Effect of electron interactions on the conductivity and exchange coupling energy of disordered metallic magnetic multilayer

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Abstract – We consider the effect of electron-electron interactions on the current-in-plane (CIP) conductivity and exchange coupling energy of a disordered metallic magnetic multilayer. We analyze its dependence on the value of ferromagnetic splitting of conducting electrons and on ferromagnetic-layers relative magnetizations orientation. We show that the contribution to the CIP conductivity and exchange coupling energy as a periodic function of the angle of magnetizations relative orientation experience $2\pi \rightarrow \pi$ transition depending on the characteristic energies, i.e. ferromagnetic splitting of the conducting electrons and the Thouless energy of paramagnetic layer.

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Introduction. – The most interesting features of a perfect metallic magnetic multilayer are the oscillatory behavior of the bilinear exchange coupling energy between ferromagnets [1,2] due to Friedel oscillations, and the large magnetoresistance in small magnetic fields [3,4].

The magnetic structure of adjoining magnetic layers in a perfect multilayered structure oscillates between ferro- and antiferro-magnetic states with increasing the spacer thickness $L$. Disorder in layers contributes to quadratic exchange coupling. It was shown that thickness fluctuations of the paramagnetic layer give rise to biquadratic exchange coupling, often leading to non-collinear magnetic ordering [5]. In refs. [6,7] the role of scattering of conducting electrons by impurities in metallic magnetic multilayers was studied. It was pointed out that in the case of small (compared to thickness of the layers) mean free path of conducting electrons, when average Friedel oscillations are exponentially suppressed, the exchange coupling energy due to random Friedel oscillations and correlation effects can have biquadratic form. Transition to the non-collinear phase in disordered structure with increasing $L$ was experimentally observed in ref. [8].

It is established that the magnetoresistance of a perfect metallic magnetic multilayered structure is related to the spin-dependent scattering of conducting electrons at the interfaces between the layers [9–11]. In the case of disorder, scattering of conducting electrons by impurities suppresses the effect of spin-dependent scattering on magnetoresistance. Moreover, it was theoretically shown that the magnitude of the magnetoresistance decreases exponentially in the case of the current-in-plane (CIP) geometry, when the thickness of the nonmagnetic spacer exceeds the mean free path of conducting electrons [10,12].

In the present paper we study the effect of electron-electron interactions on CIP conductivity and exchange coupling energy of a disordered metallic multilayered structure consisting of two ferromagnetic layers with paramagnetic spacer. We consider the case when the mean free path of conducting electrons is smaller than the thickness of the layers. It is known that in disordered conductors electron-electron interactions result in anomalous contributions to conductivity, thermodynamic quantities and negative magnetococonductivity [13,14]. The physics behind the effect of electron-electron interactions in disordered conductors is the electron scattering by the random Friedel oscillations [15]. Friedel oscillations in a magnetic multilayered structure do depend on the angle $\varphi$ between the directions of magnetization in the magnetic layers. The study of this dependence is the subject of this paper.

The scattering by Friedel oscillations exists in any type of ferromagnetic structure: itinerant ($d$-type) or localized ($f$-type). In what follows, the most relevant factors of a disordered magnetic multilayered structure are the characteristic of the disorder —the Thouless energy $D/L^2$ ($D$ is the conducting-electron diffusion constant), and ferromagnetic splitting of conducting electrons. Because of that, we consider the model of localized $s$-$f$ magnetism that
contains these parameters in the most transparent way. We propose that each ferromagnetic layer is described by the homogeneous magnetization, and that the directions of magnetizations of the different layers make an angle \( \varphi \).

The conductivity and exchange coupling energy in a magnetic multilayered structure are periodic functions of \( \varphi \). We show that depending on the ratio of the above-introduced characteristic energies, the angle-\( \varphi \)-dependent contributions of electron-electron interactions experience 2\( \pi \rightarrow \pi \) periodicity transition. The magnitudes of the contributions are estimated.

