Resource Allocation Strategies for Secure WPCN Multiantenna Multicasting Systems

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Abstract

This paper investigates a secure wireless-powered multiantenna multicasting system, where multiple power beacons (PBs) supply power to a transmitter in order to establish a reliable communication link with multiple legitimate users in the presence of multiple eavesdroppers. The transmitter has to harvest radio frequency (RF) energy from multiple PBs due to the shortage of embedded power supply before establishing its secure communication. We consider two different scenarios. In the first, the PBs and the transmitter belong to the same operator, where we formulate the resource allocation problem as the minimization of the total transmit power subject to the target secure rate constraint. The solution of this problem yields both the optimal power and energy transfer time allocation. Due to the non-convexity of this problem, we propose a two-level approach, where the inner level problem can be recast as a convex optimization framework via conic convex reformulation, while the outer level problem can be handled by using one-dimensional (1D) search. The second scenario considers the case where the transmitter and the PBs belong to different service suppliers. Hence, we formulate the resource allocation problem where we consider incentives for the PBs to assist the transmitter. This leads to the formulation of a Stackelberg game for the secure wireless-powered multiantenna multicasting system. The transmitter has to pay for the energy services from these multiple PBs in order to facilitate secure communications. In this game, the transmitter and the PB are modelled as leader and follower, respectively, in which both of them try to maximize their own utility function. The closed-form Stackelberg equilibrium of the formulated game is then derived. Finally, numerical results are provided to validate our proposed schemes.

Index Terms

Wireless powered communication networks (WPCN), Physical layer security, SWIPT, Multicasting, Stackelberg game

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I. INTRODUCTION

Wireless multicast media streaming is anticipated to be a significant component of the forthcoming 5G systems, motivated by the consumers’ desire to take advantage of high quality multimedia wireless devices (e.g., 4k hand-held devices, 3D augmented reality)\cite{1}–\cite{3}. Energy efficiency and security are major critical issues that must be addressed in the design of such systems.

Radio frequency (RF) energy harvesting and transfer techniques have recently been considered as a promising solution to the energy-constrained wireless networks\cite{4}–\cite{6}. As a recent application of RF energy harvesting and transfer techniques, wireless powered communication networks (WPCNs) have become a novel technology in wireless networking and attracted more and more attention\cite{7}. A “harvest-then-transmit” protocol was proposed for WPCNs in\cite{8}, where the wireless users harvest power from the RF signals broadcast by an access point in the downlink (DL), and then send information to the AP in the uplink (UL) by employing the harvested energy. Cooperative protocols for WPCNs were developed based on different models\cite{9}–\cite{11}. A different approach consists of deploying a dedicated wireless energy transfer (WET) network with multiple power beacons (PBs) to provide wireless charging services to the wireless terminals via the RF energy transfer technique\cite{12}, \cite{13}. Since the PBs do not require any backhaul link, the associated cost of PBs deployment is much lower, hence, it is feasible to deploy the PBs densely to guarantee network coverage for a wide range of mobile devices\cite{14}.

Security in data transmission can be addressed either by traditional crypto methods, or more fundamentally, in terms of information theoretic secure rates. The latter approach, commonly referred to as “physical-layer security,” was initially developed for the wiretap channel\cite{15}, \cite{16}, i.e., a broadcast channel with one transmitter and two sets of receivers: legitimate users and eavesdroppers. Multiantenna wiretap channels have been widely investigated in terms of secure rate region\cite{17}–\cite{21}. Some state-of-art techniques, such as artificial noise (AN) and cooperative jammer (CJ), have been designed for multiantenna transceivers, in order to introduce more interference at the eavesdroppers\cite{22}–\cite{27}. In\cite{22}, rank-one solution properties were exploited with semidefinite programming (SDP) relaxation for secure transmit beamforming. AN-assisted transmit optimization has been presented in\cite{23}, where the spatially selective AN embedded with secure transmit beamforming was designed to obtain the optimal power allocation. In\cite{24}, CJ from an external node is exploited in order to create interference at the eavesdroppers and achieve the desired target secure rate. However, it is not always possible to have an own CJ to improve the secrecy rates. Another option could be to employ a private CJ by paying a price for the jamming services. This strategy was investigated in\cite{24}, \cite{25}, where a CJ releases its jamming service depending interference caused to the eavesdropper, while the transmitter pays a certain amount to guarantee its secure communication. In this strategy, a Stackelberg game can be formulated to obtain the optimal power
allocation. In addition, cooperative cognitive radio (CR) combined with secure communications could also be modelled as a Stackelberg game to determine the optimal resource allocations [28]. In [26], [27], the secrecy rate optimization problem was posed in terms of outage secrecy rates, due to the fact that the channels are not perfectly known and are subject to random fading with known statistics. Physical-layer security techniques have also been recently developed in radio frequency identification (RFID). The design of RFID systems is a challenge due to the broadcast nature of backscatter communication, which is vulnerable to eavesdropping [29]. Simultaneous wireless information and power transfer (SWIPT) has emerged as one of most promising approaches to provide power for communication devices. SWIPT has been considered in combination with physical-layer security in a number of recent works (e.g., [30]–[33]). It is worth pointing out that the transmit power is constant in the above secure communication systems. However, the use of WET effectively makes the available transmit power a system variable in order to achieve secure communications. Thus, this research gap motivates the work in this paper.

We investigate a WPCN-assisted multiantenna secure multicasting system in which a multicast service provider (i.e., the transmitter) guarantees secure communication with legitimate users in the presence of multiple eavesdroppers by utilizing the harvested energy from the PBs. In particular, we consider two different but complementary scenarios. In the first scenario, the transmitter and the PBs are considered to belong to the same service provider. In the second, they belong to different service providers with different objectives. Accordingly, we formulate and solve the two different problems as follows:

1) **Power minimization for WPCN-aided multiantenna secure multicasting system**: This problem is formulated for the first scenario, where there is no energy trading between the transmitter and the PBs. They cooperate to minimize the transmit power for a given target secure multicast rate. The optimization problem is not jointly convex in terms of PBs’ transmit power, the fraction of time dedicated to WET, and the transmit beamforming vector. To circumvent this non-convexity issue, we consider a two-level approach. The outer level can be recast as a single-variable optimization problem with respect to the energy time allocation, in which the optimal solution can be achieved via numerical search, while the inner level remains a non-convex power minimization problem. There are different approaches for the inner level problem. In [22], semidefinite programming (SDP) relaxation was employed to solve the power minimization for the case of single legitimate user only. However, this approach will not guarantee a rank-1 solution when extending to the case of multiple users. Therefore, a rank-1 feasible solution for the original beamforming problem must be obtained from the solution of the relaxed problem, with no optimality guarantee (see, e.g., [34], [35]). Differently, we first propose a novel reformulation based on matrix transformations and convex conic optimization techniques, yielding a second-order cone programming (SOCP) solution.
which is optimal when the SDP relaxed solution satisfies the rank-one condition. Then, we propose a successive convex approximation (SCA) based SOCP scheme, which is performed iteratively to obtain the optimal transmit beamformer directly for any general case. Numerical results confirm that our proposed SCA based SOCP scheme outperforms the SDP relaxed scheme, and the proposed SOCP scheme has more computationally efficient than the SDP relaxed scheme that uses Guided Randomization method.

2) **Game theory based WPCN-aided multiantenna secure multicasting system**: In this problem, we investigate the case where the transmitter and the PBs belong to different service operators, both of which want to maximize their own benefit. Thus, an energy price must be paid by the transmitter in order to induce the PBs to provide enough WET to guarantee secure communications. In particular, we develop an energy trading framework for wireless powered secure multiantenna multicasting systems, where the strategic behavior of the transmitter and the PBs is modeled as a Stackelberg game. The transmitter acts a leader that buys energy from the PBs to achieve a desired secure multicast rate to the users. The transmitter optimizes the energy price and energy transfer time to maximize its utility function, defined as the weighted difference between revenues (proportional to the achieved secure rate) and costs of the purchased energy. The PBs are the followers, that determine their optimal transmit powers based on the energy price released from the transmitter to maximize their own profits, defined as the difference between the payment received from the transmitter and its energy cost. We obtain a closed-form solution for the Stackelberg equilibrium, in which both the PBs and the transmitter come to an agreement on the energy price, transmit power and energy time allocation.

**Related works**: Considering the first scenario, unlike [28] where an intermediate self-sustainable relay was employed to enable cooperation between a WET network and a wireless information transfer (WIT) network to guarantee secure communications subject to outage probability constraints, our work is to exploit direct energy interactions between the multicasting service provider and the PBs to facilitate secure communications of the legitimate users. Moreover, we also consider a special case with a single legitimate user and a single eavesdropper. In [36], the authors aimed to maximize the secrecy rate subject to the transmit power budget, and the optimal beamforming vector was designed based on Rayleigh quotient approach; whereas our work for this special case presents the closed-form solution of the joint design of the optimal time allocation and the optimal transmit beamforming vector to achieve the target of minimizing the transmitter’s transmit power, via the dual problem and the Karush-Kuhn-Tucker (KKT) condition.

