Gravitational form factors of the neutrino

K. L. Ng*

Department of Physics, National Taiwan University, Taipei 115, Taiwan

ABSTRACT

The gravitational properties of the neutrino is studied in the weak field approximation. By imposing the hermiticity condition, CPT and CP invariance on the energy-momentum tensor matrix element, we shown that the allowed gravitational form factors for Dirac and Majorana neutrinos are very different. In a CPT and CP invariant theory, the energy-momentum tensor for a Dirac neutrino of the same specie is characterized by four gravitational form factors. As a result of CPT invariance, the energy-momentum tensor for a Majorana neutrino of the same specie has five form factors. In a CP invariant theory, if the initial and final Majorana neutrinos have the same (opposite) CP parity, then only tensor (pseudo-tensor) type transition are allowed.

PACS numbers: 11.30.Er, 14.60.Gh

* Present address: Institute of Physics, Academia Sinica, Nankang, Taipei 115, Taiwan.
1. Introduction

If a neutrino has mass, then the question of whether the neutrino is a Dirac or Majorana type particle arise naturally. This is because the neutrino may be its own anti-particle (Majorana particle). The difference between a Dirac and Majorana neutrino is clearly exhibited in the neutral current interaction process [1], observation of neutrinoless double beta decay, and in their electromagnetic properties [2,3]. For example, a spin 1/2 Majorana neutrino can only have the anapole moment form factor if CPT invariance holds. This result was generalized to an arbitrary half integral spin Majorana fermion in Ref. [4], and an arbitrary spin Majorana fermion in Ref. [5].

However, there are relatively few discussions on the gravitational properties of a spin 1/2 fermion [6, 7]. In this paper, we extend their work by performing a complete study of the gravitational properties of the neutrino. In section 2, we present a general analysis of the energy-momentum tensor $\theta_{\alpha\beta}$ matrix element between two spin 1/2 neutrinos. Using the Dirac equation, the symmetric properties of $\theta_{\alpha\beta}$ and the energy-momentum conservation condition, we arrive at the most general expression for the gravitational form factors of the neutrino. By imposing the hermiticity condition, CPT and CP invariance on the $\theta_{\alpha\beta}$ matrix element, we obtained certain conditions on the gravitational form factors for the neutrino. We summarize the results in section 3.
2. Gravitational form factors of the neutrino

In this section we study the allowed form of the couplings for the energy-momentum tensor $\theta_{\alpha\beta}$ matrix element between two neutrino states. We carry out the analysis in the weak field approximation,

$$g_{\alpha\beta} = \eta_{\alpha\beta} + \kappa h_{\alpha\beta} \quad (2.1)$$

where $\eta_{\alpha\beta}$ is the flat space-time metric, $h_{\alpha\beta}$ is the graviton field, and $\kappa = 32\pi G$. In our paper we closely follow the notations used in Ref.

A. General analysis

Consider the invariant amplitude for the process $\nu_i \rightarrow \nu_f + g$, where $\nu_i$ and $\nu_j$ are two Dirac neutrinos with masses $m_i$ and $m_j$ ($m_i > m_f$) and $g$ is the graviton (virtual or real). The transition amplitude for this process is given by

$$< \nu_f | \theta_{\alpha\beta} | \nu_i > = \bar{u}(p_f) (\Gamma_{\alpha\beta})_{fi} u(p_i) \quad (2.2)$$

where $| \nu_i >$ and $< \nu_f |$ are the initial and final neutrino states respectively, and $(\Gamma_{\alpha\beta})_{fi}$ is the dressed vertex function that characterizes the above invariant amplitude.

Lorentz invariance implies that the vertex function in general can have twenty-four types of coupling: twelve tensor types and twelve pseudo-tensor types. The
twelve possible tensor types of coupling have the following forms: \( \eta_{\alpha\beta} \), \( q_{\alpha}q_{\beta} \), \( \{qP\}_{\alpha\beta} \), \( \{q\gamma\}_{\alpha\beta} \), \( P_{\alpha}P_{\beta} \), \( \{P\gamma\}_{\alpha\beta} \), \( \{\sigma_{\alpha\mu}q_{\mu}\}_{\alpha\beta} = \{\sigma qq\}_{\alpha\beta} \), \( \{\sigma qq\gamma\}_{\alpha\beta} \), \( \{\sigma P\gamma\}_{\alpha\beta} \), \( \{\sigma P\gamma\}_{\alpha\beta} \), where we have suppressed the Lorentz indicies, \( \{\} \) denote symmetrization over the indices \( \alpha \) and \( \beta \), \( q = p_{f} - p_{i} \), \( P = p_{f} + p_{i} \), and \( \sigma = \sigma_{\alpha\mu} = \frac{i}{2}[\gamma_{\alpha}, \gamma_{\mu}] \). The pseudo-tensor types of coupling are obtained by the addition of a \( \gamma_{5} \) factor.

