Effect of Link Flexibility on tip position of a single link robotic arm

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Abstract. The flexible robots are widely used in space applications due to their quick response, lower energy consumption, lower overall mass and operation at high speed compared to conventional industrial rigid link robots. These robots are inherently flexible, so that the kinematics of flexible robots can’t be solved with rigid body assumptions. The flexibility in links and joints affects end-point positioning accuracy of the robot. It is important to model the link kinematics with precision which in turn simplifies modelling of dynamics of flexible robots. The main objective of this paper is to evaluate the effect of link flexibility on a tip position of a single link robotic arm for a given motion. The joint is assumed to be rigid and only link flexibility is considered. The kinematics of flexible link problem is evaluated by Assumed Modes Method (AMM) using MATLAB Programming. To evaluate the effect of link flexibility (with and without payload) of robotic arm, the normalized tip deviation is found for flexible link with respect to a rigid link. Finally, the limiting inertia for payload mass is found if the allowable tip deviation is 5%.

1. Introduction
The Conventional rigid-link robots are widely used in industrial automation. However, to obtain high accuracy in the end-point position control of these robots, the weight to payload ratio of the robot must be high and the operation speed is normally quite slow. At the same time, large power supply requirements and thus considerable energy consumption is inevitable to operate these heavy-weight robots. These drawbacks greatly limit the applications of rigid-link robots. Flexible link robots have potential advantages, such as faster operation, low energy consumption, and higher load-carrying capacity for the amount of energy expended by using light-weight flexible link manipulators over rigid manipulators. The Light weighted flexible link robots, being inherently flexible, can no longer be modelled with rigid body assumptions. During the motion of such robots or due to external disturbances, the links vibrate. These vibrations must be damped out or controlled to achieve required tasks. There are two commonly used methods to discretize an infinite number of degrees of freedom systems into a finite dimensional system, Namely, Assumed Mode Method and Finite Element Method. The finite element method may require a fewer number of operations but the resulting stiffness of the link is over-estimated which intern results in unstable closed-loop responses of the original manipulator. Whereas assumed mode method is accurate with some complex mathematical formulations and consumes
less time compared to Finite Element Method. The present paper deals with estimating the deviations in the tip position of a single link robotic arm by Assumed Modes Method (AMM) using MATLAB Programming for specified joint motion (Constant Angular Velocity).

2. Literature review
Book et.al.[1] in their paper discussed about the dynamics and control of flexible manipulators. Cetinkunt, and Book [2] in their work discussed the implementation of symbolic and dynamic simulation manipulator with compliant joints and links with recursive Lagrangian-Assumed Modes formulation. The FE modelling impact on One-Link flexible Robotic Arm is illustrated in the work of Chapnik et.al. [3]. Vincentefeliu, et.al. [4] have addressed special class of single-link flexible arms, i.e. flexible mass less structures having masses concentrated at certain points of the beam.

3. Problem definition
The main objective of this paper is to evaluate the effect of link flexibility on tip position of a single link robotic arm with and without payload for a specified joint motion. Firstly tip positions are found for rigid link and using Assumed Modes Method the flexible links tip positions were found. As a case study when a joint is subjected to constant angular velocity i.e. $\theta(t)=0.5t$ radians for a simulation time of 12 sec with step size of 0.1 sec, the tip deviations with respect to rigid link were found and compared. The Assumptions made for modelling a flexible link with Assumed mode method (AMM).

- The flexible link with uniform density and flexural rigidity is considered.
- Euler-Bernoulli beam is assumed i.e. vibration of beam considered only in transverse direction.
- The deflection of the flexible link is small compared to the length of the link.
- The payload attached to the free tip of the flexible robot is a concentrated mass.

