From Byzantine Failures to Crash Failures in Message-Passing Systems: a BG Simulation-based approach

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Abstract

The BG-simulation is a powerful reduction algorithm designed for asynchronous read/write crash-prone systems. It allows a set of \((t + 1)\) asynchronous sequential processes to wait-free simulate (i.e., despite the crash of up to \(t\) of them) an arbitrary number \(n\) of processes under the assumption that at most \(t\) of them may crash. The BG simulation shows that, in read/write systems, the crucial parameter is not the number \(n\) of processes, but the upper bound \(t\) on the number of process crashes.

The paper extends the concept of BG simulation to asynchronous message-passing systems prone to Byzantine failures. Byzantine failures are the most general type of failure: a faulty process can exhibit any arbitrary behavior. Because of this, they are also the most difficult to analyze and to handle algorithmically. The main contribution of the paper is a signature-free reduction of Byzantine failures to crash failures. Assuming \(t < \min(n', n/3)\), the paper presents an algorithm that simulates a system of \(n'\) processes where up to \(t\) may crash, on top of a basic system of \(n\) processes where up to \(t\) may be Byzantine. While topological techniques have been used to relate the computability of Byzantine failure-prone systems to that of crash failure-prone ones, this simulation is the first, to our knowledge, that establishes this relation directly, in an algorithmic way.

In addition to extending the basic BG simulation to message-passing systems and failures more severe than process crashes, being modular and direct, this simulation provides us with a deeper insight in the nature and understanding of crash and Byzantine failures in the context of asynchronous message-passing systems. Moreover, it also allows crash-tolerant algorithms, designed for asynchronous read/write systems, to be executed on top of asynchronous message-passing systems prone to Byzantine failures.

Keywords: Asynchronous processes, BG simulation, Byzantine process, Distributed computability, Fault-tolerance, Message-passing system, Process crash, Read/write shared memory system, Reduction algorithm, \(t\)-Resilience, System model, Wait-freedom.
1 Introduction

What is the Borowsky-Gafni (BG) simulation and why is it important? Considering an asynchronous system where processes can crash, the \((n, k)\)-set agreement problem is a basic distributed decision task defined as follows \([11]\). Each of the \(n\) processes proposes a value, and every process that does not crash has to decide a value (termination), such that a decided value is a proposed value (validity) and at most \(k\) different values are decided (agreement). The consensus problem corresponds to the particular case \(k = 1\).

The \((n, k)\)-set agreement is fundamental because it captures the essence of fault-tolerant distributed computability issues. A central question related to asynchronous distributed computability is the following: “Can we use a solution to the \((n, k)\)-set agreement problem as a subroutine to solve the \((n', k')\)-set agreement problem, when at most \(t < \min\{n, n'\}\) processes may crash?” (“Is \((n', k')\)-set agreement reducible to \((n, k)\)-set agreement?”). The BG simulation (initially sketched in \([6]\) and then formalized in a journal version \([7]\), where, in addition, a formal definition of “reducibility” is given) answers this fundamental question. It states that the answer is “yes” if \(k' \geq k\) and “no” if \(k' < t \leq k\). As we can see, the answer “yes” does not depend on the number of processes.

To this end, the algorithm described in \([7]\) allows \((t + 1)\) processes to simulate a large number \(n'\) of asynchronous processes that communicate through read/write registers, and collectively solve a decision task, in the presence of at most \(t\) crashes. Each of the \((t + 1)\) simulator processes simulates all the \(n'\) processes. These \((t + 1)\) simulator processes cooperate through underlying objects that allow them to agree on a single output for each of the non-deterministic statements issued by every simulated process. (These underlying objects, called safe agreement objects, can be built of top of read/write atomic registers.)

Let BG(RW,C) denote the basic BG simulation algorithm \([7]\) (RW stands for “read/write communication”, and C stands for “crash failures”). BG(RW,C) is “symmetric” in the sense that each of the \(n'\) processes is simulated by every simulator, and the \((t + 1)\) simulators are “equal” with respect to each simulated process, namely, (1) every simulator fairly simulates all the processes, and (2) the crash of a simulator entails the crash of at most one simulated process. This symmetry allows BG(RW,C) to be suited to colorless tasks (i.e., distributed computing problems where the value decided by a process can be decided by any process \([17]\)). BG(RW,C) has then been extended to colored tasks (i.e., tasks such as renaming \([3]\), where a process cannot systematically borrow its output from another process). Extended BG simulation is addressed in \([14, 22]\). Algorithmic pedagogical presentations of the BG simulation can be found in \([18, 22]\). A topological view on distributed computability issues in Byzantine asynchronous message-passing systems has been recently presented in \([16, 28]\). A pedagogical topology-based presentation of the BG-simulation is given in chapter 7 of \([16]\).

What is learned from the BG simulation The important lesson learned from the BG simulation is that, in a failure-prone context, what is important is not the number of processes but the maximal number of possible failures and the actual number of values that are proposed to a decision task. An interesting consequence of the BG simulation (among several of its applications \([7]\)) is the proof that there is no \(t\)-resilient \((n, k)\)-set agreement algorithm for \(t \geq k\). This is obtained as follows. As (1) the BG simulation allows reducing the \((k + 1, k)\)-set agreement problem to the \((n, k)\)-set agreement problem in a system with up to \(k\) failures, and (2) the \((k + 1, k)\)-set agreement problem is known to be impossible in presence of \(k\) failures \([6, 19, 30]\), it follows that there is no \(k\)-resilient \((n, k)\)-set agreement algorithm.

Content of the paper: on the BG-simulation side As already indicated, the BG simulation has been explored in asynchronous systems where processes (1) communicate through atomic read/write registers \([25]\), and (2) may commit only crash failures. This paper extends it in two directions. The first is the communication model, namely, it considers that processes cooperate by sending and receiving messages via asynchronous reliable channels. The second dimension is related to the type of failures; more precisely, it considers two
types of failures: process crash failures, and the more severe process Byzantine failures. The paper presents the following contributions.

A first is an algorithm, denoted BG(MP,C), which simulates the execution of a colorless task running in an asynchronous message-passing system of \( n' \) processes, where up to \( t \) may crash, on top of an asynchronous message-passing system of \( n \) processes where up to \( t \) may crash. This simulation requires \( t < n/2 \) (which is a necessary and sufficient condition to simulate read/write registers in asynchronous message-passing systems of \( n \) processes [2]). While the number of simulated processes \( n' \) can be any integer, for the simulation to be non-trivial we consider that \( t < n' \).

A second contribution is an algorithm, denoted BG(MP,B), which simulates the execution of a colorless task running in an asynchronous message-passing system of \( n' \) processes, where up to \( t \) may crash, on top of an asynchronous message-passing system of \( n \) processes where up to \( t \) may be Byzantine [26]. This simulation requires \( t < n/3 \) (according to the task which is simulated, additional constraint on \( t \) may be needed, see [16]; see also Section 6). As in the case of BG(MP,C), and for the same reason, we consider that \( t < n' \). This algorithm has two noteworthy features: it is the first BG simulation algorithm that considers Byzantine failures, and it allows to run a crash-tolerant algorithm solving a colorless task on top of an asynchronous system prone to Byzantine failures. Both the algorithms BG(MP,C) and BG(MP,B) are genuine in the sense they do not rely on the simulation of an underlying shared memory.

While the full-information algorithm presented in [28] can be used to decide when there is a simulation between two models, the present paper is the first (to our knowledge) that allows the direct execution in the presence of Byzantine failures of any crash-tolerant algorithm that solves a colorless task. BG(MP,B) provides an algorithmic approach which complements the topology-based simulation framework of [28], and may also be of practical interest. It has the interesting property that the simulation of a message only requires a polynomial number of messages in the base system, and the increase in size of these messages, when compared to the size of the simulated message, is also polynomial. Additionally, differently from early works on Byzantine failures like [15], it does not use any cryptography-based mechanism.

**Content of the paper: on the safe agreement objects side** The core of the previous algorithms lies in new underlying safe agreement objects, which allow the \( n \) simulators to agree on the next operation executed by each of the \( n' \) simulated processes. Such a safe agreement object ensures that all the simulators produce the very same simulation. At the operational level, a safe agreement object provides processes with two operations, denoted \( \text{propose}() \) and \( \text{decide}() \), which are invoked in this order by each correct process. The termination property associated with a safe agreement object \( SA \) is the following: if no simulator commits a failure while executing \( SA\text{.propose}() \), then any invocation of \( SA\text{.decide}() \) by a non-faulty simulator terminates. Moreover, no two correct processes decide differently.

On the algorithmic side, a novelty of the paper lies in the algorithms implementing these new safe agreement objects. Differently from their read/write memory counterparts, they are not based on underlying snapshot objects [1]. They instead rely heavily on message communication patterns inspired from the reliable broadcast algorithms introduced in [8].

A last and noteworthy contribution of the paper lies in the second algorithm (which implements safe agreement in a Byzantine message-passing system). This object is the core of a simulation when one wants to execute asynchronous read/write crash-tolerant algorithms on top of asynchronous message-passing systems prone to Byzantine failures.

**Existing simulations considering Byzantine failures** Simulations of crash failures in a Byzantine system have been addressed in the context of synchronous systems [5, 29, 31]. The only articles we are aware of concerning such a simulation in asynchronous systems are [12, 16, 20]. As noticed in [4, 12] considers a restricted class of round-based deterministic algorithms. The simulation presented in [16] executes a full-information asynchronous crash-tolerant algorithm in an asynchronous Byzantine failure-prone system. The
article [20] considers an agent/host model and focuses mainly on reliable broadcast.

**Roadmap**  The paper is composed of 6 sections. Section 2 presents both the crash-prone and the Byzantine asynchronous message-passing models, and the notion of a task. Section 3 presents the structure of the simulation algorithms. Section 4 presents the simulation algorithm BG(MP,C), while Section 5 presents the simulation algorithm BG(MP,B). Finally, Section 6 addresses the computability implications of the Byzantine-tolerant simulation and its underlying safe agreement object.

## 2 Computation Models and Tasks

### 2.1 Computation models

**Computing entities** The system is made up of a set \( \mathcal{P} \) of \( n \) sequential processes, denoted \( p_1, p_2, ..., p_n \). These processes are asynchronous in the sense that each process progresses at its own speed, which can be arbitrary and remains always unknown to the other processes.

During an execution, processes may deviate from their specification. In that case, the corresponding processes are said to be faulty. A process that does not deviate from its specification is correct (or non-faulty). The model parameter \( t \) denotes the maximal number of processes that can be faulty in a given execution. Two failure types are considered below.

**Communication model** The processes cooperate by sending and receiving messages through bi-directional channels. The communication network is a complete network, which means that each process \( p_i \) can directly send a message to any process \( p_j \) (including itself). Each channel is reliable (no loss, corruption, or creation of messages), not necessarily first-in/first-out, and asynchronous (while the transit time of each message is finite, there is no upper bound on message transit times).

The macro-operation “broadcast \( \text{TYPE}(m) \)”, where \( \text{TYPE} \) is a message type and \( m \) is its content, is a shortcut for the following statement: “send \( \text{TYPE}(m) \) to each process (including itself)”.

**The process crash failure model** In the crash failure model, a process may prematurely stop its execution. A process executes correctly its algorithm until it possibly crashes. Once crashed, a process remains crashed forever. It is assumed that at most \( t \) processes may crash. If there is no specific constraint on \( t \), the corresponding model is denoted \( \text{CAMP}_{n,t}[t < n] \). When it is assumed that at most \( t < n/2 \) processes may crash, the corresponding model is denoted \( \text{CAMP}_{n,t}[t < n/2] \).

**The Byzantine failure model** A Byzantine process is a process that behaves arbitrarily: it may crash, fail to send or receive messages, send arbitrary messages, start in an arbitrary state, perform arbitrary state transitions, etc. Hence, a Byzantine process, which is assumed to send the same message \( m \) to all the processes, can send a message \( m_1 \) to some processes, a different message \( m_2 \) to another subset of processes, and no message at all to the other processes. Moreover, Byzantine processes can collude to “pollute” the computation.

It is assumed that Byzantine processes cannot control the network, hence, when a process receives a message, it can unambiguously identify its sender. As previously, \( t \) denotes the upper bound on the number of processes that may commit Byzantine failures. If there is no constraint on \( t \), the corresponding model is denoted \( \text{BAMP}_{n,t}[t < n] \). When it is assumed that at most \( t < n/3 \) processes may be faulty, the corresponding model is denoted \( \text{BAMP}_{n,t}[t < n/3] \).
2.2 Decision tasks and algorithms solving a task

Decision tasks The problems we are interested in are called decision tasks (the reader interested in a more formal presentation of decision tasks can consult the literature, e.g., [7, 19]). In every run, each process proposes a value and the proposed values define an input vector $I$, where $I[j]$ is the value proposed by process $p_j$. Let $I$ denote the set of allowed input vectors. Each process has to decide a value. The decided values define an output vector $O$, such that $O[j]$ is the value decided by $p_j$. Let $O$ be the set of the output vectors.

