A COMPACT STAR ROTATING AT 1122 Hz AND THE \( r \)-MODE INSTABILITY

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ABSTRACT

We show that \( r \)-mode instabilities severely constrain the composition of a compact star rotating with a submillisecond period. In particular, the only viable astrophysical scenario for such an object, present inside the low-mass X-ray binary associated with the X-ray transient XTE J1739–285, is that it has a strangeness content. Since previous analyses indicate that hyperonic stars or stars containing a kaon condensate are not good candidates, the only remaining possibility is that such an object is either a strange quark star or a hybrid quark-hadron star. We also discuss under which conditions submillisecond pulsars are rare.

Subject headings: equation of state — gravitational waves — instabilities — stars: interiors — stars: neutron — stars: rotation

Very recently Kaaret et al. (2007) reported evidence of an X-ray transient with a pulsed component of the emission having a frequency \( f = 1122 \pm 0.3 \) Hz. This signal is interpreted as due to the rotation of the central neutron star. As such this object would be the most rapidly rotating compact star discovered up to now. This single observation clearly needs to be confirmed, maybe by the analysis of future X-ray transients of the same object. The implications of this rapid rotation for the equation of state (EOS) and in particular for the allowed values of the mass and radius have been discussed in Lavagetto et al. (2006) and in Bejger et al. (2007). Here we discuss the problem of stability with respect to the \( r \)-modes of such a rapidly rotating object (for a review of \( r \)-modes see Andersson & Kokkotas 2001).

The possibility has been widely discussed in the literature that \( r \)-mode instabilities can very efficiently drag angular momentum from a rotating compact star if its temporal evolution on the \( \Omega-T \) plane (angular velocity and temperature) enters the \( r \)-mode instability window; see, e.g., Andersson (1998) and Friedman & Morson (1998). Therefore huge regions of the \( \Omega-T \) plane are excluded. Moreover, the size and position of that window are strictly related to the composition of the star, since it is strongly dependent on the value of the bulk and shear viscosity. It is particularly important to recall that for stars containing strangeness, such as hyperon stars (Lindblom & Owen 2002), hybrid stars (Drago et al. 2005), and strange quark stars (Madsen 2000), there is also a contribution to the bulk viscosity associated with the formation of strangeness. Due to this, the instability window splits into two parts: one which starts at temperatures larger than \( (7 \pm 3) \times 10^9 \) K (high-temperature instability window, HTIW) and a lower temperature window at temperatures smaller than \( (5 \pm 4) \times 10^8 \) K (low-temperature instability window, LTIW). Concerning the left border of the LTIW, its position is regulated by the shear viscosity and by the so-called viscous boundary layers located at the interface between the crust and the fluid composing the inner part of the star (Bildsten & Ushomirsky 2000). In particular, in bare quark stars (composed by either normal or superconducting quark matter), due to the absence of a significant crust, the left border of the LTIW extends to much lower temperatures and the LTIW has a minimum corresponding to a significantly lower temperature than in the case of stars containing a crust.

In this Letter we discuss in which region of the \( \Omega-T \) plane the compact star from which XTE J1739–285 originates is most likely located, due to its composition. Let us start by discussing the simplest possibility, i.e., that the object is a neutron star. In principle a neutron star can rapidly rotate in two cases, either if its temperature is very large, above a few MeV, or if it is recycling, spinning up due to mass accretion (see Fig. 1). Concerning the first possibility, a hot neutron star would be a newly born one, since the time needed to cool below 1 MeV is of the order of 1 minute. This is clearly not the case of the stellar object under discussion. Concerning recycling, it should take place on the left side of the instability window, located at lower temperatures (see Fig. 1). The main result of the analysis of Andersson et al. (2000), revisiting previous analyses (Levin 1999; Bildsten & Ushomirsky 2000), is that a neutron star can never spin up to a rotational period shorter than \( \approx 1.5 \) ms. This result is based on the estimates of the temperature and of the mass accretion rates of low-mass X-ray binaries (LMXBs; Brown & Bildsten 1998; Bhattacharyya 2002), indicating that temperatures lower than \( \approx 10^8 \) K cannot be reached. Therefore a submillisecond neutron star cannot be present at the center of a LMXB. It is important to remark that this conclusion is confirmed by more recent analysis, taking into account the composition stratification of the rigid crust (Glampedakis & Andersson 2006) and discussing the nonlinear development of the \( r \)-mode instability (Bondarescu et al. 2007).

