Induced plasma magnetization due to magnetic monopoles

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(Dated: October 19, 2010)

When magnetic monopoles are introduced in plasma equations, the propagation of electromagnetic waves is modified. In this work is shown that this modification leads to the emergence of a ponderomotive force which induces a magnetization of the plasma. As a result, a cyclotron frequency is induced in electrons. This frequency is proportional to the square of the magnetic charge.

PACS numbers: 14.80.Hv, 52.35.-g, 52.35.Mw
Keywords: Plasma waves; Magnetic monopoles; Plasma magnetization; Ponderomotive force.

I. INTRODUCTION

A common problem in electrodynamics and plasma physics courses, which vividly illustrates the phenomenon of propagation of waves through a plasma, is the calculation of the dispersion relation of electromagnetic transverse waves propagating in a plasma composed by fixed ions and moving electrons. Such a problem is widely used to introduce concepts like cutoff and plasma frequencies, as well as, group and phase velocity of a wave.

On the other hand, it is well known that the Maxwell’s equations become symmetric when electric and magnetic charges (magnetic monopoles) are theorized [1]. Even, in an elegant theoretical demonstration, Dirac shown that the simple existence of only one magnetic monopole in the universe leads to an explanation of the quantization of the elementary electric charge [2, 3].

Once magnetic monopoles are introduced in Maxwell’s equations, they become symmetric and it is expected that new effects appear due to their simmetry. Including magnetic charges, the equations are

$$\nabla \cdot \mathbf{E} = 4\pi \rho_e, \quad \nabla \cdot \mathbf{B} = 4\pi \rho_g, \quad (1)$$

$$\nabla \times \mathbf{B} = \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c} \mathbf{J}_e, \quad \nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} - \frac{4\pi}{c} \mathbf{J}_g, \quad (2)$$

where $\rho_e$ is the electron charge density, $\rho_g$ is the magnetic charge density, $\mathbf{J}_e$ is the electron current density, and $\mathbf{J}_g$ is the magnetic current density.

A simple test is to consider a plasma containing electric and magnetic charges. Such analysis has great theoretical [4, 5] and pedagogical [6] value, because of its simplicity, and the discussion that emerges from these studies.

As a simple theoretical exercise we show how a magnetization is induced in a plasma when magnetic charges are introduced. The problem is the following: we have a gas composed by magnetic monopoles and electrons. Suddenly, an electromagnetic wave passed through this fluid. We look for the effect in the propagation of the very well-known simplest wave modes when magnetic monopoles are included.

II. ELECTROMAGNETIC WAVES

Let us consider a plasma as a fluid which contains electrons, with charge $-e$ and mass $m_e$, and magnetic monopoles with magnetic charge $g$ and mass $m_g$. Let us suppose that there exist other particles with respective opposite electron and magnetic charges and with the same densities, which provide the total charge neutrality of this plasma [6].

The magnetic monopoles modify the well-known dispersion relation for electromagnetic waves. In this section, we show the derivation of the relation for this propagation mode considering the electric and magnetic charges as a fluids and perturbing their equilibrium velocities. In order to do this, we expand every quantity in Maxwell’s equations $\phi$ as $\phi_0 + \phi_1$, where $\phi_0$ is the zeroth order (equilibrium) value of $\phi$, and $\phi_1$ is the first order perturbation. In that sense, throughout this work, electrons have a zeroth order electron density $n_{0e}$ and a first order electron density $n_e$. In a similar way, magnetic monopoles have an zeroth order magnetic density $n_{0g}$ and a first order magnetic density $n_g$. Thus, the first order electric charge density is $\rho_e = -en_e$ and the first order magnetic density is $\rho = gn_g$. Thereby, the first order electric and magnetic current density are

$$\mathbf{J}_e = -en_{0e} \mathbf{v}_e, \quad \mathbf{J}_g = gn_{0g} \mathbf{v}_g, \quad (3)$$

where $\mathbf{v}_e$ and $\mathbf{v}_g$ are the first order electron and magnetic monopole velocities respectively. Both zeroth order velocities are null.

