Rheology of a confined granular material

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Abstract

We study the rheology of a granular material slowly driven in a confined geometry. The motion is characterized by a steady sliding with a resistance force increasing with the driving velocity and the surrounding relative humidity. For lower driving velocities a transition to stick-slip motion occurs, exhibiting a blocking enhancement with decreasing velocity. We propose a model to explain this behavior pointing out the leading role of friction properties between the grains and the container’s boundary.

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Dynamics of granular materials lacks of an established unified picture. A great diversity of mechanical and rheological behaviors were reported depending on whether they are vibrated, slowly sheared, avalanching on a surface, flowing in a hopper or falling in a chute[1]. For dense granular assemblies, experiments reveal strain localization[2], [3], non local rheological properties[4] and aging phenomena[5], [6]. This complex phenomenology could possibly be due to the presence, at a mesoscopic level, of a disordered contact force network with unraveled mechanical properties [2], [7]. Moreover, dramatic effects on the mechanical strength induced by slight changes in compaction were reported [2], [8], [9]. Another source of difficulty is the dynamical contribution of contact forces and there is, so far, only few studies on the macroscopic emergence of these aging properties due to the slow plastic deformation of contacts and the influence of surrounding humidity [3], [11], [12], [13].

Dynamical behavior of slowly driven granular materials was investigated recently by different groups both in compression and/or in shear experiments [3], [5], [8], [14], [15]. Here, we investigate the rheology of a granular assembly confined in a cylindrical column and pushed vertically from the bottom. The resistance to vertical motion as well as the blocking/unblocking transitions, reveal a phenomenology possibly shared by many confined granular assemblies, as for example, gouge sheared between two faults [16], pipe flows,
compaction under stress or dense granular paste extrusion. A previous investigation of the same display but in 2D [15] has already shown a rich phenomenology. For 3D granular assemblies, we get similar behavior but in this letter, we choose to report only on the most simple dynamical situation involving low friction grains confined in a column with rather frictional walls. Here, we have a weak coupling with dilatancy effects due to shearing at the walls and therefore, relations between the granular nature of the bulk (i.e. the stress redirection) and the solid friction properties at the walls are the most clearly revealed.

The grains are dry, non cohesive and monodisperse steel beads of diameter $d = 1.58 \text{ mm}$ piled into a vertical cylinder in duralumin of diameter $D = 36 \text{ mm}$. The column is closed at the bottom by a movable piston avoiding contact with the column (diameter mismatch is $0.5 \text{ mm}$). A force probe of stiffness $k = 40000 \text{ N.m}^{-1}$ is located under the piston and is pushed at a constant driving velocity $V$ (between $5 \text{ nm.s}^{-1}$ and $100 \text{ µm.s}^{-1}$) via a stepping motor (see inset of fig.1a). The resistance force $F$ encountered by the piston is measured as a function of time. We monitor also the relative humidity ($RH$) and the surrounding temperature. The central question here is an attempt to estimate the relative influence of the bulk mechanical properties with respect to frictional properties of the walls. To address directly this issue we built a special device (the slider) designed to apply a constant normal load ($F_N = 2 \text{ N}$) on three steel beads sliding vertically on the cylinder’s wall (see inset of fig.3a). Then, the dynamical evolution of the resistance force encountered by the piston pushing the grains is compared with the slider’s friction resistance driven in the same conditions. We observe two distinct regimes(fig.1a): for high driving velocities, the motion is characterized by a steady-sliding and a constant pushing force; for low velocities, the system undergoes a dynamic instability and then a stick-slip motion occurs. The transition between these behaviors is similar to the inertial regime of Heslot et al. [10] and details will be reported elsewhere.

