The total dominator coloring of dense, octahedral and queen’s graphs

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Abstract. Let $G$ be a graph, then the dominator coloring of $G$ is a proper coloring, in which every vertex of $G$ dominates every vertex of at least one color class. The Total dominator coloring of graph is a proper coloring with extra property that every vertex in the graph properly dominates an entire color class. The total dominator chromatic number $\chi_{td}(G)$ of $G$ is the minimum number of color classes in a total dominator coloring of it. In this paper, total dominator coloring of dense, octahedral and queen’s graphs have been discussed.

1. Introduction

Graph coloring is one of the major, well-known and well-studied problems in graph theory. [13] A graph $G$ is a triple consisting of a vertex set $V(G)$, an edge set $E(G)$ and a relation that associates with each edge two vertices called its endpoints. The chromatic number of $G$, denoted by $\chi(G)$ is the smallest number of colors required for coloring the graph $G$. The proper coloring of a graph $G$ is coloring of graph in which no two adjacent vertices must have same color. A color class is a set of vertices which have the same colors. The minimum number of colors needed for dominator coloring is known as dominator chromatic number $\chi_{d}(G)$.

All graphs in this paper are simple, nontrivial and undirected, see [5,6] for more detailed definition of graph. Graph coloring were studied by I N Maylisa, et al.[9], Anggraeni, et al.[1], total dominator coloring were studied by Bagan, et al. [2], Henning [8], Dr.A.Vijayalekshmi [14], and dominating number studied by Susilowati, et al.[12], Wangguway, et al[15], and Kurniawati S, at al[11].

To know the result of the research, we need to know a definition below.

Definition 1. [10] A total dominator coloring of a graph $G$, briefly TDC, is a proper coloring of $G$ in which each vertex of the graph is adjacent to every vertex of some color class. The total dominator chromatic number $\chi_{td}(G)$ of $G$ is the minimum number of color classes in a TDC in $G$.

Definition 2. [4] For any graph $G$, $\chi(G) \leq \chi_{d}(G) \leq \chi_{td}(G)$.

Definition 3. [7] Dense graph, noted by $[D]$ is a graph in which the number of edges is close to the maximal number of edges. Dense graph is partition of complete graph. The total dominator chromatic number of dense graph $\chi_{td}(D)$ is the minimum number of color classes in TDC in $D$. 
Definition 4. [16] Queen’s graph $Q_n$ has the squares of the $n \times n$ chessboard as its vertices, two square are adjacent if they are in the same row, column, or diagonal. The total dominator chromatic number of queen’s graph $\chi^t_d(Q)$ is the minimum number of color classes in TDC in $Q$.

Definition 5. [3] Graph $G$ is octahedral if and only if any two adjacent vertices of $G$ have two non-adjacent common neighbors. The total dominator chromatic number of octahedral graphs $\chi^t_d(P_{on})$ is the minimum number of color classes in TDC in $P_{on}$.

2. Results

In this paper we study total dominator colorings of dense, octahedral and queen’s graph. We start this section with cardinality of dense graph.

Let $V(D) = \{v_i; 1 \leq i \leq n\}$ and edge set $E(D) = \{v_iv_{i+1}; v_{i+1}v_{i+2}; v_{i+2}v_{i+4}; v_{i+3}v_{i+5}; 1 \leq i \leq n-1\}$.

$$f(x_i) = \begin{cases} 1; i \equiv 1(mod4) \\ 2; i \equiv 2(mod4) \\ 3; i \equiv 3(mod4) \end{cases}$$

Theorem 1. For any integers $n$, if $D$ is a dense graph, then

$\chi^t_d(D) = \begin{cases} \frac{n+1}{2}; n odd \\ \frac{n}{2}; n even \end{cases}$

Proof. Let $f(x_i)$ be coloring function of dense graph with $i; 1 \leq i \leq \frac{n}{2}$ and $i - \frac{n}{2}; \frac{n}{2} + 1 \leq i \leq n$. In order to proof total dominator coloring of this graph, we divide to two cases since they have different condition.

