Evolutions and Calibrations of Long Gamma-Ray-burst Luminosity Correlations Revisited

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Abstract

Luminosity correlations of long gamma-ray bursts (GRBs) are extensively proposed as an effective complementarity to trace the Hubble diagram of the universe at high redshifts, which is of great importance to explore properties of dark energy. Recently, several empirical luminosity correlations have been statistically proposed from GRB observations. However, to treat a GRB as the distance indicator, there are two key issues: the redshift evolution of luminosity correlations and their calibrations. In this paper, we choose the Amati relation, the correlation between the peak spectra energy and the equivalent isotropic energy of GRBs \(E_p - E_{iso}\) as an example and find that the current GRB data set implies that there could be a evolution of the luminosity correlation with respect to the redshift. Therefore, we propose an extended Amati relation with two extra redshift-dependent terms to correct the redshift evolution of the GRB relation. Second, we carefully check the reliability of the calibration method using the low-redshift GRB data. Importantly, we find that the low-redshift calibration method does not take whole correlations between \(\Omega_m\) and coefficients into account. Neglecting this correlation information can break the degeneracies and obtain the biased constraint on \(\Omega_m\), which is very sensitive to values of parameters for the calibration. A small shift in the parameters of the “calibrated” relation could significantly change the final constraint on \(\Omega_m\) in the low-redshift calibration method. Finally, we simulate several GRB samples with different statistical errors and find that, in order to correctly recover the fiducial value of \(\Omega_m\), using the low-redshift calibration method, we need a large number of GRB samples with high precisions.

Key words: cosmological parameters – gamma-ray burst: general

1. Introduction

Since the accelerating expansion of the universe was revealed by observations of type Ia supernovae (SNe; Riess et al. 1998; Perlmutter et al. 1999), SN plays an essential role in the study of the expansion history of the universe and the nature of dark energy. However, due to the limited intrinsic luminosity and the extinction from the interstellar medium, the maximum redshift of the SN we can currently observe is about 1.7. For most other popular probes at low redshift, such as baryon acoustic oscillations (BAO) and weak lensing, the bottleneck of the baryon acoustic oscillations \(\sim 2\). Whereas fluctuations of the cosmic microwave background (CMB) provide cosmological information at the last scattering surface \((z \sim 1089)\). Therefore, we do not have many effective methods to observe the evolution of our universe at \(2 < z < 1089\), usually referred to as the “cosmological desert.”

Gamma-ray bursts (GRBs) are the most energetic explosions after the big bang in the universe. The equivalent isotropic energy radiated in a few seconds can reach up to \(10^{48} \sim 10^{53}\) erg (see, e.g., Piran 1999; Mészáros 2002, 2006; Kumar & Zhang 2015 for recent reviews). Thanks to the extreme brightness and the immunity to dust extinction of high-energy photons in the gamma-ray band, GRBs are detectable up to a redshift of \(z \sim 10\) (Salvaterra et al. 2009; Tanvir et al. 2009; Cucchiara et al. 2011; Tanvir 2013). Therefore, in the literature, GRBs are widely proposed as complementary data of SNe, to trace the Hubble diagram of the universe at high redshifts. However, different from the consistent luminosities of SNe, which allow us to qualify an SN as an ideal distance indicator, the central engine mechanism of explosions of GRBs has not yet been clearly understood. Therefore, drawing cosmological implications from GRB observations is quite intractable. Recently, many methods have been proposed to achieve some progress, in order to treat the GRB as the distance indicator (Schaefer 2002; Dai et al. 2004; Ghirlanda et al. 2004b; Firmani et al. 2005; Liang & Zhang 2005; Schaefer 2007; Li et al. 2008, 2015; Liang et al. 2008; Wei & Zhang 2009; Schaefer 2002; Yonetoku et al. 2004; the correlation between peak energy and isotropic peak luminosity (\(\tau_{lag} - L\); Norris et al. 2000); the correlation between time variability and isotropic peak luminosity (\(V - L\); Fenimore & Ramirez-Ruiz 2000); the correlation between the peak energy of \(dP_t\) spectrum and isotropic equivalent energy \(E_p - E_{iso}\); and the Amati relation; Amati et al. 2002); the correlation between peak energy and collimation-corrected energy \(E_p - E_c\); Ghirlanda et al. 2004a); the correlation between peak energy and isotropic peak luminosity \(E_p - L\); Schaefer 2002; Yonetoku et al. 2004; the correlation between the minimum rise time of light curve and isotropic peak luminosity \(\tau_{RT} P - L\); Schaefer 2007); and some well-known multi-variable relations, such as the correlation between \(E_{iso}, E_p\), and the break time of the optical afterglow light curves \(t_b\) (Liang & Zhang 2005). The general
expression for luminosity correlations is

\[ y = a + bx, \]  

