Instantaneous Availability of Multi-component System under the Constraint of Spare Parts

Yun Wang, Pengwei Hu, Baidong Jin, Hailong Cheng and Yuhan Liu
China Academy of Aerospace Standardization and Product Assurance, Beijing, P.R.CHINA
E-mail: ywang8_09@163.com

Abstract. The availability of equipment is one of the most important indicators to measure support effectiveness of system, and spare parts supply is one of the key support resources, so it is necessary to consider the constraint of spare parts in the study of availability. As the evaluation of steady availability cannot meet the requirement of support effectiveness timely and accurately and the previous models of instantaneous availability are usually applicable to one-unit system. To solve the above problem, firstly, the whole process is divided into two sub-processes, i.e., the replace and repair process of fault system and the maintenance and turnover process of failure parts, then the influence between the two sub-processes is analysed. Secondly, a model that can compute the instantaneous availability of multi-component series system with spare parts is given. Finally, a numerical example is used to demonstrate the applicability and the performance of the proposed method.

1. Introduction
The availability of equipment indicates the degree to which the equipment or weapon system can work normally when needed, and is an important indicator to measure the support effectiveness of the equipment system. The traditional availability model theory focuses on the study of the steady-state value of system availability, referred to as steady-state availability, which is measured by the probability of availability and characterizes the degree to which the system is ultimately in availability. At present, the methods for steady-state availability are relatively mature, and the research content is mainly concentrated in the following three aspects: Firstly, according to different system structures, component life distribution, work requirements, maintenance strategies, types and quantities of maintenance resources, etc., study how to establish the availability model and find out the steady-state availability; Secondly, the existence of steady-state availability under various
research conditions is proved mathematically. Finally, the Monte Carlo simulation method is used to solve the steady-state availability.

With the increasing requirements for support efficiency, the traditional steady-state availability research has been unable to meet the current timely and accurate evaluation requirements. In this context, the research on the effectiveness of support has changed from steady-state availability to instantaneous availability. Instantaneous availability characterizes the degree to which the system is in a working or usable state at any time when it needs and starts to perform tasks, its analytical solution can be expressed as a function of the time variable t, reflecting the change trend of the system availability with time, which provides an effective way to analyze the system's mission continuity and combat readiness. The modeling of instantaneous availability generally uses two methods of stochastic process theory and simulation. For repairable systems with component failure times that follow an exponential distribution, the Markov process method is commonly used to analyze the availability of such systems. As the evaluation of the guarantee effectiveness has the particularity of considering the guarantee resources, and spare parts, as one of the most important guarantee resources, have the special behavior of consumption and replenishment. Therefore, it is of great practical significance to consider the constraints of spare parts when studying availability. A large amount of research has also been carried out at home and abroad, but the stochastic process is suitable for the system whose research object is a single component, and few components are involved. This is because with the increase in the number and types of components in the system, the system may produce a state space combination explosion, which will inevitably increase the difficulty of modeling and the workload of solving calculations. Although the simulation method can consider more factors, such as allowing the system to contain more types and numbers of components, it can only analyze the change law of the instantaneous availability of the system from the phenomenon, and cannot reveal the inherent change law of the system.

In this paper, considering the constraints of spare parts, the instantaneous availability of the series system is studied. The structure is organized as follows: Firstly, by analyzing the work and support process of the entire system, it is divided into two sub-processes of replacement and repair of the faulty system and repair and turnover of the faulty parts, and the correlation between the two sub-processes is analyzed; Secondly, the availability of the series system is converted into the calculation of the availability of the single-component system, and the calculation model of the instantaneous availability of the single-component system is established to obtain the instantaneous availability of the series system; Finally, put forward conclusions and contents to be further studied.

2. Problem Formulation

2.1 Model Assumptions
For a series system as shown in the figure, when any component in the system fails, the system stops working. If the faulty parts are available in stock, the maintenance personnel immediately remove the faulty parts and replace the faulty parts with good parts in the spare parts warehouse; otherwise, it is necessary to wait for the availability of spare parts before carrying out the replacement of the faulty parts. After the replacement is completed, the system enters the working state. At the same time, the removed faulty parts are transferred to the maintenance warehouse, and
the repair equipment immediately repairs the replaced faulty parts, and then puts them back into the warehouse as available spare parts after repair. It is assumed that there is no situation where the faulty parts are waiting for maintenance.

