Distributed Ranging and Localization for Wireless Networks via Compressed Sensing

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Abstract—Location-based services in a wireless network require nodes to know their locations accurately. Conventional solutions rely on contention-based medium access, where only one node can successfully transmit at any time in any neighborhood. In this paper, a novel, complete, distributed ranging and localization solution is proposed, which lets all nodes in the network broadcast their location estimates and measure distances to all neighbors simultaneously. An on-off signaling is designed to overcome the physical half-duplex constraint. In each iteration, all nodes transmit simultaneously, each broadcasting codewords describing the current location estimate. From the superposed signals from all neighbors, each node decodes their neighbors’ locations and also estimates their distances using the signal strengths. The node then broadcasts its improved location estimates in the subsequent iteration. Simulations demonstrate accurate localization throughout a large network over a few thousand symbol intervals, suggesting much higher efficiency than conventional schemes based on ALOHA or CSMA.

I. INTRODUCTION

Many location-based wireless services have emerged over the last decade, e.g., emergency services, tracking services, proximity advertising, and smart home monitoring [1]–[3]. Such applications require wireless nodes to obtain their own locations accurately, which is referred to as localization. Localization is also crucial in many mobile ad hoc networks. The Global Positioning System (GPS) provides a straightforward solution for localization. However, GPS may be unavailable, e.g., indoor or in between high rises, too costly or not accurate enough in many situations. In this paper, we study the problem of distributed localization by letting nodes transmit signals to and receive signals from its one-hop neighboring nodes, referred to as its neighbors.

For ease of discussion, the wireless network is assumed to be connected, and has at least three anchors, namely, nodes who know their own exact locations. Nodes who do not know their own locations to begin with are referred to as clients. In the absence of anchors, the techniques proposed in this paper will still apply, whereas the nodes can only be located subject to rigid translation, rotation and reflection. In general, a node determines its own location using location and distance information from its neighbors. The distances can be estimated based on the time of flight, signal strength or other measurements [4]–[6].

Given the neighbors’ location estimates and distances, each node can solve an optimization problem to estimate its own location [7]–[9]. Although the clients and their neighbors (except for the anchors) generally do not know their true locations at the beginning, by iteratively improving their estimates and updating their neighbors with their new location estimates, eventually all connected clients in the network are expected to converge to good estimates of their locations.

To accomplish distributed ranging and localization requires two lower-level functions: i) Each node should be able to communicate to its neighbors about its own location or its location estimate; ii) Each node should be able to measure the (approximate) distances to its neighbors.

Conventional solutions for exchanging message and ranging are based on contention-based random access, such as ALOHA or carrier-sensing multiple access (CSMA). Localization is usually solved in the network layer or above. In particular, each node can only receive useful signals from one neighbor at a time for ranging or exchanging location information. Multiple transmissions in a neighborhood collide and destroy each other. Hence successful localization requires many re-transmissions over many frame intervals.

In this work, we point out that ranging and localization are fundamentally physical-layer issues and propose novel physical-layer techniques for solving them. In particular, each node can exchange messages with multiple neighbors and can measure its distances to multiple neighbors in a single frame all at the same time, as long as transmissions from different nodes are distinguishable. In lieu of colliding with each other, simultaneous transmissions superpose at the receiver. As a toy example, consider three nodes who are neighbors of each other. In a given frame, node 2 and node 3 transmit signals $S_2$ and $S_3$, respectively. The received signal of node 1 during this frame is $a_{12} S_2 + a_{13} S_3$, where $a_{12}$ and $a_{13}$ denote the corresponding received signal amplitudes. If the signals $S_2$ and $S_3$ are sufficiently different in some sense, node 1 can estimate the amplitudes $a_{12}$ and $a_{13}$ at the same time, and can in turn infer about the corresponding distances to nodes 2 and 3. Further, if $S_2$ and $S_3$ are codewords that bear information about the location of node 2 and node 3, respectively, node 1 can also decode their locations at the same time. Zhang and Guo [10] first pointed out that the problem of exchanging messages in a wireless network is...
fundamentally a problem of compressed sensing (or sparse recovery). Compressed sensing [11–13] studies the problem of efficiently recovering a sparse signal based on relatively few measurements made through a linear system. In this work, we carry out location information exchange and ranging jointly using compressed sensing techniques.

