NEW APPROACH OF CONTROLLING CARDIAC ALTERNANS

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ABSTRACT. The alternans of the cardiac action potential duration is a pathological rhythm. It is considered to be relating to the onset of ventricular fibrillation and sudden cardiac death. It is well known that, the predictive control is among the control methods that use the chaos to stabilize the unstable fixed point. Firstly, we show that alternans (or period-2 orbit) can be suppressed temporally by the predictive control of the periodic state of the system. Secondly, we determine an estimation of the size of a restricted attractions basin of the unstable equilibrium point representing the unstable regular rhythm stabilized by the control. This result allows the application of predictive control after one beat of alternans. In particular, using predictive control of periodic dynamics, we can delay the onset of bifurcations and direct a trajectory to a desired target stationary state. Examples of the numerical results showing the stabilization of the unstable normal rhythm are given.

1. Introduction. Nonlinear dynamics and tools from chaos theory are used to understand, to characterize and to control some cardiac pathologies [2,13,21,29,37,47]. From dynamic point of view, different states of the cardiac rhythm are qualified by the steady state, the periodicity and the chaos [4]. The chaos in the cardiac system was widely investigated in low dimension [12, 20, 25, 29, 38]. Spatiotemporal chaos exists in two and three dimensions of excitable media and it has been seen in the breakup of spiral and scroll waves and it is considered as one of the principal mechanisms of cardiac fibrillation (ventricular or atrial) [1,10,14,30,34,48,49]. Many studies deal the understanding the nonlinear dynamics of ventricular fibrillation. This particular arrhythmia is generated by the loss of stability of the periodic rhythm, namely alternans, due to a rapid pacing of the cardiac tissue [2,21,24,27,29,31,43]. The spatiotemporal chaotic behaviors have been studied for cardiac alternans in one dimension [16].

Study and treatment of cardiac arrhythmia was among the most thrilling and promising among the early applications of chaos control. Aperiodic or chaotic nature of the dynamics of ventricular fibrillation suggests using the methods of control

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of chaos. Among the pioneers of this approach, Garfinkel et al. [18] have used control algorithms based on the OGY method. Control methods using external electrical stimulation are applied to alternans and irregular heart rhythms in order to recover normal rhythm [46]. Mathematically, the control is performed with small and adapted perturbations on the system parameters in order to lead a chaotic behavior to the equilibrium point. In cardiac dynamics, the most accessible system parameters available for perturbation is usually the interval between successive stimuli or the timing of the next excitation, which can be advanced or (in some situations) delayed through low-magnitude current stimulation [41, 46].

From the perspective of chaos theory, chaotic attractors contain an infinite number of unstable periodic orbits of any desired period [18, 27, 46]. Furthermore, the chaotic dynamic is ergodic: The chaotic trajectory visits the vicinity of each one of the unstable periodic orbits which may correspond to a desired system’s performance depending on some conditions [6, 32, 33, 40, 41]. This offers the possibility for controlling chaos, so when the chaotic orbit approaches to the desired unstable periodic orbit, it can be attracted and maintained on the orbit by applying small perturbations on the system [30, 32, 41]. In literature [5, 7, 8, 19, 26, 32, 33, 36, 40–42, 44], among the best known historical methods of control should be mentioned those that have been the most actively developed:

- The OGY (Ott-Grebogi-Yorke) method: It is based on the fact that a considerable change in the behavior of a chaotic system can be obtained through a very small change in one or more of its parameters [32]. It was the first technique applied to control cardiac rhythms, precisely to stabilize the aperiodic ventricular tachycardia dynamics in a rabbit cardiac muscle. So during chaos control, a perturbation is applied to vary the interbeat intervals [27] and the irregular chaotic dynamics was controlled and replaced by periodic rhythms, typically with a period-3 or period-4 rhythms. Hence, this way of perturbation prevented from having the period-1 rhythm [18, 19, 45, 46], and was better than acting on the amplitude of the signal [6].

- The time delayed feedback control (DFC or Pyragas’s method) is based on the feedback state control: The control law is calculated from the difference between the current state and the state with a time delay $T$, where $T$ represents a period of the orbit to be stabilized [36]. The state controlled by this technique converges to the desired orbit and not to an approximation, as is the case with the OGY method [26]. The advantage of this method (DFC) lies in the fact that the approximations are not used in the state feedback [5].

