Fast Model-Selection through Adaptive Design of Experiments Maximizing Information Gain

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Abstract

To perform model-selection efficiently, we must run informative experiments. Here, we extend a seminal method for designing Bayesian optimal experiments that maximize the information gained from data collected. We introduce two computational improvements that make the procedure tractable: a search algorithm from artificial intelligence and a sampling procedure shrinking the space of possible experiments to evaluate. We collected data for five different experimental designs of a simple imperfect information game and show that experiments optimized for information gain make model-selection possible—and cheaper. We compare the ability of the optimal experimental design to discriminate among competing models against the experimental designs chosen by a “wisdom of experts” prediction experiment. We find that a simple reinforcement learning model best explains human decision-making and that subject behavior is not adequately described by Bayesian Nash equilibrium. Our procedure is general and can be applied iteratively to lab, field and online experiments.

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1 Introduction

Experimentation in the social sciences is a fundamental tool for understanding the mechanisms and heuristics that underlie human behavior. At the same time, running experiments is a costly process and requires careful design in order to test hypotheses while maximizing statistical power. This experimental design process is often guided by the intuition of scientists conducting the research, and while there are many benefits in relying on the intuition of skilled researchers, there is often a lack of principled, optimal experimental design when choosing which experiment to run (Fisher, 1936; Hill, 1995). As a result, experimental games often have limited power to distinguish models (Salmon, 2001) and thus designing experiments that have optimal power to accurately and reliably distinguish between the data generating processes is critical to advance economic theory. Despite best efforts to design good experiments, we often are faced with conflicting results that stall scientific progress, which may be one consequence of under-powered or suboptimal experimental designs.

Today, social scientists are witnessing two parallel improvements in experimental design techniques. First, a number of new, online experimentation platforms have begun to address the problem of cost and efficiency of experimentation (e.g., nodeGame (Balietti, 2017), Volunteer Science (Radford et al., 2016)). These online platforms are re-tooling social science by allowing for rapidly-deployed, large-scale experiments involving participants from around the world. Second, recent advances in artificial intelligence (AI) offer computational improvements as well. A growing body of researchers from social scientists, computer scientists, and industry professionals have explored the ways in which experiment selection can be optimized in their own fields. For example, some researchers now optimize the experiments they run using statistical techniques (e.g., Thompson Sampling) to preferentially assign participants to experimental treatments in order to optimize the treatment effect. This allows researchers to easily decide which treatment among many possible treatment arms is most effective in maximizing a certain outcome, such as the response rate in a marketing campaign (Eckles & Kaptein, 2014; Letham, Karrer, Ottoni, & Bakshy, 2017). Other techniques involve optimizing the length of the experiment, the number of participants needed, or the sequence of questions in behavioral batteries (Wang, Filiba, & Camerer, 2010; Imai & Camerer, 2016; Pooseh, Bernhardt, Guevara, Huys, & Smolka, 2018). In each case, optimizing experimental design has proved fruitful, and in the current work, we continue this line of research by focusing on a different goal of scientists designing experiments. We describe the benefits of optimizing the information gained from an experiment, which allows researchers to design experiments that will best distinguish between competing models of human behavior.

The notion of information gain is intuitive, but it is seldom defined precisely. Here, information refers to bits—a mathematical quantity that encodes counterfactual knowledge and describes the
amount of uncertainty or noise in a system. In the context of experimentation, we gain information by becoming more certain about the relationships between what we predict and what we observe in the data that we collect. This is equivalent to say that we are becoming more certain about the models and theories we use to describe the world. Different theories generate different models that predict how humans will behave, and an informative experiment is one that is able to rule out models that cannot adequately explain the data that they are confronted with. This procedure is called model-selection. With an experimental design that maximizes information gain, a small batch of data may be sufficient to determine which of these competing models is best suited to describe the data observed, and—importantly—this is not necessarily true for experimental designs that do not (see Fig. 1 for a visual example of experiments with low and high information gain).

![Figure 1: Illustration of lowly vs highly informative experiments.](image)

In the following sections, we introduce an optimal design procedure that maximizes the information gained from running an experiment. Our procedure recommends which experiments to run (i.e., which parameter values to use for data collection) in order to maximally diverge the predictions of multiple competing models of human behavior. The output of this procedure takes the form of a coordinate—a point in the space of all possible experiments—that corresponds to
optimal experimental parameters. By way of example, consider an experiment that measures gambling proclivity and is parameterized by a maximum payout \((M)\) and a payout multiplier \((m)\); our protocol would output a particular experimental design in the form of a coordinate, \((M, m)\), which may have values of, for example, \(M=4.25\) and \(m=1.2\) (as opposed to \(M=4.15\) and \(m=1.3\)). While knowing the optimal experimental design is valuable for collecting informative data, finding the optimal design is a computationally intensive task. As such, we propose two methodological innovations that build on seminal methods in Bayesian optimal design (El-Gamal & Palfrey, 1996). First, we use an adaptive Gaussian Processes search algorithm to efficiently search the space of all possible experimental parameter combinations (Contal, Buffoni, Robicquet, & Vayatis, 2013). Second, we employ a sampling technique that simulates likely datasets instead of needing to span the computationally costly space of all observable datasets in order to assign likelihoods to observing different outcomes. Overall, our procedure is able to reduce the computation time by several orders of magnitude while maintaining accuracy.

There are five main parts in this article. First, we describe how to optimize experimental design for information gain, and we lay out our methodological improvements to this protocol. Second, we evaluate and quantify the extent of these improvements. Third, we use our improved procedure to find the optimal experimental design of a classic two-person imperfect information game from behavioral economics (El-Gamal & Palfrey, 1996). Fourth, we conduct an expert prediction experiment to see what experimental designs domain experts would recommend, and we compare these predictions to our algorithmic approach. We show that the experts suggested more expensive and less informative experiments. Finally, we implement and run five experiments—each with different parameterizations—to illustrate the crucial role that information plays in distinguishing competing models of behavior (in our case, three different models). The data from these experiments were best described by a model that casts decision-making as a process of playing the Bayes-Nash equilibrium, but this was not the case in every experiment we ran. This prompted us to add a fourth model of behavior in our model comparison—a simple Reinforcement Learning model (Erev & Roth, 1998)—that emerged as the best model for describing the observed data.

Adding a fourth model well illustrates how the iterative nature of this process is crucial. New theories about human behavior continue to emerge, and in this work we describe a protocol to contend with these new models by rapidly comparing them to existing models and ruling them in or out based on experiments that are designed to maximally distinguish between them. Further, by combining our optimal design procedure with a wisdom-of-experts survey, we add nuance to ongoing debates about the wisdom of the crowd and its role in experimental design.
2 Optimal Experimental Design: Theory

The notion of how informative an observation is has its roots in information theory; for instance, a bit of information is a binary unit of measure that distinguishes one state of the world from another. Using this formalism, we can compare any different models’ likelihoods of experimental outcomes based on how much relative information each distribution will provide to us. This concept has clear applications to the optimal design of experiments for testing competing hypotheses about how humans behave. All that is required is to specify competing hypotheses in the form of generative models of human behavior, assigning a certain likelihood to any given outcome of the game (likelihoods must sum up to 1). The best experiment we can run is then the experiment that will maximally distinguish the distributions of likelihood of the different competing models. Formally, this is achieved by maximizing the Kullback-Leibler (KL) divergence \( D_{KL} [P \parallel Q] \) between two distributions, \( P \) and \( Q \):

\[
D_{KL}[P \parallel Q] = \sum_{x \in X} p(x) \log \frac{p(x)}{q(x)} \quad \text{where} \quad D_{KL}[P \parallel Q] \neq D_{KL}[Q \parallel P] \tag{1}
\]

