Time-Dependent Probabilistic Tsunami Inundation Assessment Using Mode Decomposition to Assess Uncertainty for an Earthquake Scenario

Yo Fukutani, Shuji Moriguchi, Kenjiro Terada, and Yu Otake

1 College of Science and Engineering, Kanto Gakuin University, Yokohama, Japan, 2 International Research Institute of Disaster Science, Tohoku University, Sendai, Japan, 3 School of Engineering, Tohoku University, Sendai, Japan

Abstract This study presents an innovative probabilistic tsunami inundation assessment for an earthquake scenario to randomly generate tsunami inundation depth distributions by quantitatively evaluating the spatial correlation of tsunami inundation depths using singular value decomposition (SVD) derived from proper orthogonal decomposition and to evaluate the tsunami inundation depths considering the imminent occurrence of an earthquake. We found a good agreement between the evaluation results of the proposed surrogate model and the numerical results of the nonlinear long wave equations for the tsunami inundation depth distribution in Kamakura city, Japan, due to the Sagami Trough megathrust earthquake. Evaluating the spatial correlation using SVD has the advantage that the covariance matrix does not need to be defined in advance but can be defined from the data itself. We also achieved a significant reduction in the number of required tsunami propagation simulations for the probabilistic assessment and attained higher computational efficiency by extracting spatial correlations with SVD. Furthermore, we conducted a probabilistic tsunami inundation assessment focusing on a relatively short period (i.e., 50 years) considering the time-dependent occurrence probability of the target earthquake. The proposed probabilistic assessment method with mode decomposition is applicable to the general probabilistic tsunami hazard assessment by integrating it with physical stochastic slip models.

Plain Language Summary This study proposes an innovative method to evaluate the probabilistic tsunami inundation area on land considering the imminence of an earthquake occurrence. Generally, the computational cost associated with evaluating the probabilistic tsunami inundation area is high because it is necessary to consider all possible uncertainties. In this study, we apply a singular value decomposition method to a small number of inundation depth distributions to extract the characteristics of these distributions. As a result, we are able to evaluate the probabilistic tsunami inundation depth in a future 50-year period using a small number of inundation depth distributions.

1. Introduction

In the assessment of earthquake-induced tsunami hazards, there are many uncertainties in the tsunami generation, propagation, and runup processes. For example, the location and shape of the seismogenic fault, the starting point of fault rupture, the rise time in the tsunami generation process, the method used to calculate the initial water level, the governing equation of the tsunami, the tide level setting, the seafloor topography in the region of tsunami propagation, the terrestrial topography, the distribution of man-made structures (e.g., buildings and seawalls), and the roughness in the tsunami runup process all affect the final tsunami hazard assessment on land; the impacted factors include the inundation depth and inundation area. In addition, the fault parameters, including the location, width, length, slip, depth, rake, strike, and dip of the fault, all of which determine the location and shape of the seismogenic fault, vary both temporally and spatially, and these variations affect the results of tsunami hazard assessments. The probabilistic tsunami hazard assessment (PTHA) method is used to evaluate such uncertainties in tsunami hazard assessments, and a variety of PTHA approaches have been proposed since the turn of the millennium (e.g., Geist, 2002; Geist & Parsons, 2006; González et al., 2009; Grezio et al., 2017; Mori et al., 2018). PTHA methods that focus on probabilistic tsunami hazards in terms of the earthquake occurrence and coseismic sea floor displacement are specifically called seismic PTHA (SPTHIA) techniques (Lorito et al., 2015).
The main purpose of PTHA is to assess the likelihood that a given measure or metric of a tsunami hazard will be exceeded in a particular area within a given time period. The most basic outcome of such an analysis is typically expressed as a hazard curve, which shows the exceedance level of the hazard metric with the probability and probabilistic tsunami inundation area. To understand the tsunami inundation risk on populated land areas, it is necessary to perform a probabilistic tsunami inundation assessment considering such uncertainties, and PTHA can be used for the basis of engineering design (e.g., Chock, 2016). PTHA can also provide useful information for the formulation of regional community development plans, constructing various infrastructures, and conducting risk assessments in the nuclear, real estate, damage insurance, and reinsurance sectors.

In PTHA, each element of uncertainty in the tsunami hazard assessment is generally evaluated by categorizing it as either an epistemic uncertainty or an aleatory uncertainty. Epistemic uncertainty is associated with incomplete knowledge and data about the earthquake process, whereas the aleatory uncertainty is attributable to the random nature of earthquake occurrence and its effects (Annaka et al., 2007). In particular, many studies have evaluated epistemic uncertainties with logic trees (e.g., Bommer & Scherbaum, 2008; El-Husaini et al., 2018; Fukutani et al., 2015; Geist & Lynett, 2014; Geist & Parsons, 2006; Horspool et al., 2014; Jho et al., 2019; Mulia et al., 2020; Park & Cox, 2016; Priest et al., 2010; Selva et al., 2016; Thio et al., 2010). Specifically, earthquake occurrence scenarios, including their geometries and occurrence rates, are epistemic uncertainties that have the greatest impact on PTHA (Grezio et al., 2017); therefore, it is usually necessary to generate a large number of seismogenic faults to account for these uncertainties.

Unfortunately, the probabilistic tsunami inundation assessment based on a large number of seismogenic faults is usually computationally demanding and difficult because of the need to solve a large number of two-dimensional nonlinear equations. Recently, considerable research has been focused on how to reduce the computational burden of probabilistic tsunami inundation assessments and determine the most efficient method for performing the calculations. A reasonable and efficient probabilistic tsunami inundation assessment methodology will enable probabilistic tsunami inundation hazards over a wide (even global) area to be evaluated. For example, to reduce the computational burden, linear Green's function theory was applied up to the coastal wave height based on a large number of seismogenic faults, and the amplification factor (AF) method was proposed to evaluate the inundation height from the coastal wave height (Lovholt et al., 2015; Lovholt, Glimsdal, et al., 2012). The effectiveness of this method has since been demonstrated by using it to assess the uncertainty of inundation depths and validating the findings against numerical results for coastal areas in Europe (Davies et al., 2017; Glimsdal et al., 2019). The AF method is useful for quickly assessing inundation depths in inland areas where the computational burden is high but suffers from a poor accuracy at some locations because of its simplistic approach to assessing inundation depths by multiplying the coastal wave height by a coefficient. In contrast, Lorito et al. (2015) used a computationally inexpensive linear approximation, the Green's function approach, to construct a preliminary SPTHA for tsunami propagation and then applied a two-stage filtering procedure to select a reduced set of sources and calculate nonlinear probabilistic inundation maps. They proposed a unique and effective method to evaluate the probabilistic tsunami inundation depth. Volpe et al. (2019) proposed a computationally efficient approach to achieve robust, site-specific SPTHA by improving upon the method of Lorito et al. (2015) and Selva et al. (2016); however, they also used a linear equation in the tsunami propagation evaluation, which is inaccurate, and the complexity of the evaluation procedure (such as the need to apply a two-step filter) may make it unsuitable for practical applications. In addition, Rohmer et al. (2018) proposed a Bayesian procedure to infer the probability distribution of the source parameters of an earthquake to overcome the high computation time of the numerical simulator.

Among the parameters of a seismogenic fault, the parameter with the greatest influence on the tsunami inundation depth is the uncertainty of the slip distribution along the seismogenic fault. As a consequence, many methods have recently been proposed to probabilistically generate the slip distribution on the seismogenic fault for use in PTHA. Most existing studies have obtained slip samples by using a Fourier transform with random phases (e.g., Davies et al., 2015; Goda & Song, 2016; Goda et al., 2014; Li et al., 2016; Lovholt, Pedersen, et al., 2012; Mueller et al., 2014; Rohmer et al., 2018). A recent research example is the application of the Karhunen-Loève expansion (KLE) method to the slip distribution on a fault plane (Crempien et al., 2020; LeVeque et al., 2016; Mélgar et al., 2016; Sepúlveda et al., 2017). The KLE approach was initially
This study uses the POD method, which allows the data to be orthogonally decomposed similar to the KLE method and is equivalent to the principal component analysis (PCA) (e.g., Jolliffe, 1986). We further apply the singular value decomposition (SVD) method (e.g., Lanczos, 1961; Stewart, 1993), which was developed from the theory of PCA. However, previous researchers employed the KLE to generate a large number of samples for seismogenic faults. In contrast, this study directly generates a large number of tsunami inundation depth distribution samples by applying SVD to a limited number of tsunami inundation depth distributions obtained by tsunami numerical simulations based on selected seismogenic faults as the input parameters and then evaluating their eigenmodes. This method can significantly reduce the computational burden of tsunami numerical simulations for analyzing tsunami propagation. Tozato et al. (2020) also proposed a tsunami risk assessment method that focuses on the tsunami force acting on buildings under simple conditions by using the mode decomposition method and a surrogate model, but their approach differs from this study in that only two variables were analyzed and the occurrence probability of earthquakes was not considered.

There is a naturally close relationship among the KLE, PCA, and SVD methods (e.g., Kerschen et al., 2005; Liang et al., 2002; Wu et al., 2003), and this relationship will be discussed in detail in Section 2.3. In brief, the KLE and SVD differ in their treatment of the covariance matrix, which represents the spatial correlation of the data. The KLE is applied to the theoretical covariance matrix, whereas SVD is applied to the sample covariance matrix constructed from available vectors. Specifically, in previous cases where the KLE has been applied to the slip distribution of seismogenic faults, the covariance matrix, which represents the correlation of the slip distribution on the fault, was determined based on the correlation length, which was evaluated from the length and width of the fault, and the eigenmodes were identified (LeVeque et al., 2016; Melgar et al., 2016; Sepúlveda et al., 2017). On the other hand, in SVD analysis, the covariance matrix is derived from the target data sample matrix itself; therefore, there is no need to prepare the function in advance. In addition, this study has the advantage of reducing the computational burden by identifying the eigenmodes of the inundation depth distributions, thereby removing the less affected modes.

