Theory of resonant spin Hall effect

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A brief review is presented on resonant spin Hall effect, where a tiny external electric field induces a saturated spin Hall current in a 2-dimensional electron or hole gas in a perpendicular magnetic field. The phenomenon is attributed to the energy level crossing associated with the spin-orbit coupling and the Zeeman splitting. We summarize recent theoretical development of the effect in various systems and discuss possible experiments to observe the effect.

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I. INTRODUCTION

Spin-orbit interaction of electrons as a relativistic quantum mechanical effect plays important roles in a number of classes of metals and semiconductors. Spin-orbit coupling mixes different spin states and provides an efficient way to control the coherent motion of electron spins via an applied electric field. Directional motion of electron spins may circulate a spin current with a null charge current. Spin Hall effect refers to spin current or spin accumulation in the direction transverse to the applied electric field, in analogy to charge current in the Hall effect. The spin Hall effect was first proposed by Dyakonov and Perel in early of 1970\textsuperscript{1,2}. The interest of the effect was revived in recent years\textsuperscript{3,4}. The "intrinsric" spin Hall effect was proposed in 2003 by Murukami et al.\textsuperscript{5} for hole doped semiconductors and by Sinova et al.\textsuperscript{6} for two dimensional electron gas with a Rashba spin-orbit coupling. Since then studies of the spin Hall effect has evolved into a subject of intense research. Although there are debates on the precise definition of the spin current, the detect of a pure spin current continues to be a challenging issue. The spin current may lead to the formation of spin accumulation or non-uniform spin distribution at the boundaries, which may be detected experimentally\textsuperscript{7,8,9}. The spin current may also lead to a Hall voltage or charge Hall current as a reciprocal effect induced by spin current, which has been reported in diffusive metallic conductors\textsuperscript{10,11}. In addition, quantum interferences by optical means have also been reported\textsuperscript{12,13,14}. Recently it was observed that an optically injected spin current flowing through a Hall-bar system can generate an in-ward or out-ward electric current, while the Hall voltage remains zero\textsuperscript{15}. This observation renders a manifestation of the tensor-like nature of a spin current, of which both the spin polarization and the velocity are decisive factors in producing observable effects.

Resonant spin Hall effect refers to a resonant or saturated spin Hall current response to a tiny applied electric field in a 2DEG with spin-orbit coupling in the presence of a strong perpendicular magnetic field. As a result the spin Hall conductance may become divergent in a weak field limit. The resonance was attributed to the energy level crossing of the system due to the competition between the spin-orbit coupling and the Zeeman splitting or other terms. The resonant spin Hall effect was first proposed by Shen et al. in a 2DEG with a Rashba coupling\textsuperscript{16,17}. The edge spin current was studied by Bao et al.\textsuperscript{18}. The resonant spin Hall effect in a hole-doped system described by a Luttinger Hamiltonian has recently been studied by Zarea and Ullon\textsuperscript{19} and by Ma and Liu\textsuperscript{20}. At present, there have not been experimental reports on the observation of the resonant spin Hall effect or related phenomena yet, which is likely due to the combination of the difficulty in detecting the spin current or spin accumulation in the high magnetic field and the lack of experimental efforts in looking into these phenomena.

In this review we shall summarize recent theoretical works on the resonant spin Hall effect and relevant works on 2-dimensional electron or hole gases. For overviews on the general spin Hall effect, we refer readers to other recent review articles and references therein\textsuperscript{21,22,23}. The rest part of the paper is organized as follows. In section II, we review the resonant spin Hall effect in various 2-dimensional systems including the Rashba and Dresselhaus couplings of the electron system and the hole system of the Luttinger Hamiltonian. In section III, we review the edge spin current. In section IV, we discuss a close relation between the spin Hall current and the spin polarization and also discuss the effect of the disorder. We propose experiments to observe the resonant spin Hall effect.

