Topological Shiba bands in artificial spin chains on superconductors

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A major challenge in developing topological superconductors for implementing topological quantum computing is their characterization and control. It has been proposed that a p-wave-gapped topological superconductor can be fabricated with single-atom precision by assembling chains of magnetic atoms on s-wave superconductors with spin–orbit coupling. Here we analyse Bogoliubov quasiparticle interference in Mn chains, constructed atom by atom on Nb(T10), and reveal the formation of multi-orbital Shiba bands using momentum-resolved measurements. We find evidence that one band features a topologically non-trivial p-wave gap, as inferred from its shape and particle-hole asymmetric intensity. Our work is an important step towards a distinct experimental determination of topological phases in multi-orbital systems by bulk electron band structure properties only.

Topological superconductors (TSCs) in one dimension can host zero-energy Majorana bound states (MBSs) at their edges that are protected from excitations by a topological gap. With their non-Abelian exchange statistics they are candidates for topological qubits, offering ways to strong stability against decoherence. With the hope of realizing TSCs, one-dimensional (1D) magnetic nanostructures proximity-coupled to s-wave superconductors and chains of Fe atoms with helical spin order on Re(0001) that can be tailored and controlled down to each individual atom. In the case of ferromagnetically ordered nanowires, topologically non-trivial phases are predicted to be realized in the presence of Rashba spin–orbit coupling (SOC) whenever the number of spin-polarized bands crossing the Fermi energy is even. The Rashba SOC, together with the superconducting pairing, then induces gaps in the p-wave nature in these bands. For a weak interaction between the orbitals of the magnetic atoms in the wire, the low-energy electronic Shiba bands are dominated by the hybridizing Yu–Shiba–Rusinov (YSR) states of the individual atoms.

Experimental STS studies of these systems typically argued for the presence of a TSC phase using the presence or absence of zero-energy edge modes. This way of identifying a TSC phase bears the risk of confusing trivial edge states, which are ubiquitous in such systems, with MBSs. Moreover, such studies concentrated on the consequence of, rather than the cause for, TSCs, which is purely determined by the bulk band structure in the interior of the chain. To gain a better understanding of magnet–superconductor hybrid systems, experimental measurements of the bands responsible for the formation of MBSs are highly desirable. Unfortunately, it is extremely difficult to perform standard momentum–resolved measurements, such as angle-resolved photoemission spectroscopy, on nanoscale or even atomic chains on surfaces because of their sparse distribution, length variations or the impossibility to relocate chains that have been built atom by atom using scanning tunnelling microscope (STM) tip-induced atom manipulation. However, in principle, it is possible to extract the band structure from quasiparticle interference (QPI) of the electronic states inside the chain. Scattering of quasiparticles with energy E at defects or at the chain’s edges between initial and final momenta k_i and k_f, respectively, leads to interference and thereby produces modulations of the local electron density of states (LDOS) with wavelength λ, which can be directly imaged using STS. The resulting dispersions of the scattering vectors |q| = |k_i − k_f| = 2k_F are closely related to the quasiparticle band structure. Although this technique has been applied successfully to a variety of complex electron systems, such as high-temperature superconducting cuprates, Fe-based superconductors, heavy fermions or topological insulators, it has not been used so far for experimentally revealing the band structure of magnetic chains coupled to superconductors.

In this Article, we perform QPI imaging on ferromagnetic manganese (Mn) chains on the elemental superconductor niobium (Nb), a candidate for realizing 1D TSCs. The Mn chains are assembled by STM-based atom manipulation on the clean and non-reconstructed (110) surface of Nb. By changing the length in an atom-by-atom fashion, from a single Mn atom to chains comprising up to several tens of atoms, we were able to observe multi-orbital Shiba band formation inside the energy gap Δ of the superconducting Nb substrate, starting from the single impurity YSR states, and experimentally study the Shiba bands in view of their topological properties.

