Recent advances in wave energy converters based on nonlinear stiffness mechanisms*

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Abstract  Wave energy is one of the most abundant renewable clean energy sources, and has been widely studied because of its advantages of continuity and low seasonal variation. However, its low capture efficiency and narrow capture frequency bandwidth are still technical bottlenecks that restrict the commercial application of wave energy converters (WECs). In recent years, using a nonlinear stiffness mechanism (NSM) for passive control has provided a new way to solve these technical bottlenecks. This literature review focuses on the research performed on the use of nonlinear mechanisms in wave energy device utilization, including the conceptual design of a mechanism, hydrodynamic models, dynamic characteristics, response mechanisms, and some examples of experimental verification. Finally, future research directions are discussed and recommended.

Key words  wave energy, efficiency, nonlinear stiffness, hydrodynamics, bistable

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1 Introduction

With the unprecedented development of society in relation to economy, science, and technology, the massive consumption of conventional fossil energy and a series of problems associated with it, e.g., environmental pollution and resource shortages, have forced human beings to start looking for clean renewable energy. Compared with traditional fossil fuels such as coal, oil, and natural gas, renewable energy is generally characterized by less pollution and large reserves,
which is of great significance in solving the increasingly serious environmental pollution and resource shortage problems in today’s world. The pressure on the resources and environment has also brought new challenges to power systems. Using renewable energy to gradually replace traditional energy for power generation will be a trend in the development of the power industry in the future.

Among the many renewable energy sources such as wind, solar, and geothermal energy, ocean wave energy has attracted the attention of many countries because of its predictability, good continuity, and huge reserves. Wave power generation technology has been studied for many years in many countries, with great achievements. The concept of utilizing ocean waves to generate useful energy is not new, with the first patent dating back to 1799[1]. The fossil fuel crisis in the 1970s dramatically boosted research and development activities to harvest energy from ocean waves, inspired by the enormous potential of wave power, which is estimated to be 2.11 ± 0.05 TW globally[2].

1.1 Wave energy resource

The mapping of available wave energy resources is a prerequisite for the development of future power conversion projects. In recent years, numerical phase-average spectral wave models have undergone significant development with respect to their application in wave forecasting and hindcasting. The state-of-the-art third-generation wave model has been shown to be accurate for general ocean waves[3]. As a result, spectral wave models have been used to study the wave power in a large number of regional seas around the world, including the global sea[4–6] and local sea areas such as the Black Sea[7], the Spain Sea[8–9], and the South China Sea and its reef islands[10–11].

A worldwide wave energy analysis is briefly summarized. The wave power estimates were calculated based on the wave hindcast data from the Integrated Ocean Waves for Geophysical and Other Applications (IOWAGA) project of French Research Institute for the Exploitation of the Seas (IFREMER). The wave hindcast data were calculated using the spectral wave model WAVEWATCH-III, with the parameterization found in Ref. [12]. The model was forced by the European Center for Medium-range Weather Forecasts (ECMWF) reanalysis wind data. The hindcast data ranged from 1990 to 2020, with a spatial resolution of 30 arc-min and a temporal resolution of 3 h. The bulk wave parameters were output and archived at all grid points spanning the global ocean and used in this study.

The wave parameters computed by WAVEWATCH-III often consist of the significant wave height $H_s$, the peak period $T_p$, and the peak wave direction $D_{irp}$, while the wave energy period $T_e$ is hardly specified and must be computed from the wave spectrum. In most previous studies, $T_e$ was computed using a relation derived from a theoretical spectral function using a coefficient of 0.9[13–15]. This function was adopted in this study, i.e.,

$$T_e = 0.9T_p.$$  \hfill (1)

After obtaining the wave energy period $T_e$, the wave power can be calculated using a general wave energy assessment equation that accounts for the water depth[16–17]:

$$P = \frac{\pi \rho g h (H_s)^2}{16 T_e} \left( \frac{1}{\mu} + \frac{2}{\sinh(2\mu)} \right),$$  \hfill (2)

where $\rho$ indicates the density of water, and $g$ denotes the acceleration of gravity. The approximate dispersion $\mu$ is defined as follows:

$$\mu = \frac{\mu_0 (1 + \mu_0^3 e^{-(1.1 + 2.0\mu_0)})}{\sqrt{\tan \mu_0}},$$  \hfill (3)

where

$$\mu_0 = k_0 h = \frac{2\pi h}{L_0},$$  \hfill (4)
Recent advances in wave energy converters based on nonlinear stiffness mechanisms

and $L_0$ is the wave length for deep water.

Figure 1 displays the spatial distribution of the mean wave energy period. In the open ocean, the wave energy period is found to be larger than that in coastal areas. Most of the coastal regions in the northern part of America and the western part of Australia are exposed to the longest waves, with mean wave energy periods larger than 10s. Figure 2 shows the spatial distribution of the mean wave power. The wave energy resources in the Southern hemisphere are larger than those in the Northern hemisphere. The spatial variations in the wave energy resources by latitude are obvious. The highest values (over 120 kWy−1) are found in the South-Indian Ocean. The South Pacific is the second richest sea, with the mean wave power values of more than 90 kW·m−1. In the Northern hemisphere, the North Atlantic (40–60°N) is the most energetic zone, with the highest values greater than 80 kW·m−1. The richest coastal regions are the southwestern parts of South America, South Africa, and the southern coast of Australia in the Southern hemisphere. These regions are the main swell generation zones over the global ocean. In the Northern hemisphere, the North American western coast is directly exposed to 20 kW·m−1 and 60 kW·m−1, which increase in a northward direction.

