Non-local transport properties of nanoscale conductor-microwave cavity systems

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Recent experimental progress in coupling nanoscale conductors to superconducting microwave cavities has opened up for transport investigations of the deep quantum limit of light-matter interactions, with tunneling electrons strongly coupled to individual cavity photons. We have investigated theoretically the most basic cavity-conductor system with strong, single photon induced non-local transport effects; two spatially separated double quantum dots (DQD:s) resonantly coupled to the fundamental cavity mode. The system, described by a generalized Tavis-Cummings model, is investigated within a quantum master equation formalism, allowing us to account for both the electronic transport properties through the DQD:s as well as the coherent, non-equilibrium cavity photon state. We find sizeable non-locally induced current and current cross-correlations mediated by individual photons. From a full statistical description of the electron transport we further reveal a dynamical channel blockade in one DQD lifted by photon emission due to tunneling through the other DQD. Moreover, large entanglement between the orbital states of electrons in the two DQD:s is found for small DQD-lead temperatures.

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I. INTRODUCTION

In circuit quantum electrodynamics (QED), the mesoscopic analog of cavity QED, solid state qubits are coupled to superconducting microwave cavities on chip [1, 2]. Circuit QED systems combine the appealing properties of high cavity quality factors and strong vacuum microwave fields with low qubit decoherence. This has allowed for experiments in the strong coupling limit with qubit-cavity coupling exceeding the qubits decoherence rates. The strong cavity-qubit coupling together with fast, coherent manipulation of the qubits has lead to an astonishing development in the areas of quantum information processing [3, 4] and microwave quantum optics with superconducting circuits [5, 6]. Moreover, circuit QED architectures have a large potential for simulations of strongly interacting many-body systems [7] and tests of fundamental quantum physical effects [8, 9, 10].

The rapid development in circuit QED triggered investigations on nanoscale qubits and conductors coupled to microwave cavities or resonators [18, 19]. Particularly interesting are recent experiments on few-level quantum dots coherently coupled to microwave cavities [28, 29, 30, 31, 32]. These experiments open up for transport investigations of light-matter interactions in the deep quantum regime: single electrons interacting strongly and coherently with individual microwave photons. The large versatility of microwave photon state properties [19, 34, 35, 36, 37], together with the well-established controllability of quantum dot levels provide a broad scope for fundamentally important experiments.

A key feature of conductor-cavity systems is the possibility to coherently couple electrons in conductors separated up to centimeters [30]. This puts in prospect entangling macroscopically separated transport electrons, of importance for nanoscale quantum information processing and Bell inequality tests [15, 36, 37]. Moreover, this non-local feature can be harnessed for efficient heat-transfer or refrigeration over large distances [38, 39]. A first step towards these goals would be an experimental demonstration of non-local, few-photon mediated, electronic transport effects. In the present work we investigate theoretically the simplest possible strongly coupled cavity-quantum dot system where such non-local effects can be observed: two double quantum dots (DQD:s) coupled to the same transmission line cavity (See Fig. 1). We argue that dot-cavity systems with single-level or metallic dots will, in comparison, display strongly surpressed non-local effects.

The DQD:s are resonantly coupled to the same, fundamental mode of the microwave cavity. This DQD-cavity system constitutes a generalized Tavis-Cummings model [40, 50], with strong hybridization of the DQD electron and microwave photon states. Our investigation is focused on the non-local electronic transport properties. For a broad range of parameters, the current-voltage characteristics provide clear signatures of transport electrons exchanging photons. Further, for asymmetric DQD-lead couplings, noise and higher order cur-
rent fluctuations reveal a dynamical channel blockade in one DQD lifted by single photons emitted by electron-tunneling in the second DQD. In addition, we demonstrate the existence of large orbital entanglement between electrons in different DQDs.

We emphasize that the predicted non-local effects are direct consequences of the non-equilibrated, transport-induced photon state. This makes our investigation qualitatively different from earlier transport studies on pairs of two-level systems coupled via thermalized bosons \([51, 52]\). To fully account for the coherent, non-equilibrium properties of the photon state, as well as the electron tunneling through the DQDs, our investigation is carried out within the framework of a quantum master equation (QME). The approach is similar to the ones used to investigate transport through single two-level systems coupled resonantly to a photonic mode in Refs. \([23, 24, 34]\).

The article is organized as follows: In Sec. II A we introduce the model for the closed DQD-cavity system, present its Hamiltonian and discuss the eigenstates and eigenenergies. We further derive, in Sec. II B, a QME for the system when the DQDs are coupled to lead electrodes. In Sec. II C the transport properties are investigated in the regime where the temperature of the lead electrodes exceeds the DQD-cavity coupling strength. Focus is put on the non-local transport properties, calculating the non-local current-voltage characteristics, the full counting statistics. In Sec. II D we turn to the regime where the lead temperature is smaller than the DQD-cavity coupling strength. Transport signatures of coherent electron-photon interaction as well as entanglement between the electrons in the DQDs are investigated. The effect of dephasing and approaches to minimize this effect are discussed in Sec. II E.

II. SYSTEM AND METHOD

The system considered is depicted in Fig. 1. Two DQDs, denoted 1 and 2, are inserted near the endpoints of a transmission line cavity. The central conductor is capacitively coupled to the right (left) dot in DQD1(2) (see e.g. Ref. \([57]\) for a possible experimental realization). One gate and one lead electrode is further coupled to each dot in the DQDs. The leads are assumed to be in thermal equilibrium with a common temperature \(T\) and chemical potentials \(\mu_{\nu i}\), with \(\nu = L, R\) and \(i = 1, 2\) denoting to which dot the lead is coupled.

Throughout most of the paper we will consider the strong coupling limit. This implies that the DQD-cavity coupling is large compared to the DQD-lead couplings and also dominates over decoherence due to other type of system-environment interaction. Moreover, we will the DQD-lead couplings to be much stronger than the interaction with the rest of the system-environment and hence neglect decoherence from the latter. Only in last section the effect of DQD dephasing as well as DQD-relaxation and photon loss will be considered.

A. Model

The DQDs, forming singly occupied two-level systems, couple linearly to the microwave photons in the cavity. The system Hamiltonian \(\hat{H}_S\), describing the DQD-photon interaction as well as the orbital degrees freedom of the DQDs and the direct interaction between the DQD charges, is derived in Appendix A. Below we will take the DQDs to be on resonance with the fundamental mode of the transmission line cavity. Moreover, the cavity characteristic impedance \(Z_0\) is assumed to be much smaller than the resistance quantum \(R_Q = h/e^2\), relevant for regular transmission line cavities. Under these conditions the DQD-cavity system will be described by a generalized Tavis-Cummings (TC) Hamiltonian \([49]\).

\[
\hat{H}_S = \hbar \omega \hat{a}^{\dagger} \hat{a} + \sum_i \left[ \frac{\hbar \omega}{2} \hat{d}^{\dagger}_{ei} \hat{d}_{ei} - \hat{d}^{\dagger}_{gi} \hat{d}_{gi} \right] + \hbar g_0 (\hat{a}^{\dagger} \hat{d}^{\dagger}_{gi} \hat{d}_{ei} + h.c.). \tag{1}
\]

Here \(\hat{d}^{\dagger}_{gi}\) and \(\hat{d}_{gi}\) (\(\hat{d}^{\dagger}_{ei}\) and \(\hat{d}_{ei}\)) denotes the creation (annihilation) operators of the ground and exited, i.e. of the bonding and anti-bonding, state of DQDi. We have further introduced the photon creation operator \(\hat{a}^{\dagger}\) and the frequency \(\omega\) of the fundamental mode, and the DQD-cavity coupling strength \(g_0\), for simplicity taken equal for both DQDs. Note that since we have assumed single occupancy of the DQDs, the spin-degree of freedom of the DQDs will only have the effect of renormalizing tunneling rates and is hence neglected in Eq. 1 and below.

The generalized TC Hamiltonian in Eq. 1 has the form of a TC Hamiltonian for both DQDs occupied while it reduces to a Jaynes-Cummings (JC) and a harmonic oscillator (HO) Hamiltonian when one or none of the DQDs are occupied, respectively. It is of key importance for the discussion below to describe the eigenstates in the HO, JC and TC subspaces of \(\hat{H}_S\). We first note that \(\hat{H}_S\) commutes with the operator for the number of excitations \(\hat{n} = \hat{a}^{\dagger} \hat{a} + \sum_i \hat{d}^{\dagger}_{ei} \hat{d}_{ei}\). The eigenstates can then be characterized by the corresponding quantum number \(n\).

We express the eigenstates in terms of the DQD-cavity product states \(|\xi_1 \xi_2 n\rangle\), with DQDi in the state \(|\xi_i\rangle\), with \(\xi_i = 0, g, e, \) and \(p\) photons in the cavity mode.

