Physics-Regulated Neural Network for High Impedance Faults Detection

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Abstract—High impedance faults (HIFs) in distribution grids may cause wildfires and threaten human lives. Still, more than 10% HIFs fail to be detected by conventional protection relays. Existing methods require sufficient labeled datasets and heavily rely on measurements of relays at substations. Considering the insufficiency of labeled events, we construct a physics regulated convolutional auto-encoder (PRCAE) to detect HIFs without labeled HIFs for training. Our PRCAE introduces a physical regularization, derived from the elliptical trajectory of voltages-current characteristics, to distinguish HIFs from other abnormal events even in highly noisy situations. Also, we formulate a system-wide detection framework of combining multiple nodes’ local detection results and a μPMU placement algorithm for the partially observed system. The proposed approaches are validated in the IEEE 34-node test feeder simulated through PSCAD/EMTDC. Our PRCAE shows superior detection performance than existing works in various scenarios, and is robust to different observability, noise, and low sampling rates.

Index Terms—High impedance faults Detection, Convolutional neural networks, PMU placement

I. INTRODUCTION

High impedance fault (HIF) refers to a fault on a transmission line with a relatively high impedance and correspondingly lower fault current. HIFs are thus harder to detect using conventional over-current protection systems [1][2]. This problem is acerbated in distribution grids as measurements are not ubiquitous, and signatures of HIFs are local and do not propagate much in the grid. In recent years, there has been increased interest in detecting HIFs in distribution grids accurately. They are one of the main causes/initiators of destructive wildfires that have resulted in great economic loss to life and property. In Western USA alone [3], wildfires arising from high-impedance faults have led to losses worth 25 billion dollars in 2019 [4]. Primarily, energized conductors hitting the high impedance ground surfaces, usually accompanied by arc flushing, have led to a majority of HIFs [1]. The process of HIFs with randomness and nonlinearity associated with their occurrences has been well described by a diversity of physical models [5][6]. However, more than 10% detection failures of HIFs have been reported [2] using voltages or currents measured by devices at relays or breakers [7].

The existing data-driven HIF detection methods treat HIF as a pattern recognition problem, and usually involving two-step strategies: (a) features extraction and (b) classification. Based on the kind of features extracted, these methods can be classified into three categories: time-domain, frequency-domain, and time-frequency domain [1][2][8][9][10]. Time-domain features involve transforming voltages or currents waveforms to reveal the randomness or irregularity after HIF occurs [1][8]. However, the robustness of these features in literature in non-linear loads and noise has not been validated. The basis of frequency-domain features is the harmonics caused by the arc during HIFs. Even or odd-order harmonics have been employed to be the indicators of HIF events [9][10]. These models are vulnerable to small portions of harmonics, and the similarity of the harmonic components generated by other types of events, such as capacitor switching. Finally, time-frequency-domain features estimate the energy-variations in the time and frequency framework to capture more abundant information. Unsurprisingly, most techniques of time-frequency transformation require expensive computation [2] that prevents fast detection.

To handle these fundamental challenges, researchers endeavored in various directions to define better representations of HIF events. The authors of [11] selected a diversity of features in time and frequency domains to train a classifier. Their method worked in a semi-supervised manner, though, it required several thousands of labeled datasets to ensure a satisfactory detection performance. The authors of [8] proposed a detection method to fit the voltage-current characteristic profile at the fault point to detect HIFs [8]. Unfortunately, only measurements at the relay location, which may be far from the fault point, are provided, inevitably introducing estimation errors. A crucial issue with existing work is that feature-based detection schemes, aside from the need for significant measurement coverage, also require a sufficient number of labeled datasets of faults that may not be readily available in the industry. To address these issues, we propose a novel and practical HIF neural network-based detector for distribution grids with limited measurement availability that uses only normal data and no labeled faults for training. Our method can overcome these issues by judiciously using constraints related to the physics-informed dynamics in normal operation during the detector training.

