Open Spin Chains from Determinant Like Operators in ABJM Theory

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Abstract

We study the mixing problem of the determinant like operators in ABJM theory to two loop order in the scalar sector. The gravity duals of these operators are open strings attached to the maximal giant graviton, which is a D4-brane wrapping a $\mathbb{CP}^2$ inside $\mathbb{CP}^3$ in our case. The anomalous dimension matrix of these operators can be regarded as an open spin chain Hamiltonian. We provide strong evidence of its integrability based on coordinate Bethe ansatz method and boundary Yang-Baxter equation.

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1 Introduction

In recent years, a lot of progresses have been made in applying techniques of integrability to planar AdS$_5$/CFT$_4$ correspondence between IIB superstring theory on AdS$_5 \times S^5$ and four dimensional $\mathcal{N} = 4$ super Yang-Mills (SYM) theory, see [1] for a collection of reviews. Among all these notable progresses, spin chains or strings with periodic boundary condition are mostly studied and understood very well. People are also interested in non-periodic cases, including twisted boundary conditions, see for example [2,3] and open boundary conditions [4–8]. See [9,10] as reviews for these interesting topics.

In 2008, another example of AdS/CFT was proposed in [11], where the authors gave very strong evidence that type IIA string theory on AdS$_4 \times \mathbb{CP}^3$ background is dual to $\mathcal{N} = 6$ superconformal Chern-Simons matter theory (also known as Aharony-Bergman-Jafferis-Maldacena(ABJM) theory) in three dimensional spacetime with gauge group $U(N) \times U(N)$ and Chern-Simons levels $(k,-k)$. The ’t Hooft coupling of ABJM theory turns out to be $\lambda = N/k$. People usually call this dual as AdS$_4$/CFT$_3$ correspondence or ABJM/AdS$_4 \times \mathbb{CP}^3$ correspondence. Integrable structure in this setup was also extensively studied [12].

Along similar path, many studies on non-periodic integrable cases re-emerged in the context of ABJM theory [13–17]. However, there are still some potential integrable setups have not been investigated in the AdS$_4$/CFT$_3$ case, such as integrable Wilson loops [18–20] and integrability from giant gravitons [21,22] found in the $\mathcal{N} = 4$ SYM theory. In the SYM context, determinant like operators are dual to open strings attached to D-branes wrapping cycles in $S^5$. In the gravity side, such D-branes wrapping some cycles and carrying some angular momentum are usually called giant gravitons. In the context of $\mathcal{N} = 4$ SYM, integrability of open chain from giant gravitons have been studied extensively [8,21–25]. However such integrable structure from the giant gravitons in the AdS$_4$/CFT$_3$ [26,27] case has not been explored as far as we know, though the the plane wave limit in both sides are studied in [29]. In this paper, we would like to take a first step to fill these gaps. We study the anomalous dimension matrix of the determinant like scalar operators in ABJM theory up to two-loop order in the scalar sector. The anomalous dimension matrix can be viewed as the Hamiltonian of an open spin chain. Using the coordinate Bethe ansatz method, we calculate the reflection matrix for fundamental excitations of this open chain. Based on the known bulk two body S-matrix, it is not hard to verify that the boundary Yang-Baxter equations (reflection equations) are satisfied, hinting that this open spin chain is integrable.

The outline of this paper is as follows. In section 2, we introduce the determinant like scalar operators in ABJM theory. To study the mixing problem, we calculate their two point functions to two-loop order, giving the Hamiltonian of an open spin chain. In section 3, we compute the reflection matrix of this open spin chain through the coordinate Bethe ansatz method. Borrowing the two body S-matrix in the bulk from the previous result in [30], we confirm that the boundary Yang-Baxter equations (reflection equations) are satisfied. In the last section, we conclude and briefly discuss some possible problems for further studies.

