Algorithm for the formation of the internal structure of a product manufactured using FDM technology, considering the stress-strain state on the example of four-point bending

S V Shalygin, G S Russkikh

Omsk State Technical University, 11 Mira Ave., Omsk, 644050, Russia

E-mail: skb.omgtu@gmail.com

Abstract. The work is devoted to the description of an algorithm for the formation of the internal structure of a product, obtained using the FDM technology, considering the stress-strain state, based on the use of isosurfaces of equivalent stresses obtained on a pre-calculated stress-strain state of the sample in an elastic isotropic formulation. In addition, the preparation of a force structure based on isosurfaces of normal stresses, considering the problem of axial tension / compression of horizontal layers of the sample, typical for the problem of four-point bending, is described. Numerical simulations in ANSYS Workbench have obtained results showing the performance of the optimized design, using the example of the four-point bending problem. Comparison of the results of numerical modelling, optimized and monolithic samples, in elastic isotropic formulation is presented.

1. Introduction

The currently approved mathematical methods for optimizing the strength and rigidity of products obtained by 3D printing have a number of advantages. However, these optimization algorithms have little application to the most widespread and available 3D printing technology - polymer layer-by-layer deposition (FDM / FFF) technology. Recent studies devoted to this problem - the development of methods for obtaining products using the method of layer-by-layer deposition, with optimal mechanical properties from the point of view of their further operation, are presented in [1]. Issues related to the mechanical strength of samples and products made using 3D printing, considering the problems of optimizing the internal structure of products, without considering the type of loading, are considered in [2–3]. Possibility of stable implementation of printed models considering the classification of defects arising in the manufacture of parts by layer-by-layer deposition [4–5]. Theoretical studies of topological design optimization in terms of fault tolerance of 3-d printed samples made of PLA-plastic are described in [6, 7]. In view of the above, the task of developing an algorithm for forming the internal structure of a product obtained by layer-by-layer deposition of thermoplastic materials (FDM/FFF), considering the stress-strain state, is urgent. Four-point bending is considered for the following reasons, due to the fact that this type of loading is one of the special cases of the types of complex-stressed state, consideration of this type of loading hypothetically allows us to develop more complex algorithms from the presented in the paper to optimize the internal structure of products operating in a complex-stressed state.
2. The problem of four-point bending, in an elastic isotropic formulation

Please For numerical modelling, a sample according to GOST R 56805-2015 [8] from PLA (polylactide) was adopted, this standard is an analogue of the ASTM D790 – 03 standard [9] with the difference that the latter is not used for testing materials with anisotropic properties. The calculation model, which includes: sample dimensions, loading conditions and boundary conditions, was formed in ANSYS WB for Class II materials according to the scheme and is shown in Fig. 1.

![Figure 1. Four-point loading scheme for determining mechanical characteristics during bending, for materials of class II according to GOST R 56805-2015](image)

Calculation of the maximum deflection of a monolithic sample.

The maximum deflection of the sample was previously determined based on the condition of minimum allowable stresses. The permissible voltages were, in the first approximation, taken from third-party sources. [10] the design scheme is shown in Fig. 2.

![Figure 2. Calculation scheme for determining the maximum deflection of the sample based on the strength condition.](image)

To determine the maximum allowable deflection of the sample, the strength condition can be written as [11]
\[ \sigma_{\text{max}} = \frac{M_{\text{max}}}{W_x} \leq [\sigma] \]  

(1)

Where \( \sigma_{\text{max}} \) is the maximum normal stresses arising in the transverse layer of the sample; \( M_{\text{max}} \) is the maximum bending moment, which for the four-point bending problem can be written as: \( M_{\text{max}} = Q \cdot a \), where \( Q \) is the value of the transverse force for the layer with maximum stresses, and is the distance of the point of application of the force \( Q \). \( W_x \) is the moment of resistance relative to the Z axis, for a rectangular cross-section sample, the expression of which has the form:

\[ W_x = \frac{B \cdot H^2}{6} \], 

where \( B \) is the width of the sample, \( H \) is the height of the sample. In the expanded form, the expression (1) can be written:

\[ \sigma_{\text{max}} = \frac{5 \cdot Q \cdot a}{B \cdot H^2} \leq [\sigma] \]  

(2)

Under the condition of equal normal and permissible stresses \( \sigma_{\text{max}} = [\sigma] \) from expression (2), it is possible to determine \( Q_{\text{max}} \) - the maximum force at which the stresses are limiting, based on the strength condition, for a given cross section:

\[ Q_{\text{max}} = \frac{B \cdot H^2 \cdot \sigma_{\text{max}}}{6 \cdot a} \]  

(3)

