On nature of scalar and tensor mesons from the analysis of processes $\pi\pi \to \pi\pi, K\bar{K}, \eta\eta$

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Abstract

Analysis of the isoscalar $S$- and $D$-waves of processes $\pi\pi \to \pi\pi, K\bar{K}, \eta\eta$ is carried out aimed at studying the status and QCD nature of scalar and tensor mesons below 2 GeV and 2.3 GeV, respectively. Assignment of these mesons to lower scalar and tensor nonets is proposed.

Outline:

- Motivation
- Three-coupled-channel formalism
- Model-independent analysis of isoscalar-scalar sector
- Lower scalar nonets
- Model analysis of isoscalar-tensor sector
- Discussion and conclusions
1 Motivation

The study of spectrum of low-lying hadrons and their properties is very important for investigation of the confinement problem and elaboration of nonperturbative methods of QCD. Especially it concerns scalar mesons. E.g., two different nonperturbative methods of QCD (QCD sum rules and unquenched lattice calculations) give essentially distinct results for the lightest scalar glueball.

**QCD sum rules:** a lightest scalar meson with a mass below 900 MeV is rather narrow and non-$qar{q}$ state (glueball) – see, *e.g.*, [1, 2, 3].

**Lattice simulations:** The lowest mass state of a pure glue should be the $0^{++}$ with a mass of $1670 \pm 20$ MeV [4].

An assignment of the discovered scalar mesonic states to quark-model configurations is problematic up to now.

It is very important to have model-independent information on investigated states and on their QCD nature. It can be obtained only on the basis of the first principles (analyticity and unitarity) directly applied to analysis of experimental data. That approach permits us to introduce no theoretical prejudice into extracted parameters of resonances. We have already proposed such method [5]. Here we have applied it to combined analysis of experimental data on the processes $\pi\pi \rightarrow \pi\pi, K\bar{K}, \eta\eta$ in the channel with the quantum numbers $I^GJ^{PC} = 0^+0^{++}$. In the considered 3-channel case, it is turned out to be possible to use a method of the uniformizing variable which takes into account the Riemann surface structure. Considering the obtained disposition of resonance poles on the Riemann surface, obtained coupling constants with channels and resonance masses, we draw definite conclusions about nature of the investigated states.

Further we analyze the same processes in the channel with the quantum
numbers $I^G J^{PC} = 0^+ 2^{++}$ for the study of the $f_2$ mesons below 2.3 GeV. In this sector, from thirteen discussed resonances, the nine ones ($f_2(1430), f_2(1565), f_2(1640), f_2(1810), f_2(1910), f_2(2000), f_2(2020), f_2(2150), f_2(2220)$) must be confirmed in various experiments and analyses [6]. Recently in the combined analysis of $p\bar{p} \rightarrow \pi\pi, \eta\eta, \eta'\eta'$, five resonances – $f_2(1920), f_2(2020), f_2(2240)$ and $f_2(2300)$ – have been obtained, one of which ($f_2(2000)$) is a candidate for the glueball [7].

In the tensor sector, in addition to the indicated three channels, we consider explicitly also the channel $(2\pi)(2\pi)$. In the 4-channel case it is impossible to apply the uniformizing-variable method with using a simple variable. Therefore, the resonance poles are generated by the some 4-channel Breit-Wigner forms with taking into account a Blatt-Weisskopf barrier factor [8] conditioned by the resonance spins.

2 Three-coupled-channel formalism

The $S$-matrix for the 3-channel case is determined on the 8-sheeted Riemann surface. The elements $S_{\alpha\beta}$, where $\alpha, \beta = 1(\pi\pi), 2(K\bar{K}), 3(\eta\eta)$, have the right-hand cuts along the real axis of the $s$ complex plane, starting with $4m_{\pi}^2$, $4m_{K}^2$, and $4m_{\eta}^2$, and the left-hand cuts. The sheets of surface are numbered according to the signs of analytic continuations of the channel momenta

$$k_1 = (s/4 - m_{\pi}^2)^{1/2}, \quad k_2 = (s/4 - m_{K}^2)^{1/2}, \quad k_3 = (s/4 - m_{\eta}^2)^{1/2}$$

