Lattice measurement of the energy-gap
in a spontaneously broken phase

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Abstract

Using lattice simulations of a one-component ($\lambda\Phi^4$)$_4$ theory, we have measured the energy spectrum $\omega(k)$ in the broken phase at various lattice sizes. Our data show that the energy-gap $\omega(0)$ is not the ‘Higgs mass’ $M_h$ but an infrared-sensitive quantity that becomes smaller and smaller by increasing the lattice size and may even vanish in the infinite-volume limit.

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In the case of a one-component $\lambda \Phi^4$ theory, and due to the underlying ‘triviality’ of the theory in 3+1 space-time dimensions [1], the energy spectrum of the broken symmetry phase $\omega(k)$ is believed to approach a single-particle form, say $\sqrt{k^2 + M_h^2}$, in the continuum limit of quantum field theory. Equivalently, the energy-gap $\omega(0)$ is assumed to represent a good measure of the ‘Higgs mass’ $M_h$. This statement is explicitly supported by the analysis of Ref. [2], where all perturbative ambiguities in the definition of the Higgs mass are shown to be very small in the scaling region.

Thus, by approaching the continuum limit with a lattice simulation of the theory, the shape of the energy spectrum should be better and better reproduced by (the lattice version of) a single-particle form $\sqrt{k^2 + \text{const}}$. Of course, this is completely equivalent to a continuum limit of the shifted-field propagator

$$G(p) \rightarrow \frac{Z_{\text{prop}}}{p^2 + \bar{m}^2},$$

with $Z_{\text{prop}} \rightarrow 1$ for consistency with the Källen-Lehmann spectral decomposition of a ‘trivial’ theory.

These theoretical expectations have been compared with the results of model-independent lattice simulations in Refs. [3, 4]. The lattice data for the scalar propagator are well reproduced by Eq. (1) in the symmetric phase. In the broken phase, however, the fit with Eq. (1), although excellent at high momentum, becomes very poor for $p \rightarrow 0$. Equivalently, the measured energy spectrum is well reproduced by the single-particle form $\sqrt{k^2 + \bar{m}^2}$ for not too small $|k|$. However, the spectrum deviates significantly when $k \rightarrow 0$ and direct measurements of $\omega(0)$ exhibit larger and larger percentage deviations from $M_h \equiv \bar{m}$ by approaching the continuum limit [4]. Motivated by these unexpected discrepancies, we have undertaken a more systematic analysis of the energy spectrum on various $L^4$ lattices with $20 \leq L \leq 40$. Our results will be reported in the following.

The one-component ($\lambda \Phi^4$)$_4$ theory

$$S = \sum_x \left[ \frac{1}{2} \sum_\mu (\Phi(x + \hat{e}_\mu) - \Phi(x))^2 + \frac{r_0}{2} \Phi^2(x) + \frac{\lambda_0}{4} \Phi^4(x) \right]$$

(2)

is conveniently studied in the Ising limit

$$S_{\text{Ising}} = -\kappa \sum_x \sum_\mu [\phi(x + \hat{e}_\mu)\phi(x) + \phi(x - \hat{e}_\mu)\phi(x)]$$

(3)
with $\Phi(x) = \sqrt{2\kappa}\phi(x)$ and where $\phi(x)$ takes only the values $+1$ or $-1$. The broken phase is found for values $\kappa > \kappa_c \sim 0.0748$.

We performed Monte-Carlo simulations of this Ising action using the Swendsen-Wang cluster algorithm. Statistical errors can be estimated through a direct evaluation of the integrated autocorrelation time, or by using the “blocking” or the “grouped jackknife” algorithms. We have checked that applying these three different methods we get consistent results.

