Damped Bloch oscillations of cold atoms in optical lattices

A. R. Kolovsky and A. V. Ponomarev
Kirensky Institute of Physics, 660036 Krasnoyarsk, Russia

H. J. Korsch
Universität Kaiserslautern, FB-Physik, D-67659 Kaiserslautern, Germany

The paper studies Bloch oscillations of cold neutral atoms in the optical lattice. The effect of spontaneous emission on the dynamics of the system is analyzed both analytically and numerically. The spontaneous emission is shown to cause (i) the decay of Bloch oscillations with the decrement given by the rate of spontaneous emission and (ii) the diffusive spreading of the atoms with a diffusion coefficient depending on both the rate of spontaneous emission and the Bloch frequency.

I. INTRODUCTION

In 1928 Bloch predicted that the coherent motion of crystal electrons in a static electric field should be oscillatory rather than uniform [1]. Nevertheless, this phenomenon (known nowadays as Bloch oscillations), has never been observed in bulk crystals. Because of relaxation processes (scattering on lattice defects, phonons, etc.) the coherence of the system is destroyed before electrons complete one Bloch cycle. This obstacle has forced the researchers to look for other systems, where the Bloch period (which is inversely proportional to the magnitude of the static force and the lattice period) can be smaller than the characteristic relaxation time. This is realized in semiconductor superlattices, and in 1992 a direct observation of the current oscillations in semiconductor superlattices was reported [4, 5]. It should be stressed, however, that in semiconductor superlattices the relaxation time only slightly exceeds the Bloch period and, thus, in practice one always meets the regime of damped Bloch oscillations. A detailed analysis of the relaxation processes in the semiconductor system appears to be a rather complicated problem and phenomenological approaches are usually used to describe the decay of current oscillations.

Recently Bloch oscillations were observed in the system of cold atoms in an (accelerated) optical lattice [4]. This system of atomic optics mimics the solid state system where the neutral atoms and standing laser wave play the roles of electrons and crystal lattice, respectively. Unlike the solid state systems, it may show both the regimes of undamped and damped oscillations. Indeed, the main relaxation process in the system “atom in a laser field” is the spontaneous emission of photons by the excited atom and it can be well controlled by choosing an appropriate detuning from the atomic resonance. Up to now only the regime of undamped oscillations (large detuning) has attracted the attention of researchers [4, 5, 6, 7, 8, 9, 10]. However, keeping in mind an analogy with the semiconductor superlattices, the latter regime is also of considerable interest. In this present paper we study the process of decay of atomic Bloch oscillations due to the effect of spontaneous emission. In particular we address the question of the actual decay process and the dependence of the decay rate on the system parameters.

II. THE MODEL AND APPROACH

We shall take into account the spontaneous emission by using the standard approach, where its effect is characterized by a single constant $\gamma$ defined as the inverse radiative lifetime of the upper state. Namely, it is assumed that the diagonal and off-diagonal elements of the $2 \times 2$ density matrix of a fixed atom relax to their equilibrium values with the rates $\gamma$ and $\gamma/2$, respectively. When atom is in free space, its density matrix depends additionally on the position $z$ of the atom and, in general case, the resulting master equation has a rather complicated form [1]. This equation can be considerably simplified in the limit of large detuning $\delta \gg \Omega$ (here $\Omega$ and $\delta$ are Rabi frequency and detuning from the atomic resonance). In this case we can eliminate the internal structure of the atom and the master equation for the position density matrix has the form

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar} [H, \rho] - \frac{\gamma}{2} \frac{\Omega^2}{\delta^2} \int du P(u) \left( L_u^\dagger L_u \rho - 2 L_u \rho L_u^\dagger + \rho L_u^\dagger L_u \right) , \tag{1}$$

(see, Ref. [13, 14], for example) where

$$H = \frac{p^2}{2M} - \frac{\hbar \Omega^2}{\delta} \cos^2(k_L z) + F z \tag{2}$$
is the effective Hamiltonian of the system and

\[ L_u = \cos(k_L z) \exp(iu k_L z) , \quad |u| \leq 1 \]  \hspace{1cm} (3)

is the projection of the recoil operator on z-axis. The distribution \( P(u) \) of a random variable \( u \) in Eq. (1) is defined by the angle distribution for the momentum \( \hbar k_L = \hbar k_t \mathbf{n} \) of the spontaneously emitted photons, which in the case of linearly polarized light is given by \( \Phi(\mathbf{n}) = (3/8\pi)[1 - (\mathbf{n} \cdot \mathbf{e})^2] \). Note that equation (1) has the Lindblad form and, thus, \( \text{Tr} [\rho(t)] = \int dz \rho(z,z,t) = 1 \). Beside this, for linearly polarized light, \( P(u) \) appears to be almost independent of \( u \) and for most practical purposes one can set \( P(u) = 1/2 \).

