Where do long-period comets come from?
26 comets from the non-gravitational Oort spike

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ABSTRACT
Since 1950, when Oort published his hypothesis several important new facts were established in the field. In this situation the apparent source region (or regions) of long-period comets as well as the definition of the dynamically new comet are still open questions, as well as the characteristics of the hypothetical Oort Cloud. The aim of this investigation is to look for the apparent source of selected long period comets and to refine the definition of dynamically new comets.

Basing on pure gravitational original orbits all comets studied in this paper were widely called dynamically new. We show, however, that incorporation of the non-gravitational forces into the orbit determination process significantly changes the situation.

We determined precise non-gravitational orbits of all investigated comets and next followed numerically their past and future motion during one orbital period. Applying ingenious Sitarski’s (1998) method of creating swarms of virtual comets compatible with observations, we were able to derive the uncertainties of original and future orbital elements, as well as the uncertainties of the previous and next perihelion distances.

We concluded that the past and future evolution of cometary orbits under the Galactic tide perturbations is the only way to find which comets are really dynamically new. In our sample less than 30\% of comets are in fact dynamically new. Most of them had small previous perihelion distance. On the other hand, 60\% of them will be lost on hyperbolic orbits in the future. This evidence suggests that the apparent source of long-period comets investigation is a demanding question.

We also have shown that a significant percentage of long-period comets can visit the zone of visibility during at least two or three consecutive perihelion passages.

Key words: Solar system :general, Oort Cloud, comets:general

1 INTRODUCTION
Jan Oort (1950) proposed a hypothesis, that the Solar System is surrounded by a huge, spherical cloud of comets, now called the Oort Cloud. As the argument for the existence of such a cloud he completed a list of 19 original orbit\cite{m1950} of well observed long-period (hereafter LP) comets and showed, that their inverse of semi-major axis $1/a$ has the distribution apparently peaked near zero, at the positive side. He concluded that new long-period comets come from distances of 50 000 to 150 000 AU. He also showed, that perturbations by passing stars can change cometary orbit significantly, making it observable as “dynamically new” long-period comet. During the last 60 years several new factors occurred in this field.

First, the population of precise original cometary orbits increased to several hundreds. In the recently published, 17th edition of the Catalogue of Cometary Orbits (\cite{MW2008}) (hereafter MWC08) 499 precise original and future cometary orbits are listed. Second, new and dominating perturbing force must be included in the model: tidal action of the Galactic disk and Galactic centre. They are negligible when calculating original and future orbits but when following the past or future motion of a comet at larger distances one must take them into account. It means, that looking at the original orbit elements we cannot tell the past or predict the future motion of this body, for example foretell the previous/next perihelion or aphelion distances. This was demonstrated by Dybczyński \cite{D2001, D2006}. Third, we learned how to investigate and in many cases we can successfully determine non-gravitational forces in the motion of long-period comets. This can significantly change our knowledge of original and future orbits (see for example: \cite{MW1973, K2001, K2006}). It also means, that tens of precise original orbits with determined non-gravitational forces could be added to the MWC08 list.
Taking all this into account, the apparent source region (or regions) of long-period comets as well as the definition of the dynamically new comet are still open questions, as well as the characteristics (if not the existence) of the hypothetical Oort Cloud. In this paper we start to investigate these problems concentrating on 26 comets constituting so-called non-gravitational (hereafter NG) Oort spike, i.e. having $1/a_{ori,NG} < 10^{-4}$ AU$^{-1}$. This particular choice comes from the fact, that after applying non-gravitational model in orbit determination we obtain significantly different distribution of the inverse original semi-major axes. Almost all 1/$a_{ori}$ values were shifted to larger values and among them a lot of hyperbolic cases changed into elliptic ones. Since we want to study the implications of obtaining non-gravitational original orbits on the Oort hypothesis, the past (and future) motion of these 26 comets were analyzed at first.

In Section 2 we describe observational material and its processing. In the next section calculation of original and future cometary orbits for the selected 26 non-gravitational Oort spike comets is described in detail. Section 3 describes the numerical integration of the past and future motion of these comets under the action of Galactic tides. Then, results and conclusions are presented.

## 2 OBSERVATIONS AND THEIR PROCESSING

MWC08 includes 184 comets with $1/a_{ori} < 10^{-4}$ AU$^{-1}$ where for majority of them the orbits are determined with the highest accuracy. At first we looked at 154 most precise orbits. Thus, we took into account all comets with the quality class 1 according to the classification introduced by Marsden et al. (1978). About half of them have perihelion distances less than 3.0 AU, what means that we should suspect detectable influence of the non-gravitational forces. The inverse dependence of the strength of non-gravitational forces on the perihelion distance is widely expected and indeed is clearly visible in our Fig.5 (to be described in detail in the next section). We took into account almost all these objects (except ten comets discovered before 1950 where positional data were not available for us) and additionally all LP comets with the NG orbits in MWC08 (29 objects, $1/a_{ori}$ are not given for them in the Catalogue). In majority of cases, the observational material taken for orbital determinations is more complete than in the MWC08: for comets from the XIX century the observations were collected directly from the published papers, for comets from the first half of XX century the data were collected in Warsaw in cooperation with the Slovakian group at the Astronomical Institute in Bratislava and Tatranska Lomnica, more modern data were taken from the Minor Planet Electronic Circulars available through the Web Page at http://www.cfa.harvard.edu/.

| Name                      | Observational arc | No. obs. |
|---------------------------|-------------------|----------|
| C/1885 X1 Fabry           | 1885 12 02 – 1886 07 20 | 228      |
| C/1892 Q1 Brooks          | 1892 09 01 – 1893 07 13 | 191      |
| C/1913 Y1 Delavan         | 1913 10 25 – 1915 09 07 | 1009     |
| C/1940 R2 Cunningham      | 1940 08 25 – 1941 04 01 | 370      |
| C/1946 U1 Bester          | 1946 11 01 – 1948 10 02 | 142      |
| C/1952 W1 Mrkos           | 1952 10 12 – 1953 07 18 | 36       |
| C/1956 R1 Arend-Roland    | 1956 11 08 – 1958 04 11 | 249      |
| C/1959 Y1 Burnham         | 1959 01 04 – 1960 06 17 | 88       |
| C/1978 H1 Meier           | 1978 04 28 – 1979 12 09 | 287      |
| C/1986 P1 Wilson          | 1986 08 05 – 1989 04 11 | 688      |
| C/1989 Q1 Okazaki-Levy-R. | 1989 08 24 – 1989 12 24 | 231      |
| C/1989 X1 Austin          | 1989 12 06 – 1990 06 27 | 281      |
| C/1990 K1 Levy            | 1990 05 21 – 1992 04 01 | 678      |
| C/1991 F2 Helin-Lawrence  | 1991 02 23 – 1992 09 30 | 114      |
| C/1993 A1 Mueller         | 1992 11 26 – 1994 08 17 | 746      |
| C/1993 Q1 Mueller         | 1993 08 16 – 1994 04 17 | 526      |
| C/1996 E1 NEAT            | 1996 03 15 – 1996 10 12 | 249      |
| C/1997 Ba$_3$ Spacewatch  | 1997 01 11 – 2004 09 15 | 529      |
| C/1997 J2 Meunier-Dupouy  | 1997 05 05 – 1999 10 09 | 1446     |
| C/1999 Y1 LINEAR          | 1999 10 29 – 2003 07 19 | 884      |
| C/2000 SV$_3$ LINEAR      | 2000 09 05 – 2005 05 12 | 2189     |
| C/2001 Q4 NEAT            | 2001 08 24 – 2006 08 18 | 2661     |
| C/2002 E2 Snyder-Murakami | 2002 03 08 – 2003 01 08 | 941      |
| C/2002 T7 LINEAR          | 2002 10 12 – 2006 03 07 | 4451     |
| C/2003 K4 LINEAR          | 2003 05 28 – 2006 11 17 | 3658     |
| C/2004 B1 LINEAR          | 2004 01 28 – 2008 08 24 | 2075     |
Figure 1. The O-C distributions for Comet C/2004 B1 LINEAR and C/2001 Q4 NEAT. The non-weighted data are presented in the left-side column for each comet and the weighted data – in the right column where the gravitational cases are displayed in upper figures and the NG cases in lower figures. The best-fitting Gaussian distributions are shown by dots, each chosen exactly in the middle of histogram bin.

Independently for gravitational and non-gravitational orbit determination by applying

for comets before 1950:
- Chauvenet’s criterion for selection procedure + weighting,
- Bessel’s criterion for selection procedure + weighting,

for comets after 1950:
- independent Bessel’s selection for observations before and after perihelion, no weighting,
- Bessel’s criterion for selection procedure + weighting,

To reduce systematic errors in the observational material, such as the bias associated with a site as a function of time, one should consider specific procedures. For example, in the case of long sequences of observations we divided the whole observational material into a few time subintervals according to the internal structure of material, i.e. according to the existing gaps in observations.

The data of two comets with the number of observations less than 100 (C/1952 W1 and C/1959 Y1) have not been weighted. For two of the comets discovered before 1950 the Chauvenet’s and Bessel’s criterions gave the same set of residuals and for the next three comets the differences in the number of rejected residuals are very small (less than 1%). Thus, we present here results derived for more discriminative Bessel’s criterion also for comets before 1950.

One should note that our data processing resulted in reasonably low number of rejections (below 17% – for comets before 1960 and with mean percentage of rejected data of 3.4% – for comets after 1960; the only extreme case here is C/2004 B1, for which 9% of observations was rejected). The description of observational data used for orbital determination is presented in Table 1, where comets are ordered by discovery date.

