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Solution of the general multi-objective De-Novo programming problem using compensatory operator under fuzzy environment

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Abstract. This paper presents an alternative approach for the solution of the general multi-objective De-Novo Programming Problem under fuzzy environment in one step using Luhandjula’s compensatory \( \mu/+2968 \)-operator. Luhandjula used compensatory \( \mu/+2968 \)-operator to solve vector maximization problem, whereas in this paper the use of compensatory \( \mu/+2968 \)-operator has been extended to solve the general multi-objective De-Novo Programming Problem. Also the solution obtained by the proposed approach elicits an efficient solution of the problem considered under the assumption of the uniqueness of the solution. The method has been illustrated by a numerical example.

1. Introduction

One of the salient features of the traditional multi-objective optimization problem (MOOP) is trade-offs among the objectives due to the conflicting nature of the objectives. Thus under a given allocation of resources, it may not be always possible to find a single point in the feasible region, where all the objectives can be simultaneously maximized or minimized and thus trade-offs among the objectives become inevitable. De-Novo Programming Problem (DNPP) introduced by Zeleny [1] is popularly used to design an optimal system by extending existed resources (if necessary) instead of finding an optimum in a given system with fixed resources. Another interesting aspect of DNPP is that the decision-maker can obtain a trade-off free solution for its objectives. In his papers, Zeleny considered MOOPs (with maximizing type of objectives only)[1-12] where the number of objectives is equal to the number of unknowns.

But the cases where there are only minimizing type of objective functions or the general situation involving both maximizing and minimizing type of objective functions, were not addressed. Also there exists no algorithm for solving a general multi-objective De-Novo Programming Problem (MODNPP). For solving a general type of MODNPP, Li and Lee [13] introduced a two-step fuzzy approach using the concept of ideal and negative ideal points. Sasaki, et.al. [14] investigated the problem of the system reliability with fuzzy goals formulating it as DNPP and solved by Genetic Algorithm. Zhang, et.al. [15] presented an interval DNPP approach for designing optimal water resource management system mingling interval programming and DNPP. Pert FIALA [16] discussed some possible extensions of DNPP e.g. Fuzzy DNPP, Interval DNPP etc. Huang et.al. [15] used DNPP to solve Taiwan Freeway’s maintenance...
work problem. Saeedi et al. [17] determined the capacity of recovery facilities in the reverse flow system formulating it as a mixed integer non-linear programming problem with DNPP. Further works towards the enrichment of the concept of De-Novo Programming could be seen in the works of Chen [8] and Nurullah Umarusman [18]. Nurullah Umarusman [18] used Min-Max Goal Programming technique to obtain optimum solution of general DNPP under fuzzy environment to get the highest closeness to ideal solutions in two steps.

In the present paper a new method has been introduced to solve general MODNPP in one step only. The proposed approach is computationally easier for the solution of MODNPP in comparison with the existing ones and also it elicits efficient solution of the problem in one step only assuming uniqueness of such solution. Actually in this paper Luhandjula’s technique [19] for the solution of vector maximization problem using compensatory min-bounded sum operator \( \mu_\theta \) has been extended to general MODNPP. Using \( \mu_\theta \) operator, a fuzzy approach for the solution of general MODNPP has been proposed. Here the optimistic expectations of the fuzzy objectives have been taken as usual the ideal values [1], whereas the pessimistic objective values or the threshold values have been computed using Luhandjula’s comparison technique [19]. The method has been illustrated with a numerical example. It is revealed that the same solution as that of Li and Lee could be obtained by the proposed technique but with greater computational ease. A theorem has also been stated here which guarantees an efficient solution of the problem considered. With this aim the paper has been organised in four sections. Section-1 contains introduction and a brief account of the relevant contemporary works; in section-2 we actually introduce the proposed method of solution and a theorem has been stated which establishes the existence of efficient solution of the problem; section-3 illustrates the new approach with numerical example and section-4 contains the conclusion.

