Measuring the speed of gravity with laser Doppler vibrometers and silicon mechanical resonators

I Cagnani
EPS Gravitational Physics Division, 6 rue des Frères Lumière, 68200 Mulhouse, France

ivan.cagnani@gmail.com

Abstract. The old question of the speed of propagation of gravitational interactions still needs a laboratory answer. An experimental setup for direct measurement of the propagation delay in the gravitational interaction between resonating gravitationally coupled oscillators is proposed. The oscillators are designed to act like transmitter and receiver in an AM radio communication. The signal, a dynamic Newtonian field, is generated by the precise vibrations of a massive tungsten disc, in resonance with its suspension system like a trampoline oscillator. The disc is driven by a shaker locked to a GPSDO frequency reference, while its movement is tracked with a laser Doppler vibrometer and a lock-in amplifier, locked to the same frequency reference. The gravitational signal is received by an antenna, a gold-coated silicon oscillator with resonance frequency of 2 kHz and a Q of 200 000 in thermal vacuum. The vibrations of the antenna are tracked by a second laser Doppler vibrometer with a noise level of 30 fm/√Hz and a lock-in amplifier using the output of the first lock-in amplifier as reference. High-vacuum chambers and multi-stage vibration and acoustic isolation systems are required for both the generator and the antenna. Cross-correlating the data of the two oscillators, their phase difference is then measured and, using the frequency of the signal and the distance between oscillators, the speed of gravity will be calculated. Numerous measurements at various distances spanning a 300 mm range are collected and used for the determination of other parameters, including G.

1. Introduction
The speed of propagation of Newtonian gravitational fields has been questioned by physicists for centuries. Newton’s Universal Gravitation implicitly assumes an instantaneous propagation of the gravitational interaction, and using that assumption astronomers could correctly predict orbits, yet philosophically, the concept of instantaneous action-at-distance was unacceptable to many and Newton himself was notoriously skeptical. Laplace attempted a solution proposing a highly superluminar speed of gravity, but that would imply a faster-than-light transfer of information, which for modern physics is again unacceptable. We now known that stable orbits are like an undamped harmonic oscillator, or particles within a potential well: no transfer of information is needed to keep the periodic motion going steadily. A gravitational perturbation is instead an information, and it cannot propagate faster than light according to General Relativity. This last affirmation can now be tested in a room-size laboratory, using only commercial technology and without even the need for modern physics.
2. Description of the experiment

As illustrated in Figure 1, the periodic displacement of a large tungsten disc, made to oscillate along its axial direction, can generate a gravitational signal able to excite a high-Q mechanical oscillator aligned with the disc. The maximum transmission efficiency of the gravitational information is reached when the frequency of the gravitational signal is exactly equal to the resonance frequency of the antenna. Since the frequency of the gravitational signal must be equal to the frequency of the motion of the object generating the gravitational field, the frequency of the signal generator and of the antenna should be perfectly matched or at least always lie within the bandwidth of the antenna. For a high-Q antenna, such a bandwidth is very restricted.

The antenna selected for the experiment is a Phonon Transfer Resonator [1]. It is made of monocrystalline silicon and characterized by an effective mass of \( m_a = 0.3 \, g \), a resonance frequency of \( f_a = 2.0 \cdot 10^3 \, Hz \) in the highest Q flexural mode in the out of plane direction (aligned with the disc), and a Q factor of \( 2 \cdot 10^5 \).

The resonance bandwidth of the antenna is then

\[
\Delta f_a = \frac{f_a}{Q_a} = 10 \, mHz
\]  

while its damping ratio is

\[
\zeta_a = \frac{1}{2 Q_a} = 2.5 \cdot 10^{-6}
\]  

and its mechanical impedance is

\[
Z_a = \left( \frac{\omega_a^2}{Q_a^2} + \frac{\omega_a^2 - \omega_g^2}{\omega_g^2} \right)^{1/2}
\]  

where of course \( \omega_a = 2 \pi f_a \) and \( \omega_g \) is the angular frequency of the gravitational force \( F_g = \Gamma_g m_a \) driving it. When the gravitational signal driving the antenna matches perfectly the frequency of the antenna, the impedance of the antenna reaches its minimum value

\[
\omega_g = \omega_a \Rightarrow Z_a = \left( \frac{\omega_a^2}{Q_a^2} \right)^{1/2} = 3.95 \cdot 10^{-3} \, \Omega
\]  

