Selection of Input Variables in DEA using 2-Level Fractional Factorial Design

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Abstract: Data Envelopment Analysis (DEA) can be a statistics oriented, non-parametric method to gauge relative efficiency supported pre-selected Inputs and Outputs. It is a implemented mathematics based technique for measuring the relative overall performance of organisational units in which the presence of Multiple Inputs and Outputs makes evaluation difficult. In a few cases, the performance model isn’t well defined, so it’s important to pick the proper Inputs and Outputs by way of other means. We used, Morita and Avkiran proposed technique after it has been developed an Input – Output Selection Method that uses Fractional Factorial design, which is an Statistical method to locate a best and optimal combination. In this study $2^{k-p}$ Fractional Factorial design is applied to demonstrate the proposed method relates to the Manufacture of Pharmaceuticals, Medicinal Chemical and Botonical Products from the Manual of Annual Survey of Industries (ASI) 2016 – 2017.

Keywords: Data Envelopment Analysis (DEA), Decision Making Units (DMUs), CCR, BCC, Super Efficiency, Mahalanobis Distance.

I. INTRODUCTION

Companies at some stage in this Industry segment supply the lively elements employed with the aid of pharmaceutical companies to manufacture completed products, called the pharmaceutical preparations. Companies in this Industry segment furnish the active ingredients utilized by pharmaceutical corporations to manufacture completed merchandise, known as the pharmaceutical preparations. Active aspect constitute the part of a finished drug that creates the preferred effect – therapeutic or preventive for human and animals assets are vital examples of the additives produced with the aid of this industry sector. By the 1960’s synthesized chemicals both a manufactured reproduction of an organic or in natural substance or a New Chemical Entity (NCE) had become commonplace active ingredients in pharmaceuticals from vitamin drugs to hormones. Meanwhile, the bio generation revolution beginning the earliest in the 1980’s, resulted in methods of putting genetic cloth into small micro organisms. This made them miniature factories for the meeting of active drugs components like Insulin and in the process, created New Molecular Entities (NME’s) that might be patented.

By the overdue 1990’s, the primary market of this Industry the pharmaceutical industry risked high studies and development prices for the prized billions of dollars it may generate with new merchandise. Although many drug agencies have vertically manufacturing lines, a fashion grew to favour outsourcing chemical intermediates and active ingredients to smaller first-rate chemical organizations.

This category covers institutions generally engaged in manufacturing bulk natural and inorganic medicinal chemical compounds and their derivatives additionally as processing grading, grinding, Milling bulk botanical capsules and herbs.

In this paper, we are interested to show the green performance of the organizations in manufacturing pharmaceutical, Medicinal Chemicals and Botonical products and also makes use of the diagonal layout test to select output variables which is a statistical approach to find an optimal mixture to analyse the performance of the above point out Industry through the usage of the approach Data Envelopment Analysis.

Data Envelopment Analysis (DEA) is a linear programming based method for measuring the performance performance of similar kind of organisational gadgets termed as Decision Making Units (DMU’s), with Multiple Inputs and Outputs. It identifies a subset of green “Best Practice” DMUs and for closing DMUs, the importance of their non – performance is measured by comparing to a frontier comprised of the green DMUs. Data Envelopment Analysis (DEA) can be a statistics oriented, non-parametric method to evaluate relative efficiency based on pre – selected Inputs and Outputs.

Decision Making Units (DMUs) refers to the similar form of businesses that consumes a variety of same Inputs to produce a variety of equal outputs. DMU can encompass production units, Department of massive Organisations consisting of Universities, Schools, Bank Branches, Hospitals, Power Plants, Police Stations, Tax Offices etc.

The rest of the paper is organized as followings: Section II, review of literature. In Section III, a procedure of Selecting Input variables using 2 – level Fractional Factorial Design and Mahanalobis distance is presented. An empirical example is described in Section IV to illustrate the methodology presented herein. The main results of the paper are summarized in Section V.

