12C + 12C Fusion \( S^* \)-factor from a Full-microscopic Nuclear Model

Yasutaka Taniguchi*
Department of Information Engineering, National Institute of Technology (KOSEN),
Kagawa College, Mitoyo, Kagawa 769-1192, Japan and
Research Center for Nuclear Physics (RCNP), Osaka University, Ibaraki 567-0047, Japan

Masaaki Kimura†
Department of Physics, Hokkaido University, Sapporo 060-0810, Japan
Nuclear Reaction Data Centre, Hokkaido University, Sapporo 060-0810, Japan and
Research Center for Nuclear Physics (RCNP), Osaka University, Ibaraki 567-0047, Japan
(Dated: June 9, 2021)

The 12C + 12C fusion reaction plays a vital role in the explosive phenomena of the universe. The resonances in the Gamow window rule its reaction rate and products. Hence, the determination of the resonance parameters by nuclear models is indispensable as the direct measurement is not feasible. Here, for the first time, we report the resonances in the 12C + 12C fusion reaction described by a full-microscopic nuclear model. The model plausibly reproduces the measured low-energy astrophysical \( S \)-factors and predicts the resonances in the Gamow window. Contradictory to the hindrance model, we conclude that there is no low-energy suppression of the \( S \)-factor.

Introduction.— The 12C + 12C fusion reaction is a trigger and driving force of the carbon burning in massive stars [1, 2] and the X-ray superburst [3, 4]. Hence, its reaction rate is a key for understanding these explosive phenomena. In general, the reaction rate of charged particles is represented by a product of the exponentially damping factor and the astrophysical \( S \)-factor [1]. While the former represents the Coulomb penetrability, the latter carries the nuclear structural information consists of the resonant and non-resonant contributions. Because the direct measurement is quite difficult, the \( S \)-factor of the 12C + 12C reaction at low energy has been extrapolated from the measurement at higher energy. As a result, it has been the source of considerable uncertainty despite decades of studies [5, 6]. An estimate by Caughlan and Fowler (CF88) [7], a de facto standard for astrophysics simulations, assumes constant \( S \)-factor. On the contrary, Jiang et al. proposed the hindrance model [8], which asserts reducing the \( S \)-factor at low energy. It is still controversial which one is reasonable.

In addition to the uncertainty in the global behavior, the fine structure of the \( S \)-factor originating in the resonant contribution is even less understood. It is well known that low-energy resonances can significantly affect the evolution of astrophysical phenomena by increasing the reaction rate in orders of magnitude at specific temperatures [1]. For example, Cooper et al. introduced an ad-hoc resonance at 1.5 MeV inside the Gamow window of X-ray superburst [9]. In nuclear physics, such low-energy resonances in the 12C + 12C system have long been discussed [10–13]. Recently a couple of direct measurements have identified the resonances just above the Gamow window [14–17]. Furthermore, from the indirect measurement, Tumino et al. reported numerous narrow resonances in the Gamow window [18]. Contrary to the hindrance model, they proposed the reaction rate enhanced at low energy, albeit its absolute magnitude is still under debate [19].

Thus, the resonances in the 12C + 12C system are of particular interest. Since they are unlikely be measurable by the direct reactions, the study by nuclear models is indispensable. However, the description of deep sub-barrier resonances is challenging and demanding. The primary reaction channels, 12C(12C, \( \alpha \))20Ne and 12C(12C, \( p \))23Na, involve the rearrangement of many nucleons and the strong channel coupling. To mimic such complex reaction dynamics, nuclear models usually adopt phenomenological potentials [20–23], undermining their predictability at low energy. In principle, full-microscopic nuclear models without phenomenologically adjustable parameters can overcome this problem [24], but the resonant contributions have never been taken into account. Here, for the first time, we report a plausible description of the resonances by a full-microscopic nuclear model and provide an evaluation of the \( S \)-factor.

