ALTRUIST: Alternating Direction Method of Multipliers for Total Variation Regularization in Ultrasound Strain Imaging
Md Ashikuzzaman, Student Member, IEEE and Hassan Rivaz, Senior Member, IEEE

Abstract—Ultrasound strain imaging, which delineates mechanical properties to detect tissue abnormalities, involves estimating the time-delay between two radio-frequency (RF) frames collected before and after tissue deformation. The existing regularized optimization-based time-delay estimation (TDE) techniques suffer from at least one of the following drawbacks: 1) The regularizer is not aligned with tissue deformation physics due to taking only the first-order displacement derivative into account. 2) The $L_2$-norm of the displacement derivatives, which oversmoothes the estimated time-delay, is utilized as the regularizer. 3) The absolute value function’s sharp corner should be approximated by a smooth function to facilitate the optimization of $L_1$-norm. Herein, to resolve these shortcomings, we propose employing the alternating direction method of multipliers (ADMM) for optimizing a novel cost function consisting of $L_2$-norm data fidelity term, $L_1$-norm first- and second-order spatial continuity terms. ADMM empowers the proposed algorithm to use different techniques for optimizing different parts of the cost function and obtain high-contrast strain images with smooth background and sharp boundaries. We name our technique ADMM for total varying Regularization in ultrasound Strain imaging (ALTRUIST). In extensive simulation, phantom, and in vivo experiments, ALTRUIST substantially outperforms GLUE, OVERWIND, and $L_1$-SOUL, three recently-published TDE algorithms, both qualitatively and quantitatively. ALTRUIST yields 89%, 88%, and 26% improvements of contrast-to-noise ratio over $L_1$-SOUL for simulated, phantom, and in vivo liver cancer datasets, respectively. We will publish the ALTRUIST code after the acceptance of this paper at http://code.sonography.ai.

Index Terms—Ultrasound elastography, ADMM, Total variation regularization, Analytic optimization, High-contrast estimation, Boundary sharpness.

I. INTRODUCTION

Ultrasound is a widely-used medical imaging modality due to its cost-effectiveness, ease of use, non-invasiveness, and portability. Elastography [1], which entails mapping the mechanical properties of tissue, is one of the eminent diagnostic applications of ultrasound imaging. Assuming that certain pathologies such as tumor, cancer, benign lesion, cyst, etc., alter tissue elasticity, elastography makes distinction between healthy and diseased tissues by revealing their elastic contrast. Thus far, ultrasound elastography has been employed in breast tissue classification [2]–[5], liver health assessment [6]–[9], ablation monitoring [10]–[12], overseeing vascular [13]–[15] and cardiac [16]–[18] health, and numerous other clinical applications. Among the two broad classes of ultrasound elastography, the dynamic [19]–[21] one exploits small tissue deformation and often provides elasticity in terms of quantitative values. On the other hand, quasi-static elastography [2], [22] generates larger deformation with slow velocity and hence, yields high signal-to-noise ratio (SNR) and low signal correlation. This work focuses on free-hand palpation quasi-static strain imaging which entails acquiring time-series radio-frequency (RF) frames from a tissue being deformed by a quasi-static force created with a hand-held probe. The deformation field between the pre- and post-deformed frames is obtained by certain time-delay estimation (TDE) technique. The spatial derivative of the displacement field provides the strain image where a color-contrast distinguishes the pathologic tissue from the healthy one.

The non-trivial task of TDE is accomplished by tracking the displacement between the frames under consideration. Three mainstream classes of speckle-tracking techniques have been proposed: window-based, deep learning-based, and regularized optimization-based. The window-based algorithms [2], [23]–[25] segment the RF frames to a number of data blocks and finds the location of maximum correspondence between the pre- and post-deformed windows to track the displacements. The maximum data similarity is ascertained by some metric such as normalized cross-correlation (NCC) [3], [26] or zero-phase crossing [27], [28]. The window-based techniques are the most frequently-used ones since they are straightforward in nature. Nevertheless, these methods are sensitive to acquisition noise and signal decorrelation. The second class of algorithms [29]–[33] trains a deep convolutional neural network (CNN) to accomplish the TDE task. Although CNN-based techniques are recent additions to the TDE literature, they have shown promising speckle-tracking performance. However, the limited availability of clinical datasets might restrict the applicability of CNN-based TDE algorithms since they demand extensive amount of data in the training phase.

This work concerns regularized optimization- or energy-based speckle tracking techniques [7], [34]–[38] which obtain the displacement estimates by optimizing a non-linear cost function enforcing data fidelity and spatial continuity constraints. Although these algorithms yield attractive noise-suppression ability, they are computationally expensive. However, Dynamic Programming (DP) [35], [39] has resolved the issue of computational load. The integer displacement field estimated by DP is refined in Dynamic Programming Analytic Minimization (DPAM) [6] using a line-by-line optimization framework. The vertical striking artifacts introduced
by DPAM’s line-by-line minimization has been removed by Global Ultrasound Elastography (GLUE) [40] where the displacements of all samples in an RF frame are estimated simultaneously. Nevertheless, GLUE suffers from two major drawbacks. First, it formulates the regularizer by penalizing only the first-order derivative of the displacement field. This first-order regularization scheme does not coincide with the tissue deformation physics and therefore, leads to a suboptimal noise suppression and low-contrast strain estimate. Second, GLUE devises the regularization function taking the $L_2$-norms of displacement derivatives into account which often oversmoothes the displacement and strain maps. Second-order Ultrasound Elastography (SOUL) [9] has penalized both first- and second-order displacement derivatives to resolve the first limitation of GLUE. However, like GLUE, SOUL uses the $L_2$-norm to formulate the regularizer. Total Variation Regularization and Window-based TDE (OVERWIND) [41] has addressed the issue of oversmoothing by penalizing the $L_1$-norm of displacement derivatives instead of the $L_2$-norm. Although OVERWIND obtains sharper strain images than GLUE, it incorporates only the first-order derivative to design the regularizer. In addition, OVERWIND defines the $L_1$-norm in terms of a smooth approximation of the absolute value function to facilitate the simultaneous optimizations of $L_2$ data and $L_1$ continuity norms. Such an approximation hinders the optimality of OVERWIND’s displacement tracking performance.

