A Relativistic Separable Potential to Describe Pairing in Nuclear Matter

B. Funke Haas, T. Frederico, B. V. Carlson  
Departamento de Física, Instituto Tecnológico da Aeronáutica – CTA,  
12.228-900 São José dos Campos, São Paulo, Brazil

F.B. Guimarães  
EAN – Instituto de Estudos Avançados – CTA,  
12.228-840 São José dos Campos, São Paulo, Brazil

Abstract

Using the Dirac-Hartree-Fock-Bogoliubov approximation to study nuclear pairing, we have found the short-range correlations of the Dirac $^1S_0$ pairing fields to be essentially identical to those of the two-nucleon virtual state at all values of the baryon density. We make use of this fact to develop a relativistic separable potential that correctly describes the pairing fields.
I. INTRODUCTION

We have recently studied nuclear pairing in meson-exchange models of the nuclear interaction using a Dirac-Hartree-Fock-Bogoliubov (DHFB) approximation to nuclear matter \[1,2\]. An important conclusion of this study is that the short-range \(^1S_0\) pairing correlations in nuclear matter are essentially identical to the short-range correlations of the two-nucleon virtual state, due to the dominance of the virtual state in the \(^1S_0\) channel. In non-relativistic calculations, Khodel, Khodel and Clark \[3\] also noted a close similarity between the pairing gap function and the short-range vertex function of the virtual state \([I]\), which prompted them to suggest that the virtual state be used as the starting point in calculating the gap function. In our relativistic calculations, however, the similarity of the Dirac pairing fields and the vertex functions is so striking that we can claim that the two are essentially identical. We make use of this quasi-identity to develop a separable approximation to the relativistic pairing potential that correctly takes into account the high momentum contributions of the short-range two-nucleon correlations.

Another indication that a separable potential can describe nuclear pairing correlations comes from the usual assumption of a constant pairing gap in finite nucleus calculations. The analysis of the HFB equations in the zero-range limit of the two-nucleon interaction shows that the separable potential approximation becomes an exact formulation of the pairing problem in the zero-range limit as the pairing potential then becomes independent of the baryon momentum. The pairing field also become independent of the baryon momentum and vary only with respect to the baryon density of the nuclear matter, which is equivalent to a constant pairing gap assumption. Thus, the separable potential approximation for the pairing field can be seen as the simplest extension, to finite-range two-nucleon interactions and all baryon densities, of the zero-range limit of the HFB-equations. It gives the simplest improvement to the usual assumption of a constant pairing gap.

The aim of this paper is to present a rank-one, separable interaction that accurately describes the DHFB pairing field at all densities. In section II, we clarify the close relationship between the pairing field and the two-nucleon \(^1S_0\) virtual state vertex function and use this relationship to define the separable interaction. In Section III, we demonstrate the goodness of the separable interaction through comparisons with exact DHFB calculations of nuclear matter.
II. THE FORMALISM

The bound-state correlations in a two-particle system can be roughly classified as asymptotic ones and short-range ones. The asymptotic correlations are determined principally by the binding energy, while the short-range ones depend on the high-momentum components of the wave function. This can be seen by examining the manner in which a bound pair appears in the two-body T-matrix, \( T(E) \). The T-matrix satisfies the integral equation

\[
T(E) = V + V G_0(E) T(E),
\]

where \( V \) is the two-body interaction and \( G_0(E) \) the free two-body propagator. A bound state corresponds to a pole in the T-matrix at a negative value of the energy, \(-\epsilon_b\). We can decompose the T-matrix, in this case, as

\[
T(E) = \Gamma \frac{1}{-\epsilon_b - E} \Gamma^\dagger + T_c(E),
\]

where \( \Gamma \) is the bound-state vertex function and \( T_c(E) \) is the continuum component of the T-matrix (and, possibly, the contributions of other bound state poles). Substituting the decomposition in the integral equation, Eq. (1), we verify that the vertex function satisfies the equation

\[
\Gamma = V G_0(-\epsilon_b) \Gamma.
\]

The vertex function of the two-nucleon bound state is closely related to its wave function, \( \psi = G_0(-\epsilon_b) \Gamma \), which satisfies the differential equation

\[
(-\epsilon_b - H_0)\psi = (-\epsilon_b - H_0)G_0(-\epsilon_b)\Gamma = \Gamma = V G_0(-\epsilon_b) \Gamma = V \psi,
\]

where \( H_0 \) is the Hamiltonian of the free two-nucleon system.