and this will result in the maximum amplitude of vibration when stimulated by the gravitational signal

\[
\omega_g = \omega_a \Rightarrow A_a = \frac{\Delta \Gamma_g}{Z_a \omega_g}
\]
where \( \Gamma_g \) is the gravitational field strength of the generator, the intensity of the gravitational signal at the position of the antenna, then at distance \( l_a \) from the geometrical center of the disc, seen as a cylinder, along the direction of its axis. Apart from the distance \( l_a \) between the generator and the antenna, \( \Gamma_g \) depends also on the geometry and on the density \( \rho_d \) of the disc:

\[
\Gamma_g = 2 \pi G \rho_d \left\{ h_d - \left[ (h_d + l_a)^2 + (r_a^2 + r_d^2) \frac{1}{2} \right] \right\}
\]

(6)

where \( h_d \) is the thickness of the disc and \( r_d \) is its radius. The formula is valid only along the axis of the disc and considers the gravitational field as originating at a point in the geometrical center of the disc seen as a cylinder with height \( h_d \) and radius \( r_d \) [2].

The disc of the generator is made of 99.95% pure tungsten, it has a radius \( r_d \) of 175 \( \pm \) 0.1 mm, a height \( h_d \) of 10 \( \pm \) 0.01 mm and polished surfaces. It has a uniform density \( \rho_d \) of 19020 kg/m\(^3\).

Defined the disc, the value of the gravitational field strength \( \Gamma_g \) depends only on the distance \( l_a \), anyway, \( l_a \) is not constant, as the disc vibrates along the same direction. When the disc is closer to the antenna the field will be slightly more intense than when the disc is farther away. What really drives the antenna into vibration is not the field itself, which acts like the carrier of an AM radio signal, but its periodic temporal variation, acting like the envelope of an AM signal.

If we call the amplitude of the oscillation of the disc \( A_d \), we can define \( l_{a\text{max}} = l_a + A_d \) and \( l_{a\text{min}} = l_a - A_d \) the maximum and minimum distance, then we can define \( \Gamma_{g\text{min}} = \Gamma_g(l_{a\text{max}}) \) and \( \Gamma_{g\text{max}} = \Gamma_g(l_{a\text{min}}) \) the minimum and maximum of the intensity of the gravitational signal. To complete Equation 5 we can then define

\[
\Delta \Gamma_g = \Gamma_{g\text{max}} - \Gamma_{g\text{min}}
\]

(7)

For a distance of \( l_a = 250 \text{ mm} \) and an amplitude of \( A_d = 1 \cdot 10^{-5} \text{ m} \) for the oscillations of the disc, Equation 5 leads to an amplitude \( A_d = 3.3 \cdot 10^{-14} \text{ m} \) of the antenna. Being a linear system, increasing the oscillations of the disc to \( A_d = 1 \cdot 10^{-4} \text{ m} \) results in an amplitude of \( A_d = 3.3 \cdot 10^{-13} \text{ m} \) in the driven motion of the antenna, but it is not necessary. An instrument like the OFV-5000 Xtra, a single-point laser Doppler vibrometer by Polytec, can resolve a minimum displacement of \( A_{\text{min}} = 2 \cdot 10^{-14} \text{ m} \) for a bandwidth of \( \Delta f = 1 \cdot 10^{-2} \text{Hz} \) so it can be able to track every oscillation of the antenna. Considering only the intrinsic noise of the LDV, the measurement would have a \( \text{SNR} = 10 \). Of course, the motion of the antenna due to thermal noise far exceeds the steady oscillation due to the driving from the gravitational signal. Because of that, a lock-in amplifier with a large effective dynamic reserve, like model SR860 from SRS, must be used to recover the signal from the overwhelming noise. The reference signal used by the lock-in amplifier of the LDV of the antenna is the output signal of the lock-in amplifier of the LDV of the generator. There is indeed no better reference frequency than the actual movement of the generator [Figure 2].

