Wigner function evolution in a self-Kerr medium derived by entangled state representation

Li-Yun Hu\(^{1,2,3}\), Zheng-Lu Duan\(^1\), Xue-Xiang Xu\(^1\) and Zi-Sheng Wang\(^{1,2}\)

1 College of Physics and Communication Electronics, Jiangxi Normal University, Nanchang 330022, People’s Republic of China
2 Key Laboratory of Optoelectronic and Telecommunication of Jiangxi, Nanchang 330022, People’s Republic of China
E-mail: hlyun2008@126.com and hlyun2008@gmail.com

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Abstract
By introducing the thermo-entangled state representation, we convert the calculation of Wigner function (WF) of density operator to an overlap between ‘two pure’ states in a two-mode enlarged Fock space. Furthermore, we derive a new WF evolution formula of any initial state in a self-Kerr medium with photon loss and find that the photon number distribution for any initial state is independent of the coupling factor with the Kerr medium, where the number state is not affected by the Kerr nonlinearity and evolves into a density operator of binomial distribution.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction
Nonclassicality of optical fields has been a topic of great interest in quantum optics and quantum information processing [1], which is usually associated with quantum interference and entanglement. The phase space Wigner function (WF) [2, 3] of quantum states of light is a powerful tool for investigating such nonclassical effects. The WF was first introduced by Wigner in 1932 to calculate quantum corrections to a classical distribution function of a quantum-mechanical system. The partial negativity of the WF is indeed a good indication of the highly nonclassical character of the state [4] and monitors a decoherence process of a quantum state, e.g. the excited coherent state in both photon-loss and thermal channels [5, 6], the single-photon subtracted squeezed vacuum state in both amplitude decay and phase damping channels [7], and so on [8–12].

\(^{3}\) Author to whom any correspondence should be addressed.
Nonlinear interaction of light in a medium provides a very useful framework to study various nonclassical properties of quantum states of radiation. Recently, much attention has been paid to the highly $\chi^{(3)}$ nonlinear systems due to their applications, such as nondemolition measurements [13], quantum computing architectures [14], and single-particle detectors [15]. To enhance the Kerr nonlinearity, some schemes are presented by using electromagnetically induced transparency [16], and the Purcell effect [17] and the interaction of a cavity mode with atoms [18] are proposed to realize higher order nonlinearity of magnitude than natural self-Kerr interactions with negligible losses. On the other hand, decoherence is a main obstacle to implementation of schemes using Kerr nonlinearity. The influence of decoherence effects on quantum states generated as a result of the self-Kerr nonlinear interaction has been widely investigated [19–26]. For instance, Milburn and Holmes [19] solved a master equation by changing it to a Fokker–Planck (F-P) equation for the Q-function and for an initial coherent state. In addition, the evolution of a single mode of the electromagnetic field interacting with a squeezed bath in a Kerr medium is considered numerically for the Q-function [20].

Recently, another F-P equation for the WF evolution in a noisy self-Kerr medium is presented [27]. The authors numerically solved this equation assuming coherent state as an initial condition and discussed its dissipation effects. Then the influence of modal loss on the quantum state generation via cross-Kerr nonlinearity [28] is examined [29]. It is shown that correlated losses in modern realization of schemes of large cross-Kerr nonlinearity might be led to enhancement of non-classicality. In this work, we concentrate our attention on the self-Kerr medium and derive a WF evolution formula and a photon-distribution formula of any initial state in the self-Kerr medium with photon loss. As far as we are concerned, there is no report about the analytical WF evolution.

On the other hand, by using the thermo-entangled state representation (TESR) $|\eta\rangle$, we solved various master equations (MEs) to obtain density operators with an infinite operator-sum representation [30], and then revealed that the WF of the density operator can be expressed as an overlap between two pure states (see equation (21) below) [31]. This brings much convenience to calculate time evolution of WFs when quantum decoherence occurs. The merits of this approach are in three aspects: (i) using $|\eta\rangle$, one can replace the quantum Liouville equation by a Schrödinger-type equation for a ‘mock state’ $|\rho\rangle$ with a non-Hermitian Hamiltonian; (ii) the dissipation of the mixed state $\rho(t)$ in $|\eta\rangle$ representation turns out to be the evolution of initial state $\rho_0$ in the decayed entangled state $|\eta e^{-\kappa t}\rangle$ representation, accompanying with a Gaussian damping factor $e^{-\frac{1}{2} e^{-2\kappa t} |\eta|^2}$. This exhibits dissipation in an intuitive manner. Moreover, the transition $|\eta\rangle \rightarrow |\eta e^{-\kappa t}\rangle$ is governed by a two-mode squeezing operator; thus, dissipation of a system immersed in a thermal environment can be described by squeezing in the thermo-entangled state representation, a fresh view; (iii) one can identify the explicit infinite-dimensional Kraus operators of $\rho(t)$, say, for the amplitude-damping channel and for the laser process. Thus, the TESR is beneficial to quantum decoherence theory.

In our paper, we shall appeal the TESR $|\eta\rangle$ to treat the WF evolution at any initial condition in the self-Kerr medium with photon loss and present a new formula to calculate time evolution of the WF for quantum decoherence. In addition, based on the derived WF evolution formula, we shall deduce the photon number distribution for any initial state in the presence of Kerr interaction, where the photon number distribution is independent of the coupling factor $\chi$ that is relative to the Kerr medium. As examples, the WF formula is applied to the cases of initial coherent state and number state, respectively. Conclusions are involved in the last section.
2. Brief review of thermo-entangled state representation

We begin with briefly reviewing the thermo-entangled state representation (TESR). On the basis of Umezawa–Takahashi thermo-field dynamics (TFD) [32–34], we constructed the TESR in the doubled Fock space [35–37]

\[
|\eta\rangle = D(\eta)|\eta = 0\rangle = \exp\left[-\frac{1}{2}|\eta|^2 + \eta a^\dagger - \eta^* a^\dagger + a^\dagger a^\dagger\right]|0, \tilde{0}\rangle,
\]

where \( D(\eta) = e^{\eta a^\dagger - \eta^* a} \) is the displacement operator, \( a^\dagger \) is the creation operator associated with the fictitious mode, \( |0, \tilde{0}\rangle = |0\rangle|0\rangle \), and \( |0\rangle \) is annihilated by \( a \) with the relations \( [\tilde{a}, a^\dagger] = 1 \) and \( [a, a^\dagger] = 0 \). The structure of \( |\eta\rangle \) is similar to that of the EPR eigenstate shown in [35].