![Fig. 1 Global mean wave energy period distribution (color online)](image1)

![Fig. 2 Global mean wave power distribution (color online)](image2)

1.2 Wave energy convertors (WECs)

In general, there may be three stages when transferring the energy of ocean waves into electrical power (there are other uses as well), as shown in Fig. 3, including the primary, secondary, and tertiary energy conversion stages [18]. At the primary energy conversion stage, the energy of the ocean waves is transferred into mechanical power by the wave-structure interaction between the ocean waves and fixed or floating structures. At this stage, there are several working principles for transferring the power through wave-structure interaction, including the motions of floating bodies, compressed air, or overtopping (OT). At the secondary energy conversion stage, the mechanical power of the mechanical structures is converted into electrical power using power take-off (PTO) systems. These PTO systems can be direct (e.g., via a direct-drive
generator) or indirect (e.g., transferred into rotational motion by air turbines, hydraulic rams, gearboxes, or mechanical motion rectifiers). Finally, the generated electricity power is stored in an energy storage device or directly connected to the grid for the use in the tertiary energy conversion stage. However, the power quality of this conversion is usually not very good because of the complex ocean wave environment, which is affected by seasonal, environmental, climate, and other factors. Some measures should be taken to improve the power quality by utilizing power electronics technologies. From the above discussions, it can be seen that wave energy conversion requires complex systems and involves many disciplines and technologies.

The WECs at the primary energy conversion stage are among the most important devices in the entire conversion system, and directly determine how much energy can be captured from the ocean waves. Since wave energy research began, more than one thousand concepts of WECs have been proposed. There is no unique categorization method to cover all types of WEC systems. In general, WEC systems can be classified according to their deployment locations, working principles, operation modes, and device geometries. Based on the working principle of the primary capture system, WECs can be classified into three types, i.e., oscillating water column (OWC), oscillating body (OB), and OT, as shown in Fig. 4. The detailed classifications and simplifications of the WECs are listed in Table 1.

![Fig. 3 Complete wave to wire energy conversion train of a wave energy conversion device](color online)

![Fig. 4 Classification of WECs based on operation principles](color online)

| Type   | OWC | OB | OT |
|--------|-----|----|----|
| Floating structure | Owsc | Heaving | Floating structure |
| Fixed structure | | OWSC | Fixed structure |
| Typical graph | | | |

### Table 1 Wave conversion systems

1.3 Challenges

Although there have been some early commercial applications of wave energy utilization based on years of theoretical work and sea trials, it still has a number of technical bottlenecks, from the design stage to construction and maintenance. Among these challenges, a low capture
efficiency is one of the main technical bottlenecks. As discussed above, ocean wave power is transferred to mechanical power by the wave-structure interaction between ocean waves and fixed or floating structures. Thus, the system can obtain a better capture efficiency only when the floating body and incident wave have “resonance” responses. Under realistic ocean wave conditions, as shown in Fig. 1, long-period waves are dominant. Therefore, WECs, especially floating WECs, with lower natural frequencies are required. Because of the limitation on the static balance between the floating mass and buoyancy, a large scale is required for the floating body to achieve a low natural frequency. Larger-scale WECs are required to capture energy from low-frequency waves. For example, to resonate at 8 s regular waves, a heaving semi-submerged sphere would need a diameter of approximately 30 m, equivalent to a displaced volume of 7000 m$^3$. However, the large size would cause significant security and cost issues for this integrated structure. Moreover, the resonance bandwidth for a traditional linear floating WEC is narrower. Therefore, if its natural frequency is outside the range of the predominant sea wave frequency, there will be an abrupt drop in efficiency. In particular, considering real sea conditions with random characteristics, it is difficult to adequately capture the wave energy across the entire frequency band. To overcome the aforementioned drawbacks, one possible approach is to apply active control strategies to WECs. Complex conjugate control, optimum control, damping control, latching control, and reinforcement learning (RL) have been applied to various types of WECs, and their performances have been validated in laboratory or sea trials.

Another possible alternative approach is to change the stiffness characteristics associated with the natural frequency (period) of the system. A nonlinear stiffness mechanism (NSM) can achieve this goal. Among other possible engineering applications, the NSM has been widely applied in the vibration isolation problem and vibration energy harvesting, and has begun to attract attention as a promising concept for application as a feasible, efficient, and simple solution for wave energy conversion. A comprehensive literature review of NSM technology for vibration isolation and energy harvesting has been conducted. However, to the best of our knowledge, a review of the advances in WECs based on NSMs has not been reported. In this paper, studies related to the application of NSMs to wave energy devices are reviewed.

The remainder of this review is organized as follows. The conceptual designs of a variety of NSMs are described and classified in Section 2. In Section 3, the mathematical model for a nonlinear WEC is reported, which includes the governing equation, the wave energy evaluation model, and the solution methods. In Section 4, the energy capture and dynamic characteristics of different NSM technologies are reviewed, and the experimental validations are reported. In Section 5, the challenges in using NSM technologies for WECs are presented, along with some research prospects for future work. Finally, the concluding remarks are presented in Section 6.

2 Conceptual designs of NSMs

To enhance the capture efficiency and broaden the bandwidth, various NSMs have been proposed and applied to point absorber type, raft type, and pitching wave energy devices. Based on the principle used to achieve the nonlinear stiffness, the NSMs can be divided into four categories, i.e., bistable, hydrodynamic nonlinear stiffness (HNLS), vibro-impact (VI), and others. The conceptual designs for these four types of NSMs for WECs are summarized in Table 2, including the typical design concept and relevant references.

2.1 Bistable mechanisms

Bistable mechanisms are the most widely investigated NSMs in the field of WECs. Based on the idea of reducing the equivalent stiffness, some NSMs have been designed and installed on WECs to achieve the bistable property combined with a hydrostatic restoring stiffness. Unlike a traditional linear mechanism, a bistable mechanism has a double-well restoring force potential with two stable equilibria and one unstable equilibrium (see Fig. 5). Because of
the characteristics of the double-well potential, the system may have three motion patterns, i.e., low-energy local oscillations restricted in one potential well, chaotic mix-well oscillations, and periodic motion between two wells. Periodic oscillations with large amplitudes have been recognized to greatly enhance the energy harvesting performance of systems.

The bistable mechanisms used in WECs have different structural forms. A general bistable mechanism is often realized by applying an ideal spring mechanism. A typical oblique stiffness is shown in Fig. 6(a). In contrast to the use of stiffness to achieve negative stiffness, a pneumatic spring mechanism was also designed. CorPower designed a passive pneumatic machinery component called WaveSpring (see Fig. 6(b)), which is the only bistable mechanism that has so far been used commercially in wave power generation. Apart from the above-mentioned spiral spring and pneumatic spring mechanisms, the magnetic bistable mechanism is also a classical application, which is often formed by concentric circular magnets (see Fig. 6(c)).