Here, we refer to the optimal experiment as the particular set of design parameters, \( \theta \in \Theta \), that are predicted to maximally distinguish between \( n \) competing generative models of behavior in a given task. In particular, we use a one-sided form of the KL Divergence, which compares \( n - 1 \) competing models to a single model, expressed as \( I(1; \theta) \), as expressed below:

\[
I(1; \theta) = \sum_{x \in X} l_1(x; \theta) \log \left( \frac{(1 - p_1)l_1(x; \theta)}{\sum_{i=2}^{n} p_1 l_i(x; \theta)} \right) \tag{2}
\]

Using Equation 2, we assign a value for each combination of experimental parameters in \( \theta \) that corresponds to the amount of information we would expect to gain as a result of running an experiment with that particular combination of parameters.\(^1\) By computing this value for every coordinate in a grid comprised of each parameter combination, we are able to create an “information surface” where the value of each point represents the information theoretic difference between one model and \( n - 1 \) competing models (for a visual example of this, see Fig. 3).

As an illustration, consider the following example. The three authors of this paper each bet on a different model that they believe best describes human behavior in a given game. In order to determine who bet on the best model, we run an experiment. Whichever model has the highest

\(^1\)It is important to note that this difference metric is a directed, asymmetric measure. Wang et al. use a similar metric but propose using the average KL divergence (Wang et al., 2010), which can be expressed as \( I(\theta) = \sum_{i} p_i I(i; \theta) \).
relative likelihood for fitting the experimental data is considered the winning model. Obviously, to find a winner, it is crucial to avoid ties in model fitting. To do so—before running the experiment—we look at the distributions of likelihoods that each models assign to every possible game dataset. Intuitively, we can expect much of the datasets to be associated with similar likelihoods from the three models. However, there is important information encoded in where the models’ predictions diverge. Our procedure locates the experimental design(s) that accentuate the different model predictions, and this is represented by the experimental parameterization that maximizes the KL Divergence between each model’s likelihoods of observing particular datasets. We would then run that experiment and use the data collected to determine which model most likely describes the behavior we observe. Crucially, had we not run the optimal design, our ability to distinguish between—and eventually rank—the competing models might have been obscured. On the other hand, an experiment that was optimized for information gain may allow us to rule out suboptimal models of behavior after only one experiment. This, in addition to the fact that the experiment will likely be cheaper and require fewer participants, gives “information gain” an important role in understanding and optimizing experimental design.

In the following section, we will detail the specifics of the experiment we are attempting to optimize. After this introduction to the experiment, the following two sections lay out the main contributions of the current paper. The first contribution is methodological; we describe two improvements to the optimal design procedure above. Then, we describe our method for comparing the outputs from our algorithm to experimental designs recommended to us from experts in economics and behavioral economics. The final section of this paper reports results from actually running several versions of this experiment in order to compare the predictions of experts to the output from our optimal design procedure.

3 Sample Application: Game and Behavioral Models

Every study can benefit from an optimal experimental design. In the current work, we have chosen to optimize the same design used by El-Gamal & Palfrey (1996). The experiment we will use is called the Stop-Go game, and it is a simple two-player incomplete information game played over the course of three rounds. This experiment is well-suited for an optimal design procedure for at least three reasons. First, there are just two experimental parameters that define the gameplay, in addition to several model parameters. This is already enough to create a vast number of possible outcomes, which makes an optimal experiment all the more necessary to run in order to learn from the data collected. Second, we can directly compare our results against El-Gamal & Palfrey (1996). Third, the experiment itself has yet to be run in an online environment, so it will also be interesting to make a comparison of lab-vs-online behavior in the game.
In the next subsections, we provide a brief explanation of the Stop-Go game and of the behavioral models we would like to test on it. While an understanding of these details is important for this particular study, our optimal experimental design approach is fundamentally agnostic to both the specific experiment the behavioral models it is applied to.

### 3.1 Stop-Go Game Rules and Parameters to Optimize

The Stop-Go game starts with nature randomly selecting the state of the world: State $a$ with probability $\pi$ and state $b$ with probability $1 - \pi$, and this is common knowledge. Player 1 is informed about the state of the world, while Player 2 is not. Player 1 then chooses *Stop* or *Go*. *Stop* ensures that both players receive a payout of 1, regardless of the state of the world. If *Go* is selected, it is Player 2’s turn to select *Left* or *Right*. Finally, Player 1 and Player 2 are given payouts that depend on the state of the world and on one of the design parameters to optimize ($A$). Fig. 2 shows the decisions tree for one round of a two-player Stop-Go game containing the two design parameters ($A$ and $\pi$). Namely,

- $A$ is the maximum payout that either player can receive,
- $\pi$ is the probability that the state of the world is $a$.

In El Gamal & Palfrey (1996)’s experiment, $A$ was set to 3.33 and $\pi$ to 0.5, generating payoffs as shown in Table 1.

![Figure 2](image.jpg)

**Figure 2: Extensive-form representation of the game.** If Player 1 chooses *Go*, Player 2 must make a choice between *Right* and *Left*. Possible payoffs of the game are (0, 1, 2, A). The parameters to optimize are $A$ and $\pi$. 

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Table 1: **Payoff table of the game.** Assuming Player 1 chose Go, the payoffs for both players depend on the state of the world (\(a\) or \(b\)), and on the choice of Player 2 (Left or Right).

The game is played in a group of 10 players for three rounds. Each player is assigned a role (1 or 2) which is kept fixed throughout the whole game. Each round, players are rematched so that they meet a new partner of the other role ("perfect stranger"). The economic intuition for the game played over multiple rounds is about learning of a given trait of a subgroup of the population. The assumption of homogeneity of the trait within each subgroup, makes the game resemble the process of creation of stereotypes. If played within a larger population over many rounds, it could even lead to norm formation.

### 3.2 Models of Behavior and Parameters to Optimize

The three models of behavior tested in the original paper by El-Gamal & Palfrey (1996) are described below:\(^2\)

1. **Model 1**: Each individual, \(i\) plays the Bayes-Nash Equilibrium of the game, defined by \((\pi_{\text{per}}, A, \epsilon)\).

2. **Model 2**: Player 2 does not update \(\pi_{\text{per}}\) following go, and this is common knowledge.

3. **Model 3**: Individuals use fictitious play to construct beliefs about opponents’ play.

Each of the three models takes as inputs the two experimental parameters—\(A\) and \(\pi\)—described in the previous section and three model parameters: \(\alpha\) (learning speed), \(\epsilon\) (tremble rate), and \(\delta\) (accuracy in perception of probability \(\pi\)). Table\(^2\) provides a summary of both experimental and model parameters, their meanings, values, and ranges. The goal of Bayesian optimal experimental design is now to find the point in the experimental parameter space (\(A\) times \(\pi\)) where the likelihoods of three models differ the most.

\(^2\)Wording as used in El-Gamal & Palfrey (1996). See Appendix for a more detailed description.
Table 2: Parameters in the game. In order to define the optimal experiment, we need to assign likelihoods the observations that are generated under each possible combination of these parameters. In our computations, continuous parameters are approximated using the number of discrete values shown in the last column.

