Spin Texture in Doped Mott Insulators with Spin-Orbit Coupling

Shuai A. Chen, Zheng-Yu Weng, and Jan Zaanen

1 Institute for Advanced Study, Tsinghua University, Beijing 100084, China
2 Institute-Lorentz for Theoretical Physics, Leiden University, P.O. Box 950, The Netherlands

(Dated: Tuesday 15th February, 2022)

A hole injected into a Mott insulator will gain an internal structure as recently identified by exact numerics, which is characterized by a nontrivial quantum number whose nature is of central importance in understanding the Mott physics. In this work, we show that a spin texture associated with such an internal degree of freedom can explicitly manifest after the spin degeneracy is lifted by a weak Rashba spin-orbit coupling (SOC). It is described by an emergent angular momentum $J_z = \pm 3/2$ as shown by both exact diagonalization and variational Monte Carlo calculations, which are in good agreement with each other at a finite size. In particular, as the internal structure such a spin texture is generally present in the hole composite even at high excited energies, such that a corresponding texture in momentum space, extending deep inside the Brillouin zone, can be directly probed by the spin-polarized angle-resolved photoemission spectroscopy (ARPES). This is in contrast to a Landau quasiparticle under the SOC, in which the spin texture induced by SOC will not be protected once the excited energy is larger than the weak SOC coupling strength, away from the Fermi energy. We point out that the spin texture due to the SOC should be monotonically enhanced with reducing spin-spin correlation length in the superconducting/pseudogap phase at finite doping. A brief discussion of a recent experiment of the spin-polarized ARPES will be made.

CONTENTS

I. Introduction 1
   A. Background 1
   B. Main results 3

II. Models 4
   A. The $t$-$J$ and $\sigma \cdot t$-$J$ models 4
   B. Spin-orbit coupling 5

III. The Single Hole Study 5
   A. Exact diagonalization and variational ansatz in the absence of SOC ($\lambda = 0$) 5
   B. The single hole state under a weak Rashba spin-orbit coupling ($\lambda \neq 0$) 6
      1. Exact diagonalization 6
      2. Variational Monte Carlo calculation 7

IV. Discussion: Finite Doping 8

V. Concluding Remarks 10

Acknowledgments 11

A. Sign structures in the $t$-$J$ and $\sigma \cdot t$-$J$ models 12
B. One-hole wave function ansatz and emergent symmetry in the $t$-$J$ model 12
C. Rashba interaction 13
   1. Total angular momentum $J_z$ 13
   2. ED under periodic boundary condition 13
D. Variational Monte Carlo procedure 14
   1. Variational ground state at half-filling 14
   2. Variational wave functions for the $t$-$J$-R model 14
E. Spin-polarized spectral function 15
F. Fragility of spin texture in a Fermi Liquid 16

References 16

I. INTRODUCTION

A. Background

A Landau’s quasiparticle with a definite momentum, charge, and spin-1/2 has played a fundamental role in a conventional Fermi liquid. Whether a doped charge (hole) can still propagate like a Bloch wave, is important in understanding the two-dimensional (2D) Mott insulator at finite doping. The latter is widely believed to be intrinsically related to the high-$T_c$ superconductor in the cuprate [1, 2].

For a single hole injected into a quantum spin antiferromagnetic (AF) background, although earlier studies [2–7] have indicated a Landau-like quasiparticle behavior based on effective approaches, very recently it has been revealed by the unbiased exact diagonalization (ED) and density matrix renormalization group (DMRG) numerical calculations [8–10], that the doped hole will generate a composite structure by twisting the spin background via a nontrivial backflow in the $t$-$J$ model. Manifestly a net spin current will further show up due to the presence of a spin-1/2 partner around the hole, which gains a nontrivial angular momentum, $L_z = \pm 1$, or a new double degeneracy under an open boundary lattice (OBC) with a $C_4$ rotational symmetry [8].

Such a non-Landau-like novel structure of the composite hole revealed by ED and DMRG calculations can be well reproduced by the variational Monte Carlo (VMC)
by an emergent quantum number, i.e., the angular momentum $L_z$ mentioned above.

The detailed composite structure is further shown in Figs. 1(b) and (c) based on the correlators, $\langle n^h_{i_0} S^z_i \rangle$ and $\langle n^h_{i_0} J^x_{i,i+\hat{y}} \rangle$, which measure the distribution of the net $S_z = 1/2$ around the doped hole [(b)] and the corresponding circulating spin current [(c)], respectively. Here $n^h_{i_0}$ denotes the hole number at site $i_0$, $\vec{S}^z_i$ the summation of two nearest-neighboring spins along the $x$-axis, i.e., $S^z_i + S^z_{i+\hat{x}}$, with $i$ in the opposite sublattice site of $i_0$ such that a staggered oscillation is smoothed, and $J^x_{i,i+\hat{y}}$ the transverse spin current [cf. Eq. (9)]. The variational ground state [11] [cf. Eq. (7)] is obtained at a lattice of $N = L^2 = 20 \times 20$. The average distance of the $S_z = 1/2$ from the hole, $\xi_{sh}$, is shown in the inset of Fig. 1(b), which is essentially proportional to the spin-spin correlation length $\xi$ of the spin AF background that can be artificially tuned in the VMC calculation (two cases of $\xi \gg L$ and $\xi \sim 2.7$ in units of the lattice constant are shown in the main panel with $L$ denoting the sample length). Then, Fig. 1(c) further shows that the spin current around the hole gets enhanced with reducing $\xi_{sh} \sim \xi$. It is noted that here we are always focused on the one-hole case in a finite-size sample. If $L \to \infty$, a true AF long-range order $\xi \to \infty$ will set in, such that spin excitations become gapless, which may also contribute to the spin currents induced by the hole in addition to just an $S^z = 1/2$ shown in Fig. 1(a) at the finite lattice size. We do not consider this singular limit in the following.

Note that there are two chiralities of the spin currents in the one-hole degenerate ground states corresponding to the novel quantum number $L_z = \pm 1$ (only $L_z = 1$ is shown here). By contrast, the chiral spin current can be turned off with vanishing angular momentum in the so-called $\sigma \cdot t$-$J$ model, in which the doped hole restores...
the behavior of a Landau-like quasiparticle [11]. Here the sole distinction between the $t$-$J$ and $\sigma \cdot t$-$J$ models is characterized by the nontrivial backflow of spin current in the former, which is absent in the latter with the disappearance of the novelty of the hole composite, although $\xi_{\text{sh}}$ remains similar in both cases [cf. Fig. 1(b)].

It is then natural to raise the following question, i.e., how can one detect such an internal quantum structure of the composite hole experimentally? To address this question, in this paper, we introduce a weak Rashba spin-orbit coupling (SOC), which may be present in a double-layer system of the cuprate materials like Bi2212 [13]. Then we show that the hidden transverse spin current pattern appearing around the hole in Fig. 1 can manifest explicitly an emergent semiclassical spin texture lying in the $x$-$y$ plane, as illustrated in Fig. 2(a). Here a real space distribution of the correlator $\langle n_i^h S_{x,y}^{h} \rangle$ is presented with the hole site $i_0$ fixed at the center of the sample (denoted by the open circle in Fig. 2). Namely, the novel internal composite structure of the hole in the pure (without SOC) $t$-$J$ model can be effectively visualized via a frozen transverse spin texture, which is induced by a weak SOC lifting the spin degeneracy and resulting in the locking of $S_z$ with $L_z$ by a total angular momentum $J_z = L_z + S_z$. By contrast, such a transverse spin texture is absent in the $\sigma \cdot t$-$J$ model in the same weak SOC [cf. Fig. 2(b)]. In the latter, the single hole behaves like a Bloch wave without a surrounding spin current [cf. Fig. 1]. In the following, we further briefly summarize the main results obtained in the present work and their physical implications.

B. Main results

In this work, we show that the hidden spin current pattern of the spin-1/2 partner in a hole composite can manifest itself as an emergent semiclassical spin texture with the introduction of a weak Rashba SOC. It is explicitly exhibited in either the spatial space as indicated by Fig. 2(a) above and the momentum space (see below) as found by ED as well as the VMC calculation.

As shown in Fig. 2(a), such a spin twist as measured by the correlator $\langle n_i^h S_{x,y}^{h} \rangle$ is formed in the $x$-$y$ plane around the hole. Physically it reflects the underlying antiparallel spin alignments in the $x$-$y$ plane along either the $x$ or $y$ direction across the hole to facilitate the hopping. By a sharp contrast, no such a relative spin twist is shown in Fig. 2(b) for the $\sigma$-$t$-$J$ model, which is consistent with the absence of a spin current structure in Fig. 1(c). Note that $\xi \sim 2.7$ is chosen in the VMC calculation such that the distribution of the spin-1/2 partner is pulled closer to the hole to enhance the effect in Fig. 2.

The energy shifts of the ground state characterized by $J_z = \pm 3/2$ and the first excited one at $J_z = \pm 1/2$, together with the ground state energy shift in the $\sigma \cdot t$-$J$ model, are calculated by both ED and VMC methods as a function of the SOC strength $\lambda$, which are in excellent agreement as presented in Fig. 3. Here it is interesting to note that the effect of the SOC in the $\sigma \cdot t$-$J$ model is much weaker than that in the ground state energy of the $t$-$J$ model, indicating that the spin current structure substantially enhances the effect of the Rashba SOC.

Furthermore, under the rotational symmetry, a finite angular momentum $J_z$ in the $t$-$J$ model also implies a spin twist structure in momentum space, consistent with the Rashba SOC $\sim \mathbf{S}_k \cdot (\mathbf{k} \times \hat{z})$ where $\mathbf{S}_k$ denotes a single particle’s spin at momentum $\mathbf{k}$. Indeed, in Fig. 4, the spin textures of $\langle \mathbf{S}_k \rangle$ lying in the $x$-$y$ plane, characterized by a spin helicity in the momentum space, are shown for the ground state of $J_z = \pm 3/2$ and the first excited state of $J_z = \pm 1/2$, respectively, as calculated by ED. Again the spin helicity is much weaker for the $\sigma$-$t$-$J$ model [cf. Fig. 4(a)]. In Fig. 5, the ED and VMC results are further compared in consistency. Both ED and VMC calculations also indicate that the real-space spin texture in Fig. 2(a) can persist into high excited states. At a higher excitation energy, the corresponding spin texture in momentum space will be deeper inside the Brillouin zone, as shown in Fig. 6 obtained by VMC at a larger sample size. Such a hole-like quasiparticle with a spin texture in momentum space may be thus directly probed by the spin-polarized angle-resolved photoemission spectroscopy (ARPES) measurement [13].