Operating \( a \) and \( \tilde{a} \) on \( |\eta\rangle \) in equation (1) we can obtain the eigen-equations of \( |\eta\rangle \),

\[
\begin{align*}
(a - \tilde{a}^\dagger)|\eta\rangle &= \eta|\eta\rangle, \\
(a^\dagger - \tilde{a})|\eta\rangle &= \eta^*|\eta\rangle, \\
(\eta^*(a - \tilde{a}^\dagger)|\eta\rangle &= \eta|\eta\rangle.
\end{align*}
\]

Note that \([a - \tilde{a}^\dagger], (a^\dagger - \tilde{a})\) = 0; thus, \( |\eta\rangle \) is the common eigenvector of \( (a - \tilde{a}^\dagger) \) and \( (\tilde{a} - a^\dagger) \).

Using the normally ordered form of vacuum projector \( |0, \tilde{0}\rangle\langle 0, \tilde{0}| = \exp(-a^\dagger a - \tilde{a}^\dagger \tilde{a}) : \) and the technique of integration within an ordered product (IWOP) of operators [38–40], we can easily prove that \( |\eta\rangle \) is complete and orthonormal:

\[
\int \frac{d^2 \eta}{\pi} |\eta\rangle \langle \eta| = 1, \quad \langle \eta'|\eta\rangle = \pi \delta(\eta' - \eta)\delta(\eta'^* - \eta^*).
\]

It is easily seen that \( |\eta = 0\rangle \) has the properties

\[
|\eta = 0\rangle = e^{\eta a^\dagger}|0, \tilde{0}\rangle = \sum_{n=0}^\infty |n, \tilde{n}\rangle
\]

(where \( n = \tilde{n} \), and \( |n, \tilde{n}\rangle = |n\rangle \otimes |\tilde{n}\rangle = \frac{1}{\sqrt{n!}} \tilde{n} \tilde{a}^n a^\dagger |0, \tilde{0}\rangle; |n\rangle \) and \( |\tilde{n}\rangle \) denote the Fock states in the real Hilbert space \( H \) and the fictitious Hilbert space \( \tilde{H} \), respectively) and

\[
\begin{align*}
\begin{align*}
\hat{a} |\eta = 0\rangle &= \tilde{a}^\dagger |\eta = 0\rangle, \\
\hat{a}^\dagger |\eta = 0\rangle &= \tilde{a} |\eta = 0\rangle, \\
(\hat{a}^\dagger \hat{a})^n |\eta = 0\rangle &= (\tilde{a}^\dagger \tilde{a})^n |\eta = 0\rangle.
\end{align*}
\end{align*}
\]

Note that density operators \( \rho(\hat{a}^\dagger \hat{a}) \) are defined in the real space which are commutative with operators \( (\tilde{a}^\dagger \tilde{a}) \) in the tilde space.

In a similar way, we can introduce the state vector \( |\xi\rangle \) conjugated to \( |\eta\rangle \), defined as

\[
|\xi\rangle = D(\xi)e^{-a\dagger a^\dagger}|0, \tilde{0}\rangle = \exp\left(-\frac{1}{2} |\xi|^2 + \xi a^\dagger + \xi^* a^\dagger - a^\dagger a^\dagger\right)|0, \tilde{0}\rangle = (-1)^{a^\dagger a}|\eta = -\xi\rangle,
\]

which also possesses orthonormal and complete properties

\[
\int \frac{d^2 \xi}{\pi} |\xi\rangle \langle \xi| = 1, \quad \langle \xi'|\xi\rangle = \pi \delta(\xi' - \xi)\delta(\xi'^* - \xi^*).
\]
3. Master equation for a self-Kerr interaction

In the Markov approximation and interaction picture, the master equation for a dissipative cavity with the Kerr medium has the form [25, 41]

$$\frac{d\rho}{dt} = -i\chi[(a^\dagger a)^2, \rho] + \gamma(2a\rho a^\dagger - a^\dagger a^\dagger a - \rho a^\dagger a),$$

(8)

where $\gamma$ is the decaying parameter of the dissipative cavity and $\chi$ is the coupling factor depending on the Kerr medium. Milburn and Holmes [19] solved this equation by changing it to a partial differential equation for the Q-function and for an initial coherent state. Here we will solve the master equation by virtue of the entangled state representation and present the infinite sum representation of the density operator.

Operating the both sides of equation (8) on the state $|\eta = 0\rangle$, letting $|\rho\rangle = \rho |\eta = 0\rangle$ (here one should understand the single-mode density operator $\rho$ on the left of equation (8) as the direct product $\rho \otimes I$ when $\rho$ acts onto the two-mode state $|\eta = 0\rangle = e^{\sigma_0}|0, 0\rangle$, where $I$ is the identity operator in the auxiliary mode), and using equation (5) we can convert the master equation in equation (8) into the following form:

$$\frac{d}{dt} |\rho\rangle = \{ -i\chi[(a^\dagger a)^2 - (a^\dagger a)^2] + \gamma(2a\rho a^\dagger - a^\dagger a^\dagger a - \rho a^\dagger a) \} |\rho\rangle,$$

(9)

i.e. an evolution equation of state vector $|\rho\rangle$. Its solution is then of the form

$$|\rho\rangle = \exp[-i\chi t((a^\dagger a)^2 - (a^\dagger a)^2)] + \gamma t(2a\rho a^\dagger - a^\dagger a^\dagger a - \rho a^\dagger a) |\rho_0\rangle,$$

(10)

where $|\rho_0\rangle = \rho_0 |\eta = 0\rangle$, $\rho_0$ being an initial density operator. The advantage of using thermo-field notation over more traditional algebraic manipulation with superoperators is that in many situations (and, particularly, ones of our interest) it enables us to simplify, make more illustrative and less cumbersome finding solution (10) and estimation of time-dependent matrix elements. In particular, it allows us to represent in a simple form a factorization of the superoperator $\exp\{\cdots\}$ into multipliers with easily estimated actions on the number states [42].

