The quest for companions to post-common envelope binaries
IV. The 2:1 mean-motion resonance of the planets orbiting NN Serpentis

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ABSTRACT

We present 69 new mid-eclipse times of the young post-common envelope binary (PCEB) NN Ser, which was previously suggested to possess two circumbinary planets. We have interpreted the observed eclipse-time variations in terms of the light-travel time effect caused by two planets, exhaustively covering the multi-dimensional parameter space by fits in the two binary and ten orbital parameters. We supplemented the fits by stability calculations for all models with an acceptable χ². An island of secularly stable 2:1 resonant solutions exists, which coincides with the global χ² minimum. Our best-fit stable solution yields current orbital periods P₁ = 15.47 yr and P₂ = 7.65 yr and eccentricities e₁ = 0.14 and e₂ = 0.22 for the outer and inner planets, respectively. The companions qualify as giant planets, with masses of 7.0 M_Jup and 1.7 M_Jup for the case of orbits coplanar with that of the binary. The two-planet model that starts from the present system parameters has a lifetime greater than 10^6 yr, which significantly exceeds the age of NN Ser of 10^6 yr as a PCEB. The resonance is characterized by libration of the resonant variable θ₁ and circulation of ω₀ − ωᵢ, the difference between the arguments of periapse of the two planets. No stable nonresonant solutions were found, and the possibility of a 5:2 resonance suggested previously by us is now excluded at the 99.3% confidence level.

Key words. Stars: binaries: close – Stars: binaries: eclipsing – Stars: white dwarfs – Stars: individual: NN Ser – Planets and satellites: detection

1. Introduction

Planets orbiting post-common envelope binaries (PCEB) are a recent discovery, and only a few PCEB are known or suspected to harbor planetary systems. These planets are detected by the light-travel time (LTT) effect, which measures the variations in the observed mid-eclipse times caused by the motion of the planet about the common center of mass. The derived orbital periods significantly exceed those of planets orbiting single stars, because the LTT method preferably selects long orbital periods and the radial-velocity and transit methods short ones. Since eclipse-time variations can also be brought about by other mechanisms, it is necessary to prove the strict periodicity of the LTT signal to confirm its planetary origin. The orbital periods on the order of a decade represent a substantial challenge, however. For a system of circumbinary planets, one additionally has to demonstrate the secular stability of a particular solution.

The discovery of two planets orbiting the dG/dM binary Kepler 47 with semi-major axes less than 1 AU (Orosz et al. 2012) has convincingly demonstrated that close binaries can possess planetary systems, but also raised questions about their orbital co-evolution. The evolutionary history of planets orbiting PCEB may be complex: they either formed before the common-envelope (CE) event and had to survive the loss of a substantial amount of matter from the evolving binary and the passage through the ejected shell, or they were assembled later from CE-material. Even if the planets existed before the CE, their masses may have increased in the CE by accretion, making a distinction between first and second-generation origins difficult. In both cases the CE may have significantly affected the planetary orbits, and we expect that the dynamical age of the system equals the age of the PCEB, which is the cooling age of the white dwarf. The hot white dwarf in NN Ser has an age of only 10^6 yr (Parsons et al. 2010a), so the planets in NN Ser are dynamically young. Migration of planets is expected to occur in the CE, but our poor knowledge of the CE structure complicates predictions about the dynamical state of PCEB planetary systems.

Only a handful of PCEB have been suggested to contain more than one circumbinary companion. Of these, NN Ser (Beuermann et al. 2010; Horner et al. 2012) and HW Vir (Beuermann et al. 2012) have passed the test of secular stability. A final conclusion on HU Aqr (Goździewski et al. 2012; Hinse et al. 2012) is pending, and the eclipse-time variations in QS Vir are presently not understood (Parsons et al. 2010b). For a few other contenders, the data are still insufficient for theoretical modeling. In the case of NN Ser, a period ratio of the two proposed planets near either 2:1 or 5:2 was found (Beuermann et al. 2010, henceforth Paper I), with both models being stable for more than 10^6 yr. Horner et al. (2012) confirmed the dynamical stability of the proposed orbits, but suggest that further observations are vital in order to better constrain the system’s true architecture. In this paper, we report further eclipse-time observations of NN Ser and present the results of extensive stability calculations, which show that a two-planet system that starts from our best fit will be secularly stable and stay tightly locked in the 2:1 mean-motion resonance for more than 10^6 yr.

