TOPICS IN LORENTZ AND CPT VIOLATION

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This talk given at the CPT’10 meeting provides a brief introduction to Lorentz and CPT violation and outlines a few recent developments in the subject.

1. Introduction

The possibility that Lorentz violation might be manifest in nature, perhaps with attendant CPT violation, continues to attract attention from experimentalists and theorists alike. In the CPT’07 Proceedings, I outlined how the triple requirements of coordinate independence, realism, and generality lead to the conclusion that effective field theory is the appropriate framework for studying Lorentz and CPT violation. The present CPT’10 talk provides some introductory comments about this framework.

The comprehensive effective field theory incorporating General Relativity (GR) and the Standard Model can be constructed by combining all Lorentz-violating operators together with controlling coefficients to form observer-invariant terms in the Lagrange density. This theory is the Standard-Model Extension (SME). A useful limit is the minimal SME, which restricts operators to mass dimension $d \leq 4$ and is renormalizable in Minkowski spacetime. Since CPT violation in effective field theory comes with Lorentz violation, the SME also describes general CPT violation.

Many observable effects arise from the interactions of particles with the coefficients, varying with velocity, spin, flavor, and couplings. Numerous searches have been performed, but no compelling positive measurement exists to date. Some intriguing current prospects for signals include, among others, oscillations of neutrinos and neutral mesons.

Additional effects occur for spontaneous Lorentz violation because the coefficients can then fluctuate, yielding massless Nambu-Goldstone (NG) modes for the broken generators and also massive modes. The NG
modes can be identified directly with the photon in Einstein-Maxwell theory,\textsuperscript{9} the graviton in GR,\textsuperscript{11} a spin-dependent force,\textsuperscript{12} or a spin-independent force,\textsuperscript{13} or they can generate composite photons\textsuperscript{14} or gravitons.\textsuperscript{15}

2. Nonminimal terms

In the full SME with nonminimal terms, infinitely many possible Lorentz-violating operators become candidates for inclusion in the Lagrange density. As a result, enumerating these operators and determining their physical effects becomes challenging.

For operators of arbitrary mass dimension $d$, a systematic investigation has so far been performed only in the photon sector.\textsuperscript{16} This investigation studied all operators quadratic in the photon field $A_\mu$, allowing for arbitrary spacetime derivatives. The resulting explicit gauge-invariant action reveals that the number of Lorentz-violating operators grows rapidly: the minimal SME has 4 operators at $d = 3$ and 19 at $d = 4$, but 36 nonminimal ones appear at $d = 5$, 126 at $d = 6$, and the growth is cubic with $d$ at large $d$.

Each of these numerous operators produces a distinct Lorentz-violating effect on photon propagation. In some respects, the behavior of SME photons is analogous to Maxwell photons moving in an anisotropic dispersive crystal. For example, Lorentz violation can cause light to exhibit mode separation (birefringence), pulse deformation (dispersion), and direction dependence (anisotropy). Certain coefficients for Lorentz violation can be detected at leading order by studying propagation in the vacuum, while others require nonvacuum boundary conditions. The details of these effects depend on features of the specific radiation being considered, such as its frequency, polarization, and direction of travel. Surprisingly, this plethora of new effects is almost unexplored in relativity tests. No dedicated laboratory experiments have searched for these behaviors, and the existing astrophysical tests are limited to a few comparatively simple cases.

For coefficients governing leading-order birefringence in the vacuum, the most sensitive tests involve polarimetry of astrophysical sources. Birefringent effects are controlled by the ratio of the wavelength to the source distance, so the sharpest tests involve polarimetry of high-frequency radiation propagating over cosmological distances. Although still in its infancy, the polarimetry of gamma-ray bursts has already led, for example, to constraints of order $10^{-32}$ GeV$^{-1}$ on certain operators at $d = 5$.

For vacuum-nonbirefringent operators causing dispersion, interesting tests can be performed by studying the separation of a propagating pulse. The sensitivity to the corresponding coefficients is controlled by the ratio of
the pulse separation to the source distance. For cosmological sources, this 
dispersion-based sensitivity is typically many orders of magnitude weaker 
than polarimetric measurements, but nonetheless provides the best access 
to vacuum-nonbirefringent operators.

Finally, for the vast numbers of ‘vacuum-orthogonal’ operators that pro-
duce no leading-order effects on photon propagation in the vacuum, the best 
option is investigation via laboratory tests. Typical experiments with res-
onant cavities and interferometers produce sensitivities given by the ratio 
of the frequency shift to the frequency. Along with studies of astrophysical 
birefringence and dispersion, the investigations of these Lorentz-violating 
effects on light present an open experimental challenge, with a real potential 
for discovery in an area that is almost unexplored to date.