II. REVIEW OF LITERATURE

Farrel (1957) is taken into consideration to be the most influential paper on DEA. The similarly pioneering contributions were made by using CCR (1978, 1979) and CCR (1981). Banker, Charnes and Cooper (1984) and Charnes et al (1985). Banker and Morey (1986) have evaluated the relative technical and scale efficiencies of DMUs by way of mathematical programming formulations when some of the inputs and outputs are exogenously fixed and past the discretionary control of DMU personnel. A massive quantity of papers have prolonged and implemented the DEA Methodology (Coelli, 1996).Zhu (1996b) and Seiford and Zhu (1998d)
develop some of new remarkable Efficiency fashions to
determine the efficiency stability regions. Anderson and
Peterson (1993) advocate the usage of the CRS Super
Efficiency version in rating the efficient DMUs. Also, the
Super Efficiency can be used in detecting influential
observations (Wilson 1995) and in identifying the intense
efficient DMUs (Thrall, 1966). Seiford and Zhu (1999c)
examine the infeasibility of diverse Super Efficiency
fashions developed from the envelopment fashions offers
other first-rate performance models which can be utilized in
Sentivity Analysis. Hiroshi Morita and Necmi. K. Avkiran
(2008) advanced the method for selecting Inputs and
Outputs in information envelopment analysis with the aid of
designing statistical experiments. Hulya Bayrak,
Oday Jayies and Kubra Durukan demonstrated the proposed
approach of Morita and Avkiran using of Fractional
Factorial layout rk – p in information Envelopment Analysis
to selection of Outputs and Inputs.

### III. METHODOLOGY

DEA converts multiple inputs and outputs into scalar
measure of efficiency. There are essentially two varieties of
DEA Models CCR Model (Constant Returns to Scale) and
BCC Model (Variable Return to Scale). The two
envelopment versions, one involving \( \theta \) and the other
involving \( \phi \). The version involving \( \theta \) aims to produce the
observed Outputs with minimum Inputs and it is referred to as
Input Oriented Envelopment DEA Program. The other
version involving \( \phi \) refers to as an Output Oriented
Envelopment DEA Program as it aims to maximize output
production, subject to the given resource level.

| Models | Input Oriented | Output Oriented |
|--------|----------------|-----------------|
| I.CCR  | \( \text{(i) Min } \theta \) | \( \text{(ii) Max } \phi \) |
|        | \text{Subject to,} \ Y_\lambda \geq Y_0 \ |
|        | \text{X}_\lambda \leq 0 \_X_0 \quad \lambda \geq 0; \ \theta \ free \ |
|        | \text{BCC} \ |
|        | \text{(iii) Min } \theta \ |
|        | \text{Subject to,} \ Y_\lambda \geq Y_0 \ |
|        | \text{X}_\lambda \leq 0 \_X_0 \quad \lambda \geq 0; \ \theta \ free \ |
|        | \sum_{n=1}^{N} \lambda_n = 1 \ |
|        | \text{(iv) Max } \phi \ |
|        | \text{Subject to,} \ Y_\lambda \geq \phi Y_0 \ |
|        | \text{X}_\lambda \leq X_0 \quad \sum_{n=1}^{N} \lambda_n = 1 \quad \lambda \geq 0; \ \phi \ free \ |

Where \( \theta \) and \( \phi \) = Efficiency Measure
\( X = [X_1, X_2, X_3, \ldots, X_N] \) = Vector of Inputs
\( Y = [Y_1, Y_2, Y_3, \ldots, Y_N] \) = Vector of Outputs
\( \lambda = [\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_N] \) = Vector of Weights
\( Y_0 = \text{Output of the Observed DMU} \)
\( X_0 = \text{Input of the Observed DMU} \)
\( N = \text{Number of DMUs} \).

The Model (1) is an Input and Output Oriented CCR
Model. The Input and Output Oriented Envelopment Model
is solved for each DMU in the set, it gives an efficiency
score \( \theta \) or \( \phi \) and DMU weights \( \lambda_n \). If \( \theta = 1 \) and \( \phi = 1 \), then
the DMU is said to be Efficient and there is no slacks. If \( \theta < 1 \) and \( \phi > 1 \), then the DMU is Inefficient related to best
practice DMU in the sample and efficient DMUs comprise
the efficient frontier.