as follows:
The resonance representations on the Riemann surface are obtained with the help of formulas from Ref. [5], expressing analytic continuations of the matrix elements to unphysical sheets in terms of those on sheet I that have only zeros (beyond the real axis) corresponding to resonances, at least, around the physical region. Then, starting from resonance zeros on sheet I, we obtain 7 types of resonances corresponding to 7 possible situations when there are resonance zeros on sheet I only in (a) $S_{11}$; (b) $S_{22}$; (c) $S_{33}$; (d) $S_{11}$ and $S_{22}$; (e) $S_{22}$ and $S_{33}$; (f) $S_{11}$ and $S_{33}$; and (g) $S_{11}$, $S_{22}$, and $S_{33}$. E.g., the arrangement of poles corresponding to a resonance of type (g) is: each sheet II, IV, and VIII contains a pair of conjugate poles at the points that are zeros on sheet I; each sheet III, V, and VII contains two pairs of conjugate poles; and sheet VI contains three pairs of poles. A resonance of every type is represented by a pair of complex-conjugate clusters (of poles and zeros on the Riemann surface). Representation of multichannel resonances by the pole clusters gives a main model-independent effect of resonances. The cluster kind is related to nature of the state. The resonance coupled relatively more strongly to the $\pi\pi$ channel than to the $K\overline{K}$ and $\eta\eta$ ones is described by the cluster of type (a); the resonance with dominant $s\bar{s}$ component, by the cluster of type (e); the glueball, by the (g) cluster. Note that at usual representation of multichannel resonances by the simple Breit-Wigner forms, cases (d), (e), (f) and (g) practically are lost.

We can distinguish, in a model-independent way, a bound state of colourless particles (e.g., $K\overline{K}$ molecule) and a $q\bar{q}$ bound state [5, 9].
For the combined analysis of data we use the Le Couteur-Newton relations [10]:

\[
S_{11} = \frac{d(-k_1, k_2, k_3)}{d(k_1, k_2, k_3)}, \quad S_{22} = \frac{d(k_1, -k_2, k_3)}{d(k_1, k_2, k_3)}, \quad S_{33} = \frac{d(k_1, k_2, -k_3)}{d(k_1, k_2, k_3)},
\]

\[
S_{11}S_{22} - S_{12}^2 = \frac{d(-k_1, -k_2, k_3)}{d(k_1, k_2, k_3)}, \quad S_{11}S_{33} - S_{13}^2 = \frac{d(-k_1, k_2, -k_3)}{d(k_1, k_2, k_3)}. \tag{1}
\]

The Jost matrix determinant \(d(k_1, k_2, k_3)\) is the real analytic function with the only square-root branch-points at \(k_i = 0\).

In the model-independent approach that is based only on the first principles (analyticity-microcausality and unitarity) and is free from dynamical assumptions, we use the mathematical fact that a local behaviour of analytic functions determined on the Riemann surface is governed by the nearest singularities on all corresponding sheets. To take into account the branch points, we must find proper uniformizing variable. However, it is impossible to map the 8-sheeted Riemann surface onto a plane with the help of a simple function. Therefore, we neglect the influence of the \(\pi\pi\) threshold (however, unitarity on the \(\pi\pi\) cut is taken into account). This approximation means the consideration of the semi-sheets of the Riemann surface nearest to the physical region. The uniformizing variable can be chosen as [5]

\[
w = \frac{k_2 + k_3}{\sqrt{m^2 - m_K^2}}. \tag{2}
\]

It maps our model of the 8-sheeted Riemann surface onto the \(w\)-plane.

On the \(w\)-plane, the Le Couteur-Newton relations are somewhat modified:

\[
S_{11} = \frac{d^*(-w^*)}{d(w)}, \quad S_{22} = \frac{d(-w^{-1})}{d(w)}, \quad S_{33} = \frac{d(w^{-1})}{d(w)},
\]

\[
S_{11}S_{22} - S_{12}^2 = \frac{d^*(w^{*-1})}{d(w)}, \quad S_{11}S_{33} - S_{13}^2 = \frac{d^*(-w^{*-1})}{d(w)}. \tag{3}
\]
The \( d \)-function is taken as

\[
d = d_B d_{\text{res}}
\]

where the resonance part is

\[
d_{\text{res}}(w) = w^{-M} \prod_{r=1}^{M} (w + w_r^*)
\]

with \( M \) the number of resonance zeros. \( d_B \), describing the background, is

\[
d_B = \exp[-i \sum_{n=1}^{3} \frac{k_n}{m_n} (\alpha_n + i\beta_n)],
\]

where

\[
\alpha_n = a_{n1} + a_{n\sigma} \frac{s - s_{\sigma}}{s_{\sigma}} \theta(s - s_{\sigma}) + a_{nv} \frac{s - s_v}{s_v} \theta(s - s_v),
\]

\[
\beta_n = b_{n1} + b_{n\sigma} \frac{s - s_{\sigma}}{s_{\sigma}} \theta(s - s_{\sigma}) + b_{nv} \frac{s - s_v}{s_v} \theta(s - s_v).
\]

The second terms in \( \alpha_n \) and \( \beta_n \) take into account effectively possible channels below roughly 1400 MeV (mainly \( \sigma\sigma \)-channel); the third terms, many opening channels in the range of about 1.5 GeV \( (\eta\eta', \rho\rho, \omega\omega) \); \( s_v \) is their combined threshold. Moreover, the \( \pi\pi \) background is taken to be elastic up to the \( K\bar{K} \) threshold.