As an approach to the ‘Higgs mass’ we have used the method of “time-slice” variables described in Ref. [12] which has the advantage of being independent of uncontrolled theoretical assumptions. To this end, let us consider a lattice with 3-dimension $L^3$ and temporal dimension $L_t$ and the two-point correlator

$$C_1(t, 0; k) \equiv \langle S_c(t; k)S_c(0; k) + S_s(t; k)S_s(0; k) \rangle_{\text{conn}},$$

where

$$S_c(t; k) \equiv \frac{1}{L^3} \sum_x \phi(x, t) \cos(k \cdot x),$$

$$S_s(t; k) \equiv \frac{1}{L^3} \sum_x \phi(x, t) \sin(k \cdot x).$$

Here, $t$ is the Euclidean time; $x$ is the spatial part of the site 4-vector $x^\mu$; $k$ is the lattice momentum $k = (2\pi/L)(n_x, n_y, n_z)$, with $(n_x, n_y, n_z)$ non-negative integers; and $\langle ... \rangle_{\text{conn}}$ denotes the connected expectation value with respect to the lattice action, Eq. (3). In this way, parameterizing the correlator $C_1$ in terms of the energy $\omega(k)$ as

$$C_1(t, 0; k) = A \left[ \exp(-\omega(k)t) + \exp(-\omega(k)(L_t - t)) \right],$$

a mass can be defined through the lattice single-particle dispersion relation

$$m^2_{TS}(k) = 2(\cosh \omega(k) - 1) - 2 \sum_{\mu=1}^3 (1 - \cos k_\mu).$$

In the broken phase, by adopting Eq. (7) one neglects the effect of tunneling between the two degenerate vacua. For the value of $\kappa$ that we shall consider the tunneling effect is negligible for lattices as large as $20^4$, as in our case (see also the Appendix of Ref. [4]).
Eq. (1). In general, observable deviations of $m_{TS}$ from a simple constant behaviour give a measure of those contributions to the energy spectrum that go beyond a single-particle form. However, regardless of any theoretical model, $m_{TS}(0)$ defines $\omega(0)$, the energy-gap of the theory.

As a check of our simulations we started our analysis at $\kappa = 0.0740$ in the symmetric phase on a $20^4$ lattice, where high-statistics results by Montvay and Weisz [12] are available. In Fig. 1 we show the values of the time-slice mass $m_{TS}(k)$ (Eq. (8)) at several values of the 3-momentum. The shaded area corresponds to the value $\bar{m} = 0.2141(28)$ obtained from the fit to the propagator data in Ref. [4] and perfectly agrees with the result of Ref. [12]. We see that $m_{TS}$ is indeed independent of $k$ so that the energy spectrum of the symmetric phase is very well reproduced by a single-particle form as expected. Finally, we have checked two values of $k$ on a bigger $32^4$ lattice. Notice that, even for a lattice mass as small as 0.2, the numerical value of the energy-gap remains remarkably stable.

We now choose for $\kappa$ the value $\kappa = 0.076$ in the broken symmetry phase where high-statistics results by Jansen et al. [13] are available. In this case, the time-slice mass $m_{TS}(k)$ shows a distinctive behaviour as seen in Fig. 2. At higher momentum, the time-slice mass agrees well with the value of $M_h = \bar{m}$ obtained in Ref. [4] from a fit to the propagator data at high momenta. On the other hand, there are sizeable deviations when $k \to 0$.

The most striking result concerns, however, the time-slice mass at zero momentum. In Table 1 we have reported the outputs of several independent lattice simulations obtained from different random sequences (we used the pseudorandom numbers generator RANLUX [14, 15] with ‘luxury level’ 4). Regardless of the operative definition adopted for the ‘Higgs mass’, the energy-gap itself becomes smaller and smaller by increasing the lattice size (see Fig. 3). This result should be compared with the remarkable stability of $M_h \equiv \bar{m}$, as extracted from the set of the high-momentum data (see Table 2 of Ref. [4]). As one can check, Eq. (7) provides an excellent fit to the lattice data (see Fig. 4).

Thus, our lattice simulation shows that the energy-gap in the broken phase is an infrared-sensitive quantity that becomes smaller and smaller by increasing the lattice size and may even vanish in the infinite-volume limit. Quite independently of the Goldstone phenomenon, this may signal the existence of long-wavelength collective excitations of the scalar condensate. This would explain why $\omega(0)$ cannot be taken as the input definition of $M_h$ that, rather, has to be extracted from those values of $\omega(k)$ that are well reproduced by the single-particle
form $\sqrt{k^2 + \text{const.}}$.