The Model (2) is an Input and Output Oriented BCC
Model. The Input and Output Oriented Envelopment Model
is solved for each DMU in the set, it gives an efficiency
score \( \theta \) or \( \phi \) and DMU weights \( \lambda_n \). If \( \theta = 1 \) and \( \phi = 1 \), then
the DMU is said to be Efficient and there is no slacks. If \( \theta < 1 \) and \( \phi > 1 \), then the DMU is Inefficient related to best
practice DMU in the sample and efficient DMUs comprise
the efficient frontier.

#### 3.1 Super Efficiency Model:

In this paper, with the intention to rank DMUs, we use the
assessment contexts which can be acquired by means of
partitioning the set of DMUs into several degrees of
Efficiency, and rank all DMUs with two criteria: the High
and Low performers. They have an effect on all DMUs, each
Efficient and Inefficient in Ranking. Author have
proposed strategies for rating the first-class performers, as
an instance using Super – Efficiency DEA Model. When a
DMU underneath evaluation isn't always included within the
reference set of the envelopment Models, the ensuing DEA
Models are called Super – Efficiency DEA Models.

As in Charnes, Cooper and Thrall (1991), the DMUs can
be partitioned into four classes E, E’, F and N described as
follows and the extreme efficient DMUs can be identified by
the Super .efficiency Models.

- **E** – The set of Efficient DMUs
- **E’** – The set of Efficient DMUs that aren’t extreme points and
  expressed as linear combinations of the DMUs in set E.
- **F** – The set of Frontier factors (DMUs) with non zero
  slack(s) are known as Weakly Efficient.
- **N** – The set of Inefficient DMUs.

#### 3.2 Selection of Input Variables Using a 2 – Level
Fractional Factorial Design:

Based on the Efficiency score, the whole DMU’s are
categorized into two groups namely High performers and
low performers with the objective of selecting the
appropriate Input variables. The distance among the
measures is measured by the Mahalanobis distance.
3.3 Mahalanobis Distance:
In Statistics, Mahalanobis distance (MD) can be a distance measure brought by means of P.C. Mahalanobis in 1936. It's supported between variables by way of which exclusive styles are often identified and analysed. It gauges similarity of an unknown sample set to a acknowledged one. It differs from Euclidean distance therein it takes below consideration the correlations of the data set and is scale-invariant. In other words, Mahalanobis distance is a Multivariate effect size.

Formally, the MD of a Multivariate vector \( x = (x_1, x_2, ..., x_n)^T \) from a set of values with mean vector \( \mu = (\mu_1, \mu_2, ..., \mu_n)^T \) and the Covariance Matrix \( \Sigma \) is given by

\[
D_M(x) = \sqrt{(x - \mu)^T \Sigma^{-1} (x - \mu)} \quad \text{------------------(1)}
\]

The above equation is rewritten for the pattern following as

\[
\tilde{D}_M(x) = \sqrt{(x - \bar{x})^T S^{-1} (x - \bar{x})} \quad \text{---- (2)}
\]

Where, the Mean Vector and Covariance Matrix of the pattern are given as \( \bar{x} \) and \( S \) respectively.

The MD threshold is another important detail of prognostics analysis. An MD threshold value that's either too massive or too small results in false negatives or false positives, respectively. In this study, the writer considers the space of one-dimensional variables, wherein MD coincides with Welch Statistics. The Welch Statistics is given as

\[
d = \frac{\bar{h}_k - \bar{v}_l}{\sqrt{\frac{1}{n_h} \frac{1}{n_l}}} \quad \text{-----------------(3)}
\]

Where the average and variance of each sets are given as \( \bar{h}_k \) for \( n_h \) high performance, \( \bar{v}_l \) for \( n_l \) low performance.

3.4.2 - Level Fractional Factorial Design:
The intention is to locate that combo of Input variables which maximizes the distance \( d \). In our evaluation, we carried out the 2-level Factorial Design. When there are \( k \) candidates of Input variables, the overall total number of combination is \( 2^k \). Full Factorial designs perform all of \( 2^k \) combos for \( k \) candidates. On the other hand, we will outline a \( 2^{k-p} \) design to be a Fractional Factorial Design with \( k \) candidates, each at 2 levels, consisting together with \( 2^{k-p} \) runs. The first \( (k-p) \) candidates are a part of \( 2^{k-p} \) combinations as a Full Factorial Design, and the final \( p \) candidates may be generated as interactions with the first \( (k-p) \) columns. Table 1 shows an example of a Fractional Factorial Design where \( k = 7 \), \( p = 3 \), and \( x_i \) is a candidate variable. \( '+' \) shows that the variable is selected as an Input, and \( '-' \) shows that the variable is not selected as an Input. For example, the variables \( x_2, x_5, x_6 \) are decided as a selected Input variables in Run No.3.