We suppose that our results might be relevant to the series of works related to the transport properties of mesoscopic ferromagnets with domain walls, where it was experimentally [16] and theoretically [17,18] shown that the effects of electron interactions and weak localization are important.

We would like to mention that the results of this paper complement the theory of electron-electron interactions in disordered conductors [13,14] as a study of the effect of electron-electron interactions in a spatially inhomogeneous effective magnetic field that is imposed by the magnetization in layers.

**The model.** – The system of our study is a disordered metallic magnetic multilayer structure. It consists of two ferromagnetic layers of the \( f \)-type located at \( L/2 < |z| < L/2 \) and a paramagnetic layer between them (\( |z| < L/2 \)). The Hamiltonian of conducting electrons of this system is

\[
H = H_0 + H_C + H_{sf}. \tag{1}
\]

Here \( H_0 \) is the Hamiltonian of free electrons in a random field \( V(\mathbf{r}) \). Treating scattering of free electrons we use the standard diagram technique [19], which assumes \( \langle V(\mathbf{r}) \rangle = 0 \) and \( \langle V(\mathbf{r}) V(\mathbf{r}') \rangle = 1/(2\pi\nu_0\tau)\delta(\mathbf{r} - \mathbf{r}') \). Here \( \nu_0 \) is the density of states at the Fermi level per spin, \( \tau \) is the electron mean free time, and we have set \( \hbar = 1 \).

We assume the Hamiltonian of free electrons to be the same in every layer.

The second term \( H_C \) is the Hamiltonian of the Coulomb interaction between conducting electrons. \( H_C = \frac{i}{\hbar} \int \Psi_\alpha(\mathbf{r})(\mathbf{S}_i \cdot \sigma_{\alpha \beta})\Psi_\beta(\mathbf{r})d\mathbf{r}' \), where \( \Psi_\alpha(\mathbf{r}) \) and \( \Psi_\beta(\mathbf{r}) \) are the electron creation and annihilation operators, respectively. We will treat Coulomb interaction within the random phase approximation.

The third term \( H_{sf} \) is the Hamiltonian of \( s-f \) exchange in the ferromagnetic layers. At temperatures much lower than the Curie temperature one might neglect electron-magnon interaction, therefore

\[
H_{sf} = I \sum_i \Psi_\alpha(\mathbf{r}_i)(\mathbf{S}_i \cdot \sigma_{\alpha \beta})\Psi_\beta(\mathbf{r}_i) \rightarrow ISn_S \int d\mathbf{r}\Psi_\alpha(\mathbf{r})(\mathbf{n}(\mathbf{r}) \cdot \sigma_{\alpha \beta})\Psi_\beta(\mathbf{r}). \tag{2}
\]

Here \( \mathbf{n} \) is the spin of localized \( f \)-electrons, \( n_S \) is their density. \( I \) is the \( s-f \) exchange interaction. \( \mathbf{n}(\mathbf{r}) \) is the ferromagnetic-layer magnetization direction unit vector. Integration here is over the ferromagnetic layers.

Neglecting the contribution of electron-magnon interaction to the conductivity and exchange coupling energy, we assume that the Coulomb energy per electron is larger than the ferromagnetic splitting \( ISn_S \).

Finally we consider the following Hamiltonian of a disordered metallic magnetic multilayer:

\[
H = H_0 + H_C + \frac{\epsilon_{exc}}{2} \int d\mathbf{r}\Psi_\alpha^\dagger(\mathbf{r})(\mathbf{n}(\mathbf{r}) \cdot \sigma_{\alpha \beta})\Psi_\beta(\mathbf{r}). \tag{3}
\]

This Hamiltonian describes a Fermi liquid in an effective inhomogeneous magnetic field that acts only on the electron spin. This magnetic field results in spin splitting with energy \( \epsilon_{exc} = 2ISn_S \) in the ferromagnetic layers, and in zero splitting in the paramagnetic layer. The direction of this magnetic field is defined by the ferromagnetic layer magnetization direction unit vector \( \mathbf{n}(z) = \mathbf{n}_1 \) at \(-L_1/2 < z < -L/2\) and \( \mathbf{n}(z) = \mathbf{n}_2 \) at \( L_1/2 > z > L/2 \), with \( (\mathbf{n}_1 \cdot \mathbf{n}_2) = \cos \varphi \).