The rest of the paper is organized as follows. Section III presents our system model. Section III solves
the power minimization problem for the secure WPCN multiantenna multicasting system, whereas the
game theory based secure WPCN multiantenna multicasting system is investigated in Section \[IV\]. Section
\[V\] provides simulation results to validate the theoretical derivations. Finally, Section \[VI\] concludes the
paper.

A. Notations

We use the upper case boldface letters for matrices and lower case boldface letters for vectors. \((\cdot)^T\)
and \((\cdot)^H\) denote the transpose and conjugate transpose respectively. \(\text{Tr}(\cdot)\) and \(\mathbb{E}\{\cdot\}\) stand for trace of a
matrix and the statistical expectation for random variables. \(\varrho_{\text{max}}(*)\) represents the maximum eigenvalue,
whereas \(v_{\text{max}}(*)\) denotes the eigenvector associated with the maximum eigenvalue. \(A \succeq 0\) indicates that
A is a positive semidefinite matrix. \(\| * \|\) denotes the Euclidean norm of a vector. \(I\) and \((\cdot)^{-1}\) denote the
identity matrix with appropriate size and the inverse of a matrix respectively. \([x]^+\) represents \(\max\{x, 0\}\). The notation \(\succeq_{K_n}\) denotes the following generalized inequality:
\[
\begin{bmatrix}
    a \\
    b
\end{bmatrix} \succeq_{K_n} 0 \Leftrightarrow \|b\| \leq a,
\]
where \(b \in \mathbb{C}^{n-1}\) and \(K_n \subseteq \mathbb{R}^n\) is called a proper cone.

II. System Model

In this section, we consider the secure wireless powered multiantenna multicasting system as shown
in Fig. \[II\] where a transmitter broadcasts the same information to all legitimate users in the presence
of multiple eavesdroppers. Due to energy limitation at the transmitter, it is assumed that there is not
enough power supply for information transfer, and thus the transmitter must harvest energy from the PBs.
This system consists \(M\) single antenna PBs, one multiantenna transmitter equipped with \(N_T\) transmit
antennas, \(K\) single antenna legitimate users and \(L\) single antenna eavesdroppers. This secrecy model has
some potential applications, such as on-demand video broadcasting, wireless sensor networks and device
to device (D2D) communication systems. In our paper, a harvest-then-transmit protocol is considered.
Specifically, time is divided in periods of duration \(T\). Each period is split into a WET phase of duration
\(\theta T\), and a WIT phase of duration \((1-\theta)T\), where \(\theta \in (0, 1)\) is a system parameter that must be optimized.
Let \(h_{s,k} \in \mathbb{C}^{N_T \times 1}\) denote the channel coefficients between the transmitter and the \(k\)-th legitimate user,
while \(h_{e,l} \in \mathbb{C}^{N_T \times 1}\) denotes the channel coefficients between the transmitter and the \(l\)-th eavesdropper.
Also, \(g_m \in \mathbb{C}^{1 \times N_T}\) denotes the channel coefficients between the \(m\)-th PB and the transmitter. First, each
Fig. 1: WPCN for multiantenna secure multicasting system.

PB transfers the energy to the transmitter, the harvested energy during the WET phase of \( \theta T \) at the transmitter can be written as

\[
E_B = \xi \sum_{m=1}^{M} p_m \|g_m\|^2 \theta T, \tag{1}
\]

where \( p_m \) denotes the transmit power of the \( m \)-th PB, and \( 0 < \xi \leq 1 \) is the efficiency for converting the harvested energy to the electrical energy to be stored, which is assumed to be \( \xi = 1 \) in this paper. During the WIT phase of \((1 - \theta)T\), the received signal at the \( k \)-th legitimate user and the \( l \)-th eavesdropper are given by

\[
y_{s,k} = \sqrt{\frac{E_B}{(1 - \theta)T}} h_{s,k}^H s + n_{s,k}, \quad k = 1, \ldots, K,
\]

\[
y_{e,l} = \sqrt{\frac{E_B}{(1 - \theta)T}} h_{e,l}^H s + n_{e,l}, \quad l = 1, \ldots, L,
\]

where \( s \) denotes the Gaussian distributed transmit signal with unit norm, \( v \in \mathbb{C}^{N_T \times 1} \) is the normalized transmit beamformer with \( \mathbb{E}\{\|v\|^2\} = 1 \), \( n_{s,k} \) and \( n_{e,l} \) are additive white Gaussian noises (AWGNs) at the \( k \)-th legitimate user and the \( l \)-th eavesdropper with variance \( \sigma_s^2 \) and \( \sigma_e^2 \). Hence, the channel capacity of the \( k \)-th legitimate user and the \( l \)-th eavesdropper can be expressed as

\[
R_{s,k} = (1 - \theta) \log \left( 1 + \frac{\theta \sum_{m=1}^{M} p_m \|g_m\|^2 \|h_{s,k}^H v\|^2}{(1 - \theta)\sigma_s^2} \right), \quad \forall k, \tag{2}
\]
and

\[ R_{e,l} = (1 - \theta) \log \left( 1 + \frac{\theta \sum_{m=1}^{M} p_m \|g_m\|^2 |h_{e,l}^H v|^2}{(1 - \theta) \sigma_e^2} \right), \forall l, \]  

respectively. For this secure multicasting system, we have the following definition:

**Definition 1:** Multicast secrecy rate of a multicasting system with \( K \) users is defined as [35]

\[ R_K = \min_{k \in [1, K]} [R_{s,k} - \max_{l \in [1, L]} R_{e,l}]^+. \]  

In the following, we develop two resource allocation schemes depending on whether the transmitter and the PBs belong to the same service supplier.

**III. POWER MINIMIZATION FOR WPCN MULTIANTEenna SECURE MULTICASTING SYSTEM**

In this section, we consider the scenario that the transmitter and the PBs belong to the same service provider in a multicasting network. We will formulate a power minimization problem where the total transmit power at the transmitter is minimized to satisfy the target secrecy rate for all the legitimate users in the presence of multiple eavesdroppers by using the harvested energy from the PBs. We assume that the channel state information (CSI) between the transmitter and \( k \)-th user as well as \( l \)-th eavesdropper (i.e., \( h_{s,k}, \forall k \) and \( h_{e,l}, \forall l \)) is available at the transmitter. This can be achieved through different methods such as the local oscillator power leakage from the eavesdropper receivers’ RF frontend [38] or even the CSI feedback method [39]. For example, in a video broadcasting system there may be legitimate users that are entitled to receive the content and other users who have not subscribed to this content, but are still part of the system. These users obey the basic physical-layer protocol rules, which includes feeding back CSI to enable beamforming. Hence, in this case, it is practical to assume that the CSI of the eavesdroppers is known at the transmitter.

**A. Power Minimization**

The minimization of the total transmit power \(^1\) subject to a given secure multicast rate constraint can be written as:

\[ \min_{p_m, \nu, \theta} \sum_{m=1}^{M} p_m \|g_m\|^2 \frac{\theta}{(1 - \theta)} \|v\|^2, \]

\[ s.t. \quad R_K \geq \bar{R}, \quad 0 \leq p_m \leq P, \quad 0 < \theta < 1, \]  

where \( P \) is the maximum power constraint at each PB, and \( \bar{R} \) denotes the target secure multicast rate. The problem \((5)\) is not convex in terms of \( v \) and \( \theta \), and cannot be solved directly. It is obvious that from

\(^1\)The objective function in \((5)\) consists of three variables: the transmit power of the PBs (i.e., \( p_m, \forall m \)), energy transfer time allocation (i.e., \( \theta \)) and normalized transmit beamforming vector (i.e., \( v \)).
problem (5), the optimal solution of \( p_m \) is \( p_m = P \), which is the maximum transmit power at each PB. Thus, the problem (5) reduces to

\[
\min_{w,\theta} \frac{\sum_{m=1}^{M} \|g_m\|^2 \theta}{(1 - \theta)} \|w\|^2,
\]

\[s.t. \ R_K(\theta, w) \geq \bar{R}, \ 0 < \theta < 1, \tag{6}\]

where \( R_K(\theta, w) \) can be written as

\[
R_K(\theta, w) = \min_{k \in [1,K]} (1 - \theta) \left[ \log \left( 1 + \frac{\theta \sum_{m=1}^{M} \|g_m\|^2 |h_{s,k}^H w|^2}{(1 - \theta)\sigma_s^2} \right) - \max_{l \in [1,L]} \log \left( 1 + \frac{\theta \sum_{m=1}^{M} \|g_m\|^2 |h_{e,l}^H w|^2}{(1 - \theta)\sigma_e^2} \right) \right]^+,
\]

and \( w = \sqrt{P}v \).

