Precision measurement of gravity with cold atoms in an optical lattice and comparison with a classical gravimeter.

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We report on a high precision measurement of gravitational acceleration using ultracold strontium atoms trapped in a vertical optical lattice. Using amplitude modulation of the lattice intensity, an uncertainty $\Delta g/g \approx 10^{-7}$ was reached by measuring at the 5th harmonic of the Bloch oscillation frequency. After a careful analysis of systematic effects, the value obtained with this microscopic quantum system is consistent with the one we measured with a classical absolute gravimeter at the same location. This result is of relevance for the recent interpretation of related experiments as tests of gravitational redshift and opens the way to new tests of gravity at micrometer scale.

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Atom interferometry, and in general methods based on quantum interference of ultracold atoms, were largely used in recent years for gravitational physics experiments and new exciting prospects can be envisioned in the near future [1]. For example, Raman interferometry was used for precise measurements of Earth’s gravitational acceleration $g$ [2] and its gradient [3], for determining the value of the gravitational constant [4, 5], for a possible redefinition of the kg [6], and for geophysical applications [7]. Schemes based on Bloch oscillations of atoms trapped in vertical optical lattices were also used to measure gravity with the possibility of combining high sensitivity and micrometric spatial resolution [8–10]. The results of atom interferometry experiments were interpreted as tests of the isotropy of post-Newtonian gravity [11], of quantum gravity [12], and of gravitational redshift [13]. Prospects include high precision tests of the weak equivalence principle [14–15], the detection of gravitational waves [16–17], and future experiments in space [15].

So far, however, Bloch oscillation measurements had limited accuracy compared to Raman atom interferometers. Here, we present a precision measurement of gravitational acceleration $g$ with a new method based on ultracold $^{88}$Sr atoms trapped in an amplitude-modulated vertical optical lattice [19] and compare the result with the value obtained with a classical absolute gravimeter based on a Michelson interferometer with one arm including a freely-falling corner-cube. We also improved our previous observation of long-lived Bloch oscillations [1] and discuss the precision of the two methods for the determination of $g$. In addition to demonstrating the sensitivity and accuracy of this new method, our data can be interpreted as a measurement of the gravitational redshift to the Compton frequency of Sr matter waves, as suggested by H. Müller et al. [13]. Our data surpass previous Bloch oscillation measurements by one order of magnitude, making it the most precise test of the gravitational redshift based on Bloch oscillations at micrometric spatial scales. The interpretation of atom interferometer redshift tests is complicated by special relativistic time dilation since the atoms are moving [20–21], but Bloch oscillations experiments with stationary lattices provide a measurement of the purely gravitational effect.

The experimental setup is based on cooled and trapped $^{88}$Sr atoms [9] (Fig. 1). Atoms from a thermal beam are slowed in a Zeeman slower and trapped in a “blue” Magneto Optical Trap (MOT) operating on the $^1S_0-^3P_1$ resonance transition at 461 nm. The temperature is further reduced by a second cooling stage in a “red” MOT operating on the $^1S_0-^3P_1$ intercombination transition at 689 nm. This produces about $10^6$ atoms at a temperature of 0.6 $\mu$K. Since the force of gravity is comparable to the force produced by the red MOT on the atoms, the cloud of trapped atoms assumes a dish-like shape disk with a vertical size of 27 $\mu$m and a radial size of 180 $\mu$m. The atoms are adiabatically loaded in an optical lattice in 300 $\mu$s. The lattice potential is generated by a single-mode frequency-doubled Nd:YVO$_4$ laser ($\lambda_L = 532$ nm)}
delivering up to 1 W on the atoms with a beam waist of 557(7) μm. The beam is vertically aligned and retro-reflected by a mirror. The atomic sample in the lattice has a vertical RMS size of about 14 μm and a horizontal size of about 100 μm. The Bloch frequency is νB ≈ 574.3 Hz and the corresponding lattice photon recoil energy is ER ≈ 8 kHz × ħ. In typical conditions the lattice depth ranges from 2.3 to 3 ER, while the energy gap E_G at the recoil momentum k_L is EG ≈ ER. The width of the first energy band in the lattice potential is about 0.5 × E_G. Landau-Zener tunneling is negligible in these conditions. The lattice depth is stabilized by a servo loop acting on the RF signal driving an acousto-optical modulator (AOM). The same AOM is also used to add an amplitude modulation to the lattice potential. The atomic cloud can be imaged either in situ or with usual time-of-flight technique using resonant absorption imaging on a CCD camera with a spatial resolution of 5 μm. The commercial lattice laser (Coherent V–5) employed in the measurement is not frequency stabilized and a precise calibration of its frequency is then required.

