Tunneling of Macroscopic Universes

Heinz–Dieter Conradi
Institute for Theoretical Physics E, RWTH Aachen
Sommerfeldstr. 26-28, 52056 Aachen, Germany
Email: conradi@physik.rwth-aachen.de

Abstract

The meaning of ‘tunneling’ in a timeless theory such as quantum cosmology is discussed. A recent suggestion of ‘tunneling’ of the macroscopic universe at the classical turning point is analyzed in an anisotropic and inhomogeneous toy model. This ‘inhomogeneous tunneling’ is a local process which cannot be interpreted as a tunneling of the universe.

1. Timelessness and tunneling

Quantum gravity and quantum cosmology have the reputation of not describing any accessible experiments and for not having any consequences after the Planck era. However, it has been suggested that quantum cosmology could be responsible for some phenomena of everyday physics as the arrow of time and the classical appearance of our universe [1, 2], the smallness of the cosmological constant [3, 4], or the onset of inflation [5, 6].

As a prominent subject over the years, tunneling processes have been considered for different purposes in quantum cosmology. In a recent paper Dąbrowski and Larsen discussed the tunneling of a recollapsing universe at the classical turning point into an expanding state with increased scale factor [7]. Similar models have been studied e.g. also by [8] and, for the early stage of the universe, by [5, 6]. In this letter I will comment on the meaning of ‘tunneling’ in quantum cosmology and the model of [7] is substantially generalized.

The Wheeler–DeWitt equation $H\Psi = 0$ which governs quantum cosmology may be regarded as analogous to the stationary Schrödinger equation. The similarity is particularly striking for
the popular one dimensional models in which the universe is described by the scale factor $a$ only. The universe then resembles a particle in one dimension and the Wheeler–DeWitt equation reads

$$\left[-\partial_t^2 + V(a)\right] \Psi = 0,$$

with $a \in \mathbb{R}_+$. However, this similarity is misleading since, among other differences, there exists an external time parameter in ordinary quantum mechanics in contrast to quantum cosmology.

The quantum mechanical time parameter is crucial even for the most simple examples of tunneling. Consider the reflection and tunneling of a wave function at a square potential barrier. This situation is described by superposing an ingoing wave with a reflected wave in one free region while in the free region on the other side of the potential barrier the wave function consist solely of an outgoing wave, the amplitude of which determines the tunneling probability. The concepts of in– or outgoing waves are justified by either considering a wave packet by superposing different energy eigenstates the sum of which resembles a particle moving in time. (Due to time–reparametrization invariance there is only the ‘zero energy solution’ to deal with in quantum cosmology. In one dimension it is thus impossible to form wave packets.) Or more simply by observing that the crests of the solitary waves are moving with $\pm kx - \omega t = 2\pi n$.

The necessity of a time parameter is, however, clear from the onset since the very notion of tunneling assumes a state changing in time: While initially there is no particle in some region (and classically there will never be one), there is a finite probability to find one subsequently.

It is worthwhile to point out that a tunneling probability means probability of a tunneling ‘event’ of a particle and thus implies a measurement. The concept of tunneling thus presupposes both a time parameter and a theory of measurement. Concerning the first point, it does not help that there is a conserved Klein–Gordon current in quantum cosmology in more than one dimension, since its sign cannot be fixed in the absence of an external time. Denoting something as outgoing as e.g. in the definition of the ‘tunneling wave function’ would completely arbitrary fix a direction on the configuration space.

In the case of a high and broad potential barrier the situation simplifies, since the wave function in the forbidden region can be approximated by the exponentially suppressed solution only. In quantum mechanics the ratio of the (squared) wave function at the beginning and at the end of the forbidden region gives the tunneling probability. The same procedure is usually adopted in quantum cosmology as e.g. in order to calculate the tunneling probability of a bubble of false vacuum between two classically allowed regions in a ‘free lunch’ process. Accepting for the moment this interpretation, one arrives at similar conclusions in the semiclassical limit of quantum cosmology as in ordinary quantum mechanics.