For the HO subspace \(\xi_1 = \xi_2 = 0\) and hence the number of excitations is equal to the number of photons, giving the eigenstates \(|00n\rangle\). In the JC subspace with DQD1(2) occupied the eigenstate with zero excitations is \(|S^0 0\rangle = |g00\rangle\) \((|S^0 0\rangle = |g00\rangle)\). For states with a finite number of excitations the photon state and the state of the occupied DQD hybridises. The states, denoted by \(|S^\pm_n\rangle\), are superpositions of product states with \(n\) and \(n-1\) photons in the mode. For DQD1 occupied they are given by \(|S^+_n\rangle = |(g0n) \pm e(0n - 1)|/\sqrt{2}\) and for
DQD2 occupied we have \(|S^\pm_2n⟩= |0gn⟩±|0en-1⟩|/√2\). The eigenbasis in the TC subspace has a similar structure. The state with zero excitations is a product state \(|D^0⟩= |gg⟩\) and the states with one or more excitations are superpositions of product states with different number of photons. We denote the finite-excitation eigenstates by \(|D^α⟩\), with \(α = 0, ±, \) for \(n=1\) and \(D^{2γ}⟩\), with \(β, γ = ±, \) for \(n≥2\) and give their exact forms in Appendix B. The spectra of the HO, JC and TC Hamiltonians, also given in Appendix B, are shown in Fig. 2. Importantly we see in Fig. 2 that a state with \(n\) excitations has an energy \(n\hbarω+O(hg_0)\) relative to the energy of the state with zero excitations in its subspace. Moreover the TC (HO) ground state is shifted \(-\hbarω/2\) (\(\hbarω/2\)) with respect to the JC ground states.

![Fig. 2: The spectrum of the Hamiltonian in Eq. (1)](image)

B. Quantum Master Equation

The leads and their tunnel coupling to the DQDs are described by the Hamiltonians \(H_L\) and \(H_T\), respectively. The lead Hamiltonian reads \(H_L = \sum_{k,v,i} ε_k^v \hat{c}_{kvi}^\dagger \hat{c}_{kvi}\), with \(\hat{c}_{kvi}^\dagger\) creating an electron in the state with energy \(ε_k\) in the lead connected to dot \(v_i\). In the ground-excited basis the tunneling Hamiltonian is given by

\[
\hat{H}_T = \sum_{k,i} \left( t_{Li} \hat{c}_{kLi}^\dagger \left[ -\sin(θ_i) \hat{d}_{gi} + \cos(θ_i) \hat{d}_{ei} \right] + t_{Ri} \hat{c}_{kRi}^\dagger \left[ \cos(θ_i) \hat{d}_{gi} + \sin(θ_i) \hat{d}_{ei} \right] + h.c. \right),
\]

with \(t_{vi}\) denoting the energy independent lead-dot tunneling amplitude for dot \(v_i\). We have further introduced the DQD mixing angles \(\tan(θ_i) = ± t_{L,Ri}/(\sqrt{ω^2 - t_{L,Ri}^2} ± ω)\), where \(t_{L,Ri}\) denotes the interdot tunneling amplitude of DQDi. The \((+/-)\) sign here refers to the energy difference between orbitals of the left and right dots of DQDi being positive (negative).

We assume weak tunnel couplings between the dots and the leads and restrict the investigation to the sequential tunneling regime. Following the standard Born-Markov approximation scheme a quantum master equation (QME) is derived for the time evolution of the reduced density matrix \(\hat{ρ}\) of the DQD-cavity system \([53]\).

We point out that the lead-dot tunneling rates \(Γ_{∀vi} = 2\pi|t_{vi}|^2 \sum_k δ(ε - ε_k)\) must be chosen much smaller the DQD-cavity coupling strength \(g_0\). This restriction is necessary for the strong coupling condition to hold. Moreover it allows us to neglect coherences between states with an energy difference \(Δε ≥ 2h\hbar_0\), i.e. to perform a secular approximation. Considering for simplicity identical tunnel-couplings to the left and right dot in each DQD, i.e. \(Γ_{L,i} = Γ_{R,i} = Γ_i\), we can write the QME \(\frac{d}{dt}\hat{ρ} = \mathcal{L}[\hat{ρ}]\), with the Liouvillian

\[
\mathcal{L}[\hat{ρ}] = -\frac{i}{\hbar}[\hat{H}_S, \hat{ρ}] - \sum_{ν, i, ξ = e, g} \int dε' dε dτ \tilde{Γ}_{νξi}(τ) \hat{ρ} \left[ f_{νξi}(ε, τ) \hat{d}_{ξi}^\dagger (\hat{c} + \hat{S}_i) δ(ε' - \hat{H}_S) \hat{ρ} + \tilde{f}_{νξi}(ε, τ) \hat{d}_{ξi}^\dagger (\hat{c} + \hat{S}_i) δ(ε' - \hat{H}_S) \hat{ρ} \right. \\
\left. + \tilde{f}_{νξi}(ε, τ) \hat{d}_{ξi} δ(ε' - \hat{H}_S) \hat{ρ} (\hat{c} + \hat{S}_i) + \tilde{f}_{νξi}(ε, τ) \hat{d}_{ξi} δ(ε' - \hat{H}_S) \hat{ρ} (\hat{c} + \hat{S}_i) \right].
\]

Here \(f_{νξi}(ε) = f(ε - μvi)\) and \(\tilde{f}_{νξi}(ε) = 1 - f(ε - μvi)\), with \(f\) denoting the Fermi-function. We have further introduced the rates \(\tilde{Γ}_{Lξi}(θ_i) = \tilde{Γ}_{Rξi}(θ_i) = Γ_i \cos^2(θ_i)\) and \(Γ_{L,Rξi}(θ_i) = Γ_i \sin^2(θ_i)\).

We point out that electron tunneling into or out of the DQDs in the sequential secular regime can be associated with jumps between energy levels in adjacent columns in Fig. 2. This means that the most compelling physical picture will emerge when working in the eigenbasis of \(\hat{H}_S\) (see Sec. III A). By evaluating the rates for the different tunneling processes, described by Eq. (3), in this eigenbasis we can draw the following two important conclusions:

(i) The number of excitations of the system can change with 0 or \(-(+1)\) when an electron tunnels into (out of)
one of the DQDs, as indicated by the red arrows in Fig. 2. The system energy can thus increase or decrease by the energy \( \hbar \omega / 2 + \mathcal{O}(h\nu_0) \) in a tunneling event. Consequently, an electron tunneling through one of the DQDs, e.g., from the left lead into the DQD and then out to the right lead, can change the system energy by \( 0, \pm \hbar \omega + \mathcal{O}(h\nu_0) \). If a tunneling electron changes the energy with \( \mp (\hbar \omega + \mathcal{O}(h\nu_0)) \) a subsequently tunneling electron, in the same or the other DQD, can absorb (emit) this energy. This process, here referred to as the transport electrons exchanging a photon, is depicted in Fig. 3.

(ii) The rate for processes where the energy is increased or decreased for an electron tunneling into dot \( \nu \) in DQDi is proportional to \( \Gamma_{\nu f_i f_{\nu_1}}[\hbar \omega / 2 + \mathcal{O}(h\nu_0)] \) and \( \Gamma_{\nu f_i f_{\nu_1}}[-\hbar \omega / 2 + \mathcal{O}(h\nu_0)] \), respectively. Similarly, the rates for exciting or deexciting the system when electrons tunnel out of DQDi are given by \( \Gamma_{\nu f_i f_{\nu_1}}[-\hbar \omega / 2 + \mathcal{O}(h\nu_0)] \) and \( \Gamma_{\nu f_i f_{\nu_1}}[\hbar \omega / 2 + \mathcal{O}(h\nu_0)] \). Note that the rates for tunneling events in which energy is emitted or absorbed can be controlled by the chemical potentials \( \mu_{f_{\nu_1}} \) and mixing angles \( \theta_{\nu_1} \). Note further, that for \( \nu_0 = 0 \) these rates coincide with the corresponding rates for the DQDs decoupled from the cavity mode, as depicted in Fig. 3.

Based on an analytical solution to the QME (See Appendix C), we can further conclude that a well defined steady-state solution for \( \hat{\rho} \) exist for mixing angles \( \theta_1, \theta_2 > \pi / 4 \). For smaller angles and certain bias configurations the photon number can diverge. In Ref. 31, where a single DQD-cavity system was considered, it was shown that DQD mixing angles \( \theta < \pi / 4 \) can lead to population inversion and hence a cavity lasing state. Since the focus on this work is the few-photon regime we, if not otherwise explicitly stated, focus on mixing angles \( \theta_1, \theta_2 > \pi / 4 \) below.