The persistence of the spin texture in the Brillouin zone, even close to the $\Gamma$ point in Fig. 6, is a fact that is in sharp contrast to a Landau Fermi liquid. In the latter, the spin of a quasiparticle may also lock with its momentum to form a spin helicity in the momentum space by SOC. But it is a single particle picture without the spatial spin texture [cf. Fig. 2(a)], in which the SOC-induced spin helicity is expected to be only robust near the Fermi surface within a very narrow energy interval comparable with the strength of the weak SOC. Away from the Fermi energy, the spin helicity of a Landau quasiparticle can be easily washed away by scattering (cf. Fig. 8) without the protection from the many-body effect in the $t$-$J$ case as represented by a real-space spin texture associated with $J_z \neq 0$.

Finally, we argue that even though this work has mostly focused on a single hole in a finite size system, where ED and VMC calculations are available to compare with each other, one may reasonably generalize the main conclusion to the experimentally relevant regime at finite doping. Firstly, it has been shown by the VMC approach that the spin texture amplitude gets enhanced with reducing $\xi$ in the spin background (cf. the inset of Fig. 7), which is due to the fact that the composite of a hole and its spin-1/2 partner gets smaller with an enlarged spin current. In order to make quantitative comparison with the experiment [13], aside from the absolute amplitude of the spin-polarized spectral function in the inset, a relative polarization ratio of the spectral function with spin perpendicular to momentum is also shown in the main panel of Fig. 7 at a given momentum, which indeed indicates a comparable strength with the experi-
mental observation [13]. Secondly, we point out that the pairing of the doped holes with the spin texture structure obeying the time-reversal symmetry is presumably between the composite holes with the opposite pseudospin $J_\sigma = \pm 3/2$ in the presence of SOC. Those other quasiparticle excitations with distinct spin textures, which have higher energies due to the SOC splitting, are thus effectively excluded out of the pair condensate. In such a superconducting/pseudogap regime, predominantly the chiral spin texture associated with the unpaired quasiparticles of $J_\sigma = \pm 3/2$ (which exhibit the same spin texture in momentum space) will be observed by the spin-polarized ARPES measurement over a finite range of energy, which covers the extended momentum space away from the Fermi surface region. Such a many-body picture clearly illustrates why a quasiparticle excitation with spin texture is “protected”, which can survive even when the thermal energy in superconducting/pseudogap regime is dominant over the small energy scale of SOC.

By contrast, the emergent Bogoliubove/Landau quasi-particle with Fermi arc [14] should only contribute to a negligible spin texture under a weak SOC.

The rest of paper will be organized as follows. In Sec. II, the $t$-$J$ and $\sigma$-$t$-$J$ models with a Rashba SOC are introduced. In Sec. III, we focus on a single hole’s motion in a finite-size lattice and discuss the spin texture structure induced by the SOC based on both ED and VMC approaches. In Sec. IV, a qualitative discussion at a finite doping is presented. Finally, Sec. V is devoted to the concluding remarks. Various technical details and analyses are presented in Appendices.

II. MODELS

A. The $t$-$J$ and $\sigma$-$t$-$J$ models

In this work we shall examine some novel spin texture structure hidden in the quasi-hole excitation of the doped Mott insulator described by the $t$-$J$ model [1, 15], whose Hamiltonian is composed of the hopping $H_t$ and the superexchange $H_J$ terms as follows,

$$ H_t = -t \sum_{\langle ij \rangle, \sigma} c_{i\sigma}^{\dagger} c_{j\sigma} + h.c. , $$

$$ H_J = J \sum_{\langle ij \rangle} \left( \mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} n_i n_j \right) , $$

where $\langle ij \rangle$ denotes two nearest neighbor sites $i$ and $j$, and $c_{i\sigma}$ creates an electronic hole with the spin index $\sigma = \uparrow$ or $\downarrow$ at site $i$. $\mathbf{S}_i$ and $n_i$ are the spin and electron number operators, respectively, and the Hilbert space is always subject to the no double occupancy constraint

$$ n_i = \sum_{\sigma} c_{i\sigma}^{\dagger} c_{i\sigma} \leq 1 . $$

At half-filling $n_i = 1$, the system becomes the Mott insulator with the spin degrees of freedom described by the

Heisenberg Hamiltonian $H_J$. We fix the model parameters at $t/J = 3$ and $J = 1$ in the following.

The motion of the holes injected into such a system will strongly interact with the quantum spin background to acquire a Berry-phase-like emergent sign structure, i.e., the so-called phase string [16–18]. This singular effect is manifested by generating neutral spin current [8, 11] as the backflow associated with the hopping of the doped holes. An important fact is that the spin current is conserved, or the phase string created by the hopping is “irreparable”, which replaces the original fermion signs to become the new statistical signs in the $t$-$J$ model [18]. One may find a more detailed discussion on the phase string sign structure in Appendix A.

On the other hand, the phase string sign structure in the doped $t$-$J$ model can be precisely eliminated by modifying $H_t$ into the following form [19, 20]:

$$ H_{\sigma \cdot t} = -t \sum_{\langle ij \rangle} \sigma c_{i\sigma}^{\dagger} c_{j\sigma} + h.c. , $$

where a spin-dependent sign is inserted to the hopping process, while the superexchange term $H_J$ remains unchanged. It can be shown that the resulting $\sigma$-$t$-$J$ model with $H_{\sigma \cdot t} \equiv H_{\sigma \cdot t} + H_J$ will no longer generate any spin currents as the phase string is precisely “switched off” [cf. Appendix A]. Consequently an injected hole will behave simply like a Landau quasiparticle with no more internal structure associated with the backflow spin current. Thus, a comparative study of the $t$-$J$ and $\sigma$-$t$-$J$ models can provide a unique understanding of the doped Mott physics.
B. Spin-orbit coupling

In this work, we shall introduce a Rashba SOC term as a perturbation, which is given as follows [21]

\[ H_R = \lambda \sum_i i(\hat{c}^\dagger_{i\uparrow} \hat{c}_{i+\hat{y}\downarrow} + \hat{c}^\dagger_{i\downarrow} \hat{c}_{i+\hat{y}\uparrow}) - (\hat{c}^\dagger_{i\uparrow} \hat{c}_{i+\hat{x}\downarrow} - \hat{c}^\dagger_{i\downarrow} \hat{c}_{i+\hat{x}\uparrow}) + h.c. \]  \hspace{1cm} (4)

Here the parameter \( \lambda \) characterizes the Rashba SOC strength (valued in units of \( J = 1 \)) and \( \hat{x} \) (\( \hat{y} \)) denotes the unit vector along the \( x \)(\( y \)) direction. It is noted that a weak Rashba SOC in the cuprate may actually arise from the lack of the inversion symmetry in a double-layer system [13]. In the present approach, we shall neglect the interlayer \( t-J \) and SOC coupling between the double layers. Namely one is still focused on the one layer problem with only retaining an effective weak intra-layer SOC given in Eq. (4), while the other layer experiences an opposite SOC [13]. As a perturbation to the \( t-J \) or \( \sigma \cdot t-J \) model, \( H_R \) can further provide a locking between the spin and orbital angular momenta in the new Hamiltonian \( H_{t,J,R} = H_t + H_J + H_R \) or \( H_{\sigma \cdot t,J,R} = H_{\sigma,t} + H_J + H_R \).

Then the single hole problem previously explored by both ED and VMC methods can be easily generalized to the present case, which will reveal interesting properties of the doped hole as a new twisted quasiparticle in a rather unique way.

III. THE SINGLE HOLE STUDY

As stated in the Introduction, the single-hole ground state of the \( t-J \) model has exhibited some highly nontrivial phenomenon as shown by ED and DMRG [8]. Namely, the motion of the hole in the \( t-J \) model is always accompanied by a backflow spin current due to the phase string effect, which then leads to a continuous distribution of momentum for the hole as if there is an incoherent many-body component in addition to a quasiparticle component in a finite size sample. In the following, we shall examine the response of such a two-component single-hole state to a weak Rashba SOC perturbation by ED and VMC, respectively, which will then reveal the nature of the internal structure of a doped hole as an unconventional many-body composite in an explicit way.

A. Exact diagonalization and variational ansatz in the absence of SOC (\( \lambda = 0 \))

To begin with, we note that for one hole injected into a half-filled spin background in 2D, recent ED and DMRG studies in the pure \( t-J \) model have clearly demonstrated [8] that the ground state is characterized by an angular momentum \( L_z = \pm 1 \) and \( S^z = \pm 1/2 \) under OBC. In other words, the single hole eigenstates are totally four-fold degenerate, which is not only true for the ground state but also valid for the excited states as indicated by the string effect, which then leads to a continuous distribution of momentum for the hole as if there is an incoherent many-body component in addition to a quasiparticle component in a finite size sample. In the following, we shall examine the response of such a two-component single-hole state to a weak Rashba SOC perturbation by ED and VMC, respectively, which will then reveal the nature of the internal structure of a doped hole as an unconventional many-body composite in an explicit way.

To begin with, we note that for one hole injected into a half-filled spin background in 2D, recent ED and DMRG studies in the pure \( t-J \) model have clearly demonstrated [8] that the ground state is characterized by an angular momentum \( L_z = \pm 1 \) and \( S^z = \pm 1/2 \) under OBC. In other words, the single hole eigenstates are totally four-fold degenerate, which is not only true for the ground state but also valid for the excited states as indicated by the string effect, which then leads to a continuous distribution of momentum for the hole as if there is an incoherent many-body component in addition to a quasiparticle component in a finite size sample. In the following, we shall examine the response of such a two-component single-hole state to a weak Rashba SOC perturbation by ED and VMC, respectively, which will then reveal the nature of the internal structure of a doped hole as an unconventional many-body composite in an explicit way.

A. Exact diagonalization and variational ansatz in the absence of SOC (\( \lambda = 0 \))

To begin with, we note that for one hole injected into a half-filled spin background in 2D, recent ED and DMRG studies in the pure \( t-J \) model have clearly demonstrated [8] that the ground state is characterized by an angular momentum \( L_z = \pm 1 \) and \( S^z = \pm 1/2 \) under OBC. In other words, the single hole eigenstates are totally four-fold degenerate, which is not only true for the ground state but also valid for the excited states as indicated by the string effect, which then leads to a continuous distribution of momentum for the hole as if there is an incoherent many-body component in addition to a quasiparticle component in a finite size sample. In the following, we shall examine the response of such a two-component single-hole state to a weak Rashba SOC perturbation by ED and VMC, respectively, which will then reveal the nature of the internal structure of a doped hole as an unconventional many-body composite in an explicit way.

