SUSY and the Electroweak Phase Transition

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Abstract: We analyze the effective 3 dimensional theory previously constructed for the MSSM and multi-Higgs models to determine the regions of parameter space in which the electroweak phase transition is sufficiently strong for a $B + L$ asymmetry to survive in the low temperature phase. We find that the inclusion of all supersymmetric scalars and all 1-loop corrections has the effect of enhancing the strength of the phase transition. Without a light stop or extension of the MSSM the phase transition is sufficiently first order only if the lightest Higgs mass $M_h \lesssim 70$ GeV and $\tan\beta \lesssim 1.75$.

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For electroweak baryogenesis to occur it is necessary that the electroweak phase transition be sufficiently strongly first order. Otherwise, sphaleron transitions after the phase transition wash out any baryon asymmetry which may have been produced at the electroweak scale \[1\]. It is now known that this requirement is not satisfied in the minimal Standard Model \[2, 3\]. Thus, it is of interest to investigate extensions of the minimal Standard Model. Many authors have studied the order of the electroweak phase transition in the Minimal Supersymmetric Standard Model (MSSM). Most of these studies rely on a one- and two-loop finite-temperature effective potential analysis of the phase transition \[4, 5, 6, 7, 8\] in which stops were expected to make the most significant contribution from supersymmetric particles. The authors of these studies, in the limit of a large pseudoscalar Higgs mass, \(m_A \to \infty\), have identified a region of parameter space for which the transition is strong enough. This corresponds to low values of \(\tan \beta\), and values of the soft supersymmetry breaking right stop mass, \(m_{\tilde{U}}^2\), which are small or even negative. In reference \[6\] the analysis was extended for the full range of allowed values of \(m_A\). It was found that larger values of \(m_A\) are favored.

A different approach consists of separating the perturbative and non-perturbative aspects of the phase transition. This is performed through the perturbative construction of effective three dimensional theories, and a subsequent lattice analysis of the reduced theory \[2, 3, 9, 10\]. For the case in which the reduced theory contains a single light Higgs field, characterized by a Higgs self-coupling, \(\tilde{\lambda}_3\), and an effective 3D gauge coupling, \(g_3\), the condition for a sufficiently strong first order phase transition becomes \[2\]

\[x_c = \frac{\tilde{\lambda}_3}{g_3^2} \lesssim 0.04,\]  

(1)

where the quantities \(\tilde{\lambda}_3\) and \(g_3\) are functions of the various parameters appearing in the original 4D theory.

An analysis of the parameter space for the reduced theory of the Standard
Model was performed in [2, 3]. It indicated that for no value of the Higgs mass is electroweak baryogenesis possible. This demonstrates that a purely perturbative analysis is inadequate, since with that method the electroweak phase transition was found to be sufficiently strong for small values of the Higgs mass. In [11] a 3D theory for the MSSM was constructed, including Standard Model particles and additional corrections arising from gauginos, higgsinos and all squarks and sleptons. Here we use these results to explore the MSSM parameter space in order to determine the regions for which electroweak baryogenesis may occur\(^2\). This relies on the relation between the running parameters in the original 4D theory and physical parameters; this is given in [12]. A simplified version of the present analysis was performed by others [13, 14]. However only the contribution from gauge bosons, higgses and third generation quarks and squarks to the 3D reduction were included, and one-loop corrections to 4D parameters were not fully incorporated. In our work all one-loop corrections as well as contributions from all SUSY particles have been considered. This allows us to investigate the effect of the full complement of supersymmetric particles, in addition to third generation squarks, on the strength of the phase transition. We find that these effects are important and should not be neglected. We also clarify the discrepancy between the results of references [13] and [14]. In reference [14] the results agreed basically with those found in the perturbative effective potential analysis. The most favorable region of parameter space was found to be \(m_h \leq m_W\) (low \(\tan \beta\)), small stop mixing, \(m_{U_3} \leq 50\) GeV and \(m_A \geq 200\) GeV. In addition to this region, reference [13] found another region of parameter space in which arbitrary values of \(\tan \beta\) and a range of values for the pseudoscalar Higgs mass, \(40 \leq m_A \leq 80\) GeV, give a sufficiently strong phase transition. We comment on the latter region below.

