Status and Prospects of CKM Phase Determinations

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Abstract

We give an overview of direct determinations of the angles of the unitarity triangle of the CKM matrix, using CP-violating effects in $B$-meson decays. After a discussion of $B \rightarrow \pi K$ modes, which can be described efficiently through allowed regions in observable space and play an important rôle to determine $\gamma$, we turn to extractions of the $B^0_d - B^0_d$ mixing phase $\phi_d$, which equals $2\beta$ in the Standard Model, from $B_d \rightarrow J/\psi K_S$, and emphasize that it is important to determine this phase unambiguously. Finally, we focus on $B_d \rightarrow \pi^+ \pi^-$, where recent $B$-factory data point towards large penguin contributions. The question arises now how the CP-violating observables of this mode can be transformed into information on the angles of the unitarity triangle. A promising tool to achieve this goal is offered by $B_s \rightarrow K^+ K^-$, which is very accessible at hadronic $B$ experiments, and allows a determination of $\phi_d$ and $\gamma$. A variant for the $e^+ e^-$ $B$-factories is provided by $B_d \rightarrow \pi^+ K^+$, where data are already available, pointing to an exciting picture and a highly constrained allowed region in $B_s \rightarrow K^+ K^-$ observable space.

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1 Introduction

The non-conservation of the CP symmetry in weak interactions is a mystery since its
discovery through $K_L \rightarrow \pi\pi$ decays in 1964 \[1\]. Before the start of the $B$
factories, CP-violating effects could only be studied in the kaon system. In this decade, decays of
neutral and charged $B$-mesons will provide further valuable insights into CP violation, in
particular stringent tests of the Kobayashi–Maskawa mechanism of CP violation \[2\], which
introduces this exciting phenomenon to the Standard Model of electroweak interactions.
Using the “gold-plated” mode $B_d \rightarrow J/\psi K_S$ \[3\], large CP-violating effects could actually
be established in the $B$ system in 2001 \[4, 5\]. The central target of the $B$ factories is the
famous unitarity triangle of the Cabibbo–Kobayashi–Maskawa (CKM) matrix, with the
goal to overconstrain this triangle as much as possible through independent measurements
of its sides and angles. A detailed review of this topic can be found in \[6\].

The “standard analysis” to determine the apex of the unitarity triangle in the plane of
the generalized Wolfenstein parameters $\rho$ and $\eta$ \[7, 8\] employs the following ingredients,
as illustrated in Fig. 1:

- Exclusive and inclusive semi-leptonic $B$ decays caused by $b \rightarrow c(\tau, u)\ell$ quark-level
  transitions \[9\], fixing a circle with radius $R_b$ around (0, 0).
- $B_q^0 - B_q^0$ mixing ($q \in \{d, s\}$), fixing a circle with radius $R_t$ around (1, 0).
- Indirect CP violation in the neutral kaon system, $\varepsilon$, fixing a hyperbola.

Many different strategies to deal with the corresponding theoretical and experimental
uncertainties can be found in the literature. The most important ones are the simple
scanning approach \[10\], the Gaussian approach \[11\], the BaBar 95% scanning method
\[12\], the Bayesian approach \[13\], and the statistical approach developed in \[14\].
discussion of these approaches is beyond the scope of this presentation. Let us here just give typical ranges for $\alpha$, $\beta$ and $\gamma$ that are implied by these strategies:

$$70^\circ \lesssim \alpha \lesssim 130^\circ, \quad 15^\circ \lesssim \beta \lesssim 35^\circ, \quad 50^\circ \lesssim \gamma \lesssim 70^\circ.$$  \hspace{1cm} (1)

As far as the range for $\gamma$ is concerned, a particularly important constraint is provided by lower bounds on the $B_s$ mass difference $\Delta M_s$, which imply upper bounds on the side $R_t$ of the unitarity triangle \cite{15}:

$$(R_t)_{\text{max}} = 0.83 \times \xi \times \sqrt{\frac{15.0 \text{ps}^{-1}}{(\Delta M_s)_{\text{min}}}},$$  \hspace{1cm} (2)

where

$$\xi \equiv \frac{\sqrt{B_{B_s} f_{B_s}}}{\sqrt{B_{B_d} f_{B_d}}} = 1.15 \pm 0.06$$  \hspace{1cm} (3)

measures $SU(3)$-breaking effects in non-perturbative mixing and decay parameters. The strong experimental bound $\Delta M_s > 14.6 \text{ps}^{-1}$ (95\% C.L.) \cite{18} excludes already a large part in the $\pi \pi$ plane, implying in particular $\gamma < 90^\circ$. In a recent paper \cite{17}, it is argued that $\xi$ may actually be significantly larger than the conventional range given in (3), $\xi = 1.32 \pm 0.10$. In this case, the excluded range in the $\pi \pi$ plane would be reduced, shifting the upper limit for $\gamma$ closer to $90^\circ$. Hopefully, the status of $\xi$ will be clarified soon. In the near future, run II of the Tevatron should provide a measurement of $\Delta M_s$ \cite{18, 19}, thereby constraining the unitarity triangle and $\gamma$ in a much more stringent way.

After the discovery of CP violation in the $B$ system, the major goal is now the direct determination of the angles of the unitarity triangle through CP-B measurements. To this end, various approaches were proposed over recent years. Here we shall have a closer look at the following strategies: in Section 2 we discuss $B \to \pi K$ modes, which are promising to extract $\gamma$. In Section 3, we turn to the determination of the $B^0_d - \bar{B}^0_d$ mixing phase $\phi_d$, which equals $2\beta$ in the Standard Model, from the “gold-plated” decay $B_d \to J/\psi K_S$, and emphasize that it is important to determine this phase unambiguously. Another particularly interesting mode, the $B_d \to \pi^+\pi^-$ channel, is the subject of Section 4. Finally, concluding remarks and a brief outlook are given in Section 5.

