Spreading of a droplet impacting on a smooth flat surface: how liquid viscosity influences the maximum spreading time

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Abstract

Existing energy balance models, which estimate maximum droplet spreading, fail to cover from low to high Weber and Reynolds numbers and contact angles. This is mainly due to the simplified definition of the viscous dissipation term and incomplete modeling of the maximum spreading time. In this particular research, droplet impact onto a smooth sapphire surface is studied for seven glycerol concentrations between 0% - 100%, and 294 data points are acquired using high speed photography. Fluid properties such as density, surface tension, and viscosity are also measured. For the first time, we incorporate the fluid viscosity in the modeling of the maximum spreading time based on the recorded data. We also estimate the characteristic length and velocity of the viscous dissipation term in the energy balance equation. These viscosity-based characteristic scales help to formulate a more comprehensive maximum droplet spread-
ing model. Thanks to this improvement, our model successfully fits the data available in the literature for various fluids and surfaces compared to the existing models.

**Introduction**

Droplet impact onto solid surfaces is a very popular research field thanks to an extensive variety of applications with the examples including surface coating, forensic science, inkjet printing, and spray cooling. In particular, the maximum spreading ratio $\beta_{\text{max}}$, i.e., the ratio of the maximum spreading diameter $D_m$ to the initial droplet diameter $D_0$, determines the performance of these processes. As the fluid and surface properties govern droplet spreading, the following non-dimensional numbers must be considered for its modeling: the Weber number $\text{We} = \rho D_0 u_0^2 / \sigma$ (inertia/surface tension forces), the Reynolds number $\text{Re} = \rho u_0 D_0 / \eta$ (inertia/viscous forces), and the Ohnesorge number $\text{Oh} = \sqrt{\text{We}} / \text{Re}$ (viscous forces/inertia and surface tension forces). The fluid properties, i.e., the density, surface tension, and dynamic viscosity are represented by $\rho$, $\sigma$, and $\eta$, respectively, and the impact parameters are denoted by the droplet diameter upon impact and the impact speed, $D_0$ and $u_0$, respectively. When a droplet comes into contact with a solid surface, it can be assumed that there is a competition between the spreading and the viscous forces affecting the droplet dynamics. Inertia is considered as the dominating force during the initial wetting phase, in contrast to the viscosity. Moreover, although the capillarity and the inertia are the main driving forces of the early wetting dynamics, the addition of surfactants or super-spreader solutions remarkably changes the maximum spreading.

Several methods, which exist in the literature for the maximum spreading ratio estimation, are either based on empirical scaling, or energy conservation. For instance, Scheller et al. introduced the correlation presented in Eq. 1 that depends on both Reynolds and Ohnesorge numbers with two empirical coefficients, $A$ and $\alpha$:

$$\beta_{\text{max}} = A \left( \text{Re}^2 \cdot \text{Oh} \right)^\alpha.$$  \hspace{1cm} (1)
Tang et al.\textsuperscript{18} used the same scaling parameter \((\text{Re}^2 \cdot \text{Oh})\) and empirically found several coefficients for 5 different surface roughness values and 4 different fluids, e.g., water, decane, ethanol, and tetradecane. Sen et al.\textsuperscript{19} applied the same methodology on the biofuel droplets to empirically model their maximum spreading on the stainless steel substrate. On the other hand, Roisman\textsuperscript{20} came up with a semi-empirical relation (Eq.\textsuperscript{2}), which approximates the Navier-Stokes equations:

\[
\beta_{\text{max}} = 0.87 \text{Re}^{0.2} - 0.4 \text{Re}^{0.4}/\sqrt{\text{We}}. \tag{2}
\]

Other empirical models, estimating the maximum spreading ratio available in the literature, are reported in the first section of Table \textsuperscript{1}.

Another strong parameter on the droplet spreading mechanism is the contact angle \(\theta\) between the substrate and the droplet. Thus, models based on the energy balance better represent the physics of the phenomenon by including the contact angle as a parameter.\textsuperscript{21} Nevertheless, this approach requires a geometrical approximation of the droplet shape for the surface free energy estimation, as shown in Fig.\textsuperscript{1}. Moreover, the evaluation of viscous dissipation during the droplet spreading has to be accurately modeled. In this research, we apply the most common shape approximation, assuming the droplet as a cylinder with diameter \(D_m\) and height \(h_m\) at the maximum spreading, similar to these works.\textsuperscript{22–24} In order to better incorporate the liquid-solid wettability, other geometrical approximations are also considered, such as the spherical cap model\textsuperscript{25,26} or the model of the surrounding rim at the periphery of the droplets.\textsuperscript{13,27–29} These energy balance models are generally tuned to best fit specific experimental data and may fail when applied to different cases.
Figure 1: Geometrical approximations of the droplet shape for the surface free energy estimation at the maximum spreading.