### 3 Model-independent analysis of isoscalar-scalar sector

We analyzed the data on three processes \( \pi\pi \rightarrow \pi\pi, K\bar{K}, \eta\eta \) in the channel with the vacuum quantum numbers. As the data, we use the results of phase analyses which are given for phase shifts of the amplitudes \( \delta_{ab} \) and for moduli of the \( S \)-matrix elements \( \eta_{ab} = |S_{ab}| \) \( (a, b=1-\pi\pi, 2-K\bar{K}, 3-\eta\eta) \):

\[
S_{aa} = \eta_{aa} e^{2i\delta_{aa}}, \quad S_{ab} = \eta_{ab} e^{i\delta_{ab}}.
\]
If below the $\eta\eta$-threshold there is the 2-channel unitarity, then the relations

$$\eta_{11} = \eta_{22}, \quad \eta_{12} = (1 - \eta_{11}^2)^{1/2}, \quad \delta_{12} = \delta_{11} + \delta_{22} \quad (9)$$

are fulfilled in this energy region.

For the $\pi\pi$ scattering, the data from the threshold to 1.89 GeV are taken from Ref. [11]; below 1 GeV, from many works [12].

For $\pi\pi \rightarrow KK$, practically all the accessible data are used [13]. For the process $\pi\pi \rightarrow \eta\eta$, here we exploited data for the quantity $|S_{13}|^2$ from the threshold to 1.72 GeV [14].

We considered the case with all five resonances discussed below 1.9 GeV. From a variety of variants of the resonance representations by possible pole-clusters, the analysis selects the following one to be most relevant – when the $f_0(600)$ is described by the cluster of type (a); $f_0(1370)$, type (c); $f_0(1500)$, type (g); $f_0(1710)$, type (b); the $f_0(980)$ is represented only by the pole on sheet II and shifted pole on sheet III. Description of the resonances of types (a), (b) and (c) can be related to the Breit-Wigner forms. To reduce the number of adjusted parameters, we make it here, except for the $f_0(980)$.

We obtain a satisfactory description: for the $\pi\pi$-scattering from about 0.4 GeV to 1.89 GeV ($\chi^2/\text{ndf} = 202.111/(165-34) \approx 1.54$); for the process $\pi\pi \rightarrow K\overline{K}$, from the threshold to about 1.6 GeV ($\chi^2/\text{ndf} = 161.912/(120-33) \approx 1.86$); for the $|S_{13}|^2$ data of the reaction $\pi\pi \rightarrow \eta\eta$, from the threshold to 1.72 GeV ($\chi^2/\text{ndf} \approx 0.992$). The total $\chi^2/\text{ndf}$ for all three processes is $379.893/(301-42) \approx 1.46$.

The background parameters are $a_{11} = 0.183$, $a_{1\sigma} = 0.0252$, $a_{1v} = 0.0155$, $b_{11} = 0$, $b_{1\sigma} = -0.0089$, $b_{1v} = 0.04336$, $a_{21} = -0.6973$, $a_{2\sigma} = -1.427$, $a_{2v} = -5.935$, $b_{21} = 0.0447$, $b_{2\sigma} = 0$, $b_{2v} = 7.044$, $b_{31} = 0.6346$, $b_{3\sigma} = 0.3336$, $b_{2v} = 0$; $s_\sigma = 1.638 \text{ GeV}^2$, $s_v = 2.084 \text{ GeV}^2$. 


On figures 1-5, we demonstrate energy dependences of phase shifts and moduli of the matrix elements of processes $\pi\pi \rightarrow \pi\pi, K\bar{K}, \eta\eta$ compared with the experimental data.

\[
\pi + \pi \rightarrow \pi + \pi
\]

![Figure 1: The phase shift of the $\pi\pi$-scattering S-wave amplitude.](image1)

\[
\pi + \pi \rightarrow \pi + \pi
\]

![Figure 2: The module of the $\pi\pi$-scattering S-wave matrix element.](image2)

Let us indicate in Table 1 the obtained pole clusters for resonances on the complex energy plane $\sqrt{s}$, poles on sheets IV, VI, VIII and V, corresponding to the $f_0(1500)$, are of the 2nd and 3rd order, respectively.

8
Figure 3: The phase shift of the $\pi\pi \rightarrow K \bar{K}$ $S$-wave matrix element.

(this is an approximation).

