Double Moving Average Control Chart for Zero-Truncated Poisson Distribution

Y Areepong¹ and C Chananet¹,*

¹Department of Applied Statistics, Faculty of Applied Science, King Mongkut’s University of Technology North Bangkok, Bangkok, Thailand

Corresponding author: chanaphun.c@sci.kmutnb.ac.th

Abstract. The objective of this paper is to present an explicit formula for Double Moving Average (DMA) control charts when the observation follows a Zero-Truncated Poisson (ZTP) distribution. The popular characteristics of a control chart are Average Run Length (ARL0) for in-control process and Average Run Length (ARL1) for out-of-control process. Whereas ARL0 is the mean of observations taken before, a system gives an out-of-control signalled. For ARL1 is the expected number of observations taken from an out-of-control process until the control chart is shown the out-of-control signal. The ARL results indicated that the DMA control chart for Zero-Truncated Poisson distribution (DMAZTP) is superior to the MA control chart for Zero-Truncated Poisson distribution (MAZTP) for detecting all magnitudes of shifts. This explicit formula is accurate, as well as easy to be implemented and used by practitioners

1. Introduction

Statistical Process Control (SPC) charts are widely used for monitoring, controlling, measuring, and improving quality of production in many areas of applications including epidemiology and health surveillance, engineering, and others. Attribute control charts, such as p, np, c, and u control charts are important tools in statistical process control to monitor processes with discrete data. When the quality characteristic cannot be measured on a continuous scale, for instance, the number of nonconformities in a production process or in counting the number of defective products, an attribute control chart must be used. Additionally, CUSUM and EWMA control charts were also developed for attribute data (see, Page [1] and Alwan [2]). Recently, the Moving Average control chart (MA) has been introduced for both discrete and continuous processes (see, Khoo [3] and Adeoti and Olaomi [4]). To date, numerous extensions of the MA chart have been proposed. Khoo and Wong [5] extended the MA control chart to be DMA control chart by repeating the moving average of the MA statistic. They proposed this chart with normal observations and also shown the numerical results of ARL using Monte Carlo simulations. The results shown that the performance of theDMA chart is quicker to detect out-of-control signals to the MA, EWMA and CUSUM control charts for monitoring small and moderate shifts for process mean. In addition, Areepong [6] proposed the explicit formulas of ARL for moving average control chart in order to monitor the number of defective products.

In general, probability distributions often arise in practice which is of the Poisson distribution, but in which the zero value is unobserved called the zero-truncated Poisson (ZTP) distribution. This may occur in the situations when the observational apparatus becomes active when at least one event occurs, such as, the number of an increase in the incidence rate, accidents per workers in a factory, the number of surface defects in x-ray film, etc. The performance of control charts for ZTP distribution has been
investigated by several authors. The cumulative sum (CUSUM) and Shewhart control charts for ZTP were developed by Chakraborty and Kaloty [7] and Chakraborty and Singh [8], respectively. More recently, Balamuarii and Kalyanasundaram [9] have developed design and implementation procedures of CUSUM control schemes based on zero truncated Poisson distribution.

A common characteristic used for comparing the performance of control charts is Average Run Length (ARL) defined as the expected value of the number of observations taken from an in-control process until the control chart falsely signals out-of-control which is denoted by \( ARL_0 \). An \( ARL_0 \) will be regarded as acceptable if it is large enough to keep the level of false alarms at an acceptable level. The second common characteristic is the expected number of observations taken from an out-of-control process until the control chart signals that the process is out-of-control which is denoted by \( ARL_1 \). Ideally, the \( ARL_1 \) should be as small as possible. Many methods for evaluating the \( ARL_0 \) and \( ARL_1 \) for control charts have been studied in the literature. One of the most used methods is the Monte Carlo simulation technique (MC). Another method is the Markov chain approach (MCA), see Brook and Evans [10] and the recent technique to evaluate the \( ARL_0 \) and \( ARL_1 \) is based on the integral equation approach (IE), see Crowder [11]. Recently, the explicit formulas of \( ARL_0 \) and \( ARL_1 \) for MA control chart are presented by Areepong and Sukparungsee [12]. In this paper, we presented the explicit formula of Average Run Length (ARL) of the DMA control chart when observations are Zero-truncated Poisson. The rest of this paper is organized as follows. In the next section, descriptions of the characteristics of control charts are presented. The explicit formulas and the numerical results are presented in Sections 3 and 4 respectively. Finally, the discussion and some conclusions are presented in Section 5.

