Study of direct $CP$ in charmed $B$ decays and
measurement of the CKM angle $\gamma$ at Belle

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Abstract

The Belle experiment, running at the KEKB $e^+e^-$ asymmetric energy
collider during the first decade of the century, has recorded 770 fb$^{-1}$ of data at the
$\Upsilon(4S)$ resonance. A combination of recent Belle results obtained with this sam-
ple is used to perform a measurement of the CKM angle $\gamma$. We use $B^+ \to DK^+$
and $B^\pm \to D^*K^\pm$ decays where the $D$ meson ($D^0$ or $\bar{D}^0$) decays into $K_S^0\pi\pi$,
$K\pi$, $KK$, $\pi\pi$, $K_S^0\pi^0$ and $K_S^0\eta$ final states and $D^*$ decays into $D\pi^0$ and $D\gamma$.
Belle obtains the most precise $\gamma$ measurement to date, $\gamma = (68^{+15}_{-14})^\circ$. 

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1 Introduction

Two angles of the CKM unitarity triangle, $\beta$ and $\alpha$, have now been measured with high precision [1]. The determination of the third angle, $\gamma$, using $B^\pm \to DK^\pm$ decays, will require much more data than for the other angles. Its determination is however theoretically clean due to the absence of loop contributions; $\gamma$ can be determined using tree-level processes only, exploiting the interference between $b \to c\pi d$ and $b \to u\pi d$ transitions that occurs when a process involves a neutral $D$ meson reconstructed in a final state accessible to both $D^0$ and $\bar{D}^0$ decays (Fig 1). Therefore, the angle $\gamma$ provides a SM benchmark, and its precise measurement is crucial in order to disentangle non-SM contributions to other processes, via global CKM fits. The size of the interference also depends on the ratio ($r_B$) of the magnitudes of the two tree diagrams involved and $\delta_B$, the strong phase difference between them. Those hadronic parameters will be extracted from data together with the angle $\gamma$. The value of $r_B$ is the product of the ratio of the CKM matrix elements $|V_{ub}^*V_{cs}|/|V_{cb}V_{us}| \sim 0.38$ and the color suppression factor, which result in a value of around 0.1, whereas $\delta_B$ can not be precisely calculated from theory. Note that $r_B$ and $\delta_B$ can take different values for different $B$ decays: the values of $B^\pm \to DK^\pm$ and $B^\pm \to D^\ast K^\pm$ are not the same. Several different $D$ decays have been studied in order to maximize the sensitivity to $\gamma$. The archetype is the use of $D$ decays to $CP$ eigenstates, a method proposed by M. Gronau, D. London, and D. Wyler (and called the GLW method) [2]. An alternative approach was proposed by D. Atwood, I. Dunietz, and A. Soni [3]. Instead of using $D^0$ decays to $CP$ eigenstates, the ADS method uses Cabibbo-favored and doubly Cabibbo-suppressed $D$ decays. In the decays $B^+ \to [K^-\pi^+]_D K^+$ and $B^- \to [K^+\pi^-]_D K^-$, the suppressed $B$ decay is followed by a Cabibbo-allowed $D^0$ decay, and vice versa. Therefore, the interfering amplitudes are of similar magnitude, and one can expect a large $CP$ asymmetry. The main limitation of the method is that the branching fractions of those decays above are small. A Dalitz plot analysis of a three-body $D$ meson final state allows one to obtain all the information required for the determination of $\gamma$ in a single decay mode. Three-body final states such as $K_S^0\pi^+\pi^-$ have been suggested as promising modes [4] for the extraction of $\gamma$. At present the Dalitz method (or GGSZ method) has the best sensitivity to $\gamma$. 

![Feynman diagrams](image_url)
Latest Belle results using the full data sample taken at the $\Upsilon(4S)$ (corresponding to $772 \times 10^6 \, B\overline{B}$ pairs) are described in these proceedings and the resulting $\gamma$, combining these different results, is given.

