On the smallness of the cosmological constant in SUGRA models

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Abstract

In no–scale supergravity global symmetries protect local supersymmetry and a zero value for the cosmological constant. We consider the breakdown of these symmetries and present a minimal SUGRA model motivated by the multiple point principle, in which the total vacuum energy density is naturally tiny. In order to reproduce the observed value of the cosmological constant and preserve gauge coupling unification, an additional pair of $5 + 5$–plets of superfields has to be included in the particle content of the considered model. These extra fields have masses of the order of the supersymmetry breaking scale; so they can be detected at future colliders. We also discuss the supersymmetry breakdown and possible solution of the cosmological constant problem by MPP in models with an enlarged gauge symmetry.

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1 Introduction

The origin of a tiny energy density spread all over the Universe (the cosmological constant \( \Lambda \)), which is responsible for its accelerated expansion, is one of the most challenging problems nowadays. A fit to the recent data shows that \( \Lambda \sim 10^{-123} M_{Pl}^4 \sim 10^{-55} M_Z^4 \). At the same time the presence of a gluon condensate in the vacuum is expected to contribute an energy density of order \( \Lambda_{QCD}^4 \sim 10^{-74} M_{Pl}^4 \). On the other hand if we believe in the Standard Model (SM) then a much larger contribution \( \sim v^4 \sim 10^{-62} M_{Pl}^4 \) must come from the electroweak symmetry breaking. The contribution of zero–modes is expected to push the vacuum energy density even higher up to \( \sim M_{Pl}^4 \). Therefore the smallness of the cosmological constant should be regarded as a fine-tuning problem, for which new theoretical ideas must be employed to explain the enormous cancellations between the contributions of different condensates to the total vacuum energy density.

At this moment none of the available generalizations of the SM provides a satisfactory explanation for the smallness of the cosmological constant. An exact global supersymmetry (SUSY) ensures zero value for the energy density at the minimum of the potential of the scalar fields. However, in the exact SUSY limit, bosons and fermions from one chiral multiplet are degenerate. Because superpartners of quarks and leptons have not been observed yet, supersymmetry must be broken. In general the breakdown of supersymmetry induces a huge and positive contribution to the total vacuum energy density of order \( M_S^4 \), where \( M_S \) is the SUSY breaking scale. The non–observation of superpartners of observable fermions implies that \( M_S \gg 100 \text{ GeV} \).

Our basic scenario for evaluating the tiny value of the cosmological constant is to assume the existence of a second vacuum degenerate with the one in which we live. We assume that our vacuum is a softly broken supersymmetric vacuum and that the second vacuum is supersymmetric without any soft SUSY breaking terms. However we imagine that the supersymmetry in the second vacuum is broken dynamically, when the supersymmetric QCD interaction becomes non-perturbative. This happens at a much lower energy scale than \( \Lambda_{QCD} \), since the supersymmetric QCD beta function must be used, and thereby generates a small cosmological constant. This small value is then transferred to our vacuum by the assumed degeneracy.

The assumed degeneracy of the vacua is supposed to be justified by the so-called Multiple Point Principle (MPP), according to which Nature chooses values of coupling constants such that many phases of the underlying theory should coexist. On the phase diagram of the theory it corresponds to the special point – the multiple point – where many phases meet. The vacuum energy densities of these different phases are degenerate at the multiple point.
In the case of global supersymmetry, the energy density of a supersymmetric vacuum is naturally zero. However, since we are interested in the value of the cosmological constant, we must include gravity and thus local supersymmetry. In supergravity (SUGRA) models, the vacuum energy density is not naturally zero; indeed in general it is expected to be large and negative. In our MPP scenario above, prior to the dynamical SUSY breaking in the second vacuum, we require the existence of degenerate supersymmetric and non-supersymmetric vacua with vanishing energy density. In a previous application of MPP to supergravity, a supersymmetric phase in flat Minkowski space was simply assumed to exist, in addition to the phase in which we live. Since the vacuum energy density of supersymmetric states in flat Minkowski space is just zero, the cosmological constant problem was thereby solved to first approximation by assumption. The degeneracy between the supersymmetric and physical vacua was attained by fine-tuning the Kähler function of the considered SUGRA model. However this previous work corresponds to searching for only a partial solution of the cosmological constant problem and makes the whole approach look rather artificial. The situation changes dramatically if supergravity can be supplemented by a global symmetry that ensures a zero value for the cosmological constant. This is precisely what happens in no-scale supergravity.

In no-scale supergravity the supersymmetric states with zero vacuum energy density emerge automatically at low energies. But the global symmetry, which ensures the vanishing of the cosmological constant and the degeneracy of global vacua in the no-scale models, also protects supersymmetry which has to be broken in any phenomenologically acceptable theory. In this paper we explore no–scale SUGRA models in which the extended global symmetry is broken in such a way that our MPP scenario is fulfilled without any extra fine-tuning. In the next section we specify the no–scale SUGRA models, consider the breakdown of local supersymmetry in these models and discuss the connection with MPP.

The simplest model, in which the implementation of our MPP scenario does not require any extra fine-tuning, is constructed in section. In section we estimate the value of the cosmological constant in MPP inspired SUGRA models. The realization of our MPP scenario in models based on enlarged gauge symmetry groups like \[ SU(3) \times SU(2) \times U(1) \] is considered in section. Section is reserved for our conclusions and outlook.

2 No–scale supergravity

The full \((N = 1)\) SUGRA Lagrangian is specified in terms of an analytic gauge kinetic function \(f_a(\phi_M)\) and a real gauge-invariant Kähler function \(G(\phi_M, \phi_M^*)\), which depend on the chiral superfields \(\phi_M\). The function \(f_a(\phi_M)\) determines the kinetic
terms for the fields in the vector supermultiplets and the gauge coupling constants
\( R e f_a(\phi_M) = 1/g_a^2 \), where the index \( a \) designates different gauge groups. The Kähler
function is a combination of two functions

\[
G(\phi_M, \phi^*_M) = K(\phi_M, \phi^*_M) + \ln |W(\phi_M)|^2,
\]

where \( K(\phi_M, \phi^*_M) \) is the Kähler potential whose second derivatives define the kinetic terms
for the fields in the chiral supermultiplets. \( W(\phi_M) \) is the complete analytic superpotential
of the considered SUSY model. Here we shall use standard supergravity mass units:
\( \frac{M_{Pl}}{\sqrt{8\pi}} = 1 \).

The SUGRA scalar potential can be presented as a sum of \( F \)– and \( D \)–terms
\( V_{SUGRA}(\phi_M, \phi^*_M) = V_F(\phi_M, \phi^*_M) + V_D(\phi_M, \phi^*_M) \), where the \( F \)– and \( D \)–parts are given
by \[5\]-\[6\]

\[
V_F(\phi_M, \phi^*_M) = \sum_{M,N} e^G \left( G_M G^{MN} G_N - 3 \right),
\]

\[
V_D(\phi_M, \phi^*_M) = \frac{1}{2} \sum_a (D^a)^2, \quad D^a = g_a \sum_{i,j} (G_i T^a_{ij} \phi_j),
\]

\[
G_M \equiv \partial_M G \equiv \partial G/\partial \phi_M, \quad G^a_M \equiv \partial_M G \equiv \partial G/\partial \phi^*_M.
\]

In Eq. \[2\] \( g_a \) is the gauge coupling constant associated with the generator \( T^a \) of the gauge
transformations. The matrix \( G^{MN} \) is the inverse of the Kähler metric \( K_{NM} \), i.e.

\[
G_{NM} \equiv \partial_N \partial_M G = \partial_N \partial_M K \equiv K_{NM}.
\]

In order to break supersymmetry in \( (N = 1) \) SUGRA models, a hidden sector is introduced. It contains superfields \( (z_i) \), which are singlets under the SM
\( SU(3)_C \times SU(2)_W \times U(1)_Y \) gauge group. It is assumed that the superfields of the hidden
sector interact with the observable ones only by means of gravity. If, at the minimum
of the scalar potential, hidden sector fields acquire vacuum expectation values so that at
least one of their auxiliary fields

\[
F^M = e^{G/2} G^{MP} G_P
\]

is non-vanishing, then local SUSY is spontaneously broken. At the same time a massless
fermion with spin \( 1/2 \) – the goldstino, which is a combination of the fermionic partners
of the hidden sector fields giving rise to the breaking of SUGRA, is swallowed up by the
ggravitino which thereby becomes massive \( m_{3/2} = < e^{G/2} > \). This phenomenon is called
the super-Higgs effect \[7\].

