Improving students' higher order mathematical thinking skills in accelerated classes through purdue learning model

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Abstract. The acceleration program is a positive alternative for intellectually gifted students to obtain relevant education with their potential to develop optimally. The purpose of this study was to improve higher order mathematical thinking skills (HOMTS) in accelerated class students through the Purdue learning model. The research was conducted at one of the state junior high schools in Tangerang Selatan for the 2016/2017 academic year. The research method was Classroom Action Research (CAR) in two cycles. Data were collected using observation sheets, daily journals, and tests. The results revealed that the application of the Purdue learning model could increase the average HOMTS of 63.75 in the first cycle to 74.25 in the second cycle. The improvement of HOMTS includes aspects of analyzing, evaluating, and creating. Student activities including visual, oral, writing, drawing, motor, mental and emotional also increased in the very good category. In addition, the results of the study also revealed that most students gave positive responses to the Purdue learning model. The conclusion of this study is that the application of the Purdue model in mathematics learning can improve HOMTS, activities, and students' positive responses.

1. Introduction

Education in the 21st century prioritizes the importance of HOTS (high order thinking skills), including communication, collaboration, critical thinking, creative thinking, computational logic, and compassion. This capability can be developed in educational institutions. Mathematics is a subject that has the potential to develop HOTS. Mathematics is a method of logical, critical, creative, analytical, and systematic thinking that needs to be taught to students from primary to higher education. According to Abdurrahman, mathematics is a logical way of thinking, a tool for solving problems and an instrument for recognizing relationship patterns and generalizing experiences [1].

Mathematics learning in schools plays an important role in developing students’ mathematical thinking skills. In general, based on its depth and complexity, there are two levels of mathematical thinking, namely low-order thinking (LOT) and high-order thinking (HOT) [2]. Brookhart defines Higher Order Thinking (HOT) into three categories, namely transfer, critical thinking, and problem solving [3]. HOT as a transfer is the ability of students to apply their knowledge to other knowledge beyond what they have learned. HOT as critical thinking is the ability of students to consider, reflect, and make the right decisions. According to Barahal, critical thinking is defined as the art of thinking which includes reasoning, questioning and investigating, observing and describing, comparing and connecting, finding complexity, and exploring points of view. Furthermore Nitko and Brookhart, HOT as a problem solving is a process of thinking when students encounter a problem and want to solve it
but do not automatically find the right solution, so they have to use one or more higher-order thinking processes [3]. Higher-order thinking is the ability to complete tasks for which no algorithm can directly be used, requires justification and may have more than one solution [4].

HOT ability indicators are related to divergent and convergent thinking skills or creative and critical thinking. Divergent thinking is spontaneous and free flowing, whereas convergent thinking is systematic and logical [5]. Furthermore, Ayuningtyas and Rahaju argued that the HOT indicator includes analyze (differentiating, organizing, attributing), evaluate (checking, critiquing), and create (generating, planning, and producing) [6].

However, the students’ higher order mathematical thinking skill (HOMTS) was still low. The results of the PISA (Program for International Students Assessment Program) in 2018 reported that the mathematics ability of Indonesian students was ranked 73 out of 78 countries with an average score of 379 from the international average of 489 [7]. PISA test consist of six levels of mathematical ability, specifically levels 4, 5, and 6 are measuring HOMTS indicators, namely the ability to choose, compare, evaluate strategies, create concepts, generalize and use information based on investigations and modeling complex problem situations. The 2018 PISA test results show that less than 10% of Indonesian students can answer the test [8]. Furthermore, the results of pre-research on the acceleration program at a State Junior High School in Tangerang Selatan, showed that the average HOMTS of students was only 53.79 with the ability to analyze (39.00), evaluate (61.00), and creating (69.50). These results indicate that the students’ HOMTS in the accelerated class is still low.

One alternative learning model that can develop students’ HOMTS is the Purdue model. The Purdue model has learning stages that can make students active in developing HOMTS. These stages are the enrichment of the three stages of learning for gifted students, namely the stage of creative thinking skills (fluency, flexibility, originality, elaboration), the stage of solving complex problems (problem finding, clarification, problem analysis, evaluation), and the independent learning stage (independence, synthesis, implementation, effectiveness) [9]. In Indonesia, several top schools hold accelerated classes. Accelerated classes are specially designed for students who want to accelerate the learning process at a certain level of education with an accelerated learning program with a student study period of 2 years. Accelerated classes can be a place for gifted students.

