Study of 1D stranged-charm meson family using HQET

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Abstract

Recently LHCb predicted spin 1 and spin 3 states $D_{s1}^*(2860)$ and $D_{s3}^*(2860)$ which are studied through their strong decays, and are assigned to fit the $1^{3}D_{1}$ and $1^{3}D_{3}$ states in the charm spectroscopy. In this paper, using the heavy quark effective theory, we state that assigning $D_{s1}^*(2860)$ as the mixing of $1^{3}D_{1} - 2^{3}S_{1}$ states, is rather a better justification to its observed experimental values than a pure state. We study its decay modes variation with hadronic coupling constant $g_{xh}$ and the mixing angle $\theta$. We appoint spin 3 state $D_{s3}^*(2860)$ as the missing $1D_{3}$ state, and also study its decay channel behavior with coupling constant $g_{yh}$.

To appreciate the above results, we check the variation of decay modes for their spin partners states i.e. $1D_{2}$ and $1D_{2}'$ with their masses and strong coupling constant i.e. $g_{xh}$ and $g_{yh}$. Our calculation using HQET approach give mixing angle between the $1^{3}D_{1} - 2^{3}S_{1}$ state for $D_{s1}^*(2860)$ to lie in the range $\Theta \in [-1.6 \text{ radians} \leq \Theta \leq -1.2 \text{ radians}]$. Our calculation for coupling constant values gives $g_{xh}$ to lie between value $0.17 - 0.20$ and $g_{yh}$ to be 0.40. We expect from experiments to observe this mixing angle to verify our results.

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1 Introduction

Over the last decade many new heavy-light mesons have been observed by various experimental collaborations. The $D_{sJ}^*(2860)$ was first observed by the BaBar Collaboration in $D_{s1}^*(2860) \rightarrow D^0K^+, D^+K^0$ with mass $M = 2856.6 \pm 1.5 MeV$ and width $\Gamma = 48 \pm 7 MeV$ [1]. It was supposed to have natural parity states i.e. $0^+, 1^-, 2^+, 3^-$ etc. State $D_{sJ}^*(2860)$ as the $0^+$ state was ruled out after the observation of $D_{sJ}^*(2860) \rightarrow D^+K[2]$. Along with the $D^+K$ channel [2] BaBar also gives the ratio $R$ measured as $R = \frac{Br(D_{sJ}^*(2860) \rightarrow D^+K)}{Br(D_{sJ}^*(2860) \rightarrow DK)} = 1.10$. The $D_{sJ}^*(2860)$ went through extensive discussions by various theoretical models, to find a place in strange charm spectrum. Zang et al has assign the $D_{sJ}^*(2860)$ as $2^3P_0$ or $1^3D_3$ states using the $3P_0$ model[3], Colangelo et al. assign the $D_{sJ}^*(2860)$ to be $1^3D_3$ state using the heavy meson effective theory [4,5], D.M.Li et al., also favors the $D_{sJ}^*(2860)$ as the $2^3P_0$ or $1^3D_3$ state using Regge phenomenology [6]. Different approaches calculated different value of the R ratio. Heavy Quark Effective Theory predicts $R$ to be $\approx 0.39[4]$, while $3P_0$ model calculated it to be $R = 0.59$ which is deviating from the the experimental value $R = 1.10$. All these references favored $D_{sJ}^*(2860)$ as $1^3D_{3}$ state due to observed narrow decay

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width at the cost of mismatch of R with experiments. Li and Ma assign \(D_{sJ}(2860)\) to be mixing state of \(1^3D_1 - 2^3S_1\) with \(D_{s1}^*(2700)\) to be its orthogonal partner \([7]\) and obtained R= 0.8 nearly close to the experimental value.

Recently LHCb Collaboration predicted a new resonance around 2.86 GeV in the \(\overline{D}^0 K^-\) invariant mass spectrum from decay channel \(B_s^0 \rightarrow \overline{D}^0 K^-\Pi^+\), containing the mixture of spin-1 and spin-3 states components corresponding to \(D_{s1}^*(2860)\) and \(D_{s3}^*(2860)\) \([8,9]\) where the mass and width parameters are

\[
\begin{align*}
M(D_{s1}^*(2860)) &= 2859 \pm 12 \pm 6 \pm 23 MeV \\
\Gamma(D_{s1}^*(2860)) &= 159 \pm 23 \pm 27 \pm 72 MeV \\
M(D_{s3}^*(2860)) &= 2860.5 \pm 2.6 \pm 2.5 \pm 6.0 MeV \\
\Gamma(D_{s3}^*(2860)) &= 53 \pm 7 \pm 4 \pm 6 MeV
\end{align*}
\]

Here the first error is statistical error, second is the experimental systematic effects and the last one is due to model variations. LHCb observed two \(D_{sJ}^*(2860)\) states with spin 1 and spin 3.

