Electric dipole moment of the neutron in Two Higgs Doublet Models with flavor changing

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I consider contributions to the neutron electric dipole moment within Two Higgs Doublet Models without imposed restrictions on flavor changing neutral Higgs couplings. In a previous paper, I considered (hypothetical) flavor changing couplings for the Standard Model Higgs boson. In that paper I found that the obtained value of the neutron electric dipole moment were below the present experimental limit, given previous restrictions on such couplings. However, the result depended on an ultraviolet cut off $\Lambda$, parametrized as $\ln(\Lambda^2)$.

In the present paper I demonstrate how these divergences which appeared in some of the amplitudes in my previous paper are removed within a Two Higgs Doublet Model which a priori allow for (small) neutral flavor changing couplings.

If one considers the limit where the (hypothetical) heavy neutral Higgs boson $H$ with mass $M_H$ is much heavier than the light neutral Higgs boson $h$ with mass $M_h$, i.e. $M_H^2 \gg M_h^2$, then the $\ln(\Lambda^2)$ behaviour is replaced by $\ln(\tilde{M}_H^2)$, where $\tilde{M}_H$ is of order $M_H$. I have found that in this limit the result stays the same up to $M_{SM}^2/M_H^2$ corrections, where $M_{SM}$ is the mass of either the Standard Model Higgs-boson, the top-quark, or the $W$-boson.

Keywords: CP-violation, Electric dipole moment, Flavor changing Higgs.
PACS: 12.15. Lk., 12.60. Fr.

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I. INTRODUCTION

An electric dipole moment (EDM) for elementary particles is a CP-violating quantity and it gives important information on the asymmetry between matter and anti-matter in the universe. Within the Standard Model (SM), EDMs of elementary fermions are induced through the Cabibbo-Kobayashi-Maskawa (CKM) CP-violating phase. EDMs are studied also within many models Beyond the SM (BSM). For reviews on SM and BSM contributions to EDMs, see [1–4]. Explicitly, for the EDM of the neutron (nEDM = \(d_n\)) discussed in this paper, the present experimental bound is [5]

\[ d_{n}^{\text{exp}} / e \leq 2.9 \times 10^{-26} \text{ cm} . \]  

(1)

Within the SM, the nEDM is calculated to be several orders of magnitudes below the experimental bound, ranging from \(10^{-34} e \text{ cm}\) to \(10^{-31} e \text{ cm}\), depending on the considered mechanism [6–15]. Calculations of the nEDM will in general put bounds on hypothetical models BSM, and any measured nEDM significantly bigger that the SM estimate would signal New Physics.

The electric dipole moment of a single fermion has the form

\[ \mathcal{L}_{\text{EDM}} = \frac{i}{2} d_f \bar{\psi}_f \sigma_{\mu\nu} F^{\mu\nu} \gamma_5 \psi_f , \]  

(2)

where \(d_f\) is the electric dipole moment of the fermion, \(\bar{\psi}_f\) is the fermion (quark) field, \(F^{\mu\nu}\) is the electromagnetic field tensor, and \(\sigma_{\mu\nu} = i[\gamma_\mu, \gamma_\nu]/2\) is the dipole operator in Dirac space.

Many models BSM suggest possible contributions from new particles or new interactions to the nEDM [1–4, 16–25]. In the case of New Physics presence, flavor physics might give a number of useful CP-violating observables. These may occur for instance in CP-violating mesonic decays [17, 18, 26]. In such processes there might be mechanisms that generate new contributions to the electric dipole moments of quarks due to New Physics particles coupling to SM quarks (see e.g. [27]).

The properties and couplings of the Higgs boson are still not completely known. For instance, the SM Higgs might mix with (a) higher mass scalar(s) in the BSM. Some authors [28–33] have suggested that the SM Higgs boson might have flavor changing (FC) couplings to fermions which might also be CP-violating. In these papers bounds on quadratic expressions of such couplings were obtained from various processes, say, like \(K \to \bar{K}, D \to \bar{D}\), and \(B \to \bar{B}\)
mixings, and also from leptonic flavor changing decays like $\mu \to e\gamma$ and $\tau \to \mu\gamma$. In the latter case two loop diagrams of Barr-Zee type [34] for EDMs, were also considered [28, 30, 35, 36]. Extra couplings of the SM Higgs to quarks has also been considered in [37].

In a previous paper [38], I calculated two loop diagrams containing a flavor changing SM Higgs coupling (FCH) to first order, -in contrast to contributions with FCH coupling squared, as in [29, 30]. Some of the two loop diagrams were divergent and parametrised by a ultraviolet cut-off $\Lambda$ [38]. In the present paper I address the same diagrams within Two Higgs Doublet Models (2HDM). Considering emission of the soft photon by a $W$-boson as a typical example, I show how such divergences might be removed. For 2HDMs see the review by Branco et al. [39], and also more recent papers [40–51].

Some technical details from the two loop calculations are given in the Appendix.

II. FLAVOR CHANGING PHYSICAL HIGGS?

Within the framework in [28–32] the effective interaction Lagrangian for a flavor transition between fermions of the same charge due to SM Higgs boson exchange might be obtained from a six dimensional non-renormalizable Higgs type Yukawa-like interactions as shown explicitly in [30, 33]:

$$\mathcal{L}^{(D)} = -\lambda_{ij}(Q_L)_i \phi (d_R)_j - \tilde{\lambda}_{ij}(Q_L)_i \phi (d_R)_j (\phi) \phi + h.c., \quad (3)$$

where the generation indices $i$ and $j$ running from 1 to 3 are understood to be summed over; i.e. $d_j = d, s, b$ for $j = 1, 2, 3$. Further, $\phi$ is the SM Higgs $SU(2)_L$ doublet field, $(Q_L)_i$ are the left-handed $SU(2)_L$ quark doublets, and the $(d_R)_j$’s are the right-handed $SU(2)_L$ singlet $d$-type quarks in a general basis. Further, $\Lambda_{NP}$ is the scale where New Physics is assumed to appear. There is a similar term $\mathcal{L}^{(U)}$ like the one in (3) for right-handed type $u$-quarks, $u_j$, i.e. $u_j = u, c, t$ for $j = 1, 2, 3$.

