DIS in AdS'

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Abstract. We calculate the total cross section for the scattering of a quark–anti-quark dipole on a large nucleus at high energy for a strongly coupled $\mathcal{N} = 4$ super Yang-Mills theory using AdS/CFT correspondence. We model the nucleus by a metric of a shock wave in AdS$_5$. We then calculate the expectation value of the Wilson loop (the dipole) by finding the extrema of the Nambu-Goto action for an open string attached to the quark and antiquark lines of the loop in the background of an AdS$_5$ shock wave. We find two physically meaningful extremal string configurations. For both solutions we obtain the forward scattering amplitude $N$ for the quark dipole–nucleus scattering. We study the onset of unitarity with increasing center-of-mass energy and transverse size of the dipole: we observe that for both solutions the saturation scale $Q_s$ is independent of energy/Bjorken-$x$ and depends on the atomic number of the nucleus as $Q_s \sim A^{1/3}$. Finally we observe that while one of the solutions we found corresponds to the pomeron intercept of $\alpha_p = 2$ found earlier in the literature, when extended to higher energy or larger dipole sizes it violates the black disk limit. The other solution we found respects the black disk limit and yields the pomeron intercept of $\alpha_p = 1.5$. We thus conjecture that the right pomeron intercept in gauge theories at strong coupling may be $\alpha_p = 1.5$.

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SETTING UP THE PROBLEM

It is well-known that using the light-cone perturbation theory one can decompose the total scattering cross section of deep inelastic scattering (DIS) into a convolution of a light-cone wave function squared ($\Phi$) for a virtual photon to decay into a $q\bar{q}$ pair and the imaginary part of the forward scattering amplitude for the $q\bar{q}$ pair–target interaction ($N$):

$$\sigma_{tot}^{\gamma\nu A}(Q^2, x_{Bj}) = \int \frac{d^2r d\alpha}{2\pi} \Phi(r, \alpha, Q^2) d^2b \ N(r, b, Y).$$

(1)

While $\Phi$ is well known and contains only QED interactions, the QCD part is contained in $N(r, b, Y)$, which is the imaginary part of the forward scattering amplitude for the scattering of a quark-antiquark dipole of transverse size $r$ at center-of-mass impact parameter $b$ on a target, where the total rapidity of the scattering process is $Y = \ln 1/x_{Bj}$ with $x_{Bj}$ the Bjorken $x$ variable. One can therefore relate $N(r, b, Y)$ to the expectation value of a fundamental Wilson loop by writing

$$N(r, b, Y) = 1 - S(r, b, Y)$$

(2)

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1 Talk given by Yu.K. at the International Workshop on Diffraction in High-Energy Physics (Diffraction 2008) in La Londe-les-Maures, France, based on [1].
FIGURE 1. Dipole-nucleus scattering in the dipole’s rest frame. While quark and anti-quark have the same $x^3$-coordinate, we show them apart from each other for illustration purposes.

with the (real part of the) $S$-matrix

$$S(r, b, Y) = \frac{1}{N_c} \text{Re} \left\langle W \left( b + \frac{1}{2} r, b - \frac{1}{2} r, Y \right) \right\rangle. \quad (3)$$

Here $W \left( b + \frac{1}{2} r, b - \frac{1}{2} r, Y \right)$ denotes a Wilson loop formed out of a quark line at $b + \frac{1}{2} r$ and an antiquark line at $b - \frac{1}{2} r$ with the links connecting the two lines at plus and minus temporal infinities, as shown in Fig. 1. The averaging in Eq. (3), denoted by $\langle \ldots \rangle$, is performed over all possible wave functions of the target.

