An empirical test of the theoretical population corrections to the Red Clump absolute magnitude

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ABSTRACT

The mean absolute magnitude of the local red clump (RC), $M_{\lambda}^{RC}$, is a very well determined quantity due to the availability of accurate Hipparcos parallaxes for several hundred RC stars, potentially allowing it to be used as an accurate extra-galactic distance indicator. Theoretical models predict that the RC mean magnitude is dependent on both age and metallicity and, furthermore, that these dependencies are non-linear. This suggests that a population correction, $\Delta M_{\lambda}^{RC}$, based on the star formation rate (SFR) and age-metallicity relation (AMR) of the system in question, should be applied to the local RC magnitude before it can be compared to the RC in any other system in order to make a meaningful distance determination.

Using a sample of 8 Galactic open clusters and the Galactic Globular Cluster 47 Tuc, we determine the cluster distances, and hence the RC absolute magnitude in $V$, $I$ and $K$, by applying our empirical main sequence fitting method, which utilizes a large sample of local field dwarfs with accurate Hipparcos parallaxes. The 9 clusters have metallicities in the range $-0.7 \leq [\text{Fe/H}] \leq +0.02$ and ages from 1 to 11 Gyr, enabling us to make a quantitative assessment of the age and metallicity dependences of $\Delta M_{\lambda}^{RC}$ predicted by the theoretical models of Girardi & Salaris (2001) and Salaris & Girardi (2002). We find excellent agreement between the empirical data and the models in all 3 pass-bands, with no statistically significant trends or offsets, thus fully confirming the applicability of the models to single-age, single-metallicity stellar populations.

Since, from the models, $\Delta M_{\lambda}^{RC}$ is a complicated function of both metallicity and age, if this method is used to derive distances to composite populations, it is essential to have an accurate assessment of the SFR and AMR of the system in question, if errors of several tenths of a magnitude are to be avoided. Using recent determinations of the SFR and AMR for 4 systems – the LMC, SMC, Carina and the solar neighbourhood – we examine the quantity $I_{\text{obs}}^{RC} - K_{\text{obs}}^{RC}$, which is the difference between the mean magnitude of the RC in the $I$-band and the $K$-band. Comparing the theoretical predictions with the most recent observational data, we find complete agreement between the observations and the models, thus confirming even further the applicability of the population corrections predicted from theory.

Key words: stars: distances – stars: horizontal branch – open clusters and associations: general – Magellanic Clouds

1 INTRODUCTION

In recent years much work has been devoted to studying the suitability of Red Clump (RC) stars as a distance indicator. These core helium-burning stars are essentially in an identical evolutionary phase to those which make up the horizontal branch in globular clusters. However, in intermediate and higher metallicity systems only the red end of the distribution is seen, forming a clump of stars in the colour-magnitude diagram - hence the name. In an infrared study of metal-rich globular clusters, Kuchinski et al. (1995) realised that the absolute $K$-band magnitude of the horizontal branch (effectively, the RC) displayed very little cluster-to-cluster variation, and therefore suggested it could potentially be used as a standard candle for distance determinations. However, at that time, no absolute calibration of the RC magnitude was available and so distances could only be measured differentially, with respect to a specific cluster. Interest
in the RC as a potential extra-galactic distance indicator was rekindled a few years later due to the availability of accurate \textit{Hipparcos} (ESA 1997) parallaxes (and hence absolute magnitudes) for several hundred local RC stars, enabling a very precise calibration of their mean absolute magnitude. In fact, RC stars are currently the only extra-galactic distance indicator which can be tied precisely to the \textit{Hipparcos} distance scale, due to the lack of a significant number of RR Lyraes with accurate parallax determinations (see e.g. Fernley et al. 1998) and the current uncertainties in the metallicity dependence of the Cepheid period-luminosity relation (Feast 2003, and references therein).

Paczyński & Stanek (1998) made the first attempt to provide a precise measurement of the local RC absolute magnitude from \textit{Hipparcos} data by fitting the I-band magnitudes of RC stars to a Gaussian function, superimposed on a background distribution of red giant branch (RGB) stars, to find a peak value of \( M_I^{RC} = -0.28 \). This early work focussed on the I-band since, observationally, the I-band magnitudes of RC stars showed up to no dependence on colour, unlike those in the V-band, hence it was thought that the I-band magnitude of RC stars may provide a truly ‘standard’ candle. The initial value from Paczyński & Stanek (1998) was subsequently revised slightly to \( M_I^{RC} = -0.23 \) by Stanek & Garnavich (1998). This was then used by Stanek, Zaritsky & Harris (1998) to make the first direct comparison with the RC in the LMC, yielding a dereddened distance modulus of \( \mu_{0,LMC} = 18.065 \pm 0.031_{\text{statistical}} \pm 0.09_{\text{systematic}} \). Contemporaneously, an almost identical result of \( \mu_{0,LMC} = 18.08 \pm 0.03_{\text{statistical}} \pm 0.12_{\text{systematic}} \) was found by Udalski et al. (1998) using the same method. The implied LMC distance from these results is short, even by the standards of the ‘short’ distance scale, and they are in significant disagreement with the LMC distance modulus of \( \mu_{0,LMC} = 18.50 \pm 0.10 \) adopted by the \textit{Hubble Space Telescope} Extra-galactic Distance Scale Key Project (Freedman et al. 1994).

