Warped accretion disks and the unification of Active Galactic Nuclei

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ABSTRACT
Orientation of parsec-scale accretion disks in AGN is likely to be nearly random for different black hole feeding episodes. Since AGN accretion disks are unstable to self-gravity on parsec scales, star formation in these disks will create young stellar disks, similar to those recently discovered in our Galactic Center. The disks blend into the quasi-spherical star cluster enveloping the AGN on time scales much longer than a likely AGN lifetime. Therefore, the gravitational potential within the radius of the black hole influence is at best axi-symmetric rather than spherically symmetric. Here we show that as a result, a newly formed accretion disk will be warped. For the simplest case of a potential resulting from a thin stellar ring, we calculate the disk precession rates, and the time dependent shape. We find that, for a realistic parameter range, the disk becomes strongly warped in few hundred orbital times. We suggest that this, and possibly other mechanisms of accretion disk warping, have a direct relevance to the problem of AGN obscuration, masing warped accretion disks, narrow Fe Kα lines, etc.

1 INTRODUCTION
Observations show that a significant fraction, perhaps a majority, of AGN of different types are obscured by a screen of a cold dusty matter, thought to be a molecular torus-like structure with a scale between 1 to a 100 parsec (Antonucci & Miller 1985, Sazonov & Revnivtsev 2004). Given that stellar processes. Considering “large”, e.g. 100 parsec torii, and via numerical simulations of supernova explosions resulting from star formation in a self-gravitating, massive disk, they have shown that large random cold gas velocities result from the interaction of the gas with the supernova shells. However as such about 99% of the gaseous disk in the simulations is consumed in the star formation episode rather than being accreted by the black hole. Unless most of the stars formed in the disk are later somehow accreted onto the SMBH, this mechanism can only work for torii on scales much larger than the SMBH gravitational sphere of influence, i.e. where the stellar mass is much greater than \( M_{\text{BH}} \). It is also not clear whether this mechanism would work for smaller accretion/star formation rates because then supernova explosions become too rare. Finally, time variability of obscuring column depths argues for much smaller radial sizes of the “torus”, e.g. in the range 0.1 – 10 parsec (Risaliti, Elvis, & Nicastra 2002).

Here we would like to emphasize that accretion disks in AGN are very unlikely to be planar. There are several mechanisms that are capable of producing strong warps. We believe that such warps have to be an integral part of the AGN obscuration puzzle. In particular, we shall describe and calculate a disk warping mechanism driven by an axisymmetric gravitational potential.

The sequence of events that takes place in an AGN feeding cycle in our model is as following. First of all, a merger with another galaxy or a satellite, or another source of cold gas, fills the inner part of the galaxy with plenty of gas. This gas will in general have a significant angular momentum ori-
The initially flat disk will be warped with time. \( \omega \), i.e. circular orbits. Since precession rates exactly conserved (see, e.g., Krajnovic & Jaffe 2004). Stars are formed inside the accretion disk, producing a flat stellar system. This long-lived axi-symmetric (or perhaps also warped) structure will torque any orbit which is not exactly co-planar with it, resulting in a precession around the symmetry axis. Now, as the time goes on, the orientation of the angular momentum vector of the incoming gas changes, and the newly built disk is exposed to the torque from the stellar disk remnant. Different rings of the disk precess at different rates, thus the disk becomes warped.

We shall emphasize that existence of these stellar disk remnants is hardly a question based on the severe self-gravity problems for the standard accretion disks at large radii. The recent observational evidence supports this conclusion too. The best known example of such flat stellar systems is in our Galactic Center (Genzel et al. 2003) where two young stellar rings are orbiting the SMBH only a tenth of a parsec away. The orbital planes of these rings are oriented at very large angles to the Galactic plane. Such flat stellar systems are also being found elsewhere (e.g., Krajnovic & Jaffe 2004).

