Gravitational Radiation during Thorne-Żytkow object formation

S.N. Nazin, K.A. Postnov
Sternberg Astronomical Institute and
Physical Department of M.V.Lomonosov State University, Moscow, Russia
e-mail: nazin@sai.msu.su; pk@sai.msu.su

July 25, 2018

Abstract. Calculation of gravitational radiation during binary inspiral leading to possible formation of a Thorne-Żytkow (TZ) object (a neutron star inside a supergiant core) is performed. The calculations were done for polytropic density distributions with different indices n, as well as for realistic models of supergiants. A maximum frequency of the emitted gravitational waves during this process was found to range from a few to 300 initial keplerian orbital frequencies, that is from $10^{-5}$ to $0.1$ Hz, depending on the model. A dimensionless strain metric amplitude can reach $h \sim 10^{-23.5}$ for a source lying 10 kpc away from the Sun. We conclude that TZ objects forming at a rate of 1 per 500 yrs in the Galaxy could be potential astrophysical targets for space laser interferometers. Analysis of gravitational waveforms emitted during TZ-object formation could bring a unique information about stellar structure.

Key words: gravitation – relativity – binaries: close

1. Introduction

According to the modern scenario of binary star evolution (see van den Heuvel (1994) for a review), massive binary systems with mass ratios far from unity ($\sim 2.5$) or in which the mass losing star has a convective envelope must pass through a so called common envelope (CE) stage (Paczynski 1976). Considered for the first time in detail by Taam, Bodenheimer and Ostriker (1978), the the double core hydrodynamical evolution in common envelope is repeatedly calculated (see Livio 1994 for a review). The issue of the common envelope evolution is either ejection of part or all of the envelope, or coalescence of the two cores with little mass ejection leading to formation of a Thorne-Żytkow object (Thorne and Żytkow 1977).

An important result of the extensive numerical calculations is that the ratio of binding energy of the stellar envelope ejected during the CE stage to change of the system’s orbital energy, \( \alpha_{CE} = \Delta E_{env}/\Delta E_{orb} \) is of order unity. This implies a more than 100-fold shrinkage of the initial orbital separation after the spiral-in for typical parameters of high-mass X-ray binaries (HMXB) (a supergiant with a mass \( M_{sg} \sim 10 - 15M_\odot \) having a \( \sim 2 - 3M_\odot \) helium core in pair with a \( m = 1.4M_\odot \) neutron star). Thus, a crude lower limit for formation rate of TZ-objects can be taken as that for HMXB, i.e. $\sim 10^{-3}$ per year per a $10^{11}M_\odot$ Milky-Way-type galaxy. In fact, the formation rate of TZ objects in the Galaxy typically can be nearly twice as high. This means that $> 30$ formations of TZ objects per year could be observed from within a distance of 30 Mpc.

Let us estimate the frequency and amplitude of gravitational radiation emitted during this process. Consider a close binary system consisting of a neutron star orbiting a massive red (super)giant. The dynamical friction is the main drag force acting the neutron star inside the supergiant’s atmosphere:

\[
F_d(r) \sim R_G^2 \rho(r) v_r(r)^2, \tag{1}
\]

where \( r \) is radial distance to the supergiant’s center, \( \rho \) is its density, \( v_r \) is the neutron star velocity relative to the envelope, and \( R_G = 2Gm/(v_r^2 + a_s^2) \) is gravitation capture radius for the neutron star (\( G \) is the newtonian gravity constant, \( a_s \) is the sound velocity).

Hydrodynamical calculations (Livio 1994) have shown that the neutron star can reach the dense stellar core during some $10^3$ years and the final plunge inside the dense core takes less than one month. A crude estimation for
a dimensionless strain metric amplitude of gravitational radiation at the beginning of the plunge is

\[
h \sim \frac{r_g(M)}{R_c} \approx 5 \times 10^{-26} \left( \frac{M}{10 M_\odot} \right) \times \left( \frac{m}{1.4 M_\odot} \right) \left( \frac{10^{11} \text{cm}}{R_c} \right) \left( \frac{10 M \text{pc}}{D} \right),
\]

where \( M \) and \( R_c \) are mass of the star and radius of its core, respectively, \( r_g(M) = 2GM/c^2 \) is gravitational radius of mass \( M \) (\( c \) is the speed of light) and \( D \) is a distance to the object. Obviously, the amplitude must vanish as the neutron star approaches the center of the star.