Based on the Fractional Factorial design in Table 1, we calculate the performance efficiency scores by means of Model 1 (i) and Mahalanobis distance by using (3) between two performers using the selected Input variables. The analysis of Variance (ANOVA) for the Fractional Factorial Design is shown in Table 2.

**Table 1: Fractional Factorial Design for \( 2^7-3 \) and selected Inputs**

| Runs | \( x_1 \) | \( x_2 \) | \( x_3 \) | \( x_4 \) | \( x_5 \) | \( x_6 \) | \( x_7 \) | Selected Inputs | Mahalanobis Distance |
|------|--------|--------|--------|--------|--------|--------|--------|-----------------|-------------------|
| 1    | -      | -      | -      | -      | -      | -      | -      | None            | \( d_{1} \)       |
| 2    | +      | -      | -      | -      | +      | -      | +      | \( x_1, x_2, x_7 \) | \( d_{2} \)       |
| 3    | -      | +      | -      | -      | +      | +      | -      | \( x_2, x_3, x_6 \) | \( d_{3} \)       |
| 4    | +      | +      | -      | -      | +      | +      | -      | \( x_1, x_2, x_6 \) | \( d_{4} \)       |
| 5    | -      | -      | +      | -      | +      | +      | -      | \( x_3, x_5, x_6 \) | \( d_{5} \)       |
| 6    | +      | -      | +      | -      | +      | +      | -      | \( x_1, x_3, x_6 \) | \( d_{6} \)       |
| 7    | -      | +      | +      | -      | -      | +      | -      | \( x_2, x_3, x_7 \) | \( d_{7} \)       |
| 8    | +      | +      | +      | -      | -      | +      | -      | \( x_1, x_2, x_5 \) | \( d_{8} \)       |
| 9    | -      | -      | -      | +      | +      | +      | -      | \( x_4, x_6 \) | \( d_{9} \)       |
| 10   | +      | -      | -      | +      | +      | +      | -      | \( x_1, x_4, x_6 \) | \( d_{10} \)      |
| 11   | -      | +      | +      | +      | +      | +      | -      | \( x_2, x_4, x_5 \) | \( d_{11} \)      |
| 12   | +      | +      | -      | +      | -      | +      | -      | \( x_1, x_2, x_4 \) | \( d_{12} \)      |
| 13   | -      | -      | +      | +      | +      | +      | -      | \( x_3, x_4 \) | \( d_{13} \)      |
| 14   | +      | -      | +      | +      | -      | +      | -      | \( x_1, x_3, x_4 \) | \( d_{14} \)      |
| 15   | -      | +      | +      | +      | +      | +      | -      | \( x_2, x_3, x_4 \) | \( d_{15} \)      |
| 16   | +      | +      | +      | +      | +      | +      | +      | \( x_1, x_2, x_3, x_4, x_5, x_6, x_7 \) | \( d_{16} \)      |

The total sum of squares \( S_T \) is given as

\[
S_T = \sum_{i=1}^{p} (d_i - \bar{d})^2 \quad \text{----------------- (8)}
\]

The sum of squares \( S_i \) for candidate \( i \) reflects the main effect of the variable, which is the difference between '+' and '-' as

\[
S_i = 2\{d (x_i +) - \bar{d}(x_i -)\} \quad \text{----------------- (9)}
\]

\( \bar{d} \) (\( x_i \)) is the Mean of the Mahalanobis distances observed when \( x_i = + \). The residual sum of squares \( S_E \) is given by subtracting the sum of \( S_i \) from \( S_T \).
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\[ S_E = S_T - (S_1 + S_2 + S_3 + S_4 + S_5 + S_6 + S_7 + S_8 + S_9 + S_{10} + S_{11} + S_{12} + S_{13} + S_{14} + S_{15} + S_{16}) \quad \text{------ (10)} \]

