Cosmic Rays Propagation in Bose Condensed Dark Matter

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We have calculated the dispersion relation for the high energy cosmic rays (protons, electrons and neutrinos) propagating in the Bose gas as well as Bose condensate medium of the pseudo-scalar particles. For cosmic rays propagating in the Bose gas, the mass of the particle will decrease and this is proportional to the boson density in the medium. If the propagating fermion is massless then it will develop an imaginary mass and thus will be absorbed in the medium. But if the condition $m^2/m_\phi p_0 \simeq 1$ is satisfied, then the cosmic rays will propagate freely without losing energy in the Bose condensate medium.

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I. INTRODUCTION

Axions and majorons are the pseudo-scalar Goldstone bosons associated with the Peccei-Quinn and the U(1)$_B$ and U(1)$_L$ symmetry respectively and they get mass because of the spontaneous breaking of these symmetries $\partial^2$. Astrophysical and cosmological calculations allow the axion mass range $10^{-6}$ eV to $10^{-3}$ eV. Axions in this mass range would have been produced non-thermally, as a Bose condensate i.e., a classical coherent field oscillation of the axion field $\phi$. Because the axions are produced with small velocity, they behave as cold dark matter (CDM), in spite of their small masses. Also the CDM scenario for structure formation is at present the most promising with axions and neutralinos being the leading CDM candidates. If axions provide the bulk of the dark matter, they must comprise a significant fraction of the dark halo of our own galaxy $\mathcal{H}$. The existence of coherent axion mini clusters has been suggested $\mathcal{H}$. It is shown that, axion clumps were formed in the universe, when the temperature was about 1 GeV, because of the nonlinearity of the axion potential and the inhomogeneity of coherent axion oscillation of the scale beyond the horizon $\mathcal{H}$.

It is expected that, in the present universe, there exist the axion mini clusters and the axion boson stars as well as the incoherent axion gas as dark matter comprises these coherent axion clumps. Majorons of KeV mass is also proposed as the candidate for the dark matter $\mathcal{M}$. As the majorons are also pseudo-scalar particle, they can undergo Bose Einstein Condensation (BEC) and can form clusters in the galactic halo.

The discovery of ultra-high energy cosmic rays with energy above Greisen-Zatsepin-Kuzmin (GZK) cut-off ($\sim 5 \times 10^{19}$ eV) $\mathcal{G}$ are particularly mysterious since, if the cosmic rays are protons, then photopion production caused by the resonant scattering process with the microwave background should result in a rapid loss of energy. It is believed that all the high energy cosmic rays are AGNs origin. The propagation of these high energy particles in the inter galactic medium is also presently a puzzling phenomena.

In this present work we study the propagation of ultrahigh energy cosmic rays in the bosonic medium and BEC medium. The paper is organised as follows: propagation of the fermion in the Bose gas is studied in section 2. Section 3 deals with the propagation of fermion in the BEC medium. Ultra-high energy cosmic rays propagation, in Bose gas and in BEC medium is discussed in section 4. In conclusions we briefly summarize our results.

II. FERMION SELF-ENERGY IN A BOSE GAS

At finite temperature and density the physical processes take place in a heat bath of particle and anti-particles and the properties of the test particle deviate from its vacuum values. The dispersion relation satisfied by the free propagation of the particle is modified, in the presence of the heat bath. The dispersion relation of the photon and leptons are well studied in the finite temperature heat bath, particularly in astrophysical as well as in cosmological scenario. However to my knowledge the particle propagation in Bose gas and in Bose condensed matter is not yet well studied.

