Coupled with the power system through power-electronic interfaces, renewable energies including wind power and photovoltaic can control the power quickly and flexibly. In the steady-state stability analysis, by neglecting the fast dynamics of power-electronic interfaces, the renewable energy power is simplified to a static power injection model and can be described as an algebraic equation in the dynamic process. Based on this simplified model, the steady-state stability of sending-end system with mixed synchronous generator and power-electronic-interfaced renewable energy is studied. By proposing a triangular transformation model based on the classical model of power system, the steady-state stability analysis becomes feasible. The mechanism of steady-state stability is revealed, and the influence of renewable energy on the steady-state stability limit is quantitatively investigated. When the renewable energy power increases, the steady-state stability limit of the sending-end system first increases and then decreases. Reducing the power output of synchronous generator can change for a higher integration limit of renewable energy. Simulation results validate the conclusion.

1. Introduction

Coupled with the power system through power-electronic interfaces, renewable energies including wind power and photovoltaic can change the stability of power system. The dynamic equations of power system due to the introduction of power-electronic interfaces become more complex, and it is difficult to analyze the influence of renewable energy on the power system stability. In recent years, with the increasing maturity of detailed models of wind power, photovoltaic, and other renewable energies, the research on the stability of power system with large-scale renewable energy mainly relies on numerical simulation methods [1–8], namely, establishing a detailed mathematical model for simulating the dynamic process of renewable energy, and carrying out the numerical simulation through such stability analysis software as BPA, PSASP, and PSS/E. The numerical simulation method has the advantages of good model detail and high precision and can simulate the response characteristics and dynamic process of renewable energy in detail. In addition, in order to improve the accuracy of the model, uncertainties of decision variables and parameters such as generator set and load scheduling are also considered [9–11]. However, relying solely on a large number of numerical simulation calculations, it is difficult to provide a more explicit physical interpretation for the stability mechanism of power system with renewable energy, such as steady-state stability, transient stability, and voltage stability.

So far, there is still no basic principle that can intuitively explain the stability mechanism of power system with renewable energy, and the research on the stability mechanism is still in the initial stage [12–14]. Different from conventional synchronous generators, the introduction of power-electronic interfaces makes the dynamic equations describing the stability problem become more complex. It is very difficult to give the intuitive physical interpretation of
stability mechanism, even in the low-order dynamic equations.

The simulation results show that the influence of renewable energy on the rotor-angle stability is restricted by factors such as network parameters, outputs of synchronous generators, and renewable energy. The output growth of renewable energy in a certain range is beneficial to the rotor-angle stability.

Sending-end system with mixed synchronous generator, generally thermal generators, and power-electronic-interfaced renewable energy, generally wind power, widely exists in northern China and is a typical form of renewable energy exploitation. In addition, new smart hybrid energy market policies improve system reliability and other incentives [15, 16]. In this paper, the steady-state stability of this type of system is studied by using the classical model of power system. Since the response time constant of renewable energy is much less than the inertia constant of thermal generator, the renewable energy can be approximately equivalent to a constant power load in the steady-state stability analysis. By establishing the differential-algebraic equation (DAE), the mechanism of steady-state stability is revealed, and the steady-state stability limit is quantitatively analyzed in the sending-end system with renewable energy.

2. Classic Model of Sending-End System with Renewable Energy

The power response characteristic of renewable energy is the key to affect the power system stability. Compared with the conventional thermal generator, the power response of renewable energy is controlled by power-electronics interfaces, and the regulating speed is very fast. According to the measured data, the time required for the active and reactive power of double-fed wind turbines entering the steady state is approximately 0.15 s [13], while that of direct-driven wind turbines and photovoltaic generation is shorter, generally only 0.04–0.05 s [14, 17]. The response time constant of renewable energy is far less than the inertia constant of thermal generator, which makes the electromechanical transient process of power system include dynamic processes with different speed and multi-timescale characteristics.

According to the modeling theory of multi-timescale dynamic system [18–20], when the time constants of two dynamic processes vary greatly, through ignoring the fast dynamic, a reduced order dynamic model only retaining the normal rate variables can be established, and the fast dynamic process can be expressed as an algebraic equation. Therefore, when the electromechanical transient process is studied, the fast dynamic of renewable energy with variable regulating speed can be ignored, and the renewable energy is considered to quickly reach the steady state.

The simulation result of a certain sending-end system with renewable energy in China is shown in Figure 1, and it can be seen that in the dynamic process after fault, both wind power and photovoltaic power fast converge to the steady state, and there is no such phenomenon as the electromechanical oscillation of the thermal generator. Therefore, in the study of small disturbance stability, the renewable energy can be approximately equivalent to a constant power injection, and the dynamic process of renewable energy is simplified to algebraic equations [21].