A decision task is a binary relation $\Delta$ from $I$ into $O$. A task is colorless if, when a value $v$ is proposed by a process $p_j$ (i.e., $I[j] = v$), then $v$ can be proposed by any number of processes and, when a value $v'$ is decided by a process $p_j$ (i.e., $O[j] = v'$), then $v'$ can be decided by any number of processes. Consensus, and more generally $k$-set agreement, are colorless tasks. Otherwise the task is colored. Symmetry breaking and renaming are colored tasks [3, 10, 21].

Algorithm solving a task An algorithm solves a task in a $t$-resilient environment if, given any $I \in I$, (1) each correct process $p_j$ decides a value $o_j$, and (2) there is an output vector $O$ such that $(I, O) \in \Delta$ where $O$ is defined as follows. If $p_j$ decides $o_j$, then $O[j] = o_j$. If $p_j$ does not decide, $O[j]$ is set to any value $v'$ that preserves the relation $(I, O) \in \Delta$.

Considering a system of $n$ processes, a task is $t$-resiliently solvable if there is an algorithm that solves it in the presence of at most $t$ faulty processes. As an example, consensus is not 1-resiliently solvable in asynchronous crash-prone systems, be the communication medium a set of read/write registers [27], or a message-passing system [13]. Differently, renaming with $2n - 1$ new names is $(n - 1)$-resiliently solvable in asynchronous read/write crash-prone systems [9, 19], and is $t$-resiliently solvable in asynchronous crash-prone message-passing systems for $t < n/2$ [3].

3 Structure of the Simulation Algorithms

Aim Let $A'$ be an algorithm that solves a colorless decision task among $n'$ processes in the system model $\mathcal{CAMP}_{n',t}[t < n']$. The aim is to design an algorithm that simulates $A'$ in the system model $\mathcal{CAMP}_{n,t}[t < n/2]$ (resp., $\mathcal{BAMP}_{n,t}[t < n/3]$). As already indicated, the corresponding simulation algorithm is denoted $BG(MPC)$ in the first case, and $BG(MPB)$ in the second case.

Notation A simulated process is denoted $p_j$, where $1 \leq j \leq n'$. Similarly, a simulator process (“simulator” in short’) is denoted $q_i$, where $1 \leq i \leq n$. The set $\Pi$ denote the set of the simulator indexes, i.e., $\Pi = \{1, ..., n\}$.

The safe agreement objects, build in the simulation and used by the simulators, are identified with upper case letters, e.g., $SA$. The variables local to simulator $q_j$ is identified with lower case letters, and the resulting identifiers are subscripted with $j$.

Behavior of a simulator $q_i$ Each simulator is given the code of all the simulated processes $p_1$, ..., $p_{n'}$. It manages $n'$ threads, one associated with each simulated process, and executes them in a fair way.

The code of a simulated process $p_j$ contains local statements, send statements, and receive statements. It is assumed that the behavior of a simulated process $p_j$ is deterministic in the sense it is entirely defined from its local input (as defined by the task instance), and the order in which $p_j$ receives messages.

The simulation has to ensure that (1) all simulators simulate the same behavior of the set of simulated processes, and (2) a faulty simulator entails the failure of at most one simulated process. The way this is realized depends, of course, on the failure model that is considered.
4 BG(MP,C): BG in the Crash-prone Asynchronous Message-Passing Model

This section presents the algorithm BG(MP,C). As previously indicated, this algorithm simulates, in the model $CAMP_{n,t}[t < n/2]$, an algorithm $A'$ solving a task in $CAMP_{n',t}[t < n']$. It is made up of two parts: an algorithm implementing a safe agreement object, and the simulation itself, which uses several of these objects to allow the simulators to cooperate.

4.1 Safe agreement object in $CAMP_{n,t}[t < n/2]$: definition

This object type (or variants of it), briefly sketched in the Introduction, is at the core of both the BG simulation [6, 7, 14, 22], and the liveness guarantees of concurrent objects [23, 24]. It is a one-shot object that solves consensus in failure-free scenarios, and allows processes to agree with a weak termination guarantee in the presence of failures.

A safe agreement object provides each simulator $q_i$, $1 \leq i \leq n$, with two operations denoted $propose()$ and $decide()$, that $q_i$ can invoke at most once, and in this order; $propose()$ allows $q_i$ to propose a value, while $decide()$ allows it to decide a value. Considering the crash failure model, the properties associated with this object are the following ones.

- Validity. A decided value is a proposed value.
- Agreement. No two simulators decide distinct values.
- Propose-Termination. An invocation of $propose()$ by a correct simulator terminates.
- Decide-Termination. If no simulator crashes while executing $propose()$, then any invocation of $decide()$ by a correct simulator terminates.

It is easy to see that a safe agreement object is a consensus object whose termination condition is failure-dependent. Algorithms implementing safe agreement objects (or variants of it) can be found in [6, 7, 24].

4.2 Safe agreement object in $CAMP_{n,t}[t < n/2]$: algorithm

An algorithm implementing a safe agreement object in $CAMP_{n,t}[t < n/2]$ is described in Figure 1.

Local data structures Each simulator $q_i$, $1 \leq i \leq n$, manages three local data structures, namely, the arrays $values_i[1..n]$, $my\_view_i[1..n]$, $all\_views_i[1..n]$, all initialized to $[\bot, \ldots, \bot]$, where $\bot$ denotes a default value that cannot be proposed to the safe agreement object by the simulators.

- The aim of $values_i[x]$ is to contain, as currently known by $q_i$, the value proposed to the safe agreement object by the simulator $q_x$.
- The aim of $my\_view_i[x]$ is to contain, as known by $q_i$, the value proposed to the safe agreement object by the simulator $q_x$, as witnessed by strictly more than $n/2$ distinct simulators (i.e., at least a correct process).
- The aim of $all\_views_i[x]$ is to contain what to $q_i$’s knows about the view seen by $q_x$.

Algorithm: the operation propose() The algorithm implementing the operation $propose()$ invoked by a simulator $q_i$ is described at lines C01-C14 (client side) and lines C20-C22 (server side). This algorithm is made up of three parts.

First part. A simulator $q_i$ first broadcasts the message $VALUE (i, v_i)$, where $v_i$ is the value it proposes to the safe agreement object (line C01). Then, it waits until it knows that strictly more than $n/2$ simulators know its value (line C02). On its “server” side, when $q_i$ receives for the first time the message $VALUE (x, v)$, it
first saves \( v \) in \( \text{values}_i[x] \); then it forwards the received message to cope with the (possible) crash of \( q_x \) (this witnesses the fact that \( q_i \) knows the value proposed by \( p_x \), line C20\(^1\)).

\begin{verbatim}
operation propose \((v_i)\) is
(C01) broadcast VALUE \((i, v_i)\);
(C02) wait \((\text{value} \ (i, v_i) \ \text{received from strictly more than} \ \frac{n}{2} \ \text{different simulators})\);
(C03) for each \( x \in [1..n] \) do broadcast \( \text{READ} \ (i, x) \) end for;
(C04) for each \( x \in [1..n] \) do
(C05) wait \((\text{READ'} \text{ANSWER} \ (i, x, \bot) \ \text{received from strictly more than} \ \frac{n}{2} \ \text{different simulators})\);
(C06) \( \lor \ \exists w : \text{VALUE} \ (x, w) \ \text{received from strictly more than} \ \frac{n}{2} \ \text{different simulators})\);
(C07) if \((\text{predicate of line C06 satisfied})\)
(C08) then my_view\(_i[x] \leftarrow w\)
(C09) else my_view\(_i[x] \leftarrow \bot\)
(C10) end if
(C11) end for;
(C12) broadcast \( \text{VIEW} \ (i, \text{my_view}_i)\);
(C13) wait \((\text{VIEW} \ (i, \text{my_view}_i) \ \text{received from strictly more than} \ \frac{n}{2} \ \text{different simulators})\);
(C14) return().

operation decide \((i)\) is
(C15) wait \((\exists \text{a non-empty set} \ \sigma \subseteq \Pi:\)
(C16) \( \forall y \in \sigma : \left( (\text{all_views}_i[y] \neq \bot) \ \land \ (\forall z \in \Pi : (\text{all_views}_i[y][z] \neq \bot) \Rightarrow (z \in \sigma)) \right)\);
(C17) let \( \text{min}_\sigma \) be the set \( \sigma \) of smallest size;
(C18) let \( \text{res} \) be \( \min \{ \text{values}_i[y] : y \in \text{min}_\sigma \} \);
(C19) return(res).

% when the message \( \text{VALUE} \ (x, v) \) is received for the first time:
% “for the first time” is with respect to each pair of values \((x, v)\) %
(C20) \( \text{values}_i[x] \leftarrow v; \) broadcast \( \text{VALUE} \ (x, v)\).

% when the message \( \text{READ} \ (j, x) \) is received for the first time:
(C21) send \( \text{READ'} \text{ANSWER} \ (j, x, \text{values}_i[x]) \) to \( q_j \).

% when the message \( \text{VIEW} \ (x, \text{view}) \) is received for the first time:
(C22) \text{all_views}_i[x] \leftarrow \text{view}; \) broadcast \( \text{VIEW} \ (x, \text{view})\).
\end{verbatim}

Figure 1: Safe agreement object in \( \text{CAMP}_{n,t}[t < n/2] \) (code for the simulator \( q_i \))

Second part. In this part, \( q_i \) builds a local view of the values proposed by the \( n \) simulators. To this end, it first broadcasts messages \( \text{READ} \ (i, x) \), \( 1 \leq x \leq n \), to learn the value proposed by each simulator \( q_x \) (line C03). On its server side, when \( q_i \) receives such a message, it broadcasts by return its current knowledge of the value proposed by \( q_x \) (line C21).

Then, the simulator \( q_i \) builds its local view of the values that have been proposed. For each simulator \( q_x \), \( q_i \) waits until it has received from strictly more than \( \frac{n}{2} \) different simulators the very same message, namely, either the message \( \text{READ'} \text{ANSWER} \ (i, x, \bot) \), or the message \( \text{VALUE} \ (x, w) \) (lines C05-C06). In the first case, \( q_i \) considers that \( q_x \) has not yet proposed a value, while in the second case it considers that \( q_x \) proposed the value \( w \) (let us observe that, while \( q_i \) can receive both \( \text{READ'} \text{ANSWER} \ (i, x, \bot) \) and messages \( \text{VALUE} \ (x, w) \), it stops waiting as soon as it received strictly more than \( \frac{n}{2} \) of one of them) (lines C07-C10).

Third part. Finally, the simulator \( q_i \) informs the other simulators on its local view \( \text{my_view}_i[1..n] \). To this end, it broadcasts the message \( \text{VIEW} \ (i, \text{my_view}_i) \). When it has received the corresponding “acknowledgements”, \( q_i \) returns from its invocation of the operation \( \text{propose} \) (line C12-C14). (The behavior of \( q_i \) when it

\(^1\) Let us observe that the lines C01 and C20 implement a reliable broadcast of the message \( \text{VALUE} \ (i, v_i) \). Similarly, the lines C12 and C22 implement a reliable broadcast of the message \( \text{VIEW} \ (i, \text{my_view}_i) \). It is easy to see that the cost of such a reliable broadcast is \( O(n^2) \) messages.
receives a message \texttt{VIEW} \((x, \text{view})\) is similar to the one when it receives a message \texttt{VALUE} \((x, v)\). The only difference is that \(\text{values}_{i}[x]\) is now replaced by \(\text{all_views}_{i}[x]\), line \([22]\).

**Algorithm: the operation** \texttt{decide()}  

The algorithm implementing the operation \texttt{decide()} is described at lines \([15]-[19]\). It consists in a “closure” computation. A simulator \(q_i\) waits until it knows a non-empty set of simulators \(\sigma\) such that (a) it knows their views, and (b) this set is closed under the relation “has in its published view the value of” which means that the processes whose values appear in a view of a process of \(\sigma\) are also in \(\sigma\) (lines \([13]-[16]\)).

Let us observe that it is possible that, locally, several sets satisfy this property. If it is the case, \(q_i\) selects the smallest of them. Let \(\text{min}_{\sigma_i}\) be this set of simulators (lines \([17]\)). The value that is returned by \(q_i\) is then the smallest value among the the values proposed by the simulators in \(\text{min}_{\sigma_i}\) (lines \([18]-[19]\)).

### 4.3 Safe agreement object in \(\text{CAMP}_{n,t}[t < n/2]\): proof

This section proves that the algorithm presented in Figure \([\text{I}]\) implements a safe agreement object, i.e., any of its runs in \(\text{CAMP}_{n,t}[t < n/2]\) satisfies the validity, agreement, and termination properties, which define it.

