Online estimation of battery power state based on improved equivalent circuit model

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Abstract. The state-of-power (SOP) of lithium-ion batteries is an important parameter for safety control and energy recovery of electric vehicles. In the battery management system (BMS), the Equivalent-Circuit Model (ECM) is commonly used to simulate battery dynamics. However, there is always a contradiction between the complexity and accuracy of the model. A simple model usually cannot reflect all the dynamic effects of the battery, which may bring error identification to the parameters. A complex model always has too many parameters that cannot be identified, and there may be parameter divergence problems. In order to solve this problem, this paper proposes a new equivalent circuit model, that is, the equivalent circuit model of the vehicle power battery based on the Auto Regression (AR) model. Based on this model, some inaccurate parameters were found in the parameter identification process, so a parameter identification method based on the extended Kalman filter algorithm and Recursive Least Squares (RLS) was used, and a composite model estimation method was used. Comprehensive consideration of a series of restrictions such as State of Charge (SOC) limit and factory setting limit, combined with other methods such as equivalent circuit model, requires the estimated result to meet all the restrictions, to make up for some of the shortcomings of other methods, and make SOP estimation more accurate and reliable.

Keywords: lithium ion battery; state of power; equivalent circuit model; AR model; recursive least square; extended Kalman filter.

1. Introduction
Electric vehicles and hybrid vehicles are promising solutions to the oil crisis and environmental problems exacerbated by traditional vehicles. The key component of electric vehicles and hybrid vehicles is the battery. In order to meet the power requirements of vehicles, low-voltage batteries need to be connected to a large number of loaded battery packs [1, 2], and a refined battery management system is essential to ensure the safety and durability of each unit [3].

In order to accurately monitor the battery status, a battery model is needed to describe the battery dynamics, which is controlled by a series of complex electrochemical processes. The electrochemical model can accurately capture the behavior of the battery, and can accurately explain the behavior of the battery from the laws of chemical reactions, transport and kinetic phenomena [4, 5]. However, too many
inputs in the system usually have the problem of overfitting [6]. The equivalent circuit model uses electrical components that are usually voltage sources, ohmic resistors, and one or more resistor-capacity (RC) circuits to approximate battery dynamics. Compared with the electrochemical model, ECM requires less input and has better regression performance, so it has been widely used. ECM parameters can be identified offline or online. For offline methods, pulse discharge [7] and electrochemical impedance spectroscopy [8-10] are usually used. However, the values of battery parameters will vary with the operating environment, and the characteristics of battery aging are manifested in the State of health (SOH) [11-13], offline recognition is unlikely to include all these changes. Therefore, an online method that can update battery parameters in real time is better in battery parameter identification. Many methods have been proposed in recent research. In the literature [14-16], an RC circuit is used as the ECM, and the Recursive Least Squares algorithm is used to identify its parameters. An RC circuit model can simulate the main dynamic effects of lithium-ion batteries with a simple topology [17], which has the advantage of easy convergence. However, it cannot reflect the low-frequency response of the battery, such as diffusion [16-18] and some other dynamic effects, which may bring errors to the model output and parameter identification. References [10-18] used some other ECMs with more RC circuits. Their performance in simulating battery behavior is better than a simple RC circuit model, but more parameters that need to be identified greatly increase the computational difficulty and may even cause parameter divergence. In [19], the Randall model was established, and the Recursive Extended Least Squares (RELS) algorithm was used to identify online model parameters related to the SOC and SOH of the battery. However, the battery open circuit voltage is not included in Randall's model. Therefore, SOP cannot be easily obtained from this model. In order to solve the contradiction between model simplicity and accuracy and improve the accuracy of SOP prediction, this paper proposes a new ECM.

The contribution of this paper is to establish a new equivalent circuit model, using regression model to apply to the prediction model of stationary process time series. Due to its linear structure, the amount of calculation is relatively simple, it has the characteristics of online update, and has good dynamic characteristics. In addition, the AR model is completely based on data drivers, and the process data can truly reflect the aging process of internal parameters during the entire life cycle of the power battery. Therefore, the AR model is used for online dynamic estimation inside the power battery, which can better capture the changes in the aging process of the power battery. The equivalent circuit model uses capacitors, resistors, inductances and other circuit components to characterize the internal impedance and polarization of the battery. Through reasonable configuration of the circuit components, it simulates the steady state and dynamic response of the battery. It is usually used as online parameters. Identification and state estimation. In this paper, the composite model estimation method is used to comprehensively consider a series of limitations such as SOC limitation and factory setting limitation, combined with other methods such as equivalent circuit models, and requires the estimated results to meet all limitations and make up for some of the defects of other methods. In the proposed multi-state joint estimator, RLS method is used to perform parameter identification through the real-time measurement of battery current and voltage. In the process of parameter identification, the problem of inaccuracy of some parameters is found. Firstly, some parameters are calculated based on the car's one-time charge data, and then the more mature extended Kalman filter algorithm with self-correction ability and fast convergence is used to update and then perform some parameter identification, which finally improves the accuracy of parameter identification. Improved SOP estimation accuracy and battery safety performance.

