In such a condition, the Alfvén wave travels outward without dissipation (\( \dot{q}_w = 0 \)) near the PNS surface, and its amplitude evolves according to the relation (for the non-dissipation regime)

\[
\frac{\langle \delta B^2 \rangle}{4\pi} \frac{(v_\perp + v)^2}{v_\parallel} r^2 = H_{w,0} r^2, \tag{7}
\]

where \( H_{w,0} \) is the wave action at its surface, and \( v_\parallel \) is the radial velocity of the background accretion flow. Note that under the condition \( v_\perp \gg v_\parallel \), equation (7) gives \( \langle \delta B^2 \rangle \propto r^4 \) and \( \langle w \rangle \propto r^4 \) in the nondissipation regime, where again \( \langle \delta B^2 \rangle = \langle \delta B^2 \rangle / 12 \) and \( \langle w \rangle = \langle w^2 \rangle / 12 \).

As the Alfvén waves travel outward, the non-linearity, \( w \), increases because of the expansion of the radial magnetic flux tube (eq. [1]). Dissipation of the Alfvén waves eventually sets in when they reach the non-linear regime, as discussed above, and the non-linearity of the Alfvén waves is saturated at a certain level (for the dissipation regime),

\[
\langle w \rangle \approx \alpha. \tag{8}
\]

In this paper we adopt a constant \( \alpha = 0.5 \) as a standard saturation level, based on our previous results for the solar and stellar winds (SI05; SI06; Suzuki 2007), which showed that \( \langle w \rangle \approx 1 \) is more or less constant or very slowly varying as a function of \( r \) (see also, e.g., Hollweg 1973 for steady-state modeling). This simple prescription of the constant saturation level is expected to incorporate phenomenologically all the complex physical processes of Alfvén wave dissipations discussed in § 2.2, and to provide us with a reasonable heating rate.

For the steady state, the energy dissipation rate, \( \dot{q}_w \), can be obtained from the conservation of the wave action, without referring to the details of the nonlinear processes that are responsible for the dissipations. Using the relation in the dissipation region,
We also define the location of wave dissipation, the field strength, $B_{r,0}$, the saturation level, $\epsilon$, and the mass, respectively. We fix the saturation level, $\epsilon$, of perturbations, determine the wave energy in-\(\left\{v_{A} + v_{r}\right\}^{2} \sqrt{4\pi \rho} \right\} \right\).

\begin{equation}
\dot{q}_{w} = \frac{v_{A}}{v_{A} + v_{r}} \frac{\alpha^{2} B_{r}}{4\pi \rho} \frac{d}{dt} \left(\frac{(v_{A} + v_{r})^{2} \sqrt{4\pi \rho}}{2}\right) \tag{9}\end{equation}

An important point here is that we can evaluate $\dot{q}_{w}$ from the local distributions of $\rho$, $v_{r}$, and $B_{r}$.

### 2.4. Simulations

Incorporating the above formula for the heating by Alfvén waves, we perform 1D time-dependent simulations of the post-bounce evolutions of a SN core. We use the 15 $M_{\odot}$ progenitor star of Woosley & Weaver (1995) as an initial condition, and employ a numerical code (Sumiyoshi et al. 2005) to solve general relativistic hydrodynamics and neutrino transport, adding the extra heating term given by equation (9) in the energy equation. The Alfvén wave heating ($\dot{q}_{w}$) is switched on at 100 ms after core bounce. We have in mind that this is the time for the development of (magnet)convection that drives Alfvén waves (Keil et al. 1996; Akiyama et al. 2001; Masada et al. 2006). We also define the PNS surface as the position of the density cut, for the models that produce shock revival and models that do not give shock revival, respectively.

We have three parameters, $B_{r,0}$, $\epsilon$, and $\alpha$, in the above prescription. We fix the saturation level, $\alpha = 0.5$, which controls the location of wave dissipations. The other two parameters, the field strength, $B_{r,0}$, at the PNS surface and the normalized initial amplitude, $\epsilon$, of perturbations, determine the wave energy injected from the PNS surface. In the following section, we explore the conditions on these two parameters for shock revival by simulating nine models in Table 1.

### 3. RESULTS

Figure 1 displays the success or failure of shock revival in the plane of the field strength at the PNS surface, $B_{r,0}$, and the initial amplitude of velocity perturbation, $\epsilon$. Table 1 gives more detailed information on the ejected mass, $M_{ej}$, explosion energy, $E_{exp}$, and remnant ($\approx$PNS) mass, $M_{cut}$, for the models that produce shock revival under the current approximation. They are estimated at time $t = 200$ ms after core bounce, as follows. We first define the ejecta as the collection of mass shells with positive total energy, which is the sum of kinetic, internal, and gravitational energies. Then $M_{ej}$ and $E_{exp}$ are obtained as the sums of mass and total energy, respectively, of each mass shell that comprises the ejecta. In so doing, the nonrelativistic expression is employed for the energy.

