Optimization Research on Ampacity of Underground High Voltage Cable Based on Interior Point Method

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Abstract. The conservative operation method which takes unified current-carrying capacity as maximum load current can’t make full use of the overall power transmission capacity of the cable. It’s not the optimal operation state for the cable cluster. In order to improve the transmission capacity of underground cables in cluster, this paper regards the maximum overall load current as the objective function and the temperature of any cables lower than maximum permissible temperature as constraint condition. The interior point method which is very effective for nonlinear problem is put forward to solve the extreme value of the problem and determine the optimal operating current of each loop. The results show that the optimal solutions obtained with the purposed method is able to increase the total load current about 5%. It greatly improves the economic performance of the cable cluster.

1. Introduction

Compared with overhead lines, electric power cables have many advantages. For example, power cables have high performance of high reliability, require no floor space, and don’t affect the appearance of the city. With the development of urban needs, the application of underground power cable in the urban power grid is growing. Ampacity is the important operating parameter of the power cable. It depends on the maximum operating temperature of the cable insulation. For cross-linked polyethylene (XLPE) insulated cables, the ampacity is corresponding to the steady working current of the cable when the temperature of core reaches 90\textdegree C. If the capacity is too large, the service life of the cable will be shortened and the operation reliability will be reduced. If the capacity is too small, the metal material of the cable core can’t be fully utilized, and the cable transmission capacity can’t be fully utilized [1]. Therefore, accurate calculation and evaluation of ampacity is the premise of safe and efficient operation of cable. Reasonable determination of cable ampacity has important engineering significance for short-term expansion and operation design. At present, the methods of determining the cable ampacity usually include three methods: analytic calculation, numerical calculation and experiment. Analytic calculation usually adopts IEC formula and NM method [2, 3, 4]. Numerical calculation includes finite element method, boundary element method, finite difference method, finite volume method and so on [5, 6, 7]. The analytical method is simple and convenient in calculation, it is suitable for solving relatively simple problems in geometry, such as homogeneous soil with uniform thermal conductivity. For complex cases, such as uneven soil heat conduction or backfill soil, correction factors are needed to correct the calculation. The numerical method is more suitable for...
analyzing complex cable system by programming. And it can be used to deal with complicated physical conditions. The calculation results are more accurate.

In actual operation, carrying capacity is an important reference for the cable group. When the load current of any loop in the cable group does not exceed the carrying capacity, the cable group can operate safely, but the power transmission capacity of the cable isn’t fully taped. In order to improve the utilization rate of the cable and achieve the maximum transmission capacity of existing cable line, the interior point method is used to compute the best current distribution of 110 kV single core cable group under the circumstance of each core cable temperature not exceeding 90 degrees and uniform soil conductivity.

2. IEC formula for ampacity of cable group
According to IEC60287, the ampacity of the cable group can be represented by the formula (1):

$$I_i = \sqrt{\frac{(\theta - \theta_c) - W_d [0.55_t + n_i (T_2 + T_3 + T_4)]}{R_i + n_i (1 + \lambda_1) R_2 + n_i (1 + \lambda_2) R_3 / (T_3 + T_4)}}$$

Where, $I_i$ is the load current of cable “$i$” in A, $\theta$ is the running temperature of cable “$i$” in °C, $\theta_c$ is the environmental temperature of cable laying soil in °C, $W_d$ is the dielectric loss per unit length of cable insulation of cable “$i$” in watt per meter, $R_i$ is the thermal resistance per unit length between single cable core and metal sheath in kelvin meter per watt, $T_2$ is the thermal resistance per unit length of lining which is between armor and metal sheath in kelvin meter per watt, $T_3$ is the thermal resistance per unit length of outer sheath in kelvin meter per watt, $T_4$ is the thermal resistance per unit length between single buried cable surface and surrounding media in kelvin meter per watt, $R_i$ is the AC resistance per unit of cable core when the temperature of cable “$i$” reaches $\theta_i$ in $\Omega/m$, $\lambda_1$, $\lambda_2$ are the ratio of the total metal sheath losses to the total conductor losses and the total armor losses to the total conductor losses respectively.

