Indispensability of Ghost Fields and Extended Hamiltonian Formalism in Axial Gauge Quantization of Gauge Fields

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It is shown that ghost fields are indispensable in deriving well-defined antiderivatives in pure space-like axial gauge quantizations of gauge fields. To avoid inessential complications we confine ourselves to noninteracting abelian fields and incorporate their quantizations as a continuous deformation of those in light-cone gauge. We attain this by constructing an axial gauge formulation in auxiliary coordinates $x^\mu = (x^+, x^-, x^1, x^2)$, where $x^+ = x^0 \sin \theta + x^3 \cos \theta$, $x^- = x^0 \cos \theta - x^3 \sin \theta$ and $x^+ = A^0 \cos \theta + A^3 \sin \theta = 0$ are taken as the evolution parameter and the gauge fixing condition, respectively. We introduce $x^-$ independent residual gauge fields as ghost fields and accommodate them to the Hamiltonian formalism by applying McCartor and Robertson’s method. As a result, we obtain conserved translational generators $P_\mu$, which retain ghost degrees of freedom integrated over the hyperplane $x^- = \text{constant}$. They enable us to determine quantization conditions for the ghost fields in such a way that commutation relations with $P_\mu$ give rise to the correct Heisenberg equations. We show that regularizing singularities arising from the inversion of a hyperbolic Laplace operator as principal values, enables us to cancel linear divergences resulting from $(\partial^-)_-^2$ so that the Mandelstam-Leibbrandt form of gauge field propagator can be derived.

It is also shown that the pure space-like axial gauge formulation in ordinary coordinates can be derived in the limit $\theta \to \frac{\pi}{2} - 0$ and that the light-cone axial gauge formulation turns out to be the case of $\theta = \frac{\pi}{4}$.

§1. Introduction

Axial gauges $n^\mu A_\mu = 0$, specified by a constant vector $n^\mu$, have been used recently in spite of their lack of manifest Lorentz covariance. It was first found that the Faddeev-Popov ghosts decouple from the theory in the axial gauge formulations.\(^1\) Among others the case of $n^2 = 0$, namely the light-cone gauge has been extensively considered in light-front field theory (LFFT), which studies nonperturbative solutions of QCD. As a matter of fact the infinite-momentum limit is incorporated in LFFT by the change of variables $x^+_1 = \frac{2m^3}{\sqrt{2}}$, $x^-_1 = \frac{2m^3}{\sqrt{2}}^2$ so that one is able to have vacuum state composed only of particles with nonnegative longitudinal momentum and also to have relativistic bound-state equations of the Schrödinger-type. (For a good overview of LFFT see Ref. 3.)

After a considerable amount of work had been done, it was definitely discovered that the axial gauge formulations are not ghost free, contrary to what was originally expected and is still sometimes claimed. It was first pointed out by Nakanishi\(^4\) that there exists an intrinsic difficulty in the axial gauge formulations so that an indefinite metric is indispensable even in QED. It was also noticed that in order to bring per-
turbative calculations done in the light-cone gauge into agreement with calculations
done in covariant gauges, spurious singularities of the free gauge field propagator have
to be regularized not as principal values (PV), but according to the Mandelstam- Leibbrandt (ML) prescription\(^5\) in such a way that causality is preserved. Shortly
afterwards, Bassetto et al.\(^6\) found that the ML form of the propagator is realized in
the light-cone gauge canonical formalism in ordinary coordinates if one introduces
a Lagrange multiplier field and its conjugate as residual gauge degrees of freedom.
Furthermore, Morara and Soldati\(^7\) found just recently that the same is true in the
light-cone temporal gauge formulation, in which \(x^+ = 0\) and \(A_0 - A_3 / \sqrt{2} = 0\) are taken as
the evolution parameter and the gauge fixing condition, respectively. It should also
be noted that McCartor and Robertson\(^8\) showed in the light-cone axial gauge for-
mulation, where \(A_0 - A_3 / \sqrt{2} = 0\) is instead taken as the gauge fixing condition, that the
translational generator \(P_+\) consists of physical degrees of freedom integrated over
the hyperplane \(x^+ = \) constant and ghost degrees of freedom integrated over the
hyperplane \(x_1^+ = \) constant.

Because the axial gauges could be viewed as continuous deformations of the
light-cone gauge, extensions of the ML prescription outside the light-cone gauge for-
mulations have also been studied. Lazzizzera\(^9\) and Landshoff and Nieuwenhuizen\(^10\)
constructed canonical formulations for non-pure space-like case \((n^0 \neq 0, n^2 < 0)\) in
ordinary coordinates. However, in spite of so many attempts, no one has succeeded
in constructing consistent pure space-like axial gauge \((n^0 = 0, n^2 < 0)\) formulations.
This motivates us to consider constructing a pure space-like axial gauge formulation
in which the ML form of gauge field propagator is realized. To clarify the difficulties
preventing consistent quantizations to this time we obtain the pure space-like gauge
as a continuous deformation of the light-cone gauge. Thus we construct an axial
gauge formulation in the auxiliary coordinates \(x^\mu = (x^+, x^-, x^1, x^2)\), where
\[
x^+ = x^0 \sin \theta + x^3 \cos \theta, \quad x^- = x^0 \cos \theta - x^3 \sin \theta.
\]
(1.1)

Accordingly we impose
\[
A_- = A^0 \cos \theta + A^3 \sin \theta = 0
\]
(1.2)
as the gauge fixing condition. The same framework was used previously by others\(^11\)
to analyze two-dimensional models. It should be noticed that quantizations in this
framework are easier than those in LFFT. This is because we can choose \(x^-\) as the evolution parameter in the interval \(0 < \theta < \pi/4\) and construct the temporal
gauge formulation, in which the Lagrangian is regular. Whereas in the interval
\(\pi/4 < \theta < \pi/2\), \(x^+\) can be chosen as the evolution parameter to construct the axial
gauge formulation, in which fewer constraints appear than in LFFT. Furthermore
we can expect that the temporal and pure space-like axial gauge formulations in
ordinary coordinates are obtained by letting \(\theta\) tend to 0 and \(\pi/2\), respectively and
also that the light-cone temporal and axial gauge formulations are derived in the
light-cone limits \(\theta \to \pi/4 \pm 0\).

