Relativistic Photon Mediated Shocks

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A system of equations governing the structure of a steady, relativistic radiation dominated shock is derived, starting from the general form of the transfer equation obeyed by the photon distribution function. Closure is obtained by truncating the system of moment equations at some order. The anisotropy of the photon distribution function inside the shock is shown to increase with increasing shock velocity, approaching nearly perfect beaming at upstream Lorentz factors $\Gamma_+ \gg 1$. Solutions of the shock equations are presented for some range of upstream conditions. These solutions are shown to converge as the truncation order is increased.

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Relativistic shocks play an important role in essentially all classes of high-energy compact astrophysical systems. Behind these shocks the bulk energy of a collimated outflow expelled from the central engine is dissipated and converted to the radio-through-$\gamma$-ray emission observed. The micro-physics of the shock depends on the conditions in the upstream and downstream regions. Shocks that form in a region of small optical depth for Thomson scattering are mediated by collective plasma processes, and are termed collisionless shocks [1]. Various observations suggest that these shocks may provide sites for acceleration of nonthermal particles (both electrons and protons) and, at least in some systems (e.g., GRB afterglow), may somehow generate macroscopic, sub-equilibrium magnetic fields in the downstream region [2]. The latter processes may in fact be an inherent part of the shock structure [3]. The micro-physics of collisionless shocks is highly involved. Recent efforts have led to important progress in the study of collisionless shocks, e.g., Refs. [3, 4, 5, 6, 7], but despite these efforts many key issues remain as yet unresolved.

When the ratio of radiation to matter pressure in the post shock region exceeds a certain level the shock transition may become mediated by photons. Such conditions are anticipated in, e.g., internal [8, 9] and oblique [10] shocks producing the GRB prompt emission, during shock breakout in supernovae and hypernovae [11, 12], in blazars, and in accretion flows onto a black hole [13, 14, 15]. The effect of radiation on the structure of a non-relativistic, strong shock has been studied by different authors [16, 17, 18, 19, 20, 21, 22] and applied to various astrophysical systems, such as supernovae [17] and accretion flows onto neutron stars [18]. The structure and transmitted photon spectrum of a cold, radiation mediated shock was computed in Ref. [20], where it was shown that bulk Comptonization on the converging flow can give rise to a power law spectrum at high energies with a spectral index that tends to unity for large Mach numbers. The model has been generalized later to incorporate thermal effects [21, 22]. The analysis in Ref. [20] is restricted to sufficiently optically thick shocks with small upstream velocity, viz., $\beta_- \ll 1$. In this regime the scattered photon distribution is nearly isotropic at every point inside the shock and the diffusion approximation can be employed to solve the transport equation describing the evolution of the photon population across the shock [19]. Moreover, to order $\beta_-^2$ the radiation field satisfies the equation of state $P_{\text{rad}} = U_{\text{rad}}/3$, which provides a closure condition for the set of hydrodynamic equations governing the shock structure. This simplifies the analysis considerably, but renders its applicability to most high-energy compact sources of little relevance.

Several complications arise in the relativistic regime. Firstly, the optical depth across the shock, $\tau \sim c/\beta_-$, approaches unity as the shock becomes relativistic, hence photons do not experience multiple scattering. This is a consequence of the fact that the average energy a photon gains in a single scattering is large, viz., $\Delta \epsilon/\epsilon > 1$. As a result, the photon distribution function across the shock is anticipated to be highly anisotropic, rendering the diffusion approximation inapplicable. Obtaining a closure of the hydrodynamic shock equations then becomes an involved problem. Secondly, the optical depth of a fluid slab having a Lorentz factor $\Gamma > 1$ depends on the angle $\theta$ between the photon direction and the shock velocity as $d\tau = \Gamma(1-\beta \cos \theta) dx$, and needs to be properly accounted for. Thirdly, pair creation may become important if the photon energy exceeds the pair creation threshold. Sufficiently relativistic shocks may therefore become dominated by electron-positron pairs. From the physical point of view, then, a relativistic radiation-dominated shock is a complex physical phenomenon, posing an intriguing unsolved problem. In what follows we derive a coupled set of hydrodynamic equations that provide a complete description of the shock structure, starting from the general form of the transfer equation obeyed by the photon distribution function. We then propose a method to close the resultant system of equations and present solutions for some range of upstream conditions. Convergence of these solutions is verified.

