Single-quark spin asymmetries in $e^+e^- \to t\bar{t}$ and anomalous gluon couplings

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Abstract

The effect of anomalous chromoelectric couplings of the gluon to the top quark are considered in $e^+e^- \to t\bar{t}$. The total cross section, as well as $t$ and $\bar{t}$ polarizations are calculated to order $\alpha_s$ in the presence of the anomalous couplings. One of the two linear combinations of $t$ and $\bar{t}$ polarizations is CP even, while the other is CP odd. Limits that could be obtained at a future linear collider on CP-odd combinations of anomalous couplings are determined.
The discovery of a heavy top quark, with a mass of \( m_t = 175 \pm 6 \text{ GeV} \) \cite{1}, which is far larger than that of all other quarks, opens up the possibility that the top quark may have properties very different from those of the other quarks. Observation of these properties might even signal new physics beyond the standard model. Several efforts in the past few years have gone into the investigation of the potential of different experiments to study possible new interactions of the top quark. In particular, possible anomalous couplings of the top quark to electroweak gauge bosons \cite{1} and to gluons \cite{3, 4} have also been discussed. Top polarization is especially useful in such studies, because with a mass around 175 GeV, the top quark decays before it can hadronize \cite{5}, and all spin information is preserved in the decay distributions.

In this paper, we investigate the potential of \( e^+ e^- \) experiments at a future linear collider with centre-of-mass (c.m.) energies of 500 GeV or higher, to study the CP-violating anomalous chromoelectric dipole couplings of the top quark to gluons. So far, a considerable amount of earlier work on the topic of anomalous gluon couplings has concentrated on hadron colliders. But also high energy \( e^+ e^- \) experiments with sufficiently high luminosities would provide a relatively clean environment to probe the standard model for anomalous gluon couplings. While earlier efforts in the context of \( e^+ e^- \) colliders are mainly based on an analysis of the gluon distribution \cite{4} in \( e^+ e^- \rightarrow \bar{t}t g \), we look at the possible information that could be obtained from studying the total cross section, and the polarization of \( t \) and \( \bar{t} \) separately. This has the advantage over \( t \) and \( \bar{t} \) spin correlations (as for example studied in \cite{6}) that, because the polarization of only one of \( t \) and \( \bar{t} \) is analyzed by means of a definite decay channel, the other is free to decay into any channel. This leads to much better statistics compared to the case when \( t - \bar{t} \) spin correlations are considered, where definite \( t \) and \( \bar{t} \) channels have to be used as analyzers.

We find that of the three independent quantities, viz., the cross section, and the \( t \) and \( \bar{t} \) polarizations, one quantity, viz., a linear combination of the \( t \) and \( \bar{t} \) polarizations can be used to probe CP-odd chromoelectric dipole coupling.

\footnote{References to the voluminous literature on this subject can be found, for example, in \cite{3}.}
We have also considered the effect of beam polarization on the sensitivity of the measurements.

An effective $t\bar{t}g$ interaction can be written in the form

$$\mathcal{L}_{t\bar{t}g} = -g_s \left[ 7\gamma^\mu G_\mu t + \frac{\mu}{2m_t} T\sigma^{\mu\nu} G_{\mu\nu} t + \frac{id}{2m_t} 7\sigma^{\mu\nu} \gamma_5 G_{\mu\nu} t \right],$$

(1)

where

$$G_\mu = \sum_a G^a_\mu T_a; \quad T^a = \frac{1}{2}\lambda^a; \quad G_{\mu\nu} = G^a_{\mu\nu}; \quad G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu;$$

(2)

$G^a_\mu$ being the gluon field, and $\lambda^a$ being the SU(3) Gell-Mann matrices. This is the most general Lorentz- and colour-invariant trilinear interaction (additional quadrilinear terms are needed for local colour invariance, but we do not need them here). The $\mu$ and $d$ terms are the chromomagnetic and chromoelectric dipole terms, respectively. In an effective theory, these are in fact momentum-dependent form factors, and complex in general.

