Energetic-particle-modified global Alfvén eigenmodes

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15th IAEA TM on Energetic Particles
Princeton, NJ
September 5 – 8, 2017
Why study GAEs in NSTX-U?

- GAEs (and CAEs) have been linked to anomalous $T_e$ flattening at high beam power in NSTX\textsuperscript{1}
- Recently shown to be suppressed by off-axis neutral beam injection\textsuperscript{2}
- Avalanches and chirping can generate large fast ion losses
  - Presents opportunities to probe nonlinear physics

\textsuperscript{1}Stutman \textit{et al.} Phys. Rev. Lett. \textbf{102}, 115002 (2009)
\textsuperscript{2}Fredrickson \textit{et al.} Phys. Rev. Lett. \textbf{118}, 265001 (2017)
• High frequency Alfvén eigenmodes routinely excited in NSTX(-U) plasmas by neutral beam injection
  – Driven by Doppler-shifted cyclotron resonance with fast ions
    \[ \omega - \langle k_\parallel v_\parallel + k_\perp v_{Dr} \rangle = I \langle \omega_{ci} \rangle \]
• Identified as combination of compressional (CAE) and global (GAE) Alfvén eigenmodes
  – Co-/cntr-propagating \(|n| \approx 3 – 14\)
  – \(\omega_{C/GAE}/\omega_{ci} \approx 0.1 – 0.7 (\gg \omega_{TAE})\)
Perturbative GAE Properties

- Weakly damped GAEs may exist below minimum of Alfvén continuum
  - Approximate dispersion
    \[ \omega \leq \left[ k_\parallel (r) v_A(r) \right]_{\text{min}} \]
- Shear Alfvén mode: \( \delta B_\perp \gg \delta B_\parallel \)
  - In NSTX conditions, also have large compressional component
    \( \delta B_\parallel \approx \delta B_\perp \) near edge
- Perturbative assumption
  Fast ions provide drive but do not modify mode properties

\[ n = -3 \text{ GAE calculated by NOVA in NSTX}^3 \]

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\(^3\)Gorelenkov et al. Phys. Plasmas 11, 2586 (2004)
Hybrid Simulation Method

- Hybrid MHD and Particle code (HYM)\(^4\)
  - Single fluid resistive MHD thermal plasma
  - Full orbit kinetic fast ions with $\delta F$ scheme

- Initial value code in 3D toroidal geometry
- Linear fluid equations and unperturbed particle trajectories
- Self-consistent equilibrium including energetic particle effects

\[
\frac{\partial^2 \psi}{\partial z^2} + R \frac{\partial}{\partial R} \left( \frac{1}{R} \frac{\partial \psi}{\partial R} \right) = -R^2 P' - H H' - G H' + RJ_{b\phi}
\]

\[
B = \nabla \phi \times \nabla \psi + h \nabla \phi \quad h(R, z) \equiv H(\psi) + G(R, z) \quad J_{b, pol} = \nabla G \times \nabla \phi
\]

$\rightarrow$ pressure anisotropy, increased Shafranov shift, more peaked current

\(^4\)Belova et al. Phys. Plasmas 10, 3240 (2003)
HYM Physics Model

Fluid thermal plasma

\[ \rho \frac{dV}{dt} = -\nabla P + (J - J_b) \times B \]

\[ -en_b(E - \eta \delta J) + \mu \Delta V \]

\[ E = -V \times B + \eta \delta J \]

\[ \frac{\partial B}{\partial t} = -\nabla \times E \]

\[ \mu_0 J = \nabla \times B \]

\[ \frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho V) \]

\[ \frac{d}{dt} \left( \frac{P}{\rho^\gamma} \right) = 0 \]

Kinetic fast ions

\[ \frac{dx}{dt} = \nu \]

\[ \frac{dv}{dt} = \frac{q_i}{m_i} (E - \eta \delta J + v \times B) \]

\[ \delta F \text{ Scheme} \]

\[ F = F_0(E, \mu, p, \phi) + \delta F(t) \]

\[ w = \delta F / F \]

\[ \frac{dw}{dt} = - \left( \frac{F}{P} - w \right) \frac{d \ln F_0}{dt} \]

- \( \rho, V, P \) are plasma mass density, velocity, and pressure
- \( n_b, J_b \) are beam ion density and current
  - Assuming \( n_b \ll n_e \) but allowing \( J_b \approx J_{th} \)
Fast Ion Distribution