**Lemma 1.** An invocation of \texttt{propose()} by a simulator that does not crash during this invocation, terminates.

**Proof**  

Let us consider a simulator \(q_i\) that does not crash during its invocation of \texttt{propose()}. Hence, \(q_i\) broadcast the message \texttt{VALUE} \((i, v_i)\) at line \([01]\). This message is received by strictly more than \(\frac{n}{2}\) correct simulators, and each of them broadcasts this message when it receives it. It follows that \(q_i\) cannot block forever at line \([02]\).

Let us now consider the wait statement at lines \([05]-[06]\). There are two cases. Let \texttt{READ} \((i, x)\) be a message broadcast by the simulator \(q_i\) at line \([03]\).

- **Case 1:** No correct simulator ever receives a message \texttt{VALUE} \((x, \bot)\). In this case, each correct simulator \(q_y\) is such that \(\text{values}_{y}[x]\) remains always equal to \(\bot\). It follows that, when \(q_y\) receives the message \texttt{READ} \((i, x)\), it sends back to \(q_i\) the message \texttt{READ'ANSWER} \((i, x, \bot)\) (line \([21]\)). As there are strictly more than \(\frac{\nu}{2}\) correct simulators, \(q_i\) eventually receives the message \texttt{READ'ANSWER} \((i, x, \bot)\) from strictly more than \(\frac{n}{2}\) different simulators, and the predicate of line \([05]\) is then satisfied.

- **Case 2:** At least one correct simulator \(q_y\) receives a message \texttt{VALUE} \((x, v)\). In this case, \(q_y\) broadcasts the message \texttt{VALUE} \((x, v)\) when it receives it (line \([20]\)). It follows from the broadcasts issued at this line that \(q_i\) eventually receives \texttt{VALUE} \((x, v)\) from strictly more than \(\frac{\nu}{2}\) different simulators. When this occurs, the predicate of line \([06]\) is satisfied, and \(q_i\) exits the wait statement.

As this is true for any message \texttt{READ} \((i, x)\) broadcast by the simulator \(q_i\) at line \([03]\) it follows that \(q_i\) cannot remain block forever at lines \([05]-[06]\).

Let us finally consider the lines \([12]-[13]\). As the message \texttt{VIEW} \((i, \text{my_view}_i)\) broadcast by \(q_i\) at line \([12]\) is received by at least all the correct processes, and each of them broadcast it when it receives it for the first time, it follows that \(q_i\) receives the message \texttt{VIEW} \((i, \text{my_view}_i)\) from strictly more than \(\frac{n}{2}\) distinct processes, and stops waiting at line \([13]\) which concludes the proof of the lemma.

**Lemma 2.** The value returned by an invocation of \texttt{propose()} is a value that was proposed by a simulator.

**Proof**  

Let us observe that (due to its definition) the set \(\text{min}_{\sigma}\) is non-empty, and (due the first predicate of line \([16]\)) the simulator indexes \(y\) it contains are such that \(\text{values}_{i}[y] \neq \bot\). As, for any of those \(y\), \(\text{values}_{i}[y]\) is set to a non-\(\bot\) value (only once) at line \([20]\) it follows that \(q_i\) received a message \texttt{VALUE} \((y, v_y)\). Hence, the values in the variables \(\text{values}_{i}[y]\) are values proposed by the corresponding simulators \(q_y\). It follows that the value computed at line \([18]\) is a value that was proposed by a simulator, which concludes the proof of the lemma.
Lemma 3. No two invocations of decide() return different values.

Proof Let us first observe that, due to the reliable broadcast of the messages \text{VALUE} () (lines C01 and C20) and \text{VIEW} () (lines C12 and C22), and the fact that a simulator broadcast a single message \text{VALUE} (), we have:

- \((\text{values}_i(x) \neq \bot) \land (\text{values}_j(x) \neq \bot) \Rightarrow (\text{values}_i(x) = \text{values}_j(x))\).
- \((\text{all}_\text{views}_i(x) \neq \bot) \land (\text{all}_\text{views}_j(x) \neq \bot) \Rightarrow (\text{all}_\text{views}_i(x) = \text{all}_\text{views}_j(x))\).

Let us assume, by contradiction, that two simulators \(q_i\) and \(q_j\) decide different values. This means that the sets \(\text{min}_\sigma_i\) and \(\text{min}_\sigma_j\) computed at line C17 by \(q_i\) and \(q_j\), respectively, are different.

Since \(\text{min}_\sigma_i\) and \(\text{min}_\sigma_j\) are different, let us consider \(z \in \text{min}_\sigma_i \setminus \text{min}_\sigma_j\) (if \(\text{min}_\sigma_i \subseteq \text{min}_\sigma_j\), swap \(i\) and \(j\)). According to the closure predicate used at line C16, as \(z \notin \text{min}_\sigma_j\), we have \(\forall y \in \text{min}_\sigma_j : \text{all}_\text{views}_j[y][z] = \bot\). It follows that any simulator \(q_y\) such that \(y \in \text{min}_\sigma_j\) does not fulfill the condition of line C07 for \(x = z\). Consequently, \(q_y\) received at line C05 a message \text{READ}'\text{ANSWER}(y, z, \bot)\) from a set of simulators \(Q_{y,r}(z)\) of size strictly greater than \(\frac{n}{2}\). Consequently, \(q_y\) executed line C03 for \(x = z\), all the simulators \(q_k\) of \(Q_{y,r}(z)\) verified \(\text{values}_k[z] = \bot\).

When the simulator \(q_z\) stops waiting at line C02, it received messages \(\text{VALUE}(z, v_z)\) (where \(v_z\) is the value sent by \(q_z\) at line C01) from a set \(Q_{z,w}\) of strictly more than \(\frac{n}{2}\) simulators. It follows that \(Q_{y,r}(z) \cap Q_{z,w} \neq \emptyset\), consequently there is a simulator \(q_y\) that sent a message \text{READ}'\text{ANSWER}(y, z, \bot)\) to \(q_y\) and a message \(\text{VALUE}(v_y, v_z)\) to \(q_z\). Since \(\text{values}_k[z]\) is never reset to \(\bot\) after being assigned, the simulator \(q_y\) necessarily executed line C13 for \(x = z\) strictly before \(q_z\) stops waiting at line C02. Consequently \(q_y\) stopped waiting at line C02 before \(q_z\) executes line C03 for \(x = y\). It does so after receiving messages \(\text{VALUE}(y, v_y)\) (where \(v_y\) is the value sent by \(q_y\) at line C01) from a set \(Q_{y,w}\) of strictly more than \(\frac{n}{2}\) simulators \(q_k\), and each of these simulators then verifies \(\text{values}_k = v_y\). These simulators do not send \text{READ}'\text{ANSWER}(z, y, \bot)\) messages when they receive the \(\text{READ}(z, y)\) message sent by \(q_z\). Thus, it is impossible that \(q_z\) receives these messages from strictly more than \(\frac{n}{2}\) processes, it consequently cannot verify the predicate of line C05. It follows that \(q_z\) executes line C12 with \(\text{my}_\text{views}_x[y] = v_y \neq \bot\) and this entails that \(\forall k \in \Pi : \text{all}_\text{views}_k[z] \neq \bot \Rightarrow \text{all}_\text{views}_k[z][y] = v_y\).

Since \(z \in \text{min}_\sigma_i\), \(\text{all}_\text{views}_i[z] \neq \bot\), \(\text{all}_\text{views}_i[z][y] \neq \bot\). According to the predicate of line C16, this entails that \(y \in \text{min}_\sigma_i\), and since the previous reasoning holds for any \(y \in \text{min}_\sigma_j\), it shows that \(\text{min}_\sigma_i \subseteq \text{min}_\sigma_j\). It follows that, when \(q_i\) executes line C17, \(\forall y \in \text{min}_\sigma_j : \text{all}_\text{views}_i[y] \neq \bot\) and, consequently, \(\forall y \in \text{min}_\sigma_j : \text{all}_\text{views}_i[y] = \text{all}_\text{views}_j[y]\). It entails that if \(|\text{min}_\sigma_j| < |\text{min}_\sigma_i|\), then \(\text{min}_\sigma_j\) would have been chosen by \(q_i\) at line C17 which proves that \(\text{min}_\sigma_i = \text{min}_\sigma_j\) and contradicts the fact that \(q_i\) and \(q_j\) decide differently.

⚠️️ Lemma 3

Lemma 4. If no simulator crashes while executing \text{propose}(), then any invocation of \text{decide}() by a correct simulator terminates.

Proof If no simulator crashes while executing \text{propose}(), it follows from Lemma 1 that every simulator \(q_i\) that invokes \text{propose}() broadcasts a message \text{VALUE}(i, v_i) at line C01 and a message \text{VIEW}(i, my\_views_i) at line C12.

Assuming no simulator crashes while executing \text{propose}(), let \(P\) be the set of simulators that invoke \text{propose}(), and suppose that one of them, \(q_i\), invoke \text{decide}() and never terminates. This can only happen if \(q_i\) waits forever for the condition of lines C15-C16 to be fulfilled. Since eventually the messages broadcast by the simulators of \(P\) are all delivered to \(q_i\), after some finite time \(\forall y \in P : \text{all}_\text{views}_i[y] \neq \bot\). Moreover, since the views broadcast by the simulators of \(P\) are built at line C08 from the messages \text{VALUE}(\_\_\_\_) they receive, it follows that these views can contain non-\(\bot\) values only for the entries corresponding to the simulators of \(P\) (the simulators that are not in \(P\) do not send messages \text{VALUE}(\_\_\_, \_\_\_)\)). Consequently, \(p_i\) eventually verifies \(\forall y \in P : (\text{all}_\text{views}_i[y] \neq \bot) \land (\{z \in \Pi : \text{all}_\text{views}_i[y][z] \neq \bot\} \subseteq P)\). It follows that the property of lines C15-C16 is eventually true for \(\sigma = P\), which contradicts the fact that \(q_i\) never terminates its \text{decide}() operation.

⚠️️ Lemma 4
**Theorem 1.** The algorithm in Figure 1 implements a safe agreement object in CAM\(P_{n,t}[t < n/2]\).

**Proof** The proof follows from Lemma 1 (Propose-Termination), Lemma 2 (Validity), Lemma 3 (Agreement), and Lemma 4 (Decide-Termination). \(\Box\)

### 4.4 Simulation algorithm

The simulation algorithm takes as input a distributed algorithm \(A\) solving a (colorless) task in the system model CAM\(P_{n',t}[t < n']\), and simulates it in CAM\(P_{n,t}[t < n/2]\). Each simulator \(q_i\), \(1 \leq i \leq n\), is given a copy of the \(n'\) processes of \(A\), and a private input vector \(\text{input}_i[1..n']\), with one input per simulated processes \(p_j\).

The simulation consists in a fair simulation by each of the \(n\) simulators \(q_i\) of the \(n'\) simulated processes \(p_j\). To that end, each simulator manages \(n'\) threads (each simulating a process \(p_j\)), and the \(n\) threads associated with the simulation of a process \(p_j\) cooperate through safe agreement objects.

**Objects shared by the simulators** To produce a consistent simulation, for each simulated process \(p_j\), the \(n\) simulators have to agree on the same sequence of the messages received by \(p_j\). To that end, they use an array of safe agreement objects, denoted \(SA[1..n', \_]\), such that \(SA[j, sn]\) allows them to agree on the \(sn\)-th message received by the \(n'\) threads simulating \(p_j\) at each simulator \(q_i\).

**Objects managed by each simulator \(q_i\)** Each simulator manages the following data structures, with respect to each simulated process \(p_j\).

- \(\text{input}_i[j]\) contains the input of the simulated process \(p_j\), proposed by the simulator \(q_i\). (Simulators are allowed to propose different input vectors for the simulated processes).
- \(sn_i[j]\) is the sequence number (from the simulation point of view) of the next message received by the simulated process \(p_j\).
- \(\text{sent}_i[j]\) is a sequence containing messages sent by the simulated processes to the simulated process \(p_j\). It is assumed that the \(n'\) threads of \(q_i\) access \(\text{sent}_i[j]\) in mutual exclusion (when they add messages to or withdraw messages from this sequence). The symbol \(\oplus\) is used to add messages at the end of a sequence. Sometimes \(\text{sent}_i[j]\) is used as a set.
- \(\text{received}_i[j]\) is a set containing the messages received by the simulated process \(p_j\) (init. \(\emptyset\)).
- \(\text{state}_i[j]\) contains the current local state of the simulated process \(p_j\). \(\text{input}_i[j]\) is a part of \(\text{state}_i[j]\).

It is assumed that the behavior of each simulated process \(p_j\) is described by a deterministic transition function \(\delta_j()\), such that \(\delta_j(\text{state}_i[j], \text{msg})\) (a) simulates \(p_j\) until its next message reception, and (b) returns a pair. This pair is made up of the new local state of \(p_j\) plus an array \(\text{msgs}[1..n']\) where \(\text{msgs}[x]\) contains messages sent by \(p_j\) to the simulated process \(p_x\).