In such cases the interaction for the SM neutral Higgs boson $h^0$ to $d$-type quarks has the form

$$\mathcal{L}^{(D)}_Y = h^0 (d_i)_L \left( -\frac{M^{(D)}_i}{v} + Y_R^{(D)}_{ij} \right) (d_j)_R + h.c., \quad (4)$$

where $M^{(D)}_i$ is the mass matrix for $d$-type quarks, and $Y_R^{(D)}$ the Yukawa couplings beyond
the SM, related to the six dimensional operators. Explicitly:

\[(\mathcal{M}^{(D)})_{ij} = \frac{v}{\sqrt{2}} \left( \lambda_{ij} + \frac{v^2 \Lambda^2_{NP}}{2} \right) \],

and

\[(Y^{(D)}_R)_{ij} = \frac{v^2 \Lambda^2_{NP}}{\sqrt{2}} \cdot \] (6)

Here again, the indices \( i, j \) are running over the three generations. As usual \( v \) is the vacuum value 246 GeV for the SM Higgs field. The mass matrix \( \mathcal{M}^{(D)} \) may be rotated to diagonal form. However, this rotation will in general not give a diagonal \( Y^{(D)}_R \), such that the SM Higgs coupling to fermions will in general be flavor changing.

### III. Yukawa Interactions for 2HDM

The 2HDM are discussed in many papers [39–51]. By assumption, the extended Yukawa interactions for right-handed type \( d \)-quarks may then, in the most general case, be written as [44] :

\[- L^{(D)}_T = (\overline{(Q_L)^0_i})^r \left[ (\Gamma_1)_{ij}^{rs} (\Phi_1)^s + (\Gamma_2)_{ij}^{rs} (\Phi_2)^s \right] (d_R^0)^0_j + \text{h.c.} , \]

where \( i, j \) are as before generation indices running from 1 to 3 and \( r, s \) are \( SU(2)_L \) indices running from 1 to 2. The upper index 0 denotes the fields before diagonalization of the mass matrices in the quark sector. Thus the \( \Gamma \)'s are \( 2 \times 2 \) dimensional in \( SU(2)_L \) space and \( 3 \times 3 \) dimensional in generation space. The fields \( \Phi_{1,2} \) are the two Higgs fields.

For the right-handed type \( u \)-quarks one has similarly as (7):

\[- L^{(U)}_\Delta = \overline{(Q_L)^0} \left[ \Delta_1 \Phi_1 + \Delta_2 \Phi_2 \right] (u_R^0) + \text{h.c.} , \]

where the generation and \( SU(2)_L \) indices are suppressed. \( \Delta_{1,2} \) and \( \Gamma_{1,2} \) are in general complex and independent quantities. In many papers one discusses restrictions on 2HDMs to avoid flavor changing neutral currents. But in this paper the point is to study such effects.

The two Higgs doublets may for \( n = 1, 2 \) be written [39, 42] :

\[
\Phi_n = e^{i \xi_n} \left( \frac{1}{\sqrt{2}} (v_n + \rho_n + i \eta_n) \right), \quad \bar{\Phi}_n = e^{-i \xi_n} \left( \frac{1}{\sqrt{2}} (v_n + \rho_n - i \eta_n) \right), \]

where \( \Phi_n \equiv i \sigma_2 \Phi_n^* \), and \( e^{i \xi_n} \) are phase factors. One introduces the parameter \( \beta \) through

\[\tan \beta \equiv \frac{v_2}{v_1}. \] (10)
After diagonalisation of the mass matrix for the neutral fields $\rho_{1,2}$ obtained from the Higgs potential \[39\] one finds the neutral scalar mass eigenstates
\[
h = \rho_1 s_\alpha - \rho_2 c_\alpha ; \quad H = -\rho_1 c_\alpha - \rho_2 s_\alpha ,
\] (11)
and the inverted relations are:
\[
-\rho_1 = -H c_\alpha - s_\alpha h ; \quad -\rho_2 = h c_\alpha - H s_\alpha .
\] (12)
Here $s_\alpha \equiv \sin \alpha$ and $c_\alpha \equiv \cos \alpha$, where $\alpha$ is the mixing angle coming from the diagonalisation of the mass matrix of the $\rho_{1,2}$ fields. Note that in the previous paper \[38\] the SM Higgs was denoted $H$. In the present paper this symbol is reserved for the heavy neutral Higgs boson within 2HDM.