2. General Multi-Objective De-Novo Programming Problem (MODNPP)

Let us first consider an MODNPP [20]

\[
\text{Max } Z = Cx; \quad \text{Subject to } Ax \leq b; \quad pb \leq B; \quad x \geq 0
\]

where \( C \in R^{q \times n} \) and \( A \in R^{m \times n} \) are the matrices of dimensions \( q \times n \) and \( m \times n \) respectively, \( b^T = [b_1, b_2, ..., b_m]^T \in R^m \) be the m-dimensional vectors of resources, \( x^T = [x_1, x_2, ..., x_n]^T \in R^n \) be the n-dimensional vectors of decision variables. Here \( B \) be the total available budget and \( p = (p_1, p_2, ..., p_m) \in R^m \) be the vector of unit prices of m resources. Here \( Z_k(x) = C_k x = \sum_{j=1}^{n} c_{kj} x_j \) is the k-th objective where \( C_k = (c_{k1}, c_{k2}, ..., c_{kn}) \) and \( C = [C_1, C_2, ..., C_q] \). Throughout this paper row vectors and column vectors are respectively denoted by parenthesis ‘( )’ and square brackets ‘[ ]’ and the members of \( R^n \) are treated as row vectors.

The main feature of a De-Novo Program is that the resource vector \( b \) is unknown, one has to evaluate \( b \) in such problem. Under a given budgetary provision \( B \), the resources have to be determined in such a way that either all the objectives are optimized with respect to the constraints simultaneously or an efficient/non-dominated solution can be obtained.

In the system (1) only maximizing type of objectives has been considered. But in practical situations, at the same time the decision-maker may have to consider both maximizing and minimizing type of objectives. Thus a general De-Novo set-up where both maximizing and minimizing type of objectives coexist, could be represented as [13]

\[
\begin{align*}
\text{Max } & Z_k = \sum_{i=1}^{n} c_{kj} x_i \quad k = 1,2,...,l \quad \text{subject to} \quad \sum_{i=1}^{n} a_{ij} x_i - b_i \leq 0, \quad i = 1,2,...,m \\
\text{Min } & W_s = \sum_{i=1}^{n} c_{sj} x_i \quad s = 1,2,...,r
\end{align*}
\]

\[
\sum_{i=1}^{m} p_i b_i \leq B \\
x_j \geq 0, \quad j = 1,2,...,n
\]

(2)
In respect of system (2), the decision-maker maximizes each of the objectives $Z_1, Z_2, ..., Z_l$ and minimizes each of the objectives $W_1, W_2, ..., W_r$ independently subject to the given constraints (without considering the budgetary restriction) to find the ideal point for the present condition based on the present configuration and available resources. Thus the ideal point $\mathbf{a}^*$ of the system for the present state is given by $\mathbf{a}^* = (Z_1^*, Z_2^*, ..., Z_l^*, W_1^*, W_2^*, ..., W_r^*)$ where $Z_k^* = \max Z_k(\mathbf{x})$ and $W_s^* = \min W_s(\mathbf{x})$.

But there may not exist a feasible solution $\mathbf{x}^*$, determined by the present portfolio of the resources $b_i$ which satisfies $Z_k^* = Z_k(\mathbf{x}^*)$ and $W_s^* = W_s(\mathbf{x}^*)$ simultaneously. In other words, the ideal point of the system-(2) for the given portfolio of the resource $b_i$ may not correspond to a feasible solution. If such a solution (that is ideal solution) belongs to the feasible space, then the system is optimally designed. Otherwise by extending the available resources under the given budgetary provision B, our aim is to reach either to the ideal solution or to a non-dominated solution under the given budgetary provision. A feasible solution $\mathbf{x} \in X$ is said to be efficient or non-dominated for the MOOP (1) involving both maximizing and minimizing objective functions, if there is no $\mathbf{x} \in X$ so that $Z_k(\mathbf{x}) \leq Z_k(\mathbf{x})$, $\forall \mathbf{x} \in X$, $k = 1,2, ..., l$ and $W_s(\mathbf{x}) \geq W_s(\mathbf{x})$, $\forall \mathbf{x} \in X$, $s = 1,2, ..., r$ and for atleast one $k$, $Z_k(\mathbf{x}) < Z_k(\mathbf{x})$ or for atleast one $s$, $W_s(\mathbf{x}) > W_s(\mathbf{x})$ where $Z_k$’s and $W_s$’s are respectively the maximizing and minimizing objectives. As the system configuration changes, the decision-maker has to search for a new ideal point and corresponding ideal solution. In this way continuous designing of the system could be accomplished via DNPP.

