On Quantum-Assisted LDPC Decoding Augmented with Classical Post-Processing

Aditya Das Sarma1, Utso Majumder1, Vishnu Vaidya2, M Girish Chandra3, A Anil Kumar3, Sayantan Pramanik2
1Jadavpur University, 2TCS Incubation, 3TCS Research

aditya.41200@hotmail.com, utsomajumder@gmail.com, vaidya.vishnu@tcs.com, m.gchandra@tcs.com, achannaanil.kumar@tcs.com, sayantan.pramanik@tcs.com

Abstract—Utilizing present and futuristic Quantum Computers to solve difficult problems in different domains has become one of the main endeavors at this moment. Of course, in arriving at the requisite solution both quantum and classical computers work in conjunction. With the continued popularity of Low Density Parity Check (LDPC) codes and hence their decoding, this paper looks into the latter as a Quadratic Unconstrained Binary Optimization (QUBO) and utilized D-Wave 2000Q Quantum Annealer to solve it. The outputs from the Annealer are classically post-processed using simple minimum distance decoding to further improve the performance. We evaluated and compared this implementation against the decoding performance obtained using Simulated Annealing (SA) and belief propagation (BP) decoding with classical computers. The results show that implementations of annealing (both simulated and quantum) are superior to BP decoding and suggest that the advantage becomes more prominent as block lengths increase. Reduced Bit Error Rate (BER) and Frame Error Rate (FER) are observed for simulated annealing and quantum annealing, at useful SNR range - a trend that persists for various codeword lengths.

Index Terms—LDPC code, Quantum annealing, Simulated annealing, Minimum distance decoding, QUBO.

I. INTRODUCTION

Low Density Parity Check (LDPC) codes are linear block codes originally proposed in the 1960s by Gallager in his seminal doctoral work. The name reflects the fact that the parity check matrix used in LDPC coding is sparse with low density of 1s in the matrix. The performance of the LDPC codes approach theoretically described capacity limits, and therefore are very powerful. LDPC codes have established themselves as appropriate candidates for wireless systems based on multi-antenna multi-carrier transmission. Suitably designed LDPC codes are also proven to be excellent candidates for Hybrid Automatic Repeat Request (HARQ) schemes. The success and the consequent popularity of the LDPC codes over the years has resulted in support and proposals for its utilization in various applications and standards. Some examples are DVB-S2 (2nd Generation Digital Video Broadcasting via Satellite), 5G New Radio (NR) access technology standards, recent revisions of the 802.11Wi-Fi protocol family and various storage applications. Practically utilizable codes should constitute certain favourable properties, especially low encoding and decoding complexities, good waterfall regions, low error floors and flexibility in the context of getting different rates and frame lengths. There are various code designs available, starting from the pseudo-random constructions to sophisticated algebraic and graph-based techniques. See [1], [2] and [3] and some of the original references therein for more details.

Good performance of LDPC codes can be achieved with a proper choice of code and decoding algorithm. Belief Propagation algorithms, like the Sum-Product algorithm are widely used in classical LDPC decoding. The Sum-Product algorithm can be viewed as a message passing algorithm operating on the Tanner graph, which is a bipartite graph representing the parity check matrix, and consisting of variable nodes and check (or constraint) nodes. Each iteration of the algorithm can be divided into two halves. In the first half, message is passed from each check node to all adjacent variable nodes and in the second, from each variable node to its adjacent check nodes. The decoding performance is achieved through multiple iterations of the message passing along the edges of the graph, until some stopping criterion is reached. In the direction of reducing the complexity of the (regular) Sum-Product algorithm, many variants of it have been proposed in the literature, one example being, min-sum algorithm (see [1] and the references there in for details).

Currently, we are in an exciting period in Quantum Technologies. With the intermediate-scale commercial quantum computers becoming increasingly available, Quantum Information Processing is witnessing spectacular developments (see [4], [5], [6] and the relevant references there in). Before quantum processors become scalable, capable of error correction and universality [5], the current and near-term devices, referred to as the Noisy Intermediate-Scale Quantum (NISQ) [6] devices are getting explored for solving certain hard problems to achieve significant speedups over the best known classical algorithms. Promising results are already reported for solutions in the areas like, optimization, machine learning and chemistry. Apart from speedup considerations, quantum mechanical properties of superposition, entanglement and interference are being explored for solving problems differently with possible performance improvements. In the NISQ era, the hybrid quantum-classical processing has established itself as an essential combination, and this “cooperation” will continue for a long time.