In addition to the previous local data, each simulator \(q_i\) uses a starvation-free mutual exclusion lock, whose operations are denoted \(\text{mutex}_\text{in}_i()\) and \(\text{mutex}_\text{out}_i()\). This lock is used to ensure that, at any time, at most one of the \(n'\) threads of \(q_i\) access a safe agreement object. This is to guarantee that the crash of a simulator \(q_i\) entails the crash of \(at\) \(most\) \(one\) simulated process \(p_j\) (line 09). More precisely, if \(q_i\) crashes while executing \(SA[j, sn].\text{propose}().\), it can block forever only the invocations of \(SA[j, sn].\text{decide}().\), issued by the other simulators, thereby preventing the simulation of \(p_j\) from terminating.
The simulation algorithm  The algorithm describing the simulation of a process \( p_j \) by the associated thread of the simulator \( q_i \) is presented in Figure 2.

The simulators have first to agree on the same input for process \( p_j \). To this end, they use the safe agreement object \( SA[i, 0] \) (lines 01-02). Moreover, when considering all the simulated processes, it follows from the mutual exclusion lock that, whatever the number of simulated processes, a simulator \( q_i \) is engaged in at most one invocation of \( \text{propose}() \) at a time. Then, according to the decided input of \( p_j \), \( q_i \) locally simulate \( p_j \) until it invokes a message reception (lines 03-04).

After this initialization, each simulator \( q_i \) enters a loop whose aim is to locally simulate \( p_j \). To this end, \( q_i \) first determines the message that \( p_j \) will receive; this message is saved in \( \text{rec}_\text{msg} \) and added to \( \text{received}_i[j] \) (lines 07-12). When this message has been determined, \( q_i \) simulates the behavior of \( p_j \) until its next message reception (lines 13-14). Finally, if \( \text{state}_i[j] \) allows \( p_j \) to decide a value with respect to the simulated decision task, this value is decided (lines 15-17).

4.5  Proof of the simulation

The reader interested in a formal definition of the term simulation –as used here– will consult [7].

Lemma 5. The crash of a simulator \( q_i \) entails the crash of at most one simulated process \( p_j \).

Proof The only places where a simulator \( q_i \) can block is during the invocation of the safe agreement operation \( \text{decide}() \). Such invocations appear at line 02 and line 11. It follows from the termination property of the safe agreement objects that such an invocation can block forever the invoking process only if a simulator crashes during the invocation of the operation \( \text{propose}() \) on the same object. But, due to the mutual exclusion lock used at line 01 and line 10, a simulator can be engaged in at most one invocation of \( \text{propose}() \) at a time. It follows that the crash of a simulation \( q_i \) can entail the definitive halting (crash) of at most one simulated process \( p_j \).

Lemma 6. The simulation of the reception of the \( k \)-th message received by a simulated process \( p_j \), returns the same message at all simulators.

Proof The simulation of the message receptions for a simulated process \( p_j \), are executed at each simulator \( q_i \) at lines 08-11 and all the simulators use the same sequence of sequence numbers (line 07). It then follows
from the agreement property of the safe agreement object \(SA[j, sn]\), that no two simulators obtain different messages when they invoke \(SA[j, sn].\text{decide()}\), and the lemma follows. \(\square\text{Lemma 6}\)

**Lemma 7.** For every simulated processes \(p_j\), no two simulators return different values.

**Proof** The only non-deterministic elements of the simulation are the input vectors \(\text{input}_i[1..n']\) at each simulator \(q_i\), and the reception of the simulated messages.

The lines 01-02 of the simulation force the simulators to agree on the same input value for each simulated process \(p_j\), \(1 \leq j \leq n'\). Similarly, as shown by Lemma 6 for each simulated process \(p_j\), the lines 07-11 direct the simulators to agree on the very same sequence of messages received by \(p_j\). It follows from the fact that the function \(\delta_j()\) is deterministic, that any two simulators \(q_i\) and \(q_k\), that execute lines 15-16 during the same “round number” \(sn_i[j] = sn_k[j]\), are such that \(\text{state}_i[j] = \text{state}_k[j]\), from which the lemma follows. \(\square\text{Lemma 7}\)

**Lemma 8.** The sequences of message receptions simulated by each simulator \(q_i\) on behalf of each simulated process \(p_j\), define a correct execution of the simulated algorithm.

**Proof** To prove the correctness of the simulation, we have to show that

1. Every message that was sent by a simulated process to another simulated process (whose simulation is not blocked either), is received, and
2. The simulated messages respect a simulated physical order (i.e., no message is “received” before being “sent”).

Item 1 is satisfied because the messages sent by the simulated process \(p_j\) to the simulated process \(p_k\) are received (lines 09-11) in their sending order (as defined at line 04 and line 14). Hence, if \(p_k\) is not blocked (due to the crash of a simulator) it obtains the messages from \(p_j\) in their sending order.

For Item 2, let us define a (simulated) physical order as follows. For each simulated message \(m\), let us consider the first time at which the reception of \(m\) was simulated (i.e., this occurs when –for the first time– a simulator terminates the invocation of \(SA[-, -].\text{decide()}\) that returns \(m\)). A message that is decided has been proposed by a simulator to a safe agreement object before being decided (validity property). The sending time of a simulated message is defined as the first time at which \(SA[-, -].\text{propose}(m)\) is invoked by a simulator. It follows that any simulated message is sent before being received, which concludes the lemma. \(\square\text{Lemma 8}\)

**Lemma 9.** Each correct simulator \(q_i\) computes the decision value of at least \((n' - t)\) simulated processes.

**Proof** Due to Lemma 5 and the fact that at most \(t\) simulators may crash, it follows that at most \(t\) simulated processes may be prevented from progressing. As (a) by assumption the simulated algorithm \(A'\) is \(t\)-resilient, and (b) due to Lemma 8 the simulation produces a correct simulation of \(A'\), it follows that at least \((n' - t)\) simulated processes decide a value. \(\square\text{Lemma 9}\)

**Theorem 2.** Let \(A\) be an algorithm solving a decision task in \(\text{CAMP}_{n', t}[t < n']\). The algorithm described in Figure 2 is a correct simulation of \(A\) in \(\text{CAMP}_{n, t}[t < n/2]\).

**Proof** The theorem follows from Lemma 8 and Lemma 9. \(\square\text{Theorem 2}\)

### 5 BG(MP,B): BG in the Byzantine Asynchronous Message-Passing Model

This section presents an algorithm, denoted BG(MP,B), which implements the BG simulation in the Byzantine asynchronous message-passing model \(\text{BAMP}_{n, t}[t < n/3]\). To this end, an appropriate safe agreement object is first built, and then used by the simulation algorithm.
5.1 From crash failures to Byzantine behaviors

The idea is to extend the algorithm of Figure 1 to obtain an algorithm that copes with Byzantine simulators. The main issues that have to be solved are the following.

- The simulators need a mechanism to control the validity of the inputs to the safe agreement objects. (See below for the notion of a valid value.)
- The simulators must be able to check if a given simulator \( q_i \) is participating in more than one operation \( \text{propose}(\cdot) \) at the same time (on the same or several safe agreement objects). If it is the case, \( q_i \) is faulty and its definitive stop can block forever several simulated processes. Hence, such a faulty simulator has to be ignored.

To solve these issues, each safe agreement object may no longer be considered as a separate abstraction: each new instance depends on the previous ones. This is captured in the following specification customized to the Byzantine model, and, at the operational level, in the predicate \( \text{valid}(\cdot) \) used in the algorithm implementing the operation \( \text{propose}(\cdot) \).

5.2 Safe agreement in \( \text{BAMP}_{n,t} \; [t < n/3] \): definition

To cope with the previous observations, the fact that a faulty process may decide an arbitrary value, and the fact that the safe agreement objects are used to solve specific problems (a simulation in our case), the specification of the safe agreement object is reshaped as follows.

A value proposed by a process to a safe agreement object must be valid. At each correct simulator \( q_i \), the validity of a value is captured by a predicate denoted \( \text{valid}_i(j,v) \) where \( v \) is the value and \( q_j \) the simulator that proposed it. This predicate is made up of two parts (defined in Section 5.3 and Section 5.6, respectively). If \( q_j \) is correct, the predicate \( \text{valid}_i(j,v) \) eventually returns true at \( p_i \). If \( q_j \) is faulty, \( \text{valid}_i(j,v) \) returns true at \( p_i \) only if (a) the value \( v \) could have been proposed by a correct simulator and (b) to \( q_i \)'s knowledge, \( q_j \) does not participate concurrently in several invocations of \( \text{propose}(\cdot) \).

- Validity. If a correct simulator \( q_i \) decides the value \( v \), there is a correct simulator \( q_j \) such that \( \text{valid}_j(\cdot, v) \).
  (\( v \) was validated by a correct simulator.)
- Agreement. No two correct simulators decide distinct values.
- Propose-Termination. Any invocation of \( \text{propose}(\cdot) \) by a correct simulator terminates.
- Decide-Termination. The invocations by all the correct simulators of \( \text{decide}(\cdot) \) on all the safe agreement objects terminate, except for at most \( t \) safe agreement objects.

5.3 Safe agreement in \( \text{BAMP}_{n,t} \; [t < n/3] \): algorithm

The local variables \( \text{values}_i[1..n] \), \( \text{my_view}_i[1..n] \), \( \text{all_views}_i[1..n] \), and the algorithm implementing the operation \( \text{decide}(\cdot) \) are the same as in Figure 1 (lines C15-C19). The new algorithm implementing the operation \( \text{propose}(\cdot) \), and the processing of the associated messages, are described in Figure 3 and Figure 4.

This implementation uses an additional local array \( \text{answers}_i[k][j][x] \), all entries of which are initialized to “?”. The meaning of “\( \text{answers}_i[k][j][x] = v \)” (where \( v \) is a proposed value or \( \perp \) ) is the following: to the knowledge of \( q_i \), the simulator \( q_k \) answered value \( v \) when it received the message \( \text{READ}(j, x) \) sent by \( q_j \). (A simulator \( q_j \) broadcasts such a message when it needs to know the value proposed by the simulator \( q_x \); \( \perp \) means that \( q_k \) does not know this value yet.) This means that, from \( q_i \)'s point of view, the value proposed by \( q_x \), as known by \( q_k \) when it received the request by \( q_j \), is \( v \).

Lemma 10. Any two sets of simulators \( Q_1 \) and \( Q_2 \) of more than \( \frac{n+4}{2} \) elements have at least one correct simulator in their intersection.
Proof As we consider integers, “strictly more than $\frac{n+t}{2}$” is equivalent to “at least $\lfloor \frac{n+t}{2} \rfloor + 1$”.

- $Q_1 \cup Q_2 \subseteq \{p_1, \ldots, p_n\}$. Hence, $|Q_1 \cup Q_2| \leq n$.
- $|Q_1 \cap Q_2| = |Q_1| + |Q_2| - |Q_1 \cup Q_2| \geq |Q_1| + |Q_2| - n \geq 2(\lfloor \frac{n+t}{2} \rfloor + 1) - n > 2(\lfloor \frac{n+t}{2} \rfloor + 1) - n = t$.
  Hence, $|Q_1 \cap Q_2| \geq t + 1$. It follows that $Q_1 \cap Q_2$ contains at least one correct simulator.

The fact that, despite Byzantine processes, the intersection of any two simulator sets of size greater than $\frac{n+t}{2}$ have at least one correct simulator in common, is used in many places in the algorithm. This property will be used in the proof to show that the local views of the correct processes are mutually consistent.

The operation propose() The client side of the algorithm implementing the operation propose() is described in Figure 3; its server side is described in Figure 4. The client side algorithm is very close to the one of the crash failure case (Figure 1). They differ in two points.

- The message tags VALUE and VIEW (used at lines C02, C06, and C13 in Figure 1) are replaced in Figure 3 by the tags VALUE’ACK and VIEW’ACK, respectively. The role of these message tags is explained below.
- The predicate of line B05 is replaced by the predicate $\{|k : answers_i[k][i][x] = \bot\} > \frac{n+t}{2}$. This predicate states that more than $\frac{n+t}{2}$ simulators answered $\bot$ to the request message READ$(i, x)$ broadcast by $q_i$, (i.e., they did not know the value proposed by $q_x$ when they received the read request).

```plaintext
operation propose (vi) is
(B01) broadcast VALUE (i, vi);
(B02) wait (VALUE’ACK (i, vi) received from $> \frac{n+t}{2}$ different simulators);
(B03) for each $x \in [1..n]$ do broadcast READ (i, x) end for;
(B04) for each $x \in [1..n]$ do
(B05) wait $(\{|k : answers_i[k][i][x] = \bot\} > \frac{n+t}{2}) \lor$
(B06) $(\exists w : VALUE’ACK (x, w) received from $> \frac{n+t}{2}$ different simulators))
(B07) if (predicate of line B06 satisfied)
(B08) then my_view[x] ← w
(B09) else my_view[x] ← \bot
(B10) end if
(B11) end for;
(B12) broadcast VIEW (i, my_view);
(B13) wait (VIEW’ACK (i, my_view) received from $> \frac{n+t}{2}$ different simulators);
(B14) return() .
```

```plaintext
operation decide () is
(C15) wait (\exists a non-empty set $\sigma \subseteq \Pi$:
(C16) $\forall y \in \sigma : (\{all_views_i[y] \neq \bot\} \land (\forall z \in \Pi : (all_views_i[y][z] \neq \bot) \Rightarrow (z \in \sigma)));
(C17) let min_\sigma be the set $\sigma$ of smallest size;
(C18) let res be min($\{values_i[y] : y \in min_\sigma\}$);
(C19) return(res).
```

Figure 3: Safe agreement object in $BAMP_{n, t}[t < n/3]$; operation propose() of simulator $q_i$

Messages VALUE(), VALUE’VALID(), VALUE’WITNESS() and VALUE’ACK() When a simulator $q_i$ invokes the operation propose($vi$), it first broadcasts the message VALUE $(i, vi)$, and waits for $\frac{n+t}{2}$ acknowledgments (messages VALUE’ACK($i, vi$), lines B01-B02). Then, as in the crash failure case (Figure 1), it builds its local view of the values proposed to the safe agreement object (lines B03-B11). Finally, it sends its local view to all other simulators (lines B12-B13).

