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Chance-Constrained Model Predictive Control
A reformulated approach suitable for sewer networks

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Abstract
In this work, a revised formulation of Chance-Constrained (CC) Model Predictive Control (MPC) is presented. The focus of this work is on the mathematical formulation of the revised CC-MPC, and the reason behind the need for its revision. The revised formulation is given in the context of sewer systems, and their weir overflow structures. A linear sewer model of the Astlingen Benchmark sewer model is utilized to illustrate the application of the formulation, both mathematically and performance-wise through simulations. Based on the simulations, a comparison of performance is done between the revised CC-MPC and a comparable deterministic MPC, with a focus on overflow avoidance, computation time, and operational behavior. The simulations show similar performance for overflow avoidance for both types of MPC, while the computation time increases slightly for the CC-MPC, together with operational behaviors getting limited.

KEYWORDS:
Stochastic MPC; combined sewer overflow; chance-constrained; Astlingen sewer network

1 | INTRODUCTION

With the increase in heavy rains in the recent decade1, the operation of sewer systems has become more important. In sewage systems2, there are several objectives for the ideal system operation. Some of these objectives are control of the flow to the wastewater treatment plant (WWTP), and the avoidance of weir overflows. In the previous decades, Model Predictive Control (MPC) has been applied to sewage systems with fair results3-9, aiming for the ideal operation. However, the structure of sewage systems is always changing, leading to model uncertainties. In addition, the systems being intrinsically driven by rain- and dry weather inflows create a dependency on the quality of the predictions of those inflows. With the classical MPC being a deterministic method, the presence of uncertainty has not in general been included in the research on MPC for sewer networks.

While the classical MPC is deterministic, MPC methods for handling uncertainty have been developed in the past decade10-14, but have not been applied to sewer systems. Collectively these methods are referred to as Robust Model Predictive Control (RMPC) or Stochastic Model Predictive Control (SMPC), depending on their approach to uncertainty handling and each includes a wealth of methods. Briefly, the RMPCs15 are methods that aim at finding the most optimal solution that is applicable under every realization of the uncertainty; in short, they aim to operate under the worst-case scenario. The RMPC methods include approaches such as min-max MPC15 and tube-based MPC19.

The SMPCs10-14 can in brief be described as finding the most optimal solution for the most likely uncertainty realizations; allowing for tunable coverage of the uncertainty spectrum. The SMPC methods range from the finite scenario-based robust approaches13 to methods based on the probability of constraints being true11,12,16 among others10.
Given that uncertain rain forecasts contain some rain realizations, which have a very low likelihood but would have a high impact on the system, such as a cloudburst on a sunny day, we will in this work focus on a method of SMPC; given their ability to omit the least likely scenarios.

The stochastic method focused on in this paper is known as chance-constrained MPC (CC-MPC) and has previously been applied to other water systems, such as drinking water systems\cite{11} with good results. The CC-MPC method utilizes an optimization based on the expected cost of the system, together with probabilistic formulations of the constraints. The probabilistic formulations are introduced to tighten the constraints so that the performance resulting from the controller is feasible within the real constraints with a given probability.

While CC-MPC and other similar SMPC methods can utilize information of the uncertainty to generate constraint tightenings for more robust performances, it does not come without drawbacks. Given that tighter constraints mean the workspace of the controller gets smaller and in worst-case results in loss of feasibility for the CC-MPC. This can happen if the uncertainty is too large or if the desired probability for the constraints to hold is too high, resulting in overlapping constraints and infeasibility.

Another important aspect of MPC design for sewer networks, besides feasibility, is how the overflows from weirs are integrated into the design formulation. Weirs are physical structures with a binary nature of either overflowing or not, giving two different dynamics of the systems to include in the MPC design. weir overflows are usually integrated by one of three approaches: 1) they are ignored in the formulation and their occurrence means an infeasible scenario. 2) They are integrated into the constraints but excluded from the dynamics. 3) They are integrated into the dynamics and propagated through the MPC formulation.

In this paper, we will consider the third approach to the weir overflows, however, the CC-MPC mentioned earlier is not suitable for this approach, due to the overflows being defined by the original constraints (without tightening). The inclusion of overflow into the dynamics leads to another issue with the formulation of CC-MPC. With the inclusion, the constraints defining the weir elements become intrinsically feasible through the presence of overflow. This result in the probabilistic formulations of the CC-MPC method becomes insensible with probabilities larger or equal to one.

### 1.1 Main Contribution

In this paper, the main contribution is the outline of a revised formulation of the CC-MPC method, suitable for the application in sewers with weir structures and addressing the aforementioned drawbacks. The reformulation will aim to introduce sensible probabilistic constraints, suitability for the inclusion of weir structures into the dynamics, as well as the preservation of the original feasibility of the system. The paper will provide an application example and a performance evaluation of the example.

For the application of the revised CC-MPC formulation, we will utilize a model of the Astlingen benchmark Network\cite{20}, described in section 3. The Astlingen system is a representative model of a typical sewer system, designed for testing of control strategies, containing six interconnected tanks with weirs and upstream sections. The evaluation of the application will focus on the revised CC-MPC’s performance in view of the classical deterministic MPC’s performance when using the third approach to overflow integration.

In section 2 of the paper, we will first present the general MPC program for systems with overflows, and then we will discuss and outline the revised CC-MPC formulation. Section 3 contains the description of the Astlingen system, in connection to an applied example of the revised CC-MPC. In section 4, the results of simulations with the applied example are discussed.