Several previous studies related to high order thinking skills, including the research of Kutlu & Gökder, found that the Purdue model can improve students’ creative thinking skills [10]. Research by Ghasempour et al., reported that problem posing has all the criteria of a practical assignment to increase HOT skills among engineering students [11]. Winarso's findings, revealed that learning is still teacher-centered, the approach is expository, the tasks are more routine and rarely provide questions that require students to use higher order thinking skills [12].

Several previous studies generally only examined HOT in regular classes with conventional learning models, almost no research has been done regarding the increase in HOT through the Purdue model in accelerated classes at the junior high school level. The distinctive difference from this research is the development of the Purdue learning model tool for accelerated classes and the HOMTS measurement test instrument. Therefore the purpose of this study is to improve students’ HOMTS includes aspects of analyzing, evaluating, and creating. Student activities including visual, oral, writing, drawing, motor, mental and emotional, and students' positive attitudes towards the implementation of the Purdue learning model.

2. Method
This research was conducted at a State Junior High School in Tangerang Selatan, Indonesia in the 2016/2017 school year. The research subjects were 28 students of 8th grade-Acceleration. The acceleration class was chosen as the research class because the characteristics of the Purdue model were suitable for gifted students. This research uses Classroom Action Research (CAR). The research was conducted in two cycles with the Purdue model of learning intervention. Each cycle consists of four stages, namely planning, acting, observing, and reflecting. The first and second cycles were each carried out in three class meetings. The CAR design is presented in figure 1.
2.1 Pre-research stage

Before the Purdue model learning intervention was carried out, pre-research activities were carried out as follows:

2.1.1 Observation of the learning process
Researchers made observations of the learning process in 8th grade-Acceleration to be able to identify problems, find causes of problems, and find alternative solutions.

2.1.2 Interviews with subject teachers
Researchers conducted interviews with subject teachers before the implementation of cycle I began to determine the conditions and responses of students to the mathematics learning process that is usually carried out in class as well as to record the teacher’s efforts in improving students’ HOMTS abilities.

2.1.3 Providing HOMTS test
Researcher gave HOT ability test to determine the level of HOMTS ability of students in 8th grade-Acceleration. The test materials involved are comparisons, sets, transformations, flat shapes, and data presentation in 7th grade.

2.2 Research implementation stage

2.2.1 Planning
At this stage, the researcher prepared a lesson plan based on the stages of the Purdue model, made interview guidelines, observation sheets, daily journals, student worksheets, and the HOMTS test.

2.2.2 Acting
At the acting stage, researchers collaborate with field teachers in implementing learning scenarios. Researchers act as actors of action while teachers act as observers. The acting stage in table 1.

| Table 1. Acting stage in Cycle I and Cycle II. |
|-----------------------------------------------|
| Stages | Cycle I | Cycle II |

Figure 1. The design of the CAR.
Stage I:

- Fluency
- Flexibility
- Originality
- Elaboration

Researchers provide open-ended problems to students that focus on developing students' creative thinking skills.

Researchers carry out learning with an inquiry strategy that focuses on developing students' creative thinking skills.

Stage II:

- Problem Finding
- Clarification
- Problem Analysis
- Evaluation

Researchers classify students into small groups consisting of 4-5 people.

Researchers classify students into small groups that are more heterogeneous of 4-5 people.

Participants carry out learning with the STAD strategy by focusing on developing students' problem-solving abilities.

Stage III:

- Independence
- Synthesis
- Implementation
- Effectiveness

Researchers provide tasks to students in the form of worksheets to be done individually by developing previously acquired knowledge, combining ideas and integrating concepts at the independence stage to be used in other problems, applying the knowledge that has been obtained, and presenting the results of the discussion in front of the class and relating it to everyday life.

2.2.3 Observing

Observations are carried out by working with collaborator teachers to collect data and explore and document student activities and responses during the learning process.

2.2.4 Reflecting

At this stage the researcher analyzes the results of giving actions in each cycle, discusses the results of observations with the field teacher and evaluates the actions in each cycle. If the results of reflection have not reached the criteria or benchmarks for success, then the action is continued to the next cycle.