The power system of a thermal generator and a renewable energy generator transmitting power to an infinite bus (see Figure 2(a)) is defined as “renewable-thermal infinite bus sending-end system,” where a classical model is used for the thermal generator, and the transient resistance is also incorporated into the network, and then, after the star-delta transformation, the equivalent circuit (see Figure 2(b)) can be achieved. For this system, the following algebraic differential equations based on the classical model can be formulated:

\[
\begin{align*}
\dot{\delta} &= \omega_0 \omega, \\
T_{jr} \dot{\omega} &= P_m - P_e - D \omega, \\
P_e &= f(\delta, U_r, \varphi_r), \\
P_r &= g(\delta, U_r, \varphi_r), \\
Q_r &= h(\delta, U_r, \varphi_r),
\end{align*}
\]

where \(P_r\) and \(Q_r\) are the active and reactive power injections of the renewable energy, respectively; \(U_r\) and \(\varphi_r\) are the bus voltage amplitude and phase angle of the renewable energy, respectively; \(P_m\) is the prime mover power; \(P_e\) is the electromagnetic power, power angle, angular velocity, synchronous angular velocity, inertia time constant, and inner potential of thermal generator, respectively; \(D\) is the damping coefficient; \(U\) is the voltage of infinite bus.

In equation (1), the electromechanical transient process of the thermal generator with slow speed is described by differential equations, and the regulating process of the renewable energy with fast speed is simplified to algebraic equations. This model is reasonable for the analysis of the steady-state stability of power system. The main advantage of this model is that the model is simple and beneficial to the analysis of stability mechanism, while the disadvantage is that the detailed dynamic process of renewable energy is simplified.

3. Theoretical Analysis of Renewable-Thermal Infinite Bus Sending-End System

3.1. Derivation of Power Flow Equation. In the renewable-thermal infinite bus sending-end system of Figure 2(b), the electromagnetic power of thermal generator can be derived as follows:

\[
P_e = \frac{EU \sin \delta}{x_1} + E \frac{C(\delta, U_r, Q_r) - B(\delta, P_r)}{A(\delta)},
\]

where

\[
A(\delta) = x_1^2 E^2 + x_2^2 U_r^2 + 2 x_2 x_3 E U \cos \delta, \\
B(\delta, P_r) = x_3 P_r (x_1 E + x_2 U \cos \delta), \\
C(\delta, U_r, Q_r) = U (x_2 U_r^2 + x_3 U_r^2 + x_2 x_3 Q_r) \sin \delta.
\]
By solving the power flow equation, \( U_r \) can be derived as follows:

\[
U_r = \frac{F(\delta, Q_r)}{2(x_2 + x_3)^2} + \sqrt{\frac{D(\delta, P_r, Q_r)}{2(x_2 + x_3)^2}},
\]

where

\[
F(\delta, Q_r) = A(\delta) + 2x_2x_3(x_2 + x_3)Q_r,
D(\delta, P_r, Q_r) = F^2(\delta, Q_r) - 4x_2^2x_3^2(x_2 + x_3)^2(P_r^2 + Q_r^2).
\]

(5)

3.2. Necessary Conditions for the Solution of Power Flow Equation. When \( D(\delta, P_r, Q_r) \geq 0 \), equation (4) has solutions, and the following inequality is derived:

\[
P_r \leq \frac{A(\delta)}{2x_2x_3(x_2 + x_3)} = p_r^{\text{Max}}(\delta).
\]

(6)

Equation (6) is a necessary condition for the renewable-thermal infinite bus sending-end system maintaining a stable operation.

Equation (6) shows that the active power transmission capacity of renewable energy is limited by the power angle \( \delta \) of thermal generator and the reactive power \( Q_r \) of renewable energy. When the active power \( P_r \) of renewable energy increases, in order to ensure the system to maintain a stable operation, it needs to increase the reactive power \( Q_r \) or reduce the power angle \( \delta \) of thermal generator.

3.3. Triangular Transformation Model of Renewable-Thermal Infinite Bus Sending-End System. In order to facilitate the description of the problem, this paper studies the case of the reactive power \( Q_r = 0 \), namely, the operation of renewable energy is in the self-balancing state of reactive power, which is a very representative mode of the actual power system operation. Therefore, equation (6) can be simplified as follows:

\[
P_r \leq \frac{A(\delta)}{2x_2x_3(x_2 + x_3)} = p_r^{\text{Max}}(\delta).
\]

Although only the case of \( Q_r = 0 \) is studied, equation (2) is still very complex. Here, a triangular transformation is introduced to determine the relations between the generator electromagnetic power \( P_r \) and the renewable energy \( P_r \).

