Nonlinear Vibration Analysis of Generator Rotor of a Military General Mobile Power Plant

Xiaoquan Li\textsuperscript{1,*}, Jiangjiang He\textsuperscript{2}, Yuwei Zhao\textsuperscript{1} and Yuemeng Cheng\textsuperscript{1}

\textsuperscript{1}Air and Missile Defense College, Air Force Engineering University, Xi’an, China
\textsuperscript{2}The Graduate School, Air Force Engineering University, Xi’an, China
*Corresponding author e-mail:lxq389@126.com

Abstract. The generator is the core component of the military general mobile electric power plant. In general, the rotor of generator has different degrees of eccentricity which affect the operation and even cause system failure. To explore the vibration characteristics of the rotor, the rotor of generator is taken as research object. A Jeffcott rotor system with cubic terms is established and reduced to Duffing equation. The multi-scale method is used to solve the first approximate solution of the forced vibration. The amplitude-frequency, phase-frequency response equation and the resonance conditions are obtained. The influence of different mechanical parameters on the lateral vibration and the main resonance of the system are studied: eccentricity and the stiffness have little effect on the amplitude variation, but have a great influence on the stability region; the main resonance frequency and the excitation change the magnitude of the system common amplitude value; the damping not only has a great influence on the common amplitude value, but also changes the response and phase.

1. Introduction
Rotating machinery is widely used in various fields as an energy conversion device. The rotor-bearing system is the core component of rotating machinery and plays a key role in the power, energy, transportation, petrochemical and defense sectors [1]. As a core component of military mobile power stations, generators are inevitable due to mechanical problems. The literatures studied the main causes and hazards affecting the operation of rotating machinery. The results show that the mechanical failure caused by rotor eccentricity is the most prominent.

The vibration characteristics of generator rotor has become a hot topic in rotor dynamics research [2]. In general, the excitation force has a significant impact on the vibration behavior and stability [3]. Liu, etc calculated the transient vibration response of the Jeffcott rotor with critical stiffness caused by the friction crack [4]. At present, the nonlinear equations of the rotor model are solved by numerical method and analytical method. Yang, etc used the MR-K iterative method to solve the numerical solution of the strongly nonlinear rotor system, and analyzed the amplitude-frequency response of the rotor system under the support stiffness, damping and gear stiffness parameters [5]. Hou, etc considered the cubic nonlinear stiffness of the support and proposed a Duffing model [6]. In [7], the FE method is used to study the rotor asymmetry and time-varying parameters, which lead to the lateral dynamic instability of the rotor. The analytical calculation method is easier to analyze than the numerical method and has higher calculation accuracy, especially the multi-scale algorithm overcomes the harmonic balance, the average method solves the amplitude-frequency relationship.

In order to study the mechanical problems caused by rotor eccentricity, the multi-scale method is used to solve the nonlinear equation of rotor eccentric forced vibration, and the amplitude-frequency response equation, phase-frequency response equation and rotor system stability conditions are
obtained. Considering the unbalanced mass and other factors in the rotor eccentricity, the influence of mechanical parameters such as damping, eccentricity, stiffness, main resonance frequency and excitation on the lateral main resonance of the rotor system is explored.

2. Generator Rotor Nonlinear Dynamics Model

The bearing-rotor system consists of stator, rotor, shaft and bearing of generator. The diagram of rotor eccentricity as shown in figure 1 and the Jeffcott rotor model as shown in figure 2. It is assumed that the quality of the shaft is neglected, only the stiffness is obtained; The rotor is bilaterally symmetrical, and the disc is located in the middle of the span; the mass of the system is concentrated on the disc, and the bearing is equivalent to an isotropic spring-damping system in both radial directions.

![Figure 1. Diagram of rotor eccentricity](image1)

![Figure 2. Jeffcott rotor dynamics model](image2)

The equation of Jeffcott rotor as expressed:

\[
\begin{align*}
\ddot{m}x + c \dot{x} + k_x x + k_3 x^3 &= m e w^2 \cos(\omega t) \\
\ddot{m}y + c \dot{y} + k_1 y + k_3 y^3 &= m e w^2 \sin(\omega t)
\end{align*}
\]  

(1)