One challenge in wireless networks is the half-duplex constraint, where a node cannot transmit and receive useful signals at the same time over the same frequency. To resolve this, we employ the rapid on-off division duplex (RODD) signaling proposed in [10], [14], [15]. The key idea is to let each node randomly divide a frame (typically of a few hundred to a few thousand symbols) into on-slots and off-slots, where the node transmits during the on-slots and listens to the channel during the off-slots. A node receives a superposition of neighbors’ transmissions through its off-slots. The received messages can be decoded as long as the frame is sufficiently long.

The proposed physical-layer technique assumes that node transmission are synchronized. Local-area synchronization can be achieved using a common source of timing, such as a beacon signal, or using a distributed consensus algorithm [16], [17]. To maintain synchronization requires an upfront cost in the operation of a wireless network. The benefit, however, is not limited to the ease of ranging and localization, but includes improved efficiency in many other network functions. More discussions on the synchronization issue are found in [10], [14].

The proposed ranging and localization algorithm is validated through simulations. A network of one hundred Poisson distributed nodes is considered, where transmissions are subject to path loss and additive Gaussian noise. Nodes also interfere with each other. The iterative algorithm is carried out under rather realistic assumptions. Numerical results show that about 10 iterations suffice, where each iteration requires 1200 symbol transmissions. The total number of symbol transmission is about 12,000 symbols. (As a reference, one WiFi frame consists of several thousand symbols.) Thus the proposed scheme is much more efficient than random access schemes, where many more transmissions and retransmissions are needed due to collisions.

The remainder of the paper is organized as follows: Section II describes the channel and network models. The distributed ranging and localization algorithm is described in Section III. Section IV presents numerical results and Section VI concludes the paper.

II. CHANNEL AND NETWORK MODELS

A. Notation

In this paper, upper case letters denote random variables and the corresponding lower case letters denote their realizations or estimates. Vectors and matrices are denoted by bold-face letters. $E \{ \cdot \}$ denotes the expectation over the random variables within the bracket. $\mathbb{R}$ denotes the real number set. $| \cdot |$ denotes the absolute value of a number or the cardinality of a set. $\| \cdot \|$ represents the Euclidean norm of a vector.

B. Linear Channel Model

Let $\Phi = \{ Z_i \}_i$ denote the set of nodes on the two-dimensional plane. We refer to a node by its (random) location $Z_i \in \mathbb{R}^2$. Suppose all transmissions are over the same frequency band. Let time be slotted and all nodes be perfectly synchronized over each frame of $M_s$ slots. For simplicity, each slot consists of one symbol. Suppose node $Z_i$ wishes to broadcast an $l$-bit message $\omega_i$ to its neighbors. Let the signature $S_i(\omega_i)$ denote the $M_s$-symbol codeword transmitted by node $Z_i$ over symbol intervals 1 through $M_s$, whose entries take the values in $\{-1, 0, 1\}$. During slot $m$, the node $Z_i$ listens to the channel and emits no energy if the $m$-th bit of the codeword $S_i^m = 0$; otherwise, the node $Z_i$ transmits a symbol. The design of the on-off signatures will be discussed in Section III.

The physical link between any pair of nodes is modeled as a fading channel. Let the path loss satisfy a power law with exponent $\alpha$. Without loss of generality, we focus on node $Z_0$. The signal received by node $Z_0$, if it could listen over the entire frame, can be expressed as

$$\tilde{Y}_0 = \sum_{Z_i \in \Phi \setminus \{ Z_0 \}} \sqrt{\gamma h_{0i}} R_0^{-\alpha/2} S_i(\omega_i) + \tilde{W}_0,$$

where $\Phi \setminus \{ Z_0 \}$ denotes the set of all nodes except $Z_0$. $\gamma$ denotes the nominal signal-to-noise ratio (SNR), $R_0 = \| Z_0 - Z_i \|$ is the distance between the nodes $Z_0$ and $Z_i$, $h_{0i}$ denotes the small-scale fading coefficient, and $\tilde{W}_0$ is additive white noise vector whose components follow the unit circularly symmetric complex Gaussian distribution $\mathcal{CN}(0, 1)$. For any pair of nodes, e.g., $Z_i$ and $Z_j$, the channel gain between them can be expressed as $| h_{ij} |^2 R_{ij}^{-\alpha}$. Channel reciprocity is a given, i.e., $h_{ij} = h_{ji}$.