Nuclear model and resonance parameters.— To describe the low-energy resonances, we employ antisymmetrized molecular dynamics (AMD) [25], which handles the channel coupling by configuration mixing. Using the Gogny D1S nuclear density functional [26], it accurately describes the low-energy resonances of astrophysical interests such as 12C + 16O and \( \alpha + ^{28}\text{Si} \) and their compound systems [27–30]. The model wave function of AMD is a parity-projected Slater determinant of the nucleon wave packets [31],

\[
\Phi^\pi = \frac{1 + \pi P}{2} A\{ \varphi_1 \cdots \varphi_A \}, \quad (\pi = \pm), \tag{1}
\]

\[
\varphi_i = \prod_{\sigma = x, y, z} e^{-\nu_\sigma (r_\sigma - Z_{\gamma z})^2} (a_i |\uparrow\rangle + b_i |\downarrow\rangle) \eta_i, \tag{2}
\]

where each wave packet \( \varphi_i \) has the parameters; Gaussian centroid \( Z_\gamma \), width \( \nu \) and spin \( a_i \) and \( b_i \). The isospin \( \eta_i \) is
fixed to proton or neutron. Using the constraint on the inter-nuclear distance [32], all parameters are determined for each channel and inter-nuclear distance by the energy variation. In this study, we have calculated the $^{12}$C + $^{12}$C and $\alpha + ^{20}$Ne channels by using this method. Figure 1 shows several channel wave functions which have different inter-nuclear distances and orientations of nuclei. Note that the rotation and polarization of nuclei depending on the inter-nuclear distance are described naturally. As explained later, these are essential in describing the low-energy resonances. In addition, we also include the wave functions of the compound nucleus $^{24}$Mg [33]. All these channel wave functions ($\Phi_l^r$) are superposed to describe the resonances,

$$\Psi_{JM}^{J_\pi} = \sum_{iK} c_{iK} P_{MK}^{J} \Phi_{l}^{r}, \quad (3)$$

where $P_{MK}^{J}$ denotes the angular momentum projector. The resonance energy and the coefficients $c_{iK}$ are determined by the diagonalization of the Hamiltonian. The superposition of various channel wave functions significantly improves the model accuracy. First, it describes the rotational excitation of nuclei during the reaction process. Therefore, the model can explain the decays to the excited states such as $\alpha + ^{20}$Ne(2$^+$). It also accounts for the dynamical change of the Coulomb barrier height due to the nuclear deformation [34]. Second, the channel coupling is described by a full-microscopic Hamiltonian in a unified manner. Hence, the model is free from adjustable parameters. These improvements realize an accurate description of the deep sub-barrier resonances.

To evaluate the decay widths, we calculate the reduced width amplitude (RWA) [35, 36] which is the overlap between the decay channel and resonance wave functions,

$$\gamma_l^{A_1 + A_2}(a) = \sqrt{\frac{24}{A_1}} \left\langle \delta(r - a) \right\rangle \left\langle \Phi_{l_1}^{r_1}, \Phi_{l_2}^{r_2} Y_l^J \right\rangle_M P_{MK}^{J} \Phi_{l_1}^{r_1}, \quad (4)$$

where the ket is the resonance wave function, while the bra is the channel wave function decaying into two nuclei with masses $A_1$ and $A_2$ separated by the distance $a$ with the orbital angular momentum $l$. We consider four decay channels, $p + ^{23}$Na(3/2$^+$), $p + ^{23}$Na$(5/2^+_1)$, $\alpha + ^{20}$Ne(0$^+_1$) and $\alpha + ^{20}$Ne$(2^+_1)$ which are denoted by $p_0$, $p_1$, $\alpha_0$, and $\alpha_1$, respectively. The wave functions of $\alpha$, $^20$Ne, and $^{23}$Na are also calculated by AMD. For the later use, the RWAs are given as the ratio to the Wigner limit,

$$\theta_{A_1 + A_2}^2(a) = \frac{a}{3} |\gamma_l^{A_1 + A_2}(a)|^2, \quad (5)$$

where the channel radii for the $^{12}$C + $^{12}$C, $\alpha_{0,1}$, and $p_{0,1}$ channels are chosen as 6, 6 and 4 fm so that RWAs are smoothly connected to the Coulomb wave function.

