Calculation the Height between Floating Towed Body and Towed Point Based On CFD

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Abstract. In order to calculate the attitude and height of floating towed body, a special method is needed to solve this problem. Firstly, cubic spline interpolation is used to calculate the pitch angle of floating towed body in water, and then the drag and lift of floating towed body in water are calculated by CFD. Such calculation is necessary, because the pitch angle of the floating towing body is not known in advance. For the towed cable, finite difference method is used to deal with the element, and cubic spline interpolation is used to calculate the lift and drag of the towed cable element at different angles of attack. In this way, the height difference of floating towing body and the tension at the end of the towing cable can be obtained.

1. Introduction
Underwater towed body cause sinking force, but floating towed body with upper lift is relatively rare. At present, most of the rising objects are kites and planes. However, the kite is controlled by people in real time, and the aircraft is not towed by towing cables. The floating drag body itself has no power and is powered by towing cables. The floating towing body itself shows positive buoyancy in water, but in order to ensure that it can have a better height difference in motion, the lift needs to be generated by the wing.

How to calculate the floating towed body is an interesting thing. The stability of the floating towing body should be considered in the calculation. With this stable attitude, the drag and lift force of the floating towing body can be determined. However, for the calculation of the towed cable, it is necessary to discretize towed cable to calculate the tension and height difference at the end of the towed cable.

2. Outline of floating towing body
In order to ensure a good positive buoyancy, the buoyancy material of glass beads is usually used to process the floating towed body. In addition, NACA wing or flat wing can be used to support the good lift effect of the floating wing towed body. However, NACA wing lift performance is good, but its processing cost is expensive, flat wing processing cost is very cheap. Usually, the flat wing is chosen for the floating towed body. The shape of the floating drag body is shown in Fig. 1. The length is 0.8m, the diameter is 0.12m.
The floating wing trailing body has a connection with the load-bearing cable underneath. When the towing cable is in motion, the floating towed body moves accordingly, and there will be a certain range of height difference between the towing point and the floating towed body (Fig. 2). This paper will discuss the calculation method of the height in a certain speed.

3. Basic theory of fluid mechanics

Control equation. The continuity equation and momentum equation for incompressible viscous fluid are:

\[
\frac{\partial \bar{u}_i}{\partial x_i} = 0 \tag{1}
\]

\[
\rho \frac{\partial \bar{u}_i}{\partial t} + \rho \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = \rho \bar{F}_i - \frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \mu \frac{\partial \bar{u}_i}{\partial x_j} - \rho u_i' u_j' \right) \tag{2}
\]

Where, \( \rho \) is density, \( \mu \) is Viscosity Coefficient, \( \bar{p} \) is average pressure, \( \bar{F}_i \) is external force, \( \bar{u}_i \) is mean velocity, \( u_i' \) is fluctuating velocity, \( -\rho u_i' u_j' \) is reynolds stress.

Turbulence model. In order to make the equation closed, a new turbulent model equation must be introduced to link the fluctuating values with the time average in the stress terms. There, we chose the RNGk – \( \varepsilon \) equitions.
The transport equations of turbulent kinetic energy and turbulent fluctuation intensity in the $RNGk-\varepsilon$ equation are:

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_i}(\rho ku_i) = \frac{\partial}{\partial x_j}[\alpha_\text{eff} \frac{\partial k}{\partial x_j}] + G_k - \rho \varepsilon + S_k$$

(3)

$$\frac{\partial}{\partial t}(\rho \varepsilon) + \frac{\partial}{\partial x_i}(\rho \varepsilon u_i) = \frac{\partial}{\partial x_j}[\alpha_\text{eff} \frac{\partial \varepsilon}{\partial x_j}] + G_\varepsilon \frac{\varepsilon}{k} G_k - C_{2\varepsilon} \rho \frac{\varepsilon^2}{k} - R_\varepsilon + S_\varepsilon$$

(4)

Where, $\mu_{\text{eff}} = \mu + \mu_t$, $\mu_t = \rho C_\mu \frac{k^2}{\varepsilon}$, $G_k = -\rho u_j \frac{\partial u_j}{\partial x_i}$, $R_\varepsilon = \frac{C_\mu \rho \eta^3 (1 - \eta)}{1 + \beta \eta^3} \frac{\varepsilon^2}{k}$, $\eta \equiv \frac{Sk}{\varepsilon}$.

$S = \sqrt{2S_i S_j}$, $S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$, $S_k$ and $S_\varepsilon$ are user-defined source terms.

Constant, $G_{1\varepsilon} = 1.42$, $G_{2\varepsilon} = 1.68$, $C_\mu = 0.0845$, $\sigma_k = 1.0$, $\sigma_\varepsilon = 1.3$, $\eta_0 = 4.38$, $\beta = 0.012$

4. Setting for the floating towed body

Fluid computational domain. The calculation domain and boundary conditions for floating towed body are shown in Fig. 3.

Fig. 3  Computational domain and boundary setting for floating towed body

The setting of boundary conditions is one of the necessary conditions to ensure the realization of CFD. These include: speed inlet, pressure outlet, wall and symmetry plane.

(1) Inlet: Setting the magnitude and direction of the incoming velocity on the intake surface, $V_{in} = V_0$.

(2) Outlet: Set the hydrostatic pressure relative to the reference pressure point on the outflow surface, $p_{out} = 0$.

(3) Wall condition: The surface of floating towed body is set as a non slip boundary condition, $u = v = w = 0$.