3. Gravity

The key feature of Special Relativity is the isotropy of spacetime. An observ-
able background Lorentz vector or tensor implies a spacetime anisotropy of 
the vacuum and hence Lorentz violation. Similarly, a key component of GR 
is the local isotropy of spacetime. Lorentz violation in this context can be 
understood as the presence of an observable background vector or tensor 
in a local Lorentz frame.

A local Lorentz frame at a given point is a tangent spacetime to the 
spacetime manifold. Since local Lorentz violation is a property of the tan-
gent spacetime rather than the manifold, the ‘vierbein formalism’ is appropri-
ate for studies of local Lorentz violation and gravity. In this approach, 
the vierbein $e^a_\mu$ implements the conversion from local Lorentz coordinates $a$, $b$, . . . to spacetime manifold coordinates $\mu$, $\nu$, . . .

‘No-go’ result for explicit Lorentz violation. The ramifications of these 
simple observations are surprisingly broad. One powerful result is that ex-
licit Lorentz violation is incompatible with generic Riemann geometries 
and therefore with GR.\textsuperscript{2} The basic point is that explicit Lorentz violation 
occurs when the background tensors are externally prescribed, but this is 
inconsistent with the Bianchi identities for general Riemann spacetimes.

To illustrate this no-go result, suppose explicit Lorentz violation appears 
in the matter sector. The energy-momentum tensor is then nonconserved 
in most spacetimes and the equations of motion are inconsistent with the 
Bianchi identities,

$$0 \equiv D_\mu G^{\mu\nu} = 8\pi G_N D_\mu T^{\mu\nu} \neq 0 \text{ (explicit breaking).}$$

(1)

In contrast, in spontaneous Lorentz violation the background tensors are
dynamically determined along with the metric and are therefore compatible with the spacetime geometry,

\[ 0 \equiv D_\mu G^{\mu\nu} = 8\pi G_N D_\mu T^{\mu\nu} = 0 \] (spontaneous breaking). \tag{2}

The no-go result holds also for explicit Lorentz violation in the gravity sector and for Riemann-Cartan spacetimes.\(^2\) In the general case with explicit Lorentz breaking, imposing consistency with the Bianchi identities enforces an additional nondynamical constraint on the spacetimes solving the theory. The constraint often forbids any solution, but in any case it represents at best a *post hoc* assumption slicing the solution spacetimes of the theory. The no-go result also presents an obstruction to reproducing GR from a theory with explicit Lorentz violation, including theories such as ‘Lifschitz gravity’ that attempt to generate GR through the running of explicit Lorentz-violating couplings. Gravity theories in which the graviton arises from spontaneous Lorentz violation\(^\text{11}\) avoid the no-go result.

*Gravitational signals from spontaneous Lorentz violation.* Lorentz violation can occur in the pure-gravity and matter-gravity sectors. The no-go result shows it must be spontaneous, so the coefficients for Lorentz violation must originate as dynamical fields. Each coefficient field can therefore be written as the sum of the vacuum coefficient for Lorentz violation and a fluctuation. Since the breaking is spontaneous, the fluctuation includes massless NG modes and so can affect the dynamics even at low energies. The problem of solving for these modes and eliminating them to recover an effective post-newtonian gravitational theory is challenging but has been solved in both the pure-gravity\(^\text{17}\) and the matter-gravity\(^\text{13}\) sectors.

Observable effects arise from Lorentz violation in the gravitational field of the source and in the trajectory of a test body. As an example, the local gravitational acceleration experienced by a test body near the surface of the Earth acquires sidereal and annual variations that can depend on the composition of the test body and the Earth. In general, signals can appear in gravimeters (free fall and force comparison), tests of the weak equivalence principle (free fall, force comparison, and space based), exotic matter (antihydrogen, higher-generation particles, etc.), solar-system measurements (lunar laser ranging, perihelion shift, gyroscopes, etc.), binary pulsars, and various photon tests (Shapiro delay, Doppler shift, gravitational redshift, null redshift, etc.). Also, a nonzero background torsion can be understood in terms of certain coefficients for Lorentz violation, so sensitive constraints on torsion can be obtained.\(^\text{18}\) The overall prospects for new and improved searches for gravitational Lorentz violation are excellent.
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