Phase resetting and intermittent control at the edge of stability in a simple biped model generates 1/f-like gait cycle variability

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Abstract
The 1/f-like gait cycle variability, characterized by temporal changes in stride-time intervals during steady-state human walking, is a well-documented gait characteristic. Such gait fractality is apparent in healthy young adults, but tends to disappear in the elderly and patients with neurological diseases. However, mechanisms that give rise to gait fractality have yet to be fully clarified. We aimed to provide novel insights into neuro-mechanical mechanisms of gait fractality, based on a numerical simulation model of biped walking. A previously developed heel-toe footed, seven-rigid-link biped model with human-like body parameters in the sagittal plane was implemented and expanded. It has been shown that the gait model, stabilized rigidly by means of impedance control with large values of proportional (P) and derivative (D) gains for a linear feedback controller, is destabilized only in a low-dimensional eigenspace, as P and D decrease below and even far below critical values. Such low-dimensional linear instability can be compensated by impulsive, phase-dependent actions of nonlinear controllers (phase resetting and intermittent controllers), leading to the flexible walking with joint impedance in the model being as small as that in humans. Here, we added white noise to the model to examine P-value-dependent stochastic dynamics of the model for small D-values. The simulation results demonstrated that introduction of the nonlinear controllers in the model determined the fractal features of gait for a wide range of the P-values, provided that the model operates near the edge of stability. In other words, neither the model stabilized only by pure impedance control even at the edge of linear stability, nor the model stabilized by specific nonlinear controllers, but with P-values far inside the stability region, could induce gait fractality. Although only limited types of controllers were examined, we suggest that the impulsive nonlinear controllers and criticality could be major mechanisms for the genesis of gait fractality.

Keywords Gait stability · Long-range correlation · Phase resetting · Intermittent control

1 Introduction

Gait cycle variability during steady-state human walking exhibits 1/f-like fluctuation, which is referred to as the gait fractality (Hausdorff 2007). That is, a power spectral density of stride intervals is characterized by a power-law function, i.e., \( f^{-\beta} \) as a function of frequency \( f \), where \( \beta \) is typically close to but less than unity. \( \beta \) for 1/f noise is related to the Hurst exponent \( H \) as \( \beta = 2H - 1 \) (Gao et al. 2006). The power-law spectrum is equivalent to an autocorrelation function with a power-law form, \( n^{-\beta} \) or equivalently \( n^{2H-2} \) (Gao et al. 2006) as a function of the lag \( n \) for the stride number, referred to as the long-range correlation (Höll and Kantz 2015). Monotonous decay of the autocorrelation only with positive values means statistical positive persistency (\( 1/2 < H < 1 \)). Fractality can be better characterized by a detrended fluctuation analysis (DFA) (Rangarajan and Ding 2000), where the size of detrended variations at time scales of \( n \) strides is represented as \( n^{\alpha} \) using the scaling exponent \( \alpha \), where \( \alpha = H \) for a fractional Gaussian noise (Gao et al. 2006). For the gait cycle variability in healthy young adults,
the scaling exponent $\alpha (= H)$ is close to unity (Pailhous and Bonnard 1992; Hausdorff et al. 1995; Frenkel-Toledo et al. 2005).

There is evidence that gait fractality is relevant for characterizing pathological and age-related alterations in gait (Hausdorff 2007). The exponent $\alpha$, which remains close to unity in healthy young adults, tends to approach 1/2 in elderly and patients with neurological diseases including Parkinson’s disease, which corresponds to uncorrelated white noise ($\beta = 2H - 1 = 2\alpha - 1 \sim 0$) (Hausdorff et al. 1997; Hausdorff 2009). Moreover, the scaling exponent $\alpha$ could discriminate fallers and non-fallers among patients with high-level gait disorder (Herman et al. 2005). However, despite the empirical importance as a biomarker for risk of falls (Hausdorff 2007), mechanisms of gait fractality have yet to be fully clarified. Moreover, there are fewer studies that relate gait fractality to gait stability (Bruijn et al. 2013). Some studies suggest that the central nervous system is involved in the regulation of long-range correlations, as peripheral sensory loss does not alter gait fractality (Gates and Dingwell 2007).

Note that the power-law features of the human gait cycle variability have been confirmed only for a limited range in a logarithmic scale, about two decades from ten to thousands of strides, due to a limited stride numbers acquired experimentally. Moreover, we should take into account that analytical methods of signal processing, such as DFA, for estimating the exponent $\alpha$ could induce false-positive outcomes (Willson et al. 2002; Dingwell and Cusumano 2010). Thus, it is still controversial as to whether the slow decay in the correlation of stride intervals is rigorously power-law, or it is just seemingly a power-law, and a slow exponential decay, in the form of $\lambda^n$ with $|\lambda|$ close to unity, is erroneously judged as power-law (Gao et al. 2006). However, because the current study did not aim to discriminate true power-law behaviors from false-positive phenomena, we chose to regard gait dynamics as fractal, if DFA-estimated $\alpha$-values were close to unity.

The aim of this study was to gain novel insights about neuro-mechanical mechanisms of gait fractality, based on a mathematical model of bipedal gait. Several types of dynamical models for explaining the fractal features of gait have been proposed. The simplest model assumes multiple independent oscillators with different periodicities, among which locomotive central pattern generators exhibit stochastic transitions (Hausdorff et al. 1995; Ashkenazy et al. 2002). Ahn and Hogan showed that non-chaotic dynamics of a simple biped model can exhibit gait fractality by tuning a model’s parameter for achieving moderate stability (Ahn and Hogan 2013). Dingwell and colleagues have actively investigated gait fractality and proposed theoretical models (Dingwell et al. 2010). A central idea of their model is the minimum intervention principle, implemented with a goal equivalent manifold (GEM). They focus on the kinematic redundancy of walking at a given constant speed $v$, considering that this can be achieved, in a stride-to-stride basis, by all the possible combinations of the $n$-th stride length $L_n$ and the corresponding stride interval $T_n$ with the constraint of $L_n = v T_n$. Then, they consider the one-dimensional linear GEM, $L_n - v T_n = 0$, in the state space of $x_n \equiv (T_n, L_n)$ for a simple lag-1 linear feedback control system with multiplicative and additive noise terms. If the state point $x_n$ is on the GEM, the control goal of $L_n - v T_n = 0$ is achieved with a null of error, $e_n = 0$. Minimizing a cost function representing the sum of errors from GEM and from an operating point $x^* = (T^*, L^*)$ and squared strengths of the feedback control input yields a linear optimal feedback controller. Since $e_n = 0$ is satisfied as far as $x_n$ is on the GEM, the optimal controller would naturally allow a relatively large variation of $x_n$ along the GEM, if a greater emphasis is given to reducing the error $e_n$ than the other costs. As a result, in this case, dynamics of the system with the optimal controller are determined by two eigenmodes; one is along the GEM, and the other spans a remaining dimension, where the eigenvalue $\lambda$ corresponding to the GEM-mode becomes close to unity, leading to a large variation along the GEM. It is important to note that, as in Ahn and Hogan (2013), the DFA-based scaling exponent $\alpha$ close to unity in the models by Dingwell et al. is achieved by a slow exponential decay at the edge of stability, meaning that $\alpha \sim 1$ in their models is generated in the absence of long-range correlations.

