Packing Properties of Fibres in the Open-Packed Yarn Model

Abstract
Theoretical observations of the packing properties of non-compressible round fibres for two idealized modifications of the open-packed yarn model are discussed. The modifications differ in the method of arrangement of fibres within the cross-sectional ring layer. Modification I has a number of fibres regularly increasing in further layers, and Modification II has the fibres maximum packed in the layers. A procedure for obtaining the number of fibres in the layers of Modification II was proposed. The investigation showed that with the beginning of the 5th layer, the above-mentioned modifications have different packing properties. Because of additional fibres in the layers of Modification II, packing fractions in the layers and yarn obtained were greater if compared with those for Modification I. Analysis of packing properties was made up to 12 layers of the yarn model and also was done for a case of an infinitely large numbers of layers or fibres in a yarn.

Key words: packing fraction, fibres packing, yarn cross-section, yarn model, yarn structure.

Introduction
Since yarns are assemblies of fibres, it becomes important to understand how the fibres are arranged in the yarn cross-section or, in other words, are packed. The yarn geometry and different behaviour of the yarns depend, to a large extend, on the way in which the constituent fibres, i.e. staple fibres or filaments are packed within the yarns. The indices of idealised packing of fibres are widely used in the predicting of such yarn structural properties as the overall density, diameter, twist contraction, linear density, etc. The packing of fibres in a yarn cross-section is of great practical importance because not a little number of characteristics of various woven and knitted fabrics is predetermined by the yarn structure. Therefore the arrangement of fibres in the cross-section of a yarn and the characteristics of packing as an object of the yarn structural morphology have been investigated for years.

The first studies about the idealised packing of fibres in yarns were proposed by Schwarz [1, 2]. Later some aspects of yarn structure were discussed by Gracie [3], Iyer and Phatarfod [4], Hearle and Merchant [5], investigating polyamide (nylon) filament yarns and applying the open-packed model. The basic properties of the open-packed yarn model were summarised by Hearle [6]. In this study, a model of up to six layers was examined, and important parameters, for example, the maximum number of fibres capable of packing into a given layer and the total number of fibres was proposed. The essential features of the regular open-packed model were also given in [7]. Later various modifications of the packing models were used or investigated in other scientific works [8-17]. For instance, Binkevičius [9] applied the open-packed model in yarn twist contraction geometry. Zemlekov and Popov [10] studied the packing of the cross-section of multifilament yarn during axial tension. Although other investigators, for example, Morris et al. [15] used another model, i.e. the hexagonal close-packed model, it was also mentioned that the density of close packing is greater than that required for most yarns. For instance, the packing fraction of the simplest geometrical element of the close-packed model has a value of 0.906 [17], and the packing fraction of the close-packed yarn model computed for 12 ring layers is 0.898 [17]. Therefore, naturally, hexagonal model modifications with reduced packing density when compared with the conventional model are also used in papers by Neckár and Ježek [8], and Morris et al. [15]. On the other hand, the open-packed model with its modifications is another option where the packing fractions can range at different levels. For instance, the number of fibres in several layers of the open-packed model can be found [5-7, 18, 19].

Modern fields of textile applications, like the medtech, indutech, mobitech and pro-tech sectors, among others, are connected with a great variety of fibres and yarns. The yarns for textile materials, especially those for non-conventional applications, are very different in their fineness, number of fibres in the yarn cross-section, etc. Microfilament yarns, nanostructured yarns and others are widely known, for instance, dry spun carbon nanotube fibres and yarns are mentioned by Li et al. [20], and Zhang et al. [21]. Continuous twist-ed nanofibre yarns were developed by He et al. [22]. Some types of micron-sized fibres can be used as elements of yarns, plies, braids and other complex products. Therefore the number of fibres in the yarn cross-section can be much greater than in a conventional yarn structure, and the models of such yarns can also have greater numbers of cross-sectional layers when compared with the above-mentioned structures. Trends in the packing properties of fibres in such yarns where fibres can arrange themselves differently are not well known. Therefore, in the current paper, the packing properties of the open-packed yarn model are studied applying different modifications of the fibre arrangement for an unlimited range of the number of constituent fibres and layers.