Using the Dirac equation, \((\gamma_{\mu}p^{\mu} - m)u = 0\), one obtains identities which relate the various types of coupling (like the Gordon decomposition relation), and hence reduces the number of independent couplings. We collect these relations in the appendix. Thus, the energy-momentum matrix element between two Dirac neutrino states may be written as

\[
< \nu_{f}(p_{f})|\theta_{\alpha\beta}|\nu_{i}(p_{i})> = \overline{u}(p_{f})\left[T_{1fi}\eta_{\alpha\beta} + T_{2fi}q_{\alpha}q_{\beta} + T_{3fi}P_{\alpha}P_{\beta}ight. \\
+ T_{4fi}\{qP\}_{\alpha\beta} + T_{5fi}\{\sigma qq\}_{\alpha\beta} + T_{6fi}\{\sigma qP\}_{\alpha\beta} \\
+ T_{4fi}\{\gamma_{5}q_{\gamma}\}_{\alpha\beta} + T_{5fi}\{\gamma_{5}\sigma q_{\gamma}\}_{\alpha\beta} + T_{6fi}\{\gamma_{5}\sigma qP\}_{\alpha\beta}
\] \tag{2.3}

where \( T = T(q^{2}, m_{i}, m_{f}) \) and \( P = P(q^{2}, m_{i}, m_{f}) \) are the tensor and pseudo-tensor types form factors respectively.

Conservation of the energy-momentum tensor \((q^{2}\theta_{\alpha\beta} = 0)\) implies the following
relations among the form factors,

\[ T_1 + T_2 q^2 + \delta m_{f_i}^2 T_4 = 0 \] (2.4)

\[ \delta m_{f_i}^2 T_3 + q^2 T_4 = 0 \] (2.5)

\[ q^2 T_5 + \delta m_{f_i}^2 T_6 = 0 \] (2.6)

\[ P_1 + P_2 q^2 - M_{i_f} P_3 = 0 \] (2.7)

\[ i(q^2 P_5 + \delta m_{f_i}^2 P_6) - M_{i_f} P_4 = 0 \] (2.8)

\[ q^2 P_3 + 2\delta m_{f_i}^2 P_4 = 0 \] (2.9)

where \( M_{i_f} = m_i + m_f \), and \( \delta m_{f_i}^2 = m_f^2 - m_i^2 \). Lorentz invariance and energy-momentum conservation imply that for \( q^2 \neq 0 \), the general form for the energy-
momentum matrix element between two neutrinos states is given by

\[<\nu_f(p_f)|\theta_{\alpha\beta}|\nu_i(p_i)> = \overline{\nu}(p_f) \left[ T_{1fi}(\eta_{\alpha\beta} - \frac{1}{\delta m_{fi}^2} \{gP\}_{\alpha\beta} + \frac{q^2}{(\delta m_{fi}^2)^2} P_{\alpha\beta}) \right. \]

\[+ T_{2fi}\left( q_{\alpha\beta} - \frac{q^2}{\delta m_{fi}^2} \{qP\}_{\alpha\beta} + \frac{q^4}{(\delta m_{fi}^2)^2} P_{\alpha\beta} \right) \]

\[+ T_{6fi}\left( \{\sigma qP\}_{\alpha\beta} - \frac{\delta m_{fi}^2}{q^2} \{\sigma qq\}_{\alpha\beta} \right) \]

\[+ P_{1fi}\gamma_5 (\eta_{\alpha\beta} + \frac{1}{M_{fi}} \{q\gamma\}_{\alpha\beta} - \frac{q^2}{2\delta m_{fi}^2 M_{fi}} \{P\gamma\}_{\alpha\beta} + \frac{i}{2\delta m_{fi}^2} \{\sigma qq\}_{\alpha\beta}) \]

\[+ P_{2fi}\gamma_5 \left( q_{\alpha\beta} + \frac{q^2}{M_{fi}} \{q\gamma\}_{\alpha\beta} - \frac{q^4}{2\delta m_{fi}^2 M_{fi}} \{P\gamma\}_{\alpha\beta} + \frac{i q^2}{2\delta m_{fi}^2} \{\sigma qq\}_{\alpha\beta} \right) \]

\[+ P_{6fi}\gamma_5 \left( \{\sigma qP\}_{\alpha\beta} - \frac{\delta m_{fi}^2}{q^2} \{\sigma qq\}_{\alpha\beta} \right) \right] u(p_i) \]

