Black Holes and Photons with Entropic Force

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We study entropic force effects on black holes and photons. We find that application of an entropic analysis restricts the radial change $\Delta R$ of a black hole of radius $R_h$, due to a test particle of a Schwarzschild radius $R_h$ moving towards the black hole by $\Delta x$ near black body surface, to be given by a relation $R_h \Delta R = R_h \Delta x/2$, or $\Delta R/\lambda_M = \Delta x/2 \lambda_m$. We suggest a new rule regarding entropy changes in different dimensions, $\Delta S = 2\pi kD\Delta l/\lambda$, which unifies Verlinde’s conjecture and the black hole entropy formula. We also propose to extend the entropic force idea to massless particles such as a photon. We find that there is an entropic force on a photon of energy $E_x$, with $F = GMm_x/R^2$, and therefore the photon has an effective gravitational mass $m_g = E_x/c^2$.

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Introduction

Newton’s laws of motion and gravity provided a unified description of motions for observable objects on the Earth and in the sky. One important formula in the laws of motion is the second law,

$$F = ma,$$

which states that a point-like particle of mass $m$ acquires an acceleration $a$ by a force $F$ acting on that particle. Newton’s law of gravity states that two particles of masses $m$ and $M$ experience an attractive force between each other,

$$F = G \frac{mm}{r^2},$$

where $G$ is a universal gravitational constant called Newton constant, and $r$ is the distance between the two particles. Newton’s theory only states how the laws work but not why they work. Therefore the desire to seek for explanations why these laws work is beyond the scope of classical mechanism.

There has been a new conjecture\textsuperscript{4,5} to interpret the gravity as an emergent force based on knowledge of black hole thermodynamics, relativity, and quantum theory. There have been a number of discussions\textsuperscript{6} concerning the conjecture, with also some applications to expansion\textsuperscript{5}, acceleration\textsuperscript{6}, and inflation\textsuperscript{7} in cosmology. In this work, we further study some implications of the entropic force. It is found that since the entropy of a black hole is completely determined by the surface, the application of entropic force forces the radial change $\Delta R$ of a black hole of radius $R_h$, due to a test particle of a Schwarzschild radius $R_h$ moving towards the black hole by $\Delta x$ near black hole surface, to be given by a relation $R_h \Delta R = R_h \Delta x/2$, or $\Delta R/\lambda_M = \Delta x/2 \lambda_m$, where $\lambda_m$ and $\lambda_M$ correspond to the Compton wavelengths of the test particle and the black hole respectively. We find a consistent way to unify black hole properties and Verlinde’s conjecture. We also expand the entropic force idea to massless particles such as a photon. We find that there is an entropic force on this photon, and therefore the photon has an effective gravitational mass leading to blue and red shifts, and also deflection for a photon passing by a massive body.

Several essential ideas of the entropic force are related to the properties of a back hole. A black hole is a region of space from which nothing, including light, can escape. Around a black hole there is a surface or screen which marks the point of no return, called event horizon. As a black hole exhibits a remarkable tendency to increase its horizon surface area under any transformation, Bekenstein\textsuperscript{8} conjectured that a black hole entropy is proportional to the area $A_{BH}$ of its event horizon divided by the Planck area $l_P^2$, where $l_P = \sqrt{G\hbar/c^3}$ is the Planck length, $\hbar$ is the Planck constant, and $c$ is the speed of light. The entropy of a black hole is given by\textsuperscript{9,10}

$$S_{BH} = \frac{kA_{BH}}{4l_P^2},$$

where $k$ is the Boltzmann constant. The black holes create and emit particles as if they were black bodies with temperature $T_{BH}$.

The idea that the black hole information can be thought of as encoded on a boundary to the region preferably a light-like boundary like the gravitational horizon, is called the holographic principle\textsuperscript{11,12}. A test particle of mass $m$ after swallowed by a black hole increases the black hole mass and its surface. This procedure signifies information carried by the test particle lost to the increased black hole entropy or holographic screen.

The entropic force idea expands the information and holographic connection to flat non-relativistic space, not near the black hole. Verlinde postulated that the entropy

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$\Delta S$ gained on a holographic screen associated with a test particle of mass $m$ moving by a distance $\Delta x$ perpendicular towards the screen is given by

$$\Delta S = 2\pi k \frac{mc}{h} \Delta x.$$  \hspace{1cm} (4)

This equation, although a postulation, is very intuitive from the point of view that the entropy gained is proportional to the information loss of the test particle, the tendency for an increase of information bits is proportional to $\Delta x/\lambda_m$. Several papers have discussed the properties of the quantity $\Delta S/k$ \cite{2,3,13}, which is argued to be quantized in units of $2\pi$. One therefore can write

$$\Delta S = 2\pi k \frac{\Delta x}{\lambda_m}. \hspace{1cm} (5)$$

Associating the information bits with the wavelength, instead of the mass of a particle has the advantage, from quantum mechanics of view, when one tries to expand the same idea to massless particles, such as a photon. A massless particle moving towards a holographic screen should also have similar behavior. We will come back to this when we discuss entropic force on photons later.

