Investigation of sheath properties in a warm plasma with two kappa-distributed electrons and monoenergetic electron beam

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Sheath formation criterion of an electropositive plasma consisting of singly charged positive ions, two kappa-distributed electron species with different effective temperatures and a monoenergetic electron beam is investigated by the Sagdeev potential approach. Using this criterion, effects of electron beam, superthermality of electron species as well as temperature and concentration of positive ions on the sheath properties are studied numerically. It is shown that the temperature of positive ions, concentration and superthermality of electron species and presence of electron beam affect Bohm velocity of positive ions. Also, it is observed that density distribution of the charged particles and sheath thickness increase in the presence of electron beam. In addition, it is found that with increasing the ion temperature, the sheath width and density distribution of the charged particles in the sheath area decrease.

Many observations have confirmed the existence of both cool and hot electron populations with a non-Maxwell distribution function. For example, analysing data from Cassini spacecraft by Schippers et al. showed a best fit for both cool and hot electron velocity distributions with kappa distributions with low values of $\kappa$. Although the empirical evidence for the kappa distributions function dates back to the 1960s, its statistical justification occurred in the 1980s based on maximizing the q-entropy. Kappa distributions may be produced in space and astrophysical plasmas by various mechanisms such as superstatistics, the effect of shock waves, pump acceleration mechanism, and etc.

The power law one dimensional kappa velocity distribution is given by:

$$f_\kappa (v) = \left( \frac{n_0 \Gamma(\kappa)}{\Gamma(\kappa - 1/2)(\pi\kappa\theta^2)^{1/2}} \right) \left( 1 + \frac{v^2}{\kappa\theta^2} \right)^{-\kappa},$$

(1)

where $f_\kappa (v)$ is distribution function, $n_0$ is the equilibrium density, $\Gamma$ is the standard gamma function, $\kappa$ is the parameter of the superthermal particles and is known as spectral index, $\theta = \left( \frac{2\kappa - 3k_B T}{\kappa m_p} \right)^{1/2}$ denotes the effective thermal speed or most probable speed of the particles of mass $m_p$ and temperature $T$ and, $k_B$ is the Boltzmann constant. From $\theta$, we note that a well-defined value of effective thermal speed requires $\kappa > 3/2$.

Integrating the kappa distribution over velocity space, the electron number density can be written as follows:

$$n_e(\phi) = n_{0e} \left( 1 - \frac{e\phi}{(\kappa_e - 3/2)k_B T_e} \right)^{-\kappa_e + 1/2},$$

(2)

where $n_{0e}$ and $T_e$ are the equilibrium number density and temperature of electrons, respectively, $\kappa_e > 3/2$ is the spectral index and $\phi$ is the electrostatic potential.

During the last decade, kappa distribution has been used by many authors for various research purposes such as occurrence and propagation of electron-acoustic and ion-acoustic waves, dust-ion-acoustic shock waves and electromagnetic cyclotron instabilities in plasma with two kappa-distributed electron species. On the other hand, sheath formation in plasmas is one of the fundamental issues in plasma physics. Recently, there has been a lot of interest in the study of plasma sheath formation in the presence of non-Maxwellian electrons. As a result of these studies, it has been found that depending on the degree of superthermality of electrons, the
velocity of the ions at the sheath edge could be more or less than their velocity in the corresponding plasma with Maxwellian electrons. Therefore, it can be concluded that the presence of non-Maxwellian electrons makes a significant contribution to the variation of plasma sheath properties.

Since electrons have a significant effect on the plasma sheath layer, the study of this layer in the presence of an electron beam, anisotropic distributed electrons, etc. is very important. For this reason, the effect of presence of an electron beam on plasma sheath properties has been the subject of many studies. In 1990, Ingram and Braithwaite have used a quasi-neutral model to examine the nature of the plasma-sheath boundary in the presence of a high-energy monoenergetic electrons and Maxwellian electrons. The behavior of the plasma sheath boundary in the presence of an electron beam with finite temperature has been examined by Bradley and Amemiya. Pal et al. have experimentally investigated the effect of an electron beam on plasma characteristics and boundary sheath in a dc discharge plasma. Demidov et al. have discussed the sheath formation in plasmas with low density nonlocal fast electrons. Sharifian and Shokri have studied the influence of the presence of a fast monoenergetic electron beam on the temporal evolution of the ion–matrix sheath in an electropositive plasma with Maxwellian electrons. Gyergyek et al. have presented a fluid model of the sheath formation in front of a planar electron emitting electrode immersed in a plasma with Maxwellian electrons and a monoenergetic electron beam. Chekour et al. have investigated the Bohm criterion in an electronegative dusty plasmas in the presence of a fast monoenergetic electrons and Maxwellian distributed electrons and negative ions. Ou et al. have presented a sheath model to investigate the plasma-wall interaction in the fusion boundary layer in the presence of a fast monoenergetic electron beam.

