A meson-exchange $\pi N$ model up to energies $\sqrt{s} \leq 2.0$ GeV

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A meson-exchange $\pi N$ model, previously constructed using three-dimensional reduction scheme of the Bethe-Salpeter equation for a model Lagrangian involving $\pi$, $\eta$, $N$, $\Delta$, $\rho$, and $\sigma$ fields, is extended to energies up to 2 GeV by including the $\eta N$ channel and all the four stars $\pi N$ resonances up to the $F$-waves. The effects of other $2\pi$ channels are taken into account phenomenologically. The extended model gives an excellent fit to both $\pi N$ phase shifts and inelasticity parameters in all channels up to the $F$-waves. However, a few of the extracted resonance parameters differ considerably from the PDG values.

1. INTRODUCTION

Pion-nucleon scattering is one of the main sources of information for the baryon spectrum. In addition, it also plays a fundamental role in the description of nuclear dynamics for which the $\pi N$ off-shell amplitude serves as the basic input to most of the existing nuclear calculations at intermediate energies. Knowledge about the off-shell $\pi N$ amplitude is also essential in interpreting the experiments performed at the intermediate-energy electron accelerators in order to unravel the internal structure of these hadrons \cite{23}. It is hence important to have a sound theoretical description of the $\pi N$ interaction.

It is commonly accepted that Quantum Chromodynamics (QCD) is the fundamental theory of the strong interaction. However, due to the confinement problem, it is still practically impossible to derive the $\pi N$ interaction directly from QCD. On the other hand, models based on meson-exchange (MEX) pictures have been very successful in describing the $NN$ scattering. Over the past decade, MEX approach has also been applied by several groups \cite{14,15,16,17,18,19} to construct models for $\pi N$ scattering.

In previous works we have constructed several MEX $\pi N$ models within the three-dimensional reduction scheme of the Bethe-Salpeter equation \cite{20} and investigated their sensitivity with respect to various three-dimensional reduction schemes. The model Lagrangian included only $\pi$, $N$, $\Delta$, $\rho$, and $\sigma$ fields. It was found that all the resulting meson-exchange models can yield similarly good descriptions of $\pi N$ scattering data up to 400 MeV. The model obtained with the Cooper-Jennings reduction scheme \cite{10} was recently

\textsuperscript{*}This work is supported in part by the National Science Council of ROC under grant No. NSC94-2112-M002-025.
extended up to a c.m. energy of 2 GeV in the $S_{11}$ channel by including the $\eta N$ channel and a set of higher $S_{11}$ resonances $[11]$. The effects of the other $\pi\pi N$ channels like the $\sigma N$, $\rho N$, and $\pi\Delta$, instead of including them directly in the coupled-channels calculation, were taken into account by introducing a phenomenological term in the resonance propagators. An excellent fit to the t-matrix in both $\pi N$ and $\eta N$ channels was obtained. Here we further extend the model to other higher partial waves up to the $F-$waves.

2. MESON-EXCHANGE MODEL FOR $\pi N$ SCATTERING

The MEX $\pi N$ model we previously constructed was obtained by using a three-dimensional reduction scheme of the Bethe-Salpeter equation for a model Lagrangian involving $\pi, N, \Delta, \rho,$ and $\sigma$ fields. Details can be found in Ref. $[9]$. As the energy increases, two-pion channels like $\sigma N, \eta N, \pi\Delta, \rho N$ as well as a non-resonant continuum of $\pi\pi N$ states become increasingly important, and at the same time more and more nucleon resonances appear as intermediate states. In Ref. $[11]$ the $\pi N$ model constructed in $[9]$ was extended for the $S_{11}$ partial wave by explicitly coupling the $\pi$, $\eta$ and $\pi\pi$ channels and including the couplings with higher baryon resonances. For example, in the case when there is only one resonance $R$ contributing, the Hilbert space was enlarged to include a bare $S_{11}$ resonance $R$ which acquires a width by its coupling with the $\pi N$ and $\eta N$ channels through the Lagrangian

$$\mathcal{L}_I = ig^{(0)}_{\pi NR} \bar{R} \tau N \cdot \pi + ig^{(0)}_{\eta NR} \bar{R} N \eta + h.c.,$$

(1)

where $N, R, \pi,$ and $\eta$ denote the field operators for the nucleon, bare $R$, pion and eta meson, respectively. Then the full $t$-matrix can be written as a system of coupled equations,

$$t_{ij}(E) = v_{ij}(E) + \sum_k v_{ik}(E) g_k(E) t_{kj}(E),$$

(2)

where $i$ and $j$ denote the $\pi$, or $\eta$ channel and $E = W$ is the total center mass energy.

