NONEQUILIBRIUM NEUTRINO OSCILLATIONS AND PRIMORDIAL NUCLEOSYNTHESIS

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We studied nonequilibrium oscillations between left-handed electron neutrinos and nonthermalized sterile neutrinos. The exact kinetic equations for neutrinos, written in terms of neutrino density matrix in momentum space were analyzed. The evolution of neutrino density matrix was numerically calculated. This allowed to study precisely the evolution of the neutrino number densities, energy spectrum distortion and the asymmetry between neutrinos and antineutrinos for each momentum mode. Both effects of distortion and asymmetry, which cannot be accounted for correctly when working in terms of particle densities and mean energies, and the depletion of electron neutrino state have been proved considerable for a certain range of oscillation parameters. The influence of nonequilibrium oscillations on primordial nucleosynthesis was calculated. Cosmologically excluded regions for oscillation parameters were obtained.

1 Nonequilibrium neutrino oscillations

We discuss nonequilibrium oscillations between weak interacting electron neutrinos $\nu_e$ and sterile neutrinos $\nu_s$ for the case when $\nu_s$ do not thermalize till 2 MeV and oscillations become effective after $\nu_e$ decoupling. Oscillations of that type, but for the case of $\nu_s$ thermalizing before or around 2 MeV have been already discussed in literature. We have provided a proper kinetic analysis of the neutrino evolution in terms of kinetic equations for the neutrino density matrix in momentum space. The assumptions of the model are the following: (a) Singlet neutrinos decouple much earlier than the active neutrinos do: $T_{\nu_s}^F \geq T_{\nu_e}^F$, therefore, in later epochs $T_{\nu_s} \leq T_{\nu_e}$, due to the additional heating of $\nu_e$ in equilibrium in comparison with the already decoupled $\nu_s$. Hence, the number densities of $\nu_s$ are considerably less than those of $\nu_e$, $N_{\nu_s} \ll N_{\nu_e}$, (b)We consider oscillations between $\nu_s$ and $\nu_e$, according to the Majorana&Dirac mixing scheme with mixing present just in the electron

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sector $\nu_i = U_l \nu_l$, $l = e, s$

\[
\begin{align*}
\nu_1 &= \cos(\vartheta)\nu_e + \sin(\vartheta)\nu_s \\
\nu_2 &= -\sin(\vartheta)\nu_e + \cos(\vartheta)\nu_s,
\end{align*}
\]

(1)

where $\nu_s$ denotes the sterile electron antineutrino, $\nu_1$ and $\nu_2$ are Majorana particles with masses correspondingly $m_1$ and $m_2$. (c) We assume that neutrino oscillations become effective after the decoupling of the active neutrinos, $\Gamma_{osc} \geq H$ for $T \leq 2\,MeV$. This puts constraint on the neutrino mass difference: $\delta m^2 \leq 1.3 \times 10^{-7} \,eV^2$. (d) We require that sterile neutrinos have not thermalized till 2 $MeV$ when oscillations become effective. This puts the following limit on the allowed range of oscillation parameters: \[ \sin(2\vartheta)\delta m^2 \leq 10^{-7}eV^2. \]

As far as for this model the rates of expansion of the Universe, neutrino oscillations and neutrino interactions with the medium may be comparable, we have used kinetic equations for neutrinos accounting simultaneously for the participation of neutrinos into expansion, oscillations and interactions with the medium.\[10,12,13\] We have analyzed the evolution of nonequilibrium oscillating neutrinos by numerically integrating the kinetic equations for the density matrix in momentum space for the period after the decoupling of the electron neutrino till the freezing of neutron-proton ratio ($n/p$-ratio), i.e. for $2\,MeV \geq T \geq 0.3\,MeV$. We considered both resonant $\delta m^2 = m_2^2 - m_1^2 < 0$ and nonresonant $\delta m^2 > 0$ oscillations.

2 Kinetics of nonequilibrium neutrino oscillations

The kinetic equations for the density matrix of the nonequilibrium oscillating neutrinos in the primeval plasma of the Universe in the epoch previous to nucleosynthesis have the form:

\[
\frac{\partial \rho(t)}{\partial t} = H_p \frac{\partial \rho(t)}{\partial p} + i [H_o, \rho(t)] + i [H_{int}, \rho(t)] + O(H_{int}^2),
\]

(2)

where $p$ is the momentum of neutrino and $\rho$ is the density matrix of the massive Majorana neutrinos in momentum space.

