QCD Sum Rules: The Second Decade
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1 Introduction

My task in this talk, as I understand it, is two-fold. First, to describe the basic idea of the QCD sum rule approach which was – and still is – one of the most productive tools for calculating hadronic parameters from Quantum Chromodynamics. Second, I would like to try to convey the historic flavor of the exciting time when the theory was making its first steps, with a euphoric hope (you could feel it in the air) of an imminent solution of the confinement problem.

To make a proper perspective it is, perhaps, worth starting from the second point. It seems fair to say that QCD was born after the talk of Gell-Mann and Fritzsch [1] (see also ref. [2]) in which the color-octet gluons were introduced. The next step is certainly the discovery of asymptotic freedom in 1973 [3]. In the first few years, roughly speaking till 1975, the theorists’ attention was almost totally focussed on hard processes in which the short-distance physics plays the dominant role. This topic became hot and fashionable, piles of new papers appearing daily. The main achievement of this period is that people learned how to reliably isolate the short distance contributions governed by the small coupling constant and how to generalize electrodynamical perturbative calculations to non-Abelian theories.

A recent paper of Polyakov [4] presenting his understanding of the development of our field in the seventies is entitled ”A View from the Island” which gives a good idea of our place in the scientific process in Moscow in those days. The isolation was almost complete, and we could not compete with Western theorists in most fashionable directions where the results seemed to be on the surface, for obvious reasons.

Because of the total censorship preprints and journals from the West used to come with enormous delays, and our own papers could be submitted to the Western journals only after a complicated procedure of getting clearance from half a dozen of different instances, typically a waste of a few months. Publication of preprints was an adventure due to bureaucratic limitations. For instance, one and the same paper could not be issued as a preprint and, then, in a journal, and the preprint version could not be longer than 25 typewritten pages (or 35, I do not remember exactly). So we had a lot of ”fun” trying to muddle and deceive our censors by making different titles in the preprint and journal versions of one and the same work, changing the order of the authors or serializing preprint publications like a detective story in a popular newspaper (unlike the latter case, though, we had to make an impression that each successive part is not connected to the previous). Quite often these tricks created a terrible mess, to say nothing about wasted efforts. Occasionally, in the most important cases, one would risk to bypass the standard procedure by using different ”illegal” channels, mostly our Western friends. By the way, any contacts with the latter were also severely constrained. There is a famous story about one of the scientific bosses in Dubna who was instructing the Soviet participants of a conference before its opening. He said: ”Well..., we had to organize this conference, and even invite some foreigners. Unfortunately, to my deep regret, this time it will
be impossible to completely avoid contacts...” Conferences in the West were open for a handful of specially selected, through a humiliating procedure; and even those few which took place in the Soviet Union were not always accessible. I remember, for instance, that one of my colleagues from ITEP and I were not granted permission to participate in ”Neutrino –77” in Baksan.

This is a long saga, and I could easily speak for an hour on this topic, but now it is clearly time to stop. To make the long story short I will only say that making our results known was a difficult, nervous and time-consuming part of our job. This largely predetermined the choice of topics we could work on and formed in the ITEP theory group and elsewhere a very special atmosphere, now gone forever.

In the seventies ITEP had one of the best groups in the world, an excellent collection of enthusiasts whose attitude to physics was totally ”non-commercial”. People were always eager to discuss with each other every interesting scientific question emerging during the seminar talks or elsewhere, and these discussions quite often would last till midnight. You could easily find experts in any conceivable field or direction who would gladly share their knowledge with you. Our common enemies and common isolation created, as a counterreaction, very strong friendly and scientific ties, as the only way of survival, and helped develop protective values; the tacit understanding that good physics was above all was among these values.

The only drawback I can think of now, in retrospective, is the general negative attitude to field theories in the very end of sixties and the beginning of seventies when I just appeared in ITEP. The reason is obvious, of course: the influence of Landau and his discovery of the zero-charge property in QED [5] – the influence which was alive and very strong in the ITEP theory group in those days. The attitude to the field theory as to something absolutely not serious was so deeply rooted that the fact of the antiscreening of the gauge coupling constant in non-abelian theories which was reported in ITEP at least twice [6], [7] in the sixties has not been appreciated and recognized [8]. A restructuring in minds started only after the very same fact, the surprising fall-off of the gauge charge at large distances, became known from ref. [3].