The current study was motivated by the hypothesis that the minimum intervention principle can be preserved, while assuring as well long-range correlation in gait cycle variability, by an implementation based on nonlinear and, particularly, impulsive controllers. An impulsive controller is indeed compatible with the minimum intervention principle, as it acts only during very brief periods of time. Taking this into account, the present study builds upon a previously developed heel-toe footed, seven-rigid-link bipedal model in the sagittal plane (Fu et al. 2014; Yamasaki et al. 2003a). Based on an analysis of the model by Floquet theory, we have shown previously that the modeled gait, stabilized rigidly by an impedance control scheme with large values of the proportional ($P$) and derivative ($D$) gain parameters of a conventional linear feedback controller, is destabilized only in a low-dimensional eigenspace (referred to as the unstable manifold), as $P$ and $D$ decrease below and even far below critical values (Fu et al. 2014). Such a low dimensionality of the unstable manifold might be realized by virtue of “evolutionary optimal” parameter values of body segments that affect stability of mechanical dynamics of the human body, leading to the reduction in the dimension of instability (the low-dimensional unstable manifold that needs to be taken care of for achieving stability) and the increase in the controllability (the remaining high-dimensional stable
eigenspace, referred to as the stable manifold, that does not need to be stabilized actively). The low-dimensional linear instability can be compensated relatively easily by impulsive, phase-dependent actions of nonlinear controllers, namely, the phase resetting controller (Yamasaki et al. 2003a, b) and the intermittent controller (Fu et al. 2014). Those mechanisms could lead to flexible walking with joint impedance in the model being as small as that in humans, because impulsive feedback actions are silent most of the time, and thus no deviations from the steady-state trajectory in the joint movement of lower extremities are actively opposed. Note that gait dynamics with small joint impedance, that is dominantly determined by small \( P \) and \( D \) gain values, are referred to as the joint flexibility in this article. See Fu et al. (2014) for a rigorous definition of joint impedance (with other factors that contribute to the joint impedance than \( P \) and \( D \) values) for the model. Here, with this circumstance, we added noisy joint torques of white Gaussian to the model for examining \( P \)-value-dependent stochastic dynamics of the model, for small fixed values of \( D \) for simplicity.

Besides an examination of gait fractality in the model with both the phase resetting and the intermittent controllers, we analyze whether stability region of the model with these two nonlinear controllers in the \( P \)-gain parameter space, because the model with both controllers simultaneously has not been analyzed yet, although gait stability of the model with either phase resetting controller (Yamasaki et al. 2003a, b) or intermittent controller (Fu et al. 2014) separately has been examined in our previous studies, which have shown substantial expansion of stability region by the use of either phase resetting controller or intermittent controller.

An additional motivation, with an expectation of the emergence of gait fractality for small \( P \)-values, stems from the experience in a similar situation in the study of postural sway during quiet stance (Bottaro et al. 2008; Asai et al. 2009; Suzuki et al. 2012). Those studies demonstrated indeed that joint flexibility and robust postural stability can be achieved simultaneously in the intermittent control model, in contrast with the performance of conventional models with linear impedance controllers (Nomura et al. 2013). Based on this expectation, we performed simulations of the gait model with noise. In order to elucidate possible mechanisms of gait fractality, we performed numerical simulations of the model stabilized by different sets of nonlinear feedback controllers, with a large range of parameter values of \( P \) for the linear controller, in the presence of joint torque noise. Then, we analyzed the relationship between gait stability and fractality in the model and determined the conditions for the genesis of fractal gait. Note that a clarification of conditions for gait fractality could also suggest possible mechanisms for a loss of gait fractality.

2 Methods

We used a previously developed gait model (Fu et al. 2014; Yamasaki et al. 2003a, b), which is based on a multi-link rigid body system, actuated by a feedforward (FF) controller and a conventional time-continuous proportional-derivative (PD) feedback controller for tracking a prescribed periodic joint angle profile, thus implementing an impedance control scheme. Note that we consider situations in which the phase of cyclic actions of the FF controller is modified impulsively in a feedback manner, as defined below. As a consequence, the FF component of the gait control model is modulated in a nonlinear feedback manner, contradicting the name with “feedforward” of this component. In this study, instability of the impedance control scheme that uses a linear proportional and derivative controller with small feedback-gain values of \( P \) and \( D \) is supplemented in different ways, by utilizing well-timed brief activations of two types of time-discontinuous, nonlinear feedback controllers that we proposed previously (Fu et al. 2014; Yamasaki et al. 2003a, b). One is the intermittent controller (INT), which is activated briefly only at a specific gait phase (Fu et al. 2014). The other is the phase resetting controller (PR) that compensates occurrences of early and late foot-contact events due to endogenous and/or exogenous perturbations. That is, the nominal joint-angle trajectory and the corresponding joint torques generated by the FF controller may be phase-advanced or phase-delayed impulsively in response to early or late occurrences of foot-contact events, respectively (Yamasaki et al. 2003a, b; Glass 2001; Schillings et al. 2000).

Variations of stride interval in the gait model studied here are implemented using different sources. The primary cause is the noisy joint torques, making a deterministically periodic gait stochastic. With such stochasticity, one of the mechanistic causes of variability is phase resetting, where the phase advance or delay in the periodic reference joint angle profiles and the phase of FF controller induce shorting and lengthening in the succeeding gait cycle, respectively. Another cause is a relaxation process, during which a state point of the model, perturbed stochastically from a steady-state limit cycle, returns back to the limit cycle. Foot-contact events during a relaxation process, even in the absence of noise and the phase resetting mechanism, would occur at earlier or later time compared to the ones during the steady-state periodic walking precisely on the limit cycle\(^1\). Intuitively speaking, if a relaxation occurs very quickly, deviations in the gait cycle from the periodic orbit decay to zero very quickly, result-

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\(^1\) Note that the trunk-inclination angle is not a part of the joint angle trajectories. One can imagine a walking toy-robot with a trunk and legs, whose joint angles change exactly in accordance with prescribed profiles. Even in this case, the toy-robot can exhibit a variety of gait dynamics, including falls, accompanied with different time-profiles in the inclination angle of the trunk of the robot.
ing in a variability with short memory. Alternatively, if it occurs slowly, a deviation in one gait cycle might affect the deviations in the succeeding cycles. Despite the apparent simplicity of such sources of variability of the gait cycle, their interactions are not trivial at all, and thus, computational simulations can provide an understanding of the nature of such dynamics.

In the simulations of the gait control model reported in the following, the FF and PD controllers are always employed, whereas the other nonlinear feedback controllers (i.e., INT and PR) are introduced in different combinations for various P-gains values of the PD controller. We focus our analysis particularly on small P and D values, where the combined action of the FF+PD controllers is insufficient to stabilize gait. In this section, the equations of motion of the gait model, the controllers of FF, PD, INT, and PR are briefly described. Possible criticisms of oversimplified characteristics of the controllers used in the model are faced in the discussion. For full details, see our previous work (Fu et al. 2014) was used in this study. The model with seven rigid-links in the sagittal plane from our previous work (Fu et al. 2014; Yamasaki et al. 2003a, b). The following section also outlines the DFA method, which was used to characterize fractality in different model implementations.

2.1 Equations of motion of the model

The model with seven rigid-links in the sagittal plane from our previous work (Fu et al. 2014) was used in this study. The general coordinates of the model is

\[ q = (q_1, q_2, \ldots, q_9)^T \equiv (\theta, x, y, \theta_a^l, \theta_k^l, \theta_a^r, \theta_k^r)^T, \quad \tag{1} \]

where \( \theta, x \) and \( y \) are the tilt angle, horizontal and vertical positions of the head-arm-trunk (HAT) center of mass (CoM). \( \theta_a^l, \theta_k^l, \) and \( \theta_a^r, \theta_k^r \) are the ankle, knee, and hip joint angles of the left \( (i = l) \) and right \( (i = r) \) limbs. The equation of motion is represented as

\[ J(q)\ddot{q} + B(q, \dot{q}) + K(q) + G(q, \dot{q}) = U, \quad \tag{2} \]

where \( J(q), B(q, \dot{q}), K(q) \) and \( G(q, \dot{q}) \) are the inertia matrix, centrifugal and Coriolis torque, gravitational torque and the ground reaction force, respectively. \( U = (0, 0, 0, U_a^l, U_k^l, U_a^r, U_k^r, U_h^l, U_h^r)^T \) on the right-hand side represents the joint torques acting on the six joints. \( G(q, \dot{q}) \) was modeled using a nonlinear spring and damper forces acting on the heel and the toe of either foot (Fu et al. 2014; Yamasaki et al. 2003a).

