THREE-DIMENSIONAL NON-VACUUM PULSAR OUTER-GAP MODEL: LOCALIZED ACCELERATION ELECTRIC FIELD IN THE HIGHER ALTITUDES

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ABSTRACT

We investigate the particle accelerator that arises in a rotating neutron-star magnetosphere. Simultaneously solving the Poisson equation for the electro-static potential, the Boltzmann equations for relativistic electrons and positrons, and the radiative transfer equation, we demonstrate that the electric field is substantially screened along the magnetic field lines by pairs that are created and separated within the accelerator. As a result, the magnetic-field-aligned electric field is localized in higher altitudes near the light cylinder and efficiently accelerates the positrons created in the lower altitudes outward but does not accelerate the electrons inward. The resulting photon flux becomes predominantly outward, leading to typical double-peak light curves, which are commonly observed from many high-energy pulsars.

Key words: gamma rays: stars – magnetic fields – methods: numerical – stars: neutron

1. INTRODUCTION

The Large Area Telescope (LAT) on board the Fermi Gamma-Ray Space Telescope has detected 117 rotation-powered pulsars (Abdo et al. 2013). The unprecedented amount of data for these γ-ray sources allows us to study the statistical properties of the high-energy pulsars through the light curve analysis. Adopting the polar cap (PC) model (Sturrock 1971; Harding et al. 1978; Daugherty & Harding 1982; Dermer & Sturmer 1994), the slot gap (SG) model (Arons 1983; Muslimov & Harding 2004; Dyks & Rudak 2003; Harding et al. 2005), and the outer gap (OG) model (Cheng et al. 1986; Romani 1996; Cheng et al. 2000; Romani & Watters 2010), and comparing the predicted light curve morphology with the observations (Dyks et al. 2004), researchers that the SG model is geometrically favored in some cases but the OG model is favored in other cases as opposed to the lower altitude emission models such as the PC model (Romani & Watters 2010; Takata et al. 2011; Pierbattista et al. 2012, 2014; Johnson et al. 2014). Moreover, the MAGIC and VERITAS experiments reported pulsed signals from the Crab pulsar up to 400 GeV (Aleksić et al. 2011, 2012; Aliu et al. 2011, 2014), which indicates that such very-high-energy photons are probably emitted from higher altitudes to avoid the strong magnetic absorption that would arise near the polar cap (PC) surface.

In an OG, the pairs created are separated by the magnetic-field-aligned electric field, $E_\parallel$. For middle aged pulsars, pairs are mostly created near the inner boundary; thus, the outgoing particles span much longer distance in the OG than the incoming ones, resulting in an order of magnitude greater outward flux than the inward one (Figure 7 and 8 of Hirotani & Shibata 2002). On the other hand, for relatively young pulsars such as the Vela pulsar, the outward γ-ray flux dominates the inward flux by only several times (Takata et al. 2008), because the outgoing particles span a distance in the strong-$E_\parallel$ region several times longer than the incoming ones. However, such a non-negligible inward flux leads to a light curve that generally exhibits more than two peaks in a single neutron star rotation, which contradicts with observations.

Although it is unclear if the OG model does predict a dominant outward γ-ray flux for young or relatively young pulsars, they have assumed so to obtain the observed double-peaked light curves (Romani & Yadigarogh 1995; Cheng et al. 2000; Zhang & Cheng 2002; Takata & Chang 2007; Tang et al. 2008; Romani & Watters 2010; Bai & Spitkovski 2010a, 2010b; Venter et al. 2012; Pierbattista et al. 2012, 2014; Johnson et al. 2014). We are therefore motivated by the need to contrive a more consistent OG model that quantifies the outward and inward γ-ray fluxes, incorporating the screening effect of $E_\parallel$ by the created and separated charges in the gap. Extending the idea of Beskin et al. (1992), Hirotani & Shibata (1999a, 1999b) first simultaneously solved a set of the non-vacuum Poisson equation, Boltzmann equations for electrons and positrons ($e^\pm$), and the radiative transfer equation in a pulsar magnetosphere. It has been demonstrated (Takata et al. 2004; Hirotani 2006, 2013) that a strong $E_\parallel$ does arise between the null-charge surface and the light cylinder (LC), whose distance from the rotation axis is given by the light cylinder radius, $\sigma_{\ast,LC} = c/\Omega$, where $c$ designates the speed of light and $\Omega$ the rotational frequency of the neutron star.