The fluid in the shock transition layer is a mixture of baryons, $e^\pm$, pairs, and radiation. We denote by $u^\alpha = (\Gamma, \Gamma \beta)$ the 4-velocity of the mixed fluid, by $n_b$ and $n_\pm$ the number density of baryons and pairs, respec-
tively (the total number of electrons in this notation is \( n_e = n_{\text{b}} + n_{\text{p}} \)), and by \( T_{\alpha}^{\mu \alpha} \), \( T_{b}^{\mu \alpha} \), and \( T_{r}^{\mu \alpha} \) the stress-energy tensors of baryons pairs and radiation, respectively. The entire system must conserve energy, momentum and baryon number:

\[
\frac{\partial}{\partial x^\alpha} \left( n_b u^\alpha \right) = 0, \tag{1}
\]

\[
\frac{\partial}{\partial x^\alpha} \left( T_b^{\mu \alpha} + T_r^{\mu \alpha} \right) = 0. \tag{2}
\]

The energy-momentum tensor of the radiation can be expressed explicitly in terms of the photon distribution function, \( f_r(k, x^\mu) \), as

\[
T_r^{\mu \nu}(x^\mu) = \int dk^\mu k^\nu f_r(k, x^\mu) \frac{d^3k}{k^0}, \tag{3}
\]

where \( k^\mu \) denotes the 4-momentum of a photon. The distribution function satisfies a transfer equation, which we express in the form

\[
k^\mu \frac{\partial f_r}{\partial x^\mu} = n_b^i \int R(k, k_i) f_r(k_i) \frac{d^3k_i}{k_i^0} \tag{4}
\]

\[
- n_{i}^i \int R(k, k_i) f_r(k_i) \frac{d^3k_i}{k_i^0} + C_{pp}(f_r, f_{\pm}, k) + S_k.
\]

Here \( n^i = n_{b}^i + n_{p}^i + n_{\pm}^i \) is the net number density of scatterers (electrons plus positrons), as measured in the fluid rest frame (henceforth, primed quantities refer to the fluid rest frame), \( R(k, k_i) \) is the redistribution function for Compton scattering from initial state \( k_i \) to a final state \( k \), the operator \( C_{pp}(f_r, f_{\pm}, k) \) accounts for the change in \( f_r \) due to e\(^\pm\) pair creation and annihilation, and \( S_k \) is a source term associated with all other processes that create or destroy photons (e.g., free-free emission and absorption). Conservation of energy, momentum and number of quanta of the interacting pair-photon system implies

\[
\frac{\partial}{\partial x^\alpha} T_{\pm}^{\mu \alpha} = - \int k^\mu C_{pp}(f_r, f_{\pm}, k) \frac{d^3k}{k^0}, \tag{5}
\]

\[
\frac{\partial}{\partial x^\alpha} \left( n_{\pm}^i \nu^\alpha \right) = - \int C_{pp}(f_r, f_{\pm}, k) \frac{d^3k}{k^0}. \tag{6}
\]

The above set of equations augmented by appropriate boundary conditions upstream provides a complete description of the shock transition layer. To simplify the problem we ignore KN effects in eq. (4). We then have

\[
R(k', k_i') = \frac{3\sigma_T}{16\pi} \left[ 1 + (\mathbf{k}' \cdot \mathbf{k}_i')^2 \right]^2 \delta(k_0' - k_i^0) \tag{7}
\]

where \( \sigma_T \) is the cross section for Thomson scattering. Multiplying Eq. (4) by \( k^\mu \), integrating over final states \( k \), using Eq. (7), and relating \( k' \) and \( k \) through the Lorentz transformation \( k'' = \Lambda^\gamma_{\alpha} k^{\alpha} \), we obtain

\[
\frac{\partial}{\partial x^\mu} T_{b}^{\mu \nu} = - n_i^i \sigma_T \left( \Lambda_{0}^{\nu} T_{r}^{00} - \Lambda_{3}^{\nu} T_{r}^{30} \right) = n_i^i \sigma_T \Lambda_{k}^{\nu} T_{r}^{k0} \tag{8}
\]