Table 1: Pole clusters for the $f_0$-resonances.

| Sheet | II   | III  | IV   | V    | VI   | VII  | VIII |
|-------|------|------|------|------|------|------|------|
| $f_0(600)$ | $E_r$ | 678±14 | 688±16 |      | 628±17 | 618±15 |      |
|        | $\Gamma_r$ | 608±22 | 608±9  |      | 608±28 | 608±26 |      |
| $f_0(980)$ | $E_r$ | 1016±5 | 986±18 |      |      |      |      |
|        | $\Gamma_r$ | 32±8  | 59±16  |      |      |      |      |
| $f_0(1370)$ | $E_r$ |      |      | 1400±21 | 1400±20 | 1400±20 | 1400±20 |
|        | $\Gamma_r$ |      |      | 89±13  | 71±15  | 45±6  | 27±9  |
| $f_0(1500)$ | $E_r$ | 1505±22 | 1480±30 | 1505±20 | 1500±20 | 1493±27 | 1488±25 | 1505±20 |
|        | $\Gamma_r$ | 360±23 | 140±21  | 240±30 | 139±21 | 194±27 | 88±15  | 360±30 |
| $f_0(1710)$ | $E_r$ | 1704±18 | 1704±21 | 1704±32 | 1704±30 |      |      |
|        | $\Gamma_r$ | 95±14  | 105±17  | 325±26 | 325±45 |      |      |

Note a surprising result obtained for the $f_0(980)$ state. It turns out that this state lies slightly above the $K\bar{K}$ threshold and is described by a pole on sheet II and by a shifted pole on sheet III under the $\eta\eta$ threshold without an accompaniment of the corresponding poles on sheets VI and VII, as it was expected for standard clusters. This corresponds to the description of
the ηη bound state.

For now, we did not calculate coupling constants in the 3-channel approach. Therefore, for subsequent conclusions, let us mention the results for coupling constants from our previous 2-channel analysis (Table 2)[15]: $g_{1r}$ is the coupling constant of resonance ”r” with the $\pi\pi$-system; $g_{2r}$, with $K\bar{K}$. We see that the $f_0(980)$ and the $f_0(1370)$ are coupled essentially

more strongly to the $K\bar{K}$ system than to the $\pi\pi$ one, i.e., they have a dominant $s\bar{s}$ component. The $f_0(1500)$ has the approximately equal coupling constants with the $\pi\pi$ and $K\bar{K}$, which apparently could point to its dominant glueball component. In the 2-channel case, $f_0(1710)$ is represented by the cluster corresponding to a state with the dominant $s\bar{s}$ component.

Our 3-channel conclusions on the basis of resonance cluster types gen-
Figure 5: The squared module of the $\pi \pi \rightarrow \eta \eta$ S-wave matrix element.

erally confirm the ones drawn in the 2-channel analysis (besides the above surprising conclusion about the $f_0(980)$ nature).

Masses and widths of these states should be calculated from the pole positions. If to take the resonance part of amplitude as

$$T^{res} = \sqrt{s}\Gamma_{el}/(m^2_{res} - s - i\sqrt{s}\Gamma_{tot}),$$

we obtain for masses and total widths the following values (in MeV):

for $f_0(600)$, 868 and 1212;
for $f_0(980)$, 1015.5 and 64;
for $f_0(1370)$, 1407.5 and 344;
for $f_0(1500)$, 1546 and 716;
for $f_0(1710)$, 1709.6 and 276.

4 Lower scalar nonets

It is known that an assignment of the scalar mesonic states to quark-model configurations is problematic up to now, although there is a number
of interesting conjectures [16]-[21]. Let us also (on the basis of obtained results) propose a following assignment of scalar mesons below 1.9 GeV to lower nonets. First of all, we exclude from this consideration the $f_0(980)$ as the $\eta\eta$ bound state. Then we propose to include to the lowest nonet the isovector $a_0(980)$, the isodoublet $K_0^*(905)$ (or $\kappa(800)$), and two isoscalars $f_0(600)$ and $f_0(1370)$ as mixtures of the eighth component of octet and the SU(3) singlet. Note that we consider the $K_0^*(905)$ (or $\kappa$) which one has observed at analysing the $K^-\pi$ scattering [22, 23], extracted from reaction $K^-p \to K^-\pi^+n$, and at studying the decay $D^+ \to K^-\pi^+\pi^+$ [24]. Then the Gell-Mann–Okubo formula

$$3m_{f_8} = 4m_{K_0^*} - m_{a_0}$$  \hspace{1cm} (11)

gives $m_{f_8} = 0.88$ GeV. Our result for the $\sigma$-meson mass is $m_\sigma \approx 0.868 \pm 0.02$ GeV (if $m_\kappa = 0.8$, $m_{f_8} \approx 0.73$).