To solve Eq. (1), we use a Monte-Carlo method of Ref. [14]. According to this method one finds the dynamics of an arbitrary observable \( \langle A(t) \rangle = \text{Tr}[A \rho(t)] \) as an average

\[ \langle A(t) \rangle = \langle \psi(t)|A|\psi(t) \rangle \]  \hspace{1cm} (4)

over different solutions of the stochastic Schrödinger equation

\[ d\psi = \left( -\frac{i}{\hbar} H dt - \frac{\gamma}{2} \sum_{l,m}^L L_{l,m} dt + \sqrt{\gamma} L_{l,m} \xi \right) \psi . \]  \hspace{1cm} (5)

In the latter equation \( \gamma = \gamma(\Omega/\delta)^2 \) is the spontaneous emission rate and \( \xi \) is a Wiener process with \( \langle \xi \rangle = 0 \) and \( \langle \xi^2 \rangle = dt \) (i.e., \( \xi(t) \) is \( \delta \)-correlated white noise). It is worth to note that the choice of the stochastic Schrödinger equation is not unique and one can construct many different stochastic equations corresponding to the same master equation. Most often the nonlinear versions, which preserve the norm of the wave function \( \psi(t) \), are employed [15]. Here, following Ref. [14], we use a linear stochastic equation which is preferable from the numerical point of view. Then the norm is conserved only after averaging over different realizations of the random process \( \xi(t) \). This fact was used to control the statistical convergence.

### III. TIGHT-BINDING APPROXIMATION

First we shall analyze the problem in the tight-binding approximation. In this approximation the Hamiltonian (2) is substituted by the tri-diagonal matrix

\[ H_{l,m} = -\frac{\Delta}{2} (\delta_{l,m+1} + \delta_{l,m-1}) + dF \delta_{l,m} , \]  \hspace{1cm} (6)

and the recoil operator (3) by the diagonal matrix

\[ L_{l,m}(u) = (-1)^l \exp(i \pi u l) \delta_{l,m} . \]  \hspace{1cm} (7)

Here \( d = \pi/k_L \) is the period of the optical potential and the index \( l \) refers to the localized Wannier function \( |l\rangle \) associated with the \( l \)-th well of the periodic potential. In the absence of spontaneous emission (\( \gamma = 0 \)) and static force (\( F = 0 \)) the eigenfunctions of the system are Bloch waves \( |\psi_k \rangle = \sum_l \exp(idk_l|l\rangle \text{ corresponding to the energy } \epsilon(k) = -\Delta \cos(dk) \). This dispersion relation is assumed to approximate the dispersion relation of the atom in the ground Bloch band. In the presence of a static force, the quasimomentum \( \kappa \) of the wave function \( |\psi_k \rangle \) evolves according to the classical equation \( d\kappa/dt = F/\hbar \). In terms of the atomic velocity \( v = \hbar^{-1} \partial \epsilon(k)/\partial \kappa \) and coordinate \( z \) this corresponds to a periodic oscillation of the wave packet with the Bloch frequency \( \omega_B = dF/\hbar \):

\[ \langle v(t) \rangle = \frac{d\Delta}{\hbar} \sin(\omega_B t) , \quad \langle z(t) \rangle = \frac{\Delta}{F} \cos(\omega_B t) . \]  \hspace{1cm} (8)

(The coordinate and velocity operators are obviously given by the matrices \( z_{l,m} = d\delta_{l,m} \) and \( v_{l,m} = (d\Delta/2\hbar)(i\delta_{l,m+1} - i\delta_{l,m-1}) \).) Note that within the tight-binding approximation the atoms always oscillate according to a cosine law. In reality, however, the oscillations are asymmetric. Besides this, there is a decay of the oscillations due to the interband Landau-Zener tunneling [16, 17, 18, 19] — a phenomenon completely ignored by the tight-binding (and more general single-band) models. With these remarks reserved, we proceed with the analysis of the effect of spontaneous emission.

