We calculate the spectrum of ultra high energy cosmic rays produced by the decay of a superheavy dark matter population clustered in the galactic halo. To perform this calculation we start with fragmentation functions measured at LEP and evolve them to the cosmic ray energy scale using the QCD DGLAP equations. We consider Standard Model evolution and supersymmetric evolution. We also take into account many–body final states in the decay of the dark matter particles.
1 Motivation

Ultra High Energy Cosmic Rays (UHECR) are microscopic particles—protons, photons or perhaps more exotic objects—with a macroscopic energy, about 50 Joules per particle in the extreme of the present observed spectrum. They strike the upper layers of the Earth atmosphere at a rate of about one event per century and per kilometer square. Over one hundred of them have been detected so far by kilometer scale detectors and many more are expected to be seen by forthcoming observatories.

Explaining how these particles get this huge energy is a real challenge for our present understanding of the cosmos. One UHECR primary gets somehow ten million times the energy that a proton will gain at the future Large Hadron Collider (LHC) at CERN. Nature has always been at ease beating the achievements of mankind. One can envisage two broad classes of mechanisms by which nature can produce UHECRs, the so-called bottom up class models and the top-down class models.

In bottom-up models charged particles are accelerated by magnetic fields in large astrophysical sites. In top-down models particles are not accelerated but are created at birth with the huge energy typical of UHECRs.

Here we will concentrate on a particular top-down model \cite{1, 2, 3}. We will assume the UHECRs are produced by the decay of a population of superheavy dark matter particles with a lifetime longer than the age of the Universe $\tau > 10^{10}$ yr and with mass $\sim 100$ Joule/$c^2$. Theoretical motivation for these particles can be found in \cite{4}.

A superheavy dark matter population created at some stage of the early universe \cite{5, 6} will gravitationally cluster in the galactic halo. Since the length scale of the halo is around 100 kpc, UHECRs produced in the galactic halo will not have time to interact with the CMB before they reach the Earth. The absence of GZK cut-off is a genuine prediction of models where UHECRs are produced by the decay of a superheavy dark matter particle clustered in the galactic halo.

Since the halo shape is close to spherical one expects a quasi isotropic distribution of events from a halo superheavy dark matter population, which is compatible with experiments. At present it is not possible to make any strong claim about the observed angular distribution of UHECR events because of low statistics. Hopefully, future observatories like Pierre Auger will gather a large enough sample of events to settle down this issue \cite{7}.

Finally there is the question of the UHECR composition. The cosmic ray observatories cannot measure the composition of the primary flux (whether they are photons, protons or heavy nuclei) in an event by event basis. Composition can only be determined in a statistical way. Present analyses tend to favour protons as the main component in the primary flux \cite{8}. In top-down models the main component in the production site, in our case the the galactic halo, is the photon component (actually, neutrinos dominate over photons in a large range of the spectrum but their probability to be detected is too small with present or past detectors). However, on their way to the Earth, photons will interact with the low frequency radio background in the Galaxy and their total flux may
be substantially diminished so that on the Earth the total photon flux may be comparable or smaller than the baryon flux. Whether this is indeed possible, subject to the EGRET bound on the low energy γ-rays which will be created by the electromagnetic cascading, will be discussed elsewhere.

Summing up, there are three main tests to falsify or support the hypothesis of UHECRs produced by the decay of dark matter particles clustered in the galactic halo. The first one is the energy distribution of events or spectrum. The second one is the expected angular distribution of events in the sky. The third one is the composition of the primary flux. Here we will focus on the first test and, partially, on the third one. We will briefly show how quantum chromodynamics (QCD) can be used to calculate the spectrum and composition (without photon galactic processing) of the expected UHECR flux. For further details see [9, 10]. QCD was also used to calculate the spectra of UHECRs in [11, 12, 13].