2 short description of both criterions is given in 7.1

To determine the NG cometary orbit we used the standard formalism proposed by [Marsden et al. 1973] where the three orbital components of the NG acceleration acting on a comet are symmetric relative to perihelion:

\[
F_i = A_i \cdot g(r), \quad A_i = \text{const for } i = 1, 2, 3,
\]

\[
g(r) = \alpha (r/r_0)^{-m}[1 + (r/r_0)^n]^{-k},
\]

where \(F_1, F_2, F_3\) represent the radial, transverse and normal components of the NG acceleration, respectively. The exponential coefficients \(m, n, k\) are equal to 2.15, 5.093, and 4.6142, respectively; the normalization constant \(\alpha = 0.1113\) gives \(g(1 \text{ AU}) = 1\); the scale distance \(r_0 = 2.808 \text{ AU}\). From orbital calculations, the NG parameters \(A_1, A_2,\) and \(A_3\) were derived together with six orbital elements within a given time interval (numerical details are given in Królikowska 2006). This NG model assumes that water sublimates from the whole surface of an isothermal cometary nucleus. One can see in the 10th column of Table 2 that in some cases just two NG parameters were determinable with reasonable accuracy (in two cases – only one NG parameter). We decided to use in this investigation the minimal necessary number of NG parameters. For example, in the case of C/1993 A1 and C/1996 E1 the NG parameter \(A_3\) is determinable, however, the decrease of rms (relative to NG solution with \(A_1\) and \(A_2\)) is insignificant with no perceptible changes in O-C diagrams and O-C distributions (compare with online material in Królikowska 2004). The asymmetric NG model which introduces the additional NG parameter \(\tau\) – the time displacement of the maximum of the \(g(r(t - \tau))\) relative to perihelion, seems be important just for one comet, C/1959 Y1 Burnham, what makes the number of NG parameters for this comet to be equal 4.

To decide whether the derived NG solution better represents the actual motion of individual comet than the pure gravitational orbit we use three criterions:
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Figure 2. The O-C diagrams for Comet C/1997 J2 Meunier-Dupouy (weighted data). Two upper figures are given for O-C based on pure gravitational solution, two lower – for O-C based on NG solution. Residuals in right ascension are shown as black dots and in declination – as grey crosses; the moment of perihelion passage is shown by dashed vertical line.

Figure 3. The O-C diagrams for Comet C/2001 Q4 NEAT (weighted data). Two upper figures are given for O-C based on pure gravitational solution, two lower – for O-C based on NG solution (for the clarity the three last observations taken in August 2006 are not displayed). Symbols have the same meaning as in Fig. 2.

- decrease of the rms for the NG solution in comparison with the pure gravitational solution,
- smaller trends in O-C diagrams for NG orbit fitting,
- the O-C distribution of NG orbit closer to the normal one.

After inspection of the O-C diagrams and distributions we always conclude that the NG orbit derived from the weighted and selected data sets represents the actual comet motion much better than the pure gravitational orbit. This fact is visualized in Fig. 1 for two comets C/2004 B1 and C/2001 Q4. Therefore, the pure gravitational orbits are presented in this paper only for comparison and to show that such orbits could produce misleading orbital results and incorrect conclusions about future and past history of cometary dynamics. The examples of differences in the O-C diagrams between pure gravitational and NG solutions are presented for two comets in Figs. 2 and 3. For comet C/1997 J2 only pure gravitational orbit is presented in MWC08 as well as at the JPL Small-Body Database Browser (http://ssd.jpl.nasa.gov/sbdb.cgi), whereas for Comet C/2001 Q4 NG solutions are presented in both sources. Fig. 2 reveals that some trends visible in pure gravitational O-C diagram for C/1997 J2 Meunier-Dupouy are almost canceled for NG solution while the rms decreased from 0.91 to 0.67. Similar behavior was noticed for all NG solutions given in this paper even for four objects with largest perihelion distance (q > 3.0 AU) as was shown in Fig. 2 for Comet C/1997 J2.

Among investigated sample of NG Oort spike comets C/2001 Q4 NEAT presents the case of one of the most significant decrease of rms from 1.69 (pure gravitational orbit, non-weighted data) to 0.63 (NG orbit, weighted data) (Table 2, Figs. 1, 3). In this group are also C/1959 Y1 Burnham, C/1990 K1 Levy, C/1993 A1 Mueller and C/2002 T7 LINEAR.

For many comets with evidently detectable NG-effects we obtained strongly non-Gaussian O-C distribution for pure gravitational orbit whereas the O-C distribution follows the normal distribution when the NG orbit were fitted. An excellent example of such behavior is Comet C/2004 B1. The departures from the best Gaussian fitting to O-C residuals are shown in Fig. 1. One can note that the deviations of O-C distribution from normal distribution in the pure gravitational case always decreased for weighted data in comparison to non-weighted data, however in some cases this deviation is still clearly visible even for weighted data as one can see for both comets presented in Fig. 1. It seems evident that non-Gaussian distribution of O-C residuals results from ignoring of NG forces in the cometary motion. In a few cases (C/1990 K1, C/1993 A1, C/2001 Q4, C/2002 T7 and C/2003 K4), however, the O-C distributions for NG solution obtained on the basis on weighted data differ from the Gaussian distribution also. In all these five cases the residuals in NG model still display some trends too (especially around perihelion passage), though less significant than in the pure gravitational case (Fig. 3). We tested that the time-shifted g(r)-function relative to perihelion passage do not improved the O-C diagrams here. It seems that some other function than standard g(r) would be more adequate to describe the NG effects in these comets. It should be noticed, however, that the presented NG nominal orbits for these five comets are closer to actual cometary orbit than pure gravitational solutions.

And the last, important remark concerning the gravitational solutions determined from weighted data. In practice, these solutions are not purely gravitational since the time distribution of weights could compensate the NG trends. For three objects from our sample (C/2002 T7, C/2001 Q4 and C/1997 BA6) relative decreasing of rms and some time-dependence in weights are evidently
visible. For the most spectacular case (C/2002 T7) the rms of gravitational solutions decreased from $1''21$ (non-weighted data) to $0''61$ (weighted data) whereas the rms of NG solutions decreased from $0''64$ to $0''58$, respectively, and systematic trends in weights disappeared. Thus, we should keep in mind that so called gravitational solutions determined on the basis of weighted data could partially incorporate NG trends.

3 OBTAINING THE MOST PROBABLE ORIGINAL AND FUTURE ORBITS

To investigate the past and future history of each particular comet we have examined the evolution of thousands of virtual comets (VCs) orbits from the confidence region, i.e. the 6-dimensional region (7–9D region in the NG case) of orbital elements (and NG parameters), each time fully compatible with the observations. We constructed the confidence region using the Sitarski’s method of the random orbit selection (Sitarski 1998).

This method allows us to generate any number of randomly selected orbits of VC. It should be noticed that according to this random selection method the derived sample of VCs follows the normal distribution in the orbital elements space (also the rms’s fulfill the 6–9 dimensional normal statistics. Królakowska et al. 2009).

In the calculations described here we fill a confidence region with 5 000 VCs for each nominal solution (GR and NG) for each considered comet. Fig. 3 shows projections of the 6D/8D parameter space of 5 000 VCs of C/1989 X1 (upper panels) and C/1993 A1 (lower panels) onto the plane of two chosen orbital elements. Orbital cloning procedure was applied close to the observational arc (at the epoch of 1990 April 19 and 1992 October 25, respectively).

The derived swarm of VCs follows the normal distribution in the 6D space of orbital elements (8D space of orbital elements and two NG parameters in the NG case for both comets). This is visualized by four grey tints of points in Fig. 4. Each point represents a single VC, while its grey tint indicates the deviation magnitude from the nominal orbit with the confidence levels defined as follows: $< 50\%$, $50\% – 90\%$, $90\% – 99\%$, and $> 99\%$ (from the darkest to brightest points, respectively). The symbols in crowded areas heavily overlap and the brighter points are often overprinted with darker ones. One can see that the NG swarm of C/1989 X1 is more disperse than its pure gravitational swarm (upper panel of Fig. 4). Similar situation took place in almost all investigated comets except C/1993 A1 and C/2000 SV₇₆. The largest dispersion of original NG swarm within our sample of NG Oort spike comets appeared in case of comet C/1959 Y1 Burnham where four NG parameters were determined. In the C/1993 A1 case the compactness of NG swarm of VCs is higher than compactness of GR swarm though in the first case we determined two more parameters from the same data set than in the latter case (lower panel of Fig. 4). It is a consequence of radically better NG orbit fitting to data (rms=0.98) in comparison to pure gravitational orbit fitting (rms=2.49). In the majority of cases, however, the relative decreasing of rms is smaller and the NG swarm of VCs is more disperse than pure gravitational swarm, as expected when increasing parameter space dimensions.

To calculate the original and future swarm of cometary orbit each of VC was followed from its position at osculation epoch backwards and forwards until the VC reached a distance of 250 AU from the Sun. The equations of comet’s motion have been integrated numerically using the recurrent power series method (Sitarski 1989, 2002), taking into account perturbations by all the planets and including the relativistic effects. In this way we were able to obtain the nominal original/future orbit of each comet as well as the uncertainties of the derived values of orbital elements by fitting the normal distribution to each original/future cometary swarm (Królakowska 2001). Original and future semi-major axes with their uncertainties are given in Table 2 where comets are ordered by discovery date. Fig. 4 visualizes the differences between original 1/a derived for NG and GR swarms of comets. One can note that all investigated comets have $1/a_{\text{NG,GR}} < 5 \times 10^{-5}$ AU⁻¹. This means, that all these comets would belong to the first row of Table 1 in the Oort (1950) paper.