![System Failure and Maintenance Process](image)

**Figure 1.** System Failure and Maintenance Process.

For the entire process shown in Figure 1, it can be divided into two sub-processes—the replacement and repair process of the faulty system and the repair and turnover process of the faulty parts. The corresponding systems are the working system and the inventory system. The above two systems are not independent, but there are interconnections: On the one hand, in the working system, the number of available spare parts in the entire system is reduced by replacing the faulty parts. While in the inventory system, the number of available spare parts in the entire system is increased through the maintenance turnover, and the two tasks are performed simultaneously to ensure the normal operation of the working system; On the other hand, in the case of a small number of spare parts, due to the long maintenance and turnover time, a delayed supply of spare parts may occur. In this case, the inventory system cannot provide enough spare parts in time. Therefore, the replacement and repair process of the working system will be affected, and the replacement and repair time of the defective parts will be extended.

2.2 Notations.

Suppose a series system is composed of $I$ components, and any component $i$ in the series system is regarded as a subsystem. According to the previous analysis, each subsystem corresponds to two sub-processes—the replacement and repair process of the faulty system and the maintenance turnover process of the faulty parts. For a single-component working system, it is assumed that the system has only two states of operation and failure. The failure interval time and replacement time of component $i$ follow the exponential distribution with mean values $1/\lambda_i$ and $1/\mu_i$ respectively,
the corresponding spare parts inventory is $N_i$, and the maintenance and turnover time of the failed parts follow the general distribution $G_i(x)$, and there is a density function $g_i(x)$ with the mean value $\int_0^{\infty} t g_i(t) dt$. Assuming that the system has only two states of working and fault, all these random variables are independent of each other, and all the parameters of the spare parts and corresponding working parts are exactly the same. The service life distribution of the faulty parts after repair is the same as that of the new parts, and spare parts will not fail when they are spared.

3. Modeling Technique

The availability of a series system can be expressed by the availability of a single component:

$$A = \frac{1}{\sum_{i=1}^{I} A_i} (1 \leq i \leq I)$$ (1)

Through the previous analysis, we have divided the work and guarantee process of the single-component system into the replacement and repair process of the faulty system and the maintenance and turnover process of the faulty parts. Therefore, first of all, by analyzing the replacement and maintenance process of the faulty system, the status of the working system is divided into three states: normal operation, system failure and available spare parts, and system failure but spare parts are out of stock. This process is approximately regarded as a Markov process, and the state transition equations are listed, and then the relationship between the system availability at time $t$ and the spare parts satisfaction rate and the rate of repaired repairs is returned; Then, by analyzing the maintenance and turnover process of the faulty parts and using the stochastic process theory, the expressions of the satisfaction rate of the spare parts and the rate of repair and return of the faulty parts change with time are obtained; According to the above two steps, the instantaneous availability of the single-component system can be obtained, and then the instantaneous availability of the entire series system can be obtained from the above formula.

3.1 Analysis of the replacement and maintenance process of the faulty system.

For a single-component working system, it is assumed that the system has only two states of operation and failure, if spare parts are considered, the fault state can be divided into two states: with spare parts and no spare parts. Define the rate of repair and return of the faulty part as $\omega(t)$, which is determined by the maintenance and turnover process of the faulty part. When a component fails, the probability of having spare parts available is $q(t)$, and the probability of having no spare parts available is $1 - q(t)$. State 0 represents the normal operation of the system; for ease of analysis; State 1 represents system failure, this state is a transient state; State 2 indicates that there
are spare parts available, and the faulty parts are being repaired; State 3 represents out of stock at this time, waiting for the arrival of new available spare parts. The state transition diagram is as follows:

![State Transition Diagram](image)

**Figure 2.** The State Transition Diagram of A Single-Component.

Considering this process as a Markov process, the following state transition equations are obtained

\[
\begin{align*}
P_0 (t + \Delta t) &= P_0 (t) \cdot (1 - \lambda \Delta t) + P_2 (t) \mu \Delta t \\
P_2 (t + \Delta t) &= P_1 (t + \Delta t) q (t) + P_2 (t) (1 - \mu \Delta t) + P_3 (t) \omega (t) \Delta t \\
P_3 (t + \Delta t) &= P_1 (t + \Delta t) [1 - q (t)] + P_3 (t) [1 - \omega (t) \Delta t] \\
P_1 (t + \Delta t) &= P_0 (t) \cdot \lambda \Delta t \\
P_0 (t) + P_2 (t) + P_3 (t) &= 1
\end{align*}
\]