In a previous preliminary work\(^12\) it was shown that \(x^-\)-independent residual
gauge fields can be introduced as static ghost fields in the canonical temporal gauge
formulation and play roles as regulators so that the ML form of the gauge field
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The propagator can be realized. In this paper we proceed to verify that the ghost fields introduced in the temporal gauge formulation are also indispensable as a pair of canonical variables in the axial gauge formulation. It is noticed immediately that we encounter two intrinsic problems. One is that we cannot obtain the Poincaré generators for the \( x^- \)-independent ghost fields by integrating densities made of those ghost fields over the three-dimensional hyperplane \( x^+ = \text{constant} \). That implies that we cannot obtain their quantization conditions from the traditional Dirac canonical quantization procedure,\(^{13}\) which makes use of a Hamiltonian density integrated over the three-dimensional hyperplane \( x^+ = \text{constant} \). We overcome this problem by applying McCartor and Robertson's\(^8\) way of quantizing axial gauge fields. Therefore we first obtain translational generators \( P_\mu \) in the auxiliary coordinates and then obtain quantization conditions by requiring that the commutation relations with \( P_\mu \) give rise to the Heisenberg equations of the ghost fields. The other problem is that in the axial gauge formulation, the Laplace operator, which operates on the ghost fields, becomes hyperbolic, so that we have to regularize divergences resulting from its inverse. We regularize those singularities as principal values. As a consequence, linear divergences resulting from \((\partial_-)^{-2}\) are canceled so that the ML form of gauge field propagator can be derived.

The paper is organized as follows. In §2, by integrating divergence equations of the energy-momentum tensor over a suitable closed surface, we obtain the conserved translational generators including the ghost field's part integrated over the hyperplane \( x^- = \text{constant} \). Then quantization conditions are obtained by requiring that the obtained translational generators give rise to Heisenberg equations. In §3 it is shown that by defining singularities resulting from inversion of a hyperbolic Laplace operator as principal values, we can obtain the ML form of gauge field propagator. In §4 conserved parts of Lorentz transformation generators are given and section 5 is devoted to concluding remarks.

We use the following conventions:
Greek indices \( \mu, \nu, \rho, \sigma, \cdots \) will take the values +, −, 1, 2 and label the component of a given four-vector (or tensor) in the auxiliary coordinates;
Latin indices \( i, j, k, l, \cdots \) will take the values 1, 2 and label the 1, 2 component of a given four-vector (or tensor) in the auxiliary coordinates;
Latin indices \( r, s, t, \cdots \) unless otherwise stated, will take the values +, 1, 2 and label the +, 1, 2 components of a given four-vector (or tensor) in the auxiliary coordinates;
the Einstein convention of sum over repeated indices will be always used;

\[
\begin{align*}
x^\pm &= (x^\pm, x^1, x^2), \quad x_\perp = (x^1, x^2), \quad d^2 x_\perp = dx^1 dx^2, \quad d^3 x^\pm = dx^1 dx^2 dx^\pm \\
k_\pm &= (k_\pm, k_1, k_2), \quad d^3 k_\pm = dk_1 dk_2 dk_\pm
\end{align*}
\]

§2. Quantization of the axial gauge fields in the auxiliary coordinates

We begin by denoting the metric of the auxiliary coordinates\(^{12}\):

\[
\begin{align*}
g_{--} &= \cos^2 \theta, \quad g_{--} = g_{+-} = \sin^2 \theta, \quad g_{++} = -\cos^2 \theta \\
g_{-i} &= g_{i-} = g_{+i} = g_{i+} = 0, \quad g_{ij} = -\delta_{ij}, \quad (2.1)
\end{align*}
\]
\[ g^{-} = \cos 2\theta, \quad g^{+} = g^{-} = \sin 2\theta, \quad g^{+} = -\cos 2\theta \]
\[ g^{-i} = g^{+i} = g^{i+} = 0, \quad g^{ij} = -\delta_{ij}. \]  
(2.2)

In this paper we keep \( \theta \) in the interval \( \frac{\pi}{4} < \theta < \frac{\pi}{2} \) to take \( x^+ \) as the evolution parameter. Furthermore we have chosen the gauge fixing direction in such a way that it is orthogonal to the quantization plane. In fact \( x^+ \) and \( A_- \) are described in ordinary coordinates as inner products with orthogonal constant vectors \( m_\mu \) and \( n_\mu \),:
\[ x^+ = m \cdot x, \quad A_- = n \cdot A, \]  
(2.3)

where
\[ m_\mu = (m_0, m_3, m_1, m_2) = (\sin \theta, \cos \theta, 0, 0), \quad m^2 > 0, \]
\[ n_\mu = (n_0, n_3, n_1, n_2) = (\cos \theta, \sin \theta, 0, 0), \quad n^2 < 0. \]  
(2.4)

Field equations of noninteracting abelian axial gauge fields \( A_\mu \) in the auxiliary coordinates are defined by the Lagrangian
\[ L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - B(n \cdot A) \]  
(2.5)

where \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \) with \( \partial_\mu = (\partial/\partial x^+, \partial/\partial x^-, \partial/\partial x^1, \partial/\partial x^2) \) and \( B \) is the Lagrange multiplier field, that is, the Nakanishi-Lautrup field in noncovariant formulations.\(^{14}\)

It is noted that the constant vector \( n \) is described in the auxiliary coordinates as \( n^\mu = (n^+, n^-, n^1, n^2) = (0, 1, 0, 0) \) and \( n_\mu = (n_+, n_-, n_1, n_2) = (\sin 2\theta, \cos 2\theta, 0, 0) \). Thus we obtain from the Lagrangian the field equations
\[ \partial_\mu F^{\mu\nu} = n^\nu B \]  
(2.6)

and the gauge fixing condition
\[ A_- = 0. \]  
(2.7)

The field equation of \( B \),
\[ \partial_- B = 0, \]  
(2.8)

is obtained by operating on (2.4) with \( \partial_\nu \).

Before entering into details it is useful to point out that conjugate field of \( B \), which we denote \( C \) in what follows, is a hidden one. In fact it has not been introduced in the Lagrangian (2.5), but is to be introduced as an integration constant. Canonical conjugate momenta, defined by
\[ \pi^+ = \frac{\delta L}{\delta \partial_+ A_+} = 0, \quad \pi^- = \frac{\delta L}{\delta \partial_+ A_-} = F_{+-}, \quad \pi^i = \frac{\delta L}{\delta \partial_+ A_i} = F_{+i}, \quad \pi_B = \frac{\delta L}{\delta \partial_+ B} = 0, \]  
(2.9)

satisfy, by virtue of \( \partial_\mu F^{\mu\nu} = 0 \),
\[ \partial_- \pi^- + \partial_i \pi^i = 0. \]  
(2.10)

In addition, because \( A_- = 0 \), \( \pi^- \) is related with \( A_+ \) as
\[ \pi^- = -\partial_- A_+. \]  
(2.11)
It seems at first glance that we have only two independent pairs of canonical variables. If so, we have the following paradox: When we formulate the temporal gauge formulation, in which \( x^- \) is taken as the evolution parameter, we obtain three independent pairs of canonical variables in spite of the fact that the field equations are the same. Therefore it is reasonable to think that we have three independent pairs of canonical variables also in the axial gauge formulation. We notice that two integration constants are overlooked in the traditional formulations. As a matter of fact one is overlooked in the equation

\[
\pi^- = \frac{1}{\partial_-} \partial_i \pi^i.
\]  

(2.12)

The other is overlooked in the equation

\[
A_+ = -\frac{1}{\partial_-} \pi^-.
\]  

(2.13)

It turns out below that first one is nothing but \( B \), while the other is \( C \).