The ratio \(\lambda_3/g_3^2\) appearing in equation (1) depends on the param-

\(^2\)Throughout, we work in the \(\sin^2 \theta_W = 0\) approximation, which was found in ref. [2] to be adequate for the MSM.
eters in the 4D theory \((x_c = x(M_A, m_o, \mu, m_{\tilde{g}}, m_{\tilde{g}}, A, \tan \beta, T_c))\), as well as

the Standard Model gauge couplings. \(A\) is the scalar trilinear soft SUSY

breaking parameter, taken to be universal; \(\mu\) is the supersymmetric higgsino

mass parameter; \(m_o, m_{\tilde{q}}, m_{\tilde{g}}\) denote the common squark/slepton mass at the

SUSY breaking scale \(\tilde{s}\), the \(SU(2)\) gaugino, and gluino mass, respectively.

\(M_A\) is the physical pole mass of the pseudoscalar Higgs, and \(\tan \beta\) is the ratio of the vacuum expectation values of the Higgs fields in the renormalized zero

temperature theory. \(x\) also depends indirectly on the scale \(M_{SUSY}\), the scale at which the SUSY boundary conditions on the quartic Higgs couplings appearing in the Higgs potential are imposed \[15\] and at which mass parameters for squarks/sleptons are specified.

The critical temperature, \(T_c\), is defined to be the temperature at which there is a direction in field space at the origin of the Higgs potential for which the transition to the minimum of the potential in the broken phase can occur classically \[4\]. In 3D lattice calculations \[2, 10\] the critical temperature is taken to be the temperature of phase coexistence. These two values of temperature are generally close but not identical. The actual temperature at which the phase transition occurs lies between these two values. We will remark below on the circumstances under which there can be a significant difference arising from this distinction.

Throughout our analysis we will concentrate on the regions of parameter space which describe an effective theory in which there is a single light scalar and thus the bound given by equation (1) is valid. However, we mention that another possibility is a scenario in which two scalars, e.g. one Higgs and the right stop, are both nearly massless at \(T_c\) \[7\]. A lattice calculation for this extended 3D model is required before that scenario can be investigated with

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3We consider non-universal squark masses where indicated.

4We have checked that the difference in the critical temperature from the diagonalization of the Higgs mass matrix, equation (10) in \[11\] and from equation (7.9) in \[14\] is extremely small \((\leq 0.1 \text{ GeV})\) and for our purposes negligible.

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the present approach.

As is well known, the MSSM Higgs sector can be parametrized in terms of two quantities: $\tan\beta$ and the pole mass of the pseudoscalar Higgs boson, $M_A$. These are the most important parameters in determining the strength of the phase transition. We consider $M_A$ values between 40-300 GeV in order to be compatible with experimental limits and ensure the validity of the high-temperature expansion. In the figures we show results for $\tan\beta$ between 1.25 and 13.3. For larger and smaller values the conclusions are essentially the same as for the extrema of the range.

In general, the masses of all particles are taken such that the high temperature expansion is valid. The experimental constraints we impose on the masses are: for stop masses $m_{\tilde{t}_2} \gtrsim 50$ GeV, $m_{\tilde{t}_1} \gtrsim m_t$, for first and second generation squarks $m_{\tilde{q}_i} \gtrsim 200$ GeV, sleptons $m_{\tilde{l}} \gtrsim 50$ GeV, the gluino mass either \( \lesssim 1 \) GeV or \( \gtrsim 150 \) GeV \cite{16, 17}. In addition, the value of the left soft supersymmetry breaking stop mass, $m_{Q_3}$, must be such that the contribution from stops and sbottoms to the $\rho$ parameter is not too large \cite{5}.

