Languages cool as they expand: Allometric scaling and the decreasing need for new words

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We analyze the occurrence frequencies of over 15 million words recorded in millions of books published during the past two centuries in seven different languages. For all languages and chronological subsets of the data we confirm that two scaling regimes characterize the word frequency distributions, with only the more common words obeying the classic Zipf law. Using corpora of unprecedented size, we test the allometric scaling relation between the corpus size and the vocabulary size of growing languages to demonstrate a decreasing marginal need for new words, a feature that is likely related to the underlying correlations between words. We calculate the annual growth fluctuations of word use which has a decreasing trend as the corpus size increases, indicating a slowdown in linguistic evolution following language expansion. This “cooling pattern” forms the basis of a third statistical regularity, which unlike the Zipf and the Heaps law, is dynamical in nature.

Books in libraries and attics around the world constitute an immense “crowd-sourced” historical record that traces the evolution of culture back beyond the limits of oral history. However, the disaggregation of written language into individual books makes the longitudinal analysis of language a difficult open problem. To this end, the book digitization project at Google Inc. presents a monumental step forward providing an enormous, publicly accessible, collection of written language in the form of the Google Books Ngram Viewer web application. Approximately 4% of all books ever published have been scanned, making available over 10^7 occurrence time series (word-use trajectories) that archive cultural dynamics in seven different languages over a period of more than two centuries. This dataset highlights the utility of open “Big Data,” which is the gateway to “metaknowledge,” the knowledge about knowledge. A digital data deluge is sustaining extensive interdisciplinary research efforts towards quantitative insights into the social and natural sciences.

“Culturomics,” the use of high-throughput data for the purpose of studying human culture, is a promising new empirical platform for gaining insight into subjects ranging from political history to epidemiology. As first demonstrated by Michel et al., the Google n-gram dataset is well-suited for examining the microscopic properties of an entire language ecosystem. Using this dataset to analyze the growth patterns of individual word frequencies, Petersen et al. recently identified tipping points in the life trajectory of new words, statistical patterns that govern the fluctuations in word use, and quantitative measures for cultural memory. The statistical properties of cultural memory, derived from the quantitative analysis of individual word-use trajectories, were also investigated by Gao et al., who found that words describing social phenomena tend to have different long-range correlations than words describing natural phenomena.

Here we study the growth and evolution of written language by analyzing the macroscopic scaling patterns that characterize word-use. Using the Google 1-gram data collected at the 1-year time resolution over the period 1800–2008, we quantify the annual fluctuation scale of words within a given corpora and show that languages can be said to “cool by expansion.” This effect constitutes a dynamic law, in contrast to the static laws of Zipf and Heaps which are founded upon snapshots of single texts. The Zipf law, quantifying the distribution of word frequencies, and the Heaps law, relating the size of a corpus to the vocabulary size of that corpus, are classic paradigms that capture many complexities of language in remarkably simple statistical patterns. While these laws...
have been exhaustively tested on relatively small snapshots of empirical data, here we test the validity of these laws using extremely large corpora.

Interestingly, we observe two scaling regimes in the probability density functions of word usage, with the Zipf law holding only for the set of more frequently used words, referred to as the “kernel lexicon” by Ferrer i Cancho et al.14. The word frequency distribution for the rarely used words constituting the “unlimited lexicon”14 obeys a distinct scaling law, suggesting that rare words belong to a distinct class. This “unlimited lexicon” is populated by highly technical words, new words, numbers, spelling variants of kernel words, and optical character recognition (OCR) errors.

Many new words start in relative obscurity, and their eventual importance can be under-appreciated by their initial frequency. This fact is closely related to the information cost of introducing new words and concepts. For single topical texts, Heaps observed that the vocabulary size exhibits sub-linear growth with document size18. Extending this concept to entire corpora, we find a scaling relation that indicates a decreasing “marginal need” for new words which are the manifestation of cultural evolution and the seeds for language growth. We introduce a pruning method to study the role of infrequent words on the allometric scaling properties of language. By studying progressively smaller sets of the kernel lexicon we can better understand the marginal utility of the core words. The pattern that arises for all languages analyzed provides insight into the intrinsic dependency structure between words.