Let us denote the set of neighbors of node $Z_j$ as

$$\mathcal{N}(Z_j) = \{ Z_i \in \Phi : | h_{ij} |^2 R_{ij}^{-\alpha} \geq \theta, i \neq j \}$$

where $\theta$ is the threshold. The reasons for not defining the neighborhood purely based on the geometrical closeness as [7], [8] are as follows: i) The channel gain plays a key role in determining the connectivity between a pair of nodes; ii) The attenuation generated by path loss and fading can not be separated within one frame.

Transmissions for non-neighbor are accounted for as part of the additive Gaussian noise. Hence, (1) can be rewritten as

$$\tilde{Y}_0 = \sum_{Z_i \in \mathcal{N}(Z_0)} \sqrt{\gamma} U_{0i} S_i(\omega_i) + \tilde{W}_0,$$

where the channel coefficient

$$U_{0i} = h_{0i} R_{0i}^{-\alpha/2}$$

and the components of $\tilde{W}_0$ follow $\mathcal{CN}(0, \sigma^2)$. The variance $\sigma^2$ results from the additive noise $\tilde{W}_0$ as well as the aggregate interference caused by non-neighbors, which will be given in Section IV-B because it depends on the transmission scheme.
C. Node Distribution and Neighborhoods

Suppose all nodes are distributed across the two-dimensional plane according to a homogeneous Poisson point process with intensity $\lambda$, which is the frequently used to study wireless network (see [18] and references therein). The number of nodes in any region of area $A$ is a Poisson random variable with mean $\lambda A$.

We next derive the relationship between the average number of neighbors a node has and the threshold $\theta$ that defines the neighborhood. These parameters shall be used by the receiver to be described in Section III. Without loss of generality, we drop the indices of the pair of nodes of interest. By definition of a neighbor, the channel gain must satisfy $|h|^2 R^{-\alpha} \geq \theta$, i.e., $R \leq \left(\frac{|h|^2}{\theta}\right)^{1/\alpha}$. Under the assumption that all nodes form a Poisson point process, for given $|h|^2$, this arbitrary neighbor node $Z_i$ is uniformly distributed in a disc centered at node $Z_0$ with radius $\left(\frac{|h|^2}{\theta}\right)^{1/\alpha}$. Hence the conditional distribution of $R$ given $|h|^2$ is given by

$$P(R \leq r| |h|^2) = \min\left\{ 1, r^2 \left(\frac{\theta}{|h|^2}\right)^{\frac{\alpha}{2}} \right\}. \tag{5}$$

Now for every $u \geq \sqrt{\theta}$, using (5), we have

$$P(|h|^2 R^{-\alpha} \geq u^2) = E_{|h|^2} \left\{ P\left( R \leq \left(\frac{|h|^2}{u^2}\right)^{\frac{\alpha}{2}} \right) \right\}
\begin{array}{c}
= E_{|h|^2} \left\{ \left(\frac{|h|^2}{u^2}\right)^{\frac{\alpha}{2}} \left(\frac{\theta}{|h|^2}\right)^{\frac{\alpha}{2}} \right\} \\
= \frac{\theta^{\frac{\alpha}{2}}}{u^{\frac{\alpha}{2}}}, \tag{6}
\end{array}$$

Therefore, for an arbitrary neighbor $Z_i$ of node $Z_0$, the probability density function (pdf) of $|U_{0i}|$, which is the amplitude of the channel coefficient, is

$$p(u) = \begin{cases} 
\frac{4}{\alpha u^{\frac{\alpha}{2}+1}} \frac{\theta^{\frac{\alpha}{2}}}{u^{\frac{\alpha}{2}}} & u \geq \sqrt{\theta}; \\
0 & \text{otherwise}.
\end{cases} \tag{7}$$

