Fermi Liquid Theory of Ultra-Cold Trapped Fermi Gases: Implications for Pseudogap Physics and Other Strongly Correlated Phases

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We show how Fermi liquid theory can be applied to ultra-cold Fermi gases, thereby expanding their “simulation” capabilities to a class of problems of interest to multiple physics sub-disciplines. We introduce procedures for measuring and calculating position dependent Landau parameters. This lays the ground work for addressing important controversial issues: (i) the suggestion that thermodynamically, the normal state of a unitary gas is indistinguishable from a Fermi liquid (ii) that a fermionic system with strong repulsive contact interactions is associated with either ferromagnetism or localization; this relates as well to ³He and its p-wave superfluidity.

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Normal Fermi liquid theory describes low temperature fermionic quantum matter which is adiabatically connected to its non-interacting counterpart [1]. This theory, which has wide ranging implications for condensed and neutron star matter as well as nuclei and nuclear physics [2], revolves around a quantification of Landau parameters. These are viewed as a collection of molecular field contributions characterizing the interaction between renormalized (quasi) particles. Because Fermi liquid theory is a major tool for addressing many-body systems and ultracold Fermi gases are a testing ground for counterpart theories, it is essential to establish how to measure as well as calculate the associated Landau parameters. This leads to the goal of the present paper: to formulate the extension of Landau’s important theory to an atomic trapped gas. Our work incorporates the local density approximation (LDA) which is used in both our experimentally-based and theoretical analyses. In the context of the latter, we present a methodology for calculating the Landau parameters, given a microscopic many body scheme.

This paper addresses the relationship between Fermi liquid theory and two important situations: trapped gases in the presence of moderately strong attractive as well as repulsive contact interactions. For the former the ground state is a superfluid in which the pairing strength can be continuously tuned from BCS to Bose Einstein condensation (BEC) [3]. For intermediate strength attraction, associated with the “unitary” phase, of considerable interest is whether the normal state (above the transition temperature $T_c$) is a Fermi liquid or not. A body of evidence [4-6] has arisen in support of the prediction [7] that somewhat above $T_c$, an unpolarized, unitary gas contains pre-formed pairs associated with a normal state gap or pseudogap. If there is a gap in the fermionic excitation spectrum, these unitary gases would not qualify as Fermi liquids. However, this has become a more controversial point, recently when this observation was challenged [8] through fits to the measured temperature dependence of the pressure. The claim from Ref. [8] that “the low-temperature thermodynamics of the strongly interacting normal phase is well described by Fermi liquid theory”, is both important and at odds with previous inferences [3,6,9] that Fermi liquid theory is inappropriate at unitarity.

By contrast, for the case of repulsive interactions, Fermi liquid theory should be widely applicable. Nevertheless, there are on-going controversies about the microscopic nature of such a Fermi liquid. In the cold gases this question is posed in the context of considering even stronger repulsion where the central issue is whether the ground state of the Hubbard gas model will be ferromagnetic [10,11] or localized [12]. Here we argue that, in the Fermi liquid regime (at higher $T$, or weaker interactions so that there is no spontaneous symmetry breaking) one can use the behavior of the measured Fermi liquid parameters as a probe into the nature of the dominant incipient instability. Indeed, this debate in the cold gases is completely analogous to a related controversy in the prototypical Fermi liquid of condensed matter physics: ³He, which is thought to be similarly described by the repulsive Hubbard gas model. At issue is whether the Landau parameters are controlled by proximity to a ferromagnetic [13] or localization [14] instability. Further insight into ³He should also be important to the cold gas community in additional contexts, since the p-wave superfluidity of ³He at $T = 0$ can serve as a template for addressing quite generally, non-s wave (e.g., d-wave) superfluidity which indirectly arises via repulsive on-site interactions.

An understanding of precisely what does and what does not constitute a Fermi liquid is clearly lacking in the literature of ultracold Fermi gases, where we have seen competing viewpoints expressed such as between Ref. [8] and Refs. [5-6] or between Refs. [10,11] and Ref. [12], or Refs. [13] and [14]. There has been some discussion of the basis of Fermi-liquid theory, albeit limited [15] to the dynamical response in (less accessible) homogeneous systems. Fermi liquid theory is thought to be relevant to strongly polarized unitary gases, when superfluidity is destabilized. There are conjectures that a rapidly rotating unitary gas (such that superfluidity is suppressed) will also be a Fermi liquid [15], but this needs to be
systematically investigated since related analogous condensed matter systems show that pairing survives even when superfluid coherence is destroyed \cite{3}.

