A GAUSS-HERMITE EXPANSION OF THE GALACTIC GLOBULAR CLUSTER LUMINOSITY FUNCTION

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ABSTRACT

We decompose the luminosity function of Galactic globular clusters into a sum of the orthogonal Gauss-Hermite functions. This method quantifies the asymmetric third-order ($h_3$) and symmetric fourth-order ($h_4$) terms of the distribution while minimizing the effect of outliers in the data. For 138 Galactic globulars we obtain $< M_V > = -7.41 \pm 0.11$, $\sigma(M_V) = 1.24$ mag, $h_3 = 0.02 \pm 0.05$, and $h_4 = 0.06 \pm 0.05$, i.e. the core of the distribution does not differ significantly from a Gaussian. For a low-metallicity subsample of 103 globular clusters with $[Fe/H] < -0.8$, we find $< M_V > = -7.48 \pm 0.11$, $\sigma(M_V) = 1.08$ mag, $h_3 = 0.05 \pm 0.05$, and $h_4 = 0.13 \pm 0.05$.

Subject headings: globular clusters: luminosity function
1. INTRODUCTION

The Galactic globular cluster luminosity function (GCLF) has usually been represented by a Gaussian (log-normal) distribution, with mean absolute magnitude $M_V \sim -7.3$ mag, and dispersion $\sigma(M_V) \sim 1.1$ mag (Harris & Racine 1979, Hanes & Whittaker 1987, Racine & Harris 1992, Secker & Harris 1993). This simple characterization provides a reasonably good description of the GCLF expressed in terms of clusters per unit magnitude. However van den Bergh (1985) has pointed out that the observed luminosity function of globular clusters is slightly asymmetric, with a long tail extending to faint absolute magnitudes. Recent studies have suggested that the Galactic GCLF is non-Gaussian, and have attempted to model it either as a t-distribution (Secker 1992; Racine & Harris 1992), or to model the GCLF (expressed in terms of clusters per unit luminosity) as a truncated power-law model derived by assuming a mass function constructed from three power laws (McLaughlin 1993; Harris & Pudritz 1994).

In this paper we will adopt an alternative approach to characterizing the Galactic GCLF. Since the luminosity function is quite close to a Gaussian, we will decompose the observed GCLF into the sum of orthogonal Gauss-Hermite polynomials. This technique was developed by van der Marel & Franx (1993), and independently by Gerhard (1993), to describe the (continuous) line-of-sight velocity distributions of the stars in galaxies. It was subsequently used by Zabludoff, Franx, & Geller (1993) to describe the (discrete) velocity distributions in a sample of eight rich Abell clusters. The expansion into Gauss-Hermite polynomials describes the overall distribution as the sum of a simple Gaussian (the lowest order term in the expansion), along with symmetric and asymmetric components whose relative sizes allow a precise description of the departures from the simple Gaussian model. An additional benefit of this description is that the characteristic parameters of the lowest-order Gaussian are insensitive to outliers in the data. In the case of the Galactic
GCLF, it is possible that clusters in the faint tail of the distribution are dying objects such as E3 (van den Bergh 1980), in which the high abundance of binaries points to significant mass loss via evaporation. By reducing the sensitivity to these outlying points in our modeling, we hope to be able to represent more accurately the intermediate-luminosity GCLF, which may have been less affected by cluster evolution than the faint tail of the complete luminosity function.

2. METHOD AND RESULTS

Our decomposition of the luminosity function into a Gauss-Hermite series is very similar to the well-known procedure for reconstructing a function using Gram-Charlier series of type A (van der Marel & Franx 1993; Kendall & Stuart 1943). The major advantage of our parametrization using Gauss-Hermite polynomials is that it is less sensitive to outliers in the wings of the distribution than is the case when higher order moments are used to characterize the data. This is often an important advantage in astrophysical applications.

