Simulation Studies of a Total Absorption Dual Readout Calorimeter

Hans Wenzel
Fermilab, P.O. Box 500, Batavia, IL 60510-5011, USA
E-mail: wenzel@fnal.gov

Abstract. We have created a flexible, easy to use simulation framework based on GEANT 4 to perform detailed simulations of a crystal based total absorption calorimeter with dual readout. The correction to the observed scintillation signal can be determined by comparison of the scintillation signal with the beam energy as a function of the Cerenkov-to-scintillation \( C/S \) ratio. In this article we describe the features of the software we developed and show results for various crystal materials and physics lists. We show that applying an energy-independent correction results in an energy resolution of \( \approx 10\%/\sqrt{E} \) for single \( \pi^- \).

1. Introduction
In this article we describe the full GEANT 4 [1] simulation used to study the properties and performance of a homogeneous total absorption dual readout calorimeter. Such a calorimeter addresses the following principal contributions to hadron energy resolution and non-linearity:

- fluctuations in nuclear binding energy loss dominate the energy resolution resulting in a non-linear, non-Gaussian hadron response. We will demonstrate that this can be mitigated using dual readout where both Cerenkov \( C \) and scintillation \( S \) signal are read out from the same crystal and and the \( C/S \) ratio is used to correct the scintillation signal. We show that we can achieve linear and Gaussian response and improved hadronic resolution.

- Sampling fluctuations in the sharing of the shower energy between the active and passive materials in sampling calorimeters are eliminated by making the calorimeter homogeneous and totally active.

- Difference in the sampling fractions (i.e. ratio in the effective energy loss) between the different materials in the sampling calorimeters can be eliminated by making the calorimeters homogeneous.

- While leakage fluctuations due to escaping neutrinos and muons can not be avoided, tails of the hadronic shower escaping the detector can be minimized by using dense high density modern heavy metal crystals. The availability of such crystals means that full containment of hadronic showers can be achieved with a total absorption calorimeter with a volume fitting into a present collider detector. All materials are dense with a nuclear interaction length \( \lambda_I \) in a range from 21 cm (\( PW\) and \( PbF_2 \)) to 23 cm (\( BGO \)) for a complete list of crystal properties (\( BGO, PbF_2, PbWO \)) see [2].

Besides heavy crystals the other enabling technology is the development of on-detector readout devices, e.g., SiPMs, MCPs, etc., which allow that individual crystal calorimeter cells can be
organized in a highly granular configuration that would allow the use of Particle Flow Algorithms [3] to further improve the performance.

2. The Software Environment:
We have developed a flexible simulation framework CaTS (Calorimeter and Tracker Simulation) [4] to study various aspects of a dual read out calorimeter. CaTS is a stand alone GEANT 4 application which allows for a large range of studies. For example one can do detailed studies of single crystals or small test beam setups where all effects like photon propagation, photon absorption, Rayleigh scattering, optical surface properties detection efficiency etc. are taken into account and optical photons are produced and traced until they are either absorbed or reach a photo-detector. The major components of CaTS include:

(i) Detector Description: CaTS uses the XML based Geometry Description Markup Language (GDML) [5], as the primary geometry implementation language. To change the detector setup no recompilation of CaTS is necessary one only needs to select a different GDML input file. GDML allows to define shapes, to place and rotate the volumes, to describe material properties like density, material composition as well as optical properties like e.g. absorption length and refraction index and surface properties [6]. In addition GDML allows to declare volumes as sensitive and to declare what hit collections should be produced and written out in the event stream. A library of various GDML detector descriptions ranging from single crystals to test beam setups is provided in the CaTS CVS repository.

(ii) Persistency: Events are stored as Root files [7]. Reflex is used to automatically generate dictionaries for all classes that we want to write out. This makes it easy to add new classes or to extend existing classes written out in the event stream. The resulting Root files can then be analyzed via Root macros or C++ routines. Several example macros and C++ routines are provided in the CaTS CVS repository.

(iii) Input modules:
- GPS: General Particle Source [8] allows the specifications of the spectral, spatial and angular distribution of the primary source particles.
- Particle Gun is a very simple particle gun that shoots a particle of given type into a given direction with a given kinetic energy or momentum.
- HEPMC [9] provides a C++ Event Record for Monte Carlo Generators like Pythia [10].

