Multipath number estimation based on QR decomposition

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Abstract. Accurate multipath number estimation is the prerequisite for MUSIC algorithm to achieve excellent superresolution performance. To solve the problem that the existing multipath number estimation algorithms based on eigenvalue distribution usually require a large amount of computation, this paper proposes a fast multipath number estimation algorithm based on QR decomposition under the OFDM signal model. This algorithm only needs few times QR iteration decomposition to obtain the pseudo-eigenvalues of the matrix, and uses them to estimate the multipath number through two-step comparison. Due to the avoidance of accurate eigenvalues solution, compared with the traditional algorithms based on eigenvalue distribution, proposed algorithm’s computation is smaller. Simulation results showed that the proposed algorithm has better detection performance in low SNR and few snapshots than existing algorithms.

1. Introduction

Along with the continuous development of the unmanned aerial vehicle(UAV) technology and the decrease of the price, more and more people can afford UAV, which brings us a lot of entertainment and also safety risks. As an important link to ensure the safety of urban airspace, the government must realize the control of UAVs, and the first step is to realize its accurate positioning. In practical projects, target passive location is mainly achieved by solving the location equations constructed by arrival time difference(TDOA)[1][4].

When the UAV is flying, in order to make the ground pilot master the flight situation in real time, it will transmit the image signal modulated by orthogonal frequency division multiplexing(OFDM) technology to the ground, which makes passive location possible, and the TDOA of target can be obtained by subtracting two high-precision time delay estimates[5]. As most of the control requirements for UAVs are in cities, where buildings are relatively dense, the multipath effect is serious. Therefore, for UAVs’ signals, subspace algorithm is mostly used to estimate the time delay. [6] presented a MUSIC time delay estimation algorithm suitable for OFDM signals. This algorithm used channel frequency response estimation to obtain the covariance matrix, decomposed the matrix to obtain the noise subspace, then constructed the MUSIC pseudo-spectral function, and finally realized the time delay estimation through spectrum peak search. However, covariance matrix eigenvalue decomposition and peak search in [6] both lead to high computational complexity, which is not applicable in reality. In order to avoid peak search, [7] introduced root-music algorithm, which constructed polynomial using the denominator of pseudo-spectral function, and solved it to obtain the estimated value of time delay, reduced the computational complexity of algorithm. In [8], a propagator algorithm(PM)-based time delay estimation algorithm is proposed, this algorithm did not require eigendecomposition, and directly blocked the
covariance matrix to estimate the noise subspace, resulting in a significant decrease in computational complexity. Then [5] combined the algorithms of [7] and [8], proposed a PM-based root-Music time delay estimation algorithm, it solved the two main causes of high computational complexity and had better performance.

Similar to the source number estimation algorithms in the spatial spectrum estimation techniques, accurate multipath number estimation is also a parameter that must be known to realize the performance of time delay estimation in subspace algorithm. Most of the above time delay estimation algorithms assume that the number of multipath is known, but this prior condition usually cannot be obtained in the actual situation. The wrong estimation of multipath number will lead to the wrong division of signal and noise subspace and directly affect the accuracy of time delay estimation. The existing estimation algorithms like [10], [11] all require accurate eigenvalue decomposition of the covariance matrix, which undoubtedly increases the computational burden of the algorithm and is difficult to be implemented in engineering. Therefore, this paper proposed a multipath number estimation algorithm based on QR decomposition. Compared with the existing algorithms, it not only reduced the computational burden, but also improved the detection accuracy.

2. Signal Model

In this paper, we consider that there is only one target in space, and the OFDM signal emitted by it has K normal subcarriers. Then the time-domain expression of OFDM symbol is

\[ s(t) = \frac{1}{\sqrt{T}} \sum_{k=0}^{K-1} b_k e^{j2\pi f_k t} = \frac{1}{\sqrt{T}} \sum_{k=0}^{K-1} b_k e^{j2\pi f_c \left( \frac{f_k}{T} \right)} \]  

(1)

where \( T \) is the net data length of OFDM symbol, and it is also the period of fast Fourier transform (FFT) and inverse FFT, the subcarrier interval \( \Delta f = 1/T \) [12]; \( f_k \) is the frequency of the kth subcarrier; \( f_c \) is the carrier frequency of OFDM signal; \( b_k \) is the complex signal modulated on the kth subcarrier. The transmitted OFDM signals propagate in wireless multipath environment. Through the effect of channel and additive white Gaussian noise, the received signals \( y(t) \) can be expressed as

\[ y(t) = s(t) * h(t) + n(t) \]  

(2)

where \( n(t) \) is additive complex white Gaussian noise; \( h(t) \) is the time-domain shock response of a multipath wireless channel. Since the symbol period of OFDM is much shorter than the channel transformation period, the wireless channel remains unchanged within an OFDM symbol duration, so \( h(t) \) can be expressed as

\[ h(t) = \sum_{k=0}^{p-1} a_k \delta(t - \tau_k) \]  

(3)

where \( L_p \) is the number of multipath; \( a_k = |a_k| e^{j\phi_k} \) is the complex fading coefficient of kth path; \( \tau_k \) is the propagation delay of the kth path. Arrange \( \tau_k \) in ascending order, then \( \tau_0 \) is the propagation delay of the shortest path; \( \delta(\cdot) \) is the impact response function.