The fermion and pseudo-scalar interaction is given by the lagrangian density,

$$\mathcal{L}_{\text{int}} = ig_p \psi^\dagger \gamma_5 \phi \psi$$  \hfill (1)

where $\phi$ is the pseudo-scalar field and $\psi$ is the fermion field. The self energy for fermion in the presence of a heat bath is given by $\mathcal{G}$,

$$-i\Sigma(k) = \int \frac{d^4 k}{(2\pi)^4} (ig_p \gamma_5 iS_F(p-k)ig_p \gamma_5 iD(k))$$  \hfill (2)

where $S_F(p)$ and $D(k)$ are the fermion and the boson propagator. In real time formalism of finite temperature field theory the fermionic and the bosonic (scalar/pseudo-scalar) propagators are given by

$$S_F(p) = (\not{p} + m) \left( \frac{1}{(p^2 - m^2)} + i\Gamma F(p) \right)$$  \hfill (3)

and
Then the real part of the self-energy term will be given only bosons (pseudo-scalars) are there in the medium. The real part of the self energy can be expressed as
\[ \text{Re} \Sigma = -g_p^2 \int \frac{d^4k}{(2\pi)^4} \frac{\delta(k^2 - m_\phi^2) n_B(w)}{(p - k)^2 - m^2}. \] (7)
The real part of the self energy can be expressed as
\[ -\text{Re} \Sigma = a (\not{p}^\mu + b \not{\mu} \not{k}^\mu) \] (8)
where \( a \) and \( b \) are two Lorentz-invariant functions and \( u^\mu = (1, 0), p^\mu = (p^0, -\not{p}) \) and \( k^\mu = (w, -\not{k}) \). Then the coefficients \( a \) and \( b \) are given by
\[ a = \frac{(T_p - T_u p_0)}{p^2}, \] (9)
and
\[ b = \left( \frac{p_0^2}{p^2} - 1 \right) T_u - \frac{p_0}{p^2} T_p, \] (10)
where \( T_p \) and \( T_u \) are given as
\[ T_p = -\frac{1}{4} Tr(\not{p} \text{Re} \Sigma) \] (11)
and
\[ T_u = -\frac{1}{4} Tr(\not{\mu} \text{Re} \Sigma). \] (12)
In general the dispersion relation for the fermion is given by
\[ det(\not{p} - m - \Sigma) = 0 \] (13)
and this gives rise to
\[ (p_0(1 + a) + b)^2 = m^2 + 2(1 + a)^2 p^2. \] (14)
By substituting eq.(10) in eqs.(11) and (12) and the simplifying them we obtain
\[ T_p = -\frac{g_p^2}{32\pi^2 |p|} \int \frac{k^2 dk}{w_k} n_B(w_k) \left[ (p_0^2 - p^2 + m^2 - m_\phi^2) \times \ln \left| \frac{D_2 + \sigma}{D_2 - \sigma} \right| + 2\sigma \right] \] (15)
and
\[ T_u = -\frac{g_p^2}{2\pi^2} \int \frac{k^2 dk}{2w_k} (p_0 - w_k) n_B(w_k) \ln \left| \frac{D_2 + \sigma}{D_2 - \sigma} \right|, \] (16)
where \( w_k = \sqrt{k^2 + m_\phi^2} \),
\[ D_2 = p_0^2 - p^2 - m^2 - m_\phi^2 - 2p_0 w_k \] (17)
and \( \sigma = 2|p||k| \). For fermions having \( p_0 \) and \( |p| >> m, m_\phi \) and \( |k| \) the logarithmic term will vanish i.e.
\[ \ln \left| \frac{D_2 + \sigma}{D_2 - \sigma} \right| \approx 0. \] (18)
This gives
\[ T_p = -\frac{g_p^2}{32\pi^2 |p|} \int \frac{k^2 dk}{w_k} n_B(w_k) \] (19)
and \( T_u \approx 0 \). Putting \( T_p \) and \( T_u \) in eqs.(11) and (12) we obtain
\[ b = -a p_0. \] (20)
For \( \exp(\beta \sqrt{k^2 + m_\phi^2}) >> 1 \) we can write the integrand in eq.(10) as a sum. Then
\[ \int \frac{k^2 dk}{w_k} n_B(w_k) = \frac{1}{\beta^2} \sum_{n=0}^{\infty} \int_{\alpha}^{\infty} \sqrt{x^2 - \alpha^2} e^{-(n+1)x} dx = \frac{\alpha}{\beta^2} \sum_{n=0}^{\infty} K_{-\alpha}(\alpha(n+1))/n+1, \] (21)
where \( \alpha = m_\phi \beta \). In the limit \( \alpha(n+1) \to \infty \) the modified Bessel function
\[ K_{-\alpha}(\alpha(n+1)) \approx \sqrt{\frac{\pi}{2\alpha(n+1)}} e^{-(n+1)\alpha} \left( 1 + \frac{3}{8\alpha} \right). \] (22)
Putting this in eq.(21), we get
\[ \int \frac{k^2 dk}{w_k} n_B(w_k) = \sum_{n=0}^{\infty} \sqrt{\frac{\pi \alpha}{2\beta^4(n+1)^3}} e^{-(n+1)\alpha} \left( 1 + \frac{3}{8\alpha} \right). \] (23)
Keeping the leading order term in \( n \) and for \( m_\phi \beta >> 1 \) we have
\[ T_p = -\frac{g_p^2}{4m_\phi^2} n_0, \] (24)
where
where in eq.(7) we get

