Energy levels and electromagnetic transitions of some even-even and even-odd Barium Isotopes

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Abstract: In this paper, the nuclear structure of Barium nuclei has been studied by interacting boson model. The properties of eigenstates and reduce probability of electric transition and potential energy surface have been calculated. The nuclei understudy has proton number equal 56 and neutron numbers 75 and 76. The results of energy states and B(E2) values have been compared with the experimental data. These comparisons show that acceptable between the calculations and experimental data. The parameters which used in the calculation have been estimated and give best approximation. The shape of potential energy surface show that the even-even nucleus have O(6) dynamical symmetry.

Keywords: IBM; IBFM; Ba nuclei; low-lying states; B(E2) values.

1. Introduction

In the nuclear structure, more models used to clarify the nuclear structure one of these models is Interacting Boson Model (IBM). It is endorsed by Arima and Iachello to delineate the atomic structure for even-even focuses [1, 2]. It has been generally applied to the structure of yrast and stimulated state in even proton and even neutron of center focus and has fundamental accomplishment. In the IBM, the even-much center is acknowledged to be a grouping of passing on s and d bosons with cheeky power (L)= 0 and 2, in particular. This model is related with a trademark get-together structure, which allows the introduction of confining adjusts called U(5), SU(3) and O(6) [1, 3]. The accessory boson model watches out for a huge progression forward in our appreciation of nuclear structure. It offers a head Hamiltonian, masterminded outlining total nuclear properties over a wide level of centers, and is set up on rather wide numerical get-together theoretical structures, which have in like way found propelling application to issues in atomic, sub-nuclear, and high-essentialness material science [4]. The utilization of this model to distorted centers is starting at now a subject of fundamental interest and discussion. The passing on boson model (IBM-1) [5] and its advancement to the odd-A centers, the interfacing boson-
fermion model (IBFM-1) [6], have wind up being set up to give an earth shattering delineation of broadly changing classes of centers built away from shut shell structures. The even proton and even neutron of Barium focuses, Ba (Z=56), are one of the most significant centers which depicted by shape changes among circuitous and deformed. Distinctive exploratory and hypothetical examinations on the structure of significance level and electromagnetic change properties of the even-even magnificent earth isotopes had been investigated [7-17]. The purpose behind the current work is to do a Phenomenological evaluation of the even-even and even-odd Ba isotopes inside the IBM and the IBFM to give a total point of view on these isotopes in rather direct way. The aftereffects of the IBFM dazed techniques $^{132, 133}$Ba isotopes will present for imperativeness levels and changes probabilities and will isolate and the separating the central data. In like manner, the IBM-1 will apply to calculate the low-centrality levels as appeared by strategy of get-togethers (gr-, γ-and β-) and the B(E2) regard for even-even $^{132, 133}$Ba isotopes by then evaluation of the wave work structure for Ba isotopes.

2. Theory

The nuclear model (interacting boson fermion model) its structure hinders a lot of numbers of boson which is distributed in the angular orbitals L=0 and 2. Moreover, the odd nucleon proton or neutron, and M-fermions involving single-molecule circles with rakish second $j_i = j_1, j_2, j_3, j_4, ...$. The segments of the fermion precise second are the m-dimensional space of the gathering $U(m)$ with $m_j = \sum j_i(2j_i + 1)$. The fermions creation $a_i^\dagger$ and destruction $\tilde{a}_i$ administrators are for the single-molecule notwithstanding. The boson creation $b_i^\dagger$ and obliteration $\tilde{b}_i$ are administrators for the aggregate degrees of opportunity. The fermion administrators fulfill hostile to compensation relations [18-20]:

$$\{\tilde{a}_i, a_j^\dagger\} = \delta_{ij}, \{a_i^\dagger, a_j^\dagger\} = \{\tilde{a}_i, \tilde{a}_j\} = 0 \quad (1)$$

The linear finding of fermion creation $a_i^\dagger$ and destruction $\tilde{a}_j$

The Hamiltonian of Interacting Boson Fermion Model had been depend on the algebraic structure, that aim of concurrent probability of dynamical limit for even-odd nuclei. In case of the single-$j$, the m values are $m = 2j + 1$, in general, a chain of algebras is:

$$U(2j + 1) \supset SU(2j + 1) \supset SO(2j + 1) \supset SU(2) \supset O(2) \quad (2)$$

In present work, the yrast and energized state for $^{130, 131}$Ba cores have been determined by IBFM. Turn, equality and vitality of levels, the diminished likelihood of electric progress are determined. These computations contrasted and those of trial information.