Estimation of time delay by MUSIC algorithm actually uses covariance matrix composed of data received by one single array element at different time points[13]. So we substitute equation (3) into equation (2) can get the received OFDM signal as

\[ y(t) = \frac{1}{\sqrt{T}} \sum_{k=0}^{K-1} b_k \left[ \sum_{j=0}^{L_p-1} a_j e^{-j2\pi f_c \left( \frac{f_j}{T} \right) \tau_j} \right] e^{j2\pi f_c \left( \frac{f_k}{T} \right) t} + n(t) \]  

(4)

then, by sampling the received signal \( y(t) \) and performing FFT, the received data of the kth subcarrier can be obtained as
where $n_k$ is a complex white Gaussian noise with mean of zero and variance of $\sigma^2$. Then the frequency response estimation of the multipath channel of the $k$th subcarrier can be expressed as

$$H_k = \sum_{i=0}^{L_k-1} a_i e^{-j2\pi f_i T} + n_k$$

and the channel frequency response estimation vector is

$$\hat{H} = H + n = V a + n$$

where $H$ represents the real impulse response of the channel in the frequency domain, and $n$ represents the estimation error,

$$H = [H_0 \ H_1 \ \cdots \ H_{K-1}]^T$$

$$V = [v(\tau_0) \ v(\tau_1) \ \cdots \ v(\tau_{L_p-1})]^T$$

$$v(\tau_i) = [1 \ e^{-j2\pi f_i T} \ \cdots \ e^{-j2\pi f_i (K-1)T}]^T$$

$$a = [a_0 \ a_1 \ \cdots \ a_{L_p-1}]^T$$

$$n = [n_0 \ n_1 \ \cdots \ n_{K-1}]^T$$

The covariance matrix of the channel frequency response is defined as

$$R_{\hat{H}\hat{H}} = E[\hat{H}\hat{H}^H] = VR_{aa}V^H + \sigma^2 I$$

Theoretically, the multipath number $L_p$ can be easily determined by observing the eigenvalues of the covariance matrix $A$, because $R_{\hat{H}\hat{H}}$ can be represented as

$$R_{\hat{H}\hat{H}} = \sum_{k=0}^{L_p-1} \lambda_k u_k u_k^H + \sum_{k=L_p}^{K-1} \lambda_k u_k u_k^H = U_S A_S U_S^H + U_N A_N U_N^H$$

where $\lambda$ represents the eigenvalue, $A_S = \text{diag}[\lambda_0 \ \lambda_1 \ \cdots \ \lambda_{L_p-1}]$ corresponds to the signal eigenvalues, $A_N = \text{diag}[\lambda_{L_p} \ \lambda_{L_p+1} \ \cdots \ \lambda_{K-1}]$ corresponds to the noise eigenvalues, and

$$\lambda_0 \geq \lambda_1 \geq \cdots \geq \lambda_{L_p-1} \geq \lambda_{L_p} = \lambda_{L_p+1} = \cdots = \lambda_{K-1} = \sigma^2$$

that means each of the $K - L_p$ minimum eigenvalues of $R_{\hat{H}\hat{H}}$ should be equal to $\sigma^2$, so that we can determine the number of multipath $L_p$. However, in practical processing, as shown in equation (15), the estimated covariance matrix is usually obtained by using a finite number of snapshots in a finite time period, so the eigenvalues of the noise are all different[9], and we can no longer determine the number of multipath by finding how many identical eigenvalues there are.

$$R_{\hat{H}\hat{H}} = \frac{1}{P} \sum_{p=1}^{P} \hat{H}^{(p)} \hat{H}^{(p)H}$$

where $P$ is the number of snapshot; $\hat{H}^{(p)}$ is the $p$th frequency response estimation vector of the channel. It should be noted that when using MUSIC algorithm to estimate the time delay, the covariance matrix $R_{\hat{H}\hat{H}}$ must be a full-rank matrix. So the number of snapshot should be greater than the dimension of channel frequency response estimation vector $\hat{H}$, which means $P \geq K$. However, in the real situation, it is usually impossible to obtain enough snapshots. When $P < K$, $R_{\hat{H}\hat{H}}$ is no longer full rank, in this case, frequency domain smoothing pretreatment is needed to make the covariance matrix become full rank again.
3. Estimation Algorithm

Existing algorithms to estimate multipath number by the eigenvalues of covariance matrix all require eigenvalue decomposition of covariance matrix, so the computation of these algorithms is large, which is not suitable for engineering application. In order to reduce the computation of the algorithm, we use QR decomposition instead of eigenvalue decomposition.

