Exploring stable models in $f(R, T, R_{\mu\nu}T^{\mu\nu})$ gravity.

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Abstract

We examine in this paper the stability analysis in $f(R, T, R_{\mu\nu}T^{\mu\nu})$ modified gravity, where $R$ and $T$ are the Ricci scalar and the trace of the energy-momentum tensor, respectively. By considering the flat Friedmann universe, we obtain the corresponding generalized Friedmann equations and we evaluate the geometrical and matter perturbation functions. The stability is developed using the de Sitter and power-law solutions. We search for application the stability of two particular cases of $f(R, T, R_{\mu\nu}T^{\mu\nu})$ model by solving numerically the perturbation functions obtained.

1 Introduction

The study of the universe has undergone serious changes, as theoretical than observational during the last decades. The cosmology, which attaches to understand the global properties and large structures of the universe (their origin, their evolution, theirs characteristics...) is entered an era precision. This discipline has long been a highly speculative domain and reposed as much on metaphysics than physics. With the formulation in 1915 of the theory of General Relativity, a coherent theoretical framework allowing mathematicalisation of space and time has become appeared. One could then, in 1924, formulate the cosmological models based on this theory. These models of the universe, whose main characteristic is to be expanding, helped to understand many observations such as the

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recession of galaxies revealed by Edwin Hubble. During twenty years, the cosmology is confined to the description and the reconstruction of this expansion. In the second period, starting around 1948, the study of physical processes in an expanding space allowed to formulate the big bang model. This model based on Einstein’s theory of $GR$ is one of the great success stories in modern theoretical physics and it describes a universe that is isotropic and homogeneous on large scales. Recent observations of the supernovae type Ia (SNe Ia) [1], the cosmic microwave background radiation (CMBR) [2], the baryon acoustic oscillation (BAO) surveys [3], the large scale structure [4] and the weak lensing [5], clearly indicate that the universe is currently expanding with an accelerating rate. These recent observations of the universe also possess some problems for the standard model of cosmology and requires the presence of a “dark energy” component whose nature is not well understood. In order to explain the acceleration of the universe without introducing such a tantalizing source of energy, other gravitation theories have been designed. The Palatini $f(R)$ gravity is also proposed first as an alternative to dark energy in a series of works [6]-[9]. There has since been growing interest in these modified gravity theories: for the local tests of the Palatini and metric $f(R)$ gravity models see [10]-[11], and for the cosmologies of these two classes of models see [12]-[18]. A generalization of $f(R)$ theory, namely $f(R, T)$ gravity, was introduced first in Ref.[19] and then, studied in Refs.[20]-[31] with interesting results. The $f(R, T)$ gravity model depends on a source term, representing the variation of the matter energy-momentum tensor with respect to the metric. The justification for the dependence on $T$ comes from inductions arising from some exotic fluid and/or quantum effects (conformal anomaly). Actually, this induction point of view adopts or links with the known proposals such as geometrical curvature inducing matter, a geometrical description of physical forces, and a geometrical origin for the matter content of the universe.

Recently, $f(R, T)$ theory has been generalized by introducing the contribution from contraction of the Ricci tensor and energy-momentum tensor named as $f(R, T, R_{\mu\nu}T^{\mu\nu})$ gravity [33]-[34]. In this theory, the energy-momentum tensor is generally non-conserved and extra force is present as a result of non-minimal matter geometry coupling. Haghhani
et al. [33] have discussed the cosmological implications of this theory and developed the Dolgov-Kawasaki instability criterion. The problem of matter instability in $f(R, T, R_{\mu\nu}T^{\mu\nu})$ gravity is discussed by Odintsov and Sàez Gomez [34]. Sharif and Zubair [35]-[36] have presented the field equations for more general case and formulated energy conditions and the thermodynamic laws in $f(R, T, R_{\mu\nu}T^{\mu\nu})$ modified gravity. Because of the interesting aspects and the progress realized by these authors, it seems there is still room to study some motivating gravitational and cosmological aspects of such modified gravity which have not yet been studied. In the context of $f(R)$ modified theories of gravity, the stability of the Einstein static universe was also analyzed by considering homogeneous perturbations [32]. By considering specific forms of $f(R)$, the stability regions of the solutions were parameterized by a linear equation of state parameter $w = \frac{p}{\rho}$. In Ref. [37] the authors showed that the cosmological viable $f(R, T)$ dark model obtained by imposing the conservation of the energy-momentum tensor presents stability for both the de Sitter and power-law solutions. In present paper, we extend the work of Ref. [37] in the context of $f(R, T, R_{\mu\nu}T^{\mu\nu})$ modified gravity, ie we generalized the stability analysis in $f(R, T, R_{\mu\nu}T^{\mu\nu})$ gravity and analysed the viability for some specific known models. This paper is outlined in the following manner: In Section II, we briefly review the basic formalism in $f(R, T, R_{\mu\nu}T^{\mu\nu})$ modified gravity. In section III, we consider a general form of $f(R, T, R_{\mu\nu}T^{\mu\nu})$ gravity, and analyze the stability solutions, by considering homogeneous perturbations around the Hubble parameter and energy density of matter. Finally, in Section IV we present our conclusions.