From equation (7), the renewable energy \( P_r \) must be the values in \([0, p_r^{\text{Max}}(\delta)]\) for any power angle \( \delta \) in order to maintain a stable operation. Therefore, the following triangular transformation can be proposed:

\[
\sin \theta \cdot p_r^{\text{Max}}(\delta) = P_r, \quad \theta \in \left[0, \frac{\pi}{2}\right].
\]

(8)

By substituting equation (8) into equation (2), the following equation can be obtained:

\[
P_r(\delta, \theta) = a \sin \delta + b \sin(\delta - \theta) - c \sin \theta,
\]

where

\[
a = \frac{EU}{x_1} + \frac{EU}{2(x_2 + x_3)},
\]

\[
b = \frac{EU}{2(x_2 + x_3)},
\]

\[
c = \frac{x_3E^2}{2x_2(x_2 + x_3)}.
\]

(9)

Moreover, equation (8) can be simplified as follows:

\[
P_r(\delta, \theta) = (c + d + 2b \cos \delta) \sin \theta,
\]

(11)

where \( d = x_3U^2/2x_2(x_2 + x_3) \).

Obviously, the constraints are derived between \( a, b, c \), and \( d \) as follows:

\[
a > b,\]

\[
c + d \geq 2b.
\]

(12)

Equation (9) and (11) are defined as “the triangular transformation model” of renewable-thermal infinite bus sending-end system. Equation (2) and (7) are called as “the original model” of renewable-thermal infinite bus sending-end system. The following conclusions are obtained:

(a) The variable space set of the original model can be expressed as follows:

\[
M = \{(\delta, P_r) | P_r \leq p_r^{\text{Max}}(\delta), \quad \delta \in [0, \pi]\}.
\]

(13)

The variable space set of the triangular transformation model can be expressed as follows:

\[
N = \{(\delta, \theta) | \delta \in [0, \pi], \quad \theta \in \left[0, \frac{\pi}{2}\right]\}.
\]

(14)

Since any element in the set \( N \) is corresponding to the unique element in the set \( M \), it is the “one-to-one mapping” relations between the set \( M \) and the set \( N \).
(b) In the original model, \( P_e \) is a function of the two nonindependent variables \( P_r \) and \( \delta \), which satisfy the inequality constraint of equation (7); in the triangular transformation model, by converting the inequality constraint of equation (7) to the equality constraint of equation (8), \( P_e \) is transformed into a function of the two independent variables \( \theta \) and \( \delta \).

(c) Compared with the original model, the triangular transformation model is more simplified. More importantly, by transforming the inequality constraint into the equality constraint, the quantitative and analytical analysis of the steady-state stability becomes possible.

(d) In the triangular transformation model, \( P_e(\delta, \theta) \) is a curved surface in three-dimensional space, where \( \delta \in [0, \pi] \) and \( \theta \in [0, \pi/2] \).

4. Steady-State Stability Limit of Renewable-Thermal Infinite Bus Sending-End System

4.1. Steady-State Stability Limit of Thermal Generator

**Theorem 1.** In the triangular transformation model of renewable-thermal infinite bus sending-end system, for any fixed \( \theta \in [0, \pi/2] \), there exists a unique \( \delta_m(\theta) \in (0, \pi) \) which makes \( P_e(\delta, \theta) \) achieve the maximum value \( P_e(\delta_m(\theta), \theta) \); when \( \theta \) changes in \( [0, \pi/2] \), \( P_e(\delta_m(\theta), \theta) \) is monotone decreasing, and \( P_e(\delta_m(\theta), \theta) \) achieves a unique maximum value.

**Proof.** : In equation (9), by fixing the value of \( \theta \), and differentiating equation (9) with respect to \( \delta \), the following equation is obtained:

\[
\frac{dP_e(\delta, \theta)}{d\delta} = a \cos \delta + b \cos(\delta - \theta) = 0. \tag{15}\]

Equation (15) has a unique solution \( \delta_m(\theta) \) in \( (0, \pi) \), which satisfies the following equation:

\[
\sin \delta_m(\theta) = \frac{a + b \cos \theta}{\sqrt{a^2 + b^2 + 2ab \cos \theta}}, \tag{16}\]

\[
\cos \delta_m(\theta) = -\frac{b \sin \theta}{\sqrt{a^2 + b^2 + 2ab \cos \theta}}. \tag{17}\]

From (15) ~ (17), the following result is derived:

\[
\frac{d^2P_e(\delta_m(\theta), \theta)}{d\delta^2} = -\frac{b^2 \sin^2 \theta - 2ab^2 \cos^2 \theta}{\sqrt{a^2 + b^2 + 2ab \cos \theta}^3} < 0. \tag{18}\]

Therefore, \( P_e(\delta_m(\theta), \theta) \) is the unique maximum value of \( P_e(\delta, \theta) \) for \( \theta \in [0, \pi] \). By substituting equations (17) and (19) into equation (9), the maximum electromagnetic power \( P_e(\delta_m(\theta), \theta) \) is derived as a function of \( \theta \):

\[
P_e(\delta_m(\theta), \theta) = -c \sin \theta + \sqrt{a^2 + b^2 + 2ab \cos \theta}. \tag{19}\]

