Recent solar neutrino experiments have shown that both $\phi(8\text{B})$ and the neutrino flux ratio $\phi(7\text{Be})/\phi(8\text{B})$ are substantially below their standard solar model values, leading some to discount the possibility of an astrophysical solution to the solar neutrino puzzle. We test this conclusion phenomenologically and find that the discrepancies can be significantly reduced by a distinctive pattern of core mixing on timescales characteristic of $^3\text{He}$ equilibration.
The results of the $^{37}$Cl [1], SAGE/GALLEX [2], and Kamioka II/III [3] experiments are consistent with an unexpected pattern of neutrino fluxes,

\[
\phi(\text{pp}) \sim \phi^{SSM}(\text{pp}) \\
\phi(\text{7Be}) \sim 0 \\
\phi(\text{8B}) \sim 0.4\phi^{SSM}(\text{8B})
\]

where \(\phi^{SSM}\) denotes the standard solar model [1] (SSM) value. As \(\phi(\text{8B}) \sim T_{22}^c\) [5], where \(T_c\) is the solar core temperature, the required reduction in this flux can be achieved by lowering \(T_c\) to about 0.96 of the SSM value. However such a reduction leads to an elevated \(\phi(\text{7Be})/\phi(\text{8B})\) flux ratio because

\[
\frac{\phi(\text{7Be})}{\phi(\text{8B})} \sim T_c^{-10}
\]

in contradiction to the results of Eq. (1).

This qualitative expectation - that it is very difficult to simultaneously reduce the $^8$B neutrino flux and the $^7$Be/$^8$B flux ratio - has been examined and confirmed in several careful studies. These have included variations in the SSM parameters within generally accepted ranges [6], variations far outside such ranges [5,7], and explorations of nonstandard models [8]. This has led many in the field to favor nonastrophysical solutions to the solar neutrino problem, such as neutrino oscillations.

It is clear that no astrophysical solution of the solar neutrino problem will give a perfect fit to the results of existing experiments: the measurements are inconsistent with any combination of undistorted $^8$B, $^7$Be, and pp neutrino fluxes at a confidence level of about $2\sigma$ [9]. Yet it is also clear that a compelling argument for a resolution in terms of new particle physics must rest on the more dramatic discrepancy (often estimated at $5\sigma$) that exists between experiment and the flux predictions of standard and nonstandard models. Thus it is important to determine whether a nonstandard model might exist in which the naive $T_c$ dependence described above is circumvented.
If such a model exists, the associated physics could be subtle. For this reason we will try a simple-minded approach - changing the SSM phenomenologically - putting aside for the moment the deeper issue of the underlying mechanism. We consider perturbations of the Bahcall-Pinsonneault (BP) SSM (without He or metal diffusion), constrained by three conditions. First, we retain all of the standard nuclear and atomic microphysics, e.g., nuclear cross sections and opacities. This reflects our view that SSM “best values” and uncertainties are sensible chosen, and our reluctance to produce a trivial solution to the solar neutrino problem by ad hoc adjustments of parameters. Second, we require that our phenomenological changes not alter the known solar luminosity. In a steady-state model this places the following constraint on the neutrino fluxes

$$6.481 \cdot 10^{10}/\text{cm}^2\text{sec} = \phi(\text{pp}) + 0.9561 \phi(7\text{Be}) + 0.5075 \phi(8\text{B})$$  \hspace{1cm} (3)$$

where we ignore small changes due to variations in the CNO cycle contribution. In our calculations this constraint was enforced in a somewhat crude fashion, by retaining the BP temperature profile of Table VII of Ref. [4] but allowing an overall rescaling. Third, we require the model to be steady-state, demanding where appropriate equilibrium in the production and consumption of pp chain “catalysts” such as D, $^3$He, and $^7$Be.