III. NON-LOCAL TRANSPORT PROPERTIES

The main purpose of this work is to investigate non-local transport properties due to exchange of photons between tunneling electrons in different DQDs. This investigation is carried out in the regime \( \hbar \omega \gg k_B T \gg h\nu_0 \). The experiments reported in Refs. 25, 24, 37 were performed under these conditions. Moreover, for \( k_B T \gg h\nu_0 \) the occupations of the leads do not change significantly over the energy scale \( h\nu_0 \), i.e. \( f_{\nu_1}[\pm \hbar \omega / 2 + \mathcal{O}(h\nu_0)] \approx f_{\nu_1}(\pm \hbar \omega / 2) \). Then there are effectively only two energies in the leads at which the electrons can tunnel into and out of the DQDs. This simplifies the expressions for the tunneling rates into and out of the DQDs (See Sec. III) and allows to reduce the QME to an ordinary master equation (ME) (See Appendix C). We write this ME \( \dot{\rho} = [\hat{H}_S + \hat{M}_P, \rho] \), where \( \hat{P} \) is a vector with the probabilities of eigenstates of \( \hat{H}_S \) and \( \hat{M} \) is the matrix with the transition rates between these eigenstates.

For the system to display non-local transport effects it is clear that two conditions must be fulfilled: First, transport electrons in different DQDs must exchange photons. Second, for at least one of the DQDs the effective tunneling rate into the empty DQD or out of the occupied DQD must be dependent on the number of excitations in the system or the occupation of the other DQD. The most clear non-local effects will thus occur for a state-dependence such that transport becomes blocked in one of the DQDs if photons are not emitted by the other. Here we take the DQD2 to be blocked when there are no excitations in the system. This is accomplished by choosing a bias configuration similar to the one depicted in Fig. 3 i.e. such that \( f_{L_2}(-\hbar \omega / 2) = 1, f_{R_2}(\hbar \omega / 2) = 0, f_{L_2}(-\hbar \omega / 2) = f_{L_2}(\hbar \omega / 2) = 1 \).

A. Current

We first consider the currents in the DQDs as a function of the bias voltage \( V_1 \) across DQD1. The current \( I_1 \) through DQDi is determined by the populations of the eigenstates of the system and the effective tunneling rates between the DQD and its right lead. The currents \( I_1(V_1) \) and \( J_2(V_1) \) are plotted for symmetric bias, \( \mu_{L_1} = -\mu_{R_1} = eV_1 / 2 \), and \( \theta_1 = \theta_2 = \pi / 3 \) in Fig. 4.
The key feature of both current-voltage characteristics is a thermally broadened onset at $eV_1 = \hbar \omega$. In DQD1 the onset occurs when the energies at which the electron can tunnel into and out of the DQD enters the bias window. The electron tunneling through DQD1 will further excite the system to states where tunneling out of DQD2 becomes possible. Hence the onset of the current $I_2$ occurs at the same bias voltage $V_1$. In the limits $\Gamma_1/\Gamma_2 \ll 1$ and $\Gamma_1/\Gamma_2 \gg 1$ the solution to the ME, and hence the currents, can be obtained analytically (See Appendix C). Focusing on the non-locally induced current $I_2(V_1)$, for symmetric bias and $eV_1 \gg k_B T$ we obtain

$$I_2 = \frac{e \Gamma_1 \gamma^2}{1 + \gamma}, \quad \Gamma_1/\Gamma_2 \ll 1$$

$$I_2 = \frac{e \Gamma_2 \gamma^2}{[1 + 2 \cot^2(\theta_2)](1 - 2\gamma + 2\gamma^2) + \gamma^2}, \quad \Gamma_1/\Gamma_2 \gg 1,$$

where $\gamma = f_{1i}(\hbar \omega/2) \cos^2(\theta_1)$. From these expressions it is clear that the magnitude of the induced current can be made $\sim e \Gamma_1$ and $\sim e \Gamma_2$ in the limits $\Gamma_1/\Gamma_2 \ll 1$ and $\Gamma_1/\Gamma_2 \gg 1$, respectively. From the plot of the high-bias current against the asymmetry factor $\Gamma_1/\Gamma_2$ for $\theta_1 = \theta_2$ in Fig. 4 and from further investigation for $\theta_1 \neq \theta_2$, we find that the non-local effect is maximal for $\Gamma_1 \sim \Gamma_2$. We can thus conclude that the non-locally induced current, qualitatively behaving as $I_2 \sim e \Gamma_1 \Gamma_2/((\Gamma_1 + \Gamma_2)$, is considerable for a large range of the parameters $\Gamma_1, \Gamma_2, \theta_1$ and $\theta_2$.

To estimate the magnitude of the non-locally induced current we first recall that strong coupling implies the limit $\Gamma_i \ll g_0$ for the tunneling rates. In recent single DQD-cavity experiments [37, 39] fundamental frequencies $\omega/2\pi \sim 10\text{GHz}$ and DQD-cavity coupling strengths $g_0 \sim 500\text{MHz}$ were reported. This means that strong coupling requires $\Gamma_i \lesssim 100\text{MHz}$ giving non-locally induced currents of the order $I_2 \sim 0.1\text{pA}$. Importantly, currents of this magnitude have been measured in DQD-cavity systems [37].

To further put the magnitude of the non-locally induced current in perspective we briefly discuss non-local transport properties for the system with the DQD:s replaced by single-level, or metallic dots. As follows from our Ref. [32], the ME:s describing the evolution of these systems are explicitly dependent on the parameter $Z_0/R_Q$, typically much smaller than unity for regular transmission lines. Importantly, the ME:s show that the effective tunneling rate into an empty dot or out of an occupied dot is independent on the system state to first order in $Z_0/R_Q$. It thus follows that the non-locally induced current will be proportional to $(Z_0/R_Q)^2$, to first non-vanishing order. In a single-level dot- or metallic dot-cavity system the non-locally induced current will thus, in contrast to the current in a DQD-cavity system, be significantly suppressed.

\section{B. Current correlations}

Having established that photon exchange between transport electrons in the spatially separated DQD:s can result in a non-local current we, as the next natural step, investigate the mechanism behind this exchange. To this aim we consider the low-frequency correlations $S_{ij}$ between the currents in DQDi and DQDj. Current correlations are known to provide information about e.g. the effective charge, interactions and statistical properties of the charge carriers [54]. The correlations $S_{ij}$ can formally be obtained from a number-resolved version of the ME (See Appendix D). Focusing first on the cross-correlations $S_{12}$ we plot in Fig. 4 the cross-correlation Fano-factor $F_{12} = S_{12}/(e \sqrt{I_1 I_2})$ against the bias voltage $V_1$ for different asymmetry factors. Similar to the normalized currents $I_1$ and $I_2$ the cross-correlations have an onset at $eV_1 = \hbar \omega$. However, in contrast to the current $I_2$ (but similar to $I_1$) the cross-correlations have a strong dependence on the asymmetry factor $\Gamma_1/\Gamma_2$. This can be seen by considering the limits $\Gamma_1/\Gamma_2 \ll 1$ and $\Gamma_1/\Gamma_2 \gg 1$ where analytical expressions can be obtained. Above the
the asymmetry factor \( \Gamma \).

\[
F_{12} = \frac{\cos(\theta_1)[1 + \cos^4(\theta_1)]}{[1 + \cos^2(\theta_1)]^2}, \quad \Gamma_1/\Gamma_2 \ll 1
\]

\[
F_{12} = \mathcal{O}(\sqrt{\Gamma_2/\Gamma_1}), \quad \Gamma_2/\Gamma_1 \ll 1.
\]  

Equation (5)

From these expressions we see that the currents in DQD1 and DQD2 are manifestly positively correlated, \( F_{12} > 0 \) (\( \cos(\theta_1) > 0 \)), for \( \Gamma_1/\Gamma_2 \ll 1 \). The correlations are also strong, \( F_{12} \sim 1 \). In contrast, for \( \Gamma_1/\Gamma_2 \gg 1 \), the currents are essentially uncorrelated. The cross-over between the two regimes is shown in Fig. 5 for \( \theta_1 = \theta_2 \). The strong, positive correlations appearing for \( \Gamma_1 \ll \Gamma_2 \) clearly show that tunneling through DQD2 is triggered by tunneling through DQD1. The qualitatively different system behavior in the limits \( \Gamma_1/\Gamma_2 \ll 1 \) and \( \Gamma_1/\Gamma_2 \gg 1 \), respectively is also manifested in the auto-correlations \( S_{22} \).

In Fig. 6 we see that the auto-correlation Fano-factor \( F_{22} = S_{22}/(e\ell_2) \) above onset goes from a sub-Poissonian value, \( F_{22} < 1 \), to a super-Poissonian value, \( F_{22} > 1 \), as the asymmetry factor \( \Gamma_2/\Gamma_1 \) is decreased from infinity to zero. This describes a transition from anti-bunching to bunching behavior of the transport electrons 54.

