A Quasi-Static Model of Explosive Spherical Charge in Rock and Soil with Impulsive Impedance

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Abstract: The incident wave pressure acting on the wall of the spherical charge in the rock and soil is an important factor affecting the size of the explosion cavity, and the pressure of the incident wave is related to the impedance of both the explosive and the rock and soil. In this paper, based on the quasi-static model of explosive spherical charge in rock and soil, the impact impedance was introduced to calculate the size of explosion cavity in rock and soil medium. Through the field underground explosion experiment, it is found that the calculation result of the impact impedance model is 6.15% higher than the original model, and the influence rule of several geotechnical medium parameters on the cavity size was also analyzed.

1 Introduction

Explosion in rock and soil is often used in underground construction excavation, seismic exploration and agricultural water storage. When explosives explode, high-temperature and high-pressure air masses will act on the interface between rock and soil and explosives, forming shock waves that transmit in the rock and soil media, and the initial contact interface of rock-soil and explosive is squeezed by high-pressure gas to move away from the core of the explosion, and finally form a stable explosion cavity. The study of the explosion cavity is one of the important issues in the coupling mechanism of underground explosion energy and geotechnical media energy, involving important contents such as the utilization of explosion energy efficiency and the determination of blasting parameters, which have aroused widespread interest in related research fields.

Due to the complexity of the coupling between the explosion energy and the rock-soil medium, the size of the internal cavity is difficult to accurately define. At present, the research on the formation mechanism of the explosion cavity in the rock and soil mainly includes theoretical analytical methods \cite{1}–\cite{5}, numerical simulation methods \cite{6}–\cite{8}, experimental research \cite{9} and other methods. Henrych described the explosion characteristics in the soil in detail \cite{1}. Explosive explosion produces high-temperature and high-pressure detonation products, and shock waves act on the rock and soil wall causes its failure to form a cavity. The rock and soil outside the cavity are crushed (for rocks) or plastically deformed (for soil media), and when the shock wave rapidly attenuates below the elastic limit of the rock and soil, it propagates outward in the elastic region in the form of elastic wave; Lin et al. \cite{2} studied the interaction between the explosion and the rock-soil medium. After the theoretical derivation of the interaction between the explosion shock wave and the condensed medium, the initial parameters of the explosion of the rock-soil medium and the expansion of the cave wall were given. The calculation
method of AGChernikov et al. [3] theoretically estimated the size range of the fracture zone caused by
the explosion of concentrated charges in brittle materials, and the rationality of the estimated range was
verified by experiments; Chen and Wang et al. [6-7] used numerical simulation to calculate the rock's
broken zone and the explosion-squeezed clay density change; Wang et al. [9] have measured various
physical quantities of the soil after explosion after multiple explosion-to-cavity experiments, and found
that the natural density, dry density, saturation, compression modulus and cohesion of the soil decrease
with the increase of the distance from the explosion center, and the porosity ratio, compressibility
coefficient and permeability coefficient of soil increase with the increase of the distance from the
explosion center, and the effect of explosion impact on the internal friction angle of soil is not obvious.

For the theoretical analysis method of the explosion cavity size in the spherical explosive rock and
soil, currently researchers commonly used cavity seismic source model [10], quasi-static theory [11],
dynamic model [6] [11] [12] and quasi-static model [4] to calculate. The cavity seismic source model is
suitable for all homogeneous elastic media, and the radius of the elastic cavity and the equivalent
pressure of the cavity wall can be obtained; the quasi-static theory gives the upper and lower dimensions
of the explosive cavity in the soil. The experiment shows that the measured cavity radius is basically
within the limit, but the relative error is also large; the simplified quasi-static model was originally
proposed by M.F. Drukovany et al. [13] and used to calculate the explosion cavity radius of infinite long
cylindrical charge in rock and soil. Yu Chenglong, Wang Zhongqi et al. [4] established a one-dimensional
spherical explosion cavity prediction quasi-static model on the basis of M.F. Drukovany. After
comparing with the experimental and dynamic model results, it shows a high adaptability and accuracy.
However, when the pressure inside the cavity is substituted into the quasi-static prediction model of
spherical blasting cavity, the shock wave impedance matching between rock and explosive is not
considered. The explosive C-J pressure is used instead of the incident wave pressure of the shock wave
acted on the wall, which leads to the change of the wall pressure due to the difference of rock and soil
wave impedance.

