1 Introduction

In an attempt to understand the hierarchy of the fermion masses and the mixing of the quarks and leptons in term of a few fundamental parameters, the idea of extending the Standard Model (SM) by a $U(1)$ horizontal symmetry of stringy origin has been investigated in the prospect of anomaly cancellation a la Green Schwarz, whereby, the mixed anomalies of the $U(1)$ with the $SU(2)$ and $U(1)$ gauge groups are cancelled by a shift in the dilaton in 4D string theories, a suitable condition for obtaining also the canonical unification of the three gauge couplings at $M_\text{GUT} = 10^{16}$ GeV, as well as the phenomenologically successful

$$m' m_b m_e m' m_d m_{1}$$

relations without the need of a grand unified group. Besides, since with the thirty observables of the Yukawa sector one cannot uniquely determine the entries of the quark and lepton mass matrices, even by assuming that the latter are hemitian, it has become increasingly popular to assume that some or all of the entries are sufficiently suppressed with respect to others so that they can be replaced by so-called texture zeros, which reflect the nature of the underlying horizontal symmetry and give relations between the fermion masses and the mixings, like the successful $Y_{ij}' = m_{d_i m_1}$. We shall therefore start with a discussion of how to generate such phenomenologically successful symmetrical textures in the context of a broken horizontal symmetry with a minimum number of scalar fields and higher-dimensional operators.

The extension of this approach to the lepton sector gives different neutrino mass and oscillation patterns, which are able to explain some of the three neutrino mass related puzzles, such as the solar neutrino dect (SN), the atmospheric neutrino dect (AN), and the need of hot dark matter (HDM) for galaxy formation. This will be discussed in the second part of the talk.

2 Generating the texture of the quark and lepton mass matrices from an extra $U(1)$

Let us assume the existence of a family-dependent $U(1)$ symmetry at the Planck scale, with respect to which the quarks and leptons carry charges $a_i$ and $a_j$ respectively, where $i = 1;2;3$ is the generation index. Due to the electroweak symmetry breaking, the left-handed up (quark/lepton) states and the corresponding down (quark/lepton) states carry the same charges. The generation of symmetrical textures in planes on the other hand enables charges for left-handed and righthanded states.

We generate the texture of the quark mass matrices $M_u$ and $M_d$. Given the role played by the third generation we start with rank-one mass matrices and make a choice for the charges such that only the $(3,3)$ renormalizable couplings $\mathcal{F}^{\text{th}}$ and $\mathcal{F}^{\text{th}}_{2}$ are allowed, the other entries being zero as long as the symmetry is exact. This choice generates the charges of the light Higgses $h_{1,2} \to 2$ (3). On the other hand, the presence of scalar fields, some of which acquire a vev at the unification scale, and of higher-dimensional operators, as in the case of $M_\text{GUT}$ string com pactification schemes, can lead to the spontaneous breaking of the symmetry and the generation of small nonzero entries. The most economical scenario requires only one singlet field or a pair of fields, developing equal (vev’s) along a $\mathcal{D}$-at direction and carrying opposite charges $1$. These in turn can give rise to higher-order couplings $\mathcal{F}_{1}^{\text{th}} = \mathcal{F}_{2}^{\text{th}}$, where $M$ is a scale typical of these higher-dimensional operators, $\mathcal{E}$ the string unification scale $M_\text{GUT} \sim 10^{18}$ GeV or $M_\text{F}$. The power with which the scale $\mathcal{E} = \mathcal{E}_{\text{F}}$ will in the $(ij)$ entry is such as to compensate the charge of $\mathcal{F}_{1}^{\text{th}}$. Notice that when the exponent is positive (negative) only the field $\sim$ (−) can contribute. Most likely such a theory will contain also heavy Higgs multiplets $H_i$, needed for breaking the GUT symmetries and giving rise to $\mathcal{F}_{1}^{\text{th}} = \mathcal{F}_{2}^{\text{th}}$, where by $2; i$ we denote the charge of $H_i$. As a consequence one can generate mass matrices of the following type:

$$M_{\text{ud}} = \begin{pmatrix} 0 & \mathcal{E}^{2} x_{1} & \frac{1}{2} \mathcal{E}^{2} x_{1} x_{2} z & \frac{1}{2} \mathcal{E}^{2} x_{1} x_{2} z & \frac{1}{2} \mathcal{E}^{2} x_{1} x_{2} z & \frac{1}{2} \mathcal{E}^{2} x_{1} x_{2} z \\ \mathcal{E}^{2} x_{1} & 0 & \mathcal{E}^{2} x_{1} x_{2} z & \mathcal{E}^{2} x_{1} x_{2} z & \mathcal{E}^{2} x_{1} x_{2} z & \mathcal{E}^{2} x_{1} x_{2} z \\ \frac{1}{2} \mathcal{E}^{2} x_{1} x_{2} z & \mathcal{E}^{2} x_{1} x_{2} z & 0 & \mathcal{E}^{2} x_{1} x_{2} z & \mathcal{E}^{2} x_{1} x_{2} z & \mathcal{E}^{2} x_{1} x_{2} z \\ \frac{1}{2} \mathcal{E}^{2} x_{1} x_{2} z & \mathcal{E}^{2} x_{1} x_{2} z & \mathcal{E}^{2} x_{1} x_{2} z & 0 & \mathcal{E}^{2} x_{1} x_{2} z & \mathcal{E}^{2} x_{1} x_{2} z \\ \frac{1}{2} \mathcal{E}^{2} x_{1} x_{2} z & \mathcal{E}^{2} x_{1} x_{2} z & \frac{1}{2} \mathcal{E}^{2} x_{1} x_{2} z & \frac{1}{2} \mathcal{E}^{2} x_{1} x_{2} z & 0 & \mathcal{E}^{2} x_{1} x_{2} z \\ \frac{1}{2} \mathcal{E}^{2} x_{1} x_{2} z & \mathcal{E}^{2} x_{1} x_{2} z & \frac{1}{2} \mathcal{E}^{2} x_{1} x_{2} z & \frac{1}{2} \mathcal{E}^{2} x_{1} x_{2} z & \frac{1}{2} \mathcal{E}^{2} x_{1} x_{2} z & 0 \end{pmatrix} \quad \text{(2)}$$

where $j; j = 1, 2$ and $j; j = 1, 2$. Assuming two scales $\mathcal{E} = 0.2$ and $\mathcal{E} = \mathcal{E}_{\text{F}}$, where $\mathcal{Y}_{\text{ab}}$ is the Wolfenstein parameter, and the right combination of charges one can generate four sets of $M_u$ and $M_d$ textures.
A, B, C and D (Table 1 and eq. (3)) which contain ve zeros in total, without counting zeros of symmetric entries twice, and lead to the most predictive Ansatz for quark masses and mixings in agreement with experiment. A 4th set, E, with four zeros emerges from the particularly economical scenario with the two singlets $0 \sim 1$. All such ve sets can be obtained from

$$
\begin{align*}
&0 \sim 6 & 4 & 1 & 0 \sim 3 & 0 \\
&1 & 4 & 2 & 1 & 0 \sim 1 \\
M_u, & M_d, & M_t \\
&0 & 6 & 4 & 2 & 1 \\
&0 & 3 & 2 & 1 & 0
\end{align*}
$$

and Table 1.

Let us now turn to the charged lepton mass matrix $M_{\ell}$. Assuming simply the gauge symmetries of the SM the $U(1)_X$ charges of the leptons are not related to those of the quarks but adopting the philosophy that all the entries except the $(3,3)$ entry are zero before symmetry breaking leads to $a_1 = 1$. Another constraint comes from the second mass relation of eq.(1) which implies $a_1 + a_2 = 1 + 2$. This relation can be satisfied when $a_1 = 1$ and $a_2 = 2$ and leads to the GUT relation:

$$
M_{\ell}^D = M_{\ell}^U
$$

with a slight modification of the $(2,2)$ entries as in the Georgi-Jarlskog model.