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when the message \texttt{VALUE} \((j, v)\) is received from \(q_i\) for the first time:

\begin{enumerate}
\item \texttt{wait} \((\text{valid}, (j, v)); \text{broadcast \texttt{VALUE'VALID}} (j, v)\).
\end{enumerate}

when the message \texttt{VALUE'VALID} \((j, v)\) is received:

\begin{enumerate}
\item if \((\text{VALUE'VALID} (j, v) \text{ received from } > \frac{n+1}{2} \text{ different simulators}) \land \texttt{VALUE'WITNESS} (j, v) \text{ never broadcast})
\item then \text{broadcast \texttt{VALUE'WITNESS}} (j, v) \end{enumerate}

when the message \texttt{VALUE'WITNESS} \((j, v)\) is received:

\begin{enumerate}
\item if \((\text{VALUE'WITNESS} (j, v) \text{ received from } t + 1 \text{ different simulators}) \land \texttt{VALUE'WITNESS} (j, v) \text{ never broadcast})
\item then \text{broadcast \texttt{VALUE'WITNESS}} (j, v) \end{enumerate}

when the message \texttt{READ} \((j, x)\) is received from \(q_i\) for the first time:

\begin{enumerate}
\item \texttt{wait} \((\text{VALUE'ACK} (j, v) \text{ received from } > \frac{n+1}{2} \text{ different simulators});
\item \texttt{values}_i[j] ← v; \text{broadcast \texttt{VALUE'ACK}} (j, v);
\item \text{broadcast \texttt{READ'ANSWER}} (j, x, \texttt{values}_i[x]).
\end{enumerate}

when the message \texttt{READ'ANSWER} \((j, x, v)\) is received from \(q_i\) for the first time:

\begin{enumerate}
\item \texttt{if} \((\text{READ'ANSWER'WITNESS} (k, j, x, v) \text{ received from } k + 1 \text{ different simulators}) \land \texttt{READ'ANSWER'WITNESS} (k, j, x, v) \text{ never broadcast})
\item \texttt{then} \text{broadcast \texttt{READ'ANSWER'WITNESS}} (k, j, x, v) \end{enumerate}

when the message \texttt{VIEW} \((j, \text{view})\) is received from \(q_i\) for the first time:

\begin{enumerate}
\item \texttt{if} \((\texttt{VIEW'WITNESS} (j, \text{view}) \text{ never broadcast}) \land (\texttt{view}[j] ≠ ⊥)
\item \texttt{then for} \((x ∈ [1..n]) \text{ do}
\item \texttt{if} (\texttt{view}[x] ≠ ⊥)
\item \texttt{then \texttt{wait} \((\texttt{VALUE'ACK} (x, \text{view}[x]) \text{ received from } > \frac{n+1}{2} \text{ different simulators})
\item \texttt{else \texttt{wait} ((\{k : \texttt{answers}_i[k][j][x] = ⊥\}) > \frac{n+1}{2})
\item \texttt{end if}
\item \texttt{end for}
\item \texttt{broadcast \texttt{VIEW'WITNESS}} (j, \text{view})
\item \texttt{end if}
\end{enumerate}

when the message \texttt{VIEW'WITNESS} \((j, \text{view})\) is received:

\begin{enumerate}
\item \texttt{if} \((\texttt{VIEW'WITNESS} (j, \text{view}) \text{ received from } t + 1 \text{ different simulators}) \land \texttt{VIEW'WITNESS} (j, \text{view}) \text{ never broadcast})
\item \texttt{then \text{broadcast \texttt{VIEW'WITNESS}} (j, \text{view}) \end{enumerate}

On its server side, when a simulator \(q_i\) receives a message \texttt{VALUE} \((j, v)\), it first checks if this message is valid (line B15). If the message is valid, \(q_i\) broadcasts (echoes) the message \texttt{VALUE'VALID} \((j, v)\) to inform
the other simulators that it agrees to take into account the pair \((j, v)\) (line B15).

When the simulator \(p_i\) has received the message \(\text{VALUE}'\text{VALID} (j, v)\) from more than \(\frac{n+t}{2}\) simulators, it broadcasts the message \(\text{VALUE}'\text{WITNESS} (j, v)\) to inform the other processes that at least \(\frac{n+t}{2} - t = \frac{n-t}{2} \geq t + 1\) correct simulators, have validated the pair \((j, v)\).

When \(q_i\) has received the message \(\text{VALUE}'\text{WITNESS} (j, v)\) from \((t + 1)\) simulators (i.e., from at least one correct simulator) it broadcasts this message, if not yet done (lines B18-B20). This is to prevent invocations of \(\text{propose}\) from blocking forever (while waiting \(\text{VALUE}'\text{ACK} (j, v)\) messages at line B02, B06, B24 or B38), because not enough \(\text{VALUE}'\text{WITNESS} (j, v)\) messages have been broadcast. Then, if \(q_i\) has received the message \(\text{VALUE}'\text{WITNESS} (j, v)\) from more than \(\frac{n+t}{2}\) simulators, it takes \(v\) into account (writes it into \(\text{values}_i[j]\)) and sends an acknowledgment to \(q_j\) (lines B21-B23). The corresponding message \(\text{VALUE}'\text{ACK} (j, v)\) broadcast by \(q_i\) will also inform the other simulators that \(q_i\) took into account the value \(v\) proposed by \(q_j\). Hence, this message will help \(q_j\) progress at line B02 and all correct simulators progress at line B06.

**First part of the predicate valid\(_i\)(j, v)** As already indicated, the aim of this predicate is to help a simulator \(q_i\) detect if the value \(v\) proposed by the simulator \(q_j\) is valid. It is always satisfied when \(q_j\) is correct, and it can return \text{true} or \text{false} when \(q_j\) is faulty. It is made up of two sub-predicates \(P1\) and \(P2\).

- The first sub-predicate \(P1\) checks if, for the messages \(\text{VALUE} (j, \_ -)\) (from \(q_j\)) and \(\text{VALUE}'\text{VALID} (j, \_ -)\) (from more than \(t + 1\) different simulators) that \(q_i\) has received for other safe agreement objects, \(q_i\) has also received the associated messages \(\text{VIEW}'\text{WITNESS} (j, \_ -)\) from at least \((n - t)\) different simulators. This allows \(q_i\) to check if the simulator \(q_j\) is not simultaneously participating in other invocations of \(\text{propose}\) on other safe agreement objects.
- The aim of the second sub-predicate \(P2\) (defined in Section 5.6 and used in the simulation) is to allow the simulators to check that the simulation is consistent. As the present section considers safe agreement objects independently from its use in the simulation, we consider, for now, that \(P2\) is always satisfied.

If the full predicate \(\text{valid}_i(j, v)\) is never satisfied, \(q_i\) will, collectively with the other correct simulators, prevent the faulty simulator \(q_j\) from progressing with respect to the corresponding safe agreement object.

**Messages** \(\text{READ}()\), \(\text{READ}'\text{ANSWER}()\) and \(\text{READ}'\text{ANSWER}'\text{WITNESS}()\) After the value \(v_i\) it proposes to the safe agreement object has been taken into account \(\text{by} \frac{n+t}{2}\) simulators, \(q_i\) builds a local view of all the values proposed (array \(\text{my\_view}_i[1..n]\)). To this end, as in the crash failure model, \(q_i\) sends to each simulator \(q_j\) the customized message \(\text{READ} (i, x)\) (line B03). Its behavior is then similar to the one of the crash failure model (line B04-B11), where the new predicate \(\{k : \text{answers}_i[k][i][x] = \perp\} > \frac{n+2}{2}\) is now used at line B05.

When \(q_i\) receives the message \(\text{READ} (j, x)\) from the simulator \(q_j\), it first waits until it knows that the value proposed by \(q_j\) is known by more than \(\frac{n+t}{2}\) simulators (line B24). This is to check that \(q_j\) broadcast its proposed value before reading the other simulator values used to build its own view. When this occurs, \(q_i\) answers the message \(\text{READ} (j, x)\) by broadcasting the message \(\text{READ}'\text{ANSWER} (j, x, \text{values}_i[x])\) to inform all the simulators on what it currently knows on the value proposed by \(q_j\) (line B26). (Let us remind that, in the crash failure model, \(q_i\) was sending this message only to \(q_j\).

When it receives the message \(\text{READ}'\text{ANSWER} (j, x, v)\) from a simulator \(q_k\), if not yet done, \(q_i\) broadcasts the message \(\text{READ}'\text{ANSWER}'\text{WITNESS} (k, j, x, v)\). The lines B27-B31 implement a reliable broadcast [8], i.e., the message \(\text{READ}'\text{ANSWER}'\text{WITNESS} (k, j, x, v)\) is received by all correct processes or none of them, and is always received if the sender is correct. The reliable reception of this message entails the assignment of \(\text{answers}_i[k, j, x] \rightarrow v\) (line B33).

\[^2\text{A similar mechanism is used in \([8]\) to ensure that the proposed reliable broadcast abstraction guarantees that a message is received by all or none of the correct processes.}\]
Messages \textsf{VIEW()}, \textsf{VIEW’WITNESS()} and \textsf{VIEW’ACK()} Finally, as in Figure 1, the simulator \(q_i\) broadcasts its local view of proposed values to all simulators, waits until more than \(\frac{n+t}{2}\) of them sent back an acknowledgment, and returns from the invocation of propose() (lines B12-B14).

When \(q_i\) receives for the first time the message \textsf{VIEW}(j, \texttt{view})\), it realizes an enriched reliable broadcast whose aim is to assign \texttt{view} to all\_\texttt{view}_i[j]. Let us first observe that if \texttt{view}[j] = \perp, then \(q_j\) is Byzantine. If it has not yet broadcast \textsf{VIEW’WITNESS}(j, \texttt{view}) and if \texttt{view}[j] \neq \perp (line B35), \(q_i\) first checks if all the values in \texttt{view}[1..n] are consistent. From its point of view, this means that, for each simulator \(q_k\), (a) if \texttt{view}[x] = \texttt{v}, it must receive messages \textsf{VALUE’ACK}(x, \texttt{v}) from more than \(\frac{n+t}{2}\) simulators, and (b) if \texttt{view}[x] = \perp, the same predicate as in line B35 must become satisfied. This consistency check is realized by the lines B35-B41.

Finally, when \(q_i\) receives a message \textsf{VIEW’WITNESS}(j, \texttt{view})\), it does the following. First, if it has received this message from at least one correct simulator, and has not yet broadcast it, \(q_i\) does it (lines B44-B49). This part of the reliable broadcast is to prevent the correct simulators from blocking forever. Then, if it has received \textsf{VIEW’WITNESS}(j, \texttt{view}) from more than \(\frac{n+t}{2}\) simulators and has not yet assigned a value to all\_\texttt{view}_i[j], \(q_i\) does it and sends to \(q_j\) the acknowledgment message \textsf{VIEW’ACK}(j, \texttt{view}) to inform \(q_j\) that it knows its view (lines B57-B59).

5.4 A communication pattern

When considering the algorithm of Figure 4, it appears that the processing of the messages \textsf{VALUE’WITNESS()} (lines B18-B23), \textsf{READ’ANSWER’WITNESS()} (lines B28-B34), and \textsf{VIEW’WITNESS()} (lines B44-B49), follow the same generic pattern. This pattern, inspired from [8] and where \textsc{WITNESS} is used as message tag, is described in Figure 5.

\begin{figure}[h]
\centering
\begin{algorithm}
\caption{Generic communication pattern in \textit{BAMP}\textsubscript{n,t}[t < n/3]}
\begin{algorithmic}
\State \textbf{when} \textsc{WITNESS}(m) is received:
\State \hspace{1em} (GP01) \textbf{if} \ (\textsc{WITNESS}(m) \textbf{received} \textbf{from} t + 1 \textbf{different} \textbf{simulators})
\State \hspace{2em} \textbf{and} \ (\textsc{WITNESS}(m) \textbf{never broadcast})
\State \hspace{2em} (GP02) \textbf{then} broadcast \textsc{WITNESS}(m)
\State \hspace{2em} (GP03) \textbf{end if};
\State \hspace{1em} (GP04) \textbf{if} \ (\textsc{WITNESS}(m) \textbf{received} \textbf{from} > \frac{n+t}{2} \textbf{different} \textbf{simulators})
\State \hspace{2em} (GP05) \textbf{then} execute statement \emph{A}
\State \hspace{2em} (GP06) \textbf{end if};
\State \hspace{1em} (GP07) \textbf{end if}.
\end{algorithmic}
\end{algorithm}
\end{figure}

\textbf{Theorem 3.} (i) If a correct simulator executes action \emph{A}, all correct simulators do it.
(ii) If \((t + 1)\) correct simulators execute broadcast \textsc{WITNESS}(m), all correct simulators execute action \emph{A}.