As in the case of a cold plasma, the classical equations of motion for electrons and magnetic monopoles [3], at first order are

$$\frac{\partial \mathbf{v}_e}{\partial t} = \frac{-e}{m_e} \mathbf{E}_1, \quad \frac{\partial \mathbf{v}_g}{\partial t} = \frac{g}{m_g} \mathbf{B}_1. \quad (4)$$
As seen, the Lorentz force over a magnetic monopole is the same as for an electron but changing electric fields by magnetic field and vice versa.

In order to obtain the dispersion relation, the usual method is apply a Fourier transform over all the linearized equations with the form \( \exp(\imath \mathbf{k} \cdot \mathbf{r} - \Omega t) \), where \( \omega \) is the frequency and \( \mathbf{k} \) is the wavenumber of the wave. Thus, the equations of motion (4) becomes

\[
\tilde{\mathbf{v}}_e = \frac{-\imath e}{m_e \omega} \tilde{\mathbf{E}}_1, \quad \tilde{\mathbf{v}}_m = \frac{\imath g}{m_g \omega} \tilde{\mathbf{B}}_1, \tag{5}
\]

respectively. Here \( \tilde{\mathbf{v}}_e \) is the Fourier transform of \( \mathbf{v}_e \), and the same for the other quantities.

We can write the electron and magnetic current density with the help of the velocities given by Eqs. (5). Therefore, Eqs. (2) can be rewritten as

\[
\mathbf{c} \times \tilde{\mathbf{B}}_1 = \left( \frac{\omega_p^2 - \omega_m^2}{\omega} \right) \tilde{\mathbf{E}}_1, \quad \mathbf{c} \times \tilde{\mathbf{E}}_1 = \left( \frac{\omega^2 - \omega_m^2}{\omega} \right) \tilde{\mathbf{B}}_1, \tag{6}
\]

where we define the electron plasma frequency and a magnetic monopole plasma frequency respectively as

\[
\omega_p = \sqrt{\frac{4\pi e^2 n_0}{m_e}}, \quad \omega_m = \sqrt{\frac{4\pi g^2 n_0}{m_g}}. \tag{7}
\]

Finally, the dispersion relation for a transversal electromagnetic wave, with \( \mathbf{k} \cdot \tilde{\mathbf{E}}_1 = 0 \), in an electron plasma with magnetic monopoles can be obtained from Eqs. (6). This gives

\[
c^2 k^2 = \frac{1}{\omega^2} \left( \omega^2 - \omega_p^2 \right) \left( \omega^2 - \omega_m^2 \right). \tag{8}
\]

Dispersion relation (8) have been obtained previously in Ref. [3]. Notice that when magnetic monopoles are neglected, \( \omega_m = 0 \) and we recover the usual dispersion relation \( \omega^2 = \omega_p^2 + c^2 k^2 \) for electromagnetic waves in cold plasmas.

### III. Magnetization Due to Ponderomotive Force

By incorporating magnetic charges can be seen several effects in the propagation of waves through plasmas [5, 6]. We focus on the emergence of a magnetic field due to the ponderomotive force that the wave exerts on the electrons. This magnetization appears due to the magnetic charge corrections in dispersion relation [8]. Similarly to the inclusion of magnetic monopoles, it has been studied the plasma magnetization when quantum effects are considered in the calculation of the dispersion relation of an electron plasma [7, 8].

The ponderomotive force \( \mathbf{f} \) induced by the high-frequency electromagnetic waves in a plasma is given in general by \( \mathbf{f} = \mathbf{f}^{(s)} + \mathbf{f}^{(t)} \), where the ponderomotive forces \( \mathbf{f}^{(s)} \) and \( \mathbf{f}^{(t)} \) are related to the space \( (s) \) and time \( (t) \) variations of the amplitude \( |\mathbf{E}_1| \) of the electric field. This force is a nonlinear effect which origin is the inhomogeneity of the electromagnetic field. Each component is given by [9]