(iv) Physics Lists: CaTS allows to choose from all GEANT 4 Reference Physics Lists. This lists can be extended to include optical physics processes like the Cerenkov effect, Rayleigh scattering, the production of Scintillation Photons etc.. We found that FTFP_BERT is currently the best available physics list for the simulation of hadronic showers. This is also supported by studies of the CALICE collaboration [11] comparing results from various GEANT 4 models with test beam data.

(v) Sensitive Detectors and corresponding Hit collections:
- TrackerSD: is a sensitive detector class to simulate the response of a tracking detector. Each energy deposition in the detector material results in a different Tracker Hit.
- CalorimeterSD: is a sensitive detector class that registers the energy deposit of each particle traversing the sensitive volume. A hit is the sum of all energy deposits in a calorimeter cell.
- DRCalorimeterSD: is a sensitive detector class representing a dual read out calorimeter. Besides registering the energy deposit of each particle traversing the sensitive volume the number of produced Cerenkov photons is calculated for each charged particle above the Cerenkov threshold( $\beta > 1/n$). The same algorithm as in the G4Cerenkov Class is used but here only the number of produced Cerenkov photons is of interest.
StoppingCalorimeterSD: is a sensitive detector class that registers the properties of particles entering the sensitive Volume but then the particle is killed to save CPU time.

PhotonSD: sensitive detector that registers optical photons.

(vi) User Actions: examples of user actions (EventAction, RunAction, StackingAction, SteppingAction...) are provided

3. Calorimeter response before dual readout correction

For this studies we use a very simple calorimeter with the total dimension of $3 \times 3 \times 3 \ m^3$, longitudinally segmented in 1 mm thick sheets of Crystal material comprising 13/14 nuclear interaction length $\lambda_I$ for BGO/(PWO and PbF2).

**Figure 1.** Average ratio of scintillation response to available energy $S/E_{in}$ for mono-energetic particles as a function of $E_{kin}$. The calorimeter material is BGO. The Physics List is FTFP_BERT.

**Figure 2.** Average ratio of Cerenkov response to available energy $C/E_{in}$ for mono-energetic particles as a function of $E_{kin}$. The calorimeter material is BGO. The Physics List is FTFP_BERT.

**Figure 3.** Average ratio of Cerenkov and ionization response($C/S$) for mono-energetic particles as a function of $E_{kin}$. The calorimeter material is BGO. The Physics List is FTFP_BERT.
Figure 1 and 2 show the scintillation $S$ and Cerenkov response $C$ with respect to the deposited energy $E_{in}$ as a function of the kinetic energy for different mono-energetic particles. The Cerenkov and ionization response was calibrated with electrons, the response is multiplied with a constant factor so that $S(e) = E_{in}(e) = C(e)$. Both $S$ and $C$ for $e^\pm$ are linear function of $E_{in}$. For the Cerenkov response this is due to the fact that the $\beta$ ($E$) spectrum of charged tracks above the Cerenkov threshold in the electromagnetic shower is universal meaning composition and shape don’t change with $E_{in}$.

For protons and neutrons $E_{in}$ is equal to the kinetic energy $E_{kin}$ of the incident particle ($E_{in} = E_{kin}$). For charged $K$’s and $\pi$’s in addition to $E_{kin}$ the invariant mass is transformed into available energy when the particle decays ($E_{in} = E_{kin} + E_m(\pi^{\pm}, K^{\pm})$). For anti protons ($\bar{p}$) twice the invariant proton mass is released when the $\bar{p}$ annihilates with a proton in the target material ($E_{in} = E_{kin} + 2 \times E_m(p)$). We observe the response $S$ to hadrons ($p$, $\bar{p}$, $n$, $K^{\pm}$, $\pi^{\pm}$) is non-linear and lower than the response to electrons ranging in average from 0.85 to 0.95 for $E_{kin}$ between 1 and 100 GeV. The response to different hadrons is slightly different ($\approx 1\%$). The hadronic Cerenkov response $C$ is very nonlinear and compared to $S$ highly suppressed ranging from 0.4 to 0.72 for $E_{kin}$ between 1 and 100 GeV. Figure 3 shows the ratio of the average Cerenkov and ionization response $C/S$ showing the different behavior of the two signals as a function $E_{kin}$. Using the difference in the response of the two signals to correct the scintillation response is the basic idea of a dual read out calorimeter.