The energy conservation principle applied to a spreading droplet states that the sum of the kinetic energy $K_0$ and the surface free energy $S_0$ prior to the impact should be equal to the sum of the dissipated energy due to viscosity $W$, the kinetic energy $K_m$, the surface free energy $S_m$ at the maximum spreading. Since the capillary length $\lambda_c = \sqrt{\frac{\sigma}{\rho g}}$ is computed to be greater than the droplet radius, the contribution of the potential energy is neglected.\(^{30,31}\) The mathematical expression is provided in Eq. 3:

$$K_0 + S_0 = W + K_m + S_m.$$ \hspace{1cm} (3)

The terms in the energy balance equation are given as follows.\(^{22,24,32}\)

$$K_0 = \frac{\pi \rho u_0^2 D_0^3}{12},$$ \hspace{1cm} (4a)

$$S_0 = \pi D_0^2 \sigma,$$ \hspace{1cm} (4b)

$$K_m = 0,$$ \hspace{1cm} (4c)

$$S_m = \pi \sigma D_m h_m + \frac{\pi}{4} \sigma D_m^2 (1 - \cos \theta).$$ \hspace{1cm} (4d)

The sub-equations of Eq. 4 make a perfect sphere assumption for the droplet upon impact.
and a stationary cylinder at the maximum spreading, which implies $h_m = \frac{2D_0^3}{3D_m^2}$ due to the mass conservation. Furthermore, different contact angle definitions appear in the literature for Eq. (4d), equilibrium, advancing, Young’s, or contact angles at the maximum spreading.

The $W$ term stands for the viscous dissipation in the energy balance equation, which requires solving the following integral in Eq. (5):

$$W = \int_0^{t_m} \int_{V_e} \Phi dV dt,$$

where $\Phi = \tau : \nabla u$ is the viscous dissipation function, and modeled as $\Phi \approx \eta \left( \frac{u_c}{L_c} \right)^2$. Therefore, the integration results in:

$$W = \eta \left( \frac{u_c}{L_c} \right)^2 V_e t_m,$$

where $V_e = \frac{\pi D_m^2 L_e}{4}$ is the effective volume of the viscous dissipation, and $L_e$, $u_c$, and $t_m$ denote the characteristic length and velocity, and the maximum spreading time, respectively in Eq. (6). These characteristic scales represent the length and the velocity during which the viscous dissipation occurs. The maximum spreading time is defined as the time between the droplet impact and the maximum spreading, i.e., the time during which the viscous dissipation takes place. Hence, its modeling for the viscous dissipation characterization becomes particular of concern to many studies. The main models based on the energy balance approach are summarized in the second part of Table [1] together with the chosen characteristic length, velocity, and the maximum spreading time.

Despite numerous attempts, there is still no universal equation for the prediction of the maximum droplet spreading. The goal of our study is to develop a new semi-empirical model with broader applicability. Specifically, we solve the energy balance equation by introducing new terms for the maximum spreading time and the characteristic scales, which are functions
of We and Re numbers. We perform an experimental campaign to gather a large set of data to tune our semi-empirical model. Finally, we validate the applicability of our model by using available data in the literature for various types of fluids and surfaces (large ranges of We and Re numbers, and contact angles).
Table 1: Literature models for the maximum spreading ratio $\beta_{\text{max}}$ with characteristic velocity $u_c$, characteristic length $L_c$, and maximum spreading time $t_m$.

