Sieve of Prime Numbers Using Algorithms
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Abstract
This study suggests grouping of numbers that do not divide the number 3 and/or 5 in eight columns. Allocation results obtained from multiplication of numbers is based on column belonging to him. If in the Sieve of Eratosthenes the majority of multiplication of prime numbers result in a results devoid of practical benefit (numbers divisible by 2, 3 and/or 5), in the sieve of prime numbers using algorithms, each multiplication of prime number gives a result in a number not divisible to 2, 3 and/or 5.

Keywords: Column; Factor; Position; Sieve; Termination

Introduction
Sieve of prime numbers using algorithms
This paper deals with the study of odd numbers that cannot be divided with 3 and/or 5 by grouping them in eight columns, as follows:

The multiplication versions are in number of 36, their results being allocated according to columns, explained in Table 1.

Position Calculus
From the result of multiplying two numbers subtract the number assigned at position zero of the column namely one of the numbers i(p0): 7-11-13-17-19-23-29-31, the result is divided by 30. Integer obtained indicates the position of that number considering its column origin [1,2].

Formulas for determining the position
Position occupied by the result of the multiplication between

| Col.1=Col.1x8 | 2x4 | 3x5 | 6x7 |
| Col.2=Col.1x8 | 3x4 | 5x7 |
| Col.3=Col.1x5 | 2x6 | 3x8 | 4x7 |
| Col.4=Col.1x2 | 3x7 | 4x8 | 5x6 |
| Col.5=Col.1x1 | 2x7 | 3x3 | 4x4 | 5x8 | 6x6 |
| Col.6=Col.1x7 | 2x3 | 3x6 | 4x5 | 5x7 |
| Col.7=Col.1x4 | 2x5 | 3x6 | 4x6 | 5x5 | 7x8 |
| Col.8=Col.1x3 | 2x2 | 4x6 | 5x5 | 7x7 | 8x8 |

Table 1: Multiplication versions are in number of 36.

| Position | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|----------|---|---|---|---|---|---|---|---|
| 0        | 7 | 11 | 13 | 17 | 19 | 23 | 29 | 31 |
| 1        | 37 | 41 | 43 | 47 | 49 | 53 | 58 | 61 |
| 2        | 67 | 71 | 73 | 77 | 79 | 83 | 89 | 91 |
| 3        | 97 | 101 | 103 | 107 | 109 | 113 | 119 | 121 |

Table 2: Odd numbers that cannot be divided with 3 and/or 5.

| 7+31 | 5+23 | 4+19 | 2+11 | 1+7 | 6+29 | 3+17 | 2+13 | +37n |
| 8+17 | 11+31 | 8+23 | 2+7 | 10+29 | 4+13 | 6+19 | 3+11 | +41n |
| 8+19 | 7+17 | 13+31 | 12+29 | 5+13 | 4+11 | 9+23 | 2+7 | +43n |
| 6+11 | 7+13 | 16+29 | 17+31 | 9+17 | 10+19 | 3+7 | 12+23 | +47n |
| 8+13 | 18+29 | 4+7 | 14+23 | 19+31 | 10+17 | 6+11 | 11+19 | +49n |
| 22+29 | 5+7 | 8+11 | 14+17 | 19+23 | 23+31 | 9+13 | 12+17 | +53n |
| 22+23 | 18+19 | 16+17 | 12+13 | 10+11 | 6+7 | 29+31 | 27+29 | +59n |
| 7+7 | 11+11 | 13+13 | 17+17 | 19+19 | 23+23 | 29+29 | 31+31 | +61n |

Table 3: Position occupied p1 as a result of multiplication of numbers i.
In column 9 we register numbers under test up to P (max). Maxim position calculation is the integer number of the maximum number being tested radical divided by 30 [2-4].

Formulas belonging composite numbers are omitted. The algorithm uses formulas primes numbers squared correlating n=0,1,2,3,... With Pn.

Using the tables respecting the above algorithm complexity is much smaller, any multiple of prime number (which represents the number of position) has corresponding number is compound odd number and not divisible by 3 and/or 5.