In PTHA, we need to assign an occurrence probability to a target earthquake. This probability has significant impacts on the tsunami hazard curve and the probabilistic tsunami inundation depth distribution. Two basic approaches are followed to ensure that the occurrence probability is either time-independent or time-dependent. In the former method, the occurrence probability of an earthquake is assumed to follow a Poisson distribution; in the latter method, the intervals between earthquake events have a Brownian passage time (BPT) distribution, which may serve as a temporal model for time-dependent, long-term seismic forecasting (Matthews et al., 2002). The application of the time-dependent model is based on a characteristic earthquake model. In the field of PTHA, some case studies have modeled the occurrence probability of characteristic earthquakes using the BPT distribution, a time-dependent model, instead of the Poisson distribution, a time-independent model (e.g., Bayraktar & Ozer Sozdimler, 2020; Goda, 2019). By using a time-dependent occurrence model, it is possible to generate hazard curves or probabilistic tsunami inundation depth distributions that focus on a medium- to long-term period, such as the next 30 or 50 years, instead of hazard curves averaged over the long term. This means that the effect of the long time lapse since the last earthquake (and hence the imminent threat of the next earthquake) is reflected in the hazard assessment. This is an important concept when we consider the useful lives of buildings and other various structures or the life spans of humans. Therefore, in this study, we assign the BPT distribution for the occurrence probability to analyze the Sagami Trough earthquake, a characteristic event that could have a devastating impact on the Kanto region of Japan. However, we should bear in mind that it is thought that it is difficult to define a “characteristic” earthquake, and we may need to rigorously look at characteristic earthquake models and their implicit assumptions (e.g., Kagan et al., 2012).
It should be noted that in addition to seismic sources, tsunamis can be produced by a variety of other sources, such as landslides, submarine landslides, volcanic eruptions, and meteorites. However, this study focuses only on the probabilistic hazard assessment of tsunamis caused by earthquakes.

2. Methodology

2.1. Research Flow

Figure 1 shows a flowchart of the probabilistic tsunami inundation assessment method used in this study. First, we create multiple seismogenic faults that take into account various uncertainties by changing the parameters of the seismogenic faults used in the tsunami inundation simulations. Then, we calculate the tsunami inundation depth distributions in the target area from the seismogenic fault parameters by using Okada's equation and nonlinear long wave equations (continuity equation and equations of motion). Next,
we create a data matrix \( X \) comprising multiple tsunami inundation depth distributions and decompose it into inundation mode distributions by applying SVD. By linearly combining all inundation mode distributions, we can fully represent the original inundation depth distribution; moreover, by reducing the problem to the appropriate number of dimensions and constructing a surrogate model, we can nearly replicate the original inundation depth distribution and significantly reduce the computational cost. We determine each coefficient of the linear combination using Gaussian process regression so that the values of the coefficients derived from the inundation depths from the original tsunami numerical simulations can be fully reproduced. If we change the coefficients, an inundation depth distribution different from the original distribution can be achieved while maintaining the spatial correlation of the inundation depths. By applying the BPT distribution (a probability distribution of earthquake occurrence) to the large number of generated inundation depth distributions described above, we can perform time-dependent Monte Carlo simulations to evaluate the conditional tsunami hazard curves and probabilistic tsunami inundation depth distributions corresponding to the occurrence of a large earthquake in the Sagami Trough. The numbers within the two lines in Figure 1 represent the following sections, each of which describes the method in detail: Section 2.2 describes the settings of the seismogenic fault and numerical simulation; SVD and the surrogate model are introduced in Section 2.3; Section 2.4 presents Gaussian process regression and the linear combination of modes, and the time-dependent Monte Carlo simulations are portrayed in Section 2.5.

2.2. Tsunami Numerical Simulation

Given the fault parameters of an earthquake that serves as a tsunami source, we evaluate the amount of crustal movement using Okada’s equation (Okada, 1985). Then, we obtain the initial water level of the tsunami by taking the crustal movement as the change in sea level and conduct a tsunami numerical simulation by using the following nonlinear long wave equations (continuity equation and equations of motion, Equations 1–3) (Goto & Ogawa, 1982; UNESCO & IUGG/IOC Time Project, 1997):

\[
\frac{\partial \eta}{\partial t} + \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} = 0
\]  
(1)

\[
\frac{\partial M}{\partial t} + \frac{\partial M}{\partial x} \left( \frac{M^2}{D} \right) + \frac{\partial N}{\partial y} \left( \frac{MN}{D} \right) + gD \frac{\partial \eta}{\partial x} + \frac{gN^2}{D^{\frac{3}{2}}} \sqrt{M^2 + N^2} = 0
\]  
(2)

\[
\frac{\partial N}{\partial t} + \frac{\partial M}{\partial x} \left( \frac{MN}{D} \right) + \frac{\partial N}{\partial y} \left( \frac{N^2}{D} \right) + gD \frac{\partial \eta}{\partial y} + \frac{gM^2N}{D^{\frac{3}{2}}} \sqrt{M^2 + N^2} = 0
\]  
(3)

where \( \eta \) denotes the water level, \( D \) denotes the total water level, \( g \) denotes the acceleration due to gravity, \( n \) denotes the Manning coefficient, and \( M \) and \( N \) denote the fluxes in the \( x \) and \( y \) directions, respectively. We solve the governing equations by the finite-difference method in staggered grids.

2.3. SVD and a Surrogate Model

Before describing the SVD technique used in this study, we present an overview of previous studies that used the KLE and clearly discuss the differences between our approach and previous methods. The KLE (e.g., Karhunen, 1947; Loève, 1977; Schwab & Todor, 2006) is a standard approach for representing a Gaussian random field as a linear combination of the eigenvectors of a presumed covariance matrix \( \hat{C} \). Let a slip vector \( s \) with subdivided \( N \) meshes be a Gaussian random field with a desired mean slip \( \mu \in \mathbb{R}^N \) and a covariance matrix \( \hat{C} \in \mathbb{R}^{N \times N} \) expressed as:

\[
s \sim \mathcal{N} \left( \mu, \hat{C} \right)
\]  
(4)

where \( \mathcal{N} \) indicates a random variable from a multivariate normal distribution. Then, we compute the eigenvalues \( \lambda_k \) of the covariance matrix \( \hat{C} \) and corresponding normalized eigenvectors \( \psi_k \) so that the matrix of
eigenvectors $V$ (with the $k$th column $v_k$) and the diagonal matrix of eigenvalues $\Lambda$ satisfy $\hat{C} = V \Lambda V^T$. Then, the KLE can be written in matrix-vector form as (LeVeque et al., 2016):

$$s = \mu + V\Lambda^{1/2}z$$

(5)

where $z \in \mathbb{R}^N$ is a vector of independent and identically distributed random numbers $\mathcal{N}(0,1)$. In this way, the original slip vector $s$ can be decomposed. Previous studies (e.g., LeVeque et al., 2016; Melgar et al., 2016) that used the KLE first chose the desired mean $\mu$ and covariance matrix $\hat{C}$. They computed the pairwise distance between subfaults $i$ and $j$ to define the correlation matrix as follows:

$$\hat{C} = C_{ij} = \exp\left(-\left(d_{\text{strike}}(i,j)/r_{\text{strike}}\right) - \left(d_{\text{dip}}(i,j)/r_{\text{dip}}\right)\right)$$

(6)

where $d_{\text{strike}}$ and $d_{\text{dip}}$ are estimates of the distances between subfaults $i$ and $j$ in the strike and dip directions, respectively, and $r_{\text{strike}}$ and $r_{\text{dip}}$ are the correlation lengths in the corresponding directions. The correlation lengths are generally defined by the fault length and width; in this way, the correlation lengths are forced to approximate the correlation matrix before applying the KLE.

SVD is a mathematical method similar to the KLE and thus is also used in matrix decomposition. Given an $N \times M$ matrix $X \in \mathbb{R}^{N \times M}$ of rank $r \leq \min(N, M)$, the SVD theorem (e.g., Lanczos, 1961) states that orthogonal matrices $U \in \mathbb{R}^{N \times N}$ and $V \in \mathbb{R}^{M \times M}$ exist such that $X$ is factored in the following form:

$$X = U\Sigma V^T$$

(7)

If the rank of $X$ is $k = \min(m,n)$, the diagonal matrix $\Sigma(m \times n)$ has $k$ nonnegative diagonal elements arranged in a descending order, $\Sigma = \text{diag}\left(\sqrt{\lambda_1}, \sqrt{\lambda_2}, \ldots, \sqrt{\lambda_k}\right)$. These elements of the pseudodiagonal matrix are the singular values of the matrix $X$. The SVD computation can be carried out by first forming $XX^T$ or $X^TX$ and then performing an eigenanalysis by finding all the positive eigenvalues and eigenvectors. Therefore, SVD is, in principle, equivalent to PCA. While the KLE is applied to the sample covariance matrix $\hat{C}$ constructed on a theoretical basis, the SVD is applied to this function constructed from the data matrix $X$ itself. Hence, SVD may be considered even more nonparametric than the KLE since the former does not rely on any a priori assumptions of the statistical properties of the target. Consequently, SVD is convenient since we do not need the second-order statistics as we do for the KLE.

Although previous studies that used the KLE and the present study using SVD are similar in terms of the mode decomposition method, the subjects of the method are completely different. Previous studies applied the KLE to the slip distribution on the seismogenic fault plane, but our study applies SVD to the tsunami inundation depth distributions simulated by nonlinear long wave equations with multiple parameters of the seismogenic fault, which exhibit a heterogeneous slip distribution. Moreover, instead of generating a large number of seismogenic faults, we apply SVD to the distribution of tsunami inundation depths calculated from a limited number of seismogenic faults; this approach can significantly reduce the computational burden of PTHA.

The specific application of SVD to the tsunami inundation depth distribution is as follows. First, the inundation depth matrix at each mesh point $j$ calculated by the nonlinear long wave equations (Equations 1–3) is defined as $x = (x_1, \ldots, x_j)^T$. The following inundation data matrix $X$ is generated with the number of analysis cases $j$ considering the uncertainty of the seismogenic fault and the inundation depth value for each analysis case $j$ as $x_j$:

$$X = \begin{pmatrix} x_1 & \cdots & x_j \\ \vdots & \ddots & \vdots \\ x_j & \cdots & x_j \end{pmatrix} = \begin{pmatrix} x_{11} & \cdots & x_{1j} \\ \vdots & \ddots & \vdots \\ x_{ij} & \cdots & x_{ij} \end{pmatrix}$$

(8)
where \( X \) is an \( i \times j \) matrix and is often a nonsquare matrix. We apply SVD to this data matrix \( X \):

\[
X = U \Sigma V^T = \begin{pmatrix}
1 & \ldots & 1 \\
\vdots & \ddots & \vdots \\
1 & \ldots & 1
\end{pmatrix}
\begin{pmatrix}
\lambda_1 & \ldots & 0 \\
\vdots & \ddots & \vdots \\
0 & \ldots & \lambda_j
\end{pmatrix}
\begin{pmatrix}
v_1 \\
v_j
\end{pmatrix}^T
\]

