Dissipative processes in superfluid quark matter

Massimo Mannarelli
Universitat de Barcelona

massimo@ecm.ub.es

arXiv:0807.3264    arXiv:0904.3023    arXiv:0909.4486

Collaborators: M.A. Escobedo, C. Manuel, B.A. Sa’d, M.Ruggieri, R.Anglani, G.Colucci
OUTLINE

✦ QCD phase diagram
✦ Color flavor locking
✦ Superfluids
✦ Dissipative processes
✦ R-mode oscillations

Reviews:  hep-ph/0011333, hep-ph/0102047, hep-ph/0202037, 0709.4635
QCD phase diagram

T

$T_c$

QGP

Hadron gas

Nuclear liquid

Color Superconductor

$\mu$
QCD PHASE DIAGRAM

QGP

Hadron gas

T

T_c

Nuclear liquid

Color Superconductor

μ
QCD PHASE DIAGRAM

- QGP
- Color Superconductor
- Hadron gas
- Nuclear liquid
- Compact stars

Temperature ($T$) vs Chemical Potential ($\mu$)
QCD PHASE DIAGRAM

Warning: At high density ab initio calculations using QCD not available
Warning: At high density ab initio calculations using QCD not available
QCD PHASE DIAGRAM

Warning: At high density ab initio calculations using QCD not available

Confined Strong coupling Weak coupling

Hadrons CSC phase? quarkyonic phase? CFL
# Color superconductor

| Confined | Strong coupling | Weak coupling |
|----------|-----------------|---------------|

\[ \mu \]
Color superconductor

Confined  \[\xrightarrow{\text{Strong coupling}}\]  Weak coupling

\[\mu\]
Color superconductor

Confined  Strong coupling  Weak coupling

- Degenerate system of quarks
- Attractive interaction between quarks

\[ 3 \times 3 = \bar{3}_A + 6_S \]

attractive channel

\[ p, p' \simeq p_f \]
Using quarks as building blocks, one has color, flavor as well as spin degrees of freedom: the game is complicated.

QCD, allows for a zoo of colored phases and one has to single out the one with the smallest free-energy.
\[ \mu \gg m_s \]

CFL phase

Alford, Rajagopal, Wilczek hep-ph/9804403

\[ \langle \psi_{\alpha i} C \gamma_5 \psi_{\beta j} \rangle \sim \Delta \epsilon_{I\alpha\beta} \epsilon_{Iij} \]

\[ SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B \rightarrow SU(3)_{c+L+R} \times Z_2 \]
**Xtreme density**

\[ \mu \gg m_s \]

**CFL phase**

Alford, Rajagopal, Wilczek hep-ph/9804403

\[ \langle \psi_{\alpha i} C \gamma_5 \psi_{\beta j} \rangle \sim \Delta \epsilon_{I\alpha\beta} \epsilon_{Iij} \]

\[ SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B \rightarrow SU(3)_{c+L+R} \times Z_2 \]

\[ <\psi_L \psi_L> \quad <\psi_R \psi_R> \]
$\mu \gg m_s$

CFL phase

Alford, Rajagopal, Wilczek hep-ph/9804403

$\langle \psi_{\alpha i} C \gamma_5 \psi_{\beta j} \rangle \sim \Delta \epsilon_{I\alpha \beta} \epsilon_{I i j}$

$$SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B \rightarrow SU(3)_{c+L+R} \times Z_2$$

$SU(3)_L$ rotation
$\mu \gg m_s$

CFL phase

$\langle \psi_{\alpha i} C \gamma_5 \psi_{\beta j} \rangle \sim \Delta \epsilon_{I\alpha\beta} \epsilon_{Iij}$

$SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B \rightarrow SU(3)_{c+L+R} \times Z_2$
\[ \mu \gg m_s \]

CFL phase

Alford, Rajagopal, Wilczek hep-ph/9804403

\[ \langle \psi_{\alpha i} C \gamma_5 \psi_{\beta j} \rangle \sim \Delta \epsilon_{I \alpha \beta} \epsilon_{I ij} \]

\[ SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B \rightarrow SU(3)_{c+L+R} \times Z_2 \]
Xtreme density

\[ \mu \gg m_s \]

