Tau lifetime and decays

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Recent results of a high-statistics study of $\tau$ lepton properties and decays at $B$ factories are reviewed. We discuss measurements of $\tau$ lifetime, branching fractions, and spectral functions for several hadronic $\tau$ decay modes with $K_S^0$. Results of a search for lepton flavor violating $\tau$ decays as well as CP symmetry violation are briefly discussed.

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1 Introduction

The world largest statistics of $\tau$ leptons collected at $e^+e^-$ $B$ factories (Belle \cite{1, 2} and BABAR \cite{3, 4}) and LHCb \cite{5} opens new era in the precision tests of the Standard Model (SM). Basic tau properties like lifetime, mass, couplings, electric dipole moment (EDM), anomalous magnetic dipole moment and other are introduced as free parameters in the theory, or they can be calculated in the SM. Hence the former parameters should be measured experimentally as precise as possible, while the latter ones provide the unique possibility to test SM and search for the effects of New Physics (NP). An essential progress has been made in the study of the main $\tau$ properties at Belle and BABAR, namely lifetime \cite{6, 7}, mass \cite{8, 9}, EDM \cite{10}, coupling constant ratios \cite{11} have been measured with the best or competitive to the world best accuracies \cite{12}.

In the SM $\tau$ decays due to the charged weak interaction described by the exchange of $W^\pm$ with a pure vector coupling to only left-handed chirality fermions. There are two main classes of tau decays: leptonic decays $^\ast$ ($\tau^- \rightarrow \ell^- \bar{\nu}_\ell \nu_\tau$, $\tau^- \rightarrow \ell^- \bar{\nu}_\ell \nu_\tau \gamma$, $\tau^- \rightarrow \ell^- \ell'^- \bar{\nu}_\ell \nu_{\ell'}$; $\ell, \ell' = e, \mu$), and hadronic decays. Leptonic decays provide very clean laboratory to probe electroweak couplings \cite{13}, which is complementary or competitive to the precision studies of muon in the experiments with muon beams \cite{14}. Plenty of NP models can be tested and constrained in the precision studies of the dynamics of $\tau$ decays with leptons \cite{15-30}.

Hadronic decays of $\tau$ offer unique tools for the precision study of low energy QCD \cite{31}. The hadronic system is produced from the QCD vacuum via decay of the $W^-\bar{\nu}_\tau$ boson into $\bar{u}$ and $d$ quarks (Cabibbo-allowed decays) or $\bar{u}$ and $s$ quarks (Cabibbo-suppressed decays). As a result the decay amplitude can be factorized into a purely leptonic part including the $\tau$ and $\nu_\tau$ and a hadronic spectral function. Various decay modes are interesting to study precisely the structure of the hadronic spectral functions \cite{32} and measure precisely parameters of the intermediate states, testing the Wess-Zumino-Witten anomaly \cite{33}, chiral theory \cite{34, 35}, and relations to $e^+e^-$ cross sections following from the conservation of the vector current \cite{36}. Measurement of the inclusive hadronic spectral function of Cabibbo-allowed decays is important for the precision determination of $\alpha_s$ \cite{37}, while the inclusive strange hadronic spectral function is used to evaluate $s$-quark mass and $V_{us}$ element of Cabibbo-Kobayashi-Maskawa (CKM) quark flavor-mixing matrix \cite{38}. Recently lots of important results in the hadronic sector of $\tau$ physics have been obtained at Belle and BABAR \cite{39}.

In the leptonic sector CP symmetry violation (CPV) is strongly suppressed in the SM ($A_{SM}^{CP} \lesssim 10^{-12}$) leaving enough room to search for the effects of NP \cite{40}. Of particular interest are strangeness changing Cabibbo-suppressed hadronic $\tau$ decays, in which large CPV could appear from a charged scalar boson exchange \cite{41-45}.

Probability of lepton flavor violating (LFV) decays of charged leptons is extremely

$^*\text{Unless specified otherwise, charge conjugate decays are implied throughout the paper.}$
small in the Standard Model (for example $B(\tau \to \ell \nu) \sim \Delta m^2 / m^2_{W} < 10^{-54}$ [46].

Many models beyond the SM predict LFV decays with the branching fractions up to $\lesssim 10^{-7}$ [47-51]. As a result, observation of LFV is a clear signature of New Physics. $\tau$ lepton is an excellent laboratory to search for the LFV decays due to the enhanced couplings to the new particles as well as large number of LFV decay modes. Study of different $\tau$ LFV decay modes allows one to test various NP models. Huge statistics collected by Belle and BABAR was used in the searches for 48 LFV $\tau$ decays, the upper limits on the branching fractions for the most of LFV modes approach $10^{-8}$ level, which allows one to constrain the parameter spaces of many NP models [39]. Recently LHCb collaboration performed results of the search for LFV $\tau^- \to \mu^- \mu^+ \mu^-$, lepton number (LNV) and barion number violating (BNV) $\tau$ decays with proton $\tau^- \to p\mu^- \mu^-$, $\bar{p}\mu^+ \mu^-$ at the Large Hadron Collider [52].

## 2 Measurement of $\tau$ lifetime at Belle

### 2.1 Tests of lepton universality

Lepton universality in the charged lepton sector of the SM is the fundamental assumption about lepton flavor-independent structure of the charged weak interaction. It is introduced in the theory as an equality of the coupling constants for $e^-$, $\mu^-$ and $\tau^-$: $g_e = g_\mu = g_\tau$. This universality can be experimentally tested by comparing the rates of the leptonic decays: $\tau^- \to e^- \bar{\nu}_e \nu_\tau$, $\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau$ and $\mu^- \to e^- \bar{\nu}_e \nu_\mu$. The total decay width with electroweak radiative corrections of lepton $L^{-}$ ($L = \mu$, $\tau$) reads [53]:

$$
\Gamma(L^- \to \ell^- \bar{\nu}_\ell \nu_L(\gamma)) = B(L^- \to \ell^- \bar{\nu}_\ell \nu_L(\gamma)) = \frac{B(L^- \to \ell^- \bar{\nu}_\ell \nu_L(\gamma))}{\tau_L} = \frac{g_L^2 g_\ell^2}{32 M_W^4} \frac{m_L^5}{192 \pi^3} F_{\text{corr}}(m_L, m_\ell),
$$