So, our start was relatively slow. By 1974, however, we were fully submerged in this subject, and shortly after it became clear that the cavalry attacks do not help to solve the problem of confinement and that the wide-spread expectations of a Messiah who would come soon and teach us the mystery of the solution had to be tempered. The fact that instantons [9], a beautiful theoretical construction which was a breakthrough in the qualitative understanding of the QCD vacuum [10], turned out to be useless in the quantitative sense because of the infrared divergences was a serious blow. So we adopted a less ambitious approach (by ”we” I mean Valya Zakharov, Arkady Vainshtein and myself). The idea was to start from short distances where the quark-gluon dynamics was essentially perturbative and we felt ourselves on a firm ground, and then to extrapolate to larger distances (where the hadronic states are presumably formed) including non-perturbative effects ”step by step” and using some kind of an approximate procedure to extract information on
hadronic properties. Of course, this idea was rather vague at first, the program as we know it now has been crystallizing gradually.

It is rather difficult to identify the work which, for us, was the first crucial step. With some reservations I might say that the first step has been done in ref. [11]. Perhaps now the sum rule for the charmed particle photoproduction obtained in [11] does not seem very impressive but this analysis carried important elements which where later laid in the foundation of the sum rule method. A spectacular success came after we united our efforts with V. Novikov, L. Okun and M. Voloshin. It turned out that a whole variety of the charmonium parameters are predictable essentially from pure duality, and for about a year we played the game of constructing the charmonium widths and mass ratios from simple numbers. In 1977 a review report [12] was submitted. At about that time it became clear that the progress was limited; the method presented in [12] could not be reliably generalized to such typical representatives of the hadronic family as, say, $\rho$ mesons or nucleons. And the desire to get access to these hadrons was strong.

The remainder of this story, including its culmination – introduction of the gluon condensate [13] in fall 1977 – is described elsewhere [14]. The basic elements of the approach were already visible in this first work, although some new important findings, like e.g. borelization, came somewhat later. We worked at a feverish pace for the whole academic year, accumulating a large number of results for the hadronic parameters with the accuracy which we could never expect beforehand. In late spring 1978 the question of how all this wealth could be published became of prime concern to us. As usual, we had a lot of funny adventures (they can hardly be understood by the Western physicists) in preparing the manuscript, typing it and issuing preprints. Needless to say, it was a serial publication, see above. As for the journal article, Nuclear Physics was a natural candidate, but previously we had bad luck with this journal: our paper on penguins [15] was buried in the editorial office for more than two years. We were too tired, however, to invent anything new and decided to try our luck again. The report occupying the whole issue of Nuclear Physics [16] appeared a year later.

2 The Main Idea

The lagrangian of Quantum Chromodynamics is built from gluons and quarks. At short distances these degrees of freedom fully describe the dynamics, while at large distances where the hadrons are formed the quark-gluon description becomes irrelevant: the Green functions are strongly distorted and nobody knows how to calculate them from first principles. The hope which lies behind the approximate method developed in [12] is as follows. Let us assume that there exists a transitional domain of distances (stretching up to the sizes of the classical states with small spins, like $\rho$) where the effect of the infrared component of the quark and gluon Green functions can be parametrized in terms of a few vacuum condensates. If this assumption is
correct then a systematic approach can be developed allowing one to calculate the parameters of these classical states. The technical basis of the approach is the Wilson operator expansion (OPE) [17], a construction very close in spirit and ideally suiting our purposes. Indeed, the essence of OPE is separation of all field fluctuations in scales: the small scale fluctuations are accounted for explicitly in the coefficient functions; the large scale fluctuations are referred to the matrix elements of a set of (local) operators. Generally speaking, this set is infinite.

If the ultimate theory of hadrons existed it should be able to answer in full two questions: (i) what are the values of the coefficient functions, and (ii) what are the values of the matrix elements from the infinite set mentioned above. Given the exact answers one can readily extract from them the exact and complete information about all hadronic parameters.

Will the exact answers be ever known? Time will show. Our current abilities are much more modest. The coefficient functions are, obviously, approximately calculable as an expansion in the small running coupling constant. In principle, they include both perturbative and non-perturbative parts [18], but in practice we have to limit ourselves to several leading terms in the perturbative expansion. As for the matrix elements, these quantities are essentially non-perturbative, and can not be calculated in the present-day QCD. Therefore, the success of the method crucially depends on whether or not a finite truncation of the infinite set of operators is sufficient to ensure a reasonable accuracy in the domain where the classical hadrons are formed. If the convergence of OPE is good, and the first few terms in the expansion describe the correlators under consideration up to distances of order of the ρ meson size then the corresponding few matrix elements can be just parametrized. In this way we arrive at a workable substitute to the ultimate theory, which, if does not make us completely happy, still allows to investigate the important regularities of the hadronic family in a model-independent way without postponing the task till indefinite future.

