Abstract

The free electromagnetic field, solution of Maxwell’s equations and carrier of energy, momentum and spin, is construed as an emergent collective property of an ensemble of photons, and with this, the consistency of an interpretation that considers that the photons, and not the electromagnetic fields, are the primary ontology is established.

I. INTRODUCTION

This work is the third of a series of papers devoted to the construction and presentation of a consistent understanding of the quantum mechanical description of electromagnetic radiation, based on the assumption that the photons, and not the electromagnetic fields, constitute the basic ontology; that is, the photons are the objectively existent building blocks that require a quantum mechanical treatment and the electromagnetic fields are an emergent collective property of an ensemble of photons (as is, for instance, the pressure or temperature of an ensemble of molecules) and therefore a direct quantization of the fields is not necessary. This assumption may seem trivial and unnecessary to many physicists however the opposite view has been also adopted, namely, that the electromagnetic fields are the really existent objects and the photons are just a mathematical tool or “particle-like excitations”\(^1\) (quasi particles) corresponding to the normal modes of oscillations of the fields (in a similar way that phonons are used to describe lattice vibrations)
In the first paper of this series\(^2\), the commutation relations of the operators associated to the free electromagnetic field were derived from first principles (without reference to quantum field theory) and their singular character was analysed. The conclusion reached there is that we should not consider the electromagnetic fields as a primary ontology that must be treated quantum mechanically but instead, they can be thought to be a collective manifestation emerging from an ensemble of fundamental entities -the photons- with objective existence. One of the reasons for considering that the photons, and not the electromagnetic fields, are more appropriate for the quantum description of the electromagnetic phenomena is that the quantization of the lagrangian field theory for electromagnetism presents several difficulties, that can of course be solved, but are indicative that the electromagnetic fields are perhaps not the best language for the quantum mechanical description of electromagnetic radiation. One of these difficulties is, for example, that the canonical field variables (the four-vector potential) and its corresponding conjugate momentum are subject to several constraints\(^3\). Another example is that, in order to maintain relativistic covariance, every Lorentz transformation must be accompanied by a gauge transformation\(^4\). Much simplicity is gained if we accept the objective existence of the photons as the elementary entities responsible for the electromagnetic phenomena. These photons, massless relativistic particles, were the main concern of the second paper\(^5\). They are treated by quantum mechanics in the usual way except for the complications arising from their massless character that imposes a coupling of the spin states with the momentum states. For this reason there is a clear preference for treating the photons in their momentum eigenstates as compared with localized states. The elements of physical reality of the classical relativistic photon were formalized in the second paper of the series by the definition of a photon tensor \(f^{\mu\nu}\), that is most conveniently described in terms of the three-vectors \(e\) and \(b\). However these quantities are not the space components of a four-vector because they do not transform as such in a general Lorentz transformation. They should be considered as a convenient notation for the six nonvanishing components of the photon tensor \(f^{\mu\nu}\), according to the assignment given by

\[
\begin{pmatrix}
0 & e_1 & e_2 & e_3 \\
-e_1 & 0 & b_3 & -b_2 \\
-e_2 & -b_3 & 0 & b_1 \\
-e_3 & b_2 & -b_1 & 0
\end{pmatrix}.
\]

(1)

The energy \(E\), momentum \(\mathbf{P}\) and spin \(\mathbf{S}\) of a photon are related by

\[
E = c|\mathbf{P}|, \quad \mathbf{S} \times \mathbf{P} = 0,
\]

(2)
and as a consequence of its massless character (not of quantum mechanics), the energy must be related to some intrinsic frequency $\omega$. In any reference frame, we can visualize a positive or negative helicity photon of energy $E$ and spin $\hbar$, propagating with speed $c$ in a direction given by a unit vector $\mathbf{k}$, as a unit vector $\hat{\mathbf{e}}$ rotating clockwise or counterclockwise in a plane orthogonal to $\mathbf{k}$ with frequency $\omega = E/\hbar$. In the same plane we have another unit vector $\hat{\mathbf{b}} = \mathbf{k} \times \hat{\mathbf{e}}$ and with the vectors $\mathbf{e} = \omega \hat{\mathbf{e}}$ and $\mathbf{b} = \omega \hat{\mathbf{b}}$ we can build the photon tensor $f^{\mu\nu}$ whose Lorentz transformations provide the description of the photon in other reference frames.

The rotating vectors $\mathbf{e}_s(t)$, corresponding to a photon of helicity $s = \pm 1$, used to define the photon tensor can be given more conveniently in terms of the circular polarization complex vectors $\mathbf{e}_s$ defined by

$$\mathbf{e}_+ = \frac{1}{\sqrt{2}} (\hat{\mathbf{e}} + i \hat{\mathbf{b}})$$

$$\mathbf{e}_- = \frac{1}{\sqrt{2}} (i \hat{\mathbf{e}} + \hat{\mathbf{b}})$$

resulting in

$$\mathbf{e}_+(t) = \omega (\hat{\mathbf{e}} \cos \omega t + \hat{\mathbf{b}} \sin \omega t) = \left( \frac{\omega}{\sqrt{2}} \mathbf{e}_+ e^{-i\omega t} + \text{c.c.} \right)$$

$$\mathbf{e}_-(t) = \omega (\hat{\mathbf{b}} \cos \omega t + \hat{\mathbf{e}} \sin \omega t) = \left( \frac{\omega}{\sqrt{2}} \mathbf{e}_- e^{-i\omega t} + \text{c.c.} \right),$$

where c.c. stands for complex conjugation of the previous term. (Another initial position of the vector can be achieved simply by multiplying the circular polarization complex vectors by a phase, that is, $e^{-i\theta} \mathbf{e}_s$). We have then

$$\mathbf{e}_s(t) = \left( \frac{\omega}{\sqrt{2}} \mathbf{e}_s e^{-i\omega t} + \text{c.c.} \right).$$

The other vector, $\mathbf{b}_s(t)$, needed in order to build the photon tensor is simply obtained as $\mathbf{b}_s(t) = \mathbf{k} \times \mathbf{e}_s(t)$. The circular polarization complex vectors depend, of course, on the direction of propagation of the photon, $\mathbf{k}$, and could be denoted by $\mathbf{e}_s(\mathbf{k})$; however, for simplicity, we will not show this dependence explicitly in the notation. The usual orthogonality relations are

$$\mathbf{e}_s^* \cdot \mathbf{k} = 0,$$

$$\mathbf{e}_s^* \cdot \mathbf{e}_{s'} = \delta_{s,s'},$$

$$\mathbf{e}_s^* \times \mathbf{e}_{s'} = sik\delta_{s,s'},$$

$$\mathbf{k} \times \mathbf{e}_s = se_{-s}^*.$$
and for further reference we present some useful algebraic relations

\[ \epsilon_- = i \epsilon_+^* , \]  
\[ \epsilon_s \cdot \epsilon_{s'} = i(1 - \delta_{s,s'}) = i\delta_{s,-s'} , \]  
\[ \epsilon_s \times \epsilon_{s'} = sk(1 - \delta_{s,s'}) = sk\delta_{s,-s'} . \]  