II. RESONANT SPIN HALL EFFECT

In this section we discuss the theoretical studies of resonant spin Hall effect (RSHE) in various systems. We will first discuss the effect in 2-dimensional electron gas (2DEG) with a linear Rashba spin-orbit coupling, followed by a discussion of the effect in 2DEG with Dresselhaus spin-orbit coupling by examining a symmetry transformation to the Rashba coupling. We then discuss the effect in 2DEG with both Rashba and Dresselhaus couplings and the effect in 2-dimensional hole gas.
A. RSHE in 2DEG with Rashba coupling

The resonant spin Hall effect was first proposed in 2DEG with a Rashba spin-orbit coupling $\beta$. The Hamiltonian for a single electron of spin-$1/2$ in 2DEG of area $L_x \times L_y$ subject to a perpendicular uniform magnetic field $\mathbf{B} = B\mathbf{z} = \nabla \times \mathbf{A}$ and an in-plane electric field $\mathbf{E} = E\mathbf{y}$ is given by

$$H_R = \frac{1}{2m} \left( \mathbf{p} + \frac{e}{c} \mathbf{A} \right)^2 + \beta \mathbf{\hat{z}} \cdot \left( \mathbf{p} + \frac{e}{c} \mathbf{A} \right) \times \sigma$$

$$+ \frac{g_s}{2} \mu_B B \sigma_z + eEy$$

(1)

where $m, -e, g_s$ are the electron's effective mass, charge and the Lande g-factor, respectively. $\mu_B$ is the Bohr magneton, and $\sigma$ are the Pauli matrices. We choose a Landau gauge $\mathbf{A} = yB\mathbf{z}$, and consider a periodic boundary condition in the $x$ direction. We consider the case $E = 0$ first. At $\beta = 0$, the solution of the above Hamiltonian is well known and the energy spectra are given by evenly-spaced Landau levels $(n+1/2)\hbar \omega - g_s \mu_B B \sigma_z / 2$ for each spin state ($\omega = eB/nc$, $n=0,1,2,...$). At $\beta \neq 0$, the Rashba term hybridizes the two neighboring Landau levels with opposite spins, and the problem can be solved analytically with the energy levels given by $[16,24]$

$$\epsilon_{ns} = \hbar \omega \left( n + \frac{g}{2} \sqrt{(1-g)^2 + 8n\eta^2} \right),$$

(2)

where $\eta = \beta ml_b/\hbar$, and $g = g_s m/2m_e$, with $m_e$ the mass of a free electron and $l_b = \sqrt{\hbar c/eB}$ the magnetic length with $s = \pm 1$, for $n \geq 1$; and $s = 1$ for $n = 0$. The eigenstate $|n,k,s\rangle$ has a degeneracy $N_\phi = L_x L_y eB/\hbar c$, corresponding to $N_\phi$ quantum values of $p_x = k$. The two-component wavefunction is given by

$$|n,k,s\rangle = \left( \begin{array}{c} \cos \theta_{ns} \phi_{nk} \\ i \sin \theta_{ns} \phi_{n-1k} \end{array} \right)$$

(3)

where $\phi_{nk}$ is the eigenstate of the $n^{th}$ Landau level in the absence of the Rashba interaction. Only two levels of $\phi_{nk}$ with spin up and $\phi_{n-1k}$ with spin down are mixed together. For $n = 0$, $\theta = 0$, and for $n \geq 1$, $\tan \theta_{ns} = u_n - s \sqrt{1 + u_n^2}$, with $u_n = (1-g)/\sqrt{8n\eta}$. One of the key features of the solution is that the energy levels may cross, which introduces additional degeneracy of the spectrum (see Fig. 1).

Analysis shows that the additional degeneracy caused by the crossing of the Landau levels will lead to some anomalous properties related to electron spins. One of them is the resonant spin Hall effect, where the spin Hall conductance may become divergent when the Fermi surface is located at or near the crossing point $[16]$. In other words, a tiny external electric field $E$ will generate a saturated spin Hall current. This effect can be best understood in a simplified two-level model $[16]$.