In the cable group, there is mutual heat influence between different cable circuits when their current values are different, which leads to superposition effect of temperature. Therefore, the thermal effect of other cables can be equivalent to the ambient temperature rise of the cable “$i$”. The relationship between the cable current and temperature can be formed as follows [8]:

$$I_i = \sqrt{\frac{(\theta - \theta_c) - W_d [0.55_t + n_i (T_2 + T_3 + T_4)] - \Delta \theta_{int}}{R_i + n_i (1 + \lambda_1) R_2 + n_i (1 + \lambda_2) R_3 / (T_3 + T_4)}}$$

In formula (2)

$$\Delta \theta_{int} = \sum_{j=1, j \neq i}^{n} n_j \left( R_j (1 + \lambda_1 + \lambda_2) + R_3 / 2 \pi \right) \rho s_{int} d_{ij} / d_{ij}$$

Where, $\Delta \theta_{int}$ is conductor temperature reduction factor due to the heating from the neighboring cables, $I_i$ is the current of cable “$j$” in A, $W_d$ is the dielectric loss per unit length of cable insulation of cable “$j$” in watt per meter, $R_j$ is the AC resistance per unit of cable core when the temperature of cable “$j$” reaches $\theta_j$ in $\Omega/m$, $n_j$ is the number of conductors loaded in a cable. $n_j = 1$ represents single core cable and $n_j = 3$ represents three core cable. $\lambda_1$, $\lambda_2$ are the ratio of the total metal sheath losses to the total conductor losses and the total armor losses to the total conductor losses respectively for cable “$j$”. $d_{ij}$ is the center distance between cable “$i$” and cable “$j$” in meter, $d_{ij}$ is the mirror distance between cable “$i$” and cable “$j$” in meter, $\rho s$ is the thermal resistance coefficient of cable laying soil in kelvin meter per watt.
3. Optimization of ampacity based on interior point method

3.1. Optimization Objective and Constraint Function

In this paper, the maximum transmission current of the cable group is regarded as the objective function, and the running temperature of each cable not exceeding 90 degrees is regarded as the constraint condition. The variable is the current of the cable and the corresponding running temperature. The resulting problem can be expressed as follows:

Objective function and constraints are giving in (4) and (5)

\[
\max f(x) = \sum_{i=1}^{n} I_i
\]

\[
\theta_i \leq 90; \quad i = 1, 2, \ldots, n
\]

Where, \( I_i \) and \( \theta_i \) are the current and running temperature of cable \( i \) respectively, \( n \) is the total number of cables in the system.

3.2. Equivalent Transformation

By formula (2), it can be seen that with the increase of cable core temperature, the current of cable increases, but the relationship between them is not simple linear, and can be further described as:

\[
I_i^2 = \frac{(\theta_i - \theta_c) - W_{dl} \left[ 0.5T_i + n_i (T_{S2} + T_{S3} + T_{A4}) \right] - \sum_{j=1}^{n} n_j W_{dj} \ln \left( \frac{d_j}{d_j} \right)}{R_{i} T_i + n_i R_i (1 + \lambda_i) T_{S2} + n_i R_i (1 + \lambda_i + \lambda_{2j}) (T_{S3} + T_{A4})} \sum_{j=1}^{n} \frac{n_j \left[ I_j^2 R_i (1 + \lambda_i + \lambda_{2j}) \right] \frac{\rho_i}{2 \pi} \ln \left( \frac{d_j}{d_j} \right)}{R_{i} T_i + n_i R_i (1 + \lambda_i) T_{S2} + n_i R_i (1 + \lambda_i + \lambda_{2j}) (T_{S3} + T_{A4})}
\]

Obviously, the difference value \((\theta_i - \theta_c)\) between the running temperature of cable \( i \) and the ambient temperature should be less than the difference value \((\theta_0 - \theta_c)\) between the highest rated temperature and the ambient temperature. Then formula (6) can be expressed as:

\[
I_i^2 \leq \frac{(90 - \theta_c) - W_{dl} \left[ 0.5T_i + n_i (T_{S2} + T_{S3} + T_{A4}) \right] - \sum_{j=1}^{n} n_j W_{dj} \ln \left( \frac{d_j}{d_j} \right)}{R_{i} T_i + n_i R_i (1 + \lambda_i) T_{S2} + n_i R_i (1 + \lambda_i + \lambda_{2j}) (T_{S3} + T_{A4})} \sum_{j=1}^{n} \frac{n_j \left[ R_i (1 + \lambda_i + \lambda_{2j}) \right] \frac{\rho_i}{2 \pi} \ln \left( \frac{d_j}{d_j} \right)}{R_{i} T_i + n_i R_i (1 + \lambda_i) T_{S2} + n_i R_i (1 + \lambda_i + \lambda_{2j}) (T_{S3} + T_{A4})}
\]

After finishing, the inequality constraints (7) can be reduced to

\[
\begin{align*}
\frac{a_{11}}{b_1} I_1^2 + \frac{a_{12}}{b_2} I_2^2 + \cdots + \frac{a_{1n}}{b_n} I_n^2 &\leq 1 \\
\frac{a_{21}}{b_1} I_1^2 + \frac{a_{22}}{b_2} I_2^2 + \cdots + \frac{a_{2n}}{b_n} I_n^2 &\leq 1 \\
\vdots &
\frac{a_{n1}}{b_1} I_1^2 + \frac{a_{n2}}{b_2} I_2^2 + \cdots + \frac{a_{nn}}{b_n} I_n^2 &\leq 1
\end{align*}
\]