(2.10)

For the same neutrino flavor \(m_i = m_f\), the solutions for eq.(2.4) to eq.(2.9) are

\[T_4 = T_5 = 0, T_2 = -\frac{T_1}{q^2}, P_3 = 0, P_2 = -\frac{P_1}{q^2}\] and \(P_3 = -\frac{2m_i P_1}{q^2}\).

Thus the energy-momentum matrix element is reduced to

\[<\nu_f(p'_f)|\theta_{\alpha\beta}|\nu_i(p_i)> = \overline{\nu}(p'_f) \left[ T_{1fi}(\eta_{\alpha\beta} - \frac{q_{\alpha\beta}}{q^2}) + T_{3fi}P_{\alpha\beta} \right. \]

\[+ T_{6fi}\{\sigma qP\}_{\alpha\beta} + P_{1fi}\gamma_5 (\eta_{\alpha\beta} - \frac{q_{\alpha\beta}}{q^2}) \]

\[+ P_{4fi}\gamma_5 \left( (\gamma - \frac{\gamma_{\mu}\gamma^\mu}{q^2}) P\right)_{\alpha\beta} + P_{6fi}\gamma_5 \{\sigma qP\}_{\alpha\beta} \right] u(p_i) \]

(2.11)

This result agrees with Ref. [6, 7] except the \(P_4\) term. In analogy to the electromagnetic form factors, \(T_6\) is called the anomalous gravitational magnetic moment form factor, \(P_4\) the gravitational anapole moment form factor, and \(P_6\) the gravitational dipole moment form factor [6].
B. Gravitational form factors of a Dirac neutrino

The energy-momentum tensor $\theta_{\alpha\beta}$ is proportional to $p_\alpha p_\beta$ [8], where $p_\alpha = (ip_0, \mathbf{p})$, whereas the hermiticity of the energy-momentum tensor operator is given by, $\theta^\dagger_{\alpha\beta} = \eta_\alpha \eta_\beta \theta_{\alpha\beta}$. The hermiticity condition implies

$$<\nu_f(p_f)|\theta_{\alpha\beta}|\nu_i(p_i)>^\dagger = \eta_\alpha \eta_\beta <\nu_i(p_i)|\theta_{\alpha\beta}|\nu_f(p_f)>$$

(2.12)

where $\eta_\alpha = (-1, 1, 1, 1)$. As a result of hermiticity, we have

$$\gamma_0 (\Gamma_{\alpha\beta})^\dagger_{fi} \gamma_0 = \eta_\alpha \eta_\beta (\Gamma_{\alpha\beta})_{if}.$$  

(2.13)

This implies the following relations among the gravitational form factors,

$$(T_1, T_2, T_6, P_1, P_2, P_6)_{f_i}^* = (T_1, T_2, -T_6, -P_1, -P_2, P_6)_{i_f}$$  

(2.14)

and

$$(T_1, T_3, T_6, P_1, P_4, P_6)_{ii}^* = (T_1, T_3, -T_6, -P_1, P_4, P_6)_{ii}. $$

(2.15)

For the off-diagonal case, $\nu_f \neq \nu_i$, hermiticity does not put any restriction on the form factors. For the diagonal case, hermiticity requires that all the form factor are real except for $T_6$ and $P_1$. 

7
Under the CPT transformation, $\theta_{\alpha\beta} \xrightarrow{\text{CPT}} \theta_{\alpha\beta}$ and

$$\text{CPT} \langle \nu_f(p_f) | \theta_{\alpha\beta} | \nu_i(p_i) \rangle >_{\text{CPT}} = \langle \nu_i(p_i) | \theta_{\alpha\beta} | \nu_f(p_f) \rangle >. \quad (2.16)$$

