Research on Online Mathematical Models of Tandem Cold Rolling

Henan Bu1*, Xingyu Ji1 and Zhuwen Yan2
1 School of Mechanical Engineering, Jiangsu University of Science and Technology, Zhenjiang, Jiangsu, 212003, China
2 Research Department of Intelligent Manufacturing Equipment, Nanjing Institute of Technology, Nanjing, Jiangsu, 211167, China
*Corresponding author’s e-mail: hnbu520@just.edu.cn

Abstract. The tandem cold rolling process control system is an important part of the computer control system of the tandem cold mill coupled pickling line, and it is an important means to ensure the output and quality of cold rolled strip. On-line mathematical models of tandem cold rolling are the premise of formulating rolling rules, setting of rolling mill parameters and calculation of load distribution. They calculate the set values and rolling parameters required by the automation system based on the rolled piece and equipment parameters. Therefore, it is of great practical significance to study the on-line mathematical models of tandem cold rolling and to establish a model of high precision and high value-added products.

1. Introduction
Cold-rolled strip steel is made from hot-rolled strip steel and rolled by a cold-rolling mill at room temperature to achieve the purpose of improving the surface finish and dimensional accuracy of the strip and obtaining better mechanical properties. Because of its good surface quality, high dimensional precision, and good mechanical and technological properties, cold-rolled strip has been widely used in various sectors of the national economy, such as aerospace, automotive appliances, food processing, chemical construction and civil hardware. In recent years, with the rapid development of modern industrial technology, the requirements for the types, specifications and output of strip are also increasing. It is an obvious change in the overall structure of steel in industrial developed countries to increase the output of cold-rolled strip with high added value while ensuring that the strip ratio of steel material is not reduced[1,2].

With the acceleration of China's urbanization and economic development, the industrial structure will be gradually upgraded. The manufacturing industry, represented by household appliances and automobiles, is rapidly improving its production capacity, and the domestic market's demand for cold-rolled products is increasing day by day, and will maintain a long-term growth trend. With the increase of the output of cold-rolled strip, the downstream industry has put forward higher and higher requirements for its quality. Therefore, it is a hot problem to improve the thickness accuracy and shape quality of cold-rolled products[3]. This paper studies the mathematical models of the tandem cold rolling process control system, which lays a foundation for improving the precision of the tandem cold rolling process control[4].
2. Mathematical models for process control
The mathematical models and formula variables used in the setting calculation of the tandem cold rolling mill are shown in Table 1.

| Mathematical models | Formula variables |
|---------------------|-------------------|
| Rolling force       | Deformation resistance, friction coefficient, thickness, strip width, unit tension, roll flattening radius |
| Forward slip        | Roll flattening radius, thickness, deformation resistance, friction coefficient, unit tension |
| Deformation resistance | Thickness |
| Friction coefficient | Rolling length, strip speed |
| Rolling torque      | Deformation resistance, friction coefficient, thickness, strip width, unit tension, work roll radius, roll flattening radius, roll speed |
| Motor power         | Rolling torque, roll speed, work roll radius |
| Roll flattening     | Work roll radius, rolling force, strip width, thickness, deformation resistance, unit tension |
| Rolling mill modulus | Strip width, work roll radius, backup roll radius |
| Thickness gauge     | Strip width, rolling force |

2.1 Rolling force model
In the rolling process of cold rolled strip, the strip will undergo elastic deformation in addition to plastic deformation, so the deformation of cold rolled strip is divided into plastic deformation zone and elastic deformation zone, as shown in Figure. 1. The elastic deformation zone is located at the entrance and exit sides of the rolling deformation zone, and only elastic deformation of the strip occurs in this area. Among them, the inlet side is an elastic compression area, and the outlet side is an elastic recovery area. In the plastic deformation zone, the strip steel will produce permanent plastic deformation[5,6].

\[
F = F_{in} + F_{p} + F_{out}
\]  

(1)
Where, $F^p$ is the rolling force in the plastic zone, and the formula is:

$$F^p = Q_p \left( k_m - \xi \right) W \sqrt{R'(h_{in} - h_{out})}/1000$$  \hfill (2)

$$\xi = (1 - \chi) t_{in} + \chi t_{out}$$  \hfill (3)

$$Q_p = 1.08 - 1.02r + 1.79r \mu \sqrt{1 - r} \sqrt{R'/h_{out}}$$  \hfill (4)

$F^e$ is the rolling force in the elastic zone, and its formula is:

$$F^e = F^e_{in} + F^e_{out}$$

$$= \frac{2}{3} \sqrt{1 - \nu^2} \frac{h_{out}}{E} (k_m - \xi) W \sqrt{R'(h_{in} - h_{out})}$$  \hfill (5)