2 DGLAP Evolution

Let us assume that a dark matter particle $X$ population, superheavy ($M_X > 3 \times 10^{12}$ GeV) and metastable, with a lifetime larger than the age of the universe, is clustered in the galactic halo. Barring unnatural relations between the coupling constant of $X$ and the other fields the total particle multiplicity produced by its decay will be dominated by partonic decays. The flux for the primary $h$ (baryon, photon or neutrino) is proportional to the inclusive decay width

$$J_h(E) \propto \frac{1}{\Gamma_X} \frac{d\Gamma(X \to h + \ldots)}{dx} = \sum_a \int_x^1 \frac{dz}{z} \left. \frac{1}{\Gamma_a} \frac{d\Gamma_a(y, \mu^2, M_X^2)}{dy} \right|_{y=x/z} D_h^a(z, \mu^2).$$

Particle $h$ carries a fraction $x$ of the maximum available momentum $M_X/2$, and a fraction $z$ of the parton $a$ momentum. The first term in the integrand is the decay width of $X$ into parton $a$, $d\Gamma_a/dy$, which is calculable in perturbation theory; in lowest order and for 2-body decay it is proportional to $\delta(1 - y)$. The second factor, the non-perturbative $D_h^a$, is the fragmentation function (FF) for particles of type $h$ from partons of type $a$. It gives the expected mean number of particles $h$ coming from parton $a$. Fragmentation functions cannot be calculated from firsts principles but their dependence in the energy scale $\mu$ is governed by the Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) equations [14, 15],

$$\frac{\partial D_h^a(x, \mu^2)}{\partial \ln \mu^2} = \sum_b \frac{\alpha_s(\mu^2)}{2\pi} P_{ab}(x, \alpha_s(\mu^2)) \otimes D_h^b(x, \mu^2),$$

where $\alpha_s(\mu^2)$ is the strong coupling constant and $P_{ab}(x, \alpha_s)$ is the splitting function for the parton branching $a \to b$. Here the convolution of two functions $A(x)$ and $B(x)$ is defined as

$$A(x) \otimes B(x) \equiv \int_x^1 \frac{dz}{z} A(z) B\left(\frac{x}{z}\right).$$
The splitting functions can be expanded perturbatively:

\[ P_{ba}(x, \alpha_s) = P_{ba}(x) + \mathcal{O}(\alpha_s). \]  

We limit our study to leading order in \( \alpha_s \) and therefore ignore \( \mathcal{O}(\alpha_s) \) corrections to the splitting functions. It is also convenient to define the following dimensionless evolution parameter

\[ \tau \equiv \frac{1}{2\pi b} \ln \frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}, \]  

\( b \) being the coefficient in the leading order \( \beta \)-function governing the running of the strong coupling: \( \beta(\alpha_s) = -b\alpha_s^2 \). We take \( D_h^a \) to represent the sum of particle \( h \) and, if different, its antiparticle \( \bar{h} \).

In the Standard Model there are two partons species: gluons \( g \) and quarks \( q \) (for the sake of brevity we shall only consider the singlet quark, i.e., the sum of all quark and antiquark flavours). The DGLAP equations can be written as

\[
\partial_\tau \begin{pmatrix} D_q \\ D_g \\ D_s \\ D_\lambda \end{pmatrix} = \begin{pmatrix} P_{qq} & \frac{2n_F P_{qg}}{P_{gg}} & P_{sq} & \frac{2n_F P_{q\lambda}}{P_{\lambda g}} \\ P_{gq} & P_{gg} & P_{sg} & P_{g\lambda} \\ P_{qs} & 2n_F P_{qs} & P_{ss} & 2n_F P_{s\lambda} \\ P_{q\lambda} & P_{g\lambda} & P_{s\lambda} & P_{\lambda\lambda} \end{pmatrix} \otimes \begin{pmatrix} D_q \\ D_g \\ D_s \\ D_\lambda \end{pmatrix}. \]  

When supersymmetry (SUSY) is included one has in addition the gluinos and the squark singlet. The DGLAP equations are now

\[
\partial_\tau \begin{pmatrix} D_q \\ D_g \\ D_s \\ D_\lambda \end{pmatrix} = \begin{pmatrix} P_{qq} & \frac{2n_F P_{qg}}{P_{gg}} & P_{sq} & \frac{2n_F P_{q\lambda}}{P_{\lambda g}} \\ P_{gq} & P_{gg} & P_{sg} & P_{g\lambda} \\ P_{qs} & 2n_F P_{qs} & P_{ss} & 2n_F P_{s\lambda} \\ P_{q\lambda} & P_{g\lambda} & P_{s\lambda} & P_{\lambda\lambda} \end{pmatrix} \otimes \begin{pmatrix} D_q \\ D_g \\ D_s \\ D_\lambda \end{pmatrix}. \]  

We have developed a C++ code to solve the DGLAP equations for fragmentation functions. We have extended the algorithm introduced in Ref. [16] to be able to solve the SUSY equations as well as the Standard Model equations [10].