In 2HDMs one often uses the Higgs basis, where the fields $H_{1,2}$ are defined by
\[
e^{-i \xi_1} \Phi_1 = c_\beta H_1 + s_\beta H_2 ; \quad e^{-i \xi_2} \Phi_2 = s_\beta H_1 - c_\beta H_2 ,
\] (13)
where $c_\beta \equiv \cos \beta$ and $s_\beta \equiv \sin \beta$. With this definition $H_1$ has a vacuum value $v = \sqrt{v_1^2 + v_2^2}$ and $H_2$ has zero vacuum value. Thus, in this basis
\[
H_1 = \begin{pmatrix} \frac{1}{\sqrt{2}} (v + h^0 + i G^0) \\ \frac{1}{\sqrt{2}} (w + h^0 + i G^0) \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}} (R^0 + i A) \end{pmatrix}
\] (14)
where $v$ is the vacuum value 246 GeV for the SM Higgs field, and $G^+$ and $G^0$ are Goldstone fields. $H^+$ is the charged Higgs field and $A$ the neutral pseudoscalar field within 2HDMs. Now the neutral scalar fields $h^0$ and $R^0$ can be written in terms of the physical (within 2HDM) neutral scalars $h$ and $H$ as
\[
h^0 = -c_\theta H + s_\theta h ; \quad R^0 = s_\theta H + c_\theta h ,
\] (15)
where
\[
c_\theta \equiv \cos \theta , \quad \text{and} \quad s_\theta \equiv \sin \theta , \quad \text{where} \quad \theta \equiv \alpha - \beta ,
\] (16)
will be the mixing angle in the neutral Higgs sector. Assuming the SM field $h^0$ to be close to $h$, means that $\sin \theta$ is close to one.

In the Higgs basis the extended Yukawa interactions may (in matrix notation) then be written
\[
- \frac{v}{\sqrt{2}} L_Y = \overline{Q_L}^0 (M^0_{1d} H_1 + N^0_{1d} H_2) \ d_R^0 + \overline{Q_L}^0 \left( M^0_{u} \tilde{H}_1 + N^0_{u} \tilde{H}_2 \right) u_R^0 + h.c. ,
\] (17)
where
\[ M_0^d = (c_\beta \Gamma_1 + e^{i \xi} s_\beta \Gamma_2) \frac{v e^{i \xi_i}}{\sqrt{2}}, \quad N_0^d = (s_\beta \Gamma_1 - e^{i \xi} c_\beta \Gamma_2) \frac{v e^{i \xi_i}}{\sqrt{2}}, \] (18)

where \( \xi = \xi_2 - \xi_1 \) and for the \( u \)-quark case:
\[ M_0^u = (c_\beta \Delta_1 + e^{-i \xi} s_\beta \Delta_2) \frac{v e^{-i \xi_i}}{\sqrt{2}}, \quad N_0^u = (s_\beta \Delta_1 - e^{-i \xi} c_\beta \Delta_2) \frac{v e^{-i \xi_i}}{\sqrt{2}}. \] (19)

Now one transforms the mass matrices \( M_{0u}^{d,u} \) to diagonal form with matrices \( U_{R,L}^{d,u} \):
\[ M_d = (U_L^d)\dagger M_0^d U_R^d = \text{diag}(m_d, m_s, m_b), \quad N_d = (U_L^d)\dagger N_0^d U_R^d, \quad d_{R,L} = (U_{R,L}^d)\dagger d_{R,L}, \] (20)

and similarly
\[ M_u = (U_L^u)\dagger M_0^u U_R^u = \text{diag}(m_u, m_c, m_t), \quad N_u = (U_L^u)\dagger N_0^u U_R^u, \quad u_{R,L} = (U_{R,L}^u)\dagger u_{R,L}, \] (21)

for the \( u \)-quark case. The total neutral Yukawa interactions may now be written as:
\[ - v L_Y^{(d,n)} = \overline{d}_L (v + i G^0 - c_\theta H + s_\theta h) M_d d_R + \overline{d}_L (s_\theta H + c_\theta h + iA) N_d d_R + h.c \] (22)

and for the \( u \)-quark case
\[ - v L_Y^{(u,n)} = \overline{u}_L (v - i G^0 - c_\theta H + s_\theta h) M_u u_R + \overline{u}_L (s_\theta H + c_\theta h - iA) N_u u_R + h.c \] (23)

For the charged interactions one obtains
\[ - \frac{v}{\sqrt{2}} L_Y^{\text{charged}} = \overline{u}_L V_{CKM} \left( G^+ M_d + H^+ N_d \right) d_R - \overline{d}_L V_{CKM}^\dagger \left( G^- M_u + H^- N_u \right) u_R + h.c \] (24)

While the mass matrices \( M_d \) and \( M_u \) are now flavor diagonal, the matrices \( N_d \) and \( N_u \) are in general flavor non-diagonal and CP-violating. These may give contributions to the \( Y_R \)'s in eq. (11) and (19). Note that in \([40]\), a certain hierarchy within the mass matrices are assumed. In many papers on 2HDM, one assumes for instance (an) extra discrete symmetry(-ies) to simplify the theory. In \([44]\) possible restrictions on \([7]\) and \([8]\) are discussed. Here I stick to the general case.

Further, how might the 6-dimensional interaction in eq (3) obtained in 2HDM? One way might be to consider the part of the Higgs potential containing a product of four \( \Phi_1 \) or \( \Phi_2 \) Higgs fields (see for example \([42]\)):
\[ V_{2HDM}^{\Phi} = \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \frac{1}{2} \left[ \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + h.c. \right] + \left[ \lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2) \right] (\Phi_1^\dagger \Phi_2) + h.c. \] (25)
Such potentials may contribute to $Y_R$'s in (4). In (3) $\phi$ is the SM Higgs which one within 2HDM identify with $H_1$ in (14). Inserting (13) in (25), the Higgs potential (25), will contain several terms of the type $(H_1^\dagger H_1)(H_2^\dagger H_1)$. The coefficient for the sum of such terms is

$$C_\lambda = \lambda_1 s_\beta c_\beta^3 + \lambda_2 c_\beta s_\beta^3 + (\lambda_3 + \lambda_4) s_\beta c_\beta (s_\beta^2 - c_\beta^2) + \lambda_5 e^{-2i\xi} s_\beta^3 c_\beta - \lambda_6 e^{2i\xi} c_\beta^3 s_\beta$$

$$+ 2 \lambda_6 c_\beta^2 s_\beta e^{-i\xi} + \lambda_6^* (s_\beta^2 c_\beta^2 - c_\beta^4)e^{i\xi} + \lambda_7 (s_\beta^4 - s_\beta^2 c_\beta^2)e^{-i\xi} - 2 \lambda_8 c_\beta^2 s_\beta e^{i\xi}. \quad (26)$$

