Interaction of bubbles in an ultrasonic running wave

I A Aganin and A I Davletshin

Institute of Mechanics and Engineering - Subdivision of the Federal State Budgetary Institution of Science “Kazan Scientific Center of the Russian Academy of Sciences” (IME - Subdivision of FIC KazanSC of RAS) 2/31, Lobachevsky str., Kazan 420111 Russia

E-mail: aganel@gmail.com

Abstract. The influence of the initial sizes of two bubbles on their dynamics is studied under the action of a plane ultrasonic wave travelling along a straight line passing through the centers of the bubbles. The bubbles are filled with air, the liquid is water in room conditions. Initially the total volume of the two bubbles is equal to two volumes of a 5 μm bubble, the distance between the bubble centers is 30 μm. The wavelength is 5 mm, its amplitude is 0.3 bar. A mathematical model used is of fourth order of accuracy in terms of the ratio of the radii of bubbles to the distance between them. It allows for the small non-sphericity of the bubbles, the liquid viscosity and compressibility, the surface tension. It is shown that under the considered conditions three scenarios of the interaction of bubbles are possible: their collision, their divergence, and destruction of one of them due to large deformations of its surface. The ranges of a parameter characterizing the ratio of the initial radii of bubbles in which these scenarios are realized are revealed. The interaction scenarios are shown to depend on the arrangement of the bubbles.

1. Introduction
Dynamics of bubbles in acoustic fields attracts considerable scientific and applied interest. Its features are associated with such phenomena as luminescence [1], neutron emission [2], acoustic streamers [3]. Bubbles in acoustic fields are used for ultrasonic cleaning of surfaces of bodies from various contaminants [4], intensification of physical and chemical processes [5], diagnostics of diseases, drug delivery [6, 7]. In this paper, we study the features of the interaction of two gas bubbles of micron sizes in liquid in a plane ultrasonic wave propagating along a straight line passing through the centers of the bubbles. Similar problems may arise in medical applications. A mathematical model [8] is used, which is a small modification of the models of [9, 10], in which the wave acting on the bubbles is considered to be standing.

2. Problem statement and numerical technique
The dynamics of two gas (air) bubbles in liquid (water) in a plane ultrasonic running wave is considered (figure 1). The wave propagates in liquid in the positive direction of the z axis, on which the bubble centers are located. It is assumed that at the initial time \( t = 0 \) the bubbles are spherical, and their surfaces are motionless. The initial radii of the bubbles \( R_{0,1} \) and \( R_{0,2} \) are varied so that the sum of their volumes remains unchanged and equal to the sum of the volumes of two identical bubbles of radius \( R_0 = 5 \) μm. The initial distance between the centers of the bubbles is \( 6R_0 \). Radii \( R_{0,1} \) and \( R_{0,2} \) are defined as follows
\[
R_{0,1} = (2-\alpha^3)^{1/3} R_0, \quad R_{0,2} = \alpha R_0, \quad (1)
\]
\[
R_{0,1} = \alpha R_0, \quad R_{0,2} = (2-\alpha^3)^{1/3} R_0, \quad (2)
\]

The parameter \( \alpha \) varies in the range \( 0.2 \leq \alpha \leq 1 \). In the case (1), we have \( R_{0,1} > R_{0,2} \), and in the case (2), on the contrary, \( R_{0,1} < R_{0,2} \).

The room conditions are assumed: the static liquid pressure \( p_0 = 1 \) bar, the liquid density \( \rho = 1000 \text{ kg/m}^3 \). The length of the traveling wave is \( \lambda = 2\pi c_0 / \omega \) where \( \omega = 0.6\pi \text{ MHz} \), \( c_0 = 1500 \text{ m/s} \) is the sound velocity in liquid, the wave amplitude is \( p_a = 0.3 \) bar.

The surfaces of the bubbles are presented in the form of the series of spherical harmonics

\[
r_i = R_i(t) \left[ 1 + \sum_{m=2}^{N} \varepsilon_{m,i}(t) P_m(\cos \theta_i) \right],
\]

where \( i = 1, 2 \) is the bubble number, \( r_i, \theta_i \) are the radial and angular coordinates of the spherical reference system \( r_i, \varphi_i, \theta_i \) with the origin at the center of the \( i \)-th bubble, \( P_m \) is the Legendre polynomial of degree \( m \), \( \varepsilon_{m,i} = a_{m,i}/R_i \) is the dimensionless amplitude of the deviation of the shape of the bubble from the spherical shape in the form of \( P_m(\cos \theta) \), \( a_{m,i} \) is a similar but dimensional amplitude, \( N \) is the number of harmonics in the expansion.

The non-sphericity of a bubble is assumed to be small, so that \( |\varepsilon_{m,i}| < 1 \), it is accepted that \( N = 5 \). The bubble dynamics model [8] utilized in the present paper is a system of ordinary differential equations of the second order in the radii of the bubbles \( R_i \), the coordinates of their centers \( z_i \) on the \( z \) axis, and the dimensionless amplitudes of the deviations of their shapes from the spherical one \( \varepsilon_{m,i} \). It is of the fourth order of accuracy in terms of the ratio of the radii of the bubbles to the distance between them. The gas in the bubbles is assumed to be homobaric, the effects of the liquid viscosity and compressibility are taken into account approximately.