The total degree of freedom is \( \phi_T = 7 \), which is the number of runs minus 1, and the degree of freedom for each sum of squares is \( \phi_i = 1 \). Therefore the degree of freedom for the residual is given as,

\[ \phi_E = \phi_T - ( \phi_1 + \phi_2 + \phi_3 + \phi_4 + \phi_5 + \phi_6 + \phi_7 + \phi_8 + \phi_9 + \phi_{10} + \phi_{11} + \phi_{12} + \phi_{13} + \phi_{14} + \phi_{15} + \phi_{16} ) \quad \text{--- (11)} \]

The Null Hypothesis that the candidate has no effect as an Input is tested by using F Statistics, \[ F = \frac{S_i/\phi_i}{S_E/\phi_E} \quad \text{------- (12)} \]

The test rejects the Null Hypothesis at level \( \alpha \) if \( F \) – value exceeds \( \alpha \) percentile of \( F \) distribution with degrees of freedom \( (\phi_i, \phi_E) \), and the hypothesis tests is as following:

- \( H_0: \) The variable candidate has no impact on Input.
- \( H_1: \) The variable candidate has impact on Input.

This results within the optimal aggregate of Input variables.

### Table 2: ANOVA table for Fractional Factorial Design of 27 – 3

| Variables | Sum of Squares | Degrees of Freedom | Mean Sum of Squares | F Statistics |
|-----------|---------------|--------------------|---------------------|--------------|
| \( x_1 \) | \( S_1 \) | \( \phi_1 = 1 \) | \( V_1 = \frac{S_1}{\phi_1} \) | \( V_1 \) |
| \( x_2 \) | \( S_2 \) | \( \phi_2 = 1 \) | \( V_2 = \frac{S_2}{\phi_2} \) | \( V_2 \) |
| \( x_3 \) | \( S_3 \) | \( \phi_3 = 1 \) | \( V_3 = \frac{S_3}{\phi_3} \) | \( V_3 \) |
| \( x_4 \) | \( S_4 \) | \( \phi_4 = 1 \) | \( V_4 = \frac{S_4}{\phi_4} \) | \( V_4 \) |
| \( x_5 \) | \( S_5 \) | \( \phi_5 = 1 \) | \( V_5 = \frac{S_5}{\phi_5} \) | \( V_5 \) |
| \( x_6 \) | \( S_6 \) | \( \phi_6 = 1 \) | \( V_6 = \frac{S_6}{\phi_6} \) | \( V_6 \) |
| \( x_7 \) | \( S_7 \) | \( \phi_7 = 1 \) | \( V_7 = \frac{S_7}{\phi_7} \) | \( V_7 \) |
| Error | \( S_E \) | \( \phi_E = 8 \) | \( V_E = \frac{S_E}{\phi_E} \) |
| Total | \( S_T \) | \( \phi_T = 15 \) |

The following is the summary procedure for the selection of variables.

- List potential Input Output Variables
- Used Super Efficiency Model as external criteria to distinguish the performance of two groups, e.g. high and low performance. (Here the author used BCC Model and Super Efficiency Model for grouping.)
- Assign the variables to the selected 2 – level Orthogonal layout and determine the combination of selected variables used in the experiments. (To Select Input or Output Variables alone 2 – level Fractional Factorial Orthogonal layout is used and to select both Input and Output Variables then choose 3 – level Fractional Factorial design)
- Calculate the DEA Efficiency Score and Mahalanobis distance among the two groups by using the chosen variables.
- Determine the most effective combination of Input and Output variables based totally on consequences of Analysis of Variance.
- Identify the most efficient designation of statistically great variables as either an Input or an Output the usage of Mahalanobis distance.

