Simulation of cylindrical shell collapse with considering plasticity and fracture of metals

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Abstract. In this paper, in one-dimensional cylindrical formulation, the equations of mechanics of continuous medium supplemented by equations of plasticity and fracture models are numerically solved. The results of the calculation were verified on the data obtained by interferometric measurement of the free-surface velocity profile in the problem of high-speed collision of metallic samples. The model is applied to the problem of cylindrical metal shell collapse.

1. Introduction
Experiments on the collapse of cylindrical and spherical metal shells [1] are used in order to study the deformation behavior of metals under conditions of energy cumulation in a convergent shock wave. The investigation of the collapse of cylindrical and spherical shells is of great interest for both the analysis of physical processes related to the phenomena of cumulation and the solution of some engineering problems. At the present time, substantial experimental material has been accumulated, including metallographic data [1]. The simulation of this process is usually carried out in the hydrodynamic approximation [2], or using the simplest models of elastoplastic behavior of metals [3]. Also, the great interest is the solution of models with a more realistic description of the evolution of structural defects in the material, e.g. dislocations, voids, etc. In this work, the collapse of an aluminum shell of cylindrical shape is simulated. In one-dimensional plane, cylindrical formulation, the equations of continuum mechanics are solved using the numerical method [4], supplemented by the equations of the dislocation plasticity model [5, 6] and the equations of nucleation and growth of pores [7, 8]. From a comparison of the calculated data with experiment [9] on plate impact, it turns out that the model describes the deformation of aluminum. The main goal of this work is to apply the previously developed models of dislocation plasticity and fracture to solve the problem of shell collapse.

2. Continuum model
The current problem is solved in one-dimensional cylindrical formulation. In view of symmetry of the problem all non-diagonal components of tensors \( i \neq k \) are equal to zero. Total deformation of the material deformation of the material is described by three components

\[ u_{ik} - w_{ik} + W_{ik}, \] (1)
where $u_{ik}$ is the tensor of macroscopic deformations arising from the motion of substance, $w_{ik}$ is the tensor of plastic deformations arising from the motion of plasticity defects and $W_{ik}$ is the material deformation caused by the development of voids.

The complete stresses is the sum of spherical stress tensor and tensor of deviators

$$\sigma_{ik} = -P\delta_{ik} + S_{ik},$$

where $P$ is the pressure, $S_{ik}$ is the stress-deviator tensor components. The tensor of deviators is found from Hooke’s law

$$S_{ik} = 2G \left[ u_{ik} - \frac{1}{2} u_{ll}\delta_{ik} - w_{ik} + W_{ik} - \frac{1}{3} W_{ll}\delta_{ik} \right].$$

The equation of motion takes the following form due to the choice of one-dimensional cylindrical coordinate system

$$\rho \frac{dv}{dt} = \frac{\partial \sigma_{rr}}{\partial r} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r},$$

where $\rho$ is the density of matter, $v$ is the velocity of the substance.

The equation for internal energy is as follows

$$\rho \frac{dU}{dt} = \left( \frac{d u_{ll}}{dt} + \frac{d W_{ll}}{dt} \right) (-P) + S_{ik} \frac{dw_{ik}}{dt} + Q.$$

The second term in this equation is due to the work of plastic deformation connected with the movement of dislocations and third term is the energy of annihilation of dislocation. The internal energy $U$ does not take into account the change in energy due to the elastic change in shape and energy of the defects. Connection between pressure, density, temperature and internal energy is described by the wide-range interpolation equations of state [10].

The components of the macroscopic strain tensor are defined as

$$\frac{du_{rr}}{dt} = \frac{\partial v}{\partial r}, \quad \frac{du_{\phi\phi}}{dt} = \frac{v}{r}.$$  

(6)

The continuum model supposes an averaged description of the lattice defects in the form of various parameter fields, like a scalar density of dislocations, a concentration of voids, etc [5, 6]. The plastic distortion change is connected with the velocity of dislocation by the Orowan’s equation

$$\frac{dw_{ik}}{dt} = -\frac{1}{2} \sum_{\beta} (b_i^\beta n_k^\beta + b_k^\beta n_i^\beta) V_{D}^{\beta} \rho_{D}^{\beta}. $$

(7)

First multiplier is the force of the mechanical stresses acting on dislocation, second factor is the dislocation velocity, and if we will multiply this by the density of dislocation, we will have the change of plastic tensor in substance. The density of dislocation is the full length of dislocations per volume unit. Index $\beta$ runs all possible combinations of Burger’s vectors $\vec{b}$ and normal vectors $\vec{n}$ of slip planes.