We use Eq. (1) to calculate the $t\bar{t}$ total cross section and the $t$ and $\bar{t}$ polarizations to order $\alpha_s \equiv g_s^2/(4\pi)$. The extra diagrams contributing to this order are shown in Fig. 1, where the large dots represent anomalous couplings. The anomalous couplings enter in the amplitudes for soft and collinear gluon emissions. The infrared divergences in the amplitudes for soft gluon emission and in the virtual gluon corrections cancel as in standard QCD, since the anomalous terms vanish in the infrared limit. Moreover, we do not include anomalous couplings in the loop diagrams. The latter are included merely to regulate the infrared divergences. We thus study how anomalous couplings at tree-level would modify standard QCD predictions at order $\alpha_s$.

For heavy-quark production, the standard QCD one-loop corrections to the total cross section and the longitudinal spin polarization \cite{7} were calculated in closed analytic form before. Those results have been extended here to the case when anomalous couplings are present.

The total unpolarized production rate is given only in terms of the $VV$ and $AA$ parity-parity combinations for the Born contributions:

$$\sigma_{\text{Born}} \left( e^+ e^- \rightarrow \gamma, Z \rightarrow q\bar{q} \right) = \frac{1}{2} v(3 - v^2) \sigma^{VV} + v^3 \sigma^{AA},$$

(3)
where the mass parameters are $v = \sqrt{1 - \xi}$ and $\xi = 4m_q^2/s$. The $O(\alpha_s)$ unpolarized case has the cross section

$$\sigma \left( e^+ e^- \to \gamma, Z \to q\bar{q} \right) = \frac{1}{2}v(3 - v^2)\sigma^{VV}e^{VV} + v^3\sigma^{AA}c^{AA},$$  \hspace{1cm} (4)$$

where the $VV$ and $AA$ factors that multiply with the appropriate Born terms are given below.

$$c^{VV} = 1 + \frac{\alpha_s}{2\pi}C_F \left[ \tilde{\Gamma} - v \frac{\xi}{2 + \xi} \ln \left( \frac{1 + v}{1 - v} \right) - \frac{4}{v}I_2 - \frac{\xi}{v}I_3 + \frac{4}{v(2 + \xi)} - \frac{2 - \xi}{v}I_5 + \Delta_0^{VV} \right].$$ \hspace{1cm} (5)$$

In this equation, the contribution from the virtual gluon loop is denoted as

$$\tilde{\Gamma} = \left[ 2 - \frac{1 + v^2}{v} \ln \left( \frac{1 + v}{1 - v} \right) \right] \ln \left( \frac{1 + v}{v} \right) + \frac{1 + v^2}{v} \left[ \text{Li}_2 \left( -\frac{2v}{1 - v} \right) - \xi \text{Li}_2 \left( \frac{2v}{1 + v} \right) + \pi^2 \right]$$

$$+ 3v \ln \left( \frac{1 + v}{1 - v} \right) - 4.$$ \hspace{1cm} (6)$$

Here, the $q\bar{q}g$ phase-space integrals are abbreviated by $I_i$, and $\tilde{I}_i$ specify the results after the (soft) IR divergences have canceled. The explicit analytical expressions for these phase-space integrals may be found in [8]. The additional component stemming from the anomalous gluon bremsstrahlung is given by

$$\Delta_0^{VV} = \frac{8}{(2 + \xi)v} \left[ \text{Re}(\mu)(I_1 + I_4) + \frac{2}{\xi} \left( |\mu|^2 + |d|^2 \right) (I_1 - 2I_8) \right].$$ \hspace{1cm} (7)$$

For the $AA$ contribution we find:

$$c^{AA} = 1 + \frac{\alpha_s}{2\pi}C_F \left[ \tilde{\Gamma} + \frac{2}{v^3} \ln \left( \frac{1 + v}{1 - v} \right) + \frac{\xi}{v^3}I_1 - \frac{4}{v}I_2 - \frac{\xi}{v^3}I_3 - \frac{2 + \xi}{v}I_4 - \frac{2 - \xi}{v}I_5 + \Delta_0^{AA} \right],$$ \hspace{1cm} (8)$$

with the following anomalous part

$$\Delta_0^{AA} = \frac{2}{v^3} \left[ \text{Re}(\mu) \left\{ - (4 - \xi)I_1 + (2 + \xi)I_4 \right\} + \left( |\mu|^2 + |d|^2 \right) \left\{ \left( \frac{4}{\xi} + \xi - 6 \right) I_1 + \xi I_4 \right\} \right]$$