**Example**: Determination of prime numbers up to N=1001.

In parentheses are the numbers corresponding to position past according to column.

**Divisibility by 7**:

Col.1: 7+7n=7(217) – 14(427) – 21(637) – 28(847)
Col.2: 5+7n=5(161) – 12(371) – 19(581) – 26(791) – 33(1001)
Col.3: 4+7n=4(133) – 11(343) – 18(553) – 25(763) – 32(973)
Col.4: 2+7n=2(77) – 9(287) – 16(497) – 23(707) – 30(917)
Col.5: 1+7n=1(49) – 8(259) – 15(469) – 22(679) – 29(889)
Col.6: 6+7n=6(203) – 13(313) – 20(523) – 27(733) – 34(943)
Col.7: 3+7n=3(119) – 10(329) – 17(539) – 24(749) – 31(959)
Col.8: 2+7n=2(91) – 9(301) – 16(511) – 23(721) – 30(931)

**Divisibility by 11**:

Col.1: 6+11n=6(187) – 17(517) – 26(747) – 35(977)
Col.2: 11+11n=11(341) – 22(671) – 33(1001)
Col.3: 8+11n=8(253) – 19(583) – 30(913)
Col.4: 2+11n=2(77) – 13(407) – 24(737)
Col.5: 10+11n=10(319) – 21(649) – 32(979)
Col.6: 4+11n=4(143) – 15(473) – 26(803)
Col.7: 6+11n=6(209) – 17(539) – 28(869)
Col.8: 3+11n=3(121) – 14(451) – 25(781)

**Divisibility by 13**:

Col.1: 8+13n=8(247) – 21(637)
Col.2: 6+13n=6(187) – 23(697)
Col.3: 13+13n=13(403) – 26(793)
Col.4: 16+13n=16(493)
Col.5: 12+13n=12(377) – 25(767)
Col.6: 17+13n=17(527)
Col.7: 5+13n=5(169) – 18(559) – 31(949)
Col.8: 9+13n=9(289) – 26(799)

**Divisibility by 17**:

Col.1: 7+17n=7(221) – 20(611) – 33(1001)
Col.2: 7+17n=7(221) – 24(731)
Col.3: 13+17n=13(403) – 26(793)
Col.4: 16+17n=16(493)
Col.5: 12+17n=12(377) – 25(767)
Col.6: 17+17n=17(527)
Col.7: 5+17n=5(169) – 18(559) – 31(949)
Col.8: 9+17n=9(289) – 26(799)

**Divisibility by 19**:

Col.1: 8+19n=8(247) – 27(817)
Col.2: 22+23n=22(667)
Col.3: 4+19n=4(133) – 23(703)
Col.4: 8+19n=8(283) – 31(943)
Col.5: 14+19n=14(437)
Col.6: 14+23n=14(437)
Col.7: 9+19n=9(299) – 29(889)
Col.8: 12+19n=12(391) – 29(901)

**Divisibility by 23**:

Col.1: 8+19n=8(247) – 27(817)
Col.2: 22+23n=22(667)
Col.3: 4+19n=4(133) – 23(703)
Col.4: 8+19n=8(283) – 31(943)
Col.5: 14+19n=14(437)
Col.6: 14+23n=14(437)
Col.7: 9+19n=9(299) – 29(889)
Col.8: 12+19n=12(391) – 29(901)

**Divisibility By 29**:

Col.2: 8+19n=8(247) – 27(817)
Col.1: 22+29n=22(667)
Col.2: 18+19n=18(551)
Col.3: 4+19n=4(133) – 23(703)
Col.4: 8+19n=8(283) – 31(943)
Col.5: 14+19n=14(437)
Col.6: 14+23n=14(437)
Col.7: 9+19n=9(299) – 29(889)
Col.8: 12+19n=12(391) – 29(901)

**Divisibility by 31**:

Col.1: 8+19n=8(247) – 27(817)
Col.2: 22+23n=22(667)
Col.3: 4+19n=4(133) – 23(703)
Col.4: 8+19n=8(283) – 31(943)
Col.5: 14+19n=14(437)
Col.6: 14+23n=14(437)
Col.7: 9+19n=9(299) – 29(889)
Col.8: 12+19n=12(391) – 29(901)
Col.8=27+29n=27(841)
Col.8=31+31n=31(961)
Numbers not eliminated are prime numbers

Application: The Factorial Multiplying or the Method of Determining if a Number is Prime up to a Given Number

The method of grouping odd numbers according to Table 1, allows checking whether a number is prime according to the last two or five digits of position the number.