Table I lists the calculated resonance parameters in the energy range of interest (resonance energy $E_R < 4$ MeV). Note that the calculation yields many deep sub-barrier resonances. In particular, it is remarkable that some of them plausibly coincide with the observed higher-energy peaks of the S-factor at higher energies. For example, the $J^\pi = 2^+$ resonances obtained at 2.18 and 3.73 MeV are the candidates for the 2.14 and 3.8 MeV resonances identified by the direct measurements [14, 17, 37]. Furthermore, the calculation predicts three resonances within the Gamow window at 0.93, 0.94, and 1.50 MeV. This result confirms the long-standing conjecture about the existence of the low-energy resonances in the $^{12}$C + $^{12}$C system [10–12] and is in accordance with the indirect measurement [18] which reported many resonances.

The RWAs of the resonances demonstrate the importance of the microscopic treatment of channel coupling and rotational excitation. The $0^+$ resonance at 0.94 MeV has a large RWA in the $\alpha_1$ channel showing the rotational excitation of $^{20}$Ne. Moreover, the $2^+$ resonance at 2.18 MeV has sizable RWAs in the $\alpha_0$, $\alpha_1$, and $p_0$ channels and in the entrance $^{12}$C + $^{12}$C channel. Therefore, it should affect the reaction rates in both the $\alpha$ and $p$ channels. The importance of the channel coupling can also be confirmed differently. If we perform a single-channel calculation, for example, with only the $^{12}$C + $^{12}$C channel, we do not obtain any resonance in the Gamow window. It demonstrates the necessity of the full-microscopic calculation for quantitative discussion of the sub-barrier resonances.

Astrophysical S-factor.— From the resonance parameters, we evaluate the modified astrophysical S-factors ($S^*$-factors) [39]. According to the R-matrix theory [40, 41], the partial width for the $A_1 + A_2$ decay is given as,

$$\Gamma_{A_1 + A_2} = \frac{2ka}{F_1(ka)^2 + G_1(ka)^2} \frac{3\hbar^2}{2\mu^2} \theta_{A_1 + A_2}^2, \quad (6)$$

where the wave number $k$ is related with the decay $Q$-value as $Q = \hbar^2 k^2 / 2\mu$ with the reduced mass $\mu$. $F_1$ and $G_1$ are the regular and irregular Coulomb wave functions, respectively. The partial cross section at the center-of-mass energy $E$ is given as the Breit-Wigner form [40],

$$\sigma(E) = \frac{\pi \hbar^2 (2J + 1)}{2\mu E} \frac{\Gamma_1 \Gamma_{A_1 + A_2}}{(E - E_R)^2 + \Gamma^2/4}. \quad (7)$$

FIG. 1. The wave functions for the (a)–(c) $\alpha + ^{20}$Ne, and (d)–(f) $^{12}$C + $^{12}$C channels obtained by the energy variation. Inter-nuclear distance is 6.5, 5.0, and 3.5 fm from left to right.
TABLE I. The calculated resonance energies in MeV, isoscalar transition matrix elements in Weisskopf unit, and RWAs of the $0^+$ and $2^+$ resonances. The RWAs are given as the ratio to the Wigner limit (see text) and multiplied by a factor of a hundred.