2. Online modeling technology

2.1. Black box model.

To estimate the dynamic behavior and state of a lithium battery, a battery model is first established. For data-driven battery management system applications, resistance and capacitance based equivalent circuit battery models have gained more attention [6, 8, 19]. In this paper, a power battery equivalent model with black box structure is proposed. The equivalent circuit is first given, as shown in Fig.1.
Figure 1. Equivalent circuit of electric vehicle power battery

In Figure 1, $U_d$ represents the battery terminal voltage, $U_{ocv}$ represents the electromotive force of the battery, which is usually called the open circuit voltage; the internal voltage drop caused by the polarization reaction of the $U_{AR}$ battery, $R$ represents the internal resistance in the battery, and $I$ represents the current in the circuit (assuming Discharge is positive, charging is negative). $U_{AR}$ can be regarded as a "black box". This article believes that the "black box" is a time-varying parameter that cannot obtain accurate values. It varies with temperature, charge and discharge current, battery $S_S$ and $SOC$, etc. Affected by many factors, it has a nonlinear and complex time-varying structure, which is difficult to express with an accurate equivalent circuit. Therefore, it is more accurate to use a black box to characterize it; assuming that during the discharge process, it can be obtained based on Figure 1.

$$U_d = U_{ocv} - U_{AR} - IR \quad (1)$$
$$U_n = U_{AR} + IR \quad (2)$$

$U_{ocv}$ and $SOC$ have a one-to-one correspondence relationship with time. According to the above analysis, we can get

$$U_{ocv}(k) = K_0 + \frac{k_1}{SOC(k)} + K_2 SOC(k) + K_3 \ln(SOC(k)) + K_4 \ln(1 - SOC(k)) \quad (3)$$

In formula (3), $SOC(k)$ represents the value at time $k$, and $k_0, k_1, k_2, k_3, k_4$ are coefficients; what needs to be explained here is that the remaining battery power $SOC(k)$ is taken to the battery management system, in this SOP It is considered to be known in the estimation method. The variable $U_{AR}$ in formula (1) is determined by the charging and discharging current and the setting parameters. Therefore, the product of the current and internal resistance after formula (1) can be regarded as part of the $AR$ model, and the internal resistance $R$ is regarded as one of the parameters, and the internal impedance is affected by many factors such as battery current, temperature, $SOH$, and $SOC$, etc. Therefore, the slow time-varying parameter of the black box system with the power battery current as the input and $U_n$ as the output can be used. Its function expression is

$$U_n = a(I(k)) \quad (4)$$

In equation (4), the functional relationship $a(\cdot)$ can be regarded as the black box part of the system. Since the input and output in equation (4) are all time series, the $AR$ model of time series analysis is used to fit it.

$$U_n(k) = A_{r1} I(k) \cdots I(k-p) \quad (5)$$

In formula (5), $A_{r1} I(k) \cdots I(k-p)$ represents the $p+1$-order $AR$ model, which represents the battery $U_n(k)$ at time $t$ and the current value $I(k) \cdots I(k-p)$ at the previous time $p$. According to the literature [16], when the order of the AR model is appropriate, it can approach the true value infinitely. Therefore, the AR model suitable for time series analysis is more reasonable and the computational complexity is relatively low. $A_{r1} I(k) \cdots I(k-p)$ is rewritten as

$$U_n(k) = \sum_{p=0}^{l} a_p I(k-p) \quad (6)$$

2.2. Data-driven parameter identification method

The state equation of the lumped parameter battery model proposed in equation (1) can be discrete in linear discrete form.
\[ U_d(k) = U_{ocv}(k) - U_{AR}(k) - I(k)R \quad (7) \]

\[ U_d(k) = K_0 + \frac{K_1}{Z(k)} + K_2Z(k) + K_3 \ln(Z(k)) + K_4 \ln(1 - Z(k)) - \sum_{p=0}^{l} a_p I(k - p) \quad (8) \]

Equation (8) can be obtained from equations (3), (6), and (7), where the order of the \(AR\) model is generally set as [2, 6], or use the dual iterative BDT algorithm proposed in [20] to adaptively set, \(a_p\) is the \(AR\) model coefficient, and the ohmic resistance \(R\) to be identified is regarded as the \(a_0\) coefficient of the \(AR\) model, and \(Z(k)\) is the \(SOC(k)\) Abbreviation.