4 Optimal Experimental Design: Implementation

In order to determine the optimal design for maximizing information gain, we first need to enumerate every possible dataset we might observe when we run our experiment. We define as a single outcome the behavioral response observed at the smallest unit of analysis in the game, in our case a pair of players at any round. Following Fig. 2, we have then 8 possible outcomes: \(a\)-go-left, \(a\)-go-right, \(a\)-stop-left, \(a\)-stop-right, \(b\)-go-left, \(b\)-go-right, \(b\)-stop-left, and \(b\)-stop-right. The ensemble of all outcomes for all players for all rounds is a dataset (i.e. the data obtained after running the experiment once). The total number of possible datasets is equal to the number of permutations of outcomes with replacement. Even in a simple game like Stop-Go, the number of possible datasets quickly becomes very large. Considering only one round and five pairs of players, that number is 32,768. For three rounds, the total number of unique datasets is equal to \(32,768^3 \approx 3.5^{13}\). This is a great deal of likelihoods to compute! However, in this count, some datasets are “duplicates,” in the sense that they contain the exact same distribution of outcomes, but switched around different players. Depending on how the models actually assign likelihoods to outcomes (e.g., if and how they take into account the history of past outcomes, as would learning models) and on the rules for matching players in the game, the total number of likelihoods to compute can be greatly reduced. In our specific case, this is not possible.

Recall that this optimal design procedure is meant to distinguish competing models of behavior, and in order to determine the optimal experiment, we must first use each competing model to assign a likelihood to each unique dataset, for each combination of design and model parameters. In the current study, we use 3 model parameters \((\epsilon, \alpha, \delta)\), each of which needs to be integrated out when calculating the likelihood of an individual dataset. We discretized them to test 34 different values of \(\epsilon\), 34 different values of \(\alpha\), and 7 different values of \(\delta\) (see Table 2). Each value of \(\delta\) leads, on average, to 7 values of model parameter \(\pi_{per}\), hence 34*34*7*7=56,644. Doing so generates 56,644 unique combinations of model parameters for each experimental parameter. There are 2
experimental design parameters \((A, \pi)\), that we searched in a grid of \(20 \times 20 = 400\) points. The output of an optimal design procedure such as this one is an information surface that represents the multi-dimensional (in our case, two-dimensional) grid of every possible experiment and its associated information gain from taking the KL Divergence between the dataset likelihoods of the competing models (see Fig. 3 for example information surfaces). In our case, the total number of computations necessary to determine the optimal design of our experiment is equal to \(400 \times 56,644 \times 3.5^{13} \approx 8^{20}\).

Running the total number of likelihood calculations proved unfeasible even for the coarse grid we chose and on a super-computing cluster. To address this, we developed an algorithmic heuristic that leverages recent adaptive search algorithms from artificial intelligence as well as dataset sampling in order to simplify our computations. This approach generates accurate information surfaces, and more importantly, always converges on the optimal experiment.

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\subsection{4.1 Algorithmic Improvements for Optimal Design}

As described above, the protocol for maximizing the information gain in an experiment is often computationally infeasible. To address this, we introduce two improvements to the Bayesian optimal design procedure:

\begin{enumerate}
\item Gaussian Process search to adaptively select the next coordinate in the information surface to test
\item Sampling likely datasets (by selecting uniformly each model parameter) instead of spanning through all possible datasets
\end{enumerate}

The first improvement gets around the requirement to grid-search every coordinate in order to construct the information surface. There are certain regions of any landscape where having more precise information is not useful, thus we can improve the speed of the algorithm by ignoring regions where the added information gain is not meaningful in our recreation of the true information surface. The second improvement makes this search process even faster by not requiring the calculation of the model likelihoods for every possibly observed dataset. Instead, by sampling only a fraction of the possible datasets, the calculation of the KL Divergence between the model likelihoods for a given point takes dramatically less time (e.g. in our case, this amounts to sampling as little as 1.5\% of the possible datasets).

\[ \text{\footnote{In fact, we could construct the information surface for choosing the optimal design parameters only for a simpler two-player version of our game, and it took approximately 72 hours on the following super-computing cluster: Four hundreds parallel R v.3.x jobs, distributed across 56-core x86 64 Little Endian Intel(R) Xeon(R) cpus (E5-2680 v4 @ 2.40GHz; L1d cache: 32K, L1i cache: 32K, L2 cache: 256K, L3 cache: 35840K).}} \]

\[ \text{\footnote{All code is available at http://github.com/shakty/optimal-design.}} \]
(a) Original optimal design method from El-Gamal & Palfrey (1996)  
(b) Replication of method from El-Gamal & Palfrey (1996)  
(c) Improved search method using Parameter-Sampled GPUCB-PE

Figure 3: Information surfaces for experimental design. (a) Information surface originally presented in El-Gamal & Palfrey (1996) (b) Information surface generated by replicating the optimal design procedure in El-Gamal & Palfrey (1996); (c) Information surface generated using Parameter-Sampled GPUCB-PE, where the points shown represent coordinates searched by the algorithm, and the green star representing the point corresponding to the experiment that is predicted to produce the maximum information gain. Note: as described in El-Gamal & Palfrey (1996), the experiment with the maximum information is expected to occur when $A$ reaches its minimum value, which approaches (but is not equivalent to) $A = 2.0$.

4.1.1 Gaussian Process Upper Confidence Bound-Pure Exploration

Given the computationally taxing nature of this systematic grid-search technique for finding the optimal experimental design, we implemented an adaptive search technique used in artificial intelligence research to hasten the construction of the information surface. This algorithm, known as Gaussian Process Upper Confidence Bound Pure Exploration (GPUCB-PE), is a way of reconstructing the contours of a landscape in an adaptive manner, while also minimizing regret. Here regret is defined as the difference between the maximum observed value in a landscape to the global maximum value (Stefanakis, Contal, Vayatis, Dias, & Synolakis, 2014; Contal et al., 2013). Recall that each point in the experimental design landscape is a possible experiment to run and that there are certain regions of the design landscape that will maximize the information gained more than others. The GPUCB-PE algorithm will actively seek out regions on the landscape where there is a higher KL divergence between the model likelihoods while inferring the information content of the points it has not yet searched. The output from this algorithm is an information surface that identifies the most highly informative coordinate in the experimental design space while also accurately reproducing the contours elsewhere in the information surface (satisfying both the exploration and exploitation of points in the landscape).

The algorithm produces an accurate information surface faster than if it were grid-searching through every point in the landscape. It does this as follows:
1. Initialize a grid with coordinates that represent possible combinations of experimental parameters.

2. Using Equation 2, calculate the KL Divergence between the model likelihoods of the $n$ competing models at an initial number of points. These initial points can be random points in the landscape, but we chose to distribute the initial number of points using a quasi-random spacing technique known as Sobol sequencing (Sobol, 1998).

3. Proceed with the GPUCB-PE algorithm, iteratively selecting points in the landscape and calculating the information gain at each of those points, while also using Gaussian Process regression to infer the expected information gain and confidence bounds of every point that has not yet been searched. This algorithm alternates between selecting i) the point with the highest expected information gain plus its upper confidence bound and ii) the point—with within a region of eligible points—with the highest upper confidence bound (detailed in Stefanakis et al., 2014). The two types of points that this algorithm selects satisfies the exploitation and exploration of the landscape, respectively.