2 Open spin chain in ABJM theory

2.1 Determinant like operators in ABJM theory

We begin with a very brief review of determinant like operators in ABJM theory. In ABJM theory, the scalar fields $(A_1, A_2, B_1^1, B_2^1)$ transform in the fundamental representation of the $SU(4)$ R-symmetry group. We make the following identification,

$$(A_1, A_2, B_1^1, B_2^1) = (Y_1, Y_2, Y_3, Y_4).$$ (2.1)
Using the conventions of [31], the action of ABJM theory can be written as

\[ S = \int d^3x (L_{CS} + L_k - V_F - V_B), \]

\[ L_{CS} = \frac{k}{4\pi} \mu \nu \rho \sigma \text{tr} \left( A_\mu \partial_\nu A_\rho + \frac{2i}{3} A_\mu A_\nu A_\rho - \hat{A}_\mu \partial_\nu \hat{A}_\rho - \frac{2i}{3} \hat{A}_\mu \hat{A}_\nu \hat{A}_\rho \right), \]

\[ L_k = \text{tr}(-D_\mu Y^I D^\mu Y_I + i\Psi^I \gamma^\mu D_\mu \Psi_I), \]

\[ V_F = \frac{2\pi i}{k} \text{tr} \left( Y_I^J Y^J Y_K^L + Y_I^J Y^L Y_K^J + Y_I^J Y^J Y_K^L + 4Y_I^J Y^J Y^L Y_K^L \right) \tag{2.2} \]

\[ V_B = -\frac{4\pi^2}{3k^2} \text{tr} \left( Y_I^J Y^J Y_K^L Y_L^I + Y_I^J Y^L Y_K^J Y_L^I + Y_I^J Y^J Y_K^L Y_L^J + 4Y_I^J Y^L Y_K^J Y_L^J \right). \]

Covariant derivatives are defined as

\[ D_\mu Y^I = \partial_\mu Y^I + iA_\mu Y^I - iY^I \hat{A}_\mu, \quad D_\mu Y^I = \partial_\mu Y^I + i\hat{A}_\mu Y^I - iY^I A_\mu. \tag{2.3} \]

In this paper we focus on the determinant like operators

\[ O_W = \epsilon_{a_1 \ldots a_N} \epsilon^{b_1 \ldots b_N} (A_1 B_1)^{a_1}_{b_1} \ldots (A_N B_N)^{a_N}_{b_N} W^{a_N}_{b_N}, \tag{2.4} \]

with

\[ W = Y_I^J Y^J Y_K^L Y_L^I. \tag{2.5} \]

It was suggested in [29] that the dual descriptions of these operators are open strings attached to the giant graviton D4-brane wrapping a CP^2 inside CP^3. The operator with \( W = A_1 B_1 \) is dual to the D4-brane itself.

As discussed in [8], open spin chain corresponding to determinant like operators in \( \mathcal{N} = 4 \) SYM has nontrivial boundary conditions. One may expect that there are similar boundary conditions in the case of open spin chain in ABJM theory. To show this, we compute the tree level two-point function. The operator \( O_W \) and its conjugate \( \bar{O}_W \) can be rewire as

\[ O_W = \frac{1}{(N-1)!} \epsilon_{[I]_{N-1} c} \epsilon_{[K]_{N-1} b} [A]^{J}_{N-1} B^{L}_{N-1} W^a_{b}, \tag{2.6} \]

\[ \bar{O}_W = \frac{1}{(N-1)!} \epsilon_{[J]_{N-1} f} \epsilon_{[Q]_{N-1} e} [P]^{J}_{N-1} \bar{B}^{S}_{N-1} \bar{W}_d. \]

Here we use the shorthand notations

\[ A = A_1, \quad B = B_1, \quad \hat{A} = A_1^I, \quad \hat{B} = B_1^I, \quad [I]_{N-1} = I_1 \ldots I_{N-1}; \quad A^{[I]_{N-1}}_J = A^J_{I_1} \ldots A^{I_{N-1}}_{J}, \tag{2.7} \]

In the 't Hooft limit of large \( N \) with a fixed ratio \( \lambda = N/k \), we need to distinguish two cases. When \( Y^I \neq A \) and \( Y^I_{JL} \neq B \), we get

\[ \langle O_W \bar{O}_W \rangle \sim \frac{1}{(N-1)!} \epsilon_{[I]_{N-1} c} \epsilon_{[K]_{N-1} b} [A]^{J}_{N-1} B^{L}_{N-1} \epsilon_{[J]_{N-1} f} \epsilon_{[Q]_{N-1} e} [P]^{J}_{N-1} \bar{B}^{S}_{N-1} \langle W^a_{b} \bar{W}_d \rangle \]

\[ = (N-1)!^2 N (\text{tr}(WW)) \]

\[ \sim (N-1)!^2 N^{2L+2}. \tag{2.8} \]
Here we have omitted the spacetime dependence explicitly because they can be easily put back at the end of the calculation. When \( Y_{L}^{I} = A \) or \( Y_{R}^{I} = B \) the operator factorizes \([32,33]\), so the combinatorics of contractions is different. For instance, when \( W = AV \) we have

\[
O_{W} = \det A e^{i[K]_{N-1b}^{-1} b^{-i[L]_{N-1c}^{-1}} A^{-1}} V_{b}^{c},
\]

and then

\[
\langle O_{W} \bar{O}_{W} \rangle \sim N! (N-1)^{4} N^{2L} = (N-1)^{4} N^{2L+1}.
\]

