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Exact solution of stress and displacement of rotating elastic interference fit for a quill shaft of micro gas turbine

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Abstract. Interference fit is widely used in micro gas turbine to transfer large torque and offer significant cost advantages. In this paper, a interference fit model was developed to research the convection characteristics of quill shaft for micro gas turbine. The radial stress, tangential stress, radial displacement and the Von Mises stress of the quill shaft are derived by using elastic theory to analyze the influence of the contact surface pressure and angular velocity on the strength of the quill shaft. Numerical calculated results show that The radial stress of the quill shaft is identically greater than zero and positively correlated with the contact pressure and angular velocity. The direction of the tangential stress of the quill shaft is determined by the value of the contact pressure. The radial displacement of the quill shaft is a monotony decrease function of the contact pressure and a monotony increase function of the angular velocity. Generally, the maximum stress of the quill shaft happens in the inner surface of the quill shaft regardless of the angular velocity.

1. Introduction
Interference fit of coupling and shaft are widely used in high-speed or ultra-high-speed shafting for the connection and coaxial driving of generator, compressor and gas turbine since it, which can transfer large torque, are easy to produce and offer significant cost advantages and used in the design for fixation of pinions, couplings and the like on shafts in order to overcome the disadvantages of bolts or mating spline which can connect the coupling and the shaft together at low rotational speed [1]. The bolts could introduce certain unavoidable unbalances at high rotational speed, and the mating spline with an internal tooth and an external tooth has hysteretic forces and moments [2]. There is a vast literature [3-8], most of which is concerned with a laminated composite tube which is interference fitted onto other structure like a bearing bush or a hub. Such a clearance between coupling and shaft is given, that the stresses and displacements of all regions of the fit can be calculated by Lame’s solution commonly. However, the centrifugal force of the coupling and shaft should be considered at high-speed or ultra-high-speed shafting to guarantee the connection strength and safe operating, and the point of the maximum stress of the structure will be determined by the contact pressure of the interface between the coupling and shaft.
Accordingly, the aim of this paper is to develop a better calculation method used in the design of the interference fit for the quill shaft which transfer torques at the high speed, due to higher operating speeds and lighter weights for micro gas turbines.

2. Analytical model

2.1. Structure
In this study, the structure analyzed here is only the part of the interference fit of a high-speed coupling-shaft system. During operation, the torque moment is transferred from the motor shaft to the diaphragm coupling first, and then to the turbine shaft, through the interference fits [6]. In convenience, the outer and inner parts of the interference fits are named as high-speed coupling and quill shaft respectively in this paper. The interference fit of the motor shaft and left cylindrical side of the diaphragm cup-shaped coupling are enlarged, shown in Fig.1. Fig. 1(a) show the initial clearance δ between the interference fit parts before assembly. Fig.1 (b) shows the assembled high-speed coupling and quill shaft.

In addition, the inner radius of the quill shaft and the outer radius of the coupling are defined as rsi and rco before assembly. The outer radius of the shaft is equal to the inner radius of the coupling which is indicated as Rafter assembly. Then uc and us are the radial displacements of the coupling and shaft respectively. Further we stipulate that the elastic moduli E and Poisson’s ratio μ as well as the density ρ of the coupling and shaft are equal. The influences of the contact pressure of the interference fit interface p and the angular velocity ω as well as the outer radius of the quill shaft rso on the stresses and radial displacement of the quill shaft are investigated as follow to study the connection characteristics for the interference fit of the quill shaft.

