QCD radiative enhancement of the decay $b \to c \overline{c} s$.

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Abstract

A substantial enhancement is found of the $b \to c \overline{c} s$ decay rate due to the QCD interaction within the $\overline{c}s$ pair, which enhancement may amount to 30%, or more. Some general features of calculation of the QCD radiative corrections in two first orders are discussed.
The present theoretical understanding of $B$ decays conspicuously runs into a problem with explaining the experimental observation of rather low semileptonic branching ratio\textsuperscript{[1]}, which requires\textsuperscript{[2]} an enhancement of the non-leptonic decay rate by as much as $20\% - 30\%$ over the existing theoretical predictions. Therefore a more thorough theoretical study of the $B$ decays is imperative. In this note the decay $b \to c \bar{c}s$ is discussed, whose rate is suppressed by the presence of two massive charmed quarks as compared to the dominant non-leptonic decay $b \to c \bar{u}d$. An enhancement of the decay $b \to c \bar{c}s$ would somewhat relax the problem of low semileptonic branching ratio. However, such enhancement would also worsen another possible problem, which is perhaps hinted at by the experiment\textsuperscript{[3]}, that is the problem of a low average charm yield per $B$ decay\textsuperscript{[1]}. Nevertheless, within the present uncertainty of the measured average charm yield in $B$ decays the data can still accommodate a substantial enhancement of the decay $b \to c \bar{c}s$. In any case, it is important to have the potentially essential effects calculated. Here it is found that the first order QCD correction to the total rate of the decay $b \to c \bar{c}s$ due to the interaction within the quark pair $c \bar{s}$ has relative magnitude $\delta_{cs} \alpha_s/\pi$ with an unusually large coefficient: $\delta_{cs} = 4.46$ for $m_c/m_b = 0.3$, unlike the case of the decay $b \to c \bar{u}d$, where the similar factor due to the $u \bar{d}$ pair is equal to one.

The QCD radiative effects in the decay rate of the $b$ quark are usually analyzed within the leading log approximation in $\ln(m_W/m_b)$ or in the next-to-leading log approximation (see e.g. \textsuperscript{[5, 6]}). However, since, $\ln(m_W/m_b)$ is not really a sufficiently large parameter, the non-logarithmic terms may be quite essential, and it might be more reasonable to rely instead on complete calculation of the QCD radiative corrections in the first and second orders in $\alpha_s$. The error in the logarithmic terms, induced by such truncation of the series, is then not more than about $5\%$ (in the total rate), which is not larger than other uncertainties in the calculation, in particular not larger than the non-logarithmic terms.

Both the perturbative and non-perturbative effects in the inclusive decay rates of $B$ hadrons are conveniently calculated by representing\textsuperscript{[7]} the decay rate through the absorptive part of the $B$ self-energy, arising in the second order in the weak interaction. For the first-order QCD corrections to the rate of the decay $b \to c \bar{c}q_1 q_2$ one thus should consider the absorptive part arising from all possible unitary cuts of the graphs of three types shown in fig.1. (These graphs ignore the ‘penguin’ contribution, arising for $q_1 = q_2$, which

\textsuperscript{1}Clearly, it is impossible to solve simultaneously both these problems by an enhancement of the decay $b \to c \bar{c}s$ \textsuperscript{[3]}.}
is to be discussed separately.) In fact, however, it is clear that the graphs of the type in fig.1c, i.e. with gluon exchange between the bc line and the $q_2 q_1$ loop, are vanishing because of the color trace over the loop (a gluon can not interact through the loop with two colorless W bosons). The graphs of the type in fig.1a do not involve the interaction of the quarks $q_1$ and $q_2$ in the loop with gluons. Therefore the correction arising from these graphs can be adapted from the old calculations of QED corrections to the muon decay (see also [10, 11]). As to the effects of the gluon exchange within the colorless $q_2 q_1$ loop, these were discussed thus far for both quarks being massless, in which case the correction reduces to the familiar factor $\alpha_s/\pi$. This limit is justified for the case of the decay $b \to c \pi d$, but, as will be shown, is misleading for the decay $b \to c \tau s$, where the charmed quark in the loop has mass, which is not small in comparison with $m_b$.

