A Quantum Spin Hall Round Disk as a Spin Rotator and Filter

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We study theoretically the spin transport of a Quantum Spin Hall round disk. When an electron traverses the disk in virtue of the edge states, its spin’s in-plane component can be rotated by a magnetic flux through the disk. The spin rotation occurs due to the interference of two helical edge states with opposite spins, which is regarded as the Aharonov-Bohm effect in the spin space and a manifestation of the Berry phase. Besides, the disk has a spin filter effect on the tunneling current when we apply an appropriate magnetic field and gate voltage on it. The spin polarization ratio can reach 100% when the couplings between the disk and leads are weak.

I. INTRODUCTION

The geometric phase of quantum system undergoing adiabatic, cyclic evolution was first discovered by Berry and a simple geometric interpretation was given by Simon. It has a tremendous impact on various areas of physics which have triggered active researches over a quarter of century. Now it is recognized that the geometric phase is an important concept of quantum mechanism both from a theoretical perspective and potential applications. A manifestation of the Berry phase is the well-known Aharonov-Bohm (AB) effect, that an electrical charge which cycles around a magnetic flux. From the viewpoint of the symmetry between magnetic field and electric field in the Maxwell equations, Aharonov and Casher predicted that a magnetic moment acquires a phase around a charge flux line. The the Aharonov-Casher (AC) effect is weak in the neutron interference experiments, while recently an enhanced AC effect is realized in the two-dimensional electron system confined in a semiconductor quantum well with strong spin-orbit coupling. This is another great manifestation of the Berry phase.

The researches on the spin-orbit interaction systems lead to the discovery of the quantum spin Hall (QSH) materials as a new phase of the condensed matter, which can be regarded as two copies of the integer quantum Hall systems for up and down spins with the opposite chiralities. For a strip geometry, the edge states for up and down spins propagate in the opposite directions along each edge, which are called the helical edge modes. When the Fermi energy of a QSH system is in the bulk gap, only the edge modes are responsible for transport, thus the whole system can be regarded as two one-dimensional (1D) channels on both edges of the strip.

Recently, Chu et al. has predicted the AB oscillations in the magnetococonductance of a singly connected QSH disk. In this paper, we consider the spin transport of a QSH round disk connected to two normal electrodes with a vanishing spin-orbit interaction (SOI). The system is equivalent to a 1D AB ring in which the transport is carried out by the helical edge states. The up and down spin propagating in opposite directions along the edge results in a spin rotation effect under a magnetic field, which is a manifestation of the Berry phase in the spin space. Besides, because the external magnetic field lifts the spin degeneracy, a spin filter effect is found, which arises from the spin-resolved resonant tunneling.

II. THE HAMILTONIAN AND THE SPIN ROTATION MECHANISM

The effective four-band Hamiltonian proposed for a HgTe/CdTe quantum well is given by

$$H(k) = \begin{pmatrix} h(k) & 0 \\ 0 & h^*(-k) \end{pmatrix},$$

with

$$h(k) = \epsilon(k) I_{2 \times 2} + d_{\alpha}(k) \sigma^\alpha,$$

where

$$\epsilon(k) = C - Dk^2, \quad d_{\alpha}(k) = (Ak_x, -Ak_y, M(k)),$$

and

$$M(k) = M - Bk^2,$$

where we have used the basis order $|E_1+, H_1+, E_1-, H_1-\rangle = E_1^+\rangle, H_1^+\rangle, E_1^-\rangle, H_1^-\rangle$.

“$E_1^+$” and “$H_1^+$” represent the electron and hole bands, “$+^+$” and “$-^-$” represent the spin along z direction. For a strip geometry, it is well known the Hamiltonian with $M < 0$ has a pair of helical edge modes on each side of the strip. Similarly for a round disk geometry we find it also has helical edge modes along the edge, each forming an ideal 1D loop. The spin-up states travel clockwise, and the spin-down states travel anti-clockwise.

When the Fermi energy of a QSH system is in the bulk gap, only the edge modes are responsible to transport. The edge modes can be described by an effective 1D Hamiltonian,

$$H_{1D}(k) = v_F k \sigma_z,$$

where $v_F$ is the Fermi velocity and $\sigma_z$ is the Pauli matrix defined in the $(+, -)$ space. $k$ is the quasi-continuous momentum, whose positive direction is defined along the azimuthal direction. In this 1D Hamiltonian, $\sigma_z$ is a good quantum number, and the direction of the motion is correlated to the spin polarization. Obviously this Hamiltonian retains the information in the $(+, -)$ space and...
neglects the dynamics in the \((E_1, H_1)\) space, it can be used to study the properties of spin qualitatively.