In 2012, Badman et al. reported the existence of electron beams at Saturn based on Cassini observations. Also, as mentioned earlier, the analysis of Cassini data revealed the existence of two species of electrons (cool and hot) with a kappa distribution function in Saturn. These facts encourage us to study the structure of plasma sheath in a plasma consisting of fluid positive ions and two kappa-distributed electron species with different effective temperatures in the presence of a monoenergetic electron beam which, to our knowledge, has not yet been investigated. Therefore, the main focus of the current work is to discuss about the effect of presence of an electron beam on the sheath properties of a multi-component electropositive plasma containing ions and two kappa-distributed electron species with different effective temperatures. In order to do this, we first derive the sheath formation criterion for the mentioned plasma system and then we investigate the behavior of the plasma sheath characteristics in the presence of an electron beam. The results of the present work should be useful in understanding the properties of plasma sheath in space plasmas, such as Saturn’s magnetosphere and Pulsar magnetosphere where the combined presence of electron beams and excess superthermal electrons may be encountered.

This work is arranged in four sections including the introduction as the first section. In “Model and basic equations” section, the basic equations of the considered plasma are presented. Bohm sheath criterion and the plasma sheath characteristics are investigated in “Results and discussion” section, and finally, our brief conclusion is presented in “Conclusion” section.

Model and basic equations

Let us consider an electropositive plasma containing warm fluid ions and two kappa-distributed electron species with two different effective temperatures in the presence of an electron beam. Assuming the x axis to be normal to the wall and the physical parameters change only along the x (see Fig. 1), the fluid equations for the warm, collisionless ions are described as

$$\frac{d}{dx}(n_i v_i) = 0,$$  \hspace{1cm} \text{(3)}

$$\frac{m_i v_i}{dx} d\nu_i = -e \frac{d\varphi}{dx} - \frac{1}{n_i} \frac{dp_i}{dx},$$  \hspace{1cm} \text{(4)}
where \( e \) is the unit electric charge, \( \psi \) is the electrostatic potential and \( n_i, v_i, m_i \) and \( p_i \) are density, velocity, mass and pressure of positive ions, respectively. The positive ion pressure \( p_i \) is related to the ion density as follows:

\[
p_i = k_B T_i n_i^{\gamma_i} / n_{i0}^{\gamma_i - 1},
\]

where \( T_i \) is the ion temperature, \( n_{i0} \) is the ion density at the sheath edge (\( x = 0 \)) and \( \gamma_i = 3, 2, 5/3 \) for unidimensional, bidimensional, or tridimensional adiabatic flow, respectively.

The number densities of kappa-distributed electron species are expressed as:

\[
n_j = n_0 \left( 1 - \frac{e \psi}{k_B T_{ej} (k_j - 3/2)} \right)^{-k_j + 1/2},
\]

where \( j = c, h \) refer to cool and hot electron species and \( T_{ej} \) and \( n_{j0} \) are the temperature and density of electron species \( j \) at the sheath edge, and \( k_j > 3/2 \) is the spectral index of each electron species.

Similar to Ref.\(^{37}\), we assume that there is an isotropic, monoenergetic electron beam with a unidirectional velocity distribution in the plasma medium. Therefore, in addition to the fact that the velocities of all electrons are the same (called \( v_b \)), the directions of their velocities are uniformly distributed in space. As a result, the velocity distribution of the monoenergetic electrons is defined as follows:

\[
f_b (\nu) = \frac{n_{0b}}{4 \pi v_b^2} \delta (\nu - v_b),
\]

where \( n_{0b} \) is the density of the beam electrons far from the plane wall. Integrating \( f_b (\nu) \) over the velocity space, the density of the beam electrons is written as follows\(^{37}\):

\[
n_b (x) = \int_0^\infty f_b (\nu) d^3 \nu = \frac{n_{0b}}{2} \left( 1 - \sqrt{-\frac{2e \psi (x)}{m_e v_b^2}} \right).
\]

From relation (8), it is seen that \( n_b (x) \) decreases by increasing the electrostatic potential of the sheath layer. Moreover, to avoid complicating the presented fluid model, the effects of the collision of beam electrons in the presheath layer have been neglected.