In the present Letter, we numerically solve the non-vacuum OG electrodynamics in the three-dimensional (3D) magnetosphere of a typical young pulsar and demonstrate that the outward photon flux naturally dominates the inward flux by virtue of the $E_\parallel$ screening due to the separated motion of the $e^\pm$’s created. Without loss of any generality, we can assume a positive $E_\parallel$; in this case, $e^+$’s are accelerated outward while $e^-$’s are accelerated inward, forming an outward current in the OG as a part of the global current circuit. We do not solve the global current closure issue, assuming a starward return current in the magnetic polar regions. We define the magnetic coordinates in Section 2 and describe the 3D vacuum OG model in Section 3. We then propose the new 3D non-vacuum OG model in Section 4, and discuss some implications of this modern OG model in Section 5.

2. 3D MAGNETIC COORDINATES

In a rotating magnetosphere, the Poisson equation for the non-corotational potential $\Psi$ becomes

$$-\nabla^2 \Psi = 4\pi (\rho - \rho_\Omega).$$

(1)
where ρ denotes the real charge density and the Goldreich–Julian (GJ) charge density is defined by

\[ \rho_{\text{GJ}} \equiv -\frac{\Omega \cdot B}{2\pi c} + \frac{\left(\Omega \times r\right) \cdot (\nabla \times B)}{4\pi c}. \]  

(2)

The acceleration electric field can be computed by

\[ E_1 \equiv \mathbf{B} \cdot \mathbf{E}/B = -\partial \mathbf{\Psi}/\partial s, \]  

(3)

where s denotes the distance along the magnetic field line.

To specify the position in a 3D pulsar magnetosphere, it is convenient to introduce the magnetic coordinates (s, \(\theta_s\), \(\varphi_s\)), where \(\theta_s\) and \(\varphi_s\) represent the footpoints of the field lines on the neutron star (NS) surface. Their relationship with the polar coordinates is given by Equations (15)–(17) of Hirotani (2006a).

The magnetic azimuthal angle \(\varphi_s\) is defined counter-clockwise around the magnetic dipole axis; \(\varphi_s = 0\) points in the opposite direction of the rotation axis from the magnetic axis on the two-dimensional (2D) poloidal plane where both the rotation and magnetic axes reside. Thus, a negative \(\varphi_s\) represents a magnetic field line in the trailing side of a rotating magnetosphere, while a positive \(\varphi_s\) represents the same in the leading side.

As for the magnetic colatitudes \(\theta_s\), it is convenient to replace it with the dimensionless trans-field coordinate \(h\) such that

\[ h \equiv 1 - \theta_s/\theta_{s,\text{max}}, \]  

(4)

where \(\theta_{s,\text{max}} = \theta_{s,\text{max}}(\varphi_s)\) describes the PC rim, outside of which the magnetic field lines close within the LC. In what follows, we use the coordinates (s, h, \(\varphi_s\)) to specify points in the 3D magnetosphere. In an OG, \(E_1\) vanishes on the last open field line, h = 0. In the convex side of the magnetic field lines, \(E_1\) increases nearly quadratically with increasing h at each (s, \(\varphi_s\)), attains the maximum value near the central height \(h = 0.5 h_m\), then reduces to vanish above a certain height \(h_m = h_m(s, \varphi_s)\), which forms the upper boundary of the OG. Here, the gap trans-field thickness \(h_m\) corresponds to \(f\) in Cheng et al. (1986) and \(w\) in Romani (1996). Because \(E_1\) is screened by the pairs created, we obtain \(h_m \sim 0.1\) for very young pulsars like the Crab pulsar, while \(h_m > 0.5\) for middle aged pulsars like the Geminga pulsar. In the non-vacuum OG model, the upper boundary \(h_m(s, \varphi_s)\) is consistently determined from the separating motion of the charges by the Poisson and the Boltzmann equations.

To describe the magnetic field, we adopt the vacuum rotating dipole solution (Cheng et al. 2000) in the entire simulation region. Although this approximation breaks down near and outside the LC (Spitkovski 2006), it properly gives the outward/inward flux ratio at least qualitatively, because the screening of \(E_1\) takes place within the LC.

