Single interface effects dominate in exciton-condensate/normal-barrier/exciton-condensate (EC/N/EC) structures of long-barrier

Ya-Fen Hsu 1,2 and Jung-Jung Su 1,2

1 Department of Electrophysics, National Chiao Tung University, Hsinchu 300, Taiwan
2 Physics Division, National Center for Theoretical Sciences, Hsinchu, 30013, Taiwan
E-mail: yf-hsu@mx.nthu.edu.tw and jungjsu@nctu.edu.tw

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Abstract

We study theoretically the exciton-condensate/normal-barrier/exciton-condensate (EC/N/EC) structures in bilayers with a tunable relative phase \( \phi_0 \) between the two exciton condensates (ECs). It is a setup inspired initially by the superconducting Josephson junction but with a special ingredient added for bilayer systems, namely, the interlayer tunneling. Our results show that in an EC/N/EC structure of long-barrier, the single Andreev reflection at one EC/N interface dominates—as opposed to the same structure of short-barrier, in which multiple Andreev reflections can be accommodated (similar to the superconducting Josephson junctions). The single interface effect turns the other EC inert and the system can no longer be understood as a Josephson junction. The supercurrent, however, still occurs at the N/EC interface since the current conservation is still fulfilled with the assistance of the interlayer tunneling in barriers. This exotic mechanism gives rise to only a half portion from a fractional soliton of a doubled topological charge \( 2Q = \phi_0 / \pi \) (for the same relative phase \( \phi_0 \)), as opposed to a full portion fraction soliton of charge \( Q = \phi_0 / 2\pi \) in the structures of short-barriers. We predict the current phase relation for the EC/N/EC structures of long-barriers which can be tested experimentally.

1. Introduction

Ever since the proposal of spatially separating the electrons and holes in excitons (indirect excitons) for supercurrent detections [1], exploring the electrical nature of the equilibrium indirect exciton-condensate have been at the heart of two-dimensional physics [2–6]. In GaAs bilayers under strong magnetic field, the zero-bias tunneling anomaly due to coherent interlayer tunneling [6–10], the quantum Hall drag [6, 11–14], and the counterflow supercurrents [6, 10, 13, 15–20], have each attracted intense research efforts while the new excitations are continued to be reported until to date [21–27]. The quest for indirect exciton condensates has recently extended to cover novel materials [33–46], especially the topological materials [28–32] and the graphene based materials [33–46]. The latter is particularly promising because the electron–hole separation can be reduced to atomic length scale in such systems to greatly enhance the electron–hole binding. In fact, ground-breaking progress has recently been made in demonstrating the quantum Hall drag in graphene double bilayers [45, 46]. The demonstration suggests the existence of condensation and opens up a new era for the indirect exciton-condensation.

Intriguing physics can happen when employing two excitonic condensates to sandwich a normal-barrier, forming an EC/N/EC structure (figure 1(a)). The two condensates are designed to hold a constant relative phase \( \phi_0 \) that can be generated by externally applying an spatially localized in-plane magnetic fields [47], or a vertical electric bias pulse with a controllable temporal width to reach the designated phase [48]. When the enclosing normal-barrier is short, this EC/N/EC structure resembles a superconducting Josephson junction [47, 49–51]

3 The excitons in transition metal dichalcogenide (TMDC) are not in discussion here since it is not in equilibrium.
but with coherent interlayer tunneling [3, 6–10]; such structure is also referred to as the excitonic Josephson junction (EJJ). In our previous work of EJJ [52], we found exotic fractional soliton, an object that carries a topological charge of $Q \equiv \phi_0/2\pi$, which resembles fractional vortices [53–56] in $0 – \kappa$ superconducting Josephson junction [57, 58]. This type of topological objects is a potential candidate for flux- or phase-base qubits. The EC/N/EC structure in discussion can be even more advantageous than the traditional $0 – \pi$ junctions [59–63] since its relative phase is electrically accessible.