In terms of the Dirac spinor, the left-handed side of eq.(2.16) can be written as

$$\text{CPT} \langle \nu_f(p_f) | \theta_{\alpha\beta} | \nu_i(p_i) \rangle >_{\text{CPT}} = \bar{u}_{\text{CPT}}(-p_i)(\overline{\Gamma}_{\alpha\beta})_{fi}u_{\text{CPT}}(-p_f) \quad (2.17)$$

and $u_{\text{CPT}}(p)$ is the CPT conjugate of the spinor $u(p)$,

$$u_{\text{CPT}}(-p) = \gamma_0 V_T^{-1}(C\bar{u}(p))^* \quad (2.18)$$

where $V_T$ is the time-reversal matrix, $t$ denotes the transpose operation, and $\overline{\Gamma}_{\alpha\beta}$ is the vertex function describing the process $\bar{\nu}_i \rightarrow \bar{\nu}_f + g$, where $\bar{\nu}$ denotes the anti-neutrino state.

Using Eq.(2.17) and the transformation properties of the gamma matrices under the operators C and $V_T$ in Eq.(2.16), we obtain

$$CV_T(\overline{\Gamma}_{\alpha\beta})_{fi}V_T^{-1}C^{-1} = -(\Gamma_{\alpha\beta})_{if} \quad (2.19)$$

As a result of CPT invariance, we obtained the following relations among the form
factors,

\[(T_1, T_2, T_6, \bar{T}_1, \bar{T}_2, \bar{T}_6)_{fi} = (-T_1, -T_2, T_6, -P_1, -P_2, P_6)_{if}\]  

(2.20)

and

\[(T_1, T_3, T_6, \bar{T}_1, \bar{T}_4, \bar{T}_6)_{ii} = (-T_1, -T_3, T_6, -P_1, P_4, P_6)_{ii}.\]  

(2.21)

Under the CP transformation, \(\theta_{\alpha\beta} \rightarrow \eta_\alpha \eta_\beta \theta_{\alpha\beta}\),

\[CP< \nu_f(p_f)|\theta_{\alpha\beta}|\nu_i(p_i)>_{CP} = \eta_\alpha \eta_\beta < \nu_f(p_f)|\theta_{\alpha\beta}|\nu_i(p_i)> .\]  

(2.22)

The left-handed side of eq.(2.22) is given by

\[CP< \nu_f(p_f)|\theta_{\alpha\beta}|\nu_i(p_i)>_{CP} = \bar{u}_{CP}(-p'_i)(\bar{\Gamma}''_{\alpha\beta})_{fi}u_{CP}(-p'_f)\]  

(2.23)

where \(p'_\alpha = -\eta_\alpha p_\alpha = (ip_0, -p)\), \(\bar{\Gamma}''\) denotes the dressed vertex function with \(q\) and \(P\) replaced by \(q'\) and \(P'\) and

\[u_{\overline{CP}}(-p') = \gamma_0 C \bar{\nu}'(p).\]  

(2.24)

Inserting Eq. (2.23) and Eq. (2.24) into Eq. (2.22), we obtain

\[\gamma_0 C(\bar{\Gamma}''_{\alpha\beta})_{fi} C^{-1} \gamma_0 = -\eta_\alpha \eta_\beta (\Gamma_{\alpha\beta})_{fi}.\]  

(2.25)

If CP invariance holds, we obtain the following relations among the form factors,
\[(\overline{T}_1, \overline{T}_2, \overline{T}_6, \overline{P}_1, \overline{P}_2, \overline{P}_6)_{fi} = (-T_1, -T_2, T_6, P_1, P_2, -P_6)_{fi} \quad (2.26)\]

and

\[(\overline{T}_1, \overline{T}_3, \overline{T}_6, \overline{P}_1, \overline{P}_4, \overline{P}_6)_{ii} = (-T_1, -T_3, T_6, P_1, P_4, -P_6)_{ii}. \quad (2.27)\]

For the diagonal case, it follows from the CPT and CP invariance that \(P_{1ii} = P_{6ii} = 0\). That means in a CPT invariant theory, a Dirac neutrino cannot have the form factors \(P_1\) and \(P_6\) if the interaction respects CP symmetry.
C. Gravitational form factors of a Majorana neutrino

Under the CPT transformation a Majorana neutrino $\nu^M$ transforms as [9]

$$\text{CPT}|\nu^M(p, s)\rangle = \eta_{\text{CPT}}^s|\nu^M(p, -s)\rangle$$

(2.28)

where $\eta_{\text{CPT}}^s$ is a phase factor that depends on the spin of the particle, with $\eta_{\text{CPT}}^s = -\eta_{\text{CPT}}^{-s}$. Assuming CPT invariance for the energy-momentum tensor matrix element, we have

$$\text{CPT}<\nu^M_f(p_f)|\theta_{\alpha\beta}|\nu^M_i(p_i)\rangle_{\text{CPT}} = <\nu^M_i(p_i)|\theta_{\alpha\beta}|\nu^M_f(p_f)\rangle$$