Now we consider a QSH round connected to two normal electrodes without SOI, an effect similar to the AB effect is expected. But the effect here must be different from the conventional AB effect because the Hamiltonian \(\mathbf{A}\) is relative to spin. If the incident electron is polarized in the \(x\)-\(y\) plane (we assume the quantum well is grown along the \(z\) direction), the polarization direction of the outgoing current will rotate in the \(x\)-\(y\) plane when an perpendicular magnetic field is applied on it. The magnetic flux through the disk induces the spin current oscillation rather than the charge current oscillation in the conventional AB effect. The spin rotation process can be analyzed through the 1D Hamiltonian \(\mathbf{A}\). An electron injected from the left electrode polarized along positive \(x\) direction is represented as a linear combination of spin-up and spin-down waves along the \(z\) direction,

\[
\left( \begin{array}{c} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \\ \end{array} \right) = \left( \begin{array}{c} 1 \\ 0 \\ \end{array} \right) + \left( \begin{array}{c} 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \\ \end{array} \right),
\]

(3)

When they enter the disk, they propagate in virtue of the edge states of the QSH disk. The up-spin travels in the clockwise direction while the down-spin travels in the anti-clockwise direction. When a homogeneous magnetic field \(\mathbf{B} = (0, 0, B)\) is applied, the edge states can obtain additional phases correlated with the vector \(\sigma\) of spin-up and spin-down waves along the \(z\) direction, can be analyzed through the 1D Hamiltonian \(\mathbf{A}\). An electron injected from the left electrode polarized along positive \(x\) direction is represented as a linear combination of spin-up and spin-down waves along the \(z\) direction,

\[
\left( \begin{array}{c} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \\ \end{array} \right) = \left( \begin{array}{c} 1 \\ 0 \\ \end{array} \right) + \left( \begin{array}{c} 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \\ \end{array} \right),
\]

(3)

where \(L = 2\pi R_{eff}\). The phase difference between up-spin and down-spin determine the spin of the outgoing wave \(\langle \sigma_y \rangle = (2\cos(e\phi_{eff}/h) - 2\sin(e\phi_{eff}/h), 0)\), where \(\phi_{eff} = B\pi R_{eff}^2\) is the magnetic flux through the effective area enclosed by the edge states, so the spin rotation angle is given by \(\theta = \arctan(\langle \sigma_y \rangle / \langle \sigma_x \rangle) = -e\phi_{eff}/h\). Taking into account the contribution from the dynamics in the \((E_1, H_1)\) space which is omitted by the 1D Hamiltonian \(\mathbf{A}\), the spin-rotation angle is given by

\[
\theta = \theta_0 - 2\pi \frac{\phi_{eff}}{\phi_0},
\]

(5)

where \(\phi_0 = h/e\) is the magnetic flux quantum, \(\theta_0\) is the intrinsic phase shift in the absence of the magnetic field which is related to the details of the system such as the size, shape and interface. Yokoyama et al. has studied the giant spin rotation happening at the interface between a normal metal and a QSH insulator. Here we focus on the spin rotation induced by the magnetic field, so introduce \(\theta_0\) as a priori parameter.

III. NUMERICAL RESULTS

We consider a two-terminal QSH round device connected with two normal leads without SOI, as shown in Fig. (a). The disk is described by the four-band Hamiltonian \(\mathbf{H}\), the leads are described by a single-band Hamiltonian with spin degeneracy. The magnetic field \(\mathbf{B} = (0, 0, B)\) is taken into account through the Peierls substitution \(k \to k + eA/h\) with the Landau gauge \(A = (eB/B, 0, 0)\), the Zeeman splitting is ignored. In the tight-binding representation, the Hamiltonian of the whole system is given by

\[
H = H_{disk} + H_{lead} + H_c,
\]

(6)

\[
H_{disk} = \sum_{i,\alpha} V_\alpha c_{i\alpha}^+ c_{i\alpha} + \sum_{i,\alpha}\frac{D}{a^2} c_{i\alpha}^+ e^{iA_{(i-j)}(i-j)} c_{j\alpha} + \sum_{i} A_{(i+1)\alpha} c_{i\alpha}^+ c_{i\alpha} + \sum_{i} A_{(i-1)\alpha} c_{i\alpha}^+ c_{i\alpha} + \sum_{i} A_{(i\alpha)\alpha} c_{i\alpha}^+ c_{i\alpha} + \sum_{i} A_{(i\alpha)\alpha} c_{i\alpha}^+ c_{i\alpha} + \sum_{i,\alpha,\beta} (\gamma_{\alpha\beta} c_{i\alpha}^+ e^{iA_{(i-j)}(i-j)} d_{j\beta} + h.c.),
\]

