Cluster structures observed in $^{40}\text{Ca}$ from $^{12}\text{C}^{+28}\text{Si}$ scattering

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Abstract. We discuss evidences of strong cluster structures observed in nuclear reactions even at quite high excitation energies. The oscillatory angular distributions of the elastic scattering of the $^{12}\text{C}^{+28}\text{Si}$ system measured at 13 energies close to the Coulomb barrier were reproduced introducing in the scattering $S$-matrix additional Regge-pole form terms. These poles were interpreted as describing doorway states with $^{12}\text{C}^{+28}\text{Si}$ cluster structure in the composite nucleus $^{40}\text{Ca}$. The excitation energies of these states indicate that they can be part of a rotational band in $^{40}\text{Ca}$, together with similar states observed in the elastic scattering and $\alpha$-transfer reactions of the $^{16}\text{O}^{+24}\text{Mg}$ system. We propose that the presently observed, largely deformed molecular band corresponds to the hyper-deformed band in $^{40}\text{Ca}$, which has been found in a new Nilsson-model-calculation, which predicts preference of the $^{12}\text{C}^{+28}\text{Si}$ and $^{16}\text{O}^{+24}\text{Mg}$ molecular structures, in accordance with our present results.

1. Introduction

Atomic nuclei often display characteristics which can be related to clusterization. Due to the very large binding energy and compact structure of $^4\text{He}$, the alpha-clusterization is the best known example. Reaction studies have also revealed interesting aspects about the presence of clusters [1, 2] in nuclei. Elastic, inelastic scattering experiments and $\alpha$-transfer reactions realized

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with systems of light n-α nuclei, have shown anomalies called ALAS (Anomalous Large Angle Scattering) with strong oscillations in the angular distributions correlated with large peaks in the excitation functions [3]. These anomalies got mainly two interpretations: a non-resonant i.e. potential scattering description, where the angular momentum associated with the oscillations is simply the grazing angular momentum in the entrance channel. The other interpretation describes these states as molecular resonances with cluster structure in the composite system.

In previous papers [1, 2] we could explain fifteen strongly oscillating elastic scattering angular distributions of the $^{12}\text{C}+^{24}\text{Mg}$ system by adding energy-dependent resonance terms to the ‘background’ scattering matrix obtained from the ‘São Paulo potential’ (SPP) [4, 5]. These resonances belong to a rotational band, with a moment of inertia close to that of a hyperdeformed band, provided by α-cluster [6] and Nilsson [2] calculations. Similar quasi-molecular states observed previously in the $^{20}\text{Ne}+^{16}\text{O}$ reaction [7–9] fall on the same rotational band, giving support to the interpretation of $^{36}\text{Ar}$ composite system resonances.

In this work we measured complete angular distributions of the elastic scattering of the $^{12}\text{C}+^{28}\text{Si}$ system at 13 energies, between $E_{c.m.}=12.4$ to 22.4 MeV, all energies close to the Coulomb barrier [10–12]. The $^{12}\text{C}+^{28}\text{Si}$ system has been extensively studied over the past three decades for a number of reasons. It shows a strong orbiting behavior [13, 14] and demonstrates strong ALAS phenomena [15–17]. Superdeformed (SD) bands have also been discovered in the $N=Z$ nuclei $^{36}\text{Ar}$ and $^{40}\text{Ca}$ [18].

2. Experimental method

The measurements were performed at the Pelletron laboratory of the University of São Paulo, using $^{28}\text{Si}$ and $^{12}\text{C}$ beams on, respectively, $^{12}\text{C}$ and $^{28}\text{Si}$ targets. The data were obtained at center of mass energies close and above the Coulomb barrier ($V_{CB} \sim 13.0$ MeV), namely at $E_{c.m.}=12.4, 13.3, 14.0, 14.7, 16.1, 16.8, 17.5, 18.9, 19.6, 20.3, 20.9, 21.9$ and $22.4$ MeV (see fig. 1). The simultaneous measurement of the scattered projectile ($^{28}\text{Si}$ or $^{12}\text{C}$) and the recoiling target nucleus ($^{12}\text{C}$ or $^{28}\text{Si}$) allowed to cover an angular range from $\theta_{c.m.}=27^\circ$ to $164^\circ$. A strong oscillatory behavior was observed in the angular distributions of $^{12}\text{C}+^{28}\text{Si}$ system at energies above $E_{c.m.} = 16.1$ MeV and they are presented in figure 1.