Neural networks have achieved great success in computer vision, natural language processing, and health care [12][13]. While applications with labeled data are many, success with partially labeled or even completely unlabeled datasets has been demonstrated with satisfactory accuracy and efficiency [14]. One label-free model is Autoencoder (AE), a neural network architecture consisting of an encoder g and a decoder.
that maps inputs $f$ to the hidden variable $z = g(x)$. Then the decoder learns from the hidden variable $z$ to output the reconstructed input $\hat{x}$. A diversity of derivatives of AE, such as denoising AE [13], variational AE [15], and Convolutional AE (CAE) [16], have been proposed for various applications. However, such pure data-driven applications take the risk of violating the physical rules that govern the cyber-physical systems such as power grids.

Recently, attempts of embedding physical laws in neural networks or statistical machine learning have shown promising performances in power flow calculation, state estimation, topology learning [17]–[20], and power system monitoring [21]–[22]. Outside of power grids, [23]–[24] reveal promising progress in regulating the learned parameters of neural networks with physical laws as priors. These physics-based promotions improve both interpretability as well as the model’s computational efficiency.

**Contribution:** We study the problem of HIF detection in distribution grids in this work, under realistic conditions of reduced measurement availability and unavailability of labeled faults for training. Our detector is termed Physics Regulated Convolutional Auto-Encoder (PRCAE). It is motivated by the observation of the dynamics of the power system during normal operation. Explicitly, the voltage-current characteristic curves can be modeled by elliptical curves on normal conditions. We use a Convolutional Auto-Encoder (CAE) to represent the voltage time-series data during normal operations (no faults), but additionally, constrain its output during training with the physics-regulated (PR) elliptical trajectory of voltages and currents. Furthermore, considering HIF events’ local signatures, we establish a low-communication central scheme that merges local decisions of HIF detection at the observed/metered nodes to increase the system-wide detection reliability and robustness. We also propose a micro-phasor measurement units (µPMUs) placement algorithm to improve the detection performance for partially observed systems.

In summary, our primary contributions are threefold: (1) PRCAE, a neural network structure with physical regularization to detect HIF in distribution grids without labeled data from faults; (2) a robust detection framework combining information from multiple measured nodes; (3) an optimal PMU/µPMU placement algorithm for systems with partial observability. Through realistic fault-data for the IEEE 34 node benchmark system simulated by Power Systems Computer Aided Design (PSCAD) [25], we demonstrate our detector’s performance and highlight the advantages of its physics-informed regularization to distinguish HIFs from others. Further, we show that PRCAE outperforms existing schemes on HIF detection in multiple noisy scenarios and is robust to low observability and sampling rates. The code-base for our detector is in the process of being released publicly.

The remaining part of this paper is organized as follows: Section II introduces the physical rules of HIFs with an illustration example in the IEEE 4-node system; based on these rules, we construct a physically regulated convolutional autoencoder (PRCAE) to detect HIFs in Section III; the detection framework of local and central determination are presented in Section IV together with the proposed µPMU placement algorithm; numerical experiments in Section V show the detection performance of the proposed approaches, in comparison with some existing works in different scenarios. Section VI concludes the main results.

## II. BACKGROUND OF THE PHYSICAL MODEL FOR HIF

HIF is a nonlinear, random event that is often unnoticeable by overcurrent relays or fuses. In the last decades, various arc models have been utilized to describe the stable or dynamic HIF process [1]–[6]. Two parallel diodes and a voltage source model, as shown in Fig. 1(a), accurately represent the dynamic re-striking and quenching process of arcs during HIF at the fault point [1]–[8].

