Cosmology with a Nonlinear Born-Infeld Type Scalar Field

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Abstract

Recent many physicists suggest that the dark energy in the universe might result from the Born-Infeld(B-I) type scalar field of string theory. The universe of B-I type scalar field with potential can undergo a phase of accelerating expansion. The corresponding equation of state parameter lies in the range of $-1 < \omega < -\frac{1}{3}$. The equation of state parameter of B-I type scalar field without potential lies in the range of $0 \leq \omega \leq 1$. We find that weak energy condition and strong energy condition are violated for phantom B-I type scalar field. The equation of state parameter lies in the range of $\omega < -1$.

Keywords: Dark energy; Born-Infeld type scalar field; Phantom cosmology.

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1 Introduction

Evidence that the universe is undergoing a phase of accelerating expansion at the present epoch continues to grow. This not only can be inferred by accelerating dynamics in high redshift surveys of Ia type supernovae[1], but also now it is independently implied from seven cosmic microwave background experiments(including the latest WMAP)[2]. The favoured explanation for this behavior is that the universe is presently dominated by some form of dark energy density, contribution up to 70% of the critical energy density, with the remaining 30% comprised of clumpy baryonic and non-baryonic dark matter. One of the central questions in cosmology today is the origin of the dark energy. Many candidates for dark energy have been proposed so far to fit the current observations. Among these models, the most important one is a self-interacting scalar field with a potential and thereby acts as a negative pressure source, referred to as "quintessence"[3]. This paradigm has caused attentions because a wide class of models exhibits tracking behavior at late time, where the dynamics of the field becomes independent of its initial conditions in early universe. In principle, this may resolve the fine-tuning inherent problem in dark energy models purely based on a cosmological constant. The major difference among these models is that different model predicts dissimilar equation of state of the dark energy, thus different cosmology is predicted. Especially, for these models, the equations of state parameter are confined within the range of \(-1 < \omega = \frac{p}{q} < -\frac{1}{3}\), which can drive the conclusion of accelerating expansion of the universe. However, some analysis of the observation data hold that the range of the equation of state parameter may not always be greater than \(-1\). In fact, they can lie in the range of \(-1.48 < \omega < -0.72\) [4]. It is obvious that the equation of state of conventional quintessence models based on a scalar field with positive kinetic energy can not evolve into the range of \(\omega < -1\), and therefore, some authors[5] investigated phantom field models that possess negative kinetic energy and can realize \(\omega < -1\) in their evolution. It is true that the field theory with negative kinetic energy poses a challenge to the widely accepted energy condition and leads to a rapid vacuum decay[6], but it is still very important to study these models, in some sense that is phenomenologically interesting.

On the other hand, the role of tachyon field in string theory in cosmology has been widely studied[5]. It shows that the tachyon can be described by a Born-Infeld (B-I)type lagrangian resulting from string theory. It is clear that the lagrangian \(\frac{1}{\eta}[1 - \sqrt{1 - \eta g^{\mu\nu} \phi, \mu \phi, \nu}] - u(\phi)\)
is equivalent formally to the tachyon type lagrangian $\frac{1}{\eta} \sqrt{1 - g^{\mu\nu} \Phi, \mu \Phi, \nu}$ with a potential $\left[ \frac{1}{\eta} - u(\varphi) \right]$, where re-scale the scalar field as $\Phi = (\eta)^{1/2} \varphi$. In this paper we consider cosmology of B-I type scalar field. The paper is organized as follows: In sec.2, we consider B-I type Lagrangian of scalar field without a potential. We obtain $0 < \omega < 1$. In sec.3, the B-I type Lagrangian of scalar field with a potential is considered. We find that potential $u(\varphi)$ is greater than $\frac{1}{\eta}$, and the kinetic energy of B-I scalar field is smaller than $\frac{1}{3\eta}$, thus we obtain $-1 < \omega < -1/3$. In sec.4, the phantom with B-I Lagrangian is considered. We find that weak energy condition and strong energy condition are violated for phantom B-I type scalar field with potential. The equation of state parameter lies in the range $\omega < -1$. Sec.5 is Summary.

