Late time Cosmological Phase transition and Galactic Halo as Bose-Liquid

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We consider the ultra light pseudo Nambu-Goldstone boson appearing in the late time cosmological phase transition theories as a dark matter candidate. Since it is almost massless, its nature is more wave like than particle like. Hence we apply quantum mechanics to study how they form the galactic halos. Three predictions are made; (1) the mass profile $\rho \sim r^{-1.6}$, (2) there are ripple-like fine structures in rotation curve, (3) the rotation velocity times ripple’s wave length is largely galaxy independent. We compare the rotation curves predicted by our theory with the data observed.

1. Introduction — Recently, motivated by the large scale structure, an ultra light Nambu-Goldstone boson was introduced in late time cosmological phase transition theory.[1] The bottom line of the theory is that if a phase transition happens after decoupling, one can avoid the constraint imposed by isotropy of microwave back ground. If an event happens so late then the universe is already big, hence the relevant particle whose compton wave length provides the scale of interesting structure must be very light and this particle could be a dark matter candidate. [2]

The dark matter distribution is most clearly imprinted in the rotation curve. The overall ‘flatness’ of the rotation curve (RC) is equivalent to mass profile $\rho \sim r^{-2}$, which can be obtained by assuming a thermal distribution.[10] Though they are said to be ‘flat’. So neither the observed rotation curves are actually slightly increasing $\rho \sim r^{-1.6}$, giving a slowly decreasing rotation curve. However, the observed rotation curves are actually slightly increasing [10] though they are said to be ‘flat’. So neither the thermal distribution nor the infall model is completely satisfactory.

If dark matter consists of these particles whose mass is, say, $m \sim 10^{-24}eV$ and dark matter density around us is $\sim 10^{-25}g/cm^3$, then the inter-particle distance is order of $10^{-13}cm$, while the compton wave length is of 10pc order. Hence there is no point talking about the individual particle’s position and momentum. The dark matter is more like a wave than a particle, and the galactic halos are giant systems of condensed boson liquid. Here we are in the region reigned by quantum mechanics. In this context, we will consider the dark matter distribution quantum mechanically and this is the purpose of our paper. Our result will show mass profile $\rho \sim r^{-1.6}$ leading to slightly increasing rotation curve. Further more due to the system’s wave nature, rotation curve has the ripple-like fine structure. By the uncertainty principle, our theory predicts that the rotation velocity times the ripple’s wave length is galaxy independent.

2. Quantum considerations of dark matter — We consider the spherically symmetric case for simplicity and also assume that there are no inter-particle interactions apart from gravity. The classical energy of a particle moving inside a potential generated by other particles is

$$E = \frac{1}{2}mv^2 + \int_0^r dr' \frac{GmM(r')}{r'^2}, \tag{1}$$

where $M(r) = \int_0^r dr' 4\pi r'^2 \rho(r')$. Quantum mechanically, we are interested in a wave equation which governs the collective behavior of highly correlated bosonic particles. In principle, one has to work out Hartree’s self consistent procedure, but here the fact that the system is a highly correlated system of bosons simplifies the life. Assuming that all or significant fraction of particles are in the same state, we take the condensation wave function $\psi$ [11] as the Hartree’s self-consistent field. Then the density of particles is proportional to $|\psi|^2$ and we obtain a wave equation given by

$$i\hbar\partial_t \psi = -\frac{\hbar^2}{2m} \Delta \psi + GmM0 \int_0^r dr' \int_0^{r'} dr'' 4\pi r''^2 |\psi|^2 \cdot \psi(r) \tag{2}$$

Here $M0$ in $M = M0 \int dr 4\pi r^2 |\psi|^2$ is a mass parameter introduced for dimensional reasons. The equation is non-linear, hence we can not superpose two solutions, nor do we have freedom to normalize it. However, as we shall show later, this equation has a scaling symmetry which allows us to resolve the ambiguity due to the choice of $M0$. That is, the physical quantity $M$ does not depend on the choice of $M0$, which is a nontrivial property.

Another remark is on the non-relativistic treatment. (From now on we set $\hbar = c = 1$ except for emphasis.) One might wonder that when the mass of the particle is so small, whether a non-relativistic treatment is correct. The justification is two-fold. The basic reasons are (1) the virial theorem tells us $v^2 = GM/r$ is independent of individual particle mass $m$ and (2) the total mass of the system is small enough as we shall show below. When the effective coupling constant $e^2 := GmM \sim 1$ or equivalently when $M \sim 1/Gm$, the gravitational Bohr radius $r_0 \sim 1/me^2$ is equal to the compton wave length $1/m$,
and the size of the system is equal to the size of a particle. This means that particles lie on top of one another, which is nothing but a black hole system. In fact, the Schwarzschild radius is $2GM \sim 1/m$, confirming our picture. When this happens, the non-relativistic treatment breaks down. In our case, however, $1/Gm \sim 10^{-14} M_\odot$ while $M \sim 10^{12} M_\odot$ so that $v \sim c^2 \sim 10^{-2}$. Hence the non-relativistic treatment is well justified.

