Study on Necessary Condition of Malikov Criterion
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Abstract

Because of the defect that Malikov criterion may produce wrong judgment in application, the invalidation reason of Malikov criterion was analyzed, and the necessary condition for Malikov criterion being valid using a t-test method is brought forward that the anterior half measure data are inconsistent with posterior half. Only in this way, the judgment according to Malikov criterion is correct.

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1. Introduction

There are errors in all measurement unavoidably. Errors must be eliminated or reduced in order to improve the measurement precision. According to the characteristics and properties, the error can be divided into systematic error, random error and bulky error [1]. The feature of systematic error is the absolute value and symbols of error remain unchanged several times in the same conditions or error changes according to certain rules when the conditions change [2]. Systematic error mainly comes from the defect of measuring system and causes deviation between the result of measurement and true value. System error can't be found and eliminated through increasing measurement times. The systematic error makes the result of measurement deviating constant value from the true value or affecting by a certain rules if the systematic error of experimental data isn’t be treated correctly.

To different systematic error, the eliminating or reducing method is different. Only the variation discipline of systematic error in the measuring data list is judged accurately, the factors may generate systematic errors and the method which may reduce and eliminate systematic error can be found out. The symmetric method can be used to eliminate the linear systematic error, namely, the arithmetic mean of double indication of each symmetric point when the measure is arranged symmetrically.

Malikov criterion is a common method used to judge whether the measurement data containing linear systematic error in error theory and data processing [1], [3]~ [7]. Its advantage is simple discriminant, small amount of calculation and suitable for most situations, so it has been widely used. But there are a
few misjudgment in practice [8][9], or its judgment is conflict with other criterion when Malikov criterion is used. Therefore, it’s necessary to research the effective condition of Malikov criterion.

In this paper, Malikov criterion was introduced and the reason of misjudgment using Malikov criterion was analyzed, and finally a necessary condition for Malikov criterion being valid had been advanced using t-test method.
2. Malikov criterion

Systematic error can be divided into fixed value systematic error and variable value systematic error according to the different change rule. Variable value systematic error can be divided into linear systematic error, the cyclical change systematic error and complex systematic error. Linear systematic error refers to the error value which is in proportion to increase or decrease in the whole measuring process [3][8].

For measure data list \( l_1, l_2, \ldots, l_n \), its measure times is \( n \), and its average is \( x \). If the systematic error of the measure data list is \( \Delta l_1, \Delta l_2, \ldots, \Delta l_n \), and its average is \( \Delta x \). The measure data list containing no systematic error is \( l_1', l_2', \ldots, l_n' \), its average is \( x' \).

So, there is the as follow formula:

\[
\bar{x} = \bar{x} + \Delta \bar{x}, \quad l_r' = \bar{x} + \Delta l_r \quad (r = 1, 2, \ldots, n) \quad (1)
\]

The residual error of every measure is:

\[
v_r = l_r' - \bar{x} \quad (r = 1, 2, \ldots, n) \quad (2)
\]

The residual error of any measure containing no systematic error is:

\[
v_r' = l_r' - \bar{x}' \quad (r = 1, 2, \ldots, n) \quad (3)
\]

Then, we can give: \( v_r = v_r' + (\Delta l_r - \Delta \bar{x}) \) \quad (4)

When \( n \) is even number, \( K = n/2 \). When \( n \) is odd number, \( K = (n+1)/2 \). The sum of residual errors posterior to \( K \) subtracted from the sum of residual errors anterior to \( K \) gives difference \( V \):

\[
V = \sum_{i=1}^{K} v_i - \sum_{j=K+1}^{n} v_j = \sum_{i=1}^{K} v_i - \sum_{j=K+1}^{n} v_j + \sum_{j=1}^{K} (\Delta l_j - \Delta \bar{x}) - \sum_{j=K+1}^{n} (\Delta l_j - \Delta \bar{x}) \quad (5)
\]

When the measure times are sufficient, we can give:

\[
\sum_{j=1}^{K} v_i = \sum_{j=K+1}^{n} v_j = 0 \quad (6)
\]

So, formula (5) turned into:

\[
V = \sum_{j=1}^{K} v_j - \sum_{j=K+1}^{n} v_j = \sum_{j=1}^{K} (\Delta l_j - \Delta \bar{x}) - \sum_{j=K+1}^{n} (\Delta l_j - \Delta \bar{x}) \quad (7)
\]

If the difference \( V \) doesn’t equal to zero obviously, there are linear systematic errors in the measure data list. This method is called Malikov criterion.