2. Double Exponential Weighted Moving Average Control Chart

Statistical Process Control (SPC) charts are considered under the assumption that sequential observations \( X_1, X_2, ..., X_m \) of a process are identically independently distributed random variables with a distribution function \( F(x, \lambda) \), where \( \lambda \) is a control parameter. It is assumed that \( \lambda = \lambda_0 \) while the process is in-control and that \( \lambda = \lambda_1 > \lambda_0 \) when the process goes out-of-control. It is assumed that there is a change-point time \( \theta \leq \infty \) in which the parameter changes from \( \lambda_0 \) to \( \lambda_1 \). Note that \( \theta = \infty \) means that the process always remains in the in-control state.

Let \( E_o(\cdot) \) denote the expectation that the change-point from \( \lambda_0 \) to \( \lambda_1 \) for a distribution function \( F(x, \lambda) \) occurs at time \( \theta \), where \( \theta \leq \infty \). The quantity \( E_o(\tau) \) is called the Average Run Length (ARL) of the control chart for the given process.

A typical condition imposed on an \( ARL_0 \) is that:
\[
ARL_0 = E_o(\tau) = T,
\]
where \( T \) is given (usually large). A typical definition of the \( ARL_1 \) is that
\[
ARL_1 = E_i(\tau | \tau \geq 1),
\]
for the change point occurs at \( \theta = 1 \). One could expect that a sequential control chart has a near optimal performance if \( ARL_1 \) is close to a minimal value.

Let observations \( X_1, X_2, ..., X_m \) be independent random variables with Zero-truncated Poisson distribution, where \( X_i \) number of nonconforming is items in sample \( i \) of \( m \) samples of size \( n \). The Zero-truncated Poisson density function can be written as Johnson, Kemp and Kotz [13].
\[
f(X = x; \lambda) = \frac{\lambda^x}{(e^\lambda - 1)x!}
\]
for $x = 1, 2, \ldots$, where $\lambda > 0$.

The mean and variance are $E(X) = \frac{\lambda e^x}{e^x - 1}$ and $V(X) = \frac{\lambda^2}{1 - e^{-\lambda}} - \frac{1}{(1 - e^{-\lambda})^2}$, respectively.

A Moving Average control chart for Zero-truncated Poisson distribution ($MA_{ZTP}$) is defined by the following statistics:

$$MA_i = \begin{cases} \frac{X_i + X_{i-1} + \ldots + X_{i-w+1}}{i} & ; i < w \\ \frac{X_i + X_{i-1} + \ldots + X_{i-w+1}}{w} & ; i \geq w \end{cases}$$

(4)

where $w$ is the width of $MA_{ZTP}$ chart. The mean and variance of $MA_{ZTP}$ control chart are $E(MA_i) = \frac{\lambda e^x}{e^x - 1}$ and $V(MA_i) = \frac{\lambda^2}{1 - e^{-\lambda}} - \frac{1}{(1 - e^{-\lambda})^2}$.

For period $i \geq w$, the upper and lower control limits are given

$$UCL_i \ / \ LCL_i = \frac{\lambda_0 e^{h_0}}{(e^{h_0} - 1)} \pm H \sqrt{\frac{\lambda_0 + \lambda_0^2}{w} - \frac{\lambda_0^2}{(1 - e^{-\lambda_0})^2}}$$

(5)

where $H$ is a suitable control width limit.