Here $\sum_{i=1}^{n} p_{i} a_{ij}$ represents per unit cost of the product $j$. Let us denote it by $v_j$. Thus $\sum_{i=1}^{n} p_{i} a_{ij} = v_j$, $j=1,2, ..., n$. Using $v_j$, the constraints of system (2) is rewritten as $\sum_{i=1}^{n} p_{i} \sum_{j=1}^{n} a_{ij} x_j \leq \sum_{i=1}^{n} p_{i} b_{i}$ or, $\sum_{j=1}^{n} (\sum_{i=1}^{m} p_{i} a_{ij}) x_j \leq B$. That is $\sum_{j=1}^{n} v_j x_j \leq B$.

Thus using $v_j$, system (2) can be reframed as $\max Z_k = \sum_{j=1}^{n} c_{k} x_j$, $k = 1,2, ..., l$;

$\min W_s = \sum_{j=1}^{n} c_{s} x_j$, $s = 1,2, ..., r$; Subject to $\sum_{j=1}^{n} v_j x_j \leq B$; $x_j \geq 0$, $j = 1,2, ..., n$ (3)

The advantage of system (3) is that there is only one constraint involving $n$ unknowns and hence the basic feasible solutions (at most $n$) could easily be determined. Further it is to be mentioned here that in the process of restructuring of system (2) to (3), the merit of De-Novo program is no way being distorted. This is because the budgetary restriction is reflected in the constraint of system (3) and every feasible solution of system (2) is also a feasible solution of system (3) and conversely [21]. Thus the ideal point of the system (3) is also an ideal point for the system (2). Hence considering the objective functions independently together with the single constraint $\sum_{j=1}^{n} v_j x_j \leq B$, one can easily find the ideal point [5].

2.1. Proposed Approach for the Solution of General MODNPP under Fuzzy Environment Using Compensatory Operator

As mentioned earlier, there exists no general algorithm to solve the general MODNPP (2). However, in [13], Li and Lee solved a crisply defined general MODNPP in two steps in section 2 of [13] using fuzzy techniques. They actually fuzzified the crisp objectives between their absolute boundaries, the ideal values and the negative ideal values [13]. In the first step of their two-step approach, Li and Lee solved the MODNPP (3) under fuzzy environment using the non-compensatory ‘min’ operator $\lambda$ [22]. The solution obtained in step 1, however, may not be an efficient one. In step 2, they actually derived an efficient solution of the problem (3) under fuzzy environment using fully compensatory operator.

In the present paper, it has been illustrated that only by a proper choice of the aspiration levels of the objectives and hence of their membership functions, an efficient solution of the problem (3) could be obtained in one step only. Here we solved a crisp general MODNPP in one step by fuzzifying the objectives between their ideal values and some practical threshold values obtained by using Luhandjula’s comparison technique [19] and considering the compensatory min-bounded sum operator $\mu_k$ [19]. To demonstrate the new approach we consider the general MODNPP (2) and restructured it as system (3).
Let \((Z_1^*, Z_2^*, \ldots, Z_l^*, W_1^*, W_2^*, \ldots, W_r^*)\) be the ideal point of the system (3). Further, let the pessimistic values of the objective functions obtained by Luhandjula’s technique be respectively given by \(\tilde{Z}_1, \tilde{Z}_2, \ldots, \tilde{Z}_l, \tilde{W}_1, \tilde{W}_2, \ldots, \tilde{W}_r\). The linear membership functions \([2, 2, 23]\) for the objectives of (3) are constructed as follows:

\[
\mu_{z_k} = \begin{cases} 
0, & \tilde{Z}_k < Z_k \leq Z_k^*, \ k = 1, 2, \ldots, l \\
\frac{Z_k - \tilde{Z}_k}{Z_k^* - \tilde{Z}_k}, & \tilde{Z}_k < Z_k < Z_k^*, \ k = 1, 2, \ldots, l \\
1, & Z_k \geq Z_k^*
\end{cases}
\]