2 Basic Formalism in $f(R, T, R_{\mu\nu}T^{\mu\nu})$ modified gravity

In $f(R, T, R_{\mu\nu}T^{\mu\nu})$ gravity, we define the action with matter as [33]-[36]

$$S = \frac{1}{2\kappa^2} \int \sqrt{-g} dx^4 f(R, T, R_{\mu\nu}T^{\mu\nu}) + \int L_m \sqrt{-g} dx^4, \tag{1}$$

where $R$, $T$ and $R_{\mu\nu}T^{\mu\nu}$ denote respectively the Ricci scalar, the trace of energy-momentum tensor and contraction of the Ricci tensor with $T^{\mu\nu}$. The matter Lagrangian $L_m$ is con-
nected at energy-momentum tensor by the relation

\[ T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta (\sqrt{-g} \mathcal{L}_m)}{\delta g^{\mu\nu}}. \] (2)

Assumption that the matter Lagrangian \( \mathcal{L}_m \) is a function of the metric \( g_{\mu\nu} \) and not its derivatives, the energy-momentum tensor (2) takes the following form

\[ T_{\mu\nu} = g_{\mu\nu} \mathcal{L}_m - 2 \frac{\partial \mathcal{L}_m}{\partial g_{\mu\nu}}. \] (3)

Varying the action (1) with respect the components of the tensor metric \( g_{\mu\nu} \), one gets the field equations in \( f(R, T, R_{\mu\nu} T^{\mu\nu}) \) gravity given by

\[
R_{\mu\nu} f_R - \left[ \frac{1}{2} f - \mathcal{L}_m f_T - \frac{1}{2} \nabla_\alpha \nabla_\beta (f_Q T^{\alpha\beta}) \right] g_{\mu\nu} + (g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu) f_R + \frac{1}{2} \Box (f_Q T_{\mu\nu}) + 2f_Q R_{\alpha\beta} - \nabla_\alpha \nabla_\beta \left[ T_{\alpha\nu} f_Q \right] - G_{\mu\nu} \mathcal{L}_m f_Q - 2 \left( f_T g^{\alpha\beta} + f_Q R^{\alpha\beta} \right) \frac{\partial^2 \mathcal{L}_m}{\partial g^{\mu\nu} \partial g^{\alpha\beta}} \\
= \left( k^2 + f_T + \frac{1}{2} R f_Q \right) T_{\mu\nu}, \] (4)

where we setted \( Q = R_{\mu\nu} T^{\mu\nu} \) in order to simplify the expressions during this paper.

We note that when \( f(R, T, R_{\mu\nu} T^{\mu\nu}) \equiv f(R, T) \), from Eq. (4) we obtain the field equations in \( f(R, T) \) gravity [19]. In the expression (4) \( f_R, f_T \) and \( f_Q \) represents the partial derivatives of \( f \) with respect to \( R, T \) and \( Q \) respectively; \( G_{\mu\nu} \) being the Einstein tensor.