It is clear that an explosion with $E_{exp} \geq 10^{51}$ erg is obtained if the magnetic field at the PNS surface is strong, $B_{r,0} \approx 2 \times 10^{15}$ G, and if the initial amplitude of Alfvén waves is larger than a certain value, $\epsilon = (\langle B_{r,0}/c_{s}\rangle) \approx 0.2$. Note that the surface field strength is of the same order as those inferred for magnetars.

Figure 2 presents a typical Alfvén wave–driven shock revival, in which the result for model V (gray lines) is superimposed on the original result for the nonmagnetized spherically symmetric model (black lines) (Sumiyoshi et al. 2005). The trajectories of mass shells are plotted against the time from core bounce. The figure clearly demonstrates shock revival by Alfvén wave heating for the otherwise failed neutrino heating model (Sumiyoshi et al. 2005), as those discussed in many previous papers (e.g., Kotake et al. 2006 and references therein).

### Summary of Simulations

| Model | $B_{r,0}$ (G) | $\epsilon$ | Explosion | $E_{exp}$ ($\times 10^{51}$ erg) | $M_{ej}$ ($M_{\odot}$) | $M_{cut}$ ($M_{\odot}$) |
|-------|-------------|----------|-----------|-----------------|-----------------|-----------------|
| I     | $1 \times 10^{15}$ | 0.1      | No        | ...            | ...             | ...             |
| II    | $1 \times 10^{15}$ | 0.2      | No        | ...            | ...             | ...             |
| III   | $1 \times 10^{15}$ | 0.3      | No        | ...            | ...             | ...             |
| IV    | $2 \times 10^{15}$ | 0.1      | Marginal  | ...            | ...             | ...             |
| V     | $2 \times 10^{15}$ | 0.2      | Yes       | 1.2            | 0.08            | 1.38            |
| VI    | $2 \times 10^{15}$ | 0.3      | Yes       | 1.6            | 0.10            | 1.37            |
| VII   | $3 \times 10^{15}$ | 0.1      | Yes       | 0.33           | 0.04            | 1.41            |
| VIII  | $3 \times 10^{15}$ | 0.2      | Yes       | 1.5            | 0.10            | 1.38            |
| IX    | $3 \times 10^{15}$ | 0.3      | Yes       | 2.2            | 0.14            | 1.38            |

Note—$E_{exp}$, $M_{ej}$, and $M_{cut}$ give the explosion energy, ejecta mass, and PNS mass, respectively.
The localization of the Alfvén wave heating in the line panel (Fig. 3) gives a total heating rate of $3 \times 10^{31}$ erg/s. The arrow indicates the location of the shock front. Bottom: Distributions of $\frac{\delta B_{\perp}}{B_{\perp}}$ (solid line), $v_A$ (dashed line), and $v_A + v_{\nu}$ (dot-dashed line) at $t = 100$ ms for model V.

Figure 3 shows the rates of Alfvén wave heating and neutrino heating (top panel), along with the velocity distribution (bottom panel) at $t = 100$ ms. The top panel demonstrates that the Alfvén wave heating operates mainly in the vicinity of the stalled shock wave and dominates the neutrino heating. The main reason for the localization of the Alfvén wave heating in the Eulerian frame is the trapping of Alfvén waves. The propagation speed of the outgoing Alfvén wave is $v_A + v_{\nu}$ in this frame. It rapidly decreases from $2 \times 10^7$ km s$^{-1}$ at $r = 100$ km to $\approx 0$ km s$^{-1}$ at $r = 300$ km for model V (bottom panel of Fig. 3), for example. The Alfvén waves cannot travel farther outward and are trapped inside $r \lesssim 300$ km in model V, so that they spend a long time there to damp almost completely. In the Lagrangian frame, which moves at the inflow velocity of the accreting matter, on the other hand, the Alfvén waves propagate outward at the speed $v_A$. This means that the inflowing matter is heated up by the dissipation of Alfvén waves just when it reaches the vicinity of the stalled shock wave.

