A Two-period Game Theory Model in Subsidizing the Electric Vehicle Supply Chain with Used Electric Vehicle Batteries Remanufacturing Process

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Abstract. Electric vehicle batteries are usually retired when the performance drops to about 70% of factory performance. At this point, they can be remanufactured into raw materials for the production of new batteries. This paper studies a two-period vehicle supply chain consisting of government, an electric/gasoline vehicle manufacturer, a EV battery manufacturer and consumers with and without retired batteries’ remanufacturing process. The purpose is to understand government subsidy policy in these two different periods. By adopting Stackelberg game theory and Nash equilibrium, a mathematical model was developed. The study gives the optimal subsidy with and without battery remanufacturing process in different two periods. We also consider the quality and recycling rates of retired batteries, as well as the impact of raw material costs for new batteries and government environmental spending on subsidies and commodity prices such as vehicles and batteries. Meanwhile, the results show that government subsidy to EV manufacturers can be reduced when retired battery remanufacturing process exists. Increasing in retired batteries’ quality, recycling rates, and the cost of raw materials for new batteries will lead to a reduction in government subsidies; but a rising of government spending on environmental protection will lead to the need for more subsidies to electric vehicle manufacturers.

1. Introduction
Currently, electric vehicles (EVs) have become one of the future directions of the automotive industry and an excellent example of clean energy use. According to International Energy Agency [1], in 2019, the global electric car stock reached 7.2 million units. Over the five-year period (2014-19), the annual average increase was 60%. Government subsidies for EVs have played a huge role in the expansion of the EV market.
On the other aspect, with the development of EVs, the battery, one of the most important parts in an electric vehicle, is developing rapidly. Unlike gasoline vehicles (GVs), the batteries installed in EVs must be scrapped from existing EVs after a certain amount of time using. These decommissioned batteries contain a lot of resources, such as phosphorus, iron, lithium and so on. If these batteries are discarded directly, it is not conducive to environmental protection on the one hand, and not conducive to resource reuse and sustainable development on the other. Usually, they will go into the remanufacturing process, be disassembled, and the important raw materials will be extracted in order to produce new electric car batteries. In this context, with the proliferation of EVs, the increase in the number of retired batteries and their remanufacturing, it is a matter of debate whether the government's subsidy policy for EVs needs to be adjusted or whether it is possible for the government to stop subsidizing them.
Accordingly, this research article will develop a vehicle supply chain of GV/EV manufacturer, battery manufacturer and vehicle consumers, and investigate the government subsidy policy for EVs
with and without battery remanufacturing process in different periods. In detail, this research attempts to answer the following research questions: (1) What is the government's policy on subsidies for EV manufacturers at different periods in the development of EVs? (2) What about the difference in the government's subsidy policy when there is a battery remanufacturing process and when there is no battery remanufacturing process? (3) How do parameters such as the price of raw materials for batteries, the quality of scrapped batteries, and the cost of environmental protection affect government subsidies, and the price of vehicles?

The rest of the paper is organized as follows. The next section reviews some relevant papers. Section 3 describes the model and derives the optimal subsidy, vehicle prices and battery price. Section 4 conducts a numerical experiment, and some analysis of the model are carried out. Section 5 concludes this research and discusses the limitations of this research.

2. Literature review

Government subsidies, as a common means of regulating the economy, have a relatively important significance in promoting industrial development. The choice of government subsidies has always been a research hotspot, and in the face of different industries and different market structures, the optimal way of government subsidies also differs. Toshimitsu [2] constructs the Cournot duopoly model of product differentiation and investigates the optimal government subsidy policy when considering the environmental and welfare effects. Guo et al. [3] examine the impact of two government subsidy policies on social welfare and the profits of supply chain members, using a supply chain system consisting of three members: supplier, manufacturer, and government. Hattori [4] constructed a model of upstream monopolistic innovators developing cleaner production technologies and licensing them to downstream polluting firms, and discussed the optimal environmental policies of the government in R&D subsidies, adoption subsidies, and emission taxes. Chen et al. [5] studied the impact on the level of innovation and the distribution of innovation costs in a supply chain consisting of a single manufacturer and a single retailer when the government uses R&D and product subsidies, respectively.

In the field of research related to electric vehicles and batteries, more research is focused on the technical aspects, such as Tong et al. [6] and Patten et al. [7], etc. Some articles discuss the secondary use of batteries [8-10]. In the EV industry and government incentive design aspect, current research mainly focuses on the optimal strategy of government subsidies and its impact on the vehicle industry. For example, Gnann et al. used alternative vehicle diffusion and automotive infrastructure models to make analysis about market evolution of EVs in Germany [11]. Huang et al. analysed the subsidy incentive policy in a duopoly market comprising a GV supply chain and plug-in hybrid EV supply chain. A Nash equilibrium of the wholesale price in the above two supply chains is achieved when the government provides subsidies to promote the EV market based on an information condition [12]. Furthermore, Luo et al. focused on the EV supply chain model where the customer, manufacturer, and government were all included and the best EV market share rate and subsidy ceiling were figured out through a perfect information cooperative game [13]. Then, Gu et al. designed a joint GV/EV supply chain and analyses the optimal government subsidy policy for EVs under conditions of incomplete information [14]. Moreover, several studies have looked at issues related to battery recycling for electric vehicles. For example, Harper reviewed some policies and approaches to recycling and reuse lithium-ion batteries from electric vehicles [15]. Pugliaro offered an updated global perspective and provided a circular economy insight on lithium-ion battery reuse and recycling [16]. Gu et al. studied a three-period electric vehicle battery recycle and reuse closed-loop supply chain consisting of a battery manufacturer and a remanufacturer and found the optimal price about the used EV batteries [8]. A search of relevant databases reveals that few studies discuss the issue of subsidies for the recycling of used EV batteries. Although Li et al., discussed a unique deposit-refund scheme implemented in Shenzhen, China to levy recycling fees on EV sales and subsidize the retailers who collect used EV batteries [17], so far, there has been no discussion of how government subsidy policies for EVs should be implemented when there are EV battery recycling and remanufacturing processes.

