Transition to chaos of a vertical collapsible tube conveying air flow

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Abstract. “Sky dancers”, the large collapsible tubes used as advertising, are studied in this work through a simple experimental device. Our study is devoted to the nonlinear dynamics of this system and to its transition to chaos. Firstly, we have shown that after a collapse occurs, the air fills the tube at a different speed rate from the flow velocity. Secondly, the temporal intermittency is studied as the flow rate is increased. A statistical analysis shows that the chaotic times maintain roughly the same value by increasing air speed. On the other hand, laminar times become shorter, until the system reaches a completely chaotic state.

1. Introduction
Compliant-tube flows are the subject of various recent studies due to their applications in oil industry (see the pioneering works [1, 2]) and in different thermic systems [3] as well as in biological systems [4]. Their interest also comes from their great dynamical behaviour diversity, the nature and the complexity of their instabilities [5, 6]. For a free-ended tube, different circumferential modes are exhibited, as described in [7, 8]. These modes are characterized by the spatial deformations of the membrane. In these systems, a strong coupling exists between the flow inside the tube and the elastic pipe. Indeed, fluid pressure modifies, and is modified by, the cross-sectional area.

Different configurations and boundary conditions of the collapsible tubes conveying flow are largely described by Paidoussis [9]. A clamped-clamped system can lead to a static instability called divergence [10]. In comparison, a free-ended tube configuration shows a fluttering instability above a certain flow speed threshold, described by a Hopf bifurcation [9].

This last instability is seen daily when one uses a water-conveying hose to irrigate plants: for a high enough flow rate, the tube flutters. This “garden-hose instability”, as it is called, was experimentally and theoretically studied in [11, 12, 13].

A similar system is the “sky dancer” used as advertising. It is composed of a long cylindrical membrane fluctuating vertically above an air blower. The two main differences against the studies in [11, 12, 13] are as follows: (a) the sky dancer tube wall is much thinner; this lets the pipe shape show “breaks” and folds when it is fluttering, (b) the air flow is turbulent, because of both the tube diameter size and the high flow speed.

The purpose of this work is to describe the fluctuation dynamics of a vertical tube conveying air flow, and to characterize transition to chaos when the flow rate is increased. This last analysis is made statistically. The experimental system is described in section 2, and a sequence of pipe fluctuation images is shown in section 3. A statistical study of the chaotic and laminar times is
developed in section 4, followed in section 5 by a discussion. The last section is for conclusion and perspectives.

2. Experimental system

2.1. Experimental device

The tube is made of a 2.5 Mpa Young Modulus white plastic membrane, 0.1 mm in thickness. Its surface density is 0.083 kg/m² and its length $L = 39$ cm. The tube is heat-sealed in order to form a 4-cm diameter cylinder and is mounted on the external diameter of the air pump exit. We calculated the parameter $\beta = m/(m + M)$, used in these kind of systems, where $m$ is the mass per unit of length of air in the pipe, and $M$ the mass per unit of length of the pipe. In our case $M = 0.01$ kg/m, so that $\beta = 0.13$.

Figure 1 shows the experimental device. In order to capture the tridimensional instability of the pipe, the video camera is located above the system; its optical axis is centered on the pump exit. The video frequency is equal to 29.97 Hz and the frame size is 720 x 480 pixels. In order to observe sharp images, a stroboscope is placed near the video camera and pulses at the same frequency as the video camera.

![Figure 1. Experimental set-up.](image)

The air pump exit has the same external diameter as the tube. The air speed varies from 18 m/s to more than 40 m/s, corresponding to a flow rate between 0.0226 m³/s and more than 0.0502 m³/s. Air speed is stable at 3%. The Reynolds number is found to be more than 51000, so that the air flow at the pump exit is turbulent.

2.2. Experimental procedure

Once the tube is mounted on the pump exit, the air blower is switched on so the air speed is increased until the tube stands up on a stable state. We wait a few seconds to avoid transient
effects. After that, we capture a film during two minutes at least. We repeat the same procedure for higher flow velocities, until a completely chaotic regime is reached.

We extract all the images from the videos; this means 3603 images for each flow velocity. The left side of figure 2 shows two of these images. The picture on top shows the tube when it stands up; the pipe looks like a white circle on a gray background. The bottom picture represents the collapsed tube; it appears like a long white rectangle on a dark background.