A similar analysis applies to the case when \( Y_{L}^{I} \neq B \). Therefore the mixing between factorizing operators and non-factorizing operators is suppressed in the large \( N \) limit. In this paper we only consider operators with \( Y_{L}^{I} \neq A \) and \( Y_{R}^{I} \neq B \).

### 2.2 Two-loop open spin-chain Hamiltonian

We now derive the two-loop anomalous dimension matrix for determinant like operators in the ’t Hooft limit. We need to consider the mixing of the two operators

\[
W = Y_{L}^{I} Y_{J}^{I} \ldots Y_{L}^{I} Y_{R}^{I}, \quad \bar{W} = Y_{M}^{I} Y_{N}^{I} \ldots Y_{M}^{I} Y_{N}^{I}
\]

where \( Y_{L}^{I} \neq A, Y_{N}^{I} \neq \bar{A}, Y_{L}^{I} \neq B \) and \( Y_{M}^{I} \neq \bar{B} \). Keeping one \( A \) and one \( B \) uncontracted with the corresponding \( \bar{A} \) and \( \bar{B} \), we get

\[
\langle O_{W} \bar{O}_{W} \rangle_{2\text{-loop}} \sim (N-1)^{2} \langle I \rangle_{N-2} \langle K \rangle_{N-2} \langle L \rangle_{N-2} \langle \ell \rangle_{N-2} \langle |L|_{N} \rangle \langle s \rangle_{N} \langle d \rangle_{N} \langle \ell \rangle_{N} \langle s \rangle_{N} \langle d \rangle_{N} \langle \ell \rangle_{N} \langle s \rangle_{N} \langle d \rangle_{N}
\]

Contraction of the generalized Kronecker deltas give

\[
\langle \delta_{ja}^{\ell} \delta_{pg}^{s} \delta_{ik}^{d} \delta_{sd}^{a} \delta_{bd}^{c} \rangle_{2\text{-loop}} = (N-2) \langle \text{tr}(W \bar{W}) \text{tr}(\bar{A}A) \text{tr}(\bar{B}B) - \text{tr}(\bar{W}W \bar{B}B) \text{tr}(A\bar{A}) - \text{tr}(A\bar{W}W \bar{B}B) + \text{tr}(W \bar{B}B \bar{W}A\bar{A}) \rangle_{2\text{-loop}}
\]

One can check that in the large \( N \) limit the first, second and third terms in the second line give bulk, right and left boundary contributions respectively, and the contributions from other terms are suppressed. For example, one part of the leading contribution from the second term corresponds to the contraction

\[
- (N-2) \langle \text{tr}(W \bar{W} \bar{B}B) \rangle_{\text{connected},2\text{-loop},\text{tr}(A\bar{A})} \sim \frac{N^{2L+6}}{k^{2}}.
\]

Note that the combination between \( A \) and \( \bar{A} \) gives a factor \( N^{2(N-1)^{-1}} \), here the factor \( (N-1)^{-1} \) is from avoiding repeatedly counting of contractions. The Hamiltonian of the bulk part the open chain is the same as that of the closed spin chain which was derived in \([34,35]\). We need to consider the boundary contributions. We first focus on the left boundary corresponding to the term

\[
\langle \text{tr}(A\bar{A}W \bar{W}) \rangle_{2\text{-loop}} \rightarrow \langle \text{tr}(A\bar{A}Y_{L}^{I} Y_{R}^{I} Y_{M}^{I} Y_{N}^{I}) \rangle_{2\text{-loop}}.
\]

Contributions from wave function renormalization (self-interactions) are proportional to \( \delta_{N}^{I} \) and thus flavor blind. Because \( Y_{L}^{I} \neq A \) and \( Y_{N}^{I} \neq \bar{A} \), contributions from gluon exchange and fermion exchange are also flavor blind. We only need to consider contribution from sextet scalar potential \( V_{B} \). Then we get

