Recent MILC spectrum results

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We report on results from three spectrum calculations with staggered quarks: 1) a quenched calculation with the standard action for the gluons and quarks; 2) a quenched calculation with improved actions for both the gluons and quarks; and 3) a calculation with two flavors of dynamical quarks using the standard actions for the gluons and quarks.

1. INTRODUCTION

A controlled calculation of the masses of the light hadrons has been one of the major goals of lattice QCD[1]. It has become increasingly clear in recent years that in order to carry out such a calculation one must perform high statistics simulations using a range of lattice volumes in order to control finite size effects, a range of quark masses in order to extrapolate to the chiral limit, and a range of lattice spacings in order to extrapolate to the continuum limit. We have carried out such a series of simulations in the quenched approximation with staggered quarks[2]. In Sec. 2 we report on new results from this study with particular emphasis on the extrapolation to the chiral limit.

Improved actions have the potential to significantly increase our ability to perform accurate spectrum calculations[3]. In Sec. 3 we report on exploratory quenched calculations using improved actions for the gluons and the staggered quarks. Finally, in Sec. 4 we show preliminary results from a series of full QCD calculations with two flavors of staggered quarks at gauge coupling $\frac{6}{g^2} = 5.50$.

2. QUENCHED SPECTRUM WITH STAGGERED QUARKS

Over the past several years we have carried out a series of quenched spectrum calculations with staggered quarks. Calculations were performed with four values of the gauge coupling, $\frac{6}{g^2} = 5.54, 5.70, 5.85$, and 6.15. Five different lattice volumes were used at 5.70 and three at 5.85 in order to study finite volume effects. Five quark masses were used at each coupling. For $\frac{6}{g^2} = 5.70$ and 5.85, we used $a m_q = 0.16, 0.08, 0.04, 0.02$ and 0.01. At 6.15 the masses were halved, and at 5.54 they were doubled. For each coupling, lattice volume and quark mass we performed correlated fits to determine the masses. An Edinburgh plot is shown in Fig. 1.
Hadron masses calculated on the same set of gauge configurations, but with different quark masses are, of course, correlated. So, we performed correlated fits for each coupling and lattice volume to extrapolate to the chiral limit. We used a wide variety of fitting functions with terms inspired by quenched chiral perturbation theory. The first conclusion was that simple linear or quadratic fits to the nucleon and rho masses are inconsistent with our data. Good fits are possible. For example, the function $M = a + bm_q^2$ gives reasonable fits for both the nucleon and rho masses. We also obtain good fits for the nucleon with functions containing square roots or logarithms of the quark masses suggested by quenched chiral perturbation theory. Once such a fit is in hand, we can investigate the dependence of $m_N/m_\rho$ as a function of lattice spacing for fixed $m_\pi/m_\rho$. Some typical plots are shown in Fig. 2 using the above fitting form. These fits allow us to extrapolate to the continuum limit for fixed values of $m_\pi/m_\rho$. They are also proving very useful in comparing results obtained with improved actions to our high statistics results with the standard staggered action.

In Fig. 3 we plot $m_N/m_\rho$ as a function of $am_\rho(m_q = 0)$ with the ratio $m_\pi/m_\rho$ held fixed at its physical value. Fits to the rho and nucleon used in this plot were again made with the form $M = a + bm_q^{3/2} + cm_q^2$. The solid curve is a fit of the form $c_0 + c_1a^2$ to data for all four couplings, and the dotted line is a fit of the same form to data for the three weakest couplings. Here $a$ is the lattice spacing, and $c_i$ are fitted parameters. We are unable to uniquely determine the optimum fitting function for the chiral extrapolation from our data. In particular, nucleon fits with $m_q^{1/2}$ terms have confidence levels similar to those shown in Figs. 2 and 3. An $m_q^{1/2}$ terms tend to pull the value of $m_N$ down in the chiral limit, thereby reducing $m_N/m_\rho$. At present the uncertainty in the choice of fits is the largest source of error in our determination of $m_N/m_\rho$.

