A solution of the coincidence problem based on the recent galactic core black hole mass density increase

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(Dated: February 12, 2013)

A mechanism capable to provide a natural solution to two major cosmological problems, i.e. the cosmic acceleration and the coincidence problem, is proposed. A specific brane-bulk energy exchange mechanism produces a total dark pressure, arising when adding all normal to the brane negative pressures in the interior of galactic core black holes. This astrophysically produced negative dark pressure explains cosmic acceleration and why the dark energy today is of the same order to the matter density for a wide range of the involved parameters. An exciting result of the analysis is that the recent rise of the galactic core black hole mass density causes the recent passage from cosmic deceleration to acceleration. Finally, it is worth mentioning that this work corrects a wide spread fallacy among brane cosmologists, i.e. that escaping gravitons result to positive dark pressure.

I. INTRODUCTION

During last decades it has been realized that the investigation of the problems associated with the cosmological constant would provide an insight into the structure and the properties of elusive quantum gravity. A serious problem concerning the cosmological constant refers to the vast discrepancy between the value a theorist would expect and the very low value of the effective cosmological constant. As Zel’dovich [1] first noticed, the effective cosmological constant we measure is the sum of the pure geometric origin cosmological constant plus the energy density of the vacuum. It seems impossible to understand why the measured effective cosmological constant is so much smaller than the value of the vacuum energy calculated by a quantum field theorist (cosmic phase transition, quantum field zero-point energies). This puzzle challenges the inflationary scenario and the various models of quantum gravity.

The present work attempts to solve a recently emerged problem regarding the cosmological constant issue which concerns the measured cosmic acceleration. During the last decade, it has been established through different independent pieces of astronomical data that empty space, devoid of the usual matter, is anti-gravitating. It creates gravitational repulsion and gives rise to an accelerated cosmological expansion. According to our present-day understanding, this accelerated expansion could be induced by an effective cosmological “constant”-like term (vacuum \( p = -\rho \) or dark energy \( p < -\frac{1}{3} \)). Data suggest that the magnitude of the required vacuum/dark energy is quite close to the critical (closure) cosmological energy density (approximately 70\%). Why vacuum energy, which stays constant in the course of cosmological evolution, or why dark energy, which evolves with time quite differently from the normal matter, have similar magnitude with matter density just today, all being close to the value of the critical energy density?

There are several ideas in literature, though yet incomplete, that have the potential to provide solutions to cosmic acceleration problem. The present paper proposes that the negative five-dimensional pressure produced from an astrophysical brane-bulk energy outflow occurring inside all cosmic black holes is large enough to drive the measured cosmic acceleration. Assuming an RS-like cosmological brane [2], [3], [4], this total pressure (called dark pressure) arising from the sum of all negative pressures normal to the brane suffices to explain the coincidence problem, without using a negative vacuum energy or exotic fields/fluids throughout the universe. In the presented scenario dark energy “originates” from dark matter (grows from a negligible value to a significant one due to dark pressure); moreover, the recent appearance of the cosmic acceleration is correlated to the recent increase of the galactic core black hole density. An accelerating universe has already been produced by several brane-bulk energy exchange scenarios [5], [6], [7], [8], [9]. However, in all these works the whole universe should be hot enough, and therefore, these scenarios fail to explain recent acceleration.

Both astrophysical black holes in haloes and supermassive black holes at the galactic centres appear after the large scale structure of the universe, weight a portion of \( \rho_m \) and are regions where high energy interactions occur. This fact will be at the center of the proposed mechanism. Assuming that a brane cosmological model describes our universe, it is natural to expect a moderate exchange of energy between the brane and the bulk. Astrophysical black holes contain matter in an unknown form, i.e. effective quantum fluid (possibly arising from superposition of non empty black hole quantum spacetimes) and accrete continuously mass. Collapsing matter falling into a black hole accelerates,
and gets easily “thermalized” to temperatures close and above $M$ (five-dimensional fundamental Planck mass). Furthermore, it is expected portion of black hole mass to be in the form of highly energetic states close to $M$, not only due to accreting matter interactions but also due to Hawking-like particle production in the interior \[10\]. But for energy scales close to $M$, matter interactions result to graviton escape to the bulk. Therefore, energy outflow can occur in the interior of galactic halo black holes and galactic core supermassive black holes. This black hole originated exchange results to non-zero energy-momentum tensor components $T_{05}$ and $T_{50}$ and is able to provide the necessary conditions for a cosmic acceleration.

The emission of gravitons to the bulk is associated with negative dark pressure on the brane. Indeed, a brane experiences positive pressure when bulk particles fall into it. This negative pressure is the responsible quantity that provides the required amount of the measured cosmic acceleration. Although there is a small outflow in our scenario in each of the galactic black holes (consistent with black hole mass evolution and galaxy dynamics), this leakage is associated unavoidably with orders of magnitude larger dark pressure. Moreover, the emerged dark energy is not a small portion of the dark matter but of the same order with it. Note that in our scenario, before outflow starts (before galaxy formation) there may be either a zero or a very small positive non zero decelerating dark radiation term $\frac{\delta}{N} > 0$. However, this radiation term overpasses well known problems of nucleosynthesis constraints \[11\]. When negative dark pressure emerges, this radiation term is modified and finally becomes an accelerating dark energy component.

Note also that it costs almost nothing to stretch a brane that has zero total tension/cosmological constant \[12\]. Apparently spacetime is such that it takes a lot of energy to curve it, while stretching it is almost for free, since the cosmological constant is zero. This is quite contrary properties of objects from every day experience, where bending requires much less energy than stretching. The present study tries only to explain naturally the recent cosmic acceleration while assumes that there is some mechanism that sets the cosmological constant from field theory vacuum energy equal to zero (for example R-S fine tuning, holography etc.).

The paper is organized as follows. In section II the mathematical framework is presented. Here, it becomes obvious from Eq. (2.4) that the negative dark pressure II can drive acceleration. In addition, Eq. (2.18) shows that II determines also the time derivative of dark energy, resulting to the current value of $w_{DE}$. Furthermore, the question of how large should be the present value of dark pressure II, is estimated in Eq. (2.22). Such a value can easily be obtained, as it is explained in sections III and V, for moderate values of the involved astrophysical parameters. In section III the connection of the mathematical framework with the astrophysical context is given. Section IV provides various supportive theoretical aspects of the physics of the proposed mechanism. However, section IV is not crucial to the main results of the analysis. In section V numerical computations are presented in order to prove the success of the model. Results have been derived both analytically and numerically (for verification reasons) and are presented together with descriptions of the range of the involved parameters that ensure: i) recent passage from deceleration to acceleration, ii) small outflow that do not violate black hole evolution/mass, iii) nucleosynthesis bounds, and iv) universe age. Finally, section VI is dedicated to the conclusions.

II. THE FRAMEWORK: BRANE COSMOLOGY WITH 5-DIM BULK ENERGY EXCHANGE

We begin with a model described by a 5-dim Einstein-Hilbert action with matter and a 5-dim cosmological constant $\Lambda$ plus the contribution describing the brane \[1\].

$$S = \int d^5x \sqrt{-g} \left( M^3 R - \Lambda + \mathcal{L}_B^{\text{mat}} \right) + \frac{1}{2} d^4x \sqrt{-h} \left( -V + \mathcal{L}_b^{\text{mat}} \right),$$

(2.1)

where $R$ is the Ricci scalar of the five-dimensional metric $g_{AB}$ ($A, B = 0, 1, 2, 3, 5$) and $h$ is the induced metric on the 3-brane. We identify $(x, z)$ with $(x, -z)$, where $z \equiv x_5$, in order to impose the usual $\mathbb{Z}_2$ reflection symmetry of the AdS slice. Following the conventions of \[2\], we extend the bulk integration over the entire interval $(-\infty, \infty)$. $\mathcal{L}_B^{\text{mat}}$ and $\mathcal{L}_b^{\text{mat}}$ are the bulk and brane matter contents respectively. $M$ is the five-dimensional Planck mass. The quantity $V$ can include the brane tension as well as quantum contributions to the four-dimensional cosmological constant.