(7)

where \(H_{disk}, H_{lead}, H_c\) are for the disk, leads and the disk-lead coupling, respectively. \(\alpha \in \{E_1, H_1\}\) and \(\beta \in \{\uparrow, \downarrow\}\). \(c_{i\alpha}\) annihilates an \(\alpha\)-band eletron on the site \(i\). \(\delta_{x,y}(i)\) is the unit vector along the \(x(y)\) direction in the disk, and \(d_{i\beta}\) annihilates a spin-\(\beta\) electron on the site \(i\) in the leads. \(a = 3.3nm\) is the lattice constant, and \(\delta_{x,y}(i)\) is the unit vector along the \(x(y)\) direction in the disk. \(D_{x,y}\) is the on-site energy in the disk and the spin-\(\beta\) in the leads. If the interfacial spin-flip scattering is neglected, it can be estimated \(\gamma_{E_1, \uparrow} = \gamma_{H_1, \uparrow} = \gamma_{E_1, \downarrow} = \gamma_{H_1, \downarrow} = \gamma_{H_1, \downarrow} = \gamma_{E_1, \uparrow} = \gamma_{H_1, \downarrow} = \gamma_{E_1, \downarrow} = \gamma_{H_1, \uparrow} = \gamma_{H_1, \downarrow} = 0\). The on-site energy in the disk is given by

\[
V_\alpha = \begin{cases} C + M - \frac{2(D+B)}{B}, & \alpha \in \{E_1\} \\ C - M - \frac{2(D-B)}{B}, & \alpha \in \{H_1\} \\ -\infty, & \alpha \in \{\text{shadow area}\} \end{cases},
\]

(10)

which promises the effective geometry is a round disk. The on-site energy in the leads is \(V_L = C_L - \frac{4B}{a^2}\) and
$C_L = -0.1eV$ promises Fermi energy of the leads is in the conduction band. The other parameters we used are $A = 364.5meV·nm$, $B = -868meV·nm^{-2}$, $C = 0$, $D = -512meV·nm^{-2}$, $M = -10meV$. The radius of the disk is $R = 100nm$, and the width of the leads are $L = 30nm$.

We assume the left lead has a x-spin-polarized potential $V$, the x-spin-resolved Fermi energy in the leads is $E_f^x - V = E_f^{R_{eff}} = E_f$. The spin conductance with respect to the spin along $α$-axis in the right lead is defined by $G^α = \lim_{V→0} J^α/V$, where $J^α$ is the $α$-spin current in the right lead. The practical calculation bases on the lesser Green’s function in the right lead,

$$G^α(\epsilon_x) = \frac{2e^2}{h} \sum_{\epsilon_y} \text{Re} \left[ \text{Tr}(\sigma_x H_{L+\delta_x, L} G_{\epsilon_x}^{<} \right], \tag{11}$$

where the lesser Green’s function $G^{<} = G^ρ \Sigma^< L_{eff} G^ρ$, in which $G^ρ = [E - H ± i0^+]^{-1}$, and the lesser self-energy

$$\Sigma^< = \frac{2\pi e^2}{\alpha} U^+(G^ρ L_{eff} - G^ρ L_{eff}^+) U.$$

$G^ρ L_{eff} = \left(G^ρ L_{eff}\right)^+$ and $G^ρ L_{eff}^+$ is the surface Green’s function in the z-spin-up subspace for a semi-infinite lead obtained by a recursive method. $U = e^{2\theta \gamma}/4$ is a SU(2) rotation from z-spin to x-spin. Although Eq. (11) is a function of the x-coordinate, the result is independent of $\epsilon_x$ because the spin conversation in the normal leads without SOI.