Here we see the rough division of the two-nucleon correlations in the vacuum. The asymptotic correlations are determined by the singularity of the Green’s function \( G_0(-\epsilon_b) \) while the short-range ones are contained in the high momentum components of the vertex function, \( \Gamma \). This analysis can also be extended to the nucleon pairing correlations in nuclear matter. One can directly associate the bound-state vertex function \( \Gamma \) of the T-matrix with the pairing field of the HFB approximation, \( \Delta \), and the adjoint vertex function, \( \Gamma^\dagger \), with the time-reverse conjugate pairing field, \( \bar{\Delta} \). To demonstrate this, we develop the relation between \( \Gamma \) and \( \Delta \) in a formal way, by comparing the equations that define the two. (In a similar manner, the pair wave function \( \psi \) can be associated with the residue of the anomalous
propagator $F$ in the complex energy plane. However, we will not develop this association here.)

In the Dirac-Hartree-Fock-Bogoliubov (DHFB) approximation to pairing in nuclear matter, the self-consistency equation for the pairing field can be written as

$$\Delta(k) = i \sum_j \int \frac{d^4q}{(2\pi)^4} \Lambda_{j\alpha} D_{j\beta}^\alpha(k - q) F(q) B \Lambda_{j\beta}^T B \dagger,$$

where $\Lambda_{j\alpha}$ represents the meson-baryon vertex of meson $j$ (for the $\sigma$ meson, $\Lambda_{\sigma\alpha} = ig_s \mathbb{I}$, for the $\omega$ meson, $\Lambda_{\omega\alpha} = -ig_v \gamma_\alpha$, etc.), $D_{j\alpha}^\beta$ represents the propagator of meson $j$ and the Greek letters represent any necessary Lorentz and/or isospin indices. The matrix $B$ relates transposed quantities to the complex conjugates of time-reversed ones. It is given by $B = \tau_2 \otimes \gamma_5 C$, where the Pauli matrix $\tau_2$ acts in the isospin space and $C$ is the charge conjugation matrix.

The anomalous propagator $F(q)$ in the self-consistency equation, Eq. (5), is one of the number-non-conserving components of the baryon propagator $S_F(q)$, which, in the Gorkov formalism, takes the form

$$S_F(q) = \begin{pmatrix} G(q) & F(q) \\ \bar{F}(q) & \bar{G}(q) \end{pmatrix},$$

in which $G(q)$ is the usual number-conserving propagator and $F(q)$ and $\bar{G}(q)$ are the corresponding time-reversed propagators. The baryon propagator $S_F(q)$ can be written in terms of the self-energy and pairing fields, $\Sigma(q)$ and $\Delta(q)$, respectively, as

$$S_F(q) = \begin{pmatrix} \hat{\gamma} - M - \Sigma(q) + \mu \gamma_0 & \Delta(q) \\ \Delta(q) & \hat{\gamma} + M + \Sigma(q) - \mu \gamma_0 \end{pmatrix}^{-1},$$

where $\mu$ is a Lagrange multiplier used to fix the average baryon density.

The form of the $^1S_0$ pairing field can be determined from the hermiticity and antisymmetry properties of the Lagrangian density, as well as the translational, rotational and isospin symmetries of symmetric nuclear matter, as is shown in Ref. [1]. The general form obtained for the field is

$$\Delta(k) = (\Delta_S(k) - \gamma_0 \Delta_\theta(k) - \gamma_0 \vec{\gamma} \cdot \hat{k} \Delta_T(k)) \vec{\gamma} \cdot \hat{n}.$$
We can decompose the self-consistency equation for the pairing field, Eq. (5), into equations for the components of Eq. (8). After integrating over energy and angle and neglecting retardation effects, the component equations take the form,

\[
\Delta_S(k, k_F) = \frac{1}{2\pi^2} \int_0^{\Lambda} q^2 dq V_S(k, q) F_S(q, k_F),
\]

\[
\Delta_0(k, k_F) = \frac{1}{2\pi^2} \int_0^{\Lambda} q^2 dq V_0(k, q) F_0(q, k_F),
\]