(4)

and \(\mu_{w_s} = \begin{cases} 
0, & \frac{\tilde{W}_s - W_s}{W_s - \tilde{W}_s} \leq W_s \leq \tilde{W}_s, \ s = 1, 2, \ldots, r \\
\frac{\tilde{W}_s - W_s}{W_s - \tilde{W}_s}, & W_s < \tilde{W}_s \leq \frac{\tilde{W}_s - W_s}{W_s - \tilde{W}_s}, \ s = 1, 2, \ldots, r \\
1, & W_s \leq \tilde{W}_s^*
\end{cases}
\]

(5)

These membership functions indicate the degree of satisfaction of the decision-maker for the attainment of the aspiration levels of the corresponding objectives.

2.1.1. Solution of the General MODNPP Using Compensatory Min–Bounded Sum Operator Under Fuzzy Environment

Luhandjula in [19] defined an alternative operator \(\mu_{\theta}\) other than the usual Zimmermann’s operator, called min-bounded sum operator, to solve vector maximization problem. Let \(\mu_{i}, i=1,2,\ldots,q\) be the membership functions for the fuzzy objectives. Thus \(\mu_{\theta}\) is defined as follows:

\[
\mu_{\theta} = \gamma \min_i \mu_i + (1 - \gamma) \min(1, \sum_i \mu_i)
\]

\(\gamma \in (0, 1)\) is the grade of compensation between aggregated membership functions.

In this paper the use of this min-bounded sum operator \(\mu_{\theta}\) has been extended to the general MODNPP (3) and also it has been shown that an efficient solution could be obtained assuming uniqueness of such solution.

In this regard we first state some results whose validity could easily be checked following Luhandjula’s proposition 2[19].

**Theorem 2.1.** \(\bar{x}\) will be the optimal solution for

\[
\max \mu_{\theta}; \ \text{Subject to} \quad \mu_{\theta} = \gamma \min_i \mu_i + (1 - \gamma) \min(1, \sum_i \mu_i + \sum_{j=1}^r \mu_j); \ \forall x \leq: x \geq 0; \ \gamma \in (0, 1); \mu_{i} \in [0, 1]
\]

if \((\bar{x}, \bar{\lambda}, \bar{\mu})\) be the optimum solution of

\[
\max \mu_{\theta} = \gamma \bar{\lambda} + (1 - \gamma) \mu; \ \text{Subject to} \quad \bar{\lambda} \leq \min_i \mu_i; \ \mu \leq 1;
\]

\[
\mu \leq \sum_{k=1}^l \mu_k + \sum_{s=1}^r \mu_s; \ \forall x \leq B: x \geq 0
\]

\(\lambda \in [0,1]; \mu \in [0,1]; \gamma \in (0,1); \mu_k \in [0,1]; \mu_s \in [0,1]; \mu_\theta \in [0,1] \) where \(\sum_{k=1}^l \mu_k \equiv \mu_k \) and \(\sum_{s=1}^r \mu_s \equiv \mu_s \) ((4) and (5)).

**Lemma 2.1.** \((\bar{x}, \bar{\lambda}, \bar{\mu})\) is optimal for (7) then \(\bar{\lambda} = \min(\mu_k(\bar{x}), \mu_s(\bar{x}))\) and \(\bar{\mu} = \min(1, \sum_{k=1}^l \mu_k(\bar{x}) + \sum_{s=1}^r \mu_s(\bar{x}))\).

**Note**

In general, the optimum solution of (6) may not be efficient. However, the compensatory min-bounded sum operator \(\mu_{\theta}\) may also yield an efficient solution for the De-Novo Program (3) if \(\bar{x}\) is unique. This can be easily proved by the proposed theorem 2.2 with the help of theorem 2.1 and lemma 2.1.

But to prove theorem 2.2, an extension of the concept of domination of solutions to the following general type of multi-objective linear programming problem (8) where both maximizing and minimizing type of objectives are involved, is needed.