| Literature                  | Empirical models                                                                 |
|-----------------------------|----------------------------------------------------------------------------------|
| Scheller et al. (1995)      | $\beta_{\text{max}} = 0.61 \left( \text{Re}^2 \cdot \text{Oh} \right)^{0.166}$ |
| Clanet et al. (2004)        | $\beta_{\text{max}} \sim \text{We}^{0.25}$ for $\text{We}/\text{Re}^{0.8} < 1$ |
|                            | $\beta_{\text{max}} \sim \text{Re}^{0.2}$ for $\text{We}/\text{Re}^{0.8} \geq 1$ |
| Bayer et al. (2006)         | $\beta_{\text{max}} = 0.72 \left( \text{We}/\text{Oh} \right)^{0.14}$         |
| Roisman et al. (2009)       | $\beta_{\text{max}} = 0.87 \text{Re}^{0.2} - 0.4 \sqrt{\text{We}}$            |
| Sen et al. (2014)           | $\beta_{\text{max}} = 1.73 \left( \text{We}/\text{Oh} \right)^{0.14}$         |
| Clanet et al. (2004)        | $\beta_{\text{max}} = \frac{\text{Re}^{0.2} \sqrt{P}}{A + \sqrt{P}}$, $P = \frac{\text{We}}{\text{Re}^{0.4}}$ |
| Seo et al. (2015)           | $\beta_{\text{max}} = 1.27 \left( \text{Re}^2 \cdot \text{Oh} \right)^{0.122}$, valid for low-viscosity fluids than water. |
| Tang et al. (2017)          | $\beta_{\text{max}} = a \left( \text{We}/\text{Oh} \right)^b$, $a \& b$ depend on the fluid type and surface roughness |

| Literature                  | Energy models                                                                 |
|-----------------------------|--------------------------------------------------------------------------------|
| Chandra & Avedisian (1991)  | $1.9 \left[ 1 + \frac{\text{We}}{\text{Re}} \right]^{1/2} - \left( \frac{1}{3} \text{We} + 4 \right) = 0$ |
| Pasandideh-Fard et al. (1996) | $\beta_{\text{max}} = \sqrt{\frac{\text{We}}{3(1 - \cos \theta) + 4 \sqrt{\text{Re}}}}$ |
| Mao et al. (1997)           | $0.83 \left[ 1 - \cos \theta \right]^{0.2} \sqrt{\frac{\text{We}}{\text{Re}^{0.33}}} - \left( \frac{\text{We}}{\text{Re}^{1.2}} + 1 \right) \beta_{\text{max}} = 2$ |
| Ukiwe et al. (2005)         | $(\text{We} + 12) \beta_{\text{max}} = 8 + 3 \left[ 1 - \cos \theta \right]^{0.6} \sqrt{\frac{\text{We}}{\text{Re}}} \left[ \frac{\text{We}}{\text{Re}} \frac{r_c}{D} \right]^{1/2} \left( \frac{1}{3} \sqrt{\frac{\text{Re}}{\text{We}}} \cos \theta_0 \right)$ |
| Gao et al. (2014)           | $12 \left[ \frac{\text{We}}{\text{Re}} + \frac{1}{2} \right] = \left[ 3 \left[ 1 - \cos \theta \right]^{0.2} \sqrt{\frac{\text{We}}{\text{Re}}} \right]^{1/2} \left[ \frac{\text{Re}}{\text{We}} \frac{r_c}{D} \right]^{1/2} + \frac{\alpha}{\sqrt{\text{Re}}} \frac{\text{We}}{\text{Re}} \frac{r_c}{D} \sin \theta_0$ |
| Wildeman et al. (2016)      | $12 \left[ \frac{\text{We}}{\text{Re}} + \frac{1}{2} \right] = \beta_{\text{max}}^2 + 3 \left[ 1 - \cos \theta \right]^{0.2} \sqrt{\frac{\text{We}}{\text{Re}}} \left[ \frac{\text{Re}}{\text{We}} \frac{r_c}{D} \right]^{1/2} + \frac{\alpha}{\sqrt{\text{Re}}} \frac{\text{We}}{\text{Re}} \frac{r_c}{D} \sin \theta_0$ |
| Huang et al. (2018)         | $3 \left[ \frac{\text{We}}{\text{Re}} + \frac{\text{We}^*}{\sqrt{\text{Re}^*}} \right] \beta_{\text{max}}^2 + 3 \left[ 1 - \cos \theta \right]^{0.2} \sqrt{\frac{\text{We}}{\text{Re}}} \left[ \frac{\text{Re}}{\text{We}} \frac{r_c}{D} \right]^{1/2} + \frac{\alpha}{\sqrt{\text{Re}}} \frac{\text{We}}{\text{Re}} \frac{r_c}{D} \sin \theta_0$ |
| Du et al. (2021)            | $(\text{We} + 12) \beta_{\text{max}} = 8 + 3 \left[ 1 - \cos \theta \right]^{0.2} \sqrt{\frac{\text{We}}{\text{Re}}} \left[ \frac{\text{Re}}{\text{We}} \frac{r_c}{D} \right]^{1/2} + \frac{0.98}{\sqrt{\text{Re}}} \frac{\text{We}^{1.06}}{\text{Re}^{2.06}} \beta_{\text{max}}^6$ |
Experimental setup and conditions

The applied methodology and the experimental setup (see Fig. 2) are described in detail in our previous work.  