Case 1. Let be $n \geq 5$ and $n$ is odd. the dense graph consist of vertex set $V = \{v_i; 1 \leq i \leq n\}$ and edge set $E = \{v_i v_{i+1} \cup v_i v_{i+2} \cup v_{i+2} v_{i+4} \cup v_{i+4} v_{i+5}; 1 \leq i \leq n-1\}$. Then the color is assigned to the vertices by coloring function. Note that every vertices dominate some color class. From definition 1 we can conclude that’s TDC because every vertices dominate vertex in some color class. Let be $n < 5$, and $n$ is odd. from definition 2, it’s not the dense graph. So the $X^t_d(D) = \frac{n+1}{2}$, $n odd$.

For example, let be $n = 5$. The dense graph consist of vertex set $V = \{v_i; 1 \leq i \leq n\}$ and edge set $E = \{v_i v_{i+1} \cup v_i v_{i+2} \cup v_{i+2} v_{i+4} \cup v_{i+4} v_{i+5}; 1 \leq i \leq n-1\}$. Then the color is assigned to the vertices. The vertices $v_1, v_4$ assigned to color 1, vertices $v_2, v_5$ assigned to color 2, and vertices $v_3$ assigned to color 3.

Note that $v_1$ dominate vertex in color class 2 and color class 3, $v_2$ dominate vertex in color class 1 and color class 3, $v_3$ dominate vertex in color class 1 and color class 2, $v_4$ dominate vertex in color class 2 and color class 3, and $v_5$ dominate vertex in color class 1 and color class 3. From definition 1 we can conclude that’s TDC because every vertices dominate vertex in some color class. So the $X^t_d(D) = \frac{5+1}{2} = \frac{6}{2} = 3$.

Let be $n = 7$. The dense graph consist of vertex set $V = \{v_i; 1 \leq i \leq n\}$ and edge set $E = \{v_i v_{i+1} \cup v_i v_{i+2} \cup v_i v_{i+3} \cup v_{i+3} v_{i+5} \cup v_{i+5} v_{i+6}; 1 \leq i \leq n-1\}$. Then the color is assigned to the vertices. The vertices $v_1, v_5$ assigned to color 1, vertices $v_2, v_6$ assigned to color 2, vertices $v_3, v_7$ assigned to color 3, and vertices $v_4$ assigned to color 4.
Note that $v_1$ dominate vertex in color class 2, color class 3 and color class 4, $v_2$ dominate vertex in color class 1, color class 3 and color class 4, $v_3$ dominate vertex in color class 1, color class 2, color class 4, $v_4$ dominate vertex in color class 1, color class 2 and color class 3, $v_5$ dominate vertex in color class 2, color class 3 and color class 4, $v_6$ dominate vertex in color class 1, color class 3 and color class 4, and $v_7$ dominate vertex in color class 1, color class 2, and color class 4. From definition we can conclude that’s TDC because every vertices dominate vertex in some color class. So the $X_{d'}(D) = \frac{n+1}{2} = \frac{2+1}{2} = \frac{3}{2} = 4$. Therefore its proven that for any integers $n \geq 5$ and $n$ is odd, if $D$ is dense graph, then $X_{d'}(D) = \frac{n+1}{2}$.

Case 2. Let be $n \geq 6$ and $n$ is even. The dense graph consist of vertex set $V = \{v_i; 1 \leq i \leq n\}$ and edge set $E = \{v_{i}v_{i+1} \cup v_{i}v_{i+2} \cup v_{i}v_{i+4} \cup v_{i}v_{i+5} \cup v_{i}v_{i+6}; 1 \leq i \leq n - 1\}$. Then the color is assigned to the vertices by coloring function. Note that every vertices dominate some color class. From definition 1 we can conclude that’s TDC because every vertices dominate vertex in some color class.