## 2 GGSZ results

As in the GLW and ADS methods, the two amplitudes interfere if the $D^0$ and $\overline{D}^0$ mesons decay into the same final state $K_S^0\pi^+\pi^-$. Assuming no $CP$ asymmetry in neutral $D$ decays, the amplitude for $B^+ \to D[K_S^0\pi^+\pi^-]K^+$ decay as a function of Dalitz plot variables $m_+^2 = m_{K_S^0\pi^+}^2$ and $m_-^2 = m_{K_S^0\pi^-}^2$ is

$$f_{B^+} = f_D(m_+^2, m_-^2) + r_B e^{i\gamma + i\delta_B} f_D(m_-^2, m_+^2)$$

(1)

where $f_D(m_+^2, m_-^2)$ is the amplitude of the $\overline{D}^0 \to K_S^0\pi^+\pi^-$ decay. Similarly, the amplitude for $B^- \to D[K_S^0\pi^+\pi^-]K^-$ decay is

$$f_{B^-} = f_D(m_-^2, m_+^2) + r_B e^{-i\gamma + i\delta_B} f_D(m_+^2, m_-^2).$$

(2)

The $\overline{D}^0 \to K_S^0\pi^+\pi^-$ decay amplitude $f_D$ can be determined from a large sample of flavor-tagged $\overline{D}^0 \to K_S^0\pi^+\pi^-$ decays produced in the continuum $e^+e^-$ annihilation. Once $f_D$ is known, a simultaneous fit to $B^+$ and $B^-$ data allows the contributions of $r_B$, $\gamma$ and $\delta_B$ to be separated. The method has only two-fold ambiguity: $(\gamma, \delta_B)$ and $(\gamma + 180^\circ, \delta_B + 180^\circ)$ solutions cannot be distinguished. Due to the fact that $r_B$ is bound to be positive, the direct extraction of $r_B$, $\delta_B$ and $\gamma$ can be biased. To avoid these biases, the Cartesian coordinates have been introduced, $x^\pm = r_B \cos(\delta_B \pm \gamma)$ and $y^\pm = r_B \sin(\delta_B \pm \gamma)$. A combined unbinned maximum likelihood fit to the $B^+$ and $B^-$ samples with free parameters $(x^+, \, y^+)$ yields the values given in Table 1.

| Observables | $B \to DK$ | $B \to D^*K$ |
|-------------|------------|--------------|
| $x^+$       | $-0.107 \pm 0.043 \pm 0.011$ | $+0.083 \pm 0.092 \pm 0.011$ |
| $y^+$       | $-0.067 \pm 0.059 \pm 0.018$ | $+0.157 \pm 0.109 \pm 0.018$ |
| $x^-$       | $+0.105 \pm 0.047 \pm 0.011$ | $-0.036 \pm 0.127 \pm 0.011$ |
| $y^-$       | $+0.177 \pm 0.060 \pm 0.018$ | $-0.249 \pm 0.118 \pm 0.018$ |

Combining $B^\pm \to DK^\pm$ and $B^\pm \to D^*K^\pm$, the value $\gamma = (78^{+11}_{-12} \pm 4 \pm 9)^\circ$ is obtained, where the quoted uncertainties are respectively statistical, systematic and due to an imperfect knowledge of the amplitude model that describes $D \to K_S^0\pi^+\pi^-$ decays. The last source of uncertainty can be eliminated by binning the Dalitz plot (Refs. [4, 5]).
using information on the average strong phase difference between $D^0$ and $\overline{D}^0$ decays in each bin that can be determined using the quantum-correlated $\psi(3770)$ data. Such results have been published recently by CLEO-c [7]. The measured strong phase difference is used to obtain the model-independent result [8], $\gamma = (77 \pm 15 \pm 4 \pm 4)^\circ$, where the last uncertainty is due to the statistical precision of the CLEO-c results.

3 ADS results

For the ADS method, Belle has studied the $B \to D^{(*)} K$ decays where $D \to K^- \pi^+$. The observables measured in the ADS method are the ratio of the suppressed and allowed branching fractions:

$$ R_{\text{ADS}} = \frac{\Gamma(B^+ \to [K^+\pi^+]_D K^+)}{\Gamma(B^+ \to [K^+\pi^+]_D K^+)} = r_B^2 + r_D^2 + 2r_B r_D \cos \gamma \cos \delta, $$