CFL phase

Alford, Rajagopal, Wilczek hep-ph/9804403

\[ \langle \psi_{\alpha i} C \gamma_5 \psi_{\beta j} \rangle \sim \Delta \epsilon_{I\alpha\beta} \epsilon_{Iij} \]

\[ SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B \to SU(3)_{c+L+R} \times Z_2 \]

Gluons acquire a Meissner mass by Higgs mechanism

8 pseudo Nambu-Goldstone bosons

All quarks are paired; quasiparticles with a gapped excitation spectrum

No electrons

1 Nambu-Goldstone boson \( \to \) Superfluid
UNVISCID (DRY) FLUID

Continuity equation
\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0 \]

Euler equation
\[ \frac{\partial v}{\partial t} + (v \cdot \nabla)v = -\frac{\nabla p}{\rho} - \nabla \phi \]

Vorticity \( \Omega = \nabla \times v \)  
Unviscid fluid \( \Omega = 0 \)

The flow is permanently irrotational \( v = \nabla \varphi \)
**VISCOUS (WET) FLUID**

\[
\rho \left( \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right) = -\nabla p - \rho \nabla \phi + \eta \nabla^2 \mathbf{v} + \left( \frac{\eta}{3} + \zeta \right) \nabla (\nabla \cdot \mathbf{v})
\]

- **Shear viscosity** describes reaction to shear stresses
- **Bulk viscosity** describes reaction to compression/rarefaction

Using vorticity
\[
\frac{\partial \Omega}{\partial t} + \nabla \times (\Omega \times \mathbf{v}) = \frac{\eta}{\rho} \nabla^2 \Omega
\]

Vorticity generated by the shear viscosity
**Viscous (Wet) Fluid**

\[
\rho \left( \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} \right) = -\nabla p - \rho \nabla \phi + \eta \nabla^2 \mathbf{v} + \left( \frac{\eta}{3} + \zeta \right) \nabla (\nabla \cdot \mathbf{v})
\]

- **Shear viscosity** describes reaction to shear stresses.
- **Bulk viscosity** describes reaction to compression/rarefaction.

Using vorticity

\[
\frac{\partial \Omega}{\partial t} + \nabla \times (\Omega \times \mathbf{v}) = \frac{\eta}{\rho} \nabla^2 \Omega
\]

Vorticity generated by the shear viscosity.
**NON-RELATIVISTIC SUPERFLUIDS**

**Landau two-fluid theory**

\[ \rho = \rho_n + \rho_s \quad \text{and} \quad \mathbf{j} = \rho_n \mathbf{v}_n + \rho_s \mathbf{v}_s \]

The two “components” correspond to two different motions of the fluid.
Non-relativistic superfluids

Landau two-fluid theory

\[ \rho = \rho_n + \rho_s \]
\[ \mathbf{j} = \rho_n \mathbf{v}_n + \rho_s \mathbf{v}_s \]

The two “components” correspond to two different motions of the fluid

Normal component: viscous fluid
Superfluid component: unviscid fluid

Not completely correct: neglects interactions
Dissipative terms in relativistic hydrodynamics:

\[ \partial_\mu n^\mu = 0 \]
\[ \partial_\mu (T^{\mu\nu} + T_d^{\mu\nu}) = 0 \]
\[ u^\mu \partial_\mu \phi + \mu + \chi = 0 \]
**RELATIVISTIC HYDRODYNAMICS**

\[ \partial \mu n^\mu = 0 \]
\[ \partial \mu (T^{\mu \nu} + T_d^{\mu \nu}) = 0 \]
\[ u^\mu \partial \mu \phi + \mu + \chi = 0 \]

**dissipative terms**

**Close to equilibrium**

\[ \chi = -\zeta_3 \partial \mu (V^2 w^\mu) - \zeta_4 \partial \mu u^\mu \]
\[ T_d^{\mu \nu} = \kappa (\Delta^{\mu \gamma} u^\nu + \Delta^{\nu \gamma} u^\mu) (\partial_\gamma T + T u^\delta \partial_\delta u_\gamma) \]
\[ + \eta \Delta^{\mu \gamma} \Delta^{\nu \delta} \left( \partial_\delta u_\gamma + \partial_\gamma u_\delta + \frac{2}{3} g_{\gamma \delta} \partial_\alpha u^\alpha \right) \]
\[ + \Delta^{\mu \nu} (\zeta_1 \partial_\gamma (V^2 w^\gamma) + \zeta_2 \partial_\gamma u^\gamma) \]