The third condition is typically implemented locally in the SSM, while the weaker condition of global equilibrium is still compatible with a steady-state sun. Thus these conditions allow a broad class of continuously mixed suns where pp chain products ($^4$He as well as the nuclei mentioned above) are transported. The possibility of slow mixing, which would allow the “catalysts” to remain at their local equilibrium values but tend to homogenize the H and $^4$He in the mixed portion of the core, was explored many years ago [10,11] and is known not to produce the flux pattern of Eq. (1). Likewise it can be reasonably argued that the transport of D and $^7$Be is less interesting because of the requirement of very large mixing velocities: D is destroyed almost instantaneously, while the solar lifetime of $^7$Be is about 100 days. On the other hand, $^3$He is intriguing. It is produced in the pp chain by $p + p \rightarrow D + e^+ + \nu$ followed by $D + p \rightarrow ^3\text{He} + \gamma$ at a rate...
\[ r_{11} \propto X_1^2 T_7^4 \]  

where \( X_1 \) and \( T_7 \) are the local mass abundance of hydrogen and temperature (units of \( 10^7 \) K). It is consumed by the competing reactions

\[ ^3\text{He} + ^3\text{He} \rightarrow ^4\text{He} + 2p \quad (5a) \]

\[ ^3\text{He} + ^4\text{He} \rightarrow ^7\text{Be} + \gamma \quad (5b) \]

with the former being dominant. As its rate is

\[ r_{33} \propto X_3^2 T_7^{16} \]  

where \( X_3 \) is the abundance of \(^3\text{He}\), it follows that, in local equilibrium,

\[ X_3 \sim 7 \cdot 10^{-4} X_1 T_7^{-6} \]  

Thus the SSM \(^3\text{He}\) equilibrium abundance increases sharply with radius (decreasing \( T_7 \)), as does the time required to reach equilibrium, which varies approximately as \( T_7^{-10} \). For example, the time required to reach 99\% of the \(^3\text{He}\) equilibrium value at \( r \sim 0.1 \text{ R}_\odot \) is \( \sim 5 \cdot 10^6 \) years. The SSM \([4]\) predicts that today’s sun has reached \(^3\text{He}\) equilibrium for \( r \lesssim 0.27 \text{ R}_\odot \).

We introduced changes in the SSM equilibrium \(^3\text{He}\) profile, constrained by the requirement of \(^3\text{He}\) global equilibrium in the core. Such changes alter the competition between the ppI, ppII, and ppIII cycles and thus affect the luminosity. To recover the correct luminosity we adjusted the overall scale of the BP temperature profile. This procedure must then be iterated to convergence. As \(^3\text{He}\) mixing timescales are short compared to overall solar evolution, H and \(^4\text{He}\) were assumed to be homogeneous throughout the mixed portion of the core. We chose very simple, piecewise constant \(^3\text{He}\) profiles, as our goal was to determine the qualitative features of any \(^3\text{He}\) distribution consistent with Eq. (1).

Profiles that simultaneously produced a reduced \( \phi(8\text{B}) \) (we selected values near 0.4 of the SSM) and a reduced flux ratio \( \phi(7\text{Be})/\phi(8\text{B}) \) had a characteristic shape: an order-of-magnitude elevation in the \(^3\text{He}\) abundance, relative to the equilibrium value, at small radii,
and a depletion at large r. The breadth and height of the region of elevated abundance can be adjusted over some range. The corresponding temperature scale factors ranged from near 1 to about 0.93, with 0.95 being a typical value. Thus the resulting sun is a cooler one, consistent with the increase in ppI terminations demanded by Eq. (1). Some typical results are given in Table I and illustrated in Fig. 1. (Fig. 1 is, of course, a caricature: only the qualitative aspects of the $^3$He distribution have significance.)

It is readily seen why such a change moves the neutrino flux predictions towards the results of Eq. (1). First, a large fraction of the produced $^3$He is burned out of equilibrium at small r. The ppI terminations are governed by reaction (5a), which is quadratic in the $^3$He abundance, while the competing reaction (5b) is linear. Thus the rate of ppII+ppIII terminations relative to ppI terminations is reduced in direct proportion to the $^3$He excess, suppressing both the $^7$Be and $^8$B neutrino fluxes. However, when reaction (5b) does occur, short-lived $^7$Be is produced at small r, where the ambient temperature is high. This favors ppIII terminations over ppII terminations, leading to a suppressed $\phi(^7\text{Be})/\phi(^8\text{B})$ flux ratio. The combined effects of the reduced (ppII+ppIII)/ppI and enhanced ppIII/ppII branching ratios yield a somewhat reduced $^8$B neutrino flux and a significantly reduced $^7$Be flux.