Few other methods for controlling the chaotic processes of cardiac activity have been proposed in the literature. A method based on the one step linear time-delayed feedback was proposed in [9] for suppressing the pathological rhythm of period 2. A method of controlling the distributed processes of wave instability in cardiac tissues on the basis of time-delayed feedback was proposed in [39]. Ushio and Yamamoto [44] have proposed the predictive control method based on both the principle of the Pyragas’s method and on the prediction of the states of the uncontrolled system. The control law is calculated from the difference between the current state and the future state of the chaotic system. Recent development of predictive control method was proposed by Boukabou [7, 8] for controlling continuous-time chaotic systems.

The point to be emphasized is that the speed with which the heart tissue is excited mainly characterizes the action potential. Particularly, conduction velocity and AP Duration (APD) depend on one or several previous diastolic or interbeat intervals. This rate-dependence, called restitution, is an important determinant of
The stability of conduction \cite{24}. A specific cardiac strategy called Adaptive Control of Diastolic Intervals (DI) to control the duration of Action Potential (APD) alternans is proposed by Jordan and Christini \cite{27}. Specifically, they use the restitution properties of cardiac action potential duration (APD) current as a function of previous (DI). This cardiac paradigm control is effective to direct the periodic and aperiodic rhythms to the equilibrium point or normal rhythms, and it can be applied to control noisy rhythm \cite{10}. The one-dimensional map, which models the (APD) given by the precedent one $APD_{i+1} = f(APD_i)$ has the same dynamics of a partial differential equation thus providing not only qualitative information (i.e. predicting bistability, period-doubling bifurcation, chaos, . . . ), but good quantitative information with the simulation over a large range of pacing period \cite{29}.

The aim of this paper is twofold: Firstly, to obtain the control of the alternans through the one dimensional map using predictive control method, and secondly to obtain an estimation of the size of a restricted basin’s attraction of the unstable equilibrium point representing the unstable regular rhythm, stabilized by the control. The paper is organized as follows: After this introduction (section 1), the one-dimensional map model is described in the section 2. The section 3 is devoted to the development of our mathematical approach allowing the estimation of the optimal control distance, the time that must pass before the activation of the control and the influence of noise on the effectiveness of control. Finally, the control process proposed in this paper is discussed in the section 4.

2. The one-dimensional map model (APD).

2.1. Example of simulation of Beeler-Reuter model in a 1D. The effects of a periodic stimulation of the cardiac tissue have been investigated in many papers \cite{27,29,31} using cable simulations (or the response of one dimensional cable equation to a periodic stimulation) based on ionic model as Beeler-Reuter (BR) model \cite{3}. The figure 1 shows some results corresponding to the simulation of the BR model in a one dimension (1D). Assuming that a $N : M$ rhythm ($N \geq 1$, $M \geq 0$) is periodic with $N \cdot t_s$ periods \cite{24,29} containing the repeating $N : M$ cycles, each exhibiting $N$ stimulus pulses and $M$ action potentials or $M$ beats. Under low amplitude, the response of cardiac tissue is similar for each stimulus (see Fig. 1(a)). This rhythm is noted $1 : 1$, it is the equilibrium point (or period-1 orbit), where the first 1 indicates one stimulus and the second 1 indicates one beat or response. If the stimulation frequency is increased more than some critical values, the cardiac dynamic evolved into cycle (or period-2 orbit) noted that $2 : 2$. It is mentioned to as the alternans, which mean that the APD oscillates between two values. In particular, beat-by-beat the long interval of APD is still followed by the short one (see Fig. 1(b)). The rhythm $1 : 1$ became unstable via a double period bifurcation. Still increasing the frequency, the $2 : 2$ rhythm loss its stability, generating the $2 : 1$ rhythm (see Fig. 1(c)). There are another periodic-2 orbits, noted as $2N : 2$ rhythms($N > 1$).