| $J^*$ | $E_R$ | $M_{IS}$ | $\theta_0^2 \cdot 10^2$ | $\theta_2^2 \cdot 10^2$ | $\theta_4^2 \cdot 10^2$ | $\theta_{p0}^2 \cdot 10^2$ | $\theta_{p2}^2 \cdot 10^2$ |
|-------|-------|----------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| $2^+$ | 0.93  | 1.56     | 1.4                      | —                        | 3.5                      | 0.661                    | 1.7                      | 6.7                      |
|       |       |          |                          |                          |                          | 0.47                     | 0.081                    | 0.030                    |
| $0^+$ | 0.94  | 0.59     | 7.3                      | 0.20                     | —                        | 7.1                      | —                        | 10                       |
|       |       |          |                          |                          |                          | —                        | —                        | 10                       |
| $2^+$ | 1.50  | 1.04     | 2.9                      | —                        | 1.1                      | 4.0                      | 0.90                     | 0.51                     |
|       |       |          |                          |                          |                          | —                        | 16                       | 0.22                     |
| $2^+$ | 2.18  | 0.51     | 3.4                      | —                        | 1.0                      | 1.0                      | 0.19                     | 3.4                      |
|       |       |          |                          |                          |                          | 3.3                      | 0.12                     | 0.10                     |
| $0^+$ | 3.02  | 1.05     | 11.0                     | 0.26                     | —                        | 0.57                     | —                        | 0.99                     |
|       |       |          |                          |                          |                          | —                        | —                        | 4.3                      |
| $2^+$ | 3.56  | 0.23     | 1.2                      | —                        | 0.038                    | 0.056                    | 0.006                    | 0.040                    |
|       |       |          |                          |                          |                          | 0.66                     | 0.86                     | 0.001                   |
|       |       |          |                          |                          |                          | 0.029                    | 0.79                     | 0.041                   |
| $2^+$ | 3.73  | 0.41     | 8.3                      | —                        | 0.10                     | 0.066                    | 0.10                     | 0.88                     |
|       |       |          |                          |                          |                          | 0.24                     | 0.72                     | 0.028                   |
|       |       |          |                          |                          |                          | 0.043                    | 0.67                     | 0.089                   |

where $J$ and $E_R$ denote the spin and energy of a resonance and $\Gamma_1$ is the partial width in the entrance ($^{12}\text{C} + ^{12}\text{C}$) channel. The total width $\Gamma$ is estimated in the same manner as in the latest experimental analysis [16, 17], i.e., it is estimated from the $p_1$ and $\alpha_1$ widths and the linear extrapolation of the observed branching ratio [38]. The $S^*$-factor is calculated as,

$$S^*(E) = E\sigma(E)\exp(2\pi\eta + 0.46 \text{ MeV}^{-1}E),$$ \hspace{1cm} (8)

with the Sommerfeld parameter $\eta = 36/137\sqrt{\mu c^2/2E_R}$ [39].

Figure 2(a) compares the calculated and observed $S^*$-factors in the $p_1$ channel. Above the Gamow window ($E \gtrsim 2.0$ MeV), the calculated $S^*$-factor has the contributions from four resonances ($2.18, 3.02, 3.56,$ and $3.73$ MeV) and its magnitude is the order of $10^{14}$–$10^{15}$ MeV·b, which plausibly agrees with the experiments [14, 16, 17, 37, 38]. The peak positions at 2.2 and 3.8 MeV coincide with the direct $^{12}\text{C} + ^{12}\text{C}$ fusion experiments [14, 17, 37]. The calculation predicts that the resonant contributions are also present inside the Gamow window. In particular, the resonances at 0.94 and 0.95 MeV form a prominent peak. For the comparison, the calculated $S^*$-factor in the $p_0$ channel is also shown in Fig. 2(a). It has a similar shape, but the magnitude is much larger than the $p_1$ channel.

Because of the strong coupling between $p$ and $\alpha$ channels, the same resonant peaks appear in the $\alpha_1$ channel, as shown in Fig. 2(b). The peaks at 2.2 and 3.8 MeV exist in the $\alpha_1$ channel in the calculation and observed data [14, 37, 38]. The calculation yields more prominent peak at 3.0 MeV than that in the $p_1$ channel. This peak nicely coincides with the peak approximately at 3.2 MeV observed in Refs. [14, 37, 38]. It is impressive that the calculation predicts pronounced $S^*$-factor inside the Gamow window even stronger than above the window owing to the contributions from the resonances at 0.93 and 0.94 MeV.

