Thermal effects on dilepton production from $\pi - \pi$ annihilation

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Abstract

We study finite temperature effects on dilepton production from pion-pion annihilation in hot hadronic matter. The softening of the pion dispersion relation in a medium is found to enhance the production rate of dileptons with invariant masses in the region of $2m_\pi^*(T) < M < m_\rho$. On the other hand, the reduction of the pion electromagnetic form factor at finite temperature leads to a suppression of the dilepton production rate. Including both effects, we have found that the dilepton yield is slightly enhanced in the invariant mass region $M = 270 \sim 600$ MeV but is suppressed around the vector meson resonance. We further discuss the relevance of our results to recent experimental data from the S+Au collisions at CERN/SPS energies by the CERES collaboration.
Dilepton production from relativistic heavy ion collisions has continually attracted great interest for possible signals of the quark-gluon plasma that is expected to be formed in the initial stage of the collision [1–3]. Since dileptons would escape from the collision region without further interactions, they carry the information about the hot dense matter from which they are produced and, therefore, are considered as reliable probes for the hadronic matter at extreme condition. It has been suggested that a window for observing dileptons from the plasma phase exists in the invariant mass region between the $\phi$ and $J/\psi$ [2]. Above the $J/\psi$ mass, dominant contributions are from the Drell-Yan process and direct charm decay [3], while below the $\phi$ meson mass, radiative and direct decays, together with $\pi\pi$ annihilation, form the most important sources [5].

Recently, experimental data on dileptons have been measured by the CERES collaboration at the CERN/SPS [6]. In proton induced reactions, such as the 450 GeV $p$ − $Be$ and $p$ − $Au$ collisions, the low-mass dilepton spectra can be satisfactorily explained by dileptons from hadron decays. On the other hand, in the $S$ + $Au$ collision at 200 GeV/nucleon a significant enhancement of dileptons over the hadronic contribution has been observed in the invariant mass region $200 \text{ MeV} < M < 1 \text{ GeV}$. The enhancement seen in the CERES experiment is for dileptons at the central rapidity where the charge particle density is high. In another experiment by the HELIOS-3 collaboration [7], dileptons at forward rapidities, where the charge particle density is low, were measured, and the enhancement was found to be smaller. Suggestions have thus been made that the excess dileptons seen in these experiments are from pion-pion annihilation, $\pi^+\pi^- \rightarrow e^+e^-$. However, the inclusion of this contribution in model calculations cannot explain the observed dilepton yield as long as the production rate is calculated with parameters in free space [3,4].

In this letter, we study the modification of pion properties at finite temperature and its effect on dilepton production from pion-pion annihilation in hot hadronic matter. To study the pion properties at finite temperature we shall use an effective chiral Lagrangian that includes explicitly vector mesons. In the literature, two methods have been introduced to include vector mesons and photon field in the chiral Lagrangian; the massive Yang-Mills
approach [10] and the hidden gauge approach [11]. These two methods have been shown to be gauge equivalent and to have identical symmetry properties at finite temperature [12].

We shall follow the hidden gauge approach by considering the $[SU(2)_L \times SU(2)_R]_{\text{global}} \times [SU(2)_V]_{\text{local}}$ “linear” model. It is constructed with two $SU(2)$-matrix valued variables $\xi_L(x)$ and $\xi_R(x)$, which transform as $\xi_{L,R}(x) \rightarrow h(x)\xi_{L,R}g^\dagger_{L,R}$ under $h(x) \in [SU(2)_V]_{\text{local}}$ and $g_{L,R} \in [SU(2)_L,R]_{\text{global}}$. Introducing the vector meson $V_\mu$ as the gauge field of the local symmetry and the photon $B_\mu$ as an external gauge field of the global symmetry, we have the following chirally invariant Lagrangian,

$$\mathcal{L} = f_\pi^2 \text{tr} \left[ \frac{1}{2i} (D_\mu \xi_L \cdot \xi_L^\dagger - D_\mu \xi_R \cdot \xi_R^\dagger) \right]^2 + af_\pi^2 \text{tr} \left[ gV_\mu - \frac{1}{2i} (D_\mu \xi_L \cdot \xi_L^\dagger + D_\mu \xi_R \cdot \xi_R^\dagger) \right]^2 + \mathcal{L}_{\text{kin}}(V_\mu, B_\mu),$$

where $f_\pi = 93$ MeV is the pion decay constant. The covariant derivative, $D_\mu \xi_{L,R}$, is given by

$$D_\mu \xi_{L,R} = \partial_\mu \xi_{L,R} + ie\xi_{L,R}B_\mu \tau_3/2.$$  

We also add a term that explicitly breaks the chiral symmetry,

$$\mathcal{L}_{SB} = \frac{1}{4}f_\pi^2 m_\pi^2 \text{tr} (\xi_L \xi_L^\dagger + \xi_R \xi_R^\dagger).$$