Usually, the situation is more complicated than in the examples above, as e.g. in the alpha decay when one of the two classically allowed regions is restricted to a finite region in space. But while in that case it is still possible to have a purely outgoing wave outside the nucleus, this breaks down if both classically allowed regions are finite. (Dąbrowski and Larsen used this kind of potential for analyzing the tunneling probability at the classical turning point of a FRW universe with several matter sources.) In Fig. 1 the first situation is depicted by the dashed line while the case with two bounded regions is shown by the solid line. In order to calculate the tunneling probability it is thus not sufficient to consider stationary waves only. The calculation...
Fig. 1: The solid line shows the kind of potential used by Dąbrowski and Larsen. The second classically allowed region is due to domain walls and a negative cosmological constant. In order to get a ‘tunneling universe’, obviously, the ascending part of the potential is not necessary. In Sec. 2 a potential is used which is depicted by the dashed line. The decreasing is due to a positive dashed line.

is much more involved as can be seen e.g. by the deviation from the exponential decay law for the nuclear decay in a box [10]. Without the aid of the external time parameter the situation in quantum cosmology is quite unclear. One cannot even use the above mentioned comparison of the wave function at both ends of the potential barrier as a formal tool, since there will be no purely exponentially suppressed solutions.

In view of the aforementioned problems I will use the notion ‘tunneling’ only as a formal notion provided there exists a purely exponentially decaying solution: The ratio of the squared wave function at the beginning and at the end of the exponential region is defined as ‘tunneling probability’. (Note that the exponentially increasing wave function will also be a solution — ‘reversed tunneling’ — and arbitrary superpositions of this two basic solutions.)

This formal concept may only be regarded as a corresponding to a ‘real process’ if there is (at least) a time parameter and a theory of measurement in quantum cosmology. In more complicated examples as e.g. the one of Fig. 1 one furthermore has to be rather careful with the calculation of the probabilities. One may try to circumvent some of these difficulties by introducing a semiclassical time parameter e.g. due to a decoherence process or due to a Born–Oppenheimer type of approximation. Usually this point of view is put forward by mentioning that the tunneling occurs in a region where the semiclassical approximation should hold, although Kiefer and Zeh have argued that this argument is not sufficient [11]. If, however, decoherence could be used to define a semiclassical time, it is expected to simultaneously suppress the tunneling process by a Zeno-type effect [12]. I will come back to these issues in Sec. 3 when the results of the calculations in the toy model are discussed. As frequently stressed above, the notion of tunneling makes sense only if there is a time parameter. Since this is certainly not the case in the Planck era one cannot sensibly speak of ‘tunneling from nothing’ of the universe which is frequently used as a quantum alternative to the big bang.

The preceding considerations implicitly assume the so called naïve interpretation of the wave function which itself relies on the similarity of the Wheeler–DeWitt equation with the Schrödinger equation. That is, I assume the wave functions to be normalizable on the whole (unconstrained) configuration space or that at least a conditional probability can be used. Any other interpretation in quantum cosmology which works on the reduced phase space or isolates a time parameter is meaningless in the example of the FRW universe with phenomenological matter. This is because in these interpretations there is but one physical state. But even for
more realistic examples in which the time parameter is given by some function of the volume, it does not make sense to compare volumes. As already mentioned above, even if a physical time has been introduced one has to establish a theory of measurement in quantum cosmology before one could sensibly speak of tunneling.

2. An anisotropic and inhomogeneous model

The Kantowski–Sachs model as the most simple anisotropic and even inhomogeneous model has the advantage that it can be solved exactly. The homogeneous version of the model combines spherical symmetry with a translational symmetry in the ‘radial’ direction. (First the homogeneous model is discussed before the translational symmetry is relaxed, see below.) The spacelike hypersurfaces of constant times are therefore cylinders

\[ ds^2 = z^2 \, dr^2 + b^2 \, d\Omega^2 . \]  

Here \( b(t), z(t) \in \mathbb{R}_+ \); \( b(t) \) is the surface measure of the two–spheres with metric \( d\Omega^2 \) and \( z(t) \) measures the spacelike distance between them; \( r \) is the radial coordinate.