Let us now discuss the case in which the compact star contains strangeness. It has been shown, in the case of hyperon stars (Wagoner 2002; Reisenegger & Bonacic 2003) and for hybrid stars (Drago et al. 2006) (strange quark stars also have a similar behavior), that due to the large value of the bulk viscosity associated with the nonleptonic weak decays, there are two windows of instability, the LTIW and the HTIW introduced above. The HTIW does not affect significantly the angular velocity of the star because the cooling of a newborn star is so fast that there is not enough time for the \( r \)-mode instability to drag a significant fraction of the angular momentum. Therefore, the star exits the HTIW with an angular velocity close to the initial one. When the temperature drops down to a few \( \approx 10^8 \) K the star reaches the LTIW and it starts to lose angular momentum due to the \( r \)-mode instability. In Figure 1

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3 Here and in the following we are discussing the temperature of the region of the star where \( r \)-mode instabilities can develop.
we show examples of the instability windows in the cases of a pure quark star and a hybrid star. Note that the position of the LTIW depends rather strongly on the mass of the strange quark, \( m_s \). For large values of \( m_s \), the instability windows shrink considerably. In our analysis we have considered two possibilities concerning the value of \( m_s \): a small value \( \sim 100 \text{ MeV} \), of the order of the strange quark current mass, and a large value \( \sim 300 \text{ MeV} \), similar to what has been obtained in NJL-like models of quark matter for which a density-dependent constituent mass has been computed (Buballa 2005).

Concerning the left side of the LTIW, it is easy to see that it cannot be used to accommodate a submillisecond pulsar. Indeed, in the case of hybrid or hyperonic stars the left side of the LTIW is similar to the one of neutron stars, discussed concerning the minimal temperature of the LTIW is similar to the one of neutron stars, discussed. Indeed, in the case of hybrid or hyperonic stars the left side of the LTIW is similar to the one of neutron stars, discussed concerning the minimal temperature of the LTIW. It is easy to see that it cannot be used to accommodate a submillisecond pulsar. Indeed, in the case of hybrid or hyperonic stars, the left side of the LTIW is similar to the one of neutron stars, discussed above, and we can apply here the same analysis done for a neutron star and concerning the minimal temperature of the compact object at the center of a LMXB. Therefore also hybrid or hyperonic stars cannot recycle to frequencies significantly exceeding 700–800 Hz if they are to the left of the LTIW.

Moreover, in the case of quark stars the absence of viscous boundary layers implies that they can rotate rapidly only on the right side of the LTIW (a very light crust suspended over the surface of a quark star is irrelevant from the viewpoint of the \( r \)-mode instability, although it can be important for the production of X-ray bursts; see Page & Cumming 2005). In conclusion, stars containing strangeness can rotate at submillisecond periods only if they are to the right of the LTIW.

Let us now discuss the results of our analysis also considering the constraints imposed on the EOS by the mass-shedding limit. The main result of Lavagetto et al. (2006) and of Bejger et al. (2007) is that soft EOSs, and in particular the ones based on hyperonic matter or on matter in which kaon condensation takes place, are rather unlikely. In fact, not only the range of allowed masses is rather small, but moreover the only configurations satisfying the stability constraint for mass shedding turn out to be supramassive (Bejger et al. 2007), making it very difficult to use these stellar structures in a mass accretion scenario. This result confirms previous discussions of other astrophysical objects ruling out soft EOSs (Ozel 2006). On the other hand, stars containing deconfined quark matter are not excluded if the quark EOS is stiff enough (Alford et al. 2007). We can therefore conclude that the object under discussion is either a quark or a hybrid star.

We can now study the temporal evolutions of the angular velocity of the star \( \Omega \), of its temperature \( T \), and of the amplitude of the \( r \)-modes \( \alpha \) which are calculated by solving the set of differential equations given in Andersson et al. (2002, eqs. [15], [23], [24]). The only technical difference in our calculation is the inclusion of the reheating associated with the dissipation of \( r \)-modes by bulk viscosity, as discussed in various papers (Wagoner 2002; Reisenegger & Bonacic 2003; Drago et al. 2006). Due to the reheating, the time needed for the star to cool down along the right border of the LTIW is rather long. In the equation regulating the thermal evolution (Andersson et al. 2002) we have added a term,

\[
\dot{E}_{\text{br}} = 2E\tau_{\text{br}},
\]

where \( E \sim 10^{41} \alpha^2 M_{1.4} R_{10}^2 P_{12}^{-2} \) ergs is the energy of the \( r \)-modes (the mass, radius, and rotation period of the star are here expressed in units of 1.4 \( M_{\odot} \), 10 km, and milliseconds, respectively) and \( \tau_{\text{br}} \) is the bulk viscosity. Due to the large value of the bulk viscosity for strange matter at temperatures of a few \( 10^8 \text{ K} \), the related dissipation of \( r \)-modes strongly reheats the star. The trajectory in the \( \Omega-T \) plane describing the time evolution of the star follows essentially the border of the LTIW and the star keeps rotating as a submillisecond pulsar for a very long time, strongly dependent on the value of \( m_s \) and ranging from \( \sim 10^7 \text{ yr} \) when \( m_s = 100 \text{ MeV} \) to \( \sim 10^8 \text{ yr} \) when \( m_s = 300 \text{ MeV} \), as shown in Figure 2. In Figure 3 we show the temporal dependence of the \( r \)-mode amplitude. Note that
in the absence of reheating the time needed to slow down to frequencies below 1 kHz is at maximum of the order of $\sim 10^5$ yr, and can be much shorter than a year if a small value of $m_\ast$ is adopted. In the past this branch of the instability window was only discussed in connection with very young pulsars because reheating due to bulk viscosity was not taken into account.