Hence, this study aims to fill the research gap about government incentive design in EV research area especially when there is a recycle and remanufacture process for decommissioned batteries. The
objective of this paper is to design a two-period vehicle and battery supply chain model and discuss the optimal government subsidy and make recommendations accordingly.

3. Model Description
In this paper, we consider a two-period model about government subsidy to the EV supply chain. The main structure of this model is shown in Figure 1 below.

![Figure 1. Model structure.](image)

In period 1, the government subsidizes EVs sold to consumers to electric vehicle manufacturers in order to increase the market share of EVs. In period 2, a portion of the batteries reach the end of their life cycle on the EV, and these batteries are acquired, dismantled, and remanufactured to the new batteries by the battery manufacturers. In this context, the question of how government subsidies for EVs should be adjusted to maximize the profit of the entire supply chain in the second period becomes an issue for discussion. So, this research will discuss the subsidy in different two periods.

The schematic diagram for the model can be seen in Figure 1. The symbols in the model and their meanings are shown in Table 1 below.

| Symbol | Meaning |
|--------|---------|
| Subscript $EV$ | Electric Vehicle; |
| Subscript $GV$ | Gasoline Vehicle; |
| Subscript $GVNT$ | Government; |
| $\theta_{GV}$, $\theta_{EV}$ | Green preference coefficient for the GV/EV customer |
| $C_{GV}$, $C_{EV}$ | Use cost of GV/EV |
| $\pi_{P}$ | Utility of using the vehicle |
| $k$ | Consumers' willingness to buy cars, $k \in [0,1]$; |
According to Morwitz [18] and Gu et al. [14], we assume that the customers buy vehicles based on their desire to buy a car, their greenness awareness for GVs and EVs, the utility of using a car, the total using cost over the life of the car, and the purchase price of the car. Specifically, if a consumer buys a GV, the utility function is as follows

\[ U_{GVi} = (\theta_{GVi} \pi_{GV} - c_{GV})k - p_{GVi} \]  

(1)

If a consumer buys an EV, the utility function will be

\[ U_{EVi} = (\theta_{EVi} \pi_{EV} - c_{EV})k - (p_{EVi} + p_{btyi}) \]  

(2)

where \( i = 1,2 \) which means period 1 or 2. We assume that consumers who choose an EV have a greater green preference, i.e., \( \theta_{EV} > \theta_{GV} \). According to [14], we also assume that the cost of using an electric vehicle is lower than the cost of using a fuel-efficient vehicle, i.e., \( c_{EV} < c_{GV} \). Thus, we easily get \( (\theta_{EV} \pi_{EV} - c_{EV}) > (\theta_{GV} \pi_{GV} - c_{GV}) \). To facilitate the calculation, we make \( A_{GV} = (\theta_{GV} \pi_{GV} - c_{GV}) \) and \( A_{EV} = (\theta_{EV} \pi_{EV} - c_{EV}) \). In addition, we also observe that the average price of an is greater than that of a GV, and to facilitate the analysis, we decompose the whole price of an EV into two components, the battery price and the car price. So, we have \( (p_{EVi} + p_{btyi}) > p_{GVi} \). The relationship between \( U_{GVi} \) and \( U_{EVi} \) is shown in Figure 2 below.
The consumer will only choose the car that gives him more utility, and we can derive from (1) and (2) that the probability of the consumer choosing a GV is

$$P_{GV} = \frac{p_{GV} + p_{EV} - p_{CV}}{A_{GV} - A_{EV}}$$

the probability of choosing an GV is

$$P_{EV} = 1 - \frac{p_{GV} + p_{EV} - p_{CV}}{A_{GV} - A_{EV}}$$

And we have the total GV consumer’s profit is

$$\pi_{GV} = \int \left( \frac{p_{GV} + p_{EV} - p_{CV}}{A_{GV} - A_{EV}} \right) A_{GV} k - p_{GV}$$

The total EV consumer’s profit is

$$\pi_{EV} = \int \left( \frac{p_{EV} + p_{CV} - p_{EV}}{A_{EV} - A_{CV}} \right) A_{EV} k - (p_{EV} + p_{bty})$$

### 3.1. Period 1

The decision-making order is described in Figure 3 below. The government first decides the subsidy to the EV manufacturer. Then, EV manufacturer and GV manufacturer will decide the vehicle selling price and produce quantity. After that, battery manufacturer will decide the battery price. We have the quantity of GV needed in period 1 is

$$q_{GV1} = M_{EV} P_{GV1}$$

and quantity of EV needed in this period is

$$q_{EV1} = M_{EV} P_{EV1}$$

![Figure 3](image-url)