\[ n_0 = \left( \frac{m_\phi}{2\pi \beta} \right)^{3/2} e^{-m_\phi \beta} \]  \hspace{1cm} (25)

is the number density of bosons in the medium. Then the dispersion relation in eq.(14) is simplified to

\[ p_0^2 - p^2 - m^2 = P^2 (a^2 + 2a). \] \hspace{1cm} (26)

Keeping terms up to order \( a \) in the above equation the dispersion relation is

\[ p_0^2 - p^2 = m^2 - \frac{g_p^2}{2m_\phi} n_0. \] \hspace{1cm} (27)

Thus in a bosonic medium the propagating particle mass square decreases by an amount \( \delta m^2 \) is \( g_p^2 n_0 / 2m_\phi \). If the propagating fermion is massless then it is Landau damped, because it will acquire an imaginary mass. On the other hand if the condition \( m^2 = g_p^2 n_0 / 2m_\phi \) is satisfied then the fermion will propagate like a massless particle in the vacuum.

### III. FERMION SELF-ENERGY IN BEC MEDIUM

For BEC of the pseudo-scalar bosons, the number density \( n_0 \) can be expressed as

\[ n_B(k_0) = (2\pi)^3 n_0 \delta^3(k) \] \hspace{1cm} (28)

where \( n_0 \) is the particle density in the condensate medium, with zero momentum. Then putting eq.(28) in eq.(4) we get

\[ T_P = -g_p^2 \left( \frac{p_0^2 - p^2 - p_0 m_\phi}{p_0^2 - p^2 + m_\phi^2 - 2p_0 m_\phi - m^2} \right) \frac{n_0}{2m_\phi} \] \hspace{1cm} (29)

and

\[ T_U = -g_p^2 \left( \frac{p_0 - w}{p_0^2 - p^2 + m_\phi^2 - 2p_0 m_\phi - m^2} \right) \frac{n_0}{2m_\phi}. \] \hspace{1cm} (30)

Then putting the values of \( T_P \) and \( T_U \) in eq.(5) and (11) we obtain for the BEC medium

\[ a = \frac{C_P}{(p_0 - m_\phi)^2 - p^2 - m^2} \] \hspace{1cm} (31)

and

\[ b = -a m_\phi, \] \hspace{1cm} (32)

where \( C_P = g_p^2 n_0 / 2m_\phi \). For BEC medium \( b \) is proportional to the pseudo-scalar mass whereas for Bose gas it is proportional to the energy of the propagating fermion as shown in eq. (23). Putting the value of \( b \) we obtain

\[ (p_0(1 + a) - a m_\phi)^2 = m^2 + (1 + a)^2 p^2. \] \hspace{1cm} (33)

Keeping terms up to order \( a \) the dispersion relation will be

\[ p_0^2 - p^2 = m^2 + \frac{2C_p m_\phi p_0}{(p_0 - p^2 - m^2) + m_\phi^2 - 2p_0 m_\phi}. \] \hspace{1cm} (34)

