Heavy ion-acoustic rogue waves in electron-positron multi-ion plasmas

N. A. Chowdhury, A. Mannan, M. M. Hasan, and A. A. Mamun
Department of Physics, Jahangirnagar University, Savar, Dhaka-1342, Bangladesh.
*Email: nurealam1743phy@gmail.com

The nonlinear propagation of heavy-ion-acoustic (HIA) waves (HIARWs) in a four component multi-ion plasma (containing inertial heavy negative ions and light positive ions, as well as inertialess nonextensive electrons and positrons) has been theoretically investigated. The nonlinear Schrödinger (NLS) equation is derived by employing the reductive perturbation method. It is found that the NLS equation leads to the modulational instability (MI) of HIARWs, and to the formation of HIA rogue waves (HIARWs), which are due to the effects of nonlinearity and dispersion in the propagation of HIARWs. The conditions for MI of HIARWs, and the basic properties of the generated HIARWs are identified. It is observed that the striking features (viz. instability criteria, growth rate of MI, amplitude and width of HIARWs, etc.) of the HIARWs are significantly modified by effects of nonextensivity of electrons and positrons, ratio of light positive ion mass to heavy negative ion mass, ratio of electron number density to light positive ion number density, and ratio of electron temperature to positron temperature, etc. The relevancy of our present investigation to the observations in the space (viz. cometary comae and earth’s ionosphere) and laboratory (laser plasma interaction experimental devices) plasmas is pointed out.

I. LEAD PARAGRAPH

A new electron-positron, multi-ion plasma model has been considered to identify new features (instability criteria, growth rate of MI, amplitude and width, etc.) of heavy ion-acoustic rogue waves. This rogue waves are associated with nonlinear propagation of HIARWs in which the inertia (restoring force) is mainly provided by the heavy negative ions (nonextensive electron and positron temperatures) and are appeared as the solutions of NLS equation (derived here by the reductive perturbation method) in unstable parametric regime. The new striking features of these HIARWs are identified and are found to be applicable in the space and laboratories plasmas.

II. INTRODUCTION

Over the last few decades, wave dynamics in electron-positron-ion (e-p-i) plasmas is one of the major research area for the plasma physicists because of painstaking experimental observational evidence in both space (viz. early universe [1], active galactic nuclei [2], pulsar magnetosphere [3], and neutron stars [4]) and laboratories plasmas (laser-plasma interaction research [5], semiconductor plasmas [6], and other magnetic confinement systems [7]) revealed the existence of nonlinear structures (viz. solitons, envelope solitons, shocks, vortices, rogue waves etc.) in such kind of multicomponent plasmas. The existence of negative ions in the cometary comae [8] and earth’s ionosphere [9] is already established. Plasma can contain negative ions along with positive ions. The negative ions can appear in electronegative plasmas as a result of elementary processes such as dissociative or nondissociative electron attachment to neutrals [10–12]. Positive and negative ion may coexist simultaneously in neutral beam sources [13], plasma processing reactor [14], and in low-temperature laboratory experiments [15]. Similarly positrons can be generated in modern laser plasma experiments when ultra-intense laser pulse interacts with matter [16, 17].

In case of space and laboratory plasmas, all time particles do not follow Maxwellian distribution (which is a velocity distribution describing the plasma particles in a thermal equilibrium [18, 20]). Although, a large number of authors considered that plasma components are in thermal equilibrium but due to the some external disturbances (e.g. wave-particle interactions, external force fields present in natural space plasma environments, and turbulence, etc.) their assumption is no longer valid. In space and astrophysical environments, the Maxwellian distribution is no longer exist when the plasma particles move very fast compared to their thermal velocities. Non-extensive generalization of the Boltzmann-Gibbs-Shannon entropy, first recognized by Renyi [21] and subsequently proposed by Tsallis [22] has been obtained a great deal of interest during last few decades. The nonextensive distribution is generally denoted by q and in the q-nonextensive framework, the one-dimensional equilibrium distribution function $f_s(v_s)$ is given [22] by

$$f_s(v_s) = A_q \left[ 1 - (q - 1) \frac{m_s v_s^2}{2 k_B T_s} \right]^\frac{1}{q-1}.$$