For termination two digits

The calculation algorithm is:
1. Determine the position number and column it belongs;
2. Last two digits of the calculated number indicates the termination position of tested number;
3. Determine factors for termination and column number tested. I have illustrated the calculation of factors termination 10, column 1. Once calculated these factors can be used to determine of any prime numbers that belongs to the column 1, termination 10.
4. It performs testing divisibility of a number with multiples of 3 000 plus pairs of numbers factorial group to which it belongs termination corresponding column number tested.

We assign factorial group for multiplying operation positions from 0-99, as in Table 1, numbers between 7-3.001 grouped in columns. The position occupied by the result of the multiplication between any two numbers in the factorial group is a maximum six digit number. The last two digits of the number shows the termination, the rest of maximum four digits is the factor and which the position will be calculated for those termination belonging to specific column [5,6].

I1 and I2 are two numbers higher than the numbers belonging to factorial group.

Position obtained by multiplying the numbers is determined by formula:
\[ P = n_2 \times i_1(f) + n_1 \times i_2(F) \], followed by T
\[ \text{Or, } = n_1 \times i_2(f) + n_2 \times i_1(F) \], followed by T

Where:
\[ n_1, n_2 \]: represents multiples of 3000 corresponding of i1(f), respectively i2(f);
\[ i_1(f), i_2(f) \]: represents the corresponding numbers of i1 and i2 in factorial group;

\[ F \]: Factor
\[ T \]: Termination

Be: 32 999 × 32 693=1 078 836 307
\[ P=(1 078 836 307 − 7): 30=35 961 210 \]
\[ = 10 \times 2 693+10 \times 32 999+P, \text{ followed by T} \]

We calculate all the factors column 1, termination 10. The four types of multiplication corresponding col. 1 between numbers belonging to factor group, generates 400 factors with T, as follows:

\[ 7 \times 901=2 37 \times 1 711=21 67 \times 721=16 \]
\[ 307 \times 3 001=307 337 \times 811=91 367 \times 2 821=345 \]
\[ 607 \times 2 101=425 637 \times 2 911=618 667 \times 1 921=427 \]
\[ 2 707 \times 1 801=1 625 2 737 \times 2 611=2 382 2 767 \times 1 621=1 495 \]
\[ 97 \times 931=30 127 \times 2 341=99 157 \times 1 951=102 \]
\[ 397 \times 4 427 \times 1 441=205 457 \times 1 051=160 \]
\[ 697 \times 2 131=495 727 \times 541=131 757 \times 151=38 \]

\[ 2 797 \times 1 831=1 707 2 827 \times 2 411=227 2 857 \times 2 851=2 715 \]
\[ 187 \times 2 761=722 217 \times 1 771=128 247 \times 1 981=163 \]
\[ 487 \times 1 861=302 517 \times 871=150 547 \times 1 081=197 \]
\[ 787 \times 961=252 817 \times 2 971=809 847 \times 181=51 \]
\[ 2 887 \times 661=636 2 917 \times 2 671=2 597 2 947 \times 2 881=2 830 \]

\[ = 277 \times 391=36 \]
\[ 577 \times 2 491=476 \]
\[ 877 \times 1 591=465 \]
\[ 2 977 \times 1 291=1 281 \]

