DETECTABILITY OF SPACE-TIME FLUCTUATIONS IN ULTRA HIGH ENERGY COSMIC RAY EXPERIMENTS

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ABSTRACT. It is generally expected that quantum gravity affects the structure of space-time by introducing stochastic fluctuations in the geometry, and, ultimately, in the measurements of four-distances and four-momenta. These fluctuations may induce observable consequences on the propagation of ultra high energy particles, mainly in the range of energies of interest for cosmic ray physics, over large distances, leading to their detection or constraining the underlying quantum gravitational structure. We argue that the detectable effects of fluctuations may extend to energies much lower than the threshold for proton photopion production (the so-called GZK cutoff), so that lower energy observations may provide strong constraints on the role of a fluctuating space-time structure.
INTRODUCTION

Since the advent of Special and General Relativity we are acquainted to the concept that space-time is dynamically related to its content. It is however generally thought as a preexisting, background arena in which our Universe lives. However, as already recognized 45 years ago [1], quantum gravitational (QG) effects are likely to profoundly modify this picture: at the Planck scale, which is the scale at which QG becomes important, the geometry of the space-time ceases to be definite and fluctuations are expected both in its geometrical and topological structure. In fact the most ambitious QG programs do not stipulate a preexisting space-time which should instead emerge as the long (compared to Planck) distance limit of some, more fundamental, quantum structure.

Although this is known since long time, it has always been relegated to the realm of the unreacheable super-Planckian world, i.e. at energies exceeding $10^{29}$ eV.

This situation has rapidly changed in the last years with the realization that Nature provides us with probes (either UHE cosmic rays or gammas from very far, variable sources) which can feel in their travel the fundamental structure of our Universe. In this contribution we will focus on UHE particles. In this case it has been realized that the onset of the processes responsible for absorption of particles from very far sources on universal background radiation fields (of which the best studied is the Cosmic Microwave Background Radiation, CMBR) is extremely sensitive to even tiny modifications of Lorentzian empty-space propagation, and that the corresponding absorption thresholds can be modified in a way testable even from present day experiments.

In previous work [2] we focused attention to the effect of explicit modifications of Relativistic Invariance, compatible with all low energy experiments and yet capable to be verified/falsified. In the present contribution we discuss the possible effects of the intrinsic uncertainty on every measurement induced by quantum gravity and discuss in some details their implications. The results, when properly taking into account all processes affecting propagation, are striking and contrary to intuition. The threshold for absorption effects moves to lower energies with respect to the expected one\textsuperscript{1}.

\textsuperscript{1}In the following we will always use units such that $\hbar = c = 1$ unless needed.
EFFECTS OF GEOMETRY FLUCTUATIONS ON PROPAGATION OF UHE PARTICLES

It is long known that QG effects preclude an arbitrary precision in the measurement of distances (time). There are various ways of demonstrating this fact, but perhaps the simplest and most intuitive uses the operative definition of distance, i.e.: distance \( \propto N. \) of wavelengths between points. To measure a distance \( D \leq L_P \) (Planck length) \(^2\) we need wavelengths \( \lambda < L_P \), i.e. frequencies \( > M_P \) thus leading inevitably to the formation of a black-hole, where information is lost; in this sense distances \( \leq L_P \) cannot be defined and the minimum uncertainty on distances is \( \delta l \propto L_P \).

This uncertainty can be be transferred into uncertainty on momenta assuming that this is also the uncertainty on the de-Broglie wavelength of a particle:

\[
\delta p = \delta \frac{1}{\lambda} = \frac{1}{\lambda^2} \delta \lambda = \frac{L_P}{\lambda^2} = \frac{p^2}{M_P}.
\]

and similarly for energies.