2. Observations

We obtained 69 additional mid-eclipse times of the 16.8 mag binary with the MONET/North telescope at the University of Texas’ McDonald Observatory via the MONET browser-based remote-observing interface. The photometric data were taken with an Apogee ALTA E47+ 1k×1k CCD camera mostly in
white light with 10 s exposures separated by 3 s readout. In most cases photometry was performed relative to a comparison star 2.0 arcmin SSW of NN Ser, but in some nights absolute photometry yielded lower uncertainties. The eclipse light curves were analyzed with the white dwarf represented by a uniform disk occulted by the secondary star (see Backhaus et al. 2012). A seven-parameter fit was made to each individual eclipse profile of the relative or absolute source flux. The free parameters of the fit were the mid-eclipse time $T_e$, the fluxes outside and inside eclipse, the FWHM of the profile, the ingress/egress time, and the two parameters $a_1$ and $a_2$ of a multiplicative function $f = 1 + a_1(t - T_e) + a_2(t - T_e)^2$, which allowed us to model photometric variations outside of the eclipse. Such variations can be caused by effects intrinsic to the source, as the illumination of the barycenter, using the tool provided by Eastman et al. (2010) with a mean of 569.20 s and a standard deviation of 0.79 s. In columns 1–3 of Table 1 we list the cycle numbers, the observed mid-eclipse times $T_e$ in UTC, and the $1\sigma$ errors of the 69 new eclipses. Re-analysis of earlier MONET/N data led to small corrections, and we also include the seven mid-eclipse times published already in Paper I. The errors of $T_e$ vary between 0.17 and 0.83 s, depending on the quality of the individual light curves. The mean timing error is 0.38 s. All mid-eclipse times were converted from Julian days (UTC) to barycentric dynamical time (TDB) and corrected for the light travel time to the solar system barycenter, using the tool provided by Eastman et al. (2010)$^1$. These corrected times are given as barycentric Julian days in TDB in column 4 of Table 1. Together with the errors in column 3, they represent the new data subjected to the LTT fit in the Section 4.1.

$^1$ http://astroutilis.astronomy.ohio-state.edu/time/

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### Table 1. Mid-eclipse times of NN Ser measured with MONET/N.

| Cycle | JD 2450000+ | Error (days) | BJD(TDB) 2450000+ | $O-C_{c,tt}$ (s) |
|-------|-------------|--------------|-------------------|-----------------|
| (a) January/February 2010 (Paper I). | | | |
| 60489 | 5212.9431997 | 0.0000069 | 5212.9418190 | 0.20 |
| 60505 | 5215.0243170 | 0.000067 | 5215.0230965 | −0.17 |
| 60528 | 5218.0159241 | 0.000044 | 5218.0149383 | −0.23 |
| 60735 | 5244.9406264 | 0.000229 | 5244.9415257 | 0.09 |
| 60743 | 5245.9808144 | 0.000032 | 5245.9821657 | 0.02 |
| 60751 | 5247.0213676 | 0.000034 | 5247.0228067 | 0.02 |
| 60774 | 5250.0129571 | 0.000125 | 5250.0146473 | −0.16 |

(b) September 2010 to February 2013 (this work).