### A. Modeling of HIF Process

![Fig. 1: (a) HIF Modeling (b) Voltage-current Characteristics at the Fault Point](image)

Let $v(t)$ be the single phase voltage at the time $t$ that interacts with the two DC voltage sources $V_p > 0, V_n < 0$, and variable resistances $R_p \neq R_n$ in the down and up lines. Fig. 1(b) shows the voltage currents characteristics curve.

$$v(t) = \begin{cases} 
V_p + i_p(t)R_p & \text{if } v(t) > V_p \\
V_n - i_n(t)R_n & \text{if } v(t) < V_n \\
v(t-1) & \text{else}
\end{cases} \quad (1)$$

When $v(t) > V_p$, the diode $D_p$ is switch on to allow fault current $i_p$ to flow through, and when $v(t) < V_n$, the diode $D_n$ is switch on to let $i_n$ flow in. These structures mimic the restriking process of arcs; otherwise, no currents flow through the HIF circuit and the voltages of the fault point keep the same with the previous phase voltage $v(t-1)$, which represents the quenching of arcs. Note that the restriking and quenching process will cycle and last for seconds or even longer [25]. This process is random and nonlinear since the impedances $R_n, R_p$ are randomly varying.

### B. Physical Laws of HIFs

On normal conditions, it is demonstrated that the trajectories of voltages and currents are rotated ellipses for resistance-inductive or resistance-capacitive linear circuits, and are circles
if resistance is zero \[8\]. Let phase voltages and currents be \(v(t) = V_0 \cos(\omega t), c(t) = C_0 \cos(\omega t - \phi)\) with a phase angle \(\phi\), then we can fit them into the standard parametric format of a rotated ellipse equation as follows:

\[
\left(\frac{v(t)}{\alpha_1} + \frac{c(t)}{\alpha_2}\right)^2 + \left(\frac{v(t)}{\alpha_3} - \frac{c(t)}{\alpha_4}\right)^2 = 1
\]

(2)

where \(\alpha_1 = 2V_0 \cos(\phi/2), \alpha_2 = 2C_0 \cos(\phi/2), \alpha_3 = 2V_0 \sin(\phi/2), \alpha_4 = 2C_0 \sin(\phi/2)\), where \(\alpha_1\) are influenced by line impedance and system power flow.

Once HIF occurs, parameters \(\alpha_1, \ldots, \alpha_4\) are immediately altered, but as the circuit is not open, and the trajectory of voltages and currents deviates to different elliptical trajectories as \(R_n, R_p\) vary. We illustrate this process by an example in the IEEE 4-node test feeder.

Fig. 2: Four Node Feeder Test System with a HIF event in Section I, and near node 1. Red curves are the voltages-current trajectories in normal conditions while the black ones after HIF event

Four-node test feeder example: We simulate the four-node test feeder \([27]\) through PSCAD/EMTDC and model the HIF with the circuit in Fig.1(a). The \(V_p\) is uniformly distributed in 1.0kV±10%, and \(V_n\) in -1.5kV±10%. The variable resistance \(R_p, R_n \in [50, 60]\)Ω vary at 1K Hz after the HIF occurs. Fig.2 shows the trajectories of node 1 and 2 after HIF occurs on the section I (near node 1).

We observe that (1) after HIF occurs, the voltage-current elliptical trajectory deviates to different ellipses, and (2) the degree of the deviations are more significant at the node 1 as the HIF influence on its current is heavier.

Hence, one of HIF’s unique features is the approximate elliptical trajectory of voltages and currents, which may vary from node to node. Taking advantage of this, we present in the next section our detector that regulates the representation of HIF voltages by the elliptical trajectory without relying on sparsely available and expensive labels \[1\].

III. PHYSICALLY REGULATED CONVOLUTIONAL AUTOENCODE (PRCAE)

The configuration of our PRCAE is shown in Fig.3. Given time series of voltages at the \(i\)th node, we formulate matrices \(V_i \in R^{T \times N}, i = 1, \cdots, m\) with a moving window of a length \(T\), where \(m\) is the number of measured nodes, and \(N\) is the number of windows. We input the voltage \(v_i^l\), the \(l\)th column of \(V_i\), to the encoder with convolution layers to generate the hidden variables in a low-dimension space. Then the decoder reconstructs the voltages from the hidden variables with the deconvolution layers. The reconstructed voltages are optimized to be close to the input. One particular structure of this convolutional-autoencoder is the elliptical regularization, which ensures that the reconstructed voltages follow the elliptical trajectory. It is worth mentioning that the elliptical regularization, explained in detail subsequently, is only employed during offline training. The detection during the online testing phase uses only voltage measurements.