Let \(x\) and \(x^*\) be two feasible solutions of the general multi-objective linear programming

\[
\max \mu_{\theta}; \ \text{Subject to} \quad \mu_{\theta} = \gamma \min_i \mu_i + (1 - \gamma) \min(1, \sum_{k=1}^l \mu_k + \sum_{s=1}^r \mu_s); \ \forall x \leq: x \geq 0; \ \gamma \in (0, 1); \mu_{i} \in [0, 1]
\]

Theorem 2.2. Let \(x^*\) be the optimal solution of (7) then \(\mu_{\theta} = \gamma \min_i \mu_i + (1 - \gamma) \min(1, \sum_{k=1}^l \mu_k + \sum_{s=1}^r \mu_s); \ \forall x \leq: x \geq 0; \ \gamma \in (0, 1); \mu_{i} \in [0, 1]
\]

iff \((x, \bar{\lambda}, \bar{\mu})\) be the optimum solution of

\[
\max \mu_{\theta} = \gamma \bar{\lambda} + (1 - \gamma) \mu; \ \text{Subject to} \quad \bar{\lambda} \leq \min_i \mu_i; \ \mu \leq 1;
\]

\[
\mu \leq \sum_{k=1}^l \mu_k + \sum_{s=1}^r \mu_s; \ \forall x \leq B: x \geq 0
\]

\(\lambda \in [0,1]; \mu \in [0,1]; \gamma \in (0,1); \mu_k \in [0,1]; \mu_s \in [0,1]; \mu_\theta \in [0,1] \) where \(\sum_{k=1}^l \mu_k \equiv \mu_k \) and \(\sum_{s=1}^r \mu_s \equiv \mu_s \) ((4) and (5)).

**Note**

In general, the optimum solution of (6) may not be efficient. However, the compensatory min-bounded sum operator \(\mu_{\theta}\) may also yield an efficient solution for the De-Novo Program (3) if \(\bar{x}\) is unique. This can be easily proved by the proposed theorem 2.2 with the help of theorem 2.1 and lemma 2.1.

But to prove theorem 2.2, an extension of the concept of domination of solutions to the following general type of multi-objective linear programming problem (8) where both maximizing and minimizing type of objectives are involved, is needed.

Let \(x\) and \(x^*\) be two feasible solutions of the general multi-objective linear programming
Max \( Z_k = \sum_{j=1}^{n} c_{kj} x_j \), \( k = 1, 2, \ldots, l \); \( \text{Min } W_s = \sum_{j=1}^{m} c_{sj} x_j \), \( s = 1, 2, \ldots, r \); Subject to \( A x \leq b; \quad x \geq 0 \) \hspace{1cm} (8)

**Definition 2.1.** For the feasible solutions \( \bar{x} \) and \( x \) of (8), \( \bar{x} \) is said to dominate strictly \( x \) if for maximizing objectives \( Z_k(\bar{x}) \geq Z_k(x), \forall k \) and for minimizing objectives \( W_s(\bar{x}) \leq W_s(x), \forall s \) and \( Z_k(\bar{x}) > Z_k(x) \) or \( W_s(\bar{x}) < W_s(x) \) for at least one \( k \) or one \( s \) respectively.

**Theorem 2.2.** The optimum solution \( \bar{x} \) of the system (6) will be an efficient solution of (3) if \( \bar{x} \) is unique.

3. Numerical Example Illustrating the Proposed Approach

Let us consider the MODNPP [Example 2 of [13]]

\[
\begin{align*}
\text{max } Z_1 &= 2x_1 + 5x_2 + 7x_3 + x_4; \\
\text{max } Z_2 &= 4x_1 + x_2 + 3x_3 + 11x_4; \\
\text{max } Z_3 &= 9x_1 + 3x_2 + x_3 + 2x_4 \\
\text{min } W_1 &= 1.5x_1 + 2x_2 + 0.3x_3 + 3x_4; \\
\text{min } W_2 &= 0.5x_1 + x_2 + 0.73x_3 + 2x_4 \\
\text{subject to: } &2x_1 + 2x_2 + 2x_3 + 8x_4 \leq b_1; \\
&4x_1 + 4x_2 + 2x_3 + 4x_4 \leq b_2; \\
&5x_2 + 5x_4 \leq b_3; \\
&2.5x_1 + 2.5x_2 + 2.5x_4 \leq b_4; \\
&0.5b_1 + 0.25b_2 + 0.3b_3 + 0.4b_4 = 150; \\
&x_1, x_2, x_3, x_4 \geq 0
\end{align*}
\]  