Or,
\[ 11 \times 1 937=7 41 \times 227=3 71 \times 2 117=50 \]
\[ 311 \times 2 837=294 341 \times 1 127=128 371 \times 17=2 \]
\[ 611 \times 737=150 641 \times 2 027=433 671 \times 917=205 \]
\[ 2 711 \times 1 037=937 2 741 \times 2 327=2 126 2 771 \times 1 217=1 124 \]
\[ 101 \times 1 607=54 131 \times 1 697=4 161 \times 2 387=128 \]
\[ 401 \times 2 507=335 431 \times 2 597=374 461 \times 287=44 \]
\[ 701 \times 407=95 731 \times 497=121 761 \times 1 187=3 011 \]
\[ 2 801 \times 707=660 2 831 \times 797=752 2 861 \times 1 487=1 418 \]
\[ 191 \times 677=43 221 \times 2 567=189 251 \times 2 057=1 172 \]
\[ 491 \times 1 577=258 521 \times 467=81 551 \times 2 957=543 \]
\[ 791 \times 2 477=653 821 \times 1 367=374 851 \times 857=243 \]
\[ 2 891 \times 2 777=2 676 2 921 \times 1 667=1 623 2 951 \times 1 157=1 138 \]
\[ 281 \times 2 147=201 \]
\[ 581 \times 47=9 \]
\[ 881 \times 947=278 \]
\[ 2 981 \times 1 247=1 239 \]

Or,
\[ 19 \times 1 753=11 49 \times 1 843=30 79 \times 1 333=35 \]
\[ 319 \times 2 653=282 349 \times 2 743=319 379 \times 2 233=282 \]
\[ 619 \times 553=114 649 \times 643=139 679 \times 133=30 \]
2 719 × 853 = 773 2 749 × 943 = 864 2 779 × 433 = 401
109 × 223 = 8 119 × 223 = 8 139 × 1 513 = 70 169 × 2 203 = 124
409 × 1 123 = 535 439 × 2 413 = 353 469 × 1 03 = 16
709 × 2 023 = 1 246 739 × 313 = 7 769 × 1 003 = 257

If not results indicate position of N decreased by the factor F=2, the
number studied does not divide with multiples of 3000 plus pair of
numbers 7-901
(3 000 × n+307) × (3 000 × n+3001) F=307
307 × n; 3 001 × n; 3 001+3 07xn; 3 001+2+6 307xn; 3 001+3+9
307xn;.................

307 × n correspond to: 307 × (3 000 × n+3 001); 3 001 × n
correspond to: 3 001 × (3 000 × n+307);
3 001+3 07xn correspond to: 3 007 × (3 000 × n+3 001);
3 001+2+6 307xn correspond to: 6 307 × (3 000 × n+3 001);
3 001+3+9 307xn correspond to: 9 307 × (3 000 × n+301);

Extract factor F=307 out of the position number of N than check
calculation above.
(3 000 × n+307) × (3 000 × n+2 101) F=425
2 707 × n; 1 801 × n; 1 801+5 707xn; 1 801x2+8 707xn; 1 801x3+9
707xn;

For this example (p=359 612) we check these calculations:
Divisibility by:
(3 000 × n+7) × (3 000 × n+901) F=2 P – F=359 602
7 × 51 372 = 359 604 not divisible by 7 × (3 000 × n+901)
901 × 399 = 359 499 not divisible by 901 × (3 000 × n+7)
901+3 007x119 = 358 734 -/- 3 007 × (3 000 × n+901)
901x2+6 007xn = 356 215 -/- 6 007 × (3 000 × n+901)
901x3+9 007xn = 353 976 -/- 9 007 × (3 000 × n+901)
901x4+12 007x29 = 351 807 -/- 12 007 × (3 000 × n+901)
901x5+15 007x23 = 349 666 -/- 15 007 × (3 000 × n+901)
901x6+18 007x20 = 347 546 -/- 18 007 × (3 000 × n+901)
901x7+21 007x16 = 345 419 -/- 21 007 × (3 000 × n+901)
901x8+24 007x14 = 343 306 -/- 24 007 × (3 000 × n+901)
901x9+27 007x13 = 341 200 -/- 27 007 × (3 000 × n+901)
901x10+30 007x11 = 339 087 -/- 30 007 × (3 000 × n+901)
901x20+60 007x5 = 318 055 -/- 60 007 × (3 000 × n+901)
901x30+90 007x3 = 297 054 -/- 90 007 × (3 000 × n+901)
901x40+120 007x2 = 276 054 -/- 120 007 × (3 000 × n+901)
901x50+150 007x2 = 255 064 -/- 150 007 × (3 000 × n+901)