Another relation that will be later needed is

\[ \sum_{s=\pm} (\epsilon_s^*)_i (\epsilon_s)_j = \delta_{i,j} - (k)_i(k)_j , \]

where \((\epsilon_s)_j\) and \((k)_j\) are the cartesian components of the corresponding vectors in an arbitrary set of orthogonal unit vectors \((\hat{x}_1, \hat{x}_2, \hat{x}_3)\). In order to prove this, notice first that \(\sum_{s=\pm}(\epsilon_s^*)_i (\epsilon_s)_j = (\hat{e})_i(\hat{e})_j + (\hat{b})_i(\hat{b})_j\). Now, \((\hat{e})_i, (\hat{b})_i, (k)_i\) are the cartesian components of \(\hat{x}_i\) in the orthogonal set \((\hat{e}, \hat{b}, k)\) and therefore, from the scalar product \(\hat{x}_i \cdot \hat{x}_j = \delta_{i,j} = (\hat{e})_i(\hat{e})_j + (\hat{b})_i(\hat{b})_j + (k)_i(k)_j\) follows the proof.

The quantum mechanical description of a photon in a Hilbert space \(\mathcal{H} = \mathcal{H}^S \otimes \mathcal{H}^K\) corresponding to the spin and kinematic part, is most conveniently done in terms of eigenstates of fixed helicity \(s = \pm 1\) and momentum \(\mathbf{p}\) (in the direction of the unit vector \(\mathbf{k}\)), denoted by \(\varphi_{s,\mathbf{p}} = \chi^s_s \otimes \phi_{\mathbf{p}}\). For an explicit representation of these states we can choose for the three dimensional spin-Hilbert space \(\mathcal{H}^S\), the one where the spin operators take the matrix form

\[
S_x = \hbar \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad S_y = \hbar \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}, \quad S_z = \hbar \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix};
\]

that is, with the matrix elements given by

\[
(S_j)_{kl} = -i\hbar \varepsilon_{jkl} .
\]

In this space, the helicity states are

\[
\chi^k_\pm = \frac{1}{2\sqrt{1-k_x k_y - k_y k_z - k_z k_x}} \begin{pmatrix} 1 - k_x(k_x + k_y + k_z) \pm i(k_y - k_z) \\ 1 - k_y(k_x + k_y + k_z) \pm i(k_x - k_y) \\ 1 - k_z(k_x + k_y + k_z) \pm i(k_x - k_y) \end{pmatrix} .
\]

For the kinematic description we can choose the space of square integrable functions (more precisely, its rigged extension) in the position representation, where the momentum eigenstates are given by

\[
\phi_{\mathbf{p}}(\mathbf{r}) = \frac{1}{(\sqrt{2\pi\hbar})^3} \exp\left(\frac{i}{\hbar} \mathbf{p} \cdot \mathbf{r} \right) .
\]
A free photon with fixed helicity and momentum is then described by the state $\varphi_{s,\mathbf{p}} = \chi_s^k \otimes \phi_{\mathbf{p}}$, and we can use the representations given in Eqs.(18) and (19) for any explicit calculation. These states build a basis suitable for the construction of any arbitrary state for one single photon. However the most interesting physical systems involve a large, or undetermined, number of photons whose state is presented in next section.

After having summarized the main results of the previous papers we will see in this paper how the electromagnetic fields are built and emerge as an observable property of an ensemble of photons.

II. FOCK SPACE STATES FOR MANY PHOTONS

The most effective way of dealing with a quantum system consisting in many photons, or with an indefinite number of photons, is to define the Fock space of states for the system. The Fock space is built by the orthogonal sum of the vacuum space plus the Hilbert space for one photon, plus the Hilbert space for two photons and so on. In this space we define the operators $a_{s}^{\dagger}(\mathbf{p})$ and $a_{s}(\mathbf{p})$ corresponding to the creation or annihilation of a photon with momentum $\mathbf{p}$ and helicity $s = \pm 1$. The effect of these operators on a state for a system with $n$ photons having helicity and momenta $s_1\mathbf{p}_1, s_2\mathbf{p}_2, \ldots, s_n\mathbf{p}_n$ are given by

$$a_{s}^{\dagger}(\mathbf{p}) \varphi_{s_1\mathbf{p}_1, \ldots, s_n\mathbf{p}_n} = \sqrt{n+1} \varphi_{s\mathbf{p}, s_1\mathbf{p}_1, \ldots, s_n\mathbf{p}_n}, \quad (20)$$

$$a_{s}(\mathbf{p}) \varphi_{s_1\mathbf{p}_1, \ldots, s_n\mathbf{p}_n} = \frac{1}{\sqrt{n+1}} \sum_{i=1}^{n} \delta_{s, s_i} \delta(\mathbf{p} - \mathbf{p}_i) \varphi_{s_1\mathbf{p}_1, \ldots, -\delta(s_i\mathbf{p}_i), \ldots, s_n\mathbf{p}_n}, \quad (21)$$

where the symbol $-\delta(s_i\mathbf{p}_i)$ indicates that the corresponding indices are eliminated if they are present. The vacuum state $\varphi_0$ with zero photons is such that

$$a_{s}(\mathbf{p}) \varphi_0 = 0, \quad (22)$$

and an $n$ photon state is built applying the creation operator to the vacuum state,

$$\varphi_{s_1\mathbf{p}_1, s_2\mathbf{p}_2, \ldots, s_n\mathbf{p}_n} = \frac{1}{\sqrt{n!}} a_{s_1}^{\dagger}(\mathbf{p}_1) a_{s_2}^{\dagger}(\mathbf{p}_2) \cdots a_{s_n}^{\dagger}(\mathbf{p}_n) \varphi_0. \quad (23)$$

The symmetry requirements for identical boson states impose the commutation relations for the creation and annihilation operators

$$[a_s(\mathbf{p}), a_s^{\dagger}(\mathbf{p}')] = \delta_{s,s'} \delta(\mathbf{p} - \mathbf{p}'), \quad [a_s^{\dagger}(\mathbf{p}), a_{s'}^{\dagger}(\mathbf{p}')] = [a_s(\mathbf{p}), a_{s'}(\mathbf{p}')] = 0. \quad (24)$$