Considering the hardness and complexity of the some of
the important problems in the current and emerging Communication Systems, research efforts have been under way to explore Quantum Computing paradigms to solve them. Some references in this direction are [5], [7], [8], [9], [10] and [11], among many others. Needless to say, due to the present requirements of Quantum Computers (QCs), like dilution refrigerators to maintain superconducting cooling, the usage of QCs are targeted to the Centralized Data Centers (Radio Access Networks), see for example, [5] and [9]. In this paper, similar to some of the references mentioned in this paragraph, we would be considering the baseband processing, in particular the LDPC decoding (in fact, we use [7] as the starting point). The relevance of LDPC codes in modern wireless networks can be seen in the search for computationally efficient decoders and their ASIC/FPGA implementations in [7]. As a futuristic notion, it is also useful to see how Quantum Processing Unit (QPU) enhanced (or accelerated) processing together with the classical computation can be worked out to carry out some of the complex and computationally heavy processing at the data center.

It has been well established for the last few years that QCs can “naturally” solve the discrete combinatorial optimization problems. Many of these problems fall under the unifying model of Quadratic Unconstrained Binary Optimization (QUBO) (see [12]). One of the approaches to finding the solution to a QUBO formulation is to construct a physical system, typically a set of interacting spin particles (two-state particles) whose lowest energy state encodes the solution to the problem, so that solving the problem is equivalent to finding the ground state of the system. Two main approaches have been identified to find the ground state of interacting spin systems (quantum optimization) on NISQ processors [4], [5]: Quantum Annealing (QA) and Quantum Approximate Optimization Algorithms (QAOA) [13]. QA is an approximate or non-ideal implementation of Adiabatic Quantum Computing, which is an analog quantum computation. QA has been developed theoretically in the early nineties but realized experimentally in a programmable device by D-Wave Systems, nearly two decades later. A digitized version of Quantum Adiabatic Computing leads to QAOA, a gate-circuit based quantum computing.

In this paper, we have taken up the study of LDPC Decoding using Quantum Annealers similar to [7]. But, the following novelties are brought in. Keeping in mind the tandem working of Quantum and Classical computers, we have attempted to exploit the inherent randomness of the QCs and the outputs or the results of the “shots” are subjected to classical postprocessing to arrive at better inference (in particular, better decoding) performance. In this direction, instead of just picking up the minimum-energy solution as prevalent in the Quantum Computing literature, the different outputs are post-processed using simple minimum distance computation to the received codeword vector to arrive at the final decoding. This approach sets the direction to consider appropriate and more sophisticated post processing for Quantum-Enhanced baseband processing. We have taken this route to bring out a notion of diversity emerging from the shots. In fact, different outputs emerging from the shots are seen as a kind of “diversity”, which to the best of our knowledge are not interpreted this way in the existing literature. Preliminary results with length 32 and 96 rate half LDPC codes (14) demonstrate the improved performance of the quantum-enhanced decoders, even with these short lengths, over conventional Sum-Product Algorithm. The paper also spell out certain new remarks/observations about different formulations considered and is organized as follows:

In Section II, we capture aspects related to classical Sum-Product and Min-Sum algorithms, QUBO and Annealing (both Simulated and Quantum). Section III provides the details about the Proposed augmented method; results and the discussions are covered in Section IV.

II. BRIEF ELABORATION ON QUANTUM ANNEALER AND QUBO

A. D-Wave Quantum Annealer

Quantum Annealing (QA) is a metaheuristic for solving QUBO problems [15]. The adiabatic theorem of quantum mechanics states that Quantum Annealing, in a closed system, will find the final ground state encoding the solution, provided the annealing time is appropriately large compared to the inverse of the energy gap in quantum ground state. However, this does not guarantee that QA will always perform better than classical optimization algorithms, as the relative success of QA depends on the suitability of the optimization landscape to obtain an quantum advantage. D-Wave provides access to their devices which implement Quantum Annealing on Quantum hardware, through its cloud access provision Leap. Here, we are not capturing information on D-Wave Annealers, since nice documentation/information is available in their website. Also see [16] [17].