For periods $i < w$, the upper and lower control limits are given

$$UCL_i \ / \ LCL_i = \frac{\lambda_0 e^{h_0}}{(e^{h_0} - 1)} \pm H \sqrt{\frac{\lambda_0 + \lambda_0^2}{i} - \frac{\lambda_0^2}{(1 - e^{-\lambda_0})^2}}$$

(6)

The alarm time for the $MA_{ZTP}$ procedure is given by

$$\tau = \inf\{i > 0 : MA_i > UCL_i \ or \ MA_i < LCL_i \}$$

(7)

A Double Moving Average (DMA) control chart was initially introduced by [4] which is defined by the following statistics:

$$DMA_i = \begin{cases} \frac{MA_i + MA_{i+1} + MA_{i+2} + \ldots}{i} & ; i \leq w \\ \frac{MA_i + MA_{i+1} + \ldots + MA_{i-w+1}}{w} & ; w < i < 2w - 1 \\ \frac{MA_i + MA_{i+1} + \ldots + MA_{i-w+1}}{w} & ; i \geq 2w - 1 \end{cases}$$

(8)

The DMA control chart for Zero-Inflated Poisson distribution is so-called $DMA_{ZTP}$ control chart which the mean of $DMA_{ZTP}$ is $E(DMA_i) = \frac{\lambda e^x}{(e^x - 1)}$ and its variance is
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\[
V(DMA) = \begin{cases} 
\frac{1}{i^2} \left( \frac{\lambda + \lambda^2}{1 - e^{-\lambda}} - \frac{\lambda^2}{(1 - e^{-\lambda})^2} \right) \overbrace{\sum_{j=1}^{i-1} \frac{1}{j}}^{i \leq w} \\
\frac{1}{w^2} \left( \frac{\lambda + \lambda^2}{1 - e^{-\lambda}} - \frac{\lambda^2}{(1 - e^{-\lambda})^2} \right) \overbrace{\sum_{j=i-w+1}^{w-1} \frac{1}{(i-w+1)}}^{w < i < 2w-1} + \left( \frac{1}{w} \right) \\
\frac{1}{w^2} \left( \frac{\lambda + \lambda^2}{1 - e^{-\lambda}} - \frac{\lambda^2}{(1 - e^{-\lambda})^2} \right) \overbrace{\sum_{j=2w-1}^{w-1} \frac{1}{w^2}}^{i \geq 2w-1}
\end{cases}
\]

where \( w \) is the width of the \( DMA_{ZTP} \) control chart.

The upper and lower control limits are given as

\[
UCL_k / LCL_k = \begin{cases} 
\frac{\lambda_0 e^{\lambda_0}}{(e^{\lambda_0} - 1)} \pm LA_{i_{k}} & i \leq w \text{ and } k = 1, \\
\frac{\lambda_0 e^{\lambda_0}}{(e^{\lambda_0} - 1)} \pm LA_{j_{k}} & w < i < 2w-1 \text{ and } k = 2, \\
\frac{\lambda_0 e^{\lambda_0}}{(e^{\lambda_0} - 1)} \pm LA_{k_{k}} & i \geq 2w-1 \text{ and } k = 3,
\end{cases}
\]

where

\[
A_{i_{k}} = \sqrt{\lambda_0 + \frac{\lambda_0^2}{1 - e^{-\lambda_0}} - \frac{\lambda_0^2}{(1 - e^{-\lambda_0})^2} \overbrace{\sum_{j=1}^{i-1} \frac{1}{j}}^{i \leq w}} \quad A_{j_{k}} = \sqrt{\lambda_0 + \frac{\lambda_0^2}{1 - e^{-\lambda_0}} - \frac{\lambda_0^2}{(1 - e^{-\lambda_0})^2} \overbrace{\sum_{j=i-w+1}^{w-1} \frac{1}{(i-w+1)}}^{w < i < 2w-1} + \left( \frac{1}{w} \right) },
\]

\[
A_{k_{k}} = \sqrt{\lambda_0 + \frac{\lambda_0^2}{1 - e^{-\lambda_0}} - \frac{\lambda_0^2}{(1 - e^{-\lambda_0})^2} \overbrace{\sum_{j=2w-1}^{w-1} \frac{1}{w^2}}^{i \geq 2w-1}}
\]

and \( L \) is a suitable control limit width.

