Charged Lepton Flavour Violation in Littlest Higgs model with T-parity

Naveen Gaur

Abstract. The Littlest Higgs model with T-parity (LHT) belongs to the non-minimal flavour violating model. This model has new sources of flavour and CP violation both in quark and leptonic sectors. These new sources of flavour violation originates by the interaction of Standard Model (SM) fermions with heavy gauge bosons and heavy (or mirror) fermions. In this work we will present the impact of the new flavour structure of T-parity models on flavour violations in leptonic sector.

Keywords: Lepton Flavour Violation, Little Higgs model

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INTRODUCTION

Little Higgs (LH) models provides a very attractive solution to the so called \textit{little hierarchy problem} of the SM \cite{1}. These models are perturbative upto the scale of \(\sim \) 10 TeV and have relatively smaller number of parameters. Unlike Supersymmetry (SUSY), in LH models the cancellation of quadratic divergences to the Higgs mass is achieved by the particles of the same statistics. The new particles introduced in these models have mass around TeV.

In LH models, Higgs boson is kept naturally light by identifying it with the Nambu-Goldstone Boson (NGB) of a spontaneously broken global symmetry and hence it remains massless at tree level. The gauge and Yukawa interactions of NGB are introduced without generating any one loop quadratic mass divergences in Higgs mass. This is made possible by the mechanism called \textit{collective breaking of symmetries}. Two copies of the gauge group are required to achieve the \textit{collective breaking of symmetries}. Under \textit{collective breaking of symmetries}, the gauge and Yukawa interactions of Higgs are introduced such that they explicitly break the global symmetry. However the symmetry breaking is such that as long as only one set of coupling is present, enough global symmetry is preserved to protect the Higgs mass. Only when both sets of couplings are present logarithmic corrections to the Higgs mass is generated.

The most economical, in terms of the additional parameters and the particle content of the LH models, is the Littlest Higgs model \cite{2}. In Littlest Higgs model the global symmetry group is SU(5) which is broken down to SO(5) at the scale \(f \sim O(1\text{TeV})\). A subgroup : \([SU(2) \times U(1)]_1 \times [SU(2) \times U(1)]_2\) of global SU(5) is gauged to provide the gauge and Yukawa interactions in the model. In this minimal model the additional particles which appear at TeV scale are the heavy partners of the SM gauge bosons \((W_H, Z_H, A_H)\), a heavy vector like top quark (T) and the triplet scalar \((\Phi)\).

The Littlest Higgs model is very tightly constraint by the electroweak (EW) precision tests. The reason for this is the tree level contributions to the SM processes and triplet vev which breaks custodial SU(2) symmetry. These constraints requires the new physics scale to be \(f \geq 3\text{ TeV}\), which re-introduces the fine tuning and the \textit{little hierarchy problem}. To reconcile the LH models with EW precision tests, Cheng & Low \cite{3} introduced a discrete symmetry, named T-parity into these models.

T-parity forbids the tree level contributions to SM processes. The triplet vev in these model vanishes identically and custodial symmetry is restored. Hence it is easy to reconcile these models with the EW precision tests which brought down the scale of these models, consistent with experimental data, to \(f \sim 500\text{ GeV}\).

LEPTON FLAVOUR VIOLATION IN LHT

The detailed description of Littlest Higgs model with T-parity (LHT) model can be found in \cite{4}. Here we will briefly describe the features of LHT model required for the study of LFV.

All the interactions in T-parity model are proposed to conserve the discrete symmetry called T-parity. This results in the existence of a stable T-odd particle which can be identified as a possible candidate for dark matter. The T-odd gauge sector of LHT consists of \(W_H^\pm, Z_H\) and a heavy photon \(A_H\). Heavy photon \(A_H\) which is electrically neutral is the lightest T-odd heavy particle and is a suitable candidate for dark matter. The T-even fermionic sector of LHT consists of SM fermions and a T-even heavy vector like top quark \((T_+)\). The T-odd fermionic sector consists of a T-odd vector like top quark...
(T−) and three generation of mirror quarks and leptons, they are denoted by:
\[
\begin{pmatrix}
    u'_H \\
    d'_H
\end{pmatrix}, \quad \begin{pmatrix}
    V_H \\
    \delta_H
\end{pmatrix}, \quad \text{with } i = 1, 2, 3
\]
The masses of up and down type mirror fermions are equal up to the leading order in \((v/f)\). The masses of all the new particles introduced are of order \(f\). The interaction of mirror fermions with SM fermions and heavy gauge bosons introduces the new flavour interactions in the model involving two unitary matrices in quark sector \(V_{Hq}\) and \(V_{Hl}\), and two unitary matrices in lepton sector namely \(V_{Hl}\) and \(V_H\). These mirror quark and mirror lepton matrices are related via,
\[
V_{Hq}^\dagger V_{Hq} = V_{CKM}, \quad V_{Hl}^\dagger V_{Hl} = V_{PMNS}
\]
The explicit form of these matrices are given in [4]. These new mixing matrices along with mirror fermions are responsible for the new and rich flavour structure of LHT model.