In order to search for cosmological solutions we consider the corresponding form for the metric

$$ds^2 = -n^2 (t, z) dt^2 + a^2 (t, z) \gamma_{ij} dx^i dx^j + b^2 (t, z) dz^2,$$

(2.2)

where $\gamma_{ij}$ is a maximally symmetric 3-dimensional metric with $i, j = 0, 1, 2, 3$ (we use $k = -1, 0, 1$ to parameterize the spatial curvature). The five-dimensional Einstein equations are $G_{MN} = \frac{1}{M^5} T_{MN}$, where $T_{MN}$ is the total energy momentum tensor, i.e.

$$T^M_N = T^M_N \big|_{v, b} + T^M_N \big|_{m, b} + T^M_N \big|_{v, B} + T^M_N \big|_{m, B},$$

(2.3)

$$T^M_N \big|_{\text{vac}, b} = \frac{\delta (z)}{b} \text{diag} (-V, -V, -V, -V, 0),$$

(2.4)

$$T^M_N \big|_{\text{vac}, B} = \text{diag} (-\Lambda, -\Lambda, -\Lambda, -\Lambda, -\Lambda),$$

$$T^M_N \big|_{\text{matter}, b} = \frac{\delta (z)}{b} \text{diag} (-\rho, p, p, p, 0),$$

$$T^M_N \big|_{\text{matter}, B} = \text{diag} (0, 0, 0, 0, T_s^5) + \left( \frac{\nabla^2}{b^2} T_s^5 \right)(2.5)$$

Here, $T^M_N \big|_{m, B}$ denotes the energy-momentum tensor of the brane perfect cosmic fluid and $\rho, p$ are its energy density and pressure respectively. In our approach $T^M_N \big|_{m, B}$ gets zero contributions from the presence of flows.
from the brane. The off-diagonal contribution $T^o_5$ expresses the brane-bulk energy exchange flow of gravitons, while the $T^5_5$ component expresses the corresponding pressure along the fifth dimension. In order to keep predictability we seek to derive a solution that is largely independent of the bulk dynamics. Thus, any other existing bulk field contribution is considered negligible. In addition, a small energy exchange from the brane (true in the proposed mechanism) keeps the bulk largely unperturbed. The set of the Einstein equations at the location of the brane is

$$\dot{\rho} + 3\frac{a_o}{n_o}(\rho + p) = -\frac{2n_o^2}{b_o} T^0_5$$

$$\frac{1}{n_o^2}(\frac{\dot{a}_o}{a_o} + \frac{\dot{a}_o}{a_o} n_o) + \frac{k}{a_o^2} = \frac{1}{6M^3} (\Lambda + \frac{V^2}{12M^3})$$

$$-\frac{1}{144M^6} (V(3p - \rho) + \rho(3p + \rho)) = \frac{1}{6M^3} T^5_5 .$$

(2.7)

Dots indicate derivatives with respect to $t$. We indicate by the subscript "o" the value of various quantities on the brane and $T_{05}$, $T_{55}$ are the 05 and 55 components of $T_{MN}$ evaluated on the brane.

Since we are interested in a model that reduces to the Randall-Sundrum vacuum [2] in the absence of matter we require the bulk cosmological constant and the brane tension to satisfy $\Lambda + \frac{1}{12M^3} V^2 = 0$.

It is convenient to employ a coordinate frame in which $b_o = n_o = 1$ in the above equations. This can be achieved by using Gauss normal coordinates with $b(t, z) = 1$ and by going to the temporal gauge on the brane with $n_o = 1$. Thus, using $\lambda \equiv M^{-6}/144$ and $\gamma \equiv V/\beta$ and omitting the subscript $o$ for convenience in the following, we take

$$\dot{\rho} + 3(1 + w) H \rho = -T$$

$$q = 1 + H^{-2} \frac{k}{a_o^2} + H^{-2}(3w - 1) \gamma \rho +$$

$$+ H^{-2}(3w + 1) \beta \rho^2 + H^{-2} \sqrt{\beta} \Pi .$$

(2.9)

Here, $q$ is the usual deceleration parameter $q = -\frac{2}{3} H^{-2} - p = w \rho$ and $T = 2T^0_5$, $\Pi = 2T^5_5$ are the discontinuities of the zero-five and five-five components of the bulk energy-momentum tensor respectively. It is obvious from Eqs. (2.8), (2.9) that the only way to pass from a deceleration era to an accelerated cosmological phase in a flat universe ($k = 0$) is the case that the dark pressure term $\Pi$ becomes negative at some moment in the cosmic history. This is actually what happens in the studied proposal where dark pressure from zero acquires a non-zero negative value due to the leakage towards the extra dimension. As we will explain in section III this leakage causes a negative pressure in every galactic center resulting to a total value for $\Pi$ capable to produce the observed cosmic acceleration.

Defining now an auxiliary quantity $\psi$ by

$$\frac{\dot{a}}{a} = -(2 + 3w) \beta \rho^2 - (1 + 3w) \gamma \rho - \sqrt{\beta} \Pi - \psi + \lambda ,$$

(2.10)

we can rewrite equations (2.6), (2.7) in the equivalent form

$$\dot{\rho} + 3(1 + w) \frac{\dot{a}}{a} \rho = -T$$

$$\frac{\dot{a}}{a^2} = \beta \rho^2 + 2\gamma \rho - \frac{k}{a^2} + \psi + \lambda$$

$$\psi + \frac{\dot{a}}{a} \psi = 2\beta \left(\rho + \frac{\gamma}{\beta}\right) T - 2\sqrt{\beta} \frac{\dot{a}}{a} \Pi .$$

(2.13)

Here, the effective cosmological constant on the brane $\lambda = (\Lambda + V^2/12M^3)/12M^3$, as we have mentioned before, it will be set to zero, but for the time being we leave it intact. Additionally, $\gamma = \frac{4\pi G}{3}$ in order to recover standard 4-dimensional gravity.

In the special case of no-exchange ($\Pi = 0, T = 0$), $\psi$ represents the dark radiation $\rho \phi^2$, reflecting the non-zero Weyl tensor of the bulk.

In order to study further cosmic acceleration, it is convenient to consider the set of differential equations (2.14), (2.15), (2.16) for $q, \rho, \psi$, equivalent to the last system of equations (2.14), (2.15), (2.16)

$$\frac{dq}{da} = \frac{2}{a} q(q + 1) + H^{-2} \left[2(2 + 3w) \beta \rho \frac{dp}{da} + (1 + 3w) \gamma \frac{dp}{da}ight]$$

$$+ \frac{d\psi}{da} + \sqrt{\beta} \frac{d\Pi}{da}$$

(2.14)

$$\frac{dp}{da} = -\frac{1}{a} \left[3(1 + w) \rho + T H^{-1}\right] ,$$

(2.15)

where we should replace everywhere $\psi$ and $d\psi/da$ by

$$\psi = -(2 + 3w) \beta \rho^2 - (1 + 3w) \gamma \rho + H^2 q - \sqrt{\beta} \Pi + \lambda$$

(2.16)

$$\frac{d\psi}{da} = \frac{1}{a} \left[-4\psi + 2\beta \left(\rho + \frac{\gamma}{\beta}\right) T H^{-1} - 2\sqrt{\beta} \Pi \right]$$

(2.17)

and we substitute $H^2$ with the help of

$$H^2 = \frac{(3w + 1) \beta \rho^2 + (3w - 1) \gamma \rho + \frac{k}{a^2} - 2\lambda + \sqrt{\beta} \Pi}{q - 1} .$$

(2.18)

It is worth mentioning that both $\Pi$ and $T$, according to the proposed mechanism, depend on the astrophysical properties of black holes.

We are going to distinguish two cases in our numerical study of the system (2.14), (2.15), (2.16). Both are consistent with the details of the involved phenomena. In the first case, which is valid for a study of the recent cosmological time period, i.e. $z$ close to 0, we assume a constant number of typical galactic black holes and therefore, the dark pressure $\Pi$ can be modeled to be analogous to a known constant $\pi$ times the inverse Hubble volume, $\Pi = \pi H^3$ (see section III for justification). The same holds also for the outflow that can be approximated to be analogous to a known constant $\tau$ times the inverse Hubble volume,
where \( \Pi = \tau H^3 \). Therefore, the derivative \( \frac{d}{da} \) that appears in (2.14) should be replaced as follows

\[
\frac{d\Pi}{da} = -3\tau H^3(q + 1).
\]

Finally, \( H \) can be found solving exactly the cubic equation

\[
-\sqrt{3} \omega H^3 + (q-1)H^2 = (3w+1)\beta \rho^2 + (3w-1)\gamma \rho + \frac{k}{a^2} - 2\lambda.
\]

The above cubic equation for all realistic parameters has always two real positive roots associated with expanding universe and one negative leading to contracting cosmological solutions. These two roots for \( H \) are given by

\[
H = s_1 + s_2 - A/3 \quad \text{or} \quad H = \frac{1}{2}(s_1 + s_2) - A - \frac{i\sqrt{3}}{2}(s_1 - s_2),
\]

where

\[
s_1 = \left[ \eta + (\theta^3 + \eta^2)^{1/2} \right]^{1/3}, \quad s_2 = \left[ \eta - (\theta^3 + \eta^2)^{1/2} \right]^{1/3}
\]

\[
A = \frac{1 - q}{\sqrt{3} \omega}, \quad \vartheta = -\frac{1}{9} A^2
\]

\[
\eta = -\frac{1}{2} \frac{(3w+1)\beta \rho^2 + (3w-1)\gamma \rho + \frac{k}{a^2} - 2\lambda}{\sqrt{3} \omega} - \frac{1}{27} A^3.
\]

Now, the system of differential equations (2.14), (2.15) can easily be solved numerically. Thus, it is possible to test if the measured cosmic acceleration can be produced from some sensible values of \( T, \Pi \) according to our scenario.