| Cycle | JD 2450000+ | Error (days) | BJD(TDB) 2450000+ | $O-C_{c,tt}$ (s) |
|-------|-------------|--------------|-------------------|-----------------|
| 62316 | 5450.5995341 | 0.000052 | 5450.5981913 | −0.35 |
| 62339 | 5453.5916006 | 0.000067 | 5453.5903072 | −0.10 |
| 62347 | 5454.6323120 | 0.000067 | 5454.6306732 | −0.53 |
| 62462 | 5469.5925302 | 0.000049 | 5469.5899042 | 0.87 |
| 62531 | 5478.6868430 | 0.000051 | 5478.6854275 | 0.39 |
| 63403 | 5591.9955682 | 0.000030 | 5591.9953015 | 0.20 |
| 63449 | 5597.9787495 | 0.000027 | 5597.9789848 | −0.06 |
| 63457 | 5599.0193095 | 0.000027 | 5599.0196329 | 0.55 |
| 63472 | 5600.9703359 | 0.000067 | 5600.9708248 | −0.33 |
| 66709 | 6018.9138903 | 0.000027 | 6018.9138694 | 0.33 |
| 66815 | 6035.8235355 | 0.000061 | 6035.8287909 | 0.25 |
| 66893 | 6045.9694909 | 0.000055 | 6045.9750363 | −0.44 |
| 66900 | 6046.8800348 | 0.000041 | 6046.8855994 | −0.28 |
| 66908 | 6047.9260573 | 0.000031 | 6047.9262424 | −0.14 |
| 67284 | 6096.8315332 | 0.000034 | 6096.8363914 | 0.13 |
| 67330 | 6102.8155143 | 0.000035 | 6102.8200772 | −0.46 |
| 67337 | 6103.7261262 | 0.000028 | 6103.7306356 | −0.32 |
| 67352 | 6105.6774387 | 0.000036 | 6105.6818401 | −0.16 |
| 67675 | 6147.6963688 | 0.000058 | 6147.6977420 | 0.19 |
| 67698 | 6150.6884525 | 0.000041 | 6150.6895788 | −0.45 |
| 67775 | 6160.7054661 | 0.000046 | 6160.7057628 | 0.42 |
| 67928 | 6180.6093127 | 0.000034 | 6180.6082721 | 0.11 |
| 67936 | 6181.6500361 | 0.000036 | 6181.6487626 | 0.46 |
| 68028 | 6193.6182396 | 0.000097 | 6193.6160366 | −0.65 |
| 69168 | 6341.9060724 | 0.000028 | 6341.9074599 | 0.06 |
| 69575 | 6394.8458086 | 0.000020 | 6394.8500987 | 0.04 |
Figure 1 shows the $O - C_{\text{lin}}$ residuals relative to the linear ephemeris of the binary quoted in Eq. 1 and derived in Sect. 4.1 below. The data points to the left of the dashed line are from Paper I and those to the right from this work. The residuals of the individual MONET eclipse times are displayed as crosses with error bars. Overplotted are the weighted mean $O - C$ values for 19 groups of timings typically collected in the dark periods of individual months (cyan-blue filled circles). Plotting these mean values avoids cluttering up the graphs on the expanded ordinate scales in Figs. 5 and 6. All fits presented in this paper were made to the total set of 121 individual mid-eclipse times, 52 from Paper I and 69 from this work. The solid curve in Fig. 1 represents the best-fit two-planet LTT model derived in Sect. 4.1. The residuals $O - C_{\text{eff}}$ of the individual timings relative to this fit are included in column 5 of Table 1. The new feature that allows us to derive a significantly improved orbital solution within the framework of the two-planet LTT model is the detection of the upturn in $O - C_{\text{lin}}$ that commences near JD 2455650.

3. General approach

In this paper, we adopt a purely planetary explanation of the eclipse-time variations in NN Ser and represent the set of mid-eclipse times by the sum of the linear ephemeris of the binary and the LTT effect of two planets. A model that only involves a single planet was already excluded in Paper I and is not discussed again, given the very poor fit with a reduced $\chi^2$ of 40.3. Specifically, we describe the motion of the center of mass of the binary in barycentric coordinates by the superposition of two Kepler ellipses, which reflect the motion of the planets (Irwin, 1952, Eq. 1; Kopal, 1959, Eq. 8-94, Beuermann et al., 2012, Eq. 2; Paper I, Eq. 1\textsuperscript{2}). We justify the Keplerian model at the end of this section.

The central binary was treated as a single object with the combined mass of the binary components $M_{\text{bin}} = 0.646 M_\odot$ (Parsons et al. 2010a). This approach is justified, because the gravitational force exerted by the central binary on either of the planets varies only by a fraction of $10^{-7}$ or less over the binary period. Hence, the NN Ser system was treated as a triple system with the components $M_k$ with the unknown inclinations of planet $k$. The semi-major axes $a_k$ and the eccentricities $e_k$ of the individual planets are fixed as in Paper I and are not discussed again.