Usually the vacuum energy density at the minimum of SUGRA scalar potential \[2\] is
negative. To show this, let us suppose that, the Kähler function has a stationary point,
where all derivatives \( G_M = 0 \). Then it is easy to check that this point is also an extremum
of the SUGRA scalar potential. In the vicinity of this point local supersymmetry remains intact while the energy density is \(-3 < e^G >\). It implies that the vacuum energy density must be less than or equal to this value. Therefore, in general, an enormous fine–tuning must be imposed, in order to keep the total vacuum energy density in SUGRA models around the observed value of the cosmological constant [8].

Because the smallness of the parameters in a physical theory may be related to an almost exact symmetry, it is interesting to investigate what kind of symmetries could protect the cosmological constant in \(N = 1\) supergravity. It was discovered a long time ago that invariance with respect to \(SU(1,1)\) symmetry transformations results in a tree–level scalar potential which vanishes identically along some directions [4], [9]–[10]. In other words the corresponding scalar potential [2] possesses an infinite set of degenerate minima with zero vacuum energy density. The \(SU(1,1)\) structure of the \(N = 1\) SUGRA Lagrangian can have its roots in supergravity theories with extended supersymmetry \((N = 4 \text{ or } N = 8)\) [4].

The group \(SU(1,1)\) contains subgroups of imaginary translations and dilatations [10]–[11]. The invariance of the Kähler function under the imaginary translations of the hidden sector superfields

\[ z_i \rightarrow z_i + i\beta_i; \quad \varphi_\alpha \rightarrow \varphi_\alpha \]

implies that the Kähler potential depends only on \(z_i + \bar{z}_i\), while the superpotential is given by [12]

\[ W(z_i, \varphi_\alpha) = \exp \left\{ \sum_{i=1}^{m} a_i z_i \right\} \tilde{W}(\varphi_\alpha), \]

where the \(a_i\) are real. Here we assume that the hidden sector involves \(m\) singlet superfields while the observable sector comprises chiral multiplets \(\varphi_\alpha\). Since \(G(\phi_M, \bar{\phi}_M)\) is evidently invariant under the Kähler transformations [13]

\[
\begin{align*}
K(\phi_M, \bar{\phi}_M) & \rightarrow K(\phi_M, \bar{\phi}_M) - g(\phi_M) - g^*(\bar{\phi}_M), \\
W(\phi_M) & \rightarrow e^{g(\phi_M)}W(\phi_M)
\end{align*}
\]

the most general Kähler function can be written as

\[ G(\phi_M, \bar{\phi}_M) = K(z_i + \bar{z}_i, \varphi_\alpha, \bar{\varphi}_\alpha) + \ln |W(\varphi_\alpha)|, \]

where \(W(\varphi_\alpha) = \tilde{W}(\varphi_\alpha)\).

The dilatation invariance constrains the Kähler potential and superpotential further. Suppose that hidden and observable superfields transform differently

\[ z_i \rightarrow \alpha^2 z_i, \quad \varphi_\sigma \rightarrow \alpha \varphi_\sigma. \]

\[ 5 \]
Then the structure of the superpotential \( W(\varphi_\alpha) \) in phenomenologically acceptable SUGRA models is determined by the symmetry transformations (4) and (7). Indeed because the superpotential in these models contains trilinear terms, which induce masses of quarks and leptons, all terms involving \( n \) chiral superfields with \( n \geq 3 \) are forbidden by the dilatation invariance. If there is only one field \( T \) in the hidden sector, then the Kähler function is fixed uniquely by the gauge and global symmetries of the model:

\[
K(T + \bar{T}, \varphi_\sigma, \bar{\varphi}_\sigma) = -3 \ln(T + T) + \sum_\sigma C_\sigma |\varphi_\sigma|^2 \\
W(\varphi_\alpha) = \sum_{\sigma, \beta, \gamma} \frac{1}{6} Y_{\sigma\beta\gamma} \varphi_\alpha \varphi_\beta \varphi_\gamma,
\]

where \( C_\sigma \) and \( Y_{\sigma\beta\gamma} \) are constants. Here we restrict our consideration to the lowest order terms \( |\varphi_\sigma|^2 \) in the expansion of the Kähler potential in terms of observable superfields. The contribution of higher order terms to the SUGRA scalar potential is suppressed by inverse powers of \( M_{Pl} \) and can be safely ignored.

For the particular choice of the symmetry transformations (7) the part of the SUGRA scalar potential which is induced by the Kähler function of the hidden sector vanishes \[10\], i.e.

\[
V_{hid} = e^G (G_T G_T^\dagger G_T - 3) = 0.
\]

Then the full scalar potential takes the form

\[
V = \frac{1}{3} e^{2K/3} \sum_\alpha \left| \frac{\partial W(\bar{\varphi}_\alpha)}{\partial \bar{\varphi}_\alpha} \right|^2 + \frac{1}{2} \sum_a (D^a)^2,
\]

where the observable superfields are rescaled as \( \varphi_\alpha = \sqrt{\frac{C_\sigma}{3}} \bar{\varphi}_\alpha \). The potential (9) leads to a supersymmetric particle spectrum at low energies. Owing to the particular form of the Kähler potential (5) with \( k = 2 \), it is positive definite. Its minimum is reached at the points for which \( \left< \frac{\partial W(\varphi_\alpha)}{\partial \varphi_\alpha} \right> = < D^a >= 0 \). As a consequence the vacuum energy density goes to zero near global minima of the scalar potential (9). Thus imaginary translations (4) and dilatations (7) protect a zero value for the cosmological constant in supergravity \[10\].

The invariance of the Kähler function with respect to symmetry transformations (4) and (7) also prevents the breaking of local supersymmetry. In order to illustrate this, let us consider an SU(5) SUSY model with one field in the adjoint representation \( \Phi \) and with one singlet field \( S \). As before the structure of the Kähler function is completely fixed by the global symmetries (4) and (7), which result in a Kähler potential and superpotential

\[\text{In [14] a symmetry that forbids a cosmological constant in six and ten dimensional theories is discussed.}\]
of the form given by Eq. (8). The superpotential of the considered model is further constrained by the $SU(5)$ gauge symmetry:

$$W(S, \Phi) = \frac{\kappa}{3} S^3 + \lambda \text{Tr} \Phi^3 + \sigma \text{STr} \Phi^2.$$  

(10)

In the general case the minimum of the scalar potential, which is induced by the superpotential (10), is attained when $<S> = <\Phi> = 0$ and does not lead to the breakdown of local supersymmetry or of gauge symmetry. But if $\kappa = -40\sigma^3/(3\lambda^2)$ there is a vacuum configuration

$$<\Phi> = \frac{\Phi_0}{\sqrt{15}} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -3/2 & 0 \\ 0 & 0 & 0 & 0 & -3/2 \end{pmatrix}, \quad <S> = S_0,$$

$$\Phi_0 = \frac{4\sqrt{15} \sigma}{3\lambda} S_0,$$  

(11)

which breaks $SU(5)$ down to $SU(3) \times SU(2) \times U(1)$. However, along the valley (11), the superpotential and all auxiliary fields $F_i$ vanish preserving local supersymmetry and the zero value of the vacuum energy density.