In the second case which is more general, we do not assume a constant number of black holes since we are interested to include the dependence of the total mass density of black holes on the scale factor evolution. In this way it will be possible to describe the cosmological behaviour for redshifts far away from \( z = 0 \). The proposed mechanism suggests, in this second case, that \( \Pi = \widehat{\omega} \rho_{BH} \) and \( T = \tau \rho_{BH} \), where \( \widehat{\omega}, \tau \) are known constants and \( \rho_{BH} \) is the density of the relevant black holes which is a function of the scale factor. Now, the derivative \( \frac{d\Pi}{da} \) that appears in (2.14), equals

\[
\frac{d\Pi}{da} = \widehat{\omega} \frac{d\rho_{BH}}{da}.
\]

Since it is possible to know estimations concerning the evolution of \( \rho_{BH} \) as a function of \( a \) (see section V), it is possible to solve numerically the system (2.14), (2.15). In this second case, there is no need to solve any cubic equation since we can estimate \( H \) from (2.18).

Let’s now see if we can learn from these dynamic equations something about the value of dark pressure we need to have in order to explain cosmic acceleration. Equation (2.18) provides a constraint that the cosmic acceleration should satisfy. From it, we can determine the current value of \( \Pi \) as a function of the present values

\[
\Pi_0 = \left( -1 + q_0 + \frac{\Omega_{m,0}}{2} \right) \beta^{-1/2} H_0^2
\]

\[
\Leftrightarrow q_0 = 1 - \frac{\Omega_{m,0}}{2} + \Pi_0 H_0^{-2} \beta^{1/2},
\]

where we have replaced \( \rho = \Omega_{m,0} \rho_{cr} \) and \( k = 0, \lambda = 0, w = 0 \). Note that the current value of the deceleration parameter \( q_0 \) depends only on \( \Pi_0, H_0^2, \Omega_{m,0} \). Now, if we set \( \rho_0 \simeq \frac{1}{3} \rho_{cr,0} \) and for example \( q_0 = -1 \), we get

\[
\Pi_0 \simeq -\frac{11}{6} \frac{H_0^2}{\beta} \beta^{-1/2}.
\]

Such negative values can be easily realized in our scenario, arising from the proposed outflow mechanism. Therefore, it is possible without solving the differential equations to check the efficiency of the proposed mechanism using the value of \( \Pi \) which is fully determined from the involved astrophysical parameters and the value of \( M \). In this way, someone can be convinced that a small outflow in each of the galactic black holes suffices to result to the required amount of the total dark pressure and consequently to the measured cosmic acceleration. Intuitively, one could say that the geometry of the membrane universe is such that the negative dark pressure “stretches” this membrane and causes acceleration.

Since \( q \) is not directly measured, we have to express it as a function of the ratios of cosmological matter density to critical density and dark energy density to critical density. We define

\[
\Omega_m = \frac{2\gamma \rho}{H^2} = \frac{\rho}{\rho_{cr}}, \quad \Omega_{\Lambda} = \frac{\lambda}{H^2}, \quad \Omega_k = -\frac{k}{a^2 H^2}.
\]

and for the dark energy part

\[
\Omega_{DE} = \frac{\beta \rho^2 + \psi}{H^2} = \frac{\rho_{DE}}{\rho_{cr}}.
\]

Therefore, Eq. (2.12) gives

\[
\Omega_m + \Omega_{DE} + \Omega_{\Lambda} + \Omega_k = 1.
\]

Finally, the deceleration parameter can be found from

\[
q = \Omega_{DE} + \sqrt{3} \Pi H^{-2} + (1+3w) \frac{\Omega_m}{2} \left( 1 + \frac{\beta H^2}{2\gamma^2 \Omega_m} \right) - \Omega_{\Lambda}.
\]

This last equation is going to provide us the initial condition for \( q_0 = q(z = 0) \) given that \( \Pi_0 \) is provided by the astrophysical parameters, either in the first or second case discussed above. Now, the system of differential equations (2.14), (2.15) can be solved using the initial conditions \( q_0, \rho_0 = \rho(z = 0) \).

Another useful quantity used in physical cosmology is the coefficient \( w_{DE} \) of the equation of state of the dark energy. In our case the dark energy density encodes the
density required to represent the energy exchange. According to [13], $w_{DE}$ is given by
\[ w_{DE} = -1 - \frac{1}{3} \left( H^2 \right) - \frac{\Omega_{m,0}}{a^3} \left( \frac{H^2}{H_0^2} - \frac{\Omega_{m,0}}{a^3} \right). \] (2.31)

It is straightforward to prove that
\[ w_{DE} = \frac{1}{H^2} \left( \frac{\Omega_{DE,0}}{H_0^2} - \frac{1}{3} \left( \frac{1+3w}{6} \gamma^2 \Omega_m H^2 - \frac{2}{3} \sqrt{\beta} \right) \right), \] (2.32)

and therefore, the today value (for $w = 0$) is
\[ w_{DE,0} = \frac{1}{3} + \frac{\beta}{6\gamma^2} \frac{\Omega_{m,0}^2 H_0^2}{\Omega_{DE,0}} + \frac{2}{3} \frac{\sqrt{\beta} \Pi}{\Omega_{DE,0} H_0^2}. \] (2.33)

The numerical value of $w_{DE,0}$ will be a prediction for our model. This equation manifestly shows that a negative dark pressure term can easily cause not only cosmic acceleration but also the crossing of the $w_{DE} = -1$ phantom divide line. This was pointed out in a different context in [6].

Finally, an alternative useful expression that can be derived from equations (2.11), (2.12), (2.13) is the following single differential equation for $k = 0$ that depends on the energy density and which can be easily solved numerically
\[ \frac{dq}{d\rho} = 2q(q + 1)Z^{-1} - 3H\sqrt{\beta}(q + 1)\omega Z^{-1} + \frac{H^{-2}}{2(2 + 3w)} \beta \rho + (1 + 3w)\gamma - 4\psi Z^{-1} + 2\beta(\rho + \gamma)TZ^{-1}X \] \[ \left. + 2\sqrt{\beta} \Psi X \right) Z^{-1}, \] (2.34)

where
\[ Z = -3(w + 1)\rho - T \] (2.35)
\[ X = (\beta \rho^2 + 2\gamma \rho - k \omega^2 + \psi + \lambda)^{-1/2} \] (2.36)

and we replace everywhere $H$ and $\psi$ from equations (2.10), (2.21) or (2.18).

As mentioned, in the Randall-Sundrum model the effective cosmological constant $\lambda$ vanishes, and this is the value we assume in the rest of the paper. We also set $k = 0$ since we are interested on flat universes. Finally, since we are analyzing the cosmic acceleration after the large scale structure of the universe we set $w = 0$.

### III. A NOVEL PHENOMENON: BRANE-BULK ENERGY EXCHANGE INSIDE GALACTIC CORE BLACK HOLES AND/OR GALACTIC HALO BLACK HOLES

It is quite natural in the framework of brane cosmologies to expect a small energy exchange of our brane universe with the bulk space. This energy exchange phenomenon is a high energy phenomenon. The channels for energy exchange “open” when the relevant energies reach the relatively low five-dimensional fundamental Planck energy scale $M$. In the cosmological context regions where such high energy phenomena could occur are not as many.

