Mirror Matter, Mirror Gravity and Galactic Rotational Curves

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Abstract

A bigravity theory where the normal and dark matter components are coupled to separate metric fields linked to each-other with small non-derivative terms allows the Yukawa-like modification of the gravitational potential at large distances. This opens new prospects for the dark matter candidates. Namely, instead of being cold and collisionless, dark matter can be collisional and dissipative, as it occurs in the case of mirror matter that presumably does not form extended halos but is clumped similarly to visible matter. We show that the flattening of the galactic rotational curves can be explained if the typical scale of the Yukawa-like potential is about few tens of kpc and if the mass ratio between dark matter and visible matter in galaxies is about 10.

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1 Introduction

Cosmological observations show that the Universe is nearly flat with an energy density very close to the critical one: $\Omega_{\text{tot}} = \Omega_M + \Omega_\Lambda \simeq 1$. The dark energy component $\Omega_\Lambda = 0.76 \pm 0.02$ dominates today’s universe while radiation is negligible and what is left is matter (ordinary+dark) with $\Omega_M = 0.24 \pm 0.02$. In particular $\Omega_M = \Omega_B + \Omega_{DM}$, where $\Omega_B = 0.042 \pm 0.005$ and $\Omega_{DM} = 0.20 \pm 0.02$ are, respectively, the baryonic and dark matter components [1].

The standard cosmological model is in agreement with experimental results on Cosmic Microwave Background (CMB) and Large Scale Structure (LSS) that strongly support the presence of the dark matter component. Many DM candidates have been proposed in the literature axions ($m \sim 10^{-5}$ eV), neutralinos ($m \sim$ TeV), wimpzillas ($m \sim 10^{14}$ GeV) etc. However, the question why the fractions of the baryonic and dark components are so close, $\Omega_{DM}/\Omega_B \sim 5$, remains unsolved.

The most direct evidence for DM comes from galaxy rotational curves and cluster dynamics. In the Newtonian picture, far from the galactic center the velocity of rotating matter is expected to be of the form $v(r) \propto 1/\sqrt{r}$ (known as Keplerian fall-off). This is due to the fact that the gravitational potential outside the galaxy inner core (bulge) is $\phi \simeq GM/r$, where $M$ is total mass in the the bulge. Instead, one observes that outside the bulge $v$ is approximately constant. The standard way to explain this anomalous behavior is to introduce non-baryonic dark matter (as in the standard CDM paradigm) distributed in an extended spherically symmetric halo around the galaxy, as a typical profile one can take an “isothermal” mass distribution $\rho(r) \propto (1 + (r/a)^2)^{-1}$, where the scale $a$ depends on the galaxy [1]. Since gravity is universal (no distinction between visible and dark matter), any point-like source (dark or ordinary) will generate the same potential

$$\phi(r) = \frac{G}{r}(M_1 + M_2) \quad (1)$$

where $M_1$ and $M_2$ is, respectively, the the mass of the visible and dark component. In the CDM scenario it is also assumed that the DM particles are collisionless. As a consequence of this assumption, N-body numerical simulations tend to predict that DM clustering form singular profiles (cusps) $\rho(r) \propto 1/r^\alpha$, with $\alpha = 1 \div 1.5$, [2, 3, 4, 5] quite different from the isothermal profiles. For most galaxies these cusp profiles can hardly reproduce the observed rotational curves [7], in particular, they provide for a steep growth of rotational curves at small distance.

An alternative scenario is the mirror matter [8], according with our Universe is made of two similar gauge sectors. In other words, parallel to our sector of the ordinary particles (O-) and interactions described by the Standard Model, there exists a