4 PAST AND FUTURE MOTION UNDER THE GALACTIC TIDE

4.1 Force model

Following Fouchard et al. (2003) and others, we used a set of equations of motion in the vicinity of the Sun, accounting for Galactic disk tide and Galactic centre tide. Preško (1998, 1999) argued that for relatively small time intervals (say ten million years) it is not necessary to include the Galactic centre term, as its influence manifests significantly on considerably longer time intervals. However, to maintain compatibility with other works in the field and taking into account, that including Galactic centre term into the model of Galactic tides is not very time consuming we decided to use such a full model. During all integrations we performed calculations in both: simplified and full Galactic tide models, just to confirm the above-mentioned conclusions and observe the difference in some

![Figure 4](image-url)
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Table 2. Original and future semi-major axes derived from pure gravitational nominal solutions (columns 2–3) and NG nominal solutions (columns 4–5) where the number of NG parameters determined for NG solutions is given in the column 10. The rms’s and number of residuals are given in the columns 6–7 and 8–9, respectively, where ‘GR’ refers to gravitational nominal solution. If there are two rows for a particular comet, the first describes results with weighting and the second without weighting. Single row (comets means) that weighting was applied.

| Name                          | gravitational solutions | NG solutions | mNGGR | mNGNG | res. | res. | NG | number of par. |
|-------------------------------|-------------------------|--------------|-------|-------|------|------|----|---------------|
|                               | 1/\(a_{\text{ori}}\) | \(1/a_{\text{ori}}\) | \(1/\text{a}_{\text{NG}}\) | \(1/a_{\text{NG}}\) |       |      |    |               |
|                               | in units of 10^{-6} AU^-1 | in units of 10^{-6} AU^-1 |       |       |      |      |    |               |
| C/1885 X Fabry                | -2.73 ± 11.28           | -256.73 ± 11.28 | 61.04 ± 17.17 | -103.42 ± 41.33 | 3.92 | 3.58 | 394 | 390            |
| C/1892 Q1 Brooks              | -57.34 ± 12.52          | -568.73 ± 12.52 | 58.07 ± 50.88 | -486.86 ± 23.28 | 3.20 | 2.80 | 343 | 334            |
| C/1913 Y1 Delavan             | +22.14 ± 1.30           | +564.11 ± 1.30 | +52.57 ± 4.23 | +486.84 ± 4.23 | 2.06 | 2.00 | 1874 | 1860           |
| C/1940 R2 Cunningham          | -60.00 ± 17.66          | -1630.62 ± 17.67 | +51.66 ± 17.30 | -1494.22 ± 36.52 | 1.78 | 1.49 | 678 | 670            |
| C/1946 U1 Bester              | +5.72 ± 4.97            | +32.53 ± 4.97 | +25.41 ± 5.25 | +52.21 ± 5.25 | 1.53 | 1.35 | 257 | 253            |
| C/1952 W1 Mkos                | -1.40 ± 21.69           | -298.66 ± 21.69 | -0.07 ± 85.78 | -41.39 ± 106.94 | 1.14 | 1.08 | 61 | 61             |
| C/1956 R1 Aренд-Roland        | -97.97 ± 5.46           | -603.51 ± 5.46 | +10.37 ± 11.42 | -579.69 ± 9.97 | 1.68 | 1.39 | 461 | 458            |
| C/1859 Y1 Burnham             | -104.38 ± 5.02          | -609.92 ± 5.03 | +7.83 | -511.87 ± 10.59 | 1.68 | 1.41 | 449 | 449            |
| C/1978 H1 Meier               | -138.32 ± 15.93         | -589.97 ± 15.94 | +2.09 | +123.88 | -286.65 ± 33.46 | 3.00 | 1.60 | 146 | 146            |
| C/1986 P1 Wilson              | +24.52 ± 2.73           | +1027.96 ± 2.73 | +89.23 | -1810.74 ± 8.09 | 1.22 | 1.18 | 549 | 549            |
| C/1899 Q1 Okazaki-Levy-R.     | -12.14 ± 14.07          | +182.62 ± 14.02 | +2.90 | +22.24 | -80.49 ± 27.32 | 1.84 | 1.30 | 458 | 452            |
| C/1989 X1 Austin              | -6.47 ± 12.66           | +188.30 ± 12.66 | +54.58 | +24.90 | +93.84 ± 33.52 | 1.86 | 1.46 | 441 | 438            |
| C/1990 K1 Levy                | +17.59 ± 1.70           | -857.12 ± 1.71 | +104.71 | +767.22 ± 2.35 | 1.35 | 1.11 | 1364 | 1361           |
| C/1991 F2 Helin-Lawrence       | +26.02 ± 1.65           | -848.67 ± 1.65 | +114.44 | +790.43 ± 3.51 | 1.93 | 1.21 | 1302 | 1302           |
| C/1993 A1 Mueller             | +14.64 ± 2.13           | -114.67 ± 2.13 | +24.90 | -89.76 ± 12.25 | 0.89 | 0.86 | 212 | 212            |
| C/1993 Q1 Mueller             | +11.26 ± 3.99           | -508.85 ± 3.99 | +66.26 | -240.08 ± 2.64 | 2.42 | 2.03 | 1488 | 1489           |
| C/1996 E1 NEAT                | +46.62 ± 2.04           | -62.43 ± 2.04 | +12.24 | +615.20 ± 23.40 | 0.97 | 0.92 | 1047 | 1041           |
| C/1997 BA_s, Spacewatch       | +43.35 ± 4.05           | +354.93 ± 4.04 | +18.42 | +530 | +379.24 ± 7.89 | 1.01 | 0.77 | 484 | 484            |
| C/1998 J2 Meunier-Dupouy      | +43.35 ± 4.05           | +354.93 ± 4.04 | +18.42 | +530 | +379.24 ± 7.89 | 1.01 | 0.77 | 484 | 484            |
| C/1999 Y1 LINEAR              | +49.92 ± 0.88           | +350.42 ± 0.88 | +47.35 | +345.69 ± 1.46 | 0.61 | 0.54 | 1474 | 1479           |
| C/2000 SV_74 LINEAR           | +52.00 ± 0.43           | +85.55 ± 0.43 | +92.31 | +54.88 | +0.60 | 1.11 | 739 | 737            |
| C/2001 Q4 NEAT                | +46.66 ± 0.39           | -89.12 ± 0.39 | +94.77 | +60.83 ± 0.75 | 1.03 | 0.94 | 4356 | 4356           |
| C/2002 T7 LINEAR              | +42.95 ± 2.31           | +425.73 ± 2.31 | +89.59 | +241.69 ± 2.79 | 0.61 | 0.57 | 1863 | 1863           |
| C/2004 B1 LINEAR              | +34.64 ± 0.20           | +841.63 ± 0.20 | +35.55 | +690.98 ± 0.68 | 0.53 | 0.43 | 3777 | 3758           |
| C/2006 E2 Snyder-Murak        | +34.66 ± 0.22           | +841.61 ± 0.22 | +38.01 | +660.07 ± 0.61 | 0.58 | 0.58 | 3783 | 3783           |

specific cases. It appeared, that while qualitative evolution is almost the same, the quantitative results may differ. Some examples of the significant differences in the output from these two models are given in Figs 5 and 7.

We use here two different frames. The rotating frame is defined as a heliocentric frame with the \(x'\) axis in the radial direction pointing toward the Galactic center, the \(y'\) axis pointing along the Sun's velocity, and \(z'\) completing a right-handed system. The nonrotating frame \(Oxyz\) coincides with this rotating one at \(t = 0\), but keeps its direction fixed. Basing on \citet{Heisler & Tremaine 1986}, one can write the formula for the tidal force from the disk and the centre of the Galaxy in the rotating galactic frame \(Ox'y'z'\) as follows:

\[
\mathbf{F} = \frac{GM_B}{r^3} \mathbf{r'} + (A - B)(3A + B)\mathbf{x'}^2 +
-(A - B)\mathbf{y'}^2 + [4\pi G\rho_o - (2B^2 - A^2)]\mathbf{z'}^2,
\]

where \(A\) and \(B\) are the Oort galactic constants, \(G\) is the gravity
constant, \( r \) is the heliocentric distance, \( \rho_0 \) is the local disk matter density and \( M_B \) is the central mass (the Sun and planets), expressed in solar masses. Let \( G_1, G_2 \) and \( G_3 \) be such that:

\[
G_1 = -(A - B)(3A + B);
\]

\[
G_2 = (A - B)^2;
\]

\[
G_3 = 4\pi G\rho_0 - 2(B^2 - A^2);
\]

then the equations of motion in rectangular coordinates (\( Oxyz \) inertial frame) are:

\[
\frac{d^2x}{dt^2} = -\frac{GM_B}{r^3}x - G_1x'\cos(\Omega_0t) + G_2y'\sin(\Omega_0t);
\]

\[
\frac{d^2y}{dt^2} = -\frac{GM_B}{r^3}y - G_1y'\sin(\Omega_0t) - G_2x'\cos(\Omega_0t);
\]

\[
\frac{d^2z}{dt^2} = -\frac{GM_B}{r^3}z - G_3z;
\]

where \( \Omega_0 \) is the angular velocity of the Sun about the Galactic center and as consequence the angular velocity of the rotating frame with respect to the nonrotating one and

\[
x' = x\cos(\Omega_0t) + y\sin(\Omega_0t),
\]

\[
y' = -x\sin(\Omega_0t) + y\cos(\Omega_0t).
\]

Substituting this into the equations of motion and remembering that \( G_3 = -G_1 \) (all contemporary models of Galactic tides adopt \( B^2 = A^2 \), see [Fouchard et al. 2005] as the most recent review of the different tide models and their calculation methods) we obtain:

\[
\frac{d^2x}{dt^2} = -\frac{GM_B}{r^3}x - G_1x\cos(2\Omega_0t) - G_1y\sin(2\Omega_0t);
\]

\[
\frac{d^2y}{dt^2} = -\frac{GM_B}{r^3}y - G_1x\sin(2\Omega_0t) + G_1y\cos(2\Omega_0t);
\]

\[
\frac{d^2z}{dt^2} = -\frac{GM_B}{r^3}z - G_3z;
\]

which is the set we integrate numerically. We used \( G_1 = -7.0702403 \times 10^{-10} \), and \( G_3 = 4\pi G\rho_0 \), where \( \rho_0 \), is the local Galactic mater density in solar masses per cubic parsec. The model limited to the disk tide can be obtained by putting \( G_1 = 0 \), and we used it in parallel, to monitor the separate influence of the Galactic centre term. Two examples of such a comparison of different dynamical models are presented in Figs. 6 and 7. As it concerns the adopted value of the local Galactic mater density \( \rho_0 \), we followed most of the contemporary papers in the field, adopting \( \rho_0 = 0.100 \, M_\odot \, pc^{-3} \). To observe the influence of a possible error in this value we repeated the calculations for several comets using three significantly different values: \( \rho_0 = 0.050, \rho_0 = 0.100 \) and \( \rho_0 = 0.150 \, M_\odot \, pc^{-3} \). An example of such a comparison is presented in Fig. 8 which is significantly magnified to make the differences visible.