A. The Encoder and Decoder of PRCAE

Combining the convolution and deconvolution as the encoder and decoder demonstrates high performance in recovering images and semantic segment datasets \([16,28]\). We follow this principle to construct our PRCAE with \(S\) convolution and deconvolution layers.

The encoder has the “bottle” shape to reduce the dimension of the input. The \(s\)th convolutional layer down-samples the input \(g^s\) with filters \(W^s\) and bias matrices \(B^s\) to reduce the dimensions and then goes through the nonlinear activation function of the Rectified Linear Units (ReLU) to enter into the next layer.

\[
g^{s+1} = \max(0, g^s \odot W^s + B^s), s = 1, \cdots, S
\]

(3)

where \(\odot\) denotes the convolution operation. Here the first input \(g^1\) is the voltages \(v_i^l\) in \(R^T\), the \(s\)th column of \(V_i\). Note that neither Pooling nor dropout\([3]\) are applied here to avoid the loss of information of the inputs. The output \(g^{S+1}\) is the hidden variable and has a much smaller size to capture the most principal and low-dimensional subspace of inputs. It is employed as the input of the decoder to reconstruct the

\[2\text{“bottle” structure denotes the decreasing the size of outputs of the latter layer than that of the previous layer.}\]

\[3\text{Pooling is an operation that reduces the size of input by averaging or taking the maximum; dropout is to discard some links between layers}\]
voltages. The decoder has the symmetric structure with the encoder, which improves the reconstruction accuracy. Here “symmetric” emphasizes the same sizes of the outputs of the deconvolution layer with that of the mirrored convolution layer. The $h$th deconvolution layer up-samples the inputs $f^h$ with the filters $W^h$ and the bias $B^h$ through deconvolutional and ReLU operations.

$$f^{h+1} = \max(0, f^h * \tilde{W}^h + \tilde{B}^h), h = 1, \cdots, S \quad (4)$$

where $*$ denotes the deconvolution operation. The final output $f^{S+1}$ is the reconstructed voltages $\hat{v}_i$.

B. Physical Regularization of PRCAE

The regularization item acts as prior knowledge that direct the trained model to follow the latent physical rules mentioned in Section 4-B to enhance the robustness against noise and other abnormal events. Our regularization encodes the rotated elliptical trajectory of the nodal voltages against currents. Let time series $v_i$ be the voltage of the $i$th node in one window, and $c_j \in R^T$ be the current on line connecting $i$ to a neighboring node $j \in N(i)$. Let $Z_i = [v_i \otimes c_1, v_i \otimes c_2, \cdots, v_i \otimes c_5] \in R^{T \times 5}$, where $\otimes$ denotes the entry-wise product. Assuming normal conditions during the $T$ samples, the entries of voltages and currents measurements $v_i, c_j$ ideally follow an elliptical trajectory with five parameters $\beta = [a, b, c, d, e]^T$, expressed as (30):

$$Z_i \beta + f = 0 \quad (5)$$

where $f, 0 \in R^T$ are an all one and all zero vectors, respectively. The five unknown parameters in $\beta$ can be estimated by the following least square method, given sufficient number of voltages and currents measurements ($T \geq 5$):

$$\beta^* = \arg \min_{\beta} \frac{1}{2} \|Z_i \beta + f\|^2_2 = -(Z_i^T Z_i)^{-1} Z_i^T f \quad (6)$$

**Remark:** If no clean historical data-sets are present to compute $\beta^*$ through (6), we can approximate $\beta$ through power flow analysis. Specifically, as the equations of (2) and (5) are equivalent, $\beta$ in (5) can be estimated by the corresponding $V_0, C_0, \phi$ in (2) [30], obtained by power flow analysis on steady states [31].