In general, the potential $v_{ij}$ is a sum of non-resonant ($v_{ij}^B$) and bare resonance ($v_{ij}^R$) terms,

$$v_{ij}(E) = v_{ij}^B(E) + v_{ij}^R(E).$$

(3)

The non-resonant term $v_{ij}^B$ for the $\pi N$ elastic channel contains contributions from the s- and u-channel, pseudovector Born terms and t-channel contributions with $\omega, \rho,$ and $\sigma$ exchange. The parameters in $v_{ij}^B$ are fixed from the analysis of the pion scattering phase shifts for the $S-$ and $P-$waves at low energies ($W < 1300$ MeV) $[9]$. In channels involving $\eta$, $v_{ij}^B$ is taken to be zero since the $\eta NN$ coupling is very small.

The bare resonance contribution

$$v_{ij}^R(E) = \frac{h^{(0)}_{ijR} \bar{h}^{(0)}_{ijR}}{E - M^{(0)}_R},$$

(4)

where $h^{(0)}_{ijR}$ and $M^{(0)}_R$ denote the bare vertex operator for $\pi/\eta + N \rightarrow R$ and bare mass of $R$, respectively, arises from the excitation and de-excitation of the resonance $R$. The matrix elements of the potential $v_{ij}^R(E)$ can be symbolically expressed in the form

$$v_{ij}^R(q, q'; E) = \frac{f_i(\tilde{\Lambda}_i, q; E) g_i^{(0)}(0) f_j(\tilde{\Lambda}_j, q'; E)}{E - M^{(0)}_R + \frac{i}{2} \Gamma^{(0)}_R(E)},$$

(5)
where \( q \) and \( q' \) are the pion (or eta) momenta in the initial and final states, and \( g^{(0)}_{i/j} \) is the resonance vertex couplings. As in [9], we associate with each external line of the particle \( \alpha \) in a Feynman diagram a covariant form factor \( F_\alpha = [n_\alpha \Lambda_\alpha^4/(p_\alpha^2 - m_\alpha^2)^2]^{n_\alpha} \), where \( p_\alpha, m_\alpha, \) and \( \Lambda_\alpha \) are the four-momentum, mass, and cut-off parameter of particle \( \alpha \), respectively, and \( n_c = 10 \). As a result, \( f_i \) depends on the product of three cut-off parameters.

In Eq. (5) we have included a phenomenological term \( \Gamma_{\pi N}^{(0)}(E) \) in the resonance propagator to account for the \( \pi\pi N \) decay channel. Therefore, our ”bare” resonance propagator already contains some renormalization or ”dressing” effects due to the coupling with the \( \pi\pi N \) channel. With this prescription we assume that any further non-resonant coupling mechanisms with the \( \pi\pi N \) channel are small. The form of \( \Gamma_{\pi N}^{(0)}(E) \) can be found in [11] and is characterized by two parameters, a cut-off \( \Lambda_R \) and the \( 2\pi \) decay width at the resonance \( \Gamma_{\pi N}^{(0)} \). Consequently, one isolated resonance will contain five free parameters, \( M_R^{(0)}, \Gamma_{\pi N}^{(0)}(\Lambda_R, g_0^{(0)} \) and \( g_j^{(0)} \). The generalization of the coupled channels model to the case of \( N \) resonances with the same quantum numbers is then given by

\[
v_{ij}^{R}(q, q'; E) = \sum_{n=1}^{N} v_{ij}^{Rn}(q, q'; E),
\]

with free parameters for the bare masses, \( 2\pi \) widths, coupling constants, and cut-off parameters for each resonance.