In fact, fading coefficient vector $G_i = \{|h_{ij}|^2\}_j$ for all $j \neq i$ can be regarded as an independent mark of node $Z_i$, so that $\Phi = \{(Z_i, G_i)\}_i$ is a marked Poisson point process. Let $\tilde{\Phi} = \{(Z_i, G_i)\}_i \setminus \{(Z_0, G_0)\}$ denote the pair set $\Phi$ excluding the pair $(Z_0, G_0)$. By the Campbell’s theorem [18], $\Phi$ is also marked Poisson point process with intensity $\lambda$, the average number of neighbors of node $Z_0$ can be obtained using (7) as

$$c = E_{\tilde{\Phi}} \left\{ \sum_{(Z_i, G_i) \in \tilde{\Phi}} 1\left( |h_{0i}|^2 R_{0i}^{-\alpha} \geq \theta \right) \right\}
\begin{array}{c}
= 2\pi \lambda \int_0^\infty \int_0^\infty 1 \left( \tilde{h} r^{-\alpha} \geq \theta \right) re^{-\tilde{h}} dr d\tilde{h} \\
= \frac{2}{\alpha} \pi \lambda \theta^{-\frac{\alpha}{2}} \Gamma \left(\frac{2}{\alpha}\right), \tag{8}
\end{array}$$

where $1(\cdot)$ and $\Gamma(\cdot)$ are the indicator function and Gamma function, respectively.

III. THE LOCALIZATION PROBLEM

It is assumed that every node in the network has already discovered its set of neighbors defined by (2), e.g., by using techniques proposed in [10] or [19]. That is to say, the node has acquired each neighbor’s network interface address (NIA). It is fair to assume that the node also knows the codebook used by each neighbor for transmission. This is easily accomplished, e.g., by letting each node generate its codebook using a pseudo random number generator with its NIA as the seed.

The network $\Phi$ consists of a subset $\Phi_0$ of anchors, who know their own locations, and a subset $\Phi_c$ of clients, who wish to estimate their locations. The problem of localization is to let all nodes in the network estimate their respective locations through some transmissions between neighboring nodes. We are only interested in clients who are connected to the dominant component of the network.

IV. THE PROPOSED DISTRIBUTED ALGORITHM

In this section, we describe an iterative, distributed ranging and localization algorithm carried out by all nodes in the network. As discussed in Section II, node transmissions are synchronized in each local area.

The algorithm is roughly describes as follows: Each iteration corresponds to two frame intervals, during which a node may transmit two codewords to describe the two coordinates of its location, respectively. During the same two frame intervals, the node also receives the superposition of its neighbors’ transmissions. The node then decodes the location information from its neighbors as well as infers the distances to the neighbors based on the received signal strengths. Using the locations of and distances to the neighbors, the node can estimate its own location, which it then broadcasts to its neighbors in the subsequent iteration. Note that the anchors always broadcast their actual locations. Although the clients do not know their own locations initially, it is expected that nodes near the anchors will first locate themselves with some accuracy, which, in turn, help their neighbors to locate themselves. After sufficient number of iterations, all clients in the network shall have a good estimate of their locations, subject to uncertainty due to noise, interference, and node connectivity.

In the following, we first discuss each step of the localization algorithm in detail, and then summarize the overall procedure using a flow chart (Fig. 1).

A. Quantization of Location

Suppose the network is confined to the area $[0, A] \times [0, A]$ on the plane. Although the physical location of a node is described by two real numbers on $[0, A]$, the node can only communicate with its neighbors with finite precision due to noise and interference. Let the node quantize its location $z$ to $(\omega, \nu)$, where $\omega$ and $\nu$ are $l$-bit strings. The quantization step size is equal to $\Delta = 2^{-l}A$. It suffices to quantize to precision finer than the expected localization error.
B. Encoding

Let the two quantized coordinates of node $Z_i$'s location, denoted by $(\omega_i, \nu_i)$ be sent over two separate frames. Following [10], we let node $Z_i$ randomly and independently generate a codebook of $2^l$ codewords, each consisting of $M_s$ symbols. Specifically, let the $2^l M_s$ entries of the codebook be independent and identically distributed, where each entry is equal to 0 with probability $1-q$ and is equal to 1 and $-1$ with probability $q/2$ each. To broadcast coordinates $(\omega_i, \nu_i)$ to its neighbors, node $Z_i$ transmits over the first frame interval the $\omega_i$-th codeword from its codebook, denoted by $S_i(\omega_i)$, and transmits $S_i(\nu_i)$ over the second frame interval.

C. The Received Signals

Without loss of generality, we focus on the signals received by node $Z_0$ and assume its neighbors are indexed by $1, \ldots, K$, where $K = |\mathcal{N}(Z_0)|$. The total number of signatures owned by all the neighbors is $N = 2^K$. Node $Z_0$ basically receives a superposition of $K$ codewords, one chosen from each codebook. What complicates the matter somewhat is the fact that $Z_0$ can only listen to the channel through its own off-slots due to the half duplex constraint.