With all the appealing coherent properties of the EC/N/EC structure of short-barrier (excitonic Josephson junction), it is necessary to ask how would the length of the barrier change the previous prediction. It so turns out that in the EC/N/EC structure of long-barrier ($d_f > \xi$), the supercurrent is not a result of the coherence between the two N/EC interfaces (as in the superconducting Josephson junctions), but is a single N/EC interface effect. It is known that there is Andreev reflection at the N/EC interface [18–20, 64], but instead of forming Andreev bound states between the two N/EC interfaces, the supercurrent in the EC/N/EC structure of long-barrier is formed by a single Andreev reflection at one interface—plus the single-particle tunneling in the normal-barrier to fulfill the charge conservation. It is therefore a property of only one N/EC interface and does not require coherence between the two N/EC interfaces. The distinct mechanism gives rise to distinct topology. In our calculation of a EC/N/EC structure of long-barrier, a half portion of a fractional soliton with twice the topological charge ($2\phi_0/2\pi$) is found, as oppose to a full fractional soliton with charge $Q = \phi_0/2\pi$ as in an EJJ. We also obtain the corresponding current phase relations (CPRs) in EC/N/EC structure of long barrier that can serve as a clear experimental evidence to tell when coherence between two interfaces is missing and the single interface physics takes over.

2. Theoretical method

We use the developed lattice model [52, 65–67] and tailor it to describe specifically our EC/N/EC structures. Consider that the electrons can only be in either top or bottom layer, the wave function of the eigenstate can be generally expressed as:

$$|\Psi\rangle = \prod_X [u(X)c_{X1}^\dagger + v(X)\exp[i\phi(X)]c_{X2}^\dagger] |0\rangle,$$

where $X$ labels the lattice site in the x-direction. The behavior in y-direction is assumed to be translationally invariant. Notice that in our notation, $u(X)$, $v(X)$ are real numbers and the phase information is contained in $\phi(X)$. The creation operator $c_{X(1,2)}^\dagger$ represents the creation of an electron at position $X$ in the top (bottom) layer. The vacuum state $|0\rangle$ indicates the state of no electron in either layer. By performing an SU(2) to O(3) mapping, the exciton-condensate system is transformed into a classical spin-dynamics problem with the pseudospin defined as:

$$\vec{m}(X) = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta).$$

Here $\theta = 2 \arccos u(X)$ and $\phi = \phi(X)$. In this representation, the in-plane magnetization $(m_x, m_y)$ and the out-of-plane magnetization $m_z$ correspond to the excitonic coherence and the population imbalance, respectively. The excitonic system is now readily described by the Landau–Lifshitz–Gilbert (LLG) equation.
In this section, we demonstrate the effect of the barrier on EC structures described under the above scheme. For easy analytic presentation, we will from now on use the continuous varying field instead of the discrete $x$. As illustrated in figure 1, the system with length $L$ is composed of two exciton condensates (EC1 and EC2), each with a length of $(L - d_J)/2$, and a barrier length $d_J$ in between. In EC2 in particular, an external phase $\phi_{\text{ex}}$ can be controlled externally and is the key to tuning the phase of the excitonic coherence on each site with its neighbors. The third term is the interlayer tunneling energy where $n$ is the density and $\Delta_s$ is the interlayer tunneling strength. Note that this term is essentially the in-plane Zeeman energy, $-1/2 \int d\vec{r} \ n \ \vec{m} \cdot \vec{h}$ with the Zeeman field $\vec{h} = \Delta_s (\cos \phi_{\text{ex}}, \sin \phi_{\text{ex}}, 0)$ breaks the U(1) symmetry and aligns the pseudospins in the direction of $\phi_{\text{ex}}$. This $\phi_{\text{ex}}$ can be controlled externally and is the key to tuning the phase of the excitonic phase. Using the above energy functional, we perform numerical calculations of the LLG equation for each discretized position $X$. Notice that the most important spatial variable $\phi(X)$ is actually the average phase over a lattice site. That means that the rapid spatial phase fluctuation, e.g. merons, have been averaged out. That will in effect, cause a reduction of $\rho_\phi$.