**Remark 1:** Problem (6) may be infeasible, depending on the value of \( \bar{R} \) and on the channel vectors \( h_{s,k} \) and \( h_{e,l} \). However, the feasibility can be ensured with probability of 1 for the case where the users’ and eavesdroppers’ channels are drawn randomly and independently from a continuous distribution with full-rank covariance matrix (e.g., a proper Gaussian non-degenerate \( N_T \)-variate distribution) and the number of antennas \( N_T \) is not smaller than \( K + L \) (e.g., in the massive multiantenna systems).

It can be observed that even when problem (6) is feasible, it still still nonconvex. In order to solve this problem, we considered a two-level approach, where the inner level can be handled by employing the convex conic reformulation for a given energy transfer time allocation factor \( \theta \), whereas the outer level consists of a line search over the parameter \( \theta \) in \((0,1)\). Now, we rewrite the problem (6) into the following two levels:

1) Inner level:

\[
f(\theta) = \min_w \frac{\theta(\sum_{m=1}^{M} \|g_m\|^2)}{1 - \theta} \|w\|^2, \ s.t. \ R_K(w) \geq \bar{R}, \tag{7}\]

2) Outer level:

\[
\min_{\theta} f(\theta), \ s.t. \ 0 < \theta < 1. \tag{8}\]

We first solve the inner problem (7) for a given \( \theta \), which can be equivalently modified as

\[
\min_w \|w\|^2
\]

\[s.t. \ \log \left( 1 + \frac{|h_{s,k}^H w|^2}{\sigma_s^2} \right) - \log \left( 1 + \frac{|h_{e,l}^H w|^2}{\sigma_e^2} \right) \geq \bar{R}, \ \forall \ (k,l) \tag{9}\]

\( ^{2}\)In this case, it is always possible to use zero-forcing beamforming to all eavesdroppers, such that for sufficiently large \( P \) any target secrecy rate \( \bar{R} \) can be achieved. The condition of sufficient transmit power \( P \) can be guaranteed because of the fact that the PBs and the transmitter belong to the same service provider and they can work together on an adaptive power transfer policy (i.e., \( P \) is adjusted according to the required solution \( w \) such that \( P = \|w\|^2 \)).
where \( \bar{\sigma}_s^2 = \frac{(1-\theta)\sigma_s^2}{\theta \sum_{m=1}^M \| g_m \|^2} \), \( \bar{\sigma}_e^2 = \frac{(1-\theta)\sigma_e^2}{\theta \sum_{m=1}^M \| g_m \|^2} \), and \( \bar{R} = \frac{R}{1-\theta} \). The problem (9) is not convex in terms of \( w \), and still cannot be solved efficiently. In order to circumvent this non-convex issue, we can introduce a new semi-definite matrix \( Q_s = ww^H \), and the problem (9) can be relaxed as

\[
\min_{Q_s \succeq 0} \text{Tr}(Q_s),
\]

s.t.
\[
\frac{1}{\bar{\sigma}_s^2} \text{Tr}(h_{s,k}h_{s,k}^H Q_s) - \frac{2\bar{R}}{\bar{\sigma}_e^2} \text{Tr}(h_{e,l}h_{e,l}^H Q_s) \geq 2\bar{R} - 1, \quad \forall (k,l).
\]

The above problem is a convex optimization framework, which can be solved by using interior-point methods [40]. It is not always possible to expect that the optimal solution of (10) attains the optimum of the original problem (9). However, the SDP relaxation will be tight if the optimal solution of (10) is of rank-one. Therefore, we characterize the rank property of the SDP relaxed solution via the following proposition:

**Proposition 1:** [34, Theorem 1] Provided the problem (10) is feasible, the optimal solution of (10) must satisfy the following rank inequality:

\[
\text{rank}(Q_s) \leq \min(K, \sqrt{KL})
\]

With Proposition 1, we are able to identify the tightness of SDP relaxed solution via the following lemma

**Lemma 1:** [34, Corollary 1] Provided the problem (10) feasible, it is guaranteed that (10) can yield a rank-one solution which exactly solves the problem (9) when either of the following conditions is satisfied:

1) \( K = 1 \) and \( L \geq 1 \).
2) \( 1 < K \leq 3 \), and \( L = 1 \).

By exploiting Proposition 1 and Lemma 1 if rank \( (Q_s) \) satisfies the rank-one condition in Lemma 1, we can employ the eigen-decomposition for \( Q_s \) to obtain the optimal transmit beamformer \( w \). Otherwise, we need to use a rank reduction algorithm to tackle this problem [41]. However, when rank-one condition in Lemma 1 is satisfied, we can also consider the following theorem to directly solve the problem (9).

**Theorem 1:** The problem (9) can be reformulated into the following convex optimization framework when the rank-one condition in Lemma 1 is satisfied.

\[
\min_{t \geq 0, w} t,
\]

\[
s.t. \begin{bmatrix} t \\ w \end{bmatrix} \succeq_{K(N_T+1)} \mathbf{0}, \begin{bmatrix} \frac{1}{\bar{\sigma}_s} w^H h_{s,k} \\
\frac{2\bar{R}}{\bar{\sigma}_e} w^H h_{e,l} \\
(2\bar{R} - 1) \frac{1}{2} \end{bmatrix} \succeq_{K_3} \mathbf{0}, \quad \forall (k,l).
\]

**Proof:** Please refer to Appendix A.

By exploiting Theorem 1, it is easily verified that (12) is a convex optimization problem, which can be solved by using interior-point methods [40]. Thus, the optimal transmit power of each PB can be adaptively
updated as $P_{\text{opt}} = \|w\|^2$, and the optimal transmit beamforming vector $v_{\text{opt}}$ can also be easily achieved. Now, we consider the computation complexity of solving problem (12). According to the analysis of the basic complexity elements in [42], problem (12) includes one second-order cone (SOC) constraint with dimension $N_T + 1$, $KL$ SOC constraints with dimension $N_T$, and one linear constraint. Thus, its computation complexity can be given by $O\left(\sqrt{2KL + 3n[KLN_T^2 + (N_T + 1)^2 + 1 + n^2]}\right) \ln\left(\frac{1}{\epsilon}\right)$, where $n = O(N_T + 1)$, and $\epsilon > 0$ denotes the accuracy requirement.

A special case: Consider the case with a single legitimate user and a single eavesdropper only. The closed-form solution can be derived by exploiting Lagrange dual problem and KKT conditions. For notational convenience, we replace the channel notations $h_{s,k}$ and $h_{e,l}$ by $h_s$ and $h_e$, respectively. The following lemma is introduced:

**Lemma 2:** The optimal solution to (7) with only single legitimate user and single eavesdropper is given by

$$w_{\text{opt}} = \sqrt{P_{\text{opt}}}v_{\text{opt}}, \quad v_{\text{opt}} = \frac{\bar{w}}{\|\bar{w}\|_2}, \quad w = v_{\max}\left(\frac{1}{\sigma_s^2}h_sh_s^H - \frac{2\hat{R}}{\sigma_e^2}h_eh_e^H\right),$$

$$P_{\text{opt}} = \alpha_{\text{opt}}(2\hat{R} - 1), \quad \alpha_{\text{opt}} = \frac{1}{\theta_{\max}\left(\frac{1}{\sigma_s^2}h_sh_s^H - \frac{2\hat{R}}{\sigma_e^2}h_eh_e^H\right)}.$$ (13)

**Proof:** See Appendix B.

Now, it is natural that a question may arise with regard how to tackle with the problem (9) when the rank-one condition in Lemma 1 is not satisfied. Hence, we consider a SCA based scheme to reformulate the problem (9) for any general case, yielding an SOCP. We equivalently rewrite the problem (9) by introducing a new set of variables $(x_{s,k}, y_{s,k}, b_{s,k})$, $\forall k$ as

$$\min_{w,b_{s,k}} \|w\|^2,$$

s. t. $x_{s,k}^2 + y_{s,k}^2 \geq b_{s,k}$,

$$(2\hat{R} - 1)\sigma_s^2 + \frac{2\hat{R}\sigma_s^2}{\sigma_e^2}|w^Hh_{e,l}|^2 \leq b_{s,k},$$

$$x_{s,k} = \Re\{w^Hh_{s,k}\}, \quad y_{s,k} = \Im\{w^Hh_{s,k}\},$$ (14a)

$$x_{s,k}^2 + y_{s,k}^2 = q_{s,k}^Tq_{s,k} \approx \|u_{s,k}\|^2 + 2\sum_{i=1}^2 u_{s,k}^{(n)}[q_{s,k}(i) - u_{s,k}^{(n)}(i)],$$ (15)
where the parameter vector $u^{(n)}_{s,k}(i)$ can be updated $u^{(n+1)}_{s,k} = q^{(n)}_{s,k}$, $\forall k$ at the $(n+1)$-th iteration. From this update, it is easily verified that $u^{(n)}_{s,k}$ can be determined by $q^{(n-1)}_{s,k}$. Thus, (15) can be given by

$$\|u^{(n)}_{s,k}\|^2 + 2\sum_{i=1}^{2}u^{(n)}_{s,k}[q_{s,k}(i) - u^{(n)}_{s,k}(i)] \geq b_{s,k}. \quad (16)$$