The luminosity, $L_A$, of Alfvén waves at the PNS surface can be estimated under the assumption of spherical symmetry, as follows:

$$L_A = \rho_0 (\delta v_{\perp})^2 v_A 4\pi r_0^2 \times 2 \times 10^{52} \text{ erg s}^{-1} \left( \frac{\rho_0}{10^{11} \text{ g cm}^{-3}} \right)^{1/2} \left( \frac{c_{s,0}}{0.1c} \right)^2 \left( \frac{\epsilon}{0.2} \right)^2 \times \left( \frac{B_{r,0}}{2 \times 10^{15} \text{ G}} \right) \left( \frac{r_0}{50 \text{ km}} \right)^2.$$  

This implies that the emission of Alfvén waves for $\approx 50$ ms gives an energy injection of $\approx 10^{54}$ erg. In most cases, almost all the energy of Alfvén waves is absorbed, thanks to the trapping of Alfvén waves just mentioned. This is also confirmed by comparison with the simulation results. For example, the direct integration of $4\pi r^2 d\rho d\phi d\theta$ for the snapshot at $t = 100$ ms of model V (Fig. 3) gives a total heating rate of $3.2 \times 10^{52}$ erg s$^{-1}$ (this is slightly larger than the estimate of eq. [10] because the PNS surface, $r_0$, is a little bit larger than 50 km), which indicates that the Alfvén wave luminosity is completely used for heating the stalled shock.

The regions with fast accretion velocities are preferentially heated up and eventually start to move outward, provided $L_A$ is sufficiently large. Once the stagnated shock wave is relaunched, the heating is reduced, because Alfvén waves become untrapped again. For larger $L_A$ the shock revival occurs earlier, and the duration of the heating, $\Delta t_A$, is shorter. As a result, $E_{\text{exp}}$ (roughly $\propto L_A \Delta t_A$) is not very sensitive to $B_{r,0}$ and $\epsilon$. In fact, although $L_A$ of model IX is larger than that of model V by more than a factor of 3, the difference in $E_{\text{exp}}$ is less than a factor of 2. In this sense, the Alfvén wave mechanism is self-regulating. Interestingly, the acoustic wave mechanism is also claimed to be self-regulating (Burrows et al. 2006), although the regulating mechanism is different; the generation of the acoustic waves continues until the shock is revived and matter ceases to accrete.

Model VII is exceptional among the explosion cases, giving a very small explosion energy. The Alfvén wave heating operates in a very outer region in this case, because Alfvén waves become nonlinear, $\langle \delta B_{\perp} \rangle / B_{\perp} > \alpha$, only after crossing the shock wave, owing to the large $B_{r,0}$ and small $\epsilon$. A sizable fraction of $L_A$ leaks out of the stalled shock wave, and a tiny amount is ejected with a quite small $E_{\text{exp}}$.

The models with $B_{r,0} = 1 \times 10^{15}$ G produce no explosion. This is first because $L_A$ itself is small (mainly models I and II), owing to the small $B_{r,0}$, and second because the dissipation of Alfvén waves occurs too early (mainly model III). Note in particular that $L_A$ of model III is larger than those of the explosion cases V and VII. In this case, Alfvén waves dissipate in the inner region and the temperature is increased there. As a result, the energy deposited by Alfvén waves is mostly converted to neutrino emission in this case.

4. DISCUSSION

In this section, we discuss in more detail the validity of the assumptions and approximations employed for the background magnetic field and the formulation of Alfvén wave propagation in this paper.

4.1. Magnetic Field

Since we have seen that strong magnetic fields ($B_{r,0} \gtrsim 2 \times 10^{15}$ G), which are of magnetar scale and much larger than those of ordinary radio pulsars, are necessary to revive the stalled prompt shock by Alfvén wave heating, one may think that the mechanism considered in this paper is only applicable to magnetar-forming supernovae, and that Alfvén waves are not a major ingredient of ordinary SN explosions. We cautiously note, however, that this strong magnetic field is not required for ordinary neutron stars as we observe them, but for the PNSs in their very infancy. As a matter of fact, there is speculation that the magnetic fields of nascent PNSs may be temporarily very strong and then decrease to the “normal” value ($\approx 10^{12}$ G) by energy release occurring during the SN explosion and later evolution (Wheeler et al. 2002). If this is true, the Alfvén wave mechanism may work in a larger population of core-collapse SNe.

The origin of such strong magnetic fields is still controversial. One possibility is referred to as the fossil origin hypothesis: the strong magnetic field in compact stars is simply a consequence of the compression of the magnetic field that already exists in OB progenitors prior to gravitational collapse. In fact, several magnetic massive stars have been observed to have magnetic fields with an average dipole-field strength of $\approx 1000$ G (Neiner et al. 2003; Hubrig et al. 2006; Donati et al. 2006). The total magnetic flux of
these stars is comparable to that of a typical magnetar (Ferrario & Wickramasinghe 2006), which implies that an additional generation and/or amplification of magnetic fields will not be necessary to obtain a highly magnetized compact remnant for these stars.

Another possibility is an amplification of weak magnetic fields by the MRI (Akiyama et al. 2003; Masada et al. 2006), in which stellar rotation plays a key role, winding poloidal fields and driving the instability. In this scenario toroidal magnetic fields are efficiently produced, whereas for a fossil origin we expect the radial component of magnetic field to be dominant.