2 Extracting $\gamma$ from $B \to \pi K$ Decays

If we employ flavour-symmetry arguments and make plausible dynamical assumptions, $B \to \pi K$ decays allow determinations of $\gamma$ and hadronic parameters with a “minimal” theoretical input \cite{20, 21}. Alternative approaches, relying on a more extensive use of theory, are provided by the recently developed “QCD factorization” \cite{33} and “PQCD” \cite{34} approaches, which allow furthermore a reduction of the theoretical uncertainties of the flavour-symmetry strategies discussed here.
Table 1: CP-conserving $B \rightarrow \pi K$ observables as defined in \([1]-[3]\). For the evaluation of $R$, we have used $\tau_{B^+}/\tau_{B^0_d} = 1.060 \pm 0.029$. The data refer to \([35, 36]\).

| Observable | CLEO       | BaBar     | Belle     | Average    |
|------------|------------|-----------|-----------|------------|
| $R$        | 1.00 ± 0.30 | 1.08 ± 0.16 | 1.23 ± 0.26 | 1.10 ± 0.14 |
| $R_c$      | 1.27 ± 0.47 | 1.27 ± 0.24 | 1.33 ± 0.37 | 1.29 ± 0.21 |
| $R_n$      | 0.59 ± 0.27 | 1.09 ± 0.42 | 1.42 ± 0.68 | 1.03 ± 0.28 |

Table 2: CP-violating $B \rightarrow \pi K$ observables as defined in \([1]-[3]\). For the evaluation of $A_0$, we have used $\tau_{B^+}/\tau_{B^0_d} = 1.060 \pm 0.029$. The data refer to \([36, 37]\).

| Observable | CLEO       | BaBar     | Belle     | Average    |
|------------|------------|-----------|-----------|------------|
| $A_0$      | 0.04 ± 0.16 | 0.05 ± 0.07 | 0.07 ± 0.09 | 0.05 ± 0.07 |
| $A_c^0$    | 0.37 ± 0.32 | 0.00 ± 0.23 | 0.05 ± 0.26 | 0.14 ± 0.16 |
| $A_n^0$    | 0.02 ± 0.10 | 0.05 ± 0.07 | 0.09 ± 0.13 | 0.05 ± 0.06 |

2.1 General Features and Observables

As can be seen by looking at the corresponding Feynman diagrams, $B \rightarrow \pi K$ modes may receive contributions from penguin and tree-diagram-like topologies, where the latter bring the CKM angle $\gamma$ into the game. Because of the small CKM factor $|V_{us}V_{ub}^*/(V_{ts}V_{tb}^*)| \approx 0.02$, the QCD penguin topologies play actually the dominant rôle, despite their loop suppression. As far as electroweak (EW) penguins are concerned, they contribute in colour-suppressed form to $B^0_d \rightarrow \pi^- K^+$ and $B^+ \rightarrow \pi^+ K^0$, and are hence expected to play a minor rôle in these modes. On the other hand, EW penguins may also contribute in colour-allowed form to $B^+ \rightarrow \pi^0 K^+$ and $B^0_d \rightarrow \pi^0 K^0$, and may here compete with tree-diagram-like topologies.

Relations between the $B \rightarrow \pi K$ amplitudes that are implied by the $SU(2)$ isospin flavour symmetry of strong interactions suggest the following combinations to probe $\gamma$: the “mixed” $B^\pm \rightarrow \pi^\pm K$, $B_d \rightarrow \pi^\pm K^\pm$ system \([21]-[24]\), the “charged” $B^\pm \rightarrow \pi^\pm K$, $B^\pm \rightarrow \pi^0 K^\pm$ system \([25]-[27]\), and the “neutral” $B_d \rightarrow \pi^0 K$, $B_d \rightarrow \pi^\mp K^\pm$ system \([27, 28]\). Correspondingly, we may introduce the following sets of observables \([27]\):

\[
\begin{align*}
\{ R \\
A_0
\end{align*}
\equiv \frac{\text{BR}(B_d^0 \rightarrow \pi^- K^+) \pm \text{BR}(\overline{B_d^0} \rightarrow \pi^+ K^-)}{\text{BR}(B^+ \rightarrow \pi^+ K^0) + \text{BR}(B^- \rightarrow \pi^- K^0)} \tau_{B^+}/\tau_{B^0_d}
\] (4)

\[
\begin{align*}
\{ R_c \\
A_c^0
\end{align*}
\equiv 2 \frac{\text{BR}(B^+ \rightarrow \pi^0 K^+) \pm \text{BR}(B^- \rightarrow \pi^0 K^-)}{\text{BR}(B^+ \rightarrow \pi^+ K^0) + \text{BR}(B^- \rightarrow \pi^- K^0)}
\] (5)

\[
\begin{align*}
\{ R_n \\
A_n^0
\end{align*}
\equiv \frac{1}{2} \frac{\text{BR}(B_d^0 \rightarrow \pi^- K^+) \pm \text{BR}(\overline{B_d^0} \rightarrow \pi^+ K^-)}{\text{BR}(B_d^0 \rightarrow \pi^0 K^0) + \text{BR}(\overline{B_d^0} \rightarrow \pi^0 K^0)}
\] (6)

The experimental status of these observables is summarized in Tables 1 and 2, where the CLEO numbers refer to \([37, 38]\), and those of BaBar and Belle to the recent updates.
reviewed in [37]. Moreover, there are stringent constraints on CP violation in $B^\pm \to \pi^\pm K$:

$$A_{\text{CP}}(B^\pm \to \pi^\pm K) = \begin{cases} 0.17 \pm 0.10 \pm 0.02 & \text{(BaBar)} \\ -0.46 \pm 0.15 \pm 0.02 & \text{(Belle)} \end{cases} \quad (7)$$

Let us note that a very recent preliminary study of Belle indicates that the large asymmetry in (7) is due to a $3\sigma$ fluctuation [38]. Within the Standard Model, a sizeable value of $A_{\text{CP}}(B^\pm \to \pi^\pm K)$ could be induced by large rescattering effects. Other important indicators for such processes are branching ratios for $B \to KK$ decays, which are already strongly constrained by the $B$ factories, and would allow us to take into account rescattering effects in the extraction of $\gamma$ from $B \to \pi K$ modes [24, 25, 27, 39]. Let us note that also the QCD factorization approach [33] is not in favour of large rescattering processes. For simplicity, we shall neglect such effects in the discussion given below.

Interestingly, already CP-averaged $B \to \pi K$ branching ratios may lead to non-trivial constraints on $\gamma$ [22, 25], provided the corresponding $R(c,n)$ observables are found to be sufficiently different from one. The final goal is, however, to determine $\gamma$. Let us first turn to the charged and neutral $B \to \pi K$ systems in the following subsection.

### 2.2 The Charged and Neutral $B \to \pi K$ Systems

The starting point of our considerations are relations between the charged and neutral $B \to \pi K$ amplitudes that follow from the $SU(2)$ isospin symmetry of strong interactions. Assuming moreover that the rescattering effects discussed above are small, we arrive at a parametrization of the following structure [27] (for an alternative one, see [29]):

$$R_{c,n} = 1 - 2r_{c,n} (\cos \gamma - q) \cos \delta_{c,n} + \left(1 - 2q \cos \gamma + q^2\right) r_{c,n}^2$$

$$A_{0,c,n} = 2r_{c,n} \sin \delta_{c,n} \sin \gamma \quad (8)$$

Here $r_{c,n}$ measures – simply speaking – the ratio of tree to penguin topologies. Using $SU(3)$ flavour-symmetry arguments and data on the CP-averaged $B^\pm \to \pi^\pm \pi^0$ branching ratio [20], we obtain $r_{c,n} \sim 0.2$. The parameter $q$ describes the ratio of EW penguin to tree contributions, and can be fixed through $SU(3)$ flavour-symmetry arguments, yielding $q \sim 0.7$ [23]. In order to simplify (8) and (9), we have assumed that $q$ is a real parameter, as is the case in the strict $SU(3)$ limit; for generalizations, see [27]. Finally, $\delta_{c,n}$ is the CP-conserving strong phase between trees and penguins.