The correlations in word use can also be author and topic dependent. Bernhardsson et al. recently introduced the “metabook” concept19,20, according to which word-frequency structures are author-specific: the word-frequency characteristics of a random excerpt from a compilation of everything that a specific author could ever conceivably write (his/her “metabook”) should accurately match those of the author’s actual writings. It is not immediately obvious whether a compilation of all the metabooks of all authors would still conform to the Zipf law and the Heaps law. The immense size and time span of the Google n-gram dataset allows us to examine this question in detail.

Results

Longitudinal analysis of written language. Allometric scaling analysis23 is used to quantify the role of system size on general phenomena characterizing a system, and has been applied to systems as diverse as the metabolic rate of mitochondria22 and city growth23–29. Indeed, city growth shares two common features with the growth of written text: (i) the Zipf law is able to describe the distribution of city sizes regardless of country or the time period of the data26, and (ii) city growth has inherent constraints due to geography, changing labor markets and their effects on opportunities for innovation and wealth creation27–29. Just as vocabulary growth is constrained by human brain capacity and the varying utilities of new words across users14, the correlations in word use can also be author and topic dependent.

We construct a word counting framework by first defining the quantity \( f_i(t) \) as the number of times word \( i \) is used in year \( t \). Since the number of books and the number of distinct words grow dramatically over time, we define the relative word use, \( u_i(t) \), as the fraction of the total body of text occupied by word \( i \) in the same year

\[
f_i(t) \equiv u_i(t)/N_w(t),
\]

where the quantity \( N_w(t) = \sum_{i=1}^{N_v(t)} u_i(t) \) is the total number of distinct words used while \( N_v(t) \) is the total number of distinct words digitized from books printed in year \( t \). Both the \( N_w \) (“types” giving the vocabulary size) and the \( N_v \) (“tokens” giving the size of the body of text) are generally increasing over time.

The Zipf law and the two scaling regimes. Zipf investigated a number of bodies of literature and observed that the frequency of any given word is roughly inversely proportional to its rank14, with the frequency of the \( z \)-ranked word given by the relation

\[
f(z) \sim z^{-\zeta},
\]

with a scaling exponent \( \zeta = 1 \). This empirical law has been confirmed for a broad range of data, ranging from income rankings, city populations, and the varying sizes of avalanches, forest fires30 and firm size31 to the linguistic features of noncoding DNA32. The Zipf law can be derived through the “principle of least effort,” which minimizes the communication noise between speakers (writers) and listeners (readers)16. The Zipf law has been found to hold for a large dataset of English text14, but there are interesting deviations observed in the lexicon of individuals diagnosed with schizophrenia33. Here, we also find statistical regularity in the distribution of relative word use for 11 different datasets, each comprising more than half a million distinct words taken from millions of books4.

Figure 1 shows the probability density functions \( P(f) \) resulting from data aggregated over all the years \((A,B)\) as well as over 1-year periods as demonstrated for the year \( t = 2000 \) (C,D). Regardless of the language and the considered time span, the probability density functions are characterized by a striking two-regime scaling, which was first noted by Ferrer i Cancho and Solé14, and can be quantified as

\[
P(f) \sim \begin{cases} f^{-x_+}, & \text{if } f < f_x \text{ ["unlimited lexicon"]} \\ f^{-x_-}, & \text{if } f > f_x \text{ ["kernel lexicon"]} \end{cases}
\]

These two regimes, designated “kernel lexicon” and “unlimited lexicon,” are thought to reflect the cognitive constraints of the brain’s finite vocabulary14. The specialized words found in the unlimited lexicon are not universally shared and are used significantly less frequently than the words in the kernel lexicon. This is reflected in the kink in the probability density functions and gives rise to the anomalous two-scaling distribution shown in Fig. 1.