We begin with FFs at the energy scale \( \mu_0 = M_Z \) and evolve them to the final energy scale \( \mu = M_X \) using Eq. (1) and also Eq (7) if SUSY is included. For the initial fragmentation function of baryons, \( D^b_a(x, M_Z^2) \), we adopt the fit performed in Ref. [11] to LEP hadronic data [17]. For photons and neutrinos we generate initial data at the Z peak using the QCD Monte Carlo event generator HERWIG [18]. Comparison with LEP data shows that although HERWIG overproduces baryons at high \( x \) [11, 19], its photon and meson output at the Z peak matches the experimental spectra remarkably well. Since neutrinos mainly come from charged pion and kaon decays, one can thus be confident in taking the HERWIG generated FF for neutrinos as the initial condition for the evolution. There is also a sizable contribution from heavy flavour decays to the neutrino spectrum at high \( x \) which is explicitly taken into account by HERWIG.

A (s)parton is not included in the evolution as long as the energy scale is lower than its mass; when the threshold for its production is crossed, it is added to the evolution equations with an initially vanishing FF and it is assumed to be a relativistic particle.
In the SM case we evolve the \( q \) and \( g \) initial fragmentation functions from \( M_Z \) to \( M_t \), the top quark mass, with the number of flavours set to \( n_F = 5 \), and then evolve from \( M_t \) to \( M_X \) with \( n_F = 6 \).

In the SUSY case we evolve the \( q \) and \( g \) initial fragmentation functions from \( M_Z \) to the supersymmetry breaking scale \( M_{\text{SUSY}} > M_t \) using the SM equations to obtain \( D^h_i(x, M^2_{\text{SUSY}}) \), with \( i = q, g \). Then we take \( D^h_i(x, M^2_{\text{SUSY}}) \), \( i = q, g \), and \( D^j_i(x, M^2_{\text{SUSY}}) = 0 \), \( j = s, \lambda \), and evolve them from \( M_{\text{SUSY}} \) to \( M_X \) using the SUSY equations. All spartons are taken to be degenerate with a common mass \( M_{\text{SUSY}} \).

\section{Ultra High Energy Cosmic Ray Spectrum}

We can now translate the calculated fragmentation functions into the expected cosmic ray spectrum in order to confront the observational data. In the previous Section (2) we have briefly shown how to calculate quark singlet and gluon functions for the SM, and quark singlet, gluon, squark singlet and gluino functions for SUSY. In the absence of a specific model for the different branching ratios we weight all the (s)parton contributions evenly. For 2-body decay of \( X \), \( x = 2E/M_X \) the flux of particle \( h \) is given by \([9]\):

\[ E^3 J^{\text{halo}}(E) = B x^3 D^h(x, M^2_X). \] (8)

We have multiplied the flux by \( E^3 \), as is usual, to emphasise the structure in the spectrum near the GZK energy. The normalisation factor \( B \) is common for the galactic halo flux of baryons, neutrinos and photons, and determines the quantity \( n_X/\tau_X \) (see Eq. [1]).

Let us now compare the calculated cosmic ray flux to the published data from Fly’s Eye \([20]\), AGASA \([21]\), Haverah Park \([22]\) and Yakutsk \([23]\). In Ref. \([24]\) these data have been carefully assessed for mutual consistency and appropriate adjustments made to the energy calibration. Its authors recommend adoption of the following standard differential energy spectrum \textit{below} the GZK energy, in the range \( 4 \times 10^{17} \text{eV} < E < 6.3 \times 10^{18} \text{eV} \):

\[ J(E) = (9.23 \pm 0.65) \times 10^{-33} \text{ m}^{-2}\text{s}^{-1}\text{sr}^{-1}\text{eV}^{-1} \left( \frac{E}{6.3 \times 10^{18} \text{eV}} \right)^{-3.20 \pm 0.05}, \] (9)