It should be noticed however that we do not have any guiding principles to specify those integration constants as far as we confine ourselves to the traditional Hamiltonian formalism. Note that Dirac’s canonical quantization procedure is also not helpful to this matter, because it cannot solve the problem of how to define the antiderivative \((\partial_-)^{-1}\). We overcome this problem by making use of known solutions of (2.4) and (2.7), namely the temporal gauge solutions. By observing that if we extrapolate the temporal gauge solutions, they are also ones in the axial gauge formulation, we verify that the ghost fields \( B \) and \( C \) are indispensable canonical variables also in the axial gauge formulation. To attain this we have to obtain the Hamiltonian and other translational generators in the axial gauge formulation.

It was shown in the previous paper \(^{12}\) that \( A_\mu \) satisfying (2.6), (2.7) and canonical quantization conditions in the temporal gauge formulation are described as

\[
A_\mu = a_\mu - \frac{\partial_\mu}{\partial_-} a_- + \Gamma_\mu
\]  

(2.14)

where \( a_\mu \) are the physical photon fields and thus satisfy free massless d’Alembert equation together with

\[
\partial^\mu a_\mu = 0, \quad \partial_i a_i = 0.
\]  

(2.15)

\( \Gamma_\mu \) stands for the following residual gauge degrees of freedom

\[
\Gamma_\mu = -\frac{n_\mu}{\partial_1^2 + n^2 \partial_2^2} B - \frac{\partial_\mu}{\partial_1^2 + n^2 \partial_2^2} \left( C - n^2 x^- B - \frac{n^2 n_+}{\partial_1^2 + n^2 \partial_2^2} \partial_+ B \right)
\]  

(2.16)

where

\[
\partial_1^2 = \partial_1^2 + \partial_2^2
\]  

(2.17)

and \( C \) is the field introduced as the \( x^- \)-independent integration constant. Furthermore the antiderivative \( \frac{1}{\partial_-} \) in (2.14) is defined by

\[
(\partial_-)^{-1} f(x^-) = \frac{1}{2} \int_{-\infty}^{\infty} dy^- \varepsilon(x^- - y^-) f(y^-)
\]  

(2.18)
which imposes, in effect, the principal value regularization. It should be noticed
that Laplace operator $\partial^2 + n^2 \partial^2$ becomes hyperbolic because $n^2 = \cos^2 \theta < 0$
in the axial gauge formulation, so that its inverse gives rise to singularities. We regularize
them as the principal values in next section. Now that we have the expression of $A_\mu$
described in terms of physical operators $a_\mu$ and ghost operators $B$ and $C$, we turn to
obtaining conserved generators in the auxiliary coordinates by applying McCartor
and Robertson’s procedure.

The symmetric energy-momentum tensor is given by

$$\Theta^{\mu\nu} = -F^{\mu\sigma} F^{\nu}_\sigma + \frac{g^{\mu\nu}}{4} F^{\rho\sigma} F_{\rho\sigma} - n^\nu B A^\mu. \quad (2.19)$$

We denote hereafter its physical part by small letters as in the following

$$\Theta^{\mu\nu}_{\text{physical part}} = \theta^{\mu\nu}, \quad (2.20)$$

where

$$\theta^{\mu\nu} = -f^{\mu\sigma} f^{\nu}_\sigma + \frac{g^{\mu\nu}}{4} f^{\rho\sigma} f_{\rho\sigma} \quad (2.21)$$

with

$$f^{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu. \quad (2.22)$$

From the divergence equation

$$\partial_\nu \Theta^{\mu\nu} = 0, \quad (2.23)$$

we obtain

$$\oint \Theta^{\mu\nu} d\sigma_\nu = 0, \quad (2.24)$$

where the integral is taken over a closed surface.

It is useful to remark here that we have to resort to a nonstandard way when
we derive conserved generators from (2.24) in the axial gauge formulation. This is
because the integral $\int_{-\infty}^{\infty} \partial_\nu \Theta^{\mu\nu}_- dx^-$ does not vanish, although $x^-$ is one of space
coordinates. In fact the ghost fields do not depend on $x^-$ and $A_\mu$ in (2.14) depend
explicitly on $x^-$. This reflects the fact that $\int d^3 x^- \Theta^{\mu-}_+$ are not well-defined. There-
fore we have to retain $\Theta^{\mu-}_-$. For the transverse directions we are justified to assume
that the integral $\int_{-\infty}^{\infty} \partial_i \Theta^{\mu,i}_+ dx^i$ vanishes. (Here repeated indices do not imply sum
over $i$.) Therefore as the closed surface we employ one shown in Fig. 1, whose bounds
$T$ and $L$ are let tend to $\infty$ after calculations are finished. It is remarked that we can
use the surface even in the limit $\theta \to \pi/2 - 0$ in contrast with one used in Refs. 15)
and 8). It is straightforward to obtain

$$0 = \int d^2 x_\perp \left( \int_{-L}^{L} dx^- \left[ \Theta^{\mu+}_{\mu+}(x) \right]^{x^+ = T}_{x^+ = -T} + \int_{-T}^{T} dx^+ \left[ \Theta^{\mu-}_{\mu-}(x) \right]^{x^- = L}_{x^- = -L} \right) \quad (2.25)$$

where

$$\left[ \Theta^{\mu+}_{\mu+}(x) \right]^{x^+ = T}_{x^+ = -T} = \Theta^{\mu+}_{\mu+}(x)_{x^+ = T} - \Theta^{\mu+}_{\mu+}(x)_{x^+ = -T},$$

$$\left[ \Theta^{\mu-}_{\mu-}(x) \right]^{x^- = L}_{x^- = -L} = \Theta^{\mu-}_{\mu-}(x)_{x^- = L} - \Theta^{\mu-}_{\mu-}(x)_{x^- = -L}. \quad (2.26)$$
By integrating by parts in the transverse directions, it can be shown that products of physical operators and ghost operators vanish, so that physical part and ghost part decouple in (2.25). This result reflects the fact that the physical parts are conserved by themselves in the noninteracting theory. It can be also shown that parts of ghost terms, which are not well-defined in the limit $L \to \infty$, cancel among themselves. After getting the decoupled expression, we take the limit $L \to \infty$. This limit enables us to discard the physical operators $\theta_{\mu}^-(x)$, which is usually done when one obtains conserved physical generators in the noninteracting axial gauge theory. Finally we take the limit $T \to \infty$. In this stage we assume that we can integrate remained ghost terms in the $\Theta_{\mu}^-(x)$ by parts in $x^+$ direction. This assumption is justified because $x^+$ is not different from the transverse variables $x^i$ for the ghost fields. As a consequence we obtain