### IV. DATA DESCRIPTION

The data analysed in this paper is a secondary data of Manufacturing Pharmaceuticals, Medicinal Chemical and Botanical Products taken from the published Manual of Annual Survey of Industries (ASI) 2016 – 2017. The data provides the information about the Number of Employees, Number of Factories in Operation, Fixed Capital, Gross Value of Plant and Machinery, Materials Consumed, Fuel Consumed, Wages and Salaries, Net Value Added, Total Output, Income, Profit and Gross Capital Information for each States (i.e., 35 – Including Union Territories) during the year 2016 - 2017. Here, the author considered only the 24 States (Including Union Territories) as DMUs by considering only the positive values of the variables and each DMU is characterized by the following Inputs and Outputs as shown in Table 3. (Appendix 1)

#### 4.1 EMPIRICAL ANALYSIS:

Based on the data structure, the results of the BCC Model in respect of data structure are given below

### Table 4: Efficiency Score, Peers And Rank By BCC Model

| S. No | State/UT/Division | Efficiency Score | Status   | Peer Weights | Peer Count | Rank |
|-------|-------------------|------------------|----------|--------------|------------|------|
| 1     | Andhra Pradesh    | 1                | Efficient|              |            | 2    |
| 2     | Assam             | 1                | Efficient|              | 2          | 4    |
From the above table, the author could not categorize the DMUs in groups because of the tie ranks. Assam, Gujarat and Rajasthan with Rank 4, Similarly, Dadar& Haveli, Delhi, Jammu and Kashmir, Madhya Pradesh and Uttar Pradesh with Rank 5. So, for instance using Super – Efficiency DEA Model the tie ranks were solved and the DMUs were categorized in two groups namely High Performance and Low Performance by considering the set of extreme efficient DMUs in High Performance and the set of inefficient DMUs in Low Performance as shown in the below Table 5.

| S. No | State/UT/Division | Efficiency Score | Status          | Rank |
|-------|-------------------|------------------|-----------------|------|
| 1     | Andhra Pradesh    | 1.6818           | Extreme Efficient | High Performance |
| 2     | Assam             | 1.8113           | Extreme Efficient | High Performance |
| 3     | Bihar             | 0.8821           | Inefficient     | Low Performance |
| 4     | Chhattisgarh      | 1                | Efficient       | High Performance |

### Table 5: Efficiency Score by Super Efficiency Model

| S. No | State/UT/Division | Efficiency Score | Status          |
|-------|-------------------|------------------|-----------------|
| 1     | Andhra Pradesh    | 1.6818           | Extreme Efficient |
| 2     | Assam             | 1.8113           | Extreme Efficient |
| 3     | Bihar             | 0.8821           | Inefficient     |
| 4     | Chhattisgarh      | 1                | Efficient       |
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| 5 | Dadra and Nager Haweli | 1.3029 | Extreme Efficient | High Performance |
|---|-----------------------|--------|------------------|-----------------|
| 6 | Delhi                 | 1.1993 | Efficient        |                 |
| 7 | Goa                   | 1.8803 | Extreme Efficient| High Performance|
| 8 | Gujarat               | 1.3534 | Extreme Efficient| High Performance|
| 9 | Haryana               | 10.5115| Extreme Efficient| High Performance|
| 10| Himachal Pradesh      | 0.8266 | Inefficient      | Low Performance |
| 11| Jammu and Kashmir     | 1.5722 | Extreme Efficient| High Performance|
| 12| Karnataka             | 0.72   | Inefficient      | Low Performance |
| 13| Kerala                | 1.0457 | Efficient        |                 |
| 14| Madhya Pradesh        | 0.8307 | Inefficient      | Low Performance |
| 15| Maharashtra           | 0.8135 | Inefficient      | Low Performance |
| 16| Odisha                | 0.6101 | Inefficient      | Low Performance |
| 17| Puducherry            | 0.7678 | Inefficient      | Low Performance |
| 18| Rajasthan             | 1.1045 | Efficient        |                 |
| 19| Sikkim                | 2.4227 | Extreme Efficient| High Performance|
| 20| Tamil Nadu            | 0.5373 | Inefficient      | Low Performance |
| 21| Telegana              | 0.8074 | Inefficient      | Low Performance |
| 22| Uttar Pradesh         | 1.9017 | Extreme Efficient| High Performance|
| 23| Uttarakhand           | 0.7991 | Inefficient      | Low Performance |
| 24| West Bengal           | 0.9029 | Inefficient      |                 |

The author constructed two performers, High Performance and Low Performances based at the value of Super Efficiency Scores and it is shown in Table 6 (Appendix 2). Based on the Fractional Factorial layout in Table 1, the author calculated the Efficiency Scores and Mahalanobis distance between the two performers using selected Inputs and shown in Table 7 and Table 8 indicates the analysis of variance of the data, where the significance level is shown as the p – value. Thus, the candidate variable of $X_1$, $X_2$, $X_3$, $X_4$, $X_5$, $X_6$ and $X_7$ has no impact on Input.