We consider a plane-parallel shock in the \((y, z)\) plane (the flow is moving in \(x\)-direction), and solve the above equations in the shock frame, where the system is assumed to be in a steady state. For the range of upstream conditions explored here the downstream temperature is well below \( m_e c^2 \), and so the pairs are sub-relativistic. To a good approximation we then have \( T_r^{\mu \nu} = n_{\pm} m_e c^2 u^\mu u^\nu \). Since the shock is radiation dominated we can neglect the pressure contributed by the particles to get \( T_{b}^{\mu \nu} = m_e c^2 n_b^i u^\mu u^\nu \). Using the latter result and the transformation \( T_{r}^{00} = \Lambda_{0}^{0} (-\beta) \Lambda_{3}^{0} (-\beta) T_{r}^{\mu \nu} \), the zeroth component of Eq. (8) yields an equation of motion for the fluid Lorentz factor:

\[
J \frac{d^2 \gamma}{d\tau} = \beta T_{r}^{00} = \frac{\beta^2}{1 + \beta^2} \left[ (1 + \beta^2) T_{r}^{00} - \beta (T_{r}^{00} + T_{r}^{0x}) \right], \tag{9}
\]

where \( d\tau = \sigma_T n_b^i \Gamma dx \) is the angle-averaged optical depth for Thomson scattering of a fluid slab of thickness \( dx \), and \( J = m_e c^2 n_b^i u^x \) is the integral of Eq. (11). To complete the set of equations a closure relation is needed. Far upstream \((x = -\infty)\) and downstream \((x = +\infty)\) the radiation field is in equilibrium, and the equation of state \( 3T_{r}^{xx} = T_{r}^{00} \) applies. However, within the shock transition layer this relation no longer holds. It is convenient to define a dimensionless function \( \xi \) that measures the deviation from isotropy at any given point:

\[
T_{r}^{xx} = \frac{1}{3} (1 + \xi) T_{r}^{00}. \tag{10}
\]

In general \( \xi \) is a function of the shock velocity and its derivatives, that is, \( \xi = \xi(u^\nu, u^\nu_{\nu}) \). Owing to relativistic boosting the radiation field is expected to be beamed preferentially in the direction of fluid motion. Consequently we anticipate values between \( \xi = 0 \) (complete isotropy) and \( \xi = 2 \) (perfect beaming in the \(x\) direction).
at any given point in the shock transition layer. Let us denote by $C_1$ and $C_2$ the integrals of the zeroth and $x$ components of Eq. (2), respectively. These integration constants are determined by the conditions upstream the shock, at $x = -\infty$. From Eqs. (1), (2) and (10) we obtain the net photon flux in the local rest frame of the fluid:

$$T_{r}^{0x} = -\frac{1 + \xi}{1 - \beta} \left[ \frac{J}{\Gamma} + m_e c^2 n_{\pm} \beta \right] 
- \frac{C_1 \left( 1 + \xi + \beta^2 \right)}{1 + \xi} + \frac{C_2 \left( 1 + \xi + 3 \beta \right)}{1 + \xi}. \tag{11}$$

It can be readily shown that for $\xi = 0$ the flux $T_{r}^{0x}$ has a root at $\beta = \beta_+$, and $\beta = \beta_-$, where $\beta_+ (\beta_-)$ is the velocity of the downstream fluid, as determined from the shock jump conditions. It is also seen that the comoving photon flux has a singular point at $\beta^2 = (1 + \xi)/\beta$. Since $T_{r}^{0x}$ must be finite everywhere inside the shock it implies that the numerator on the R.H.S of Eq. (11) must vanish at the critical point. For upstream velocity $\beta_- > 1/\sqrt{3}$ we can then readily constrain $\xi$. For illustration, we present in Fig. 1 plots of $\xi_\rho$, the value of $\xi$ at the singularity, for cases where the energy density of pairs can be neglected in Eq. (11). The presence of pairs is not expected to change the results significantly. The maximum value of $\xi$ inside the shock must lie between $\xi_\rho$, and $2$. As seen from Fig. 1, $\xi$ approaches the maximal value $\xi = 2$ as the shock becomes relativistic, implying a highly beamed radiation field inside the shock.