The exponent \( x_- \) and the corresponding rank-frequency scaling exponent \( \zeta \) in Eq. (2) are related asymptotically by14

\[
x_- \approx 1 + 1/\zeta,
\]

with no analogous relationship for the unlimited lexicon values \( x_- \) and \( \zeta_- \). Table 1 lists the average \( x_- \) and \( x_+ \) values calculated by aggregating \( x \) values for each year using a maximum likelihood estimator for the power-law distribution26. We characterize the two scaling regimes using a crossover region around \( f_x \approx 10^{-3} \) to distinguish between \( x_- \) and \( x_+ \): (i) \( 10^{-9} \leq f \leq 10^{-8} \) corresponds to \( x_- \) and (ii) \( 10^{-3} \leq f \leq 10^{-2} \) corresponds to \( x_+ \). For the words that satisfy \( f_x > f_x \), that comprise the kernel lexicon, we verify the Zipf scaling law \( \zeta \approx 1 \) (corresponding to \( x_+ \approx 2 \)) for all corpora analyzed. For the unlimited lexicon regime \( f \leq f_x \), however, the Zipf law is not obeyed, as we find \( x_- \approx 1.7 \). Note that \( x_- \) is significantly smaller in the Hebrew, Chinese, and the Russian corpora, which suggests that a more generalized version of the Zipf law41 may be needed, one which is slightly language-dependent, especially when taking into account the usage of specialized words from the unlimited lexicon.

The Heaps law and the increasing marginal returns of new words. Heaps observed that vocabulary size, i.e. the number of distinct words, exhibits a sub-linear growth with document size18. This observation has important implications for the “return on investment” of a new word as it is established and becomes disseminated throughout the literature of a given language. As a proxy for this return, Heaps studied how often new words are invoked in lieu of preexisting competitors and examined the linguistic value of new words and ideas by analyzing the relation between the total number of words printed in a body of text \( N_w \) and the number of these which are distinct \( N_v \), i.e. the vocabulary
The marginal returns of new words, \( \partial N_w/\partial N_u \), quantifies the impact of the addition of a single word to the vocabulary of a corpus on the aggregate output (corpus size).

For individual books, the empirically-observed scaling relation between \( N_u \) and \( N_w \) obeys

\[ N_w \sim (N_u)^b, \quad (5) \]

with \( b \leq 1 \), with Eq. (5) referred to as “the Heaps law”. It has subsequently been found that Heaps’ law emerges naturally in systems that can be described as sampling from an underlying Zipf distribution. In an information theoretic formulation of the abstract concept of word cost, B. Mandelbrot predicted the relation

\[ b = 1/\zeta \] in 1961, where \( \zeta \) is the scaling exponent corresponding to \( \alpha_+ \), as in Eqs. (3) and (4). This prediction is limited to relatively small texts where the unlimited lexicon, which manifests in the \( \alpha_- \) regime, does not play a significant role. A mathematical extension of this result for general underlying rank-distributions is also provided by Kornišov using an infinite urn scheme, and extended to broader classes of heavy-tailed distributions recently by Gnedin et al.