2.3 Data collection techniques

Data collection using test and non-test techniques. HOMTS data collection used an essay test equipped with a rubric (0 - 4) adapted from Bosch [13]. Non-test, namely daily journal, used to determine student responses to the Purdue learning model. Observation sheets are used to determine the extent to which students are active in the learning process. Interviews were conducted to determine firsthand the condition of students and to find out an overview of the learning process. Meanwhile, field notes are used to record findings during learning that are not observed in observation.

2.4 Research instrument

The instrument developed was the HOMS test in the form of an essay of 8 items that measured indicators of analysis, evaluation, and creation. The research instrument must have quality from the aspects of validity and reliability. The validity of the contents of the HOMTS test was determined using the Content Validity Ratio (CVR) method from Lawshe (1975). The determination of content validity involved 13 expert in the field of mathematics, including 5 mathematics lecturer and 8 mathematics teachers. CVR formula [14].

\[
CVR = \frac{(N_e - N/2)}{N/2}
\]

Where CVR is the content validity ratio, \(N\) is the total number of panellist, dan \(N_e\) is the number of panellist in indicating essential. Furthermore empirical validity was obtained through HOMST testing to 56 students of class 8th at junior high schools. Validity testing uses the product moment formula.
$r_{xy} = \frac{N(\sum XY) - (\sum X)(\sum Y)}{\sqrt{[N(\sum X^2) - (\sum X)^2][N(\sum Y^2) - (\sum Y)^2]}}$ (2)

Where $r_{xy}$ is the correlation coefficient between item score (X) and total score (Y). The formula (2) for $\alpha = 0.05$, item is valid if the $r_{xy}$ ≥ $r_{table}$ and if $r_{xy} < r_{table}$ is the item is not-valid.

The reliability coefficient of the HOMTS test was carried out using the Alpha Cronbach formula.

$$r_{11} = \left(\frac{k}{k-1}\right)\left(1-\frac{\sum \sigma_i^2}{\sigma^2}\right)$$ (3)

Where $r_{11}$ is the reliability coefficient, $k$ is the number of items, $\sum \sigma_i^2$ is the number of the item score variance and $\sigma^2$ is the total variance. If $r_{11} > 0.70$ from the formula (3), reliability is good [15].

2.5 Benchmarks of achievement

The benchmarks for the cycle of this study are as follows: (1) The HOTMS test result at the end of each cycle has obtained ≥70% the student's score reaches minimal mastery ≥ 75; (2) The average percentage of student learning activities reaches ≥ 75%; (3) The average percentage of students' positive responses in learning the Purdue model reaches ≥ 80%.

3. Results and Discussion

3.1 Results of instrument validation

The results of the content validity and empirical validity of the HOMTS test items in Table 2.

| Indicators | Items | CVR | Min-CVR | $r_{obs}$ | $r_{table}$ | Decision |
|------------|-------|-----|---------|-----------|-------------|----------|
| Analysis   | Item 01 | 0.69 | 0.54   | 0.680     | 0.294       | Valid    |
|            | Item 02 | 0.85 | 0.54   | 0.641     | 0.294       | Valid    |
|            | Item 03 | 0.85 | 0.54   | 0.679     | 0.294       | Valid    |
|            | Item 04 | 0.69 | 0.54   | 0.613     | 0.294       | Valid    |
| Evaluation | Item 05 | 0.69 | 0.54   | 0.576     | 0.294       | Valid    |
|            | Item 06 | 1.00 | 0.54   | 0.725     | 0.294       | Valid    |
| Create     | Item 07 | 0.69 | 0.54   | 0.407     | 0.294       | Valid    |
|            | Item 08 | 0.69 | 0.54   | 0.733     | 0.294       | Valid    |

The results of the analysis in Table 2 reveal that 8 items are valid with a reliability coefficient of $r_{11} = 0.710$ or classified as high either category. Thus, the instrument can be used to capture HOMTS data.

3.2 Students' higher order mathematical thinking skills (HOMTS)

Results of the students' HOMTS score in the pre-Cycle, Cycle I and Cycle II in Table 3.

| Statistik   | Pre-cycle | Cycle I | Cycle II |
|-------------|-----------|---------|----------|
| Minimum score | 25.00 | 25.00 | 50.00 |
| Maksimum score | 81.25 | 87.50 | 100.00 |
| Average ($\bar{x}$) | 53.79 | 63.79 | 74.25 |
| % Student mastery (≥75) | - | 53.57 | 71.43 |
| % Student not mastery (<75) | - | 46.63 | 28.57 |
The results of the analysis in table 3 reveal that overall, the average HOMTS score increased from 63.79 in Cycle I to 74.25 in Cycle II. Achievements in cycle II has reached > 70% of students who get a score of 75. Furthermore, the average HOTS score at the end of Cycle I and Cycle II according to the indicators is presented in figure 2.