In this paper, we concentrate on the linear gravitational warping effect to investigate its main features. Taking the simplest case of a warp produced by a massive ring, we calculate the gravitational warping torque. Starting from a thin test accretion disk, we calculate its time dependent shape. Under quite realistic assumptions, the disk becomes strongly warped in some 10\(^{-3}\) years. Since the warping is gravitational in nature, gaseous and clumpy molecular disks alike are subject to such deformations. We speculate that in the non-linear stage of the effect, a clumpy warped disk will form a torus-like structure.

## 2 TORQUES IN A LINEAR REGIME

The basic physics of the effect is very simple and has been explained by e.g., Binney 1992 in his consideration of the galactic warps (most galactic disks are somewhat warped). Consider a nearly circular orbit of radius \( R \) for a test particle in an axi-symmetric potential (e.g. a central point mass plus a flat disk). Let the axis \( z \) to be perpendicular to the plane of the disk. The particle makes radial and vertical oscillations with slightly different frequencies, and the difference is the precession frequency, \( \omega_p \). Due to the symmetry, the \( z \)-projection of the angular momentum of the particle is exactly conserved (see, e.g., §3.2 in Binney & Tremaine 1987). Thus the angle between the angular momentum of the particle and the \( z \)-axis, \( \beta \), is a constant. The line of the nodes (the line over which the two planes intersect), however, precesses.

Now consider a “test-particle” disk initially in the same plane as the circular orbit. The disk is a collection of rings, i.e. circular orbits. Since precession rates \( \omega_p \) are different for different \( R \) rings turn around the \( z \)-axis on unequal angles. The initially flat disk will be warped with time.

### 2.1 Gravitational torques between two rings

Let us calculate the gravitational torque between two rings with radii \( R_1 \) and \( R_2 \), inclined at angle \( \beta \) with respect to each other. We work in two rigid coordinate systems, \( x, y, z \) and \( x', y', z' \), centers of which are at the SMBH. For convenience, we place the first ring in the \( z = 0 \) plane, whereas the second ring is at \( z' = 0 \) plane. Angle \( \beta \) is obviously the angle between the axes \( z \) and \( z' \). Further, the axes \( x \) and \( x' \) are chosen to coincide with the line of the nodes.

The total torque exerted by the second ring on the first one is

\[
\tau_{21} = G \sigma_1 R_1 \sigma R \int_0^{2\pi} d\phi_1 \int_0^{2\pi} d\phi_2 \frac{[\vec{r}_1 \times \vec{r}_2]}{|\vec{r}_1 - \vec{r}_2|^3}
\]

(1)

where \( \sigma_1 = M_1/2\pi R_1 \) and \( \sigma = M/2\pi R \), with \( M_1 \) and \( M \) being the masses of the two rings, respectively. The integration goes over angles \( \phi_1 \) and \( \phi_2 \) that are azimuthal angles in the respective frames of the rings. From this expression it is immediately clear that co-planar rings do not exert any torque on each other, \( [\vec{r}_1 \times \vec{r}_2] = 0 \). In addition, if \( \beta = \pi/2 \), the integral vanishes as well because for each \( \vec{r}_1 \) the opposite sides of the second ring (e.g. \( \phi \) and \( \phi + \pi \)) make equal but opposing contributions.

It is also possible to show that due to symmetry the torque’s \( z \)-projection vanish. Thus we only have the \( \tau_{21,x} = -\tau_{12,x} \) component, meaning that the angular momentum vector of the ring will rotate without changing its magnitude. Also, if \( M_1 \gg M \), then one can neglect warping of the first ring.