Grid partition. Because of the complex shape of the floating towed body and the long time required to divide the structured grid, the unstructured grid is used to set the boundary layer grid around the floating wing towing body (Fig. 4).
5. Calculation of lift and resistance for floating towed body

Floating wing towed body is symmetrical in shape and weight, so its roll angle is considered to be zero when moving underwater, and its pitch angle needs to be calculated in advance, so the lift resistance can be calculated first. In order to find out its elevation angle, the force analysis of the floating wing drag body is carried out (Fig. 5).

\[ F_x - R = 0 \]  \hspace{0.5cm} (5)  
\[ V + L - G - F_y = 0 \]  \hspace{0.5cm} (6)  
\[ M_A = M_V + M_L - M_G - M_R \]  \hspace{0.5cm} (7)

Where, \( F_x \) is \( F \) in X direction, \( F_y \) is \( F \) in Y direction, \( V \) is buoyancy, \( M_V \) is Buoyancy moment on drag point, \( G \) is gravity, \( M_G \) is gravity moment on drag point, \( L \) is lift, \( M_L \) is lift moment on drag point, \( R \) is resistance, \( M_R \) is resistance moment on drag point, \( \alpha \) is pitching angle.

We get \( M_A \) at different \( \alpha \) (Table 1).

| \( \alpha \) (º) | 5 | 10 | 15 | 20 |
|------------------|---|----|----|----|
| \( M_A \) (Nm)   | 0.3 | 0.2 | -0.1 | -0.3 |
Using cubic spline interpolation, getting $M_A = 0$, $\alpha = 13.5$.

Then, $F_x = 75.2N$, $F_y = 219.8N$

6. Calculation of height and resistance of floating towed body

When the $F_x$ and $F_y$ are calculated, the tension and height of the towing cable have to be calculated.

Because the towing cable is flexible, it will be deformed by the force. Therefore, the angle of attack of the towing cable at different place is different. The finite difference method is used to process the towing cable into finite element, and the stress analysis is carried out for each element. The force, rotation angle and coordinate point of the unknown position are obtained from the initial point.

In order to meet the needs of the project and basically meet the actual situation, the following assumptions are made:

a) Telescopic overlook of cable finite element under tension;
b) The towing cable is flexible and can not transfer bending moment
c) The flow field is only considered as a two-dimensional flow.
d) The hydrodynamic coefficient on the finite element of the towing cable is equivalent to a towing cable that acts on an infinite length.

Force analysis of finite element of the towing cable is shown in Fig. 6.

![Fig. 6 Force analysis of finite element of towing cable](image)

According to the decomposition of the tangential force and the normal force, get:

$$ (T + dT) \sin\left(\frac{d\phi}{2}\right) + T \sin\left(\frac{d\phi}{2}\right) = Dds + pds \cos \phi $$

(8)

$$ (T + dT) \cos\left(\frac{d\phi}{2}\right) + pds \sin \phi = Fds + T \cos\left(\frac{d\phi}{2}\right) $$

(9)

There, $T$ is force, $D$ is the normal force of finite element of the towing cable, $F$ is the tangential force of finite element of the towing cable, $p$ is the weight of finite element of the towing cable. Neglecting the small amount of the two order in equality (8), get:

$$ 2T \sin\left(\frac{d\phi}{2}\right) = (D + p \cos \phi)ds $$

(10)
Because \( \frac{d\phi}{2} \) is a small quantity, then \( \sin\left(\frac{d\phi}{2}\right) \approx \frac{d\phi}{2} \), get:

\[
ds = \frac{T}{D + p\cos\phi} d\phi
\]

The equation (11) is replaced by the equation (9), get

\[
dT = \frac{F - p\sin\phi}{D + p\cos\phi} T d\phi
\]

By equation (11) and (12), the towing force and rotation angle of the towing cable in different positions can be obtained.

In addition:

\[
dx = ds \times \cos\phi
\]

\[
dy = ds \times \sin\phi
\]

The coordinates of the corresponding position points can be obtained by equation (13) and (14).

The length of the towing cable is discretized, and the iterative calculation can be carried out. Computational fluid dynamics and finite difference method are used to calculate \( D \), \( F \).

It is necessary to explain that the empirical formula calculation \( D \) and \( F \) do not consider the mutual interference between the tangential velocity and the normal velocity, and the calculation results are not correct enough. At present, CFD technology has been developed more mature, and can be directly calculated. In addition, CFD can only calculate tangential forces and normal forces under limited attack angles. In order to meet the need of actual attack angle calculation, the three spline interpolation method is applied to solve them.

In this study, the diameter of the towing cable is 6mm and the length is 20m.

From the foregoing, the tension of the floating towed body has been calculated, and then the finite difference method is used to deal with the towing cable element. Finally, the resistance and height of floating towed body are calculated by the method mentioned above (Table 2).

### Table 2. Resistance and height of floating towed body

| \( v \) (m/s) | 1     | 2     | 5     | 7     |
|--------------|-------|-------|-------|-------|
| \( R \) (N)  | 35.2  | 52.8  | 264.4 | 414.6 |
| \( H \) (m)  | 15.2  | 13.2  | 10.5  | 8.2   |

7. **Summary**

CFD is used to calculate the tangential force and normal force of the towline element. The relationship between tangential velocity and normal velocity is considered in this method. The finite difference method is adopted to superimpose the streamline microelement, and the resistance of the whole cable can be calculated. The method of cubic spline interpolation and CFD is used to calculate the pitch angle of the floating towing body when it is stable. This method has good universality and can quickly calculate the drag and height of floating towing body.
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