Effect of Degenerated Particles on Internal Bremsstrahlung of Majorana Dark Matter

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Abstract

Gamma-ray generated by annihilation or decay of dark matter can be its smoking gun signature. In particular, gamma-ray coming from internal bremsstrahlung of dark matter is promising since it can be a leading emission of sharp gamma-ray. However if thermal production of Majorana dark matter is considered, the derived cross section for internal bremsstrahlung becomes too small to be observed by future gamma-ray experiments. We consider a framework to achieve an enhancement of the cross section by taking into account degenerated particles with dark matter. We find that the enhancement of about order one is possible without conflict with the dark matter relic density. Due to the enhancement, it would be tested by the future experiments such as GAMMA-400 and CTA.

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1 Introduction

Although the existence of dark matter (DM) is crucial, its properties like mass and interactions are not completely known yet. There are a lot of theoretical DM candidates such as Weakly Interacting Massive Particle (WIMP), axion, gravitino and asymmetric DM. In particular, WIMP is the most promising DM candidate and experiments regarding DM are focusing on WIMP detection.

Indirect detection is one of the ways, which is an attempt to identify DM properties by observing cosmic-rays, gamma-rays and neutrinos from the galaxy since DM may annihilate or decay into this kind of the Standard Model (SM) particles. Although known astrophysical sources of gamma-rays induce only smooth spectra, exploring sharp gamma-ray peak coming from the galaxy is extremely important because such a line like spectrum can be generated by DM annihilation or decay. Thus DM model which can induce strong sharp gamma-ray is fascinating from experimental viewpoint. In fact, the gamma-ray excess from the galactic center have been reported by several papers [1,2] and [3–5]. The former implies the DM mass to be 130-135 GeV with its partial annihilation cross section $\sigma v \sim 10^{-27}$ cm$^3$/s into gamma-ray. The latter has found rather mild excess around 10 GeV which is caused by 30-60 GeV of DM mass with a specific mode such as a pair of bottoms or taus. The partial cross section should be $\sigma v \sim 3 \times 10^{-26}$ cm$^3$/s that is very close to the cross section for the observed relic density.

Representative of processes inducing keen gamma-ray is two-body annihilation such as $\chi \chi \to \gamma \gamma$ and $\chi \chi \to \gamma Z$. The energy of the emitted photon is kinematically determined as $E_\gamma = m_\chi$ for $\gamma \gamma$ channel or $E_\gamma = m_\chi \left(1 - m_Z^2/(4m_\chi^2)\right)$ for $\gamma Z$ channel. The cross section for these processes are typically small because of loop suppression. Another process of sharp gamma-ray is internal bremsstrahlung $\chi \chi \to f\bar{f}\gamma$ [6–10]. This process becomes leading for gamma-ray when the two-body process $\chi \chi \to f\bar{f}$ is chirally-suppressed. The gamma-ray spectrum of internal bremsstrahlung can be excessively sharp if the mediated particle mass is not far from the DM mass. This is because (anti-)fermion with soft energy is produced in the final state and most energy of the DM is taken away by the other two particles. In particular for real scalar DM, more passionate gamma-ray can be induced than Majorana DM case due to d-wave suppression of the two-body process [13–16]. On the other hand for Majorana DM, the annihilation cross section for $\chi \chi \to f\bar{f}$ is p-wave suppressed. It is known that the cross section for internal bremsstrahlung $\chi \chi \to f\bar{f}\gamma$ cannot be enough large to be detected in near future if thermal production of DM is concerned. This is because once the interaction strength is fixed in order to obtain the observed relic density of DM, the cross section for internal bremsstrahlung is also determined and is not large enough.

In this paper, we consider enhancement of internal bremsstrahlung process for Majorana DM due to increase of DM effective degree of freedom taking into account degener-
ated particles with DM. Assuming that the degenerated particles weakly interact with the SM particles, the effective annihilation cross section would be smaller than the standard annihilation cross section. Thus the coupling between DM and the SM particles should be larger in order to satisfy the observed DM relic density. As a result, the cross section for internal bremsstrahlung is also enhanced. In addition, frameworks which lead such a small mass splitting and more concrete models will be also referred in the later part of the paper.