(9)

To solve the MODNPP (9) of example 3.1, we first recast it in the following form using the procedure as mentioned in section 2.

\[
\begin{align*}
\text{max } Z_1 &= 2x_1 + 5x_2 + 7x_3 + x_4; \\
\text{max } Z_2 &= 4x_1 + x_2 + 3x_3 + 11x_4; \\
\text{max } Z_3 &= 9x_1 + 3x_2 + x_3 + 2x_4 \\
\text{min } W_1 &= 1.5x_1 + 2x_2 + 0.3x_3 + 3x_4; \\
\text{min } W_2 &= 0.5x_1 + x_2 + 0.73x_3 + 2x_4 \\
\text{subject to: } &3x_1 + 4.5x_2 + 1.5x_3 + 7.5x_4 \leq 150; \\
&x_1, x_2, x_3, x_4 \geq 0
\end{align*}
\]  

(10)

For the solution of the problem (10), we first calculate the ideal points. For this, five LPPs are solved considering each objective independently subject to the single constraint. The solutions obtained with corresponding objective values are furnished below:

\[
\begin{align*}
Z_1^* &= 700; \quad x_1 = 0, x_2 = 0, x_3 = 100, x_4 = 0 \\
Z_2^* &= 300; \quad x_1 = 0, x_2 = 0, x_3 = 100, x_4 = 0 \\
Z_3^* &= 450; \quad x_1 = 50, x_2 = 0, x_3 = 0, x_4 = 0 \\
W_1^* &= 30; \quad x_1 = 0, x_2 = 0, x_3 = 100, x_4 = 0 \\
W_2^* &= 25; \quad x_1 = 50, x_2 = 0, x_3 = 0, x_4 = 0
\end{align*}
\]

Thus the ideal point is \( \alpha = (Z_1^*, Z_2^*, Z_3^*, W_1^*, W_2^*) = (700, 300, 450, 30, 25) \). To a decision-maker, attainment of objective values equal to those of ideal values is the most expected one. So in the construction of Zimmermann type of membership function for the fuzzy objectives, the ideal values are the natural choice of optimistic values or upper limits of the corresponding objective functions. Now to obtain a practical threshold value for the objectives, we apply Luhandjula’s technique and the calculations are shown in Table 1.
Table 1. Calculation of Pessimistic Object Values.

| Solutions | Maximizing Objective Values | Minimizing Objective Values |
|-----------|-----------------------------|----------------------------|
|           | $x_1$ $x_2$ $x_3$ $x_4$ $Z_1$ $Z_2$ $Z_3$ $W_1$ $W_2$ |                         |
|           | 0 0 100 0 700 300 100 30 73 |                         |
|           | 0 0 100 0 700 300 100 30 73 |                         |
|           | 50 0 0 0 100 200 450 75 25 |                         |
|           | 0 0 100 0 700 300 100 30 73 |                         |
|           | 50 0 0 0 100 200 450 75 25 |                         |

Corresponding Pessimistic Objective Values
(\(\hat{Z}_1\)) (\(\hat{Z}_2\)) (\(\hat{Z}_3\)) (\(\hat{W}_1\)) (\(\hat{W}_2\))

Now the membership functions for the maximizing and minimizing objective functions are constructed using the relations (4) and (5) of section 2.1 as follows:

For Maximizing Objectives:
\[
\mu_{Z_1}(x) = \begin{cases} 
0, & Z_1 < 100 \\
\frac{Z_1 - 100}{600}, & 100 \leq Z_1 < 700; \\
1, & Z_1 \geq 700. 
\end{cases}
\]
\[
\mu_{Z_2}(x) = \begin{cases} 
0, & Z_2 < 200 \\
\frac{Z_2 - 200}{100}, & 200 \leq Z_2 < 300; \\
1, & Z_2 \geq 300. 
\end{cases}
\]
\[
\mu_{Z_3}(x) = \begin{cases} 
0, & Z_3 < 100 \\
\frac{Z_3 - 100}{350}, & 100 \leq Z_3 < 450; \\
1, & Z_3 \geq 450. 
\end{cases}
\]

For Minimizing Objectives
\[
\mu_{W_1}(x) = \begin{cases} 
0, & W_1 \geq 75 \\
\frac{75 - W_1}{45}, & 30 \leq W_1 < 75; \\
1, & W_1 < 30. 
\end{cases}
\]
\[
\mu_{W_2}(x) = \begin{cases} 
0, & W_2 \geq 73 \\
\frac{73 - W_2}{48}, & 25 \leq W_2 < 73; \\
1, & W_2 < 25. 
\end{cases}
\]

Now introducing Luhandjula's \(\mu_\theta\)-operator, the system (10) under fuzzy environment becomes
\[
\max \mu_\theta = \gamma \lambda + (1 - \gamma) \mu
\]
Subject to \(\lambda \leq \frac{1}{600}(2x_1 + 5x_2 + 7x_3 + x_4 - 100); \lambda \leq \frac{1}{100}(4x_1 + x_2 + 3x_3 + 11x_4 - 200); \lambda \leq \frac{1}{350}(9x_1 + 3x_2 + x_3 + 2x_4 - 100); \lambda \leq \frac{1}{45}(75 - 1.5x_1 - 2x_2 - 0.3x_3 - 3x_4); \lambda \leq \frac{1}{40}(73 - 0.5x_1 - x_2 - 0.73x_3 - 2x_4); \mu \leq 1; \mu \leq \frac{1}{600}(2x_1 + 5x_2 + 7x_3 + x_4 - 100) + \frac{1}{100}(4x_1 + x_2 + 3x_3 + 11x_4 - 200) + \frac{1}{350}(9x_1 + 3x_2 + x_3 + 2x_4 - 100) + \frac{1}{45}(75 - 1.5x_1 - 2x_2 - 0.3x_3 - 3x_4)
+ \frac{1}{48}(73 - 0.5x_1 - x_2 - 0.73x_3 - 2x_4); 3x_1 + 4.5x_2 + 1.5x_3 + 7.5x_4 \leq 150; \lambda \in [0,1]; \mu \in [0,1]; \\
y \in (0,1); \mu_k \in [0,1]; \mu_3 \in [0,1]; \mu_4 \in [0,1]; \quad x_1, x_2, x_3, x_4 \geq 0

Solving the above system again by using LINGO software, the following results are obtained:

\mu_3 = 0.5, \lambda = 0.5, \mu = 1, x_1 = 25, x_2 = 0, x_3 = 50, x_4 = 0; \quad \text{max} Z_1 = 400, \text{max} Z_2 = 250, \text{max} Z_3 = 275, \text{min} W_1 = 52.5, \text{min} W_2 = 49

The results obtained by our proposed approach and that of Li and Lee’s two step approach are identical.

4. Conclusion

In the present treatise a new approach for the solution of the general MODNPP under fuzzy environment has been introduced which yields efficient solution to the problems in one step only. The proposed technique could be used for the solution of a class of problems, like academic resource allocation problems, portfolio analysis, product – mix problems etc., which could be easily converted into multi-objective De-Novo pattern under a given budgetary provision. The proposed method is computationally easier than the existing ones for the solution of MODNPP. The construction of the proposed method is such that they could be successfully applied for the solution of MODNPP involving even large number of objectives, constraints as well as variables. Finally, apart from the computational simplicity, the proposed method requires less processing time in comparison with the existing ones because through the proposed method, the general MODNPP can be solved in one step only. To make the problem more flexible, instead of crisp coefficients, fuzzy, type2 fuzzy coefficients can be considered and solution procedure can be investigated.

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