B. QUBO

The concept of a QUBO formulation is fundamental to utilizing a Quantum Annealer to solve a given optimization problem.

Let \( f : B^n \rightarrow R \) be a quadratic polynomial with \( q_i \in B = \{0, 1\} \) for \( 1 \leq i \leq n \):

\[
f_{\bar{a}}(x) = \sum_{i=1}^{n} \sum_{j=1}^{i} \alpha_{ij} q_i q_j \tag{1}
\]

The QUBO problem then consists of finding \( q^* \) such that:

\[
q^* = \text{argmin}_{q \in B^n} f_{\bar{a}}(q) \tag{2}
\]

The QUBO form of (1) can be written, separating the linear and quadratic terms, and noting that \( q_i^2 = q_i \), and setting \( \alpha_i = \alpha_{ii} \), as:

\[
f_{\bar{a}}(q) = \sum_{i=1}^{n} \alpha_i q_i + \sum_{i=1}^{n} \sum_{j=1}^{i} \alpha_{ij} q_i q_j \tag{3}
\]

\( \alpha_i \) is called the the bias of the variable \( q_i \), and \( \alpha_{ij} \) is called the bias/coupling of the quadratic term \( q_i q_j \).
Any optimization problem that we wish to solve with the QA, must first be formulated as a QUBO problem. We discuss the QUBO formulation of LDPC decoding in the next section.

III. PROPOSED APPROACH

The flowchart given in Fig. 1 summarizes the proposed solution approach. In the following sub-sections requisite details are elaborated.

A. Encoding

- To implement the LDPC encoding, we consider a valid parity matrix $H$ and the corresponding generator matrix $G$.
- For a randomly generated message $m$, codeword $c$ corresponding to $m$ is obtained by multiplying $c$ with the generator matrix $G$.

$$mG = c$$

where the multiplication is mod-2.
- To simulate the effect of the channel on the transmission of the codeword, we add Additive White Gaussian Noise (AWGN) to the transmitted codeword, to obtain the received signal $r$.

$$r = c + n$$

where $n \sim \mathcal{N}(0, \sigma^2)$. We can adjust SNR by adjusting the variance $\sigma$.

B. Decoding

- To decode the received signal $r$, we first put in place the corresponding QUBO formulation. The QUBO for $r$ is composed of two parts:
  1) Distance Metric: Let binary variable $q_i$ represent the $i^{th}$ bit of the decoded codeword. We compute the expectation of $q_i$ given the received symbol $r_i$, as $P(r_i = 1 | q_i)$. For an AWGN Channel with Binary Phase Shift Keying (BPSK) Modulation, this quantity, as given in [7], is:

$$Pr(q_i = 1 | r_i) = \frac{1}{1 + \exp \frac{2r_i}{\sigma^2}}$$

We expect that the transmitted codeword is “proximal” to the received signal. Therefore, to find the transmitted codeword, we seek to minimize the following Distance Metric $\delta$ that computes the proximity of a codeword to the received information:

$$\delta = \sum_{i=1}^{n} (q_i - Pr(q_i = 1 | r_i))^2$$

A minimum of (7) is an estimate of the transmitted codeword, computed with the quantities $Pr(q_i = 1 | r_i)$ alone.

2) Constraint Satisfaction Metric: The LDPC constraints ensure that the modulo-2 sum at each check node $c_n$ is 0. These equality constraints need to be incorporated into an objective function that can be minimized. We implement this with the following function. For each check node $c_i$, one can define LDPC satisfier function (see also [7]):

$$L_{sat}(c_i) = ((\Sigma_{j: h_{ij}=1} q_j) - 2L_e(c_i))^2$$

Through minimization of the above function, we can force the sum at that check node to be even: that is, force the modulo-2 sum at that node to zero. $L_e(c_i)$ is implemented with additional ancillary qubits. Next enters the Constraint Satisfaction Metric $L$:

$$L = \sum_{i} L_{sat}(c_i)$$

Minimizing $L$ would result in the satisfaction of the LDPC constraints at the check nodes.