$F^e$ is the rolling force in the plastic zone, and the formula is:

Where, $F$ is rolling force, kN; $k_m$ is average deformation resistance, MPa; $W$ is strip width, mm; $R'$ is roll flattening radius, mm; $h_{in}$ and $h_{out}$ are strip entrance thickness and exit thickness respectively, mm; $F^e_{in}$ is rolling force in elastic compression zone, kN; $F^e_{out}$ is rolling force in elastic recovery zone, kN; $\nu$ is Poisson's ratio, its value is 0.3; $E$ is elastic modulus of work roll, its value is $21700 \times 9.80665$ MPa; $\chi$ is tension influence coefficient; $t_{in}$ and $t_{out}$ are entrance unit tension and exit unit tension respectively, MPa; $r$ is reduction rate; $\mu$ is coefficient of friction.

The calculation model of roll flattening radius is:

$$R' = R \left[ 1 + \frac{16(1-\nu^2) F \times 1000}{\pi E W \cdot \Delta h_{eq}} \right]$$  \hfill (6)

Where,

$$\Delta h_{eq} = \left( \sqrt{\Delta h_{out}^e} + \sqrt{\Delta h_{in}^p} + \sqrt{\Delta h_{in}^e} \right)^2$$  \hfill (7)

$$\sqrt{\Delta h_{out}^e} = \frac{1 - \nu^2}{E} h_{out}(k_m - t_{out})$$  \hfill (8)

$$\sqrt{\Delta h_{in}^p} + \sqrt{\Delta h_{in}^e} = \sqrt{h_{in} - h_{out} + \frac{1 - \nu^2}{E} h_{out}(k_m - t_{out})}$$  \hfill (9)

Where, $R$ is roll initial radius, mm; $\Delta h_{eq}$ is equivalent reduction, mm; $\Delta h_{in}^p$ is reduction in plastic zone, mm; $\Delta h_{out}^e$ is reduction in elastic compression zone, mm; $\Delta h_{in}^e$ is reduction in elastic recovery zone, mm; $k_{out}$ is exit deformation resistance, MPa.

2.2 Rolling torque model

The rolling torque model is:

$$G = \left[ (k_m - \xi) W R (h_{in} - h_{out}) Q_G + t_{in} W R h_{in} - t_{out} W R h_{out} \right]/1000 + \Delta G_L$$  \hfill (10)

Where,

$$\xi = a \cdot t_{in} + b \cdot t_{out}$$  \hfill (11)

$$Q_G = 1.05 - 0.85 \cdot r + (0.07 + 1.32 \cdot r) \sqrt{1 - r} \mu \sqrt{R'/h_{out}}$$  \hfill (12)

$$\Delta G_L = a_G v_R + b_G$$  \hfill (13)

Where, $G$ is rolling torque, N·m; $R$ is roll radius, mm; $Q_G$ is external friction influence coefficient of rolling torque; $\Delta G_L$ is compensation for mechanical losses, N·m; $a$ and $b$ are influence coefficient of entrance tension and exit tension respectively; $v_R$ is roll speed, m/min; $a_G$ and $b_G$ are mechanical losses coefficients.
2.3 Motor power model
The motor power model is:

\[ P = C_P \frac{1}{\eta} \frac{v_R G}{R} \times \frac{1}{60} \]  

(14)

Where, \( P \) is motor power, kW; \( C_P \) is power adaptive learning coefficient; \( \eta \) is motor efficiency.

2.4 Rolling mill elastic modulus model
The rolling mill elastic modulus model is:

\[ M = M_0 + a_M (W - W_0) + b_M (D_{WR} - D_{WR0}) + c_M (D_{IR} - D_{IR0}) + d_M (D_{BUR} - D_{BUR0}) \]  

(15)

Where,

\[ M_0 = \frac{F_{j+1} - F_j}{S_{j+1} - S_j} \]  

(16)

Where, \( M \) is rolling mill stiffness coefficient, kN/mm; \( M_0 \) is rolling mill stiffness reference value, kN/mm; \( D_{WR} \) is work roll diameter, mm; \( D_{IR} \) is intermediate roll diameter, mm; \( D_{BUR} \) is backup roll diameter, mm; \( S_i \) is rolling mill bounce, mm; \( a_M \sim d_M \) are model coefficients; \( S_j \) and \( F_j \) are rolling mill standard curve points.

The rolling mill stiffness curve is shown in Figure 2. The reference value of rolling mill stiffness \( M_0 \) can be obtained by means of rolling mill standard curve, and the standard curve data is collected in the process of mill calibration.