$$- \frac{4}{\xi} (2 - \xi)I_8 + \frac{4}{\xi}I_9 \right].$$ \hspace{1cm} (9)$$

The remaining $VA$ and $AV$ parts are identical and only contribute to the spin-dependent cross section. In the absence of anomalous couplings ($\mu = d = 0$), the cross section for
longitudinally polarized quarks of helicity $\pm \frac{1}{2}$ is given by

$$\sigma \left( e^+ e^- \rightarrow \gamma, Z \rightarrow q(\lambda_{\pm}) \bar{q} \right) = \frac{1}{2} v (3 - v^2) \sigma^{VV} c^{VV} + v^3 \sigma^{AA} c^{AA} \pm v^2 \sigma^{VA} c^{VA}. \quad (10)$$

The multiplication factors $c^{ij}$ are expressed in terms of phase-space integrals of type $S_i$:

$$c^{VA, AV}_{\pm} = 1 + \frac{\alpha_s}{2 \pi} C_F \left[ \bar{\Gamma} + \frac{\xi}{v} \ln \left( \frac{1 + v}{1 - v} \right) + \Delta^{VA, AV}_{\mu=d=0} \right], \quad (11)$$

where

$$\Delta^{VA, AV}_{\mu=d=0} = \frac{1}{2} \left[ (4 - \xi) S_1 - (4 - 5 \xi) S_2 - 2(4 - 3 \xi) S_4 - \xi (1 - \xi) (\bar{S}_3 + \bar{S}_5) + \xi (S_6 - S_7) - 2 S_8 + (2 - \xi) S_9 + (6 - \xi) S_{10} - 2 S_{11} + 2(1 - \xi) (2 - \xi) S_{12} \right]. \quad (12)$$

The full analytic forms for the $S$ integrals are too lengthy to be exhibited here. Most of them are compiled in Ref. [8], except for the four additional integrals:

$$S_{14} = \frac{2}{\xi} - \frac{2 + \sqrt{\xi}}{2(2 - \sqrt{\xi})} - \ln(2 - \sqrt{\xi}) + \frac{1}{2} \ln \xi - \frac{1}{2}, \quad (13)$$

$$S_{15} = \frac{1}{32} \xi^3 \left[ \frac{1}{2} \ln \xi - \ln(2 - \sqrt{\xi}) \right] - \sqrt{\xi} \left( 4 + \frac{1}{2} \xi + \frac{1}{4} \xi^2 \right) + \frac{1}{8} \left( 7 - \frac{1}{2} \xi \right) \xi + \frac{1}{3}, \quad (14)$$

$$S_{16} = -\frac{\xi^4 (4 - \xi)}{512 (2 - \sqrt{\xi})^2} + \frac{1}{16} \left( \frac{3}{16} \xi - 1 \right) \ln(2 - \sqrt{\xi}) - \frac{3}{8} \xi \left( 1 + \frac{3}{8} \xi \right) + \frac{1}{128} \xi^3 \left[ 4 - \xi + (16 - 3 \xi) \ln \xi \right] \frac{1}{4} \left( \frac{7}{3} - \frac{1}{2} \xi + \frac{1}{16} \xi^2 \right) + \frac{1}{24} \xi^3 \right] \quad (15)$$

$$S_{17} = \frac{1}{32} \xi^3 \left[ (6 - \xi) \left[ -\frac{1}{2} \ln \xi + \ln(2 - \sqrt{\xi}) \right] + \frac{1}{2} \xi \left( 4 - \xi \right) \right] \frac{\xi^3}{(2 - \sqrt{\xi})^2} \quad (16)$$

Including spins for the quark or the antiquark introduces additional spin-flip terms in the $O(\alpha_s)$ $c$ factors given in Eqs. [8], [8] and [11]. For longitudinal quark polarization we find

$$c^{ij}_{\pm} = \frac{1}{2} \left[ c^{ij} \pm \frac{\alpha_s}{2 \pi} C_F \Delta^{ij}_{S} \right]. \quad (17)$$

