On the low-$x$ behavior of nuclear shadowing

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Abstract

We calculate the $x$ dependence of nuclear shadowing at moderate values of $Q^2$ by using HERA diffractive data. We show that no decrease of shadowing occurs down to very low $x$ ($x \simeq 10^{-4}$).

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The possibility of studying nuclear shadowing (\textit{i.e.} the depletion of bound nucleon structure functions ($F_2^A$) with respect to the free nucleon ones ($F_2^N$)) at HERA \cite{1} has prompted new interest on this subject. Different models and interpretations have been suggested in the past years to explain nuclear shadowing \cite{2}. A nuclear HERA program would allow extending the experimental investigation to very low $x$, thus offering the chance of a deeper understanding of the phenomenon.

One recent interesting prediction \cite{3} is that, at low $x$ and for moderately large $Q^2$ ($Q^2 \sim 10 \text{ GeV}^2$) shadowing stops its rise, starts decreasing and eventually vanishes. This effect arises from the different low-$x$ behavior of the inclusive and the diffractive structure functions. In fact, at $x \ll 1$, whereas $F_2(x, Q^2) \sim (1/x)^\Delta(Q^2)$, with a ‘hard’ intercept $\Delta(Q^2) \approx 0.3 - 0.4$, the shadowing term $\Delta F_2^\text{sh} \equiv F_2^A - F_2^N$, which is proportional to the diffractive structure function, is dominated by soft physics and has an asymptotic $x$ dependence of the form $\Delta F_2^\text{sh} \sim (1/x)^2\Delta(\mu^2)$, with $\Delta(\mu^2) \approx 0.1$, where $\mu^2$ is the typical scale of soft interactions.

While the argument above is rather general and correct, the specific finding of Ref. \cite{3}, \textit{i.e.} that at $Q^2 \sim 10 \text{ GeV}^2$ shadowing reaches its largest value already at $x \approx 10^{-2}$ and then begins to decrease towards smaller $x$, is based on some restrictive assumptions. We shall call $x_0$ the $x$ value where shadowing attains its maximum. The value of $x_0$ is found in \cite{2} to be almost independent of $Q^2$ (for $Q^2 > 5 \text{ GeV}^2$) and of the atomic mass. The prediction of \cite{3} is that in the region between $x = 10^{-4}$ and $x = 10^{-2}$ shadowing should be a decreasing function of $x$ towards $x \to 0$. It is evident that such a behavior should easily be observable at HERA.

The results of Ref. \cite{3} rely on two assumptions:

1) the shadowing term, which is related to the diffractive structure function, is given by a simple power–like parameterization

$$\Delta F_2^\text{sh} \sim x^{2-2\alpha^F(0)}$$

with $\alpha^F(0) \approx 1.1$;

2) the mass $M^2$ of the diffractively produced states is kept fixed and equal to $Q^2$.

Both these assumptions are questionable. The first one is true only asymptotically, as $x \to 0$, and cannot be used to draw any conclusion about the $x$-region between $10^{-4}$
and $10^{-2}$. The second assumption is not valid when the dynamics is dominated by the triple pomeron contribution, which provides a nonvanishing high–mass tail to the distribution of the diffracted states.

The purpose of this letter is to carry out a more precise phenomenological analysis of nuclear shadowing, by relying on the diffractive structure function measurements of HERA. Our main concern will be with the $x$–dependence of shadowing, rather than with its absolute normalization, which is at present unpredictable due to experimental and theoretical uncertainties. Our calculation, which does not make use of the assumptions of Ref. [3], leads to the conclusion that the onset of the decrease of shadowing is likely to be around or smaller than $x_0 = 10^{-4}$ and thus hardly reachable at HERA. In other terms, we predict that no decrease of shadowing will be observed at HERA.

Let us start from the well-known relation between nuclear shadowing and diffraction dissociation established long ago by Gribov [4, 5]. In virtual-photon–nucleus scattering the nuclear cross section is given by [2, 6]

$$\sigma^{\gamma^* A} = A \sigma^{\gamma^* N} - 4\pi \frac{A-1}{A} \int dM^2 \frac{d^2\sigma^D}{dM^2dt} \bigg|_{t=0} \int d^2b |\Phi(k_L, b)|^2 + \ldots, \quad (2)$$

where $d^2\sigma^D/dM^2dt$ is the $\gamma^* N$ diffraction dissociation cross section ($M^2$ being the invariant mass of the excited hadronic states), and the longitudinal form factor $\Phi$, which is function of the impact parameter $b$ and of the longitudinal momentum of the recoil proton $k_L = x m_N (1 + M^2/Q^2)$, is related to the nuclear density $\rho_A$ by