From the previous study it can be speculated that it is a spin 3 resonance of \(D_{sJ}^*(2860)\) that belongs to \(1^3D_3\) state, with a narrow width \(\Gamma = 53 MeV\). Theoretically, R value can be matched with the experimental value, considering its contribution coming from spin \(1\) \(D_{s1}^*(2860)\) resonance. Comparing the earlier theoretical mass predictions, \(D_{s1}^*(2860)\) can be assumed to fit in \(1^3D_1\) state of 1D family or can be a mixture of \(1^3D_1\) and \(2^3S_1\) states. Assigning \(D_{s1}^*\) as a mixing state of \(1^3D_1 - 2^3S_1\) may be a better justification than assigning it as a pure state, because the R value calculated here matches with the experimental data. By choosing suitable mixing angle \((\theta)\), the calculated R value is better justified with the experimental value. X.H.Zhong by chiral quark model \([10,11]\) studied the \(D_{sJ}^*\) state as the \(1^3D_3\) state with some \(1^3D_2 - 1^1D_2\) mixing. Wang \([12]\) tried to reproduce the experimental value R= 1.10 with some suitable hadronic coupling constants, by including chiral symmetry breaking corrections in heavy quark effective theory. Besides these studies, Vijande et al., also assign \(D_{sJ}^*(2860)\) to be the multi-quark exotic state as \(c\bar{s} - c\bar{n}\bar{s} \bar{\pi}\) \([13]\). Stephen Godfrey by adopting the pseudoscalar emission decay model\([14]\), Qing-Tao Song by QPC model \([15]\) studied \(D_{sJ}^*\) as \(1^3D_1 - 2^3S_1\). Various predictions are made to study the mixing effects in \(D_{sJ}^*\) state \([16,17,18,19]\).

In Particle data Group \([20]\) 1S and 1P stranged charm states are nicely described, but information for other states is still missing. Thus \(D_{s1}^*(2860)\) and \(D_{s3}^*(2860)\) can be fitted in 1D state family. The strange meson states with \(J^P\) states predicted by various theoretical model is gathered in Table[I].

Table[I] shows the state and decay modes of 1D family. Here it is seen, that complete information about the states and decay modes is still missing, thus more theoretical and experimental efforts are to be made. In this paper, we study the strong decays and R values for 1D family using heavy quark effective approach and analyze the mixing effects in such states. In the past years, HQET has been successful in assigning suitable \(J^P\) states to the observed D and B-mesons using their decays widths in terms of coupling constants. We use HQET approach to study \(D_{s1}^*(2860)\) as a pure state, and also as a mixture of \(1^3D_1\) and \(2^3S_1\) states and would rather classify them on the basis of their \(J^P\) state analyzing their strong decays channels and hadronic strong coupling constants \(gxh\) and \(gyh\). \(D_{s3}^*(2860)\) is assigned the \(1^3D_3\) position in 1D strange charm mesons. To complete the 1D family, we also try to study the behavior of their spin partners i.e. \(1D_2\) and \(1D_2\) which are still missing experimentally.

The paper is divided in the following sections. Section 2 describes the heavy quark effective theory formalism used for the strong decays. Section 3 discusses the members of 1D family. In
Table 1: Theoretically predicted masses

| $J^P(2s+1L)$ | GI[21] (MeV) | PE[22] (MeV) | EFG[23] (MeV) | Expt.[20] (MeV) |
|--------------|-------------|-------------|--------------|----------------|
| 0$^-$($^1S_0$) | 1979        | 1965        | 1969         | 1968           |
| 1$^-$($^3S_1$) | 2129        | 2113        | 2111         | 2112           |
| 0$^+$($^3P_0$) | 2484        | 2487        | 2509         | 2318           |
| 1$^+$($^1P_1$) | 2459        | 2535        | 2536         | 2460           |
| 1$^+$($^3P_1$) | 2556        | 2605        | 2574         | 2536           |
| 2$^+$($^3P_2$) | 2592        | 2581        | 2571         | 2573           |
| 1$^-$($^3D_1$) | 2899        | 2913        | 2913         | 2859           |
| 2$^-$($^2D_2$) | 2900        | 2900        | 2931         | -              |
| 2$^-$($^3D_2$) | 2926        | 2953        | 2961         | -              |
| 3$^-$($^3D_3$) | 2917        | 2925        | 2871         | 2860           |

this section, all the four states with their decay modes in terms of their couplings are described in different subsections. To appreciate the experimental value of $R$, various mixing effects in terms of mixing angle theta is studied. We finally conclude our results in section 4.