It was soon pointed out that models of core helium-burning stars predict that the RC luminosity is dependent on both age and metallicity, causing differences of up to 0.6 mag in the mean value of \( M_I^{RC} \) between different stellar populations (Cole 1998; Girardi et al. 1998). The implication therefore, is that any distance derivation to an extra-galactic system should first account for population differences between the local RC and the RC of the system in question, for the results to be meaningful. In an attempt to address this problem, Udalski (2000) examined the metallicity dependence of \( M_I^{RC} \) in the solar neighbourhood from the empirical data. He found a weak dependence of \( M_I^{RC} \) on [Fe/H] at the level of \( \sim 0.13 \) mag dex\(^{-1} \), and consequently derived an I-band correction of \(-0.07\) mag for the LMC. Subsequent work has focussed on tightening the constraints on the metallicity dependence of \( M_I^{RC} \) in several pass-bands, whilst at the same time, investigating the effects of age.

Based on the models of Girardi et al. (2000), predicted mean RC magnitudes for a large range of ages (0.5–12 Gyr) and metallicities (\(-1.7 \leq [\text{Fe/H}] \leq 0.2\)) are given by Girardi & Salaris (2001, hereafter GS01, V and I-band) and Salaris & Girardi (2002, hereafter SG02, K-band). In order to test these predictions the effects of age and metallicity must be examined separately. To this end, a number of recent studies have examined small samples of Galactic open clusters, each of single age and chemical composition, enabling a comparison of the observed RC magnitudes with those predicted from theory (Sarajedini 1999; Twarog, Anthony-Twarog & Bricker 1999; GS01; Grocholski & Sarajedini 2002). Qualitatively, these studies find broad agreement with the models in all 3 pass-bands (V, I and K), confirming that ‘population corrections’ to the RC absolute magnitude are necessary before it can be used as an accurate extragalactic distance indicator. It is important to note here that the metallicity dependence found by Udalski (2000) is derived from local stars in the disc and cannot be regarded as universally applicable since, if we accept the need for population corrections at whatever level, the star formation history and age-metallicity relation are likely to be key factors in determining the metallicity dependence of the RC magnitude. In fact, GS01 model the RC in the solar neighbourhood and reproduce very well the metallicity dependence found by Udalski, giving further weight to the applicability of the models.

In this work, we give a more quantitative assessment of the accuracy of the theoretical predictions for the RC population corrections, over a range of ages and metallicities. We also aim to improve on previous work in several ways. Firstly, determination of the RC absolute magnitude for a particular cluster requires knowledge of the cluster distance. All previous studies involving samples of galactic open clusters have used distances which have been derived using methods based on fitting the cluster sequences to theoretical isochrones. For example, Sarajedini (1999) employs cluster distances which are derived relative to that of M67, the distance to M67 having been determined by fitting to theoretical isochrones. These methods can potentially introduce systematic errors which depend on the particular choice of isochrones. In this work, we employ the purely empirical MS-fitting method, based on a large sample of local field dwarfs with accurate \textit{Hipparcos} parallaxes, developed in Percival, Salaris & Kilkenny (2003, hereafter Paper I), thus removing a potentially large source of systematic error. Furthermore, we ensure complete consistency in the analysis by studying V-, I- and K-band data simultaneously, utilizing the same distance modulus, age estimate and reddening for each cluster in all 3 bands.

Secondly, using Monte Carlo methods, we quantify the effects of sampling on the luminosity of the RC in clusters of different ages and metallicities, in terms of its (empirically) measured magnitude and the likely error. The same simulations also allow us to quantify any offsets between mode, median and mean values for the RC absolute magnitude, \( M_I^{RC} \). It should be noted that the model values tabulated in GS01 and SG02 are the predicted mean values for \( M_I^{RC} \) whilst, from empirical data, generally the median of the distribution is measured. These values are expected to differ since the distribution of magnitudes in the RC is not symmetric – the mode of the magnitude distribution is centred on the zero age horizontal branch, and the distribution has a tail towards brighter magnitudes as the RC stars evolve to higher luminosities.

A third important element in this work, particularly when considering the population effects in the LMC, is the inclusion of 4 clusters with ages \( \leq 2 \) Gyr. These young clusters are important since, for populations with recent star formation (like the LMC), a large fraction of RC stars have ages in the range 1–2 Gyr. The models predict large, and
rapid, changes in $M^{RC}$ in all 3 pass-bands for stars with ages $< 2$ Gyr and so it is vitally important to test these predictions empirically, and quantitatively assess their validity.

The layout of the paper is as follows: In Section 2 we review the basic method of using the RC as a distance indicator, in Sect. 3 we give details of the clusters used in this study and in Sect. 4 we describe the methods involved in deriving cluster distances, and determining the RC magnitude, both empirically and from the theoretical models. Sect. 5 presents our results and in Sect. 6 we discuss some implications of these results and make our conclusions.