Finally, we combine the two components with Lagrange weights $W_1$ and $W_2$, to compose the final QUBO. Minimizing the QUBO in general tends to minimize both the composite components. We can prioritize the minimization of one component over the other with a high choice.
for the Langrange weight for that component relative to the other. We have experimented with variations on $W_1$, keeping $W_2$ fixed at 1.0. The resulting QUBO is:

$$F = W_1 \delta + W_2 L$$  \hspace{1cm} (10)

- The QUBO is then passed to the D-Wave annealer. Several samples are collected by running the annealer multiple times.
- Valid codewords (codewords that satisfy LDPC constraints) are filtered out from the samples and then minimum distance decoding is performed with the received signal to obtain the final decoded codeword.

As can be seen from the above description this QA-based framework doesn't require message passing iterations typically used to perform LDPC decoding with classical BP algorithms. Instead, a Quantum Annealer implemented on real Quantum hardware "naturally settles" to the optimal state for the QUBO, thereby performing the LDPC decoding.

IV. EXPERIMENTAL RESULTS

Decoding was performed on LDPC parity matrices of dimensions (32, 16) and (96, 48), using quantum and simulated annealing, and classical Belief Propagation algorithms (see [14]). Quantum methods provide an inherent mode of diversity, due to its stochastic nature, giving different outputs for the same $r$, for different runs of the experiment. This advantage is not available for classical BP algorithms, which are deterministic in nature. In other words, for successive runs of the experiment, using the same $r$ results in different outputs due to the inherent randomness in quantum information processing. On the other hand, it is trivial to observe that the same output, and not the "different copies" of information related to the transmitted codeword. Of course, this benefit is coming because of the use of Quantum Computers.

A. Results for fixed SNR channel

For this scenario, different SNRs are considered for experimentation. For each SNR, the BER and FER estimate is obtained with $10^6$ Monte Carlo iterations. The term "fixed" refers to the fact that the SNR remains the same for all these $10^6$ "transmissions". Based on the four plots in Fig. 2, the following observations are evident:
In the moderate SNR regime, Quantum Annealing (QA) and Simulated Annealing (SA) perform better than the classical BP.

- At lower SNRs, performance of QA and SA is close to the performance of classical BP.
- However, a sharp drop is seen in BER, as well as in FER, around 7.5 to 8 dB range for both simulated and quantum annealing. When SNR reaches 10dB, the noise becomes small enough such that all the methods achieve the similar BER and FER.

In the limited amount of studies we carried out using two short codewords, Simulated Annealing performs slightly better than Quantum Annealing. It is to be noted that the QA results are obtained from the actual D-Wave Annealer, and these realistic machines do have imperfections ("noisy behavior") at present. Of course, as remarked earlier, both SA and QA performed better than classical BP.

B. Results for time-varying SNR

In order to simulate a time-varying SNR function and observe how the proposed approach performs in this case, the following procedure was undertaken and observations were recorded.

- For each of the 1000 codewords transmitted, the SNR is varied. In our experimentation, the samples have been drawn from the normal distribution with $\mu=5$, $\sigma=2$. A realization of the SNR is depicted in Fig. 3.
- It is again observed that simulated annealing has the highest fraction of correct codewords decoded, followed by quantum annealing and classical belief propagation, as given in Table I.

In this paper, we have just considered a time-varying SNR to assess the performance of the proposed methodology. In the direction of considering the more realistic scenarios, we are in the process of implementing complex-baseband processing with the Rayleigh fading channel. The possible modifications to QUBO formulation for this case is also envisaged.

The results for both fixed and time-varying SNR demonstrated the correct functionality of the QUBO formulation of the LDPC decoding augmented with post-processing which exploits the special diversity mentioned. Experimentations with longer codewords may bring out the beneficial aspects of the proposed approach compared to classical counterparts. Elaborating further, it is expected that the Quantum Computers, including the Annealers will only improve in terms of number of qubits, quality of the qubits, the connectivity between them, etc. They can then not only accommodate larger-sized problems (for instance, longer code lengths of practical importance, etc), but also naturally solve them with better performance and speed compared to the fully classical counterparts (both Simulated Annealing and the variants of Sum-Product). Additionally, the right integration of classical and quantum computing systems may result in useful energy savings as well [18].

V. CONCLUSION & FUTURE WORK

Classical post-processing assisted quantum annealing is proposed for LDPC decoding which exploits the stochastic nature of quantum computers to arrive at improved solutions at SNRs of practical relevance, when compared with classical BP decoding. Unlike classical BP decoding, iterations are not required for this QA-based approach. The physical nature of the quantum computing allows natural settlement to the lowest energy state, which in most cases is the optimal solution. Post-processing was based on minimum distance decoding schemes, this can be extended to more refined methods. There is plenty of scope for expanding this work for different baseband processing techniques in future, including different channels such as Rayleigh fading channel.