Unlike ideal, pneumatic, and magnetic springs, Shi et al. proposed a novel torsion bistable mechanism for a raft-type WEC based on a cam roller mechanism (see Fig. 7(a)). However, because of the large motions of an ocean wave energy harvesting system, the limiting constraints on the geometrical and physical characteristics of the mechanical compression springs will significantly affect the performance improvement of the nonlinear stiffness in practical implementations. Besides, some of the previous bistable mechanisms have disadvantages, including the relative difficulty in actual installation operations. With this motivation, Zhang
et al.\cite{48} proposed a simple and effective bistable mechanism for a raft-type WEC (see Fig. 7(b)). The novel bistable structure is realized by a stretch spring arranged between two floating bodies, which can be easily installed and implemented in an ocean WEC device. Furthermore, introducing the classical oblique stiffness model, Zhang et al.\cite{27} proposed an oblique spring bistable mechanism and applied it to a two-module raft-type WEC (see Fig. 7(c)).

As seen in Fig. 5, a WEC may only capture a large amount of wave energy if the OB has large motions between two potential wells. However, the OB may be restrained in one potential well by the potential barrier when the OB is excited under a low-amplitude wave. This drawback limits the energy capture robustness of bistable mechanisms in complex wave environments. To improve the performance of the traditional bistable mechanism in low-wave excitations, a novel bistable mechanism has been proposed. Based on the shapes of the potential energy curves, the bistable mechanisms can be divided into four categories, i.e., traditional bistable, multi-stable, adaptive bistable, and improved bistability, the details of which are listed in Table 3.
### Table 3  Four typical bistable structures

| Type                     | Traditional bistable | Multi-stable | Improved bistable | Adaptive bistable |
|--------------------------|----------------------|--------------|-------------------|-------------------|
| Typical graph            | ![Typical graph](image) | ![Typical graph](image) | ![Typical graph](image) | ![Typical graph](image) |
| Ref.                     | Ref. [52]            | Ref. [65]    | Ref. [54]         | Ref. [76]         |
| Refs.                    | [27], [34]–[38], [40]–[45], [48], [50]–[53], [55]–[64] | [31], [47], [65]–[66] | [32], [54] | [39], [46], [49], [76] |

Younesian and Alam [65] proposed a bistable mechanism with two rigid links and two oblique stiffnesses. By adjusting the geometric parameters of the bistable mechanism, this structure can be achieved in different multi-stable states from bistable to four-stable, and the potential barrier can be reduced by increasing the number of steady states. Li and Jing [57] proposed an X-structured device and applied it to a point-absorber WEC, as shown in Fig. 8. This X-structured device could achieve bistable or quasi-zero stiffness properties. In contrast to the multi-stable mechanism, some scholars have added an auxiliary stiffness device to the traditional bistable mechanism to directly reduce the barrier. Based on the traditional magnetic bistable mechanism proposed by Zhang et al. [41], Zhao et al. [45] and Xi et al. [55] proposed improved magnetic bistable mechanisms by adding auxiliary magnetic rings, as shown in Fig. 9. These auxiliary magnetic rings could provide nonlinear stiffness, which effectively reduced the large negative stiffness near the unstable equilibrium point and thus reduced the potential barrier. More importantly, the improved bistable mechanism proposed by Xi et al. [55] can effectively reduce the potential barrier, ensuring that the distance between two potential wells does not decrease, which can achieve large motions under a lower wave amplitude. Very recently, Liu et al. [54] proposed an improved bistable mechanism by replacing two traditional inclined springs [77] with two oblique springs. Zhang et al. [76] proposed a novel adaptive bistable mechanism that was realized using two symmetrically oblique main springs, together with two auxiliary springs. The potential function could be adaptively adjusted to lower the potential barrier near the unstable equilibrium position.

![Fig. 8 Bistable X-structured electromagnetic WEC](image)
Recent advances in wave energy converters based on nonlinear stiffness mechanisms

A prototype design of an improved magnetic bistable mechanism (1—rod, 2—liner bearing, 3—flange plate, 4—end cap, 5—sleeve, 6—inner support ring, 7—outer support ring, 8—outer magnetic ring, 9—inner magnetic ring)

A schematic diagram of two MBGs, (a) the traditional type and (b) the novel type. (1—rigid rod, 2—linear bearing, 3—end cap, 4—ring flange, 5—internal magnetic ring, 6—external magnetic ring, 7—hoop, 8—internal back-up ring, 9—external back-up ring, 10—lower internal magnetic ring, 11—lower external magnetic ring)

Fig. 9 Improved magnetic bistable mechanisms: (a) structure with three pairs of magnetic rings\(^{[45]}\) and (b) structure with two pairs of magnetic rings\(^{[55]}\) (color online)

The mechanisms discussed above were applied to an OB WEC with rigid motions. Le et al.\(^{[53]}\) proposed a bistable mechanism for a flexible WEC based on the buckled beam theory, as shown in Fig. 10. The structure was similar to the skeleton of a reptile. It bent rather than contracted the hose by stretching the spine, which produced a bistable effect under the action of waves.

Fig. 10 Buckled sea wave device\(^{[53]}\) (color online)

2.2 HNLS

In contrast to the additional NSMs discussed above, HNLS is achieved by designing an OB with a specific shape. Roveda\(^{[67]}\) used the immersed variable volume (IVV) method to obtain an HNLS. The nonlinear stiffness was achieved by changing the shape of a spar buoy WEC, as shown in Fig. 11(a). The hydrodynamic negative spring mechanism proposed by Gradowski et al.\(^{[28]}\) was achieved by changing the air volume of an OWC WEC, as shown in Fig. 11(b). The hydrodynamic negative spring method applied to the OWC involved widening the tube inside the floater, filling the space with seawater on the downward cycle, and transferring it back to the sea on the upward cycle.