Consequently, the observables $R_{c,n}$ and $A_{0,c,n}$ depend on the two “unknowns” $\delta_{c,n}$ and $\gamma$. If we vary them within their allowed ranges, i.e. $-180^\circ \leq \delta_{c,n} \leq +180^\circ$ and $0^\circ \leq \gamma \leq 180^\circ$, we obtain an allowed region in the $R_{c,n}$–$A_{0,c,n}$ plane [24, 30]. If the measured values of $R_{c,n}$ and $A_{0,c,n}$ should lie outside this region, we would have an immediate signal for new physics. On the other hand, if the measurements should fall into the allowed range, $\gamma$ and $\delta_{c,n}$ could be extracted. In this case, $\gamma$ could be compared with the results of alternative strategies and the values implied by the “standard analysis” of the unitarity triangle discussed in Section I, whereas $\delta_{c,n}$ provides valuable insights into hadron dynamics, thereby allowing tests of theoretical predictions.
Figure 2: The allowed regions in the $R_c - A_c^0$ plane: (a) corresponds to $0.20 \leq r_c \leq 0.28$ for $q = 0.68$, and (b) to $0.51 \leq q \leq 0.85$ for $r_c = 0.24$; the elliptical regions arise if we restrict $\gamma$ to the Standard-Model range specified in (10). In (c) and (d), we show the contours for fixed values of $\gamma$ and $|\delta_c|$, respectively ($r_c = 0.24$, $q = 0.68$).

In Fig. 2, we show the allowed regions in the $R_c - A_c^0$ plane for various parameter sets. The crosses represent the averages of the experimental results given in Tables 1 and 2. If $\gamma$ is constrained to

$$50^\circ \lesssim \gamma \lesssim 70^\circ,$$

which corresponds to the “Standard-Model” range for $\gamma$ implied by the fits of the unitarity triangle (see [1]), a considerably more restricted range arises in the $R_c - A_c^0$ plane. The contours in Figs. 2 (c) and (d) allow us to read off straightforwardly the preferred values for $\gamma$ and $|\delta_c|$, respectively, from the measured observables [30]. Interestingly, the present data seem to favour $\gamma \gtrsim 90^\circ$ (see also [1]), which would be in conflict with (10). Moreover, they point towards $|\delta_c| \lesssim 90^\circ$; factorization predicts $\delta_c$ to be close to $0^\circ$ [33]. If future, more accurate data should really yield a value for $\gamma$ in the second quadrant, the discrepancy with (10) may actually be larger than $90^\circ$. As we shall see in Section 4, new data on CP violation in $B_d \to \pi^+\pi^-$ would allow us to accommodate also this picture [30]. In the latter case, the Standard-Model expressions (8) and (9) would receive corrections due to new physics, so that also the extracted value for $\gamma$ would not correspond to the Standard-Model result. More detailed and explicit discussions of such new-physics scenarios can be found in [41].

The allowed regions and contours for the neutral $B \to \pi K$ system in the $R_n - A_n^0$ plane look very similar to those shown in Fig. 2 [30]. Unfortunately, the experimental
situation in the neutral $B \to \pi K$ system is still rather unstable. For instance, in the recent Belle update \cite{37}, the central value for $R_n$ has moved from 0.6 to 1.4. In the neutral strategy, another interesting observable is provided by the mixing-induced CP asymmetry of $B_d \to \pi^0 K_S$, which arises in the corresponding time-dependent rate asymmetry of the following kind \cite{6}:

$$\frac{\Gamma(B_q^0(t) \to f) - \Gamma(B_{\bar{q}}^0(t) \to \bar{f})}{\Gamma(B_q^0(t) \to f) + \Gamma(B_{\bar{q}}^0(t) \to \bar{f})} = A_{\text{dir}}^{\text{CP}}(B_q \to f) \cos(\Delta M_q t) + A_{\text{mix}}^{\text{CP}}(B_q \to f) \sin(\Delta M_q t).$$

(11)

In the Standard Model, we expect \cite{21}

$$A_{\text{mix}}^{\text{CP}}(B_d \to \pi^0 K_S) = A_{\text{mix}}^{\text{CP}}(B_d \to J/\psi K_S),$$

(12)

which may well be affected by new-physics contributions, preferably to the $B_d \to \pi^0 K_S$ amplitudes. Concerning the extraction of $\gamma$ from neutral $B \to \pi K$ decays discussed above, this CP asymmetry would allow us to take into account possible rescattering effects in an exact manner \cite{27}, i.e. without using flavour-symmetry arguments.

### 2.3 The Mixed $B \to \pi K$ System

After the discussion of the charged and neutral $B \to \pi K$ systems in Subsection 2.2, the mixed $B \to \pi K$ system consisting of $B^\pm \to \pi^\pm K$, $B_d \to \pi^\mp K^\pm$ can be described straightforwardly by just making appropriate replacements of variables: first, we have $r_{c,n} \to r$, where the determination of $r$ requires the use of factorization to fix a colour-allowed amplitude $T$ \cite{21, 22, 23, 22, 23}, or a measurement of $B_s \to \pi^\pm K^\mp$ and $U$-spin arguments \cite{12}. Second, we may set $q \to 0$, as EW penguins contribute only in colour-suppressed form \cite{21, 23}. We obtain then the following expressions:

$$R = 1 - 2 r \cos \gamma \cos \delta + r^2$$

$$A_0 = 2 r \sin \delta \sin \gamma,$$

(13)

(14)

which are symmetric under $\gamma \leftrightarrow \delta$. If we vary the CP-conserving strong phase $\delta$ and the CP-violating angle $\gamma$ within their physical ranges, we obtain an allowed region in the $R$--$A_0$ plane, as shown in \cite{30}. The experimental data fall well into the allowed region, but do not yet allow us to draw further definite conclusions. At present, the situation in the charged and neutral $B \to \pi K$ systems appears to be more exciting.

### 2.4 Other Recent $B \to \pi K$ Analyses

A study complementary to the one of the allowed regions in observable space discussed above was performed in \cite{31}, where the allowed regions in the $\gamma$--$\delta_{c,n}$ planes implied by $B \to \pi K$ data were explored. Another recent $B \to \pi K$ analysis can be found in \cite{32}, where the $R_{(c)}$ were calculated for given values of $A_0^{(c)}$ as functions of $\gamma$, and were compared with the $B$-factory data. Making more extensive use of theory than in the flavour-symmetry
strategies discussed in Subsections 2.2 and 2.3, several different avenues to extract $\gamma$ from $B \to \pi K$ modes are provided by the QCD factorization approach [33], which allows also a reduction of the theoretical uncertainties of the flavour-symmetry approaches, in particular a better control of $SU(3)$-breaking effects. In order to analyse $B \to \pi K$ data, also a set of sum rules relating CP-averaged branching ratios and CP asymmetries of $B \to \pi K$ modes may be useful [13].