The maximum gravitational radiation output during the spiral-in is expected to occur at two-folded circular frequency of the neutron star orbital motion. Here we will calculate gravitational wave emission as the neutron star plunges into a model star with a polytropic density distribution. We will also consider the case of a realistic density profile in giant stars.

2. Quadrupole gravitational radiation from spiralling-in neutron star

The transverse-traceless components of the metric perturbations are connected with the second time derivatives of the transverse-traceless components of the system’s quadrupole moment, \( \mathcal{I}^{TT}_{ij} \) (see Misner, Thorne, Weeler 1973):

\[
h^{TT}_{ij} = -\frac{2G}{c^4 D} \frac{d^2 \mathcal{I}^{TT}_{ij}}{dt^2},
\]

so we need to calculate how the \( \mathcal{I}^{TT}_{ij} \) changes along the neutron star trajectory.

2.1. Quadrupole moment of the system

We consider a point mass \( m \) in a circular orbit with a radius \( r(t) < R \) inside a spherical star of mass \( M \) and radius \( R \). For our model calculations we will assume the inner structure of the giant star not to change during the spiral-in. Lagrangian mass comprised inside the orbit is \( M(r) = 4\pi \int_r^R \rho(x)x^2 dx \).

The traceless part of the second moment of the system’s mass density \( \rho \) computed in a Cartesian coordinate system centered on the binary barycenter is

\[
\mathcal{I}_{ij}(t) = \int \rho(t)(x_i x_j - \frac{1}{3} \delta_{ij} x_i x_j) d^3 x.
\]

We assume the binary orbit to lie in the XY coordinate plane. We introduce a polar angle \( \varphi \) between the line connecting centres of the bodies and OX axis. Symmetry relative to the orbital plane implies \( \mathcal{I}_{xz} = \mathcal{I}_{yz} = 0 \). Assuming the neutron star does not influences the matter distribution inside the supergiant (which can be the case for close enough spiralling-in binaries, see Taam et al. (1978)) and the giant star rotation as a whole around the barycenter, the non-zero components of the quadrupole moment are

\[
\mathcal{I}_{xx} = (\cos^2 \varphi - \frac{1}{3}) \mu a^2
\]

\[
\mathcal{I}_{yy} = (\sin^2 \varphi - \frac{1}{3}) \mu a^2
\]

\[
\mathcal{I}_{zz} = -\frac{1}{3} \mu a^2
\]

\[
\mathcal{I}_{xy} = \frac{\sin 2 \varphi}{2} \mu a^2
\]

where

\[
\mu \equiv \frac{mM}{m+M} \text{ is a reduced mass}.
\]

This is the well-known result for two point masses in the orbit.

2.2. Trajectory of the neutron star

Equations of motion of the neutron star (treated as a point mass) subjected to the dynamical friction force can be written as

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial q^a} \right) - \frac{\partial L}{\partial q^a} = -\frac{\partial D}{\partial q^a}
\]

Here \( L \) is Lagrange’s function of the point in a gravity field

\[
L = \frac{m}{2} (\dot{r}^2 + r^2 \dot{\varphi}^2) + \frac{GmM(r)}{r}
\]
and $D$ is a dissipative function:

$$D = R_c^2 \rho v^3 = R_c^2 \rho(r) \left[ \dot{r}^2 + r^2 \dot{\varphi}^2 \right]^{3/2}$$

(12)

Energy losses due to dynamical friction only are taken into account in the dissipative function. The gravitational radiation reaction contributes unsignificantly as it is of the order $(v/c)^5$. We assume no change of the neutron star mass (see Chevalier (1993) for possible issues of accretion onto spiraling-in neutron star).