(1)

$$
F_{\text{corr}}(m_L, m_\ell) = f(x) \left( 1 + \frac{3}{5} \frac{m_L^2}{M_W^2} \right) \left( 1 + \frac{\alpha(m_L)}{2\pi} \frac{25}{4} \frac{1}{\pi^2} \right),
$$

(2)

$$
f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x, \ x = m_\ell / m_L,
$$

(3)

where $m_L (m_\ell)$ and $g_L (g_\ell)$ are mass and coupling constant of initial (final) lepton, $\tau_L$ is lifetime of initial lepton, $M_W$ and $\alpha(m_L)$ are $W^-$ boson mass and fine-structure constant at the energy scale of $m_L$. Taking into account that [12]:

$$
B(\mu^- \to e^- \bar{\nu}_e \nu_\mu(\gamma)) = B(\mu^- \to e^- \bar{\nu}_e \nu_\mu) +
$$

$$+ B(\mu^- \to e^- \bar{\nu}_e \nu_\mu(\gamma)) = B(\mu^- \to e^- \bar{\nu}_e \nu_\mu(\gamma)) = B(\mu^- \to e^- \bar{\nu}_e \nu_\mu e^+ e^-) = 1,
$$

(4)

the $g_\tau / g_e$ and $g_\tau / g_\mu$ ratios of the coupling constants can be extracted:

$$
\frac{g_\tau}{g_e} = \sqrt{B(\tau^- \to \mu^- \bar{\nu}_\mu(\gamma)) \frac{\tau_\mu m_\mu^3}{\tau_\tau m_\tau^3} F_{\text{corr}}(m_\mu, m_\ell) \frac{m_\mu^5}{m_\tau^5} F_{\text{corr}}(m_\tau, m_\mu)},
$$

(5)
\[
g_{\tau} = \sqrt{\mathcal{B}(\tau^{-} \rightarrow e^{-}\bar{\nu}_{\mu}\nu_{\tau}(\gamma)) \frac{\tau_{\mu} m_{\mu}^{5} F_{\text{cor}}(m_{\mu}, m_{e})}{\tau_{\tau} m_{\tau}^{5} F_{\text{cor}}(m_{\tau}, m_{e})}}. \tag{6}
\]

As it is seen from Eq. 5 and 6 precise measurement of the branching fraction of leptonic \(\tau\) decay, \(\tau\) mass, and \(\tau\) lifetime are necessary for the accurate tests of lepton universality. According to the last report from Heavy Flavor Averaging Group (HFAG) \[54\] lepton universality was confirmed in the \(g_{\tau}/g_{e}\) and \(g_{\tau}/g_{\mu}\) ratios with the accuracy of about 0.2%:

\[
g_{\tau}/g_{e} = 1.0024 \pm 0.0021, \quad g_{\tau}/g_{\mu} = 1.0006 \pm 0.0021
\]

However, recently LEP Electroweak Collaboration published results on the test of lepton universality in \(W\)-boson decays \[55\], and the ratio of the branching fraction of \(W^{-}\)-boson decay to \(\tau^{-}\bar{\nu}_{\tau}\) to the average branching fraction of \(W^{-}\)-boson decay to \(\mu^{-}\bar{\nu}_{\mu}\) and \(e^{-}\bar{\nu}_{e}\) was found to be 2.6 standard deviations away from unity:

\[
\frac{2\mathcal{B}(W^{-} \rightarrow \tau^{-}\bar{\nu}_{\tau})}{\mathcal{B}(W^{-} \rightarrow \mu^{-}\bar{\nu}_{\mu}) + \mathcal{B}(W^{-} \rightarrow e^{-}\bar{\nu}_{e})} = 1.066 \pm 0.025.
\]

So, the improvement of the accuracy of the test of lepton universality with leptonic decays of \(\tau\) is still rather actual task. The reduction of the uncertainty of tau lifetime to the negligible level will allow one to test lepton universality in \(g_{\tau}/g_{e}\) and \(g_{\tau}/g_{\mu}\) ratios with the accuracy of about 0.1% even with the current values of the \(\tau\) mass and branching fraction uncertainties.

### 2.2 Measurement of \(\tau\) lifetime at Belle

This analysis \[6\] is based on the statistics with the luminosity integral of \(\int Ldt = 711 \text{ fb}^{-1}\) collected with the Belle detector at the KEKB asymmetric-energy \(e^{+}e^{-}\) collider operating at the \(\Upsilon(4S)\) resonance and 60 MeV below. The data sample comprises \(653 \times 10^{6} \tau^{+}\tau^{-}\) pairs. Events where both taus decay to three charged pions and neutrino were selected: \(e^{+}e^{-} \rightarrow \tau^{+}\tau^{-} \rightarrow (\pi^{+}\pi^{+}\pi^{-}\bar{\nu}_{\tau}, \pi^{+}\pi^{-}\pi^{-}\nu_{\tau})\) (or shortly \((3\pi)^{+} - (3\pi)^{-}\)). At the asymmetric-energy \(e^{+}e^{-}\) collider the angle between \(\tau^{+}\) and \(\tau^{-}\) in laboratory frame is smaller than 180°, so the \(\tau^{+}\tau^{-}\) production point can be calculated from the intersection of two trajectories defined by the \(\tau\)-lepton decay vertices and their momentum directions, see Fig. \[1\]. The position of beam interaction point is not needed at all in this method. \(\tau\) momentum direction is determined with the two-fold ambiguity in the center-of-mass system (c.m.s.), for the analysis average axis is used, see Fig. \[2\]. In this method lifetimes of \(\tau^{-}\) and \(\tau^{+}\) can be measured separately to test CPT symmetry conservation.