2 Framework

In the heavy quark limit $m_Q >> \Lambda_{QCD} >> m_q$, $Q\overline{q}$ system can be effectively studied using Heavy quark effective theory. According to this theory, heavy quark acts like static color source with spin $s_Q$, which due to heavy flavor symmetry, interacts only with the light degree of freedom having spin $s_l$ through the exchange of soft gluons. This picture can be compared with that of hydrogen atom [24]. The basic idea is that in a $Q\overline{q}$ system, heavy quark plays the role of a nucleus and the light quark plays the role of an electron. This $Q\overline{q}$ system can be categorized in doublets in relation to the total conserved angular momentum i.e. $s_l = s_{\overline{q}} + L$, where $s_{\overline{q}}$ and $L$ are the spin and orbital angular momentum of the light anti-quark respectively. For $L=0$(S-wave) the doublet is represented by $(D, D^*)$ with $J_{s_l}^P = (0^-, 1^-)_{1/2}$, which for $L=1$(P-wave) , there are two doublets represented by $(D_0^0, D_1)$ and $(D_1^*, D_2)$ with $J_{s_l}^P = (0^+, 1^+)_{1/2}$ and $(1^+, 2^+)_{3/2}$ respectively. Two doublets of $L=2$ (D-wave) are represented by $(D_1^*, D_2)$ and $(D_2^*, D_3)$ belonging to $J_{s_l}^P = (1^-, 2^-)_{3/2}$ and $(2^-, 3^-)_{5/2}$ respectively. These doublets are described by the effective super-field $H_a, S_a, T_a, X_a, Y_a$ [25], where the field $H_a$ describe the $(D, D^*)$ doublet i.e. S-wave, $S_a$ and $T_a$ fields represents the P-wave doublets $(0^+, 1^+)_{1/2}$ and $(1^+, 2^+)_{3/2}$ respectively. D-wave doublets are represented by the $X_a$ and $Y_a$
From the chiral lagrangian terms

\[ H_a = \frac{1 + \gamma}{2} \{ P_{a\mu} \gamma^\mu - P_{a5} \} \] (1)

\[ S_a = \frac{1 + \gamma}{2} \{ P_{a\mu} \gamma_\mu \gamma_5 - P_{a5} ^* \} \] (2)

\[ T^\mu_a = \frac{1 + \gamma}{2} \{ P_{2a} \gamma_\mu - P_{1a} \sqrt{\frac{3}{2}} \gamma_5 [g^{\mu\nu} - \frac{\gamma^\nu (\gamma^\mu - v^\mu)}{3}] \} \] (3)

\[ X^a = \frac{1 + \gamma}{2} \{ P_{2a} \gamma_5 \gamma_\mu - P_{1a} \sqrt{\frac{3}{2}} [g^{\mu\nu} - \frac{\gamma^\nu (\gamma^\mu + v^\mu)}{3}] \} \] (4)

\[ Y^\mu_\alpha = \frac{1 + \gamma}{2} \{ P_{3a} \gamma_\sigma - P_{2a} \sqrt{\frac{5}{3}} [g^{\mu\nu} - \frac{g_5^\alpha \gamma_\sigma}{5} - \frac{g_5^\alpha \gamma_\sigma}{5}] \} \] (5)

The light pseudoscalar mesons are described by the fields \( \xi = \exp \frac{i M}{\Lambda} \). The pion octet is introduced by the vector and axial combinations \( V = \frac{1}{2} \xi \partial^\mu \xi^\dagger + \xi^\dagger \partial^\mu \xi \) and \( A = \frac{1}{2} \xi \partial^\mu \xi^\dagger - \xi^\dagger \partial^\mu \xi \). We choose \( f_\pi = 130 MeV \). Here, all traces are taken over Dirac spinor indices, light quark \( SU(3)_V \) flavor indices \( a = u, d, s \) and heavy quark flavor indices \( Q = c, b \) [25]. The Dirac structure of chiral Lagrangian has been replaced by velocity vector \( v \). At the leading approximation, the heavy meson chiral lagrangian \( L_{HH}, L_{SH}, L_{TH}, L_{XH}, L_{XS}, L_{XT}, L_{YH}, L_{YS}, L_{YT} \) for the two-body strong decays to light pseudoscalar mesons can be written as:

\[ L_{HH} = g_{HH} Tr \{ \bar{H}_a H_b \gamma_\mu \gamma_5 \Lambda^\mu_{ba} \} \] (6)

