Gluon and ghost propagators from the viewpoint of general principles of quantized gauge field theories

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We discuss the possible form of gluon and ghost propagators in the infrared region of Yang-Mills theory in the covariant gauge from the viewpoint of general principles of quantized gauge field theories.

1. INTRODUCTION

Recent studies of the coupled Schwinger-Dyson (SD) equations for Yang-Mills theory have shown\textsuperscript{[3]} that the gluon propagator $D_T$ and ghost propagator $\Delta$ in the Landau gauge exhibit the power law behavior with a critical exponent $\kappa$

$$D_T(Q^2) := F(Q^2)/Q^2 \cong A(Q^2)^{2\kappa - 1}, \quad (1)$$

$$\Delta(Q^2) := G(Q^2)/Q^2 \cong B(Q^2)^{-\kappa - 1}, \quad (2)$$
in the infrared (IR) limit $Q^2 \to 0$ for Euclidean momentum $Q^2 > 0$. Surprisingly, the gluon propagator is IR enhanced for $\kappa > 1/2$, while the ghost propagator is IR suppressed. However,

1. The precise value of $\kappa$ is unknown even if the IR power law is correct. The transverse gluon propagator vanishes for $\kappa > 1/2$, while it has a non-zero limit for $\kappa = 1/2$.
2. There is no analytical method to connect the IR asymptotic solution to the UV one, although numerical methods exist. There is no guarantee for uniqueness of the solution obtained under the specific Ansatz.
3. There is no argument for the analytic continuation from Euclidean region to Minkowski region.

The purpose of this work\textsuperscript{[3]} is to search the solution which is consistent with the general principles of quantized gauge field theory:

- Non-perturbative multiplicative renormalizability
- Analyticity
- Spectral condition
- Poincaré group structure

without using the SD equations.

2. Multiplicative renormalizability

Non-perturbative multiplicative renormalizability for the gluon form factor, $F_0(k^2, \Lambda^2, \alpha_0, \lambda_0) = Z_3 \mu^2, \Lambda^2) F_R(k^2, \mu^2, \alpha, \lambda)$, yields the RG equation:

$$\left[ \mu \frac{\partial}{\partial \mu} + \beta(\alpha) \frac{\partial}{\partial \gamma^2} - 2\lambda \gamma(\alpha) \frac{\partial}{\partial \lambda} + 2\gamma(\alpha) \right]$$

$$\times F_R \left( \frac{k^2}{\mu^2}, \alpha, \lambda \right) = 0. \quad (3)$$

where $\alpha := g^2/(4\pi)$, $\beta(\alpha) := \mu \left. \frac{\partial \gamma(\alpha)}{\partial \mu} \right|_{\mu = 0}$, $\gamma(\alpha) := \mu \left. \frac{\partial \lambda}{\partial \mu} \right|_{\mu = 0}$, and $\gamma(\alpha) := \frac{1}{2} \mu \left. \frac{\partial \lambda}{\partial \mu} \right|_{\mu = 0}$ for a gauge parameter $\lambda$.

We consider the propagator along the ray in the cut $k^2$ plane (See Figure 1). The asymptotic freedom (i.e., validity of perturbation theory for large $k^2$) and the general solution of the RG equation lead to the asymptotic form for $[k^2] \to \infty$ along the ray:

$$F(k^2) \cong C(\lambda) \left( \ln \frac{k^2}{\mu^2} \right)^{-\gamma(\lambda)/\beta_0},$$

$$D_T(k^2) \cong -C(\lambda) k^{-2} \left( \ln \frac{k^2}{\mu^2} \right)^{-\gamma(\lambda)/\beta_0},$$

where $C(\lambda) > 0$ with $\beta(\alpha) = -\frac{\delta_0}{2} \alpha^2 + \cdots$, and $\gamma(\alpha) = -\frac{\delta_0}{4\pi} \alpha + \cdots$. Both functions $F(k^2)$ and $D_T(k^2)$ vanish as $|k^2| \to \infty$ along the ray, since $\gamma(\lambda)/\beta_0 = 13/22 > 0$. 

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3. Analyticity

Suppose i) the complex function \( f(z) \) is analytic in the whole complex plane \( z := k^2 \) except for the positive real axis \( z > 0 \), ii) \( f(z) \) vanishes asymptotically along any ray in the cut complex plane: \( |f(z)| \to 0 \) as \( |z| \to \infty \), and iii) \( f(k^2) \) is real on the real axis \( k^2 < s_{\text{min}} \) (at least for the space-like region \( k^2 < 0 \)). Then, for a reference point \( k^2 \), the dispersion relation follows

\[
f(k^2) = \frac{1}{\pi} \int_0^\infty dz \frac{Im f(z + i\epsilon)}{z - k^2}.
\]  

We apply this relation to the form factor and the propagator.