In this paper, we propose using the alternating direction method of multipliers (ADMM) to optimize a novel cost function consisting of $L_2$-norm data fidelity term, $L_1$-norm first- and second-order continuity terms. The major advantages of the proposed algorithm are twofold. First, it penalizes the $L_1$-norms of both first- and second order derivatives to ensure proper contrast and boundary sharpness simultaneously. Second, ADMM allows using different methods for optimizing different parts of the cost function, and therefore, an inexact approximation of the absolute value function is no longer necessary. Unlike previous works [42]–[44] employing ADMM in the reconstruction of tissue elasticity modulus, this work applies ADMM in ultrasound strain imaging for optimizing a $L_1$-norm (alternatively known as total variation) regularizer containing both first- and second-order displacement derivatives. We name our technique ALTRUIST; ADMM for total L_1 variance Regularization in ultrasound Strain imaging. We have validated ALTRUIST with simulated, phantom, and in vivo liver cancer datasets. Similar to our previous work [7], [9], [45], the ALTRUIST code will be published at http://code.sonography.ai after the acceptance of this paper.

### II. Methods

Let $I_1 \in \mathbb{R}^{m \times n}$ and $I_2 \in \mathbb{R}^{m \times n}$ be two time-series RF frames collected from a tissue being deformed by an external or internal force. Our goal is to obtain the axial strain map between $I_1$ and $I_2$. To that end, we calculate the initial axial and lateral displacement fields $\delta$ and $l$ using DP [39]. The most vital step is to refine this initial displacement estimate using a TDE algorithm. We first summarize one such previously published TDE technique. Then we present ALTRUIST, in detail.

#### A. Second-Order Ultrasound Elastography (SOUL)

Along with data fidelity, SOUL [9] imposes both first- and second-order continuity constraints to formulate a non-linear cost function. An efficient optimization strategy is adopted to minimize the aforementioned cost function and obtain the refinement deformation fields $\Delta a$ and $\Delta l$. SOUL originally penalizes the $L_2$-norm of the displacement derivatives to formulate the regularizer which often blurs the inclusion boundary. Therefore, this paper uses the $L_1$ version of SOUL (L1-SOUL) as a comparison technique where the $L_1$-norm of the displacement derivative is penalized instead of $L_2$-norm. In L1-SOUL, the absolute value function’s sharp corner is smoothed in the same fashion as [41] to make the optimization of $L_1$-norm tractable.

#### B. ADMM for Total Variation Regularization in Ultrasound Strain Imaging (ALTRUIST)

$L_1$-SOUL approximates the sharp corner of the absolute value function with a differentiable function to facilitate the simultaneous optimizations of $L_2$ data norm and $L_1$ continuity norm. This inexact approximation of $L_1$ leas does not allow the TDE technique reach its full potential. In addition, the regularization scheme of L1-SOUL treats every sample equally which hinder a sharp transition at inclusion edges. In contrast, ALTRUIST exploits the power of alternating minimization strategy to facilitate the use of different techniques for optimizing $L_2$- and $L_1$-norms. In addition, ADMM handles every RF sample individually and adaptively imposes different levels of smoothing based on the spatial distribution of elasticities, allowing sharp strain transitions at target boundaries.

The cost function of ALTRUIST is formulated as Eq. 2.

### Algorithm 1: Workflow of the ALTRUIST algorithm

**Input:** Pre- and post-deformed RF frames $I_1$ and $I_2$, DP initial estimate $d$

**Output:** Total displacement field $d + \Delta d$, axial strain image

1. Formulate the cost function $C$ containing $L_2$-norm data and $L_1$-norm first- and second-order continuity terms (Eq. 5);
2. Split $\Delta d$ into two variables $\Delta d$ and $\nu$ and devise a constrained optimization problem (Eq. 13);
3. Formulate the augmented Lagrangian corresponding to the constrained problem (Eq. 14);
4. Alternatively solve for $\Delta d$ and $\nu$ following an iterative scheme (Eqs. 15 - 17);
5. Add optimal $\Delta d$ to $d$ for obtaining the final displacement field;
6. Differentiate the axial displacement field spatially to find the axial strain image;
\[
D' = \begin{bmatrix}
I'_{2,a}(1,1) & I'_{2,l}(1,1) & 0 & \cdots & 0 & 0 \\
0 & 0 & I'_{2,a}(1,2) & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & I'_{2,a}(m,n) & I'_{2,l}(m,n)
\end{bmatrix}
\]

(1)

\[
C(\Delta a_{1,1}, \ldots, \Delta a_{m,n}, \Delta l_{1,1}, \ldots, \Delta l_{m,n}) = \frac{1}{2} \| D_I(i, j, a_{i,j}, l_{i,j}, \Delta a_{i,j}, \Delta l_{i,j}) \|_2^2 + \| D_R \Delta d + D_R d + \mathcal{E} \|_1
\]

(2)

where \( d \in \mathbb{R}^{2mn \times 1} \) and \( \Delta d \in \mathbb{R}^{2mn \times 1} \) stack the DP initial estimates and the fine-tuning displacement estimates, respectively. \( \mathcal{E} \in \mathbb{R}^{(8mn+2n) \times 1} \) contains the adaptive regularization terms [7], [9]. \( D_I(i, j, a_{i,j}, l_{i,j}, \Delta a_{i,j}, \Delta l_{i,j}) \) forms the data constancy term and is defined as follows.

\[
D_I(i, j, a_{i,j}, l_{i,j}, \Delta a_{i,j}, \Delta l_{i,j}) = I_1(i, j) - I_2(i + a_{i,j} + \Delta a_{i,j}, j + l_{i,j} + \Delta l_{i,j})
\]

(3)

We approximate \( I_2 \) by its first-order Taylor series expansion to remove the non-linearity present in the data function:

\[
I_2(i + a_{i,j} + \Delta a_{i,j}, j + l_{i,j} + \Delta l_{i,j}) \approx I_2(i + a_{i,j}, j + l_{i,j}) + \Delta a_{i,j} I'_{2,a}(i, j) + \Delta l_{i,j} I'_{2,l}(i, j)
\]