By introducing the following operators:

$$K_0 = a^\dagger a - \hat{a}^\dagger \hat{a}, \quad K_\tau = \frac{a^\dagger a + \hat{a}^\dagger \hat{a} + 1}{2}, \quad K_- = a\hat{a},$$

(11)

which satisfy $[K_0, K_\tau] = [K_0, K_-] = 0$, we can rewrite equation (10) as

$$|\rho\rangle = \exp[-i\chi t(K_0(2K_\tau - 1) + \gamma t(2K_\tau - 2K_\tau + 1)) |\rho_0\rangle$$

$$\times \exp\left[\frac{-2t(\gamma + i\chi K_0)}{\gamma + i\chi K_0} \left(K_\tau + \frac{-\gamma}{\gamma + i\chi K_0} K_\tau \right)\right] |\rho_0\rangle.$$ (12)

With the aid of the operator identity [44]

$$e^{[A + B]t} = e^{iA} \exp[\sigma B(1 - e^{-\lambda\tau})/\tau]$$

$$= \exp[\sigma B(1 - e^{-\lambda\tau})/\tau] e^{iA},$$

(13)

which is valid for $[A, B] = \tau B$, and noticing $[K_\tau, K_-] = -K_-$, we can reform equation (12) as

$$|\rho\rangle = \exp[i\chi tK_0 + \gamma t] \exp[\Gamma_\tau K_\tau] \exp[\Gamma_- K_-] |\rho_0\rangle,$$

(14)

where

$$\Gamma_\tau = -2t(\gamma + i\chi K_0), \quad \Gamma_- = \frac{\gamma(1 - e^{-2t(i\chi K_0)})}{\gamma + i\chi K_0}.$$ (15)
From equation (14) we can obtain the infinite operator-sum form of \( \rho(t) \) (see appendix A):

\[
\rho(t) = \sum_{m,n,l=0}^{\infty} M_{m,n,l}^{\dagger} \rho_0 M_{m,n,l},
\]

where the two operators \( M_{m,n,l} \) and \( M_{m,n,l}^{\dagger} \) are respectively defined as

\[
M_{m,n,l} \equiv \sqrt{\frac{\Lambda_{m,n}}{l!}} e^{-i\chi t(m-n)} |m\rangle \langle m| a^{\dagger},
\]

\[
M_{m,n,l}^{\dagger} \equiv \begin{cases} 
\sqrt{\frac{\Lambda_{m,n}}{l!}} e^{-i\chi t(m-n)} |n\rangle \langle n| a^{\dagger} & \text{if } l \text{ is even} \\
\text{dagger} & \text{if } l \text{ is odd}
\end{cases},
\]

(17)

where \( \Lambda_{m,n} \) is defined by

\[
\Lambda_{m,n} = \frac{\gamma(1 - e^{-2i\chi(m-n)})}{\gamma + i\chi (m-n)}.
\]

(18)

Although \( M_{m,n,l} \) is not Hermite conjugate to \( M_{m,n,l}^{\dagger} \), the normalization still holds \( \sum_{m,n,l=0}^{\infty} M_{m,n,l}^{\dagger} M_{m,n,l} = 1 \), see appendix B [43], i.e. they are trace-preserving in a general sense, so \( M_{m,n,l} \) and \( M_{m,n,l}^{\dagger} \) may be named the generalized Kraus operators.

4. Evolution of WF for the self-Kerr channel

The Kerr medium is one of the simplest nonlinearity, which shall allow us to investigate the full time-dependent WF dynamics with or without a quantum noise. In this section, we consider WF’s time evolution in the self-Kerr medium channel. For this purpose, we shall derive a new expression of WF in the TESR. According to the definition of WF of density operator \( \rho \),

\[
W(\alpha, \alpha^*) = \text{Tr}[\Delta(\alpha, \alpha^*) \rho],
\]

(19)

where \( \Delta(\alpha, \alpha^*) \) is the single-mode Wigner operator [2, 44], whose explicit normally ordered form is [45]

\[
\Delta(\alpha, \alpha^*) = \frac{1}{\pi} : e^{-2(a^{\dagger} - \alpha^*)(a - \alpha)} : = \frac{1}{\pi} D(2\alpha)(-1)^{a^{\dagger}a}.
\]

(20)

By using \( \langle \tilde{n}| \tilde{m} \rangle = \delta_{n,m} \) and noting (6) as well as \( |\rho\rangle = |\rho|\eta = 0 \), we can reform equation (19) as

\[
W(\alpha, \alpha^*) = \sum_{m,n}^\infty \langle n, \tilde{n} | \Delta(\alpha, \alpha^*) \rho | m, \tilde{m} \rangle
\]

\[
= \frac{1}{\pi} \langle \eta = 0 | D(2\alpha)(-1)^{a^{\dagger}a} | \rho \rangle
\]

\[
= \frac{1}{\pi} \langle \eta = -2\alpha | (-1)^{a^{\dagger}a} | \rho \rangle
\]

\[
= \frac{1}{\pi} \langle \xi = 2\alpha | \rho \rangle,
\]

(21)

where equation (21) is the WF formula in thermo-entangled state representation, with which the WF of density operator is simplified as an overlap between two ‘pure states’ in the enlarged Fock space, rather than using the ensemble average in the system-mode space. This will bring much convenience to calculate the time evolution of WFs when quantum decoherence occurs.
Projecting (14) on the entangled state representation \( \frac{1}{\pi} \langle \xi_{2\omega} | \) and inserting the completeness relation (7), we find

\[
W(\alpha, \alpha^*, t) = 4 \int \frac{d^2 \beta}{\pi} G(\alpha, \beta, t) W(\beta, \beta^*, 0),
\]

where \( W(\alpha, \alpha^*, t) \) and \( W(\beta, \beta^*, 0) \) are the anytime WF and the initial WF, respectively, and

\[
G(\alpha, \beta, t) = \langle \xi_{2\omega} | \exp \{ i \chi t K_0 + \gamma t \} \exp [\Gamma ; K_z] \exp [\Gamma^* K_{-}] | \xi_{2\omega} \rangle.
\]