(1)

where \( x = \log E_p \) (or \( \log V, \log \eta_{bkg}, \log \eta_{RT} \)) and \( y = \log L \) (or \( \log E_{iso}, \log E_o \)). Determinations of coefficients \( a \) and \( b \) of correlations could tell us whether these empirical luminosity correlations are independent of the redshift, which is crucial for treating a GRB as the distance indicator to investigate the evolution of our universe. In addition, some new models for GRBs have been proposed in recent years, such as the spectro-temporal multi-component model (Guiriec et al. 2011, 2013, 2015a, 2015b, 2016a, 2016b). Interestingly, this model could result in a new time-resolved relation: the relation between the luminosity of the non-thermal component, \( L_{nTH} \), and its corresponding \( \nu_F \) spectral peak energy in the rest-frame, \( E_{peak,i}^{NT, rest} \) (i.e., \( E_{i}^{nTH} = E_{peak,i}^{NT, rest} \) relation).

More importantly, determinations of coefficients can also show the reliability of the calibration of GRB luminosity correlations directly. In the literature, there are two methods to calibrate the luminosity correlations: the “self-calibration” method, which is based on the global fitting technique using all GRB data to constrain the cosmological parameters and the coefficients of luminosity correlations simultaneously (Schaefer 2002; Li et al. 2008), and the “low-redshift calibration” method, which uses the low-redshift GRB data to obtain the constraints on coefficients of luminosity correlations and directly extrapolate them to high redshifts to constrain the cosmological parameters using the high-redshift GRB data (Liang et al. 2008; Wei & Zhang 2009; Wei 2010; Ding et al. 2015; Liu & Wei 2015; Lin et al. 2016; Wang et al. 2016).

Apparently, these two methods are seriously dependent on the assumption that empirical luminosity correlations are universal and do not evolve with respect to redshift. In practice, for the Amati relation, Li (2007) first revealed that the systematically significant variations of the intercept \( a \) and the slope \( b \) with the cosmological redshift. Later, Basilakos & Perivolaropoulos (2008) investigated the same issue for several other luminosity correlations. Due to the limited quality of GRB samples, there was no statistically significant evidence for the evolution of the calibration parameters. In addition, as a cosmological probe, the evolution of GRB luminosity correlations may result in the overestimate or underestimate of cosmological parameters (Dainotti et al. 2013). More recently, in Lin et al. (2015), the Amati relation was reinvestigated using low-redshift \((z < 1.4)\) and high-redshift \((z > 1.4)\) GRBs, and it was found that the coefficients of the Amati relation in low-redshift GRBs differs from those of high-redshift GRBs at a more than 3\( \sigma \) confidence level. Therefore, they finally concluded that long GRBs might not be standard candles. Following this topic, Lin et al. (2016) further examined the possible redshift dependence of several other luminosity correlations and found that only for the \( E_p-E_\nu \) relation, the low-redshift GRB could give the similar constraints on the coefficients with those from the high-redshift GRB within a 1\( \sigma \) confidence level.