Multi-orbital YSR states of Mn atoms

We used a clean Nb(110) crystal as a substrate for the chains (Methods and Supplementary Note 1), as this exhibits the largest gap, Δ = 1.51 meV (ref. 45), among all elemental superconductors and therefore provides an improved effective energy resolution with respect to previous experiments. To enhance our energy resolution further, we indented our W tip into the substrate to a depth of several nanometres so that a superconducting cluster was
formed on the tip apex. We then deposited single Mn atoms, which appear as ~90-nm-high protrusions in Fig. 1a. We measured clear superconductor–insulator–superconductor (SIS) tunnelling (grey spectrum, upper panel of Fig. 1d) on the bare Nb substrate with the Nb covered tip. All features of the sample’s LDOS appear shifted by an energy given by the gap of the superconducting tip, \( \Delta \), and below \( \Delta \) (Supplementary Note 3). Therefore, the coherence peak of Nb is visible in the following (bottom, Fig. 1d) and above \( \Delta \). Tunnelling spectra measured on top of the Mn atoms (red spectrum) show four pairs of additional peaks inside this gap, named \( \alpha_+ \), \( \beta_+ \), \( \gamma_+ \), and \( \delta_+ \), where the peaks above (+) and below (−) \( E_F \) appear at bias voltages symmetrically around zero. They are assigned to the multiplet of YSR states of the Mn atom due to the five \( d \) orbitals that induce the magnetism \( 30,31,44 \). We numerically deconvolute the measured SIS spectra \( 45 \) in the following (bottom, Fig. 1d) such that the presented data directly correspond to the LDOS of the sample, as is common for STS experiments (Supplementary Note 2). The spatial structure of the YSR states can be determined using \( \text{d}/\text{d}V \) maps (Methods) at the peak energies (Fig. 1c); these resemble the associated shapes of the relevant orbitals to a good approximation \( 30,31,44 \). We thus attribute the state \( \alpha \), which is strongest in intensity and closest to the coherence peak, to the atomic \( d_{z^2} \) orbital, the \( \beta \) state to the in-plane \( d_{xy} \) orbital, the \( \gamma \) state to the \( d_{x^2-y^2} \) orbital, and the \( \delta \) state, which is closest to \( E_F \) to \( d_{xz} \) with the \( y \) axis pointing along the [110] direction (Fig. 1c). Note, that the \( \alpha \) peak has a strongly increased intensity with respect to its partner \( \gamma \). This particle–hole asymmetry in the spectral weight of the YSR state is a well-known effect and is related to additional spin-independent scattering of the substrate electrons off the corresponding Mn electrons \( 32,33,35,36 \). We now turn to the construction of such Mn chains and the investigation of their electronic properties.

### Confined Bogoliubov–de Gennes quasiparticles in Mn chains

The Nb(110) surface allows for STM tip-induced atom manipulation \( 34 \) (Methods). We were thus able to construct atomically well-defined linear chains without any defects (Fig. 1b), which is crucial for enabling the free propagation of quasiparticle waves. The chains are built along the [001] direction and consist of a defined number \( N \) of Mn atoms on nearest-neighbouring four-fold coordinated hollow adsorption sites with a distance of \( d = 329.4 \) pm, as sketched in Fig. 1c. All magnetic moments in such Mn chains align ferromagnetically, as determined by spin-polarized STM measurements \( 37 \). To observe the formation of bands emerging from the YSR states of the Mn atoms, we recorded \( \text{d}/\text{d}V \) spectra along the longitudinal axis through the centre of each constructed Mn chain (dashed line, top panel of Fig. 2a); these are called \( \text{d}/\text{d}V \) line profiles in the following. The deconvoluted \( \text{d}/\text{d}V \) line profiles are shown in Fig. 2 for three chains of different length (data for all chains from \( N = 1 \) to \( N = 36 \) are provided in Supplementary Note 5). Energetically sharp states with a defined number of LDOS maxima \( n_e \) along the chains are formed inside the gap \( \Delta \). The number of maxima starts from \( n_e = 1 \) for the state at an energy of \( E_F \approx +0.5 \) meV and then increases in integer steps towards negative energies approaching the coherence peak, where the energy \( E_n \) of these states depends on the length \( N \) of the chain. We interpret these states as signatures of QPI of the Bogoliubov–de Gennes (BdG) quasiparticles confined in the superconductor–magnet hybrid, which we explain further in the following. As shown in Supplementary Note 5, by plotting the \( \text{d}/\text{d}V \) signal averaged along the entire chain against length \( N_c \), we can observe a continuous shift of \( E_F \) from the energy of the \( \alpha \) YSR state for \( N = 1 \) (the single atom in Fig. 1d) to \( E_F = +0.5 \) meV for \( N > 10 \). This provides strong evidence that the corresponding band, which