(2.29)

For a Majorana neutrino, the left-hand side of Eq. (2.29) can be written as

$$\text{CPT}<\nu^M_f(p_f)|\theta_{\alpha\beta}|\nu^M_i(p_i)\rangle_{\text{CPT}} = \pi_{PT}(p_f)|\Gamma_{\alpha\beta}^f|u_{PT}(p_i)$$

(2.30)

where $u_{PT}(p) = \gamma_0 V_T^{-1} u^*(p)$. This implies that

$$V_T^{-1}(\Gamma_{\alpha\beta}^f)_fiV_T = (\Gamma_{\alpha\beta})_{ij}.$$  

(2.31)

Using the transformation properties of the gamma matrices under the operator $V_T$ in Eq.(2.31), then as a result of CPT invariance, we obtain the following relations among the form factors,
\[(T_1, T_2, T_6, P_1, P_2, P_6)_{fi} = (T_1, T_2, T_6, P_1, P_2, P_6)_{if}\] (2.32)

and

\[(T_1, T_3, T_6, P_1, P_4, P_6)_{ii} = (T_1, T_3, T_6, P_1, -P_4, P_6)_{ii}.\] (2.33)

For the same neutrino species, CPT invariance implies that \(P_4 = 0\), that is a Majorana neutrino cannot have the gravitational anapole moment form factor.

Under CP transformation, a Majorana neutrino transforms as

\[CP|\nu^M(p, s) > = \eta^*_{CP}|\nu^M(-p, s) >\] (2.34)

where \(\eta^*_{CP}\) is the CP parity of the Majorana neutrino with \(\eta^*_{CP} = \pm i\). Assuming CP invariance we have

\[CP < \nu^M_f(p_f)|\theta_{\alpha\beta}|\nu^M_i(p_i) > = \eta_\alpha \eta_\beta < \nu^M_f(p_f)|\theta_{\alpha\beta}|\nu^M_i(p_i) >.\] (2.35)

The left-hand side of Eq. (2.35) can be written as

\[CP < \nu^M_f(p_f)|\theta_{\alpha\beta}|\nu^M_i(p_i) > = \pi_P(p_f') (\Gamma'_{\alpha\beta})_{fi} u_P(p_i')\] (2.36)

where \(u_P(p') = \gamma_0 u(p)\). Using Eq.(2.35) and Eq.(2.36), we obtain

\[\eta^i \eta^j \gamma_0 (\Gamma'_{\alpha\beta})_{fi} \gamma_0 = \eta_\alpha \eta_\beta (\Gamma_{\alpha\beta})_{fi}\] (2.37)

where \(\eta_{CP} = i\eta\). As a result of CP invariance, we obtain the following relations
among the form factors,

\[ \eta^i \eta^f (T_1, T_2, T_6, P_1, P_2, P_6)_{fi} = (T_1, T_2, T_6, -P_1, -P_2, -P_6)_{fi} \quad (2.38) \]

and

\[ (T_1, T_3, T_6, P_1, P_4, P_6)_{ii} = (T_1, T_3, T_6, -P_1, -P_4, -P_6)_{ii}. \quad (2.39) \]

We observe that the amplitude for the process \( \nu_i^M \rightarrow \nu_f^M + g \) depends on the relative CP parity of the initial and final neutrino states. For instance, if \( \eta^i \eta^f = 1 \), a Majorana neutrino has a tensorial type of transition form factors, while for \( \eta^i \eta^f = -1 \), a Majorana neutrino has a pseudo-tensorial type of transition form factors.

3. Summary

It is shown that the invariant amplitude for the process \( \nu_i \rightarrow \nu_f + g \) is characterized by six gravitational form factors (three tensor and three pseudo-tensor types). The hermiticity condition requires that four of the form factors are real. As a result of CPT and CP invariance, a Dirac neutrino of the same species has four gravitational form factors. A Majorana neutrino has five form factors (no gravitational anapole moment form factor) as a result of CPT invariance, which is agree
with Ref. [7] result. In a CP invariant theory, if the initial and final Majorana neutrinos have the same (opposite) CP parity, then only tensor (pseudo-tensor) type transition are allowed. For the same neutrino species, the energy-momentum matrix element for a Majorana neutrino is characterized by tensor couplings only [7].