The closest analogy I can think of to this method is the propagation of external objects injected at \( \tau = 0 \) in a medium. The medium structure is complicated and essentially we have no idea how to obtain it at the microscopic level. The motion of the injected objects at \( \tau \to \infty \) depends, generally speaking, on unknown details of what is going on in the medium reacting on the presence of the external objects. If, however, the characteristic frequency \( \omega \) of the external objects is much larger than that of the medium, we can consider the propagation of the objects during the time interval \( \tau = \text{several units} \times \omega^{-1} \). This is a large time interval with regards to the external objects, so that they have enough time to form a stationary state. On the other hand, for the medium this is a short time; it remains intact, and the external objects perceive only its averaged (coarse) characteristics.

The external objects are the valence quarks produced by external currents we pick up at will, and the medium is the QCD vacuum. The main problem is the fact that there is no obvious parameter which could be fine-tuned to ensure a large ratio of two frequencies: that of the valence quarks to the typical frequency of the
vacuum fluctuations. We have to rely on numerical parameters whose existence is not clear \textit{a priori}. Moreover, it may well happen that in some channels a favorable numerical enhancement exists while it is absent in the others. As a matter of fact, this is exactly what we have discovered \cite{19}: not all hadrons (even with small spins) are alike the classical $\rho$ mesons or nucleons. There are deep distinctions in the hadronic family. This is, probably, the most important qualitative finding obtained within the sum rule approach, which escaped attention of the lattice people. This finding served also as an initial impetus for the introduction of the instanton liquid model. I will return to this issue later, and now let me only mention that the reasons explaining the non-universal behavior of different low-lying hadronic states are qualitatively understood \cite{19}.

Before dwelling on the technical realization of the idea it is instructive to discuss the place the method occupies in the catalog of existing approaches to the hadronic physics. The sum rule method is admittedly approximate, it requires a certain amount of guesswork and can not be formalized in the same sense as, say, the solution of the Schrödinger equation. At the same time, it is not a model. Any model necessarily requires \textit{ad hoc} assumptions, and the accuracy of the corresponding predictions (estimates) can not be controlled inside the model itself. In the sum rule method, once the rules of the game are accepted (the condensates are introduced once and forever) there is no freedom left; they tell you themselves whether this or that particular problem is solvable and what accuracy should be expected. In a sense, they must be compared to the lattice calculations. The strength of the latter is that they can indefinitely improve with time. The virtue of the sum rule method is that it is analytic, simple and is open for the qualitative analysis where one can easily see what stems from what.

\section{Implementation of the Idea}

In order to sketch the basic technical elements of the method let us discuss the problem of charmonium. This example is singled out for two reasons. First, the theoretical situation here is somewhat simpler and cleaner than in other cases. The second motivation is historical (it should be, perhaps, considered as the main at this Conference, with the focus on the history of QCD). The first estimates of the gluon condensate have been obtained from the charmonium sum rules \cite{9,16}. The latter work devoted to the $0^-$ charmonium level, $\eta_c$, has a particularly dramatic history. The point is that the $\eta_c$ particle was first found experimentally at a wrong place (2.83 GeV).

The value of the gluon condensate has been first extracted from the analysis of $J/\psi$, and then applied in the $0^-$ channel. The result of this investigation was formulated as a categoric prediction for the $\eta_c$ mass, $(M_{\eta_c})_{\text{theor}} = 3.00 \pm 0.03$ GeV. The later discovery of the genuine $\eta_c$ state with mass 2.98 GeV \cite{21} was one of the greatest successes of QCD and the strongest impetus for further work on the sum
The $1^-$ charmonium state, $J/\psi$ particle, is produced from the vacuum by the current
\[ j_\mu = \bar{c} \gamma_\mu c \]
where $c$ is the charmed quark field. Now, let us consider the correlation function
\[ \Pi_{\mu\nu}(x) = <0 | T\{j_\mu(x)j_\nu(0)\} | 0 >. \] (1)
If $x$ is sufficiently small $\Pi_{\mu\nu}$ is given by the sum of the Feynman graphs like those presented on Fig. 1 where the solid line is the quark propagator while the dashed line denotes the gluon propagator.