As follows from the explicit form of Eq. (5), the recoil operator randomly changes the atomic quasimomentum. Thus any narrow distribution of the quasimomentum (which is usually considered as a prerequisite for observing Bloch oscillations) will be smeared over the entire Brillouin zone and the oscillations should decay. The following
simple “classical” model helps to understand the details of the decay process. Let us consider an ensemble of classical particles with the Hamiltonian

$$H = -\Delta \cos(dp/h) + Fz$$  \hspace{1cm} (9)$$

affected additionally by noise (random recoil kicks) so that the conditional probability is

$$W(p, p', t) = (2\pi\gamma t)^{-1/2} \exp\left\{-\frac{[d(p - p' - Ft)/h]^2}{4\gamma t}\right\} .$$  \hspace{1cm} (10)$$

Then the mean velocity $\langle v(t) \rangle = \langle \partial H/\partial p \rangle$ decays as

$$\langle v(t) \rangle = v_0 \exp(-\gamma t) \sin(\omega_B t) , \quad v_0 = d\Delta/h$$  \hspace{1cm} (11)$$

and the mean squared velocity relaxes to $v_{st}^2 = v_B^2/2$.

The solid line in Fig. 1 shows the dynamics of $\langle v(t) \rangle$ calculated on the basis of equations (9) - (11), where as an initial condition we choose $\rho(0) = |\psi(0)\rangle \langle \psi(0)|$ with $\psi(0) \sim \sum \exp(-l^2/100)/l$ and set $\Delta = 1, h = 1$, and $d = \pi$ for simplicity. It is seen that the solid line in Fig. 1 closely follows the theoretical prediction (11) (dashed line), i.e. the decay rate of Bloch oscillations is given by the rate $\gamma$ of spontaneous emission. Simultaneously with the decay of the mean velocity, the mean squared velocity relaxes to its equilibrium value $v_{st}^2 = v_B^2/2$, which corresponds to a uniform distribution of the atoms over the Brillouin zone (so-called recoil heating, see inset in Fig. 1).

Since Bloch oscillations decay after a transient time $t \sim 1/\gamma$ it might be naively thought that in the stationary regime $t \gg 1/\gamma$ there is no difference between the cases $F \neq 0$ and $F = 0$. Although this is true for the momentum distribution, the difference appears when we analyze the atomic dynamics in coordinate space. Indeed, according to the “classical model” (9) the velocity correlation function $R_t(\tau) = \langle v(t+\tau)v(t) \rangle$ does not depend on $t$ in the stationary regime and obeys

$$R(\tau) = v_{st}^2 \exp(-\gamma\tau) \cos(\omega_B\tau) .$$  \hspace{1cm} (12)$$

Substituting Eq. (12) into the equation for the mean squared displacement,

$$\langle z^2(t) \rangle = \langle \left( \int_0^t v(t')dt' \right)^2 \rangle \approx 2t \int_0^\infty R(\tau)d\tau ,$$  \hspace{1cm} (13)$$

we obtain

$$\langle z^2(t) \rangle \sim Dt , \quad D = 2v_{st}^2 \frac{\gamma}{\omega_B^2 + \gamma^2} .$$  \hspace{1cm} (14)$$

Thus the static force suppresses diffusion caused by the recoil heating. As an example, Fig. 2 shows numerical results (obtained on the basis of the tight-binding model) for the mean squared displacement $\langle \Delta z^2(t) \rangle = \langle z^2(t) \rangle - \langle z(t) \rangle^2$ for some values of $F$ (solid lines) together with the diffusion law $\langle \Delta z^2(t) \rangle = Dt$ (dashed lines) where the diffusion coefficient $D$ is obtained from (14). Concluding this section we stress that Eq. (14) refers only to the stationary regime $t \gg 1/\gamma$. In the opposite limit the atomic motion is essentially oscillatory (large static force) or ballistic (weak force) and $\langle \Delta z^2(t) \rangle \sim t^2$.

