QUANTUM KALUZA–KLEIN COMPACTIFICATION

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Abstract. Kaluza–Klein compactification in quantum field theory is analysed from the perturbation theory viewpoint. Renormalisation group analysis for compactification size dependence of the coupling constant is proposed.

Introduction

Development of the string theory increased interest to field theoretical and quantum mechanical models in high dimensions \( D \geq 5 \). The consistency of string theory need it to be formulated in either \( D = 10 \) for supersymmetric strings or \( D = 26 \) for bosonic ones, while “everyday” physics is four-dimensional. From the other hand most interesting field theoretical models can be consistently formulated at dimensions not exceeding four. Compactification of extra dimensions existent in string theory to have four-dimensional physics at the low energies conciliates these facts.

It would be interesting to explain this mechanism dynamically, at least in some approximation at low energies. Much optimism is inspired by the progress in understanding non-perturbative strings and in special ADS/CFT correspondence.

In the present work, however, we address a different question: what is the effect of the compactification on the level of quantum field theory?

It is known that certain quantum field theories/gravities can serve as low energy effective actions for string theories. Since the string description must not enter in contradiction with low energy field theoretical one, one can limit oneself to study of the last.

As an important example can serve ten-dimensional IIA string model whose low energy field theory is IIA supergravity. As it is believed string theory for large couplings \( g_{\text{string}} \) results in an eleven-dimensional model (M-theory). Its low energy effective field theory model is \( D = 11 \) supergravity. Therefore, stringy corrections for large \( g_{\text{string}} \) in IIA supergravity must led to the eleven-dimensional theory (see [5] for a recent review). It is, however, easier to see instead the transition from higher dimensional model to lower one.

In classical physics, compactification of a \( (D + p) \)-dimensional model to \( D \) dimensions is given by “confining” some \( p \) spacial dimensions of original space-time to form a compact manifold.

From the \( D \)-dimensional point of view the spectrum of the compactified model consists of a light field which corresponds to constant or zero mode on the compact directions and an infinite number of massive fields corresponding to non-constant in the compact direction modes or Kaluza–Klein (KK) modes. Masses of KK-modes

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are proportional to inverse compactification size $M = R^{-1}$ (i.e. the typical size of the compact dimensions). In the limit of strong compactification KK-modes acquire large masses and do not propagate, and decouple at the classical level.

In quantum description, however, “virtual” KK-particles can contribute even for energies less than their masses, becoming significant as they are approached.

In the limit of zero compactification size one expects to have all the KK-modes decoupled also in quantum theory since their masses become infinite. In fact, the fields not only do not simply decouple in this limit but they may also produce additional divergences. Also, extra divergences appear even for finite compactification sizes due to infinite number of fields. These divergences are natural reflection of the fact that usually similar models in higher dimensions are more divergent.

As one can see these divergences can be eliminated in the framework of the standard renormalisation procedure and one is left with renormalised physical parameters which depend on the size of the compact space.

In actual paper we are going to consider a simple (toy) model to illustrate above ideas. We will consider $D + 1$-dimensional $\phi^3$-model compactified (on a circle) to a $D$-dimensions.

The structure of the paper is as follows. In the next section we consider compactified $D + 1$-dimensional $\phi^3$ model, and in the second one its $D$-dimensional one-loop effective action. We also analyse the renormalisation group dependence of the effective $D$-dimensional coupling on the compactification mass $M$. In the Appendix we give some properties of $\zeta$ and $\Gamma$-functions used in the body of the paper and describe the computation of the effective action.

1. Compactified $\phi^3$ model

Let us consider $D + 1$ dimensional $\phi^3$ model described by the following classical action

$$ S = \int d^{D+1} \vec{x} \left( \frac{1}{2} (\partial_\mu \vec{\phi})^2 - \frac{1}{2} m^2 \vec{\phi}^2 - \frac{\lambda}{3!} \vec{\phi}^3 \right), $$

where bar always refer to $D + 1$-dimensional quantities. We compactify this model along $D$-th spatial direction by requiring equivalence of $x^D \equiv \theta \simeq \theta + 2\pi R$, where $R \equiv M^{-1}$ is the size of compactification.