Now we are ready to study the effect of electron interaction on the CIP conductivity and exchange coupling energy between the ferromagnetic layers. We will be interested in its dependence on the angle between the directions of magnetization \( \varphi \), ferromagnetic splitting energy \( \epsilon_{exc} \), and paramagnetic thickness \( L \).

**Conductivity.** We use Kubo linear response formalism in order to study the effect of the electron interaction on conductivity. Treating electron interaction as a perturbation, we come up with corrections to the conventional Drude formula for conductivity. We study only the first-order perturbation correction. The magnetic-field-dependent or, in our case, magnetization-dependent correction to conductivity comes from the interaction between two electrons in the triplet state (the so-called the Hartree-type correction) [13,14]. The disorder-averaged correction can be described with the help of a Feynman diagram, shown in fig. 1A. Black squares in the figure stand for the diffusion ladder that represents electron’s charge and spin densities propagation. Calculations of the presented diagrams for the homogeneous magnetic field are carried out in ref. [13]. We make use of the bilinear representation of the diffusion ladder in order to calculate these diagrams for an inhomogeneous magnetic field. Our calculations show that in the case of the CIP geometry all diffusion ladder combinations that appear in the figure can be simplified in to the following expression:

\[
\delta\sigma(z) = I (F(z))^2 \frac{e^2}{8\pi^2} \times \int_{-\infty}^{\infty} d\omega \frac{d}{d\omega} \left( \omega \coth \left( \frac{\omega}{2T} \right) \right) \text{Tr} \mathcal{D}_{\alpha\beta\alpha\beta}(z,0,-i\omega), \tag{4}
\]

where \( F(z) = F_{fer}(\sqrt{|z| - L/2}) + F_{param}(L/2 - |z|) \) is the electron interaction constant that is discussed in ref. [13].

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The angle-dependent part of the CIP conductance is given by the expression \[13\] with 

\[\delta G \equiv \int dz \delta \sigma (z)\] as a sum over the paramagnetic and ferromagnetic layers.

**Exchange coupling energy.** The contribution to the multilayer thermodynamic potential, which depends on the relative orientation of the ferromagnetic magnetizations, is also due to the interaction between two electrons in the triplet state. This correction is presented in fig. 1B and given by the expression \[13\]

\[
\frac{\delta \Omega (\varphi)}{S} = \sum_{\omega_n | \tau < 1} \left| \omega_n \right| \int \frac{d^2 q}{(2\pi)^2} \int dz F(z) \text{Tr} D_{\mu \nu}^{\alpha \beta} (z, \omega_n),
\]

\[\text{(5)}\]

where \(\omega_n = 2\pi n T\) is the Matsubara frequency, \(S\) is the area of the system. The angle-dependent part of \(\delta \Omega (\varphi)\) determines the exchange coupling energy of the adjoint ferromagnetic layers.

**Diffusion ladder.** The angle \(\varphi\) dependence in expressions \(4\) and \(5\) is due to the diffusion ladder. The graphical equation for diffusion ladder \(D_{\mu \nu}^{\alpha \beta} (z, z', q, \omega_n)\) is shown in fig. 1C, and in the case of diffusion approximation \(|\omega_n / \tau < 1, \epsilon_{\text{exc}} / \tau < 1, |q| v_F / \tau < 1\), with \(v_F\) being the Fermi velocity) it satisfies the following differential equation:

\[
\left( -D \frac{d^2}{dz^2} + Dq^2 + |\omega_n| \right) D_{\mu \nu}^{\alpha \beta} + i \frac{\epsilon_{\text{exc}}}{2} n(z) (\sigma_{\alpha \gamma} D_{\mu \nu}^{\beta \gamma} - D_{\mu \nu}^{\beta \alpha} \sigma_{\gamma \nu}) \text{sign} \omega_n = \delta(z-z') \delta_{\alpha \beta} \delta_{\mu \nu}.
\]

\[\text{(6)}\]