2.3 VI and other NSMs

The VI (see Fig. 12)\(^{[29,68–71]}\) WEC (VIWEC) is realized using a VI mass-spring-damper PTO system. The spring stop mechanism and internal mass block induce VI events. In contrast to the concrete nonlinear mechanisms discussed above, some scholars have also achieved nonlinear stiffness using a theoretical mathematical model. Zheng et al.\(^{[30]}\) employed a nonlinear hardening spring model, and Wilson et al.\(^{[72]}\) adopted cubic stiffness for a point absorber WEC.
In this section, OB type WECs with only one degree of freedom (DOF) are taken as an example to review the theoretical dynamic model. For OB type WECs, the OB acts as the “absorber” to capture the wave energy and convert it as the kinetic energy of the body in the first step. Then, the PTO system connected with the oscillator converts the kinetic energy into electricity. This section will briefly introduce the dynamic model for the OB type WECs, wave energy evaluation model, and the numerical solving method.

### 3.1 Governing equation of WECs with nonlinear stiffness

The governing equation of motion for an OB type WEC with the NSM in the time domain can be described by the following Cummins’ equation\(^{78}\):

\[
(m + \mu_\infty)\ddot{X}(t) + \int_{-\infty}^{t} h(t - \tau)\dot{X}(\tau)d\tau + k_s X(t) + F_p + F_N = \Re \left( F_e e^{-i\omega t} \right),
\]

where \(m\) is the generalized mass, \(\mu_\infty\) is the added mass matrix at the infinite frequency \(\omega = \infty\). \(X\) denotes the displacement of the floater. \(k_s\) denotes the hydrostatic restoring stiffness. \(h(t)\)
Recent advances in wave energy converters based on nonlinear stiffness mechanisms

is the impulse response function of the radiation wave force, which could be determined using the radiation damping coefficient matrix $\lambda(\omega)$ by\textsuperscript{[79]}

$$h(t) = \int_0^\infty \lambda(\omega) \cos(\omega t) d\omega. \quad (6)$$

The term $F_P$ indicates the force of the PTO, which usually can be expressed as

$$F_P = C \cdot \dot{X}, \quad (7)$$

where $C$ is the damping coefficient of the PTO system.

The term $F_N$ represents the force of the NSM, the specific expression of $F_N$ is determined by the structure of the nonlinear mechanism.

The term $F_e(t)$ on the right hand of Eq. (5) represents the wave excitation force. For regular wave, $F_e(t)$ can be written as

$$F_e(t) = F e^{-i\omega t}. \quad (8)$$

Real sea state is a complex random process which is usually represented by irregular waves. Irregular waves can be approximately represented as a superposition of the infinite regular wave components satisfying the distribution rule of a wave spectrum. Therefore, the wave excitation force on a floater in irregular waves can be calculated by superimposing infinite regular wave components of different frequencies with different random phases as follows:

$$F_{ir}(t) = \sum_{i=1}^{\infty} A_{\omega_i} F(\omega_i) \sin(\omega_i t + \theta_i + \varepsilon_i), \quad (9)$$

where $\omega_i$ represents the circular frequency of the $i$th regular wave component, $A_{\omega_i}$ denotes the amplitude of the $i$th regular wave component, $\theta_i$ is the initial phase of the wave excitation force for the $i$th regular wave component, and $\varepsilon_i$ is the random phase which is a real number in the interval $(0, 2\pi)$.

### 3.2 Wave energy evaluation

The capture width ratio is the key indicator for evaluating the efficiency of the WECs, which is defined as the quotient of the absorption power divided by the transported energy on the cross-section of the buoy. First, for the linear waves, the incident wave power per meter of the wave-front length for a regular harmonic wave with a finite water depth can be calculated by\textsuperscript{[80]}

$$E_0^r = \frac{1}{2} \rho g A^2 V_g, \quad (10)$$

where $V_g$ indicates the group velocity. For a finite water depth, $V_g$ satisfies

$$V_g = \frac{\omega}{2k} \left( 1 + \frac{2kd}{\sinh(2kd)} \right), \quad (11)$$

where $d$ indicates the water depth, and $k$ denotes the wave number satisfying the disperse relationship $\omega^2 = gk \tanh(kd)$.

For irregular waves, the time-averaged incident wave power can be calculated by using the spectral analysis method as follows\textsuperscript{[81]}:

$$E_{ir}^r = \int_0^\infty \rho g V_g S(\omega) d\omega, \quad (12)$$

where $S(\omega)$ is the wave spectrum function.
After the dynamic responses of the buoy are obtained, the average wave power extracted by the PTO can be calculated by the work done of the damping force as follows:

$$E_r = \frac{1}{T_0} \int_{t}^{t+T_0} C \ddot{X}^2 dt. \quad (13)$$

Therefore, the capture width ratios can be written as

$$C_W = \begin{cases} \frac{E_r}{E_r^0} & \text{for regular waves}, \\ \frac{E_{ir}}{E_{ir}^0} & \text{for irregular waves}. \end{cases} \quad (14)$$

### 3.3 Solving method

Nonlinear WEC models are generally simulated through time-domain numerical integration, and the Runge-Kutta (RK) method, as a numerical method, is adopted to solve the nonlinear ordinary differential equations (5). However, it is inconvenient and time-consuming for the researcher to calculate the convolution integral in Eq. (5) directly. For this reason, the state space model method is utilized to replace the convolution integral\(^\text{[79]}\).

For the convolution integral in Eq. (5), we denote

$$y(t) = \int_{t}^{\infty} h(t-\tau) \ddot{X}(\tau) d\tau. \quad (15)$$

The convolution integral term can be approximately represented by the state space model

$$\dot{U}(t) = AU(t) + BX(t), \quad y(t) = CU(t), \quad (16)$$

where \(U\) is the added state vector whose dimension is dependent on the identified dimension of the state space model. The next work is to determine the coefficient matrices \(A, B, \text{and } C\). The frequency-domain identification method based on the hydrodynamic data \(\mu\) and \(\lambda\) without using the infinite-frequency added mass proposed by Taghipour et al.\(^\text{[79]}\) is adopted.