This is the starting point for our considerations [3]: we assume that

a) Energies and momenta fluctuate independently (but, only for simplicity, maintaining rotational invariance, i.e. the space components of momenta fluctuate in the same way) and b) the relation between energy and 3-momentum (dispersion relation) also fluctuates independently:\(^3\)

\[
E = \bar{E} + \alpha \frac{\bar{E}^2}{M_P} \quad p = \bar{p} + \beta \frac{\bar{p}^2}{M_P} \quad E^2 - p^2 = m^2 + \gamma \frac{\bar{E}^3}{M_P}\]

where \( \bar{E}, \bar{p} \) are the values obtained averaging over a large number of measurements, and \( \alpha, \beta, \gamma \) are random variables with zero mean and variance \( \approx 1 \); the form of their distribution is not important and for definiteness we assume it gaussian. Equation 2 expresses the fact that each measurement of energy and momenta is undetermined up to an uncertainty that grows with (energy) momenta so that an infinite precision cannot be attained. On the other hand each interaction (through conservation of four-momentum) is a measurement, so that a particle propagating in a medium will have in each interaction a slightly different momentum. In particular, particle having

\(^2\)We remind that \( L_P = (\frac{\hbar G}{c^3})^{\frac{1}{2}} \approx 10^{-23} \) cm is the Schwarzschild radius of a mass \( M_P \approx 10^{28} \) eV.

\(^3\)This last assumption is essentially equivalent to considering only non conformal metric fluctuations [5].
average momenta below a certain absorption threshold would have a finite, non-zero probability to fluctuate above threshold and be absorbed.

We now compute the effect of such fluctuations on UHE Cosmic Rays, and for definiteness we consider the propagation of UHE protons in the relic $3^oK$ radiation. In this case the absorption process is $p\gamma_{3^oK} \rightarrow \pi N$ with a threshold at $\approx 5 \cdot 10^{19}$ eV. We compute the threshold for this process by stipulating independent fluctuations for each initial and final particle, since typical scales of interactions are much larger than Planck ones. Solving the conservation equations supplemented by the dispersion relations one then obtains a distribution of threshold values, instead than a definite one as in the normal case.

To better clarify this case, consider a simplified (but still physical) example, and assume that only relevant fluctuations act on the dispersion relation of the UHE proton: this position is likely to introduce a relatively small error since fluctuations increase with energy and final particles have lower energy. The conservation-dispersion equations are then as in [2] with the solution for the threshold (neglecting pion mass):

$$\gamma \frac{p_0^3}{m_p^2 M_P^3} \frac{p_{th}^3}{p_0^3} + \frac{p_{th}}{p_0} - 1 = 0$$

(3)

where $p_{th}$ is the threshold (with fluctuations) and $p_0$ the normal, Lorentz invariant one. Clearly when $\gamma = 0$ the threshold is equal to the normal one; on the other hand the coefficient of the cubic term is very large ($O(10^{13})$ for the case we consider) so, as soon as $\gamma \leq -10^{-13}$, which happens almost exactly in the 50% of the cases for any reasonable, symmetric distribution with zero mean and unit variance, there is no solution, i.e. the reaction is not allowed. On the other hand, for $\gamma > 10^{-13}$, an approximate solution is given by

$$p_{th} \approx m_p^2 M_P^3 \gamma \frac{1}{4} \approx 10^{-15} \gamma^{-\frac{1}{4}} \text{ eV.}$$

(4)

Therefore, when $\gamma > 0$, the probability of having a given threshold will peak around $\gamma \approx 1$ i.e. $p_{th} \approx 2 \times 10^{-15}$ eV; for instance for a gaussian distribution we obtain a distribution of probability as in Figure 1 in the 50% of the cases in which the threshold exists. In the case in which energies, momenta and dispersion relation of all particles fluctuate independently the shape of the distribution is unchanged, only the probability that the interaction is kinematically forbidden decreases to $\approx 30\%$ since for fixed initial momenta the final ones can fluctuate in a way to make the reaction allowed.
Figure 1: Distribution of probability of thresholds in the 50% of cases in which the interaction is allowed.

It is therefore clear that during propagation in the CMBR fluctuations of the space-time affect UHE particles in such a way that in a sizable fraction of cases the effective threshold is moved to lower energies $\approx 2 \times 10^{15} \text{eV}$.