In the IBFM odd-A centers are depicted similar to a mixed game plan of conveying bosons and fermions, the possibility of dynamical adjusts must be summarized. Under the impediment, that both the boson and fermion states have incredible saucy power, the different social occasion binds should contain the upheaval bundle O(3) for boson and SU(2) for fermion as subgroup [21,22].

$$U^B(6) \supset ... ... ... O^B(3) \quad (3)$$
$$U^F(m) \supset ... ... ... SU^F(2)$$

If one of subgroups of $U^B(6)$ is isomorphic to one of the subgroups of $U^F(m)$, the boson and fermion bundle chains can be joined into a regular boson-fermion pack chain. Right when the Hamiltonian is composed similarly as Casimir invariants of the joined boson-fermion pack chain, dynamical boson-fermion equity rises. The odd-A centers are delineated by the coupling of the odd fermionic semi atom to a total boson place. The total Hamiltonian involves three segments and is given by the going with condition [23, 24]:

$$H = H_b + H_f + V_{BF} \quad (4)$$

The one body fermion contained in the $H_f$ term and can be written as:

$$H_F = \sum_{j\mu} \epsilon_j a_{j\mu}^\dagger \tilde{a}_{j\mu} \quad (5)$$
The $\gamma_{2013}$ are the energies of quasi-particle and $a_{jm}^\dagger \tilde{a}_{jm}$ is the creation (annihilation) operator for the quasi-particle in the Eigen function $|jm\rangle$.

And $V_{BF}$ is describes of the interaction quasi-nucleon with the core (even-even) nucleus that’s mean $V_{BF}$ represented the interaction between fermion and the core nucleus [25-28]:

$$V_{BF} = \sum_j A_j \left[ (d^+ \times d^0)^{(0)} \times (a_j^+ \times \tilde{a}_j)^{(0)} \right]_0 + \sum_{j\neq j'} \Gamma_{jj'} Q^{(2)} \times (a_j^+ \times \tilde{a}_j)^{(2)} \times (d^+ \times d^0)^{(0)} \times (a_{j'}^+ \times \tilde{a}_{j'})^{(0)}$$

(6)

where $Q^{(2)}$ is the quadrupole operator for core nucleus.

The $A_j$, $\Gamma_{jj'}$ and $\Lambda_{jj'}$ are parameters defined in the following equations:

$$A_j = A_0 \sqrt{2j + 1}$$

$$\Gamma_{jj'} = \sqrt{5} \Gamma_0 (\nu_j \nu_{j'} - \nu_j \nu_{j'}) Q_{jj'}$$

$$\Lambda_{jj'} = -\sqrt{5} A_0 \left[ \frac{(\nu_j \nu_{j'} + \nu_j \nu_{j'}) Q_{jj'} \beta_{jj'} + (\nu_j \nu_{j'} + \nu_j \nu_{j'}) Q_{jj'} \beta_{jj'}}{2j + 1} \right]$$

(7)

The $A_0$ is the monopole communication;

$\Gamma_0$ is the quadrupole communication;

$\Lambda_0$ is the trading of a semi molecule with one of the two fermions framing a boson.

The dynamical boson-fermion evenness related with as far as ability and the single fermion (odd nucleon) involving single-molecule circles with turn $j = 1/2, 3/2, 5/2$. For this situation, the fermion space is disintegrated into a pseudo-orbital part with $K = 0, 2$ and a pseudo-turn part with $s = 1/2$ [29].

3. Theoretical Calculations and Discussion

3-1 Low-Lying States

In this section, the results of energy states for even-even and even-odd Barium nuclei will be described. The Barium nuclei have proton number 56 and neutron numbers 75 and 76. All calculation achieved by interacting boson model for even –even Barium nucleus. Also, the calculations of even-odd Barium nucleus are achieved by interacting boson fermion model. The framework codes which are used in the present work did not distinguishes between the proton or neutron boson and fermion. Moreover, the free parameters in Hamiltonian are tried to keep in the minimum number for analysis the calculated energy levels of Ba nucleus. The Hamiltonian adopt in the calculations can be expressed [30];