\[ \text{The integral of the latter } \rho(r) \text{ does not converge, therefore we need a cut-off radius } L \text{ quite bigger than the visible galactic size.} \]
hidden sector (M-) that is an exact duplicate of the ordinary sector with particles and interactions having exactly the same characteristics; the two sectors are connected by the common gravity (see for review [9]). Therefore, if the mirror sector exists, then the Universe should contain along with ordinary particles (electrons, nucleons, photons, etc.) also their mirror partners with exactly the same masses (mirror electrons, mirror nucleons, mirror photons, etc.). Mirror matter is invisible to us because it does interact with ordinary photons and naturally constitutes a dark matter candidate. One should stress that the fact that O- and M-sectors have the same micro-physics, does not imply that their cosmological evolutions should be the same. Indeed, if mirror particles had the same temperature in the early universe as ordinary ones, this would be conflict with Big Bang Nucleosynthesis (BBN). The BBN limit on the effective number of extra neutrinos implies that the temperature of the mirror sector $T'$ must be at least about twice smaller than the temperature $T$ of the ordinary sector allowing mirror baryons to be a viable candidate for dark matter. In particular, the mirror dark matter scenario would give the same pattern of LSS and CMB as the standard CDM if $T'/T \lesssim 0.2$ [10].

In addition, the baryon asymmetry of the Universe can be generated via out-of-equilibrium $B-L$ and $CP$ violating processes between ordinary and mirror particles [11] whose mechanism could explain the intriguing puzzle of the correspondence between the visible and dark matter fractions in the Universe, naturally predicting the ratio $\Omega_{DM}/\Omega_B \sim 1 \div 10$ [12].

However, in contrast to the collisionless CDM, mirror baryons obviously constitute collisional and dissipative dark matter. Therefore, one expects that mirror matter undergoes a dissipative collapse and thus DM (mirror)in galaxies has a distribution similar to the the visible matter instead of producing extended quasi-spherical CDM halos. Thus, for the DM sector as well for the luminous sector, an exponential profile $\rho(r) \propto e^{-r/r_0}$ is expected. As a result, the distribution of the dark matter is more compact in the center of the galaxy and is not extended as in the CDM halos. If gravity is universal in the two sectors, the mirror dark matter hypothesis gets into difficulties in explaining the flat rotational curves of galaxies. However, if each sector has its own gravitational field and there is a non-derivative mixing term then gravity gets modified at large distance in way that rotational curves can be explained in terms of mirror DM. A detailed discussion of how gravity is modified is given in separate paper [13, 14, 15]; for what concerns us the bottom line is that the non-derivative interaction between the two gravitational fields leads to a modified Newtonian potential. Namely, an ordinary test mass (matter of type 1) at distance $r$ from a source of matter ordinary + dark matter (matter of type 2) ($M_1$ and $M_2$),

\footnote{The particle mixing phenomena between ordinary and mirror sectors were discussed in the literature for photons [16], neutrinos [17], neutrons [18], etc., as well as possible common gauge interactions between two sectors [19].}
instead of (1) is the subject to the following potential

\[ \phi(r) = \frac{G}{2r} (M_1 + M_2) + \frac{Ge^{-\frac{r}{r_g}}}{2r} (M_1 - M_2), \]

(2)

where \( G \) is the Newton constant and \( r_g \) is the graviton inverse mass. Notice that at small distance \( r \ll r_g \) gravity is not modified: an ordinary particle feels only the influence of luminous ordinary matter \( M_1 \) through the standard Newton potential:

\[ \phi(r \ll r_g) \approx \frac{G M_1}{r} \]

(3)

At large distance \( r \gg r_g \) the interaction with dark matter kicks in and effectively the particle “feels” the sum of all matter \( M_1 + M_2 \) with a halved Newton constant \( G \):

\[ \phi(r \gg r_g) \approx \left( \frac{G}{2} \right) \frac{M_1 + M_2}{r} \]

(4)

This is the key result that enables us to reproduce the observed rotational curves of galaxies.

## 2 Galaxy Rotational Curves

A rotational curve describes the velocity of gravitionally bounded galactic objects (typically stars and interstellar gases) as a function of the distance \( r \) from the center. Most of matter a galaxy (e.g. a disk galaxy) is concentrated in the internal region, called bulge. In the simplest schematization, a galaxy is modelled as an inner spherically symmetric constant density region; the picture can be refined taking a matter density profile that follows approximately the luminosity profile which decreases exponentially with \( r \) moving from the center to the outer region:

\[ \rho(r) = \rho_0 e^{-r/r_0}, \quad \rho_0 = \frac{m}{8\pi r_0^3}, \]