To solve the Poisson equation and the \(e^\pm\) Boltzmann equations, we adopt 300 bins in s direction (from the PC surface \(s = 0\) to \(s = 3\sigma_{\text{LC}}\) along each magnetic field line), 72 bins in h direction (from the lower boundary \(h = 0\) to \(h = 3h_{\text{max}}\)), and 96 bins in \(\varphi_s\) direction (from \(\varphi = -\pi\) to \(\varphi = \pi\); here, \(h_{\text{max}}\) refers to the maximum value of \(h_m(s, \varphi_s)\). The outer boundary of the gap is determined as the free boundary at which \(E_1\) vanishes. For example, if the OG is vacuum (\(\rho = 0\)) and thin (\(h_{\text{max}} \ll 1\)), the outer boundary is located at the inflection point where the poloidal magnetic field configuration changes from convex to concave (Equation (68) of Hirotani 2006a). If the OG is non-vacuum or thick, we must solve the Poisson equation to find the outer boundary. Although there is no physical

\[ \text{Figure 1. Acceleration electric field in the 3D vacuum OG model. The maximum value of } E_1 \text{ in the trans-field direction is projected on the last open field line surface at each } (s, \varphi_s). \]
Figure 2. Gamma-ray pulse profiles above 91 MeV at three discrete viewing angles for the 3D vacuum OG model. The thick curves represent the outward gamma-ray fluxes, while the thin ones represent the inward fluxes. Photons are emitted not only within the light cylinder, but also outside of it by the positrons escaped from the gap. The ordinate ranges from $0$ to $4 \times 10^{-4} \text{MeV s}^{-1} \text{cm}^{-2} \text{deg}^{-1}$.

a greater gradient there compared to the lower altitudes or in the trailing side, as indicated by Figure 1 of Hirotani (2014). This result forms a striking contrast to the standard OG models, which extends the 2D solution of $E_\parallel$ on the poloidal plane (i.e., at $\varphi_0 = 0$) into the toroidal direction (i.e., to $\varphi_0 \neq 0$ regions). It means that we must solve the Poisson equation fully three dimensionally even in the vacuum case.

Using this $E_\parallel$, we can solve the Boltzmann equations for $e^\pm$'s, and the emissivity distribution in the 3D magnetosphere. Note that in a vacuum OG model, the Poisson equation is solved separately from the Boltzmann equations or the radiative transfer equations. The particles are accelerated up to the Lorentz factors $\gamma \sim 3 \times 10^7$ and efficiently emit GeV photons via the curvature process mainly within the OG, and less efficiently up-scatter the magnetospheric IR–UV photons into TeV after escaping from the OG, where the IR–UV photons are mostly emitted by the secondary pairs created outside the OG via the synchrotron process.

Before escaping from the gap, typical inward-migrating electrons run $0.3\sigma_{LC}$, while typical outward-migrating positrons run $0.7\sigma_{LC}$. As a result, outward flux becomes only a few times stronger than the inward flux. Therefore, the light curve in Figure 2 generally exhibits more than two peaks in a single NS rotation, which contradicts with the majority of gamma-ray observations.

4. 3D NON-VACUUM OUTER GAP MODEL

Let us next consider the screening effect due to the separating motion of the pairs created in the gap. We solve Equations (43)–(55) in Hirotani (2013) under the boundary conditions that $e^-$'s or $e^+$'s do not enter across either the inner or the outer boundaries. Pairs are mainly created when the inward curvature $\gamma$-rays collide with the outward thermal X-rays emitted from the NS surface. The pairs created in the gap are separated to screen $E_\parallel$ to a small amplitude so that the pairs can be marginally separated. In this case, the real charge density has the same spatial gradient as the GJ charge density (Goldreich & Julian 1969) along the magnetic field, as indicated by Figure 5 of Hirotani (2006a). We neglect the magnetic field deformation due to the magnetospheric currents, adopting the same magnetic field geometry as in Section 3.

Because of this screening effect, $E_\parallel$ becomes very weak in the middle and lower altitudes, $s < 0.77\sigma_{LC}$, as Figure 3 shows. It is also found that the regions that emit photons in phases P1 and P2 (i.e., in $50^\circ < \varphi_0 < 80^\circ$ and $-105^\circ < \varphi_0 < 45^\circ$ in Figure 3) have greater $E_\parallel$ than other regions. The gap trans-field thickness becomes $0.11 < h_m < 0.13$ in most portions of the gap.

As a result of this $E_\parallel$ screening, the outward photon flux dominates the inward one, as demonstrated by the light curves in Figure 4. This is because the pairs are mostly created in the middle or lower altitudes, $s < 0.77\sigma_{LC}$, which indicates that
the positrons experience an efficient acceleration in the strong $E_\parallel$ region in the higher altitudes while the electrons do not. Therefore, the light curve is dominated by the outward photons, which are emitted from the northern OG into the southern hemisphere, and it tends to exhibit a double-peak pulse profile for a wide range of $\zeta$.