**Figure 3.** Decision making in period 1.

Based on backward induction and Stackelberg game model, we first consider the profit for the battery manufacturer. With equation (4), the profit for the battery manufacturer is the profit for each battery multiplied by the demand for batteries (also $q_{EV1}$):

$$\pi_{bty1} = q_{EV1} \left( p_{bty1} - c_{ntr} \right)$$

$$= \frac{M_{EV}}{A_{EV} - A_{CV}} p_{bty1}^2 + \frac{M_{EV}(A_{EV} - A_{CV} + c_{ntr} - p_{EV1} + p_{CV1})}{A_{EV} - A_{CV}} p_{bty1} + \frac{c_{ntr} M_{EV}(A_{EV} - A_{CV} + p_{EV1} - p_{GV1})}{A_{EV} - A_{CV}}$$

Based on previous assumptions, we have $A_{GV} - A_{EV} < 0$. Along with the quadratic function about $p_{bty1}$, when condition $\frac{\partial \pi_{bty1}}{\partial p_{bty1}} = 0$ is satisfied, $\pi_{bty1}$ will obtain the maximum value. Therefore, we have the optimal battery price

$$p_{bty1} = \frac{1}{2} \left( A_{EV} - A_{CV} + c_{ntr} - p_{EV1} + p_{GV1} \right)$$

Now we discuss GV and EV manufacturer. The profit of the fuel car manufacturer is the demand for GVs multiplied by the profit per GV:
\[ \pi_{GV1} = q_{GV1}(p_{GV1} - c_{GV1}) = \frac{1}{2} \left( \frac{1}{A_{GV} - A_{EV}} - \frac{2}{A_{GV}} \right) M_y p_{EV1}^2 \]

\[ = + \frac{M_y (A_{EV}(A_{EV}+2c_{GV1})+A_{GV}(c_{GV1}+c_{EV1})-c_{EV1})}{2(A_{EV}-A_{GV})} M_y p_{EV1}^2 \]

\[ = \frac{c_{GV1} M_y (A_{EV} - A_{GV} + c_{EV1})}{2(A_{EV} - A_{GV})} \]

The EV manufacturer's profit is the demand for EVs multiplied by the sum of the profit per EV and the subsidy per EV given by the government:

\[ \pi_{EV1} = q_{EV1}(p_{EV1} - c_{EV1} + S_{ev1}) = \frac{2(A_{EV} - A_{GV})}{M_y} \]

\[ + \frac{M_y (A_{EV} - A_{GV} + c_{EV1}) + A_{EV} (c_{EV1} - c_{GV1} - c_{EV1} + 2S_{ev1})}{2(A_{EV} - A_{GV})} p_{EV1} \]

\[ = 0 \]

It is easy to find that \( \frac{1}{A_{GV} - A_{EV}} - \frac{2}{A_{GV}} < 0 \) and \( A_{EV} - A_{GV} < 0 \). With Nash Equilibrium, \( \pi_{GV1} \) and \( \pi_{EV1} \) will get the optimal value when both \( \frac{\partial \pi_{GV1}}{\partial p_{GV1}} = 0 \) and \( \frac{\partial \pi_{EV1}}{\partial p_{EV1}} = 0 \) hold at the same time:

\[ \frac{\partial \pi_{GV1}}{\partial p_{GV1}} = M_y (A_{EV} (A_{EV} + 2c_{GV1} - p_{GV1}) + A_{EV} (c_{EV1} + c_{EV1} - c_{GV1} - p_{EV1} + 2p_{GV1})) = 0 \]

\[ \frac{\partial \pi_{EV1}}{\partial p_{EV1}} = M_y (A_{EV} - A_{EV} + c_{EV1} - c_{EV1} - 2p_{EV1} + p_{EV1} - c_{EV1}) = 0 \]

We are able to get

\[ p_{GV1}^* = \frac{A_{EV} (3A_{GV} + 4c_{GV1}) + A_{EV} (c_{GV1} - 3A_{GV} + 2c_{EV1} + c_{EV1})}{8A_{EV} - 5A_{GV}} \]

\[ + \frac{M_y (A_{EV} (A_{EV} - A_{EV} + c_{EV1} - c_{EV1} + 2S_{ev1}) + A_{EV} (2c_{GV1} - 4c_{EV1} - 4c_{EV1} + 4S_{ev1}))}{8A_{EV} - 5A_{GV}} \]

Substituting the optimal battery price \( p_{EV1}^* \), GV price \( p_{GV1}^* \) and EV price \( p_{EV1}^* \) into the demand of GV and EV, we can update \( q_{EV1} \) and \( q_{GV1} \) to the following equation.

\[ q_{EV1} = \frac{(2A_{EV} - A_{EV}) M_y (A_{EV} (3A_{GV} - 4c_{GV1}) + A_{EV} (c_{GV1} - 3A_{GV} + 3c_{EV1} + c_{EV1}))}{2(8A_{EV} - 5A_{GV}) (A_{EV} - A_{GV}) A_{GV}} \]

\[ q_{GV1} = \frac{(2A_{EV} - A_{EV}) M_y (A_{EV} (3A_{GV} - 4c_{GV1}) + A_{EV} (c_{GV1} - 3A_{GV} + 3c_{EV1} + c_{EV1}))}{2(8A_{EV} - 5A_{GV}) (A_{EV} - A_{GV}) A_{GV}} \]