Methods and materials
An open-packed yarn model composed of equal, non-compressible and circular fibres is analysed. Figure 1 (see page 58) shows a typical scheme of such a yarn model in which the total gaps between fibres in the layers are specially exhibited. In this model, the fibres are arranged in concentric ring layers, four of which are shown in the scheme. The thickness of each layer equals the fibre diameter. As shown in the model given, a single fibre is in the centre of the yarn. In the nearest to the central fibre layer, six fibres are arranged. The next 12 fibres are located in the 3rd layer. A peculiarity of this layer lies in the arrangement of fibres, they only
touch the circle, which restricts the surface of the 2nd layer. The analogous nature of the arrangement of fibres is shown for the 4th layer. It is necessary to note that the total gaps inside the 3rd and 4th layers \((i=3, i=4)\) are less when compared to the fibre diameter. For \(i \geq 4\), two modifications of the open-packed yarn model, i.e. Modification I and Modification II were studied.

Modification I has a regular number of fibres in the layers. For such a case, the number of fibres in the current layer \(i\), when \(i \geq 2\), is:

\[
n_i = K \times (i-1) \tag{1}\]

where \(K\) is the coefficient of proportionality \((K=6)\).

Table 1. Parameters of the arrangement of maximum packed fibres in the layers of the open-packed yarn model.

| Layer \(i\) | Distance \(l_{ABi}\) | Distance \(l_{ADi}\) | Angle \(\alpha_i\) (rad) | Ratio \(2\alpha_i/\alpha_i\) |
|------------|-------------------|-------------------|-----------------|-----------------|
| 1          | –                 | –                 | –               | –               |
| 2          | \(2r_f\)          | \(3r_f\)          | 1.0473          | 6.0             |
| 3          | \(4r_f\)          | \(5r_f\)          | 0.5057          | 12.4            |
| 4          | \(6r_f\)          | \(7r_f\)          | 0.3351          | 18.8            |
| 5          | \(8r_f\)          | \(9r_f\)          | 0.2500          | 25.1            |
| 6          | \(10r_f\)         | \(11r_f\)         | 0.2001          | 31.4            |
| 7          | \(12r_f\)         | \(13r_f\)         | 0.1674          | 37.5            |
| 8          | \(14r_f\)         | \(15r_f\)         | 0.1443          | 43.5            |
| 9          | \(16r_f\)         | \(17r_f\)         | 0.1265          | 49.7            |
| 10         | \(18r_f\)         | \(19r_f\)         | 0.1096          | 57.3            |
| 11         | \(20r_f\)         | \(21r_f\)         | 0.1020          | 61.6            |
| 12         | \(22r_f\)         | \(23r_f\)         | 0.0895          | 70.2            |
| ...        | ...               | ...               | ...             | ...             |
| Infinitely large value | Infinitely large value | Infinitely large value | 0.000 | Infinitely large value |

This modification has a number of fibres in the yarn

\[
n = 1 + 3 \times i \times (i-1) \tag{2}\]

where \(i\) is the number of layers.

Modification II has a non-regular number of fibres in the layers. For this modification, the layers are maximum packed by fibres. Although the fibres are packed in each layer up to the maximum value, the precondition for the non-compressibility of fibres in the transversal direction is also applied, as was used for the modification previously mentioned.

In view of the large numbers of fibres available in yarns of some structures, both of the two modifications were studied in a range between 1 and 12 layers. Additionally a case of a yarn with an infinitely large number of layers or fibres was also examined. For the yarn model, all distances were computed using the fibre radius \(r_f\).

**Results and discussion**

Geometrical parameters of closely packed fibres

To obtain the geometrical parameters of Modification II, a schematic of the 1st \((i=1)\) and 2nd \((i=2)\) layers (Figure 2) was used as a basis. In this schematic, the fibres of \(i=2\) are shown as maximum packed within the layer.