In this paper, we choose the Amati relation as an example to investigate the evolutions and calibrations of GRB luminosity correlation with the latest GRB observational data (Liu & Wei 2015). First, we divide the whole GRB sample into two bins \((z < 1.4, \text{and} \ z > 1.4)\). In each bin, we use the GRB data to constrain the coefficients in the \( \Lambda \)CDM framework to check whether they are independent of the redshift. Then, we propose an extended Amati relation by introducing two extra terms to characterize the evolutions with respect to the redshift. Second, we carefully check the reliability of the calibration method. In practice, we use the low-redshift GRB data \((z < 1.4)\) to calibrate the coefficients of the GRB luminosity correlation and directly extrapolate them to high-redshift events. Then we use this “calibrated” Amati relation to constrain the matter density parameter \( \Omega_m \) with the high-redshift GRB data. Finally, we simulate several samples of GRB data with high precision to investigate whether these empirical luminosity correlations and the redshift are independent of the redshift. The reason for choosing \( z = 1.4 \) as the threshold is that the maximal redshift of SNe Ia data samples we are using for cosmological studies is usually around \( z = 1.4 \). The universe below \( z = 1.4 \) has already been well studied by using the SNe
then we constrain the luminosity correlation coefficients $a$ and $b$ by maximizing the D’Agostini’s likelihood (D’Agostini 2005):
\[
\mathcal{L}_D(a, b, \sigma_{\text{int}}) \propto \prod_i \frac{1}{\sqrt{\sigma_{\text{int}}^2 + \sigma_y^2 + b^2 \sigma_x^2}} \\
\times \exp \left[ -\frac{(y_i - a - bx_i)^2}{2(\sigma_{\text{int}}^2 + \sigma_y^2 + b^2 \sigma_x^2)} \right],
\]
\[
\chi^2_D(a, b, \sigma_{\text{int}}) = \sum_i \ln(\sigma_{\text{int}}^2 + \sigma_y^2 + b^2 \sigma_x^2) \\
+ \sum_i \frac{(y_i - a - bx_i)^2}{\sigma_{\text{int}}^2 + \sigma_y^2 + b^2 \sigma_x^2},
\]
where $\sigma_{\text{int}}$ is the intrinsic scatter, which represents any other unknown uncertainties except for the observational statistical ones. Here, in order to test the feasibility of D’Agostini’s likelihood, we constrain the two parameters of the Amati relation using all GRB data (138 GRB) in the flat $\Lambda$CDM framework first. The minimum chi-squared value is $\chi^2_{\text{min}} = 134.91$ and $\chi^2_{\text{min}} / \text{dof} = 0.98$. We also plot the comparison between the GRB data and the theoretical prediction of the best-fit values in Figure 1. All of these results show that the D’Agostinis’ likelihood is feasible to constrain parameters. Finally, after getting constraints on $a$ and $b$, in Figure 2, we plot the two-dimensional marginalized constraints in the plane of $(a, b)$ from the low-redshift (blue contours) and high-redshift (red contours) GRB data, respectively.

In our analysis, we use emcee,\footnote{https://pypi.python.org/pypi/emcee} introduced by Foreman-Mackey et al. (2013), a Python module that uses the Markov chain Monte Carlo (MCMC) method to get the best-fit values and their uncertainties of parameters $a$, $b$, and $\sigma_{\text{int}}$ by generating sample points of the probability distribution. The best-fit values with 1$\sigma$ errors are shown in Table 1. We also plot the results in Figure 2 (the upper right panel), in which the blue and red contours denote the results on the parameters of Amati relation from the low-redshift and high-redshift GRB subsamples, respectively. Apparently, these two constraints do not match very well. The best-fit values are more than $3\sigma$ away. When considering the uncertainties, the tension between two fitting results is less than a $2\sigma$ confidence level, which implies that the parameters of the Amati relation might be redshift-dependent. The result is similar to those in some other works (Lin et al. 2015).

2.2. Redshift Evolution of the Amati Relation

Because the statistically obvious evolution of the luminosity correlation has been revealed, we should be careful to use the GRB data as the distance indicator for cosmological studies. Because all of these investigations are based on the assumption that the luminosity correlation does not vary with respect to the redshift, i.e., both $a$ and $b$ in Equation (2) should be constant. To solve this problem, inspired by the study for the evolution of light-curve fitting parameters in SN observations (Wang & Wang 2013; Wang et al. 2014) and that for the evolution of the GRB intrinsic luminosity (Yu et al. 2015), we introduce two extra redshift-dependent terms to characterize the redshift evolution of the luminosity correlation. Since most redshifts of GRB are much greater than unity, rather than the linear expression in Wang & Wang (2013) and Wang et al. (2014) and the logarithmic parametrization in Yu et al. (2015), we...
Constraints on the Intrinsic Scatters

The Amati relation: high-z Extended low-z

Note.

Relation Bins Amati relation: high-z

Amati relation:

extended Amati relation

Global Fit

Global Fit

In the low-redshift calibration method, we obtain the luminosity distance of the low-redshift GRB sample (z < 1.4) by using Equation (4). Then we calibrate the luminosity correlation by maximizing the D’Agostini’s likelihood (Equation (5)) for the 59 low-redshift GRBs. The constraints on the luminosity correlation coefficients are shown in Table 1. Then we directly extrapolate these coefficients to high redshifts. Therefore, we construct the Hubble diagram for the 79 high-redshift GRBs based on the calibrated luminosity correlation and investigate cosmological implication from this Hubble diagram in the context of standard ΛCDM model. In this framework, the luminosity distance is determined by Equation (4), and the corresponding distance modulus is

\[ \mu(z) = 5 \log \frac{d_L(z)}{Mpc} + 25. \]