2.2 The desired joint angle profiles

Joint angles of the model are represented by six elements of the coordinate \( q \), and denoted by

\[ \hat{q} \equiv (q_4, \ldots, q_9)^T. \quad \tag{3} \]

Time profiles of the desired joint angles or the desired joint angle trajectories, denoted by \( \hat{q}^d(\phi) \) as a function of the gait phase \( \phi \) or \( \hat{q}^d(t) \) as a function of time \( t \), are defined for \( \hat{q} \) as

\[ \hat{q}^d(\phi) \equiv (q_4^d(\phi), \ldots, q_9^d(\phi))^T \quad \tag{4} \]

where the gait phase \( \phi \) takes a value in \( [0, T_c] \) with \( T_c = 1.135 \) (sec) being the gait period of the steady-state walking, or equivalently

\[ \hat{q}^d(t) \equiv (q_4^d(\text{mod}T_c(t + \phi_0)), \ldots, q_9^d((\text{mod}T_c(t + \phi_0))^T, \quad \tag{5} \]

for an appropriate initial phase \( \phi_0 \) of the desired joint angle trajectory, where \( \text{mod}T_c \) takes a modulo \( T_c \) of the argument. Note that if \( \phi \) and \( t \) are related as \( \phi = \phi(t) \equiv \text{mod}T_c(t + \phi_0) \). \( \hat{q}^d(\phi) \) or \( \hat{q}^d(t) \) was obtained as the Fourier expanded series of a human walking motion-captured data as detailed in Fu et al. (2014), Yamasaki et al. (2003a).

2.3 FF controller

A benefit of the model used here with the desired joint angle trajectory \( \hat{q}^d(\phi) \) is that the model can walk stably when the time courses of all of the six joints in Eq. (2) are kinematically constrained by \( \hat{q}^d(\phi) \). With those constraints, only three kinematic degrees of freedom \( (q_1, q_2, q_3) \) and their derivatives \( (\dot{q}_1, \dot{q}_2, \dot{q}_3) \) are remained as dynamic variables (Yamasaki et al. 2003a). A forward dynamic simulation of Eq. (2) with such kinematic constraints, after its transient dynamics, generates the corresponding time profiles of \( (q_1^s(\phi), q_2^s(\phi), q_3^s(\phi)) \). In this way, we obtained a kinematics of the model during the steady-state periodic gait as \( \ddot{q}(\phi) \equiv (q_4^s(\phi), q_5^s(\phi), q_3^s(\phi), q_4^d(\phi), \ldots, q_9^d(\phi))^T \), and the corresponding ground reaction force \( G(\ddot{q}(\phi), \dot{q}(\phi)) \). The inverse dynamics solution of Eq. (2) for the six joints was used as the outputs of the FF controller (Fu et al. 2014) as follows.

\[ U_{ff}(\phi) = J(\ddot{q}(\phi))\dot{q}(\phi) + B(\ddot{q}(\phi), \dot{q}(\phi)) + K(\ddot{q}(\phi)) + G(\ddot{q}(\phi), \dot{q}(\phi)). \quad \tag{6} \]

The equation of motion with the FF controller is formulated as

\[ J(q(t))\ddot{q}(t) + B(q(t), \dot{q}(t)) + K(q(t)) + G(q(t), \dot{q}(t)) = U_{ff}(\phi(t)). \quad \tag{7} \]
In this study, we considered the periodic trajectory of the state point defined as \( \phi(t) = (\theta(t), \dot{\theta}(t)) \) in the 18-dimensional state space of Eq. (7), referred to as \( \gamma \). By definition, \( \gamma \) is always a periodic solution of Eq. (7), regardless of its stability, although the FF controller alone cannot stabilize \( \gamma \) (Fu et al. 2014).

### 2.4 PD feedback controller (FF+PD).

As in Fu et al. (2014), the PD controller generates the feedback torques defined as

\[
U_{fb}(t) = P(\ddot{q}(t+\phi_0) - \dot{\phi}(t)) + D(\dddot{q}(t+\phi_0) - \ddot{\phi}(t)),
\]

or equivalently

\[
U_{fb}(\phi) = P(\ddot{q}(\phi) - \dot{\phi}(\phi)) + D(\dddot{q}(\phi) - \ddot{\phi}(\phi)),
\]

where

\[
P = \text{diag}(P_a, P_h, P_a, P_h),
\]

\[
D = \text{diag}(D_a, D_h, D_a, D_h).
\]

The equation of motion with the FF+PD controller is formulated as

\[
J(q)\ddot{q} + B(q, \dot{q}) + K(q) + G(q, \dot{q}) = U_{ff}(\phi) + U_{fb}(\phi).
\]

Because \( \gamma \) is a solution of Eq. (7), it is also a solution of Eq. (10). The FF+PD controller can stabilize the limit cycle \( \gamma \), if \( P \) and \( D \) are large enough, as in the typical impedance control. In this study, each element of \( D \)-vector was fixed commonly at small values, as \( D_a = D_h = 10 \) Nm/rad which are insufficient for stability of the model only with FF+PD controllers, throughout the study. This was simply for reducing the time necessary for exploring parameter-dependency of the model. \( P \)-gains were altered systematically in the range of \( P_h \in [0, 1000] \) Nm/rad, \( P_a \in [0, 1500] \) Nm/rad, and \( P_l \in [0, 1500] \) Nm/rad.

### 2.5 INT feedback controller (FF+PD+INT)

The INT feedback controller provides feedback torques only during brief period, lasting \( w = 0.1 \) second, after the heel contact within double support phase of the gait (Fu et al. 2014). It is used together with FF+PD controllers, typically when \( P \)-\( D \) gains of the PD controller are small and located outside the linear stability regions of the model only with FF+PD. Because the INT controller for each leg is activated only in the brief period within each gait cycle, it contributes to reducing the overall joint impedance.

The INT controller exploits the stable manifold \( W^s(\gamma) \) of unstable \( \gamma \) as the solution of Eq. (10) for small gains of PD controller, where \( W^s(\gamma) \) was obtained locally around \( \gamma \) using the monodromy matrix (a linearized Poincaré map) with its stable eigenvalues (Floquet multipliers with absolute values less than unity) and the corresponding eigenvectors (Fu et al. 2014). Let \( \gamma(\phi) \) be a state point on \( \gamma \) at phase \( \phi \), which means \( \gamma = \bigcup_{\phi} \gamma(\phi) \). Then, we can define \( W^s(\gamma(\phi)) \) as the intersection between \( W^s(\gamma) \) and a Poincaré section of \( \gamma \) passing through \( \gamma(\phi) \) for the linearized Poincaré map. The INT controller, when it is activated, aims to force the state point off the limit cycle \( \gamma \) at the phase \( \phi \) toward \( W^s(\gamma(\phi)) \), not directly to \( \gamma(\phi) \), using the feedback torque defined as

\[
U_{int}(\phi) = P^+(q_\text{ref}(\phi) - q) + D^+(\dot{q}_\text{ref}(\phi) - \dot{q}),
\]

where \((q_\text{ref}(\phi), \dot{q}_\text{ref}(\phi)) \in W^s(\gamma(\phi)) \) with the gains of \( P^+ \) and \( D^+ \) defined as

\[
P^+ = \text{diag}(P_{a^+}, P_{h^+}, P_{a^+}, P_{h^+}),
\]

\[
D^+ = \text{diag}(D_{a^+}, D_{h^+}, D_{a^+}, D_{h^+}).
\]