Writing \( p_0^2 - p^2 - m^2 = \delta m^2 \), we get

\[ \delta m^2 = \frac{2C_p (m_\phi p_0 - m^2)}{\delta m^2 + (m_\phi^2 - 2p_0 m_\phi)}. \] \hspace{1cm} (35)

It is a quadratic equation in \( \delta m^2 \), so \( \delta m^2 \) has two solutions. But the physical solution is the one, which goes to zero for \( C_p \) goes to zero. So the value of \( \delta m^2 \) is given by

\[ \delta m^2 = -\frac{2C_p (p_0 - m_\phi^2)}{2p_0 - m_\phi}. \] \hspace{1cm} (36)

For \( p_0 \gg m_\phi \) the above relation will be

\[ \delta m^2 = -\frac{C_p}{p_0} \left( p_0 - \frac{m_\phi^2}{2p_0} - \frac{m_\phi^2 + m_\phi}{2} \right) \] \hspace{1cm} (37)

Now the dispersion relation for the fermion propagating in the BEC medium is

\[ p_0^2 - p^2 = m^2 - \frac{g_p^2 n_0}{2m_\phi} \left( 1 - \frac{m_\phi^2}{m_\phi p_0} - \frac{m_\phi^2}{2p_0} + \frac{m_\phi^2}{2p_0} \right) \] \hspace{1cm} (38)

where \( n_0 \) is the number density of the pseudo-scalar particles in the medium. Comparison of eq.(7) with eq.(38) shows that, we do not have a pure BEC medium, because the second term in the right hand side of eq.(38) i.e. \( g_p^2 n_0 / 2m_\phi \) comes from the Bose gas contribution. Again the eq.(38) clearly shows that, for a massless fermion propagating in the BEC medium, the particle will develop a negative mass square value of magnitude \( g_p^2 n_0 (1 + m_\phi / 2p_0) / 2m_\phi \) and will be absorbed in the medium. Now let us compare the contributions of different terms (within the bracket of eq.(38)). For high energy particle \( p_0 \gg m_\phi \), the term \( m_\phi / 2p_0 \) almost does not contribute. For particle with \( p_0 \gg m_\phi \) the term \( m_\phi^2 / 2p_0 \) is also small. So only the first and the second terms with in the bracket are contributing. Thus we have

\[ p_0^2 - p^2 \approx m^2 - \frac{g_p^2 n_0}{2m_\phi} \left( 1 - \frac{m_\phi^2}{m_\phi p_0} \right). \] \hspace{1cm} (39)

Let us consider different situations, which might arise depending on the mass and energy of the propagating particle. For massive fermions if \( m_\phi^2 / m_\phi \gg p_0 \) condition is satisfied then the particle mass will be increased by an amount \( g_p^2 n_0 m^2 / 2m_\phi^2 p_0 \) and the BEC contribution will dominate over the Bose gas one. On the contrary if \( m_\phi^2 / m_\phi << p_0 \), then, the particle mass will be reduced by an amount \( g_p^2 n_0 / 2m_\phi \) which is purely due to the Bose gas nature of the medium and BEC contribution is negligible. For \( p_0 \approx m^2 / m_\phi \), the particle will propagate like a free particle in the BEC medium without loosing any energy.
IV. COSMIC RAYS PROPAGATION

Active galactic nuclei are found to be the origin of the highest energy cosmic rays, like photons, protons, electrons and neutrinos [10]. The recent discovery of cosmic rays events above Greisen-Zatsepin-Kuzmin (GZK) cut-off ($\sim 5 \times 10^{19} \text{eV}$) [7] by the AGASA [11], Fly’s Eye [12], Haverah Park [13] and Yakutsk collaborations [14] is an outstanding puzzle in cosmic rays physics. If these cosmic rays are protons with energy more than GZK cut-off then, interaction with the cosmic microwave background radiation (CMBR), would cause rapid loss of energy by photopion production and consequently depletion of the observed flux of these particles [8]. For every mean free path $\sim 6 \text{Mpc}$ of travel, the proton loses 20% of its energy on average [15]. Since AGNs are hundreds of megaparsecs away, the energy requirement for a proton which arrives at earth with a super GZK energy is extremely high. Presently we do not have a reliable theoretical model to explain the origin and propagation of such high energy cosmic rays in the galactic medium. On the other hand ultra-high energy neutrinos will escape first and bring us first-hand information regarding the source. Also these AGN neutrinos can be used to study the properties of neutrinos themselves [10].