Finally, the potential in the sheath \( \psi \) is determined by the Poisson equation:

\[
\frac{d^2 \psi}{dx^2} = \frac{e}{\varepsilon_0} (n_c + n_h + n_b - n_i),
\]

where \( \varepsilon_0 \) is the electric permittivity of free space. The quasineutrality condition at the sheath edge of such a multi-component plasma is \( n_{0b} = n_{0c} + n_{0h} + n_{0b} \).

We normalize the physical quantities in (3)–(6), (8) and (9) with the following normalized variables:

\[
N_i = n_i / n_{i0}, \quad N_j = n_j / n_{j0}, \quad N_h = n_h / n_{h0}, \quad \sigma_j = T_{ej} / T_{ec}, \quad x = x / \lambda_D, \quad u_i = v_i / c_i, \quad u_c = v_c / c_c, \quad \delta = n_{0b} / n_{0c}, \quad \delta_h = n_{0h} / n_{0c}, \quad \sigma_i = T_i / T_{ec}, \quad \phi = -e \psi / k_B T_{ec}, \quad \delta_i = n_{0i} / n_{0c}
\]

where \( j = c, h, \lambda_D = (\varepsilon_0 k_B T_{ec} / e^2 n_{0c})^{1/2} \) and \( c_c = (k_B T_{ec} / m_c)^{1/2} \).

Using the above mentioned normalized variables for a unidimensional adiabatic ion flow, the basic equations of our fluid model can be written as follows:

\[
\frac{d(N_{i0})}{d\xi} = 0,
\]

\[
\frac{dN_i}{d\xi} - \frac{3}{2} \left( \frac{\sigma_i}{\delta_i^2} \right) \frac{dN_i^2}{d\xi},
\]

\[
\frac{d^2 \phi}{d\xi^2} = \frac{1}{\delta_i} \left[ N_i - \left( 1 + \frac{\phi}{\sigma_c (k_c - 2)} \right)^{-k_c + 1/2} - \delta \left( 1 + \frac{\phi}{\sigma_h (k_h - 2)} \right)^{-k_h + 1/2} - \frac{\delta_h}{2} \left( 1 - \sqrt{\frac{2\phi}{mu_c^2}} \right) \right],
\]

where \( m = m_c / m_i \).

**Results and discussion**

In this section, we are going to analytically derive the sheath formation criterion for an electropositive plasma consisting of singly charged positive ions and two kappa-distributed electron species in the presence of a monoenergetic electron beam. Without knowing this criterion, it will not be possible to use the normalized basic equations to numerically study the sheath structure in the considered plasma.

Multiplying the normalized Poisson equation by \( (d\phi / d\xi) \) and integrating, we obtain
\[
\left( \frac{d\phi}{d\xi} \right)^2 = \left( \frac{d\phi}{d\xi} \right)_{\xi=0}^2 - 2S(\phi, u_{0i}),
\]

where

\[
S(\phi, u_{0i}) = -u_{0i} \int_{\phi_0}^{\phi} \eta \frac{d\phi}{u_i} \left( 1 + \frac{\phi}{\kappa} \left( \frac{1}{\kappa - \frac{3}{2}} \right) \right)^{-\kappa_i + 3/2} \phi \sigma_c \left( \frac{1 + \frac{\phi}{\kappa} \left( \frac{1}{\kappa - \frac{3}{2}} \right)}{\sigma_c \left( \frac{1 + \frac{\phi}{\kappa} \left( \frac{1}{\kappa - \frac{3}{2}} \right)}{\delta_0} \right)} \right)^{-\kappa_i + 3/2} \phi \delta_0 + \left( \frac{\delta_b \phi - \delta_b \phi}{3\delta_0} \sqrt{\frac{2}{mu_i^2} \phi^{3/2}} \right)_{\phi_0}^\phi,
\]

is the Sagdeev potential and \(\phi_0\) and \(u_{0i}\) are the normalized electrostatic potential and normalized velocity of the positive ions at the sheath edge (\(\xi = 0\)), respectively.