The outputs of both lock-in amplifiers must then be cross-correlated in data analysis to measure the phase difference of the antenna with respect to the generator. The measured quantity has two components

\[
\psi_{\text{tot}} = \psi_{\text{res}} + \psi_g
\]

(8)

one of them is given by theory: the phase of a harmonic resonator at resonance is always \( \psi_{\text{res}} = \frac{\pi}{2} \), lagging 90° from the driving force, the other component is the aim of the quest: if gravity propagated at infinite speed, \( \psi_g \) would be zero independently of the distance \( l_a \) or the frequency \( \omega_g \) of the signal, but assuming a speed of propagation equal to the speed of light \( c \)

\[
v_g = c \Rightarrow \psi_g = \frac{l_a \omega_g}{c}
\]

(9)

In the case of \( l_a = 250 \text{ mm} \), the component \( \psi_g = 1 \cdot 10^{-5} \text{ s} = 6 \cdot 10^{-4} \text{°} \).
Figure 2. General schematics of the experiments, with connections. The DNF gravitational signal is represented in green, the laser of the LDV in red, connections in blue are electrical (analog or digital).

Figure 3. The generator is a tungsten disc connected like a trampoline, with wires, to the wall of its own vacuum chamber. The dimensions of the wires are exaggerated in the drawing for visibility. In 3a, the front view, an optical fibre connects the head of the LDV (not illustrated) to the lens, allowing the measurement of the vibrations of the disc in high vacuum. The lens is held in place by a special vibration damping mount. In 3b, the back view, an electrical cable connects the shaker, in orange, with the its controller external to the chamber (not illustrated). In 3c, the side view, the attachment points of the wires are indicatively represented. The thickness of the disc is exaggerated for visibility.

3. Signal generation and noise management

As illustrated in Figure 2, a GPS-locked and rubidium-locked frequency reference, like the model FS740 by Stanford Research Systems, generates a signal with an Allan Deviation of \(1 \cdot 10^{-14}\) over an integration time of 3.5 days, and a phase noise of -150 dBc/Hz for the same period. The signal is set to the desired frequency of the gravitational signal, after a precise measurement of the selected resonant mode of the antenna in the actual experimental conditions. The instrument allows a frequency resolution of 1 \(\mu\)Hz.

The signal of the frequency generator will be amplified by the controller of the mechanical transducer. The duty of the transducer is sustaining a fixed level of kinetic energy in the highest Q vibration traversal mode of the mechanical system consisting of the tungsten disc, the suspension
wires connecting the disc to the wall of the vacuum chamber [Figure 3] and the transducer itself. The transducer can be a calibration-grade shaker, a high-power vibration speaker, or a device using a different actuation technology, as long as the conversion of the electric signal into mechanical force, imparted toward the center of the disc, can be well controlled in intensity, frequency and phase.

The suspension wires of the disc act like parallel springs in a mass-spring system: their total spring constant is the sum of the individual ones. The total spring constant required is:

\[ k_{\text{tot}} = m d \omega_g^2 = 2.89 \times 10^9 \frac{N}{m} \]  

(10)

The required length of each wire, \( l_w \), is given by \( E \), the Young’s modulus of its material, \( A \), its cross-section area, and \( n \), the number of wires:

\[ l_w = \frac{E A n}{k_{\text{tot}}} \]  

(11)

As illustrated in Figure 3, a total of 24 wires are used for suspending the disc, so \( n = 24 \). Considering it is easier to weld tungsten with tungsten, tungsten wire is the obvious choice. Its Young’s modulus is 411 GPa. For a wire diameter of 1.5 mm, the length of each wire should be

\[ l_w = 7.68 \times 10^{-3} \text{ m} \]  

(12)

The resulting natural frequency of the generator is 2000.008 Hz. Cutting the wires with 0.5 micron of tolerance, the frequency of the generator can be fine-tuned to be precisely at the center of the measured resonance bandwidth of the antenna.

The vacuum chamber of generator is a cylindrical chamber made of copper for emf radiation shielding and thermal conductivity. It has a base hosting two ion pumps and a counterweight under the base. The whole system is suspended over damped springs on top of a table provided with active vibration isolation. Active isolation has the advantage of being able to isolate vibration sources on top of the table in addition to ground vibration. All the surfaces of the generator that can transfer acoustic energy to the surrounding air are covered with layers different highly effective acoustic damping products like Moongel® made by RTOM Corporation and Sorbothane® made by Sorbothane, Inc.

The same approach to acoustic and vibration isolation used for the generator applies to the antenna, with the fundamental difference that the antenna does not generate energy of any kind, but it is extremely receptive to external energy inputs.

The antenna is a silicon oscillator clamped vertically to a pure copper structure specially designed for thermal and vibration management of the highest Q mode of the oscillator. Its high-vacuum
chamber is also made of copper for electromagnetic isolation (alternatively, a chamber of mu-metal can be used). An optical flange allows the connection of the optical fibre of the LDV, while a lens holder is fixed inside to point the lens of the LDV to one of the two pads of the silicon oscillator. The chamber is also surrounded by layers of different acoustic isolation materials like Moongel® and Sorbothane®.