IV. DECOHERENCE BY RECOIL HEATING

The results reported in the previous section can be also viewed as a decoherence process. Indeed, in the single-band approximation the dynamics of the system is characterized by the density matrix $\rho(t) = \sum_{n,m} \rho_{n,m}(t)|m\rangle \langle n|$. Substituting Eqs. (9)-(11) into the master equation (1) and taking into account that $P(u) \approx 1/2$ we obtain

$$\frac{d\rho_{n,m}(t)}{dt} \approx -\frac{i\Delta}{2\hbar} \sum_{\pm,\mp} (\rho_{n\pm1,m} - \rho_{n,m\pm1}) - i\omega_B(n - m)\rho_{n,m} - \gamma(1 - \delta_{n,m})\rho_{n,m} .$$  \hspace{1cm} (15)$$

The last term in this equation causes the decay of off-diagonal matrix elements. Thus the density matrix of the system tends to be diagonal in the basis of the localized Wannier states. As an illustration to this statement Fig. 3 shows the absolute values of $\rho_{n,m}(t)$ as a gray-scaled map for the parameters of Fig. 1 and $t = 0$ and $t = 5T_B$. It is seen in the figure that at $t > 1/\gamma$ only the diagonal and nearest to diagonal matrix elements $\rho_{n,n+1} = \rho_{n+1,n}$ have non-negligible
values. This property of the “stationary” density matrix allows us to obtain Eq. (14) of the previous section without appealing to the classical model [3]–[10]. The derivation is as follows.

First, we estimate the off-diagonal matrix elements $\rho_{n+1,n}$. The formal solution for these elements reads

$$
\rho_{n+1,n}(t) = \frac{i\Delta}{2\hbar} \exp(-i\omega_B t - \gamma t) \int_0^t dt' (\rho_{n+1,n+1} - \rho_{n,n}) \exp(i\omega_B t' + \gamma t')
$$

(16)

(here we neglect all off-diagonal elements except the ones nearest to the main diagonal). Because in the stationary regime the characteristic rate of change of the diagonal matrix elements is much smaller than $\omega_B$, we obtain

$$
\rho_{n+1,n}(t) \approx \frac{i\Delta}{2\hbar} \frac{\rho_{n+1,n+1} - \rho_{n,n}}{i\omega_B + \gamma}.
$$

(17)

Next, substituting this estimate into the equation for the diagonal matrix elements yields the rate equation

$$
\frac{d\rho_{n,n}(t)}{dt} = \left( \frac{\Delta}{2\hbar} \right)^2 \frac{2\gamma}{\omega_B^2 + \gamma^2} (\rho_{n+1,n+1} - 2\rho_{n,n} + \rho_{n-1,n-1})
$$

(18)

Finally, approximating $(\rho_{n+1,n+1} - 2\rho_{n,n} + \rho_{n-1,n-1})/d^2$ by $\partial^2 \rho/\partial z^2$ we end up with the diffusion equation

$$
\frac{\partial \rho}{\partial t} = D \frac{\partial^2 \rho}{\partial z^2}
$$

(19)

with the same diffusion coefficient $D$ as given in Eq. (14).

To summarize, the recoil heating reduces the coherence length of the atomic wave function to that of the localization length of the Wannier function which, in turn, is of the order of the optical lattice period $\pi/k_L$. Obviously, under this condition a direct observation of Bloch oscillations is impossible. However, they show up indirectly as a ”correction” to the diffusion coefficient $D$.

V. BEYOND THE SINGLE-BAND RESULTS

We proceed with the analysis of Bloch oscillation on the basis of Eqs. (3) – (5), i.e. beyond the tight-binding (single-band) approximation. It is convenient to use scaled variables, where the length is measured in units of the laser wave-length $(z \rightarrow k_L z)$, the energy in units of the recoil energy $E_R = \hbar^2 k_L^2 / 2M$, and the time is scaled on the basis of the recoil frequency $(t \rightarrow E_R t / \hbar)$. Then the Hamiltonian (3) takes the form

$$
H = p^2 - U \cos^2(z) + Fz,
$$

(20)

where $p = -i\partial/\partial z$, $U = (\hbar^2 \delta / \beta) / E_R$, and $F$ is the scaled static force. In what follows we restrict ourselves by considering two different values for amplitude of the optical potential: $U = 1$, which we shall refer to as case (a), and case (b), where $U = 4$. The band spectrum of the system in these two cases is depicted in Fig. 4. It is seen in the figure that for $U = 1$ there is an essential deviation of the ground-Bloch-band dispersion relation from the cosine law, while for $U = 4$ the relation $\epsilon_0(\kappa) = -\Delta \cos(\pi \kappa)$ holds with good accuracy. The more important difference of two spectra, however, is the size of the energy gap between the ground and “first excited” bands. Indeed, according to the Landau-Zener theory this gap defines the probability of interband tunneling, which decreases exponentially as a function of squared energy gap [1]. Thus, in the case (b) the rate of Landau-Zener tunneling is negligible in comparison with case (a).