By substituting equation (17) into equation (11), the renewable energy \( P_e(\delta_m(\theta), \theta) \) corresponding to the maximum electromagnetic power \( P_e(\delta_m(\theta), \theta) \) is derived as a function of \( \theta \):

\[
P_r(\delta_m(\theta), \theta) = (c + d) \sin \theta - \frac{2b^2 \sin^2 \theta}{\sqrt{a^2 + b^2 + 2ab \cos \theta}} \tag{20}\]

Furthermore, when \( \theta \) changes in \( [0, \pi/2] \), the following results can be proved:

(a) Differentiate equation (19) with respect to \( \theta \):

\[
\frac{dP_e(\delta_m(\theta), \theta)}{d\theta} = -\frac{ab \sin \theta}{\sqrt{a^2 + b^2 + 2ab \cos \theta}} < 0. \tag{21}\]

Therefore, \( dP_e(\delta_m(\theta), \theta)/d\theta \) is always less than 0 for \( \theta \in [0, \pi/2] \), and \( P_e(\delta_m(\theta), \theta) \) is monotone decreasing for \( \theta \in [0, \pi/2] \).
(b) Differentiate equation (20) with respect to \( \theta \):

\[
\frac{d^2 P_r(\delta_m(\theta), \theta)}{d\theta^2} = -(c + d)d\sin \theta - \frac{b^2k(\theta)(a^2 + b^2 + 2ab\cos \theta) + 6ab^2\sin^4 \theta}{(a^2 + b^2 + 2ab\cos \theta)^{5/2}},
\]

where \( k(\theta) = 4(a^2 + b^2)\cos 2\theta + ab\cos \theta (8 - 6\sin^2 \theta) \).

It can be proved that \( d^2 P_r(\delta_m(\theta), \theta)/d\theta^2 < 0 \) for all \( \theta \in [0, \pi/2] \), and \( d^2 P_r(\delta_m(\theta), \theta)/\theta d\theta > 0 \) when \( \theta = 0 \), and \( d^2 P_r(\delta_m(\theta), \theta)/\theta d\theta < 0 \) when \( \theta = \pi/2 \).

Therefore, \( d^2 P_r(\delta_m(\theta), \theta)/\theta d\theta \) is monotone decreasing for \( \theta \in [0, \pi/2] \), and the equation \( d^2 P_r(\delta_m(\theta), \theta)/\theta d\theta = 0 \) has a unique solution which is the unique maximum value of \( P_r(\delta_m(\theta), \theta) \) for \( \theta \in [0, \pi/2] \).

Here, the proof of \( d^2 P_r(\delta_m(\theta), \theta)/d\theta^2 < 0 \) is not given in detail, and only the proof steps are presented as follows:

According to equation (22), \( d^2 P_r(\delta_m(\theta), \theta)/d\theta^2 < 0 \) for \( \theta \in [0, \pi/2] \) is equivalent to the following inequality:

\[
f(\theta) = -(c + d)d\sin \theta (a^2 + b^2 + 2ab\cos \theta)^{5/2} - b^2k(\theta)(a^2 + b^2 + 2ab\cos \theta) - 6ab^2\sin^4 \theta < 0.
\]

By expanding \([a^2 + b^2 + 2ab\cos \theta]^5\) and considering the constraint of \( a > b \), the following inequality can be proved for \( \theta \in [0, \pi/2] \):

\[
(a^2 + b^2 + 2ab\cos \theta)^5 \geq \left( a^2 + b^2 \right)^{5/2} + 4ab^2(a^2 + b^2)\cos \theta \cdot 2.
\]

(24)

By substituting equation (24) into equation (23), the following inequality can be proved for \( \theta \in [0, \pi/2] \):

\[
f(\theta) \leq g(\theta),
\]

where

\[
g(\theta) = -(c + d)d\sin \theta (a^2 + b^2)^{5/2} - b^2k(\theta)(a^2 + b^2) - 6ab^2\sin^4 \theta.
\]

(25)

(26)

By substituting \((a^2 + b^2)^{5/2} \geq [(a + b)/2]^2)^{5/2}\) into (26) and considering the constraints of \( c + d \leq 2b \) and \( a > b \), it can be proved that \( g(\theta) < 0 \) for all \( \theta \in [0, \pi/2] \). Therefore, \( d^2 P_r(\delta_m(\theta), \theta)/d\theta^2 < 0 \) is proved.