\textbf{Proof} Proof of (i). Let \(p_t\) be a correct process that executes \emph{A}. It follows from line GH05 that it has received the message \textsc{WITNESS}(m) from more than \(\frac{n+t}{2}\) different simulators. As \(n > 3t\), \(\left\lceil \frac{n+t}{2} \right\rceil + 1 \geq 2t + 1\), \(p_t\) received the message \textsc{WITNESS}(m) from at least \((t + 1)\) correct simulators. It then follows from lines GH01-GH02 that all correct simulators broadcast \textsc{WITNESS}(m) and, consequently, each correct simulator receives \textsc{WITNESS}(m) from at least \((n - t)\) simulators. The proof follows from \(n - t > \frac{n+t}{2}\).

Proof of (ii). If \((t+1)\) correct simulators broadcast \textsc{WITNESS}(m), the predicate of line GH01 is eventually satisfied at every correct simulator. As \(n - t > \frac{n+t}{2}\), it follows that the predicate of line GH05 will also be satisfied at each correct simulator, which concludes the proof. \(\square\)

5.5 Safe agreement object in \textit{BAMP}\textsubscript{n,t}[t < n/3]: proof

This section proves that the algorithm presented in Figures 3 and 4 implements a safe agreement object in the presence of Byzantine simulators, i.e., any of its runs in \textit{BAMP}\textsubscript{n,t}[t < n/3] satisfies the validity, agreement, and termination properties that define this object.
Propose-termination

Lemma 11. Let \( q_i \) be a correct simulator. If the predicate \( \text{valid}_i(j, v_i) \) eventually becomes satisfied at the correct simulators \( q_j \), then the invocation of \( \text{propose}(v_i) \) by \( q_i \) terminates.

Proof. A correct simulator \( q_i \) can be blocked forever in a wait statement (1) at line \( B[02] \) (2) at lines \( B[05]-B[06] \) or (3) at line \( B[13] \). We show that, if the predicate \( \text{valid}_i(j, v_i) \) is eventually satisfied at the correct simulators \( q_j \), \( p_i \) cannot block forever in the invocation of \( \text{propose}(v_i) \).

- wait instruction at line \( B[02] \)
  
  Simulator \( q_i \) first broadcasts the message \( \text{VALUE}(i, v_i) \) (line \( B[01] \)), then waits for \( \text{VALUE}’\text{ACK} \) messages from more than \( \frac{n+t}{2} \) different simulators. When a correct simulator \( q_j \) receives \( \text{VALUE}(i, v_i) \) for the first time, it waits until \( \text{valid}_j(i, v_i) \) becomes satisfied. By assumption, this happens. Simulator \( q_j \) then broadcasts \( \text{VALUE}’\text{VALID}(i, v_i) \). It follows that each of the at least \( (n-t) \) correct simulators broadcasts the message \( \text{VALUE}’\text{VALID}(i, v_i) \).

As \( n-t > \frac{n+t}{2} \), it follows that each correct simulator \( q_j \) receives the message \( \text{VALUE}’\text{VALID}(i, v_i) \) from more than \( \frac{n+t}{2} \) simulators and broadcasts the message \( \text{VALUE}’\text{WITNESS}(i, v_i) \).

According to Theorem 3, \( q_j \) updates \( \text{values}_j[i] \) with \( v_i \) and broadcasts \( \text{VALUE}’\text{ACK}(i, v_i) \) (lines \( B[21]-B[23] \)). The correct simulator \( q_i \) will then receive the message \( \text{VALUE}’\text{ACK}(i, v_i) \) from at least \( n-t > \frac{n+t}{2} \) simulators. Hence, it cannot block forever at line \( B[02] \).

- wait instruction at line \( B[05]-B[06] \)

In this waiting statement, \( q_i \) waits until either \( |\{ k : \text{answers}_i[k][i][j] = \bot \}| > \frac{n+t}{2} \) becomes true, or until it receives \( \text{VALUE}’\text{ACK}(j, w) \) from more than \( \frac{n+t}{2} \) different simulators.

  - If \( q_j \) is a correct simulator that invoked \( \text{propose}(j, w) \), the reasoning is the same as above. Consequently, \( q_i \) will receive \( \text{VALUE}’\text{ACK}(j, w) \) from at least \( n-t > \frac{n+t}{2} \) different simulators.

  - If \( q_j \) is faulty or never invokes \( \text{propose}(j, w) \), \( q_i \) may never receive \( \text{VALUE}’\text{ACK}(j, w) \) from more than \( \frac{n+t}{2} \) different simulators. We will show that, in this case, the wait predicate \( |\{ k : \text{answers}_i[k][i][j] = \bot \}| > \frac{n+t}{2} \) eventually becomes true.

We first show that, if a correct simulator receives \( \text{VALUE}’\text{ACK}(j, w) \) from more than \( \frac{n+t}{2} \) different simulators, then all correct simulators do receive \( \text{VALUE}’\text{ACK}(j, w) \) from more than \( \frac{n+t}{2} \) different simulators. If a correct simulator receives \( \text{VALUE}’\text{ACK}(j, w) \) from more than \( \frac{n+t}{2} \) different simulators, at least \( (t+1) \) correct simulators broadcast it. Every correct simulator will then receive the message \( \text{VALUE}’\text{ACK}(j, w) \) from at least \( (t+1) \) different simulators and, if not already done, broadcasts it (lines \( B[24]-B[25] \)). All correct simulators will then receive the message \( \text{VALUE}’\text{ACK}(j, w) \) from at least \( n-t > \frac{n+t}{2} \) different simulators.

According to the previous observation, let us consider the case in which no correct simulator ever receives the message \( \text{VALUE}’\text{ACK}(j, w) \) from more than \( \frac{n+t}{2} \) different simulators. A correct simulator \( q_k \) assigns a non-\( \bot \) value to \( \text{values}_k[j] \) only if it receives \( \text{VALUE}’\text{WITNESS}(j, w) \) from more than \( \frac{n+t}{2} \) different simulators (line \( B[22] \)), or if it receives \( \text{VALUE}’\text{ACK}(j, w) \) from more than \( \frac{n+t}{2} \) different simulators (line \( B[25] \)). If a correct simulator receives \( \text{VALUE}’\text{WITNESS}(j, w) \) from more than \( \frac{n+t}{2} \) different simulators, according to Theorem 3 all correct simulators receive \( \text{VALUE}’\text{WITNESS}(j, w) \) from more than \( \frac{n+t}{2} \) different simulators, and broadcast the message \( \text{VALUE}’\text{ACK}(j, w) \). Because no correct simulator ever receives \( \text{VALUE}’\text{ACK}(j, v_j) \) messages from more than \( \frac{n+t}{2} \) different simulators, no correct simulator \( q_k \) will ever assign a non-\( \bot \) value to \( \text{values}_k[j] \) (line \( B[22] \)).

When a correct simulator receives a \( \text{READ}(i, j) \) message from \( q_i \), it waits until it has received \( \text{VALUE}’\text{ACK}(i, v_i) \) messages from more than \( \frac{n+t}{2} \) different simulators (line \( B[24] \)). The reasoning above (first item) shows that this will eventually become true.
Every correct simulator \( q_k \) will then broadcast \( \text{READ'}\text{ANSWER}(i,j,\perp) \). This will cause all correct simulators to broadcast messages \( \text{READ'}\text{ANSWER'}\text{WITNESS}(k,i,j,\perp) \), which will be received by the simulator \( q_i \). This will then assign \( \perp \) to \( \text{answers}_i[k][i][j] \) for at least \( n-t > \frac{n+1}{2} \) different values of \( k \). Consequently, it will not remain blocked at lines B05-B06.

- wait instruction at line B13.

As simulator \( q_i \) broadcasts its view with a message \( \text{VIEW}(i,\text{view}) \) (line B12), every correct simulator checks if this view is consistent when it receives it (lines B36-B41). Let us first consider the entries \( \text{view}[j] \) such that \( \text{view}[j] = w \neq \perp \). This means that \( q_i \) has received \( \text{VALUE'}\text{ACK}(j,w) \) from more than \( \frac{n+1}{2} \) different simulators. All the correct simulators then receive the same message from a sufficient number of different simulators and do not remain blocked at line B38 (Theorem 3).

Let us now consider the entries \( \text{view}[j] \) such that \( \text{view}[j] = \perp \). Simulator \( q_i \) assigned \( \perp \) to \( \text{view}[j] \) because it received \( \text{READ'}\text{ANSWER'}\text{WITNESS}(k,i,j,\perp) \) from more than \( \frac{n+1}{2} \) different simulators (lines B32-B33). According to Theorem 3, all the correct simulators \( q_x \) will also receive \( \text{READ'}\text{ANSWER'}\text{WITNESS}(k,i,j,\perp) \) from more than \( \frac{n+1}{2} \) different simulators, and will assign \( \perp \) to \( \text{answers}_x[k][i][j] \). They will thus not remain blocked at line B39.

All the correct simulators will then broadcast the message \( \text{VIEW'}\text{WITNESS}(i,\text{view}) \) (line B42). By Theorem 3, they will all send \( \text{VIEW'}\text{ACK}(i,\text{view}) \) to \( q_i \). This will allow \( q_i \) to terminate its invocation of \( \text{propose}(i,v_i) \), which concludes the proof of the lemma.

\( \square \) Lemma 11

**Lemma 12.** Let \( v_1, \ldots, v_x, \ldots \) be the values proposed by a correct simulator \( q_i \) to a sequence of safe agreement objects. If \( q_i \) does not invoke \( \text{propose}() \) operations concurrently and \( \text{valid}_j(i,v_x) \) is eventually satisfied at every correct simulator \( q_j \), then \( \text{valid}_j(i,v_{x+1}) \) is also eventually satisfied at \( q_j \).

**Proof** We consider here that the sub-predicate \( P2 \) is always satisfied, and thus consider only the sub-predicate \( P1 \). Let us recall that \( P1 \) states that, for every message \( \text{VALUE}(i,-) \) that \( q_j \) received from \( q_i \), and for every message \( \text{VALUE'}\text{VALID}(i,-) \) that \( q_j \) received from at least \( t+1 \) different simulators, it has also received the corresponding messages \( \text{VIEW'}\text{WITNESS}(i,-) \).

By hypothesis, \( \text{valid}_j(i,v_x) \) is eventually satisfied at the correct simulator \( q_j \). Once \( q_i \) broadcasts the message \( \text{VALUE}(i,v_x) \), \( q_j \) only needs to receive the corresponding \( \text{VIEW'}\text{WITNESS}(i,\text{view}) \) for \( P1 \) to be satisfied. By Lemma 11, \( q_i \) terminates its invocation of \( \text{propose}(i,v_x) \), from which we conclude that it received \( \text{VIEW'}\text{ACK}(i,\text{view}) \) from more than \( \frac{n+1}{2} \) different simulators (line B13). A correct simulator sends such a message only if it has received \( \text{VIEW'}\text{WITNESS}(i,\text{view}) \) from more than \( \frac{n+1}{2} \) different simulators (lines B47-B48). According to Theorem 3, all the correct simulators also broadcast it (lines B44-B45). The correct simulator \( q_j \) then receives them from more than \( \frac{n+1}{2} \) different simulators. The predicate \( \text{valid}_j(i,v_{x+1}) \) is then eventually satisfied at \( q_j \).

\( \square \) Lemma 12

**Decide-termination**

**Lemma 13.** If a correct simulator terminates its invocation of \( \text{decide}() \), then all correct simulators terminate their invocation of \( \text{decide}() \).

**Proof** Suppose, by way of contradiction, that the invocation of \( \text{decide}() \) by a correct simulator \( q_i \) terminates, and that the invocation of \( \text{decide}() \) by another correct simulator \( q_j \) does not.

The invocation of \( \text{decide}() \) by \( q_i \) can terminate only if the predicate at lines C15-C16 is satisfied. Let \( q_k \) be any simulator in the set \( \sigma \) defined at line C15. We show that \( \text{all_views}_i[k] = \text{view} \) implies that we eventually have \( \text{all_views}_j[k] = \text{view} \), and thus that \( q_j \) must decide.