Now, we need to solve for the optimal amount of government subsidies paid to the EV manufacturer. The government's main concerns are the profits of battery manufacturer, profits of EV and GV manufacturers, the total utility of consumers, the government's own spending on environmental protection for GV, and the spending on subsidies for EVs, that is

\[ \pi_{GVNT} = \pi_{EV1} + \pi_{GV1} + \pi_{EV1} + U_{EV1} + U_{GV1} = q_{GV1} C_{environ} - q_{EV1} S_{ev1} \]

\[ = K_1 S_{ev1} + K_2 S_{ev1} + K_3 \]

where

\[ K_1 = \frac{(16 A^2_{EV} - 20 A_{EV} A_{GV} + 5 A^2_{EV}) (2 M_y - 1) S_{ev1}}{4 (8 A_{EV} - 5 A_{GV}) (2 A_{EV} - A_{GV}) A_{EV} A_{GV}} \]

\[ K_2 = \frac{(16 A^2_{EV} (1 + 2 M_y) - 14 A_{EV} A_{GV} + 9 A_{EV} + 14 C_{environ} + 12 C_{environ} - 16 C_{environ} + 12 C_{environ})}{4 (8 A_{EV} - 5 A_{GV}) (2 A_{EV} - A_{GV}) A_{EV} A_{GV}} \]
Otherwise, the optimal government subsidy will be the total revenue from the sale of batteries minus the cost of producing the batteries, minus the cost of remanufacturing:

\[ \pi_{\text{GVT}} = \frac{16A_{\text{EV}}(1+6M_T)+5A_{\text{GV}}(2c_{\text{GV}})-3c_{\text{GV}}+c_{\text{envir}}-2(3c_{\text{GV}}+6c_{\text{GV}} (c_{\text{envir}} + c_{\text{recycled}}) + s_{\text{attn}}(2c_{\text{envir}} + 5c_{\text{attn}})M_T - 3A_{\text{GV}}(1+2M_T) + 2c_{\text{EV}}(c_{\text{envir}} + c_{\text{attn}} - 2(3c_{\text{GV}},M_T - s_{\text{attn}}(2c_{\text{envir}} + 5c_{\text{attn}})M_T) + 2A_{\text{EV}}(3c_{\text{GV}} - 3c_{\text{EV}} - 5c_{\text{attn}}(3c_{\text{envir}} + 9c_{\text{environ}})M_T + 4(18c_{\text{EV}}) + 1c_{\text{EV}} - 2(3c_{\text{GV}} + 19c_{\text{EV}} + 9c_{\text{environ}})M_T) + 4A_{\text{EV}}(c_{\text{envir}} + 2(3c_{\text{GV}} + 2c_{\text{environ}})(c_{\text{envir}} + 2c_{\text{environ}}) + 4A_{\text{EV}}(-2c_{\text{envir}} - 1c_{\text{EV}} - 2(3c_{\text{GV}} + 4c_{\text{environ}})(c_{\text{envir}} + 2(3c_{\text{GV}} + 2c_{\text{environ}})M_T)
\]

\[ K_3 = \frac{8l_{\text{BAE}}A_{\text{EV}}(A_{\text{EV}} - A_{\text{GV}})A_{\text{GV}}}{(18)} \]

We have \( K_3 < 0 \) as well. So, when \( \frac{\partial \pi_{\text{GVT}}}{\partial S_{\text{ev1}}} = 0 \), we can find the optimal subsidy \( S_{\text{ev1}}^* \) as follows:

\[ S_{\text{ev1}}^* = \frac{16A_{\text{EV}}(1+2M_T) + 5A_{\text{GV}}(2c_{\text{GV}} - 3c_{\text{EV}} - 10c_{\text{attn}} + 3c_{\text{envir}})M_T + 2A_{\text{EV}}(3c_{\text{GV}} - 3c_{\text{EV}} - 5c_{\text{attn}}(3c_{\text{envir}} + 9c_{\text{environ}})M_T + 4(18c_{\text{EV}}) + 1c_{\text{EV}} - 2(3c_{\text{GV}} + 19c_{\text{EV}} + 9c_{\text{environ}})M_T) + 4A_{\text{EV}}(c_{\text{envir}} + 2(3c_{\text{GV}} + 2c_{\text{environ}})(c_{\text{envir}} + 2c_{\text{environ}}) + 4A_{\text{EV}}(-2c_{\text{envir}} - 1c_{\text{EV}} - 2(3c_{\text{GV}} + 4c_{\text{environ}})(c_{\text{envir}} + 2(3c_{\text{GV}} + 2c_{\text{environ}})M_T)
\]

\[ \frac{\partial \pi_{\text{GVT}}}{\partial S_{\text{ev1}}} = 0, \quad \pi_{\text{GVT}} \text{ can obtain a maximum value, and solving the equation} \]