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**Fig. 1** Multi-orbital YSR states of Mn atoms and construction of Mn chains. **a**, STM image of single Mn atoms adsorbed on a clean Nb(110) surface. Scale bar, 2 nm. **b**, Series of STM images obtained during the construction process of Mn chains (\( N \) is indicated) using atom manipulation. **c**, Sketch of the uppermost layer of Nb atoms (brown) and the positions of Mn atoms (red) arranged in a linear chain along the [001] direction. The lattice constant \( a \) of Nb is indicated. **d**, Top: \( \text{d}/\text{d}V \) spectra taken using a superconducting tip averaged over the centre of a single Mn atom and over a clean area of the substrate. Bottom: the same data after numerical deconvolution. The energetic positions of the sample \( \Delta \) and tip \( \Delta \) gaps are indicated by dashed vertical lines and of the pairs of multi-orbital YSR peaks by Greek letters. **e**, \( \text{d}/\text{d}V \) maps around a single Mn atom shown in the STM image (topography) evaluated at the indicated bias voltages according to the YSR-multiplet energies. The panels each have dimensions of \( 2 \times 2 \) nm\(^2\). Parameters: \( V = 50 \) mV, \( I = 50 \) pA (a), \( V = -6 \) mV, \( I = 200 \) pA (b) and \( V_{\text{stat}} = -6 \) mV, \( I_{\text{stat}} = 1 \) nA, \( V_{\text{mod}} = 20 \) \( \mu \)V (d, e).
hosts these BdG quasiparticles, is a Shiba band formed by hybridizing \( \alpha \)-YSR states, and we will consequently name it the \( \alpha \)-YSR band in the following. Most strikingly, there is a region of width \( 2\Delta_{\text{ind}} \approx 360 \mu \text{eV} \) around \( E_{\text{f}} \) where the \( \text{d}I/\text{d}V \) intensity is strongly reduced, resembling an induced \( p \)-wave gap in the \( \alpha \)-YSR band, which will be substantiated in the following.

In addition to the \( \text{d}I/\text{d}V \) line profiles taken along the longitudinal axis of each chain, we mapped the whole \( \text{d}I/\text{d}V \) signal in a region covering the chain and its vicinity (we call these \( \text{d}I/\text{d}V \) maps in the following). The \( \text{d}I/\text{d}V \) maps taken at selected energies of one of the longest chains, \( \text{Mn}_{34} \), are shown in Fig. 3. The previously discussed states of the \( \alpha \)-YSR band, marked by the number of maxima \( n_{\alpha} \), are localized predominantly inside the area on top of the chain, which is enclosed by white dashed lines. We also observe additional confined BdG states with a different number of maxima, called \( n_{\alpha} \) in the following, which are mainly located outside this area on both sides along the chain. These thus have very weak intensity in the chain centres. Their maxima in intensity appear slightly offset in the \([110]\) direction from the longitudinal chain axis, at a distance and direction that matches the extent and orientation of the lobes of the \( \delta \)-YSR state observed for the single Mn atoms (Fig. 1e). It is thus very plausible that these states derive from an additional Shiba band formed by the hybridization of the \( \delta \)-YSR states, which we name the \( \delta \)-YSR band in the following.