QR decomposition is a common matrix decomposition algorithm, it can decompose a matrix into the product of an orthogonal matrix and an upper triangular matrix[14]. According to [15], the eigenvalues of the matrix can be calculated through QR iterative decomposition. Its main idea is to use iterative similarity transformation to get better matrix with the same eigenvalues. For the matrix $A$, it can be represented as

$$A = QR$$

in equation (16), $Q$ is a orthogonal matrix, and $Q^t = Q^*$; $R$ is a non-negative diagonal upper triangular matrix. Let $A_1 = A$, and we have

$$A_j = Q_j R_j, \quad j = 1, 2, \cdots n$$

$$A_{j+1} = R_j Q_j, \quad j = 1, 2, \cdots n$$

where $A_j$, each matrix in the sequence is similar to the original matrix $A$, and have the same eigenvalues as $A$. The matrix sequence finally converges to an upper triangular matrix, and the eigenvalues of $A$ are distributed on its diagonal in the decreasing order.

However, our aim is not to calculate all the matrix’s eigenvalues accurately, but to estimate the multipath number of the signal, so we only need to distinguish the signal eigenvalues and noise eigenvalues by the relative size of the eigenvalues. From the property of QR iterative decomposition, we know that during the iteration, the size of elements on $A_j, j = 1, 2, 3, \cdots n$ diagonal gradually move towards the eigenvalues, we can think of the elements along the diagonal of $A_j$ as approximate eigenvalues of the matrix. And it can also reflect the relative size of the eigenvalues, which provides the possibility to reduce the computational burden of the algorithm. On the other hand, after a large number of simulation experiments, it can be found that there are two main characteristics in the relationship between signal eigenvalues and noise eigenvalues: 1.Compared with the signal eigenvalue, the noise eigenvalue is very small, and there is big difference between the two; 2. The difference between noise eigenvalues is very small. So, we can first use $n$ times QR iterations to get the approximate eigenvalues on the diagonal of matrix $A_{n+1}$, and then distinguish the signal eigenvalues and noise eigenvalues according to the two characteristics between them.

Substitute $R_{n+1}$ into equation (17) as $A_j$ and decompose it through $n$ times QR iterations, arrange the diagonal elements of $A_{n+1}$, namely the approximate eigenvalues of the matrix, in descending order, and the first $L_p$ maximum values are corresponding multipath signals. In order to make full use of the relationship between signal eigenvalues and noise eigenvalues, we set the amplitude of change of adjacent eigenvalues as

$$\Delta \lambda_i = \frac{\lambda_i}{\lambda_{i+1}}$$

$$\Delta \lambda_i, i = 1, 2, \cdots, K - 1$$ represents the difference between two adjacent eigenvalues. According to the relationship between signal eigenvalues and noise eigenvalues described above, the maximum $\Delta \lambda_i$ should appear at the point where $\lambda_i$ is the last signal eigenvalue and $\lambda_{i+1}$ is the first noise eigenvalue,
then the number of multipath \( L_p = i \) can be determined. However, when the signal-to-noise(SNR) is small or the number of multipath is large, the difference between the last signal eigenvalue and the first noise eigenvalue decreases, the maximum \( \Delta \lambda_i \) does not necessarily appear in the position corresponding to the number of multipath, so this algorithm is no longer reliable.

We note that the SNR and multipath number does not affect the feature of small difference between noise eigenvalues, so \( \Delta \lambda_i = L_p - 1, L_p + 2, \ldots, K - 1 \) is always concentrated near the value 1. We take this as the judgment criterion and assume that if both successive \( \Delta \lambda_i \) are less than 1.5, let \( L = j \) to ensure \( \hat{\lambda}_L \) is the maximum noise eigenvalue with the maximum probability. Then the ratio formula of all approximate signal eigenvalues and the maximum approximate signal eigenvalues is introduced, which is defined as the amplitude of change of eigenvalues

\[
\Delta \lambda_{ii} = \frac{\hat{\lambda}_i}{\hat{\lambda}_i}
\]  

(19)

