More about excited bottomonium radiative decays

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Radiative decays of bottomonium are revisited, focusing on contributions from higher-order relativistic effects. The leading relativistic correction to the magnetic spin-flip operator at the photon vertex is found to be particularly important. The combination of $O(v^4)$ effects in the nonrelativistic QCD action and in the transition operator moves previous lattice results for excited $\Upsilon$ decays into agreement with experiment.

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In a recent paper [1] we studied bottomonium radiative decays using lattice nonrelativistic QCD (NRQCD) [2]. It was shown that robust signals for transitions among the ground state and the first two excited states could be obtained using multiexponential fitting techniques [3]. The qualitative features of the decay amplitudes were in agreement with phenomenological expectations but quantitatively the values obtained were considerably larger than those determined by experiment.

In Ref. [1] the $b$ quarks were described using an $O(v^4)$ lattice NRQCD action and only the simplest magnetic spin-flip transition operator was used. In this work higher-order relativistic effects in the action and additional terms in the transition operator are investigated. It is expected that the hindered transitions, which involve states with different principal quantum numbers, are very sensitive to relativistic corrections. In comparison to Ref. [1], the additional effects considered in this work substantially reduce the amplitudes for excited $\Upsilon$ decays and bring them into line with experimental values.

The general setup for this study is the same as in Ref. [1]. The gauge fields come from a 2+1-flavor dynamical simulation done on a $32^3 \times 64$ lattice by the PACS-CS collaboration [3]. An ensemble of 192 configurations is used. The light quark parameters are such that the pion mass is near physical at 156 MeV. Landau link tadpole improvement is implemented with a value of 0.8463 for the link. The $b$-quark bare mass is 1.945 (in lattice units) and a stability parameter $n$ of 4, in line with Ref. [5], is used.

The three-point functions for vector to pseudoscalar transitions are constructed using a sequential source method. Starting with a vector (pseudoscalar) operator at the source $t_s$, the quark propagator is evolved to a time $T$ at which a pseudoscalar (vector) operator is applied. This quantity is then evolved backward in time. At intermediate times $t_s < t' < T$ a transition operator is inserted and evolution is continued to complete the quark antiquark loop at the source.

With a vector operator at the source and pseudoscalar at the sink the three-point function is expected to have the form

$$ G_{oo'}^{(VP)}(t'; T) = \sum_{n,n'} c_o(n) A_{nn'}^{(VP)} c_{o'}(n') \times e^{-E_n(t'-t_s)} e^{-E_{n'}(T-t')} $$

where the subscripts $o, o'$ indicate the type of operator (local or smeared) that is used. As in Ref. [1], three types of operators are used: a local operator, a wavefunction smeared operator [6] and an operator with the smearing applied twice [5]. See Ref. [1] for a description of the smearing function and parameters.

The overlap coefficients $c$ and simulation energies $E$ are the same ones that appear in the two-point function. The quantity $A_{nn'}^{(VP)}$ is the matrix element of the transition operator between the vector state $n$ and the pseudoscalar state $n'$. The three-point function with pseudoscalar source and vector sink has the same form with $V$ and $P$ labels reversed. The matrix elements $A_{nn'}^{(VP)}$ are related to those appearing in [1] by $A_{vn'}^{(VP)} = A_{n'v}^{(VP)}$. The matrix elements can be determined by fitting the $t'$ dependence of the three-point function for a fixed $T$ using overlap coefficients and energies obtained from a fit to two-point correlators.

The calculations in Ref. [1] were done using an NRQCD action including terms to $O(v^4)$. In this work $O(v^6)$ terms are also considered. The complete action can be found, for example, in Ref. [7]. Here we display only the spin-dependent terms linear in chromoelectric and chromomagnetic fields which are relevant for subsequent discussion (see Ref. [7] for detailed explanation of notation):

$$ \delta H^{(4)} = -\frac{c_4 g}{2M_0} \sigma \cdot \hat{B} - \frac{c_3 g}{8M_0^2} \sigma \cdot (\hat{\Delta} \times \hat{E} - \hat{E} \times \hat{\Delta}), $$