2 The Model with B-I Lagrangian $\frac{1}{\eta} \left[ 1 - \sqrt{1 - \eta g^{\mu\nu} \varphi, \mu \varphi, \nu} \right]$

In 1934[7], Born and Infeld put forward a theory of non-linear electromagnetic field. The lagrangian density is

$$L_{BI} = b^2 \left[ 1 - \sqrt{1 - \left( \frac{1}{2b^2} \right) F_{\mu\nu} F^{\mu\nu}} \right] \quad (2-1)$$

The lagrangian density for a B-I type scalar field is

$$L_S = \frac{1}{\eta} \left[ 1 - \sqrt{1 - \eta g^{\mu\nu} \varphi, \mu \varphi, \nu} \right] \quad (2-2)$$

Eq.(2-2) is equivalent to the tochyon lagrangian $[-V(\varphi) \sqrt{1 - g^{\mu\nu} \varphi, \mu \varphi, \nu} + \Lambda]$ if $V(\varphi) = \frac{1}{\eta}$ and cosmological constant $\Lambda = \frac{1}{\eta} \left( \frac{1}{\eta} \right)$ is two times as "critical" kinetic energy of $\varphi$ field. The lagrangian (2-2) possesses some interesting characteristics, it is exceptional in the sense that shock waves do not develop under smooth or continuous initial conditions and because nonsingular scalar field solution can be generated[8]. When $\eta \to 0$, by Taylor expansion, Eq.(2-2) approximates to the lagrangian of linear scalar field.

$$\lim_{\eta \to 0} L_S = \frac{1}{2} g^{\mu\nu} \varphi, \mu \varphi, \nu \quad (2-3)$$

A quantum model of gravitation interacting with a lagrangian (2-2) of B-I type scalar has been considered by us. We obtained the Wheeler-Dewitt equation of B-I scalar field and found the wave function of the universe. An inflationary universe, with the largest possible vacuum energy and the largest interaction between the particles of B-I scalar field[9], is predicted.

Next we consider classical cosmology. For the spatially homogeneous scalar field, Eq.(2-2)
becomes

\[ L_S = \frac{1}{\eta} \left[ 1 - \sqrt{1 - \eta \dot{\varphi}^2} \right] \quad (2-4) \]

In the spatially flat Robertson-Walker metric \( ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2) \), Einstein equation \( G_{\mu\nu} = KT_{\mu\nu} \) can be written as

\[ \left( \frac{\dot{a}}{a} \right)^2 = \frac{K}{3} T^0_0 \quad (2-5) \]

\[ 2 \ddot{a} + \left( \frac{\dot{a}}{a} \right)^2 = KT^1_1 = KT^2_2 = KT^3_3 \quad (2-6) \]

Substituting Eq.(2-5) into Eq.(2-6), we get

\[ \frac{\ddot{a}}{a} = -\frac{K}{3} (T^0_0 - 3T^1_1) \quad (2-7) \]

where

\[ T^\mu_\nu = \frac{\delta^{\mu\nu} - \frac{1}{2} \eta g^{\mu\nu} \varphi \cdot \varphi \cdot \varphi}{\sqrt{1 - \eta g^{\mu\nu} \varphi \cdot \varphi \cdot \varphi}} - \delta^\mu_\nu L_S \quad (2-8) \]

The energy density \( \rho_s = T^0_0 \) and pressure \( p_s = -T^i_i \) are defined as following:

\[ \rho_s = T^0_0 = \frac{\dot{\varphi}^2}{1 - \eta \dot{\varphi}^2} - L_s \quad (2-9) \]

\[ p_s = -T^i_i = \frac{1}{\eta} \left[ 1 - \sqrt{1 - \eta \dot{\varphi}^2} \right] \quad (2-10) \]

where the upper index "." denotes the derivative with respect to \( t \).