To begin with, we note that for one hole injected into a half-filled spin background in 2D, recent ED and DMRG studies in the pure \( t-J \) model have clearly demonstrated [8] that the ground state is characterized by an angular momentum \( L_z = \pm 1 \) and \( S^z = \pm 1/2 \) under OBC. In other words, the single hole eigenstates are totally four-fold degenerate, which is not only true for the ground state but also valid for the excited states as indicated by the string effect, which then leads to a continuous distribution of momentum for the hole as if there is an incoherent many-body component in addition to a quasiparticle component in a finite size sample. In the following, we shall examine the response of such a two-component single-hole state to a weak Rashba SOC perturbation by ED and VMC, respectively, which will then reveal the nature of the internal structure of a doped hole as an unconventional many-body composite in an explicit way.
finite spectral weight $Z_k$, we may call the hole state a Landau-like quasiparticle.

However, instead of Eq. (5), a novel single-hole state wave function has been found [11] in good agreement with the above ED and DMRG results [8]. Mathematically it is realized by replacing the bare hole creation operator $c$ in Eq. (5) with $\hat{c}$:

$$c_{i\sigma} \rightarrow \hat{c}_{i\sigma} \equiv c_{i\sigma} e^{-i\hat{\Omega}_i},$$

which leads to a new single-hole ansatz state [11, 12]

$$|\Psi\rangle_{1h} = \sum_i \varphi_h(i) c_{i\sigma} e^{-i\hat{\Omega}_i} |\phi_0\rangle.$$

Here the nontrivial phase shift operator takes the form

$$\hat{\Omega}_i = \sum_l \text{Im} \ln(z_i - z_l) n_{l\downarrow},$$

where $n_{l\downarrow}$ denotes the $\downarrow$-spin operator acting on $|\phi_0\rangle$ with $z_i$ as the complex coordinate of site $i$. It encodes the mutual braiding between the hole and the $\downarrow$-spins in the background spins, as illustrated in Figs. 1, to give rise to a nontrivial angular momentum $L_z = \pm 1$ and a net spin current [11]

$$J^\mu_z = i(S^+_i S^-_j - S^-_i S^+_j).$$

The numerical ED and DMRG (up to $8 \times 8$) results [8] for the $t$-$J$ model can be well reproduced by such a variational ground state ansatz in Eq. (7), with the hole wave function $\varphi_h(i)$ being the variational parameter in the VMC calculation [11]. In Eq. (6), a minus sign is chosen in $\hat{\Omega}_i$, which defines one possible chirality of the ground states. One may also choose a positive sign, i.e. $e^{i\hat{\Omega}_i}$, to accordingly give rise to a degenerate ground state with the opposite chirality. A detailed symmetry analysis of the wave function in Eq. (7) is given in Appendix B.

By contrast, the Landau-like wave function in Eq. (5) is shown [11] to be a good ground state for the $\sigma$-$t$-$J$ model, in which the phase-string sign structure is switched off without creating a spin-current backflow during the hopping process [cf. Fig. 1(c)] even though the spin $S_z = \pm 1/2$ has a similar distribution around the hole as in the $t$-$J$ case [cf. Fig. 1(b)]. Obviously the two wave functions in Eq. (5) and Eq. (7) cannot be adiabatically connected to each other, indicating the qualitative distinction and non-perturbative nature of the single-hole state in comparison with the conventional Landau paradigm. Crucially, the nonlocal phase structure in Eq. (6) persists through the whole spectrum of the single-hole eigenstates irrespective of the excited energy. One may also continuously tune the spin-spin correlation length $\xi$ in $|\phi_0\rangle$ variationally (see Appendix D) to simulate the case of an arbitrary finite $\xi$, which should be self-consistently realized in the ground state at finite doping of holes [22, 23]. The detailed structures of the single-hole state as variationally obtained at $\xi \gg L$ and $\xi \sim 2.7$ are illustrated in Figs. 1(b) and 1(c).

After the above brief outline of the single hole state in the pure case, in the following, one can straightforwardly generalize the above calculations to $\lambda \neq 0$ and study $H_{t,J-R}$ by the VMC approach in comparison with the ED results. We will see that the exotic non-Landau-like features of the doped hole will further show up.

### B. The single hole state under a weak Rashba spin-orbit coupling ($\lambda \neq 0$)

#### 1. Exact diagonalization

Let us first start with the ED study under OBC, which is to be compared with a VMC study based on an analytic ground state ansatz below. In contrast to the ED under the periodic boundary condition (PBC) given in Appendix C2, the ground state degeneracy of the $t$-$J$ model (without SOC) under the OBC (with $C_4$ symmetry) has been shown to be reduced to four-fold characterized by the orbit angular momentum $L_z = \pm 1$ and spin $S_z = \pm 1/2$ [11]. Once $\lambda$ is switched on in $H_{t,J-R}$, $S_z$ and the angular momentum $L_z$ are no longer conserved separately. Instead, the total angular momentum $J_z = L_z + S_z$ will remain as a good quantum number (cf. Appendix C1), with the ground states characterized by $J_z = L_z + S_z = \pm 3/2$ with two-fold remaining degeneracy by ED. To show the overall strength of the SOC, the change of the ground state energy as a function of $\lambda$ is shown in Fig. 3, in which (a) is for ED and (b) is for VMC.

![FIG. 5.](image-url)
The energy splitting between $J_z = \pm 3/2$ and $J_z = \pm 1/2$ is clearly shown here. By contrast, the SOC effect for a quasiparticle in the $\sigma$-$t$-$J$ model (blue triangles) is much reduced. A similar result of the ED calculation under PBC can be found in Appendix C.2.

To quantitatively characterize the spin texture induced by $\lambda \neq 0$, one may define a “helicity” as the projection of the averaged spin $\langle \mathbf{S}_k \rangle$ at the direction perpendicular to the momentum $\mathbf{k}$ lying in the $x$-$y$ plane:

$$h \equiv \sum_{\mathbf{k}} \langle \mathbf{S}_k \rangle \cdot (\mathbf{k} \times \hat{\mathbf{z}}) / |\mathbf{k}|,$$

where $\mathbf{S}_k = \frac{1}{2} \mathbf{c}_k \sigma \mathbf{c}_k$ with $\mathbf{c}_k = [\mathbf{c}_{k\uparrow}, \mathbf{c}_{k\downarrow}]^T$. It measures the strength of locking between the spin and orbit associated with the single hole in the momentum space as shown in Fig. 4(a), with the sign $\chi = \text{sign}(h)$ depicting the chirality of the corresponding spin texture in Figs. 4(b) and (c). Here the helicity $h$ for $J_z = \pm 3/2$ (the ground states) shows a significantly larger value as compared to that of the first excited state at $J_z = \pm 1/2$ in Figs. 4(b) and (c), which both possess the same chirality $\chi$ at $\lambda > 0$. The sign of $\chi$ does not depend on the sign of $J_z$ but only on the sign of $\lambda$ as shown in Fig. 4(a).

It is easy to understand that the four spin textures in Figs. 4(b)-(c) and their counterparts at $\lambda < 0$ just correspond to the eight degenerate eigenstates of the $t$-$J$ model at $\lambda \to 0$ in the thermodynamic limit or under PBC [8, 11], which are split into ground state and the first excitations in a finite lattice under OBC, and further split at $\lambda \neq 0$ with distinct chiralities. On the other hand, the helicity $h$ for the $\sigma$-$t$-$J$ model is indeed much reduced as shown in Fig. 4(a).

2. Variational Monte Carlo calculation

The above ED calculation shows that by introducing a weak Rashba SOC to lift the separate $S_z$ and $L_z$ degeneracy, the eigenstate degeneracy associated with the hole will be reduced to two with the locking of $S_z$ and $L_z$ in the $t$-$J$ model. Specifically, the total angular momentum $J_z \equiv L_z + S_z$, which remains to be a good quantum number, will characterize the composite hole $\hat{c}$ as a pseudo-spin in replacing $S_z = \pm 1/2$. Namely, the Rashba SOC will select a new quantum number $J_z$, which leads to a chiral spin texture. A detailed symmetry analysis is given in Appendix C.1.

In the following, we shall employ a variational wave function to understand the underlying physics involving the many-body response to a weak SOC in the one-hole-doped Mott insulator. By generalizing the previous four-fold degenerate wave functions for the pure $t$-$J$ model, one may construct the following time-reversal invariant new ground states in terms of the linear combinations of $|\phi_0\rangle$:

$$|\Psi_R\rangle_{1h} = \sum_i [\varphi^\dagger_h(i) e^{-i\xi_i \cdot \mathbf{c}_i \uparrow} + \varphi^\dagger_h(i) e^{i\xi_i \cdot \mathbf{c}_i \uparrow}] |\phi_0\rangle,$$

where $\varphi^\dagger_h$ and $\varphi^\dagger_h$ are the variational parameters. One may also construct other types of ansatz by choosing a different factor $e^{i\xi_i \cdot \mathbf{c}_i \uparrow}$ in Eq. (11), which variationally produces excited states. The VMC results are presented as follows [see Appendix D.2 for the detail].

A spin texture around the hole has been clearly found to lie in the $x$-$y$ plane in Fig. 2(a) with $J_z = \pm 3/2$ for the $t$-$J$ model (with $\lambda = 0.01$). Here the spin texture emerges out of the hidden spin current structure of the non-Landau “twisted” hole in Fig. 1(a) via SOC. Its scale can be fitted by $\xi_{sh} \simeq \xi$ (cf. the inset of Fig. 1(b)) with $\xi_{sh}$ being the spin-hole correlation length. In Fig. 2, a finite spin-spin correlation length $\xi \sim 2.7$ in $|\phi_0\rangle$ has been used in the VMC. By comparison, one can examine the spin texture in a Landau-like quasiparticle in the $\sigma$-$t$-$J$ model, i.e., by turning off the phase string effect in the $t$-$J$ model. Without creating a spin current backflow, a doped hole will propagate like a conventional Bloch wave in the $\sigma$-$t$-$J$ model. At the same strength of $\lambda = 0.01$, there is no more internal spin texture structure around the hole [see the right panel of Fig. 2(b)]. Here the variational wave function of the $\sigma$-$t$-$J$ model is given by

$$|\Psi\rangle_{\sigma-t-J-R} = \sum_i [\varphi^\dagger_h(i) c_i \uparrow + \varphi^\dagger_h(i) c_i \uparrow] |\phi_0\rangle,$$

which is different from those of the $t$-$J$ model by simply switching off the phase shift operator $\hat{\Omega}$ in Eq. (11).