Where, \( m, e, \) and \( \omega \) are the mass, eccentricity, and rotational speed, respectively; \( c, k_1, k_3 \) are damping, linear stiffness, nonlinear stiffness; \( k_1, k_3 \) is independent [3]. In addition, \( x \) and \( y \) represent displacements in both radial directions, respectively. The equation can be transformed into a general form considering the displacement \( u \) as follows:

\[
\ddot{m}u + c \dot{u} + k_1 u + k_3 u^3 = m e w^2 \cos(\omega t)
\]  

(2)

The equation (2) dimensionless can be expressed as:

\[
\ddot{X} + 2\zeta \dot{X} + X + \alpha X^3 = \Omega^2 \cos(\Omega \tau)
\]  

(3)

3. Main Resonance Analysis

The multi-scale method is used to study the forced vibration under harmonic excitation. The rotor forced vibration equation (2) can be rewritten as:

\[
\ddot{u} + \omega_n^2 \dot{u} = -\epsilon(2\zeta\omega_n \dot{u} + \beta u^3) + F \cos(\omega t)
\]  

(4)

Where, \( F = e \omega^2 \) is the system excitation, \( \zeta > 0, \beta = k_3/m \) can be positive or negative. The tuning parameter is introduced instead of the excitation frequency \( \omega \).

\[
\omega = \omega_n + \epsilon \sigma
\]  

(5)

Where, \( \sigma = 0 \) (1). when \( \sigma = 0 \), no matter how small the excitation, it will cause unbounded vibration.

\[
\begin{align*}
\zeta \omega_n &= \mu, \mu = 0(1) \\
F &= e f, f = 0(1)
\end{align*}
\]  

(6)
Under the condition of (5)(6), the Equation (4) is rewritten as:

\[ \ddot{u} + \omega_n^2 u = -\varepsilon\left[2\mu u + \beta u^3 + F\cos(\omega_n t + \varepsilon \sigma)\right] \quad (7) \]

It takes only two time-scales to study the first approximation of the solution.

\[ u(\tau, \varepsilon) = u_0(T_0, T_1) + \varepsilon u_1(T_0, T_1) + \cdots \quad (8) \]

Substituting equation (7) into equation (4), comparing the coefficients of the same power to obtain approximate linear partial differential equations of various orders:

\[ \varepsilon^0 : D_0^2 u_0 + u_0 = 0 \quad (9) \]

Let the general solution of equation (8) be:

\[ u_0 = A(T_1) \exp(i\omega_n T_0) + \bar{A}(T_1) \exp(-i\omega_n T_0) \quad (10) \]

Substituting equation (10) into (9) and expressing \( \cos(T_0 + \sigma T_1) \) in complex form, there are:

\[ D_0^2 u_0 + \omega_n^2 u_0 = -[2i\omega_n (A + \varepsilon A) + 3\beta A^2 \bar{A}] \exp(i\omega_n T_0) \]
\[ -\beta A^2 \exp(3i\omega_n T_0) + \frac{1}{2} f \exp[i(\omega_n T_0 + \sigma T_1)] + cc \quad (11) \]

Where, \( cc \) represents the conjugate complex of the preceding term. If \( A \) is chosen as the solution of the equation, then the duration term in equation (11) is eliminated.

\[ 2i\omega_n (A + \varepsilon A) + 3\beta A^2 \bar{A} - \frac{1}{2} f \exp[i(\omega_n T_0 + \sigma T_1)] = 0 \quad (12) \]

To solve equation (12), denote \( A \) as

\[ A = \frac{1}{2} a \exp(i\gamma) \quad (13) \]

Where, \( a \) and \( \gamma \) are both real functions of \( T_1 \). Substituting equation (13) into equation (12), and dividing it into real and imaginary parts.

\[ \begin{cases} D_a = -\mu a + \frac{1}{2} f \sin(\sigma T_1 - \gamma) \\ aD_a = \frac{3}{8} \frac{\beta}{\omega_n} a + \frac{1}{2} f \cos(\sigma T_1 - \gamma) \end{cases} \quad (14) \]

An approximate solution of the equation is obtained from equation (14):

\[ u_0 = a \cos(\omega_n T_1 + \gamma) + \alpha(\varepsilon) \quad (15) \]

Equation (14) can be transformed into an autonomous differential equation by assuming the parameter \( \phi = \sigma T_1 - \gamma \) and considering the steady-state, there is \( \dot{a} = \dot{\phi} = 0 \).