**where**

\[ w^\mu = - (\partial^\mu \varphi + \mu u^\mu) \]
\[ \Delta^{\mu \nu} = g^{\mu \nu} - u^\mu u^\nu \]
Bulk viscosity depends on the low energy spectrum of the theory.

| Bulk Viscosity Phase | Authors and Ref.          | ArXiv Number    |
|----------------------|---------------------------|-----------------|
| CFL phase            | Manuel and LLanes         | 0705.3909       |
| Contribution of phonons | MM and Manuel              | 0909.4486       |
| Contribution of kaons | Alford et al.              | nucl-th/0701067 |
|                      | Alford et al.              | 0707.2389       |
| Spin 1 phase         | Sa’d et al.                | astro-ph/0607643|
|                      | Wang et al.                | 1006.1293       |
| 2SC phase            | Alford and Schmitt         | nucl-th/0608019 |

Large amplitude behavior of bulk viscosity: talk by Alford
Fluid of quasiparticles (phonons) moving on the top of the superfluid background

Effective Lagrangian (low energy, asymptotic densities)  Son hep-ph/0204199

\[ \mathcal{L}_{\text{eff}} = \frac{3}{4\pi^2} \left[ (\partial_0 \varphi - \mu_q)^2 - (\partial_i \varphi)^2 \right]^2 \]
Fluid of quasiparticles (phonons) moving on the top of the superfluid background

**Effective Lagrangian** (low energy, asymptotic densities)  
Son hep-ph/0204199

\[ \mathcal{L}_{\text{eff}} = \frac{3}{4\pi^2} \left[ (\partial_0 \varphi - \mu_q)^2 - (\partial_i \varphi)^2 \right]^2 \]

**Scale separation**

\[ \varphi(x) = \bar{\varphi}(x) + \phi(x) \]
Fluid of quasiparticles (phonons) moving on the top of the superfluid background

Effective Lagrangian (low energy, asymptotic densities) Son hep-ph/0204199

\[ \mathcal{L}_{\text{eff}} = \frac{3}{4\pi^2} \left[ (\partial_0 \varphi - \mu_q)^2 - (\partial_i \varphi)^2 \right]^2 \]

Scale separation

\[ \varphi(x) = \bar{\varphi}(x) + \phi(x) \]
Fluid of quasiparticles (phonons) moving on the top of the superfluid background

Effective Lagrangian (low energy, asymptotic densities) \( \text{Son hep-ph/0204199} \)

\[
\mathcal{L}_{\text{eff}} = \frac{3}{4\pi^2} \left[ (\partial_0 \varphi - \mu_q)^2 - (\partial_i \varphi)^2 \right]^2
\]

Scale separation

\[
\varphi(x) = \bar{\varphi}(x) + \phi(x)
\]

- superfluid
- phonon
- bulk
- long-wavelength fluctuations
Fluid of quasiparticles (phonons) moving on the top of the superfluid background

**Effective Lagrangian** (low energy, asymptotic densities) Son hep-ph/0204199

\[ \mathcal{L}_{\text{eff}} = \frac{3}{4\pi^2} \left[ (\partial_0 \varphi - \mu_q)^2 - (\partial_i \varphi)^2 \right]^2 \]

\varphi(x) = \bar{\varphi}(x) + \phi(x)

**Scale separation**

\[ S[\varphi] = S[\bar{\varphi}] + \frac{1}{2} \int d^4x \left. \frac{\partial \mathcal{L}_{\text{eff}}}{\partial (\partial_\mu \varphi) \partial (\partial_\nu \varphi)} \right|_{\bar{\varphi}} \partial_\mu \phi \partial_\nu \phi + \cdots \]
Fluid of quasiparticles (phonons) moving on the top of the superfluid background

Effective Lagrangian (low energy, asymptotic densities) Son hep-ph/0204199

$$\mathcal{L}_{\text{eff}} = \frac{3}{4\pi^2} \left[ (\partial_0 \varphi - \mu_q)^2 - (\partial_i \varphi)^2 \right]^2$$

Scale separation

$$\varphi(x) = \bar{\varphi}(x) + \phi(x)$$

Phonon's action

$$S[\phi] = \frac{1}{2} \int d^4x \sqrt{-g} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$
Fluid of quasiparticles (phonons) moving on the top of the superfluid background