The overall agreement of the energies between the ED and VMC calculations are further illustrated in Fig. 3, including the energy split between $J_z = \pm 3/2$ and $J_z = \pm 1/2$ in the $t$-$J$ model and the ground state energy shift in the $\sigma$-$t$-$J$ model. One may see the contributions of the SOC to the variational energy shifts in Fig. 3(b) as a
function of $\lambda$, which are in excellent agreement with the ED results [Fig. 3(a)]. For the $t$-$J$ model, in which a Landau’s quasiparticle picture can be recovered at $\lambda = 0$ [11], one finds a drastically weakened SOC effect at the same $\lambda$ as compared to the $t$-$J$ model. As pointed out above, the latter only differs from the former by the phase string sign structure as represented by the backflow spin current accompanying the hopping. Similar contrast has been also clearly shown by ED under PBC as presented in Appendix C2. Therefore, the phase string effect indeed significantly amplifies the weak Rashba SOC effect in the $t$-$J$ model.

Furthermore, given the spatial spin texture around the hole in Fig. 2(a), a corresponding spin texture described by the average spin $\langle S_k \rangle$ in momentum space is expected by the rotational symmetry, which has been already illustrated in, say, Fig. 4(b) at $J_z = \pm 3/2$. In Fig. 5, the spin-momentum-locking spin textures obtained by ED and VMC are presented together for comparison, in which $\langle S_k \rangle$ in the ground state are shown in (a) and (c) for a $4 \times 4$ lattice.

In Figs. 5(b) and (d), we further present the ED and VMC results for the spin-polarized spectral function $I_{\text{max}}(k)$, which is defined in Eq. (E8) of Appendix E at each given momentum $k$ with the maximal magnitude of the spectral function via properly adjusting the excited energy. In other words, $I_{\text{max}}(k)$ can effectively detect the maximal spin texture in momentum space involving the excited states, which is experimentally relevant. In Fig. 6, the spin texture in momentum space is calculated by VMC in a larger $(8 \times 8)$ lattice such that more momentum points are available, where the spin vortex structure is clearly illustrated in the full Brillouin zone, or in other words, persists over the high energy towards the $\Gamma$ point.

Finally, we emphasize that it is the intrinsic phase string that causes the hole dressed by a surrounding spin current of the $S = 1/2$ partner and the weak Rashba SOC only lifts the degeneracy by inducing a spin texture by breaking the spin rotational symmetry. If one starts with the large-U Hubbard model, the high order corrections to the $t$-$J$ model may be at the same order as $\lambda$, but they do not break the spin rotational symmetry like the SOC does and it has been shown [25] that the phase string effect still present. The present conclusion should thus be expected to remain unchanged.

IV. DISCUSSION: FINITE DOPING

For the single hole case studied in the previous section, the spin-1/2 partner forms a spin current vortex surrounding the hole as illustrated in Fig. 1, which is then “frozen” into a spin texture lying in the $x$-$y$ plane via a weak SOC as shown in Fig. 2(a). The excellent agreement between the ED and VMC methods clearly indicates that the single-hole ansatz wave function well captures the intrinsic structure of the doped hole in a finite size lattice that cannot be adiabatically connected to a Landau-like quasiparticle, not only in the absence of SOC but also in the presence of a weak Rashba SOC.

Recently, the VMC approach has been further generalized to the two-hole states in the absence of SOC [26, 27]. By forming a tightly bound pair, the chiral spin current gets cancelled and the ground state becomes non-degenerate with angular momentum $L_z = 2$ and a $d$-wave pairing symmetry [cf. Ref. 27], which is also consistent with the ED and DMRG result [8]. According to the above discussion, in the absence of the chiral spin current, a weak SOC usually should not be able to induce a spin texture in a pairing state of holes in contrast to a single hole case. Therefore, at finite doping, the spin texture associated with a doped hole is expected to appear only when the unpaired holes are present in an excited state.

Two questions remain to be answered. One concerns the SOC effect in the thermodynamic limit, e.g., in the AF long-range ordered state with $N = L^2 \to \infty$ and $\xi \to \infty$, where ED certainly does not apply. The VMC calculation indicates that the SOC effect should get diminished continuously as the spin current associated with the spin-1/2 spreads over in space with the increase of $\xi_{sd} \approx \xi$. However, in a true AF long-range ordered state, the gapless spin excitations from the spin background can be further excited to form an even stronger spin currents around the hole. In such a limit, the strong spin fluctuations may even cause the self-localization of the doped hole [11], leading to a very singular effect in response to an SOC, which is beyond the present VMC approach at a finite size. On the other hand, the spin texture as the internal structure of the doped hole discussed in this work is expected to be robust if $\xi$ becomes finite in the spin background with $L \gg \xi$. What we have
explored here is actually about how a hole moving in a spin background of finite $\xi$. For example, the ED results of a single hole on a $4 \times 4$ lattice may be roughly understood as a finite doping $\sim 1/16$ system. As a matter of fact, the variational single hole state in Eq. (7) may be created in a spin background $|\phi_0\rangle$ whose spin-spin correlation length $\xi$ can be artificially tuned (cf. Appendix D1). For instance, to better illustrate the spin texture around the hole in Fig. 2(a), a variational $|\phi_0\rangle$ with a finite $\xi \sim 2.7$ has been used. Since $\xi_{sh} \simeq \xi \ll L$ in this case, the spin texture as the integral part of the hole composite is clearly shown in Fig. 2(a). In other words, if a finite doping mainly changes the spin background into a short-range AF state (or spin liquid) with the doped hole still described by $\delta_{i\sigma}$ in Eq. (6), the above account of the Rashba SOC effect should be valid in the leading order of approximation.

So the second question will be whether the present study of the single-hole behavior under the Rashba SOC can be meaningfully generalized to the quasiparticle excitation at finite doping, where a short-range AF state with finite $\xi$ will set in. Namely, a single-particle excitation above the ground state $|\Psi_G\rangle$ (which may be either a superconducting or pseudogap state at finite doping, see below) may be constructed by

$$|\psi\rangle_{qp} = \sum_i \varphi_i(i)\delta_{i\sigma}|\Psi_G\rangle,$$

(13)

similar to the one-hole state in Eq. (7) with replacing the “vacuum” state $|\phi_0\rangle$ by $|\Psi_G\rangle$. We shall delay the discussion of such a second question for a moment. Let us first assume that Eq. (13) is indeed valid, in which $|\Psi_G\rangle$ may be still treated as a short-ranged AF state $|\phi_0\rangle$ with the spin correlation length $\xi$ tunable. Then one may further make an estimation of the strength of the SOC effect on the quasiparticle excitation as follows.

The spin-polarized spectral function corresponding to the quasiparticle excitation in Eq. (13) will be then approximately treated similar to that of the single hole case under the SOC (cf. Appendix E). The spin texture profile, determined by VMC in the momentum space based on $I_{\text{max}}(k)$, is similar to Fig. 6 with the magnitude depending on $\xi$ and $\lambda$. In Fig. 7, a relative strength of the spin texture at $k = (\pi/2, \pi/2)$ is shown as a function of $\xi$ at various $\lambda$’s, which is defined by [13]

$$\text{pol}(\omega_c, k) = \frac{A_{\sigma_+}(\omega_c, k) - A_{\sigma_+}(\omega_c, k)}{A_{\sigma_+}(\omega_c, k) + A_{\sigma_+}(\omega_c, k)},$$

(14)

in terms of the spin-polarized ARPES spectral function $A_{\sigma}(\omega, k)$ at energy $\omega$ with spin index $\sigma$ [defined in Eq. (E1) in Appendix E]. Note that here $\sigma_\perp$ denotes the spin polarization perpendicular to $k$ ($\sigma_\perp = -\sigma_\parallel$) and $\omega_i$, the frequency at which the intensity reaches its maximum. As Fig. 7 shows, pol($\omega_c, k$) is roughly flat in a range of finite $\xi$ with the strength monotonically dependent on $\lambda$, which is comparable to the experimental measurement in the cuprate [13]. Note that pol here only measures the relative spin polarization perpendicular to $k$. In the inset of Fig. 7, the total strength of the spectral function $|I_{\text{max}}|$ (cf. Appendix E) at the same $k = (\pi/2, \pi/2)$ is also shown as a function of $\xi$ as $\xi \rightarrow \infty$, which eventually approaches zero at the AF long-ranged order limit. Physically it may be understood as the spin current carried by the spin-1/2 partner of the hole composite becomes less and less concentrated around the hole to result in the reducing SOC effect at large $\xi$ and $L$.

Now let us come back to address the above second question about the single-particle excitation at finite doping. We point out that the single-hole ground state ansatz [Eq. (7)] can be actually regarded as the few hole special cases [11, 12, 26, 27] of a general ground state ansatz previously proposed to describe the SC and pseudogap phase at finite doping [22, 23]:

$$|\Psi_G\rangle = \hat{D}^{\dagger} |\text{RVB}\rangle,$$

(15)

in which the AF vacuum state $|\phi_0\rangle$ is self-consistently replaced by $|\text{RVB}\rangle$, which is AF short-range-ordered at a finite doping. Here the $N^\text{th}$ holes are paired up and created by

$$\hat{D} = \sum_{ij} g_{ij} \bar{c}_{i\uparrow} c_{j\downarrow},$$

(16)

where $\bar{c}$ defined by Eq. (6) describes the doped holes as twisted (non-Landau) quasiparticles. Here $|\Psi_G\rangle$ is intrinsically superconducting at $T = 0$ in a finite concentration of holes. The excitations of free unpaired spinons in $|\text{RVB}\rangle$ can eventually destroy the SC phase coherence, resulting in the so-called lower pseudogap phase (LPP) or spontaneous vortex phase, where the pairing in terms of the twisted quasiparticles, i.e., $\langle \hat{D} \rangle \neq 0$ is still present, but the true Cooper pairing order parameter vanishes [22, 23].