In order to get a vacuum where local supersymmetry is broken, one should violate dilatation invariance in the superpotential. Eliminating the singlet field from the considered $SU(5)$ model and introducing a mass term for the adjoint representation, we get the superpotential

$$W(\Phi) = M_X \text{Tr} \Phi^2 + \lambda \text{Tr} \Phi^3.$$  

(12)

The scalar potential of the resulting model is given by Eq. (9). It has a few degenerate vacua with vanishing vacuum energy density. For example, in the scalar potential there exist a minimum where $<\Phi> = 0$ and another vacuum, which has a configuration similar to Eq. (11) but with $\Phi_0 = \frac{4\sqrt{15}}{3\lambda} M_X$. In the first vacuum the $SU(5)$ symmetry and local supersymmetry remain intact, while in the second one the auxiliary field $F_T$ acquires a vacuum expectation value and a non-zero gravitino mass is generated:

$$<|F_T|> \simeq \left\langle \frac{|W(\Phi)|}{(T + \bar{T})^{1/2}} \right\rangle = m_{3/2} \left\langle (T + \bar{T}) \right\rangle,$$

$$m_{3/2} = \left\langle \frac{|W(\Phi)|}{(T + \bar{T})^{3/2}} \right\rangle = \frac{40}{9} \frac{M_X^3}{\lambda^2 \left\langle (T + \bar{T})^{3/2} \right\rangle},$$  

(13)

although the vacuum expectation value of $T$ is undetermined at tree level, since the hidden sector scalar potential is flat. As a result, local supersymmetry and gauge symmetry are broken in the second vacuum. Nevertheless the invariance of the low energy effective Lagrangian of the observable sector under the transformations of global supersymmetry
is preserved (see Eq. (9)). When $M_X$ goes to zero the dilatation invariance, as well as SU(5) symmetry and local supersymmetry in the second vacuum, are restored.

This simple SU(5) model with the superpotential (12) illustrates how the degenerate vacua required for the application of MPP to supergravity are naturally realized in no-scale supergravity. In the second vacuum local supersymmetry is broken, as is supposed to be the case in the physical vacuum in which we live. It is usually supposed that local supersymmetry breaking induces SUSY breaking terms. However there are no such terms in this no-scale SUGRA model and global supersymmetry is unbroken in both vacua.

3 Minimal MPP inspired SUGRA model

The no-scale SUGRA model with the superpotential (12) is not viable from the phenomenological point of view, due to the absence of global supersymmetry breaking in the observable sector for all vacua. This raises the question of whether it is possible to construct a phenomenologically acceptable model based on broken global symmetries (11) and (17), which realises our MPP scenario without extra fine–tuning. We need to generate soft SUSY breaking terms that break global supersymmetry in the observable sector of the physical vacuum. These soft terms are generally characterised by the gravitino mass scale, which must then be of order the electroweak scale. This required small value of the gravitino mass $m_{3/2}$ of course constitutes the gauge hierarchy problem, whose solution was the original motivation for no-scale models with a flat hidden sector scalar potential. In this paper we concentrate on the hierarchy problem associated with the tiny value of the cosmological constant and do not explicitly address the solution of the gauge hierarchy problem. We shall simply assume there is a weak breaking of the dilatation invariance of the hidden sector superpotential characterised by an hierarchically small parameter $\kappa$.

In fact, we take the hidden sector to include two superfields, $T$ and $z$, that transform differently under dilatations

\[ T \rightarrow \alpha^2 T, \quad z \rightarrow \alpha z, \quad \varphi_\alpha \rightarrow \alpha \varphi_\alpha \]  \hspace{1cm} (14)

and imaginary translations

\[ T \rightarrow T + i \beta, \quad z \rightarrow z, \quad \varphi_\alpha \rightarrow \varphi_\alpha. \]  \hspace{1cm} (15)

In Eq. (14)–(15) $\varphi_\alpha$ represent the observable superfields. The hidden sector superfield $z$ transforms similarly to $\varphi_\alpha$ under the global symmetry transformations (13)–(15). It

\footnote{An enormous mass hierarchy ($m_{3/2} \ll M_{Pl}$) can appear due to a non-perturbative source of local supersymmetry breaking.}
plays a role analogous to the SU(5) adjoint field $\Phi$ in Eq. (12) and appears in the full superpotential of the model:

$$W(z, \varphi_\alpha) = \kappa \left( z^3 + \mu_0 z^2 + \sum_{n=4}^{\infty} c_n z^n \right) + \sum_{\sigma, \beta, \gamma} \frac{1}{6} Y_{\sigma\beta\gamma} \varphi_\sigma \varphi_\beta \varphi_\gamma,$$

(16)

The bilinear mass term for the superfield $z$ and the higher order terms $c_n z^n$ in the superpotential (16) spoil the dilatation invariance. But, as we noticed in section 2, such a breakdown of the symmetry protecting the cosmological constant may preserve a zero value of the vacuum energy density in all global minima of the scalar potential of the model, if the structure of the Kähler potential remains intact. It may also give rise to the spontaneous breakdown of local supersymmetry in the physical vacuum. Furthermore we require a locally supersymmetric vacuum with zero cosmological constant in our MPP scenario. We note that the conditions for the existence of such a vacuum are that the superpotential $W$ for the hidden sector and its derivatives should vanish\(^3\) at the corresponding minimum of the scalar potential:

$$\left\langle W(z) \right\rangle = \left\langle \frac{\partial W(z)}{\partial z} \right\rangle = 0.$$

(17)

So we restrict our considerations to breakdowns of dilatation invariance which result in a global minimum of the SUGRA scalar potential at $z = 0$, because it represents a vacuum where local supersymmetry remains intact. According to Eq. (12) there is no global minimum at $z = 0$, if the superpotential (16) contains a term proportional to $z$ or terms which are inversely proportional to a power of $z$. Terms involving negative powers of the superfields are not present in the superpotentials of the simplest SUSY models like the minimal supersymmetric standard model (MSSM) and the next to minimal supersymmetric standard model. A term proportional to $z$ can be forbidden by a gauge symmetry of the hidden sector, if $z$ transforms non–trivially under the corresponding gauge transformations, as in the case of our toy $SU(5)$ model (12).

Because the dilatation invariance is broken explicitly, one may expect the appearance of bilinear and higher order terms in the superpotential of the observable sector. Some of them are potentially dangerous. For instance, the inclusion of the bilinear terms $\mu_{\alpha\beta} \varphi_\alpha \varphi_\beta$ leads to the so–called $\mu$–problem in the simplest SUSY models. Actually in the MSSM, the SM gauge symmetry allows only one bilinear term $\mu H_1 \epsilon H_2$ where $H_1$ and $H_2$ are Higgs doublets. From dimensional considerations it is obvious that the corresponding mass parameter $\mu$ should be of order of the Planck scale, because this is the only scale

\(^3\)The vanishing of $W$ implies that the last term in the expression for $V_F(\phi_M, \phi_M^*)$ (see Eq. (2)), which led to the negative energy density, vanishes. Taking into account that the Kähler metric of the hidden sector is positive definite, one can prove that the absolute minimum of the scalar potential (2) is achieved when the derivative of $W$ vanishes [3].
characterising SUGRA theories. At the same time the correct pattern of electroweak symmetry breaking requires $\mu$ to be in the TeV range. In order to avoid a “new hierarchy” problem, the dilatation invariance should not be spoilt in the part of the superpotential (16) that includes observable superfields.