3 Extracting $\phi_d$ from $B_d \to J/\psi K_S$

3.1 The “Gold-Plated” Mode $B_d \to J/\psi K_S$

Since penguin topologies enter in $B_d \to J/\psi K_S$ essentially with the same weak phase as tree-diagram-like contributions, i.e. the phase difference is doubly Cabibbo-suppressed, we obtain to a very good approximation [3, 6]:

$$A_{\text{dir}}^{\text{CP}}(B_d \to J/\psi K_S) = 0, \quad A_{\text{mix}}^{\text{CP}}(B_d \to J/\psi K_S) = -\sin \phi_d,$$

where the CP-violating weak $B_d^0 \to B_d^+$ mixing phase $\phi_d$ is given by $2\beta$ in the Standard Model. After important first steps by the OPAL, CDF and ALEPH collaborations, the $B_d \to J/\psi K_S$ mode (and similar decays) led eventually to the observation of CP violation in the $B$ system in 2001 [4, 5]. The present status of $\sin 2\beta$ is given as follows:

$$\sin 2\beta = \begin{cases} 0.75 \pm 0.09 \pm 0.04 & \text{(BaBar [44])} \\ 0.82 \pm 0.12 \pm 0.05 & \text{(Belle [45])} \end{cases},$$

yielding the average of

$$\sin 2\beta = 0.78 \pm 0.08,$$

which agrees very well with the results of the CKM fits (see [11]), $0.5 \lesssim \sin 2\beta \lesssim 0.9$.

In the LHC era, the experimental accuracy of the measurement of $\sin 2\beta$ may be increased by one order of magnitude [46]. In view of such a tremendous accuracy, it will then be important to obtain deeper insights into the theoretical uncertainties affecting (15), which are due to penguin contributions. A possibility to control them is provided by the $B_s \to J/\psi K_S$ channel [17]. Moreover, also direct CP violation in $B \to J/\psi K$ modes allows us to probe such penguin effects [48, 49]. So far, there are no experimental indications for non-vanishing CP asymmetries of this kind.

3.2 Unambiguous Determination of $\phi_d$

Although the agreement between (17) and the results of the CKM fits is striking, it should not be forgotten that new physics may nevertheless hide in $A_{\text{mix}}^{\text{CP}}(B_d \to J/\psi K_S)$. The point is that the key quantity is actually $\phi_d$, which is fixed through $\sin \phi_d = 0.78 \pm 0.08$ up to a twofold ambiguity,

$$\phi_d = \left(51^{+8}_{-7}\right)^\circ \lor \left(129^{+7}_{-8}\right)^\circ.$$

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Here the former solution would be in perfect agreement with the range implied by the CKM fits, $30^\circ \lesssim \phi_d \lesssim 70^\circ$, whereas the latter would correspond to new physics. The two solutions can be distinguished through a measurement of the sign of $\cos \phi_d$: in the case of $\cos \phi_d = +0.6 > 0$, we would conclude $\phi_d = 51^\circ$, whereas $\cos \phi_d = -0.6 < 0$ would point towards $\phi_d = 129^\circ$, i.e. new physics.

There are several strategies on the market to resolve the twofold ambiguity in the extraction of $\phi_d$ [50]. Unfortunately, they are rather challenging from a practical point of view. In the $B \to J/\psi K$ system, $\cos \phi_d$ can be extracted from the time-dependent angular distribution of the decay products of $B_d \to J/\psi \to \ell^+ \ell^- K^* \to \pi^0 K_S$, if the sign of a hadronic parameter $\cos \delta$ involving a strong phase is fixed through factorization [51]. Let us note that analyses of this kind are already in progress at the $B$ factories [52].

### 3.3 Aspects of New Physics in $B \to J/\psi K$

The preferred mechanism for new physics to manifest itself in CP-violating effects in $B_d \to J/\psi K_S$ is through $B_d^0 - \overline{B}_d^0$ mixing, which arises in the Standard Model from the well-known box diagrams. However, new physics may also enter at the $B \to J/\psi K$ amplitude level. Employing estimates borrowed from effective field theory suggests that the effects are at most of $\mathcal{O}(10\%)$ for a generic new-physics scale $\Lambda_{\text{NP}}$ in the TeV regime. In order to obtain the whole picture, a set of appropriate observables can be introduced, using $B_d \to J/\psi K_S$ and its charged counterpart $B^\pm \to J/\psi K^\pm$ [49]. So far, these observables do not yet indicate any deviation from the Standard Model.

In the context of new-physics effects in the $B \to J/\psi K$ system, it is interesting to note that an upper bound on $\phi_d$ is implied by an upper bound on $R_b \propto |V_{ub}/V_{cb}|$, as can be seen in Fig. 1. To be specific, we have

$$\sin \beta_{\text{max}} = R_{b_{\text{max}}}^\text{max},$$

yielding $(\phi_d)_{\text{max}}^{\text{SM}} \sim 55^\circ$ for $R_{b_{\text{max}}}^\text{max} \sim 0.46$. As the determination of $R_b$ from semi-leptonic tree-level decays is very robust concerning the impact of new physics, $\phi_d \sim 129^\circ$ would require new-physics contributions to $B_d^0 - \overline{B}_d^0$ mixing. As we will see in the subsequent section, an interesting connection between the two solutions for $\phi_d$ and constraints on $\gamma$ is provided by CP violation in $B_d \to \pi^+ \pi^-$ [31].

Concerning the search for new physics, many other promising strategies can now be performed in practice at the $B$ factories. An important example is the decay $B_d \to \phi K_S$, where – within the Standard Model – mixing-induced CP violation is to a good accuracy equal to the one in $B_d \to J/\psi K_S$ [18, 53, 57], and direct CP violation is expected to be small. In analogy to the $B \to J/\psi K$ system [19], also here $B^\pm \to \phi K^\pm$ decays should be considered as well in order to obtain the whole picture [53]. Let us note that the $B$ factories have already observed these modes, and that first measurements of $A_{\text{CP}}(B_d \to \phi K_S)$ were reported very recently by BaBar and Belle [56]. In the future, the experimental situation will improve dramatically.

At this conference, a much more detailed discussion of $B$ physics beyond the Standard Model was given by David London [57]. Let us now focus on the decay $B_d \to \pi^+ \pi^-$. 