\[
\Delta_T(k, k_F) = \frac{1}{2\pi^2} \int_0^{\Lambda} q^2 dq V_T(k, q) F_T(q, k_F),
\]

where \(k = |\vec{k}|\) and \(q = |\vec{q}|\). The pairing potentials, \(V_S, V_0\) and \(V_T\) (given in the Appendix), are functions of the coupling constants and meson masses, while \(\Lambda\) is a cutoff in the baryon momentum. The Fermi momentum \(k_F\) is defined through its conventional relation to the baryon density, that is, \(\rho_B = \gamma k_F^3/3\pi^2\), where \(\gamma\) is 2 for nuclear matter and 1 for neutron matter. The components of the anomalous propagator are defined as

\[
F_S(k) = \frac{1}{8} \text{Tr}[\vec{\tau} \cdot \hat{n} F(k)],
\]

\[
F_0(k) = \frac{1}{8} \text{Tr}[\gamma_0 \vec{\tau} \cdot \hat{n} F(k)],
\]

\[
F_T(k) = -\frac{1}{8} \text{Tr}[\gamma_0 \vec{\gamma} \cdot \hat{k} \vec{\tau} \cdot \hat{n} F(k)].
\]

These are evaluated in the Appendix in the case in which their negative-energy Dirac sea contributions are neglected.

The components of the anomalous propagator, \(F_S, F_0\) and \(F_T\) carry the information of the density-dependent nucleon self-energy and pairing mean-fields. The pairing potentials \(V_S, V_0\) and \(V_T\) also possess a density dependence associated with the retardation terms, which account for the finite meson propagation velocity. In Eqs. (9), these retardation terms are neglected since their effects on the pairing fields have been found to be small [1]. The density dependence remaining in the self-consistent pairing equations, Eqs. (9), is thus entirely contained in the components of the anomalous propagator.

When the baryon density \(\rho_B\) tends to zero, both the self-energy \(\Sigma(q)\) and the pairing field \(\Delta(q)\) also tend to zero. The anomalous propagator \(F(q)\) can then be well approximated by

\[
F(q) \approx -\frac{1}{\hat{q} - M + \mu \gamma_0 + i\eta} \Delta(q) \frac{1}{\hat{q} + M - \mu \gamma_0 - i\eta}, \quad \rho_B \rightarrow 0,
\]

an approximation that becomes exact at zero density. The vacuum limit of the pairing field thus satisfies the homogeneous equation.
\[ \Delta(k) = i \sum_j \int \frac{d^4q}{(2\pi)^4} \Lambda_j \alpha \beta \beta(k - q) \left( \frac{1}{\mu \gamma_0 + \not{q} - M + i\eta} \Delta(q) \right) \frac{1}{\mu \gamma_0 - \not{q} - M + i\eta} B \Lambda^T \beta \beta \beta(k - q), \quad \rho_B \to 0. \]  

(12)

Turning now to the Bethe-Salpeter equation, we write the bound-state vertex function as \( \Gamma(k, P) \), where \( P \) is the center-of-mass four momentum of the baryon pair and \( k \) is their relative four momentum. The Bethe-Salpeter equation for the vertex function, in the ladder approximation, is often written as

\[ \Gamma(k, P) = i \sum_j \int \frac{d^4q}{(2\pi)^4} \Lambda_{j\alpha} \Lambda_{j\beta} \beta \beta \beta(k - q) G_0(P/2 + q) \Gamma(q, P), \]  

(13)

where the free baryon propagator \( G_0(q) \) is given by

\[ G_0(q) = \frac{1}{\not{q} - M + i\eta}. \]  

(14)

We can rewrite this in a matrix form, making use of the matrix \( B \), as

\[ \Gamma(k, P) B^\dagger = i \sum_j \int \frac{d^4q}{(2\pi)^4} \Lambda_{j\alpha} \Lambda_{j\beta} \beta \beta \beta(k - q) G_0(P/2 + q) \Gamma(q, P) B^\dagger G_0(P/2 - q) B \Lambda^T \beta \beta \beta. \]  

(15)

Comparing this expression with the self-consistency equation for the vacuum pairing field, Eq. (12), we find the two are identical if we

- evaluate the vertex function in the center-of-mass frame, taking
  \[ P_\alpha = 2\mu \delta_{\alpha0}, \]  
  (16)

where \( \mu \) is the Lagrange multiplier of the Gorkov propagator \( S_F(q) \) of Eq. (7), and

- associate the pairing field and the vertex function as
  \[ \Delta(k) = \Gamma(k) B^\dagger, \]  
  (17)

where we now suppress the center-of-mass dependence of the vertex function.