In order to use the time series method to identify battery model parameters, equation (8) should be rewritten as:

\[
\begin{align*}
\gamma(k) &= \alpha_{\text{rls}}(k) \beta_{\text{rls}}(k) \\
\alpha_{\text{rls}}(k) &= \left[ 1, \frac{1}{Z(k)} Z(k) \ln(Z(k)) \ln(1 - Z(k)) I(k) I(k - 1) \cdots I(k - p) \right] \\
\beta_{\text{rls}}(k) &= [K_0, K_1, K_2, K_3, K_4, a_0, a_1, \ldots, a_p]
\end{align*}
\]

\(\alpha_{\text{rls}}(k)\) and \(\beta_{\text{rls}}(k)\) are the data matrix and parameter matrix at the \(k\)th sampling time.

Aiming at the real-time measurement of battery current and voltage, the \(RLS\) is modeled online using the optimal forgetting factor algorithm. Since the function relationship between the open circuit voltage of lithium-ion batteries and \(SOC\) (3) mainly changes with battery aging, it is almost unchanged during a charge and discharge cycle, which does not meet the actual application, so this article assumes the parameters in formula (3) \([k_0, k_1, k_2, k_3, k_4]\) are constant in one charge and discharge cycle. Through the analysis of the parameters obtained from the online identification of the initial overall parameter matrix \(\beta_{\text{rls}}(k)\), it is found that the data change trend of the parameters \([k_0, k_1, k_2, k_3, k_4]\) fluctuates too much, and the last moment of its change The results of the parameters \([k_0, k_1, k_2, k_3, k_4]\) obtained through linear fitting of the battery charge data are close. The experimental results obtained are shown in Figure 2. The blue curve is the result of online recognition, and the red straight line is the result of the parameter \([k_0, k_1, k_2, k_3, k_4]\) obtained by linear fitting through the battery charge data first, so the value of the parameter \([k_0, k_1, k_2, k_3, k_4]\) is determined as the battery charge data Figure 3 shows the schematic diagram of the online recognition of some parameters of the battery model based on the results of curve fitting.
In the online identification of some parameters of the battery model, $U_n$ is updated using the current mature extended Kalman filter algorithm with self-correction capability and fast convergence. The specific algorithm is as follows:

$$U_n(k + 1) = a_0 I(k + 1) + a_1 I(k) + \cdots + a_p I(k - p + 1)$$  \hspace{1cm} (10)

$$U_n(k + 1) = U_n(k) + a_0 \Delta I(k + 1) + a_1 \Delta I(k) + \cdots + a_p \Delta I(k - p + 1)$$  \hspace{1cm} (11)

Where $\Delta I(k) = I(k) - I(k - 1)$

$$U_n(k) = U_n(k - 1) + a_0 \Delta I(k) + a_1 \Delta I(k - 1) + \cdots + a_p \Delta I(k - p)$$  \hspace{1cm} (12)

$$U_n(k) = U_n(k - 1) + [a_0 \ a_1 \cdots a_p] [\Delta I(k) \ \Delta I(k - 1) \cdots \Delta I(k - p)]$$  \hspace{1cm} (13)

$$U_n(k) = U_n(k - 1) + CI$$

$$\{ U_n(k) = U_n(k - 1) + CI$$

$$U_d(k) = U_{ocv}(k) - U_n(k)$$

Where $I = [\Delta I(k) \ \Delta I(k - 1) \cdots \Delta I(k - p)], C = [a_0 \ a_1 \cdots a_p]$

The execution process of the extended Kalman filter algorithm:

State calculation is divided into time update and filter update

Time update: $\hat{U}_n(k|k-1) = \hat{U}_n(k-1|k-1) + CI$  \hspace{1cm} (15)