Our technique is very similar to the procedure used by Zabludoff, Franx, and Geller (1993). Given a sample of $N$ absolute magnitudes, $M_i$, $1 < i < N$, the Gauss-Hermite moments $h_j$ of the luminosity distribution are given by:

$$h_j = \frac{2\sqrt{\pi}}{N\sigma(M_V)} \sum_{i=1,N} \frac{\exp(-x^2/2)}{\sqrt{2\pi}} H_j(x_i),$$

where $H_j$ are the Hermetian polynomials given by:

$$H_0(x) = 1$$

(1)
\[ H_1(x) = \sqrt{2}x \]  
(3)

\[ H_2(x) = \frac{1}{\sqrt{2}}(2x^2 - 1) \]  
(4)

\[ H_3(x) = \frac{1}{\sqrt{6}}(2\sqrt{2}x^3 - 3\sqrt{2}x) \]  
(5)

\[ H_4(x) = \frac{1}{\sqrt{24}}(4x^4 - 12x^2 + 3). \]  
(6)

The parameter \( x_i \) is defined by:

\[ x_i = \frac{M_i - <M_V>}{\sigma(M_V)}. \]  
(7)

For any choice of the parameters \(<M_V>\) and \(\sigma(M_V)\) the Gauss-Hermite series converges to the true function. In practise one chooses \(<M_V>\) and \(\sigma(M_V)\) in such a way that the series converges quickly, so that the lowest order term approximates the series as well as possible.

As shown in van der Marel and Franx (1993), choosing \(<M_V>\) and \(\sigma(M_V)\) so that the zeroth order term is effectively the best-fitting Gaussian is equivalent to solving for \(<M_V>\) and \(\sigma(M_V)\) in the system of equations: \(\{h_1 = 0; h_2 = 0\}\). This choice of mean luminosity and dispersion for the best-fitting Gaussian then fixes the third order (asymmetric) and fourth order (symmetric) terms in the expansion, \(h_3\) and \(h_4\). This expansion could, in principle, be extended to higher orders but we have found that inclusion of terms up to the fourth order is sufficient to adequately describe the observed luminosity distribution.

In order to apply this procedure to the analysis of the Galactic globular cluster luminosity function, we used data from the recent compilation of Djorgovski (1993) which assumes \(M_V(\text{RR}) = +0.6\), and took subsamples from this listing corresponding to (a) all clusters, (b) low-metallicity clusters with \([Fe/H] < -0.8\), (c) clusters with distances from the Galactic center, \(R\), in the range \(3 < R(\text{kpc}) < 30\), and (d) clusters with \([Fe/H] < -0.8\) and \(3 < R(\text{kpc}) < 30\). The results obtained by decomposing the luminosity distributions of these data using Gauss-Hermite polynomials are summarized in Table 1, and illustrated
in Figures 1–4. An estimate of the standard error of $h_3$ and $h_4$ for $\sim 100$ objects has been given by Zabludoff, Franx, & Geller (1993) as $\sigma(h_3) \approx \sigma(h_4) \approx 0.06$. It should be emphasized that this estimate is a relatively crude approximation and, like Zabludoff et al. (1993), we rely on Monte Carlo simulations to estimate the final uncertainties on these parameters. The confidence intervals from these simulations are shown in Figure 5. Note that the confidence intervals are not circular (i.e, estimates of $h_3$ and $h_4$ are correlated). These correlations appear to be somewhat stronger than predicted from the correlation matrix given in van der Marel and Franx (1993), although the internal correlations in this procedure are still much weaker than those inherent in decomposition of a distribution into the sum of two Gaussians.