In this paper, the formation method of the closed explosion cavity of the spherical charge was studied.
Considering the influence of the explosive detonation product and the impedance of the geotechnical
medium, the wall pressure of the quasi-static model was discussed, and the spherical explosion cavity
that does not consider the impact impedance effect was further discussed. The prediction quasi-static
model is improved, and the prediction results of the model are compared with the experimental results
to analyze the influence of different geotechnical parameters in the model on the prediction results.

2. Introducing a quasi-static model of impulse impedance

2.1 Calculation of wall pressure

When the shock wave is transmitted from the explosive detonation product to the rock and soil medium,
a transmission shock wave must be generated in the rock and soil medium, and at the same time, a
reflected wave propagating to the explosive detonation product is also generated at the interface. When
the wave impedance of the rock and soil medium is greater than explosive wave impedance, the reflected
wave is a shock wave; when the rock and soil medium wave impedance is smaller than the explosive
wave impedance, the reflected wave is a sparse wave. This paper considers the case of reflected sparse
waves [2]. When the explosive shock wave impedance is greater than the shock wave impedance of the
medium, the sparse wave is reflected in the detonation product. The pressure and particle velocity at the
interface are determined by the incident wave, the reflected wave and the transmitted wave.

If the detonation wave reaches the interface and starts to reflect, there is no forward sparse wave, and
the reflected wavefront is the C-J stable state of the detonation wave, then the area reached by the
reflected wave is the backward simple wave region, and the detonation product is assumed to satisfy the
isentropic equation

\[ p = A \rho^\gamma \]  

(1)

In the formula: \( p \) is the pressure, \( A \) is a constant, \( \rho \) is the density, and \( \gamma \) is the expansion index.
It can be solved as
\[ \frac{dx}{dt} = u - c, \quad u + \frac{2}{\gamma - 1} c = \text{const} \quad (2) \]

In the formula: \( u \) is the medium particle velocity; \( c \) is the medium wave velocity.

So, at the interface, there is

\[ u_b + \frac{2}{\gamma - 1} c_b = u_j + \frac{2}{\gamma - 1} c_j \quad (3) \]

That is,

\[ u_b = u_j + \frac{2c_j}{\gamma - 1} \left( 1 - \frac{c_b}{c_j} \right) \quad (4) \]

Where: \( u_b \) and \( c_b \) are the medium particle velocity and medium wave velocity on the interface, respectively, \( u_j \) and \( c_j \) are the medium particle velocity and wave velocity on the C-J surface, respectively.

The wave velocity relationship is

\[ c = \sqrt{\frac{\gamma p}{\rho}} \quad (5) \]

According to the isentropic state equation (1) and the wave velocity relationship equation (5), the relationship between the interface and the C-J surface medium wave velocity can be obtained

\[ \frac{c_b}{c_j} = \left( \frac{p_b}{p_j} \right)^{\frac{\gamma - 1}{2\gamma}} \quad (6) \]

In the formula: \( p_b \) is the pressure of the interface medium, \( p_j \) is the pressure of the C-J surface medium, that is, the explosive pressure of the explosive.

Considering that:

\[ c_j = \frac{\gamma}{\gamma + 1} D_j, \quad u_j = \frac{1}{\gamma + 1} D_j \quad (7) \]

Where: \( D_j \) is the velocity of the detonation wave.