Now the field $H_2^\dagger$ at space-time $z_2$ in such expressions might be contracted with the field $H_2$ at space-time $z_1$ in (17). This makes the field contraction ($C$):

$$C \left( H_2(z_1)(H_2(z_2))^\dagger \right) = D_{H_2}(z_1 - z_2). \quad (27)$$

Then one obtains an effective 6-dimensional interaction

$$-\frac{v}{\sqrt{2}} L_6^{(D)} = \left( (Q_L(z_1))^0 \right) [M_d H_1(z_1)$$

$$+ C_\lambda N_d D_{H_2}(z_1 - z_2) H_1(z_2)((H(z_2_1))^\dagger H_1(z_2))] (d_R)^0(z_1) + h.c. . \quad (28)$$

So far, this is not a local operator. The propagator $D_{H_2}(z_1 - z_2)$ contains a propagator $D_h$ for the light neutral Higgs, and a part $D_H$ for the heavy neutral Higgs $H$. However, if the SM Higgs is close to the light Higgs $h$, then $s_\theta$ is close to one, and thereby the scalar $R_0$ is close to the heavy Higgs $H$. The part containing the heavy Higgs $H$ interaction is then for $M_H^2 \gg M_h^2$:

$$D_H(z_1 - z_2) \simeq -\frac{\delta^{(4)}(z_1 - z_2)}{M_H^2} (\sin \theta)^2 , \quad (29)$$

making the $H$-part of the interaction local in this limit, and giving the following contribution to $Y_R$ in (4) and (6):

$$\left( (Y_R^{(D)})_{ij} \right)_H = \frac{v^3(\tilde{\lambda}_{ij})_H}{\sqrt{2} \Lambda_{NP}^2} \simeq -C_\lambda (N_d)_{ij} \frac{v^2}{\sqrt{2} M_H^2} (\sin \theta)^2 . \quad (30)$$

The non-local $h$-part $D_h$ would be a term corresponding to a higher order diagram. This term is shown in Fig. 4 and shortly discussed at the end of section V.

**IV. NEDM GENERATED FROM A FC HIGGS COUPLING**

In this section I give a short summary of the results from the previous paper [38]. The reason being that the diagrams calculated in that paper are also relevant in 2HDM's.
First, the relation between the nEDM and EDMs of light $u$- and $d$-, and even $s$ quarks are often given by the formula

$$d_n = \gamma_u d_u + \gamma_d d_d + \gamma_s d_s , \quad (31)$$

similar to a corresponding magnetic moment formula. In the strict valence quark limit,

$$\gamma_u = -\frac{1}{3} , \quad \gamma_d = \frac{4}{3} , \quad \gamma_s = 0 , \quad (32)$$

while recent lattice calculations [52, 53] give

$$\gamma_u = -0.22 \pm 0.03 , \quad \gamma_d = 0.74 \pm 0.07 , \quad \gamma_s = 0.008 \pm 0.010 . \quad (33)$$

In [38], two classes of diagrams for EDMs of light quarks, shown in Fig. 1 and Fig. 2, were considered. These diagrams are obtained from the flavor non-diagonal interaction in (4), completed by SM interactions. But these diagrams also give contributions within 2HDM with flavor change, as explained in the next section.

![Diagram](image)

**FIG. 1:** The first class of diagrams contain the FCH coupling and the big Higgs-top coupling proportional to the top mass $m_t$. For the first three diagrams, there are also corresponding diagrams where the $W$-boson is replaced by an unphysical (Goldstone) Higgs-boson within Feynman gauge. In this figure $H$ may denote the SM Higgs $h^0$. Further, within the 2HDMs $H$ may denote the lightest neutral Higgs boson $h$ or the heavier neutral Higgs boson $H$.

The various loop contributions to the EDM from Fig. 1 and Fig. 2 can be written

$$\mathcal{M}(f \to f\gamma)(a) = A \left( \bar{f} \sigma \cdot F P_R f \right) . \quad (34)$$
FIG. 2: A class of diagrams containing the FCH coupling and the big $\text{HWW}$ coupling proportional to $M_W$. Additional graphs with the W replaced by an unphysical (Goldstone) Higgs within Feynman gauge has to be added. In this figure $H$ may denote the SM Higgs $h^0$, or within the 2HDM the lightest neutral Higgs $h$, or the heavier neutral Higgs $H$.

With interchanged order of Higgs and W-bosons, the contributions have the form:

$$\mathcal{M}(f \to f\gamma)_{(c)} = A^* (\bar{d} \sigma \cdot F P_L d) .$$  \hspace{1cm} (35)

Using (34) and (35), one finds that the electric dipole moment is equal to

$$(d_f)_{2\text{-loop}} = 2 \text{Im}(A) .$$  \hspace{1cm} (36)

There is also a contribution to the magnetic moment (i.e the gyromagnetic quantity $(g - 2)$) given by $2 \text{Re}(A)$.