The total sample of all 138 clusters deviates from a Gaussian at barely the $1\sigma$ level, but the GCLF of the low-metallicity ($N = 103$ objects) subsample deviates from the Gaussian with over $2\sigma$ confidence. This suggests that the form of the GCLF might be metallicity-dependent. However, the evidence for this is rather weak, since the overlapping $2\sigma$ confidence intervals for the complete and low-metallicity subsamples seen in Figure 5 suggests that the two luminosity functions differ at only the $\sim 1\sigma$ level. This agrees with the result obtained from a Kolmogorov-Smirnov test, which suggests that these two distributions do not differ significantly. In order to test whether the observed values of $h_3$ and $h_4$ are sensitive to the value $M_V(RR) = +0.6$ assumed by Djorgovski (1993) in tabulating his globular cluster sample, we repeated the calculations of $h_3$ and $h_4$ for cluster sample assuming $M_V(RR) = 0.30[Fe/H] + 0.94$ as advocated by Sandage (1993), and $M_V(RR) = 0.15[Fe/H] + 1.01$ as found by Carney et al. (1992). We find that the assumption of a metallicity-dependent $M_V(RR)$ affects the parameters $h_3$ and $h_4$ at the $\Delta h_3 = \Delta h_4 = 0.01$ level, and is thus negligible in comparison with the confidence intervals shown in Figure 5.
Since the analysis procedure described in this work is only suited to the study of samples of $N \simeq 100$ objects, probing finer subdivisions within this sample is difficult. A sample of low-metallicity clusters with $3 < R(\text{kpc}) < 30$ ($N = 71$ objects) appears to deviate from a Gaussian at about $1.5\sigma$ confidence.

3. DISCUSSION

The results presented in the previous section suggest that deviations from the canonical log-normal GCLF might be most pronounced amongst the low-metallicity subsample of the Galactic globular cluster population. The dominant higher order component in our expansion of the low-metallicity GCLF is the symmetric $h_4$ term. However, there is also evidence for the presence of a small asymmetric contribution from a non-zero $h_3$ term. The size of the $h_3$ and $h_4$ terms are small enough that the overall low-metallicity luminosity function remains quite well described by a Gaussian. It is interesting to compare these results with those obtained by Secker (1992), who investigated a subsample of 100 globular clusters from the tabulation of Harris et. al. (1991). This sample included globulars with Galactocentric distances in the range $2 < R(\text{kpc}) < 35$ that have reddenings $E_{B-V} < 1.0$. These constraints on $R$ and $E_{B-V}$ were chosen in order (1) to select a sample of globulars that would allow comparison with the globular cluster population in M31, and (2) to be insensitive to incompleteness caused by absorption. Secker (1992) concludes that a Gaussian fits the GCLF rather well (it cannot be excluded at the $2\sigma$ level), but that a t-distribution gives a slightly better fit to their data. This is partly due the extra degree of freedom in the t-distribution model [the value of the likelihood function for the best-fitting Gaussian and t-distribution models given in Secker (1992) are very similar], but Secker’s overall conclusion that the GCLF is slightly non-Gaussian is quite consistent with our results. Secker’s preferred t-distribution has slightly wider wings than an equivalent Gaussian, in good
agreement with our detection of a small positive $h_4$ term in the GCLF. In our analysis we have made no attempt to exclude globulars on the basis of visibility to an external observer. Furthermore, our sample includes a number of distant Palomar-type halo clusters and bulge clusters that were excluded by Secker in order to avoid skewing his model fits. The use of Gauss-Hermite polynomials in modelling the GCLF allows the total sample (including outliers) to be used without having to discard any data, which is a major advantage in this context. Since the Central Limit Theorem (Kendall & Stuart 1979) suggests that many observed processes in nature will have underlying distributions that are Gaussians, we feel that there is considerable appeal in a technique that allows one to accurately model the GCLF as a Gaussian with small perturbations, rather than as a model distribution chosen without a close connection to an underlying physical or statistical process.