3. Data analysis and results

The starting point in our calculations was a double-folding, deep optical potential, also called São Paulo Potential (SPP) [4, 5], where the Pauli non-locality results in energy dependence of the potential depth. The imaginary part of the potential has the same form factor as the real part, and the normalizations of the real and imaginary parts are, respectively, $N_r$ and $N_i$. In SPP, Fermi-Dirac type density distributions are used with diffuseness $a = 0.56$ fm and the normalization are $N_r=1.0$ and $N_i = 0.78$. These are values obtained through the systematic study of many systems [19] with SPP. The angular distributions calculated with this strongly absorbing, deep potential present no oscillations. For the energies below and near the Coulomb barrier, at $E_{c.m.}=12.4, 13.3, 14.0$ and $14.7$ MeV no oscillations were observed in the angular distributions and they could be well fitted by the SPP. The optical model analysis clearly does not reproduce the strong oscillations at energies above $E_{c.m.} = 16.1$ MeV and we have added Regge poles [20] to the optical model S-matrix $S^0_l$, as shown below:

$$ S_l = S^0_l \left(1 + \frac{i De^{2i\Phi}}{(l - l_0) - \frac{i\Delta_l}{2}}\right), \quad (1) $$

where $S^0_l$ is the optical model ‘background’ S-matrix, and $D$, $\Phi$ and $\Delta_l$ are the amplitude, phase and width, respectively, of a pole centered at $l_0$. The second term in eq. (1) is related
to a resonance centered in $l_0$ in the angular momentum space. For a complete analysis we also used angular distributions found in the literature [21, 22].

The data were fitted using the Monte-Carlo method, varying $D$, $\Phi$, $\Delta l$, and $l_0$. The angular distribution of $E_{c.m.} = 16.1$ MeV could be well reproduced with a single pole with $l_0 = 8\hbar$ and width $\Delta l = 1.47\hbar$. All the other angular distributions needed 2 poles with different values of $l_0$ in order to obtain a good fit. As a matter of fact, for most of cases $\Delta l$ was very small and the only term affected in the $S$-matrix was $S_{l_0}$, the term at the resonance $l_0$-value.

![Figure 1](image.png)

Figure 1. The elastic angular distributions for the $^{12}\text{C}+^{28}\text{Si}$ system. The dashed and solid lines are fits without and with the introduction of Regge poles in the $S$-matrix form, respectively.

Figure 1 shows the fits obtained for the $^{12}\text{C}+^{28}\text{Si}$ system using the São Paulo Potential without poles (dashed line) and introducing the Regge poles (solid line). In figure 1 the $l_0$ values of the poles are indicated for every angular distribution. Thus poles with $l_0 = 8, 10$ and $13\hbar$ are needed to reproduce the angular distributions between energies $E_{c.m.} = 16.1$ and $21.9$ MeV. For energies $E \geq 22.4$ MeV [22], there are no measurements at backward angles ($\theta_{c.m.} \geq 100$ degrees) and the fits become worse. The determination of $l_0$ at these energies is ambiguous.