In three-dimensional space, the geometric equation of Theorem 1 is as follows: for any fixed \( \theta_0 \in [0, \pi/2] \), \( P_r(\delta, \theta_0) \) is the intersection curve of the plane \( \theta = \theta_0 \) with the curved surface \( P_r(\delta, \theta_0) \), and the curve has a unique maximum value for \( \delta \in (0, \pi) \); when \( \theta_0 \) changes in \([0, \pi/2] \), these unique maximum values constitute a continuous curve which is monotone decreasing. Therefore, the curve can be called as the "ridge" of the curved surface \( P_r(\delta, \theta_0) \); meanwhile, the corresponding renewable energy \( P_r(\delta, \theta_0) \) of each point on the "ridge" also constitutes a continuous curve which has a unique maximum value for \( \theta_0 \in [0, \pi/2] \).

According to Theorem 1 and the "one-to-one mapping" relationship between the set \((\delta, \theta)\) and the set \((\delta, P_r)\), any renewable energy \( P_r = P_m \) in the set \((\delta, P_r)\) of the original model is corresponding to a unique \( P_r(\delta, \theta) \) in the set \((\delta, \theta)\) of the triangular transformation model, and \( P_r(\delta_m(\theta), \theta) \) is the unique maximum value of \( P_r(\delta_m(\theta), \theta) \) for \( \delta \in (0, \pi) \). Therefore, in the original model, \( P_r(\delta_m(\theta), \theta) \) is the steady-state stability limit of the thermal generator when the renewable energy \( P_r = P_m \). Moreover, when \( \theta \) increases from 0 to \( \pi/2 \), \( P_r(\delta_m(\theta), \theta) \) monotonically increases to a maximum value \( P_r(\delta_m(\theta_M), \theta_M) \) and then decreases monotonically, and \( P_r(\delta_m(\theta), \theta) \) is always monotone decreasing. Therefore, in the original model, when the renewable energy \( P_r \) increases from 0 to \( P_r(\delta_m(\theta_M), \theta_M) \), the steady-state stability limit of the thermal generator is always decreasing; when the renewable energy \( P_r \) is more than \( P_r(\delta_m(\theta_M), \theta_M) \), equation (6) does not hold, namely, the renewable-thermal infinite bus sending-end system cannot maintain a stable operation. Therefore, Theorem 1 is equivalent to the following conclusion in the set \((\delta, P_r)\) of the original model.

**Conclusion 1.** In the renewable-thermal infinite bus sending-end system, the renewable energy \( P_r \) has a maximum value of \( P_r^{\text{Max}} \); when the renewable energy \( P_r \) increases from 0 to \( P_r^{\text{Max}} \), the steady-state stability limit of the thermal generator is always decreased, and when the renewable energy \( P_r \) is more than \( P_r^{\text{Max}} \), the renewable-thermal infinite bus sending-end system cannot maintain a stable operation.

Note that, in equation (19), we have

\[
P_r(\delta_m(0), 0) = a + b.
\]

(27)

When \( \sqrt{a^2 + b^2} > c \), \( P_r(\delta_m(\theta), \theta) > 0 \) for all \( \theta \in [0, \pi/2] \), namely, the steady-state stability limit of the thermal generator is always more than 0 for all \( \theta \in [0, \pi/2] \); when \( \sqrt{a^2 + b^2} \leq c \), there exists a unique \( \theta_1 \in (0, \pi/2) \) satisfying \( P_r(\delta_m(\theta), \theta) = 0 \), namely, the steady-state stability limit of the thermal generator is less than or equal to 0 for \( \theta \geq \theta_1 \).

Obviously \( \sqrt{a^2 + b^2} \leq c \) when \( x_1 \times x_2 \) is big enough; thus, the physical meaning of \( \sqrt{a^2 + b^2} \leq c \) is that the electrical distance from the renewable energy to the thermal generator is far less than the electrical distance from the renewable energy to the infinite bus.

The following conclusion can be achieved.

**Conclusion 2.** In the renewable-thermal infinite bus sending-end system, the steady-state stability of the thermal generator changes with the electrical distance from the renewable energy to the thermal generator, when the
renewable energy is close enough to the thermal generator to satisfy the network parameter relationship of $\sqrt{a^2 + b^2} \leq c$, the steady-state stability limit of the thermal generator can be reduced to 0 with the increase of the renewable energy, which directly leads to the steady-state instability of the system.

4.2. Total Steady-State Stability Limit of Renewable-Thermal Infinite Bus Sending-End System. The total transmission power of renewable-thermal infinite bus sending-end system is defined as follows:

$$ P_{\Sigma}(\delta, \theta) = P_e(\delta, \theta) + P_r = a \sin \delta + b \sin(\delta + \theta) + d \sin \theta. $$

(28)

**Theorem 2.** In the triangular transformation model of renewable-thermal infinite bus sending-end system, for any fixed $\theta \in [0, \pi/2]$, there exists a unique $\delta_\Sigma(\theta) \in (0, \pi)$ which makes $P_{\Sigma}(\delta, \theta)$ achieve the maximum value $P_{\Sigma}(\delta_{\Sigma}(\theta), \theta)$; when $\theta$ changes in $[0, \pi/2]$, $P_{\Sigma}(\delta_{\Sigma}(\theta), \theta)$ achieves a unique maximum value, and $P_{\Sigma}(\delta_{\Sigma}(\theta), \theta)$ is monotone increasing.

