Unidirectional Heat Transfer into a Rotating Cylinder Containing R-134a Refrigerant

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Abstract. An analysis was conducted on the feasibility of designing a unidirectional heat transfer cylindrical wall suitable for microgravity conditions. The heat transfer wall is unidirectional in the sense that heat is only allowed to flow in one direction, which is made possible by the principles of phase-change heat transfer (PCHT). The wall is to consist of two concentric cylindrical walls and is to direct heat from the outer wall to the inner wall radially. A mathematical model is presented for the proposed unidirectional heat transfer wall. Preexisting heat transfer correlations were utilized to calculate the heat transfer rate of the unidirectional heat transfer wall with R-134a as the working fluid. The present study demonstrates that the unidirectional heat transfer wall can switch from a heat conductor to a thermal insulator by adjusting the loading of the working fluid within the walls of the proposed device.

1. Introduction
One of the challenges of designing a thermal control system for space missions is the presence of extreme temperature fluctuations in space. For this, phase-change heat transfer (PCHT) that are highly coveted in engineering due to the high heat transfer rates can be a key tool. However, in the application of space missions, separation of the liquid and vapor phases does not occur due to the absence of gravity. In lieu of the gravitational force, the present study suggests that a centrifugal force is utilized for the separation of phases. The design of the unidirectional heat transfer walls in figure 1 consists of a double-walled cylindrical tube, in which fluid is contained within the wall. The heat transfer wall is designed to enclose an environment to be maintained at a lower temperature than the temperature of the surroundings. When the device remains still, the vapor covers the inside surface of the outer wall, leading to insulating the wall. When the device is rotated axially, the centrifugal force separates liquid and vapor phases of the working fluid allowing heat transfer from outside to inside.

Figure 1. Cross-sectional view of the unidirectional heat transfer wall.
In the preliminary stages of research, efforts were made to find literature that modeled systems similar to that of the proposed unidirectional heat transfer wall. A topic of particular interest was the modeling of axially rotating heat pipes. Rotating heat pipes are similar in that they also utilize both PCHT and rotational motion to achieve effective heat transfer. Rotating heat pipes are PCHT devices that are commonly implemented on rotating machinery, such as high-speed drills, motors, and compressors [1-3]. The principles of rotating heat pipes provided conceptual insight into the design of the unidirectional heat transfer wall. However, their mathematical models were not suitable for the purposes of the present research. Therefore, the present research focused on formulating a suitable model for the unidirectional heat transfer wall—a model that serves as a proof of concept for the feasibility of the heat transfer wall’s design. The calculation of the heat transfer coefficient provides insight into the range of adjustability in performance of the unidirectional heat transfer wall.

2. Model Development

The inner and outer walls of the device are assumed to spin at the same angular velocity in our model. No-slip condition for both the inner and outer walls allows the working fluid to be stationary relative to the walls and consequently leads to pool boiling of the working fluid. Figure 2 shows the orientation of the Cartesian coordinate system. The liquid velocity in the x-direction is assumed to be negligible. The working fluid is R-134a. The heat transfer analysis of the present study is limited to the space between the inner and outer walls of the system.

![Figure 2. Diagram of the coordinate system of the unidirectional heat transfer wall.](image)

As liquid velocity is not of concern in modeling the system, simple existing Nusselt correlations for boiling and condensation can be used in calculating the heat transfer coefficient of the unidirectional heat transfer wall. In respects to analysis of evaporation of the liquid film, no forced convection is assumed in the x-direction – heat transfer in the liquid film is assumed to be of natural convection alone. Marto [4] reported on a correlation formulated by Korner for natural convection in nucleate pool boiling:

\[
Nu_e = 0.133Ra^{0.375}
\]

where \( Ra \) is the Rayleigh number based on the centrifugal acceleration \( a \) and the liquid film thickness \( \delta \). This Nusselt correlation allows for the calculation of the Nusselt number for the process of evaporation for the heat transfer wall. This Nusselt number is subsequently used to calculate the evaporation heat transfer coefficient, \( h_e \).

Gerstmann et al. [5] suggested an analysis of the thickness of the condensate film for horizontal downward facing horizontal plates. Such an analysis provides a order of magnitude estimate to the relative size of the condensate droplets. The film thickness was determined to be relatively small, which gives rationale to conclude that the effects of the returning condensate drops on the liquid film are negligible.