(5)

where \( r = \sqrt{x^2 + y^2 + z^2} \) (the galactic plane lies in the xy plane), \( m \) is the the galaxy total mass and \( \rho_0 \) is the bulge size. The total visible mass of a galaxy can be estimated from the the bulge size \( \rho_0 \) and the velocity \( (v_0) \) in the region where the plateau starts as

\[ m \simeq \frac{r_0 v_0^2}{2G}, \]

(6)

In a different cylindrical symmetry modellization the galaxy is taken to be two-dimensional with a disc surface density

\[ \sigma(r_c) = \sigma_0 e^{-r_c/r_0}, \quad \sigma_0 = \frac{m}{2\pi r_c^2}, \]

(7)
Here $r_c = \sqrt{x^2 + y^2}$. A fully realistic modellization of the matter distribution is an intermediate between the spherically and cylindrical symmetric cases. For instance, due to the rotational symmetry along the $z$ axis, the exponential fall-off will depend not only by the distance from the center but also on $z$.

$$\rho(r) = \rho_0 e^{-\frac{r}{r_0}} f(z/z_0), \quad (8)$$

where $r_0$ and $z_0$ are of order of the bulge size (we will refer to it as ”f-modulated”). When $f = 1$ we have spherical symmetry. In general, a good approximation is $f(z/z_0) = \text{sech}^2(z/z_0)$.

Consider now the radial velocity distribution $v(r)$ for matter (gas, stars, etc.) rotating around the galactic center. According the Newtonian theory, $v(r)$ is expected to fall-off as $1/\sqrt{r}$ at large distances. Observations show a totally different behavior: asymptotically, there is a plateau (flattening). In the standard paradigm to reconcile theory and experiments a new kind of “dark” matter is introduced providing galaxies with an invisible spherically symmetric halo. The rotational velocity $v^2$ has then two contributions: one from the visible matter distributed according to (8) and the second from the dark component.

$$v^2 = v^2_{\text{vis}} + v^2_{\text{dm}}, \quad (9)$$

The distribution profiles strongly depend on the nature of dark matter, a common parametrization is

$$\rho_{\text{dm}}(r) = \frac{\rho_0}{(r/r_s)^{\alpha}[1 + (r/r_s)^{\beta}]^{(\beta-\gamma)/\alpha}},$$

where $r_s$ is a characteristic length scale. The values for $\alpha, \beta, \gamma$ are shown in the table (IT=Isothermal, NWF=Navarro-Frenk-White and M=Moore). By using the IT model one can nicely fit many galaxy rotational curves, even those dominated by the dark matter as the dwarf and low surface brightness (LSB) galaxies. The trouble with IT model is that, when N-body simulations are used to predict DM profiles, singular distributions are produced, containing a cusp when $r \to 0$. For instance in the NFW model $\rho \sim 1/r$, while in the M model $\rho \sim 1/r^{1.5}$ (see also [6]). These profiles are shown in Figures. (1) and (2). Figures (3) and (4) compare the visible matter contribution to the total velocity for profiles with different symmetry. In the cylindrical case the bump at low distance from the center is enhanced. Finally, in figures (15) and (5) the visible and dark contribution in the IT model both for spherical and cylindrical symmetric profile. Let us now discuss how the rotational curves are reproduced using the potential (2). In order to grasp basic the idea, let us imagine a galactic object moving along a circular orbit of radius $r$ around the center;
its velocity is determined by equating the centrifugal force with the radial component of gravitational pull $g(r)$ derived from the potential (2)

$$
\frac{v^2(r)}{r} = g(r) = G \left[ \frac{M_1 + M_2}{2r^2} + \frac{M_1 - M_2}{2r^2} \left( 1 + \frac{r}{r_g} \right) e^{-\frac{r}{r_g}} \right].
$$

Notice that if $M_1 = M_2$ we are back to the Newtonian theory; on the other hand when $M_1 \neq M_2$ and $r \gg r_g$ we get

$$
g_m(r) \simeq \left( \frac{G}{2} \right) \frac{m(M_1 + M_2)}{r^2},
$$

effectively we have a Newtonian force with the Newton constant $G$ replaced by $G/2$. When $r \ll r_g$ a ordinary test mass $m$ will feel a force only from visible matter sources

$$
F_m(r) = G \frac{mM_1}{r^2}.
$$

Summarizing, when $r \ll r_g$, the influence of dark matter on visible matter is negligible and the behavior is Newtonian. In the opposite limit: $r \gg r_g$, the force is also
Figure 5: Different contributions to the rotational curves in IT dark matter halo (cylindrical approximation): (dashed), invisible (dotted) and total (solid), with $r_0 = 8$ kpc, $r_v = 3$ kpc and $M_{dm} = 5M_v$.