The right column of figure 2 shows each left image binarized. All the pixels whose values are above a threshold are set to 255 (white), while the pixels whose values are below the same threshold become 0 (black). The cut-off value is constant for all the images. This value is chosen so that the tube boundaries in the binarized images match the boundaries in the original pictures. A threshold equal to 55 was set up for this experiment.

In order to analyze the transition to chaos, the system evolution is reduced to a two states time series: the stable state is when the tube stands up and the chaotic state when the tube fluctuates. In order to discriminate these two states, we count the white pixels contained inside a circle centered on the blower exit. The circle radius is chosen so that the steady state corresponds to an image where all the white pixels are included inside this circle. In our experiment, a suitable value is \( r = 4.7 \) cm. We calculate for each image the proportion \( p \) of the white pixels inside the circle; for instance, \( p = 1 \) for the figures on top (stable state), and \( p = 0.294 \) \((p < 1)\) for the bottom pictures (chaotic state). The evolution of \( p \) as a function of time will be shown in section 4.

![Figure 2](image-url) Raw pictures (left side) and binarized pictures (right side) for both tube states: stable state (top) and chaotic state (bottom). The proportion \( p \) of white pixels in the circle was found to be equal to \( p = 1 \) and \( p = 0.294 \) respectively.

3. Description of the fluttering behaviour

Figure 3 illustrates the tube dynamics at the threshold of the instability. In this section, the images are shown from a side view. Two consecutive pictures are separated by the time interval \( \Delta t = 33 \) ms.
In the first picture, a constriction appears at the base of the tube. This constriction moves downstream with time, as shown on the second picture. The third picture shows that the tube section located above the constriction point falls down. Below it, the tube is air-filled, while it is empty on top. This break point rises up, as shown in the following images, and a straight line can be drawn indicating the fold position: the constriction speed is roughly constant. Finally, when the fold reaches the top of the tube, it stands up again in a stable state.

It is important to note that the constriction velocity is different from the air speed. In this case, the air speed was 18.8 m/s, while the constriction point travelled at a speed of 1.5 m/s. In section 5 we will see that this velocity is related to the dynamical pressure needed to push the break point upwards.

![Image](image.png)

**Figure 3.** Time evolution of the fluctuation of 35 cm-length tube at a $v = 18.8$ m/s ($C = 0.0236$ m$^3$/s). Each picture is separated by the video time $\Delta t = 33$ ms, and the straight line shows how the constriction point travels downstream with a constant velocity (different from the air speed).

4. Chaos transition characterization

As explained in the section 2.2, the pictures are processed in order to extract the proportion $p$ of white pixels inside a centered circle. Figure 4 shows the evolution of $p$ over time for four air speed values: $v = 18.8, 19.9, 23.3$ and 25 m/s (corresponding to flow rates $C = 0.0236, 0.0250, 0.0293$ and 0.0314 m$^3$/s respectively). For $v = 18.8$ m/s, the tube raises up, but it is important to note that it is sensitive to perturbations. For example, approaching an object to the exit of the tube makes it bend and fluctuate. After this, the tube may or may not come back to its equilibrium position. The stable state shown in the first curve of figure 4 was sustained during more than 4 minutes without perturbation.

When the velocity is increased ($v = 19.9$ m/s, $C = 0.0250$ m$^3$/s), chaotic fluctuations are generated which alternate with the stable state. The binarization of the function $p$ was made so that the corresponding points were set to zero for $p < 0.96$, while the points are set to one for $p > 0.96$. This threshold allows us to discard few white pixels which appear outside the circle because of light inhomogeneities. Moreover, if at most four isolated pixels with values one (or zero) appear in a region of zeros (or ones respectively), these isolated values are set to zeros (or ones). This is particularly convenient to delimit properly the highly fluctuating chaotic regions.

For higher flow rates, the pipe fluctuations increase. For the air speed $v = 26$ m/s ($C = 0.0327$ m$^3$/s), only three very short stable regions appear in the temporal evolution of $p$ during two minutes, so that the mean value $\bar{p}$ of the binarized function is nearly zero for this value. Figure 5 shows the evolution of the function $\bar{p}$ as a function of air speed.