From (14), it is found that \(S(\phi_0, u_{0i}) = 0\) and \(dS(\phi_0, u_{0i})/d\phi = 0\). Therefore, the sheath edge is an extremum point for the Sagdeev potential. However, it should be mentioned here that the concept of sheath edge resulting from the Bohm criterion is suitable for collisionless plasmas\(^{4}\). Maximizing \(S\) in the sheath edge, we obtain

\[
\frac{\partial^2 S(\phi_0, u_{0i})}{\partial \phi^2} = \frac{1}{u_{0i}^2 - 3\sigma_i} - \frac{\delta_b}{4\delta_0} \sqrt{\frac{2}{mu_i^2} \phi_0} = \frac{1}{\delta_0} (A + B) \leq 0,
\]

or

\[
u_{0i} \geq \left( 3\sigma_i + \frac{\delta_i}{A + B + \frac{\delta_b}{4} \sqrt{\frac{2}{mu_i^2} \phi_0}} \right)^{1/2},
\]

where

\[
A = -\frac{1}{\sigma_c} \kappa \left( \frac{1}{\kappa - \frac{3}{2}} \right) \left( 1 + \frac{\phi_0}{\kappa c \left( \frac{1}{\kappa - \frac{3}{2}} \right)} \right)^{-\kappa_i + 1/2},
\]

and

\[
B = -\frac{\delta}{\sigma_h} \kappa \left( \frac{1}{\kappa - \frac{3}{2}} \right) \left( 1 + \frac{\phi_0}{\kappa c \left( \frac{1}{\kappa - \frac{3}{2}} \right)} \right)^{-\kappa_i + 1/2}.
\]

Using the normalized quasineutrality condition at the sheath edge, \((N_i + N_h + N_b = N_i)_{\phi=\phi_0}\), it is easily found that

\[
\delta_i = \left( 1 + \frac{\phi_0}{\sigma_c \left( \frac{1}{\kappa - \frac{3}{2}} \right)} \right)^{-\kappa_i + 1/2} + \delta \left( 1 + \frac{\phi_0}{\sigma_h \left( \frac{1}{\kappa - \frac{3}{2}} \right)} \right)^{-\kappa_i + 1/2} + \frac{\delta_b}{2} \left( 1 - \frac{2\phi_0}{mu_i^2} \right)^{1/2}.
\]

Relation (16) is the sheath formation criterion for an electropositive plasma consisting of warm positive ions, two species of kappa-distributed electrons and a monoenergetic electron beam. From (16), it is found that the Bohm velocity of positive ions at the sheath edge depends on different plasma parameters such as temperature and initial (unperturbed) density of ions and electron species, spectral index of each electron species, concentration and velocity of the electron beam and electrostatic potential at the sheath edge. Examining Eq. (16) for some specific cases can be useful. For example, ignoring the presence of electron beam and assuming \(\phi_0 = 0\), relation (16) leads to the ion velocities at the sheath edge of an electropositive plasma with two species of kappa-distributed electrons. Also, assuming \(\delta_0 = 0\) in the limit \(\kappa \rightarrow \infty\), the modified sheath criterion (Eq. (16)) reduces to \(u_{0i} \geq (1 + 3\sigma_i)^{1/2}\) for \(\phi_0 = 0\) which is the sheath criterion for an ion-electron plasma with warm ions and Maxwellian-distributed electrons. Finally, assuming that the ions are cold and the electrons are in thermal equilibrium \((\kappa \rightarrow \infty)\), Eq. (16) will lead to the well-known Bohm criterion \(u_{0i} \geq 1\) in the absence of the electron beam.

Using Eqs. (10)–(12) and (16), one can study the plasma sheath properties of an electropositive plasma consisting of warm fluid positive ions and two species of kappa-distributed electrons with different effective
temperature in the presence of a monoenergetic electron beam. Before doing so, we are going to investigate the behavior of the modified Bohm criterion with different plasma parameters.

Figure 2 shows variation of minimum velocity of the positive ions at the sheath edge $u_{0i}$ with the electron beam velocity $u_e$ and effective temperature ratio of hot-to-cool electron species $\sigma_h$ for $\phi_0 = 0.008$, $\kappa_c = 2.4$, $\kappa_h = 4$, $\sigma_i = 0.02$, $m = 1/1836$, $\delta = 0.2$ and $\delta_b = 0.001$. Hereafter, to prevent the divergence of the second term in (16), we assume that $\phi_0$ takes a nonzero but infinitesimal value in our numerical calculation. It is seen that an increase in beam velocity has a negligible effect on the ion velocities at the sheath edge. Also, similar to plasma with two nonextensive electron species\cite{27}, it is observed that $u_{0i}$ increases by increasing the effective temperature ratio of hot-to-cool electron species.