Here \(D = \frac{1}{4} v_F^2 T\) is the diffusion constant. We assume it to be the same in every layer. We notice that in expressions \(4\) and \(5\) only trace of the diffusion ladder appears. Since the trace is invariant under unitary transformations, we may choose the ferromagnetic-layers magnetization direction unit vector in the form convenient for further calculations: in \(-L_1/2 < z < -L/2\) the direction is \(n = (1, 0, 0)\), and in \(L_1/2 > z > L/2\) it is \(n = (\cos \varphi, 0, \sin \varphi)\).

The solution of equation \(6\) has to satisfy the boundary conditions \(\frac{d}{dz} D_{\mu \nu}^{\alpha \beta}(z, z', q, \omega_n) = 0\) at \(z = \pm L/2\), and the continuity conditions at \(z = \pm L/2\).

\(2\pi \rightarrow \pi\) transition. – Here we present our main results by skipping all derivations that will be given in the appendix to the paper. We consider only the angle-dependent parts of \(4\) and \(5\).

The characteristic energies of a magnetic multilayer are the Thouless energy of the paramagnetic layer \(D/L^2\) and the ferromagnetic splitting energy \(\epsilon_{\text{exc}}\). We present results for the cases of large ferromagnetic thickness \(d = L_1/2 - L/2; \epsilon_{\text{exc}} d^2 / D > 1\) and for small temperatures: \(\epsilon_{\text{exc}} \gg T\).

Our main finding is the following: depending on the ratio of the Thouless energy and ferromagnetic splitting energy, contribution to the CIP conductance \(4\) and exchange coupling energy between ferromagnetic layers \(5\) might have \(2\pi\) or \(\pi\) periodicity as a function of \(\varphi\). \(2\pi \rightarrow \pi\) transition occurs when the ratio \(\epsilon_{\text{exc}} L^2 / D\) increases.

i) In the case of large Thouless energy \(\epsilon_{\text{exc}} L^2 / D < 1\) the leading term in the angle-dependent parts of the CIP conductance and exchange coupling energy is proportional to \(\cos \varphi\). One can show that the ferromagnetic layers give the main contribution to the conductance as well as to the exchange coupling energy:

\[
\delta G \simeq -F_{\text{fer}} \frac{\epsilon^2}{32\pi^2} \cos \varphi,
\]

\[\text{(7)}\]

\[
\frac{\delta \Omega (\varphi)}{S} \simeq \frac{-\epsilon_{\text{exc}} \cos \varphi}{2(4\pi)^2 D} \left(0.06 F_{\text{par}} + 0.7 F_{\text{fer}} \ln \left(\frac{D}{\epsilon_{\text{exc}} L^2}\right)\right).
\]

\[\text{(8)}\]

The correction to conductance originating from the paramagnetic layer is reduced by a factor of \(\sqrt{\epsilon_{\text{exc}} L^2 / D}\).

ii) In the limit of \(\epsilon_{\text{exc}} L^2 / D > 1\) the leading term in \(4\) and \(5\) is \(\cos^2 \varphi\), and it is independent of the ferromagnetic splitting energy \(\epsilon_{\text{exc}}\):

\[
\delta G \simeq -(0.7 F_{\text{fer}} + 0.03 F_{\text{par}}) \frac{\epsilon^2}{4\pi^2} \cos^2 \varphi,
\]

\[\text{(9)}\]

\[
\frac{\delta \Omega (\varphi)}{S} \simeq \frac{(3F_{\text{fer}} + F_{\text{par}})}{2(4\pi)^2 D} \left(\frac{D}{L^2}\right)^2 \cos^2 \varphi.
\]

\[\text{(10)}\]
The physics behind this transition is the following. According to ref. [13] the magnetic-field–dependent contributions to the conductivity and exchange coupling energy are due to the interaction of electrons with total spin \( J = 1 \) and projection \( J_z = \pm 1 \). In our case when the direction of \( \mathbf{n}(\mathbf{r}) \) varies in space, the projections \( J_z = 0, \pm 1 \) are defined only locally.