\[ L_{SH} = g_{SH} Tr \{ \bar{H}_a \gamma_\mu \gamma_5 \Lambda^\mu_{ba} \} + h.c. \] (7)

\[ L_{TH} = \frac{g_{TH}}{\Lambda} Tr \{ \bar{H}_a T_b^\mu (i D_\mu A + i \partial A_\mu)_{ba} \gamma_5 \} + h.c. \] (8)

\[ L_{XH} = \frac{g_{XH}}{\Lambda} Tr \{ \bar{H}_a X^\mu_{ba} (i D_\mu A + i \partial A_\mu)_{ba} \gamma_5 \} + h.c. \] (9)

\[ L_{XS} = \frac{g_{XS}}{\Lambda} Tr \{ \bar{S}_a X^\mu_{ba} (i D_\mu A + i \partial A_\mu)_{ba} \gamma_5 \} + h.c. \] (10)

\[ L_{XT} = \frac{1}{\Lambda^2} Tr \{ \bar{T}_a X^\mu_{ba} \gamma_\mu [1^T D_\mu, D_\nu] A_\lambda + k^2 [1^T (D_\mu D_\lambda A_\nu + D_\nu D_\lambda A_\mu)]_{ba} \gamma_\lambda \gamma_5 \} + h.c. \] (11)

\[ L_{YH} = \frac{1}{\Lambda^2} Tr \{ \bar{Y}_a Y^\mu_{ba} \gamma_\mu [1^H D_\mu, D_\nu] A_\lambda + k^2 [1^H (D_\mu D_\lambda A_\nu + D_\nu D_\lambda A_\mu)]_{ba} \gamma_\lambda \gamma_5 \} + h.c. \] (12)

\[ L_{YS} = \frac{1}{\Lambda^2} Tr \{ \bar{S}_a Y^\mu_{ba} \gamma_\mu [1^S D_\mu, D_\nu] A_\lambda + k^2 [1^S (D_\mu D_\lambda A_\nu + D_\nu D_\lambda A_\mu)]_{ba} \gamma_\lambda \gamma_5 \} + h.c. \] (13)

\[ L_{YT} = \frac{g_{YT}}{\Lambda} Tr \{ \bar{T}_a X^\mu_{ba} (i D_\nu A + i \partial A_\nu)_{ba} \gamma_5 \} + h.c. \] (14)

From the chiral lagrangian terms \( L_{HH}, L_{SH}, L_{TH}, L_{XH}, L_{YH} \), the two body strong decay of \( Q \bar{Q} \) system to final state light pseudo-scalar mesons \( M (\Pi, \eta, K) \) can be described as
\( (1^{-}, 2^{-}) \rightarrow (0^{-}, 1^{-}) + M \)

\[
\Gamma(1^{-} \rightarrow 0^{-}) = C_{M} \frac{4g_{X}^{2}}{3\Pi f_{H}^{2} A^{2}} M_f \left[ p_{M}^{3}(m_{M}^{2} + p_{M}^{2}) \right]
\]

\[
\Gamma(1^{-} \rightarrow 1^{-}) = C_{M} \frac{2g_{X}^{2}}{3\Pi f_{H}^{2} A^{2}} M_f \left[ p_{M}^{3}(m_{M}^{2} + p_{M}^{2}) \right]
\]

\[
\Gamma(2^{-} \rightarrow 1^{-}) = C_{M} \frac{2g_{X}^{2}}{3\Pi f_{H}^{2} A^{2}} M_f \left[ p_{M}^{3}(m_{M}^{2} + p_{M}^{2}) \right]
\]

\( (2^{-}, 3^{-}) \rightarrow (0^{-}, 1^{-}) + M \)

\[
\Gamma(2^{-} \rightarrow 1^{-}) = C_{M} \frac{4g_{X}^{2}}{15\Pi f_{H}^{2} A^{4}} M_f \left[ p_{M}^{7} \right]
\]

\[
\Gamma(3^{-} \rightarrow 0^{-}) = C_{M} \frac{4g_{X}^{2}}{35\Pi f_{H}^{2} A^{4}} M_f \left[ p_{M}^{7} \right]
\]

\[
\Gamma(3^{-} \rightarrow 1^{-}) = C_{M} \frac{16g_{X}^{2}}{105\Pi f_{H}^{2} A^{4}} M_f \left[ p_{M}^{7} \right]
\]