The equation of motion of scalar field \( \varphi \) is

\[ \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\nu} \left[ \sqrt{-g} g^{\mu\nu} \varphi \cdot \varphi \cdot \varphi \cdot \varphi \cdot \varphi \right] = 0 \quad (2-11) \]

The scalar field \( \varphi \) only depends on \( t \). From Eq.(2-11), we can obtain

\[ \dot{\varphi} = \frac{c}{\sqrt{a^6 + \eta c^2}} \quad (2-12) \]

where \( c \) is integral constant. When \( a(t) = 0 \), the kinetic energy \( \dot{\varphi}^2 = \frac{1}{\eta} \) is critical maximum.

From Eqs.(2-5),(2-7),(2-9),(2-10),(2-12) we get

\[ \left( \frac{\dot{a}}{a} \right)^2 = \frac{K}{3\eta} \left[ 1 + \eta c^2 a^{-6} - 1 \right] \quad (2-13) \]

\[ \ddot{a} = -\frac{K}{3\eta} \left[ 2 - \frac{2a^6 - \eta c^2}{a^3 \sqrt{a^6 + \eta c^2}} \right] \quad (2-14) \]
From Eq.(2-14) we can find that $\ddot{a}$ is always smaller than zero, no matter what the value of $a(t)$ is. When $a \to \infty$, then $\ddot{a} \to 0$. It shows that the universe starts with decelerated regime and gradually enters the zero acceleration. From Eq.(2-13) we get

$$\dot{a} = \sqrt{\frac{K}{3\eta}} \left[ a^2 \sqrt{1 + \eta c^2 a^{-6} - a^2} \right]$$  \hspace{1cm} (2 - 15)$$

By Eq.(2-15), then we can find that the minimum of $a(t)$ is zero, assume $a(t) |_{t=0} = 0$. When $a(t) \ll 1$, Eq.(2-15) can be approximated as

$$\sqrt{a}\dot{a} \approx \sqrt{\frac{Kc}{3\sqrt{\eta}}}$$ \hspace{1cm} (2 - 16)$$

$$a^{3/2} \approx \frac{3}{2} \sqrt{\frac{Kc}{3\sqrt{\eta}}} t$$ \hspace{1cm} (2 - 17)$$

$$a(t) \sim t^{2/3}$$ \hspace{1cm} (2 - 18)$$

For the energy density $\rho_\varphi$ (2-9) and pressure $p_\varphi$ (2-10) of B-I scalar field, there is no violation of the strong energy condition. In universe with B-I scalar field without potential, there is no a phase of accelerating expansion.

From Eqs.(2-9)(2-10)and (2-12), we have

$$\omega = \frac{p_\varphi}{\rho_\varphi} = \frac{a^3}{\sqrt{a^6 + \eta c^2}}$$  \hspace{1cm} (2 - 19)$$

and can see

$$0 \leq \omega \leq 1$$  \hspace{1cm} (2 - 20)$$

In Sec.3, it is different that we find the universe accelerating expansion.

## 3 The Model with Lagrangian of $\frac{1}{\eta}[1 - \sqrt{1 - \eta g^{\mu\nu} \varphi, \mu, \nu}] - u(\varphi)$

From Eq.(2-9) and lagrangian, we can get

$$T_{0}^{0} = \rho_\varphi = u - \frac{1}{\eta} + \frac{1}{\eta \sqrt{1 - \eta \varphi^2}}$$  \hspace{1cm} (3 - 1)$$

$$-T_{i}^{i} = p_\varphi = \frac{1}{\eta}[1 - \sqrt{1 - \eta \varphi^2}] - u$$  \hspace{1cm} (3 - 2)$$

When $u > 0$, so we can always find $\rho_\varphi > 0$.