In a similar way, distances \(l_{ABi}\) & \(l_{ADi}\), and angle \(\alpha_i\) can be specially shown for further layers of the model. It was found that

\[
l_{ABi} = 2 \times r_f \times (i-1) \tag{3}\]

and

\[
l_{ADi} = r_f \times \left(4 \times i^2 - 8 \times i + 3\right)^{1/2} \tag{4}\]

when \(i \geq 2\).

Since

\[
\cos \left(\frac{\alpha_i}{2}\right) = \frac{l_{ADi}}{l_{ABi}} \tag{5}\]

we have

\[
\alpha_i = 2 \times \arccos \left(\frac{4 \times i^2 - 8 \times i + 3}{2 \times (i-1)}\right)^{1/2} \tag{6}\]

The geometrical parameters computed according to Equations (2), (3) and (6) are given in Table 1.
As was expected from the schematic given in Figure 2, distances $l_{ab}$ and $l_{oa}$ tend to increase with different intensity (see Table 1) when the number of layers $i$ increases. Distances $l_{ab}$ and $l_{oa}$ differ minimally when parameter $i$ has the greatest value. On the other hand, the greatest difference between $l_{ab}$ and $l_{oa}$ was shown for the case of two layers (see Figure 2). Therefore the value of angle $\alpha$ is the greatest for $i=2$, i.e. $\alpha_2 = 1.0473$ rad, decreasing up to zero for an infinitely large number of layers. Since angle $\alpha$ is shown between two lines AB and AC, which connect the axes of two maximum packed adjacent fibres of the current layer ($i=2$) with the axis of the central fibre ($i=1$), it is possible to use this value in computations of the number of fibres in layer $n_i$. Having this intention, ratio $2\pi/\alpha_i$ was obtained for each layer (Table 1), varying from 6.0 to 70.2 for layers $i=2$ and $i=12$, respectively. Because of the assumption about the non-compressibility of fibres, in further calculations, the whole numbers of ratio $2\pi/\alpha_i$ were used. Table 1 also shows that for an infinitely large value of $i$, parameters $l_{ab}$ & $l_{oa}$ and $2\pi/\alpha_i$ have infinitely large values.

### Numbers of fibres

Summarised results of fibres for Modification I and Modification II of the open-packed model are presented in Table 2.

The current layer radius was computed as

$$R_i = 2 \times r_i \times (i - 1). \tag{7}$$

To show the main trends, the numbers of fibres are given separately for layers ($n_i$) and for the whole yarn ($n$). Figure 1 shows that both of the two modifications have identical numbers of fibres for the model up to four layers. However, these results are different from the $5^j$th layer, as shown in Figures 3 and 4, where different total gaps between fibres in the layers can be obtained.

Additional fibres of Modification II are marked in Figure 4. For instance, when $i=12$, parameter $n_i$ for Modification I and Modification II is 60 and 70, respectively. When $i=12$, the above-mentioned modifications have numbers of fibres in the yarn of 397 and 410, respectively.

In Figure 5 (see page 60), the trend in the growth of a difference between these numbers as index $\Delta n$ is shown for the yarn model examined up to 12 layers.

For infinitely large numbers of $i$ and $t$, these modifications have infinitely large numbers of $n_i$ and $n$.

#### Packing fractions

At first, the packing properties of fibres in the yarn model layers were studied. For layer $i$, the packing fraction is