![FIG. 1: Landau levels of an electron as functions of effective Rashba coupling $\eta = aml_b/\hbar$ for $g = g_s m/2m_e = 0.1$. Arrows indicate those level crossings giving rise to resonant spin Hall conductance in a weak electric field. [From Ref.([16])]](image)

Let’s consider two states $|0, +1\rangle$ and $|1, -1\rangle$ and the effective Hamiltonian $H_R$ in the presence of an electric field $E$ is reduced to

$$H_{\text{reduced}} = \left( \begin{array}{cc} \Delta \epsilon & v_0 \\ v_0 & -\Delta \epsilon \end{array} \right)$$

(4)

where $\Delta \epsilon = \epsilon_{0,+1} - \epsilon_{1,-1}$, and $v_0 = -eEl_\eta \sin \theta_{1,-1}$. A key feature of this model is that the electric field generates a non-zero off-diagonal element. Near the crossing point the electron spins in the two levels are almost opposite. The external electric field hybridizes the two levels and opens an energy gap. Each mixed state carries a finite spin Hall current in the opposite direction. When the lower energy states are occupied, a finite spin Hall current circulates once the energy gap is opened by the external field. A detailed analysis shows that at zero temperature and weak field limit, the spin Hall conductance $G_s = \frac{e}{4\pi} \frac{\delta \epsilon}{\Delta \epsilon}$, where $\delta \epsilon$ is the filling factor at the two levels. At low $T$, as the magnetic field approaches the resonant point $B \to B_0$, $G_s \to -e\nu \sqrt{2B_0} \sqrt{(1+g)/(1-g)}$, and $\int G_s dB \to -e\nu \sqrt{2B_0} g/(4\pi(1+g)) \ln [\hbar \omega_0/k_B T]$. At the resonant point and low temperature, $G_s \propto 1/E$.

B. RSHE in 2DEG with Dresselhaus coupling

In III-V compounds such as GaAs and InAs, the spin-orbit interaction also generates a Dresselhaus coupling in systems with bulk inversion asymmetry, which is given by $H_D^{so} = \alpha (p_x \sigma_y - p_y \sigma_x)$. The Rashba coupling and Dresselhaus coupling plays different roles in the spin Hall effect. In this subsection, we examine the symmetries related to the Rashba and Dresselhaus couplings. We consider a 2D spin-orbit coupling, $V_{so}^{2D} = H_{so}^{R} + H_{so}^{D}$.

Interchange symmetry of the two couplings. Under the unitary transformation, $\sigma_x \to \sigma_y$, $\sigma_y \to \sigma_x$, $\sigma_z \to -\sigma_z$. No such symmetry is present for the Rashba coupling.
\( \sigma_z \rightarrow -\sigma_z \), the Rashba and Dresselhaus couplings are interchanged:\(^{25}\)

\[
\begin{align*}
\alpha(\Pi_x \sigma_x - \Pi_y \sigma_y) & \rightarrow \alpha(\Pi_x \sigma_y - \Pi_y \sigma_x); \\
\beta(\Pi_x \sigma_y - \Pi_y \sigma_x) & \rightarrow \beta(\Pi_x \sigma_x - \Pi_y \sigma_y); \\
g_s & \rightarrow -g_s.
\end{align*}
\] (5, 6, 7)

Therefore a system with Rashba coupling \( \beta \), Dresselhaus coupling \( \alpha \), and Lande g-factor \( g_s \) is mapped on to a system with Rashba coupling \( \beta \), Dresselhaus coupling \( \alpha \), and Lande g-factor \( -g_s \). In particular, a system with only Dresselhaus coupling can be mapped on to a system with only Rashba coupling and an opposite sign in \( g_s \). At the symmetric point \( \alpha = \beta \), \( V_{zz}^{2D} \) is invariant under the transformation. \( \alpha = -\beta \) is another symmetric point under the transformation, \( \sigma_x \rightarrow -\sigma_y, \sigma_y \rightarrow -\sigma_x, \sigma_z \rightarrow -\sigma_z \).