**Multi-orbital YSR-band dispersions**

To extract the dispersion of the \( \alpha \)- and \( \delta \)-YSR bands, we analyse the numbers of maxima \( n_{\alpha} \) and \( n_{\delta} \) of the confined BdG quasiparticles in dependence on energy and chain length. We recall that the Shiba bands are induced by local magnetic coupling between the chain and the superconductor, and the associated BdG quasiparticles will experience strong confinement in an effective 1D potential well of length \( L = Na \) formed by the magnetic chain, because they cannot exist in the bulk superconductor. Therefore, the BdG quasiparticles with initial momenta \( k_{i} \) are scattered at the chain’s ends to momenta \( k_{f} \) by a scattering vector of length \( |q| = |k_{f} - k_{i}| \). This scattering happens with high probability in the same band. The resulting interference becomes visible as a standing wave in the LDOS(\( E \)), which is approximately proportional to the \( \text{d}I/\text{d}V \) signal measured at \( V = E/\epsilon \). Constructive interference is reached whenever \( q = \pm 2n\pi/L = \pm 2n\pi/Na \), with \( n \in \mathbb{N} \). By this selection rule, only a discrete set of quasi-particle states of the Shiba bands have non-zero LDOS. The LDOS(\( E \)) then equals \( \sum_{n}\phi_{n}|\delta (E - E_{n}(q))| \) with the confined states \( \phi_{n} = \sin (n \pi x/Na) \) (here \( x = 0 \) corresponds to one of the ends of the chain). We therefore fit the \( \text{d}I/\text{d}V \) line profiles of Fig. 2 with a linear combination \( \text{d}I/\text{d}V(E = \epsilon V, x) = \sum_{n=1}^{18} c_{n}(E) (|\phi_{n}(x)|)^{2} \). Unlike the delta functions, the coefficients \( c_{n}(E) \) take into account the strongly different intensities of the confined BdG quasiparticle states. We restrict the analysis to \( n \leq 18 \) (for details see Supplementary Note 4 and ref. 43). They exhibit pronounced peaks at the energetic positions of the confined BdG quasi-particle state with scattering vector \( q = \pm 2n\pi/Na \) in the chain and therefore relate \( E \) and \( q \) of the quasiparticles. This method enables us to approximate the dispersion of the scattering
The strongest peaks in the coefficients $c_n$ plotted against their energies and scattering vectors $q/2 = n\pi/N$ extracted from the $dI/dV$ line profiles of Fig. 2 of all chains with $14 < N < 36$. Colours indicate the respective intensities of the peaks. The induced gap $\Delta_{ind}$ is indicated on the right side of the panel. The inset shows the central region with adjusted contrast. The $\alpha$- and $\delta$-YSR bands are indicated by Greek letters. 

b. Fit of the band dispersion from the Shiba chain model (solid line) to the manually evaluated experimental data of the $\alpha$-YSR band (dots; Supplementary Note 4). Parameters of the model (Methods): $\Delta_1 = 1.5$ meV, $A = 3.1$, $B = 2.35$, $\xi = 0.77$ nm, $k_A = 0.69$ a/Å, $k_F = 0.14$ a/Å. For the dashed line, the $p$-wave nature of the gap was artificially set to zero ($k_F = 0$).

c. Experimentally detectable particle contributions of the band dispersions from the Shiba chain model for the topological phase (left) using the same parameters that fit to the experimental data in a and b, at the topological phase transition using $A = 3.6$ (centre) and in a trivial phase using $A = 3.9$ (right).

We can identify a dominant nearly parabolic band, which is assigned to the $\alpha$-YSR band, with a band maximum at $E_\alpha \approx +0.5$ meV ($q = 0$) and negative effective mass. Note that we can also resolve the particle–hole partner of this band with a minimum at $E_\alpha \approx -0.5$ meV ($q = 0$) and a positive effective mass, which is compulsory for a BdG quasiparticle band of a superconductor with non-vanishing particle weight, but which was barely visible in the original data representation of Fig. 2. In the region around $E_\alpha$, the $\alpha$-YSR band is gapped by $2\Delta_{ind} \approx 360$ meV. Importantly, the intensity of the $\alpha$-YSR band has a comparatively strong particle–hole asymmetry, similar to the $\alpha$-YSR state of the single Mn atom (Fig. 1d). Zooming into the gap region close to $q = 0$, we can also resolve the $\delta$-YSR band, which has a Dirac-like dispersion with almost vanishing difference of particle and hole weights. The $\delta$-YSR band crosses $E_\delta$ at $q/2 = \pm 0.1\pi/a$ without opening a gap within our experimental energy resolution. This band, which is mainly localized alongside the chain (Fig. 3 and Supplementary Note 4), appears with a weak intensity in the analysis of the $dI/dV$ line profiles that are taken on top of the chains. Still, there is some faint intensity of the corresponding BdG quasiparticles that are barely visible in the original data of Fig. 2 within the induced gap. In the following, we will provide evidence of the $p$-wave nature of the gap $2\Delta_{ind}$ in the $\alpha$-YSR band and possible conclusions about the topological character of this band.