Finally, Fig. 2(c) shows the total $S^*$-factor as the sum of the $S^*$-factors in all $\alpha$ and $p$ channels. Here, the total $S^*$-factors in the $\alpha$ and $p$ channels are again estimated in the same manner with Refs. [16, 17]. The figure also shows the estimations by CF88 [7] and the hindrance model [8] in addition to the data from Refs. [14, 16, 17, 38]. Similar to the $S^*$-factors in the $p_1$ and $\alpha_1$ channels, the calculation plausibly describes the observed global behavior. Furthermore, as we have already repeated, the calculation predicts resonance contributions at very low energy. Consequently, the calculated $S^*$-factor has prominent peaks in the Gamow window and does not show the low-energy hindrance. Hence, the low-energy enhancement of the $S$-factor, for example, that postulated by Cooper et al. [9], is conceivable enough.

A novel probe for the resonances.— Finally, we propose the isoscalar transitions as a novel experimental probe to identify the resonances predicted in this study. The transition matrix element from the ground state to a resonance is defined as,

$$M_{IS} = \langle J^\pi, J_z = 0 | M_{IS} | ^{24}\text{Mg}(g.s.) \rangle,$$ \hspace{1cm} (9)

where the bra is the resonance wave function with the spin-parity $J^\pi$, while the ket is the ground state of $^{24}\text{Mg}$. $M_{IS}$ is either of the isoscalar monopole or quadrupole operators depending on the spin-parity of the resonance,

$$M_{IS} = \left\{ \begin{array}{ll}
\sum_{l=1}^{4} r_l^2 Y_{20}(\hat{r}_l), & J^\pi = 0^+ \\
\sum_{l=1}^{4} r_l^2 Y_{20}(\hat{r}_l), & J^\pi = 2^+.
\end{array} \right.$$ \hspace{1cm} (10)

In recent years, it has been extensively discussed that the isoscalar transitions from the ground state to the cluster resonances are considerably enhanced [42–45]. As listed in Table I, the present calculation predicts that all the resonances have large transition strengths comparable with the Weisskopf unit. Therefore, the reactions that induce isoscalar transitions should strongly populate these resonances. The most promising and feasible reaction may be the $\alpha$-inelastic scattering, $^{24}\text{Mg}(\alpha, \alpha')^{24}\text{Mg}^*$. Note that this reaction bypasses the Coulomb barrier, and hence, can easily access the deep sub-barrier resonances. It is encouraging that a couple of experiments have already been conducted [46–49], and several candidates of the resonances are observed in the energy range of our interest, although the spin-parity and decay
Cauldrons in the cosmos

The AMD, a microscopic nu-
12
12
onances in the

observed total

for the comparison. Panel (c) compares the calculated and

uncover the resonances in the Gamow window. Consequently, the calculation suggested no low-
energy suppression of the $S^*$-factors above the Gamow window. Furthermore, it predicted several resonances inside the Gamow window, which create low-energy $S^*$-factor peaks. Consequently, the calculation suggested no low-energy suppression of the $S^*$-factors above the Gamow window. Finally, we propose the $^{24}\text{Mg}(\alpha, \alpha^{'})^{24}\text{Mg}^*$ reaction experiment as the most promising bypass to access the deep sub-barrier resonances.

This work was supported by JSPS KAKENHI Grant Number 19K03859, the collaborative research program at Hokkaido University, and the COREnet program in RCNP, Osaka University. Numerical calculations were performed using Oakforest-PACS at the CCS, University of Tsukuba.