The model with positive cosmological constant \( \Lambda \) and pressureless dust is considered here. It is well known that it is only the presence of matter which renders the disklike singularity \((z \to 0)\) from a mere coordinate singularity which indicates the incompleteness of the model into a curvature singularity. The dust is described by the parameter

\[ z_m = \rho z b^2 = \text{const} \]  

(analogous to \( a_m = \rho a^3 \) in the FRW model). For homogeneous models this approach is essentially equivalent to a more sophisticated one which starts from a Lagrangian for the dust degrees of freedom, see e.g. [13] and the literature cited therein. Although dust as a matter source in quantum cosmology is unsatisfactory, it does here lead to a toy model which is calculable and complete. In addition, the present context of a macroscopic universe justifies this phenomenological description.

The following account of the classical dynamics of this well known model is rather cursory. More details can be found e.g. in [14, 15] and the literature cited therein. That time parameter is used which is equivalent to conformal time in the FRW model \((N = b)\). The dynamics is determined by the Hamiltonian constraint

\[ z\dot{b}^2 + 2z\dot{b}b + b^2 \left( z - z_m - \Lambda z b^2 \right) = 0 , \]  

(written in the configuration variables and their velocities) and by one of the equations of motion

\[ \dot{b}^2 + b - b_m - \frac{1}{3} \Lambda b^3 =: \ddot{b} - \ddot{V}_b (b) = 0 . \]

Here the equation of motion for \( b(t) \) has already been integrated, with \( b_m \) as an arbitrary constant of motion. The equation of motion for the scale factor of the closed FRW model is identical to Eq. (3), \( \dot{a}(t) = \dot{b}(t) \), but the corresponding constant \( a_m \) is fixed by the matter content. In the Kantowski–Sachs model the matter content instead fixes the constant of integration of \( z(t) \) as indicated by the notation.

The classically forbidden regions in this model are determined by \( \ddot{V}_b (b) \leq 0 \). Note that there is no forbidden region on configuration space due to the Hamiltonian (3) since its kinetic
term is indefinite. Consequently, the above defined forbidden region is defined for each classical solution separately, because each solution is uniquely represented by the ‘effective mass’ \( b_m \). There are two classically allowed regions only if the cosmological constant is positive and smaller than the Einstein value: \( 0 < \Lambda < 4/(9b_m^2) \). This potential which is depicted by the dashed line in Fig. 1 is similar to the situation in [1], see the solid line in the same figure, except that it is not increasing for large values of \( b \).

The one–dimensionality of Eq. (4) suggests, furthermore, the possibility of an inhomogeneous \( z(t, r) \). Geometrically, this looks reasonable because a cylinder is defined only by the homogeneity of the surface measure \( b \) while an inhomogeneous spacelike distance \( z \) between the two–spheres would not deform it. It turns out that an inhomogeneous \( z(t, r) \) is even dynamically consistent for pressureless dust as a matter content, as was first noticed by Ellis [16]. A rigorous proof is possible by using the equations of motion for the general spherically symmetric model as is performed in the appendix. There it follows directly, since \( z' \) is always tied to \( b' \) and thus does not appear in the Kantowski–Sachs model. Due to this absence of any spatial derivatives, states at different points do not interact: The Hamiltonian (3) remains thus essentially unchanged

\[
\dot{z}(t, r)b^2(t) + 2\dot{z}(t, r)b(t)b(t) + b^2(t)\left(z(t, r) - z_m(r) - \Lambda z(t, r)b^2(t)\right) = 0, \quad (5)
\]

One thus has a one parameter set of homogeneous solutions \( b = b(t) \) which then determines uniquely the solution \( z = z(t, r) \) the inhomogeneity of which is due to the inhomogeneity of the dust.

In the recollapsing region one gets a qualitatively correct picture of the dynamics by considering the exact solutions for \( \Lambda = 0 \):

\[
b(t) = b_m \sin^2\left(\frac{t}{2}\right) \quad \text{and} \quad z(t, r) = K(r) \cot\left(\frac{t}{2}\right) + z_m(r) \left(1 - \frac{t}{2} \cot\left(\frac{t}{2}\right)\right), \quad (6)
\]

with \( t \in [0, 2\pi] \) and where \( K \) is a physically meaningless ‘constant’ of time–integration (different functions of \( K \) are identified by a redefinition of the radial coordinate and by simultaneously changing the dust potential). The other classically allowed region for large \( b \) can be approximated by considering only the cosmological term in the potential. Obviously, the inhomogeneity of \( z(t, r) \) is due to inhomogeneous dust \( z_m(r) \).