Filter update: $\hat{U}_n(k|k) = \hat{U}_n(k|k-1) + K(k) \{ U_d(k) - (U_{ocv}(k) - \hat{U}_n(k|k-1)) \}$  \hspace{1cm} (16)

Kalman gain: $K(k) = P(k|k-1)(P(k|k-1) + R(k))^{-1}$  \hspace{1cm} (17)

Variance calculation is divided into time update and filter update

Time update: $P(k|k-1) = P(k-1|k-1) + Q(k-1)$  \hspace{1cm} (18)

Filter update: $P(k|k) = (I - K(k)H(k))P(k|k-1)$  \hspace{1cm} (19)

$$Q(k) = K(k)H(k)K(k)$$  \hspace{1cm} (20)

$$R(k) = H(k) - P(k|k-1)$$  \hspace{1cm} (21)

$$H(k) = H(k) + \{ U_d(k) - (U_{ocv}(k) - \hat{U}_n(k|k-1)) \}^2$$  \hspace{1cm} (22)

Equations (10)-(13) are the derivation process of the state equation, where the equation (11) can be obtained from the equations (6) and (10), the equation (14) is the state equation and the observation equation, and the equations (15)-(19) It is the main formula in the traditional EKF algorithm. Now the state value at time $k - 1$ is used to predict time $k$ in advance. The prior state update formula is (15),
and the prior covariance update formula is (17). At time $k$, the new measurement $U_d(k)$ is obtained, the posterior state update formula is (16), the posterior covariance update formula is (19), and the update method for $Q$ and $R$ in this paper is (20)-(22).

**Figure 3.** Parameter identification based on recursive least squares

In Figure 3, $ff$ is the forgetting factor of RLS, $\hat{U}_n(k)$ is the estimated value of the total voltage drop inside the battery in the battery equivalent circuit model, and the parameter $c(k) = [a_0, a_1, \ldots, a_p]$.

3. A data-driven multi-state joint estimation method

3.1. SOC-based current estimation

In order to make the battery performance safer and more reasonable when the actual SOC is close to its design limit, we should control its discharge current and maximize the charge. The actual SOC is close to its minimum design current limit $Z_{mn}$, otherwise the battery will be over-discharged; another one hand, we should control its charging current and make it so that when the actual SOC is close to its maximum design limit $Z_{max}$, otherwise the battery will be overcharged. The rule equation and related description of the current current estimation and calculation using the SOC limited method are as follows

$$Z(k+1) = Z(k) - \frac{\eta \Delta t(k)}{C_n}$$

(23)

Therefore, the maximum discharge current $I_{dis,max}^{z,short}(k+1)$ and the minimum charge current $I_{chg,min}^{z,short}(k+1)$ under the SOC limit can be calculated by (24) and (25) are calculated.

$$I_{dis,max}^{z,short}(k+1) = \frac{Z(k) - Z_{min}}{\eta \Delta t} \frac{C_n}{\Delta t}$$

(24)

$$I_{chg,min}^{z,short}(k+1) = \frac{Z(k) - Z_{max}}{\eta \Delta t} \frac{C_n}{\Delta t}$$

(25)

$C_n$ represents the maximum usable capacity of the battery, $\eta$ represents the Coulomb efficiency, and $\Delta t$ sampling period. The method based on SOC limitation is usually used in combination with other peak power estimation methods.

3.2. Model-based current estimation

For safe and durable operation, the operating current and voltage of the lithium-ion battery must be limited to the window, the battery power (discharge is positive, charge is negative). This paper applies model-based current capability estimation, as shown in equation (26).

$$U_d(k+1) = U_{ocv}(Z(k+1)) - U_n(k+1)$$

(26)
Since \( Z(k + 1) \) itself is a function of the current \( I \), and \( U_{ocv} \) is a non-linear function of \( Z(k) \), the peak current cannot be directly calculated by equation (26). The Taylor series expansion method can be used to linearize formula (26) to obtain formula (27).