2 Framework

2.1 Non-degenerated Case

Internal bremsstrahlung of Majorana DM is briefly reviewed here. We consider the Yukawa interaction

\[ \mathcal{L} = y \varphi \chi P_R f + \text{H.c.}, \]  

(1)

where \( \chi \) is a Majorana DM, \( \varphi \) is an electromagnetically charged scalar mediator and \( f \) is a light fermion in the SM which is typically regarded as a charged lepton. Although the DM can couple to three generations of the SM fermions in general, we consider the interaction with only one generation for simplicity. The DM is stabilized by a symmetry like \( \mathbb{Z}_2 \) parity. In the above case, \( \mathbb{Z}_2 \) assignment should be odd for the DM \( \chi \) and the mediator \( \varphi \) and even for the fermion \( f \). Since it is possible to choose either of left or right chirality, we fixed to the right chirality here. When the fermion mass is much lighter than DM mass \( m_f / m_\chi \ll 1 \), the annihilation cross section into \( f \bar{f} \) is calculated as

\[ \sigma v_{f\bar{f}} = \frac{|y|^4}{48\pi m_\chi^2} \frac{1 + \mu^2}{(1 + \mu)^4} v^2, \]  

(2)

where \( \mu = m_\varphi^2 / m_\chi^2 \) is the mass ratio between the DM and the mediator, and \( v \) is the relative velocity of DM. Thermal relic density of the DM is determined by this cross section to satisfy the observed relic density \( \Omega h^2 \approx 0.120 \) [17]. The required strength of the Yukawa coupling for the several fixed mass ratio \( m_\varphi / m_\chi \) is shown in the left panel in Fig. 1. As one can see from the figure, order one Yukawa coupling is needed for the certain relic density. When the mass ratio between the DM and the mediator is \( m_\varphi / m_\chi = 1.01 \), the required Yukawa coupling becomes rather small since the co-annihilation with the mediator is effective. In this case, the DM mass which is less than 170 GeV gives too tiny relic density and cannot satisfy the appropriate relic density. There is a small damp around 40 GeV for \( m_\varphi / m_\chi = 1.1 \). This is because the co-annihilation with \( \varphi \) which is the s-channel \( Z \) boson mediated process is dominant. The region of the DM mass more than a few TeV tends to be ruled out by perturbativity of the coupling.

As one can see from Eq. (2), the cross section is suppressed by the relative velocity squared (p-wave) due to the helicity suppression. Because of that, this channel is
extremely suppressed at the present universe since the averaged DM relative velocity is roughly $v \sim 10^{-3}$. As a result, three body process $\chi \chi \rightarrow f \gamma$ becomes important for indirect detection of DM. There are three diagrams which contribute to the process. Among them, a photon emitted from the virtual intermediate particle $\varphi$ may induce sharp spectrum when the $\varphi$ mass is not far from the DM mass. In the limit of $m_f/m_\chi \rightarrow 0$, the cross section for internal bremsstrahlung $\chi \chi \rightarrow f \gamma$ is calculated as

$$
\sigma v_{f\gamma} = \frac{\alpha_{em} |y|^4}{64\pi^2 m_\chi^2} \left[ (\mu + 1) \left\{ \frac{\pi^2}{6} - \log^2 \left( \frac{\mu + 1}{2\mu} \right) - 2\text{Li}_2 \left( \frac{\mu + 1}{2\mu} \right) \right\} + \frac{4\mu + 3}{\mu + 1} 
+ \frac{(4\mu + 1)(\mu - 1)}{2\mu} \log \left( \frac{\mu - 1}{\mu + 1} \right) \right].
$$

This cross section is numerically estimated with the Yukawa coupling satisfying the observed relic density and is shown in the right panel in Fig. 1. In the low mass region for $m_\varphi/m_\chi = 1.01$, since the co-annihilation of $\varphi$ is dominant, the Yukawa coupling becomes small and the cross section for internal bremsstrahlung becomes also feeble. For larger mass ratio, although the required Yukawa coupling can be sizable, the cross section becomes small since the cross section strongly depends on the mass ratio $m_\varphi/m_\chi$ which decreases with roughly $m_\chi^4/m_\varphi^4$. The excluded region of the cross section by Fermi-LAT and H.E.S.S. is shown as the light yellow [9], and the prospected upper bounds of the future gamma-ray experiments GAMMA-400 and CTA are described by the black lines [18, 19]. Although the evaluation of the limits have been done for internal bremsstrahlung plus gamma-ray lines [9], this limit would be applied as a good approximation. The maximal cross section is about $\sigma v_{f\gamma} \approx 2 \times 10^{-28}$ cm$^3$/s which is about order one below the GAMMA-400 and CTA prospects.