The new parameters of LHT model which are relevant for the study of LFV decays are the symmetry breaking scale \((f)\), mirror lepton masses \((m_{Hl})\), mixing angles \((\theta_{ij}^f)\) and phases \((\delta_{ij}^f)\) of the mirror leptonic sector. These were tabulated in [8]: \(f, m_{H1}^f, m_{H2}^f, m_{H1}^l, \theta_{12}^f, \theta_{13}^f, \theta_{23}^f, \delta_{12}^f, \delta_{13}^f, \delta_{23}^f\).

There have been many studies of both quark and lepton flavour sector within the context of LH models [5, 7]. Without T-parity the LH models are Minimal Flavour Violating (MFV) models and hence the contribution of LH to the hadronic flavour violating processes comes out to be small. In addition there are no new phases and hence no new source of CP violation. In the leptonic sector of LH models due to triplet vev there was a possibility of writing down Lepton Number violating [6, 7] interactions which then could give rise to LFV. In LH model, Higgs triplet is essentially responsible for LNV and LFV and there is no new flavour structure in the model. The situation get changed substantially in LHT model which has additional flavour structure. The introduction of T-parity not only makes the LH models more consistent with EW precision tests but also give rise to new flavour structure described by new flavour mixing matrices. This makes the LHT model a non-MFV model which not only has a much richer flavour structure but also has new weak phases for CP violating studies. Extensive studies of these in the case of quark sector has been done [4]. In quark sector SM processes still play dominant role in most of the interactions although the presence of additional weak phases can have interesting consequences. The situation is very different in the case of leptonic sector. The smallness of active neutrino mass forces SM to have an observably small LFV in charged lepton sector. The new flavour structure of LHT model could provide much larger contribution to LFV processes which can be observed in future experiments. The absence of QCD in leptonic sector allows one to make very clean predictions for LFV processes. LFV within the context of LHT model was studied in [8, 9].

**RESULTS**

LFV in LHT model was discussed for the first time in [9] but a detailed analysis including all the LFV processes was done in [8]. The estimation of \((g-2)_{\mu}\) in LHT model was done in [8, 9]. It was found that muon anomalous magnetic moment can not provide any useful constraints on LHT parameters.

A study of radiative LFV modes \(\ell_i \rightarrow \ell_j \gamma\), where \(i \neq j\) was done in [8, 9]. These studies showed that LHT can give substantial contribution to the radiative LFV processes. In that work [9] absence of correlation between various radiative decays i.e. \(\mu \rightarrow e\gamma, \tau \rightarrow (\mu, e)\gamma\) was also emphasized. This indicates that these three modes can provide independent probes to the lepton flavour sector of the model [9]. It was shown that the present limits on \(\mu \rightarrow e\gamma\) could provide very stringent constraints on LHT model, furthermore the experimental prospects of this mode seems very promising as MEG will soon improve the prediction of this mode by two orders in magnitude. Although radiative modes involving tau lepton, namely \(\tau \rightarrow (\mu, e)\gamma\) are not strongly correlated to \(\mu \rightarrow e\gamma\), but the existing constraints on radiative LFV tau decays from B-factories are weak and hence do not provide any further constraints on LHT parameter space. The situation could change in future as SuperB factories, that are expected to probe these decays upto the accuracy of \(\sim 10^{-9}\).