The absolute distance between two ring elements with the respective angles \( \phi_1 \) and \( \phi_2 \) is

\[
|\vec{r}_1 - \vec{r}_2|^2 = R_1^2 + R^2 - 2 R_1 R \cos \lambda,
\]

(2)

where

\[
\cos \lambda = \cos \beta \sin \phi_1 \sin \phi_2 + \cos \phi_1 \cos \phi_2.
\]

(3)

Without going into tedious detail, we write the torque expression separating out the leading radial dependence and the integral over angles, which we label \( I(\delta, \beta) \):

\[
\tau_{21,x} = \frac{G M_1 M R_1 R}{(R_1^2 + R^2)^{3/2}} I(\delta, \beta),
\]

(4)

with

\[
I(\delta, \beta) \equiv \sin \beta \int_0^{2\pi} \frac{d\phi_1}{2\pi} \int_0^{2\pi} \frac{d\phi_2}{2\pi} \frac{\sin \phi_1 \sin \phi_2}{[1 - \delta \cos \lambda]^{3/2}}
\]

(5)

and

\[
\delta \equiv \frac{2 R_1 R}{R_1^2 + R^2}.
\]

(6)

Total angular momentum of the second ring is \( L = M \Omega_R R^2 \). Recall that \( L_x = L \cos \beta = \text{const} \), whereas the component of \( L \) in the plane of the first ring \( (z = 0) \) precesses. We chose it to be initially in the \( y \)-direction, so that \( L_y(t = 0) = L \sin \beta \) (cf. equation (12) below). The precession frequency for the second ring is then

\[
\omega_p = \frac{\tau_{12,x}}{L_y} = \frac{\tau_{12,x}}{M \Omega_R R^2 \sin \beta},
\]

(7)

In general, the integral in equation (5) is calculated numerically, but for \( \delta \ll 1 \) one can decompose \( [1 - \delta \cos \lambda]^{-3/2} \approx 1 + 3/2 \delta \cos \lambda \) and obtain \( I(\delta, \beta) \approx \)
(3/8)δ sin 2β. When this approximation holds, that is when $R \gg R_1$ or $R \ll R_1$, the precession frequency for the second ring is

$$\frac{\omega_p}{\Omega_K} \approx -\frac{3M_1}{4M_{BH}} \cos \beta \frac{R^3 R_1^2}{[R^2 + R_1]^{5/2}}.$$ (8)

Few estimates can now be made. First of all, $\omega_p$ reaches a maximum at radius $R_\text{m}$ at which $R_\text{m}/R_1 = \sqrt{3/7}$. The maximum growth rate of the warp is thus

$$\max |\omega_p| \approx 0.085 \Omega_K \frac{M_1}{M_{BH}}.$$ (9)

$M_1/M_{BH}$ may be expected on the order of a percent or so since this is when the accretion disks become self-gravitating (e.g., Nayakshin & Cuadra 2004), and the resulting stellar mass would probably be of that order too. In this case one notices that to produce a sufficiently large warp one has to wait for at least $\sim 10 \times \Omega_{BH}/M_1 \sim 1000 \Omega_K^{-1}$ or $\sim 150$ orbital times at the radius of the maximum warp $R_\text{m}$. While this is a fairly long time, it is still much shorter than the corresponding disk viscous time (cf., e.g., Fig. 2 in Nayakshin & Cuadra 2004).

The asymptotic dependence of precession frequency on radius $R$ is

$$\omega_p \rightarrow -\frac{3M_1}{4M_{BH}} \cos \beta \frac{R^3}{R_1^3} \Omega_K \quad \text{for} \quad R \ll R_1,$$ (10)

$$\omega_p \rightarrow -\frac{3M_1}{4M_{BH}} \cos \beta \frac{R_1^2}{R^2} \Omega_K \quad \text{for} \quad R \gg R_1.$$ (11)

A thin massive stellar ring would thus only warp a range of radii, leaving the portions of the disk much internal and also much external to it unaffected.

### 2.2 Test disk warping

We now want to calculate the shape of a light non-self-gravitating disk warped by a massive ring $R_1$. The disk is treated as a collection of rings with different radii $R$ and negligible mass. We assume that the mass of the whole disk, $M$, is negligible in comparison with the mass of the ring, $M \ll M_1$, and that therefore the massive ring’s orientation (angular momentum) does not change with time.