(9)

where \( U \) is an \( i \times j \) orthonormal matrix containing the left-singular vectors \( u_j \), \( \Sigma \) is an \( j \times j \) pseudodiagonal and semipositive definite matrix with diagonal entries containing the singular values \( \lambda_j \), and \( V \) is an \( j \times j \) orthonormal matrix containing the right-singular vectors \( v_j \). In relation to the covariance matrix of \( X \), the information about the spatial correlation between the meshes is aggregated into a left-singular vector \( u_j \) and a singular value \( \lambda_j \). The left-singular vectors \( u_j \) refer to the eigenmodes \( j \) of the tsunami inundation depth distributions. The contribution rate \( c_j \) for a particular mode \( j \) using the singular value \( \lambda_j \) for each mode \( j \) is calculated as follows:

\[
c_j = \frac{\lambda_j}{\sum_{k=1}^{n} \lambda_k}
\]

(10)

where \( n \) is the number of analysis cases. From Equation 9, the column vector \( x_j \) of inundation depths for a case \( j \) can be transformed as follows:

\[
x_j = \sum_{k=1}^{N} u_k \left( \lambda_k v_{jk} \right) = \sum_{k=1}^{N} \left( \lambda_k v_{jk} \right) u_k = \sum_{k=1}^{N} \left( \sigma_{jk} \right) u_k
\]

(11)

where \( N \) is the number of all modes and \( \sigma_{jk} \) is expressed as the coefficient of the \( j \)th case for mode \( k \) multiplied by the singular value and the right-singular vector. Equation 11 demonstrates that the column vector \( x_j \) of the inundation depth can be represented as a linear sum of each mode value. Considering the influence of each mode represented by the singular value \( \lambda \), we can generate a surrogate model with reduced dimensionality by removing only the low-impact modes based on Equation 10. By generating the surrogate model as a linear sum of the high-impact modes from 1 to \( R \), we can obtain the following equation:

\[
x_j = \sum_{k=1}^{R} \left( \sigma_{jk} \right) u_k
\]

(12)

We can randomly generate a number of inundation depth distributions considering the probability distribution of the input parameters by expressing the coefficient \( \sigma_{jk} \) of the column vector \( u_k \) up to mode \( R \) as a response to the input parameters. The coefficient \( \sigma_{jk} \) estimates its distribution by a Bayesian estimation technique using a Gaussian process, as explained in the next section.

### 2.4. Gaussian Process Regression

A Gaussian process is a collection of random variables such that the joint distribution of every finite subset of random variables is multivariate normal. Let \( f(x) = f \) be the function of any \( n \) points \( x = (x_1, \ldots, x_n) \); then, a Gaussian process is completely defined by its mean function \( m(x) \) and covariance function \( k(x, x_*) \) for a real process \( f \) that is multivariate (Rasmussen & Williams, 2006):

\[
p(f \mid x) = \mathcal{N}(f \mid m(x), k(x, x_*))
\]

(13)

\[
m(x) = \mathbb{E}[f(x)]
\]

(14)

\[
k(x, x_*) = \text{cov}(f(x), f(x_*)) = \mathbb{E}[(f(x) - m(x))(f(x_*) - m(x_*))]
\]

(15)
where $p(\cdot)$ is a probability density function. We apply a prior distribution to the mean function $m(x)$ and the covariance function $k(x, x)$ and evaluate its posterior predictive distribution by Bayesian estimation. In this study, we estimate the mean function $m(x)$ of the coefficient $\alpha_{jk}$ corresponding to each mode.

### 2.4.1. Prior Distribution

Usually, we take the mean function to be zero due to a lack of prior knowledge. The covariance function $k(x, x)$, also known as the kernel function, has been proposed in many different ways depending on the application, but in this study, we utilize the commonly used Gaussian kernel represented by Equation 16.

The covariance function specifies the covariance between pairs of random variables:

$$k(x, x) = \exp\left(-\beta (x - x)^T (x - x)\right)$$

The covariance is almost unity between variables whose corresponding inputs are very close and decreases as their distance in the input space increases, $\beta$ is the characteristic length-scale parameter of the Gaussian process and defines the smoothness of the function; in this study, we adopt $\beta = 1.0$. For more discussion about how such factors affect the prediction, the reader is referred to Rasmussen and Williams (2006). Then, we can generate a random Gaussian vector with the mean and the covariance function as follows:

$$p(f | x) = \mathcal{N}(f | 0, k(x, x))$$

Let $f(x) = \mathbf{f}$ be the function of $\alpha_{jk}$ corresponding to each mode obtained from SVD as the training points $x = (x_1, \ldots, x_j)$ for case $j$ of the tsunami inundation analysis, and let $f(x_*) = \mathbf{f}_*$ be the prediction function of $\alpha_{jk}$ for any $n$ test points $x_* = (x_1^*, \ldots, x_n^*)$. Then, the joint prior distribution of $f$ and $f_*$ due to the Gaussian process prior is also multivariate normal:

$$p(\mathbf{f}, \mathbf{f}_* | x, x_*) = \mathcal{N}\left(0, \begin{bmatrix} K(x, x) & K(x, x_*) \\ K(x_*, x) & K(x_*, x_*) \end{bmatrix}\right)$$

where $K(x, x)$ is a matrix with $m$ and $n$ components of the Gaussian kernel $k(x_m, x_n)$. It should be noted that we perform regression analysis by using a noise-free Gaussian process so that the estimates pass through the value of $\alpha_{jk}$ obtained by SVD at the data point during the tsunami inundation analysis.

### 2.4.2. Posterior Predictive Distribution

Using Bayesian inference (see Appendix A for more details), the posterior predictive distribution $f_*, f | \mathbf{f}$ that follows the prediction $f_*$ given the training function $\mathbf{f}$ is:

$$p(f_* | f, X_*, X) = \mathcal{N}(f_* | m_*, V_*)$$

$$m_* = K(x_*, x)^T K(x, x)^{-1} f$$

$$V_* = K(x_*, x_*) - K(x_*, x)^T K(x, x)^{-1} K(x, x_*)$$

where $m_*$ is the expected value and $V_*$ is the covariance function of the posterior distribution. We can generate the distribution values $f_*$ from the joint posterior distribution by evaluating the mean and covariance function from Equations 19–21 and generate the samples accordingly. The details of the derivation of Gaussian process regression have been provided in many past studies (e.g., Kuss, 2006; Rasmussen & Williams, 2006).

In this study, we let $f$ be the functional value of $\alpha_{jk}$ corresponding to each mode. Given the fault parameters representing the uncertainty of the seismogenic fault by a Monte Carlo simulation and finding the value of $m_*$ corresponding to its random value from the obtained function, we can evaluate the value of $\alpha_{jk}$ corresponding to each fault parameter. Using this $\alpha_{jk}$, we can randomly generate the tsunami inundation depth distribution by linearly combining each mode value.
2.5. Time-Dependent Occurrence Probability Model

To evaluate the tsunami hazard curves and probabilistic tsunami inundation depth distributions due to earthquake-induced tsunamis in a given area, it is necessary to consider the probability of each target earthquake. As explained in the Introduction, several methods have been proposed to evaluate the earthquake occurrence probability. The Sagami Trough earthquake, the target of this study, has been considered a characteristic earthquake, and its interval of occurrence has been specified (Earthquake Research Committee, 2014). Therefore, we use the following time-dependent model, the BPT distribution:

\[ P(t) = \frac{\mu}{2\pi\alpha^2 t^3} \exp \left( -\frac{(t - \mu)^2}{4\mu\alpha^2 t} \right) \]  

where \( t \) is the elapsed time since the last earthquake, \( \mu \) and \( \alpha \) are the first- and second-order parameters of the distribution, respectively, and \( \mu \) is defined as the mean interval between active years for the earthquake. By conducting time-dependent Monte Carlo simulations using the occurrence probability model of the target earthquake represented by Equation 22, we can conduct a probabilistic uncertainty analysis in the target region for the next \( \Delta T \) years. In each time-dependent Monte Carlo simulation, we determine whether an earthquake occurs each year over the period spanning \( \Delta T \) years, and we update the BPT distribution every year following Equation 23:

\[ P(t, \Delta T) = \frac{\int_{t}^{t+\Delta T} p(t) dt}{\int_{t}^{t+\infty} p(t) dt} \]  

where \( p(t) \) is the probability density function for the earthquake recurrence interval, \( t \) is the elapsed time since the last major earthquake, and \( \Delta T \) is the exposure period (Erdik et al., 2004). For the specific time-dependent Monte Carlo simulation methodology, see the case analysis in Section 3.4.

3. Application to the Sagami Trough Earthquake in Japan

Here, the theoretical framework in Chapter 2 is applied to the Sagami Trough earthquake, which is assumed to occur in the vicinity of major metropolitan areas in Japan. It should be noted that the purpose of the investigations presented below is not to show the exact tsunami hazard values in the target area but only to highlight the research flow of the proposed methodology as a case study based on the theoretical framework.

The Philippine Sea plate subducts beneath the North American plate along the Sagami Trough, causing the accumulation of strain along the plate boundary and generating great earthquakes. The Sagami megathrust earthquake is assumed to occur along the Sagami Trough (Cabinet Office, 2013; Earthquake Research Committee, 2004, 2014). Figure 2 shows the region where such a megathrust earthquake could occur: the red line indicates the area that could contain the maximum source of the Sagami Trough earthquake, which is feared to have the potential to cause a large tsunami in the near future (Okumura et al., 2017; Satake, 2015; Yamao et al., 2015). The moment magnitude (Mw) of the possible maximum source of the Sagami Trough earthquake is Mw 8.7, which was published by the Cabinet Office (2013). Additionally, the regions ruptured during the 1923 Taisho and 1703 Genroku Kanto earthquakes are shaded in yellow, while the purple area was ruptured only in the 1703 event.

In Section 3.1 of this chapter, we will discuss the earthquake source setting considering various uncertainties. In Section 3.2, we present the numerical setting of the tsunami simulation. Examples of the application of SVD to two- and three-variable cases are provided in Section 3.3. The results are verified by the surrogate
model generated by SVD in Section 3.4, and time-dependent Monte Carlo simulations are conducted in Section 3.5. Finally, the tsunami hazard curves and probabilistic tsunami inundation assessment for Kamakura city, Japan, are presented in Section 3.6.

### 3.1. Earthquake Source Setting

In this study, we consider the uncertainties in the earthquake Mw, fault depth, and asperity location.

We consider the Mw uncertainties with a variation of Mw ± 0.1 in addition to the base Mw. We achieved the variation of Mw ± 0.1 by uniformly modifying the overall slip of the entire fault. Specifically, Mw +0.1 and Mw −0.1 were changed by applying factors of 1.4 and 0.7 to the base case, respectively. We name this factor the slip scale factor.