The number of $Z_0$’s off-slots, denoted by $M$, has binomial distribution, whose expected value is $E\{M\} = M_s(1-q)$. Let the matrix $S \in \mathbb{R}^{M \times N}$ contain the columns of the signatures from all neighbors of node $Z_0$, observable during the $M$ off-slots of node $Z_0$. To ensure the expected value of the $l_2$ norm of each column in $S$ to be 1, we normalize the signature matrix $S$ by $\sqrt{M_s(1-q)q}$.

We focus on the first frame received by $Z_0$ as the second frame is of identical form. The received signal of $Z_0$ during its $M$ off-slots based on (3) can be expressed as

$$Y_0 = \sqrt{\gamma_s} S X + W_0,$$

where

$$\gamma_s = \gamma M_s (1-q) / \sigma^2,$$

$W_0$ consists of circularly symmetric complex Gaussian entries with unit variance, and $X$ is a binary $N$-vector for indicating which $K$ signatures are selected to form the sum in (3) as well as the signal strength of each neighbor. Specifically,

$$X_{(i-1)2^l+j} = U_{0i} \mathbf{1}(\omega_i = j)$$

for $i = 1, \ldots, K$ and $j = 1, \ldots, 2^l$. For example, consider $K = 3$ neighbors with $l = 2$ bits of information each, where the messages are $\omega_1 = 1, \omega_2 = 3$ and $\omega_3 = 2$, the vector $X$ can be expressed as

$$X = [U_{01} 0 0 0 0 0 U_{02} 0 0 U_{03} 0 0]^T.$$

The sparsity of the signal is exactly $2^{-l}$.

D. The Effect of Interference

As aforementioned, the impact of interference from non-neighbors is accounted for in the variance ($\sigma^2$) of the additive noise term in channel model (3). The variance $\sigma^2$, or rather the signal-to-interference-plus-noise ratio $\gamma_s$ in model (9), will be used by the decoder. Once the coding and transmission schemes have been described in Section IV-B, $\sigma^2$ can be obtained as follows.

We first derive the aggregate interference caused by neighbors of node $Z_0$ in each time slots as

$$E_{\Phi_q} \left\{ \sum_{(Z_i, U_i) \in \Phi_q} \gamma |h_{0i}|^2 R_{ii}^{-1} \left( |h_{0i}|^2 R_{ij}^{-1} < \theta \right) \right\}$$

$$= 2\pi \lambda q \gamma \int_0^\infty \int_0^\infty h r^{-\alpha} \mathbf{1} \left( h r^{-\alpha} < \theta \right) e^{-h} dhrdh$$

$$= 2\pi \lambda q \gamma \int_0^\infty \int_0^\infty r^{-\alpha+1} \left[ 1 - (\theta r^{-\alpha} + 1) e^{-\theta r^{-\alpha}} \right] dr$$

$$= \frac{4}{\alpha (\alpha - 2)} \pi \lambda q \gamma \theta^{\frac{1}{\alpha}} \Gamma \left( \frac{2}{\alpha} \right)$$

where $\Phi_q$ is an independent thinning of $\Phi$ with retention probability $q_c$ constructed by those transmitting nodes in each time slots, which is still an independent marked Poisson point process, but with intensity $\lambda q_c$. Therefore, the variance is

$$\sigma^2 = \frac{4}{\alpha (\alpha - 2)} \pi \lambda q \gamma \theta^{\frac{1}{\alpha}} \Gamma \left( \frac{2}{\alpha} \right) + 1.$$  