In application to the real disks, it should be remembered that any attempt to warp a disk will be resisted by disk viscous forces (e.g., Bardeen & Petterson 1974) that transfer the angular momentum through the disk. However, we are interested in the outermost regions of AGN accretion disks where they “connect” to the galaxy. On these, a tenth of a parsec to 100 parsec scales (the range depends on $M_{BH}$), the usual α-disk viscosity (Shakura & Sunyaev 1973) becomes ineffective. The viscous scale becomes too long and the disks are believed to be prone to self-gravity (e.g., Paczynski 1978, Shlosman & Begelman 1989, Collin & Zahn 1990, Goodman 2001). Therefore we are justified in neglecting the restoring viscous forces. A much more important effect will be the restoring force from the self-gravity of the disk that is being warped, but we defer the study of this and other non-linear effects to a future paper.

It is convenient to work with angle $\beta$, already introduced, and one additional angle, $\gamma$. Recall that angle $\beta$ is the local angle of the ring’s tilt to the z-axis and it remains constant in the test particle regime. Angle $\gamma$ is needed to introduce the projections of the unit tilt vector normal to the ring, $\hat{l}(R,t)$, on the $x$ and $y$ axes (Pringle 1990):

$$\hat{l} = (\cos \gamma \sin \beta, \sin \gamma \sin \beta, \cos \beta).$$ (12)

Angle $\gamma$ therefore describes the precession of each ring around the z-axis. With the chosen coordinate system, at time $t = 0$, $\gamma = \pi/2$ (and we get, in accord with conventions of Shlosman & Begelman 1989, $L_y(0) = L \sin \beta$, $L_z(0) = 0$). As each of the disk rings precesses,

$$\gamma = \pi/2 + \omega_p t.$$ (13)

It is useful to write the expression for the unit tilt vector in the ($x'$, $y'$, $z'$) coordinate system rigidly bound to the initial disk plane:

$$l'_x = \sin \beta \cos \gamma,$$ (14)

$$l'_y = -\cos \beta \sin \beta (1 - \sin \gamma),$$ (15)

$$l'_z = \cos^2 \beta + \sin^2 \beta \sin \gamma.$$ (16)

Note that if $\beta = 0$, $l'_x = l'_y = 0$, e.g. the rings are never tilted with respect to the $x' = z'$ axis, as it should. Also, when $\gamma = \pi/2, \hat{U}$ indeed coincides with the z'-axis, i.e. the disk is flat.

Using equations (14-16), we can now calculate the shape of the warped disk in that system given the function $\gamma(R,t)$. To accomplish this, one first introduces the azimuthal angle $\phi$ on the surface of the ring. The coordinates of the points on the ring, $\vec{r}'$, are then given by equations 2.2 and 2.3 in Pringle (1990). The corresponding coordinates in the primed system of reference are easily obtained by $x' = (\hat{r} \hat{e}'_x)$, etc., where $\hat{e}'_x$ and so on are the unit coordinate vectors of the primed system. The result is:

$$x'/R = \cos \beta \cos \gamma \sin \phi + \sin \gamma \cos \phi,$$ (17)

$$y'/R = -\cos \beta \cos \gamma \cos \phi + \sin \phi (\cos^2 \beta \sin \gamma + \sin^2 \beta),$$ (18)

$$z'/R = -\sin \beta [\cos \gamma \cos \phi + (1 - \sin \gamma) \cos \beta \sin \phi].$$ (19)

