Engineering a Bosonic AdS/CFT Correspondence

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Abstract

We search for a possible bosonic (i.e. non-supersymmetric) string/gauge theory correspondence by using ten-dimensional IIB and 0B strings as a guide. Our construction is based on the low-energy bosonic string effective action modified by an extra form flux. The closed string tachyon can be stabilized if the AdS scale $L$ does not exceed certain critical value, $L < L_c$. We argue that the extra form may be non-perturbatively generated as a soliton from 3-string junctions, similarly to the known non-perturbative (Jackiw-Rebbi-'tHooft-Hasenfratz) mechanism in gauge theories. The stable $\text{AdS}_{13} \times S^{13}$ solution is found, which apparently implies the existence of a 12-dimensional AdS-boundary conformal field theory with the $\text{SO}(14)$ global symmetry in the large $N$ 't Hooft limit. We also generalize the conjectured bosonic AdS/CFT duality to finite temperature, and calculate the ‘glueball’ masses from the dilaton wave equation in the AdS black hole background, in various space-time dimensions.

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1 Introduction

The (perturbatively defined, quantized) string theories can be either bosonic or supersymmetric. The (open and closed) bosonic strings naturally live in twenty-six dimensions, whereas the supersymmetric strings are ten-dimensional. There is growing evidence that all five different supersymmetric strings are related via dualities, while they also appear as certain limits of a single supersymmetric (non-string) theory called M-Theory that is essentially non-perturbative and has stable (BPS) branes [1]. Given the existence of unique theory underlying all strings, there should exist a connection between the supersymmetric M-theory and the bosonic strings too. This raises a question whether the primordial theory must be supersymmetric, and if not, one should then distinguish between the supersymmetric (say, eleven-dimensional) M-theory and the unified theory (U-Theory) of all strings and branes. In other words, supersymmetry may not be fundamental but rather of dynamical origin in certain compactifications of bosonic U-Theory. It should then be possible to generate superstrings from bosonic strings.

In this paper we take this idea seriously and investigate some of its implications from the viewpoint of a conjectured bosonic AdS/CFT correspondence. The obvious objections against bosonic U-Theory are (i) the apparent absence of fermions and (ii) the existence of a tachyon in the bosonic string theory, either open or closed. To further justify our approach, we first briefly address these fundamental issues.

The principal possibility of generating fermions from bosons in exactly solvable two-dimensional quantum field theories (like sine-Gordon) is well known since 1975 [2]. This may also apply to the bosonic string in its world-sheet formulation given by a two-dimensional conformal quantum field theory, as was first noticed by Freund in 1984 [4]. Freund considered special dimensional reductions of the bosonic string theory in 26 dimensions on 16-tori down to 10 uncompactified dimensions, and he argued that the type-I open superstring theory with the $SO(32)$ gauge group can be generated this way, with spacetime fermions originating as solitons via the standard (Frenkel-Goddard-Olive) vertex operator construction [4]. This implies the existence of a connection between the bosonic strings and supersymmetric M-Theory since the latter is related to the type I superstrings via the known chain of dualities [4].

The world-sheet mechanism of generating spacetime fermions from bosons can be complemented by the spacetime mechanism, which is known in the (perturbatively) bosonic quantum gauge field theories with the $SU(2)$ gauge group since 1976 due to

\[ \text{2See, e.g., ref. [3] for a recent review.} \]
Jackiw and Rebbi, Hasenfratz and ’t Hooft 5. They argued that the bound state of a ’t Hooft-Polyakov monopole interacting with a free particle in a half-integral representation of the unbroken $SU(2)_{\text{diag}}$ symmetry is fermionic, whereas it is bosonic in the case of an integral representation. Very recently, this field theory mechanism was generalized to string theory by David, Minwalla and Núñez 6 who argued about the non-perturbative appearance of fermions in various (perturbative) bosonic string theories from 3-string junctions in higher spacetime dimensions.

The significance of the open bosonic string tachyon was well understood during the last two years after the crucial observation by Sen 7 who first noticed that the open string vacuum can be viewed as the closed string vacuum with an unstable D25-brane. It has then become evident that there exists a stable minimum of the tachyon potential in the open bosonic string theory at the value equal to minus the tension of that D25-brane. The tachyon instability of the closed bosonic string theory may be removed in the strong coupling limit of this theory, as was recently argued by Horowitz and Susskind in the framework of hypothetical 27-dimensional Bosonic M-Theory they proposed.

The crucial feature of all those M-Theory type constructions is the presence of an extra background form flux. In the supersymmetric M-Theory the presence of the 3-form is, of course, dictated by eleven-dimensional supersymmetry. In Bosonic M-Theory, the existence of a three-form in 27 dimensions is postulated. The recently proposed non-supersymmetric type 0A string theory in ten dimensions may also be interpreted as a type IIA string in the background of a Ramond-Ramond 2-form. The tachyon instability of the type 0A string apparently implies the tachyon decay into the stable type IIA vacuum.

In this paper we study the low-energy effective action of closed bosonic strings in the background of a hypothetical gauge $(n-1)$ form $A$. The perturbative spectrum of a closed bosonic string has only one such (Kalb-Ramond) form with $n = 3$. As was argued above, it is, however, possible that higher-$n$ antisymmetric tensor fields (including the self-dual ones) may be generated as solitons, similarly to the bound states involving instantons of $SU(N)$ gauge theories in even spacial dimensions or instantons of $SO(N)$ gauge theories in $(4n+1)$ spacial dimensions (that are especially interesting for 26-dimensional bosonic strings), at least for sufficiently large $N$ (cf. ref. 8). Since any field theory containing gravity is expected to be holographic, the bosonic AdS/CFT correspondence may apply at large $N$.