To connect these findings to the properties of the photon exchange we perform a careful investigation of the ME in the limit \( \Gamma_1 \ll \Gamma_2 \). The processes contributing to transport quantities to leading order in the asymmetry parameter \( \Gamma_1/\Gamma_2 \) are depicted in Fig. 6. From this scheme it is apparent that the states \( |D^0\rangle \) and \( |S_{22}^0\rangle \) will have occupations \( \mathcal{O}(1) \), while the other states have occupation \( \mathcal{O}(\Gamma_1/\Gamma_2) \). The system will thus spend most of its time in the states \( |D^0\rangle \) and \( |S_{22}^0\rangle \) and will occasionally be excited out of this subspace by a tunneling event in DQD1, from the state \( |S_{22}^0\rangle \) to any of the states \( \{|D^0\rangle \} \). The system can from here go back and forth between \( \{|D^0\rangle \} \) and \( |S_{22}^0\rangle \) an arbitrary number of times before relaxing to \( |D^0\rangle \). This will occur on a time-scale \( \sim 1/\ell_2 \). Each tunneling event in DQD1 which excites the system to \( \{|D^0\rangle \} \) will hence be followed by one or more tunneling events in DQD2 during a short time-window \( \sim 1/\ell_2 \). The electrons in DQD2 are thus transported in cascades induced by randomly occurring tunneling events in DQD1 with separation \( 1/\ell_2 \gg 1/\Gamma_2 \). This mechanism, commonly referred to as dynamical channel blockade 55, 56, explains both the positive cross-correlations, \( S_{12} > 0 \), and the bunching of electrons in DQD2, \( F_{22} > 1 \). Importantly, each cascade in DQD2 is initiated by the emission of a single photon emitted by an electron tunneling through DQD1. Our investigation thus supports the physical picture where single photons are exchanged between the DQDs for \( \Gamma_1 \ll \Gamma_2 \).

C. Full counting statistics

To obtain a complete picture of the elementary processes of the charge transport for \( \Gamma_1 \ll \Gamma_2 \) we consider the full transport statistics. The statistics is most clearly visualized via the cumulant generating function (CGF) \( \mathcal{F} \) which can be obtained analytically above onset (See

\[ FIG. 5: \text{Upper panel: Cross-correlation Fano factor } F_{12} = S_{12}/(e\ell_1) \text{ for } \theta_1 = \theta_2 = \pi/3 \text{ and temperature } k_B T = 0.05\hbar \omega. \text{ Middle panel: Cross-correlation Fano factor above onset as a function of asymmetry parameter } \Gamma_1/\Gamma_2 \text{ for } \theta_1 = \theta_2 = \pi/3 \text{ and } k_B T = 0.05\hbar \omega. \text{ Lower panel: Auto-correlation Fano factors } F_{22} = S_{22}/\ell_2 \text{ above onset as a function of asymmetry parameter for mixing angles } \theta_1 = \theta_2 = \pi/3 \text{ and } k_B T = 0.05\hbar \omega. \]
FIG. 6: (a) Schematic of processes in ME contributing to transport quantities to first order in the asymmetry parameter $\Gamma$. The tunneling processes in DQD1 (slow processes) are marked by blue arrows and the tunneling processes in DQD2 (fast processes) are marked by red arrows. (b) Tunneling processes in DQD2. Process (1) and (2) describe the tunneling into and out of the DQD as the system goes back and forth between $|S_0^2\rangle$ and $|D^0\rangle$. Processes (3) and (4) describe the tunneling processes where the system relaxed from $|S_0^2\rangle$ to $|D^0\rangle$.

Appendix D) and is given by

$$F(\chi_1, \chi_2) = -\Gamma_T \left( 1 + \cos^2(\theta_1) \right) - \sqrt{\sin^2(\theta_1) + 4 \cos^2(\theta_1) e^{i\chi_1} (\sin^2(\theta_1) + \cos^2(\theta_1) y)}$$

$$y = (\cos^2(\theta_2) + \sin^2(\theta_2) e^{i\chi_2}) \sum_{n=0}^{\infty} \frac{z^n}{z_0} e^{i n \chi_2},$$

Here $z = \cos^2(\theta_2)/(1 + \cos^2(\theta_2))$, $z_0 = 1 + \cos^2(\theta_2)$ and $\chi_1$ is the counting field for charge transfer in DQDi.

IV. SPECTRAL FINE STRUCTURE AND ENTANGLEMENT

It is interesting to investigate what qualitatively new physical effects come into play in the regime where the thermal broadening in the leads is much smaller than the DQD-cavity coupling strength, i.e. $k_B T \ll \hbar \omega_0$. In this regime the ME-description used in the previous section is no longer valid and we need to consider the full QME of Eq. (3).

A. Transport properties

We first demonstrate that the structure on the scale $\sim \hbar \omega_0$ in the spectrum of the generalized TC Hamiltonian appear in the transport properties of the system for $k_B T \leq \hbar \omega_0$. This fine structure, a manifestation of coherent electron-photon interaction, appear already in the average current and we therefore focus on this quantity. Moreover, we consider the simplest possible parameter regime by taking DQDs with identical mixing angles $\theta_1 = \theta_2 = \theta$, lead-DQD tunneling rates $\Gamma_1 = \Gamma_2 = \Gamma$ and with left (right) leads having the same chemical potential, i.e. $\mu_{i1} = \mu_{i2} = \mu_i$. This will give the same current $I = I_1 = I_2$ in DQD1 and DQD2. This cur-
rent $I$ is readily obtained from a numerical solution of Eq. (3) (See Appendix D for details). In Fig. 7 we plot $I(V)$ for symmetric bias voltage $\mu_L = -\mu_R = eV/2$ and $\theta = \pi/3$ for temperatures ranging from $k_B T \sim h\omega_0$ down to $k_B T \ll h\omega_0$. We clearly see how the single step onset at $eV = h\omega$ is split up into several smaller steps, spaced $\sim h\omega_0$ as the temperature is decreased. These steps can directly be attributed to the structure of the spectrum of the generalized TC Hamiltonian. They are a consequence of eigenstates with energy splittings $\sim h\omega_0$ becoming populated at different bias voltages. It is here interesting to note that signatures of the JC spectrum was found in the frequency-dependent current auto-correlations in the transport through a system with only one DQD coupled to the cavity mode [24].

B. Transport-induced entanglement

A natural question to ask when considering two coupled, spatially separated, DQD:s is to what extent their orbital degrees of freedom become entangled by the exchange of cavity photons. The object of interest, describing the properties of the electronic state with one electron in each DQD, is the reduced two-particle density matrix $\hat{\rho}_T$. The reduced density matrix, of dimension $4 \times 4$, is formally obtained by first projecting the total system density matrix $\hat{\rho}$ onto the TC subspace and then tracing out the photonic degrees of freedom. As follows from the structure of Eq. (3) and the TC-eigenstates (Appendix B), the reduced density matrix can be written as a sum of the four diagonal components in the singlet-triplet basis,

$$\hat{\rho}_T = \rho_{gg} |gg\rangle \langle gg| + \rho_{ee} |ee\rangle \langle ee| + \rho_S |S\rangle \langle S| + \rho_T |T\rangle \langle T|$$

where $|S(T)\rangle = (|eg\rangle - (+) |ge\rangle)/\sqrt{2}$. The entanglement of $\hat{\rho}_T$ is conveniently quantified via the concurrence [8], ranging from 1 for a maximally entangled state to 0 for a non-entangled, separable state. For a density matrix on the form in Eq. (7) the concurrence $C(\hat{\rho}_T)$ takes on the simple form

$$C = \max\{|\rho_S - \rho_T| - 2\sqrt{\rho_S \rho_T}, 0\}$$

To determine if entanglement can be induced by photon exchange we first consider the scheme in Fig. 8 displaying the lowest energy states with the TC-subspace well resolved. By noting that the lowest excited TC-states $|D^{+1}\rangle$ are written $|D^{+1}\rangle = |S0\rangle$ and $|D^{-1}\rangle = (1/\sqrt{2})(|gg1\rangle \pm |T0\rangle)$ it is clear that a selective population of any of the $|D^{+1}\rangle$ states would give an electronic state with a large singlet (S) or entangled triplet (T) component. To demonstrate such a selective population we choose bias voltages $V_i$ and dot-level positions such that the chemical potentials $\mu_i L$ and $\mu_i R$ obey the relations

$$\frac{\hbar \omega}{2} - \frac{\sqrt{2} h\omega_0}{2} < \mu_{Li} < \frac{\hbar \omega}{2} - h\omega_0,$$

$$-\frac{\hbar \omega}{2} > \mu_{Ri} > -\frac{\hbar \omega}{2} - (\sqrt{2} - 1) h\omega_0.$$  

For $k_B T \ll h\omega_0$ then only $|D^{-1}\rangle$ of the excited TC-states becomes populated.