By dividing the absolute value of the $S_{l_0}$ by the absolute value of the background $S_0^0$ (eq. (1)) $|[S_l]|/|S_0^0|$ for each $l = l_0$ and for each energy we can determine the effect of the poles individually. In this way, deviations from unity of the $|[S_l]|/|S_0^0|$ will indicate the effect of a pole centered in $l_0$. The center of mass energy of the $^{12}\text{C}+^{28}\text{Si}$ system was transformed into excitation energy in $^{40}\text{Ca}$ by summing $Q = 13.35$ MeV and $|[S_l]|/|S_0^0|$ was plotted as a function of the excitation energy (see figure 2(a)). The existence of peaks is suggested on this figure, which could correspond to resonances with cluster structure of $^{12}\text{C}+^{28}\text{Si}$, with $J^\pi = 8^+, 10^+, 13^-$ and $15^-h$, respectively, at $E_{c.m.} = 19.0, 20.6, 22.0$ and $23.6$ MeV and excitation energies of $32.4$, $34.0$, $35.3$ and $37.0$ MeV in the compound nucleus $^{40}\text{Ca}$. The $J = 15h$ and $18h$ states are represented in figure 2(c). Figure 2 compares $|[S_l]|/|S_0^0|$ with backward angle cross sections of the elastic scattering excitation functions of the $^{12}\text{C}+^{28}\text{Si}$ [15, 16] and the $^{16}\text{O}+^{24}\text{Mg}$ [23, 25] systems found in the literature. We indicate by vertical dashed lines the approximate energy position of our peaks in $|[S_l]|/|S_0^0|$ for each $l_0 = J$, and our resonance energies are in agreement with the structures observed in the excitations functions. For $J = 18h$ we adopted the energy found by Ost [16].

Ost et al [16, 17] have fitted backward angle angular distributions of the $^{12}\text{C}+^{28}\text{Si}$ elastic scattering using Legendre polynomials of $J = 13, 15$ and $20(19)h$ at $E_{c.m.} = 21.5, 23.2$ and $33.6$ MeV, and $J = 18h$ at $26.0$ and $30.2$ MeV, which correspond to $E^*(^{40}\text{Ca}) = 34.9, 36.6, 39.4, 43.6$ and $47.0$ MeV of excitation energy in $^{40}\text{Ca}$. Similar measurements (excitation functions and angular distributions on the peaks) have been performed for the elastic scattering of the
$^{16}$O+$^{24}$Mg system, and for the $\alpha$-transfer reaction $^{24}$Mg($^{16}$O,$^{12}$C)$^{28}$Si at forward and backward angles, respectively. The $^{24}$Mg($^{16}$O,$^{12}$C)$^{28}$Si forward angular distributions [23, 24] indicated molecular states with $J=20, 21, 23$ and $25\hbar$ at $E_{c.m.}=27.6, 28.2, 31.2$ and $34.2$ MeV. The backward angular distributions of [23, 25] yielded $J=15, 20, 21, 26\hbar$ at $E_{c.m.}=21.6, 27.8, 28.0$ and $36.2$ MeV. Transforming the $E_{c.m.}$ into excitation energy in the compound nucleus $^{40}$Ca, through the addition of $Q=16.176$ MeV, the following results were obtained for the $^{16}$O+$^{24}$Mg entrance channel: molecular states with $J = 15, 20, 21, 23, 25$ and $26\hbar$ at excitation energies of $37.8, 44.0(43.8), 44.2, 47.0(47.4), 50.4$ and $52.4$ MeV.

We interpret the five poles added to the $S$-matrix in our analysis, as well as the other states observed in the $^{12}$C+$^{28}$Si and $^{16}$O+$^{24}$Mg systems as corresponding to molecular resonances with di-nuclear cluster structure in the $^{40}$Ca compound nucleus. In figure 3 we plot their excitation energies against the $J(J+1)$ values and we observe a rotational band with a clear linear dependence,

$$E_J^* - E_0 = E_{rot} = \frac{J(J+1)\hbar^2}{2I},$$

where $E_J^*$ is the excitation energy of a state with spin $J$ in the $^{40}$Ca nucleus, $E_0 = 29.8(7)$ MeV is the band head energy and $I = 15.3(9)\hbar^2$/MeV = $6.0(4) \times 10^5$ MeV fm$^2$/c$^2$ is the moment of inertia. The superdeformed (SD) band, observed in $^{40}$Ca [18] and the grazing angular momenta

![Figure 2](image-url)

**Figure 2.** (a) and (e): $|S_1|/|S_0|$ for the $^{12}$C+$^{28}$Si system, as a function of the excitation energy in $^{40}$Ca. (b), (c) and (d): Excitation functions for $^{12}$C+$^{28}$Si [15, 16] and $^{16}$O+$^{24}$Mg [23–25].