If the energy-gap $\omega(0)$ vanishes in an infinite volume, as our data suggest, the same conclusion holds in a spontaneously broken continuous $O(N)$ symmetry for the energy spectrum of the singlet Higgs field. Therefore in the Standard Model there would be unexpected long-range forces that survive after coupling the scalar fields to gauge bosons. In view of the importance of the issue, we hope and expect that our numerical results for the energy-gap will be checked (and/or challenged) by other groups.
REFERENCES

[1] R. Fernandez, J. Fröhlich, and A. D. Sokal, *Random Walks, Critical Phenomena, and Triviality in Quantum Field Theory* (Springer-Verlag, Berlin, 1992).

[2] M. Lüscher and P. Weisz, Nucl. Phys. B295, 65 (1988).

[3] P. Cea, M. Consoli, and L. Cosmai, Mod. Phys. Lett. A13, 2361 (1998), hep-lat/9805005.

[4] P. Cea, M. Consoli, L. Cosmai, and P. M. Stevenson, Mod. Phys. Lett. A14, 1673 (1999), hep-lat/9902020.

[5] I. Montvay and G. Münster, *Quantum Fields on a Lattice* (Cambridge University Press, 1994).

[6] R. H. Swendsen and J.-S. Wang, Phys. Rev. Lett. 58, 86 (1987).

[7] N. Madras and A. D. Sokal, J. Statist. Phys. 50, 109 (1988).

[8] C. Whitmer, Phys. Rev. D29, 306 (1984).

[9] H. Flyvbjerg and H. G. Petersen, J. Chem. Phys. 91, 461 (1989).

[10] B. Efron, *Jackknife, the Bootstrap and Other Resampling Plans* (SIAM Press, Philadelphia, 1982).

[11] B. A. Berg and A. H. Billoire, Phys. Rev. D40, 550 (1989).

[12] I. Montvay and P. Weisz, Nucl. Phys. B290, 327 (1987).

[13] K. Jansen, T. Trappenberg, I. Montvay, G. Münster, and U. Wolff, Nucl. Phys. B322, 698 (1989).

[14] M. Lüscher, Comput. Phys. Commun. 79, 100 (1994), hep-lat/9309020.

[15] F. James, Comp. Phys. Commun. 79, 111 (1994).
TABLE I: The energy-gap, the magnetization, and the susceptibility for $\kappa = 0.076$ at various lattice sizes. The reported data refer to independent simulations.

| lattice size | #configs. | $\omega(0)$  | $<|\phi|>$  | $\chi$     |
|--------------|-----------|--------------|-------------|------------|
| $20^4$       | 7500K     | 0.3912(12)$^a$ | 0.30158(2)$^a$ | 37.85(6)$^a$ |
| $24^4$       | 3950K     | 0.3820(47) | 0.301592(20) | 37.66(8)   |
| $24^4$       | 2750K     | 0.3756(53) | 0.301594(24) | 37.55(9)   |
| $32^4$       | 820K      | 0.3438(125) | 0.301567(31) | 37.74(21)  |
| $32^4$       | 620K      | 0.3558(150) | 0.301569(28) | 37.72(28)  |
| $32^4$       | 1000K     | 0.3353(135) | 0.301593(27) | 37.73(20)  |
| $40^4$       | 375K      | 0.2940(182) | 0.301564(24) | 37.69(37)  |
| $40^4$       | 240K      | 0.3051(262) | 0.301601(35) | 38.13(32)  |

$^a$From Ref. [13]
FIG. 1: The data for the time-slice mass Eq. (8) at different values of the 3-momentum. The shaded area represents the value \( \bar{m} = 0.2141(28) \) obtained in Ref. [4] from the fit to the propagator data and perfectly agrees with the value 0.2125(10) of Ref. [12].
FIG. 2: The time-slice mass Eq. (8) for several values of the spatial momentum and different lattice sizes. The open circle at zero momentum is the result of Ref. [13]. Our zero-momentum values are weighted averages of the corresponding measurements shown in Table I. The shaded area represents the value $\bar{m} = 0.42865(456)$ obtained from the propagator data that are well fitted by Eq. (1), see Ref. [4].
FIG. 3: The measured energy-gap for $\kappa = 0.076$ at different lattice sizes. The reported values are the weighted averages of the results in Table I. The value for $L = 20$ is from Ref. [13].
FIG. 4: The lattice data for the connected correlator Eq. (4) as a function of $t$ at $k = 0$. The reported data refer to $24^4$, $32^4$, and $40^4$ lattices in the broken phase at $\kappa = 0.076$. The solid line is the fit with Eq. (7).