Recent research efforts using stochastic master equation techniques

| Corpus (1-grams) | \( \text{Min}[N_u] \) | \( b(U = 0) \) | \( \langle x_- \rangle \) | \( \langle x_+ \rangle \) | \( \zeta \) | \( \beta \) |
|------------------|-------------------|----------------|----------------|----------------|--------|--------|
| Chinese          | 35, 394           | 0.77 ± 0.02    | 1.49 ± 0.15    | 1.91 ± 0.04    | 1.10 ± 0.05 | 0.20 ± 0.01 |
| English          | 42, 786, 702      | 0.54 ± 0.01    | 1.73 ± 0.05    | 2.04 ± 0.06    | 0.96 ± 0.06 | 0.19 ± 0.01 |
| English fiction  | 13, 184, 111      | 0.49 ± 0.01    | 1.68 ± 0.10    | 1.97 ± 0.04    | 1.03 ± 0.04 | 0.18 ± 0.01 |
| English GB       | 38, 956, 621      | 0.44 ± 0.01    | 1.71 ± 0.07    | 2.02 ± 0.05    | 0.98 ± 0.05 | 0.17 ± 0.01 |
| English US       | 5, 821, 340       | 0.51 ± 0.01    | 1.70 ± 0.08    | 2.03 ± 0.06    | 0.97 ± 0.06 | 0.18 ± 0.01 |
| English 1M       | 42, 778, 968      | 0.53 ± 0.01    | 1.71 ± 0.04    | 2.04 ± 0.06    | 0.96 ± 0.06 | 0.25 ± 0.01 |
| French           | 34, 198, 362      | 0.52 ± 0.01    | 1.69 ± 0.06    | 1.98 ± 0.04    | 1.02 ± 0.04 | 0.26 ± 0.01 |
| German           | 2, 274, 842       | 0.60 ± 0.01    | 1.63 ± 0.16    | 2.02 ± 0.03    | 0.98 ± 0.03 | 0.27 ± 0.01 |
| Hebrew           | 9, 482            | 0.47 ± 0.01    | 1.34 ± 0.09    | 2.06 ± 0.05    | 0.94 ± 0.05 | 0.35 ± 0.01 |
| Russian          | 6, 944, 366       | 0.65 ± 0.01    | 1.55 ± 0.17    | 2.04 ± 0.06    | 0.96 ± 0.06 | 0.28 ± 0.01 |
| Spanish          | 1, 777, 563       | 0.51 ± 0.01    | 1.61 ± 0.15    | 2.07 ± 0.04    | 0.93 ± 0.04 | 0.26 ± 0.01 |
to model the growth of a book have also predicted this intrinsic relation between Zipf’s law and Heaps’ law.\textsuperscript{12,17,34}

Figure 2 confirms a sub-linear scaling \((b < 1)\) between \(N_u\) and \(N_w\) for each corpora analyzed. These results show how the marginal returns of new words are given by

\[
\frac{\partial N_u}{\partial N_w} \sim (N_w)^{(1-b)/b},
\]

which is an increasing function of \(N_w\) for \(b < 1\). Thus, the relative increase in the induced volume of written languages is larger for new words than for old words. This is likely due to the fact that new words are typically technical in nature, requiring additional explanations that put the word into context with pre-existing words. Specifically, a new word requires the additional use of preexisting words as a result of both (i) the explanation of the content of the new word using existing technical terms, and (ii) the grammatical infrastructure necessary for that explanation. Hence, there are large spillovers in the size of the written corpus that follow from the intricate dependency structure of language stemming from the various grammatical roles.\textsuperscript{39,40}

In order to investigate the role of rare and new words, we calculate \(N_u\) and \(N_w\) using only words that have appeared at least \(U_c\) times. We select the absolute number of uses as a word use threshold because a word in a given year cannot appear with a frequency less than \(1/N_w\), hence any criteria using relative frequency would necessarily introduce a bias for small corpora samples. This choice also eliminates words that can spuriously arise from Optical Character Recognition (OCR) errors in the digitization process and also from intrinsic spelling errors and orthographic spelling variations.

Figures 3 and 4 show the relational dependence of \(N_u\) and \(N_w\) on the exclusion of low-frequency words using a variable cutoff \(U_c = 2^n\) with \(n = 0 \ldots 11\). As \(U_c\) increases the Heaps scaling exponent increases from \(b \approx 0.5\), approaching \(b \approx 1\), indicating that core words are structurally integrated into language as a proportional background. Interestingly, Altmann et al.\textsuperscript{41} recently showed that “word niche” can be an essential factor in modeling word use dynamics. New niche words, though they are marginal increases to a language’s lexicon, are themselves anything but “marginal” – they are core words within a subset of the language. This is particularly the case in online communities in which individuals strive to distinguish themselves on short timescales by developing stylistic jargon, highlighting how language patterns can be context dependent.