For the chosen parameters, as seen in Fig. 8 only five states of the generalized TC-model contributes to transport. For this case the QME can be solved exactly. Importantly, the steady-state solution gives a reduced two-particle density matrix $\hat{\rho}_T$ with only $\rho_S$ and $\rho_T$ nonzero. As is clear from Eq. (8) the resulting concurrence is finite. For a symmetric parameter setting, i.e. for $\theta_1 = \theta_2 = \theta$, $\Gamma_1 = \Gamma_2 = \Gamma$ and $\mu_{i1} = \mu_{i2} = \mu_i$ this concurrence is given by

$$C = \frac{\cos^4(\theta)}{2[\cos^4(\theta) + \sin^4(\theta)]},$$

showing that the concurrence can reach up to $C = 1/2$ for $\theta \ll 1$. We stress that for the chosen parameters there is no bound on $\theta$ in order to have a well defined solution to the QME.
Having confirmed the existence of large entanglement $C \lesssim 1/2$ we consider the effect of finite solving and modified bias voltage $V = V_1 = V_2$. Solving numerically the QME, the resulting concurrence $C(V)$ is plotted in Fig. 9 for different temperatures. For low temperatures $k_B T \ll \hbar g_0$ the entanglement has an onset when $|D^{-1}|$ is populated, with the concurrence given by Eq. (10). Increasing the voltage further $|D^1|$ is populated as well, decreasing the concurrence due to the finite probability for both entangled triplet, $\mu_T$ and singlet, $\rho_S$, electronic states, clear from Eq. (8). For even larger bias all TC-states $|D^r\rangle$ have finite population and the entanglement disappears. From Fig. 9 it is also clear that increasing the temperature smears the $C(V)$ curve and successively suppresses the entanglement, reaching a separable state at $k_B T \approx \hbar g_0/2$.

We can thus conclude that both in the high-bias and high-temperature regimes, where the system can be described by a ME, the entanglement is zero. To clarify the generality of this observation we investigated the concurrence in all regimes where we could solve the ME analytically (see Appendix C). In these regimes we could formally prove the absence of entanglement. Moreover, we considered the concurrence obtained numerically for a broad range of other system parameters in the ME-regime but did not find any entanglement. We thus conclude that it is highly probable that electrons in the two DQDs can only be entangled for temperatures $k_B T \ll \hbar g_0$, in biasing regimes where TC-states with the same number of excitations are selectively populated.

We stress that our investigation mainly aims at demonstrating the existence of entanglement. We do not analyze how or even if it can be detected by transport measurements. Moreover, we have not made a full investigation over the entire parameter space to identify the regime with the largest entanglement.

V. DEPHASING AND RELAXATION EFFECTS

So far we have neglected dephasing and relaxation effects in the DQD:s as well as loss of cavity photons. From the recent single DQD experiments \cite{37,39} it is clear that the dephasing rate $\Gamma_D$ is much larger than the rates $\Gamma_R$ and $\kappa$ for relaxation and cavity loss, respectively. We thus focus on the effect of dephasing on the results presented above.

Dephasing can qualitatively be accounted for by adding a term to the Liouvillian in Eq. (4). Here $\hat{L}_i = \hat{a}_i^\dagger \hat{a}_{ci} - \hat{a}_{gi}^\dagger \hat{a}_g$ and the dephasing is taken independent, with the same rate $\Gamma_D$, for the two DQDs. An investigation of dephasing in all parameter regimes is beyond the scope of the present article. However, we stress that for strong dephasing, $\Gamma_D \gg g_0$, coherent superpositions between excited and ground states in the DQD:s are suppressed. The steady state solution of Eqs. (3) and (11) is diagonal in the basis of the DQD-cavity product states $|\xi_1\xi_2\rangle$, with $\xi_1 = 0, g, e$ and $p$ the number of photons. As a consequence, electrons and photons are decoupled and the non-local transport effects as well as the DQD entanglement appearing in the regime $\hbar g_0 \gg k_B T$ are suppressed.

Importantly, in both single DQD-experiments \cite{37,39} the dephasing is found to be strong, with $\Gamma_D \sim 1$ GHz, substantially larger than the coupling strength $g_0/2\pi \sim 100$ MHz. It is thus necessary to consider ways to increase $g_0$ and/or suppress $\Gamma_D$, in order to approach the strong coupling limit $g_0 \gg \Gamma_D$ where the non-local effects discussed above are fully developed. First and foremost, the coupling $g_0$ can be increased substantially by increasing the fundamental frequency ($g_0 \propto \omega$), simply by making a shorter cavity. Importantly, since we consider an isolated cavity, $\omega$ is not limited by requirements of an external microwave circuitry. The limit is instead set by the energy gap of the superconducting cavity material, of the order of hundreds of GHz for large gap superconductors as e.g. Nb ($\omega/2\pi \approx 10$ GHz in \cite{37,39}). Second, unconventional transmission line cavities, with a central conductor consisting of e.g. Josephson junctions or SQUIDS, \cite{61,62} can have characteristic impedances $Z_0 \sim 1k\Omega$. This gives coupling strengths $g_0 \sim 0.1\omega$, one order of magnitude larger than for conventional transmission lines. Third, since Refs. \cite{37,39} are the first experiments on DQD:s in cavities, there is probably room for for optimizing the circuit design, further suppressing the dephasing. Taken together, the strong coupling limit of our proposal is arguably within reach experimentally. Moreover, the relaxation rate in $\Gamma_D$ was estimated to $\Gamma_R \sim 100$ MHz, one order of magnitude smaller than the dephasing rate. Hence, in the strong coupling limit $g_0 \gg \Gamma_D$, relaxation is expected to be negligible. In addition, the cavity loss rate $\kappa$ in $\Gamma_D$ was already much smaller than $g_0$, sug-

![Graph showing entanglement as a function of bias voltage](image)

FIG. 9: Concurrence $C$ as a function of bias voltage for different temperatures for $g_0 = 0.1\omega$, $\mu_L + \mu_R = -0.12\hbar\omega$ and $\theta_1 = \theta_2 = \pi/6$. 
VI. CONCLUSION

In conclusion we have theoretically investigated the non-local transport properties of a DQD-cavity system. We have found that the photons emitted by electrons tunneling in one DQD can assist transport of electrons through the other DQD, giving a strong non-locally induced current and large cross-correlations between currents in the two DQDs. Moreover, in the low temperature regime, \( k_B T \ll \hbar g_0 \), we have demonstrated that signatures of the TC-spectrum will appear in the I-V characteristics and that the orbital degrees of freedom of electrons in the two DQDs can become entangled. Importantly, our work provides a theoretical framework for investigations of non-local electronic transport properties in cavity-coupled nanoscale conductors. The analysis can readily be modified to study transport through other nanoscopic two-level systems coupled to cavities, e.g. superconducting single electron transistors and spin qubits, and other nanoscopic two-level systems coupled to cavities, e.g. superconducting single electron transistors and spin qubits.

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Appendix A

We here derive the Hamiltonian for the DQD-transmission line cavity system. The first step is to describe the DQDs within the standard constant-interaction model. Then only the excess dot charges will interact capacitively with the cavity. This means that the total Hamiltonian of the system, \( \hat{H}_S \), becomes the sum of the Hamiltonian for the orbital states of the DQDs, \( \hat{H}_O \), and the Hamiltonian for the cavity, the dot charges and their interactions, \( \hat{H}_C \). For the orbital part \( \hat{H}_O \) we consider DQDs formed by two tunnel coupled quantum dots with a single active spin-degenerate level in each dot. The orbital part of the Hamiltonian, in the localized basis of the DQDs, then has the form

\[
\hat{H}_O = \sum_{i=1,2} \frac{\Delta_i}{2} (\hat{d}_i^\dagger \hat{d}_{Li} - \hat{d}_{Ri}^\dagger \hat{d}_i + t_{Li} \hat{d}_i^\dagger \hat{d}_{Ri} + t_{Ri} \hat{d}_{Ri}^\dagger \hat{d}_i) + \frac{\Delta_i}{2} (\hat{d}_i^\dagger \hat{d}_{Li} - \hat{d}_{Ri}^\dagger \hat{d}_i + t_{Li} \hat{d}_i^\dagger \hat{d}_{Ri} + t_{Ri} \hat{d}_{Ri}^\dagger \hat{d}_i)
\]  

(12)

with \( \Delta_i \) being the energy difference between the bare energies of the orbitals in the left and right dot of DQDi. We recall that \( t_{Li,Ri} \) denotes the interdot tunneling amplitude of DQDi and note that the creation operators \( \hat{d}_i^\dagger, \hat{d}_i \) are related to the eigenbasis creation operators of the orbital states of the DQDs.

The Hamiltonian \( \hat{H}_C \) is derived within the framework of circuit QED. Following the procedure of Refs. [32, 65, 66] we start from the classical Lagrangian of a circuit representation of the system, including the capacitances of the dots. The transmission line is modelled by a single LC-circuit. This will describe the physics of one finite-frequency (the fundamental mode) \( \nu_f \) and the zero-frequency mode. The circuit diagram is shown in Fig. 10, where \( C_{G1}, C_{G2}, C_{LRi} \) and \( C_i, V_{Qvi} \) denote the capacitances and gate voltages of DQDi and \( L_0, C_0 \) the total inductance and total capacitance to ground of the central conductor.