3. The invalidation reason analysis of Malikov criterion

If there are linear systematic errors in the measure data list \( l_1, l_2, \ldots, l_n \), and because the systematic errors are linear systematic errors, there is the formula(8):

\[
\Delta l_r = \delta_0 + (r-1)\delta \quad (8)
\]

In above formula, \( \delta_0 \) is the constant systematic error; \( \delta \) is the step difference of linear systematic errors. When \( n \) is even number, we can give:

\[
l_r = l_r' + \Delta l_r = l_r' + \delta_0 + (r-1)\delta \quad (9)
\]

\[
V = \sum_{i=1}^{\frac{n}{2}} (l_i - \bar{x}) - \sum_{j=\frac{n}{2}+1}^{n} (l_j - \bar{x}) = \sum_{i=1}^{\frac{n}{2}} [l_i' - \bar{x} + (i-1)\delta] - \sum_{j=\frac{n}{2}+1}^{n} [l_j' - \bar{x} + (j-1)\delta] \quad (10)
\]

By formula (9), we can get:

\[
\bar{x} = \frac{1}{n} \sum_{j=1}^{n} l_j = \frac{1}{n} \sum_{r=1}^{n} [l_r' + \delta_0 + (r-1)\delta] = \bar{x}' + \delta_0 + \frac{n-1}{2} \delta \quad (11)
\]
\[ V = \sum_{i=1}^{n/2} [l_i' - \bar{x} + (i-1)\delta] - \sum_{j=\lceil n/2 \rceil}^{n} [l_j' - \bar{x} + (j-1)\delta] = \sum_{i=1}^{n/2} l_i' - \sum_{j=\lceil n/2 \rceil}^{n} l_j' + \sum_{i=1}^{n/2} (i-1)\delta - \sum_{j=\lceil n/2 \rceil}^{n} (j-1)\delta \]  

(12)

If the systematic errors were eliminated, the average of anterior and posterior half data respectively is:

\[ \bar{x}_q = \frac{2}{n} \sum_{i=1}^{n/2} l_i', \quad \bar{x}_h = \frac{2}{n} \sum_{j=\lceil n/2 \rceil}^{n} l_j' \]  

(13)

Then formula (12) turned into:

\[ V = \frac{n}{2} (\bar{x}_q - \bar{x}_h) - \frac{n^2}{4} \delta \]  

(14)

When \( n \) is odd number, we also can give:

\[ V = \frac{n+1}{2} (\bar{x}_q - \bar{x}_h) - \frac{n^2 - 1}{4} \delta \]  

(15)

In formula (14) and (15), there are only random errors in \( \bar{x}_q \) and \( \bar{x}_h \). When measure time is large enough, random errors counteract each other. So we should give:

\[ \bar{x}_q = \bar{x}_h = \bar{x} \]  

(16)

Then formula (14) turned into:

\[ V = -\frac{n^2}{4} \delta \]  

(17)

Therefore, if the difference \( V = 0 \), it also means \( \delta = 0 \), namely there aren’t linear systematic errors in the measure list. If the difference \( V \) doesn’t equal to zero obviously, namely there are big linear step difference between the measure data, so there are linear systematic errors in the measure list.

But in actual measure, the measure time is limited, generally \( n \leq 10 \), so \( \bar{x}_q \neq \bar{x}_h \), it can be seen that the difference \( V \) doesn’t equal to zero obviously from formula (14) and (15) even if \( \delta = 0 \). Therefore the measure list will be misjudged using Malikov criterion.

On the other hand, whether linear systematic errors exist in the measure data list when \( V = 0 \)? In this way, we premise there are linear systematic errors in the measure data list, namely \( \delta \neq 0 \). When \( n \) is even number, we can give the below formula from formula (14):

\[ V = \frac{n}{2} (\bar{x}_q' - \bar{x}_h') - \frac{n^2}{4} \delta = 0 \]  

(18)

So we can give:

\[ \bar{x}_q' - \bar{x}_h' = \frac{n}{2} \delta \]  

(19)

\[ \bar{x}_q = \frac{1}{n/2} \sum_{i=1}^{n/2} l_i' = \frac{1}{n/2} \sum_{i=1}^{n/2} [l_i' + \delta_0 + (i-1)\delta] = \bar{x}_q' + \delta_0 + \frac{1}{2} (\frac{n}{2} - 1)\delta \]  