For example, let be $n < 6$, and $n$ is even. The dense graph consist of vertex set $V = \{v_i; 1 \leq i \leq n\}$ and edge set $E = \{v_{i}v_{i+1} \cup v_{i}v_{i+2} \cup v_{i}v_{i+3}; 1 \leq i \leq n - 1\}$. Then the color is assigned to the vertices. the vertiches $v_1, v_3$ assigned to color 1, and vertices $v_2, v_4$ assigned to color 2.

Note that $v_1$ dominate vertex in color class 2, $v_2$ dominate vertex in color class 1, $v_3$ dominate vertex in color class 2, $v_4$ dominate vertex in color class 1. The vertices can’t dominate vertex on some color class. from definition 1 it’s can’t be called TDC.
For example, let be $n = 6$. The dense graph consist of vertex set $V = \{v_i; 1 \leq i \leq n\}$ and edge set $E = \{v_i v_{i+1} \cup v_i v_{i+2} \cup v_i v_{i+3} \cup v_i v_{i+5}; 1 \leq i \leq n - 1\}$. Then the color is assigned to the vertices. The vertices $v_1, v_4$ assigned to color 1, vertices $v_2, v_5$ assigned to color 2, and vertices $v_3, v_6$ assigned to color 3.

Note that $v_1$ dominate vertex in color class 2 and color class 3, $v_2$ dominate vertex in color class 1 and color class 3, $v_3$ dominate vertex in color class 1 and color class 2, $v_4$ dominate vertex in color class 2 and color class 3, $v_5$ dominate vertex in color class 1 and color class 3, $v_6$ dominate vertex in color class 1 and color class 2. From definition 1 we can conclude that’s TDC because every vertices dominate vertex in some color class. So the $X_1^t(D) = \frac{n}{2} = \frac{6}{2} = 3$.

Let be $n = 8$. The dense graph consist of vertex set $V = \{v_i; 1 \leq i \leq n\}$ and edge set $E = \{v_i v_{i+1} \cup v_i v_{i+2} \cup v_i v_{i+3} \cup v_i v_{i+5} \cup v_i v_{i+6} \cup v_i v_{i+7}; 1 \leq i \leq n - 1\}$. Then the color is assigned to the vertices. The vertices $v_1, v_5$ assigned to color 1, vertices $v_2, v_6$ assigned to color 2, vertices $v_3, v_7$ assigned to color 3, and vertices $v_4, v_8$ assigned to color 4.

Note that $v_1$ dominate vertex in color class 2, color class 3 and color class 4, $v_2$ dominate vertex in color class 1, color class 3 and color class 4, $v_3$ dominate vertex in color class 1, color class 2, and color class 4 $v_4$ dominate vertex in color class 1, color class 2 and color class 3, $v_5$ dominate vertex in color class 2, color class 3 and color class 4, $v_6$ dominate vertex in color class 1, color class 3 and color class 4, $v_7$ dominate vertex in color class 1, color class 2, and color class 4, and $v_8$ dominate vertex in color class 1, color class 2 and color class 3. From definition 1 we can conclude that’s TDC because every vertices dominate vertex in some color class. So
the $X_t^d(D) = \frac{n}{2} = \frac{8}{2} = 4$. Therefore its proven that for any integers $n \geq 6$ and $n$ is even, if $D$ is dense graph, then $X_t^d(D) = \frac{n}{2}$.

**Theorem 2.** For any integers $n=2,3$, if $Q$ is a queen’s graph, then $X_t^d(Q_{2,n}) = 4$.

**Proof.** In order to proof total dominator coloring of this graph, we devide to two cases since they have different condition. **Case 1.** Let be $n=2$. The queen’s graph consist of vertex set $V = \{v_i; 1 \leq i \leq n\}$ and edge set $E = \{v_i v_{i+1} \cup v_i v_{i+2} \cup v_i v_{i+3}; 1 \leq i \leq n-1\}$. Then the color is assigned to the vertices. The vertices $v_1$ assigned to color 1, vertices $v_2$ assigned to color 2, vertices $v_3$ assigned to color 3, and vertices $v_4$ assigned to color 4.