### Tight-binding model for Shiba bands

We model the Shiba bands by a low-energy model based on ref. 10 for YSR impurities in a superconducting host with effective Rashba SOC (Methods). We derive Shiba band structures that closely resemble the experimental $\alpha$- and $\delta$-YSR bands (Fig. 4b,c and Supplementary Figs. 6 and 7) using parameters that are consistent (Supplementary Note 4) with the energies and particle–hole asymmetries of the respective single-atom YSR states from which the bands emerge (Fig. 1d). In particular, Fig. 4b and Fig. 4c (left) show the corresponding fit to the $\alpha$-YSR band. Originating from the non-degenerate single-impurity YSR states (Fig. 1), the $\alpha$-YSR band is non-degenerate itself. A symmetry analysis of the system corroborates this result. We therefore conclude that the gap opened in the $\alpha$-YSR band is due to a $p$-wave contribution to the pairing mechanism. From the model calculations, we find by direct computation that the topological index $\nu'$ of the $\alpha$-YSR band is non-trivial. Generally, the topological index can also be derived from the experimental data assuming a weak superconducting pairing ($\Delta_{ind} \ll$ Shiba band width) and if the band features a strong asymmetry in its respective particle–hole weight (Supplementary Note 7), both of which are fulfilled for the $\alpha$-YSR band. We extrapolate to the fictitious case of a gapless band structure by setting the effective SOC to zero (dashed line, Fig. 4b) and count the number of Fermi level crossings $c = 1$ between $k = 0$ and $k = \pi$. This directly relates to Kitaev’s $Z_2$-invariant $M = (-1)^c = -1$ for this band. We thus find that the $\alpha$-YSR band carries a non-trivial topological index. By contrast, band structures with trivial gaps (Fig. 4c, centre and right), which can be simulated by changing the coupling of the individual impurity to the superconductor (parameterized by $A$; Methods), would have a strongly different appearance, in conflict with the experiment.

We suppose that the $p$-wave pairing in the $\alpha$-YSR band is induced by a Rashba-like SOC caused by the breaking of inversion symmetry at the Nb(110) surface46. Inversion symmetry-breaking naturally has a stronger effect on the $d_{z^2}$ orbitals pointing along the surface normal as compared to the in-plane oriented $d_{xy}$ orbitals46, which could explain the absence of a measurable gap in the...
δ-YSR band. In addition, the superconducting order parameter of a Shiba chain in the presence of SOC becomes $k$-dependent and thus can prevent the opening of a measurable gap at $k$ values where $\Delta_k$ vanishes (Supplementary Note 6 and Supplementary Fig. 7). We argue, however, that it is unlikely that the $p$-wave pairing in the δ-YSR band is exactly zero, but rather is just too small to be resolved experimentally.

Discussion

We finally discuss the possible emergence of topological edge states from the observed Shiba bands. MBs are, in principle, expected to form at the chain’s ends if there is an odd number of topological bands, all bands are gapped and if the chain is long compared to the localization length of the MBs $\xi_M$ (refs.[1,10]). The question arises whether the zero-energy mode observed in Fig. 3 could be the MB of the $\alpha$-YSR band. Zero-energy states with a similar spatial distribution have been attributed to topological MBs in previous work[19,20]. However, in the system investigated here, these states clearly do not belong to the $\alpha$-YSR band, because MBs arising from this band would have a modulation of $q_{\alpha,1}/2 \approx 0.2\pi/a$—that is, equal to the scattering vector at the gap minimum—imprinted on their LDOS[19,20] (Supplementary Fig. 6b). The zero-energy state in Fig. 3 is modulated with only half of this wavevector and we can therefore exclude that it is the zero-energy MB stemming from the $\alpha$-YSR band. Instead, because the δ-YSR band remains gapless up to experimental resolution, some confined BdG quasiparticles of this band are usually located inside the gap $\Delta_{\delta}$ of the $\alpha$-YSR band. For certain lengths of the chain, they can be located very close to $E_F$ as is the case, for example, for the state with $n_s=3$ of the $M_n$ chain marked in the $dI/dV$ maps in Fig. 3.

Using our knowledge about the $\alpha$-YSR band dispersion, we are able to estimate the expected MBs localization length via $\xi_M = h\nu_F/\Delta_{\delta}$ from the experimental value of the induced gap $\Delta_{\delta} \approx 180$ μeV and the Fermi velocity $\nu_F$, which is evaluated from the slope of the $\alpha$-YSR band close to $E_F$ via $\nu_F = 2\hbar^{-1}\partial E/\partial q(0) \approx 1\times 10^4$ m s$^{-1}$, yielding $\xi_M \approx 3.8$ nm (Supplementary Note 4 and Supplementary Fig. 4). Therefore, in our longest chains ($N=36$ of length $L=Na \approx 12$ nm, the expected MBs of the $\alpha$-YSR band should have only little overlap. However, their apparent absence in our data would be explained by hybridization with states of the $\delta$-YSR band or with states of potentially experimentally invisible Shiba bands formed by the hybridizing YSR states of the other orbitals. We emphasize that the presence of additional unresolved bands may change the total topological phase of the chain, which is the product of the individual topological indices of all relevant bands. We cannot experimentally exclude that additional topologically non-trivial bands originating from the $d_{\alpha,1}$, $d_{\alpha,2}$ or $d_{\alpha,3}$ orbitals exist, as tunnelling into these in-plane orbitals is strongly suppressed (Fig. 1d), which explains the invisibility of these bands in Fig. 4. Hybridizations between such bands generally destroy the topological protection of MBs, except under very particular conditions[35]. This implies additional constraints for realizing MBs in multi-orbital magnetic chains on a superconductor platform. Thus, progress can be made in the atomic-scale tuning of the sub-gap band structure, which may enable the design of robust topologically superconducting phases in future experiments.