\footnote{This can be checked at two-loop order by a simple large \( N \) counting.}
\[ H_{\text{left}} = \frac{\lambda^2}{2} \left( \frac{1}{2} \delta^{N_l}_{j_l} + 2 \delta^{M_l}_{j_l} \delta^{N_1}_{j_1} - \delta^{M_1}_{N_1} \delta^{N_l}_{j_1} + C \delta^{N_1}_{j_l} \delta^{M_1}_{j_1} \right). \] (2.16)

Here the normalization is fixed by comparing with bulk Hamiltonian from sextet scalar potential. The constant \( C \) comes from the contributions from gluon exchange, fermion exchange and self-interactions. An analogous discussion applies to the right boundary. We will show in Appendix A that the anomalous dimension of the operator with \( W = (A_2 B_2)^L \) is zero in the large \( N \) limit, which allows us to determine the sum of the constant \( C \) and a similar constant from the right boundary. At the end the total Hamiltonian is given by

\[
\begin{align*}
H &= \lambda^2 \sum_{l=2}^{2L-3} \left( \mathbb{I} + \frac{1}{2} \mathbb{P}_{l,l+2} \mathbb{K}_{l,l+1} + \frac{1}{2} \mathbb{P}_{l,l+2} \mathbb{K}_{l+1,l+2} \right) Q^A_1 Q^B_1 \\
&\quad + \lambda^2 Q^A_1 \left( \mathbb{I} + \frac{1}{2} \mathbb{K}_{1,2} - \mathbb{P}_{1,3} + \frac{1}{2} \mathbb{P}_{1,3} \mathbb{K}_{1,2} + \frac{1}{2} \mathbb{P}_{1,3} \mathbb{K}_{2,3} \right) Q^B_2 \\
&\quad + \lambda^2 Q^B_2 \left( \mathbb{I} + \frac{1}{2} \mathbb{K}_{2L-1,2L} - \mathbb{P}_{2L-2L,2L} + \frac{1}{2} \mathbb{P}_{2L-2L,2L} \mathbb{K}_{2L-2L,2L} + \frac{1}{2} \mathbb{P}_{2L-2L,2L} \mathbb{K}_{2L-2L,2L} \right) \\
&\quad + \lambda^2 (\mathbb{I} - Q^A_2) Q^A_1 Q^B_2 + \lambda^2 (\mathbb{I} - Q^B_2) Q^A_1 Q^B_2
\end{align*}
\] (2.17)

where the trace operator \( \mathbb{K} \) and permutation operator \( \mathbb{P} \) are defined as

\[
(\mathbb{K}_{ij})_{j,i, j} = \delta_{i,j} \delta_{j,i}, \quad (\mathbb{P}_{ij})_{j,i, j} = \delta_{i,j} \delta_{j,i},
\] (2.18)

and the \( Q \) operators are defined as

\[
Q^\phi |\phi\rangle = 0, \quad Q^\phi |\psi\rangle = |\psi\rangle, \quad \text{for} \ \psi \neq \phi.
\] (2.19)

Half of the \( \frac{1}{2} \mathbb{K}_{1,2} (\frac{1}{2} \mathbb{K}_{2L-1,2L}) \) term in (2.17) comes from the third (second) term in (2.13), and another half comes from the first term in (2.13).

### 3 Integrability from coordinate Bethe ansatz

In this section we discuss the integrability of the above open spin chain in the framework of coordinate Bethe ansatz. The reflection equations are necessary conditions for the integrability of the open spin chain Hamiltonian. We want to know whether the boundary reflection matrices satisfy the reflection equations or not.

The vacuum of this open chain is chosen to be

\[
W = (A_2 B_2) \cdots (A_2 B_2).
\] (3.1)

The one-particle excitations include

- bulk odd site \((A_2 B_2) \cdots (A_1 B_2) \cdots (A_2 B_2)\)
- bulk even site \((A_2 B_2) \cdots (B_1 A_2) \cdots (A_2 B_2)\)
- left boundary \((B_1^\dagger B_2) \cdots (A_2 B_2)\)
- right boundary \((A_2 B_2) \cdots (A_2 A_1^\dagger)\)

(3.2)\), (3.3)\), (3.4)\), (3.5)\), (3.6)\), (3.7)\)
We denote the open chain as $(1)(2) \cdots (x) \cdots (L)$ with every site $(x)$ containing two fields. Then the above excitations can be simply denoted as

$$
| x \rangle_{A_1}, 2 \leq x \leq L,
| x \rangle_{B_1}, 1 \leq x \leq L,
| x \rangle_{B_1}, 1 \leq x \leq L - 1,
| x \rangle_{A_1}, 1 \leq x \leq L,
$$

(3.8)

where $| 1 \rangle_{B_1}$ is the left boundary excitation state and $| L \rangle_{A_1}$ is the right boundary excitation state while all others are bulk one-particle excitation state.