![Figure 5. Queen’s Graph with (2,2) vertex](image)

Note that $v_1$ dominate vertex in color class 2, color class 3, and color class 4, $v_2$ dominate vertex in color class 1, color class 3, and color class 4, $v_3$ dominate vertex in color class 1, color class 2, and color class 4, and $v_4$ dominate vertex in color class 1, color class 2, and color class 3. From definition 1 we can conclude that’s TDC because every vertices dominate vertex in some color class. So the $X_t^d(Q_{2,2}) = 4$.

**Case 2.** Let be $n=3$. The queen’s graph consist of vertex set $V = \{v_i; 1 \leq i \leq n\}$ and edge set $E = \{v_i v_{i+1} \cup v_i v_{i+2} \cup v_i v_{i+3} \cup v_i v_{i+4} \cup v_i v_{i+5} \cup v_i v_{i+6}; 1 \leq i \leq n-1\}$. Then the color is assigned to the vertices. The vertices $v_1,v_4$ assigned to color 1, vertices $v_2,v_5$ assigned to color 2, vertices $v_3$ assigned to color 3, and vertices $v_6$ assigned to color 4.

![Figure 6. Queen’s Graph with (2,3) vertex](image)

Note that $v_1$ dominate vertex in color class 2, color class 3, and color class 4, $v_2$ dominate vertex in color class 1, color class 3, and color class 4, $v_3$ dominate vertex in color class 1, color class 1, color class 1, color class 1, color class 1, color class 1,
class 2, and color class 4, and $v_4$ dominate vertex in color class 2, color class 3, and color class 4, $v_5$ dominate vertex in color class 1, color class 3, and color class 4, and $v_6$ dominate vertex in color class 1, color class 2, and color class 3. From definition 1 we can conclude that’s TDC because every vertices dominate vertex in some color class. So the $X_d^1(Q_{2,3}) = 4$. Therefore its proven that for any integers $n=2,3$, if $Q$ is queen’s graph, then $X_d^1(Q_{2,n}) = 4$.

**Theorem 3.** For any integers $n \geq 4$, if $Q$ is queen’s graph, then $X_d^1(Q_{2,n}) = n$.

**Proof.** In order to proof total dominator coloring of this graph, we devide to two cases since they have different condition.

![Figure 7](image_url)

**Figure 7.** Queen’s Graph with (2,4) vertex

**Case 1.** Let be $n = 4$. The queen’s graph consist of vertex set $V = \{v_i; 1 \leq i \leq n\}$ and edge set $E = \{v_iv_{i+1} \cup v_iv_{i+2} \cup v_iv_{i+3} \cup v_iv_{i+4} \cup v_iv_{i+5} \cup v_iv_{i+6} \cup v_iv_{i+7}; 1 \leq i \leq n - 1\}$. Then the color is assigned to the vertices. the vertices $v_1,v_6$ assigned to color 1, vertices $v_2,v_5$ assigned to color 2, vertices $v_3,v_8$ assigned to color 3, and vertices $v_4,v_7$ assigned to color 4.

![Figure 8](image_url)