Finally, the operator corresponding to the number of photons with helicity $s$ and with momentum within $d^3\mathbf{p}$ centered in $\mathbf{p}$ is given by
\[ N_s(p) = a_s^\dagger(p) a_s(p) , \]  

(25)

and the operator for the total number of photons in the system is

\[ N = \sum_s \int d^3p \ N_s(p) . \]  

(26)

The creation and annihilation operators are not only useful for the representation of the state of many photons but they can also be used to represent any observable of a multi-photon system. This is so because any operator can be given in terms of the spectral decomposition involving projectors in their eigenstates. Now if we represent these eigenstates in terms of the Fock basis generated by the application of creations operators to the vacuum state, we finally obtain the operator expressed with annihilation and creation operators. The fact that every observable can be given in terms of creation and annihilation operators will be relevant in next section where we will try to discover relevant observables by considering the simplest construction of hermitian operators with the nonhermitian operators \(a_s^\dagger(p)\) and \(a_s(p)\).

III. MANY PHOTON OBSERVABLES AND THE CONSTRUCTION OF THE ELECTROMAGNETIC FIELD

If we accept that the photons have objective existence and that each of them carries momentum \(p\), energy \(E = c|p| = \hbar \omega\) and spin \(\pm \hbar\) in the direction of propagation \(k\), we can build the total momentum, energy and spin of a system of many non interacting photons, simply as the sum of the corresponding contribution of each photon. The total energy, momentum and spin are then

\[ H = \sum_s \int d^3p \ \hbar \omega \ N_s(p) , \]  

(27)

\[ P = \sum_s \int d^3p \ p \ N_s(p) , \]  

(28)

\[ S = \int d^3p \ \hbar k \ (N_+(p) - N_-(p)) . \]  

(29)

The assumption that the photons are non interacting is a very good approximation because photons couple only to charged particles and the leading contribution photon-photon interaction, corresponding to a “box graph” Feynman diagram, is of fourth order in perturbation theory and can therefore be ignored. All the observables above are given in terms of the
number operator $N_s(p) = a^\dagger_s(p)a_s(p)$; however we can expect that besides this number operator there is another relevant hermitian operator related to the creation and annihilation operator. This expectation comes from the fact that a non hermitian operator like $a^\dagger_s(p)$ or $a_s(p)$ is related to two independent hermitian operators (this is similar to the case of complex numbers that contain two independent real numbers). Now, the number operator is just one of them (actually it is the operator modulus squared of $a_s(p)$) and we can expect the existence of another relevant operator, or observable of a multi-photon system, corresponding to the hermitian or antihermitian part of $a_s(p)$. Therefore we expect an observable of the form

$$\sum_s \int d^3p \ (f(s, p, E, r, t) \ a_s(p) \pm \text{h.c.}) ,$$

where h.c stands for “hermitian conjugate” of the previous term. Notice that the spin, momentum and energy of each individual photon are integrated and therefore this observable is related to the ensemble of photons as a collective property. Now we can make some considerations in order to make an educated guess of the form that the function $f(s, p, E, r, t)$ can take. First, this function must involve the elements of physical reality of the contributing photons that are formalized by the two orthogonal rotating vectors $e_s(t)$, given in Eq.(7), and $b_s(t) = k \times e_s(t)$; therefore we have two choices: $e_s e^{-i\omega t}$ and $(k \times e_s)e^{-i\omega t}$. Next we can expect that the space dependence of the function will be the same as the space dependence of the photon state in the position representation as given in Eq.(19). For the energy dependence, we don’t have any argument suggesting a particular form. Consistent with all this, we propose the two hermitian operators

$$E(r, t) = \frac{1}{2\pi \hbar} \sum_s \int d^3p \ \sqrt{\omega} \left( i \ a_s(p) \ e_s e^{\frac{i}{\hbar}(p \cdot r - Et)} + \text{h.c.} \right) ,$$

$$B(r, t) = \frac{1}{2\pi \hbar} \sum_s \int d^3p \ \sqrt{\omega} \left( i \ a_s(p) \ (k \times e_s) e^{\frac{i}{\hbar}(p \cdot r - Et)} + \text{h.c.} \right) .$$

The symbols used to denote these operators suggest that they correspond to the electromagnetic fields but, of course, we haven’t yet given any argument supporting this. That these operators are indeed the electromagnetic fields, will be established when we derive relations among them and with the total energy, momentum and spin. Before doing this, it is convenient to define another operator such that its time and space derivatives result in the two operators above. This is,

$$A(r, t) = \frac{c}{2\pi \hbar} \sum_s \int d^3p \ \frac{1}{\sqrt{\omega}} \left( a_s(p) \ e_s e^{\frac{i}{\hbar}(p \cdot r - Et)} + \text{h.c.} \right) ,$$
and we have

\[ \mathbf{E}(\mathbf{r}, t) = -\frac{1}{c} \partial_t \mathbf{A}(\mathbf{r}, t) , \quad \mathbf{B}(\mathbf{r}, t) = \nabla \times \mathbf{A}(\mathbf{r}, t) . \quad (34) \]

The operators in Eqs.(31, 32, 33) have linear dependence on the creation and annihilation operators whereas the total energy, momentum and spin given in Eqs.(27, 28, 29) have a quadratic dependence on them. Therefore we can expect that the total energy, momentum and spin will be related to \textit{products} of the operators above. In fact, we can show that

\[ H = \frac{1}{8\pi} \int d^3 \mathbf{r} \ (\mathbf{E}^2 + \mathbf{B}^2) , \quad (35) \]

\[ \mathbf{P} = \frac{1}{8\pi c} \int d^3 \mathbf{r} \ (\mathbf{E} \times \mathbf{B} - \mathbf{B} \times \mathbf{E}) , \quad (36) \]

\[ \mathbf{S} = \frac{1}{8\pi c} \int d^3 \mathbf{r} \ (\mathbf{E} \times \mathbf{A} - \mathbf{A} \times \mathbf{E}) . \quad (37) \]

These are the usual expressions for the energy, momentum and spin of the electromagnetic fields found in any electrodynamic text, except that here we allow for the possible non-commutation of the fields. The time dependence of the fields in Eqs.(31, 32, 33) is cancelled in the combinations of the integrands in Eqs.(35, 36, 37) and the resulting operators are conserved. The proof that the integrals in Eqs.(35, 36, 37) lead to the operators in Eqs.(27, 28, 29) is given in the appendix.