Samples of various glycerol concentrations $\omega_G$ ranging between 0% - 100% are tested. The pre-impact parameters, such as droplet diameter and impact velocity are controlled by the needle size and its position, respectively, and 294 data points are acquired ($57 < \text{We} < 460$ and $4 < \text{Re} < 9200$). The drops are released on a smooth sapphire substrate. A detailed overview of the samples is given in Table 2.

Table 2: Material properties.

| Sample name (label)       | $\omega_G$ [wt%] | $\sigma$ [mN/m] | $\rho$ [kg/m$^3$] | $\eta$ [mPa·s] |
|---------------------------|------------------|-----------------|-------------------|---------------|
| Water (W)                 | 0                | 71.3            | 996.8             | 1.0           |
| Aqueous Glycerol (AG-30)  | 30               | 69.9            | 1071.4            | 2.5           |
| Aqueous Glycerol (AG-37)  | 37               | 68.6            | 1090.2            | 3.3           |
| Aqueous Glycerol (AG-43)  | 43               | 68.3            | 1105.3            | 4.1           |
| Aqueous Glycerol (AG-58)  | 58               | 66.9            | 1143.9            | 8.7           |
| Aqueous Glycerol (AG-70)  | 70               | 66.5            | 1177.4            | 19.7          |
| Glycerol (GLY)            | 99.9             | 62.9            | 1260.3            | 1021.5        |

The droplet diameter and its impact speed are computed from the high-speed camera images by an in-house MATLAB code having a simplified particle tracking velocimetry algorithm. This code is also used for the determination of the maximum spreading parameters. The image recording properties are also summarized in Table 3. Additionally, the circularity of the droplets just before the impact is ensured based on another MATLAB routine to disregard the shape effects. For the experimental consistency, every impact condition is repeated more than 3 times and the environmental conditions are kept constant at the temperature of $21 \pm 1^\circ\text{C}$ and at the atmospheric pressure. The substrate is always cleaned after each drop.
Table 3: Image recording properties.

| Frame rate [fps] | Frame size [px] | Shutter time [µs] | Resolution [px/mm] | Gray-scale level [bit] |
|------------------|-----------------|-------------------|--------------------|-----------------------|
| 12000            | 896×288         | 1/35000           | 60                 | 10                    |

to disregard the influence of the surface roughness. Glycerol concentration change due to air humidity is prevented by releasing some additional droplets prior to each measurement.43,44

Evaluation of the literature models

Fig. 3 presents how some of the existing $\beta_{\text{max}}$ models based on the energy balance approach perform on our experimental data. Since the empirical models become ineffective for the hydrophobic and hydrophilic surfaces due to the lack of contact angle dependence, miscellaneous models based on the energy balance approach are proposed to have a better physical insight on the droplet spreading dynamics. However, it is clear that these models only succeed in the limited regions and fail to complement all of the viscosity regimes. That is, if the models predict the maximum spreading ratio well for the low viscous liquids, they tend to underestimate it for the high viscous fluids, or vice versa. A very early example of the $\beta_{\text{max}}$ models based on the energy balance cannot perform well as shown in Fig. 3a, which could be the result of selecting the improper characteristic scales. Pasandideh-Fard et al.22 changed the characteristic scales and as a consequence of that, Fig. 3b represents a better agreement between the experimental results and the model for the low viscous liquids. By just adding the peripheral area of the cylinder to the surface free energy term, Ukiwe & Kwok23 slightly modified the model (Fig. 3c). Yet, since the viscous dissipation term in those correlations are adjusted only for the low viscous fluids, they still underestimate the maximum droplet spreading of the high viscous ones. Hence, Du et al.24 proposed different characteristic scales and tuned the maximum spreading time based only on the high viscous fluids, such as aqueous glycerol and silicon oil (viscosity range of 35.5 - 220 mPa·s). Hence, their model better perform only for the high viscous fluid data. On the other hand, they modeled the maximum spreading time as a function of We number only. We show in the next section that the fluid viscosity also impacts the maximum spreading time, which should be defined as a function of both We and Re numbers. The performance details of each model is discussed in Table 5.
Figure 3: Performance of the several existing models on our experimental data: (a) Chandra & Avedisian, (b) Pasandideh-Fard et al., (c) Ukiwe & Kwok, and (d) Du et al. The color bar is defined based on the glycerol concentration of the samples (see Table 2). The error bands (10-30 %, in gray color) help to present the accuracy of the models.