It turns out that $u$-quark contributions are suppressed by a factor of order $(m_b/m_t)^2 \sim 10^{-3}$ compared to the analogous $d$-quark contributions. Therefore the $u$-quark dipole moment $d_u$ due to diagrams in Fig. 1 are suppressed, and therefore neglected. Thus, the $d$-quark dipole moment contributions dominate, and other contributions are neglected \cite{38}.

Summing all contributions from diagrams in Fig. 1 and Fig. 2 I obtained the dominant contribution in the bare case (before QCD corrections) \cite{38}:

$$\left(\frac{d_d}{e}\right)_{\text{Tot}} = (f(u_t) C_A + f_{F\text{fin}}) F_2 \text{Im}[Y_R(d \to b) V_{td}^* V_{tb}] .$$ \hspace{1cm} (37)

Here the $V$’s are CKM matrix elements in the standard notation. The constant $F_2$ sets the overall scale of the EDMs obtained from our two loop diagrams:

$$F_2 = \frac{g_W^3}{M_W \sqrt{2}} \left(\frac{1}{16 \pi^2}\right)^2 = \frac{2M_W^2}{v^3} \left(\frac{1}{16 \pi^2}\right)^2 \approx 6.94 \times 10^{-22} \text{ cm} ,$$ \hspace{1cm} (38)
where \( v = 246 \text{ GeV} \) is the electroweak symmetry breaking scale, and where I have used the conversion relation \( 1/(200\text{MeV}) = 10^{-13} \) cm. Furthermore,

\[
f(u) = \frac{u}{4(u-1)} \left( \frac{u \ln(u)}{u-1} + \frac{1}{3}(8u-11) \right),
\]

where numerically

\[
u_t \equiv \left( \frac{m_t}{M_W} \right)^2 \simeq 4.65 ; \quad f(u_t) = 4.83.
\]

The UV divergence is parametrized through the quantity

\[
C_\Lambda \equiv \ln \left( \frac{\Lambda^2}{M_W^2} \right) + \frac{1}{2},
\]

where \( \Lambda \) is the UV cut-off. Numerically, \( C_\Lambda \) is \( \sim 5.5 \) to 9.4 for \( \Lambda \sim 1 \) to 7 TeV.

The quantity \( f_{\text{Fin}} \simeq -7.7 \), is the sum of the completely finite diagrams and also the finite part of diagrams containing a divergence. We note that because \( V_{td}^* V_{tb} \) is complex, there will be an EDM even if \( Y_R(d \to b) \) is real!

Using (37), the lattice values in (33) and absolute value of \( V_{td}^* V_{tb} \) from [54], one may write my result for the nEDM in the following way, as shown in [38]:

\[
d_n/e \simeq N(\Lambda) \times \left\{ \frac{|Y_R(b \to d)|}{|Y_R(b \to d)|_{\text{Bound}}} \cdot \text{Im} \left[ \frac{Y_R(d \to b)}{|Y_R(b \to d)|} \cdot \left( \frac{V_{td}^* V_{tb}}{|V_{td}^* V_{tb}|} \right) \right] \right\} \times 10^{-26} \text{ cm},
\]

where I have scaled the result with the bound from [29, 30]:

\[
|Y_R(d \to b)| \leq 1.5 \times 10^{-4} \equiv |Y_R(d \to b)|_{\text{Bound}}.
\]

The function \( N(\Lambda) \) is is plotted as a function of \( \Lambda \) in Fig. 3 for the bare case (at the renormalization scale \( \mu = \mu_\Lambda \), blue curve) and with QCD corrections (at the hadronic scale \( \mu = \mu_h \simeq 1 \text{ GeV} \), red curve, as explained in [38])

Now, the maximal value of the parenthesis \( \{ \ldots \} \) in (42) is \( = 1 \). Thus, if the bound for \( Y_R(d \to b) \) in (43) is saturated, the plot for the function \( N_\Lambda \) in Fig. 3 shows that when the cut-off \( \Lambda \) is stretched up to 20 TeV, the bound for nEDM in (1) is reached in the bare case, while the perturbative QCD-suppression tells [38] that the value of the nEDM can at maximum be of order one tenth of the experimental bound for \( \Lambda \) up to 20 TeV. If the bound for \( |Y_R(d \to b)| \) is reduced, and also \( \Lambda \) is reduced, my value for nEDM will be accordingly smaller.
FIG. 3: The quantity $N(\Lambda)$, in units $10^{-26} \text{ cm}$, as a function of cut-off $\Lambda$ (in TeV). The blue (upper) curve is for the bare case, and the red (lower) curve is for the case when the suppressing QCD corrections are included.

One observes that, including QCD corrections, and using (43) the nEDM can at most be of order $0.4 \times 10^{-26} e \text{ cm}$. Thus, I found (for $\Lambda$ up to 20 TeV) a bound on the coupling $Y_R(d \to b)$ comparable to the previous bound, but not directly on the absolute value given as in [29, 30].

V. THE NEDM IN THE 2HDM

All diagrams in Fig. 1 and Fig. 2 will also contribute within 2HDM without restrictions as given in (22) and (23), and one obtains diagrams both with light $h$- and heavy $H$-exchanges. Exchange of the pseudoscalar $A$ (as defined in eq. (14)) does not contribute to the order I work, because it does not couple to the mass matrices $M_u$ and $M_d$ in (22) and (23). For all contributions with exchange of $h$ and $H$, the amplitudes are equal, but have opposite signs according to the eqs. (22) and (23). Explicitly, for $X = h$ and $X = H$ exchanges I find the
effective contribution
\[ [Y_R(d \to b)]_N = (N_{d})_{bd} \cos \theta \sin \theta, \tag{44} \]
where \( \theta \) is defined in (16) and (\( N_{d})_{bd} \) in (18). Within 2HDMs, given by (22) and (23), eq. (44) may be inserted in (42) to obtain a bound on the right hand side of the right hand side of (22).