$$0 = \int d^3x^- [\theta^+_r]_{x^+=\infty}^{x^-=-\infty} + \int d^3x^+ \left[ B \frac{1}{\partial^2_{\perp} + n^2 \partial^2_{\parallel}} \partial_r C \right]_{x^-=-\infty}^{x^+=\infty}, \quad (r = +, 1, 2)$$

$$0 = \int d^3x^- [\theta^-_r]_{x^+=\infty}^{x^-=-\infty} - \frac{1}{2} \int d^3x^+ \left[ B(x) \frac{n^2}{\partial^2_{\parallel} + n^2 \partial^2_{\perp}} B(x) \right]_{x^-=-\infty}^{x^+=\infty}. \quad (2.27)$$
Hence from these we obtain the conserved generators:

\[
P_r = \int d^3x^- \theta^+(x) + \int d^3x^+ B(x) \frac{1}{\partial_+^2 + n^2\partial_+^2} \partial_r C(x), \quad (r = +, 1, 2) \quad (2.29)
\]

\[
P_- = \int d^3x^- \theta^+(x) - \frac{1}{2} \int d^3x^+ B(x) \frac{n_-}{\partial_+^2 + n^2\partial_+^2} B(x). \quad (2.30)
\]

Now we can derive axial gauge quantization conditions in the auxiliary coordinates by requiring

\[
[P_i, B(x)] = -i\partial_i B(x), \quad [P_i, C(x)] = -i\partial_i C(x), \quad (2.31)
\]

\[
[P_i, a_r(x)] = -i\partial_i a_r(x), \quad (r = -, 1, 2) \quad (2.32)
\]

It is straightforward to deduce from \(P_i\) in (2.29) that commutation relations

\[
[B(x), B(y)] = [C(x), C(y)] = 0 \quad (2.33)
\]

\[
[B(x), C(y)] = -[C(x), B(y)] = -i(\partial_+^2 + n^2\partial_+^2)\delta^{(3)}(x^+ - y^+) \quad (2.34)
\]

yield (2.31). However it is difficult to deduce quantization conditions for the \(a_\mu\) from the expression for \(P_i\) in (2.29). Thus we rewrite its physical part \(p_i\) into the canonical expression by making use of the fact that integration of spatial divergence term over the whole three dimensional space vanishes. Then further use of integration by parts in the transverse directions, divergence relations \(\partial_i a_i = 0\) and \(\partial^+ a_+ + \partial^- a_- = 0\) and identity \((\partial^+)^2 = \partial_+^2 - n^2\partial_+^2\) for the physical fields enables us to have

\[
p_i = \int d^3x^- \left( \frac{\partial_+^2}{\partial_+^2 - n^2\partial_+^2} \partial^+ a_- \partial_i a_- + \partial^+ a_j \partial_i a_j \right). \quad (2.35)
\]

We see from this that in the axial gauge formulation \(a_+\) is not canonically independent variable. In fact it is described as

\[
a_+ = \frac{1}{n_-} \left( n_+ a_- - \frac{\partial_-}{\partial_-^2 - n^2\partial_+^2} \partial^+ a_- \right), \quad (2.36)
\]

It is now easy to deduce that the equal \(x^+\)-time commutation relations

\[
[a_r(x), a_s(y)] = [\partial^+ a_r(x), \partial^+ a_s(y)] = 0, \quad (r, s = -, 1, 2) \quad (2.37)
\]

\[
[a_r(x), \partial^+ a_s(y)] = i \frac{\partial_+^2 - n^2\partial_+^2}{\partial_+^2} \delta^{(3)}(x^- - y^-), \quad (2.38)
\]

\[
[a_i(x), \partial^+ a_j(y)] = i \left( \delta_{ij} - \frac{\partial_i \partial_j}{\partial_+^2} \right) \delta^{(3)}(x^- - y^-), \quad (i, j = 1, 2) \quad (2.39)
\]

give rise to (2.32).

To be complete we note that the physical parts \(p_-\) and \(p_+\) are described in terms of canonically independent variables, respectively as

\[
p_- = \int d^3x^- \left( \frac{\partial_-^2}{\partial_-^2 - n^2\partial_+^2} \partial^+ a_- \partial_- a_- + \partial^+ a_j \partial_- a_j \right), \quad (2.40)
\]
\[
p_+ = \frac{-1}{2n_-} \int d^3 x \left( (\partial^+ - n_+ \partial_-) a_i (\partial^+ - n_+ \partial_-) a_i + \partial^+ a_- \frac{\partial^2}{\partial^2 - n^2 \partial^2} \partial^+ a_- + \partial_i a_- \partial_i a_- - \partial_- a_- \frac{2n_+ \partial^2}{\partial^2 - n^2 \partial^2} \partial^+ a_- \right).
\]

It follows from these that \( P_- \) and \( P_+ \) work for \( a_{\mu} \) and \( B \) as the generators. For the \( C, P_+ \) works as the generator, while \( P_- \) gives rise to a nonstandard commutation relation

\[
[P_-, C(x)] = -i n^2 B(x), \quad (2.42)
\]

but this is necessary to generate the Heisenberg equation of \( A_{\mu} \):

\[
[P_-, A_{\mu}(x)] = -i \partial_- A_{\mu}(x).
\]  

**§3. Roles of ghost fields to make up the ML form of propagator**

We begin by describing the constituent fields in terms of creation and annihilation operators. Because the physical fields satisfy commutation relations \((2.37) \sim (2.39)\) and divergence relations \( \partial_i a_i = 0 \) and \( \partial^+ a_+ + \partial^- a_- = 0 \), we can express them as follows

\[
a_-(x) = \frac{1}{\sqrt{2(2\pi)^3}} \int \frac{d^3 k}{\sqrt{k^+ k_- k_\perp}} \{ a_1(k_-) e^{-ik \cdot x} + a_1^\dagger(k_-) e^{ik \cdot x} \}, \quad (3.1)
\]

\[
a_+(x) = \frac{-1}{\sqrt{2(2\pi)^3}} \int \frac{d^3 k}{\sqrt{k^+ k_- k_\perp}} \{ a_1(k_-) e^{-ik \cdot x} + a_1^\dagger(k_-) e^{ik \cdot x} \}, \quad (3.2)
\]

\[
a_i(x) = \frac{1}{\sqrt{2(2\pi)^3}} \int \frac{d^3 k}{\sqrt{k^+}} \epsilon_i^{(2)}(k) \{ a_2(k_-) e^{-ik \cdot x} + a_2^\dagger(k_-) e^{ik \cdot x} \}, \quad (3.3)
\]

where

\[
k^+ = \sqrt{k_-^2 - n^2 k_\perp^2}, \quad k_\perp = \sqrt{k_1^2 + k_2^2}, \quad k_+ = \frac{n_+ k_- - k^+}{n_-}.
\]  