After solving the coupled channel equations, the next task is the extraction of the physical (or ”dressed”) masses, partial widths, and branching ratios of the resonances. It is well-known this procedure is definitely model dependent, because the background and the resonance contributions can not be separated in a unique way. In this work, we employ the procedure used in Ref. [3] where, in the case of pion-nucleon elastic scattering with only one resonance contributing, the full t-matrix is written as follows,

\[
t_{\pi N}(E) = t_{\pi N}^B(E) + t_{\pi N}^R(E),
\]

where

\[
t_{\pi N}^B(E) = v_{\pi N}^B + v_{\pi N}^B g_0(E) t_{\pi N}(E), \quad t_{\pi N}^R(E) = v_{\pi N}^R + v_{\pi N}^R g_0(E) t_{\pi N}(E).
\]

The ”background” \( t_{\pi N}^B \) includes contributions not only from the background rescattering but also from intermediate resonance excitation. This is compensated by the fact that the resonance contribution \( t_{\pi N}^R \) now contains only the terms that start with the bare resonance vertex. In terms of self-energy and vertex functions, one obtains the result [12]

\[
t_{\pi N}^R(E) = \frac{\tilde{h}_{\pi R}^R(E) \tilde{h}_{\pi R}^{(0)}(E)}{E - M_R^{(0)}(E) - \Sigma_R(E)},
\]

where

\[
\tilde{h}_{\pi R}(E) = (1 + g_0(E) t_{\pi N}^R(E)) h_{\pi R}^{(0)} \quad \text{and} \quad \Sigma_R(E) = h_{\pi R}^{(0)} g_0 \tilde{h}_{\pi R}(E).
\]

\( \tilde{h}_{\pi R}(E) \) describes the dressed vertex of \( R \rightarrow \pi N \) [3]. \( \Sigma_{\pi R}^R \) is the self-energy of the dressed \( R \) arising from one-pion intermediate states and \( \Sigma_R(E) = \Sigma_{\pi R}^R + \Sigma_{R}^R \) with \( \Sigma_{R}^R(E) = 2i\Gamma_{\pi N}^{R} \).
The information about the physical mass and the total width of the resonance \( R \) are contained in the dressed resonance propagator given in Eq. (9). The complex self-energy \( \Sigma_R(E) \) leads to a shift from the real "bare" mass to a complex and energy-dependent value. We characterize the resonance by energy-independent parameter that is obtained by solving the equation

\[ E - M_R^{(0)} - Re \Sigma_R(E) = 0. \] (11)

The solution of this equation, \( E = M_R \), corresponds to the energy at which the dressed propagator in Eq. (9) is purely imaginary. The "physical" or "dressed" mass and the width of the resonance is then defined by,

\[ M_R = M_R^{(0)} + Re \Sigma_R(M_R), \quad \Gamma_R = -2 Im \Sigma_R(M_R). \] (12)

When there are \( N \) resonances contributing in the same channel, Eq. (8) can be generalized to take the form of

\[ \pi N(E) = B \pi N(E) + N \sum_{i=1}^{N} t_{\pi N}^{R_i} \pi N(E). \] (13)

The contribution from each resonance \( R_i \) can be expressed in terms of the bare \( h_{\pi R_i}^{(0)} \) and dressed \( h_{\pi R_i} \) vertex operators as well as the resonance self energy derived from one-pion \( \Sigma_{1\pi}^{R_i}(E) \) and two-pion \( \Sigma_{2\pi}^{R_i}(E) \) channels, that is

\[ t_{\pi N}^{R_i}(E) = \frac{h_{\pi R_i}(E)h_{\pi R_i}^{(0)}}{E - M_{R_i}^{(0)} - \Sigma_{1\pi}^{R_i}(E) - \Sigma_{2\pi}^{R_i}(E)}, \] (14)

where \( M_{R_i}^{(0)} \) is the bare mass of \( R_i \). The vertices for resonance excitation are obtained, in analogous to Eqs. (8-10), from the following two equations:

\[ \bar{h}_{\pi R_i}(E) = (1 + g_0(E) t_{\pi N}^{B_i}(E))h_{\pi R_i}^{(0)} \] (15)

\[ t_{\pi N}^{B_i}(E) = v_i^B(E) + v_i^B(E) g_0(E) t_{\pi N}^{B_i}(E), \] (16)

where \( v_i^B(E) = v_{\pi N}^B + \sum_{j \neq i}^{N} v_{\pi N}^{R_j}(E) \). The one-pion self-energies corresponding to Eq. (10), is \( \Sigma_{1\pi}^{R_i} = h_{\pi R_i}^{(0)} g_0 \bar{h}_{\pi R_i} \). We wish to emphasize that in the formulation we present above, namely in Eqs. (13-16), the \( N \) resonances are treated in a completely symmetrical way and the self-energy and the dressing of any resonance receive contributions from all other resonances.