Consider the (infinite) set of $D$-dimensional fields $\phi_n(x)$ which is Fourier transform with respect to the $D$-th coordinate $\theta$

$$ \tilde{\phi}(\vec{x}) = (2\pi R)^{-1/2} \sum_{n=-\infty}^{+\infty} \phi_n(x) e^{iMn\theta}, $$

where $M = R^{-1}$ is the energy scale of compactification, and fields $\phi_n$ are given by inverse Fourier transform

$$ \phi_n(x) = (2\pi R)^{-1/2} \int_0^{2\pi R} d\theta \tilde{\phi}(\vec{x}) e^{-iMn\theta}. $$
In terms of fields $\phi_n$ action (1.4) look as follows

\[ S = \int_{M_D} \, d^Dx \left( \sum_{n \geq 0} \left( \frac{1}{2} \partial \phi_n \partial \phi_n^* - \frac{1}{2} (m^2 + M^2 n^2) \phi_n \phi_n^* \right) - \frac{\lambda}{3!} \sum_{n,n'} \phi_n \phi_{n'} \phi_{n+n'}^* \right). \]

The $D$-dimensional coupling $\lambda$ in eq.(1.4) is related to $D+1$-dimensional one $\bar{\lambda}$ by compactification size dependent rescaling:

\[ \lambda = \frac{\bar{\lambda}}{\sqrt{2\pi R}} = \sqrt{\frac{M^2}{2\pi}} \bar{\lambda} \]

The rescaling take place due to dependence of coupling dimensionality on the space-time dimension. Indeed, dimensions of the scalar field $\phi$ and cubic coupling $\lambda$ in $D$ space-time dimensions are respectively (in mass unities):

\[ [\phi] = D^2 - 1 \]
\[ [\lambda] = 3 - \frac{D}{2} \]

Thus, the coupling must acquire a $\sim M^{1/2}$ factor while descending one dimension.

Now, the action for zero-mode field $\phi(x) \equiv \phi_0(x)$ only is just the naive $D$-dimensional scalar field action, i.e. one that one would have in $D$ dimensions. Beyond this standard $D$-dimensional part there are also terms for higher KK-modes $\phi_n, (n \neq 0)$ and ones responsible for their interaction with $\phi$. Making such separation in $D$-dimensional fields and KK-modes explicit one can rewrite the action (1.4) in the following form

\[ S_{D+1} = S_D(\phi) + S'(\phi, \phi_n, \phi_n^*), \quad n > 0 \]

where actions $S_D(\phi)$ and $S'(\phi, \phi_n, \phi_n^*)$ are given by the following $D$-dimensional Lagrangians

\[ L_D(\phi) = \frac{1}{2} (\partial \phi)^2 - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{3!} \phi^3 \]

and

\[ L' = \sum_{n>0} \left( \frac{1}{2} |\partial \phi|^2 - \frac{M^2 n^2}{2} |\phi_n|^2 - 2\lambda \phi_n \sum_{n>0} |\phi_n|^2 \right) + \frac{\lambda}{3!} \sum_{n,n'} \phi_n \phi_{n'} \phi_{n+n'}^*, \]

where the primed sum is taken over all values of $n$ and $n'$ which satisfy $n, n' \neq 0$ and $n \neq n'$.

From the explicit form of eqs (1.9,1.10) one can see that fields $\phi_n$, $n \neq 0$ are charged with respect to $U(1)$ group acting as $\phi_n \rightarrow e^{-i\alpha n} \phi_n$. This transformation corresponds to shifts (rotations) in $D$-th (compact) direction, and leaves the action (1.8) invariant. Gauging this symmetry give rise to $U(1)$ KK gauge field $A_\mu$.

In the compactification limit: $R \rightarrow 0 \ (M \rightarrow \infty)$ the KK modes acquire infinite masses and as we mentioned are expected to decouple. In this limit KK modes do not propagate anymore, but due to their interaction with remaining $D$-dimensional fields they can produce a non-vanishing contribution. In fact this contribution is divergent, divergencies being accomplished also by the infinite number of KK-fields.

The extra divergencies of compactified model are easily explained by the fact that index divergence in higher dimensions is worse than in lower.
To evaluate the effect of compact KK-modes on $D$-dimensional theory let us compute the effective action for zero mode $\phi$, in one-loop approximation.

2. The Effective Action

The effective action for the field $\phi$ is given by the following equation:

$$e^{i S_{\text{eff}}(\phi)} = e^{i S_D} \prod_{n > 0} d\phi_n d\phi_n^* e^{i \int d^D x L'(\phi, \phi_n, \phi_n^*)}.$$  \hspace{1cm} (2.1)

In what follows we will consider compactification size to be small. The presence of the compactification size in the model introduces a new scale parameter. In fact one can identify this scale with the cut off one, but we will not do this at the moment.