In an interesting paper, Harris and Pudritz (1994) have recently attempted to model the observed luminosity function of globular clusters (expressed in terms of clusters per unit luminosity) by using a discontinuous multiple-component power-law model for the globular cluster mass spectrum (under the assumption of constant mass-to-light ratio). This simple model is appealing because the shape of the bright end of the globular cluster mass spectrum appears to be similar in form in giant and supergiant galaxies. This shape can be approximated by a two component power-law model with spectral index $\alpha = 1.6$ to $1.7$ between $10^{5.2}$ and $10^{6.5} M_\odot$, steepening to $\alpha = -3.2$ beyond $10^{6.5} M_\odot$. Harris and Pudritz (1994) and McLaughlin (1993) show that the mass spectrum of giant molecular clouds can also be approximately described by a two-component power-law model with spectral index $\alpha \sim 0.15$ between $10^4 - 10^{5.5} M_\odot$, and $\alpha \sim 1.7$ at higher masses. Harris and Pudritz argue that if the rate of globular cluster destruction is negligible, and if the progenitors of globular clusters are dense gaseous cores embedded in giant molecular clouds and HII regions with mass spectra similar to those seen in giant molecular clouds at the current epoch, then a “universal” discontinuous three-component power-law mass-spectrum
can be defined between $10^4 M_\odot$ and $10^{6.5} M_\odot$ by simply joining the molecular cloud/HII region mass spectrum with the globular cluster mass spectrum seen in giant and supergiant galaxies. The GCLF expressed in terms of clusters per unit magnitude resulting from this universal mass-spectrum model is approximately Gaussian. While Harris and Pudritz’ model has the disadvantage of being discontinuous in the mass spectrum, and has a relatively large number of parameters, the good agreement between the near-Gaussian GCLF predicted by their model and the observed near-Gaussian GCLF is intriguing. However if the weak ($\sim 1\sigma$) evidence presented in the previous section for a metallicity dependence in the GCLF is confirmed, this would be a potentially serious problem for this model, since a metallicity-dependent GCLF seems difficult to reconcile with Harris & Pudritz’ assumption of a universal mass spectrum at all epochs. Furthermore, is not clear that dense gaseous cores embedded in giant dust-free metal-poor gas clouds in the turbulent halo of the proto-Galaxy will have the same mass spectrum as that presently exhibited by the dusty molecular clouds in the Galactic disk. It is also not clear that the masses of globular clusters should be proportional to those of their parent clouds. Finally, a potentially serious difficulty may lie in the assumption of negligible cluster destruction. Calculations by Hut & Djorgovski (1992) suggest that $\sim 4\%$ of the total Galactic globular cluster population is destroyed per Gyr as a result of relaxation-driven evaporation and shocking by the combined effects of interactions with the Galactic disk and bulge. We note in passing that a Schechter function provides an excellent fit to the Harris & Pudritz universal mass-spectrum model, except at the very low-mass end of the spectrum. A primordial Schechter-law mass-function, combined with systematic destruction of low-mass globulars, might result in a present-epoch mass-spectrum similar in form to the Harris and Pudritz three power-law model.

If cluster destruction is occurring, the small deviations from the canonical Gaussian distribution measured by $h_3$ and $h_4$ can be used to gain some insight into the shape of
the primordial GCLF. In simple models, wherein the cluster halo is formed through an amalgamation of distinct (Gaussian) cluster populations in an early merger, observed $h_3$ and $h_4$ values may restrict the relative populations, mean luminosities, and luminosity dispersions of objects in the merging sub-clumps. For example, in the simple two-subclump model of Zabludoff, Geller, & Franx (1992), a superposition of two Gaussian sub-clumps results in $h_3 \sim 0$ and $h_4 \sim 0.15$ if the two populations have identical mean luminosities but the second sub-clump has a substantially larger luminosity dispersion than the first clump, while containing a factor of $\sim 5$ times fewer objects. Such models are probably too simplistic to realistically reproduce the formation of the globular cluster halo, although they do provide some insight into the higher order moments that might be expected to result from the combination of similarly shaped globular cluster luminosity distributions in a merger.
4. SUMMARY AND CONCLUSIONS