The dynamical equations were numerically solved in order to obtain arrays of $r(t)$, $\varphi(t)$ and their time derivatives up to the second order, which were then used for computing the second time derivatives of the traceless quadrupole moment $\tilde{\mathcal{I}}_{ij}(t)$.

The density distribution inside the core was modelled by polytropic distributions with indices $n$ (so that the pressure is $P \propto \rho^{1+1/n}$) ranging from 0 to 2.5. We also apply our calculations to a realistic density profile inside a 16 $M_\odot$ red giant (Taam 1979), and to the extreme case of density distribution inside a 15 $M_\odot$ supergiant just prior to the core collapse (Weaver et al. 1979), which approximately corresponds to a polytropic distribution with $n = 3$.

In Fig. 2 we present the evolution of the two-folded orbital frequency (the main frequency of the gravitational wave radiation) for the neutron star,

$$f_{GW} = 2f_{orb} =$$

$$= 6.4 \times 10^{-4} \text{(Hz)} \left( \frac{\dot{\varphi}}{\pi} \right) \sqrt{\frac{M/M_\odot}{(R/R_\odot)^3}}$$

(13)

as a function of time (in units of the initial keplerian time) for different polytropes. Here $\dot{\varphi}$ denotes the dimensionless time derivative. The top and right frames are shown for $M = 2M_\odot$, $R = 10^{10}$ cm. It is clearly seen from the figure that the larger $n$, the higher final frequency can be and the longer time the star remains in a keplerian orbit at the outer less denser layers. The oscillations seen for low $n$ reflect elliptical character of the initially circular motion suddenly subjected to a drag force. Note that for polytropes with $n > 1$ the plunge to the final frequency occurs very rapidly, so the gravitational wave radiation will have a burst-like character for dense configurations (see the next section).

Evolution of the gravitational wave frequency $f_{GW}$ with time for the case of density distribution inside Taam (1979) and Weaver et al. (1979) red giants is shown in Fig. 2.

In Fig. 3 we present the trajectory of the neutron star final plunge in a red giant with extreme density distribution (Weaver et al. 1979).

The neutron star trajectory inside the innermost region $a < 0.2R$ for Weaver et al.’s red giant is shown in
2.3. Gravitational radiation

The dimensionless waveforms $h_+$ and $h_\times$ of the metric strain were calculated in Newtonian quadrupole approximation (see e.g. Kochanek et al 1990). As an example, we calculated $h_+$ and $h_\times$ as seen from a distance $D$ for the face-on orientation of the binary relative to the line of sight, so that

$$D h_+ = \frac{G}{c^3} (\dot{\mathcal{S}}_{xx} - \dot{\mathcal{S}}_{yy});$$

$$D h_\times = 2 \frac{G}{c^3} \dot{\mathcal{S}}_{xy}.$$  

By measuring mass in units of the stellar mass $M$, radial distance in units of the stellar radius $R$, time in units of the initial free-fall time

$$t_{ff} = \sqrt{\frac{R^3}{GM}} \approx 1.6 \times 10^3 \, \text{(s)},$$

we can rewrite these equations in the form

$$\frac{D}{r_g(M)} h_+ = \frac{1}{4} \frac{r_g(M) \mu}{R} \frac{M}{M} (\bar{\Psi}_{xx} - \bar{\Psi}_{yy}),$$

$$\frac{D}{r_g(M)} h_\times = \frac{1}{2} \frac{r_g(M) \mu}{R} \frac{M}{M} \bar{\Psi}_{xy},$$

where $\Psi_{ij}$ denotes the dimensionless components of the quadrupole moment $\mathcal{S}_{ij}$ (cf. intuitive estimation \(\text{(2)}\)).