Variation of $u_{0i}$ with the spectral index of cool $\kappa_c$ and hot $\kappa_h$ electron species is shown in Fig. 3 for $\phi_0 = 0.008$, $\sigma_h = 10$, $\sigma_i = 0.02$, $\delta = 0.2$ and $\delta_b = 0.001$. Hereafter, to prevent the divergence of the second term in (16), we assume that $\phi_0$ takes a nonzero but infinitesimal value in our numerical calculation. It is seen that an increase in beam velocity has a negligible effect on the ion velocities at the sheath edge. Also, similar to plasma with two nonextensive electron species\cite{27}, it is observed that $u_{0i}$ increases by increasing the effective temperature ratio of hot-to-cool electron species.

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Effect of hot-to-cool electron initial densities $\delta$ and the beam density $\delta_b$ on the modified sheath formation criterion is indicated in Fig. 4 for $\kappa_c = 2.4$, $\kappa_h = 4$ and the other parameters of Fig. 3. One can see that any increase
Figure 4. Ion velocities at the sheath edge as a function of $\delta$ and $\delta_b$ for $\phi_0 = 0.008$, $\sigma_h = 10$, $\sigma_i = 0.02$, $m = 1/1836$, $\kappa_c = 2.4$ and $\kappa_h = 4$.

Figure 5. Spatial distribution of (a) normalized density of positive and negative particles and (b) normalized sheath potential for $\phi_0 = 0.008$, $\sigma_h = 10$, $\sigma_i = 0.02$, $m = 1/1836$, $\kappa_h = 4$, $\delta = 0.2$, $\delta_b = 0.002$ and different values of $\kappa_c$. 
in density of hot electron species leads to an increase in the Bohm velocity of positive ions but the beam density has a different effect on the behavior of $u^0_i$ and as can be seen, with increasing $\delta_b$, the Bohm velocity decreases.

Variation of density distribution of the charged particles ($N_i$ and $N^\sim = N_c + N_h + N_b$) and electrostatic potential of the sheath region with the spectral index of cool electron species ($\kappa_c$) is illustrated in Fig. 5a,b, respectively for $\phi_0 = 0.008$, $\sigma_h = 10$, $\sigma_i = 0.02$, $\delta = 0.2$, $\kappa_h = 4$ and $\delta_b = 0.002$. Here, due to the similarity of the results, we have ignored the study of variation in $N_i$, $N^\sim$ and $\phi$ with $\kappa_h$. Figure 5a shows density distribution of negative particles $N^\sim$ decrease with $\kappa_c$. Given the fact that the number of fast electrons decreases with increasing the spectral index of electron species (e.g., $\kappa_c$), the decrease in negatively charged particle densities is understandable. Normalized density distribution of positive ions also decreases with increasing $\kappa_c$. As explained, any increase in $\kappa_c$ causes a decrease in the population of energetic electrons. The result is a reduction in the number of electrons reaching the cathode and thus the number of ions. Moreover, Fig. 5b shows that the potential of the sheath also increases as $\kappa_c$ increases which is due to decrease of floating potential with $\kappa_c$.

Now, we are going to investigate the effect of presence of monoenergetic electron beam ($\delta_b$) on the properties of the sheath region of an ion-electron plasma with two species of kappa-distributed electrons. Figure 6 indicates variation of density distribution of positive ions with the beam density for the same parameters of Fig. 5. This figure shows that an increase in $\delta_b$ leads to an increase in density distribution of positive ions. Also, this figure shows variation of density distribution of negatively charged particles $N^\sim$ with different values of $\delta_b$ which also increases with increasing $\delta_b$. This can be explained by the fact that increasing the beam density increases the number of electrons reaching the cathode and thus increases the cathode potential. Therefore, the density of ions also increases with increasing $\delta_b$. These results are consistent with those reported in Refs. 35,36 for a plasma with Maxwellian electrons and monoenergetic electron beam.