Simultaneous formulas (4)(7) can be obtained

\[ u_b = \frac{D_j}{\gamma + 1} \left( 1 + \frac{2\gamma}{\gamma - 1} \left( 1 - \left( \frac{p_b}{p_j} \right)^{\frac{\gamma - 1}{2\gamma}} \right) \right) \quad (8) \]

The mass conservation equation in the basic relation of shock wave one-dimensional propagation is

\[ D_s - u_0 = v_0 \frac{u_1 - u_0}{v_0 - v_1} \quad (9) \]

The energy conservation equation is

\[ \rho \frac{u_1^2 - u_0^2}{D_s - u_0} = \frac{1}{2}(e_1^2 - e_0^2) \quad (10) \]

In the formula: \( D_s \) is the steady propagation velocity of shock wave, \( u, \nu, \rho, e, \) and \( p \) are medium velocity, specific volume, density, internal energy and pressure, respectively. Subscripts \( 0 \) and \( 1 \) represent the medium parameters before and after shock wave, respectively.

Combining equations (9) and (10) to eliminate \( D_s - u_0 \), the relationship between the velocity and pressure of the medium particle at the rock-soil interface of the explosive shock wave can be obtained.

\[ u_{mb} = \sqrt{(p_{mb} - p_{m0})(v_{m0} - v_{mb})} \quad (11) \]

Or satisfying

\[ p_{mb} = \rho_{m0} D_{mb} u_{mb} \quad (12) \]

In the formula, the subscripts \( m0 \) and \( mb \) represent the shock wave front and back parameters, respectively.

According to the interface continuity condition \( u_b = u_{mb} \), \( p_b = p_{mb} \), ignoring the value of \( p_{m0} \), it can be obtained that

\[ u_b = \sqrt{p_b (v_{m0} - v_{mb})} \quad (13) \]

\[ p_b = \rho_{m0} D_{mb} u_b \quad (14) \]

Tait derived the pressure density equation of state suitable for rock and soil under explosive shock load [14].

\[ p_b = p_0 + \frac{\rho u_b^2}{\gamma} \left( \frac{\rho}{\rho_0} \right)^\gamma - 1 \quad (15) \]

The simultaneous equations (13)(14)(15) can obtain the initial pressure \( p_b \) of the transmitted shock.
wave at the interface and the interface particle velocity $u_0$.

### 2.2 Introducing a quasi-static model of wall pressure

Yu and Wang proposed a spherical charge prediction quasi-static model based on the one-dimensional cylindrical charge quasi-static model \(^4\). This model fully considers the stress state of the rock and soil. The tangential stress under the condition of the spherical cavity expansion in the elastic medium is 0, the hoop stresses are equal, and the cohesive granular incompressible medium model is introduced to obtain the radius of the blast cavity and the radius of the plastic zone in the rock and soil.

The radius of the burst cavity is:

$$ b^* = a_0 \left( \frac{p_I}{k \left( \sigma_0 + \frac{6 \rho}{k(1+\psi)} \right)} \right)^{\frac{1}{3}} \sqrt{\frac{2 \mu}{3 \sigma_0}} 
$$

In the formula,

$$ L = \frac{\mu}{\sigma_0(1+\ln \frac{\sigma_0}{\rho})} 
$$

The radius of the plastic zone is

$$ b_0 = \left( \frac{\sigma_0}{\sigma_0} \right)^{\frac{1}{2}} b^* 
$$

The final pressure inside the cavity is

$$ p_m = -\frac{k}{\rho} + (\sigma_0 + \frac{k}{\rho} \sigma_0)^{\frac{1}{2}} \frac{4\rho}{k(1+\psi)} 
$$

In these formulae: $a_0$ is the radius of the spherical charge, $k$ is the cohesive force, $\psi$ is the friction coefficient, $\sigma_0$ is the compressive strength of the medium, $\sigma_0$ is the tensile strength of the medium, and $\mu$ is the shear modulus.