\[
U_d(k + 1) = U_{ocv}(Z(k + 1)) - U_n(k + 1)
= U_{ocv}(Z(k) - I(k + 1) \frac{\eta \Delta t}{C_n}) - \sum_{p=0}^{j} a_p I(k - p + 1)
= U_{ocv}(Z(k)) - I(k + 1) \frac{\eta \Delta t}{C_n} \frac{\partial U_{ocv}}{\partial Z} \bigg|_{z=z_k} - R_1(Z(k), I) \frac{\eta \Delta t}{C_n} - a_0 I(k + 1) - \sum_{p=1}^{j} a_p I(k - p + 1)
= U_{ocv}(Z(k)) - I(k + 1) \frac{\eta \Delta t}{C_n} \left( -\frac{k_1}{z(k)^2} + K_2 + \frac{k_3}{1-z(k)} - R_4(Z(k), I) \frac{\eta \Delta t}{C_n} \right) - a_0 I(k + 1) - \sum_{p=1}^{j} a_p I(k - p + 1)
\]

Ignoring the first-order residual term \( R_1(\cdot) \), the peak current based on voltage limitation can be obtained, as shown in equations (28) and (29).

\[
im_{\text{dis,max}}(k + 1) = \frac{U_{ocv}(Z(k)) - U_{d,max} - \sum_{p=1}^{j} a_p I(k-p+1)}{R_4 Z(k)^2 + K_2 + \frac{k_3}{1-z(k)}}
\]

\[
im_{\text{chg,min}}(k + 1) = \frac{U_{ocv}(Z(k)) - U_{d,min} - \sum_{p=1}^{j} a_p I(k-p+1)}{R_4 Z(k)^2 + K_2 + \frac{k_3}{1-z(k)}}
\]

The peak current calculated by formula (28) and formula (29) is obtained under the voltage limit, \( U_{d,max} \) is the maximum limit voltage of the battery itself, \( U_{d,min} \) is the minimum cut-off voltage of the battery itself, \( \nim_{\text{dis,max}}(k + 1) \) is the maximum discharge current of the next sampling interval under the voltage limit, \( \nim_{\text{chg,min}}(k + 1) \) is the minimum charge current of the next sampling interval under the voltage limit.

Once the current model-based current estimate is calculated, the current estimate with all constraints is calculated as:

\[
im_{\text{dis,short}}(k + 1) = \min(I_{\text{max}}, \nim_{\text{dis,max}}(k + 1), \nim_{\text{dis,short}}(k + 1))
\]

\[
im_{\text{chg,short}}(k + 1) = \max(I_{\text{min}}, \nim_{\text{chg,min}}(k + 1), \nim_{\text{chg,short}}(k + 1))
\]

\( I_{\text{max}} \) and \( I_{\text{min}} \) are the design limits of the maximum discharge current and the minimum charge current, respectively. Under all restrictions, \( \nim_{\text{chg,short}}(k + 1) \) and \( \nim_{\text{dis,short}}(k + 1) \) are the minimum charge current and maximum discharge current, respectively.

3.3. SOP estimation under multiple constraints

3.3.1. Short-term SOP estimation. The power capability estimate can be calculated as follows:

\[
P_{\text{dis,short}}(k + 1) = \min(P_{\text{max}}, U_d(k + 1) \nim_{\text{dis,short}}(k + 1))
\]

\[
P_{\text{chg,short}}(k + 1) = \max(P_{\text{min}}, U_d(k + 1) \nim_{\text{chg,short}}(k + 1))
\]

Among them, \( P_{\text{max}} \) and \( P_{\text{min}} \) are the power design limits of the battery, \( P_{\text{max}} \) represents the discharge power capacity of the design limit, and \( P_{\text{min}} \) represents the charge power capacity of the design limit. Then, using the terminal voltage calculation equation (27), the calculation method of the model-based power capability estimation can be obtained.

\[
P_{\text{max}}^{\text{dis,short}}(k + 1) = \min\left(P_{\text{max}}, U_{ocv}(Z(k)) - \nim_{\text{max}}(k + 1) \frac{\eta \Delta t}{C_n} \left( -\frac{k_1}{z(k)^2} + K_2 + \frac{k_3}{1-z(k)} - a_0 \nim_{\text{max}}(k + 1) - \sum_{p=1}^{j} a_p I(k - p + 1) \nim_{\text{max}}(k + 1) \right) \right)
\]
\[
P_{\text{min}}^{\text{charg,short}}(k+1) = \max\left(P_{\text{min}}, (U_{\text{ocv}}(Z(k))-I_{\text{min}}^{\text{charg,short}}(k+1) + \frac{\eta \Delta t}{c_n} \left(- \frac{K_1}{Z(k)^2} + \frac{K_2}{Z(k)} + \frac{K_3}{1-Z(k)}\right) - \frac{K_4}{1-Z(k)} \right) - a_p \sum_{p=1}^{j} a_p l(k-p+1) I_{\text{min}}^{\text{charg,short}}(k+1)
\]