![Figure 1. The interference fit between the coupling and quill shaft.](image)

2.2. Modeling
Suppose σr, σθ and u are the radial stress, tangential stress and the radial displacement. As shown in Fig. 2, the rotating disk is subjected to the external pressure and internal pressure, and the radial equilibrium of a rotating disk can be expressed as

\[
\frac{d^2 u}{dr^2} + \frac{1}{r} \frac{du}{dr} - \frac{u}{r^2} = -\left(1 - \mu^2\right) \frac{\rho \omega^2}{E} \frac{r}{r^2}
\]  

(1)
Suppose \( \varepsilon_r \) and \( \varepsilon_\theta \) are the radial strain and tangential strain, the total strains consist of elastic and thermal strains. The elastic strains are related to stresses through Hooke’s Law of plane stress problems, so the strain-displacement relation can be given as

\[
\begin{align*}
\varepsilon_r &= \frac{1}{E} (\sigma_r - \mu \sigma_\theta) \\
\varepsilon_\theta &= \frac{1}{E} (\sigma_\theta - \mu \sigma_r)
\end{align*}
\] (2)

The strain-displacement relation can be written as

\[
\begin{align*}
\varepsilon_r &= \frac{du}{dr} \\
\varepsilon_\theta &= \frac{u}{r}
\end{align*}
\] (3)

![Figure 2. The Rotating annular disk.](image)

Then the solution of equation (1) can be derived as

\[
u = u_i + u^* = C_1 r + \frac{C_2}{r} - \left(1 - \mu^2\right) \frac{\rho \omega^2 r^3}{8}
\] (4)

Where \( C_1 \) and \( C_2 \) are the integration constants and determined by the boundary conditions of the interference fit. \( U_1 \) is homogeneous solution of the equation, and \( u^* \) is particular solution of the equation. The radial stress and tangential stress of the quill shaft can be written as

\[
\sigma_r = C_1 + \frac{C_2}{r^2} - \frac{3 + \mu}{8} \rho r^2 \omega^2
\] (5)
\[ \sigma_r = C_1 - \frac{C_2}{r^2} - \frac{1+3\mu}{8} \rho r^2 \omega^2 \]  

(6)

2.3. Stress

The boundary conditions of the interference fit of the quill shaft can be introduced as follow to guarantee the equations above have a unique solution.

\[ \sigma_r (r = R) = -p \]  

(7)

\[ \sigma_r (r = r_h) = 0 \]  

(8)

Where contact pressure \( p \) can be given by

\[ p = \frac{M}{2\pi f L R^2} \]  

(9)

Where \( M \), \( f \) and \( L \) are torque, static friction coefficient of the interface of the interference and length of the interference fit respectively. Then the radial stress \( \sigma_r \) and tangential stress \( \sigma_\theta \) of the interference fit of the quill shaft can be expressed as

\[ \sigma_r = \frac{p R^2}{R^2 - r_i^2} \left( 1 - \frac{r_i^2}{r^2} \right) + \frac{(3 + \mu) \rho \omega^2}{8} \left( R^2 + r_i^2 - \frac{R^2 r_i^2}{r^2} - r^2 \right) \]  

(10)

\[ \sigma_\theta = -\frac{p R^2}{R^2 - r_i^2} \left( 1 + \frac{r_i^2}{r^2} \right) + \frac{(3 + \mu) \rho \omega^2}{8} \left( R^2 + r_i^2 + \frac{R^2 r_i^2}{r^2} - \frac{1+3\mu}{3} r_i^2 \right) \]  

(11)

Suppose \( \sigma_s \) is yield strength of the material of the quill shaft, the radial stress \( \sigma_r \) and tangential stress \( \sigma_\theta \) can be normalized as

\[ \Sigma_r = \sigma_r / \sigma_s \]  

(12)

\[ \Sigma_\theta = \sigma_\theta / \sigma_s \]  

(13)

2.4. Displacement

The radial displacement \( u \) of the interference fit of the quill shaft can be written as

\[ u = -\frac{p R^2}{E (R^2 - r_i^2)} \left[ (1 - \mu) r^2 + (1 + \mu) r_i^2 \right] + \frac{(1 + \mu)(3 - 2\mu) \rho \omega^2 r}{8 E (1 - \mu)} \left[ (1 - 2\mu) \left( R^2 + r_i^2 \right) + \frac{R^2 r_i^2}{r^2} - \frac{2\mu^2 - 1}{3 - 2\mu} r^2 \right] \]  