4. Research methodology
Considering a single rigid link robotic arm as shown in the Figure 1, the tip position of the link can be easily expressed by Equation 1.

\[(X_0, Y_0) = (L \cos(\theta t), L \sin(\theta t))\] (1)

Considering the flexible link robotic arm as shown in the Figure 2, the flexible link undergoes deformation in motion due to the flexibility of the link. One can observe that a point on this link
has a deviation \( y(x,t) \) from the un-deformed position. Therefore the motion of the point, related to \( y(x,t) \), is not completely determined by joint angle \( \theta \) and it can also be concluded that an infinite number of \( \theta \)'s needed to describe the motion of the entire link. This paper attempts to compute the effect of flexibility on tip position of a single robotic arm at a given time instant for a given joint motion with and without payload using Assumed Mode Method (AMM) and compute the effect of variation of payload on the tip position. Table 1 illustrates the input parameters for modeling the link. The tip position is computed using Equation 1 in MATLAB.

| Link input parameters                  | Link     |
|----------------------------------------|----------|
| Length of the link \( (L) \)          | 0.5m     |
| Width of the Link \( (w) \)           | 0.04m    |
| Depth of the cross section of link \( (d) \) | 0.002m  |
| Cross Section Area \( (A) \)          | \( w \times d \text{ m}^2 \) |
| Moment of inertia \( (I) \)           | \( (w \times d^3)/12 \text{ m}^4 \) |
| Youngs Modulus of the material \( (E) \) | \( 7 \times 10^4 \text{ N/mm}^2 \) |
| Density \( (\rho) \)                  | 2700 kg/m\(^3\) |
| Payload mass \( (M_p) \)              | 1 kg     |
| Payload Inertia \( (J_p) \)           | 1 to 5 kgm\(^2\) |
| Joint Motion \( (\theta(t)) \)        | 0.5t radians |

4.1. Calculation of Tip position with constant angular velocity

4.1.1. Case I: Motion of rigid link without payload at link tip. When the link is subjected to a joint motion, the tip of the single rigid link starts at \( X_0 = 0.5 \text{ m} \), at \( t = 0 \text{ sec} \) and reaches \( X_0 = 0.48 \text{ m} \), in 12 sec. The resultant figures in 3 to 5 provide the positions of tip of the link in x and y axis and variation of link length as the joint is moved with constant angular velocity.

4.1.2. Case II: Motion of rigid link with payload at link tip. When the link tip is provided with payload mass and subjected to above motion, as the link is assumed to be rigid there will not be any change in tip positions of the link as shown in figures 3 to 5.

4.1.3. Motion of a Flexible Link. The position of link tip is not independent of payload for joint motion of flexible link as in rigid link. This results in complex kinematics for flexible link. One of the widely used methods to solve flexible links is Assumed Mode Method (AMM) Consider a flexible link robot with body attached reference frame XOY and fixed reference frame \( X_0OY_0 \) as shown in figure 2. The different notations being used in subsequent sections are enumerated below:
When the link undergoes angular motion the tip of the link vibrates which dies down with time. Therefore the position of any point and the tip of the link can be defined as

\[ P_0(x, t) = A_R P(x, t) \]  

(2)

where \( A_R \) the rotation matrix.

\[
A_R = \begin{bmatrix}
\cos(\theta) & -\sin(\theta) \\
\sin(\theta) & \cos(\theta)
\end{bmatrix}
\]

The position vector of any point on link with body attached frame of reference is given by

\[
P(x, t) = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y(x, t) \end{bmatrix}
\]

(3)
The position vector along the length of the link depends on the lateral deformation ‘y’ of the link at that section at a given time. Value of y can be found by Assumed Mode Method (AMM). The problem of flexible link can be solved assuming it as a Euler-Bernoulli’s cantilever beam with a payload ‘M_P’ at the tip of the beam undergoing free vibration, the governing equation to represent the vibration of link can be written as follows

\[ EI \frac{\partial^4 y(x,t)}{\partial x^4} + \rho A \frac{\partial^2 y(x,t)}{\partial t^2} = 0 \]  

(4)