Since Alfvén waves carry energy along field lines, we are interested only in open magnetic flux tubes that extend beyond the stalled shock wave. As the simplest configuration, we have considered radial magnetic fields and neglected the toroidal component in this paper. As mentioned above, the approximation is justified if the magnetic field is of fossil origin and the progenitor is a slow rotator. Even if the progenitor core is a rapid rotator, our models will be still applicable to the polar region, where the effect of rotation is not strong and the toroidal magnetic fields are less important, although the radial dependence of the field strength may need elaboration.

In the equatorial region of a rapidly rotating supernova core, on the other hand, the situation is much more complicated. Field lines are not directed radially in general, and some of them may be closed, as expected for the dipole configuration. In addition, the continuous downward advection of magnetic fields may cause reconnections in the PNS, which in turn will open up some field lines again. In any case, the toroidal component will be dominant over the radial component (Burrows et al. 2007), and we need to include the effects of these spiral magnetic fields as well as rotation itself in discussing Alfvén wave heating in supernova cores quantitatively, which will be a future task.

4.2. Alfvén Wave Propagation

The treatment of Alfvén waves in this paper is admittedly a crude approximation. We employ the nonrelativistic, steady-state, and WKB approximations for describing the propagation of Alfvén waves. Among the assumptions made, nonrelativity is adequate for the Alfvén waves launched from the PNS surface, since the relativistic corrections are indeed minor outside the PNS. The steady-state approximation is also reasonable, because the Alfvén transit time is shorter than the expansion time of accreting matter; while the expansion timescale of the ejecta is ~50–100 ms, the time for the Alfvén wave to travel from the PNS surface (r ≈ 50 km) to the wave trapping region around the stalled shock (r ≈ 200 km) is ~10 ms for v_A ≈ 2 × 10^4 km s^{-1}.

The WKB approximation is acceptable if the wavelength is shorter than the scale height of the background. The typical period of Alfvén waves generated in the PNS is supposed to be τ ∼ 1 ms, corresponding to the dynamical timescale. Then the wavelength becomes λ ≈ v_A τ ∼ 2 × 10^4 km s^{-1} × 10^{-3} s ∼ 20 km. Since the density scale height is shortest near the PNS surface and is H_p ≈ 30 km, Alfvén waves might be partially reflected there due to the deformation of the wave shape. In a more detailed study, this effect should be taken into account, together with the wave pressure ignored in this paper.

5. CONCLUSION

In this paper we have studied matter heating by Alfvén waves in a postbounce supernova core and its implications for shock revival. In order to elucidate the essential features of the mechanism quantitatively, we have performed a couple of 1D dynamical simulations, neglecting rotation and toroidal magnetic fields, but employing the Alfvén wave heating rate based on our model of the nonlinear damping of Alfvén waves.

We have found that if the surface magnetic field strength is ≥2 × 10^{15} G and if the surface velocity fluctuation is ε ≥ 0.2, which corresponds to (δv_{rA}) ≥ 6 × 10^4 km s^{-1}, the stalled shock is revived by the Alfvén wave heating, with a canonical explosion energy, E_{exp} ≥ 10^{51} erg. The current mechanism is self-regulating in the sense that the explosion energy is not very sensitive to the surface field strength or initial velocity fluctuation, as long as they satisfy the above conditions. The above strong magnetic field is not a requirement for ordinary neutron stars as observed, but for the PNSs in their infancy. If magnetic fields decay through their subsequent evolution (Wheeler et al. 2002), Alfvén waves may play an important role in a larger population of core-collapse SNe.

It has also been found that wave trapping is essential in localizing the Alfvén wave heating in the vicinity of the stalled shock wave, as well as in regulating the explosion energy. In fact, if the magnetic field is weaker (ε < 2 × 10^{15} G), the Alfvén wave heating takes place much closer to the PNS, because the Alfvén wave becomes nonlinear earlier on, at smaller radii. Then no shock revival occurs, because the dissipated energy is mostly lost by neutrino cooling; neutrino emissions are enhanced in this case. On the other hand, if the initial velocity fluctuation is smaller (ε ≤ 0.2), with a stronger magnetic field, ≥3 × 10^{15} G, most of the Alfvén waves propagate through the stalled shock region depositing only a small amount of energy, which then results in a weak explosion, with E_{exp} < 10^{51} erg. It is noted, however, that even in these cases the Alfvén wave heating will still be important in supplementing the neutrino heating.

As a first step, the models presented in this paper leave much room for sophistication. Among other things, as mentioned repeatedly, rotation and toroidal magnetic fields should be somehow taken into account. In reality, the Alfvén wave mechanism probably works in cooperation with these processes. This will require multidimensional numerical modeling, and will be addressed in a future work.

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