In general, we would expect a brane-bulk energy exchange through:

1. High energy interactions in some accretion disks and more importantly inside galactic centres/galactic black holes leading to energy loss to the bulk due to the production of gravitons from high energetic accelerated particles.

2. Gravitational attraction of a portion of the gravitons that were escaped into the bulk or gravitational accretion of bulk matter to brane black hole.

3. Attraction of bulk matter from the whole brane.

4. Decay of very massive scalars and/or fermions.

The third type of exchange can be seriously studied only if the bulk matter content is known in detail, see for example [14], and consequently only if we are sure about the geometry of the bulk space, its anisotropies and the motion of our brane in it. Various different approaches to describe bulk dynamics/matter can be found in [15], [16], [17], [18], [19].

The fourth exchange mechanism works only for very massive particles like light supersymmetric particles with masses above 1TeV. This option to produce the measured acceleration is not very generic (see [21]).

The second exchange mechanism regarding attraction of the escaping gravitons contributes to dark pressure, but only to a small amount at late times [8], for which we are interested in. On the other hand, the attraction of bulk matter from the gravitational field of brane black holes may be not negligible for significant values of bulk matter density. Nevertheless, since we are interested to investigate energy exchange without losing predictability we have assumed that the matter energy density of bulk fluid is small or zero.

Our proposed mechanism considers a homogeneous distribution of galactic black holes on a brane. The study of a Swiss cheese-like brane world model with bulk energy exchange in each black hole (extension of work [22]) certainly would be a more precise modeling. The case is analogous to a swiss-cheese like brane world model with Schwarzschild-de Sitter black holes finally resulting to a dust FRW cosmology with an overall cosmological constant arising from each black hole contribution. However, the modifications of this more precise modeling are expected to be small. Therefore, the total brane-bulk energy exchange under consideration will be

\[ T = T_e, \] (3.37)

where $T_e$ represents the outflow of energy due to the production of escaping gravitons from high energetic particles inside BHs. Note also that in the present paper
we will not consider the possibility of an existing considerable amount of primordial black holes today, and therefore we will not study further scenarios with such black holes.

Last years it became evident that every nearby massive galaxy possesses a central black hole with mass proportional to that of the galaxy spheroid. This implies that they also possess an Active Galactic Nuclei (AGN) \cite{23}. In addition, there are evidences for the existence of a large amount of extra-galactic exposure at TeV energies \cite{24, 25} and some of it can be associated to the presence of galactic black holes and galactic core supermassive black holes.

It is certainly a safe assumption that in the accretion discs and more importantly in the interiors of galactic black holes and galactic core supermassive black holes various particles as electrons and protons can be thermalised/accelerated to energies around $M$ or above. Particle acceleration starts in the accretion discs outside the horizon and increases as the particle crosses it. Consequently, particle collisions become capable to produce gravitons escaping to the bulk space.

Moreover, assuming a black hole with physics that respects unitarity in its interior, it is acceptable to use the picture of an effective quantum fluid that fills the black hole and does not concentrate at the singular center (otherwise there will be information loss from the exactly thermal Hawking radiation). This effective fluid may be on a high temperature below or close to $M$. At these energies it is possible \cite{8, 9} to obtain rapid energy favored production of bulk gravitons from collisions of energetic baryon matter. In a hot plasma the production rate per 3-volume is the thermal average of the cross section times the lost energy of the particles. Therefore, the total energy loss rate due to bulk graviton radiation is \cite{9, 8}

\[
\Delta \rho_{pls} = 0.112 \frac{\Theta^4}{2M^2} \rho_{pls} = 0.112 \frac{\pi^2}{60} \frac{\Theta^8}{M^3}, \tag{3.38}
\]

where $\Theta$ is the temperature and $\rho_{pls}$ is the total energy density of the hot regions. The second equation in \eqref{3.38} is derived assuming a relativistic plasma with $g_* = 106.75$ the effective number of the relativistic degrees of freedom.

In order to proceed to a rough estimation of the mean outflow energy rate, an effective mean black hole plasma energy-mass density $\rho_{BH}^{pls}$ is assumed inside brane black holes expressed with the help of an effective mean temperature $\Theta_{mean}$. The following expression will be used

\[
\rho_{BH}^{pls} \simeq g_* \frac{\pi^2}{30} \Theta_{mean}^4. \tag{3.39}
\]

Now, let $\Delta \rho_{tot}$ be the leakage of energy from the total volume of the warm plasma of a black hole. In order to evaluate $T_e$ we have to add all these leakages from all galactic halo black holes and all black holes at the galactic central regions and divide with the Hubble volume $H^{-3}$, thus $T_e = H^3 \sum \Delta \rho_{tot}$. Since the total volume in the universe of warm black hole plasma is $N_{BH} V_{BH}$, we get

\[
T_e \simeq 0.112 \frac{\pi^2}{60} \frac{\Theta_{mean}^8}{M^3} \left[ N_{haloBH} V_{haloBH} + N_{coreBH} V_{coreBH} \right] H^3 \text{ or } \tag{3.39}
\]

\[
T_e \simeq 0.112 \frac{\Theta_{mean}^4}{2M^3} \left[ N_{haloBH} M_{haloBH} + N_{coreBH} M_{coreBH} \right] H^3 \text{ or } \tag{3.40}
\]

\[
T_e \simeq 0.112 \frac{\Theta_{mean}^4}{2M^3} \left( \rho_{haloBH} + \rho_{coreBH} \right) = \frac{\tilde{\tau}}{\rho_{BH}} \left( \rho_{haloBH} + \rho_{coreBH} \right) \tag{3.41}
\]

We assume the existence of $N_{haloBH}$ galactic halo black holes with mean mass $M_{haloBH}$ and $N_{coreBH}$ galactic central regions carrying a supermassive black hole with a mean value equal to $M_{coreBH}$. Since the mean value of mass density $\rho_{BH}^{pls}$ are very different among a typical halo black hole and a typical supermassive core black hole, we substitute $V_{haloBH} = M_{haloBH} (\rho_{BH}^{pls})^{-1}$ and $V_{coreBH} = M_{coreBH} (\rho_{BH}^{pls})^{-1}$. Although for simplicity in the above formulæ the temperature appears as a common mean value, in reality $\Theta_{mean}$ can be different between halo and core black holes, something that has been considered in the numerical study of the solutions.

The magnitude of the three dimensional pressure inside the black hole is equal to the magnitude of the pressure of the effective fluid. Since our collapsing fluid is not an ideal fermi gas, we adapt an index $\tilde{\tau}$ for determining the three dimensional pressure in the interior of both halo and core black holes, i.e.

\[
\Pi = \xi (\rho_{BH}^{pls})^{\tilde{\tau}}. \tag{3.42}
\]

The constant $\xi$ is determined by the thermal characteristics of the fluid and it can also be understood as a measure of the ratio of pressure to energy density at the center of black hole. Since the aim is to determine the dark pressure towards the fifth dimension we should divide the three dimensional pressure with the characteristic kinetic length scale $L$ of the plasma towards the bulk $\rho_{BH}^{pls} / L$. This length $L$ has been proven in \cite{8} as $L = \frac{M^3}{\rho_{BH}^{pls}}$. This is the reason why in Eq. \eqref{3.38} the outflow is analogous to $\frac{\rho_{BH}^{pls} M^3}{L^3}$. Therefore we get

\[
\Pi = -\xi \left( \frac{\rho_{BH}^{pls}}{\rho_{BH}^{pls}} \right)^{\tilde{\tau} + 1} \frac{M^3}{L^3}. \tag{3.43}
\]

Note that the last expression for $\tilde{\tau} = 1$ reduces to the dark pressure estimated in \cite{8}.

The phenomenon under discussion most importantly results to the appearance of a negative pressure orthogonal to the fifth dimension. At the position of the brane the five-dimensional pressure $\Pi = 2T^5$ equals the momentum flux carried from the bulk to the brane. Because of momentum conservation this pressure equals the opposite of the momentum flux carried by the escaping gravitons from the brane to the bulk. Therefore, $\Pi < 0$. This
negative sign is a subtle point missed in the analysis in \textsuperscript{3}.