Next, we explain the uncertainty of the fault depth. Historical records show that earthquakes rarely occur exactly on the actual plate boundary; thus, there is uncertainty about the depth at which an earthquake will occur. Therefore, in addition to the original case, we generated two cases in which the depth was uniformly changed to +2 and −1 km throughout the entire fault. Although this is based on the Earthquake Research Institute (2012), which indicates the depth of the plate boundary of the Sagami Trough and estimates that an earthquake would occur with an error of ∼2 km, this set value should be updated as appropriate in accordance with the latest seismological findings. This point is also discussed later in the section describing the probability distribution setting of fault depths. It is noted that the reason for setting the value of −1 km is that some fault parameters reach the seafloor when set shallower than 1 km.

We changed the asperity location as follows. The Sagami Trough megathrust earthquake encompasses 6,149 subfaults, and we established three levels of slip for each subfault, that is, superslip, large slip, and the back-ground region, to satisfy a moment magnitude of Mw 8.7 for the entire fault according to the formula by the Earthquake Research Committee (2017). The amount of slip in each region was determined by the following procedure. First, we obtained the average amount of slip on the fault \( a_D \) using the seismic moment \( oM \) and the moment magnitude \( wM \) with the following formula proposed by Kanamori (1977):

\[
M_w = \log_{10} \frac{9.1}{1.5} wM - \log_{10} \frac{9.1}{1.5} oM
\]  

(24)

\[
D_a = \frac{M_w}{\mu S}
\]  

(25)

where \( \mu \) denotes the rigidity of the subduction zone (Pa), \( D_a \) denotes the average slip amount (m), and \( S \) denotes the seismogenic fault area (m²). It is noted that \( \mu \) in Equation 25 represents a different variable from \( \mu \) in Equation 22 that represents the mean recurrence interval of the earthquake. Davies and Griffin (2020) used two rigidity models (constant and depth-varying models) combined with three slip models to investigate the impact of rigidity on the PTHA and concluded that the tsunami hazard off the shore of Australia was insensitive to the chosen rigidity model. Based on their results, we set the rigidity to be constant, as other tsunami hazard studies often assume a constant value (e.g., Butler et al., 2017; Kalligeris et al., 2017). The Japan Society of Civil Engineers (2016) estimated that the rigidity of the plate boundaries surrounding the islands of Japan is \( 3.5 \times 10^{10} \) Pa when the fault depth is shallower than 20 km, \( 6.5 \times 10^{10} \) Pa when the fault depth is deeper than 20 km, and \( 5.0 \times 10^{10} \) Pa when the fault depth reaches 20 km. Considering the fault depth of the Sagami Trough, we set the rigidity to \( 5.0 \times 10^{10} \) Pa.

Using the fault area, rigidity, and seismic moment of the Sagami Trough earthquake, we calculated the average slip along the entire fault; in addition, we set the superslip area to \( 4D_a \) and the large-slip area to \( 2D_a \). We set the total area of the superslip and large-slip regions to be ∼20% of the total fault area and calculated the seismic moment \( M_s \) of both regions. Then, we estimated the average slip of the background region \( D_b \) in conjunction with the area of the background region \( S_b \):

\[
D_b = \frac{M_s - M_a}{\mu S_b}
\]  

(26)
As evident above, the background region is the area where the total area of the superslip and large-slip areas are subtracted from the entire fault area.

The Cabinet Office (2013) studied three cases of the Sagami Trough megathrust earthquake for the purpose of developing regional disaster prevention plans and creating tsunami hazard maps, as shown in Figure 3, with different locations of superslip and large-slip areas; these cases are also adopted for this study. The three cases focus on the western, central, and eastern parts of the Sagami Trough megathrust earthquake. Taking the lessons learned from the occurrence of the 2011 Tohoku earthquake, superslip and large-slip areas were placed in the shallow part of the seismogenic fault (Kodaira et al., 2012; Satake et al., 2013; Sun et al., 2017). The superslip area was placed at depths of 0–10 km, while the large-slip area was located in the shallow region at depths of 0–20 km. The tsunami generated by the superslip area in the case focusing on the western part of the fault would cause extensive damage to Kamakura city, which is the target of the assessment in this study.
study due to its location on the fault. In contrast, the tsunamis generated by the superslip areas in the cases focused on the central and eastern parts of the fault would affect the Pacific coastline of Chiba Prefecture (east of Sagami Bay and Tokyo Bay), whereas we assume these tsunamis would have little impact on Kamakura city. Table 1 shows the average slip, large slip, superslip, and background slip of each case on the seismogenic fault. The moment magnitude for each case is Mw 8.7.

The total number of cases of constructed faults for the Sagami Trough megathrust earthquake is 27, including three cases for earthquake Mw, three cases for fault depth, and three cases for asperity location. The other fault parameters (i.e., dip, rake, and strike) were set according to the sources based on information published by the Cabinet Office (2013) in Japan created from the crustal structure of the plate boundaries.

It should be noted that since the SVD technique used in this study was applied to the tsunami inundation depth distribution, there is no restriction on the method for setting the slip distribution on seismogenic faults. In this study, we used the data published by the Cabinet Office (2013), so we set only three levels of the slip distribution; nevertheless, for example, we could have used samples from seismogenic faults generated by random generation models.

### 3.2. Tsunami Numerical Simulation

Table 2 shows the numerical tsunami conditions for the case analysis targeting Kamakura city in Japan. Using the initial water level as an input value evaluated using the theory of Okada (1985) (see Figure S1), we performed tsunami numerical simulations via the continuity equation (Equation 1) and nonlinear shallow water equations (Equations 2 and 3) with a time interval of 0.6 s and grid spacings of 270–90–30–10 m (see Figure S2). We set up the spatial grids and the time interval to satisfy the Courant-Friedrichs-Lewy condition. The tide level in the coastal area of Kamakura city fluctuates in the range of T.P. +0.85 m to −0.65 m based on Kanagawa Prefecture (2015). T.P. is the abbreviation of Tokyo Peil, which is the mean sea level in Tokyo Bay, and T.P. +0.85 m is the mean monthly highest water level along Sagami Bay. Tidal conditions may also affect the tsunami inundation depth (Adams et al., 2015; Mofjeld et al., 2007), but in this study, we set a single tidal level. Since tsunami wave heights in excess of 15 m have been calculated in coastal areas due to variations in the asperity position of earthquake faults, we determined that the range of tide levels is relatively small when considering tsunami inundation assessments on land. Therefore, we set the constant tide level of T.P. +0.85 m for simplicity, which is the mean monthly highest water level, based on the concept of assuming the dangerous side when assessing the tsunami risk. We used the topographic and Manning’s coefficient data published

| Asperity location | (a) West | (b) Center | (c) East |
|-------------------|---------|-----------|---------|
| Moment magnitude Mw | 8.7     |           |         |
| Average slip $D_a$ (m) | 9.63    | 9.49      | 9.42    |
| Large slip $2D_a$ (m) | 19.27   | 18.99     | 18.83   |
| Superslip $4D_a$ (m) | 38.53   | 37.98     | 37.67   |
| Background slip (m) | 4.69    | 4.73      | 4.75    |

Table 1

| Calculation condition | Calculation condition |
|-----------------------|-----------------------|
| Governing equation    | 2D nonlinear shallow water equations (Tohoku University TUNAMI model) |
| Numerical integration method | Staggered leap-frog finite-difference method |
| Initial condition     | Initial water level calculated from the fault parameters using the theory of Okada (1985) |
| Boundary condition    | Radiation boundary condition |
| Coordination system   | Cartesian coordinate system |
| Stability criterion   | Courant-Friedrichs-Lewy condition |
| Tidal setting         | T.P. + 0.85 m |
| Mesh size $\Delta X, \Delta Y$ | 270 m (Domain 1) – 90 m (Domain 2) – 30 m (Domain 3) – 10 m (Domain 4) |
| Time step $\Delta T$  | 0.6 s |
| Calculation time      | 3 h |
by the Cabinet Office (2013) in our numerical simulations and did not consider any infrastructure. We performed tsunami numerical simulations for the first 3 h after the earthquake so that the maximum tsunami inundation depth could be properly evaluated while considering the effects of tsunami reflection and amplification.

Figure 4 shows a map focusing on the target area of Kamakura city and the distribution of the maximum tsunami inundation depth in the base scenario of the Sagami Trough seismogenic fault (Figure 3a). A large part of Kamakura city could be inundated by a tsunami, which would cause extensive damage.

Figure 4. Target area (Kamakura city, Kanagawa Prefecture) and distribution of the maximum tsunami inundation depth in the case where the superslip and large-slip areas are located on the western part of the Sagami Trough seismogenic fault (Figure 3a). A large part of Kamakura city could be inundated by a tsunami, which would cause extensive damage.

3.3. Singular Value Decomposition

3.3.1. In the Case of Two Variables

First, we show the results for the case involving two variables: Mw and fault depth. Figure 5 shows the distributions of the maximum tsunami inundation depths for the target area simulated for nine combinations of source parameters with three values of Mw and three fault depths. The south side of the city is Sagami Bay, which faces the Pacific Ocean, and tsunamis strike directly from the bay with high tsunami inundation depths exceeding 10 m in the coastal areas. However, there is a clear trend: the larger Mw is, the larger the tsunami inundation area and depth; moreover, the relationship between the fault depth and the tsunami inundation area is not clear, but the tsunami inundation area tends to be slightly larger as the fault depth increases.

We generated a data matrix $X$ for the nine tsunami inundation depth distributions and decomposed the matrix into singular vectors following Equation 9. Figure 6 shows the spatial distribution of column vectors $u_j$ ($j = 1, \ldots, 9$) comprising the left-singular vector. These column vectors are called modes, and we can identify nine mode distributions corresponding to tsunami inundation depths. Positive values in the figure are shown in red, while negative values are shown in blue (representing a positive correlation of the inundation depth between meshes of the same sign and a negative correlation of the inundation depth between meshes of different signs, respectively). The distribution of the first mode (Mode 1) shows that the overall values are negative and thus positively correlated between the meshes. The distribution of the second mode (Mode 2) shows that the values are positive in coastal areas and negative in inland areas; thus, there is an inverse correlation between the
coastal and inland areas. The distributions from the third mode (Mode 3) onward are complex, with sparsely distributed positive and negative values. Nevertheless, the original inundation depth distribution can be reproduced by multiplying the distribution of each mode by a coefficient and taking a linear sum. Figure 7 shows the contribution rate $c_j$ calculated in Equation 10, where $n$ is the number of analysis cases; thus, $n = 9$. We found that $\sim$79.5% of the total can be represented in the first mode, 96.0% in the first through third modes, 98.5% in the first through fifth modes, and 99.5% in the first through seventh modes.