(23)

It is convenient to calculate the matrix element in (23) according to the two-mode Fock space. Thus, the \( \langle \xi_{2\omega} | \) is expanded as

\[
\langle \xi | = \langle 0, 0 | \sum_{m,n=0}^{\infty} \frac{\alpha^m \alpha^*_n}{m!n!} H_{m,n}(\xi, \xi)e^{-|\xi|^2/2}.
\]

(24)

By using the two-mode Fock state \( |m, \bar{n}\rangle = a^m \bar{n} |0, 0\rangle \), we get

\[
\langle \xi | m, \bar{n}\rangle = H_{m,n}(\xi^*, \xi)e^{-|\xi|^2/2}/\sqrt{m!\bar{n}!},
\]

(25)

where \( H_{m,n}(\xi^*, \xi) \) is the two-variable Hermite polynomials \([46, 47]\). Inserting the complete relation \( \sum_{m,n=0}^{\infty} |m, \bar{n}\rangle \langle m, \bar{n}| = 1 \), after a long but straight calculation, WF’s evolution is given by (see appendix C)

\[
W(\alpha, \alpha^*, t) = \sum_{m,n=0}^{\infty} C_{m,n}(\alpha, \alpha^*, t) E_{m,n},
\]

(26)

where \( \Lambda_{m,n} = \gamma (1 - e^{-2\gamma + \gamma(m+n)}) \),

\[
C_{m,n}(\alpha, \alpha^*, t) = \frac{e^{-i\chi t (m^2 - n^2)} - i\gamma (m+n) e^{-2|\alpha|^2}}{m!n!(\Lambda_{m,n} + 1) (\Lambda_{m,n} + 2)/2} H_{m,n}(2\alpha^*, 2\alpha),
\]

(27)

and

\[
E_{m,n} = 4 \int \frac{d^2 \beta}{\pi} W(\beta, \beta^*, 0) e^{2\Lambda_{m+n-1,n} |\beta|^2} H_{m,n} \left( \frac{2\beta}{\sqrt{\Lambda_{m,n} + 1}}, \frac{2\beta^*}{\sqrt{\Lambda_{m,n} + 1}} \right).
\]

(28)

It is obvious that, when \( \chi = 0 \), the case of photon loss, \( \Lambda_{m,n} \rightarrow (1 - e^{-2\gamma}) = T \) and equation (26) just does reduce to (see appendix E)

\[
W(\alpha, \alpha^*, t) = \frac{2}{T} \int \frac{d^2 \beta}{\pi} \exp \left( -\frac{2}{T} |\alpha - \beta e^{-\gamma t}|^2 \right) W(\beta, \beta^*, 0),
\]

(29)

which is just the evolving formula of WF for the amplitude-damping channel. While for \( \gamma = 0 \), without photon-loss, equation (26) reduces to

\[
W(\alpha, \alpha^*, t) = \sum_{m,n=0}^{\infty} \frac{\exp [-i\chi t (m^2 - n^2)]}{m!n!e^{2|\alpha|^2}} H_{m,n}(2\alpha^*, 2\alpha)
\]

\[
\times 4 \int \frac{d^2 \beta}{\pi} e^{-2|\beta|^2} H_{m,n}(2\beta, 2\beta^*) W(\beta, \beta^*, 0).
\]

(30)

At the end of this section, we should mention that using this overlap between the TESR and the ‘mock state’ corresponding to density operator (14), as well as the completeness relation of the TESR, one can not only easily obtain the relation between the any time WF and the initial time WF but also get the evolution of the characteristic function by noting that \( \chi_{S}(\lambda, \lambda^*) = \text{tr} \{ e^{i\lambda \sigma_3 - \lambda^* \sigma_1} \} = \sum_{n=0}^{\infty} \langle n | e^{i\lambda \sigma_3 - \lambda^* \sigma_1} | n\rangle = \langle \eta = -\lambda | \rho \rangle \). Thus, our method can also be directly applied to obtain the time evolution of other distribution functions, such as the P-function and Q-function.
5. Photon number distribution in the presence of Kerr interaction

Now we consider photon number (PN) distribution in the presence of the Kerr medium. According to the TFD, we can reform the PN
\[ p(n) = \langle n | \rho | n \rangle = \sum_{m=0}^{\infty} \langle n, \tilde{n} | m, \tilde{m} \rangle \]
which is converted to the matrix element \[ \langle n, \tilde{n} | \rho \rangle \] in the context of thermodynamics. Then using the completeness of \[ \langle \xi | \rho \rangle \] and equation (31) as well as equation (21), we have
\[ p(n) = \int \frac{d^2 \xi}{\pi} \frac{\pi}{e^{\frac{2}{\gamma}} - 1} \sum_{m,n} F_{m,n} \beta \left( \frac{4}{2e^{2\gamma t} - 1} \right) \]
where \[ F_{m,n} = \int \frac{d^2 \alpha e^{-2|\alpha|^2}}{m! n! (A_{m,n} + 1)^{m+n+1}} H_{m,n}(2\alpha, 2\beta) = \frac{\pi}{4} \delta_{m,s} \delta_{n,s}. \]

Then substituting equations (34) and (28) into (33) yields
\[ p(s) = \frac{4(-1)^s e^{2yt}}{(2e^{2yt} - 1)^{s+t}} \int \frac{d^2 \beta}{2e^{2yt} - 1} \exp \left\{ - \frac{2|\beta|^2}{2e^{2yt} - 1} \right\} \]
which is just the same as the photon number distribution of density operator evolving in the amplitude-damping quantum channel (\[ \chi = 0 \]) [31]. From equation (35), it is easy to see that, for any initial state, the photon number distribution \[ p(s) \] is independent of the coupling factor \[ \chi \] that is relative to the Kerr medium, as expected, according to [26, 50].