Finally,

\[
\Pi = -\xi \left[ \left( \frac{\rho_{\text{BH}}}{M^3} \right)^{\gamma+1} N_{\text{haloBH}} V_{\text{haloBH}} + \left( \frac{\rho_{\text{coreBH}}}{M^3} \right)^{\gamma+1} N_{\text{coreBH}} V_{\text{coreBH}} \right] H^3
\]

\[
\Pi = -\xi \left[ \left( \frac{\rho_{\text{haloBH}}}{M^3} \right)^{\gamma} N_{\text{haloBH}} M_{\text{haloBH}} + \left( \frac{\rho_{\text{coreBH}}}{M^3} \right)^{\gamma} N_{\text{coreBH}} M_{\text{BHcore}} \right] H^3
\]

\[
\Pi = \omega H^3 \tag{3.44}
\]

\[
\Pi = -\xi \left( \frac{\rho_{\text{haloBH}}}{M^3} \right)^{\gamma} \rho_{\text{haloBH}} + \left( \frac{\rho_{\text{coreBH}}}{M^3} \right)^{\gamma} \rho_{\text{coreBH}} \tag{3.45}
\]

Equation (3.45) holds for the case where the mass density of the core black holes is dominant.

It should be noticed that the proposed outflow mechanism has no similarity with scenarios that set the density of the plasma equal to the density of the overall cosmological fluid which cools as the universe expands. For example in \textsuperscript{9}, \textsuperscript{8} the whole universe has to be thermalised in temperatures close to the fundamental Plank scale which is not true at late times of the evolution. In our work the thermalised fluid is in the interiors of black holes and leaks towards the bulk. Furthermore, the pressure $T_{55}$ of the fluid is not of the same order with the $T_{05}$ leakage as in \textsuperscript{8} since a non ideal gas quantum fluid is expected/assumed inside black holes.

Based on the above discussion we can directly see the connection of the present energy density of the universe $\rho_0$ to the observed dark energy. Namely, the outflow energy rate and the dark pressure are

\[
T_{c,0} = 0.112 \frac{\Omega_{\text{mean}}^4}{2 M^3} \varepsilon \rho_0, \tag{3.46}
\]

\[
\Pi_0 = -\xi \left( g_s \frac{\pi^2}{30} \Omega_{\text{mean}}^4 \right)^{\gamma} \frac{1}{M^3} \varepsilon \rho_0, \tag{3.47}
\]

while the current cosmic acceleration becomes

\[
q_0 = 1 - \Omega_{\text{mean},0} \frac{2}{g_s} \frac{\pi^2}{30} \Omega_{\text{mean}}^4 \frac{1}{12 M^3} H_0^2 \varepsilon \rho_0. \tag{3.48}
\]

The quantity $\varepsilon$ is the portion of the present black hole mass density $\rho_{\text{BH},0}$ relative to the present cosmic mass density $\rho_0$. In section V it will be demonstrated that even for the most conservative values of all the involved parameters such negative values of $q_0$ can be achieved.

IV. ADDITIONAL SUPPORT OF THE PROPOSED MECHANISM

In this section various physical aspects of the proposed mechanism are presented.

A. The proposed mechanism and the gravitational collapse on the brane

In this subsection estimations are presented concerning the evolution of a spherical collapse in the brane scenario presented above. Our goal is to describe quantitatively the expected behavior of temperature rise as the collapse of a fluid proceeds. In our case, strong quantum gravity corrections are not necessary since our intention is to describe the collapse up to the point where the outflow becomes significant. This happens for temperatures close to the fundamental Planck scale which can be relatively low.

The spherical gravitational collapse on a brane with a realistic brane-bulk energy exchange will now be analyzed. The interior of the collapsing spherical region undergoing an Oppenheimer-Snyder collapse will be described by the brane cosmological metric (2.2) presented above, with nonzero $T_{05}, T_{55}$. Therefore the evolution has to be a contracting solution of the system of the brane cosmological equations (2.11), (2.12) and (2.13). Now, the energy density, the dark radiation and the dark pressure concern the plasma in the interior of black hole/collapsing region. Thus, the system of differential equations that the evolution of the collapsing region should respect is

\[
\dot{\rho}_{\text{pls}} + 3 \left( \rho_{\text{pls}} + p_{\text{pls}} \right) \frac{\dot{R}}{R} = -T_{\text{pls}}, \tag{4.49}
\]

\[
\frac{\dot{R}}{R^2} = \beta \rho_{\text{pls}}^2 + 2 \gamma \rho_{\text{pls}} - \frac{\kappa}{R^2} + \psi, \tag{4.50}
\]

\[
\dot{\psi} + 4 \frac{\dot{R}}{R} \psi = 2 \beta \left( \rho_{\text{pls}} + \frac{\gamma}{\beta} \right) T_{\text{pls}} - 2 \sqrt{\beta} \frac{\dot{R}}{R} \Pi_{\text{pls}}, \tag{4.51}
\]

where $\kappa$ characterizes the spatial topology of the collapsing shell (with most interesting case $\kappa = 1$). Here, the scale factor $R(t)$ of the collapse region is related to the proper radius $r$ from the center of the cloud through $r = R \chi / (1 + \kappa \chi^2/4)$, where $\chi$ is the comoving coordinate and the dot denotes a proper time derivative. The energy outflow and dark pressure are given by

\[
T_{\text{pls}} \simeq 1.68 \frac{1}{\pi^2} \frac{1}{M^3} \rho_{\text{pls}}^2 \simeq 0.112 \frac{\pi^2}{30} \frac{\rho_{\text{mean}}^4}{M^3}, \tag{4.52}
\]

\[
\Pi_{\text{pls}} = -\xi \left( g_s \frac{\pi^2}{30} \Omega_{\text{mean}}^4 \right)^{\gamma+1} \frac{1}{M^3}. \tag{4.53}
\]

Collapsing plasma has been assumed to have an equation of state deviated from this of an ideal gas. The relation between the energy density and the temperature (local thermodynamic equilibrium) is given by the ansatz

\[
\rho_{\text{pls}} \simeq g_s \frac{\pi^2}{30} \Omega_{\text{mean}}^4 \simeq \sigma \Omega_{\text{mean}}^4. \tag{4.54}
\]

Pressure is expressed as $p_{\text{pls}} = \frac{\kappa}{\beta} \rho_{\text{pls}}$. Realistic quantum fluids can effectively be described by an equation of state with deviations from ideal gas behavior \textsuperscript{20}.

Therefore, in order to study the temperature evolution as the collapse continues, we have to find solution of the
following system of differential equations

\[ \dot{\Theta}_{\text{mean}} + \frac{3}{4} \Theta_{\text{mean}} \left( 1 + \xi \Theta_{\text{mean}}^{4(\gamma - 1)} \right) \frac{R}{R} - 0.014 \frac{\Theta_{\text{mean}}^5}{M^3} = 0 \]  
\[ (4.54) \]

\[ \frac{R^2}{R^2} = \beta \sigma^2 \Theta_{\text{mean}}^8 + 2 \gamma \sigma \Theta_{\text{mean}}^4 - \frac{\kappa}{R^2} + \psi \]  
\[ (4.55) \]

\[ \psi + 4 \frac{R}{R} = 0.112 \beta \sigma \left( \Theta_{\text{mean}}^4 + \frac{\Theta_{\text{mean}}^8}{\beta} \right) \frac{\Theta_{\text{mean}}^8}{M^3} \]
\[ + 2 \sqrt{\beta} \xi \Theta_{\text{mean}}^{\gamma+1} \frac{R}{R} \Theta_{\text{mean}}^{4(\gamma+1)} \frac{1}{M^3} \]  
\[ (4.56) \]

The above system of the first and third equation can be solved numerically without difficulties. This study shows that for expected parameters \( \frac{R}{M} < 1 \) and for \( R < 0 \), which is the case of spherical collapse, we can get \( \dot{\Theta}_{\text{mean}} > 0 \), which is what we want to prove. A typical solution of this system is shown in Fig. 1, where time \( t \) is measured in GeV\(^{-1} \) and temperature \( \Theta \) in GeV.

![FIG. 1: Temperature rise during a collapse for \( M = 10^4 \) GeV](image)

Note that we are not interested here to find a static exterior for the above described collapsing spherical region \[27, 28, 29].

**B. The proposed mechanism and the Hawking-like radiation**

An interesting work about the physics inside the collapsing horizon of collapsing shells or the interior of black holes accreting matter is this of Greenwood, Stojkovic \[10\]. In this work, Hawking radiation was studied as seen by an infalling observer. Based on functional Schrödinger formalism it is possible to calculate radiation in Eddington-Finkelstein coordinates which are not singular at the horizon. In these coordinates Hawking radiation does not diverge on the horizon. The estimated occupation numbers at any frequency, as measured by an observer crossing the horizon, were found to increase as the distance from the black hole center decreases. The spectrum is not thermal and therefore there is no well-defined temperature measured by the observer. Although this work does not refer to brane black holes, we expect similar qualitative behavior for this case too. Therefore, the above discussion suggests that an observer entering the horizon encounters/interacts with more and more highly energetic particles which can escape easily to the bulk or cause through their interactions energy loss to the bulk. Estimations presented in \[10\] are not valid for distances close to the black hole centre where strong backreaction effects have to be considered. However, in our case energy loss can start inside and near the horizon for temperatures close to \( M \).