3The ’t Hooft-Polyakov monopole is well known to preserve the diagonal subgroup $SU(2)_{\text{diag}}$ of $SU(2)_{\text{space}} \times SU(2)_{\text{isospin}}$. 4
2 Action and equations of motion

Our starting point is the following field theory action in arbitrary even (Euclidean) spacetime dimensions $D$:

$$S = \frac{1}{2\kappa^2} \int d^Dx \sqrt{\tilde{g}} \left\{ e^{-2\Phi} \left[ \tilde{R} - \frac{1}{12} \tilde{H}^2 + 4(\tilde{\partial}\Phi)^2 \right] + \frac{1}{2n!} \tilde{F}^2 \right\}. \quad (1)$$

Here $\tilde{g}_{MN}$ denotes a metric, $M = 1, 2, \ldots , D$, in the string frame, $\tilde{R}$ is the associated Ricci scalar, $\Phi$ is a dilaton,

$$\tilde{F}^2 = \tilde{g}^{M_1N_1} \cdots \tilde{g}^{M_nN_n} F_{M_1\ldots M_n} F_{N_1\ldots N_n}, \quad (2)$$

and

$$\tilde{H}^2 = \tilde{g}^{M_1N_1} \cdots \tilde{g}^{M_3N_3} H_{M_1\ldots M_3} H_{N_1\ldots N_3}, \quad (3)$$

in terms of the field strength $F = dA$ of the gauge $(n-1)$ form $A$ and the field strength $H = dB$ of the Kalb-Ramond two-form $B$.

The action (1) differs from the standard closed bosonic string effective action in the string frame [11] by the last term mimicking the RR-type contributions in the type-IIB string theory. Thus, at the very bottom line, we just study the field theory (1). We hope, however, that this field theory may be connected to full bosonic string theory (see sect. 1).

The metric $g$ in the Einstein frame is related to the string frame metric $\tilde{g}$ via the Weyl transformation

$$g_{MN} = e^{\frac{4}{D-2}(\Phi_0 - \Phi)} \tilde{g}_{MN}, \quad (4)$$

where $\Phi_0$ denotes the expectation value of the dilaton. We define as usual $\phi := \Phi - \Phi_0$, so $\phi$ has the vanishing expectation value. As is demonstrated in the next section, the inclusion of a tachyon does not necessarily cause any damage, so we can add the standard tachyonic action [11] to our action in the Einstein frame and thus obtain

$$S_E = \frac{1}{2\kappa^2} \int d^Dx \sqrt{g} \left\{ R - \frac{1}{12} e^{-8\phi/(D-2)} H^2 - \frac{4}{D-2}(\partial\phi)^2 - \frac{1}{2n!} e^{\frac{2(D-2)}{D-2}} \phi F^2 \right\} - \frac{1}{2} \int d^Dx \sqrt{g} e^{-2\phi} \left[ (\partial T)^2 + m^2 T^2 \right], \quad (5)$$

where $\kappa := \kappa_0 e^{\Phi_0}$, $m^2 < 0$ is the negative mass squared of the tachyon $T$ and the contractions in $F^2$ and $H^2$ are performed with the Einstein frame metric $g$.

The equations of motion derived from the action (5) are
Einstein’s equation,
\[ R_{MN} = \frac{4}{D-2} \partial_M \phi \partial_N \phi + \kappa^2 e^{-2\phi} \left[ \partial_M T \partial_N T + \frac{m^2}{D-2} g_{MN} T^2 \right] + \frac{1}{2n!} e^{2(D-2n)/(D-2)} \phi \left[ n F_{MP_2...P_n} F_{NP_2...P_n} - \frac{n+1}{D-2} g_{MN} F^2 \right] + \frac{1}{12} e^{-8\phi/(D-2)} \left[ 3 H_{MP_2P_3} H_{NP_2P_3} - \frac{2}{D-2} g_{MN} H^2 \right], \] (6)

Maxwell’s equation for the \( n \)-form and the Kalb-Ramond field strength,
\[ \partial_M \left( \sqrt{g} e^{2(D-2n)/(D-2)} \phi F^{MP_2...P_n} \right) = 0 \text{ and} \]
\[ \partial_M \left( \sqrt{g} e^{-8\phi/(D-2)} H^{MP_2P_3} \right) = 0, \] (7) (8)

the tachyonic field equation,
\[ \frac{1}{\sqrt{g}} e^{2\phi} \partial_M \left( \sqrt{g} e^{-2\phi} g^{MN} \partial_N T \right) - m^2 T = 0, \] (9)

as well as the dilatonic field equation,
\[ \Delta \phi + \frac{D-2}{4} \kappa^2 e^{-2\phi} \left[ (\partial T)^2 + m^2 T^2 \right] = \frac{D-2n}{8n!} e^{2(D-2n)/(D-2)} \phi F^2 - \frac{1}{12} e^{-8\phi/(D-2)} H^2, \] (10)

where the Laplace-Beltrami operator \( \Delta \), \( \text{viz.} \)
\[ \Delta \phi = \frac{1}{\sqrt{g}} \partial_M \left( \sqrt{g} g^{MN} \partial_N \phi \right), \] (11)

has been introduced.