3. QUENCHED SPECTRUM WITH IMPROVED STAGGERED QUARKS

Up to now, most of the effort aimed at improving the quark action has focused on Wilson quarks; however, some time ago Naik proposed adding third nearest neighbor hopping terms to the staggered quark action, adjusted to remove the order $a^2$ errors in the lattice quark propagator[4]. We are experimenting with a tadpole...
improved version of this action to carry out a quenched spectrum calculation with gauge configurations generated using a tadpole-Symanzik improved gauge action\cite{5}. We have generated configurations at three different couplings, $\beta_{pl} = 6.8$, 7.1 and 7.4, on lattices ranging in size from $8^3 \times 16$ to $16^3 \times 32$. We have measured the spectrum on these lattices with both the tadpole improved Naik action and the standard Kogut-Susskind action. In Fig. 4 we show the Edinburgh plot for the Naik (crosses) and standard staggered (diamonds) quark actions on $16^3 \times 32$ lattices at $\beta_{pl} = 7.4$. For comparison, we also show quenched spectrum results with the standard gauge and staggered fermion actions (fancy plusses) at $6/g^2 = 5.54$. This coupling corresponds to approximately the same lattice spacing, as measured by the rho mass, as the improved action at $\beta_{pl} = 7.4$. One sees from Fig. 4 that the improved gluon action does lead to lower values of the nucleon to rho mass ratio at small values of $m_{\pi}/m_\rho$, than the standard gluonic action. However, the Naik term has little effect at the values of the coupling constants and quark masses that we have investigated to date.

An important test of improved staggered quark actions is the extent to which they restore flavor symmetry. Our preliminary results indicate that the mass difference between the $\pi$ and $\pi_2$ does decrease with the improved gluon action, but that the Naik term has little impact. Although our tests of the tadpole improved Naik action have not been very encouraging at the parameters studied to date, it should be noted that this action has the advantage of significantly hastening the rate at which thermodynamic quantities approach the continuum limit for free quarks. One might therefore hope that it would lead to a significant improvement in high temperature QCD calculations, and the Bielefeld group has shown that this is the case\cite{6}. It would therefore seem that this action should be studied in greater detail.

4. SPECTRUM WITH TWO FLAVORS OF STAGGERED QUARKS AT $6/g^2 = 5.50$

As part of our study of the decay constants of heavy-light mesons, we are generating a set of lattices with two flavors of Kogut-Susskind quarks at gauge coupling $6/g^2 = 5.50$. We are using quark...
masses $m_q = 0.10, 0.05, 0.025$ and 0.0125. All lattices have a time dimension $N_t = 64$. We use spatial volumes $20^3$ for the two lighter masses, and $24^3$ for the two heavier ones. (The use of larger spatial volumes for the heavier quark masses was dictated by the peculiarities of the CM5 on which the heavier quark mass calculations were performed). We have calculated the spectrum using the first 1,000 equilibrated molecular dynamics time units for each quark mass. The Edinburgh plot is shown in Fig. 5. We have included quenched data for gauge couplings $6/g^2 = 5.70, 5.85$ and 6.15 for comparison. The data indicates a somewhat faster fall off of $m_N/m_\rho$ than the quenched data at $6/g^2 = 5.70$, which has a comparable lattice spacing. This is to be expected because lighter dynamical quarks will lead to a greater renormalization of the coupling.

Figure 5. The Edinburgh plot for full QCD with two flavors of staggered quarks at $6/g^2 = 5.50$.

In Fig. 6 we plot the masses of the $\pi$ and the $\pi_2$ as a function of $am_\rho$ to illustrate the extent of flavor symmetry breaking in this data set. We also include very preliminary results from a set of runs at $6/g^2 = 5.60$ on $24^3 \times 64$ lattices with quark masses 0.01, 0.02, 0.04 and 0.08. These results are from approximately 600 equilibrated trajectories at each mass.

Figure 6. $m_\pi$ and $m_{\pi_2}$ as a function of $am_\rho$ for full QCD at $6/g^2 = 5.50$ and 5.60. The 5.60 results are quite preliminary.

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