![Figure 3](image-url)

**Figure 3.** Resonances obtained in this work (crosses), in [16, 17] (full diamonds) and in the $^{16}$O+$^{24}$Mg system [23–25] (circles), as a function of $J(J+1)$. 

16O+$24$Mg system, and for the $\alpha$-transfer reaction $^{24}$Mg($^{16}$O,$^{12}$C)$^{28}$Si at forward and backward angles, respectively. The $^{24}$Mg($^{16}$O,$^{12}$C)$^{28}$Si forward angular distributions [23, 24] indicated molecular states with $J=20, 21, 23$ and $25\hbar$ at $E_{c.m.}=27.6, 28.2, 31.2$ and $34.2$ MeV. The backward angular distributions of [23, 25] yielded $J=15, 20, 21, 26\hbar$ at $E_{c.m.}=21.6, 27.8, 28.0$ and $36.2$ MeV. Transforming the $E_{c.m.}$ into excitation energy in the compound nucleus $^{40}$Ca, through the addition of $Q=16.176$ MeV, the following results were obtained for the $^{16}$O+$^{24}$Mg entrance channel: molecular states with $J = 15, 20, 21, 23, 25$ and $26\hbar$ at excitation energies of $37.8, 44.0(43.8), 44.2, 47.0(47.4), 50.4$ and $52.4$ MeV.

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$l_g$ for $^{12}$C+$^{28}$Si are also included in fig. 3. The moment of inertia of the nucleus in the SD band is $\mathcal{I} = 8.07h^2/\text{MeV}=3.1 \times 10^5 \text{ MeV fm}^2/\text{c}^2$. Thus the molecular rotational band has a much larger deformation than the SD band, indicating a possible hyperdeformation in $^{40}$Ca. The grazing angular momenta $l_g$ were calculated using SPP and they fall on a parallel line below the molecular band, showing that they have similar moments of inertia. However, as $J \leq l_g$, the molecular states are more compact or internal than the grazing situation. The facts that $J \leq l_g$ and that the molecular resonances of $^{12}$C+$^{28}$Si and $^{16}$O+$^{24}$Mg lie on the same rotational band indicate that there is a strong cluster structure mixing in the composite system $^{40}$Ca.

4. Theoretical interpretation
The hyperdeformed state and other shape isomers of the $N = Z$ nuclei can be determined theoretically from different models. Here we refer to a symmetry-adapted method, which provides not only the shape isomers, but also their possible clusterizations [26]. It consists of two parts. First the shape isomers are obtained from a Nilsson-calculation by requiring the self-consistency of the quadrupole shape [2, 27, 28]. This procedure is based on the application of the quasi-dynamical U(3) symmetry, which coincides with the real U(3) symmetry in the simple cases when this latter one is valid too [29]. Due to to this fact the U(3) selection rule for the determination of the allowed clusterizations can be extended to the quasi-dynamical symmetry [30–32]. Therefore, in the second step, the allowed clusterizations can be obtained both for the binary [33, 34] and for the ternary configurations [35, 36]. Work is in progress to carry out this kind of investigations systematically for the shape isomers of the $^{40}$Ca nucleus; the preliminary results indicate a hyperdeformed state, which is similar to the suggestion of the experimental band, and has the preferred cluster configurations of $^{12}$C+$^{28}$Si and $^{16}$C+$^{24}$Mg.

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