6. Evolution of quantum states

Quantum phase space distributions are useful computational tools enabling one to transcribe operator equations into c-number language. Among them, the Wigner distribution function description of quantum states of light is a powerful tool to investigate nonclassical effects,
such as quantum interference and entanglement. For more discussions about the nonclassical states of light propagating in Kerr media, readers can refer to [50]. In this section, as the applications of the WF evolution formula, we take two special initial states as examples.

(1) The coherent state $|z\rangle$ is known as the most classical among all the pure states, whose WF is given by $W(\beta, \beta^*) = \frac{1}{\pi} e^{-2|\beta|^2}$; thus, substituting it into equation (28) yields (see appendix G)

$$E_{m,n} = \frac{1}{\pi} \left( \Lambda_{m,n} + 1 \right) \frac{m!n!}{\Lambda_{m,n} + 1} e^{(\Lambda_{m,n} - 1)|z|^2} e^{m+n},$$

(36)

so

$$W(\alpha, \alpha^*, t) = \frac{e^{-2|\alpha|^2}}{\pi} \sum_{m,n=0}^{\infty} \frac{z^m \alpha^* n}{m!n!} e^{-i \chi t (m^2 - n^2) - \gamma (m + n)}$$

$$\times e^{(\Lambda_{m,n} - 1)|z|^2} H_{m,n}(2\alpha^*, 2\alpha),$$

(37)

which is an explicit expression of the evolution of WF for any initial state. In particular, when $\gamma = 0$, without the dissipation, equation (37) reduces to

$$W(\alpha, \alpha^*, t) = \frac{e^{-|\alpha|^2}}{\pi} \sum_{m,n=0}^{\infty} \frac{z^m \alpha^* n}{m!n!} e^{-i \chi t (m^2 - n^2)} H_{m,n}(2\alpha^*, 2\alpha),$$

(38)

which is identical to equation (7) in [27], where equation (7) is used to make the numerical calculation since it is much more rapid than the other expression (6). In addition, from equation (37) one can see that the WF can be obtained very quickly when the dissipation cannot be negligible. Further when $\chi t = 2\pi$, equation (38) just returns to the WF of the initial coherent state.

Figure 1 presents the plots of the WF for different parameters $\chi t$ and $\alpha$. From figure 1, one can see that the WF turns into an ellipse and becomes a non-Gaussian squeezed state in an appropriate direction after some time of interaction with the Kerr medium. Interference fringes with the negative part can be seen clearly from the plots of WF. In other words, at the beginning of evolution the squeezing effect dominates and then the interference effect appears. In addition, we should point out that without damping ($\gamma = 0$) the plots of WF are very similar for small $|\alpha| \approx 1$ and large $|\alpha| \gg 1$, while for $\gamma \neq 0$ (i.e. including amplitude damping) and too small amplitude, the quantum effects shall be quickly exhausted due to the decoherence interaction. These points can be seen from the explicit expression in equation (37). In order to see clearly the squeezing effect and the interference fringes during the WF evolution in the presence of dissipation, parameters $\chi t$ and $\alpha$ are chosen as $\chi t = 0 \rightarrow 0.2$ and $\alpha = 2$ in the illustrative figure 1. It should be noted that at greater times $\chi t$ the coherent effects disappear.

(2) Another example is the number state, where the WF of the number state $|s\rangle$ is given by

$$W_s(\beta, \beta^*, 0) = \frac{1/s!}{\pi} e^{-2|\beta|^2} H_{s,s}(2\beta, 2\beta^*)$$

$$= \frac{(-1)^s}{\pi} e^{-2|\beta|^2} L_s(4|\beta|^2),$$

(39)

substituting it into (28) and using the generating function of two-variable Hermite polynomials (see equation (F.3)), we have

$$E_{m,n} = \frac{s! \Lambda_{m,n} (\Lambda_{m,n} + 1)^{m+1}}{(s - m)!} \delta_{m,n},$$

(40)
thus, the evolution of WF for $|s\rangle$ is

$$W_s(\alpha, \alpha^*, t) = \sum_{m=0}^{s} \binom{s}{m} e^{-2m\gamma t} (1 - e^{-2\gamma t})^{s-m} W_m(\alpha, \alpha^*, 0), \quad (41)$$

which is the WF of the number state in the photon-loss channel and indicates that the number state is not affected by the Kerr nonlinearity. In particular, when $\gamma = 0$, or $t = 0$, equation (41) just reduces to the WF of number state. From equation (41), on the other hand, it is found that the number state evolves into a density operator of binomial distribution (a mixed state) if $e^{-2\gamma t}$ is a binomial parameter. In fact, we can give a clear physical picture for that the evolution of the initial number state is insensitive to the value of $\chi$. For simplicity, we consider the case without damping ($\gamma = 0$). For the initial number state $|s\rangle \langle s|$, using a unitary evolution operator $U = \exp(-iHt) = \exp(-i(a^\dagger a)^2t)$ then its evolution state is given by $U |s\rangle \langle s| U^\dagger = \exp(-i(s^2t)) |s\rangle \langle s| \exp(i(s^2t)) = |s\rangle \langle s|$, which shows that the initial number state keeps invariant under the unitary transformation. This case is true for the number distribution (35) of any initial state.
7. Conclusions

In summary, by converting WF for quantum state into an overlap between two ‘pure states’ in a two-mode enlarged Fock space, we investigate the WF evolution of any initial condition in the self-Kerr medium with photon loss and present a new formula for calculating time evolution of the WF for quantum decoherence. Based on the derived WF evolution formula, in addition, we discuss the photon number distribution for any initial state in the presence of Kerr interaction. It is found that the photon number distribution is independent of the coupling factor $\chi$ in correlation with the Kerr medium, as expected by the people. As applications, furthermore, the two cases of initial coherent state and number state are considered. It is shown that the coherent state can be squeezed due to the presence of the Kerr medium, while the number state is not affected by the Kerr nonlinearity and evolves into a density operator of binomial distribution (a mixed state) with $e^{-2\gamma t}$ being a binomial parameter. Our method can be applied to solving some other master equations and present the time evolution of quasi-probability distribution functions (such as the WF, Q-function and P-function) by using the overlap relation between TESR $|\eta\rangle$ or $|\xi\rangle$ and ‘mock state’ $|\rho\rangle$.