Here the normalization constant is

$$A_q = \frac{n_s \sqrt{\frac{\Gamma \left( \frac{1}{1-q} \right)}{\frac{1}{1-q} - \frac{1}{2}}} \sqrt{\frac{m_s (1-q)}{2 \pi k_B T_s}}}{\Gamma \left( \frac{1}{1-q} - \frac{1}{2} \right)}, \text{ for } 0 < q < 1,$$

where $n_s$, $m_s$, $v_s$, and $T_s$ are the equilibrium number density, mass, thermal speed, and temperature of the energetic particle species $s$. The thermal speed $v_s$ of the
nonextensive particle species $s$ is defined as $(2k_B T_s/m_s)$, where $k_B$ is the familiar Boltzmann constant. Nonextensive plasmas are found in cosmological and astrophysical scenarios (viz. stellar polytropes [23], hadronic matter and quark-gluon plasma [23], dark-matter halos [26], Earths bow-shock [27], magnetospheres of Jupiter and Saturn [28], etc.) as well as laboratory applications like nanomaterials, micro-devices, and micro-structures [29], etc.

The propagation of wave packets in a nonlinear, dispersive medium is appeared to the modulation of their wave amplitudes, because of the interaction between high and low frequency modes, parametric wave coupling, the nonlinear self-interaction of the carrier wave modes, and such kind of system is governed by the NLS equation which admits interesting rational solution named rogue waves (also familiar as freak waves, extreme waves, killer waves, and monster waves) solution in the modulational-unstable region. The rogue waves were first observed in ocean [30], now it can be seen in plasmas, atmospheric physics, optics, stock market crashes [31], and super-fluid helium [32].

Recently Eslami et al. [33] investigated the stability of IAWs in present of q-nonextensive distributed electron and positron plasmas where they found nonextensive parameter plays a considerable effects to modify stability conditions of the IAWs. Bacha et al. [34] analyzed the rogue waves may be notably affected by electron nonextensivity depending on whether the parameter $q$ is positive or negative. In multi-ion plasma rogue wave is examined by El-Labany et al. [35] they found that within certain negative ion mass the rogue wave cannot propagate and energy concentration in rogue wave pulses largely depends on negative ion mass and density. Jannat et al. [36] studied Gardner solitons in a multi-ion plasma system with nonextensive electrons and positrons. The aim of the present work is, by employing reductive perturbation method a NLS equation is derived to study rogue waves in multi-ion plasma with nonextensive electron and positron.

The paper is organized in the following fashion: The model equations are presented in Sec. III. By using reductive perturbation technique, we derived a NLS equation which governs the slow amplitude evolution in space and time is given in Sec. IV. The stability of HIAWs and rogue waves are presented in Sec. V. The conclusion is provided in Sec. VI.

III. THE MODEL EQUATIONS

We consider an unmagnetized four component plasma system comprising of inertial light positive ions, heavy negative ions, as well as inertial nonextensive electrons and positrons. At equilibrium, the charge neutrality condition can be expressed as $Z_1 n_1 + n_{p0} = Z_2 n_{20} + n_{e0}$, where $n_{s0}$ is the unperturbed number densities of the species $s$ $(s = 1, 2, e, p)$, for light positive ions, heavy negative ions, electrons, and positrons, respectively) and $Z_1 (Z_2)$ is the charge state of positive light ion (heavy negative ion). The normalized basic equations governing the dynamics of the IAWs in our considered plasma system are given by

$$\frac{\partial n_1}{\partial t} + \frac{\partial}{\partial x}(n_1 u_1) = 0,$$  

(1)

$$\frac{\partial u_1}{\partial t} + u_1 \frac{\partial u_1}{\partial x} = -\frac{\partial \phi}{\partial x},$$  

(2)

$$\frac{\partial n_2}{\partial t} + \frac{\partial}{\partial x}(n_2 u_2) = 0,$$  

(3)

$$\frac{\partial u_2}{\partial t} + u_2 \frac{\partial u_2}{\partial x} = \alpha \frac{\partial \phi}{\partial x},$$  

(4)

$$\frac{\partial^2 \phi}{\partial x^2} = \mu_e n_e - \mu_p n_p - n_1 + (1 - \mu_e + \mu_p)n_2.$$  

(5)