For completeness we have to specify the Kähler potential in our MPP inspired SUGRA model. It is fixed as follows

$$K(\phi_M, \phi_M^*) = -3 \ln \left[ T + \bar{T} - |z|^2 - \sum_\alpha \zeta_\alpha |\varphi_\alpha|^2 \right] +$$

$$+ \sum_{\alpha, \beta} \left( \frac{\eta_{\alpha\beta}}{2} \varphi_\alpha \varphi_\beta + h.c. \right) + \sum_\beta \xi_\beta |\varphi_\beta|^2,$$

(18)

where $\zeta_\alpha$, $\eta_{\alpha\beta}$, $\xi_\beta$ are some constants. The kinetic terms of the scalar fields, which are induced by the first term on the right hand side of Eq. (18), are invariant under the isometric transformations of the non–compact $SU(N, 1)$ group [16], where $N$ is the number of chiral superfields in the model. This symmetry can be derived from extended ($N \geq 5$) supergravity theories [17]. The Yukawa interactions in the superpotential (16) and D–terms in the scalar potential break $SU(N, 1)$ symmetry explicitly, in such a way that only invariance under the dilatations and imaginary translations can be realized in phenomenologically viable $N = 1$ SUGRA models. Exactly this type of SUGRA model was discussed in section 2. The Kähler potential (18) can be easily reproduced, if one expands the first term in Eq. (18) in powers of $|z|^2$ and $|\varphi_\alpha|^2$. Thus, in the limit when $\eta_{\alpha\beta}$, $\xi_\beta$ and $\zeta_\alpha$ go to zero, the invariance under the symmetry transformations (14)–(15) is restored, protecting supersymmetry and a zero value of the cosmological constant.

In section 2 we demonstrated that the violation of dilatation invariance does not necessarily cause the breaking of global supersymmetry at low energies. This is the reason why we include extra terms in the Kähler potential of our SUGRA model. We allow the breakdown of the dilatation invariance in the Kähler potential of the observable sector only. The part of $K(\phi_M, \phi_M^*)$ involving hidden sector superfields is responsible for the cancellation of the negative contribution to the total vacuum energy density coming from the term $-3e^G$ in the scalar potential (2). Therefore any variations in the Kähler potential of the hidden sector may spoil the vanishing of the vacuum energy density in global minima. For example, if the factor in front of the logarithm in Eq. (18) is greater than $-3$ then SUGRA scalar potential is not positive definite and the total energy density tends to be huge and negative.

In order to avoid cumbersome calculations, we introduce the simplest set of terms breaking the dilatation invariance in the Kähler potential. All the terms are bilinear with respect to observable superfields and do not depend on the hidden sector fields. Higher order terms are irrelevant for our study, since their contribution to the low energy effective
potential is suppressed by inverse powers of $M_{Pl}$. Additional terms which are proportional to $|\varphi_\alpha|^2$ normally appear in minimal SUGRA models \cite{18}–\cite{20}. The other terms $\eta_{\alpha\beta}\varphi_\alpha\varphi_\beta$ introduced in the Kähler potential \cite{18} give rise to effective $\mu$ terms after the spontaneous breakdown of local supersymmetry, solving the $\mu$ problem \cite{21}.

In the limit when $\xi_\beta$ and $\eta_{\alpha\beta}$ vanish while $\zeta_\alpha \to 1$, we return back to the SUGRA scalar potential of the form \cite{9}. In this case, the scalar potential of the hidden sector becomes

$$V(T, z) = \frac{1}{3(T + \overline{T} - |z|^2)^2} \left| \frac{\partial W(z)}{\partial z} \right|^2.$$  \hfill (19)

The minima of the scalar potential \cite{19} are attained at the stationary points of the hidden sector superpotential. In the simplest case when $c_n = 0$, the superpotential \cite{16} has two extremum points at $z = 0$ and $z = -\frac{2\mu_0}{3}$. At these points the scalar potential \cite{19} achieves its absolute minimal value i.e. zero. In the first vacuum where $z = -\frac{2\mu_0}{3}$, local supersymmetry is broken and the gravitino gets a non–zero mass:

$$m_{3/2} = \left\langle \frac{W(z)}{(T + \overline{T} - |z|^2)^{3/2}} \right\rangle = \frac{4\xi\mu_0^3}{27 \left\langle \left( T + \overline{T} - \frac{4\mu_0^2}{9} \right)^{3/2} \right\rangle}.$$  \hfill (20)

In the second minimum, the vacuum expectation value of the superfield $z$ and the superpotential of the hidden sector vanish. Therefore the conditions \cite{17} are fulfilled automatically and local supersymmetry remains intact. If the high order terms $c_n z^n$ are present in Eq. \cite{16}, the scalar potential of the hidden sector may have many degenerate vacua with vanishing vacuum energy density, where the gravitino may remain massless or gain a non–zero mass.

The main disadvantage of the scenario considered above is related with the degeneracy between bosons and fermions in the observable sector, which is preserved in the limit $\xi_\beta, \eta_{\alpha\beta} \to 0$ despite the breakdown of local supersymmetry. In the general case, when both $\xi_\alpha$ and $\zeta_\alpha$ have non–zero values, the situation changes dramatically. Since, by construction, the dilatation invariance is only broken in the part of the Kähler potential \cite{18} containing observable superfields, it does not affect the scalar potential of the hidden sector which is still described by Eq. \cite{19}. As a result our MPP scenario is realized without any extra fine–tuning.

Nevertheless the shape of the effective scalar potential of the observable sector, in the vacua where the super-Higgs effect takes place, alters significantly. The structure of the soft SUSY breaking terms in the considered model, which is discussed in the Appendix, allows us to write the effective potential of the observable sector \cite{32} in a compact form:  

\footnote{This form of the scalar potential can be established in a straightforward way in the limit when all $\zeta_\alpha$ go to zero ($x_\alpha \to \infty$). Then the Kähler metric of the observable superfields $K_{\alpha\beta}$ is diagonal and does not}
Although global supersymmetry is softly broken, the effective potential of the scalar fields (21) is still positive definite and vanishes near its global minima. It follows that the spontaneous breakdown of electroweak symmetry cannot be naturally arranged in our model, because normally it results in negative vacuum energy density, i.e. the minimum of the scalar potential with broken $SU(2)_W \times U(1)_Y$ symmetry ought to be deeper than the vacuum where gauge invariance is preserved and the doublet Higgs fields vanish ($<H_1>=<H_2>=0$). Moreover in the simplest MPP inspired SUGRA model discussed above, the mechanism for the stabilization of the vacuum expectation value of the hidden sector field $T$ remains unclear. As a result the gravitino mass (see Eq. (20)) and the supersymmetry breaking scale are not fixed in the physical vacuum.

However all these problems cannot be addressed in the framework of the simplest MPP inspired SUGRA model. In order to get a self–consistent solution, one has to include all perturbative and non–perturbative corrections to the considered SUGRA Lagrangian, which should depend on the structure of the underlying theory. If we take into account the evolution of the soft scalar masses, then their renormalization group flow might provide a radiative mechanism for electroweak symmetry breaking [22]. We hope that an underlying renormalizable or even finite theory can be found, which sheds light on the origin of the terms that spoil the global symmetry protecting the cosmological constant in our SUGRA model. It should also ensure the stabilization of the vacuum expectation values of the hidden sector fields and the supersymmetry breaking scale.

4 Cosmological constant in MPP inspired SUGRA models

We now assume that a phenomenologically viable MPP inspired SUGRA model of the type just discussed can be constructed. That is to say, we assume the existence of a phase with electroweak gauge symmetry breaking induced by soft SUSY breaking terms degenerate with a second phase, in which the low–energy limit of the considered theory is described by a pure supersymmetric model in flat Minkowski space. Non-perturbative effects in the observable sector may lead to supersymmetry breakdown in the second vacuum state (for recent reviews see [23]–[24]). Then in compliance with our MPP philosophy, we require the degeneracy of the vacua after all non-perturbative effects are included.

depend on the hidden sector superfields, which makes the computation of the SUGRA scalar potential relatively simple.
The non-perturbative effects in simple SUSY models, like the minimal supersymmetric standard model (MSSM), are extremely weak. Our strategy is to estimate these effects in the second vacuum and thereby estimate the energy density in the second (almost supersymmetric) phase. This value of the cosmological constant can then be interpreted as the physical value in our phase, by virtue of MPP.