Thus in the first order in $\alpha_s$ the rate of the decay $b \to c \tau s$ can be written as

$$\Gamma_{c\tau s} = \Gamma_{c\tau s}^{(bare)} \left(1 + \frac{\alpha_s}{\pi} [\delta_{bc} + \delta_{cs} + \delta_{penguin}] \right),$$

(1)

where $\Gamma_{c\tau s}^{(bare)}$ is the rate without any QCD corrections, $\delta_{bc}$ arises from gluon exchange on the $bc$ line, $\delta_{cs}$ is due to gluon interactions within the $\tau s$ loop, and $\delta_{penguin}$ is due to effects of the penguin type. It is the goal of the present note to calculate the correction factor $\delta_{cs}$.

Starting with the relevant term in the weak Lagrangian of the form

$$L_{int} = 2\sqrt{2} G_F V_{cb} V_{q_2 q_1} (\bar{c}_L \gamma_\mu b_L) (\bar{q}_1^L \gamma_\mu q_2^L)$$

(2)

and parametrizing the spectral density of the current $j_\mu = (\bar{q}_1^L \gamma_\mu q_2^L)$ as

$$\sum_n \langle 0 | j_\mu (-q) | n \rangle \langle n | j_\mu (q) | 0 \rangle = -\frac{3}{8\pi} A(q^2) \left(q^2 g_{\mu\nu} - q_\mu q_\nu\right) + \frac{3}{8\pi} B(q^2) q_\mu q_\nu,$$

(3)

one can write the total decay rate of $b \to c \tau_2 q_1$ as

$$\Gamma_{c\tau_2 q_1} = 6\Gamma_0 m_b^{-8} \int_{(m_{b1}+m_{c1})^2}^{(m_b-m_c)^2} \left\{ A(q^2) q^2 (m_b^2 + m_c^2 - q^2) + \frac{1}{2} (A(q^2) + B(q^2)) \right\} \sqrt{\lambda(m_b^2, q^2, m_c^2)} dq^2,$$

(4)

In particular, this explains why there is no first-order correction proportional to $\alpha_s \ln(m_W/m_b)$: The graphs of the types in fig.1a (gluon exchange on the $bc$ line) and in fig.1b (gluon exchange within the loop) contain renormalization of the $V - A$ currents, which is not logarithmic.
where $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$ is the standard kinematical function, and

$$\Gamma_0 = \frac{3G_F^2 |V_{cb}|^2 m_b^5}{192\pi^3}$$

is the lowest-order parton decay rate with massless quarks in the final state.

It should be emphasized that the equation (4) is applicable to calculation of only the effects associated with the gluon exchanges within the $q_2 q_1$ loop, i.e. the ones, which are discussed in this note, and does not include the effects of gluon exchange on the $bc$ line (like the one shown in fig.1a) or the effects of gluon exchange between the loop and the $bc$ line, which arise starting from order $\alpha_s^2$. In the absence of QCD radiative effects the form factors $A$ and $B$ are readily calculable. In the case when $q_2 = c$ and $q_1 = s$, so that $m_1 = 0$ and $m_2 = m_c$, one finds

$$A_0 = \frac{2}{3} \left( \frac{q^2 - m_c^2}{q^2} \right)^2 \left( 1 + \frac{m_c^2}{2q^2} \right),$$

$$B_0 = \frac{m_c^2}{q^2} \left( \frac{q^2 - m_c^2}{q^2} \right)^2.$$ (6)

Using these expressions in eq.(4) and integrating over $q^2$ gives the well known result for the ‘bare’ rate of the decay $b \to c \tau s$ in eq.(1):

$$\Gamma_{\tau s}^{(bare)} = \Gamma_0 I \left( \frac{m_c}{m_b}, \frac{m_c}{m_b} \right),$$ (7)

where

$$I(x, x) = \sqrt{1 - 4x^2} (1 - 14x^2 - 2x^4 - 12x^6) + 24x^4 (1 - x^4) \ln \left( \frac{1 + \sqrt{1 - 4x^2}}{1 - \sqrt{1 - 4x^2}} \right).$$ (8)