To proceed, the constraint (14b) can be equivalently reformulated into the following SOC

$$\left[\begin{array}{c}(2\hat{R} - 1)\frac{1}{2}\sigma_s \\ \frac{2\hat{R}}{\sigma_e}w^H h_{e,l}\end{array}\right]^2 \leq b_{s,k} \Rightarrow \left[\begin{array}{c}(2\hat{R} - 1)\frac{1}{2}\sigma_s \\ \frac{2\hat{R}}{\sigma_e}w^H h_{e,l}\end{array}\right] \leq \frac{(b_{s,k} + 1)}{2} \quad (17)$$

**Remark 2:** The convexity of the term $q^T_{s,k}q_{s,k}$ and the first-order Taylor approximation ensures that the right hand side in (13) bounds the left side in each iterative procedure. In other words, the optimal solution of the problem with the approximate constraint in (16) definitely belongs to the feasible set of the original optimization problem at each iteration. Also, due to the above update, in the $(n+1)$ step, the approximation in (16) holds with equality. In addition, the gradients of both sides with respect to the optimization variables in (16) are also the same at the $(n+1)$-th iteration, which can prove that the solution of the iterative procedure satisfies the KKT conditions of the original problem (9). This fact has been proved in [43].

By exploiting Remark 2, the problem (9) takes the following form at the $n$-th iteration

$$\min_{w,b_{s,k}} \|w\|^2, \quad s.t. \ (16), (17), (14c), \ \forall (k,l). \quad (18)$$

Based on the above discussion, an iterative algorithm to approximately solve the problem in (9) is summarized as in **Algorithm 1**.

**Algorithm 1: Successive convex approximation to solve (9)**

1) **Initialization:** Randomly generate $u^{(0)}_{s,k}$, $\forall k$ to make (18) feasible

2) **Repeat**
   a) **Solve** (18).
   b) **Set** $u^{(n+1)}_{s,k} = q^{(n)}_{s,k}$, $\forall k$.
   c) **Set** $n := n + 1$.

3) **Until** required accuracy is achieved or the maximum number of iterations is reached.

From **Algorithm 1** the initialized vector $u_{s,k}$ is given by random generation to guarantee the feasibility of (18), which can be updated at each iteration until $u^{(n+1)}_{s,k} = q^{(n)}_{s,k}$ holds when the algorithm converges. In addition, it is guaranteed that **Algorithm 1** converges to a locally optimal solution (quite close to the
globally optimal solution) [44], [45].

Next, we focus on the outer level problem in (8). This is a single-variable optimization problem with respect to $\theta$, which is rewritten as follows:

$$\min_{\theta} f(\theta), \quad s.t. \ 0 < \theta < 1. \tag{19}$$

We can show that $f(\theta)$ is a convex optimization problem with respect to $\theta$ via the following lemma:

**Lemma 3:** (19) is a convex problem with respect to $\theta$.

**Proof:** See Appendix C. 

By exploiting Lemma 3, the optimal energy transfer time allocation factor (i.e., $\theta^{\text{opt}}$) can be obtained by using one-dimensional (1D) single-variable search (e.g., golden search) in the interval $\theta \in [0, 1]$. Then, the optimal power allocation policy can be achieved.

**IV. Game theory based Secure WPCN Multiantenna Multicasting System**

In the previous section, we considered that the transmitter and the PBs belong to the same service provider, in which they try to work together to achieve their common target. However, this is not always the case so that, in this section, we consider the opposite scenario where the transmitter and the PBs are from two different service providers. Both parties want to maximize their own benefit. To model this scenario, we assume that the transmitter will have to pay for the energy services from the PBs, whereas the PBs will consider this payment as incentives to provide wireless energy transfer service. Obviously, being able to decide what price to pay for the energy service, the transmitter can take a leading role in dictating the energy trading interaction. This fits very well the model of a *Stackelberg* game, which motivates us to use this game theory to optimize both parties’ benefit. In this game model, the transmitter (leader) first pays for the harvested energy with an energy price to maximize its utility function. Then, the PBs (followers) optimize their transmit powers based on their released energy price to maximize their individual utility function.

**A. Stackelberg Game Formulation**

Let $\lambda$ denote the energy price that the transmitter will pay to the PBs. The total payment of the transmitter to the $M$ PBs, denoted by $Q_M$, is written as

$$Q_M = \lambda T \sum_{m=1}^{M} p_m \| \mathbf{g}_m \|^2, \tag{20}$$

where $p_m$ denotes the transmit power of the $m$th PB. Without loss of generality, we can assume $T = 1$.

We now define the utility function of the transmitter as follows:

$$U_M = \mu R_K - Q_M, \tag{21}$$
where $\mu > 0$ is the weight per a unit of secrecy throughput, by which the transmitter uses to convert the achievable secrecy rate $R_K$ into the equivalent revenue. Therefore, the leader game for the transmitter can be formulated as

$$\max_{\lambda, \theta, v} U_M, \ s.t. \ 0 < \theta < 1, \ \lambda \geq 0.$$  \hspace{1cm} (22)

At the same time, each PB can be modelled as a follower that wants to maximize its own revenue function, which is defined as follows:

$$U_{PB,m} = \theta (\lambda p_m \|g_m\|^2 - F_m(p_m)),$$  \hspace{1cm} (23)

where $F_m(p_m)$ is used to model the cost of the $m$-th PB per unit time for wirelessly charging the transmitter with the transmit power $p_m$. In this paper, we consider the following quadratic model for the cost function of the PBs:

$$F_m(x) = A_m x^2 + B_m x$$  \hspace{1cm} (24)

where $A_m > 0$ and $B_m > 0$ are the constants that can be different for each PB. Thus, the follower game of $m$-th PB is given by

$$\max_{p_m} U_{PB,m}, \ s.t. \ p_m \geq 0.$$  \hspace{1cm} (25)

Both (22) and (25) form a *Stackelberg* game for this secure WPCN multiantenna multicasting system, where the transmitter (leader) announces an energy price, and then the PBs (followers) optimize the transmit power based on the released energy price to maximize their individual revenue functions. The solution of this *Stackelberg* game can be obtained by investigating the *Stackelberg* equilibrium, where the transmitter and the PBs come to an agreement on the energy price, the transmit power of each PB and the time fraction of energy transfer duration. Note that the deviation of either the transmitter or the PBs from the *Stackelberg* equilibrium will introduce a loss in their revenue functions.

### B. Solution of The Proposed Stackelberg Game

In order to derive the solution of this game, the well-known *Stackelberg* equilibrium concept can be defined as follows:

**Definition 2:** Let $(\theta^{opt}, \lambda^{opt})$ denote the solutions of problem (22) while $\{p^{opt}_m\}$ represents the solution of problem (25) (here, the brackets $\{\}$ indicate a vector that include all $p_m$’s with $\forall m$). Then, the triple-

\[1\]

The quadratic model has been commonly used in the energy market to model the energy cost \[46\].
variable set \((\theta^{\text{opt}}, \lambda^{\text{opt}}, p_m^{\text{opt}})\) is a Stackelberg equilibrium of the formulated game provided that the following conditions are satisfied

\[
U_M(\theta^{\text{opt}}, \lambda^{\text{opt}}, \{p_m^{\text{opt}}\}) \geq U_M(\theta, \lambda, \{p_m^{\text{opt}}\}), \tag{26}
\]

\[
U_{PB,m}(\theta^{\text{opt}}, \lambda^{\text{opt}}, p_m^{\text{opt}}) \geq U_{PB,m}(\theta^{\text{opt}}, \lambda^{\text{opt}}, p_m), \quad \forall m. \tag{27}
\]

for \(0 < \theta < 1\), \(\lambda \geq 0\), and \(p_m \geq 0\), \(\forall m\).