Simulator \( q_i \) assigns \( \text{view} \) to \( \text{all_views}_i[k] \) at line B48. This can happen only because \( q_i \) received \( \text{VIEW'}\text{WITNESS}(k,\text{view}) \) messages from more than \( \frac{n+1}{2} \) different simulators. According to Theorem 3, \( q_j \)
eventually receives enough \text{VIEW’WITNESS}(k, view) messages and also assigns view to all_views\(_j[k]\). Simulator \(q_j\) will then also have to decide.

\textbf{Lemma 14.} The invocations of \text{decide()} by all the correct simulators on all the safe agreement objects terminate, except for at most \(t\) safe agreement objects.

\textbf{Proof} Suppose, by way of contradiction, that there are \(t+1\) safe agreement objects such that at least one correct simulator never terminates its invocation of \text{decide}(). By Lemma 13 there must be \((t+1)\) different safe agreement objects in which no correct simulator terminates its invocations of \text{decide}().

The invocation of the \text{decide()} operation by a correct simulator \(q_i\) on a safe agreement object can only be blocked at lines 15–16 if the corresponding predicate is never satisfied. This can happen if (1) there is no simulator \(q_j\) such that all_views\(_i[j]\) \(\neq \bot\) or, (2) for every non-empty set of simulators \(\sigma\), there are two simulators \(q_y \in \sigma\) and \(q_z\) such that all_views\(_i[y]\)[\(z\)] \(\neq \bot\) and all_views\(_i[z]\) = \(\bot\). Because a correct simulator \(q_i\) invokes \text{propose()} before invoking \text{decide}(), case (1) cannot happen; we always have all_views\(_i[i]\) \(\neq \bot\). We then consider case (2).

Case (2) can happen if \(q_x\) starts an invocation of \text{propose()} and communicates its proposed value to other processes, but does not terminate its invocation by communicating its view. Because there are at most \(t\) faulty simulators, by the pigeonhole principle, there must be a faulty simulator \(q_z\) that prevents \(q_i\) from deciding on two different safe agreement objects.

A correct simulator \(q_k\) broadcasts a \text{VALUE’VALID}(z, –) after receiving a \text{VALUE}(z, –) message only if the predicate valid\(_k(z, –)\) is satisfied (line H15). Due to the predicate valid\(_k(z, –)\), this is true only if \(q_k\) received \text{VIEW’WITNESS}(z, –) messages from at least \((n-t)\) different simulators, each of these messages corresponding to the all the \text{VALUE}(z, –) and \text{VALUE’VALID}(z, –) messages that it has previously received (see the definition of the predicate \(P1\) of valid\(_k(z, –)\)).

Let \text{propose}(v_1) be the invocation of \text{propose()} by \(q_x\) on the first safe agreement object on which \(q_i\) is blocked, and \text{propose}(v_2) the one on the second safe agreement object on which \(q_i\) is blocked. Because there is a simulator \(q_y \in \sigma\) such that all_views\(_i[y]\) \(\neq \bot\) in the two invocations of \text{decide()} by \(q_i\), in both cases, more than \(\frac{n+t}{2}\) different simulators have broadcast a \text{VIEW’WITNESS}(y, –) message (line B48). Both sets include correct simulators. They must then have received \text{VALUE’ACK}(z, v_1) and \text{VALUE’ACK}(z, v_2) from more than \(\frac{n+t}{2}\) different simulators (line B38). Again, both sets include correct simulators that must have received \text{VALUE’WITNESS}(z, v_1) and \text{VALUE’WITNESS}(z, v_2) from more than \(\frac{n+t}{2}\) different simulators (line B21).

In order to broadcast a \text{VALUE’WITNESS}(z, –) message, a correct simulator must either (a) receive \text{VALUE’WITNESS}(z, –) messages from at least \(t+1\) different simulators (line H18), or (b) receive \text{VALUE’VALID}(z, –) messages from more than \(\frac{n+t}{2}\) different simulators (line B16). The first correct simulator that broadcasts a \text{VALUE’WITNESS}(z, –) message must then have received \text{VALUE’VALID}(z, –) messages from more than \(\frac{n+t}{2}\) different simulators.

According to Lemma 10 there is a least one correct simulator \(q_\ell\) that broadcasts both \text{VALUE’VALID}(z, –) messages (line H15). In order to do so, the predicate valid\(_\ell(z, v_2)\) must have been verified at the time that \(q_\ell\) broadcast the \text{VALUE’VALID}(z, v_2) message. It must then have received the \text{VIEW’WITNESS}(z, view) messages that correspond to \(v_1\) from more than \(\frac{n+t}{2}\) different simulators. According to Theorem 3, \(q_i\) must then also have received these messages from more than \(\frac{n+t}{2}\) different simulators and assigned view to all_views\(_i[z]\) (line B48) in the instance that corresponds to the invocation of \text{propose}(v_1) by \(q_x\), a contradiction that concludes the proof of the lemma.

\textbf{Agreement}

\textbf{Lemma 15.} For any simulator \(q_x\) and any correct simulator \(q_i\), if \(q_i\) assigns a non-\(\bot\) value \(v\) to values\(_i[x]\), then (1) no value \(v’ \neq v\) is ever assigned to values\(_j[x]\) by a correct simulator \(q_j\) and (2) each such correct simulator \(q_j\) eventually assigns \(v\) to values\(_j[x]\).
Suppose that there exists a value \( v' \neq v \) such that there is a correct simulator \( q \) that assigns \( v' \) to \( \text{values}_k[x] \). Suppose that \( q \) is the first process to do so. It follows that \( q \) received \( \text{VALUE}'\text{WITNESS} \ (x, v') \) messages from strictly more than \( \frac{n+1}{2} \) different simulators (line B21 or line B24).

Consider the first correct simulator that broadcasts a \( \text{VALUE}'\text{WITNESS} \ (x, v') \) message. In order to do so, it must have received \( \text{VALUE}'\text{VALID} \ (x, v') \) messages from strictly more than \( \frac{n+1}{2} \) different processes (lines B16-B17). However, the first correct simulator that broadcasts a \( \text{VALUE}'\text{WITNESS} \ (x, v') \) message must also have received \( \text{VALUE}'\text{VALID} \ (x, v) \) messages from strictly more than \( \frac{n+1}{2} \) different processes. There must then be a correct simulator that sent both \( \text{VALUE}'\text{VALID} \ (x, -) \) messages. The only place a correct simulator can send a \( \text{VALUE}'\text{VALID} \ (x, -) \) message is at Line 15 and it does so only once for each simulator \( q \), a contradiction which concludes the proof of the lemma.

**Proof** Let \( q_k \) be the first simulator that assigns \( v \) to \( \text{values}_k[x] \). Since \( q_k \) executes line B22 it received strictly more than \( \frac{n+1}{2} \) \( \text{VALUE}'\text{WITNESS} \ (x, v) \) messages from different simulators. At least \( t + 1 \) correct simulators consequently sent this message to all processes at line B17 or at line B19. By Theorem 3, every correct simulator \( q_j \) consequently eventually receives such a message from each correct simulator and assigns \( v \) to \( \text{values}_j[x] \).

Lemma 16. *For any simulators \( q_k, q_\ell, q_x \) and any correct simulator \( q_i \), if \( q_i \) assigns a non-\( \perp \) value \( v \) to \( \text{values}_i[x] \), then (1) \( \) no value \( v' \neq v \) is ever assigned to \( \text{values}_i[x] \) by a correct simulator \( q_j \) and (2) each such correct simulator \( q_j \) eventually assigns \( v \) to \( \text{values}_j[x] \).*

**Proof** The proof is the same as for Lemma 15

Lemma 17. *For any simulator \( q_x \) and any correct simulator \( q_i \), if \( q_i \) assigns a non-\( \perp \) value \( \text{view} \) to all \( \text{views}_i[x] \), then (1) no value \( \text{view}' \neq \text{view} \) is ever assigned to all \( \text{views}_j[x] \) by a correct simulator \( q_j \) and (2) each such correct simulator \( q_j \) eventually assigns \( \text{view} \) to all \( \text{views}_j[x] \).*

**Proof** The proof is the same as for Lemma 15

Lemma 18. *No two invocations of \( \text{decide()} \) return different values.*

**Proof** Let us recall that the algorithm implementing the operation \( \text{decide()} \) is described at lines C15-C19. Let \( q_i \) and \( q_j \) be two correct simulators. According to Lemmas 15-17 we have:

- \( \text{values}_i[x] \neq \perp \) \( \land \) \( \text{values}_j[x] \neq \perp \) \( \Rightarrow \) \( \text{values}_i[x] = \text{values}_j[x] \).
- \( \text{values}_i[x] \neq \perp \) \( \land \) \( \text{values}_j[x] \neq \perp \) \( \Rightarrow \) \( \text{values}_i[x] = \text{values}_j[x] \).
- \( \text{all_views}_i[x] \neq \perp \) \( \land \) \( \text{all_views}_j[x] \neq \perp \) \( \Rightarrow \) \( \text{all_views}_i[x] = \text{all_views}_j[x] \).

Let us assume, by contradiction, that \( q_i \) and \( q_j \) decide different values. This means that the sets \( \text{min}_\sigma_i \) and \( \text{min}_\sigma_j \) computed at line C17 by \( q_i \) and \( q_j \), respectively, are different.

Since \( \text{min}_\sigma_i \) and \( \text{min}_\sigma_j \) are different, let us consider \( z \in \text{min}_\sigma_i \setminus \text{min}_\sigma_j \) (if \( \text{min}_\sigma_i \subseteq \text{min}_\sigma_j \), swap \( i \) and \( j \)). According to the closure predicate used at line C16, as \( z \notin \text{min}_\sigma_j \), we have \( \forall y \in \text{min}_\sigma_j : \text{all_views}_j[y][z] = \perp \).

It follows that \( q_j \) received \( \text{VIEW}'\text{WITNESS} \ (y, \text{all_views}_j[y]) \) messages (with \( \text{all_views}_j[y][z] = \perp \)) from a set of simulators \( Q_{j,vw} \) of size strictly larger than \( \frac{n+1}{2} \) (the subscript \( vw \) stands for “view witness”). The correct simulators of \( Q_{j,vw} \) sent these messages after checking at line B39 that a set \( Q_{j,vw,r} \) of strictly more than \( \frac{n+1}{2} \) reliably broadcast (thanks to the mechanism of lines B26 to B33) a \( \text{READ}'\text{ANSWER} \ (y, z, \perp) \) message. The correct simulators of \( Q_{j,vw,r} \) sent these messages at line B26 after they received \( \text{VALUE}'\text{ACK} \ (y, v_y) \) messages from a set \( Q_{y,w} \) of strictly more than \( \frac{n+1}{2} \) simulators (the subscript \( w \) stands for “witness”). Each correct simulator \( q_y \) of \( Q_{y,w} \) had \( \text{values}_k[y] = v_y \) when it sent this message and it happens strictly before the first correct simulator sends a \( \text{READ}'\text{ANSWER} \ (y, z, \perp) \) message.
Since $z \in \text{min}_\sigma$, the correct simulator $q_i$ received $\text{VIEW’WITNESS}(z, \text{all_views}_z[z])$ messages from a set $Q_{i,vw}$ of strictly more than $\frac{n+2}{2}$ simulators. The correct simulators of $Q_{i,vw}$ sent these messages after the check of the values at lines B38–B39.

Suppose that some of them verified the predicate of line B39 for $x = y$. It entails that a set $Q_{i,vw,r}$ of strictly more than $\frac{n+4}{2}$ simulators reliably broadcast a $\text{READ’ANSWER} (z, y, \perp)$. The correct simulators of $Q_{i,vw,r}$ sent this message after receiving at line B24 $\text{VALUE’ACK}(z, v_z)$ messages from a set $Q_{z,w}$ of strictly more than $\frac{n+4}{2}$ simulators. This happens strictly before the first $\text{READ’ANSWER} (z, y, \perp)$ message is sent by a correct simulator. Since $|Q_{i,vw,r}|, |Q_{j,vw,r}| > \frac{n+4}{2}$, $Q_{i,vw,r} \cap Q_{j,vw,r}$ contains at least a correct simulator $p_k$.

Simulator $p_k$ thus broadcast a $\text{READ’ANSWER} (y, z, \perp)$ message and a $\text{READ’ANSWER} (z, y, \perp)$ message (line B26). It then had $\text{views}_k[z] = \perp$ before broadcasting the $\text{READ’ANSWER} (y, z, \perp)$ message and $\text{views}_k[y] = \perp$ before broadcasting the $\text{READ’ANSWER} (z, y, \perp)$. Because of the first instruction of line B25, this is impossible, and thus each correct process that sends a $\text{VIEW’WITNESS}(z, \text{all_views}_z[z])$ message ended the wait instruction of lines B38–B39 by verifying the predicate of line B39. This entails that $\forall x \in \Pi : \text{all_views}_x[z] \neq \perp \Rightarrow \text{all_views}_x[z][y] \neq \perp$. Consequently, $\text{all_views}_x[z][y] \neq \perp$.