(4)

where \( I'_{2,a} \) and \( I'_{2,l} \), respectively, denote the axial and lateral derivatives of \( I_2 \) at \((i + a_{i,j}, j + l_{i,j})\). Now the cost function can be formulated as follows.

\[
C(\Delta a_{1,1}, \ldots, \Delta a_{m,n}, \Delta l_{1,1}, \ldots, \Delta l_{m,n}) = \frac{1}{2} \| \Xi - D' \Delta d \|_2^2 + \| D_R \Delta d + D_R d + \mathcal{E} \|_1
\]

(5)

where \( D' \) and \( \Xi \) are defined by Eqs. 1 and 6, respectively.

\[
\Xi = \begin{bmatrix}
g_{1,1} & g_{1,2} & g_{1,3} & \cdots & g_{m,n}
\end{bmatrix}^T
\]

(6)

Here, \( g_{i,j} \) is defined as follows.

\[
g_{i,j} = I_1(i, j) - I_2(i + a_{i,j}, j + l_{i,j})
\]

(7)

\( D_R \in \mathbb{R}^{(8mn+2n) \times 2mn} \) embeds the weighted first- and second-order derivative operators in axial and lateral directions, and is defined as:

\[
D_R = \begin{bmatrix}
D_{f,a} \\
D_{f,l} \\
D_{l,a} \\
D_{l,l} \\
D_{2,a} \\
D_{2,l}
\end{bmatrix}
\]

(8)

where \( D_{f,a} \in \mathbb{R}^{2n \times 2mn} \) is given by: \( D_{f,a} = \begin{bmatrix}
D_{f,a,1} \\
D_{f,a,2} \\
\vdots \\
D_{f,a,n}
\end{bmatrix} \). Here, \( O \in \mathbb{R}^{2n \times 2n} \) is a zero matrix and \( D_{f,a,1} \in \mathbb{R}^{2n \times 2n} \) is defined as: \( D_{f,a,1} = diag(\gamma, 0, \gamma, 0, \ldots, \gamma, 0) \).

where \( \gamma \) denotes a continuity weight. \( D_{l,a} \in \mathbb{R}^{2mn \times 2mn} \) represents the first-order derivative in the axial direction and

is defined as:

\[
D_{l,a} = \begin{bmatrix}
O & O & O & \cdots & O \\
0 & 0 & 0 & \cdots & O \\
-\beta_{1,a} & -\beta_{1,a} & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
O & \cdots & O & -\beta_{1,a} & -\beta_{1,a}
\end{bmatrix}
\]

(9)

where \( \beta_{1,a} \in \mathbb{R}^{2n \times 2n} \) is defined as: \( \beta_{1,a} = diag(\alpha_1, \alpha_1, \alpha_1, \ldots, \alpha_1, \alpha_1) \).

Here, \( \alpha_1 \) and \( \beta_1 \) refer to the axial and lateral regularization parameters, respectively. \( D_{l,l} \in \mathbb{R}^{2mn \times 2mn} \) denotes the first-order derivative in the lateral direction and is given by: \( D_{l,l} = diag(B_{l,1}, B_{l,1}, B_{l,1}, \ldots, B_{l,1}) \)

where \( B_{l,1} \in \mathbb{R}^{2n \times 2n} \) is defined as:

\[
B_{l,1} = \begin{bmatrix}
0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & \cdots & 0 \\
-\beta_2 & 0 & \beta_2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
O & \cdots & -\beta_2 & 0 & \beta_2 \\
O & \cdots & 0 & -\beta_2 & 0
\end{bmatrix}
\]

(10)

Here, \( \beta_2 \) and \( \beta_2 \) respectively, denote the axial and lateral continuity weights. \( D_{2,a} \in \mathbb{R}^{2mn \times 2mn} \) formulates the second-order derivative in the axial direction and is defined as:

\[
D_{2,a} = \begin{bmatrix}
O & O & O & O & \cdots & O \\
B_{2,a} & -B_{2,a} & B_{2,a} & O & \cdots & O \\
O & B_{2,a} & -B_{2,a} & -B_{2,a} & \cdots & O \\
\vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\
O & \cdots & B_{2,a} & -B_{2,a} & B_{2,a} & O \\
O & \cdots & O & -B_{2,a} & B_{2,a} & O
\end{bmatrix}
\]

(11)

where

\[
B_{2,a} = diag(\theta_1, \theta_1, \theta_1, \ldots, \theta_1, \lambda_1)
\]

Here, \( \theta_1 \) and \( \lambda_1 \) stand for the axial and lateral regularization weights, respectively.

\( D_{2,l} \in \mathbb{R}^{2mn \times 2mn} \) presents the second-order derivative in the lateral direction which is defined as follows. \( D_{2,l} = diag(B_{2,l}, B_{2,l}, B_{2,l}, \ldots, B_{2,l}) \)
where

\[B_{2,l} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & 0 & \cdots & 0 \\
\theta_2 & 0 & -2\theta_2 & 0 & \theta_2 & \cdots & 0 \\
0 & \lambda_2 & 0 & -2\lambda_2 & \cdots & \text{signal-to-noise ratio (SNR), contrast-to-noise ratio (CNR), and strain ratio (SR). RMSE is defined as follows.}
\end{bmatrix}\]

\[\theta_2 \text{ and } \lambda_2, \text{ respectively, denote the axial and lateral continuity parameters. Now we split } \Delta d \text{ into two variables } \Delta d \text{ and } \nu \text{ and impose } D_R \Delta d + D_R d + E = \nu. \text{ Then the optimization problem is transformed to:}
\]

\[\begin{align*}
(\Delta d, \nu) &= \arg \min_{\Delta d, \nu} \frac{1}{2} \|E - D' \Delta d\|_2^2 + \|\nu\|_1 \\
\text{s.t.} \quad D_R \Delta d + D_R d + E &= \nu \quad (13)
\end{align*}\]

We convert the constrained optimization problem into an unconstrained one by formulating the augmented Lagrangian:

\[\begin{align*}
(\Delta d, \nu) &= \arg \min_{\Delta d, \nu} \frac{1}{2} \|E - D' \Delta d\|_2^2 + \|\nu\|_1 + \\
&\quad \frac{\zeta}{2} \|D_R \Delta d + D_R d + E - \nu + u\|_2^2 \quad (14)
\end{align*}\]

where \(\zeta\) is a tunable parameter and \(u\) is the Lagrange multiplier. Now Eq. 14 is solved alternatively to obtain the optimal solutions for \(\Delta d\) and \(\nu\):