The number densities of electrons and positrons following $q-$ distribution are

$$n_e = [1 + (q - 1)\phi]^{(1+\lambda q)/4},$$  

$$n_p = [1 - (q - 1)\sigma\phi]^{(1+\lambda q)/4},$$  

(6)

where $q$ is the nonextensive parameter describing the degree of nonextensivity, i.e. $q = 1$ corresponds to Maxwellian distribution, whereas $q < 1$ refers to the superextensivity, and the opposite condition $q > 1$ refers to the subextensivity. Substituting Eq. (6) into Eq. (5) and expanding up to third order, we get

$$\frac{\partial^2 \phi}{\partial x^2} = \mu_e - \mu_p - n_1 + \lambda n_2 + \gamma_1 \phi + \gamma_2 \phi^2 + \gamma_3 \phi^3 + \cdots,$$  

(7)

where

$$\lambda = 1 - \mu_e + \mu_p,$$  

$$\gamma_1 = \frac{(\mu_e + \mu_p)^2}{2} (q + 1),$$  

$$\gamma_2 = \frac{(\mu_e - \mu_p)^2}{8} (q + 1)(3 - q),$$  

$$\gamma_3 = \frac{(\mu_e + \mu_p^2)}{48} (q + 1)(q - 3)(3q - 5).$$  

and

$$\sigma = T_e/T_p,$$  

$$\alpha = \frac{Z_1 m_1}{Z_2 m_2},$$  

$$\mu_e = \frac{n_{e0}}{n_{10}},$$  

$$\mu_p = \frac{n_{p0}}{n_{10}}.$$  

In the above equations, $n_1 (n_2)$ is the number density of light positive ions (heavy negative ions) normalized by its equilibrium value $n_{10} (n_{20})$; $u_1 (u_2)$ is the positive (negative) ion fluid speed normalized by $C_1 = (Z_1 k_B T_e/m_1)^{1/2}$, and $\phi$ is the electrostatic wave potential normalized by $k_B T_e/e$ (with $e$ being the magnitude of an electron charge and $k_B$ is the Boltzmann constant).

$m_1 (m_2)$ is the rest mass of light positive ion (heavy negative ion), respectively; $T_e$ and $T_p$ is the temperature of electrons and positrons respectively. The time and space variables are normalized by $\omega_{pi}^{-1} = (m_1/4\pi Z_1^2 e^2 n_{10})^{1/2}$ and $\lambda D_1 = (k_B T_e/4\pi Z_1 e^2 n_{10})^{1/2}$, respectively.
method. The independent variables are stretched as the dependent variables as

\[v_n = \xi_n, \quad n = 1, 2, \ldots\]

Then we can write a general expression for the dependent variables as

\[n_1 = 1 + \sum_{m=1}^{\infty} \epsilon^{(m)} \sum_{l=-\infty}^{\infty} n_1^{(m)}(\xi, \tau) \exp[il(kx - \omega t)],\]

\[u_1 = \sum_{m=1}^{\infty} \epsilon^{(m)} \sum_{l=-\infty}^{\infty} u_1^{(m)}(\xi, \tau) \exp[il(kx - \omega t)],\]

\[n_2 = 1 + \sum_{m=1}^{\infty} \epsilon^{(m)} \sum_{l=-\infty}^{\infty} n_2^{(m)}(\xi, \tau) \exp[il(kx - \omega t)],\]

\[u_2 = \sum_{m=1}^{\infty} \epsilon^{(m)} \sum_{l=-\infty}^{\infty} u_2^{(m)}(\xi, \tau) \exp[il(kx - \omega t)],\]

\[\phi = \sum_{m=1}^{\infty} \epsilon^{(m)} \sum_{l=-\infty}^{\infty} \phi_l^{(m)}(\xi, \tau) \exp[il(kx - \omega t)],\]

where \(k\) and \(\omega\) are real variables representing the carrier wave number and frequency respectively. Since \(n_1, u_1, n_2, u_2, \) and \(\phi\) must be real, all elements in Eq. (9) satisfy the reality condition \(A^{(m)}_l = A^{(m)*}_l\), where the asterisk denotes the complex conjugate. The derivative operators in the above equations are treated as follows:

\[\frac{\partial}{\partial t} \rightarrow \frac{\partial}{\partial t} - \epsilon v_g \frac{\partial}{\partial \xi} + \epsilon^2 \frac{\partial}{\partial \tau},\]

\[\frac{\partial}{\partial x} \rightarrow \frac{\partial}{\partial x} + \epsilon \frac{\partial}{\partial \xi} + \epsilon^2 \frac{\partial}{\partial \tau}.\]