The very last step in our process generates Table 9 which shows Sum of the Mean of the Mahalanobis distance (MD) for each variable at every level. For example, when variable $X_1$ is selected as an Input, the Sum of the Mean of MD is 6.9080 and if $X_1$ is not selected as an Input then the Sum of the Mean of MD is 4.0935. Thus, given that the greatest value of MD for variable $X_1$ is 6.9080 and it have to be decided on as an Input. Maximum values of the sum of the mean are indicated in bold font in Table 9. Thus we pick five variables as Inputs namely, $X_1$ (Number of Employees), $X_2$ (Number of Factories in Operation), $X_4$ (Gross Values of Plants and Machinery), $X_5$(Materials Consumed) and $X_6$ (Fuels Consumed). Then run the DEA Model 1(i) the usage of the chosen Inputs.

Table 7: Selected Input Variables and Mahalanobis Distance

| Runs | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | Selected Inputs | Mahalanobis Distance |
|------|-------|-------|-------|-------|-------|-------|-------|------------------|----------------------|
| 1    | 1     | -     | -     | -     | -     | -     | -     | None             | 0.7564               |
| 2    | 1     | 1     | -     | -     | -     | -     | -     | $x_1, x_3, x_7$  | 7.5618               |
| 3    | 1     | 1     | 1     | -     | -     | -     | -     | $x_2, x_5, x_6$  | 6.1733               |
| 4    | 1     | 1     | 1     | 1     | -     | -     | -     | $x_1, x_2, x_5, x_7$ | 6.0529             |
| 5    | 1     | 1     | 1     | 1     | 1     | -     | -     | $x_3, x_5, x_6, x_7$ | 4.8482             |
| 6    | 1     | 1     | 1     | 1     | 1     | 1     | -     | $x_1, x_3, x_4$  | 5.0002               |
| 7    | 1     | 1     | 1     | 1     | 1     | 1     | 1     | $x_2, x_3, x_7$  | 3.9544               |
| 8    | 1     | 1     | 1     | 1     | 1     | 1     | 1     | $x_1, x_2, x_3, x_5$ | 6.4989             |
| 9    | 1     | 1     | 1     | 1     | 1     | 1     | 1     | $x_4, x_6, x_7$  | 3.8209               |
| 10   | 1     | 1     | 1     | 1     | 1     | 1     | 1     | $x_1, x_4, x_5, x_6$ | 12.671              |
| 11   | 1     | 1     | 1     | 1     | 1     | 1     | 1     | $x_2, x_3, x_5, x_7$ | 4.4877              |
| 12   | 1     | 1     | 1     | 1     | 1     | 1     | 1     | $x_1, x_2, x_4$  | 8.1002               |
| 13   | 1     | 1     | 1     | 1     | 1     | 1     | 1     | $x_3, x_4, x_5$  | 4.3502               |
| 14   | 1     | 1     | 1     | 1     | 1     | 1     | 1     | $x_1, x_3, x_4, x_7$ | 3.6715               |
AIS, V.

CONCLUSION

The author considered an Input Selection approach that applies a 2 – level Fractional Factorial design, Mahalanobis Distance and ANOVA. The Concept of Super Efficiency Model is used to differentiate between the two set of performers namely high Performance and Low Performance. Variables are selected from the effects of ANOVA to Maximize the Mahalanobis Distance between the two performers. The writer unearths an powerful variable aggregate Method from a constrained quantity of experiments and proven the effectiveness of this new approach using a secondary information pertains to the Manufacture of Pharmaceutical Medicinal Chemical and Botanical products published in the Manual of Annual Survey of Industries (ASI) 2016 – 2017. The Selected Input variable measures the performance performance in a better way and gives a more potent optimality.

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