Orthogonal Gauss-Hermite functions, which minimize the effects of outliers in the tails of the observed distribution, show that the GCLF is well represented by a Gaussian with $<M_V> = -7.41 \pm 0.11$ mag and $\sigma(M_V) = 1.24$ mag. Various subsamples of the data can also be well represented as Gaussians with small higher-order perturbations. The dominant higher-order term in the Gauss-Hermite expansion is a symmetric $h_4 \sim 0.1$ term. Subsamples of the data based on metallicity and Galactocentric radius do not differ from each other at high levels of statistical significance. In particular, the luminosity function of metal-poor halo clusters does not appear to differ markedly from that of metal-rich disk globulars, although there is some evidence for a weak effect at the $\sim 1\sigma$ level. These results suggest that, for most applications, a simple Gaussian description of the GCLF is an adequate representation of the data. Non-Gaussian representations of the GCLF, such as that resulting from a t-distribution (Secker 1992) or from the multiple power-law mass-spectrum model of Harris & Pudritz (1994), require more free parameters, and do not result in substantial improvements to the modelling of the luminosity function. The Harris & Pudritz (1994) description of the GCLF is appealing because it results from a well-defined physical model, but it is not yet clear that the mass-spectrum of giant molecular clouds at the current epoch should be similar to the mass spectrum of globular cluster progenitors at the epoch of cluster formation.

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Fig. 1.— (Top) The distribution of all 138 Galactic globular clusters in our sample shown as a histogram, along with the reconstructed distribution obtained from the Gauss-Hermite decomposition. (Bottom) The reconstructed distribution (dark solid line), along with the $h_0$ (thin solid line), asymmetric $h_3$ (dashed line), and symmetric $h_4$ (dotted line) components.

Fig. 2.— As for Figure 1, except for the 103 Globular clusters with $[Fe/H] < -0.8$.

Fig. 3.— As for Figure 1, except for the 96 Globular clusters with $3 < R(\text{kpc}) < 30$.

Fig. 4.— As for Figure 1, except for the 71 Globular clusters with $[Fe/H] < -0.8$ & $3 < R(\text{kpc}) < 30$.

Fig. 5.— The 2σ confidence regions expected from a Gauss-Hermite decomposition of a sample of 100 objects. The upper ellipse shows the confidence region for a distribution with $h_3 = 0.05$, $h_4 = 0.13$ (point A), corresponding to our low-metallicity subsample of globular clusters. The lower ellipse shows the confidence region for a perfect gaussian with $h_3 = h_4 = 0$ (point C). Point B corresponds to $h_3 = 0.02$, $h_4 = 0.06$ (our derived parameters for the total sample of 138 globular clusters). The confidence regions were estimated from 1000 Monte Carlo simulations, assuming $M_V = -7.4$ mag and $\sigma(M_V) = 1.2$ mag.
Table 1. PARAMETERS FOR SUBSAMPLES OF GLOBULAR CLUSTERS

| Subsample                        | N  | $< M_V >$ | $\sigma(M_V)$ | $h_3$   | $h_4$  |
|----------------------------------|----|-----------|----------------|---------|--------|
| All                              | 138| $-7.41 \pm 0.11$ | 1.24           | 0.02 $\pm$ 0.05 | 0.06 $\pm$ 0.05 |
| $[Fe/H] < -0.8$                  | 103| $-7.48 \pm 0.11$ | 1.08           | 0.05 $\pm$ 0.05 | 0.13 $\pm$ 0.05 |
| $3 < R(\text{kpc}) < 30$         | 96 | $-7.46 \pm 0.12$ | 1.19           | 0.02 $\pm$ 0.06 | 0.07 $\pm$ 0.06 |
| $[Fe/H] < -0.8 \& 3 < R(\text{kpc}) < 30$ | 71 | $-7.52 \pm 0.12$ | 1.03           | $-0.01 \pm 0.07$ | 0.10 $\pm$ 0.07 |