The individual parity-parity combinations are

$$(2 + \xi) v \Delta^{VV}_{S} = 8 \text{Im} (\mu^d) \left[ -2 \left( 1 - \frac{2}{\xi} \right) S_1 + \left( 1 - \frac{4}{\xi} \right) S_8 - S_9 - S_{10} + S_{11} + 2 \left( 1 - \frac{4}{\xi} \right) S_{13} + \frac{4}{\xi} (S_{15} + S_{17}) \right]$$
\[ + \text{Im}(d) \left[ 8(1 - \xi)S_1 - \left( 8 - 3\xi(2 - \xi) \right)S_2 + \left( 8 + \xi(2 - 3\xi) \right)S_4 \\
-\xi(2 + \xi)S_6 - 4S_8 + 4(1 - \xi)S_9 - 4(3 + \xi)S_{10} + 4S_{11} \\
+ \xi(2 - \xi)S_{14} \right], \quad (18) \]

\[ v^3 \Delta_{S}^{AA} = 4 \text{Im}(\mu^*d) \left[ \frac{2}{\xi}(1 - \xi)(2 - \xi)S_1 + \left( 5 - \frac{4}{\xi} \right)S_8 - (1 - \xi)(S_9 + S_{10}) + S_{11} \\
+ 2 \left( 3 - \frac{4}{\xi} \right)S_13 - 2 \left( 1 - \frac{2}{\xi} \right)S_{15} - \frac{4}{\xi}S_{16} - 2 \left( 1 - \frac{4}{\xi} \right)S_{17} \right] \\
+ \text{Im}(d) \left[ 2\xi S_1 - (1 - \xi)(4 - \xi)(S_2 - S_4) - \xi(1 - \xi)S_6 - (2 - \xi)S_8 \\
+(2 - 3\xi)S_9 - (6 - 5\xi)S_{10} + (2 + \xi)(S_{11} + 2S_{13}) + \xi(1 - \xi)S_{14} \right], \quad (19) \]

\[ v^2 \Delta_{S}^{VA,AV} = \text{Re}(\mu) \left[ - \xi(S_1 + S_7) - 2S_8 + (2 - \xi)(S_9 + S_{10}) - 2S_{11} \right] \\
\pm 2i \text{Im}(\mu) \left[ (2 - \xi)S_1 - \xi S_{10} - 2S_{13} \right] \\
+ \left( |\mu|^2 + |d|^2 \right) \left[ - (4 - \xi)S_1 - \xi S_7 + 4S_8 + \xi S_9 + (4 - \xi)S_{10} - 4S_{11} \right]. \quad (20) \]

Using charge conjugation in the final state, one can readily obtain the corresponding expressions for (longitudinal) antiquark polarization. In the following, we denote the antiquark results by an additional bar, \( \bar{\sigma}_{ij} \):

\[ \bar{\Delta}_{S}^{VV} = \Delta_{S}^{VV}, \quad (21) \]
\[ \bar{\Delta}_{S}^{AA} = \Delta_{S}^{AA}, \quad (22) \]

where the following identities hold

\[ \bar{\Delta}_{S}^{VA} = -\Delta_{S}^{VA} = (\bar{\Delta}_{S}^{AV})^*, \quad (23) \]

with \( \bar{\Delta}_{S}^{AV} = -\Delta_{S}^{AV} \). \quad (24)

Considering the above expressions, we can construct the following combinations of polarization asymmetries of \( t \) and \( \bar{t} \),

\[ \Delta\sigma^{(+)} = \frac{1}{2} \left[ \sigma(\uparrow) - \sigma(\downarrow) - \bar{\sigma}(\uparrow) + \bar{\sigma}(\downarrow) \right], \quad (25) \]
\[ \Delta \sigma^{(-)} = \frac{1}{2} \left[ \sigma(\uparrow) - \sigma(\downarrow) + \bar{\sigma}(\uparrow) - \bar{\sigma}(\downarrow) \right], \] (26)