Figure 5 shows the dynamics of an atomic wave packet for $\gamma = 0$, $U = 1$, and $F = 0.025$. As expected the atoms perform a periodic oscillations with Bloch period $T_B = 2/F$. It is also seen in the figure that during each Bloch cycle (when the wavepacket is reflected at its leftmost position) a fraction of probability is “emitted” in the negative direction. In terms of the Bloch-band spectrum this effect corresponds to Landau-Zener tunneling between the ground and upper bands, discussed above. Experimentally, this phenomenon was observed in Ref. [3] and its complete theoretical description is given in Ref. [11] by using the formalism of resonance (metastable) Wannier-Stark states. Because of tunneling the probability $P(t)$ to find the atom in any finite interval (larger than the amplitude of Bloch oscillations) exponentially decreases with time. In what follows, we quantify this process by the increment $\nu$, which we refer to as the depletion constant [20]. In numerical simulation the depletion constant was found by approximating the function

$$
P(t) = \int_{-20\pi}^{20\pi} dz |\psi(z,t)|^2
$$

(21)
by an exponential function. This gives \( \nu = 2.5 \times 10^{-4} \) and \( \nu < 10^{-6} \) for the cases (a) and (b), respectively. For the sake of future reference the solid lines in Fig. 3 show the dynamics of the mean atomic velocity \( \langle v(t) \rangle \), calculated as

\[
\langle v(t) \rangle = \int_{-20\pi}^{20\pi} \, dz \psi^*(z, t) \left( -i \frac{\partial}{\partial z} \right) \psi(z, t),
\]

and the wave packet dispersion \( \langle \Delta z^2(t) \rangle = \langle z^2(t) \rangle - \langle z(t) \rangle^2 \). To compensate the decrease of probability, both \( \langle v(t) \rangle \) and \( \langle \Delta z^2(t) \rangle \) are normalized by dividing by \( P(t) \). The asymmetry of the velocity oscillations obviously reflects the deviation of the actual dispersion relation \( \epsilon_0(\kappa) \) from the cosine dispersion relation. The dashed lines in the figure correspond to the tight-binding approximation of Sec. 3 where, to take into account the asymmetry of the oscillations, the tri-diagonal matrix (3) is substituted by few-diagonal matrix with off-diagonal matrix elements given by the coefficients of Fourier transform of \( \epsilon_0(\kappa) \). A reasonable coincidence is noticed.

We proceed with the case of nonzero rate of the spontaneous emission. First we study the dependence of the depletion constant \( \nu \) on \( \hat{\gamma} \). These studies are summarized in Table 4. It is seen from the table that the spontaneous emission strongly enhances the “tunneling” decay of the system. This can be understood by noting that the recoil operator may “kick out” the atom from the ground Bloch band. (More formally, given \( \psi(z) \) belonging to the subspace of Hilbert space spanned by Bloch waves with zero band index, \( L_u \psi(z) \) generally does not belong to this subspace.)

Because the rate of Landau-Zener tunneling increases with the band index, this causes a faster decay of \( P(t) \).

Since the depletion of the ground band introduces an additional decay mechanism, it might be naively expected that the decay of Bloch oscillations should be faster than it is predicted by the tight-binding model. However, the real situation appears to be inverse. This is illustrated in Fig. 5 where the dashed lines are predictions of the tight-binding model and the solid lines show actual behaviour of the system. (The system parameters are the same as in Fig. 3 but \( \hat{\gamma} = 0.01 \).) The explanation for this effect is given below. As follows from the explicit form of the recoil operator (3), it multiplies the Bloch wave by a plane wave with a wave vector varying from zero to that of the size of the Brillouin zone. Within the single band approximation this always results in a change of the quasimomentum \( \kappa \) of Bloch wave by the same value. In the reality, however, only small values of the wave-vector (“weak kicks”) cause a simple change of the quasimomentum. The large values (“strong kicks”), as explained above, may remove the system from the ground band. As a result, the redistribution of the quasimomentum \( \kappa \) over the entire Brillouin zone is slowed down and the oscillations of the mean velocity (normalized by survival probability \( P(t) \)) decays slower.