4 Extracting Weak Phases from $B_d \to \pi^+\pi^-$

4.1 The Penguin Problem

In contrast to $B_d \to J/\psi K_S$, the relevant penguin parameter is not doubly Cabibbo suppressed in the $B_d \to \pi^+\pi^-$ decay amplitude, leading to the well-known “penguin problem” in $B_d \to \pi^+\pi^-$. If we had negligible penguin contributions, the corresponding CP-violating observables were given as follows [6]:

$$A_{\text{dir}}^{\text{CP}}(B_d \to \pi^+\pi^-) = 0, \quad A_{\text{mix}}^{\text{CP}}(B_d \to \pi^+\pi^-) = \sin(2\beta + 2\gamma) = -\sin 2\alpha,$$

where we have also used the unitarity relation $2\beta + 2\gamma = 2\pi - 2\alpha$. We observe that actually the phases $2\beta = \phi_d$ and $\gamma$ enter directly in the $B_d \to \pi^+\pi^-$ observables, and not $\alpha$. Consequently, since $\phi_d$ can be fixed straightforwardly through $B_d \to J/\psi K_S$, we may use $B_d \to \pi^+\pi^-$ to probe $\gamma$. This is advantageous to deal with penguins and possible new-physics effects, as we will see below.

Measurements of the CP-violating $B_d \to \pi^+\pi^-$ observables are already available:

$$A_{\text{dir}}^{\text{CP}}(B_d \to \pi^+\pi^-) = \begin{cases} -0.02 \pm 0.29 \pm 0.07 & \text{(BaBar [58])} \\ -0.94^{+0.31}_{-0.25} \pm 0.09 & \text{(Belle [59])} \end{cases}$$

$$A_{\text{mix}}^{\text{CP}}(B_d \to \pi^+\pi^-) = \begin{cases} 0.01 \pm 0.37 \pm 0.07 & \text{(BaBar [58])} \\ 1.21^{+0.27+0.13}_{-0.38-0.16} & \text{(Belle [59])} \end{cases}$$

Unfortunately, the BaBar and Belle results are not fully consistent with each other. This discrepancy will hopefully be resolved soon. Forming nevertheless the naive averages of the numbers in (21) and (22) yields

$$A_{\text{dir}}^{\text{CP}}(B_d \to \pi^+\pi^-) = -0.48 \pm 0.21, \quad A_{\text{mix}}^{\text{CP}}(B_d \to \pi^+\pi^-) = 0.61 \pm 0.26.$$

Direct CP violation at this level would require large penguin contributions with large CP-conserving strong phases. A significant impact of penguins on $B_d \to \pi^+\pi^-$ is also indicated by data on $B \to \pi K, \pi\pi$ decays, as well as by theoretical considerations [33, 34, 60]. Consequently, it is already evident that the penguin contributions to $B_d \to \pi^+\pi^-$ cannot be neglected.

4.2 Possible Solutions of the Penguin Problem

There are many approaches to deal with the penguin problem in the extraction of weak phases from CP violation in $B_d \to \pi^+\pi^-$; the best known is an isospin analysis of the $B \to \pi\pi$ system [61], yielding $\alpha$. Unfortunately, this approach is very difficult in practice, as it requires a measurement of the $B_d \to \pi^0\pi^0$ branching ratio. However, useful bounds may already be obtained from experimental constraints on this branching ratio [62, 63].

Alternatively, we may employ the CKM unitarity to express $A_{\text{mix}}^{\text{CP}}(B_d \to \pi^+\pi^-)$ in terms of $\alpha$ and hadronic parameters. Using $A_{\text{dir}}^{\text{CP}}(B_d \to \pi^+\pi^-)$, a strong phase can be
eliminated, allowing us to determine \( \alpha \) as a function of a hadronic parameter \(|p/t|\), which is, however, problematic to be determined reliably \[31, 33, 34, 35, 36, 37\].

A different parametrization of \( B_d \to \pi^+\pi^- \), involving a parameter \( e^{i\theta} \equiv T/P \) and \( \phi_d = 2\beta \), is employed in \[67\], where, moreover, \( \alpha + \beta + \gamma = 180^\circ \) is used to eliminate \( \gamma \), and \( \beta \) is fixed through the Standard-Model solution \( \sim 26^\circ \) implied by \( A^{\text{mix}}_{\text{CP}}(B_d \to J/\psi K_S) \). Provided \( d \) is known, \( \alpha \) can be extracted from the CP-violating \( B_d \to \pi^+\pi^- \) observables. To this end, \( SU(3) \) flavour-symmetry arguments and plausible dynamical assumptions are used to fix \( |P| \) through the CP-averaged \( B^\pm \to \pi^\pm K \) branching ratio. On the other hand, \( |T| \) is estimated with the help of factorization and data on \( B \to \pi\ell\nu \). Refinements of this approach were presented in \[68\].

Another strategy to deal with penguins in \( B_d \to \pi^+\pi^- \) is offered by \( B_s \to K^+K^- \). Using the \( U \)-spin symmetry of strong interactions, \( \phi_d \) and \( \gamma \) can be extracted from the corresponding CP-violating observables \[69\]. In the following discussion, we shall put a particular emphasis on a variant for the \( e^+e^- \) \( B \)-factories, where \( B_s \to K^+K^- \) is replaced by \( B_d \to \pi^+\pi^\pm \) \[70\]. We may then already use the \( B \)-factory data to explore allowed regions in observable space and to extract weak phases and hadronic parameters \[30\].

### 4.3 The \( B_d \to \pi^+\pi^- \), \( B_s \to K^+K^- \) System

As can be seen from the corresponding Feynman diagrams, \( B_s \to K^+K^- \) is related to \( B_d \to \pi^+\pi^- \) through an interchange of all down and strange quarks. The decay amplitudes read as follows \[69\]:

\[
A(B_d^0 \to \pi^+\pi^-) = C \left[ e^{i\gamma} - de^{i\theta} \right] \\
A(B_s^0 \to K^+K^-) = \left( \frac{\lambda}{1 - \lambda^2/2} \right) C' \left[ e^{i\gamma} + \left( 1 - \frac{\lambda^2}{\lambda^2} \right) d' e^{i\theta} \right],
\]

where the CP-conserving strong amplitudes \( de^{i\theta} \) and \( d' e^{i\theta} \) measure, sloppily speaking, ratios of penguin to tree amplitudes in \( B_d^0 \to \pi^+\pi^- \) and \( B_s^0 \to K^+K^- \), respectively. Using these general parametrizations, we obtain expressions for the direct and mixing-induced CP asymmetries of the following kind:

\[
A_{\text{CP}}^{\text{dir}}(B_d \to \pi^+\pi^-) = \text{fct}(d, \theta, \gamma), \quad A_{\text{CP}}^{\text{mix}}(B_d \to \pi^+\pi^-) = \text{fct}(d, \theta, \gamma, \phi_d) \quad (26)
\]

\[
A_{\text{CP}}(B_s \to K^+K^-) = \text{fct}(d', \theta', \gamma), \quad A_{\text{CP}}^{\text{mix}}(B_s \to K^+K^-) = \text{fct}(d', \theta', \gamma, \phi_s \approx 0). \quad (27)
\]