Next we explain how the EC/N/EC structures are described under the above scheme. For easy analytic presentation, we will from now on use the continuous varying field instead of the discrete $x$. As illustrated in figure 1, the system with length $L$ is composed of two exciton condensates (EC1 and EC2), each with a length of $(L - d_J)/2$, and a barrier length $d_J$ in between. In EC2 in particular, an external phase $\phi_{\text{ex}} = \phi_0$ is introduced by applying an interlayer voltage pulse. For convenience, we define the left edge of EC2 to be the origin of $x$ so the barrier is now located at $-d_J < x < 0$ and EC1 at $-(L + d_J)/2 < x < -d_J$; in the $y$-direction the behavior is assumed to be translationally invariant. With this notation, the external phase in equation (4) is then:

$$\phi_{\text{ex}} = \phi_0 \Theta(x),$$

where $\Theta(x)$ is the Heaviside step function. Meanwhile, superfluid density $\rho_\phi$ in the barrier is set to zero:

$$\rho_\phi(x) = \rho_{\phi_0} \Theta(-d_J - x) + \rho_{\phi_0} \Theta(x).$$

These configurations will enter the energy functional and affect the pseudospin evolution described by LLG equation. On the other hand, if only the lowest-energy steady-state is of interest, it can also be approached by minimizing the energy functional with respect to the phase $\phi$. In the condensate regime, we have $m_x \sim 0$ and the phase derivative of the energy function yields the modified sine-Gordon equation (MSGE):

$$\lambda^2 \nabla^2 \phi - \sin(\phi - \phi_0) = 0.$$ 

Here $\lambda$ is the Josephson length defined as

$$\lambda = \sqrt{2 \rho_\phi / n \Delta_s}.$$ 

The above MSEG is a variation of the ordinary sine-Gordon equation (OSGE), with an additional $\phi_0$. While each OSGE exhibits the lowest excitation state of a one-soliton solution of $4 \arctan[\exp(x/\lambda)]$, the MSEG gives rise to a fractional one-soliton solution that exists in an EC/N/EC structure with $d_J = 0$ [52]:

$$\phi(x) = \begin{cases} 4 \arctan[e^{-x - x_0}/\lambda], & x < 0, \\ 4 \arctan[e^{x + x_0}/\lambda] - 2\pi + \phi_0, & x > 0, \end{cases}$$

where $x_0 = -\lambda \ln[\tan(\phi_0/8)]$. Finally, notice that the Josephson length $\lambda$ characterizes the size of the ordinary one-soliton and it also serves as the length scale of the structure in the discussion.

3. Results and discussions

In this section, we demonstrate the effect of the barrier on EC/N/EC structures by solving for the static solution to the discretized LLG equation. The parameters used in the equation are: $\beta = 0.02E_0$, $\rho_0 = 0.005E_0$ and $\Delta_s = 10^{-6}E_0$. Here $E_0$ is the characteristic energy scale of the system, the Coulomb energy, $E_0 \equiv e^2/\epsilon l \sim 7$ meV where $l \sim 15–18$ nm [6] is the magnetic length. These are the typical numbers in quantum Hall bilayers [70]. Especially, the values for $\beta$ and $\rho_0$ are derived from the mean-field calculation. The tunneling strength $\Delta_s$ usually ranges from $10^{-6}E_0$ to $10^{-4}E_0$ in these systems according to the tunneling experiment [10, 71, 72]. Here $\Delta_s = 10^{-6}E_0$ is chosen for numerical convenience. For the length scales, both the coherence length $\xi$ and the
The Josephson length $\lambda$ characterize the systems. The correlation length $\xi$ obtained in the quantum Hall bilayer experiment is roughly 200 nm (equivalent to $\sim 10\lambda$) [73]. Since the excitonic superfluid loses its coherence after traveling in the barrier over a distance $\xi$, the correlation length also corresponds to the pixel size in the lattice model [70]. The Josephson length $\lambda$, on the other hand, is roughly $250/\lambda$ for the $\Delta_s$ chosen above. Note that the choice of $\Delta_s$ is not crucial. Physics of different parameters, including $\Delta_s$, can be accessed by scaling the system with $\lambda$. Finally, the supercurrents calculated later are usually in comparison with a more universal scale: $j_0 \equiv e \rho_0 / h \lambda \sim 2 \text{nA} \ \mu\text{m}^{-1}$. In the following, we discuss the EC/N/EC structures of long-barrier ($d_f < \lambda$) including both cases of short ($L < \lambda$) and long ($L > \lambda$) total structure length.

Figure 2(a) shows the typical phase profile for an EC/N/EC structure of long barrier but with short structure length (specifically, $d_f = 0.08 \lambda$ and $L = 0.8 \lambda$). We first notice that the phase profiles show non-differentiable rises in the vicinity of the N-EC2 interface then the profiles saturate smoothly to the maxima. Although our approach cannot uniquely identify excitonic Andreev reflection, these rises do correspond to the sudden increase of supercurrent at the interface and are also consistent with the known effect of excitonic Andreev reflection [18–20, 64, 74]. In the following we provide our picture on the EC/N/EC structures of long-barrier.