We use the numerical method to fit Ω_m and the corresponding \( \chi^2 \) is

\[ \chi^2 = \sum_i \frac{[\mu_{obs}(z_i) - \mu_{th}(z_i; \Omega_m)]^2}{\sigma_{\mu,i}^2}. \]

where \( \sigma_{\mu,i} \) is the total uncertainty of distance modulus for the \( i \)th GRB and it is propagated from the uncertainties of \( S_{bol} \), \( E_{p,i} \), \( E_{iso} \)

\[ \sigma_{\mu,i}^2 = \left( \frac{5}{2} \right) \left[ \sigma_{E_{iso}}^2 + \left( \frac{\sigma_{S_{bol}}}{S_{bol} \ln 10} \right)^2 \right]. \]

where

\[ \sigma_{\log E_{iso}}^2 = \sigma_a^2 + \left( \frac{z \sigma_a}{1 + z} \right)^2 + \left( \frac{\sigma_b \log \frac{E_{p,i}}{300 \text{ keV}}}{1 + z} \right)^2 + \left( \frac{z \sigma_b}{1 + z} \cdot \log \frac{E_{p,i}}{300 \text{ keV}} \right)^2 + \sigma_{\text{int}}^2. \]

First, we consider the original Amati relation. In the upper right panel of Figure 3 (red dashed line), we show the one-dimensional distribution of the matter density parameter \( \Omega_m \) obtained from the 79 high-redshift GRB data by using the low-redshift calibration method. If we use the best-fit values \( a = 52.75 \) and \( b = 1.60 \) to calibrate the high-redshift GRB data, we obtain the 68% C.L. constraint on the matter density

### Table 1

| Relation | Bins     | \( \sigma_{\text{int}} \) | \( N \)   | \( \chi_{\text{min}} \) | \( a \)     | \( b \)     | \( \alpha \) | \( \beta \) |
|----------|----------|---------------------------|----------|--------------------------|------------|------------|-------------|------------|
| low-z    | 0.40 ± 0.05 | 59                        | 55.51    | 52.75 ± 0.06              | 1.60 ± 0.10 | ...        | ...         | ...        |
| high-z   | 0.32 ± 0.04 | 79                        | 76.18    | 52.95 ± 0.05              | 1.30 ± 0.12 | ...        | ...         | ...        |
| Global Fit | 0.35 ± 0.03 | 138                      | 134.83   | 52.62 ± 0.07              | 1.49 ± 0.07 | ...        | ...         | ...        |
| low-z    | 0.40 ± 0.05 | 59                        | 55.15    | 52.42 ± 0.35              | 1.46 ± 0.29 | 0.72 ± 0.75 | 0.14 ± 0.64 | ...        |
| high-z   | 0.32 ± 0.04 | 79                        | 74.06    | 53.42 ± 0.46              | 0.08 ± 1.01 | -0.67 ± 0.64 | 1.73 ± 1.42 | ...        |
| Global Fit | 0.35 ± 0.03 | 138                      | 134.65   | 52.52 ± 0.16              | 1.59 ± 0.18 | 0.33 ± 0.40 | -0.31 ± 0.32 | ...        |

Note. For comparison, we also show the constraints using the global fitting method.

select two mild formulas

\[ a \rightarrow A = a + \alpha \frac{z}{1 + z}, \quad b \rightarrow B = b + \beta \frac{z}{1 + z} \]

to avoid extreme results at high redshifts. Therefore, the extended Amati relation can be expressed as

\[ \log \frac{E_{iso}}{\text{erg}} = \left( a + \frac{\alpha z}{1 + z} \right) + \left( b + \frac{\beta z}{1 + z} \right) \log \frac{E_{p,i}}{300 \text{ keV}}, \]

where \( a, b, \alpha, \) and \( \beta \) are four new coefficients in the relation, which should be obtained from the GRB data.

Following the same procedures for analyzing the evolutions of \( a \) and \( b \) presented in the Section 2.1, we also investigate the evolutions of the luminosity correlation coefficients \( a, b, \alpha, \) and \( \beta \) using the GRB sample in the same low-redshift and high-redshift subsamples, respectively. The best-fit values with 1σ errors are shown in Table 1. Figure 2 (the lower left six panels) shows the results regarding evolutions of luminosity correlation coefficients \( a, b, \alpha, \) and \( \beta \). Since we have two parameters to describe the redshift dependence, compared with the result of original Amati relation, the tension between constraints of parameters from low-z and high-z GRB data are alleviated, most of which are consistent within the 1σ confidence level. Of course, these two extra free parameters could bring the large uncertainties and make the constraints weaker.