For heavy quarks the small values of $x$ do not necessarily imply that we have to consider large euclidean momenta in $\Pi_{\mu\nu}(Q)$. Indeed, even if $Q \sim 0$ the characteristic distance between the points of emission of the heavy quarks and their absorption is of order $x \sim (2m_c)^{-1} \ll \Lambda_{QCD}^{-1}$. Moreover, if the loop integrations are saturated in the domain of large virtual momenta we can use the bare expressions for the quark and gluon propagators, $(p^2 + m_c^2)^{-1}$ and $(k^2)^{-1}$, respectively.

The fact that the loop integrals are saturated at large virtual momenta does not mean that there is no contribution coming from the small momenta, $k \leq \Lambda_{QCD}$. It is clear that here, at $k \leq \Lambda_{QCD}$, we make a gross mistake by calculating the Feynman graphs with the bare propagators.

What can be done under the circumstances? Let us exclude the domain of small $k$ from the calculation of the perturbative correction of Fig. 1b. In order to define what we actually exclude we will need to introduce the normalization point $\mu$. For $|k| > \mu$ we use the bare gluon propagator, for $|k| < \mu$ we do not know what to use. Let us observe, however, that the momentum flowing along the quark line is large, and, hence, we can approximate $k < \mu$ by $k = 0$ in the quark propagator; what is left of the gluon line is just the integral over the unknown gluon Green function over the domain $|k| < \mu$. Graphically we can depict this contribution as on Fig. 2. In this way the gluon condensate shows up. The result obviously reduces to
\[ < \alpha_s(A^a_\mu A^a_\mu^\mu) > m_c^2 f(Q^2/m_c^2) \]
where $f$ is a calculable function determined by the lines which are far off shell. In the case at hand $f$ vanishes due to the gauge invariance. The vertex of emission of the soft gluons should contain $k_a k_\beta$ in the sum of three graphs (Fig. 3). Then the contribution of Fig. 3 can be written as follows:
\[ < \alpha_s G^{2\alpha\beta}_\mu > \mu F(Q^2/m_c^2) \] (2)
where $G_{\alpha\beta}$ is the gluon field operator and the subscript $\mu$ reminds us of the normalization point. In the general case any Feynman graph can be prepared in this way,
and we arrive at the following decomposition

\[
i \int e^{ixT} \{j(x)j(0)\} = \sum_n C_n(Q, m_c, \mu) O_n(\mu)
\]

where the set \( \{O_n\} \) includes all local gauge invariant operators expressible in terms of the gluon fields and the fields of the light quarks. Eq. (3) is a concise form of the Wilson operator expansion. The coefficients \( C_n(\mu) \), by construction, include only the domain \( |k| > \mu \).

So far we have done nothing particularly interesting. The procedure of separation of the loop momenta described above is quite universal and is applicable in any theory, say QED, or two-dimensional \( \sigma \) model. It is merely an identical transformation. Moreover, it looks rather awkward in QED since, instead of directly calculating the Feynman graphs, it prescribes first to do the separation in the integrands and then to calculate the very same graphs in the separated form.

In QCD this is more than a simple reshuffling. Indeed, we suspect (actually we are sure) that at \( |k| < \mu \) the Green functions are different from their bare expressions. The details of the infrared behavior are not known, but a few integrals can be parametrized. What is absolutely crucial is that the difference between the genuine propagators and smooth extrapolations from the perturbative domain is enormous (see Fig. 4). Moreover, the transition from the perturbative to non-perturbative regime is very sharp. These two facts are not derivable in the present-day theory; we learned of them indirectly, by accumulating empirical evidence. They make the sum rule approach technically manageable and ensure predictive power.

Indeed, under the circumstances the coefficients \( C_n \) are approximately determined by the standard perturbation theory. The fact that we have to "cut a hole" in the Feynman graphs (to discard the domain \( |k| < \mu \) ) is not important numerically since \( \mu^2 \) can be chosen sufficiently small in the characteristic hadronic scale set by \( <4\pi^2G^2>^{1/2} \). At the same time, under such choice of \( \mu \), the vacuum condensates are due to non-perturbative effects, to a good approximation. These effects are so violent, that the small perturbative background is totally negligible, which implies in turn that the vacuum parameters \( <O_n> \) are practically \( \mu \) independent, and there is no need to bother about the precise definition of the \( \mu \)-separation procedure (Fig. 4). Needless to say, that this is great luck; in other models the situation may be less favorable. (As an example where this element of luck becomes a parametrically exact assertion let me mention the \( O(N) \) \( \sigma \) model. One can show \( ^{22} \) that in the leading \( 1/N \) approximation the condensate parameters in this model are \( \mu \) independent. At the same time, in the next-to-leading \( 1/N \) order they are very sensitive to the \( \mu \) dependence. A straightforward generalization of the QCD sum rules, quite predictive in the leading \( 1/N \) approximation, becomes useless at the level of \( 1/N \).)