Because of these controversies and confusion, an important contribution of this paper is to establish the signatures of Fermi liquid and non-Fermi liquid behavior. In the process, we will also establish the thermodynamic signatures of pseudogap effects. Figure 1 is devoted to a comparison of a number of Fermi liquid properties with those of a non-Fermi liquid. To represent the latter, we choose a unitary gas superfluid, for which there should be no disagreement that normal Fermi liquid theory fails. Figure 1a presents plots of the calculated density profiles for the Fermi liquid (FL) and non-Fermi liquid at the same temperature $T = 0.5T_c$, where $T_c$ is the transition temperature of the unitary superfluid. For concreteness, in the FL case, we presume a repulsive contact interaction of moderate size and use Hartree-Fock theory (described below) to compute the profile. As an experimental calibration, for the unitary case both above and below $T_c$, the profiles are in very good agreement \cite{10} with the data. Despite their evident similarity, the FL and non-Fermi liquid density profiles contain very different physics which must be carefully extracted.

These differences are best achieved by studying derived physical properties. In this paper we focus on unpolarized Fermi gases. As a necessary first step, we compute the compressibility $\kappa$, the spin susceptibility and the entropy from the density profiles. Within the LDA, the chemical potential is $\mu(r) = \mu(r = 0) - (1/2) m \omega_r^2 r^2$, where $m$ is the bare mass of the fermions and $\omega_r$ is the trap frequency. Then

$$ \frac{dn}{d\mu} = -\frac{2}{m\omega_r^2} \frac{dn}{dr}, \quad (1) $$

which, in turn, allows one to measure $\kappa \equiv n^{-2} \frac{dn}{d\mu}$. From the profiles of a slightly polarized gas (with polarization $\lesssim 5\%$) one can arrive at the Pauli susceptibility $\chi_s \equiv \frac{\partial \delta n}{\partial \delta \mu} \approx \frac{\delta n}{\delta \mu}$. Here $\delta n = n_1 - n_2 \ll n$ and $\delta \mu = \mu_1 - \mu_2$ and one requires that $\mu_2$ and $n_2$ be established separately. Measurements of $\chi_s$ have a firm basis in the past success of LDA approaches to polarized Fermi gases with attractive interactions \cite{17}. One requires that the densities be small at the trap edge and that (for attractive interactions) low polarizations at $T$ slightly away from zero are considered so that the phase separated state (with discontinuous densities) is not encountered.

Techniques for extracting the entropy are directly based on those discussed elsewhere \cite{18}, albeit modified somewhat. We consider two clouds at nearby temperatures $T_1$ and $T_2$ and label associated quantities of each cloud with subscripts 1 and 2. Then by locating the points $\mu_1(T_1, r) = \mu_2(T_2, r)$ on the two clouds \cite{18} and using the Gibbs-Duhem relation,

$$ \left( \frac{\Delta F}{\Delta T} \right)_\mu = s(T_2) + \frac{1}{2} \left( \frac{ds(T_2)}{dT} \right)_\mu \Delta T \quad (\text{in general}) $$

$$ = s \left( \frac{T_1 + T_2}{2} \right) \quad (\text{Fermi liquids}), \quad (2) $$

we note that the extracted $s(r)$ agrees very well with the exact $s(r)$ provided one uses the averaged $T$. With care, one should be able to choose the separation between $T_1$ and $T_2$ large enough so that noise effects do not affect the extraction of $s(r)$.

We show in the inset to Fig. 1b a plot of s(r) calculated \cite{19} at unitarity as compared with the FL case, for $T/T_c = 0.5$. In the main body of Fig. 1b we compare this with the counterpart unitary plots just below the transition temperature ($T/T_c = 0.9$) and in the normal phase at $T/T_c = 1.1$. The total entropy involves a bosonic as well as a fermionic contribution \cite{19}. A salient feature of $s(r)$ in these unitary superfluids is that it is spatially non-monotonic. This behavior is associated with a pairing gap which is largest at the trap center and decreasing with increasing $r$. That this non-monotonicity in $s(r)$ is found at $T/T_c = 0.9$ suggests the pairing gap is still large at these high temperatures; because the transition is second order this pairing gap must persist into the normal phase, as a “pseudogap” \cite{3} \cite{5} \cite{20} asso-
associated with non-condensed pairs. Indeed, one sees that the curves for $T/T_c = 1.1$ exhibit an (albeit, slightly) anomalous non-monotonic entropy profile.

Similar non-monotonic behavior can also be found in the spin susceptibility $\chi_s$ of an unpolarized unitary Fermi gas which is plotted in Fig. (1c) for $T/T_c = 1.1$ (and in the inset at $T/T_c = 0.5$) compared to that of the same Fermi liquid as discussed in Fig. (1a). These two non-monotonicities contrast with their counterparts for the Fermi liquid case shown in the figures. The suppression of $\chi_s$ at the trap center of the normal unitary gas can similarly be traced to singlet non-condensed pairs \[ 7 \], or pseudogap effects.

