Mesoscopic Correlation with Polarization of Electromagnetic Waves

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Mesoscopic correlations are observed in the polarization of microwave radiation transmitted through a random waveguide. These measurements, supported by diagrammatic theory, permit the unambiguous identification of short, long, and infinite range components in the intensity correlation function, as well as an additional frequency-independent component.

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Mesoscopic transport of both classical and quantum mechanical waves is characterized by the degree of non-local intensity or current correlation which reflects the closeness to the Anderson localization threshold. Correlation is generally treated using scalar waves with the electromagnetic (EM) polarization or electron spin accounted for by doubling the density of states. The vector nature of EM radiation is exhibited in coherent correlations, which determine fluctuations of intensity correlation with polarization rotation. This enables an unambiguous separation of the intensity correlation function, \(C(\Delta \theta_S, \Delta \theta_D)\), into that of the correlation function with displacement of the source and detector, \(C(\Delta \theta_S, \Delta \theta_D)\). In each component, \(C\) depends upon a given degree of freedom only through its dependence upon the square of the field correlation function, \(F \equiv |F_E|^2\). When expressed in terms of polarization rotations of the source and detector, this gives,

\[
C(\Delta \theta_S, \Delta \theta_D) = F(\Delta \theta_S) F(\Delta \theta_D)
+ A_2' [F(\Delta \theta_S) + F(\Delta \theta_D)]
+ A_3' [1 + F(\Delta \theta_S) + F(\Delta \theta_D)]
+ F(\Delta \theta_S) F(\Delta \theta_D). \tag{1}
\]

The leading contributions to \(A_2'\) and \(A_3'\) are of order \(1/g\) and \(1/g^2\), respectively, where \(g\) is the dimensionless conductance.

The field correlation function with polarization rotation can readily be found for multiply scattered waves when the transmitted field is completely depolarized. In this case, the average intensity is independent of the polarization of the field at the source or detector and cross-polarized fields are uncorrelated. When the polarization of the detector is rotated by \(\Delta \theta_D\) from an initial direction \(\theta_D\), the detected field \(\mathbf{E}(\theta_D + \Delta \theta_D)\) may be expressed in terms of the vector sum of the field along \(\theta_D\), \(\mathbf{E}(\theta_D)\), and the field perpendicular to this direction, \(\mathbf{E}(\theta_D + 90^\circ)\), as follows,

\[
\mathbf{E}(\theta_D + \Delta \theta_D) = \mathbf{E}(\theta_D) \cos \Delta \theta_D + \mathbf{E}(\theta_D + 90^\circ) \sin \Delta \theta_D.
\]

The field correlation function for the normalized average intensity, \(\langle |\mathbf{E}(\theta_D)|^2 \rangle = 1\), is therefore,

\[
F_E(\Delta \theta_D) = \langle \mathbf{E}(\theta_D) \mathbf{E}^*(\theta_D + \Delta \theta_D) \rangle = \cos \Delta \theta_D.
\]

We begin with the conjecture, borne out by diagrammatic calculations and measurements reported below, that the structure of the correlation function of normalized intensity with regard to rotations in the polarization of source and detector, \(C(\Delta \theta_S, \Delta \theta_D)\), is analogous to that of the correlation function with displacement of the source and detector, \(C(\Delta \theta_S, \Delta \theta_D)\). In each component, \(C\) depends upon a given degree of freedom only through its dependence upon the square of the field correlation function, \(F \equiv |F_E|^2\). When expressed in terms of polarization rotations of the source and detector, this gives,

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cornered from the topological structure of the underlying di-
tion waves in a detailed diagrammatic study [15]. The rela-
tionship of Ref. [14] has been generalized to vector
elds, in which nonuniversal “$C_i$” contributions to
C are negligible, $A_1 = A'_3$, $A_2 = A'_2 + A'_3$, and $A_3 = A'_2$. The analysis of Ref. [14] has been generalized to vector
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$A_1 = A_3$, which follows from Eq. (1), can be under-
stood from the topological structure of the underlying di-
grams, and the equal weights of the two $A_2$ terms follow
from reciprocity. In the diffusive regime in the absence of
absorption, to order $1/g^2$,

$$A'_2 = \frac{2}{3g}, \quad A'_3 = \frac{2}{15} g^3,$$

where $g = 2 \times k^2 \ell/3\pi L$, which includes both polarizations. Here, $k$ is the wave number, $\ell$ is the transport
mean free path, $A$ and $L$ are the cross-section area and
length of the sample, respectively. Values of $A'_2$ and $A'_3$
slightly depend on absorption [14, 15]. In our experi-
ment, $L/L_a = 3.6$, where $L_a$ is the absorption length.
This gives $A'_2 = 0.57/g$ and $A'_3 = 0.113/g^3$.