The expected phase-averaged spectrum is plotted for $\zeta = 110^\circ$ in Figure 5. For comparison, we plot the spectrum of the outward and inward photons as the thick and thin curves, respectively. It is also confirmed from the figure that the gamma-ray flux is predominantly outward. As a result of the superposition of the curvature emission from different places with varying $E_\parallel$, between 0.16 GeV and 1.6 GeV, the $\nu F_\nu$ spectrum becomes a power law with an index of 0.68, which is consistent with the Fermi observations of young pulsars. Since $E_\parallel$ depends not only on ($s$, $\phi_*$), but also quadratically on $h$, it is essential to consider the superposition of the emission spectra along different magnetic field lines. The intrinsic luminosity of the magnetospheric emission is $1.7 \times 10^{36}$ erg s$^{-1}$ above 160 MeV, while it is $1.3 \times 10^{35}$ erg s$^{-1}$ below 160 MeV. The heated PC luminosity is $3.3 \times 10^{32}$ erg s$^{-1}$. For comparison, the spin-down luminosity is $4.4 \times 10^{37}$ erg s$^{-1}$, and the cooling NS X-ray luminosity is $1.2 \times 10^{33}$ erg s$^{-1}$. The heated PC flux becomes greater than the magnetospheric X-ray flux if $\zeta < 45^\circ$ or $\zeta > 135^\circ$.

5. DISCUSSION

In summary, we have numerically examined the pulsar outer gaps by solving a set of the Poisson equation, the $e^\pm$ Boltzmann equations, and the radiative transfer equation from 0.005 eV to 20 TeV. Applying the method to a young pulsar with a 3 kyr age, we find that the acceleration electric field $E_\parallel$ is substantially...
screened by the separating motion of the pairs created, and that the γ-ray flux becomes predominantly outward due to the localization of \( E_{\gamma} \) in higher altitudes. To reproduce the observed double-peak light curves, it is essential to solve the outer gap three dimensionally, taking account of this screening effect.

As Figure 4 indicates, the trailing peak has a long tail until the rotational phase of \( \sim 100 \) degrees. This is due to emission from the side of the magnetosphere that is most trailing (\( \phi_s < -90^\circ \)) from very high altitude (\( \sigma_{\gamma LC} < s < 3\sigma_{\gamma LC} \)). Since the actual strength and direction of such emissions depend on the magnetic field configuration near the LC, it could be possible to constrain the magnetic field configuration there. Figure 4 also shows that there is a strong emission component around \( \phi_s \sim -20^\circ \) (i.e., before P2). This component is suppressed if we consider a very thin OG (e.g., \( h_m < 0.05 \)), as suggested in the standard OG or SG models. However, if we solve the set of Maxwell-Boltzmann equations, we obtain \( h_m \sim 0.12 \) and the broad light curves as presented. This means that we cannot simultaneously reproduce the observed flux and sharp pulses by the current particle accelerator models.

Let us briefly discuss an implication when the minimal cooling scenario with a heavy element envelope breaks down. If the NS envelope contains light elements with mass greatly exceeding \( 10^{-16} M_\odot \), the higher NS surface temperature (Page et al. 2004) leads to a reduction of \( h_m \) and hence the OG luminosity. On the contrary, if the cooling is dominated by neutrino emission via the direct Urca process, the resultant rapid cooling (in the initial \( \sim 100 \) yr) (Negreiros et al. 2014; Coelho et al. 2014) will lead to an increase of \( h_m \) and hence luminosity. In either case, we expect that \( \dot{E}_{\parallel} \) is localized in higher altitudes in the same way as demonstrated in this Letter, by virtue of the strong negative feedback effects in the OG electrodynamics (Hirotani 2006b).

Since the optical depth for photon-photon pair creation is around unity, the TeV photons created via the synchrotron-self-Compton process cannot be easily absorbed by the magnetospheric IR–UV photons, as indicated by the dashed and thick solid curves in Figure 5. It suggests that we may expect relatively strong pulsed emissions around TeV from the pulsars of which inverse-Compton and photon-photon-absorption optical depths are around unity, which is typical for young pulsars with ages around several thousand years. We will discuss this possibility in a separate paper.

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