(14)
And the displacement can be normalized as

\[ U = u / r_0 \]  \hspace{1cm} (15)

2.5. Maximum Torque Load Capacity

Deal with torque moment transfer case, the equilibrium equation in tangential direction can be expresses as

\[ \frac{d \tau_{r\theta}}{dr} + 2 \frac{\tau_{r\theta}}{r} = 0 \]  \hspace{1cm} (16)

And the shear stress-strain and shear strain-displacement can be expressed as

\[ \gamma_{r\theta} = \frac{2(1 + \mu)}{E} \tau_{r\theta} \]  \hspace{1cm} (17)

\[ \gamma_{r\theta} = \frac{du_{\theta}}{dr} \frac{u_{\theta}}{r} \]  \hspace{1cm} (18)

Then the relation between the tangential displacement and shear strain are

\[ u_{\theta} = C_3 r + \frac{C_4}{r} \]  \hspace{1cm} (19)

\[ \gamma_{r\theta} = -\frac{2C_4}{r^2} \]  \hspace{1cm} (20)

\[ \tau_{r\theta} = -\frac{EC_4}{r^2(1 + \mu)} \]  \hspace{1cm} (21)

Where C3 and C4 are the integration constants and determined by the boundary conditions of the interference fit. Under the maximum torque transfer case, the shear stressed can be expressed as follows

\[ \tau_{r\theta} = fP \]  \hspace{1cm} (22)
Where $f$ is the friction coefficient of the contact surface of the interference fit.

2.6. The Von Mises Stress

The radial stress, tangential stress and shear stress of the quill shaft can be condensed into an equivalent stress by using the Von Mises stress, it can be expressed as

$$
\sigma_c = \sqrt{\sigma_{\theta}^2 + \sigma_r^2 - \sigma_{\theta}\sigma_r + 3\tau^2}
$$

(23)

This formula can be used to justify whether this stress reached the limit yielding of material and it can be normalized as

$$
\Sigma_c = \sigma_c / \sigma_s
$$

(24)

3. Numerical analysis

This section studies the influences of the contact pressure $p$ and angular velocity $\omega$ as well as the radius of the quill shaft $r$ on the radial stress $\Sigma_r$, tangential stress $\Sigma_\theta$, displacement $U$ and the Von Mises stress $\Sigma_c$ of the interference fit. Extensive numerical computations have been performed with the following data in Table 1.

| E (Pa)     | $\rho$ (kg m$^{-3}$) | $\mu$ | $P$ (kW) | $\sigma_s$ (Pa) | $f$ | $R$ (m) | $r_i$ (m) |
|------------|-----------------------|-------|----------|-----------------|-----|---------|-----------|
| 2.1×10$^{11}$ | 7.8×10$^3$           | 0.3   | 100      | 9.6×10$^8$      | 0.12| 0.046   | 0.025     |

3.1. Radial stress

The radial stresses of the quill shaft $\Sigma_r$ are shown in Figure 3 for the angular velocity $\omega=5000$ rad/s. The radial stress of the inner surface of the quill shaft $\Sigma_r(r = r_i)$ is identically equal zero regardless of the contact surface pressure $p$, while the radial stress of the outer surface of the quill shaft $\Sigma_r(r = R)$ and the radial stress of the middle of the quill shaft $\Sigma_r(2r = R + r_i)$ are both positively correlated with the contact surface pressure $p$. For the contact surface pressure $p = 0$, the radial stress of the inner surface of the quill shaft $\Sigma_r(r = r_i)$ and the radial stress of the outer surface of the quill shaft $\Sigma_r(r = R)$ are equal to zero, but the radial stress of the middle of the quill shaft $\Sigma_r(2r = R + r_i)$ is greater than them due to the influence of the angular velocity $\omega$.