Effect of beam density on the electrostatic potential of the sheath region is depicted in Fig. 7. From this figure one can see that an increase in the beam density causes a decrease in the potential of sheath. Also, it is seen that the sheath thickness increases by increasing $\delta_b$ which is in agreement with the result of Refs. 35,36. The reason

Figure 6. Spatial distribution of the normalized density of positive ions and negatively charged particles for $\kappa_c = 2.4$ and different values of $\delta_b$. The other parameters are the same with Fig. 5.

Figure 7. Spatial distribution of the normalized sheath potential for $\kappa_c = 2.4$ and different values of $\delta_b$. The other parameters are the same with Fig. 5.
behind this is that the potential of the wall increases by increasing $\delta_b$ and as a result the sheath width decreases. Therefore, it can be concluded from Figs. 6 and 7 that density distribution of negatively charged particles and positive ions in the sheath increases in the presence of electron beam while the potential of the sheath region decreases.

At the end of this section, we study the effect of temperature of positive ions on the sheath properties of an electropositive plasma containing monoenergetic electron beam, warm positive ions and two kappa-distributed electron species. Figure 8a,b show the effect of positive ion temperatures on the density distribution of cool and hot electron species in the sheath region for $\delta = 0.2$, $\delta_b = 0.005$, $\kappa_c = 2.4$, $\kappa_h = 4$, $\phi_0 = 0.008$, $\sigma_h = 10$ and different values of $\sigma_i$. It is seen that density of both electron species decreases as the ion temperature increases. In addition, Fig. 9 shows that the behavior of density distribution of the negatively charged particles for different values of $\sigma_i$ is similar to Fig. 8 and $N_-$ decreases by increasing the positive ion temperatures. It is also seen that the positive ion density $N_i$ is falling, too, which is due to the ion acceleration. A similar result has previously been reported in Refs. 43,45 for a warm Maxwellian plasma.

Finally, the effect of temperature of positive ions on the normalized potential of the sheath region of a plasma with two kappa-distributed electrons and electron beam is depicted in Fig. 10. It is seen that the normalized potential increases by increasing $\sigma_i$ in the presence of electron beam. Also, in confirming the results of previous works, it is seen that the sheath width decreases with increasing ion temperature. The reason for this is that as the temperature of the ions increases, $u_{i0}$ and consequently the velocity of the ions hits the cathode increases. As a result, the sheath thickness decreases by increasing $\sigma_i$. Therefore, it can be concluded that density distribution of the charged particles decreases by increasing $\sigma_i$ while electrostatic potential of the plasma sheath region increases.

**Conclusion**

Sheath formation criterion was investigated in a collisionless electropositive plasma with a monoenergetic electron beam by using the hydrodynamic equations. It was assumed that the plasma consisting of singly charged positive ions with finite temperature and two species of kappa-distributed electrons with different effective temperatures. Using Sagdeev potential approach, the modified Bohm criterion was derived and it was shown that the ion velocity at the sheath edge depends on various plasma parameters such as spectral index, concentration and temperature of each electron species, density and velocity of electron beam and concentration and temperature of...
the positive ions. Sheath region structure was also investigated in the presence of electron beam and warm positive ions. It was found that the presence of electron beam causes the density distribution of the charged particles in the sheath region as well as the sheath thickness to increase but the potential of the sheath to decrease. Also, it was shown that an increase in the temperature of positive ions leads to a decrease in the density distribution of ions and both cool and hot electron species while the potential of the sheath region increases. Moreover, it was found that an increase in spectral index of the electron species leads to a decrease (an increases) in density distribution of the charged particles (electric potential of the sheath region).