3. Calibrations of Luminosity Correlations

3.1. Low-redshift Calibration

As shown in Section 2.2, the tension between the constraints on parameters of the original Amati relation from the low-redshift and high-redshift GRB subsamples is more than the 1σ confidence level, while the constraints are consistent within the 1σ confidence level for the extended Amati relation. However, this does not mean that we can safely use the GRB data for cosmological studies. We need to check the calibration of GRB luminosity correlations further. Usually, we have two calibration methods. One is to calibrate luminosity correlations with low-redshift GRB samples in the context of the flat ΛCDM model and then extrapolate these obtained coefficients to the high-redshift range. The other method is the global fitting analysis using the MCMC technique in which luminosity correlation parameters and cosmological parameters are simultaneously fitted on the same weight in the context of the flat ΛCDM model (Li et al. 2008). Therefore, here we use these two methods to calibrate the extended Amati relation and compare the constraints on the cosmological parameters.
Based on this result, we can see that this obtained constraint on $W_m$ is far away from the fiducial value of $W_m = 0.308$, which is used to constrain the parameters $a$ and $b$ from the low-$z$ GRB data for calibration. The significance of this inconsistence is at about a $2\sigma$ confidence level.

Then, we use the extended Amati relation to do the calculations, following the same procedures. In the lower right panel of Figure 3 (red dashed line), we also show the one-dimensional distribution of $W_m$ obtained from the 79 high-redshift GRB data by using the low-redshift calibration method. The obtained constraint on $W_m$ is

$$W_m = 0.24 \pm 0.12,$$

at a 68% confidence level, which is consistent with previous works (Wang et al. 2016). This result is very consistent with the fiducial value of the matter density parameter, which also implies that the extended Amati relation might be better than the original one.

### 3.2. Global Fitting Method

The second method is the global fitting method, in which we calculate the matter density parameter $W_m$ and luminosity correlation coefficients $(a, b, \alpha, \beta)$ simultaneously with the same weight. The constraints on these coefficients from all-redshift GRB data are also listed in Table 1.

Similar to the discussion in Section 2, we start with the original Amati relation. In Table 1 and Figure 4, we show that constraints on the parameters of the Amati relation from the low-$z$ GRB data in the low-redshift calibration method and from all GRB data in the global fitting method are more or less consistent within a $1\sigma$ confidence level. As we know, the constraint on the matter density parameter is strongly correlated with these coefficients of the GRB relation. Therefore, the constraint on $W_m$ using the global fitting method is quite different from that using the low-redshift...
acquisition from the low-redshift GRB data alone in the contours with the crossing values.

The points represent the best-fitting parameters of the Amati relation could ease the tension of constraints on the matter density parameter. This means that the low-redshift calibration method gives the strongly biased and overestimated constraint on the matter density parameter, which cannot be trusted.

In the two-dimensional contour (lower left panel) of Figure 3, we add a vertical line, which denotes the best-fit value of $\alpha = 0.72$ obtained from the low-redshift GRB data alone in the low-redshift calibration method. This line crosses the $1\sigma$ and $2\sigma$ contours with the crossing values $0.11 \leq \Omega_m \leq 0.36$ and $0.05 \leq \Omega_m \leq 0.76$. We find that these values are quite similar to the $1\sigma$ and $2\sigma$, lower and upper limits, of the constraint on the matter density parameter by using the low-redshift calibration method (Equation (13)). This analysis implies that in the low-redshift calibration method, the strong correlations between $\Omega_m$ and coefficients of the extended Amati correlation are not fully taken into account in the calculation. Neglecting this information could break the strong degeneracies between $\Omega_m$ and correlation coefficients by force and then give a quite stringent constraint on the matter density parameter.

Furthermore, in the low-redshift calibration method, we find that the final constraint on $\Omega_m$ is very sensitive to how the relation is calibrated. Here, we assume three sets of parameters to calibrate the extended Amati relation:

(a) the best-fit values obtained from the low-z GRB data;
(b) the best-fit values obtained from all GRB data;
(c) $a = 52.40$, $b = 1.45$, $\alpha = 0.10$, and $\beta = 0.14$, which is quite similar to the first case (a), except for the $\alpha$.

Based on Table 1, we can easily know that these three cases are consistent with each other at the $1\sigma$ confidence level. Now we use the low-redshift calibration method to constrain $\Omega_m$, which is shown in the lower right panel of Figure 3 ((a) the red dashed line, (b) green dashed–dotted line, and (c) black dotted line, respectively). Case (a) is the standard case in the low-redshift calibration method and obtains the stringent constraint on $\Omega_m$ (see Equation (13)). However, if we use case (b) to calibrate the relation, the obtained constraint on the matter density parameter is quite different from that in case (a):

$$\Omega_m = 0.58 \pm 0.12 \text{ (68\% C.L.)}.$$  

The small shift of the parameters of the “calibrated” relation could significantly change the final constraint on $\Omega_m$ in the low-redshift calibration method, which implies that the constraint on $\Omega_m$ is very sensitive to the values of parameters for the calibration, when we do not fully take the strong correlations among parameters into account and underestimate the statistical errors. When we use case (c), the situation becomes worse. The obtained constraint on $\Omega_m$ is weaker and only has a lower limit of $\Omega_m > 0.71$ at a 95% confidence level.