As usual Dirac’s quantization scheme is used by turning the variables \( b, z \) and their conjugate momenta into operators which satisfy the standard commutation relations. The Hamiltonian constraint (3) is turned into an operator which annihilates the physical states: \( \hat{H}\Psi = 0 \), the so called Wheeler–DeWitt equation. In the configuration space representation it reads

\[
-2zb^2\hat{H}\Psi = \left[z^2\partial_z^2 + k z \partial_z - 2zb\partial_b \partial_z + z^2b^2 \left(1 - \Lambda b^2\right) - zz_m b^2\right]\Psi(b, z) = 0, \quad (7)
\]

where \( \partial_z \) and \( \partial_b \) are the partial derivatives with respect to \( z \) and \( b \), respectively. Factor ordering is partially left open as indicated by the parameter \( k \). The Laplace–Beltrami ordering is given by \( k = 1 \). This equation can exactly be solved for a more general potential which contains arbitrary functions of \( b \) multiplied by \( z \) and \( z^2 \) [14].
Although \( z(r) \) is a field it suffices to consider the minisuperspace Wheeler–DeWitt equation (7) at each point \( r \) since there are no partial derivatives with respect to \( r \) (and because of the simple structure of the solutions). Quantum fluctuation are expected to result in interaction between different points. But this effect can only be considered in the context of a more general model as is common to this kind of problems.

In order to solve the Wheeler–DeWitt equation it is convenient to introduce the operator

\[
\hat{b}_m := -\frac{1}{b} \partial_z^2 + (b - \Lambda b^3/3) \text{ since } [\hat{b}_m, \hat{H}] = 0.
\]

As indicated by the notation, the eigenvalues of this operator is the effective mass for \( b \). The eigenvalue equation \( \hat{b}_m \Psi_{b_m} = b_m \Psi_{b_m} \) can easily be solved, and one finally ends up with the following set of exact solutions for the Wheeler–DeWitt equation [14],

\[
\Psi_{b_m}(z, b) = \sqrt{\frac{b^{k-1}}{|V_{b_m}(b)|}} \exp \left\{ \pm i \left( z \sqrt{b V_{b_m}(b)} + \frac{z_m}{2} \int^b b \sqrt{V_{b_m}(b')} \, db' \right) \right\}.
\] (8)

Alternatively, one can consider the solutions which are given by a superposition of the exponential with plus and minus sign to form e.g. the ‘cos’ and the ‘sin’ (the ‘cosh’ and ‘sinh’ in the forbidden region). The integral in the exponential can be expressed in terms of elementary function for \( b_m = 0 \) (that is for large values of \( b \)) and for vanishing \( \Lambda \) (that is for small values of \( b \)). Neither case is appropriate here.

In order to analyze the formal tunnel probability one has to calculate the ratio

\[
\text{prob} := |\Psi(b_1)|^2 / |\Psi(b_2)|^2
\]

where \( b_1 \) and \( b_2 \) denote the beginning and the end of the classically forbidden region, respectively. While, the prefactor of the wave function is divergent at the borderline of the classically allowed region it cancels in \( \text{prob} \), since \( \tilde{V}_{b_m}(b) \) possesses three distinct, simple zeros (the third one is negative and has no physical significance). The first term in the exponent vanishes for \( b = b_1, b_2 \). There remains the second term in the exponential which is a definite integral between \( b_1 \) and \( b_2 \). This integral remains finite since at both limits of integration the integrand is approximately \( 1/x \) and the infinities at the two boundaries cancel each other.

If one is interested in a ‘tunneling’ from the recollapsing region into the region right from the potential barrier, one has to choose the minus sign in the exponent of the wave function. (Note that the Hartle–Hawking wave function has the opposite sign [15] and describes thus a ‘tunneling’ into the recollapsing region.)