The proof of Theorem 2 is easier than that of Theorem 1. Here is no longer given the detailed proof of Theorem 2, and only some relevant expressions are given as follows:

$$ \sin \delta_{\Sigma}(\theta) = \frac{a + b \cos \theta}{\sqrt{a^2 + b^2 + 2ab \cos \theta}} $$

(29)

$$ \cos \delta_{\Sigma}(\theta) = \frac{b \sin \theta}{\sqrt{a^2 + b^2 + 2ab \cos \theta}} $$

(30)

$$ P_{\Sigma}(\delta_{\Sigma}(\theta), \theta) = d \sin \theta + \sqrt{a^2 + b^2 + 2ab \cos \theta}, $$

(31)

$$ P_r(\delta_{\Sigma}(\theta), \theta) = (c + d) \sin \theta + \frac{2b^2 \sin^2 \theta}{\sqrt{a^2 + b^2 + 2ab \cos \theta}} $$

(32)

$$ \frac{d^2 P_{\Sigma}(\delta_{\Sigma}(\theta), \theta)}{d \theta^2} = -d \sin \theta - abh(\theta) < 0, $$

(33)

where $abh(\theta) = \cos \theta(a^2 + b^2 + 2ab \cos \theta) + ab \sin^2 \theta/(a^2 + b^2 + 2ab \cos \theta)^{3/2}$.

For $\theta \in [0, \pi/2]$, $P_{\Sigma}(\delta_{\Sigma}(\theta), \theta)$ has a unique maximum value and $P_r(\delta_{\Sigma}(\theta), \theta)$ is monotone increasing. Therefore, similar to the proof of Theorem 1, Theorem 2 can be proved.

In three-dimensional space, the geometric explanation of Theorem 2 is as follows: for any fixed $\theta_0 \in [0, \pi/2], P_{\Sigma}(\delta, \theta_0)$ is the intersection curve of the plane $\theta = \theta_0$ with the curved surface $P_{\Sigma}(\delta, \theta)$, and the curve has a unique maximum value for $\theta_0 \in (0, \pi)$; when $\theta_0$ changes in $[0, \pi/2]$, these unique maximum values constitute the “ridge” of the curved surface $P_{\Sigma}(\delta, \theta)$ which is a continuous curve achieving a unique maximum value for $\theta_0 \in [0, \pi/2]$, and the corresponding renewable energy $P_r(\delta, \theta_0)$ of each point on the “ridge” constitutes a continuous curve which is monotone increasing for $\theta_0 \in [0, \pi/2]$.

According to the “one-to-one mapping” relationship between the set $(\delta, \theta)$ and the set $(\delta, P_r)$, Theorem 2 is equivalent to the following conclusion in the set $(\delta, P_r)$ of the original model.

**Conclusion 3.** In the renewable-thermal infinite bus sending-end system, when the renewable energy $P_r$ increases from 0, the total steady-state stability limit of the renewable-thermal infinite bus sending-end system increases first, and then, after reaching the maximum value at a certain value of the renewable energy $P_r$, begins to decrease.

4.3. Features of Steady-State Stability Limits. According to (19) and (20), when the thermal generator reaches the steady-state stability limit, the total transmission power of the renewable-thermal infinite bus sending-end system is as follows:

$$ P_{\Sigma}(\delta_{m}(\theta), \theta) = P_e(\delta_{m}(\theta), \theta) + P_r(\delta_{m}(\theta), \theta) = d \sin \theta + p(\theta) - \Delta(\theta), $$

(34)

where

$$ \Delta(\theta) = \frac{2b^2 \sin^2 \theta}{\sqrt{a^2 + b^2 + 2ab \cos \theta}}, $$

(35)

$$ p(\theta) = \sqrt{a^2 + b^2 + 2ab \cos \theta}, $$

and (19) and (20) are expressed as follows:

$$ P_e(\delta_{m}(\theta), \theta) = -c \sin \theta + p(\theta), $$

(36)

$$ P_r(\delta_{m}(\theta), \theta) = (c + d) \sin \theta - \Delta(\theta). $$

(37)

According to (31) and (32), when the total transmission power reaches the total steady-state stability limit, the electromagnetic power of the thermal generator is as follows:

$$ P_e(\delta_{\Sigma}(\theta), \theta) = P_{\Sigma}(\delta_{\Sigma}(\theta), \theta) - P_r(\delta_{\Sigma}(\theta), \theta) = -c \sin \theta + p(\theta) - \Delta(\theta). $$