### 1.2 Notation

In this paper, the following notation is utilized. Bold font is utilized to indicate vectors, while a bullet • represents a subset or set of a function’s variables. For a stochastic variable \( X \), the expectation and variance are denoted \( E\{X\} \) and \( \sigma_X^2 \) respectively, while \( Pr\{X \leq x\} \) and \( \Phi(x) \) are the probability function and cumulative distribution function (CDF) respectively for a given value \( x \). The notation \( X \sim F \) indicates that \( X \) is following a given distribution \( F \). The weighted quadratic norm of \( x \) is denoted by \( ||x||_A^2 = x^T A x \), while the minimum and maximum of a given function \( f(x) \) are denoted \( \underline{f} \) and \( \overline{f} \) respectively. The notation \( \Delta T \) and the subscript \( k \) indicate the sampling time and the sample number respectively. Variables written with the letters \( V \) and \( q \) are used to indicate volume and flow respectively, while the letters \( u \) and \( D \) indicate the control (flow) and delayed flows. The superscripts \( in \), \( out \), and \( w \) indicate the inflow, outflow, and weir overflow respectively. The prefix \( u \) is used for \( 10^{-6} \).
FIGURE 1 An illustration of the nature of weirs where the weir flow $q_{k,i}^w$ is zero when the switching function $T(\bullet)$ is negative, and following a given non-negative weir function $t_w(T)$, when the switching function is positive (here shown for a linear $t_w(T)$).

2 STOCHASTIC MPC WITH WEIR ELEMENTS

In sewer systems, weirs are a common structural element to encounter, providing passive redirection of flows. A weir flow is the overflowing water, for example from a tank that is filled to the brim while water is still flowing in. Similarly, when the tank is not full, the weir flow is zero.

For systems with weirs or weir-like structures, they have a binary nature originating from each of the weirs. If we consider a weir function $t_w(T)$ describing the weir flow, according to some switching function $T(\bullet)$, then the weir flow’s binary nature can be described by (3). The switching function $T(\bullet)$ describes some capacity related to the specific weir, such as water above the brim of the aforementioned tank. Here the $\bullet$ input is the set of variables specific to each individual weir, affecting whether or not an overflow occurs. An example of the binary nature of weirs is shown in Figure 1, for a linear weir function $t_w(T)$. The binary nature can easily be observed, by noting that the flow is zero when the switching function $T_i(\bullet)$ is negative, and otherwise follows the given function $t_w$ depending on the switching function.

The general deterministic formulation of MPC for systems with weirs can be formulated as below.

\[
J = \min_u f(x, u, z^{ref}, w, q^w) \tag{1}
\]
\[
x_{k+1} = h_{proc}(x_k, u_k, w_k, q^w_k), \quad x_0 = x_{ini} \tag{2}
\]
\[
q_{k,i}^w = \begin{cases} t_{w,i}(T_i(\bullet)), & T_i(\bullet) \geq 0 \\ 0 & \forall i \in \{1 : N_w\} \tag{3}
\end{cases}
\]
\[
g(x_k, w_k, u_k, q^w_k) \leq \bar{g} \tag{4}
\]

Where $x$ corresponds to the system states, $u$ is the control of the system, $w$ is the rain inflow into the system, and the weir flow $q_{k,i}^w$ corresponds to the ith weir element out of $N_w$ at time $k$ and is always non-negative. While $x_{ini}$ is the system’s initial state and $z^{ref}$ is the reference for the desired behavior.

As mentioned earlier, we will in this work consider the stochastic MPC method CC-MPC. The CC-MPC method is based on the assumption that the stochastic distributions of the initial state and all inputs are known within the prediction horizon so that the stochastic properties of any predicted state and constraints can be derived and taken into account.

When using the CC-MPC method to handle uncertainty, the cost function in (1) is rewritten as the expectation of the given function, $E\{f(\bullet)\}$. The inequality constraints in (4) are reformulated as the probability of the constraints holds with a given probability, $Pr\{g_i(\bullet) \leq \bar{g}_i\} \geq \alpha$. The equality constraints, such as the process equation in (2) and the weir definitions in (3), are substituted into the cost function (1) and inequality constraints (4) so that the constraints are only inequalities, and the only state the system is explicitly depending on is the initial state $x_0$. For general stochastic equality constraints, further substitution might be applied.

Due to the presence of weirs with (3), the resulting probability functions become meaningless as will be shown later. Therefore, we will reformulate the CC-MPC formulation, such that the inclusion of weir structures gives a sensible expression of the probabilistic formulation.
2.1 Revised CC-MPC Formulation

In our revised formulation of CC-MPC, we will formulate how to include weir structures in the probabilistic formulation, but we will also consider the feasibility of the program, as well as overflow determination. For simplicity of notation, we will substitute the state $x_k$ in the formulation with the propagated process equation from Eq. (2), such that the cost function and constraints are only dependent on the initial state $x_0$ and the variables across the prediction horizon. For the reformulation, we maintain the same formulation of the cost function as mentioned above for standard CC-MPC and given below

$$J = \min_{u} \left\{ f(x_0, u, z^{ref}, w, q^w) \right\}$$  \hspace{1cm} (5)

Where $q^w$ is written for clarity of the presence of weirs.

The formulation of the probability constraints used in CC-MPC to describe the inequality constraints, can cover sets of constraints\cite{11,12} or individual constraints\cite{13,14}, known as joint or marginal probability constraints respectively. The correct approach to the probability constraints depends on the specific system and the correlation between constraints. Given that the weir overflows relate to individual structures and times, we will in this work, consider the marginal approach. Where the approach to the reformulation of the probabilistic inequality constraints depends on the specific constraint, rather than a collection of constraints. For simplicity, we apply the marginal approach to all of the constraints, thereby assuming the set of marginal probabilities to be a fair approximation of the correlated joint probability constraints\cite{11}. For the discussion and application, we will define the marginal probability $a_i$ directly, not considering the corresponding joint equivalent.