We now return to the relation between Heaps’ law and Zipf’s law. Table I summarizes the \(b\) values calculated by means of ordinary least squares regression using \(U_c = 0\) to relate \(N_u(t)\) to \(N_w(t)\). For \(U_c = 1\) we find that \(b \approx 0.5\) for all languages analyzed, as expected from Heaps’ law, but for \(U_c \geq 8\) the \(b\) value significantly deviates from 0.5, and for \(U_c \geq 1000\) the \(b\) value begins to saturate approaching unity. Considering that \(\alpha_u = 2\) implies \(\zeta \approx 1\) for all corpora, Figures 3 and 4 show that we can confirm the relation \(b(U_c) \approx 1/\zeta\) only for the more pruned corpora that require relatively large \(U_c\). This hidden feature of the scaling relation highlights the underlying structure of language, which forms a dependency network between the common words of the kernel lexicon and their more esoteric counterparts in the unlimited lexicon. Moreover, the function \(\partial N_u/\partial N_w \sim (N_w)^{b-1}\) is a monotonically decreasing function for \(b < 1\), demonstrating the decreasing marginal need for additional words as a corpora grows. In other words, since we get more and more “mileage” out of new words in an already large language, additional words are needed less and less.

**Corpora size and word-use fluctuations.** Lastly, it is instructive to examine how vocabulary size \(N_u\) and the overall size of the corpora \(N_u\) affect fluctuations in word use. Figure 5 shows how \(N_u(t)\) and \(N_w(t)\) vary over time over the past two centuries. Note that, apart from the periods during the two World Wars, the number of words printed, which we will refer to as the “literary productivity”, has been increasing over time. The number of distinct words (vocabulary size) has also increased reflecting basic social and technological advancement.\textsuperscript{8}

To investigate the role of fluctuations, we focus on the logarithmic growth rate, commonly used in finance and economics

\[
r(t) = \ln f(t + \Delta t) - \ln f(t) = \ln \left( \frac{f(t + \Delta t)}{f(t)} \right),
\]

to measure the relative growth of word use over 1-year periods, \(\Delta t = 1\) year. Recent quantitative analysis on the distribution \(P(t)\) of word use growth rates \(r(t)\) indicates that annual fluctuations in word use deviates significantly from the predictions of null models for language evolution.\textsuperscript{5}

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**Figure 2 | Allometric scaling of language.** Scatter plots of the output corpora size \(N_u\) given the empirical vocabulary size \(N_w\) using all data \((U_c = 0)\) over the 209-year period 1800–2008. Shown are OLS estimation of the exponent \(b\) quantifying the Heaps’ law relation \(N_u \sim [N_w]^b\).
We define an aggregate fluctuation scale, $\sigma_r(t|\nu_c)$, using a frequency cutoff $\nu_c = 1/\text{Min}[\nu_r(t)]$ to eliminate infrequently used words. The quantity $\text{Min}[\nu_r(t)]$ is the minimum corpora size over the period of analysis, and so $1/\text{Min}[\nu_r(t)]$ is an upper bound for the minimum observed frequency for words in the corpora. Figure 6 shows $\sigma_r(t|\nu_c)$, the standard deviation of $r(t)$ calculated across all words that satisfy the condition $f_r(t) \geq \nu_c$ for words with lifetime $T_r \approx 10$ years, using $\nu_c = 1/\text{Min}[\nu_r(t)]$. Visual inspection suggests a general decrease in $\sigma_r(t|\nu_c)$ over time, marked by sudden increases during times of political conflict. Hence, the persistent increase in the volume of written language is correlated with a persistent downward trend what could be thought of as the “system temperature” $\sigma_r(t|\nu_c)$: as a language grows and matures it also “cools off.”