Substituting Eqs. (9) and (10) into Eqs. (1) – (4) and (7), then the first-order approximation \(m = 1\) with the

FIG. 1: Showing the variation of \(P/Q\) against \(k\) for different values of \(q\). (a) For \(q = \text{positive}\). (b) For \(q = \text{negative}\). Generally all the figures are generated by using these values \(\alpha = 0.5, \sigma = 0.3, \mu_c = 0.7,\) and \(\mu_p = 0.5\).

FIG. 2: Variation of \(P/Q\) against \(k\) for different values of \(\mu_c\). (a) For \(q = 1.5\). (b) For \(q = -0.7\).

IV. DERIVATION OF THE NLS EQUATION

In order to investigate the modulation of the HIAWs in our considered plasma system, we will derive the NLS equation by employing the reductive perturbation method.

\[\begin{align*}
\xi &= \epsilon(x - v_g t), \quad \tau = \epsilon^2 t, \\
n_1 &= 1 + \sum_{m=1}^{\infty} \epsilon^{(m)} \sum_{l=-\infty}^{\infty} n_1^{(m)}(\xi, \tau) \exp[il(kx - \omega t)], \\
u_1 &= \sum_{m=1}^{\infty} \epsilon^{(m)} \sum_{l=-\infty}^{\infty} u_1^{(m)}(\xi, \tau) \exp[il(kx - \omega t)], \\
n_2 &= 1 + \sum_{m=1}^{\infty} \epsilon^{(m)} \sum_{l=-\infty}^{\infty} n_2^{(m)}(\xi, \tau) \exp[il(kx - \omega t)], \\
u_2 &= \sum_{m=1}^{\infty} \epsilon^{(m)} \sum_{l=-\infty}^{\infty} u_2^{(m)}(\xi, \tau) \exp[il(kx - \omega t)], \\
\phi &= \sum_{m=1}^{\infty} \epsilon^{(m)} \sum_{l=-\infty}^{\infty} \phi_l^{(m)}(\xi, \tau) \exp[il(kx - \omega t)],
\end{align*}\]
The second-order approximation ($l = 1$) yields the following relation

\[-i\omega n_{11}^{(1)} + iku_{11}^{(1)} = 0, \quad -i\omega n_{21}^{(1)} + iku_{21}^{(1)} = 0,
\]

\[-i\alpha k^{2}/(1 + \alpha\lambda) \phi_{1}^{(1)} = 0, \quad -i\alpha k^{2}/(1 + \alpha\lambda) \phi_{1}^{(1)} = 0,
\]

\[n_{11}^{(1)} = \frac{k^{2}}{\omega^{2}} \phi_{1}^{(1)}, \quad u_{11}^{(1)} = \frac{k}{\omega} \phi_{1}^{(1)},
\]

\[n_{21}^{(1)} = -\frac{\alpha k^{2}}{\omega^{2}} \phi_{1}^{(1)}, \quad u_{21}^{(1)} = -\frac{\alpha k}{\omega} \phi_{1}^{(1)},
\]

we thus obtain the dispersion relation for HIAWs

\[\omega^{2} = \frac{k^{2}(1 + \alpha\lambda)}{k^{2} + \gamma_{1}}.
\]

The second-order equation when ($m = 2$) reduced equations with ($l = 1$) are

\[n_{11}^{(2)} = \frac{k^{2}}{\omega^{2}} \phi_{1}^{(2)} + \frac{2ik(v_{g}k - \omega)}{\omega^{3}} \partial \phi_{1}(1),
\]

\[u_{11}^{(2)} = \frac{k}{\omega} \phi_{1}^{(2)} + \frac{i(v_{g}k - \omega)}{\omega^{2}} \partial \phi_{1}(1),
\]

\[n_{21}^{(2)} = -\frac{\alpha k^{2}}{\omega^{2}} \phi_{1}^{(2)} - \frac{2ik(v_{g}k - \omega)}{\omega^{3}} \partial \phi_{1}(1),
\]

\[u_{21}^{(2)} = -\frac{\alpha k}{\omega} \phi_{1}^{(2)} - \frac{i(1 + \alpha\lambda) - \omega^{2}}{k(1 + \alpha\lambda)} \partial \xi,
\]

whereas the second-order approximation ($m = 2$) with the first harmonic ($l = 1$) gives

\[v_{g} = \frac{\omega(1 + \alpha\lambda - \omega^{2})}{k(1 + \alpha\lambda)}.
\]