As a final confirmation that the fields in Eqs.(31,32) are indeed the free electromagnetic fields, we can see that they satisfy Maxwell’s equations.

\[ -\nabla \times \mathbf{E} = \frac{1}{c} \partial_t \mathbf{B} , \quad (38) \]

\[ \nabla \times \mathbf{B} = \frac{1}{c} \partial_t \mathbf{E} , \quad (39) \]

\[ \nabla \cdot \mathbf{E} = 0 , \quad (40) \]

\[ \nabla \cdot \mathbf{B} = 0 . \quad (41) \]

We have indeed,

\[ -\nabla \times \mathbf{E} = -\frac{1}{2\pi \hbar} \sum_s \int d^3 \mathbf{p} \ \sqrt{\omega} \left( i a_s(\mathbf{p}) \frac{i}{\hbar} (\mathbf{p} \times \epsilon_s) e^{i(\mathbf{p} \cdot \mathbf{r} - \mathbf{E} t)} + \text{h.c.} \right) \]

\[ = \frac{1}{2\pi \hbar} \sum_s \int d^3 \mathbf{p} \ \sqrt{\omega} \left( a_s(\mathbf{p}) \frac{E}{\hbar c} (\mathbf{k} \times \epsilon_s) e^{i(\mathbf{p} \cdot \mathbf{r} - \mathbf{E} t)} + \text{h.c.} \right) \]

\[ = \frac{1}{c 2\pi \hbar} \sum_s \int d^3 \mathbf{p} \ \sqrt{\omega} \left( a_s(\mathbf{p}) (\mathbf{k} \times \epsilon_s) i \partial_t e^{i(\mathbf{p} \cdot \mathbf{r} - \mathbf{E} t)} + \text{h.c.} \right) \]

\[ = \frac{1}{c} \partial_t \mathbf{B} , \quad (42) \]
and the other equations are proved in similar way.

With the fields given in terms of the creation and annihilation operators, we can now prove the commutation relations that were derived in reference (2) from general principles. Let us calculate first the commutation relations among two cartesian components of the electric field.

\[
[E_i(r_1, t_1), \ E_j(r_2, t_2)] = \frac{1}{(2\pi\hbar)^2} \sum_s \sum_{s'} \int d^3p \int d^3p' \sqrt{\omega\omega'} \left[ (i \ a_s(p) \ (\epsilon_s)_i \ e^{\frac{i}{\hbar}p\cdot(r_1-E_1t_1)} - i \ a_s^\dagger(p) \ (\epsilon^*_s)_i \ e^{\frac{i}{\hbar}p\cdot(r_1-E_1t_1)}) ,
                     (i \ a_{s'}(p') \ (\epsilon_{s'})_j \ e^{\frac{i}{\hbar}(p'-r_2-E't_2)} - i \ a_{s'}^\dagger(p') \ (\epsilon^*_{s'})_j \ e^{\frac{i}{\hbar}(p'-r_2-E't_2)}) \right]
= \frac{1}{(2\pi\hbar)^2} \sum_s \sum_{s'} \int d^3p \int d^3p' \sqrt{\omega\omega'} \left[ (a_s(p), a_{s'}^\dagger(p')) (\epsilon_s)_i (\epsilon^*_{s'})_j e^{\frac{i}{\hbar}(p\cdot(r_1-E_1t_1) + p'\cdot(r_2-E't_2))} \right]
+ \left[ a_s^\dagger(p), a_{s'}(p') \right] (\epsilon_s)_i (\epsilon_{s'})_j e^{-\frac{i}{\hbar}(p\cdot(r_1-E_1t_1) - p'\cdot(r_2-E't_2))}
= \frac{1}{(2\pi\hbar)^2} \int d^3p \omega (\delta_{i,j} - (k)_i (k)_j) e^{\frac{i}{\hbar}(p\cdot(r_1-r_2) - E(t_1-t_2))} - \text{c.c.})
= \frac{-2i}{(2\pi\hbar)^2} \int d^3p \omega (\delta_{i,j} - (k)_i (k)_j) e^{\frac{i}{\hbar}(p\cdot(r_1-r_2))} \sin(\omega(t_1-t_2)) .
\]

The last result was obtained doing a variable change \( p \to -p \) in the integration of the “c.c” term. This result can be written with help of the singular function

\[
D(\rho, \tau) = \frac{-1}{(2\pi\hbar)^3} \int d^3p \ e^{\frac{i}{\hbar}p\cdot\rho} \frac{\sin(\omega\tau)}{\omega}
= \frac{-1}{8\pi^2c} \left[ \delta(\rho - c\tau) - \delta(\rho + c\tau) \right] ,
\]

where \( \rho = |\rho| \). In order to prove the second expression we can write the integration over \( d^3p \) in polar coordinates with the “third” axis along the vector \( \rho \). The angle integrations can be easily performed and the \( p = |p| \) integration results in the Dirac distributions. In this second representation we see that the functions \( D(\rho, \tau) \) has support on the light cone, a fact of physical relevance. With this, the commutator for the cartesian components of the electric field is

\[
[E_i(r_1, t_1), \ E_j(r_2, t_2)] = -i4\pi\hbar c^2 \left( \delta_{i,j} \frac{1}{c^2} \partial_{1i} \partial_{2j} + \partial_{r_1,i} \partial_{r_2,j} \right) D(\mathbf{r}_1 - \mathbf{r}_2, t_1 - t_2) .
\]

For the calculation of the commutator between the cartesian components of the magnetic field we use the corresponding polarization vectors but we obtain the same result,

\[
[B_i(r_1, t_1), \ B_j(r_2, t_2)] = -i4\pi\hbar c^2 \left( \delta_{i,j} \frac{1}{c^2} \partial_{1i} \partial_{2j} + \partial_{r_1,i} \partial_{r_2,j} \right) D(\mathbf{r}_1 - \mathbf{r}_2, t_1 - t_2) ,
\]
and the remaining commutator is calculated in similar fashion,

\[ [E_i(r_1, t_1), B_j(r_2, t_2)] = i4\pi\hbar c \varepsilon_{ijk} \partial_{t_1} \partial_{r_{1,k}} D(r_1 - r_2, t_1 - t_2). \]  

(46)

The singular character of these commutators was discussed in the first paper with the conclusion that the treatment of the electromagnetic fields as quantum mechanical observables is perhaps not very meaningful. Regardless of its physical relevance, we can calculate commutation relations among any observables build with the creation and annihilation operators using the basic commutators given in Eq.(24). For instance we can easily calculate the commutators of the fields with the total number of photons

\[ [E(r, t), N] = \frac{1}{2\pi\hbar} \sum_s \int d^3p \sqrt{\omega} \left( i a_s(p) \epsilon_s e^{\pm(p \cdot r - E t)} - \text{h.c.} \right), \]  

(47)

\[ [B(r, t), N] = \frac{1}{2\pi\hbar} \sum_s \int d^3p \sqrt{\omega} \left( i a_s(p) (k \times \epsilon_s) e^{\pm(p \cdot r - E t)} - \text{h.c.} \right), \]  

(48)

\[ [A(r, t), N] = \frac{c}{2\pi\hbar} \sum_s \int d^3p \frac{1}{\sqrt{\omega}} \left( a_s(p) \epsilon_s e^{\pm(p \cdot r - E t)} - \text{h.c.} \right). \]  

(49)

Notice, as a curiosity, that the fields are related to the hermitian part of the creation operators and the commutators above are related to the anti-hermitian part.