In the relation for masses of nonet

$$m_\sigma + m_{f_0(1370)} = 2m_{K_0^*(905)},$$  \hspace{1cm} (12)

the left-hand side is about 26 $\%$ bigger than the right-hand one if to take our mass values.

The next nonet (maybe, of radial excitations) could be formed of the isovector $a_0(1450)$, the isodoublet $K_0^*(1430 - 1450)$, and of the $f_0(1500)$ and $f_0(1710)$ as mixtures of the eighth component of octet and the SU(3) singlet, the $f_0(1500)$ being mixed with a glueball which is dominant in this state. From the Gell-Mann–Okubo formula we obtain $m_{f_8} \approx 1.45$ GeV. In second formula

$$m_{f_0(1500)} + m_{f_0(1710)} = 2m_{K^*(1450)},$$  \hspace{1cm} (13)

the left-hand side is about 12 $\%$ bigger than the right-hand one.
Though the Gell-Mann–Okubo formula is fulfilled for both nonets rather satisfactorily, however, the breaking of 2nd relation (especially for the lowest nonet) tells us that the $\sigma - f_0(1370)$ and $f_0(1500) - f_0(1710)$ systems get additional contributions absent in the $K^*_0(905)$ and $K^*_0(1450)$, respectively.

Note that the 3-channel analysis indicates on a non-simple picture of mixing the states $f_0(1370)$ and $f_0(1710)$ with the wide $f_0(1500)$ and $f_0(600)$. The $f_0(1370)$ is coupled more strongly to the $\eta\eta$ channel than to the $\pi\pi$ and $K\bar{K}$ ones; the $f_0(1710)$, to the $K\bar{K}$ than $\pi\pi$ and $\eta\eta$ ones, whereas if these states were the pure $s\bar{s}$ ones, they would be described by clusters of type ($e$), and their coupling constants with the $K\bar{K}$ and $\eta\eta$ channels would be congruent numbers for each state.

5 Model analysis of isoscalar-tensor sector

We analyze the isoscalar D-waves of the processes $\pi\pi \rightarrow \pi\pi, K\bar{K}, \eta\eta$ in the 4-channel approach with the explicit account of the channel $(2\pi)(2\pi)$ ($i=4$), too. The Jost matrix determinant $d(k_1, k_2, k_3, k_4)$ is taken as

$$d = d_Bd_{res}.$$  \hspace{1cm} (14)

The 4-channel Breit-Wigner form for the resonance part of the $d$-function is taken in the form ($\rho_{rj} = 2k_i/\sqrt{M_r^2 - 4m_j^2}$):

$$d_{res}(s) = \prod_r \left[ M_r^2 - s - i \sum_{j=1}^4 \rho_{rj}^5 R_{rj} f_{rj}^2 \right],$$  \hspace{1cm} (15)

where $f_{rj}^2/M_r$ is the partial width, $R_{rj}$ is a Blatt-Weisskopf barrier factor \cite{8}

$$R_{rj} = \frac{9 + \frac{9}{4}(\sqrt{M_r^2 - 4m_j^2} r_{rj})^2 + \frac{1}{16}(\sqrt{M_r^2 - 4m_j^2} r_{rj})^4}{9 + \frac{9}{8}(\sqrt{s - 4m_j^2} r_{rj})^2 + \frac{1}{16}(s - 4m_j^2 r_{rj})^4}.$$  \hspace{1cm} (16)
with radii of 0.955 Fermi for all resonances in all channels, except for $f_2(1270)$, $f_2'(1525)$ and $f_2(1950)$ for which they are: for $f_2(1270)$, 1.496, 0.704 and 0.604 Fermi respectively in channels $\pi\pi$, $K\bar{K}$ and $\eta\eta$; for $f_2'(1525)$, 0.576 and 0.584 Fermi in channels $K\bar{K}$ and $\eta\eta$, and for $f_2(1950)$, 0.178 Fermi in channel $K\bar{K}$.

The background is parameterized by

$$d_B = \exp \left[ -i \sum_{n=1}^{3} \frac{2k_n}{\sqrt{s}} \right] \left( a_n + ib_n \right). \quad (17)$$

To take into account an influence of opening channels in the range of $\sim 1.5$ GeV ($\eta\eta'$, $\rho\rho$, $\omega\omega$), $a_1$ and $b_n$ ($n = 1, 2, 3$) are taken in the form:

$$a_1 = \alpha_{11} + \frac{s - 4m_K^2}{s} \alpha_{12} \theta(s - 4m_K^2) + \frac{s - s_v}{s} \alpha_{10} \theta(s - s_v), \quad (18)$$

$$b_n = \beta_n + \frac{s - s_v}{s} \gamma_n \theta(s - s_v), \quad (19)$$

where $s_v \approx 2.274$ GeV$^2$ is the combined threshold of channels $\eta\eta'$, $\rho\rho$, $\omega\omega$.