The relatively high computational requirements of time-domain integration are not compatible with applications where a lot of simulations are needed. Spectral domain linearization has also been proposed to some nonlinear effects. The harmonic balance method (HBM), as a nonlinear frequency domain method, is widely used to solve the nonlinear dynamic equations\(^\text{[82]}\). By using the Ogilvie relationship, the governing equation (5) can be rewritten as a matrix form as follows:

$$m \ddot{X} + k_s X + F_P + F_N = F_e + F_R. \quad (17)$$

The radiation force \(F_R\) is related to the additional inertia force and the radiation damping force. For the linear motion \(\zeta\), it can be written as\(^\text{[83]}\)

$$F_R = - (\mu(\omega) \dot{\zeta} + \lambda(\omega) \ddot{\zeta}). \quad (18)$$

Now, we assume that the nonlinear motion can be approximated as a finite sum of harmonics as follows:

$$X(t) = a_0 + \sum_{j=1}^{N} x_j = a_0 + \sum_{j=1}^{N} a_j \cos(\omega_j t) + b_j \sin(\omega_j t), \quad (19)$$
where $a_0$ is the initial excursion which might appear in the bistable system, $\omega_j = j\omega$, $\omega$ is the fundamental frequency consistent with the excitation frequency, and $a_j$ and $b_j$ are the undetermined component amplitudes of the $j$th harmonic.

Subsequently, the radiation force can be considered as the sum of a series of wave forces radiated by the multi-frequency harmonic motion, and the radiation force for multi-frequency harmonics can be expressed as

$$ F_{R} = -\left( \sum_{j=1}^{N} (\mu(\omega_j)\ddot{x}_j + \lambda(\omega_j)\dot{x}_j) \right). $$

The damping force $F_P$ and the stiffness force $F_N$ are also functions of harmonics, and thus the undetermined component amplitudes $a_j$ and $b_j$ can be determined by the harmonic balance.

Substituting the displacement equation (19) and all force expressions into the nonlinear motion equation (17), then using the trigonometric functions to convert the equation into a linear combination of harmonics of different frequencies.

Ignoring harmonics higher than order $N$, we will obtain $N$ sinus terms of different frequencies and $N$ cosine terms of different frequencies on the left side of the equation and a wave force term on the right side of the equation. Based on harmonic balance, we obtain a solvable nonlinear algebraic equation set made of $2N$ equations with $2N$ unknown coefficients.

4 Dynamics, energy harvesting performance, and underlying mechanism

The introduction of an NSM into a WEC brings some unique nonlinear dynamic characteristics, which are quite different from those without these nonlinear mechanisms. In fact, it is a nonlinear dynamic characteristic for improving the energy harvesting performance of the WEC, although the underlying mechanism is very complicated and is still not fully understood.

4.1 Dynamics

It is well-known that for a linear system, a single-frequency input will lead to a dynamic response with the same frequency as the input. However, for a WEC with an NSM, even for a single frequency input, the dynamic response of the system is rather complicated, and the random wave excitation is not mentioned.

For a point absorber WEC with a nonlinear bistable mechanism, an example of the motion characteristics of the device under regular waves of different frequencies is shown in Fig. 13. These results were obtained by Zhang et al.\[33\]. It is obvious that the motion of a linear WEC is sinusoidal under regular wave excitation, bearing the features of an elliptical displacement-velocity trajectory and a single frequency component in the frequency domain. However, for a (conventional or adaptive) bistable WEC, the dynamic response has multi-frequency components and is not sinusoidal even for a monochromatic input.

The dynamic response characteristics of a bistable WEC are affected by both the frequency and the amplitude of the ocean waves. For example, as shown in Fig. 13, when the wave frequency changes from $\omega^* = 0.2$ to $\omega^* = 0.74$, the displacement response of an adaptive bistable WEC varies from multi-frequency periodic motion to random chaotic motion. For a given wave frequency (such as $\omega^* = 0.2$), increasing the wave amplitude from $A^* = 0.1$ to $A^* = 0.3$ alters the motion of the conventional bistable WEC from intra-well motion to periodic inter-well motion.

In addition, the dynamic response of the WEC with a nonlinear bistable mechanism is also affected by the configuration and physical parameters of the bistable system. As summarized in Section 2, the bistable mechanism can be realized using different physical setups such as an ideal spring, pneumatic cylinders, and magnet systems (see Fig. 6). Although all these setups can provide a bistable mechanism, the nonlinear forces have different expressions, which may
Fig. 13  Motion characteristics of floaters for different types of WECs at (a) $\omega^* = 0.2$ and $A^* = 0.1$; (b) $\omega^* = 0.2$ and $A^* = 0.3$; and (c) $\omega^* = 0.74$ and $A^* = 0.1$ for only adaptive bistable WECs. The figures were obtained from Figs. 9(a), 9(b), and (14) in Ref. [33]
Recent advances in wave energy converters based on nonlinear stiffness mechanisms

affect the dynamics of the bistable WEC. To the best of the authors’ knowledge, there is a lack of comprehensive comparisons between these different physical setups.

For the most frequently investigated spring-type bistable mechanism, two key parameters, i.e., the stiffness values of two oblique springs and their installation distance, determine the dynamic response of the bistable WEC. A non-dimensional parameter, $\gamma$, is defined as the ratio between the half of the installation distance between two oblique springs, $l$, and their original length, $l_0$, i.e., $\gamma = l/l_0$. This parameter determines the height of the potential barrier and magnitude of the negative stiffness caused by the bistable mechanism, as shown in Fig. 14. Overall, a smaller value of $\gamma$ corresponds to a higher potential barrier and a larger value of the negative stiffness near the unstable equilibrium position.

![Spring-type bistable mechanism](image)

**Fig. 14** Spring-type bistable mechanism, where $k_0$ is the stiffness of the spring, $2l$ is the installation distance between two oblique springs, $l_0$ is the original length of the spring, and $\gamma = l/l_0$.

As shown in Fig. 15, for a given wave amplitude and frequency, the dynamic response of the bistable WEC is significantly affected by the non-dimensional parameter, $\gamma$. As $\gamma$ increases from 0.01 to 0.6, the nonlinear feature of the motion becomes less significant, approximating a linear WEC.

![Graphs showing effects of $\gamma$](image)

**Fig. 15** Effects of $\gamma$ on (a) the equivalent stiffness and (b) the corresponding potential of the bistable mechanism. The figure was obtained from Ref. [63] (color online).

