Apparent Lorentz violation through spacetime-varying couplings

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In this talk, we explore the relation between smoothly varying couplings and Lorentz violation. Within the context of a supergravity model, we present an explicit mechanism that causes the effective fine-structure parameter and the effective electromagnetic $\theta$ angle to acquire related spacetime dependences. We argue that this leads to potentially observable Lorentz violation and discuss some implications for the Standard-Model Extension.

1 Introduction

Originally proposed by Dirac,\(^\text{1}\) the idea of spacetime-dependent couplings has remained the subject of a variety of experimental and theoretical investigations. As a result of current claims of observational evidence for a time variation of the electromagnetic coupling\(^\text{2}\) and the recent realization that such effects are a natural consequence of many unified theories;\(^\text{3}\) this idea has regained considerable interest.\(^\text{4}\) Although there has been substantial theoretical progress in the subject, a realistic underlying theory allowing concrete predictions is presently still lacking. For the identification of high-precision tests, it is therefore important to determine generic physical effects caused by spacetime-dependent couplings.

In models with varying couplings, the usual spacetime symmetries described by the Poincaré group can be effected. For example, translational invariance is normally lost in such models. In this talk, we argue that varying couplings can also lead to violations of the remaining spacetime symmetries associated with the Lorentz group, a point not widely appreciated. Intuitively, this can be understood when the effective vacuum is interpreted as a spacetime-varying medium, in which, for example, isotropy can be lost, so that certain rotations, which are contained in the Lorentz group, may no longer be associated with a symmetry transformation.

A partial motivation for this study is provided by the extreme sensitivity of experimental Lorentz tests and by recent progress in the understanding of a general Lorentz- and CPT-violating extension of the Standard Model;\(^\text{5}\) a framework that includes all possible coordinate-invariant Lorentz- and CPT-
breaking interactions. It describes the low-energy limit of possible Lorentz and CPT violation at a more fundamental level, such as strings, nontrivial spacetime topology, and realistic noncommutative field theories. The Standard-Model Extension (SME) has provided the basis for numerous experimental investigations involving hadrons, protons and neutrons, electrons, photons, muons, and cosmic-ray physics. These studies place tight constraints on possible violations of Lorentz and CPT symmetry. We also remark that in this context, the inverse line of reasoning has already been discussed: certain constant parameters in the SME are equivalent to spacetime-dependent masses.

As part of our analysis, we construct a classical cosmological solution in the framework of the pure $N = 4$ supergravity in four spacetime dimensions demonstrating how the fine-structure parameter $\alpha$ and the electromagnetic $\theta$ angle can acquire related spacetime dependences. Although this model is known to be unrealistic in detail, it is a limit of the $N = 1$ supergravity in 11 dimensions, which is contained in M-theory. It could therefore yield valuable insight into generic features of a promising candidate fundamental theory. Moreover, a smoothly varying $\theta$ angle can be associated with a Lorentz-breaking Chern-Simons-type interaction. Our explicit mechanism for a varying $\theta$ in the context of a consistent supergravity model therefore sheds some light on the usual theoretical difficulties associated with this term and how they may be avoided.

2 Cosmology

When only one graviphoton, $F_{\mu\nu}$, is excited and Planck units are adopted, the bosonic lagrangian for the pure $N = 4$ supergravity in four dimensions is

$$L = \sqrt{g} \left( -\frac{1}{2} R - \frac{1}{4} M F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} N F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{\partial_{\mu} A \partial^{\mu} A + \partial_{\mu} B \partial^{\mu} B}{4B^2} \right),$$

(1)

where $g_{\mu\nu}$ represents the metric, $\tilde{F}^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}/2$, and

$$M = \frac{B(A^2 + B^2 + 1)}{(1 + A^2 + B^2)^2 - 4A^2}, \quad N = \frac{A(A^2 + B^2 - 1)}{(1 + A^2 + B^2)^2 - 4A^2}. \quad (2)$$

The conventional complex scalar denoted by $Z$, which contains an axion and a dilaton, is related to $A$ and $B$ via a canonical transformation, such that $B$ can be identified with the string-theory dilaton.