**IV. ELECTROMAGNETIC FIELDS OF AN ENSEMBLE OF PHOTONS**

In this section we will see some simple examples of the electromagnetic fields associated to some multi-photon systems. The quantity that we must calculate is the expectation value of the fields in the quantum state describing the system of photons. The first simplest case is the vacuum, with zero photons, described by the state \( \varphi_0 \). It follows from Eq.(22) that any operator with the form given in Eq.(30) will have zero vacuum expectation value; therefore

\[ \langle \varphi_0, E(r, t) \varphi_0 \rangle = \langle \varphi_0, B(r, t) \varphi_0 \rangle = 0. \]  

(50)

This is of course expected; however what may be surprising is that the square of the fields have nonvanishing vacuum expectation values. In the case of the electric field, for example, the field in Eq.(31) has two terms and the square of it will have four terms, but three of them have vanishing vacuum expectation value due to Eq.(22). The remaining term is

\[ \langle \varphi_0, E^2(r, t) \varphi_0 \rangle = \frac{1}{(2\pi\hbar)^2} \sum_s \sum_{s'} \int d^3p \int d^3p' \sqrt{\omega\omega'} \epsilon_s \cdot \epsilon_{s'}^* e^{\pm(p \cdot r - E t)} e^{-\mp(p' \cdot r - E' t)} \langle \varphi_0, a_s(p) a_{s'}^\dagger(p') \varphi_0 \rangle. \]  

(51)
From the commutation relation of the creation operators we have $a_s(p) a^\dagger_{s'}(p') = a^\dagger_{s'}(p') a_s(p) + \delta_{ss'}\delta(p - p')$ and the first term has vanishing vacuum expectation value. Considering also the orthogonality relation of the polarization vectors, we finally get the nonzero, even diverging, vacuum expectation value

$$\langle \varphi_0, E^2(r, t) \varphi_0 \rangle = \frac{1}{(2\pi \hbar)^2} \frac{1}{2} \int d^3p \omega . \quad (52)$$

This non-vanishing value indicates that there are fluctuations of the electric field in vacuum. The result is divergent because we have calculated the fluctuation at one point. In a more realistic situation, where we calculate the fluctuations averaging the field in a small region, we would get a finite value. The electromagnetic fluctuations of the vacuum is a quantum effect with empirical manifestations in the Lamb shift or in the Casimir force.

Let us now consider the system made of $n$ photons in the same state with fixed helicity and momentum (this is of course possible because the photons are bosons). The quantum state of the system is then

$$\varphi_{n(s_1p_1)} = \frac{1}{\sqrt{n!}} \left(a^\dagger_{s_1}(p_1)\right)^n \varphi_0 , \quad (53)$$

where the index $n(s_1p_1)$ means an $n$ times repetition of $s_1p_1$. In order to calculate the expectation value of the electromagnetic field for this state, we need the expectation values of the creation and annihilation operators. However these also vanish:

$$\langle \varphi_{n(s_1p_1)} , a^\dagger_s(p) \varphi_{n(s_1p_1)} \rangle = \langle \varphi_{n(s_1p_1)} , a_s(p) \varphi_{n(s_1p_1)} \rangle = 0 , \quad (54)$$

and similarly $\langle \varphi_{n(s_1p_1)} , a^\dagger_s(p) \varphi_{n(s_1p_1)} \rangle = 0$. Therefore we have

$$\langle \varphi_{n(s_1p_1)} , E(r, t) \varphi_{n(s_1p_1)} \rangle = \langle \varphi_{n(s_1p_1)} , B(r, t) \varphi_{n(s_1p_1)} \rangle = 0 . \quad (55)$$

It may be surprising that the electromagnetic field for an exact number of photons vanish (as before, the square of the fields do not vanish) however we should consider that such a state with an exact number of photons is physically very rare because photons interact readily with matter being created and absorbed. A system much closer to physical reality is a system of photons with an indefinite number of photons described by a superposition of the states given in Eq.(53),

$$\psi = \sum_n C_n \varphi_{n(s_1p_1)} . \quad (56)$$

For this state we have
\[ \langle \psi , a_s^\dagger (p) \psi \rangle = \sum_n \sum_{n'} C_n^* C_{n'} \langle \varphi_n (s_1 p_1) , \varphi_{n'} (s_1 p_1) \rangle = \\
\sum_n \sum_{n'} C_n^* C_{n'} \sqrt{n' + 1} \langle \varphi_n (s_1 p_1) , \varphi_{n'} (s_1 p_1) \rangle = \sum_n C_n^* C_{n-1} \sqrt{n} \delta_{s_1,s} \delta(p_1 - p) , \quad (57) \]

\[ \langle \psi , a_s (p) \psi \rangle = \sum_n \sum_{n'} C_n^* C_{n'} \langle a_s^\dagger (p) \varphi_n (s_1 p_1) , \varphi_{n'} (s_1 p_1) \rangle = \\
\sum_n \sum_{n'} C_n^* C_{n'} \sqrt{n + 1} \langle \varphi_{n'} (s_1 p_1) , \varphi_{n''} (s_1 p_1) \rangle = \sum_n C_n^* C_{n+1} \sqrt{n + 1} \delta_{s_1,s} \delta(p_1 - p) , \quad (58) \]

and with this the electric field expectation value is

\[ \langle \psi , E(r,t) \psi \rangle = \frac{\sqrt{\omega_1}}{2\pi \hbar} \left( i \sum_n C_n^* C_{n+1} \sqrt{n + 1} \epsilon_{s_1} e^{i\vec{p}_1 \cdot \vec{r} - E_1 t} \right. \\
\left. - i \sum_n C_n^* C_{n-1} \sqrt{n} \epsilon_{s_1} e^{-i\vec{p}_1 \cdot \vec{r} - E_1 t} \right) . \quad (59) \]

The second sum can be written as the complex conjugation of the first and therefore we have

\[ \langle \psi , E(r,t) \psi \rangle = \frac{\sqrt{\omega_1}}{2\pi \hbar} \left( i \sum_n C_n^* C_{n+1} \sqrt{n + 1} \epsilon_{s_1} e^{i\vec{p}_1 \cdot \vec{r} - E_1 t} + \text{c.c.} \right) . \quad (60) \]

The electromagnetic field of an indefinite number of photons, all with the same helicity and momentum, is a plane wave with circular polarization. The quantum state where all photons are in the same one photon state of fixed helicity and momentum can be seen as a Bose-Einstein condensate that can be maintained, even at high temperature, because the photons do not interact (more precisely, their interaction can be neglected because it is a fourth order perturbation effect).

