I. INTRODUCTION

Plasmas with impurity particles, so called dusty or complex plasmas, have a manifold of applications, ranging from applied to basic research problems. Due to this wide range of applicability, there has been a steadily increasing interest in the physics of dusty plasmas (see Ref. [1] and references therein). The above mentioned spectrum of applications covers, e.g., planetary rings, lightning discharges in smoke contaminated air, fusion plasmas, low-temperature laboratory plasmas, and processing plasmas in the semi-conductor industry. To be specific, by a dusty plasma one usually means a three component plasma consisting of electrons, ions, and dust which is considered to be significantly heavier than the ions [2]. The charge of the dust is assumed to range from a few electron charges to thousands. For astronomical applications it is often important to also include the dynamics of neutral particles [3]. Dusty plasmas contains novel physical phenomena, such as dust acoustic waves [4], dust ion-acoustic waves [5], dusty plasma crystals [6, 7], and dust lattice waves [8], all of which have been experimentally verified, see Refs. [9] and references therein. In fact, due to the (in general) relatively low phase velocity of dusty plasma waves, these plasmas are useful for probing basic properties of plasma excitations.

Lately there has also been an increasing interest in quantum plasma physics [10, 11, 12, 13, 14, 15, 16], in particular the nonlinear aspects of such systems. Such plasmas are in general typical for condensed matter environments, where the density of the electron gas is high [13, 17], giving a considerable influence from the wave function structure of the electrons. Moreover, the spin properties of plasmas has been investigated recently by means of quantum hydrodynamical models [10, 13, 16] and also by the use of spin kinetic models [20, 21, 22]. Even in a regime considered as classical, the effects of spin may give a nontrivial influence on the dynamics of an electron plasma [23]. Furthermore, spin and the intrinsic magnetic moment of the constituent particles are an essential part of magnetic fluids or ferrofluids [24]. A ferrofluid is a mixture of nanosized magnetic particles suspended in a liquid. In this article we will consider a model where magnetic dust particles are suspended in an electron-ion plasma, which can be said to be the plasma analogue of a ferrofluid. Such systems has recently been investigated both theoretically, considering single particle dynamics [25, 26, 27, 28, 29] and experimentally [30, 31]. Here we will consider sufficiently low frequency phenomena so that we may use a hydrodynamical model to describe the dynamics of the dust particles.

In Section II we present the governing equation and show that the model satisfies an energy conservation law in which magnetization transport is included. We then linearize the equations in Section III to obtain the general dispersion relation. For the case of a static external magnetic field we consider modes propagating perpendicular to the magnetic field as well as kinetic Alfvén waves. We show that these modes exhibit instabilities. Finally, in Section IV we summarize and draw our conclusions.

II. A MAGNETIZED DUST MODEL

We here consider a three component plasma consisting of electrons, positive ions and negatively charged dust particles (denoted by subscripts e, i and d respectively). The dust is assumed to be magnetized and have a charge $-Ze$ (where $e$ is the elementary charge). The magnetization of the dust can be assumed to be due to quantum mechanical spin, or from a macroscopic magnetization of the dust grains themselves. The size of the dust particles in, e.g., astronomical environment ranges from a few nanometers to about 100 microns and the weight from about $10^{-15}$g to $10^{-5}$g [1, 2]. In the laboratory for example Ref. [31] uses grains with size 4.5µm and mass $\sim 10^{-14}$g. Hence the dust is much heavier than the ions, $m_d \gg m_i$.

In the framework of the multi-fluid theory the dynamics is governed by the continuity and momentum conservation equations. The continuity equation is given by