4. After each new point has been searched, determine if the landscape satisfies the stopping rule, which is based on the similarity between the current reconstruction of the landscape and the previous reconstruction. If the two landscapes are repeatedly more than 99.9% similar to each other according to a modified Spearman Rank Correlation, the algorithm stops running and the final information surface is output.

By adopting this method, we are able to reconstruct an information surface without needing to span the grid of all possible experiments.

4.1.2 Sampling Parameters to Sample Datasets

In addition to implementing GPUCB-PE search to hasten the construction of the information surface, we introduced a technique for sampling all model parameters in every possible dataset. That is, we compute the Bayesian optimal experiment to run without needing to perform days-long computations on super-computing clusters. We will refer to this modification to the GPUCB-PE algorithm as Parameter-Sampled GPUCB-PE.

In order to understand why Parameter-Sampled GPUCB-PE speeds the search process up so much, it is important to understand the vast number of calculations that go into producing one value in the information surface we are trying to construct. One coordinate in that landscape represents a unique combination of experimental parameters (e.g. payout amount and probability...
of receiving the payout). The values for these parameters are among the inputs to a model that generates likelihoods that a participant makes a given decision in a given round. In addition to the experimental parameters, the model takes as inputs model parameters that represent attributes of the participant in the experiment (e.g. how often errors are made, how risk averse they are, etc.). In order to calculate the likelihood of a particular dataset under a particular experimental design, and given that we do not know the true value of these parameters in a population,\(^6\) we have to span through all combinations of model parameters in order to properly assign a likelihood to a given dataset (and subsequently do the same for every dataset).

Through Parameter-Sampled GPUCB-PE, we avoid the need to span all \(N\) possibly-observed datasets by uniformly sampling \(n_s \ll N\) combinations of model parameters and using them as inputs for simulating participant behavior. Practically, sampling entails the following steps:

1. Draw uniformly at random a parameter value for each of the model parameters.\(^7\)
2. Insert the sampled parameters inside each of the model likelihood functions to obtain simulated game outcomes for all participants for all rounds.
3. Repeat Step 1. and 2. a sufficiently high number of times \(K\).
4. Merge the sample of simulated datasets by all models and compute the relative likelihoods as the fraction of times each of them occurred in the sample.

In our experiment, we sampled \(K = 10,000\) datasets per search—several orders of magnitude less than what was required before. Not only does this approach reduce the number of computations by several orders of magnitude, we also found that sampling any number of datasets greater than 1,000 was sufficient to 1) accurately reconstruct the “true” information landscape, 2) make the algorithm satisfy the stopping rule, and 3) most importantly, find the coordinate of the optimal design on the information surface (see appendix for further analysis). This is likely due to the particular shape of our information surface as well as the complexity of the Stop-Go game itself. That is, if the models used to construct the information surface had assigned similar likelihoods of observing certain datasets, the algorithm would require even more samples in order to properly distinguish the models and find the optimal experiment. Lastly, while we use this Parameter-Sampled GPUCB-PE approach in order to find the maximally informative experimental design, this algorithm can be used to create an analogous “information surface” corresponding to any measure that we may want to optimize (e.g. experiment cost, treatment effect, etc.).

Finally, it should be noted that the Parameter-Sampled GPUCB-PE has no prior knowledge of neither the underlying models nor their behavioral assumptions; even so, it is able to reliably

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\(^6\) Nor is it useful to expect that we could, given that all models are abstractions of an observed behavior.

\(^7\) Non-uniform sampling can be used when the experimenter has prior over the distribution of model parameters.
reconstruct information surfaces, proving itself a very versatile approach to create informative experiments in any domain knowledge.

### 4.2 Performance Evaluation

We compared the effectiveness of our Parameter-Sampled GPUCB-PE modification to a grid search of the landscape, and a summary of the results is shown in Fig. 4. The algorithmic improvements we have made to the optimal design procedure laid out in El-Gamal & Palfrey (1996) to construct an information surface that maintains the same general contours as well as precisely the same global maximum value as the original algorithm. This allows us to rapidly understand the role that particular combinations of experimental design parameters play in determining how informative an experiment is. Furthermore, it is able to do it considerably faster than the traditional Grid Search approach, here evaluated by iteratively constructing grids of larger size (see inset in Fig. 4). Note that Grid approach can be insufficient for finding the precise optimal design because the exact coordinate of the maximum value might be between the cells of the grid. The implications of this are discussed further in Section 7.

![Comparison of Algorithm Performance](image)

**Figure 4:** Comparison of algorithm performance measured by regret minimization. Parameter-Sampled GPUCB-PE almost immediately finds the global maximum of the information surface; Grid Search systematically takes longer to uncover the point with the highest information gain, performing even worse than a Random search.
5  Wisdom of Experts

There are a number of ways to assess the effectiveness of the optimal experimental design protocol we laid out above. As we have previously described, our algorithm finds the Bayesian optimal experiment that maximally distinguishes competing models of behavior. However, a persistent question that arises in this approach is whether or not the output from this optimal design procedure would even be useful to experimentalists in academia or industry. Is the algorithm simply generating outputs that would be obvious to a trained experimenter? To answer this question, we sought out experts in economics and behavioral sciences, and we asked them to complete a survey where they provided us an estimate of what experiment they would run if they were in control of the design.

In a brief survey, we first described the task: these experts were to suggest parameters for an experimental design that would maximally distinguish the likelihoods of four different models of behavior. We then collected demographic information about the respondents (gender, field of expertise, academic position, country of employment). Then, we described the Stop-Go game, including the same language and figures from Section 3 and Fig. 2 (for precise wording, see Appendix). After describing the four models the experiment was meant to distinguish, we asked the participants to suggest to us the precise values for $A$ and $\pi$ that would isolate and distinguish the different models from one another; these were presented as text boxes that were constrained by the possible parameters for $A$ and $\pi$. The participants then reported their confidence in each of those predictions (from 1 - not confident to 5 - very confident) as well as which model they believed would eventually perform best after the experiment was ultimately run.

5.1 Recruitment Strategy

In order to assemble a group of experts who were reasonably-suited to perform this task, we sought out researchers who were familiar with common behavioral models, Nash Equilibrium, experimental design, and we preferred to look for researchers who were currently faculty in a department related to Behavioral Economics. From this criteria, we chose to email researchers from the following areas:

- Authors of work presented at the CODE@MIT, a conference on digital experimentation (2014, 2015, 2016, and 2017).

- Attendees of the “Behavioral/Micro” course at the National Bureau of Economic Research (NBER) Summer Institute (2012, 2013, 2014, 2015, 2016, and 2017)
• Authors of work presented at the Economic Science Association’s North American annual conference (2015, 2016, and 2017)

• Boston-area faculty in Economics (including Harvard University, Massachusetts Institute of Technology, Northeastern University, Boston University, Boston College, University of Massachusetts Amherst, and University of Massachusetts Boston).

This process generated 811 names, all of whom were sent an initial email on 12/01/2017. This email served to introduce the project, provide information about Northeastern University’s IRB protocol, provide a link to the survey itself, and allow them to opt-out of further communication. After one week, a reminder email was sent to the ones who had not completed the survey. After another week, a thank-you email was sent to those participants who had completed the survey. This protocol was adapted from methods used in DellaVigna & Pope (2016). In total, 148 participants began the survey (i.e., opened the browser window and agreed to the informed consent), but 55 participants finished the survey.