where \( \sigma(\uparrow) \), \( \sigma(\uparrow) \) refer respectively to the cross sections for top and antitop with positive helicity, and \( \sigma(\downarrow) \), \( \sigma(\downarrow) \) are the same quantities with negative helicity. Of these, \( \Delta \sigma^{(+)} \) is CP even and \( \Delta \sigma^{(-)} \) is CP odd. This is obvious from the fact that under C, \( \sigma \) and \( \bar{\sigma} \) get interchanged, while under P, the helicities of both \( t \) and \( \bar{t} \) get flipped. Consequently, \( \sigma \) and \( \Delta \sigma^{(+)} \), both nonzero in standard QCD, receive contributions from combinations of anomalous couplings which are CP even, viz., \( \text{Im}(\mu^2 + |d|^2) \) and \( \text{Re}(\mu) \). On the other hand, \( \Delta \sigma^{(-)} \) vanishes in standard QCD, and in the presence of anomalous couplings it depends only on the CP-odd variables \( \text{Im}(\mu^*d) \) and \( \text{Im}(d) \). That \( \Delta \sigma^{(-)} \) depends on the imaginary parts rather than the real parts of a combination of couplings follows from the fact that it is even under naive time reversal \( T_N \), i.e., reversal of all momenta, without change in helicities, and without interchange of initial and final states (as would have been required by genuine time reversal). As a consequence, it is odd under \( CPT_N \), and imaginary parts of couplings have to appear in order to avoid conflict with the CPT theorem.

We concentrate on the CP-odd combination of top and antitop polarizations, since the CP-even combination would get contributions from higher-order QCD as well any new interactions beyond the standard model. The CP-violating contribution cannot get nonzero contribution from QCD in any order, and therefore a nonzero chromoelectric moment would signal new physics beyond the standard model.

Fig. 2a shows a three-dimensional plot of \( \Delta \sigma^{(-)} \) as a function of \( \text{Im}(\mu^*d) \) and \( \text{Im}(d) \) at \( \sqrt{s} = 500 \text{ GeV} \). Fig. 2b shows a similar plot for \( \sqrt{s} = 1000 \text{ GeV} \).

We use our expressions to obtain simultaneous 90\% confidence level (CL) limits that could be obtained at a future linear collider with an integrated luminosity of 50 \( \text{fb}^{-1} \). We do this by equating the magnitude of the difference between the values for a quantity with and without anomalous couplings to 2.15 times the statistical error expected. Thus, the limiting values of \( \text{Im}(\mu^*d) \) and \( \text{Im}(d) \) for an integrated luminosity \( L \) and a top detection efficiency of \( \epsilon \) are obtained from

\[
\epsilon L \left| \Delta \sigma^{(-)}(\mu, d) - \Delta \sigma^{(-)}_{\text{SM}} \right| = 2.15 \sqrt{L \left| \sigma_{\text{SM}}(\uparrow) + \bar{\sigma}_{\text{SM}}(\uparrow) \right|}. \] (27)
In the above expressions, the subscript “SM” denotes the value expected in the standard model, with \( \mu = d = 0 \). We use \( \epsilon = 0.1 \) in our numerical estimates. For the running of the strong coupling, we choose \( \alpha_s^{(5)}(M_Z) = 0.118 \) (with \( M_Z = 91.178 \) GeV) in the modified minimal subtraction scheme and use the appropriate conditions to match for six active flavours.

Eq. (27) is used to obtain contours in the plane of \( \text{Im}(\mu^*d) \) and \( \text{Im}(d) \), shown in Figs. 3a and 3b for \( \sqrt{s} = 500 \) GeV and 1000 GeV, respectively. The contours are presented for different \( e^- \) longitudinal beam polarizations \( P_- \). In Fig. 3, the allowed regions are the bands lying between the upper and lower straight lines.

A conclusion that can be drawn from Fig. 3 is that a large left-handed polarization leads to increase in sensitivity.

For a fixed energy, there is only one CP-violating quantity which can be measured, viz., \( \Delta \sigma^{(-)} \). This cannot give independent limits on the two quantities \( \text{Im}(\mu^*d) \) and \( \text{Im}(d) \). However, if a measurement of \( \Delta \sigma^{(-)} \) is made at two c.m. energies, a relatively narrow allowed range can be obtained, allowing independent limits to be placed on both \( \text{Im}(\mu^*d) \) and \( \text{Im}(d) \). This is demonstrated in Fig. 4. In fact, the improvement in the limit on \( \text{Im}(\mu^*d) \) in going from \( \sqrt{s} = 500 \) GeV to \( \sqrt{s} = 1000 \) GeV is considerable. The possible limits are

\[
-0.8 < \text{Im}(\mu^*d) < 0.8, \quad -11 < \text{Im}(d) < 11. \tag{28}
\]