If supersymmetry breaking takes place in the second vacuum, it is caused by the strong interactions. Indeed, even in the pure MSSM, the beta function of the strong gauge coupling constant $\alpha_3$ exhibits asymptotically free behaviour ($b_3 = -3$). As a consequence $\alpha_3(Q)$ increases in the infrared region and one can expect that the role of non-perturbative effects is enhanced. Since in the minimal SUGRA model the kinetic functions essentially do not depend on the hidden superfields ($f_a(z_m) \simeq \text{const}$), the values of the gauge couplings at the high energy scale and their running down to the scale $M_S \simeq m_{3/2}$ are the same in both vacua. Below the scale $M_S$ all superparticles in the physical vacuum decouple and the corresponding beta functions change ($\tilde{b}_3 = -7$). Using the value of $\alpha_3^{(1)}(M_Z) \approx 0.118 \pm 0.003$ and the matching condition $\alpha_3^{(2)}(M_S) = \alpha_3^{(1)}(M_S)$, one finds the strong coupling in the second vacuum

$$\frac{1}{\alpha_3^{(2)}(M_S)} = \frac{1}{\alpha_3^{(1)}(M_Z)} - \frac{\tilde{b}_3}{4\pi} \ln \frac{M_S^2}{M_Z^2}. \quad (22)$$

In Eq. (22) $\alpha_3^{(1)}$ and $\alpha_3^{(2)}$ are the values of the strong gauge couplings in the physical and second minima of the SUGRA scalar potential.

At the scale $\Lambda_{SQCD}$, where the supersymmetric QCD interactions become strong in the second vacuum

$$\Lambda_{SQCD} = M_S \exp \left[ \frac{2\pi}{b_3 \alpha_3^{(2)}(M_S)} \right], \quad (23)$$

the supersymmetry may be broken dynamically due to non-perturbative effects. If instantons generate a repulsive superpotential [23], [25]–[26] which lifts and stabilizes the vacuum valleys in the scalar potential, then a generalized O’Raifeartaigh mechanism gives rise to a non-zero positive value for the cosmological constant

$$\Lambda \simeq \Lambda_{SQCD}^4. \quad (24)$$

In Fig. 1 the dependence of $\Lambda_{SQCD}$ on the SUSY breaking scale $M_S$ is examined. Because $\tilde{b}_3 < b_3$ the QCD gauge coupling below $M_S$ is larger in the physical minimum than in the second one. Therefore the value of $\Lambda_{SQCD}$ is much lower than the QCD scale in the Standard Model and diminishes with increasing $M_S$. When the supersymmetry breaking

---

$^5$The gauge couplings obey the renormalization group equations $\frac{d\log \alpha_i(Q)}{d\log Q^2} = \frac{b_i \alpha_i(Q)}{4\pi}$, where $\alpha_i(Q) = g_i^2(Q)/(4\pi)$. 

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scale in our vacuum is of the order of $1 \text{ TeV}$, we obtain $\Lambda_{\text{SQCD}} = 10^{-26} M_{\text{Pl}} \simeq 100 \text{ eV}$. This results in an enormous suppression of the total vacuum energy density ($\Lambda \simeq 10^{-104} M_{\text{Pl}}^4$) compared to say an electroweak scale contribution in our vacuum $v^4 \simeq 10^{-62} M_{\text{Pl}}$. From the rough estimate of the energy density [24], it can be easily seen that the measured value of the cosmological constant is reproduced when $\Lambda_{\text{SQCD}} = 10^{-31} M_{\text{Pl}} \simeq 10^{-3} \text{ eV}$. The appropriate values of $\Lambda_{\text{SQCD}}$ can therefore only be obtained for $M_S = 10^3 - 10^4 \text{ TeV}$. However the consequent large splitting within SUSY multiplets would spoil gauge coupling unification in the MSSM and reintroduce the hierarchy problem, which would make the stabilization of the electroweak scale rather problematic.

A model consistent with electroweak symmetry breaking and cosmological observations can be constructed, if the MSSM particle content is supplemented by an additional pair of $5 + \bar{5}$ multiplets. These new bosons and fermions would not affect gauge coupling unification, because they form complete representations of $SU(5)$ (see for example [27]). In the physical vacuum these extra particles would gain masses around the supersymmetry breaking scale. The corresponding mass terms in the superpotential are generated after the spontaneous breaking of local supersymmetry, due to the presence of the bilinear terms $[\eta(5 \cdot \bar{5}) + h.c.]$ in the Kähler potential of the observable sector [21]. Near the second minimum of the SUGRA scalar potential the new particles would be massless, since $m_{3/2} = 0$. Therefore they give a considerable contribution to the $\beta$ functions ($b_3 = -2$), reducing $\Lambda_{\text{SQCD}}$ further. In this case the observed value of the cosmological constant can be reproduced even for $M_S \simeq 1 \text{ TeV}$ (see Fig. 1).

Unfortunately achieving dynamical SUSY breaking at the scale $\Lambda_{\text{SQCD}}$ is actually not at all easy. The situation is different depending on whether the number of flavours $N_f$ is larger or smaller than the number of colours $N_c$. In the MSSM and its simplest extensions, where $N_c = 3$ and $N_f = 6$, the generated superpotential has a polynomial form [24], [28]. The absolute minimum of the SUSY scalar potential is then reached when all the superfields, including their F- and D-terms, acquire zero vacuum expectation values preserving supersymmetry in the second vacuum. This result throws some doubt on our estimations of the value of the cosmological constant, which is based on Eq. (24).

But the above disappointing facts concerning dynamical SUSY breaking were revealed in the framework of pure supersymmetric QCD, where all Yukawa couplings were supposed to be small or even absent. At the same time the t–quark Yukawa coupling in the MSSM is of the same order of magnitude as the strong gauge coupling at the electroweak scale. Therefore it might change the results of the SUSY breaking studies drastically leading, for example, to the formation of a quark condensate that breaks supersymmetry.
5 Implementation of the MPP in the models with extended gauge symmetry

The breakdown of supersymmetry in the observable sector can be more easily achieved in models with an extended strong interaction gauge sector. Here we restrict our consideration to the class of models based on $SU(N)$ gauge symmetry groups. Since the extension of the gauge sector of the SM is already a very strong assumption, we prefer to limit the particle content of the model as much as possible. In particular it is worth combining the spontaneous breakdowns of the enlarged gauge symmetry and local supersymmetry, as takes place in our toy $SU(5)$ model (12), rather than introducing two separate sectors for this purpose. It seems that the simplest gauge extension of the MSSM, for which the dynamical supersymmetry breaking occurs at low energies independently of the values of the Yukawa couplings, should include at least three $SU(3)$ gauge groups. If the quarks of each generation are coupled to the gauge bosons of just their own distinct $SU(3)$, then the criterion $N_c > N_f$ is satisfied. We here consider a model with an $\left[ SU(3) \right]^3$ gauge symmetry, as in the family replicated gauge group or anti-grand unification model [2], [29].

As in this model, we take the corresponding three $\left[ SU(3) \right]^3$ gauge coupling constants to be equal and we denote their value at the scale $Q$ by $g_{33}(Q)$.