It will appear two different action potentials which alternate but the size of the range of $t_s$ over which we see that any two different $2N : 2$ rhythms ($N > 1$) will be different, and the range of APD encountered over these two ranges of $t_s$ will be also different. Otherwise, over the value of $t_s$ decreases, over the range of the $2N : 2$ rhythms stability decreases. These cycles rapidly loses their stability since the dynamics becomes very complex \cite{31}. In \cite{24,29}, it has been proved that although the alternans and periodic-2 orbits or $2N : 2$ rhythms ($N > 1$) have regular behavior, they correspond to pathological rhythms, in fact they generate
Figure 1. The time evolution of transmembrane action potential using cable simulations (Beeler-Reuter model) for a periodically paced cell: (a) the response 1:1, (b) the presence of alternans 2:2, (c) the response 2:1, (d) the irregular response. APD means Action Potential Duration, DI means Diastolic Interval.
immediately irregular chaotic rhythms (see Fig. 1(d)). For the 4:2 rhythm, which is a period-doubling of the 2:1 rhythm, we have two action potentials that would be separated by a blocked beat (See Fig. 1(d)), which is not the case in alternans (i.e., 2:2 rhythm). Similarly, there is a period-doubling on the map from 3:1 to 6:2 rhythms, and for the voltage waveform, we see that there is an alternation in the APD. These two (APs) are separated by two blocked beats, which is not the case in alternans (i.e., 2:2 rhythm).

2.2. Description of the one-dimensional map model. The figure 1 shows that electrical stimulations applied at a regular time intervals \( t_s \) generate an action potential depending on some conditions (state of cells, amplitude, duration and frequency of the stimulus [48]). At arbitrary \( t_s \), the current duration of any given action potential is given by the previous one [29]:

\[
APD_{i+1} = A - B_1 \exp \left( \frac{APD_i - nt_s}{\tau_1} \right) - B_2 \exp \left( \frac{APD_i - nt_s}{\tau_2} \right)
\]  

(1)

Where \( APD_{i+1} \) is the APD of \((i+1)st\) action potential, and let \( n \) a parameter block in the production of an action potential on condition that \((nt_s - APD_i > DI_{\text{min}})\). The constants \( A, B_1, B_2, \tau_1, \tau_2 \) are related to the heart electrophysiological constraints defined in [29]: \( A = 270 \text{ ms}, B_1 = 2441 \text{ ms}, B_2 = 90.02 \text{ ms}, \tau_1 = 19.60 \text{ ms}, \tau_2 = 200.5 \text{ ms}, \) and \( DI_{\text{min}} = 53.5 \text{ ms} \). The equation (1) is called the one dimensional map model. This one is used in order to control the dynamics of the action potential in our model. The dynamics of the one dimensional map model has been graphically studied in [29]. When the stimulation frequency \( t_s \) decreases from the value 400 ms to the value of 25.1 ms with increment equal to 0.1 and iteration starting from initial condition \( APD_1 = 240 \text{ ms} \). One obtains the following sequence rhythms: \([1:1 \rightarrow 2:2(\text{alternans}) \rightarrow 2:1 \rightarrow 4:2 \rightarrow \text{chaos} \rightarrow 3:1 \rightarrow 6:2 \rightarrow \text{chaos} \rightarrow 4:1 \rightarrow 8:2 \rightarrow \text{chaos} \rightarrow 5:1 \rightarrow 10:2 \rightarrow \text{chaos} \rightarrow 6:1 \rightarrow 12:2 \rightarrow \text{total chaos}]\). The bifurcation diagram (see Fig. 2) shows the different dynamics of the system.

2.3. Determination of unstable fixed point of map. The purpose is to use the predictive control to direct the periodic rhythm (or period-2 orbit) to the \( N : 1 \) rhythm, \( N \geq 1 \) (or period-1 orbit). So it is necessary to know the unstable
equilibrium point value producing alternans. If we consider $t_s = 302 ms$ (see Fig. 2) the rhythm 1 : 1 (or equilibrium point) loses its stability, alternans 2 : 2 (or periodic-2 orbit) occurs and the APD oscillates between two values $APD_1^* = 204.9 ms$ and $APD_2^* = 197.3 ms$ (see Fig. 3). Therefore the objective is to direct the 2 : 2 rhythm into 1 : 1 rhythm. For $t_s = 302 ms$, the unstable fixed point value of 1 : 1 rhythm $APD^*$ satisfies the following equation:

$$f(APD^*) = APD^*$$  (2)

On the iteration interval $I = [0, 270]$ we use the dichotomy method [15] to find an approximation of the fixed point $APD^*$ and we obtain: For $t_s = 302 ms$, $APD^* \simeq APD_{12} = 201.2306 ms$ is the approached value of the unstable fixed point (or 1 : 1 unstable rhythm $|APD^* - APD_{12}| < 10^{-5}$)

