The Mach-Zehnder and the Teleporter

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Abstract

We suggest a self-testing teleportation configuration for photon q-bits based on a Mach-Zehnder interferometer. That is, Bob can tell how well the input state has been teleported without knowing what that input state was. One could imagine building a “locked” teleporter based on this configuration. The analysis is performed for continuous variable teleportation but the arrangement could equally be applied to discrete manipulations.

One problem with teleportation experiments as they are currently performed [1–3] is that Victor (the verifier) must examine the teleported state to determine if the machine is working. Victor prepares the original input state and is the only person who knows its identity. Because of the imperfect nature of experiments even Victor must be careful not to be tricked in deciding if some level of teleportation has occurred. In principle one might imagine checking the teleporter is working once, then leaving it to go, but in practice machines drift. Thus it would be useful if a constant, straightforward assessment of the teleportation could be carried out without prior knowledge of the input.

Consider first the set-up shown schematically in Fig.1(a). Basically we place a teleporter in one arm of a Mach-Zehnder interferometer, inject a single photon state, in an arbitrary polarization superposition state into one port, then use the interference visibility at the output ports to characterize the efficacy of teleportation. The beauty of such a set-up is the visibility does not depend on the input state, so we can assess how well the teleporter is working without knowing what is going into it. Let us see how this works.

The input for one port of the interferometer is in the arbitrary polarization superposition state

$$|\phi\rangle_a = \frac{1}{\sqrt{2}}(x|1,0\rangle + y|0,1\rangle)$$

where $|n_h, n_v\rangle \equiv |n_h\rangle_h \otimes |n_v\rangle_v$, $n_h$ and $n_v$ are the photon number in the horizontal and vertical polarizations respectively, and $|x|^2 + |y|^2 = 1$. The input of the other port is in the
vacuum state $|\phi\rangle_b = |0,0\rangle$. The Heisenberg picture operators for the four input modes (two spatial times two polarization) are $a_h$ and $a_v$ (superposition), and $b_h$ and $b_v$ (vacuum). We propagate these operators through the Mach-Zehnder (including the teleporter). After the first beamsplitter we can write

$$c_{h,v} = \frac{1}{\sqrt{2}}(a_{h,v} + b_{h,v})$$
$$d_{h,v} = \frac{1}{\sqrt{2}}(a_{h,v} - b_{h,v})$$

(2)

One of the beams ($c$) is then teleported using the continuous variable method discussed in Reference [4,5]. The individual polarization modes of $c$ are separated using a polarizing beamsplitter. Each mode is then mixed on a 50:50 beamsplitter with a correspondingly polarized member of an entangled pair of beams. The entangled pairs may come from two separate 2-mode squeezers or alternatively a single polarization/number entangler could be used [4]. Amplitude and phase quadrature measurements are carried out respectively on the two output beams for each mode either through homodyne detection [4] or parametric amplification [5]. A classical channel for each of the polarization modes is formed from these measurements which are passed to the reconstruction site where they are used to displace the corresponding entangled pair for each mode. The output $c_T$ is formed by combining the two displaced polarization modes on a polarizing beamsplitter. Under conditions for which losses can be neglected the output from the teleporter is

$$c_{h,v,T} = \lambda c_{h,v} + (\lambda \sqrt{H} - \sqrt{H-1}) f_{h,v,1}^\dagger + (\sqrt{H} - \lambda \sqrt{H-1}) f_{h,v,2}$$

(3)

where $\lambda$ is the feedforward gain in the teleporter, the $f_{h,v,i}$ are vacuum inputs to the 2-mode squeezer providing the entanglement for the teleporter (see Fig.2(a)) and $H$ is the parametric gain of the squeezer. The fields are recombined in phase at the final beamsplitter giving the outputs

$$a_{h,v,\text{out}} = \frac{1}{\sqrt{2}}(c_{h,v,T} + d_{h,v})$$
$$b_{h,v,\text{out}} = \frac{1}{\sqrt{2}}(c_{h,v,T} - d_{h,v})$$

(4)

The photon counting rates of the two arms have expectation values

$$<a_{\text{out}}^\dagger a_{\text{out}}> = \langle \phi| a_{\text{out}}^\dagger + a_{\text{out}}(a_{h,\text{out}}^\dagger + a_{\text{out}}) |\phi\rangle |\phi\rangle_b$$
$$= 0.25(1 + \lambda)^2 + (\lambda \sqrt{H} - \sqrt{H-1})^2$$
$$<b_{\text{out}}^\dagger b_{\text{out}}> = \langle \phi| b_{\text{out}}^\dagger + b_{\text{out}}(b_{h,\text{out}}^\dagger + b_{\text{out}}) |\phi\rangle |\phi\rangle_b$$
$$= 0.25(1 - \lambda)^2 + (\lambda \sqrt{H} - \sqrt{H-1})^2$$