$$\Phi(k_L, b) = \int dz \rho_A(b, z) e^{ik_L z}. \quad (3)$$

The dots in eq. (2) represent the higher rescattering terms, which are non negligeable for heavy nuclei. The simplest way to take them approximately into account is [7] to introduce an eikonal factor $e^{-\sigma_{\text{eff}} T(b)/2}$ in the integration over the impact parameter in (2). $T(b)$ is the nuclear thickness

$$T(b) = \int_{-\infty}^{+\infty} dz \rho_A(b, z); \quad (4)$$

$\sigma_{\text{eff}}$ is the effective cross section for the interaction of the diffracted states with the nucleon, given by

$$\sigma_{\text{eff}} = 16\pi \frac{1}{\sigma^{\gamma^* N}} \left. \frac{d\sigma^D}{dt} \right|_{t=0}. \quad (5)$$
At small $x$ this cross section turns out to be almost $x$ independent. In practical calculations it will be taken as a constant (see below).

In terms of the inclusive structure functions per nucleon $F_{2}^{N,A}$, and of the diffractive structure function

$$\begin{align*}
F_{2}^{D(4)}(\beta, Q^2, x_B, t = 0) &= \frac{4\pi^2 \alpha_{em}}{Q^2} \frac{d\sigma^{D}}{d\beta dQ^2 dx_B dt} \bigg|_{t=0},
\end{align*}$$

(6)

where ($W^2$ is the squared center-of-mass energy of the $\gamma^* N$ system)

$$x_B \equiv \frac{M^2 + Q^2}{W^2 + Q^2}, \quad \beta \equiv \frac{x}{x_B},$$

(7)

eq. (6) becomes ($k_L = m_N x_B$)

$$\begin{align*}
F_{2}^{A}(x, Q^2) &= F_{2}^{N}(x, Q^2) \\
&- 4\pi \frac{A - 1}{A^2} \int dx_B F_{2}^{D(4)}(\beta, Q^2, x_B, t = 0) \int d^2 b e^{-\frac{2m_B T}{b}} |\Phi(x_B, b)|^2,
\end{align*}$$

(8)

In principle, the diffractive structure function $F_{2}^{D(4)}$ can be obtained from the experiment. However, what the present experimental analyses provide is only the diffractive structure function integrated over $t$

$$\begin{align*}
F_{2}^{D(3)}(\beta, Q^2, x_B) &= \int_{0}^{[t]_{max}} d|t| F_{2}^{D(4)}(\beta, Q^2, x_B, t),
\end{align*}$$

(9)

with $[t]_{max} \simeq 0.5$ GeV$^2$.

The ZEUS parametrization [8] for $F_{2}^{D(3)}$ has the form

$$\begin{align*}
F_{2}^{D(3)} &= F_{v}^{D(3)} + F_{s}^{D(3)} \\
&= A x_B^{-a} \left[\beta(1 - \beta) + \frac{C}{2} (1 - \beta)^2\right]
\end{align*}$$

(10)

where $a = 1.30 \pm 0.08^{+0.08}_{-0.14}$, $A = 0.018 \pm 0.001 \pm 0.005$, $C = 0.57 \pm 0.12 \pm 0.22$. The exponent $a$ is found to be essentially independent of $\beta$. A more recent preliminary analysis [9] gives a smaller value for $a$: $a \sim 1.1 - 1.2$ in the $Q^2$ range $10 - 20$ GeV$^2$. In eq. (10) we separated a ‘valence’ part $F_{v}^{D(3)} \propto \beta(1 - \beta)$ and a ‘sea’ part $F_{s}^{D(3)} \propto (1 - \beta)^2$. In the language of the color dipole model [11], the valence corresponds to the lowest Fock state ($q\bar{q}$) of the virtual photon, whereas the sea corresponds to higher Fock states ($q\bar{q}g...$), which represent the triple pomeron contribution of Regge theory. Notice that
factorization breaking effects, predicted in Ref. [11], which would imply a non–universal flux factor for the valence and sea components, are not yet observable.