Next, we estimated the coefficients $\alpha_{jk}$ for the parameters (fault depth and slip scale factor) using the Bayesian estimation method with the Gaussian process described in Section 2.4. Figure 8 shows the coefficients $\alpha_{jk}$ corresponding to each mode estimated by Equations 19–21. The red dots indicate the values of $\alpha_{jk}$ calculated from the tsunami numerical simulation results. The smooth surface estimated using Gaussian process regression passing through the red points is the posterior distribution $\hat{m}_r$. Estimation by noise-free Gaussian process regression shows that the response to the input parameters passes completely through all of the red dots. The $\alpha_{jk}$ values corresponding to the first mode show a clear tendency to decrease as the slip scale factor increases, but the relationship with the fault depth is insensitive. The second modes show a similar tendency, but the coefficients $\alpha_{jk}$ corresponding to each mode after the third mode do not show a clear relationship between the slip scale factor and the fault depth.

Once the coefficients $\alpha_{jk}$ corresponding to each mode are determined, the tsunami inundation depth distribution can be generated randomly using the surrogate model represented by Equation 12.

Initially, we generated the fault depth and slip scale factor by using random numbers and took them as the input parameters. Based on the criteria of Japan Society of Civil Engineers (2016), which states that the
slip amount of a seismogenic fault should include an error of \( \pm 0.1 \) for \( M_w \) when considering previous earthquakes with the same area, we established a lognormal distribution with a mean of 1.0 and a log standard deviation of 0.35 so that \( M_w \) varied by \( \pm 0.1 \). Next, we evaluated the fault depth with a normal distribution (mean \( \pm 0.0 \) km and standard deviation of 2.0 km). The standard deviation was set to 2.0 km in this study based on the report of the Earthquake Research Institute (2012), which reported the depth to the upper boundary of the Philippine Sea plate. The degrees of uncertainty in these parameters should be evaluated in a variable manner depending on the recently acquired knowledge about seismogenic faults and tectonic plates.

We determined the coefficients \( \alpha_{jk} \) using random numbers following the abovementioned probability distribution of the input parameters and randomly generated the inundation depth distribution using the surrogate model represented by Equation 12. The Monte Carlo simulation samples have tsunami inundation depth distributions similar to those obtained in Figure 5 from the numerical simulation using the nonlinear long wave equations (see Figure S3). The verification of these results by the surrogate model will be discussed in detail in the next section. Figure 9a shows the average inundation depth in each mesh evaluated by using 10,000 random numbers, and Figure 9b shows the maximum inundation depth in each mesh sampled by 10,000 simulations. The mean

---

**Figure 6.** Spatial distributions of the column vectors \( \mathbf{u}_j \) \((j = 1, \ldots, 9)\) comprising the left-singular vector in singular value decomposition. Positive values are shown in red, and negative values are shown in blue (representing positive correlations of the inundation depth between meshes of the same sign and a negative correlation of the inundation depth between meshes of different signs). The original inundation depth distribution can be reproduced by multiplying the distribution of each mode by a coefficient and taking a linear sum.

**Figure 7.** Contribution rate and cumulative contribution rate of each mode. We find that \( \sim 79.5\% \) of the total can be represented in the first mode, \( 96.0\% \) in the first through third modes, \( 98.5\% \) in the first through fifth modes, \( 99.5\% \) in the first through seventh modes, and \( 100\% \) in the first through ninth modes (all modes).
values in Figure 9a are similar to the inundation depth distribution for the control case in Figure 5, and the maximum values in Figure 9b are similar to the inundation depth distribution for the Mw 8.8 case in Figure 5, indicating that we were able to reproduce the spatial distribution of tsunami inundation depths produced from the numerical analysis by using the surrogate model.

Figures 10a and 10b show the tsunami exceedance probability curves representing the relationship between the inundation depth and exceedance probability for Point A and Point B, respectively. We compare the results for all modes at Points A and B with those for each mode \(j\) \((j = 1, 2, 3, 5)\). For Point A, the results for the first mode and up to the second mode deviate from the other results, but the results for up to the third mode and fifth mode are similar to those for all modes. On the other hand, for Point B, only the results for the first mode deviate from those for other modes, while the results for up to the second mode and fifth modes are almost the same as the results for all modes. These findings indicate that the number of modes to be linearly combined varies depending on the target points, but the shape of the hazard curve seems to be stable when the cumulative contribution to each mode is \(\sim 95\%\) or more.
3.3.2. In the Case of Three Variables

In this section, we present the results for the case involving three variables: the Mw, fault depth, and asperity location. The numerical simulation results for the nine cases in which the superslip and large-slip areas are located on the western part of the Sagami Trough seismogenic fault are shown in Figure 5. We performed nine additional numerical simulations with varying Mw and fault depth settings for each case of the Sagami Trough earthquake with asperities located in the central and eastern areas. Since we have already presented the insensitivity of the fault depth, in Figure 11, we show only the results of nine tsunami numerical simulations for changes in the Mw and asperity location. According to the asperity locations on the seismogenic fault and their locations in relation to Kamakura city, the inundation depth and inundation depth distribution are both larger in the case where the asperities are placed on the western part, followed by the case where the asperities are located in the center, and the inundation depth and inundation area are smaller in the case where the asperities are placed on the eastern part of the fault.

Likewise, we generated a data matrix $X$ for the 27 cases of the tsunami inundation depth distribution and decomposed the matrix into singular vectors. Figure 12 shows the spatial distribution of column vectors $u_j$ ($j = 1, \ldots, 18$) comprising the left-singular vector. We can identify 27 cases of mode distributions corresponding to the tsunami inundation depth, but only 18 cases are illustrated as an example. As in the case of two variables, the distribution of the first mode (Mode 1) shows that the overall values are negative and thus positively correlated between the meshes, and the distribution of the second mode (Mode 2) shows that the values are positive in coastal areas and negative inland areas; thus, there is an inverse correlation between the coastal and inland areas. However, unlike the case involving two variables, the third mode (Mode 3) clearly shows that the values are negative around coastal areas and positive inland areas, again...
demonstrating that there is an inverse correlation between the coastal and inland areas. The distributions after the fourth mode (Mode 4) are complex, with sparsely distributed positive and negative values.

Figure 13 shows the contribution rate. We find that $\sim 63.2\%$ of the total can be represented in the first mode, $83.9\%$ in the first through third modes, $86.9\%$ in the first through fourth modes, $91.7\%$ in the first through seventh modes, and $96.8\%$ in the first through fourteenth modes.

We estimated the coefficients $\alpha_{jk}$ using the Bayesian estimation method with Gaussian process regression. In this case, however, we cannot display the regression results due to the use of three variables. We generated the fault depth and slip scale factor input parameters in the same way as we did in the case involving two variables. In contrast, for the asperity location, we considered the following values depending on the location of the asperity on the seismogenic fault: $-1.0$ for the asperity in the western part of the fault, $0.0$ for the asperity in the central area, and $+1.0$ for the asperity in the eastern part. Suppose we assume that, the superslip area and the large-slip area are one-dimensionally located only in the shallow part of the fault. In that case, we can represent the variation in the asperity location by generating uniform random numbers from $-1.0$ to $+1.0$. The Monte Carlo simulation samples have tsunami inundation depth distributions similar to those obtained from the results using the nonlinear long wave equation in Figure 11 (see Figure S4). The verification of these results by the surrogate model will be discussed in detail in the next section. Figure 14a shows the average inundation depth, and Figure 14b shows the maximum inundation depth in each mesh sampled by 10,000 simulations. The mean values in Figure 14a are similar to the inundation depth distribution for the cases in Figure 11, where the asperities are located in the central area, and the maximum values in Figure 14b are similar to the inundation depth distribution for the Mw 8.8 case in Figure 5, where the asperities are located in the western part, indicating that we were able to reproduce the spatial distribution of the tsunami inundation depth produced by numerical analysis using the surrogate model.

Figure 11. Maximum tsunami inundation depth distributions from the tsunami numerical simulations for different Mw values (Mw 8.6, 8.7, and 8.8) and asperity locations (western, central, and eastern parts of the Sagami Trough seismogenic fault).
Figure 12. Spatial distributions of the column vectors $u_j$ ($j = 1, \ldots, 18$) comprising the left-singular vector in singular value decomposition. Positive values are shown in red, and negative values are shown in blue (representing a positive correlation of the inundation depth between meshes of the same sign and a negative correlation of the inundation depth between meshes of different signs). The original inundation depth distribution can be reproduced by multiplying the distribution of each mode by a coefficient and taking a linear sum.
Figures 15a and 15b show the tsunami exceedance probability curves for Point A and Point B, respectively. Compared to the case involving two variables, the shapes of the hazard curves are significantly altered, reflecting the effect of changing the spatial distribution of slip. Hence, the variation in the spatial slip distribution is once again found to have a significant impact on the PTHA. We compare the results for all modes at Points A and B with those for each mode \( j \in \{1, 2, 3, 4, 7, 14\} \). For Point A, the results for the first mode and up to the second mode deviate from the other results, but the results for up to the third mode and up to the fourteenth mode are similar to those for all modes. On the other hand, for Point B, the results for up to the second mode and up to the fourteenth mode show a similar trend to those for all the modes; nevertheless, the values fluctuated more in the areas with a low exceedance probability and a high tsunami inundation depth. The shape of the hazard curve is relatively stable when the cumulative contribution to each mode reaches \( \sim 85\% \) for the three variables.

3.4. Verification of the Simulation Results by the Surrogate Model

To verify whether the surrogate model can reproduce the results of the tsunami numerical simulations, we performed additional numerical tsunami simulations with settings other than those of the previous 27 cases and compared the results with those of the surrogate model.

First, we performed numerical simulations for the two cases shown in Table 3. In case 1, we created a seismogenic fault for the purpose of verifying the surrogate model in the case involving two variables: Mw and fault depth. In case 2, we used seismogenic faults to verify the surrogate model in the case involving three variables: the Mw, fault depth, and varying asperity position.

For case 1, we varied the Mw and fault depth to Mw 8.75 and +1.0 km, respectively. Figure 16 shows (a) the numerical simulation result for case 1, (b) the evaluation results obtained by the surrogate model, and (c) the differences between the results yielded by the numerical simulation and surrogate model. The maximum value of the difference within the computational domain is 1.89 m, and the minimum value is \(-1.32\) m. We see that the inundation depth and inundation depth distribution of the surrogate model are effectively representative of the numerical tsunami simulation results, although there are some areas of increasing error in the northern part of the city.