- Equilibrium distribution $F_0 = F_1(v)F_2(\lambda)F_3(p_\phi, v)$
  - Energy $\mathcal{E} = \frac{1}{2}m_iv^2$
  - Trapping parameter $\lambda = \mu B_0/\mathcal{E} \approx \mathcal{E}_\perp B_0/\mathcal{E} B$
    - Passing: $0 < \lambda < 1 - \epsilon$
    - Trapped: $1 - \epsilon < \lambda < 1 + \epsilon$
  - Canonical angular momentum $p_\phi = -q_i\psi + m_iRv_\phi$

\begin{align*}
F_1(v) &= \frac{1}{v^3 + v_c^3} \quad \text{for } v < v_0 \\
F_2(\lambda) &= \exp \left( -\frac{(\lambda - \lambda_0)^2}{\Delta \lambda^2} \right) \\
F_3(p_\phi, v) &= \left( \frac{p_\phi - p_{\min}}{m_iR_0v - q_i\psi_0 - p_{\min}} \right)^\alpha \quad \text{for } p_\phi > p_{\min}
\end{align*}

- NSTX: $v_0/v_A \lesssim 5$, $v_c \approx v_0/2$, $\lambda_0 = 0.7$, $\Delta \lambda = 0.3$, $\alpha = 6$
- Simulations expand to $v_0/v_A = 2 - 6$, $\lambda_0 = 0.1 - 0.9$
• GAE frequency changes dramatically with $v_0/v_A$ for all $n$
  – Can change by $20 - 50\%$, or $100 - 500$ kHz
• Change is continuous to at least $\Delta v_0 = 0.1 v_A$ resolution
  – Uncharacteristic of excitation of distinct MHD modes with discrete frequencies

• Sign of change consistent with resonance condition
EP Effects on Frequency

- Two independent ways EPs can affect mode frequency
  - **Equilibrium** Changes to EP distribution can modify self-consistent equilibrium, shifting continuum
  - **Phase Space** Moving EP in phase space changes range of accessible resonances

- \( J \equiv n_b v_0 / n_e v_A \propto J_{\text{beam}} / J_{\text{plasma}} \) characterizes EP influence
- Varying \( n_b / n_e \) and \( v_0 / v_A \) independently distinguishes frequency dependence due to equilibrium changes alone from total change in frequency
- Calculating equilibrium *without* EP effects while varying \( v_0 / v_A \) isolates frequency dependence on EP phase space
  - Computed with same total current as self-consistent case
  - EP pressure absorbed into thermal pressure
Self-Consistent Equilibrium Simulations II

- EP-induced changes to equilibrium not large enough to account for change in frequency
  - Red squares: simulations with \( n_b/n_e = 0.053 \), varying \( v_0/v_A \)
    - Slope \( \frac{d\omega}{dJ} \) is total effect of equilibrium and EP phase space
  - Blue circles: simulations with \( v_0/v_A = 5.5 \), varying \( n_b/n_e \)
    - Slope \( \left( \frac{\partial \omega}{\partial J} \right)_{EQ} \) is effect due to changes in equilibrium

\[
\frac{d\omega}{dJ} = 1.03
\]

\[
\left( \frac{\partial \omega}{\partial J} \right)_{EQ} = -0.27
\]

\[
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\]

\[
\left( \frac{\partial \omega}{\partial J} \right)_{EQ} = -0.13
\]
MHD Equilibrium Simulations

- EP phase space produces most of change in frequency
- Green triangles: simulations with $n_b/n_e = 0.053$, varying $v_0/v_A$ with a single “MHD” equilibrium
  - Slope $\left( \frac{\partial \omega}{\partial J} \right)_{EP}$ is effect due to changes in EP phase space
- Equilibrium and EP phase space effects are nearly linear

$$\frac{d\omega}{dJ} \approx \left( \frac{\partial \omega}{\partial J} \right)_{EQ} + \left( \frac{\partial \omega}{\partial J} \right)_{EP} = n_e v_A \left[ \frac{1}{v_0} \frac{\partial \omega}{\partial n_b} + \frac{1}{n_b} \frac{\partial \omega}{\partial v_0} \right]$$

\[ n = 6 \text{ cntr-GAE} \]

\[ \left( \frac{\partial \omega}{\partial J} \right)_{EP} = -1.14 \]

\[ \frac{d\omega}{dJ} = 1.03 \]

\[ \left( \frac{\partial \omega}{\partial J} \right)_{EQ} = -0.27 \]

\[ n = 9 \text{ co-GAE} \]

\[ \left( \frac{\partial \omega}{\partial J} \right)_{EP} = 1.12 \]

\[ \frac{d\omega}{dJ} = 1.03 \]

\[ \left( \frac{\partial \omega}{\partial J} \right)_{EQ} = -0.13 \]
• Modes are not localized near minimum of Alfvén continuum
  – Frequencies are 100 – 250 kHz below nearest minimum
• Modes intersect continuum substantially, yet do not have singular structure
• Structure is closest to $m = 0$, while frequencies are far from this branch