Naturally, an elementary quasiparticle excitation can be constructed, based on $|\Psi_G\rangle$, as composed of the unpaired twisted quasiparticle given in Eq. (13). In particular, since the twisted quasiparticles are paired up via $\hat{D}$ in $|\Psi_G\rangle$, the latter may be reasonably considered to be a “vacuum” as far as $\bar{c}$ is concerned in Eq. (13), which thus resembles a $|\phi_0\rangle$ just like the single-hole state in Eq. (7) except for a finite $\xi$. Furthermore, turning on the Rashba SOC in $H_{\text{SC/LPP}}$ with $\lambda \neq 0$, there will be an additional important effect occurring in the SC/LPP at finite doping, which can further enhance the spin texture structure found in the single hole case, besides the reducing $\xi$ discussed above, as to be outlined below.

Note that in the above SC/LPP ansatz states, the pairing between the twisted quasi-holes of $\delta_{i\sigma}$ with $\sigma = \uparrow, \downarrow$ will be modified by the SOC with the degenerate ground states characterized by a time-reversal pair of new good quantum numbers: $J_z = 3/2$ and $J_z = -3/2$, i.e.,

$$\hat{D} \rightarrow \sum_{ij} g_{ij} \bar{c}_{i,J_z=3/2} \bar{c}_{j,J_z=-3/2}.$$

(17)
As we have seen, the twisted hole states with both \( J_z = \pm 3/2 \) have the same chirality of the spin texture. In Eq. (17), the low-lying excitations will be mainly dominated by those unpaired quasiparticles of \( J_z = \pm 3/2 \) with the same spin texture, where the other excited states with opposite chirality in the single hole problem should no longer be stable to contribute to the spin-polarized \( I_{\text{max}}(\mathbf{k}) \) shown in Fig. 6. Thus, \( I_{\text{max}} \) is expected to be further enhanced due to the fact that the twisted quasiparticles of \( J_z = \pm 3/2 \) are uniquely singled out via the pairing condensation in \( |\Psi_C\rangle \) to exclude all those excitations of opposite spin textures split by the SOC.

**V. CONCLUDING REMARKS**

As the precursor of a non-Fermi liquid, it has been found recently [11, 27] that a hole injected into a quantum AF spin background will behave like a non-Landau quasiparticle. It is a hole composite in which the neighboring spin background will get “twisted” by the phase string sign structure to facilitate the hopping of the hole. In the single-hole ansatz state (7), such a many-body effect is well captured by the nonlocal phase shift factor \( e^{-i\hat{\Omega}_i} \). In particular, such a novel structure can become explicitly manifested if an extra \( S_z = \pm 1/2 \) (introduced by the hole doped into the spin singlet background) is nearby in a finite system, which will form a spin current swirling around the hole like in an atom with a nontrivial angular momentum \( L_z = \pm 1 \).

Then we have shown in this work that by introducing a weak Rashba SOC to lift the spin-1/2 degeneracy associated with the doped hole, the spin current will be turned into a semiclassical chiral spin texture lying in the \( x-y \) plane in both real and momentum space, which is characterized by a new quantum number \( J_z = L_z + S_z \). It is important to emphasize that a weak SOC does not change the essential internal structure of the hole composite, but mainly affects the “soft” spin current contributed by the degenerate \( S_z = \pm 1/2 \), which are locked with \( L_z \) due to breaking the spin rotational symmetry by SOC. In this sense, the present VMC approach is perturbative-like based on the good (variational) solutions in the pure \( t-J \) models. In particular, the semiclassical spin texture formed by the spin-1/2 around the hole in Fig. 2(a) is merely a “fingerprint” that explicitly reveals the underlying quantum many-body hole-spin entanglement, which already exists in the absence of SOC. A recent proposal of the so-called rotational ARPES spectra [28] may be also used to probe such an internal spin structure associated with the doped hole.

By using ED and VMC methods, such a spin texture pattern characterized by the new quantum number \( J_z \) has been confirmed by both exact numerics and the variational wave function. The excellent agreement between the ED and VMC results shows that the hole composite for the \( t-J \) model has properly captured the essence of a doped Mott insulator in response to a perturbation of the SOC. On the other hand, by precisely turning off the phase string in the \( \sigma \cdot t-J \) model, both ED and VMC calculations also demonstrate the disappearance of the novelty of the non-Landau quasiparticle, with a substantially weakened spin texture entirely induced by the Rashba SOC for a Landau-like quasiparticle.

Furthermore, we have argued that such a chiral spin texture in the \( \sigma \cdot t-J \) model may become directly observable at finite doping in the superconducting/pseudogap regime, where the majority of the doped holes form a paired condensate, which effectively selects the hole composites with opposite quantum numbers \( J_z = \pm 3/2 \) in a weak Rashba interaction with the same chiral spin texture. In particular, by making the spin AF background short-ranged, the hole composite becomes “atom-like” as a truly bound entity such that the spin texture around the hole is both amplified by the Rashba effect and independent of the lattice size. Consequently, the low-lying quasiparticles excited from the paired condensate will be

**FIG. 8. Spin texture configuration due to a weak Rashba SOC as determined by the spin-polarized spectral function \( I_{\text{max}}(\mathbf{k}) \) for (a) a free Fermi gas, and (b) a Fermi liquid. The spin textures are fragile in the Fermi liquid once the scattering between Laudau’s quasiparticle is taken into consideration (cf. Appendix F for the detail).**
predominately characterized by the same spin texture structure in momentum space, which may be well observed by the spin-polarized ARPES in momentum space, extending deep inside the Brillouin zone away from the Fermi energy. Of course, we caution that the present discussion of the finite doping phase is mostly based on a generalization of the single hole problem. A further microscopic study at the same level as dealing with the emergent Bogoliubove/Landau quasiparticle with Fermi arc [14] will be desirable to describe the spin texture structure under a weak SOC.

It is important to note that even though a similar spin texture in momentum space (but not in the real space) commensurate with the Rashba SOC can be simply found in a noninteracting band structure model as shown in Fig. 8(a) (cf. Appendix F), the spin texture is actually well protected only within a very narrow energy regime near the Fermi surface decided by the SOC strength λ. As illustrated in Fig. 8(b), the spin texture at momenta away from the Fermi surface becomes much fragile in a Fermi liquid theory due to a general scattering effect on the quasiparticles once the excitation energy is higher than a small scale set by the SOC splitting energy (cf. Appendix F).

We comment on a recent experimental result of the spin-polarized ARPES experiment on the high-$T_c$ cuprate of the Bi-2212 compound [13] in the pseudogap regime with a $J \sim 120$ meV. The dependence of the in-plane spin polarization on the quasiparticle momentum observed in the experiment seems to be compatible with a weak Rashba SOC strength as studied in this work, which presumably comes from the double interlayer structure [13]. It is an important observation that the spin polarization extends deep towards the Γ point of the Brillouin zone, which is hard to understand from a conventional Fermi liquid theory. As discussed above, the spin texture found in the spin polarization measurement should be restricted by a characteristic energy scale due to the SOC of a single quasiparticle, which is rather small (e.g., $\lambda \sim 0.01J \sim 1.2$ meV) and determines a very narrow regime in the momentum space near the Fermi energy. By contrast, the spin texture of the new quasiparticle in a doped Mott insulator is more intrinsically associated with the internal structure of the doped hole, which is present even for the high energy excitations. An entirely different mechanism due to quantum many-body correlations is at work here.

Finally, along with the SOC that breaks the spin rotation, we point out another important effect involving the Zeeman field, which also breaks the spin rotational symmetry. Such a Zeeman effect acts on the whole neutral spin background, in contrast to SOC mainly coupling a hole with its surrounding spins. Although no spin texture is expected, the Zeeman effect that polarizes spins can exert strong influence on the charge degree of freedom via an enhanced spin current due to the enlarged net spin $\hat{S}_z$. Such an important effect due to phase string is currently under study elsewhere.

ACKNOWLEDGMENTS

We acknowledge stimulating discussion with Jingyu Zhao. This work is supported by MOST of China under Grant No. 2017YFA0302902.
Appendix A: Sign structures in the t-J and σ·t-J models

To be self-contained, in the following we briefly outline the phase-string sign structure in the t-J model and the absence of it in the σ·t-J model [16–18].

In the t-J model, the Fermi sign structure vanishes at half-filling due to the no-double-occupancy constraint or the “Mottness”. Upon doping, a reduced Fermi sign structure starts to recover with the increase of doping concentration of holes. Specifically, the partition function for the t-J model on a square lattice at an arbitrary doping can be expressed by

$$Z_{t-J} = \sum_c \tau_c Z[c] ,$$

where the summation is over $c$, denoting the set of closed loops of the spatial trajectories of all spins and holes. Accompanying a non-negative weight $Z[c] \geq 0$, each path $c$ is associated with a sign factor $\tau_c$ given by

$$\tau_c \equiv (-1)^{N^1_h[c]} \times (-1)^{N^0_h[c]} .$$

Here the Berry’s phase-like $\tau_c$ replaces the usual Fermion signs in a partition function of a fermion system due to an explicit restriction of the Mottness. In $\tau_c$, $(-1)^{N^1_h[c]}$ resembles a conventional Fermi sign structure now only associated with the doped holes, while $(-1)^{N^0_h[c]}$ depends on the total parity of exchanges between the holes and $\downarrow$-spins for the closed path set $c$, which is known as the phase-string sign structure.

By contrast, in the σ·t-J model, the phase-string sign structure can be simply switched off such that the partition function is reduced to

$$Z_{\sigma·t-J} = \sum_c (-1)^{N^0_h[c]} Z[c] ,$$

where the non-negative weight $Z[c]$ remains the same as in the t-J model, with the same Fermi sign structure of the holes: $(-1)^{N^0_h[c]}$. At half-filling, the t-J and σ·t-J models both reduce to the Heisenberg model.

Therefore, the distinction between the t-J and σ·t-J models only emerges upon doping, which is solely distinguished by the phase-string sign structure $(-1)^{N^0_h[c]}$ in Eq. (A2). As it turns out, such a phase string is very singular, which prevents a perturbative treatment of the t-J model, even in the one-hole-doped case. In the next appendix, we shall discuss a duality transformation and an emergent symmetry in the one-hole ground state, which are entirely due to the phase string effect in a non-perturbative way.

Appendix B: One-hole wave function ansatz and emergent symmetry in the t-J model

The Mott physics in the pure t-J model is crucially associated with an emergent phase string sign structure as outlined above, which will result in a non-perturbative duality transformation and a new type of ground state ansatz [11, 12]. In the following, we concentrate on such a one-hole problem from the perspective of symmetry.