The Lagrangian of the circuit is given by

\[
\mathcal{L} = \sum_{i=1,2} \left( \frac{C_i \dot{\phi}_i - \delta_{i1} \dot{\phi}_{Ri} - \delta_{i2} \dot{\phi}_{Li}}{2} + \frac{C_{0} \dot{\phi}_i^2}{4} \right) + \frac{C_{LRi} \dot{\phi}_i \dot{\phi}_{Ri}}{2} + \sum_{\nu} \frac{C_{Qvi} (\phi_{\nu} - V_{Qvi})^2}{2} + \frac{C_{G1} \dot{\phi}_1^2 + C_{G2} \dot{\phi}_2^2}{2} - \frac{(\phi_1 - \phi_2)^2}{2L_0},
\]  

(14)

where \( \phi_{\nu i} \) and \( \phi_i \) denote the phases of node \( \nu \) and \( i \), respectively (see Fig. 10). The zero- and finite-frequency normal modes, describing the electrostatics and -dynamics of the circuit, respectively, are obtained from the Euler-Lagrange equations. By rewriting the Lagrangian in terms of these normal modes, performing a Legendre transformation and a canonical quantization of
the fundamental mode a quantum Hamiltonian, \( \hat{H}_C \), is obtained. By further writing the excess charges of the dots as \( e(d_{\nu i}d_{\nu i} - n_{gvi}) \), where \( n_{gvi} \) denotes the gate-induced charges, we get \( \hat{H}_C = \hat{H}_I + \hat{H}_{D10} + \hat{H}_{D11} \), with

\[
\hat{H}_I = \hbar \omega \hat{a}^\dagger \hat{a} + \sum_{\nu i} \left[ \frac{e^2(d_{\nu i}^1d_{\nu i} - n_{gvi})^2}{2C_{vi}} + \lambda_{\nu i}(\hat{a}^\dagger \hat{a})^2 \right] \\
\hat{H}_{D10} = \sum_{\nu \mu} U_{\nu 1\mu 2}(d_{\nu 1}^1d_{\nu 1} - n_{gvi})(d_{\mu 2}^1d_{\mu 2} - n_{g\mu 2}) \\
\hat{H}_{D11} = \sum_{\nu \mu} \frac{\lambda_{\nu i}\lambda_{\mu 2}}{2\hbar \omega}(d_{\nu 1}^1d_{\nu 1} - n_{gvi})(d_{\mu 2}^1d_{\mu 2} - n_{g\mu 2}).
\]

(15)

In the total Hamiltonian \( \hat{H}_S = \hat{H}_C + \hat{H}_I \), the part \( \hat{H}_D \) and \( \hat{H}_I \) has the standard form for a few-level dot system linearly coupled to a bosonic mode (see e.g. Ref. [68]), with \( C_{vi} \) and \( \lambda_{\nu i} \) denoting the effective self-capacitances of the dots and the coupling strengths between the photons of the fundamental mode and the dot charges, respectively. The part \( \hat{H}_{D10} \) contains cross terms, \( \sim d_{\nu i}^1d_{\nu i}d_{\mu 2}^1d_{\mu 2} \), describing direct non-local coupling between the dot charges. The coupling strengths, \( U_{\nu 1\mu 2} \), depend parametrically only on the capacitances of the circuit in Fig. 10 i.e. not on the inductance \( L_0 \), and would thus remain unchanged if this inductance was short-circuited. This means that \( \hat{H}_{D10} \) describes purely electrostatic, or capacitive, coupling. The part \( \hat{H}_{D11} \), just as \( \hat{H}_{D10} \), describes direct non-local coupling between the dot charges. However, in contrast to \( \hat{H}_{D10} \) the coupling strengths in \( \hat{H}_{D11} \), \( \lambda_{\nu i}\lambda_{\mu 2}/\hbar \omega \), depend parametrically on the inductance \( L_0 \). This part is therefore electrodynamic.

As the next step we approximate the amplitudes leading from Eqs. (12) and (15) to Eq. (1), under the conditions described in the main text. To do this we use the relations \( \lambda_{\nu i} \propto \sqrt{Z_0/RQ} \omega \) (\( Z_0 = \sqrt{L_0/C_0} \)), \( \omega \sim 1/\sqrt{L_0C_0} \), and \( U_{\nu 1\mu 2} \sim e^2/C_0 \) for the parameters in \( \hat{H}_C \). First, noting that \( Z_0 \ll RQ \) and hence \( \lambda_{\nu i}/\hbar \omega \ll 1 \) justifies a rotating-wave approximation, which amounts to neglecting all terms of \( O(\lambda_{\nu i}/\omega) \) in \( \hat{H}_S \), e.g. the counter-rotating terms. Second, for the DQD:s resonant with the cavity mode the direct capacitive interaction will scale as \( U_{\nu 1\mu 2}/\lambda_{\nu i}(\mu i) \sim \sqrt{Z_0/RQ} \) and can thus be neglected. Moreover, we considered a full description for the transmission-line DQD circuit, including all of the cavity modes, the higher frequency modes would be off-resonant with a detuning \( \Delta E \gtrsim \hbar \omega \). The corrections due to this off-resonant interaction would then scale as \( \lambda_{\nu i}/\hbar \omega \) and therefore be negligible. It should be noted that for the resonance condition to hold the tunneling amplitudes and detunings between the left and right dot-orbitals the DQD:s must be chosen such that \( |\Delta| = 2\sqrt{\hbar \omega} \), \( \lambda_{\nu i}/\hbar \omega \). We further point out that the DQD-cavity coupling strengths is \( g_i = \sin(2\theta_i)(\lambda_{Ri} - \lambda_{Li})/2 \). Thus, for the case of iden-

tical coupling strengths \( g_1 = g_2 = g_0 \), considered in the main text, the mixing angles \( \theta_1 \) and \( \theta_2 \) cannot be tuned independently for fixed \( \lambda_{Li} \) and \( \lambda_{Ri} \).

Appendix B

We here give the explicit form for the eigenstates with a finite number of excitations in the TC-subspace and all eigenenergies of \( \hat{H}_S \). The former are given by

\[
|D^{01}\rangle = |S0\rangle, \quad |D^{\pm 1}\rangle = |gg1\rangle \pm |T0\rangle/\sqrt{2}
\]

(16)

and for \( n \geq 2 \)

\[
|D^{+n}\rangle = \sqrt{n-1}|ggn\rangle - \sqrt{n}|eedn - 2\rangle, \\
|D^{\pm n}\rangle = \sqrt{n}|ggn\rangle \pm \sqrt{n-1}|eedn - 2\rangle \pm |Tn - 1\rangle/\sqrt{2}, \\
|D^{-n}\rangle = |Sn - 1\rangle,
\]

(17)

with \( |S(T)n\rangle = (|egn\rangle + (-)^n|gen\rangle)/\sqrt{2} \). The eigenenergies are

\[
\epsilon_{00n} = \hbar \omega(n + 1)
\]

for the HO subspace,

\[
\epsilon_{S0n} = \hbar \omega/2, \quad \epsilon_{S^0n} = \hbar \omega(n + 1/2) \pm \sqrt{n}\hbar g_0, \quad n \geq 1
\]

(19)

for the JC subspaces, and

\[
\epsilon_{D0n} = 0, \quad \epsilon_{Dn} = \hbar \omega/2, \quad \epsilon_{D\pm 1} = \hbar \omega \pm \sqrt{2}\hbar g_0, \\
\epsilon_{D\pm \pm n} = n\hbar \omega \pm \sqrt{2}(2n - 1)g_0, \quad \epsilon_{D\pm \pm n} = n\hbar \omega, \quad n \geq 2
\]

(20)

for the TC subspace.

Appendix C

In this Appendix we explain how the QME in Eq. (3) can be reduced to a ME in the limit \( \hbar g_0 \ll k_B T \), give the explicit form of the ME and solve it in three limiting cases. We start by pointing out that in the secular regime \( \Gamma_i \ll g_0 \), considered here, only coherences between degenerate states, i.e. \( |+ - n\rangle \) and \( |-- + n\rangle \) need to be taken into account in the QME. Moreover, only the diagonal elements \( \langle S^0 n|\hat{\rho}|S^0 n\rangle \) couple to the coherences. As pointed out in the text, for \( g_0 \ll k_B T \), the QME becomes independent on \( g_0 \). This introduces additional symmetries in the QME, with two important consequences: (1) The coherences \( \langle + - n|\hat{\rho}|+ + n\rangle \) and \( \langle + - n|\hat{\rho}|+ - n\rangle \) couple with opposite signs to \( \langle S^+ n|\hat{\rho}|S^+ n\rangle \) and \( \langle S^+ n|\hat{\rho}|S^- n\rangle \). (2) For several pairs of diagonal elements of \( \hat{\rho} \), only the sums of the elements couple to the
other diagonal elements. In particular, this holds for the sum $(S_i^+ n | \hat{\rho} | S_i^+ n) + (S_i^- n | \hat{\rho} | S_i^- n)$, to which the coherences, according to (1), do not contribute. As a result of (1) and (2), the coherences decouple from the diagonal elements of the QME, allowing us to reduce it to a standard ME.