3. Controlling alternans with predictive algorithm. Controlling the transmembrane potential by varying the excitation interval may not be sufficient to control the alternans [13]. Our objective is to lead the alternans (or periodic orbit) to a desired unstable normal rhythm (or unstable fixed point) by applying small perturbation to the APD values. Generally a periodic orbit does not have to visit the vicinity of the desired unstable fixed point as does a chaotic trajectory. Before activating the predictive control algorithm, we will firstly determine a periodic trajectory (or a cycle) and the size of the restricted attraction basin of the unstable equilibrium point.

3.1. The mathematical approach. Let the nonlinear one-dimensional map defined by the flow $f$ of class $C^1$. Let $I$ an invariant part of $\mathbb{R}$ with $f$, such as:

$$APD_{i+1} = f(APD_i) + u(i)$$  (3)

The iterate $APD_i$ is a state of the phase space $I$ and $u(i)$ is the control function applied to the one-dimensional map of APD (1).

The control value $u(i)$ is equal to zero when the map generates periodic dynamics and the control $u$ is activated when the uncontrolled predicted state $APD_{i+1}$ enters in the restricted attraction basin of the target fixed point $APD^*$. After carrying
out the asymptotic convergence of the desired fixed point, the control value will become zero. In the predictive control method, the control law \( u(i) \) is determined by the difference between the current state and the uncontrolled predicted state and then added as a control parameter to the system. The formula of control \( u \) is given by [44]:

\[
u(i) = K \left[ (APD_i)_p - APD_i \right]
\]

(4)

To stabilize an unstable equilibrium point of the periodic one-dimensional map of APD, it is necessary to determine the feedback control gain \( K \). Using \( f(APD_i) \) as a prediction of the fixed point to be stabilized. The predicted state uncontrolled \((APD_i)_p\) is given by:

\[
(APD_i)_p = f(APD_i) = APD_{i+1}
\]

(5)

The control is determined by the following relationship:

\[
u(i) = \begin{cases} 
K \left[ (APD_{i+1}) - (APD_i) \right] & \text{if } d_i < \varepsilon \\
0 & \text{elsewhere}
\end{cases}
\]

(6)

With \( d_i = |APD_i - APD_{i-1}| \) is called “the control distance” and \( \varepsilon \) is the optimal control distance. When the system evolves, it is possible to estimate the distance between the point of the attractor, corresponding to the current state, and the desired trajectory. This distance changes over time. So for chaotic dynamics a sufficiently small value of this “control distance” \( d_i \), the control can be applied. However, for controlling periodic dynamics, it is important to estimate this distance. Hence \( \varepsilon \) must be properly estimated for an effective activation of control. Before activating the control of the periodic dynamics, \( \varepsilon \), the distance between two periodic points \( APD^*_1 \) and \( APD^*_2 \), is determined.

We propose the estimation of the optimal control distance \( \varepsilon \), this last one defines the size of the restricted attraction’s basin of the desired unstable fixed point,

\[
\varepsilon = |APD^*_1 - APD^*_2|
\]

(7)

Let \( \Delta(APD(i)) = |APD_{i+1} - APD_i| \)

\[
u(i) = K \Delta(APD(i))
\]

(8)

The controlled map model will be given by:

\[
APD_{i+1} = f(APD_i) + K \Delta(APD_i)
\]

(9)

The linearization of the equation (9) around the unstable fixed point \( APD^* \) is given by:

\[
\delta APD_{i+1} = J^* \delta APD_i
\]

(10)

Assuming that \( J \) is the Jacobian of uncontrolled map (1) at the unstable fixed point \( APD^* \),

\[
J = \left[ \frac{\partial f}{\partial APD_i} \right]_{APD^*}
\]

(11)

then:

\[
J^* = K \left[ \left[ \frac{\partial f}{\partial APD_i} \right]_{APD^*} - 1 \right] + \left[ \frac{\partial f}{\partial APD_i} \right]_{APD^*}
\]

(12)

The map stabilizes around a desired fixed point by the predictive control if the gain \( K \) value satisfies the following inequality control:

\[
|J^*| < 1,
\]

(13)
avec $K \in ]-1,0[$

In the following section, some results obtained from numerical simulations are shown in order to validate the theoretical approach proposed in our work.