$$\Phi_i = \frac{A_i}{A_{fi}}. \tag{8}$$

### Table 2. Data of fibres for Modification I and Modification II of open-packed yarn model.

| Layer $i$ | Layer radius $R_i$ | Number of fibres in layer $n_i$ | Number of layers in yarn $l$ | Number of fibres in yarn $n$ |
|-----------|--------------------|---------------------------------|-------------------------------|-------------------------------|
|           |                    | I     | II   | I     | II   | I      | II      |
| 1         | 0                  | 1     | 1    | 1     | 1    | 1      | 1       |
| 2         | $2r_i$             | 6     | 6    | 2     | 7    | 7      |         |
| 3         | $4r_i$             | 12    | 12   | 3     | 19   | 19     |         |
| 4         | $6r_i$             | 18    | 18   | 4     | 37   | 37     |         |
| 5         | $8r_i$             | 24    | 25   | 5     | 61   | 62     |         |
| 6         | $10r_i$            | 30    | 31   | 6     | 91   | 93     |         |
| 7         | $12r_i$            | 36    | 37   | 7     | 127  | 130    |         |
| 8         | $14r_i$            | 42    | 43   | 8     | 169  | 173    |         |
| 9         | $16r_i$            | 48    | 49   | 9     | 217  | 222    |         |
| 10        | $18r_i$            | 54    | 57   | 10    | 271  | 279    |         |
| 11        | $20r_i$            | 60    | 61   | 11    | 331  | 340    |         |
| 12        | $22r_i$            | 66    | 70   | 12    | 397  | 410    |         |
| ...       | ...                | ...   | ...  | ...   | ...  | ...    | ...     |

| Infinitely large value | Infinitely large value | Infinitely large value | Infinitely large value | Infinitely large value | Infinitely large value |

**Figure 3. Cross-section for Modification I of open-packed yarn model with six layers ($t=6$).**

**Figure 4. Cross-section for Modification II of open-packed yarn model with six layers ($t=6$).**
At first, the packing properties of fibres in the yarn model layers were studied. Packing fractions and values of \( \Phi \) for Modification I, the quantity of fibres in the yarn of 397 and 410, respectively. Because of different differences between the modifications in values of the packing fractions \( \Phi \), they are in the range of +1.7 and +6.0 %.

Finally the packing properties of fibres in yarn were examined. The cross-sectional area of fibres in yarn \( \Sigma A_i \) and cross-sectional area of yarn \( \Sigma A_s \), are:

\[
\Sigma A_i = \pi r_i^2 \quad \text{and} \quad \Sigma A_s = \pi (R_t + r_i)^2 - \pi r_i^2,
\]

where \( t \) is the number of layers in the yarn. In the case of Modification II, the quantity of \( \Phi \) is not stable if \( i \geq 4 \). Because of additional fibres in layers, the values of \( \Phi \) are greater if compared with those results of Modification I. When \( i \geq 4 \), differences between the modifications in values of the packing fractions \( \Phi \) are in the range of +1.7 and +6.0 %.

As in the previous case of the investigation, it is clear that \( \Phi \) equals the same constant when \( n \) increases up to an infinitely large value:

\[
\Phi = \frac{3x}{4x - 1}.
\]

The main parameters of the packing of fibres in yarn for Modification I and Modification II are presented in Table 4. For Modification I, the packing fraction \( \Phi \) decreases from 1.000 for monofilament yarn (\( t = 1 \)) to 0.750 for yarn with 12 layers. Modification II has values of \( \Phi \) in the range between 1.000 and 0.775. When \( t \geq 4 \), differences in \( \Phi \) values obtained between the modifications were from +1.6 to +3.3 %.

When Modification I has an infinitely large number of layers \( t \), Equation (19) shows that

\[
\Phi = \lim_{t \to \infty} \frac{3x^2 - 3x + 1}{4x - 1} = \lim_{t \to \infty} \frac{3(3i^2) + (1/i^2)}{4 - (4(i^2) + (1/i^2)} = 0.750
\]

This result can also be obtained when the number of fibres in yarn \( n \) has an infinitely large value. The positive root of \( t \) from Equation (2) is

\[
t = \frac{1}{2} \left( \frac{4x - 1}{3x} \right)^{1/2}
\]

Therefore Equation (18) acquires the following form:

\[
\Phi = \frac{3x}{4x - 1}.
\]

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It can be also noted that the above-mentioned limiting value of $\Phi$ conforms to the value 0.750 obtained earlier for Modification I when $i = 12$ (see Table 4).