Let us begin with

$$
| k \rangle_{B_1} = \sum_{x=1}^{L} f_{B_1}(x)| x \rangle_{B_1},
$$

(3.9)

where

$$
f_{B_1}(x) = F_{B_1} e^{ikx} + \tilde{F}_{B_1} e^{-ikx}.
$$

(3.10)

On the states $| x \rangle_{B_1}$, the Hamiltonian acts as follows

$$
H| x \rangle_{B_1} = \lambda^2 (2| x \rangle_{B_1} - | x + 1 \rangle_{B_1} - | x - 1 \rangle_{B_1}),
$$

(3.11)

when $2 \leq x \leq L - 1$, and

$$
H| 1 \rangle_{B_1} = \lambda^2 (| 1 \rangle_{B_1} - | 2 \rangle_{B_1}),
$$

(3.12)

$$
H| L \rangle_{B_1} = \lambda^2 (2| L \rangle_{B_1} - | L - 1 \rangle_{B_1}).
$$

(3.13)

So we get

$$
H| k \rangle_{B_1} = \lambda^2 \sum_{x=2}^{L-2} (2f_{B_1}(x) - f_{B_1}(x - 1) - f_{B_1}(x + 1))| x \rangle_{B_1}
+ \lambda^2 (f_{B_1}(1) - f_{B_1}(2))| 1 \rangle_{B_1} + \lambda^2 (2f_{B_1}(L) - f_{B_1}(L - 1))| L \rangle_{B_1}.
$$

(3.14)

Then equation

$$
H| k \rangle_{B_1} = E(k)| k \rangle_{B_1},
$$

(3.15)

leads to the following dispersion relation

$$
E(k) = \lambda^2 (2 - 2 \cos k),
$$

(3.16)

and

$$
f_{B_1}(1) = f_{B_1}(0),
$$

(3.17)

$$
f_{B_1}(L + 1) = 0.
$$

(3.18)

Since the reflections of $B_1$ excitation at both sides are diagonal, we define the left reflection coefficient to be

$$
K_{L,B_1} = F_{B_1}/\tilde{F}_{B_1},
$$

(3.19)

and the right reflection coefficient to be

$$
K_{R,B_1} = e^{2ik(L-1)} F_{B_1}/\tilde{F}_{B_1}.
$$

(3.20)

\footnote{We have taken into account that for every excitation, there are $L - 1$ bulk sites.}
They are determined by eqs. (3.17) and (3.18), respectively. The results are

\[ K_{L,B_1} = e^{-ik}, \]
\[ K_{R,B_1} = -e^{-4ik}. \]  

For the other three excitations, the computations are similar. So we only list the action of the Hamiltonian, obtained boundary conditions and reflection coefficients. For \(|x\rangle_{A_1}, 2 \leq x \leq L\) we have

\[ H|\langle x |_{A_1} = \lambda^2(2|x\rangle_{A_1} - |x+1\rangle_{A_1} - |x-1\rangle_{A_1}), \quad 3 \leq x \leq L - 1 \]  
\[ H|\langle 2 |_{A_1} = \lambda^2(2|2\rangle_{A_1} - |3\rangle_{A_1}), \]  
\[ H|\langle L |_{A_1} = \lambda^2(|L\rangle_{A_1} - |L-1\rangle_{A_1}). \]

This gives

\[ f_{A_1}(1) = 0, f_{A_1}(L + 1) = f_{A_1}(L), \]  
which leads to

\[ K_{L,A_1} = -e^{-2ik}, K_{R,A_1} = e^{-3ik}. \]  

For \(|x\rangle_{B_1}, 1 \leq x \leq L - 1\), we have

\[ H|\langle x |_{B_1} = \lambda^2(2|x\rangle_{B_1} - |x+1\rangle_{B_1} - |x-1\rangle_{B_1}), \quad 2 \leq x \leq L - 2 \]  
\[ H|\langle 1 |_{B_1} = \lambda^2(|1\rangle_{B_1} - |2\rangle_{B_1}), \]  
\[ H|\langle L-1 |_{B_1} = \lambda^2(2|L-1\rangle_{B_1} - |L\rangle_{B_1}). \]

this leads to

\[ f_{B_1}(1) = f_{B_1}(0), f_{B_1}(L) = 0, \]
then

\[ K_{L,B_1} = e^{-ik}, K_{R,B_1} = -e^{-2ik}. \]  