We thus conclude that the self-consistency equation for the vacuum pairing field is identical to the ladder approximation to the Bethe-Salpeter equation in the center-of-mass frame.

An analysis of the symmetry and antisymmetry properties that we expect of the center-of-mass Bethe-Salpeter vertex function, \( \Gamma(k) \), leads us to the following form,

\[ \Gamma(k) B^\dagger = \left( \Gamma_s(k) - \gamma_0 \Gamma_0(k) - \gamma_0 \gamma \cdot k \Gamma_T(k) \right) \gamma \cdot \hat{n}, \]  

(18)
which is identical to that of the pairing field $\Delta(k)$ given in Eq. (8). We substitute this vertex function in the Bethe-Salpeter equation, Eq. (15), integrate over energy and angle, and neglect retardation effects and the contribution of the negative energy states, just as we have done in the self-consistency equation of the pairing field. We find,

\[
\Gamma_S(k) = \int_\Lambda V_S(k,q) \frac{q^2}{4\pi^2} \frac{E_q \Gamma_S - M \Gamma_0}{E_q(M_B - 2E_q)} dq,
\]

\[
\Gamma_0(k) = \int_\Lambda V_0(k,q) \frac{q^2}{4\pi^2} \frac{(2q^2 - M_B E_q) \Gamma_0 + M M_B \Gamma_S}{M_B E_q(M_B - 2E_q)} dq,
\]

\[
\Gamma_T(k) = -\int_\Lambda V_T(k,q) \frac{q^2 k}{4\pi^2} \frac{g(M_B \Gamma_S - 2M \Gamma_0)}{M_B E_q(M_B - 2E_q)} dq,
\]

where $k = |\vec{k}|$, $q = |\vec{q}|$, $\Lambda$ is a cutoff in the baryon momentum, and $E_q = \sqrt{q^2 + M^2}$. The potentials $V_S$, $V_0$, and $V_T$ are, of course, the same as those of the self-consistency equations for the pairing field, Eqs. (11), and are given in the Appendix. The center-of-mass energy of the bound two-nucleon pair, that is, the bound two-nucleon mass, $M_B$, is given by

\[
M_B = 2\mu = 2M - \epsilon_B.
\]

In the Appendix, the equivalence between the vacuum pairing equation and the ladder approximation to the Bethe-Salpeter equation is once again demonstrated, by reducing the explicit form of the components of the pairing equation, Eqs. (11), to the components of the Bethe-Salpeter equation, Eqs. (19), in the vacuum limit.

The above expressions are valid for a bound two-body system in the vacuum. If the pole of the $T$-matrix corresponds to a virtual state (anti-bound state), as in the case of the physical $^1S_0$ two-nucleon channel, one must redefine the integrands through an analytic continuation into the second sheet of the complex energy plane \[6\]. The analytical continuation introduces an additional term, which takes into account the discontinuity of the propagator across the cut in the energy plane. In the components of the Bethe-Salpeter equation, Eqs. (19), in which the negative-energy states have been neglected, the analytically continued equations can be obtained through the substitution,

\[
\frac{1}{M_B - 2E_q} \rightarrow \frac{1}{M_v - 2E_q} - 2\pi i \delta(M_v - 2E_q),
\]

where $M_v$ is the mass of the two-nucleon virtual state,

\[
M_v = 2M - \epsilon_v = \sqrt{M^2 - |q_v|^2},
\]

with $q_v = -i|q_v|$. 

