EFFECT OF MODULE VARIATION ON A 100WATT HORIZONTAL AXIS WIND TURBINE SPUR GEAR DRIVE

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ABSTRACT

Spur gears are simple to design and construct to actualize their usage in power transmission as well as for speed reduction or increase. The aim of this research work is to determine the effect of module variation on a 100Watt horizontal axis wind turbine (HAWT) spur gear drive. The solution is to determine and appropriately select gear parameter values based on design considerations and the significance is for enhanced machine reliability and balance with economic production. The method used involved the application of gear design principles, modelling the gears with AutoCAD, module variation and evaluation of induced bending stress at the gears root; gear diameter, tooth thickness and face width become bigger with choosing higher modules for cutting tooth size, with the largest diameter of 168.4mm and 80mm for driver and driven gear respectively and face width of 60mm from using a module of 6mm. Although, the total torque exerted remains constant at 7.9Nm and 3.8Nm for a 100W HAWT. The tangential force in meshed operation of the gears and induced bending stress kept reducing as module increased. The highest tangential force is 282.4N and corresponding induced bending stress is 26.1N/mm^2 based on American Gear Manufacturers Association (AGMA) standard and the range of data analyzed, at the lowest module of 2mm. Selecting lower modules means that higher bending stresses will be induced at the gears root, but smaller production cost and more compact system with less space requirement. The lower bending stresses with increasing module will support higher load capacity of the gears due to the increasing face width and enhance the reliability of the HAWT in respect of its performance.

KEYWORDS: Module, Spur Gear, Power Transmission, Bending Stress, Wind Turbine

INTRODUCTION

Where there is the need to transmit power from one point to another, a number of solution options are usually available depending on the suitability. Power transmission can be achieved by the use of mechanical, pneumatic, hydraulic, and electrical methods. Mechanical method of power transmission includes the use of gear, belt, and chain drives. The concern of this research is on spur gear drives. The objectives include minimum module determination, design and modelling of the gears with AutoCAD, evaluation of the induced bending stress, and gear size with module variation. The justification includes facilitating gear analysis, enabling design choice in terms of space and compactness, as well as preventing mechanical failure and promoting economic decision in production. There are different types of gears including spur, helical, worm and wheel, and bevel gears. In today’s world of industrialization, gears are the major means for mechanical power transmission system, and in most industrial rotating machinery (Dewanjii, 2016; Patil, 2017). Gears can be used to transmit power from one shaft to another and can also be used to change the direction of motion. For instance, bevel gears, worm and wheel gears can be applied where shafts are at right angles, while spur gears can be used to transmit power between two parallel shafts. Helical gears have helix angle since their teeth are cut along a helix, but spur gears have zero helix angle since their teeth are straight or parallel to the gears’ axis. The advantages of spur gears over helical gears are their ease to produce and assemble, as well as high efficiency. But helical gears can support more load, high speed and power, they work quietly, and power transmission can be done with shafts not being parallel. Spur gears have simple design, more efficient power transfer, whereas helical gears support high speed, high power mechanical systems with silent operation (Suvo, 2009). Again, gears can be used to increase torque while speed is being reduced, or in the reverse order. Usually, when gears are used to reduce torque or increase speed, the gear ratio is less than unity and when they are used to increase torque, which is in gear reduction applications, the gear ratio is greater than 1. If an output gear of a gear train rotates more slowly than the input gear, the gear train is called a speed reducer and has increased torque (Radhakrishna, Nidigonda, and Nasaiah, 2017).

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A number of parameters are used to define a gear system to be designed. These properties include, module, pressure angle, tooth thickness or circular thickness, number of teeth, reference diameter, tip diameter, root diameter, centre distance and face width (Zhang, Finger and Behrens, 2010). Considering the torque acting on the gears, the allowable bending stress for the material, ratio of face width to module and number of teeth, the minimum module for cutting the gear teeth can be determined (equation 1). Normally, in gear design the angle which lies between the tooth profile and a line radiating through the pitch point is called the pressure angle. A tangent to an involute tooth profile such as that of a spur gear usually makes an angle with a line radiating through the pitch point of the gear. This angle is called the pressure angle and it is usually set to $20^\circ$, but generally different types of pressure angles including $14.5^\circ$ and $25^\circ$ exist (Radhakrishna et al, 2017; Subramanian and Srinivasan, 2014). The gears would be mounted on shafts, and the shaft design results for 100 Watt horizontal axis wind turbine (HAWT) which this research follows and tailored to is given in Table 1.

| Table 1: Shaft Design Results for a 100W HAWT |
|-----------------|-----------------|
| Power, P (W)    | 100             |
| Main Shaft Rotational Speed, $N_1$ (rpm) | 381             |
| Main Shaft Design Torque, $T_d$ (Nm)     | 2.62            |
| Main Shaft Brake Torque, $T_{br}$ (Nm)   | 5.28            |
| Main Shaft Total Torque, $T_T$ (Nm)      | 7.92            |
| Output Shaft Rotational Speed, $N_2$ (rpm) | 800             |
| Output Shaft Torque, $T_o$ (Nm)          | 1.26            |
| Output Shaft Brake Torque, $T_{bo}$ (Nm)  | 2.52            |
| Output Shaft Total Torque, $T_{to}$ (Nm)  | 3.78            |
| Main Shaft Diameter, $D_1$, (mm)         | 15.7            |
| Output Shaft Diameter, $D_2$, (mm)       | 5.6             |