Since $z \in \text{min}_r$, $\text{all_views}_z[z] \neq \perp$ and thus $\text{all_views}_z[z][y] \neq \perp$. According to the predicate of line C16 this entails that $y \in \text{min}_r$, and since the previous reasoning holds for any $y \in \text{min}_r$, it shows that $\text{min}_r \subseteq \text{min}_i$. It follows that, when $q_i$ executes line C17 $\forall y \in \text{min}_r : \text{all_views}_i[y] \neq \perp$ and, consequently, $\forall y \in \text{min}_r : \text{all_views}_i[y] = \text{all_views}_i[y]$. It entails that if $|\text{min}_r| < |\text{min}_i|$, then $\text{min}_r$ would have been chosen by $q_i$ at line C17 which proves that $\text{min}_r = \text{min}_i$ and contradicts the fact that $q_i$ and $q_j$ decide differently.

**Lemma 19.** If a correct simulator $q_i$ decides the value $v$, there is a correct simulator $q_j$ such that $\text{valid}_j(-, v)$.

**Proof**

Let $v$ be the value decided by a correct simulator $q_i$. Value $v$ has then be proposed by a simulator $q_j$ such that $\text{all_views}_i[j] \neq \perp$ (definition of $\sigma$ at lines 15–16 and choice of value at line C18). In order to assign a non-$\perp$ value to $\text{all_views}_i[j]$, $q_i$ must have received $\text{VIEW’WITNESS}(j, -)$ messages from more than $\frac{n+2}{2}$ different simulators (lines B47–B48), and consequently from at least one correct simulator. Consider the first correct simulator $q_k$ that has broadcast a $\text{VIEW’WITNESS}(j, -)$ message. Before sending it, it must have assigned a non-$\perp$ value to $\text{values}_s[j]$ (lines B35–B42). It then has received either (a) $\text{VALUE’WITNESS}(j, -)$ messages from more than $\frac{n+4}{2}$ different simulators or (b) $\text{VALUE’ACK}(j, -)$ messages from more than $\frac{n+4}{2}$ different simulators.

In case (a), consider the first correct simulator $q_k$ that has broadcast a $\text{VALUE’WITNESS}(j, -)$ message. In order to do so, it must have received $\text{VALUE’VALID}(j, -)$ messages from more than $\frac{n+4}{2}$ different simulators (lines B16–B17). The predicate $\text{valid}_k(j, v)$ must have been satisfied at the simulators that broadcast these messages (line B15). In case (b), the first correct simulator that has broadcast a $\text{VALUE’ACK}(j, -)$ message must first have received $\text{VALUE’WITNESS}(j, -)$ messages from more than $\frac{n+4}{2}$ different simulators (lines B21–B23). The situation is then similar to Case (a).

**Theorem 4.** The algorithms described in Figure 3 and Figure 4 implement a safe-agreement object in $\text{BAMP}_{n,t}[t < n/3]$.

**Proof** The proof follows from the previous lemmas.

### 5.6 Simulation algorithm and its proof in $\text{BAMP}_{n,t}[t < n/3]$

**Simulation algorithm** When we consider the simulation algorithm described in Figure 2, we observe that the $n$ simulators communicate only through safe agreement objects. It follows that the same algorithm works
in $\mathcal{BAMP}_{n,t}[t < n/3]$, when the crash-tolerant safe agreement objects are replaced by Byzantine-tolerant safe agreement objects previously described. Two things remain to be done: define the specific sub-predicate $P_2$ of the predicate $\text{valid}()$, and do a specific proof of this algorithm (i.e., a proof based on the specification of the Byzantine-tolerant safe agreement objects defined in Section 5.2).

Sub-predicate $P_2$  As far as $P_2$ is concerned we have the following. Let us consider the simulator $q_i$ that invokes $\text{valid}_i(j,v)$, with respect to the simulation of a process $p_x$. In the simulation algorithm, the parameter $v$ is the message $\text{msg}$ that $q_j$ proposes to a safe agreement object from which will be decided the next message to be received by the simulated process $p_x$ (lines [8][9] of Figure 2). $P_2$ checks, from $q_i$’s local point of view, that, if the message $v$ has been sent in the simulation, then it has not yet been consumed, i.e., $(v \in \text{sent}_i[x]) \Rightarrow (v \notin \text{received}_i[x])$.

Proof of the simulation algorithm in $\mathcal{BAMP}_{n,t}[t < n/3]$

Lemma 20. The simulation of at most $t$ simulated processes can be blocked.

Proof  The only places where a correct simulator $q_i$ can block is during the invocation of the safe agreement operation $\text{decide}()$. Such invocations appear at line [02] and line [11].

Because the invocations by all the correct simulators of $\text{decide}()$ on all the safe agreement objects terminate, except for at most $t$ safe agreement objects (Lemma 14), the simulation of at most $t$ simulated processes can be blocked.  $\Box$ Lemma 20

Lemma 21. The simulation of the reception of the $k$-th message received by a simulated process $p_j$, returns the same message at all correct simulators.

Proof  The simulation of the message receptions for a simulated process $p_j$, are executed at each correct simulator $q_i$ at lines [08][11], and all the correct simulators use the same sequence of sequence numbers (line [07]). It then follows from the agreement property of the safe agreement object $\text{SA}[j, sn]$, that no two correct simulators obtain different messages when they invoke $\text{SA}[j, sn].\text{decide}()$, and the lemma follows.  $\Box$ Lemma 21

Lemma 22. For every simulated processes $p_j$, no two correct simulators return different values.

Proof  The only non-deterministic elements of the simulation are the input vectors $\text{input}_i[1..n']$ at each simulator $q_i$, and the reception of the simulated messages.

The lines [01][02] of the simulation force the correct simulators to agree on the same input value for each simulated process $p_j$, $1 \leq j \leq n'$. Similarly, as shown by Lemma 21, for each simulated process $p_j$, the lines [07][11] direct the simulators to agree on the very same sequence of messages received by $p_j$. It follows from the fact that the function $\delta_j()$ is deterministic, that any two correct simulators $q_i$ and $q_k$, that execute lines [15][16] during the same “round number” $\text{sn}_i[j] = \text{sn}_k[j]$, are such that $\text{state}_i[j] = \text{state}_k[j]$, from which the lemma follows.  $\Box$ Lemma 22

Lemma 23. The sequences of message receptions simulated by each simulator $q_i$ on behalf of each simulated process $p_j$, define a correct execution of the simulated algorithm.

Proof  To prove the correctness of the simulation, we have to show that

1. Every message that was received by a simulated process was sent by another simulated process,
2. Every message that was sent by a simulated process to another simulated process (whose simulation is not blocked either), is received, and
3. The simulated messages respect a simulated physical order (i.e., no message is “received” before being “sent”).
Item 1 follows from Lemma 19 and from the definition of \( P_2 \). Item 2 is satisfied because the messages sent by the simulated process \( p_j \) to the simulated process \( p_k \) are received (lines 09 and 11) in their sending order (as defined at line 04 and line 14). Hence, if \( p_k \) is not blocked (due to a faulty simulator) it obtains the messages from \( p_j \) in their sending order.

For Item 3, let us define a (simulated) physical order as follows. For each simulated message \( m \), let us consider the first time at which the reception of \( m \) was simulated (i.e., this occurs when for the first time a simulator terminates the invocation of \( SA[-, -].\text{decide()} \) that returns \( m \)). A message that is decided has been proposed by a simulator to a safe agreement object before being decided (validity property). The sending time of a simulated message is then the first time at which \( SA[-, -].\text{propose}(m) \) is invoked by a simulator. It follows that any simulated message is sent before being received, which concludes the lemma.

**Lemma 24.** Each correct simulator \( q_i \) computes the decision value of at least \((n' - t)\) simulated processes.

**Proof** Due to Lemma 20 and the fact that at most \( t \) simulators may be byzantine, it follows that at most \( t \) simulated processes may be prevented from progressing. As (a) by assumption the simulated algorithm \( A' \) is \( t \)-resilient, and (b) due to Lemma 23 the simulation produces a correct simulation of \( A' \), it follows that at least \((n' - t)\) simulated processes decide a value.

**Theorem 5.** Let \( A \) be an algorithm solving a decision task in \( CAMP_{n', t}[t < n'] \). The algorithm described in Figure 2 in which Byzantine-tolerant safe agreement objects are used, is a correct simulation of \( A \) in \( BAMP_{n, t}[t < n/3] \).

**Proof** The theorem follows from Lemma 23 and Lemma 24.

Additionally, the reader can easily check that the simulation of a message only requires a polynomial number of messages in the base system, and the increase in size of these messages, when compared to the size of the simulated message, is also polynomial.

### 6 Implications of the Simulation

**BG-simulation in Byzantine message-passing systems** A main result of this paper is a signature-free distributed algorithm that solves BG-simulation in Byzantine asynchronous message-passing systems. In addition to being the first algorithm that solves BG-simulation in such a severe failure context, the proposed simulation algorithm has noteworthy applications as shown below.

**From Byzantine-failures to crash failures in message-passing systems** The simulation presented here allows the execution of a \( t \)-resilient crash-tolerant algorithm in an asynchronous message-passing system where up to \( t \) processes may be Byzantine. A feature that is sometimes required from a Byzantine-tolerant algorithm solving a task (not usually considered in the crash failure case) is that the value decided by any correct process should be based only on inputs of correct processes. This prevents Byzantine processes from “polluting” the computation with their inputs. A way to guarantee that an input has been proposed by a correct process is to check that it has been proposed by at least \((t + 1)\) different processes. Assuming that in any execution at most \( m \) values are proposed, this constraint translates as \( n - t > mt \).

In the case of the simulation presented in Section 5 this requirement can easily be satisfied by adding a first step of computation before the start of the simulation. Simulators first broadcast their input. They then echo every value that they receive from more than \( t + 1 \) different simulators, and consider these values (and only these values) as valid inputs. An input considered valid by a correct simulator is then eventually considered valid by all correct simulators, and the only inputs allowed in the simulation are inputs of correct simulators. Because we consider colorless tasks, the choice of output is done in the same way as in the original BG-simulation: a simulator can adopt the output of any simulated process that has decided a value.
The possible Byzantine behaviors are restrained by the underlying Byzantine-tolerant safe agreement objects used in the simulation. Surprisingly, this shows that, from the point of view of the computability of colorless tasks and assuming $n > (m + 1)t$ (this requirement always implies $n > 3t$ when at least two different values can be proposed), Byzantine failures are equivalent to crash-failures. This provides us with a new understanding of Byzantine failures and shows that their impact can be restricted to the much simpler crash-failure case.

From wait-free shared memory to message-passing  

The proposed simulation can be combined with previous works to further extend the scope of the result. Consider an algorithm $A_0$ that solves a colorless task, where $m > 1$, in a wait-free read/write memory system of $t + 1$ processes, denoted $CARW_{t+1,t}[0]$. Using the basic BG-simulation [6], this algorithm can be transformed into an algorithm $A_1$ that works in the $t$-resilient read/write memory system of $(m + 1)t + 1$ processes, in which at most $t$ can crash. This model is denoted $CARW_{(m+1)t+1,t}[0]$. Using an implementation of a read/write memory in a crash-prone message-passing system in which a majority of processes are correct [2], we obtain an algorithm $A_2$ which work in $CAMP_{(m+1)t+1,t}[0]$ (message-passing system system of $(m + 1)t + 1$ processes, in which at most $t$ can crash; notice that $m > 0 \Rightarrow (m + 1)t + 1 > 2t$). Finally, using the simulation presented in this paper, we obtain Byzantine-tolerant algorithm $A_3$ which works in $BAMP_{(m+1)t+1,t}[0]$ (message-passing system of $(m + 1)t + 1$ processes, of which at most $t$ can be Byzantine; notice that $m > 1 \Rightarrow (m + 1)t + 1 > 3t$).

These transformations show that, as far as the computability of colorless tasks that admit up to $m > 1$ different input values is concerned, an $n$-process Byzantine-prone message-passing system, in which up to $t < n/(m + 1)$ processes can be Byzantine, is equivalent to a wait-free shared memory system of $t + 1$ processes, which at most commit crash failures. When considering colorless tasks with $m > 1$, a figure relating these transformations is depicted in Figure 6. Differently from the full-information algorithm presented in [28], the simulation presented in the present paper (along with [6] and [2]) allows a direct transformation of any wait-free shared-memory algorithm that solves a colorless task into a message-passing Byzantine-tolerant algorithm.

$$A_0 \text{ in } CARW_{t+1,t}[0]$$

BG simulation [6]

$$A_1 \text{ in } CARW_{(m+1)t+1,t}[0]$$

ABD simulation [2]

$$A_2 \text{ in } CAMP_{(m+1)t+1,t}[0]$$

This paper

$$A_3 \text{ in } BAMP_{(m+1)t+1,t}[0]$$

Figure 6: From crash in read/write to Byzantine in message-passing (with $m > 1$)

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