**C. The proposed mechanism and the Fuzzball approach**

A fluid description is certainly a phenomenological picture which is traditionally followed in relativistic cosmology/astrophysics. Here we will attempt to discuss microscopically the reason why such a description can be based on fundamental physics. It is expected that in reality in the interior and most certainly near the centers of black holes the notion of classical spacetime is replaced by another not well understood “quantum” spacetime. The full treatment is still unknown; nonetheless it is expected that an effective description of the black hole interior with the help of a “non perfect” fluid without infinite density could be a fair approximation. Adopting this effective approach we assume that at the center of the black hole, the density is large but finite and equals to \( \rho_{\text{pl}}(r = 0) \). Therefore, if the physics in the interior of astrophysical black holes was known, one could in principle be able to reproduce an effective description estimating a mean value of the plasma density \( \rho_{\text{pl}}^{BH} = \frac{4\pi}{V_{\text{BH}}} \int_0^R \rho_{\text{pl}}(r)^2 dr. \) The value of the effective radial dependent plasma density \( \rho_{\text{pl}}(r) \) as well as the effective central finite value \( \rho_{\text{pl}}(0) \) would then be determined by quantum gravity.

Although in pure general relativity such an expression has no meaning since the energy density becomes infinite and the spacetime description breaks at the center, new ideas arising from string theory possibly allow an effective quantum statistical description of the black hole interior. A promising approach for addressing questions regarding physics inside black holes is the fuzzball proposal \[30, 31, 32, 33, 34, 35, 36\]. According to this view, the infinite “throt” that a classical geometrical description exhibits near the singularity is replaced by a long finite throat which ends in a quantum fuzzy cap. The fuzzball conjecture claims that the astrophysical black holes are described by microstates which all behave like
the ones that have been constructed for extremal black holes in string theory. The bound states in string theory are not in general Planck sized or string sized, but have a size that grows with the degeneracy of the bound state. To make a big black hole a large number of elementary quanta need to be placed together. Regarding the size of the bound state one may think that this is equal to string or Planck scale $l_{pl}$. However, if this was true we would get the traditional picture of brane black holes with all matter placed at the singularity (then hawking radiation becomes exactly thermal leading to loss of unitarity). The correct picture is that the size of the bound state increases with the number of quanta in the bound state. In the fuzzball approach the size of the bound state $R \sim N^a l_{pl}$ has been proven to be equal to the black hole horizon radius that we would find for the classical geometry which has the mass and charge carried by these $N$ quanta. $N$ is some count of the quanta and $a$ depends on what quanta are being bound together.

The fuzzball theory suggests two important elements regarding the physics of the brane black holes interior. First, the matter content is distributed all over the interior, a fact that allows an effective description with a quantum statistical fluid described by a non conventional equation of state. Second, some of the quanta are free to tunnel into the bulk not due to Hawking black hole evaporation but due to the absence of microstate horizons and the brane-bulk geometry associated with a “small” value of $M$. Hawking radiation is due to fractional brane-antibranes annihilations, while outflow is the result of tunneling of string quanta of fractional and non fractional branes-antibranes towards the bulk space.

Let us think in more detail what may happens inside a black hole. If we increase the energy density of a collection of branes to very large values, it becomes entropically favorable to produce a large number of sets of mutually BPS branes and anti-branes. These branes “fractionate” each other, resulting to entropy that grows more rapidly as a function of energy compared to that of radiation or a Hagedorn type string or brane gas. Therefore, in the case of astrophysical black holes it is expected that after the beginning of the collapse energy density grows and matter reaches a Hagedorn phase of strings. Although this pressureless phase keeps its energy nearly constant (there are already significant open outflow channels) thanks to the continuing collapse the energy density increases further. Finally, we end up to an even higher energy scale phase with a soup of many fractional and less non fractional branes.

In the two charge system NS1-P bound state there is a string that loops $n_1$ times around $S^1$ (radius $R$) with a momentum charge $P$ which is bound to the string in the form of traveling waves on the NS1 brane. The number of states that contribute more to the entropy is approximately equal to $\exp(\sqrt{n_1 l_{pl}})$. These states are fractional with a length $L_T$ equal to the classic geometry horizon (if we add one more charge). These fractional states have a low temperature/average energy (equal to Hawking temperature if we add one more charge) given by

$$T_H = \frac{\sqrt{n_1 l_{pl}}}{L_T} ,$$

where the total length of the string is large and equal to

$$L_T = 2\pi R n_1$$

since in realistic astrophysical black holes $n_1$ can be very large.

However, in the black hole interior there are also fewer states with large temperature/energy because 1) $L_T$ can be very small since $R$ is very small, while $n_1$ is also small for non fractional states, 2) branes need a large time (evaporation timescale) to fractionate to very large lengths. These non fractional states tunnel immediately to the bulk space as long as

$$M \leq \frac{\sqrt{n_1 l_{pl}}}{2\pi R \sqrt{n_1}} .$$

Now the disappearing states to the bulk due to tunneling are continuously replaced in the high energy density regions of the interior at the cost of the collapsing matter’s energy density. Thus, we have a non vanishing flow of energy towards the bulk.

In the three charge system there are $n_3$ NS5 branes and $n_1$ NS1 branes that define a system with a momentum charge $P$. Therefore, the bound system of these branes generate an “effective string” with a total winding number $n_1 n_3$. All the above discussion for the two charge system and all relevant expressions remain the same replacing everywhere $n_1$ with $n_1 n_3$.

Apart from the outflow originated by these states in the black hole interior, there are two more outflow open channels. As we have previously mentioned, a portion of collapsing matter is still in the string/brane gas phase which is a very hot phase that certainly can leak to the bulk space. In addition, there must be a non negligible outflow from the portion of the collapsing matter that is between the string/brane phase and the electroweak energy scale ($\sim$TeV) as long as its local temperature is close or larger than $M$.

In summary, the reasoning that ensures outflow is the observation that astrophysical black holes are not non-perturbative configurations composed of wrapped strings or branes living at the Planck regime or M-theory landscape. They are objects created dynamically from collapsing matter initially respecting our $U(1)$ vacuum. This matter unavoidably gets compressed to smaller and smaller volumes until it reaches very high energy scales where outflow is not negligible and unavoidable.

To close this section, it is important to mention that although the fuzzball proposal is helpful in order to understand the microscopic processes of the outflow, the proposed mechanism operates based only on two sensible requirements: first, the existence of Schwarzschild-like black hole solutions on the brane with nonzero $T_3$,.
proved in ref. [37] and used in a more general form here, and second, the existence of a non conventional quantum fluid in the interior of black holes or better the validity of an effective description of the interior with such a fluid, something that sounds natural since quantum states have to be important at the horizon, otherwise thermal Hawking radiation would lead to information loss.

V. AMOUNT OF PRODUCED COSMIC ACCELERATION

The goal of the present work is to estimate for the proposed brane-bulk energy exchange mechanism the amount of the produced present cosmic acceleration for various values of the relevant parameters. This section presents the numerical results of our study.