Where,
After above transformation, the original problem can be equivalently transformed into the following nonlinear programming problem:

$$\max_{x} f(x) = \sum_{i=1}^{n} I_i$$

$$s.t. \quad g(x) \leq 0$$

Where,

$$g_i(x) = \frac{\partial}{\partial x_i} I_i^2 + \frac{\partial}{\partial x_i} I_k^2 + \ldots + \frac{\partial}{\partial x_n} I_n^2 - 1$$

$$x = [I_1, I_2, \ldots, I_n]^T$$

### 3.3 Calculation Procedure of Interior Point Method

Interior point method has the advantages of fast computation speed, good convergence and strong robustness. It can effectively deal with inequality constraints and other advantages. And it is widely applied to solve nonlinear programming problems. The essence of it is that the constrained programming problem is transformed into an unconstrained programming problem by Lagrange method. The KKT formula of the original problem is solved by Newton method and the optimal solution is iterated to the optimal solution [9].

The maximum value problem is transformed into a minimum value problem. The relaxation variable is introduced to transform the inequality constraint into an equality constraint, and the original problem can be expressed as:

$$\min_{x} f(x) = -\sum_{i=1}^{n} I_i$$

$$s.t. \quad g(x) + l = 0$$

$$l \geq 0$$

Construct a related Lagrange function:

$$L(x, y, l, \lambda) = f(x) - y^T (g(x) + l) - \lambda^T l$$

The KKT equations with perturbation factors $\mu > 0$ are derived from the first order optimality conditions:
\begin{align}
L_{x} &= \nabla f(x) - \nabla g(x) v = 0 \\
L_{y} &= g(x) + l = 0 \\
L_{\mu} &= L \gamma e - \mu e = 0 \\
(l; y; \gamma) &\geq 0
\end{align}
\tag{9}

Where, \( l \) is relaxation variable, \( v \) and \( \tau \) Lagrange multipliers, \((L, \gamma)\) are all diagonally matrices, and \( e = [1, \ldots, 1]^T \).

The perturbation method is applied to solve the perturbed KKT equations, and the modified equation is obtained:

\begin{align}
\begin{cases}
(\nabla^2 g(x)) v - \nabla^2 f(x) \Delta x &= L_{0} \\
\nabla g(x)^T \Delta x + \Delta \gamma &= -L_{y0} \\
\n\gamma \Delta x + L \Delta \gamma &= -L_{\mu}
\end{cases}
\tag{10}
\end{align}

In (10), \( L_{0}, L_{y0}, L_{\mu} \) denote the residuals of the perturbed KKT equations, \( \nabla^2 f(x) \) and \( \nabla^2 g(x) \) are Hessian matrices of \( f(x) \) and \( g(x) \).

The proposed interior point method algorithm may be summarized as follows:

Step 0: Initialization; set initial value of an iterative counter \( k = 0 \), maximum iterations \( k_{\text{max}} = 50 \), centering parameter \( \sigma \in (0, 1] \), and calculation tolerance \( \epsilon = 10^{-6} \). Choose relaxation variable \( l > 0 \) and dual variable \( y > 0 \).

While \( k < k_{\text{max}} \) DO:

Step 1: compute complementary gap \( C_{\text{gap}} = \mu^T v \):

If gap \( (C_{\text{gap}} \leq \epsilon) \), then stop computation and output optimal solution.

Step 2: compute the perturbed factor \( \mu = \sigma \frac{C_{\text{gap}}}{v} \)

Step 3: solve the correction equation (10), and obtain the correction of variables

\( \Delta v^k = [\Delta x, \Delta \gamma, \Delta \mu]^T \)

Step 4: determine the maximum step size of the original and dual variables

\( \alpha_p = 0.9995 \min \left\{ \frac{L_{0}}{L_{y}}, 1 \right\}, \alpha_d = 0.9995 \min \left\{ \frac{L_{y0}}{L_{y}}, 1 \right\} \)

Step 5: update the primal and dual variables by

\[
\begin{bmatrix}
x^{(k+1)} \\
y^{(k+1)} \\
l^{(k+1)}
\end{bmatrix} =
\begin{bmatrix}
x^{(k)} \\
y^{(k)} \\
l^{(k)}
\end{bmatrix} +
\begin{bmatrix}
\alpha_p \Delta x \\
\alpha_d \Delta y \\
\alpha_p \Delta \mu
\end{bmatrix}
\]

Step 6: set \( k = k + 1 \)

END DO

Step 7: Print “Computation does not converge!” and stop.