Contracting Eq. (4) with respect the tensor metric \( g^{\mu\nu} \), one gets the relation between the Ricci scalar \( R \) and the trace \( T \) of the energy-momentum tensor,

\[
(f_R + \mathcal{L}_m f_Q) R + \nabla_\alpha \nabla_\beta (f_Q T^{\alpha\beta}) + 4 \mathcal{L}_m f_T - 2f + 3 \Box f_R + \frac{1}{2} \Box (f_Q T) + 2f_Q R_{\alpha\beta} T^{\alpha\beta} - 2 g^{\mu\nu} (f_T g^{\alpha\beta} + f_Q R^{\alpha\beta}) \frac{\partial^2 \mathcal{L}_m}{\partial g^{\mu\nu} \partial g^{\alpha\beta}} = \left( k^2 + f_T + \frac{1}{2} R f_Q \right) T. \] (5)

By taking the divergence of the gravitational field equations (4), we obtain the covariant divergence of the energy-momentum tensor as

\[
\nabla_{\mu} T^{\mu\nu} = \frac{2}{(2 + R f_Q + 2 f_R)} \left[ \nabla_\mu (f_Q R^{\sigma\nu} T_{\sigma\nu}) + \nabla_\nu (\mathcal{L}_m f_T) - \frac{1}{2} (f_Q R_{\rho\sigma} + f_T g_{\rho\sigma}) \nabla_\nu T^{\rho\sigma} - G_{\mu\nu} \nabla^\mu (f_Q \mathcal{L}_m) - \frac{1}{2} T_{\mu\nu} \left( \nabla^\mu (R f_Q) + 2 \nabla^\mu f_T \right) \right]. \] (6)
Eq. (6) show that in $f(R, T, R_{\mu\nu}T^{\mu\nu})$ theory the matter energy-momentum tensor is generally not conserved, and this non-conservation determines the appearance of an extra-force acting on the particles in motion in the gravitational field. However in such theory we take often important aspect to guarantee the conservation of energy-momentum tensor.

We assume at follows the matter content of universe as perfect fluid which can be written as

$$T_{\mu\nu} = (\rho + p)u_{\mu}u_{\nu} - pg_{\mu\nu},$$

where $\rho$ and $p$ are the energy density and the pressure of the ordinary matter, respectively and the flat Friedmann-Robertson-walker (FRW) space time with the element line

$$ds^2 = dt^2 - a(t)^2[dx^2 + dy^2 + dz^2].$$

However in present study, we choose the Lagrangian density $L_m = -\rho$. We recall that the expression of the Lagrangian density is not unique, this expression depends on nature of matter source of the universe [19]. Therefore within the consideration that the Lagrangian density $L_m = -\rho$ does not depend on the metric tensor, the term $\frac{\partial^2 L_m}{\partial g^\mu\nu \partial g^{\alpha\beta}}$ vanish and the field equations (4) becomes

$$R_{\mu\nu}f_R - \left[\frac{1}{2} + p + f_T - \frac{1}{2} \nabla_\alpha \nabla_\beta (T_{\alpha\beta})\right]g_{\mu\nu} + (g_{\mu\nu} - \nabla_\mu \nabla_\nu) f_R + \frac{1}{2} R_{\mu\nu} + 2f_Q R_{\alpha\mu}(T_{\alpha\nu}) - \nabla_\alpha \nabla_{(\mu} f_{\nu)} + G_{\mu\nu} f_Q \rho = \left(k^2 + f_T + Rf_Q \right) T_{\mu\nu}.$$  

To simplify, we interest at the general form of model $f(R, T, Q) = R + f(T) + f(Q)$, thus from the field equations (9) the modified Friedmann equations takes respectively the following forms

$$3H^2 = \frac{1}{1 + \rho f_Q} \left[ (k^2 + 2f_T)\rho + \frac{1}{2} f(T, Q) + \frac{3}{2} H \partial_t [(p - \rho)f_Q] - \frac{3}{2} (3H^2 - \dot{H}) \rho f_Q - \frac{3}{2} (3H^2 + \dot{H})p f_Q \right].$$

$$-2\dot{H} - 3H^2 = \frac{1}{1 + \rho f_Q} \left[ k^2 p + f_T(p - \rho) - \frac{1}{2} f(T, Q) + \frac{1}{2} (\dot{H} + 3H^2) \rho f_Q + \frac{1}{2} (3H^2 - \dot{H})p f_Q + 2H \partial_t [(p + \rho)f_Q] + \frac{1}{2} \partial_{tt} [(p - \rho)f_Q] \right].$$
where we shall assume separable algebraic functions of the form \( f(T, Q) = f(T) + f(Q) \).