Figure 1: Contours satisfying the observed DM relic density on DM mass and Yukawa coupling plane (left panel). Comparison of cross section for internal bremsstrahlung with gamma-ray experiments (right panel).
2.2 Degenerated Case

We consider $k$ degenerated Majorana fermions with DM. The Yukawa interaction is given by

$$\mathcal{L} = \sum_{i=1}^{k} y_i \chi_i \bar{P}_R f + H.c.,$$

(4)

where $\chi_1$ is the lightest and DM with the mass $m_1$, $\chi_i$ ($i \neq 1$) are assumed to be degenerated with the DM whose mass is denoted as $m_i$ ($i \neq 1$). In the following, we may express the DM $\chi_1$ with $m_1$ as just $\chi$ with $m_\chi$. We can choose the diagonal base for the mass matrix of $\chi_i$. When the DM is degenerated with the particles, the co-annihilation with these particles has to be taken into account to evaluate thermal relic density of DM. Following the ref. [20], the effective annihilation cross section is given by

$$\sigma_{\text{eff}} v = \sum_{i=1}^{k} \sum_{j=1}^{k} g_i g_j g_{\text{eff}} \sigma_{ij} v \left(1 + \Delta_i\right)^{3/2} \left(1 + \Delta_j\right)^{3/2} e^{-\left(\Delta_i + \Delta_j\right)x},$$

(5)

where $g_{\text{eff}}$ is the effective degree of freedom

$$g_{\text{eff}} = \sum_{i=1}^{k} g_i \left(1 + \Delta_i\right)^{3/2} e^{-\Delta_i x},$$

(6)

and $\Delta_i \equiv (m_i - m_1)/m_1$ is the mass difference between the DM and the other particles, $g_i = 2$ is the degree of freedom for each Majorana particle $\chi_i$, and $x = m_1/T$. If there are a lot of degenerated particles, the effective degree of freedom $g_{\text{eff}}$ increases. In addition, if the degenerated particles have sizable interactions with the SM particles, the derived relic density of the DM is more reduced due to its larger effective annihilation cross section $\sigma_{\text{eff}} v$. On the other hand, if they weakly interact with the SM particles, the effective annihilation cross section becomes smaller than the standard cross section of the DM. Thus a larger coupling of the DM is required to satisfy the observed relic density. We consider the latter case for enhancement of Yukawa coupling for the DM.

Here we give some numerical results for the enhancement of the cross section for internal bremsstrahlung due to the degenerated Majorana fermions. The effective cross section has been calculated by microMEGAs [21]. The mass degeneracy dependence of the cross section for internal bremsstrahlung is shown in Fig. 2 where the strength of the Yukawa coupling is fixed to $|y_i/y_1| = 0.1$ for the left panel and 0.01 for the right panel, and we assumed that all degenerated particles have same masses for simplicity. The mass ratio between the DM $\chi$ and the charged scalar mediator $\varphi$ is fixed to $m_\varphi/m_\chi = 1.1$. This should not far from 1 to generate sharp gamma-ray peak of internal bremsstrahlung. As one can see, if the number of the particles increase and they strongly degenerate, the cross section for internal bremsstrahlung is enhanced. According to the figures, it is possible to obtain around order one enhancement. Note that the dependence of the DM mass is almost negligible.
Figure 2: Mass degeneracy dependence of cross section for internal bremsstrahlung where mass of $\varphi$ is fixed to $|y_i/y_1| = 0.1$ for left panel and 0.01 for right panel.

Figure 3: Comparison of cross section for internal bremsstrahlung with gamma-ray experiments where the mass ratios are fixed to $m_\varphi/m_\chi = 1.1$ and $m_i/m_\chi = 1.01$.