Equation (14) is squared and added to obtain:

\[ [\mu^2 + (\sigma - \frac{3}{8} \frac{\beta}{\omega_n} a^2)]a^2 = \frac{f^2}{4\omega_n^2} \]

\[ \phi = \tan^{-1}\left(\frac{-\mu}{\sigma - \frac{3}{8} \frac{\beta}{\omega_n} a^2}\right) \quad (17) \]

Equation (16) is an implicit function equation that responds to amplitude \( a \) as a tuning parameter \( \sigma \) and excitation amplitude \( f \). Equation (16) is a frequency response equation, and equation (17) is a phase frequency response equation. The amplitude frequency curve is shown in figure 3.
Figure 3. Amplitude-frequency response of steady-state main resonance

It can be seen that for a fixed excitation frequency ($\sigma$), the primary resonance may be unique, or there may be three cases: similar to the case where the autonomous system has multiple equilibrium solutions, the true implementation of multiple steady-state primary resonances depends on consistent with the Sommerfeld effect in terms of its stability and initial system conditions [8].

Substituting equations (5) and $\varphi = \sigma T_1 \beta$ into equation (15) yields a first approximation of the steady-state solution as:

$$\hat{u}_0 = a \cos(\omega f + \varepsilon \sigma t - \varphi) + o(\varepsilon)$$

Where, $a$ and $\varphi$ are constants. Therefore, the steady-state response is exactly the same as the excitation frequency, and the system response phase difference is $-\varphi$.

4. Calculation Result and Analysis

Figure 3 shows the bending of the nonlinear frequency response curve. When $\beta > 0$, the system has hard characteristics, while when $\beta < 0$, the system has soft characteristics. PMS generator with high power density, torque density, large power density, wide range of constant torque, and good low-speed hard characteristics. Using the weak magnetic control strategy, the rotor system shows soft characteristic, linear stiffness $k_1 > 0$, nonlinear stiffness $k_3 < 0$. The influence of nonlinear disturbance on the system is shown in figure 4. With the decrease of $\beta$, the response frequency characteristic curve moves to the right. The unstable region also decreases.

The multi-value of the nonlinear common amplitude frequency response of the rotor system will lead to the jump phenomenon. As shown in figure 5. In the range of some excitation frequencies, the response of the system has three values, two of which are stable and the other is unstable, which causes the amplitude jump and bifurcation of the system to occur in this region. Starting from point 1, the speed slowly decreases and a jump occurs between point 3 and point 4. On the other hand, if the value will gradually increase from point 5, the jump from point 6 to point 2 occurs. The jumping phenomenon is a non-linear phenomenon. In the case of the generator rotor, when the frequency is reduced, the response amplitude jumps to a lower value, the saddle-junction bifurcation and the jumping phenomenon occur, and the fatigue damage caused by the rotor, the shaft, and the like is caused.
Figure 4. Nonlinear term affects the amplitude-frequency response of rotor vibration

Figure 5. Jump phenomenon of main amplitude-frequency response

There is an unstable region in the resonance region of the rotor system (solid line represents the stable solution in figure 5, and the dashed line represents the unstable solution). When the excitation frequency $\omega$ increases or decreases, the stable equation of motion changes from a single value to a multi-value and then to a single value. When the parameters of the system reach the critical value, the stability of the solution changes abruptly, which leads to the sudden change of the response of the rotor system.

In order to discuss the influence of the rotor system parameters on the resonance response, the response curves of the rotor system under different parameters are analyzed and studied.

1) Influence of relative mass eccentricity on resonance of rotor system

As one of the important factors affecting the motion characteristics of generator rotor system, it is of great significance to study the influence of mass eccentricity on the dynamic behavior of the rotor system. When the rotor has mass eccentricity and rotates at constant speed, the rotor is subjected to centrifugal force. If the eccentricity of the generator is 0.01 mm, 0.05 mm and 0.08 mm respectively, the common amplitude frequency response of the rotor is shown in figure 6 and the other calculated parameters are constant. It can be seen that with the increase of eccentricity, the corresponding amplitude-frequency characteristic curve moves to the left, and the resonance region and unstable region expand, and the resonance amplitude does not change. The rotor system tends to stabilize the system, ideally the stator, rotor and the shaft should be concentric.