The alarm time for the \( DMA_{ZTP} \) procedure is given by

\[
\tau = \inf \{ i > 0 : DMA_i > UCL_k \text{ or } DMA_i < LCL_k \}.
\]

3. The Explicit Formula for \( DMA_{ZTP} \)

Let \( ARL = n \), then

\[
\frac{1}{ARL} = \left( \frac{1}{n} \right) P(\text{o.o.c. signal at time } i \leq w) + \left( \frac{1}{n} \right) P(\text{o.o.c. signal at time } w < i < 2w-1) + \\
\left( \frac{n-(2w-2)}{n} \right) P(\text{o.o.c. signal at time } i \geq w)
\]

\[
= \frac{1}{n} \left[ \sum_{i=1}^{w} \left\{ P \left[ i \sum_{j=1}^{\lambda_0} \frac{\lambda_0 e^{\lambda_0}}{(e^{\lambda_0} - 1)} + LA_{i_k} \right] + P \left[ i \sum_{j=1}^{\lambda_0} \frac{\lambda_0 e^{\lambda_0}}{(e^{\lambda_0} - 1)} < LA_{i_k} \right] \right\} \right] + \\
\]
\[
\frac{1}{n} \sum_{j=1}^{n-w-1} \left[ P \left( \frac{\lambda_j e^{x_j}}{(e^{x_j}-1)} > \frac{\lambda_0 e^{x_0}}{(e^{x_0}-1)} + LA_{j|k} \right) + P \left( \frac{\lambda_j e^{x_j}}{(e^{x_j}-1)} < \frac{\lambda_0 e^{x_0}}{(e^{x_0}-1)} - LA_{j|k} \right) \right] \\
+ \left( \frac{n-(2w-2)}{n} \right) \left[ P \left( \frac{\lambda_j e^{x_j}}{(e^{x_j}-1)} > \frac{\lambda_0 e^{x_0}}{(e^{x_0}-1)} + LA_{j|k} \right) + P \left( \frac{\lambda_j e^{x_j}}{(e^{x_j}-1)} < \frac{\lambda_0 e^{x_0}}{(e^{x_0}-1)} - LA_{j|k} \right) \right] \cdot \left( n-(2w-2) \right)
\]

The explicit formula of \( ARL \) is

\[
ARL = 1 - \sum_{j=1}^{n-w-1} \left[ P \left( Z > \frac{\lambda_j e^{x_j}}{(e^{x_j}-1)} - LA_{j|k} - \frac{\lambda_0 e^{x_0}}{(e^{x_0}-1)} \right) \right] + \left( 2w-2 \right), \quad (12)
\]

when

\[
A_{j|k} = \sqrt{\frac{1}{w^2} \left( \frac{\lambda_{j+1} + \lambda_{j+2}}{1-e^{-\lambda_j}} - \frac{\lambda_0}{(1-e^{-\lambda_0})^2} \right) \left( \sum_{i=j}^{j+1} \frac{1}{j-w+1} \left( \frac{1}{w} \right) \right)},
\]

\[
A_{j|k} = \sqrt{\frac{1}{w^2} \left( \frac{\lambda_{j+1} + \lambda_{j+2}}{1-e^{-\lambda_j}} - \frac{\lambda_0}{(1-e^{-\lambda_0})^2} \right) \left( \sum_{i=j}^{j+1} \frac{1}{j-w+1} \left( \frac{1}{w} \right) \right)}.
\]

4. Numerical Results

In this section, the numerical results of \( ARL \) for \( DMA_{ZTP} \) control chart were calculated from Equation 12 as shown in Tables 1, 2. The parameter values of \( DMA_{ZTP} \) control chart were chosen by given desired \( ARL_0 = 370 \) and 500, in-control parameter \( \lambda_0 = 3, 5 \) and shift parameters \( (\delta) \) are varied from 0.1 to 5 where \( \lambda_0 = \lambda_0 + \delta \). For the process of a small change it was found that the control chart was effective as \( w \) increased. For an example, in Table 1, when \( \delta = 0.1, ARL_0 = 370 \) and 500, \( DMA_{ZTP} \) chart with \( w = 25 \)
showed the best performance due to the given minimum ARL. However when the moderate and large change process, as the DMAZTP control chart was found to be effective for small w. For an example, in Table 1, when \( \delta = 5.0, ARL_0 = 370, \) and 500, DMAZTP control chart with \( w = 2 \) showed the best performance. Additionally, it showed similar results as in Table 2. The comparison of control charts between MAZTP and the DMAZTP control charts is presented in Tables 3 and 4. In Table 3, the parameter values of DMAZTP and MAZTP control charts were chosen by given desired ARL\(_0 = 370, \) in control parameter \( \lambda_0 = 5, 10 \) and given desired ARL\(_0 = 500 \) in Table 4. The results showed that for all value of \( w \) the DMAZTP control chart performed better than the MAZTP control chart for all magnitude of shifts.