Fig. 2(a) shows the spin conductances as functions of the magnetic field for a fixed Fermi energy $E_f = -0.8meV$. The oscillations on $G^x$, $G^ρ$ means a spin rotation in the x-y plane. The spin rotation angle is defined by $θ = \arctan(G^ρ/G^x)$. Fig. 2(b) shows the rotation angles as functions of the magnetic field for various Fermi energy $E_f = -8, 0, 4, 8meV$. As a comparison, the red solid line shows the analytical result from Eq. (6) with the value assignments $θ_0 = -0.43\pi$ and $R_{eff} = R$. The numerical curves follow the same tendency with the analytical line. The fluctuations on the numerical curves are caused by the evolution in the $(E_1, H_1)$ space which is not included in Eq. (6). The deviations between the numerical curves and the analytical line are because the effective radius of the edge states is smaller than the disk radius. We use Eq. (6) fitting the numerical curves to get $R_{eff}/R = 0.98, 0.91, 0.85, 0.75$ for $E_f = -8, 0, 4, 8meV$. In the energy regime we study, a smaller effective radius is deduced for a higher Fermi energy, which is consistent with the results from Chu et al. The edge states with higher energy are wider and penetrate more towards the center of the disk, so the enclosed area have a smaller effective radius $R_{eff}$. The phenomenon of the spin rotation angle depending on the Fermi energy is called the dispersibility of the spin rotation effect, on the contrary to the nondispersibility of the conventional AB effect in which the conductance oscillation is independent of the energy of the particles. The dispersibility here is due to the effective 1D ring formed by the edge state has a energy-dependent area.

A similar spin rotation has been reported in the scalar AB experiment with Neutrons, but the result is explained unnecessarily by the phase shift difference of the AB effect. It can be explained by a classical spin precession instead. We emphasize the spin rotation effect we discover here is a topological effect which can not be explained by the classical spin precession. It is a manifestation of Berry’s topological phase in the spin space, due to the phase interference of the different z-spin components, bearing an analogy to the rotation of linearly polarized light in a helically wound optical fiber.

To see the transport process in the disk detailedly, we
calculate the pseudo-spin density distribution \( S(\mathbf{i}) \) in Fig. 1 as a response of the x-spin-polarized potential in the left lead. The pseudo-spin is defined in the disk region as \( \hat{S}_\alpha = I_2 \otimes \sigma_\alpha \), where \( I_2 \) is the unit matrix acting on the \((E_1, H_1)\) space and \( \sigma_\alpha \) is the Pauli matrix acting on the \((+, -)\) space. This definition is just the “spin” defined by Yokoyama et al.\(^{15}\). We note here that the real spin is not a good quantum number in the QSH insulator because of the spin-orbit coupling. So the pseudo-spin is used to distinguish the two states of a Kramers doublet. We observe the pseudo-spin density distribution in the disk region as a response of the x-spin-polarized potential in the lead. The pseudo-spin is defined in the disk region as \( \hat{S}_\alpha G_{\alpha \beta} \). (\( \alpha = x, y, z \))

\[
S_\alpha(\mathbf{i}) = -i Tr \left[ \hat{S}_\alpha G_{\alpha \beta} \right]. (\alpha = x, y, z) \tag{12}
\]

We can see that the pseudo-spin density concentrates on the edge of the disk, which ensures the transport is born by the edge states. Although the incident electrons are polarized along x direction, the local spin on the edge is mainly polarized in z direction, and the upper edge and the lower edge are occupied by the opposite polarized z-spins. The edge states with opposite z-spins acquire different phase shifts when they travel along the opposite edges, resulting in an overall spin rotation in the x-y plane after superposition at the interface to the right lead. So it is the manifestation of AB effect in the spin space. Besides, we can observe the x- and y-spin wave along the edge, which is also arising from the interference of the z-spin-up and -down wavefunctions because a part of the electron wave is reflected at the interfaces to the leads, forming a standing wave along the edge and interfering with each other.

IV. SPIN FILTER EFFECT

The QSH round disk has a spin filter effect for the spin in the z direction when a magnetic field lifts the spin degeneracy. The mechanism for lifting the spin degeneracy is not the tiny Zeeman splitting but the angular momentum of the edge state. The edge state travels closely to the boundary of the disk, so it has a large angular momentum which strongly couples to the external magnetic field. We can estimate the energy levels for the edge states of a QSH round disk. Zhou et al. has solved the dispersions of the edge states of a QSH strip exactly.\(^27\)

\[
E_\pm(k) = -\frac{MD}{B} \pm A \sqrt{\frac{B^2 - D^2}{B^2}} k, \tag{13}
\]

where \( E_+(E_-) \) derives from the upper (lower) block of the Hamiltonian (1). For a large round disk, an analytical solution gives the quantization condition \( k \approx (n + 1/2)/2R_{eff} \). When a magnetic field \( \mathbf{B} = (0, 0, B) \) is applied, the energy level shift in the linear order of \( B \) is given by

\[
\Delta E_\pm = \frac{e}{2} (\mathbf{r} \times \mathbf{v}) \cdot \mathbf{B} \approx \pm \frac{eB}{2h} \frac{\partial E}{\partial k} R_{eff}. \tag{14}
\]