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The equations for the components of the vertex-function then become

\[
\Gamma_S(k) = \Gamma_S(k_v) - \frac{|q_v|}{\pi} \left\{ V_s(k, q) \left( E_q \Gamma_S + M \Gamma_0 \right) \right\}_{q=q_v}, \\
\Gamma_0(k) = \Gamma_0(k_v) + \frac{|q_v|}{\pi} \left\{ V_0(k, q) \left( \frac{2q^2 - M_v E_q}{M_v} \Gamma_0 - M M_v \Gamma_s \right) \right\}_{q=q_v}, \\
\Gamma_T(k) = \Gamma_T(k_v) + \frac{|q_v|}{\pi} \left\{ k V_T(k, q) \frac{M_v \Gamma_S + 2 M \Gamma_0}{M_v} \right\}_{q=q_v},
\]

The functions \( \Gamma_s(k_v), \Gamma_0(k_v) \) and \( \Gamma_T(k_v) \) are given by Eqs.(19) with the mass of the bound state \( M_B \) substituted by that of the virtual state \( M_v \).

As stated in the introduction, we have found that the momentum dependence of the components of the pairing field \( \Delta \), calculated with various sets of interaction parameters, shows a very small variation over a wide range of values of the baryon density and a wide range of cut-offs (\( \Lambda \) between 2.5 fm\(^{-1} \) and 15.0 fm\(^{-1} \)) \[2\]. As an example, in Fig. 1 we show the momentum dependence of the principal components of the pairing field, \( \Delta_s(k) \) and \( \Delta_0(k) \), for several values of the Fermi momentum, which were obtained using the \( \sigma - \omega \) parameters of Ref. \[7\] with a momentum cutoff of \( \Lambda = 10 \) fm\(^{-1} \). Note that the curves denoted by Fermi momenta zero correspond to the components of the vertex function of the \( ^1S_0 \) virtual state in the vacuum. The small variation in the components as a function of the Fermi momentum suggests the use of the momentum dependence at one value of the density. Use of the vacuum momentum dependence is particularly convenient, since in this case a direct correspondence exists between the self-consistent pairing field and the Bethe-Salpeter vertex function. A direct comparison between the numerical solutions for the vertex function, \( \Gamma(k) \), and for the pairing field, \( \Delta(k) \), gives us the following very good approximate relations between the components of the two fields,

\[
\Delta_s(k, k_F) \approx d(k_F) \Gamma_s(k), \quad \Delta_0(k, k_F) \approx d(k_F) \Gamma_0(k), \quad \Delta_T(k, k_F) \approx d(k_F) \Gamma_T(k),
\]

where \( d(k_F) \) is a density-dependent factor determining the overall magnitude of the pairing fields.

We emphasize that, in the nonrelativistic approach of Khodel, Khodel and Clark, the virtual state vertex function serves only as a good starting point for calculating the pairing field \[3\], while in our relativistic approach, the two are essentially identical in form. We can explain the difference between the two approaches by analyzing the effective gap function for the positive energy DHFB states, which takes the form
\[ \Delta_g(k) = \Delta_0 \frac{M^*}{E^*_k} - \Delta_S + \Delta_T \frac{k^*}{E^*_k}, \quad (24) \]

where \( E^* = \sqrt{k^*^2 + M^*^2} \) and the effective momentum and mass of the quasi-nucleons, \( k^* \) and \( M^* \), are defined in the Appendix. The gap function \( \Delta_g(k) \) is the quantity whose role is closest to that of the non-relativistic pairing field \[1\). Like the non-relativistic self-energy, the gap function is the difference between two larger relativistic quantities, \( \Delta_0 \) and \( \Delta_S \). (\( \Delta_T \) is two to three orders of magnitude smaller than the other components.) The effective mass \( M^* \) that appears in the gap function varies strongly with the nuclear matter density. Although the form of the components of the Dirac pairing field remain almost constant with density, the gap function changes, due to its dependence on \( M^* \). The density independence of the form of the relativistic pairing field thus does not carry over to the nonrelativistic field.