(20)

\[ \bar{x}_h = \bar{x}_h' + \delta_0 + \frac{1}{2} (\frac{3n}{2} - 1)\delta \]  

(21)

Then, the below formula can be give:

\[ \bar{x}_q' - \bar{x}_h' = \bar{x}_q - \bar{x}_h - \frac{n}{2} \delta \]  

(22)

From formula (19), it can be give:

\[ \bar{x}_q' = \bar{x}_h = \bar{x} \]  

(23)
This shows the measure data average of anterior and posterior half is equal. There are linear systematic errors in the measure data list according to the foregoing hypothesis. Because of the accumulation of linear systematic errors, the below formula can be given consequentially:

\[ \bar{x}_q \neq \bar{x}_h \neq \bar{x} \]  \hspace{1cm} (24)

The formula (24) and (23) is inconsistent. Therefore there aren’t linear systematic errors in the measure data list when \( V = 0 \).

According the above analysis, there is a possible that the measure data list will be misjudged using Malikov criterion when the difference \( V \) doesn’t equal to zero obviously.

4. The necessary condition of Malikov criterion

According to above analysis, the premise of Malikov criterion is \( \bar{x}_q = \bar{x}_h = \bar{x} \), namely the average of anterior and posterior half data which the systematic errors were eliminated are equal. If there are linear systematic errors in the measure data list, the average of anterior and posterior half data can’t be equal because of the cumulative systematic errors. Namely the anterior and posterior half data should be inconsistent. Then, the judgement according to Malikov criterion is correct.

According to \( t \)-test of the error theory, the measure data list, its anterior and posterior half data should have the equal standard deviation in the same condition [9]. If the degree of freedom of anterior and posterior half data respectively are \( v_q = n_q - 1 \), \( v_h = n_h - 1 \), so the standard deviation of \( \bar{x}_q \) and \( \bar{x}_h \) respectively are: \( s_q = \frac{\sigma_q}{\sqrt{n_q}} \), \( s_h = \frac{\sigma_h}{\sqrt{n_h}} \), the standard deviation of \( \bar{x}_q - \bar{x}_h \) is:

\[ s = \sqrt{s_q^2 + s_h^2} \]  \hspace{1cm} (25)

Then the degree of freedom of \( s \) is: \( v = n_q - 1 + n_h - 1 = n - 2 \)

If variable \( t \) is:

\[ t = \frac{\bar{x}_q - \bar{x}_h}{s} \]  \hspace{1cm} (26)

Thus \( t \) is the \( t \) distribution variable which obeys the \( t \) distribution. In \( t \)-test method, the value of significance level \( \alpha \) is usually chosen for 0.01 or 0.05. The value of \( t_\alpha \) can be get from \( t \) – distribution table according to \( v \) and \( \alpha \).

According to \( t \)-test method, if \(|t| < t_{0.05}\), it means that the difference between two average isn’t significant; if \( t_{0.05} \leq |t| < t_{0.01} \), it means that the difference between two average is significant; and if \(|t| \geq t_{0.01}\), it means that the difference between two average is significant greatly. So \( t \)-test method can be used to judge whether the anterior and posterior half data are consistent. If \(|t| < t_{0.05}\), it means that the anterior half data are consistent with posterior half data because the difference between them isn’t significant. If \(|t| \geq t_{0.05}\), it means that anterior half data are inconsistent with posterior half data because the difference between them is significant.

Therefore, it should be the necessary condition of Malikov criterion that the anterior and posterior half data are inconsistent. In this way, Malikov criterion should be described as: when the difference \( V \) between residual errors sum of anterior and posterior half data doesn’t equal to zero obviously, and simultaneity the anterior half data are inconsistent with posterior half data, there are linear systematic errors in the measure data list.
5. Conclusion

Linear systematic errors don’t exist in the measure data list when the difference $V$ between residual errors sum of anterior and posterior half data equals to zero. And there is a possible that the measure data list will be misjudged with Malikov criterion when the difference $V$ doesn’t equal to zero obviously. Because the premise of Malikov criterion is the anterior half data should be inconsistent with posterior half data. Thus it should be the necessary condition of Malikov criterion that the anterior half data are inconsistent with posterior half data. Only in this way, the judgment according to Malikov criterion is correct.

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