In order to derive $F_{2}^{D(4)}(\beta, Q^2, x_F, t = 0)$ from the measured $F_{2}^{D(3)}(\beta, Q^2, x_F)$, eq. (10), we assume a simple peripheral $t$-dependence of the form

$$F_{v,s}^{D(4)}(\beta, Q^2, x_F, t) = F_{v,s}^{D(4)}(\beta, Q^2, x_F, t = 0) e^{-\left(B_{v,s} + 2\alpha'_{IP} \log \frac{1}{x_F}\right) |t|},$$

where $\alpha'_{IP} \simeq 0.5$ and the slopes $B_{v,s}$ are borrowed from hadron scattering and real photoproduction. We use $B_s \simeq 6$ GeV$^{-2}$, $B_v \simeq 12$ GeV$^{-2}$.

In eq. (8) $F_{2}^{D(3)}(\beta, x_F, Q^2)$ is integrated over $x_F$ between $x$ and 1. However, due to the selection of the rapidity gap events, there is an experimental upper cutoff on $x_F$: $x_F^c = 0.04$. We checked that in the $x$ region of interest the large–$x_F$ tail neglected in the integration is irrelevant.

For consistency, we used the ZEUS parametrization for $F_{2}^{N}(x, Q^2)$ [12]. Our predictions do not depend on the parametrization adopted for $F_{2}$ (the same results are obtained using MRS(R1) [13]). As for the nuclear part of the calculation, we used a Fermi–type nuclear density.

The two main sources of uncertainty in our calculation are: i) the effective cross section $\sigma_{\text{eff}}$ (a constant value, specified below, is used); ii) the large experimental error on $a$, the exponent of the so-called pomeron flux, see eq. (10). As we shall see, these uncertainties prevent us from predicting the absolute amount of shadowing, although they do not affect the qualitative features of the $x$–dependence of shadowing, that we are most interested in. Both the normalization and the $x$–dependence of shadowing depend very little on the other parameters appearing in the calculation. In particular we checked that even a large variation of the slopes $B_{v,s}$ changes by no more than few percent the predicted shadowing.

Let us come now to the results.

In Fig. 1 we plot the ratio $F_{2}^{A}/F_{2}^{N}$ for calcium. The diffractive structure function used in the calculation is given by the ZEUS parametrization (10), which is valid at moderately large $Q^2$ ($Q^2 \simeq 10 – 30$ GeV$^2$). We use the central values of the ZEUS fit for the coefficients $A$ and $C$, quoted after eq. (10). We allow the exponent of the pomeron flux $a$ to vary around a value ($a = 1.2$) which is smaller than the central value
\((a = 1.3)\) of the ZEUS analysis \([8]\) and closer to the most recent finding \((a \simeq 1.1 - 1.2)\) \([9]\). For comparison, the value used in Ref. \([8]\) is 1.20. For the effective cross section we take \(\sigma_{\text{eff}} = 10\ \text{mb}\). The \(x\) range shown in figure is the one allowed by the present experimental fits on the diffractive structure function. No decrease of shadowing occurs down to \(x \simeq 10^{-4}\).

In Fig. 2 we set \(a = 1.2\) and we vary the coefficient \(C\) of the sea component of the diffractive structure function \((10)\) within the errors of the ZEUS fit \([8]\). We take \(\sigma_{\text{eff}} = 10\ \text{mb}\). Again, the shadowing curve is at most (when \(C\) is small) rather flat towards \(x = 10^{-4}\) but does not exhibit any sensible decrease.

Our main finding is therefore that the onset of shadowing saturation is at much lower \(x\) than argued in Ref. \([8]\) \((x_0 \lesssim 10^{-4})\) and is outside the range of investigation of HERA experiments. Only a combination of unlikely circumstances \((a \text{ and } C \text{ very small, } a \lesssim 1.1, C \simeq 0, \text{ that is a pomeron flux less singular than the Donnachie–Landshoff one and an almost vanishing sea component in the diffractive structure function})\) would produce a visible decrease of nuclear shadowing in the region above \(x = 10^{-4}\).

In Fig. 3 we illustrate the dependence of our results on the effective cross section \(\sigma_{\text{eff}}\), for two nuclei (carbon and calcium). The shadowing curves with two choices of \(\sigma_{\text{eff}}\) (12 and 15 mb) and \(a = 1.2\) in eq. \((10)\) are shown. We see that the value of \(\sigma_{\text{eff}}\) affects the absolute normalization of nuclear shadowing but not its \(x\)-dependence. For light nuclei, such as carbon, not even the normalization depends on \(\sigma_{\text{eff}}\). Thus the theoretical uncertainty related to the choice of \(\sigma_{\text{eff}}\) does not spoil our conclusions about the \(x\) behavior of the shadowing curve.