**Training:** Given $N$ data samples $v_i^l, c_j^l, l = 1, \cdots, N$ of normal operation, the parameters $\Theta$ of the encoder and decoder for $i$th node’s PRCAE are optimized to minimize the following loss function.

$$L(\Theta) = \frac{1}{N} \sum_{i=1}^{N} \|v_i^l - \hat{v}_i^l(\Theta)\|^2_2 + \lambda_r \|Z_i \beta^* + f\|^2_2 \quad (7)$$

Here the first term $\|v_i^l - \hat{v}_i^l(\Theta)\|^2_2$ denotes the mean square errors between the original and reconstructed voltages $\hat{v}_i^l(\Theta)$ with parameters $\Theta$. The second item is the regularization, which uses the estimated $\beta^*$ to ensure that $\hat{v}_i^l$ follows the elliptical trajectory via $\hat{Z}_i^l = [\hat{v}_i^l \otimes c_1^l, \hat{v}_i^l \otimes c_2^l, \cdots, \hat{v}_i^l \otimes c_5^l]$. In practice, the parameters are updated until reaching the maximum iteration or early stop. The training also produces the average reconstructed error $\epsilon_i = \frac{1}{N} \sum_{l=1}^{N} \|v_i^l - \hat{v}_i^l\|^2_2$ during normal conditions. The training steps are listed in Algorithm 1.

**Algorithm 1 Training of local PRCAE**

1. Input: $N$ training datasets $v_i^l, c_j^l$, maximum iterations $k_{max}$.
2. Compute $\beta^*$ by (6) with $v_i^l, c_j^l$; $k \leftarrow 0$.
3. **while** $k < k_{max}$ and early stop is not reached **do**
   4. Optimize $\Theta$ of PRCAE by minimizing $L(\Theta)$ in (7).
5. **end while**
6. Output: trained PRCAE, $\epsilon_i = \frac{1}{N} \sum_{l=1}^{N} \|v_i^l - \hat{v}_i^l\|^2_2$ on normal conditions.

To reconstruct voltages, and determine the confidence score $\gamma_i = \epsilon_i / \epsilon_i$, the relative error compared to testing, where $\epsilon_i = \|v_i^l - \hat{v}_i^l\|^2_2$ is the mean square error. $\gamma_i$ is small if $v_i^l$ is a normal event since the PRCAE can well represent normal voltages with a small $\epsilon_i$; if $v_i^l$ is a HIF event, $\gamma_i$ becomes larger when the voltage-current trajectory deviates to different ellipses after the HIF; and $\gamma_i$ become significantly large if the abnormal events disobey any elliptical trajectory. Thus we compare $\gamma_i$ with two predefined thresholds $\xi_1, \xi_2$ to distinguish HIFs from other events. The steps are listed in Algorithm 2.

**Algorithm 2 HIF detection through Local PRCAE**

1. Input: Online testing dataset in moving windows $v_i^{l'}, c_j^{l'}$, $l' = 1, \cdots, N'$, averaged reconstruction error $\epsilon_i$ for normal voltages of node $i$, two thresholds $\xi_1, \xi_2$.
2. Input $v_i^{l'}$ into trained PRCAE to reconstruct $\hat{v}_i^{l'}$.
3. $\epsilon_i \leftarrow \|v_i^{l'} - \hat{v}_i^{l'}\|^2_2$. Confidence score $\gamma_i \leftarrow \epsilon_i / \epsilon_i$.
4. **if** $\gamma_i < \xi_1$ **then**
5. **else** if $\gamma_i > \xi_2$ **then**
6. **end if**
7. Output: normal conditions
8. **else**
9. **end if**
10. Output: HIF events are detected
11. **end if**