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References
1. Barbosa, D. D. & Kurth, W. S. On the generation of plasma waves in Saturn's inner magnetosphere. J. Geophys. Res. 98, 9351 (1993).
2. Sittler, E. C. J., Ogilvie, K. W. & Scudder, J. D. Survey of low energy plasma electrons in Saturn's magnetosphere: Voyagers 1 and 2. Geophys. Res. 88, 8847 (1983).
3. Young, D. T. et al. Composition and dynamics of plasma in Saturn's magnetosphere. Science 307, 1262 (2005).
4. Schippers, P. et al. Multi-instrument analysis of electron populations in Saturn's magnetosphere. J. Geophys. Res. 113, A07208 (2008).
5. Olbert, S. Summary of experimental results from MIT detector on IMP-1. In Physics of the Magnetosphere (eds Carovillano, R. L. et al.) 641 (Springer, 1968).
6. Vasyliunas, V. M. A survey of low-energy electrons in the evening sector of the magnetosphere with OGO 1 and OGO 3. J. Geophys. Res. 73, 2839 (1968).
7. Tsallis, C. Possible generalization of Boltzmann-Gibbs statistics. J. Stat. Phys. 52, 479 (1988).
8. Schwadron, N. et al. Superposition of stochastic processes and the resulting particle distributions. Astrophys. J. 713, 1386 (2010).
9. Hanel, R., Thurner, S. & Gell-Mann, M. Generalized entropies and the transformation group of superstatistics. *Proc. Natl. Acad. Sci. USA* **108**, 6390 (2011).
10. Zank, G. P. *et al.* Particle acceleration at perpendicular shock waves: Model and observations. *J. Geophys. Res.* **111**, A06108 (2006).
11. Fisk, L. A. & Gloeckler, G. The case for a common spectrum of particles accelerated in the heliosphere: Observations and theory. *J. Geophys. Res.* **119**, 8733 (2014).
12. Hellberg, M. A., Mace, R. L., Baluku, T. K., Kourakis, I. & Saini, N. S. Comment on “Mathematical and physical aspects of kappa velocity distribution”. *Phys. Plasmas* **16**, 094701 (2009).
13. Baluku, T. K., Hellberg, M. A. & Mace, R. L. Electron acoustic waves in double-kappa plasmas: Application to Saturn’s magnetosphere. *J. Geophys. Res.* **116**, A04227 (2011).
14. Baluku, T. K. & Hellberg, M. A. Ion acoustic solitons in a plasma with two-temperature kappa-distributed electrons. *Phys. Plasmas* **19**, 012106 (2012).
15. Singh, S. V., Devanandhan, S., Lakhina, G. S. & Bharuthram, R. Effect of ion temperature on ion-acoustic solitary waves in a magnetized plasma in presence of superthermal electrons. *Phys. Plasmas* **20**, 012306 (2013).
16. Alama, M. S., Masud, M. M. & Mamun, A. A. Effects of bi-kappa distributed electrons on dust-ion-acoustic shock waves in dusty superthermal plasma. *Chin. Phys. B* **22**, 115202 (2013).
17. Alama, M. S., Masud, M. M. & Mamun, A. A. Cylindrical and spherical dust-ion-acoustic modified gardner solitons in dusty plasmas with two-temperature superthermal electrons. *Plasma Phys. Rep.* **39**, 1013 (2013).
18. Alinejad, H., Mahdavi, M. & Shahmansouri, M. Modulation instability of ion-acoustic waves in a plasma with two-temperature kappa-distributed electrons. *Astrophys. Space Sci.* **352**, 571 (2014).
19. Saberian, E.E.-K.A. & Afari-Ghazi, M. Large amplitude dust-acoustic solitons in a plasma consisting of superthermal electron-positron pairs and dust particulates. *The Open Plasma Phys. J.* **8**, 8 (2015).
20. Lazar, M., Yoon, P. H. & Eliaison, B. Electromagnetic cyclotron instabilities in bi-Kappa distributed plasmas: A quasilinear approach. *Phys. Plasmas* **24**, 042110 (2017).
21. Ali Shan, S., Khan, D. & Saleem, H. Kappa distributed trapped electrons, drift wave instability and nonlinear structures in O-H plasma of ionosphere. *Eur. Phys. J. Plus* **135**, 580 (2020).
22. Guo, R. The electron acoustic waves in plasmas with two-kappa-distributed electrons at the same temperatures and immobile ions. *Phys. Plasmas* **28**(8), 082105. https://doi.org/10.1063/5.0057693 (2021).
23. Gougam, L. A. & Tribeche, M. Debye shielding in a nonextensive plasma. *Phys. Plasmas* **18**, 062102 (2011).
24. Liu, Y., Liu, S. Q. & Zhou, L. Bohm criterion in a dust plasma with nonextensive electrons and cold ions. *Phys. Plasmas* **20**, 043702 (2013).