The amplitude of the second-order harmonics are found to be proportional to $|\phi_{1}^{(1)}|^{2}$

\[n_{12}^{(2)} = C_{1}|\phi_{1}^{(1)}|^{2}, \quad n_{10}^{(2)} = C_{6}|\phi_{1}^{(1)}|^{2},
\]

\[u_{12}^{(2)} = C_{2}|\phi_{1}^{(1)}|^{2}, \quad u_{10}^{(2)} = C_{7}|\phi_{1}^{(1)}|^{2},
\]

\[n_{22}^{(2)} = C_{3}|\phi_{1}^{(1)}|^{2}, \quad n_{20}^{(2)} = C_{8}|\phi_{1}^{(1)}|^{2},
\]

\[u_{22}^{(2)} = C_{4}|\phi_{1}^{(1)}|^{2}, \quad u_{20}^{(2)} = C_{9}|\phi_{1}^{(1)}|^{2},
\]
\( \Phi(2) = C_5 |\phi_1^{(1)}|^2 \), \( \phi_3^{(2)} = C_{10} |\phi_1^{(1)}|^2 \), \( \phi_2^{(2)} = C_5 |\phi_1^{(1)}|^2 \), \( \phi_2^{(2)} = C_{10} |\phi_1^{(1)}|^2 \), (16)

where

\[
C_1 = \frac{3k^4 + 2C_5 \omega^2 k^2}{2\omega^4}, \\
C_2 = \frac{k^3 + 2kC_5 \omega^2}{2\omega^3}, \\
C_3 = \frac{3\alpha^2 k^4 - 2\alpha k^2 \omega^2 C_5}{2\omega^4}, \\
C_4 = \frac{\alpha^2 k^3 - 2\alpha C_5 \omega^2}{2\omega^3}, \\
C_5 = \frac{2\gamma \omega^4 + 3\lambda \alpha^2 k^4 - 3k^4}{2\omega^2 k^2 - 2\omega^2(4k^2 + \gamma_1) + 2\alpha \lambda \omega^2 k^2}, \\
C_6 = \frac{2\nu_g k^3 + \omega k^2 + C_{10} \omega^3}{v_g^2 \omega^3}, \\
C_7 = \frac{k^2 + C_{10} \omega^2}{v_g^2 \omega^2}, \\
C_8 = \frac{2\nu_g \alpha^2 k^3 + \omega \alpha^2 k^2 - \alpha C_{10} \omega^3}{v_g^2 \omega^3}, \\
C_9 = \frac{\alpha^2 k^2 - \alpha C_{10} \omega^2}{v_g \omega^2}, \\
C_{10} = \frac{2\gamma \nu_g^2 \omega^3 + 2\lambda \nu_g \alpha^2 k^3 + \omega \lambda \alpha^2 k^2 - 2\nu_g k^3 - \omega k^2}{\omega^3(1 + \alpha \lambda - \gamma_1 v_g^2)).
\]

Finally, the third harmonic modes \( (m = 3) \) and \( (l = 1) \) and with the help of Eqs. (12) – (16), give a system of equations, which can be reduced to the following NLS equation:

\[
i \frac{\partial \Phi}{\partial \tau} + P \frac{\partial^2 \Phi}{\partial \xi^2} + Q |\Phi|^2 \Phi = 0,
\]

where \( \Phi = \phi_1^{(1)} \) for simplicity. The dispersion coefficient \( P \) is

\[
P = \frac{3}{2} \left( \frac{\nu_g}{\omega} - \frac{1}{k} \right) v_g.
\]
and the nonlinear coefficient $Q$ is given by
\[
Q = \frac{\omega^2}{2k^2(1 + \alpha \lambda)} \left[ 2\gamma_2(C_5 + C_{10}) + 3\gamma_3 - \frac{k^2(C_1 + C_6)}{\omega^2} \right. \\
- \frac{2k^3(C_2 + C_7)}{\omega^3} - \frac{\alpha \lambda k^2(C_3 + C_8)}{\omega^2} \left. - \frac{2\alpha \lambda \kappa^3(C_4 + C_9)}{\omega^3} \right].
\]