(38)

Equations (31) and (32) are expressed as follows:

$$ P_{\Sigma}(\delta_{\Sigma}(\theta), \theta) = d \sin \theta + p(\theta), $$

(39)

$$ P_r(\delta_{\Sigma}(\theta), \theta) = (c + d) \sin \theta + \Delta(\theta). $$

(40)

By comparing equations (34)–(37) with (38)–(40), the following features can be found:

(a) For any $\theta \in [0, \pi/2]$, the electromagnetic power and the total transmission power cannot reach their own steady-state stability limits at the same time, and when one of them reached, the others always $\Delta(\theta)$ less than its own steady-state stability limit.

(b) For any $\theta \in [0, \pi/2]$, when the thermal generator reaches the steady-state stability limit, the total transmission power always $\Delta(\theta)$ less than the total
steady-state stability limit; meanwhile, when the total transmission power reaches the total steady-state stability limit, the electromagnetic power always $\Delta(\theta)$ less than the steady-state stability limit of the thermal generator. 

(c) For any $\theta \in (0, \pi/2]$, $\delta_2(\theta) < \pi/2 < \delta_{cm}(\theta)$ according to equations (17) and (30).

(d) According to equations (9), (11), and (28), it can be proved that, for any $\theta \in (0, \pi/2]$, when the power angle of the thermal generator is decreased from $\delta_{cm}(\theta)$ to $\delta_2(\theta)$, $P_r(\delta, \theta)$ monotonically decreases from $-\sin \theta + p(\theta)$ to $-\sin \theta + p(\theta) - \Delta(\theta)$, $P_r(\delta, \theta)$ monotonically increases from $a + d\sin \theta - \Delta(\theta)$ to $(c + d)\sin \theta + \Delta(\theta)$, and $P_r(\delta(\theta), \theta)$ monotonically increases from $d\sin \theta + p(\theta) - \Delta(\theta)$ to $d\sin \theta + p(\theta)$. In this process, by reducing $P_r(\delta, \theta)$ from its own steady-state stability limit by $\Delta(\theta)$, $P_r(\delta, \theta)$ can be increased by $\Delta(\theta)$, and the total transmission power can be increased by $\Delta(\theta)$ reaching the total steady-state stability limit. Thus, the following conclusion can be achieved in the original model.

**Conclusion 4.** In the renewable-thermal infinite bus sending-end system, assuming that the thermal generator is operating in the steady-state stability limit state, by reducing the electromagnetic power $P_e$ and increasing more renewable energy $P_r$, the total transmission power $P_2$ can be increased until the total steady-state stability limit state. From the former state to the latter state, the reduction of the electromagnetic power $P_e$ can be exchanged for two times of renewable energy $P_r$.

Conclusion 1, Conclusion 3, and Conclusion 4 reveal a complete mechanism of steady-state stability of renewable-thermal infinite bus sending-end system: the increase of renewable energy causes the steady-state stability limit of the thermal generator decreased, and the total steady-state stability limit first increased and then decreased; reducing the electromagnetic power away from its own steady-state stability limit can be exchanged for a higher transmission capacity of the renewable energy.

### 5. Simulation and Discussion

#### 5.1. Renewable-Thermal Infinite Bus Sending-End System

In the renewable-thermal infinite bus sending-end system, select the infinite bus voltage $U = 1.0$ p.u. and the generator inner potential $E = 1.2$ p.u., and according to the numerical relationship between $\sqrt{a^2 + b^2}$ and $c$, select four sets of network parameters (see Table 1) for simulation analysis. The following simulation results can be obtained:

(a) The change of the steady-state stability limit of the thermal generator with the renewable energy is shown in Figure 3.

The simulation result in Figure 3 is in agreement with Conclusion 1: the steady-state stability limit of the thermal generator is always decreased when the renewable energy $P_r$ increases, and when the renewable energy $P_r$ is increased to a certain value, in order to maintain the stable operation of the thermal generator, the renewable energy must be reduced.

(b) The generator power angle relationships under parameter 3 and parameter 4 are shown in Figures 4 and 5, respectively.

The simulation result in Figures 4 and 5 is in agreement with Conclusion 2: when $\sqrt{a^2 + b^2} = c$, the steady-state stability limit of the thermal generator can reach 0 with the increase of the renewable energy; when $\sqrt{a^2 + b^2} < c$, the generator steady-state stability limit can be less than 0 with the increase of the renewable energy.

(c) The change of the total steady-state stability limit with the renewable energy is shown in Figure 6, and the change of the total transmission power with the generator power angle under parameter 1 is shown in Figure 7.