The operators \( a_{\lambda} (k_-) \) and \( a_{\lambda}^\dagger (k_-) \) \((\lambda = 1, 2)\) are normalized so as to satisfy the usual commutation relations,

\[
[a_{\lambda} (k_-), a_{\lambda'} (q_-)] = 0, \quad [a_{\lambda} (k_-), a_{\lambda'}^\dagger (q_-)] = \delta_{\lambda \lambda'} \delta^{(3)}(k_- - q_-)
\]

and \( \epsilon_i^{(2)}(k) \) is a physical polarization vector given by

\[
\epsilon_\mu^{(2)}(k) = (0, 0, \frac{k_2}{k_\perp}, -\frac{k_1}{k_\perp}).
\]  

It should be noted here that with the help of another physical polarization vector

\[
\epsilon_\mu^{(1)}(k) = \left( \frac{k_\perp}{k_-}, 0, -\frac{k^+ k_1}{k_- k_\perp}, -\frac{k^+ k_2}{k_- k_\perp} \right)
\]  

\]

\[
\]

\[
\]

\[
\]

\[
\]
we can express the physical part \( u_\mu \equiv a_\mu - \frac{\partial}{\partial \omega} a_- \) in the compact form

\[
 u_\mu(x) = \frac{1}{\sqrt{2(2\pi)^3}} \int \frac{d^3 k_+}{\sqrt{k_+^2 + n^2 k_+^2}} \sum_{\lambda=1}^{2} \epsilon^{(\lambda)}(k)\{a_\lambda(k_-)e^{-ik\cdot x} + \text{h.c.}\}
\]

and that the polarization vectors satisfy

\[
 k^\mu \epsilon^{(\lambda)}(k) = 0, \quad n^\mu \epsilon^{(\lambda)}(k) = 0, \quad (\lambda = 1, 2)
\]

\[
 \sum_{\lambda=1}^{2} \epsilon^{(\lambda)}_\mu(k) \epsilon^{(\lambda)}_\nu(k) = -g_{\mu\nu} + \frac{n_\mu k_\nu + n_\nu k_\mu}{k_-} - n^2 \frac{k_\mu k_\nu}{k_-^2}.
\]

We expand \( B \) and \( C \) in terms of zero-norm creation and annihilation operators as follows

\[
 B(x) = \frac{1}{\sqrt{(2\pi)^3}} \int \frac{d^3 k_+}{\sqrt{k_+^2 + n^2 k_+^2}} \theta(k_+)(k_+^2 + n^2 k_+^2)\{B(k_+)e^{-ik\cdot x} + B^\dagger(k_+)e^{ik\cdot x}\}|_{x^- = 0},
\]

\[
 C(x) = \frac{i}{\sqrt{(2\pi)^3}} \int d^3 k_+ \sqrt{k_+^2 + n^2 k_+^2} \{C(k_+)e^{-ik\cdot x} - C^\dagger(k_+)e^{ik\cdot x}\}|_{x^- = 0},
\]

where

\[
 [B(k_+), C^\dagger(q_+)] = [C(k_+), B^\dagger(q_+)] = -\delta^{(3)}(k_+ - q_+),
\]

and all other commutators are zero. We note here that limiting the \( k_+ \)-integration region to be \((0, \infty)\) is indispensable to incorporate the ML form of gauge field propagator and that by choosing the suitable vacuum it is always possible\(^{12}\) so that we can solve the problem, pointed out by Haller,\(^{16}\) namely the problem that the canonical commutation relations cannot distinguish the PV and ML prescriptions. We define the vacuum state and physical space \( V_P \), respectively by

\[
 B(k_+)|\Omega\rangle = C(k_+)|\Omega\rangle = 0,
\]

\[
 V_P = \{ |\text{phys}\rangle \mid B(k_+)|\text{phys}\rangle = 0 \}.
\]

Now we can calculate the \( x^+ \)-ordered gauge field propagator

\[
 D_{\mu\nu}(x-y) = \langle \Omega | \{\theta(x^- - y^+)A_\mu(x)A_\nu(y) + \theta(y^- - x^+)A_\nu(y)A_\mu(x)\}|\Omega\rangle
\]=

\[
 = \frac{1}{(2\pi)^4} \int d^4 q D_{\mu\nu}(q)e^{-iq\cdot(x-y)}.
\]

It is straightforward to show that its physical part is described as

\[
 D_{\mu\nu}^p(q) = \frac{i}{q^2 + i\epsilon} \left( -g_{\mu\nu} + \frac{n_\mu q_\nu + n_\nu q_\mu}{q_-} - n^2 \frac{q_\mu q_\nu}{q_-^2} \right) - \delta_{\mu+\delta_{\nu+}^\dagger} \frac{i}{q_-^2},
\]

where \( q^2 = -n^2 q_+^2 + 2n_+q_- q_- + n^2 q_-^2 - q_+^2 \) with \( n^2 = n_- \). We investigate in detail how the ghost fields play roles as regulators. In the case that \( \mu = i \) and \( \nu = j \) we obtain the following ghost contribution

\[
 \langle \Omega | T \{ \Gamma_i(x)\Gamma_j(y) \} |\Omega\rangle = \frac{1}{(2\pi)^4} \int d^4 q D^g_{ij}(q)e^{-iq\cdot(x-y)}
\]
where

\[ D_{ij}^0(q) = q_i q_j \int_0^\infty dk_+ \left[ \delta'(q_-) \frac{n^2}{n^2 k_+^2 + q_+^2} \left( \frac{i}{k_+ - q_+ - i\epsilon} - \frac{i}{k_+ + q_+ - i\epsilon} \right) \right. \]

\[ - \delta(q_-) \frac{2n^2 k_+}{(n^2 k_+^2 + q_+^2)^2} \left( \frac{i}{k_+ - q_+ - i\epsilon} + \frac{i}{k_+ + q_+ - i\epsilon} \right) \]. \quad (3.19) \]