Online content

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Methods

Experimental procedures. All experiments were performed in a home-built low-temperature STS facility at a base temperature of \( T = 0.32 \) K (ref. 3). A clean Nb(110) surface was obtained by flashing the single crystal to \( T > 2.700 \) K (ref. 3). Single Mn atoms were deposited successively while keeping the substrate cooled below \( T = 7 \) K. This resulted in a statistical distribution of adatoms on the surface, as shown in Fig. 1a.

STM images were obtained by maintaining a constant-current \( I \) while applying a bias voltage \( V \) across the tunnelling junction. For the measurement of \( dI/dV(V) \) spectra, the tip was stabilized at bias voltage \( V_{\text{sta}} \) and current \( I_{\text{sta}} \). Subsequently, the feedback loop was opened and the bias voltage was swept from \(-4\) mV to \(+4\) mV. The differential tunnelling conductance, \( dI/dV \), was measured using a standard lock-in technique with a small modulation voltage \( V_{\text{mod}} \) (t.m.s.) of modulation frequency \( f = 4.142 \) kHz added to the bias voltage. The \( dI/dV \) line profiles and maps were acquired by recording multiple \( dI/dV \) spectra along a line or grid, respectively. The 1D chains were assembled using lateral atom manipulation techniques at low tunnelling resistances of \( R \approx 30–60 \) k\( \Omega \).

Tight-binding model for Shiba bands. Pientka et al. have developed a model\(^{30} \) for a chain of YSR impurities embedded in a superconducting substrate, which describes weakly interacting impurities at energies close to the Fermi level. We have extended the model to include single-particle scattering by a YSR impurity. This is an essential step to describe the experimentally observed particle–hole asymmetry.

The BdG Hamiltonian of a chain consisting of \( n \) YSR impurities that are spin-polarized out of plane is

\[
H = \epsilon_p \mathbf{r}^2 + \Delta_\mathbf{r}^2 + \sum_{j=1}^{n} \left[ V_{\mathbf{r}} \mathbf{r}^2 - J_\sigma \mathbf{r}^2 \right] \delta \left( \mathbf{r} - j a (1, 0, 0)^T \right),
\]

where \( a \) is the distance between the impurities, \( \Delta_\mathbf{r} \) denotes the superconducting \( s \)-wave pairing, \( J \) is the exchange coupling between the magnetic impurity and the superconductor, \( V_{\mathbf{r}} \) is the non-magnetic scattering at the impurity and \( r' = 1_{i \times 2} \otimes r \) and \( \mathbf{r}'' = 1_{i \times 2} \otimes r \) (with Pauli matrices \( r \) ) act on the Nambu space in the basis \( \{ |c^+ (r), c (\mathbf{r})\} \) and \( \mathbf{r} \) is the Kronecker product.

Supplementary information

The analysis codes that support the findings of the study are available from the corresponding author upon reasonable request. Source data are provided with this paper.

Data availability

Data that support the findings of this paper are available from the corresponding author upon reasonable request.

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Author contributions

L.S., P.B., R.W. and J.W. conceived the experiments. L.S. and P.B. performed the measurements and analysed the experimental data together with J.W. T.P. derived the effective low-energy Shiba model. L.S. performed the numerical simulations using the effective low-energy Shiba model and the fitting to the experimental data. D.C., E.M. and S.R. performed numerical simulations, which were essential for the understanding of the system. L.S. prepared the figures. L.S. and J.W. wrote the manuscript. All authors contributed to the discussions and to correcting the manuscript.

Competing interests

The authors declare no competing interests.

Additional information

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