I. Numerical analysis with time-dependent black hole cosmic density. First we analyze the more general and interesting case of time dependent black hole cosmic mass density (referred in section II as second case) considering astrophysical estimates reported in [38]. With the help of them we can describe the core black hole density evolution with the following relation (valid for $z < 2$

$$\log_{10}(\rho_{coreBH}|_{z=0}) = -\mu z + \log_{10}(\rho_{coreBH}|_{z=0}) \quad (5.60)$$

Since $\rho_{coreBH}|_{z=0} = 4.3 \times 10^5 M_\odot Mpc^{-3}$ is the current galactic core black hole matter density and $\rho_{coreBH}|_{z=2} = 1.5 \times 10^9 M_\odot Mpc^{-3}$ is the density at redshift $z = 2$ we obtain

$$\mu = \frac{\log_{10}(\rho_{coreBH}|_{z=0}) - \log_{10}(\rho_{coreBH}|_{z=2})}{2} \quad (5.61)$$

Equation (5.60) shows that when the redshift $z$ decreases (cosmic matter density decreases), the energy density of black holes increases. Therefore, from (5.61) we see that the absolute value of dark pressure increases for the late stages of cosmic evolution. Equations (2.30), (2.22) show that $q, w_{DE}$ get progressively negative values. Based on the expression (5.60) it is possible to estimate the dependence on the scale factor of dark radiation and dark pressure from Eqs. (3.41), (3.45). The numerical investigation of (2.14), (2.15) reveals that for a wide range of the parameters $\gamma, M$ it is always possible to find a range for the mean temperature $\Theta_{mean}$ that results to cosmological solutions with current cosmic acceleration $q < 0$, with $w_{DE}$ around -1, and equally importantly with a deceleration era that only currently becomes acceleration. Table 1 presents some representative results, while Fig. 2 shows the evolution of the deceleration parameter $q$ and $w_{DE}$ for $\gamma = 0.05$, $M = 50$ TeV. Results in Table 1 reveal that it is possible to get dark acceleration for reasonable values of $M$ and $\Theta_{mean}$. For values $M > 10^3$ TeV there is a need for very large temperatures in the interior of black holes. Another remark is that the index $\gamma$ has to be less than unity and this would be connected with the physical properties of the assumed quantum fluid in the interior of the black holes. Also note that the values of $\Pi$ are orders of magnitude larger than $T$.

It is also worth mentioning that both $T, \Pi$ are zero before large scale structure since black holes have not appeared yet. Only after the large scale structure and the growth of a significant population of astrophysical black holes the mechanism is able to result to cosmic acceleration. The latter observation provides a natural solution to the coincidence problem.

The auxiliary field $\psi$ which is the basic component of the dark energy can also be estimated during the cosmic evolution ($\psi$ appears in equation (2.12)). Numerical results show that $\psi$ decreases during the evolution from $z = 2$ to 0 with typical values in the range $2.5 \times 10^{-83}$ GeV$^2 < \psi < 5 \times 10^{-84}$ GeV$^2$, while $2\gamma \rho$ also decreases and takes values in the region $2.8 \times 10^{-83}$ GeV$^2 < 2\gamma \rho < 4.5 \times 10^{-84}$ GeV$^2$. It is now apparent that the small outflow that produces the values of dark pressure shown in Table 1, is associated with values of $\psi$ comparable with $\gamma \rho$ values, and consequently modify non trivially the cosmic expansion and cosmic acceleration through equations (2.72), (2.10). Note that in the absence of outflow (in our case before the structure formation) the usual braneworld cosmology holds with the well known solution $C/a^4$ for $\psi$, which is the so called dark radiation. Assuming a typical ansatz for the law that describes the non linear increase of the galactic core black hole mass density from $z = 8$ (approximate moment of the formation of first galaxies) to $z = 2$ (moment that observations suggest a known value for $\rho_{coreBH}$) it was possible to show that starting from a zero or from a small positive dark radiation, dark energy $\psi$ increases to the values described above (for $z \leq 2$) which are capable to drive cosmic acceleration. Moreover, this positive radiation term $C/a^4 > 0$ at $z = 8$ is small enough to overpass well known problems of nucleosynthesis constraints [11].

In order to confirm these numerical results, the analytical solution for the system of differential equations (2.11) - (2.13) has been derived. The analytic solution that will be given corresponds to the case $T = 0$. Indeed this approximation is valid since for all the interesting cosmological parameters (which make the scenario successful) the contribution of the terms containing $T$ is negligible compared to the other terms. The solution is

$$\psi(a) = \frac{H_0^2}{a^4} \left( \Omega_{DE,0}^{1/2} - \beta H_0^2 \Omega_{m,0}^{3/2} - \frac{\sqrt{3} \xi (\rho_{BH}^B)^{\gamma}}{12 M^5} \rho_{BH,0} \right) \times \left\{ 10^{\mu (1 - \frac{\mu}{2})} \left[ \left( \frac{\mu \ln 10}{a} \right)^3 - 2 + \frac{2 \mu \ln 10}{a} - \frac{\mu \ln 10}{a} \right] - \frac{1}{a^7} \left( \frac{\mu \ln 10}{a} \right)^3 - 2 + \frac{2 \mu \ln 10}{a} - \frac{\mu \ln 10}{a} \right\} (5.62)$$

$$\rho(a) = \frac{H_0^2 \Omega_{m,0}}{2 \gamma a^3} \quad (5.63)$$

$$H^2(a) = \beta \rho^2 + 2 \gamma \rho + \psi, \quad (5.64)$$
where $\rho_{\text{BH,0}} = \rho_{\text{coreBH}}|_{z=0}$ and $E_i$ is the exponential integral function. Based on this solution the results shown in Table 1 are verified.

II. Numerical analysis with a certain number of black holes. A different useful approach although less general, is to study the behavior of our mechanism for the recent epoch $z \sim 0$. If we are not interested to investigate the early time evolution of the cosmic acceleration but just to explore parameter combinations that provide values $w_{DE,0} \simeq -1$, it is correct to set in the relevant differential equations (2.14), (2.15) constant values for $T$, $\Pi$. These values are estimated taking into account the present values of astrophysical data (number of galaxies, number of black holes etc.). The black hole mass density decreases due to the cosmic expansion and increases due to matter accretion but for the current time period of interest ($z < 1$) the cosmic matter density rate is three orders of magnitude larger than the black hole density rate $\frac{d\rho_{\text{BH}}}{dt} \ll \frac{d\rho}{dt}$ [39]. Therefore it is a fair approximation to assume for the recent cosmic evolution, constant values for $T$, $\Pi$ in the differential equations (2.14), (2.15).

Based on the derived expressions (3.40), (3.44) for the cosmic energy outflow $T_{e,0}$ and the associated pressure $\Pi_0$ we will consider various cases for the black hole matter content in order to evaluate the cosmic acceleration.

First we will consider as an extreme case a matter content with a large amount of black holes in halos suggested in [40]. In this case, we assume a universe with $10^{11}$ halos and $10^{10}$ large black holes per halo. Further we set as a crude mean mass for a halo black hole a value equal to $M_{\text{haloBH}} = 10^2 M_\odot$. Consequently, we estimate a total mass in the form of halo black holes equal to $N_{\text{haloBH}} M_{\text{haloBH}} = 10^{23} M_\odot$ (these numbers were taken from [40], however, note that the assumption appeared in [40] that all dark matter consists of black holes is not necessary or related to the present paper). Galactic core black holes contribute much less in this case, i.e. there are $10^{11}$ supermassive black holes each with a mean mass $10^7 M_\odot$, i.e. $N_{\text{coreBH}} M_{\text{BH,core}} = 10^{18} M_\odot$. Therefore, in this extreme case all the cosmic acceleration comes from halo black holes. We can get $w_{DE,0} \simeq -1$ for various combinations of the parameters, see Table 2.

It is more safe to assume that the mass density of galactic core black holes is larger than the density of halo black holes. Taking $N_{\text{coreBH}} M_{\text{coreBH}} = 10^{18} M_\odot$, again there are plenty of numerical solutions of (2.14), (2.15) for various parameters giving acceleration $w_{DE,0} \simeq -1$. Similarly, assuming a more conservative case where $N_{\text{coreBH}} M_{\text{coreBH}} = 10^{15} M_\odot$ it is easy to find many numerical solutions of equations (2.14), (2.15) resulting to the required cosmic acceleration. Such representative results are shown in Table 2. Results shown in Table 2 reveal that the various quantities are of the same order with the corresponding quantities of Table 1 as expected.