Finally for \(|x\rangle_{A_1}, 1 \leq x \leq L\), we have

\[ H|\langle x |_{A_1} = \lambda^2(2|x\rangle_{A_1} - |x+1\rangle_{A_1} - |x-1\rangle_{A_1}), \quad 2 \leq x \leq L - 1 \]  
\[ H|\langle 1 |_{A_1} = \lambda^2(2|1\rangle_{A_1} - |2\rangle_{A_1}), \]  
\[ H|\langle L |_{A_1} = \lambda^2(|L\rangle_{A_1} - |L-1\rangle_{A_1}). \]

This gives

\[ f_{A_1}(0) = 0, f_{A_1}(L) = f_{A_1}(L + 1), \]  
and

\[ K_{L,A_1} = -1, K_{R,A_1} = e^{-3ik}. \]  

With the order of the excitations as \(A_1, B_1^\dagger, A_1^\dagger, B_1\), the left reflection matrix is

\[ K_L = \begin{pmatrix} -e^{-2ik} & e^{-ik} \\ e^{-ik} & -1 \end{pmatrix}, \]
and the right reflection matrix is

\[ K_R = \begin{pmatrix} e^{-3ik} & -e^{-4ik} \\ -e^{-3ik} & e^{-2ik} \end{pmatrix}. \]
The two reflection matrices are diagonal in the chosen natural basis. This is quite different from the results in [17], where the reflection matrices are anti-diagonal in the same basis. Also notice that each excitation always has Dirichlet boundary condition on one end of the open chain, and Neumann boundary condition on the other end. This is different from the SYM case [33,34] where the boundary conditions are always left-right symmetric. The S-matrix in ABJM theory can be found in [30]. It satisfies the Yang-Baxter equation

\[ S_{12}(k_1, k_2)S_{13}(k_1, k_3)S_{23}(k_2, k_3) = S_{23}(k_2, k_3)S_{13}(k_1, k_3)S_{12}(k_1, k_2). \] (3.40)

Now we are ready to check the reflection equations. It can be straightforward to verify that reflection equations are satisfied

\[ K_{L2}(k_2)S_{12}(k_1, -k_2)K_{L1}(k_1)S_{21}(-k_2, -k_1) = S_{12}(k_1, k_2)K_{L1}(k_1)S_{21}(k_2, k_1)K_{L2}(k_2), \] (3.41)

\[ K_{R2}(-k_2)S_{21}(k_2, -k_1)K_{R1}(-k_1)S_{12}(k_1, k_2) = S_{21}(-k_2, -k_1)K_{R2}(-k_1)S_{12}(k_1, -k_2)K_{R2}(-k_2). \] (3.42)

The \( \frac{1}{2}K_{1,2} \) and \( \frac{1}{2}K_{2L-1,2L} \) terms in the Hamiltonian (2.17) have no effect in the above calculation. To understand their role in the coordinate Bethe ansatz, one needs to consider impurities \( \phi \phi^\dagger \). Although not shown here, we have checked that the \( \frac{1}{2}K_{1,2} \) and \( \frac{1}{2}K_{2L-1,2L} \) terms in the Hamiltonian are necessary in the construction of the eigenstates involving \( \phi \phi^\dagger \) scattering and the above bound states using coordinate Bethe ansatz.

### 4 Conclusions and discussions

We have obtained the two-loop Hamiltonian of the open spin chain corresponding to the determinant like operators in ABJM theory which are dual to open strings attached to D4-branes wrapping cycles in \( \mathbb{CP}^3 \). The Hamiltonian is different from the periodic spin chain only in the boundary terms. Using the coordinate Bethe ansatz, we present strong evidence that the Hamiltonian may be integrable. In other words, the giant graviton may provide integrable boundary conditions for the open string. It is possible to go beyond the two loop order to an all loop prediction which is similar to previous studies in the SYM context [21,23] using symmetries as the guide, and could even further to solve the full open string spectrum through boundary thermodynamical Bethe ansatz and/or Y-system which have already been done in the SYM case [24,25]. To have a more solid ground for integrability of our two loop Hamiltonian, it would be better to have an algebraic Bethe ansatz construction [37] as people have done in the SYM theory [38].