\[ S_{\text{ev1}}^* = \frac{16A_{\text{EV}}(1+2M_T) + 5A_{\text{GV}}(2c_{\text{GV}} - 3c_{\text{EV}} - 10c_{\text{attn}} + 3c_{\text{envir}})M_T + 2A_{\text{EV}}(3c_{\text{GV}} - 3c_{\text{EV}} - 5c_{\text{attn}}(3c_{\text{envir}} + 9c_{\text{environ}})M_T + 4(18c_{\text{EV}}) + 1c_{\text{EV}} - 2(3c_{\text{GV}} + 19c_{\text{EV}} + 9c_{\text{environ}})M_T) + 4A_{\text{EV}}(c_{\text{envir}} + 2(3c_{\text{GV}} + 2c_{\text{environ}})(c_{\text{envir}} + 2c_{\text{environ}}) + 4A_{\text{EV}}(-2c_{\text{envir}} - 1c_{\text{EV}} - 2(3c_{\text{GV}} + 4c_{\text{environ}})(c_{\text{envir}} + 2(3c_{\text{GV}} + 2c_{\text{environ}})M_T)
\]

Therefore, when \( S_{\text{ev1}}^* \) is less than the government subsidy budget \( S \), the optimal subsidy is \( S_{\text{ev1}}^* \). Otherwise, the optimal government subsidy will be \( S \).

3.2. Period 2

In this period, period 2 as can be seen in Figure 4, a fraction of batteries reaches the end of their life cycle using on the EV. We assume that there are \( q_{\text{dis}} = \gamma q_{\text{EV1}} \) batteries that will be recycled.

Similar to period 1, the utility for consumers who buy and use GV and EV are \( U_{\text{EV2}} = (\theta_{\text{EV}} - C_{\text{EV}})k - p_{\text{EV2}} = A_{\text{EV}}k - p_{\text{EV2}} \) and \( U_{\text{EV2}} = (\theta_{\text{EV}} - C_{\text{EV}})k - (p_{\text{EV2}} + p_{\text{by2}}) = A_{\text{EV}}k - (p_{\text{EV2}} + p_{\text{by2}}) \). We can also obtain the probability that a consumer chooses an EV and a GV as \( P_{\text{GV2}} = \frac{P_{\text{by2}*P_{\text{EV2}} - P_{\text{GV2}}}}{A_{\text{EV}} - A_{\text{GV}}} \) and \( P_{\text{EV2}} = 1 - \frac{P_{\text{by2}*P_{\text{EV2}} - P_{\text{GV2}}}}{A_{\text{EV}} - A_{\text{GV}}} \). In this period, battery manufacturer’s profit is the total revenue from the sale of batteries minus the cost of producing the batteries, minus the cost of buying used batteries from consumers and the cost of remanufacturing:

\[ \pi_{\text{by2}} = \theta_{\text{by2}} \gamma_{\text{by2}} - (\theta_{\text{EV2}} - \lambda)q_{\text{by2}} c_{\text{rec}} - q_{\text{by2}}(\beta P_{\text{by2}} + c_{\text{rec}}) = \left( \frac{M_T(A_{\text{EV}} - A_{\text{GV}} + s_{\text{attn}} - p_{\text{EV2}} - p_{\text{by2}})}{A_{\text{EV}} - A_{\text{GV}}} - \beta \gamma q_{\text{EV2}} P_{\text{by2}} \right)
\]

\[ = \left( \frac{M_T(A_{\text{EV}} - A_{\text{GV}} + s_{\text{attn}} - p_{\text{EV2}} - p_{\text{by2}})}{A_{\text{EV}} - A_{\text{GV}}} - \beta \gamma q_{\text{EV2}} P_{\text{by2}} \right) + \left( \frac{M_T(A_{\text{EV}} - A_{\text{GV}} + s_{\text{attn}} - p_{\text{EV2}} - p_{\text{by2}})}{A_{\text{EV}} - A_{\text{GV}}} + \beta \gamma q_{\text{EV2}} P_{\text{by2}} \right) \]
As $A_{GV} - A_{EV}$, $\pi_{by2}$ will achieve the maximum profit when $\frac{\partial \pi_{by2}}{\partial P_{by2}} = 0$ holds. Through solving

$$p_{by2}^* = \frac{1}{2} \left( A_{EV} - A_{GV} + c_{itr} - p_{EV2} + p_{GV2} + \frac{\beta y(A_{EV} + A_{GV})q_{EV1}}{M_y} \right) \quad (21)$$

Similar with period 1, we have the profit for the GV manufacturer

$$\pi_{GV2} = q_{GV2}(P_{GV2} - c_{GV})$$

$$= \left( \frac{1}{2} \left( \frac{1}{A_{GV} - A_{EV}} - \frac{2}{A_{GV}} \right) M_y P_{GV2} + \frac{M_y(A_{EV}(A_{EV} + 2c_{GV} + c_{itr} - A_{GV} - c_{GV} + p_{EV2})) + \beta y A_{EV}(A_{EV} - A_{GV})q_{EV1}}{2(A_{EV} - A_{GV})A_{GV}} \right) p_{GV2}$$

$$+ \frac{1}{2} c_{GV}(\beta y q_{EV1} - M_y(A_{EV} - A_{GV})q_{EV2}) \quad (22)$$