**Signs of the couplings.** Under the transformation, \( \sigma_x \rightarrow -\sigma_x, \sigma_y \rightarrow -\sigma_y, \sigma_z \rightarrow \sigma_z \), we have \( \alpha \rightarrow -\alpha \) and \( \beta \rightarrow -\beta \). The eigenenergy spectrum is invariant under the simultaneous sign changes of the two couplings. The eigenenergy spectrum is even in \( \alpha \) if \( \beta = 0 \) and is even in \( \beta \) if \( \alpha = 0 \).

**Charge conjugation.** Under the charge conjugation transformation, \( -e \rightarrow e \), the magnetic moment of the carrier also changes its sign, or effectively \( g_s \rightarrow -g_s \) in Eq. (4). This transformation is equivalent to the flip of the external magnetic field \( B \rightarrow -B \). Therefore, a system of hole carriers has the same physical properties as the corresponding electron system except for possible directional changes in the observables.

**C. RSHE in 2DEG with Rashba and Dresselhaus couplings**

The Landau levels of 2DEG with a pure Rashba coupling in the absence of an electric field was obtained over forty years ago by Rashba, and studied by a lot of authors.\(^ {21} \) The Landau levels of 2DEG with a pure Dresselhaus coupling can be obtained by symmetry as discussed in the last subsection. However, the problem becomes more complicated in a system with both the Rashba and Dresselhaus couplings. The truncation and the perturbation approximations have been applied to study the system and the response to an electric field.\(^ {22, 23} \) This problem has recently been solved by D. Zhang.\(^ {24} \) By introducing two boson operators and using a unitary transformation, an exact solution can be expressed as an infinite series in terms of the free Landau levels and other parameters. Suing the result obtained from the exact solution it was confirmed that a crossing between the energy eigenstates occur for larger Rashba coupling, and the degeneracy may lead to the resonant spin Hall conductance, which is consistent with the results obtained by using the perturbation theory and truncation approximation.\(^ {16, 17, 18} \)

**D. RSHE in 2D hole gas**

The electronic structure of hole-doped systems near the \( \Gamma \) point is well described by the Luttinger Hamiltonian

\[
H_L = \frac{1}{2m}(\gamma_1 + \frac{5}{2}\gamma_2)p^2 - \frac{\gamma_2}{m}(p \cdot J)^2
\] (8)

where \( J \) is the total angular momentum operator of \( j = 3/2 \). If the system breaks also the bulk inversion symmetry, a Rashba term \( \beta(J \times p) \cdot \hat{z} \) may be included in \( H_L \). The Rashab term introduces an energy splitting of the heavy holes in the 2-dimensions, and is reduced to the form of the so-called cubic Rashba term, \( \gamma(k_x^2 \sigma_z + k_y^2 \sigma_x) \). In the presence of a perpendicular magnetic field, both the Luttinger model and cubic Rashba model can be solved analytically. Recently, Zarea and Ulloa\(^ {25} \) and Ma and Lin\(^ {26} \) have obtained exact solutions in two dimension, and used the Kubo-Greenwood formula to calculate the spin Hall conductance. Neglecting the Zeeman splitting, they found that the spin Hall conductance approaches to \( 9e/8\pi \) in the limit of a weak magnetic field and low density of carriers, consistent with the result at the zero field. This is in contrast to the linear Rashba coupling model of \( H_R \), where the spin Hall conductance vanishes in the absence of the Zeeman coupling.\(^ {27} \) When the Zeeman splitting is taken into account in the cubic Rashba system, it was found that a resonant spin Hall effect may occur when the Fermi surface crosses through the crossing point of the spectra as the magnetic field decreases.\(^ {28} \)

**III. EDGE SPIN CURRENT AND RESONANT SPIN HALL EFFECT**

It was well known that the edge state provides an alternative approach to understand the quantum Hall effect in 2DEG. Here we discuss the edge spin current and the resonant spin Hall effect in the Rashba system described by \( H_R \) with a periodic boundary condition in the \( x \) direction and two edges along the \( y \)-direction. The velocity operator in the Hilbert subspace of \( y_0 = p_x b_x^2/\hbar \) can be written as \( v_y = (l_y^2/\hbar) (\partial H/\partial y_0) \) and thus the spin current operator

\[
j_s^z = l_y^2 (\partial H/\partial y_0 \sigma_z + \sigma_z \partial H/\partial y_0)/2.
\] (9)