3. RESULTS AND DISCUSSIONS

In Fig. 1, we compare our results for the real and imaginary parts of the \( t \)-matrix in some selected channels in \( S-, P-, D- \) and \( F- \) waves up to 2 GeV c.m. energy with the experimental data as obtained in the SAID partial wave analysis \[13\]. One sees that we are able to describe the data very well.

The bare and physical resonance masses, and widths extracted according to Eq. (12) are presented in Table 1. Even though our model describes the data for \( t \)-matrix well as
Figure 1. The best fit of the real and imaginary parts of the $\pi N$ scattering t-matrix using dynamical MEX model (solid curves). The dashed curves give the background contribution. Experimental data are the results of the partial wave analysis from Ref. [13].

seen in Fig. 1, the resonance properties we extract as given in Table 1 do show several differences when compared with PDG values [14]. The most notable ones are that (1). we require two resonances not listed in PDG: $S_{11}(1878)$ and $D_{13}(1946)$; (2). the masses and widths we obtain for the 2nd and 3rd resonances in $S_{31}$ and $P_{11}$ deviates substantially from the PDG values. The PDG values for $(M_{R}, \Gamma_{R})$ for these resonances are $S_{31}$: $(1900 \pm 50, 190 \pm 50)$, $(2150 \pm 100, 200 \pm 100)$ and $P_{11}$: $(1710 \pm 30, 180 \pm 100)$, $(2125 \pm 75, 260 \pm 100)$; (3). the width we obtain for $F_{15}(2000)$ is only 58 MeV which is much smaller that the PDG value of $490 \pm 310$ MeV.

4. SUMMARY

We have extended our previously constructed meson-exchange $\pi N$ model to energies up to 2 GeV by including the $\eta N$ channel and all the four stars $\pi N$ resonances up to the $F$–waves. The effects of other $2\pi$ channels are taken into account phenomenologically. We have treated, in any given channel, all the contributing resonances in a completely symmetrical manner such that every resonance is dressed by the presence of all other resonances. The extended model gives an excellent fit to both $\pi N$ phase shifts and inelasticity parameters in all channels up to the $F$–waves. However, a few of the resonance parameters differ substantially from the PDG values. This $\pi N$ model will be applied to evaluate the contribution of the pion cloud to the photopion reactions up to 2 GeV c.m. energy as was done in Ref. [3] so that the photoexcitation strengths of all resonances below 2 GeV can be reliably extracted.
Table 1
Bare $M_R^{(0)}$ and physical $M_R$ resonance masses and total width in MeV.

| $N^*$ | $M_R^{(0)}$ | $M_R$ | $\Gamma$ | $M_R^{(0)}$ | $M_R$ | $\Gamma$ | $M_R^{(0)}$ | $M_R$ | $\Gamma$ |
|-------|-------------|-------|---------|-------------|-------|---------|-------------|-------|---------|
| $S_{11}$ | 1559 | 1520 | 130 | 1727 | 1678 | 200 | 1803 | 1878 | 508 |
| $S_{31}$ | 1654 | 1616 | 160 | 1796 | 1770 | 430 | 2118 | 1942 | 416 |
| $P_{11}$ | 1612 | 1418 | 436 | 1798 | 1803 | 508 | 2196 | 2247 | 1020 |
| $P_{33}$ | 1425 | 1233 | 132 | 1575 | 1562 | 216 | 1856 | 1827 | 834 |
| $D_{13}$ | 1590 | 1520 | 94 | 1753 | 1747 | 156 | 1972 | 1946 | 494 |
| $D_{33}$ | 1690 | 1650 | 260 | 2100 | 2092 | 310 | — | — | — |
| $D_{15}$ | 1710 | 1670 | 154 | 2300 | 2286 | 532 | — | — | — |
| $F_{15}$ | 1748 | 1687 | 156 | 1928 | 1926 | 58 | — | — | — |
| $F_{37}$ | 1974 | 1916 | 338 | — | — | — | — | — | — |

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