Note that the explicit \( x^- \) dependence gives rise to the factor \( \delta'(q_-) \). Note also that there is no on mass-shell condition among ghost field’s momenta \( k_+, k_1, k_2 \) so that there remains a \( k_+ \)-integration. As a consequence there arise singularities resulting from the inverse of the hyperbolic Laplace operator. Nevertheless, when we regularize the singularities as the principal values, the integral on the first line of (3.19) turns out to be well-defined. In fact we can rewrite its integrand as a sum of simple poles:

\[ \frac{n^2}{n^2 k_+^2 + q_+^2} \left( \frac{i}{k_+ - q_+ - i\epsilon} - \frac{i}{k_+ + q_+ - i\epsilon} \right) = \frac{n^2}{n^2 q_+^2 + q_+^2} \left( \frac{i}{k_+ - q_+ - i\epsilon} - \frac{i}{k_+ - a} \right), \quad (3.20) \]

where \( a = \frac{q_+}{\sqrt{-q_+}} \), and from direct calculations we obtain

\[ \int_0^\infty dk_+ \left( \frac{i}{k_+ - q_+ - i\epsilon} - \frac{i}{k_+ + q_+ - i\epsilon} \right) = -\pi \text{sgn}(q_+), \quad (3.21) \]

\[ \text{P} \int_0^\infty dk_+ \left( \frac{1}{k_+ - a} - \frac{1}{k_+ + a} \right) = 0, \quad (3.22) \]

where \( \text{sgn}(q_+) \) is obtained because we have limited the \( k_+ \)-integration region to be \((0, \infty)\). On the other hand, the integral on the second line of (3.19) yields a linear divergence, which is seen by rewriting its integrand as a sum of simple and double poles:

\[ \frac{2n^2 k_+}{(n^2 k_+^2 + q_+^2)^2} \left( \frac{i}{k_+ - q_+ - i\epsilon} + \frac{i}{k_+ + q_+ - i\epsilon} \right) = \frac{2n^2 q_+}{(n^2 q_+^2 + q_+^2)^2} \]

\[ \times \left( \frac{i}{k_+ - q_+ - i\epsilon} + \frac{i}{k_+ + q_+ - i\epsilon} \right) - \frac{1}{a (n^2 q_+^2 + q_+^2)^2} \]

\[ \times \left( \frac{i}{k_+ - a} - \frac{i}{k_+ + a} \right) - \frac{1}{n^2 q_+^2 + q_+^2} \left( \frac{i}{(k_+ - a)^2} + \frac{i}{(k_+ + a)^2} \right), \quad (3.23) \]

where integrations of the first and second terms on the right hand side are evaluated by the help of (3.21) and (3.22). However we cannot regularize a linear divergence resulting from the double pole by the PV prescriptions. We show below that this linear divergence is necessary to cancel a corresponding one in the physical part. For later convenience we rewrite the linearly diverging integration in the form

\[ \text{P} \int_0^\infty dk_+ \left( \frac{1}{(k_+ - a)^2} + \frac{1}{(k_+ + a)^2} \right) = \text{P} \int_{-\infty}^\infty dk_+ \frac{1}{(k_+ - a)^2}. \quad (3.24) \]
Substituting (3.21), (3.22) and (3.24) into (3.19) yields

\[ D_{ij}^\theta(q) = \frac{n_q^2 q_j q_j}{q^2 + i\epsilon} \delta'(q_-) \pi \text{sgn}(q_+) - \frac{i n_q^2 q_j q_j q_j}{q^2 + i\epsilon} \delta(q_-) \int_{-\infty}^{\infty} dk_+ \frac{1}{(k_+ - a)^2}, \]

(3.25)

where we have made use of the identity

\[ \frac{1}{q^2 + i\epsilon} \delta'(q_-) = - \frac{1}{n_q^2 q_+ + q_-^2} \delta'(q_-) + \frac{2n_q q_+}{(n_q^2 q_+ + q_-^2)^2} \delta(q_-). \]

(3.26)

Thus as the sum of (3.17) and (3.25) we obtain

\[ D_{ij}(q) = \frac{i}{q^2 + i\epsilon} \left( - g_{ij} - n_q^2 q_j q_j - i n_q^2 q_j \pi \text{sgn}(q_+) \delta'(q_-) - n_q q_j \delta(q_-) \int_{-\infty}^{\infty} dk_+ \frac{1}{(k_+ - a)^2} \right). \]

(3.27)

Now we can demonstrate that the linear divergence resulting from \( \frac{1}{q^2 + i\epsilon} \) is canceled by the final term of (3.27) when we restore \( D_{ij}(x) \) by substituting (3.27) into (3.16).

We first change the integration variable from \( k_+ \) to \( q_- = \frac{n_q^2}{m_q}(k_+ - a) \) to rewrite the linear divergence term of (3.27) as follows

\[ \int_{-\infty}^{\infty} dk_+ \frac{n_+}{(k_+ - a)^2} = -n^2 \int_{-\infty}^{\infty} dq_- \frac{1}{q_-^2}. \]

(3.28)

This enables us to show easily that the following \( k_- \)-integration does not give rise to any divergences:

\[
\int_{-\infty}^{\infty} dq_- \frac{1}{q_-^2 + i\epsilon} \left( \frac{1}{q_-^2} - \delta(q_-) \int_{-\infty}^{\infty} dq_- \frac{1}{q_-^2} \right) e^{-iq_-x^-} \\
= \int_{-\infty}^{\infty} dq_- \frac{-e^{-iq_-x^-}}{q_-^2 (q_-^2 + i\epsilon)} + \frac{1}{n_q^2 q_+^2 + q_-^2} \int_{-\infty}^{\infty} dq_- \frac{1}{q_-^2}. 
\]

(3.29)

In fact, we can rewrite the second line further as

\[
\int_{-\infty}^{\infty} dq_- \frac{-e^{-iq_-x^-} - \frac{1}{q_-^2}}{q_-^2 (q_-^2 + i\epsilon)} + \int_{-\infty}^{\infty} dq_- \frac{1}{q_-^2} \left( \frac{1}{n_q^2 q_+^2 + q_-^2} + \frac{1}{q_-^2 + i\epsilon} \right) \\
= \int_{-\infty}^{\infty} dq_- \left( \frac{-e^{-iq_-x^-} - \frac{1}{q_-^2}}{q_-^2 (q_-^2 + i\epsilon)} + \frac{2n_+ q_+ + n_q^2}{(n_q^2 q_+^2 + q_-^2)(q_-^2 + i\epsilon)} \right). 
\]

(3.30)

We see that the last integrals diverge at most logarithmically, but logarithmic divergences can be regularized by the PV prescriptions, so that there arise no divergences from (3.30). This verifies that the following identity effectively holds:

\[
\frac{1}{q_-^2} + i\pi \text{sgn}(q_+) \delta'(q_-) - \delta(q_-) \int_{-\infty}^{\infty} dq_- \frac{1}{q_-^2} \\
= \text{Pf} \frac{1}{q_-^2} + i\pi \text{sgn}(q_+) \delta'(q_-) = \frac{1}{(q_- + i\text{sgn}(q_+))^2} 
\]

(3.31)
where Pf denotes Hadamard’s finite part. It follows that we have the ML form of gauge field propagator:

\[
D_{ij}(q) = \frac{i}{q^2 + i\epsilon} \left( -g_{ij} - \frac{n^2 q_i q_j}{(q_+ + i\text{sgn}(q_+))^2} \right). \tag{3.32}
\]