Following the works by Laughlin,\(^ {29} \) Halperin,\(^ {29} \) and MacDonald and Streda.\(^ {30} \) the total spin Hall current in the filled Landau level \((n, s)\) can be expressed as\(^ {18} \)

\[
(j_s^z)_{n,s} = \int_{-L/2}^{+L/2} \frac{dy_0}{4\pi} \left( \frac{E_{n,s}(y_0)}{\partial y_0} \right) \langle \tau | \sigma_z | \tau \rangle - \sum_{n',s'} \frac{1}{8\pi} \int_{-L/2}^{+L/2} \frac{dy_0}{\partial y_0} (E_{n,s}(y_0) - E_{n',s'}(y_0)) \times
\]

\[
((\langle \tau | \partial y_0 | \tau' \rangle \langle \tau' | \sigma_z | \tau \rangle - \langle \tau | \sigma_z | \tau' \rangle \langle \tau' | \partial y_0 | \tau \rangle) (10)
\]
where \( \tau' = (n', y_0, s') \) and \( E_{n,s}(y_0) \) are the energy spectrum. Without the spin-orbit coupling the energy eigenstate satisfies \( \langle \tau | \sigma_z | \tau' \rangle = \delta_{n,n'} \delta_{s,s'} \), so that
\[
(j^x)_{n,s} = -seV/4\pi,
\]
which is only determined by the difference of charge voltage at the two edges,
\[
E_{n,s}(L/2) - E_{n,s}(-L/2) = -eV.
\] (12)

Similar to the case for the charge Hall current\(^{28,29}\), the inclusion of impurities and Coulomb interactions in the Hamiltonian does not affect the above result. The spin Hall conductance displays a series of plateaus in the quantum Hall regime,
\[
G_s = (1 - (-1)^n)e/8\pi
\] (13)
corresponding to the quantum Hall conductance, \( G_c = ne^2/h \). In the presence of spin-orbit coupling the states with different spins will be mixed together, and the spin gradually deviates from the 2- to 4-direction and the spin Hall conductance varies with the effective Rashba coupling or magnetic field through tuning the energy gap between the two states especially near the Fermi level. The total spin Hall conductance can be calculated numerically as a function of 1/B for a fixed chemical potential assuming that the voltage drops only near the edges hence the bulk state does not contribute to the total spin Hall current. The charge Hall conductance is found to be quantized as expected, and the spin Hall conductance is of the order of e/4\pi or 10^{-3} - 10^{-4}e/4\pi depending on if odd or even number of Landau levels are occupied. The spin Hall conductance is a function of the effective spin-orbit coupling, which varies with the magnetic field. The resonant peak appears only when the two degenerate bulk Landau levels cross over a special value of chemical potential with decreasing the magnetic field. The spin Hall conductance for a fixed density of charge carriers can be abstrated from the results for a fixed chemical potential if \( L \gg l_b \). The values of the spin Hall conductance are consistent with the bulk theory for the fully filled Landau levels.\(^{18}\)

IV. DISCUSSIONS

In this section we discuss an exact relation between the spin Hall current and spin polarization and the effect of the disorder to the resonant spin Hall effect. We briefly discuss the future experimental serach of the resonant spin Hall effect.

A. Spin Hall current and spin polarization

For a Rashba system described by \( H_R \) at \( B = 0 \), it is now agreed that the spin Hall conductance vanishes when the disorder effect is taken into account.\(^{22}\) This may be understood by calculating the commutator
\[
d\sigma_x/dt = [H_R, \sigma_x]/i\hbar = -(4m\beta^2/\hbar)J_x - g_s\mu_B B \sigma_y/\hbar.
\] (14)