**Acknowledgments**

This work is supported by the National Science Council of the R.O.C research grant NSC-81-0208-M-002-518.
Appendix

In this appendix we present the identities among the various types of coupling that were employed in our calculation.

Tensor type couplings

\[ \overline{u}_f \{ \sigma qq \} u_i = i \overline{u}_f \{ qP - M_{ij}q\gamma \} u_i \]  \hspace{1cm} (A.1)

\[ \overline{u}_f \{ \sigma qP \} u_i = i \overline{u}_f \{ PP - M_{ij}P\gamma \} u_i \]  \hspace{1cm} (A.2)

\[ \overline{u}_f \{ \sigma q\gamma \} u_i = i \overline{u}_f (\{ q\gamma \} - 2\delta m_{fi}\eta_{\alpha\beta}) u_i \]  \hspace{1cm} (A.3)

\[ \overline{u}_f \{ \sigma PP \} u_i = i \overline{u}_f \{ qP - \delta m_{fi}P\gamma \} u_i \]  \hspace{1cm} (A.4)

\[ \overline{u}_f \{ \sigma Pq \} u_i = i \overline{u}_f (qq - \{ \delta m_{fi}q\gamma \}) u_i \]  \hspace{1cm} (A.5)

\[ \overline{u}_f \{ \sigma P\gamma \} = i \overline{u}_f (\{ P\gamma \} - 2M_{ij}\eta_{\alpha\beta}) u_i \]  \hspace{1cm} (A.6)

Pesudo-tensor type couplings

\[ \overline{u}_f \gamma_5 \{ \sigma qq \} u_i = i \overline{u}_f \gamma_5 \{ Pq + \delta m_{fi}q\gamma \} u_i \]  \hspace{1cm} (A.7)

\[ \overline{u}_f \gamma_5 \{ \sigma qP \} u_i = i \overline{u}_f \gamma_5 \{ PP + \delta m_{fi}P\gamma \} u_i \]  \hspace{1cm} (A.8)

\[ \overline{u}_f \gamma_5 \{ \sigma q\gamma \} u_i = i \overline{u}_f \gamma_5 (\{ q\gamma \} + 2M_{ij}\eta_{\alpha\beta}) u_i \]  \hspace{1cm} (A.9)
\[ \bar{\pi}_f \gamma_5 \{\sigma P P\} u_i = i \bar{\pi}_f \gamma_5 \{q P + M_{i f} P \gamma\} u_i \]  \hspace{1cm} (A.10)

\[ \bar{u}_f \gamma_5 \{\sigma P q\} u_i = i \bar{u}_f \gamma_5 (qq + \{M_{i f} q \gamma\}) u_i \]  \hspace{1cm} (A.11)

\[ \bar{\pi}_f \gamma_5 \{\sigma P \gamma\} = i \bar{\pi}_f \gamma_5 \{P \gamma\} + 2 \delta m_{f i} \eta_{\alpha \beta} u_i \]  \hspace{1cm} (A.12).

where \( \delta m_{f i} = m_f - m_i \) and \( M_{i f} = m_i + m_f \).

REFERENCES

1. B. Kayser and R.E. Shrock, Phys. Lett. 112B, 137 (1982).

2. B. Kayser, Phys. Rev. D, 26, 1662 (1982); J. F. Nieves, Phys. Rev. D 26, 3152 (1982).

3. S.M. Bilenky and S.T. Petcov, Rev. Mod. Phys. 59, 671 (1987).

4. E.E. Radescu, Phys. Rev. D 32, 1266 (1985).

5. F. Boudjema, C. Hamzaoui, V. Rahel and H.C. Ren, Phys. Rev. Lett 62, 852 (1989).

6. I. Yu. Kobzarev and L.B. Okun, Sov. Phys. JETP 16, 1343 (1963).

7. A. Khare and J. Oliensis, Phys. Rev. D 29, 1542 (1984).

8. The energy-momentum tensor \( \theta_{\alpha \beta} \) is proportional to \( p_{\alpha} p_{\beta} \), where \( p \) can be either \( p_i \) or \( p_f \). See S. Weinberg, Gravitation and Cosmology, John Wiley & Sons, New York 1972.
9. B. Kayser and A. Goldhaber, Phys. Rev. D 28, 2341 (1983)