5. Conclusion
The explicit formula was derived for the ARL in a Double Moving Average control chart for observations from a zero-truncated Poisson distribution. This formula was found to be accurate and easy to use for a computer program. Thus, it is suggested that the explicit formula for the ARL of DMAZTP control chart can be applied to an empirical data and real-world situation applications for a variety of data processes such as in medical, economics, finance, agriculture, environmental, etc. These issues should be addressed in future research. Furthermore, this explicit formula for ARL can be developed for other control charts such as Triple Moving Average control chart, etc.

| \( \delta \)  |
| 2  | 3  | 5  | 10 | 15 | 25 |
|------|----|----|----|----|----|
| 0.0  | 370.398 | 370.398 | 370.398 | 370.398 | 370.398 |
| 0.1  | 288.537 | 270.868 | 226.125 | 127.005 | 78.233 | 57.505 |
| 0.3  | 148.310 | 104.791 | 51.878 | 22.116 | 25.721 | 41.406 |
| 0.5  | 73.857 | 41.445 | 17.253 | 15.059 | 21.837 | 28.346 |
| 0.7  | 39.421 | 19.199 | 8.995 | 5.894 | 9.795 | 16.614 |
| 0.9  | 23.035 | 10.457 | 4.173 | 2.052 | 2.852 | 41.406 |
| 1.0  | 18.204 | 8.175 | 5.894 | 2.852 | 41.406 |
| 2.0  | 4.345 | 3.084 | 6.467 | 3.084 | 3.084 | 3.084 |
| 3.0  | 2.529 | 2.516 | 2.749 | 2.757 | 2.757 | 2.757 |
| 4.0  | 1.944 | 2.024 | 2.052 | 2.052 | 2.052 | 2.052 |
| 5.0  | 1.632 | 1.667 | 1.669 | 1.669 | 1.669 | 1.669 |

| \( \delta \)  |
| 2  | 3  | 5  | 10 | 15 | 25 |
|------|----|----|----|----|----|
| 0.0  | 370.398 | 370.398 | 370.398 | 370.398 | 370.398 |
| 0.1  | 288.537 | 270.868 | 226.125 | 127.005 | 78.233 |
| 0.3  | 148.310 | 104.791 | 51.878 | 22.116 | 25.721 |
| 0.5  | 73.857 | 41.445 | 17.253 | 15.059 | 21.837 |
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| 0.9  | 23.035 | 10.457 | 4.173 | 2.052 | 2.852 |
| 1.0  | 18.204 | 8.175 | 5.894 | 2.852 | 41.406 |
| 2.0  | 4.345 | 3.084 | 6.467 | 3.084 | 3.084 |
| 3.0  | 2.529 | 2.516 | 2.749 | 2.757 | 2.757 |
| 4.0  | 1.944 | 2.024 | 2.052 | 2.052 | 2.052 |
| 5.0  | 1.632 | 1.667 | 1.669 | 1.669 | 1.669 |

Table 1. ARL\(_0\) and ARL\(_1\) for DMAZTP Control Chart when Given \( \lambda_0 = 3. \)
Table 2. $ARL_0$ and $ARL_1$ for $DMA_{ZTP}$ Control Chart when Given $\lambda_0 = 5$.

| $ARL_0 = 370$ |  |
| --- | --- |
| **Shift** | 2 | 3 | 5 | 10 | 15 | 25 |
| $\delta$ | | | | | | |
| 0.0 | 370.398 | 370.398 | 370.398 | 370.398 | 370.398 | 370.398 |
| 0.1 | 321.097 | 307.338 | 270.135 | 171.943 | 110.199 | **70.027** |
| 0.3 | 207.458 | 157.396 | 85.583 | 30.858 | **28.498** | 43.640 |
| 0.5 | 123.123 | 74.213 | 30.566 | **17.231** | 23.887 | 35.792 |
| 0.7 | 73.348 | 37.338 | **14.506** | 14.543 | 20.627 | 24.093 |
| 0.9 | 45.424 | 20.591 | **8.985** | 12.977 | 16.195 | 15.777 |
| 1.0 | 36.400 | 15.837 | **7.666** | 12.146 | 14.076 | 13.273 |
| 2.0 | 7.528 | **3.789** | 4.749 | 5.687 | 5.686 | 5.685 |
| 3.0 | 3.568 | **2.935** | 3.550 | 3.615 | 3.615 | 3.615 |
| 4.0 | **2.491** | 2.505 | 2.650 | 2.653 | 2.653 | 2.653 |
| 5.0 | **2.020** | 2.091 | 2.112 | 2.112 | 2.112 | 2.112 |