The profit for the EV manufacturer

$$\pi_{EV2} = q_{EV2}(P_{EV2} - c_{EV} + S_{ev2})$$

$$= \left( \frac{1}{2} \left( \frac{1}{A_{EV} - A_{EV}} - \frac{2}{A_{EV}} \right) M_y P_{EV2} + \frac{M_y(A_{EV}(A_{EV} + 2c_{EV} + c_{itr} - A_{EV} - c_{EV} + p_{EV2})) + \beta y A_{EV}(A_{EV} - A_{EV})q_{EV1}}{2(A_{EV} - A_{EV})A_{EV}} \right) p_{EV2}$$

$$+ \frac{1}{2} c_{EV}(\beta y q_{EV1} - M_y(A_{EV} - A_{EV})q_{EV2}) \quad (23)$$

With Nash Equilibrium, $\frac{1}{A_{EV} - A_{EV}} - \frac{2}{A_{EV}} < 0$ and $A_{GV} - A_{EV} < 0$, when both formula $\frac{\partial \pi_{GV2}}{\partial P_{GV2}} = 0$ and formula $\frac{\partial \pi_{EV2}}{\partial P_{EV2}} = 0$ hold, that is

$$\left( \frac{1}{A_{GV} - A_{EV}} - \frac{2}{A_{EV}} \right) M_y P_{GV2} + \frac{M_y(A_{EV}(A_{EV} + 2c_{EV} + c_{itr} - A_{EV} - c_{EV} + p_{EV2})) + \beta y A_{EV}(A_{EV} - A_{EV})q_{EV1}}{2(A_{EV} - A_{EV})A_{EV}} = 0 \quad (24)$$

The results are

$$p_{GV2}^* = \frac{A_{EV}(4c_{EV}M_y + A_{EV}(3M_y - \beta y q_{EV1})) + A_{EV}(\beta y q_{EV1} - 3M_y) + M_y(c_{itr} + c_{EV} - 2c_{EV} + 2S_{ev2}))}{(8A_{EV} - 5A_{GV})M_y}$$

$$\frac{A_{EV}(4c_{EV}M_y + A_{EV}(3M_y - \beta y q_{EV1})) + A_{EV}(\beta y q_{EV1} - 3M_y) + M_y(c_{itr} + c_{EV} - 2c_{EV} + 2S_{ev2}))}{(8A_{EV} - 5A_{GV})M_y}$$

$$p_{EV2}^* = \frac{A_{EV}(4c_{EV}M_y + A_{EV}(3M_y - \beta y q_{EV1})) + A_{EV}(\beta y q_{EV1} - 3M_y) + M_y(c_{itr} + c_{EV} - 2c_{EV} + 2S_{ev2}))}{(8A_{EV} - 5A_{GV})M_y}$$

With (21) and (25), we have updated $q_{GV2}$ and $q_{EV2}$ as

$$q_{GV2} = \frac{(2A_{EV}(4c_{EV}M_y + A_{EV}(3M_y - \beta y q_{EV1})) + A_{EV}(\beta y q_{EV1} - 3M_y) + M_y(c_{itr} + c_{EV} - 2c_{EV} + 2S_{ev2}))}{(8A_{EV} - 5A_{GV})M_y}$$

$$q_{EV2} = \frac{(2A_{EV}(4c_{EV}M_y + A_{EV}(3M_y - \beta y q_{EV1})) + A_{EV}(\beta y q_{EV1} - 3M_y) + M_y(c_{itr} + c_{EV} - 2c_{EV} + 2S_{ev2}))}{(8A_{EV} - 5A_{GV})M_y}$$

In this period, the government subsidy is

$$\pi_{GVN2} = \pi_{by2} + \pi_{EV2} + \pi_{GV2} + U_{GV2} + U_{EV2} - q_{GV2}c_{envir} - q_{EV2}S_{ev2}$$

$$= K_S s_{ev2} + K_S S_{ev2} + K_6 \quad (28)$$

where

$$\frac{\partial \pi_{GVN2}}{\partial P_{GVN2}} = 0$$
\[
K_4 = \frac{(16A_{EV} - 20A_{EV}A_{GV} + 5A_{GV}^2)(2M_{tr} - 1)S_{ev2}}{8(A_{EV} - 5A_{GV})^2(A_{EV} - A_{GV})}
\]  
(29)

\[
K_5 = \frac{16A_{EV}A_{GV}(M_{tr}(20c_{EV} - 9A_{GV} + 14c_{GV} + 20c_{tr} + 4(21A_{GV} + 12c_{GV} - 16c_{EV} + 12c_{tr} - 9c_{environ}))M_{tr}}{6A_{EV}^2 + 2A_{EV}A_{GV} + A_{GV}^2 + 2A_{GV}M_{tr} + 2c_{tr} + 2c_{environ} + 2c_{tr} + 2c_{environ}}
\]  
\[
\frac{4(8A_{EV} - 5A_{GV})^2(A_{EV} - A_{GV})}{M_{tr}}
\]  
(30)

\[
\begin{align*}
K_6 &= \frac{16A_{EV}^2 + 2A_{EV}A_{GV} + A_{GV}^2 + 2A_{GV}M_{tr} + 2c_{tr} + 2c_{environ} + 2c_{tr} + 2c_{environ}}{6A_{EV}^2 + 2A_{EV}A_{GV} + A_{GV}^2 + 2A_{GV}M_{tr} + 2c_{tr} + 2c_{environ} + 2c_{tr} + 2c_{environ}}
\end{align*}
\]  
(31)