Figure 1 shows the value of $x_c$, for the case of no squark mixing, as a function of the pseudoscalar Higgs mass for values of $\tan\beta$ ranging from 1.25 to 13.3. We have fixed the other parameters to be $m_o = 50$ GeV, $m_{\tilde{g}} = 50$ GeV, $m_{\tilde{g}} = \frac{g_s}{4\pi} m_{\tilde{g}}$, $M_{\text{weak}} = m_t$, $M_{\text{SUSY}} = 10^{12}$ GeV \cite{7}. For large values of $\tan\beta$, there is no value of $M_A$ for which $x_c$ fulfills the condition given by equation (1), and $x_c$ varies very little as a function of $M_A$. However, for low

\begin{footnote}{In the dimensional reduction procedure, the explicit dependence on all Yukawa couplings was kept. For the numerical results presented here only the top Yukawa coupling is kept. We have explicitly checked that even for large values of $\tan\beta$ the bottom Yukawa coupling can be neglected.}

\begin{footnote}{The adequate suppression of non-renormalizable terms must also be verified.}

\begin{footnote}{In our approximation the masses of sleptons and of the first and second generation squarks are fixed by $m_o$, $m_{\tilde{g}}$, $m_{\tilde{g}}$, through the renormalization group running; they are constant as we vary $\tan\beta$ and $M_A$. However, due to their dependence on the renormalization group running of the top Yukawa coupling, the left and right stop masses change as we move on the curves plotted in figure 1.}

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values of $\tan\beta$ and large enough values of $M_A$, $x_c$ can be small enough for the phase transition to be sufficiently first order. The strong dependence of $x_c$ on the value of the pseudoscalar Higgs mass, for low $\tan\beta$, arises basically through the dependence of the quantity $\tilde{\lambda}_3$ in equation (1) on the mixing angle $\theta$ (see equation (18) in [11]). It is easy to see to lowest order the same dependence on $M_A$ arising in finite temperature effective potential analysis [9]. This qualitative dependence of the strength of the phase transition on $\tan\beta$ and $M_A$ was observed in [5, 6, 8, 14]. Varying the parameters $A$, $\mu$, $m_{\tilde{g}}$, $m_{\tilde{t}_2}$, $m_o$, $m_t$ either increases $x_c$ slightly or has a negligible effect.

We have also compared the results of our general analysis to those obtained with the simplifying approximations used in [13, 14], in which some supersymmetric particles are neglected. In all cases we have kept the full one-loop corrections to the 3D couplings, in contrast to [13, 14]. Figure 2 shows the variation of $x_c$ with $M_A$ for three different cases and for two values of $\tan\beta$. The solid line corresponds to our general analysis, as given above. The dashed line is the result when only third generation squarks are included; the gluino and electroweak gaugino thermal screening contribution to the 3D masses of the squarks is excluded. The dotted line corresponds to the case in which we include all squarks and sleptons, but ignore all gaugino contributions to the three dimensional theory. Although the effect of the right stop on the strength of the phase transition is greater than that of any other sfermion, the ensemble of sfermions neglected in [13, 14] significantly strengthens the phase transition. As expected from the work of [6, 7, 14], we find that the reduction of the right stop soft supersymmetric breaking mass decreases $x_c$. However, the decrease is less important than it appears when only the contribution from third generation squarks [14] is included.

We have also compared the two cases in which only third generation squarks were included with and without thermal screening arising from the gluino and gaugino. The differences in the values of $x_c$ for this case are negligible.

For given squark masses at the SUSY scale, the running masses at low
energies are reduced as the gluino mass decreases. As a result of these lower masses the sfermions’ favorable impact on the phase transition is increased. Thus light gluinos can be helpful in providing a sufficiently strong EW phase transition, and the low values of $\tan\beta$ required for the phase transition lead to chargino masses in an acceptable range in the light gaugino scenario \[18\].

An important point, which has been overlooked in the previous literature, is that for some regions of parameter space it may be incorrect to conclude from the above analysis that the phase transition is not sufficiently strongly first order. As noted previously the actual transition temperature is somewhat higher than $T_c$ as defined above. For some values of $M_A$ and $\tan\beta$, $x_c$ depends strongly on the temperature. This happens when the mixing angle $\theta$, which diagonalizes the 3D Higgs mass matrix at finite temperature, varies rapidly for temperatures near the critical temperature. A rapid variation of the mixing angle occurs when the temperature is such that the diagonal elements of the 3D Higgs mass matrix become nearly degenerate. If the critical temperature for the phase transition is close to the value of the temperature where this rapid variation occurs, then the value of $\theta$ and consequently of $x_c$ are very sensitive to the transition temperature. Note that in general our procedure of integrating out the heavier Higgs is not compromised when this phenomenon occurs because the eigenvalues remain well-separated. Rather, our inability to analyze this region of parameter space arises from our inability (with present techniques) to obtain a sufficiently accurate determination of the phase transition temperature.