The first term in the right side of Eq.2 describes the effect of expansion, the second is responsible for oscillations, the third accounts for forward neutrino scattering off the medium.\[14\] $H_o$ is the free neutrino Hamiltonian:

\[
H_o = \begin{pmatrix}
\sqrt{p^2 + m_1^2} & 0 \\
0 & \sqrt{p^2 + m_2^2}
\end{pmatrix},
\]

(3)

\[\text{bThe transitions between different neutrino flavours were proved to have negligible effects.}\]
while $H_{int} = \alpha V$ is the interaction Hamiltonian, where $\alpha_{ij} = U_{ie}^* U_{je}$, $V = G_F \left( \pm L - Q/M_W^2 \right)$, and in the interaction basis has the form

$$H_{int}^{LR} = \begin{pmatrix} V & 0 \\ 0 & 0 \end{pmatrix}. \quad (4)$$

The first ‘local’ term in $V$ accounts for charged- and neutral-current tree-level interactions with medium protons, neutrons, electrons and positrons, neutrinos and antineutrinos. It is proportional to the fermion asymmetry of the plasma $L = \sum_f L_f$, which is assumed of the order of the baryon one

$$L_f \sim \frac{N_f - \bar{N}_f}{N_{\gamma}} T^3 \sim \frac{N_B - \bar{N}_B}{N_{\gamma}} T^3 = \beta T^3. \quad (5)$$

The second ‘nonlocal’ term arises as an $W/Z$ propagator effect $Q \sim E_\nu T^3$. The two terms have different temperature dependence and an interesting interplay between them during the cooling of the Universe is observed. The last term in the Eq. describes the weak interactions of neutrinos with the medium.

We have analyzed the evolution of the neutrino density matrix assumed that oscillations become noticeable after electron neutrinos decoupling. So, the neutrino kinetics down to 2 $MeV$ does not differ from the standard case, i.e. electron neutrinos maintain their equilibrium distribution, while sterile neutrinos are absent. Then the last term in the kinetic equation can be neglected. The equation results into a set of coupled nonlinear integro-differential equations for the components of the density matrix. We have numerically calculated the evolution of the neutrino density matrix for the temperature interval $[0.3, 2.0]$ $MeV$. The oscillation parameters range studied is $\delta m^2 \in \pm [10^{-10}, 10^{-7}]$ $eV^2$ and $\vartheta \in [0, \pi/4]$. The baryon asymmetry $\beta$ was taken to be $3 \times 10^{-10}$.

3 Nucleosynthesis with nonequilibrium oscillating neutrinos

We analyzed the influence of nonequilibrium oscillations on the primordial production of $^4He$. The effect of oscillations on nucleosynthesis has been discussed in numerous publications. Here we provided a detail kinetic calculations of helium abundance for the case of nonequilibrium oscillations in medium. The

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\textsuperscript{c} For example, for the weak reactions of neutrinos with electrons and positrons $e^+ e^- \leftrightarrow \nu_i \bar{\nu}_j$, $e^\pm \nu_j \rightarrow e^\pm \nu'_i$ it was explicitly written in Refs. 10,12.

\textsuperscript{d} For the case of vacuum neutrino oscillations these equations were analytically solved in Ref. 12.
kinetic equation describing the evolution of the neutron number density in momentum space $n_n$ for the case of oscillating neutrinos $\nu_e \leftrightarrow \nu_s$ was numerically integrated for the temperature range of interest $T \in [0.3, 2.0]$ MeV.

4 Results and conclusions

Our numerical analysis showed that the nonequilibrium oscillations can considerably deplete the number densities of electron neutrinos (antineutrinos), distort their energy spectrum and produce neutrino-antineutrino asymmetry that may grow at the resonant transition and may change considerably the evolution of neutrino ensembles. The effects of nonequilibrium oscillations on nucleosynthesis may be considerable for certain range of oscillation parameters. The results of our study are as follows:

(a) As far as oscillations become effective when the number densities of $\nu_e$ are much greater than those of $\nu_s$, $N_{\nu_e} \gg N_{\nu_s}$, the oscillations tend to reestablish the statistical equilibrium between different oscillating species. As a result $N_{\nu_e}$ decreases in comparison to its standard equilibrium value.

The depletion of the electron neutrino number densities due to oscillations to sterile ones leads to an effective decrease in the weak processes rates, and thus to an increase of the freezing temperature of the $n/p$-ratio and corresponding overproduction of the primordially produced $^4$He.

(b) For the case of strongly nonequilibrium oscillations the distortion of the energy distribution of neutrinos may be considerable. The evolution of the distortion is the following: First the low energy part of the spectrum is distorted, and later on this distortion concerns neutrinos with higher and higher energies (Fig. 1).