To compute effective action $S_{\text{eff}}$ at least in the framework of the standard perturbation theory one needs first to regularise path integral (2.1). During this calculation we use dimensional regularisation scheme and perform Wick rotation: $x_0 \rightarrow ix_0$, to deal with Euclidean path integral.

KK-mode propagators look as standard Euclidean scalar propagators (in momentum representation):

$$D_n(p) = \frac{1}{p^2 + M_n^2}.$$  \hspace{1cm} (2.2)

There are also two interaction terms. The first one is $\phi$-KK-mode interaction:

$$2\lambda \phi \sum_{n > 0} |\phi_n|^2,$$  \hspace{1cm} (2.3)

and the second one is KK-mode self-interaction term:

$$\lambda \sum_{n,n' \neq 0} \phi_n \phi_{n'} \phi_{n+n'}^*.$$  \hspace{1cm} (2.4)

Since we are considering one-loop approximation only the $\phi$-KK-interaction (2.3), is relevant.

Typical one-loop diagram with $N$ “legs” $\phi(q_i)$ ($i = 1, \ldots, N$), produces the following regularised contributions (see Appendix A):

$$G_N = \frac{\lambda^N}{(2\sqrt{\pi})^D(N - 1)!} M^{D-2N} \left[ f^{(0)}_N(\tilde{D}) + M^{-2} f^{(2)}_N(\tilde{D}; q_1, \ldots, q_N) + O((q/M)^4) \right],$$  \hspace{1cm} (2.5)

where $\tilde{D} \rightarrow D$ is the (complex) dimension which regularises the theory.

Functions $f^{(i)}_N$ in eq. (2.3) have the following structure,

$$f^{(i)}_N = \zeta (2N + i - \tilde{D}) \Gamma \left( N + \frac{i - D}{2} \right) P_{(i)}(q),$$  \hspace{1cm} (2.6)

where $P_{(i)}(q)$ is a (homogeneous) polynomial of the $i$-th degree in external momenta $q_l, l = 1, \ldots, N$.

As one can expect, the right side of eq. (2.3) is divergent in the limit $\tilde{D} \rightarrow D$. Let us analyse this divergence and find the counter-terms necessary for its cancellation.

\footnote{As usual in dimensional regularisation computations we assume the coupling to be of the form $\lambda = \lambda_0 \kappa^{D - D/2}$, where $\kappa$ and $\lambda_0$ are respectively a mass dimensional “unity” and dimensionless coupling.}
The UV divergencies in eq. (2.5) manifest themselves as a potential singularity of the factor
\[ \zeta(2N + i - \bar{D})\Gamma\left(N + \frac{i - \bar{D}}{2}\right) \]
as \(\bar{D}\) goes to \(D\).

From the properties of \(\zeta\) - and \(\Gamma\)-functions (Appendix B) one can deduce that the singularity in eq. (2.7) occurs when quantity \(N + \frac{i - \bar{D}}{2}\) is either 0 (then one has singularity in \(\Gamma\)-function times regular \(\zeta\)) or \(\frac{1}{2}\) (when, oppositely, one has regular \(\Gamma\) times singular \(\zeta\)). As one can see, the latter of these two cases can be met for odd dimension \(D\) while the former happens when dimension is even.

Concerning the compactification mass value there may be two essentially different situations. The first one is when the compactification mass is below the cut-off scale, then a physically meaningful value can be fixed for it. In the second situation the compactification mass is beyond the cut-off it is physically infinite and one meets additional renormalisable divergences due to \(M \to \infty\). Their elimination by standard renormalisation of fields, masses, couplings, and mean vacuum field value brings us to usual scalar field model and one cannot speak on the compactification size dependence since it is physically infinite. We are interested mainly in the first situation.