This model does not consider the wave impedance effect of explosives on the rock and soil wall. Density is usually used to roughly judge the relationship between the wave impedance of explosives and rock and soil. Under normal circumstances, the rock and soil are relatively soft. According to the wave impedance formula $Z = \rho c$, the rock and soil wave impedance is less than the wave impedance of the explosive, so the C-J pressure of the explosive acting on the rock and soil wall will reflect sparse waves, making the transmission wave pressure of the rock and soil wall less than the C-J pressure of the explosive, so the rock and soil wall pressure is brought into the model Will be closer to the real situation.

Replace $p_I$ with $p_b$, it can be obtained that

$$ b'_* = a_0 \left( \frac{p_b}{k \left( \sigma_0 + \frac{6 \rho}{k(1+\psi)} \right)} \right)^{\frac{1}{3}} \sqrt{\frac{2 \mu}{3 \sigma_0}} 
$$

The plastic zone radius $b_0$ also changes accordingly

$$ b'_0 = \left( \frac{\sigma_0}{\sigma_0} \right)^{\frac{1}{2}} b'_* 
$$

### 3. Experimental analysis

#### 3.1 Introduction to the experiment

To accurately verify the feasibility of the model, the underground charge explosion cavity size measurement was carried out. The soil of the experimental site is mainly composed of pebbles, breccias, pebbles, semi-cemented silt, and silty clay within a depth of 30m. The buried depth of 20m is dominated by silt, mixed with a small amount of round gravel, pebbles and other hard rocks. Its physical and mechanical indicators are given by indoor geotechnical tests ($E_s$ is the compressive modulus), and the characteristic value of bearing capacity $f_{ak}$ based on on-site testing. The indoor experimental results and the “Engineering Geology Handbook” (5th edition) are given, the compressive strength $\sigma_*$ and the tensile strength $\sigma_0$ are given by querying the “Rock and Soil Mechanics Parameters Manual”. The mechanical parameters are shown in Table 1.
Table 1 Soil medium parameters

| $\sigma_1$/ MPa | $\sigma_0$/ MPa | $\mu$/ GPa | $k$  | $\varphi$/ kPa | $\rho_{\text{soil}}$/ (kg · m$^{-3}$) | $c_0$/ m/s | $E_s$/ MPa | $f_{ak}$/ kPa |
|-----------|-----------|--------|-----|------------|---------------------------|-------|--------|-------------|
| 9         | 1         | 0.204  | 0.7 | 12         | 2000                      | 1323  | 9.8    | 180         |

30kg spherical TNT was used as a spherical explosion source, with a radius of 0.1638m and a drilling depth of 20m. To avoid the impact of the backfill material on the underground blast cavity, the backfill material used a special plain concrete that matches the impedance of the original soil and was poured in the hole. The central detonation was performed by a detonator inserted in the explosive center in advance. The experimental schematic diagram is shown in Figure 1, and the experimental site diagram is shown in Figure 2.

Figure 1 Schematic diagram of the experiment

Figure 2 The charging process of the experimental site
3.2 Analysis of experimental results
After the detonation was completed, the on-site personnel drilled holes on the side of the charge hole to the position of the explosion cavity. First the camera was used to detect the measurement hole, and then the lidar was used to measure the cavity size in the hole. The detection process is shown in Figure 3. XYZ represents the relative position in space. The effect of the cavity formation can be clearly seen, and the wall of the cavity (Figure 3(c)) still has massive rock and soil compacted by the explosion impact.