In the “unitary” gauge

$$\xi_L^\dagger(x) = \xi_R(x) \equiv \xi(x) = \exp(i\pi(x)/f_\pi),$$

and with $a = 2$, this effective Lagrangian is known to give the universality of $\rho$-couplings ($g_{\rho\pi\pi} = g$), the KSRF relations ($m_\rho^2 = 2f_\pi^2 g_{\rho\pi\pi}^2$), and the $\rho$ meson dominance of the pion electromagnetic form factor ($g_{\gamma\pi\pi} = 0$). The Lagrangian has been extended to include the anomalous interactions and axial vector mesons. However, these effects are not included in present calculations.

First we consider the modification of the pion dispersion relation in hot hadronic matter which is in thermal equilibrium at temperature $T$. In effective Lagrangian approaches at
finite temperature, we assume that known hadronic interactions can be extrapolated to finite temperature and describe the interactions among particles in hot hadronic matter. The temperature effect on the properties of pions can then be studied via thermal loop-corrections which are due to interactions with particles in the hot matter. We include only one-loop diagrams as the hadronic matter is rather dilute at temperatures considered here.

The dispersion relation is found by locating the poles of the propagator and is determined by an equation of the form

\[ \omega^2 = \vec{k}^2 + m^2_{\pi} + \Pi(\omega, \vec{k}), \]  

(5)

where \( \omega \) and \( \vec{k} \) are the pion energy and momentum, respectively. The self-energy \( \Pi \) includes the modification of pion properties due to interactions with particles in the hot matter. While the real part gives its dispersion relation, the imaginary part of the self-energy determines the absorption of pions in hot matter.

The self-energy of a pion at finite temperature is calculated from diagrams in fig. 1 at the one-loop order. The contributions from fig. 1.a \( (\Pi^{(a)}) \) and 1.b \( (\Pi^{(b)}) \) are given, respectively, by

\[ \Pi^{(a)}(\omega, \vec{k}) = 2g^2 T \sum_n \int \frac{d^3p}{(2\pi)^3} \frac{(p_\mu - k_\mu)(g^{\mu\nu} - (p^\mu + k^\mu)(p^\nu + k^\nu)/m^2_{\pi})(p_\nu - k_\nu)}{(p^2 - m^2_{\pi})(p^2 + k^2 - m^2_\rho)}, \]

\[ \Pi^{(b)}(\omega, \vec{k}) = T \sum_n \int \frac{d^3p}{(2\pi)^3} \left( \frac{5 m^2_{\pi}}{6 f^2_{\pi}} + \frac{1}{3 f^2_{\pi}}(p^2 + k^2) \right) \frac{1}{(p^2 - m^2_\pi)}, \]

(6)

where \( k \) and \( p \) are the external and loop pion four momenta, respectively, and \( p^2 = p_0^2 - \vec{p}^2 \) with \( p_0 = 2\pi nTi \). Eq. (7) is now solved self-consistently with

\[ \Pi(\omega, \vec{k}) = \Pi^{(a)}(\omega, \vec{k}) + \Pi^{(b)}(\omega, \vec{k}). \]

(7)

We find that the pion dispersion relation is slightly modified at finite temperature, and the effective mass of a pion, which is defined as the pole position of the pion propagator in medium, decreases with the temperature, \( m^*_\pi \approx 133 \text{ MeV} \) at \( T = 160 \text{ MeV} \). A similar result has been obtained in massive Yang-Mills approach [13].
Using the temperature-dependent pion dispersion relation and effective mass, we have calculated the dilepton production rate, \( R = \frac{dN^{e^+e^-}}{dx^4} \), from pion-pion annihilation in hot hadronic matter. For simplicity, we carry out the calculation in the rest frame of the virtual photon. The production rate of dileptons with vanishing three momentum in hot hadronic matter is then given by \[ d^4R \bigg|_{\vec{q}=0} = \frac{\alpha^2}{3(2\pi)^4} \frac{|F_\pi(M)|^2}{(e^{M/2T} - 1)^2} \sum_k k^4 \left| \frac{d\omega}{dk} \right|^{-1} \] where \( k = |\vec{k}| \) and \( M \) is the dilepton invariant mass. The momentum \( k \) and energy \( \omega \) of the pion are related by its dispersion relation in the medium, and the last factor in the above equation takes into account this effect. The sum over \( k \)'s is restricted by \( \omega(k) = M/2 \). The pion electromagnetic form factor is denoted by \( F_\pi(M) \) and is given by \[ |F_\pi(M)|^2 = \frac{m_\rho^4}{(M^2 - m_\rho^2)^2 + m_\rho^2 \Gamma_\rho^2}, \] with the rho meson width \( \Gamma_\rho = 152 \text{ MeV} \).