(3)

and

$$ A_{\text{ADS}} = \frac{\Gamma(B^- \to [K^+\pi^-]_D K^-) - \Gamma(B^+ \to [K^-\pi^+]_D K^+)}{\Gamma(B^- \to [K^+\pi^-]_D K^-) + \Gamma(B^+ \to [K^-\pi^+]_D K^+)} = 2r_B r_D \sin \gamma \sin \delta / R_{\text{ADS}}, $$

(4)

where $r_D$ is the ratio of the doubly Cabibbo-suppressed and Cabibbo-allowed $D^0$ decay amplitudes and $\delta$ is the sum of strong phase differences in $B$ and $D$ decays: $\delta = \delta_B + \delta_D$. The latest ADS analysis [9] of $B^\pm \to D K^\pm$ decays with $D^0$ decaying to $K^+\pi^-$ and $K^-\pi^+$ (and their charge-conjugated partners) uses the full $\Upsilon(4S)$ data sample recorded by the Belle experiment. The signal yield obtained is $56^{+15}_{-14}$ events, which corresponds to the first evidence for an ADS signal (with a significance of 4.1$\sigma$); the ratio of the suppressed and allowed modes and asymmetry are summarized in Table 2. The use of two additional decay modes, $D^* \to D\pi^0$ and $D^* \to D\gamma$, provides an extra handle on the extraction of $\gamma$ as explained in Ref. [10] and illustrated by the predictions of the ADS observables [11] from the values of $(\gamma, \delta_B$ and $r_B$) obtained with the GGSZ method (shown in Fig. 2). This effect (larger ratio for $B^\pm \to D^* K^\pm$ with $D^* \to D\gamma$ and opposite asymmetry between both $B^\pm \to D^* K^\pm$ channels) is becoming visible in the most recent results from Belle [12].

Table 2: Results of the Belle ADS analyses.

| Mode                          | $R_{\text{ADS}}$          | $A_{\text{ADS}}$          |
|-------------------------------|---------------------------|---------------------------|
| $B^\pm \to D K^\pm$           | $0.0163^{+0.0044+0.0009}_{-0.0041-0.0013}$ | $-0.39^{+0.26+0.04}_{-0.28-0.03}$ |
| $B^\pm \to D^* K^\pm$, $D^* \to D\pi^0$ | $0.0100^{+0.008+0.001}_{-0.007-0.002}$ | $+0.4^{+1.1+0.2}_{-0.7-0.1}$ |
| $B^\pm \to D^* K^\pm$, $D^* \to D\gamma$ | $0.036^{+0.014+0.0012}_{-0.012-0.002}$ | $-0.51^{+0.33+0.08}_{-0.29-0.06}$ |
Figure 2: Predictions (from the world averages ($\gamma$, $\delta_B$ and $r_B$) values obtained with the GGSZ method) and measurements of the ADS observables for $B^\pm \to D K^\pm$ (left), $B^\pm \to D^* K^\pm$ with $D^* \to D \pi^0$ (center) and $B^\pm \to D^* K^\pm$ with $D^* \to D \gamma$ (right).

4 GLW results

As alluded earlier, the other interesting class of modes are the ones where the $D^0$ decays into $CP$ eigenstates [2] such as $K^+ K^-$, $\pi^+ \pi^-$ ($CP$-even eigenstates) and $K^0 S\pi^0$, $K^0 S\eta$ ($CP$-odd eigenstates). To extract $\gamma$ using the GLW method, the following observables sensitive to $CP$ violation are used: the asymmetries

$$A_{CP} = \frac{\Gamma(B^- \to D_{CP}^+ K^-) - \Gamma(B^- \to D_{CP}^- K^-)}{\Gamma(B^- \to D_{CP}^+ K^-) + \Gamma(B^- \to D_{CP}^- K^-)} = \pm \frac{2r_B \sin \delta_B \sin \gamma}{1 + r^2_B \pm 2r_B \cos \delta_B \cos \gamma}$$

and the ratios

$$R_{CP} = \frac{2\Gamma(B^- \to D_{CP}^+ K^-) + \Gamma(B^+ \to D_{CP}^+ K^+)}{\Gamma(B^- \to D^0 K^-) + \Gamma(B^+ \to D^0 K^+)} = 1 + r^2_B \pm 2r_B \cos \delta_B \cos \gamma$$