Consequently, we have four observables at our disposal, depending on six “unknowns”. However, since \( B_d \to \pi^+\pi^- \) and \( B_s \to K^+K^- \) are related to each other by interchanging all down and strange quarks, the \( U \)-spin flavour symmetry of strong interactions implies

\[
d' e^{i\theta'} = d e^{i\theta}.
\]

Using this relation, the four observables in \[26\] and \[27\] depend on the four quantities \( d, \theta, \phi_d \) and \( \gamma \), which can hence be determined \[69\]. The theoretical accuracy is only limited by the \( U \)-spin symmetry, as no dynamical assumptions about rescattering processes have
to be made. Theoretical considerations give us confidence into \(28\), as it does not receive \(U\)-spin-breaking corrections within factorization \(69\). Moreover, we may also obtain experimental insights into \(U\)-spin breaking \(69, 71\).

The \(U\)-spin arguments can be minimized, if the \(B_{0d} - \overline{B}_{0d}\) mixing phase \(\phi_d\), which can be fixed through \(B_d \to J/\psi K_S\), is used as an input. The observables \(A_{\text{CP}}^{\text{dir}}(B_d \to \pi^+\pi^-)\) and \(A_{\text{CP}}^{\text{mix}}(B_d \to \pi^+\pi^-)\) allow us then to eliminate the strong phase \(\theta\) and to determine \(d\) as a function of \(\gamma\). Analogously, \(A_{\text{CP}}^{\text{dir}}(B_s \to K^+K^-)\) and \(A_{\text{CP}}^{\text{mix}}(B_s \to K^+K^-)\) allow us to eliminate the strong phase \(\theta'\) and to determine \(d'\) as a function of \(\gamma\). The corresponding contours in the \(\gamma-d\) and \(\gamma-d'\) planes can be fixed in a theoretically clean way. Using now the \(U\)-spin relation \(d' = d\), these contours allow the determination both of the CKM angle \(\gamma\) and of the hadronic quantities \(d, \theta, \theta'\); for a detailed illustration, see \(69\). This approach is very promising for run II of the Tevatron and the experiments of the LHC era, where experimental accuracies for \(\gamma\) of \(O(10^9)\) \(18, 72\) and \(O(1^9)\) \(46\) may be achieved, respectively. It should be emphasized that not only \(\gamma\), but also the hadronic parameters \(d, \theta, \theta'\) are of particular interest, as they can be compared with theoretical predictions, thereby allowing important insights into hadron dynamics. For other recently developed \(U\)-spin strategies, the reader is referred to \(42, 47, 73\).

### 4.4 The \(B_d \to \pi^+\pi^-\), \(B_d \to \pi^\mp K^\pm\) System

Since \(B_s \to K^+K^-\) is not accessible at the \(e^+e^-\) \(B\)-factories operating at \(\Upsilon(4S)\), data are not yet available. However, as can be seen by looking at the corresponding Feynman diagrams, \(B_s \to K^+K^-\) is related to \(B_d \to \pi^\mp K^\pm\) through an interchange of spectator quarks. Consequently, we have

\[
A_{\text{CP}}^{\text{dir}}(B_s \to K^+K^-) \approx A_{\text{CP}}^{\text{dir}}(B_d \to \pi^\mp K^\pm) \quad (29)
\]

\[
\text{BR}(B_s \to K^+K^-) \approx \text{BR}(B_d \to \pi^\mp K^\pm) \frac{\tau_{B_s}}{\tau_{B_d}}. \quad (30)
\]

For the following considerations, the quantity

\[
H \equiv \frac{1}{\epsilon} \left| \frac{C'}{C} \right|^2 \frac{\text{BR}(B_d \to \pi^+\pi^-)}{\text{BR}(B_s \to K^+K^-)} \quad (31)
\]

is particularly useful \(70\), where

\[
\epsilon \equiv \frac{\lambda^2}{1 - \lambda^2}, \quad (32)
\]

and \(C\) and \(C'\) are the normalization factors introduced in \(24\) and \(25\), respectively. Using \(29\) and \(30\), as well as factorization to estimate \(U\)-spin-breaking corrections to

\[
C' = C. \quad (33)
\]

\(H\) can be determined from the \(B\)-factory data as follows:

\[
H \approx \frac{1}{\epsilon} \left( \frac{f_K}{f_\pi} \right)^2 \frac{\text{BR}(B_d \to \pi^+\pi^-)}{\text{BR}(B_d \to \pi^\mp K^\pm)} = \begin{cases} 7.3 \pm 2.9 & \text{(CLEO)} \\ 8.8 \pm 1.5 & \text{(BaBar)} \\ 6.8 \pm 1.7 & \text{(Belle)} \end{cases} \quad (34)
\]
Figure 3: The dependence of $C \equiv \cos \theta \cos \gamma$ on $d$ for values of $H$ consistent with (34). The “circle” and “square” with error bars represent the predictions of the QCD factorization [33] and PQCD [74] approaches, respectively, for the Standard-Model range (10) of $\gamma$.

If we use the $U$-spin relation (28) and the amplitude parametrizations in (24) and (25), we obtain

$$H = 1 - 2d \cos \theta \cos \gamma + d^2.$$

Consequently, $H$ allows us to determine $C \equiv \cos \theta \cos \gamma$ as a function of $d$, as shown in Fig. 3. We observe that the rather restricted range $0.2 \lesssim d \lesssim 1$ is implied by the data, demonstrating the importance of penguins in $B_d \to \pi^+\pi^-$. Moreover, the experimental curves are not in favour of a Standard-Model interpretation of the theoretical predictions for $d e^{i\theta}$ within the QCD factorization [33] and PQCD [74] approaches. Interestingly, agreement could be achieved for $\gamma > 90^\circ$, as the circle and square in Fig. 3, calculated for $\gamma = 60^\circ$, would then move to positive values of $C$ [30, 70].