So the energy dispersions for the edge states are approximately given by

\[
E_\pm(n) = -\frac{MD}{B} + A \sqrt{\frac{B^2 - D^2}{B^2}} \left( \frac{2n + 1}{2R_{eff}} \right) \pm \frac{eB}{2h} R_{eff}. \tag{15}
\]

For a disk with \( R = 100nm \), the dispersions calculated by the tight-binding method are shown in Fig. 3, which are in agreement with Eq. (15) very well. The solid lines are from the upper block \( h(k) \) and the dashed lines are from the lower block \( h^*(-k) \) in Hamiltonian (1). The energy levels are linear functions of the magnetic field, the spin-up levels and spin-down levels shift oppositely and cross each other. The red arrows indicate a series of crossing points. From the energy dispersions Eq. (15), we can estimate the crossing points locate at \( \phi_{eff}(n) = n\phi_0/2 \). In Fig. 3 the crossing points shift right when the energy is higher, that means the effective radius of a higher-energy edge state is smaller, that is consistent with the deductions from Fig. 2b) and Ref. \(^{16}\).

The lifting of the z-spin degeneracy in a magnetic field can induce a spin filter effect. If we consider the round disk as a quantum dot, the tunneling current through it must be z-spin polarized. We use the non-equilibrium Green’s function to calculate the z-spin resolved conductance. Still for the geometry in Fig. 1(a), the left lead has a higher Fermi energy than the right lead, \( E_{fL}^R - V = E_{fR} \equiv E_f \), and the Fermi energy is spin-independent so that the incoming current is unpolarized. The z-spin-resolved conductance is defined by \( G_{\alpha}^z = \lim_{V \to 0} \frac{I_{\alpha}}{V} \).
where \( J_z^\uparrow (J_z^\downarrow) \) is the z-spin-up(down) component of the outgoing current in the right lead. The practical calculations are based on the lesser Green’s function,

\[
G_\alpha^z(x) = \frac{2e^2}{h} \sum_{i_y} \text{Re} \left[ T r (\sigma_z^x H_{i+\delta_x} G_{i,i+\delta_x}^z) \right],
\]

(16)

\[
\sigma_1^z = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \sigma_2^z = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.
\]

(17)

Same to Eq. (16), the results are independent of \( i_x \) because of the spin conservation in the leads. The charge conductance \( G_z = G_1^z + G_2^z \) and the z-spin conductance \( G^z = G_1^z - G_2^z \). The z-spin-resolved conductances as functions of the Fermi energy are shown in Fig. 4(a) with three different disk-lead coupling \( \gamma \) in the Hamiltonian \( H_0 \). The magnetic field is fixed at \( B = 0.025T \), and the Fermi energy can be adjusted by a gate voltage in experiments. The spin-resolved resonant tunneling happens when the Fermi energy is aligned with any energy level of the disk. We define the spin polarization ratio of the conductance \( P = G^z / I \), which is shown in Fig. 4(b) for the corresponding \( \gamma \). For a smaller \( \gamma \), the conductance peaks are narrower, so the separation of the spin-up and spin-down conductances is clearer, thus the spin polarization ratio is larger. At some Fermi energy, the spin polarization ratio reaches \( \pm 100\% \) when \( \gamma = 0.2 \). At this time, only one edge state with a specific z-spin takes charge of the transport, so the outgoing current in the right lead is totally polarized, the QSH round disk exhibits a perfect spin filter effect.

V. CONCLUSION

In conclusion, we study the spin transport of a QSH round disk device connected to two normal electrodes without SOI. An in-plane spin rotation effect is discovered as a manifestation of the AB effect in the spin space. This phenomenon arises from the interference of the z-spin-up and -down edge states which acquire different phase shifts when traveling in the opposite directions. The spin rotation angle is related to the magnetic flux through the effective area enclosed by the helical edge states.

Besides the spin rotation effect, the QSH round disk has a z-spin filter effect when the bias voltage is spin-independent. The effect is due to the electron’s tunneling through the disk via its discrete spin-separating energy levels when a magnetic field lifts the spin degeneracy of the system. This effect is notable when the couplings between the disk and leads are weak. For a small disk-lead coupling, the z-spin polarization ratio of the outgoing current can reach \( \pm 100\% \) if an appropriate gate voltage and magnetic field are applied.

The two effects are both induced by the helical edge states of the QSH material, they are not sensitive to the size and the shape of the disk as long as the circumference of the disk is smaller than the coherent length of the edge states. We hope the effects discovered here can help designing multi-functional spintronic devices and show technological applications in the quantum computing and microelectronics.

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Note added.—After the completion of this work, we became aware of a recent preprint by J. Maciejko et. al., which proposes a spin transistor based on the spin rotation effect (spin AB effect).

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