We also perform a similar check in the original Amati relation. The constraints on the matter density parameter using cases (a) and (b) to calibrate the relation are shown in the upper right panel of Figure 3 ((a) the red dashed line and (b) green dashed–dotted line, respectively). We obtain the same conclusion that the constraint on $\Omega_m$ is very sensitive to the values of parameters for the calibration in the low-redshift calibration method.
3.3. Simulation Results

For the time being, the number of GRB data we can use for possible cosmological studies are very small, when comparing with various SNe Ia data sets. There are still very large intrinsic scatter and large statistical errors on data in the current GRB data set, as we discuss above. In order to check how large the biased constraint on the matter density parameter is, using the low-redshift calibration method, we simulate the mock GRB data with high precisions.

To get the simulation data of GRBs, we first plot the distribution of the redshift $z$ and the observed quantity peak energy $E_p$ of the 138 GRBs sample (Liu & Wei 2015) in Figure 5 with the blue histogram. Then we find two distribution functions to trace the histogram of redshift $z$ and $E_p$. For the redshift $z$, we assume that the distribution function has the form

$$p(z) \propto z e^{-z}. \quad (16)$$

We also assume that the distribution of peak energy $E_p$ is gamma distribution

$$p(x; \alpha, \lambda) = \frac{x^\alpha}{\Gamma(\alpha)} e^{-\lambda x}, \quad (17)$$

where $\alpha$ and $\lambda$ are parameters and the gamma function is

$$\Gamma(\alpha) = \int_0^\infty e^{-t} t^{\alpha-1} \, dt. \quad (18)$$

We put the distribution function of $E_p$ in Figure 5 (right panel) in the form of

$$p(E_p/100) \propto \frac{1}{\Gamma(\alpha)} (E_p/100)^\alpha e^{-E_p/100}. \quad (19)$$

We also use the distribution of $E_p,i$ to generate the mock GRB data, and find that the obtained constraints on the parameters of relation and the matter density parameter are consistent with that by using the distribution of $E_p$.

We use the fiducial values of the extended Amati relation, $a = 52.52$, $b = 1.59$, $\alpha = 0.33$, $\beta = -0.31$, obtained from all-redshift GRB data by using the global fitting calibration method in the flat $\Lambda$CDM framework. (Different choices of fiducial values do not affect the final results.) Based on these two distribution functions, Equations (16) and (19), we could obtain the simulation data for both redshift $z$ and peak energy $E_p$, and then get the simulation data of bolometric fluence $S_{bol}$ according to Equation (8). For the real GRB data, we find that the uncertainties of $E_p$ and $S_{bol}$ have the mean relative error of 28% and 18%, respectively (Liu & Wei 2015). Therefore, here we assume the simulation data of $E_p$ and $S_{bol}$ have the same relative errors. Furthermore, in the process of the simulation, we also include the information of the intrinsic scatter and choose $\sigma_{ \text{int}} = 0.355$ obtained from the calibration method using all current GRB data. Finally, the $\chi^2$ equation becomes

$$\chi^2(a, b, \alpha, \beta) = \sum_i \frac{(y_i - A - Bx_i)^2}{\sigma_i^2 + B^2 \sigma_{\text{int}}^2}. \quad (20)$$

where $\sigma_i = \sqrt{\sigma_{\text{int}}^2 + \sigma_y^2}$, which includes the intrinsic scatter.

First, we use the current precision ($\sigma_{E_p}, \sigma_{S_{bol}}$) to simulate the 138 GRB data to constrain the coefficients of the extended Amati relation and obtain the similar best-fit values and error bars of coefficients with those from the current real GRB data, which means that we could safely use this mock data for further calculations.