### 2.3 An example

To illustrate the results, we calculate the precession rates $\omega_p(R)$ for the following case, $R_1 = 2, M_1 = 0.01 M_{BH}, \beta = \pi/4$. The resulting shape of the disk in the original un-warped disk plane is plotted in Figure 1. The time is in units of $\Omega_K^{-1}$ at $R = 1$, i.e. $t = 1$ corresponds to time $t = 1400, r_{pc}^3 / M_{BH}^{-3/4}$ years, where $r_{pc}$ is the distance in units of parsec and $M_8 = M_{BH}/10^8 M_\odot$. Note that the warp is strongest around radius $R \sim R_1 = 2$, as expected. The inner disk is hardly tilted, which is not surprising given that $\omega_p(R) \rightarrow 0$ for small $R$ (equation 10). Same is true for the outer radii, where the small tilt of the original plane can be seen on the edges of the disk (we do not calculate the tilt beyond $R = 5.2$ and hence the original disk plane is still seen on the edges of the Figure).

### 3 DISCUSSION

#### 3.1 Obscuration of the central engine in AGN

We believe that the gravitational disk warping due to non-spherical mass distribution within the SMBH sphere of in-
Figure 1. Snapshots of the shape of a massless accretion disk warped by a stellar ring of radius $R = 2$ inclined at angle $\beta = \pi/4$ with respect to the disk. The snapshots are for four different time as indicated on the top of each panel. The time unit is $1/\Omega_K (R = 1) = 1400 r_{pc}^3 M_{\odot}^{-1/2}$ years. At times larger than those used in the Figure, the disk becomes warped so much that, looking from its initially non-warped plane, the plane equation $z'(x', y')$ becomes a multiple-valued function for some $(x', y')$. In reality non-linear effects will limit the growth of the warp.

The non-linear evolution

In general, the non-linear stage of the evolution of the system of disks or rings of stars and molecular gas is far from a thin disk as long as collisions are of minor importance. We have already explored the non-linear evolution of collisionless systems with N-body codes and the results will be reported elsewhere. When two massive disks warp each other, mixing occurs when $\gamma - \pi/2$ becomes large, and the resulting configuration reminds that of a torus.

fluence is a common occurrence in real AGN. It is hard to see why AGN disks should always form in one plane and even why they should be planar when they are born (Phinney 1989). The time scales for development of strong warps are short or comparable to characteristic AGN lifetimes (which are thought to be in the range of $10^7 - 10^8$ years). We thus expect that the outer edges of the disk will obscure a significant fraction of the sky as seen from the central source and hence be directly relevant to the unification schemes of AGN (Antonucci 1993).

Figure 2 shows the warped disk surface from Figure 1 at different times as seen from the origin of coordinates. The vertical axis shows $\cos \theta \equiv z'/\sqrt{(x')^2 + (y')^2 + (z')^2}$ and the horizontal one shows $\phi$ defined previously. The shape of the disk at different times is shown with different colors. At time $t = 0$ the disk is flat and its projection is simply $\cos \theta = 0$ for all $\phi$. At later times the rings of the disk near $R = R_1 = 2$ precess and bulge out of the initial disk plane. With time the fraction of the sky obscured from the central engine becomes greater than a half. In reality non-linear effects and interaction of the disk with AGN winds and radiation, etc., will become important in shaping the disk for significant warps.
Figure 2. The accretion disk surface as seen from the SMBH location for four different times: $t = 100, 400, 800, 3200$ for the black, green, red and yellow dots, respectively. Note that at the largest time most of the available solid angle is obscured by the strongly warped disk.

Now, inelastic dissipative collisions between gas clumps will eventually become important. Relaxation of a collisionless N-body system leads to non-circular orbits, and hence different disk rings will start to overlap. Collisions should then tend to destroy the random motions and should establish a common disk “plane”. However, such a disk will normally be warped itself. Therefore inelastic dissipative collisions do not necessarily turn off the obscuration in our model. Further, if molecular gas clumps are constantly supplied from the outside and come with fluctuating angular momentum, the disk may never arrive in a flat thin configuration (see also Phinney 1984; Nenkova, Ivezić, & Elitzur 2002 and Risaliti, Elvis, & Nicoastro 2002) convincingly argued that the AGN obscuring medium must not be uniform in density. This does not contradict our model at all since AGN accretion disks on large (e.g. 0.1 pc and beyond) scales are self-gravitating unless the accretion rates are tiny (e.g., Shlosman & Begelman 1989).