As shown in [38] some diagrams with exchange of only one neutral Higgs-boson have divergent parts. In the following I will for illustrative purposes consider in some detail the case where a soft photon is emitted from a \( W \)-boson, as shown in the right panel of Fig. 1. Then the result for this diagram is proportional to the two loop integral tensor:

\[ T_{\mu \nu}^W(X) = \int \int \frac{d p \, d r \, (2r + p)_{\mu} p_{\nu}}{(r^2 - M_W^2)(r^2 - m_t^2)((r + p)^2 - m_t^2)(p^2 - M_X^2)} , \tag{45} \]
where \( M_X \) is the mass of either the light or heavy Higgs, i.e \( X = h, H \). Here the integral over \( dp \) diverges.

In the limit where \( \sin \theta \) is close to one, the \( h \)-part can be written as in [38] :

\[ T_{\mu \nu}^W(h) = \frac{g_{\mu \nu}}{4 m_t^2} \left( \frac{1}{16 \pi^2} \right) \left( C_A \cdot p_W(u_t) + t_{WFin}^L + t_{WFin}^N \right) , \tag{46} \]
where \( C_A \) is given by (41) and

\[ p_W(u) = \frac{1}{(u - 1)} \left( -1 + \frac{uln(u)}{u - 1} \right) , \tag{47} \]
where \( u_t \) is the mass ratio in (40). Furthermore,

\[ p_W(u_t) \simeq 0.26 \ , \ t_{WFin}^L \simeq -2.8 \ , \ \text{and} \ t_{WFin}^N \simeq -1.1 . \tag{48} \]
Here \( t_{WFin}^L \) is the finite term following the logarithmic divergence, and \( t_{WFin}^N \) is a completely finite term, as explained in the Appendix. For other diagrams there are similar expressions as (48), but with other numbers. Completely finite diagrams have only a term similar to \( t_{WFin}^N \), for instance the two first (from left) diagrams in Fig. 1.

The individual loop integrals for \( X = h \) or \( X = H \) alone have divergent parts. But within 2HDM, one observes from eqs. (15), (22) and (23) that the terms with exchange of \( h \) and \( H \) will have opposite signs due to the Cabibbo-like mixing of \( h \) and \( H \), and one obtains a GIM-like cancellation of the divergences. Thus, I use

\[ \frac{1}{(p^2 - M_h^2)} - \frac{1}{(p^2 - M_H^2)} = \frac{(M_h^2 - M_H^2)}{(p^2 - M_h^2)(p^2 - M_H^2)} \ . \tag{49} \]
and obtain the total amplitude for exchanges of both $h$ and $H$:

$$
\Delta T_{\mu\nu}^W = T_{\mu\nu}^W(h) - T_{\mu\nu}^W(H) = (M_h^2 - M_H^2) S_{\mu\nu}^W ,
$$

(50)

where

$$
S_{\mu\nu}^W = \int \int \frac{d p d r (2r + p)_\mu p_\nu}{(r^2 - M_W^2)^2((r + p)^2 - m_t^2)(p^2 - M_h^2)(p^2 - M_H^2)} ,
$$

(51)

is finite.

As shown in the Appendix, this loop integral contains logarithmic and dilogarithmic functions of masses of the top quark, the $W$-boson and the neutral Higgs bosons $h, H$. In the limit $M_H^2 \gg M_h^2$, I find that

$$
S_{\mu\nu}^W \sim \frac{\ln(M_H^2)}{M_H^2} .
$$

(52)

I have found the leading result replacing eq. (37) can be written:

$$
\Delta T_{\mu\nu}^W = \frac{g_{\mu\nu}}{4m_t^2} \left( \frac{1}{16\pi^2} \right)^2 \left( (\widetilde{C}_H)^W \cdot p_W(u_t) + t_{WFin}^L + t_{WFin}^N \right) + \mathcal{O}(M_{SM}/M_H^2) ,
$$

(53)

where corrections are of order $(M_{SM}/M_H)^2$, where $M_{SM}$ is either $m_t$, $M_h$ and/or $M_W$. Here one might expect that the divergent term $C_\Lambda$ from (46) is replaced by a finite term where the cut-off $\Lambda$ is replaced by just the heavy neutral Higgs mass $M_H$. But it turns out that the $(\widetilde{C}_H)^W$ is a bit more complicated, as shown in the Appendix:

$$
(\widetilde{C}_H)^W = \ln \left[ \left( \frac{(\widetilde{M}_H)^W}{M_W} \right)^2 \right] + \frac{1}{2} ,
$$

(54)

(up to corrections of order $(M_{SM}/M_H)^2$ as mentioned above) where

$$
(\widetilde{M}_H)^W = M_H e^{\alpha_W} , \quad \alpha_W = \frac{(\ln u_t)^2}{4(1 - 1/u_t - (\ln u_t))} , \quad e^{\alpha_W} \simeq 0.45 ,
$$

(55)

and where $u_t$ is given in (40). The term $t_{WFin}^L$ in (53) corresponds to the same $t_{WFin}^L$ as in (46) and given in (61) in the Appendix. It is easy to see that the $t_{WFin}^L$’s are the same if one uses the mathematical trick given in (77) in the Appendix. The term $t_{WFin}^N$ is trivially the same (up to corrections of order $(M_{SM}/M_H)^2$). For other diagrams, where the soft photon is emitted by a quark $q = b, t$, say, the factor $e^{\alpha_W}$ will be replaced by a similar factor $e^{\alpha_q}$ of the same order of magnitude. Now the result given by (41), and (53)-(55) can be completed with similar expressions for the rest of digrams in Figs. 1 and 2. Then the final result will
be as in \[38\], \textit{i.e.} as in eq. (42) and Fig. 3, with the cut-off \(\Lambda\) replaced by a mass \(\tilde{M}_H\) of order \(M_H\) (depending on the various \(\alpha\)'s similar to \(\alpha_W\) in \(55\)).