The simulation result in Figures 6 and 7 is in agreement with Conclusion 3: when the renewable energy increases, the total steady-state stability limit of the renewable-thermal infinite bus sending-end system increases first and then begins to decrease after reaching the maximum value.

(d) In three-dimensional space, the curve of $P_r(\delta, P_r)$ in the original model and the curve of $P_r(\delta, P_r)$ in the triangular transformation model are shown in Figures 8(a) and 8(b), respectively, and the curve of $P_r(\delta, P_r)$ in the original model and the curve of $P_r(\delta, P_r)$ in the triangular transformation model are shown in Figures 9(a) and 9(b), respectively.

After mapping to the triangular transformation model, the properties of $P_r$ and $P_e$ remain unchanged, and the “ridges” of $P_r$ and $P_e$ in both models show the same characteristics.

(e) When the thermal generator and the total transmission power, respectively, reach their own steady-state stability limit, the changes of $P_e$, $P_2$, and $P_r$ with $\theta$ are shown in Figure 10.

The simulation result in Figure 10 is in agreement with not only Conclusion 1 and Conclusion 3 but also Conclusion 4: when the electromagnetic power is less than its own steady-state stability limit, the renewable energy can be increased. With the reduction of electromagnetic power and the increase of renewable energy, the total transmission power reaches the total steady-state stability limit where the increase of renewable energy is two times of the reduction of electromagnetic power.
5.2. Practical Power System. Here, a regional transmission power grid in China is employed as a study object of steady-state stability. As a typical long-distance sending-end system with large-scale renewable energy and thermal generators, the regional transmission power grid contains the thermal power installed capacity of 7770 MW, the wind power installed capacity of 4160 MW, and the power supply load of 3380 MW and connects with the main power grid through two 500 kV and three 220 kV transmission lines, where the length of each 500 kV transmission line is about 260 km.

Adopting the numerical simulation method, the total steady-state stability limit when the wind power $P_w$ changes from 0 MW to 4160 MW (see Figure 11).

The numerical simulation result in Figure 11(a) is in agreement with Conclusion 3: when the wind power $P_w$ is 2500 MW, the total steady-state stability limit $P_{TL}$ reaches the maximum value 4670 MW; when the wind power $P_w$ changes from 0 MW to 2500 MW, the total steady-state stability limit $P_{TL}$ is increased from 3820 MW to 4670 MW, which is increased by 850 MW; when the wind power $P_w$ changes from 2500 MW to 4160 MW, the total steady-state stability limit $P_{TL}$ is decreased from 4670 MW to 3420 MW, which is decreased by 1250 MW. The total steady-state stability limit $P_{TL}$ keeps increasing in a fairly long period with the increase of the wind power $P_w$.

The numerical simulation result in Figure 11(b) is in agreement with Conclusion 4: the decrease of electromagnetic power from 7430 MW to 3490 MW can be exchanged for a greater increase of wind power $P_w$ from 0 MW to 4160 MW, especially in the rising stage of the total steady-state stability limit in Figure 11(a).
The simulation result of another regional transmission power grid in China (see Figure 12) is also in agreement with Conclusion 3 and Conclusion 4. This regional transmission power grid contains the thermal power installed capacity of 6250 MW, the wind power installed capacity of 3100 MW, and the power supply load of 1550 MW and connects with the main power grid through for 500 kV transmission lines, where the length of each 500 kV transmission line is about 180 km. A total of seven regional wind-thermal bundled power grids in China are employed as a study object of
steady-state stability, and the simulation results are in agreement with the conclusions.

6. Conclusions

The steady-state stability of sending-end system with renewable energy is studied by using the classical model of power system. Based on the differential-algebraic equation in which the renewable energy is simplified to static power injection model, and by proposing a triangular transformation model, the mechanism of steady-state stability of sending-end system with renewable energy is revealed theoretically, and the influence of renewable energy on the steady-state stability limit is analyzed in detail.

In the sending-end system with renewable energy, when the renewable energy increases from 0, the steady-state stability limit of thermal generator is always decreased; however, the total steady-state stability limit of sending-end system is first increased and then decreased. In the renewable-thermal infinite bus sending-end system, by reducing the electromagnetic power from its own steady-state stability limit, the transmission capacity of the renewable energy can be increased by two times of the reduction of electromagnetic power, and the total transmission capacity can be increased by one time of the reduction of electromagnetic power. Reducing the electromagnetic power in a certain range can improve the steady-state stability of sending-end system and get a higher transmission capacity of renewable energy. Simulation results in the test system and practical systems validate the conclusions.

This paper only studies the case that the reactive power of renewable energy is zero, and it can be proved that the same conclusions of the steady-state stability can be achieved when the power factor of renewable energy is in the vicinity of 1.0.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors state that there are no conflicts of interest at the time of publication.

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