(35)

3.3.2. Long-term SOP estimation. To realize continuous power capability estimation, a current calculator must be established first, assuming that the input current of the system is constant from sampling time \(k\) to \(k+L\), and \(I((k+L)) = I(k)\) in the sampling interval of \(L \times \Delta t\), Where \(L\) is an integer and depends on the requirements of the on-board energy management unit. In some cases, 15 seconds of continuous power is required to accelerate, brake or climb. In this way, \(L\) is calculated as \(15/\Delta t\). However, in this article, \(L\) is set to 30, which means we need 30 seconds of continuous power capability. Note that the sampling interval in this article is 1 second. Therefore, the maximum discharge current \(I_{\text{dis}, \text{max}}^{z, \text{long}}(k+L)\) and the minimum charge current \(I_{\text{ch, min}}^{z, \text{long}}(k+L)\) under the SOC limit can be expressed by (37) and (38) are calculated. At the same time, the peak current based on voltage limit can be obtained by (38), as shown in equations (39) and (40).

\[
I_{\text{dis}, \text{max}}^{z, \text{long}}(k+L) = \frac{Z(k) - Z_{\text{min}}}{\eta \Delta t / c_n}
\]

(36)

\[
I_{\text{ch, min}}^{z, \text{long}}(k+L) = \frac{Z(k) - Z_{\text{max}}}{\eta \Delta t / c_n}
\]

(37)

\[
U_d(k+L) = U_{\text{ocv}}(Z(k+L)) - U_n(k+L)
\]

\[
= U_{\text{ocv}}(Z(k)) - l(k+L) \frac{\eta \Delta t}{c_n} - \sum_{p=0}^{j} a_p l(k+L)
\]

\[
= U_{\text{ocv}}(Z(k)) - l(k+L) \frac{\eta \Delta t}{c_n} \left(- \frac{K_1}{Z(k)^2} + \frac{K_2}{Z(k)} + \frac{K_3}{1-Z(k)}\right) - \frac{K_4}{1-Z(k)} - \sum_{p=0}^{j} a_p l(k+L)
\]

(38)

Ignoring the first-order residual term \(R_1(\cdot)\), the peak current based on voltage limitation can be obtained, as shown in equations (39) and (40).

\[
I_{\text{dis}, \text{max}}^{z, \text{long}}(k+L) = \frac{U_{\text{ocv}}(Z(k)) - U_{\text{d,max}}}{R + \left(\eta \Delta t / c_n \left(- \frac{K_1}{Z(k)^2} + \frac{K_2}{Z(k)} + \frac{K_3}{1-Z(k)}\right) + \sum_{p=0}^{j} a_p \right)}
\]

(39)

\[
I_{\text{ch, min}}^{z, \text{long}}(k+L) = \frac{U_{\text{ocv}}(Z(k)) - U_{\text{d,min}}}{R + \left(\eta \Delta t / c_n \left(- \frac{K_1}{Z(k)^2} + \frac{K_2}{Z(k)} + \frac{K_3}{1-Z(k)}\right) + \sum_{p=0}^{j} a_p \right)}
\]

(40)

Therefore, the current current estimate with all limitations is calculated as:

\[
i_{\text{max}}^{z, \text{long}}(k+L) = \min(I_{\text{max}}, I_{\text{dis}, \text{max}}^{z, \text{long}}(k+L), I_{\text{ch}, \text{min}}^{z, \text{long}}(k+L))
\]

(41)

\[
i_{\text{min}}^{z, \text{long}}(k+L) = \max(I_{\text{min}}, I_{\text{dis}, \text{max}}^{z, \text{long}}(k+L), I_{\text{ch}, \text{min}}^{z, \text{long}}(k+L))
\]

(42)

The power capability estimate can be calculated as follows:

\[
P_{\text{max}}^{z, \text{long}}(k+1) = \min(P_{\text{max}}, U_d(k+1) I_{\text{max}}^{z, \text{long}}(k+L))
\]

(43)

\[
P_{\text{min}}^{z, \text{long}}(k+1) = \max(P_{\text{min}}, U_d(k+1) I_{\text{min}}^{z, \text{long}}(k+L))
\]

(44)