The following criteria were applied to select signal \((3\pi)^{+} - (3\pi)^{-}\) events: six charged pions with zero net charge and no other tracks are found; the thrust value
in the c.m.s. is greater than 0.9; three pions (triplet) with ±1 net charge in each hemisphere, separated by the plane perpendicular to the thrust axis in the c.m.s.; there are no additional $K_0^S, \Lambda,$ and $\pi^0$ candidates; absolute value of the transverse momentum of the 6π system is greater than 0.5 GeV/$c$; the mass of 6π system should satisfy the requirement 4 GeV/$c^2 < M(6\pi) < 10.25$ GeV/$c^2$; the pseudomass of each triplet of pions is $\sqrt{M_h^2 + 2(E_{beam} - E_h)(E_h - P_h)} < 1.8$ GeV/$c^2$, $h = (3\pi)^-$, $(3\pi)^+$; each triplet vertex-fit quality fulfill $\chi^2 < 20$; the distance $(dl)$ of the closest approach of $\tau^-$ and $\tau^+$ trajectories in laboratory frame satisfy $dl < 0.03$ cm. Finally 1148360 events were selected with the background contamination of about 2%, the dominant background comes from the continuum $e^+e^- \rightarrow q\bar{q}$ ($q = u, d, s$) events.

The probability density function (p.d.f.) for the measured $\tau$ decay length distribution is written in the form:

$$P(x) = N \int e^{-x'/\lambda_{\tau}}R(x-x';\vec{P})dx' + N_{uds}R(x;\vec{P}) + B_{cb}(x),$$

(7)

where $x = \ell/(\beta_{\tau}\gamma_{\tau})$ is normalized $\tau$ decay length, $N$ is normalisation constant, $\lambda_{\tau}$ is estimator of $c\tau_{\tau}$ and $c\tau_{\tau} = \lambda_{\tau} + \Delta_{corr}$ ($\Delta_{corr}$ is determined from MC), $N_{uds}$ is contribution of background from $e^+e^- \rightarrow q\bar{q}$ ($q = u, d, s$) (predicted by MC), $B_{cb}(x)$ is p.d.f. contribution to describe background from $e^+e^- \rightarrow q\bar{q}$ ($q = c, b$) (fixed from
MC), $R(x; \vec{P})$ is detector resolution function (see Fig. 3), parametrized by:

$$R(x; \vec{P}) = (1 - 2.5x) \cdot \exp \left( -\frac{(x - P_1)^2}{2\sigma^2} \right),$$

$$\sigma = P_2 + P_3|x - P_1|^{1/2} + P_4|x - P_1| + P_5|x - P_1|^{3/2}. \quad (8)$$

$\lambda_\tau$, $N$ and $\vec{P} = (P_1, ..., P_5)$ are free parameters of the fit. From the fit of experimental

![Figure 3: Distribution of the difference](image)

![Figure 4: Distribution of the experimental measured decay length values, line shows result of the fit](image)

Entries / 10 $\mu$m

Pull

Figure 3: Distribution of the difference between the reconstructed and true $\tau$ decay length values, line shows result of the fit (points with errors), result of the fit is shown by black line. Distribution of the residuals solid line. Red histogram and solid line show the MC prediction and parametrisation for the $e^+e^- \rightarrow q\bar{q}$ ($q = u, d, s$) and two-photon backgrounds, blue histogram and solid line show the MC prediction and parametrisation for the $e^+e^- \rightarrow q\bar{q}$ ($q = c, b$) background. Distribution of the residuals for the fit is also shown.

Figure 4: Distribution of the experimentally measured decay length (points with errors), result of the fit is shown by black solid line. Red histogram and solid line show the MC prediction and parametrisation for the $e^+e^- \rightarrow q\bar{q}$ ($q = u, d, s$) and two-photon backgrounds, blue histogram and solid line show the MC prediction and parametrisation for the $e^+e^- \rightarrow q\bar{q}$ ($q = c, b$) background. Distribution of the residuals for the fit is also shown.

data $\tau$ lifetime estimator is obtained to be $\lambda_\tau = 86.53 \pm 0.16 \mu m$, applying MC correction $\Delta_{corr} = 0.46 \mu m$ the $\tau$ lifetime (multiplied by speed of light) value is $c\tau_\tau = 86.99 \pm 0.16 \mu m$, where the error is statistical only. The result of the fit is demonstrated in Fig. 4. The main sources of systematic uncertainties are summarized in Table 1. The obtained results for the product of the lifetime and speed of light and for the lifetime are:

$$c\tau_\tau = (86.99 \pm 0.16(stat) \pm 0.10(syst)) \mu m.$$
Table 1: Systematic uncertainties of $c\tau_\tau$.

| Source                              | $\Delta c\tau_\tau$ ($\mu$m) |
|-------------------------------------|-------------------------------|
| Silicon vertex detector alignment   | 0.090                         |
| Asymmetry fixing                    | 0.030                         |
| Fit range                           | 0.020                         |
| Beam energy, ISR, FSR               | 0.024                         |
| Background contribution             | 0.010                         |
| $\tau$-lepton mass                  | 0.009                         |
| **Total**                           | **0.101**                     |

Figure 5: Summary of $\tau$ lifetime measurements.
\[ \tau_\tau = (290.17 \pm 0.53 \text{(stat)} \pm 0.33 \text{(syst)}) \text{ fs}. \]

Belle result on \( \tau_\tau \) and previous measurements are shown in Fig. 5. For the first time the upper limit on the relative \( \tau_\tau \) difference between \( \tau^+ \) and \( \tau^- \) was measured to be:

\[ |\tau_{\tau^+} - \tau_{\tau^-}|/\tau_{\text{average}} < 7.0 \times 10^{-3} \text{ at } 90\% \text{ CL.} \]

With the new \( \tau_\tau \) Belle result the \( g_\tau/g_e \) and \( g_\tau/g_\mu \) ratios were recalculated to revise lepton universality:

\[ g_\tau/g_e = 1.0031 \pm 0.0016, \quad g_\tau/g_\mu = 1.0013 \pm 0.0016. \]

It is seen that the uncertainty of the ratios was improved by a factor of about 1.3 in comparison with the last HFAG result [54], and now the \( g_\tau/g_e \) ratio is almost 2 standard deviations away from unity.