2 THE RC AS A DISTANCE INDICATOR – BASIC METHOD

In order to utilize the Red Clump as an extragalactic distance indicator, once the apparent magnitude of the RC, $m^{RC}$, is measured, the distance modulus, $\mu_0 = (m - M)_0$, to a particular galaxy is given by:

$$\mu_{0,\text{galaxy}} = m^{RC} - M^{RC} + A_\lambda + \Delta M^{RC}_\lambda$$

(1)

where $m^{RC}$ is the apparent magnitude of the RC in a particular pass-band, $M^{RC}$ is the ‘zero-point’ absolute RC magnitude given by the local RC stars, $A_\lambda$ is the extinction, and $\Delta M^{RC}_\lambda$ is the population correction to the RC magnitude.

The most recent determinations of the local RC absolute magnitude, $M^{RC}$, are given by Alves et al. (2002), and are $0.73 \pm 0.03, -0.26 \pm 0.03$ and $-1.6 \pm 0.03$ mag in, respectively, the $V$, $I$ and $K$ pass-bands. $\Delta M^{RC}_\lambda$, the population correction, is defined as the difference between the local RC absolute magnitude and that of the RC of the system in question i.e. $\Delta M^{RC}_\lambda = M^{RC}_\lambda (\text{local}) - M^{RC}_\lambda (\text{galaxy})$. As we have already seen, theory predicts that $\Delta M^{RC}_\lambda$ is dependent on both age and metallicity.

To test for and quantify these dependencies, we need to employ a sample of galactic open clusters i.e. simple populations, each of single age and single metallicity, so that the effects of age and metallicity can be disentangled. From eq. 1, it can be seen that the population correction, $\Delta M^{RC}_\lambda$, for a particular cluster will be given by:

$$\Delta M^{RC}_\lambda = \mu_{0,\text{cluster}} - m^{RC} + M^{RC} + A_\lambda$$

(2)

In this equation, $m^{RC}$, $M^{RC}$ and $A_\lambda$ are known quantities from the literature (see sect. 3 for full details) and the distance moduli to the individual clusters, $\mu_{0,\text{cluster}}$ are to be determined.

As already noted, all previous studies have relied on cluster distances derived from MS fitting to theoretical isochrones (i.e. using stellar models), which potentially introduces systematic errors. The aim of this work is to quantify the population corrections via a completely model independent method by utilizing our purely empirical MS-fitting procedure (Paper I) to derive the cluster distances. This method employs a sample of 54 local unevolved field dwarfs with accurate Hipparcos parallaxes (errors are generally $< 5\%$) for which we acquired new $BVIC$ photometry. From fitting to this sample in both the $(B - V)$ and $(V - I)$ colour planes we derive a Hyades distance modulus of $(m - M)_0 = 3.33 \pm 0.06$ – exactly reproducing the distance directly measured from parallaxes of the Hyades stars (Perryman et al. 1998). Hence we are able to tie the absolute magnitude of the RC in each of the open clusters in our sample to the Hipparcos distance scale. Since the absolute magnitude of the local RC is also determined from Hipparcos parallaxes, systematic errors are minimized and we can provide a completely self-consistent assessment of the age and metallicity dependencies for the RC population corrections.

3 CHOOSING THE CLUSTER SAMPLE

The two most important criteria in choosing open clusters for inclusion in this work are that, firstly, each cluster must have a very well defined main sequence in the HR-diagram so that ridge lines can be derived and, secondly, the photometry (which is all existing in the literature) must go deep enough to apply our empirical MS-fitting method. This method uses only the unevolved lower portion of the MS in the fitting procedure, to ensure that there are no evolutionary effects – hence the cluster photometry must extend down to at least $M_V = 7.0$. Since we have $B$, $V$ and $I_C$ photometry for our field dwarf sample used in the MS fitting, we can utilize clusters with either $(B - V)$ or $(V - I)$ photometry, or, indeed, both.

We took, as a starting point, the sample of 8 open clusters from Sarajedini (1999, hereafter S99), however we discounted NGC 6791 and NGC 6819 as their main sequence photometry was not deep enough to apply our MS-fitting method (see Section 4.1 for more details). Thus we included M67, NGC 188, NGC 7789, Berkeley 39, NGC 2204 and Mel 66 – the sources of photometry for these 6 clusters are as listed in S99, their Table 1. We added 2 more young clusters for which there is deep photometry available in the literature, namely NGC 2204 (Kassis et al. 1997, $(B - V)$ and $(V - I)$ data) and NGC 2420 (Anthony-Twarog et al. 1990, $(B - V)$ data only).

Cluster abundances (and their quoted errors) were taken from the catalogue of Gratton (2000), ensuring that the metallicities for the sample clusters and for the field dwarfs used in the MS-fitting are on an homogeneous scale (see discussion in Paper I). We took cluster reddenings from Twarog, Ashman & Anthony-Twarog (1997), assuming a $0.02$ mag error in $E(B - V)$ for all. Absolute ages (and associated errors) were estimated from comparison of the dereddened cluster main lines, and their turn off absolute magnitudes, with Girardi et al. (2000) isochrones - the absolute magnitude of the turn off having been determined using the distance moduli derived in this paper. We note here that our derived ages do not differ significantly from those found by S99, who used the Bertelli et al. (1994) isochrones to estimate cluster ages.