To write the explicit form of the ME it is convenient to first introduce a shorthand notation for the diagonal elements of $\hat{\rho}$, i.e. the probabilities for the eigenstates of $H_\alpha$. The probabilities, or the sums of probabilities, for states the HO, JC and TC subspaces are denoted by

\[
P_{00}^n = (00n | \hat{\rho} | 00n),
\]

\[
P_{S_i}^n = \delta_{n0} (S_i^0 | \hat{\rho} | S_i^0) + (1 - \delta_{n0}) \sum_{a=\pm} \langle S_i^a n | \hat{\rho} | S_i^a n \rangle
\]

respectively. By further introducing vectors $P_X = (P_{X0}^0, P_{X1}^1, P_{X2}^2, \ldots)^T$, with $X = 00, S_1, S_2$, containing the probabilities for states with one or both DQD:s unoccupied and vectors $P_D = (P_D^0, P_D^1, P_D^2, \ldots)^T$ and $P_{D^{+-}} = (P_{D^{+-0}}, P_{D^{+-1}}, P_{D^{+-2}}, \ldots)^T$ containing the probabilities for states with both DQD:s occupied, the ME can be written

\[
\frac{d}{dt} \begin{pmatrix} P_X \\ P_D \end{pmatrix} = \begin{pmatrix} M_X \\ M_D \end{pmatrix}.
\]

The submatrices in $M$ above and below the diagonal are here given by

\[
(M_{00}^{S_0})_{nm} = \sum_{j=0,1} \delta_{nj} G_j^0, \quad (M_{S_1}^{Z})_{nm} = \sum_{j} \delta_{nm+j} J_j^Y G_j^1, \quad x_{nj}^D = \frac{4n+3-2j}{4(2n+1)}, \quad x_{nj}^{D^{+-}} = \frac{n+j}{2n+1}, \quad x_{nj}^{D^{-+}} = \frac{1}{4}
\]

\[
(M_{S_i}^{D0})_{1m} = \delta_{1m} G_i^0, \quad (M_{S_i}^{D^1})_{1m} = \delta_{1m} G_i^1/2, \quad (M_{S_i}^{D1})_{1m} = \delta_{1m} G_i^1/4 (M_{D0})_{n1} = \delta_{n1} G_i^0.
\]

and

\[
(M_{S_i}^{S_0})_{nm} = \sum_{j=0,1} \delta_{nm-j} G_j^1, \quad (M_{S_i}^{H})_{nm} = \sum_{j=0,1} \delta_{nm-j} G_j^2, \\
y_{nj}^D = \frac{4n+3+2j}{4(2n+1+2j)}, \quad y_{nj}^{D^{+-}} = \frac{n+j}{2n+1+2j}, \quad y_{nj}^{D^{-+}} = \frac{1}{2}
\]

\[
(M_{D^{+-}})^{S_0} = \delta_{1n} G_i^0/2 + \delta_{n2} G_i^1/2, \quad (M_{D^{+-}})^{S_1} = \delta_{n1} G_i^0/4 + \delta_{n2} G_i^1/4,
\]
respectively. Here $G_0^0 = \sum_i \Gamma_{vqi} f_{vqi} (-\hbar \omega / 2)$, $G_1^0 = \sum_i \Gamma_{vqi} f_{vqi} (\hbar \omega / 2)$, $G_0^1 = \sum_i \Gamma_{vqi} f_{vqi} (-\hbar \omega / 2)$, $G_1^1 = \sum_i \Gamma_{vqi} f_{vqi} (\hbar \omega / 2)$ and we use the index convention $i = 2$ and $2 = 1$. The submatrices on the diagonal in $M$ are diagonal with elements such that the sum of every column in $M$ is zero. This structure ensures that the ME conserves probability.

We find the steady-state solution to the ME in Eq. (22) analytically in three limiting cases:

(i) For $\theta_1 = \theta_2$, symmetric bias voltages $eV_1, eV_2 \gg \hbar \omega$ applied across both DQD1 and DQD2.

(ii) For the bias condition of Sec. III with $\Gamma_1 \gg \Gamma_2$ and symmetric bias voltage across DQD1. Here we calculate the distribution to zeroth order in $\Gamma_1 / \Gamma_2$.

(iii) For the bias condition of Sec. III with $\Gamma_2 \gg \Gamma_1$ and symmetric bias voltage across DQD1. The distribution is here calculated to first order in $\Gamma_1 / \Gamma_2$.

The solution for case (i) is used to derive the stability condition in Sec. IV while the solutions for the cases (ii) and (iii) are used to obtain the analytical expression for the current in DQD2, correct to first order in $\Gamma_2 / \Gamma_1$ and $\Gamma_1 / \Gamma_2$, respectively. (See Sec. IV.

To find the solution of the ME in case (i) we use a general property of the ME. This property states that $P_n^S_1$ and $P_n^S_2$ will couple only to the probabilities for states with both DQD:s unoccupied having $n$ or $n + 1$ excitations and for states with both DQD:s occupied having $n$ or $n - 1$ excitations (See Fig. 2). In turn the probabilities for these states will couple to $P_{n+k}^S_1$ and $P_{n+k}^S_2$, with $k = -1, 0, 1$. Two coupled second order difference equations are thus obtained for the probabilities in $P_n^S_1$ and $P_n^S_2$. For the special conditions (i) these difference equations become particularly simple: Introducing the vector $P_n = (P_n^S_1, P_n^S_2)^T$ these equation be written

\[
\sin^4(\theta) M_0 P_1 = \left(2 \cos^4(\theta) M_0 + [1 + \cos^2(\theta)] B \right) P_0
\]

\[
\sin^4(\theta) M_1 P_2 = \left[\cos^4(\theta) M_1 + \cos^4(\theta) M_0 + \frac{3B}{2} \right] P_1
\]

\[
-2 \cos^4(\theta) M_0 P_0
\]

\[
\sin^4(\theta) M_n P_{n+1} = \left[\cos^4(\theta) M_n + \sin^4(\theta) M_{n-1} + \frac{3B}{2} \right] P_n
\]

\[
- \cos^4(\theta) M_n - 1 P_{n-1}, \quad n \geq 2
\]

with

\[
B = \Gamma_1 \Gamma_2 \left( \begin{array}{cc} 1 & -1 \\ -1 & 1 \end{array} \right),
\]

\[
M_n = \left( \begin{array}{cc} \Gamma_1^2 / 2 + \Gamma_2^2 b_n & \Gamma_1 \Gamma_2 (1/2 + b_n) \\ \Gamma_1 \Gamma_2 (1/2 + b_n) & \Gamma_2^2 / 2 + \Gamma_1^2 b_n \end{array} \right),
\]

and

\[
b_n = \frac{(4n + 3)(4n + 1)}{4(2n + 1)[4n + 1 + 2 \cos^2(\theta)]} + \frac{1}{4}
\]

\[
+ \frac{n(n + 1)}{2(2n + 1)[n + \sin^2(\theta)]}.
\]

Together with the condition $P_n \to 0$ for $n \to \infty$ Eq. (29) has the solution $P_n = \cot^{4n+2}(\theta)(1/2)^2 P_{11}^{11}$ independent on $\Gamma_1$ and $\Gamma_2$. This solution is then used to find the probabilities for the states with both DQD:s occupied and unoccupied, respectively. Requiring that the solution to Eq. (23) is normalized we get

\[
P_{00}^S = \cot^4(\theta)x_0, \quad P_{01}^S = \cot^2(\theta)x_0,
\]

\[
P_{10}^S = 2\cot^{4n+2}(\theta)x_0, \quad n \geq 1
\]

\[
P_{11}^S = x_0, \quad P_{12}^S = 2P_{01}^D = 2\cot^4(\theta)x_0,
\]

\[
P_{22}^D = 2P_{12}^D = 2\cot^4(\theta)x_0 \quad n \geq 2
\]

with $x_0 = [1 - \cot^4(\theta)]/[1 + \cot^2(\theta) + \cot^4(\theta)]^2$. It is clear from Eq. (29) that a well defined solution, or equivalently a solution with non-infinite mean number of excitations, exists if $\theta > \pi/4$. On physical grounds we argue that there exists a more general stability condition applying also for mixing angles $\theta_1 \neq \theta_2$. We start by considering the case when $\theta_1 = \theta_2 > \pi/4$ and one of the mixing angles is increased. From the discussion below Eq. (3) it is clear that this will increase absorption relative to emission of photons by tunneling electrons. The mean number of excitations will then be decreased and the distribution must therefore still be convergent. Since all pairs of mixing angles $\theta_1, \theta_2 > \pi/4$ can be reached this way it follows that a well defined solution exists for all of them. The mean number of excitations will also decrease if the bias voltage is decreased. The stability condition, $\theta_1, \theta_2 > \pi/4$, must therefore also hold for finite bias voltages. This conclusion is further supported by numerical investigations.