An extensive study of the correlations of LFV processes in LHT and its comparison with SUSY models was done in [8]. A summary of their results is given in Table 1. In their work they gave the results for LFV modes having three charged leptons in the final state \(i.e. \mu/\tau \rightarrow \ell_i\ell_j\ell_k\) and their correlation with other LFV modes. Their results show that the prediction of the ratio of the rates of \(\mu^{-} \rightarrow e^{-}e^{+}e^{-}\) to \(\mu \rightarrow e\gamma\) in LHT model can be substantially different from SUSY models. It is well known that in SUSY dipole operators give the dominant contribution to these modes. The LFV tau decay modes, like \(\tau \rightarrow \ell_i\ell_j\ell_k\) in SUSY, receives contributions from dipole operators and Higgs mediated scalar operators. It is evident from the results given in Table 1 that in SUSY, Higgs mediated contributions can be dominant for modes having muons in final state. In SUSY the modes dominated by dipole contributions show very strong correlation between \(\tau \rightarrow \ell_i\ell_j\ell_k\) and \(\tau \rightarrow \ell_i\gamma\) (with \(i = e, \mu\)) which tends to get relaxed for Higgs mediated contributions. But the predictions of these ratios in LHT
are strikingly different from SUSY models even if we include Higgs contributions.

Another notable difference in SUSY and LHT model predictions of LFV processes comes while correlating processes \( \tau \to \ell_i \ell_j \ell_k \) i.e. tau decays having three leptons in final state. For this purpose following ratios were constructed in [8]:

\[
R_1 = \frac{Br(\mu^- \to e^- e^+ e^-)}{Br(\mu^- \to e^- \gamma)} ,
\]

\[
R_2 = \frac{Br(\tau^- \to e^- e^+ e^-)}{Br(\tau^- \to e^- \mu^+ \mu^-)} ,
\]

\[
R_3 = \frac{Br(\tau^- \to \mu^- \mu^+ \mu^-)}{Br(\tau^- \to \mu^- e^+ e^-)}
\]

As can be seen from table 1 these ratios can be substantially different in SUSY and LHT. The reason for these differences in LHT and SUSY lies in the mechanism responsible for LFV in these models. Whereas dipole operators are responsible for radiative modes \( (\ell_i \to \ell_j \gamma) \) in both these models, in SUSY, LFV in \( \mu / \tau \to \ell_i \ell_j \ell_k \) arises due to photon mediated dipole operators and Higgs mediated scalar operators. The Higgs mediated scalar operators can be dominant in decays involving tau lepton. In SUSY both of these contributions come from penguin or self energy diagrams. On the other hand in LHT the dipole contributions can be almost completely neglected in comparison with the Z-penguin and box diagrams. The dipole dominance gives a relatively stable prediction for \( R_1 \) in SUSY whereas this ratio can change a bit in LHT where \( \mu^- \to e^- e^+ e^- \) is not dominated by dipole contributions. \( R_2 \) and \( R_3 \) for LHT model are of order one and do not change much on changing the model parameters. This is because in LHT the Z-penguin and box diagram contributions are nearly equal for the decays of type \( \tau \to \ell_i \ell_j \ell_k \) and hence the ratios of these processes is stable. In the case of SUSY, Higgs mediated diagrams give rise to scalar operators that can alter the predictions of the mode having muons in final state \( (\tau^- \to \mu^- \mu^+ \mu^-) \) as opposed to LHT.

In summary, LHT model has the structure to provide LFV which can be observed in future experiments. LFV processes, if observed, can also be used to distinguish the models responsible for these processes.

| TABLE 1. Comparison of various ratios of branching ratios in the LHT model and in the MSSM using the dipole and Higgs mediated contributions [8]. |
|-----------------|-----------------|-----------------|-----------------|
| ratio           | LHT             | MSSM (dipole)   | MSSM (Higgs)    |
| \( Br(\mu^- \to e^- e^+ e^-) \) | 0.4...2.5       | \( \sim 6 \cdot 10^{-3} \) | \( \sim 6 \cdot 10^{-3} \) |
| \( Br(\mu^- \to e^- \gamma) \) | 0.4...2.3       | \( \sim 2 \cdot 10^{-3} \) | 0.06...0.1       |
| \( Br(\tau^- \to e^- e^+ e^-) \) | 0.3...1.6       | \( \sim 2 \cdot 10^{-3} \) | 0.02...0.04      |
| \( Br(\tau^- \to e^- \mu^+ \mu^-) \) | 1.3...1.7       | \( \sim 5 \)     | 0.3...0.5        |
| \( Br(\tau^- \to \mu^- e^+ e^-) \) | 1.2...1.6       | \( \sim 0.2 \)   | 5...10           |

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