$ARL_0 = 500$

| 0.0 | 500.619 | 500.619 | 500.619 | 500.619 | 500.619 | 500.619 |
| 0.1 | 430.487 | 411.046 | 358.701 | 222.074 | 136.655 | **78.076** |
| 0.3 | 271.632 | 203.525 | 107.559 | 34.831 | **29.635** | 44.206 |
| 0.5 | 157.133 | 92.865 | 36.427 | **17.942** | 24.322 | 36.812 |
| 0.7 | 91.415 | 45.375 | 16.408 | **14.878** | 21.176 | 25.170 |
| 0.9 | 55.403 | 24.349 | **9.717** | 13.292 | 16.805 | 16.494 |
| 1.0 | 43.948 | 18.477 | **8.147** | 12.475 | 14.647 | 13.827 |
| 2.0 | 8.360 | **3.947** | 4.844 | 5.874 | 5.872 | 5.871 |
| 3.0 | 3.772 | **2.992** | 3.652 | 3.728 | 3.728 | 3.728 |
| 4.0 | 2.578 | **2.564** | 2.728 | 2.730 | 2.730 | 2.730 |
| 5.0 | **2.073** | 2.144 | 2.169 | 2.169 | 2.169 | 2.169 |

Note that: bold values are the minimum of $ARL_1$.

Table 3. $ARL_1$ for $DMA_{ZTP}$ and $MA_{ZTP}$ Control Chart, Given $ARL_0 = 370$.

| $\lambda_0$ | $\delta$ | 0.1 | 0.3 | 0.5 | 0.7 | 0.9 | 1.0 | 1.1 | 1.3 | 1.5 | 2.0 |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| $\lambda_0 = 5$ | | DMA | 321.097 | 207.458 | 123.123 | 73.348 | **45.424** | 36.400 | 29.536 | 20.181 | **14.455** | 7.528 |
| w = 2 | MA | 326.903 | 235.996 | 161.04 | 108.433 | 73.835 | 61.423 | 51.424 | 36.786 | 27.045 | 14.031 |
| w = 5 | DMA | **270.135** | 85.583 | 30.566 | 14.506 | 8.985 | 7.666 | 6.813 | 5.860 | 5.387 | **4.749** |
| MA | 318.299 | 195.638 | 110.065 | 63.111 | 38.262 | 30.518 | 24.739 | 17.044 | 12.458 | 7.665 |
| w = 15 | DMA | 110.199 | 28.498 | 23.887 | 20.627 | 16.195 | 14.079 | 12.268 | 9.629 | **7.967** | 5.686 |
| MA | 292.907 | 123.668 | 53.244 | 28.106 | 18.248 | 15.664 | 13.885 | 11.690 | 10.418 | 8.581 |
| w = 25 | DMA | **70.027** | 43.640 | 35.792 | 24.093 | 15.777 | 13.273 | 11.527 | 9.307 | **7.881** | 5.686 |
| MA | 271.786 | 91.996 | 39.233 | 24.156 | 18.940 | 17.598 | 16.632 | 15.238 | 14.107 | 11.317 |
| $\lambda_0 = 10$ | | DMA | **346.029** | 274.218 | 199.705 | 140.293 | 98.056 | 82.264 | 69.288 | 49.862 | **36.655** | 18.719 |
| w = 2 | MA | 349.333 | 295.783 | 238.205 | 186.085 | 143.313 | 125.561 | 110.04 | 84.829 | 65.922 | 36.780 |
| w = 5 | DMA | **314.672** | 150.591 | 66.596 | 32.772 | 18.515 | 14.667 | 12.015 | 8.364 | **7.185** | 5.605 |
| MA | 344.417 | 264.678 | 184.678 | 124.688 | 84.426 | 69.942 | 58.295 | 41.333 | 30.171 | 15.617 |
| w = 15 | DMA | **171.322** | 37.966 | 26.513 | 24.044 | 21.875 | 20.489 | 18.921 | 15.648 | **12.796** | 8.519 |
| MA | 329.197 | 195.222 | 103.733 | 58.033 | 35.782 | 29.232 | 24.502 | 18.468 | 15.051 | 11.186 |
| w = 25 | DMA | **103.49** | 46.121 | 42.263 | 35.946 | 27.380 | 23.393 | 19.960 | 14.967 | **11.987** | 8.390 |
| MA | 315.610 | 155.397 | 74.003 | 41.635 | 28.205 | 24.610 | 22.128 | 19.081 | 17.346 | 14.870 |