\( (1^{-}, 2^{-}) \rightarrow (0^{+}, 1^{+}) + M \)

\[
\Gamma(2^{-} \rightarrow 1^{+}) = C_{M} \frac{2g_{X}^{2}}{5\Pi f_{H}^{2} A^{2}} M_f \left[ p_{M}^{5} \right]
\]

\[
\Gamma(2^{-} \rightarrow 0^{+}) = C_{M} \frac{4g_{X}^{2}}{15\Pi f_{H}^{2} A^{4}} M_f \left[ p_{M}^{5} \right]
\]

\[
\Gamma(1^{-} \rightarrow 1^{+}) = C_{M} \frac{2g_{X}^{2}}{3\Pi f_{H}^{2} A^{2}} M_f \left[ p_{M}^{5} \right]
\]

\( (2^{-}, 3^{-}) \rightarrow (0^{+}, 1^{+}) + M \)

\[
\Gamma(3^{-} \rightarrow 1^{+}) = C_{M} \frac{4g_{X}^{2}}{15\Pi f_{H}^{2} A^{4}} M_f \left[ p_{M}^{5}(m_{M}^{2} + p_{M}^{2}) \right]
\]

\[
\Gamma(2^{-} \rightarrow 1^{+}) = C_{M} \frac{8g_{X}^{2}}{75\Pi f_{H}^{2} A^{4}} M_f \left[ p_{M}^{5}(m_{M}^{2} + p_{M}^{2}) \right]
\]

\[
\Gamma(2^{-} \rightarrow 0^{+}) = C_{M} \frac{4g_{X}^{2}}{25\Pi f_{H}^{2} A^{4}} M_f \left[ p_{M}^{5}(m_{M}^{2} + p_{M}^{2}) \right]
\]

In the above expressions of decay width, \( M_i, M_f \) stands for initial and final meson mass, \( g_X \) and \( g_Y \) are hadronic coupling constants, \( \Lambda \) is the chiral symmetry breaking scale = 1GeV, \( p_{M} \) and \( m_{M} \) is the final momentum and mass of the emitted light pseudo-scalar meson. The coefficient \( C_{\Pi^{\pm}}, C_{K^{\pm}}, C_{K^{0}}, C_{\bar{K}^{0}} = 1, C_{\Pi^{0}} = \frac{1}{3} \) and \( C_{\eta} = \frac{2}{3} or \frac{1}{6} \) [25]. Different values of \( C_{\eta} \) corresponds to the initial state being \( c\bar{\tau}, c\bar{d} \) or \( c\bar{s} \) respectively.
3 Numerical Results

OZI allowed two-body strong decays of 1D strange charm family are calculated using the heavy quark effective approach given in section 2. Partial and total decay widths of these states are studied and compared with the experimental values. OZI allowed decay channels for $D_{s1}^*(2860)$ and $D_{s3}^*$ are $DK, D^*K, D_s\eta$ and $D_s^*\eta$, and for their spin partners $1D_{s2}$ and $1D_s'$ decay modes are $D^*K, D_s^*\eta, D(2400)K$ and $D_s^*(2317)\eta$. For this, we take input parameters as the initial masses for $D_{s1}^*$ and $D_{s3}^*$, given by the LHCb [8,9] and 2890 MeV and 2900 MeV for their spin partner states $1D_{s2}$ and $1D_s'$ respectively. Heavy Quark Effective Theory shows that, decay widths also depend on the strong hadronic coupling $g_{XH}, g_{YH}, g_{XS}$ and $g_{YS}$. The strong couplings have been constrained to be with 0 and 1 [26] but their experimental information is still missing. In the next subsections, we calculated two of these coupling constants i.e. $g_{XH}$ and $g_{YH}$ using the decay widths and other available experimental data.

3.1 $D_{s1}^*(2860)$

$D_{s1}^*(2860)$ was first observed by BaBar collaboration and in 2014 its spin, mass and decay width was confirmed by LHCb. In this subsection, heavy quark effective theory is adopted to reproduce the experimental data given by these collaborations. The coupling constant $gxh$ is obtained and $D_{s1}^*(2860)$ state are assigned as the $1^-$ member of the 1D charm family. Assuming it to be the pure $1D 1^-$ state, we calculated the total and partial decay widths of $D_{s1}^*(2860)$ using the decay width formulae given in section 2 in terms of their hadronic coupling constants. These partial decay widths and ratios are tabulated in Table 2. Along with the partial decay widths, we also studied the ratios such as

$$R = \frac{Br(D_{s1}^*(2860) \to D^*K)}{Br(D_{s1}^*(2860) \to DK)}$$

$$R_1 = \frac{Br(D_{s1}^*(2860) \to D_s\eta)}{Br(D_{s1}^*(2860) \to DK)}$$

| Theory       | DK       | $D^*K$   | $D_s\eta$ | $D_s^*\eta$ | Total | $\Gamma(D^*K)/\Gamma(DK)$ | $\Gamma(D_s\eta)/\Gamma(DK)$ |
|--------------|----------|----------|-----------|-------------|-------|---------------------------|-------------------------------|
| Our          | 2865.45$gxh^2$ | 693.135$gxh^2$ | 508.189$gxh^2$ | 85.70$gxh^2$ | 4152.48$gxh^2$ | 0.24 | 0.177 |
| Experimental | 159      | 1.10     |           |             |       |                           |                               |