$$\frac{\partial n_s}{\partial t} + \nabla \cdot (n_s \mathbf{v}_s) = 0,$$

where the subscript $s$ denotes the different species ($s = e, i, d$), $n_s$ is the number density and $\mathbf{v}_s$ is the fluid velocity of particle type $s$. The momentum conservation for
the dust reads
\[ m_d n_d \left( \frac{\partial}{\partial t} + v_d \cdot \nabla \right) v_d = -Z_d e n_d (E + v_d \times B) - k_B \nabla (T_e n_e) + M_\alpha \nabla B_\alpha, \quad (2) \]
where \( n_d \) is the number density of the dust, \( k_B \) is Boltzmann’s constant, \( T_e \) is the dust temperature and \( \mathbf{M} \) is the magnetization. We use Einstein’s summation convention for greek indices. The last term in the equation above is usually neglected when considering plasmas, but for microsized dust grains in a plasma the mutual magnetic dipole interaction can be of importance for laboratory conditions [30]. We will consider perturbations slow compared to the plasma frequencies of the ions and the electrons. Neglecting the momentum of these particles we obtain
\[ 0 = -e n_e (E + v_e \times B) - k_B \nabla (T_e n_e), \quad (3) \]
\[ 0 = Z_i e n_i (E + v_i \times B) - k_B \nabla (T_i n_i), \quad (4) \]
where \( T_{e(i)} \) is the temperature of the electrons (ions) and \( Z_e \) is the charge of the ions. Note, we neglect the spin of the electron and the protons. The magnetization of the dust is assumed to be orders of magnitude larger. The equations above are coupled to Maxwell’s equations
\[ \nabla \cdot \mathbf{E} = -\frac{e}{\epsilon_0} (n_e - Z_i n_i + Z_d n_d), \quad (5) \]
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (6) \]
and
\[ \nabla \times \mathbf{B} = \mu_0 \nabla \times \mathbf{M} - e \mu_0 (n_e v_e - Z_i n_i v_i + Z_d n_d v_d). \quad (7) \]
A closed system of equations for the dust can be derived. To do this we start by adding the momentum Eqs. (3) and (4) and assume that quasi-neutrality holds \( Z_i n_i \approx n_e + Z_d n_d \) to obtain
\[ -e Z_d n_d e = -e (n_e v_e - Z_i n_i v_i) \times B - k_B \nabla (T_i n_i + T_e n_e). \quad (8) \]
Solving this for the electric field and inserting it into the momentum equation for the dust (2) yields
\[ m_d n_d \left( \frac{\partial}{\partial t} + v_d \cdot \nabla \right) v_d = -e (n_e v_e - Z_i n_i v_i + Z_d n_d v_d) \times B - k_B \nabla (T_d n_d + T_i n_i + T_e n_e) + M_\alpha \nabla B_\alpha. \quad (9) \]
Using Eq. (7) we obtain
\[ m_d n_d \left( \frac{\partial}{\partial t} + v_d \cdot \nabla \right) v_d = \nabla \times \left( \frac{\mathbf{B}}{\mu_0} - \mathbf{M} \right) \times \mathbf{B} \quad (10) \]
\[ -k_B \nabla \left[ \left( T_e + \frac{T_i}{Z_i} \right) n_e + \left( T_d + \frac{Z_d T_i}{Z_i} \right) n_d \right] + M_\alpha \nabla B_\alpha \]
where the quasi-neutrality condition has been used to rewrite the thermal pressure terms and we will assume that \( (T_e + T_i/Z_i) n_e \ll (T_d + Z_d T_i/Z_i) n_d \) in order to obtain a closed set of equations. This condition can of course be relaxed, introducing new thermal effects. However, the principle dynamics of the complex plasma will not be significantly affected. Using some vector identities we can finally write the continuity and momentum equations for the dust
\[ \frac{\partial n_d}{\partial t} + \nabla (n_d v_d) = 0, \quad (11a) \]
and
\[ m_d n_d \left( \frac{\partial}{\partial t} + v_d \cdot \nabla \right) v_d = \mathbf{B} \cdot \nabla \left( \frac{\mathbf{B}}{\mu_0} - \mathbf{M} \right) - \nabla \left[ \frac{B^2}{2 \mu_0} - \mathbf{M} \cdot \mathbf{B} + k_B \left( T_d + \frac{Z_d T_i}{Z_i} \right) n_d \right]. \quad (11b) \]
respectively. Using Eqs. (10), (11d) and (11b) we can derive the time evolution equation for the magnetic field
\[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (v_d \times \mathbf{B}) + \nabla \times \left\{ \frac{\nabla \times (\mathbf{B} - \mu_0 \mathbf{M})}{\mu_0 Z_d e n_d} \times \mathbf{B} \right\}. \quad (11c) \]
The magnetization is taken to be proportional to the density of the dust particles and in the direction of the magnetic field
\[ \mathbf{M} = \mu_d (B, T_d) n_d \mathbf{B}. \quad (11d) \]
This model includes the case where the particles have an intrinsic magnetic moment that will be aligned with an applied magnetic field, which is what we have in mind. However, it also includes the case where the dust particles have no net magnetic moment. The occurrence of an external magnetic field will induce a magnetization in the dust particles which yields a macroscopic magnetization of the fluid. A magnetization of the dust could also arise from spinning of the dust particles (with charges attached to the surface) which yields a diamagnetic response to an applied field. However, the magnetic dust-dust interaction due to this can often be neglected. See Ref. [28] for a more detailed discussion. Our model can be compared with a ferrofluid which is a colloidal suspension of magnetic particles in a liquid. In an ionic ferrofluid the magnetic particles are kept apart by repulsive electrostatic forces, see e.g. Refs. [24, 31, 32, 33, 34]. Due to the high density and the correspondingly high collision frequency, we note that free currents can typically be neglected in ionic ferrofluids. By contrast, the simultaneous existence of free currents and magnetic dipole moment have been shown to be significant in dusty plasmas [31]. A more general assumption than Eq. (11d) would be to assume that the magnetization is also dependent on the electron and ion temperatures since collisions with these particles may change the magnetization.