Note that: bold values are the minimum of $ARL_1$. 

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Table 4. ARL$_A$ for DMA$_{	ext{ZIP}}$ and $MA_{ZIP}$ Control Chart, Given ARL$_0$ = 500.

| $\lambda_0$ | $w$ | $\delta$ | control chart | 0.1 | 0.3 | 0.5 | 0.7 | 0.9 | 1.0 | 1.1 | 1.3 | 1.5 | 2.0 |
|-------------|-----|---------|---------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $\lambda_0 = 5$ | $w = 2$ | DMA | 430.487 | 271.632 | 157.133 | 91.415 | 55.403 | 43.948 | 35.314 | 23.684 | 16.672 | 8.360 |
|              | MA | 438.718 | 311.040 | 208.039 | 137.384 | 91.885 | 75.800 | 62.954 | 44.360 | 32.163 | 16.181 |
| $w = 5$ | DMA | 358.701 | 107.559 | 36.427 | 16.408 | 9.717 | 8.147 | 7.141 | 6.036 | 5.503 | 4.844 |
|              | MA | 426.512 | 255.351 | 139.675 | 78.028 | 46.173 | 36.403 | 29.179 | 19.675 | 14.098 | 7.676 |
| $w = 15$ | DMA | 136.655 | 29.635 | 24.322 | 21.176 | 16.805 | 14.647 | 12.770 | 9.998 | 8.248 | 5.872 |
|              | MA | 390.476 | 157.292 | 64.462 | 32.400 | 20.157 | 17.010 | 14.871 | 12.283 | 10.830 | 8.852 |
| $w = 25$ | DMA | 78.076 | 44.206 | 36.812 | 25.170 | 16.494 | 13.827 | 11.967 | 9.626 | 8.143 | 5.871 |
|              | MA | 360.431 | 114.152 | 45.298 | 26.287 | 19.925 | 18.343 | 17.234 | 15.705 | 14.529 | 11.728 |
| $\lambda_0 = 10$ | $w = 2$ | DMA | 465.879 | 364.485 | 261.182 | 180.473 | 124.165 | 103.385 | 86.439 | 61.330 | 44.476 | 21.995 |
|              | MA | 470.583 | 394.751 | 314.225 | 242.412 | 184.371 | 160.552 | 139.87 | 106.597 | 81.943 | 44.990 |
| $w = 5$ | DMA | 421.362 | 194.451 | 82.880 | 39.313 | 21.396 | 16.641 | 13.394 | 9.545 | 7.579 | 5.738 |
|              | MA | 463.578 | 351.011 | 240.565 | 159.477 | 106.113 | 87.169 | 72.054 | 50.268 | 36.117 | 18.000 |
| $w = 15$ | DMA | 219.679 | 41.544 | 27.210 | 24.478 | 22.383 | 21.055 | 19.531 | 16.261 | 13.327 | 8.827 |
|              | MA | 441.839 | 254.549 | 130.862 | 70.778 | 42.157 | 33.857 | 27.916 | 20.427 | 16.260 | 11.686 |
| $w = 25$ | DMA | 122.898 | 46.941 | 42.921 | 36.974 | 28.528 | 24.467 | 20.908 | 15.636 | 12.454 | 8.672 |
|              | MA | 422.349 | 199.421 | 90.477 | 48.384 | 31.312 | 26.820 | 23.753 | 20.057 | 18.021 | 15.318 |

Note that: bold values are the minimum of ARL.$_A$

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