Summary.— We have investigated the low-energy resonances in the $^{12}\text{C} + ^{12}\text{C}$ fusion reaction, which is of astrophysical interest. The AMD, a microscopic nu-
clear model, firstly realized a quantitative description of the low-energy resonances by handling the channel coupling and nuclear rotation without adjustable parameters. It successfully described observed resonances and behavior of the $S^*$-factors above the Gamow window. Furthermore, it predicted several resonances inside the Gamow window, which create low-energy $S^*$-factor peaks. Consequently, the calculation suggested no low-energy suppression of the $S^*$-factors above the Gamow window. Finally, we propose the $^{24}\text{Mg}(\alpha, \alpha^{'})^{24}\text{Mg}^*$ reaction experiment as the most promising bypass to access the deep sub-barrier resonances.

This work was supported by JSPS KAKENHI Grant Number 19K03859, the collaborative research program at Hokkaido University, and the COREnet program in RCNP, Osaka University. Numerical calculations were performed using Oakforest-PACS at the CCS, University of Tsukuba.

* taniguchi-y@di.kagawa-nct.ac.jp
† masaaki@nucl.sci.hokudai.ac.jp
[1] C. E. Rolfs and W. S. Rodney, Cauldrons in the cosmos (University of Chicago Press, Chicago, 1988).
[2] S. E. Woosley, A. Heger, and T. A. Weaver, Rev. Mod. Phys. 74, 1015 (2002).
[3] A. Cumming and L. Bildsten, The Astrophysical Journal 559, L127 (2001), arXiv:0107213 [astro-ph].
[4] T. E. Strohmayer and E. F. Brown, The Astrophysical Journal 566, 1045 (2002).
[5] B. B. Back, H. Esbensen, C. L. Jiang, and K. E. Rehm, Reviews of Modern Physics 86, 317 (2014).
[6] C. Beck, A. M. Mukhamedzhanov, and X. Tang, The European Physical Journal A 56, 87 (2020).
[7] G. R. Caughlan and W. A. Fowler, Atomic Data and Nuclear Data Tables 40, 283 (1988).
[8] C. L. Jiang, K. E. Rehm, B. B. Back, and R. V. F. Janssens, Physical Review C 75, 015803 (2007).
[9] R. L. Cooper, A. W. Steiner, and E. F. Brown, The Astrophysical Journal 702, 660 (2009).
[10] E. Almqvist, D. A. Bromley, and J. A. Kuehner, Physical Review Letters 4, 515 (1960).
[11] R. G. Stokstad, Z. E. Switkowski, R. A. Dayras, and R. M. Wieland, Physical Review Letters 37, 888 (1976).
[12] S. K. Korotky, K. A. Erb, S. J. Willett, and D. A. Bromley, Physical Review C 20, 1014 (1979).
[13] M. Freer, Reports on Progress in Physics 70, 2149 (2007).
[14] T. Spillane, F. Raiola, C. Rolfs, D. Schirrmann, F. Strieder, S. Zeng, H. W. Becker, C. Bordeau, L. Galianella, M. Romanu, and J. Schweitzer, Physical Review Letters 98, 122501 (2007).
[15] B. Bucher, X. D. Tang, X. Fang, A. Heger, S. Almaraz-Calderon, A. Longi, A. D. Ayangeakaa, M. Beard, A. Best, J. Browne, C. Cahillane, M. Couder, R. J. DeBoer, A. Kontos, L. Lamm, Y. J. Li, A. Long, W. Lu, S. Lyons, M. Notani, D. Patel, N. Paul, M. Pignatari, A. Roberts, D. Robertson, K. Smith, E. Stech, R. Taiwar, W. P. Tan, M. Wiescher, and S. E. Woosley, Physical

![Image of graph showing calculated and observed modified astrophysical $S^*$-factors in the (a) $p_1$ and (b) $p_0$ channels. The calculated $S^*$-factors in the $p_0$ and $p_0$ channels are also shown for the comparison. Panel (c) compares the calculated and observed total $S^*$-factors with the evaluations [7, 8]. Open symbols indicate upper limits.](attachment:image.png)