Next, using the expression for maximum deflection in the pure four-point bending problem [11]

\[ f_{\text{max}} = \frac{E I \cdot Q \cdot l^2}{24} \left[ 3a^3 - \frac{4a^3}{l} \right] \]  

(4)

Where \( E \) is Young’s modulus; \( I \) - the moment of inertia of the cross section \( \frac{I}{12} \) in this case, relative to the Z axis; \( l \) is the length between the supports. Substituting the expression into the formula of the maximum deflection (4), we obtain the expression (3):

\[ f_{\text{max}} = \frac{B \cdot H^2 \cdot E I \cdot \sigma_{\text{max}} \cdot l^2}{144 \cdot a} \left[ \frac{3a^3}{l} - \frac{4a^3}{l^3} \right] \]  

(5)

Numerically solving expression (5) in accordance with the parameters for this problem, we obtain \( f_{\text{max}} = 0.008155 \). We get the maximum allowable deflection = 0.8 mm. To account for internal 3D printing defects, we introduce a safety factor equal to 0.8 \( \sigma_{\text{max}} \). Then, according to expression (5), the maximum deflection should not exceed 0.7 mm. A plot of normal stress isosurfaces was obtained from the QE calculation by internal means of ANSYS Mechanical and imported into the CAD system in STL format. Fig. 3.
Figure 3. A plot of isosurfaces of equivalent stresses for a monolithic sample of PLA plastic working for bending.

Then the internal structure of the sample was constructed according to the following algorithm:
1) Construction of elements of the internal structure of the sample parallel to the isosurfaces and ensuring the strength of the sample under the condition of maximum tensile stresses along the X axis
2) Construction of the central part providing strength and rigidity in the axial cross-section of the sample, based on the hypothesis of equal deformations.
3) Determination of the pitch and the required area of the remaining internal filling elements, considering the proportionality of stresses, and Pareto's law.

Construction of the internal structure of the sample working on tension/compression along the X axis

Let us assume the following assumptions, let each of the elements of the internal structure of the sample be equidistant at an infinitesimal length of the normal stress isosurface obtained from the CE analysis. The amount of material providing tensile strength can be determined in accordance with the normal stresses on each isosurface, for this it is necessary to determine the cross section providing strength based on the problem of axial compression stretching. Fig. 4.

Figure 4. A scheme for determining the longitudinal force in accordance with normal stresses in a monolithic sample.

The strength condition of axial tension compression can be written as [11]:

$$\sigma_i = \frac{|N_i|}{A_i} \leq [\sigma] \quad (6)$$

Where $N_i$ is the longitudinal tensile / compressive force; for the cross-sectional area providing strength, $A_i$ is the cross-sectional area of the conditional layer in which $\sigma_i$ acts; i = 8 this number is taken in accordance with the number of finite elements, the sample along the Y axis, to ensure the accuracy of determining the value of normal stresses. The cross-sectional area based on this condition,
we obtain equal: 
\[ A_i = \frac{1}{8} \cdot B \cdot H = \frac{1}{8} \cdot 0.015 \cdot 0.04 = 0.00075 \text{ m}^2. \]

Longitudinal tension/compression force for each conditional layer: 
\[ N_i = \sigma_i \cdot A_i. \]

Next, it is possible to determine the minimum cross-sectional area for each layer using the condition of axial compression tension, (6), focusing already on the selection of the cross-section based on the allowable stresses for the material. In this case, expression (6) will take the form: 
\[ A_i = \frac{N_i}{\sigma_i}. \]

Given the known side of the section \( b \) equal to the width of the sample, we will find the second one based on the required area 
\[ A_i = b_i \cdot h_i; \]

\[ b_i = A_i / h_i. \]

The production of samples is assumed by the method of layer-by-layer deposition, as a result of which the values obtained are rounded up to a multiple of the nozzle diameter of the additive installation. For eight layers, the obtained values are presented in Table 1.

**Table 1.** The longitudinal tension/compression force depending on the normal stresses acting on each layer.

| Layer Number, \( i \) | Longitudinal forces \( N_i \), (N) | Cross-section height providing strength \( h_i \), mm | The cross-section height providing strength, considering the multiplicity of the nozzle of the additive installation \( h_i \), mm |
|------------------------|-------------------------------------|---------------------------------|-------------------------------------------------|
| 1                      | 68.94                              | 0.8480                          | 0.8                                             |
| 2                      | 45.90                              | 0.5646                          | 0.6                                             |
| 3                      | 22.85                              | 0.2811                          | 0.4                                             |
| 4                      | 0.19                               | 0.0023                          | 0                                               |
| 5                      | -11.71                             | 0.1141                          | 0                                               |
| 6                      | -34.76                             | 0.4276                          | 0.4                                             |
| 7                      | -46.28                             | 0.5693                          | 0.6                                             |
| 8                      | -69.33                             | 0.8527                          | 0.8                                             |

The outer shell of the sample, in which the maximum stresses in the middle zone operate, has a thickening, \( h_i = 0.8 \) mm, which is obtained by considering the combination of two adjacent isosurfaces to ensure the manufacturability of printing. Fig. 5.