![Figure 2. HOMTS Score in Cycle I and Cycle II according to the indicators.](image)

Based on the results of the analysis in figure 2, it shows that after the Purdue model learning intervention, the overall HOMTS indicator analyzing, evaluating, and creating an average increase from Cycle I to Cycle II. Examples of questions and answers to analyzing indicators in figure 3.

| Cycles | Questions                                                                 | Answer |
|--------|---------------------------------------------------------------------------|--------|
| Cycle I| "Show that the linear equation  
g1: 3x + 11y - 18 = 0,  
g2: 5x = 30 - y, and  
g3: 2x + 40 = 10y are the lines of a right triangle. Explain the concept used!" |        |
| Cycle II| "Ahmad left for school at 6:15 a.m., but was 20 minutes late from school time. The next day, Ahmad left 15 minutes early, apparently he was still five minutes late from school time. It is known that the travel time from home to school is always constant. Construct the equation for the line for the problem. Conclude at what time Ahmad has to leave home so he won't be late!" |        |

![Figure 3. Examples of questions and answers to analyzing indicators.](image)
HOMTS increase in analyzing indicators is supported by providing guidance that is more focused on passive students during learning to be more active in expressing opinions, questions, and responses to classmates' presentations. Giving worksheets during Cycle I made students gradually get used to solving problems. This is facilitated by the development stage of complex problem solving in worksheets which requires students to do problem finding, clarification, problem analysis, and evaluation of a problem. The findings of this study are similar to those of Prasetyani et al., that problem-based learning can train students to use higher order thinking skills that require students to analyze, evaluate, and also be creative [16].

The achievement of the evaluation indicator score was 84 in Cycle II which was classified as high. The increase in Cycle II was supported by the provision of more problems to evaluate accuracy of the data for each stage. Examples of questions and answers to evaluating indicators in figure 4.

| Cycles | Questions | Answer |
|--------|-----------|--------|
| Cycle I | "We know that line II with the equation \((5x + 3y) - 2a (x - y) = 3 \) and line I2 with the equation \((4y - 5x) + a (y - x) = a \). Is it true that the value of \(a\) must be \(-3\frac{1}{2}\) in order for the two lines to be expressed as two parallel lines? Give an explanation of each step used!" | ![Worksheet Example](image1) |
| Cycle II | "Coordinates A (-3, 2), B (3, 5), and C (4, -2). If the lines AB intersect at point C, is it true that the line \(2x + y - 6 = 0\) is the line that passes through point C and is perpendicular to the line AB? Give an explanation of each step used!" | ![Worksheet Example](image2) |

Figure 4. Examples of questions and answers to evaluating indicators.

The HOMTS score for the creative indicator has increased by 70 in cycle I to 78 in cycle II. Even though it has increased, some students are still not used to creating truly original ideas. As in cycle II questions, when students are asked to create ideas in arranging daily life problems related to straight line equations, some students have not been able to define the variables used correctly so that the problems they make do not represent straight line equations. One of the factors causing the lack of optimal creative indicators is that students in the acceleration class before this research were conducted, had a habit of working on questions in a drill, procedurally, and directly used the formula when working on the questions so that students' thought processes could not be seen.

The increase in the average HOMTS score of students cannot be separated from the improvements in the Purdue model of the learning process as a result of reflection on Cycle I. Several improvements were made, including providing opportunities for students to review learning materials, involving students who were less active in expressing opinions. The researcher also made improvements to the little group discussion stage with the STAD cooperative technique. Furthermore, the independent study
stage, the researcher used the small group activity technique with two students in pairs. Examples of questions and answers to creating indicators are presented in Figure 5.

| Cycles  | Problem                                                                 | Answer                                                                 |
|---------|-------------------------------------------------------------------------|------------------------------------------------------------------------|
| Cycle I | "Make a straight line equation that has a positive slope, then graph it on the coordinate plane!" |                                                                         |
| Cycle II| "Given the straight line equation $y = 20 + 5x$. Create your ideas to create stories in everyday life that illustrate the equation of these straight lines!" |                                                                 |

Figure 5. Examples of questions and answers to creating indicators.