The QCD radiative corrections due to interactions within the $\tau s$ loop are expressed through the radiative corrections to the form factors $A$ and $B$ in the spectral density in eq.(3). The calculation of the $O(\alpha_s)$ corrections to the spectral density with unequal masses of quarks has been done in connection with the QCD sum rules both for the longitudinal form factor $B$ [12, 13, 14] and for the transversal one $A$ [13]. For the case, relevant here, where one of the quarks is massless the result reads as

$$A(q^2) = A_0(q^2) \left[ 1 + \frac{4 \alpha_s}{3 \pi} \left( f_1 \left( \frac{q^2}{m_c^2} \right) + \frac{2q^2}{2q^2 + m_c^2} f_2 \left( \frac{q^2}{m_c^2} \right) \right) \right]$$

$$B(q^2) = B_0(q^2) \left[ 1 + \frac{4 \alpha_s}{3 \pi} \left( f_1 \left( \frac{q^2}{m_c^2} \right) - 1 \right) \right]$$ (9)
with

\[ f_1(z) = \frac{13}{4} + 2 \text{Li}\left(\frac{1}{z}\right) + \ln z \ln \frac{z}{z-1} - \frac{3}{2} \ln(z-1) + \ln \frac{z}{z-1} + \frac{1}{z} \ln(z-1) + \frac{1}{z-1} \ln z \]  

(10)

and

\[ f_2(z) = -\frac{5}{2} - \frac{1}{z} - \frac{1}{z-1} + \left(\frac{z-1}{z}\right) \left(\frac{3}{2} + \frac{1}{2z}\right) \ln(z-1) + \frac{z}{(z-1)^2} \ln z, \]  

(11)

where \( \text{Li}(x) = -\int_0^x \frac{\ln(1-t)}{t} dt \) is the standard dilogarithm function\(^3\).

The integral in eq.(4) with the radiatively corrected values of the form factors can be easily done numerically, thus giving the value of the correction factor \( \delta_{cs} \) in the equation (1). The results of such calculation are shown in fig.2 in terms of the behavior of \( \delta_{cs} \) as a function of \( m_c/m_b \). In particular, at a ‘reference’ point \( m_c/m_b = 0.3 \) one finds \( \delta_{cs} = 4.46 \), which is significantly larger than the analogous correction factor for the \( b \to c \tau d \): \( \delta_{\tau d} = 1 \).

If one uses the value \( \alpha_s = 0.3 \), then the discussed correction is about 30%. However, a closer inspection of the integral for the correction in eq.(4) shows that the integrand has a maximum at \( q^2 \approx 2m_c^2 \approx 0.2m_b^2 \) for the discussed here range of values of \( m_c/m_b \). Therefore the appropriate value of \( \alpha_s \) can in fact be larger. Naturally, a quantitative clarification of this point requires a higher order calculation\(^4\). It can be also noted that the enhancement of the contribution of relatively low values of \( q^2 \) is due to the logarithmic growth of the function \( f_1(z) \) in eq.(11) in the threshold region \( z \to 1 \), which is a consequence of the ‘hybrid’ anomalous dimension\(^1\) of the current (\( \tau \Gamma \)’s).

We therefore conclude, that the decay \( b \to c \tau s \) is enhanced by about 30% or more by the correction proportional to \( \delta_{cs} \). To assess the resulting fraction of this decay in the total decay rate one should also take into account the corrections with \( \delta_{bc} \) and \( \delta_{\text{penguin}} \) and measure the result against, say, the semileptonic mode \( b \to c l \nu \) with \( l = e \) or \( l = \mu \), whose rate contains only the QCD correction associated with the \( bc \) line i.e. with \( \delta_{bc}^{(l)} \).