The dilepton production rate from pion-pion annihilation is shown in fig. 2 for \( T = 160 \text{ MeV} \). The result obtained with the modified pion dispersion relation (dashed line) is compared with that calculated using the dispersion relation in free space (dotted line). There are two prominent effects due to changes of the pion properties in hot hadronic matter. First, the threshold of dilepton production from pion-pion annihilation is lowered because of the reduction of the pion mass at finite temperature. Secondly, the dilepton production rate is enhanced in the invariant mass region, \( 2m_\pi(T) < M < m_\rho \), and shows a peak at \( M \sim 350 \text{ MeV} \), which is due to the softening of the pion dispersion relation in medium. The latter, however, does not have any effect on dileptons with invariant masses near the vector meson resonance.

\[ ^1 \text{In principle the formfactor is given by the more sophisticated Gounaris-Sakurai formula } \] However, since we are only interested in relative changes due to in medium corrections, the simpler form used here is sufficient.
To ensure gauge invariance, we need to include the temperature dependence of the pion electromagnetic form factor as pointed out in Refs. [16] for the case of finite density. The temperature dependence of the form factor involves the modification of the rho-pion-pion vertex, the change in rho meson properties, and the correction to the rho-photon coupling in hot matter. These effects have been studied in Ref. [17] with the same Lagrangian, and it has been shown that the production rate of dileptons with invariant masses around the vector resonance is suppressed. We have repeated the calculation of Ref. [17] by including the finite pion mass which was neglected in the previous calculation. The result with the temperature-dependent pion electromagnetic form factor is shown in fig. 2 by the dash-dotted line. The modification of the pion electromagnetic form factor at finite temperature is seen to reduce the dilepton production rate. The result obtained with both the form factor and the dispersion relation evaluated at finite temperature is given by the solid line. Compared to the free case (dotted line) the combined medium effects yield a very small enhancement close to the two pion threshold and a strong reduction in the rho meson mass region. The ratio between the yield at the two pion threshold and that at the rho meson mass, however, is enhanced by a factor of 4 due to medium modifications of the pion properties.

The above calculations have been done for dileptons with vanishing three momentum. To compare with experimental data, we need to extend the calculations to dileptons with finite three momentum. For free pions the production rate for dileptons with a momentum $\vec{q}$ is given by

$$\frac{d^4 R}{dM dq^3} \bigg|_{q^3 \neq 0} = \exp\left[-\frac{(\sqrt{q^2 + M^2} - M)/T}{\sqrt{1 + q^2/M^2}}\right]\left(\frac{dR}{dMdq^3} \bigg|_{q^3 = 0}\right).$$  \hspace{1cm} (10)

In medium the result is nontrivial, and we have thus assumed that the above equation remains valid. This seems to be reasonable as dominant contributions to the pion electromagnetic form factor at finite temperature are momentum-independent and also the pion in-medium dispersion relation is not very different from the free one. By factorizing the momentum dependence in this way one can easily integrate over the dilepton three-momentum, and the result is shown in fig. 3. Because of the finite momentum effect, the dilepton spec-
trum obtains a quite different shape; the production rate in the low invariant mass region is reduced, and the peak near $M \sim 2m_\pi$ disappears. Even though the production rate is enhanced for dileptons in the invariant mass region near the two pion threshold, the absolute yield is now less than that of the vector resonance peak.

In summary, we have studied the pion properties in hot hadronic matter and the dilepton production rate from pion-pion annihilation with the modified pion properties. It is shown that the production rate is suppressed in the invariant mass region near the vector meson resonance but slightly enhanced at $M \sim 2m_\pi$. The suppression mainly comes from the reduction of the pion electromagnetic form factor but is recovered near the two pion threshold due to modifications of the pion dispersion relation in hot matter. After integrating over the space-momentum of the virtual photon we obtain a quite different shape of the dilepton spectrum compared with that of zero space-momentum dileptons. Here, we have assumed the same momentum dependence as that for free pions but this approximation might be too simple to take into account the full effects due to finite dilepton momenta. More work is thus needed to include more rigorously the finite momentum effect.

The dilepton production rate from the $\pi - \pi$ contribution has been calculated using a relativistic transport model and compared with the data from S+Au collisions at the CERN/SPS [8]. The model calculation shows, however, a disagreement with the experimental data unless effects due to the reduced vector meson masses in medium are included. The estimated dilepton yield with free vector meson masses is about a factor $4 \sim 6$ less than the measured one in the low invariant mass region of $M = 200 \sim 600$ MeV but about a factor 2 more near the vector meson resonance. A similar result has also been obtained in hydrodynamic model calculations including the quark-hadron phase transition [9]. In this calculation, the most important contribution to dilepton production comes from the hadronic component of the mixed phase at $T_c = 160$ MeV.