Dybczyński (2006) have shown, that none of the known stars could significantly influence the cometary motion recently (during last 10 million years, or so) and in similar time interval in the fu-
ture. This time interval is comparable with orbital period of an elliptic comet with the semi-major axis of 50 000 AU and thus it is close to the timescale of the longest past or future cometary motions analyzed in this paper. Dybczynski (2006) searched for real stellar perturbers in all available catalogues, selected several stars with maximum possible influence on the Oort cloud comets and showed that comparing with Galactic perturbations their influence is negligible. He also presented a discussion on the completeness of the stellar data (besides positions, both parallax and radial velocity are necessary). In short: we cannot be sure that some stars passed (or will pass) in the vicinity of the Sun during 20 million years time interval, centred at present. But these stars are absent in our catalogues either because they are too small (e.g. brown dwarf, see for example: Research Consortium on Nearby Stars 2003) or move very fast, and now are too far, or both. But only relatively massive stars (of order of one solar mass or more) and moving rather slowly (less than 30 km s\(^{-1}\)) can significantly change cometary orbit, even with the aphelion at 100 000 AU. For more detailed discussion see Dybczynski (2006).

For the reasons explained above, and taking into account, that precise calculation of (even very weak) stellar perturbations would be time consuming, we decided to omit them in our model. Perhaps in future, especially if some important star will be discovered (and its parallax and radial velocity will be measured, see Dybczynski & Kwiatkowski 2003) we will include its gravitational influence on the Sun and a comet into our model.

### 4.2 Numerical Integration

The original and future orbit calculations produced an output in the form of a set of 5001 (nominal orbit plus 5000 VCs) barycentric orbits for each comet under consideration. To observe how Galactic perturbation action varies depending on small initial condition changes we separately followed numerically all 5001 VCs, and also several stars with maximum possible influence on the Oort cloud comets and showed that comparing with Galactic perturbations their influence is negligible. He also presented a discussion on the completeness of the stellar data (besides positions, both parallax and radial velocity are necessary). In short: we cannot be sure that some stars passed (or will pass) in the vicinity of the Sun during 20 million years time interval, centred at present. But these stars are absent in our catalogues either because they are too small (e.g. brown dwarf, see for example: Research Consortium on Nearby Stars 2003) or move very fast, and now are too far, or both. But only relatively massive stars (of order of one solar mass or more) and moving rather slowly (less than 30 km s\(^{-1}\)) can significantly change cometary orbit, even with the aphelion at 100 000 AU. For more detailed discussion see Dybczynski (2006).

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4.2 Numerical Integration

The original and future orbit calculations produced an output in the form of a set of 5001 (nominal orbit plus 5000 VCs) barycentric orbits for each comet under consideration. To observe how Galactic perturbation action varies depending on small initial condition changes we separately followed numerically all 5001 VCs, in general, one orbital period to the past and to the future. We used the well known, widely used and tested routine RA15 (Everhart 1985) with the force model described above (with the parameter LL=12 to obtain the highest possible accuracy on 64-bit computers). Since the Galactic tide is relatively weak, the automatically adjusted integration step was generally very large so the speed of the calculation allowed us to follow individually such a big number of VCs for each comet.

Since we are interested mainly in the past motion of a comet and the apparent source of the observed long-period comets we followed their past motion for one orbital period. Having the opportunity to investigate also the future motion of these bodies we decided to perform similar calculations for all of them going one orbital period to the future. Going further (both back and forth) seems useless, since a lot of comets have their previous and/or next perihelion distances deep in the Planetary System. Since there are no means to calculate precise planetary perturbations for such bodies say ten million years from now, we cannot follow their motion “behind” previous and/or next perihelion point. On the other hand, when a comet have large previous/next perihelion distance, their subsequent motion do not change our knowledge on the apparent source of the observed long-period comets, or their future.

The situation complicates when past or future orbit of particular VC is hyperbolic. We have to stop numerical integration at some heliocentric distance, which we call escape limit. To define this limit it seems useful to recall contemporary studies on the outer border of the Oort cloud. Among recent reviews in this field we mention a chapter in Comets II book by Dones et al. (2004) and a book by Fernandez (2005). Both present consistent conclusions that while Oort placed the outer border of the cometary cloud at 5–15\(\times\)10\(^5\) AU current understanding of cometary dynamics makes this border much smaller, say 2–8\(\times\)10\(^3\) AU for the aphelion distance and 1–4\(\times\)10\(^2\) AU for semi-major axis. Additionally, the majority of the observed Oort spike comets have the semi-major axes in the range of 20 000–50 000 AU for pure gravitational orbits but this significantly decreases to the value less than 20 000 AU if the NG forces are included in the dynamical model. Some discussion of this effect is also in Section 5.2.

We decided to be very careful (and a little conservative) so we decided to adopt the escape limit to be equal 120 000 AU. This is the maximum heliocentric distance (or maximum aphelion distance) at which we still follow every VCs motion. This was also
applied for very elongated elliptic orbits – if the particular VC goes further than 120,000 AU from the Sun we call it escaping and stopped the numerical integration.

This particular choice of the escaping border was unfortunate for a handful of comets, where small number of VCs traveled a little bit further than 120,000 AU, but then return to the Sun. We examined closely all such cases and if the necessary increase in the outer limit seemed acceptable we changed it to obtain homogeneous swarm of returning VCs for that particular comet. Such a procedure was applied for example for C/1996 E1, where increasing the outer limit up to 140,000 AU allowed all 5001 VCs to return to their previous perihelion among Solar System planets when calculating its past motion. To obtain the same effect for C/1988 X1 one should extend the outer limit up to 260,000 AU, what, in the light of arguments included in both papers quoted above, seems problematic. At such a large heliocentric distance the comet motion became chaotic, see Fig. 1 in Dybczyński (2001).

Due to the fact, that some VCs swarms consists of both returning and escaping ones, the rules of stopping the integration were sometimes more complicated: the escaping part of the swarm was stopped at the 120,000 AU and the rest of VCs was allowed to reach previous/next perihelion. There were also a synchronous variant of calculation stopping: for returning swarms all VCs were stopped when the nominal one reached previous/next perihelion and for escaping or “mixed” swarms, all VCs were stopped when the fastest escaping VC reached 120,000 AU. This was done to obtain more homogeneous swarm to analyze, where all VCs are stopped at the same time.

5 RESULTS
5.1 Overview

Let us start with the context in which our results should be interpreted. In his paper Dott (1951) treated original semi-major axes as definitive for describing past motion of a comet. In the same time he stated, that all comets with 1/a_{ori} < 1 x 10^{-2} AU^{-1} should be treated as dynamically new, i.e. they are visiting the observable region for the first time. Such a conclusion came from two assumptions. First, the only possible perturbation source are stellar perturbations, what means that going to the past we should follow original, unperturbed orbit as long as a comet reaches the heliocentric distance large enough to be perturbed by passing stars. Second, all observed long-period comets were significantly perturbed by stars (if not, their keplerian, original orbits bring them exactly to the same perihelion distance at previous passage and no one can call them “dynamically new”).

Today, none of this two assumptions seem to be fully correct. As we noted in the Introduction, a comet motion now cannot be treated as unperturbed and the dominating (if not the only one) perturbing force is the differential, gravitational action of our Galaxy. While Galactic tides perturb the semi-major axis practically on the negligible level during one orbital period, the most important part of the past and future cometary orbit evolution manifests in the eccentricity changes.

As a result, while semi-major axis remains almost unchanged, the perihelion distance can change significantly, even during one orbital period both in past and future motion of course. Taking C/1913 Y1 as typical example, one can note that the original semi-major axis of this comet a_0 = 19022.723 AU and one orbital period before it was equal 19022.794 AU. At the same time original perihelion distance q_0 = 1.10 AU and one orbital period earlier it was equal 2.30 AU. Small changes in the semi-major axis under the Galactic perturbations is a well known fact. In the averaged problem of the disk tide the semi-major axis is even an integral of motion (see Breiter et al. (1996) for details), Prezeau (1996, 1999) showed that slow secular changes in the semi-major axis due to the action of the Galactic centre can be observed on the much longer time intervals than considered here. It means that looking only at V_{ori} of a comet nobody can say what was its previous (or next) perihelion distance.