At small \( \epsilon_{exc} \) the leading contribution is due to the interaction of electrons with total spin \( J = 1 \) and projections \( J_z = \pm 1 \), and the dependence of the conductivity and of the exchange coupling energy is proportional to \( \cos \varphi \). In the case of large \( \epsilon_{exc} \) diffusion modes with total spin \( J = 1 \) and projection \( J_z = \pm 1 \) do not penetrate into ferromagnetic layers and their contributions to the conductivity and the exchange coupling energy are suppressed. The main contribution in this case is due to the diffusion mode with \( J = 1 \), \( J_z = 0 \) and is proportional to \( \cos^2 \varphi \). Let us note, that in this case the electron-magnon interaction, which creates \( J_z = \pm 1 \) diffusion modes in ferromagnetic layers might be neglected even if the Coulomb energy per electron is smaller than the ferromagnetic splitting.

At temperatures \( T > D/\hbar^2 \) the contribution to the conductance decreases as

\[
\delta G = -\frac{e^2}{4\pi^2} \frac{0.2L^2}{D} [F_{par} + 5F_{fer}] \cos^2\varphi, \quad (11)
\]

and the exchange coupling energy decreases with the temperature as \( \exp(-L/\sqrt{8\pi T/D}) \).

In the Fermi-liquid theory the interaction constants are considered as some parameters. In the case of weakly non-ideal 3D Fermi gas \( F = \kappa^2/(2p_F^2)\ln(1 + 4p_F^2/\kappa^2) \), where \( \kappa \) and \( p_F \) are the inverse screening length and the Fermi momentum, respectively [13]. For the Coulomb interaction \( F_{par} \) and \( F_{fer} \) are both positive. In both limits, i) and ii), the contribution to conductance has a minimum at \( \varphi = 0 \). The exchange coupling energy has a minimum at \( \varphi = \pi/2 \) in the first limit, and at \( \varphi = \pi/2 \) in the second limit.

Let us estimate the value of exchange coupling energy. Taking \( D = 10 \) cm\(^2\)/s, \( \epsilon_{exc} = 300 \) K \( \times \) \( k_B \) (where \( k_B \) is the Boltzmann constant), we obtain that crossover occurs at the paramagnetic layer thickness \( L = \sqrt{\hbar D/\epsilon_{exc}} = 5 \) nm. Plugging these values into eqs. (8) and (10), and assuming \( \delta F_{fer} \sim F_{par} \sim 1 \), we obtain

\[
\frac{\delta \Omega(\varphi)}{S} \sim 0.5 \times 10^{-3} \text{erg/cm}^2.
\]

The correction to conductance is of the order of percents of \( \frac{\delta G}{G} \).

**Conclusion.** – In the present paper we have considered the properties of a disordered magnetic metallic multilayer in the case of small electron mean free path compared to the paramagnetic layer thickness. The angle \( \varphi \) dependence of the CIP conductance and of the exchange coupling energy is determined by the electron-electron interactions. Depending on the ratio of the Thouless and ferromagnetic splitting energies these quantities might be \( 2\pi \) as well as \( \pi \) periodic functions of \( \varphi \).

Obtaining our results, we have neglected the effect of spin-orbit scattering. This scattering smears out the magnetic dependence of electron interaction corrections to thermodynamic and kinetic properties [13]. However, this effect become crucial for the paramagnetic thickness (larger than the spin relaxation length due to spin-orbital scattering) when the values of calculated quantities already become too small.

We would like to point out that in ref. [7] the contribution to the exchange coupling energy due to the interaction was considered only in the paramagnetic layer. In this paper we have shown that the contribution of ferromagnetic layers is larger than that of the paramagnetic layer.