With some care in the calculations we can generalize this result. Let \( \varphi_{n_1, n_2, \ldots n_k} \) be the state corresponding to \( n_k \) photons with helicity and momentum \( s_k, p_k \). Let us consider a superposition of these states

\[ \psi = \sum_{n_1, n_2, \ldots} C_{n_1, n_2, \ldots} \varphi_{n_1, n_2, \ldots} . \quad (61) \]

Then, the electric field for this ensemble of photons is

\[ \langle \psi , E(r,t) \psi \rangle = \frac{1}{2\pi \hbar} \left( i \epsilon_{s_1} \sqrt{\omega_1} e^{i\vec{p}_1 \cdot \vec{r} - E_1 t} \sum_{n_1, n_2, \ldots} C_{n_1, n_2, \ldots}^* C_{n_1+1, n_2, \ldots} \sqrt{1 + n_1 + n_2 + \cdots} \right. \\
\left. + i \epsilon_{s_2} \sqrt{\omega_2} e^{i\vec{p}_2 \cdot \vec{r} - E_2 t} \sum_{n_1, n_2, \ldots} C_{n_1, n_2, \ldots}^* C_{n_1, n_2+1, \ldots} \sqrt{1 + n_1 + n_2 + \cdots} \right. \\
\left. + \cdots + \text{c.c.} \right) . \quad (62) \]

So we have the electromagnetic field given as a combination of plane waves related to the different values of helicity and momentum that are, macroscopically, associated to the normal modes of oscillation. The electromagnetic manifestations of general multi-photon systems is a very extensive subject, quantum optics, for which there are excellent books\(^6\).
V. CONCLUSIONS

In this work we have seen how the electromagnetic fields can be construed as an emergent property of an ensemble of photons, and with this, we have shown the consistency of an interpretation of physical reality where the photons are assigned objective existence in opposition to another interpretation where the electromagnetic fields are the primary ontology and the photons are denied physical existence. In order to avoid misunderstanding it should be clearly stated that in this interpretation the physical reality of the fields is not denied. Fields exist but are not the primary ontology. One interesting consequence of this interpretation is that the electromagnetic fields and the potential field become the same ontological character, that is, they exist on the same footing, as is apparent in Eqs. (31,32,33), and this is supported by the Aharonov-Bohm effect that requires the objective existence of the potential field. Another support for this interpretation comes from the interaction of electromagnetic radiation with matter as is described by QED. It is relevant to notice that the best description of the interactions is given in terms of Feynman diagrams that contain “photon lines” and not fields. Perturbation theory in QED clearly favours the ontology adopted in this work.

In this work and in the preceding ones\textsuperscript{2,5} we have presented a model for the photon, first as a relativistic massless particle with its elements of physical reality described by a photon tensor $f^{\mu\nu}$, and after, as a quantum system with the usual particle observables. The massless character of the photon requires that the photon energy must transform like a frequency and therefore the famous relation $E = h\nu$ is a consequence of special relativity and not of quantum mechanics. Another consequence of the massless character is that the spin and the linear momentum of the photon must be coupled and this establishes a clear preference of the momentum eigenstates for the quantum mechanical description of photons.

The wave-particle duality for the photon was clarified\textsuperscript{5} and should not be confused with the two conflicting interpretations mentioned above that identify the primary ontology with the photons in our choice and with the fields in the other case. So, in this interpretation, it is wrong to think that the photon is the particle-like duality partner of the wave-like electromagnetic field. Another confusion analysed is the erroneous identification of Maxwell’s equations for the electromagnetic fields with Schrödinger’s equation for the photon. Indeed, it was shown\textsuperscript{5} that the intended derivation of Maxwell’s equations from Schrödinger’s equation is erroneous.

In order to be able to accept the interpretation defended in this series of papers, it is necessary to show that the electromagnetic fields, solution of Maxwell’s equations and
carriers of energy, momentum and spin, can be construed as an emergent collective property of an ensemble of photons. This was the main purpose of this work.

VI. APPENDIX

Let us prove that the integration in Eq.(35) is the total energy as given in Eq.(27)

$$\frac{1}{8\pi} \int d^3r \left( E^2 + B^2 \right) = \frac{1}{8\pi} \int d^3r \left( \frac{1}{(2\pi\hbar)^2} \sum_s \sum'_{s'} \int d^3p \int d^3p' \sqrt{\omega \omega'} \right.$$

$$\left\{ \left( \hat{a}_s(p) \epsilon_s \ e^{i\pi(p \cdot r - Et)} - \hat{a}^+_s(p) \epsilon^*_s \ e^{-i\pi(p \cdot r - Et)} \right) \cdot \left( \hat{a}_{s'}(p') \epsilon_{s'} \ e^{i\pi(p' \cdot r - E't')} - \hat{a}^+_{s'}(p') \epsilon^*_{s'} \ e^{-i\pi(p' \cdot r - E't')} \right) + \left( \hat{a}_s(p) \ (k \times \epsilon_s) \ e^{i\pi(p \cdot r - Et)} - \hat{a}^+_s(p) \ (k \times \epsilon^*_s) \ e^{-i\pi(p \cdot r - Et)} \right) \cdot \left( \hat{a}_{s'}(p') \ (k' \times \epsilon_{s'}) \ e^{i\pi(p' \cdot r - E't')} - \hat{a}^+_{s'}(p') \ (k' \times \epsilon^*_{s'}) \ e^{-i\pi(p' \cdot r - E't')} \right) \right\} .$$

(63)