3 A stable non-supersymmetric solution

We now observe that at \( n = D/2 \) (that is why we wanted an even-dimensional space-time) the coupling between the dilaton \( \phi \) and the field strength \( F \) in eq. (7) vanishes. The first term on the right hand side of the dilaton equation (10) vanishes at \( n = D/2 \), too. The same situation takes place in the type IIB supergravity in ten dimensions with the self-dual RR five-form field strength, where dilaton also decouples. We use the IIB superstring theory as our guide in the nonsupersymmetric case (see also ref. [12] as regards the similar case of type 0B strings in ten dimensions).

By using the standard (Freund-Rubin) compactification Ansatz [13] for our metric and the field strength \( F \), we find that the \( D \)-dimensional spacetime of the form
AdS$_{D/2} \times S^{D/2}$ with the rest of the fields set to zero is a solution to the equations of motion of Sect. 2, namely,

\begin{align}
g &= g_{\text{AdS}_{D/2}} \oplus g_{S^{D/2}} , \\
\phi &= 0 , \\
T &= 0 , \\
H &= 0 , \\
F &= Q \text{vol}_{S^{D/2}} ,
\end{align}

(12)

where vol$_{S^{D/2}}$ is the volume (top) form on the sphere S$^{D/2}$, and $Q$ is a real constant.

The similar AdS$_5 \times S^5$ solution (with a non-vanishing RR flux) \cite{14} to the type-IIB supergravity equations of motion is believed to hold in the full type-IIB superstring theory, i.e. to all orders in $\alpha'$, being supported by unbroken supersymmetry \cite{15}. In our bosonic case the classical solution (12) may be modified after taking into account possible bosonic string corrections in higher orders with respect to $\alpha'$.

The stability of the field configuration (12) under small fluctuations,

\begin{align}
g_{MN} &\mapsto g_{MN} + h_{MN} , \quad H \mapsto H + \delta H , \quad F \mapsto F + f , \\
\phi &\mapsto \phi + \delta \phi , \quad T \mapsto T + \delta T ,
\end{align}

(13)

to the first order can be demonstrated as follows.

We first take a look at the right hand side of the Einstein equation (6) and observe that it is quadratic in the Kalb-Ramond field strength as well as in the scalar fields and their derivatives. This means that the variation of these terms to the first order about the background (12) vanishes. We are now left with merely the second line as the right hand side of the Einstein equation (6). We can now simply ‘borrow’ a recent analysis of stability of Freund-Rubin compactifications in non-dilatonic gravity theories \cite{16} where the stability of the configuration (12) for $g_{MN}$ and $F$ was shown to the first order in the variations. Adding a Kalb-Ramond field, a dilaton and a tachyon does not change the geometry, at least in the given approximation.

In our case, the results of ref. \cite{16} are not enough because of the additional scalars and the Kalb-Ramond field, which have to be considered separately. We begin with the fluctuations of the Kalb-Ramond field strength $H$. Inserting field variations (13) into the equation of motion (7) together with the background (12) results in the only nonvanishing equation

\begin{align}
\partial_M (\sqrt{g} \delta H^{MP_2P_3}) = \partial_M (\sqrt{g} \delta [M \delta B^{P_2P_3}]) ,
\end{align}

(14)
or, in the language of differential forms,

\[ d \star d (\delta B) = 0, \]  

(15)

up to higher order terms in the fluctuations. In eq. (15) we have introduced \( \delta B \) by \( \delta H = d \delta B \), and we have used the fact that a variation \( \delta \) and a derivative \( d \) commute. Equation (15) is just the equation of a free form field. Hence, by the same reasoning as in ref. [16], we conclude that the background value \( H = 0 \) is stable under small fluctuations to the first order.

Varying the dilaton equation (10) yields

\[
(g - h)^{MN} \nabla_M \nabla_N (\phi + \delta \phi) + \\
\frac{D-2}{4} \kappa^2 e^{-2(\phi + \delta \phi)} [(g - h)^{MN} \nabla_M (T + \delta T) \nabla_N (T + \delta T) + m^2 (T + \delta T)^2] = \\
- \frac{1}{12} e^{-8(\phi + \delta \phi)} (g - h)^{M_1 N_1} \ldots (g - h)^{M_3 N_3} (H + \delta H)_{M_1 M_2 M_3} (H + \delta H)_{N_1 N_2 N_3}.
\]

(16)

Being evaluated about the background (12), this gives us a fluctuation equation on \( \delta \phi \) in the form

\[ \Delta (\delta \phi) = 0, \]  

(17)

up to terms quadratic in the fluctuations. The Laplace operator in this equation is supposed to be associated with the background metric of eq. (12). Since this background is a product, the Laplace operator in \( D \) dimensions splits as

\[ \Delta = \Delta_{\text{AdS}_{D/2}} + \Delta_{S_{D/2}}. \]  

(18)

The fluctuations can be expanded, as usual, in the spherical harmonics of \( S_{D/2} \) as follows:

\[ \delta \phi = \sum_I \varphi^I (x) Y^I (y), \quad \text{where} \quad x \in \text{AdS}_{D/2} \quad \text{and} \quad y \in S^{D/2}, \]  

(19)

while the eigenvalue equation \( \Delta_{S} Y^I (y) = - \lambda^I Y^I (y) \) with \( \lambda \geq 0 \) holds. Inserting eqs. (18) and (19) into eq. (17) (and neglecting the higher order terms) yields

\[ \Delta_{\text{AdS}} \varphi^I = \lambda^I \varphi^I. \]  

(20)

This implies the stability of the vanishing dilaton background under small fluctuations to the first order, because of the non-negativity of all ‘masses’ \( \lambda^I \). They all have to
satisfy the *Breitenlohner-Freedman* (BF) bound in the AdS space \[1\] (see eq. (24) below).