From Eqs.(3-1)and (3-2), we have

$$\rho_\varphi + 3p_\varphi = \frac{2}{\eta} - 2u + \frac{3\eta \varphi^2 - 2}{\eta \sqrt{1 - \eta \varphi^2}}$$  \hspace{1cm} (3 - 3)$$
When potential is greater than $\frac{1}{\eta}$ ($\frac{1}{\eta}$ is two times as "critical" kinetic energy of $\phi$ field). And the kinetic energy of $\phi$ field evolves to region of $\dot{\phi}^2 < \frac{2}{3\eta}$, we have $\rho + 3p_{\phi} < 0$ from Eq.(3-3). The universe undergoes a phase of accelerating expansion.

From Eqs.(3-1) and (3-2), we also can get

$$p_{\phi} + \rho_{\phi} = \frac{\dot{\phi}^2}{\sqrt{1 - \frac{\dot{\phi}^2}{\eta^2}}} > 0$$

(3-4)

Eq.(3-4) could be written as

$$\omega = \frac{p_{\phi}}{\rho_{\phi}} > -1$$

(3-5)

when $\phi$ approximation zero $p_{\phi} = -\rho_{\phi}$. The universe is dominated by the potential. It will undergo inflation phase. In next section, we consider the case that the kinetic energy term is negative.

4 The Model with Lagrangian of $\frac{1}{\eta} [1 - \sqrt{1 + \eta g^{\mu\nu} \phi_{,\mu} \phi_{,\nu}} - u(\phi)]$

In this section we consider the case that the kinetic energy term is negative.

$$L = \frac{1}{\eta} [1 - \sqrt{1 + \eta g^{\mu\nu} \phi_{,\mu} \phi_{,\nu}} - u(\phi)]$$

(4-1)

The Energy-moment tensor is

$$T^\mu_\nu = -\frac{g^\mu_\nu \phi_{,\mu} \phi_{,\nu} + \rho}{\sqrt{1 + \eta g^{\mu\nu} \phi_{,\mu} \phi_{,\nu}}} - \delta^\mu_\nu L$$

(4-2)

From Eq.(4-2), we have

$$\rho = T^0_0 = \frac{1}{\eta \sqrt{1 + \eta \dot{\phi}^2}} - \frac{1}{\eta} + u$$

(4-3)

$$p = -T^i_i = \frac{1}{\eta} - \frac{\sqrt{1 + \eta \dot{\phi}^2}}{\eta} - u$$

(4-4)

From Eqs.(4-3) and (4-4), we get

$$\rho + p = -\frac{\dot{\phi}^2}{\sqrt{1 + \eta \dot{\phi}^2}}$$

(4-5)

It is clear that the equation of static $\omega < -1$ is completely confirmed by Eq.(4-5) and it accords with the recent analysis of observation data. We also can get

$$\rho + 3p = \frac{2}{\eta} \left[ \frac{-2 - 3\eta \dot{\phi}^2}{\sqrt{1 + \eta \dot{\phi}^2}} \right] - 2u$$

(4-6)
It is obvious that $\rho + 3p < 0$. Eq.(4-6) shows that the universe is undergoing a phase of accelerating expansion. The model of phantom B-I scalar field without potential $u(\varphi)$ is hard to understand. In this model we can always find $\rho = \frac{1}{\eta \sqrt{1 + \eta \dot{\varphi}^2}} \frac{1}{\eta} < 0$ and $\left(\frac{\dot{a}}{a}\right)^2 = \frac{K}{3} \rho < 0$. It is unreasonable apparently. However, in the model of phantom B-I scalar field with potential $u(\varphi)$, when $u(\varphi) > \frac{1}{\eta} - \frac{1}{\eta \sqrt{1 + \eta \dot{\varphi}^2}}, \rho$ is always greater than zero. In phantom B-I scalar model with a potential $u(\varphi)$, we also find the strong and weak energy condition always failed from Eqs.(4-5) and (4-6).