To account for the phase string effect in the one hole case, a many-body phase shift factor $e^{\pm i\Omega_i}$ produced by the doped hole has to be introduced [11, 12] in Eq. (6), with $\Omega_i$ defined in Eq. (8). The four-fold degenerate one-hole ground states can be constructed with $|\Psi_G\rangle_{1,2}$ by removing a $\downarrow$-spin and $|\Psi_G\rangle_{3,4}$ by removing an $\uparrow$-spin from the half-filling ground state $|\phi_0\rangle$ as follows:

$$|\Psi\rangle_{1,2} = \sum_i \varphi_{1,2}(i) e^{\mp i\Omega_i} c_i \downarrow|\phi_0\rangle ,$$

$$|\Psi\rangle_{3,4} = \sum_i \varphi_{3,4}(i) e^{\mp i\Omega_i} c_i \uparrow|\phi_0\rangle .$$

Here the variational parameters $\varphi_{1,2,3,4}(i)$ only depend on the hole’s position. The variational ansatzes in Eqs. (B1) and (B2) divide the single-hole Hilbert space into four sectors. The VMC simulation and analytic analysis have shown [11], consistent with the ED and DMRG results, that besides $S_z = 1/2$ for $|\Psi_G\rangle_{1,2}$ and $S_z = -1/2$ for $|\Psi_G\rangle_{3,4}$, each of them acquires an additional double degeneracy with nontrivial orbital momenta $L_z = \pm 1$, respectively, for a bipartite lattice of $N = 2M \times 2M$.

The phase shift factors $e^{\pm i\Omega_i}$ in Eqs. (B1) and (B2) characterize the mutual entanglement between the hole and spins in the AF background $|\phi_0\rangle$ to precisely keep track of the phase-string structure. Explicitly, we can make a choice $\Omega_i = \sum_i \text{Im} \ln(z_i - z_i^\dagger)$, as in Eq. (8) for a 2D system. It represents a mutual duality transformation, which cannot be perturbatively obtained based on a bare hole state created by $c_i \sigma_\uparrow$ on $|\phi_0\rangle$. With the nontrivial many-body effect encoded in $e^{\mp i\Omega_i}$, the resulting $\varphi_{1,2,3,4}(i)$ are consequently treated as the single-hole wave functions in the VMC calculation, which have turned out to be a very good approximation [11].

Given the single-particle nature, nevertheless, $\varphi_{1,2,3,4}(i)$ are not simply Bloch wave functions. By a symmetry analysis, we show some intrinsic topological relations among them in the following.

The anti-unitary time reversal symmetry $T$ flips spin direction by

$$Tc_\uparrow T^{-1} = c_\downarrow , \quad Tc_\downarrow T^{-1} = -c_\uparrow , \quad TiT^{-1} = -i .$$

Then by noting the time-reversal symmetry in the ground states, one has

$$T|\Psi_G\rangle_1 = |\Psi_G\rangle_3 ,$$

which requires

$$\varphi_3(i) = -\varphi_1^*(i) e^{i \sum z_i \text{Im} \ln(z_i - z_i^\dagger)} ,$$

with the opposite $S_z = \pm 1/2$ and $L_z = \pm 1$. Similarly, $T$ transforms the state $|\Psi_G\rangle_2$ into $|\Psi_G\rangle_4$, which results in

$$\varphi_4(i) = -\varphi_2^*(i) e^{i \sum z_i \text{Im} \ln(z_i - z_i^\dagger)} .$$
TABLE I. List of the quantum numbers of four degenerate ground states $|\Psi_G\rangle_{1,2,3,4}$ in the pure single-hole doped $t$-$J$.

| $|\Psi_G\rangle_i$ | $c_1^\dagger c_2^\dagger$ | $L_z$ | $J_z$ | Relation with $|\Psi_G\rangle_1$ |
|------------------|------------------|--------|--------|-------------------|
| $|\Psi_G\rangle_1$ | $e^{-i\theta_1}$ | $+\frac{1}{2}$ | $+1$ | $+1$ | $+\frac{1}{2}$ | $G|\Psi\rangle_1$ |
| $|\Psi_G\rangle_2$ | $e^{i\theta_1}$ | $+\frac{1}{2}$ | $-1$ | $-\frac{1}{2}$ | $G|\Psi\rangle_1$ |
| $|\Psi_G\rangle_3$ | $e^{-i\theta_1}$ | $-\frac{1}{2}$ | $-1$ | $-\frac{1}{2}$ | $G|\Psi\rangle_1$ |
| $|\Psi_G\rangle_4$ | $e^{i\theta_1}$ | $-\frac{1}{2}$ | $+1$ | $+\frac{1}{2}$ | $G|\Psi\rangle_1$ |

Furthermore, one may also define a unitary symmetry $Z_2$ generated by the spin flip operation $R$, which is defined

$$Rc_1R^{-1} = c_1, \quad Rc_2R^{-1} = -c_2,$$

with no extra sign change in the imaginary unit $i$. Then,

$$R|\Psi_G\rangle_1 = |\Psi_G\rangle_4$$

leads to

$$\varphi_4(i) = -e^{-i\sum_{\sigma} \text{Im log}(z_{\sigma}-z_i)} \varphi_1(i),$$

where $|\Psi_G\rangle_1$ and $|\Psi_G\rangle_4$ have the same $L_z = -1$ and opposite $S_z = \pm 1/2$. Similarly, $R$ transforms the state $|\Psi_G\rangle_2$ into $|\Psi_G\rangle_3$. Table B lists the corresponding quantum numbers of the degenerate ground states $|\Psi_G\rangle_{1,2,3,4}$ and the actions of time reversal $T$ and spin flip symmetries $R$.

Appendix C: Rashba interaction

1. Total angular momentum $J_z$

The presence of the Rashba interaction breaks conservation of the angular momentum $L_z$ and total spin $S_z$. However, the total angular momentum $J_z = L_z + S_z$ remains as a good quantum number. Here we give a proof. We aim to prove $UH_{t-J-R}U^\dagger = H_{t-J-R}$ for $t$-$J$-$R$ Hamiltonian with $U = \exp(-i\frac{\pi}{2} J_z)$ generated by $J_z$. For Rashba SOC term $c_j^\dagger c_{j+\hat{y}}$ in Eq. (4),

$$U c_j^\dagger c_{j+\hat{y}} U^\dagger = e^{-i\frac{\pi}{2}(L_z+S_z)} c_j^\dagger c_{j+\hat{y}} e^{i\frac{\pi}{2}(L_z+S_z)}$$

$$= e^{-i\frac{\pi}{2} S_z} c_j^\dagger R(\hat{y}) c_{j+\hat{y}} e^{i\frac{\pi}{2} S_z}$$

$$= -ic_j^\dagger R(\hat{y}) c_{j+\hat{y}}.$$  \hspace{1cm} (C1)

where $R(\hat{y})$ is the site by rotating site $i$ counterclockwisely 90 degrees. Similarly, for term $c_j^\dagger c_{j+\hat{y}}^\dagger$, we have

$$U c_j^\dagger c_{j+\hat{y}} U^\dagger = -ic_j^\dagger R(\hat{y}) c_{j+\hat{y}}.$$  \hspace{1cm} (C2)

Combination of Eqs. (C1) and (C2) demonstrates the invariance of Rashba interaction under $U$,

$$U H_R U^\dagger = H_R.$$  \hspace{1cm} (C3)

The hopping and superexchange terms obviously conserve the total momentum $J_z$. Therefore, we prove that total momentum $J_z$ is a good quantum number for the $t$-$J$-$R$ model.

FIG. 9. Two-component structure of the one-hole ground states of the $t$-$J$ model with a weak SOC ($\lambda = 0.006J$) obtained by ED under PBC in a $4 \times 4$ lattice: (a) The total spin $S = 1/2$ is locked with four total momenta at $k_0 = (\pm \pi/2, \pm \pi/2)$; (b) Corresponding to one of the $k_0$'s in (a) (the full arrow), a “spin current” pattern of $\langle J^z_{k,k'} \rangle$ in Eq. (C4) indicates a hidden many-body effect (see text).

2. ED under periodic boundary condition

We present the ED result under the periodic boundary condition (PBC) in the following. Generally, without SOC, there are eight-fold degenerate ground states distinguished by momenta $k_0 = (\pm \pi/2, \pm \pi/2)$ with total $S^z = \pm 1/2$ in the pure one-hole state [8]. The Rashba SOC will lift the spin degeneracy such that the spin $S = 1/2$ will be locked with the four momenta $k_0$ to give rise to a spin texture of $\langle S_k \rangle$ lying in the $x$-$y$ plane as illustrated in Fig. 9(a), which is obtained by ED with a very small SOC strength $\lambda = 0.006J$ (see below). Here, corresponding to a ground state with a given total momentum, say, $k_0 = (\pi/2, \pi/2)$, $\langle S_k \rangle$ is mainly concentrated at $k_0$ as indicated by the solid arrow in Fig. 9(a) similar to a free quasiparticle.

However, different from the free quasiparticle picture, in the present ground state, a nontrivial “spin current” pattern is also found in the momentum space as shown in Fig. 9(b), which is defined by the correlator

$$\langle J^z_{k,k'} \rangle \equiv i \langle S_{k'}^+ S_{k'}^- - S_{k'}^+ S_{k}^- \rangle.$$  \hspace{1cm} (C4)

Here $\langle J^z_{k,k'} \rangle$ illustrates correlations between spins at the neighboring momenta (with $k \neq k'$), which is present even in the absence of the SOC. The nontrivial spin current pattern in Fig. 9(b) only disappears in the half-filling due to fully frozen charge degree of freedom and the absence of phase string effect. Hence, $\langle J^z_{k,k'} \rangle \neq 0$ represents an incoherent component of the one-hole ground state, which manifests as a broaden momentum distribution of the single hole as a partial momentum can be carried away by the backflow spin current such that the single-particle momentum $k$ is no longer conserved [8]. This represents the so-called incoherent component violating the Landau’s one-to-one correspondence between the momentum $k$ of the hole and the total momentum $k_0$. The
latter should be always conserved under PBC due to the translational invariance of the total system including all spins and the hole. However, due to the spin current, a partial momentum can be continuously transferred to the spin background from the hole or vice versa a partial momentum can be continuously transferred to spins and the hole. However, due to the spin current, a partial momentum can be continuously transferred to the spin background from the hole or vice versa to result in a non-Landau quasiparticle state, which is essentially a many-body effect [8].