Figure 6: Different contributions to the rotational curves IT dark matter halo (spherical approximation): visible (dashed), invisible (dotted) and total (solid), with $r_0 = 8$ kpc, $r_v = 3$ kpc and $M_{dm} = 5M_v$.

Newtonian, though a test particle feels the total mass $M_1 + M_2$ and the effective Newton constant is $G/2$. In the crossover region $r \sim r_g$ there is a sizeable deviation from Newtonian theory due to the presence of type 2 matter, resulting in an enhancement of $v^2$ as shown in Figs. (7) and (8) for point-like distributions.

Figure 7: Rotational curves $v(r)$ for point-like distribution: Newtonian (solid) and modified potential for $M_2 = 10M_1$ and $r_g = 15$ kpc (dashed). The enhancement in the shape for modified gravity can explain the observed flattening.

Figure 8: Normalized point-like rotational curves $v(r)r^{1/2}$ ($M_2 = 10M_1$ and $r_g = 15$ kpc). The flat behaviour at large scale is in fact a slow fall-off as $r^{-1/2}$.

For extended objects one has the following integral expression for the acceleration

\[
g(r) = \int_{\text{Disk}} d^3r' \left[ G \left( \frac{\rho_1(r') + \rho_2(r')}{2|\mathbf{r} - \mathbf{r}'|^3} \right) (\mathbf{r} - \mathbf{r}') + G \frac{\rho_1(r') - \rho_2(r')}{2|\mathbf{r} - \mathbf{r}'|^3} (\mathbf{r} - \mathbf{r}') \left( 1 + \frac{|\mathbf{r} - \mathbf{r}'|}{r_g} \right) e^{-\frac{|\mathbf{r} - \mathbf{r}'|}{r_g}} \right], \tag{14}
\]
where $\rho_{1,2}(\mathbf{r})$ are the density distribution for the matter of type 1 and 2, respectively, $\mathbf{r}$ is the position of the test mass. For the benefit of reader we have collected in a table the relevant expressions for different profiles discussed.

|            | Spherical                                      | Cylindrical                         | f-modulated                          |
|------------|-----------------------------------------------|-------------------------------------|--------------------------------------|
| $d^3\mathbf{r}'$ | $2\pi r'^2 \sin \vartheta \, dr' d\vartheta$ | $r' \, dr' d\vartheta$ | $dx \, dy \, dz$                     |
| $\mathbf{r} - \mathbf{r}'$ | $\hat{\mathbf{r}}'(r - r' \cos \vartheta)$ | $\hat{\mathbf{r}}'(r - r' \cos \vartheta)$ | $(x - x', y, z)$                     |
| $\rho_1(\mathbf{r})$ | $(M_1/8\pi r_1^3)e^{-r/r_1}$ \hspace{2mm} | $(M_1/2\pi r_1^2)e^{-r/r_1}$ \hspace{2mm} | $\rho f_1(z/z_1)e^{-r/r_1}$ \hspace{2mm} |
| $\rho_2(\mathbf{r})$ | $(M_2/8\pi r_2^3)e^{-r/r_2}$ \hspace{2mm} | $(M_2/2\pi r_2^2)e^{-r/r_2}$ \hspace{2mm} | $\rho f_2(z/z_2)e^{-r/r_2}$ \hspace{2mm} |

Here, $M_1$ and $M_2$ are the total mass of the visible and dark component of the galaxy respectively; $r_{1,2}$ and $z_{1,2}$ are their bulge sizes and $\rho_{f1,f2} = (\int d^3\mathbf{r}' \, f_{1,2}(z/z_0)e^{r'/r_{1,2}})^{-1}$. For both the spherical and cylindrical symmetry case, $\vartheta$ is the angle between $\mathbf{r}$ and $\mathbf{r}'$. By symmetry, the only non-vanishing component $g(r)$ of $\mathbf{g}$ is along $\mathbf{r}$. Therefore, the circular rotational velocity, $v(r)$, is given by

$$v(r) = \sqrt{rg(r)}$$

(15)