If the constraint does not contain a weir element, meaning that no weir overflow is defined by this particular constraint, then the direct probabilistic approach from CC-MPC can be utilized to handle the uncertainty. Below is shown the probabilistic rewriting of the ith inequality constraint (6), into the quantile function-based constraint (8), with arrows indicating the order of steps in the process.

$$g_i(\bullet) \leq \bar{g}_i$$  \hspace{1cm} (6)

$$\rightarrow \Pr\{g_i(\bullet) \leq \bar{g}_i\} \geq \alpha$$  \hspace{1cm} (7)

$$\rightarrow \Phi^{-1}_{\bar{g}_i}(\alpha) \leq \bar{g}_i$$  \hspace{1cm} (8)

The quantile function in the resulting constraint is based on the distribution of the original constraint. Given the optimization variables are contained within the quantile function, this generally becomes more difficult to solve optimization-wise, as the constraints get more complex and nonlinear. Depending on the exact constraint distribution, then by utilizing a suitable standardization of the constraint distribution, this might be simplified so that the optimization variables are independent of the quantile function. The approach of such standardization trick is distribution-type specific; if e.g. the constraint distribution is defined purely by its expectation and variance as in the case of the Gaussian distribution, then the standardization can be done as shown in Eq. (9) for the constraint in Eq. (8). In the rest of our discussions of the reformulation of the CC-MPC, in our notation we will assume the Gaussian standardization is applicable for the probabilistic constraints, with comments on the general unstandardized case.

$$E\{g_i(\bullet)\} \leq \bar{g}_i - \sigma_{g_i(\bullet)}\Phi^{-1}(\alpha)$$  \hspace{1cm} (9)

If the constraint $g_i(\bullet, q^w_{k,i})$ does define a weir overflow $q^w_{k,i}$, then the direct probabilistic approach results in a meaningless probability. This is due to the weir element making the constraint intrinsically feasible, by counteracting the breaching of the constraint with the weir flow, as demonstrated below:

$$g_i(\bullet, 0) \leq \bar{g}_i \rightarrow g_i(\bullet, q^w_{k,i}) \leq \bar{g}_i, \quad q^w_{k,i} = 0$$  \hspace{1cm} (10)

$$g_i(\bullet, 0) > \bar{g}_i \rightarrow g_i(\bullet, q^w_{k,i}) = \bar{g}_i, \quad q^w_{k,i} > 0$$  \hspace{1cm} (11)

$$\rightarrow \Pr\{g_i(\bullet, q^w_{k,i}) \leq \bar{g}_i\} = 1$$  \hspace{1cm} (12)

where regardless of which parameters $\bullet$ the constraint depends on, the weir function $t_w$ and switching function $T(\bullet)$ of the weir overflow will be depending on the same parameters so that the constraint holds.

For this reason, including the above constraint in the optimization formulation is redundant. In order for achieving a statistical bound on the overflow generation, we instead turn to the probability of keeping the weir overflow $q^w_{k,i}$ non-positive, where we
can see this is related to its switching function \( T(\bullet) \), as shown below.

\[
\begin{align*}
Pr\{q_{k,i}^w \leq 0\} &= Pr\{T_i(\bullet) \leq 0\} \geq \gamma \\
\rightarrow \quad \Phi_{T_i(\bullet)}^{-1}(\gamma) &\leq 0 \\
\rightarrow \quad E\{T_i(\bullet)\} &\leq -\sigma_{T_i(\bullet)}\Phi^{-1}(\gamma)
\end{align*}
\]

This allows us to formulate probabilistic constraints for both types of inequality constraints with or without weir elements, as were shown in (9) and (15) for the standardized case, and (8) and (14) for the general case.

### 2.2 Feasibility

In the above, we only considered handling the uncertainty such that a given solution would be feasible in the real system with known probability. This leads to probabilistic restrictions on the inequality constraints, but these restrictions will also lead to more rain scenarios causing infeasibility during computations. By utilizing slack variables with a suitable cost term in the cost function, we can restore the original feasibility of the constraints by the following approach, while keeping the probabilistic restrictions, when possible.

\[
\begin{align*}
E\{g_i(\bullet)\} &\leq \bar{g}_i + s_k - \sigma_{g_i(\bullet)}\Phi^{-1}(\alpha) \\
E\{T_i(\bullet)\} &\leq c_k - \sigma_{T_i(\bullet)}\Phi^{-1}(\gamma) \\
0 &\leq s_k, c_k
\end{align*}
\]

Where the constraints without weirs are given by (16) and the weir defining constraints is given by (17). For the constraints without weirs (16), an extra constraint is necessary to preserve the original constraint of the system:

\[
0 = E\{h(x_0, u, z^{ref}, w, q)\}
\]

The addition of an upper limit to slack variables \( s_k \) ensures that the found solution does not intentionally breach the physical constraints of the system, mathematically. The definition of the upper bound is dependent on the interpretation of the tightening of the constraint, \( \sigma_{g_i(\bullet)}\Phi^{-1}(\alpha) \) for the Gaussian case, preserving the physical constraint for the expectation. A similar simple general bound could be \( (\Phi_{g_i(\bullet)}^{-1}(\alpha) - E\{g_i(\bullet)\}) \).

Using the above versions of the constraints, the formulation of the optimization program for feasible CC-MPC can be written as the following:

\[
J = \min_{u,k,s} E\{ f(x_0, u, z^{ref}, w, q) \} + l(c, s) \\
0 = E\{h(x_0, u, w, q)\} \\
E\{g_i(\bullet)\} &\leq \bar{g}_i + s_k - \sigma_{g_i(\bullet)}\Phi^{-1}(\alpha) \\
E\{T_i(\bullet)\} &\leq c_k - \sigma_{T_i(\bullet)}\Phi^{-1}(\gamma) \\
s_k &\leq \sigma_{g_i(\bullet)}\Phi^{-1}(\alpha) \\
0 &\leq s_k, c_k
\]

where the additional function \( l(c, s) \) in the cost functions is the cost term of the slack variables, penalizing their usage.