Thus in QCD we arrive at a pragmatic version of OPE \( ^{18} \). The corresponding simplified prescription reads: calculate the coefficient functions perturbatively, and hide all non-perturbative effects (and only them) in the vacuum expectation values. The anomalous dimensions, if present, should be taken into account, of
course. It is worth emphasizing again that strictly speaking this prescription is only approximate. The Wilson OPE does not discriminate perturbative against non-perturbative, rather it discriminates small $k$ against large $k$.

Returning to the correlator (1) and applying the approach described above to the graphs of Fig. 1 (see also Fig. 3) we get

\[ \Pi_{\mu\nu}(q) = (q^2 g_{\mu\nu} - q_\mu q_\nu)\Pi(q), \]

\[ \Pi(q) = F^{(0)}_0(q/m_c) + \alpha_s F^{(1)}_0(q/m_c) + \frac{<\alpha_s G^2>}{m_c^4} F_4(q/m_c) + \ldots \]  

(4)

where $F$ are dimensionless functions whose subscript indicates the (normal) dimension of the corresponding operator. Notice that the $F^{(1)}_0$ term contains, generally speaking, an $O(\mu^4)$ part whose origin is associated with the constraint in the Feynman integral. It must be compensated by a similar contribution coming from $<G^2>$. Both are numerically small and are neglected in eq. (4).

If all terms were known we could find the sum and then the position of the pole and its residue exactly. The position of the pole is the $J/\psi$ mass.

Since the vacuum condensates are not calculable (at least, now) we have to truncate the sum. To keep the theoretical part of eq. (4) under control it is desirable to limit ourselves to a few condensate parameters. The proliferation of these parameters would make the analysis unmanageable and would be equivalent, in essence, to a model dependence. Needless to say, that this was not what we wanted.

With a few terms in the right-hand side there is no way to reach the pole. In other words, the evolution of the $\bar cc$ pair in the vacuum medium from zero to infinity can not be traced within our approach. On the other hand, if the evolution time is not infinite the correlator $\Pi$ is saturated not only by the lowest-lying state in the given channel, $J/\psi$, but by excited states as well. Thus, there is a conflict of interests. To single out the ground state with the given quantum numbers we need large values of $x$. To reduce the theoretical calculation to a few vacuum condensates describing the average vacuum characteristics we need relatively small values of $x$. Is there a window where both requirements are met?

The answer to this question is not universal and depends on what hadronic channel is under consideration. For $J/\psi$, as well as for classical hadrons like $\rho$ or nucleon, the fiducial domain does exist, as has been shown in ref. [16] and later investigations [23]. Now, by comparing the theoretical expression for $\Pi(q)$ in the fiducial domain with its phenomenological counterpart we extract the parameters of the $J/\psi$ meson.

Since the existence of the window can not be guaranteed by fine-tuning of a parameter and relies on a subtle numerical balance different tricks aimed at minimizing the contamination due to the excited states and enhancing the role of the condensates from the standard set are very helpful, especially for the light-quark mesons. One such trick is the borelization procedure suggested in [16]. First, it factorially suppresses the operators with high dimensions; second, it simultaneously transforms
the standard dispersion relation in an exponential integral where the weight of the excited states is exponentially small. A close strategy [24] in the light-quark channels is considering the correlation function $\Pi(x)$ directly in the coordinate space, instead of analyzing $\Pi(Q)$ in the momentum space. Fig. 5 illustrates the existence of the window and the quality of the postdiction for the $\rho$ meson mass and residue. The vertical axis presents the ratio of the correlation function induced by the current $\bar{u}\gamma_\mu d$ to that for the bare quarks (solid line). The wavy line is the same ratio obtained theoretically with the gluon and quark condensates included. The arrows $A$ and $B$ show the boundaries of the window (the fiducial domain).

4 Current Status

Unlike QED Quantum Chromodynamics is deprived of a truly small expansion parameter like $\alpha = 1/137$. Even $1/N_c$, the last hope of many theorists, is not that small (I remind that $N_c$ is the number of colors in the multicolor version of ’t Hooft [25]). Unfortunately (or, perhaps, fortunately) we have to deal with the genuine strong coupling regime. Moreover, QCD is a notoriously rich theory responsible for an incredible diversity of phenomena in the hadronic world. Therefore I dare to conjecture that iterative procedure allowing one, at least in principle, to calculate the hadronic parameters with arbitrary accuracy, will not be found in the near future. It is not high precision which we should target but, rather, high reliability of theoretical predictions and full understanding and control over possible uncertainties.