To obtain Newton’s second law, Verlinde associates temperature $T$ in the term $TdS$ with the Unruh \cite{14} temperature $T_u$ by

$$T = T_u = \frac{ha}{2\pi ck}, \hspace{1cm} (6)$$

where $a$ denotes the acceleration experienced by the test particle.

For the isolated system composed of a holographic screen and a test particle, the tendency for an increase of the entropy of the system gives rise of an emergent force $F$ on the test particle, i.e.,

$$F \Delta x = T \Delta S, \hspace{1cm} (7)$$

Thus one immediately gets Newton’s second law

$$F = ma. \hspace{1cm} (8)$$

A system of total mass $M$ within the holographic screen is assumed to be related to the total information bits $N$ by the equipartition rule \cite{3}

$$Mc^2 = E = \frac{1}{2} kTN, \hspace{1cm} (9)$$

where $N$ is the number of quantized bits on the holographic screen

$$N = \frac{A}{l_p^2} = \frac{Ac^3}{Gh}. \hspace{1cm} (10)$$

This means that the area of the holographic screen has to be quantized in units of the Planck area $l_p^2$. One then obtains

$$T = \frac{2Mc^2}{kN} = \frac{GM}{R^2} \frac{h}{2\pi kc}, \hspace{1cm} (11)$$

in which $A = 4\pi R^2$ is adopted. Substituting this into Eq. (6), one arrives at the surface acceleration

$$a = \frac{2\pi ck}{h} T = \frac{GM}{R^2}. \hspace{1cm} (12)$$

Thus Eq. (8) changes into

$$F = G \frac{Mm}{R^2}, \hspace{1cm} (13)$$

which is the famous Newton’s law of gravity.

Alternatively, substituting Eq. (11) into Eq. (7) together with the Verlinde conjecture Eq. (5), one still arrives at the gravity law Eq. (13), without using the Unruh temperature Eq. (6).

Note that the sizes of the holographic screen containing a fixed total energy $E = Mc^2$ are different when a test particle is at different locations $R$ and $r$ because the force are $GMM/r^2$ and $GMM/r'$, respectively. This is a situation which can be applied to the situation for a test particle on the Earth and a test particle on Mars in the solar system. A consistent picture can be obtained by noticing that the Unruch temperatures at these two locations are also different $T_R/T_r = r^2/R^2$, and identifying the sizes of the holographic screens felt by the test particle depend on the location given by $A_R = 4\pi R^2$ and $A_r = 4\pi r^2$, The equipartition rule still holds.

**Black Hole from Entropic Consideration**

For a black hole, its entropy is completely determined by the black hole surface $A_{BH}$. A radial increase $\Delta R$ of a black hole with horizon radius $R_H$ causes an increase of entropy

$$\Delta S_{BH} = \frac{8\pi R_H}{4l_p^2} \Delta R. \hspace{1cm} (14)$$

If the black hole horizon is the holographic screen of a test particle of $m$ just outside and moves a distance $\Delta x$, one would have, according to Verlinde’s conjecture,

$$\frac{2\pi kR_H}{l_p^2} \Delta R = \frac{2\pi kmc}{h} \Delta x, \hspace{1cm} (15)$$

which leads to

$$R_H \Delta R = \frac{l_p^2 \Delta x}{\lambda_m} = \frac{2Gm}{c^2} \frac{\Delta x}{2}. \hspace{1cm} (16)$$

Since $R_h = 2Gm/c^2$ is the Schwarzschild radius of black hole with mass $m$, one obtains a relation

$$R_H \Delta R = R_h \Delta x/2. \hspace{1cm} (17)$$
There is a question on how to understand this relation. In the special case $m = M$, $R_H = R_h$, one therefore obtains $\Delta R = \Delta x/2$ to have a consistent picture. This can be understood from the point of view that the change $\Delta x$ in Eq. (5) is linear in one dimension, but $\Delta R$ in Eq. (14) causes a change for a two dimensional surface and therefore should be a factor 2 less. For $m$ not equal to $M$, in general, one should set the entropy change in $\Delta S$ to be in units of $2\pi k$ after a displacement $\Delta x$, and the entropy change in $\Delta S_{BH}$ to be in units of $4\pi k$ after a change in radius $\Delta R$. If $\Delta S$ is quantized in unit $2\pi k$ as suggested in Refs. [2, 3, 13], $\Delta S_{BH}$ should quantized in units of $4\pi k$.

We can re-write the relation Eq. (17) as

$$\Delta R = \frac{\Delta x}{\lambda_M} = \frac{\Delta x}{2\lambda_m}. \tag{18}$$

The above equation indicates that the number of entropy in basic units recorded in $\Delta R$ by the holographic screen with two dimensional degrees of freedom should be half of that lost by the test particle moving $\Delta x$ with only one dimensional degree of freedom. Then we find the equivalence between the entropy acquainted by the black hole during a horizon radial increase $\Delta R$ and that in Verlinde’s conjecture, i.e., Eq. (15).