Such a pattern of $^3$He burning can only arise if there is core mixing on a timescale characteristic of $^3$He equilibration. In fact, the profile of Fig. 1 suggests a rather specific mixing mechanism. First, there must be a relatively rapid downward flow of $^3$He-rich material from large r; the speed must be sufficient to take a mass element well past the usual equilibrium point, into a region where the rapidly decreasing local lifetime of $^3$He final results in sudden $^3$He ignition. This mass element, now depleted in $^3$He and buoyant because of the energy release, must return to large r sufficiently slowly to allow the p+p reaction to replenish the $^3$He. This flow is depicted in Fig. 2. As we are assuming a steady-state process in which any mass element is roughly equivalent to any other, each mass element must, on average, remain within a radial shell bounded by r and r+dr for a time proportional to the mass dM(r) contained within that shell. This condition would be satisfied if the slow upward flow is broad with a local velocity inversely proportional to dM(r) - the kind of flow that
would result from displacement from below. Such upward flow will produce a positive \(^3\)He gradient, as in the SSM; but the upward flow must be sufficiently fast to keep the \(^3\)He below its local equilibrium value to prevent burning at large \(r\). To keep the circulation steady, the rapid downward flow clearly must be localized, e.g., perhaps in narrow plumes.

We would like to stress that we are not proposing this as a solution to the solar neutrino puzzle. But we are suggesting that arguments against an astrophysical solution based on the naive \(T_c\) dependence of neutrino fluxes are likely overstated. We have sketched how the naive expectations might be circumvented by core mixing.

Yet there are amusing aspects of the mixing that we would like to explore further, with the understanding that our comments are quite speculative:

1) Although core mixing on the timescale for \(^3\)He equilibration has been considered previously, including in the early work of Shaviv and Salpeter [11], we believe the possibility of different upward and downward flow velocities has not been explored. In Table I we give estimates of these velocities for a various profiles of the type illustrated in Fig. 1.

The downward plume velocity (taken to be constant) is fixed by the condition that a mass element with the necessary \(^3\)He abundance will be swept to the appropriate point before burning commences. Defining the onset of burning as a depletion of the \(^3\)He to 80\% of its initial value at large \(r\), we find transport times \(\tau_\downarrow = (2-12) \times 10^6\) years, or velocities on the order of 10-100 m/y.

The temperature and volume of the mass element will increase as \(^3\)He burning proceeds under the condition of constant pressure, resulting in an upward acceleration due to the buoyancy. We lack a sufficiently detailed physical picture to model this: clearly the temperature trajectory will depend on a competition between energy generation and thermal transport, with the later depending on the plume geometry. Thus we have depicted this part of the trajectory in Fig. 2 by a dashed line. Qualitatively the rising temperature will increase the suddenness of the \(^3\)He burning and further suppress the ppII/ppIII branching ratio, relative to the estimates of Table I.

As the rising mass element is now depleted in \(^3\)He, when it cools it should be similar to,
and merge with, the surroundings. We envision the subsequent upward flow as described above - slow and global, proportional to \( dM(r) \) - and have checked whether the necessary accumulation of \(^3\text{He}\) could then occur, given the constraint that the produced \(^3\text{He}\) not burn at large \( r \). The results are somewhat interesting. For profiles similar to Fig. 1 in which the mixing was confined to the inner core, \( r \lesssim 0.2 \, \text{R}_\odot \), this can be achieved only if the slow, upward flow (the solid part of the upward trajectory in Fig. 2), begins at relatively large \( r \). It then becomes complicated to explain how the overall flow can be viewed as one where every mass element within the mixed core cycles in an equivalent way. On the other hand, if the mixed region extends to large \( r \), 0.25-0.30 \( \text{R}_\odot \), the slow upward flow dominates the mixed core, beginning typically at \( r \sim 0.1 \, \text{R}_\odot \). The results in Table I are of this class, with the starting point for the upward flow defined by the requirement that at least 80% of the produced \(^3\text{He}\) survives unburned. The corresponding times required for the flow fall in the range \( \tau_\uparrow = (4.2-15.2) \times 10^7 \) years, roughly an order of magnitude longer than the corresponding \( \tau_\downarrow \).