The current results of $\Phi$ values could be considered as a limit for special types of yarns, for example, made from a great number of nanofibers.

## Conclusions

An attempt has been made to analyse the packing properties of non-compressible round fibres in yarn for two idealised modifications, i.e. Modification I and Modification II of the open-packed yarn model. These modifications of the model may be termed as open-packed structures with respect to ring layers situated around the central fibre. However, Modification I represents a case of a model for which the number of fibres in further layers regularly increases. Meanwhile Modification II is based on the assumption of a maximum packed structure within each layer.

To obtain the number of fibres in the layers of Modification II, a procedure of computing of ratio $2\pi \alpha / r$, for the current layer $i$ of the model was proposed. The analysis showed that with the beginning of the $5^{th}$ layer, the above-mentioned modifications differ in packing properties. For Modification I and Modification II, the structures with 12 layers have 397 and 410 fibres in the yarn, respectively. Because of additional fibres in the 5th-12th layers of Modification II, the packing fractions of fibres in the layers $F_i$ are 1.7-6.0 % greater compared with those of Modification I. Moreover parameter $F_i$ of Modification I has a constant value 0.750 for layers with $i \geq 2$. Because of the same reason of additional fibres, values of the packing fraction of fibres in yarn $\Phi$ for Modification II were 1.6-3.3% greater when compared with data of Modification I. For an infinitely large number of layers, $t$, parameter $\Phi$ of Modification I has a limiting value of 0.750. This result was also confirmed for a case of an infinitely large number of fibres in yarn $n$.

### References

1. Schwarz ER. *Textiles and the microscope*. McGraw-Hill, 1934.

2. Schwarz ER. Certain aspects of yarn structure. *Textile Research Journal* 1951; 21: 125-136.

3. Gracie PS. Twist geometry and twist limits in yarns and cords. *Journal of the Textile Institute* 1960; 51 (7): 271-288.

4. Iyer KB and Phatarfod RM. Some aspects of yarn structure. *Journal of the Textile Institute* 1965; 56 (5): 225-247.

5. Hearle JWS and Merchant VB. Relation between specific volume, count and twist of spun nylon yarns. *Textile Research Journal* 1963; 33 (6): 417-424.

6. Hearle JWS, Grosberg P and Backer S. *Structural mechanics of fibres, yarns and fabrics*. Wiley-Interscience, 1969.

7. Sokolov GV. *Theory of twisting of fibrous materials* (in Russian). Light Industry, 1977.

8. Neckář B and Ježek H. Influence exerted by the spinning system and the fibre properties on staple fibre yarns (in German). *Melliand Textilberichte* 1965; 66 (7): 481-485.

9. Binkevicius A. Predicting of physical and mechanical properties of union yarns (in Russian). Ph. D Thesis, Kaunas Polytechnic Institute, Kaunas, 1985.

10. Zemlekov VI and Popov LN. Packing of cross-section of multifilament yarn during axial tension (in Russian). *Technology of Textile Industry* 1988; 3: 11-14.

11. Neckář TB. Relation between compression and filling up of fibrous configurations (in Czech). *Textil* 1989; 44 (10): 366-370.

12. Iyer PB, Sreenivasan S, Patel GS, Iyer KRK and Patil NB. Effect of yarn geometry and fibre properties on tensile behaviour of cotton yarns swelled and stretched in aqueous zinc-chloride. *Journal of Applied Polymer Science* 1991; 42 (11): 2915-2922.

13. Van Langenhove L. Simulating the mechanical properties of a yarn based on the properties and arrangement of its fibres. Part I: The finite element model. *Textile Research Journal* 1997; 67: 263-268.

14. Neckář TB. Yarn fineness, diameter and twist. *Fibres & Textiles in Eastern Europe* 1998; 6 (4): 20-23.

15. Morris PJ, Merkin JH and Rennell RW. Modelling of yarn properties from fibre properties. *Journal of the Textile Institute* 1999; 40 (3): 322-335.