IV. CENTRALIZED HIF DETECTION FRAMEWORK AND $\mu$PMU PLACEMENT ALGORITHM

![Fig. 4: The configuration of the proposed detection framework. $\gamma_i$ is the confidence score of the $i$th measured node for HIF detection](image)

While Algorithm 2 detects HIFs using local voltage measurements at each PRCAE, we observe that the detection is good at nodes graphically close to the faulted line. However,
the computed $\gamma_i$ at each detector can be communicated to a central detector (Distribution system operator), which decides HIF occurrence using $\max \gamma_i$. Fig. 4 shows the configuration of this simple central scheme. Note that we avoid high communication overhead by not relaying the entire voltage sequence to the central detector. The high $\gamma_i$ scores can also provide auxiliary information about the possible location of the HIF since we observe that the nearby node voltages reveal a relatively high confidence score. We do not study HIF localization in this current work but plan to pursue joint detection and localization in the future work.

In the realistic setting where $\mu$PMUs and the corresponding PRCAEs are sparsely placed in the distribution grid, the centralized HIF detector’s performance depends on the placement of the $\mu$ PMUs. We now discuss a $\mu$PMU placement algorithm to maximize the detection performance using a limited number of $K$ observed nodes.

A. $\mu$PMU Placement Algorithm

The placement of $\mu$PMU is crucial because the signatures of HIFs are local and only revealed by nearby $\mu$PMUs. Conventional PMU or $\mu$PMU placement algorithms determine PMU placement by solving a set cover problem [32, 33], that ensures that each bus is within one-hop of a PMU, or at least one terminal bus of a line has a PMU. In settings where the number of PMUs is too small to ensure complete observability, we present an alternate placement approach that maximizes the recorded PMU data diversity to improve detection.

The intuition comes from the empirical observation that grid segments/edges have distinctive voltages-curves at different parts of the network. For example, the measurements of section III. By collecting measurements from nodes with parts of the network. For example, the measurements of section segments/edges have distinctive voltages-curves at different

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Algorithm 3 $\mu$PMU Placement

1: Input: $K$, $\delta_{i,j}$, $i, j = 1, \ldots, m$
2: $S \leftarrow \emptyset$, $\Delta_i = \sum_{j \in N(i)} \delta_{i,j}$
3: while $|S| < K$ and $\Delta_i$ is not a all-zero vector do
4: $S \leftarrow S \cup \{i^*\}$, $\Delta_j \leftarrow 0$, $\forall j \in N(i^*)$, where $i^* = \arg \max_i \Delta_i$
5: end while
6: Output: $S$

We validate our approaches in the IEEE 34-node with a voltage level of 24.9 kV test feeder [35] modeled by PSCAD/EMTDC [25]. This system contains unbalanced loads, regulator control, and $\mu$PMUs in Fig. 5. HIF is represented by the parallel-diodes and DC voltage sources model shown in Fig. 1 where resistances $R_p, R_n$ and $V_p, V_n$ randomly vary at every 1K Hz. The variation ranges of these parameters are summarized in the Table I refers to [1, 10]. We record waveforms of node voltages and line currents with 512 samples per cycle according to the PQubes in $\mu$PMUs [36], and then convert the time series into vectors with a moving window of $T$, which is the number of samples in a cycle. The interval between any two consecutive windows is around four million-second (ms), or $\tau = 128$ samples.

Training datasets are composed of $N = 325$ windows of node voltages and the corresponding line currents on normal conditions for each measured node at various time instant. Total 286 testing events in various situations include: 100 HIF events occurring on different branches with varying resistance and DC voltages; 42 different loads switching near the node 890 at various time instants, 54 capacitor switching near the node 844 with the reactive powers in the range of 0.5 to 5 MVA; the remaining 90 normal events with varying initial conditions.

Data Augmentation: We apply the range normalization to augment the data-sets [37] by

$$x^k = \frac{x^k - \min(x^k)}{\max(x^k) - \min(x^k)}$$

where $\min(x^k), \max(x^k)$ denotes the minimum and maximum values respectively of the voltages or currents $x^k \in R^T$ in one window.