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### A Vacuum of the open chain

In this appendix, we show that the anomalous dimension of the operator

\[ O_0 = \epsilon_{a_1...a_N}b_1^{b_1}...b_N^{b_N}(A_1B_1)^{a_1}_{b_1}...(A_1B_1)^{a_{N-1}}_{b_{N-1}}((A_2B_2)^L)^{a_N}_{b_N} \] (A.1)

\footnote{A non-supersymmetric flavored ABJM theory was constructed in [30], where the corresponding reflection matrices are diagonal.}
is suppressed in the large \(N\) limit with \(\lambda = N/k\) fixed. As discussed in \[39\], at two-loop order the contribution from bosonic D-terms, gluon exchange, fermion exchange from fermionic D-terms and self-interactions cancel for operators in the \(SU(2) \times SU(2)\) sector, and the fermionic F-terms do not contribute to the anomalous dimension. We only need to consider the contributions from bosonic F-terms \[31\]

\[
V_{bos}^{\text{F}} = -\frac{16\pi^2}{k^2} \text{tr}(A_i^{\dagger}B_j^{\dagger}A_k^{\dagger}A_iB_jA_k - A_i^{\dagger}B_j^{\dagger}A_k^{\dagger}A_iB_jA_k + B_i^{\dagger}A_j^{\dagger}B_k^{\dagger}A_kB_i - B_i^{\dagger}A_j^{\dagger}B_k^{\dagger}B_kA_iB_i).
\]

(A.2)

Using (2.13) one can check that the anomalous dimension of \(O_0\) is subleading in \(1/N\).

References

[1] N. Beisert et al., Review of AdS/CFT Integrability: An Overview, Lett. Math. Phys. 99 (2012) 3–32, [1012.3982].

[2] N. Beisert and R. Roiban, Beauty and the twist: The Bethe ansatz for twisted \(N = 4\) SYM, JHEP 08 (2005) 039, [hep-th/0505187].

[3] N. Beisert and R. Roiban, The Bethe ansatz for \(Z_S\) orbifolds of \(N = 4\) super Yang-Mills theory, JHEP 11 (2005) 037, [hep-th/0510209].

[4] B. Chen, X. J. Wang and Y. S. Wu, Integrable open spin chain in super Yang-Mills and the plane wave / SYM duality, JHEP 02 (2004) 029, [hep-th/0401016].

[5] B. Chen, X. J. Wang and Y. S. Wu, Open spin chain and open spinning string, Phys. Lett. B 591 (2004) 170–180, [hep-th/0403004].

[6] O. DeWolfe and N. Mann, Integrable Open Spin Chains in Defect Conformal Field Theory, JHEP 04 (2004) 035, [hep-th/0401041].

[7] T. Erler and N. Mann, Integrable open spin chains and the doubling trick in \(N = 2\) SYM with fundamental matter, JHEP 01 (2006) 131, [hep-th/0508064].

[8] D. Berenstein and S. E. Vazquez, Integrable open spin chains from giant gravitons, JHEP 06 (2005) 059, [hep-th/0501078].

[9] K. Zoubos, Review of AdS/CFT Integrability, Chapter IV.2: Deformations, Orbifolds and Open Boundaries, Lett. Math. Phys. 99 (2012) 375–400, [1012.3998].

[10] S. J. van Tongeren, Integrability of the \(AdS_5 \times S^5\) superstring and its deformations, J. Phys. A47 (2014) 433001, [1310.4854].

[11] O. Aharony, O. Bergman, D. L. Jafferis and J. Maldacena, \(N = 6\) superconformal Chern-Simons-matter theories, M2-branes and their gravity duals, JHEP 10 (2008) 091, [0806.1218].

[12] T. Klose, Review of AdS/CFT Integrability, Chapter IV.3: \(N = 6\) Chern-Simons and Strings on \(AdS_4 \times CP^3\), Lett. Math. Phys. 99 (2012) 401–423, [1012.3999].

[13] S. He and J.-B. Wu, Note on Integrability of Marginally Deformed ABJ(M) Theories, JHEP 04 (2013) 012, [1302.2208].

[14] H.-H. Chen, P. Liu and J.-B. Wu, Y-system for \(\gamma\)-deformed ABJM theory, JHEP03(2017) 133, [1611.02804].
[15] H. H. Chen and J. B. Wu, Finite-size Effect for Dyonic Giant Magnons in CP$^3_{\beta}$, Phys. Lett. B769, (2017) 90–99, 1612.04513.