Source: Ayadju and Ujevwerume, 2019

MATERIALS AND METHODS
The method used involved the application of gear design principles, and modelling of the gears with AutoCAD. Pitch and minimum module (Radhakrishna, et al, 2017) for cutting the gears were determined using equation (1). Input and output gear diameters were determined, and gear module was consistently varied and the induced bending stress at the gears root was evaluated in accordance with acceptable standards (Prabhakaran, Balaji, and Kumar, 2017).

THEORY/CALCULATION
The minimum module for cutting the teeth is given as

$$m_{od} \geq 1.26 \left( \frac{M_t}{\sigma_b \varphi_m Z} \right)^{1/3}$$  

where, $Z$ = number teeth; $M_t = (7.92 \times 10^3)$Nm = $T_T$ (Table 1) taken for safety; $\sigma_b = 145$N/mm$^2$ = allowable bending stress; $\varphi_m = b/m_{od} = 10$ (assumed); $b$ = face width; $m_{od}$ = module; $y = 0.3$. Considering mild steel with Brinell hardness number (BHN) = 130; For BHN 130 ≤ 350. and life cycle ≥ $10^7$ (Radhakrishna et al, 2017).

Tooth pitch, reference diameter, tooth thickness, face width, and tangential force acting on teeth of driver and driven gear in meshed operation are given in equations (2), (3), (4), (5) and (6) accordingly,

$$p = \pi m_{od}$$  

$$D = Z m_{od}$$  

$$t = \pi m_{od}/2$$  

$$b = \varphi_m m_{od}$$  

(Radhakrishna et al, 2017)

$$F_t = T*2/D$$  

or

$$F_t = 2 \times T/ m_{od} \times Z$$ (Patil, 2017),

where, $m_{od} = 2mm =$ module used; $p =$ pitch (mm); $t =$ tooth thickness; $D =$ reference diameter; $F_t =$ tangential force; $T =$ torque; $Z =$ number of teeth.

Stresses are developed on the tooth of the gears when in mating contact, and tooth failure is the major factor that causes breakdown of the system which uses the spur gear (Gavali and Satav, 2018). Analytically, a gear tooth root stress can be evaluated using existing theories (Sivakumari and Michael, 2018). The induced bending stress at the tooth root when the meshed gears are in operation can be found using American Gear Manufacturers Association (AGMA) standard in equation (7) and Lewis equation (8) respectively, given as

$$\sigma_b = \left( \frac{F_t}{b \times m_{et} \times y} \right) \times (K_r \times K_s \times K_y \times (0.93 K_m))$$

(7)
where, $K_v = 1.89 = $dynamic velocity factor; $K_o = 1 = $over load factor; $K_s = 0.75 = $size factor; $K_m = 1.4 = $load distribution factor; $m_n = normal module$

\[ \sigma_b = \frac{F_t}{m} \cdot b \cdot y \]  

(8)

where, $y = 0.3 = Lewis factor; m = module (Prabhakaran, Balaji and Kumar, 2017; Patil, 2017). Torque is given as

\[ T = \frac{F_t}{r} \]

where, $r = radius of gear (that is reference diameter divided by 2)$. 

MINIMUM MODULE DETERMINATION

Using equation 1, considering 28 teeth and 13.3 teeth driver and driven gears respectively, then

\[ m_{od} = 1.1 \text{mm} \quad \text{and} \quad m_{od} = 1.4 \text{mm} \]

take 2mm as minimum module.

Pitch, Diameter, Thickness, Face Width

From equation (2), when the minimum module is applied then,

\[ p = \pi \cdot 2 = 6.3 \text{mm} \]

and considering the 28 teeth and 13.3 teeth gears in respect of equation (3), the diameters will be

\[ D_1 = 28 \cdot 2 = 56 \text{mm} \quad \text{and} \quad D_2 = 13.3 \cdot 2 = 26.6 \text{mm} \]

$D_1$ and $D_2$ are driver and driven gear diameters respectively. Figure 1 shows the AutoCAD 3-D model of the gears using a module of 2mm.