For \(K\) iterations \{\n\begin{align*}
\Delta d &\leftarrow \arg \min_{\Delta d} \left\{\frac{1}{2} \|E - D' \Delta d\|_2^2 + \right. \\
&\left. \frac{\zeta}{2} \|D_R \Delta d + D_R d + E - \nu + u\|_2^2 \right\} \quad (15)
\end{align*}\n
\begin{align*}
\nu &\leftarrow \arg \min_{\nu} \frac{1}{2} \|\nu - (D_R \Delta d + D_R d + E + u)\|_2^2 + \frac{1}{\zeta} \|\nu\|_1 \quad (16)
\end{align*}\n
\begin{align*}
u &\leftarrow u + D_R \Delta d + D_R d + E - \nu \quad (17)
\end{align*}\n
\}

The quadratic cost function in Eq. 15 is optimized in the similar fashion as SOUL [9]. The Lagrange multiplier is updated in Eq. 17. The classic form of the cost function in Eq. 16 is optimized using the shrinkage function:

\[\nu \leftarrow S_\zeta(D_R \Delta d + D_R d + E + u) \quad (18)\]

where the shrinkage function \(S_\zeta(.)\) is defined as follows.

\[S_\zeta(.) = \text{sign}(\cdot) \max\{|\cdot| - \frac{1}{\zeta}, 0\} \quad (19)\]

The optimal refinement field is added to the DP initial estimate to find the final displacement field. A spatial differentiation of the final displacement estimate is performed to obtain the strain map. The workflow of ALTRUIST is summarized in Algorithm 1.

C. Ultrasound Simulation and Data Acquisition

We simulated two phantoms containing three layers with varying elasticities. The target-background elastic contrast for the first and second layer phantoms are high and low, respectively. In addition to the layer phantoms, we simulated a homogeneous phantom containing a stiff inclusion. The real validation experiments were performed with one set of breast phantom and two sets of in vivo liver cancer data.

1) Simulated Layer Phantoms: Two homogeneous phantoms, each containing a stiff layer, were compressed by 4% using closed-form equations as described in [7]. The elastic moduli of the background and target tissue layers, respectively, were set to 20 kPa and 40 kPa for the first phantom and 20 kPa and 22.86 kPa for the second phantom. The ultrasound simulation software Field II [46] was used to generate the RF frames, setting the center and sampling frequencies to 7.27 MHz and 40 MHz, respectively. Random Gaussian noise with 20 dB peak SNR (PSNR) was added to the RF data to mimic real data acquisition environment.

2) Hard-inclusion Simulated Phantom: A uniform phantom of 4 kPa Young’s modulus containing a hard inclusion of 40 kPa elastic modulus was simulated. A 1% compression was applied to the simulated phantom using the finite element (FEM) package ABAQUS (Providence, RI). Pre- and post compressed RF frames were simulated with Field II, setting the transmit and sampling frequencies to 7.27 MHz and 100 MHz, respectively. We corrupted the RF data with added Gaussian noise of 24 dB PSNR.

3) Experimental Breast Phantom: An experimental breast phantom (Model 059, CIRS: Tissue Simulation & Phantom Technology, Norfolk, VA) with Young’s modulus of 20±5 kPa was compressed with an L3-12H linear array probe. The hard inclusion’s elasticity modulus was at least twice as large as that of the background. Time-series RF frames were collected using an Alpinion E-Cube R12 research ultrasound machine setting the transmit and sampling frequencies to 10 MHz and 40 MHz, respectively.

4) In vivo Liver Cancer Datasets: The in vivo datasets were collected at the Johns Hopkins Hospital (Baltimore, MD) from two liver cancer patients before undergoing open surgical RF thermal ablation. The compression was performed with free hand using a VF 10-5 linear array probe. RF frames were collected from the livers being deformed with an Antares Siemens research ultrasound system. The center and sampling frequencies were set to 6.67 MHz and 40 MHz, respectively. All procedures aligned with the ethics approval obtained from the Institutional Review Board. In addition, both patients provided written consent to this in vivo study. Further details of this experiment can be found in [6].

D. Quantitative Metrics

The strain imaging quality of ALTRUIST is compared to those of GLUE, OVERWIND, and L1-SOUL using Mean Structural SIMilarity (MSSIM) [47], root-mean-square error (RMSE), signal-to-noise ratio (SNR), contrast-to-noise ratio (CNR), and strain ratio (SR). RMSE is defined as follows.
TABLE I: MSSIM and RMSE values for the simulated high-contrast layer phantom dataset.

|        | MSSIM   | RMSE    |
|--------|---------|---------|
| GLUE   | 0.05    | $3.4 \times 10^{-3}$ |
| OVERWIND | 0.39   | $2.6 \times 10^{-3}$ |
| $L_1$-SOUL | 0.71   | $2.6 \times 10^{-3}$ |
| ALTRUIST | **0.92** | **$2.4 \times 10^{-3}$** |

TABLE II: MSSIM and RMSE values for the simulated low-contrast layer phantom dataset.

|        | MSSIM   | RMSE    |
|--------|---------|---------|
| GLUE   | 0.004   | $2.9 \times 10^{-3}$ |
| OVERWIND | 0.06   | $1.1 \times 10^{-3}$ |
| $L_1$-SOUL | 0.23   | $1.3 \times 10^{-3}$ |
| ALTRUIST | **0.71** | **$7.05 \times 10^{-4}$** |

TABLE III: MSSIM and RMSE values for the hard-inclusion simulated phantom.

|        | MSSIM   | RMSE    |
|--------|---------|---------|
| GLUE   | 0.19    | $1.4 \times 10^{-3}$ |
| OVERWIND | 0.43   | $9.93 \times 10^{-4}$ |
| $L_1$-SOUL | 0.59   | $9.46 \times 10^{-4}$ |
| ALTRUIST | **0.66** | **$9.35 \times 10^{-4}$** |

TABLE IV: SNR, CNR, and SR for the hard-inclusion simulated phantom dataset. SNR and SR are calculated between the white target and black background windows depicted in Fig. 1(a) of the Supplementary Material, whereas SNR is obtained from the background window only.

|        | SNR   | CNR   | SR    |
|--------|-------|-------|-------|
| GLUE   | 15.58 | 11.10 | 0.20  |
| OVERWIND | 28.51 | 24.19 | 0.15  |
| $L_1$-SOUL | 61.36 | 21.12 | 0.14  |
| ALTRUIST | **63.77** | **39.96** | **0.13** |

III. RESULTS

All four techniques’ parameter sets were carefully tuned for each dataset individually to ensure best achievable visual results. In addition, the differentiation kernel length was set to 3 RF samples for all validation experiments to prevent an artificial oversmoothing of strain images.