When the content of matter is considered as ordinary matter with equation of state \( p = w \rho \), we evaluate the expression \( Q = R_{\mu \nu}T^{\mu \nu} \) as

\[
Q = -3\rho \left[ (1 + w)\dot{H} + (1 + 3w)H^2 \right].
\]  

(12)

Eqs. (10) and (11) can be reformulated in the following forms

\[
3H^2 = \kappa_{\text{eff}}^2 (\rho + \rho_{DE}) = \kappa_{\text{eff}}^2 \rho_{\text{eff}},
\]

(13)

\[-2\dot{H} - 3H^2 = \kappa_{\text{eff}}^2 (p + p_{DE}) = \kappa_{\text{eff}}^2 p_{\text{eff}},\]

(14)

where

\[
\kappa_{\text{eff}}^2 = \frac{\kappa^2}{1 + \rho f_Q},
\]

(15)

\[
\rho_{DE} = \frac{1}{\kappa^2} \left[ 2f_T \rho + \frac{1}{2} f(T, Q) + \frac{3}{2} H \partial_t [(p - \rho)f_Q] - \frac{3}{2} (3H^2 - \dot{H}) \rho f_Q - \frac{3}{2} (3H^2 + \dot{H}) p f_Q \right],
\]

(16)

\[
p_{DE} = \frac{1}{\kappa^2} \left[ f_T (p - \rho) - \frac{1}{2} f(T, Q) + \frac{1}{2} (\dot{H} + 3H^2) \rho f_Q + \frac{1}{2} (3H^2 - \dot{H}) p f_Q + 2H \partial_t [(p + \rho)f_Q] + \frac{1}{2} \partial_{tt} [(\rho - p)f_Q] \right],
\]

(17)

respectively.

From the conservation law, the effective energy density \( \rho_{\text{eff}} \) evolves as

\[
\frac{d(\kappa_{\text{eff}}^2 \rho_{\text{eff}})}{dt} + 3H \kappa_{\text{eff}}^2 (\rho_{\text{eff}} + p_{\text{eff}}) = 0.
\]

(18)

Assuming the ordinary matter as dust and by making use of Eqs. (13) and (14), (18) can be rewritten as

\[
f_{QQ} \dot{Q} + (3 + 2H - \frac{\dot{H}}{H} + \frac{\dot{\rho}}{\rho}) f_Q - \frac{1}{3H} (2\kappa^2 + 4f_T + \frac{1}{\rho} f(T, Q) - \frac{6}{\rho} H^2) = 0.
\]

(19)
One can rewrite the above equation using the redshift $z = \frac{1}{a} - 1$ as follows

$$\frac{d^2 H}{dz^2} + \frac{1}{H} \left( \frac{dH}{dz} \right)^2 + \left[ 1 - \frac{5}{1 + z} + \frac{f_Q}{3\rho_0 H^2 (1 + z)^4} \right] \frac{dH}{dz} + \frac{1}{9H^3 f_Q} \left[ \frac{2(\kappa^2 + 2f_T)}{\rho_0 (1 + z)^5} + \frac{1}{\rho_0^2 (1 + z)^8} (f(T, Q) - 6H^2) \right] + \frac{(H - 3)}{3\rho_0 (1 + z)^5} \frac{f_Q}{f_Q Q} - \frac{3H}{(1 + z)^2} = 0 \tag{20}$$

Within the specific choice of $f(R, T, Q)$ model we can solve numerically Eq.\,(20) to analyze the evolution of cosmological parameters \cite{29} such as Hubble parameter $H(z)$, deceleration parameter $q(z)$, equation of state of dark energy $\omega_{DE}(z)$ and Ricci scalar $R(z)$ versus $z$.

## 3 Stability analysis in $f(R,T,Q)$ gravity

In this section, we study the linear stability of the model $f(R, T, Q) = R + f(T) + f(Q)$ using the power law and de Sitter solutions. We will be interested to the perturbation of both the geometrical and matter parts of the modified equations of motion and we consider the ordinary matter as dust. For this, we focus our attention to the Hubble parameter for what concerns the geometry and the energy density of the ordinary content (dust) concerning the matter of the background, and perform the perturbation about them as \cite{38-39}

$$H(t) = H_b(t)(1 + \delta(t)), \quad \rho(t) = \rho_b(t)(1 + \delta_m(t)), \tag{21}$$