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Appendix A. Derivation of equation (16)

In order to obtain the infinite operator-sum form of $\rho(t)$ from equation (14), using the completeness relation of Fock state in the enlarged space $\sum_{m,n=0}^{\infty} |m, \tilde{n}\rangle \langle m, \tilde{n}| = 1$ and noting $a^\dagger |n\rangle = \sqrt{\frac{(m+n)!}{n!}} |n+l\rangle$, we have

$$|\rho\rangle = e^{i\gamma t} |\tilde{n}\rangle e^{\Gamma K_1} |\rho_0\rangle$$

$$= e^{i\gamma t} \sum_{m,n=0}^{\infty} |m, \tilde{n}\rangle \langle m, \tilde{n}| e^{\Gamma K_1} |\rho_0\rangle$$

$$= \sum_{m,n=0}^{\infty} e^{-i\gamma t (m^2-n^2) - \gamma t (m+n)} |m, \tilde{n}\rangle \langle m, \tilde{n}| e^{\Lambda_{m,n}} |\rho_0\rangle,$$

where $\Lambda_{m,n}$ is defined in equation (18).

Furthermore, using the relations

$$\langle n| \eta = 0\rangle = |\tilde{n}\rangle,$$

$$|m, \tilde{n}\rangle = |m\rangle \langle n| \eta = 0\rangle,$$

we find

$$\langle m, \tilde{n}| d^\dagger \rho_0 a^\dagger |\eta = 0\rangle = \langle m| d^\dagger \rho_0 a^\dagger |\eta = 0\rangle$$

$$= \langle m| d^\dagger \rho_0 a^\dagger (|\tilde{n}\rangle \eta = 0\rangle$$

$$= \langle m| d^\dagger \rho_0 a^\dagger |n\rangle.$$
Thus, equation (A.1) becomes
\[
|\rho\rangle = \sum_{m,n,l=0}^{\infty} \frac{L_{m,n}^l}{l!} e^{-i \chi t(m^2-n^2) - i \gamma t(m+n)} |m, n\rangle \langle m| a^l \rho_0 a^l |n\rangle
\]
\[
= \sum_{m,n,l=0}^{\infty} \sqrt{(n+l)! (m+l)!} \frac{\Lambda_{m,n}^l}{\sqrt{n! m! l!}} e^{-i \chi t(m^2-n^2) - i \gamma t(m+n)} |m, n\rangle \rho_0 |n+m,l\rangle, \tag{A.4}
\]
where \(\rho_0 |m+n,l\rangle \equiv \langle m+l| \rho_0 |n+l\rangle\). Using equation (A.3) again, we see
\[
|\rho\rangle = \sum_{m,n,l=0}^{\infty} \frac{\sqrt{(n+l)! (m+l)!}}{\sqrt{n! m! l!} \Lambda_{m,n}^l} e^{-i \chi t(m^2-n^2) - i \gamma t(m+n)} \rho_0 |m+n,l\rangle |m\rangle \langle n| \eta = 0. \tag{A.5}
\]
After depriving \(\eta = 0\) from both sides of equation (A.5), the solution of master equation (8) appears as an infinite operator-sum form:
\[
\rho(t) = \sum_{m,n,l=0}^{\infty} \sqrt{(n+l)! (m+l)!} \frac{\Lambda_{m,n}^l}{n! m! l!} e^{-i \chi t(m^2-n^2) - i \gamma t(m+n)} |m, n\rangle \langle m| a^l \rho_0 a^l |n\rangle. \tag{A.6}
\]
Note that the factor \((m-n)\) appears in the denominator of \(\Lambda_{m,n}\) (see equation (18)), (this is originated from the nonlinear term of \((a^l a)^2\)) so that it is impossible to move all \(n\)-dependent terms to the right of \(a^l \rho_0 a^l\). Fortunately, we can formally express equation (A.6) as equation (16).

**Appendix B. Proof of normalization for the generalized Kraus operators**

Using the operator identity \(e^{a^l a^l} =: \exp[(e^\lambda - 1) a^l a]\); and the IWOP technique, we can prove that
\[
\sum_{m,n,l=0}^{\infty} \mathcal{M}_{m,n,l}^l M_{m,n,l}
\]
\[
= \sum_{n,l=0}^{\infty} \frac{(n+l)!}{n!} \frac{1 - e^{-2 i \gamma l}}{l!} e^{-2 i \gamma l} |n+l\rangle \langle n+l|
\]
\[
= \sum_{n,l=0}^{\infty} \frac{(1 - e^{-2 i \gamma l})}{l!} a^l \rho_0 a^l |n\rangle \langle n| a^l
\]
\[
= \sum_{l=0}^{\infty} \frac{(1 - e^{-2 i \gamma l})}{l!} : \exp\left[(e^{-2 i \gamma l} - 1) a^l a^l\right] (a^l a)^l : = 1, \tag{B.1}
\]
from which one can see that the normalization still holds, i.e. they are trace preserving in a general sense, so \(M_{m,n,l}\) and \(\mathcal{M}_{m,n,l}^l\) may be named the generalized Kraus operators.