In the physical vacuum the $\left[ SU(3) \right]^3$ gauge symmetry must be broken down to $SU(3)_C$, which is associated with the SM strong interactions. This can be simply arranged if the considered theory includes multiplets in the bi–fundamental representation, which transform as a triplet with respect to one $SU(3)$ and as an anti–triplet under another $SU(3)$ symmetry. In the models based on $SU(3)_a \times SU(3)_b \times SU(3)_c$ there can be six bi–fundamental representations: $\Phi_{ab}, \Phi_{ac}, \Phi_{ba}, \Phi_{bc}, \Phi_{ca}, \Phi_{cb}$ where the indices $a, b$ and $c$ correspond to the three different $SU(3)$ gauge groups and the corresponding quark generations. If the superfields $\Phi_{ij}$ acquire vacuum expectation values

$$\Phi_{ij} = \Phi_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad i, j = a, b, c$$

(25)

then below the energy scale $\Phi_0$ the $\left[ SU(3) \right]^3$ gauge group reduces to the diagonal subgroup corresponding to the usual QCD $SU(3)_C$ symmetry. It follows that the QCD gauge coupling constant $g_3(Q)$ is then related to the $\left[ SU(3) \right]^3$ gauge coupling constant at the scale $\Phi_0$:

$$g^{(1)}_{33}(\Phi_0 + \varepsilon) = \sqrt{3} g^{(1)}_3(\Phi_0 - \varepsilon).$$

(26)
The desired pattern of \([SU(3)]^3\) gauge symmetry breaking can be obtained in the no-scale SUGRA model with superpotential

\[
W = \mu_X \left[ \Tr (\Phi_{ab} \Phi_{ba}) + \Tr (\Phi_{ac} \Phi_{ca}) + \Tr (\Phi_{bc} \Phi_{cb}) \right] + \\
+ k \left[ \Tr (\Phi_{ab} \Phi_{bc} \Phi_{ca}) + \Tr (\Phi_{ba} \Phi_{ac} \Phi_{cb}) \right] + \hat{W}(\varphi_\sigma)
\]  \hspace{1cm} (27)

and Kähler potential

\[
K(\phi_M, \phi_M^*) = -3 \ln \left[ T + \mathcal{T} - \sum_{i,j} |\Phi_{ij}|^2 \right] + \hat{K}(\varphi_\sigma, \varphi_\sigma^*)
\]  \hspace{1cm} (28)

where \(\hat{W}(\varphi_\sigma)\) and \(\hat{K}(\varphi_\sigma, \varphi_\sigma^*)\) depend on the Higgs, quark and lepton superfields \(\varphi_\sigma\). The model possesses two degenerate minima, where \(\Phi_0 = -\frac{\mu_X}{k}\) and \(\Phi_0 = 0\) respectively. In the first vacuum \(\Phi_0 = -\frac{\mu_X}{k}\) local supersymmetry and \([SU(3)]^3\) gauge symmetry are broken. The breaking of global supersymmetry can be induced at low energies as well, if the part of the Kähler potential \(\hat{K}(\varphi_\sigma, \varphi_\sigma^*)\) is not invariant under the dilatation transformations. For example, in the simple case considered where \(\hat{K}\) does not depend on \(T\) and \(\Phi_{ij}\), i.e.

\[
\hat{K}(\varphi_\sigma, \varphi_\sigma^*) = \sum_{\alpha,\beta} \left( \frac{\eta_{\alpha\beta}}{2} \varphi_\alpha \varphi_\beta + \text{h.c.} \right) + \sum_\sigma \xi_\sigma |\varphi_\sigma|^2,
\]  \hspace{1cm} (29)

the scalar components of the observable superfields \(\varphi_\sigma\) gain a universal mass which coincides with the gravitino mass \(m_{3/2} = \frac{3\mu_X^3}{k^2 < (T + \mathcal{T}) - 18(\mu_X/k)^2>^{3/2}}\) (see Eq. (25) where \(x_\alpha \to \infty\)). For simplicity, we take \(k\) of order unity in the following.

In the second vacuum \((\Phi_0 = 0)\) supersymmetry and gauge symmetries are left unbroken. The gauge couplings of each \(SU(3)\) grow with decreasing energy scale developing a Landau pole much below \(\mu_X\). At low energies, where the \(SU(3)\) gauge interactions become very strong \((E \simeq \Lambda_{SQCD})\), non-perturbative effects induce a sizable instanton contribution \(W_{\text{inst}}\) to the effective superpotential (see [25]) that takes the form

\[
W = W_{\text{inst}} + h_t(\hat{H}_u \hat{Q})\hat{t}^c + h_b(\hat{H}_d \hat{Q})\hat{b}^c + h_\tau(\hat{H}_d \hat{L})\hat{\tau}^c,
\]

\[
W_{\text{inst}} \simeq \frac{\Lambda_{SQCD}^7}{(Q_\alpha t^c)\epsilon_{\alpha\beta}(Q_\beta b^c)}.
\]  \hspace{1cm} (30)

For simplicity we only keep superfields belonging to the third generation in the superpotential [30], together with the Higgs doublets \(H_u\) and \(H_d\). In Eq. (30) \(\alpha\) and \(\beta\) are \(SU(2)\) indices labelling the components of the \(SU(2)\) doublet \(Q_\alpha\), whereas \(\epsilon_{\alpha\beta}\) is the completely antisymmetric tensor. The non-perturbative superpotential \(W_{\text{inst}}\) gives rise to supersymmetry breaking. Indeed, in a vacuum where supersymmetry is preserved, all
the auxiliary fields $F_i$ have to be zero. The vanishing of $F_{H_u}$ implies that the vacuum expectation value of either $<Q>$ or $<t^c>$ is zero. At the same time with the superpotential $W_{inst}$ as well as $F_Q$ and $F_{t^c}$ are singular when $<Q>=0$ or $<t^c>=0$. Therefore it is not consistent to assume that supersymmetry is preserved in the vacuum, but non-perturbative instanton effects must break the supersymmetry and give rise to a non–zero vacuum energy density $\Lambda \simeq \Lambda_{SQCD}^4$.

So far the gauge kinetic function in the considered model has not been specified. In contrast with the simplest MPP inspired models, a constant gauge kinetic function in this particular gauge extension of the SM does not allow us to reproduce the observed value of the cosmological constant. In realistic scenarios the supersymmetry breaking scale in the physical vacuum has to be above a few hundred GeV, restricting the permitted range of $\mu_X$ from below. Assuming that $T$ gets a vacuum expectation value around unity (i.e. $T \sim M_{Pl}$), the scale of $\left[SU(3)^3 \right]$ symmetry breaking ought to be higher than $10^{13}$ GeV but should not exceed $M_{Pl}$. In order to get a phenomenologically acceptable value for the vacuum energy density in the second minimum, which according to MPP coincides with the cosmological constant in our vacuum, we require $\Lambda_{SQCD} \simeq 10^{-3}$ eV. Hence the $SU(3)$ gauge couplings $g_{33}^{(2)}$ at the scale $\mu_X$ are required to be in the vicinity of 0.4 in the second vacuum (see Fig. 2). However, the value of the $SU(3)$ gauge couplings in the physical vacuum $g_{33}^{(1)}$ just above the scale $\mu_X$ is considerably larger than $g_{33}^{(2)}(\mu_X)$, as one can see from Fig. 3.

Thus, in order to obtain an appropriate value of $\Lambda_{SQCD}$, the $SU(3)_i$ gauge couplings in the second vacuum have to be two or three times smaller than in the physical one. This can be achieved if the gauge kinetic function depends quite strongly on the vacuum expectation values of the bi–fundamental multiplets $\Phi_{ij}$. The simplest gauge kinetic function for the gauge group $SU(3)_a$, which is invariant under gauge symmetry transformations, imaginary translations and dilatations, can be written as

$$f_a(\phi_M) = f_a^0 + \sum_{i,j} f_{ij}^a |\Phi_{ij}|^2 \left(\frac{T + \bar{T}}{T + \bar{T}}\right).$$

When we take $f_a^0 \simeq 6.28$, i.e. $(g_{33}^{(2)}(M_{Pl}))^2 = 1/f_a^0 \simeq 0.16$, the gauge couplings of $\left[SU(3)^3 \right]$ blow up near the scale $\Lambda_{SQCD} \simeq 10^{-3}$ eV, inducing a suitable value of the vacuum energy density. In the physical vacuum the gauge couplings $g_{33}^{(1)}(M_{Pl})$ differ from $g_{33}^{(2)}(M_{Pl})$, because the bi–fundamental multiplets acquire non–zero vacuum expectation values. If the second term in Eq. (31) takes the value $(-4.9)$ in the physical vacuum, the measured value of $\alpha_3^{(1)}(M_Z)$ is reproduced using Eq. (26). This value can be obtained with all the parameters $f_a^0$ and $f_{ij}^a$ of the same order of magnitude, provided that $\Phi_0 \simeq M_{Pl}$.