We performed a large number of least-squares fits in search of the global $\chi^2$ minimum, using the Levenberg-Marquardt minimization algorithm implemented in mpfit of IDL (Marquardt 1963; Markwardt 2009). The time evolution of all models that achieved a $\chi^2$ below a preset limit (Sect. 4.1) was followed numerically until they became either unstable or reached a lifetime of $10^8$ yr. Most models that fit the data developed an instability within a few hundred years or less and fewer than 200 survived for $10^8$ yr, allowing us to calculate the evolution of all models with an acceptable overall CPU-time requirement. We used the hybrid symplectic integrator in the mercury6 package (Chambers 1999), which allows one to evolve planetary systems very efficiently with high precision over long times. The model treats the central object and the planets as point masses and angular momentum and energy are conserved to a high degree of accuracy. Time steps of 0.1 yr were used, which is not adequate for the treatment of close encounters, but such incidents do not occur in the successful models (see Beuermann et al. 2012 for more details).

These dynamical model calculations provide us with information on the complex time variations of the orbital parameters, but also allow us to test the validity of the Keplerian assumption in fitting the data. Figure 2 shows the calculated variation of the mid-eclipse times for the first 500 yr of our best-fit longlived hypothesis, are unfortunately extremely improbable. Accurate eclipse-time measurements over a sufficiently large number of orbital periods could, in principle, provide information on the inclinations of the gravitationally interacting orbits, but this is presently not feasible given the long periods of the planets in NN Ser. The measured orbital periods and amplitudes of the LTT effect yield minimum masses, with the true masses scaling as $1/\sin i_k$, where $i_k$ is the unknown inclination of planet $k$. The semi-major axes depend on the total system mass, implying a minor dependence on $i_k$.

Even with the new data, the least-squares fits of the LTT model permit a wide range of model parameters. As in Paper I, we required, therefore, that an acceptable model provides a good fit to the data and fulfills the side condition of secular stability. Formally, a minimum lifetime of only $10^6$ yr is required, the cooling age of the white dwarf and the age of NN Ser as a PCEB, but most models in the vicinity of the best fit reached more than $10^8$ yr, suggesting that a truly secularly stable solution exists.

\textsuperscript{2} Eq. 1 of Paper I contains a misprinted sign in the last bracket.
model, starting from the Keplerian fit. The LTT effect is displayed folded over the 15.47-yr orbital period of the outer planet, which contributes 87% of the signal. The first two orbits (30 yr) agree closely and are indistinguishable from the Keplerian fit, but in the long run the signal changes due to the dynamical evolution. The orbital phase interval covered in Fig. 2 is the same as in Fig 5 (top panel), where the data and the Keplerian fit are displayed. We conclude that in this special case fitting the data by a Keplerian model is justified. Data trains that extend over more orbits or involve more massive companions will require a dynamical model.

4. Results

Our least-squares fit to the 121 mid-eclipse times involves twelve free parameters, two for the linear ephemeris of the binary, the epoch $T_{\text{bin}}$ and period $P_{\text{bin}}$, and five orbital elements for each planet. With all parameters free, the number of degrees of freedom of the fit is therefore 109. The five parameters for planet $k$ are the orbital period $P_k$, the eccentricity $e_k$, the argument of periapse $\omega_k$, measured from the ascending node in the plane of the sky, the time $T_k$ of periapse passage, and the amplitude of the eclipse time variation, $K_{\text{bin},k} = a_{\text{bin},k} \sin i_k/c$, with $a_{\text{bin},k}$ the semi-major axis of the orbit of the center of mass of the binary about the common center of mass of the system, $i_k$ the inclination of the planet’s orbit against the plane of the sky, and $c$ the speed of light. The arguments of periapse of the center of mass of the binary (with the index ‘bin’) and that of the planet differ by $\pi$. For clarity, we use the indices $k=i$ or $k=o$ for the inner and outer planets, respectively.

We fitted a total of 101,618 models to the data, adopting three lines of approach. Run 1 with 81,552 models is a grid search in the $e_o$,$e_i$ plane. In Run 2 with 10,598 models, we started again from the $e_o$,$e_i$ grid values, but subsequently optimized the eccentricities along with the other parameters. Run 3 with 9,468 mod-

There is no official nomenclature for the naming of exoplanets. At the time of Paper I, the A&A editor considered assigning the first planet discovered around a binary the small letter ‘b’ as illogical and suggested that the two planets orbiting the binary NN Ser should be named ‘c’ and ‘d’, ‘ab’ being the binary components. In the nomenclature proposed by Hessman et al. (2010), the planets in NN Ser would be labeled (AB)b and (AB)c. Since neither convention has been officially adopted, we prefer the neutral designations ‘outer’ and ‘inner planet’, which avoids any misunderstanding, as long as there are only two planets.
A grid search in the $e_i$, $i$, plane was performed to study the properties of selected solutions, in particular, models in the vicinity of the best fit. It also includes models with a circular outer orbit, as advocated in Paper I, but dismissed now and not discussed further in this paper.