(5)

In the limit of very strong entanglement squeezing ($\sqrt{H} - \sqrt{H-1} \to 0$) we find from Eq. 3 that $c_{h,v,T} \to c_{h,v}$ for unity gain ($\lambda = 1$), i.e. perfect teleportation. For the same conditions (and only for these conditions) the visibility of the Mach-Zehnder outputs,

$$V = \frac{<a_{\text{out}}^\dagger a_{\text{out}}> - <b_{\text{out}}^\dagger b_{\text{out}}>}{<a_{\text{out}}^\dagger a_{\text{out}}> + <b_{\text{out}}^\dagger b_{\text{out}}>}$$

(6)
goes to one, indicating the state of the teleported arm exactly matches that of the unteleported arm. Notice that the expectation values (Eq.5), and thus the visibility, do not depend on the actual input state (no dependence on $x$ and $y$). Hence we can demonstrate that the teleporter is operating ideally even if we do not know the state of the input. Classical limits can be set by examining the visibility obtained with no entanglement ($H = 1$). In Fig.3 we plot the visibility versus feedforward gain in the teleporter for the cases of no entanglement (0%), 50% entanglement squeezing and 90% entanglement squeezing. Maximum visibility in the classical case is 0.42. Increasing entanglement leads to increasing visibility.

It is known that using a single mode squeezed beam, divided in half on a beamsplitter (see Fig.2(b)), instead of a true 2-mode squeezed source (which exhibits Einstein, Podolsky, Rosen (EPR) correlations), can still produce fidelities of teleportation higher than the classical limit for coherent state inputs. Loock and Braunstein [6] have recently contrasted various single mode and 2-mode squeezing schemes on the basis of their fidelity. It is educational to examine how well the single squeezer teleporter performs in our single photon Mach-Zehnder. The input output relation for a single squeezer teleporter is

$$c_{h,v,S} = \lambda c_{h,v} + \frac{1}{\sqrt{2}}((\lambda\sqrt{H} - \sqrt{H-1})f_{h,v,1} + (\sqrt{H} - \lambda\sqrt{H-1})f_{h,v,1} + \lambda f_{h,v,2} + f_{h,v,2}) \tag{7}$$

The expectation values for the outputs then become

$$<a^{\dagger}_{\text{out}} a_{\text{out}}> = 0.25(1 + \lambda)^2 + 0.5(\lambda\sqrt{H} - \sqrt{H-1})^2 + 0.5\lambda^2$$

$$<b^{\dagger}_{\text{out}} b_{\text{out}}> = 0.25(1 - \lambda)^2 + 0.5(\lambda\sqrt{H} - \sqrt{H-1})^2 + 0.5\lambda^2 \tag{8}$$

On Fig.3 we also present the visibility as a function of gain for the single squeezer case with squeezing of 87.5%. The squeezing is picked such that the average coherent state unity gain fidelity is the same as for the 50% squeezed 2-mode entanglement (the criteria used in Ref. [6]). The performance of the single squeezer teleporter is clearly inferior. Although achieving a better visibility than the classical teleporter it never exceeds, or equals, for any gain, the performance of the 50% squeezed 2-mode teleporter. The maximum visibility of the 2-mode teleporter is 25% higher. We conclude that the entanglement of the single squeezer is not as useful for teleportation as might be suggested by the coherent state average fidelity measure.

In the experiments we have imagined so far the level of visibility has been determined not only by the ability of the teleporter to reproduce the input states of the photons (the mode overlap) but also the efficiency with which input photons to the teleporter lead to correct output photons (the power balance). It is of interest to try to separate these effects. We can investigate just state reproduction if we allow attenuation to be applied to beam $d$, thus “balancing” the Mach-Zehnder by compensating for the loss introduced by the teleporter (see Fig.1(b)). The attenuated beam $d$ becomes

$$d_{h,v,A} = \sqrt{\eta}d_{h,v} + \sqrt{1 - \eta}g_{h,v} \tag{9}$$

where $g$ is another vacuum field and $\eta$ is the intensity transmission of the attenuator. The expectation values of the outputs (using 2-mode entanglement) are now