### 3 Hadronic \( \tau \) decays with \( K_S^0 \) at Belle

#### 3.1 Measurement of the branching fractions

The analysis [56] is based on the data sample with the luminosity integral of \( \mathcal{L} = 669 \text{ fb}^{-1} \) which comprises 615 million \( \tau^+\tau^- \) events. One inclusive decay mode \( \tau^- \rightarrow K_S^0 X^- \nu_\tau \) and 6 exclusive hadronic tau decay modes with \( K_S^0 (\tau^- \rightarrow \pi^- K_S^0 \nu_\tau, \tau^- \rightarrow \bar{K}^- K_S^0 \nu_\tau, \tau^- \rightarrow \pi^- K_S^0 \bar{K}_S^0 \pi^0 \nu_\tau, \tau^- \rightarrow \bar{K}^- K_S^0 \bar{K}_S^0 \pi^0 \nu_\tau, \tau^- \rightarrow \pi^- K_S^0 \pi^0 \nu_\tau, \tau^- \rightarrow \pi^- K_S^0 K_S^0 \pi^0 \nu_\tau) \) were studied. After the standard Belle \( \tau\tau \) preselection criteria [56] the following selection criteria are applied: the thrust value in the c.m.s. is greater than 0.9; event is separated into two hemispheres separated by the plane perpendicular to the thrust axis; in the tag hemisphere one-prong tau decay is required (decays with \( e, \mu \) or \( \pi/K(n \geq 0)\pi^0 \)); in the signal hemisphere particular hadronic final state is required; \( K_S^0 \) candidate is reconstructed from a pair of oppositely charged pions with \( \pi^+\pi^- \) invariant mass within the range \( 0.485 \text{ GeV}/c^2 < M_{\pi\pi} < 0.511 \text{ GeV}/c^2 \), decay length in the \( r-\varphi \) plane satisfying \( 2 \text{ cm} \leq L_{\perp} \leq 20 \text{ cm} \), and \( z \)-distance between the two helices at the \( \pi^+\pi^- \) vertex position before the fit is required to be \( \Delta Z_{1,2} < 2.5 \text{ cm} \); \( \pi^0 \) candidate is reconstructed from a pair of \( \gamma \)'s with normalized \( \gamma\gamma \) invariant mass satisfying \( -6 < (m_{\gamma\gamma} - m_{\pi^0})/\sigma_{\gamma\gamma} < 5 \); charged kaon/pion identification parameter \( \mathcal{P}_{K/\pi} = \mathcal{L}_K/(\mathcal{L}_\pi + \mathcal{L}_K) \) is required to be \( \mathcal{P}_{K/\pi} > 0.7(< 0.7) \) for kaon (pion); the total energy of extra photons (not from the signal \( \pi^0 \) candidates) in laboratory frame should satisfy \( \sum E_{\gamma}^\text{LAB} < 0.2 \text{ GeV} \). For the inclusive mode \( \tau^- \rightarrow K_S^0 X^- \nu_\tau \) the number of signal events is obtained from the fit of the \( \pi^+\pi^- \) invariant mass distribution of \( K_S^0 \) candidate. Number of selected events \( (N^\text{data}) \) as well as background admixture \( (N^\text{bg}/N^\text{data}) \) for each decay mode are shown in Table 2. The main non-\( \tau\tau \) background
comes from $e^+e^- \to q\bar{q}$ ($q = u, d, s, c$) events. To take into account cross-feed background six decay modes are analysed simultaneously and number of signal events of $i$-th mode is calculated according:

$$N_i^{\text{sig}} = \sum_j (E^{-1})_{ij} (N_j^{\text{data}} - N_j^{\text{bg}}), \quad i = 1 \div 6,$$

where $E_{ij}$ is $6 \times 6$ efficiency matrix (diagonal elements of $E_{ij}$ are shown in Table 2), $N_i^{\text{sig}}$ numbers for all modes are also summarized in Table 2.

For the $\tau^- \to \pi^- K_S^0 \nu_\tau$, $\tau^- \to K^- K_S^0 \nu_\tau$, $\tau^- \to \pi^- K_S^0 \pi^0 \nu_\tau$ and $\tau^- \to K^- K_S^0 \pi^0 \nu_\tau$ modes with large statistics lepton tag is applied and normalisation to the two-lepton events ($\tau^+ \to e^+ \nu_\tau$, $\tau^\pm \to \mu^\pm \nu_\tau$) (or shortly $e - \mu$) is used to calculate branching fractions:

$$B_i = \frac{N_i^{\text{sig}}}{N_{e-\mu}^{\text{sig}}} \frac{B_e B_\mu}{B_e + B_\mu},$$

where $N_i^{\text{sig}}$ is the number of $e - \mu$ events, $B_i$ is the branching fraction of $\tau^- \to \ell^- \nu_\tau$, $\ell = e, \mu$. To increase statistics for the remaining $\tau^- \to \pi^- K_S^0 K_S^0 \nu_\tau$ and $\tau^- \to \pi^- K_S^0 K_S^0 \pi^0 \nu_\tau$ modes one-prong decay tag and luminosity normalisation method are used:

$$B_i = \frac{N_i^{\text{sig}}}{2\mathcal{L} \sigma_{\tau\tau} B_{1-\text{prong}}},$$

where $\sigma_{\tau\tau} = (0.919 \pm 0.003) \text{nb}$ [57] is the cross section of $\tau^+\tau^-$ production, $B_{1-\text{prong}} = (85.35 \pm 0.07)\%$ [12] is the one-prong decay branching fraction. The branching fractions for one inclusive and six exclusive $\tau$ hadronic decay modes with $K_S^0$ measured at Belle are summarized in Table 3. The branching fractions for $\tau^- \to K^- K_S^0 \nu_\tau$ and $\tau^- \to K^- K_S^0 \pi^0 \nu_\tau$ are measured for the first time at the $B$ factories. The results on branching fractions of all studied decay modes are consistent with the previous experiments and have better precision.