The invariant momentum dependence of the components of the pairing fields suggests a separable form for an effective potential, in which each of the components of the interaction, \( V_S, V_0 \) and \( V_T \), is proportional to the corresponding component of the vertex function, \( \Gamma_S, \Gamma_0, \) and \( \Gamma_T \). Such potentials are well known from the nonrelativistic treatment of the two-nucleon problem. It is such a separable form of the non-relativistic pairing potential that Khodel, Khodel and Clark suggest as a starting point for calculating the nonrelativistic gap function \[3\]. Here, we use a form appropriate for the components of the relativistic pairing field that also preserves the symmetry in the two momenta, \( k \) and \( q \), of the potential, taking

\[
\begin{align*}
V_S(k,q) &= \frac{\lambda_S}{4} \Gamma_S(k)\Gamma_S(q), \\
V_0(k,q) &= \frac{\lambda_0}{4} \Gamma_0(k)\Gamma_0(q), \\
kV_T(k,q) &= \frac{\lambda_T}{4} \Gamma_T(k)\Gamma_T(q),
\end{align*}
\]

\( (25) \)

where the potential strengths, \( \lambda_S, \lambda_0, \) and \( \lambda_T, \) are obtained through the simultaneous solution of Eqs.(19) and (25), in the case of a bound state, or Eqs.(21) and (25), in the case of a virtual state. We note that, although we have included the equation for the tensor component \( \Gamma_T \) in Eqs. (23) and (25), we have neglected this component in our numerical calculations. The inclusion of the tensor term offers little further calculational difficulty but, being extremely small, has little effect on the rest of the calculation.

For any given meson-exchange potential, we have shown in Ref. [2] that the size of the pairing gap, for low baryon densities, is correlated with the energy of the virtual state of the T-matrix, \( \epsilon_v \). The value of \( \epsilon_v \), in turn is related to the baryon momentum cut-off, \( \Lambda \), in the integrals of the self-consistency equations. Consequently, fixing \( \epsilon_v \) at its physical value
in the Bethe-Salpeter equation for $\Gamma$ determines a (usually unique) value for $\Lambda$.

The separable potential introduces two additional parameters, $\lambda_S$ and $\lambda_0$, when the tensor term is neglected. These parameters, together with the momentum cutoff, $\Lambda$, are fixed by requiring that the energy of the two nucleon state and the ratio of the vertex functions be equal to the respective ones obtained by solving the Bethe-Salpeter equation with a meson-exchange potential. Together, the parameters $\lambda_S$, $\lambda_0$ and $\Lambda$ introduce an ambiguity in the determination of the separable potential. Our calculations have shown that, within a wide range of values of the cutoff $\Lambda$, we can find values of $\lambda_S(\Lambda)$ and $\lambda_0(\Lambda)$ that fix the virtual state energy and the vertex functions at their physical values. We regard this as a positive feature of the separable potential, as it permits us to fix one of the parameters, usually $\Lambda$, at an arbitrary, and possibly convenient, value, and then solve for the others.

The only task remaining to complete the definition of the separable pairing potential is to extend the potential, which we have defined in the vacuum, to finite values of the baryon density. This is done by observing that, as the retardation terms in the pairing potentials have been neglected, the dependence of the pairing field on the baryon density is completely determined by the density dependence of the components of the anomalous density, $F_S$, $F_0$ and $F_T$, in Eq.(9). Consequently, the separable pairing potential for finite baryon densities must be the same as the potential in the vacuum. The self-consistency equations for the components of the pairing field are thus given by Eqs. (9) and (25), with the parameters $\lambda_S$, $\lambda_0$, and $\Lambda$ (and, eventually, $\lambda_T$ as well) fixed by the vacuum solution.

III. RESULTS AND CONCLUSION

We have compared calculations of the pairing fields obtained using a separable interaction with the usual DHFB results for various sets of $\sigma - \omega$ interaction parameters. The relativistic separable potential describes well both the gap parameter, $\Delta_g$, and the two large components, $\Delta_0$ and $\Delta_s$, at all baryon densities. It also reproduces the behavior of the pairing field $\Delta$ for increasing values of the momentum cut-off $\Lambda$ in the self-consistency equations.

In Fig. 2 we show the particular results for the $\sigma - \omega$ interaction of Bouyssy et.al. [7], for a cut-off of $\Lambda = 3.8$ fm$^{-1}$, obtained by adjusting the virtual state energy to its physical value. This potential has been chosen because its vacuum gap function, which is the nonrelativistic two-nucleon vertex function, is in good agreement with the nonrelativistic two-nucleon vertex function calculated with the Bonn-B potential [8]. We note a difference of at most a few percent between the components of the relativistic fields obtained in exact and separable
calculations. The differences in the two gap functions are even smaller. Similar results are obtained for other sets of $\sigma - \omega$ interaction parameters \[2\].