We are somewhat surprised that simple flow patterns could qualitatively produce the \(^3\text{He}\) burning profile depicted in Fig. 1, as the latter was deduced from Eqs. (1) phenomenologically and without regard for physical plausibility. In fact, the resulting preference for larger mixed cores, ones encompassing most of the region where the SSM \(^3\text{He}\) gradient has been established, is a rather pleasing result. Of course, our inability to model the small \( r \) region where \(^3\text{He}\) is burned is an important caveat.

2) The possibility of flows in the background of a positive \(^3\text{He}\) gradient raises the old issue of the “solar spoon” instability first discussed by Dilke and Gough [12]. These authors pointed out that the SSM is unstable to large amplitude perturbations because the energy released by enhanced \(^3\text{He}\) burning can exceed the work against gravity required to force a mass element at large \( r \) through the denser material below. The solar spoon has been most often discussed as a trigger for periodic core mixing [13].

In the case of the continuous flow postulated in 1), the core would remain homogenized in H and \(^4\text{He}\) while still permitting a \(^3\text{He}\) gradient, an amusing variation on the solar spoon.
The plume flow we have described would then be essentially adiabatic. Large-scale adiabatic flow that would allow the sun to produce the required luminosity more efficiently (i.e., by burning at a lower temperature) has a certain attractiveness. On the other hand, one clearly needs to explain how the plumes maintain their chemical identity as they descend through the $^3\text{He}$-poor surroundings.

Speculations about a persistent convective core [14] could be relevant to the question of how the plumes are first generated. The core of the early sun is believed to be convectively unstable prior to the establishment of equilibrium in the pp and CNO cycles: $\eta = \text{d} \log \epsilon / \text{d} \log T$, where $\epsilon$ is the energy generation rate, is initially in excess of the critical value of about 5.0. While the initial instability is due to the out-of-equilibrium burning of $^{12}\text{C}$ to $^{14}\text{N}$, Roxburg [14] has suggested that $^3\text{He}$ transport by convective overshooting might help to maintain the conditions for convection up to present times. We suspect that the flow postulated in 1) would be convectively unstable in the region where the $^3\text{He}$ is being burned, as the disequilibrium is so similar to the early sun. The fact that the flow is both driven by and maintains the disequilibrium opens up the possibility of persistent mixing: if one introduces a perturbation to speed the mixing, this should diminish the $^3\text{He}$ gradient and energy production, thus slowing the mixing. That is, the mixing, if otherwise viable, could prove to be stable to perturbations.

3) Such a mixed core will have other astrophysical consequences. For example, galactic evolution models [15,16] predict $^3\text{He}$ abundances in the presolar nebula and in the present interstellar medium (ISM) that are substantially (i.e., a factor of five or more) in excess of the observationally inferred values. This enrichment of the ISM is driven by low-mass stars in the red giant phase, when the convective envelope reaches a sufficient depth to mix the $^3\text{He}$ peak, established during the main sequence, over the outer portions of the star. The $^3\text{He}$ is then carried into the ISM by the red giant wind. This difficulty prompted Galli et al. [15] to suggest an enhanced $^3\text{He}+^3\text{He}$ cross section, which would suppress the main sequence $^3\text{He}$ peak, as the solution most compatible with observation. While the galactic evolution of $^3\text{He}$ is clearly a complex problem, it is interesting that the mixing we have discussed lowers
the main sequence $^3$He abundance at large $r$.

The envisioned mixing will also change thermal and composition gradients and thus the core sound speed, affecting the helioseismology [17]. If helioseismology can rule out the postulated mixing, it would suggest that astrophysical solutions to the solar neutrino problem would have to be more exotic than the steady-state models considered here.

As the core mixing lowers $T_c$ and enhances ppi burning, it should slow stellar evolution somewhat. However, it is generally believed [11] that mixing confined to the core ($r \lesssim 0.4R_\odot$) will not affect main sequence and red giant evolution to a degree that would be apparent in the color-magnitude diagram.