In the case when $\Phi_0 \simeq M_{Pl}$ the gauge symmetry, global and local supersymmetry are
all broken just below the Planck scale in the physical vacuum. As can be seen from Fig. 3, the $SU(3)$ gauge couplings then take the value $g_{33}^{(1)}(M_{Pl}) \simeq 0.85$. This is consistent with the critical value of the gauge coupling constant obtained from lattice calculations \cite{30}, for which three phases of the regularised $SU(3)$ gauge theory coexist, i.e. for which the corresponding vacuum states have the same energy density in agreement with our MPP philosophy. Similar results were obtained for the $SU(2)$ and $U(1)$ gauge couplings in the family replicated gauge group model \cite{2}, \cite{29}, using the measured values of $\alpha_2(M_Z)$ and $\alpha_1(M_Z)$ as inputs. We note that a phenomenologically successful structure for the quark and lepton mass matrices can be naturally generated from the chiral gauge charges in the family replicated gauge group model \cite{31}.

6 Summary and concluding remarks

In supergravity the cosmological constant problem can be alleviated by imposing an extra global symmetry. In particular the invariance under imaginary translations and dilatations, which are subgroups of $SU(N,1)$, leads to the vanishing of the vacuum energy density in the no-scale SUGRA models. At the same time these symmetries, which naturally arise in theories with extended supersymmetry ($N \geq 5$), preserve local supersymmetry which must however be broken in any phenomenologically acceptable theory. We have argued that the breakdown of these global symmetries protecting the cosmological constant does not necessarily result in a non–zero vacuum energy density. In particular, violation of dilatation invariance in the superpotential of no–scale models may give rise to the spontaneous breakdown of local supersymmetry, and still preserve a zero value for the energy density in the vacua of these models.

All global minima of the SUGRA scalar potential \cite{2} in the no–scale models, where the invariance with respect to dilatations is spoiled in the superpotential, are degenerate. Normally the set of global minima in the considered class of models includes vacua with broken and unbroken local supersymmetry. In the vacua where local supersymmetry remains intact, the gravitino mass goes to zero and the conditions (17) are fulfilled automatically. According to our MPP scenario the SUGRA scalar potential must possess at least two degenerate vacua in which $m_{3/2} = 0$ and $m_{3/2} \neq 0$ respectively. In one of them, where $m_{3/2}$ has a non–zero value, local supersymmetry is broken in the hidden sector at the high energy scale ($\sim 10^{10} - 10^{12}$ GeV), inducing a set of soft SUSY breaking terms for the observable fields. In the other vacuum ($m_{3/2} = 0$) the low energy limit of the considered theory is described by a pure supersymmetric model in flat Minkowski space. The energy density and all auxiliary fields $F^M$ of the hidden sector vanish in this second
Although the breakdown of dilatation invariance in the superpotential of no–scale SUGRA models ensures the degeneracy of vacua, where $m_{3/2} = 0$ and $m_{3/2} \neq 0$ respectively, the particle spectrum remains supersymmetric at low energies in all vacua. Thereby none of these vacua can be the physical one. Nevertheless a minimal SUGRA model has been constructed, where our MPP scenario is realized without any extra fine–tuning. It is based on broken $SU(N, 1)$ symmetry. The hidden sector of the minimal MPP inspired SUGRA model contains two superfields $T$ and $z$, which transform differently under imaginary translations and dilatations. We allowed the breakdown of dilatation invariance in the superpotential of the hidden sector and in the part of the Kähler potential which contains the observable superfields. The $SU(N, 1)$ structure of the Kähler potential of the hidden sector guarantees the vanishing of the cosmological constant in all the global minima of the scalar potential in the model. Owing to the breakdown of dilatation invariance in the hidden sector superpotential, a set of degenerate vacua with broken and unbroken local supersymmetry emerges. Meantime we maintain dilatation invariance in the observable sector superpotential, preventing the appearance of bilinear and high order terms involving observable superfields in the rest of the superpotential and thereby eliminating the $\mu$–problem. Finally, due to a suitable breakdown of dilatation invariance in the Kähler potential of the observable sector, effective $\mu$–terms and a set of soft SUSY breaking terms are generated in the vacua where local supersymmetry is spontaneously broken.

In spite of the vanishing of the vacuum energy density in all global minima of the tree level scalar potential of the MPP inspired SUGRA models, the value of the cosmological constant may differ from zero. This occurs if non–perturbative effects in the observable sector give rise to the breakdown of supersymmetry in the second vacuum (phase). Our MPP philosophy then requires that the phase in which local supersymmetry is broken in the hidden sector has the same energy density as a phase where supersymmetry breakdown takes place in the observable sector. If the gauge couplings at high energies are identical in both vacua, the value of the energy density in the second vacuum can be estimated relatively easily. It is positive definite and determined by the scale where the $SU(3)_C$ gauge interactions become strong. The numerical analysis carried out in the framework of the pure MSSM has revealed that the corresponding scale is naturally low ($\Lambda_{SQCD} \lesssim 10^{-25} M_{Pl}$) for a reasonable choice of the supersymmetry breaking scale, $M_S \gtrsim 1$ TeV, in the first (physical) vacuum. Moreover the introduction of an extra pair of $5 + \bar{5}$ multiplets reduces this scale down further, so that the energy density of the second phase approaches the observed value of the cosmological constant even when $M_S \simeq 1$ TeV.

The crucial idea is then to use MPP to transfer the energy density or cosmological con-
stant from this second vacuum into all other vacua, especially into the physical one in which we live. In such a way we have suggested an explanation of why the observed value of the cosmological constant is positive and takes on the tiny value it has. The MPP scenario with additional $5 + 5$ multiplets of matter and supersymmetry breaking scale in the TeV range can be tested at the LHC or ILC.

The trouble with the MPP prediction for the value of the cosmological constant is that it is not clear if the required dynamical supersymmetry breaking actually takes place in the framework of the simplest SUSY extensions of the SM, which describe the observable sector of SUGRA models at low energies. On the other hand, the dynamical breakdown of supersymmetry can be attained in SUSY models with an extended gauge sector for the strong interactions similar to that in the family replicated gauge group model \textsuperscript{[2]}, \textsuperscript{[29]}, \textsuperscript{[31]}. But, in order to obtain the appropriate value of the cosmological constant in this case, the gauge couplings in the first and second vacua should differ considerably. Therefore one has to admit a dependence of the gauge kinetic function on the chiral superfields, which are responsible for the breaking of the enlarged gauge symmetry down to $SU(3)_C \times SU(2)_W \times U(1)_Y$. Then, if local supersymmetry and the extended gauge symmetry are broken near the Planck scale, the gauge couplings in the second vacuum can be smaller than in the physical one by a factor of 2, which allows us to reproduce the observed value of the cosmological constant. In the first vacuum where we live the SM is valid up to the Planck scale. It has recently been pointed out that the enormous hierarchy between the electroweak and Planck scales might also be explained by MPP \textsuperscript{[29]}, \textsuperscript{[32]} in the SM.

Although MPP provides an attractive explanation for the smallness and sign of the cosmological constant in $(N = 1)$ supergravity, we have not been able to present a fully self–consistent model. The no–scale models discussed above possess one defect. Namely, the mechanism for the stabilization of the vacuum expectation value of the hidden sector field $T$ and the SUSY breaking scale remains unclear.