Though for the potential [2] Gauss theorem does not apply, the deviation is not numerically relevant as one can see from Fig. [9]. In the model presented it is natural to consider that matter of type 1 and type 2 distributed in the same way. Indeed, each type of matter have the same non-gravitational interactions and the sectors 1 and 2 couple only gravitationally. The very same formation mechanism for visible galaxies (type 1) will also form the invisible (type 2) ones giving the same matter distributions up to a rescale of relevant parameters. The scale $R_2$ is set simply by imposing that $\rho_2$ is related to $\rho_1$ by a dilatation factor $\alpha$ fixed by the invisible to visible mass.

**Figure 9:** Comparison between rotational curve (spherical) computed with numerically method (solid) and using the Gauss theorem (dashed). The result is similar ($M_2 = 10M_1$, $r_1 = 3$ kpc, $r_2 = 5.4$ kpc, $r_d = 15$ kpc).

**Figure 10:** Rotational curves with modifed potential: comparison between spherical (dashed) and f-modulated (solid) approximation ($r_g = 20$ kpc, $M_2 = 10M_1$, $r_1 = 3$ kpc and $r_2 = 5.4$ kpc).
ratio $\beta = M_2/M_1$. Then it follows that $R_2 = \alpha R_1$, with $\alpha = (M_2/M_1)^{1/4} = \beta^{1/4}$. For instance, if $R_1 = 3$ kpc and $\beta = 10$, then $R_2 \approx 5.4$ kpc. For the cylindrical approximation, a similar discussion tell us that $\alpha = \sqrt{3}$.

As a result, matter of type 2 plays the role of the dark matter with an additional welcome benefit: there is no clash between the actual halo shape (i.e. that fits the observed rotational curves) and the dark matter density profile produced by numerical simulations. The invisible, dark, matter of type 2 has a distribution similar to visible one and the observed rotational curves is reproduced by the non-Newtonian character of the gravitational potential at intermediate distances.

A large analysis varying the main parameter of the theory are shown in Figs. (10), (11), (12), (13), (17) and (18). Both variations of $M_2/M_1$ and $r_g$ can determine the flattening of the rotational curve for a large range of distances. Another relevant term is the sizes of the bulges of both matter distribution. It is interesting to note (see, Figs. (15) and (16)) that the bigravity rotational curves can approximately reconstruct a shape similar to the shape obtained with the classical IT model. Notice that our model requires dark matter two times more abundant than the matter needed in the CMD model. This amount is the same even for cluster dynamics and cosmology because, at large distance, gravity is Newtonian and reproduces the same results of the standard CDM with $M_{DM} \approx M_B$.

The value of the parameter $r_g$ stays in the range $10 \div 20$ kpc, that is the typical size of a galaxy ($M \sim 10^{11} M_\odot$).

![Figure 11: Different behaviors of rotational curves (Spherical approximation) varying $M_2/M_1$ mass ratio: 1 (dotted), 5 (dashed) and 10 (solid) ($r_g$ = 20 kpc, $r_1 = 3$ kpc and $r_2 = 5.4$ kpc). Notice that, for $M_1 = M_2$, we obtain once again the Newtonian shape.]

![Figure 12: Different behaviors of rotational curves (Spherical approximation) varying the $r_g$: 10 kpc (dotted), 20 (dashed) and 40 (solid) ($M_2/M_1 = 10$ kpc and $r_1 = 10$ kpc).]

Finally, we show a comparison with the experimental data relative to the NGC 2403 galaxy (see Figs. (19) and (20)). Observe that in this disk galaxy the cylindrical shape approximation reproduce with high accuracy the profile observed, while the
spherical approximation underestimates the values of velocity at low distances.