In the case when the compactification mass \(M\) is kept fixed below the cut-off scale the structure of the UV divergencies look as follows,
\[ \Delta Z_N^{(i)} = \frac{\lambda^N}{(2\sqrt{\pi})^D(N-1)!} P_{(i)}(q) \cdot \left(\frac{1}{2\varepsilon}\right) + \text{finite terms}, \]

\[ N + \frac{i - \bar{D}}{2} = 0, \text{ for even dimension } D, \text{ and, respectively}, \]
\[ \Delta Z_N^{(i)} = \frac{\lambda^N M^{-1}}{(2\sqrt{\pi})^D(N-1)!} P_{(i)}(q) \cdot \left(\frac{\sqrt{\pi}}{\varepsilon}\right) + \text{finite terms}, \]
\[ N + \frac{i - \bar{D}}{2} = \frac{1}{2}, \text{ for odd one, also } \varepsilon \equiv \bar{D} - D. \]

In what follows let us consider the coupling constant renormalisation due to presence of the compact extra dimension. In our notations the coupling \(\lambda\) renormalises as follows,
\[ \lambda_R = \lambda Z^{(0)}_3 (Z^{(2)}_2)^{-3/2}. \]

As one can immediately see the coupling acquires an infinite renormalisation only in dimensions \(D = 5\) and \(D = 6\). In other cases both \(Z^{(0)}_3\) and \(Z^{(2)}_2\) are finite.

The case of five dimensions is particular due to compactification mass dependence of divergent terms. This leads to compactification mass dependence of the renormalisation procedure. As a result one cannot define in five dimensions “physical” coupling for all values of \(M\) simultaneously, but must re-renormalise it for each particular value of \(M\).

In other dimensions \((D \neq 5)\) the renormalisation is either finite or it is compactification mass independent, as it is in \(D = 6\).

Consider now, dimensions different from \(D = 5, 6\). For dimensions four and less, eq. (2.10) is regular and one just has,
\[ \lambda_R = \lambda \left(1 - \frac{\lambda^2}{4(2\sqrt{\pi})^D M^{D-6} \zeta(6 - D)\Gamma(3 - D/2)}\right). \]
The same is true (and regular) also for odd dimensions greater than six. For even dimensions greater than six, however, eq. (2.11) has singularity in $\Gamma$-function which is cancelled by zeroes of $\zeta$-function. Computation the limit yields,

$$
\lambda_R = \lambda \left( 1 - \frac{(-1)^n \lambda^2}{4(2\sqrt{n})^{D/2} n!} M^{D-6} \xi'(6-D) \right), \quad D \equiv 2n > 6
$$

Consider now in more details the most interesting case $D = 6$, where the model is yet renormalisable but not superrenormalisable. Renormalisation group equation gives,

$$
M \frac{\partial \lambda}{\partial M} = \beta(\lambda),
$$

where $\beta(\lambda)$ is Calan–Simanzik $\beta$-function. It can be computed from the equation,

$$
\beta(\lambda) = \kappa \frac{\partial \lambda}{\partial \kappa}.
$$

Using eqs. (2.10) and (2.3) one has for $\beta$-function,

$$
\beta(\lambda) = -\frac{\lambda^3}{(2\sqrt{\pi})^6}.
$$

Thus solution to (2.13) yields,

$$
\lambda^2(M) = \frac{\lambda^2(M_0)}{1 - \frac{2\lambda^2(M_0)}{(2\sqrt{\pi})^6} \log(M/M_0)},
$$

where $M_0$ is the value of the compactification mass parameter where $\lambda(M_0)$ was computed.

Eq. (2.16) gives the dependence of the effective coupling $\lambda$ on compactification mass parameter $M$. This equation is valid in the approximation $p \ll M$. The behaviour of the coupling is of such nature that it exhibits a singularity at $M \to \infty$ (or compactification size $R \to 0$).

This singularity could be expected since there are additional logarithmic divergences of contributing Feynman diagrams as $M$ goes to infinity.

As a conclusion one cannot compactify seven-dimensional scalar field model to six-dimensional one in a way smooth in the compactification size. Renormalisation group behaviour shows that at small values of $R = M^{-1}$ the coupling constant rises disabling the perturbative analysis.

This is in contrast with the so called large mass decoupling problem [6, 7, 8, 9] (see also [10]).

From the other hand, there maybe such a situation for special choice of model and compact manifolds when the coupling’s behaviour is asymptotically free at large $M$, in this case one can safely reach the compactification limit.

**Discussions.** We found the compactification mass (inverse of compactification size) dependence of the $D$-dimensional coupling of the model resulting in compactification of the $D + 1$ dimensional scalar field model. We considered the simplest possible case: scalar field and $U(1)$ compactification. However, this work can be extended to more complicated cases both as field content and as the type of compactification, including various compactifications of $D = 11$ and $D = 10$ supergravity theories [11].