The results of lidar scanning are shown in Figure 4(a). Because the compacted cavity shell falls off during the punching, which affects the measurement. The measured data in the vertical direction (Z direction) is less, and the data range is narrow (the Z direction measurement range is 0 ~ 250mm), which cannot fit a circular curve in the vertical direction. The data point with the largest measurement radius (at Z=140mm) in the horizontal direction (XY plane) is selected for fitting, and some irregular pointed “spikes” can be obviously seen from this data point, which are the anisotropy of rock and soil materials, and the crack propagation caused by uneven force. The fitting result is shown in Figure 4(b), and the fitting radius is 0.667m. According to previous experience, the cavity radius is generally 3 to 5 times of the charging radius. Compared with the calculation results of the original quasi-static model and the quasi-static model with impact impedance introduced, as shown in Table 2, both calculation methods are larger than the measured values. This is because the explosion cavity obtained by the actual underground explosion is not a regular spherical shape, but an irregular ellipsoid with a large bottom and a small head [15]. The main influencing factors are three points: (1) the position of the explosive is closer to the ground surface, and stretching waves reflected from the ground surface elongate the cavity; (2) the soil stress on the upper and lower sides of the explosive ball is inconsistent, and the stress on the upper side is less than the lower side; (3) the inconsistent mechanical properties of the filling material and the original soil will also affect the expansion direction of the blast cavity.

After the impulse impedance is introduced, the recalculation of the pressure on the contact surface of the explosive and the rock-soil has a certain influence on the calculation results of the blast cavity radius. Compared with the measured value, the calculation deviation of the quasi-static model with impact impedance is 6.15% less than that of the original quasi-static model.

Table 2 Comparison of the measured blast cavity radius and the results of the two calculation methods

| Wall pressure /GPa | Burst cavity radius /m | Deviation from actual measured value |
|-------------------|-----------------------|-------------------------------------|
| Actual measured value | 0.667                      |                                    |
| Original quasi-static model | 21 0.804                | 20.54%                             |
| Quasi-static model introduced impact resistance | 13 0.763                  | 14.39%                             |

(a) The measurement position of the hole  (b) Camera detection
4. Analysis of the influence of quasi-static model parameters with impulse impedance

4.1 Analysis of the impact of compressive and tensile strength

To analyze the influence of compressive and tensile strength on the radius of the cavity and plastic zone, the single variable method is used for analysis based on the complexity of the geotechnical medium and the rationality of the change of the medium parameters. As can be seen from Figure 5(a), the greater the compressive strength, the smaller the radius of the cavity formed by the explosives of the corresponding energy destroying the rock and soil medium. In the range of 1~20Mpa, the cavity radius $b'$ is reduced from 1.911m to 0.596m, a decrease of 69%. The radius of the plastic zone $b'_0$ increased from 1.901m to 2.730m, an increase of 43%. The preliminary analysis suggests that the increase in compressive strength increases the energy utilization rate of the explosive outward diffusion, making the cavity smaller and the plastic zone increases. It can be seen from Figure 5(b) that increasing the tensile strength does not affect the size of the cavity radius. The tensile strength hardly affects the cavity formation stage, but it has a greater impact on the radius of the plastic zone. In the range of 1~2Mpa, the radius of the plastic zone $b'_0$ is reduced from 2.53m to 1.76m, a decrease of 29%, which is related to its own calculation method. Due to the symmetry of the explosion effect, the tensile strength mainly affects the circumferential fracture of the medium, and the greater the tensile strength, the smaller the ring fracture radius, and the smaller the radius of the plastic zone.
Figure 5 The influence of compressive and tensile strength of rock and soil media on cavity radius $b^*_{0}$ and plastic zone radius $b_0^*$. 

4.2 Analysis of the influence of shear modulus

In the process of many calculations, it is found that the cohesion force has little effect on the calculation of the cavity size. Here, we will focus on the analysis of the influence of the shear modulus. It can be seen from Figure 6 that the shear modulus affects the cavity radius $b^*_{0}$ and the plasticity radius $b_0^*$ of the zone has a positive correlation. The shear modulus is in the range of 0.262Gpa to 1.06Gpa, and the cavity radius $b^*_{0}$ increased from 0.763m to 1.117m, an increase of 46.4%, while the radius of the plastic zone $b_0^*$ increased from 2.53m to 3.70m, an increase of 46.2%, and the increase rate of these two are almost equal.