25. Hatami, M. M. Nonextensive statistics and the sheath criterion in collisional plasmas. *Phys. Plasmas* **22**, 013508 (2015).
26. Safa, N. N., Ghomi, H. & Niknam, A. R. Effect of the q-nonextensive electron velocity distribution on a magnetized plasma sheath. *Phys. Plasmas* **21**, 082111 (2014).
27. Borgehain, D. R., Saharia, K. & Goswami, K. S. Behavior of plasma sheath with nonextensively distributed two-temperature electrons and isothermal ions. *Phys. Plasmas* **23**, 122113 (2016).
28. Hatami, M. M. & Tribeche, M. Sheath properties in two-temperature non-maxwellian electron plasmas. *IEEE Trans. Plasma Sci.* **46**, 668 (2018).
29. Arghand-Hesar, A., Esfandyari-Kalejah, A. & Akbari-Moghanjoughi, M. Effects of a monoenergetic electron beam on the sheath formation in a plasma with a q-nonextensive electron velocity distribution. *Phys. Plasmas* **24**, 063504 (2017).
30. Basnet, S., Patel, A. & Khanal, R. Electronegative magnetized plasma sheath properties in the presence of non-Maxwellian electrons with a homogeneous ion source. *Plasma Phys. Control. Fusion* **62**, 115011 (2020).
31. Moulick, R., Garg, A. & Kumar, M. Sheath formation in the presence of non-extensive electron distribution. *Concr. Plasma Phys.* **61**(8), 1–10. https://doi.org/10.1002/cppl.202100047 (2021).
32. Ingram, S. G. & Braitwaite, N. S. J. The plasma-sheath boundary with fast monoenergetic electrons. *J. Phys. D: Appl. Phys.* **23**, 1646 (1990).
33. Bradley, J. W. & Aminyari, H. Plasma-sheath boundary in plasmas containing thermal and beam electrons. *J. Phys. Soc. Jpn.* **63**, 3295 (1994).
34. Pal, A. R., Boruah, D., Bailung, H. & Chutia, J. Influence of low energy ion beam on sheath characteristics in plasma. *Phys. Lett. A* **305**, 419 (2002).
35. Demidov, V. I., DeJoseph, C. A. Jr. & Kudryavtsev, A. A. Anomalously high near-wall sheath potential drop in a plasma with nonlocal fast electrons. *Phys. Rev. Lett.* **95**, 215002 (2005).
36. Sharifian, M. & Shokri, B. Effects of fast monoenergetic electrons on the ion dynamics near the cathode in a pulsed direct current plasma sheath. *Phys. Plasmas* **15**, 033503 (2008).
37. Gygergky, T., Kovacic, J. & Cercek, M. Potential formation in front of an electron emitting electrode immersed in a plasma that contains a monoenergetic electron beam. *Phys. Plasmas* **17**, 083504 (2010).
38. Chekov, S., Tahraoui, A. & Zaham, B. Effects of fast monoenergetic electrons on the generalized Bohm criterion for electron-nega- tive dusty plasma. *Phys. Plasmas* **19**, 033502 (2012).
39. Ou, J., Zhao, X. & Gan, C. Plasma-sheath interaction in an electrostatic sheath of plasma containing a monoenergetic electron beam. *Phys. Plasmas* **23**, 043510 (2016).
40. Badman, S. V. *et al.* Cassini observations of ion and electron beams at Saturn and their relationship to infrared auroral arcs. *J. Geophys. Res.* **117**, A01211 (2012).
41. Kundu, S. K., Ghosh, D. K., Chatterjee, J. & Das, R. Shock waves in a dusty plasma with positive and negative dust, where electrons are superthermally distributed. *Bulg. J. Phys.* **38**, 409 (2011).
42. Valentini, H.-B. Sheath formation in low-pressure discharges. *Plasma Sour. Sci. Technol.* **9**, 574 (2000).
43. Das, G. C., Singha, B. & Chutia, J. Characteristic behavior of the sheath formation in thermal plasma. *Phys. Plasmas* **6**, 3685 (1999).
44. Bohm, D. *The characteristics of electrical discharges in magnetic fields*, edited by Gurthy A. & Wakerling R. K. (McGraw-Hill, 1949).
45. Ou, J. & Yang, J. Properties of a warm plasma collisional sheath in an oblique magnetic field. *Phys. Plasmas* **19**, 113504 (2012).
46. Fernandez Palop, J. I., Ballesteros, J., Hernandez, M. A., Morales Crespo, R. & Borrego del Pino, S. Sheath structure in electronegative plasmas with finite positive ion temperature. *J. Appl. Phys.* **95**, 4585 (2004).

**Author contributions**
M. M. Hatami proposed the concept, performed the analytical calculation as well as numerical simulations and analyzed numerical data.

**Competing interests**
The author declares no competing interests.

**Additional information**
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