E. Decoding via Sparse Recovery

Algorithm 1: The Message-Passing Decoding Algorithm

1. Input: $S, Y, \gamma_s, q$.
2. Initialization:
3. $\Lambda \leftarrow -\log(2^l - 1)$, $L \leftarrow \sum_k |\partial \mu|$, $L_2 \leftarrow \sum_k |\partial \mu|^2$
4. $R \leftarrow 2^{-l\gamma_s / 2} Y - S \cdot 1$
5. $\tilde{m}_{0k} \leftarrow 0$ for all $k, k$
6. Main iterations:
7. for $t = 1$ to $T$ do
8. for all $k$, $k$ with $s_{0k} \neq 0$ do
9. $m_{0k}^{(t)} \leftarrow \tanh \left( \frac{4}{\alpha} + \sum_{\nu \in \partial_k} \mu \tanh^{-1} \tilde{m}_{0k}^{(t-1)} \right)$
10. end for
11. $Q^t \leftarrow \frac{1}{L_2} \sum_k |\partial \mu| \sum_j \partial_{\partial \mu} \left( m_{0j}^{(t)} \right)^2$
12. $A^t \leftarrow \left[ \frac{4}{\alpha} + \sum_{\nu \in \partial_k} \mu \tanh^{-1} \left( 1 - Q^t \right) \right]$
13. for all $k$, $k$ with $s_{0k} \neq 0$ do
14. $m_{0k}^{(t)} \leftarrow \tanh \left( A^t s_{0k} (r_{0k} - \sum_{\nu \in \partial_k} s_{\nu k} m_{\nu k}^{(t)}) \right)$
15. end for
16. end for
17. $m_k \leftarrow \tanh \left( \frac{4}{\alpha} + \sum_{\nu \in \partial_k} \mu \tanh^{-1} \tilde{m}_{\nu k}^{(T-1)} \right)$ for all $k$
18. Output: for $i = 1, \ldots, K$,
19. $w_i = \arg \max_{j \in \{1, \ldots, 2^l\}} m_{(i-1)2^l+j}$
20. $u_i = \max_{j \in \{1, \ldots, 2^l\}} m_{(i-1)2^l+j}$

In this subsection, we discuss how to process the received signal $Y_0$ described by (9) to recover signal $X$, which
contains location messages from all neighbors as well as the corresponding amplitudes that indicate their distances. The signature matrix $S$, the signal-to-interference-plus-noise ratio $\gamma_s$, as well as the duty cycle of the signatures $q$ are all inputs to the decoder.

To recover the sparse signal $X$ based on observations made through linear model $u$ is fundamentally a problem of compressed sensing. There have been a number of algorithms [20], [21] developed in the literature to solve the problem, the complexity of which are often polynomial in the size of the codebook. In this paper, we augment the iterative algorithm proposed in previous work [10]. The difference is that in [10], the decoder only outputs the support of $X$, whereas in this work, we also estimate the amplitudes of the non-zero elements of $X$. The iterative message-passing algorithm is based on belief propagation. The computational complexity is in the order of $O(MNq)$, the same order as the complexity of other two other popular sparse recovery algorithms [20], [21].

The message-passing algorithm, described as Algorithm 1, solves an inference problem on a Forney-style bipartite factor graph that represents the model $u$. Since $u$ is complex valued, we divide it into the real and imaginary parts, which share the same bipartite graph. Let us focus on one of them:

$$y_\mu = \sqrt{\gamma_s} \sum_{k=1}^{N} s_{\mu k} x_k + u_\mu,$$  \hspace{1cm} (15)

where $\mu = 1, \ldots, M$ and $k = 1, \ldots, N$ index the measurements (the off slots within one frame) and the “input symbols,” respectively. $x_k$ (resp. $y_\mu$) corresponds to the value of the symbol node (resp. measurement node). For every $(\mu, k)$, there is a link between symbol node $k$ and measurement node $\mu$ if $s_{\mu k} \neq 0$. For convenience, let $\partial \mu$ (resp. $\partial k$) denote the subset of symbol nodes (resp. measurement nodes) connected directly to measurements node $\mu$ (resp. symbol node $k$), called its neighborhood. Also, let $\partial \mu \backslash \mu$ denote the neighborhood of measurement node $\mu$ excluding symbol node $k$ and let $\partial k \backslash k$ be similarly defined.

In each iteration of Algorithm 1, every symbol node $k$ first computes messages to pass to the measurement nodes connected to them; then every measurement node $\mu$ computes messages to pass to their corresponding symbol nodes. After $T$ iterations (typically about 10 iterations suffice), Algorithm 1 outputs two sets of estimates. One set consists of the position of the largest element of each of $K$ sub-vectors, corresponding to the location message $\omega_i$ from neighbor $Z_i$; the other set consists of the corresponding amplitude ($|u_i|$) of the largest element of each of $K$ sub-vectors, which shall be used to infer about the distance to neighbor ($Z_i$).