5.2 Past and future evolution - some statistics

As it was described in sections 2, 3 and 4 we very carefully determined orbits for 26 observed long-period comets using sophisticated observations treatment and NG force model. Than, at osculat-
of VCs, that one can call dynamically new, basing on previous statistics.

or three deciles at 10%, 50% (i.e. median), and 90%. In case of mixed swarms the mean values or deciles of Q and q are given for returning part of the VCs swarm. For comparison we included the osculating perihelion distance in the second column. Last two columns present the value of $1/\Delta v_{\text{ini}}$ and the percentage of VCs, that one can call dynamically new, basing on previous q statistics.

| Comet            | $q_{\text{osc}}$ | Number of VCs | Q $10^3$ AU | q AU | $1/\Delta v_{\text{ini}}$ $10^{-6}$ AU$^{-1}$ | % of VCs dyn. new |
|------------------|------------------|---------------|-------------|------|---------------------------------|-----------------|
| C/1885 X1        | 0.64             | 4984$^a$      | 17           | 1    | 24.2 – 32.7 – 50.9               | 0.72 – 0.95 – 4.3 | 61.0 ± 17.2 | 4.6 |
| C/1892 Q1        | 0.98             | 3960$^a$      | 1041         | 645  | 15.7 – 28.1 – 68.6               | 1.00 – 1.20 – 28.16 | 58.1 ± 50.9 | 30.8 |
| C/1913 Y1        | 1.10             | 5001          | 0            | 0    | 34.5 – 38.0 – 42.5               | 1.87 – 2.30 – 3.14 | 52.6 ± 4.2  | 0   |
| C/1940 R2        | 0.37             | 4886$^a$      | 115          | 5    | 27.1 – 38.3 – 63.1               | 0.46 – 0.93 – 13.5 | 51.7 ± 17.3 | 11.5 |
| C/1946 U1        | 2.41             | 4847$^a$      | 154          | 0    | 62.1 – 78.2 – 102.3              | 19.7 – 89.7 – 517 | 25.4 ± 5.2  | 94.3 |
| C/1951 W1        | 0.78             | 2137          | 2864$^a$     | 2464 | 13.4 – 28.9 – 78.7               | 0.79 – 1.03 – 27.0 | -0.1 ± 85.8 | 63.2 |
| C/1956 R1        | 0.32             | 1463          | 3538$^a$     | 909  | 62.3 – 90.1 – 113.8              | 0.94 – 18.0 – 93.2 | 10.4 ± 11.4 | 86.3 |
| C/1959 Y1        | 0.50             | 2245          | 2756$^a$     | 2498 | 9.3 – 20.5 – 62.9               | 0.10 – 0.47 – 15.5 | 2.1 ± 108.1 | 59.6 |
| C/1978 H1        | 1.14             | 5001          | 0            | 0    | 21.2 – 24.9 – 29.9               | 0.73 – 0.91 – 1.00 | 80.5 ± 10.7 | 0   |
| C/1986 P1        | 1.20             | 5001          | 0            | 0    | 44.9 – 47.6 – 50.7               | 2.67 – 3.11 – 3.72 | 42.0 ± 2.0  | 0   |
| C/1989 Q1        | 0.64             | 4411$^a$      | 590          | 130  | 27.2 – 42.3 – 78.6               | 0.41 – 0.55 – 33.4 | 42.9 ± 22.2 | 24.4 |
| C/1989 X1        | 0.35             | 4974$^a$      | 27$^b$       | 0    | 37.5 – 49.1 – 70.7               | 1.7 – 5.9 – 49.4 | 40.8 ± 9.9  | 25.9 |
| C/1990 K1        | 0.94             | 5001          | 0            | 0    | 17.9 – 19.1 – 20.6               | 0.909 – 0.915 – 0.920 | 104.7 ± 5.7 | 0   |
| C/1991 F2        | 1.52             | 2971$^a$      | 2030         | 0    | 96.6 – 121 – 141                | 214 – 805 – 2012 | 14.2 ± 4.0  | 100 |
| C/1993 A1        | 1.94             | 5001          | 0            | 0    | 30.5 – 31.9 – 33.4               | 2.40 – 2.48 – 2.59 | 62.7 ± 2.3  | 0   |
| C/1993 Q1        | 0.97             | 1461          | 3540$^a$     | 72   | 83.4 – 106.5 – 122.7             | 75 – 396 – 957 | 12.2 ± 6.1  | 99.8 |
| C/1996 E1        | 1.36             | 5001$^a$      | 0            | 0    | 55.7 – 66.5 – 82.5               | 2.78 – 9.72 – 46.3 | 30.2 ± 4.6  | 35.5 |
| C/1997 BAg       | 3.44             | 5001$^a$      | 0            | 0    | 60.1 – 62.8 – 65.8               | 15.9 – 19.5 – 24.7 | 31.8 ± 1.2  | 95.3 |
| C/1997 J2        | 3.05             | 5001          | 0            | 0    | 44.8 ± 0.9                      | 2.801 ± 0.017 | 44.6 ± 0.9  | 0   |
| C/1999 Y1        | 3.09             | 5001          | 0            | 0    | 42.2 ± 0.8                      | 5.82 – 6.11 – 6.46 | 47.3 ± 0.9  | 0   |
| C/2000 SV+3      | 3.54             | 5001          | 0            | 0    | 217 ± 0.2                       | 3.791 ± 0.009 | 92.3 ± 0.8  | 0   |
| C/2001 Q1        | 0.96             | 5001          | 0            | 0    | 32.3 – 32.9 – 33.5               | 1.875 ± 0.057 | 60.9 ± 0.5  | 0   |
| C/2002 E2        | 1.47             | 5001          | 0            | 0    | 18.9 – 22.3 – 27.1               | 1.55 – 1.62 – 1.77 | 89.6 ± 12.4 | 0   |
| C/2002 T7        | 0.61             | 5001          | 0            | 0    | 94.0 – 96.2 – 98.6               | 142 ± 19 | 20.7 ± 0.4  | 100 |
| C/2003 K4        | 1.02             | 5001          | 0            | 0    | 65.1 ± 1.1                      | 11.7 – 13.8 – 16.2 | 30.7 ± 0.5  | 25.1 |
| C/2004 B1        | 1.60             | 5001          | 0            | 0    | 56.3 ± 1.1                      | 2.49 – 3.17 – 4.06 | 35.6 ± 0.7  | 0   |

$^a$ – this part of swarm includes the nominal orbit,  $^b$ – 12 VCs have 200 000 AU < $Q$ < 320 000 AU, and 3 VCs - $Q$ between 0.4 – 6.1 × 10$^6$ km $^c$ – we used 140 000 AU as the outer border (just 6 VCs have $Q$ > 120 000 AU).

Figure 12. The past swarm of VCs for the comet C/1991 F2. Here 2971 returning VCs (black dots) were stopped at previous perihelion and 2030 escaping VCs (dark-gray or red dots) were stopped when crossed 120 000 AU. Note that due to the spread distribution of returning VCs, lots of escaping VCs are overprinted with each other.

Table 3 presents the most important result of this paper. For each comet we show, how many VCs return to previous perihelion (column [R]), how many cross the adopted escape border at 120 000 AU ([E] column) and additionally how many of escaping VCs have formally hyperbolic orbits at the end of our calculation ([H] column, which presents subset of [E] column). Next three columns are discussed below. Last column shows the percentage of VCs, that one can call dynamically new, basing on previous perihelion distance statistics, i.e. we calculate the part of dynamically new VCs by adding the number of returning VCs with $q_{\text{osc}} > 15$ AU
### Table 4. The future distributions of the NG swarms of VCs in terms of returning [R], escaping [E], including hyperbolic [H] VC numbers. Aphelion and perihelion distances as well as the eccentricity are described either by a mean value for the normal distributions, or three deciles at 10%, 50%, and 90%. In case of mixed swarms the mean values or deciles of Q, q and e are given for returning part of the VCs swarm. For comparison we included the osculating perihelion distance in the second column. Last column present the value of $1/a_{\text{ret}}$.

| Comet          | $q_{\text{osc}}$ AU | Number of VCs | eccentricity at 120 000 AU | Q $10^3$AU | q AU | $1/a_{\text{ret}}$ $10^{-6}$AU$^{-1}$ |
|---------------|-----------------|----------------|---------------------------|-----------|-----|--------------------------|
| C/1885 X1     | 0.68            | 6              | 4993$^{a}$ 4969$^{b}$     | 1.000037-1.000101-1.000206 | -   | -                        | -102.3± 41.1 |
| C/1892 Q1     | 0.98            | 0.00501 5001   | 1.000049±0.000039         | -         | -   | -886.0± 23.2             |
| C/1913 Y1     | 1.10            | 5001 0       | 0.9999154±0.0000059       | 21.7-23.0-24.6 | 0.945-0.975-0.997 | 86.8± 4.2          |
| C/1940 R2     | 0.37            | 0              | 1.000759±0.000034         | -         | -   | -1492± 36                 |
| C/1946 U1     | 2.41            | 5001 0       | 0.999765-0.999808-0.999819 | 34.0-38.3-44.0 | 3.07-3.67-5.17 | 52.2± 5.3          |
| C/1952 W1     | 0.78            | 1509 3492$^{c}$ | 3229$^{a}$ 0.9999506-1.0000196-1.0000540 | 12.3-28.6-75.6 | 0.13-0.71-7.61 | -38.9±106.7       |
| C/1956 R1     | 0.32            | 0              | 1.000048±0.000014         | -         | -   | -579.7± 10.0             |
| C/1959 Y1     | 0.50            | 0              | 1.00106±0.000024          | -         | -   | -286.3± 33.4             |
| C/1978 H1     | 1.14            | 0              | 1.00306±0.000041          | -         | -   | -1025± 8                 |
| C/1986 P1     | 1.20            | 5001 0       | 0.9990777±0.000028        | 2.606±0.008 | 1.202057±0.000001 | 767.2± 2.3        |
| C/1989 Q1     | 0.64            | 4950$^{a}$ 51 6 | 0.9990031-0.999336-0.999402 | 17.3-24.9-43.0 | 0.67-0.76-1.97 | 80.6± 26.5        |
| C/1989 X1     | 0.35            | 0              | 1.00055±0.000025          | -         | -   | -368.0± 5.4              |
| C/1990 K1     | 0.94            | 0              | 1.001618±0.0000013        | -         | -   | -790.4± 3.5              |
| C/1991 F2     | 1.52            | 0              | 1.001911-1.000258-1.000340 | -         | -   | -922.5± 7.3              |
| C/1993 A1     | 1.94            | 0              | 1.000362±0.0000015        | -         | -   | -407.5± 2.6              |
| C/1993 Q1     | 0.97            | 0              | 1.000746-1.000974-1.001242 | -         | -   | -204.1± 23.3             |
| C/1996 E1     | 1.36            | 5001 0       | 0.999504±0.0000073        | 5.32-5.42-5.53 | 1.34466±0.000004 | 368.8± 5.4        |
| C/1997 BA6    | 3.44            | 5001 0       | 0.998618±0.0000059        | 4.966±0.021 | 3.432091±0.0000066 | 402.5± 1.7        |
| C/1997 T2     | 3.05            | 151 4850$^{c}$ | 0.9999319±0.0000032$^{b}$ | 124.134-145 | 4.568±0.063$^{b}$ | 19.4± 0.9        |
| C/1999 Y1     | 3.09            | 5001 0       | 0.9989332±0.0000045       | 5.783±0.024 | 3.08617±0.000003 | 345.7± 1.5        |
| C/2000 SV$_{34}$ | 3.54            | 0              | 1.0001075±0.0000036       | -         | -   | -54.4± 0.6               |
| C/2001 Q4     | 0.96            | 0              | 1.000550±0.000001         | -         | -   | -695.4± 0.5              |
| C/2002 E2     | 1.47            | 0              | 1.0005871±0.0000065       | -         | -   | -441.6± 2.8              |
| C/2002 T7     | 0.61            | 0              | 1.0015129±0.0000045       | -         | -   | -660.0± 1.1              |
| C/2003 K4     | 1.02            | 0              | 1.00120±0.000015          | -         | -   | -125.1± 0.8              |
| C/2004 B1     | 1.60            | 0              | 1.00292±0.000008          | -         | -   | -606.0± 0.7              |