where $H_b(t)$ and $\rho_b(t)$ satisfies the modified Friedmann equations \(\text{(I)}\) and \(\text{(II)}\) and denote respectively the Hubble parameter and the energy density of the ordinary matter of the background. $\delta(t)$ and $\delta_m(t)$ being, the perturbation functions about the geometry and the matter, respectively. Regarding the form of the $f(R, T, Q)$ model that we considered in this section, the novelty here is the effect coming from $f(T, Q) = f(T) + f(Q)$. Then, we develop the model $f(T, Q)$ as function of two variable $T$ and $Q$ in a series of $T_b = \rho_b$ and $Q_b = -3\rho_b (\dot{H}_b + H_b^2)$ respectively, as follows

$$f(T, Q) = f(T_b, Q_b) + \frac{\partial f}{\partial T}(T - T_b) + \frac{\partial f}{\partial Q}(Q - Q_b) + \mathcal{O}^2, \tag{22}$$
where $O^2$ term includes all the terms proportional to the higher powers of $T$, $Q$ respectively. From the standard conservation law of the energy-momentum tensor, we deduct the energy density of the ordinary matter at background as

$$\rho_b(t) = \rho_0 e^{-\frac{3}{2} \int H_b(t) dt}, \quad (23)$$

with $\rho_0$ the constant of integration. Considering that the ordinary matter is essentially the dust, the first modified Friedmann equations (10) yields

$$3H^2 = \frac{1}{1 + \rho f_Q} \left[(k^2 + 2f_T)\rho + \frac{1}{2} f(T, Q) - \frac{3}{2} H\partial_t(\rho f_Q) - \frac{3}{2} (3H^2 - \dot{H})\rho f_Q \right]. \quad (24)$$

By substituting the Eqs. (21) and (22) in (24), one gets the following differential equation

$$\ddot{\delta} + \dot{\delta} \left[\frac{-2}{3\rho_b H_b} f_Q^b + \left(4\rho_b H_b + \frac{\dot{\rho}_b}{\rho_b} + \frac{\dot{H}_b}{H_b}\right) f_Q^b - 3f_Q^{bQQ}\left(\dot{\rho}_b \dot{H}_b + \rho_b \dot{H}_b^2 + 2\rho_b H_b \dot{H}_b\right)\right] + \frac{H_b}{\rho_b} \dddot{\delta}_m +$$

$$\delta \left[-\frac{4}{3\rho_b} - \frac{2}{\rho_b} f_Q^b + f_Q^b \left(-6 + 9H_b^2 - \frac{H_b^2}{H_b^2} + 2\frac{\dot{\rho}_b}{\rho_b} \frac{\dot{H}_b}{H_b} + 3\frac{\dot{\rho}_b}{\rho_b} \frac{\dot{H}_b}{H_b} + 2\frac{\dot{H}_b}{H_b} + 3\dot{H}_b\right) - 3f_Q^{bQQ}\left(\frac{\dot{\rho}_b}{H_b} \frac{\dot{H}_b^2}{H_b} + 3H_b \dot{H}_b \dot{\rho}_b + 2\dot{\rho}_b H_b^3 + \frac{\rho_b}{H_b} \frac{\dot{H}_b}{H_b} + 3\frac{\dot{\rho}_b}{\rho_b} \frac{\dot{H}_b}{H_b} + 3\frac{\dot{\rho}_b}{\rho_b} \frac{\dot{H}_b}{H_b} + 2\frac{\dot{\rho}_b}{\rho_b} H_b^2 + 4\frac{\rho_b}{H_b} H_b^2 \frac{\dot{H}_b}{H_b}\right)\right] +$$

$$\delta_m \left[\frac{2\kappa^2}{9\rho_b H_b^2} - \frac{1}{\rho_b} f_Q^b + \frac{5}{9\rho_b H_b^2} f_T^b + \frac{1}{9H_b^2} f_T^{bTT} + f_Q^{bQQ} \left(2H_b^2 - \frac{\dot{H}_b^2}{H_b^2} + 4\frac{\dot{\rho}_b}{\rho_b} \frac{\dot{H}_b}{H_b} + 4\frac{\dot{H}_b}{H_b} + 4\frac{\dot{\rho}_b}{\rho_b} H_b + \frac{\dot{H}_b}{H_b} + 3\dot{H}_b\right) - 3f_Q^{bQQ}\left(\frac{\dot{\rho}_b}{H_b} \frac{\dot{H}_b + 3H_b \dot{H}_b \dot{\rho}_b + H_b^3 \dot{\rho}_b + \rho_b \frac{\dot{H}_b}{H_b} + \rho_b \dot{H}_b + 2\rho_b H_b^2 + 2\rho_b \frac{\dot{H}_b}{H_b}\right)\right] = 0. \quad (25)$$