### 2.3 Overflow Approximation

So far, we have been considering an optimization formulation with a dynamic description of the weir overflows included in it. Given the nature of weir overflows being binary as seen in (3) and therefore not being convex, the inclusion of the dynamic can lead the optimization program to be computational heavy. One approach to deal with this is to treat the weir overflows as additional optimization variables and penalize their utilization. This has previously been shown in practice to be a reasonable approximation of weir flows.

Given that overflows cannot be negative, a constraint for this need to be added when using this approach. Another aspect is the determination of the value of the overflow for approximation; Given that we are minimizing the overflow, we need a constraint telling us the minimum size of the overflow. A fitting constraint for this is the original constraint containing the overflow, due to it being its very definition. We can utilize the expectation of this constraint, to achieve a description of the overflow size and
still take care of the uncertainty. Based on the added constraint shown below, our approximated overflow can be considered the expected overflow of the system in some sense.

\[
E\{g_i(\bullet, q^w)\} \leq \bar{g}_i
\]

\[
0 \leq q^w_k
\]

With the approximation approach we have utilized, we can formulate the optimization program as below. The cost function now includes a penalty term on the overflow variables. This term is a penalty on the accumulated overflow volumes at each sample in the predictions.

\[
J = \min_{u, z, q^w} E\{f(x_0, u, z^{ref}, w, q)\} + l(c, s) + \sum_{k=0}^{N} M^T_k q^w_k
\]

\[
0 = E\{h(x_0, u_k, w_k, q^w_k)\}
\]

\[
E\{g_i(\bullet)\} \leq \bar{g}_i + s_k - \sigma_{g_i(\bullet)} \Phi^{-1}(\alpha)
\]

\[
E\{T_i(\bullet)\} \leq c_k - \sigma_{T_i(\bullet)} \Phi^{-1}(\gamma)
\]

\[
s_k \leq \sigma_{g_i(\bullet)} \Phi^{-1}(\alpha)
\]

\[
E\{g_i(\bullet, q^w_k)\} \leq \bar{g}_i
\]

\[
0 \leq s_k, c_k, q^w_k
\]

3 | MODEL & COST

In this section, we will outline an example of the application of the revised CC-MPC formulation. For clarity, we will first outline the design model of the deterministic MPC followed by the stochastic counterpart. The system considered is a linear model of the Astlingense sewer network illustrated in Figure 2. The Astlingen system consists of 10 catchment areas connected to a system of 6 controllable tanks and 4 independent weirs, all capable of flooding the nearby river by overflows. For the cost function of the MPC given below, we will utilize a mix of linear and quadratic cost terms, including the overflow approximation approach, discussed previously.

\[
J = \min_{u_k, q^w_k} \sum_{k=0}^{N} ||\Delta u_k||^2_R + Q^T z_k + W^T V^w_k
\]

\[
V^w_k = \Delta T \sum_{i=0}^{k} q^w_i
\]

where the cost is minimized over an N step prediction horizon on the system. The first term in the cost function \(||\Delta u_k||^2_R\) is a quadratic penalty on the control change \(\Delta u\), penalizing changes from the current operation. The second term \(Q^T z_k\) is a linear cost on the output objectives \(z\), providing penalties to undesired changes in system behaviors. The output objectives \(z\) correspond to the following system objectives:

- Maximizing flow to WWTP
- Minimizing flow to the environment

The third term \(\frac{1}{\Delta T} W^T V^w_k\) is a linear penalty on the accumulated overflow volume at time \(k\). Where the volumes \(V^w_k\) at time \(k\) are defined by (36), as the sums of each overflow until and including time \(k\) for each element separately.

The system can be considered to consist of tanks, pipes with weirs, and delay pipe elements. The state vector \(x_k\) then consists of the tank volumes \(V_k\) and the pipe delays \(D_k\), which will be described individually in the following discussion. If the sizes of the delays are in multiples of the sampling time, then they can be considered a cascade of delays \(D_{k,i}\) each of the size of the sampling time. The dynamics of the tanks and delays are described by the following equations:

\[
V_{k+1,i} = V_{k,i} + \Delta T(q^{in}_{k,i} - q^{out}_{k,i} - q^{w}_{k,i})
\]

\[
D_{k+1,i} = q^{in}_{k,i}
\]
The outflows of each element are described by the equations below, where V, P, and D indicate the type of element; tank volume, pipe flow, and delay flow respectively.

\[
\begin{align*}
q_{out,V}^{k,i} &= u_{k,i} \\
q_{out,P}^{k,i} &= q_{in}^{k,i} - q_{w}^{k,i} \\
q_{out,D}^{k,i} &= D_{k,i}
\end{align*}
\]  

(39)  

(40)  

(41)

The inflow \(q_{in}^{k,i}\) of the ith element, given below, is dependent on the connections of the elements in the system as shown in Table 1. Where the ith tank is denoted by \(T_i\), pipe i from catchment i by \(p_i\), and the n minute delay point to tank i is given by \(T_i:n\).

\[
q_{in}^{k,i} = w_{k,i} + \sum_{j \in U} u_{k,j} + \sum_{j \in Q^V} q_{out,V}^{k,j} + \sum_{j \in Q^P} q_{out,P}^{k,j} + \sum_{j \in Q^D} q_{out,D}^{k,j}
\]

(42)

Where the j denotes the flows of the subsets \(U, Q^V, Q^P, Q^D\) of all control flows, passive tank outflows, pipe outflows, and delay outflows respectively. The variable \(w_{k,i}\) indicates the rain inflow to the system part, described by \(w_{k,i} = A_i I_{k,i} + q_{i,dwf}\), where \(A_i\) is the catchment area, \(I_{k,i}\) is the rain intensity and \(q_{i,dwf}\) is the dry weather flow.