\[
\mathbf{f}^{(s)} = \frac{\epsilon - 1}{16\pi} \nabla |\mathbf{E}_1|^2, \quad \mathbf{f}^{(t)} = \frac{\mathbf{k}}{16\pi \omega^2} \frac{\partial (\omega^2 (\epsilon - 1))}{\partial \omega} \frac{\partial |\mathbf{E}_1|^2}{\partial t}, \tag{9}
\]

where \( \epsilon = c^2 k^2 / \omega^2 \) is the dielectric function of the propagation mode. Using the dielectric function \( \epsilon \) of the dispersion relation (5), we can calculate the ponderomotive force for the high-frequency electromagnetic wave. It is straightforward to obtain an expression for the ponderomotive force related to time variations

\[
\mathbf{f}^{(t)} = - \frac{\omega_p^2 \omega_m^2}{8\pi \omega^3} \frac{\partial |\mathbf{E}_1|^2}{\partial t}. \tag{10}
\]

Notice that the effects of magnetic charges in the ponderomotive force are in \( \omega_m \). When \( \omega_m = 0 \) the ponderomotive force \( \mathbf{f}^{(t)} \) is null.

The effect of ponderomotive force of the electromagnetic wave is to push the electrons locally. Thus, it creates a slowly varying electric field \( \mathbf{E}_s \) such that \( \mathbf{f} = -e n_0 \mathbf{E}_s = -e n_0 \nabla \phi_s - (e n_0 / c) \partial t \mathbf{A}_s \) [7, 8]. Using the ponderomotive forces (10), we can identify the slowly varying vector potential as

\[
\mathbf{A}_s = \frac{e\omega_p^2 \omega_m^2}{8\pi \omega^3 e n_0} |\mathbf{E}_1|^2. \tag{11}
\]

Owing to the well-known relation \( \mathbf{B} = \nabla \times \mathbf{A} \), the vector field (11) induces a slowly varying magnetic field \( \mathbf{B}_s \)

\[
\mathbf{B}_s = \frac{e\omega_p^2 \omega_m^2}{2m_e c^2} \nabla \times (\mathbf{k} |\mathbf{E}_1|^2). \tag{12}
\]

This magnetic field interacts with electrons and induces an electron cyclotron frequency \( \Omega_{cs} = -e \mathbf{B}_s / m_e c \). Taking the approximation \( \nabla \times (\mathbf{k} |\mathbf{E}_1|^2) \approx k |\mathbf{E}_1|^2 / L \), where \( L \) is the scale length of \( |\mathbf{E}_1|^2 \) [7, 8]. The induced electron cyclotron frequency is given by

\[
\Omega_{cs} = -\frac{\omega_p^2 k}{2\omega^2 L} V_0, \tag{13}
\]

where \( V_0 = \epsilon |\mathbf{E}_1| / m_e \omega \) is the electron quiver velocity. Notice that when magnetic monopoles are neglected, \( \Omega_{cs} = 0 \) and there is no magnetization.

### IV. Conclusions

Starting from the inclusion of magnetic charges in Maxwell’s equations, we have studied the simple problem of propagation of electromagnetic waves in a cold electron plasma. We have shown that when magnetic
monopoles are introduced in an electron plasma, the dispersion relation \( [5] \), and thereby the propagation of electromagnetic waves, is modified because of the magnetic monopole plasma frequency \( [7] \).

Due to the presence of the magnetic monopoles, the plasma is magnetized with a magnetic field given by Eq. \( [12] \), which is of \( y^2 \) order. However, it can interact with the electrons inducing an electron cyclotron frequency. The frequency \( [13] \) is only due to the ponderomotive force related to time variation of the field intensity and which is induced by magnetic charges. It depends on the magnetic monopole plasma frequency \( \omega_m \) and on the propagation mode \( (8) \) through \( \omega \). Finally, notice that the magnetization and the cyclotron motion decrease when the frequency of the electromagnetic wave increase.

The above calculations of the dispersion relation and of the plasma magnetization are very simple. Therefore, they are useful as a pedagogical tool for teaching the recognition of basic phenomena arising of magnetic charges in plasmas, and of how these simple effects can bring new insights of the electron plasma dynamics at classical level.

**Acknowledgments**

P. S. M. thanks to CONICyT, Chile Doctoral Fellowship for their financial support.

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