$$ \delta H^{(6)} = -\frac{c_2 g}{8M_0^2} \left\{ \hat{\Delta}^{(2)} , \sigma \cdot \hat{B} \right\} - \frac{c_4 g}{64M_0^3} \left\{ \hat{\Delta}^{(2)} , \sigma \cdot (\hat{\Delta} \times \hat{E} - \hat{E} \times \hat{\Delta}) \right\}, $$

where $\delta H^{(4)}$ gives the $O(v^4)$ chromomagnetic coupling...
and spin-orbit terms and $\delta H^{(6)}$ their leading relativistic corrections. The fields are tadpole-improved and in our calculations the action coefficients take the tree-level values. The transition operator should be constructed in a way which is consistent with the action. This can be achieved by replacing in (2) and (3) the SU(3) color electric and magnetic fields by external electromagnetic fields and the strong coupling constant by the charge. See for example Ref. [8].

For an M1 decay, the photon momentum, polarization vector and quark spin operator should be mutually orthogonal. The explicit choice made for the operators used in our lattice simulation is given in Table I. The normalization is such that in the infinite mass limit the transition operator should be constructed in a way which is consistent with the action. This can be achieved by replacing in (2) and (3) the SU(3) color electric and magnetic fields by external electromagnetic fields and the strong coupling constant by the charge. See for example Ref. [8].

Table I: Transition operators used in calculating three-point functions. The momentum $k$ is chosen to have a component only in the one direction and $\Delta_{1k} \equiv (\Delta_1 e^{i k \cdot \mathbf{x}} + e^{i k \cdot \mathbf{x}} \Delta_1)$.

| Magnetic | Electric |
|----------|----------|
| $O(v^4) \equiv \sum_x \sigma_3 e^{i k \cdot \mathbf{x}}$ | $\frac{1}{3 \sqrt{3}} \sum_x \sigma_3 \Delta_{1k}$ |

Results for the three-point transition matrix elements are given in Table II for different source-sink time separations and different values of recoil momentum. The calculations are done with the $O(v^4)$ action and the leading magnetic operator $\sigma_3$. The values for $T - t_s$ equal to 19 and 27 are from Ref. [1]. For the excited to ground state transitions, there is very good agreement between results using different time separations. However, using a smaller time separation $T - t_s = 15$ allows for a better determination of the $2 \rightarrow 2$ transition and we use only this for the present study.

Changing the action from $O(v^4)$ to $O(v^6)$ results in a decrease in the strength of the spin-dependent interactions. This is evidenced by a decrease in the mass difference between $\Upsilon$ and $\eta_b$ states [3]. For our calculation, done at a single lattice spacing of about 0.09 fm the $\Upsilon - \eta_b$ mass difference is 56(1) MeV and 24(3) MeV for 1S and 2S states respectively using the $O(v^4)$ action. These values are reduced to 43(2) MeV and 14(2) MeV when $O(v^6)$ effects are included. It is expected that inclusion of radiative corrections to the coefficients of the NRQCD action would raise these values somewhat [9]. For reference, the experimental values are $69.3 \pm 2.8$ MeV (from the Particle Data Group [10]) and $59.3 \pm 1.9^{+2.4}_{-1.4}$ MeV (from Adachi et al. [11]) for 1S. For 2S there are claims of $48.7 \pm 2.3 \pm 2.1$ MeV [12] and $24.3^{+3.5}_{-1.5}$ MeV [13] for the spin splitting.

Decreasing the strength of the spin-dependent interactions leads to greater overlap of wavefunctions with the same principal quantum number and decreases the overlap of states with different principal quantum numbers. This expectation is reflected in the lattice sim-

Figure 1: The matrix elements for decay of $\Upsilon$ to $\eta_b$ with the same principal quantum number, as a function of momentum. Shown are $O(v^4)$ (circles) and $O(v^6)$ (squares) action results with the leading $O(v^4)$ magnetic operator. Triangles include the relativistic correction to the magnetic operator in the $O(v^6)$ calculation.

Figure 2: The matrix elements for decay of an excited $\Upsilon$ to the $\eta_b$ ground state, as a function of momentum. Shown are $O(v^4)$ (circles) and $O(v^6)$ (squares) action results with the leading $O(v^4)$ magnetic operator. Triangles include the relativistic correction to the magnetic operator in the $O(v^6)$ calculation.
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Figure 3: The matrix elements for decay of an excited
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on the wavefunctions leads to a relative negative sign
of the transition amplitude compared to excited
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decay. Figure 3 shows that changing the action from
O(v^4) (circles) to
O(v^6) (squares) results in a decrease in the
magnitude of the amplitude but including the relativis-
tic correction to the operator (triangles) has the opposite
effect.

