Towards nonlinear tuning system of stiffness compensator

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Abstract. In this paper, one of the tuning systems for the stiffness compensator based on permanent magnets with changing of the axial distance between magnets has been described. The dependency of the traction force of the anchor rotation angle relatively the basis in the stiffness compensator has been described. Research & Development was performed under the scholarship of the President of the Russian Federation, order № 231 as of April 3, 2018.

1. Introduction

Application of the power installations not only in industry, but in every type of the vehicles, inevitably leads to vibration. The vibration negatively influences reliability, lifetime of the machines, buildings, apparatuses where they are installed, and also the systems of the automatic control systems [1, 4, 14, 16]. Frequently, the vibration is the reason of the accidents. The most acute problem is vibration protection in the automotive, shipbuilding, where the internal combustion engines (ICE) are used as power installation.

The low-frequency oscillations are produced by the ICE that are mostly harmful for human and lead to different diseases. Nowadays the fight with mechanical oscillations (vibration) is one of the important issues.

There are many ways to decrease the vibrations: the dynamic balancing of the engines, the dynamic suppressors of the oscillations, the active vibroprotection systems with additional vibration source. The most common systems are performed in the form of rubber-metal shock absorbers. However, these simple and reliable vibroisolator are ineffective, because the decreasing of their rigidity factor in order to reduce transferred dynamic forces increases the relative movements of the ICE and the equipment is coupled to the engine. This is lack of vibroisolators with floating segment of the zero stiffness.

The application of the vibroisolation devices with the floating segment of the zero stiffness [1, 2, 5, 9, 15] is the most perspective method of the suppression of the vibration level. In this type of the vibroisolators with a rigid element, the tunable stiffness compensators with a falling power characteristic (the negative stiffness factor) are connected in parallel. Nowadays many types of the compensators have been developed, however their designs do not correspond to modern requirements for vibroisolation. Among the known compensators, the most effective compensator is the electromagnetic stiffness compensator (ESC) [2, 3, 11, 12]; it corresponds most fully to the requirements for the ideal vibration isolation both for constant in magnitude and for arbitrarily changing loads.
2. Calculation and simulations

The design of the ESC is the two counter connected electromagnets. This design provides the falling power characteristic, which allows correcting the stiffness of the vibration isolator as a whole.

For well operation of the vibroisolation suspend with correction of the stiffness under the changing loads, the compensator is equipped with the special tuning system. To provide the floating of the zero stiffness segment during the changing of the forces, the tuning system redistributes the electromagnet coils voltage. In [6] it was established that a law of the change of the voltage on the electromagnets during the movement $\Delta U(\Delta x)$ can be: linear or nonlinear as shown in Figures 1 and 2. In [6, 7] it is shown that the nonlinear law of the changing electromagnets voltage can be approximated by quadratic dependency:

$$\Delta U(\Delta x) = a_2 \cdot (\Delta x)^2,$$

where $a_2$ - the constant factor.

Let's change of the variables in (1) to variables $\Delta U \rightarrow f$, $\Delta x \rightarrow x$:

$$f(x) = a_2 \cdot (x)^2.$$

where $f(x)$ – the initial continuous function;

$x$ – the independent variable.

The investigation of the vibroisolation system in both cases of the reconstruction system showed that the use of the linear adjustment system is preferable. In addition, it is much easier to implement the linear version of the regulator. Thus we can proceed from the quadratic approximated nonlinear law of the voltage dependency from the relative displacement of the vibrating and protected objects to the linear law. The vibration system is working under small enough relative displacement of the vibrating and protected object. So let's consider the non linear law, which is shown in Figure 2 in the segment $[a, b]$ close to the origin point. After performing some calculations of the nonlinear law of the voltage changing the displacement in given segment, it is possible to note that the law is appropriate for dependency, which is described by eq. 2, but with $a_2 \approx 1$. So the nonlinear law in segment $[a, b]$ is described by:

$$f(x) = x^2.$$

Now we can perform approximation to obtain the linear dependence by using the Weierstrass theorem [10]: if the function $f(x)$ is continuous on the interval $[a, b]$, then, as little as a positive number $\epsilon$, there is a polynomial of sufficiently high degree $m$, whose deviation from the given function $f(x)$ on the segment $[a, b]$ is less than $\epsilon$, that is, for all points $x \in [a, b]$, the inequality is:

$$\left|f(x) - Q_m(x)\right| < \epsilon$$

Figure 1. Linear law of the voltage changing from displacement changing.

Figure 2. Nonlinear law of the voltage changing from displacement changing.
where \( Q_m(x) \) - the continuous approximating generalized polynomial;
\( \varepsilon \) - the quadratic deviation (deviation from zero).

For the function (3), the polynomial of best approximation is:
\[ Q_1(x) = a_1 \cdot x + a_0 \]  
(5)
where \( a_0, a_1 \) - constant coefficients.

The first order part in segment \([a, b]\) is:
\[ Q_1(x) = x^2 - T_2(x), \]  
(6)
where \( T_2(x) \) - the Chebyshev polynomial.

Indeed, according to the meaning of the problem, the difference is:
\[ f(x) - Q_1(x) = x^2 - a_1 \cdot x - a_0. \]  
(7)
This polynomial is polynomial of the best approximation, which has minimal deviation from zero in a given segment, i.e. the Chebyshev polynomial.

Consequently, the polynomial:
\[ Q_2(x) = x^2 - a \cdot x - a_0 \]  
(8)
has deviation from zero in the segment \([a, b]\).

The required coefficients in the equation of the polynomial (8) can be calculated from the following expression:
\[ Q_2(x) = T_2(x) = \left( \frac{b-a}{2} \right)^2 \cdot T_2 \left( \frac{x - \frac{b+a}{2}}{\frac{b-a}{2}} \right). \]  
(9)

After substitution of the equation (8) with the resulting coefficients into expression (6), we obtain:
\[ Q_1(x) = x - a_0. \]  
(10)
We can use the variables acting in the compensator tuning system are the voltage increment \( \Delta U \) and the increment \( \Delta x \):
\[ \Delta U(\Delta x) = \Delta x - a_0. \]  
(11)
The characteristic, which approximately describes the dependency of the voltage changing from the displacement changing, is shown in figure 3 (plot 2). From figure 3 we can see that inside of the segment \([a, b]\), the characteristic can be describe by equation 11, i.e. the polynomial of the first order with less deviations from the given curve (plot 1) has been obtained in [6, 8, 13]. Consequently, the dependency of the voltage changing of the one electromagnet from displacement changing of the segment \([a, b]\) can be considered as linear. And the deviation from zero can be calculated by expression:
\[ \varepsilon = \left( \frac{b-a}{2} \right)^2 \cdot \frac{1}{2} = \frac{1}{8} \cdot (b-a)^2. \]  
(12)
It is possible to note that the deviation is realized with three points:
\[ Q_1(a) = Q_1\left( \frac{a+b}{2} \right) = Q_1(b). \]  
(13)
From figure 3, we can see that the plot of the function (11) geometrically represents the middle parallel between the secant passing through the extreme points and the tangent parallel to this secant.
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