The value of the temperature at which this rapid variation occurs depends strongly on the value of the pseudoscalar Higgs mass. As $M_A$ increases this temperature also increases; for $M_A \gtrsim 100$ GeV it is well-separated from the phase transition temperature. Moreover, the extent of the variation of $x_c$ is less for larger $\tan\beta$. Figure 3 shows the dependence of $\theta$ on the temperature for $M_A = 40$ GeV; the solid line corresponds to $\tan\beta = 1.25$ and the dashed line to $\tan\beta = 13.3$, keeping all other parameters fixed. Figure 4 is the same
for $M_A = 300$ GeV. Figure 5 shows $x_c$ as a function of temperature close to $T_c$, for $M_A = 40$ GeV. A 5 GeV variation in the temperature around $T_c$ induces, for $M_A = 40$ GeV, a change in the value of $x_c$, $\Delta x_c \sim 0.13$, while for $M_A = 300$ GeV, $\Delta x_c \sim 0.005$. Thus the possibility of large uncertainty in the mixing angle is relevant only for low values of $tan\beta$ and $M_A$. Since $M_A \lesssim 100$ GeV is already ruled out experimentally [19] for the MSSM in the $tan\beta$ region of interest, this possibility cannot enlarge the viable region of parameter space in the MSSM and we do not pursue it further. However we note that this phenomenon may play a role in the discrepancy between the conclusions of [13, 14] for $M_A \lesssim 100$ GeV.

We now turn to implications of these constraints for the mass of the lightest Higgs. The light curves in figure 6 indicate contours of constant $M_h$ in the $tan\beta - M_A$ plane, using the results given in [12] to relate the running parameters of the MSSM analysis to the physical parameters and taking $m_{\tilde{t}_2} \sim 180$ GeV, $m_{\tilde{t}_1} \sim 320$ GeV. Due to approximations in the RG analysis and our ignorance with respect to the mass of the stop, one should attach a few-GeV uncertainty to these curves. The region of $tan\beta - M_A$ which preserves a baryon asymmetry generated at the weak scale is below and to the right of the thick line, which corresponds to $x_c = 0.04$. We conclude that unless $M_h \lesssim 70$, electroweak baryogenesis is not viable in the MSSM.

Given that experimental Higgs limits already nearly exclude such a light Higgs, we briefly mention three alternate extensions of the SM which might allow electroweak baryogenesis. In order to increase the strength of the phase transition one must either increase the $\phi^3 T$ term in $V_{\text{eff}}$, or decrease the coefficient of the $\phi^4$ term, or both. The first possibility is employed in theories which are fine-tuned such that one or more scalars in addition to the usual Higgs scalar are nearly massless at the phase transition temperature. Terms in $V_{\text{eff}}$ cubic in $\phi$ arise from bosonic mass-squareds which are proportional

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8This choice of parameters maximizes the region in $tan\beta - M_A$ giving $x_c \leq 0.04$. 
to $\phi^2$. Such contributions are enhanced if there is a precise cancelation of
the thermal contributions to the mass, e.g., from a negative mass-squared
at the SUSY scale [7]. Other scalars such as additional Higgs or sneutrinos
could in principle be employed to serve a similar purpose, although the stop
is particularly natural and has 3 color degrees of freedom as well. Non-
perturbative effects in such scenarios cannot be analyzed without further
lattice calculations. For the light stop scenario, the 3D theory which must
be analyzed on the lattice is considerably more complicated than the one
relevant to the SM [3] and the theories analysed here. The relevant 3D
lattice calculation for the light stop scenario must include SU(3) as well as
SU(2) gauge interactions, and one of the scalars couples to both SU(2) and
SU(3) gauge bosons, as well as to the other scalar. See ref. [14, 11] for the
full 3D reduction with two light scalars.