3. Results

The results of calculations have shown that with the initial data and the conditions of the action of a plane ultrasonic traveling wave on two bubbles, taken in this paper, three scenarios of their interaction are realized: collision of bubbles, destruction of bubbles due to large deformations of the surface of one of them, and unlimited divergence of bubbles. In the literature, one more scenario is known in which the bubbles travel for a fairly long time in the direction of propagation of the wave, being at an equal distance from each other [9], but under the conditions of this work it is not implemented. It should be noted that here, unlike other papers, possible destruction of bubbles due to large deformations of their surfaces is analyzed.

Figure 2 illustrates the first scenario of bubble interaction, when bubbles eventually approach and collide. In the presented case, they behave almost identically, so that the influence of the wave phase on their interaction is unessential. Distortions of the bubble surfaces remain small until collision.

Figure 3 illustrates the second scenario of bubble interaction, when one of the bubbles (the first one) is destroyed. The destruction is due to the increase in the distortion of its sphericity in the form of the second harmonic. An increase in the magnitude of the dimensionless amplitude of the bubble
sphericity distortion $\varepsilon_{m,i}$ in the form of any harmonic to the value 1 is accepted as a criterion for destruction.

**Figure 2.** Changes in the radii of the bubbles $R_i$ (the curves for the two bubbles graphically coincide) (a), in the positions of their centers $z_i$ on the $z$ axis (thick solid curves) (b) and in the dimensionless amplitudes of the deviations of their shapes from the spherical one $\varepsilon_{m,i}$ (c) in the case of bubble collision ($R_{0,1} = R_{0,2} = 5 \mu m$). In figure (b): thin solid curves are the change of the closest-to-each-other points of the surfaces of bubbles located on the $z$ axis.

**Figure 3.** The same as in figure 2, but for the case of destruction of one of the bubbles ($\alpha = 0.625$, $R_{0,1} \approx 6.032 \mu m > R_{0,2} = 3.125 \mu m$).

**Figure 4.** The same as in figure 2, but for the case of bubble divergence ($\alpha = 0.25$, $R_{0,1} \approx 6.283 \mu m > R_{0,2} = 2.25 \mu m$). In figure (b): the dashed lines (for bubble 1 it graphically coincides with the solid line) are the displacements of similar but single bubbles.

Figure 4 characterizes the third scenario of interaction, when the bubbles diverge (the distance between them increases unlimitedly). It can be seen that the effect of the interaction on the larger bubble is insignificant.

Figure 5 characterizes the ranges of the parameter $\alpha$ in which each of the three identified scenarios of bubble interaction is realized. It is seen that for $0 < 1 - \alpha < 0.438$ the "rearrangement" of bubbles at the beginning of their interaction does not affect the scenario of their interaction.
0.438 < 1 − \alpha < 0.6, the bubble collision is realized earlier if \( R_{0,1} < R_{0,2} \). At 0.6 ≤ 1 − \alpha < 0.8, the bubbles collide in the case of \( R_{0,1} < R_{0,2} \) and diverge at \( R_{0,1} > R_{0,2} \).

**Figure 5.** Dependence on 1 − \alpha of the number of lengths of the traveling wave \( K \), at which the bubbles collide (solid lines) or the destruction of one of them takes place (dashed line). The vertical dotted line is the left boundary of the region of variation of the parameter 1 − \alpha, in which the distance between the bubbles increases with time unlimitedly in the case (1). Curves 1, 2 were obtained for \( \alpha \) determined by (1) \( (R_{0,1} > R_{0,2}) \) and (2) \( (R_{0,1} < R_{0,2}) \), respectively.

**Acknowledgments**
The work was supported by the Russian Science Foundation (grant No. 17-11-01135).

**References**
[1] Putterman S J and Weninger K P 2000 Sonoluminescence: How Bubbles Turn Sound into Light *Annu. Rev. Fluid Mech.* **32** 445
[2] Taleyarkhan R P, West C D, Cho J S, Lahey R T (jr), Nigmatulin R I and Block R C 2002 Evidence for Nuclear Emissions During Acoustic Cavitation *Science* **295** 1868
[3] Parlitz U, Mettin R, Luther S, Akhatov I, Voss M and Lauterborn W 1999 Spatio-temporal dynamics of acoustic cavitation bubble clouds *Phil. Trans. R. Soc. Lond. A* **357** 313–34
[4] Mason T J 2016 Ultrasonic cleaning: An historical perspective *Ultrasonics Sonochemistry* **29** 519–23
[5] Suslick K S 1990 Sonochemistry *Science* **247** 1439–45
[6] Miller D L and Quddus J 2000 Diagnostic ultrasound activation of contrast agent gas bodies induces capillary rupture in mice *Proceedings of the National Academy of Sciences of the United States of America* **97** 10179–84
[7] Seemann S, Hauff P, Schultz-Mosgau M, Lehmann C, and Reszka R 2002 Pharmaceutical evaluation of gas-filled microparticles as gene delivery system *Pharmaceutical Research* **19** 250–57
[8] Aganin I A and Davletshin A I 2017 Dynamics of two gas bubbles in liquid in an ultrasonic traveling wave *Proceedings of the Mavlyutov Institute of Mechanics* **12** no 1 33–9
[9] Aganin A A and Davletshin A I 2009 Simulation of interaction of gas bubbles in a liquid with allowing for their small asphericity *Matematichesko modelirovanie* **21** no 6 89–102
[10] Aganin A A, Davletshin A I and Toporkov D Yu 2014 Dynamics of cavitation bubbles arranged in a line in an intense acoustic wave *Computational Technologies* **19** no 1 3–19