These 8 open clusters span a metallicity range of $-0.44 \leq [\text{Fe/H}] \leq +0.02$ and have ages from 1–7.5 Gyr. We also included in the sample the metal-rich Galactic Globular Cluster 47 Tuc ([Fe/H] = −0.7), since we have an empirically determined MS-fitting distance (and hence an age estimate) for this cluster which is also tied to the Hipparcos distance scale (Percival et al. 2002). Photometry for 47 Tuc is from Stetson’s ‘secondary’ standards available through the Canadian Astronomy Data Centre web site (http://cadcwww.hia.nrc.ca/standards/ – see Stetson 2000 for details). This extends the age range of the cluster sample
Table 1. Cluster data – metallicities from Gratton (2000), reddenings from Twarog et al. (1997). Cluster ages, dereddened distance moduli and absolute RC magnitudes in $V$, $I$ and $K$ are those derived in this work.

| Cluster   | [Fe/H]     | $E(B-V)$ | Age in Gyr | $(m-M)_0$ | $M_{R}^{RC}$ | $M_{I}^{RC}$ | $M_{K}^{RC}$ |
|-----------|------------|----------|------------|-----------|--------------|--------------|--------------|
| M67       | +0.02 ± 0.06 | 0.04 ± 0.02 | 4.0 ± 0.5 | 9.60 ± 0.99 | 0.820 ± 0.115 | −0.203 ± 0.103 | −1.614 ± 0.097 |
| NGC 2477  | 0.00 ± 0.08 | 0.23 ± 0.02 | 1.0 ± 0.3 | 10.74 ± 0.08 | 0.915 ± 0.115 | −0.030 ± 0.012 | −1.304 ± 0.095 |
| NGC 188   | −0.03 ± 0.06 | 0.09 ± 0.02 | 6.0 ± 0.5 | 11.17 ± 0.08 | 0.988 ± 0.106 | −0.091 ± 0.092 | −1.584 ± 0.095 |
| NGC 7789  | −0.13 ± 0.08 | 0.29 ± 0.02 | 1.6 ± 0.3 | 11.22 ± 0.07 | 0.800 ± 0.100 | −0.140 ± 0.085 | −1.574 ± 0.095 |
| Be 39     | −0.15 ± 0.06 | 0.11 ± 0.02 | 7.5 ± 1.0 | 12.97 ± 0.09 | 0.954 ± 0.112 | −0.107 ± 0.100 | −1.434 ± 0.094 |
| NGC 2204  | −0.38 ± 0.08 | 0.08 ± 0.02 | 1.7 ± 0.3 | 13.12 ± 0.08 | 0.481 ± 0.113 | −0.435 ± 0.100 | −1.676 ± 0.093 |
| Mel 66    | −0.38 ± 0.06 | 0.14 ± 0.02 | 4.5 ± 0.5 | 13.22 ± 0.07 | 0.846 ± 0.098 | −0.197 ± 0.084 | −1.584 ± 0.095 |
| 47 Tuc    | −0.70 ± 0.10 | 0.04 ± 0.02 | 11.0 ± 1.4 | 13.25 ± 0.07 | 0.690 ± 0.096 | −0.149 ± 0.081 | −1.280 ± 0.072 |
| NGC 2420  | −0.44 ± 0.06 | 0.05 ± 0.02 | 2.0 ± 0.3 | 11.94 ± 0.07 | 0.498 ± 0.118 | $$^*$$ | −1.676 ± 0.099 |

4 METHOD

4.1 Empirical MS fitting

Before the MS-fitting method can be applied to derive the cluster distance, the MS ridge line must be determined. The procedure we adopted consists of making 0.2 mag cuts in $V$ across the MS and then plotting histograms in colour ($E(B-V)$) to find the peak value. This method clearly identifies the ridge line for a cluster and also clearly shows the location of a binary sequence, if present. The resultant points are fitted to a polynomial (usually cubic) function.

Cluster distances are derived using our empirical MS-fitting method, which we briefly outline here. The method is fully described in Paper I, to which we refer the interested reader for more details. In Paper I we presented new $BV$ (Johnson-Cousins) photoelectric photometry for a sample of 54 field dwarfs with accurate Hipparcos parallaxes, which span a metallicity range of $-0.4 \leq [\text{Fe/H}] \leq +0.3$. Absolute magnitudes for these stars are in the range $5.4 \leq M_V \leq 7.0$, thus they are unevolved zero-age MS stars and their location in a Colour-Magnitude diagram is totally insensitive to any age differences between the clusters which we are fitting to and field dwarf sample.

The basic MS-fitting method consists of constructing a template MS from the field dwarf sample by applying colour shifts to the individual stars to account for the differences in metallicity between the field stars and the cluster. This template is then shifted in $V$ to match the dereddened and extinction-corrected cluster main line, the difference in magnitude being equal to the absolute distance modulus $(m-M)_0$.

The procedure used to calculate the magnitude of the metallicity dependent colour shifts for the field dwarfs relies on first establishing that the shape of the MS is insensitive to $[\text{Fe/H}]$ in the narrow range of metallicities and magnitudes we are dealing with. Next, we determine the colour that each field dwarf would have at a fixed magnitude of $M_V = 6$, which we call $(B-V)_{M_V=6}$ (or $(V-I)_{M_V=6}$), using the slope of the Hyades MS as a reference slope. Finally, we calculate the derivative $\delta (B-V)_{M_V=6}/\delta [\text{Fe/H}]$ (or $\delta (V-I)_{M_V=6}/\delta [\text{Fe/H}]$). Because of the constant shape of the MS across the metallicity range, this derivative is appropriate for the whole magnitude range spanned by the field dwarfs, and hence colour shifts are applied to each star in the sample, at their observed magnitudes, to construct a template MS at the metallicity of the cluster in question.