The other strategy to enhance the phase transition is to decrease the
coefficient of the $\phi^4$ term, i.e., $\tilde{\lambda}_3$ in the 3D theory. This may be possible
either in non-SUSY multi-Higgs doublet models or in the NMSSM (minimal
SUSY augmented with a gauge singlet Higgs). Non-SUSY multi-Higgs theo-
ries have some advantage in this regard, since the Higgs potential of the 4D
theory is not fixed by gauge couplings at the SUSY scale. It is of course also
constrained by non-observation of a Higgs particle and the requirement that
the broken-symmetry vacuum is the minimum of the T=0 theory. A dis-
advantage of non-SUSY theories is the absence of sfermions, which enhance
the strength of the transition, as we saw above. Finally, the NMSSM has
sfermions and also more freedom in the Higgs sector, so the lightest Higgs
in the 4D theory may be acceptably heavy even with a small $\tan\beta$, without
requiring a large $M_A$ [20, 21]. For this theory in particular the $\theta$ dependence
noted above may prove important in the analysis.

In conclusion, we have employed existing lattice calculations to analyze
the electroweak phase transition in the MSSM, including non-perturbative
as well as perturbative thermal effects. We include all 1-loop corrections
and integrate out all gauginos and sfermions. Although we find qualitative agreement with the results of refs. [3, 14], we find that inclusion of all sfermions and D-term couplings is important quantitatively, because of their large multiplicity. This enhances the strength of the phase transition in the relevant regime of parameters. We find that the MSSM provides sufficient suppression of sphaleron transitions in the broken phase only for values of \( \tan \beta \lesssim 1.75 \), unless the right stop soft-SUSY breaking mass is less than 50 GeV. In the latter case coefficients of non-renormalizable terms become large, signaling the onset of the breakdown of our analysis when there are two light scalars at the phase transition. In this regime our method does not apply, although purely perturbative analysis leads one to expect that the strength of the phase transition is enhanced [3, 4]. The strength of the phase transition increases as the mass of the pseudoscalar Higgs increases. \( M_A \) can be as low as 100 GeV, for \( \tan \beta \sim 1.25 \), and still give \( x_c \lesssim 0.04 \), even assuming universal soft supersymmetry breaking masses at the SUSY breaking scale (i.e., without the light stop scenario). The region of parameter space potentially supporting electroweak baryogenesis requires the lightest physical Higgs mass \( M_h \) to be \( \lesssim 70 \) GeV. This is very close to being experimentally excluded. We commented on possible alternatives to the MSSM in which electroweak baryogenesis could be compatible with experimental constraints on the Higgs mass.

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Figure 1: Plot of $x_c$ vs. $M_A$ for several different values of $\tan\beta$. The solid line corresponds to $\tan\beta = 13.3$, the dashed line to $\tan\beta = 1.75$, the dashed-dot line to $\tan\beta = 1.5$, and the dotted line to $\tan\beta = 1.25$. 
Figure 2: Plot of $x_c$ vs. $M_A$ for $\tan\beta = 1.5$ and 1.75 including all supersymmetric particles (solid line), only third generation squarks (dashed lines) and all squarks and sleptons but not gaugino corrections (dotted line).
Figure 3: Plot of $\theta$ vs. $T$ for $M_A = 40$ GeV. The solid line corresponds to $\tan\beta = 1.25$, $T_c = 63.3$ GeV, the dashed line is for $\tan\beta = 13.3$, $T_c = 75.6$ GeV.
Figure 4: Plot of $\theta$ vs. $T$ for $M_A = 300$ GeV, $T_c = 60, 75.6$ GeV. The solid line corresponds to $\tan \beta = 1.25$, the dashed line to $\tan \beta = 13.3$. 
Figure 5: Plot of $x_c$ vs. $T$ for $tan\beta = 1.25$, $M_A = 40$ GeV, $T_c = 60.8$ GeV.
Figure 6: Contours of constant $M_h = 70, 60, 45$ from top to bottom in the $\tan\beta$ vs $M_A$ plane. The thick line gives the constraint for a sufficiently strong first order phase transition, $x_c = 0.04$. 