Or,
29 × 2 183 = 21 59 × 1 073 = 21 89 × 2 363 = 70
329 × 83 = 9 359 × 1 973 = 236 389 × 263 = 34
629 × 983 = 206 659 × 2 873 = 631 689 × 1 163 = 267
2 729 × 1 283 = 1 167 2 759 × 173 = 159 2 789 × 1 463 = 1 360
119 × 53 = 2 149 × 143 = 7 179 × 2 633 = 157
419 × 953 = 133 449 × 1 043 = 156 479 × 533 = 85
719 × 1 853 = 444 749 × 1 943 = 485 779 × 1 433 = 372
2 819 × 2 153 = 2 023 2 849 × 2 243 = 2 130 2 879 × 1 733 = 1 663
209 × 1 523 = 106 239 × 2 813 = 224 269 × 503 = 45
509 × 2 423 = 411 539 × 713 = 128 569 × 1 403 = 266
809 × 323 = 87 839 × 1 613 = 451 869 × 2 303 = 667
2 909 × 623 = 604 2 939 × 1 913 = 1 874 2 969 × 2 603 = 2 576
299 × 593 = 59
599 × 1 493 = 298
899 × 2 393 = 717
2 999 × 2 693 = 692

Grouping numbers from left of multiplying operation according
to the above model, in this case numbers on the right have a constant
growth rate, which allows for relatively simple determination of
them. Perform tests to see if number N is prime or not, using position
formulae, as follows:

Divisibility by:
(3 000 × n+7) × (3 000 × n+901) F=2
7 × n; 901 × n; 901+3 007xn; 901x2+6 007xn; 901x3+9
007xn;.................

7xn correspond to: 7 × (3 000 × n+901); 901xn correspond to: 901
× (3 000 × n+7);
901+3 007xn correspond to: 3 007 × (3 000 × n+901);
For termination five digits

The calculation algorithm is:

Pas.1: Determine the position number and column it belongs;

Pas.2: Last five digits of the calculated number indicates the termination position of tested number;

Pas 3: Determine factors for termination and column number tested. I have illustrated the calculation of factors termination 001 10, column 1;

Pas.4: We divisibility test the formulas for calculating factorial.

For pair of numbers 31 – 397

31 × (3 000 000xn+1 161 397) p=12+31 × n; divisibility by 31

3 031 × (3 000 000xn+1 800 397) p=1 819+3 031 × n -//- 3 031

6 031 × (3 000 000xn+2 439 397) p=1 819+3 085+6 031 × n -//- 6 031

9 031 × (3 000 000xn+3 078 397) p=1 819+3 085 × 2+1 278+9 031 × n -//- 9 031

12 031 × (3 000 000xn+3 717 397) p=1 819+3 085 × 3+1 278 × (2)!+12 031 × n -//- 12 031

15 031 × (3 000 000xn+4 356 397) p=1 819+3 085 × 4+1 278 × (3)!+15 031 × n -//- 15 031

18 031 × (3 000 000xn+4 995 397) p=1 819+3 085 × 5+1 278 × (4)!+18 031 × n -//- 18 031

2 997 031 × (3 000 000xn+639 522 397) p=1 819+3 085 × 998+1 278 × (997)!+2 997 031 × n divisibility by 2 997 031

3 000 031 × (3 000 000xn+640 161 397) p=1 819+3 085 × 999+1 278 × (998)!+3 000 031 × n divisibility by 3 000 031

3 003 031 × (3 000 000xn+640 800 397) p=1 819+3 085 × 1 000+1 278 × (999)!+3 003 031 × n divisibility by 3 003 031