5.2 Expert Prediction Results

For full survey results, see Table 3 in Appendix 1, but we report the major descriptive results here. The median response time was approximately 11 minutes. The mean response time was 105 minutes, although after removing participants who took over 6 hours to complete the survey (we assume that they were not actively working on the survey for that amount of time), this value became closer to 16 minutes. Of the 18% of participants who began the survey, 39% ended up finishing, making for a 7% response rate overall.

This survey generated three main takeaways. First, and most importantly, there appeared to be little agreement among the various respondents as to what was the optimal experiment to run, shown in Fig. 5a. This is validated by the low median self-reported confidence in their predictions about both $A$ and $\pi$ (median of “2 - somewhat not confident” for both $A$ and $\pi$). Second, while the estimates spanned much of the experimental design space, the modal value was $\pi = 0.5$ and $A = 6.0$. This corresponds to a point on our information surface that would be expected to generate less information gain than the optimal point. The effect of running this suboptimal experiment might be that the competing models would appear to be less distinct, at which point we would need more data in order to distinguish the most likely model. Third, this modal value suggests an interesting mechanism for how experts choose experimental designs when they are uncertain. While the most commonly chosen value for $\pi$ (0.5) was the same as the optimal value, the experts surveyed were inclined to recommend a more expensive experiment as being the optimal one ($A = 6.0$). That is, experts appeared to believe that giving research subjects a higher reward in
an experiment would elucidate greater differences among the four competing models.\(^8\) Indeed, it is generally accepted in experimental economics that greater rewards can increase participants’ effort and attention during an experiment (Knez & Camerer, 1994; Hertwig & Ortmann, 2001), even if some contrary evidence is also found (Kachelmeier & Towry, 2005). However, according to the landscape generated by our theory of optimal design, this should not affect the informativeness of our experiment.

From these results, we chose to include the modal value of \(\pi = 0.5\) and \(A = 6.0\) as an experiment to test when we eventually tested this optimal design procedure, along with four additional designs shown in Fig. 5. These designs included the original experiment run in El-Gamal & Palfrey (1996), the point that our algorithm determined had the maximum information gain, the corresponding point expected to have the minimum information gain, and a point corresponding to a medium information gain between the maximum and minimum.

![Figure 5](image)

(a) Experts’ optimal experiment predictions (n=55). (b) Final description of experiments run.

**Figure 5: Expert predictions and experiments chosen.** (a) Raw data from the experts survey showing both the modal prediction of \(A = 6.0\) and \(\pi = 0.5\), but more importantly, the inherent noise and uncertainty that the respondents showed; (b) These five points in the experimental design space were ultimately the ones that were tested on Amazon Mechanical Turk. Note: the experiment with highest information gain is when \(\pi = 0.5\) but approaches \(A = 2.0\). We report results from the design \((\pi = 0.5, A = 2.0)\) as human participants would be unable to distinguish \(A = 2.000\) from \(A = 2.001\), for example.

\(^8\)Note: the optimal experiment we report—\(A \approx 2.0\) and \(\pi \approx 0.5\), as shown in Figure 3—is generated from an information surface comparing three models. The same coordinate is the optimal experiment when we include all four models. See Fig. [10] in the Appendix for the full information surface.
6 Experiment Results

After selecting experiments to conduct from the original El-Gamal & Palfrey (1996) paper, our own algorithmic extension, and the predictions of experts in behavioral economics, we ran five experiments on January 19, 2018 on Amazon Mechanical Turk. This experiment was designed and implemented on the nodeGame platform for group experimentation online (Balietti, 2017). Each of the five experimental designs were run using 14 participants instead of 10, to take into account of possible dropouts during an experiment (Arechar, Gächter, & Molleman, 2018; Stewart, Chandler, & Paolacci, 2017). In the end, we had a very low rate of dropouts, only in two sessions we had one dropout. In our setup, dropouts have been replaced by computer bots that automatically chose the most common action played by previous players who found themselves in the same situation. For our analysis, we did not use any data generated directly by bots, by participants matched with bots, or by participants matched with a human player who has previously played with a bot. We then computed the predicted likelihood of observing the data collected according to the three models, and plotted them as likelihood odds ratios in Fig. 6.

![Figure 6: Likelihood odd ratios of models according to real participants behavior.](image)

We used a bootstrapping procedure to simulate the likelihood generated by datasets with fewer observations (matches) than the total actually collected to see the stability of these likelihoods. The final likelihood odds for each of the models is the value of the rightmost data point in each panel.

Heuristically, one can think of the plots where the likelihood odds are more different as containing more information. As such, it is encouraging to note that experimental designs (3.33, 0.5) and (2.0, 0.5) are the ones where the likelihoods of the different models diverge the most, corresponding to regions of high information in our information surface. We report the same data mapped onto the information surface in Fig. 7.
Intuitively, what we see here validates our expectation of the amount of information gain—the amount of divergence—between the likelihoods of the different models.

6.1 Iterating the Procedure

What we see after collecting data from several experiments is that Model 1—the model predicting that humans play the Stop-Go game by finding the Bayes-Nash equilibrium—is overall the model that best fits the respondents’ data. That is, in experiments where an assessment about the most likely model can be made, Model 1 is best suited to describe the observed data.
However, as shown in Fig. [7], there exist points in the information surface where Model 1 is outperformed by another model at predicting observed human behavior. This is the case when $A = 6.0$ and $\pi = 0.50$ (Model 3 is the most likely model to describe the data from this experiment), and it suggests that participants possibly change behavior based on the value of $A$. However, this interpretation poses a problem when trying to build parsimonious theories of human behavior because a well-constructed model should be able to incorporate changes to experimental parameters—in this case a higher reward value, $A$. It is natural to ask whether there exists a better model, one with greater explanatory power that is able to describe data from a wide range of experimental parameters. To answer this question, we repeated our optimal design protocol, and we included a fourth model that casts human behavior with a Roth-Erev Reinforcement Learning model (Erev & Roth, 1998).

Learning theories have become increasingly popular for describing human decision-making across a number of settings (Salmon, 2001). Recent reviews of the effectiveness of this model have shown that it is well-suited to describe human behavior in a number of settings, especially those where learning from experience is necessary in order to maximize a payout (Erev & Roth, 2014). As such, we chose to add this model in our model comparisons. Further model competition to put under test could include models based upon Experience-Weighted Attraction learning (Camerer & Ho, 1999; Ho, Wang, & Camerer, 2008), and it is left for future research.

![Figure 8](image-url)

**Figure 8:** Likelihood odd ratios of models according to real participants behavior including a fourth Model. After including Model 4—a Reinforcement Learning model—Model 1 is no longer the best-suited for describing this data.