4. Calculation result analysis

Take the YJLW03 64/100 kV parallel buried single core XLPE cable which is widely used in Shanghai area as an example, an illustration of the sequence corresponding to cable configuration is shown in Fig. 1 [10].
Soil temperature is 20 °C, and soil thermal resistance coefficient equals 1. Each loop is vertically arranged from top to bottom by A, B and C, and the subscript of the letter indicates the loop number. The cross sectional area of the single core XLPE cable is 400 mm², and the structural parameters are shown in Table 1.

### Table 1. Parameters of 110kV single core cables

| Structure                  | Diameter /mm | Outer diameter /mm | Thickness /mm | Conduction coefficient /W·(m·K)^{-1} |
|----------------------------|--------------|--------------------|---------------|--------------------------------------|
| conductor                 | 23.8         | —                  | —             | 400                                  |
| inner semiconductor        | —            | —                  | 1.3           | 1                                    |
| insulation layer           | —            | —                  | 17.5          | 1                                    |
| outer semiconductor        | —            | —                  | 1.2           | 0.2                                  |
| water barrier layer        | —            | —                  | 5.0           | 1                                    |
| aluminum bellows           | —            | —                  | 2.0           | 238                                  |
| outer sheath               | —            | —                  | 4.0           | 0.17                                 |
| cable                      | —            | 85.8               | —             | —                                    |

The current value and the core temperature optimized by the interior point method, and the ampacity and the core temperature of cables calculated by the IEC formula method are shown in Table 2.

### Table 2. Comparison of ampacities and optimal current and cables core temperature of 7 loops

| Loop number | IEC formula method | Interior point method |
|-------------|--------------------|-----------------------|
|             | Cable core temperature /C | Capacity | Cable core temperature /C | Optimized current |
|             | A phase | B phase | C phase | A phase | B phase | C phase |
| 1           | 72.0     | 76.7    | 76.2    | 84.0     | 90.0    | 89.0    | 327.4A |
| 2           | 79.0     | 84.3    | 83.2    | 84.0     | 90.0    | 88.6    | 292.1A |
| 3           | 82.8     | 88.4    | 87.1    | 84.0     | 90.0    | 88.5    | 276.7A |
| 4           | 84.0     | 90.0    | 88.4    | 84.0     | 90.0    | 88.5    | 272.3A |
| 5           | 82.8     | 88.4    | 87.1    | 84.0     | 90.0    | 88.5    | 276.7A |
| 6           | 79.0     | 84.3    | 83.2    | 84.0     | 90.0    | 88.6    | 292.1A |
| 7           | 72.0     | 76.7    | 76.2    | 84.0     | 90.0    | 89.0    | 327.4A |
| Total number of cable group/A | 5918 | 6195 |
As shown in Table 2, when the cable groups operate under current capacity, the fourth loop B cable located at the center of the cable group has the highest cable temperature, reaching a maximum operating temperature of 90 °C. In the whole cable group, the cable core temperature from high to low is B phase, C phase and A phase. The core temperature of each cable in the cable group is symmetrical distribution with the fourth loop which is the center, the maximum cable core temperature difference between the 7 loops cable groups is 18 °C. The average temperature of the cable core is 82 °C.

When each loop cable operates under the unequal current optimized by the interior point method, the B phase of the 7 loops cable all reach the highest running temperature of 90 °C. The maximum temperature difference of the 7 loops cable core is reduced to 6 °C, and the average temperature of the cable core of the 7 loops is increased to 87.5 °C. The higher the overall average temperature of the cable core, the higher the utilization ratio of the cable, and the better the overall transport capacity of the cable group. The total current of the whole cable group is increased by 4.68% compared with the current carrying capacity when the 7 loops cable operate with the optimized current. In addition, comparing the optimal running current of each loop, it is found that the current of the first loop and the seventh loop near the outside of the cable group is the largest. The running current of the loop closed to the center is smaller than the current of the loop which locate outside of the cable group, this is because the loop near the outside has better heat dissipation condition, but the heat dissipation condition is poor in the middle of the cable group.

5. Conclusion
In this paper, the interior point method is used to solve the optimal operation of HV cable group. Through the calculation of the cable group with equal spacing as an example, the following conclusions are obtained:

1. When the cable group runs under current capacity, the mutual heat effect between cables makes the temperature in the middle of the cable group higher and the temperature of edge cable lower. The bottleneck effect of the intermediate cable leads to the failure of maximize the transmission capacity of the whole cable group.
2. When the cable group runs under optimized current, the current of the loop near the outside is larger because of the good heat dissipation condition. The average temperature of the cable core increases to 87.5 °C, and the overall current delivery capacity of the cable cluster is increased by 4.68%.

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