Upper limits on the branching fractions of two $\tau$ decay modes with two $K_S^0$ obtained recently at $BABAR$ [58] are shown in Table 3 as well.

| Mode | $K_S^0 X^-$ | $\pi^- K_S^0$ | $K^- K_S^0$ | $\pi^- K_S^0 \pi^0$ | $K^- K_S^0 \pi^0$ | $\pi^- K_S^0 K_S^0$ | $\pi^- K_S^0 K_S^0 \pi^0$ |
|------|-------------|--------------|-------------|-------------------|------------------|---------------------|---------------------------|
| $N_{i}^{\text{data}}$ | $397806 \pm 631$ | $157836 \pm 541$ | $32701 \pm 295$ | $26605 \pm 208$ | $8267 \pm 109$ | $6684 \pm 96$ | $303 \pm 33$ |
| $N_{i}^{\text{bg}}$ (\%) | $4.20 \pm 0.46$ | $8.86 \pm 0.05$ | $3.55 \pm 0.07$ | $5.60 \pm 0.10$ | $2.43 \pm 0.10$ | $7.89 \pm 0.24$ | $11.6 \pm 1.60$ |
| $N_{i}^{\text{sig}}$, $10^3$ | $3947 \pm 7$ | $1793 \pm 5$ | $319.3 \pm 1.8$ | $833.6 \pm 7.0$ | $322.6 \pm 4.5$ | $244.7 \pm 3.3$ | $21.05 \pm 1.40$ |
| $\varepsilon_{\text{det}}$ (\%) | $9.66$ | $7.09$ | $6.69$ | $2.65$ | $2.19$ | $2.47$ | $0.82$ |
| $(\Delta B_i^R)_{\text{syst}}$ (\%) | $2.4$ | $2.5$ | $4.0$ | $3.9$ | $5.2$ | $4.4$ | $8.1$ |

Table 2: Results of the event selection, detection efficiency and systematic uncertainties of the branching fractions.
The fractions (significances) of three contributions in $B_K$ are shown in Fig. 6. The invariant masses of the $M$ in the other mass spectra. To determine the observed contributions simultaneous (54 ± 0.07) × 10^{-3} Belle

$K^- K^0 \pi^0 \nu_\tau$ (2.00 ± 0.22 ± 0.20) × 10^{-5} Belle

$K^- K^0 K^0 \pi^0 \nu_\tau$ < 4.0 × 10^{-7} at 90% CL $BABAR$

$K^- K^0 K^0 \pi^0 \nu_\tau$ < 6.3 × 10^{-7} at 90% CL $BABAR$

$K^0 S K^0 S \pi^0 \nu_\tau$ = (9.15 ± 0.01 ± 0.15) × 10^{-3} Belle

$\pi^- K^0 S \nu_\tau$ (4.16 ± 0.01 ± 0.08) × 10^{-3} Belle

$K^- K^0 S \nu_\tau$ (7.40 ± 0.07 ± 0.27) × 10^{-4} Belle

$\pi^- K^0 S \pi^0 \nu_\tau$ (1.93 ± 0.02 ± 0.07) × 10^{-3} Belle

$K^- K^0 S \pi^0 \nu_\tau$ (7.48 ± 0.10 ± 0.37) × 10^{-4} Belle

$\pi^- K^0 S K^0 S \nu_\tau$ (2.33 ± 0.03 ± 0.09) × 10^{-4} Belle

$\pi^- K^0 S K^0 S \pi^0 \nu_\tau$ (2.00 ± 0.22 ± 0.20) × 10^{-5} Belle

Table 3: Summary of the branching fractions of the $\tau$ decay modes with $K^0_S$ from Belle [56] and $BABAR$ [58]. The first uncertainty is statistical and the second one is systematic.

### 3.2 Study of mass spectra in $\tau^- \rightarrow \pi^- K^0 S K^0 S \pi^0 \nu_\tau$ sample

The invariant masses of the $K^0_S K^0_S \pi^0$ and $\pi^- K^0_S$ subsystems for $\tau^- \rightarrow \pi^- K^0_S K^0_S \pi^0 \nu_\tau$ events are shown in Fig. 6. Clear peak at the mass of about 1280 MeV/c^2 is seen in the $K^0_S K^0_S \pi^0$ mass distribution as well as some bump around 1420 MeV/c^2, taking into account the quantum numbers of the hadronic final state observed structures are associated with the intermediate $f_1(1285)(J^{PC} = 1^{++})$ and $f_1(1420)(J^{PC} = 1^{++})$ pseudovector mesons. The $\pi^- K^0_S$ distribution exhibits clear signal from the intermediate $K^{*-}(892)$ vector meson. No additional resonancelike structures are observed in the other mass spectra. To determine the observed contributions simultaneous fit of the $M(K^0_S K^0_S \pi^0)$ and $M(\pi^- K^0_S)$ mass spectra is performed. The result of the fit is also shown in Fig. 6. The fractions (significances) of three contributions in $\tau^- \rightarrow \pi^- K^0_S K^0_S \pi^0 \nu_\tau$ are extracted to be (34 ± 5)% (12σ), (12 ± 3)% (4.8σ) and (54 ± 6)% (7.8σ) for the $f_1(1285)$ $\pi^- \nu_\tau$, $f_1(1420)$ $\pi^- \nu_\tau$ and $K^{*-}(892)K^0_S \pi^0 \nu_\tau$ mechanisms, respectively. With obtained fractions, products of the branching fractions for the subprocesses are calculated (the first uncertainty is statistical and the second one is systematic):

$B(\tau^- \rightarrow f_1(1285) \pi^- \nu_\tau) \cdot B(f_1(1285) \rightarrow K^0_S K^0_S \pi^0) = (0.68 \pm 0.13 \pm 0.07) \times 10^{-5}$,