\( P_{\text{h}}^+D_{\text{h}}^+P_{\text{a}}^+D_{\text{a}}^+ \) and \( P_{\text{h}}^+D_{\text{h}}^+ \) are the gains of the INT feedback controller acting on the ankle, knee and hip joints, respectively, of the left and right limbs (Fu et al. 2014). \( U_{int} \) is supplemented to the right-hand side of Eq. (10) intermittently only during on-period defined as \([\phi_{on}L, \phi_{on}L + w]\) for \( w \) seconds. That is, the equation of motion with FF+PD+INT controllers is described as

\[
J(q)\ddot{q} + H(q, \dot{q}) = U_{ff}(\phi) + U_{fb}(\phi) + U_{int}(\phi),
\]

during each on-period of the INT controller, i.e., when \( \phi \in [\phi_{on-L}, \phi_{on-L} + w] \) or \( \phi \in [\phi_{on-R}, \phi_{on-R} + w] \), and

\[
J(q)\ddot{q} + H(q, \dot{q}) = U_{ff}(\phi) + U_{fb}(\phi),
\]

otherwise during the off-period of INT controller. In Eqs. (12) and (13), \( H(q, \dot{q}) \equiv B(q, \dot{q}) + K(q) + G(q, \dot{q}). \)

In this study, \( \phi_{on-L} \) and \( \phi_{on-R} \) were set to the timings of 1.5 ms after every heel-contact event of the left and right feet, respectively (Fu et al. 2014).

### 2.6 PR controller (FF+PD+PR)

The PR controller used in this study resets the phase \( \phi \) of the desired joint angle trajectory, and thus also the phase of FF controller, impulsively at each heel-contact. During steady-state walking of the model along the prescribed \( \gamma \), heel-contact events of the left (right) foot occur periodically with the gait period \( T_c \), for which the phases of left (right) heel-contact events are fixed at \( \phi_L(\phi_R) \). During transient gait or in the presence of noise that perturbs the state point from \( \gamma \), the phase of every heel-contact may deviate from the reference phases, i.e., \( \phi_L \) and \( \phi_R \). The PR controller shifts the
phase $\phi$ of the desired joint angle trajectory and the corresponding FF output to compensate such deviations. That is, when a heel-contact of the left (or right) foot is detected, the phase $\phi$ of the desired trajectory is reset to $\phi_L(\phi_R)$ (Yamasaki et al. 2003b; Aoi et al. 2010). Thus, the amount of phase resetting is $\Delta(\phi) = \phi - \phi_L$ for a left heel-contact, and it is $\Delta(\phi) = \phi - \phi_R$ for a right heel-contact. If a heel-contact occurs earlier (later) than the reference timing, $\Delta(\phi) < 0$ ($\Delta(\phi) > 0$), by which the gait phase is advanced (delayed) from $\phi$ to $\phi - \Delta(\phi)$, which is equal to either $\phi_L$ or $\phi_R$ for both of advanced and delayed resettings. Note that the FF controller defined above, with PR controller, is no longer feedforward, because the cyclic phase of FF output is reset by the same amount when the desired joint angle trajectory is reset. It has been shown that the PR mechanism can increase gait stability and largely reduce the impedance necessary for stability (Yamasaki et al. 2003b; Aoi et al. 2010).

### 2.7 FF+PD+INT+PR

When both of the PR and INT controllers were used together with FF+PD, the sequence of performing two time-discontinuous control actions was set as follows. First, we detected a heel-contact event, and then, phase resetting of the desired joint trajectory and the FF output. The INT controller was switched on 1.5 ms after the PR was performed. The order of onsets of the PR and the INT controllers (and a detail of the small time difference taken for the simulations) did not affect dynamics of the model, but they should be defined for clarity.

### 2.8 Torque noise

Stochastic fluctuation of the gait was induced by adding a set of six independent series of white Gaussian torque noise (with an identical standard deviation $\sigma = 0.003$ Nm) acting separately on each of the six joints. Because the gait model in this study focused only on the recovery against small perturbations when the desired joint-angle trajectory and thus the deterministic gait cycle were altered actively only by the phase resetting controller, the stride interval fluctuation could not be large enough in comparison with the experimental data during human walking. However, we intended to use large noise amplitude as much as possible.

### 2.9 DFA

The standard procedure of DFA (Peng et al. 1994, 1995) was applied for the stride interval time series data $(x(i))_{i=0}^{N-1}$ of length $N$. We obtained an integrated time series $y(k) = \sum_{i=0}^{l-1} (x(i) - \bar{x})$, where $\bar{x}$ is the sample mean of the series. Then $\{y(k)\}_{i=1}^{N}$ was divided into equal segments of length $n$ without overlapping, so the segment number was $M = N/n$. In each segment of length $n$ with an index $l$ ($l = 1, \ldots, M$), a least square polynomial $p_n^{(l)}(k)$ as the trend of the segment was fitted to the data. Square of deviations in the segment $l$ was summed up to obtain $f_n^{(l)} = \sum_{k=0}^{n-1} (y(k) - p_n^{(l)}(k))^2$. The root of average square deviation of all segments was then calculated as $F(n) = [M^{-1} \sum_{l=1}^{M} f_n^{(l)}]^{1/2}$. For each $n$, the corresponding $F(n)$ was calculated, finally the scaling exponent $\alpha$ was obtained by the linear fitting of $\log F(n)$ and $\log n$. Moreover, the intercept value for the vertical $F(n)$-axis of the plot was determined as $c$, by which we obtained an approximation of the plot as $\log F(n) = \alpha \log(n) + c$, or $F(n) = 10^{c2^n}$.

Longer data length is preferred to obtain reliable results. Typically in the literature, 1-h walk including about 3000 strides are tested experimentally (Hausdorff et al. 1996). Here we adopt the higher standard of 3,000 strides for every set of $P$-gains to quantify the scaling exponent. The orders of the polynomial ranging from 1 to 3 were examined for many preliminary sets of $P$-gains, for each of which we obtained 10 simulated trials of 3,000 strides to calculate a mean of 10 exponent values. We found that the exponent was not altered quantitatively for the different orders. Thus, we used the linear fitting (the order 1) for calculating the exponent for the wide-range parameter study, in which the exponent was calculated from only one trial of 3,000 strides for each $P$-gain due to significant computation time necessary for the simulations.

### 2.10 Numerical simulation

Numerical simulations of the model were performed by integrating equations of motion simply using the Euler-Maruyama method (Higham 2001) with a time step of $\Delta t = 10^{-5}$ seconds. The robustness of this numerical simulation method was tested by time step ten times larger and smaller. For each simulation, an initial state $(q(0), \dot{q}(0))$ and an initial phase $\phi_0$ were specified, in which the initial state was set close to the limit cycle such that $|q(0) - q(\phi_0)| < \epsilon$ was satisfied for a given $\phi_0$. $\epsilon$ was set as $10^{-3}$ rad on the HAT tilt angle. See Fu et al. (2014) and its supplemental material.

In the model with PR and INT controllers in the presence of noise, we considered that a heel contacted on the ground if the ground reaction force remained non-zero continuously more than 20 time steps. PR was performed immediately after a detection of heel-contact. Then, with a latency of 1.5 ms (150 time steps), the INT controller was switched on only for the brief time interval of $w = 0.1$ seconds.