First, it can be observed that problem (25) is convex with respect with \(p_m\) for given values of \(\lambda\) and \(\theta\). Thus, the optimal solution is obtained as in the following theorem:

**Theorem 2:** For given \(\lambda\) and \(\theta\), the optimal solution to (25) is given by

\[
p_m^{\text{opt}} = \begin{cases} \frac{\lambda \|g_m\|^2 - B_m}{2A_m}, & \lambda > \frac{B_m \|g_m\|^2}{2A_m} \\ 0, & \lambda \leq \frac{B_m \|g_m\|^2}{2A_m}. \end{cases} \tag{28}
\]

**Proof:** The proof of this theorem is achieved by equating the first derivative of (23) to zero.

From Theorem 2 we can deduce the following remark:

**Remark 3:** The optimal power allocation \(p_m^{\text{opt}}\) can only be obtained under the condition that the energy price \(\lambda\) is greater than threshold \(\frac{B_m \|g_m\|^2}{2A_m}\). Thus, we divide the PBs into two sets, namely, the active and non-active PBs. The PBs who can transfer the power to the transmitter and help determine the achievable secrecy rate by using the harvested energy are called active PBs. The remaining ones are non-active PBs. Generally, according to (28), we can determine these active PBs and re-index them for the total power computation. In our paper, for convenience, we assume that all PBs are active and considered to be involved in the WET phase of \(\theta T\).

By exploiting Theorem 2 and Remark 3 we replace \(p_m\) in (22) with (28), problem (22) now becomes

\[
\max_{\lambda, \theta} \mu(1 - \theta) \left[ \log \left[ 1 + \left( \lambda \sum_{m=1}^{M} \frac{\|g_m\|^4}{2A_m} - \sum_{m=1}^{M} \frac{B_m \|g_m\|^2}{2A_m} \right) t_s \right] 
- \log \left[ 1 + \left( \lambda \sum_{m=1}^{M} \frac{\|g_m\|^4}{2A_m} - \sum_{m=1}^{M} \frac{B_m \|g_m\|^2}{2A_m} \right) t_e \right] 
\right]
- \theta \lambda^2 \sum_{m=1}^{M} \frac{\|g_m\|^4}{2A_m} + \theta \lambda \sum_{m=1}^{M} \frac{B_m \|g_m\|^2}{2A_m}
\]

s.t. \(0 < \theta < 1\), \(\lambda \geq 0\) \(\tag{29}\)

where

\[
t_s = \min_k t_{s,k} = \min_k \frac{\theta |h_{s,k}^H v|^2}{(1 - \theta) \sigma_s^2}, \quad \forall k, \quad t_e = \max_l t_{e,l} = \min_l \frac{\theta |h_{e,l}^H v|^2}{(1 - \theta) \sigma_e^2}, \quad \forall l.
\]

Note that before solving problem (29), the normalized transmit beamforming vector \(v\) can be obtained by employing a similar approach as in Section III Thus, we solve for the optimal solution to the energy
price $\lambda$ and the energy transfer time allocation $\theta$ only in leader level game (29). The problem (29) is not jointly convex in terms of $\theta$ and $\lambda$. It is extremely hard to find their optimal solutions simultaneously due to the complexity of the objective function in (29). In order to address this issue, we first derive the closed-form solution for the optimal $\lambda$ with a given value $\theta$. Then, the optimal value for $\theta$ can be achieved through numerical analysis. In order to derive the closed-form solution of $\lambda$, we set $C_M = \sum_{m=1}^{M} \frac{\|g_m\|^4}{2A_m}$, $D_M = \sum_{m=1}^{M} B_m \|g_m\|^2$, and the objective function to (29) can be expressed as

$$U_M(\theta, \lambda) = \mu (1 - \theta) \left[ \log \left( 1 + \left( \lambda C_M - 2D_M \right) t_s \right) - \log \left( 1 + \left( \lambda C_M - 2D_M \right) t_e \right) \right] - \theta \lambda^2 C_M + 2\theta \lambda D_M. \quad (30)$$

**Lemma 4:** (30) is a concave function with respect to $\lambda$.

**Proof:** See Appendix D. \qed

By exploiting Lemma 4, we can claim that (29) is a convex problem. Now, we derive the closed-form solution of $\lambda$. In order to obtain the optimal solution to $\lambda$, let the first order derivative of (30) equal to zero, we have

$$\frac{\mu (1 - \theta) C_M}{1 + (\lambda C_M - D_M) t_s - D_M t_s} - \frac{\mu (1 - \theta) C_M}{1 + (\lambda C_M - D_M) t_e - D_M t_e} - 2\theta C_M \lambda + 2\theta D_M = 0. \quad (31)$$

Set $x = \lambda C_M - D_M$, and after a few of mathematical simplifications, we arrive at

$$x^3 + ax^2 + bx + c = 0, \quad (32)$$

where

$$a = \frac{(t_s + t_e) - 2D_M t_s t_e}{t_s t_e}, \quad b = \frac{(D_M t_s - 1)(D_M t_e - 1)}{t_s t_e}, \quad c = -\mu (1 - \theta) M(t_s - t_e).$$

It is easily observed that (32) is a cubic equation, which can be solved in terms of closed-form solution of $x$ by using Cardano’s formula [47],

$$x_{\text{opt}} = e^{j \angle x_1} \sqrt{|x_1|} + e^{j \angle x_2} \sqrt{|x_2|} - a/3, \quad (33)$$

where $\angle$ denotes the phase angle of an complex random variable, and

$$x_1 = -\frac{q}{2} + \sqrt{\Delta}, \quad x_2 = -\frac{q}{2} - \sqrt{\Delta},$$

$$\Delta = \frac{p^3}{27} + \frac{q^2}{4}, \quad p = -\frac{a^2}{3} + b, \quad q = \frac{2a^3}{27} - \frac{ab}{3} + c.$$

Thus, we obtain the optimal energy price as

$$\lambda_{\text{opt}} = \frac{x_{\text{opt}} + D_M}{C_M}. \quad (34)$$
We have already obtained the optimal energy price of the transmitter for a given $\theta$. Now, the optimal energy transfer time allocation is derived in the following. We substitute the closed-form expression (34) into (30), thus, the following optimization problem can be written with respect to $\theta$,

$$
\max_{\theta} U_M(\theta, \lambda_{\text{opt}}), \text{s.t. } 0 < \theta < 1.
$$

(35)

The problem (35) can be efficiently handled by using 1D search to obtain the optimal energy transfer time solution $\theta_{\text{opt}}$ as follows:

$$
\theta_{\text{opt}} = \arg \max_{\theta \in (0,1)} U_M(\theta, \lambda_{\text{opt}}).
$$

(36)

We have completed the derivation of the Stackelberg equilibrium $(p_{\text{opt}}^m, \lambda_{\text{opt}}, \theta_{\text{opt}})$ for the formulated Stackelberg game, which are shown in (28), (33) and (36).

V. NUMERICAL RESULTS

In this section, we provide the simulation results to validate the proposed schemes. We consider the secure multiantenna multicasting system that consists of three legitimate receivers (i.e., $K = 3$) and two eavesdroppers ($L = 5$), where the transmitter is wirelessly powered by five PBs ($M = 5$). It is assumed that the transmitter is equipped with eight transmit antennas (i.e., $N_T = 8$), whereas the others consist of single antenna. We employ the path loss channel model $\sqrt{A d_x^{-\alpha}}$, where $A = 10^{-3}$. The path loss exponent is set to $\alpha = 3$. Distance variable $d_x$ can be replaced with $d_s$, $d_e$, and $d_{PB}$ according to different channel coefficients, representing the distance between the transmitter and the legitimate users, the eavesdroppers as well as the PBs, respectively. In our simulation, we choose $d_s = d_e = 2$ m, and $d_{PB} = 5$ m unless specified. The target secrecy rate is set to be $\bar{R} = 2$ bps/Hz. The noise powers at the legitimate users and the eavesdroppers are set as $\sigma_s^2 = \sigma_e^2 = 10^{-8}$ mW.

First, we evaluate the minimized transmit power obtained from our proposed scheme in Section III against the distance between PBs and the transmitter (i.e., $d_{PB}$). Fig. 2 shows the result with general case when the rank-one condition in Lemma 1 is not satisfied. From this result, one can observe that our proposed SCA scheme outperforms the SDP scheme in [35]. This is owing to a fact that our proposed scheme can achieve a optimal solution via Algorithm 1 whereas the SDP scheme cannot satisfy the rank-one condition such that the relaxed solution cannot achieve optimality. Fig. 3 evaluates the results of Theorem 1, where it is observed that the SOCP yields the same performance with the proposed SCA scheme and SDP scheme, which confirms that the correctness and accuracy of Theorem 1 Fig. 4 shows the impact of the energy time allocation. From this figure, we observe that the proposed SOCP scheme with the fixed time allocation based scheme (i.e., $\theta = 0.5$) that obviously requires more transmit power than the proposed SOCP scheme with optimal energy time allocation, this is owing to a fact that our
Fig. 2: Comparison of transmit power between the proposed scheme and SDP scheme versus distance between PBs and transmitter.