This implies that our knowledge of black holes is consistent with Verlinde’s conjecture. One can also re-derive the black hole properties, such as the Schwarzschild radius and the entropy formula, from an entropic viewpoint without Newtonian mechanism and relativity.

Our knowledge of black holes is based on classical mechanism, relativity, and quantum theory. Reversely, one may also take Eq. (18) as a basic Ansatz, instead of Verlinde’s conjecture, to re-derive the second law of motion and the law of gravity, based on knowledge of black holes.

At this point we should like to suggest an entropic framework with a rule regarding entropy changes in different dimensions. The entropy change caused by a linear displacement $\Delta l$ in units of its Compton wavelength $\lambda$ is given by

$$\Delta S = 2\pi k D \Delta l / \lambda, \tag{19}$$

where $D$ is the dimensional degree of freedom of the object under consideration. For example in our analysis $D = 1$ for a test particle and $D = 2$ for a holographic screen. If $\Delta S$ is quantized in unit $2\pi k$ as suggested in Refs. [2, 3, 13], the entropy change for a $D$ dimensional object is then given by $2\pi k D$.

This new rule unifies Verlinde’s conjecture and the entropy change of black hole by a radial change $\Delta R$. It extends Verlinde’s conjecture to a more general case where the resultant change of entropy is encoded in a multi-dimensional surface due to a linear displacement.

**Entropic Force for Photon**

So far the entropic force has been considered for massive particles. The behaviors of the photon in gravitational environments cannot be handled in classical mechanism as the photon mass is zero. A photon should have similar property in carrying information just like a massive particle should. Eq. (5) already provided a hint how entropic force can be generalized to photon since a photon with energy $E_\gamma$ has a Compton wavelength $\lambda = h/c/E_\gamma$ from quantum theory. To this end we propose that the entropy increase on the screen for a photon moves towards a holographic screen is

$$\Delta S = 2\pi k \frac{\Delta x}{\lambda}. \tag{20}$$

From $F\Delta x = T \Delta S$ one gets

$$F = \frac{2\pi k T}{\lambda} = \frac{2\pi k T}{h c} E_\gamma, \tag{21}$$

where $T$ is the temperature of the holographic screen from Eq. (11). Thus we obtain the gravitational force on the photon

$$F = \frac{G M \gamma E_\gamma}{R^2 c^2} = G \frac{M m_\gamma}{R^2}, \tag{22}$$

as if the photon has a gravitational mass $m_\gamma$, from the relation $E_\gamma = m_\gamma c^2$. Such a conclusion is the same as that from the equivalence principle of general relativity, therefore it gives the same predictions of the gravitational red/blue shift and bending of light by gravity. The entropic analysis brings also new insights on entropic force for photon. Similarly one can generalize to all massless particles.

**Discussion**

We have studied some implications of entropic force proposed recently by Verlinde. We find that the size of and temperature on the holographic screen felt by a test particle depend on the distance of the test particle away from the center of the energy contained inside the screen.

When applied the entropic analysis to a black hole, we find that since the entropy is completely determined by the surface, the change of radius $\Delta R$ of a black hole, due to a test particle moving towards the black hole by $\Delta x$ near the black body surface, is given by $\Delta R = R_h \Delta x/2R_H$, from which one has $R_H \Delta R = R_h \Delta x/2$ or $\Delta R/\lambda_M = \Delta x/2\lambda_m$. Such a relation implies a consistency between Verlinde’s conjecture and the entropy acquainted by the black hole during a radial increase. We suggested a new measure for entropy change in units of $2\pi k D$ for a $D$ dimensional object. For example with $D = 1$ and $D = 2$ for a linear movement of a test particle and a black hole surface, respectively.

When expanding the entropic force idea to massless particles such as a photon, we find that there is an entropic force on a photon of energy $E_\gamma$, with $F = GMm_\gamma/R^2$, and therefore the photon has an effective
gravitational mass $m_{\gamma} = E_{\gamma}/c^2$ leading to blue and red shifts, and also bend of light for a photon passing by a massive body.

From our analysis it is clear that the holographic principle, the concept of Unruh temperature and the equipartition rule offer natural derivations for the second law of motion and the gravity law in Newton’s theory, the properties of black holes, and also the gravitational effect on photons. Therefore we may consider the entropic framework as an axiomatic system in parallel to Newtonian mechanism, Lagrangian mechanism, or Hamiltonian mechanism. It is a new language for the conventional knowledge of classical mechanism. However, there are new perspectives beyond the classical mechanism from this new framework, as have been seen by the gravitational effect on photons revealed in this work. Therefore we are optimistic that the entropic framework can offer us insights to understand physics from a new way.

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