For other cases we omit detailed demonstrations, because the calculations are similar. In the case that \( \mu = + \) and \( \nu = i \) we obtain the following ghost contribution

\[
D_{+i}^g(q) = q_i \int_{0}^{\infty} dk_+ \left[ \delta(q_-) \frac{n^2}{n^2 k_+^2 + q_+^2} \left( \frac{i}{k_+ - q_+ - i\epsilon} - \frac{i}{k_+ + q_+ - i\epsilon} \right) \right.
\]

\[
+ \delta'(q_-) \frac{n^2 k_+}{n^2 k_+^2 + q_+^2} \left( \frac{i}{k_+ - q_+ - i\epsilon} + \frac{i}{k_+ + q_+ - i\epsilon} \right) - \left. \delta(q_-) \frac{2n^2 k_+ q_+^2}{(n^2 k_+^2 + q_+^2)^2} \left( \frac{i}{k_+ - q_+ - i\epsilon} - \frac{i}{k_+ + q_+ - i\epsilon} \right) \right], \tag{3.33}
\]

where the integrals on the first line give rise to the imaginary term of \( \frac{1}{q_+ + i\text{sgn}(q_+)} \). Hence we obtain

\[
D_{+i}(q) = i \frac{q_i}{q^2 + i\epsilon} \left( \frac{n^2 q_i}{q_+ + i\text{sgn}(q_+)} - \frac{n^2 q_i q_i}{(q_+ + i\text{sgn}(q_+))^2} \right). \tag{3.34}
\]

For the case that \( \mu = \nu = + \) we obtain the following ghost contribution

\[
D_{++}^g(q) = \int_{0}^{\infty} dk_+ \left[ \delta'(q_-) \frac{n^2 k_+^2}{n^2 k_+^2 + q_+^2} \left( \frac{i}{k_+ - q_+ - i\epsilon} - \frac{i}{k_+ + q_+ - i\epsilon} \right) \right.
\]

\[
- \delta(q_-) \frac{2n^2 k_+ q_+^2}{(n^2 k_+^2 + q_+^2)^2} \left( \frac{i}{k_+ - q_+ - i\epsilon} + \frac{i}{k_+ + q_+ - i\epsilon} \right) \right], \tag{3.35}
\]

so that we have

\[
D_{++}^g(q) = \frac{1}{q^2 + i\epsilon} \left( 2n^2 q_+ \delta(q_-) \pi \text{sgn}(q_+) \right.
\]

\[
+ n^2 q_+^2 \delta'(q_-) \pi \text{sgn}(q_+) - q_+^2 \delta(q_-) \int_{-\infty}^{\infty} dq_- \frac{i}{q_-^2} \right) \tag{3.36}
\]

Here it is noted that the last term in (\ref{3.36}) also cancels the linear divergence resulting from the contact term in (3.17). In fact it possesses a contact term, as is seen from

\[
\delta(q_-) \frac{q_+^2}{q^2 + i\epsilon} = \delta(q_-) \left( 1 + \frac{n^2 q_+^2}{q^2 + i\epsilon} \right). \tag{3.37}
\]

Therefore, combining it with the corresponding one of (3.17) in the inverse Fourier transform we get

\[
\frac{1}{2\pi} \int_{-\infty}^{\infty} dq_- \left( \delta(q_-) \int_{-\infty}^{\infty} dq_- \frac{1}{q_-^2} - \frac{1}{q_-^2} \right) e^{-q_- x_-} = \frac{1}{2} \int_{0}^{\infty} dq_- \frac{1 - \cos q_- x_-}{q_-^2} = \frac{|x_-|}{2}, \tag{3.38}
\]
where the Fourier transform of the last term is $-\frac{1}{2} \left( \frac{1}{(q_- + i\epsilon)^2} + \frac{1}{(q_- - i\epsilon)^2} \right)$. Thus we obtain

$$D_{++}(q) = \frac{i}{q^2 + i\epsilon} \left( -g_{++} + \frac{2n_+ q_+}{q_- + i\epsilon \text{sgn}(q_+)} - \frac{n^2 q_+^2}{(q_- + i\epsilon \text{sgn}(q_+))^2} \right)$$

$$- \delta_{\mu+\nu+} \frac{i}{2} \left( \frac{1}{(q_- + i\epsilon)^2} + \frac{1}{(q_- - i\epsilon)^2} \right). \quad (3.39)$$

This demonstrates that, owing to the ghost fields, the linear divergences are eliminated even in the most singular component of $x^+$-ordered propagator.

§ 4. Conserved Lorentz transformation generators

We begin by pointing out that the Lorentz transformation generators $M^{\mu\nu}$ are not all conserved because of the non-covariant gauge fixing term of the Lagrangian (2.5). It gives rise to the non-symmetric term of the symmetric energy-momentum tensor (2.14), which in turn gives rise to nonvanishing terms of the divergence equations of the angular momentum density

$$M^{\mu\nu\sigma} = x^\mu \Theta^{\nu\sigma} - x^\nu \Theta^{\mu\sigma}. \quad (4.1)$$

In fact we obtain

$$\partial_\nu M^{\mu\nu\sigma} = B(n^\nu A^\mu - n^\mu A^\nu) \quad (4.2)$$

where the terms on the right hand side do not vanish in case that one of the indices takes the value $-$, because $n^\mu = (0, 1, 0, 0)$. Therefore we consider obtaining conserved parts $M^i_-$ of the non-conserved Lorentz transformation generators $M^i$.

Because the terms on the right hand side of (4.2) are proportional to the field $B$, we can expect at least that physical parts of the $M^i_-$ are conserved. Moreover there might exist other conserved terms made of the ghost fields. As clues to derive those conserved parts, we make use of the following identity derived by subtracting the terms on the right hand side from the divergence equations (4.2):

$$\partial_\nu M^{\mu\nu\sigma} - B(n^\nu A^\mu - n^\mu A^\nu) = 0. \quad (4.3)$$

We repeat almost the same procedure as in § 3. Integrating the identities over the closed surface depicted in Fig. 1 and assuming that the integrals $\int_{-\infty}^{\infty} \partial_1 M^{\mu\nu} dx^1$ vanish allows us to obtain the following identities

$$0 = \int d^2 x_\perp \left( \int_{-L}^{L} dx^- \left[ M^{\mu\nu+}(x) \right]_{x^+=T}^{x^+=-T} + \int_{-T}^{T} dx^+ \left[ M^{\mu\nu-}(x) \right]_{x^-=-L}^{x^-=L} 

- \int_{-L}^{L} dx^- \int_{-T}^{T} dx^+ B(x)(n^\nu A^\mu(x) - n^\mu A^\nu(x)) \right). \quad (4.4)$$