When we do this, Model 1 is no longer the best model for describing the data (see Fig. 8). Reinforcement learning explains the data significantly better than any of the other model, across all
five experimental designs. This illustrates another advantage of our optimal design approach: it can be iterated, allowing for sequential testing of new models. That is, we facilitate the comparison of predictions from new models against the predictions of the most likely model currently available. The reason for this is simple, although it can be easily overlooked; the data collected after running an experiment can be used to fit the most likely parameters of the competing models. In the current study, each model had three parameters: $\epsilon$, $\alpha$, and $\delta$ (see Appendix for model details). There exists a value for each parameter, under each model, that maximizes the model’s likelihood of explaining the data. Because of this, we can assign posterior probabilities to different values of each model parameter. By doing this, we can find not only the most likely model that describes human behavior but also the most likely parameters of this model. After, we can restart our optimal design procedure using our estimates of the most likely model parameters, reconstruct an information surface, and run new experiments that test our newly parameterized model against a new competing model. Through this process, we found that the new fourth model outperformed the other three models in every experiment that we ran. The reinforcement model reliably and robustly describes the data from five different experiment configurations better than the static equilibrium predictions. This also suggests that the reinforcement learning model is quite insensitive to the parameterization of the experiment, indicating its suitability for describing behavior in this experiment. On the other hand, observing such a strong fit in an experiment as simple as the Stop-Go Game indicates that this model can (and should) be tested in new domains in order to assess its effectiveness across a number of settings. In a way, this finding is iconic and represents the fact that, as technology and scientific understanding of human behavior co-evolve, newer better, more biologically and socially plausible models of behavior will emerge. Roth-Erev Reinforcement Learning was one such model that has proven to be effective in describing behavior.

7 Discussion

The goal of this paper was to investigate the notion of information gain in experimentation. Specifically, we sought to improve upon a classic optimal experimental design approach, where the optimal experiment is defined as the experiment that maximizes the KL Divergence between the likelihoods of multiple competing models under a given experimental design. In addition to illustrating and improving upon existing methods, we further asked whether domain experts would predict optimal experimental designs that corresponded to those generated algorithmically. Lastly, we illustrated the potential that optimizing experiments for information gain has by adding an additional model into the protocol. In a way, this is highlighting the adaptive nature of optimizing experimental design for information gain—it allows for the evaluation of new models of behavior as they are invented.
By implementing a modified version of the adaptive search GPUCB-PE algorithm, we were able to actively construct an information landscape—a surface that represents how much information gain is associated with each possible combination of experimental design parameters. In doing so, we also eliminated the computationally costly task of computing model likelihoods for all possibly-observable datasets during the optimal design process. This was achieved by sampling model parameters and simulating likely-observed datasets; while this process produced more noisy estimates of the information gain associated with each experimental design, this noise quickly disappears through repeated (GPUCB-PE) search on the information surface. In practical terms, this reduced the task of finding the optimal experiment from 72 hours of computation on supercomputing cluster to approximately an hour on a standard laptop, which suggests that this procedure could be run in real-time.

The information landscape we constructed using Parameter-Sampled GPUCB-PE was then used to inform which experimental designs to test in our replication of the Stop-Go Game (ElGamal & Palfrey, 1996). However, we supplemented this list of experimental designs with insight from another source: domain experts. We conducted a survey reaching out to over 800 experts in economics and behavioral economics to assess whether their domain expertise could associate to optimal predictions about the most informative experiment to run. The expert prediction experiment showed that there was little consensus among the experts about which experimental design was optimal. Interestingly, the experts generally lacked confidence in their predictions of the optimal experimental design, and this also appeared to make them inclined to advocate for an experiment with higher payouts to the participants. In reality, however, the optimal experiment was predicted to be one with the smallest payout.

Ultimately, we ran five experiments in an attempt to distinguish between competing models of behavior. We began our optimal design procedure with three competing models; after we collected data, we found that one model was well-equipped to explain data from several experiments, but it was not the most likely model across every experiment we ran. For this reason, we chose to introduce a fourth model in our model comparison—a simple reinforcement learning model. Model 4 ended up being the model that was best able to explain the observed data across each of the five experiments. This finding is key, and it demonstrates the potential benefits of an optimal design procedure like the one described here; researchers are able to quickly introduce new competing models that can be anything from subtle tweaks to an existing model to an entirely new model. All that is needed is to add the new model to the model comparison procedure, find the experimental design that is expected to be most informative, and collect a new batch of data to test the models. Given that it is unlikely researchers will discover one best model for describing human behavior, there is a constant drive towards improving upon existing models, exploring for novel ones, and actively comparing them throughout the scientific process. Our fourth model outperforms the initial
three that were tested, illustrating this point.

Beyond the current applications of Parameter-Sampled GPUCB-PE, the benefits of rapidly assigning expected information gain to different experimental designs have a very wide reach. While the current approach is meant for distinguishing between generative models of human behavior and picking the right experiment to separate the models, it can be adapted to accommodate different goals in experimentation like optimizing the number of participants, optimizing for effect size, or optimizing for monetary cost. Looking ahead, this is a step in the direction of optimizing social network experiments where the particular network topology is the experimental design to test, a notoriously challenging computational task (Phan & Airoldi, 2015; Parker, Gilmour, & Schormans, 2017) that could be eased using the methods described here.

8 Conclusion

Scientists who study human behavior use a number of behavioral models in order to understand the mechanisms and decision processes underlying choice behavior. As such, the role of experimentation in behavioral sciences is paramount for performing model-selection, that is, distinguishing and evaluating the effectiveness of different models for describing human behavior. Using the algorithmic tools described in this paper, experimenters can now rapidly determine the exact experimental design that would maximally distinguish between the competing models of behavior. As a proof of principle, we used this tool to replicate a classic experiment known as the Stop-Go Game under several different parameterizations. First, however, we solicited the opinions of experts in economics and behavioral economics about which experimental design would contribute the most information in distinguishing competing models of behavior. The results offer some nuance to the “wisdom of the crowd” debate, in part illustrating that the domain expertise of the crowd may not be enough to construct an information surface, let alone find the true maximum point in that landscape. We find that the simple reinforcement learning model we consider robustly describes the data from all parameterizations of our experiment better than Bayesian Nash equilibrium predictions. This contributes to the growing body of work suggesting that reinforcement learning is good at describing human behavior (Feltovich, 2000; Gale, Binmore, & Samuelson, 1995; Sarin, Vahid, et al., 2001; Pershtman & Pakes, 2012) by adding experimental results for repeated imperfect information games.

In the future, it is not difficult to imagine an automated online experimentation platform that assigns the parameters of an experiment, runs an optimal experiment, re-updates the model predictions, and evaluates the different competing behavioral models. In a way, this places more emphasis on the scientist’s role as a theorist, integrating results from optimally-designed experiments into new models that capture the commonalities and peculiarities of human behavior.
Acknowledgements

The authors acknowledge Mahmoud El-Gamal for his helpful correspondences, and Stephanie W. Wang for useful comments on the design and implementation. This work was supported in part by the Office of Naval Research (N00014-16-1-3005 and N00014-17-1-2542) and the National Defense Science & Engineering Graduate Fellowship (NDSEG) Program.

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9 For Online Publication: Appendix

9.1 Parameter-Sampled GPUCB-PE

Figure 9: Relationship between the number of sampled datasets per search in Parameter-Sampled GPUCB-PE (1000, 5000, or 10,000), the number of iterations needed to converge, and the regret (each simulated 200 times). Sampling more datasets tends to require more searches in order for the algorithm to converge, while also minimizing regret more than if fewer datasets were sampled. Note: the differences in regret in this plot are due to the fact that sampling fewer datasets produces a relatively-higher Mean Squared Error, but crucially, each of these simulations converged to the true global maximum ($\pi = 0.5$ and $A = 2.0$).