V. STABILITY AND ROGUE WAVES

The nonlinear evolution of the HIAWs typically depends on the coefficients of dispersion term $P$ and nonlinear term $Q$ which are function of the various plasma parameters such as $\alpha$, $\sigma$, $\mu_e$, $\mu_p$, and $q$. Thus, these plasma parameters are significantly controlled the stability conditions of the HIAWs. If $PQ < 0$, HIAWs are modulationally stable and for the case $PQ > 0$, HIAWs are modulationally unstable against external perturbations and simultaneously when $PQ > 0$ and $k_{MI} < k_c$, the growth rate ($\Gamma_g$) of MI is given by
\[
\Gamma_g = |P| k_{MI}^2 \left( k_{MI}^2 - 1 \right). \quad (18)
\]

Here $k_{MI}$ is the perturbation wave number and the critical value of the wave number of modulaton $k_c = \sqrt{2QP\Phi_0^2/P}$, where $\Phi_0$ is the amplitude of the carrier waves. Hence the maximum value $\Gamma_{g\text{max}}$ of $\Gamma_g$ is obtained at $k_{MI} = k_c/\sqrt{2}$ and is given by $\Gamma_{g\text{max}} = |Q|\Phi_0^2$.

Therefore, we have investigated the stability of the profile by depicting the ratio of $P/Q$ versus $k$ for different plasma parameters. When $P/Q < 0$, HIAWs are modulationally stable, while $P/Q > 0$, HIAWs will be modulationally unstable against external perturbations. When $P/Q \to \pm \infty$, the corresponding value of $k(= k_c)$ is called critical or threshold wave number for the onset of MI. Figure 1 shows the variations of $P/Q$ with $k$ for possible three ranges $-1 < q < 0$, $0 < q < 1$, and $q > 1$ of the nonextensive parameter $q$. It is observed from both Figs. 1(a) and 1(b) that for small $k$ there is stable region (HIAWs are modulational stable and dark envelope solitons exist), where unstable region (HIAWs are modulational unstable and bright envelope solitons exist) is found for large $k$. The critical value $k_c$ is increases (decreases) with an increase in the value of nonextensive parameter $q$ for the ranges $q > 0$ ($q < 0$) which can be observed from Figs. 1(a) and 1(b), respectively. So the range of nonextensive parameter (greater than or less than zero) plays a vital role for controlling the stability of the HIAWs.

We are now investigating the effects of the ratio of electron to light positive ion number density (via $\mu_e$) on stability conditions of HIAWs which are depicted in Figs. 2(a) and 2(b). Nonextensive electrons play a fascinating role to control the stability of HIAWs structure. Within the range $q > 0$, increasing values of electron number density, the critical value $k_c$ is shifted to the higher values that means the instability domain is occurred at the higher values of $k$ in Fig. 2(a). Similar behaviour is observed for the case $q < 0$, the critical value $k_c$ is occurred at higher wave number with increasing of electrons number density in Fig. 2(b). So excess number of electrons are provided more restoring force which maximize the stability region without depending on the values of nonextensive parameter.

The effects of temperature parameter (via $\sigma$) on stable and unstable regions can be shown from Figs. 3(a) and 3(b), respectively. The variation of $P/Q$ with $k$ for different values of $\sigma$ is depicted in Figs. 3(a) and 3(b). It is observed that the critical value ($k_c$) is shifted towards higher (lower) values as electron temperature increases for the range of $q > 0$ ($q < 0$). The range of nonextensive parameter change the order of the variation of critical value ($k_c$). Thus, electron temperature plays a crucial role to change the stability of the wave packets.

To highlight the effects of the ratio of positron to light positive ion number density (via $\mu_p$) on the growth rate of MI is depicted in Fig. 4. The growth rate of the MI is so much sensitive to change the values of $\mu_p$. It is observed from Figs. 4(a) and 4(b) that an increase of $\mu_p$ value the maximum growth rate is increased (decreased) for $q > 0$ ($q < 0$), respectively. So the nonextensive positrons (via $\mu_p$) can be recognized to enhance or suppress the MI growth rate of our considered plasma model.