**Appendix C. Derivation of equation (26)**

Using equation (25) and (A.1) as well as (18), equation (23) can be rewritten as
Using the integral expression of two-mode Hermite polynomials, Appendix D. Derivation of equation (C.2)

\[ G(\alpha, \beta, t) = \sum_{m,n=0}^{\infty} e^{-|\xi|^{2} + |\eta|^{2}} \langle \xi_{2\alpha}, \eta_{2\beta} | m, n, \tilde{\alpha}, \tilde{\beta} | \rangle_{2} e^{\lambda_{m,n,\alpha\beta}} \]

\[ = \sum_{m,n=0}^{\infty} \frac{e^{-2|\alpha|^{2}}}{\sqrt{m!n!}} e^{-i\sqrt{m^{2} - n^{2}} - \sqrt{m+n}} H_{m,n}(2\alpha^{*}, 2\alpha) \sum_{l=0}^{\infty} \frac{A_{l,m,n}}{l!} \langle m, n | d_{l}^{|} | \xi_{2\beta} \rangle \]

\[ = e^{-2|\beta|^{2} - 2|\alpha|^{2}} \sum_{m,n=0}^{\infty} \frac{e^{-i\sqrt{m^{2} - n^{2}} - \sqrt{m+n}}}{m!n!} H_{m,n}(2\alpha^{*}, 2\alpha) \]

\[ \times \sum_{l=0}^{\infty} \frac{A_{l,m,n}}{l!} H_{m+n,l}(2\beta, 2\beta^{*}). \]  

\text{(C.1)}

Further using a new sum formula (see appendix D)

\[ \sum_{l=0}^{\infty} \frac{x^{l}}{l!} H_{m+n,l}(x, y) = \frac{e^{z}e^{y}}{(z + 1)^{(m+n+2)/2}} H_{m,n} \left( \frac{x}{\sqrt{z} + 1}, \frac{y}{\sqrt{z} + 1} \right); \]

\text{(C.2)}

thus, equation (C.1) can be recast into the following form:

\[ G(\alpha, \beta, t) = \sum_{m,n=0}^{\infty} C_{m,n}(\alpha, \alpha^{*}, \beta) e^{\frac{2|\alpha|^{2}}{\sqrt{\Lambda_{m,n} + 1}} H_{m,n} \left( \frac{2\beta}{\sqrt{\Lambda_{m,n} + 1}}, \frac{2\beta^{*}}{\sqrt{\Lambda_{m,n} + 1}} \right)}, \]

\text{(C.3)}

where \( C_{m,n}(\alpha, \alpha^{*}, \beta) \) is defined in (27). Substituting equation (C.3) into (22) yields (26) and (28).

\section*{Appendix D. Derivation of equation (C.2)}

Using the integral expression of two-mode Hermite polynomials,

\[ H_{m,n}(\xi, \eta) = (-1)^{n} e^{\xi^2} \int \frac{d^{2}z}{\pi} z^{m} e^{\eta z} \exp \left[ -|z|^{2} + \xi \eta - \eta^{2} \right]. \]  

\text{(D.1)}

we have

\[ \sum_{l=0}^{\infty} \frac{x^{l}}{l!} H_{m+n,l}(x, y) = \sum_{l=0}^{\infty} \frac{\alpha^{l}}{l!} (-1)^{l} e^{\alpha y} \int \frac{d^{2}z}{\pi} z^{m+l} e^{\eta z} \exp \left[ -|z|^{2} + \alpha z - \eta z^{*} \right] \]

\[ = e^{\alpha y} \left( -1 \right)^{n} \int \frac{d^{2}z}{\pi} \sum_{l=0}^{\infty} \frac{(-\alpha |z|^{2})^{l}}{l!} z^{m+l} \exp \left[ -|z|^{2} + \alpha z - \eta z^{*} \right] \]

\[ = e^{\alpha y} \left( -1 \right)^{n} \int \frac{d^{2}z}{\pi} \sum_{l=0}^{\infty} \frac{(-\alpha |z|^{2})^{l}}{l!} \exp \left[ -|z|^{2} + \alpha z - \eta z^{*} \right] \]

\[ = e^{\alpha y} \left( -1 \right)^{n} \int \frac{d^{2}z}{\pi} \sum_{l=0}^{\infty} \frac{(-\alpha |z|^{2})^{l}}{l!} \exp \left[ -|z|^{2} + \frac{\alpha z - \eta z^{*}}{\sqrt{\alpha + 1}} \right] \]

\[ = \frac{e^{\alpha y}}{(\alpha + 1)^{(m+n+2)/2}} \sum_{l=0}^{\infty} \frac{(-\alpha |z|^{2})^{l}}{l!} \exp \left[ -|z|^{2} + \frac{\alpha z - \eta z^{*}}{\sqrt{\alpha + 1}} \right] \]

\[ = \frac{e^{\alpha y}}{(\alpha + 1)^{(m+n+2)/2}} H_{m,n} \left( \frac{x}{\sqrt{\alpha + 1}}, \frac{y}{\sqrt{\alpha + 1}} \right); \]

\text{(D.2)}

thus, we have completed the proof of (C.4).
Appendix E. Derivation of equation (29)

When $\chi = 0$, $\Lambda_{m,n} \rightarrow (1 - e^{-2\gamma t}) = T$, and

$$C_{m,n}(\alpha, \alpha^*, t) = \exp \left[ -\gamma t (m + n) \right] m! n! (T + 1)^{m+n+2}/2 H_{m,n}(2\alpha^*, 2\alpha) e^{-2|\alpha|^2},$$

(E.1)

we have

$$\sum_{m,n=0}^{\infty} C_{m,n}(\alpha, \alpha^*, t) H_{m,n}(2\beta, 2\beta^*) = \frac{e^{-2|\alpha|^2}}{T + 1} \sum_{m,n=0}^{\infty} \frac{(e^{-2\gamma t})^{m+n}}{m! n!} H_{m,n}(2\alpha^*, 2\alpha) H_{m,n}(2\beta, 2\beta^*).$$

(E.2)

Using the following formula:

$$\sum_{m,n=0}^{\infty} \frac{s^n t^m}{m! n!} H_{m,n}(x, y) H_{m,n}(\alpha, \beta) = \frac{1}{1 - st} \exp \left[ sx\alpha + ty\beta - (xy + \alpha\beta) st \right],$$

(E.3)

equation (E.2) can be rewritten as

$$\int d^2\alpha \pi e^{-|\alpha|^2} \frac{1}{1 - st} \exp \left[ \frac{4e^{-2\gamma t}/(s+st)}{T+1} (\alpha^* \beta + \alpha\beta^*) - \frac{4e^{-2\gamma t}}{T+1} \frac{|\alpha|^2 + |\beta|^2}{T+1} \right] W(\beta, \beta^*, 0) = \text{rhs of equation (29)}.$$  