![Figure 4.](image)

**Figure 4.** Initiation of predictive control of the alternans 2 : 2, (a) after 838 beats of alternans, (b) after only one beat of alternans. After control is initiated in figure (a) or (b), $APD_n$ alternates around $APD^*$ as the asymptotic stability of 1 : 1 rhythm is performed.

At $t_s = 302 \text{ ms}$, there is bistability between two rhythms \{2 : 2 ↔ 2 : 1\} (See Fig. 5) in which the interval [42 ms, 248.5 ms] is the basin attraction of the alternans 2 : 2, and the interval [248.6 ms, 270 ms] is the basin attraction of the normal rhythm 2 : 1. When the alternans is suppressed by the predictive control, the interval [42 ms, 248.5 ms] will be the basin attraction of the desired normal rhythm 1 : 1.

3.2. **Numerical results.** To stabilize the unstable equilibrium point (or 1 : 1 unstable rhythm) with the value $APD^* = 201.2306$ ms at $t_s = 302$ ms, the gain
Figure 5. Bifurcation diagram (APD<sub>i</sub> vs. t<sub>s</sub>) without control for t<sub>s</sub> = 200 − 400 ms. At each t<sub>s</sub>, the map (1) was iterated 20000 times and the first 19800 iterates discarded to suppress transients due to initial conditions. Increment in t<sub>s</sub> was 0.1 ms, APD<sub>1</sub> = 198 ms.

Figure 6. Bifurcation diagram (APD<sub>i</sub> vs. t<sub>s</sub>) with predictive control for t<sub>s</sub> = 200 − 400 ms. At each t<sub>s</sub>, the controlled map (3) was iterated 20000 times and the first 19800 iterates discarded to suppress transients due to initial conditions. Increment in t<sub>s</sub> was 0.1 ms, APD<sub>1</sub> = 198 ms, ε = 7.7 ms, K = −0.2.

value K must be choosen from the interval K ∈ ]−1, 0[. Suitable condition values are given in the Table 1, starting iteration from arbitrary initial condition from the basin of attraction [42 ms, 248.5 ms] (see Fig. 4(a)) and from the restricted
Figure 7. Initiation of predictive control of periodic rhythm 6 : 2, (a) after 2 beats, (b) after 1 beat. After control is initiated in Fig(a) or (b), $APD_n$ alternates about $APD^*$ as the asymptotic stability of 6 : 1 rhythm is performed.

attraction’s Basin $[197.3 \, ms, 205.05 \, ms]$ (see Fig.4(b)). In Fig. 4(a), after 838 beats of alternans, the state $APD_{838}$ is in the restricted attraction’s basin of the unstable fixed point $APD^*$ (or 1 : 1 unstable rhythm). The value of $u(838) = 0.8999 \, ms$, and the asymptotic stability is performed on the fixed point with the value $APD^* = $

| K   | $\varepsilon$ | $APD_1$ |
|-----|---------------|---------|
| -0.1| 9 ms          | 240 ms  |
| -0.1| 7.7 ms        | 198 ms  |

Table 1. Example of parameter values $K$, $\varepsilon$ and $APD_1$ for the stabilisation of the unstable equilibrium point (or 1 : 1 unstable rhythm).
201.2 ms. However, in Fig. 4(b), only one beat is enough to detect the restricted attraction’s basin of the unstable fixed point $APD^*$ (or 1 : 1 unstable rhythm). Applying small perturbation to the state dynamics $APD_2$, such that $APD_2 = 204.3$ ms and $u(2) = 0.6303$ ms. After 23 beats, the unstable 1 : 1 rhythm will be stabilize asymptotically.