Table 2: Calculated partial and total decay widths of $D_{s1}^*(2860)$ as pure $(1^3D_1)$

It can be seen from the Table 2 that our calculated R value does not matches with the experimental value 1.10. Same has also been calculated by various theoretical models like Ref[15] gives $\Gamma(D^*K)/\Gamma(DK) = 0.46t0.70$ and $\Gamma(D_s\eta)/\Gamma(DK) = 0.10t0.14$, Ref[18] gives $\Gamma(D^*K)/\Gamma(DK) = 12.5t0.6$ and $\Gamma(D_s\eta)/\Gamma(DK) = 0.30t0.14$ and Ref[5] gives $\Gamma(D^*K)/\Gamma(DK) = 0.06$ and $\Gamma(D_s\eta)/\Gamma(DK) = 0.23$. As it can be seen that R value i.e. $\Gamma(D^*K)$ calculated by our HQET approach and by other theoretical approaches [15,18,5] does not matches with the experimental R value i.e. 1.10. As R is independent of couplings, so to justify the experimental value of R, we include the mixing of the states. Here we constraint the hadronic coupling constant $gxh$ from the literature. According to this scheme, state $D_{s1}^*(2860)$ is assumed...
to be the mixture of $2^3S_1$ and $1^3D_1$ states with $D_s(2700)$ to be its orthogonal partner satisfying the relation

$$
\begin{pmatrix}
D_{s1}(2S) \\
D_{s1}(2860)
\end{pmatrix}
= 
\begin{pmatrix}
\cos\theta & \sin\theta \\
-\sin\theta & \cos\theta
\end{pmatrix}
\begin{pmatrix}
2^3S_1 \\
1^3D_1
\end{pmatrix}
$$

where $\theta$ is the mixing angle. Effect of variation of total decay width of $D_{s1}(2860)$ state with coupling constant for different mixing angle is shown in Figure1 to Figure4. We have seen this variation for some typical values of mixing angle at $\theta = 0^\circ$, $\theta = -30^\circ$ and for $\theta = -60^\circ$ and $\theta = -80^\circ$ where $\theta = 0^\circ$ correspond to non mixing i.e. pure $1^3D_1$.

Figure 1: Decay widths of $D_{s1}(2860)$ for $\theta = 0^\circ$  
Figure 2: Decay widths of $D_{s1}(2860)$ for $\theta = -30^\circ$  
Figure 3: Decay widths of $D_{s1}(2860)$ for $\theta = -60^\circ$  
Figure 4: Decay widths of $D_{s1}(2860)$ for $\theta = -80^\circ$

Figure1 - Figure4 shows that $DK$ is the main decay channel of this state. Apart from $DK$, $D^*K$ and $D_s\eta$ are also important decay channels of $1^3D_1$, whereas the calculated decay width for $D_s^*\eta$ is found to be small. Dominance of $D_s^*\eta$ decay channel enhances with more mixing effect. $R$ ratio defined in the section 1 now depends on the mixing angle and strong coupling constants $g_{xh}$ and $ghh$. Fixing $ghh = 0.17$ [25] variation of $R$ value with the mixing angle is seen in figure 5. This figure shows that experimental $R$ value 1.10 can be achieved theoretically corresponding to the mixing angle $-1.6 \leq \theta \leq -1.2$ radians. For this range of mixing angle our hadronic coupling constant comes out to be $0.17 \leq g_{xh} \leq 0.20$. This variation of $g_{xh}$ can be shown in Figure 6.