Figure 6 Influence of the shear modulus of rock and soil media on the cavity radius $b^*_{0}$ and the plastic zone radius $b_0^*$

5. Conclusion

Through the underground explosion experiment and the analysis of the influence of geotechnical media parameters, the following conclusions are obtained: (1) When the explosive wave impedance is greater than the rock and soil wave impedance, the interface reflects sparse waves, and the transmission wave pressure is less than the detonation wave pressure, in this case, using the transmitted wave pressure as the initial interface pressure is closer to the real situation; (2) In the experiment, the accuracy of the blasting radius of the quasi-static model with impact impedance is 6.15% higher than that of the original
quasi-static model, which has good applicability and can be used for engineering calculations; (3) Parameter influence analysis shows that, the increase in compressive strength will hinder the expansion of the cavity, but it will also increase the energy transfer efficiency of the plastic zone outside the cavity and increase the radius of the plastic zone; the increase of tensile strength does not affect the cavity size, but has a negative correlation with the radius of plastic zone; the increase of the shear modulus will increase the radius of the cavity and the radius of the plastic zone at the same time, and their increase rates are almost equal.

References
[1] Henrych J.(1987) Explosive dynamics and its application. Science Press, Beijing.
[2] Lin D.C., Bai C.H.,(2007) explosion earthquake effect. Geology Press, Beijing.
[3] Chernikov, A.G., Sher, E.N. (1990) A Quasistatic Model of a Confined Explosion of a Concentrated Charge in a Bed and in a Block. Journal of Mining Science,26(4):355-362.
[4] Yu,C.L.,Wang,Z.Q.,Han,W.G.(2017)A prediction model for amplitude-frequency characteristics of blast-induced seismic waves.Geophysical Prospecting.voll 3,Issue 3:219–224.
[5] Li,P.Y., Wang,Z.Q.,Xu,Q.,Liang,X.X.(2019)Calculation method for characteristic size of blasting cavity of finite length cylindrical charges in soil.Explosion and Shock Waves. Issue 39(12)
[6] Chen,W.X.,Wang,M.Y.,Qian,Y.H.,Fang,Y.G.,Wu,H.(2012)Dynamic model and numerical simulation for hard rock near explosion cavity.Chinese Journal of Rock Mechanics and Engineering.262(S1):3091-3097.
[7] Wang,Z.Q.,Zhang,Q.,Bai,C.H.(2001)Numerical simulation on variation of density of the soil compacted by explosion.Chinese Journal of Geotechnical Engineering, Issue 3:350-353.
[8] Ambrosini, D.,Luccioni, B.(2020)Effects of underground explosions on soil and structures.Underground Space,Issue 4.
[9] Wang,H.L.,Ye,C.L.(2011)Variation pattern of soil physico-mechanical performance by explosive enlargement.Rock and Soil Mechanics,Issue 9:2617-2622
[10] Jeffreys,H.(1931)On the Cause of Oscillatory Movement in Seismograms.Geophysical Journal International, Issue 1(2):407 ~ 416.
[11] Wang,H.L.,Feng,C.G.,Hou,Z.X.,Wang,L.Q.(2001)The Calculation on Radius of Explosion Cavity in Soil.Explosive Materials, Issue 3.
[12] Holmberg,R.,Persson,P.A.(1978)The Swedish approach to contour blasting.Proc.IVth Conf. on Exp. and Blasting Tech,ISEE,New Orleans,LA: pp113-127.
[13] Drukovanyi,M.F.,Kravtsov,V.S.,Chernyavskii,Y.E.,Shelenok,V.V.(1976) Calculation of fracture zones created by exploding cylindrical charges in ledge rocks.Journal of Mining Science, Issue (3).
[14] Cole,R.(1948)Underwater explosions.Princeton University Press, Princeton city.
[15] Težak,D.,Stankovi´ć,S.,Kova´c,Ivan.(2019)Dependence models of borehole expansion on explosive charge in spherical cavity blasting.Geosciences, Issue 9(9):383