Our goal here is to find the expectation value of the Wilson loop $\langle W \rangle$ in Fig. 1 using the AdS/CFT correspondence. According to AdS/CFT prescription [2] the expectation value of the Wilson loop is given by the classical Nambu-Goto action of the open string in AdS$_5$ space $\langle W \rangle \sim e^{iS_{NG}}$ with the worldsheet of the string connecting to the Wilson loop at the boundary of AdS$_5$. We will model the nucleus by a smeared shock wave in AdS$_5$ with the following metric [3] (note that in this approximation there is no $b$-dependence, which we will henceforth suppress in the arguments of $S$ and $N$)

$$ds^2 = \frac{L^2}{z^2} \left[-2 dx^+ dx^- + \frac{\mu}{a} \theta(x^-) \theta(a - x^-) z^4 dx^- z dx_\perp^2 + dx_\perp^2 + dz^2 \right]. \quad (4)$$

Here $\mu = p^+ \Lambda^2 A^{1/3}$ and $a \approx 2 R \frac{A}{p^+} \sim A^{1/3}$, where the nucleus of radius $R$ has $A$ nucleons in it with $N_c^2$ valence gluons each. $p^+$ is the light cone momentum of each nucleon and $\Lambda$ is the typical transverse momentum scale. We thus need to extremize the string worldsheet in the background of Eq. (4).

**STATIC SOLUTION**

One can argue that for a large enough nucleus the string configuration should be static [1]. Parameterizing the static string as $X^\mu(t, x) = (t, x, 0, z(x))$ with $x \in \left[ -\frac{r}{2}, \frac{r}{2} \right]$ and
\( z(x = \pm r/2) = 0 \) we write the Nambu-Goto action as (\( \lambda \) is \( 't \) Hooft coupling)

\[
S_{NG}(\mu) = -\frac{\sqrt{A}}{2\pi} \int_{0}^{\pi} dt \int_{-r/2}^{r/2} dx \frac{1}{z^2} \sqrt{(1+z'^2) \left( 1 - \frac{\mu}{2a} z^4 \right)}. \tag{5}
\]

The classical string configuration extremizing the action (5) was found in [1] having the shape of a hanging string. In fact one finds six different extremal configurations characterized by different results for the maximum of the 5th string coordinate \( z_{\text{max}} \):

\[
z_{\text{max}} = c_0 r \sqrt{\frac{1}{3m\Delta}} + \Delta, \tag{6}
\]

with

\[
\Delta = \left[-\frac{1}{2m} + \sqrt{\frac{1}{4m^2} - \frac{1}{27m^3}} \right]^{1/3} \exp \left[\frac{2\pi n}{3}\right]. \tag{7}
\]

We defined \( \frac{\mu^2}{a} = s^2 = p^+ \Lambda^2/2 \) and \( m = c_0^4 r^4 s^2 \). The index \( n \) in Eq. (7) takes on values \( n = 0, 1, 2 \), and taking principal and secondary square roots in Eq. (6) makes the total number of solutions for \( z_{\text{max}} \) equal to six. We pick the right solution by imposing the physical requirement that \( N(r = 0, Y) = 0 \) (color transparency) and \( N(r \to \infty, Y) \to 1 \) (black disk limit).

**RESULTS AND CONCLUSIONS**

The solution which naturally satisfies the above requirements is given by Eqs. (6) and (7) with \( n = 1 \) and taking the secondary square root in Eq. (6). Writing the result as \( z_{\text{max}} = -i\rho_{\text{max}}(r, s) \), using the obtained solution in

\[
S(r, Y) = \text{Re} \left[ \frac{\langle W \rangle_{\mu}}{\langle W \rangle_{\mu \to 0}} \right] = \text{Re} \left[ e^{i[S_{NG}(\mu) - S_{NG}(\mu \to 0)]} \right] \tag{8}
\]

to find the S-matrix [2, 4], we obtain the dipole-nucleus scattering amplitude [1] \((s \sim e^Y)\)

\[
N(r, s) = 1 - \exp \left\{-\frac{\sqrt{\lambda} a}{\pi c_0 \sqrt{2}} \left[ \frac{c_0^2 r^2}{\rho_{\text{max}}(r, s)} + \frac{2}{\rho_{\text{max}}(r, s)} - 2\sqrt{s} \right] \right\}. \tag{9}
\]

Here \( c_0 \equiv \Gamma^2 \left( \frac{1}{4} \right) / (2\pi)^{3/2} \). The amplitude in Eq. (9) is plotted in Fig. 2 as a function of dipole size \( r \) for a range of different energies \( s \).