Before analyzing the experimental consequences of these distributions, it is interesting to derive the above effects in a more intuitive way, that can be most easily displayed in the case of pair production by UHE $\gamma$ on CMBR: $\gamma(q)\gamma_{30}K(k) \rightarrow e^+e^-$. The four-momentum of the initial photon will fluctuate as in eq. 2, and we neglect the fluctuations on the low energy one. We further assume that once fluctuations in four-momenta are taken into account, then one can use normal Lorentz transformation to compute momenta in different frames ([4]). Then one can define, starting from the momenta in Lab. frame for the above reaction, a frame in which the space component of the total momenta of the initial particles is $k_i + q_i = 0$, i.e. the “rest” frame. Clearly the velocity of this frame with respect to the Laboratory will be affected by the fluctuations of the energy and momentum:

$$\beta \approx 1 - 2 \frac{k}{q} + (\alpha - \beta) \frac{\bar{q}}{M_p}$$  \hspace{1cm} (5)
and this parameter becomes unphysical when $\beta > 1$; given the form of fluctuations, this happens in almost exactly the 50% of the cases as above.

Using this parameter we can now compute the total energy of the initial state in the “rest” frame; the reaction then is allowed if this energy is larger than the sum of the masses of the final particles$^4$:

$$E_{CM} \approx \left( k\bar{q} - (\alpha - \beta)\frac{\bar{q}^3}{2M_P} \right)^{\frac{1}{2}} \geq 2m_e$$

from which

$$\bar{q} \geq \bar{q}_{thr} \approx \left( \frac{2m_e^2 M_P}{\alpha - \beta} \right)^{\frac{1}{3}} \approx 10^{-12} (\alpha - \beta)^{-\frac{4}{3}} \text{eV}$$

which has the same form of the analogous threshold (eq. 4) in the case of UHE proton propagation, displaying in a clear and intuitive way the origin of the fluctuations for the absorption thresholds.

**EXPERIMENTAL CONSEQUENCES**

To analyze the detectable consequences of the above findings we need to consider the propagation of UHE particles (protons). In each interaction, for energies $> 2 \cdot 10^{15}$ eV, the probability of being absorbed is $P_{int} \approx 70\%$ and of escaping $P_{esc} = 1 - P_{int}$. We further assume that the cross section (and interaction length $\lambda$) for absorption is unchanged with respect to the L.I. case (although in a different region of energy) at least far above the new threshold. Therefore the effective interaction length is slightly increased $\lambda_{eff} \approx \frac{\lambda}{P_{int}}$, and for propagation over distances $L >> \lambda$ we have $P^T_{esc} = P_{esc} \frac{L}{\lambda} \rightarrow 0$. Therefore, although in a single interaction $\approx 30\%$ of particles escape absorption, in the long run essentially all are absorbed although with a slightly increased interaction length.

The situation is essentially the same as for the GZK cutoff in the L.I. approach, the striking difference being that the GZK feature is now moved to much lower energies, and there is no more anything special with $10^{20}$ eV.

Is this prediction testable? Clearly it is easily falsifiable: if the GZK feature would be detected, than the above is wrong, and this implies that

$^4$If also momenta of final particles are allowed to fluctuate in fact this condition could be somewhat weakened, leading to a larger probability of interaction.
some of the assumptions are untenable, for instance the fluctuations might be 'conformal' with much smaller effects, or their variance $< 10^{-13}$ or again proportional to $(p/M_P)^\alpha$ with $\alpha > 2.3$. In all these cases this would give important hints for QG model building.

On the other hand if future experiments will not find the GZK feature where normally expected, then our prediction is that the entire extragalactic component of CRs above $\approx 2 \cdot 10^{15}$ eV, not limited to those exceeding $10^{20}$ eV, arrives at our detectors from within a (slightly enlarged) GZK horizon. This prediction might have observable effects, for instance in terms of anisotropy of UHECR sources; however a detailed propagation study is needed. On the other hand particles that suffered interaction would pile up around $10^{15}$ eV, but for protons their detectability is questionable, due to the much larger abundance of galactic CRs. However photons produced in the decay of secondary neutral pions would initiate a cascade on CMBR and pile up at GeV energies, with a predicted flux orders of magnitude larger than the EGRET experimental limit, and already excluded; but this conclusion relies on L.I. decay lengths and cascade development estimates, which could be modified by fluctuations, so again a detailed propagation analysis has to be performed. In relation to this, it is interesting to notice that the use of standard Lorentz transformations on fluctuating momenta could in principle allow to take into account space-time fluctuations within standard propagation codes.

References

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