Concerning the six-dimensional interaction in (3), it will for the local part also be proportional to \(N_d\) as shown in (30), and also suppressed by \((v/M_H)^2\). The non-local part given by exchange of the light \(h\)-boson will be of one order higher. The corresponding loop diagram is proportional to

\[ T_{\mu\nu}^{W6} = S_{\mu\nu}^W(M_H = M_h) = g_{\mu\nu} \frac{1}{(16\pi^2)^2} \frac{1}{4M_W^4} \times 0.0149 \]

(56)

where standard numerical values for masses for SM particles has been used. The contribution from this diagram should also be multiplied by \((\cos \theta)^2 C_\Lambda v^2\), and will be small \((\cos \theta\) is small when \(h\) is close to \(h^0\)).

![Diagram generated in the 2HDM. The blob denotes the interaction in (25). The dashed lines are Higgses and the crosses in the end of two of these denotes the Higgs VEV.](image.png)

**FIG. 4:** Diagram generated in the 2HDM. The blob denotes the interaction in (25). The dashed lines are Higgses and the crosses in the end of two of these denotes the Higgs VEV.

**VI. CONCLUSIONS**

In the present paper I have shown that the divergences appearing for some EDM diagrams with flavor changing Higgs in my previous paper [38] is removed when extending the analysis to Two Higgs Doublet Models allowing for flavor changes by neutral scalars. I find that the result [38] stays unchanged up to corrections of order \((M_h/M_H)^2\) when the cut off \(\Lambda\) is replaced by a quantity \(\tilde{M}_H\) related to the heavy neutral Higgs mass \(M_H\).
It is also found that the six dimensional interaction in (3) used in (38) will also be proportional to the non-diagonal \( N_d \)-matrix in (17), (20) and (22). In general this coupling must be small to avoid too big flavor changing neutral currents. The bound on \( Y_R (d \to d) \) found in the previous paper (38) will then give the result (bound) for the effective flavor changing quantity \( (N_d)_{bd} \cos \theta \sin \theta \). The \( N_d \) might be non-zero for 2HDMs without a priori imposed restrictions. Also the contributions from the six dimensional interaction in (3) involving \( C_\lambda \) from the Higgs potential reduces to a quantity proportional to \( N_d \).

As the purpose of this paper have been to show how the divergences appearing in (38) are removed in a 2HDM without imposed restrictions, my analysis is different from the one by Jung and Pich (4), who consider Barr-Zee type of diagrams.

VII. APPENDIX

If the soft photon is emitted from the \( W \)-boson as in the left diagram in Fig. (I) then the left sub-loop containing the Higgs boson is logarithmically divergent. The result of the divergent part of (45) can be written

\[
T_{\mu\nu}(h) = \frac{g_{\mu\nu}}{4} 2! \int_0^1 dx \int_0^{(1-x)} dy \int \frac{dr}{(r^2 - M_W^2)^2(r^2 - m_b^2)} \left( I_2(R) + R \cdot I_3(R) \right) ,
\]

(57)

where the quantity \( R \) depends on the squared loop momentum \( r^2 \) For \( n = 2, 3 \) :

\[
I_n(R) = \int \frac{dp}{(p^2 - R)^n}.
\]

(58)

Then for cut-off regularization :

\[
(I_2(R) + R \cdot I_3(R)) = \frac{i}{16\pi^2} \left( \ln(\Lambda^2/R) - \frac{3}{2} \right) ,
\]

(59)

where \( \Lambda \) is the cut-off, and \( x \) and \( y \) are Feynman parameters, and

\[
R \equiv B - x(1-x)r^2 ; \quad B \equiv m_b^2 + x(M_W^2 - m_b^2) + y(M^2_R - m_b^2) .
\]

(60)

One may split up

\[
\left( \ln(\Lambda^2/R) - \frac{3}{2} \right) = \left( \ln(\Lambda^2/M_W^2) - \frac{3}{2} \right) + \ln(M_W^2/R) ,
\]

(61)

where the first term corresponds to \( C_\Lambda \) in (41), and the \( \ln(M_W^2/R) \) term correspond to \( t_{WF_{in}}^L \). There is also a finite term \( t_{WF_{in}}^N \) corresponding to an completely finite extra term
\( \sim 1/R \) not shown in (57). We also note that the one loop integral
\[
K_W = \int \frac{dr}{(r^2 - M_W^2)^2(r^2 - m_i^2)} = -\frac{i}{16\pi^2 M_W^2} p_W(u), \tag{62}
\]
where \( p_W(u) \) defined in (47) is the proportionality factor for the divergent term \( C_A \) in (41).