Using Eq. (9) one can easily show that at lower energies and/or small dipole sizes it reduces to

\[
N(r, s) \bigg|_{r^2 s \ll 1} = 1 - \exp \left\{-\frac{\sqrt{\lambda} c_0}{2\pi \sqrt{2}} r^2 s^{1/2} \Lambda \Lambda A^{1/3} \right\}. \tag{10}
\]

Hence for very small dipoles \( N \sim s^{1/2} \). Identifying this behavior with a single graviton exchange in the bulk, and hence with the single pomeron exchange in the gauge theory, we obtain the pomeron intercept of \( \alpha_p - 1 = 1/2 \) or, equivalently,

\[
\alpha_p = 1.5. \tag{11}
\]
At high energies Eq. (9) gives
\[ N(r,s) \bigg|_{r^2 s \gg 1} = 1 - \exp \left\{ - \frac{\sqrt{\lambda}}{\pi \sqrt{2}} r^{1/3} \Lambda A^{1/3} \right\}. \] (12)

One can see that the amplitude \( N(r,s) \) becomes energy-independent! Defining the saturation scale \( Q_s \) by requiring that \( N(r = 1/Q_s, s) = o(1) \) we get from Eq. (12) \( Q_s \sim \sqrt{\lambda} \Lambda A^{1/3} \). Thus the saturation scale is independent of energy! This conclusion appears to agree with the results of [5].

The only problem with the \( n = 1 \) solution for \( z_{\text{max}} \) is that it does not map onto Maldacena’s vacuum string configuration from [2] in the \( \mu \to 0 \) limit. While it is not clear whether such mapping is a necessary requirement, we point out that the \( n = 2 \) solution for \( z_{\text{max}} \) does map onto the vacuum solution from [2] in the \( \mu \to 0 \) limit. The dipole amplitude given by the string configuration given by \( z_{\text{max}}^{n=2} \) is plotted in Fig. 3.

One can show that at lower energies and/or small dipole sizes the dipole amplitude given by \( z_{\text{max}}^{n=2} \) is (note the cosine!)
\[ N(r,s) \bigg|_{r^2 s \ll 1} = 1 - \cos \left\{ - \frac{\sqrt{\lambda} c_0^2}{2 \pi} r^{3/2} s^{1/3} \Lambda A^{1/3} \right\}. \] (13)

At very small-\( r \) this gives \( N(r,s) \sim s^2 \). Identifying this with a double-pomeron (graviton) exchange as the small-\( r \) expansion starts from a quadratic term in the Nambu-Goto
action (diffractive dominance) we see that in this case the pomeron intercept is given by $\alpha_P = 2$ in agreement with the results of [6]. The problem with this solution is the oscillations shown in Fig. 3 which can also be seen from Eq. (13). Strictly speaking there is nothing wrong with $N$ going above one (i.e. above the black disk limit) as the real constraint is $0 \leq \sigma_{\text{inel}} \propto 2N - N^2$ leading to $N \leq 2$. We note that $N > 1$ in the regions where the elastic cross section contribution dominates in the total cross section, in agreement with [6]. However the oscillations in Fig. 3 appear to have no clear physical origin and have no analogue in perturbative calculations: this gives us a reason to doubt the validity of this solution. In [1] it is shown how a modification of the prescription in Eq. (8) leads to a more physical $N(r,s)$ for the solution given by $z_{\text{max}}^{n=2}$ (and which is qualitatively similar to the $n = 1$ solution): however the resulting $N$ is identically 0 for a range of small $r$, making the determination of the pomeron intercept impossible. To uniquely determine which one of the $n = 1$ and $n = 2$ solutions is the true prediction of string theory one probably has to calculate quantum corrections to the above results.

We conclude by displaying the saturation line found in this work in Fig. 4 along with a possible matching of our results on the well-known perturbative behavior.

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