Next we examine the quantitative role of SOC. Figure 10(a) shows the change of the ground-state energy $\Delta E_{0}^{1st}(\lambda) = E_{0}^{1st}(\lambda) - E_{0}^{1st}(0)$ as a function of $\lambda$. For the $t$-$J$ model, the linear-$\lambda$ dependence (red line) and a finite value of its first derivative over $\lambda$ at $\lambda \to 0$ [cf. the inset of Fig. 10(a)] strongly suggest that the spin texture in Fig. 9 at a very small $\lambda = 0.006J$ is not completely new as induced by the SOC, but should rather exist already in the pure $t$-$J$ model to give rise to the first-order perturbative contribution. Namely, a weak $\lambda$ here mainly selects and picks up a particular linear recombination state from the pure one-hole (2 $\times$ 4 = 8 fold) degenerate ground states in forming the spin texture in Figs. 9(a) and 9(b).

Indeed, by turning off the phase string effect or the spin current structure in the $t$-$J$ model to result in a Landau-like quasiparticle in the $\sigma$-$t$-$J$ model, the change in ground state energy due to SOC becomes significantly weakened as compared shown by Fig. 10(a) and the inset (blue triangles). There is no more linear-$\lambda$ term. As a matter of fact, to make the spin texture amplitude comparable, the SOC strength $\lambda$ has to be enhanced by two orders of magnitude in $\sigma$-$t$-$J$ model as illustrated in Fig. 10(b). Here the intensity $|I_{\text{max}}(k)|$ of the spin-polarized spectral function [defined in Eq. (E8) in Appendix E] is shown in Fig. 10(b) at $k = (\pi/2, \pi/2)$.

Appendix D: Variational Monte Carlo procedure

1. Variational ground state at half-filling

At half filling, both $t$-$J$ and $\sigma$-$t$-$J$ models are reduced to a pure spin-1/2 Heisenberg system. At present, the best variational ground state is the so-called Liang-Doucot-Anderson bosonic “resonating valence bond” (RVB) state [24] as

$$|\phi_{0}\rangle = \sum_{\nu} \omega_{\nu}|\nu\rangle,$$  \hfill (D1)

where each pair of spins in a dimer covering configuration $|\nu\rangle$ from different sublattices

$$|\nu\rangle = \sum_{\langle\sigma\rangle (i,j)\in\nu} \epsilon_{\sigma,\sigma_{j}} c_{i\sigma}^{\dagger} \cdots c_{N\sigma_{N}}^{\dagger}|0\rangle,$$  \hfill (D2)

forms a singlet pairing forced by a Levi-Civita symbol $\epsilon_{\sigma,\sigma_{j}}$, or, equivalently the Marshall sign [29]. During the calculations, the non-negative amplitude $\omega_{\nu}$ can be factorized as $\omega_{\nu} = \prod_{(i,j)\in\nu} h_{ij}$, where $h_{ij}$ is a non-negative function depending on sites $i$ and $j$. To obtain a ground state for AF spin Heisenberg model, we have to optimize all factors $h_{ij}$. Conversely, with predetermined $h_{ij}$, one can artificially tune the background spin correlations $\xi$. For example, in Fig. 7, we choose a form of $h_{ij}$,

$$h_{ij} = r_{ij}^{-\alpha},$$  \hfill (D3)

where $r_{ij}$ being the Manhattan distance $|x_{i} - x_{j}| + |y_{i} - y_{j}|$ with $(x_{i}, y_{i})$ being a coordinate of site $i$. The spin correlation length $\xi$ gradually decreases as the parameter $\alpha$ in Eq. (D3) increases. For example, a state with $\alpha = 2$ shows an AF long range order $\xi \gg L$, as shown in Fig. 1, while a short range ordered state with $\xi \sim 2.7$ can be obtained by setting $\alpha = 4$.

2. Variational wave functions for the $t$-$J$-$R$ model

Based on the ground states in Eqs. (B1) and (B2), the wave function $\varphi_{1,2,3,4}(i)$ can be determined by VMC. For example, $\varphi_{1,2}(i)$ will be obtained by diagonalizing the following effective Hamiltonian $H_{\text{eff}}$ as the wave function for the composite quasiparticle $\hat{c}_{\downarrow}$ defined in Eq. (6):

$$H_{\text{eff}} = - \sum_{(i,j)} \tilde{t}_{ij}^{\dagger} \hat{c}_{i\downarrow}^{\dagger} h_{ij} + h.c.,$$  \hfill (D4)

where $\tilde{t}_{ij}^{\dagger}$ represents an effective hopping integral

$$\tilde{t}_{ij}^{\dagger} = t \sum_{\sigma} \langle \phi_{0}\rangle \hat{c}_{i\sigma}^{\dagger} \epsilon_{\sigma}^{\epsilon_{\sigma}} \hat{c}_{j\sigma} \hat{c}_{i\downarrow} (\phi_{0}) .$$  \hfill (D5)

Here $\tilde{t}_{ij}^{\dagger}$ generally takes a complex value due to the phase shift factor of a twisted particle $\hat{c}_{i\downarrow} \equiv e^{+i\phi_{i\downarrow}} c_{i\downarrow}$. Thus,
it is a Harper-Hofstadter-like Hamiltonian with a non-uniform flux in Eq. (D4) that breaks the translational symmetry for $\varphi_{1,2}(i)$. Such an emergent flux can induce exotic consequences beyond the simple tight-binding model. In such an effective Hamiltonian, $\mathbf{h}_1^i$ creates a ‘twisted’ hole with the wave function $\varphi_{1,2}(i)$. Two sectors are available, denoted as $|\Psi_{1,2}\rangle$ respectively in Eq. (B1), which correctly reproduce two ground states. The degeneracy is protected by a time reversal symmetry for uniform flux in Eq. (D4) that breaks the translational symmetry for $\varphi_{1,2}(i)$.

Now let us turn on the SOC with $\lambda$, which will select one chiral spin pattern with total momentum $J_z = \frac{3}{2}$ or $-\frac{3}{2}$, such that $|\Psi_{R,updp}\rangle$ produces ground states. If we choose $|\Psi_2\rangle$ to be $|\Psi_{G,2}\rangle$ with $J_z = \frac{3}{2}$ then $|\Psi_2\rangle$ must have a quantum number $L_z = 2$ with $J_z = -\frac{3}{2}$ which is the first excited state of the $t$-$J$ model. If we start with $|\Psi_{G,4}\rangle$ as a choice for $|\Psi_4\rangle$, then $|\Psi_2\rangle$ is the first excited state of the $t$-$J$ model with $L_z = -2$. The two ground states can be transformed to each other by time-reversal symmetry $T$ in Eq. (B3). Technically, as proved by previous ED results [8], non-degeneracy between $|\Psi_{G,1,2,3,4}\rangle$ and first excited states accidently originates from the open boundary condition, which leads to dramatically large energy gain induced by SOC as compared to the $\sigma$-$t$-$J$-$R$ model in Fig. 3(a) and (b).

Appendix E: Spin-polarized spectral function

The spin-polarized angle-resolved photoemission spectroscopy (ARPES) can be used to detect the spin texture of a quasiparticle (hole) excitation. The normal spectral function for a single quasihole of spin $\sigma$ is defined by $A_\sigma(\omega, k) = \sum_n \langle \Psi_G|c_{k\sigma}^\dagger|n\rangle\langle n|c_{k\sigma}|\Psi_G\rangle \delta(\omega - E_n + E_G)$

$$A_\sigma(\omega, k) = \frac{1}{\pi} \text{Im} \sum_n \frac{\langle \Psi_G|c_{k\sigma}^\dagger|n\rangle\langle n|c_{k\sigma}|\Psi_G\rangle}{\omega - E_n + E_G - i\eta},$$

where $|\Psi_G\rangle$ is the ground state with energy $E_G$ and $|n\rangle$ denotes an eigenstate of energy $E_n$ with an extra hole created by $c_{k\sigma}^\dagger$ on $|\Psi_G\rangle$. In Eq. (E1), $\eta$ is a broadening introduced to represent energy resolution. Based on $A_\sigma(\omega, k)$, we may further extract information on spin polarization, for example, by focusing on the intensity of the scattered electrons that are parallel or perpendicular to the momentum $k$. One may define

$$c_{k,\sigma}^\dagger = \frac{1}{\sqrt{2}} \left( c_{k\uparrow}^\dagger + i e^{i\theta_k} c_{k\downarrow}^\dagger \right),$$

$$c_{k,\sigma}^\dagger = \frac{1}{\sqrt{2}} \left( c_{k\downarrow}^\dagger - i e^{i\theta_k} c_{k\uparrow}^\dagger \right),$$

to create holes respectively with spin polarization at $S_{\sigma \downarrow} = (\cos \theta_k, \sin \theta_k, 0)$ and $S_{\sigma \uparrow} = -S_{\sigma \downarrow}$ perpendicular to momentum $k$ lying in the $x$-$y$ plane with $e^{i\theta_k} = \frac{k_x + ik_y}{|k|^2}$. Similarly, the two operators

$$c_{k,\sigma}^\dagger = \frac{1}{\sqrt{2}} \left( c_{k\uparrow}^\dagger + e^{i\theta_k} c_{k\downarrow}^\dagger \right),$$

$$c_{k,\sigma}^\dagger = \frac{1}{\sqrt{2}} \left( c_{k\downarrow}^\dagger - e^{i\theta_k} c_{k\uparrow}^\dagger \right),$$

create holes with spin polarization along $S_{\sigma \downarrow} = (\sin \theta_k, -\cos \theta_k, 0)$ and $S_{\sigma \uparrow} = -S_{\sigma \downarrow}$ parallel to the momentum $k$. Then the spin polarization $I_{\perp}(\omega, k)$ at the direction perpendicular to momentum $k$ takes the form

$$I_{\perp}(\omega, k) = A_{\sigma \downarrow}(\omega, k) - A_{\sigma \uparrow}(\omega, k).$$
and $I_\parallel (\omega, k)$ at direction parallel to momentum $k$:
\[
  I_\parallel (\omega, k) = A_{\parallel}(\omega, k) - A_{\perp}(\omega, k) .
\]  
(E7)

Finally one may define an intensity vector $I_{\max}(k)$
\[
  I_{\max}(k) = I_\parallel(\omega_c, k)e_{k,\parallel} + I_\perp(\omega_c, k)e_{k,\perp} ,
\]  
(E8)

where $e_{k,\parallel}$ ($e_{k,\perp}$) is the unit vector parallel (perpendicular) to momentum $k$ and $\omega_c$ denotes the frequency at which $\sqrt{I_\parallel(\omega, k)^2 + I_\perp(\omega, k)^2}$ reaches its maximum value $|I_{\max}(k)|$. The quantity $I_{\max}(k)$ as probed by the spin-polarized ARPES experiment can effectively characterize the spin texture structure in the momentum space, away from the Fermi energy and deep inside the Brillouin zone.