2) Influence of damping coefficient on resonance of rotor system

In the rotor system, the damping has an obvious effect on the total amplitude value, and the damping of the system can reduce the main total amplitude value. Figure 7 shows the effect of the damping $\mu$ on the resonance response of the rotor system. When free vibration, the amplitude value is infinite, the frequency response curve is composed of two branches, and the amplitude frequency of the skeleton line and the steady state main resonance is consistent with the soft characteristic. With the increase of the damping ratio $\zeta$, the resonance peak amplitude of the corresponding amplitude-frequency characteristic curve is obviously reduced. When $\zeta=0$, the Equation (18) has a number of $\phi=n\pi$, and $n$ is an integer. It can be seen that the response is either the same as the excitation or the same phase, or the phase difference. When there is damping, the amplitude is a finite value, the damping is smaller, while the amplitude is the larger, therefore, $\phi=\sin^{-1}\left(\frac{3}{8}\frac{\beta}{\omega_n}a_n\right)$, so that changes the phase difference of the response. The damping ratio has no effect on the skeleton curve of the frequency response of the steady-state motion of the rotor system, and the resonance region does not move when the damping ratio changes.
3) Influence of stiffness on resonance of rotor system

Stiffness determines the ability of the rotor to resist elastic deformation when it is subjected to force. Figure 8 shows the effect of linear stiffness $k_1$ on the frequency response curve of the rotor system. It can be seen that with the decrease of stiffness, the corresponding amplitude-frequency characteristic curve moves to the left, the amplitude of vibration increases, and the resonance region and the unstable region expand at the same time. The proper supporting stiffness of the generator can reduce the lateral vibration of the rotor.

4) Influence of main resonance frequency on resonance of rotor system

Through the analysis of the frequency factor in the system, it can be seen that the main resonance occurs under the internal excitation of the system. Figure 9 shows the response curves at different principal resonance frequencies. It can be seen that with the decrease of the main resonance frequency, the corresponding amplitude-frequency characteristic curve shifts to the left, and the amplitude of the vibration increases gradually. At the same time, the resonance region and the unstable region are enlarged, and the amplitude of the resonance changes greatly.

5. Conclusion

When an unbalanced centrifugal force is caused by eccentricity, the following conclusions can be drawn: (1) according to the characteristics of the PMS generator, the nonlinear characteristics of the rotor system are soft; (2) the influence of nonlinear term, eccentricity and linear stiffness on rotor system has the same law, that is, the resonance amplitude change is consistent, all of them can affect the change of unstable region; (3) the change of damping, primary resonance frequency can cause the
great change of resonance amplitude of rotor system, and the change of primary resonance frequency will cause the change of skeleton line, while the change of damping and excitation will not change the skeleton line of amplitude-frequency response.

6. References

[1] Zhao L K, and Ma Z Y 2013 J. Dynamic analysis for rotor system with rub-impact of a hydraulic generating set under unbalanced magnetic pull. Journal of Vibration and Shock, 32(8):48-54 (In Chinese).

[2] Carrella A, Friswell M I, Zotov A, Ewins D J, and Tichonov A 2009 J. Using nonlinear springs to reduce the whirling of a rotating shaft. Mechanical Systems & Signal Processing, 23(7):2228-2235.

[3] Zhu C S, and Chen Y J 2006 J. Vibration Characteristics of Aeroengine's Rotor System During Maneuvering Flight. Acta Aeronautica Et Astronautica Sinica, 27(5):835-841.

[4] Liu Z, and Wang J J 2016 J. Transient vibration characteristics of a rotor with breathing crack. Journal of Vibration and Shock, 35(7):233-240 (In Chinese).

[5] Yang L, Yang S P, and Yang Y T 2018 J. Dynamic behavior of a locomotive nonlinear rotor system. Journal of Vibration and Shock, 37(15):33-42 (In Chinese).

[6] Hou L, Chen Y, Fu Y, and Li Z 2015 J. Nonlinear response and bifurcation analysis of a duffing type rotor model under sine maneuver load. International Journal of Non-Linear Mechanics, 78:133-141.

[7] Dakel M, Baguet S, and Dufour R 2014 J. Steady-state dynamic behavior of an on-board rotor under combined base motions, Journal of Vibration and Control, 20 (15): 2254–2287.

[8] Xia M L, and Sun Q P 2016 J. Thermomechanical responses of nonlinear torsional vibration with NiTi shape memory alloy alternative stable states and their jumps. Journal of the Mechanics & Physics of Solids, 102:257-276.