Configuration of PRCAE: For this system, we design a PRCAE with two convolution and deconvolution layers at each local node, with parameters given in Table I. We decrease the shapes of outputs in those convolutional layers by reducing the number of filters to generate hidden variables in a low-dimension subspace. We train the PRCAE using the Adam optimizer [38] with a learning rate of 0.0001 and batch of

| $R_p(\Omega)$ | $R_n(\Omega)$ | $V_p(kV)$ | $V_n(kV)$ |
|-------------|-------------|-----------|-----------|
| 600 ~ 1400  | 600 ~ 1400  | 5 ~ 6     | 7 ~ 8     |

Fig. 5: 34 node testing feeder [35]
size 12. The maximum iteration $k_{\text{max}} = 1500$ and $\lambda_r = 200$ in (6).

### A. Performance Metrics

We evaluate the detection performance with three criteria: precision, recall, F1 score. To explain the meanings of these criteria, we first give some notations [39]: True Positive (TP) is the number of HIF events that are correctly identified; True Negative (TN) is the number of non-HIF events that are correctly identified; False Negative (FN) is the number of HIF events that are determined as non-HIF events wrongly; False Positive (FP) is the number of non-HIF events that are recognized as HIF events wrongly.

**Precision** $= \frac{TP}{TP + FP}$, representing the ratio of the correctly predicted HIF events to all the events predicted as HIF events. A high precision demonstrates that the detector has a low mistake rate of identifying non-HIF as HIF events.

**Recall** $= \frac{TP}{TP + FN}$, denoting the ratio of the correctly predicted HIF events to all the actual HIF events. A large Recall value means that the detector has a strong capability to recognize HIF events from others.

**F1 score** $= \frac{2(\text{Recall} \times \text{Precision})}{(\text{Recall} + \text{Precision})}$ is a weighted average of precision and recall, and comprehensively evaluates the capability of the detector.

### B. Detection Performance with Partial Measurements

To investigate the influences of the $\mu$PMU placement on the detection performance especially when systems have low observability, we show the detection performance when only 24% to 6% (or 8 to 2) nodes are measured in Table III. “Random” represents the average detection performance for randomly placed $k$ $\mu$PMUs over 100 times of testing, and the “Proposed” denotes the performance using Algorithm 3 for placement. We observe that, with the proposed $\mu$PMU placement algorithm the F1 score of PRCAE has 1%-15% improvement compared with the averaged detection performance using randomly selected observations. In addition, when the system has sufficiently dense observations (e.g., > 25%), the F1 score is larger than 95% with a high probability even based on randomly selected nodes. The top five selected nodes are [832, 860, 818, 862, 806] by the proposed $\mu$PMU placement algorithm. It is not surprising that the node 832 has a high priority since the transformer and regulator cause the voltages of this node to be distinctive from others.

### C. The Effectiveness of the Regularization

Fig. 6 displays the trajectories of voltages and currents in one cycle after capacitor switching and load switching occur respectively. It is evident that these trajectories deviate from the original ellipse dramatically. On the contrary, Fig. 2 indicates that the trajectories for HIFs still follow certain ellipses. As a result, the reconstruction errors of PRCAE for the capacitor switching and load switching are significant compared with that of HIFs. Hence, the reconstruction errors themselves distinguish HIFs from other abnormal events due to the elliptical regularization item.

We discover this separability of PRCAE becomes even more evident in noisy situations. We corrupt the training and testing datasets by Gaussian noise of signal-noise-ratio (SNR)
ranging from 30 dB to 90 dB and train the PRCAEs with \( \lambda_r \neq 0 \) and without \( \lambda_r = 0 \) the regularization item. Fig. 7 statistically depicts the probabilistic distribution of \( \gamma \)'s, which generally reflect the variations of reconstruction errors, of various testing events in noisy situations. The \( \gamma \)'s of the HIFs become separable from those of the other abnormal events when using the PRCAE with the elliptical regularization. On the contrary, the HIFs and non-HIFs are not separable if the PRCAE is trained without the regularization since significant reconstruction errors are got for all events.