*a* – this part of swarm includes the nominal orbit, 
*b* – in this case statistics are for synchronous swarm (see text).

The following table presents the number of VCs that are returning or escaping from the Sun for a specific comet. The numbers are given in terms of Q, q and e, which are the aphelion, perihelion, and eccentricity, respectively. The table also includes the mean and standard deviation of these values, as well as the median value and the 10th, 50th, and 90th percentiles.

- **C/1885 X1**: Returns 6 VCs with an aphelion distance of 1.000037 AU, a perihelion distance of 0.999950 AU, and an eccentricity of 0.000084. The mean value of $1/a_{\text{ret}}$ is -102.3 AU.
- **C/1892 Q1**: Returns 0.00501 VCs with an aphelion distance of 0.000049 AU, a perihelion distance of 0.999915 AU, and an eccentricity of 0.000039. The mean value of $1/a_{\text{ret}}$ is -886.0 AU.

This table shows that the majority of the VCs return close to the Sun, with a few VCs escaping at distances beyond 10 AU. The eccentricity values indicate that the orbits are generally hyperbolic, with some exceptions showing a slight elliptical nature.

The table also includes a column for the $1/a_{\text{ret}}$ values, which represent the semi-major axis of the comet's orbit after returning. These values are given in AU and are calculated as $1/a_{\text{ret}} = (1/a_{\text{ret}})^{-1}$. The values range from 606 AU (1/a_{\text{ret}} = 0.0000032) to 3.54 AU (1/a_{\text{ret}} = 0.30000001).
of 1509 returning VCs and 3492 escaping (with 3229 hyperbolas among them, including the nominal orbit). A synchronous plot of all 5001 VCs for this comet is depicted in Fig. 14. It seems that this comet will be lost in the future. The similar fate can be attributed for next 16 comets having 100% of escaping, hyperbolic VCs and for C/1885 X1 (over 99% escaping hyperbolic VCs).

Only 6 comets, C/1913 Y1, C/1946 U1, C/1986 P1, C/1996 E1, C/1997 BA6 and C/1999 Y1 have 100% of returning VCs and all of them will have the future perihelion distance deep among Solar System planets. The future motion of these comets seems to be unpredictable due to possible planetary perturbations.

Two special cases of future motion: C/1989 Q1 and C/1997 J2 are discussed in the next section.

Summarizing, from our sample of 26 comets 17 should be treated as lost in the future, one (C/1997 J2) may remain bound to the Solar System on large elliptical orbit and 8 will return in the future, pretending they are coming from the so called inner Oort Cloud, and having perihelion distances well inside the sphere of visibility.

5.3 Individual cases

5.3.1 C/1885 X1 Fabry

This is the oldest comet investigated in this paper. Its past motion is almost clear. When dealing with different 5000 VC swarms of this comet, sometimes we obtain just few (or a dozen) of VCs with hyperbolic past orbit. In every case, there is also some, very small, percentage of VCs, which have their aphelia slightly above the outer limit of the heliocentric distance, adopted in this paper as the limit for returning comet (120 000 AU). This discrepancy shows the importance of NG forces applied at the stage of orbit determination (Dybczyński 2001, 2006 used pure gravitational orbits). It is really impressive if we keep in mind that gravitational original orbit of eight comets from our sample are hyperbolic and only four of them might be called dynamically new when NG original orbit is properly accounted for.

As it was mentioned earlier, we also studied the future motion of all 26 comets in our sample, in an analogous manner. The results of this part of investigation are summarized in Table 4. The description is much simpler here, because we have only one comet, C/1952 W1, with significantly mixed swarm of VCs, consisting widely adopted threshold value for significant planetary perturbations) varying from 25% to 63%.

Performing a very simple (while really impressive) statistics, one can sum all VCs for all 26 comets and calculate the percentage of VCs with previous perihelion distance larger than 15 AU. The result is 32.9%. Statistically this is the percentage of dynamically new comets among observed long-period comets, basing on our sample of 26 comets.

This value is significantly smaller than obtained by Dybczyński (2001; 2006), who claimed that about 50% of long-period comets with semi-major axes larger than 10 000 AU could be treated as dynamically new. This discrepancy shows the importance of NG forces applied at the stage of orbit determination (Dybczyński 2001, 2006 used pure gravitational orbits). It is really impressive if we keep in mind that gravitational original orbit of eight comets from our sample are hyperbolic and only four of them might be called dynamically new when NG original orbit is properly accounted for.

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Where do long-period comets come from?

5.3.1 C/1885 X1 Delavan

Previous perihelion passage of this comet is determined with the highest accuracy of a group of comets discovered before 1970. Our calculations show that C/1885 X1 was previously inside inner Solar System and lived it in an orbit of $q_{\text{ori}} = 14$ AU and $Q_{\text{ori}} = 60,000$ AU. The $1/a_{\text{ori}} = (-102.0 \pm 41.1) \times 10^{-6}$ AU$^{-1}$ here, so this comet is almost certainly lost from the Solar System.

5.3.2 C/1913 Y1 Delavan

Previous perihelion passage of this comet is determined with the highest accuracy of a group of comets discovered before 1970. Our calculations show that C/1913 Y1 was previously inside inner Solar System and lived it in an orbit of $q_{\text{ori}} < 3$ AU and $Q_{\text{ori}} < 43,000$ AU (Table 3). This is the second comet in our sample which was originally considered by Oort as dynamically new object. It seems worth to mention here, that such a small previous perihelion distance was obtained as a result of non-gravitational forces incorporated in this paper for the original and future orbit determination. Pure gravitational model gives the previous perihelion value of the order of 150 AU for this comet.

The future motion of C/1913 Y1 is also determinable with very high accuracy. In contrary to C/1885 X1 here all VCs are returning with the distribution of the next perihelion and aphelion distances only slightly departing from the Gaussian one. However, by fitting to Gaussian one can derive the values of $0.97$ AU $\pm 0.02$ AU and $(23.1 \pm 1.1) \times 10^{3}$ AU for next perihelion and aphelion distance, respectively (compare with Table 3). The heliocentric motion of all VCs for one past and one future orbital periods is shown in Figure 17.

If we demand, that the Oort cloud comet should move outside the sphere of planetary perturbations (before it is observed for the first time) C/1913 Y1 do not fulfill this condition either in the past or in future.

5.3.3 Comets C/1892 Q1, C/1952 W1 and C/1959 Y1

These three old comets have the past swarms of VCs widely spread. Apart from a large number of escaping VCs, the significant part of them have hyperbolic orbits. While in all cases we can call them ‘dynamically new’ with rather large degree of certainty, the exact provenience of these comets is hard to be determined, we cannot exclude even their interstellar origin. The probability of such a case was extensively discussed by Hughes (1991).

C/1952 W1 is the only comet in our sample which have formal value of $1/a_{\text{ori}} < 0$ (see Table 3). The future motion of C/1892 Q1 and C/1959 Y1 show evident hyperbolic ejection while for C/1952 W1 64% of VCs are escaping on hyperbolic orbits but on the other hand some 30% of VCs will return, sometimes with very small perihelion distances (see Table 3).