**Figure 8.** Queen’s Graph with (2,5) vertex

Note that $v_1$ dominate vertex in color class 2, color class 3, and color class 4, $v_2$ dominate vertex in color class 3 and color class 4, $v_3$ dominate vertex in color class 1 and color class 2, $v_4$ dominate vertex in color class 1, color class 2, and color class 3, $v_5$ dominate vertex in color class 1, color class 3, and color class 4, $v_6$ dominate vertex in color class 3 and color class 4, $v_7$ dominate vertex in color class 1 and color class 2, and $v_8$ dominate vertex in color class 1, color class 2, and color class 4. From definition 1 we can conclude that’s TDC because every vertices dominate vertex in some color class. So the $X_d^1(Q_{2,4}) = 4$. 
Case 2. Let be \( n = 5 \). The queen’s graph consist of vertex set \( V = \{v_i; 1 \leq i \leq n\} \) and edge set \( E = \{v_iv_{i+1} \cup v_iv_{i+2} \cup v_iv_{i+3} \cup v_iv_{i+4} \cup v_iv_{i+5} \cup v_iv_{i+6} \cup v_iv_{i+7} \cup v_iv_{i+8} \cup v_iv_{i+9}; 1 \leq i \leq n - 1\} \). Then the color is assigned to the vertices. the vertices \( v_1, v_6 \) assigned to color 1, vertices \( v_2, v_5 \) assigned to color 2, vertices \( v_3, v_9 \) assigned to color 3, vertices \( v_4, v_8 \) assigned to color 4, and vertices \( v_7, v_{10} \) assigned to color 5.

Note that \( v_1 \) dominate vertex in color class 2, color class 3, and color class 4, \( v_2 \) dominate vertex in color class 3 and color class 4, \( v_3 \) dominate vertex in color class 1 and color class 2, \( v_4 \) dominate vertex in color class 1, color class 2, and color class 5, \( v_5 \) dominate vertex in color class 1, color class 2, and color class 3 and color class 4, \( v_6 \) dominate vertex in color class 1, color class 2, and color class 3, and color class 5, \( v_7 \) dominate vertex in color class 1, color class 2, and color class 4, \( v_8 \) dominate vertex in color class 1 and color class 3, and color class 4, \( v_9 \) dominate vertex in color class 1, color class 2, and color class 5, and \( v_{10} \) dominate vertex in color class 1, color class 2, and color class 4. From definition 1 we can conclude that’s TDC because every vertices dominate vertex in some color class. So the \( X_{TDC}^{f}(Q_{2,5}) = 4 \). Therefore its proven that for any integers \( n \geq 4 \), if \( Q \) is queen’s graph, then \( X_{TDC}^{f}(P_{an}) = n \).

Theorem 4. For any integers \( n = 3 \), if \( P_o \) is octahedral graph, then \( X_{TDC}^{f}(P_{an}) = 3 \).

Proof. Let be \( n = 3 \). The octahedral graph consist of vertex set \( V = \{v_i; 1 \leq i \leq n\} \) and edge set \( E = \{v_iv_{i+1} \cup v_iv_{i+2} \cup v_iv_{i+3} \cup v_iv_{i+4} \cup v_iv_{i+5} \cup v_iv_{i+6} \cup v_iv_{i+7} \cup v_iv_{i+8} \cup v_iv_{i+9}; 1 \leq i \leq n - 1\} \). Then the color is assigned to the vertices. the vertices \( v_1, v_6 \) assigned to color 1, vertices \( v_2, v_5 \) assigned to color 2, and vertices \( v_3, v_4 \) assigned to color 3.

Note that \( v_1 \) dominate vertex in color class 2 and color class 3, \( v_2 \) dominate vertex in color class 1 and color class 3, \( v_3 \) dominate vertex in color class 1 and color class 2, \( v_4 \) dominate vertex in color class 1, color class 2, and color class 3, \( v_5 \) dominate vertex in color class 1 and color class 2, \( v_6 \) dominate vertex in color class 1 and color class 2, and color class 3. From definition 1 we can conclude that’s TDC because every vertices dominate vertex in some color class. So the \( X_{TDC}^{f}(P_{o3}) = 3 \). Therefore its proven that for any integers \( n = 3 \), if \( P_o \) is octahedral graph, then \( X_{TDC}^{f}(P_{an}) = 3 \).

Figure 9. Octahedral Graph

Theorem 5. For any integers \( n \geq 2 \), if \( P_o \) is octahedral graph, then \( X_{TDC}^{f}(shack(P_{o3^v}, v, k)) = 3 \).