A. Simulated Layer Phantoms

Fig. 1 shows the axial strain results for the high- and low-contrast layer phantoms. For both phantoms, GLUE obtains the noisiest strain images. OVERWIND yields a better noise suppression performance. In addition, the OVERWIND strains present clearer layer boundaries than GLUE. However, the estimated elastic contrast between different tissue layers is not satisfactory. $L_1$-SOUL exhibits better target-background contrast than GLUE and OVERWIND. Moreover, it yields a sharper layer edge. Nevertheless, the performance of $L_1$-SOUL in the uniform tissue regions is not up to the mark. ALTRUIST produces the best strain images by ensuring smoothness in the uniform tissue region and sharp transition at the layer boundaries. The advantage of ALTRUIST is even more evident in case of the low-contrast phantom since it is a highly challenging scenario. The MSSIM and RMSE values reported in Tables I and II support our visual inference.

B. Hard-inclusion Simulated Phantom

Fig. 1 of the Supplementary Material depicts the FEM and the estimated axial strain images for the hard-inclusion simulated phantom. GLUE provides a noisy strain estimate with low target-background contrast. OVERWIND yields a clearer inclusion boundary than GLUE, while improving the contrast. However, the background strain imaging performance of OVERWIND is unsatisfactory. $L_1$-SOUL resolves this issue of OVERWIND by providing a smoother background and preserving the contrast. It is worth mentioning that $L_1$-SOUL exhibits spurious edges inside the stiff inclusion which stem from the estimation noise. In addition, a small region on the right of the inclusion edge is slightly broken. ALTRUIST substantially outperforms GLUE, OVERWIND, and $L_1$-SOUL both in the inclusion boundary and the uniform regions. It is apparent from the MSSIM and RMSE values in Table III that ALTRUIST shows the maximum resemblance to the FEM strain. The SNR, CNR, and SR values reported in Table IV corroborate the superiority of ALTRUIST over the other three techniques.

In addition to a single CNR value, we show the histogram of CNR values (see Fig. 4(a)) for 120 combinations (6 target and 20 background) of target and background strain windows. The histogram demonstrates that ALTRUIST occupies most of the high CNR values. The paired t-test indicates that ALTRUIST is significantly better than GLUE, OVERWIND, and $L_1$-SOUL with $p$-values of $7.08 \times 10^{-33}$, $7.35 \times 10^{-16}$, and $1.62 \times 10^{-23}$, respectively.

C. Breast Phantom Dataset

The axial strain results for the experimental breast phantom dataset have been reported in Fig. 2. GLUE exhibits extensive background noise and blurred inclusion boundary. Although the inclusion edge is clearer in the OVERWIND strain, the background tissue region appears to be dark which might originate from the underestimation of strain. $L_1$-SOUL

\[ \text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n} \sum_{j=1}^{m} (\hat{s}_{i,j} - s_{i,j})^2}{mn}} \] (20)

where $\hat{s}_{i,j}$ and $s_{i,j}$ stand for the estimated and ground truth strains corresponding to the RF sample at $(i,j)$. SNR, CNR, and SR are defined as:

\[ \text{SNR} = \frac{\hat{s}_{b}}{\sigma_b}, \text{SNR} = \frac{s_{b}}{\sigma_b}, \text{SR} = \frac{s_{t}}{s_{b}} \] (21)

where $\hat{s}_{b}$ and $s_{b}$ denote the mean background and target strains, respectively. $\sigma_b$ and $\sigma_t$ refer to the standard deviations of the background and target strain windows, respectively.
outperforms GLUE and OVERWIND in terms of both edge-clarity and target-background contrast. However, the background, especially in the deep tissue regions, is still noisy and the inclusion edge is not sufficiently sharp. In addition, the target-background contrast has room for improvement. ALTRUIST resolves the issues associated with GLUE, OVERWIND, and L1-SOUL by providing the highest contrast, smooth background, and sharp target-background boundary. The quantitative values reported in Table V substantiate this visual judgement.

The histogram of 120 CNR values (see Fig. 4(b)) shows that ALTRUIST obtains majority of the high CNR values. The paired $t$-test confirms that ALTRUIST significantly outperforms GLUE, OVERWIND, and L1-SOUL with $p$-values of $6.21 \times 10^{-49}$, $3.04 \times 10^{-36}$, and $2.39 \times 10^{-19}$, respectively.

**Table V:** Quantitative results for the breast phantom dataset. CNR and SR are calculated incorporating the blue target and red background windows shown in Fig. 2(a). SNR is calculated on the background window.

|       | SNR  | CNR  | SR  |
|-------|------|------|-----|
| GLUE  | 7.27 | 6.30 | 0.38|
| OVERWIND | 15.76 | 12.00 | 0.45|
| L1-SOUL | 14.83 | 14.49 | 0.30|
| ALTRUIST | **25.85** | **27.28** | **0.18** |

**D. In vivo Liver Cancer Datasets**

Fig. 3 shows the B-mode and the axial strain estimates obtained by GLUE, OVERWIND, L1-SOUL, and ALTRUIST. The patient one’s B-mode image shows an echogenic contrast between the tumor and the healthy tissue. On the other hand, the echogenic difference between the target and the background is negligible in case of patient two. For both
FIG. 2: Results obtained from the experimental breast phantom. Columns 1 to 5 correspond to the B-mode image and the axial strain images produced by GLUE, OVERWIND, L1-SOUL, and ALTRUIST, respectively.