The integration over \( r \) leads to Dirac distribution and the products of the polarization vectors are given by the relations in Eqs.(8-14). Then

$$\frac{1}{8\pi} \int d^3r \left( E^2 + B^2 \right) = \frac{1}{8\pi} \int d^3r \left( \frac{1}{(2\pi\hbar)^2} \sum_s \sum'_{s'} \int d^3p \int d^3p' \sqrt{\omega \omega'} \right.$$

$$\left\{ \left( \hat{a}_s(p) \ a_{s'}(p') i\delta_{s,-s'} (2\pi\hbar)^3 \delta(p + p') e^{-i(\omega + \omega')t} \right) + \left( \hat{a}^+_s(p) \ a^+_s(p') \delta_{s,s'} (2\pi\hbar)^3 \delta(p - p') \right) + \left( \hat{a}^+_s(p) \ a_{s'}(p') \delta_{s,s'} (2\pi\hbar)^3 \delta(p - p') \right) + \left( \hat{a}_s(p) \ a^+_s(p') (-i) \delta_{s,-s'} (2\pi\hbar)^3 \delta(p + p') e^{i(\omega + \omega')t} \right)$$

$$\left. + \left( \hat{a}^+_s(p) \ a^+_s(p') \ s's' \delta_{s,-s'} (2\pi\hbar)^3 \delta(p + p') e^{-i(\omega + \omega')t} \right) + \left( \hat{a}_s(p) \ a^+_s(p') \ s's' \delta_{s,s'} (2\pi\hbar)^3 \delta(p - p') \right) + \left( \hat{a}^+_s(p) \ a_{s'}(p') \ s's' \delta_{s,s'} (2\pi\hbar)^3 \delta(p - p') \right) + \left( \hat{a}^+_s(p) \ a^+_s(p') \ s's' \delta_{s,-s'} (2\pi\hbar)^3 \delta(p + p') e^{i(\omega + \omega')t} \right) \right\}$$

$$= \sum_s \int d^3p \ h\omega \ \frac{1}{2} \left( a_s(p) a^+_s(p) + a^+_s(p) a_s(p) \right)$$

$$= \sum_s \int d^3p \ h\omega \ \left( N_s(p) + \frac{1}{2} |a_s(p), a^+_s(p)| \right)$$

$$= \sum_s \int d^3p \ h\omega \ N_s(p) + C_\infty 1 .$$

(64)

We obtain the wanted result if we ignore, as all authors do, the identity operator multiplied by an infinite constant. There are many hand waving arguments for doing it however this remains unsatisfactory. Let us prove now that the integration in Eq.(36) is the total momentum as given in Eq.(28)
\[
\frac{1}{8\pi c} \int_0^\beta \mathbf{r} \cdot (\mathbf{E} \times \mathbf{B} - \mathbf{B} \times \mathbf{E}) = \frac{1}{8\pi c} \int_0^\beta \mathbf{r} \cdot \frac{1}{(2\pi \hbar)^2} \sum_s \sum_{s'} \int_0^\beta \mathbf{p} \int_0^\beta \mathbf{p}' \sqrt{\omega\omega'} \\
\left[ \left( i a_s(\mathbf{p}) \mathbf{e}_s e^{\frac{i}{\hbar}(\mathbf{r} \cdot \mathbf{E})} - i a^{\dagger}_s(\mathbf{p}) \mathbf{e}_s^* e^{\frac{i}{\hbar}(\mathbf{r} \cdot \mathbf{E})} \right) \times \right. \\
\left( i a_{s'}(\mathbf{p}') \mathbf{e}_{s'} e^{\frac{i}{\hbar}(\mathbf{r}' \cdot \mathbf{E})} - i a^{\dagger}_{s'}(\mathbf{p}') \mathbf{e}_{s'}^* e^{\frac{i}{\hbar}(\mathbf{r}' \cdot \mathbf{E})} \right) \\
- \left( i a_s(\mathbf{p}) \mathbf{k} \times \mathbf{e}_s \right) e^{\frac{i}{\hbar}(\mathbf{p} \cdot \mathbf{r} - Et)} - i a^{\dagger}_s(\mathbf{p}) \mathbf{k} \times \mathbf{e}_s^* e^{\frac{i}{\hbar}(\mathbf{p} \cdot \mathbf{r} - Et)} \times \\
\left( i a_{s'}(\mathbf{p}') \mathbf{e}_{s'} e^{\frac{i}{\hbar}(\mathbf{p}' \cdot \mathbf{r} - E't')} - i a^{\dagger}_{s'}(\mathbf{p}') \mathbf{e}_{s'}^* e^{\frac{i}{\hbar}(\mathbf{p}' \cdot \mathbf{r} - E't')} \right) \right] . \tag{65}
\]

The integration over \( \mathbf{r} \) leads to Dirac distribution and the products of the polarization vectors are given by the relations in Eqs.(8-14). Then

\[
\frac{1}{8\pi c} \int_0^\beta \mathbf{r} \cdot (\mathbf{E} \times \mathbf{B} - \mathbf{B} \times \mathbf{E}) = \frac{1}{8\pi c} \int_0^\beta \mathbf{r} \cdot \frac{1}{(2\pi \hbar)^2} \sum_s \sum_{s'} \int_0^\beta \mathbf{p} \int_0^\beta \mathbf{p}' \sqrt{\omega\omega'} \\
- a_s(\mathbf{p}) a_{s'}(\mathbf{p}') (-i) \mathbf{k} \mathbf{e}_{s,s'} (2\pi \hbar)^3 \delta(\mathbf{p} + \mathbf{p}') e^{-i(\omega + \omega')t} \\
+ a_s(\mathbf{p}) a^{\dagger}_{s'}(\mathbf{p}') \mathbf{k} \mathbf{e}_{s,s'} (2\pi \hbar)^3 \delta(\mathbf{p} - \mathbf{p}') e^{i(\omega + \omega')t} \\
+ a^{\dagger}_s(\mathbf{p}) a_{s'}(\mathbf{p}') \mathbf{k} \mathbf{e}_{s,s'} (2\pi \hbar)^3 \delta(\mathbf{p} + \mathbf{p}') e^{i(\omega + \omega')t} \\
+ a_s(\mathbf{p}) a_{s'}(\mathbf{p}') \mathbf{k} \mathbf{e}_{s,s'} (2\pi \hbar)^3 \delta(\mathbf{p} - \mathbf{p}') e^{-i(\omega + \omega')t} \\
- a^{\dagger}_s(\mathbf{p}) a_{s'}(\mathbf{p}') (-i) \mathbf{k} \mathbf{e}_{s,s'} (2\pi \hbar)^3 \delta(\mathbf{p} + \mathbf{p}') e^{i(\omega + \omega')t} \\
+ a_s(\mathbf{p}) a^{\dagger}_{s'}(\mathbf{p}') \mathbf{k} \mathbf{e}_{s,s'} (2\pi \hbar)^3 \delta(\mathbf{p} - \mathbf{p}') e^{-i(\omega + \omega')t} \\
+ a^{\dagger}_s(\mathbf{p}) a_{s'}(\mathbf{p}') (-i) \mathbf{k} \mathbf{e}_{s,s'} (2\pi \hbar)^3 \delta(\mathbf{p} + \mathbf{p}') e^{-i(\omega + \omega')t} \\
+ a_s(\mathbf{p}) a^{\dagger}_{s'}(\mathbf{p}') \mathbf{k} \mathbf{e}_{s,s'} (2\pi \hbar)^3 \delta(\mathbf{p} - \mathbf{p}') e^{i(\omega + \omega')t}) \\
= \sum_s \int_0^\beta \mathbf{p} \frac{1}{2} (a_s(\mathbf{p}) a^\dagger_s(\mathbf{p}) + a^\dagger_s(\mathbf{p}) a_s(\mathbf{p}) + i(a_s(\mathbf{p}) a_s(-\mathbf{p}) e^{-i2\omega t} - a^\dagger_s(\mathbf{p}) a^\dagger_s(-\mathbf{p}) e^{i2\omega t})) \\
= \sum_s \int_0^\beta \mathbf{p} \left( N_s(\mathbf{p}) + \frac{1}{2} [a_s(\mathbf{p}), a^\dagger_s(\mathbf{p})] \right) \\
= \sum_s \int_0^\beta \mathbf{p} \mathbf{p} N_s(\mathbf{p}) . \tag{66}
\]