The mean value $\bar{p}$ can be seen as the proportion of laminar times with respect to the whole measure duration. As the system is stable for $v = 18.8$ m/s ($C = 0.0236$ m$^3$/s), $\bar{p} = 1$. When the air flow is increased, $\bar{p}$ decreases. For $v = 22.5$ m/s ($C = 0.0283$ m$^3$/s), $\bar{p} = 0.503$: it can be considered that half of time, the tube fluctuates while the other half, the tube stands up. The function $\bar{p}$ reaches the value 0.002 for $v = 26$ m/s ($C = 0.0327$ m$^3$/s): at this flow rate, the system appears in a permanent chaotic state.
Figure 4. Time evolution of white pixel proportion $p$ inside the circle drawn in figure 2. When $p = 1$, the tube stands up, otherwise it is fluctuating. From top to bottom, flow velocity is: $v = 18.8, 19.9, 23.3$ and $25$ m/s (corresponding to flow rates $C = 0.0236, 0.0250, 0.0293$ and $0.0314$ m$^3$/s respectively). The oscillating curves correspond to the temporal variation of $p$, while the rough points correspond to the binarized values of $p$. Time is in seconds.

Figure 5. Evolution of the mean value $\bar{p}$ of the binarized value of the proportion $p$, as a function of the air speed $v$ (m/s) and flow rate $C$ (m$^3$/s). When $v = 18.8$ m/s ($C = 0.0236$ m$^3$/s), the tube stands up ($\bar{p} = 1$), but as the flow velocity is increased, the tube fluctuations increase and the function $\bar{p}$ decreases until it reaches $\bar{p} = 0.002$ for $v = 26$ m/s ($C = 0.0327$ m$^3$/s).
Figure 6. Cumulated histograms of: chaotic states (left column) and laminar state (centre and right column). Left and centre columns: semi-log graphics, right column: log-log graphic. For a better clarity, the y-labels are on top of each column. The different flow speed values are, from top to bottom: \( v = 19.9, 23.3 \) and \( 25 \) m/s \( (C = 0.0250, 0.0293 \) and \( 0.0314 \) m\(^3\)/s respectively). The chaotic times are best interpolated by an exponential law, while laminar time histograms appear as a product of an exponential with an algebraic law. The first points are best interpolated by an exponential law (centre column) while the intermediate points by an algebraic equation (right column).

In order to characterize this transition to chaos, we analyze statistically the laminar and chaotic times as usually done in temporal dynamical \[14\] and spatiotemporal \[15, 16, 17\] systems. We plotted in figure 6 the cumulated histograms of the chaotic times (left column) and those of the laminar durations (centre and right column). The chaotic time distributions are shown in semi logarithmic scale and can be interpolated as a straight line. So, the fit by a function of the type \( N(\tau > \tau_{\text{turb}}) = N_0 \exp(\tau / T_{c-\text{turb}}) \) gives a value for a characteristic chaotic time \( T_{c-\text{turb}} \) whose value can be calculated for each air speed.

The laminar time distributions are shown in the centre column in a semi logarithmic scale and in the right column in a log-log scale. The exponential law (centre column) fits better the first distribution points (short times), while the algebraic law (right column) interpolates better the intermediate values. In the case of the laminar regions, we calculated a characteristic time \( T_{c-\text{lam}} \) as for the chaotic times, as well as the exponent \( \alpha \) of the algebraic law \( N(\tau > \tau_{\text{lam}}) = N_0 \tau^\alpha \). The evolutions of these quantities are shown in figure 7.

It can be observed on the first plot (a) that the chaotic characteristic times do not have a monotonous evolution. While \( T_{c-\text{turb}} = 1.96 \) s for \( v = 19.9 \) m/s \( (C = 0.0250 \) m\(^3\)/s), it reaches a maximum \( T_{c-\text{turb}} = 2.41 \) s for \( 20.5 \) m/s \( (C = 0.0258 \) m\(^3\)/s) and a minimum \( T_{c-\text{turb}} = 1.43 \) s for \( v = 22.5 \) m/s \( (C = 0.0283 \) m\(^3\)/s). We conclude that the fluctuation durations \( T_{c-\text{turb}} \) of the tube do not increase with time but are varying around a constant value (calculated as 1.94 s).
The quantities deduced from the laminar time distributions present a more evident evolution with respect to $v$. Although fluctuations are present particularly in the interval $[22, 24]$ m/s ($[0.0276, 0.302]$ m$^3$/s), the characteristic laminar time $T_{c-lam}$ (figure 7-b) decreases from $T_{c-turb} = 1.68$ s for $v = 19.9$ m/s ($C = 0.0250$ m$^3$/s) to 0.31 s for $v = 25$ m/s ($C = 0.0314$ m$^3$/s). For the same air speed values the laminar exponent decreases from $-0.63$ to $-1.40$ (figure 7-c). Although we could not characterize clearly the distribution type for the laminar times, we put in evidence that these laminar regions are shorter and shorter. The system transits towards chaos via a reduction of laminar times and not by an increase of fluctuation regions.