With the exception for the occurrence of the magnetization due to the magnetic moment of the dust, \( \mathbf{M} \), these equations are the same as Hall-MHD theory [32, 33]. Moreover, it should be noted that the structure of the
system of Eqs. (11a)-(11c) is the same as one get from
the magnetized ideal MHD model of Refs. [10, 34],
if that system is extended to include the Hall current.
In that case naturally the dust density and velocity in-
stead will refer to the ion density and velocity. Although
Eqs. (11a)-(11c) can describe an electron-ion plasma, it
should be noted that the physics for that case is different
in several respects: Firstly, in the case of an electron-
ion plasma, it is the lighter species, the electrons, that
contribute to the magnetization, due to their magnetic
moments being larger than those of the ions. Secondly
we stress that the validity conditions of the model have
no simple correspondence between the dust dominated
plasma and the electron-ion plasma case. In what fol-
lows, we will mainly be concerned with the dusty plasma
applications.

Next we want to find an energy conservation law for
the Eqs. (11). In order to do this we must specify an
equation of state for the system. For simplicity we choose
the simple model

$$\frac{P}{P_0} = \left(\frac{n_d}{n_{d0}}\right)^\gamma,$$

for the total pressure $P = k_B(T_d + Z_dT_i/Z_i)$, where $P_0$
and $n_{d0}$ are the equilibrium pressure and density respec-
tively. With this equation of state the energy conserva-
tion becomes

$$\frac{\partial W}{\partial t} + \nabla \cdot \mathbf{P} = 0,$$  

where

$$W = \frac{m_d n_d v_d^2}{2} + \frac{P}{\gamma - 1} + \frac{B^2}{2\mu_0} - \mathbf{B} \cdot \mathbf{M}$$

is the energy per volume and

$$P = \frac{m_d n_d v_d^2}{2}v + \frac{\gamma P}{\gamma - 1}v - (\mathbf{B} \cdot \mathbf{M})v$$

$$- \left[ v \times \mathbf{B} + \frac{1}{Z_d e n_d} (\nabla \times \mathbf{H}) \times \mathbf{B} \right] \times \mathbf{H}$$

is the flow of energy out of the region. In Eq. (14) the
first term is the kinetic energy per volume, the second
term is the energy density from the pressure, the third
term is the energy stored in the magnetic field and the
last term is the energy in each volume element due to the
magnetic moment of the dust particles. Similarly, in Eq.
(15), the first three terms are the flow of kinetic, pressure
and magnetic energy density that follows the flow of each
volume element. The last term is the Poynting vector
which is modified by the inclusion of the Hall-term and
the magnetization.