The data for the $\pi\pi$ scattering are taken from an energy-independent analysis by B. Hyams et al. [11]. The data for the processes $\pi\pi \to K\bar{K}, \eta\eta$ are taken from works [25]. We obtained ten resonances $f_2(1270)$, $f_2(1430)$, $f_2'(1525)$, $f_2(1580)$, $f_2(1730)$, $f_2(1810)$, $f_2(1950)$, $f_2(2000)$, $f_2(2240)$ and $f_2(2410)$. We will not discuss the last resonance because there are not practically data. We need it for satisfying unitarity.

On figures 6-9, we demonstrate results from our fitting to data (in the Argand plot units).

We obtain a reasonable description (the total $\chi^2/\text{ndf} = 162.577/(168 - 66) \approx 1.59$) with the values of parameters of $f_2$-resonances shown in Table 3.
For the background we find:
\[ \alpha_{11} = -0.0785, \alpha_{12} = 0.0345, \alpha_{10} = -0.2342, \beta_1 = -0.06835, \gamma_1 = -0.04165, \beta_2 = -0.981, \gamma_2 = 0.736, \beta_3 = -0.5309, \gamma_3 = 0.8223. \]

6 Discussion and conclusions

- In combined 3-channel model-independent analysis of data on processes \( \pi \pi \rightarrow \pi \pi, K \bar{K}, \eta \bar{\eta} \) in the channel with \( I^G J^{PC} = 0^+0^{++} \), an additional confirmation of the \( \sigma \)-meson with mass 0.868 GeV is obtained. This mass value rather accords with prediction \( m_\sigma \approx m_\rho \) on the basis of mended symmetry by Weinberg [26]. In works [27]-[32] evidences of the existences of the \( \sigma \)-meson have been given, too.

- The \( f_0(1370) \) and \( f_0(1710) \) have the dominant \( s \bar{s} \) component. Conclusion about the \( f_0(1370) \) quite well agrees with the one of work of Crystal Barrel Collaboration [33] where the \( f_0(1370) \) is identified as \( \eta \bar{\eta} \) resonance in the \( \pi^0 \eta \bar{\eta} \) final state of the \( \bar{p}p \) annihilation at rest. Conclusion about the \( f_0(1710) \) is quite consistent with the experimental
facts that this state is observed in $\gamma\gamma \rightarrow K_S K_S$ \cite{34} and not observed in $\gamma\gamma \rightarrow \pi^+\pi^-$ \cite{35}.

• Indication for $f_0(980)$ to be the $\eta\eta$ bound state is obtained. From point of view of quark structure, this is the 4-quark state. Maybe, this is consistent somehow with arguments of Refs. \cite{36, 37} in favour of the 4-quark nature of $f_0(980)$.

Remembering a dispute \cite{38, 39} whether the $f_0(980)$ is narrow or not, we agree rather with the former. Of course, it is necessary to make analysis of other relevant processes, first of all, $J/\psi$ and $\phi$ decays.

• As to the $f_0(1500)$, we suppose that it is practically the eighth component of octet mixed with a glueball being dominant in this state. Its biggest width among enclosing states tells also in behalf of its glueball nature \cite{18}.

• An assignment of the scalar mesons below 1.9 GeV to lower nonets is proposed. Note that this assignment moves a number of questions and
Figure 8: The squared module of the $\pi\pi \rightarrow K\bar{K}$ D-wave matrix element.

does not put the new ones. Now an adequate mixing scheme should be found.

- We do not obtain $f_2(1640)$, $f_2(1910)$, $f_2(2150)$, $f_2(2010)$, however, we see $f_2(1450)$ and $f_2(1730)$ which are related to the statistically-valued experimental points.

- Usually one assigns to the first tensor nonet the states $f_2(1270)$ and $f_2'(1525)$. To the second nonet, one could assign $f_2(1601)$ and $f_2(1767)$ though for now the isodoublet member is not discovered. If one takes for the isovector of this octet the state $a_2(1730)$ and if the $f_2(1601)$ is almost its eighth component, then, on the basis of the Gell-Mann–Okubo formula, we would expect this isodoublet mass at about 1.635 GeV. Then the relation for masses of nonet would be well fulfilled. Note that in the Particle Data Group issue [6] is indicated an experiment [40] in which one has observed the strange isodoublet with yet indefinite remaining quantum numbers and with mass $1.629 \pm 0.007$ GeV.
The states $f_2(1963)$ and $f_2(2207)$ together with the isodoublet $K_2^*(1980)$ could be put into the third nonet. Then in the relation for masses of nonet