For case 2, we varied the asperity location with the control case of Mw 8.7 and depth \( \pm 0.0 \) km. The asperities were placed between the western and central parts of the Sagami Trough seismogenic fault (see Figure S5). We determined that the coefficient of the asperity location in the surrogate model is \(-0.5\), and we then...
estimated the coefficients $\alpha_{jk}$. In the previous section, the coefficients were set to $-1.0$ for the asperity in the western part of the fault, $0.0$ for the asperity in the central area, and $+1.0$ for the asperity in the eastern part of the fault. If we assume that the asperity is located one-dimensionally in the shallow part of the fault, we can set the coefficient to $-0.5$, which is between $-1.0$ and $0.0$, because we consider the fault model in which the asperity is located between the western and central parts of the fault. Figure 17 shows (a) the numerical simulation result for case 2, (b) the evaluation results obtained by the surrogate model, and (c) the differences between the results yielded by the numerical simulation and surrogate model. In the western region of the calculation domain, the results of the surrogate model show a tendency for the inundation depth to be $\sim 5$ m lower. The maximum difference in the computational domain (excluding the aforementioned region) is $3.32$ m, and the minimum difference is $-2.25$ m. Compared with case 1, the trend of the inundation depth distribution of the surrogate model is generally in agreement with the numerical simulation results, although the error is larger in some areas. Nevertheless, for future research, we will need to devise a method to treat the variation in the asperity position as a variable.

As mentioned above, the results of the surrogate model are generally consistent with the results simulated directly from the physical equations, although some areas with large errors in the model were observed.

### 3.5. Time-Dependent Monte Carlo Simulation

We can obtain the tsunami hazard curve and probabilistic tsunami inundation depth distribution under an assumed earthquake occurrence by multiplying the exceedance probability distribution of the tsunami inundation depth evaluated at each mesh point by the occurrence probability of the target earthquake. A complete PTHA should initially consider all possible earthquakes, but in this study, we evaluate the conditional tsunami hazard curve and probabilistic tsunami inundation depth distribution because we consider only the Sagami Trough megathrust earthquake, which is most likely to affect the target site. In the following, we will present the results using the surrogate model for the three variables up to modes $j = 1 \sim 4$ (cumulative contribution rate of $\sim 86.9\%$), for which the shape of the tsunami exceedance probability curves is stable.

The Earthquake Research Committee (2014) assessed the probability of an “M8-class earthquake in the Sagami Trough region” from multiple perspectives and ultimately estimated that the average interval between earthquakes is 180–590 years based on the topographic and geological data. They estimated the value of $\alpha$ in the BPT distribution to represent the variability of the earthquake interval as $\alpha = 0.24$ by using the maximum likelihood method and referencing the value used in the evaluation of active faults. Moreover, the average interval for only earthquakes with a magnitude equivalent to or greater than that of the Genroku Kanto earthquake is estimated to be

---

**Table 3**

| Case | Mw  | Depth (km) | Asperity location |
|------|-----|------------|-------------------|
| 1    | 8.75| +1.0       | West              |
| 2    | 8.7 | ±0.0       | Between West and Center |
~2,300 years. We carried out a probabilistic tsunami inundation assessment using the aforementioned values, although the Sagami Trough megathrust earthquake is considered larger than the Genroku Kanto earthquake because the former includes an “M8-class earthquake in the Sagami Trough region.”

Figure 18 shows the shapes of the BPT distributions for different mean values of the earthquake interval. The shape of the BPT distribution differs greatly, and this difference could have a great impact on a tsunami hazard assessment limited to a specific period of time (e.g., the next 30 or 50 years).

We next performed a time-dependent Monte Carlo simulation using the BPT distribution to generate conditional tsunami hazard curves, which represent the exceedance probability of the tsunami inundation depth within ΔT years and the probabilistic tsunami inundation depth distribution for the target area. The time-dependent Monte Carlo simulation was performed by the following procedure.

First, we determine an earthquake occurrence every year during ΔT years. In the first year, we randomly determine an earthquake occurrence based on the initial BPT distribution. If an earthquake occurs, the probability distribution of the inundation depth evaluated by the surrogate model is used to generate the inundation depths by assigning random numbers; these values are kept, and a second Monte Carlo simulation is executed. If no earthquake occurs, we use Equation 23 to update the probability distribution of the initial BPT distribution, and based on the updated probability distribution, we determine an earthquake occurrence in the second year by using random numbers. If an earthquake occurs in the second year, we similarly generate the inundation depths by assigning random numbers using the probability distribution of inundation depths evaluated by the surrogate model; these values are recorded, and another Monte Carlo simulation is performed. If no earthquake occurs in the second year, the probability distribution of the BPT distribution is updated again according to Equation 23, and based on the updated probability distribution, we determine an earthquake occurrence in the third year by using a random number. The above procedure is repeated for ΔT years, and if no earthquake occurs after ΔT years, we record the inundation depth at
the target point as 0 m and move on to a second round of Monte Carlo simulations, for which we revert the BPT distribution in the first year of the second Monte Carlo simulation to the initial probability and repeat the above simulation cycle.

In this study, we generated conditional tsunami hazard curves representing the exceedance probability of the tsunami inundation depth within the next $\Delta T$ years by repeating 100,000 Monte Carlo simulations for $\Delta T$ years. It should be noted that while Goda (2019) performed a time-dependent analysis considering the generation of two earthquakes in a $\Delta T$ year, in this study, for simplicity, we moved on to the subsequent simulation cycle once one earthquake occurred in a $\Delta T$ year. The average interval between the target earthquakes is a minimum of 180 years; nevertheless, for an assessment evaluating the next 50 years, this limitation is not expected to have a significant impact on the assessment results.

Since the most recent M8-class earthquake that occurred in the Sagami Trough region was the 1923 Taisho Kanto earthquake and the elapsed time was 97 years as of 2020, we set the initial value of elapsed years from the last event to $t = 97$. The red lines in Figure 19 show the tsunami inundation hazard curves evaluated by the time-dependent Monte Carlo simulation within the next 50 years ($\Delta T = 50$). Figure 19a shows the results for point A, and Figure 19b shows the results for point B. As an example, we consider the results for point A.

In the case of the mean value of $\mu = 590$ years, we could not draw a hazard curve because there was no case involving a great earthquake within the next 50 years. On the other hand, when we set the shortest mean interval of occurrence to $\mu = 180$ years, the number of earthquake occurrences within the next 50 years was naturally the highest, and the maximum exceedance probability was $\sim$23.2% within the next 50 years; the inundation depth with a 10% probability of exceedance within the next 50 years was $\sim$8.46 m, and the maximum inundation depth within $\sim$17.76 m. When we set $\mu = 300$ years, the maximum exceedance probability was 0.16% within the next 50 years; the maximum inundation depth with a 0.1% probability of exceedance was $\sim$7.7 m, and the maximum inundation depth was 15.16 m.

The above analysis reveals that accurately determining the mean occurrence interval of characteristic earthquakes is important for performing a reliable time-dependent probabilistic analysis because the tsunami hazard curve changes significantly depending on the setting of the mean earthquake occurrence interval. In addition, we note that the BPT distribution of the earthquakes in the Sagami Trough was set at $\alpha = 0.24$ by the Earthquake Research Committee (2014) and that considering the effect of the variation in $\alpha$ is also necessary.

Finally, we compare the results of the time-dependent Monte Carlo simulation using the BPT distribution with those based on a Poisson distribution, which is not time dependent in terms of the earthquake occur-

![Figure 18](image1.png)

**Figure 18.** Differences in the distribution of the occurrence probability (Brownian passage time [BPT] distribution) of the Sagami Trough megathrust earthquake due to differences in the mean value $\mu$ of the earthquake interval. The Earthquake Research Committee (2014) stated that the mean interval of M8-class earthquakes along the Sagami Trough varies from 180 to 590 years, and thus, the BPT distributions are drawn with $\mu = 180$ and 590 years.

![Figure 19](image2.png)

**Figure 19.** Fifty-year frequency of exceedance curves for each average year in the Brownian passage time (BPT) distribution and Poisson distribution for (a) Point A and (b) Point B in Figure 14.
The Poisson distribution is used when only the mean value of the earthquake interval is known, and this distribution is described by the following equation:

\[ P(t) = \frac{\lambda^t}{t!} \exp^{-\lambda} \]  

where \( t \) is the elapsed time since the last earthquake and \( \lambda \) is the average return period. We ran 100,000 Monte Carlo simulations for 50 years using the Poisson distribution. The results are shown as black lines in Figure 19. Compared to the results obtained using the BPT distribution shown as red lines, we can see that the variation in the exceedance probability curves is extremely small, even when the mean value of \( \mu \) changes. We also find that if we consider short intervals of earthquake occurrence, such as \( \mu = 180 \) or 200 years, the probability of occurrence is underestimated. Thus, not considering the imminence of earthquake occurrence leads to an underestimation of the hazard of the future earthquake occurrence.

**3.6. Probabilistic Tsunami Inundation Assessment**

Figure 20 shows the inundation depth distributions with 39%, 10%, 5%, and 2% probabilities of exceedance within the next 50 years; these distributions were evaluated by conducting time-dependent Monte Carlo simulations at all mesh points. In the long-term average evaluation, a 50-year 39% probability corresponds to the occurrence of an earthquake approximately once every 100 years, whereas a 50-year 10% probability corresponds to approximately one event every 500 years, a 50-year 5% probability corresponds to approximately one earthquake every 1,000 years, and a 50-year 2% probability corresponds to approximately one event in 2,500 years. These relationships can be calculated by the following equation:

\[ P = 1 - (1 - \rho)^r \]
where \( P \) is the return period when averaged over the long term (e.g., once in 500 or 1,000 years), \( r \) is the target period for the hazard assessment (e.g., the next 30 or 50 years), and \( p \) is the occurrence probability during the target period.

As a matter of course, the smaller the exceedance probability is, the wider the inundation area and the greater the inundation depth, and vice versa. In the Sagami Trough earthquake region, 97 years have passed since the last earthquake (as of 2020), and thus, the next earthquake is imminent, indicating that even under a 50-year 39% exceedance probability, the inundation area will spread over large areas of the city. The Design for Tsunami Loads and Effects in the ASCE 7–16 Standard stipulates that the maximum considered tsunami should be considered for a tsunami that is likely to occur approximately once every 2,500 years (Chock, 2016), which is equivalent to the tsunami corresponding to Figure 20d.

Finally, we derive a confidence interval around the central hazard estimates from Monte Carlo simulations using Greenwood’s formula (Greenwood, 1926), taking \( \mu = 180 \) years and a 50-year 2% probability as an example. Figure 21a shows the tsunami hazard curve (the central estimates) and its confidence interval for Point B in Figure 14. In this study, we considered the 95% confidence interval. Figure 21b shows the 95% confidence intervals for the 50-year 2% probability inundation map. Based on this map, we can consider the uncertainties around the central hazard estimates.