9.2 Expert Prediction Survey

In order to query the 811 experts in behavioral economics, we implemented a survey on Qualtrics for two weeks in early December of 2017. The survey was approved by Northeastern IRB (#17-11-20). Below is the text used in the email recruitment for the survey and the survey itself.

9.2.1 Recruitment Email Text

Dear colleague:

We are researchers at the Network Science Institute at Northeastern University, and we would like to invite you to participate in a web-based online survey.

This study is about experimental design, and we are especially interested in how experts design experiments and make predictions about an experiment’s outcomes. This survey is anonymous and is intended for academics in Economics and related disciplines. It is intended to take no more than 7 minutes.
Click here to start the survey.

On behalf of the Collaborative Social Systems Lab at Northeastern University, thank you very much for your time!

Sincerely,

Christoph Riedl, Assistant Professor and Principle Investigator  
Stefano Balietti, Post-doc and Co-Principle Investigator  
Brennan Klein, PhD Student and Research Assistant

9.2.2 Survey Text

Introduction: Imagine you are a researcher who wants to study human behavior in a simple, two-player, repeated interaction game played over the course of three rounds. To do this, you decide to run an experiment that seeks to uncover a model that best describes human behavior. You believe that there are four candidate models that may describe human behavior, and you are attempting to find out which of them best explains the behavior of participants in this game. We will ask you to choose values for two experimental parameters, $A$ and $p$, for the game that you believe will maximally distinguish the predictions of the four models. That is, you want to design the experiment such that it has the highest power to decide which model is most likely. We will then ask you to report which of the four models you think will best describe participants behavior in this game. First, we will ask you a few brief questions about your demographic information and education background. Next, we will describe a simple incomplete information game played in pairs.

The Game: In this game, there are two possible states of the world, $a$ and $b$. The game starts with nature selecting state $a$ with probability $p$ and state $b$ with probability $1 - p$ ($p$ is common knowledge). Player 1 is informed about the state of the world, while Player 2 is not. Player 1 then chooses to Stop or Go. Stop ensures that both players receive a payout of 1, regardless of the state of the world. If Go is selected, it is Player 2’s turn to select Left or Right. Finally, depending on the state of the world, Player 1 and Player 2 are given payouts according to those in Fig. 2. This game is repeated for three rounds.

Your task is to choose values for the experimental parameters $A$ and $p$.

- $A$ is the maximum payout that either player can receive
- $p$ is the probability that the state of the world is $a$

The values you choose for $A$ and $p$ should maximally distinguish the predicted behavior of participants under each of the four models. That is, you want to pick values for $A$ and $p$ such that the predictions generated by the four models will be most different. On the following page, you will see descriptions of the four models.

---

9Here, we asked participants to provide their current academic position, the country of their current academic institution, up to three research specialties, and their gender.
The Models:

- **Model Q**: Each player plays the Bayes-Nash Equilibrium of the game.
  In this model, each player predicts the other’s actions by forming beliefs about $p$ (which determines the state of the world) and updating those beliefs after each round of play, based on the outcomes they have observed.

- **Model R**: Similar to Model Q, but Player 2 does not update their beliefs about the state of the world.
  In this model, each player predicts the other’s actions by forming beliefs about $p$ (which determines the state of the world) and updating those beliefs after each round of play, based on the outcomes they have observed.

- **Model S**: Each player engages in fictitious play to construct beliefs about their opponents’ actions.
  In this model, each player keeps track of the state of the world and the actions taken by each player in every round they have played. They use this information to make predictions about $p$ and what actions they should take in the current round.

- **Model T**: Each player engages in Roth-Erev Reinforcement Learning.
  In this model, a player is more likely to repeat an action if it had a positive outcome in the past. The propensity to select an action is a discounted sum of reinforcements obtained from playing the same action previously.

**Your Prediction**: Each of the four models assigns different likelihoods to the actions of Player 1 and Player 2. In order to distinguish between the four models, you will need to choose the experimental design where the predictions of the four models are most different. This way, the data that you collect in your experiment will isolate one model as being more likely than the other three.

- What is your prediction for the best value of $A$? The values of $A$ range from $2.0 \leq A \leq 6.0$.
  (text box)

- How confident are you in your prediction about $A$? (from 1 to 5)

- What is your prediction for the best value of $p$? The values of $p$ range from $0.0 < A < 1.0$.
  (text box)

- How confident are you in your prediction about $p$? (from 1 to 5)

- Please rank the following four models based on which model do you think will best describe the participants’ behavior in the experiment you’ve proposed. Here, a rank of 1 is the most likely, and 4 is the least likely. NOTE: The names of the models (Q, R, S & T) have been randomized and should not reflect the model’s ranking at all.

- How confident are you in this ranking? (from 1 to 5)

- Optional: In the text box below, please provide 2-3 sentences about why you chose the model that you did.
### 9.2.3 Survey Data

|                      | Partial Completion | Full Completion |
|----------------------|--------------------|-----------------|
| **Response Rate**    | 0.18               | 0.07            |
| **Primary Field (select up to 3)** |                    |                 |
| Behavioral Economics | 0.30               | 0.29            |
| Game Theory          | 0.17               | 0.16            |
| Microeconomics       | 0.12               | 0.10            |
| Labor Economics      | 0.07               | 0.08            |
| Developmental Economics | 0.04              | 0.02            |
| Information Economics| 0.04               | 0.04            |
| Econometrics         | 0.03               | 0.03            |
| Financial Economics  | 0.03               | 0.04            |
| Business Economics   | 0.03               | 0.03            |
| Theoretical Economics| 0.02               | 0.01            |
| Macroeconomics       | 0.01               | 0.03            |
| International Economics | 0.01              | 0.01            |
| Other                | 0.13               | 0.16            |
| **Academic Rank**    |                    |                 |
| Assistant Professor  | 0.39               | 0.42            |
| Associate Professor  | 0.25               | 0.13            |
| PhD Student          | 0.16               | 0.15            |
| Postdoctoral Researcher | 0.12              | 0.11            |
| Other                | 0.06               | 0.16            |
| Prefer not to say    | 0.02               | 0.03            |
| **Gender**           |                    |                 |
| Male                 | 0.71               | 0.73            |
| Female               | 0.27               | 0.24            |
| Prefer not to say    | 0.02               | 0.03            |
| Other                | 0.00               | 0.00            |
| **Modal Parameter Value** |                |                 |
| \(\pi\)             | 0.50               | 0.50            |
| A                    | 6.00               | 6.00            |
| **Total Response Times** |                  |                 |
| Median               | –                  | 10.93 min.      |
| Mean                 | –                  | 105.02 min.     |
| Mean (removing > 6hr.) | –                 | 16.23 min.      |

Table 3: Data about experts who responded to our survey (811 contacted).
Figure 10: Information surface generated by comparing four models. By taking the average KL Divergence between the likelihoods generated by each model we tested (including Model 4—the basic reinforcement learning model), we find an optimal experimental design that is consistent with the optimal experiment found in Figure 3, at approximately $A \approx 2$ and $\pi \approx 0.5$.

### 9.3 StopGo Experiment

This experiment was approved by Northeastern IRB (#13-13-09). We conducted seven sessions of five experimental designs and ran 2 additional sessions to compensate for 2 dropouts happened in the original sessions.
9.4 Models of Behavior

We implemented the same models that were tested in El-Gamal & Palfrey (1996):

**Model 1:** Each individual, \( i \) plays the Bayes-Nash Equilibrium of the game, defined by \( (\pi_{\text{per}}, A, \epsilon) \).