$$M_{f_2(1963)} + M_{f_2(2207)} = 2M_{K^*(1980)}, \tag{20}$$

doing the left-hand side is only 5.3% bigger than the right-hand one. If one consider $f_2(1963)$ as the eighth component of octet, then the Gell-Mann–Okubo formula

$$M_{a_2} = 4M_{K^*(1980)} - 3M_{f_2(1963)} \tag{21}$$

gives $M_{a_2} = 2.031$ GeV. This value practically coincides with the one (2.03 GeV) for $a_2$-meson obtained on the basis of the recent data [41]. This state is interpreted [7] as a second radial excitation of the $1^{-2^{++}}$-state on the basis of consideration of the $a_2$ trajectory on the $(n, M^2)$ plane ($n$ is the radial quantum number of the $q\bar{q}$ state).

As to $f_2(2017)$, the ratio of the $\pi\pi$ and $\eta\eta$ partial widths is in the limits obtained in Ref.[7] for the tensor glueball on the basis of the
Table 3: The $f_2$-resonance parameters (all in the MeV units).

| State  | $M$       | $f_{r1}$  | $f_{r2}$  | $f_{r3}$  | $f_{r4}$  | $\Gamma_{tot}$ |
|--------|-----------|-----------|-----------|-----------|-----------|----------------|
| $f_2(1270)$ | 1275.1±1.8  | 470.9±5.4  | 201.5±11.4 | 89.5±4.76 | 22.6±4.6  | >212           |
| $f_2(1430)$ | 1450.8±18.7 | 128.3±45.9 | 562.3±142 | 32.7±18.4 | 8.2±65    | >230           |
| $f_2'(1525)$ | 1535±8.6    | 28.6±8.3   | 253.8±78  | 92.7±11.5 | 41.4±160  | >76            |
| $f_2(1565)$ | 1601.4±27.5 | 75.5±19.4  | 315±48.6  | 388.9±27.7| 127±199   | >170           |
| $f_2(1730)$ | 1724.4±5.7  | 78.8±43    | 289.5±62.4| 460.3±54.6| 107.6±76.7| >181           |
| $f_2(1810)$ | 1766.5±15.3 | 129.5±14.4 | 259±30.7  | 469.7±22.5| 90.3±90   | >177           |
| $f_2(1950)$ | 1962.8±29.3 | 132.6±22.4 | 333±61.3  | 319±42.6  | 65.4±94   | >119           |
| $f_2(2000)$ | 2017±21.6   | 143.5±23.3 | 614±92.6  | 58.8±24   | 450.4±221 | >299           |
| $f_2(2240)$ | 2207±44.8   | 136.4±32.2 | 551±149   | 375±114   | 166.8±104 | >222           |
| $f_2(2410)$ | 2429±31.6   | 177±47.2   | 411±196.9 | 4.5±70.8  | 460.8±209 | >169           |

1/N-expansion rules. However, the $K\bar{K}$ partial width is too large for the glueball. This question requires an additional investigation.

• Finally we have $f_2(1450)$ and $f_2(1730)$ with the rather unusual properties. These are non-$q\bar{q}$ states and non-glueball. Since one predicts that masses of the lightest $q\bar{q}g$ hybrids are bigger than the ones of lightest glueballs, maybe, these states are the 4-quark ones.

Yu.S. and R.K. acknowledge support provided by the Bogoliubov – Infeld Program. M.N. were supported in part by the Slovak Scientific Grant Agency, Grant VEGA No. 2/3105/23; and D.K., by Grant VEGA No. 2/5085/99.

References

[1] V. Elias et al., Nucl. Phys. A 633, 279 (1998).

[2] S. Narison, Nucl. Phys. B 509, 312 (1998).
[3] L.S. Kisslinger and M.B. Johnson, Phys. Lett. B 523, 127 (2001).

[4] C. Michael, Nucl. Phys. A 655, 12 (1999).

[5] D. Krupa, V.A. Meshcheryakov, Yu.S. Surovtsev, Nuovo Cim. A 109, 281 (1996).

[6] S. Eidelman et al. (PDG), Phys. Lett. B 592, 1 (2004).

[7] V.V. Anisovich et al., Int. J. Mod. Phys. A 20, 6327 (2005); hep-ph/0506133.

[8] J. Blatt and V. Weisskopf, Theoretical nuclear physics, Wiley, N.Y., 1952.

[9] D. Morgan and M.R. Pennington, Phys. Rev. D 48, 1185 (1993).