In this respect the situation with the QCD sum rules, if not ideal, is at least satisfactory. The spectrum of applications of the method is very broad now. There are very few problems left in the low-energy hadronic physics not analyzed within the QCD sum rules. In some instances the analysis fails to produce reasonable results, and the challenge is to understand why. What is even more important, the sum rules can be used in both directions: each new hadronic channel is a potential source of information about the QCD vacuum; at the same time whatever new we learn of the vacuum we can immediately translate this knowledge in new predictions, say, in the glueball sector.

The standard set of the condensates which are in use now essentially reduces to five: two gluon condensates, two quark and one mixed,

$$\langle \alpha_s G^2 \rangle, \langle g^3 G^3 \rangle, \langle \bar{\psi} \psi \rangle, \langle \bar{\psi} \Gamma \psi \bar{\psi} \Gamma \psi \rangle, \langle \bar{\psi} \sigma_{\mu\nu} G_{\mu\nu} \psi \rangle. \tag{5}$$

True, these parameters enter the sum rules on uneven footing. The cubic gluon condensate, as a rule, plays an insignificant role. The quartic quark condensate is not treated as independent. Instead, invoking factorization (which is, in turn, justified by large $N_c$ arguments) one reduces it to the square of $\langle \bar{\psi} \psi \rangle$. Although
it is quite obvious that deviations from factorization should be present at a certain level numerous attempts to detect them theoretically yielded no conclusive evidence. Finally the mixed quark-gluon condensate shows up at a noticeable level mainly in the baryon sum rules. As a matter of fact, its numerical value has been extracted first from the analysis in the nucleon channel [26].

Using essentially three phenomenological numbers plus the quark masses as input the QCD practitioners were able to understand an enormous wealth of data referring to the low-energy hadronic physics. For this Conference I intended to prepare a list of the most spectacular predictions (postdictions) illustrating this statement. It turned out impossible for two reasons. First, it became rather boring. Whatever parameter you randomly choose from the Rosenfeld Tables (say, the $\rho$ meson mass, or $g_A$ – the nucleon axial constant – or the pion charge radius, or...) you look in the Tables, then in the corresponding papers, and then you find total agreement within the expected theoretical accuracy. More seriously, the number of works reporting on remarkable achievements is so large that making a short list of “most important” results would, probably, be unfair, the more so that all results quoted would require comments, and the ”short list” would become not so short very soon. A general idea of the range of problems solved can be obtained from the review paper [27] and the Reprint Volume [28]. It would be quite in order to have a book covering different versions of the method existing today and all major applications. Alas, such book is not written yet. The best I can do now is to try to catalog the main directions in which the sum rule method has been developing.

It started from the masses and residues of the low-lying hadronic states. All conceivable combinations – light quarks, quarks and gluons (exotics), glueballs, heavy quarks, light and heavy quarks; mesons and baryons – are considered and analyzed. At the second stage it was understood how to treat more complicated static properties, such as magnetic moments, charge radii, the axial and vector coupling constants for baryons, etc. Form factors at intermediate values of the momentum transfer and the light-cone wave functions were next logical steps. In the mature phase the QCD sum rules were applied to very sophisticated processes which are so far beyond reach of other approaches. The nucleon structure functions and the weak non-leptonic and radiative decays are just two examples immediately coming to one’s mind. I bring my apologies to my colleagues for the fact that I failed to compile even an incomplete list of references at this point. The interested reader might turn to the Reprint Volume [28].

Surprisingly, the method is applied now far beyond the scope it was originally designed for. Let me mention, for instance, recent attempts to expand it to cover the cases of the nuclear matter [29] and/or non-zero temperatures and densities [24]. This is a promising although complicated direction where the theoretical situation is so volatile that I do not risk to discuss it here in more detail. A less unexpected field where the sum rules successfully compete [30] with other approaches is the Effective Heavy Quark Theory.

As it often happens the most intriguing problems – the epicenter of the current
efforts – are those where the sum rule approach fails. One failure is quite obvious: the large-spin hadrons \[31\]. Indeed, the latter have parametrically large sizes and a ”sausage-like shape” (growing with spin) and, therefore, it is quite clear that the basic idea of the method – extrapolation from short to intermediate distances – is not applicable. Practically we have to stop at \[S = 2\]. Higher spins are the realm of stringy models or theories (if and when they are created, of course).