The thermal effects on dilepton production from the early stage of the slowly expanding hot dense hadronic matter or from the hadronic component of the mixed phase can be easily estimated from eq. (10) and fig. 3. Our results indicate that the production rate of dileptons
near the vector meson mass will be suppressed by the reduction of the pion electromagnetic form factor at finite temperature. The thermal effect on the pion dispersion relation also shows an enhancement of dilepton productions with invariant masses $M = 270\text{ MeV} \sim 600\text{ MeV}$ and brings the shape of the dilepton spectrum close to the one measured. However, thermal effects may not be enough to explain the excess dileptons observed in this low invariant mass region.

In the future, we need to include properly the expansion dynamics of the hot matter that is formed in high energy nucleus-nucleus collisions to obtain a more realistic determination of the dilepton production rate. We should also include corrections due to the experimental acceptance. In particular, the transverse momentum cut, applied separately to the electrons and positrons in experimental measurements, affects significantly the dilepton yield near the two pion threshold. Finally, we have not included the effect of finite baryon density on the dilepton production rate, which is expected to be important as well in experiments at the CERN/SPS. We plan to study consistently both the temperature and the density effect on dilepton production in high energy heavy ion collisions.

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REFERENCES

[1] E. L. Feinberg, Nuovo Cimento 34A (1976) 39.

[2] E. V. Shuryak, Phys. Lett. B 78 (1978) 150.

[3] P. V. Ruuskanen, Nucl. Phys. A 544 (1992) 169c;  
J. Kapusta, Nucl. Phys. A 566 (1994) 45c.

[4] C. M. Ko and L. H. Xia, Phys. Rev. Lett. 62 (1989) 1595;  
M. Asakawa, C. M. Ko and P. Lévai, Phys. Rev. Lett. 70 (1993) 398;  
M. Asakawa and C. M. Ko, Phys. Lett. B 322 (1994) 33.

[5] K. Kajantie, J. Kapusta, L. McLerran and A. Mekijan, Phys. Rev. D 34 (1986) 2746.  
C. Gale and P. Lichard, Phys. Rev D 49 (1994) 3338.  
C. Song, C. M. Ko and C. Gale, Phys. Rev. D 50 (1994) R1827.

[6] G. Agakichiev et. al., CERES Collaboration, CERN preprint CERN PPE/95-26 (1995);  
J. P. Wurm for the CERES Collaboration, in Proc. Quark Matter '95, January 9-13, 1995; Nucl. Phys. A, to be published.

[7] M. Masera for the HELIOS-3 Collaboration, in Proc. Quark Matter '95, January 9-13, 1995; Nucl. Phys. A, to be published.

[8] G. Li, C. M. Ko and G. E. Brown, Texas A&M University Preprint (1995).

[9] D. K. Srivastava, B. Sinha and C. Gale, McGill University Preprint (1995).

[10] U.-G. Meißner, Phys. Rep. 161 (1988) 213.

[11] M. Bando, T. Kugo and K. Yamawaki, Phys. Rep. 164 (1988) 217.

[12] S. H. Lee, C. Song and H. Yabu, Phys. Lett. B 341 (1995) 407.

[13] Chungsik Song, Phys. Rev. D 49 (1994) 1556; Phys. Lett. B 329 (1994) 312.

[14] C. Gale and J. Kapusta, Phys. Rev. C 35 (1987) 2107.
L. H. Xia, C. M. Ko, L. Xiong, and J. Q. Wu, Nucl. Phys. A 485 (1988) 721.

[15] G. J. Gounaris and J. J. Sakurai, Phys. Rev. Lett. 21 (1968) 244.

[16] C. L. Korpa and S. Pratt, Phys. Rev. Lett. 64 (1990) 1502.
    M. Asakawa, C. M. Ko, P. Lévai and X. J. Qiu, Rev. C 46 (1992) R1159.
    M. Herrmann, B. L. Friman and W. Nörenberg, Nucl. Phys. A 560 (1993) 411.

[17] C. Song, S. H. Lee, and C. M. Ko, Phys. Rev. C, to be published.
**Figure Captions**

**Fig. 1:** One loop diagrams for the pion self-energy. The dotted and solid lines denote the pion and $\rho$ meson, respectively.

**Fig. 2:** The dilepton production rate from pion-pion annihilation in hot hadronic matter at $T = 160$ MeV. The dotted line is the result obtained without medium effects while the dashed line is that including modifications of the pion dispersion relation. The dash-dotted line is the production rate obtained by only taking into account the effect of pion electromagnetic form factor at finite temperature, and the solid line is the result when both effects are included.

**Fig. 3:** The dilepton production rate after integrating over the dilepton three momentum. The result obtained without medium effects (dotted line) is compared with that obtained with modifications of the pion dispersion relation and electromagnetic form factor at finite temperature (solid line).
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