The proposed scheme can achieve an optimal energy time allocation by numerical search (i.e., $\theta = \theta^{\text{opt}}$). Similar behaviours are observed in Fig. 5 which is obtained for the special case of single user and single eavesdropper. This figure shows that the derived closed-form solution in Lemma 2 matches well with the numerical results obtained from a convex optimization tool, which validates the accuracy of this closed-form solution. In Fig. 6, we compare the run time of our proposed scheme and the SDP relaxation with Guided Randomization [48] versus the number of transmit antenna $N_T$. From this result, one can observe that our proposed scheme consumes less time than SDP with Guided Randomization, which implies that the proposed algorithm has a lower computational complexity.

Next, we validate the equilibrium of the proposed Stackelberg game. In order to support the derived Stackelberg equilibrium, we first evaluate the utility function of the transmitter versus the energy transfer price $\lambda$ with a fixed energy transfer time allocation $\theta$ in Fig. 7. From this figure, it is observed that the revenue function is concave, which validates the proof of convexity shown in Lemma 4. In this figure, it also can be shown that the optimal utility function of the transmitter can be obtained via optimal energy transfer price $\lambda^{\text{opt}}$ in [54] and it matches the numerical search with different given $\theta$, which confirms the

3Guided Randomization is employed to tackle with the scenario that the SDP relaxed solution is not rank-one.
optimal closed-form solution of the energy transfer price $\lambda$. Also, as $\theta$ increases, the utility function of the legitimate transmitter is decreasing, and the optimal value $\lambda$ shifts to the left. In addition, the revenue function of the transmitter versus energy transfer time allocation (i.e., $\theta$) with optimal energy price $\lambda^{opt}$ is shown in Fig. 8. From this figure, it is shown that the revenue function is concave with respect to $\theta$, which validates (36). Moreover, there exists a optimal utility transfer time (i.e., $\theta^{opt}$) via numerical search with the optimal energy price. As $\mu$ increases, the optimal value slightly shifts to the left.

Then, we evaluate the transmitter revenue function performance of the proposed Stackelberg game. Fig. 9 and Fig. 10 show the revenue function of the transmitter versus the number of PBs. From both figures, we can observe that this utility is improved with increasing of the number of PBs and $\mu$. From Fig. 9, increasing the number of the eavesdroppers can have more significant impact on the revenue than increasing the number of the legitimate users. In Fig. 10, the revenue is decreased when the distance between the source and PBs is increased from 5m to 6.5m. This is because the nearer the PBs to the transmitter, the higher the transferred energy efficiency between them, which reduces the transmitter’s payments to the PBs for their wireless energy services. Fig. 11 shows the optimal energy transfer time $\theta$ versus the number of PBs. It is observed that $\theta$ decreases as either the number of the PBs or $\mu$ increase. The same behavior is observed when the distance between the source and PBs (i.e., $d_{PB}$) decreases.
Fig. 4: Comparison of transmit power between the proposed scheme with optimal $\theta$ and fixed $\theta$ versus distance between PBs and transmitter.

Fig. 5: Transmit power versus distance between PBs and transmitter with special case.
Fig. 6: Run time versus the number of transmit antenna $N_T$.

Fig. 7: The utility function of transmitter $U_M$ versus energy transfer price $\lambda$
Fig. 8: The utility function of transmitter $U_M$ versus energy transfer time allocation $\theta$

Fig. 9: The utility function of transmitter $U_M$ versus No. of PBs with different number of legitimate users and eavesdroppers
Fig. 10: The utility function of transmitter $U_M$ versus No. of PBs with different $d_{PB}$

Fig. 11: Optimal energy transfer time allocation $\theta$ versus No. of PBs
shows the optimal energy price versus the number of PBs. The price decreases as the number of PBs increases. Besides, the larger $\mu$, the higher optimal energy price needs to be paid. It can also be seen from this figure that the decrease of the distance between the transmitter and PBs can also reduce the optimal energy price. This is because the shorter the distance between the source and PBs, the more energy harvested by the transmitter for the same power transmitted by the PBs, such that a lower energy price can be paid by the transmitter.

![Optimal energy price versus No. of PBs](image)

**Fig. 12:** The optimal energy price versus No. of PBs.

**VI. CONCLUSION AND FUTURE WORK**

In this paper, we investigated the secure wireless-powered multiantenna multicasting system, which consists of a dedicated wireless powered networks with multiple PBs and a multiantenna secure multicasting system. We first investigated the power minimization problem to obtain the optimal power and energy transfer time allocation when both wireless power and secure information networks belong to the same service supplier. As this problem is not convex, we proposed a two-level approach, where the inner level problem can be relaxed as a convex optimization framework via iterative conic convex reformulation (i.e., SCA based SOCP scheme), while the outer level problem can be handled by using an 1D search. Next, we formulate a *Stackelberg* game for the case when the WET and secure WIT networks belong
to different service suppliers. We formulated the energy trading process between the transmitter and the PBs as a Stackelberg game, in which the transmitter plays a leader role and pays a price for the energy services from the PBs to guarantee the required security, and optimizes the energy price and the energy transfer time to maximize its utility function. Meanwhile, the PBs are modelled as the followers that determine their optimal transmit powers based on the released energy price to maximize their own utility function. The Stackelberg equilibrium have been derived in terms of closed-form solution, where both the transmitter and the PBs come to an agreement on the energy price, transmit power and energy transfer time. Simulation results have been provided to validate the proposed schemes. For future works, we can consider a more challenging scenario such that the PBs will not stay silent during the WIT phase but rather transmit artificial noise to interfere with the eavesdroppers. This would change the dynamic of the optimization problems and may require different design/solutions.

APPENDIX

A. Proof of Theorem 1

When the solution $Q_s$ satisfies rank-one conditions in Lemma 1, the SDP relaxed problem (10) can be equivalently modified as

$$\min_\mathbf{w} \|\mathbf{w}\|^2,$$

s.t. $\frac{g}{\sigma_e^2} |\mathbf{w}^H \mathbf{h}_{e,l}|^2 + (2 \bar{R} - 1) \leq \frac{1}{\sigma_s^2} |\mathbf{w}^H \mathbf{h}_{s,k}|^2, \forall (k, l). \tag{37}$$

The problem (37) is still not convex in terms of the beamforming vector $\mathbf{w}$. Thus, we propose a novel reformulation method by applying the following inequality relations in (37),

$$\begin{bmatrix} a \\ \mathbf{b} \end{bmatrix} \succeq_K \mathbf{0} \Leftrightarrow \|\mathbf{b}\| \leq a, \tag{38}$$

such that (37) can be reformulated into the following SOCP:

$$\min_{t, \mathbf{w}} t,$$

s.t. $\begin{bmatrix} t_1 \\ \mathbf{w} \end{bmatrix} \succeq_{K_{N+1}} \mathbf{0}, \begin{bmatrix} \frac{1}{\sigma_s} \mathbf{w}^H \mathbf{h}_{s,k} \\ \frac{g}{\sigma_e} \mathbf{w}^H \mathbf{h}_{e,l} \\ (2 \bar{R} - 1)^\frac{1}{2} \end{bmatrix} \succeq_{K_3} \mathbf{0}, \forall k, \forall l. \tag{39}$$

Thus, this completes the proof of Theorem 1.
B. Proof of Lemma 2

The power minimization problem (9) with only single legitimate user and eavesdropper can be rewritten as

\[ \min_w w^H w \]
\[ s.t. \quad \log \left( 1 + \frac{|h_s^H w|^2}{\sigma_s^2} \right) - \log \left( 1 + \frac{|h_e^H w|^2}{\sigma_e^2} \right) \geq \bar{R}. \]  

(40)

Now we equivalently modify (40) as

\[ \min_{v, P} P v^H v \]
\[ s.t. \quad v^H (I + \frac{P}{\sigma_s^2} h_s h_s^H) v \geq 2 \bar{R}, \quad v^H v = 1, \quad P \geq 0. \]  

(41)

In order to achieve the optimal solution \((P^{opt}, v^{opt})\), we consider the Lagrange dual problem to (40), which can be expressed as

\[ L(w, \mu) = w^H w + \alpha \left[ 2 \bar{R} \left( 1 + \frac{1}{\sigma_s^2} w^H h_s h_s^H w \right) - \left( 1 + \frac{1}{\sigma_e^2} w^H h_e h_e^H w \right) \right], \]

(42)

where \(\alpha \geq 0\) is the Lagrange multiplier associated with the secrecy rate constraint. The corresponding dual problem can be given by

\[ \max_{\alpha \geq 0} \alpha (2 \bar{R} - 1), \quad s.t. \quad Y = I - \alpha \left( \frac{1}{\sigma_s^2} h_s h_s^H - \frac{1}{\sigma_e^2} h_e h_e^H \right) \succeq 0. \]

(43)