The results of a numerical simulation in the case \( U = 4 \) are shown in Fig. 5. Peak-like behaviour of the dispersion at short times time origin is a short-time transient phenomenon depending on the initial condition. (In principle, this peak can be removed by an appropriate choice of the initial wave packet.) Ignoring this transient phenomenon, the overall dynamics of the system is now closer to that predicted by the tight-binding model. This is actually not surprising, because the tight-binding approximation is more reliable for larger values of the energy gap.

VI. SUMMARY

We analyzed Bloch oscillations of neutral atoms in the presence of spontaneous emission. It is shown that random recoil kicks (coming from spontaneously emitted photons) cause a decay of Bloch oscillations through the dephasing of Bloch waves. The decay of the mean atomic velocity is accompanied by a diffusive spreading of the atoms in configuration space. We studied both of these processes by using the tight-binding approximation and by direct numerical simulation of the wave packet dynamics.

Within the tight-binding model, the increment of Bloch oscillations decay is proven to coincide with the rate of spontaneous emission \( \hat{\gamma} \) which, in turn, is given by the product of the excitation probability of the upper level and its natural width. The diffusion coefficient depends additionally on the value of the Bloch frequency \( \omega_B \) (which is proportional to magnitude of the static force) and is smaller for larger values of the Bloch frequency. Thus, the static force suppresses the spreading of the atoms caused by the recoil heating.

Direct numerical simulation of the system dynamics confirms qualitatively these results of the tight-binding model. However, because of the failure of the single-band approximation, the real situation appears to be more complicated. It is shown that the recoil kicks depletes the ground Bloch band with a rate depending on the (scaled) spontaneous emission rate \( \hat{\gamma} \) and the size of the energy gap following the ground band. Surprisingly, the depletion of the Bloch bands affects the decay of Bloch oscillations in a counter-intuitive way: it slows down the decay of the oscillations.

[1] F. Bloch, Z. Phys 52, 555 (1928).
[2] J. Feldmann et al., Phys. Rev. B 46, 7252 (1992).
TABLE I: Depletion constant $\nu$ for different rate of spontaneous emission $\tilde{\gamma}$. The value of the static force is $F = 0.025$.

| $\tilde{\gamma}$ | 0.0000 | 0.0001 | 0.0005 | 0.0010 | 0.0050 |
|------------------|--------|--------|--------|--------|--------|
| $U = 1$          | $2.5 \times 10^{-4}$ | $6.0 \times 10^{-4}$ | $3.0 \times 10^{-3}$ | $1.0 \times 10^{-2}$ |
| $U = 4$          | $< 10^{-6}$ | $1.5 \times 10^{-4}$ | $1.5 \times 10^{-3}$ | $0.9 \times 10^{-2}$ |
FIG. 1: Tight-binding model. Dynamics of the mean velocity (solid line) and the mean squared velocity (inset) for $\tilde{\gamma} = 0.05$ and $F = -0.1$. The dashed line corresponds to Eq. (11).

FIG. 2: Tight-binding model. Dispersion of the wave packet as a function of time for $\tilde{\gamma} = 0.05$. The slopes of the dashed lines are given by values of the diffusion coefficient (14).
FIG. 3: Absolute values of the density matrix elements $\rho_{n,m}(t)$ as a gray-scaled map for $t = 0$ (a) and $t = 5T_B$ (b). The parameters are the same as in Fig. 1.

FIG. 4: Bloch band spectrum of the system (20) for $F = 0$ and $U = 1$ (a) and $U = 4$ (b).
FIG. 5: Coherent (undamped) Bloch oscillations. Absolute value of the atomic wave function is shown as a gray-scaled map. The system parameters are $U = 1$ and $F = 0.025$.

FIG. 6: Undamped Bloch oscillations. The mean atomic velocity (a) and the dispersion of the wave packet (b) are shown as the functions of time. Parameters are the same as in Fig. 4. The dashed lines show the results of the single-band approximation.
FIG. 7: Damped Bloch oscillations. The same as in Fig. 5 but for $\tilde{\gamma} = 0.01$.

FIG. 8: Damped Bloch oscillations. The same as in Fig. 6 but for $U = 4$. (Weak oscillations of the velocity is an artificial fact.