For the conventional bistable mechanism (see Fig. 14 or Table 3), the height of the potential barrier is unchanged during the motion, which may cause intra-well oscillation under small-wave excitation. As a result, in a sea region where the variation in the wave height is significant, a WEC with a conventional bistable mechanism may experience intra-well motion, as well as a poor energy harvesting performance, for a considerable amount of time. To tackle this problem, some modified bistable mechanisms have been proposed, such as multi-stable mechanisms,
adaptive bistable mechanisms, and improved bistable mechanisms (see Table 3). An illustration of the variation in the potential function with displacement is shown in Fig. 16.

As shown in Fig. 16(a), a multi-stable mechanism can be realized through proper physical design. Compared with the conventional bistable mechanism, the multi-stable mechanism has more stable equilibrium positions, which increases the complexity of the motion. A thorough investigation of the dynamics of the multi-stable mechanism can be found in Ref. [65], whereas there is no systematic comparison with its conventional bistable counterpart. The adaptive bistable mechanism, by introducing two auxiliary springs (see Table 3), can automatically adjust its potential barrier during motion (see Fig. 16(b)), which makes it more adaptable to a highly variable sea environment. A systematic comparison between the adaptive bistable mechanism and its conventional counterparts is presented in Ref. [33]. As an example, Figs. 16(a) and 16(b) clearly show that for a given wave amplitude, the adaptive bistable mechanism exhibits inter-well motion, whereas its conventional bistable counterpart exhibits intra-well oscillation. The improved bistable mechanism featured a lower potential barrier, which greatly improved the capture width ratio and frequency bandwidth at low excitations.

Apart from the bistable mechanism, an HNLS (or HNGS) has also been investigated by some researchers [28, 67]. Consider an OWC device with an HNGS mechanism as an example. As shown in Fig. 17, the hydrodynamic negative stiffness is realized by the volume of the chamber between the red dashed line and the inner wall of the floater. The hydrostatic restoring force,
Recent advances in wave energy converters based on nonlinear stiffness mechanisms

Fig. 17 Potential functions for different types of modified bistable mechanisms: (a) multi-stable mechanism (refer to Fig. 2(a) in Ref. [65]); (b) adaptive bistable mechanism (refer to Fig. 3(a) in Ref. [33]); and (c) improved bistable mechanism (IB-WEC; refer to Fig. 7(a) in Ref. [54]) (color online)

\[ F_{HS} = -\rho g S_F x_3 + \rho g (S_{HNGS} - S_{OWC})(x_3 - x_9), \]  \hspace{1cm} (21)

where \( S_F = \pi (R_F^2 - R_{OWC}^2) \), \( S_{HNGS} = \pi R_{HNGS}^2 \), and \( S_{OWC} = \pi R_{OWC}^2 \).

The first term in Eq. (27) represents the hydrostatic restoring force for the OWC with no HNGS mechanism, i.e., corresponding to a positive stiffness. The second term exists because of the expansion of the inner chamber at the floater region, i.e., the variation of the radius from \( R_{OWC} \) to \( R_{HNGS} \). This second term provides a negative stiffness, the magnitude of which can be altered by changing the cross section of the inner chamber, i.e., \( S_{HNGS} \). A comparison of the relative heave motion amplitudes of the OWC with and without the HNGS mechanism is shown in Figs. 18 and 19. It can be seen that the OWC with an HNGS mechanism (where \( \alpha < 1 \)) has a larger amplitude of relative motion than its counterpart without an HNGS mechanism (where \( \alpha = 1 \)). It can also be found that there is an optimum value for the hydrodynamic negative stiffness, for example, \( \alpha = 0.8 \) in Fig. 19.

Fig. 18 Illustration of the OWC with the HNGS mechanism, where \( x_3 \) and \( x_9 \) represent the heaving displacements of the floater and inner water columns, respectively. The figure was obtained from Ref. [28] (color online)

Fig. 19 Heave motion amplitude of the OWC with the HNGS mechanism, where \( \alpha = 1 - (S_{HNGS} - S_{OWC})/S_F \). The figure was obtained from Ref. [28] (color online)
The VI mechanism is another type of NSM (see Fig. 12). For this type of mechanism, the dynamics are related to the displacement of the inner mass relative to the outer mass. As shown in Fig. 12, the interaction force $f_{\text{int}}$ is given by

$$f_{\text{int}} = \begin{cases} 
  k_1 z_r + k_2 (z_r - G) + c \dot{z}_r, & z_r \geq G, \\
  k_1 z_r + c \dot{z}_r, & G > z_r > -G, \\
  k_1 z_r + k_2 (z_r + G) + c \dot{z}_r, & z_r \leq -G,
\end{cases} \tag{22}$$

where $z_r$ and $\dot{z}_r$ are the relative displacement and velocity between the floater and the inner mass, respectively. The definitions of the other parameters can be found in Fig. 12.

Figure 20 shows an example of the dynamic response of a WEC with a VI mechanism. It can be seen that the existence of the VI mechanism shifts the peak of the relative RAO to a higher frequency region and broadens the bandwidth of the dynamic response (although the peak value is lowered as a result of the physical constraints).

**Fig. 20** Some results for the WEC with the VI mechanism: (a) buoy RAO, (b) mass RAO, (c) relative RAO, (d) average power, (e) capture width ratio, and (f) peak-to-average power ratio, where $H = 0.8$ m, $M_m = 2000$ kg, $c = 1100$ N·s·m$^{-1}$, $G = 0.8$ m, $k_1 = 10,000$ N·m$^{-1}$, and $k_2 = 250,000$ N·m$^{-1}$. For the case of no impact, $k_2 = 0$ N·m$^{-1}$ was used. The physical constraint was computed by $h/H$, and the impact boundary was computed by $2G/H$. The figure was obtained from Ref. [70] (color online)

### 4.2 Energy-harvesting performance

The ultimate purpose of adopting an NSM in a WEC is to enhance the energy-harvesting performance. Because a WEC with a nonlinear stiffness mechanism is a multi-parameter nonlinear system, its energy harvesting performance can be affected by different wave and system parameters such as the wave amplitude, the wave period, the PTO damping, and the parameters of the nonlinear stiffness system. Therefore, researchers need to choose a near-optimal set of parameters to optimize the energy harvesting of nonlinear WECs.