Within this model, one can determine a simple classical solution. To this end, we set $F_{\mu\nu}$ to zero for the moment and assume a flat ($k = 0$), homogeneous
and isotropic Universe. In comoving coordinates, the associated metric has the usual Friedmann-Robertson-Walker (FRW) line element
\[ ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2), \]
where \( a(t) \) denotes the cosmological scale factor. The above assumptions also imply that \( A \) and \( B \) can depend on \( t \) only. For a more realistic situation, the known matter content of the Universe needs to be modeled. An often employed approach is to include the energy-momentum tensor of dust, \( T_{\mu\nu} \).

If \( u^\mu \) is the unit timelike vector orthogonal to the spatial surfaces and \( \rho(t) \) is the energy density of the dust, it follows that \( T_{\mu\nu} = \rho u_\mu u_\nu \), as usual. In the present context, this type of matter arises from the fermionic sector of our supergravity model. Since the scalars \( A \) and \( B \) do not couple to the fermion kinetic terms, we will take \( T_{\mu\nu} \) as conserved separately.

In this model, the dependences of \( A \) and \( B \) on a parameter time defined by \( \tau = \sqrt{3/4 \text{arcoth}(\sqrt{3}c_n/4c_1 t + 1)} \) are given by
\[ A = \pm \lambda \tanh \left( \frac{1}{\tau} - \frac{1}{\tau_0} \right) + A_0, \quad B = \lambda \text{sech} \left( \frac{1}{\tau} - \frac{1}{\tau_0} \right), \]
where \( \sqrt{3c_n/4c_1}, \lambda, 1/\tau_0, \) and \( A_0 \) are integration constants. It can be verified that both \( A \) and \( B \) tend to constant values as \( t \to \infty \). It follows that in our supergravity cosmology the axion \( A \) and the dilaton \( B \) become fixed despite the absence of a dilaton potential. This essentially results from the conservation of energy.

3 Spacetime-varying couplings

We proceed by considering localized excitations of \( F_{\mu\nu} \) in the scalar background given by (4). Since experimental investigations are often performed in spacetime regions small on a cosmological scale, it is appropriate to work in local inertial frames.

In the presence of a nontrivial \( \theta \)-angle, the conventional electrodynamics lagrangian in inertial coordinates can be taken as
\[ \mathcal{L}_{\text{em}} = -\frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} - \frac{\theta}{16\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}, \]
where \( e \) is the electromagnetic coupling. Comparison with our supergravity model shows that \( e^2 \equiv 1/M \) and \( \theta \equiv 4\pi^2N \). Note that \( M \) and \( N \) are functions of the axion-dilaton background (4), so that \( e \) and \( \theta \) acquire related spacetime dependences in an arbitrary local inertial frame.
The spacetime dependence of both $\alpha = e^2/4\pi$ and $\theta$ can be relatively complicated and can vary qualitatively with the choice of model parameters. Figure (1) depicts relative variations of $\alpha$ in comoving coordinates for $1/\tau_0 = 0$. The fractional look-back time to the big bang is defined by $1 - t/t_n$, where $t_n$ denotes the present age of the Universe. The solid line corresponds to no time variation. Each broken line is associated with a set of nontrivial choices for $\lambda$, $\sqrt{3c_n/4c_1 t_n}$, and $A_0$. Parameter sets consistent with the Oklo constraints$^{22}$ are marked with an asterisk. Note the qualitative differences in the various plots, the nonlinear features, and the sign change for $\dot{\alpha}$ in the two cases with positive $A_0$. Also shown in Fig. (1) are the recent experimental results$^9$ obtained from measurements of high-redshift spectra over periods of approximately $0.6t_n$ to $0.8t_n$ assuming $H_0 = 65 \text{ km/s/Mpc}$ and $(\Omega_m, \Omega_\Lambda) = (0.3, 0.7)$.
4 Lorentz violation