And,

397 × (3 000 000xn+2 403 031) p=318+397 × n divisibility by 397

3 397 × (3 000 000xn+234 031) p=265+3 397 × n -//- 3 397

6 397 × (3 000 000xn+1 065 031) p=265+3 006+6 397 × n -//- 6 397

9 397 × (3 000 000xn+1 896 031) p=265+2 006 × 2+1 662+9 397 × n -//- 9 397

12 397 × (3 000 000xn+2 727 031) p=265+2 006 × 3+1 662 × (2)!+12 397 × n -//- 12 397

15 397 × (3 000 000xn+3 558 031) p=265+2 006 × 4+1 662 × (3)!+15 397 × n -//- 15 397

18 397 × (3 000 000xn+4 389 031) p=265+2 006 × 5+1 662 × (4)!+18 397 × n -//- 18 397

2 997 397 × (3 000 000xn+829 572 031) p=265+2 006 × 998+1 662 × (997)!+2 997 397 × n divisibility by 2 997 397

3 000 397 × (3 000 000xn+830 403 031) p=265+2 006 × 999+1 662 × (998)!+3 000 397 × n divisibility by 3 000 397

3 003 397 × (3 000 000xn+831 234 031) p=265+2 006 × 1 000+1 662 × (999)!+3 003 397 × n divisibility by 3 003 397

Or, pair of numbers 331 – 1 297

331 × (3 000 000xn+2 755 297) p=304+331 × n divisibility by 331

3 331 × (3 000 000xn+994 297) p=1 104+3 331 × n -//- 3 331

6 331 × (3 000 000xn+2 233 297) p=1 104+3 609+6 331 × n -//- 6 331

9 331 × (3 000 000xn+3 472 297) p=1 104+3 609 × 2+2 478+9 331 × n -//- 9 331

12 331 × (3 000 000xn+4 711 297) p=1 104+3 609 × 3+2 478 × (2)!+12 331 × n -//- 12 331

15 331 × (3 000 000xn+5 950 297) p=1 104+3 609 × 4+2 478 × (3)!+15 331 × n -//- 15 331

18 331 × (3 000 000xn+7 189 297) p=1 104+3 609 × 5+2 478 × (4)!+18 331 × n -//- 18 331

And,

1 297 × (3 000 000xn+342 331) p=148+1 297 × n divisibility by 1 297

4 297 × (3 000 000xn+1 773 331) p=2 540+4 297 × n -//- 4 297

7 297 × (3 000 000xn+3 204 331) p=2 540+5 254+7 297 × n -//- 7 297

10 297 × (3 000 000xn+4 635 331) p=2 540+5 254 × 2+2 862+10 297 × n -//- 10 297

13 297 × (3 000 000xn+6 066 331) p=2 540+5 254 × 3+2 862 × (2)!+13 297 × n -//- 13 297

16 297 × (3 000 000xn+7 497 331) p=2 540+5 254 × 4+2 862 × (3)!+16 297 × n -//- 16 297

19 297 × (3 000 000xn+8 928 331) p=2 540+5 254 × 5+2 862 × (4)!+19 297 × n -//- 19 297

Conclusion

Number testing is done with all the 400 pairs of numbers in the
group factorial. Factorial multiplication process has as principle of
calculation pairs of numbers that belong to the factorial group unique
to each termination and column.

References
1. Canfield ER, Erdos P, Pomerance C (1983) On a problem of Oppenheim concerning Factorisatio Numerorum. J Number Theory 17: 1-28.
2. Davis JA, Holdridge DB (1983) Factorization sings the quadratic sieve algorithm. Advances in Cryptology 2: 103-113.
3. Lehmer DH, Powers RE (1931) on factoring large numbers. Bull her Math Soc 37: 770-776.
4. Miller JCP (1975) on factorisation with a suggested new approach. Math Comp 29: 155-772.
5. Pomerance C, Wagstaff SS (1983) Implementation of the continued fraction algorithm. Cow Numerantium 37: 99-118.
6. Morrison MA, Brillhart J (1975) A method of factoring and the factorization of F, Math Comp 29: 183-205.

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