3.3 Student activities

The average percentage of student activity in the Purdue model learning in Cycle I and Cycle II is presented in Figure 6.

Figure 6. Percentage of student activity in Cycle I and Cycle II.

The results of the analysis in Figure 6 reveal that there is an increase in the average percentage of student activity from the first cycle of 67.62% to the second cycle of 80%. 80% of student activity achievement in cycle II met the criteria, namely ≥75%. This shows that the Purdue model of learning can improve student learning activities.
Other findings reveal that the increase in oral activities and motor activities is still low in Cycle II. The aspect of oral activities is the activity of students in presenting the results of group discussions, asking questions, and responding to friends' opinions. Although in this aspect, some students in Cycle I were still passive, and were just starting to be brave enough to express their opinions in their presentations in Cycle II. The findings of this study are similar to the findings of Respati et al., that students in schools administering the acceleration program have several symptoms of negative behavior, including students who lack communication, experience tension, lack of association and do not like sports lessons. Accelerated classes only accelerate the cognitive development of students, but do not accelerate the affective and psychomotor aspects [17].

Furthermore, drawing activities also increased in Cycle II. Drawing activities represent student activities in drawing straight line equation graphs. Aspects of visual activities, mental activities and emotional activities also improved quite well in cycle II. The improvement of the visual aspects of activities was supported, among others, by paying attention to material explanations during the presentation. The improvement of mental activities is supported by involving students voluntarily to review material at previous meetings. The increase in emotional activities is supported by the enthusiasm of students during the Purdue model learning. Likewise, writing activities that capture student activities in noting important things and making summaries. By integrating STAD techniques and small group activities in the Purdue model of learning, it supports students to have a sense of responsibility to carry out these activities individually. The findings of this study are similar to those of Hiriza that found cooperative learning type STAD, able to build cooperation in groups, dare to ask questions if there are difficulties in learning, actively answer questions from teachers and dare to come forward to present group learning outcomes [18].

3.4 Student responses
The average percentage of student responses to the Purdue model learning in figure 7.

![Figure 7](image)

Based on the results in figure 7, it shows that students' positive responses to the Purdue model of learning increased from cycle I to cycle II. Based on daily journals, the average percentage of positive responses of 80% in cycle II is in accordance with the benchmarks of success, success ≥80%. This shows that the Purdue model of learning can increase students' positive responses. Positive responses, including students' statements that learning is more fun, challenging, and making students more active. The research findings also revealed that students' neutral and negative responses to the Purdue model of learning decreased from cycle I to cycle II.

Researcher also found that the attitudes of accelerated class students tended to be more individualized and spent more time learning cognitive aspects. This is in line with Wardhani's findings, that students who take accelerated classes feel they do not have free time for activities outside school hours such as extracurricular activities because they have to follow tight schedules and
subject matter, resulting in a lack of student social interaction both at school and in the environment [19]. However, students in accelerated classes have advantages in terms of task commitment. Most of the students were able to complete assignments well and on time. This can be seen during the learning process, where some students at the beginning of the Purdue model of learning feel bored working on worksheets, but they are still able to be responsible for their respective assignments. The research findings are similar to those of Nawantara and Arofah’s research that the level of assignment commitment of accelerated class students is higher than the level of assignment commitment of regular class students [20].

4. Conclusion
Overall, the Purdue model learning can increase students’ HOMTS by 63.79 in Cycle I to 74.25 in Cycle II. These improvements include mathematical thinking skills in the aspects of analyzing, evaluating, and creating. Mathematical thinking skills that are more prominent are the skills to analyze and evaluate. Student activity in the accelerated class has increased with the Purdue model of learning. Increased activities include visual activities, oral, writing, drawing, motor, mental, and emotional activities. The most prominent activities are mental and emotional activities. Meanwhile, activities that are classified as less prominent are oral and drawing activities. Students give a positive response to the Purdue learning model. Most of the students stated that learning with the Purdue model was more fun and they were challenged to complete the tasks in the student worksheet. This study suggests that teachers can use and adapt the design of the Purdue learning model by integrating HOMTS-based assessments and mathematics learning media and technology.

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