For a WEC with a conventional bistable mechanism realized by springs, pneumatic cylinders, or magnets,[27,34–38,40–45,48,50–53,55–64], the overall trend is that the introduction of a bistable mechanism increases the power captured by the WEC if inter-well motion occurs. The maximum power captured by the bistable WEC increases with a decrease in the non-dimensional parameter, and the corresponding frequency is also shifted to a low-frequency region. The energy harvesting performance of the conventional bistable WEC is also affected by the wave
Recent advances in wave energy converters based on nonlinear stiffness mechanisms

Recent advances in wave energy converters based on nonlinear stiffness mechanisms

frequency and amplitude. In general, for a given set of bistable parameters, there exists a desirable wave amplitude region where the energy-harvesting performance of a bistable WEC is good. However, the energy harvesting performance may be worse if the wave amplitude is not large enough to ensure the inter-well oscillation. In addition, the improvement in the energy harvesting by the bistable mechanism is not significant for a fairly large wave amplitude.

As mentioned earlier, several concepts for modified bistable mechanisms have been proposed to further improve the energy-harvesting performance of a WEC. It was shown in Ref. [65] that a nonlinear multi-stable mechanism broadens the frequency bandwidth of a WEC and the bandwidth of the PTO damping coefficient. The improved bistable mechanism [54] enables the WEC to achieve higher efficiency than its linear and conventional bistable counterparts at low frequencies, and enhances its robustness against the PTO damping detuning and sea state changes, as shown in Fig. 21. One of the most important features for an adaptive bistable mechanism [33] is that a WEC with such a nonlinear mechanism can exhibit a good energy

![Fig. 21](image_url)  
Capture width ratios for three different significant wave heights $H_s^*$ in the wave frequency $\omega_p^*$ and PTO damping $C^*$ space domain; LN-WEC: linear WEC; CB-WEC: conventional bistable WEC; and IB-WEC: improved bistable WEC. The figures were obtained from Fig. 9 in Ref. [54] (color online)
harvesting performance under relatively low-energy sea conditions, improving its adaptability in a sea state with high variations. An example of a comparison of linear, conventional bistable, and adaptive bistable WECs is shown in Fig. 22. It can be seen that for proper system parameters, the adaptive bistable WEC has a good energy harvesting performance over the entire frequency range considered, compared with its linear and conventional bistable counterparts.

![Fig. 22](image)

**Fig. 22** Power capture width ratios of three types of point absorber WECs: linear WEC, conventional bistable WEC, and adaptive bistable WEC. The figure was obtained from Fig. 2 in Ref. [39] (color online)

A comparison of the power values captured by WECs with and without the HNGS mechanism is shown in Fig. 23. It can be seen that by introducing the HNGS mechanism, the power captured by the WEC is almost doubled, demonstrating the advantages of the HNGS mechanism.

![Fig. 23](image)

**Fig. 23** Power captured by the WEC for different values of $H_s$ and $T_p$ (a) without and (b) with the HNGS mechanism. The figures were obtained from Ref. [28] (color online)

An illustration of the energy-harvesting performance of a WEC with a VI mechanism is shown in Fig. 24. Figures 24(b) (no impact occurs) and 24(c) (impact occurs) correspond to the same wave frequency but different initial conditions. It can be seen that the occurrence of impact significantly increases the power captured by the WEC from 1 kW to 8.5 kW.

### 4.3 Underlying mechanism related to nonlinear stiffness

Understanding the underlying mechanism related to the effect of the nonlinear stiffness on the energy harvesting performance of a WEC is the basis for the proper design of the system parameters. Most of the previous studies were conducted using the following steps: 1) conceptually design the NSM, 2) establish the motion equation of a WEC with an NSM, 3) numerically solve the equation of motion and calculate the power, and 4) explore how the energy
Recent advances in wave energy converters based on nonlinear stiffness mechanisms

Fig. 24  Time traces of the buoy, mass and relative displacement, instantaneous power, and average power with (a) $\omega = 1$ rad·s$^{-1}$, (b) and (c) $\omega = 2.2$ rad·s$^{-1}$, and (d) $\omega = 3$ rad·s$^{-1}$. The other parameters are $H = 0.8$ m, $M_m = 2000$ kg, $c = 1100$ N·s·m$^{-1}$, $G = 0.8$ m, $k_1 = 10000$ N·m$^{-1}$, and $k_2 = 250000$ N·m$^{-1}$. (b) and (c) illustrate the multi-stability phenomena of the VIWEC system with the initial conditions of $\{z_{b0}, \dot{z}_{b0}, z_{m0}, \dot{z}_{m0}\}' = [0, 0, 0, 0]'$ and $\{z_{b0}, \dot{z}_{b0}, z_{m0}, \dot{z}_{m0}\}' = [0, 0, 0, 3]'$, respectively (color online)

harvesting performance is affected by different wave and system parameters. A depth of the exploration of the underlying mechanism leading to the improvement of the energy harvesting performance is still very limited.

One such attempt was made by Zhang et al.\[39\], who explored the mechanism related to broadband energy harvesting using an adaptive bistable WEC. The underlying mechanism can be explained in two ways. As shown in Fig. 25, the first explanation is that for a given wave amplitude, the broadband energy harvesting is due to the fact that the adaptive bistable mechanism introduces both negative (at low wave frequencies) and positive (at high wave frequencies) stiffness to the device. The second explanation is that it is due to the fact that an inherent “phase control” feature is observed for the adaptive bistable mechanism, especially at a low wave frequency region, as shown in Fig. 26. As well-known, in the optimum condition for energy harvesting, the phase of the velocity matches that of the wave excitation force. It can be observed from Fig. 26 that the phase of the float’s velocity is closer to that of the wave excitation force for an adaptive bistable WEC than for its linear counterparts.