Figure 5: Variation of $R$ value with mixing angle

Figure 6: Variation of coupling constant $g_{xh}$ with mixing angle

For these calculated values of mixing angle and coupling constant, partial and total decay width are again studied. Total width $\Gamma$ comes out to be 159 MeV, which matches very well with the experimental. Other partial decay widths are listed in Table3. $R$ value shown in column 7 comes out to be 0.91 which now is close to the the experimental observed value.

| Theory | $DK$ (MeV) | $D^*K$ (MeV) | $D_s\eta$ (MeV) | $D_s^*\eta$ (MeV) | Total (MeV) | $\frac{D^*K}{DK}$ | $\frac{D_s\eta}{DK}$ |
|--------|------------|--------------|-----------------|------------------|-------------|----------------|----------------|
| Our    | 60.61      | 55.26        | 11.85           | 18.66            | 146.39      | 0.91           | 0.19           |
| Experimental | 159      | 1.10         |

Table 3: Calculated partial and total decay widths of $D_{s1}^*(2860)$ as a mixture of $(1^3D_1)$ and $2^3S_1$ states

### 3.2 $D_{s3}^*(2860)$

Considering $D_{s3}^*(2860)$ as the $1^3D_{s3}$, its decay channels and partial decay widths are presented in Table4. Decay widths are listed in terms of hadronic coupling constant $g_{yhh}$. Figure7 shows the variation of the partial and total decay width with this coupling constant.

7
| Theory | $DK$ (MeV) | $D^*K$ (MeV) | $D_s\eta$ (MeV) | $D_s^*\eta$ (MeV) | Total (MeV) |
|--------|-----------|-------------|----------------|-----------------|------------|
| Our    | 249.18$g\nu h^2$ | 96.39$g\nu h^2$ | 24.72$g\nu h^2$ | 4.54$g\nu h^2$ | 374.846$g\nu h^2$ |
| Experimental |               |             |               |                 |            |

Table 4: Calculated partial and total decay widths of $D_{s3}^*(2860)$ as $1^3D_{s3}$

Figure 7: shows the variation of partial decay widths of $D_{s3}^*(2860)$ as the $1^3D_{s3}$ state with hadronic coupling.

This picture clearly shows that $DK$ is the dominant decay mode of $D_{s3}^*(2860)$. Other important decay channels are $D^*K$, $D_s\eta$ with $D_s^*\eta$ contributing least. Computing it with the experimental value of total decay width $\Gamma = 53 MeV$, coupling constant $g\nu h$ comes out to be $0.40$. These partial decay widths can be used to calculate the ratio $R$.

$$R = \frac{\Gamma(D_{s3}^*(2860) \to D^*K)}{\Gamma(D_{s3}^*(2860) \to DK)} = 0.38$$ (29)

$$R1 = \frac{\Gamma(D_{s3}^*(2860) \to D_s\eta)}{\Gamma(D_{s3}^*(2860) \to DK)} = 0.03$$ (30)

These ratios are compared with previous valued predicted by various theoretical models as in Ref[5] $\Gamma(D^*K)$ = 0.39 and $\Gamma(D_s\eta) = 0.13$, Ref[10] gives $\Gamma(D^*K) = 0.43$ and $\Gamma(D_s\eta) = 0.11$ and Ref[7] gives $\Gamma(D^*K) = 0.8$ and $\Gamma(D_s\eta) = 0.05$

3.3 $1D_{s2}$ and $1D_{s2}'$

$1D_{s2}$ is the spin partner of the $D_{s1}^*(2860)$ belonging to $J^P$ as $2^-_s$ state, and $1D_{s2}'$ state belongs to $J^P_s$ to $2^-_s$. These both states are still unknown in the charm meson spectrum.

As shown in Table 1, their masses have been already predicted by various theoretical models [21,22,23]. Taking their masses to be within the allowed range 2800 MeV to 3000 MeV, variation of their total OZI allowed two body strong decay width have been plotted with respect to mass and coupling constant in Figure 8 and Figure 9.

Figure 8: Variation of $1D_{s2}$ with its mass and coupling constant $g\nu h$

Figure 9: Variation of $1D_{s2}'$ with its mass and coupling constant $g\nu h$

Using the hadronic couplings obtained in previous subsections $g\nu h = 0.20$ and $g\nu h = 0.40$ partial and total decay widths of these states are listed in first column of Table 5. Also, these two states can mix through spin-orbit interaction or by some other mechanism and physically $D_{s2}'$ and $D_{s2}$ can be represented as the linear combination of $3D_2$ and $1D_2$ states as

$$\begin{pmatrix} 1D(2^-) \\ 1D'(2^-) \end{pmatrix} = \begin{pmatrix} \cos\theta_{1D} & \sin\theta_{1D} \\ -\sin\theta_{1D} & \cos\theta_{1D} \end{pmatrix} \begin{pmatrix} 1^3D_2 \\ 1^1D_2 \end{pmatrix}$$
As pure states

\( \theta = 0^\circ \)