The final result then is given by:

\[
\text{prob} = (b_1/b_2)^{(k-1)} \exp\{-z_m F(b_m, \Lambda)\},
\]

where the number \( F(b_m, \Lambda) \) is the definite integral. Inserting some simple values: \( b_m = 1, \Lambda = 3/8 \) gives \( b_1 = \sqrt{5} - 1 \) and \( b_2 = 2 \); the evaluation of the integral gives: \( F(b_m, \Lambda) \approx 5.106 \). Choosing furthermore the Laplace–Beltrami factor ordering, \( k = 1 \), one finds

\[
\text{prob} \approx \exp\{-5.106z_m(r)\}.
\] (9)

The simple structure of this result relies on the semiclassical structure of the exact wave function plus the canceling of the prefactor in \( \text{prob} \).

3. Results and remarks

A remarkable feature of the above ‘tunneling probability’ is the way it depends on the matter content: The logarithm of \( \text{prob} \) depends linearly on \( z_m \). Apart from factor ordering effects the
tunneling probability equals one for the vacuum model. Since the matter does not influence the tunneling barrier one has the analogue of a particle with mass $z_m$ (squared) running against a potential barrier. However, different points $r$ in the Kantowski–Sachs model will ‘tunnel’ individually since they do not interact. This situation is analogues to a cloud of non–interacting particles. In more realistic models one might think of weakly coupled points (perhaps galaxies), which nevertheless will behave mainly independent in the ‘tunneling process’. This local process clearly cannot be interpreted as a tunneling of a whole universe.

How is this process to be interpreted? Due to the timeless nature of quantum cosmology, and in particular since a semiclassical time might not be defined at the turning point, both ‘universes’ (that is both classically allowed regions) exist ‘simultaneously’. In this case one might interpret the tunneling as a quantum wormhole. However, the same argument of timelessness tells one, that there is no classical observer at the turning point, no incoming and no outgoing wave. The world is completely quantum and consequently one cannot speak of a tunneling process or of wormholes. If, on the contrary, the tunneling occurs as a change ‘in time’, e.g. defined by decoherence, the variable $b$ in some space regions changes from $b_{\text{max}}$ of the recollapsing solution to $b_{\text{min}}$ of the solution right from the potential barrier, with $b_{\text{min}} > b_{\text{max}}$. This is similar to the ‘free lunch’ process in the very early universe. These tunneling regions behave dynamically different from their environment since they are now expanding instead of recollapsing and because of the sudden change of the $b$-variable. This results in a destruction of the cylinder geometry of the universe.

It has been emphasized by Dąbrowski and Larsen that this tunneling process does not require any change in the matter content as e.g. a change of the vacuum of the involved fields. However, Rubakov has suggested that the matter content may be changed due to the very tunneling process. He considered a scalar field, conformally coupled to the FRW model, and observed that the tunneling probability is enhanced with growing particle content. Probably, this remark will apply too when only small parts of the universe are involved. However, the calculation for the Kantowski–Sachs model is much more involved than that for the FRW model.

A comparison of prob for the Kantowski–Sachs and for the FRW model shows another difference than that of inhomogeneity. Starting from the Wheeler–DeWitt equation for the FRW model $\left[\partial^2 a + \frac{1}{3} \Lambda a^6 + a^4 - a_m a^3\right]\Psi = 0$ one gets (in WKB approximation)

$$prob_{\text{FRW}} = \exp \left\{-2 \int_{a_1}^{a_2} a^3 \tilde{V}(a, a_m, \Lambda) da\right\},$$

(10)

where $\tilde{V}$ is defined as in the Kantowski–Sachs model. This result is quite different from the expression for the Kantowski–Sachs model, since in the FRW model $a_m$ is determined by the matter content. The analogue for the tunneling in the FRW model is that of a particle running against a barrier the width of which is fixed by the matter content.

It was shown in that one can insert a Kantowski–Sachs cylinder between two FRW half–spheres. The ‘tunneling probability’ in this compact model changes drastically at the

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1 In this way the model automatically takes into account that the vacuum model does not possess forbidden regions: It describes the dynamical regions of a black hole (in de Sitter space).

2 A quantum wormhole is formally defined by an Euclidean region between two separated classical regions.
borderline between the different parts. However, since the dust content in the FRW model fixes the constant of integration $b_m$ of the Kantowski–Sachs region, the tunneling probability in the Kantowski–Sachs region is a function of the FRW dust, too. The functional dependence is even similar in both regions.