![Figure 5. The outer shell of the optimized specimen](image)

Internal elements providing strength based on the condition of axial tension along the X axis are built on the basis of the isosurfaces presented in Fig. 3, the thickness of each \( i \)-element corresponds to the height of the minimum cross-section \( h_i \), considering the rounding of the latter to a thickness that ensures manufacturability. The internal structure of the elements providing strength based on the condition of axial tension along the X axis is shown in Fig. 6.
Construction of the central part of the sample providing rigidity

Based on the hypothesis of equal deformations, the central part of the optimized structure should ensure the rigidity and strength of the sample under these loading conditions. To determine the cross-section of the rod, it is possible to use the strength and stiffness conditions for the problems of axial compression stretching, considering the loading by two forces, the algorithm is described in detail [12].

Determination of the step and the required area of the remaining internal filling elements. To ensure strength and rigidity, considering the already obtained width of the central element, it is proposed to take the bearing area of the remaining elements considering the proportionality of stresses and Pareto's law [13]. Let 20% of the area of an unoptimized sample provide its strength and rigidity, on the basis of which the sum of the cross sections of all vertical elements should be at least 20% of the sample area. The step distribution of vertical structures is proposed to be proportional to the step of the isosurfaces of normal stresses, which in turn is equal to $b \sim (0.15...0.18) l$ - these values correspond to the stress distribution step of the non-optimized sample. To connect the previously constructed elements, to ensure flexural rigidity, it is proposed to project the isosurface of normal stresses directly onto the central plane of the sample in its longitudinal section. Based on this, it is possible to construct splines connecting the vertical projections of the planes of the elements obtained by calculating the total area of the distribution according to Pareto's law, and isosurfaces of equivalent stresses. Further, to ensure a uniform distribution of elastic characteristics throughout the optimized structure, we use the partitioning step for the cross-sectional areas of vertical structures obtained earlier. We will divide each of the spline projections of stress isosurfaces into proportional distances. Having connected the obtained points, we have the "power" structure of the sample (Fig. 7), hypothetically providing strength and rigidity under these loading conditions, considering the criterion of minimum stiffness of the axial cross-section, to ensure proportional stresses relative to the non-optimized sample.
After obtaining the main internal structure, the central part of the sample was modified by adding an elliptical hole to reduce stress concentrators. Also, the sides of the sample working to transfer the force from the punches and supports were additionally reinforced with elements based on a parabolic spline to ensure rigidity. The final view is shown in Fig. 8.

![Figure 8. The final view of the internal structure of the optimized sample.](image1)

For verification, we use numerical modelling of monolithic and optimized samples in ANSYS Mechanical using the example of a four-point bend. Both samples were numerically modelled in ANSYS WB. Load movement of the upper punches according to the calculated values obtained as a result of calculations (1) ... (5). The KE calculation was carried out in an isotropic formulation, the sample material from the standard libraries ANSYS WB PLA (polylactide) with elastic characteristics Young's modulus $E=3.04 \times 10^9$ Pa, Poisson's ratio $\mu=0.40890$. Material of supports and loading punch structural steel with elastic characteristics Young's modulus $E=2 \times 10^{11}$ Pa, Poisson's ratio $\mu=0.30000$. The contacts “sample-support”, “sample loading punch” are sliding, with friction, allowing large movements. The speed of movement of the punch is 1mm/min. The optimized sample was subjected to a similar loading. The diagram of equivalent stresses of the sample is shown in Fig. 9 (a,b). It can be seen that the stress distribution has become uniform, the main points where the stresses are highest are stress concentrators resulting from the construction of the geometry of the internal structure.

![Figure 9. a Plot of normal stresses of a monolithic specimen](image2)

![Figure 9.b Plot of normal stresses of a specimen with an optimized internal structure.](image3)

### 3. Findings

The nature of the work of the sample material can be represented in the form of two safety margin plots Fig. 10. for a monolithic sample (bottom). It can be seen that a significant part of the material does not carry a load, for a sample with an optimized internal structure, the distribution of material in the body of the sample provides strength and rigidity of almost the entire volume. Numerical data for the final comparison of the two samples are presented in Table 2.
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The results of numerical simulation show the operability of the selected algorithm. The analysis of the data obtained allows us to conclude about the advantage of an optimized sample: with a decrease in mass, the strength and rigidity of the sample are preserved. This structure can be created when using FDM printing. For the practical application of the proposed technique, it is necessary to consider the anisotropy and plastic deformations that occur during the loading of the specimen.

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