As discussed above, it is possible to adjust the strengths $\lambda_S$ and $\lambda_0$ of the separable interaction to fit the experimental value of the two-nucleon virtual state energy as a function of the cutoff $\Lambda$. For each value of the $\Lambda$, the resulting values of the strengths $\lambda_S$ and $\lambda_0$ yield a separable potential which carries the physical information necessary for the correct evaluation of the gap parameter at all values of the density. Since the properties of the virtual state dominate the physics of $^1S_0$ pairing, we would expect the pairing fields that result to be independent of the cutoff used. We can verify the extent to which this is true by comparing the results obtained for $d(k_F)$ of Eq. (23), the common factor determining the overall magnitude of the pairing fields, at different values of the momentum cutoff $\Lambda$. We do this in Fig. 3, where we display the factor $d(k_F)$ as a function of the Fermi momentum for several values of the cutoff $\Lambda$. We find the form of the factor $d(k_F)$ to be fairly constant but its magnitude to decrease between 5 and 10 % as the momentum cutoff $\Lambda$ increases from 3.3 fm$^{-1}$ to 10 fm$^{-1}$.

In Ref. \[2\], a strong correlation between the magnitude of the pairing gap and the energy of the two-nucleon virtual state was observed in all the meson potentials studied there. The correlation is reproduced, for all values of the cut-off, when using the corresponding separable potential. In Fig. 4, we compare the correlation for the gap parameter, $\Delta_g$, obtained with the separable interaction with the exact result, using the $\sigma - \omega$ parameters of Ref. \[7\]. The agreement between the two is excellent, confirming once again the goodnes of the separable approximation to the pairing interaction.

We thus conclude that a separable interaction obtained by exploiting the close relationship between the Dirac pairing field and the virtual state vertex function furnishes an excellent description of the Dirac pairing fields at all values of the baryon density. We next plan to implement a configuration-space version of the separable interaction in a DHFB code for finite nuclei calculations. We expect that such a potential will describe the short-range pairing correlations with greater accuracy than those obtained with the usual constant gap assumption.
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APPENDIX: THE ANOMALOUS PROPAGATOR AND THE PAIRING POTENTIAL

The functions $F_S$, $F_0$ and $F_T$ in Eq.(6) are given by [1],

$$F_S(k) = \left\{ \Delta_S[-\alpha + \beta + E_F^* k^2 + M^* k^2 + k^2] + \Delta^*_S(\Delta^2_S - \Delta^2_0 + \Delta^2_T k^2) \right\} / (4\beta\omega_-),$$

$$F_0(k) = \left\{ \Delta_0(\alpha - \beta + E_F^* k^2 + M^* k^2 + k^2) + \Delta^*_0(\Delta^2_S - \Delta^2_0 + \Delta^2_T k^2) \right\} / (4\beta\omega_-),$$

and

$$F_T(k) = \left\{ -k\Delta_T(-\alpha + \beta + E_F^* k^2 + M^* k^2 + k^2) - k\Delta^*_T(\Delta^2_S - \Delta^2_0 + \Delta^2_T k^2) \right\} / (4\beta\omega_-),$$

where the asterisk indicates complex conjugation and $k = |\vec{k}|$. The quantity $\omega_-$ is

$$\omega_- = \sqrt{\alpha - \beta}$$

where
\[ \alpha = E_F^* + k^* + M^* + |\Delta S|^2 + |\Delta_0|^2 + |\Delta T|^2 , \]
\[ \beta = 2\sqrt{\beta_1^2 + \beta_2^2 + \beta_3^2 + \beta_4^2} , \]
\[ \beta_1 = -\text{Im}(\Delta_T \Delta_0^*) , \]
\[ \beta_2 = E_F^* M^* + \text{Re}(\Delta_S \Delta_0^*) , \]
\[ \beta_3 = E_F^* k^* + \text{Re}(\Delta_S \Delta_T^*) , \]
\[ \beta_4 = |M^* \Delta_T + k^* \Delta_0| . \]

The effective momentum and mass of the baryons, \( k^* \) and \( M^* \), are given in terms of the components of the nucleon self-energy,
\[ \Sigma(\vec{k}) = \Sigma_S(k) - \gamma_0 \Sigma_0(k) - \vec{\gamma} \cdot \vec{k} \Sigma_V(k) , \]
as,
\[ k^* = (1 + \Sigma_V(k))k \quad \text{and} \quad M^* = M + \Sigma_s(k) . \]

The Fermi energy is
\[ E_F^* = \Sigma_0(k) + \mu , \]
where \( \mu \) is the chemical potential.