For a vertically pushed granular assembly, the driving force exerted by the piston is screened. To evaluate this effect, the mean resistance force is measured as a function of the packing height (see fig.1b). For this dataset the driving velocity $V$ corresponds to a steady and continuous sliding of the grains. The resistance force $F$ increases very rapidly with the column’s height $H$. This strong resistance to motion is due to the horizontal redirection of stresses in association with solid friction at the side walls. Following the standard Janssen’s screening picture[9], [17], the force $F$ exerted by the grains on the piston can be modelled by the relation:

$$F = \rho g \lambda \pi R^2 \times \epsilon (\exp(\frac{H}{\lambda}) - 1)$$

where $\rho$ is the mass density of the granular material, $R$ is the cylinder radius and $g$ the acceleration of gravity. The length $\lambda = R/2K\mu$ is the so-called effective screening length, where $K$ is the Janssen’s parameter rendering the average horizontal redirection of vertical stresses and $\mu$ can either be the dynamic or the static coefficient of friction of beads at the cylinder’s wall. When $\epsilon = +1$, friction is fully mobilized downwards (our pushing experiment) and when $\epsilon = -1$, friction is fully mobilized upwards. It is easily seen from (1) that when $\epsilon = +1$, any slight change in $\mu$ or $K$ is exponentially amplified
with a drastic influence on the pushing force $F$. In the case of steel beads, we found that, starting from a dense or a loose packing, the final average steady state compacity $\bar{\nu}$ does not change; we have $\bar{\nu} \approx 62.5\%$ for all velocities and relative humidities $RH$ tested. In the steady state regime, the experimental data obtained for a given pushing velocity $V$ can be fitted with relation (1) by adjusting only one parameter $p+1 = K \times \mu_d$ where $\mu_d$ is the dynamic coefficient of friction at velocity $V$. For the relative humidity $RH = 42\%$, we obtain $p+1 = 0.140 \pm 0.001$ at $16 \mu m.s^{-1}$ and $p+1 = 0.146 \pm 0.001$ at $V_{up} = 100 \mu m.s^{-1}$. As a check of consistency, we performed the following dynamical experiment. First, the granular column is pushed upwards in order to mobilize the friction forces downwards and to reach the steady state compacity. Starting from this situation, the friction forces are reversed at the walls by moving the piston downwards at a constant velocity $V_{down} = 16 \mu m.s^{-1}$. Following relation (1), this procedure should imply a change of $\epsilon$ from 1 to $-1$, and consequently, the dynamical force on the piston should decrease from $F_{+1}$ to $F_{-1}$. In fig.1b the pushing force $F_{-1}$ is measured for different packing heights $H$. Injecting the preceding value of $p_{+1}(16 \mu m.s^{-1})$ into (1) with $\epsilon = -1$, we check on fig1b, that the theoretical expectation agrees quite well with the experimental data of $F_{-1}$ versus $H$. Note that in a previous study it was found that the Janssen's picture has a general tendency to underestimate the stress below a granular column [9]. But in the present situation, with low friction steel beads, this model though elementary, seems a good base for analysis. Nevertheless, a question is still that the fitting parameters $p = K \times \mu$ extracted from the model are unable to sort between what comes out from wall-bead interactions ($\mu$) and what comes out from bulk properties ($K$). Actually, from series of static Janssen experiments we extracted $K\mu_s$. Independently, the static coefficient of friction $\mu_s$ of our steel beads on duralumin was measured (in the short time limit) using the sliding angle of a three beads tripod. We get $\mu_s = 0.170 \pm 0.005$ and $K = 1.08 \pm 0.05$ is extracted from this procedure. This $K$ value is consistent with previous measurements done on a granular column at this compacity[9]. Note that, if we tentatively assume a constant value for $K$ in static and dynamic experiments, the dynamical coefficient $\mu_d$ can be extracted from the measured values of $p_{+1}$. For instance, for $RH = 42\%$ we extract the values, $\mu_d(16\mu m.s^{-1}) = 0.130 \pm 0.005$ and $\mu_d(100\mu m.s^{-1}) = 0.135 \pm 0.005$. This would imply a slight increase of the bead/wall friction with the driving velocity. This result is going to be directly tested in the following, using the three beads slider device.