Therefore, for long-term prediction, the power capability estimate can be calculated as follows:
\[\begin{align*}
    p_{\text{max}}^{\text{dis,long}}(k + L) &= \min \left( P_{\text{max}}, (U_{\text{ocv}}(Z(k)) - \sum_{p=0}^{i} a_p^{\text{dis,long}}(k + L) x_{\text{max}} \right) \\
    &\quad - \frac{\eta_{\text{L}} z(k)}{c_n} \left( -\frac{K_1}{z(k)^2} + \frac{K_2}{z(k)} - \frac{K_3}{1 - z(k)} \right) - \sum_{p=0}^{i} a_p^{\text{dis,long}}(k + L) x_{\text{max}} \right) \\
    p_{\text{min}}^{\text{chg,short}}(k + L) &= \max \left( P_{\text{min}}, (U_{\text{ocv}}(Z(k)) - \sum_{p=0}^{i} a_p^{\text{chg,short}}(k + L) x_{\text{min}} \right) \\
    &\quad - \frac{\eta_{\text{L}} z(k)}{c_n} \left( -\frac{K_1}{z(k)^2} + \frac{K_2}{z(k)} - \frac{K_3}{1 - z(k)} \right) - \sum_{p=0}^{i} a_p^{\text{chg,short}}(k + L) x_{\text{min}} \right) \\
\end{align*}\]

3.3.3. Other factors battery SOP factors. In addition to voltage and current limitations, battery SOP is also affected by the following conditions. If the battery temperature is too high, lower the \( S_S \) to reduce heat generation. According to the results of online parameter identification, if the internal resistance is too large, the BMS should reduce the battery SOP and notify to check the battery connection or replace the aging battery. If the insulation resistance is too low, the battery SOP drops to zero and the BMS reports an insulation failure. The flow chart of the SOP prediction algorithm in this article is as follows:

![Flow chart of SOP estimation based on AR equivalent circuit model](image)

4. Algorithm verification and comparison

4.1. Data Sources
In order to get closer to the actual vehicle battery discharge characteristics, this paper selects the data collected by the A123 battery in the open data set of the University of Maryland CALCE battery experiment measurement [21] under the dynamic stress test (DST). The validity and applicability of the multi-state joint estimator SOP algorithm of the AR equivalent circuit model is verified. The battery parameters are shown in Table 1:

| Types     | Nominal voltage | Nominal capacity | Charge/discharge cut-off voltage | Maximum discharge current |
|-----------|-----------------|------------------|----------------------------------|--------------------------|
| LiFePO4   | 3.3V            | 1.1Ah            | 3.6V/2.0V                        | 30A                      |
The DST operating condition was proposed by the Advanced Battery Association of America [22], which is mainly used to study the transient characteristics of batteries, so it is closer to the transient random discharge characteristics of vehicle power batteries.

4.2. Model accuracy verification and comparison

4.2.1. Model accuracy verification and comparison without noise. In order to verify the accuracy of the AR equivalent circuit model proposed in this paper, this article uses the AR equivalent circuit model and the traditional Thevenin model to estimate the terminal voltage under DST conditions. The result is shown in Figure 5.

![Figure 5. Accuracy comparison results of AR model and Thevenin model](image)

Figure 5. Accuracy comparison results of AR model and Thevenin model

In Figure 5, $U_d^Z$ represents the real terminal voltage, $U_d^{AR}$ is the terminal voltage estimated by the AR model proposed in this paper, and $U_d^{RC}$ is the terminal voltage curve estimated by the Thevenin model. It can be clearly seen from Fig. 6 that the AR model proposed in this paper has higher estimation accuracy and higher stability, which is closer to the real terminal voltage.

4.2.2. Model accuracy verification and comparison under white noise. Furthermore, in order to verify and compare closer to reality, we add (60db) Gaussian white noise to the current $I(k)$ and the terminal voltage $U_d(k)$, and use the noise-containing current and voltage data to compare the equivalent The accuracy of the circuit AR model is verified, and the accuracy is compared with the traditional Thevenin model. The verification and comparison results are shown in Figure 7, and the error comparison with the actual measured data is shown in Figure 8.