### 4.1. Grid search in the $e_i$, $i$-plane

The grid search in the $e_i$, $i$ plane covers eccentricities from zero to 0.40 in steps of 0.01, with an extension to $e_i = 0.70$ in steps of 0.05. The ten other parameters were optimized, starting from values chosen randomly within conservative limits. Figure 3 (left panel) shows the resulting $\chi^2$ distribution for $e_i < 0.3$. For each grid point, the figure displays the best $\chi^2$ value of typically 50 model fits. The minimum is attained at $e_i, i = 0.15, 0.22$ with $\chi^2_{\text{min}} = 95.4$. In the extension of the grid to $e_i = 0.70$, no model fit with $\chi^2 < 150$ was found. The contour lines refer to increments $\Delta \chi^2$ of +1.0, +2.3, +4.6, and +13.8, corresponding to confidence levels of 40%, 68.3%, 90.0%, and 99.9% for two degrees of freedom. Color coding displays the lowest $\chi^2$ as dark red, fading into white at the 99.9% level. The 68.3% contour encloses a substantial fraction of the $e_i$, $i$ plane, indicating that the data permit fits with a wide range of eccentricities and appropriate adjustment of the remaining parameters. Notably, those describing the inner planet are not well defined by the data alone, and the independent stability information is required for further selection.

We calculated the temporal evolution of all models with $\chi^2 < 110$ (99.93% confidence level), starting from the fit parameters and requiring that the lifetime $\tau$ exceeds the age of NN Ser of $10^6$ yr. We found that an island of secularly stable models is located close to the minimum $\chi^2$, establishing an internal consistency between the fits to the data and the results of the stability calculations. The requirement of $\tau > 10^6$ yr imposes severe constraints on the orbital parameters of the successful models and, not surprisingly, the minimum $\chi^2$ of the stable models is slightly inferior to that of the unconstrained fits. The best stable model has $\chi^2_{\text{min}} = 95.6$, eccentricities $e_i, i = 0.14, 0.21$, and a lifetime $\tau = 5 \times 10^6$ yr. The center panel of Fig. 3 shows the lifetime distribution in the $e_i$, $i$-plane, with the peak lifetime in each bin displayed. The detailed structure of the lifetime distribution is complex. In particular, the appearance of secularly stable solutions outside the main island of stability is reminiscent of a skerry landscape. These solutions fit the data less well and disappear if the $\chi^2$ limit is reduced or, to stay in the picture, the sea level is raised. This is shown in the right-hand panel of Fig. 3, where only solutions with $\tau > 10^6$ yr and $\chi^2 < 97.9$ are retained (68.3% confidence). The efficiency of the lifetime selection is demonstrated by the reduced size of the island of stability, which covers only a fraction of the $\chi^2$-space delineated by the 1-$\sigma$ contour level in the left-hand panel (solid curve). Its size decreases a bit further if the eccentricities are also optimized as done in Run 2.

We combined the results of Runs 1 and 2 to investigate the spread of the fit parameters for the solutions that pass the lifetime criterion, irrespective of their position in the $e_i$, $i$ plane. Figure 4 shows the distributions of the orbital periods $P_o$ and $P_i$ and the period ratio $P_o / P_i$ for model fits with $\chi^2 < 109.4$ and $\chi^2 < 97.9$, corresponding to the selections in the center and right-hand panels of Fig. 3, respectively. For the more lenient $\chi^2$ limit, 169 of 173 solutions yield nearly identical periods and a period ratio of 2.022 with a standard deviation $\sigma = 0.025$. The remaining four solutions interestingly have a period ratio of 2.512 with $\sigma = 0.026$. The requirement of long-term stability implies that resonant solutions are selected, preferably the 2:1 case, but also 5:2 for a few fits. Other period ratios do not occur and no nonresonant secularly stable model was found in the entire parameter space. The stability island, together with all long-lived outliers (dark red) in the center panel of Fig. 3, represents the 2:1 case, while the four 5:2 solutions lie in the light red region around $e_o, e_i = 0.12, 0.14$ on a $\chi^2$ ridge. The most longlived of the four has $\tau = 6.7 \times 10^6$ yr with $\chi^2 = 106.0$ and the best-fitting
Table 2. Observed and derived parameters of the Keplerian two-planet LTT model for NN Ser.