To this purpose, part of the lattice light is sent to a home–built self-referenced Ti:Sa optical frequency comb. Due to residual frequency instabilities on the time scale of the Bloch frequency measurement, the uncertainty in the laser frequency is ≈ 100 MHz. The correction for the index of refraction of the Sr cloud and the background gas in the vacuum chamber is negligible. The vertical alignment of the lattice was checked with a precision of 0.5 μrad, corresponding to a relative uncertainty of 10 ppb on g, by overlapping the downward laser beam with the reflection from the surface of water in a glass container inserted in the beam. A tiltmeter with a resolution of 1.7 μrad attached to the optical table was employed to check the alignment stability during the measurements.

After loading ⁸⁸Sr atoms in the vertical lattice, the trap depth is modulated sinusoidally. When the modulation frequency matches an integer harmonic of the Bloch frequency, atoms start to tunnel in neighbor lattice sites giving rise to a net increase in the spatial vertical atomic distribution which is observed in situ by resonant absorption imaging (see Fig. 2). The lifetime in the vertical lattice is about 20 s. This allows us to apply amplitude modulation to the lattice for ~10 s resulting in an increased quality factor observed on the resonance at ν_m = 5 × ν_B. With a typical amplitude modulation depth of the order of 7% we estimate a tunneling rate J/ℏ = 0.75. The recording time for a whole resonance spectrum is about 1 hour and leads to a maximum resolution of 150 ppb in ν_B. The value of g is given by g = 2hν_B/(m_Srλ_L), where m_Sr is the mass of ⁸⁸Sr atoms and h the Planck constant which are both known with a relative uncertainty of ~ 5 × 10⁻⁸.

An important contribution to systematic shifts in gravity measurements with trapped neutral atoms is due to the lattice light itself. Both the intensity and the wavevector of the lattice beam, which results from the interference of two counter-propagating Gaussian beams, yield space-dependent terms to the potential U_tot(z) = U_s(z) + U_t(z)cos(2k(z)z) − mgz, where U_s(z) and U_t(z) depend on the squared sum and on the product of the two beam field amplitudes, respectively. This additional dependence of the potential along the vertical direction gives rise to two correction terms in the typical Bloch formula g = 2hν_B/(m_Srλ_L) + ∆g_ν + ∆g_k given by the spatial derivative of the potential and the spatial derivative of the difference between the Gouy phase [22] for the two beams at the position of the atomic cloud z_slot. The shift introduced by these two extra terms is estimated by a precise determination of the geometry of the incoming and the reflected trapping beams and the position of the cloud with respect to the beam waist, with a relative uncertainty of 1%. Moreover, an independent determination of the transverse beam size at z_slot has been done by measuring the axial and radial atomic trap oscillation frequencies through parametric heating technique [23, 24]. For typical experimental parameters, the two terms are

![Diagram](image-url)

**FIG. 2**: Spectrum recorded by modulation of the lattice depth at the 5th harmonic of the Bloch frequency for 10.4 s with a modulation depth of 7%. The red line is a fit of experimental data with a sinc function.

| Effect                      | Correction | Uncertainty |
|-----------------------------|------------|-------------|
| Lattice wavelength          | 0          | 2           |
| Lattice beam vertical align. | 0          | 0.1         |
| Stark shift (beam geometry) | 14.3 ÷ 17.3| 0.4         |
| Experiment timing           | 0          | 0.2         |
| Tides                       | -1.4 ÷ 0.9 | <0.1        |
| Height difference           | 4.3        | 0.2         |
| Refraction index            | 0          | <0.01       |
| Fundamental constants       | 0          | 0.7         |
| Systematics total           | 17.2 ÷ 22.5| 2.2         |

**TABLE I**: Systematic corrections and their associated uncertainties (× 10⁻⁷) for the gravity measurement with ⁸⁸Sr atoms in the amplitude modulated optical lattice.
\[ \Delta g_{U} = 1.53(3) \times 10^{-5} \text{m/s}^2 \text{ and } \Delta g_{E} = 1.0(2) \times 10^{-8} \text{m/s}^2. \]

Tidal effects were evaluated and removed from the raw data using the same algorithm and potential model used for the absolute gravimeter data processing. The peak-to-peak effect of tides at our site is of the order of \(2 \times 10^{-6}\text{m/s}^2\). Since each measurement lasts about 1 hour the variation of \(g\) during a single measurement due to tides is below \(10^{-7}\text{m/s}^2\) (i.e. below 10 ppb).

We checked also for a possible dependence from magnetic field gradients by performing a set of measurements applying a quadrupole magnetic field (from the MOT coils) up to \(B = 40\) gauss/cm. The effect on \(\nu_{T}\) is smaller than the statistical error. All the other sources of systematic shift in the measurement we evaluated (spurious higher harmonics of amplitude modulation, Bloch-Siegert shift) are far below the current accuracy level. Table I summarizes the systematic shifts for the gravity measurements with the amplitude modulation technique. The values of individual shifts depend on the experimental conditions; the quoted uncertainties are typical values.