$$<a^{\dagger}_{\text{out}} a_{\text{out}}> = 0.25(\sqrt{\eta} + \lambda)^2 + (\lambda\sqrt{H} - \sqrt{H-1})^2$$

$$<b^{\dagger}_{\text{out}} b_{\text{out}}> = 0.25(\sqrt{\eta} - \lambda)^2 + (\lambda\sqrt{H} - \sqrt{H-1})^2 \tag{10}$$
In Fig. 4 we plot visibility versus gain, using the attenuation $\eta$ to optimize the visibility ($\eta \leq 1$). Now we can always achieve unit visibility for any finite level of entanglement by operating at gain $\lambda_{opt} = \sqrt{\frac{H-1}{H}}$ and balancing the interferometer by setting $\eta = \lambda_{opt}^2$. The high visibility is achieved because at gain $\lambda_{opt}$ the teleporter behaves like pure attenuation. That is the photon flux of the teleported field is reduced, but no “spurious photons” are added to the field. Thus, at this gain, all output photons from the teleporter are in the right state, but various input photons are “lost”. This effect does not occur for the single squeezer teleporter (also plotted in Fig. 4) whose performance is not improved by balancing the interferometer, further emphasizing its lack of useful entanglement.

So far we have considered test arrangements in which a teleported field is compared with one which is not teleported. However the result of Eq. 10 suggests a self testing arrangement for a teleporter. Suppose we place a teleporter in both arms of the interferometer as portrayed in Fig. 1(c). Writing an expression for the teleported beams $d$ similar to Eq. 3 we find the expectation values of the outputs are now

$$
< a_{out}^\dagger a_{out} > = \lambda^2 + 2(\lambda\sqrt{H} - \sqrt{H-1})^2 \\
< b_{out}^\dagger b_{out} > = 2(\lambda\sqrt{H} - \sqrt{H-1})^2
$$

(11)

where we have assumed the gains of the two teleporters are the same. By monitoring the “dark” output port ($b_{out}$) it may be possible to keep the system “locked” to maximum visibility, without any knowledge of the input state or requiring the destruction of the output state ($a_{out}$). Once again, under low loss conditions, unit visibility is achieved for gain $\lambda_{opt}$ as illustrated in Fig. 5. The added complexity of using two teleporters may be justified in practice by the greater versatility of this system.

In conclusion, we have examined a Mach-Zehnder arrangement for testing the efficacy of single photon qubit teleportation. The major advantage of this arrangement is it doesn’t require the tester to know the input state of the photon. We have contrasted the results obtained with no entanglement, single mode entanglement and true 2-mode entanglement using continuous variable teleportation. The highest visibilities are always achieved with 2-mode entanglement. We have also suggested that a “locked” teleporter could be constructed using a generalization of the testing scheme. We have only examined here the case where losses can be neglected. Losses reduce visibilities but the general trends discussed here remain the same.
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FIGURES

FIG. 1. Schematics of various Mach-Zehnder plus teleporter arrangements.

FIG. 2. Schematic of the two types of entanglement used for teleportation. NDOPO stands for non-degenerate optical parametric oscillator and DOPO stands for degenerate optical parametric oscillator. A separate pair of entangled beams is needed to teleport each of the two polarization modes. Alternatively type II polarization entanglement could be used [4].

FIG. 3. Visibility versus gain for various levels of 2-mode entanglement (0%, 50% and 90%) and 87.5% single mode squeezing (single squeezer).

FIG. 4. Visibility versus gain with “attenuation balancing” for various levels of 2-mode entanglement (0%, 50% and 90%) and 87.5% single mode squeezing (single squeezer).

FIG. 5. Visibility versus gain for self-testing teleporter for various levels of 2-mode entanglement (0%, 50% and 90%).
\[
\begin{align*}
\sqrt{H}f_{h,v,1} &+ \sqrt{H-1}f_{h,v,2}^+ \\
\sqrt{H}f_{h,v,2} &+ \sqrt{H-1}f_{h,v,1}^+
\end{align*}
\]

(a) 2-mode entanglement

\[
\begin{align*}
1/\sqrt{2} (\sqrt{H}f_{h,v,1} &+ \sqrt{H-1}f_{h,v,1}^+) \\
+1/\sqrt{2} f_{h,v,2} &-1/\sqrt{2} f_{h,v,2}
\end{align*}
\]

(b) Single squeezer entanglement

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