5.3.4 C/1986 P1 Wilson

This is very difficult comet to investigate because of its splitting into two fragments. According to MWC08, the main component A of this comet exhibits NG acceleration in the time interval 1986 August 05 – 1989 April 11. Therefore, we also determined the NG effects from the whole observational arc (see Table 3) and next starting from these nominal orbits (GR and NG) the past and future orbital evolution have been analyzed. It turns out that the fragment B was a small piece of parent body and had ceased all activity or had disintegrated after April 1989. To estimate the possible uncertainty connected with this splitting we tried to find the nominal orbits from two separate sets of data: before and after splitting, but...
the exact date of this break-up is not clear. During comet’s observation on 26 October 1987 because of the likely connection with a brightness of outburst in that time. Thus, we decided to determine the past orbit of C/1986 P1 from the time interval 1986 August 05 – 1987 July 04 (next astrometric observation was taken three months later on October 2) and the future orbit from the time interval 1987 October 26 – 1989 April 11 (114 observations). We derived values of $(53.14 \pm 4.14) \times 10^{-3}$ AU$^{-1}$ and $(30.12 \pm 1.84) \times 10^{-2}$ AU$^{-1}$ for $1/a_{ori,NG}$ and $1/a_{ori,GR}$ respectively, where NG solution was determined with rms $=1.41$ and pure gravitational solution – with rms $=1.43$. In this approach we derived $37.65_{-3.4}^{+1.2} \times 10^{3}$ AU and $1.90_{-0.22}^{+0.38}$ AU for previous aphelion and perihelion passage, respectively, where by adding the lower and upper errors to nominal value the deciles of 10% and 90% can be derived and directly compared with respective values in Table 2. By comparing both results of previous perihelion swarms we stated that the comet C/1986 P1 were deeply inside inner solar system with perihelion not larger than 4 AU and aphelion less than $5 \times 10^{3}$ AU.

For the data taken after the splitting the NG effects were indeterminable. The VCs swarm of future motion is the most compact among considered objects (larger than 300 AU).

5.3.5 C/1989 Q1 Okazaki-Levy-Rudenko and C/1989 X1 Austin

Both comets were observed in short time intervals (four and six months, respectively) thus the past and future perihelion swarms of VCs are relatively disperse what gives the formal chance of only about 25% that these comets are dynamically new. Comet C/1989 Q1 have a great majority (99%) of returning VCs with previous perihelion distance of the order of 1 AU but there is a long tail in its distribution, consisting of 51 escaping VCs, including 6 hyperbolas. Thus, the resulting q-deciles are 0.41 AU–0.55 AU–33.4 AU (at 10%, 50%, and 90%) and taking limit $q_{previous}=15$ AU for dynamically new comets the probability of 24.4% was formally obtained that this comet is dynamically new.

Similarly, the q deciles of 1.7 AU–5.9 AU–49.4 AU (at 10%, 50%, and 90%) give the chance of 25.9% that comet C/1989 X1 is dynamically new.

5.3.6 C/1990 K1 Levy

This comet may serve as an excellent illustration, how the incorporation of NG forces into the process of orbit determination shrinks the semi-major axis and significantly shorten the time interval of the past motion. When we use pure gravitational solution for this comet (see the on-line extended table.), only 3644 VCs are classified as returning while 1357 as escaping but elliptical. For the slowest returning VC it takes almost $15 \times 10^{6}$ years to reach the previous perihelion. If we want all VCs to become returning, the ’escape limit’ should be increased up to 190,000 AU what seems unacceptable. But it is worth to mention that in that case the slowest VC takes over 27 million years to pass the previous perihelion.

5.3.7 C/1991 F2 Helin-Lawrence and C/1993 Q1 Mueller

These comets seem to be classical jumpers from the Oort Cloud with largest $\Delta q = q_{previous} - q_{ori}$ among considered objects (larger than 300 AU).

Since the astrometric set of data for C/1991 F2 consists of 114 observations this comet represents a limit case for weighting. We decided to show the past and future motion on the basis of weighted observations (Tables 3 and 4). It should be noted however, that in the non-weighted data case the past swarm of VCs are almost all returning (4968 of 5001 VCs) with q range: 22.3 AU–71.9 AU–340.8 AU (three deciles at 10%, 50%, and 90%) what gives 98.8% of dynamically new VCs. Thus, the main conclusion that this comet is dynamically new remains unchanged, independently of data treating.

The past swarm of comet C/1993 Q1 is dominated by escaping VCs with maximum of aphelion distance $\sim$150,000 AU (compare with returning part of swarm given in Table 4). Our results suggest that this comet at previous perihelion was extremely far from the Sun.

Both comets will be probably lost in the future – they leave the solar system on hyperbolic orbits.

5.3.8 C/1996 E1 NEAT

Because only 6 VCs of this comet crossed the adopted escape limit in the past motion we decided to increase this limit up to 14,000 AU when performing backward numerical integration. Then, all 5001 past VCs were returning and 74% of them had $q<15$ AU. After the observed passage through the Solar System its nominal semi-major axis is drastically reduced from 54,000 AU down to 2600 AU and it will pass the perihelion 1.3 AU from the Sun.
5.3.9 Comets with large observed perihelion distance

In our sample there are four comets with relatively large osculating perihelion distances: C/1997 BA$_6$ Spacewatch, C/1997 J2 Meunier-Dupouy, C/1999 Y1 LINEAR and C/2000 SV$_3$ LINEAR. First three might be grouped together to show different dynamical histories of these comets in spite of their very similar original perihelion distances (from 3.05 AU to 3.44 AU) and rather similar original semi-major axes (within the range from 21 000 AU to 31 500 AU, Table 3). One can see in Table 3 that the previous and future perihelion passages are well determined for all of them. Thus, we can state that only the first one (C/1997 BA$_6$ Spacewatch) having previous perihelion distance around 20.0 AU±3.3 AU seems be dynamically new, whereas remaining two comets were deep in the inner Solar System during the past perihelion passage (Table 3). This may serve as an additional argument, that even having all parameters of the original orbit one cannot judge what was the previous aphelion or perihelion distance of a particular comet. While the strength of the Galactic perturbations depends mainly on the semi-major axis value, it can be significantly different, depending on the orientation of the orbit with respect to the Galactic equator. There exist another point, from which C/1997 J2 Meunier-Dupouy is also an interesting comet. Its previous perihelion distance (2.80 AU) was slightly smaller than the original one (3.05 AU), what means that this comet was discovered during the “growing” phase of its perihelion distance evolution under the influence of the Galactic tides (two more comets in our sample, C/1978 H1 and C/1990 K1 have also smaller previous perihelion distance than the observed one). The statistical analysis of the q evolutionary phase of long-period comets at their discovery epoch was performed for example by Matese & Lissauer (2003). They concluded that most comets should be observed during the “decreasing” phase but noted several opposite situations. It is worth to mention however, that their analysis was based on pure gravitational cometary original orbits. Future history of C/1997 J2 is more uncertain. It can be seen from Table 3 that next VCs aphelion distances are slightly larger than our assumed standard outer border: 97% of VCs are classified as escaping, but formally all of them are elliptic, see Fig. 15. By shifting the escape border to 170 000 AU we can obtain all VCs returning and approximately Gaussian distribution of aphelion distance ($Q = 136.3 ± 8.4$ thousand AU) and non-Gaussian for perihelion distance with values of 5.6 AU, 7.9 AU, 15.9 AU for 1$^{st}$, 5$^{th}$ and 9$^{th}$ decile of $q$, respectively. But so large aphelion distance means that this comet will probably leave the Solar System. As it can be noted from Table 3 two remaining comets, C/1997 BA$_6$ Spacewatch and C/1999 Y1 LINEAR, will have a very similar future evolution. Both of them in the subsequent perihelion passage will enter the inner Solar System in orbits with perihelion distances between 3 AU and 4 AU and aphelion of ∼5 000 AU–6 000 AU.

Fourth comet with large osculating perihelion distance, C/2000 SV$_3$ LINEAR, have significantly shorter original semi-major axis of about 10 800 AU and its previous perihelion passage was below 4 AU. However, it seems that C/2000 SV$_3$ LINEAR will leave the solar system on a hyperbolic orbit.

5.3.10 C/2001 Q4 NEAT and C/2002 T7 LINEAR

These comets were at great distance from the Sun when discovered (comet C/2001 T7: over 11 AU, C/2002 T7: ~7 AU) and were almost stellar in appearance. At maximum brightness both became easy binocular objects (4 mag. and 2.5–3 mag, respectively) and as consequence were observed long after perihelion passages. Such a long observational arcs allowed to determine their orbits with highest precision and we obtained very accurate results with Gaussian distribution of VCs parameters both for past and future motion.

Both comets have past swarms of VCs fully returning and compact, however they differ dramatically in previous perihelion distance. Comet C/2001 Q4 was at previous perihelion close to the Sun (1.87 AU±0.06 AU), whereas C/2002 T7 passed the previous perihelion at a distance of 142 AU±19 AU from the Sun. Thus, from this pair, only C/2002 T7 seems to be dynamically new.

While for these comets different form of the $g(r)$ functions might be more appropriate, we are convinced that the main conclusions remains the same.

Both comets are leaving the Solar System on hyperbolic orbits.

6 CONCLUSIONS

The aim of our research is to refine widely quoted opinions about the apparent source of the long period comets and a popular habit to call some of them dynamically new by looking on the original inverse semi-major axis value only.

In this paper, the first one of a planned series, we touch the most important group of comets, 26 long-period comets with $1/a_{ori} < 10^{-4}$ AU$^{-1}$ for which we can obtain very precise orbits including NG forces in the dynamical model. The importance of this sample comes from the fact that for almost all of these comets we obtained significantly smaller semi-major axes due to NG force incorporation in the process of orbit determination, but still all of them constitute the NG Oort spike, i.e have $1/a_{ori,NG} < 10^{-4}$ AU$^{-1}$.

Having such a precise orbits we applied ingenious Sitarski’s (1998) method of creating swarms of VCs, all compatible with observations. This allowed us first to obtain original and future orbits parameters with their uncertainties and then to propagate all individual VCs one orbital period to the past and to the future, by means of strict numerical integration of the equation of motion with the Galactic tide action included.

Note, that our neglecting of stellar perturbations is based on well documented arguments (Dybczyński 2003), but the incorporation of NG forces into the orbit determination makes these arguments even stronger. All but one of comets in our sample (and many others, see Królikowska 2006) appear to have significantly smaller semi-major axes what implies that their orbital period becomes drastically shorter. This makes a strong stellar perturbation by an undiscovered massive and slow moving star extremely improbable.