The inequality constraints are formulated below, where the upper and lower limits of the tank volumes, the pipe outflow, control flow, and the weir overflows are stated for time k. The constraints are based on the individual elements i of the system. Not all of the constraints are applicable for all types of elements, e.g., (43) are only applicable for tanks. The values of each
Inflow occurrence of overflow. With the corresponding switching and weir function given by (48).

Where the expected tank volume, delay flow, element inflow, tank outflow, pipe outflow, and delay outflow are given by (50)-(55).

Both the rewritten cost function and the later inequality constraints depend on the expectation of the system’s subpart equations. The cost function of the revised CC-MPC can then be written as:

$$J = \min_{u,c,s,q} \sum_{k=0}^{N} E \{ ||\Delta u_k||_R^2 + Q^T z_k + W^T V^w_k \} + W^T c + W^T s$$  \hspace{1cm} (49)$$

### 3.1 Stochastic Model

The revised formulation of CC-MPC with overflow handling presented earlier can now be applied to the system described above. The revised formulation of CC-MPC with overflow handling can now be applied to the system described above. The cost function of the revised CC-MPC can then be written as:

$$J = \min_{u,c,s,q} \sum_{k=0}^{N} E \{ ||\Delta u_k||_R^2 + Q^T z_k + W^T V^w_k \} + W^T c + W^T s$$  \hspace{1cm} (49)$$

Both the rewritten cost function and the later inequality constraints depend on the expectation of the system’s subpart equations. Where the expected tank volume, delay flow, element inflow, tank outflow, pipe outflow, and delay outflow are given by (50)-(55) respectively.

$$E\{V_{k+1,i}\} = E\{V_{k,i}\} + \Delta T (E\{q_{k,i}^{in}\} - E\{q_{k,i}^{out,V}\} - q_{k,i}^w)$$  \hspace{1cm} (50)$$

$$E\{D_{k+1,i}\} = E\{q_{k,i}^{in}\}$$  \hspace{1cm} (51)$$

$$E\{q_{k,i}^{in}\} = E\{w_{k,i}\} + \sum_{j \in Q_i^t} E\{q_{k,j}^{out,V}\} + \sum_{j \in Q_i^p} E\{q_{k,j}^{out,P}\} + \sum_{j \in Q_i^p} E\{q_{k,j}^{out,D}\}$$  \hspace{1cm} (52)$$

$$E\{q_{k,i}^{out,V}\} = u_{k,i}$$  \hspace{1cm} (53)$$

$$E\{q_{k,i}^{out,P}\} = q_{k,i}^{in} - q_{k,i}^w$$  \hspace{1cm} (54)$$

$$E\{q_{k,i}^{out,D}\} = E\{D_{k,i}\}$$  \hspace{1cm} (55)$$

### Table 1: Inflows to the different elements of the systems

| Element | Inflow |
|---------|--------|
| T1      | $q_{k,T1}^{in} = q_{k,T1}^{out,D}$ |
| T2      | $q_{k,T2}^{in} = w_{k,2}$ |
| T3      | $q_{k,T3}^{in} = w_{k,3} + q_{k,T3}^{out,D}$ |
| T4      | $q_{k,T4}^{in} = w_{k,4} + q_{k,T4}^{out,D}$ |
| T5      | $q_{k,T5}^{in} = w_{k,5}$ |
| T6      | $q_{k,T6}^{in} = w_{k,6} + q_{k,T6}^{out,D}$ |
| T3:5    | $q_{k,T3:5}^{in} = q_{k,T3:5}^{out,D}$ |
| T3:10   | $q_{k,T3:10}^{in} = q_{k,T3:10}^{out,D}$ |
| T3:15   | $q_{k,T3:15}^{in} = u_{k,6} + q_{k,p8}^{out,P}$ |
| T4:5    | $q_{k,T4:5}^{in} = q_{k,T4:5}^{out,D}$ |
| T4:10   | $q_{k,T4:10}^{in} = q_{k,p7}^{out,P}$ |

| Element | Inflow |
|---------|--------|
| p7      | $q_{k,p7}^{in} = u_{k,7}$ |
| p8      | $q_{k,p8}^{in} = u_{k,8}$ |
| p9      | $q_{k,p9}^{in} = u_{k,9}$ |
| p10     | $q_{k,p10}^{in} = u_{k,10}$ |
| T1:5    | $q_{k,T1:5}^{in} = u_{k,2} + q_{k,T1:5}^{out,D}$ |
| T1:10   | $q_{k,T1:10}^{in} = u_{k,4} + q_{k,T1:10}^{out,D}$ |
| T1:15   | $q_{k,T1:15}^{in} = u_{k,5} + q_{k,T1:15}^{out,D}$ |
| T1:20   | $q_{k,T1:20}^{in} = q_{k,p10}$ |
| T6:5    | $q_{k,T6:5}^{in} = q_{k,T6:5}^{out,D}$ |
| T6:10   | $q_{k,T6:10}^{in} = q_{k,T6:10}^{out,D}$ |
| T6:15   | $q_{k,T6:15}^{in} = q_{k,p9}^{out,P}$ |
In the following paragraphs, the formulation of each inequality constraint is given in the context of the corresponding subpart. The resulting formulation of the lower constraint of the tank volume is given by:

$$V_{k,i} \Phi^{-1}(j) - s_{j,k} \leq E\{V_{k,i}\} - \Delta T q_{k,i}^{w}$$

$$0 \leq s_{j,k} \leq \sigma_{V_{k,i}} \Phi^{-1}(a)$$

(56)

(57)

where \(j\) indicates the specific constraint. The probabilistic formulation of the upper constraint of the tank volume is then defined by the switching function as written in (58). The constraint for overflow approximation is given by (59).

$$E\{V_{k,i}\} \leq \bar{V}_{i} - \sigma_{V_{k,i}} \Phi^{-1}(\gamma) + c_{k}$$

$$E\{V_{k,i}\} - \Delta T q_{k,i}^{w} \leq \bar{V}_{i}$$

$$0 \leq c_{k}$$

(58)