TABLE VI: SNR, CNR, and SR for the in vivo liver cancer datasets. CNR and SR are calculated utilizing the target and background windows shown in Figs. 3(a) and 3(f), whereas SNR values are calculated on the background windows only.

| Patient 1 | Patient 2 |
|-----------|-----------|
|           | SNR       | CNR   | SR  | SNR       | CNR   | SR  |
| GLUE      | 27.56     | 18.60 | 0.39| 28.15     | 14.27 | 0.47|
| OVERWIND  | 42.30     | 20.46 | 0.40| 35.90     | 19.29 | 0.48|
| L1-SOUL   | 52.83     | 24.23 | 0.31| 35.66     | 20.72 | 0.47|
| ALTRUIST  | 61.09     | 30.58 | 0.24| 41.82     | 25.96 | 0.46|

patients, the strain images produced by all four techniques clearly show the pathologic region. In addition to extensive noise in the background, GLUE strain images lack edge clarity. Although OVERWIND yields better noise suppression and edge-preserving ability than GLUE, it is still noisy and suffers from insufficient target-background contrast. Moreover, both GLUE and OVERWIND underestimate the strain in shallow tissue regions in case of patient 2. L1-SOUL partially resolves the issues of GLUE and OVERWIND providing a smoother background and darker tumor region. For both patients, ALTRUIST substantially outperforms the pervious techniques in terms of target-background contrast, smoothness in the homogeneous region and sharpness at the edges which is substantiated by the quantitative metric values reported in Table VI.

The high frequencies of ALTRUIST in high CNR values of the histograms (see Figs. 4(c) and 4(d)) demonstrate the superiority of ALTRUIST throughout the strain images. The paired t-tests corroborate this inspection by showing that ALTRUIST is statistically better than GLUE, OVERWIND, and L1-SOUL with p-values of $9.85 \times 10^{-22}$, $1.21 \times 10^{-8}$, and $1.07 \times 10^{-2}$, respectively, for patient 1 and $9.08 \times 10^{-16}$, $9.57 \times 10^{-15}$, and $3.96 \times 10^{-8}$, respectively, for patient 2.

IV. DISCUSSION

The penalty function formulated for this work consists of $L_1$-norm continuity term and $L_2$-norm data term. Ultrasound RF data generally contains additive Gaussian noise. Since $L_2$-norm provides the maximum-likelihood estimator in such a scenario, the data term is devised in terms of the $L_2$-norm of amplitude differences. Fig. 5 justifies our selection of the data term by demonstrating that the histogram of the residual between the pre-deformed and the warped post-deformed frames approximates a Gaussian distribution.

One of the strengths of ALTRUIST is that it incorporates the shrinkage function to make discrete decision on the level of continuity at each RF sample. This task is vital to ensure continuity in the uniform region and sharp discontinuity at the boundaries. ALTRUIST determines the desired derivative values for the next iteration by shrinking the current iteration’s displacement derivatives. Fig. 6 illustrates this fact for the low-contrast layer phantom dataset by showing how ALTRUIST’s iterations gradually converge to the correct level of smoothness.

ALTRUIST substantially outperforms the other three techniques in terms of visual contrast and sharpness. However, like $L_1$-SOUL, it slightly distorts the elastic structure above the hard inclusion in Fig. 1 of the Supplementary Material. In addition, ALTRUIST exhibits a spurious edge on the inclusion’s left in Fig. 2(e). A data-driven combination of mechanical-based and continuity constraints is a potential solution to these problems. However, it is beyond the scope of this work and calls for further research.

As the first step of applying ADMM in ultrasonic strain imaging, this work obtains the optimal parameter sets associated with different techniques using an ad hoc technique. As
Although ALTRUIST successfully obtains high-quality strain images, it is based on ADMM which demands explicit optimizations of both data and prior functions. Therefore, ADMM might not be suitable for handling certain penalty functions which incorporate hard-to-optimize priors. Plug-and-Play (PnP) [49] is an extension of ADMM which can potentially deal with such a situation. Because PnP converts objectives into actions and obtains the optimal solution from consensus equilibrium instead of solving the inverse problem explicitly. Our future work involves incorporating PnP to optimize a novel cost function consisting of \( L^2 \) data norm and deep learning-based denoising prior.

V. CONCLUSION

This paper proposes ALTRUIST, a novel algorithm for TDE in ultrasonic strain imaging. ALTRUIST incorporates ADMM for analytically optimizing a robust penalty function containing \( L^2 \) data fidelity norm, \( L^1 \) first- and second-order displacement...
derivative norms. ADMM ameliorates the issues regarding simultaneous optimization of $L_2$ data and $L_1$ continuity terms by allowing the employment of different techniques for optimizing different components of the cost function. In addition, the optimization of the $L_1$-norm using the shrinkage operator introduces adaptive smoothing to the displacement estimates, yielding data-driven decisions on the level of continuity at each sample. As such, ALTRUSTIST exhibits substantially superior performance over previous strain imaging techniques.

ACKNOWLEDGMENT

This work is supported in part by Natural Sciences and Engineering Research Council of Canada (NSERC) Discovery Grant. Md Ashikuzzaman holds PBEEE and B2X Doctoral Research Fellowships awarded by the Fonds de Recherche du Québec - Nature et Technologies (FRQNT). We thank Drs. E. Boctor, M. Choti and G. Hager for allowing them to use the liver datasets, and Dr. M. Mirzaei for his help tuning the parameters of OVERWIND and for providing the simulated FEM phantom.