The rogue wave (rational solution) of the NLS Eq. (17) in the unstable region ($PQ > 0$) can be written as
\[
\Phi(\xi, \tau) = \sqrt{\frac{2P}{Q}} \left[ \frac{4(1 + 4iP\tau)}{1 + 16P^2\tau^2 + 4\kappa^2} - 1 \right] \exp(i2P\tau), \quad (19)
\]

The solution (19) anticipates the concentration of the HIAWs energy into a small region that is caused by the nonlinear behavior of the plasma medium (see Figs. 5 and 6). The HIARWs are so much sensitive to change any plasma parameter. The effect of the ratio of light positive ion mass to heavy negative ion mass (via $\alpha$) on the HIARWs is shown in Fig. 5(a) and 5(b). For the range $q > 0$ ($q < 0$), the amplitude and width of the HIARWs are increased with the increase of $\alpha$ which can be seen in Figs. 5(a) and 5(b). However, the HIARWs amplitude decreases with the increase of the heavy negative ion mass (via $\alpha$). Physically, the increasing of the heavy negative ions masses lead to dissipate the energy from the system and reduce the nonlinearity that makes the HIARWs amplitude shorter. Finally, changing direction of HIARWs amplitude and width are remain invariant, irrespective of $q$ values. Exact similar fashion can be observed in Fig. 6, which is depicted against $q$. For the range $q > 0$ ($q < 0$), the amplitude and width of HIARWs is decreased (decreased) with the increase of $q$ values.

It can be deduced from Figs. 2 and 5 that the direction of the variation of $k_c$ and wave profile (light and
width) is totally independent on the nonextensive parameter ranges. For the ranges of \( q > 0 \) and \( q < 0 \), with an increase of the ratio of electron to positive light ion number density (via \( \mu_e \)) and the ratio of light positive ion to heavy negative ion mass (via \( \alpha \)), the values of \( k_c \) and wave profile (height and width) are increased which can be observed from Figs. 2 and 5, respectively.

VI. CONCLUSION

In summary a NLS equation has been derived to describe the small-amplitude HIAWs in an unmagnetized multi-component plasma consisting of inertial light positive ions, heavy negative ions, as well as inertialess nonextensive electrons and positrons. The analysis of the NLS equation reveals the existence of both stable and unstable region. The critical value \( k_c \) which determined the stability/instability region of HIAWs, is totally depend on various plasma parameters such as nonextensive parameter, electron number density, and electron temperature. Every plasma parameters plays a vital role to change the critical value \( k_c \). The MI of HIAWs can also lead to the formation of rogue waves in the unstable region in which a large amount of energy is concentrated in relatively small area in space and time. The height and width of rogue waves are greatly depended on the ratio of light positive ion to heavy negative ion mass which is observed in our present analysis. A number of observations clearly disclosed the existence of nonextensive electrons and positrons in various natural space environment and laboratory plasmas. We are optimistic that the finding of our present investigation should be useful to understand the nonlinear phenomena in both space (cometary comae and earth’s ionosphere) and laboratory plasmas (laser plasma) which containing of inertial light positive ions and heavy negative ions, as well as inertialess nonextensive electrons and positrons.

Acknowledgements

N. A. Chowdhury is grateful to the Bangladesh Ministry of Science and Technology for awarding the National Science and Technology (NST) Fellowship.
Rep. 42, 678 (2016).
[37] I. Koutrakis and P.K. Shukla, Nonlinear Proc. Geophys. 12, 407 (2005).
[38] R. Fedele, H. Schamel, and P.K. Shukla, Phys. Scr. 98, 18 (2002).
[39] R. Fedele and H. Schamel, Eur. Phys. J. B 27, 313 (2002).
[40] R. Fedele, Phys. Scr. 65, 502 (2002).
[41] Shalini, N. S. Saini, and A. P. Misra, Phys. Plasmas 22, 092124 (2015).
[42] A. Anikiewicz, N. Devine, and N. Akhmediev, Phys. Lett. A 373, 3997 (2009).
[43] H. G. Abdelwahed, E. K. El-Shewy, M. A. Zahran, and S. A. Elwakil, Phys. Plasmas 23, 022102 (2016).