The pairing potentials that take into account the exchange of the mesons \( \sigma \) and \( \omega \) are given by
\[ V_S(k,q) = \frac{2}{kq} \left[ \frac{-1}{8} g_\sigma^2 \theta_\sigma + \frac{1}{2} g_\omega^2 \theta_\omega \right] , \]
\[ V_0(k,q) = \frac{2}{kq} \left[ \frac{1}{8} g_\sigma^2 \theta_\sigma + \frac{1}{4} g_\omega^2 \theta_\omega \right] , \]
\[ V_T(k,q) = \frac{1}{2k^2q} g_\sigma^2 \phi_\sigma \]
where the functions \( \theta_\sigma \) and \( \phi_\sigma \) are
\[ \theta_\sigma = \ln \left( \frac{H_\sigma + 2kq}{H_\sigma - 2kq} \right) , \]
with
\[ H_\sigma = (\omega_0(k) - \omega_0(q))^2 - k^2 - q^2 - m_\sigma^2 . \]

and
\[ \phi_\sigma = 1 - \frac{H_\sigma}{4kq} \theta_\sigma . \]
Here \( a \) indicates the mesons of the model, with \( a = \sigma, \omega \).

In the limit of zero baryon density, we have

\[ k^* \to k, \quad M^* \to M, \quad \text{and} \quad \mu \to E_F^* \to M_B/2, \]

so that

\[ \alpha \to \frac{M_B^2}{4} + E_k^2, \quad \beta \to M_B E_k, \quad \text{and} \quad \omega_- \to \frac{1}{2} |M_B - 2E_k|, \]

where \( E_k = \sqrt{k^2 + M^2} \). The components of the pairing field tend to zero. The zero baryon density limit of the pairing self-consistency equations are then

\[
\Delta_S(k) = \int_0^{\Lambda} V_S(k,q) \frac{q^2}{4\pi^2} \frac{E_q \Delta_s - M \Delta_0}{E_q (M_B - 2E_q)} dq,
\]

\[
\Delta_0(k) = \int_0^{\Lambda} V_0(k,q) \frac{q^2}{4\pi^2} \frac{(2q^2 - M_B E_q) \Delta_0 + M M_B \Delta_S}{M_B E_q (M_B - 2E_q)} dq,
\]

\[
\Delta_T(k) = -\int_0^{\Lambda} V_T(k,q) \frac{q^2 k}{4\pi^2} \frac{q (M_B \Delta_S - 2M \Delta_0)}{M_B E_q (M_B - 2E_q)} dq,
\]

which have exactly the same form as the equations for the components of the vertex function \( \Gamma \) given in Eqs.(19).
FIG. 1. The momentum dependence of the two large components of the Dirac pairing field, $\Delta_S$ and $\Delta_0$, are shown at several values of the nuclear matter density, using the parameters of Ref. [7]. The curve labeled as zero density corresponds to the vertex function of the virtual state.

FIG. 2. The pairing fields obtained in exact and separable DHFB calculations, using the $\sigma - \omega$ parameters of Ref. [7], are shown. The two large components of the Dirac pairing field, $\Delta_S$ and $\Delta_0$, and the gap parameter, $\Delta_g$, are shown as functions of the baryon density, for $\Lambda = 3.8$ fm$^{-1}$. 
FIG. 3. The factor $d(k_F)$ determining the overall magnitude of the pairing fields, calculated using the $\sigma - \omega$ parameters of Ref. [7], is shown for several values of the momentum cutoff $\Lambda$.

FIG. 4. The magnitude of the pairing gap as a function of the energy-equivalent momentum of the two-nucleon virtual state from exact and separable DHFB calculations, using the $\sigma - \omega$ parameters of Ref. [7], are shown. The positive and negative values of the abscissa correspond to bound and virtual two-nucleon states, respectively.