REFERENCES

[1] J. Ophir, S. K. Alam, B. Garra, F. Kallel, E. Konofagou, T. Krouskop, and T. Varghese, “Elastography: Ultrasonic estimation and imaging of the elastic properties of tissues,” Proceedings of the Institution of Mechanical Engineers, Part H: Journal of Engineering in Medicine, vol. 213, no. 3, pp. 203–233, 1999.
[2] T. J. Hall, Y. Zhu, and C. S. Spalding, “In vivo real-time freehand palpation imaging,” Ultrasound in Medicine & Biology, vol. 29, no. 3, pp. 427–435, 2003.
[3] A. Nahiyani and M. K. Hasan, “Hybrid algorithm for elastography to visualize both solid and fluid-filled lesions,” Ultrasound in Medicine & Biology, vol. 41, no. 4, pp. 1058–1078, 2015.
[4] J. Jiang and T. Hall, “A coupled subsample displacement estimation method for ultrasound-based strain elastography,” Physics in medicine and biology, vol. 60, pp. 8347–8364, 10 2015.
[5] M. Ashikuzzaman, T. J. Hall, and H. Rivaz, “Adaptive data function for robust ultrasound elastography,” in 2020 IEEE International Ultrasonics Symposium (IUS), 2020, pp. 1–4.
[6] H. Rivaz, E. M. Boctor, M. A. Choti, and G. D. Hager, “Real-time regularized ultrasound elastography,” IEEE Transactions on Medical Imaging, vol. 30, no. 4, pp. 928–945, 2011.
[7] M. Ashikuzzaman, C. J. Gauthier, and H. Rivaz, “Global ultrasound elastography in spatial and temporal domains,” IEEE Trans. Ultrasonics, Ferroelectrics, and Frequency Control, vol. 66, no. 5, pp. 876–887, 2019.
[8] A. Tadj, G. Cloutier, J. UN. M. Szeveryenyi, and C. B. Sirlin, “Ultrasound elastography and mr elastography for assessing liver fibrosis: part 2. diagnostic performance, confounders, and future directions,” American journal of roentgenology, vol. 205, no. 1, pp. 33–40, 2015.
[9] M. Ashikuzzaman, A. Sadeghi-Naini, A. Samani, and H. Rivaz, “Combining first- and second-order continuity constraints in ultrasound elastography,” IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control, vol. 68, no. 7, pp. 2407–2418, 2021.
[10] T. Varghese et al., “Elastographic measurement of the area and volume of thermal lesions resulting from radiofrequency ablation: pathologic correlation,” American journal of roentgenology, vol. 181, no. 3, pp. 701–707, 2003.
[11] H. Rivaz et al., “Ablation monitoring with elastography: 2d in-vivo and 3d ex-vivo studies,” in International Conference on Medical Image Computing and Computer-Assisted Intervention, 2008, pp. 458–466.
[12] A. Mariani, W. Kwiecinski, M. Pernot, D. Balvay, M. Tanter, O. Clement, C. Cuenod, and F. Zinzindohoue, “Real time shear waves elastography monitoring of thermal ablation: in vivo evaluation in pig livers,” Journal of Surgical Research, vol. 188, no. 1, pp. 37–43, 2014.
[13] C. L. De Korte et al., “Characterization of plaque components with intravascular ultrasound elastography in human femoral and coronary arteries in vitro,” Circulation, vol. 102, no. 6, pp. 617–623, 2000.
[14] H. Li, J. Porée, M.-H. R. Cardinal, and G. Cloutier, “Two-dimensional affine model-based estimators for principal strain vascular ultrasound elastography with compound plane wave and transverse oscillation beamforming,” Ultrasonics, vol. 91, pp. 77–90, 2019.
[15] R. L. Maurice, J. Ohayon, Y. Frétigny, M. Bertrand, G. Soulez, and G. Cloutier, “Noninvasive vascular elastography: Theoretical framework,” IEEE transactions on medical imaging, vol. 23, no. 2, pp. 164–180, 2004.
[16] H. Chen, T. Varghese, P. S. Rahko, and J. Zagzebski, “Ultrasound frame rate requirements for cardiac elastography: Experimental and in vivo results,” Ultrasonics, vol. 49, no. 1, pp. 98–111, 2009.
[17] M. Strachinaru et al., “Cardiac shear wave elastography using a clinical ultrasound system,” Ultrasound in medicine & biology, vol. 43, no. 8, pp. 1596–1606, 2017.
[18] E. E. Konofagou, J. D’hooge, and J. Ophir, “Myocardial elastography—a feasibility study in vivo,” Ultrasound in medicine & biology, vol. 28, no. 4, pp. 475–482, 2002.
[19] T. Gallot, S. Catheline, P. Roux, J. Brum, N. Benech, and C. Negrier, “Passive elastography: Shear-wave tomography from physiological-noise correlation in soft tissues,” Ultrasonics, Ferroelectrics and Frequency Control, IEEE Transactions on, vol. 56, pp. 1122 – 1126, 07 2011.
[20] R. Rosado-Mendez, L. Carlson, K. Woo, A. Santos, Q. Guerrero, M. Palmeri, H. Feltovich, and T. Hall, “Quantitative assessment of cervical softening during pregnancy in the rhesus macaque with shear wave elasticity imaging,” Physics in Medicine and Biology, vol. 63, 03 2018.
[21] M. D. Horeh, A. Asif, and H. Rivaz, “Analytical minimization-based
regularized subpixel shear-wave tracking for ultrasound elastography,”
IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control,
vol. 66, no. 2, pp. 285–296, 2019.
[22] J. Ophir, I. Céspedes, H. Ponneckanti, Y. Yazdi, and X. Li, “Elastography:
a quantitative method for imaging the elasticity of biological tissues,”
Ultrasonic Imaging, vol. 13, pp. 111–34, 1991.
[23] J. Luo and E. E. Konofagou, “A fast normalized cross-correlation
calculation method for motion estimation,” IEEE Trans. Ultrasonics,
Ferroelectrics, andFreq. Cont., vol. 57, no. 6, pp. 1347–1357, 2010.
[24] M. G. Kibria and M. K. Hasan, “A class of kernel based real-time
elastography algorithms,” Ultrasonics, vol. 61, pp. 88–102, 2015.
[25] M. Mirzaei, A. Asif, M. Fortin, and H. Rivaz, “3d normalized cross-
correlation for estimation of the displacement field in ultrasound elastography,” Ultrasonics, vol. 102, p. 106053, 2020.
[26] R. Zahiri-Azar and S. Salcudean, “Motion estimation in ultrasound