The time dependent term and the term involving the commutator give a vanishing contribution upon integration because they are odd under the transformation \( \mathbf{p} \rightarrow -\mathbf{p} \). Finally, let us prove that the integration in Eq.(37) is the total spin as given in Eq.(29)

\[
\frac{1}{8\pi c} \int_0^\beta \mathbf{r} \cdot (\mathbf{E} \times \mathbf{A} - \mathbf{A} \times \mathbf{E}) = \frac{1}{8\pi} \int_0^\beta \mathbf{r} \cdot \frac{1}{(2\pi \hbar)^2} \sum_s \sum_{s'} \int_0^\beta \mathbf{p} \int_0^\beta \mathbf{p}' \sqrt{\omega\omega'} \\
\left[ \left( i a_s(\mathbf{p}) \mathbf{e}_s e^{\frac{i}{\hbar}(\mathbf{r} \cdot \mathbf{E})} - i a^{\dagger}_s(\mathbf{p}) \mathbf{e}_s^* e^{\frac{i}{\hbar}(\mathbf{r} \cdot \mathbf{E})} \right) \times \right. \\
\left( a_{s'}(\mathbf{p}') \mathbf{e}_{s'} e^{\frac{i}{\hbar}(\mathbf{r}' \cdot \mathbf{E})} + a^{\dagger}_{s'}(\mathbf{p}') \mathbf{e}_{s'}^* e^{\frac{i}{\hbar}(\mathbf{r}' \cdot \mathbf{E})} \right) \\
- \left( a_s(\mathbf{p}) \mathbf{e}_s e^{\frac{i}{\hbar}(\mathbf{r} \cdot \mathbf{E})} + a^{\dagger}_s(\mathbf{p}) \mathbf{e}_s^* e^{\frac{i}{\hbar}(\mathbf{r} \cdot \mathbf{E})} \right) \times \\
\left( i a_{s'}(\mathbf{p}') \mathbf{e}_{s'} e^{\frac{i}{\hbar}(\mathbf{r}' \cdot \mathbf{E})} - i a^{\dagger}_{s'}(\mathbf{p}') \mathbf{e}_{s'}^* e^{\frac{i}{\hbar}(\mathbf{r}' \cdot \mathbf{E})} \right) \right] . \tag{67}
\]
The integration over $r$ leads to Dirac distribution and the products of the polarization vectors are given by the relations in Eqs. (8-14). Then

$$\frac{1}{8\pi c} \int d^3 r \quad (E \times A - A \times E) = \frac{1}{8\pi} \frac{1}{(2\pi \hbar)^2} \sum_s \sum_{s'} \int d^3 p \int d^3 p' \sqrt{\frac{\omega}{\omega'}} (\quad$$

$$i \ a_s(p) \ a_{s'}(p') \ s k \delta_{s,-s'} \ (2\pi \hbar)^3 \delta(p + p') e^{-i(\omega + \omega')t}$$

$$+ i \ a_s(p) \ a_{s'}^\dagger(p') \ (-i) s k \delta_{s,s'} \ (2\pi \hbar)^3 \delta(p - p')$$

$$- i \ a_s^\dagger(p) \ a_{s'}(p') \ i s k \delta_{s,s'} \ (2\pi \hbar)^3 \delta(p - p')$$

$$- i \ a_s^\dagger(p) \ a_{s'}^\dagger(p') \ s k \delta_{s,-s'} \ (2\pi \hbar)^3 \delta(p + p') e^{i(\omega + \omega')t}$$

$$- i \ a_s(p) \ a_{s'}(p') \ s k \delta_{s,-s'} \ (2\pi \hbar)^3 \delta(p + p') e^{-i(\omega + \omega')t}$$

$$+ i \ a_s(p) \ a_{s'}^\dagger(p') \ (-i) s k \delta_{s,s'} \ (2\pi \hbar)^3 \delta(p - p')$$

$$- i \ a_s^\dagger(p) \ a_{s'}(p') \ i s k \delta_{s,s'} \ (2\pi \hbar)^3 \delta(p - p')$$

$$+ i \ a_s^\dagger(p) \ a_{s'}^\dagger(p') \ s k \delta_{s,-s'} \ (2\pi \hbar)^3 \delta(p + p') e^{i(\omega + \omega')t})$$

$$= \sum_s \int d^3 p \ s h k \frac{1}{2} \left( a_s(p) \ a_s^\dagger(p) + a_s^\dagger(p) \ a_s(p) \right)$$

$$= \sum_s \int d^3 p \ s h k \left( N_s(p) + \frac{1}{2} [a_s(p), a_s^\dagger(p)] \right)$$

$$= \sum_s \int d^3 p \ s h k \ N_s(p) .$$

**ACKNOWLEDGMENTS**

We would like to thank H. Mártil, O. Sampayo, and A. Jacobo for challenging discussions. This work received partial support from “Consejo Nacional de Investigaciones Científicas y Técnicas” (CONICET), Argentina.
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