Here we will see how the propagation of these ultra-high energy cosmic rays (protons, electrons and neutrinos) are affected by axion and majoron dark matter in the galactic halos.

A. Bose gas

Let us consider ultra-high energy cosmic rays proton propagation in the axion gas. Then for proton-axion coupling $g_p \sim 10^{-10}$, $m_a = m_a \approx 10^{-3} \text{eV}$ [10] and axion number density $n_0 \approx 3 \times 10^{13}/\text{cm}^3$ we obtain $g_p^2 n_0/2m_a \approx 1.2 \times 10^{-18} \text{eV}^2$ and this is much smaller than proton mass square $m_p^2$. Also this term is much smaller than electron mass square $m_e^2$ (we assume the electron-axion coupling same as the proton-axion coupling). So the axion gas has no effect on the propagation of the cosmic rays protons and/or electrons. Secondly let us consider the neutrino propagation in the majoron gas then, taking neutrino-majoron coupling $g_\nu \sim 10^{-4}$, $m_J \approx 1 \text{KeV}$ [10] and the majoron density same as the axion density; we obtain $g_\nu^2 n_0/2m_J \approx 1.2 \times 10^{-12} \text{eV}^2$. For non-zero neutrino mass the majoron gas has also no effect on the neutrino propagation.

B. BEC medium

For proton propagating in the axion Bose condensate, we have $m_p^2/m_a \approx 10^{21} \text{eV}$ which is above the GZK cut-off. For proton energy $p_0 << 10^{21} \text{eV}$ the proton mass will increase by an amount $g_p^2 n_0 m_a^2/(2p_0 m_p^2)$. Similarly for electron propagating in the BEC of axion, we have $m_e^2/m_a \approx 0.25 \times 10^{15} \text{eV}$. In this case also for electron energy $p_0 < 0.25 \times 10^{15} \text{eV}$ the electron mass will increase in the medium and for $p_0 >> 0.25 \times 10^{15} \text{eV}$ the mass will decrease. If $m_e^2/m_a p_0 \approx 1$, then the Bose gas contribution will be cancelled by the BEC contribution and the proton will propagate like a free particle in the vacuum, without loosing its energy.

For neutrino propagating in the majoron condensate medium, we have $m_\nu^2/m_J \approx 10^{-1} \text{eV}$, for taking neutrino mass $m_\nu = 10^{-6} \text{eV}$ and $m_J = 10^{3} \text{eV}$. The cosmic ray neutrinos have energy as high as $10^{21} \text{eV}$. So $m_\nu^2/m_J p_0 \approx 10^{-22}$; hence only the majoron Bose gas will contribute for the neutrino mass and the neutrino mass term in eq. (38) will be $(m^2 - g_\nu^2 n_0/2m_J)$. This reduction in neutrino mass is very small to account for.

V. CONCLUSIONS

We have calculated the dispersion relation for the fermion propagating in the Bose gas and the BEC of the pseudo-scalar particles. It shows that, if the fermion is massless then, it will develop an imaginary mass both in the Bose gas and in BEC medium, which is proportional to the number density of the bosons (Bose condensate) in the medium. For cosmic rays of energy below $m^2/m_\phi$, the mass of the fermion will increase, on the other hand for energy above $m^2/m_\phi$ the mass will decrease. Interesting situation arises, when $m^2/m_\phi p_0 \approx 1$. In this situation the Bose gas contribution will be cancelled by the BEC contribution and the particle will propagate freely without loosing its energy. But we found that the contribution of the axion gas or BEC of axion to proton mass is very small. Similarly majoron has also very small contribution to the neutrino mass. This small contributions are solely due to the low number density of these particles in the galactic halo.

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