Boundary Conditions: Since, the equation of motion Eq.4 involves a second order derivative with time and a fourth order derivative with x, two initial conditions and four boundary conditions are needed for finding a unique solution for y(x, t) and they are given in following Equations 5 to 10

\[ y(x,t) = 0 \]  
[5]

\[ \frac{\partial y(x,t)}{\partial t} \bigg|_{t=0} = y_i \]  
[6]

\[ y(x,t) \bigg|_{x=0} = 0 \]  
[7]

\[ \frac{\partial y(x,t)}{\partial x} \bigg|_{x=0} = 0 \]  
[8]

\[ EI \frac{\partial^2 y(x,t)}{\partial x^2} \bigg|_{x=L} = -J_L \frac{\partial^2}{\partial t^2} \left( \frac{\partial y(x,t)}{\partial x} \bigg|_{x=L} \right) \]  
[9]

\[ EI \frac{\partial^3 y(x,t)}{\partial x^3} \bigg|_{x=L} = -M_L \frac{\partial^2}{\partial t^2} (y(x,t) \bigg|_{x=L}) \]  
[10]

The solution of equation 4 can be expressed as follows

\[ y(x,t) = \sum_{j=1}^{n} C_{1,j} \sin(\omega_j t) \{ (\cos(\beta_j x) - \cosh(\beta_j x)) - \alpha(\sin(\beta_j x) - \sinh(\beta_j x)) \} \]  

(11)

where,

\[ \alpha = \frac{-\beta_j^3 \cos(\beta_j L) - \beta_j^3 \cosh(\beta_j L) + \frac{M_L}{\rho} \beta_j^4 \sin(\beta_j L) - \frac{M_L}{\rho} \beta_j^4 \sinh(\beta_j L)}{-\beta_j^3 \sin(\beta_j L) - \beta_j^3 \sinh(\beta_j L) - \frac{M_L}{\rho} \beta_j^4 \cos(\beta_j L) + \frac{M_L}{\rho} \beta_j^4 \cosh(\beta_j L)} \]  

(12)

\[ C_{1,j} = \left[ \frac{1}{\int_0^L \phi_i^2(x)dx + \frac{M_L}{\rho} \phi_i^2(L)} \right]^{1/2} \]  

(13)

On substituting the Equation 12 and 13 in Equation 11 and taking x=L, the transverse deflection of a tip with respect to body fixed frame of the link can be obtained. Using Equation 4,10, deviations in the tip position of flexible link are calculated as a normalized deviation of the tip with respect to link length.

\[ \text{Deviation in tip position\%} = \left( \frac{|\text{Deviation}|}{\text{Link Length}} \right) \times 100\% \]  

(14)

4.1.4. Case III: Motion of a Flexible Link with no payload at the link tip It is assumed that link tip is free without any payload, the link is subjected to specified joint motion as follows and the tip position of the link is found and resultants graphs are shown in Figures 6 to 8.
4.1.5. Case IV: Motion of a Flexible Link with payload at the link tip  
It is assumed that link tip is with a payload $M_L=1$kg with inertia $J_L=1$kgm$^2$ at the tip, the link is subjected to specified joint motion and the tip position of the link is shown in figures 9 to 11. The tip position of the flexible link is found by varying the payload inertia from $J_L=1$ to 5 kgm$^2$ with keeping payload mass constant.

5. Results and Discussions
The Figures 1, 2 and 3 gives the percentage deviation in the link length, X Position and Y Position of the flexible link for $\theta = 0.5t$ radians with payload no mass and with payload of varying inertia. From the above figures it can be inferred that the flexible link without the payload has negligible tip deviation. Whereas, with payload of $M_L=1$kg and maximum payload inertia of $J_L=4$ kgm$^2$, the tip deviation is found to be less than 5%.
6. Conclusions
The effective tip positions and link lengths for a flexible link with and without tip payload mass for different payload inertias are compared with that of rigid link for constant angular velocity as joint motion. Following are the conclusions drawn from above work

- The link length variation has no effect of flexibility in the case of tip carrying no payload and for the case of link carrying tip payload mass, the deviation in link length is observed to be less than 0.2% till its inertia becomes 4 kgm$^2$ and a drastic change in link length variation is observed from 5 kgm$^2$ limiting the allowable payload inertia to 4 kgm$^2$.

- The maximum percentage deviation in X and Y-position of tip with no payload mass is observed to be less than 1% and that for the case of link carrying tip payload mass, they are less than 5% until its inertia becomes 4 kgm$^2$ and a drastic change in link length variation is observed from 5 kgm$^2$ limiting the allowable payload inertia as 4 kgm$^2$.

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