(2)

Obtaining

\[
\begin{align*}
\frac{dP_0 (t)}{dt} &= -\lambda P_0 (t) + \mu P_2 (t) \\
\frac{dP_2 (t)}{dt} &= \lambda q (t) P_0 (t) - \mu P_2 (t) + \omega (t) P_3 (t) \\
\frac{dP_3 (t)}{dt} &= \lambda [1 - q (t)] P_0 (t) - \omega (t) P_3 (t) \\
P_0 (t) + P_2 (t) + P_3 (t) &= 1
\end{align*}
\]

(3)

Initial value
\[
\begin{align*}
P_0(0) &= 1 \\
P_1(0) &= 0 \\
P_2(0) &= 0 \\
P_3(0) &= 0
\end{align*}
\] (4)

Obtaining
\[
\begin{align*}
P_0(t) &= e^{-\lambda t} + \mu e^{-\lambda t} \int_0^t P_2(\tau) e^{\lambda \tau} d\tau \\
P_3(t) &= \lambda e^{-\int_0^t \omega(\tau) d\tau} \left[ \int_0^t [1 - q(\tau)] P_0(\tau) e^{\int_0^\tau \omega(\xi) d\xi} d\tau \right] + \int_0^t e^{\lambda \tau} \int_0^\tau P_2(\xi) e^{\lambda \xi} d\xi d\tau \\
P_0(t) + P_2(t) + P_3(t) &= 1
\end{align*}
\] (5)

Let \( P_0(t), P_3(t) \) be denoted by \( P_2(t) \), there is
\[
\begin{align*}
P_0(t) &= e^{-\lambda t} + \mu e^{-\lambda t} \int_0^t P_2(\tau) e^{\lambda \tau} d\tau \\
P_3(t) &= \lambda e^{-\int_0^t \omega(\tau) d\tau} \left[ \int_0^t [1 - q(\tau)] \left( e^{-\lambda t} + \mu e^{-\lambda t} \int_0^\tau P_2(\xi) e^{\lambda \xi} d\xi \right) \int_0^\tau \omega(\xi) d\xi d\tau \right]
\end{align*}
\] (6)

3.2 Analysis of maintenance and turnover process of faulty parts

Definition \( d(x)(0 < x < t) \) represents the rate at which the replaced defective parts enter the maintenance and turnover process, then the number of defective parts entering the maintenance and turnover process during time \( (x, x+\Delta x) \) is \( d(x)\Delta x \). If the stay time of the faulty parts in the maintenance cycle is less than \( t-x \), it can be considered that the faulty parts leave the maintenance cycle before time \( t-x \), and the probability of leaving is \( P(x) = G(t-x) \). Therefore, the number of the faulty parts that enter the maintenance cycle in time \( (x, x+\Delta x) \) and leave before time \( t \) is \( d(x)\Delta x G(t-x) \). Since the time \( x \) for the faulty parts to enter the maintenance and turnover process is uniformly distributed in \( (0,t) \), the number of faulty parts leaving the maintenance and turnover process before time \( t \) is \( \frac{\Delta x}{t} \int_0^t d(x) G(t-x) dx \), the integral can be obtained that the number of faulty parts leaving the maintenance and turnover process before time \( t \) within the
period of \( (0,t) \) is 
\[
\int_0^t \left[ \int_0^t d(x)G(t-x)dx \right]d\tau + \int_0^t d(x)G(t-x)dx
\]
can be calculated, its derivative is 
\[
\int_0^t d(x)g(t-x)dx, \quad \int_0^t d(t-x)g(x)dx
\]
can be obtained after transformation, this is the rate at which the repaired part returns, which is \( w(t) \), where \( d(t) = P_0(t)\lambda \).

\[
q(t) = \sum_{k=0}^{N} e^{-\int_0^t d(x)[1-G(t-x)]dx} \left\{ \int_0^t d(x)[1-G(t-x)]dx \right\}^k \frac{k!}{k!}
\]

From this, we can get:
\[
\begin{align*}
\omega(t) &= \int_0^t d(t-x)g(x)dx \\
d(t) &= P_0(t)\lambda \\
q(t) &= \sum_{k=0}^{N} e^{-\int_0^t d(x)[1-G(t-x)]dx} \left\{ \int_0^t d(x)[1-G(t-x)]dx \right\}^k \frac{k!}{k!}
\end{align*}
\]