\( \Delta \lambda_{ii}, i = 1, 2, \ldots, K \) decreases as \( i \) increases. Substitute \( \hat{\lambda}_L \) into equation (19) and get \( \Delta \lambda_{ii} = \hat{\lambda}_i / \hat{\lambda}_i \), which represents the ratio of almost maximum noise eigenvalue to maximum signal eigenvalue. The reason why it is almost the maximum noise eigenvalue is that sometimes \( L \) may be greater than \( L_p + 1 \), and then \( \hat{\lambda}_L \) is no longer the maximum noise eigenvalue, but \( \hat{\lambda}_i \) is still the larger one in the noise eigenvalues. Accordingly, we consider to construct the decision threshold based on \( \Delta \lambda_{ii} \) and compare it with \( \Delta \lambda_{ii} \) to distinguish two kinds of eigenvalues. Compared with the difference between noise eigenvalues, there is a big difference between noise eigenvalues and signal eigenvalues, so we take \( n \Delta \lambda_{ii} \) as the threshold. Through experiments, we find that the algorithm performs best when \( n = 3 \), so we set the threshold as

\[
\eta = 3 \Delta \lambda_{ii}
\]

(20)

Compare \( \Delta \lambda_{ii} \) with threshold \( \eta \), when \( \Delta \lambda_{ii} \) is less than \( \eta \) for the first time, the corresponding \( i \) is considered to be the position of the first noise eigenvalue, and the multipath number \( L_p = i - 1 \).

4. Simulation

Based on the MATLAB platform, in the wireless OFDM signal model, the algorithm in this paper is compared with Akaike information theoretic criteria(AIC), Rissanen minimum descriptive length criteria(MDL) and the algorithms in [10], [11] for performance. We set the target at 300m visible from the receiving station, and the first reaching path of the signal is the direct path. The parameters of the signal are set as follows: carrier frequency \( f_c = 2.4\text{GHz} \); bandwidth \( B = 20\text{MHz} \); the number of subcarrier \( K = 64 \); FFT period \( T = 3.2\mu\text{s} \); and the noise is white Gaussian noise. In the simulation experiment, the channel frequency response covariance matrix is decomposed iteratively by QR for 4 times, and the number of snapshot \( P = 10 \).

4.1 Simulation 1

The purpose of simulation 1 is to compare the accuracy of each algorithm in estimating the number of multipath in the presence of different number of multipath. According to the actual situation, we assume that the difference between each multipath is 15m, which is converted into time means the delay of each path increases by 50ns as a unit. The SNR of the received signal equals 5dB, 200 simulation experiments were conducted to calculate the average. The experimental result is as follows
According to Figure 1, under the same conditions, when the number of multipath $L_p$ is greater than 3, the accuracy of comparison algorithms decreases significantly. When $L_p = 4$, the estimation accuracy of the algorithms proposed in [10] and [11] has been reduced to 69% and 0% respectively, can no longer provide a reliable estimation, but the proposed algorithm in this paper can still maintain 100% accuracy. When $L_p = 5$, the accuracy of proposed algorithm also higher, reaching 97%, compared with AIC and MDL (87%). Therefore, when the number of multipath is large, the performance of proposed algorithm is still good, and it has stronger robustness than other algorithms.

4.2 Simulation 2
The purpose of simulation 2 is to compare the estimated performance of each algorithm under different SNR conditions. Fixed multipath number $L_p = 4$, and it can be seen from simulation 1 that when $L_p = 4$, the algorithm in [11] is completely unable to provide the correct estimate value, so it is no longer used as a comparison algorithm in simulation 2. Now we assume SNR decreases from 15dB to -5dB by step -1dB, 200 simulation experiments are conducted to calculate the average accuracy of estimation. The experimental result is as follows

As can be seen from Figure 2, with the continuous decrease of SNR, the accuracy of each algorithm also decreases. When SNR < 7dB, the estimation accuracy of the algorithm proposed in [10] has fallen below 70%. When SNR < 5dB, the estimation accuracy of AIC and MDL also began to decline significantly, and when SNR = -5dB, their estimation accuracy dropped to 22.5% and 10.5%
respectively, which obviously could not provide reliable estimation results. However, the estimation accuracy of algorithm in this paper decreases slowly with the decrease of SNR, when SNR = -5dB, its accuracy can still be 77.5%. So the proposed algorithm has strong anti-noise performance.

5. Conclusion
In this paper, a multipath number estimation based on QR decomposition is proposed under the OFDM signal model. Through a few QR iteration decomposition of the covariance matrix of the channel frequency response estimation, we can get the pseudo-eigenvalues and construct the amplitude of change of adjacent eigenvalues and the amplitude of change of eigenvalues, then estimate the number of multipath based on the relationship between signal eigenvalues and noise eigenvalues. This algorithm has small computation, strong anti-noise ability and robustness. Simulation results show the effectiveness of the proposed algorithm, that is, the detection performance of the proposed algorithm at low SNR and few snapshots is better than the existing algorithms. However, the algorithm in this paper assumes that it is in white noise environment, next step we will explore its performance and improve it under color noise.

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