As the system is dependent on the processes of both evaporation and condensation, Nusselt correlations for both processes must be obtained in order to calculate the overall heat transfer coefficient of the wall. In the study of Nusselt correlations for condensation, most models stem from the analytical model of condensation of a laminar film on a vertical surface, as presented by Nusselt [6]:

\[
Nu_c = 0.023Ra^{0.8}
\]
\[ N_u_c = 0.943 \left[ \frac{T_c h' f g a (\rho_l - \rho_g)}{k_l \nu_l (T_{sat} - T_i)} \right]^{\frac{1}{2}} \] (2)

where \( h' f g \) is a modified expression given for the latent heat of vaporization, \( L_c \) is the characteristic length, \( a \) is the centrifugal acceleration, \( \rho_l \) and \( \rho_v \) are the density of liquid and vapor, respectively, \( k_l \) is the liquid thermal conductivity, \( \nu_l \) is the liquid kinematic viscosity, and \( T_{sat} \) and \( T_i \) are the saturation and inner wall temperature:

\[ h' f g = h_f g (1 + 0.68 J a) \] (3)

where \( h_f g \) is the latent heat of vaporization and \( J a \) is the Jacob number. This condensation can be written in a general form, which is as follows:

\[ N_u_c = c \left[ \frac{T_c h' f g a (\rho_l - \rho_g)}{k_l \nu_l (T_{sat} - T_i)} \right]^n \] (4)

where the values of \( c \) and \( n \) account for the different geometric configurations of condensation. The values of \( c \) and \( n \) can be obtained analytically; however, for more complex configurations, these values are often found empirically. In modeling the unidirectional heat transfer wall, the geometric configuration for the process of condensation was assumed to be a horizontal downward facing plate. Gerstmann et al. [5] reported on correlations for laminar film condensation on the underside of horizontal and inclined surfaces. The paper provides the following correlation for laminar film condensation on horizontal downward facing plates:

\[ N_u_c = \begin{cases} 0.69 R a^{0.200}, & 10^6 < R a < 10^8 \\ 0.81 R a^{0.195}, & 10^8 < R a < 10^{10} \end{cases} \] (5)

where the characteristic length used in the analysis is defined as the following:

\[ L_c = \left( \frac{\sigma}{a (\rho_l - \rho_v)} \right)^{\frac{1}{2}} \] (6)

The Nusselt number allows for the calculation of the condensation heat transfer coefficient, \( h_c \). A notable observation of the characteristic length used in this analysis is that the characteristic length is independent of any physical dimensions of the system. Rather, the characteristic length is a function of the acceleration, respective densities, and surface tension \( \sigma \) of the fluid.

Once the heat transfer coefficients for both evaporation and condensation are obtained, the heat flux can be calculated for the system. The overall heat flux of the boiling/condensation model is expressed as the product of the overall heat transfer coefficient and the outer and inner wall temperature difference:

\[ q'' = U (T_o - T_i) \] (7)

\( U \) is defined as the combined heat transfer coefficient of the system, where \( h_e \) and \( h_c \) are the heat transfer coefficients for evaporation and condensation, respectively:

\[ U = \left( \frac{1}{h_e} + \frac{1}{h_c} \right)^{-1} \] (8)
3. Adjustable Heat Transfer Rate

The heat transfer characteristics and performance were investigated using the present model. The inner and outer walls rotate at a constant angular velocity; however, the inner and outer wall accelerations are proportional to their respective diameters. Note that the acceleration of the outer wall is greater than that of the inner wall, as the larger radius of the outer wall produces a larger tangential acceleration. Taking this into account, the inner wall acceleration was used for all evaporation calculations, while the outer wall acceleration was used for all condensation calculations. The acceleration on the outer wall were set to be $a_o = 1000 \text{ m/s}^2$. A wall temperature differential, $\Delta T$ was considered in determining the temperatures of the inner and outer walls. For instance, a $\Delta T$ value of 5 °C corresponds to constant wall temperatures assumed at values of 30 °C and 20 °C for the outer and inner walls, respectively (given a saturation temperature of 25 °C).

Table 1 provides the thermophysical properties of R-134a, and the dimensions of the heat transfer wall used in this study. All properties were assumed to be at saturation conditions, for both evaporation and condensation. For this particular analysis, properties of R-134a at a saturation temperature of 25 °C were used. Properties were obtained from Cengel et al. [7].