Even for small spins there exist clear-cut examples where the interaction of the valence quarks and gluons with the vacuum fluctuations is so peculiar (and strong) that the average vacuum characteristics are not enough to adequately describe how the corresponding hadrons are formed. Such a situation occurs in \[0^\pm\] quark mesons and, especially in \[0^\pm\] glueballs \[19\]. In these channels it is absolutely essential to know the details of the vacuum fluctuations, not just average densities. Hence, they will serve as the most sensitive testing ground for any future ”fundamental” theory of hadrons. It is remarkable – and I would like to emphasize it again – that in these channels we find a new scale in the physics of low-lying states, and this scale is significantly larger than that we got used to in the classical mesons and baryons. It is also worth noting that these are just the channels where the \[1/N_c\] counting also fails. \[\dagger\]

The best what can be done to-day is to develop particular models of the QCD vacuum. At least two of them are on the market: the instanton liquid model \[32\] and the stochastic vacuum model \[33\]. Both have been originally motivated by the QCD sum rules but are much more advanced in specifying the dominant vacuum fields. According to \[32\] the typical vacuum fields are given, in a reasonable approximation, by distorted instantons which still do not loose their individuality. The second model postulates the dominance of the stochastic type fluctuations with a finite correlation length. The detailization of the vacuum structure is the strength of these models – they are applicable even in those problems where there is no hope of using the standard QCD sum rules – but, simultaneously, this is their weakness. The concrete assumptions the authors had to make are not derivable so far from first principles rendering the corresponding results model-dependent and vulnerable to all sorts of critical remarks scattered in the literature.

There is something mysterious in the instanton-liquid model. On one hand, if you will attentively follow the model and do the calculations you will observe that literally speaking it leaves no space for the window like on Fig. 5. The quark and gluon condensates come out so large that they enter the game before the contamination dies off. On the other hand, the full curve which you could get in the model nicely repeats our theoretical prediction up to \[\sim 1.5\text{ fm}\]! It is clear that this aspect calls for an immediate explanation.

I have to add that there is an ongoing controversy in the current literature over

\[\dagger\]Let me parenthetically mention a problem where the sum rules are supposed to work very well, and still the corresponding prediction lies a factor of 3 higher than the experimental number, far beyond the error bars. I mean the \[J/\psi \rightarrow \eta_c \gamma\] decay. I would bet that the theory is right and experiment wrong.
several aspects of the QCD sum rule approach. Such issues as the value of the basic gluon condensate, factorization, the status of the pragmatic version of OPE, the possible role of higher condensates, etc. are vividly debated. This controversy is not necessarily a negative fact. On the contrary, it shows that the method is still in its active stage, the development goes on, new ideas continue to appear – naturally – along with new question marks. Unfortunately, I have no time here to dwell on the essence of the arguments and explain, say, why I believe that the original estimate of the gluon condensate from charmonium is quite accurate or why factorization should hold. Those who are interested can look through my comments in the Reprint Volume [28].

5 A Recent Example

In this part of my talk, just to amuse the audience and illustrate the virtues and drawbacks of the method, I will show how it works in an almost back-of-the-envelope estimate of the non-factorizable terms in the weak non-leptonic decays [34]. The estimate is not very precise; hopefully, it is valid up to a factor of two, but it is a very important phenomenological issue, and I am sure it will take quite a while before something like that appears on the lattice.

Let me remind you that the non-factorizable terms in the decays like $\bar{B}^0 \rightarrow D^+ \pi^-$ are reducible to the amplitude

$$< D\pi | \tilde{O}_2 | \bar{B} >$$

where

$$\tilde{O}_2 = 2(e\Gamma_{\mu}t^a b)(\bar{d}\Gamma^{\mu}t^a u),$$

the tilde appears due to historical reasons, $t^a$ is the color matrix while $\Gamma$ is the $V - A$ matrix. Within the naive factorization this amplitude vanishes. Of course, the perturbative gluon exchange between the heavy and the light quark brackets eliminates this zero, but the result is so small numerically that it can be safely neglected.

Still, color has to be exchanged between the brackets because otherwise the light quarks can not form the pion. The job has to be done by a soft gluon.

