Light Loop Echoes and Blinking Black Holes

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Radiation emitted near a black hole reaches the observer by multiple paths; and when this radiation varies in time, the time-delays between the various paths generate a “blinking” effect in the observed light curve \(L(t)\) or its autocorrelation function \(\xi(T) = \langle L(t)L(t-T) \rangle\). For the particularly important “face-on” configuration (in which the hole is viewed roughly along its spin axis, while the emission comes roughly from its equatorial plane – e.g. from the inner edge of its accretion disk, or from the violent flash of a nearby/infalling star) we calculate the blinking in detail by computing the time delay \(\Delta t_j(r_\ast,a)\) and magnification \(\mu_j(r_\ast,a)\) of the \(j\)th path \((j = 1, 2, 3, \ldots)\), relative to the primary path \((j = 0)\), as a function of the emission radius \(r_\ast\) and black hole spin \(0 \leq a/M \leq 1\). The particular geometry and symmetry of the nearly-face-on configuration enhances and “protects” the blinking signal, making it more detectable and more independent of certain astrophysical and observational details. The effect can be surprisingly strong: e.g. for radiation from the innermost stable circular orbit (“ISCO”) of a black hole of critical spin \((a_{\text{cr}}/M \approx 0.853)\), the \(j = 1, 2, 3\) fluxes are, respectively, 27\%, 2\% and 0.1\% of the \(j = 0\) flux.

Light rays are bent as they pass through curved regions of spacetime. To date, physicists have only detected rays with tiny bending angles \((\ll 2\pi,\text{ even in the famous “strong-lensing” systems, where galaxies appear to be stretched into banana-shaped arcs on the sky})\). On the other hand, rays that pass very near a black hole can experience large bending angles, and even be bent into “light loops” that circle the hole once or more before proceeding to the observer \cite{1}
(see Fig. 1). Detection of such highly bent rays would provide an unprecedented test of strong-field general relativity, and a precious new window onto the physics and astrophysics near black holes.

Previous authors have suggested various ways to look for these light loops observationally \cite{2,3,4,5}. In this paper, we investigate a different strategy. We start from the idea that the emission from an intrinsically time-varying source very near a black hole will reach the observer by multiple paths; and the time-delay between the different paths will induce a characteristic “blinking” signal in the observed light curve \(L(t)\) or its auto-correlation function \(\xi(T) = \langle L(t)L(t-T) \rangle\). From here, we are led to focus on the “face-on” or “right-angle” configuration (in which the hole is viewed roughly along its spin axis, while the emission comes roughly from its equatorial plane – e.g. from the inner edge of its accretion disk, or from the violent flash of a nearby/infalling star). As we shall explain, a variety of mathematical, astrophysical and observational considerations point to this configuration as being of special importance when it comes to detecting blinking black holes: just as the nearly-straight-line configuration of Fig. 1a is the ideal geometry for ordinary gravitational lensing, the nearly-face-on configuration of Fig. 1b may be regarded as the ideal geometry for blinking black holes. For this configuration, we compute the blinking signal in detail (by computing the time delay \(\Delta t_j\) and magnification \(\mu_j\) of each light loop relative to the primary light path) as a function of: (i) the distance \(r_\ast\) between the hole and the source, and (ii) the spin \(0 \leq a/M \leq 1\) of the hole. The blinking signal can be surprisingly strong, and we hope it may be detectable.

To see how we might try to detect light-loops, it is useful to start by understanding why, at first glance, the task seems practically impossible! Consider the standard gravitational lensing configuration, in which the lens is nearly aligned between the source and the observer, and far away from both (Fig. 1a). If the lens is a non-spinning (Schwarzschild) black hole of mass \(M\), the observer sees an infinite series \((j = 0, 1, 2, \ldots)\) of concentric Einstein rings on the sky \cite{1}: the outer \((j = 0)\) ring is the ordinary
one, while the inner \((j \geq 1)\) rings are due to light loops with bending angles \(\alpha_j \approx 2\pi j\). The \(j \geq 1\) rings are extremely dim relative to the \(j = 0\) ring. To see this, note that the bending angle \(\alpha\) depends on the impact parameter \(b\) as: \(\alpha(b) \approx 4M/b\) (for small \(\alpha\)) and \(\alpha(b) \approx \ln[3.48M/(b - b_{\text{crit}})]\) (for large \(\alpha\)) \([3]\), where \(b_{\text{crit}} = 3\sqrt{3}M\). Given this, standard lensing analysis \([3]\) implies that, in the limit of perfect source/lens/observer alignment, the magnification \(\mu_j\) of the \(j\)th image (\(j \geq 1\)) relative to the 0th image (rather than the unlensed image) is

\[
\mu_j \approx 9[MD_S/D_LD_LS]^{3/2}e^{-2\pi j}
\]