III. Astrophysical constraints. It is worth emphasizing that all these estimated values of energy loss $T_{e,0}$ appeared in Tables 1 and 2 are small values that do not

| assumption | assumption | assumption | output $T_{e,0}$ (GeV) | output $\Pi_0$ (GeV$^5$) | output $w_{DE,0}$ |
|------------|------------|------------|------------------------|------------------------|------------------|
| $\gamma$   | $M$ (TeV)  | $\Theta_{\text{mean}}$ (GeV) | $10^{-8}$               | $10^{-98}$             | $10^{-71}$       |
| 0.21       | 10         | $3.5 \times 10^{-10}$         | $10^{-103}$             | $10^{-71}$             | -1               |
| 0.18       | 10         | $10^{-13}$                     | $10^{-118}$             | $10^{-71}$             | -1               |
| 0.13       | 10         | $10^{-9}$                      | $10^{-104}$             | $10^{-69}$             | -1               |
| 0.07       | 50         | $10^{-13}$                     | $10^{-116}$             | $10^{-69}$             | -1               |
| 0.05       | 50         | $10^{-12}$                     | $10^{-109}$             | $10^{-68}$             | -1               |
| 0.01       | 100        | $10^{-10}$                     | $10^{-109}$             | $10^{-68}$             | -1               |
| 0.001      | 120        | $10^{-10}$                     | $10^{-109}$             | $10^{-68}$             | -1               |

Table 1: Summary of results for various values of the parameters consistent with today acceleration.
cause any astrophysical inconsistency on galaxy evolution or black hole dynamics. The rest of this section is devoted to the explanation of the absence of any conflict with the known observational characteristics of the galaxies and of their black holes. Thus, two astrophysical constraints are considered for the case where the outflow occurs at the centers of core black holes. The other case where galactic halo black holes dominate the energy outflow will be considered later.

A first bound can be found demanding that the lifetime of a galactic core black hole loosing energy according to our scenario is larger than the typical lifetime of such black holes. The first bound can be met, for example, for a halo black hole (see Table 2) with \( \Theta_{\text{mean}} \sim 10^{-9} \text{GeV} \) and \( M = 50 \text{TeV} \), is associated with an energy loss rate equal to \( 10^{34} \text{ erg sec}^{-1} \). Such losses are orders of magnitude smaller numbers compared to accretion rates. Therefore, they cannot alter significantly the black hole mass and make impossible the violation of any measured relations between central galactic black hole mass and galactic halo mass or of the observed expression of black hole density as a function of redshift (eq. (5.60) that was used in the present section).

Next, let’s study the constraints regarding the extreme case where galactic halo black holes [44, 45] dominate the energy outflow \( T_{e,0} \). The first bound demands that the time duration required for a black hole of mass \( M_{BH} \) to lose all its rest energy be larger than the maximum lifetime of a typical galactic halo black hole \( t_{\text{haloBH}} \sim 10^{10} \) years. The first bound can be easily met; for example, for a halo black hole (see Table 2) with \( \Theta_{\text{mean}} \sim 10^{-12} \text{GeV} \) the estimated lifetime is \( 10^{9} \) years!

Finally, for halo black holes the second bound can be studied demanding the net energy gain due to accretion of mass minus the radiated energy be larger than the energy loss to extra dimensions. Now, we have to distinguish two types of galactic halo black holes. Black holes that are part of a binary system have usually an efficiency \( L/L_\odot \sim 0.1 \), while the conversion efficiency is around 0.01 to 0.1. Thus the net energy gain is expected to be close to Eddington accretion, which in this case is \( L_\odot \sim 10^{38} \text{ erg sec}^{-1} \). For halo black holes with mass \( M_{\text{haloBH}} \sim 10^6 \text{M}_\odot \) and with \( \Theta_{\text{mean}} \sim 10^{-12} \text{GeV} \) the loss rate is equal only to \( 10^{17} \text{ erg sec}^{-1} \). On the other hand, galactic halo black holes that do not belong to a binary system cannot be observed since they do not accrete matter and there is no accretion disk to radiate. Therefore, in this case it is not possible to know their properties and apply the second bound.

It is worth mentioning that our cosmology solves also the problem of the age of the universe. This is known to be true for braneworld models with non zero \( T, \Pi \) due to the produced cosmic acceleration [46].

| astrophysical observation | assumption | output | output | output |
|---------------------------|------------|--------|--------|--------|
| \( N_{BH} M_{BH} \)       | 0.1        | \( 10^{-10} \) | \( 10^{-105} \) | \( -10^{-69} \) |
| \( N_{coreBH} M_{BH,core} = 10^{12} \text{M}_\odot \) | 0.07       | \( 10^{-14} \) | \( 10^{-121} \) | \( -10^{-69} \) |
| \( N_{coreBH} M_{BH,core} = 10^{15} \text{M}_\odot \) | 0.026      | \( 10^{-8} \) | \( 10^{-101} \) | \( -10^{-69} \) |
| \( N_{haloBH} M_{haloBH} = 10^2 \text{M}_\odot \) | 0.2        | \( 4.5 \times 10^{-12} \) | \( 10^{-106} \) | \( -10^{-69} \) |

Table 2: Summary of results for \( M = 50 \text{TeV} \) and for various values of the parameters consistent with today acceleration.
tribution Π from all the dark pressures from all the centers of galaxies suffice to provide the measured cosmic acceleration. An exciting outcome is that the recent passage from the deceleration to the acceleration era happens due to the recent increase of the galactic core black hole mass density. Based on the derived expressions we have shown that it is easy to get the expected negative values of cosmic acceleration $w_{DE,0} \approx -1$ and dark energy $Ω_{DE,0} = 0.7$ even for conservative values of all the relevant parameters, i.e. for small values of the mean temperature $Θ_{\text{mean}}$ in the interior of the black holes, for small values of galactic core black holes masses and for values of the five-dimensional Planck mass $M$ several decades of TeVs. In our mechanism, outflow is associated unavoidably with a large dark pressure. The magnitude of the produced dark pressure is connected with that of dark radiation through the equation of state of the quantum fluid in the interior of a black hole. Qualitatively one can immediately check the efficiency of producing the observed cosmic acceleration estimating the required amount of dark pressure shown in equations (2.25), (2.26) or (3.18). This value of dark pressure can naturally be realized in our case based on the known astrophysical data and the assumed temperature of the effective fluid. Of course, in order cosmic acceleration to be proven, the system of differential equations (2.14), (2.15) has to be solved, and indeed it is seen that the energy exchange along with the dark pressure give an order one effect on cosmological scales.

The proposed mechanism has several advantages: i) it is independent of the bulk matter and consequently retains predictability, ii) the associated values of $ψ$ in the Hubble evolution (2.12) originate from the brane black hole astrophysical phenomenon of energy outflow $T$ and its associated pressure $Π$ along the fifth dimension, and not from the motion or the position of the brane in the bulk, thus again retaining predictability, iii) the mechanism is “on” at present times and “off” at the early stages of the cosmic evolution explaining naturally coincidence problem, iv) it relates the amount of the produced acceleration with the present matter content, and v) it produces easily cosmic acceleration for sensible values of the relevant parameters.

However, the most interesting and worth mentioning finding is the fact that sensible and safe values of outflow, as those appeared in Table 2, suffice to result to the observed cosmic acceleration. A first reason behind this outcome is the large number of galaxies in the universe. Another reason is that the additive effect of all outflows and associated much larger dark pressures from each galactic center result to a non negligible kinetic effect, i.e. acceleration due to the geometry of the setup. Finally, the dark pressure drives towards acceleration from earlier times of the cosmic evolution and not just today. Since energy density of the galactic core black holes increases as redshift decreases at recent times ($z < 2$), dark pressure becomes stronger driving the passage to the acceleration era.

In summary, the novelties of the present article are: 1) correction of a wide spread mistake among the brane cosmologists that outflow is associated with a positive and not negative dark pressure, 2) the presentation of a new astrophysical origin mechanism ofbrane-bulk energy exchange, 3) new braneworld solutions describing the evolution of brane equations with non zero $T_{65}$, $T_{55}$, 4) new gravitational collapse solution on a brane with non zero $T_{65}$, $T_{55}$, 5) numerical results estimating the produced cosmic acceleration, and 6) correlation of the measured acceleration to the recent rise of the galactic core black hole cosmic energy density.

The calculations of the scenario could have failed for various reasons: if a very high temperature was needed in the interior of the black hole, or if a small fundamental Planck scale or a large $T^3$ was needed conflicting of course with galactic dynamics, or if for the given variation of the cosmic black hole density as a function of redshift the cosmic evolution failed to posses a long deceleration era accompanied by a recent acceleration one. However, the concrete and conservative numerical values used lead the scenario to success.

One interesting point worth to be raised is that the proposed mechanism could work together with various studies that suggest solutions to the cosmological problem. For example, there are recent holographic ideas capable to explain why the cosmological constant should be almost zero [47], [48]. However, it appears to have a difficulty to explain naturally the cosmic equation of state. Therefore, the present work together with all type of holographic explanations can provide a complete solution to the general problem of the cosmological constant value.

VII. ACKNOWLEDGEMENTS

We would like to thank V. Charmandaris, T. Harko, E. Kiritsis and I. Papadakis for useful discussions and comments.

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