The penguin effect is negative and is about 3% - 5% in magnitude\(^5\). The term \( \delta_{bc} \) is also negative, but due to the charmed quark mass its magnitude is somewhat smaller than that of the negative \( \delta_{bc}^{(l)} \). In effect the term with \( \delta_{\text{penguin}} \) approximately cancels against

\(^3\)The correction to the longitudinal form factor \( B \) coincides with the correction\(^1\) for scalar or pseudoscalar density up to an additive constant, corresponding to the normalization of the (pseudo)scalar operator in order \( \alpha_s \). This constant is fixed\(^13\)\(^14\) unambiguously for the longitudinal part of the vector or axial current. I am thankful to P. Ball for pointing out to me the papers\(^13\),\(^14\), where this point is clarified.

\(^4\)It has been recently argued\(^16\) that the natural normalization scale for \( \alpha_s \) in \( \delta_{bc} \) is also quite low.
the difference $\delta_{bc} - \delta_{bc}^{(l)}$ in the ratio $\Gamma_{cs}/\Gamma_{cl\nu}$ and the net $O(\alpha_s)$ correction is dominated by the large $\delta_{cs}$. Therefore it is quite possible that a sizeable part of the existing theoretical deficiency of nonleptonic decays of $B$ can be eliminated by a 30% or larger enhancement of the decay $b \to c\bar{\tau}s$.

In order to completely quantify the issue of QCD radiative effects in the $B$ decay rates and to possibly achieve an accuracy of about 5% in theoretical predictions for the rate of each inclusive mode a complete calculation in second order in $\alpha_s$ is needed. Though, no attempt of such calculation is done in this note, I would like to conclude with a simple general remark concerning a calculation of the $O(\alpha_s^2)$ corrections to the inclusive decay rates of the $b$ quark by the unitary cuts of graphs similar to those in fig. 1, albeit in the next order of the QCD perturbation theory. Also in that order one can split the graphs into few classes. One class is where the gluons are attached only to the quarks on the $bc$ line. For the dominant decay $b \to c\tau d$ this correction cancels in the ratio $\Gamma_{cs d}/\Gamma_{cl\nu}$. Another is where the gluon corrections are fully contained within the $t_2 q_1$ loop, which are reduced to the corrections to the form factors $A$ and $B$ in the spectral density \cite{1}. For the pair of massless quarks $\bar{u}d$ this can be read off the corresponding calculation \cite{5} for $e^+ e^-$ annihilation into light hadrons. Third class is where one gluon is exchanged on the $bc$ line and the other within the $t_2 q_1$ loop and is thus a product of the first-order corrections. Finally, because of the color trace over the loop, the gluon exchange between the $bc$ line and the $t_2 q_1$ loop gives a non-vanishing result only when there are two gluons exchanged, each starting on the $bc$ line and ending on the $t_2 q_1$ loop, an example of such graph is shown in fig. 3. It is with the latter graphs that the terms proportional to $(\alpha_s \ln(m_W/m_b))^2$ and to $\alpha_s^2 \ln(m_W/m_b)$ are associated and which may also contain large non-logarithmic terms.

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When this paper was finished, a revised version of Ref.\cite{6} appeared, where a similar estimate of 30% enhancement of $\Gamma_{cs}$ was found. I thank P. Ball and V.M. Braun for pointing out to me their revised estimate. As discussed in this note, the actual enhancement can in fact be larger due to a larger value of $\alpha_s$ at the relatively low relevant invariant mass of the $\tau s$ pair.

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Figure 1: Three types of graphs, whose unitary cuts describe the first QCD radiative corrections to the inclusive decay rate $b \rightarrow c\bar{q}_2 q_1$. The small filled circles represent the $W$ boson propagators and the dashed lines correspond to gluons. The gluon vertices can be anywhere on the $bc$ line (a), quark lines in the loop (b), or one vertex anywhere on the $bc$ line and the other vertex on either line in the loop (c).
Figure 2: The correction factor $\delta_{cs}$ in eq.(1), arising from the gluon exchange within the $\tau s$ loop, vs. the mass ratio $m_c/m_b$. The range of $m_c/m_b$ shown well covers the ratio of the actual quark masses with the existing uncertainty.

Figure 3: The only type of graphs in the second order in $\alpha_s$ with gluon exchange between the $bc$ line and the $\bar{q}_2 q_1$ loop, which gives contribution to the inclusive decay rate. Each of the two gluons should start anywhere on the $bc$ line and end on either of the quark lines in the loop.