III. THE DISPERSION RELATION

We linearize Eqs. (11) and Fourier decompose. Fur-
thermore, the coordinate system is defined so that $\mathbf{B}_0 =$
$B_0\mathbf{z}$ and $\mathbf{k} = k_x\mathbf{x} + k_z\mathbf{z}$. For simplicity we choose an
isothermal pressure model. This gives the dispersion rela-
tion

$$\begin{vmatrix}
\omega^2 - k_x^2 (\tilde{V}_{dA}^2 - V_{dB}^2) - k^2 \tilde{V}_{dA}^2 & -i\frac{\omega}{\omega_{cd}}(k^2 \tilde{V}_{dA}^2 - k^2 V_{dB}^2) & -k_x k_z \tilde{V}_{dA}^2 \\
&&
\omega^2 - k_x^2 \tilde{V}_{dA}^2 & -i\frac{\omega}{\omega_{cd}} k_x k_z \tilde{V}_{dA}^2 & -k_x k_z \tilde{V}_{dA}^2 \\
&&
-i\frac{\omega}{\omega_{cd}} k_x k_z V_{dM} & -k_x^2 V_{dA}^2 & -\omega^2 - k_x^2 V_{dA}^2 
\end{vmatrix} = 0,$$  

where $\tilde{V}_{dA} = V_{dA} - V_{dM}$, $\tilde{V}_{dA} = V_{dA} - V_{dB}$
and we have defined

$$V_{dA}^2 = \frac{k_B}{m_d} \left( T_d + \frac{Z_d T_i}{Z_i} \right)$$

$$V_{dA}^2 = \frac{B_0^2}{\mu_0 m_d n_{d0}}$$

$$V_{dM}^2 = \frac{\mu_0 B_0}{m_d}$$

$$V_{dB}^2 = \frac{\partial \mu_0 B_0}{\partial B_0} \frac{B_0}{m_d}$$

The velocity $V_{dA}$ is a generalized thermal speed for the
dust, $V_{dA}$ is the dust Alfvén speed and $V_{dM}$ and $V_{dB}$
are related to the magnetization of the dust. The frequency,
$\omega_{cd} = Z_d e B_0 / m_d$ is the cyclotron frequency of the dust.
The dispersion relation, Eq. (16), is in general a third
degree polynomial in $\omega^2$. Specifically, for $\omega \ll \omega_{cd}$
the three roots to the dispersion relation are the fast and slow
magnetosonic modes, and the shear Alfvén wave [38].

To see the implications of the derived dispersion rela-
tion a couple of special cases are now considered. For a
wave propagating perpendicular to the magnetic field
\( k = k_z \hat{x} \) the dispersion relation is obtained from Eq. (10) and reads
\[
\omega^2 = k_z^2 \left[ \tilde{V}_{dA}^2 + \tilde{V}_{da}^2 - V_{dB}^2 \right].
\] (18)
To get a qualitative description of the instability condition for Eq. (18) we assume that the dust and the ions are in thermal equilibrium so that we may assume \((T_d + Z_d T_i / Z_i) = N T_d\) where \(N > 1\) is a constant. Typically we have \(Z_d / Z_i\) ranging from unity to a few thousands. Furthermore we assume that the spins are thermally distributed (see e.g. Ref. [35]) the magnetization per density can be written
\[
\mu_d = \tilde{\mu}_d \tanh \left( \frac{\tilde{\mu}_d B}{k_B T_d} \right).
\] (19)
In the case that the dust particles have a high total spin number the tanh-function in Eq. (19) corresponding to Fermi-Dirac statistics should be replaced by the Langevin-function, corresponding to Maxwell-Boltzmann statistics. The difference between these functions are relatively small, however, and thus we will use Eq. (19) for the remainder of this paper. We note that the magnetic moment \(\tilde{\mu}_d\) can be several orders of magnitude larger than the Bohr magneton. The condition for instability, Eq. (18), then becomes
\[
\frac{B_0}{\mu_0 n_{d0} \tilde{\mu}_d} - N \frac{\tilde{\mu}_d B_0}{k_B T_d} = \left[ 1 - \left( \frac{\tilde{\mu}_d}{\tilde{\mu}_d} \right)^2 \right] > 0,
\] (20)
where \(\tilde{\mu}_d = \mu_d(B_0)\) in accordance with Eq. (19). Note that this equation implies that more of the magnetic dipoles are re-oriented towards the lower energy state when the magnetic field strength is increased. For this to apply, the relaxation time to reach the lower energy state must be shorter than the wave period time. In case the opposite ordering holds, the fraction of particles in the different energy states remain constant during a wave period, and consequently the term proportional to \(V_{dB}^2\) in Eqs. (16) and (17) should be dropped, which correspond to neglecting the fourth and last term in Eq. (20). The difference in the dispersion relation, depending on whether the magnetic dipoles have time to change during a wave period or not, turns out to be relatively small, however. For definiteness, we will stick to the case where the relaxation time of the magnetic dipoles is sufficiently fast for Eq. (20) to apply for the rest of this work.