When fitting to the cluster main lines to derive the cluster distance, weighted errors are used for the field dwarfs which include both photometry errors and magnitude errors due to error on the parallax (where $\Delta M_V = 2.17(\Delta \pi/\pi)$). We also account for errors on the metallicity of the both the field dwarfs and the cluster, and the best-fitting distance is found by minimizing $\chi^2$. For clusters with photometry in both colour indices, dereddened distances are taken to be an average from the $(B-V)$ and $(V-I)$ fits, which we note here are always consistent with each other, within their respective errors.

The fitting procedure is illustrated in Figure 1 which shows the $V/(B-V)$ CMD for NGC 2477 (data from Kassis
et al. 1997). The left panel shows the dereddened and extinction-corrected cluster photometry overlaid with the derived ridge line for the MS. The RC stars are contained within the box spanning $11.1 \leq V_0 \leq 12.1$ and $0.85 \leq (B - V)_0 \leq 1.1$. The panel on the right shows the cluster ridge line, shifted in $V$-magnitude to match the template MS created from the field dwarf sample, hence yielding the best-fitting distance modulus. The data points are the template MS, which has been created from the field dwarf sample by 'correcting' their colors to match the metallicity of the cluster ([Fe/H] = 0.0 in this case), as described above. The dereddened distance moduli determined by this method for all the clusters in the sample are listed in Table 1.

4.2 RC magnitudes - observed

Apparent RC magnitudes, $m_{\lambda}^{RC}$, are taken as listed in S99 ($V$- and $I$-bands only) for M67, NGC 188, NGC 7789, Berkeley 39, NGC 2204 and Mel 66. For NGC 2204 and NGC 2420 we calculated the RC magnitude using exactly the same method employed by S99, i.e. finding the median magnitude of the stars contained in a box centered on the RC. As noted by S99, this median value is virtually insensitive to the exact location of the box, a fact confirmed by us from Monte Carlo simulations of the RC (see Sect. 4.3). $K$-band RC magnitudes, determined using the same method, are taken from Grocholski & Sarajedini 2002 (note that these are listed as extinction corrected absolute magnitudes, $M_{\lambda}^{RC}$, so we have used their quoted reddening and distance modulus values to calculate the apparent $K$ magnitudes, $m_{\lambda}^{RC}$).

Thus, for each cluster the absolute RC magnitude, $M_{\lambda}^{RC}$, is given by:

$$M_{\lambda}^{RC}(\text{cluster}) = m_{\lambda}^{RC} - A_{\lambda} - (m - M)_0$$

where $m_{\lambda}^{RC}$ is the apparent RC magnitude as described above, $A_{\lambda}$ is the extinction and $(m - M)_0$ is the dereddened distance modulus as found in Sect. 4.1.

The population correction, $\Delta M_{\lambda}^{RC}$, is then given by

$$\Delta M_{\lambda}^{RC} = M_{\lambda}^{RC}(\text{local}) - M_{\lambda}^{RC}(\text{cluster})$$

where $M_{\lambda}^{RC}(\text{local})$ are the values given by Alves (2002), for $V$, $I$ and $K$ (see Sect. 2).

4.3 RC magnitudes - theoretical

Mean RC magnitudes generated from the Girardi et al. (2000) models are tabulated in GS01 ($V$ and $I$) and SG02 ($K$) for a range of ages and metallicities (see Sect. 1). These effectively correspond to cluster RC magnitudes, $M_{\lambda}^{RC}(\text{cluster})$, since each tabulated value represents the RC magnitude for a single age and metallicity.

For the mean magnitude of the local RC, $M_{\lambda}^{RC}(\text{local})$, we use the values calculated by GS01 and SG02. GS01 and SG02 employ a complete population synthesis algorithm which, using the stellar models of Girardi et al. (2000), produces a synthetic CMD, from which the RC luminosity function can be extracted. The solar neighbourhood was modelled using the Star Formation Rate (SFR) and Age Metallicity Relation (AMR) from Rocha-Pinto et al. (2000a,b). We will discuss the effects of the particular choice of SFR and AMR in Section 6. The model population correction is then calculated as for the empirical data, i.e. $\Delta M_{\lambda}^{RC} = M_{\lambda}^{RC}(\text{local}) - M_{\lambda}^{RC}(\text{cluster})$. It is important to note at this point that we are not relying on the absolute values for the RC magnitudes from the models since the population correction is a differential quantity, i.e. it is the difference between the local RC magnitude and the RC magnitude of the population under scrutiny.

The tabulated model values give the mean properties of the RC stars – i.e. the mean magnitude of all stars in the simulation which are still undergoing core helium burning. RC stars start their lives on the ZAHB, producing a peak in their luminosity function, and then evolve on shorter timescales to brighter magnitudes. Consequently, the mode of the magnitude distribution is offset from the mean value, in the sense that the mean magnitude is brighter. Obviously, from the simulations, all the RC stars are accounted for and any value can be measured – mean, median or mode. For real data sets the RC is often sparsely populated, due to poor sampling, and the empirical median values derived as described in Sect. 4.2 reflect the mode rather than the mean magnitude.