We investigate the case of a specific simple example $u = u_0 = const$ and $u_0 - \frac{1}{\eta} = A (A > 0)$. Eq.(4-3) becomes

$$\rho = \frac{1}{\eta \sqrt{1 + \eta \dot{\varphi}^2}} + A \frac{1}{\eta} \quad (4-7)$$

It is clear that there is $\rho > 0$ from Eq.(4-7). Substituting Eq.(4-7) into Einstein equation, we have

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{K}{3} \left[\frac{1}{\eta \sqrt{1 + \eta \dot{\varphi}^2}} + \frac{A}{\eta}\right] \quad (4-8)$$

The equation of motion of phantom field is

$$\frac{1}{\sqrt{-g}} \partial \frac{\partial}{\partial x^\mu} \left[\sqrt{-g}g_{\mu \nu} \varphi, \mu \right] = 0 \quad (4-9)$$

The field $\varphi$ only depends on $t$. We can obtain from Eq.(4-9)

$$\varphi = \frac{C}{\sqrt{a^6 - \eta C^2}} \quad (4-10)$$

where $C$ is integrate constant. Substituting Eq.(4-10) into Eq.(4-8), the Eq.(4-8) becomes

$$\dot{a} = \sqrt{\frac{K a^2}{3\eta} \left[\sqrt{1 - \eta C^2 a^{-6}} + A\right]} \quad (4-11)$$

The smallest $a_{min} = (\eta C^2)^{1/6}$ from Eq.(4-11), the universe is non-singular. When the universe scalar approximation $a_{min}$, Eq.(4-11) becomes

$$\dot{a} = \sqrt{\frac{KA}{3\eta} a} \quad (4-12)$$

$$a \sim \exp \left(\sqrt{\frac{KA}{3\eta}} t\right) \quad (4-13)$$

When $a \to \infty$, Eq.(4-11) becomes

$$\dot{a} = \sqrt{\frac{K(A+1)}{3\eta} a} \quad (4-14)$$
\[ a \sim \exp \left( \sqrt{\frac{K(A+1)}{3\eta}} t \right) \quad (4-15) \]

In our phantom model with potential, the universe is always undergoing a phase of inflation and gradually enters the more accelerated expansion in late time.

5 Summary

We consider cosmological solution of B-I type scalar field without the potential and come to the conclusion that the equation of state parameter \( 0 < \omega < 1 \). However, in the dark energy models of canonical B-I scalar field with potential, the universe is undergoing a phase of accelerating expansion if the potential rolls down to the minimum which is greater than \( \frac{1}{\eta} \) while scalar field evolves into region of \( \dot{\varphi}^2 < \frac{2}{3\eta} \). Correspondingly the equation of state parameter \( \omega \) is always greater than \(-1\). This model admits a late time attractor solution that leads to an equation of state \( \omega = -1 \). The lagrangian of B-I type scalar field with negative kinetic energy also is considered by us. In the phantom B-I scalar model, the universe is undergoing a phase of accelerating expansion and the equation of state parameter \( \omega \) is always smaller than \(-1\). It accords with the recent analysis of the observation that the equation of state parameter of dark energy might be smaller than \(-1\).

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References

1. S. Perlmutter et al., Ap.J, 565(1999);
   J.L. Tonry et al., astro-ph/0305008
2. C.L. Bennett et al., astro-ph/0302207;
   A. Melhiori and L. Mersini, C. J. Odmann, and M. Thodden, astro-ph/0211522
   D.N. Spergel et al., astro-ph/0302209
   N.W. Halverson et al., Ap.J, 568, 38(2002);
   C.B. Netterfield et al., astro-ph/0104460
   P. de Bernardis et al., astro-ph/0105296
   A.T. Lee et al., astro-ph/0104460
   R. Stompor, astro-ph/0105062
3. J.A.Frieman and I.Waga, Phys.Rev.D 57, 4642(1998);
   R.R.Caldwell, R.Dave and P.J.Steinhardt, Phys.rev.Lett 80, 1582(1998).