**Appendix A: Derivations.** – Here we describe the main steps of the derivation. We solve the differential equation (6) for the diffusion ladder. According to expressions (4) and (5) the solutions of interest are those with \( z \) and \( z' \) lying in the same layer.

1) Let us first derive the diffusion ladder when \( z' < -L/2 \) in the ferromagnetic layer. It is convenient to present the solution in the paramagnetic layer \((|z| < L/2)\) in the following form:

\[
\delta^\alpha_\mu = A^\alpha_\mu e^{-Qz} + U^\alpha_\gamma B^\gamma_\mu U^\gamma_\nu e^{Qz}.
\]

And in the ferromagnetic layer at \(-L/2 < z' < z < -L/2\) the solution that satisfies the boundary condition at \( z = -L/2 \) is conveniently presented in the following form:

\[
\begin{align*}
F^\alpha_\mu & = F_0^\alpha_\mu + \sum_\pm P^\alpha_\mu \gamma \epsilon_\gamma^\beta_\lambda \epsilon_\lambda^\nu_\zeta e^{Qz} (z + \frac{L_1}{2}) \\
& + P^\alpha_\mu \gamma \epsilon_\gamma^\beta_\lambda \epsilon_\lambda^\nu_\zeta e^{Qz} (Q_1 (z + \frac{L_1}{2}), (A.2)
\end{align*}
\]

where

\[
\begin{align*}
F_0^\alpha_\mu & = \sum_\pm \frac{P^\alpha_\mu \gamma P^\mu_\nu}{DQ} e^{-Qz} (z + \frac{L_1}{2}) \cosh Q_1 (z + \frac{L_1}{2}) \\
& + \frac{P^\alpha_\mu \gamma P^\mu_\nu}{DQ_1} e^{-Qz} (z + \frac{L_1}{2}) \cosh Q_1 (z + \frac{L_1}{2}), (A.3)
\end{align*}
\]

where \( P_\pm = (1 \mp \sigma_z)/2 \) are projectors on the spin-up and -down states, respectively; \( Q = \sqrt{q^2 + |\omega_n|/D}, Q_1 = \sqrt{q^2 + (|\omega_n| + i\epsilon_{exc} \sigma_y)|\omega_n|/D}, \) and \( U = \exp(i\varphi \sigma_y/2) \) is the matrix of rotation along the \( y \)-axis. \( A, B, C, E, M \)
\[
\int \frac{\text{Tr} F_{\mu \nu}^{\alpha} (z, q, \omega_n)}{W_+} \, dz = -\frac{4\Lambda_1}{Q} \left[ \frac{\text{Re} (1 + \Lambda) [\Lambda^* + \cos \varphi]}{W_+} + (\Lambda \to -\Lambda, \Lambda_1 \to -\Lambda_1) \right] \frac{1}{D} \left[ \frac{\sinh (2Qd)}{Q} + 2d \right] + \frac{8Q}{D} \text{Re} \left[ \frac{2 + \Lambda + \Lambda_1 + (2\Lambda_1 + \Lambda_1 + \Lambda) \cos \varphi}{2Q_1 (Q^2 - Q_1^2) W_+} + (\Lambda \to -\Lambda, \Lambda_1 \to -\Lambda_1) \right],
\]
(A.5)

\[
\int \frac{\text{Tr} D_{\mu \nu}^{\alpha} (z, q, \omega_n)}{W_+} \, dz = -\frac{L \text{Re} \Lambda^* (1 + \Lambda)}{2DQW_+} \left[ \left( \cos \varphi + \Lambda_1 \right) + (1 + \Lambda_1 \cos \varphi) \frac{\sinh QL}{QL} \right] - \frac{L \text{Re} \Lambda^* (1 - \Lambda)}{2DQW_-} \times \left[ \left( \cos \varphi + \Lambda_1 \right) + (1 - \Lambda_1 \cos \varphi) \frac{\sinh QL}{QL} \right] + \Lambda_1 L \left[ \frac{\text{Re} \left[ (1 + \Lambda^*) (1 + \Lambda \cos \varphi) \right] \frac{\sinh QL}{QL} - \text{Re} \left[ (1 + \Lambda^*) (A + \cos \varphi) \right]}{2DQW_+} \right]
\]
(A.7)