Model development

The previous comparison illustrates that the models available in the literature do not sufficiently include the impact of the fluid viscosity on the determination of the maximum spreading ratio. In our model, we start from the definition of the viscous dissipation term (Eq. 6) and introduce new characteristic length and velocity, and the maximum spreading time, which are functions of both We and Re numbers to better account for the viscous nature of the fluid.
The maximum spreading time $t_m$

The maximum spreading time has been modeled in several ways in the literature. Chandra and Avedisian\textsuperscript{21} modeled $t_m$ as the period of time for the droplet to reach zero from its maximum value $D_0$ at a constant velocity $u_0$, i.e., $t_m = D_0/u_0$. Pasandideh-Fard et al.\textsuperscript{22} estimated it with an additional coefficient of $8/3$ to satisfy the conservation of mass assuming the average droplet height to be $D_0/2$. Later, Lee et al.\textsuperscript{36} claimed that $t_m$ better scales with the maximum spreading diameter $D_m$ and propose $t_m = \frac{\sigma}{\sigma_{ref}} D_m u_0$, while Huang et al.\textsuperscript{37} used $t_m = D_m/2u_0$ to better fit their experimental data. Wildeman et al.\textsuperscript{9} suggested another model including both the maximum spreading and initial droplet diameter: $t_m = ((\beta_{max} - 1)D_0/2u_0$. More recently, Lin et al.\textsuperscript{45} presented an empirical correlation (Eq. 7) for the dimensionless time that fits their experimental data over wide ranges of We numbers and contact angles:

$$t_m/\tau'_{i} = a \text{We}^b,$$

where $a = 0.92$ and $b = -0.43$ are empirically determined constants and the modified capillary-inertial time is defined as $\tau'_{i} = \sqrt{\rho D_m^3/8\sigma}$.\textsuperscript{14,46} Nonetheless, in a recent publication, Du et al.\textsuperscript{24} have computed an error of 30% with those constants in the viscous regime. Hence, they follow the same linear regression analysis to find new constants, which are better fitting their experimental data, and they end up with slightly different values ($a = 1.47$ and $b = -0.44$).

In Fig. 4a, the two models of the normalized maximum spreading time $t_m/\tau'_{i}$ are plotted with the current experimental data as a function of We number. As our data points are colored based on the glycerol mass fraction, the viscosity dependence of the maximum spreading time becomes obvious. In other words, the scattering of the data is not random, but follows a trend: low viscous fluids stay below the curve of Lin et al.\textsuperscript{45} whereas high viscous samples locate at the upper side of that curve. In addition, the curve of Du et al.\textsuperscript{24} fits the trend for the higher viscosity fluids, which more closely matches their data set. Thus, to correctly address the influence of the viscous forces, we propose to include a dependence also on Re number:

$$t_m/\tau'_{i} = a \text{We}^b \text{Re}^c,$$

using the empirically determined constants $a = 2$, $b = -0.45$, and $c = -0.09$ in Eq. 8. Consequently, the maximum spreading time model based on Eq. 8 completely fits our experimental data within the uncertainty limits, as shown in Fig. 4b.
The characteristic length $L_c$ & characteristic velocity $u_c$