Since this cooling pattern could arise as a simple artifact of an independent identically distributed (i.i.d) sampling from an increasingly large dataset, we test the scaling of $\sigma_r(t|\nu_c)$ with corpora size. Figure 7(A) shows that for large $\nu_r(t)$, each language is characterized by a scaling relation $\sigma_r(t|\nu_c) \sim \nu_r(t)^{-\beta}$, with language-dependent scaling exponent $\beta \approx 0.08$–0.35. We use $\nu_c = 10/\text{Min}[\nu_r(t)]$, which defines the frequency threshold for the

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**Figure 3 | Pruning reveals the variable marginal return of words.** The Heaps scaling exponent $b$ depends on the extent of the inclusion of the rarest words. For a given corpora and $\nu_c$ value we make a scatter plot between $N_w(t|\nu_c)$ and $\nu_r(t|\nu_c)$ using words with $u_r(t) \geq \nu_c$. (Panel Inset) We use OLS estimation to estimate the scaling exponent $b(\nu_c)$ for the model $N_w(t|\nu_c) \sim [N_r(t|\nu_c)]^b$ to show that $b(\nu_c)$ increases from approximately 0.5 towards unity as we prune the corpora of extremely rare words. Our longitudinal language analysis provides insight into the structural importance of the most frequent words which are used more times per appearance and which play a crucial role in the usage of new and rare words.
inclusion of a given word in our analysis. There are two candidate null models which give insight into the limiting behavior of \( \beta \). The Gibrat proportional growth model predicts \( \beta = 0 \) and the Yule-Simon urn model predicts \( \beta = 1/2 \). We observe \( \beta < 1/2 \), which indicates that the fluctuation scale decreases more slowly with increasing corpora size than would be expected from the Yule-Simon urn model prediction, deducible via the “delta method” for determining the approximate scaling of a distribution and its standard deviation \( \sigma \).

To further compare the roles of the kernel lexicon versus the unlimited lexicon, we apply our pruning method to quantify the dependence of the scaling exponent \( \beta \) on the fluctuations arising from rare words. We omit words from our calculation of \( \sigma_j(U_c) \) if their use \( u_i(t) \) in year \( t \) falls below the word-use threshold \( U_c \). Fig. 7(B) shows that \( \beta(U_c) \) increases from values close to 0 to values less than 1/2 as \( U_c \) increases exponentially. An increasing \( \beta(U_c) \) confirms our conjecture that rare words are largely responsible for the fluctuations in a language. However, because of the dependency structure between words, there are residual fluctuation spillovers into the kernel lexicon likely accounting for the fact that \( \beta < 1/2 \) even when the fluctuations from the unlimited lexicon are removed.

Figure 4 | Pruning reveals the variable marginal return of words. The Heaps scaling exponent \( b \) depends on the extent of the inclusion of the rarest words. For a given corpora and \( U_c \) value we make a scatter plot between \( N_w(t | U_c) \) and \( N_u(t | U_c) \) using words with \( u_i(t) > U_c \) using the same data correspondence as in Fig. 3. (Panel Inset) We use OLS estimation to estimate the scaling exponent \( \beta(U_c) \) for the model \( N_w(t | U_c) \sim [N_u(t | U_c)]^{\beta} \) to show that \( \beta(U_c) \) increases from approximately 0.5 towards unity as we prune the corpora of extremely rare words. Our longitudinal language analysis provides insight into the structural importance of the most frequent words which are used more times per appearance and which play a crucial role in the usage of new and rare words.
A size-variance relation showing that larger entities have smaller characteristic fluctuations was also demonstrated at the scale of individual words using the same Google n-gram dataset\(^9\). Moreover, this size-variance relation is strikingly analogous to the decreasing growth rate volatility observed as complex economic entities (i.e. firms or countries) increase in size\(^{42,44–48}\), which strengthens the analogy of language as a complex ecosystem of words governed by competitive forces.