Note that the above relation remains valid in the presence of non-magnetic disorder. In a steady state the expectation value of \( d\sigma_x/dt \) should vanish, and we see immediately from the above relation that the spin Hall current vanishes in 2DEG described by a linear Rashba coupling in the absence of magnetic field.\(^{22}\) This can also be understood from the point of view of spin force as examined by Shen\(^{31}\) and by Jin et al.\(^{32}\). The spin-orbit coupling and the Zeeman exchange coupling induce a spin force, which contains two parts: the transverse force on a moving electron spin, \( f = 4m^2\beta^2 J^z \hat{z} \), with \( J^z \) the spin current tensor component carried by the electron, and a spin force within the plane induced by the Zeeman exchange coupling. The latter is relevant to the spin polarization, \( g = -2\beta g_s m \mu_B B [\sigma_x \hat{x} + \sigma_y \hat{y}] \). If the disorder potential \( V_{\text{disorder}} \) is taken into account, in a steady state, the spin force must reach at balance,
\[
\frac{1}{i\hbar} \left\langle \left[ e^\mathbb{A}/c, H + V_{\text{disorder}} \right] \right\rangle = \left\langle f + g \right\rangle = 0,
\] (15)

where \( \mathbb{A} \) is the spin gauge vector potential caused by the spin-orbit coupling. This result is independent of the non-magnetic disorder and interaction because the spin gauge field commutes with non-magnetic potential \( V_{\text{disorder}} \). In this way we have established a relation between spin current and spin polarization,
\[
\langle J^x \rangle = -\frac{g_s\mu_B}{2m^*} B \langle \sigma_y \rangle
\]
\[
\langle J^y \rangle = -\frac{g_s\mu_B}{2m^*} B \langle \sigma_x \rangle
\] (16)

It becomes clear that the spin Hall current vanishes in the case of \( B = 0 \) or \( g = 0 \), which is consistent with previous results based on the vertex correction calculations or non-equilibrium Green’s function calculations.\(^{22}\) Note that the extrinsic or disorder contributions are implicitly included in discussion of the spin force since it is required to reach the balance or equilibrium for the system.

B. Disorder effect and spin polarization

The effects of disorder in 2DEG with Rashba coupling in a strong magnetic field is not well understood at present. Nevertheless, it seems reasonable to assume that the spin-orbit coupling does not change the effects of disorder qualitatively. The strong magnetic field ensures extended states in the Landau levels when the disorder is not sufficiently strong as evidenced by the experimentally observed quantization of the charge Hall conductance. One may assume that the disorder gives rise to broadening of the Landau level so that the extended...
states in a Landau levels are separated in energy from those in the next one by localized states. Inspection of the spin-orbit coupling shows that Laughlin’s gauge argument still holds, and each Landau level with its extended states completely filled contributes an amount of $e^2/h$ to the charge Hall conductance. Thus one may conclude that the quantum Hall conductance remains intact with the spin-orbit interaction, except at the special degeneracy point.

The disorder effect may be studied by numerical methods. Recently Bao and Shen used the exact solutions of 2DEG in the absence of the spin-orbit coupling and the disorder as the base function to numerically solve the single particle problem by a truncation approximation. They considered a short-range disorder potential $U(x, y) = \sum_i u_i \delta(x-x_i)\delta(y-y_i)$ where $u_i \in (-u/2, u/2)$ and $(x_i, y_i)$ is distributed randomly in finite size of the system. They found that the impurity scattering tends to suppress the spin Hall conductance. However, if the disorder is not so strong the resonance remains intact.

C. Experiment to observe resonant spin Hall effect

As we discussed in this brief review, the theoretical studies predict a resonant spin Hall effect in two-dimensional electron or hole systems with spin-orbit coupling in a strong magnetic field and at low temperatures. It is likely due to the difficulty in detecting the spin current in these extreme conditions that no experiments have been carried out to observe the resonant effect. It is our view that the resonance should be measurable. The most promising way to observe the effect is to detect the spin polarization or the spin susceptibility. As we discussed in Section IV B, the spin polarization is proportional to the spin Hall current. The resonant effect may be best observed in the rapid change of the direction of spin polarization as magnetic field sweeps through the resonant field in the Rashba systems. Since the change of the spin polarization in the system only occurs at the Landau levels near the Fermi energy, the effect will be most pronounced if the resonance occurs at the lower Landau levels.

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