Up to now we have discussed only the transition due to
magnetic spin-flip operators. There are additional con-	ributions that have to be considered, that is, due to
the electric terms. These are the electromagnetic coun-
terparts of the spin-orbit terms in (2) and (3) (see also
Eq. (2) in Ref. [8]). These terms are suppressed by a
factor of (photon energy/\Me) relative to the magnetic
terms at the same order in \(v^2\). They make no contri-
bution to the transition matrix element at zero photon
momentum. For vector to pseudoscalar transitions be-
tween states with the same principal quantum number,
where the mass difference is small, their effect is not im-
portant. For excited state to ground state transitions
they make a non-negligible contribution.

Table III gives our final results for excited state decay
amplitudes interpolated (for 2S decays) or extrapolated
(for 3S decays) to the physical momentum. The values in
the first two rows were obtained using an NRQCD action
with terms up to the indicated order but only the lead-
ing magnetic spin-flip transition operator. For the third
and fourth rows all terms in the transition operator up to
the order of those included in the NRQCD action were

Figure 3 shows the comparison of the calculations for
the case of excited
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decaying to the ground state
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Here going from
O(v^4) (circles) to
O(v^6) (squares) leads to a decrease of the amplitude in line with the notion that the overlap of wavefunctions is decreased. What is surprising is that including the relativistic correction to the magnetic spin-flip (triangles) leads to such a large additional decrease of the amplitude. For the decay of an excited
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the situation is more complicated. The effect of spin-dependent interactions on the wavefunctions leads to a relative negative sign of the transition amplitude compared to excited
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effect.

Table II: Three-point matrix elements from simultaneous fits to 12 correlation functions with different source-sink separation.

| \(T - t_s\) | \(A^{(VP)}_{11}\) | \(A^{(PV)}_{21}\) | \(A^{(PV)}_{12}\) | \(A^{(VP)}_{22}\) |
|---|---|---|---|---|
| 15  | 0.916(2) | -0.068(3) | -0.050(3) | 0.071(5) |
| 19  | 0.915(2) | -0.068(2) | -0.050(4) | 0.071(4) |
| 27  | 0.916(2) | -0.068(3) | -0.051(6) | 0.071(4) |
| 15  | 0.907(2) | -0.060(6) | -0.047(5) | 0.082(5) |
| 19  | 0.907(1) | -0.062(6) | -0.047(7) | 0.079(5) |
| 27  | 0.907(2) | -0.061(5) | -0.048(8) | 0.079(5) |
| 15  | 0.877(1) | -0.031(4) | -0.040(5) | 0.103(6) |
| 19  | 0.877(1) | -0.031(4) | -0.041(6) | 0.102(5) |
| 27  | 0.878(2) | -0.029(5) | -0.039(7) | 0.100(5) |

Figure 3: The matrix elements for decay of an excited
\(\eta_b\) to the
\(\Sigma\) ground state, as a function of momentum. Shown are \(O(v^4)\) (circles) and \(O(v^6)\) (squares) action results with the leading \(O(v^4)\) magnetic operator. Triangles include the
relativistic correction to the magnetic operator in the \(O(v^6)\) calculation.
evaluated. The difference between the first and third rows is due to the $O(v^4)$ electric term. The difference between the second and fourth rows is dominated by the $\text{spin-dependent terms of } O(v^6)$ NRQCD may suggest that it would be beneficial to avoid nonrelativistic approximations altogether. The construction and use of relativistic lattice actions for heavy quarks is the subject of ongoing investigations [17, 18]. The comparison of nonrelativistic and relativistic approaches to excited bottomonium radiative decays would be a worthwhile direction for future work.

Finally, we note that the large changes in the excited state decay amplitudes found in going from $O(v^4)$ to $O(v^6)$ NRQCD may suggest that it would be beneficial to avoid nonrelativistic approximations altogether. The construction and use of relativistic lattice actions for heavy quarks is the subject of ongoing investigations [17, 18]. The comparison of nonrelativistic and relativistic approaches to excited bottomonium radiative decays would be a worthwhile direction for future work.

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