All simulations were performed on the workstation features 2 Intel Xeon E5-2687W (10 cores each at 3.10 GHz), with a 64 GB RAM. The operation system was CentOS 6.5, with Intel compiler 13.1 and job scheduler Lava 1.0.6. Typically, one trial of parameter scanning $(P_a, P_b, P_h)$ were varied between 0 and 1500 Nm/s with fixed $D$-gains at 10

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Fig. 1 Dynamics of the gait model actuated by FF+PD controllers with and without endogenous white Gaussian torque noise ($\sigma = 0.003 \text{ Nm}$). Gains of the PD controller were set to $(P_k, P_a, P_h) = (1300, 1300, 900)$. Left-2nd-trace: A sample process of stochastic (with noise) steady-state series of 2,000 stride intervals of the model with FF+PD. Left-3rd-trace: magnification of the left-2nd-trace for the first 150 strides, indicated by the thick line below the left-2nd-trace. Sequence of red squares beneath the magnified series visualizes a degree of persistency of the series, in which each red square was plotted if every three consecutive intervals exhibited patterns of increase–decrease or decrease–increase. Densely distributed red squares imply long-short-type alternations in stride intervals. A sequence of small red squares on the lower side of the slice for Fig. 1-left-3rd-trace, which is the magnified series for the first 150 strides of Fig. 1-left-2nd-trace indicated by the horizontal thick bar, are depicted to visualize a degree of the alternations, where a small red square was plotted or not plotted (white blank) for every three consecutive data (triplet) depending on the alternation pattern of the three data points. That is, if a triplet was composed of two increments with opposite signs, i.e., an increase and then a decrease or a decrease and then an increase, a small red square was plotted below the central data point of the triplet. On the other hand, if a triplet was composed of non-alternating patterns, i.e., an increase followed by another increase or a decrease followed by another decrease, no square was plot-
Fig. 1 Dynamics of the gait model actuated by FF+PD+INT controllers with and without endogenous white Gaussian torque noise ($\sigma = 0.003$ Nm). Gains of the PD controller were set to $(P_k, P_a, P_h) = (1000, 600, 600)$. See caption of Fig. 1

Figure 1-left-3rd-trace, red squares appear quite regularly and frequently along time, with less continuous blanks, reflecting the frequent stride-to-stride alternations of the stride intervals. Figure 1-left-4th-trace is a sample of transient deterministic dynamics without noise (corresponding to the impulse response), exhibiting a slow decay. From this impulse response, one could confirm that the alternating behavior was inherent in the model only with FF+PD.

Figure 1-left-top-trace shows an integrated series of the zero-mean transformed stride interval data on the 2nd trace with a black line, and the other integrated series for the other nine simulated stride interval data with gray lines. Because of the short-period alternations with the slow decay, sums of positive and negative stride intervals could not be accumulated. Thus, the integrated series did not exhibit a Brownian-motion-like diffusion, but they stayed near the origin, resulting in the anti-persistent correlation (i.e., $0 < \alpha < 1/2$). Figure 1-right-top is a DFA plot obtained from the ten simulated sample paths of the stride intervals (Fig. 1-left-top-trace), from which we estimated as $\alpha \sim 0.009$, which was almost zero, and the intercept value $c \sim -3.9$. See Discussion for a possible mechanism of the alternating behavior in the model with FF+PD controllers. Gray dotted curves in Fig. 1-left-top-trace, which look like upward and downward step functions in this case, are the theoretical estimate of standard deviations (STD) of the integrated series as a function of the stride number $n$, $\text{STD}(n) = 10^n n^\alpha$ for $\alpha \sim 0.009$ and $c \sim -3.9$. Because of the very small value of $\alpha$, STD did not grow large as $n$, but it stayed at a constant value near $10^{-4}$.

Figure 1-right-middle, below the DFA plot, shows a stability region of the model with FF+PD controllers on the $P_k$-$P_a$ parameter plane, for which $P_h$ for the hip was fixed at 900 Nm/rad. This panel shows that the model could perform stable walking for any parameter set $(P_k, P_a)$ located in either light or dark gray region, while it fell down before the model successfully achieved 100 gait cycles for parameter sets in the black region. The parameter set $(P_k, P_a) = (1300, 1300)$ with $P_h = 900$ used for Fig. 1-left is indicated by the red arrow-head, and it is located deeply inside the stability region.

Separation of the stability region into the light and dark gray regions was performed here and in this sequel, because the steady-state gait of the model for the parameter sets located in the dark gray region was apparently deviated from the one for the parameter sets located in the light gray region. Let us explain how the separation was performed more in detail for a later purpose. It was performed in an empirical way. We first note that, for any large P-gain values, time-profiles of the joint angles during the steady-state walking were identical with those of the prescribed joint angle trajectories. For such cases, a time-profile of the trunk-inclination was periodic, and it did not alter for different P-gains as far as they were located in the light gray region. That is, steady-state solutions of the model for those cases were identical, i.e., all of them could be considered as the limit cycle solution $\gamma$. However, for P-gains in the dark gray region, time-profiles of the joint angles during the steady-state walking were deviated from the prescribed joint angle trajectories, and moreover, time-profiles of the trunk-inclination were also deviated from...
the one for the light gray region. We shall refer those gait dynamics deviated from $\gamma$ to as $\gamma'$. For the separation, a parameter set was colored by the dark gray, if a difference between maximum or minimum trunk-inclination angle of $\gamma'$ during one gait cycle and maximum or minimum of $\gamma$ was greater than a threshold (0.01 rad). Although this criterion for the separation was empirical and inaccurate, it could capture alterations from $\gamma$ to $\gamma'$ to some extent, which might be associated with bifurcation phenomena of $\gamma$, i.e., instability of $\gamma$ and generations of qualitatively different solutions. In this sense, the border between the light and dark gray regions could be considered as an edge of stability of $\gamma$. Note that the
solutions represented as $\gamma'$ might not be unique, and there might be multiple solutions bifurcated from $\gamma$, some of them might coexist. In this sense, the edge of stability might not be a simple bifurcation set on the $P$-plane. Detailed analysis of such bifurcations is out of range of the current study.

Figure 1-right-bottom shows a distribution of $\alpha$ of the model with FF+PD controllers on the $P_k-P_a$ parameter plane (for fixed $P_h$ at 900 Nm/rad). This parameter plane is the same as the stability region in Fig. 1-right-middle. In this panel, $\alpha$-value is color-coded, where blue for small $\alpha$ close to zero, green for medium $\alpha$ near 1/2, and yellow for large $\alpha$ close to unity. The parameter set $(P_k, P_a) = (1300, 1300)$ with $P_h = 900$ used for Fig. 1-left and right-top is indicated by the red arrow-head, as in Fig. 1-right-middle for the stability region. The small $\alpha$-values close to zero was common throughout the stability regions on the $P_k-P_a$ plane, as confirmed by the blue-color-coded throughout the plane. That is, the model stabilized by the linear impedance control with FF+PD controllers could never exhibit gait fractality, regardless of the $P$ values and the degree of stability.

Figure 5a-1st-row (stability regions) and Fig. 5b-1st-row ($\alpha$-value distributions) show a set of $P_k-P_a$ planes, similar to Fig. 1-right-middle and Fig. 1-right-bottom, for various values of $P_h$ for the hip joint, for the model with FF+PD controllers. In the 1st row of Fig. 5a and 5b, the $P_k-P_a$ planes shown in Fig. 1-right-middle and right-bottom are indicated by red frame boxes. As shown in our previous study (Fu et al. 2014) and presented here as Fig. 5a-1st-row, stability region of the model on the $P_k-P_a$ plane disappeared for $P_h$-value smaller than 600 Nm/rad, although it re-appeared for $P_h$ near 100 Nm/rad. The $\alpha$-values were close to zero across the values of $P_h$, as confirmed by the fact that all $P_k-P_a$ planes were colored by blue. Thus, once again, the model stabilized by the linear impedance control with FF+PD controllers could never exhibit gait fractality, regardless of the $P$ values and the degree of stability.