Regarding the matter perturbation function, we obtain the differential equation

$$\dddot{\delta}_m(t) + 3H_b(t) \ddot{\delta}(t) = 0. \quad (26)$$

By making use of Eqs. (25) and (26), we obtain the differential equation
\[ \ddot{\delta}_m + \dot{\delta}_m \left[ -2 \frac{\dot{H}_b}{H_b} - \frac{2}{3 \rho_b H_b} f^b_Q + \left( 4 \rho_b H_b + \dot{\rho}_b + \frac{\dot{H}_b}{H_b} \right) f^b_{QQ} \right] + \dot{\delta}_m \left[ -2 \frac{\dot{H}_b^2}{H_b^2} + \frac{\dot{H}_b}{H_b} - 3 \frac{\dot{H}_b}{\rho_b} - \frac{4}{3 \rho_b^2} + \right. \\
\left. \left. -3 f^b_{QQQ} \left( \dot{\rho}_b \dot{H}_b + \dot{H}_b^2 \dot{\rho}_b + 2 \rho_b H_b \dot{H}_b \right) \right] \right] = 0. \tag{27} \]

Once the numerical solution of the equation (27) be done, we can perform the evolution of the geometrical perturbation function \( \delta(t) \) through the equation (26). To illustrate the viability (stability) of the \( f(R, T, Q) \) model, we interest in next subsection at the evolution of geometrical and matter perturbation functions \( \delta \) and \( \delta_m \) within the de Sitter and power-law solutions. To do, we consider two specific actions [33]

1. \( f(R, T, Q) = R + \alpha Q, \tag{28} \)
2. \( f(R, T, Q) = R + \alpha Q + \beta \sqrt{T}, \tag{29} \)

where \( \alpha \) and \( \beta \) are constant parameters. These models have been studied in [33] to analyze the evolution and dynamics of the universe for the above with and without energy conservation. Recently, the energy conditions and the thermodynamic laws of these particular models have been investigated by the authors of the references [35]–[36].
3.1 Stability of de Sitter solutions

In de Sitter solutions, the Hubble parameter for the background is a constant and one has $H_b(t) = H_0$. Thus the equations (23) and (27) reduces to

$$\rho_b(t) = \rho_0 e^{-3H_0 t},$$

(30)

$$\ddot{\delta}_m + c_1 \dot{\delta}_m + c_2 \delta_m + c_3 \delta_m = 0,$$

(31)

where

$$c_1 = -\frac{2}{3\rho_b H_0} f_Q + H_0 f_{QQ}^b (4\rho_b - 3) - 3H_0^2 \dot{\rho}_b f_{QQQ}^b,$$

(32)

$$c_2 = -\frac{3H_0^2}{\rho_b} - \frac{4}{3\rho_b^2} - \frac{2}{\rho_b} f_Q - 6f_{QQ}^b - 6H_0^3 \dot{\rho}_b f_{QQQ}^b,$$

(33)

$$c_3 = \frac{2\kappa^2}{9\rho_b H_0^2} - \frac{1}{\rho_b} f_Q + \frac{5}{9\rho_b H_0 f_T^b} f_Q^b + \frac{1}{9H_0^2} f_{TT}^b - 10H_0^2 f_{QQ}^b - 3H_0^3 \dot{\rho}_b f_{QQQ}^b.$$

(34)

3.2 Stability of power-law solutions

In this fact, we are interested to the cosmological solutions of the form $a(t) \propto t^n \Rightarrow H_b(t) = \frac{n}{t}$. For this, the equations (23) and (27) are becomes

$$\rho_b(t) = \rho_0 t^{-3n},$$

(35)

$$\ddot{\delta}_m + c'_1 \dot{\delta}_m + c'_2 \delta_m + c'_3 \delta_m = 0,$$

(36)

where

$$c'_1 = \frac{2}{t} - \frac{2}{3\rho_0 n} f_Q^{3n+1} f_T^b + \left(\frac{4\rho_0 n}{f_T^{3n+1}} - \frac{(3n + 1)}{t}\right) f_{QQ}^b + \frac{3n^2 \rho_0}{t^{3n+3}} (1 - n) f_{QQQ}^b.$$