Figure 4 shows the radial stresses of the quill shaft $\Sigma_r$ for the contact pressure $p = 0.1 \sigma_s$. The radial stress of the inner surface of the quill shaft $\Sigma_r(r = r_i)$ and the radial stress of the inner surface of the quill shaft $\Sigma_r(r = r_i)$ are constant regardless of the angular velocity, while the radial stress of the middle of the quill shaft $\Sigma_r(2r = R + r_i)$ increases due to the growth of the angular velocity $\omega$.
Figure 3. The radial stress of the quill shaft $\Sigma r$ for the different radii and the angular velocity $\omega=5000$ rad/s.

Figure 4. The radial stress of the quill shaft $\Sigma r$ for the different radii and the contact pressure $p=0.1\sigma_s$.

3.2. Tangential stress

The tangential stresses of the quill shaft $\Sigma \theta$ are shown in Figure 5 for the angular velocity $\omega=5000$ rad/s. The magnitudes and directions of the tangential stresses of the inner surface, outer surface and middle of the quill shaft relate to the contact pressure $p$.

In Figure 6, the tangential stress of the inner surface of the quill shaft $\Sigma \theta(r = r_i)$, the tangential stress of the outer surface of the quill shaft $\Sigma \theta(r = R)$ and the tangential stress of the middle of the quill shaft $\Sigma \theta(2r = R + r_i)$ are all positively correlated with the angular velocity $\omega$ for the contact surface pressure $p = 0.1\sigma_s$.

Figure 5. The tangential stress of the quill shaft $\Sigma \theta$ for the different radii and the angular velocity $\omega=5000$ rad/s.
### 3.3. Radial Displacement

Figure 7 shows the radial displacements of the quill shaft $U$ for the angular velocity $\omega = 5000 \text{ rad/s}$. The radial displacement of the inner surface of the quill shaft $U(r = r_i)$, the radial displacement of the outer surface of the quill shaft $U(r = R)$ and the middle of the quill shaft $U(2r = R + r_i)$ are all negatively correlated with the contact surface pressure $p$. The radial displacement of the inner surface of the quill shaft $U(r = r_i)$ is maximum, the radial displacement of the outer surface of the quill shaft $U(r = R)$ is minimum.

In Figure 8, the radial displacements increase scientifically when angular velocity $\omega$ is growing for the contact pressure $p=0.1\sigma$.

**Figure 6.** The tangential stress of the quill shaft $\Sigma \theta$ for the different radii and the contact pressure $p=0.1\sigma$.

**Figure 7.** The radial displacement of the quill shaft $U$ for the different radii and the angular velocity $\omega=5000 \text{ rad/s}$. 
3.4. Von Mises Stress

The Von Mises stresses of the quill shaft $\Sigma_c$ are shown in Figure 9 for the angular velocity $\omega=5000$ rad/s. The maximum stress of the quill shaft happens in the contact surface of the interference fit for the contact pressure $p=0.01\sigma_s$. However, the maximum stress of the quill shaft happens in the inner surface of the quill shaft when the contact pressure $p$ is equal to 0.05$\sigma_s$ or 0.1$\sigma_s$. The surface of the maximum stress of the quill shaft is determined by the contact surface pressure $p$. Figure 10 shows the Von Mises stresses of the quill shaft $\Sigma_c$ for the contact pressure $p=0.1\sigma_s$. The maximum stress of the quill shaft always happens in the inner surface of the quill shaft regardless of the angular velocity $\omega$.

Figure 8. The radial displacement of the quill shaft $U$ for the different radii and the contact pressure $p=0.1\sigma_s$.

Figure 9. The Von Mises stress of the quill shaft $\Sigma_c$ for the different contact pressures and the angular velocity $\omega=5000$ rad/s.
4. Conclusion

Two determine parameters: the contact pressure and angular velocity should be considered for the design of the interference fit of the quill shaft in micro gas turbine. The radial stress of the quill shaft is identically greater than zero and positively correlated with the contact press and angular velocity. The direction of the tangential stress of the quill shaft is determined by the value of the contact pressure. The radial displacement of the quill shaft is a monotony decrease function of the contact pressure and a monotony increase function of the angular velocity. Generally, the maximum stress of the quill shaft happens in the inner surface of the quill shaft regardless of the angular velocity.

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