Finally, as regards tachyonic fluctuations, their computation due to the first term in eq. (9) is quite similar to the dilaton case considered above. In particular, the expansion of the tachyonic field perturbation $\delta T$ in the spherical harmonics,

$$
\delta T = \sum_I t^I(x)Y^I(y)
$$

has the same form as that of eq. (19). Hence, we have

$$
\Delta(\delta T) = \sum_I \left[ \Delta_{\text{AdS}}t^I(x) - \lambda^I t^I(x) \right] Y^I(y).
$$

The extra term linear in $T$ in eq. (9), upon evaluation about the background configuration (12) and dropping the higher order terms, gives rise to a nonvanishing contribution from the mass term. After taking all terms together, we find the following fluctuation equation for a tachyon:

$$
0 = \Delta(\delta T) - m^2(\delta T) = \sum_I \left[ \Delta_{\text{AdS}}t^I(x) - (\lambda^I + m^2) t^I(x) \right] Y^I(y).
$$

To get a stable tachyon configuration, we have to satisfy the BF bound for all modes $I$, i.e.

$$
\lambda^I - |m^2| \geq m_{\text{BF}}^2 = -\frac{1}{4L^2} \left( \frac{D}{2} - 1 \right)^2,
$$

where we have explicitly indicated that the tachyonic mass parameter $m^2$ is negative.

Equation (24) apparently implies a restriction on the scale parameter $L$ of the AdS space, which is related to the parameter $Q$ in the Freund-Rubin compactification Ansatz (12) for the gauge field strength by

$$
Q^2 = \frac{2(D-2)}{L^2}.
$$

At this point we can study how the tachyon becomes dangerous when we make a transition from $\text{AdS}_{D/2} \times S^{D/2}$ to flat spacetime, i.e. towards the standard bosonic string theory, in the limit $L \to \infty$. Then the solution $\text{AdS}_{D/2} \times S^{D/2}$ of Einstein’s equation tends to $D$—dimensional Euclidean spacetime. Because of eq. (25) the field strength $F$ vanishes in this limit. But what happens to the tachyon? The (fixed) tachyon mass satisfies the BF bound until it is reached by increasing of $L$. Further
growth of $L$ gives rise to a violation of the BF bound by the tachyon mass, which thus renders the whole spacetime unstable. We can thus consider $L$ (or $Q$, respectively) as a moduli parameter of a family of the stable AdS×$S$ type solutions. We expect that the description of the theory in terms of the action (3) ceases to hold at the critical point where the BF bound is reached. This apparently indicates on the existence of a phase transition at certain critical value $L_c$ (cf. ref. [19]).

We should also point out another interesting feature of field theory in AdS space and its description in terms of the dual CFT. Consider a scalar field $\Phi$ of mass squared $m^2$ in a $d + 1$-dimensional AdS spacetime. The BF bound arises from the formula

$$\Delta = \frac{d}{2} \pm \sqrt{\frac{d^2}{4} + L^2 m^2}$$

that determines the parameter $\Delta$ in the propagator of $\Phi$. This parameter should be a real number satisfying

$$\Delta \geq \frac{d}{2} - 1$$

in order to ensure normalizability of the scalar modes. Thus we see that the BF bound is the necessary condition for the reality of $\Delta$. Given $m^2 L^2 > -(d/2)^2 + 1$, the normalizability condition forces us to choose the plus sign in eq. (26). Otherwise, both signs in eq. (26) are allowed. As we approach the critical point $L_c$, the two representations of the theory by the propagators with the two possibilities for $\Delta$, respectively, join at $L_c$.

Under the AdS/CFT duality [14, 15] the parameter $\Delta$ translates into the conformal dimension of the dual field, while the normalizability condition goes into the positivity (unitarity) condition on the two-point functions of the dual CFT as a quantum field theory in the sense of Wightman [18]. In general, this is not enough to ensure positivity (unitarity) of the CFT in question. For example, a symmetric four-point function $\langle ABAB \rangle$ can be expanded into the conformal partial waves as

$$\langle ABAB \rangle \sim \sum_C (f^C_{AB})^2 \int dx dy \langle ABC(x) \rangle \langle C(x)C(y) \rangle^{-1} \langle C(y)AB \rangle. \quad (28)$$

The conformal group representation of $C$ should be unitary while the coupling constant squared $(f^C_{AB})^2$ should be positive.

Summarizing above, a tachyon can be stabilized in the AdS×$S$ type background (with the other fields given above) provided the AdS scale does not exceed the certain value determined by the negative mass squared of the tachyon. Unfortunately, this fact simultaneously restricts the applicability of our result to the bosonic string theory
because the low-energy bosonic string effective action is valid only if $\alpha'$ is small enough to suppress string loop corrections, while the spacetime curvature has to be small too, in order to suppress strong gravity effects. The small curvature is needed to stabilize the tachyon in the sense of the BF bound, whereas the (negative) mass of the tachyon is proportional to $(\alpha')^{-1}$. The way out of this difficulty may be fine tuning to get a domain, where $L$ is large enough while $\alpha'$ is sufficiently small to let the low-energy effective action be valid while not spoiling the BF bound. This reasoning does not seem to be far fetched when comparing it with the type-0 string theory where a tachyon can be stabilized for sufficiently small AdS radii in the dual CFT description [19].

4 Bosonic AdS/CFT correspondence

Having established the existence of a stable solution of the form $\text{AdS}_{D/2} \times S^{D/2}$, it is natural to speculate about the existence of a bosonic AdS/CFT correspondence between gravity in the bulk and a conformal field theory on the boundary, in a certain (with small spacetime curvature) limit. This correspondence may be just a sign of a more fundamental holographic duality between bosonic strings in the bulk and conformal field theory on the AdS boundary. The action of the AdS isometry group on the AdS boundary is identified with the conformal group.