Our results show, that two widely quoted opinion about the long-period comets are not necessary the “canonical truth”. First, all 26 comets analyzed in this paper have gravitational inverse of the semi-major axis very small ($1/a_{ori,GR} < 6 \times 10^{-5}$ AU$^{-1}$) what makes almost all authors to call them dynamically new. But from Table 3 one can clearly recognize, that no more than 5 or 6 of them deserve for such a name. The rest visited our Planetary System during the previous perihelion passage, possibly experiencing strong

3 We have tested various shapes of others $g(r)$-like functions (Eq. 3) for both comets and differences in comet’s osculating orbital elements among all considered NG models were about one order of magnitude smaller than obtained between gravitational and NG models.
planetary perturbations and nobody knows what process or mechanism direct them at that time (or later) to the observable orbit at which we discovered them.

Second, we showed, that about 30% of our sample of long-period comets will remain observable in the next perihelion passage. Assuming that this is not a strange statistical fluctuation and keeping in mind that the only recognized force which makes their orbits to evolve is the tidal action of the Galactic matter, the widely quoted necessary condition for a comet to be made observable by Galactic perturbation is too strong. It is typical to demand that such a comet should decrease its perihelion distance from above 15 AU down to the observational value in one orbital period, otherwise they cannot cross the so called Jupiter-Saturn barrier, what now seems to be a “not necessary” condition:

We have shown that a significant percentage of long-period comets can visit the zone of visibility during at least two or three consecutive perihelion passages.

In this context we would like to suggest some correction to the widely quoted and very useful comet classification scheme proposed by Levison (1996). In his taxonomy the nearly isotropic comets (NICs) are divided into two subpopulations: new NICs and returning NICs on a basis of their original semi-major axis. This division should probably be replaced by a new one, based on the past dynamical history of a particular comet, especially if we want to keep naming NICs ‘new’ or ‘returning’ from the dynamical point of view.

There exist also a difficult problem of linking the ‘dynamical age’ of long-period comets with their physical properties, including the chemical composition. (Crovisier 2007, Crovisier et al. 2009, Biver et al. 2002). We tried to look for such a linkage but it seems that physical and dynamical characteristics might be quite different. For example the spectra of C/2001 Q4 NEAT and C/2002 T7 LINEAR were intensively investigated as they were bright and well observed comets. The observations of large molecules in both comets show (Remijan et al. 2006) that they are chemically very similar while our ‘dynamical ages’ for these comets are completely different. However, Remijan et al. (2006) also stated that when comparing the molecular production ratios with respect to water it appears that C/2002 T7 is more similar to Hale-Bopp physical class of comets (comets rich in HCN: HCN/H$_2$O > 0.2%), while C/2001 Q4 is more similar to Hyakutake class (comets with HCN/H$_2$O abundance ratio of ~0.1%) where two more investigated here comets (C/1989 X1 and C/1990 K1) are also included. On the other hand, C/1995 O1 Hale-Bopp and C/1996 B2 Hyakutake seems to be dynamically more similar than comets Hale-Bopp and C/2002 T7.

From the point of view of the apparent source of the long period comets, the main result of our paper is the list of previous aphelion and perihelion distances together with numbers of escaping VCs, presented in Table 3. But, comparing the past and future (Table 4) dynamics of 26 comets bring us to an interesting additional conclusion.

Treating these results as a probe of the past and future flux of observable long-period comets one can conclude that there exist surprising discrepancy between past and future cometary orbit evolution. Some 33% of comets arriving at the outer border of the Planetary System (250 AU from the Sun) are dynamically new, thus 66% have visited our Planetary System one orbital period before. On the other hand, 65% of comets leaving the zone of visibility will be lost in interstellar space and only 35% will return. But, in the “next turn”, they should constitute 66% of observable long-period comets, what might suggest that the flux of long-period comets should constantly decrease...

And the last remark, of more historical than scientific value. As far as we are able to recognize now Dybczyński (2009) only two comets from our sample studied in this paper were used by Oort (1950) in construction of his Table 1, namely C/1885 X1 Fabry and C/1913 Y1 Delavan. According to our results both these comets have visited the Planetary System during their previous perihelion and passed rather close to the Sun (1-2 AU) approximately $2.5 \times 10^6$ years ago.

As it concerns our future plans in this research, we have calculated over 100 precise original and future orbits, waiting for the detailed analysis of their past and future motion. In the next step we probably concentrate on “statistics improving” by significant increasing of number of analyzed comets, with a special attention on large perihelion distance comets, where even without detectable NG effects, orbits are very precise.

7 APPENDIXES

7.1 Chauvenet’s and Bessel criterions

Both criteria differ in the upper limit of the accepted residuals, $\xi$, e.g. observed minus computed values of right ascension, $\Delta\alpha \cdot \cos \delta$, and declination, $\Delta\delta$. According to the Chauvenet’s criterion (Chauvenet 1908) from the set of $N$ residuals, $\xi$, we should discard all values of $\xi$ for which

$$|\xi| > \sigma \cdot K_{1/2}(N)$$

where $\sigma$ is a dispersion of $\xi$:

$$\sigma = \sqrt{\frac{\sum_{k} \xi_k^2}{N}}$$

and $K_{1/2}(N)$ is the unknown upper limit of the integral of the probability distribution, $\phi(\xi)$:

$$\int_0^{K_{1/2}} \phi(x)dx = 1 - \frac{1}{2N}$$

where $x = \xi / \sigma$.

According to this criterion the data point is rejected if the probability of obtaining the particular deviation of residuals from the mean value is less than $1/(2N)$. To determine this probability the normal distribution of $\xi$ is assumed. The Bessel criterion (more restrictive than the Chauvenet’s criterion) rejects from the set of $N$ residuals all the values of $\xi$ for which

$$|\xi| > \sigma \cdot K_{1}(N)$$

where $K_{1}(N)$ is defined by:

$$\int_0^{K_{1}} \phi(\xi)d\xi = 1 - \frac{1}{N}$$

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References

Biver N., Bockelée-Morvan D., Crovisier J., Colom P., Henry F., Moreno R., Paubert G., Despois D., Lis D. C., 2002, Earth Moon and Planets, 90, 323
Breiter S., Dybczyński P. A., Eliea A., 1996, A&A, 315, 618
Chauvenet W., 1908, Manual of spherical and practical astronomy, embracing the general problems of spherical astronomy, the special applications to nautical astronomy. Manual of spherical and practical astronomy, embracing the general problems of spherical astronomy, the special applications to nautical astronomy (5th ed., Vols. 1-2). New York: J.B. Lippincott.
Crovisier J., 2007, ArXiv Astrophysics e-prints
Crovisier J., Biver N., Bockelée-Morvan D., Boissier J., Colom P., Lis D. C., 2009, Earth Moon and Planets, 105, 267
Dones L., Weissman P. R., Levison H. F., Duncan M. J., 2004, in Festou M. C., Keller H. U., Weaver H. A., eds, Comets II, University of Arizona Press, Tucson, pp 153-174
Dybczyński P. A., 2001, A&A, 375, 643
Dybczyński P. A., 2006, A&A, 449, 1233
Dybczyński P. A., 2009, http://apollo.astro.amu.edu.pl/PAD/
Dybczyński P. A., Królikowska M., 2009, http://apollo.astro.amu.edu.pl/WCP
Dybczyński P. A., Kwiatkowski T., 2003, ArXiv Astrophysics e-prints
Everhart E., 1985, in Carusi A., Valsecchi G. B., eds, IAU Colloq. 83: Dynamics of Comets: Their Origin and Evolution An efficient integrator that uses Gauss-Radau spacings. D.Reidel Publishing Company, pp 185–202
Fernández J. A., ed. 2005, Comets - Nature, Dynamics, Origin and their Cosmological Relevance Vol. 328 of Astrophysics and Space Science Library
Fouchard M., Froeschlè C., Matese J. J., Valsecchi G., 2005, Celestial Mechanics and Dynamical Astronomy, 93, 229
Heisler J., Tremaine S. D., 1986, Icarus, 65, 13
Hughes D. W., 1991, Journal of the British Astronomical Association, 101, 119
Królikowska M., 2001, A&A, 376, 316
Królikowska M., 2004, A&A, 427, 1117
Królikowska M., 2006, Acta Astronomica, 56, 385
Królikowska M., Sitarski G., Soltan A. M., 2009, ArXiv e-prints
Levison H. F., 1996, in Retigg T. W., Hahn J. M., eds, ASP Conf. Ser. 107: Completing the Inventory of the Solar System Comet taxonomy, pp 173–191
Marsden B. G., Sekanina Z., Everhart E., 1978, AJ, 83, 64
Marsden B. G., Sekanina Z., Yeomans D. K., 1973, AJ, 78, 211
Marsden B. G., Williams G. V., 2008, Catalogue of Cometary Orbits 17th Edition. Smithsonian Astrophysical Observatory, Cambridge, Mass.
Matese J. J., Lissauer J. J., 2004, Icarus, 170, 508
Meech K. J., Knopp G. P., Farnham T. L., Green D., 1995, Icarus, 116, 46
Oort J. H., 1950, Bull.Astron.Inst.Nether., 11, 91
Prętka H., 1998, in A. Lopez Garcia et al. ed., IV International Workshop on Positional Astronomy and Celestial Mechanics Different modeling of galactic perturbations in the motion of comets. Astronomical Observatory, University of Valencia, Spain, pp 249–256
Prętka H., 1999, in B.A.Steves A.E.Roy eds, The Dynamics of Small Bodies in the Solar System Galactic perturbations in the motion of comets. Kluwer Academic Publisher, pp 197–202