5. Discussion
Various authors [3, 18, 19] observed that the flutter instability was sensitive to finite disturbances and had a different threshold when the flow speed was decreased. They showed that an undisturbed system present the hysteresis, which suggests that in some cases, the Hopf bifurcation is subcritical. In our experiment, the system never comes back at a rest state by decreasing the flow velocity. Once the flow rate is low enough, the gravity makes the tube fall down and the tube does not reach anymore a stable state. The existence of a subcritical bifurcation could explain our difficulty to analyze the laminar times distribution. Moreover, it was observed that some folds could form and remain near the pipe exit after a fluctuation, which influences the dynamics (generally provoking a stabilizing effect). Plastic deformations were also observed in [10].

During the last twenty years, studies about the nonlinear behaviour of collapsible tubes conveying flows have been performed. Rousselet and Herrmann [20] drew the limit cycle amplitude of the free end of a cantilevered pipe as a function of the parameter $\beta$ (given in the section 2.1). For our value $\beta = 0.13$, they found a limit cycle with an amplitude $A/D \approx 0.15$, which is very small compared to our experimental results. Modarres-Sadeghi, Paidoussis and Semler [21] found that for this value of $\beta$, the system should flutter in a plane. Nevertheless, we repeat that our experiment differs from all the works cited in the references.

As already mentioned in the introduction, the usual differential equation cannot be applied
to this system because when the tube folds, its shape presents breaks so that the spatial
derivatives are not continuous. The studies performed in [11, 12, 13] are realized with tubes
whose wall thickness is more than 0.79 mm, which is almost an order of magnitude higher than
our experimental value. Moreover, the small tube diameters (12.70 mm at most) in these works
guarantees that the flow at the tube entry is laminar. It is not the case in our experiment.

We think that as the tube wall is very thin, we should not consider it as an elastic structure in
the physical sense. We rather consider the tube wall as a thin weighty impermeable boundary;
that makes the tube bend. The mechanism that may explain the stand-up wave velocity value
is as follows.

To a first approximation, the constriction point shuts off the tube during a fluctuation. So,
the air flow below the constriction may be brought largely to rest, whereas the air flow through
the blower continues unabated. This causes the pressure to rise in the tube section between
the blower and the constriction point, until this pressure is able to push the constriction point
upwards. That is, the dynamical pressure $\frac{1}{2} \rho v^2$ should be greater than the pressure exerted by
the weight of the membrane piece that shuts off the air flow. As an order of magnitude, we
calculated the weight of a membrane disk-shaped piece, which has the same radius as the tube,
that would block the air exit. The data in section 2.1 permit to find that the corresponding
pressure exerted by the plastic piece on the bottom air cylinder is 0.8 Pa. In order to push this
plastic disk, the velocity corresponding to the dynamical pressure must reach the value of 1.2
m/s, which is slightly lower than our experimental value.

This mechanism permits to find a stand-up wave velocity which is of the same order
of magnitude than in our experiment. Nevertheless, this first approximation makes us
underestimate the stand-up wave velocity. By considering the entire folded section weight over
the constriction point, we should find indeed a greater dynamical pressure able to push the
constriction point upwards. This comment is left for further investigations.

6. Conclusion and perspectives
This work belongs to an effervescent area which is the nonlinear dynamics of collapsible tubes
conveying fluid. It differs in several points from previous studies; but it permits to study chaos
transition through a simple experimental system; this is a reduced model of a “sky dancer”. We
showed that the wave that makes the tube stand up after a collapse has a velocity which is related
to the growing pressure in the closed, inflating tube section, located between the blower end and
the constriction point. It was also shown that the fluctuation times were roughly constant as
the air flow speed is increased. On the contrary, the decrease of laminar times constitutes the
essential ingredient in the transition to chaos. In a future work we may study the same system
in a different configuration, in order to analyze the dynamical pressure role in the stand-up
wave. Moreover, we will describe the tube trajectory in a phase space as done in recent studies
[22, 23, 24, 25], in order to perform the analysis of transition to chaos from a different point of
view.

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