images using time domain cross correlation with prior estimates,” IEEE Trans. Biomed. Eng., vol. 53, no. 10, pp. 1990–2000, 2006.
[27] X. Chen, M. Zohdy, E. SY, and M. O’Donnell, “Lateral speckle tracking
using synthetic lateral phase,” IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control, vol. 61, pp. 540–550, 2004.
[28] S. Ara et al., “Phase-based direct average strain estimation for elastography,” Ultrasonics, Ferroelectrics and Frequency Control, IEEE Transactions on, vol. 60, pp. 2266–2283, 11 2013.
[29] M. G. Kibria and H. Rivaz, “Gluenet: Ultrasound elastography using
convolutional neural network,” in Simulation, Image Processing, and Ultrasound Systems for Assisted Diag. and Nav., 2018.
[30] Z. Gao, S. Wu, Z. Liu, J. Luo, H. Zhang, M. Gong, and S. Li,
“Learning the implicit strain reconstruction in ultrasound elastography
using privileged information,” Med. im. analysis, vol. 58, 2019.
[31] S. Wu, Z. Gao, Z. Liu, J. Luo, H. Zhang, and S. Li, “Direct reconstruction
of ultrasound elastography using an end-to-end deep neural network,” in International Conference on Medical Image Computing and Computer-Assisted Intervention. Springer, 2018, pp. 374–382.
[32] B. Peng, Y. Xian, Q. Zhang, and J. Jiang, “Neural network-based
motion tracking for breast ultrasound strain elastography: An initial
assessment of performance and feasibility,” Ultrasonic Imaging, p. 01617346209002527, 2020.
[33] A. K. Z. Tehrani and H. Rivaz, “Displacement estimation in ultrasound
elastography using pyramidal convolutional neural network,” IEEE Trans. Ultrasonics, Ferroelectrics, Freq. Contr., pp. 1–1, 2020.
[34] M. Ashikuzzaman and H. Rivaz, “Denoising rf data via robust principal component analysis: Results in ultrasound elastography,” in 42nd IEEE EMBC, 2020, pp. 2067–2070.
[35] J. Jiang and T. J. Hall, “A generalized speckle tracking algorithm for
ultrasonic strain imaging using dynamic programming,” Ultrasonic in Medicine & Biology, vol. 35, no. 11, pp. 1863 – 1879, 2009.
[36] M. Ashikuzzaman and H. Rivaz, “Incorporating multiple observations in global ultrasound elastography,” in 42nd IEEE EMBC, 2020, pp. 2007–10.
[37] X. Pan, J. Gao, S. Tao, K. Liu, J. Bai, and J. Luo, “A two-step optical flow method for strain estimation in elastography: Simulation and phantom study,” Ultrasonics, vol. 54, no. 4, pp. 990–996, 2014.
[38] M. T. Islam et al., “A new method for estimating the effective poisson’s
ratio in ultrasound poroelastography,” IEEE transactions on medical imaging, vol. 37, no. 5, pp. 1178–1191, 2018.
[39] H. Rivaz, E. Doctor, P. Foroughi, R. Zellars, F. Gichtinger, and G. Hager,
“Ultrasound elastography: A dynamic programming approach,” IEEE Transactions on Medical Imaging, vol. 27, no. 10, pp. 1373–1377, 2008.
[40] H. S. Hashemi and H. Rivaz, “Global time-delay estimation in ultrasound
elastography,” IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control, vol. 64, no. 10, pp. 1625–1636, 2017.
[41] M. Mirzaei, A. Asif, and H. Rivaz, “Combining total variation regularization with window-based time delay estimation in ultrasound elastography,” IEEE Transactions on Medical Imaging, vol. 38, no. 12, pp. 2744–2754, 2019.
[42] S. Mohammed, M. Honarvar, Q. Zeng, H. Hashemi, R. Rohling, P. Koziolowski, and S. Salcudean, “Multifrequency 3d elasticity reconstruction
withstructured sparsity and admm,” arXiv preprint arXiv:2111.12179, 2021.
[43] C. F. Otesteanu, V. Vishnevsky, and O. Goksel, “Fem-based elasticity re-
construction using ultrasound for imaging tissue ablation,” International journal of computer assisted radiology and surgery, vol. 13, no. 6, pp. 885–894, 2018.
[44] S. Mohammed, M. Honarvar, P. Koziolowski, and S. Salcudean, “2d
elasticity reconstruction with bi-convex alternating direction method of
multipliers,” in 2019 IEEE 16th International Symposium on Biomedical Imaging (ISBI 2019). IEEE, 2019, pp. 1683–1687.
[45] M. Ashikuzzaman, C. Belasso, M. G. Kibria, A. Bergdahl, C. J. Gauthier, and H. Rivaz, “Low rank and sparse decomposition of ultrasound color
flow images for suppressing clutter in real-time,” IEEE Transactions on Medical Imaging, vol. 39, no. 4, pp. 1073–1084, 2020.
[46] J. Jensen, “Field: A program for simulating ultrasound systems,” Medical and Biological Engineering and Computing, vol. 34, pp. 351–352, 1996.
[47] Z. Wang, A. Bovik, H. Sheikh, and E. Simoncelli, “Image quality assessment:
from error visibility to structural similarity,” IEEE Transactions on Image Processing, vol. 13, no. 4, pp. 600–612, 2004.
[48] P. C. Hansen, “The l-curve and its use in the numerical treatment of
inverse problems,” Comput. Inv. Problems in Electrocard., 2001.
[49] S. V. Venkatakrishnan, C. A. Bouman, and B. Wohlberg, “Plug-and-play priors for model based reconstruction,” in 2013 IEEE Global Conference on Signal and Information Processing. IEEE, 2013, pp. 945–948.
In this Supplementary Material, we report the axial strain estimates obtained from the hard-inclusion FEM phantom.

I. RESULTS

Fig. 1 shows the ground truth and the axial strain estimates for the hard-inclusion simulated phantom with added Gaussian noise. It is evident that ALTRUIST substantially outperforms GLUE, OVERWIND, and \textit{L1-SOUL} in terms of target-background contrast, edge-clarity, and background homogeneity.
Fig. 1: Axial strain results obtained from the simulated FEM phantom with added Gaussian noise. (a)-(e) correspond to the FEM strain and the axial strain images produced by GLUE, OVERWIND, $L_1$-SOUL, and ALTRUIST, respectively.