Measurements of the dependence of the field and in-
tensity correlation functions upon polarization rotation
for microwave radiation transmitted through a random
dielectric sample are made with identical conical horns
positioned 40 cm in front and behind the sample (Fig. 1).
Linearly polarized microwave radiation is launched from
horn $S$ and a linearly polarized component of the trans-
mitted field is captured by horn $D$. The polarization of in-
cident and detected waves can be rotated in the $xy$-plane
by rotating the corresponding horn about its axis ($z$-
axis). Because of the conical shape of the horns, the an-
gular distribution of the incident and detected radiation
do not depend strongly on the rotation of the horns. As
a result, the variation in the detected field at a given fre-
cquency for a given sample realization is mainly due to the
shift in the polarization of the field at the source and de-
tector. The sample is composed of 0.95-cm-diameter alu-
mina spheres with refractive index 3.14 embedded within
Styrofoam spheres of diameter 1.9 cm and refractive in-
dex 1.04. The sample with an alumina volume fraction of 0.068 is contained in a 7.3-cm-diameter copper tube

![FIG. 1: Schematics of the experimental setup: $S$ (source) and $D$ (detector) microwave horns are positioned symmetrically in front and behind the sample and can be rotated about their axes ($z$-axis) thus rotating the polarizations of the incident and detected waves.](image)
found directly from $C(90,90)$ for an ensemble of 44,000 realizations, $A_3 = 0.029 \pm 0.001$. From the fit, we obtain $A_1 + A_2 = 0.286 \pm 0.005$ and $A_2 + A_3 = 0.293 \pm 0.004$. Hence $A_2 = 0.264 \pm 0.006$ and $A_1 = 0.022 \pm 0.010$. The dashed curve in Fig. 3b is $A_3 + A_2 \cos^2 \Delta \theta_D$ with the $A_2$ and $A_3$ found above. The difference between the experimental data points and the curve represents the loss of correlation due to the redistribution of intensity with the horn rotation.

The coefficients $A_i$ for arbitrary frequency shift can be determined from spectral measurements for three orientations of the source and detector as follows, $A_3 = C(90,90)$, $A_2 = C(0,90) - A_3$, and $A_1 = C(0,0) - |F_E(0,0)|^2 - 2A_2 - A_3$. We have utilized the above procedure in the ensemble of 44,000 sample realizations, to find $A_i(\Delta \nu)$ for frequency shifts $\Delta \nu$ up to 100 MHz, that is almost 100 times the Thouless frequency $\Delta \nu_{Th} = 6D/2\pi L^2 \approx 1$ MHz. To compensate for the loss of correlation in $C(0,90)$ due to the redistribution of intensity, the corresponding frequency correlation function was multiplied by a constant to make it equal to the $A_2 + A_3$ at zero frequency shift. The resulting dependencies are shown by the squares in Fig. 4. $A_2(\Delta \nu)$ and $A_3(\Delta \nu)$ are predicted [17] to fall asymptotically as $A_2(\Delta \nu) \sim 1/(\Delta \nu)^{1/2}$ and $A_3(\Delta \nu) \sim 1/(\Delta \nu)^{3/2}$. We find, however, that $A_3$ exhibits a nonzero correlation for $\Delta \nu \gg \Delta \nu_{Th}$, that largely exceeds error bars, and that $A_2$ is shifted from the predicted curve by a nearly constant value. Figs. 4a and 4b show that agreement with theory is obtained for $A_{2,3}$, if frequency-independent contributions with the values $A_2(\infty) = -0.015$ and $A_3(\infty) = 0.007$ are incorporated, respectively, in $A_2$ and $A_3$. We do not observe an asymptotic background in $A_1$ (Fig. 4c). The observed $A_1(\Delta \nu)$, however, falls faster than predicted by diagrammatic theory. To calculate $A_1(\Delta f)$, we have modified the theory for $A_3(\Delta f)$ [17].

After subtracting the asymptotic values we find $A_3(0) - A_3(\infty) = A_3'(0) = 0.022 \pm 0.001$ and $A_2(0) - A_2(\infty) = A_2'(0) + A_3'(0) = 0.279 \pm 0.004$ at zero frequency shift. Note that $A_3'(0) = A_1(0)$ holds within the error bars. Using Eq. (3) the value for $A'_3(0)$ yields $g = 2.27 \pm 0.05$ for the dimensionless conductance. This differs from the estimate, reflecting effects of localization, internal reflection, and finite scatterer density. Using this value, Eq. (3) predicts $A'_3(0) = 0.252$, and hence $A'_2(0) + A'_3(0) = 0.274$, which is also consistent with observations.

It is tempting to associate the frequency-independent background term with the $C_0$ term of Ref. [13]. A preliminary calculation for point dipoles in a tube geometry with $N$ transverse channels [15] shows $C_0$ to be...
FIG. 4: Coefficients $A_3$ (a), $A_2$ (b), and $A_1$ (c) plotted versus frequency shift. The long-dashed curves represent predictions of mesoscopic theory [17] for $A_3$, $A_2$, and $A_1$ with $g = 2.27$ and $L/L_s = 3.6$. The frequency-independent contributions with the values $A_3(\infty) = 0.007$ and $A_2(\infty) = -0.015$ have been added, respectively, to the predicted $A_3$ and $A_2$ to obtain the solid curves and agreement with the data.

A frequency-independent component, which may be related to the short-duration interaction of the wave with regions of limited spatial extent near the input and output surfaces of the sample. The simple form of the three contributions to the intensity correlation function with polarization rotation allows an unambiguous separation of correlation into short, long, and infinite range contributions. This method is readily applicable to optical measurements and can be used to determine the degree of intensity correlation in a sample and hence the closeness to the localization threshold. Analogous correlations can also be expected in electron spin propagating in a random potential.

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