Next, we use the low-redshift calibration method to constrain the matter density parameter from this mock GRB data. In practice, we select the low-redshift GRB samples with $z < 1.4$ to constrain those four coefficients of the extended Amati relation. The results are shown in Figure 6. The obtained best-fit values perfectly recover the fiducial ones of parameters of the GRB relation. In Section 3.2, we have proven that a small shift of the parameters of the “calibrated” relation could significantly change the final constraint on $\Omega_m$ from the real GRB data in the low-redshift calibration method. Therefore, here we want to check this using the mock GRB data. We assume the values of parameters (52.805, 2.408, $-0.020$, $-1.939$), which show a small shift from the best-fit fiducial values and safely lay in the $1\sigma$ contour (red points in Figure 6), to calibrate the extended Amati relation. Then we use the mock high-redshift GRB data to constrain the matter density parameter, based on this “calibrated” relation, and obtain the
68% C.L. constraint:
\[ \Omega_m = 0.65 \pm 0.15, \]
which is far away from the fiducial value \( \Omega_m = 0.308 \). This result tells us that even a small shift of coefficients in the 1\( \sigma \) contour, the constraint on the matter density parameter with the current precision of GRB data could still be significantly biased using the low-redshift calibration method. The constraint on \( \Omega_m \) is very sensitive to the values of parameters for the calibration, when we do not fully take the strong correlations among parameters into account and underestimate the statistical errors in the low-redshift calibration method.

In Figure 7, we show the obtained biased values of the matter density parameter from the mock GRB data with different precisions using the low-redshift calibration method. We can see that when GRB data have larger samples with the current precision or the relative errors of GRB data decreases, \( \Omega_m \) will be more and more close to the fiducial value.

4. Conclusions and Discussions

Thanks to the extremely high power of the explosion, GRBs are proposed as promising candidates to trace the Hubble diagram of the universe in the high-redshift range. In recent years, several popular luminosity correlations have been statistically concluded from GRB observations. People have also made great efforts to standardize them as cosmological distance indicators. In general, there are two key issues extensively discussed in the literature when we use these correlations for cosmology. The first issue is whether luminosity correlations evolve with redshift. The other one is which methods used to calibrate luminosity correlations work most effectively. In this paper, we take the Amati relation as an example and use the current GRB data in the redshift range \([0.0331, 8.1]\) to investigate these two issues. Here we summarize our main conclusions in more detail.

1. We divide the whole GRB data into two redshift bins and constrain the coefficients \( a \) and \( b \) of the original Amati relation in each bin, respectively. Based on the MCMC method, we find that the constraints on both the intercept \( a \) and the slope \( b \) are quite different from the low-redshift and high-redshift GRB data, respectively. The tension is at more than a 3\( \sigma \) level, which implies that the parameters of the original Amati relation could be redshift-dependent.

2. We introduce two extra redshift-dependent terms to characterize the redshift evolution of the luminosity correlation. Interestingly, we find that the tension between constraints of parameters from low-z and high-z GRB data are alleviated. This extended Amati relation is more or less helpful to correct the redshift evolution of GRB relation.

3. Besides the evolution of the Amati correlation, the calibration is also very important for using GRB data in cosmological studies. We first check the constraint on the matter density parameter using the low-redshift calibration method and obtain \( \Omega_m = 0.24 \pm 0.12 \) (68% C.L.), which is consistent with other works.

4. However, we could also use the global fitting method to calibrate the GRB data, in which we use all the GRB data to constrain coefficients of the extended Amati relation and the cosmological parameters simultaneously. In this case, we find that, due to the current poor precision of the GRB data, the constraint on \( \Omega_m \) is very weak, which is quite different from that by using the low-redshift calibration method.

5. After our careful checks, we find that the low-redshift calibration method does not take all of the correlations between \( \Omega_m \) and coefficients into account. Neglecting the correlation information can break the degeneracies between \( \Omega_m \) and coefficients. Therefore, the obtained constraint on \( \Omega_m \) is totally biased. A small shift in the parameters of the “calibrated” relation could significantly change the final constraint on \( \Omega_m \) in the low-redshift calibration method, which implies that the constraint on
\( \Omega_m \) is very sensitive to the values of parameters for the calibration.

6. In order to investigate the large biased constraint on \( \Omega_m \), using the low-redshift calibration, we simulate the mock GRB data with different precisions. We find that the mock data with current precision will give a significantly biased result using the low-redshift calibration method, even we assume a small shift in the parameters of the "calibrated" relation. When GRB data include larger samples with the current precision or the relative errors of GRB data decrease, the constraint on \( \Omega_m \) is more and more close to the fiducial value.