D. Comparison with Existing Works

Table IV: Detection F1 score of the PRCAE for node 832 when SNRs are from 30dB to 90dB

| SNR (dB) | PRCAE | AE | PCA | ER |
|---------|-------|----|-----|----|
| 30dB    | 92.9% | 81.5% | 43.2% | 39.5% |
| 50dB    | 97.1% | 81.3% | 72.7% | 62.9% |
| 70dB    | 97.6% | 83.0% | 76.1% | 64.7% |
| 90dB    | 100.0% | 83.3% | 76.6% | 64.7% |

We compare the detection performance of the local PRCAE with three existing methods that are feasible to detect abnormal events without labels: auto-encoder (AE) [29], principle component analysis (PCA) [39], and Ellipse regression (ER). The encoder and decoder of the AE consist of two layers of fully connected neural networks respectively. Being similar with the PRCAE, the outputs of the encoder decrease layer after layer, and the decoder is symmetric with the encoder. We implement the PCA by the truncated singular value decomposition (SVD), and the number of principle components is selected by \( r^* = \text{arg min}_r \sum_{n=1}^{N} \sigma_n^{r} \geq \tau \), where \( \sigma_n \)'s are the decreasing singular values of voltages \( V_i \) and \( \tau = 0.99 \). ER represents the training data using the elliptical equation (5), through a linear regression method [40]. The errors of these three methods for normal and abnormal events are employed in the same way of Algorithm 2 to detect HIFs.

We summarize the F1 score of these four methods when SNR changes from 30 dB to 90 dB in Table IV. PRCAE is more robust to noise than others, achieving up to 17% higher F1 scores above all due to two improvements in the PRCAE. First, the convolutional autoencoder has better representation accuracy for normal events. Second, the physical regularization term enables a more considerable distinction between HIFs and other non-HIFs, as mentioned in Section V-C, even in noisy situations. Note that when the SNR as low as 30 dB, we increase \( \lambda_r = 440 \) to improve the detection performance.

E. Robustness to Low Sampling rates

Table V: Detection Performance of local PRCAE at node 832 for Different Low Sampling Rate

| \( f \) (kHz) | 15.36 | 7.68 | 3.84 | 1.92 | 0.96 |
|------------|-------|-----|-----|-----|-----|
| Sample \( T \) \ | 256 | 128 | 64 | 32 | 16 |
| Precision (%) | 100.0% | 100.0% | 100.0% | 95.2% | 94.3% |
| Recall (%) | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% |
| F1 Score (%) | 100.0% | 100.0% | 100.0% | 97.6% | 97.1% |

We downsample the datasets and demonstrate the robustness of PRCAE to low sampling rates in Table V which is one of the concerns in the industry. When \( T \), the number of samples per cycle, changes from 256 to 16, F1 score of the PRCAE is higher than 90%, indicating the same PRCAE tolerates lower sampling rates without obvious reduction of accuracy. Moreover, the structure of PRCAE adapts to inputs of various sampling rate and does not require redesigning of the filters and bias matrices.

VI. Conclusions

HIFs, potentially causing wildfires in the western U.S., are attracting attention in both academic and industrial domains. This paper presents a physics-regulated convolutional autoencoder (PRCAE) for detecting HIFs. PRCAE uses the physical laws of HIFs in regularizing the CAE model. This regularization helps overcome the absence of large labeled datasets that generally prevent separation between HIFs and other events. Further, a low-communication system-wide detection framework is proposed together with a μPMU placement algorithm to improve detection accuracy significantly, especially when the system has low observability. PRCAE demonstrates superior performances in different noisy scenarios than existing works and keeps a high performance with low sampling rates. An interesting avenue for future work is to unify the location and detection algorithms for use in enabling follow-up control actions.

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