**Effective Lagrangian** (low energy, asymptotic densities)  
\[
\mathcal{L}_{\text{eff}} = \frac{3}{4\pi^2} \left[ (\partial_0 \varphi - \mu_q)^2 - (\partial_i \varphi)^2 \right]^2
\]

**Scale separation**

\[
\varphi(x) = \bar{\varphi}(x) + \phi(x)
\]

**Phonon’s action**

\[
S[\phi] = \frac{1}{2} \int d^4x \sqrt{-g} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi
\]

**Acoustic metric**

\[
g_{\mu\nu} = \eta_{\mu\nu} + (c_s^2 - 1) v_\mu v_\nu
\]

---

**Son hep-ph/0204199**
Phonon contribution

phonon dispersion law \[ \epsilon_p = c_s p + B p^3 + \mathcal{O}(p^5) \]

\[ \partial_t N_{\text{ph}} + \text{div}(N_{\text{ph}} \mathbf{v}_n) = -\frac{\Gamma_{\text{ph}}}{T} \mu_{\text{ph}} \]

\[ B > 0 \quad \phi \rightarrow \phi \phi \]
\[ B < 0 \quad \phi \phi \rightarrow \phi \phi \phi \]
**Phonon contribution**

Phonon dispersion law \[ \epsilon_p = c_s \rho + B \rho^3 + \mathcal{O}(\rho^5) \]

\[ \partial_t N_{ph} + \text{div}(N_{ph} v_n) = -\frac{\Gamma_{ph}}{T} \mu_{ph} \]

\[ B > 0 \quad \phi \rightarrow \phi\phi \]
\[ B < 0 \quad \phi\phi \rightarrow \phi\phi\phi \]

\[ \zeta_1 = -\frac{T}{\Gamma_{ph}} \frac{\partial N_{ph}}{\partial \rho} \left( N_{ph} - S \frac{\partial N_{ph}}{\partial S} - \rho \frac{\partial N_{ph}}{\partial \rho} \right) = -\frac{T}{\Gamma_{ph}} I_1 I_2 \]

\[ \zeta_2 = \frac{T}{\Gamma_{ph}} \left( N_{ph} - S \frac{\partial N_{ph}}{\partial S} - \rho \frac{\partial N_{ph}}{\partial \rho} \right)^2 = \frac{T}{\Gamma_{ph}} I_2^2 \]

\[ \zeta_3 = \frac{T}{\Gamma_{ph}} \left( \frac{\partial N_{ph}}{\partial \rho} \right)^2 = \frac{T}{\Gamma_{ph}} I_1^2 \]
Phonon contribution

phonon dispersion law \( \epsilon_p = c_s p + B p^3 + O(p^5) \)

\[
\partial_t N_{ph} + \text{div}(N_{ph} v_n) = -\frac{\Gamma_{ph}}{T} \mu_{ph}
\]

Notice that \( \zeta_1 = \zeta_2 \zeta_3 \) the system tends toward the state where bulk viscosity does not lead to dissipation
**PHONONS IN CFL**

Low temperatures \( T \lesssim 0.01 \text{ MeV} \)

In CFL \( B < 0 \)

Conformal limit

\[
\zeta_1 = \zeta_2 = 0 \quad \zeta_3 \sim \frac{\mu^6}{T \Delta^8}
\]

Conformal breaking due to \( m_s \)

\[
\zeta_1 \sim \frac{m_s^2 \mu^7}{T \Delta^8} \quad \zeta_2 \sim \frac{m_s^4 \mu^8}{T \Delta^8}
\]
Mutual friction

Force between the superfluid component and the normal component mediated by phonon-vortex interaction

\[
\rho_s \frac{d\mathbf{v}_s}{dt} = -\frac{\rho_s}{\rho} \nabla p - \rho_s \nabla \phi - \mathbf{F}^N
\]

\[
\rho_n \frac{d\mathbf{v}_n}{dt} = -\frac{\rho_n}{\rho} \nabla p - \rho_n \nabla \phi + \mathbf{F}_N^N + \eta \nabla^2 \mathbf{v}_n
\]
Mutual friction

Force between the superfluid component and the normal component mediated by phonon-vortex interaction

\[
\rho_s \frac{d\mathbf{v}_s}{dt} = -\frac{\rho_s}{\rho} \nabla p - \rho_s \nabla \phi - F^N
\]

\[
\rho_n \frac{d\mathbf{v}_n}{dt} = -\frac{\rho_n}{\rho} \nabla p - \rho_n \nabla \phi + F^N + \eta \nabla^2 \mathbf{v}_n
\]
Forces acting on a Vortex