Before discussing the possible astrophysical scenarios for a submillisecond pulsar inside a LMXB, it is important to remark on two constraints that a realistic model should fulfill. First, a LMXB is an old object, with a typical age of $10^7$–$10^8$ yr, which is the time needed by the companion (having a low mass) to fill its Roche lobe and to start accreting onto the neutron star. Therefore we need to provide a mechanism allowing the central object inside the LMXB to rotate rapidly while being so old. Second, submillisecond pulsars are certainly rather rare, and in particular there is no evidence of a uniform distribution of pulsar rotational frequencies extending from a few hundred Hz up to more than a kHz. Therefore a realistic model should also indicate why most of the compact stars will not be detected rotating at a submillisecond period. As we will show, these two constraints can be satisfied in two different astrophysical scenarios: a first scenario in which an old hybrid or quark star is accelerated up to frequencies exceeding 1 kHz by mass accretion, and a second scenario in which the quark or hybrid star is born with a submillisecond period and it is now spinning down by way of the r-mode instability (see Fig. 4).

The possible realization of these scenarios depends on two main ingredients:

1. the value of $m_\ast$, which, as we have already shown, regulates the magnitude and position of the LTIW;
2. the cooling rate of the central region of the star which determines the inner temperature $T$ of a star which is accreting material from the companion (Miralda-Escudé et al. 1990).

Concerning the cooling rate, if strange quark matter is present in a compact star, direct URCA processes are possible and therefore the cooling is (generally) fast. It turns out from Miralda-Escudé et al. (1990) that in this case the inner temperature is $\sim 5 \times 10^7$ K for a mass accretion rate of $\dot{M} = 10^{-10} M_\odot$ yr$^{-1}$ and that the temperature scales as $T \propto M_\odot^{1/6}$. Another possibility recently proposed is that, due to the formation of diquark condensates, URCA processes are strongly suppressed and the cooling turns out to be slow (Blaschke et al. 2000). At the same time, bulk viscosity can still be large enough to suppress the r-mode instability (Blaschke & Berdermann 2007). In the following we discuss both possibilities, either of a fast or of a slow cooling.

Let us consider first the case in which $m_\ast = 300$ MeV.—The first scenario, i.e., the star spun up by accretion, can indeed be realized if $T \geq 108$ K (see bottom panel of Fig. 1). Such a temperature can be reached via reheating due to mass accretion. In the case of fast cooling a rather large value of the mass accretion rate is needed, $\dot{M} \sim 10^{-8} M_\odot$ yr$^{-1}$, and this stringent request can explain why submillisecond pulsars are rare. A model in which the cooling is slow is excluded, because it would be extremely easy to reaccelerate the star to very large frequencies and therefore submillisecond pulsars would not be rare.

Also the second scenario in which the star is spinning down due to r-mode instabilities is possible. As shown in Figure 2,
the time spent by the star above 1122 Hz is of the order of 108 yr and it is therefore compatible with the typical age of LMXBs. In this case submillisecond pulsars are rare because only a (small) fraction of newborn compact stars can rotate with submillisecond periods (see, e.g., the discussions in Andersson & Kokkotas 2001 and Perna et al. 2008). In this scenario the distribution of millisecond and submillisecond pulsars is determined by the mass of the star at birth: if the mass is large enough, strange quark matter can form, the bulk viscosity of nucleonic matter cannot damp $r$-modes, and the star can be a submillisecond pulsar at birth; if the mass of the star is small, only the first scenario is possible and it requires a slow cooling. Our results are summarized in Table 1.

In conclusion, the two astrophysical scenarios proposed to explain a submillisecond pulsar inside a LMXB can both be realized by a quark or a hybrid star if $m_s \sim 300$ MeV. Instead, if the $m_s$ is small, only the first scenario is possible and it requires a slow cooling. Our results are summarized in Table 1.

The main uncertainties in our analysis are due to the possible existence of other damping mechanisms, taking place on the left side of the LTIW. For instance magnetic fields can be important to suppress $r$-mode instabilities (Rezzolla et al. 2000), but their effect is probably negligible for frequencies exceeding $\sim 0.35 \Omega_\star$, if the internal magnetic field is not larger than $10^{16}$ G. Obviously, even larger uncertainties exist concerning quark matter. As discussed above, the bulk viscosity of quark matter strongly depends on the strange quark mass and on the possible formation of a diquark condensate (Alford & Schmitt 2007). Clearly, our analysis can provide much needed constraints on the EOS of quark matter.

Finally, let us stress that the outcome of our analysis is that a compact star rotating at a submillisecond period inside a LMXB can only be a quark or a hybrid star. Future observations will be important to clarify if the object at the center of XTE J1739–285 constitutes indeed an example.

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