For a given height $H$ of beads ($M = 380g$ i.e. $H = 4.3R$), we study extensively how the pushing force depends on the driving velocity $V$ and on the surrounding relative humidity ($RH$). We worked in the range $35\% < RH < 75\%$, and also in dry air ($RH < 3\%$). Note that except for the dry situation, we did not have a mean of regulation of this last parameter ($RH$) but we record its values around the experimental set-up. All data are shown on a same series for similar humidity values (within 3%). As it was already mentioned, the motion is characterized by a steady sliding above a critical velocity (fig.1a). The mean force level in this regime increases slowly with velocity but surprisingly strongly with $RH$ (fig.2a). For example at $V = 100 \mu m.s^{-1}$, the resistance force is raised by $35\%$ for a change of $RH$ from $53\%$ to $72\%$. Now we perform the same series of experiment but with the three beads slider, in order to test directly the wall/bead friction properties. At
a given RH, we indeed observe velocity strengthening for the sliding of individual steel beads, corroborating qualitatively the general trend observed on the granular column. But now, we may go one step further by testing directly the possibility of a quantitative agreement within the Janssen’s model. If we compare these data to the values of μ extracted from (1) (see fig.3a), assuming the static value of \( K = 1.08 \pm 0.05 \) for all velocities, we observe that the increase of \( \mu \) with \( V \) is significantly less important in the case of the granular column than what is directly measured using the slider device. Actually, if we suppose a logarithmic increase of \( \mu \) with velocity \( V \), \( \mu \sim b \times \log(V) \), we find, at RH = 40%, \( b = (2.7 \pm 0.2) \times 10^{-2} \) for the slider and \( b = (1.2 \pm 0.2) \times 10^{-2} \) in the case of the granular column; in dry conditions, we find \( b = (2.4 \pm 0.2) \times 10^{-2} \) for the slider and \( b = (1.1 \pm 0.2) \times 10^{-2} \) in the case of the granular column. It means that the increase of \( F \) with \( V \) cannot be entirely attributed to friction effects at the walls, and that the dynamics may have also an effect on force transmission (i.e. on \( K \)). In the framework of a Janssen’s analysis it would mean that \( K(V) \) would slightly decrease when velocity increases. Using a simple Hertz law to estimate contact interactions, we find the depths of penetration of steel beads in duralumin to be around \( \delta \approx 30\text{nm} \), whereas in the slider case, we estimated \( \delta \approx 1\mu\text{m} \) which is the order of duralumin roughness. Therefore, it is also possible that contacts are not both in the same loading regime and then friction laws could be slightly different. Importantly, we have also found that an increase of humidity has quite a strong influence on the friction properties (fig.2a). Using the inverted Janssen’s model (eq. (1)), when assuming the redirection parameter \( K \) unchanged by humidity, we recover that the dependence on RH in the slider experiment is consistent with the enhancement of the friction forces measured in the granular column. In a future series of experiments, we will try to bridge the gap to controlled values of humidity close to 100%.

Now let us consider slow driving velocities where the system undergoes a dynamical instability. A stick-slip motion occurs (see fig.1a) with a narrow gaussian distribution of slip force amplitudes. In fig.2b, we display the mean maximum and mean minimum resistance forces (resp. \( F_{\text{max}} \) and \( F_{\text{min}} \)) as a function of the driving velocity, for \( m = 380g \) of beads (height \( H = 7.7\text{cm} \)), and relative humidity RH = 45 ± 3%. The mean amplitude of the slip events \( \Delta F = F_{\text{max}} - F_{\text{min}} \) increases strongly when velocity reaches values as small as 5 nm.s\(^{-1} \). We propose a model where this enhanced blocking effect can be simply interpreted by an aging effect of the contacts at the side walls. Friction coefficients of solid on solid contacts are known to evolve logarithmically with waiting time \( t \) [18]: \( \mu_s(t) = \mu_s^0 + \beta_s \log(t) \). According to fig.2a, we observe no noticeable variation of \( F_{\text{min}} \) with velocity, for given height and RH. Therefore, we will consider in the following \( F_{\text{min}} \) to be a constant. Starting at the onset of blocking \( t = 0 \), the force exerted by the force probe during a stick event is \( F(t) = F_{\text{min}} + kVt \). The time elapsed during a sticking event is:

\[
t_{\text{stick}} = \frac{F_{\text{max}} - F_{\text{min}}}{k \times V}.
\] (2)