In the presence of charged matter described by a 4-current $j^\nu$, the equations of motion for $F_{\mu\nu}$ become

$$\frac{1}{e^2} \partial_\mu F^{\mu\nu} - \frac{2}{e^2} (\partial_\mu e) F^{\mu\nu} + \frac{1}{4\pi^2} (\partial_\mu \theta) \tilde{F}^{\mu\nu} = j^\nu. \quad (6)$$

Note that in the limit of spacetime-independent $e$ and $\theta$, the usual inhomogeneous Maxwell equations are recovered. In the present context, however, the last two terms on the left-hand side of Eq. (6) lead to apparent Lorentz violation despite being coordinate invariant. This fact becomes particular transparent on small cosmological scales, where $\partial_\mu M$ and $\partial_\mu N$ are approximately constant, and hence, select a direction in the local inertial frame. As a consequence, particle Lorentz covariance is violated. It is important to note that this is not a feature of the particular coordinate system chosen. Once $\partial_\mu M$, for example, is nonzero in one local inertial frame associated with a small spacetime region, it is nonzero in all local inertial frames associated with that region.

By contrast, such Lorentz-violating effects are absent in conventional FRW cosmologies that fail to generate spacetime-dependent scalars. Although global Lorentz symmetry is usually broken, local Lorentz-symmetric inertial frames always exist. It is also important to note that the above mechanism for generating Lorentz-breaking effects is not a unique feature of our supergravity model. Equation (6) shows that any similarly implemented smooth spacetime dependence of $e$ and $\theta$ on cosmological scales can lead to such effects. This suggests that this type of apparent Lorentz breaking could be a common feature of models incorporating spacetime-varying couplings. We remark in passing that the present type of Lorentz violation differs conceptually from attempts to deform Lorentz symmetry.

An equivalent form of the electrodynamics lagrangian (5) in a local inertial frame can be obtained via an integration by parts:

$$L'_{em} = -\frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} - \frac{\partial_\mu \theta}{8\pi^2} A_\nu \tilde{F}^{\mu\nu}. \quad (7)$$

The the second term on right-hand side of Eq. (7) gives a Chern-Simons-type contribution to the action. Such gradient-photon couplings have been studied previously in various contexts. A Chern-Simons-type term is also contained in the SME, and one can identify $(k_{AF})_{\mu} \equiv e^2 \partial_\mu \theta/8\pi^2$. The presence of a nonzero $(k_{AF})_{\mu}$ in (7) shows explicitly Lorentz and CPT violation at the lagrangian level. The situation in which $e$ and $(k_{AF})_{\mu}$ are constant has been discussed extensively in the literature. In this limit, lagrangian (7)
becomes translationally invariant. However, the associated conserved energy fails to be positive definite, which usually leads to instabilities in the theory. The question arises how this problem is avoided in the present context of a positive-definite supergravity model.27

Although in most models a Chern-Simons-type term is assumed to arise in an underlying framework, its treatment at low energies usually involves a constant nondynamical \((k_{AF})_\mu\). In the present context, however, \((k_{AF})_\mu\) is associated with the dynamical degrees of freedom \(A\) and \(B\). Excitations of \(F_{\mu\nu}\) therefore lead to perturbations \(\delta A\) and \(\delta B\) in the axion-dilaton background (4). As a result, the energy-momentum tensor \((T^b)_{\mu\nu}\) of the background receives an additional contribution, \((T^b)_{\mu\nu} \to (T^b)_{\mu\nu} + \delta(T^b)_{\mu\nu}\). It can be demonstrated\(^{21}\) that this contribution does indeed compensate the negative-energy ones associated with a nonzero \((k_{AF})_\mu\).