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References

Ade, P. A. R., Aghanim, N., Arnaud, M., et al. 2016, A&A, 594, A13
Amanullah, R., Lidman, C., Rubin, D., et al. 2010, ApJ, 716, 712
Amati, L., Frontera, F., Tavani, M., et al. 2002, A&A, 390, 81
Band, D., Matteson, J., Ford, L., et al. 1993, ApJ, 413, 281
Basilakos, S., & Perivolaropoulos, L. 2008, MNRAS, 391, 411
Betoule, M., Kessler, R., Guy, J., et al. 2014, A&A, 568, A22
Cucchiara, A., Levan, A. J., Fox, D. B., et al. 2011, ApJ, 736, 7
D’Agostini, G. 2005, arXiv:physics/0511182
Dai, Z. G., Liang, E. W., & Xu, D. 2004, ApJL, 612, L101
Dainotti, M. G., Cardone, V. F., Piedipalumbo, E., & Capozziello, S. 2013, MNRAS, 436, 82
Ding, X. H., Li, Z. X., & Zhu, Z. H. 2015, JMPD, 24, 7
Fenimore, E. E., & Ramirez-Ruiz, E. 2000, arXiv:astro-ph/0004176
Firmani, C., Avila-Reese, V., & Ghirlanda, G. 2006, MNRAS, 372, L28
Firmani, C., Ghisellini, G., Ghirlanda, G., & Avila-Reese, V. 2005, MNRAS, 360, L1
Foreman-Mackey, D., Hogg, D. W., Lang, D., & Goodman, J. 2013, PASP, 125, 925
Ghirlanda, G., Ghisellini, G., & Firmani, C. 2006, NJPh, 8, 123
Ghirlanda, G., Ghisellini, G., & Lazzati, D. 2004a, ApJ, 616, 331
Ghirlanda, G., Ghisellini, G., Lazzati, D., & Firmani, C. 2004b, ApJL, 613, L13
Guiriec, S., Connaughton, V., Briggs, M. S., et al. 2011, ApJL, 727, L33
Guiriec, S., Daigne, F., Hascoët, R., et al. 2013, ApJ, 770, 32
Guiriec, S., Gonzalez, M. M., Sacahui, J. R., et al. 2016a, ApJ, 819, 79
Guiriec, S., Kouveliotou, C., Asano, K., et al. 2016b, ApJL, 831, L8
Guiriec, S., Kouveliotou, C., Daigne, F., et al. 2015a, ApJ, 807, 148
Guiriec, S., Mochkovitch, R., Piran, T., et al. 2015b, ApJ, 814, 10
Kumar, P., & Zhang, B. 2015, PRe, 561, 1
Li, H., Xia, J. Q., Liu, J., et al. 2008, ApJ, 680, 92
Li, L. X. 2007, MNRAS, 379, L55
Li, Z., Ding, X., & Zhu, Z. H. 2015, PhRvD, 91, 083010
Liang, E. W., & Zhang, B. 2005, ApJ, 633, 611
Liang, N., Xiao, W. K., Liu, Y., & Zhang, S. N. 2008, ApJ, 685, 354
Lin, H. N., Li, X., & Chang, Z. 2016, MNRAS, 459, 2501
Lin, H.-N., Li, X., Wang, S., & Chang, Z. 2015, MNRAS, 453, L128
Liu, J., & Wei, H. 2015, GReGr, 17, 141
Mészáros, P. 2002, ARA&A, 40, 137
Mészáros, P. 2006, RPPh, 69, 2259
Norris, J. P., Marani, G. F., & Bonnell, J. T. 2000, ApJ, 534, 248
Perlmutter, S., Aldering, G., Goldhaber, G., et al. 1999, ApJ, 517, 565
Piran, T. 1999, PhR, 314, 575
Riess, A. G., Filippenko, A. V., Challis, P., et al. 1998, AJ, 116, 1009
Salvaterra, R., Delia Valle, M., Campana, S., et al. 2009, Natur, 461, 1258
Schaefer, B. E. 2002, ApJL, 583, L67
Schaefer, B. E. 2007, ApJL, 660, 16
Suzuki, N., Rubin, D., Lidman, C., et al. 2012, ApJ, 746, 85
Tanvir, N. R. 2013, arXiv:1307.6156
Tanvir, N. R., Fox, D. B., Levan, A. J., et al. 2009, Natur, 461, 1254
Wang, F. Y., Dai, Z. G., & Liang, E. W. 2015, NewAR, 67, 1
Wang, F. Y., Qi, S., & Dai, Z. G. 2011, MNRAS, 415, 3423
Wang, J. S., Wang, F. Y., Cheng, K. S., & Dai, Z. G. 2016, A&A, 585, A68
Wang, S., Li, Y. H., & Zhang, X. 2014, PhRvD, 89, 063524
Wang, S., & Wang, Y. 2013, PhRvD, 88, 043511
Wei, H. 2010, JCAP, 08, 020
Wei, H., & Zhang, S. N. 2009, EPJC, 63, 139
Yonetoku, D., Murakami, T., Nakamura, T., et al. 2004, ApJ, 609, 935
Yu, H., Wang, F. Y., Dai, Z. G., & Cheng, K. S. 2015, ApJS, 218, 13