Further possible explanations for \(\beta < 1/2\) is that language growth is counteracted by the influx of new words which tend to have growth spurts around 30–50 years following their birth in the written corpora\(^9\). Moreover, the fluctuation scale \(\sigma_r(t|f_c)\) is positively influenced by adverse conditions such as wars and revolutions, since a decrease in \(N_u(t)\) may decrease the competitive advantage that old words have over new words, allowing new words to break through. The globalization effect, manifesting from increased human mobility

Figure 5 | Literary productivity and vocabulary size in the Google Inc. 1-gram dataset over the past two centuries. (A) Total size of the different corpora \(N_u(t|U_c)\) over time, calculated by using words that satisfy \(u_i(t) \geq U_i = 16\) to eliminate extremely rare 1-grams. (B) Size of the written vocabulary \(N_w(t|U_c)\) over time, calculated under the same conditions as (A).

Figure 6 | Non-stationarity in the characteristic growth fluctuation of word use. The standard deviation \(\sigma_r(t|f_c)\) of the logarithmic growth rate \(r(t)\) is presented for all examined corpora. There is an overall decreasing trend arising from the increasing size of the corpora, as depicted in Fig. 5(A). On the other hand, the steady production of new words, as depicted in Fig. 5(B) counteracts this effect. We calculate \(\sigma_r(t|f_c)\) using the relatively common words that meet the criterion that their average word use \(\langle f \rangle\) over the entire word history \(T_i\) (using words with lifetime \(T_i \geq 10\) years) is larger than a threshold \(f_c = 1/\text{Min}[N_u(t)]\) (see Table I).
during periods of conflict, is also responsible for the emergence of new words within a language.

Discussion
A coevolutionary description of language and culture requires many factors and much consideration.

We find that the word frequency distribution $P(f)$ is characterized by two scaling regimes. While frequently used words that constitute the kernel lexicon follow the Zipf law, the distribution has a less steep scaling regime quantifying the rarer words constituting the unlimited lexicon. Our result is robust across languages as well as across other data subsets, thus extending the validity of the seminal observation by Ferrer i Cancho and Solé14, who first reported it for a large body of subsets, thus extending the validity of the seminal observation by our result is robust across languages as well as across other data subsets, thus extending the validity of the seminal observation by Ferrer i Cancho and Solé14, who first reported it for a large body of subsets, thus extending the validity of the seminal observation by.

Here we analyzed the macroscopic properties of written language using the Google Books database1. We find that the word frequency distribution $P(f)$ is characterized by two scaling regimes. While frequently used words that constitute the kernel lexicon follow the Zipf law, the distribution has a less steep scaling regime quantifying the rarer words constituting the unlimited lexicon. Our result is robust across languages as well as across other data subsets, thus extending the validity of the seminal observation by Ferrer i Cancho and Solé14, who first reported it for a large body of subsets, thus extending the validity of the seminal observation by Ferrer i Cancho and Solé14, who first reported it for a large body of subsets, thus extending the validity of the seminal observation by.

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Also, it is during the “high-pressure” low productivity years that new words tend to emerge more frequently.

Interestingly, the appearance of new words is more like gas condensation, tending to cancel the cooling brought on by language expansion. These two effects, corpus expansion and new word “condensation,” therefore act against each other. Across all corpora we calculate a size-variance scaling exponents $0 < \beta < 1/2$, bounded by the prediction of $\beta = 0$ (Gibrat growth model) and $\beta = 1/2$ (Yule-Simon growth model).

In the context of allometric relations, Bettencourt et al. note that the scaling relations describing the dynamics of cities show an increase in the characteristic pace of life as the system size grows, whereas those found in biological systems show decrease in characteristic rates as the system size grows. Since the languages we analyzed tend to “cool” as they expand, there may be deep-rooted parallels with biological systems based on principles of efficiency. Languages, like biological systems demonstrate economies of scale (\(b < 1\)) manifesting from a complex dependency structure that mimics a hierarchical “circulatory system” required by the organization of language and the limits of the efficiency of the speakers/writers who exchange the words.

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