As \(K_4 < 0\), we have the optimal subsidy in period 2

\[
S_{ev2}^* = \frac{16A_{EV}A_{GV}(M_{tr}(20c_{EV} - 9A_{GV} + 14c_{GV} + 20c_{tr} + 4(21A_{GV} + 12c_{GV} - 16c_{EV} + 12c_{tr} - 9c_{environ}))M_{tr}}{6A_{EV}^2 + 2A_{EV}A_{GV} + A_{GV}^2 + 2A_{GV}M_{tr} + 2c_{tr} + 2c_{environ} + 2c_{tr} + 2c_{environ}}
\]  
(32)

Like period 1, when \(S_{ev2}^*\) is less than the government subsidy budget \(S\), the optimal subsidy will be \(S_{ev2}\). Otherwise, the subsidy will be \(S\).

4. Numerical example and analysis

4.1. Numerical Example

According to [1,19,20], we have initial values for parameters of GV, EV and the market as listed in Table 2 below:

| Parameter       | Value |
|-----------------|-------|
| \(c_{envir}\)   | 4000  |
| \(c_{environ}\) | 5000  |
| \(c_{EV}\)      | 250   |
| \(c_{EV}\)      | 250   |
| \(c_{GV}\)      | 1000  |
| \(c_{GM}\)      | 15000 |
| \(\pi_V\)       | 100000|
| \(M_{P}\)       | 100000|
| \(\beta\)       | 0.7   |
| \(\gamma\)      | 0.5   |
| \(S\)           | 15000 |

In period 1, we get the optimal subsidy \(S_{ev1} = 13327\). The GV selling price is \(p_{GV1} = 20863\), the EV price is \(p_{EV1} = 21768\) and the EV battery price is \(p_{EV1} = 18547\). And the probability that the consumer will choose a GV is \(P_{GV1} = 0.1920\) and the probability of choosing an EV is \(P_{EV1} = 0.4279\). Based on the market size, we have sales of GVs at \(q_{GV1} = 192810\); sales of EVs at \(q_{EV1} = 427860\).
In period 2, we get the optimal subsidy $S_{EV2} = 2784.7$. The GV selling price is $p_{GV2} = 21732$, the EV price is $P_{EV2} = 29292$ and the EV battery price is $P_{bEV2} = 13402$. And the probability that the consumer will choose a GV is $P_{GV2} = 0.2214$ and the probability of choosing an EV is $P_{EV2} = 0.3835$. Based on the market size, we have sales of GVs at $q_{GV2} = 221390$; sales of EVs at $q_{EV2} = 383480$.

4.2. Analysis

Now, we discuss the relationship between $\beta, \gamma, c_{ntr}, c_{emtr}$ and subsidy, prices and consumer’s purchase probabilities of GV/EV. $\beta$ and $\gamma$ are only valid in the second period. So, we will discuss relationships about $\beta$ and $\gamma$ in only period 2 and relationships about $c_{ntr}$ and $c_{emtr}$ in both period 1 and 2.

4.2.1. Relationship between $\beta$ and Parameters. We keep the initial values set in Table 2 and set the quality of used batteries as $0 < \beta < 1$. Changes in $\beta$ will only affect the parameters associated with the second period.

As can be seen in Figure 5, the higher quality of used batteries $\beta$, battery manufacturer needs to pay more to the EV used battery holders. And the government just need to pay less subsidy to the EV manufacturer in order to achieve Nash Equilibrium. Then, as can be seen in Figure 6, there will be a big price increase for the EVs and a light price increase for the GVs, but the price of new batteries will fall. Moreover, as shown in Figure 7, with the quality of decommissioned batteries continues growing, government subsidies are no longer necessary (when $\beta > 0.63$), the vehicle prices will be lower, but the battery price will be higher.

4.2.2. Relationship between $\gamma$ and Parameters. This section discusses the relationship between recycling rate of used batteries $\gamma$ and parameters. As can be seen from Figure 8, Figure 9, Figure 10, the relationships are similar to the relationship between $\beta$ and parameters.
4.2.3. Relationship between $c_{ntr}$ and Parameters. This section discusses the relationship between battery material cost $c_{ntr}$ and other parameters. As the change of $c_{ntr}$ will have impact on both periods, we will discuss the effects separately.

In period 1, as can be seen in Figure 11, Figure 12 and Figure 13, with the increasing battery material cost $c_{ntr}$, the government will pay less subsidy to the EV manufacturer. But the vehicle price, the battery price will be increased especially the EV battery price. In this case, although more people would still choose to buy an EV, the probability of people considering buying a GV tends to increase and the probability of considering buying an EV tends to decrease.

In period 2, as can be seen in Figure 14, Figure 15, Figure 16, with the increasing battery material cost, subsidy falls as well. Unlike period 1, the price of EVs has a slight decrease. The trends about other parameters’ relationship are generally consistent with period 1. However, by comparing Figure 13 and Figure 16, it appears that more people will choose not to purchase a car during period 2. Furthermore, when the subsidy is bottomed to 0, the prices of vehicles, batteries and the purchase probabilities remain essentially the same or change very slightly.
4.2.4. Relationship between $c_{\text{envir}}$ and Parameters. This section discusses the relationship between environmental protection costs $c_{\text{envir}}$ and relevant parameters. Same again, a change in parameter $c_{\text{envir}}$ will likewise affect the trend change in the parameters for both periods.