with the spectrum flattening at higher energies as \( J(E) \propto E^{-2.75 \pm 0.2} \) up to the GZK energy, and extending further to at least \( 3 \times 10^{20} \text{eV} \) \([24]\). Thus the UHECR spectrum can naturally be interpreted \([20]\) as the superposition of the ‘low energy’ component \([4]\), and the new ‘flat’ component that extends into the post-GZK region. The former is presumably galactic in origin (consistent with the detection of anisotropy at \( \sim 10^{18} \text{eV} \) \([25, 26]\)), while the latter is interpreted \([3]\) as produced by the decay of a superheavy particle population in the galactic halo. Taking baryons to be the dominant primary UHECRs as indicated by experiment, the total flux is

\[ E^3 J(E) = \frac{k}{E^m} + B x^3 D^{\text{baryon}}(x, M^2_X), \] (10)

\[ 4 \]
where the values of $k$ and $m$ can be read off Eq. (3). Note that since $D^\text{baryon}$ and $D^\gamma$ have a similar shape, taking photons to be the primaries would just alter the normalisation $B$. In Fig. 1 we plot the best SM evolution fit to the cosmic ray data while in Fig. 2 we plot the best SUSY evolution fit.

![Figure 1: The best SM evolution fit to the cosmic ray data with a decaying particle mass of $10^{12} \text{ GeV}$. The dotted line indicates the extrapolation of the power-law component from lower energies, while the dashed line shows the decay spectrum; the solid line is their sum.](image)

The assumption of 2-body decay may be rather naive for a superheavy particle like a crypton \cite{4} which is expected to decay through very high-order non-renormalisable operators. Many-body decay distributes the total energy $M_X$ among several particles and thus flattens the spectrum. We assume that many-body effects are purely kinematical and hence can be encapsulated in the phase space of the decay. Let $\rho_n(z)$ be the probability density that one parton carries off a fraction $z$ of the total available energy per parton $M_X/2$. For $n \geq 3$ we get \cite{4}

$$\rho_n(z) \propto z(1-z)^{n-3}. \quad (11)$$
Figure 2: The best SUSY evolution fit to the cosmic ray data with a decaying particle mass of $5 \times 10^{12}$ GeV and $M_{SUSY} = 400$ GeV. The dotted line indicates the extrapolation of the power-law component from lower energies, while the dashed line shows the decay spectrum; the solid line is their sum.

To leading order in $\alpha_s$ the particle flux is then given by

$$E^3 J_{\text{halo}}(E) = B x^3 \int_x^1 \frac{dz}{z} \rho_n \left( \frac{x}{z} \right) D^b(z, M_X^2).$$

(12)

In particular if $D^b(x, M_X^2) \propto (1-x)^{a(M_X^2)}$ as $x \to 1$, the differential particle flux decreases as $J_{\text{halo}}(x) \propto (1-x)^{a(M_X^2)+n-2}$.

For many-body $X$ decays the total flux is

$$E^3 J(E) = \frac{k}{E^m} + B x^3 \int_x^1 \frac{dz}{z} \rho_n \left( \frac{x}{z} \right) D^\text{baryon}(z, M_X^2),$$

(13)

where $n$ is the number of partons into which $X$ decays. In Fig. 3 we plot evolved SUSY spectra for different values of $n$. 

6
Figure 3: Cosmic ray data compared with SUSY evolved spectra for a decaying particle mass of $10^{13}$ GeV with $M_{\text{SUSY}} = 400$ GeV, and many-body decays to $n$ partons: $n = 2$ (solid line), $n = 8$ (dotted line) and $n = 16$ (dashed line).

## 4 Conclusions

We have calculated the UHECR spectra of baryons, photons and neutrinos expected from the decay of superheavy dark matter particles of mass $M_X \sim 10^{12}$ GeV which are clustered in the galactic halo. The spectrum for every primary particle is given by FFs at the scale $M_X$. We have calculated these FFs using QCD DGLAP equations. We have also taken into account the possibility of many-body decay in the decay of $X$. The shape of the fragmentation spectrum (of either baryons or photons) fits rather well the new component of ultra-high energy cosmic rays extending beyond the GZK energy.

## ACKNOWLEDGEMENTS

The work here outlined was performed in collaboration with Subir Sarkar. I would like to acknowledge support by a Marie Curie Fellowship No. HPMF-CT-1999-00268.
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