It is customary to use units of \( e \text{ cm}^{-1} \) for the electric dipole moment. In our case of the chromoelectric moment, we can express the above limits in terms of an analogous unit, viz., \( g_s \text{ cm}^{-1} \):

\[
|\text{Im}(\mu^*d)| < 9.2 \times 10^{-17} g_s \text{ cm}^{-1}, \quad |\text{Im}(d)| < 1.3 \times 10^{-15} g_s \text{ cm}^{-1}. \tag{29}
\]

These limits may be compared with the limits obtainable from gluon jet energy distribution in \( e^+e^- \rightarrow t\bar{t}g \). While our proposal for the CP-even case seems to fare worse, for

\begin{footnote}
\text{It is common to indicate the number of active flavours as superscript of } \alpha_s. \text{ For practical purposes, one usually selects bottom production as reference. Here, our choice for } \alpha_s^{(5)}(M_Z) \text{ translates to } \alpha_s^{(6)}(M_t = 172.1 \text{GeV}) = 0.10811
\end{footnote}
the CP-odd case, our proposal can be competitive. It should however be emphasized that in the case of the CP-odd couplings, we are proposing the measurement of a genuinely CP-violating quantity, whereas the analysis in [4] is merely based on the energy spectrum resulting from both CP-odd and CP-even couplings. In case of $\Delta \sigma^{(-)}$, the dependence on $e^-$ beam polarization is rather mild.

We should also emphasize that we have taken the same integrated luminosity, viz. 50 pb$^{-1}$, for c.m. energies of 500 GeV and 1000 GeV. It is usually assumed that linear colliders operating at higher energies would simultaneously increase their luminosities to off-set the drop in cross sections with energy. With a higher integrated luminosity for 1000 GeV, our limits would be considerably better than what we have obtained above with a somewhat conservative approach.

It is worthwhile noting that we have used a rather moderate value of $\epsilon = 0.1$ for top detection and polarization analysis. A better efficiency would lead to an improvement in the limits, as would a higher overall luminosity.

We have not considered the effect of initial-state radiation in this work. We have also ignored possible effects of collinear gluon emission from one of the decay products of $t$ or $\bar{t}$. A complete analysis should indeed incorporate these effects, as well as a study of $t$ and $\bar{t}$ decay distributions which can be used to measure the polarizations. However, we do not expect our conclusions to change drastically when these effects are taken into account.

In summary, we have examined the capability of single quark polarization in $e^+e^- \rightarrow t\bar{t}$ to measure or put limits on the top chromoelectric dipole coupling. The CP-violating combination of top and antitop polarizations is sensitive to the anomalous chromoelectric coupling, and can yield a limit of the order of 1 on a CP-odd combination of anomalous couplings.
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Figure Captions

Fig. 1: Additional Feynman diagrams contributing to $\sigma(e^+e^- \rightarrow \gamma, Z \rightarrow t\bar{t})$ that account for anomalous gluon couplings at $O(\alpha_s)$. The large dots represent anomalous $t\bar{t}g$ insertions according to the effective action Eq. (1).

Fig. 2: Surface plots displaying the dependence of the polarization asymmetry $\Delta\sigma^{(-)}$ on $\text{Im}(\mu^*d)$ and $\text{Im}(d)$ with initial electron beam polarization $P_- = -1$ and c.m. energies (a) $\sqrt{s} = 500$ GeV, (b) $\sqrt{s} = 1000$ GeV.

Fig. 3: Contour plots showing the allowed regions for $\Delta\sigma^{(-)}$ with 90% confidence level (integrated luminosity $L = 50$ fb$^{-1}$ and top detection efficiency $\epsilon = 0.1$). Representative c.m. energies are (a) $\sqrt{s} = 500$ GeV and (b) $\sqrt{s} = 1000$ GeV for various longitudinal electron polarizations.

Fig. 4: Intersecting area resulting from two independent $\Delta\sigma^{(-)}$ measurements at $\sqrt{s} = 500$ GeV, 1000 GeV.
Figure 1
Figure 2a
Figure 2b

$\Delta \sigma^{(-)}$

$\sqrt{s} = 1000$ GeV

$P_{-} = -1$
Figure 3a
\[ \sqrt{s} = 1000 \text{ GeV} \]

Figure 3b
Figure 4