To estimate the effect let us consider the correlation function

$$A^\beta = \int d^4x < D | T\{\tilde{O}_2(x), A^\beta(0)\} | \bar{B} > e^{iqx}$$

where $A^\beta$ is an auxiliary axial current annihilating the pion,

$$A^\beta = \bar{u}\gamma^\beta\gamma^5d$$

and $q$ is an external momentum flowing through $A^\beta$. We assume it to be euclidean, neither too small nor too large. (This is the intermediate domain we always speak
about). Now, let us consider for simplicity the limit in which the masses of the D and B mesons are close to each other, an assumption which is very helpful technically. Then the correlator (8) is trivially calculable,

$$A^\beta = -\frac{i}{4\pi^2} \frac{q^\alpha q^\beta}{q^2} < D | \bar{c}\Gamma^\mu t^a g\tilde{G}^a_{\alpha\mu} b | B >$$

(9)

plus terms suppressed by powers of $1/q^2$. Here $\tilde{G}^a_{\alpha\mu}$ is the (dual) gluon field strength tensor.

For the express evaluation we omit the terms of the higher order in $q^{-2}$ on the right-hand side. The only operator retained describes the effect of the soft gluon absorption somewhere in the gluon medium surrounding the heavy quark in the corresponding heavy meson. If the heavy quarks are treated non-relativistically then one component of the operator dominates over the others, namely, $\alpha = 0$ (in the quark rest frame) and $\Gamma^\mu \rightarrow \gamma^i \gamma^5$. In this approximation

$$\bar{c}\Gamma^\mu t^a g\tilde{G}^a_{\alpha\mu} b \rightarrow -g\bar{\sigma}\vec{H}^a t^a,$$

(10)

where $\vec{\sigma}$ represents the Pauli spin of the heavy quark while the $\vec{H}^a$ is the chromomagnetic field operator.

Although we are unable at the moment to calculate the average chromomagnetic field inside heavy mesons from first principles, it is – nevertheless – known phenomenologically, an analog of the gluon condensate (5). Indeed, $g\bar{\sigma}\vec{H}^a t^a$ is the leading term splitting the masses of $B^*$ and $B$. Hence, the matrix element on the right-hand side of eq. (9) reduces to

$$m_{\sigma H}^2 \equiv \frac{3}{4}(M_{B^*}^2 - M_B^2).$$

Saturating eq. (9) by the pion contribution we find

$$< D^+ \pi^- | \hat{O}_2 | \bar{B}^0 > \sim \frac{i}{4\pi^2 f_\pi} m_{\sigma H}^2 (M_B^2 - M_D^2).$$

(11)

It is convenient to normalize this prediction to the factorizable part associated with $O_2$. The ratio of the non-factorizable to factorizable parts is

$$r \sim -\frac{N_c m_{\sigma H}^2}{4\pi^2 f_\pi^2} \sim -1.$$  

(12)

It is worth noting that the expression for $r$ quoted above is $O(N_c^0)$ since $f_\pi^2 \sim N_c$, as was expected, of course. The result (12) will set the scale for all future estimates of the deviations from factorization which, eventually, will have better accuracy.
6 Conclusions

Sometimes people say that only in the hard processes – like the logarithmic evolution of the structure functions – QCD is a true success while in the realm of the genuinely hadronic physics it has not shed much light. Very few facts which were totally beyond our comprehension before 1972 are understood now analytically.

This statement is correct only in part. Yes, the full analytic solution of the soft hadronic physics is still lacking. However, the QCD sum rules do provide a new insight, both in quantitative and qualitative issues. They are extremely successful in correlating essentially every parameter from the Rosenfeld tables, dozens of them, to three or four vacuum expectation values. They present a useful, and in many instances, unique tool for reliable estimates in the low-energy domain. They predict the occurrence of a new large scale in certain channels with the vacuum (or ”almost” vacuum) quantum numbers. Certainly, they do not explain how the infrared soup is cooked, but taking this fact for granted, they skillfully utilize the recipe.

The QCD sum rules are engineered as an approximate computational scheme in the regime of strong coupling – QCD at intermediate distances. The method can not be used iteratively, in order to achieve arbitrary accuracy, like the $\alpha$ expansion in QED. What is important, however, is the fact that the accuracy of the approximations done can be controlled within the method itself. Using this approach we get a qualitative idea of the structure of the QCD vacuum which will stay with us irrespectively of further developments. This idea is immediately translatable in predictions concerning new regularities in the hadronic family.

Finally, let me notice that the method is very flexible and can easily accommodate new information appearing on the market, as was, for instance, the case with the large $N_c$ expansion. It can be used in conjunction with other approaches, both analytic and numerical (heavy quark expansion, lattice calculations, etc.).

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