Proof. In order to proof total dominator coloring of this graph, we devide to two cases since they have different condition.

Case 1. Let be \( n = 2 \). The octahedral graph consist of vertex set \( V = \{v_i; 1 \leq i \leq n\} \) and edge set \( E = \{v_iv_{i+1} \cup v_iv_{i+2} \cup v_iv_{i+3} \cup v_iv_{i+4} \cup v_iv_{i+5}; 1 \leq i \leq n - 1\} \). Then the color is assigned to the vertices. the vertices \( v_1, v_6, v_9 \) assigned to color 1, vertices \( v_2, v_5, v_8, v_{10} \) assigned to color 2, vertices \( v_3, v_4, v_7, v_{11} \) assigned to color 3.
Note that $v_1$ dominate vertex in color class 2 and color class 3, $v_2$ dominate vertex in color class 1 and color class 3, $v_3$ dominate vertex in color class 1 and color class 2, $v_4$ dominate vertex in color class 1 and color class 2, $v_5$ dominate vertex in color class 1 and color class 3, $v_6$ dominate vertex in color class 2, $v_7$ dominate vertex in color class 2 and color class 3, $v_8$ dominate vertex in color class 1 and color class 3, $v_9$ dominated vertex in color class 2 and color class 3, $v_{10}$ dominate vertex in color class 1 and color class 3, and $v_{11}$ dominate vertex in color class 1 and color class 2. From definition 1 we can conclude that’s TDC because every vertices dominate vertex in some color class. So the $X_t(shack(P_{o^2}, v, 2)) = 3$.

**Case 2.** Let be $n = 3$. The octahedral graph consist of vertex set $V = \{v_i; 1 \leq i \leq n\}$ and edge set $E = \{v_iv_{i+1}\cup v_i_v_{i+2}\cup v_i\cup v_{i+3}\cup v_i\cup v_{i+4}\cup v_i\cup v_{i+5}; 1 \leq i \leq n - 1\}$. Then the color is assigned to the vertices, the vertices $v_1, v_6, v_9, v_{12}, v_{16}$ assigned to color 1, vertices $v_2, v_5, v_8, v_{10}, v_{13}, v_{15}$ assigned to color 2, vertices $v_3, v_4, v_7, v_{11}, v_{14}$ assigned to color 3.

Note that $v_1$ dominate vertex in color class 2 and color class 3, $v_2$ dominate vertex in color class 1 and color class 3, $v_3$ dominate vertex in color class 1 and color class 2, $v_4$ dominate vertex in color class 1 and color class 2, $v_5$ dominate vertex in color class 1 and color class 3, $v_6$ dominate vertex in color class 2, $v_7$ dominate vertex in color class 2 and color class 3, $v_8$ dominate vertex in color class 1 and color class 3, $v_9$ dominate vertex in color class 2 and color class 3, $v_{10}$ dominate vertex in color class 1 and color class 3, $v_{11}$ dominate vertex in color class 1 and color class 2, $v_{12}$ dominate vertex in color class 2 and color class 3, $v_{13}$ dominate vertex in color class 1 and color class 3, $v_{14}$ dominate vertex in color class 1 and color class 2, $v_{15}$ dominate vertex in color class 1 and color class 3, and $v_{16}$ dominate vertex in color class 2 and
color class 3. From definition 1 we can conclude that’s TDC because every vertices dominate vertex in some color class. So the $X^d_t(shack(P_{o3^n}, v, k)) = 3$. Therefore its proven that for any integers $n \geq 2$, if $P_{o}$ is octahedral graph, then $X^d_t(shack(P_{o3^n}, v, k)) = 3$.

3. Conclusion
In this paper total dominator coloring of dense, octahedral, and queen’s graph have been discussed.

Open Problem 1
Determine the total dominator coloring in other special graph and other operations.

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