This behavior is natural, as far as neutrino oscillations affect first low energy neutrinos, $\Gamma_{\text{osc}} \sim \delta m^2/E_\nu$. The naive account of this effect by shifting the effective temperature and assuming the neutrino spectrum of equilibrium form gives wrong results for the case $\delta m^2 < 10^{-7}$eV$^2$.

The effect of the distortion on primordially produced helium is as follows. An average decrease of the energy of active neutrinos leads to a decrease of the weak reactions rate, $\Gamma_w \sim E_\nu^2$ and subsequently to an increase in the freezing temperature and the produced helium. On the other hand, there exists an energy threshold for the reaction $\tilde{\nu}_e + p \rightarrow n + e^+$. And in case when, due

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4 This effect was discussed for a first time by Dolgov, but unfortunately, as far as the case of flavour neutrino oscillations were considered and the energy distortion for that case was shown to be negligible, it was not paid the necessary attention it deserved. In paper by Kirilova, it was first shown that for the case of $\nu_e \leftrightarrow \nu_s$ vacuum oscillations this effect is considerable.
Figure 1: The figure shows the energy distortion of active neutrinos $x^2 \rho_{LL}(x)$, where $x = E_\nu/T$, for the case of nonequilibrium neutrino oscillations, $\delta m^2 = -10^{-8}$, $\delta = \pi/8$ at different temperatures: $T = 1$ MeV (a), $T = 0.7$ MeV (b), and $T = 0.5$ MeV (c).

to oscillations, the energy of the relatively greater part of neutrinos becomes smaller than that threshold the $n/p$-freezing ratio decreases The numerical analysis showed that the latter effect is less noticeable compared with the previously described ones.

(c) Other interesting effect revealed by our approach is the generation of asymmetry between $\nu_e$ and their antiparticles. The possibility of an asymmetry generation due to oscillations was discussed in many papers. Our approach allowed precise description of the asymmetry and its evolution, as far as working with the self-consistent kinetic equations for neutrinos in momentum space enabled us to calculate the behavior of asymmetry at each momentum. The calculated result may differ considerably from the rough estimations made by working with neutrino mean energy and with the integrated quantities like particle densities and the energy densities. The asymmetry effect is noticeable only for the resonant case. Even when the asymmetry is assumed initially negligibly small (of the order of the baryon one), i.e., $\sim 10^{-10}$, it may be considerably amplified at resonant transition due to different interactions of neutrinos and antineutrinos with the CP-odd medium. The value of the asymmetry may grow by several orders of magnitude, oscillating and sign changing. Even in case when the value of the asymmetry does not become considerable enough to have some direct noticeable effect, on primordial nucleosynthesis for example, the asymmetry term at the resonant transition determines the evolution of the neutrino density matrix. It effectively suppresses the resonant transitions of active neutrinos (antineutrinos) thus weakening neutrino depletion at resonance. For some model parameters this effect consists 20% of the previously discussed.

The asymmetry calculations showed a slight predominance of neutrinos over antineutrinos, leading to decrease of helium. The greater effect of the
The curves represent the evolution of the neutron number density relative to nucleons $X_n(t) = N_n(t)/(N_\nu + N_n)$ for the nucleosynthesis model with vacuum nonequilibrium oscillations and for the case of nonequilibrium oscillations in medium, $\delta m^2 = -10^{-8}$, $\vartheta = \pi/8$. For comparison the curve corresponding to the standard nucleosynthesis model is shown.

asymmetry is, however, the change in the evolution of the neutrino ensembles during and after the resonance resulting to a relative increase of both neutrino and antineutrino particle densities. This may lead to a noticeable underproduction of helium (up to 10% relative decrease).

The total effect of nonequilibrium neutrino oscillations is overproduction of helium in comparison to the standard value. The results of the numerical integration are illustrated on Fig. 2, in comparison with the vacuum case and the standard nucleosynthesis without oscillations.

From numerical integration for different oscillation parameters we have obtained constant $^4\text{He}$ contours. We have used the 4% relative increase in the primordially produced helium to obtain the exclusion region for the oscillation parameters (Fig. 3).

For the cases when the energy distortion and asymmetry are considerable we have obtained an order of magnitude stronger constraints than the cited in literature. Therefore, in conclusion we would like to stress once again, that
in case of nonequilibrium neutrino oscillations working with the exact kinetic equations for the density matrix of neutrinos in momentum space is necessary.

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