$B(\tau^- \rightarrow f_1(1420) \pi^- \nu_\tau) \cdot B(f_1(1420) \rightarrow K^0_S K^0_S \pi^0) = (0.24 \pm 0.05 \pm 0.06) \times 10^{-5}$,

$B(\tau^- \rightarrow K^*(892)^- K^0_S \pi^0 \nu_\tau) \cdot B(K^*(892)^- \rightarrow K^0_S \pi^-) = (1.08 \pm 0.14 \pm 0.15) \times 10^{-5}$.
Figure 6: Invariant mass of the $K^0_S K^0_S \pi^0$ (left) and $\pi^- K^0_S$ (right) subsystems for the $\tau^- \rightarrow \pi^- K^0_S K^0_S \pi^0 \nu_\tau$ events. Points with errors are experimental data, the hatched histogram is background from $\tau^- \rightarrow \pi^- K^0_S K^0_S \pi^- \nu_\tau$, and the shaded (yellow) histogram is the continuum $q\bar{q}$ background. Solid line is the result of the fit by the $(f_1(1285) \pi^- + f_1(1420) \pi^- + K^*^- K^0_S) \nu_\tau$ model for signal events. The $(f_1(1285) \pi^- + f_1(1420) \pi^-) \nu_\tau$ and $K^*^- K^0_S \nu_\tau$ contributions are shown by the dashed (red) and dotted (green) lines respectively.

4 Search for lepton flavor violating $\tau$ decays at LHCb

In the analysis the statistics of about 1 fb$^{-1}$ collected by LHCb at the proton-proton c.m.s. energy $\sqrt{s} = 7$ TeV in 2011 is used. The inclusive $\tau^-$ production cross section at LHCb is $\sigma_{inc}(\tau^-) = 80$ µb (80% of $\tau^-$ come from $D_s^- \rightarrow \tau^- \bar{\nu}_\tau$ decay). LFV decay $\tau^- \rightarrow \mu^- \mu^+ \mu^-$ and the LNV and BNV ($|\Delta(B - L)| = 0$) decays $\tau^- \rightarrow \bar{\nu}_\mu \mu^+ \mu^-$ and $\tau^- \rightarrow p\mu^- \mu^-$ were studied. The decay $D_s^- \rightarrow \phi \pi^-$ followed by $\phi \rightarrow \mu^+ \mu^-$ (or shortly $D_s^- \rightarrow \phi (\mu^+ \mu^-) \pi^-$), which has similar experimental signature is used for the calibration and normalization.

The following selection criteria are applied: good tracks with transversal momenta satisfying $p_T^{trk} > 0.3$ GeV/c; the transversal momentum of the combined three-track system is required to be $p_T^{3trk} > 4$ GeV/c; decay length of the $\tau$ candidate satisfy $\lambda_\tau > 100$ µm; angle between 3-track momentum and radius vector of $\tau$ decay vertex should be small; invariant mass of $\mu^+ \mu^-$ satisfy $M_{\mu^+ \mu^-} > 0.45$ GeV/c$^2$ to suppress background from the $D_s^- \rightarrow \eta(\rightarrow \mu^+ \mu^- \gamma) \mu^- \bar{\nu}_\mu$ decay.

For the further analysis the region around $\tau$ mass $(m_\tau - 20$ MeV/c$^2) < M_{inv} < (m_\tau + 20$ MeV/c$^2)$ ($\pm 2\sigma_M$) is blinded (so called blinded analysis). After all selections each $\tau$ candidate is characterized by a probability to be signal or background according to the values of three likelihood parameters: $M_{3body}$ includes geometrical properties to identify displaced 3-body $\tau$ decays, $M_{PID}$ is particle identification based on the
information from the detector subsystems: RICH, ECAL, Muon [5], $M_{\text{inv}}$ is invariant mass of $\tau$ decay products. The range for each classifier, $M_{3\text{body}}$ (varies from -1 to 1) and $M_{\text{PID}}$ (varies from 0 to 1), is subdivided into 6 and 5 unequally spaced bins respectively. And $M_{\text{inv}}$ within $\pm 20$ MeV/$c^2$ mass window around $\tau$ mass is subdivided into 8 equally spaced bins to extract number of signal events. Result of the fit of the data sidebands for particular bins in $M_{3\text{body}}$ and $M_{\text{PID}}$ for the three decay modes are shown in Fig. 7. The branching fraction of signal decay is calculated according:

$$B_{\text{sig}} = B(D_s^- \to \phi (\mu^+\mu^-)\pi^-) \frac{f_{D_s}}{B(D_s^- \to \tau^-\bar{\nu}_\tau)} \frac{\epsilon_{\text{det}}^{\text{trg}} \epsilon_{\text{norm}}^{\text{trg}}}{\epsilon_{\text{det}}^{\text{sig}} \epsilon_{\text{norm}}^{\text{sig}}} \frac{N_{\text{sig}}}{N_{\text{norm}}},$$

where $N_{\text{sig}}$ ($N_{\text{norm}}$) is number of selected signal (normalisation) events, $\epsilon_{\text{det}}^{\text{trg}}$, $\epsilon_{\text{det}}^{\text{sig}}$ ($\epsilon_{\text{norm}}^{\text{trg}}$, $\epsilon_{\text{norm}}^{\text{sig}}$) are trigger and detection efficiency for signal (normalisation) events, $f_{D_s} = 0.78 \pm 0.05$ is fraction of $\tau$’s from $D_s^-$ decays. The obtained upper limits on the branching fractions:

$$B(\tau^- \to \mu^-\mu^+\mu^-) < 8.0 \times 10^{-8} \text{ at } 90\% \text{ CL},$$
$$B(\tau^- \to \bar{\nu}_\mu\mu^+\mu^-) < 3.3 \times 10^{-7} \text{ at } 90\% \text{ CL},$$
$$B(\tau^- \to \bar{\nu}_\mu\mu^-\mu^-) < 4.4 \times 10^{-7} \text{ at } 90\% \text{ CL}.$$

Figure 8 summarizes the upper limits on the branching fractions of 48 LFV modes obtained at $B$ factories. After the LHCb upgrade [59] it will be possible to improve the upper limit on $B(\tau^- \to \mu^-\mu^+\mu^-)$ in comparison with the current best limit obtained by Belle [60]. For the first time the upper limits on the branching fractions of the LNV and BNV $\tau^- \to \bar{\nu}_\mu\mu^+\mu^-$ and $\tau^- \to \bar{\nu}_\mu\mu^-\mu^-$ decays were measured at LHCb.