The advantage of evaluating tsunamis within a limited period of time in the future, such as the next 50 years, is that it provides a realistic hazard assessment that considers an imminent earthquake; thus, we can consider the useful lives of buildings and other various structures and the life spans of humans. Hence, the results of time-dependent tsunami hazard assessments should be used as the probabilistic tsunami hazard data for community planning in the field of disaster prevention, constructing various infrastructures, and conducting risk assessments in the fields of nuclear power, real estate, and damage insurance or reinsurance.

4. Conclusion

In this study, we generated random distributions of the tsunami inundation depth by applying SVD to a limited number of tsunami inundation depths and evaluating the spatial correlation of those inundation depths, and we proposed a method to evaluate tsunami hazard curves and probabilistic tsunami inundation depths within a certain area considering the imminent occurrence of an earthquake. Based on the good agreement between the samples generated by the proposed surrogate model and the nonlinear long wave equations for the inundation depth distribution in Kamakura city due to the Sagami Trough megathrust earthquake, we can conclude that the proposed probabilistic tsunami inundation assessment using the proposed method is sufficiently accurate and practical.

Fundamentally, PTHA is computationally demanding because it assumes a large number of earthquakes and requires the solution of a large number of nonlinear equations for tsunami propagation. However, in this study, we achieved a significant reduction in the number of required tsunami propagation calculations by using the SVD method, which is an efficient statistical method for extracting spatial correlations of the data.
evaluation of spatial correlation using SVD differs from that using the KLE in that it is not necessary to define a covariance function beforehand, as the function can be defined from the data itself. In addition, we carried out a probabilistic tsunami inundation assessment focusing on a relatively short period of 50 years considering the time-dependent occurrence probability of seismogenic faults, which can provide the probabilistic tsunami hazard data for building risk assessments considering the useful lives of buildings and other various structures or risk assessments in the fields of nuclear power, real estate, and damage insurance or reinsurance.

We have shown the method of the time-dependent probabilistic tsunami inundation assessment of the uncertainty of earthquake scenarios, but some controversial issues remain. First, PTHA is generally based on the consideration of a large number of seismogenic faults and their occurrence probabilities that may affect the target area. In this study, we evaluated only conditional tsunami hazard curves and inundation depth distributions for a single earthquake scenario—the Sagami Trough megathrust earthquake. In the future, it is essential to cover multiple faults. In particular, we will use fault models generated from a physical stochastic slip model and fault models with varying parameters such as depths and runs, while in this study, the slip distribution of faults is generated simply in one dimension. If we consider a large number of uncertainties, a large number of models are generated, and the computational load is high. However, the computational load is expected to be significantly reduced by using the proposed method of applying SVD to the inundation depths. Second, we applied SVD to the inundation depth distributions for nine cases (involving two variables) and 27 cases (involving three variables), but a substantial amount of the data regarding the original inundation depths needs to be verified. In the future, we will compare the probabilistic inundation depth distributions evaluated by the nonlinear long wave equations from a larger number of seismic faults with the stochastic inundation depth distributions obtained from SVD to further discuss the necessary amount of the original data for PTHA. Third, we have presented the results for only three variables, namely, the slip amount (Mw), the fault depth, and the asperity location, but we need to perform a multivariate analysis that also considers the other variables of the seismogenic fault, such as the slip angle and strike of the fault. The above issues will be left for a future study.

Appendix A: Posterior Predictive Distribution Using Bayesian Inference

Based on Bayesian inference, a posterior distribution of interest parameters \( \mathbf{f} \) given data \( \mathbf{D} \) is calculated as follows:

\[
p(\mathbf{f} \mid \mathbf{D}) = \frac{p(\mathbf{D} \mid \mathbf{f})p(\mathbf{f})}{p(\mathbf{D})} \tag{A1}
\]

where \( p(\mathbf{f}) \) is the prior distribution over the parameters and \( p(\mathbf{D} \mid \mathbf{f}) \) is the likelihood of the given data. Note that formally, the Bayes’ rule follows directly from the definition of conditional probability. \( p(\mathbf{D}) \) is independent of the interest parameters and given by:

\[
p(\mathbf{D}) = \int p(\mathbf{D} \mid \mathbf{f})p(\mathbf{f})d\mathbf{f} \tag{A2}
\]

Then, we can derive:

\[
p(\mathbf{f} \mid \mathbf{D}) \propto p(\mathbf{D} \mid \mathbf{f})p(\mathbf{f}) \tag{A3}
\]

If we have a Gaussian prior and likelihood, this ensures that the posterior will also be Gaussian.

Here, the regression model assumes that \( \mathbf{y} = \mathbf{f} + \mathbf{e} \) where the homoscedastic additive observational error follows a normal distribution \( \mathcal{N}(\mathbf{e} \mid 0, \sigma_n^2) \), such that \( p(\mathbf{y} \mid \mathbf{f}) = \mathcal{N}(\mathbf{y} \mid \mathbf{f}, \sigma_n^2) \). It should be noted that we perform regression analysis by a noise-free Gaussian process \( \sigma_n^2 = 0 \) so that the estimates pass through the value of \( \mathbf{f} \) obtained by SVD at the data point during the tsunami inundation analysis. Given the training points \( \mathbf{x} \), the likelihood of \( \mathbf{f} \) becomes:
\[ p(y \mid f) = \mathcal{N}(y \mid f, 0) \]  

(A4)

The prior distributions are expressed by Equation 18. Since the likelihood and prior are both multivariate normal distributions, the posterior distribution of \( f \) can be calculated analytically based on Bayesian inference Equation A3:

\[
p(f \mid y) \propto p(y \mid f)p(f)
\]

(A5)

\[
= \mathcal{N}(y \mid f, 0)\mathcal{N}(f \mid 0, k(x, x))
\]

(A6)

where \( K = k(x, x) \). The posterior of \( f \) can be used to compute the posterior predictive distribution of \( f \), for any test point \( x \). The predictive distribution of \( f \) is obtained by integration over the posterior uncertainty:

\[
p(f_\ast \mid y, x) = \int p(f_\ast \mid f, x, x) p(f \mid y) \, df
\]

(A7)

where the first term of the righthand side describes the dependency of \( f_\ast \) on \( f \) and the second term is Equation A5. The conditional distribution of \( f_\ast \mid f \) can be obtained from the joint distribution Equation 18:

\[
p(f_\ast \mid f, x, x) = \mathcal{N}(f_\ast \mid m_\ast, V_\ast)
\]

(A8)

\[
m_\ast = K(x, x) K(x, x)^{-1} f
\]

(A9)

\[
V_\ast = K(x, x) - K(x, x) K(x, x)^{-1} K(x, x)
\]

(A10)

For a more detailed derivation of the Gaussian process regression, see Rasmussen and Williams (2006) and Kuss (2006).

**Conflict of Interest**

The authors declare no conflicts of interest relevant to this study.

**Data Availability Statement**

Data sets for the original fault parameters of the Sagami trough earthquake are available in Cabinet Office (2013) retrieved from http://www.bousai.go.jp/kaigirep/chuobou/seremon/shutochokkajishinmodel/. The earthquake fault parameters we constructed, and some of the simulation results described in the paper uploaded to Zenodo (http://doi.org/10.5281/zenodo.4731474).

**References**

Adams, I. M., LeVeque, R. J., & González, F. I. (2015). The pattern method for incorporating tidal uncertainty into probabilistic tsunami hazard assessment (PTHA). *Natural Hazards*, 76(1), 19–39. https://doi.org/10.1007/s11069-014-1482-z

Annaka, T., Satake, K., Sakakiyama, T., Yanagisawa, K., & Shuto, N. (2007). Logic-tree approach for probabilistic tsunami hazard analysis and its applications to the Japanese coasts. *Pure and Applied Geophysics*, 164, 577–592. https://doi.org/10.1007/s00024-006-0174-3

Bayraktar, H. B., & Ozer Sozdinler, C. (2020). Probabilistic tsunami hazard analysis for Tuzla test site using Monte Carlo simulations. *Natural Hazards and Earth System Sciences*, 20(6), 1741–1764. https://doi.org/10.5194/nhess-20-1741-2020

Bommer, J., & Scherbaum, F. (2008). The use and misuse of logic trees in probabilistic seismic hazard analysis. *Earthquake Spectra*, 24, 997–1009. https://doi.org/10.1193/1.2977755

Butler, R., Walsh, D., & Richards, K. (2017). Extreme tsunami inundation in Hawaii from Aleutian Alaska subduction zone earthquakes. *Natural Hazards*, 85, 1591–1619. https://doi.org/10.1007/s11069-016-2650-0

Cabinet Office. (2013). *The committee of the model for Tokyo metropolitan earthquake (in Japanese)*. Retrieved from http://www.bousai.go.jp/kaigirep/chuobou/seremon/shutochokkajishinmodel/

Chock, G. (2016). Design for tsunami loads and effects in the ASCE 7-16 standard. *Journal of Structural Engineering*, 142, 04016093. https://doi.org/10.1061/(ASCE)ST.1943-515X.0001565

Crempien, J., Urrutia, A., Benavente, R., & Ciefuegos, R. (2020). Effects of earthquake spatial slip correlation on variability of tsunami potential energy and intensities. *Scientific Reports*, 10, 8399. https://doi.org/10.1038/s41598-020-65412-3

**Acknowledgments**

The authors thank the anonymous reviewers who provided us with valuable comments and helped improve the manuscript. This research was partially supported by funding from the Nuclear Safety Research Institute, Chubu Electric Power Co., Inc. Kenjiro Terada, Shuji Moriguchi, Yu Otake, and Yo Fukutani conceived and designed the study. Yo Fukutani performed the experiments, analyzed the data, and wrote the manuscript.
Davies, G., & Griffin, J. (2020). Sensitivity of probabilistic tsunami hazard assessment to far-field earthquake slip complexity and rigidity depth-dependence: Case study of Australia. *Pure and Applied Geophysics*, 177, 1521–1548. https://doi.org/10.1007/s00240-019-02999-w

Davies, G., Griffin, J., Lovholt, F., Glimsdal, S., Harbitz, C., Thio, H., et al. (2017). A global probabilistic tsunami hazard assessment from earthquake sources. *Geological Society, London, Special Publications*, 456, 219–244. https://doi.org/10.1144/SP456.5

Davies, G., Horspool, N., & Miller, V. (2015). Tsunami inundation from heterogeneous earthquake slip distributions: Evaluation of synthetic-source models. *Journal of Geophysical Research: Solid Earth*, 120, 6431–6451. https://doi.org/10.1002/2015JB012272

Earthquake Research Committee. (2004). Long-term evaluation of seismicity along Sagami Trough (in Japanese). Headquarters for Earthquake Research Promotion. Retrieved from http://www.jishin.go.jp/main/chousa/04agu_sagami/index.htm