The behavior of the normal unitary gas, thus, shows a continuous evolution from the superfluid properties (say, measured at $T/T_c \approx 0.5$) to above $T_c$. This is the sense in which pseudogap effects (manifestly inconsistent with Fermi liquid theory) will be evident in thermodynamics. This is in contrast to the claim, based on a quadratic temperature dependence of the pressure \[ 8 \] that “the normal phase of the unitary Fermi gas. . . [has] thermodynamic properties [which are] . . . well described by Fermi liquid theory, unlike high-$T_c$ copper oxides.”

The same quadratic signature \[ 8 \] can also be found theoretically for a range of $T$ in strict BCS theory, where FL theory obviously fails. Thus, we infer that power law dependences in the pressure \[ 8 \] that “the normal phase of the unitary Fermi gas can similarly be traced to singlet non-condensed pairs. Indeed, one sees that the $m^*$, $\chi_s$ profiles, which reflects the $T < T_c$ physics, is the more relevant feature, a consequence but not a proof of the existence of a pseudogap \[ 4 \] and the concomitant failure of Fermi liquid theory. Interestingly, a number of the theories with which these experiments \[ 8 \] are favorably compared also contain pseudogap effects.

In proper Fermi liquids the compressibility, spin susceptibility and entropy enable one to experimentally extract the spatially dependent Landau parameters \[ 1 \] and effective mass $m^*$. In condensed matter Fermi liquids $m^*$ is most readily deduced from the linear (temperature) term in the specific heat \[ 1 \]. In cold gases, we argue that it is best obtained from the linear in temperature term in the local entropy density $s(r)$:

$$ s(r) = \frac{m^* k_F(r) T}{3}. \tag{3} $$

Here we set $\hbar = 1$ and $k_B = 1$. Fermi liquid theory and Eq. (3) are precise only if $T \ll T_F$. As the local $T_F(r)$ decreases, this procedure will fail towards the trap edge. Thus expressions including only leading order $T$ corrections in $s(r)$ (or equivalently $T^2$ terms in $P$ or $\mu$, etc.) must be constrained to $T \ll T_F(r)$. Moreover, in a Galilean invariant system \[ 1 \], $F^*_1$ satisfies

$$ \frac{m^*(r)}{m} = 1 + \frac{F^*_1(r)}{3}. \tag{4} $$

and these arguments apply to a trapped gas within the LDA. The Landau parameter $F^*_0$ can be obtained from

$$ \frac{dn}{d\mu} = \left( \frac{dn_0}{d\mu} \right) \frac{m^*/m}{1 + F^*_0}. \tag{5} $$

Here $dn_0/d\mu = -\sum_k (\partial f(\omega)/\partial \omega)|_{\omega=E_k^2}/2m-\mu$ is the known corresponding quantity for a non-interacting Fermi gas at the same chemical potential and $f(x)$ is the Fermi distribution function. Given $dn/d\mu(r)$ and $m^*(r)/m$, $F^*_0(r)$ is then known. Finally, the third Landau parameter to be quantified is $F^*_0$, which is related to the Pauli susceptibility

$$ \chi_s = \frac{m^*/m}{1 + F^*_0}. \tag{6} $$

Here $\chi_{\delta} = d\delta n_0/d\delta \mu$ is the spin susceptibility of the non-interacting Fermi gas.

We consider now a repulsive contact interaction in order to gain more understanding of the stable phases of the Hubbard Fermi gas Hamiltonian. This bears on recent \[ 10, 12 \] and longstanding controversies \[ 13, 14 \] in the literature. If the repulsion is not too large, one expects the system to be in a Fermi liquid phase. Our methodology for computing the associated Landau parameters is based on first computing the trap profiles $n_\sigma(r)$ from a microscopic theory and secondly following the experimental protocols outlined above to determine the LDA-derived $F^*_1$, $F^*_0$, and $F^*_0$. The specifics of a microscopic theory are tested by comparing the calculated Landau parameters with their experimental counterparts.

In LDA the density is $n_\sigma(r) = \sum_k \epsilon_{k_\sigma} G_\sigma(k, \omega_n, \mu(r))$. Here $G_\sigma$ is the Green’s function, $\omega_n$ is the fermion Matsubara frequency, and the fermion self-energy $\Sigma_\sigma(k, \omega_n, \mu(r))$ is chosen to correspond, for example, to a nearly ferromagnetic \[ 13 \] or nearly localized \[ 14 \] theory.