REFERENCES

[1] S. J. Johnson, Iterative Error Correction: Turbo, Low-Density Parity-Check and Repeat-Accumulate Codes. Cambridge University Press, 2009.
[2] B. Janakiram, M. G. Chandra, S. Harihar, B. Adiga, and P. Balamuralidhar, “On the usage of projective geometry based ldpc codes for wireless applications,” in 2009 7th International Conference on Information, Communications and Signal Processing (ICICS), 2009, pp. 1–5.
[3] J. H. Bae, A. Abotabl, H.-P. Lin, K.-B. Song, and J. Lee, “An overview of channel coding for 5g nr cellular communications,” APSIPA Transactions on Signal and Information Processing, vol. 8, p. e17, 2019.
[4] S. Pramanik and M. G. Chandra, “Quantum-assisted graph clustering and quadratic unconstrained d-ary optimisation,” 2021.
[5] M. Kim, D. Venturelli, and K. Jamieson, “Leveraging quantum annealing for large mimo processing in centralized radio access networks,” in Proceedings of the ACM Special Interest Group on Data Communication, ser. SIGCOMM ’19. New York, NY, USA: Association for Computing Machinery, 2019, p. 241–255. [Online]. Available: https://doi.org/10.1145/3341302.3342072
[6] J. Preskill, “Quantum Computing in the NISQ era and beyond,” *Quantum*, vol. 2, p. 79, Aug. 2018. [Online]. Available: https://doi.org/10.22331/q-2018-08-06-79

[7] S. Kasi and K. Jamieson, *Towards Quantum Belief Propagation for LDPC Decoding in Wireless Networks*. New York, NY, USA: Association for Computing Machinery, 2020. [Online]. Available: https://doi.org/10.1145/3372224.3419207

[8] M. Kim, S. Mandrà, D. Venturelli, and K. Jamieson, *Physics-Inspired Heuristics for Soft MIMO Detection in 5G New Radio and Beyond*. New York, NY, USA: Association for Computing Machinery, 2021, p. 42–55. [Online]. Available: https://doi.org/10.1145/3447993.3448619

[9] M. Kim, S. Kasi, P. A. Lott, D. Venturelli, J. Kaewell, and K. Jamieson, “Heuristic quantum optimization for 6g wireless communications,” *IEEE Network*, vol. 35, no. 4, pp. 8–15, 2021.

[10] J. Choi, S. Oh, and J. Kim, “Quantum approximation for wireless scheduling,” *Applied Sciences*, vol. 10, no. 20, 2020. [Online]. Available: https://www.mdpi.com/2076-3417/10/20/7116

[11] F. Ahmed and P. Mähönen, “Quantum computing for artificial intelligence based mobile network optimization,” in *2021 IEEE 32nd Annual International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC)*, 2021, pp. 1128–1133.

[12] G. A. Kochenberger and F. W. Glover, “A unified framework for modeling and solving combinatorial optimization problems: A tutorial,” 2006.

[13] E. Farhi, J. Goldstone, and S. Gutmann, “A quantum approximate optimization algorithm applied to a bounded occurrence constraint problem,” 2015.

[14] “https://www.uni-kl.de/channel-codes/channel-codes-database.”

[15] I. Hen and F. M. Spedalieri, “Quantum annealing for constrained optimization,” *Physical Review Applied*, vol. 5, no. 3, p. 034007, 2016.

[16] T. Kadowaki and H. Nishimori, “Quantum annealing in the transverse Ising model,” *Phys. Rev. E*, vol. 58, pp. 5355–5363, Nov 1998. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevE.58.5355

[17] A. B. Finnila, M. Gomez, C. Sebenik, C. Stenson, and J. D. Doll, “Quantum annealing: A new method for minimizing multidimensional functions,” *Chemical physics letters*, vol. 219, no. 5-6, pp. 343–348, 1994.

[18] S. Kasi, P. A. Warburton, J. Kaewell, and K. Jamieson, “A cost and power feasibility analysis of quantum annealing for nextg cellular wireless networks,” 2021. [Online]. Available: https://arxiv.org/abs/2109.01465