(59)

From (53), we know that the tank outflows are controlled and therefore deterministic, this gives us that the lower limit is the same as in (45). For the upper limit, the probabilistic formulation is given by (61) and (62).

$$u_{k,i} \leq \beta E\{V_{k,i}\} - \beta \sigma_{V_{k,i}} \Phi^{-1}(a) + s_{j,k}$$

$$0 \leq s_{j,k} \leq \beta \sigma_{V_{k,i}} \Phi^{-1}(a)$$

(61)

(62)

The limits on the pipes with weirs are given by (63) and (64) for the lower limit, and by (65)-(67) for the upper limit.

$$\sigma_{q_{k,i}^{out.p}} \Phi^{-1}(a) - s_{j,k} \leq E\{q_{k,i}^{out.p}\}$$

$$0 \leq s_{j,k} \leq \sigma_{q_{k,i}^{out.p}} \Phi^{-1}(a)$$

$$E\{q_{k,i}^{in}\} - q_{k,i}^{w} \leq \bar{q}_{i}^{out.p}$$

$$E\{q_{k,i}^{in}\} \leq \bar{q}_{i}^{out.p} + c_{j,k} - \sigma_{q_{k,i}^{in}} \Phi^{-1}(\gamma)$$

$$0 \leq c_{j,k}$$

(63)

(64)

(65)

(66)

(67)

Given that there is, per definition, no uncertainty in optimization variables, the constraints on the control and weir overflow are deterministic and are therefore the same as in (46) and (47).

### 3.2 Benefits and costs

The utilization of the approximation method discussed above has some significant drawbacks as previously discussed in [22]. The main drawbacks are the loss of design freedoms in the weighting of the cost function. These come from the extra weights on the aggregated overflow volume has to be relatively higher than the main terms of the cost functions, and have hierarchically weightings depending on their relative placement in the systems.

These design restrictions on the weightings limit the flexibility of the control with regard to the planning of overflow countermeasures. While the revised CC-MPC does not change these drawbacks, it might give a possible remedy for the hierarchical weightings requirement. If a given weir overflow in the system is more attractive to society than weir overflow further down the system (e.g. downstream is a bathing area). Then by having a higher probability guarantee (\(a, \gamma\) in [9], [15], and section 3.1) on the downstream part than on the specific upstream overflow, the downstream constraints will be less likely to cause an overflow, if it is possible to avoid.

The revised CC-MPC formulation has the drawbacks of introducing more optimization variables and inequality constraints, even without the weir overflow approximation. These drawbacks arise from the conserved feasibility through the slack variables and the constraints on these for elements without weirs. The revised CC-MPC also has the clear benefits of conserving feasibility but more importantly giving statistical constraints on overflow generation, similar to the CC-MPC formulation for systems without internal overflow description.

### 3.3 Variance of Constraints

Given the assumption of the variance of the probabilistic constraints exist, and that the probabilistic constraints are scalar, the variance of each constraint is also scalar. We can utilize this feature to derive a computationally simple method for computing
TABLE 2 Cost function weighting of accumulated overflow volume $W$, showing a higher cost for upstream elements.

|   | T1       | T2       | T3       | T4       | T5       | T6       | p7       | p8       | p9       | p10      |
|---|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
|   | $10000\frac{1}{\Delta T}$ | $5000\frac{1}{\Delta T}$ | $5000\frac{1}{\Delta T}$ | $5000\frac{1}{\Delta T}$ | $10000\frac{1}{\Delta T}$ | $10000\frac{1}{\Delta T}$ | $10000\frac{1}{\Delta T}$ | $15000\frac{1}{\Delta T}$ | $5000\frac{1}{\Delta T}$ |

the variance for the constraints. Firstly, we need to define the variance of each constraint.

$$\sigma^2_{\Delta_{k,i}} = \sigma^2_{\Delta_{k-1,i}} + \Delta T^2 \sigma^2_{\Delta_{k-1,i}}$$

$$\sigma^2_{D_{k,i}} = \sigma^2_{\Delta_{k-1,i}}$$

$$\sigma^2_{q_{in,P}} = \sigma^2_{\Delta_{k-1,i}}$$

$$\sigma^2_{q_{in,D}} = \sigma^2_{\Delta_{k-1,i}}$$

$$\sigma^2_{q_{out}} = \sigma^2_{\Delta_{k-1,i}} + \sum_{j\in\Omega^{p}} \sigma^2_{q_{in,P}} + \sum_{j\in\Omega^{D}} \sigma^2_{q_{in,D}}$$

where each source of uncertainty is assumed independent both temporally and spatially. This gives equations for the variances, which is a linear model of the initial state variance and the rain inflow variances as shown in (73). Utilizing this, all of the constraints discussed above can be combined into a matrix inequality covering the entire prediction horizon as shown in (74)-(76).

$$\sigma^2 = \Theta \sigma^2_{\Delta} + \Gamma \sigma^2_{W}$$

$$\Omega_{u} u + \Omega_{g} q_{in} \leq \Omega_{\text{const}} + \Omega_{c} s + \Omega_{\text{w}} \Phi^{-1}(y)$$

$$\Omega_{\text{const}} = \Omega_{c} E\{x_{0}\} + \Omega_{\text{w}} E\{w\}$$

$$\sigma_{\text{Diag}} = \sqrt{\text{diag}(\sigma^2)} \in \mathbb{R}^{nxn}$$

where the $\Omega_{u}$ matrices contain the terms from every constraint regarding the corresponding predicted variables $y$. The $\Omega_{r}$ matrix contains the quantile terms’ sign convention, while the $\Omega$ vector contains the constant constraint terms, such as tank limits, $\Phi$. 