Velocity of dislocation is described by

$$m_0 \frac{dV_{D}^{\beta}}{dt} = \left[ \sum_{i=1}^{N} \sum_{k=1}^{N} S_{ik} b_i^{\beta} n_k^{\beta} \pm \frac{b}{2} \right] \left[ 1 - \frac{V_{D}^{\beta}}{et} \right]^{3/2} - B V_{D}^{\beta},$$

(8)

where $b$ is the modulus of Burger’s vector, $B$ is the phonon-friction coefficient, linearly increasing with temperature. The term in the left-hand side describes the change of momentum of
dislocation \((m_0\) is the rest mass of dislocation per unit of length). The first term on the right-hand side is the forces acting on dislocation from mechanical stresses. Dislocation will begin to move when these forces will be more than yield strength

\[ Y = Y_0 + A_1 G b \sqrt{\sum_\beta \rho_\beta^\beta}. \]  

(9)

The \(bY/2\) term in equation (8) takes into account the resistance to the motion of dislocation from interaction with other dislocations and with point defects, sign of this term is always opposite to the acting forces. The second term in the right-hand side is description of interaction of moving dislocation with phonons and electrons of atom in lattice. The dependence of yield strength on dislocations is considered in a Taylor relation form, equation (8).

The change of density of mobile dislocations is described by the sum of their sources and their immobilization and annihilation at motion

\[ \frac{d\rho_\beta^D}{dt} = \frac{\eta}{\varepsilon_D} \frac{B}{\sqrt{1 - (V_D^\beta/\varepsilon_D)^2}} - k_a b |V_D^\beta| \rho_D^\beta (2\rho_D^\beta + \rho_I^\beta) - V_1 \left( \rho_D^\beta - \rho_0 \right) \sqrt{\rho_I^\beta}. \]  

(10)

Coefficient of phonon drag multiplied by velocity of dislocations and the density of dislocation is equal to the power that is dissipated by the given group of dislocations in unit volume of substance. This expression multiplied by the relation of \(\eta = 0.1\) and divided by the energy required to produce unit length of dislocation line \(\varepsilon_D = 8eV/b\) give us the rate of production of dislocation in the matter. \(\eta\) is the part of dissipated energy, which is spent by the generation of new dislocations. Value of the \(\eta\) was found from calorimetric experiments. The second term describes the process of annihilation of dislocations with opposite sign of Burger’s vectors. The temperature dependence of annihilation factor \(k_a\) was found from theoretical estimations.

The change of density of immobilized dislocations is described by

\[ \frac{d\rho_I^\beta}{dt} = V_1 \left( \rho_D^\beta - \rho_0 \right) \sqrt{\rho_I^\beta} - k_a b |V_D^\beta| \rho_D^\beta \rho_I^\beta. \]  

(11)

In the theoretical model of fracture we assume for simplicity that all voids in material have a spherical form and the same mean radius. Number of defects in material is changed by two mechanisms. First of them is the nucleation of new voids, and second is the activation of already existing voids in the material.

The rate of nucleation of new defects is described by the equation

\[ \frac{dn_1}{dt} = \frac{c}{a^4} \exp \left( \frac{-W_c}{3k_B T} \right). \]  

(12)

Here the first multiplier is the vibration frequency of lattice, where \(c\) is the speed of sound and \(a\) is the average inter-atomic distance. The second term is the probability of the nucleation of new defects, \(W_c = 16\pi\gamma^3\chi/P^2\) is the work of tensile forces against surface tension. The dimensionless reduction factor \(\chi\) differs this expression from the case of liquids.

In addition to the nucleation, the voids can initially exist in material. The already existing pores are distributed in the material exponentially by their surface area. The surface area determines the surface energy of voids and is proportional to the square of its radius. During the tension of material the critical radius is decreased. If the current radius of voids becomes less than the critical radius \(R_c = 2\gamma/(-P)\), a voids will begin to grow. The Heaviside function shows that the concentration of pre-existing voids can only increase. \(n_0\) is the total concentration of
the initial voids; $R_0$ is the size distribution parameter such that the average radius of the initial voids is equal to $R_0/(2\sqrt{\pi})$. It all result in the following equation

$$\frac{dn_2}{dt} = \frac{2n_0}{\sqrt{\pi}R_0}\exp\left(-\frac{(R_0)^2}{R_0}\right)\frac{dR_0}{dt}. \quad (13)$$

Combination of nucleation and activation of voids describes the total increase rate of the active voids concentration

$$\frac{dn}{dt} = \left[\frac{dn_1}{dt} + \frac{dn_2}{dt}\right](1 - \alpha) - n(\nabla \cdot \nu), \quad (14)$$

here $(1 - \alpha)$ multiplier takes into account that the voids can't arise inside the existing ones, in last term in the right-hand part is the vector product, this term takes into account the kinematic expansion or compression of matter.