In the absence of supersymmetry it is tempting to use type IIB superstrings as a guide. As is usual in the AdS/CFT correspondence [14, 15], our stable solution $\text{AdS}_{D/2} \times S^{D/2}$ should be identified with the near horizon limit of a black $p$-brane solution, with $p = (\frac{D}{2} - 2)$. Though this p-brane is not a BPS object since there is no supersymmetry to be partially broken, the cancellation of gravitational attraction and electromagnetic repulsion, needed for stability of the $p$-brane (see also eq. (29) below), seems to be the natural substitute for the BPS property.

We are supposed to set $D = 26$ in the case of bosonic strings, which implies that our gauge field strength form $F$ should be a 13-form that can also be self-dual in 26 dimensions, similarly to the analogous 5-form gauge field strength of IIB superstrings. The boundary of the 13-dimensional AdS space is 12-dimensional, so our considerations imply the existence of a 12-dimensional conformal field theory with a global $SO(14)$ symmetry, where we have used the fact that $S^{13} = SO(14)/SO(13)$. It may not be accidental that this 12-dimensional conformal gauge field theory is directly related to F-Theory [20] whose existence was motivated by the self-duality of type IIB superstrings!
Imposing self-duality on $F$ in our non-supersymmetric setting may be dangerous due to gravitational anomalies, since we have no mechanism at hand to cancel them. In the non-supersymmetric type 0B theory the self-duality is needed in order to obtain conformal invariance of the boundary theory up to two loop order \[12\]. The gravitational anomalies may cancel in the type 0B string theory, if the dual field theory is an orbifold of $\mathcal{N} = 4$ supersymmetric Yang-Mills theory.

To establish a connection between the gauge coupling and the AdS radius $L$ in our bosonic case, we mimick the supersymmetric AdS/CFT correspondence \[14, 15\] with IIB superstrings in the bulk. Let us consider the magnetic charge of $N$ D11-branes \[1\] stacked ‘on the top of each other’. On the one hand, this charge is given by \[1\]

$$g_{11} = N\tau_{11} \sqrt{16\pi G_N}, \quad (29)$$

where $\tau_{11}$ is the D11-brane tension and $G_N$ is Newton’s constant. They are related to ‘stringy’ constants as follows \[1\]:

$$\tau_{11} = \frac{\sqrt{\pi}}{16\kappa} (4\pi^2\alpha')(11-p)/2 = \frac{\sqrt{\pi}}{16\kappa}, \quad \kappa = \sqrt{8\pi G_N} = 2\pi g_{\text{string}}, \quad (30)$$

where $g_{\text{string}}$ is the closed bosonic string coupling constant. On the other hand, the magnetic charge is just an integral of the field strength $F$ over a sphere surrounding the D11-brane in the transverse directions. The Freund-Rubin Ansatz \[12\] now implies

$$g_{11} = \frac{1}{\sqrt{16\pi G_N}} \int_{S^{13}} F = \frac{Q}{\sqrt{16\pi G_N} \Omega_{13}}, \quad (31)$$

where $\Omega_{13} = 2\pi^7/6!$ denotes the volume of $S^{13}$. Having identified eqs. (29) and (31), we use the relations (30) to derive

$$Q = \frac{N\tau_{11} 16\pi G_N}{\Omega_{13}} = \frac{90}{\pi^{11/2}} Ng_{\text{string}}, \quad (32)$$

The Yang-Mills coupling constant of the gauge fields living in the worldvolume of the D11-brane cluster is related to the D11-brane tension as

$$g_{\text{YM}}^2 = \tau_{11}^{-1}(2\pi\alpha')^{-2}. \quad (33)$$

This relation together with eq. (30) implies a connection between the Yang-Mills coupling constant and the string coupling constant in the form

$$g_{\text{string}} = \frac{1}{8\pi^{3/2}} \alpha'^2 g_{\text{YM}}^2. \quad (34)$$

\[4\] We define Dp-branes as the spacetime $(p + 1)$ dimensional submanifolds where open bosonic strings can end (cf. ref. \[1\]).
We recall that the extremal 11-brane solution is described by the metric
\[ ds^2 = H^{-1/6} d\bar{x}^2 + H^{1/6} (dr^2 + r^2 d\Omega^2_{13}) , \]  
(35)
where we have introduced the notation \( d\bar{x}^2 = dt^2 + \sum_{j=1}^{11} (dx^j)^2 \) and the metric on the to the 11-brane world volume transverse space \( dr^2 + r^2 d\Omega^2_{13} \). The ‘warp’ factor \( H(r) \) becomes
\[ H(r) \equiv 1 + \frac{Q}{4\sqrt{3}r^{12}} \sim 0 \to \frac{Q}{4\sqrt{3}r^{12}} = \frac{90}{32\sqrt{3}\pi^4} N\alpha'^2 g_{YM}^2 r^{-12} \]  
(36)
in the near horizon limit \( r \to 0 \). In the last equality we have also plugged in eqs. (32) and (34). We now introduce the string length \( l_{\text{string}}^2 = \alpha' \), and obtain the following metric of the D11-brane in the near horizon approximation:
\[ ds^2 = \frac{r^2}{L^2} d\bar{x}^2 + \frac{L^2}{r^2} dr^2 + L^2 d\Omega^2_{13} , \]  
(37)
where
\[ L^{12} = \frac{Q}{4\sqrt{3}} = \frac{90}{32\sqrt{3}\pi^4} N g_{YM}^2 l_s^4 . \]  
(38)
Equation (37) just describes AdS_{13}(L) \times S^{13}(L). In contrast to eq. (25), in the near horizon approximation, \( L \) grows monotonically with \( Q \).