In case that neither index takes the value $-$, integrating by parts in the transverse directions shows that physical part and ghost part decouple and that the parts of
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Therefore we obtain, in the limits $L \to \infty$ and $T \to \infty$

\[ M^{ij} = \int d^3 x (x^i \partial^j - x^j \partial^i) + \int d^3 x^* B(x^i \partial^j - x^j \partial^i) \frac{1}{\partial^2_+ + n^2 \partial^2_+} C, \quad (4.5) \]

\[ M^{+i} = \int d^3 x (x^+ \partial^i - x^i \partial^+) \]
\[ + \int d^3 x^* B(x^i \partial^+ - x^+ \partial^i) \frac{1}{\partial^2_+ + n^2 \partial^2_+} C + \frac{x^i}{2} B \frac{n_+ n_-}{\partial^2_+ + n^2 \partial^2_+} B \]. \quad (4.6) \]

In case that one of the indices takes the value $-$, it happens that $x^-$-independent ghost operators are multiplied by $x^-$ in the first and third terms of (4.4), which vanish when integrated by $x^-$. It also happens that $x^-$-independent ghost operators are multiplied by $(x^-)^2$ in the second term of (4.4), which vanish because $[(x^-)^2]_{x^-=L, L = 0}$. It follows that we obtain conserved parts $M^c_-$, whose ghost parts do not involve the coordinate $x^-$ at all:

\[ M^c_- = \int d^3 x (x^r \partial^- - x^- \partial^r) \]
\[ + \int d^3 x^* \left( x^r B \frac{1}{\partial^2_+ + n^2 \partial^2_+} \partial^- C - \frac{x^r}{2} B \frac{n^2}{\partial^2_+ + n^2 \partial^2_+} \right). \quad (4.7) \]

Now that we have the Lorentz transformation generators we can calculate their commutation relations. They turn out to be as follows

\[ [P_+, M^{\mu \nu}] = i(g_{\mu}^{\prime \mu} P^\nu - g_{\nu}^{\prime \nu} P^\mu), \quad (4.8) \]

\[ [P_-, M^{r s}] = 0, \quad (4.9) \]

\[ [P_-, M^{r -}] = -i \left( P^r + \int d^3 x^+ x^r B \frac{n_+ n_-}{\partial^2_+ + n^2 \partial^2_+} \theta^2 + B \right), \quad (4.10) \]

\[ [M^{ij}, M^{\mu \nu}] = -i(g^{ij} M^{\mu \nu} - g^{ij} M^{\mu \nu} + g^{ij} M^{\mu s} - g^{ij} M^{js}), \quad (4.11) \]

\[ [M^{+i}, M^{-j}] = -i g^{+ij} M^{ij}, \quad (4.12) \]

\[ [M^{+i}, M^{r -}] = -i(g^{+i} M^{r -} - g^{i r} M^{r -} - g^{+i} m^{r}) \]
\[ - i \int d^3 x^+ x^r \left( B^2 \frac{\partial^{i +}}{\partial^2_+ + n^2 \partial^2_+} C \right. \]
\[ + B \frac{n_+ n_-}{\partial^2_+ + n^2 \partial^2_+} (x^i - \frac{\partial^i}{\partial^2_+ + n^2 \partial^2_+}) \theta^2 B \right), \quad (4.13) \]

\[ [M^{-r}, M^{s -}] = -i(g^{-r} M^{s -} - g^{s r} M^{r -} + g^{-r} m^{s}) \]
\[ + \frac{i}{2} \int d^3 x^* \left( x^s B \frac{n_+ n_-}{\partial^2_+ + n^2 \partial^2_+} x^r B - x^r B \frac{n^2}{\partial^2_+ + n^2 \partial^2_+} x^s B \right), \quad (4.14) \]
where \( p^r, m^{ir} \) and \( m^{rs} \) denote the physical parts of the corresponding generators and \( M^{\mu\nu} \) are understood to be the conserved parts in the case that one of the indices takes the value \(-\). It is noted that owing to the operator identity

\[
\int d^3 x^+ B (x^+ \partial^i - x^j \partial^+)(x^i \partial_x^2 + \frac{1}{n^2 \partial_x^2 + n^2 \partial_x^2}) B = \frac{g^{ij}}{2} \int d^3 x B \frac{1}{\partial_x^2 + n^2 \partial_x^2} B
\]

the commutator (4.12) holds exactly in spite of the fact that both \( M^{+i} \) and \( M^{+j} \) possess the bilinear term of \( B \). We see that commutation relations (4.10), (4.13) and (4.14) differ from corresponding ones in manifestly covariant formulations but the Poincaré algebra is recovered in the physical space.

§5. Concluding remarks

In this paper we have shown that \( x^- \)-independent ghost fields can be introduced as regulator fields in the axial gauge formulation of the noninteracting abelian gauge fields by extending the Hamiltonian formalism à la McCartor and Robertson. We have shown that the ghost fields successfully subtract the linear divergences resulting from the antiderivative \((\partial_-)^{-2}\) so that the ML form of gauge field propagator can be realized. Especially, we have shown that for this cancellation to work it is necessary that Eq. (3.28) holds. It follows from this that a pure space-like axial gauge formulation in the ordinary coordinates, namely the case of \( \theta = \frac{\pi}{2} \) has to be defined as the limit \( \theta \rightarrow \frac{\pi}{2} - 0 \), which enables us to define the otherwise ill-defined left hand side of Eq. (3.28) unambiguously. In contrast with the case of \( \theta = \frac{\pi}{2} \) the light-cone axial gauge formulation can be defined as the case of \( \theta = \frac{\pi}{4} \). This is because, by virtue of \( n^2 = 0 \), we do not have the linear divergences resulting from \((\partial_-)^{-2}\) and from the square of the inverse of a hyperbolic Laplace operator, except for the contact term in the most singular component of the gauge field propagator.

Because consistent pure space-like axial gauge quantization conditions are not given to this time, we have extrapolated the solutions in the temporal gauge formulation and checked that they are also solutions in the axial gauge formulation. Now that \( A_\mu \) described in (2.14) are verified to be the solutions in the axial gauge formulation, we can calculate their commutation relations. For example it can be shown that equal \( x^+ \)-time commutation relation of \( A_+ \) with \( A_i \) is well-defined and described as

\[
[A_+(x), A_i(y)]|_{x^+_y^+} = -\frac{i}{2} [x^+ - y^+] \delta(2)(x_- - y_-).
\]

(5.1)

It remains to be shown whether equal \( x^+ \)-time commutation relations given by \( A_\mu \) in (2.14) can be consistent pure space-like axial gauge quantization conditions, which are needed to quantize interacting gauge fields. We leave these tasks for subsequent studies.
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