All voids in the stabilized state have the same average radius

$$\left(\frac{dR}{dt}\right)_P = R \left(2\sigma_\tau^{\text{max}} / \gamma\right)^{1/3} - 1 \left(bV_D\rho_D\sqrt{6}\right). \quad (15)$$

If the tensile stresses $\sigma_\tau^{\text{max}}$ acting on the void will be greater than the yield point, a plastic flow begins around this void

$$\sigma_\tau^{\text{max}} = \frac{G}{K} - 3G\frac{\gamma}{R} + P(G/K + 1/2) \frac{2G - \gamma/R}{2G}, \quad (16)$$

where $G$ is the shear modulus, $K = \mu\rho^2$ is the bulk modulus, $\gamma$ is the surface tension.

Radius of the void will be increase due to the plastic deformation. Velocity of dislocations is described like in equation (8). But the total density of dislocation is the sum of macroscopic dislocation density and dislocation density in the void vicinity that originates from intensive plastic deformation in the course of voids growth $\rho_D = \rho_D^0 + \rho_D^\nu$,

$$\frac{d\rho_D^\nu}{dt} = \frac{\eta}{\varepsilon_D}(b\sigma_\tau^{\text{max}}V_D)(\rho_D^0 + \rho_D^\nu) + b\rho_D^\nu(\rho_D^0 + \rho_D^\nu) + \frac{\pi c_n}{2} \left(\frac{4\pi R^2b}{a_c^3}\right)^{1/2} \exp\left(-\frac{\pi\varepsilon_S a_c}{2k_B T}\right). \quad (17)$$

The rate of dislocation nucleation has the second effective source, which is the voids surface. The vibration frequency of crystal lattice on the surface area of a voids is multiplied into possibility of emission of loop-dislocation from the surface area of the voids; $\varepsilon_S$ is the formation energy of dislocation from void surface per unit length; $a_c = \varepsilon_S/(b\sigma_\tau^{\text{max}})$ is the critical radius of the emitted semi-circular dislocations.

Then the equation for mean radius looks like:

$$\frac{dR}{dt} = \left(\frac{dR}{dt}\right)_P - \left(R - \frac{R_0^3}{R^2}\right)\frac{dn}{dt}. \quad (18)$$

Second term in equation for mean radius corrects for the difference between the same mean radius and the critical radius for activation of the pre-existing voids.

3. Results

For verification of the numerical simulation result we consider the problem with the internal radius of shell much larger than the thickness of the shell. And the problem of collision of metal plates is solved. The impactor with thickness equal to 0.4 mm and speed of motion equal 640 m/s per sec hits the target with thickness equal 2.9 mm. The hit excites the shock wave in the sample. Back-side surface of the sample starts to move, when the shock-wave will come to the surface. In the real experiments, the velocity of back-side surface was measured by VISAR (velocity interferometer system for any reflector—interferometric system for measuring the velocity of rapidly occurring physical phenomena in media with reflective surfaces) by Kanel.
Figure 1. Verification of the metal behavior model by comparison of rear surface velocity histories: calculated results (solid curve) and experimental data [9]; impactor thickness is 0.4 mm, target thickness is 2.9 mm, and impact velocity is 640 m/s.

Figure 2. (a) The experimental results with a shell made of steel. Outer diameter = 106.6 mm, the wall thickness = 5.7 mm, a pressure in the shock wave equal to \( \approx 40 \) GPa [1]. (b) The results of numerical simulation with a aluminum shell. Outer diameter is 46 mm, the wall thickness is 3 mm, a pressure in the shock wave equal to \( \approx 40 \) GPa.

Figure 3 shows how full stresses in the shell change in time. Difference of a shock-wave profile along the radial axis is due to the collapsing of metal shell. At the beginning, the explosive impulse compresses the shell. After 1.6 \( \mu s \), when explosive impulse ends, spalling process starts in the shell due to the shock-wave exit on both sides of the shell. Plastic strain is presented in figure 4. The parts of graph with oscillations are explained by the spalling process. Before the fracture of material, the maximal plastic deformation is realized near the internal side of the shell. Figure 5 shows the dislocation density along the radial axis. Maximum of this density is
Figure 3. Full stress vs thickness of shell at the different moments of time.

Figure 4. Plastic deformation vs thickness of shell at the different moments of time.

Figure 5. Total density of dislocation as a function of thickness of shell at the different moments of time.

obtained on the external side of the shell. Spalling process leads to relaxation of substance, and dislocation density becomes smaller.

4. Conclusion
This paper presents the application of the previously developed dislocation plasticity model and model of fracture to description of the one-dimensional cylindrically symmetric problem, such
as the cylindrical shell collapse. This model can be further used for selection of adequate sizes of cylindrical shells, as well as shock impulse values for various metals (assuming the constants of the models are correct) for further practical experiments on shell collapsing.

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