The comparison with gamma-ray experiments is shown in Fig. 3 where the ratio of the Yukawa coupling is fixed to $|y_i/y_1| = 0.1$ for the left panel and $|y_i/y_1| = 0.01$ for the right panel. One can see that the cross section for internal bremsstrahlung is enhanced and testable by the next future gamma-ray experiments [18, 19].

3 Other Aspects

3.1 Possibilities of Mass Degeneracy

It is worth mentioning how to realize the degenerated Majorana fermions. A simple realization is mass splitting of a Dirac fermion by spontaneous breaking of an extra $U(1)$ symmetry [22]. Let us consider a Dirac fermion $\psi$ and a complex scalar $\Sigma$. The charge of the extra $U(1)$ symmetry is assigned as $Q$ for $\psi$ and $2Q$ for $\Sigma$. The mass term of the
Dirac fermion and the Yukawa interaction between $\psi$ and $\Sigma$ is expressed as

$$L = -m\bar{\psi}\psi - \left(\frac{y}{2}\Sigma^\dagger\bar{\psi}\psi + \text{H.c.}\right).$$

(7)

After the $U(1)$ symmetry breaking by vacuum expectation value of $\Sigma$, the mass matrix of the fermion is given by

$$L = -\frac{1}{2} \left( \begin{array}{c} \psi_L \\ \psi_R \end{array} \right) \left( \begin{array}{cc} \delta & m \\ m & \delta \end{array} \right) \left( \begin{array}{c} \psi_L \\ \psi_R \end{array} \right) + \text{H.c.},$$

(8)

where $\delta = y\langle \Sigma \rangle/2$. Thus the Dirac fermion splits to two Majorana fermions with the masses $m \pm \delta$. Since typically $\delta$ is regarded to be smaller than the bare mass $m$, the degenerated two Majorana fermions are naturally obtained.

Another way is to introduce flavor symmetries [23–25], which are well-known to derive predictive models for the neutrino oscillation data. If two Majorana fermions $N_1$ and $N_2$ are identified to the doublet of $S_3$ group; $(N_1, N_2)^T \sim 2$ as the simplest realization, one can straightforwardly obtain the completely degenerated mass term by subtracting the trivial singlet of $S_3$ group as

$$\left( \begin{array}{c} N_1 \\ N_2 \end{array} \right)_2 \otimes \left( \begin{array}{c} N_1 \\ N_2 \end{array} \right)_2 \supset M_N(N_1 N_1 + N_2 N_2)_1.$$ 

(9)

A small mass difference is expected to be generated by radiative corrections or considering higher dimensional operators. The above realizations of mass degeneration are just examples, and the other realizations are possible.

### 3.2 Concrete Models

The DM in the framework can be typically identified as the right-handed neutrino. Thus the above framework can be easily realized in many models including the right-handed neutrinos. One of the economical concrete examples is a model with radiatively induced neutrino masses, for example Ma model [26] that generates neutrino masses at one-loop level, Krauss-Nasri-Trodden model [27], Aoki-Kenemura-Seto model [28, 29] at three-loop level. In fact, internal bremsstrahlung has been discussed in the above models [30, 31]. The right-handed neutrinos are included and taking order one Yukawa coupling is possible without conflict with generation of the certain neutrino masses because there are a loop suppression factor and the other couplings. Since the DM within the models always link to the neutrino mass scale and mixing, and some processes of the lepton flavor violation, these constraints should be concerned and may affect to our analysis.

### 4 Conclusions

Identifying DM is one of primary issues. Looking for a line like gamma-ray coming from the galaxy is important for DM search since some DM candidates can generate such
kind of gamma-ray. In particular, internal bremsstrahlung of DM shows an interesting sharp gamma-ray spectrum. We have considered internal bremsstrahlung for Majorana DM with degenerated particles. When the degenerated particles have the Yukawa coupling which is smaller than that for the DM, we need a larger Yukawa coupling to satisfy the correct relic density of DM. As a result, the cross section for internal bremsstrahlung has been enhanced. More than order one enhancement has been achieved for some parameter sets, and it can be testable by the future gamma-ray experiments.

Theoretical realization of mass degeneration with an extra $U(1)$ and a flavor symmetry also has been discussed. The DM we have considered here can be identified as the TeV scale right-handed neutrino. The TeV or electroweak scale right-handed neutrinos are included in some models with radiative neutrino masses. Thus our framework discussed here naturally works in these models.

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