(E.4)

Thus, equation (26) becomes

$$W(\alpha, \alpha^*, t) = 4 \int \frac{d^2\beta}{\pi} e^{\frac{T+1}{T+1}|\beta|^2} \sum_{m,n=0}^{\infty} C_{m,n}(\alpha, \alpha^*, t) H_{m,n} \left( \frac{2\beta}{\sqrt{T+1}}, \frac{2\beta^*}{\sqrt{T+1}} \right) W(\beta, \beta^*, 0)$$

$$= \frac{2}{T} e^{-2|\alpha|^2} \int \frac{d^2\beta}{\pi} \left[ \frac{T - 1}{T + 1} |\beta|^2 + \frac{2e^{-2\gamma t}/(s+st)}{T+1} (|\alpha|^2 + |\beta|^2) \right] W(\beta, \beta^*, 0)$$

(E.5)

Appendix F. Derivation of equation (34)

Using the relation between the Hermite polynomial and Laguerre polynomial,

$$L_m(x, y) = \frac{(-1)^m}{m!} H_{m,m}(x, y),$$

(F.1)

we can recast the left of equation (34) into the following form:

$$F_{m,n} = \frac{1}{s!} \int \frac{d^2\alpha}{\pi} e^{-4|\alpha|^2} (-1)^s s! L_s(4|\alpha|^2) H_{m,n}(2\alpha^*, 2\alpha)$$

$$= \frac{1}{s!} \int \frac{d^2\alpha}{\pi} e^{-4|\alpha|^2} H_{s,t}(2\alpha^*, 2\alpha) H_{m,n}(2\alpha^*, 2\alpha)$$

$$= \frac{1}{4s!} \int \frac{d^2\alpha}{\pi} e^{-|\alpha|^2} H_{s,t}(\alpha^*, \alpha) H_{m,n}(\alpha^*, \alpha).$$

(F.2)
Further using the generating function of $H_{m,n}(\epsilon, \epsilon)$,

$$H_{m,n}(\epsilon, \epsilon) = \frac{\partial^{m+n}}{\partial t^m \partial t'^n} \exp[-tt' + \epsilon t + \epsilon t']|_{t=t'=0}, \quad (F.3)$$

and the integration formula,

$$\int \frac{d^2 \zeta}{\pi} e^{(-|\zeta|^2 + x\zeta + z\zeta^*)} = -\frac{1}{\zeta} e^{-\frac{\zeta}{\pi}}, \quad \text{Re}(\zeta) < 0, \quad (F.4)$$

we have

$$\int \frac{d^2 \alpha}{\pi} e^{-|\alpha|^2} H_{m',n'}(\alpha^*, \alpha) H_{m,n}(\alpha^*, \alpha)$$

$$= \frac{\partial^{m+n}}{\partial t^m \partial t'^n} \frac{\partial^{m+n}}{\partial t^m \partial t'^n} \exp[-|\alpha|^2 + (t + \tau)\alpha^* + (t' + \tau')\alpha]|_{t=t'=\tau=\tau'=0}$$

$$= \frac{\partial^{m+n}}{\partial t^m \partial t'^n} \frac{\partial^{m+n}}{\partial t^m \partial t'^n} \exp[t\tau' + \tau t']|_{t=t'=\tau=\tau'=0}$$

$$= m!n!\delta_{m',n'} \delta_{e',n}; \quad (F.5)$$

thus, equation (F.2) becomes the right-hand side of equation (34).

**Appendix G. Derivation of equation (36)**

For this purpose, from equation (28) we have

$$E_{m,n} = 4 \int \frac{d^2 \beta}{\pi^2} e^{-2|\beta|^2} H_{m,n}(2x\beta, 2x\beta^*) \exp[-2|\beta - z|^2], \quad (G.1)$$

where we have set

$$y = \frac{1 - \Lambda_{m,n}}{1 + \Lambda_{m,n}}, \quad x = \frac{1}{\sqrt{\Lambda_{m,n} + 1}} \quad (G.2)$$

Using equations (F.3) and (F.4), equation (G.1) can be recast into the following form:

$$E_{m,n} = 4e^{-2|z|^2} \frac{\partial^{m+n}}{\partial t^m \partial t'^n} e^{-tt'} \int \frac{d^2 \beta}{\pi^2} \exp \left[ - (y + 1)|\beta|^2 + 2\beta(x t + z^*) + 2\beta^*(x' t' + z) \right]|_{t=t'=0}$$

$$= e^{-2|z|^2} \frac{\partial^{m+n}}{\partial t^m \partial t'^n} e^{-tt'} \int \frac{d^2 \beta}{\pi^2} \exp \left[ - \frac{y + 1}{2} |\beta|^2 + \beta(x t + z^*) + \beta^*(x' t' + z) \right]|_{t=t'=0}$$

$$= \frac{1}{\pi} \frac{2}{y + 1} e^{-2|z|^2} \frac{\partial^{m+n}}{\partial t^m \partial t'^n} \exp \left[ - \left( 1 - \frac{2x^2}{y + 1} \right) tt' + \frac{2xz}{y + 1} t + \frac{2xz^*}{y + 1} t' \right]|_{t=t'=0} \quad (G.3)$$

Note that $1 - \frac{2x^2}{y + 1} = 0, \quad y + 1 = \frac{1 - \Lambda_{m,n}}{1 + \Lambda_{m,n}} + 1 = \frac{2}{\Lambda_{m,n} + 1}, \quad -\frac{2x}{y + 1} = \Lambda_{m,n} - 1$; then, we find
\[ E_{m,n} = \frac{1}{\pi} \frac{2}{y + 1} e^{-\frac{\pi y}{2}} \left( \frac{2xz}{y+1} \right)^m \left( \frac{2xz^*}{y+1} \right)^n \]
\[ = \frac{1}{\pi} \left( \Lambda_{m,n} + 1 \right) e^{\frac{\pi \Lambda_{m,n}}{2}} |z|^{2m} z^m z^{*n}. \quad \text{(G.4)} \]

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