We are interested in a stationary solution $\psi(r,t) = e^{-iEt/\hbar}\psi(r)$. After scaling

$$r = r_0 \hat{r}, \quad \psi = r_0^{-3/2} \hat{\psi}, \quad E = \frac{\hbar^2 \epsilon}{2m},$$

where $r_0 = \hbar^2/2GM_0m^2$, we can rewrite (2) in terms of the radial wave function $u(r)$,

$$u'' + (\epsilon - \int_0^{\hat{r}} d\hat{r}' \frac{1}{\sqrt{\hat{r}^2}} \int_0^{\hat{r}'} d\hat{r}'' u^2(\hat{r}''))u(\hat{r}) = 0,$$

where $\hat{\psi}(\hat{r}) = \frac{1}{\sqrt{4\epsilon}} \frac{u(\hat{r})}{\hat{r}}$. We have solved this equation numerically. Figures 1(a)-(b) are the stationary solutions and corresponding velocity curves $\hat{v}(r) = v(r)/v_0 = (\int_0^{\hat{r}} u^2 d\hat{r}/\hat{r})^{1/2}$ where $v_0 = \sqrt{GM_0}/r_0$.

Apart from mass profile and ripple’s existence, our theory has one quantitative prediction which could prove or disprove it with more precise measurements of rotation curves. Rotation velocity times the ripple wave length is largely Galaxy independent.

This is nothing but re-phrasing the uncertainty principle. More precisely,

$$v_{rotation}\lambda_{ripple} = \frac{1}{\sqrt{2}} \frac{1}{m^2} f(\hat{r}),$$

where $f(\hat{r})$ is a slowly increasing dimension-less function given by

$$f(\hat{r}) = \left[ \frac{\int_0^{\hat{r}} d\hat{r} u^2(\hat{r})}{\sqrt{\hat{r}(V(\hat{r}) - V(\hat{r}))}} \right],$$

where $V(\hat{r}) = V(r)/m \lambda_0^2$ and $r_1$ is $r_t$ is the effective classical turning point. If we further assume that the potential changes very slowly as we change $n >> 1$, we get the eigenvalue distribution $\epsilon_n \sim \epsilon_0 + (n + \beta)^{2n/(2+\alpha)}$ for $V \sim r^\alpha$. Numerical study shows that without including visible matter, $\alpha = 0.44$ for $n=6$, and 0.45 for $n=5$. When we compare with data we have to include visible matter and this leads to $\alpha \sim 0.39$. See figure 3(c).

We are interested in a higher state with nodes rather than the ground state, because there is no way for the system to lose its energy to relax to its ground state, since there is no dissipation mechanism. Let’s define the node number $n$ as the number of nodes plus 1. For $n$ greater than 4, the rotation curve resembles the observed ones and the characteristic features do not change as we increase the number of nodes. The numerical study show that mass profile $\rho \sim r^{-1.56} r >> 1$ for $n=6$, and the power depends on $n$ very weakly.

We can ‘understand’ (not derive) behaviors qualitatively by simple semi-classical analysis [12]. Since $M(r) \sim rv^2$, flatness of rotation curve is equivalent to $M'(r) \sim u^2$ being constant. Usual WKB method gives

$$u(r) \sim \frac{1}{\sqrt{pr}} \cos(\int_0^r p_dr - C\pi),$$

where $p_r = \sqrt{2m(\epsilon - V(u))}$. Since the potential is increasing very slowly as a function of $r$, so is $1/\sqrt{pr}$, hence it gives a minor modification to the ‘over-all flat’ cosine curve. This explains the overall flat but slowly increasing aspect of the rotation curves. Also notice that the last wave is almost as long as the sum of all the others since $p_r$ near the ‘turning point’ is singular. The ripple like structure is an inevitable phenomena predicted in this approach. It is a direct consequence of using the wave theory under the one state (or small number of states) dominance assumption.

Next we observe that as we vary the initial condition $\hat{\psi}_0$, the scale free quantities $\epsilon$ and $\hat{M} := \int u^2 dr$ have a simple dependence on it. Namely

$$\hat{M} = 20.3068 \sqrt{\epsilon}, \quad \epsilon = 1.4900\hat{\psi}_0 \quad \text{for} \quad n = 5$$

The coefficients depend on $n$, e.g., for $n=6$ (5 nodes), they are given by 24.408 and 1.5385 respectively. To understand this we have proved that equation (2) has the following symmetry.

Scaling symmetry : if $\psi(r,\epsilon)$ is a solution, so is $\lambda^2 \psi(\lambda r, \lambda^2 \epsilon)$. The relation (5) is just a corollary of this rather interesting property. Also, by choosing $\lambda$ such that the norm of $\psi$ is equal to one, one can equate $M_0$ and $\hat{M}$. This is not convenient in practical numerical integration, however. We will discuss this property when we discuss the data fitting.