where \(D_S\) is the observer-source distance, \(D_L\) is the observer-lens distance, and \(D_{LS}\) is the lens-source distance. This expression seems discouraging for two reasons: (i) the factor in square brackets looks tiny because in ordinary lensing, \(D_L\), \(D_S\) and \(D_{LS}\) are enormous relative to the Schwarzschild radius \(2M\) of the lens; and (ii) the factor \(e^{-2\pi j}\) says that to see highly bent rays, we must pay an exponential price (as the bending angle \(\alpha\) increases), the magnification of the corresponding image is suppressed by \(e^{-\pi}\). But before getting discouraged, note that we can improve the situation dramatically via the following two tricks. First, if we bring the source near the lens, so that \(D_{LS} \approx M\), and hence \(D_L \approx D_S\), then the factor in square brackets will be \(O(1)\). Second, if we switch from the straight-line configuration of Fig. 1a to the right-angle configuration of Fig. 1b, then instead of successive images being suppressed by \(e^{-2\pi} \approx 0.0019\), they are only suppressed by \(e^{-\pi} \approx 0.043\).

Nature may be kind enough to provide astronomical systems that take advantage of these two tricks. For example, a black hole is often surrounded by an accretion disk whose inner edge \([7]\) lies near the hole’s innermost stable circular orbit or “ISCO”. We use Boyer-Lindquist coordinates and introduce \((\lambda)\). We use Boyer-Lindquist coordinates and choose units with \(c = G = M = 1\) so that all quantities become dimensionless and \(0 \leq a \leq 1\). Light rays are the null geodesics of this metric. Along any such geodesic \(x^\mu(\lambda)\), with tangent vector \(p^\mu = dx^\mu/d\lambda\), there are 3 conserved quantities: the energy \(E = -p_t\), the axial angular momentum \(L_z = p_z\), and the Carter constant \(Q = p_\phi^2 + \cos^2(\theta)[p_r^2/\sin^2(\theta) - a^2p_\phi^2]\). We rescale the affine parameter \(\lambda\) by a constant so that \(E = 1\). Let us start by imagining that a source in a nearly circular equatorial orbit around the black hole emits a flash that is isotropic in the rest frame of the source. The null geodesics connecting the flash at \((r = r_*, \theta = \pi/2)\) to the face-on observer at \((r = \infty, \theta = 0)\) form an infinite series labeled by a non-negative integer \((j = 0, 1, 2, \ldots)\). Along the \(j\)th geodesic the polar angle \(\theta\) varies by a total amount \(\int |d\theta| = (2j + 1)\pi/2\), as shown in Fig. 1b; the azimuthal angle also varies \((\int |d\phi| \neq 0)\), but we do not need to compute this variation in order to predict the blinking signal in the face-on limit. The \(j\)th geodesic is characterized by vanishing axial angular momentum \(L_z = 0\), and a positive Carter constant \(Q_j(r_*, a) > 0\), which is determined by the requirement that \(r \text{ and } \theta\) obey the relevant first integral of the geodesic equation \([11] [12]\)

\[
\frac{|d\theta|}{\sqrt{Q + a^2 \cos^2 \theta}} = \frac{|dr|}{\sqrt{r^4 + (a^2 - Q)r^2 + 2(a^2 + Q)r - a^2Q}}
\]
as well as the boundary conditions described above. In practice, we must solve for \( Q_j(r, a) \) numerically. In doing so, note that when \( j \geq 1 \) and \( r_j \) is sufficiently large, the geodesic initially heads inward (\( dr/dt < 0 \)), reaches a radial turning point at \( R(r) = 0 \), and then heads outward (\( dr/dt > 0 \)) to the observer.

Given \( Q_j(r, a) \), we use Eqs. (180,185,186) in Sec. 62 of [13] to find the observed time delay \( \Delta t_j(r, a) \) between the \( j \)th and 0th flashes. We can also use \( Q_j(r, a) \) to compute \( \mu_j \), the ratio between the observed energy flux in the \( j \)th flash and the 0th flash, as follows. The observed energy flux in the \( j \)th flash is the product of its surface brightness \( I_j \) and its apparent angular size \( d\Omega_j \). But, for the face-on observer, each copy of the flash (\( j = 0, 1, 2, \ldots \)) has the same surface brightness \( I = \int_0^\infty I_0 dv_0 \), where \( I_0 \) is the specific intensity. To see this, first note that \( I_0 \) is the same in all Lorentz frames and conserved along a photon geodesic [14]. Next note that the ratio \( \nu_0/\nu_c \) between the observed frequency of a photon (\( \nu_c \)) and the frequency it had in the rest frame of the equatorial circularly-orbiting emitter (\( \nu_0 \)) depends on \( L_z \), but not on \( Q \). So, for our face-on observer, who only receives photons with \( L_z = 0 \), the ratio is \( j \)-independent. In other words, there is no relative redshift between the various copies of the flash received by the face-on observer. Since \( I_0 \) was isotropic in the emitter’s rest frame, \( I_0 \) and \( I \) are also \( j \)-independent.[11] Thus, \( \mu_j \) is just the ratio \( d\Omega_j/d\Omega_0 \) between the apparent size of the \( j \)th flash (\( d\Omega_j \)) and the 0th flash (\( d\Omega_0 \)), which may be calculated, given \( Q_j(r, a) \), as explained in [2,13]. See Figures 2 and 3.