### 3.2 FF+PD+INT controllers

Figure 2-left-2nd-trace shows a simulated sample data of stride intervals for the model with FF+PD+INT controllers, when the $P$-gains of the PD controller were set to $(P_k, P_a, P_h) = (1000, 600, 600)$. This set of $P$-gains was considerably smaller than the set of gains used in Fig. 1 for the model with FF+PD controllers. Indeed, the FF+PD controllers alone with this set of small $P$-gains could not stabilize the gait dynamics (Fig. 5a-1st-row). In this way, the INT controller supplemented instability in the model only with FF+PD for small $P$-values, and broadened the stability region substantially in $P$-planes (Fig. 2-right-middle and Fig. 5a-2nd-row), as shown in Fu et al. (2014).
Despite the expansion of stability region, an addition of INT controller did not contribute to the emergence of fractal gait, because the short-period alternation was still present as in the model with FF+PD, as shown in Fig. 2-left-2nd-trace and its magnification in Fig. 2-left-3rd-trace with successive appearances of the red small squares. Because of the short-period alternation of the intervals, integrated stride interval time series for 10 sample paths did not exhibit diffusion-like behaviors as shown in Fig. 2-left-top-trace, leaving the \(\alpha\)-values unchanged near zero as shown by the DFA plot (Fig. 2-right-top). Figure 2-right-middle and Fig. 2-right-bottom are, respectively, the stability region and the \(\alpha\)-value distribution of the model with FF+PD+INT controllers on the \(P_h\)-\(P_a\) parameter plane (\(P_h\) for the hip was fixed at 600 Nm/rad). The parameter set \((P_h, P_a) = (1000, 600)\) with \(P_h = 600\) Nm/rad used for Fig. 2-left is indicated by the red arrow-head in each \(P_h\)-\(P_a\) plane. As shown in Fig. 2-right-bottom, the \(\alpha\)-values were small throughout the \(P_h\)-\(P_a\) plane for \(P_h = 600\), and also for other \(P_h\) values (Fig. 5b-2nd-row). Thus, although INT controller could expand stability region, it did not contribute to generation of gait fractality.

See Discussion for a possible mechanism of the alternating behavior in the model with torque noise and the FF+PD+INT controllers, which might be different from that in the model with FF+PD controllers, as can be expected from the non-oscillatory deterministic transient dynamics of the model (the impulse response) shown in Fig. 2-left-4th-trace.
3.3 FF+PD+PR controllers

3.3.1 Cases with large $P$-gains

Figure 3a-left-2nd-trace and 3rd-trace exemplifies a time series of stride intervals for the model with FF+PD+PR controllers, when the $P$-gains of the PD controller were set to $(P_k, P_a, P_h) = (1300, 1300, 900)$. This set of $P$-gains is exactly the same as the set of gains used in Fig. 1 for the model with FF+PD controllers. Even for these large values of $P$-gains, the stride interval fluctuation became white-noise-like, when the PR controller was appended to FF+PD controllers, as confirmed by more irregular patterns of the red squares in Fig. 3a-left-3rd-trace, rather than regular and frequent appearances of the red squares in Fig. 1-left-3rd-trace and Fig. 2-left-3rd-trace for the model with FF+PD and FF+PD+INT controllers. Moreover, the deterministic impulse response became non-oscillatory with a quick decay. Because long-short alternations of the stride intervals in the stochastic dynamics were substantially suppressed, the integrated stride intervals could exhibit Brownian-motion-like diffusions as in Fig. 3A-left-top-trace. The $\alpha$-value estimated from those diffusive processes was close to 1/2 (Fig. 3a-right-top), with $c \sim -4.8$. The curve of $STD(n) = 10^n \alpha$ for this case is plotted on the integrated series in Fig. 3a-left (the top trace), which is close to $\sqrt{n}$ as in the standard Brownian motion.

Figure 3a-right-middle and a-right-bottom is the stability region and the $\alpha$-value distribution of the model with FF+PD+PR controllers on the $P_k$-$P_a$ parameter plane ($P_h$ for the hip was fixed at 900 Nm/rad). The parameter set $(P_k, P_a) = (1300, 1300)$ with $P_h = 900$ Nm/rad used for Fig. 3a-left is indicated by the red arrow-head in each plane. Comparing the stability region for this case with that for the model with FF+PD (Fig. 1-right-middle), and also comparing among 1st, 2nd, and 3rd rows of Fig. 5A, it is apparent that the PR controller broadened stability region in the $P$-gain parameter space more effectively than the INT, as expected by our previous studies (Yamasaki et al. 2003a,b). As shown in Fig. 3A-right-bottom, the $\alpha$-exponents near 1/2 (green color) were widely distributed over the $P_k$-$P_a$ plane for $P_h = 900$, and also for other $P_h$ values (Fig. 5b-3rd-row). In this way, the PR controller could expand stability region, and contributed to suppression of the anti-persistency ($\alpha < 1/2$) that was caused by alternations of the stride intervals, which made series of stride intervals close to white noise.

3.3.2 Cases with $P$-gains at the edge of stability

Figure 3b-left-2nd-trace and 3rd-trace shows a simulated series of stride intervals for the model with FF+PD+PR controllers, when the $P$-gains of the PD controller were set to $(P_k, P_a, P_h) = (700, 300, 700)$. Figure 3b-right-middle represents a stability region of the model with FF+PD+PR controllers on the $P_k$-$P_a$ plane ($P_h$ for the hip was fixed at 700 Nm/rad). The stability region for this small $P_h$-gain was still as large as that for the large $P_h$-gain in Fig. 3a-right-middle, which covered large part of the $P_k$-$P_a$ plane. The parameter set $(P_k, P_a) = (700, 300)$ with $P_h = 700$ Nm/rad used for Fig. 3b-left is indicated by the red arrow-head in the plane, which is located near the edge of stability, namely, near the border between the light and dark-gray stability regions. For this set of $P$-gains, the series of stride intervals exhibited slow fluctuation. Patterns of the red squares in Fig. 3b-left-3rd-trace containing successive blanks imply that a long (or short) interval tends to be followed successively by another long (or short) interval. The deterministic impulse response was non-oscillatory with a slow decay. Because of the positively correlated appearance of successive intervals, integrated stride intervals exhibited larger diffusions than the standard Brownian motion, as shown in Fig. 3b-left-top-trace. The DFA-estimated $\alpha$-exponent was about 0.79 (Fig. 3b-right-top), with $c \sim -4.5$. The curve of $STD(n) = 10^n \alpha$ for this case is plotted on the integrated series in Fig. 3b-left-top-trace, which is roughly proportional to $n$. Note that the range of the vertical axis of Fig. 3b-left-top-trace is 10 times larger than that of Fig. 3A-left-top-trace.

Figure 3b-right-bottom, as well as Fig. 3a-right-bottom, represents distributions of $\alpha$-exponents on the $P_k$-$P_a$ planes for the model with FF+PD+PR. The parameter set $(P_k, P_a) = (700, 300)$ with $P_h = 700$ Nm/rad used for Fig. 3b-left is indicated by the red arrow-head in the plane. One can observe that the $\alpha$-exponents took large values between 0.7 and 1.0 (orange-yellow arcs) for areas located along the edge of stability, i.e., the border between the light and dark stability regions shown in Fig. 3b-right-middle, as well as Fig. 3a-right-middle, at the side of the light gray area. This is also the case for the other $P_k$-$P_a$ planes with different values of $P_h$ (Fig. 5b-3rd-row). That is, the long-range positive correlation was observed for the $P$-gains in the model with FF+PD+PR, located near the edge of stability of $\gamma$.