We tested for the offset between mean and median magnitudes by first simulating RCs at ages from 1–10 Gyr in $V$, $I$ and $K$. This was done by populating the Girardi et al. (2000) isochrones for solar metallicity using a Salpeter IMF, and then taking all stars still in their core helium-burning phase. These stars are all within 1.0 mag of the ZAHB and, in the particular simulations used, we generate a total parent sample of 4000–5000 RC stars. (We note here that the particular choice of IMF does not influence the results at all, since the evolutionary time-scale for these stars is very short, and so the mass is nearly constant in the region of interest).

We then randomly extracted subsamples of 5 and 50 stars from the parent sample and calculated the median magnitude in all 3 pass-bands for each subsample. This process was repeated 70 times and the resultant median magnitudes plotted as histograms. The peak of each distribution (i.e. for the subsamples of 5 and the subsamples of 50) yields the most likely measured median value for a real data set, and the dispersion (i.e. 1-$\sigma$) of the distribution represents the error associated with the (small) sample size. The median values were found to be identical (within 0.01 mag) for the simulated subsamples of 5 and 50, and whilst the errors are obviously larger for the smaller sample size, they are surprisingly stable – errors are generally no more than 0.05 mag for a sample of 5, except for the 1 Gyr simulation which produces a 1-$\sigma$ error of $\sim 0.1$ mag. However, this is not a cause for concern here, as all the young clusters used in this study have a well populated RC, which produces much smaller errors.

The mean–median offsets are 0.04–0.05 mag for ages > 2 Gyr in all 3 filters, whilst at 1 Gyr, the offset is 0.07–0.08 mag. Our empirical RC absolute magnitudes, $M_{\lambda}^{RC}$, were therefore adjusted using the appropriate mean–median offset before calculating the population corrections, $\Delta M_{\lambda}^{RC}$, and comparing with the theoretical values. The empirical values of $M_{\lambda}^{RC}$ are listed in Table 1 – the quoted errors include the error on the cluster distance modulus, the statistical error on the measured RC magnitude, described above, and the error induced by a 0.02 mag uncertainty on the cluster reddening, added in quadrature.
Figure 2. The RC population correction, $\Delta M^{RC}_{\lambda}$, in $V$, $I$ and $K$, plotted against age. The lines are the Girardi et al. (2000) models at [Fe/H] of: -0.7 (solid), -0.4 (dotted), 0.0 (short–dash), +0.2 (long–dash). The data points are the empirically derived values for 9 (in $V$), 8 (in $I$) and 6 (in $K$) clusters, with error bars as detailed in the text.

5 RESULTS

Fig. 2 shows the empirically derived population corrections, $\Delta M^{RC}_{\lambda}$, in $V$ (9 pts), $I$ (8 pts) and $K$ (6 pts) overplotted with the theoretical models of Girardi et al. (2000) for [Fe/H] between −0.7 and +0.2. The data points are our empirically determined values and the vertical error bars include the error on our distance moduli (which includes error on cluster metallicity), the statistical error on the RC magnitude (detailed in Sect. 4.3), and the error on the RC magnitude induced by uncertainty in the cluster reddening (0.02 mag), all added in quadrature. The most important feature of this plot is the fact that the empirically determined population corrections follow the trend of the theoretical models in all 3 pass-bands, particularly at ages < 2 Gyr, where the models predict a strong dependence on age.

We stress again that we are not relying on absolute values from the models, which may differ from the empirical ones (e.g. for local RC magnitude, see GS01). The models are always being used differentially, remembering that the population correction, $\Delta M^{RC}_{\lambda}$, is the difference between the local RC magnitude and the RC magnitude of the cluster in question.

In order to quantify the goodness-of-fit between the models and the empirical data, we calculated the residuals of $\Delta M^{RC}_{\lambda}$ in the sense model – observed. These residuals are plotted in Figs. 3 and 4 against, respectively, age and metallicity. The vertical error bars (i.e. in magnitude only) are as in Fig. 2, but have been amended to account for the error on the cluster age. It can be seen from Fig. 2 that the error on the cluster age will cause an asymmetric error on $\Delta M^{RC}_{\lambda}$, particularly for ages < 2 Gyr, and this is reflected in the error bars in Figs. 3 and 4. These plots clearly show that the residuals in $\Delta M^{RC}_{\lambda}$ display no statistically significant trends with either age or metallicity in any of the 3 pass-bands.