We are now in a position to see the AdS/CFT correspondence at work in our setting. Let’s define the ’t Hooft coupling,
\[ \lambda = g_{YM}^2 N , \]  
(39)
and consider the large \( N \) limit, \( N \to \infty \) with \( \lambda \) fixed, which implies \( g_{YM}^2 \to 0 \) and hence, \( g_{\text{string}} \to 0 \) too. This means that we can restrict ourselves to bosonic string trees. To approach the strong ’t Hooft coupling limit, \( \lambda \to \infty \), with a fixed length scale \( L \), we must have \( l_{\text{string}}^2 = \alpha' \to 0 \). This physically means that the description of string theory in terms of particles, i.e. in terms of the low-energy effective action (4), becomes reliable. Thus we arrive at the situation quite similar to the conventional AdS/CFT correspondence in the supersymmetric type IIB context, but without supersymmetry.

## 5 Glueball masses from higher dimensions

It is rather straightforward to consider the thermodynamics of this 12-dimensional conformal field theory by relating it to the near extremal stable 11-brane. Perhaps
even more exciting is the use of the conjectured bosonic AdS/CFT duality at finite temperature, in order to get predictions for the ‘glueball’ masses as eigenvalues of the dilaton wave equation in the AdS black-hole geometry near the horizon. This may shed light on a high temperature expansion of the lattice QCD in higher (than four) spacetime dimensions.

To calculate the $0^{++}$ glueball spectrum via Witten’s method \cite{Witten}, we have to replace the AdS factor of our spacetime by an AdS black hole of mass $M$. The metric of such a $D/2-$dimensional black hole reads

$$ds^2_{\text{black hole}} = V(r) dt^2 + \frac{1}{V(r)} dr^2 + r^2 d\Omega_{n-1}^2,$$  \hspace{0.5cm} (40)

where the function $V(r)$ is given by

$$V(r) = \frac{r^2}{L^2} + 1 - \frac{w_n M}{r^{n-2}}.$$ \hspace{0.5cm} (41)

We use the notation \cite{Bekenstein-Hawking}

$$w_n = \frac{16\pi G_N}{(n-1)\Omega_{n-1}}, \quad \text{and} \quad n = \frac{D}{2} - 1,$$ \hspace{0.5cm} (42)

where $G_N$ is the gravitational (Newton) coupling constant, and $\Omega_{n-1}$ is the area of a unit sphere in $(n-1)$ dimensions.

Being a monotonic function, $V'(r) > 0$, it has exactly one zero (root) that we call a horizon $r_+$: $V(r_+) = 0$.

The inverse (Bekenstein-Hawking) temperature of this black hole is \cite{Bekenstein-Hawking}

$$\beta_0(r_+) = \frac{4\pi r_+ L^2}{nr_+^2 + (n-2)L^2}. \hspace{0.5cm} (43)$$

In the large black hole mass limit, $M \to \infty$, the one in $V(r)$ can be dropped, so that we have

$$V(r) \approx \frac{r^2}{L^2} - \frac{w_n M}{r^{n-2}}.$$ \hspace{0.5cm} (44)

After rescaling the coordinates as

$$r = \left( \frac{w_n M}{L^{n-2}} \right)^{1/n} \rho \quad \text{and} \quad t = \left( \frac{w_n M}{L^{n-2}} \right)^{-1/n} \tau,$$ \hspace{0.5cm} (45)

the black hole metric \cite{rescaling} takes the form

$$ds^2_{\text{black hole}} = \left( \frac{\rho^2}{L^2} - \frac{L^{n-2}}{\rho^{n-2}} \right) d\tau^2 + \left( \frac{\rho^2}{L^2} - \frac{L^{n-2}}{\rho^{n-2}} \right)^{-1} d\rho^2 + \left( \frac{w_n M}{L^{n-2}} \right)^{2/n} \rho^2 d\Omega_{n-1}^2.$$ \hspace{0.5cm} (46)

\footnote{See, e.g., ref. \cite{Bekenstein-Hawking} for details.}
We now observe that the sphere in the large mass limit tends to the \((n-1)\) dimensional Euclidean space multiplied by a factor \(\rho^2\). Further rescaling of the coordinates as \(\tau \mapsto L^2 \tau\) and \(x_j \mapsto L x_j\) gives us the black hole metric

\[
\frac{1}{L^2} ds_{\text{black hole}}^2 = \left(\rho^2 - \frac{L^n}{\rho^{n-2}}\right) d\tau^2 + \left(\rho^2 - \frac{L^n}{\rho^{n-2}}\right)^{-1} d\rho^2 + \rho^2 \sum_{j=1}^{n-1} dx_j^2 .
\] (47)

Having replaced the AdS factor in \(\text{AdS}_{D/2} \times S^{D/2}\) by the black hole metric, we arrive at the full metric in the form

\[
\frac{1}{L^2} ds_{\text{full}}^2 = \left(\rho^2 - \frac{L^n}{\rho^{n-2}}\right) d\tau^2 + \left(\rho^2 - \frac{L^n}{\rho^{n-2}}\right)^{-1} d\rho^2 + \rho^2 \sum_{j=1}^{n-1} dx_j^2 + d\Omega_{n+1}^2 .
\] (48)