Next, instead of a flash, let the emission have arbitrary (perhaps unknown or stochastic) time variation. If the \( j = 0 \) photons reach the face-on observer with light curve \( L_0(t) \), then the full light curve, including light loops, is \( L(t) = \sum_{j=0}^\infty \mu_j L_0(t - \Delta t_j) \), where \( \mu_0 = 1 \) and \( \Delta t_0 = 0 \); and if the emission is characterized by autocorrelation function \( \xi_0(T) = \langle L_0(t) L_0(t - T) \rangle \), then the observed autocorrelation function, including light loops, is \( \xi(T) = \langle L(t) L(t - T) \rangle = \sum_{j,k=0}^\infty \mu_j \mu_k \xi_0[T + (\Delta t_k - \Delta t_j)] \).

Given a promising astronomical source, these formulae for \( L(t) \) and \( \xi(T) \) correspond to two strategies to search for blinking (see Fig. 4). (i) Given (theoretical or empirical) information about the emitted light curve (\( \propto L_0(t) \)), one can construct a family of blinking light curves \( L(t, r_\ast, a, M) \) that may be correlated/fitted to the data, much as the LIGO experiment uses “matched filtering” to search its noisy data for predicted gravitational waveforms. (ii) Alternatively, we can search for blinking in the auto-correlation function \( \xi(T) \). This is better for sources that exhibit continuous and random variability, rather than short well-separated bursts; and it has the advantage that \( \xi(T) \) needn’t be measured on a flare-by-flare basis – rather, one can accumulate better statistics over time (\( e.g. \) over many flares, or many observations).

The symmetry of the face-on configuration makes the blinking more robust and independent of certain astrophysical and observational details, in two ways. First, since \( \Delta t_j \) and \( \mu_j \) depend on the radius \( r_\ast \), but not on the azimuthal angle \( \varphi_\ast \) of the emission, the face-on light curve is sensitive to the total emission from the equatorial ring of radius \( r_\ast \), not its \( \varphi_\ast \) profile. Second, there is no relative redshift between the various paths (\( j = 0, 1, 2, \ldots \)), so each blink \( L_j(t) \) is a shifted copy of the primary \( L_0(t) \); to compute the effect, we don’t need to know the frequency spectrum of the source, or the frequency band of the detector. Further from the face-on view, the time delays, magnifications and redshifts of the various paths are increasingly \( \varphi_\ast \)-dependent, and the blinking features in \( \xi(T) \) are increasingly smeared out.

Refs. [2, 5] consider a source that circularly orbits in the equatorial plane near a black hole and emits with luminosity that is constant (or long-lived relative to the orbital period), and calculate how the observed light curve \( L(t) \) oscillates with the orbital period. (In [2] the source is a star; in [5] it is a hotspot orbiting in the accretion disk.) This oscillation (which we call “time-dependent lensing” or “TDL”) is complementary to our blinking signal in several respects. TDL is due to the source’s \( \varphi_\ast \)-motion and \( \varphi_\ast \)-localization, not its intrinsic variability; by contrast, blinking is due to the source’s intrinsic variability, not its \( \varphi_\ast \)-motion or \( \varphi_\ast \)-localization. In the face-on configuration, where we have argued that blinking is optimal, TDL vanishes; and in the edge-on configuration, where TDL is strongest, blinking is smeared out (in \( \xi(T) \).)

![FIG. 3: Time delay \( \Delta t_j(r, a) \) and magnification \( \mu_j(r, a) \) of the \( j = 1, 2, 3 \) paths relative to the \( j = 0 \) path, as a function of emission radius \( r_\ast > r_{\text{ph, eq}} \) for holes that are non-spinning (\( a/M = 0 \), red dotted curves) and maximally spinning (\( a/M = 1 \), black solid curves). The solid grey curve in Panel a shows the orbital period (for \( a/M = 1 \)); again note that it is considerably longer than the blink separation.](image)
If the observer is sufficiently face-on, blinking dominates over TDL; if the observer is sufficiently non-face-on (e.g., radio, optical, X-ray, or gamma-ray) data sets? Might gravitational wave detectors help us to locate suitable black hole systems? It may even be worth mentioning that the blinking effect is not restricted to electromagnetic emission: if a supernova explodes near a black hole, we might see blinking in its neutrino signal; or if both (stellar-mass) compact objects merge near a (supermassive or intermediate-mass) black hole, we might see blinking in their gravitational wave signal. Is black hole blinking detectable? We hope this paper will encourage further consideration of this important question.

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