4. Spectral condition

From the assumptions: 1) Poincare group structure (representation), 2) spectral condition, and 3) completeness condition, the spectral representation for the renormalized gluon propagator follows:

\[
D_T(k^2) = \int_0^\infty dp^2 \frac{\rho(p^2)}{p^2 - k^2},
\]  

where \( \rho(k^2) = \pi^{-1} \text{Im} D_T(k^2 + i\epsilon) \). The spectral weight function \( \pi \rho(k^2) = \text{Im} D_T(k^2 + i\epsilon) = -k^{-2} \text{Im} F(k^2 + i\epsilon) \), for \( k^2 > 0 \), has the asymptotic behavior: \( \rho_{\text{as}} \equiv -4\pi (k^2)^{-1} \gamma_0 \Phi(\lambda) |\beta_0^{-1}| C_\Phi(\lambda) \left( \ln \frac{k^2}{|\mu|^2} \right)^{-\gamma_0/\beta_0-1} \) for \( k^2 \to \infty \). Here the anomalous dimension \( \gamma_0 \) is gauge dependent. The sign of the asymptotic discontinuity \( \rho_{\text{as}} \) is determined by the ratio \( \gamma_0/\beta_0 \), it is negative for \( \gamma_0/\beta_0 > 0 \).

5. Superconvergence for gluon

For \( \gamma_0^A/\beta_0 > 0 \), the gluon propagator has an unsubtracted renormalized dispersion relation

\[
D_T(k^2) = \int_0^\infty dp^2 \frac{\rho(p^2)}{p^2 - k^2}.
\]  

The renormalized \( \rho \) is a function of \( p^2, \mu, g_R, \lambda_R \) where \( \mu \) is the renormalization point and \( \lambda \) is the gauge parameter. In the similar way, the gluon dressed function or form factor \( F = -k^2 D_T \) has the renormalized dispersion relation

\[
F(k^2) = \frac{\lambda}{\lambda_*} - \int_0^\infty dp^2 \frac{\rho(p^2)}{p^2 - k^2}.
\]  

Two relations are compatible if and only if

\[
\int_0^\infty dk^2 \rho(k^2, \mu^2, g_R, \lambda_R) = \frac{\lambda}{\lambda_*}.
\]  

This is the superconvergence relation in the generalized Lorentz gauge with an initial gauge-fixing parameter \( \lambda \) and the fixed point \( \lambda_* \).
We put the reference point $k^2$ on the Euclidean region, i.e., $Q^2 = - k^2 > 0$. The gluon form factor vanishes in the Euclidean IR limit $Q^2 \downarrow 0$:

$$F(0) = \frac{\lambda}{\lambda_s} - \int_0^\infty dp^2 \rho(p^2) = 0, \quad (9)$$

and the gluon propagator converges to a constant

$$D_T(-Q^2) = \int_0^\infty dp^2 \rho(p^2) + O(Q^2). \quad (10)$$

The spectral function has the form

$$\rho(s) = Z \delta(s - M^2) + \tilde{\rho}(s),$$

where $Z \delta(s - M^2)$ corresponds to a pole at $p^2 = M^2$ and $\tilde{\rho}(s)$ is the contribution of the continuous spectrum from more than two particle states, namely, $\text{supp} \rho(s) \in [(2M)^2, \infty)$. Thus, we have shown [3] that, for gluon with massive spectrum, the power-series expansions for the gluon propagator and the form factor can be well-defined for small Euclidean momenta $Q^2$ for any gauge parameter. Note that $0 < D_T(0) < \infty$ corresponds to $\kappa = 1/2$. (The Gribov limit is $\kappa = 1$.)

6. Superconvergence for ghost?

We recall the result of [3]. The renormalized ghost propagator $\Delta_{FP}$ has a dispersion relation in the arbitrary gauge. However, the form factor $G := - k^2 \Delta_{FP}$ has a dispersion relation only in the Landau gauge $\lambda = 0$. Hence, the superconvergence relation for the ghost holds,

$$0 = \int_0^\infty dk^2 \rho_{FP}(k^2, \mu^2, q^2_R, 0),$$

only in the Landau gauge $\lambda = 0$. For $\lambda \neq 0$, however, the superconvergence relation does not hold, since the unsubtracted dispersion relation exists only for the propagator, not for the form factor, in sharp contrast with the gluon case.

If the spectral function $\rho$ has singularities accumulating toward the origin $p^2 = 0$, however, we must replace an integration contour in Figure 2 by another contour in Figure 3 to avoid the origin. We conclude that the superconvergence for ghost does not hold even in the Landau gauge [3].

7. CONCLUSIONS

From the viewpoint of general principles of QFT, the transverse gluon can be massive and short range, while the FP ghost is singular and long-range. The IR critical exponent of gluon is $\kappa = 1/2$, since the gluon propagator behaves like

$$D_T(Q^2) \equiv \text{const.} + O(Q^2). \quad (12)$$

The ghost propagator has a negative and non-integer exponent. It fulfills a sufficient condition for color confinement due to Kugo and Ojima.

$$\lim_{Q^2 \to 0} [Q^2 \Delta(Q^2)]^{-1} = \lim_{Q^2 \to 0} [G(Q^2)]^{-1} = 0. \quad (13)$$

Supposing the existence of IR fixed point for the gluon–ghost–antighost coupling constant (without dynamical quarks), as suggested from the SD equations [4], we have

$$\Delta(Q^2) \equiv (Q^2)^{-3/2}. \quad (14)$$

Thus the ghost is expected to be a carrier of confinement in the covariant gauge.

REFERENCES

1. R. Alkofer and L. von Smekal, Phys. Rept. 353, 281 (2001).
2. R. Oehme and W. Zimmermann, Phys. Rev. D 21, 471 (1980). Phys. Rev. D 21, 1661 (1980).
3. K.-I. Kondo, hep-th/0303251
4. W.J. Xu, hep-th/9607045