3. Comparison with observations — We now compare the rotation curves predicted in our theory and the actual observations. First we have to include the visible matter’s effect. See figure 3(a),(b). Equation (2) has to be modified to be

$$i\partial_t \psi = -\frac{1}{2m} \Delta \psi + Gm \int_0^r \frac{dr'}{r'^2} \int_0^{r'} \frac{d\psi}{\sqrt{2\pi}} 4\pi (M_0|\psi|^2 + \rho_{vis}) \cdot \psi$$

Figure 2 (a),(b)

Figure 3 (a),(b)
We choose the effective density of visible matter as the Plummer’s potential for the bulge and a Yukawa type exponential decay for the disk. By including the visible matter’s contribution, the rotation curve becomes more flat, but the wavelength of the ripple does not change.

Figure 3 (a),(b),(c),(d)

The ripple structure is manifest for relatively few galaxies. The data of Rubin et.al showed that in any galaxy there are fluctuations that could indicate the fine structure, but most of the data is plagued with error bars that wash out the signal of the fine structure. The data is good enough for the rotation velocity’s magnitude but too crude for the fine structure. Among 63 galaxies listed in [10], there were about five galaxies which have relatively small error bars and show ripples. These are NGC2998, NGC1357, NGC4594, NGC1620, NGC801. Galaxy NGC2998’s data is particularly interesting for us and we plot the data and the theoretical curve in Figure 3(d). There are four ripples in the NGC2998’s data but we suspect that out side the measured region there can still be density peaks and nodes that could lead to further ripples. However as n increases, the total galaxy mass increases; and for large enough n, Ω exceeds 1, violating bias of present theoretical community. For example, if n is larger than 7, the total mass of the halo is bigger than $10^{13}M_\odot$. Hence there is some restriction on n. Here we take the minimal choice n=5 (four nodes).

By measuring the ripple wavelength and the rotation velocity, we can deduce the total mass M of the Galaxy and the constituent particle mass m. For convenience we choose $\hat{\psi}_0 = 0.2$. Comparing the velocity and wavelength of, say, the second ripple, we have two relations,

$$7r_0 = 8\text{kpc}, \text{ and } 0.32v_0 = 204\text{km/sec}. \quad (10)$$

Solving these two, we get

$$m = 3.3 \times 10^{-23} eV, \text{ and } M_0 = 0.69 \times 10^{12} M_\odot. \quad (11)$$

Using the numerically calculated value $\hat{M} = 8.73575$, 

$$M = \hat{M} \cdot M_0 = 5.9 \times 10^{12} M_\odot \quad (12)$$

The central density of dark matter is given by

$$\rho_c = 1/4\pi (\hat{\psi}_0^2 (M_0/r_0^3) = 1.08 \times 10^{-22} g/cm^3. \quad (13)$$

Therefore we can weigh the Galaxy by looking at the rotation curve only! In the model adopted, the visible matter’s total mass is $2.7 \times 10^{11} M_\odot$. For other galaxies, by similar analysis the prediction of eq. (8) is roughly satisfied. More massive galaxies have larger rotational velocity hence shorter ripple wavelength. For future data analysis we give the general formula for $m, M$ with n=5.

$$m = \frac{8\text{kpc} \cdot 204\text{km/sec}}{\lambda} v \cdot 2.7 \times 10^{-23} eV \quad (14)$$

Now we show that above quantities are independent of the choice of the initial condition $\hat{\psi}_0$ or equivalently independent of the choice of $M_0$. Recall that under the scaling, $\hat{\psi} \rightarrow \lambda^2 \hat{\psi}, \hat{r} \rightarrow \hat{r}/\lambda$ and $\epsilon \rightarrow \lambda^2 \epsilon$ must follow. Therefore had we chosen $\lambda^2 \hat{\psi}_0$ instead of $\hat{\psi}_0$ above, we would get $7\text{kpc}/\lambda = 8\text{km/sec}$ and $0.32v_0\lambda = 200\text{km/sec}$, so that we get the same $m$ and $1/\lambda$ times $M_0$ above. On the other hand $M = \int d^3\hat{r} |\hat{\psi}|^2$ must scale by $\lambda^4/\lambda^3$, therefore $M = \hat{M} \cdot M_0$ is invariant under the scaling. The same argument shows $\rho_c$ is scale free. hence we can choose large $\hat{\psi}_0$ to save computing time. Lastly, had we chosen n larger than 7, total mass for halo is larger than $10^{13} M_\odot$.

4. Conclusion — In this paper we investigated the consequence of abundant existence of Nambu-goldstone bosons appearing in late time phase transition. Since the particle wavelength is far larger than the inter-particle distances, galactic halos made of this species are highly correlated Bose liquid. We set up a version of non-linear Shrödinger equation describing the collective behavior of the system. Three predictions are made; (1)mass profile $\rho \sim r^{-1.6}$, (2)there are ripple-like fine structures in rotation curve, (3) rotation velocity times ripple’s wave length is largely galaxy independent. We compare the rotation curves predicted by our theory with the data observed. We fine-tuned the mass of the particle to fit the data for halo and determine the total mass of the galaxy NGC2998. Among 62 rotation curve data of Rubin et.al., there is a one curve where all the details of the predictions are realized, but most of other data is not accurate enough to definitely confirm the the theory. We suggest that more precise measurement of rotation curve is highly worth-while to be done. If future observation shows the ripple structures in many galaxies in the way predicted in this paper, it would be a strong indication that late-time phase transition is real.

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