\[
\int \frac{\text{Tr} F_{\mu \nu}^{\alpha} (z, q, |\omega_n|)}{W_+} \, dz = \frac{-4\Lambda_1}{Q} \left[ \frac{\text{Re} (1 + \Lambda) [\Lambda^* + \cos \varphi]}{W_+} + (\Lambda \to -\Lambda, \Lambda_1 \to -\Lambda_1) \right] \frac{1}{D} \left[ \frac{\sinh (2Qd)}{Q} + 2d \right] + \frac{8Q}{D} \text{Re} \left[ \frac{2 + \Lambda + \Lambda_1 + (2\Lambda_1 + \Lambda_1 + \Lambda) \cos \varphi}{2Q_1 (Q^2 - Q_1^2) W_+} + (\Lambda \to -\Lambda, \Lambda_1 \to -\Lambda_1) \right],
\]

where we have used the differential operator \( \hat{R} \) for ferromagnetic layer (A.2) and the independent part we obtain the trace of the diffusion ladder for ferromagnetic layer (A.2).

ii) The same procedure can be applied for the case when \( z' \) is in the paramagnetic layer. Now the solution of eq. (6) when \( z \) is in the paramagnetic layer is conveniently presented in the following form:

\[
D_{\mu \nu}^{\alpha \beta} = A_{\mu \nu}^{\alpha \beta} e^{-Qz} + U_{\mu \nu}^1 B_{\mu \alpha}^{\gamma \delta} U_{\nu \lambda} e^{Qz} + \frac{e^{-Qz' - \Lambda}}{2QD} \delta_{\alpha \beta} \delta_{\mu \nu}.
\]
(A.6)

In this case the continuity conditions for the rotated diffusion ladder (A.6) \( \hat{D}_{\mu \nu}^{\alpha \beta} \equiv U_{\mu \nu}^1 \hat{D}_{\mu \alpha}^{\gamma \delta} U_{\nu \lambda}^1 \) at \( z = L/2 \) are the same as (A.4). At \( z = -L/2 \) continuity conditions for the diffusion ladder (A.6) change as

\[
\hat{L}(Q) P_{\mu \lambda}^{\alpha} D_{\nu \lambda}^{\beta} P_{\mu \nu}^{\alpha} = \hat{L}(Q_1) P_{\mu \lambda}^{\alpha} D_{\nu \lambda}^{\beta} P_{\mu \nu}^{\alpha} = \hat{L}(Q_1) P_{\mu \lambda}^{\alpha} D_{\nu \lambda}^{\beta} P_{\mu \nu}^{\alpha} = 0.
\]

Solving for matrices \( A, B \) and subtracting the angle-independent part, we obtain the diffusion ladder for the paramagnetic layer (A.6), whose trace is

\[
\text{see eq. (A.7) above}
\]

The results (7)–(11) will be recovered if one plugs the diffusion ladder traces (A.5) and (A.7) into the expressions for the conductivity (4) and exchanges the coupling energy (5). The integrals over the frequency in the resulting expressions can be evaluated as follows: in the case of \( \epsilon_{\text{exc}} L^2 / D > 1 \) the main contribution is due to frequencies \( |\omega_n| < \epsilon_{\text{exc}} \), where \( \Lambda \approx \text{Re} \Lambda \approx \exp (-QL) \).

In this case expressions (4) and (5) are even functions of \( \varphi \). In the region from \( \epsilon_{\text{exc}} L^2 / D < 1 \) the main contribution is due to \( |\omega_n| > \epsilon_{\text{exc}} \) in this case \( \Lambda \approx |A|^2 \), and expressions (4) and (5) have periodicity \( 2\pi \).
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