3.3 System comprehensive availability analysis.

Substitute (8) into (6)
\[
\begin{align*}
P_0(t) &= e^{-\lambda t} + \mu e^{-\lambda t} \int_0^t P_2(\tau) e^{\lambda \tau} d\tau \\
P_3(t) &= \lambda e^{\int_0^t \omega(\tau)d\tau} \left[ e^{-\lambda t} + \mu e^{-\lambda t} \int_0^t P_2(\xi) e^{\lambda \xi} d\xi \right] e^{\int_0^t \omega(\xi)d\xi} d\xi
\end{align*}
\]
to obtain a system of equations about \( P_2(t) \), substitute the system of equations into
\[
P_0(t) + P_2(t) + P_3(t) = 1
\]
to find \( P_2(t) \); then bring \( P_2(t) \) back to (5) to solve \( P_0(t), P_3(t) \).

Therefore, the probability that the large system composed of the working system and the inventory system can work normally at time \( t \), that is, the instantaneous availability is
\[
A^i(t) = P_0^i(t)
\]

That is, the instantaneous availability \( A^i(t) \) of each single-component system is obtained; then all \( A^i(t) \) is brought into formula (1), then the instantaneous availability of the entire series system is obtained.
4. Numerical Solution

We shall sketch a numerical procedure for determining the correct values of $P_0(t)$.

(1) When $t = 0$, there is

$$P_0(t) = P_0(0) = 1$$

$$P_2(t) = P_2(0) = 0;$$

$$P_3(t) = P_3(0) = 0$$

(2) Take $\Delta t = 1$, and substitute $t = 0, \Delta t = 1$ into (2) to get a set of initial value $P_0(t + \Delta t), P_2(t + \Delta t), P_3(t + \Delta t)$;

(3) Take $t = t + \Delta t$, and substitute it into (2) to get a new set of $P_0(t + \Delta t), P_2(t + \Delta t), P_3(t + \Delta t)$, let it be $P_0^*(t + \Delta t), P_2^*(t + \Delta t), P_3^*(t + \Delta t)$;

(4) Repeat step (3), if $|P_0^*(t + \Delta t) - P_0(t + \Delta t)| \leq e, |P_2^*(t + \Delta t) - P_2(t + \Delta t)| \leq e, |P_3^*(t + \Delta t) - P_3(t + \Delta t)| \leq e$, the algorithm ends. Among them, $e$ represents the tolerance limit, generally take $10^{-3} \sim 10^{-4}$.

(5) At this time, multiple values of $P_0(t)$ (that is, $A(t)$) are obtained, and the curve of $A(t) - t$ is drawn to obtain the instantaneous availability curve of the single-component system.

Since in actual engineering, the system generally works intermittently and shuts down, the failure rate changes periodically with time. Therefore, next we discuss how to solve the instantaneous availability model whose failure rate follows a periodic distribution. According to the relevant knowledge of Fourier series, the failure rate function satisfies the convergence theorem, so its Fourier series expansion is

$$\lambda(t) = \frac{a_0}{2} + \sum_{n=1}^\infty \left(a_n \cos \frac{n\pi t}{l} + b_n \sin \frac{n\pi t}{l}\right), (t \in C)$$

Where
\[ a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} \, dx \quad (n = 0, 1, 2, \ldots) \]
\[ b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} \, dx \quad (n = 1, 2, 3, \ldots) \]
\[ C = \left\{ t \mid \lambda(t) = \frac{1}{2} \left[ \lambda(t^-) + \lambda(t^+) \right] \right\} \]

Thus, when the failure rate follows a periodic distribution over time, its function is first expanded into a Fourier series, and then the instantaneous availability model is sufficient.

5. Conclusions
This paper gives the calculation method of the instantaneous availability of the series system considering the constraints of spare parts. The work and support process of the entire system is divided into two sub-processes of replacement and repair of the faulty system and maintenance and turnover of the faulty parts, and the interrelationship between the two sub-processes is analyzed; Secondly, the availability of the series system is converted to the calculation of the availability of the single-component system. Using Markov and stochastic process theory, the instantaneous availability of the series system is obtained.

However, this model only considers a series system composed of multiple identical components, and is suitable for a series system or parallel system composed of multiple different components; In addition, this study assumes that there is no situation where the faulty parts are waiting for repair, and in practice there are often situations where the faulty parts are waiting for repair due to factors such as shortage of guarantee resources or management. In future research, the above issues will be considered in order to reflect the actual situation more accurately.