The most discrepant point in the $V$- and $I$-bands (i.e. with the largest residuals) is Melotte 66. This is the only cluster for which there is no ($B-V$) data for the main sequence and so the distance modulus has been derived from the ($V-I$) data alone. We note, however, that there is a small amount of photoelectric $UBVRI$ photometry for this cluster from Twarog, Anthony-Twarog & Hawarden (1995).
Comparison of the photoelectric data with the CCD data used in this study (from Kassis et al. 1997) indicates that there may be small zero-point offsets in the CCD photometry which would have the effect of making the \((V-I)\) colours too red by 0.04–0.05 mag. If this were the case, the derived distance would be too short by \(\sim 0.2\) mag, causing a similar discrepancy in the RC magnitude, and therefore accounting for the discrepancy between the model and observed values for the population correction. Removing this cluster from our analysis, the mean residuals in \(\Delta M_{\text{RC}}^V\) are; +0.03 \(\pm 0.07\) in \(V\), +0.07 \(\pm 0.06\) in \(I\), and 0.00 \(\pm 0.07\) in \(K\), where the quoted errors are the 1-\(\sigma\) errors. Hence we find no statistically significant offsets in \(V\) and \(K\) and a very marginal offset in \(I\), significant only at the 1.2-\(\sigma\) level.

We note that if we included Melotte 66 in this analysis the mean residuals in \(V\) and \(I\) would become +0.05 \(\pm 0.09\) and +0.09 \(\pm 0.06\) respectively (those in \(K\) would remain unchanged) and hence our conclusions would essentially remain unaltered.

6 DISCUSSION AND CONCLUSIONS

A vital factor in determining the RC population corrections is the absolute magnitude of the local RC, for which we have very well determined observational values. Deriving the same quantities from the models requires knowledge of the local SFR and AMR, which are used as input parameters in the population synthesis algorithm (see GS01 for details). We checked the effects of assuming a different SFR and/or AMR by modelling the local RC using the AMRs of Carlberg et al. (1985) and of Edvardsson et al. (1993). We also tested the effect of using a constant star formation rate. In all cases, the model values for the mean RC magnitude, \(M_{\text{RC}}^\text{KS}\), were indistinguishable from those found when assuming the Rocha-Pinto et al. (2000a,b) relationships. This is largely because the local RC is dominated by stars with ages < 2 Gyr (see in depth discussion in GS01) and the different AMRs all predict similar metallicities at these young ages. Hence the precise details of the star formation history do not significantly affect the local RC magnitude.

The local RC simulated in GS01 (using the Rocha-Pinto 2000 SFR and AMR) yields a straight mean metallicity of \([\text{Fe}/\text{H}] = -0.04\), whilst the observed RC stars in the solar neighbourhood appear to have a mean \([\text{Fe}/\text{H}]\) of \(\sim -0.18\) (see, e.g. Alves 2000). Metallicities used by Alves for the local RC stars are from McWilliam (1990) – the same ones used by Udalski (2000) to derive a metallicity dependence for \(M_{\text{RC}}^\text{KS}\). This offset between the model and observed mean metallicity for the RC stars has already been noticed and discussed in GS01. McWilliam himself notes that his metallicities for the RC stars may be systematically too low, especially for the highest metallicities, due to the (then) lack of suitable model atmospheres for super-solar metallicities. Luck & Challener (1995) determined abundances for 55 G and K field giants from high resolution spectra – 45 of these stars being in common with the McWilliam (1990) sample. Luck & Challener find a constant offset of 0.12 dex between their abundances and those of McWilliam, in the sense that the McWilliam (1990) values are lower. From the most recent catalogue of \([\text{Fe}/\text{H}]\) determinations for FGK stars by Cayrel de Strobel, Soubiran & Ralite (2001), we made a comparison of all the stars in the McWilliam (1990) sample which had abundance determinations by other authors – this yielded a sample of 89 stars with 152 measurements from different sources. The average difference between the McWilliam abundances and all the others in the literature is 0.13 dex, with a dispersion of 0.15 dex. However it is evident that the McWilliam metallicities are increasingly discrepant at the highest metallicities. Splitting the sample into 2 groups with \([\text{Fe}/\text{H}]_{\text{other}} < 0.2\) and \([\text{Fe}/\text{H}]_{\text{other}} > 0.2\) and comparing with the McWilliam values reveals an offset of 0.10 dex for the lower metallicity group and an offset of 0.31 dex for the higher metallicity group, both with a dispersion of 0.14 dex, in the sense that the McWilliam values are lower. Hence, the apparent discrepancy between the average metallicity of the local RC predicted from the models and that derived from observational data, appears to be a problem with the zero-point of the metallicity scale derived by McWilliam (1990).

The results of our analysis fully confirm the theoretical population corrections obtained by GS01 and SG02 for single-age, single-metallicity stellar populations, which are the building blocks for computing corrections to composite systems. Remarkably, the minimum of the RC luminosity for ages around 1 Gyr predicted by theory, corresponding to the transition from electron degenerate to non-degenerate helium cores, is also clearly observed. This is a further positive test for the accuracy of the physics employed in the stellar models. A larger sample of clusters with accurate multicolour photometry for the lower MS could confirm whether the small offset in the \(I\)-band is real. For use as an accurate distance indicator, \(K\) is the best single pass-band to use as the effects of reddening are minimised and, provided ages are less than \(\sim 7\) Gyr, the metallicity dependence is very small.