**Magnus force**
\[
F^M = \kappa \rho_s (v_s - v_L) \times \hat{z}
\]

**Friction force**
\[
F^N = D(v_n - v_L) + D' \hat{z} \times (v_n - v_L)
\]

**Standard hydrodynamic force**

**Scattering of phonons off vortices**
Forces acting on a Vortex

- **Magnus force**
  \[ \mathbf{F}^M = \kappa \rho_s (\mathbf{v}_s - \mathbf{v}_L) \times \hat{z} \]

- **Friction force**
  \[ \mathbf{F}^N = D(\mathbf{v}_n - \mathbf{v}_L) + D' \hat{z} \times (\mathbf{v}_n - \mathbf{v}_L) \]

- **Standard hydrodynamic force**

- **Scattering of phonons off vortices**

- **Mutual friction**
**Forces acting on a Vortex**

- **Magnus force**
  \[ F^M = \kappa \rho_s (v_s - v_L) \times \hat{z} \]

- **Friction force**
  \[ F^N = D(v_n - v_L) + D' \hat{z} \times (v_n - v_L) \]

- **Mutual friction**

**Standard hydrodynamic force**

**Scattering of phonons off vortices**

**Elastic scattering off vortices**

\[
\frac{d\sigma}{d\theta} = \frac{c_s}{2\pi E} \frac{\cos^2 \theta}{\tan^2 \frac{\theta}{2}} \sin^2 \left( \frac{\pi E}{\Lambda} \right)
\]
Forces acting on a Vortex

**Magnus force**

\[ F^M = \kappa \rho_s (v_s - v_L) \times \hat{z} \]

**Friction force**

\[ F^N = D(v_n - v_L) + D'\hat{z} \times (v_n - v_L) \]

**Mutual friction**

- Elastic scattering off vortices
  \[ \frac{d\sigma}{d\theta} = \frac{c_s}{2\pi} \cos^2 \theta \tan^2 \frac{\theta}{2} \sin^2 \frac{\pi E}{\Lambda} \]

- Inelastic scattering on vortices
  work in progress
  see the talk by Anglani

**Standard hydrodynamic force**

**Scattering of phonons off vortices**
R-mode instability

Modes retrograde in the corotating frame (but prograde in the inertial frame) are unstable.

See also Andersson, Kokkotas gr-qc/0010102

Lindblom, astro-ph/0101136
Dissipative processes and stars oscillations

R-mode instability

Modes retrograde in the corotating frame (but prograde in the inertial frame) are unstable.
See also Andersson, Kokkotas gr-qc/0010102

R-mode oscillations difficult to damp in CFL stars
Madsen, Phys. Rev. Lett. 85, 10 (2000)

Emitting gravitational radiation the star quickly spins down
**Dissipative Processes and Stars Oscillations**

**R-mode instability**

Modes retrograde in the corotating frame (but prograde in the inertial frame) are unstable.

See also Andersson, Kokkotas gr-qc/0010102

R-mode oscillations difficult to damp in CFL stars

Madsen, Phys. Rev. Lett. 85, 10 (2000)

Emitting gravitational radiation the star quickly spins down

**CFL**

**R-mode instability**

Gravitational Radiation

Lindblom, astro-ph/0101136

**Dissipative processes effective for** $\nu \lesssim 1$ Hz

see however 1005.1163
DISSIPATIVE PROCESSES AND STARS OSCILLATIONS

R-mode instability

Modes retrograde in the corotating frame (but prograde in the inertial frame) are unstable. See also Andersson, Kokkotas gr-qc/0010102

R-mode oscillations difficult to damp in CFL stars
Madsen, Phys. Rev. Lett. 85, 10 (2000)

Emitting gravitational radiation the star quickly spins down

dissipative processes effective for $\nu \lesssim 1$ Hz

For more details see the talk by Sedrakian

Lindblom, astro-ph/0101136

CFL

see however 1005.1163
Summary

- CFL is a superfluid
- Contribution of phonons to the bulk viscosity coefficients of CFL
- For rotating superfluids one has to include the mutual friction force
- Damping of star oscillations, especially gravitationally unstable r-modes