In most of the previous works applying the energy balance approach for the droplet spreading, the impact speed is used in Eq. 6 as the characteristic velocity, i.e., $u_c = u_0$. Nevertheless, Yonemoto et al.\textsuperscript{32} and Du et al.\textsuperscript{24} approximated the characteristic velocity as $u_c = 3/8 u_0$ as a simplification of the radial velocity. Zhang et al.\textsuperscript{13} expressed that the lamella spreading speed depends on the droplet impact speed and increased contact angle positively affects the lamella spreading speed. In a similar way, several length scales have been considered to estimate the characteristic length $L_c$. For instance, Chandra and Avedisian\textsuperscript{21} assumed the characteristic length as the rim thickness $h_m$. On the other hand, Pasandideh-Fard et al.\textsuperscript{22} postulated that length to be equal to the boundary layer thickness, in which the viscous dissipation occurs since the previous model overestimates $\beta_{\text{max}}$ by 40%. Yonemoto et al.\textsuperscript{32} proposed taking the harmonic average of the possible droplet shapes at the end of the spreading process (see Fig. 1), thus ending up with $L_c = h_m/3$. Ruiter et al.\textsuperscript{47} stated that the expected boundary layer thickness always stays smaller than the rim thickness. Further, the rim thickness increases with viscosity, but decreases with increasing impact speed. Mao et al.\textsuperscript{35} defined low and high viscosity regimes separately, and claim that these regimes cannot be characterized only with Re number, i.e., viscosity.

Under the light of this aforementioned literature information, we see that previous works have selected different characteristic length and characteristic velocity based on various postulations, which are also summarized in Table 1. These inconsistencies are coming from the fact that $L_c$ and $u_c$ are case specific and have never been measured explicitly and separately. Consequently, due to the lack of direct measurements of these scales, we want to combine and represent them via one term to avoid any misinterpretation. Therefore, we first write...
the characteristic length and velocity using the following expressions:

\[ L_c = \mathcal{L} \cdot h_m, \quad (9a) \]

\[ u_c = u \cdot u_0. \quad (9b) \]

When Eq. 9 is inserted into Eq. 6, the viscous dissipation term becomes:

\[ W = \frac{36\pi}{8} \frac{D_0}{12D_m} \frac{u^2}{\mathcal{L}} \frac{We}{Re} \frac{D_0}{D_m} \beta_{max}^5 t_m, \quad (10) \]

Then, we model the combination of \( u \) and \( \mathcal{L} \) in order to include the dependence on the fluid properties (\( \eta \) and \( \sigma \)) as follows:

\[ \frac{u^2}{\mathcal{L}} = We^w Re^r, \quad (11) \]

where \( w = -0.32 \) and \( r = 0.24 \) are determined by the linear regression analysis. To further elaborate, the negative sign of \( w \) indicates that an increase in the surface tension increases \( u_c \), whereas the positive sign of \( r \) implies the amplifying influence of the viscosity on \( L_c \). Owing to the small values of those exponents, the inverse effect of \( u_0 \) on \( u_c \) coming from the negative sign of \( w \) vanishes.

**The maximum spreading ratio \( \beta_{max} \)**

Substituting Eq. 8 and Eq. 11 into Eq. 10, we obtain the final form of the viscous dissipation term. Using Eq. 3, the final expression of the maximum spreading becomes:

\[ 3.18 \frac{We^{0.72}}{Re^{0.86}} \beta_{max}^{6.5} + 3(1 - \cos \theta) \beta_{max}^3 - (We + 12) \beta_{max} + 8 = 0. \quad (12) \]

After solving Eq. 12 over broad ranges of \( We \) and \( Re \) numbers, we obtain the map shown in Fig. 5a. In the viscous regime, where \( We \gg Re \), the maximum spreading ratio mainly depends on \( Re \) number and the effect of \( We \) number is negligible. Likewise, for the low viscous fluids, such as water with \( Re \gg We \), there is a stronger dependence on the \( We \) number and a negligible effect of \( Re \) number. These observations, plotted in Fig. 5b & 5c, are also in good accordance with the scaling laws proposed by Clanet et al.: \( \beta_{max} \approx Re^{0.2} \) for high viscous fluids (\( We/Re^{0.8} > 1 \)) and \( \beta_{max} \approx We^{0.25} \) for water droplet on hydrophobic surfaces.
Figure 5: Applicability of Eq. 12 over large ranges of We and Re numbers: (a) We−Re−β_{max} contour map with Oh_{W} = 0.0022 and Oh_{GLY} = 2.26 (gray dashed line represents the line at We/Re^{0.8} = 1 and the correlation in Eq. 43 is applied to exclude the splashing region). Comparison of our model with the scaling laws: spreading behavior of (b) glycerol droplet on a smooth surface (dashed line for Re^{0.2}) and (c) water droplet on a super-hydrophobic surface (dashed-dotted line for We^{0.25}).