| Parameter | Island solutions | Best fit |
|-----------|------------------|----------|
| (a) Observed parameters: | | |
| Period $P_1$ (yr) | 15.482 ± 0.027 | 15.473 |
| Period $P_2$ (yr) | 7.647 ± 0.058 | 7.653 |
| Period ratio | 2.025 ± 0.018 | 2.022 |
| Eccentricity $e_1$ | 0.142 ± 0.011 | 0.144 |
| Eccentricity $e_2$ | 0.223 ± 0.019 | 0.222 |
| LTT amplitude $K_{lin}$ (s) | 27.65 ± 0.12 | 27.68 |
| LTT amplitude $K_{rad}$ (s) | 4.32 ± 0.22 | 4.28 |
| Argument of periapse $\omega_{1,lin}$ (°) | 317.0 ± 6.4 | 318.4 |
| Argument of periapse $\omega_{2,lin}$ (°) | 48.0 ± 2.6 | 47.5 |

$\chi^2$ for 109 degrees of freedom: <97.9, 95.56

$\tau$ for 109 degrees of freedom and $\tau > 10^5$ yr.

Notes. (1) For the island of stability in the righthand panel of Fig. 3 with $\chi^2 < 97.9$ and $\tau > 10^5$ yr, including the 1-σ errors. (2) For the best fit with $\chi^2 = 95.56$ for 109 degrees of freedom and $\tau > 10^5$ yr.

4.2. Temporal evolution of the planetary system of NN Ser

In this section, we explore the temporal evolution of the secularly stable two-planet models that start with orbital elements defined by the Keplerian fits to the data. We find that all long-lived models of Fig. 3 (central and right-hand panels) adhere to the 2:1 mean-motion resonance. This holds for the models in the stability island, but also for the solutions that correspond to the skerries surrounding it. Their isolated character probably results from the increasing difficulty of finding a set of parameters that secures resonance, as the start parameters deviate from their optimal values. In all these solutions, the mean-motion resonant variable $\Theta_1 = \lambda_1 - 2 \lambda_2 + \omega_1$, a function of the planet longitudes $\lambda_1$ and $\lambda_2$, librates about zero and the secular variable $\Delta \omega = \omega_1 - \omega_2$ circulates. This agrees with the expected behavior of a two-planet system with a mass ratio $M_2/M_1 > 1$ (Michtchenko et al. 2008).

Figure 7 shows the evolution of selected parameters for the best-fitting model of Table 2 over the first 3050 yr of the lifetime, which exceeds $10^8$ yr. The two principal periods of the system (Rein & Papaloizou 2009) are 105 yr and 736 yr. Both are prominent in the librations of $\Theta_1$. In the shorter period, low-amplitude anti-phased oscillations of the semi-major axes occur (not shown). In the longer period, $\Delta \omega$ circulates, anti-phased oscillations of the eccentricities take place, and the minimum separation between the two planets recovers. The protection mechanism due to the 2:1 resonance always keeps the separation above 2.15 AU, effectively limiting the mutual gravitational interaction (bottom right panel). As an illustration, we show the orbits at the time of peak eccentricity $\epsilon_1$ in Fig. 8. The system is locked deep in the 2:1 mean-motion resonance, as demonstrated by the near-zero mean values of $\Theta_1$, averaged over successive 736-yr intervals. For the first 20,000 yr of the evolution, the 26 values of $\langle \Theta_1 \rangle$ can be represented by an underlying linear and a superposed sinusoidal variation with a period of 3450 yr and an amplitude of 0.86°. The linear component has a fitted slope of $\langle \Theta_1 \rangle = -0.2 \pm 1.7 \times 10^{-6}$ deg yr$^{-1}$, entirely consistent with zero. All long-lived solutions that start from fits in the stability island behave similarly to the best-fit model, in the sense that all of them have $\Theta_1$ librating and $\Delta \omega$ circulating. This also holds for the long-lived outliers in the central panel of Fig. 3 that have $\tau > 10^5$ yr. They differ in the secular period, which ranges from 330 to 1700 yr.