3.4 FF+PD+INT+PR controllers

Figure 4-left-2nd-trace and 3rd-trace represents a simulated time series of stride intervals for the model with FF+PD+INT+PR controllers, for $(P_k, P_a, P_h) = (300, 300, 300)$. As confirmed by the arrow-head in Fig. 4-right-middle, this set of $P$-gains is located near the edge of stability. For this set of $P$-gains with INT+PR controllers, the series of stride intervals exhibited apparent slow fluctuation, with patterns of the red squares in Fig. 4-left-3rd-trace contain successive blanks, as in Fig. 3B-left-3rd-trace. The deterministic impulse response was non-oscillatory with a slow decay, also as in Fig. 3B-left-4th-trace. Because of the positive persistency in the intervals, integrated stride intervals exhibited
quite large diffusion as shown in Fig. 4-right-top. The \( \alpha \)-exponent was estimated as about 0.99 (Fig. 4-right-top), with \( c \approx -4.6 \). The curve of \( STD(n) = 10^n n^\omega \) for this case is plotted on the integrated series in Fig. 4-left-top-trace can be well approximated by the straight line as the function of \( n \).

Figure 4-right-bottom shows a distribution of \( \omega \)-exponents on the \( P_k-P_a \) planes for the model with FF+PD+INT+PR controllers. The parameter set \( (P_k, P_a) = (300, 300) \) with \( P_h = 300 \text{ Nm/rad} \) used for Fig. 4-left is indicated by the red arrow-head in the plane. Together with Fig. 5a-4th-row and b-4th-row, the use of both INT and PR controllers dramatically broadened the stability region in the \( P \)-space, by which the model could establish stable gait for very small \( P-D \) gains, i.e., with very flexible joint dynamics. The stability regions with the prescribed \( \gamma \) (the light gray regions) occupied larger areas than those for the model with FF+PD+PR, as compared with the light gray regions between the 3rd and 4th rows of Fig. 5a. As a result, the border between the light and dark gray regions marched into the small \( P \)-gains regimes, wherein the \( \alpha \) values between 0.8 and 1.0 were widely distributed (the orange-yellow regions in Fig. 5b-4th-row) along the edge of stability. Note that the orange-yellow arcs in the \( P_k-P_a \) planes for \( P_h \in \{600, 1000\} \) for the model with FF+PD+INT+PR controllers (Fig. 5b-4th-row) are not necessarily accompanied with the corresponding arc-shaped borders between the light and dark gray stability regions. This might be due to the inaccurate criterion for detecting instability of the limit cycle \( \gamma \), but there might be undetected bifurcation curves near the orange-yellow arcs in Fig. 5b-4th-row.

4 Discussion

4.1 Summary

The simulations of a simple gait model with torque noise, stabilized by different combinations of controllers, demonstrated that the model is capable of producing fractal gait patterns, provided that it incorporates a phase resetting (PR) mechanism. This PR mechanism appropriately shifts the reference gait motion and the corresponding feedforward torques, and the gains of the proportional-derivative (PD) feedback controller, which should be used with the PR controller, are tuned at the edge of stability. Moreover, the combined use of the intermittent (INT) feedback controller, with the PD and PR controllers, allowed remarkable decrease in critical gains of the PD controller, which is necessary for achieving stability, and broadened the gain-parameter region that exhibits gait fractality. In other words, neither the model stabilized only by the impedance control, even at the edge of linear stability, nor the model stabilized by the nonlinear controllers but with \( P \)-values far inside the stability region could ever generate gait fractality. Although only limited types of controllers were examined, we suggest that the impulsive nonlinear controllers with the PR condition, which is the criticality condition, and the INT condition, are the major mechanisms responsible for gait fractality.

This study also highlighted several scenarios that can induce a loss of gait fractality in the model. The most common effect is the large \( P \)-gains, with or without the PR and INT controllers. The lack of the PR controller could also cause a loss of gait fractality. Even with the activation of the PR controller, large \( P \)-gains lead to a small value of the exponent, close to 1/2. Finally, the disactivation of the INT controller, while leaving the PR controller active, is likely to reduce the area with gait fractality in the \( P \)-gains parameter space, inducing the possible loss of gait fractality. Thus, the results of this study suggest that pathological gait, characterized by joint-rigidity and/or loss of fractality (Hausdorff et al. 1997; Hausdorff 2009), is caused by dysfunction in some of the aforementioned conditions.

4.2 Theoretical considerations

The alternation between long and short stride intervals in the model with FF+PD controllers could be interpreted based on the linearized analysis as performed in our previous study (Fu et al. 2014). It is associated with the fact that the dominant mode (the least stable mode), characterized by Floquet multipliers for analyzing local stability of the limit cycle with FF+PD without noise, was a pair of complex conjugates with an amplitude close to unity and its argument greater than \( \pi/2 \) (see Fig. 4 of Fu et al. 2014). That is, for the pair of multipliers, denoted by \( \lambda = r e^{ \pm j \psi} \), the amplitude \( r > 0 \) was less than but close to 1, and \( \pi/2 \leq \psi \leq \pi \) for the argument. In this case, a general solution of this two-dimensional dominant mode, denoted by \( \xi_n \) representing the small deviation from the limit cycle as a function of the stride number \( n \) in an appropriate coordinated system can be written as

\[
\xi_n = r^n \left[ \begin{array}{c} \cos \psi & \sin \psi \\ -\sin \psi & \cos \psi \end{array} \right]^n \xi_1, \tag{14}
\]

where \( \xi_1 \) is an initial deviation. The simplest case is with \( \psi = \pi \), yielding

\[
\xi_n = r^n \left[ \begin{array}{cc} -1 & 0 \\ 0 & -1 \end{array} \right]^n \xi_1 = (-r)^n \xi_1, \tag{15}
\]

which alternates between positive and negative signs of \((-r)^n\), corresponding to a slowly damped period-2 oscillation with long and short intervals around the gait period. This situation is similar to, but simpler than the case for the linearized model with FF+PD controllers. For arguments \( \pi/2 \leq \psi \leq \pi \), which is exactly the case for the linearized
model with FF+PD controllers, Eq. (14) did not represent the period-2 oscillation, but it exhibited short-period alternations, oscillating roughly between long and short intervals.

The mechanism of short-period alternations in the model with FF+PD+INT might not be the same as that in the model with FF+PD, which can be seen from the non-oscillatory deterministic impulse response. For an in-depth understanding, we notice that the limit cycle \( \gamma \) of the model with FF+PD is unstable for \( P \)-gains smaller than a set of critical gains (Fu et al. 2014). Note also that the model only with FF+PD is equivalent to the model with FF+PD+INT during “off-period” of the INT controller. In the process of destabilization of stable dynamics in the model only with FF+PD for large \( P \)-gains, as \( P \)-values decrease, the aforementioned pair of complex conjugate Floquet multipliers \( \lambda = re^{\pm j\psi} \), located near the unit circle, moves out the unit circle, i.e., the amplitude \( r \) becomes greater than 1. For typical \( P \)-values in the model with FF+PD+INT, those multipliers are located near the real axis in the right-half plane, i.e., \( \psi \sim 0 \) (see Fig. 4 of Fu et al. 2014). This destabilized slowly oscillatory mode forms the low-dimensional unstable manifold of \( \gamma \), along which the model during “off-period” of the INT falls away from \( \gamma \). The alternation for the model with FF+PD+INT was due to the random positioning of the state point located at either side of the unstable manifold that is separated by the stable manifold, at every timing of the onset of “on-period” of the INT. Note that each of brief actions of the INT during each brief “on-period” brings the state point close to the stable manifold, by which the state point during “off-period” of the INT moves transiently along the high-dimensional stable manifold (slowly convergent dynamics), contributing to the stabilization of the unstable dynamics in the model only with FF+PD (Fu et al. 2014).