16. Zimliki DA, Kennedy JM and Hirt DE. Determining mechanical properties of yarns and two-ply cords from single-filament data. Part I: Model development and predictions. *Textile Research Journal* 2000; 70: 991-1004.

17. Petrulis D and Petruityte S. Properties of close packing of filaments in yarn. *Fibres & Textiles in Eastern Europe* 2003; 11 40 (1): 16-20.

18. Porwal PK, Beyerlein IJ and Phoenix SL. Statistical strength of twisted fibre bundles with load sharing controlled by friction length scales. *Journal of Mechanics of Materials and Structures* 2007; 2 (4) 773-791.

19. Chattopadhyay R. Advances in textile yarn production. In *Technical textile yarns*. *Industrial and medical applications*. Editors R. Alagirusamy, A. Das. Woodhead Publishing Limited. 2010, pp. 3-55.

20. Li Y-L, Kinloch LA and Windle AH. Direct spinning of carbon nanotube fibers from chemical vapor deposition synthesis. *Science* 2004; 304 (5688): 276-278.

21. Zhang M., Atkinson KR and Baughman RH. Multifunctional carbon nanotube yarns by downsizing: an ancient technology. *Science* 2004; 306 (5700): 1358-1361.

22. He J, Zhou Y, Qi K, Wang L, Li P and Cui S. Continuous twisted nanofiber yarns fabricated by double conjugate electrospinning. *Fibers and Polymers* 2013; 14 (11): 1857-1863.

Table 4. Packing of fibres in yarn for Modification I and Modification II of open-packed yarn model.

| Number of layers in yarn $i$ | Cross-sectional area of yarn $\Sigma_i$ | Cross-sectional area of fibres in yarn $\Sigma_{ai}$ | Packing fraction of fibres in yarn $\Phi$ | Difference, % |
|-------------------------------|----------------------------------------|------------------------------------------------|----------------------------------------|--------------|
| 1                             | $\pi r_i^2$                            | $\pi r_i^2$                                      | $\pi r_i^2$                            | $\pi r_i^2$  |
| 2                             | $2\pi r_i^2$                           | $2\pi r_i^2$                                     | $2\pi r_i^2$                           | $2\pi r_i^2$  |
| 3                             | $2\pi r_i^2$                           | $2\pi r_i^2$                                     | $2\pi r_i^2$                           | $2\pi r_i^2$  |
| 4                             | $4\pi r_i^2$                           | $4\pi r_i^2$                                     | $4\pi r_i^2$                           | $4\pi r_i^2$  |
| 5                             | $8\pi r_i^2$                           | $8\pi r_i^2$                                     | $8\pi r_i^2$                           | $8\pi r_i^2$  |
| 6                             | $16\pi r_i^2$                          | $16\pi r_i^2$                                    | $16\pi r_i^2$                          | $16\pi r_i^2$  |
| 7                             | $32\pi r_i^2$                          | $32\pi r_i^2$                                    | $32\pi r_i^2$                          | $32\pi r_i^2$  |
| 8                             | $64\pi r_i^2$                          | $64\pi r_i^2$                                    | $64\pi r_i^2$                          | $64\pi r_i^2$  |
| 9                             | $128\pi r_i^2$                         | $128\pi r_i^2$                                   | $128\pi r_i^2$                         | $128\pi r_i^2$  |
| 10                            | $256\pi r_i^2$                         | $256\pi r_i^2$                                   | $256\pi r_i^2$                         | $256\pi r_i^2$  |
| 11                            | $512\pi r_i^2$                         | $512\pi r_i^2$                                   | $512\pi r_i^2$                         | $512\pi r_i^2$  |
| 12                            | $1024\pi r_i^2$                        | $1024\pi r_i^2$                                  | $1024\pi r_i^2$                        | $1024\pi r_i^2$  |

The packing fraction of fibres in yarn $\Phi$ for Modification II has a limiting value of 0.750. This result was also confirmed for a case of an infinitely large number of fibres in yarn $n$. 

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