In addition, we calculated averaged over orientation of the binary waveforms (Kochanek et al. 1990)

$$\langle h_+^2 \rangle \propto \frac{4}{15} |\bar{\Psi}_{xx} - \bar{\Psi}_{zz}|^2 + \frac{1}{15} |\bar{\Psi}_{yy} - \bar{\Psi}_{zz}|^2 + \frac{2}{15} |\bar{\Psi}_{xy}|^2,$$

$$\langle h_\times^2 \rangle \propto \frac{1}{21} |\bar{\Psi}_{xx} - \bar{\Psi}_{yy}|^2 + \frac{1}{6} |\bar{\Psi}_{xy}|^2.$$  

Temporal behaviour of these quantities for polytropes are shown in Fig.4. For more concentrated configurations \((n > 1)\), a burst-like behaviour corresponding to the rapid final plunge is clearly seen.

Fig.3 and 5 shows the $h_+$ polarization of the gravitational wave calculated for a face-on orientation of the system relative to the line of sight, and the corresponding angle averaged metric strains $\langle h^2 \rangle$ for cases of Taam’s and Weaver et al’s red giants.

2.4. A more realistic treatment of the neutron star spiral-in

We have assumed so far that the stellar structure is not changed by the spiralling-in neutron star, which is of cause a very crude zero approximation used in order to get simple estimations of the effect. However, the qualitative behaviour of the frequency and amplitude of generated gravitational radiation is expected to not differ significantly in more accurate calculations. The problem itself is of course a complicated three-dimensional hydrodynamical one, but as an illustration we performed more precise 1.5-dimensional calculations of the neutron star spiral-in in case of Taam’s model.

We assumed after Taam (1979) that the entire common envelope is rotating rigidly (by action of effective convective angular momentum transfer) with a spatially constant angular velocity defined at each moment as $v_e = J_e r_{NS}/I_e$, where $J_e$ is the total angular momentum transferred from the orbit to the envelope rotation, $v_e$ the envelope circular velocity, $r_{NS}$ the position of the neutron star and $I_e$ the moment of inertia of the common envelope. The relative velocity of the neutron star is $v_r = v_{NS} - v_e$. Thus, instead of (12) the dissipative function is

$$D = \frac{G^2 M_{NS}^2}{(v_e^2 + a_s^2)^2} \rho(r) v_r^3.$$  

The precise value of sound velocity $a_s$ is a complicated problem and can be accurately calculated only under fully hydrodynamical treatment. We assume after Chevallier (1993) the motion of the neutron star in the envelope to be mildly supersonic with a Mach number of the order 3 – 1.4 in the central regions. Therefore, during the calculations we neglect by the sound velocity in the outer
Fig. 5. $h_+$ – polarization of the gravitational wave (a) and the corresponding averaged over binary orientation metric strain amplitude $\langle h^2 \rangle$ (b), emitted during spiral-in of a 1.4 $M_\odot$ neutron star inside Taam’s (1979) model of red giant density distribution ($M = 15M_\odot$, $R = 4.2 \times 10^{12}$ cm), as a function of time in units of $\sqrt{R^3/GM}$.

Fig. 6. The same as in Fig.5 for Weaver et al.’s (1979) density distribution inside a 16$M_\odot$ red giant with a radius of $3.9 \times 10^{13}$ cm.

layers of the giant, and take it to be $a_s \sim v_r$ in the inner regions after $v_r$ has been equal to the keplerian velocity for the Lagrangian mass $M(r_{NS})$.

As expected, the calculations yielded similar dependences of $f_{GW}$ and $h$ on time (see Fig.3, a, b), with the major difference being in time the neutron star takes to reach the final plunge. In this case that time is much larger (as the dragging force is much less), and is about $10^3$ years, in agreement with the cited hydrodynamical calculations. The final plunge starts at a distance from the center of 15% the initial radius, and lasts about several days. The GW frequency and amplitude prove to be higher than in the simplest model case considered above by a half an order and one order of magnitude, respectively.

3. Discussion and conclusions

In the present paper we calculated gravitational wave emission during formation of a Thorne-Żytkow object – a red giant star with a neutron star core, which must be formed under certain conditions during evolution of massive binary systems.

To estimate the gravitational wave emission, we made a number of simplifying assumptions:
1. no (or weak) influence of the spiralling-in neutron star to the density distribution inside the giant star;
2. the dynamical friction is the only dragging force;
3. no accretion induced mass change of the neutron star.