**Appendix F: Fragility of spin texture in a Fermi Liquid**

In a Fermi liquid, the spin texture induced by Rashba SOC is fragile at momentum away from the Fermi surface. In this appendix, we give a brief discussion.

The Landau's Fermi liquid theory asserts one-to-one correspondence between a free fermi gas system and an interacting one. The interaction as scattering between quasiparticles manifests as a self energy $\Sigma(\omega, k)$ in the Green’s function of electrons $c_\sigma$
\[
  G_\sigma(\omega, k) = \frac{1}{G_0^{-1}(\omega, k) - \Sigma(\omega, k)} = \frac{1}{\omega - \epsilon_0(k) - \mu + \text{Re}\Sigma(\omega, k) - i\Gamma(\omega, k)} ,
\]  
(F1)

where $G_0(\omega, k) = [\omega - (\epsilon_0(k) - \mu)]^{-1}$ denotes the propagator for the non-interacting Hamiltonian $H_0$, which generally involves both nearest and next-nearest neighbor hopping, $t$ and $t'$ with a dispersion relation $\epsilon_0(k)$,
\[
  \epsilon_0(k) = -2t(\cos k_x + \cos k_y) + 4t' \cos k_x \cos k_y .
\]  
(F2)

$\mu$ is a chemical potential which determines the doping concentration. The imaginary part $\Gamma(\omega, k)$ in Eq. (F1) of the self energy $\Sigma(\omega, k)$ dominates the lifetime of a quasiparticle excitation. A significant consequence is that to the leading order, $\Gamma(\omega, k)$ depends on the second power of the exciting energy,
\[
  \Gamma(\omega, k) = c\omega^2 ,
\]  
(F3)

with a coefficient $c$ determined by quasiparticles’ scatterings.

A sufficiently weak Rashba SOC interaction, on a reasonable assumption, exerts no influence on the self-energy. In other words, we shall introduce eigenmodes $d_{k,\pm}^0$ to diagonalize the free part $H_0 + H_R$,
\[
  d_{k,\pm}^0 = \frac{1}{\sqrt{2}}(c_{k,\uparrow} \pm e^{i\varphi_k} c_{k,\downarrow}) ,
\]  
(F4)

which, consequently, modify the Green’s functions
\[
  G_+(\omega, k) = \frac{1}{G_0^{-1}(\omega, k) - \Delta_k - i\Gamma(\omega, k)} ,
\]  
(F5)

\[
  G_-(\omega, k) = \frac{1}{G_0^{-1}(\omega, k) + \Delta_k - i\Gamma(\omega, k)} .
\]  
(F6)

Here $\Delta_k = 2\lambda \sqrt{\sin^2 k_x + \sin^2 k_y}$ is the energy splitting induced by SOC in $H_R$, and $e^{i\varphi_k} = \frac{\sin k_x + \sin k_y}{\sqrt{\sin^2 k_x + \sin^2 k_y}}$.

Follow the procedure presented in Sec. E and we can obtain the spin polarization perpendicular to the momentum $k$
\[
  I_\perp(\omega_c, k) = A_{\parallel}(\omega_c, k) - A_{\perp}(\omega_c, k) = 2\cos(\theta_k - \varphi_k) \frac{2\Delta_k}{c_{k}(c_{k}^2 + \Delta_k)} ,
\]  
(F7)

where $\omega_c$ is defined in Eq. (E8) to relate to the maximal spectral function $I_{\max}(k)$ and $e^{i\theta_k} = \frac{k_x + ik_y}{|k|}$. Since $\Delta_k$ is very small, a quasiparticle with momentum away Fermi surface towards $\Gamma$ point makes a vanishing contribution, which is depicted in Fig. 8 with parameters $c = 0.005, \lambda = 0.01J$ and $t' = 0.2t$.

[1] P. W. Anderson, The resonating valence bond state in La2CuO4 and superconductivity, Science 235, 1196 (1987).
[2] P. A. Lee, N. Nagaosa, and X.-G. Wen, Doping a Mott insulator: Physics of high-temperature superconductivity, Rev. Mod. Phys. 78, 17 (2006).
[3] B. I. Shraiman and E. D. Siggia, Mobile Vacancies in a Quantum Heisenberg Antiferromagnet, Phys. Rev. Lett. 61, 467 (1988).
[4] S. Schmitt-Rink, C. M. Varma, and A. E. Ruckenstein, Spectral Function of Holes in a Quantum Antiferromagnet, Phys. Rev. Lett. 60, 2793 (1988).
[5] C. L. Kane, P. A. Lee, and N. Read, Motion of a single hole in a quantum antiferromagnet, Phys. Rev. B 39, 6880 (1989).
[6] G. Martinez and P. Horsch, Spin polarons in the $t$-$J$ model, Phys. Rev. B 44, 317 (1991).
[7] Z. Liu and E. Manousakis, Spectral function of a hole in the $t$-$J$ model, Phys. Rev. B 44, 2414 (1991).
[8] W. Zheng, Z. Zhu, D. N. Sheng, and Z.-Y. Weng, Hidden spin current in doped Mott antiferromagnets, Phys. Rev. B 98, 165102 (2018).
[9] Z. Zhu, D. N. Sheng, and Z.-Y. Weng, Intrinsic translational symmetry breaking in a doped Mott insulator,
phys. Rev. B 98, 035129 (2018), arXiv:1707.00068 [cond-mat.str-el].
[10] Z. Zhu and Z.-Y. Weng, Quasiparticle collapsing in an anisotropic t-J ladder, phys. Rev. B 92, 235156 (2015), arXiv:1409.3241 [cond-mat.str-el].
[11] S. Chen, Q.-R. Wang, Y. Qi, D. N. Sheng, and Z.-Y. Weng, Single-hole wave function in two dimensions: A case study of the doped Mott insulator, Phys. Rev. B 99, 205128 (2019).
[12] Q.-R. Wang, Z. Zhu, Y. Qi, and Z.-Y. Weng, Variational wave function for an anisotropic single-hole-doped t-J ladder, arXiv e-prints , arXiv:1509.01260 (2015), arXiv:1509.01260 [cond-mat.str-el].
[13] K. Gotlieb, C.-Y. Lin, M. Serbyn, W. Zhang, C. L. Smallwood, C. Joziwicki, H. Eisaki, Z. Hussain, A. Vishwanath, and A. Lanzara, Revealing hidden spin-momentum locking in a high-temperature cuprate superconductor, Science 362, 1271 (2018).
[14] J.-H. Zhang, S. Li, Y. Ma, Y. Zhong, H. Ding, and Z.-Y. Weng, Phenomenological single-particle Green’s function for the pseudogap and superconducting phases of high-Tc cuprates, Physical Review Research 2, 023398 (2020), arXiv:1905.04862 [cond-mat.str-el].
[15] F. C. Zhang and T. M. Rice, Effective Hamiltonian for the superconducting Cu oxides, Phys. Rev. B 37, 3759 (1988).
[16] D. N. Sheng, Y. C. Chen, and Z. Y. Weng, Phase String Effect in a Doped Antiferromagnet, Phys. Rev. Lett. 77, 5102 (1996).
[17] Z. Y. Weng, D. N. Sheng, Y.-C. Chen, and C. S. Ting, Phase string effect in the t-J model: General theory, Phys. Rev. B 55, 3894 (1997).
[18] K. Wu, Z. Y. Weng, and J. Zaanen, Sign structure of the t–J model, Phys. Rev. B 77, 155102 (2008).
[19] Z. Zhu, H.-C. Jiang, Y. Qi, C. Tian, and Z.-Y. Weng, Strong correlation induced charge localization in antiferromagnets, Scientific Reports 3, 2586 (2013), arXiv:1212.6634 [cond-mat.str-el].
[20] Z. Zhu, H.-C. Jiang, D. N. Sheng, and Z.-Y. Weng, Nature of strong hole pairing in doped Mott antiferromagnets, Scientific Reports 4, 5419 (2014), arXiv:1312.6893 [cond-mat.str-el].
[21] E. I. Rashba, Theory of electrical spin injection: Tunnel contacts as a solution of the conductivity mismatch problem, Phys. Rev. B 62, R16267 (2000).
[22] Z.-Y. Weng, Mott physics, sign structure, ground state wavefunction, and high-Tc superconductivity, Frontiers of Physics 6, 370 (2011), arXiv:1110.0546 [cond-mat.supr-con].
[23] Z.-Y. Weng, Superconducting ground state of a doped Mott insulator, New Journal of Physics 13, 103039 (2011), arXiv:1105.3027 [cond-mat.supr-con].
[24] S. Liang, B. Doucot, and P. W. Anderson, Some New Variational Resonating-Valence-Bond-Type Wave Functions for the Spin-½ Antiferromagnetic Heisenberg Model on a Square Lattice, Phys. Rev. Lett. 61, 365 (1988).
[25] L. Zhang and Z.-Y. Weng, Sign structure, electron fractionalization, and emergent gauge description of the Hubbard model, Phys. Rev. B 90, 165120 (2014), arXiv:1406.6867 [cond-mat.str-el].
[26] S. Chen, Z. Zhu, and Z.-Y. Weng, Two-hole ground state wavefunction: Non-BCS pairing in a t–J two-leg ladder, Phys. Rev. B 98, 245128 (2018).
[27] J.-Y. Zhao, S. A. Chen, H.-K. Zhang, and Z.-Y. Weng, Two-Hole Ground State: Dichotomy in Pairing Symmetry, arXiv e-prints, arXiv:2106.14898 (2021), arXiv:2106.14898 [cond-mat.str-el].
[28] A. Bohrdt, E. Demler, and F. Grusdt, Rotational resonances and regge-like trajectories in lightly doped antiferromagnets, Phys. Rev. Lett. 127, 197004 (2021).
[29] A. Auerbach, Interacting Electrons and Quantum Magnetism (Springer-Verlag, 1994).