4. R.R.Caldwell, Phys.Rev.Lett. B 545, 23(2002);
   V.Faraoni, Int.J.Mod.Phys. D 11, 471(2002);
   S.Nojiri and S.D.Odintsov, hep-th/0304131 [hep-th/0306212]
   E.schulz and M.White, Phys.Rev.D 64, 043514(2001);
   T.Stachowiak and Szydlowski, hep-th/0307128
   G.W.Gibbons, hep-th/0302199
   A.Feinstein and S.Jhingan, hep-th/0304069

5. S.M.Carroll, M.Hoffman, M.Teodden, astro-th/0301273
   Y.S.piao, R.G.Cai, X.M.Zhang and Y.Z.Zhang, hep-ph/0207143
   J.G.Hao and X.Z.Li, hep-th/0305207
   S.Mukohyama, Phys.Rev.D 66,024009(2002);
   T.Padmanabhan, Phys.Rev.D 66, 021301(2002);
   M.Sami and T.Padmanabhan, Phys.Rev.D67, 083509(2003);
   G.Shiu and I.Wasserman, Phys.Lett.B 541,6(2002);
   L.kofman and A.Linde, hep-th/020512;
   H.B.Benaoum, hep-th/020140
   L.Ishida and S.Uehara, hep-th/0206102
   T.Chiba, astro-ph/0206298
   T.Mehen and B.Wecht, hep-th/0206212
   A.Sen, hep-th/0207105
   N.Moeller and B.Zwiebach, JHEP 0210, 034(2002);
   J.M.Cline, H.Firouzjahi and P.Martineau. hep-th/0207156
   S.Mukohyama, hep-th/0208094
   P.Mukhopadhyay and A.Sen, hep-th/020814;
   T.Okunda and S.Sugimoto, hep-th/0208196
   G.gibbons, K.Hashimoto and P.Yi, hep-th/0209034
   M.R.Garousi, hep-th/0209068
   B.Chen, M.Li and F.Lin, hep-th/0209222
   J.Luson, hep-th/0209255
   C.kin, H.B.Kim and O.K.Kwon, hep-th/0301142
   J.M.Cline, H.Firouzjahi and P.Muhtneau, hep-th/0207156
   G.Felder, L.Kofman and A.Starobinsky, JHEP 0209, 026(2002);
   S.Mukohyama, hep-th/0208094
   G.A.Diamandis, B.C.Georgalas, N.E.Mavromatos, E.Pantonopoulos, hep-th/0203241
G.A.Diamandis, B.C.Georgalas, N.E.Mavromatos, E.Pantonopoulos, I.Pappa, [hep-th/0107124]
M.C.Bento, O.Bertolami and A.A.Sen, hep-th/020812;
H.Lee, et.al., [hep-th/0210221]
M.Sami, P.Chingangbam and T.Qureshi, [hep-th/0301140]
F.Leblond, A.W.Peet, [hep-th/0305059]
J.G.Hao and X.Z.Li, Phys.Rev.D 66, 087301(2002);
X.Z.Li and X.H.Zhai, Phys.Rev.D 67, 067501(2003).

6. G.Felder, L.Kofman and A.Starobinsky, [hep-th/0208019]
   G.W.Gibbons, hep-th/031117; [hep-th/0302199]
   A.Frolov, L.Kofman and A.Starobinsky, [hep-th/0204187]
   A.Sen, [hep-th/0204143] hep-th/0209122 hep-th/0203211

7. M.Born and Z.Infeld, Proc.Roy.Soc A 144, 425(1934).

8. H.P.de Oliveira, J.Math.Phys. 36, 2988(1995).

9. H.Q.Lu, T.Harko and K.S.cheng, Int.J.Modern.Phys.D 8,625(1999);
   H.Q.Lu et.al., Int.J.Theory.Phys.42, 837(2003).