In order to verify the wide applicability of the new model (Eq. 12), we try to apply it on numerous experimental data points available in the literature. As presented in Fig. 6, whose legend is separately reported in Table 4, these measurements cover large spectra of contact angles and fluid viscosities (1 < We < 1283, 4 < Re < 35000, and 5.6° < θ < 140° & θ ≈ 160°). More in detail, Fig. 6a depicts the performance of our model on the current experimental data, from which a good agreement is anticipated. Furthermore, it is plotted on the data from literature (276 data points) in Fig. 6b, which proves its inclusiveness over quite miscellaneous cases.
Figure 6: Assessment of the new model (Eq. 12) on: (a) current experimental data (b) a very wide range of literature data. The detailed legend is given in Table [4]. Current experimental data (empty circles) and the data from [33] (solid squares) are colored based on the glycerol concentration. The error bands (10-30 %, in gray color) help to present the accuracy of the model.

Table 4: The details of the legend given in Fig. 6.

| Fluid          | Surface     | Marker | Ref. | Fluid          | Surface     | Marker | Ref. |
|----------------|-------------|--------|------|----------------|-------------|--------|------|
| Heptane        | Stainless steel | △      | [22] | Paraffin wax   | Aluminium   | △      | [22] |
| Tin            | Al₂O₃       | ▽      | [22] | Tin            | Stainless steel | △      | [22] |
| Water          | Beeswax     | ◇      | [22] | Water          | Cellulose acetate | □    | [22] |
| Water          | Glass       | ⚫      | [22] | Water          | Steel       | ◯      | [22] |
| Zinc           | Stainless steel | ○     | [22] | Water          | Glass       | △      | [37] |
| Water          | Parafilm    | △      | [27] | Diesel         | Aluminium   | △      | [45] |
| Blood          | Stainless steel | △     | [22] | Water          | Meule       | △      | [49] |
| Water          | Pietra serena | △     | [19] | Water          | Savonnières | ▼     | [49] |
| Water          | Steel       | ▽      | [19] | Water-glycerol | Acrylic     | △      | [50] |
| Water-glycerol | Glass       | △      | [19] | Water          | Acrylic     | △      | [50] |
| Water          | Glass       | △      | [50] | Isopropanol    | Wax         | △      | [51] |
| Isopropanol    | Smooth glass | △      | [51] | Isopropanol    | Wax         | ▼      | [51] |
| Water          | PVC         | △      | [51] | Water          | Rough glass | △      | [51] |
| Water          | Smooth glass | △      | [51] | Water          | Wax         | ☆      | [51] |
| Ethanol        | Silicon rubber | △  | [32] | Water          | Polycarbonate | △ | [32] |
| Water          | Silicon rubber | ▽  | [32] | Biofuel        | Stainless steel | △  | [19] |
To evaluate the performance of a model, the following mean error formula is employed:

$$\text{mean} \left( \frac{\beta_{\text{max,exp}} - \beta_{\text{max,model}}}{\beta_{\text{max,exp}}} \right) \times 100\%.$$  \hfill (13)

Table 5 quantifies the performances of several models from literature based on the aforementioned data. To clearly demonstrate the viscosity effects, we split our data as water, aqueous glycerol, and glycerol. Although some models seem to outperform our model, especially at low viscous regimes, they all fail for the glycerol data (see Fig. 3).

Our model proves its comprehensiveness by having the mean error being less than 15% for all cases, except the one with very high contact angle ($\theta \approx 160^\circ$). This could be due to neglecting the low surface energy to be accounted for the super-hydrophobicity. On the other hand, only 2.5% mean error is observed for the data of Sen et al., which has biofuel as working fluid possessing $\theta = 5.6^\circ$ on the stainless steel surface.