Fig. 25  Power capture width ratios for the linear point absorber WEC with different values of the additional stiffness $K^*_a$. The figure was obtained from Fig. 8 in Ref. [39] (color online)
Fig. 26  Time series of the wave excitation force and velocity of floaters for both linear and adaptive bistable WECs. It is noted that the amplitude of the wave excitation force is scaled down or up (i.e., not the actual amplitude) for the convenience of comparison with the velocity. The figures were obtained from Fig. 9 in Ref. [39] (color online)

The mechanism revealed above is only for the WECs with one DOF. For the multi-DOF WECs with nonlinear stiffness, its mechanism was also revealed by Zhang et al. [27] using the potential energy and Lissajous trajectories (as shown in Fig. 27). In comparison with Fig. 27(a) and Fig. 27(b) for linear and nonlinear raft type WECs, the potential energy well changes from circle shape to ellipsoid surface with introducing the NSM. The major axis of the projection surface with ellipse shape of the potential energy well is close to the modal of the relative pitch motion of the two-raft WEC. Because of the flattening potential energy well, the trajectory of the nonlinear WEC is flatter than that of the linear WEC which indicates that the relative pitch motions are close to anti-phase with the explanation for the Lissajous trajectory. The amplitudes of the pitch motions are also larger than that of the linear system. The larger amplitudes and approximate anti-phase of the pitch motions are the reasons for the efficient enhancement of nonlinear raft-type WECs.

Fig. 27  Potential energy and diagrams of the Lissajous trajectories in the parametric space ($\beta_1, \beta_2$) for (a) linear system and (b) nonlinear system. The figures were obtained from Fig. 17 in Ref. [27] (color online)
5 Challenges and future work

Previous work has shown that an NSM, especially a bistable mechanism, has great potential for improving the energy harvesting performance of a WEC, which may be of great significance for speeding up the development and commercialization of wave energy devices. However, there are still some challenges that need to be addressed when it comes to the application of nonlinear stiffness mechanisms in wave energy devices in real engineering scenarios.

5.1 Nonlinear hydrodynamic model

A high-fidelity and computationally efficient numerical model is necessary to explore the dynamics and optimum system parameters of a WEC with nonlinear stiffness. Most of the current work is performed under the assumption of linear hydrodynamics, which ignores wave nonlinearities (in an intermediate sea state) and the nonlinearities caused by fluid viscosities and the relatively large amplitude of the device motion. This failure to consider the nonlinear hydrodynamics may lead to an overestimation of the power captured by the device.

Although computational fluid dynamics can, in principle, consider all the nonlinear hydrodynamics, it poses a great challenge in terms of computational resources. As a result, a nonlinear potential flow model with different complexities may be used (see Refs. [84]–[87] as examples of WECs without an NSM). It is desirable to explore how different nonlinear hydrodynamics affect the dynamics and energy harvesting performance of a wave energy device with an NSM.

5.2 Experimental work

Experimental tests are important for exploring the dynamics of wave energy devices with NSMs, as well as for validating numerical models. Very little experimental work has been done on wave energy devices with NSMs, which makes the benefits of using nonlinear stiffness in wave energy harvesting slightly less convincing. In addition, some conceptually feasible NSMs may not be easy to implement in experimental testing. For example, the side offset of two oblique springs (used by many researchers to realize the bistable mechanism) is always a problem.

Experimental testing can benefit from the exploitation of nonlinear stiffness in several aspects. First, the experimental results naturally contain all the nonlinear hydrodynamics and serve as benchmark data for the validation of the numerical model. Second, when performing an experimental test, the physical realization of the NSM must be considered. Third, it is necessary to estimate the possible mechanical friction of the NSM.

5.3 Integration of nonlinear stiffness in wave-to-wire model

An emerging trend in the development of numerical models for WECs is the wave-to-wire model, which models the entire chain of energy conversion from the hydrodynamic interaction between the ocean waves and the WEC to the electricity feed into the grid. Integrating the NSM into a wave-to-wire model has several benefits. First, most previous studies considered the PTO using a constant damping coefficient, which significantly simplified the real PTO dynamics. The wave-to-wire model helps to explore how real PTO dynamics may alter the dynamic behavior, as well as the energy harvesting performance of the NSM. Second, the wave-to-wire model makes it possible to determine whether the introduction of nonlinear stiffness has additional effects when feeding electricity into the grid. Third, the wave-to-wire model with an NSM serves as a powerful tool for evaluating the effectiveness and applicability of nonlinear stiffness in WECs.

5.4 Novel conceptual design

Apart from the conventional bistable system (whose potential barrier is unchanged), there are some modified bistable mechanisms such as the adaptive bistable mechanism, which can further improve the energy harvesting performance of a nonlinear WEC. To accelerate the employment of wave energy devices with NSMs, it is very important to keep developing novel concepts for both improving the energy harvesting performance and enhancing the robustness.

Most of the existing relative studies focus on developing an NSM for a point absorber WEC[40–42,45,49,54,56,59,61–62], whereas little attention has been paid to other types[27,50]. It is
also desirable to propose some novel concepts for other types of WECs such as hinged multi-module WEC\cite{35}, surging WEC, and oscillating water column WEC. In addition to the above review for NSMs used in independent wave energy conversion systems, some novel concepts for hybrid WEC systems are also worth paying attention to, such as hybrid WEC-breakwater\cite{88} and hybrid WEC-very large floating structure (WEC-VLFS).

5.5 Robustness in real sea state

As a strongly nonlinear system, the energy harvesting performance of a wave energy device with an NSM depends on the wave and system parameters. Although significant progress has been made under regular wave conditions, the dynamic behavior of a nonlinear WEC under irregular wave conditions may exhibit different features. For example, compared with regular waves (i.e., a monochromatic input), irregular wave conditions have the features of randomness, multi-frequency input, and different energy distributions. How these unique features of irregular waves affect the dynamic characteristics and energy-harvesting performance remains unclear.

Another concern is the reliability of the components used to realize the NSM over a long period of time. Tackling the problems of fatigue, mechanical breakdown, and erosion is an unavoidable challenge.

6 Concluding remarks

An overview of the application of the NSM in a WEC is provided. Typically, three different types of NSMs (i.e., the bistable, hydrodynamic negative stiffness, and VI mechanisms) are investigated in terms of conceptual design, mathematical modeling, dynamics, energy harvesting performance, and the underlying mechanism. Overall, an NSM shows great potential for improving the efficiency of WECs, which may be a possible solution for promoting their commercialization.

Because of the inherent complexity of a nonlinear stiffness mechanism and the multi-parameter features of a WEC, there are still many problems that remain unclear and deserve further investigation. Some potential topics for future studies are also briefly summarized, such as nonlinear hydrodynamic models, experimental validations, and wave-to-wired models.

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