Let us discuss next the generation of Dirac neutrino mass terms $M_{\nu} = M_{\nu}^C N_i$ where $N_i = 1, 2, 3$ are the three righthanded neutrino states which are present in most GUT's. Since these are of the same type as the mass terms in the quark and charged lepton sector it is natural to adopt again the same approach. Then, because the charges $a_i$ have been fixed through the charged lepton Ansatz, $M_{\ell}^D = M_{\ell}^U$. Further, one for the choice $a_1 = 1$

and $a_2 = 2$ one obtains the other well known GUT relation:

$$
M_{\ell}^D = M_{\ell}^U \quad \text{or} \quad M_{\ell}^D = M_{\ell}^D;
$$

On the other hand, the heavy Majorana mass terms $M_R N_i^T N_j$, which are needed in the $(6 \times 6)$ neutrino mass matrix in order to suppress the otherwise unacceptable large masses for the $\nu_i$; $\nu_j$ need not be generated in the same way. In compact string models, due to the absence of large Higgs representations, righthanded neutrinos do not get tree-level mass terms, so all entries in $M_R$ are due to nonrenormalizable operators $N_i^T H Q_j N_j$ $(k; l = 1, 2, 3)$ whose scale is of the order of $M_{\ell}^0 = M_P$ multiplied by some orbifold suppression factor $C' < 10^{-4}$:

$$
R = C' \frac{< H >}{M_P} \sim 10^9 \times 10^{13} \text{GeV};
$$

Nothing is a priori known concerning the particular texture of $M_R$, or the existence of hierarchy in this sector. One can therefore follow either of the two paths: Assuming no hierarchy and use the operators $N_i^T H Q_j N_j$ to generate a nonsingular Majorana mass texture

$$
0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0
$$

as in refs.[3,6,8], or obtain the entries along the same principle which was used to generate the hierarchy in the quark and charged lepton sector, refs.[9-11]. In any case the spectrum of the light neutrino states $\nu_i$ depends very delicately on the $(2,3)$ and/or the $(1,3)$ entry of $M_{\ell}^D$ and the symmetry of $M_R$, as will be shown next.

3 The neutrino mass spectrum

Starting from the Ansatz $M_{\ell}^D \sim 1$ or $M_{\ell}^D \sim M_{\ell}^D$ one is led, upon diagonalisation of the reduced $(3 \times 3)$ light neutrino mass matrix

$$
M_{\ell}^D = M_{\ell}^D
$$

to the quadratic seesaw spectrum:

$$
m_\nu : m : m^{'}, \quad (z_8 : z^4 : 1) \quad m;
$$

where

$$
z = \frac{m^2}{R} \quad \text{if} \quad M_{\ell}^D = M_{\ell}^U;
$$

or,

$$
z = \frac{m^2}{R} \quad \text{if} \quad M_{\ell}^D = M_{\ell}^D;
$$

otherwise some of the standard neutrinos will be too heavy and will have to decay as required by various astrophysical and cosmological considerations.

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**Table 1: Possible sets of $M_u, M_d$ textures corresponding to different choices of $U(1)$ charges and classified according to eq.(3).**

| Set | ~ | ~ | ~ |
|-----|---|---|---|
| A   | 1 | 1 | 0 |
| B   | 1 | 0 | 1 |
| C   | 1 | 1 | 1 |
| D   | 0 | 1 | 1 |
| E   | 1 | 1 | 1 |
and the lepton mixings:

$$\mathcal{M} = \begin{pmatrix} \theta_1^2 & \theta_2 & \theta_3 \\ \theta_1 & \theta_4 & \theta_5 \\ \theta_2 & \theta_5 & \theta_6 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \theta_1^2 & \theta_2 & \theta_3 \\ \theta_1 & \theta_4 & \theta_5 \\ \theta_2 & \theta_5 & \theta_6 \end{pmatrix}^{-1},$$

where \( \theta_i \) are the mixing angles and \( M \) is the mass matrix. Then, depending upon the position of the zeros in \( M \), the m uon neutrino is heavier than the tau neutrino or, they are mass degenerate and \( \sin^2 2 \theta = 0 \). (1).

Large \( e \) mixing is on the other hand typical of the quark texture solutions \( B \) and \( C \). As a matter of fact, m ass-degenerate neutrinos and large \( e \) mixing angles emerge m aterially from the ve-zero texture solutions \( A \) than from the four-zero solution \( B \) which requires very particular conditions of strong hierarchy in \( m \) to give m ass de gen erate neutrinos. This may be so because the more texture zeros there are in the quark m ass m atrices the more the sym m etries of the righthanded neutrino sector can enhance the light neutrino spectrum.

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