Let us now come back to the decay $B_d \to \pi^+\pi^-$ and its CP-violating observables, as parametrized in (24). As we have already noted, $\phi_d$ entering $A_{\text{CP}}^{\text{mix}}(B_d \to \pi^+\pi^-)$ can be fixed through $A_{\text{CP}}^{\text{dir}}(B_d \to J/\psi K_S)$, yielding the twofold solution in (18). We may now employ $H$ as an additional observable to deal with the penguins. Applying (28), we obtain $H = \text{fct}(d, \theta, \gamma)$, as given explicitly in (35). We may then eliminate the penguin parameter $d$ in (26) through $H$. If we vary the remaining parameters $\theta$ and $\gamma$ within their physical ranges, i.e. $-180^\circ \leq \theta \leq +180^\circ$ and $0^\circ \leq \gamma \leq 180^\circ$, we obtain an allowed region in the $A^{\text{dir}}_{\text{CP}}(B_d \to \pi^+\pi^-)$–$A^{\text{mix}}_{\text{CP}}(B_d \to \pi^+\pi^-)$ plane. If the measured observables should lie within this region, we may extract $\gamma, \theta$ and $d$ [30, 74]. The value for $\gamma$ may then be compared with results of other strategies or the “Standard-Model” range [14], whereas the hadronic parameters $\theta$ and $d$ allow us to test theoretical predictions.

In Fig. 4, we show the allowed regions in the $A^{\text{dir}}_{\text{CP}}(B_d \to \pi^+\pi^-)$–$A^{\text{mix}}_{\text{CP}}(B_d \to \pi^+\pi^-)$ plane for the two solutions of $\phi_d$ and various values of $H$ [30], as well as the contours arising for fixed values of $\gamma$. We observe that the experimental averages, represented by the crosses, overlap nicely with the SM region for $\phi_d = 51^\circ$, and point towards $\gamma \sim 50^\circ$. In this case, not only $\gamma$ would be in accordance with the results of the CKM fits described in Section 1, but also the $B^0_d$–$\bar{B}^0_d$ mixing phase $\phi_d$. On the other hand, for $\phi_d = 129^\circ$, the experimental values favour $\gamma \sim 130^\circ$, and have essentially no overlap with the SM region. Since a value of $\phi_d = 129^\circ$ would require CP-violating new-physics contributions to $B^0_d$–$\bar{B}^0_d$
Figure 4: Allowed region in the $A_{\text{mix}}^{\text{CP}}(B_d \to \pi^+\pi^-)$–$A_{\text{dir}}^{\text{CP}}(B_d \to \pi^+\pi^-)$ plane for (a) $\phi_d = 51^\circ$ and various values of $H$, and (b) $\phi_d = 129^\circ$ ($H = 7.5$). The SM regions arise if we restrict $\gamma$ to $[\pi/2] (H = 7.5)$. Contours representing fixed values of $\gamma$ are also included.

Figure 5: $|A_{\text{dir}}^{\text{CP}}(B_d \to \pi^+\pi^-)|$ as a function of $\gamma$ for various values of $A_{\text{mix}}^{\text{CP}}(B_d \to \pi^+\pi^-)$ ($H = 7.5$). In (a) and (b), $\phi_d = 51^\circ$ and $\phi_d = 129^\circ$ were chosen, respectively. The shaded region arises from a variation of $A_{\text{mix}}^{\text{CP}}(B_d \to \pi^+\pi^-)$ within $[0, +1]$. The corresponding plots for negative $A_{\text{mix}}^{\text{CP}}(B_d \to \pi^+\pi^-)$ are shown in (c) and (d) for $\phi_d = 51^\circ$ and $\phi_d = 129^\circ$, respectively. The bands arising from the experimental averages in $[23]$ are also included.
mixing, as we have seen in Subsection 3.3, also the \( \gamma \) range in (10) may no longer hold in this case, as it relies on a Standard-Model interpretation of the experimental information on \( B_{d,s}^0 \rightarrow B_{d,s}^0 \) mixing. In particular, also values for \( \gamma \) larger than 90° could then in principle be accommodated.

In order to put these observations on a more quantitative basis, we show in Fig. 5 the dependence of \( |A_{\text{dir}}^{\text{CP}}(B_d \rightarrow \pi^+\pi^-)| \) on \( \gamma \) for given values of \( A_{\text{mix}}^{\text{CP}}(B_d \rightarrow \pi^+\pi^-) \) [30]. An interesting difference arises, if we consider positive and negative values of the mixing-induced CP asymmetry. In the former case, we find that the solution \( \phi_d = 51° \) being in agreement with the CKM fits allows us to accommodate conveniently the Standard-Model range (10), whereas we obtain a gap around \( \gamma \sim 60° \) for \( \phi_d = 129° \). On the other hand, if we consider negative values of \( A_{\text{mix}}^{\text{CP}}(B_d \rightarrow \pi^+\pi^-) \), both solutions for \( \phi_d \) could accommodate (10), and the situation would not be as exciting as for a positive value of \( A_{\text{mix}}^{\text{CP}}(B_d \rightarrow \pi^+\pi^-) \). Interestingly, a positive value is now favoured by the data. Taking into account the experimental averages given in (23), we obtain the bands in Figs. 5 (a) and (b), yielding the following solutions for \( \gamma \):

\[
28° \lesssim \gamma \lesssim 74° \ (\phi_d = 51°), \quad 106° \lesssim \gamma \lesssim 152° \ (\phi_d = 129°).
\]  

(36)

In the future, the experimental uncertainties will be reduced considerably, i.e. the experimental bands will become much narrower, thereby providing significantly more stringent results for \( \gamma \), as well as the hadronic parameters.

Let us now turn to the theoretical uncertainties affecting this approach. In the determination of \( H \) through (34), corrections to the \( U \)-spin relation (33) are taken into account through factorization, leading to the \( f_K / f_\pi \) factor. Moreover, in order to replace \( \text{BR}(B_s \rightarrow K^+K^-) \) through \( \text{BR}(B_d \rightarrow \pi^+K^\pm) \), \( SU(3) \) arguments and plausible dynamical assumptions have to be used. Once measurements of the direct and mixing-induced CP asymmetries of \( B_s \rightarrow K^+K^- \) are available, \( H \) can be circumvented, as we have seen in Subsection 4.3. The second kind of uncertainties enters through (28). As we have already noted, this relation is not affected by \( U \)-spin-breaking corrections within factorization [39], since the relevant decay constants and form factors cancel in the corresponding ratios of decay amplitudes. The impact of non-factorizable effects can be described by

\[
\xi_d \equiv d'/d, \quad \Delta \theta \equiv \theta' - \theta.
\]

(37)

In [30], formulae are given to take into account these parameters in an exact manner, allowing us to explore their impact. The dominant effects are due to \( \xi_d \), whereas \( \Delta \theta \) plays a minor rôle. A detailed analysis of the sensitivity of the plots shown in Figs. 3 and 6 on \( \xi_d \) was performed in [31]. Concerning the ranges for \( \gamma \) given in (36), the impact of a variation of \( \xi_d \) within [0.8, 1.2] is rather moderate, yielding

\[
(28 \pm 2)° \lesssim \gamma \lesssim (74 \pm 6)° \ (\phi_d = 51°), \quad (106 \pm 6)° \lesssim \gamma \lesssim (152 \pm 2)° \ (\phi_d = 129°).
\]

(38)

Let us finally make a few comments on factorization. Here we have \( \theta_{\text{fact}} \sim 180° \), implying \( A_{\text{CP}}^{\text{dir}}(B_d \rightarrow \pi^+\pi^-) \sim 0 \). The Belle data are not in accordance with such a picture, favouring large penguin effects and a large strong phase \( \theta \), where the sign of
Figure 6: Allowed region in the $A_{\text{CP}}^\text{mix}(B_s \to K^+K^-) - A_{\text{CP}}^\text{dir}(B_s \to K^+K^-)$ plane for (a) $\phi_s = 0^\circ$ and various values of $H$, and (b) $\phi_s^{\text{NP}} = 30^\circ$ ($H = 7.5$). The SM regions arise if $\gamma$ is restricted to $(-40, 40)$ ($H = 7.5$).