5 Summary

Our analysis suggests that couplings varying on cosmological scales can be obtained as simple solutions of theories beyond the Standard Model and that such couplings may generically lead to local particle Lorentz violation. As an illustration, we have constructed a classical cosmological solution within the pure \(N = 4\) supergravity in four dimensions that exhibits spacetime-varying electromagnetic couplings \(\alpha\) and \(\theta\) and establishes the resulting Lorentz and CPT breaking. In this model, a Chern-Simons-type term is generated but the usual associated stability difficulties are circumvented.

1. P.A.M. Dirac, Nature (London) 139, 323 (1937).
2. J.K. Webb et al., Phys. Rev. Lett. 87, 091301 (2001).
3. See, e.g., E. Cremmer and J. Scherk, Nucl. Phys. B 118, 61 (1977); A. Chodos and S. Detweiler, Phys. Rev. D 21, 2167 (1980); W.J. Marciano, Phys. Rev. Lett. 52, 489 (1984); T. Damour and A.M. Polyakov, Nucl. Phys. B 423, 532 (1994).
4. For a review, see, e.g., J.-P. Uzan, hep-ph/0205340. For recent models, see, e.g., L. Anchordoqui and H. Goldberg, hep-ph/0306084; E.J. Copeland et al., hep-ph/0307299.
5. V.A. Kostelecký and R. Potting, Phys. Rev. D 51, 3923 (1995); D. Colladay and V.A. Kostelecký, Phys. Rev. D 55, 6760 (1997); 58, 116002 (1998).
6. V.A. Kostelecký and S. Samuel, Phys. Rev. D 39, 683 (1989); 40, 1886 (1989); Phys. Rev. Lett. 63, 224 (1989); 66, 1811 (1991); V.A. Kostelecký and R. Potting, Nucl. Phys. B 359, 545 (1991); Phys. Lett. B 381, 89 (1996); Phys. Rev. D 63, 046007 (2001); V.A. Kostelecký, M. Perry, and
7. F.R. Klinkhamer, Nucl. Phys. B 578, 277 (2000);
8. S.M. Carroll et al., Phys. Rev. Lett. 87, 141601 (2001); Z. Guralnik et al., Phys. Rev. D 65, 091701(R) (2002); A. Anisimov et al., hep-ph/0106356; C.E. Carlson et al., Phys. Lett. B 518, 201 (2001).
9. KTeV Collaboration, H. Nguyen, in V.A. Kostelecký, ed., CPT and Lorentz Symmetry II, World Scientific, Singapore, 2002; OPAL Collaboration, R. Ackerstaff et al., Z. Phys. C 76, 401 (1997); DELPHI Collaboration, M. Feindt et al., preprint DELPHI 97-98 CONF 80 (1997); BELLE Collaboration, K. Abe et al., Phys. Rev. Lett. 86, 3228 (2001); FOCUS Collaboration, J.M. Link et al., hep-ex/0208034.
10. D. Colladay and V.A. Kostelecký, Phys. Lett. B 344, 259 (1995); Phys. Rev. D 52, 6224 (1995); Phys. Lett. B 511, 209 (2001); V.A. Kostelecký and R. Van Kooten, Phys. Rev. D 54, 5585 (1996); V.A. Kostelecký, Phys. Rev. Lett. 80, 1818 (1998); Phys. Rev. D 61, 016002 (2000); 64, 076001 (2001); N. Isgur et al., Phys. Lett. B 515, 333 (2001).
11. L.R. Hunter et al., in V.A. Kostelecký, ed., CPT and Lorentz Symmetry, World Scientific, Singapore, 1999; D. Bear et al., Phys. Rev. Lett. 85, 5038 (2000); D.F. Phillips et al., Phys. Rev. D 63, 111101 (2001); M.A. Humphrey et al., Phys. Rev. A 62, 063405 (2000); V.A. Kostelecký and C.D. Lane, Phys. Rev. D 60, 116010 (1999); J. Math. Phys. 40, 6245 (1999); R. Bluhm et al., Phys. Rev. Lett. 88, 090801 (2002); hep-ph/0306190.