### 4. Hadronic Response

The hadronic Scintillation and Cerenkov response can be expressed in the following way:

\[ ScintillationResponse : S/E_{in} = f_{em} + (1 - f_{em}) h_s \tag{1} \]

\[ CerenkovResponse : C/E_{in} = f_{em} + (1 - f_{em}) h_C, \tag{2} \]

where $f_{em}$ is the electromagnetic fraction in the cascade and $h_s$ and $h_C$ are the hadronic responses for Scintillation and Cerenkov readout respectively. We are using two different definitions to calculate $f_{em}$ one is the traditional method of using the total energy of $\pi^0$ in the hadronic shower to estimate $f_{em}(\pi^0)$. We prefer to define $f_{em}(em)$ as the energy deposited by $e^-$, $e^+$ or $\gamma$ in the cascade. Combining equations 2 and 1 results in:

\[ E_{in} = S \left[ \frac{(1 - h_s) - C/S(1 - h_s)}{h_s - h_C} \right] \tag{3} \]

Where: $h_s > h_c$. If $h_s$ and $h_C$ are about the same, then $S$ and $C$ are about the same and the equations are degenerate and a dual read out correction would not help. But it appears that $h_s$ is large compared to $h_C$ as is shown in Figure 4.

### 5. Dual readout correction and hadronic energy resolution

Figure 5 shows the average $S/E_{in}$ vs. $C/S$ for simulated data of mono-energetic single $\pi^\pm$ and $e^\pm$ of (1,2,5,10,20,50,100) GeV kinetic energy. The response to $e^\pm$ is at $S/E_{in} = C/S = 1$. A 3$^{rd}$ degree polynomial $F_{corr}(C/S)$ is superimposed and the fitted function is used to apply the dual readout correction to the Scintillation response $S_{corr} = S/F_{corr}(C/S)$. We found that the correction function varies only weakly with energy. Here we used one correction function for all particle energies. Further improvement of the energy resolution and elimination of remaining non-linearity might be possible using energy dependent corrections. Also different particles probe different regimes in $C/S$ (see Figure 3) and hence are sensitive to the imperfections of the correction function in different ways. This might also be mitigated by using energy dependent corrections.
Figure 4. Hadronic scintillation ($h_s(em)$, $h_s(\pi^0)$) and hadronic Cerenkov response ($h_c(em)$, $h_c(\pi^0)$) as a function of $E_{kin}$ of incident charged $\pi^-$. 

Figure 5. Dual readout correction function $f_{corr}(C/S)$ as estimated with mono-energetic single $\pi^\pm$ and $e^\pm$ of (1.2.5.10,20,50,100) GeV kinetic energy. The result of fitting a 3rd degree polynomial is superimposed. The calorimeter material is BGO. The Physics List is FTFP.BERT.

Figure 6. Relative hadronic resolution function $\sigma_{\pi^-}(E_{kin}) = C_1 + C_2 / \sqrt{E_{kin}}$ [%] as estimated with mono-energetic single $\pi^\pm$. A simple linear fit gives the following parameter values: $C_1 = 0.23 \pm 0.08\%$ and $C_2 = 9.55 \pm 0.15\%$. The calorimeter material is BGO. The Physics List is FTFP.BERT. This is before detector effects like noise or threshold cuts are taken into account.

Comparing $S_{corr}$ with $E_{in}$ for $\pi^-$ of various kinetic energies and fitting Gaussian functions to the resulting distributions one can obtain the energy resolution. Figure 6 shows the relative resolution $\sigma_{\pi^-}(E_{kin}) / \sqrt{E_{kin}}$. A simple linear fit gives the following parameter values: $C_1 = 0.23 \pm 0.08\%$ and $C_2 = 9.55 \pm 0.15\%$. Table 1 lists the fit results for various crystal materials and physics lists. In all cases using the energy-independent correction results in an energy resolution of $\approx 10% / \sqrt{E}$ for single $\pi^-$. Figure 1 shows $S_{corr}/E_{in}$ vs. $E_{kin}$. The non-linearity at lower energies are much reduced and at higher energies the response is linear. At higher energies the response to all particles is nearly identical and reproduces the deposited
energy $E_{in}$. Again further improvement especially at lower energies might be possible using energy dependent corrections.