Earthquake Research Committee. (2014). Long-term evaluation of seismicity along Sagami Trough (2nd version) (in Japanese) (p. 81). Headquarters for Earthquake Research Promotion. Retrieved from https://www.jishin.go.jp/main/chousa/14aprag_sagami/

Earthquake Research Committee. (2017). Tsunami prediction method by characterizing the source fault (tsunami recipe) (in Japanese) (p. 38). Headquarters for Earthquake Research Promotion. Retrieved from https://www.jishin.go.jp/main/chousa/17jan_tsunami-recipe.pdf

Earthquake Research Institute. (2012). Special project for mitigation of metropolitan earthquakes (in Japanese). University of Tokyo, National Research Institute for Earth Science and Disaster Resilience, Disaster Prevention Research Institute, & Kyoto University. Retrieved from http://www.eri.u-tokyo.ac.jp/shuto/report/soukatsu/0120327.pdf

El-Hussain, I., Omira, R., Alhabsi, Z., Baptista, M., Deif, A., & Mohamed, A. (2018). Probabilistic and deterministic estimates of near-field tsunami hazards in northeast Oman. *Geoscience Letters*, 5, 30. https://doi.org/10.1186/s40562-018-0129-4

Erdik, M., Demircioglu, M., Sesetyan, K., Cakti, E., & Siyahi, B. (2004). Earthquake hazard in Marmara Region, Turkey. *Soil Dynamics and Earthquake Engineering*, 24, 605–631. https://doi.org/10.1016/j.soildyn.2004.04.003

Fukutani, Y., Suppasri, A., & Imamura, F. (2015). Stochastic analysis and uncertainty assessment of tsunami wave height using a random source parameter model that targets a Tohoku-type earthquake fault. *Stochastic Environmental Research and Risk Assessment*, 29(7), 1763–1779. https://doi.org/10.1007/s00477-014-0966-4

Geist, E. (2002). Complex earthquake rupture and local tsunami. *Journal of Geophysical Research*, 107. https://doi.org/10.1029/2000JB00139

Geist, E., & Lynett, P. (2014). Source processes for the probabilistic assessment of tsunami hazards. *Oceanography*, 27, 86–93. https://doi.org/10.5670/oceanog.2014.43

Geist, E., & Parsons, T. (2006). Probabilistic analysis of Tsunami Hazards. *Natural Hazards*, 37, 277–314. https://doi.org/10.1007/s11069-005-4646-z

Glimsdal, S., Lovholt, F., Harbitz, C., Romano, F., Lorito, S., Oreilffe, S., et al. (2019). A new approximate method for quantifying tsunami maximum inundation height probability. *Pure and Applied Geophysics*, 176, 3227–3246. https://doi.org/10.1007/s00240-019-02091-w

Goda, K. (2019). Time-dependent probabilistic tsunami hazard analysis using stochastic rupture sources. *Stochastic Environmental Research and Risk Assessment*, 33, 341–358. https://doi.org/10.1007/s00477-018-1634-x

Goda, K., Mai, P., Yasuda, T., & Morl, N. (2014). Sensitivity of tsunami wave profile and inundation simulations to earthquake slip and fault geometry for the 2011 Tohoku earthquake. *Earth, Planets and Space*, 66, 105. https://doi.org/10.1186/1880-5981-66-105

Goda, K., & Song, J. (2016). Uncertainty modeling and visualization for tsunami hazard and risk mapping: A case study for the 2011 Tohoku earthquake. *Stochastic Environmental Research and Risk Assessment*, 30, 2271–2285. https://doi.org/10.1007/s00477-015-1146-x

González, F. I., Geist, E. L., Jaffe, B., Kanoğlu, U., Moffield, H., Synolakis, C. E., et al. (2009). Probabilistic tsunami hazard assessment at Sea-side, Oregon, for near- and far-field seismic sources. *Journal of Geophysical Research*, 114, C11023. https://doi.org/10.1029/2008JC005132

Goto, C., & Ogawa, Y. (1982). Tsunami numerical simulation with Leap-frog scheme. Tohoku University.

Greenwood, M. (1926). The natural duration of cancer. *Reports on Public Health and Medical Subjects*, 33, 1–26.

Grezieo, A., Babeyko, A., Baptista, M. A., Behrens, J., Costa, A., Davies, G., et al. (2017). Probabilistic tsunami hazard assessment: Multiple sources and global applications. *Reviews of Geophysics*, 55(4), 1158–1198. https://doi.org/10.1002/2017RG000579

Horspool, N., Prananto, I., Griffin, J., Latief, H., Natawidjaja, D. H., Kongko, W., et al. (2014). A probabilistic tsunami hazard assessment for Indonesia. *Natural Hazards and Earth System Sciences*, 14(11), 3105–3122. https://doi.org/10.5194/nhess-14-3105-2014

Japan Society of Civil Engineers (2016). The method of tsunami risk assessment for nuclear power plants (in Japanese). Retrieved from https://committees.jsce.or.jp/ceofnp/node/84

Jho, M. H., Kim, G. H., & Yoon, S. B. (2019). Construction of logic trees and hazard curves for probabilistic tsunami hazard analysis. *Journal of Korean Society of Coastal and Ocean Engineers*, 31(2), 62–72. https://doi.org/10.9765/ksoco.2019.31.2.62

Jolliffe, I. T. (1986). Principal component analysis. Springer.

Kagan, Y. Y., Jackson, D. D., & Geller, R. J. (2012). Characteristic earthquake model, 1884-2011, RIP. *Seismological Research Letters*, 83, 951–951. https://doi.org/10.1785/0220120107

Kalligratis, N., Montoya, L., Ayca, A., & Lynett, P. (2017). An approach for estimating the largest probable tsunami from far-field subduction zone earthquake. *Natural Hazards*, 89, 233–253. https://doi.org/10.1007/s11069-017-2961-9

Kanagawa Prefecture. (2015). The 10th tsunami inundation assessment study group, tsunami inundation forecast (Commentary) (in Japanese). Retrieved from http://www.pref.kanagawa.jp/uploaded/attachment/774761.pdf

Kanomori, H. (1977). The energy release in great earthquakes. *Journal of Geophysical Research*, 82(20), 2981–2987. https://doi.org/10.1029/JB082i020p02981

Karschunen, K. (1947). *Ubber lineare methoden in der Wahrscheinlichkeitsrechnung*. Universität Heidelberg.

Kerschen, G., Golinval, J. C., Vakakis, A., & Bergman, L. (2005). The method of proper orthogonal decomposition for dynamical characterization and order reduction of mechanical systems: An overview. *Nonlinear Dynamics*, 41, 147–169. https://doi.org/10.1007/s11071-005-1667-2

Kodaira, S., No, T., Nakamura, Y., Fujijbara, T., Kailo, Y., Miura, S., et al. (2012). Coseismic fault rupture at the trench axis during the 2011 Tohoku-oki earthquake. *Nature Geoscience*, 5, 646–650. https://doi.org/10.1038/ngeo5147

Kuss, M. (2006). Gaussian process models for robust regression, classification, and reinforcement learning (PhD thesis, pp. 1–189). Darmstadt University of Technology.

Lanczos, C. (1961). *Linear differential operators*. Van Nostrand.

LeVeque, R., Waagan, K., González, F., Rim, D., & Lin, G. (2016). Generating random earthquake events for probabilistic tsunami hazard assessment. *Pure and Applied Geophysics*, 173, 3671–3692. https://doi.org/10.1007/s00240-016-1357-1

Li, L., Switzer, A. D., Chan, C. H., Wang, Y., Weiss, R., & Qiu, Q. (2016). How heterogeneous coseismic slip affects regional probabilistic tsunami hazard assessment: A case study in the South China Sea. *Journal of Geophysical Research: Solid Earth*, 121(8), 6250–6272. https://doi.org/10.1002/2016JB013111
Yamao, S., Esteban, M., Yun, N. Y., Mikami, T., & Shibayama, T. (2015). Estimation of the current risk to human damage life posed by future tsunamis in Japan. In M. Esteban, H. Takagi, & T. Shibayama (Eds.), 2011 Great East Japan Earthquake: Reconstruction and restoration: Insights and assessment after 5 years (pp. 469–485). Springer International Publishing. https://doi.org/10.1007/978-3-319-58691-5_27

Mueller, C., Power, W., Fraser, S., & Wang, X. (2014). Effects of rupture complexity on local tsunami inundation: Implications for probabilistic tsunami hazard assessment by example. Journal of Geophysical Research: Earth, 120(1), 488–502. https://doi.org/10.1002/2014JB011301

Mulla, I. E., Ishibie, T., Satake, K., Giusman, A. R., & Murotani, S. (2020). Regional probabilistic tsunami hazard assessment associated with active faults along the eastern margin of the Sea of Japan. Earth, Planets and Space, 72(1), 123. https://doi.org/10.1186/s40623-020-01256-5

Okada, Y. (1985). Surface deformation due to shear and tensile faults in a half-space. Journal of Geophysical Research: Solid Earth, 32(8), 1985. https://doi.org/10.1007/s11069-009-9453-5

Rasmussen, C. E., & Williams, C. K. I. (2006). Gaussian processes for machine learning. MIT Press.

Tozato, K., Kotani, T., Hatazo, R., Takase, S., Moriguchi, S., Terada, K., & Otake, Y. (2020). Tsunami risk assessment using spatial modes extracted from results of numerical analysis (in Japanese). Transactions of the Japan Society for Computational Engineering and Science, 2020, 2000003. https://doi.org/10.1186/jcesc.2020.2000003

UNESCO, & IUGG/IOC Time Project. (1997). Numerical method of tsunami simulation with the leap-flog scheme (IOC Manuals and Guides No. 35). UNESCO.

Völpe, M., Lorito, S., Selva, J., Tonini, R., Romano, F., & Brizuela, B. (2019). From regional to local SPTHA: Efficient computation of probabilistic tsunami inundation maps addressing near-field sources. Natural Hazards and Earth System Sciences, 19(3), 455–469. https://doi.org/10.5194/nhess-19-455-2019

Wu, C. G., Liang, Y. C., Lin, W. Z., Lee, H. P., & Lim, S. P. (2003). A note on equivalence of proper orthogonal decomposition methods. Journal of Sound and Vibration, 265(5), 1103–1110. https://doi.org/10.1006/jsvi.2002.4430

Yamao, S., Esteban, M., Yun, N. Y., Mikami, T., & Shibayama, T. (2015). Estimation of the current risk to human damage life posed by future tsunamis in Japan. In M. Esteban, H. Takagi, & T. Shibayama (Eds.), Handbook of coastal disaster mitigation for engineers and planners (pp. 257–275). Butterworth-Heinemann. https://doi.org/10.1016/b978-0-12-801069-0.00013-7