4 | RESULTS & DISCUSSION

In the previous sections, we have introduced the revised CC-MPC formulation, in this section, we will focus on analyzing the difference between the performance of the revised CC-MPC and the classical deterministic MPC applied to the Astingen model introduced earlier. In the simulations, the examples of the MPC designs given in section 3 are used with a prediction horizon of a 100 min. and a 5 min. sampling time. The weights of each objective in the cost function have the following values; 2 for minimizing flow to nature, −1 for maximizing flow to WTTP, and 0.01 for the change in control flow. The higher the absolute weight is the higher the priority, while a negative weight indicates maximization, instead of minimization in the objective. The weighting of the accumulated overflow volume is given in table 2, where it can be seen the weights vary accordingly to the placemen of the overflows in the system, as described previously. The usages of the slack variables are weighted uniformly with 100.

Several scenarios with varying parameters have been run during the simulations. The profiles for the rain inflows in simulations were all step rains, where the rain intensity was varied from 0.5 to 6 $\mu m/s$ (1.8 to 21.6 $mm/h$), and the rain duration varying from a half-hour to five hours, with 0.5 $\mu m/s$ and half-hour intervals. For the revised CC-MPC, the probability guarantee was equal across all constraints and was varied between scenarios, with values of 90%, 80%, and 70% respectively. The deterministic MPC is assumed to have perfect forecasts of the rain inflow, while the revised CC-MPCs are operating with uncertainties following a truncated Gaussian distribution, with the expectation being the actual rain inflow. The size of the uncertainty for the CC-MPC (the standard variation $\sigma$), was chosen as a third of the expectation plus a constant deviation of 0.01 $\mu m/s$, to avoid zero uncertainty. The truncated distribution of the uncertainty was assumed non-negative and below three $\sigma$ above the expectation, resulting in all realizations of the inflows being within two times the expected non-zero value. A realization of a rain scenario can be seen in Figure 3 with the actual rain and bounds on the uncertainty, included.
FIGURE 3 Realization of the rain forecast of a rain scenario with 3.5 $\mu$m/s intensity and a three and a half-hour rain duration. Showing the uncertain prediction around the actual step inflow.

FIGURE 4 The difference of Maximum computation time during each rain scenario simulation of the perfect MPC and the revised CC-MPC computed as MPC - revised CC-MPC

TABLE 3 Resume of Maximum Computation times

|                  | MPC  | 70% RCC-MPC | 80% RCC-MPC | 90% RCC-MPC |
|------------------|------|-------------|-------------|-------------|
| Mean Time [sec.] | 0.1111 | 0.2660      | 0.2568      | 0.2561      |
| Min Time [sec.]  | 0.0712 | 0.1535      | 0.1574      | 0.1565      |
| Max Time [sec.]  | 0.3680 | 1.0293      | 0.6689      | 0.7609      |

4.1 | Computation Time

From Figure 4 we can observe the difference in the maximum computation time between the deterministic MPC and the revised CC-MPC with the three chosen probability guarantees. It can be observed that in general, the revised CC-MPCs are around 0.1 seconds slower than the deterministic MPC, with no apparent trends in the falls and rises of the differences in the computation times. The occasional difference in computation time of both types of controllers is due to numerical variations. In Table 3, the mean, minimum, and maximum of the maximum computation times for all scenarios are given, providing a quick resume and confirming the general slowness of the CC-MP. Further, we can see that in the worst-case CC-MPC is around 2-3 times slower than MPC, and in the best case still slower than the average MPC scenario.

4.2 | Weir Overflow

In this part, we focus on the results of the simulation to do with weir overflows. In Figure 5, one of the simulations results is shown, showing the overflow over time for both the MPC with perfect forecast and the revised CC-MPC with 90% probability...
bound on the constraints. We can observe that the experienced overflow of the system is identical between the two controllers. This is further supported by the percentage differences between the controllers for each rain scenario, shown in Figure 7. Here we can see that in general, the difference is approximately zero, but also that a few scenarios have larger differences. These differences are due to the accumulated differences in control action between the controllers. For the CC-MPCs, their stricter constraints provide more conservative actions than the MPC, which combined with finite prediction horizons can lead to the CC-MPCs performing better than MPC when not all of the future rain is included in the horizon at each step. The CC-MPCs conservatism, therefore, makes it more robust towards the unknown future rain, by providing what happens to be a better initial state than the MPC for the next time step.

The total volume of weir overflow of the deterministic MPC can be seen in Figure 6. By comparing the total overflow volume of Figure 6 to the percentage differences of Figure 7, we can observe that the larger differences occurred, when the overflow volume was small for the MPC. Thus making small divergences due to the control strategy, relatively big in percentages.

4.3 | WWTP

In this part, we focus on the results of the simulation to do with the amount of water sent to the wastewater treatment plant of the system. In Figure 8 and Figure 9, we can observe the volumetric difference and the percentage difference in wastewater sent to the treatment plant respectively.

We can see from the volume difference, that the deterministic MPC has an outflow, which is generally larger than the outflows of the revised CC-MPC, by somewhat constant volume bias around 120 m³ depending on the probability bound. We can observe from the percentage difference that while the bias is constant, the percentage difference primarily decreases with the duration of the rain, and not with the intensity of the rain. We can further see that the decrease in percentage difference corresponds to the increase in the outflow, depicted for the deterministic MPC in Figure 10.

4.4 | Scenario example

In this section, we will focus on a representative simulation and the operational behavior across the system. The corresponding realization of the rain and the total overflow historic were already shown earlier in Figure 3 and Figure 5. From Figure 11, we can observe the computational difference given as the cost difference. Here we can observe that the cost of the revised CC-MPC is always higher than for the deterministic MPC, with revised CC-MPC having a constant additional cost. It can also be observed that the revised CC-MPC with the highest probability bound has the highest cost, as one would expect.
FIGURE 7 Percentage difference in total overflow experience, between revised CC-MPC and MPC with perfect knowledge.