Next we consider the kinetic Alfvén type of waves. In this case the ordering \(k_z \ll k_x, V_{da} \ll V_{dA}\) and \(\omega \sim k_z V_{dA}\) applies. The dispersion relation can then be approximated by
\[
\frac{\omega^2}{k_z^2 V_{dA}^2} =
\frac{\omega^2 k_z^2}{k_d^2} \frac{(\tilde{V}_{dA}^2 - \tilde{V}_{dB}^2) \tilde{V}_{dA}^2 + V_{dM}^2 (\tilde{V}_{dA} + \tilde{V}_{da} - V_{dB})}{(V_{dA}^2 + \tilde{V}_{da}^2 - V_{dB}^2)} (\omega^2 - k_z^2 V_{dA}^2) + k_z^2 V_{da}^2
\] (21)
For \(V_{dM}, V_{dB}^2 \rightarrow 0\) this reduces to the well known kinetic Alfvén waves [39]. Analyzing Eq. (21) it is seen that the wave-mode can be unstable provided the numerator of the second term of the right hand side is negative, i.e. we obtain the instability condition
\[
(V_{dA}^2 - \tilde{V}_{dB}^2) \tilde{V}_{dA}^2 + V_{dM}^2 (\tilde{V}_{dA} + \tilde{V}_{da} - V_{dB}) < 0
\] (22)
Assuming once more that the dust and the ions are in thermal equilibrium we get
\[
\frac{B_0}{\mu_0 n_{d0} \tilde{\mu}_d} - \frac{\tilde{\mu}_d B_0}{k_B T_d} + \frac{(N - 1) \tilde{\mu}_d B_0}{N k_B T_d} \tanh^2 \left( \frac{\tilde{\mu}_d B_0}{k_B T_d} \right) < 0.
\] (23)
The conditions for instabilities Eqs. (20) and (22) have been plotted in Fig. 1. Note that in order to have instabilities we need to have sufficiently high densities and/or sufficiently low temperatures.

We here give the following simplified picture of why the instability of this type can occur. The volume elements of the plasma are electrically neutral since the electron and ion background will screen any excess electrical charge. Further, the magnetization of a volume element is in the direction of the magnetic field and the different volume elements will attract each other like small magnets. Consider now the magnetic flux through a surface with normal parallel to \(B\). If the oscillations have \(k_z = 0\) then the magnetic flux through the surface will not change, and hence there will be no build up of magnetization. If, on the other hand, the oscillations occur perpendicular to the field there can be a local build up of the magnetic field. This can for sufficiently low temperature and high density cause the plasma to collapse similarly to the case of the Jeans instability [38, 40].

\[ IV. \] SUMMARY AND CONCLUSION

In the present paper we have put forward a Hall MHD type of model with an intrinsic magnetization. We have shown that the magnetized Hall MHD model is energy conserving, and presented the expressions for energy density and the energy flux. A set of equations of this type could describe different types of systems: An ordinary electron-ion plasma, in which case the magnetization would be due to the electron spin, or - as emphasized here - a three component plasma containing electrons, ions and heavy dust particles.

As the next step, we have investigated the linear modes and the stability properties of the homogeneous system. The general dispersion relation has been derived, describing the fast and slow magnetosonic modes and the shear Alfvén waves, as modified by the Hall current and the magnetization of the system. Due to the magnetization, the homogeneous system may be unstable, as predicted already from an magnetized ideal MHD type of model [38]. The main new finding from the stability analysis in this paper, is that inclusion of the effects due to the Hall current extends the unstable region of parameter space,
as described by Fig. 1. The wave that first becomes unstable turns out to be the magnetized version of the kinetic Alfvén wave.

For the effects of magnetization to be significant, we need relatively high density plasmas, and/or low temperatures. For the case of electron-ion plasmas, these can be found in astrophysical environments, such as the interior of white dwarf stars and pulsars.

