Mass dependence in vector–meson electroproduction

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Abstract

We demonstrate that the explicit mass dependence of the exponent in the power–like energy behavior of the vector–meson production cross-section in the processes of virtual photon interactions with a proton $\gamma^* p \rightarrow V p$ obtained in the off–shell extension of the approach based on unitarity is in a quantitative agreement with the high–energy HERA experimental data.
Introduction

Besides the most known studies of DIS at low $x$ important measurements of cross-sections of the elastic vector meson production were performed in the experiments H1 and ZEUS at HERA [1, 2]. As it follows from these data the integral cross section of the elastic vector meson production increases with energy in a way similar to the $\sigma_{\gamma^*p}^{tot}(W^2, Q^2)$ dependence on $W^2$ [3]. It appeared also that the growth of the vector–meson electroproduction cross–section with energy is steeper for heavy vector mesons and when the virtuality $Q^2$ increases.

In this note we show that the approach based on the off-shell extension of the $s$–channel unitarity (cf. [4] and references therein) and its application to the elastic vector meson production in the processes $\gamma^*p \rightarrow Vp$ allows in particular to consider mass dependence of these processes. It appears that the obtained mass and $Q^2$ dependencies are in a quantitative agreement with the high–energy HERA experimental data.

1 Vector–meson electroproduction

The extension of the $U$–matrix unitarization for the off-shell scattering was considered in [3]. It was supposed that the virtual photon fluctuates into a quark–antiquark pair $q\bar{q}$ and this pair can be treated as an effective virtual vector meson state in the processes with small Bjorken $x$. There were considered limitations the unitarity provides for the $\gamma^*p$–total cross-sections and geometrical effects in the energy dependence of $\sigma_{\gamma^*p}^{tot}$. In particular, it was shown that an assumption of the $Q^2$–dependent constituent quark interaction radius leads to the following asymptotical dependence: $\sigma_{\gamma^*p}^{tot} \sim (W^2)^{\lambda(Q^2)}$, where $\lambda(Q^2)$ will be saturated at large values of $Q^2$. This result is valid when the interaction radius of the virtual constituent quark is rising with virtuality $Q^2$. The form corresponding to the virtual constituent quark interaction radius was chosen as following

$$r_{Q^2} = \xi(Q^2)/m_Q.$$  \hspace{1cm} (1)

Thus, the dependence on virtuality $Q^2$ comes through the dependence of the intensity of the virtual constituent quark interaction $g(Q^2)$ and the $\xi(Q^2)$, which determines the quark interaction radius (in the on-shell limit $g(Q^2) \rightarrow g$ and $\xi(Q^2) \rightarrow \xi$).

The reason for the rising interaction radius of the virtual constituent quark with virtuality $Q^2$ might be of a dynamical nature and it could originate from the emission of the additional $q\bar{q}$–pairs in the nonperturbative structure of a constituent quark. In this approach constituent quark consists of a current quark and
the cloud of quark–antiquark pairs of the different flavors [5]. Available experimental data are consistent with the $\ln Q^2$–dependence of the radius of this cloud. The introduction of the $Q^2$ dependence into the interaction radius of a constituent quark which in this approach consists of a current quark and the cloud of quark–antiquark pairs of the different flavors is the main issue of the off–shell extension of the model, which provides at large values of $W^2$

$$
\sigma_{\gamma^* p}(W^2, Q^2) \propto G(Q^2) \left( \frac{W^2}{m_Q^2} \right)^{\lambda(Q^2)} \ln \frac{W^2}{m_Q^2}, \quad (2)
$$

where

$$
\lambda(Q^2) = \frac{\xi(Q^2) - \xi}{\xi(Q^2)}. \quad (3)
$$

The value and $Q^2$ dependence of the exponent $\lambda(Q^2)$ is related to the interaction radius of the virtual constituent quark. The value of parameter $\xi$ in the model is determined by the slope of the differential cross–section of elastic scattering at large $t$ [6] and from the $pp$–experimental data it follows that $\xi = 2$. From the data for $\lambda(Q^2)$ obtained at HERA the “experimental” $Q^2$–dependence of the function $\xi(Q^2)$ has been calculated [3]:

$$
\xi(Q^2) = \frac{\xi}{1 - \lambda(Q^2)}. \quad (4)
$$

The rise of the function $\xi(Q^2)$ is slow and consistent with $\ln Q^2$ extrapolation:

$$
\xi(Q^2) = \xi + a \ln \left( 1 + \frac{Q^2}{Q_0^2} \right),
$$

where $a = 0.172$ and $Q_0^2 = 0.265$ GeV$^2$.