The reference value for local gravitational acceleration \(g\) was provided by an absolute gravimeter based on a Michelson interferometer with one arm including a freely-falling corner-cube (FG5, Micro-g LaCoste). The measurement was performed in the same laboratory at a distance of 1.15 m from the atomic probe position. The difference in height of 14(5) cm together with the estimated vertical gravity gradient value \(g_{zz} = -3.09 \times 10^{-6}\text{m/s}^2\) at the laboratory site was taken into account in the data analysis. The result is \(g_{FG5} = 9.804921609(84) \text{m/s}^2\)

FIG. 3: (color online). Measurements of \(g\) using the amplitude modulation technique. Each experimental point is corrected for the systematic effects presented in Tab. I. The red dashed line represents the weighted mean of the 21 measurements. The blue solid line is the value obtained with the classical absolute FG5 gravimeter.

\[ g_{\text{classical}} = 9.8049232(14) \text{ m/s}^2 \]

\[ g_{FG5} = 9.804921609(84) \text{ m/s}^2 \]

Fig. 3 presents a set of 21 determinations of \(g\) with \(^{88}\)Sr atoms. The error bars are given by the quadrature sum of the statistical errors coming from the fit of the amplitude modulation resonance and the uncertainty on systematic corrections. The standard deviation is \(\sigma = 110\) ppb with a \(\chi^2 = 30\). The resulting statistical uncertainty is \(\sigma \times \sqrt{\chi^2/(n-1)} = 140\) ppb. The weighted mean of our data is \(g_{E} = 9.8049232(14) \text{ m/s}^2\), in good agreement with the value obtained using the FG5 gravimeter.

With minor modifications of the experimental procedure, in this work we also determined \(g\) by measuring the frequency of the Bloch oscillations of the atoms in the vertical optical lattice. Thanks to a better vacuum and taking advantage of the lattice modulation method to reduce the initial momentum distribution of the atoms in the lattice, we considerably improved the visibility and duration of the oscillations and, as a consequence, the frequency resolution compared with previous experiments. After the transfer of ultracold atoms in the vertical optical lattice, an amplitude modulation burst with typical duration of 120 cycles at \(\nu_{m} \approx \nu_{B}\) is applied. The quantum phase of the atomic wavefunction induced by the amplitude modulation gives rise to an interference effect in time of flight image of the atomic cloud which results in an enhanced visibility of the Bloch oscillations peaks.

FIG. 4: (color online). Long-lived Bloch oscillations for \(^{88}\)Sr atoms in the vertical lattice under the influence of gravity. Each picture shows one Bloch cycle in successive time-of-flight absorption images giving the momentum distribution at the time of release from the lattice. Displayed are the first (a), the 2900th (b), the 7500th (c), and the 9800th (d) Bloch cycle.

\[ \Delta g_{U} = 1.53(3) \times 10^{-5} \text{m/s}^2 \text{ and } \Delta g_{E} = 1.0(2) \times 10^{-8} \text{m/s}^2. \]

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loration technique, however, we observed a considerably larger scattering in repeated measurements, mainly due to initial position instability of the atomic trap and also higher dependence on the timing of the experiment. The value for $g$ obtained with the Bloch oscillation technique is indeed $g_{\text{Bloch}} = 9.80488(6) \text{ m/s}^2$, which is consistent with the measurement discussed above but is affected by a larger uncertainty of 6 ppm.

It is important to notice that the amplitude modulation technique for gravity measurement allows further improvements of both accuracy and sensitivity. In fact our result is mainly limited by the lattice wavelength stability and by the Stark shift. The first effect could be lowered by using a tunable laser and locking it to an atomic line. For instance, if the wavelength is stabilized within 1 MHz, the uncertainty of this effect might be reduced by two orders of magnitude. The second main systematic effect could be reduced either by using a blue-detuned trapping laser [22] or by increasing the lattice beam waist. Also, the sensitivity could be increased by using higher harmonic amplitude modulation frequencies or by modulating for a longer time.

In conclusion, we performed an accurate measurement of gravitational acceleration using ultracold atoms trapped in a vertical optical lattice. The result is in agreement at 140 ppb level with an independent determination obtained with a classical FG5 gravimeter. This result represents an improvement by an order of magnitude over previous results [8, 28] and is of interest as a test of the gravitational redshift [13]. Moreover we demonstrated the validity of the amplitude modulation technique [12] for the measurement of forces with high spatial resolution [10]. We also observed persistent Bloch oscillation up to 17 s which represents the longest coherence time observed to date. This result might also have important applications in precision measurements in conjunction with nondestructive cavity QED techniques to probe atomic momentum oscillations [29].

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