Now I consider the finite loop integral in (51) with both \( h \) and \( H \) included. Doing Feynman parametrisation for the \( dp \)-integration one obtains
\[
S_{\mu \nu}^W = \frac{g_{\mu \nu}}{4} S^W, \quad S^W = 2! \left( \frac{i}{16\pi^2} \right) \int_0^1 \frac{dx}{x(1-x)} \int_0^{(1-x)} dy \int_0^{(1-x-y)} dz J_Q^W, \tag{63}
\]
plus terms suppressed by \( 1/M_H^2 \). This integral is finite. Note that the term \( \sim 1/R \) mentioned below (61) is not included. Here
\[
J_Q^W \equiv \int \frac{dr}{(r^2 - M_W^2)^2(r^2 - m_i^2)(r^2 - Q)} = \frac{i}{16\pi^2} \left[ \frac{-1}{(m_i^2 - M_W^2)(Q - M_W^2)} + \frac{m_i^2}{(m_i^2 - M_W^2)^2} \left( \frac{\ln(Q/M_W^2)}{(Q - M_W^2)} - \frac{\ln(Q/m_i^2)}{(Q - m_i^2)} \right) \right], \tag{64}
\]
where
\[
Q = \frac{1}{x(1-x)} \left( m_b^2 + x(m_i^2 - m_b^2) + y(M_H^2 - m_b^2) + z(M_H^2 - m_b^2) \right). \tag{65}
\]
Integrating over \( z \) gives a suppression factor of order \( 1/M_H^2 \). Changing variables, one obtains an integral over \( Q \) with \( dz = x(1-x)dQ/M_H^2 \) :
\[
\int_0^{(1-x-y)} dz J_Q^W = \frac{i}{16\pi^2} \frac{x(1-x)}{(M_H^2 - m_b^2)} (f_W(Q) - f_W(Q_0)), \tag{66}
\]
where
\[
f_W(Q) \equiv \frac{1}{(m_i^2 - M_W^2)} \left( -\ln \left( \frac{Q - M_W^2}{M_W^2} \right) + \frac{m_i^2}{(m_i^2 - M_W^2)} \left( \text{dilog} \left( \frac{Q}{m_i^2} \right) - \text{dilog} \left( \frac{Q}{M_W^2} \right) \right) \right). \tag{67}
\]

Here, the dilogarithmic function is in our case defined as
\[
\text{dilog}(z) = \int_1^z dt \frac{\ln(t)}{(1-t)} = \text{Li}_2(1-z). \tag{68}
\]
Further,
\[
Q_1 = \frac{1}{x(1-x)}(m_b^2 + x(m_i^2 - m_b^2) + y(M_H^2 - m_b^2) + (1-x-y)(M_H^2 - m_b^2)) ,
\]
and
\[
Q_0 = \frac{1}{x(1-x)}(m_b^2 + x(m_i^2 - m_b^2) + y(M_H^2 - m_b^2)) = \frac{B}{x(1-x)}, \tag{69}
\]
where \( B \) is defined in (60).
Now the quantity \( S \) in (63) may be split up as:

\[
S^W = \left( \frac{i}{16\pi^2} \right)^2 2! \int_0^1 dx \int_0^{(1-x)} dy \left[ f^W(Q_1) - f^W(Q_0) \right] = S^W_1 - S^W_0. \tag{70}
\]

Here the quantity \( S^W_1 \) contains a term \( ln(M^2_H) \) corresponding to the divergent term \( ln(\Lambda^2) \) in (38) and (16). In order to find \( S^W_1 \) explicitly I use the asymptotic property for \( Z \to \infty \)

\[
dilog(Z) \to -\frac{1}{2} (ln(Z))^2. \tag{71}
\]

Therefore, one obtains for \( M^2_H \gg M^2_H \)

\[
\left( -dilog\left( \frac{Q_1}{M^2_W} \right) + dilog\left( \frac{Q_1}{m_t^2} \right) \right) \to \ln\left( \frac{M^2_H}{m_t M_W} \right) \cdot \left( \ln\left( \frac{M^2_H}{m_t M_W} \right) + \ln(\sigma) \right), \tag{72}
\]

where

\[
\sigma = \frac{(1 - x - y)}{x(1 - x)}. \tag{73}
\]

Then one obtains

\[
S^W_1 = \left( \frac{1}{16\pi^2} \right)^2 \frac{1}{M^2_H(m_t^2 - M^2_W)} \left[ -\left( \ln\left( \frac{M^2_H}{M^2_W} \right) + \frac{1}{2} \right) \right.
\]

\[
+ m_t^2 \ln(m_t^2/M^2_W) \left( \ln\left( \frac{M^2_H}{m_t M_W} \right) + \frac{1}{2} \right), \tag{74}
\]

which may be manipulated into

\[
S^W_1 = \left( \frac{1}{16\pi^2} \right)^2 \frac{(\hat{C}_H)^W \cdot p_W(u_t)}{M^2_H M^2_W}, \tag{75}
\]

where \((\hat{C}_H)^W\) is given in (54) and (55).

The term \( S^W_0 \) in (70) contains the \( ln(R) \) term in (61), and is given by

\[
S^W_0 = \left( \frac{i}{16\pi^2} \right)^2 2! \int_0^1 dx \int_0^{(1-x)} dy f^W(Q_0). \tag{76}
\]

To see this clear, instead of using (61), one may use a trick by rewriting \( I_2(R) \) in (57), (58) and (59) as

\[
I_2(R) = 2 \int_R^{\Lambda^2} d\rho \int \frac{dp}{(p^2 - \rho)^3}. \tag{77}
\]

Also, one observes that \( R = -x(1 - x)(r^2 - Q_0) \).
I thank Svjetlana Fajfer for useful comments. I am supported in part by the Norwegian research council (via the HEPP project).

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