**Model 2:** Player 2 does not update \( \pi_{\text{per}} \) following \textit{go}, and this is common knowledge.

**Model 3:** Individuals use fictitious play to construct beliefs about opponents’ play.

### 9.4.1 Model 1

Model 1 is characterized by five canonical cases \( (A, B, C, D, \text{ and } E) \) which correspond to particular combinations of parameters.

\[
\hat{\pi}_{\text{per}} = \frac{A}{2 + A}
\]

**Case A:** \( \pi_{\text{per}} > \hat{\pi}_{\text{per}} \), \( \epsilon \leq \frac{2\hat{\pi}_{\text{per}}(1 - \pi_{\text{per}})}{\hat{\pi}_{\text{per}} + \pi_{\text{per}} - 2\hat{\pi}_{\text{per}}\pi_{\text{per}}} \), and \( \epsilon \leq \frac{2}{A} \)

\[
p^I_a = \frac{A(1 - \pi_{\text{per}})}{2\pi_{\text{per}}} - \frac{[(A + 2)\pi_{\text{per}} - A]/2}{(1 - \epsilon)2\pi_{\text{per}}}
\]

\[
p^I_c = 1
\]
\[ q^I = \frac{1}{1 - \epsilon} \left[ \frac{A - 1}{A} - \epsilon/2 \right] \]

**Case B:** \( \pi_{\text{per}} > \hat{\pi}_{\text{per}}, \epsilon > \frac{2\hat{\pi}_{\text{per}}(1 - \pi_{\text{per}})}{\hat{\pi}_{\text{per}} + \pi_{\text{per}} - 2\hat{\pi}_{\text{per}}\pi_{\text{per}}}, \) and \( \epsilon \leq \frac{2}{A} \)

\[ p_a^I = 1 - p_b^I = 1 - q^I = 0 \]

**Case C:** \( \pi_{\text{per}} \leq \hat{\pi}_{\text{per}}, \epsilon \leq \frac{2\hat{\pi}_{\text{per}}(1 - \pi_{\text{per}})}{\hat{\pi}_{\text{per}} + \pi_{\text{per}} - 2\hat{\pi}_{\text{per}}\pi_{\text{per}}} \)

\[ p_a^I = 1 \]
\[ p_b^I = \frac{2\pi_{\text{per}}}{A(1 - \pi_{\text{per}})} + \frac{[(A + 2)\pi_{\text{per}} - A]\epsilon/2}{(1 - \epsilon)A(1 - \pi_{\text{per}})} \]
\[ q^I = \frac{1}{2} \]

**Case D:** \( \pi_{\text{per}} \leq \hat{\pi}_{\text{per}}, \epsilon > \frac{2\hat{\pi}_{\text{per}}(1 - \pi_{\text{per}})}{\hat{\pi}_{\text{per}} + \pi_{\text{per}} - 2\hat{\pi}_{\text{per}}\pi_{\text{per}}} \)

\[ p_a^I = 1 - p_b^I = 1 - q^I = 1 \]

**Case E:** \( \pi_{\text{per}} > \hat{\pi}_{\text{per}} \)

\[ p_a^I = p_b^I = q^I = 1 \]

### 9.4.2 Model 2

The behavior predicted under Model 2 is as follows:

**Case A:** \( 2\pi_{\text{per}} > A(1 - \pi_{\text{per}}) \)

\[ p_a^{II} = \begin{cases} 1 & \text{if } \epsilon A/2 > 1 \\ 0.5 & \text{if } \epsilon A/2 = 1 \\ 0 & \text{if } \epsilon A/2 < 1 \\ \end{cases} \]

\[ p_b^{II} = \begin{cases} 1 & \text{if } 2(1 - \epsilon/2) > 1 \\ 0.5 & \text{if } 2(1 - \epsilon/2) = 1 \\ 0 & \text{if } 2(1 - \epsilon/2) < 1 \\ \end{cases} \]

\[ q^{II} = 1 \]

**Case B:** \( 2\pi_{\text{per}} < A(1 - \pi_{\text{per}}) \)

\[ p_a^{II} = \begin{cases} 1 & \text{if } (1 - \epsilon/2)A > 1 \\ 0.5 & \text{if } (1 - \epsilon/2)A = 1 \\ 0 & \text{if } (1 - \epsilon/2)A < 1 \\ \end{cases} \]

\[ p_b^{II} = q^{II} = 0 \]
Case C: $2\pi_{\text{per}} = A(1 - \pi_{\text{per}})$

$p^{II}_a = 1.0$
$p^{II}_b = 0.5$
$q^{II} = 0.5$

9.4.3 Model 3

The behavior predicted under Model 3 is as follows:

$emp_t = emp + \{num_{left}|num_{go}\} / num_{go} + 1$

$empa_t = empa + \{num_{go}|num_{game_a}\} / num_{go} + 1$

$empb_t = empb + \{num_{go}|num_{game_b}\} / num_{go} + 1$

Making Player 2’s updated belief at round $t$ equivalent to $emp\pi_t$ below:

$emp\pi_t = \frac{empa_t\pi_{\text{per}}}{empa_t\pi_{\text{per}} + empb_t\pi_{\text{per}}}$

Such that:

$p^{III}_{a,t} = \begin{cases} 
1 & \text{if } (1 - emp_t)A > 1 \\
0.5 & \text{if } (1 - emp_t)A = 1 \\
0 & \text{if } (1 - emp_t)A < 1 \\
\end{cases}$

$p^{III}_{b,t} = \begin{cases} 
1 & \text{if } 2emp_t > 1 \\
0.5 & \text{if } 2emp_t = 1 \\
0 & \text{if } 2emp_t < 1 \\
\end{cases}$

$q^{III}_t = \begin{cases} 
1 & \text{if } 2emp\pi_t > (1 - emp\pi_t)A \\
0.5 & \text{if } 2emp\pi_t = (1 - emp\pi_t)A \\
0 & \text{if } 2emp\pi_t < (1 - emp\pi_t)A \\
\end{cases}$

9.4.4 Calculating the Likelihood of Every Possible Dataset

The likelihood of a particular move for a player is defined as:

$obs_p^{M}_{a,t} = (1 - \epsilon_t) p^{M}_{a,t} + \epsilon_t / 2$

$obs_p^{M}_{b,t} = (1 - \epsilon_t) p^{M}_{b,t} + \epsilon_t / 2$

$obsq^{M}_t = (1 - \epsilon_t) q^{M}_t + \epsilon_t / 2$
Using these values, the likelihood for a particular model, $M \in \{I, II, III\}$, is defined by:

$$like^M_T = \int_{\epsilon_t} \int_{\alpha} \int_{\delta} \int_{\pi} \int_{\delta} \left( \prod_{t=1}^{T} \prod_{i=1}^{n} like(\text{action}_i^t | M; \epsilon_0, \alpha, \pi_{\text{per}}) \right) d\pi_{\text{per}} \text{prior}(d\epsilon_0, d\alpha, d\delta)$$

(3)

Using these likelihood functions in equation 3, the long process of computing all possible datasets begins. For each coordinate in a discrete grid of $A$ and $\pi$—from [2, 6] and [0.2, 0.8] respectively—we compute the likelihood of each Model (1, 2, 3) at that coordinate. This, in essence, allows us to see which design parameters will optimally distinguish our competing models.