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2 Here and on we drop subscript $R$ at $\lambda$ since the only $\lambda$ we deal with is $\lambda_R$.

3 I am grateful to Dr. R. Ruskov, who pointed my attention to this problem.
More complicated models may contain fields which realise non-singlet representations of the $D + p$-dimensional Lorentz group, which must be reduced under compactification to representations of $D$ dimensional Lorentz group. This reduction was considered in the famous Slansky’s review [12].

It is important that compactified model may be field theoretically consistent in spite the non-renormalisability of the original non-compactified one. Therefore one may expect a phase transition with compactification size as order parameter where the model pass to the $D + p$-dimensional phase. This is exactly the point where the $D$-dimensional perturbation theory fails. Thus one may think about $D + p$-model as a non-perturbative extension of the $D$-dimensional one. As we have shown on the toy model considered in the paper the renormalisation group approach can be applied to study the compactification size dependence of the model.

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Appendix A. Computation of the Effective Action

One-loop Feynman diagram with $N$ external legs $\phi(q_i)$ produces the following contribution to effective action:

\[
G_N(q_1, \ldots, q_N) = \frac{\lambda^N}{N!} \sum_{n>0} \int \frac{d^D p}{(2\pi)^D} \times \\
\times [((p^2 + M^2n^2)((p + Q_1)^2 + M^2n^2) \ldots ((p + Q_{N-1})^2 + M^2n^2)]^{-1} \\
\text{+ permutations}(q_1, \ldots, q_N),
\]

where

\[
Q_i \equiv \sum_{k=1}^{i} q_k
\]

In the framework of dimensional regularisation scheme, $\bar{D}$ is the complex dimension. The limit of “cut-off” removing is $\bar{D} = D$. As usual, $\lambda = \lambda_0 \mu^{N(D/2-3)}$ where $\lambda_0$ is dimensionless coupling constant and $\mu$ is the mass scale.

Since we are considering masses $M$ to be large we can expand integral (A.1) in powers of $q/M$. Due to Lorentz invariance only even power terms will be present in the expansion.

The first two terms in this expansion are given by

\[
G_N^{(0)} = \lambda^N \sum_{n=1}^{\infty} \int \frac{d^D p}{(2\pi)^D} \left( \frac{1}{p^2 + M^2n^2} \right)^N,
\]
for the zeroth term and by
\begin{equation}
G_N^{(2)} = \sum_{n=1}^{\infty} \int \frac{d^D p}{(2\pi)^D} \left( \sum_i Q^\mu_i Q^\nu_i \left( \frac{1}{p^2 + M^2 n^2} \right)^{N-1} \frac{\partial^2}{\partial p_\mu \partial p_\nu} \left( \frac{1}{p^2 + M^2 n^2} \right) + \sum_{i \neq j} Q^\mu_i Q^\nu_j \left( \frac{1}{p^2 + M^2 n^2} \right)^{N-2} \frac{\partial}{\partial p_\mu} \left( \frac{1}{p^2 + M^2 n^2} \right) \frac{\partial}{\partial p_\nu} \left( \frac{1}{p^2 + M^2 n^2} \right) \right) + \text{permutations}(q_1, \ldots, q_N),
\end{equation}
for next one.

Computation of the integrals (A.3, A.4) yields, respectively,
\begin{equation}
G_N^{(0)} = \lambda^N \frac{M^{\bar{D}-2N} f_N^{(0)}}{(2\sqrt{\pi})^D (N-1)!} = \lambda^N \frac{M^{\bar{D}-2N} \zeta(2N - \bar{D}) \Gamma(N - \bar{D}/2)}{(2\sqrt{\pi})^D (N-1)!},
\end{equation}
and
\begin{equation}
G_N^{(2)} = \lambda^N \frac{M^{\bar{D}-2(N+1)} f_N^{(2)}}{(2\sqrt{\pi})^D (N-1)!} = \lambda^N \frac{M^{\bar{D}-2(N+1)} 1}{2N} \zeta(2N + 2 - \bar{D}) \Gamma(N + 2 - \bar{D}/2) P_{(2)}(q),
\end{equation}
where
\begin{equation}
P_{(2)}(q) = \frac{1}{N!} \sum_{\text{perm}(q_1, \ldots, q_N)} \left( 4 \sum_{i \neq j} (Q_i \cdot Q_j) \left( \frac{1}{N + 1} \right) - \sum_i Q^2_i \left( \frac{1}{2} + \frac{3}{N + 1} \right) \right).
\end{equation}

Next terms can be computed in a similar manner, but we are interested only in those terms which are of the type contained in the classical Lagrangian (1.1).