Among these four observables, $A_{CP}$ and $R_{CP}$, only three are independent (since $A_{CP} R_{CP} = -A_{CP} R_{CP}$). Recently, Belle updated the GLW analysis using their final data sample of $772 \times 10^6 B\bar{B}$ pairs [12]. These results include the two $B$ decays: $B^\pm \to D^0 K^\pm$ and $B^\pm \to D^{*0} K^\pm$, where $D^{*0} \to D^0 \pi^0$ and $D^0 \gamma$ (the latter modes are shown for the first time in this conference, shown Fig. 5). The signs of the $A_{CP}$ and $A_{CP}$-asymmetries (Eq. 5) should be opposite (as shown by the predictions illustrated in Fig. 4 obtained by the CKMfitter group [11]), which is now confirmed by the Belle experiment (Table 3).
Figure 3: Signal for the $B^\pm \rightarrow D^* h^\pm$ decays from Belle ADS analysis. The plotted variable, $\Delta E$, peaks at zero for signal decays. On the right plots, $[K^+\pi^-]_D K^-$ components are shown (by thicker dashed curves (red)) for $D^* \rightarrow D\pi^0$ (top) and $D^* \rightarrow D\gamma$ (bottom).

5 γ combination from Belle measurements

We combine the available Belle observables of the $D^{(*)}K$ system obtained for the GGSZ method (model-dependent results shown in Table 1), the ADS method (Table 2) and the GLW method (Table 3) using the frequentist procedure also exploited in Ref. [1]. The 1−CL curves obtained with for the angle $\gamma$ as well as for the hadronic parameters ($\delta_B$ and $r_B$) of $B \rightarrow DK$ mode are shown in Fig. 6 and the 68% C.L. intervals are summarized in Table 4. Belle obtains the most precise $\gamma$ measurement to date: $\gamma = (68^{+15}_{-14})^\circ$.

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Table 3: Compilation of $R_{CP}$ and $A_{CP}$ results for $CP$-even and $CP$-odd decay modes from Belle.

| Observables | $B \to D K$ | $B \to D^* K$ |
|-------------|------------|--------------|
| $R_{CP}^+$  | 1.03 ± 0.07 ± 0.03 | 1.19 ± 0.13 ± 0.03 |
| $R_{CP}^-$  | 1.13 ± 0.09 ± 0.05 | 1.03 ± 0.13 ± 0.03 |
| $A_{CP}^+$  | +0.29 ± 0.06 ± 0.02 | −0.14 ± 0.10 ± 0.01 |
| $A_{CP}^-$  | −0.12 ± 0.06 ± 0.01 | +0.22 ± 0.11 ± 0.01 |

Figure 4: Predictions (from the world averages ($\gamma$, $\delta_B$ and $r_B$) values obtained with the GGSZ method) and measurements for the GLW observables for $B^\pm \to DK^\pm$ (top) and $B^\pm \to D^* K^\pm$ (bottom).

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Figure 5: Signals for $B^\pm \to D^* (D\pi^0) K^\pm$ decays: the left (right) figures are for $B^-$ ($B^+$) decays and the top (bottom) figures are for CP-even (CP-odd) eigenstates.

Figure 6: $1-\text{CL}$ curves for $\gamma$ (left), $r_B$ (center) and $\delta_B$ (right) from the Belle $D^{(*)}K$ results. The green curve is for the GGSZ results, the blue for GGSZ and ADS results using $\delta_D$ from mixing and CLEO-c measurements, the red for GGSZ, ADS and GLW results.
Table 4: Confidence intervals (68% C.L.) for the angle $\gamma$ and the hadronic parameters of $DK$ ($\delta_B$ and $r_B$) obtained by the combination of the $D \to D^{(*)}K$ results of the Belle collaboration.

| Method           | $\gamma$ (°) | $\delta_B$ (°) | $r_B$          |
|------------------|--------------|----------------|---------------|
| GGSZ             | $82^{+18}_{-23}$ | $141^{+27}_{-36}$ | $0.168^{+0.064}_{-0.064}$ |
| GGSZ+ADS         | $68\pm22$    | $123^{+27}_{-33}$ | $0.104^{+0.020}_{-0.021}$ |
| GGSZ+ADS+GLW     | $68^{+15}_{-14}$ | $116^{+18}_{-21}$ | $0.112^{+0.014}_{-0.015}$ |