The first assumption is the most severe, as during the spiral-in phase the whole envelope of the red giant can be lost. However, for enough tight initial configurations
(case of Taam’s model) the density profile inside the star changes no more than an order of magnitude to the end of the spiral-in stage (Taam 1979). This means that under more realistic conditions the neutron star takes more time to reach the red giant’s centre, so our estimates should be considered as lower limits. This is illustrated by calculations of the neutron star spiral-in inside Taam’s (1979) red giant model assuming rigid rotation of the common envelope.

The second assumption seems to be justified by extensive numerical calculations of the common envelope evolution. The third assumption is valid while the radiation pressure prevents matter from accreting onto the neutron star at a rate exceeding the Eddington limit \( \sim 10^{-8} M_\odot/\text{yr} \) (see discussion of another possibility in Chevalier 1993).

As is seen from Fig. 7, the maximum value of the dimensionless strain metric amplitude \( h \) during neutron star spiral-in toward the center of a compact (\( \sim R_\odot \)) polytropic configuration can reach \( \sim 10^{-24} \) at a frequency of \( \sim 10^{-2} - 10^{-1} \) Hz, assuming the source lying at 10 Mpc away from the Sun. Real density distributions yield approximately the same amplitudes but at much lower frequencies \( \sim 5 \times 10^{-5} - 2 \times 10^{-4} \) Hz (Fig. 7), which can only marginally enters into the sensitivity range for the planned LISA detector (Laser Interferometer Space Antenna; see Hills & Bender (1995) for more detail).

However, the situation is not excluded when the neutron star enters a dilute envelope of a \( 16 M_\odot \) envelope, makes it to be dynamically unbound but does penetrate into the inner 2-3 \( M_\odot \) dense core to form a TZ object. Then the frequency of the GW could well fall into the range \( 10^{-3} - 10^{-2} \), where the LISA detector is the most sensitive.

To conclude, we stress the importance of hydrodynamical modeling of processes considered in the present paper. Analysis of gravitational wave signal emitted during formation of a TZ-object can bring a unique direct information about density distribution inside the star, which is not available by other methods.

Acknowledgements
The authors acknowledge Profs. Leonid Grishchuk and Kip Thorne for stimulating discussions, Prof. Vladimir Lipunov and Dr. Mike Prokhorov for useful suggestions, and the anonymous referee for valuable notes. The work is partially supported by ESO CE&E grant A-02-079, RFFI grant 94-02-04049 and INTAS grant 93-3364.

References

Hils, D. & Bender, P.L., 1995, ApJ Lett. in press
Chevalier, R.A., 1993, ApJ 411, L33.
Kochanek, C.S. et al. 1990, ApJ. 358, 81.
Livio, M., 1994, in “Interacting Binaries” (eds. S.N.Shore, M. Livio and E.P.J. van den Heuvel), Springer: Berlin, chapter 2.
Misner, C.W., Thorne, K.S., and Weeler, J.A., 1973, Gravitation. W.H.Freeman & Co.: San Francisco.
Paczyński, B., 1976, in “Structure and Evolution of Close Binary Systems” (eds. P.Eggleton, S.Mitton and J.Wheelan), Reidel: Dordrecht, p.75.
Taam, R.E., 1979, ApJ Letters 20, 29.
Taam, R.E., Bodenheimer, P., and Ostriker, J.P., 1978, ApJ 222, 269.
Thorne, K.S., 1988, in “300 Years of Gravitaiton”, Ed. S.W.Hawking and W.Israel, Cambridge Univ. Press.
Thorne, K.S. and Zytkowski, A.N. 1977, ApJ 212, 832.
vanden Heuvel, E.P.J., 1994, in “Interacting Binaries” (eds. S.N.Shore, M. Livio and E.P.J. van den Heuvel), Springer: Berlin, chapter 3.
Weaver, T., Zimmerman, G. and Woosley, S., 1978, ApJ 225, 1021.