The AdS black hole configuration can also be obtained from non-extremal branes in the near horizon limit. To see this, let’s consider the non-extremal \((D/2-2)\)-brane configuration in our \(D\)-dimensional setting \((n = D/2 - 1, \text{ as above})\),

\[
ds^2 = f(r) H^{-2/n}(r) dt^2 + H^{-2/n}(r) \sum_{j=1}^{n-1} (dx^j)^2
\]

\[
+ f^{-1}(r) H^{2/n}(r) dr^2 + H^{2/n}(r) r^2 d\Omega_{n+1}^2 ,
\]

with \(H(r) = 1 + (h/r)^n \rightarrow h^n/r^n\) for \(r \rightarrow 0\) and \(f(r) = 1 - (r_0/r)^n\), where \(r_0\) and \(h\) are real parameters, \(r_0 = 0\) corresponds to the extremal situation, while \(h\) is related to the radius of the forthcoming AdS-type space. We find

\[
ds^2 = f(r) \left(\frac{h}{r}\right)^{-2} dt^2 + \left(\frac{h}{r}\right)^{-2} \sum_{j=1}^{n-1} (dx^j)^2 + f^{-1}(r) \left(\frac{h}{r}\right)^2 dr^2 + h^2 d\Omega_{n+1}^2
\]

\[
= V(r) dt^2 + V^{-1}(r) dr^2 + \frac{r^2}{h^2} \sum_{j=1}^{n-1} (dx^j)^2 + h^2 d\Omega_{n+1}^2 ,
\] (50)

where

\[
V(r) = \frac{r^2}{h^2} - \frac{r_0^n}{h^2 r^{n-2}} .
\] (51)

This just gives us an \((n + 1)\)-dimensional AdS black hole multiplied by an \((n + 1)\)-sphere after identifying \(w_n M\) with \(r_0^n/h^2\) and \(h\) with \(L\) (cf. our eqs. (40) and (44)).

The classical equation of motion for the dilaton \(\phi\) in this black hole background is similar to eq. (20),

\[
\Delta_{\text{black hole}} \varphi = \lambda^T \varphi ,
\] (52)
though this time with the metric (47). We only consider the s-wave mode on the sphere, so that $\lambda^l = 0$. We now look for solutions to eq. (52), which are square integrable over the black hole spacetime and correspond to a fixed momentum in the boundary theory. We further demand that the solutions of interest are independent of $\tau$. This gives rise to the following equation on $\phi$:

$$\Delta_{\text{black hole}} \phi = \partial_{\rho} \left( [\rho^{n+1} - \rho L^n] \partial_{\rho} \phi \right) + \rho^{n-3} \sum_{j=1}^{n-1} \partial_j^2 \phi = 0.$$ (53)

The standard (‘hedgehog’) Ansatz for solutions to this equation is given by

$$\phi(\rho, \vec{x}) = f(\rho) \exp \left( i \vec{k} \cdot \vec{x} \right),$$ (54)

where $\vec{x} = (x_1, \ldots, x_{n-1})$ and $\vec{k}$ is a definite momentum in the boundary theory with $\vec{k}^2 = -m^2$ (of Euclidean signature!). Inserting the Ansatz (54) into eq. (53) and using the fact that $\sum \partial_j^2 \phi = -k^2 \phi$, we find the ordinary second-order differential equation

$$\rho^4 \left[ 1 - \left( \frac{L}{\rho} \right)^n \right] f''(\rho) + \rho^3 \left[ n + 1 - \left( \frac{L}{\rho} \right)^n \right] f'(\rho) + m^2 f(\rho) = 0,$$ (55)

where the primes denote differentiation with respect to the argument $\rho$.

It is convenient to introduce the dimensionless variable $x = \rho/L$ and the dimensionless mass parameter $\tilde{m} = m/L$ in units of the AdS scale $L$, as well as define $y(x) = f(\rho)$. The equation (55) now takes the form

$$x^4 \left[ 1 - \left( \frac{1}{x} \right)^n \right] y''(x) + x^3 \left[ n + 1 - \left( \frac{1}{x} \right)^n \right] y'(x) + \tilde{m}^2 y(x) = 0.$$ (56)

Our goal is a computation of those values of $\tilde{m}^2$ that lead to square integrable and regular (at the horizon, near $x = 1$) solutions. Since we are unable to solve eq. (56) in a closed form, we determine the relevant solutions numerically, in the form of power series defined inside the convergence domains around the singular points of the differential equation (56). The solutions given by the power series must coincide in the overlaps of the convergence domains. To this end we observe that eq. (56) has singular points at $x = 0, \infty$ and the $n^{th}$ roots of unity. Therefore, we only have three real (physical) singular points at $x \geq 0$. We do not consider the root $x = 0$, since we expect a spacetime singularity there. Hence, we are left with the two singular points, $x = 1$ and $x = \infty$.

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6This looks a little bit odd, but if one follows the derivation of eq. (51) with all the rescalings done, one can check that $k_i$ and, therefore, $m$ all have dimension of $L$ indeed.