3 Discussion and Conclusion

Since DM is more abundant will start collapsing before ordinary matter and the latter will fall in the potential well of DM. The dark mirror matter is collisional and, differently from CDM, undergoes a dissipative collapse creating structures (e.g. galaxies, etc.) similar to the visible ones, i.e., with the same mass distribution. Since the gravitational force between sector 1 and 2 is effective when \( r \gtrsim r_g \sim 10 \div 20 \) kpc, after an initial stage of mutual interaction, as the collapse proceeds, the inner
structures of DM and ordinary matter are formed independently. On the contrary one can argue that for the outer region of galaxies, the interaction between ordinary and dark matter is important in the angular momentum exchange giving rising to a common galactic plane.

Besides the flattening of the rotational curves, a good DM matter candidate should also reproduce some properties that have observed in galaxies of different size and type [6, 7].

Typically, to fit the DM matter profile a constant inner core is required. In our model, dark and visible matter follow the same distribution and an inner core region with an exponential profile $\rho(r) \propto e^{-r/r_D}$ ($r_D$ is the DM bulge size) naturally exists like for the visible galaxy. Moreover, in the standard paradigm, to fit the data it is
assumed that DM is present only in a region with $r < L$, and the cut-off radius $L$ is given by the following phenomenological law $L = 100(M_{DM}/10^{11}M_\odot)^{1/3}$ kpc.

The latter can also be obtained in our model if make the rather natural assumption that $L \propto r_D$. Indeed, we have an exponential DM distribution, with total dark mass given by $M_D = \rho_D 8\pi r_D^3$ as a result $L \propto M_D^{1/3}$.

Finally, observations show that in a galaxy the DM amount is inversely proportional to the visible mass. This property is not surprising in our model where the graviton Compton length $r_g$ is of order $10 \div 20$ kpc: for scales $r > r_g$ Gravity is Newtonian and in a galaxy with an optical radius greater than $30 \div 40$ kpc the rotational curve shows that resembles the Keplerian fall; this is interpreted in standard paradigm as the observation that bigger galaxies have less DM.

Intriguingly, recent astrophysical observations indicates for dark matter a peculiar behaviour in galaxy cluster collisions. In the Bullet cluster [20], the dark matter component of the cluster shows a collisionless behaviour; instead, in the Abell 520 cluster [21] there is some evidence for an important contribution from self-interaction. In mirror dark matter models those observations are not surprising [22].

In conclusion if the Universe is made of two separate sectors, one visible and one hidden, each of them can have its own gravity and the two metrics can interact leading to a large distance modification of the gravitational force. At small distances, where the gravitational interaction between the different sectors is effectively shut off, the behaviour is Newtonian. The mirror matter as dark matter, together with Lorentz breaking can explain the flattening of galaxy rotational curves. This model assumes dark matter and modification of gravity. According to this proposal dark and visible matter have mass distribution avoiding the tension between observed rotational curves and extended CMD halo distributions predicted by N-body numerical simulations.

It is worth to point out that, at large cosmological distance both sectors interact each-other with effective Newton constant $G/2$. It follows that the observed Hubble constant implies that the total energy density of the Universe is twice bigger than in the standard cosmology in which the Newton constant at the same cosmological distance remains the canonical $G$. In this scenario, instead of $\rho_{cr} = 3H_0^2/8\pi G$ implied for the standard cosmology, we must have $\rho_{cr} = 3H_0^2/4\pi G$.

The mirror matter model with modified potential does not exclude the possibility of direct (non-gravitational) interaction between the normal and dark matter components, for example via photon kinetic mixing [13]. This also makes possible the direct detection via such interaction [23] and can explain the DAMA results [24]. In our model this possibility becomes higher because of the twice bigger DM amount in galaxy and the related bigger velocity of dark particles. Let us remark also that our scenario can have interesting implications also for the neutrino-mirror neutrino (active-sterile) oscillations [17] and can make more attractive the case of the exact

\footnote{For IT models cut-off is needed to have a finite the total DM mass.}
mirror parity when the ordinary and mirror neutrinos are degenerate in mass and maximally mixed. But their oscillations inside the galaxy should suffer a MSW-like suppression because of different gravitational potential felt by active and sterile neutrino species. Far outside the galaxy where the gravity becomes universal oscillations instead can go in full strength.

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