First we recall that in superconducting Josephsons junctions, the Josephson effect can be understood as the formation of supercurrents in an EC/N/EC structure of short-barrier. When a right-going electron in the bottom layer is Andreev-reflected to top layer, an exciton is generated and moves rightward, and vice versa. The process of infinite times of such reflection between the two N-EC boundaries transfer excitons from EC1 to EC2. (b) Tunneling-assisted supercurrent in an EC/N/EC structure of long-barrier. A right-going electron in the bottom layer is Andreev-reflected at the N-EC2 interface. The reflected electron in the top layer then moves leftward until it comes back to the bottom layer through single-particle tunneling. A loop of electron flow is completed and the charge conservation is fulfilled.

The coherence length is obtained by experiments detailed in chapter 6 of Spielman’s PhD Thesis. This value is widely used in the literature which includes [73].
What happens is that each right-going electron in the bottom layer is Andreev-reflected at the N-EC2 interface. The reflected electron in the top layer then moves leftward until it comes back to bottom layer through single-particle tunneling. It is important to note that the total charge is conserved throughout the process—the supercurrent thus produced is physical.

With the physical picture in mind, we are now ready to discuss the detailed structure of the phase profile. The maximum values of the phase profiles are much less than the assigned $\phi_0$, and even more interestingly, they do not depend monotonically on $\phi_0$. Such behavior can be understood by simplifying the modified-sine-Gordon equation. Since $\phi \ll \phi_0$, the $\phi$ term in $\sin(\phi - \phi_0)$ can be dropped in equation (6) so the solution should be well described by a simple quadratic function as:

$$\phi(x) \sim \sin(\phi_0 \Theta(x))(C_1(x/\lambda) + (1/2)(x/\lambda)^2),$$

where we have implicitly employed the boundary conditions. The $\sin \phi_0$ dependence in the solution above gives rise to the non-monotonic behavior when ranging $\phi_0$ from 0 to $\pi$. With $J_e = (\epsilon_p/\hbar) \nabla \phi$, the CPRs inherit the $\sin \phi_0$ dependence and thus:

$$J_e \propto \sin \phi_0.$$  \hspace{1cm} (10)

Indeed, the numerically calculated CPR (red circle in figure 2(b)) is perfectly fitted by the $\sin \phi_0$ curve (blue dash line) and that confirms our simple picture.

For long EC/N/EC structures of long-barrier, the phase profiles again show non-differentiable rises, as plotted in figure 4(a) for $d_f = 0.08\lambda$ and $L = 11.2\lambda$. For any given $\phi_0$, the right wing of the phase profile behaves like the segment of $x > x_0'$ in the ordinary one-soliton being shifted leftward until its left end hits the origin, with $x_0' = -\lambda \ln[\tan(\phi_0/4)]$. More explicitly, the phase profile can be read off as:

$$\phi(x) = \begin{cases} 
0, & x < 0, \\
4 \arctan[e^{x+x_0'/\lambda_0} - 2\pi + 2\phi_0], & x > 0.
\end{cases}$$  \hspace{1cm} (11)

The corresponding CPR is:

$$J_e = \frac{2\epsilon_p\phi_0}{\hbar \lambda} \sin \left( \frac{\phi_0}{2} \right)$$  \hspace{1cm} (12)

In figure 4(b) we show the CPR again from both the LLG calculation (red circle) and the $\sin(\phi_0/2)$ fitting derived from the above picture (blue dash line). The two again reach great agreements.

With the success of describing the CPR by our naive picture in both short (equation (10) and figure 2(b)) and long structure (equation (12) and figure 4(b)) regimes, the distinct behaviors in the two regimes lead us to wonder if a transition happens from the short to the long structure, or a crossover. To answer the question, we monitor how CPR evolves with increasing structure length $L$, as plotted in figure 5(a). Starting from a short structure ($L = 0.8\lambda$) as a $\sin \phi_0$ curve, the CPR skews gradually until it finally turns into a $\sin(\phi_0/2)$ curve. The evolution is a crossover. It is also interesting to note that this $\sin(\phi_0/2)$ to $\sin(\phi_0)$ crossover reminds us of the dirty-to-clean crossover in superconductor [76]. Their similar behaviors may not be a coincidence. In EC/N/EC structures, the total transmission through interlayer tunneling increases when increasing the structure length $L$.  