The tooth thickness from applying values in equation (4) is

\[ t = \pi \cdot 2 \]

(9)

The face width from equation (5) is

\[ b = 10 \cdot 2 = 20 \text{mm} \]

Tangential Force and Induced Bending Stress

The tangential force from equation (6) becomes

\[ F_t = 7.92 \cdot 10^3 \cdot 2/56 = 282.4 \text{N} \]

Using equations (7) and (8), the induced bending stress obtained based on the minimum module of 2mm,

\[ \sigma_b = \frac{282.4}{2 \cdot 2 \cdot 0.5} \cdot (1.89 \cdot 1 \cdot 0.75 \cdot (0.93 \cdot 1.4) = 26.1 \text{N/mm}^2 \]

\[ \sigma_b = 282.4 \cdot 2 \cdot 2 \cdot 0.3 = 23.5 \text{N/mm}^2 \]

are 26.1 N/mm$^2$ and 23.5 N/mm$^2$ respectively.

Results for the various parameters can be re-evaluated for different values of modules above the minimum module of 2mm using the stated equations.

RESULTS AND DISCUSSION

Table 2: Spur Gear Results for Various Modules

| Module (mm) | Pitch (mm) | Driver Gear, $D_1$ (mm) | Tooth Thickness (mm) | Face Width (mm) | Tangential Force (N) | Main Total Torque (Nm) | Shaft Total Torque (Nm) | Output Shaft Total Torque (Nm) |
|------------|------------|-------------------------|----------------------|-----------------|---------------------|------------------------|-------------------------|---------------------------------|
| 2.0        | 6.3        | 56.1                    | 26.7                 | 3.2             | 20.0                | 282.4                  | 7.9                     | 3.8                             |
| 2.5        | 7.9        | 70.4                    | 33.4                 | 4.0             | 25.0                | 225.0                  | 7.9                     | 3.8                             |
| 3.0        | 9.4        | 83.8                    | 39.8                 | 4.7             | 30.0                | 189.0                  | 7.9                     | 3.8                             |
| 3.5        | 10.9       | 97.1                    | 46.1                 | 5.5             | 35.0                | 163.1                  | 7.9                     | 3.8                             |
| 4.0        | 12.6       | 112.3                   | 53.3                 | 6.3             | 40.0                | 141.1                  | 7.9                     | 3.8                             |
| 4.5        | 14.1       | 125.7                   | 59.7                 | 7.1             | 45.0                | 126.0                  | 7.9                     | 3.8                             |
| 5.0        | 15.7       | 139.9                   | 66.5                 | 7.9             | 50.0                | 113.2                  | 7.9                     | 3.8                             |
| 5.5        | 17.3       | 154.2                   | 73.2                 | 8.7             | 55.0                | 102.7                  | 7.9                     | 3.8                             |
| 6.0        | 18.9       | 168.4                   | 80.0                 | 9.5             | 60.0                | 94.1                   | 7.9                     | 3.8                             |
Table 3: Tangential Force and Induced Bending Stress

| Module (mm) | Tangential Force (N) | Bending Stress (N/mm²), AGMA | Bending Stress (N/mm²), Lewis |
|------------|---------------------|-----------------------------|-------------------------------|
| 2.0        | 282.4               | 26.1                        | 23.5                          |
| 2.5        | 225.0               | 13.3                        | 12.0                          |
| 3.0        | 189.0               | 8.1                         | 7.0                           |
| 3.5        | 163.1               | 4.9                         | 4.4                           |
| 4.0        | 141.1               | 3.3                         | 2.9                           |
| 4.5        | 126.0               | 2.3                         | 2.1                           |
| 5.0        | 113.2               | 1.7                         | 1.5                           |
| 5.5        | 102.7               | 1.3                         | 1.1                           |
| 6.0        | 94.1                | 1.0                         | 0.9                           |

Gear size changes have been established with varying module. It is observed from Table 2 that the gear diameter and face width become bigger with higher modules for cutting tooth size, with the largest diameter of 168.4mm and 80mm for driver and driven gear respectively and face width of 60mm from using a module of 6mm. Although, the total torque exerted remains constant at 7.9Nm and 3.8Nm (Table 2) for a 100W HAWT, the tangential force in meshed operation of the gears and induced bending stress keep reducing as module increases (Table 3). The highest tangential force is 282.4N and corresponding induced bending stress is 26.1N/mm² based on AGMA and the range of data analyzed, and occurs at the lowest module of 2mm (Table 3). The results will enable the appropriate selection of gear parameter values using design considerations including size, space, weight, stress, reliability, cost, etc. Increasing gear size for the same number of teeth and gear ratio will require more material to produce the gears, though induced bending stresses will be lower. Selecting higher modules means higher bending stresses will be induced at the gears root, less production cost, more compact system and less space requirement. The lower bending stresses with increasing module will support higher load capacity of the gears due to the increasing face width and enhance the reliability of the HAWT in respect of its performance.

CONCLUSION

It was observed that gear size increased with increasing module, the largest diameters being 168.4mm and 80mm for driver and driven gear respectively, and face width of 60mm from using a module of 6mm for cutting the teeth. Nevertheless, the total torque exerted remains constant for the 100Watt HAWT with reduction in the tangential force and induced bending stress as module was increased. A maximum bending stress of 26.1N/mm² was found at the minimum module of 2mm where the gear diameters are smallest. These results will enable proper selection of gear parameter values that support balance of the machine reliability, space and economic production.

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