The vacuum chamber is bolted on top of its ion pump and both are displaced by a motorized delay line used as a linear stage positioner like model 8MT160-300 by Standa Ltd. The main characteristic of the positioner must be the straightness, and the rigidity and stability when not in motion. The delay line is bolted to a desktop active vibration isolator like model 1TS-140 by Standa Ltd. placed on top an optical table with active vibration isolation like model T1020CK by Thorlabs, Inc.

On top of the table of the antenna, several sensors are also hosted to monitor the noise levels along with the controllers for the linear positioner, the controller of the ion pump, the controller of the LDV, the lock-in amplifier of the antenna and a DAQ with SSD storage.

The need for vibration and acoustic isolation can be easily quantified: as specified before, an oscillation of $A_d = 1 \times 10^{-5} \text{ m}$ in the generator corresponds to an oscillation of $A_a = 3.3 \times 10^{-14} \text{ m}$ of the antenna, then the gravitational transmission has an attenuation of $\text{Loss}_g = -170 \text{ dBm}$ (where m indicates metres not millivolts). If we desire a SNR=10 the loss of the vibrational and acoustic counterparts of the gravitational transmission must be $\text{Loss}_v = -190 \text{ dBm}$ and $\text{Loss}_s = -190 \text{ dBm}$. To realize this, it is necessary to use a system of multiple isolation layers. For example, to minimize the transmission of vibrations, it is possible to apply at least four active mechanical filters providing each a $\text{Loss}_{\text{active}} = -40 \text{ dBm}$ and rely on passive filters, including springs, dampers and natural loss in the ground propagation between tables, for the remaining attenuation $\text{Loss}_{\text{passive}} = -30 \text{ dBm}$, which is within reason. In practice, both the generator and the antenna must be placed on top their own desktop active vibration isolator and each active isolator must be placed on top its own optical table with active isolation.

The required acoustic isolation of the antenna implicates a similar split of the attenuation budget between the generator and the antenna. Acoustic absorption and deflection technologies are very
mature anyway, and high frequency sound, like high frequency vibration, is easier to attenuate, so in this case the only minor challenge is how to keep the layers of sound damping materials from becoming unacceptably thick. The solution anyway is focusing the damping on the required frequency, even using, if necessary, materials with a non-linear response at 2000 Hz.

The procedures for calibration, data taking, data analysis and the determination of gravitational parameters including the value of the universal gravitational constant will be soon the subject of a further publication by the author, while a third paper could be focused on results of numerical multiphysics simulations and preparatory tests.

4. Conclusions
The current level of commercial LDV technology has already enabled gravitational measurements that were impossible until recently without designing and assembling custom laser measurement devices. The prospective of an improving dimensional resolution in laser Doppler vibrometry [3] will allow new measurement scenarios in experimental gravity, advancing basic research on fundamental constants and forces.

Acknowledgments
I deeply thank all the collaborators, professors, industry experts and friends who have supported me in this project with technical and scientific advice, review and encouragement.

References
[1] Serra E Bonaldi M Borrielli A Conti L Pandraud G Sarro P M 2014 Selective coating deposition on high-Q single-crystal silicon resonators for the investigation of thermal noise statistical properties Procedia Engineering 87 pp 1485-1488 (10.1016/j.proeng.2014.11.579)
[2] Tatum J B 2018 Classical Mechanics 5 p 13 University of Victoria’s website (Lecture notes http://astrowww.phys.uvic.ca/~tatum/celmechs.html)
[3] Rembe C Kadner L Giesen M 2016 Approaching attometer laser vibrometry Review of Scientific Instruments 87 102503 (https://doi.org/10.1063/1.4964625)
[4] Chen Y T Cook A 1993 Gravitational experiments in the laboratory, Cambridge University Press
[5] Astone P et Al 1998 Experimental study of the dynamic Newtonian field with a cryogenic gravitational wave antenna The European Physical Journal C 5 651-664 (https://doi.org/10.1007/s100529800987)
[6] Ferretti B 2003 On the gravitational field (A suggestion about a possible experimental research) (arXiv:gr-qc/0307030)
[7] Ferretti B 2005 On the gravitational field. A suggestion about a possible experiment (arXiv:gr-qc/0507120)