12. H. Dehmelt et al., Phys. Rev. Lett. 83, 4694 (1999); R. Mittleman et al., Phys. Rev. Lett. 83, 2116 (1999); G. Gabrielse et al., Phys. Rev. Lett. 82, 3198 (1999); R. Bluhm et al., Phys. Rev. Lett. 82, 2254 (1999); Phys. Rev. Lett. 79, 1432 (1997); Phys. Rev. D 57, 3932 (1998).
13. B. Heckel, in V.A. Kostelecký, ed., CPT and Lorentz Symmetry II, World Scientific, Singapore, 2002; R. Bluhm and V.A. Kostelecký, Phys. Rev. Lett. 84, 1381 (2000); E.O. Itan, JHEP 0306, 016 (2003); hep-ph/0308151.
14. S. Carroll, G. Field, and R. Jackiw, Phys. Rev. D 41, 1231 (1990).
15. V.A. Kostelecký and M. Mewes, Phys. Rev. Lett. 87, 251304 (2001); Phys. Rev. D 66, 056005 (2002); H. Müller et al., Phys. Rev. D 67, 056006 (2003).
16. J.A. Lipa et al., Phys. Rev. Lett. 90, 060403 (2003).
17. C. Adam and F.R. Klinkhamer, Nucl. Phys. B 657, 214 (2003); V.A. Kostelecký and A.G.M. Pickering, Phys. Rev. Lett. 91, 031801 (2003); A.P. Baeta Scarpelli et al., Phys. Rev. D 67, 085021 (2003); H. Belich
Jr. et al., Phys. Rev. D 68, 025005 (2003).
18. V.W. Hughes et al., Phys. Rev. Lett. 87, 111804 (2001); R. Bluhm et al., Phys. Rev. Lett. 84, 1098 (2000).
19. R. Lehnert, gr-qc/0304013, to appear in Phys. Rev. D; for other threshold investigations, see, e.g., T.J. Konopka and S.A. Major, New J. Phys. 4, 57 (2002); D. Mattingly, T. Jacobson, and S. Liberati, Phys. Rev. D 67, 124012 (2003).
20. E. Cremmer and B. Julia, Nucl. Phys. B 159, 141 (1979).
21. V.A. Kostelecký, R. Lehnert, and M.J. Perry, astro-ph/0212003.
22. T. Damour and F. Dyson, Nucl. Phys. B 480, 37 (1996); Y. Fujii et al., Nucl. Phys. B 573, 377 (2000); K. Olive et al., hep-ph/0205269.
23. G. Amelino-Camelia, Phys. Lett. B 510, 255 (2001); J. Magueijo and L. Smolin, Phys. Rev. Lett. 88, 190403 (2002); D.V. Ahluwalia-Khalilova, gr-qc/0212128; D. Grumiller et al., Ukr. J. Phys. 48, 329 (2003); R. Schützhold and W.G. Unruh, gr-qc/0308049.
24. See, e.g., D. Harari and P. Sikivie, Phys. Lett. B 289, 67 (1992); S.M. Carroll, Phys. Rev. Lett. 81, 3067 (1998).
25. M. Li and X. Zhang, hep-ph/0209093, to appear in Phys. Lett. B; see also O. Bertolami et al., Phys. Lett. B 395, 178 (1997).
26. R. Jackiw and V.A. Kostelecký, Phys. Rev. Lett. 82, 3572 (1999); C. Adam and F.R. Klinkhamer, Nucl. Phys. B 607, 247 (2001); and references therein.
27. A constant timelike \((k_{AF})_\mu\) also violates microcausality. The supergravity cosmology may avoid this, but a complete analysis of this lies outside our present scope.
28. V.A. Kostelecký and R. Lehnert, Phys. Rev. D 63, 065008 (2001); R. Lehnert, in V.A. Kostelecký, ed., \textit{CPT and Lorentz Symmetry II}, World Scientific, Singapore, 2002 (hep-th/0201238).