7We used the standard software Maple VI for calculations on computer.
5.1 The singular point \( x = \infty \)

Changing the variables as \( \xi = 1/x \) and \( \eta(\xi) = y(x) \) transforms eq. (54) to the form
\[
(1 - \xi^n)\eta''(\xi) + \xi^{-1}(1 - n - \xi^n)\eta'(\xi) + \tilde{m}^2\eta(\xi) = 0.
\]
(57)
The characteristic equation associated with eq. (54) at \( \xi = 0 \) is given by
\[
\nu(\nu - 1) + (1 - n)\nu = 0,
\]
(58)
and it has two solutions, \( \nu = 0 \) and \( \nu = n \). We only need \( \nu = n \) since the corresponding solution to eq. (54) decays for \( x \to \infty \) like \( x^{-n} \) and is, therefore, normalizable, whereas the other solution corresponding to \( \nu = 0 \) is not. By using the Ansatz
\[
\eta(\xi) = \xi^n \sum_{\mu \geq 0} b_{\mu} \xi^\mu,
\]
(59)
we get the following recursion relations for the coefficients \( b_{\mu} \):
\[
b_{\mu} = \frac{1}{\mu(\mu + n)} \left( \mu^2 b_{\mu - n} - \tilde{m}^2 b_{\mu - 2} \right).
\]
(60)
With the initial value \( b_0 = 1 \) we thus obtain the whole power series for \( \eta \), which converges inside the circle \( |\xi| < 1 \), or equivalently, \( |x| > 1 \).

5.2 The singular point \( x = 1 \)

The characteristic equation associated with eq. (54) at \( x = 1 \) reads
\[
\nu(\nu - 1)n + \nu n = 0,
\]
(61)
and it has a double root at \( \nu = 0 \). Hence, eq. (54) possesses a pure power series solution,
\[
y(x) = \sum_{\mu \geq 0} c_{\mu} (x - 1)^\mu.
\]
(62)
The second independent solution must have a logarithmic contribution, \( \log(x - 1) \), which is not normalizable in the vicinity of 1. Plugging the power series (62) into eq. (54), we arrive at the recursive relations for the coefficients \( c_{\mu} \),
\[
c_{\mu} = -\frac{1}{n\mu^2} \sum_{j=1}^{n} \left\{ \binom{n}{j} (\mu - j)(\mu - j + n) + \tilde{m}^2 \binom{n-3}{j-1} 
+ \binom{n}{j+1} (\mu - j)(\mu - j - 1) \right\} c_{\mu - j}.
\]
(63)
The convergence radius of the power series defined by these coefficients equals the distance to the next singular point of the differential equation, i.e. the minimum of \{1, 2\sin(\pi/n)\}.
5.3 Mass eigenvalues of the dilaton wave equation

We now compare the two power series in the overlap region of their convergence domains, in order to determine those values of the parameter $\tilde{m}^2$, for which the two power series calculated above define linearly dependent functions. In equivalent terms, we look for zeros of the Wronskian as a function of $\tilde{m}^2$,

$$
Wronskian(\tilde{m}^2) = \det \left[ \begin{array}{c}
\sum_{\mu \geq 0} c_{\mu}(x - 1)^{\mu} \\
\sum_{\mu \geq 0} b_{\mu} x^{-n+\mu}
\end{array} \right] \left[ \begin{array}{c}
\frac{\partial}{\partial x} \left( \sum_{\mu \geq 0} c_{\mu}(x - 1)^{\mu} \right) \\
\frac{\partial}{\partial x} \left( \sum_{\mu \geq 0} b_{\mu} x^{-n+\mu} \right)
\end{array} \right] \bigg|_{x = x_0},
$$

where the dependence of the right hand side upon $\tilde{m}^2$ is encoded in the coefficients $c_{\mu}$ and $b_{\mu}$. The reference point $x_0$ is supposed to belong to the overlap of the convergence domains. According to the general theory of ordinary differential equations, the exact zeros of Wronskian are independent upon a choice of the reference point $x_0$.

Though we cannot solve the recurrence relations analytically to all orders, we can compute any given number of the coefficients on the computer and truncate the power series at some finite number $\mu_{\text{max}}$. Of course, because of the truncation, the positions of the zeros of the Wronskian (64) are then only approximately independent of the reference point $x_0$. For example, when taking $x_0$ near the boundary of a convergence domain, the approximation is becoming bad and the zeros of the Wronskian are getting shifted. Given a sufficiently high cutoff $\mu_{\text{max}}$, the approximation of the power series by finite polynomials appears to be good enough for our purposes. We just plug these finite polynomials into the Wronsky determinant (64), with $x_0$ being sufficiently far away from the boundary of the convergence domains, and then we numerically determine the zeros $\tilde{m}^2$. The results of our computation of the first four zeros for various dimensions $n$ are summarized in Table 1.

We also verified that the zeros calculated in Table 1 are stable against sufficiently small variations of the reference point $x_0$ and the cutoff $\mu_{\text{max}}$. 
Table 1: Glueball masses from higher dimensions

| n  | 1st zero | 2nd zero | 3rd zero | 4th zero |
|----|----------|----------|----------|----------|
| 2  | 4.15     | 16.21    | 37.08    | 65.17    |
| 3  | 7.41     | 24.96    | 52.56    | 90.21    |
| 4  | 11.58    | 34.54    | 68.98    | 114.91   |
| 5  | 16.49    | 44.73    | 85.54    | 138.92   |
| 6  | 22.10    | 55.59    | 102.45   | 162.70   |
| 7  | 28.37    | 67.09    | 119.80   | 185.54   |
| 8  | 35.31    | 79.26    | 137.67   | 210.61   |
| 9  | 42.90    | 92.09    | 156.09   | 235.03   |
| 10 | 51.12    | 105.57   | 175.09   | 259.89   |
| 11 | 59.97    | 119.71   | 194.69   | 285.22   |
| 12 | 69.43    | 134.50   | 214.90   | 311.09   |
| 13 | 79.52    | 149.93   | 235.74   | 337.50   |

Our results for $n = 4$ coincide with those of refs. [13, 23].

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