![Figure 7. Average ratio of dual readout corrected scintillation response to available energy $S_{corr}/E_{in}$ for monoenergetic particles as a function of $E_{kin}$.](image)

Table 1. Relative hadronic resolution function $\sigma(E) / E = C_1 + C_2 \sqrt{E}$ as estimated with monoenergetic single $\pi^-$ for various calorimeter materials and GEANT 4 physics lists. $C_1$ and $C_2$ were estimated with a simple linear fit.

| Material | Physics List   | $C_1 [%]$ | $C_2 [%]$ |
|----------|----------------|-----------|-----------|
| BGO      | FTFP_BERT      | 0.23 ± 0.08 | 9.55 ± 0.15 |
| BGO      | QGSP_BERT      | 1.16 ± 0.5  | 8.6 ± 1.0  |
| PbWO     | FTFP_BERT      | 0.89 ± 0.4  | 8.1 ± 0.9  |
| PbF2     | FTFP_BERT      | 0.84 ± 0.3  | 10.6 ± 0.5 |

6. Conclusion

We created a flexible, easy to use simulation framework. It allows detailed studies of single Crystals and full detector setups. Full GEANT 4 simulation supports the homogeneous, total absorption, dual readout concept and predicts excellent energy resolution of $\approx 10\% / \sqrt{E}$ for single $\pi^-$ before detectors effects like noise and threshold cuts are taking into account. While the concept has not yet been demonstrated in a full size (containing hadronic showers) test beam setup there has been a lot of development recently by the GEANT 4 collaboration to improve the modeling of hadronic showers. Good agreement of CALICE test beam data with GEANT 4 simulation provides confidence in the GEANT 4 simulations. Also the availability of detailed test beam data will allow to further improve the models in the future.

[1] The two main reference papers for GEANT 4 are published in Nuclear Instruments and Methods in Physics Research A 506 (2003) 250-303, and IEEE Transactions on Nuclear Science 53 No. 1 (2006) 270-278. http://www.GEANT4.org/GEANT4/.

When not specified otherwise, results were obtained using GEANT4 9.5 (patch-01) released 20 March 2012 and the FTFP_BERT reference physics lists.

[2] Rihua Mao, Liyuan Zhang and Ren-Yuan Zhu, *Crystals for the HHCAL Detector Concept*, CALOR2012, Santa Fe, June 2012.
[3] Magill, Steve, *Use of Particle Flow Algorithms in a Dual Readout Crystal Calorimeter*, CALOR2012, Santa Fe, June 2012.

[4] http://home.fnal.gov/~wenzel/CaTS.html.

[5] http://lcgapp.cern.ch/project/simu/framework/GDML/gdml.html.

[6] Since GEANT 4.9.3 look up tables exist which describe various surface types and crystal surface conditions as measured by: Martin Janecek and William W. Moses, *Measuring Light Reflectance of BGO Crystal Surfaces* (LBNL-1790E (2009)) [http://escholarship.org/uc/item/83v3b0gf](http://escholarship.org/uc/item/83v3b0gf).

[7] Rene Brun and Fons Rademakers, *ROOT - An Object Oriented Data Analysis Framework*, Proceedings AIHENP’96 Workshop, Lausanne, Sep. 1996, Nucl. Inst. & Meth. in Phys. Res. A 389 (1997) 81-86. See also [http://root.cern.ch/](http://root.cern.ch/).

[8] [http://reat.space.qinetiq.com/gps/](http://reat.space.qinetiq.com/gps/).

[9] [http://lcgapp.cern.ch/project/simu/HepMC/](http://lcgapp.cern.ch/project/simu/HepMC/).

[10] [http://home.thep.lu.se/~torbjorn/Pythia.html](http://home.thep.lu.se/~torbjorn/Pythia.html).

[11] Marco Ramilli, *Hadronic models validation in GEANT4 with CALICE highly granular calorimeters*, CALOR2012, Santa Fe, June 2012.