In the limiting case (ii) the relation between probabilities for states with DQD2 unoccupied, e.g. $P_{00}^S$ and $P_{01}^S$, are entirely determined by the tunneling in DQD1. Similarly he tunneling in DQD1 entirely determines the the relation between probabilities for states with DQD2 occupied. The tunneling in DQD2 only effects the total probability for DQD2 being occupied. The main steps in the solution of the ME in this limit are most clearly visualized by rewriting $M P = 0$ as

\[
\begin{pmatrix}
M_{00}^D & M_{01}^D & M_{10}^D & M_{11}^D
\end{pmatrix}
\begin{pmatrix}
P_{20}^D \\
P_{21}^D
\end{pmatrix} = 0,
\]

where the vectors $P_{20}^D$ and $P_{21}^D$ contain the probabilities for states with DQD2 unoccupied and occupied, respectively. The matrix $M$ further contains the transition rates in $M$ divided by $\Gamma_1$. The starting point of the derivation is to note that for $\Gamma_2 = 0$ the matrix is block-diagonal, with blocks $M_{ii}^{(0)}$, and that det($M_{ii}^{(0)}$) = 0. It then follows that the eigenvalue zero of the matrix
\(\tilde{M}\) is doubly degenerate for \(\Gamma_2 = 0\). To find the solution \(\tilde{P}^{(0)}\) to Eq. (30) to zeroth order in \(\Gamma_2/\Gamma_1\), i.e. the limit of \(\tilde{P}\) as \(\Gamma_2/\Gamma_1 \to 0\), we must therefore apply degenerate perturbation theory generalized to ME matrices. The first step in this procedure is to find the two linearly independent solutions to Eq. (30), i.e. the eigenvectors corresponding to the eigenvalue zero of \(\tilde{M}\). Setting \(\Gamma_2 = 0\) in the equation the two linearly independent eigenvectors acquire the forms \(\tilde{P}_0 = (\tilde{P}^{(0)}_0 0)^T\) and \(\tilde{P}_1 = (0 \tilde{P}^{(0)}_1)^T\), where \(\tilde{P}^{(0)}_0\) and \(\tilde{P}^{(0)}_1\) fulfill the equations \(M^{(0)}\tilde{P}^{(0)}_0 = 0\) and \(M^{(1)}\tilde{P}^{(0)}_1 = 0\), respectively. Importantly these equations can readily be solved analytically as they give difference equations similar to Eq. (26). The next step is to express \(\tilde{P}^{(0)}\) as a normalized linear combination of these vectors, i.e. \(c_0\tilde{P}_0 + c_1\tilde{P}_1\), to which \(\tilde{P}\) tend as \(\Gamma_2/\Gamma_1 \to 0\). To do this we define the projectors \(\mathcal{P} = \tilde{P}_0(U_0,0) + \tilde{P}_1(0 U_1)\) and \(\mathcal{Q} = 1 - \mathcal{P}\). Here \((U_0,0)\) and \((0 U_1)\), with \(U_i = (1 \ 1 \ ... )\), are the left eigenvectors to the eigenvalue zero in \(\tilde{M}\) normalized so that \(U_i\tilde{P}_{21} = 1\). Then by applying the steps presented in Ref. [69] the equation \(\mathcal{P}\mathcal{M}(\gamma/P\mathcal{P}) = 0\) for \(\tilde{P}^{(0)}\) is obtained. In turn this equation gives

\[c_0U_0M_0\tilde{P}_{20} + c_1U_1M_{01}\tilde{P}_{21} = 0,\]

which together with the normalization condition \(c_0 + c_1 = 1\), determines \(c_0\) and \(c_1\). The different element in \(\tilde{P}^{(0)}\) can then be written

\[P^n_{00} = \frac{\eta\gamma^2}{[1 - 2\gamma + 2\gamma^2][1 + 2\cot^2(\theta_1)] + \gamma^2},\]

\[P^n_{10} = \frac{\eta[1 - \gamma^2][1 + 2\cot^2(\theta_1)]}{(1 - \gamma + \gamma^2)[1 + 2\cot^2(\theta_1)] + \gamma^2},\]

and \(\eta = (1 - 2\gamma)/(1 - \gamma + \gamma^2)\). With the probabilities in Eq. (32) and the rates between DQD2 and its right lead for the corresponding states it is straightforward to derive the second expression for the current through DQD2 in Eq. [5].

To solve the ME in case (iii) we first recall, from the main text, that only the state and transitions depicted in Fig. [6] contribute to the transport quantities to first order in \(\Gamma_1/\Gamma_2\). To find the probability distribution to \(\mathcal{P}\) first order in \(\Gamma_1/\Gamma_2\) only the additional transitions \(\{|P^{(1)}\}\} \to |S_20\rangle\) need to be taken into account. Thus, the distribution is obtained by solving an effective ME

\[M_r\tilde{P}_c = 0.\]

We get

\[P^0_D = \frac{1}{1 + \frac{\Gamma_1}{\Gamma_2}} - \frac{\Gamma_1}{\Gamma_2} \frac{2[1 + \cos^2(\theta_1)]}{\sin^2(\theta_1)} \gamma^2\]

\[+ \frac{\Gamma_1}{\Gamma_2} \frac{[1 + \cos^2(\theta_1)]}{\sin^2(\theta_1)} (1 + \gamma)^2\]

\[P^0_{S_0} = \frac{\gamma}{1 + \gamma} - \frac{\Gamma_1}{\Gamma_2} \frac{[1 + \cos^2(\theta_1)]}{\sin^2(\theta_1)} (1 + \gamma)^2\]

\[P^0_{S_1} = \frac{\Gamma_1}{\Gamma_2} \frac{[1 + \cos^2(\theta_1)]}{\sin^2(\theta_1)} (1 + \gamma)\]

These probabilities are used to obtain the first expression for current through DQD2 in Eq. [5].

Appendix D

In this Appendix we present the derivation of currents, current correlations as well as the full statistics of charge transfer across the DQDs. Following the procedure of Refs. [77] we rewrite the QME of Eq. (8) in the n-resolved form and Fourier transform it with respect to the number of electrons having tunneled through DQD1 and DQD2. The QME then transforms to \(d\rho/dt = \mathcal{L}(\chi_1, \chi_2)\rho\), where the counting fields \(\chi_1\) and \(\chi_2\) are the conjugate variables to the number of electrons having tunneled through DQD1 and DQD2. The eigenvalue of \(\mathcal{L}(\chi_1, \chi_2)\) tending to zero as \(\chi_1, \chi_2 \to 0\) is the long time limit cumulant generating function \(\mathcal{F}(\chi_1, \chi_2)\). The currents and the noise are obtained from the first and second derivatives of \(\mathcal{F}(\chi_1, \chi_2)\), i.e. \(I_i = \epsilon\partial_i\mathcal{F}|_{\chi_1 = \chi_2 = 0}\) and \(S_{ij} = \epsilon^2\partial_i\partial_j\mathcal{F}|_{\chi_1 = \chi_2 = 0}\). These quantities can conveniently be accessed via the eigenvalue problem \(\mathcal{L}(\chi_1, \chi_2)[\rho(\chi_1, \chi_2)] = \mathcal{F}(\chi_1, \chi_2)\delta(\chi_1, \chi_2)\). In the present paper this full QME-approach is used only to calculate the current in Fig. [6].

In the ME limit the eigenvalue problem above reads

\[M(\chi_1, \chi_2)\chi(\chi_1, \chi_2) = \mathcal{F}(\chi_1, \chi_2)\chi(\chi_1, \chi_2)\]

We use this equation to calculate the noise plotted in Fig. [8]. We also use the equation to obtain the CGF of Eq. (6), i.e. the CGF to first order in \(\Gamma_1/\Gamma_2\) for the bias condition of DQD2 described in Sec. III with DQD1 in the high-bias regime. To do this we write det \([M(\chi_1, \chi_2) - \mathcal{F}(\chi_1, \chi_2)] = 0\) as

\[\det[M_e(\chi_1, \chi_2) - \mathcal{F} M_{r-e}(\chi_1, \chi_2)] = 0.\]

Here \(M_{e-r}, M_{r-e}\) and \(M_e\) are matrices describing the transitions to, from and within the subspace of states not included in the Fig. [6]. Importantly, \((M_{e-r})_{nm} \propto \mathcal{O}(\Gamma_1/\Gamma_2)\) and \((M_{r-e})_{nm} \propto \mathcal{O}(1)\). This means that

\[\det[M_e(\chi_1, \chi_2) - \mathcal{F}^{(1)}(\chi_1, \chi_2)] = 0,\]
where $F^{(1)}(\chi_1, \chi_2)$ is the CGF to first order in $\Gamma_1/\Gamma_2$. Dropping terms of $O(\Gamma_1/\Gamma_2)^2$ in Eq. (30) and solving for $F^{(1)}$ then gives Eq. (41).

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