## 5 CP violation in $\tau$ decays at $B$ factories

Recent studies of CPV in the $\tau^- \to \pi^- K_S (\geq \pi^0) \nu\tau$ decays at $BABAR$ [61] as well as in the $\tau^- \to K_S \pi^- \nu\tau$ decay at Belle [62] provide complementary information about
sources of CPV in these hadronic decays.

The decay-rate asymmetry $A_{\text{CP}} = \frac{\Gamma(\tau^+ \to \pi^+ K_S^0 \nu_\tau) - \Gamma(\tau^- \to \pi^- K_S^0 \bar{\nu}_\tau)}{\Gamma(\tau^+ \to \pi^+ K_S^0 \nu_\tau) + \Gamma(\tau^- \to \pi^- K_S^0 \bar{\nu}_\tau)}$ was studied at BABAR with the $\tau^+\tau^-$ data sample of $\int Ldt = 476 \text{ fb}^{-1}$. The obtained result $A_{\text{CP}} = (-0.36 \pm 0.23 \pm 0.11)\%$ is about 2.8 standard deviations from the SM expectation $A_{\text{CP}}^0 = (+0.36 \pm 0.01)\%$.

At Belle CPV search was performed as a blinded analysis based on a 699 fb$^{-1}$ data sample. Specially constructed asymmetry, which is a difference between the mean values of $\cos \beta \cos \psi$ for $\tau^-$ and $\tau^+$ events, was measured in bins of $K_S^0\pi^-$ mass squared ($Q^2 = M^2(K_S^0\pi)$):

$$A_{i}^{CP}(Q_i^2) = \frac{\int \cos \beta \cos \psi \left(\frac{d\Gamma_{\tau^-}}{d\omega} - \frac{d\Gamma_{\tau^+}}{d\omega}\right) d\omega}{\frac{1}{2} \Delta Q_i^2 \left(\frac{d\Gamma_{\tau^-}}{d\omega} + \frac{d\Gamma_{\tau^+}}{d\omega}\right) d\omega} \simeq \langle \cos \beta \cos \psi \rangle_{\tau^-} - \langle \cos \beta \cos \psi \rangle_{\tau^+},$$

where $\beta$, $\theta$ and $\psi$ are the angles, evaluated from the measured parameters of the final hadrons, $d\omega = dQ^2 d\cos \theta d\cos \beta$. In contrary to the decay-rate asymmetry the introduced $A_{i}^{CP}(Q_i^2)$ is already sensitive to the CPV effects from the charged scalar boson exchange $[44]$. No CP violation was observed and the upper limit on the CPV parameter $\eta_S$ was extracted $|\text{Im}(\eta_S)| < 0.026$ at 90% CL. Using this limit parameters of the Multi-Higgs-Doublet models $[42, 43]$ can be constrained as
\[ |\text{Im}(X Z^*)| < 0.15 \frac{M_{H^\pm}^2}{(1 \text{ GeV}^2/c^4)}, \]
where \( M_{H^\pm} \) is the mass of the lightest charged Higgs boson, the complex constants \( Z \) and \( X \) describe the coupling of the Higgs boson to leptons and quarks respectively.

The other more complicated and most powerful method to extract CPV parameter at \( e^+e^- \) factories is an unbinned maximum likelihood fit of events in the full phase space. The main idea of this method is to consider events where both taus decay to the particular final states. One \( \tau^\pm \) (signal side) decays to particular hadronic final state (for example \( \tau^\pm \rightarrow (K\pi)^\mp \nu \)) and the other \( \tau^\pm \) (tag side) decays to some well investigated mode with large branching fraction. As a tag decay mode we can take \( \tau^\pm \rightarrow \pi^\pm \pi^0 \nu \) having the largest branching fraction, it also serves as spin analyser, which allows one to be sensitive to the spin dependent part of the differential decay width of signal decay using effect of spin-spin correlation of taus \([63]\). In the technique we analyze \( e^+e^- \rightarrow \tau^+\tau^- \rightarrow ((K\pi)^\mp \nu, \pi^\pm \pi^0 \nu) \) (or shortly \( ((K\pi)^\mp, \pi^\pm \pi^0) \)) events in the 12-dimensional phase space. The probability density function is constructed from the total differential cross section, which is a sum of spin independent term and spin-spin correlation term.

To write the total differential cross section we follow the approach developed in \([64, 65]\). The differential cross section of \( e^+e^- \rightarrow \tau^+\tau^- \rightarrow ((\vec{\zeta}^+)\tau^- (\vec{\zeta}^+)\) reaction in the center-of-mass system (c.m.s.) is given by formula \([63]\):

\[
\frac{d\sigma(\vec{\zeta}^+, \vec{\zeta}^+)}{d\Omega} = \frac{\alpha^2}{64E_\tau^2} \beta_\tau (D_0 + D_{ij} \vec{\zeta}^+ \vec{\zeta}^+) \quad D_0 = 1 + \cos^2 \theta + \frac{1}{\gamma^2_\tau} \sin^2 \theta \quad D_{ij} = \begin{pmatrix} (1 + \frac{1}{\gamma^2_\tau}) \sin^2 \theta & 0 & \frac{1}{\gamma^2_\tau} \sin 2\theta \\ 0 & -\beta^2_\tau \sin^2 \theta & 0 \\ \frac{1}{\gamma^2_\tau} \sin 2\theta & 0 & 1 + \cos^2 \theta - \frac{1}{\gamma^2_\tau} \sin^2 \theta \end{pmatrix}
\]