$A_{\text{CP}}^\text{dir}(B_d \to \pi^+\pi^-)$ implies $0^\circ < \theta < 180^\circ$. On the other hand, the BaBar data are still consistent with factorization. Interestingly, the expressions for $H$ and $A_{\text{CP}}^\text{mix}(B_d \to \pi^+\pi^-)$ depend only on $\cos \theta$. In contrast to $\sin \theta$, $\cos \theta$ is not very sensitive to deviations of $\theta$ from $\theta_{\text{fact}} \sim 180^\circ$. If we introduce $c \equiv -\cos \theta$ and $c' \equiv -\cos \theta'$, $H$ allows us to calculate $d$ and $A_{\text{CP}}^\text{mix}(B_d \to \pi^+\pi^-)$ as functions of $\gamma$. Using then the experimental value of the mixing-induced CP asymmetry, $\gamma$ and $d$ can be extracted. This determination is interestingly very stable with respect to $\theta, \theta' \neq 180^\circ$. In the case of $H = 7.5$ and $A_{\text{CP}}^\text{mix}(B_d \to \pi^+\pi^-) \sim 0$, we obtain the following solutions \cite{30}:

$$\gamma \sim 86^\circ \vee 160^\circ \ (\phi_d = 51^\circ), \quad \gamma \sim 40^\circ \vee 130^\circ \ (\phi_d = 129^\circ), \quad (39)$$

with hadronic parameters

$$d \sim 0.4 \vee 0.2 \ (\phi_d = 51^\circ), \quad d \sim 0.6 \vee 0.3 \ (\phi_d = 129^\circ). \quad (40)$$

Since theoretical estimates prefer $d \sim 0.3$, as we have seen above, the solutions for $\gamma$ larger than $90^\circ$ would be favoured. This is in accordance with the situation in Fig. 3.

### 4.5 Allowed Regions in $B_s \to K^+K^-$ Observable Space

In analogy to the analysis of the $B_d \to \pi^+\pi^-$ mode discussed in Subsection 4.4, we may also use $H$ to eliminate $d'$ in $A_{\text{CP}}^\text{dir}(B_s \to K^+K^-)$ and $A_{\text{CP}}^\text{mix}(B_s \to K^+K^-)$. If we vary then $\theta'$ and $\gamma$ within their physical ranges, i.e. $-180^\circ \leq \theta' \leq +180^\circ$ and $0^\circ \leq \gamma \leq 180^\circ$, we obtain an allowed region in the $A_{\text{CP}}^\text{mix}(B_s \to K^+K^-) - A_{\text{CP}}^\text{dir}(B_s \to K^+K^-)$ plane \cite{30}, as shown in Fig. 3. There also the impact of a non-vanishing value of $\phi_s$, which may be due to new-physics contributions to $B_s^0 - \overline{B_s^0}$ mixing, is illustrated. If we constrain $\gamma$ to $(-40, 40)$, even more restricted regions arise. The allowed regions are remarkably stable with respect to variations of $\xi_d$ and $\Delta \theta$ \cite{30}, and represent a narrow target range for run II of the Tevatron and the experiments of the LHC era, in particular LHCb and BTeV.
5 Conclusions and Outlook

Due to the efforts of the BaBar and Belle collaborations, CP violation could recently be established in the $B$ system through the “gold-plated” mode $B_d \rightarrow J/\psi K_S$, thereby opening a new era in the exploration of CP-violating phenomena. The world average $0.78 \pm 0.08$ of $\sin \phi_d$ determined through $B_d \rightarrow J/\psi K_S$ and similar modes agrees now well with the Standard Model, but leaves a twofold solution for $\phi_d$, given by $\phi_d = (51^{+8}_{-7})^\circ \lor (129^{+7}_{-8})^\circ$. The former solution is in accordance with the picture of the Standard Model, whereas the latter would point towards new-physics contributions to $B^0_d - \overline{B^0_d}$ mixing.

Another key element in the phenomenology of $B$ physics is given by $B \rightarrow \pi K$ decays, complemented by data on $B \rightarrow \pi \pi$. These modes can be described efficiently through allowed ranges in observable space, allowing a straightforward comparison with experiment, and play an important rôle to obtain information on $\gamma$. Interestingly, the data on CP-averaged $B \rightarrow \pi K$ branching ratios seem to favour $\gamma > 90^\circ$, which would be in conflict with the Standard-Model interpretation of lower bounds on $\Delta M_s/\Delta M_d$, pointing towards $\gamma < 90^\circ$. It is still too early to draw definite conclusions, but the picture is expected to improve significantly in the future.

Recent experimental results indicate large penguin effects in the $B_d \rightarrow \pi^+\pi^-$ mode. The question arises now how direct and mixing-induced CP violation in this channel can be translated – in an experimentally feasible way – into results for angles of the unitarity triangle. Here we have discussed one possibility to achieve this goal in more detail, using $B_d \rightarrow \pi^\pm K^\mp$ to control the penguin effects. Following these lines, we obtain $28^\circ \leq \gamma \leq 74^\circ \ (\phi_d \sim 51^\circ) \lor 106^\circ \leq \gamma \leq 152^\circ \ (\phi_d \sim 129^\circ)$, where the former solution would be in agreement with the Standard-Model picture. Unfortunately, the present BaBar and Belle results on CP violation in $B_d \rightarrow \pi^+\pi^-$ are not fully consistent with each other. This issue will hopefully be settled soon. Once measurements of the CP-violating observables of $B_s \rightarrow K^+K^-$ are available, a more elegant determination of $\gamma$ is possible, which is also cleaner from a theoretical point of view. Within the Standard Model, the presently available data on the ratio of the CP-averaged $B_d \rightarrow \pi^\pm K^\mp$ and $B_d \rightarrow \pi^+\pi^-$ branching ratios imply already a very constrained target region in $B_s \rightarrow K^+K^-$ observable space for run II of the Tevatron and the experiments of the LHC era. It will be exciting to see whether these experiments will actually hit the Standard-Model region.

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