In the limit $\beta_- \ll 1$ we expand $\Gamma$ in powers of $\beta$ and $T_{r}^{0x}$ in powers of $\beta$ and $\xi$. By employing Eqs. (11) and (10) and noting that the average photon energy is well below the pair production threshold, in which case we can set $n_{\pm} = 0$, we recover, to order $O(\beta^2)$, the result derived in Ref. [20]:

$$\frac{d}{d\tau} \mu = \frac{2}{\mu^2} - 4(1 + \pi_-) \mu + \frac{1}{2} + 4\pi_-, \tag{12}$$

where $\mu = \beta/\beta_+$ is the normalized fluid velocity and $\pi_- = (T^{xx}/J)_\rho$ is the ratio of radiation pressure and ram pressure far upstream. It can also be readily shown that to this order $T_{r}^{0x} = dT^{xx}/d\tau$, implying that in this limit photon transport is indeed a diffusion process.

In the relativistic case the particle content of the shocked fluid may be dominated by $e^{\pm}$ pairs. The pair density can be computed from Eq. (6) once the collision term $C_{pp}$ is known. However, the latter depends on the energy distribution of photon, the calculation of which is beyond the scope of this paper. As a rough estimate of the pair density we use the equilibrium value,

$$n'_\pm = 8\pi(2\pi)^{1/2} \left( \frac{m_e c}{\hbar} \right)^3 \left( \frac{kT}{m_e c^2} \right)^{3/2} \exp(-m_e c^2/kT). \tag{13}$$

To complete our treatment we need to derive an equation describing the change in $\xi$ across the shock. We do so by integrating Eq. (3) over $k^2$ and $\phi$. Defining $I(\mu) = \int (k^3 f dk g d\phi$, and likewise $I'(\mu') = \int (k_1^3 f_1 dk g_1 d\phi_1$, recalling that $I(\mu) = \Gamma^3 (1 + \beta \mu')^4 I'(\mu')$, and that $d\mu'/d\tau = (\mu' - 1)(\Gamma \beta)^{-1} d\Gamma/d\tau$, we arrive at,

$$\Gamma^2 (\beta + \mu') \frac{dI'}{d\tau} + 4\Gamma \frac{dT'}{d\tau} \mu'(1 + \mu'/\beta)I' + I' = \frac{T_{r}^{0x}}{2} \left[ 1 + \frac{(3\mu'^2 - 1)}{8} \right] + \kappa_{pp}(\mu'),$$

where $n'_s \sigma_T \kappa_{pp} = \Gamma^3 (1 - \beta \mu)^3 \int (k^3)^2 C_{pp} dk g d\phi$. We solve Eq. (14) by expanding $I'$ and $\kappa_{pp}$ in terms of Lagendre polynomials:

$$I'(\mu') = \frac{1}{2} \Sigma \eta_k P_k(\mu'); \quad \kappa_{pp}(\mu') = \frac{1}{2} \Sigma \kappa_k P_k(\mu'). \tag{15}$$

Note that with this normalization $T_{r}^{0x} = \eta_0$, $T_{r}^{0x} = \eta_1/\beta$ and $\xi = 2\eta_2/5\eta_0$. Substituting Eq. (15) into Eq. (14) yields an infinite set of ODE’s for the unknown variables $\eta_n$. To simplify the analysis we keep only the terms $\zeta_0$ and $\zeta_1$ in the expansion for $\kappa_{pp}$ which, using Eq. (5), can be expressed as $\zeta_0 = \Gamma^2 (\beta dT^xx/d\tau - dT_{xx}^0/d\tau)$ and $\zeta_1 = \Gamma^2 (\beta dT_{xx}^0/d\tau - dT_{xx}^0/d\tau)$. A closure relation can be obtained by truncating the expansion of $I'(\mu')$ in Eq. (14) at some order $n$. It can be shown that the resultant set of equations has singularities at certain values of $\beta$, that are independent of the conditions upstream. The root in the denominator of Eq. (11), which can alternatively be derived using the moment equations, is one of these points. As seen, the full transfer equation, Eq. (14), has a singularity only at the point $\beta = -\mu'$ (corresponding to $\mu = 0$ in the shock frame), where the optical depth tends to infinity. This implies that any physical shock solution of Eq. (14) must automatically