![Figure 3. $q''$ as a function of $\delta$, given $\Delta T$: ○, 3 °C; ▽, 5 °C; □, 7 °C.](image)

![Figure 4. $U$ as a function of $\delta$, given $\Delta T$: ○, 3 °C; ▽, 5 °C; □, 7 °C.](image)

Table 1. Dimensions of heat transfer wall and thermophysical properties of saturated R-134a.

| Parameters                        | Notation | Value | Units    |
|-----------------------------------|----------|-------|----------|
| outer wall diameter               | $D_o$    | 0.100 | m        |
| inner wall diameter               | $D_i$    | 0.090 | m        |
| saturation temperature            | $T_{sat}$| 25    | °C       |
| thermal conductivity (liquid)     | $k_l$    | 8.33 $\times 10^{-2}$ | W m$^{-1}$K$^{-1}$ |
| volume expansion coefficient      | $\beta$  | 3.24 $\times 10^{-3}$ | K$^{-1}$ |
| density (liquid)                  | $\rho_l$ | 1207  | kg m$^{-3}$ |
| density (vapor)                   | $\rho_g$ | 32.34 | kg m$^{-3}$ |
| enthalpy of vaporization          | $h_{fg}$ | 177,800 | J kg$^{-1}$ |
| dynamic viscosity (liquid)        | $\mu_l$  | 2.012 $\times 10^{-4}$ | kg m$^{-1}$s$^{-1}$ |
| kinematic viscosity (liquid)      | $v_l$    | 1.667 $\times 10^{-7}$ | m$^2$s$^{-1}$ |
| surface tension                   | $\sigma$ | 8.08 $\times 10^{-3}$ | N m$^{-1}$ |
| specific heat (liquid)            | $c_{p,l}$| 1427  | J kg$^{-1}$ K$^{-1}$ |
| thermal diffusivity (liquid)      | $\alpha$ | 4.836 $\times 10^{-8}$ | m$^2$s$^{-1}$ |

Figures 3 and 4 summarize the results of the heat flux and heat transfer coefficient calculations. $q''$ and $U$ values are given for various $\Delta T$ values, as a function of the thickness $\delta$ of the liquid layer. The graphs show that the heat transfer characteristics of the system can be adjusted significantly by varying $\Delta T$. 
The graphs show that for the given temperature differentials, the heat flux was calculated to be on the order of 10,000 W/m². This validates the hypothesis that the boiling/condensation system will produce a high heat flux for the heat transfer wall. One thing we must note is that as the no-slip condition for the liquid and vapor were applied to the inner and outer walls of the system, we assumed zero relative velocity of the liquid and of the vapor in the tangential direction. When the outer wall only spins while the inner wall is stationary, we will expect to achieve higher heat transfer performances, as the tangential velocities of the liquid and vapor will not be negligible, which increases heat transfer by mode of forced convection.

In isolating just one individual curve from the graph, we can see that varying the liquid film thickness $\delta$ has a significant effect on the associated heat transfer coefficient and heat flux of the heat transfer wall. This means that indeed, the thermal resistance of the system can be adjusted by modification of fluid loading. The thermal resistance can also be adjusted with the modification of rotational speed and wall temperature differential, which is demonstrated by the graphs.

4. Conclusions
By calculating the heat flux of the unidirectional heat transfer wall for various conditions, we were able to validate that the device can operate as both a heat conductor and a thermal insulator. Many assumptions had to be made in developing a model for the heat transfer wall. A summary of future studies of the heat transfer wall is as follows:

- In respects to fluid mechanics, the liquid and vapor were assumed to be stationary in the tangential direction. This was assumed under the no slip boundary condition applied to the working fluid within the system. However, this may not be the case for a real physical system. Further studies will be made to account for the associated velocities of the working fluid, with subsequent investigation into how the fluid mechanics of the system affects the convective heat transfer of the system.
- The effects of returning condensate drop on the liquid evaporation film were neglected in our model. Although the relative size of drops is predicted to be small, the velocity of the returning condensate drops is expected to enhance convective heat transfer performance to some degree. The study of returning condensate drops will be thoroughly studied by experimental means.
- Further study will be conducted on the thickness of the condensate liquid film, and how this film thickness varies with fluid loading. This is necessary to be able to quantify the amount of working fluid needed in the system to produce the associated evaporation and condensation liquid film thicknesses.

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