All models considered thus far involved prograde coplanar edge-on orbits. We calculated a few models with different inclinations $i_1$ and $i_2$ of the inner and outer planet as a first step toward a more comprehensive study of NN Ser’s planetary system. A common tilt in the planetary orbits, which enhances the planetary masses is limited to 25°, beyond which instability in...
Fig. 7. Temporal evolution of selected osculating orbital elements of a model, starting from the best-fit Keplerian parameters of Table 2. The first 3050 yr of the greater than 10^8 yr lifetime are displayed. The left-hand panels show the eccentricities and arguments of periapse for the motion of the center of mass of the binary, the top righthand panel shows the resonant angle Θ_1, and the bottom righthand panel the minimum separation between the two planets for successive orbits of the inner planet.

Fig. 8. Astrocentric orbits for t = 471 − 487 yr after the begin of the calculation. The locations of the periapses are marked ‘P’, the solid dots indicate conjugation, and the open circles opposition. Orbital motion is counter-clockwise.

5. Summary and discussion

We have presented 69 new mid-eclipse times of NN Ser, which cover the minimum and subsequent recovery of the O–C_in residuals expected from our previous model in Paper I. The data allowed us to derive a significantly improved two-planet model for NN Ser based on the interpretation of the observed eclipse-time variations in terms of the LTT effect. Combined with extensive stability calculations, we find that the only model that simultaneously fits the data and is secularly stable involves two planets locked in the 2:1 mean-motion resonance. We did not find any good fits that are nonresonant and stable. Apart from differences in the periods and mass ratio, the NN Ser system bears resemblance to the HD 128311 planetary system (Sándor & Kley 2006; Rein & Papaloizou 2009). The best fit implies eccentricities e_o = 0.14 and e_i = 0.22 for the outer and inner planets, respectively, and a mass ratio M_o/M_i = 4 for coplanar orbits. As expected theoretically for a system with a more massive outer planet (Michtchenko et al. 2008), the temporal evolution of the model that starts from our best fit involves a libration of the resonant variable Θ_1 and a circulation of Δω. Preliminary stability calculations for tilted orbits suggest that deviations from coplanarity with the binary orbit cannot be large.

Eclipse-time variations in PCEB seem to be ubiquitous (e.g. Beuermann et al. 2012; Parsons et al. 2010b; Qian et al. 2012, and references therein) and the plausibility of explaining them by the LTT effect has increased by the definitive discovery of planets orbiting non-evolved close binaries with KEPLER (e.g. Doyle et al. 2011; Orosz et al. 2012; Welsh et al. 2012). That the orbital periods in the two types of systems differ by one to two orders of magnitude is a selection effect, because the LTT effect increases with the orbital period, while the radial velocities and transit probabilities decrease. Contrary to radial velocities measured by line shifts, however, the interpretation of the eclipse-time variations in terms of a displacement of the binary along the line of sight is not unique, since eclipse-time variations can also be produced by mechanisms internal to the binary (e.g. Applegate 1992). Most authors, however, consider this action as inadequate for explaining the magnitude of the observed variations (e.g. Brinkworth et al. 2006; Chen 2009; Watson & Marsh 2010). Doubts about the LTT interpretation nevertheless remain, raised by such problematic cases as HU Aqr (Goździewski et al. 2012; Horner et al. 2011; Hinse et al. 2012) and QS Vir (Parsons et al. 2010b). Resolving these cases remains an important task. Presently, NN Ser represents the best documented case in favor of the LTT hypothesis with an agreement between data and
model at the 100 ms level, and it is also the prime contender for an observational proof of the required strict periodicity of the signal, given enough time.

The evolutionary history of planetary systems orbiting PCEB may be complex. Planets either existed before the CE and had to pass through the envelope or they were formed in it. In the former case, the orbit of any pre-existing planet is severely affected by the loss of typically more than one half of the mass of the central object. In the latter case, formation depends critically on the conditions in the expanding envelope. In both scenarios, migration may drive a planet pair into resonance, but predicting the properties of the emerging post-CE system is a difficult task. Establishing the structure of systems like NN Ser may provide the observational basis for such a program.

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