(37)
Figure 1: The figures show the evolution of the perturbation functions $\delta_m$ and $\delta$ within the de Sitter solutions. The graph at the left side of the figure represents the evolution of these perturbation functions for the model $f(R, T, Q) = R + \alpha Q$, while the one at the right side shows the evolution of the perturbation functions for the model $f(R, T, Q) = R + \alpha Q + \beta \sqrt{T}$. The graphs are plotted for $\rho_0 = 1$, $\alpha = 3$, $\kappa = 1.5$, $H_0 = 0.3$ and $\beta = 1.5$.

\begin{align}
  c_2' &= -\frac{3n^2}{\rho_0 t^{2-3n}} - \frac{4}{3\rho_0^2} t^{6n} - \frac{2}{\rho_0} \left( \frac{n}{3t^{2-3n}} + t^{3n} \right) f_Q^b + f_QQ^b \left( -6 - \frac{1}{t^2} (5 - 9n) + \frac{1}{t^4} (n^2 - 3n^3) + \frac{4n^2 \rho_0}{t^{3n+4}} \right) , \\
  -3\rho_0 n f_{QQQ}^b &\left( \frac{6n^3}{t} + 9n^2 t^{3n-4} + \frac{1}{t^{3n+4}} (3n^4 - n + 2) + \frac{6}{t^{3n+3}} + \frac{5n^3}{t^{3n+6}} \right) , \\
  (38)
\end{align}

\begin{align}
  c_3' &= \frac{1}{9\rho_0 n^2} (2\kappa^2 + 5f_T^b) t^{3n+2} - \frac{t^{3n} f_T^b}{\rho_0} + \frac{t^2}{9n^2} f_{TT}^b + \left( - \frac{1}{t} - \frac{1}{t^2} (10n^2 - 9n + 1) + \frac{8n^2}{t^4} \right) f_{QQ}^b , \\
  -3f_{QQQ}^b &\left( \frac{3n^2}{t^3} + \frac{\rho_0}{t^{3n+4}} (9n^3 + 2n^2 - 2n) - \frac{3n^4 \rho_0}{t^{3n-2}} - \frac{n \rho_0}{t^{3n+2}} - \frac{2 \rho_0}{t^{3n+1}} \right) . \\
  (39)
\end{align}

Regarding the evolution of the perturbation functions plotted in figures 1 and 2, we see that when the universe expands i.e the time evolves, the matter and geometric perturbation functions $\delta_m$ and $\delta$ converge, respectively. We conclude that the both models considered presents stability through the convergence of the geometrical and matter perturbation functions $\delta_m$ and $\delta$ within the de Sitter and power-law solutions.
Figure 2: The figures show the evolution of the perturbation functions $\delta_m$ and $\delta$ within the power-law solutions. The graph at the left side of the figure represents the evolution of these perturbation functions for the model $f(R, T, Q) = R + \alpha Q$, while the one at the right side shows the evolution of the perturbation functions for the model $f(R, T, Q) = R + \alpha Q + \beta \sqrt{T}$. The graphs are plotted for $\rho_0 = 1$, $\alpha = 0.1$, $\kappa = 1.5$, $H_0 = 2$, $n = 0.5$ and $\beta = 1.5$.

4 Conclusion

In this present paper, we have performed the stability analysis in gravitational theory in which matter is coupled to geometry, with the effective Lagrangian of the gravitational field being given by an arbitrary function of the Ricci scalar $R$, the trace of the matter energy-momentum tensor $T$, and the contraction of the Ricci tensor with the matter energy-momentum tensor $Q = R_{\mu\nu}T^{\mu\nu}$. By choosing the general form $f(R, T, Q) = R + f(T) + f(Q)$, we obtained the modified FRW field equations. In order to check the viability of this model, we establish the differential equation of matter and geometric perturbation functions and we perform its stability taking into account the de Sitter and power-law cosmological solutions. To illustrate how these perturbation functions ($\delta_m$ and $\delta$) can constrain the $f(R, T, Q)$ gravity, we have taken two particular cases, namely
\[ f(R, T, Q) = R + \alpha Q \] and \[ f(R, T, Q) = R + \alpha Q + \beta \sqrt{T}. \] We see that for the both considered solutions, the models considered presents stability through the convergence of the geometric and matter perturbation functions \( \delta \) and \( \delta_m \). The higher derivatives \( f(R, T, Q) \) gravity is very complex and can open a new perspective on the very early stages of the evolution of the Universe.

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