F. Distance Estimation

Each node estimates its distances to neighbors using the signal strength estimates produced by the decoder. Again, we focus on node $Z_0$. For simplicity, we assume that node $Z_0$ knows or can estimate (through training) the fading gain $|h_{0i}|^2$ between itself and each neighbor $Z_i$. Based on the relation between the distance and the received signal amplitude given by (4), the distance between neighboring nodes $Z_0$ and $Z_i$ can be estimated as

$$r_{0i} = \left( \frac{u_{0i}^2}{|h_{0i}|^2} \right)^{-1/\alpha},$$  \hspace{1cm} (16)

where $u_{0i}$ is the estimated channel coefficient ($|u_0|$ the received signal amplitude) of $U_0$, in (4).

G. Location Estimation via Convex Optimization

Using the procedures described in Sections IV-E and IV-F, each node decodes the locations of all its neighbors (or their current estimates), as well as the approximate distances to those neighbors. The node then estimates its own location as the point on the plane that is the most consistent with all information the node has collected.

Let us again focus on node $Z_0$. The node has acquired $(z_i, u_{0i})$ for all neighbors $Z_i \in N(Z_0)$. We let the location estimate of $Z_0$ be $z_0 \in \mathbb{R}^2$ which minimizes the following error:

$$\sum_{Z_i \in N(Z_0)} \|z_0 - z_i\|^2 - r_{0i}^2. \hspace{1cm} (17)$$

If all estimates were perfect, then $r_{0i} = \|z_0 - z_i\|$, so that the error is equal to 0. In general, the estimates are imperfect due to noise and interference, as well as possibly lack of network connectivity.

The preceding minimization problem is non-convex. Following (17), we use relaxation to turn the problem into a second order cone programming (SOCP) problem [22]. Relaxing the equality constraints in to “greater than or equal to” inequality constraints, we obtain the following convex problem in an SOCP form as

$$\text{minimize} \sum_{Z_i \in N(Z_0)} t_{0i} \quad \text{subject to} \quad y_{0i} \geq \|z_0 - z_i\|^2 \quad \text{and} \quad t_{0i} \geq |y_{0i} - r_{0i}^2| \quad \text{for all} \ Z_i \in N(Z_0). \hspace{1cm} (18)$$

This optimization problem can be solved efficiently to yield an estimate $(z_0)$ of the location of node $Z_0$.

H. Summary of the Algorithm

In the flow chart depicted in Fig. 1, we summarize the procedure every node executes in a distributed manner to accomplish network-wide localization. At the beginning, all clients assume they are located at the origin $(0, 0)$. Every node generates a random codebook with $2^l$ codewords, each of $M_s$ symbols. The codewords of node $Z_i$ are $S_i(0), \ldots, S_i(2^l - 1)$.

Localization is carried out iteratively in two stages. In each iteration, all anchors transmit their quantized location coordinates. In the first stage (i.e., in the first $T_1$ iterations), a client transmits its quantized coordinates if it has heard from three or more neighbors in the previous iteration. In the second
Initialization: Generate $2^l$ random on-off codewords $S_0(0), S_0(1), \ldots, S_0(2^l - 1)$. Set location estimate $z_0$ to $(0, 0)$.

Quantize the two coordinates of $z_0$ to $l$-bit strings $(\omega_0, \nu_0)$.

If iteration number less than $T_1$ and the node has heard from fewer than 3 neighbors, do not transmit; otherwise, transmit 2 frames: $S_0(\omega_0)$ followed by $S_0(\nu_0)$.

During on-off transmission, receive the superposed signal $Y_0$ through off-slots; decode the location $z_i = (\omega_i, \nu_i)$ from each neighbor $Z_i \in \mathcal{N}(Z_0)$; estimate the strength $|u_{0i}|$ of each received codeword.

For each neighbor $Z_i \in \mathcal{N}(Z_0)$, infer its distance $r_{0i}$ from $|u_{0i}|$.

Using $z_i$ and $r_{0i}$ for all neighbors as input, solve convex programming \cite{18} for location estimate $z_0$.

Repeat if not enough iterations.

Fig. 1. The flow chart of the algorithm carried out by node $Z_0$.