The slip occurs when \( F(t) \) reaches the maximum force sustainable by the granular material at time \( t \), given by (1) with \( \epsilon = +1 \). The aging properties of the friction at the wall are included in the time evolution of the static coefficient of friction \( \mu_s(t) \). Then we
write \( F(t_{\text{stick}}) = F_{\text{max}} \), i.e.:

\[
F_{\text{max}} = \frac{\rho g \pi R^3}{2K(\mu_s^0 + \beta_s \log(t_{\text{stick}}))} \\
\times \left( \exp(2K(\mu_s^0 + \beta_s \log(t_{\text{stick}}))) \frac{H}{R} - 1 \right).
\] (3)

This exponential amplification of the logarithmic aging, due to stress redirection at the walls, gives an effective power-law: \( F_{\text{max}} \sim t_{\text{stick}}^\alpha \), with \( \alpha = \frac{2\log(e) HK\beta_s}{R} \). On fig.3a, for \( RH = 45 \pm 3\% \), we display \( \mu_s \) extracted from (3) as a function of the time of stick. We assume \( K = 1.08 \) independent both of the waiting time and the driving velocity. We actually observe a logarithmic aging for waiting times \( \sim 3000\text{s} \), with a coefficient \( \beta_s = 1.8 \times 10^{-2} \pm 2 \times 10^{-3} \), value consistent with many previous reports\(^{[18]}\); in the last decade, aging is strongly increased and we have \( \beta_s \approx 6 \times 10^{-2} \). Note that our experiment is not a “clean” aging experiment since the applied loads and the shear forces are not constant in time and along the vertical direction. Furthermore and consistently with the finding of refs\(^{[6]}\) and\(^{[11]}\), we clearly observe that the aging properties are strongly affected by a variation of the relative humidity \( RH \) (see inset of fig.2a).

In conclusion, we investigated the dynamical behavior of a granular column pushed vertically from the bottom. This model experiment is suited to understand the rheology of slowly driven granular assemblies in confined geometries. Overall, the pushing force data are analyzed consistently using an inverted Janssen’s law. At such slow driving velocities we show that, all the non trivial dynamical properties exhibited by the granular rheology (including a strong dependence on relative humidity) can be dominantly attributed to the dynamical properties of solid on solid friction. In addition, the model seems to indicate the presence of a dynamical structural effect induced in the bulk at higher driving velocities.

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Figure 1: a: Resistance force versus the displacement of the stepping motor for 380 g steel beads in a duralumin cylinder, for 45% RH and for 2 velocities: $V = 30 nm.s^{-1}$ (stick-slip regime), and $V = 100 \mu m.s^{-1}$ (steady-sliding regime) shifted by +5N; the inset is a sketch of the experimental display. b: Mean force in the steady-sliding regime for $V_{up} = 16 \mu m.s^{-1}$ (squares) and for $V_{down} = 16 \mu m.s^{-1}$ (triangles) as a function of the height of beads; the lines are the fits according to (1); the dotted line is the hydrostatic curve.
Figure 2: a: Mean force in the steady-sliding regime as a function of velocity for 380g of steel beads in a duralumin cylinder and for several RH (< 3% (circles), 40% (squares), 53% (diamonds), 66% (down triangles), and 72% (up triangles)). b: $F_{\text{min}}$(circles) and $F_{\text{max}}$(squares) in the stick-slip regime as a function of velocity for 380g of steel beads in a duralumin cylinder and for $RH = 48\%$; the inset shows the variation of $\Delta F = F_{\text{max}} - F_{\text{min}}$ with RH for $V = 50mm.s^{-1}$.
Figure 3: a: Dynamic coefficient of friction as a function of velocity for the slider (filled symbols) and for the granular column (empty symbols), for $RH = 40\%$ (circles) and $RH < 3\%$ (squares); the inset shows the slider, a constant normal load is applied on the beads by the way of leaf springs. b: Static friction coefficient as a function of stick time for 380g of steel beads in a duralumin cylinder and $RH = 45\%$. 

Ovarlez et al. Fig.3