[10] K.J. Le Couteur, Proc. Roy. Soc. A 256, 115 (1960); R.G. Newton, J. Math. Phys. 2, 188 (1961); M. Kato, Ann. Phys. 31, 130 (1965).

[11] B. Hyams et al., Nucl. Phys. B 64, 134 (1973); ibid. 100, 205 (1975).

[12] A. Zylbersztejn et al., Phys. Lett. B 38, 457 (1972); P. Sonderegger and P. Bonamy, in Proc. 5th Intern. Conf. on Elementary Particles, Lund, 1969, paper 372; J.R. Bensinger et al., Phys. Lett. B 36, 134 (1971); J.P. Baton et al., Phys. Lett. B 33, 525, 528 (1970); P. Baillon et al., Phys. Lett. B 38, 555 (1972); L. Rosselet et al., Phys. Rev. D 15, 574 (1977); A.A. Kartamyshev et al., Pis’ma v Zh. Eksp. Teor. Fiz. 25, 68 (1977); A.A. Bel’kov et al., Pis’ma v Zh. Eksp. Teor. Fiz. 29, 652 (1979).

[13] W. Wetzel et al., Nucl. Phys. B 115, 208 (1976); V.A. Polychronakos et al., Phys. Rev. D 19, 1317 (1979); P. Estabrooks, Phys. Rev. D 19,
2678 (1979); D. Cohen et al., Phys. Rev. D 22, 2595 (1980); G. Costa et al., Nucl. Phys. B 175, 402 (1980); A. Etkin et al., Phys. Rev. D 25, 1786 (1982).

[14] F. Binon et al., Nuovo Cim. A 78, 313 (1983).

[15] Yu.S. Surovtsev, D. Krupa, and M. Nagy, Eur. Phys. J. A 15, 409 (2002).

[16] J. Lánik, Phys. Lett. B 306, 139 (1993).

[17] N.A. Törnqvist, hep-ph/0204215.

[18] V.V. Anisovich et al., Nucl. Phys. Proc. Suppl. A56, 270 (1997).

[19] F.E. Close and N.A. Törnqvist, J. Phys. G 28, R249 (2002).

[20] P. Minkowski and W. Ochs, Eur. Phys. J. C 9, 283 (1999); hep-ph/0209223; hep-ph/0209225.

[21] M.K. Volkov and V.L. Yudichev, Yad. Fiz. 65, 1701 (2002).

[22] S. Ishida et al., Prog. Theor. Phys. 98, 621 (1997).

[23] D.V. Bugg, Phys. Lett. B 572, 1 (2003).

[24] E.M. Aitala et al., Phys. Rev. Lett. 89, 121801 (2002).

[25] S.J. Lindenbaum and R.S. Longacre, Phys. Lett. B 274, 492 (1992); R.S. Longacre et al., Phys. Lett. B 177, 223 (1986).

[26] S. Weinberg, Phys. Rev. Lett. 65, 1177 (1990).

[27] V.V. Anisovich, Yu.D. Prokoshkin and A.V. Sarantsev, Nucl. Phys. Proc. Suppl. A56, 270 (1997); V.V. Anisovich, D.V. Bugg and A.V. Sarantsev, Phys. Rev. D 58, 111503 (1998).
[28] N.A. Törnqvist and M. Roos, Phys. Rev. Lett. 76, 1575 (1996).

[29] S. Ishida et al., Progr. Theor. Phys. 95, 745 (1996); ibid. 98, 621 (1997).

[30] M. Svec, Phys. Rev. D 53, 2343 (1996).

[31] R. Kamiński, L. Leśniak, and B. Loiseau, Eur. Phys. J. C 9, 141 (1999).

[32] L. Li, B.-S. Zou, and G.-lie Li, Phys. Rev. D 63, 074003 (2001).

[33] C. Amsler et al., Phys. Lett. B 355 425 (1995).

[34] S. Braccini, Proc. Workshop on Hadron Spectroscopy, Frascati Physics Series XV, 53 (1999).

[35] R. Barate et al., Phys. Lett. B 472, 189 (2000).

[36] N.N. Achasov, Nucl. Phys. A 675, 279c (2000).

[37] M.N. Achasov et al., Phys. Lett. B 438, 441 (1998); ibid. 440, 442 (1998).

[38] B.S. Zou and D.V. Bugg, Phys. Rev. D 48, R3948 (1993).

[39] D. Morgan and M.R. Pennington, Phys. Rev. D 48, 5422 (1993).

[40] V.M. Karnaukhov, C.Coca, and V.I. Moroz, Yad. Fiz. 63, 652 (2000).

[41] A.V. Anisovich et al., Phys. Lett. B 452, 173 (1999); ibid., 452, 187 (1999); ibid., 517, 261 (2001).