**Appendix B. Zeta and Gamma Function Properties**

In this Appendix we review some properties of Gamma and Zeta functions borrowed from [13], which are relevant for us.

**Gamma function.** For Re $s > 0$ Gamma function is defined by the integral
\begin{equation}
\Gamma(s) = \int_0^{\infty} t^{s-1} e^{-t} dt.
\end{equation}

It can be analytically continued to the values Re $\leq 0$. On the negative part of real axis, however this function has singularities (simple poles) at negative integer point as well as at the zero point.

We are mainly interested in function’s behaviour at zero point:
\begin{equation}
\Gamma(\varepsilon) \approx \frac{1}{\varepsilon} - \gamma + O(\varepsilon),
\end{equation}
where $\gamma \approx 0.577216$ is the Euler constant. At negative points poles will be cancelled by zeroes of
Zeta function. Zeta function $\zeta(s)$ is defined by the series

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-s}. \quad (B.3)$$

This defines zeta-function for $\text{Re } s > 1$. Analytic continuation to $\text{Re } s \leq 1$ has a pole at $s = 1$. Near the singularity point $\zeta(s)$ behaves like

$$\zeta(s) = \frac{1}{s-1} + \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \gamma_n (s-1)^n, \quad (B.4)$$

where

$$\gamma_n = \lim_{m \to \infty} \left\{ \sum_{k=1}^{m} \frac{(\log k)^n}{k} - \frac{(\log m)^{n+1}}{n+1} \right\}. \quad (B.5)$$

Zeta-function, also, have zeroes at even negative integer values of $s$:

$$\zeta(-2n) = 0, \quad n \in \mathbb{Z}^+. \quad (B.6)$$

These zeroes just compensate all poles of Gamma function in product

$$\zeta(2s)\Gamma(s), \quad (B.7)$$

except one for $s = 0$ since

$$\zeta(0) = -1/2. \quad (B.8)$$

Thus two singular points are $s = 0$ and $s = 1/2$ and

$$\zeta(2\varepsilon)\Gamma(\varepsilon) = -\frac{1}{2\varepsilon} + \frac{1}{2} \gamma + \log 2\pi + O(\varepsilon), \quad (B.9)$$

and

$$\zeta(1 + \varepsilon)\Gamma(1/2 + \varepsilon/2) = \sqrt{\frac{\pi}{\varepsilon}} + \sqrt{\pi} \left( \gamma + \frac{1}{2} \psi(1/2) \right) + O(\varepsilon), \quad (B.10)$$

where $\psi(1/2) \approx -1.96351$.

Also,

$$\int_{0}^{\infty} \left( \frac{1}{t + M^2 n^2} \right)^{\beta} t^{\alpha} dt = (M^2 n^2)^{\alpha+1-\beta} \Gamma(\alpha+1)\Gamma(\beta-\alpha-1) \Gamma(\beta), \quad (B.11)$$

is used in Appendix A.

**References**

[1] M.B. Green J.H. Schwarz and E. Witten. *Superstring Theory*. Cambridge University Press.

[2] J. Maldacena. The Large N limit of Superconformal Field Theories and Supergravity. hep-th/9711200.

[3] G.G. Gubser, I.R. Klebanov, and A.M. Polyakov. Gauge Theory Correlators from Noncritical String Theory. hep-th/9802109.

[4] E. Witten. Anti de Sitter Space and Holography. hep-th/9612203.

[5] E. Kiritsis. Introduction to Superstring Theory. hep-th/9709006.

[6] T. Appelquist, and J. Carazzone. *Phys. Rev.*, D11: 2856, 1975.

[7] E. Witten. *Nucl. Phys.*, B104: 445, 1976.

[8] S. Weinberg. *Phys. Lett.*, B91: 51, 1980.

[9] B.A. Ovrut, and H.J. Schnitzer. *Phys. Rev.*, D21: 3369, 1980.

[10] J.C. Collins. *Renormalization*. Cambridge University Press.

[11] C. Sochichiu. Work in progress.

[12] R. Slansky. Group Theory for Unified Model Building. *Phys. Rep.*, 79(1):1–128, 1981.
[13] I.S. Gradshteyn, and I.M. Ryzhik *Table of Integrals, Series & Products*. Academic Press. 1994.

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