$$H = H_B + H_F + V_{BF}. $$

To calculate the energy states for even-even Ba nucleus, the PHINT [31] code has been used. For even-odd Ba nucleus, the ODDA code has been used [31]. All parameters which are used in the PHINT and ODDA to calculate the $gr, \gamma$- and $\beta$- bands for even –even and 1, 2 and 3 for odd $A$ are presented in the tables 1 and 2. These parameters in MeV unit, excepted $N$ and $\nu^2_j$. 
Table 1. The parameters which are used in the PHINT and ODDA codes to calculate the energy levels. All parameters in MeV unit, excepted N

| Parameters | IBMFM | IBM |
|------------|-------|-----|
| BFE        | 0.010 | 6   |
| BFQ        | 0.000 | 0.059|
| BFM        | 0.009 | 0.056|
| N          |       | 0.042|

Table 2. The addition parameters which used in the ODDA code. $\varepsilon_j$ parameter in MeV unit and $\nu^2_j$ without unit.

| Parameters | $^{133}$Ba $^{3p_{1/2}}$ | $^{2f_{5/2}}$ | $^{3p_{1/2}}$ |
|------------|---------------------------|----------------|----------------|
| $\varepsilon_j$ | 1.36                      | 1.064          | 1.048          |
| $\nu^2_j$    | 0.37                      | 0.48           | 0.52           |

The energy states for gr-, $\gamma$- and $\beta$- bands of even-even Ba nucleus and for even-odd Ba nucleus are shown in the Figure 1. In this figure, the calculations results are compared with experimental data. Furthermore, the calculations are in reasonable with the experimental [32] data for both nuclei. The states with "( )" in gr-, $\gamma$-, $\beta$-, 1, 2 and 3-bands correspond to cases for which the energy, spin and/or parity of the corresponding states are not well confirm experimentally.

Figure 1: (Color online) the calculations of PHINT and ODDA compared with experimental data [32] for $^{132,133}$Ba nuclei.
3-2 Reduce Probability Electric Transition $B(E2)$ Value

The second property of nuclear structure is electric reduced probability transition. This property will be explained in details with empirical values. Furthermore, it is given a good test of nuclear structure and the wave function of the nuclear model. The electric transition operators can be expressed as sum of two parts, the first part is for the boson of the eigenstate \( j = 1/2, 3/2, \) and second part only on the fermion eigenstate \([18,19]\), the fermion can be moved in \( j = 1/2, 3/2, 5/2 \) sub-orbital.

The \((eB)\) values represented on the effective charge and calculated form experimental data and it tabulated in the table (3). The selection rules which used in the our estimated for the effective charge \((e_f)\) values for fermion are \(\Delta \sigma_1 = \Delta \sigma_2 = \Delta \sigma_3 = 0\) and \(\Delta (\tau_1 + \tau_2) = \pm 1\) transitions. It is for \(\Delta \sigma_1 = \Delta \sigma_2 = \Delta \sigma_3 \neq 0, \Delta (\tau_1 + \tau_2) = \pm 1\) transitions, \(e_B (\alpha_2) = e_f (f_2)\) allowed, and thus expected to be weaker, which are for \(e_B \neq e_f\). At \(e_B \neq e_f\) allowed only, the effective charge \((e_f)\) can be reproduced from the experimental \(B(E2; I_i \rightarrow I_f)\) and it express as \([18]\).

\[
B(E2; (N, 1, 0), (\tau_1, \tau_2), L_i, J_i \rightarrow (N + 1, 0, 0), (\tau_1, \tau_2)f, L_f, J_f) = (\alpha_2 - f_2)^2 \frac{2N(N+1)}{2(N+1)(N+2)}
\]

\((e_f)\) represented on the effective charge for fermion and it is tabulated in table (3). The \(B(E2)\) results are calculated and compared with experimental value and tabulated in table (4).