Without loss of generality, let one unit of distance be 1 meter. Consider a wireless network of 100 nodes. We randomly generate the true locations of client nodes according to a uniform distribution on the square $[0, 50] \times [0, 50]$. The nodes form a Poisson point process in the square conditioned on the node population. Suppose the path-loss exponent $\alpha = 3$. The threshold of channel gain to define neighborhood is set to $\theta = 10^{-3}$. It means that if the transmit power for a node one meter away is 30 dB, then the SNR attenuates to 0 dB at $= 10^{3/\alpha} = 10$ meters in the absence of fading, i.e., the coverage of the neighborhood of a node is typically a circle of radius 10 meters. According to \cite{8}, a node near the center of the square (without boundary effect) has on average $c \approx 11$ neighbors. Here we use the nominal SNR in the following simulations, indeed the SNR will decrease at most 30 dB at the revived node due to the path loss in a neighborhood circle. Each node quantizes its location to 8 bits per coordinate, i.e., about 0.4 meters in precision. The node then encodes each quantized coordinate to an on-off codeword of 600 symbols. Each frame is thus of 600 symbol intervals, and each iteration is of 1,200 symbol intervals.

Reproduced from \cite{8}. Figs. 2 and 3 demonstrate the location results for a network of 100 nodes in two scenarios. The true locations of clients and anchors are depicted using $\circ$ and $\circ$ respectively. The estimated node locations are depicted using $\ast$. Solid lines indicate the
error between the true locations and the estimated locations. In Fig. 2 the network consists of 16 anchors forming a 4 × 4 lattice, where the remaining 84 clients are Poisson distributed. In Fig. 3 the network consists of 25 anchors and 75 clients, all Poisson distributed.

In Fig. 2 with the assistance of 16 anchors forming a lattice, all clients in the convex hull of their neighbors can accurately estimate their own locations. The error is typically a small fraction of a meter. The estimated locations are less accurate for clients near the boundaries because they generally have fewer neighbors. This is thus a boundary effect.

In Fig. 3 with the assistance of 25 randomly placed anchors, the accuracy of the location estimates is about the same as in Fig. 2 for nodes near the center of the network. The boundary effect is slightly more pronounced in Fig. 3 than in Fig. 2 although more anchors are adopted. This is mainly because fewer anchors are found near the lower left and lower right corners.

Let the average localization error of all clients be calculated as

$$\frac{1}{|\Phi|} \sum_{i=1}^{||\Phi||} \|Z_i - z_i\|$$

where $Z_i$ and $z_i$ denote the true location and the estimated location of node $Z_i$, respectively. Figs. 4 and 5 are based on the same network realization. In Fig. 4 the average localization error decreases monotonically with the number of iterations. In the first stage, i.e., iterations 1 through 7, we only let clients who have heard from three or more neighbors transmit. The location error improves quickly in the first stage. Here we determine the number of iterations in the first stage in a global manner (in practice this should be determined a priori): If no new client join the set of nodes who have heard three or more neighbors in an iteration, we shall move to the second stage.

As shown in Fig. 5 the number of clients within 1 meter localization error rises quickly in the first stage. Starting from iteration 8, the number decreases for a few iterations until it rises again. The reason for this is as follows: At the beginning of stage 2, as we let clients who have only heard from two or fewer neighbors in the previous iteration transmit their location estimates, most clients’ estimate improves in the expense of some nodes who already have very accurate estimates. With additional iterations, all clients eventually converge to good estimates of their locations as shown in Fig. 4 and Fig. 5.

Fig. 6 shows the average localization error of the proposed algorithm versus SNR. The average localization error decreases sharply with the SNR in the noise-dominant regime (SNR below 30 dB). As the SNR rises above 30 dB, interference dominates, so that the performance barely improves as the SNR further increases.

VI. CONCLUDING REMARKS

In this paper, we have proposed a complete, distributed, iterative solution for ranging and localization in wireless
networks. Importantly, it is recognized that local-area functions such as ranging and localization are best addressed in the physical layer to exploit the broadcast and multiaccess nature of the wireless medium. The proposed coding and estimation techniques based on compressed sensing have shown to be highly effective through simulations. In particular, each iteration requires 1,200 symbol transmissions and about 10 iterations suffice. Note that a single WiFi frame typically consists of several thousand symbols. Thus the proposed scheme is considerably more efficient than conventional schemes based on random access, where many more transmissions and retransmissions are needed due to collisions. It would of course be interesting to evaluate the algorithm under more realistic scenarios, e.g., using a software radio implementation. This is left to future work.

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Fig. 6. Average localization error vs SNR, 100 nodes, including 16 anchors forming a 4×4 lattice.