![Figure 2. Rolling mill stiffness curve](image)

2.5 Forward slip model
During the rolling process, the phenomenon that the exit speed of the rolled piece \( v_h \) is greater than the linear velocity of the roll \( v \) is called the forward slip phenomenon\[7,8\]. According to the definition of forward slip, the actual value of forward slip can be expressed as:

\[ S = \frac{v_h - v}{v} \times 100\% \]  

(17)

The forward slip model uses the Fink formula, as shown in equation (18).

\[ S = \left( \frac{2R'}{h_{out}} \cos \phi_h - 1 \right) \left( 1 - \cos \phi_h \right) \times 100 \]  

(18)

Where,
\[ \phi_n = \sqrt{\frac{h_i^p}{R'}} \tan \left( \tan^{-1} \frac{1}{2} \sqrt{\frac{R'}{h_{out}^p}} \cos^{-1} \left( \frac{1 - h_i^p - h_{out}^p}{2R'} \right) \right) \]

\[ -1 \left( \frac{4\mu}{h_{out}^p} \right) \ln \left( \frac{h_i^p}{h_{out}^p} \frac{1 - t_{out} / k_{out}}{1 - t_{in} / k_{in}} \right) \]

\[ h_i^p = h_{in} + \frac{1}{E} h_{in} (k_{in} - t_{in}) \]  \hspace{1cm} (20)

\[ h_{out}^p = h_{out} + \frac{1}{E} h_{out} (k_{out} - t_{out}) \]  \hspace{1cm} (21)

Where, \( S \) is forward slip rate, \%; \( \phi_n \) is neutral angle, rad; \( h_i^p \) and \( h_{out}^p \) are entrance and exit thickness of plastic zone respectively, mm; \( k_{in} \) and \( k_{out} \) are entrance deformation resistance and exit deformation resistance respectively, MPa.

2.6 Thickness gauge model

The rolling mill thickness gauge model is:

\[ S_e = S_{e0} + g_e(W - W_o) \]  \hspace{1cm} (22)

The rolling mill reference value obtained from rolling mill standard curve is:

\[ S_{e0} = \frac{S_{e1} - S_{e2}}{F_{j1} - F_j}(F - F_j) + S_{ej} \]  \hspace{1cm} (23)

The equivalent rolling mill bounce corresponding to the zero roll gap is:

\[ S_{eZ} = \frac{S_{e1} - S_{e2}}{F_{Z1} - F_Z}(F - F_Z) + S_{eZ} \]  \hspace{1cm} (24)

Where, \( g_e \) is model coefficient; \( F_z \) is no-load zero rolling force, kN; \( g_e \) is no-load zero roll gap, mm.

3. Conclusion

In this paper, the online mathematical models in the tandem cold rolling process control system are studied in depth, and a high-precision rolling force model, rolling torque model, motor power model, rolling mill elastic modulus model, forward slip model, and thickness gauge model are established. The models guarantee the smooth progress of the setting of rolling mill parameters, calculation of load distribution and formulation of rolling regulations.

Acknowledgments

This study was financially supported by the National Natural Science Foundation of China (No.: 51804133) and the Natural Science Foundation of Jiangsu Province (No.: BK20180977, BK20181024).

References

[1] S.C. Li, L.W. Guo, L. G, et al. (2013) Research on mathematical model adaptive in tandem cold rolling. Metallurgical automation, S2: 321-323.

[2] A. Bemporad, D. Bernardini, F.A. Cuzzola, et al. (2010) Optimization-based automatic flatness control in cold tandem rolling. Journal of Process Control, 20: 396-407.

[3] H.N. Bu, Z.W. Yan, D.H. Zhang, et al. (2016) Rolling-schedule multi-objective optimization based on influence function for thin-gauge steel strip in tandem cold rolling. Scientia Iranica, 23: 2663-2672.

[4] N. Venkata Reddy, G. Suryanarayana. (2001) A Set-up Model for Tandem Cold Rolling Mills. Journal of Materials Processing Technology, 116: 269-277.
[5] M. Mashayekhi, N. Torabian, M. Poursina. (2011) Continuum damage mechanics analysis of strip tearing in a tandem cold rolling process. Simulation Modelling Practice and Theory, 19: 612–625.

[6] J.S. Wang, Z.Y. Jiang, A.K. Tieu, et al. (2005) Adaptive calculation of deformation resistance model of online process control in tandem cold mill. Journal of Materials Processing Technology, 162-163: 585-590.

[7] Y.G. Dong, J.F. Song. (2016) Research on the characteristics of forward slip and backward slip in alloyed bar rolling by the round-oval-round pass sequence. International Journal of Advanced Manufacturing Technology, 87: 3605-3617.

[8] M. Poursina, M. Rahmatipour, H. Mirmohamadi. (2015) A new method for prediction of forward slip in the tandem cold rolling mill. International Journal of Advanced Manufacturing Technology, 78: 1827-1835.