### Table 3. The parameters which used in the \(B(E2)\) values calculations for \(^{130,131}\)Ba nuclei. These parameters are in eb unit.

| Isotope | N | \(\alpha_2\) |
|---------|---|--------------|
| \(^{132}\)Ba | 6 | 0.1193 |
| \(e_B(eb)\) | \(E_f(eb)\) |
| \(^{133}\)Ba | 0.119 | -0.320 |

### Table 4. The calculated \(B(E2)\) values and experimental data \([32]\) in \(e^2b^2\) unit.

| \(J_i \rightarrow J_f\) | \(^{132}\)Ba | \(^{133}\)Ba |
|------------------------|------------|------------|
|                         | IBM-1      | EXP.       | IBM-1      | EXP.       |
| \(2^+_1 \rightarrow 0^+_1\) | 0.1712     | 0.1710    | 3/2^-_1 \rightarrow 1/2^-_1 | 0.0850 | <0.0726 |
| \(2^+_2 \rightarrow 0^+_1\) | 0.0072     | 0.0131    | 3/2^-_2 \rightarrow 1/2^-_1 | 0.1463 |
| \(2^+_2 \rightarrow 2^+_1\) | 0.2233     | 0.5102    | 3/2^-_2 \rightarrow 3/2^-_1 | 0.0254 |
| \(4^+_1 \rightarrow 2^+_1\) | 0.2233     | 0.2101    | 5/2^-_2 \rightarrow 1/2^-_1 | 0.0000 |
| \(6^+_1 \rightarrow 4^+_1\) | 0.2271     | --        | 5/2^-_2 \rightarrow 1/2^-_1 | 0.1601 |
| \(8^+_1 \rightarrow 6^+_1\) | 0.2018     | --        | 5/2^-_1 \rightarrow 3/2^-_1 | 0.0000 |
| \(10^+_1 \rightarrow 8^+_1\) | 0.1532     | --        | 5/2^-_2 \rightarrow 3/2^-_1 | 0.0110 |
| \(12^+_1 \rightarrow 10^+_1\) | 0.0854     | --        | 7/2^-_1 \rightarrow 3/2^-_1 | 0.1706 |
3.3. Contour Shape

The interfacing boson model was at first written as far as creation and obliteration boson administrators; its mathematical translation fit as a fiddle factors is typically done by presenting the inborn sound state, it is communicated as a boson condensate [25]:

$$|N, \beta, \gamma\rangle = 1/\sqrt{N!} \left(b^+_1\right)^N |0\rangle$$  \hspace{1cm} (9)

The $|0\rangle$ represent on the boson vacuum, and

$$b^+_c = (1 + \beta^2)^{-1/2} \left\{ s^+ + \beta \left[ \cos \gamma (d^+_0 + \sqrt{1/2} \sin \gamma (d^+_z + d^+_\gamma)) \right] \right\}$$  \hspace{1cm} (10)

The number of bosons out the closed shell of proton and neutron are $N$. The $\beta$ boundary is identified with the hub disfigurement of the core, while $\gamma$ measures the deviation from pivotal evenness which decides the mathematical state of the core. There are $\beta \geq 0$ and $0 \leq \gamma \leq \pi/3$ in this study. The calculation of the expectation value of the Hamiltonian (1) on the intrinsic boson condensate state give the expres sed of potential energy surface and it can be expressed as (8):

$$E(N, \beta, \gamma) = \langle N, \beta, \gamma | H | N, \beta, \gamma \rangle / \langle N, \beta, \gamma | N, \beta, \gamma \rangle$$

$$= \frac{Ne_d \beta^2}{(1 + \beta^2)} + \frac{N(N+1)}{(1 + \beta^2)^2} \left( \alpha_1 \beta^4 + \alpha_2 \beta^3 \cos 3\gamma + \alpha_3 \beta^2 + \alpha_4 \right)$$  \hspace{1cm} (11)

In the U(5), SU(3), and O(6) dynamical limits which correspond to the large $N_b$, $\beta_{min} = 0$, $\sqrt{2}$, and 1, respectively. Figure 2 show that the calculated potential energy surfaces for the even-even Ba nucleus with $N=75$. From this figure, the even-even Ba isotope is deformed and has $\gamma$-unstable characters.

Figure 2. PES for even $^{132}$Ba isotope
4. Conclusion
The nuclear structure of Barium with even proton and even/odd neutron has been studied using PHINT code for IBM and ODDA code for IBFM. The eigenstates and reduce electric probability of transition haven calculated. All free parameters have been tried to keep in the minimum number. The calculations are in reasonable with experimental data. The energy band $\gamma_1, \gamma_2, \beta_1, 1, 2$ and $3$ bands are predicted by frameworks. The potential energy surface, contour shape, show that the even-even Barium nucleus have O(6) dynamical symmetry.

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