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Appendix

Here we discuss the structure of the soft SUSY breaking terms which appear in the physical vacuum in a low energy effective Lagrangian of the MPP inspired SUGRA model with superpotential \( W \) and \( \mathcal{K} \)-ähler potential given by Eq. (18). In order to compute the effective scalar potential, one has to substitute vacuum expectation values for \( T \) and \( z \) as well as for their auxiliary fields (3), taking into account that only \( F_T \) acquires a non-zero vacuum expectation value. Then one expands the full SUGRA scalar potential (1) in powers of observable fields, taking the flat limit \( M_{Pl} \to \infty \) but \( m_{3/2} \) is kept fixed. In the considered limit hidden sector superfields are decoupled from the low-energy theory. The only signal they produce is a set of terms that break the global supersymmetry of the low-energy effective Lagrangian of the observable sector in a soft way [19], [33], i.e. without inducing quadratic divergences. All non-renormalizable terms vanish in the flat limit since they are suppressed by inverse powers of \( M_{Pl} \). Thus one is left with a global SUSY scalar potential \( V_{SUSY} \) plus a set of soft SUSY breaking terms \( V_{soft} \), i.e.

\[
V_{eff}(y_\alpha, y_\alpha^*) = V_{SUSY} + V_{soft},
\]

\[
V_{SUSY} = \sum_\alpha \left| \frac{\partial W_{eff}(y_\beta)}{\partial y_\alpha} \right|^2 + \frac{1}{2} \sum_a (D^a)^2,
\]

\[
V_{soft} = \sum_\alpha m_\alpha^2 |y_\alpha|^2 + \left[ \sum_{\alpha, \beta} \frac{1}{2} B_{\alpha\beta} y_\alpha y_\beta + \sum_{\alpha, \beta, \gamma} \frac{1}{6} A_{\alpha\beta\gamma} y_\alpha y_\beta y_\gamma + h.c. \right].
\]

In Eq. (32) \( y_\alpha \) are canonically normalized scalar fields

\[
y_\alpha = \tilde{C}_\alpha \varphi_\alpha , \quad \tilde{C}_\alpha = \xi_\alpha \left( 1 + \frac{1}{x_\alpha} \right) , \quad x_\alpha = \frac{\xi_\alpha < (T + \overline{T} - |z|^2) >}{3 \zeta_\alpha}.
\]

When \( M_{Pl} \to \infty \) the effective superpotential, which describes the interactions of observable superfields at low energies, only contains bilinear and trilinear terms

\[
W_{eff} = \sum_{\alpha, \beta} \frac{\mu_{\alpha\beta}}{2} \varphi_\alpha \varphi_\beta + \sum_{\alpha, \beta, \gamma} \frac{h_{\alpha\beta\gamma}}{6} \varphi_\alpha \varphi_\beta \varphi_\gamma ,
\]

\[
\mu_{\alpha\beta} = m_{3/2} \eta_{\alpha\beta} (\tilde{C}_\alpha \tilde{C}_\beta)^{-1} , \quad h_{\alpha\beta\gamma} = \frac{Y_{\alpha\beta\gamma} (\tilde{C}_\alpha \tilde{C}_\beta \tilde{C}_\gamma)^{-1}}{< (T + \overline{T} - |z|^2)^{3/2} >}.
\]

The complete set of soft SUSY breaking terms involves: gaugino masses \( M_\alpha \), masses of scalar components of observable superfields \( m_\alpha \), trilinear \( A_{\alpha\beta\gamma} \) and bilinear \( B_{\alpha\beta} \) scalar couplings associated with Yukawa couplings and \( \mu \)-terms in the superpotential [34]. Three types of soft SUSY breaking parameters \( m_\alpha^2 \), \( A_{\alpha\beta\gamma} \) and \( B_{\alpha\beta} \) appear in the scalar potential (32). In the vacua, where local SUSY is broken and the gravitino gains a non-zero mass
These parameters are given by

\[ m_\alpha = m_{3/2} \frac{x_\alpha}{(1 + x_\alpha)}, \]
\[ B_{\alpha\beta} = m_\alpha + m_\beta, \]
\[ A_{\alpha\beta\gamma} = m_\alpha + m_\beta + m_\gamma. \]

(35)

The structure of the soft SUSY breaking terms given above permits to rewrite the effective scalar potential (32) in a more compact form (21). It is worth emphasizing that the expressions for the soft SUSY breaking parameters obtained above would not change if the hidden sector of our model had many superfields \( z_i \). The soft scalar masses \( m_\alpha \) in the low energy effective Lagrangian maintain the splitting between bosons and fermions within one supermultiplet. According to Eq. (35), the masses of the superpartners of the ordinary quarks and leptons are set by the parameter \( \xi_\alpha \) and the vacuum expectation value of the superpotential of the hidden sector (or \( \kappa \)), which spoil the dilatation invariance. In other words the qualitative pattern of the sparticle spectrum in the considered SUGRA model depends on the extent to which the symmetry protecting the cosmological constant is broken. Assuming that \( \xi_\alpha, \zeta_\alpha, \mu_0 \) and \( \langle T \rangle \) are all of order unity, the phenomenologically acceptable value of the supersymmetry breaking scale \( M_S \sim 1 \text{ TeV} \) can only be obtained for extremely small values of \( \kappa \simeq 10^{-15} \).

Explicit expressions for the gaugino masses are not included in Eq. (35) because their values are determined by the gauge kinetic functions \( f_a(T, z) \) that has not been specified yet. A canonical choice for the kinetic function in minimal supergravity \( f_a(T, z) = \text{const} \) corresponds to \( M_a = 0 \). In order to avoid a conflict with chargino and gluino searches at present and former colliders, we need gaugino masses in the few hundred GeV range. Therefore we assume a mild dependence of \( f_a(T, z) \) on the hidden sector fields, which is strong enough to induce appreciable gaugino masses but weak enough to ensure that the gauge couplings in the physical and supersymmetric vacua do not differ significantly.

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\(^{6}\)In the most general case a complete set of expressions for the soft SUSY breaking parameters can be found in [35–36].
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Figure captions

Fig. 1. The value of $\log [\Lambda_{SQCD}/M_{Pl}]$ versus $\log M_S$. The thin and thick solid lines correspond to the pure MSSM and the MSSM with an additional pair of $5 + \bar{5}$ multiplets respectively. The dashed and dash-dotted lines represent the uncertainty in $\alpha_3(M_Z)$. The upper dashed and dash-dotted lines are obtained for $\alpha_3(M_Z) = 0.124$, while the lower ones correspond to $\alpha_3(M_Z) = 0.112$. The horizontal line represents the observed value of $\Lambda^{1/4}$. The SUSY breaking scale $M_S$ is measured in GeV.

Fig. 2. The value of the vacuum energy density as a function of the overall $[SU(3)]^3$ gauge coupling at the scale $\mu_X$ in the second vacuum. The dash-dotted and thick curves represent the dependence of the energy density on $g^{(2)}_{33}(\mu_X)$ for $\mu_X = M_{Pl}$ and $\mu_X = 10^{13}$ GeV respectively. The horizontal solid line corresponds to the observed value of the cosmological constant $\Lambda$.

Fig. 3. The dependence of the overall $[SU(3)]^3$ gauge coupling $g_{33}(\mu_X)$ on the scale $\mu_X$. The upper solid curve represents $g^{(1)}_{33}(\mu_X)$ and is obtained by the extrapolation of $\alpha_3(M_Z)$ up to the scale $\mu_X$ in the physical vacuum. The lower thick line represents the values of $g^{(2)}_{33}(\mu_X)$ that allow us to fit the vacuum energy density in the second vacuum to its phenomenologically acceptable value $\Lambda \simeq 10^{-123} M_{Pl}^4$. The scale of the $[SU(3)]^3$ symmetry breaking $\mu_X$ is given in GeV.
$\log[\Lambda_{SQCD}/M_{Pl}]$

Fig. 1

$\log[\Lambda/M_{Pl}^4]$

Fig. 2
$g_{33}(\mu_X)$

![Graph showing the relationship between $\mu_X$ and $g_{33}(\mu_X)$](image)

**Fig. 3**