[16] N. Bai, H. H. Chen, X. C. Ding, D. S. Li and J. B. Wu, Integrability of Orbifold ABJM Theories, JHEP11 (2016) 101, 1607.06543.

[17] N. Bai, H.-H. Chen, S. He, J.-B. Wu, W.-L. Yang and M.-Q. Zhu, Integrable Open Spin Chains from Flavored ABJM Theory, JHEP 08 (2017) 001, 1704.05807.

[18] N. Drukker and S. Kawamoto, Small deformations of supersymmetric Wilson loops and open spin-chains, JHEP07 (2006) 024, hep-th/0604124.

[19] N. Drukker, Integrable Wilson loops, JHEP 10 (2013) 135, 1203.1617.

[20] D. Correa, J. Maldacena and A. Sever, The quark anti-quark potential and the cusp anomalous dimension from a TBA equation JHEP 08 (2012) 134, 1203.1913.

[21] D. M. Hofman and J. M. Maldacena, Reflecting magnons, JHEP 11 (2007) 063, 0708.2272.

[22] D. Berenstein, D. H. Correa and S. E. Vazquez, Quantizing open spin chains with variable length: An Example from giant gravitons, Phys. Rev. Lett. 95 (2005) 191601, hep-th/0502172.

[23] C. Ahn and R. I. Nepomechie, The Zamolodchikov-Faddeev algebra for open strings attached to giant gravitons, JHEP 05 (2008) 059, 0804.4036.

[24] Z. Bajnok, R. I. Nepomechie, L. Palla and R. Suzuki, Y-system for Y = 0 brane in planar AdS/CFT, JHEP 08 (2012) 149, 1205.2060.

[25] Z. Bajnok, N. Drukker, A. Hegeds, R. I. Nepomechie, L. Palla, C. Sieg et al., The spectrum of tachyons in AdS/CFT, JHEP 03 (2014) 055, 1312.3900.

[26] D. Berenstein and D. Trancanelli, Three-dimensional N = 6 SCFT’s and their membrane dynamics, Phys. Rev. D78 (2008) 106009, 0808.2503.

[27] D. Giovannoni, J. Murugan and A. Prinsloo, The Giant graviton on AdS$_4$ x CP$^3$ - another step towards the emergence of geometry, JHEP 12 (2011) 003, 1108.3084.

[28] Y. Lozano, J. Murugan and A. Prinsloo, A giant graviton genealogy, JHEP 08 (2013) 109, 1305.6932.

[29] C. Cardona and H. Nastase, Open strings on D–branes from ABJM, JHEP 06 (2015) 016, 1407.1764.

[30] C. Ahn and R. I. Nepomechie, Two-loop test of the N = 6 Chern-Simons theory S-matrix, JHEP 03 (2009) 144, 0901.3334.

[31] M. Benna, I. Klebanov, T. Klose and M. Smedback, Superconformal Chern-Simons Theories and AdS(4)/CFT(3) Correspondence, JHEP 09 (2008) 072, 0806.1519.

[32] D. Berenstein, Shape and holography: Studies of dual operators to giant gravitons, Nucl. Phys. B675 (2003) 179-204, hep-th/0306090.

[33] V. Balasubramanian, D. Berenstein, B. Feng and M.-x. Huang, D-branes in Yang-Mills theory and emergent gauge symmetry, JHEP 03 (2005) 006, hep-th/0411205.

[34] J. A. Minahan and K. Zarembo, The Bethe ansatz for superconformal Chern-Simons, JHEP 09 (2008) 040, 0806.3951.
[35] D. Bak and S.-J. Rey, *Integrable Spin Chain in Superconformal Chern-Simons Theory*, JHEP **10** (2008) 053, [0807.2063].

[36] J. B. Wu, *Notes on Integrable Boundary Interactions of Open SU(4) Alternating Spin Chains*, Sci. China Phys. Mech. Astron. **61** (2018) 070011, [1711.04087].

[37] N. Bai, H.-H. Chen, H. Ouyang, and J.-B. Wu, *work in progress*.

[38] R. I. Nepomechie, *Revisiting the Y=0 open spin chain at one loop*, JHEP **11** (2011) 069, [1109.4366].

[39] C. Kristjansen, M. Orselli and K. Zoubos, *Non-planar ABJM Theory and Integrability*, JHEP **03** (2009) 037, [0811.2150].