FIG. 2: Fluid velocity, comoving photon flux $T_{r}^{0x}$, and anisotropy parameter $\xi$ as functions of optical depth, for matters dominated (left panel) and radiation dominated (right panel) fluid far upstream. The blue lines in both panels are solutions obtained to second order and the red lines to third order (see text for further details).
The value at the singularity that leaves the solution for equations. This is demonstrated already in Eq. (11) where pass through the singular points of the moment equations. This is different in some respects from the critical points of a hydrodynamic system that fix physical parameters of a transonic solution (see Ref. [24] for a detailed account). We were able to find physical shock solutions of the moment equations that pass smoothly through these singular points, for mildly relativistic upstream velocities ($\Gamma_-\sim$ a few). Full details will be given elsewhere. For the cases studied below we find that the solutions converge as the order of truncation is increased. An example is shown in Fig. 2, where the fluid velocity $\beta$, comoving photon flux $T_\gamma^0$, and anisotropy parameter $\xi$ are plotted as functions of the angle averaged optical depth, $\tau$, for upstream velocity $\beta_- = 0.7$. Second order (blue lines) and third order (red lines) solutions are compared in each panel. As seen, for the radiation dominated upstream condition (right panel) the agreement is perfect. For the matter dominated condition the convergence is slower. This is in part a consequence of the larger anisotropy (larger values of $\xi$ and $T_\gamma^0$) in the latter example. We have obtained also higher order solutions for this case and found nearly complete convergence at fourth order. Shock profiles computed for upstream Lorentz factor $\Gamma_- = 2$ are exhibited in fig 3. The ultrarelativistic regime requires analysis of higher order terms and is currently under study.

From the above analysis it is evident that an appreciable fraction of the shock energy should be converted, via bulk Comptonization, to high energy photons having energies well in excess of the thermal peak. If the optical depth in the upstream region is not too large then these photons would escape before being thermalized. Consequently, a non-thermal spectral component appears to be an inherent feature of relativistic radiation dominated shocks and requires no particle acceleration, as in the case of collisionless shocks. The transmitted spectrum should extend up to an energy of $\sim m_\nu c^2$, as measured in the shock frame, above which it will be suppressed by KN effects. In the frame of the observer the high energy cutoff of the emitted spectrum will be boosted by a factor $\Gamma_d$, the shock Lorentz factor. For GRBs this implies cutoff energy that can exceed 50 MeV or so in the observer frame. This mechanism can therefore account for the spectra observed in most GRBs. Detailed calculations of the transmitted spectrum are underway (Bromberg & Levinson, in preparation).

Relativistic photon mediated shocks can provide a means for producing nonthermal spectra also in purely leptonic fireballs, the consideration of which is motivated by recent post-SWIFT discoveries of a shallow afterglow phase at early times [25]. According to some interpretations (e.g., Ref. [26]) these observations indicate prolonged activity of the central engine that, if true, implies that $\gamma$-rays are emitted during the prompt phase with very high efficiency. Such episodes can be most naturally explained as resulting from photon mediated shocks in a pure electron-positron plasma. Further discussions can be found in Refs. [26, 27].

As seen from Figs 2 the scale of a photon mediated shock is a few Thomson depths, which for parameters typical to most compact, relativistic systems is several orders of magnitudes larger than any microphysical scale (e.g., the skin depth and Larmor radius) associated with collisionless shocks. Thus, unlike collisionless shocks, photon mediated shocks cannot accelerate particles to nonthermal energies. This has direct implications for production of cosmic rays and VHE neutrinos in those systems. A particular example is the recent proposal that failed GRBs may be prodigious sources of TeV neutrinos [28]. The idea is based on the ad hoc assumption that internal collisionless shocks that form in the shocked outflow accelerate protons to very high energies. However, the large Thomson depth anticipated in the region where the shocks form [28] should render them radiation dominated, as our analysis indicates, and so they may not be able to provide the required sites for the acceleration of protons. Thus, the effects of radiation domination are likely to have a strong influence on the predictions presented in [28].

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For the solutions of mildly relativistic shocks presented below we find that the energy density of pairs is anyhow negligible.