The constraint in (43) implies that the matrix \(Y\) have at least one zero eigenvalue. On the other hand, the solution of \(\alpha\) can be the maximum value that satisfies the positive semidefinite constraint in (43), which leads to

\[ \alpha^{opt} = \frac{1}{\ell_{max} \left( \frac{1}{\sigma_s^2} h_s h_s^H - \frac{2 \bar{R}}{\sigma_e^2} h_e h_e^H \right)}. \]

(44)

The problem (40) can be formulated as a convex optimization problem. Hence, the strong duality holds between the original problem (40) and the corresponding dual problem (43). The required minimum power to achieve the secrecy rate constraint is

\[ P^{opt} = \alpha^{opt} (2 \bar{R} - 1). \]  

(45)

On the other hand, it is easily verified that the optimal \(w\) lies in the null space of \(Y\)

\[ \tilde{w} = v_{\max} \left( \frac{1}{\sigma_s^2} h_s h_s^H - \frac{2 \bar{R}}{\sigma_e^2} h_e h_e^H \right), \quad v^{opt} = \frac{\tilde{w}}{\|\tilde{w}\|_2}. \]

(46)

Hence, the optimal solution to (40) can be expressed as

\[ w^{opt} = \sqrt{P^{opt}} v^{opt}. \]  

(47)
C. Proof of Lemma 3

In order to show that \( f(\theta) \) is a convex problem, there are two conditions to be satisfied: convex objective function and convex constraints [40]. First, we rewrite \( f(\theta) \) as follows:

\[
f(\theta) = \min_{\theta} \frac{\theta (\sum_{m=1}^{M} ||g_m||^2)}{1 - \theta} ||w||^2,
\]

\[
\text{s.t. } (1 - \theta) \log \left( \frac{1 - \theta + \theta t_{s,k}}{1 - \theta + \theta t_{e,l}} \right) \geq R, 0 < \theta < 1, \forall k, \forall l. \tag{48}
\]

It is easily verified that the objective function in (48) is convex with respect to \( \theta \). Then, we show that the secrecy constraint in (48) is convex. Let \( f_{k,l}(\theta) = (1 - \theta) \log \left( \frac{1 - \theta + \theta t_{s,k}}{1 - \theta + \theta t_{e,l}} \right) \), where \( t_{s,k} = \frac{|h_{s,k}^e w|^2}{\sigma^2_e} \) and \( t_{e,l} = \frac{|h_{e,l}^s|^2}{\sigma^2_e} \). Hence, the remaining part we only show that \( f_{k,l}(\theta) \) is a concave function with respect to \( \theta \), which can be equivalently written as

\[
f_{k,l}(\theta) = (1 - \theta) \log \left( \frac{1 - \theta + \theta t_{s,k}}{1 - \theta + \theta t_{e,l}} \right) = (1 - \theta) \log \left( \frac{1 - \theta + \theta t_{s,k}}{1 - \theta + \theta t_{e,l}} \right).
\]

(49)

Let \( z = 1 - \theta, (0 \leq \theta \leq 1) \), \( f_{k,l}(z) \) can be rewritten as

\[
f_{k,l}(z) = z \log \left( \frac{1 - t_{s,k}}{z} + t_{s,k} \right) - z \log \left( \frac{1 - t_{e,l}}{z} + t_{e,l} \right). \tag{50}
\]

Then, we consider the first derivative of \( f_{k,l}(z) \),

\[
\frac{\partial f_{k,l}(z)}{z} = \frac{1}{\ln 2} \left[ \left( \ln \left( \frac{1 - t_{s,k}}{z} + t_{s,k} \right) + \frac{-t_{s,k}}{(1 - t_{s,k}) z + t_{s,k}} \right) - \left( \ln \left( \frac{1 - t_{e,l}}{z} + t_{e,l} \right) + \frac{-t_{e,l}}{(1 - t_{e,l}) z + t_{e,l}} \right) \right]. \tag{51}
\]

Furthermore, the second derivative of \( f_{k,l}(z) \) is given by

\[
\frac{\partial^2 f_{k,l}(z)}{z^2} = \frac{1}{\ln 2} \left[ \frac{-t_{s,k}^2}{(1 - t_{s,k}) z + t_{s,k}} z - \frac{-t_{e,l}^2}{(1 - t_{e,l}) z + t_{e,l}} z \right]. \tag{52}
\]

Let \( g(t) = \frac{-t^2}{[(1-t)z+t]^2} \), the first derivative of \( g(t) \) is given by

\[
\frac{\partial g(t)}{t} = \frac{-2t[(1-t)z+t]}{[(1-t)z+t]^3 z} = \frac{-2zt}{[(1-t)z+t]^3 z} < 0. \tag{53}
\]

It is easily verified that (53) holds since \( z = 1 - \theta \in (0, 1) \), thus, \( g(t) \) is a monotonically decreasing function of \( t \). Due to \( t_{s,k} > t_{e,l} \), it is easily obtained that \( \frac{\partial^2 f_{k,l}(z)}{z^2} < 0 \). In other words, \( f_{k,l}(\theta) \) is a concave function with respect to \( \theta \).
D. Proof of Lemma 4

We first derive the first-order derivatives of (30), which is written as

$$\frac{\partial U_M(\lambda)}{\partial \lambda} = \frac{1}{\ln 2} \left[ \frac{\mu (1 - \theta) t_s C_M}{1 + (\lambda C_M - 2 D_M) t_s} - \frac{\mu (1 - \theta) t_e C_M}{1 + (\lambda C_M - 2 D_M) t_e} \right] - 2 C_M \lambda + 2 D_M. \tag{54}$$

Then, the second-order derivatives of (30) is given by

$$\frac{\partial^2 U_M(\lambda)}{\partial^2 \lambda} = \frac{1}{\ln 2} \left[ \frac{\mu (1 - \theta) C_M^2 (t_e^2 - t_s^2)}{[1 - (\lambda C_M - 2 D_M) t_s]^2 [1 - (\lambda C_M - 2 D_M) t_e]^2} \right] - 2 C_M < 0. \tag{55}$$

The above inequality holds since $t_s - t_e > 0$ to guarantee the minimum achievable secrecy rate is greater than zero. Thus, (30) is a concave function.

REFERENCES

[1] N. D. Sidiropoulos, T. N. Davidson, and Z.-Q. Luo, “Transmit beamforming for physical-layer multicasting,” IEEE Transactions on Signal Processing, vol. 54, no. 6, pp. 2239–2251, Jun. 2006.

[2] Z. Xiang, M. Tao, and X. Wang, “Coordinated multicast beamforming in multicell networks,” IEEE Trans. Wireless Commun., vol. 12, no. 1, pp. 12–21, Jan. 2013.

[3] Z. Xiang, M. Tao, and X. Wang, “Massive mimo multicasting in noncooperative cellular networks,” IEEE J. Sel. Area. Comm., vol. 32, no. 6, pp. 1180–1193, Jun. 2014.

[4] L. Varshney, “Transporting information and energy simultaneously,” in Proc. 2008 IEEE Int. Symp. Inf. Theory, pp. 1612–1616, July, 2008.

[5] P. Grover and A. Sahai, “Shannon meets tesla: Wireless information and power transfer,” in Proc. 2010 IEEE Int. Symp. Inf. Theory, pp. 2363–2367, Jun., 2010.

[6] R. Zhang and C. K. Ho, “MIMO broadcasting for simultaneous wireless information and power transfer,” IEEE Trans. Wireless Commun., vol. 12, pp. 1989–2001, May 2013.

[7] S. Bi, Y. Zeng, and R. Zhang, “Wireless powered communication networks: an overview,” IEEE Wireless Commun., vol. 23, no. 2, pp. 10–18, Apr. 2016.

[8] H. Ju and R. Zhang, “Throughput maximization in wireless powered communication networks,” IEEE Trans. Wireless Commun., vol. 13, no. 1, pp. 418–428, Jan. 2014.

[9] H. Ju and R. Zhang, “User cooperation in wireless powered communication networks,” in Proc. IEEE GLOBECOM, pp. 1430–1435, Dec. 2014.

[10] H. Chen, X. Zhou, Y. Li, P. Wang, and B. Vucetic, “Wireless-powered cooperative communications via a hybrid relay,” in Proc. IEEE Information Theory Workshop (ITW), pp. 666–670, Nov. 2014.

[11] H. Chen, Y. Li, J. L. Rebelatto, B. F. Ucha-Filho, and B. Vucetic, “Harvest-then-cooperate: Wireless-powered cooperative communications,” IEEE Trans. Signal Process., vol. 63, no. 7, pp. 1700–1711, Apr. 2015.

[12] K. Huang and X. Zhou, “Cutting the last wires for mobile communications by microwave power transfer,” IEEE Commun. Mag., vol. 53, no. 6, pp. 86–93, Jun. 2015.

[13] K. Huang and V. K. N. Lau, “Enabling wireless power transfer in cellular networks: Architecture, modeling and deployment,” IEEE Trans. Wireless Commun., vol. 13, no. 2, pp. 902–912, Feb. 2014.