FIGURE 8 The difference in wastewater volume sent to the treatment plants, between the revised CC-MPC and the MPC with perfect knowledge.

FIGURE 9 The percentage difference in wastewater volume sent to the treatment plants, between the revised CC-MPC and the MPC with perfect knowledge.

FIGURE 10 The total outflow volume of the deterministic MPC with perfect knowledge.
The tank operational behavior of the system can be observed in Figure 12, with the tank outflow controllers displayed in Figure 13. From the tank volumes, we can once again see that the different MPCs agree on the optimal amount of volume exceeding the tanks. We can also observe that the revised CC-MPCs find a steady-state volume, which is higher than the steady-state volume of the deterministic MPC. This can further be observed by the control flows, where we can see that the control flows of both the deterministic MPC and the revised CC-MPC with 90% probability bound stay below the individual physical control constraint bounds of the control flows. We can see that the deterministic MPC, in general, operates slightly higher than the revised CC-MPC, and as expected can operate on the constraint bound. While the graph only depicts individual physical control constraint bounds, it is still interesting to note how far the steady-state operation of the revised CC-MPC is operating from the constraint bounds, due to the stochastic restraints. It can also be noted that the difference in operation between the two MPCs, first occurs after the rain has ended and not before, where rain was forecasted to happen. This indicates the difference is due to cost priority of the steady-state operation.

4.5 | Thoughts on probability bounds

Based on the results discussed, we can comment on the choice of the probability bounds $\alpha$ and $\gamma$. We have seen that regardless of the chosen value, the general behavior across the different scenarios does not change, when considering overflow percentage difference or treated wastewater volume. We have further seen, as one would expect, that the differences between the revised CC-MPC and the deterministic MPC converge towards zero as the probability bounds decrease towards the 50% that corresponds to a deterministic MPC with no restriction of the constraints. A similar tendency was observed with the cost difference.

We can therefore see that higher choices of the probability bounds result in a performance further from the performance of the deterministic MPC, when the MPCs all have an expected forecast identical to the actual rain inflow, with the higher probability bounds providing larger gaps to the limits of the constraints.

5 | CONCLUSIONS

In this paper, we have presented a revised formulation of Chance-Constrained MPC (CC-MPC) inspired for application in sewer networks. The main aspect of the reformulation focuses on preserving feasibility and introducing overflow handling of binary structures. The mathematical formulation and reasoning behind the revised CC-MPC has been stated and applied to a model of the Astlingen sewer system for testing the method. A comparison of the performance of the revised CC-MPC and deterministic MPC was based on simulations with idealized step rain inflows as the perturbation of the Astlingen system. From the results of the simulations, it was shown that the weir overflow avoidance of the revised formulation provides similar results as the MPC.
with a perfect forecast. Indicating the revised CC-MPC as an alternative to MPC, when a perfect forecast is not achievable. The results also showed a trade-off with regards to the worst-case computation time, which in general increased slightly.

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TABLE A1 Astlingen System Data - Tanks

| Parameter | $V_{T1}$ | $V_{T2}$ | $V_{T3}$ | $V_{T4}$ | $V_{T5}$ | $V_{T6}$ |
|-----------|----------|----------|----------|----------|----------|----------|
| Value: 700 m$^3$ | 1000 m$^3$ | 2600 m$^3$ | 500 m$^3$ | 500 m$^3$ | 600 m$^3$ |

| Parameter | $\bar{u}_{T1}$ | $\bar{u}_{T2}$ | $\bar{u}_{T3}$ | $\bar{u}_{T4}$ | $\bar{u}_{T5}$ | $\bar{u}_{T6}$ |
| Value: 271.28 L/s | 140 L/s | 190 L/s | 80 L/s | 39 L/s | 175 L/s |

| Parameter | $\beta_{T1}$ | $\beta_{T2}$ | $\beta_{T3}$ | $\beta_{T4}$ | $\beta_{T5}$ | $\beta_{T6}$ |
| Value: 387.54 μs$^{-1}$ | 140 μs$^{-1}$ | 73.08 μs$^{-1}$ | 160 μs$^{-1}$ | 78 μs$^{-1}$ | 291.67 μs$^{-1}$ |

| Parameters | $A_{T1}$ | $A_{T2}$ | $A_{T3}$ | $A_{T4}$ | $A_{T5}$ | $A_{T6}$ |
| Value: 33.00 ha | 22.75 ha | 18.00 ha | 6.90 ha | 15.60 ha | 32.55 ha |

| Parameters | $q_{dwf}^{p7}$ | $q_{dwf}^{p8}$ | $q_{dwf}^{p9}$ | $q_{dwf}^{p10}$ | $q_{dwf}^{p10}$ |
| Value: 12.51 L/s | 10.56 L/s | 8.71 L/s | 2.88 L/s | 12.69 L/s | 21.26 L/s |

TABLE A2 Astlingen System Data - Pipes

| Parameters | $q_{out,P}^{p7}$ | $q_{out,P}^{p8}$ | $q_{out,P}^{p9}$ | $q_{out,P}^{p10}$ |
| Value: 0.0855 m$^3$/s | 0.48533 m$^3$/s | 0.12917 m$^3$/s | 0.20367 m$^3$/s |

| Parameters | $A_{p7}$ | $A_{p8}$ | $A_{p9}$ | $A_{p10}$ |
| Value: 4.75 ha | 28.00 ha | 6.90 ha | 11.75 ha |

| Parameters | $q_{p7}$ | $q_{p8}$ | $q_{p9}$ | $q_{p10}$ |
| Value: 6.56 L/s | 5.18 L/s | 4.43 L/s | 3.14 L/s |

APPENDIX

A ASTLINGEN DATA

In this appendix, the parameter values of the Astlingen system is given. The units are given in SI units, hectares (ha) and liters (L).