While it is relatively straightforward to use more sophisticated approaches \[ 11 \], in order to focus on the central physics, we apply Hartree-Fock theory \[ 11, 21 \]. Here $\Sigma_\sigma = g n_\sigma$, where $g = 4\pi \hbar^2 a/m$, $\sigma = -\sigma$, and $a$ is the two-body s-wave scattering length. The trap profiles correspond to $n_\sigma(r) = \frac{1}{\pi} \int \frac{dn(r)}{d\mu} f \left( \frac{\epsilon_{k_r}^2}{2m} + g n_\sigma(r) - \mu_\sigma(r) \right)$. We consider $a/a_{\text{Stoner}} = 0.6$ and 0.9 at $T/T_F = 0.02$ and 0.01, where $a_{\text{Stoner}}$ is the critical scattering length (Stoner instability) at which $\chi_s$ diverges. From the profiles, one obtains $m^*$, $\chi_s$, and $\chi_{\delta}$. The corresponding spatial dependences associated with each of these are shown on the left column of Fig. 2 where we focus on the results at the trap center. Using Eqs. (4), (5), and (6), the three Landau parameters $F^*_1$, $F^*_0$, and $F^*_0$ are obtained and plotted in the right column of Fig. 2. We see that $m^*/m$ is only slightly larger than 1 so $F^*_1$ is small. For repulsive interactions, $F^*_0$ is expected to be positive and $F^*_0$ is expected to be negative, as is consistent with Fig. 2 panels (e) and (f).

Interestingly, one had naively applied these same FL procedures to the attractive case (where we argue because of the pseudogap, Fermi liquid theory is not appropriate) one would find that $F^*_0$ is positive and $F^*_0$ is negative. Therefore, even though the trap profiles for the particle density are not qualitatively different for both cases, the derived Landau parameters make it possible to distinguish between very different physical situations.

One can see from Fig. 2d and (f) that as $a \rightarrow a_{\text{Stoner}}$ $F^*_0 \rightarrow -1$ while $F^*_1$ and the effective mass remain finite. In
contrast, for an almost localized liquid \cite{12}, $F_s^1$ or $m^*/m$ will diverge at the counterpart critical interaction strength. While this leads to a divergence in $\chi_s$, $F_0^a$ does not approach $-1$ \cite{14}. For an unknown ground state, this Fermi liquid analysis in the limit that the interaction strength is slightly reduced, can be used to provide important hints about the nature of the more strongly correlated phase.

Thus, future measurements of the Landau parameters associated with the repulsive Hubbard gas studied in Ref. \cite{10} should be able to shed light on a current controversy \cite{12} over whether a ferromagnetic ground state has been observed. There are, however, some complications in these experiments. The Feshbach resonance which tunes the repulsive interactions between fermions is associated with a molecular bound state lying below the particle continuum which is populated \cite{10} at the 25\% level. These molecules derive from molecule-molecule- or molecule-molecule scattering which may lead to systematic errors in determining $dn/d\mu$ and $\chi_s$. To resolve this issue, one has to either reduce the population of molecules or develop a more sophisticated theory to take them into account.

An important contribution of this paper has been to establish how, despite previous claims \cite{8}, pseudogap effects do enter into thermodynamical measurements. When a pseudogap is present, thermodynamical variables above $T_c$ will show a smooth evolution from their behavior deep in the superfluid phase; this contrasts with BCS theory, where the normal state thermodynamics do not reflect their superfluid counterparts. While there will be a thermodynamical feature of the phase transition \cite{9} one should see this smooth evolution in anomalous spatial dependences for example of the spin susceptibility. The experimental complexity needed to search for these pseudogap effects is comparable to that in Refs. \cite{8,22}. In this paper, following Ref. \cite{10} we have also mapped out a new direction for the field of ultra-cold trapped Fermi gases which, via simulations of the (repulsive) Fermi Hubbard gas, can serve to establish where ferromagnetism, localization and $p$ wave superfluidity are stable. This should be facilitated by analyzing simultaneous measurements and calculations of appropriate Landau parameters.

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\begin{figure}[h]
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\includegraphics[width=\textwidth]{figure2}
\caption{(Color online) The extracted (a) $m^*/m$, (b) $(dn/d\mu)/(dn_0/d\mu)$, and (c) $\chi_s/\chi_{s0}$ and the Landau parameters (d) $F_1^s$, (e) $F_0^a$, and (f) $F_0^b$. The input data are from the Hartree-Fock theory with $a/d_{\text{Stoner}} = 0.6$ (black dashed line) and 0.9 (red solid line) at $T/T_F = 0.02$ (whose $n(r)$ and $\mu(r)$ are shown as insets) and 0.01.}
\end{figure}