From Figure 17, Figure 18 and Figure 19, in period 1, with the increasing environment protection cost for the GVs, the government will have to pay more subsidy to the EV manufacturer, although the changes in price trends of vehicles and batteries are small. At the same time, there is no change in the probability of consumers choosing different cars. While in the second period, the relationship and trend of the parameters are almost the same as in the first period. And in period 2, as can be seen in Figure 20, Figure 21 and Figure 22, government will pay less subsidies to the EV manufacturer than period 1. Fewer people will select EVs than in the first period, while more people will choose to buy the GVs.

![Figure 17. $c_{\text{envir}}$ vs Subsidy in period 1.](image1)

![Figure 18. $c_{\text{envir}}$ vs Prices in period 1.](image2)

![Figure 19. $c_{\text{envir}}$ vs Probabilities in period 1.](image3)

![Figure 20. $c_{\text{envir}}$ vs Subsidy in period 2.](image4)

![Figure 21. $c_{\text{envir}}$ vs Prices in period 2.](image5)

![Figure 22. $c_{\text{envir}}$ vs Probabilities in period 2.](image6)

4.3. Results and Discussion

The above sections first show a numerical example for this two-period vehicle supply chain model with EV battery remanufacturing, and the discuss the relationships between some key input parameters and some decision parameters. To summarise, we have found that,

1. The relationship between the quality of used batteries and the relevant decision parameters was essentially indistinguishable from the relationship between the battery recovery rate and the relevant decision parameters. As these two parameters increase, the government will no longer need to provide more subsidies to EV manufacturers. At the same time, the price of batteries will decrease, but the price of cars will increase, and at the same time fewer people will choose EVs. This is mainly due to that in this case the market plays a more important role than the government regulation.

2. The higher the price of raw materials for EV batteries, the less government subsidies need to be provided to EV manufacturers. At this point, the prices of all commodities are rising, except for the price of the second period of EVs. In this case, the number of people buying EVs was decreasing, and the number of people buying GVs was increasing, even though more people in the market still preferred EVs. This is mainly because the cost of EV batteries grows faster at this time, making the total price of buying an EV grow larger. At the same time, in the second period, there is a reduction in the price of the battery due to the remanufacturing process of the used battery, which ultimately has a
compensating effect on the price of the car. This could be a reason why there is a slight decrease in
EV prices in the second period.
(3) The increase in environmental protection cost will significantly increase government subsidies
to EV manufacturers. As subsidies increase, the price of batteries, vehicles and the probability that
consumers choose a GV or EV remains essentially unchanged or change little.
(4) Regardless of the increase in the price of raw materials for EV batteries and the increase in
environmental protection cost, the government will always pay less in subsidies in the second period
than in the first period. This is also due to the recycling and remanufacturing process of retired
batteries, which reduces the price of batteries to a certain extent, forming a certain compensation
effect on the entire automotive supply chain.

5. Conclusions
In order to fill the gaps in the study of vehicle supply chain and government incentive design and
subsidy policy, we designed a discrete two-period game theory model that includes government,
GV/EV manufacturers, EV battery manufacturer and vehicle customers in subsidizing the EV
manufacturer with or without scrapped EV batteries remanufacturing process in period 1 and 2.
With the goal of Nash Equilibrium, the optimal subsidy given by the government, the optimal
vehicle prices, battery price and consumers’ vehicle purchase probabilities were derived, and
numerical examples are proposed in both 2 period. In the literature, we found that (1) a great deal of
research on EVs and EV batteries are in the field of technology and engineering; (2) There are some
articles on EV subsidies. But few articles consider studies related to government subsidy policies
when there is a recycle and remanufacturing process of retired batteries, after the early development
period of EVs. Our research fills the gap in this part of the study.

Based on our results, the research questions are able to be answered as follows: First, the optimal
government subsidy is related to the quality of the retired battery, the recycling rate of the battery, the
price of the raw material for the batteries, and the amount of money the government spends on
environmental protection. At the same time, the trend of government subsidies is roughly the same in
the first and second periods; Second, when there is the remanufacturing process for used EV batteries,
the government will spend less on subsidies for EV manufacturers; Third, the quality and the
recycling rate of EV used batteries, and the natural material cost of EV batteries are negatively
correlated on government subsidies, while the environmental protection cost is positively correlated
with government subsidies. These parameters are positively correlated with vehicle selling price.

To conclude, in this study, we attempt to fill the gaps in the literature on EV subsidy policy
research. By giving attention to study the impacts of retired batteries recycling rate and quality, the
cost of raw materials for EV batteries, and the government's environmental spending with and without
retired batteries remanufacturing process, we have learned about the optimal government subsidy
policy, the optimal pricing decision policy for vehicles and battery manufacturers. Furthermore, to
study this problem in more depth, we will examine the details of use benefits and analyze their
impacts on subsidy allocation and will also look at optimal investment strategy in the EV
infrastructure and ancillary services.

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