[14] C. Zhong, G. Zheng, Z. Zhang, and G. K. Karagiannidis, “Optimum wirelessly powered relaying,” IEEE Signal Process. Lett., vol. 22, no. 10, pp. 1728–1732, Oct. 2015.

[15] A. D. Wyner, “The wire-tap channel,” Bell Syst. Tech. Journ., vol. 54, pp. 1355–1387, Jan. 1975.
[16] I. Csiszár and J. Körner, “Broadcast channels with confidential messages,” *IEEE Trans. Inform. Theory*, vol. 24, pp. 339–348, May 1978.

[17] T. Liu and S. Shamai, “A note on the secrecy capacity of the multiple-antenna wiretap channel,” *IEEE Trans. Inform. Theory*, vol. 55, no. 6, pp. 2547–2553, Jun. 2009.

[18] A. Khisti and G. W. Wornell, “Secure transmission with multiple antennas I: The MISOME wiretap channel,” *IEEE Trans. Inform. Theory*, vol. 56, no. 7, pp. 3088–3104, Jul. 2010.

[19] A. Khisti and G. W. Wornell, “Secure transmission with multiple antennas II: The MIMOME wiretap channel,” *IEEE Trans. Inform. Theory*, vol. 56, no. 11, pp. 5515–5532, Nov. 2010.

[20] J. Li and A. P. Petropulu, “On ergodic secrecy rate for gaussian MISO wiretap channels,” *IEEE Trans. Wireless Commun.*, vol. 10, pp. 1176–1187, Apr. 2011.

[21] S. A. A. Fakoorian and A. L. Swindlehurst, “Full rank solutions for the MIMO gaussian wiretap channel with an average power constraint,” *IEEE Trans on Signal Process.*, vol. 61, no. 10, pp. 2620–2631, May 2013.

[22] Q. Li and W.-K. Ma, “Optimal and robust transmit designs for MISO channel secrecy by semidefinite programming,” *IEEE Trans. Signal Process.*, vol. 59, no. 8, pp. 3799–3812, Aug. 2011.

[23] Q. Li and W.-K. Ma, “Spatially selective artificial-noise aided transmit optimization for MISO multi-eves secrecy rate maximization,” *IEEE Trans. Signal Process.*, vol. 61, no. 10, pp. 2704–2717, May 2013.

[24] Z. Chu, K. Cumanan, Z. Ding, M. Johnston, and S. Le Goff, “Secrecy rate optimizations for a MIMO secrecy channel with a cooperative jammer,” *IEEE Trans. Vehicular Technol.*, vol. 64, no. 5, pp. 1833–1847, May 2015.

[25] Z. Chu, K. Cumanan, Z. Ding, M. Johnston, and S. L. Goff, “Secrecy rate optimization for a MIMO secrecy channel based on stackelberg game,” in *Proc. European Signal Processing Conference (EUSIPCO), Lisbon, Portugal*, pp. 126–130, Sept. 2014.

[26] Z. Chu, K. Cumanan, Z. Ding, M. Johnston, and S. Le Goff, “Robust outage secrecy rate optimizations for a MIMO secrecy channel,” *IEEE, Wireless Commun. Lett.*, vol. 4, no. 1, pp. 86–89, Feb. 2015.

[27] Z. Chu, H. Xing, M. Johnston, and S. L. Goff, “Secrecy rate optimizations for a MISO secrecy channel with multiple multiantenna eavesdroppers,” *IEEE Trans. Wireless Commun.*, vol. 15, no. 1, pp. 283–297, Jan. 2016.

[28] A. Al-Talabani, Y. Deng, A. Nallanathan, and H. X. Nguyen, “Enhancing secrecy rate in cognitive radio networks via stackelberg game,” *IEEE Trans. Commun.*, vol. 64, no. 11, pp. 4764–4775, Nov. 2016.

[29] Q. Yang, H. M. Wang, Y. Zhang, and Z. Han, “Physical layer security in MIMO backscatter wireless systems,” *IEEE Trans. Wireless Commun.*, vol. 15, no. 11, pp. 7547–7560, Nov. 2016.

[30] L. Liu, R. Zhang, and K.-C. Chua, “Secrecy wireless information and power transfer with MISO beamforming,” *IEEE Trans. Signal Process.*, vol. 62, no. 7, pp. 1850–1863, Apr. 2014.

[31] D. W. K. Ng, E. S. Lo, and R. Schober, “Robust beamforming for secure communication in systems with wireless information and power transfer,” *IEEE Trans. Wireless Commun.*, vol. 13, no. 8, pp. 4599–4615, Aug. 2014.

[32] M. Khandaker and K. Wong, “Masked beamforming in the presence of energy-harvesting eavesdroppers,” *IEEE Trans. Inf. Forensics Security*, vol. 10, pp. 40–54, Jan 2015.

[33] Z. Chu, Z. Zhu, M. Johnston, and S. Y. L. Goff, “Simultaneous wireless information power transfer for MISO secrecy channel,” *IEEE Trans. Vehicular Technol.*, vol. 65, no. 9, pp. 6913–6925, Sept. 2016.

[34] Q. Li and W. K. Ma, “Multicast secrecy rate maximization for MISO channels with multiple multi-antenna eavesdroppers,” in *Proc. IEEE, ICC, Tokyo, Japan*, pp. 1–5, Jun. 2011.

[35] K. Cumanan, Z. Ding, M. Xu, and H. V. Poor, “Secrecy rate optimization for secure multicast communications,” *IEEE J. Sel. Topics Signal Process.*, vol. 10, no. 8, pp. 1417–1432, Dec. 2016.

[36] S. Shafiee and S. Ulukus, “Achievable rates in gaussian MISO channels with secrecy constraints,” in *Proc. IEEE ISIT, Nice, France*, pp. 2466–2470, Jun. 2007.

[37] Y. Wu, X. Chen, C. Yuen, and C. Zhong, “Robust resource allocation for secrecy wireless powered communication networks,” *IEEE Commun. Lett.*, vol. 20, no. 12, pp. 2430–2433, Dec. 2016.
[38] A. Mukherjee and A. L. Swindlehurst, “Detecting passive eavesdroppers in the MIMO wiretap channel,” in *IEEE ICASSP, Tokyo, Japan*, Mar. 2012.

[39] G. Geraci, M. Egan, J. Yuan, A. Razi, and I. B. Collings, “Secrecy sum-rates for multi-user MIMO regularized channel inversion precoding,” *IEEE Trans. Commun.*, vol. 60, no. 11, pp. 3472–3482, Nov. 2012.

[40] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, UK: Cambridge University Press, 2004.

[41] Y. Huang and D. P. Palomar, “Rank-constrained separable semidefinite programming with applications to optimal beamforming,” *IEEE Trans. Signal Process.*, vol. 58, no. 2, pp. 664–678, Feb. 2010.

[42] K. Y. Wang, A. M. C. So, T. H. Chang, W. K. Ma, and C. Y. Chi, “Outage constrained robust transmit optimization for multiuser MISO downlinks: Tractable approximations by conic optimization,” *IEEE Trans. Signal Process.*, vol. 62, no. 21, pp. 5690–5705, Nov. 2014.

[43] B. R. Marks and G. P. Wright, “A general inner approximation algorithm for nonconvex mathematical programs,” *Oper Res.*, vol. 26, no. 4, pp. 681–683, 1978.

[44] L. N. Tran, M. F. Hanif, A. Toli, and M. Juntti, “Fast converging algorithm for weighted sum rate maximization in multicell MISO downlink,” *IEEE Signal Process. Lett.*, vol. 19, no. 12, pp. 872–875, Dec. 2012.

[45] L. N. Tran, M. F. Hanif, and M. Juntti, “A conic quadratic programming approach to physical layer multicasting for large-scale antenna arrays,” *IEEE Signal Process. Lett.*, vol. 21, no. 1, pp. 114–117, Jan. 2014.

[46] A. H. Mohsenian-Rad, V. W. S. Wong, J. Jatskevich, R. Schober, and A. Leon-Garcia, “Autonomous demand-side management based on game-theoretic energy consumption scheduling for the future smart grid,” *IEEE Trans. Smart Grid*, vol. 1, no. 3, pp. 320–331, Dec. 2010.

[47] X. Huang, J. He, Q. Li, Q. Zhang, and J. Qin, “Optimal power allocation for multicarrier secure communications in full-duplex decode-and-forward relay networks,” *IEEE Commun. Lett.*, vol. 18, no. 12, pp. 2169–2172, Dec. 2014.

[48] J. Liao, M. R. A. Khandaker, and K. K. Wong, “Robust power-splitting SWIPT beamforming for broadcast channels,” *IEEE Commun. Lett.*, vol. 20, no. 1, pp. 181–184, Jan. 2016.