$n = 9$ continuum for the “MHD” equilibrium, including EP pressure as part of total plasma pressure. Calculated by NOVA with $|m| \leq 22$
Mode Structure of co-GAEs

• Mode structure does not change qualitatively with frequency
  – Slight changes: peak location moves gradually inwards, mode becomes slightly elongated

• Frequency changes $\approx 20\%$ from $\omega/\omega_{ci} = 0.24$ to $0.29$ ($\Delta\omega = 125$ kHz) due to $15\%$ change in $v_0/v_A$

• Remains dominated by $m = 0$ harmonic
Inferred Dispersion

- Frequencies are near local shear Alfvén dispersion
  - \( \omega \approx k_{\parallel} v_A \pm 20\% \) evaluated at mode location
  - \( k_{\parallel} \) calculated as peak of FT of flux surface average of \( \delta B_\perp \)
    - \( k_{\parallel} \approx (n - m/q) / R_0 \) is an unreliable approximation here
- co-GAEs are well-fit by dispersion \( \omega = 1.57 k_{\parallel} v_A - 0.17 \)
- cntr-GAEs have larger spread, likely due to larger range of \( m \)
- Accurate dispersion must include EP nonperturbatively
Resonance Implies Frequency Change

- Time evolution of particle weights identifies resonant particles
- Combination of dispersion and resonance yields frequency dependence on $v_{\parallel}^{res}$

$$\omega = \frac{\langle \omega_{ci} \rangle}{l + \langle v_{\parallel} \rangle / v_A}$$

where $l = -\text{sign} \ k_{\parallel}$

- Unresolved: how does $v_{\parallel}^{res}$ depend on $v_0 / v_A$ quantitatively?
Resonant particles move to higher energy as $v_0/v_A$ increases.

In $\langle \omega_\phi \rangle, \langle \omega_{ci} \rangle$ coordinates, particles with largest weights cluster around resonances: $\omega - n \langle \omega_\phi \rangle - p \langle \omega_\theta \rangle = l \langle \omega_{ci} \rangle$

Frequency of mode is being set by location of resonant particles in phase space – characteristic of EPMs.
Experimental Clues

- Experimental analysis\(^5\) of many NSTX discharges shows cntr-GAE frequency decrease with increasing \(|n|\)
  - Opposite trend expected from dispersion
    \[ \omega = -k v_A \propto -n \]
  - Resonance condition more important than dispersion for frequency?
- Provides clues but not confirmation without measurements of \(m\)

\(^5\)Tang et al. 2017 EU/US Transport Task Force (Williamsburg, VA, April 25-28)
Other Nonperturbative Solutions

- Many examples of low frequency nonperturbative modes exist
  - Fishbone, E-GAM, RSAE cascades, RTAE, EPMs, etc
- Numerical results may indicate first high frequency solution
  - Key difference: excitation through Doppler-shifted cyclotron resonance vs Landau resonance
- EP-GAE shares characteristics of low frequency cases
  - Frequency tracks characteristic frequencies of EP motion
  - Prone to large frequency variation
  - Mode structure is robust against frequency change
Summary

- Self-consistent hybrid simulations reveal strong EP modifications to GAEs in NSTX conditions
- Frequency changes significantly and continuously with $v_0/v_A$
  - Sign and magnitude of change are consistent with the resonance condition
  - Mostly due to changes in EP phase space, not EP-induced changes to equilibrium
- Mode structure does not change substantially with frequency
  - MHD eigenmodes would have different mode numbers $(m, s)$ associated with large frequency changes
- Results may indicate a new, high frequency EPM
  - Energetic-particle-modified global Alfvén eigenmode (EP-GAE)
Open Questions and Future Work

- Derive EP-GAE dispersion nonperturbatively
- Demonstrate transition from perturbative GAE to EP-GAE
- Experimental identification in NSTX-U
  - Are the high frequency shear Alfvén modes routinely excited in NSTX actually EP-GAEs or GAEs?
- Explore impact on $T_e$ flattening theories
  - How sensitive are existing theories of enhanced electron heat transport\(^6,^7\) to properties of EP-GAE vs GAE modes?

Paper submitted to Phys. Plasmas
Contact jlestz@pppl.gov for preprint

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\(^6\)Kolesnichenko et al. Phys. Rev. Lett. 104, 075001 (2010)
\(^7\)Gorelenkov et al. Nucl. Fusion 50, 084012 (2010)