We can make some numerical estimations for the parameters $X = \bar{\mu}_d B_0/(k_B T_d)$ and $Y = B_0/(\mu_0 n_{d0} \bar{\mu}_d)$. From Ref. [30] we find that the magnetic moment per particle can be of the order $10^{-12} m^{-2} A^{-1}$ and magnetic induction of the order 0.1T. Furthermore, we assume that the dust temperature is low $T_d \sim 1 K$. The density of particles is taken to be $n_{d0} \sim 10^{12} m^{-3}$ as in Ref. [28]. This gives us $X \approx 10^{10}$ and $Y \approx 10^6$. Comparing this with Figs. 1 we see that the instabilities considered here are not possible to detect in current experiments. As shown e.g. by Refs. [30, 31], however, magnetic dipole effects can still be of significance in dusty plasmas.

The model developed here should be considered only as a first step since it does not account for potentially important effects of more elaborate models. These include the two-fluid model, where spin up and spin down populations are considered as different species [23], the kinetic description [22] and models including nearest neighbor interactions [29, 30], which is important in the strongly coupled regime.

Acknowledgments

This research was supported by the European Research Council under Contract No. 204059-QPQV, and the Swedish Research Council under Contract No. 2007-4422.
[19] M. Marklund and G. Brodin, in New Aspects of Plasma Physics - Proceedings of the 2007 ICTP Summer College on Plasma Physics, eds. P. K. Shukla, L. Stenflo and B. Eliason (World Scientific, Singapore, 2008).

[20] S. C. Cowley, R. M. Kulsrud and E. Valeo, Phys. Fluids. 29, 430 (1986).

[21] R. M. Kulsrud, E. J. Valeo, and S. C. Cowley Nucl. Fusion 26, 1443 (1986).

[22] G. Brodin, M. Marklund, J. Zamanian, Á. Ericsson, and P. L. Mana, Phys. Rev. Lett. 101, 245002 (2008).

[23] G. Brodin, M. Marklund, and G. Manfredi, Phys. Rev. Lett. 100, 175001 (2008).

[24] R. E. Rosensweig, Ferrohydrodynamics (Dover Publications, INC., Mineola, New York, 1985).

[25] V. M. Mal’nev, E. V. Martysh, and V. V. Pan’kiv, Ukr. J. Phys. 53 8 (2008).

[26] V. M. Mal’nev, E. V. Martysh, V. V. Pan’kiv, S. V. Koshevaya, and A. N. Kotsarenko, Ukr. J. Phys. 51 9 (2006).

[27] G. Uchida, U. Konopka, and G. E. Morfill, Phys. Rev. Lett. 93, 155002 (2004).

[28] V. N. Tsytovich, N. Sato, and G. E. Morfill, New. J. Phys. 5, 43, (2003).

[29] S. V. Vladimirov, G. E. Morfill, V. V. Yaroshenko, and N. F. Cramer, Phys. Plasmas 10, 7 (2003).

[30] V. V. Yaroshenko, G. E. Morfill, D. Samsonov, and S. V. Vladimirov New J. Phys. 5 18 (2003).

[31] D. Samsonov, S. Zhdanov, G. Morfill, and V. Steinberg, New. J. Phys. 5, 24 (2003).

[32] M. J. Lighthill, Philos. Trans. R. Soc. London, Ser. A 252, 397 (1960).

[33] E. A. Witalis, IEEE Trans. Plasma Sci. 14, 842 (1986).

[34] G. Brodin, L. Stenflo and P. K. Shukla, Sol. Phys. 236, 285 (2006).

[35] E. Bringuier, and A. Bourdon, Phys. Rev. E 67, 011404 (2003).

[36] G. Wang, J. P. Huang, Chem. Phys. Lett. 421, 544 (2006).

[37] I. Szalai, and S. Dietrich, J. Phys.: Condens. Matter 20, 204122 (2008).

[38] G. Brodin and M. Marklund, Phys. Rev. E 76, 055403(R) (2007).

[39] A. Hasegawa and C. Uberoi, The Alfvén wave (DOE Review Series – Advances in Fusion Science and Engineering, U.S. Department of Energy, Washington D.C., 1982).

[40] L. Herrera and N. O. Santos, Phys. Rep. 286, 53 (1997).

[41] N. Shukla, P. K. Shukla, G. Brodin, and L. Stenflo, Phys Plasmas 15, 044503 (2008).

[42] J. N. Heyman, R. Kersting, and K. Untertrainer, Appl. Phys. Lett. 84, 3984 (2004).

[43] G. Manfredi and P. A. Hervieux, Appl. Phys. Lett. 91, 061108 (2007).