![Figure 6. Comparison of estimated deviation between AR model and Thevenin model](image)
Figure 7. Accuracy comparison results of AR model and Thevenin model under Gaussian white noise

In Figure 7, the red solid line represents the actual measured terminal voltage $U_{d2}$; the green dashed line represents the terminal voltage curve estimated by the AR model under the interference of Gaussian white noise; the blue dashed line is the terminal voltage curve obtained based on the traditional Thevenin model; from Figure 7 it can be seen from the partially enlarged image of, that in a noisy environment, the AR model proposed in this paper can better reflect the actual battery electrical characteristics, and is more stable than the traditional Thevenin model, with less fluctuation, and better robustness. Figure 8 shows the error image of the AR model proposed in this article and the traditional Thevenin model to estimate the terminal voltage. It can be seen more intuitively that under the same white noise interference, the AR model proposed in this paper can approximate the true terminal voltage value with higher accuracy, and the error is basically below 0.01V, while the error of the traditional model in a noisy environment The maximum value is higher than 0.2V, and the fluctuation is small, and the robustness is better.

4.3. SOP estimator performance

4.3.1. Verification and comparison without noise. In order to evaluate the SOP estimation method, we should define the SOC design limit of the battery, as well as other design limits set by the battery manufacturer, such as voltage, power, and current. For different control strategies, the SOC limit may be different. From Figure 9-12, we find that the charge-discharge current capacity estimation calculated by the multi-constraint method includes two parts. One is estimation based on SOC, and the other is estimation method based on model. When the SOC is close to its upper limit design, the SOC-based method can make the battery work in a safe manner, effectively avoiding the safety hazard of overcharging. When the SOC is close to its lower design limit, the SOC-based method can also make it possible to operate the battery in a safe manner, effectively avoiding the potential safety hazards of over-discharge. Therefore, the SOC-based method can avoid over-discharge and over-charge. When the SOC approaches its design limit, the current current estimate may fluctuate. This can be found in Figure
13 and Figure 14. It can be noticed in the figure that, in some cases, when the current capacity estimate based on the model exceeds its design limit, the current design limit will take effect.

**Figure 9.** Comparison of charging current capability estimation (30s)

**Figure 10.** Comparison of discharge current capacity estimation (30s)

**Figure 11.** Charging current capability under multiple constraints

**Figure 12.** Charging power estimation results
It can be seen from Figure 11-14 that the SOP of the battery is very similar to the current waveform, and the charge and discharge current of the battery can reflect the SOP state of the battery to a certain extent. At the end of battery discharge, the power of the lithium battery decays faster than the current decay. At this time, the current in the battery cannot reflect the SOP state of the battery. The main reason is the sharp increase in the internal resistance of the battery at the end of the discharge. At this time, if only the current is used to guide the operation of the whole vehicle, it will cause the battery to withstand a large inrush current, which will affect the system safety and battery life.
Figure 16. Estimation results of discharge current capability for different durations

Figure 15-16 shows that as the continuous sampling interval decreases, the absolute value of the current current estimate increases significantly. When the continuous sampling interval is 30s, it is greater than 50s to 120s. In addition, due to the power requirement for a longer continuous time, the current current estimation is greatly reduced, especially the estimation of the current charge current, which may cause potential safety hazards.

4.3.2. Verification and comparison under white noise. In a noisy environment, the current $I(k)$ and terminal voltage $U_d(k)$ are respectively added (60db) Gaussian white noise), and the predicted charge and discharge current and power images are as follows:

Figure 17. Estimation results of charging current capability under Gaussian white noise

Figure 18. The estimation result of charging power capability under Gaussian white noise
It can be seen from Figure 17-20 that the SOP and current waveforms of the battery are still very similar, and the charge and discharge current of the battery can reflect the SOP state of the battery to a certain extent. In summary, it can be verified that the SOP estimator based on the equivalent circuit AR model proposed in this paper has high estimation accuracy and strong robustness no matter in an ideal environment or in a noisy environment.

5. Conclusion
This paper proposes a new equivalent circuit model, which uses the AR model for online dynamic estimation inside the power battery, which can better capture the changes in the aging process of the power battery and improve the accuracy of the model. The composite model estimation method is used to comprehensively consider a series of restrictions such as SOC limit and factory setting limit, combined with other methods such as equivalent circuit model, requires the estimated result to meet all the restrictions, to make up for some of the shortcomings of other methods, and it is found in the parameter identification process To solve the problem of inaccuracy of some parameters, it is proposed to calculate some parameters through the battery charge data first, and then use the current mature extended Kalman filter algorithm with self-correction ability and fast convergence to update and then carry out some parameters. The identification finally improves the accuracy of parameter identification, the estimation accuracy of SOP and the safety performance of the battery. The robustness under ideal environment and noise environment verifies the practicability of the algorithm in this paper.

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