![Figure 4](image)

**Figure 4.** (a) Phase profiles in a long EC/N/EC structure of long-barrier ($d_f = 0.08\lambda$ and $L = 11.2\lambda$) for various $\phi_0$. (b) The corresponding current phase relations (CPR). Red circles show the CPR and the blue dash line is its $\sin(\phi_0/2)$ fit. The two are in great consistency.
This can be analog to the increase in transmission coefficient $D$ of the superconducting Josephson junction in the clean–to–dirty crossover (see figure 3 in [76]).

Next we summarize the crossover behavior for different $\phi_0$, in particular that of $\phi_0 < \pi/2$ and of $\pi/2 < \phi_0 < \pi$. Here we pick $\phi_0 = \pi/2$ and $\phi_0 = \pi$ for easy comparison. In the main content of figure 5 (b) we plot $I_s$ versus $L$ for the two designated $\phi_0$. For $\phi_0 = \pi/2$, the curve ascends progressively as $L$ increases from zero, and saturates when $L \sim 3\lambda$. For $\phi_0 = \pi$, however, the supercurrent is essentially negligible until $L$ hits $3\lambda$, then it rapidly rises to saturation. The two saturation values can be read off and compared with those for $\phi_0 = 0.5\pi$ and for $\phi_0 = \pi$ in EC/N/EC structures of long-barrier. The inset shows that for EC/N/EC structures of short-barrier for comparison. The maximum of the supercurrent are larger in EC/N/EC structures of long-barrier than those of short-barrier because of the non-differentiable rise in the phase profiles in structures of long-barrier.

At the end of the results section, we would like to comment briefly on our LLG approach. It can essentially be understood as the Ginzburg–Landau theory. As shown in the method section, this approach reduces the complicate electron-wave functions to the local value of order parameter. It is therefore very efficient in obtaining the self-consistent result, which is non-trivial in the presence of interlayer tunneling. We also note that if the disorder becomes important or the information of the actual electron-wave function is required, we will have to step back to the microscopic description of scattering [74] or the non-equilibrium Green’s function method [19]. However, we are confident that for the current quality of bilayer and when the interlayer tunneling still plays an important role, the CPRs we predicted is fairly reliable. Next we comment on the experimental realization of EC/N/EC structures. Among all proposed systems for excitonic condensation, GaAs-based quantum Hall bilayer remains to be the most developed one. The Josephson length in this system typically ranges from 4.5 to 45 $\mu$m, while the system size can easily reach hundreds of microns. This system is ready for what we have proposed in this manuscript. Aside from the GaAs-based bilayers, graphene, especially the double bilayer graphene [36, 38–40, 42, 44–46] is another potential candidate for realizing EC/N/EC structures. Graphene is relatively clean and small (typically around 10 $\mu$m), and is most suitable for studying the quantum coherence. Moreover, the graphene bilayer, or other novel two-dimensional materials each carries exotic properties including chirality and spin–orbit coupling, etc, that can be inherited by the excitons therein. This is a land yet to be explored. We expect that the exotic nature will bring in exciting twists to the physics in EC/N/EC structures.

4. Summary

In this paper, we investigate the exciton–condensate/normal-barrier/exciton–condensate structures (EC/N/EC) of long-barrier by solving the pseudospin LLG equation. In a structure as such, the supercurrent is created by Andreev reflection at one of the N/EC interfaces and fulfills current conservation with the assistance of the coherent interlayer tunneling. The corresponding phase profile exhibits a half portion of a fractional soliton with doubled topological charge $2Q = \phi_0/\pi$. When varying structure length, we find that the CPRs skews gradually from a sin $\phi_0$ curve to a sin($\phi_0/2$) as increasing the length from short- to long structure limit. In the calculation,
the crossover happens at $L \sim 3 \lambda$. We believe that the finding summarized here on the EC/N/EC structures of long barrier, together with our previous discussion [52] on the same structure of short-barrier can shed light on the realization of excitonic-based structures and fractional solitons—which might lead to new types of quantum qubits.

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ORCID iDs

Jung-Jung Su @ https://orcid.org/0000-0001-6337-7540

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