Considering the weak dependence of the shape of shadowing on the theoretical ingredients of the present calculation (the effective cross section for multiple rescattering, the hadronic slopes and the longitudinal form factor), we can say that our prediction is dictated essentially by the experimental measurements of \(F_2^D\) and therefore is a model independent result. Moreover, the qualitative behavior of shadowing that we found is stable against a large variation of the exponent of the pomeron flux and of the size of the sea diffractive structure function. Hence, any more precise determination of the diffractive structure function should not change our main finding.
Finally, we just mention that results qualitatively similar to those reported above can be obtained by using $F_2^{D(3)}$ evaluated in BFKL–type models, such as the one of Ref. [11]: only the absolute size of shadowing turns out to be larger.

In conclusion, let us summarize our results. We studied the low-$x$ behavior of nuclear shadowing at moderately large $Q^2$. We carried out a model independent calculation by using the experimental data on the diffractive structure function and the well established relation between diffraction and shadowing. No restrictive assumptions were made. Our conclusion is that, to the best of the present experimental knowledge, no decrease of shadowing occurs above $x \simeq 10^{-4}$, that is in the kinematic region accessible at HERA. Obviously, if the present experimental results on the different low-$x$ behavior of $F_2^N$ and $F_2^{D(3)}$ will be confirmed, a decrease of shadowing does take place, but only at very small $x$, beyond the HERA range.

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References

[1] M. Arneodo et al., in Future Physics at HERA, G. Ingelman, A. De Roeck and R. Klanner eds., DESY, 1996, p. 887.

[2] V. Barone, M. Genovese, N.N. Nikolaev, E. Predazzi and B.G. Zakharov, Z. Phys. C58 (1993) 541.
S. Kumano, Phys. Rev. C48 (1993) 2016.
D. Kharzeev and H. Satz, Phys. Lett. B327 (1994) 361.
For an extensive review and an exhaustive list of references, see M. Arneodo, Phys. Rep. 240 (1994) 301.

[3] B. Kopeliovich and B. Povh, in Future Physics at HERA, G. Ingelman, A. De Roeck and R. Klanner eds., DESY, 1996, p. 959; Z. Phys. A356 (1997) 967; Phys. Lett. B367 (1996) 329.

[4] V.N. Gribov, Sov. Phys. JETP 29 (1969) 483.

[5] V.A. Karmanov and L.A. Kondratyuk, JETP Lett. 18 (1973) 266.

[6] V. Barone and M. Genovese, in Future Physics at HERA, G. Ingelman, A. De Roeck and R. Klanner eds., DESY, 1996, p. 938.

[7] J. Kwieciński, Z. Phys. C45 (1990) 461.

[8] M. Derrick et al. (ZEUS), Z. Phys. C68 (1995) 569.

[9] M. Grothe (ZEUS), contribution to DIS 97, Chicago, IL, 1997.

[10] N.N. Nikolaev and B.G. Zakharov, Z. Phys. C49 (1991) 607; C53 (1992) 331; C64 (1994) 631.

[11] M. Genovese, N.N. Nikolaev and B.G. Zakharov, J. Exp. Theor. Phys. 81 (1995) 625; Phys. Lett. B380 (1996) 213; B378 (1996) 347.

[12] M. Arneodo and M. Vreeswijk (ZEUS), private communication.

[13] A.D. Martin, R.G. Roberts and W.J. Stirling, Phys. Lett. B387 (1996) 419.
Figure captions

Fig. 1 The ratio $F_2^A(x)/F_2^N(x)$ for calcium in the $Q^2$ range 10-30 GeV$^2$. The three curves correspond to three different values of the exponent of the pomeron flux in the ZEUS fit of the diffractive structure function eq. (10): $a = 1.15$ (dotted line), $a = 1.20$ (dashed line), $a = 1.25$ (solid line). The other parameters used are specified in the text.

Fig. 2 The ratio $F_2^A(x)/F_2^N(x)$ for calcium in the $Q^2$ range 10-30 GeV$^2$. The solid curve is obtained using $C = 0.57$ in the ZEUS fit, see eq. (10). The dashed curves correspond to $C = 0.57 \pm 0.25$ (errors added in quadrature). The other parameters are specified in the text.

Fig. 3 The ratio $F_2^A(x)/F_2^N(x)$ for carbon (upper pair of curves) and calcium (lower pair of curves) in the $Q^2$ range 10-30 GeV$^2$. The solid and dashed lines correspond to $\sigma_{\text{eff}} = 10$ mb and $\sigma_{\text{eff}} = 15$ mb, respectively.
$\frac{F_2^A}{F_2^N}$
\[ \frac{F_2^A}{F_2^N} \]