where \( \vec{\zeta}^\pm \) is polarisation vector of \( \tau^\pm \) in the \( \tau^\pm \) rest frame (unitary vector along \( \tau^\pm \) spin direction). Asterisk marks parameters measured in the associated \( \tau \) rest frame. \( \alpha, E_\tau, \gamma_\tau = E_\tau/M_\tau, \beta_\tau = P_\tau/E_\tau \) and \( \theta \) are fine structure constant, energy, Lorentz factor, velocity of \( \tau \) (in the units of \( c \)) and polar angle of \( \tau^- \) momentum direction, respectively. Signal differential decay width is written in the form:

\[
\frac{d\Gamma(\tau^+ (\vec{\zeta}^+) \rightarrow (K\pi)^\mp \nu)}{dm_{K\pi}^2 d\Omega_{K\pi} d\Omega_{\pi}} = (A_0 + \eta_{CP} A_1) + (\vec{B}_0 + \eta_{CP}\vec{B}_1) \vec{\zeta}^+ + (A_0 + \eta_{CP} A_1) - (\vec{B}_0 + \eta_{CP}\vec{B}_1) \vec{\zeta}^+ ,
\]

where \( \eta_{CP} \) is CPV sensitive parameter; \( m_{K\pi}, \Omega_{K\pi} \) are \( K\pi \) invariant mass and solid angle of the \( K\pi \) system in the \( \tau \) rest frame, \( \Omega_{\pi} \) is solid angle of the charged pion in the \( K\pi \) rest frame. \( A_1 \) and \( A_2 \) are form factors of the spin-independent part; \( \vec{B}_0 \) and \( \vec{B}_1 \) are form factors of the spin-dependent part of the differential decay width.
The $\tau^\pm (\bar{\zeta}^*) \to p^\pm (K^*) \nu(q^*) \to \pi^\pm (p_1^\pi) \pi^0 (p_2^\pi) \nu(q^*)$ decay width reads (with unimportant for this analysis total normalization constant $\kappa_\rho$):

$$
\frac{d\Gamma(\tau^\pm \to \pi^+\pi^-0\nu)}{dm^2_{\pi\pi}d\Omega_\rho d\Omega_{\pi}} = A' \mp B'\zeta^*, \tag{10}
$$

$$
Q^* = p_1^\rho - p_2^\rho, \quad K^* = p_1^\rho + p_2^\rho, \quad m^2_{\pi\pi} = K^*2,
$$

$$
A' = \kappa_\rho W(m^2_{\pi\pi})(2(q,Q)Q_0^2 - Q^2q_0^*), \quad B' = \kappa_\rho W(m^2_{\pi\pi})(Q^2\bar{K}^* + 2(q,Q)\bar{Q}^*),
$$

$$
W(m^2_{\pi\pi}) = |F_\pi(m^2_{\pi\pi})|^2 p^*_\rho(m^2_{\pi\pi})\bar{p}_\pi(m^2_{\pi\pi}) , \quad p^*_\rho = \frac{M_\tau}{2} \left(1 - \frac{m^2_{\pi\pi}}{M_\tau^2}\right),
$$

$$
\bar{p}_\pi = \sqrt{(m^2_{\pi\pi} - (m_\pi + m_\pi')^2)(m^2_{\pi\pi} - (m_\pi - m_\pi')^2)} \times \frac{2m_{\pi\pi}}{m^2_{\pi\pi}} \tag{11}
$$

where $p^*_\rho$, $\Omega^*_\rho$ are momentum and solid angle of $\rho$ meson in the $\tau$ rest frame, $\bar{p}_\pi$, $\bar{\Omega}_\pi$ are momentum and solid angle of charged pion in the $\rho$ rest frame, $F_\pi(m^2_{\pi\pi})$ is pion form factor [66].

As a result the total differential cross section for $((K\pi)^\pm, p^\pm)$ events can be written as [63]:

$$
\frac{d\sigma((K\pi)^\pm, p^\pm)}{dm^2_{K\pi}d\Omega_{K\pi}d\Omega_{\pi}d\Omega_\rho d\Omega_{\pi}} = \frac{\alpha^2\beta_\tau}{64E^2_\tau} \left(\mathcal{F} + \frac{\eta_{\text{CP}}}{\bar{\eta}_{\text{CP}}} \mathcal{G}\right), \tag{12}
$$

$$
\mathcal{F} = D_0A_0A' - D_1B_0B', \quad \mathcal{G} = D_0A_1A' - D_1B_1B'.
$$

Experimentally we measure particle parameters in the c.m.s., hence visible differential cross section is given by:

$$
\frac{d\sigma((K\pi)^\pm, p^\pm)}{dp_{K\pi}d\Omega_{K\pi}d\Omega_{\pi}d\Omega_\rho} = \sum_{\Phi_1, \Phi_2} \frac{d\sigma((K\pi)^\pm, p^\pm)}{dm^2_{K\pi}d\Omega_{K\pi}d\Omega_{\pi}d\Omega_\rho} \left|\frac{\partial((\Omega^*_\rho, \Omega^*_{\pi}, \Omega_{\pi})}{\partial(p_{K\pi}, \Omega_{K\pi}, p_\rho, \Omega_\rho)}\right|, \tag{13}
$$

where the summation is done over the unknown $\tau$ direction, which is determined with two-fold ambiguity by $\Phi_1$ and $\Phi_2$ angles. They are calculated using parameters measured in the experiment [65].

As a result $\eta_{\text{CP}}$ can be extracted in the simultaneous unbinned maximum likelihood fit of the $((K\pi)^-, p^+)$ and $((K\pi)^+, p^-)$ events in the 12-dimensional phase space. The advantage of this method is that we use the whole information recorded in the experiment to extract CPV parameter. More over this, with this method it is possible to study CPV in the spin-dependent part of the decay width, i.e. we can test NP models with the nontrivial $\tau$ spin-dependent effects. Similar method is used at $e^+e^-B$ factories to measure Michel parameters in leptonic $\tau$ decays [67].
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