The inclusion of heavy vector meson production into this scheme is straightforward: the virtual photon fluctuates before the interaction with proton into the heavy quark–antiquark pair which constitutes the virtual heavy vector meson state. After an interaction with a proton this state turns out into a real heavy vector meson.

Integral exclusive (elastic) cross–section of vector meson production in the process $\gamma^* p \rightarrow V p$ when the vector meson in the final state contains not necessarily light quarks can be calculated directly:

$$
\sigma_{\gamma^* p}^V(W^2, Q^2) \propto G_V(Q^2) \left( \frac{W^2}{m_Q^2} \right)^{\lambda_V(Q^2)} \ln \frac{W^2}{m_Q^2}, \quad (5)
$$

Integral exclusive (elastic) cross–section of vector meson production in the process $\gamma^* p \rightarrow V p$ when the vector meson in the final state contains not necessarily light quarks can be calculated directly.
where
\[ \lambda_V(Q^2) = \lambda(Q^2) \frac{\bar{m}_Q}{\langle m_Q \rangle}. \] (6)

In Eq. (6) \( \bar{m}_Q \) denotes the mass of the constituent quarks from the vector meson and \( \langle m_Q \rangle \) is the mean constituent quark mass of system of the vector meson and proton system. For the on–shell scattering we have a familiar Froissart–like asymptotic energy dependence
\[ \sigma_{V \gamma p}^V(W^2, Q^2) \propto \frac{\xi^2}{m_Q^2} \ln^2 \frac{W^2}{m_Q^2}. \] (7)

It is evident from Eq. (5) that \( \lambda_V(Q^2) = \lambda(Q^2) \) for the light vector mesons. In the case when the vector meson is very heavy, i.e. \( \bar{m}_Q \gg m_Q \) we have
\[ \lambda_V(Q^2) = \frac{5}{2} \lambda(Q^2). \]

We conclude that the respective cross–section rises faster than the corresponding cross–section of the light vector meson production, e.g. Eq. (6) results in
\[ \lambda_{J/\Psi}(Q^2) \simeq 2 \lambda(Q^2). \]

To perform a fit to the high–energy HERA experimental data [1, 2] we have chosen the functional dependence of \( G_V(Q^2) \) in the form
\[ G_V(Q^2) = g \left( 1 + \frac{Q^2}{Q_0^2} \right)^{-a}. \] (8)

The agreement of Eqs. (5) and (7) with experiment when the function \( G_V(Q^2) \) has the form of Eq. (8) is illustrated by the Figs. (1-4). The values of the three adjustable parameters \( g, Q_0^2 \) and \( a \) are given in the Table 1.

**Conclusion**

A quantitative agreement with the high–energy HERA experimental data on elastic vector–meson electroproduction is in favor of relation (6) which provides explicit mass dependence of the exponent in the power–like energy dependence of cross–sections. It means that the dependence of the constituent quark interaction radius in the form \( r_{Q^*} = \xi(Q^2)/m_Q \) on its mass and virtuality has an experimental support and corresponding non–universal energy dependence predicted in [3] does not contradict to the high–energy experimental data on elastic vector–meson electroproduction.
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| Meson | $g$, $\mu$b | $Q_0^2$ | $\alpha$ |
|-------|-------------|----------|---------|
| $\rho$ | $1.22 \cdot 10^{-3}$ | 0.66 | 2.84 |
| $\omega$ | $1.20 \cdot 10^{-4}$ | 0.71 | 2.52 |
| $\phi$ | $1.11 \cdot 10^{-4}$ | 0.76 | 2.87 |
| $J/\psi$ | $7.87 \cdot 10^{-6}$ | 0.86 | 1.87 |

Table 1: The values of the adjustable parameters

Figure 1: Energy dependence of the elastic cross–section of exclusive $\rho$–meson production.
Figure 2: Energy dependence of the elastic cross-section of exclusive $\omega$–meson production.

Figure 3: Energy dependence of the elastic cross-section of exclusive $\phi$–meson production.
Figure 4: Energy dependence of the elastic cross-section of exclusive $J/\psi$ production.