Furthermore, the bistability $\{2 : 2 \leftrightarrow 2 : 1\}$ will be replaced by the bistability $\{1 : 1 \leftrightarrow 2 : 1\}$ (See Fig. 6). Comparing these two following bifurcation diagrams (See Fig 5 and Fig 6), we will see that the effectiveness of predictive control to suppress the alternans 2 : 2 and stabilizing the unstable normal rhythm 1 : 1. When the $2N : 2$ rhythm, $N > 1$ begins to oscillate from the first iterations between two very close points (like the 6 : 2, 8 : 2 and 12 : 2 rhythm), the predictive algorithm is activated in the first iteration in the both cases the global or the restricted attraction’s basin. For example, at $t_s = 100.6$ ms, the periodic rhythm 6 : 2 is suppressed and the unstable 6 : 1 rhythm will stabilize asymptotically after five or six beats for an arbitrary initial condition from the global attraction’s basin $[42 \text{ ms}, 270 \text{ ms}]$ (see Fig.7(a)), or from the restricted attraction’s basin $[200.12 \text{ ms}, 202.08 \text{ ms}]$ (see Fig.7(b)), such that the optimal value’s distance of control is $\varepsilon = 1.96$ ms. We give the suitable gain value for controlling the periodic rhythm 6 : 2 in the Table 2.

| $K$ | $\varepsilon$ | $APD_1$ |
|-----|---------------|---------|
| -0.4 | 1.96 ms | 240 ms |
| -0.4 | 1.96 ms | 202 ms |

Table 2. Example of parameter values $K$, $\varepsilon$ and $APD_1$ for controlling the periodic rhythm 6 : 2.

3.3. Influence of noise. In the presence of an external noise, the control is then turned off and one must wait until another trajectory again enters the vicinity of the
Figure 9. Bifurcation diagram ($APD_i$ vs. $\varepsilon$) with predictive control for $\varepsilon = 0 - 15 \text{ ms}$. At each $\varepsilon$, the controlled map (3) was iterated 20000 times and the first 19800 iterates discarded to suppress transients due to initial conditions. Increment in $\varepsilon$ was 0.1 ms, $t_s = 302 \text{ ms}$, $APD_1 = 240 \text{ ms}$, $K = -0.1$.

desired unstable fixed point to reactivate the control. However, it should be noted that the noise does not affect the behavior of the system once it is checked. Tolerable noise is estimated relative to absolute parameters and must be considerably less than its equivalent in terms of relative uncertainty [7]. For example: at $t_s = 302 \text{ ms}$, the noisy alternans is suppressed for an arbitrary initial condition $APD_1 = 240 \text{ ms}$, the gain control value $K = -0.1$, the optimal distance of control $\varepsilon = 9 \text{ ms}$. The control will be activated after 101 beats by applying small and continue perturbations, $APD_n$ converges to the 1 : 1 rhythm as the stability is performed (See Fig. 8).

4. Conclusion and discussion. In this work, we investigate the control of the periodic rhythms of the one-dimensional map of (APD). This one gives the duration of the cardiac action potential (APD) current as a function of the previous one. The alternans is often seen just before the onset of malignant ventricular arrhythmias such as ventricular tachycardia (VT) or ventricular fibrillation (VF). An obvious thought is that if there is a causal relationship between the alternans and the arrhythmias, then getting rid of the alternans might prevent eventual emergence of the (VT) and (VF). A method of nonlinear predictive control is proposed to direct the periodic trajectory towards the restricted attraction’s basin of the unstable equilibrium point and to control alternans $2 : 2$ and $2N : 2$ rhythms, $N > 1$.

In the case of predictive control of chaos, we use an arbitrary infinitesimal “control distance” to control the chaotic trajectory. Furthermore, the time after activation of the control of chaos may be long. There was a problem to find the moments of activating of control method to ensure control of unstable fixed points. The solution that we propose precisely solves this problem. If we iterate an arbitrary initial condition from the restricted basin of attraction, the controlled trajectory will not visit the vicinity of each one of the unstable fixed point in order to close to a desired
unstable fixed point. Thus, the value of the control distance is determined carefully (see Figure 9). Therefore, in this case, the elapsed time before activating the control is zero. This developed algorithm for the cardiac control is effective to direct the periodic rhythms to the equilibrium point or normal rhythms, and it can be applied to control noisy rhythm. The very satisfactory numerical results argue in favor of the efficiency of the proposed method. Moreover, it is forecasted to generalize our results to dimensions greater than 1 by drawing on recent works [11,35].

The suppression of cardiac alternans has important clinical implications. The alternans in the cardiac rhythm often forerun life-threatening arrhythmias and lead to a risk factor for sudden death. If a control algorithm similar to the one proposed in the present study would be incorporated in a cardiac pacemaker, alternans rhythms might be suppressed and a route to a fatal arrhythmia curtailed.

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