Table 5: Performance evaluation (based on Eq. 13) of all the maximum spreading models on various experimental data ($1 < \text{We} < 1283$, $4 < \text{Re} < 35000$, and $5.6^\circ < \theta < 140^\circ$ & $\theta \approx 160^\circ$). First three rows represent current measurements comprise water, aqueous glycerol, and glycerol to point out the viscosity effect. The models successfully estimating $\beta_{\text{max}}$ for the low viscous fluids could not perform well for the high viscous fluids, or vice versa.

| Mean error [%] | Models |
|----------------|---------|
|                | Chandra & Avedisian [21] | Scheller & Bousfield [12] | Pasandideh et al. [22] | Tang et al. [18] | Ukiwe & Kwol [23] | Roisman [20] | Sen et al. [19] | Du et al. [24] | New model |
| Water          | 30.9% | 2.3% | 3.0% | 4.4% | 3.3% | 2.1% | 9.0% | 7.7% | 3.0% |
| Aqueous glycerol | 22.8% | 2.2% | 8.9% | 20.6% | 9.7% | 1.5% | 19.2% | 9.1% | 2.9% |
| Glycerol       | 20.5% | 5.8% | 41.4% | 51.7% | 42.6% | 13.3% | 164.9% | 5.1% | 3.2% |
| Pasandideh et al. [22] | 31.9% | 11.2% | 5.7% | 16.1% | 5.6% | 9.8% | 17.5% | 12.1% | 7.2% |
| Huang et al. [31] | 34.9% | 12.5% | 12.3% | 6.9% | 9.4% | 9.7% | 16.3% | 16.0% | 11.6% |
| Jadidbonab et al. [48] | 6.0% | 14.5% | 19.9% | 42.1% | 22.0% | 13.9% | 20.8% | 1.2% | 8.0% |
| Lee et al. [49] | 27.9% | 4.2% | 7.3% | 12.8% | 7.4% | 4.9% | 22.1% | 11.2% | 5.9% |
| Santiago et al. [50] | 37.9% | 30.8% | 19.2% | 7.5% | 8.3% | 39.1% | 30.1% | 22.1% | 14.7% |
| Sikalo et al. [33] | 27.8% | 17.2% | 6.9% | 5.8% | 6.0% | 9.4% | 14.2% | 19.2% | 9.3% |
| Yonemoto et al. [32] | 29.9% | 12.6% | 13.0% | 18.4% | 12.8% | 10.8% | 21.9% | 13.6% | 10.4% |
| Sen et al. [19] | 38.0% | 15.2% | 11.8% | 14.6% | 8.1% | 12.7% | 15.1% | 24.2% | 11.3% |
| Zhao et al. [33] | 33.7% | 5.8% | 4.8% | 18.0% | 4.3% | 6.1% | 1.3% | 8.5% | 2.5% |
| Moist et al. [34] | 15.5% | 22.6% | 32.3% | 51.3% | 46.0% | 23.8% | 35.6% | 13.9% | 19.1% |
Conclusions

After a thorough literature study, it is seen that the available maximum spreading models are only applicable to the limited fluid-surface combinations and there is a lack of general application. In particular, the models tuned for the low viscous fluids underestimate the maximum droplet spreading for the high viscous fluids, whereas others overestimate it for the low viscous fluids while correctly predicting the high viscous ones. Hence, we come up with a more comprehensive model having a physical insight, which works in both high and low viscous regimes, and satisfyingly matches abundant literature data over broad ranges of $1 < \text{We} < 1283$ and $4 < \text{Re} < 35000$ numbers, as well as various surface wettability conditions ($5.6^\circ < \theta < 140^\circ$ and $\theta \approx 160^\circ$).

An extensive experimental campaign, composed of 294 data points in the viscosity range of $1 - 1021 \text{ mPa} \cdot \text{s}$, is carried out. We first discover the viscosity effect in the maximum spreading time model, which improves the viscous dissipation term. Afterwards, we model the maximum spreading of the droplet following the energy balance approach by having the characteristic length and characteristic velocity written as functions of both We and Re numbers. To validate the new model, we apply it on 276 data points from 10 previous studies. This evaluation reveals its unprecedented performance among the existing models. The mean error is less than 15% for all the aforementioned conditions, except the super-hydrophobic case. Even though there could be better models for specific data sets, our model is the most inclusive one, proving good accordance over a wide variety of fluids and surfaces. Thanks to its high accuracy in the predictions of the maximum spreading of the droplet, the performance of the processes such as surface coating and inkjet printing, can be highly enhanced. It should be remarked that the model developed in our work uses the cylindrical shape approximation at maximum droplet spreading. Therefore, the predictions can deviate for the extreme droplet shapes, which can be overcome by improved shape approximations in future studies. In addition, for further improvements, we recommend an experimental study to directly measure the velocity profile during the droplet spreading for more appropriate modeling of the characteristic length and characteristic velocity, e.g., Particle Image Velocimetry measurements.

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