It has been argued in the last section that the calculation of $\text{prob}$ is not spoiled by the divergences of the wave function (8). One can even get rid of these divergences by considering superpositions. (In contrast to the delta–functional like solutions (8) it is e.g. possible to consider ‘free waves’, the form of which reveals nothing of classically forbidden regions [17].) This is to be done in any case since usually a universe is not represented by one eigenfunction but by a wave packet. The above calculation of $\text{prob}$ is nonetheless necessary since the forbidden regions are defined for fixed values of $b_m$ only. With other words, every single component of the superposition ‘tunnels’ independently. Moreover, in more complicated models there might not be a sharply peaked wave packet at the classical turning point due to interference effects between the ‘incoming’ and the ‘outgoing’ part [18]. This is in agreement with the point of view that there is no tunneling because the wave function at this point is completely quantum.

There might be thus two effects of quantum cosmology at the classical turning point of the universe: The breakdown of the semiclassical approximation and a local change into an expanding state. One might try to circumvent the breakdown of the semiclassical approximation by considering decoherence. If this worked (Kiefer and Zeh have expressed their doubts that it does [11]), the increased classicality of the universe would further suppress the tunneling probability [12]. Both quantum effects would then be lost simultaneously, by the same token.

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**Appendix**

In order to prove that an inhomogeneous $z(t, r)$ is possible in the Kantowski–Sachs model with dust one has to discuss the equations of motion of the general spherical symmetric model. Its metric can be parametrized by

$$ds^2 = -N^2(t, r)dt^2 + L^2(t, r)dr^2 + R^2(t, r)d\Omega^2$$

(11)

where a vanishing shift–function has been chosen. One nevertheless has to satisfy the super-momentum constraint

$$0 = -L \left( \frac{\dot{R}}{N} \right) + \frac{R'}{R}(LR).$$

(12)

Here and in the following all equations are presented with the fields and their velocities instead of their momenta. According to this constraint a homogeneous surface measure $R(t, r) \to R(t) = b(t)$ necessitates a homogeneous lapse function $N(t, r) \to N(t)$. In the following the
The gravitational Hamiltonian reads
\[
0 = \frac{L \dot{R}^2}{R^2} - 2 \frac{\dot{R}^2}{R^2} (LR) - 2R \left( \frac{R'}{L} \right)' + \frac{R'^2}{L} - L + \Lambda LR^2 ,
\] (13)
and the equation of motion are given by
\[
\ddot{R} = \frac{\dot{R}^2}{2R} - \frac{R}{2} + \frac{3RR'^2}{2L^2} + \frac{\Lambda R^3}{2} ,
\] (14)
\[
(LR)' = \frac{\dot{R}}{R} (LR)' - \frac{L \dot{R}^2}{R} + R^2 \left( \frac{R'}{L} \right)' + \Lambda LR^2 + R \left( \frac{RR'}{L} \right)' .
\] (15)

The homogeneity requirement \( R(t, r) \rightarrow R(t) = b(t) \) obviously leads to an equation of motion for \( b(t) \) which is independent of any other variable. One can thus insert a solution of it into one of the other equations in order to determine \( L(t, r) \). Furthermore, since \( L' \) does only appear in combination with \( R' \) all partial derivatives cancel and the equations reduce to that of the inhomogeneous Kantowski–Sachs spacetime.

A non–vanishing pressure of a matter source would lead to an interaction of neighboring points. Thus, only incoherent or pressureless dust might serve as a matter source in this model. Since according to the Bianchi identities the dust flow is geodesic one may consider radial moving dust. Furthermore, by the other part of the Bianchi identities one obtains matter conservation \( \rho(t, r) L(t, r) R^2(t, r) = c_m(r) \). This general case is known as the Tolman model and in the case \( R(t, r) \rightarrow b(t) \) one gets the inhomogeneous Kantowski–Sachs model. In the above equations there will be an additional term in the Hamiltonian (the potential is supplemented by \(-c_m(r)\)) but the equations of motion remain unchanged by the dust. Obviously, there are no further complications to the inhomogeneity argument due to dust.

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