4.3 Physiological plausibility and limitations of the model

The gait model used here is certainly a simplification from many points of view, and thus one may wonder to which extent it is physiologically plausible. In particular, it considered a steady-state gait only in the sagittal plane, unlike others (Roos and Dingwell 2010), and this could be a significant limitation of validity, as walking instability might be related to lateral instability (Bauby and Kuo 2000).

Another issue is related to the physiological significance of the periodic reference joint-angle trajectory and the corresponding torque outputs of the FF controller. In our opinion, this may be interpreted as the walking-rhythm and force-related output signals produced by central pattern generators (CPGs) (Minassian et al. 2017). A major theory considers a CPG model with a common rhythm generator that controls the operation of the pattern formation (PF) circuitry responsible for motoneuron activation (McCrea and Rybak 2008).

The biological significance of the PR is imperative for gait fractality in our model. The PR in response to perturbations during human walking has been characterized by several studies (Schillings et al. 2000; Nessler et al. 2016; Funato et al. 2016), including ours (Kobayashi et al. 2000). Moreover, a number of animal locomotion studies have been performed to reveal neural mechanisms of phase of CPGs and/or walking rhythm reset to clarify feedback and feedforward pathways responsible for PR, including cutaneous feedback (Qoll-Houssaini et al. 1993), the vestibulospinal tract (Russell and Zajac 1979), and basal ganglia-related efferents (Takakusaki et al. 2003). Thus, although each PR in the model is simply triggered by foot-contact, the amount of physiological PR might be determined by integrating information at spinal and supraspinal levels. In this regard, if we assume that the normal function of PR is to induce gait fractality, the clinical evidence about a loss of gait fractality in patients with neurological diseases at supraspinal (Hausdorff et al. 1997; Hausdorff 2009) but not at peripheral level (Gates and Dingwell 2007), implies the importance of supraspinal mechanisms to tune the amount of PR.

Finally, it is important to note that feedback transmission delays were not taken into account for any of the controllers in our model. In particular, latencies for the PR and INT controllers might be too short for the physiological feedback control. However, an argument in favor of such simplification is that feedforward neural mechanisms for anticipating foot-contacts during gait could compensate such delays (Pisotta and Molinari 2014). In any case, investigation of a detailed model with delays should be a major issue in future developments of this study. Indeed, the intermittent control model for stabilizing quiet standing in our previous studies (Botaro et al. 2008; Asai et al. 2009; Suzuki et al. 2012) has taken feedback time delay into account and showed a key role played by delay-induced instability in the intermittent control paradigm. That is, the feedback delays might not necessarily be harmful for the gait control system, but they could be beneficial for generating a novel type of stability that allows relatively large movement fluctuations (Nomura et al. 2013).

One limitation of our model is related to the fact that the joint-angle trajectory is prescribed and only its phase is modified by the PR. As a result, the overall gait stability, i.e., the corresponding basin of attraction, is rather limited. Such limitation implies that the variance of stride intervals generated by the model is significantly smaller than the experimental data, also because the small size of the basin of attraction limits the noise intensity that can be injected in the model without inducing instability. In perspective, how-
ever, this could be overcome by another computational layer that addresses human capability to adapt to changing environments by updating an original plan of action.

4.4 Criticality

Criticality of the gait control system might be a key mechanism for fractality of human gait, because it plays a key role in most of the previous studies, including (Ahn and Hogan 2013), series of studies by Dingwell et al. (e.g., Dingwell et al. (2010), Gates et al. (2007)), and our current study. Note that criticality might be naturally equivalent to energetic efficiency, because it often arises with small feedback gains.

The achievement by Dingwell et al. (2010) is that they introduced a linear optimization process to explain how the parameter values of the model are tuned at the edge of stability. In particular, the minimum intervention principle plays the role for determining parameter values at the stability edge in a way to resolve a trade-off between minimalizations of task-related errors and energy cost. In contrast to this approach with a linear optimal controller, we suggest that the nonlinear impulsive controllers (PR+INT) with the small-gain impedance controller provide an alternative implementation of the minimum intervention principle, without explicitly solving an optimization problem. In fact, we explored mechanical energy consumed by the feedback controllers (except the FF controller) as a function of $P$-gains for different combinations of the controllers (results not shown), and obviously, confirmed less energy consumptions for small $P$-gains, stabilized by the PR+INT controllers. This means that the gait with small $P$-gains at the edge of stability with fractality, which is stabilized by the PR+INT controllers, is energetically more efficient than the gait with large $P$-gains with and without the PR+INT controllers. Thus, we could state that gait fractality implies energetic efficiency. However, consider also that most of the energy during walking is consumed by the FF controller, and the $P$-dependent differences in energy consumption among the models with different feedback controllers and different $P$-gains are tiny and virtually negligible, because of small amplitude of joint angle variability from the steady-state trajectory.

It is worthwhile to mention that emergence of fractal gait near the edge of stability in this study is reminiscent of a recent report showing that energetic costs are minimized by the control at stability’s edge during expert stick balancing (Milton et al. 2016). Moreover, the best-fit model parameters for the stick balancing experiment with a long-range correlation using the INT control model in our previous study were also located at the edge of stability (Yoshikawa et al. 2016; Morasso et al. 2019). Therefore, criticality, fractality and energetic efficiency might be commonly interrelated in the control of gait and posture.

4.5 Uncontrolled manifold and stable manifold

The GEM used in the model of Dingwell et al. (2010) is similar to the Uncontrolled Manifold (UCM) used for analyzing movement control with kinematic redundancy for a given motor task (Latash et al. 2002). Recently, we discussed a geometrical similarity between the UCM and a stable manifold used for the intermittent postural control in the model with a multi-link inverted pendulum (Suzuki et al. 2012) for analyzing postural sway during quiet stance (Suzuki et al. 2016). We showed that the UCM with a task of postural maintenance and the stable manifold of the upright equilibrium point for the double pendulum model with no active feedback control are configured very similarly. Because of this, fluctuation along the UCM cannot be distinguished from that along the stable manifold.

We speculate that gait cycle variability in our model could be dominated by dynamics along the stable manifold for most of the gait cycle duration, during which neither the PR nor the INT controller were activated. Although it is not easy to confirm the speculation, if this assumption was assumed true, similar discussion to the posture-case can be made for the GEM and the stable manifold of our model. Although the GEM and the stable manifold are defined in completely different spaces, existence of both uncontrolled and controlled manifolds might provide a basis for dealing with a trade-off-type decision of whether or not to make an active effort for achieving a motor task. Switching between such dynamics with and without active control could be an alternative nonlinear mechanism for gait fractality (Nomura et al. 2013), in addition to the criticality of the system.

4.6 Stability with flexibility

A novel contribution of this study, in addition to the fractal issue, is that the combined use of the PR and INT controllers, which were proposed separately in our previous work (Yamasaki et al. 2003a, b; Fu et al. 2014), to complement the action of the FF+PD controllers, was shown to support gait stability with extremely small joint impedance (i.e., small $P$ and $D$ parameters). For example, the gains of $P_a = P_k = P_h = 100 \text{Nm/rad}$ and $D_a = D_k = D_h = 10 \text{Nm/s/rad}$ corresponding to the parameter point located in the stability region of Fig. 5a-4th-row are quite comparable with experimentally estimated joint stiffness for the lower extremities during human walking (Shamaei et al. 2013a, b).

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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