It is evident from Fig. 2 that, in general, there is no simple relationship between the population correction in a given passband and metallicity or age. Also, it is only in the \(K\)-band, and specifically for metallicities close to solar and ages lower than a few Gyr, that the population corrections are very small (see SG02 for a discussion on this point). This means that there is no way, a priori, to determine generic empirical population corrections that are a function of only the metallicity of the population under scrutiny (this point has already been discussed in depth by GS01). In the case of composite stellar populations it is necessary to have a good determination of their Star Formation Rate (SFR) and Age Metallicity Relationship (AMR) and apply population synthesis algorithms in order to obtain the appropriate corrections (alternatively, the corrections can be derived using the simpler analytical method described in GS01). The accuracy of the derived corrections will depend entirely on the accuracy of the determined SFR and AMR, since they are combinations of the single-age, single-metallicity corrections, which we have now proved to be accurate. For systems where no precise AMR and SFR are available, the use of empirical calibrations – which are necessarily of limited validity since they are dependent on the particular properties of the calibrating sample – can be very dangerous because it can lead to errors of several tenths of a magnitude on the derived distance moduli.

The very limited range of validity of empirical corrections based only on the stellar metallicity is illustrated by the valuable data published by Pietrzyński, Gieren & Udall-
ski (2003, hereafter PGU03). These authors present new near-infrared $J$- and $K_S$-band data for several fields in the LMC, SMC, and the Fornax and Carina dwarf galaxies and, in addition, utilize $I$-band data in the literature for the same systems. PGU03 are therefore able to determine accurately the $K$- and $I$-band brightness of the RC in these 4 systems and they go on to derive empirical corrections to the RC brightness, based solely on differences in mean metallicity. After applying these corrections (which are normalized to the LMC mean metallicity) to the RCs in the SMC, Carina, Fornax and the solar neighbourhood, PGU03 look at the difference between the $I$ and $K$ brightness of the RC for each system. If the corrections are adequate, one expects that this quantity will be constant for all the systems, however PGU03 find a discrepancy of 0.23 mag for the solar neighbourhood value, which they ascribe to zero-point errors in the photometry of the local RC stars.

This difference between the $I$- and $K$-band absolute magnitude of the RC is an ideal parameter on which we can test the theoretical models further. The predicted metallicity dependencies for the population correction, $\Delta M_R^{\text{RC}}$, are very different between the 2 passbands, that in $I$ being much stronger than in $K$. Furthermore, the dependencies work in opposite directions – for increasing metallicity, $\Delta M_R^{\text{RC}}$ becomes smaller, whilst $\Delta M_K^{\text{RC}}$ becomes larger – and are not constant with increasing age. Therefore, as an experiment, we examined the difference between the $I$-band and $K$-band magnitude of the RC predicted from theory for the LMC, SMC, Carina and the solar neighbourhood. It is important to realise that in case of the theoretical population corrections, consistency with the local RC is automatically achieved, since the corrections are computed relative to the solar neighbourhood values. We have adopted the values for the LMC and SMC provided by SG02; for Carina, we have simulated the RC using the same population synthesis algorithm, assuming the SFR used in GS01 and adopting the most recent determination of the AMR from Tolstoy et al. (2003). We did not consider the case of Fornax simply because existing determinations of the SFR are highly uncertain, thus preventing an accurate assessment of the appropriate theoretical population corrections.

Figure 5 shows a comparison of the observed difference between the $I$- and $K$-band brightness of the RC, for the LMC, SMC, Carina and the solar neighbourhood, and the corresponding values predicted from theory. (We note here that the $K_S$-band magnitudes from PGU03 have been transformed to the Bessell & Brett system, which are those used in the theoretical models). The result of the comparison is excellent, with a good agreement, within the errors, between theory and observations. This confirms the accuracy of the theoretical corrections, which simultaneously reproduce the LMC, SMC and Carina data and satisfy the solar neighbourhood values without the need to invoke photometric errors of the order of 0.20 mag. As a consequence, with the theoretical population corrections one obtains full consistency between the RC $I$- and $K$-band distances to the LMC derived from the empirical values tabulated by PGU03; in fact, by using the local RC calibration by Alves et al. (2002) we obtain, respectively, $(m - M)_{0,I} = 18.94 \pm 0.06$ and $(m - M)_{0,K} = 18.46 \pm 0.03$ (the errors include both the error on the reddening and the error on the local RC magnitudes). For the SMC, again using the observed values tabulated by PGU03, we obtain $(m - M)_{0,I} = 18.90 \pm 0.06$ and $(m - M)_{0,K} = 18.88 \pm 0.03$. The difference in the RC distance moduli, $\Delta M_{\text{SMC},LMC}^{\text{RC}}$, is equal to $0.41 \pm 0.08$ in $I$ and $0.42 \pm 0.04$ in $K$, which compares well with the value of $0.50 \pm 0.08$ obtained from the comparison of the Tip of the Red Giant Branch magnitudes found by PGU03.

Significantly, the LMC distance moduli we find above, derived individually from the $I$- and $K$-band data, are fully in agreement with the LMC distance modulus determined from the multiwavelength method of Alves et al. (2002). This method imposes the constraint that the derived distance modulus must be the same in all the passbands used – solving simultaneously in several passbands then yields both distance modulus and reddening. Using $V$, $I$ and $K$ data, Alves et al. (2002) find $(m - M)_{0,LMC} = 18.493 \pm 0.033$, $\pm 0.03$. Most recently, Salaris et al. (2003) have applied the same method to the field surrounding the LMC cluster NGC 1866 and found $(m - M)_{0,LMC} = 18.53 \pm 0.07$ from HST $V$ and $I$ data.

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