The source of warping potential does not have to be a thin stellar ring or a disk: it may be any non-spherical distribution of stars in the SMBH vicinity that retains a non-zero quadrupole moment; it can also be a second (smaller) super-massive black hole during a merger of two galaxies.

Both the collision-dominated (Krolik & Begelman 1988) and the stellar-feedback inflated torii (Wada & Norman 2002) share a common starting point: the torus is the result of some internal disk physics. If our interpretation applies, AGN torii lend their existence to the way in which the cold gas arrives in the central part of the galaxy. Compared with the model of Krolik & Begelman (1988), large random speeds for the cold gas are not required in our model. Although the gas may be quite high up away from the original plane of the disk, the disk is locally coherent and thin. High speed elastic collisions between molecular clouds are thus not needed to explain obscuration of AGN in our model.

There are also two other mechanisms potentially able to produce warped disks at parsec and beyond scales around AGN. As mentioned in the X-ray binaries context, accretion disks develop twists and warps due to instabilities driven by X-ray heated wind off the disk surface (Schandl & Meyer 1994). Same can be achieved by the radiation pressure force from the central source (Pringle 1996). However, it seems that the majority of disks in X-ray binaries are either not warped or warped not strongly enough (Ogilvie & Dubus 2001) to provide the large obscuration needed for the AGN unification schemes. In contrast, the warping mechanism discussed here is not applicable to X-ray binary systems, and hence it may be natural that AGN disks are stronger warped than X-ray binary disks on appropriately scaled distances from the center.

3.3 The stellar disks in Sgr A*  

The two young stellar disks discovered recently in Sgr A* present a challenge to the usual star formation modes because the gas densities required to avoid tidal shearing are many orders of magnitude larger than the highest densities observed anywhere in the galactic molecular clouds. On the other hand, star formation inside a massive accretion disk is a long expected outcome of the self-gravitational instability of such disks (e.g. Paczynski 1978; Shlosman & Begelman 1983; Collin & Zahn 1999; Goodman 2003). As such, the young stars in the Sgr A* star cluster therefore are a first example of star formation in this extreme environment. It is very likely that the star formation efficiency and the initial mass function (IMF) are quite different in the immediate AGN vicinity and elsewhere in a galaxy. The “astro-archeology” of Sgr A* can be used to study these issues.

The gravitational warping effect discussed in this paper constrains the time-averaged mass of the outer ring, $M_{\text{outer}}$, since the moment of its creation, assumed to be $t = 2 \times 10^6$ years. The inner stellar ring is rather well defined (Genzel et al. 2003), and we thus estimate $\omega_{\text{f}}$ for the inner ring to be smaller than $\pi/4$. Taking the radius of the inner stellar ring to be $R = 3'' \approx 4 \times 10^3 R_S$ for the GC black hole, and for the outer, $R_S = 5''$, and using equation $8$ with $\cos \beta = 1/4$, one obtains $M_{\text{outer}} < 10^5 M_\odot$. Preliminary numerical N-body simulations show this limit may be even smaller.

3.4 Other implications for AGN  

There are clearly other observational implications of gravitationally warped disks. For example, Narrow Fe Kα lines, observed in many Seyfert galaxies, can be explained with X-ray reflection off such warped cold disks. In addition, warped disks will yield different coherent paths for maser amplification than flat disks do, which may be a part of the explanation for the complexity of the observed AGN maser emission.
4 CONCLUSIONS

We argued that clumpy self-gravitating accretion disks in AGN are generically strongly warped. We believe such warping should be an integral part of the explanation for the AGN unification schemes. Our model provides arguably the easiest way to obscure the central engine without the need to lift cold gas clouds off the disk plane high up via elastic collisions, supernova explosions or winds.

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