In summary we have argued that the naive $T_c$ dependence of solar neutrino fluxes could be circumvented in models where $^3$He is transported into the core. Thus the $T_c$ argument by itself is not sufficient to rule out an astrophysical solution to the solar neutrino problem, even if one limits the discussion to steady-state models with conventional microphysics. We then pointed out that the $^3$He profile consistent with Eq. (1) is suggestive of a rather unusual steady-state mixing pattern involving rapid filamental flow downward and a slow, broad restoring flow upward. Whether such flow could occur in the sun is entirely speculative, and the consistency of the resulting solar model with helioseismology is an open question. Some of the issues raised by the hypothesized mixing are reminiscent of such “closet skeletons” as the SSM $^3$He instability, an early convective core, and galactic $^3$He evolution.

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TABLE I. Modified $^3$He profiles. The inner (enhanced $^3$He) and outer (depleted) portions of the mixed core are denoted by $\Delta r_I$ and $\Delta r_O$ and specified in units of $R_\odot$. The third column gives both the absolute and normalized (relative to equilibrium, in parentheses) $^3$He mass fractions in the inner core. The temperature $T_c$, $\phi(^8\text{B})$, and $\phi(^7\text{Be})$ results are normalized to SSM values. $X_{^3\text{He}}^{eq}$ is the equilibrium $^3$He mass fraction at the outer edge of the mixed core. $\tau_\downarrow$ and $\tau_\uparrow$ are the transit times for sinking and rising mass elements (see text).

| $\Delta r_I$ | $\Delta r_O$ | $X_{^3\text{He}}^I \times 10^{-3}$ | $T_c$ | $\phi(^8\text{B})$ | $\phi(^7\text{Be})$ | $X_{^3\text{He}}^{eq} \times 10^{-3}$ | $\tau_\downarrow \times 10^7\text{ y}$ | $\tau_\uparrow \times 10^7\text{ y}$ |
|---|---|---|---|---|---|---|---|---|
| 0.000-0.020 | 0.020-0.23 | 0.334 (12.3) | 0.949 | 0.40 | 0.24 | 1.62 | 0.21 | 14.9 |
| 0.000-0.031 | 0.031-0.31 | 0.219 (6.3) | 0.932 | 0.40 | 0.33 | 13.3 | 0.85 | 4.2 |
| 0.010-0.025 | 0.025-0.26 | 0.269 (8.7) | 0.940 | 0.40 | 0.29 | 4.01 | 0.43 | 7.0 |
| 0.020-0.025 | 0.025-0.22 | 0.329 (12.0) | 0.953 | 0.41 | 0.26 | 1.32 | 0.28 | 15.2 |
| 0.020-0.031 | 0.031-0.27 | 0.246 (7.6) | 0.940 | 0.41 | 0.32 | 4.74 | 0.64 | 5.7 |
| 0.031-0.035 | 0.035-0.24 | 0.305 (10.0) | 0.952 | 0.40 | 0.31 | 1.88 | 0.52 | 11.3 |
| 0.039-0.045 | 0.045-0.27 | 0.250 (7.0) | 0.948 | 0.40 | 0.39 | 4.44 | 1.16 | 6.0 |
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FIGURES

FIG. 1. The dashed line gives the SSM equilibrium $^3\text{He}$ mass fraction rescaled for a core temperature $T_c=0.940 \ T_c^{SSM}$. For the same temperature, the solid line is a modified $^3\text{He}$ profile producing an equivalent $^3\text{He}$ burning rate, the correct luminosity, and the neutrino fluxes listed in the third row of Table I.

FIG. 2. A schematic of the circulation of a mass element that would produce a $^3\text{He}$ burning profile qualitatively similar to Fig. 1. The portion of the solid line with the downward arrow represents a $^3\text{He}$-rich plume descending toward the core; the remainder represents the slow upward flow resulting from displacement from below. The dashed line represents the process of $^3\text{He}$ ignition, buoyancy, and subsequent cooling. This portion of the circulation has not been modeled numerically.
$T = 0.940 T_{SSM}$

mixed region

$X_3$
$T = 0.940 \, T^{SSM}$

- **downward plume**
- **slow upward flow**
- $^3\text{He burning}$