As mixed states

\( \theta = -39^\circ \)

| Decay Channel | As pure states (MeV) | As mixed states (MeV) |
|---------------|----------------------|----------------------|
| \( D^*K \)   | 61.37                | 36.02                |
| \( D^*_2 \eta \) | 16.69                | 11.62                |
| \( D(2400)K \) | -7.5 \times 10^{-8}  | -1.5 \times 10^{-4}  |
| \( D^*_s(2317) \eta \) | 0.0037               | 0.0014               |
| Total         | 78.06                | 47.65                |

Table 5: Calculated partial and total decay widths of \( 1D_{s2} \) and \( 1D'_{s2} \). First section is calculated by taking them as pure states and the second section includes mixing scheme into account.

where \( \theta_{1D} \) is the mixing angle. In general the mixing angle between the \( ^3L_l \) and \( ^1L_l \) in heavy quark limit is given by \( \theta_{1D} = \arctan \sqrt{L/(L+1)} \). For this case the mixing angle corresponds to \( L = 2 \) and comes out to be \( 39.2^\circ \sim 39^\circ \). In Table 5, the last column give partial decay widths by taking this mixing into account.

4 Conclusion

Due to advancement in high energy accelerators, large amount of information is available on heavy-light charm and bottom mesons. These information motivates theorists to explore more about these heavy-light mesons. These D and B meson states are studied by observing their decaying behavior, masses, their \( J^P \) states, coupling constants, branching ratios etc. Many models like Heavy quark effective theory, Quark pair creation model, Potential models etc are framed to study these heavy-light mesons. Recently, LHCb predicted spin 1 and spin 3 strange charm mesons. In this paper, we use the heavy quark effective approach to study the recently observed spin 1 and spin 3 strange charm states. This theory treat the heavy quark as static and provide lagrangian and decay widths formulas to the available states. This theory has adequately studied the previously determined experimental states and successfully allotted their positions in the charm and bottom spectroscopy.

Observation of spin 1 and spin 3 resonances of \( D^*_s(2860) \) by LHCb has clearly indicated that there are two different states of \( D^*_{sJ}(2860) \). In the last 5 years, various theoretical models [5-7,10-19] studied \( D^*_{sJ}(2860) \), favored it as \( 1^3D_3 \) state with narrow decay width. From the LHCb data, \( D^*_{s3}(2860) \) state with \( \Gamma = 53MeV \) can be correlated with this \( D^*_{sJ}(2860) \) state. We too studied the decay behavior of \( D^*_{s3}(2860) \) assuming it to be in the \( 1^3D_3 \) state and calculated the hadronic coupling constant \( g_{yh} = 0.40 \). This value can be compared with the one obtained by Wang \( g_{yh} = 0.52 \) [25].

We also studied the remaining spin 1 observed state by LHCb \( D^*_s(2860) \), assuming it to be pure \( 1^3D_{1s} \) state and to be a mixture of \( 1^3D_1 \) and \( 2^3S_1 \) state. We study its decay channels \( (D^*K, DK, D^*_s\eta, D_s\eta) \) and R value \( (\frac{DK}{DK}) \) calculated for the pure state (R=0.48) which does not lie within the given experimental data (R=1.10). So we adopted it to be as a mixture of radially excited \( 2^3S_1 \) and orbitally excited \( 1^3D_1 \). Using this interpretation, decay widths and R value depends on mixing angle (\( \theta \)) and coupling constant \( g_{yh} \). We studied the variation of partial widths with coupling constant \( g_{yh} \) for some fixed values of mixing angle (\( \theta \)) which shows \( DK \) is the dominant
decay channel. In the variation of R value with mixing angle ($\theta$), experimental R value favors the large mixing angle. This large mixing angle implies the predominance of $2^3S_1$ state for $D_{s1}^*$ (2860).

We obtained $R=0.91$ corresponding to the mixing angle $-1.6 \leq \theta \leq -1.2$. Along with this mixing angle, we constrain the coupling constant $g_{xh}$ to be lying in the range $0.17 \leq g_{xh} \leq 0.20$. This obtained coupling value is close to the value given by Wang $g_{xh} = 0.19$ in Ref[25].

Using these coupling constants, we also calculated the decay behavior of the spin partners of these states $1D_2$ and $1D_{2s}'$. These states are studied using two ways, first by considering them as pure states and secondly by taking their mixing into account. In both cases $D^*K$ is the dominating decay channel. Decay width for $1D_{2s}'$ as a pure state comes to be small indicating the presence of other decay modes. As we have only considered the decays to pseudo-scalar mesons, so there may be a possibility that decays to light vectors mesons may also be present for this state.

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6 References

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