Zero sound in a single component fermion - Bose Einstein Condensate mixture

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The resonant dynamics of mediated interactions supports zero-sound in a cold atom degenerate mixture of a single component fermion gas and a Bose-Einstein condensate (BEC). We characterize the onset of instability in the phase separation of an unstable mixture and we find a rich collective mode structure for stable mixtures with one undamped mode that exhibits an avoided crossing and a Landau-damped mode that terminates.

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Recent successes in cold atom fermion cooling have opened up possibilities to study Fermi-liquid in depth, beyond the realm of fermion superfluidity. Cold atom fermions are cooled ‘sympathetically’ by using a Bose-Einstein condensate (BEC) as a refrigerator. The BEC mixture, if it is cooled below 10% of the Fermi-temperature, would result in the first neutral Fermi-liquid/boson superfluid mixtures to be investigated in the laboratory since the *He–*He systems. Cold atom technology is ideally suited for revisiting interesting features of the Fermi-liquid/boson superfluid phase diagram whose microscopic description has proven to be a considerable challenge in the case of the strongly coupled *He–*He systems. For instance, a fermion-boson Feshbach resonance can vary the mediated interactions that play a crucial role in the phase separation of *He–*He.

In this letter, we show that a dilute fermion gas of single spin atoms immersed in a BEC exhibits a genuine collective Fermi-liquid behavior by supporting a long-lived zero-sound excitation due to the BEC-mediated fermion-fermion interactions. The observation of this mode would present a clear manifestation of mediated interactions since the Pauli exclusion-principle prevents bosons significantly exceed the range of the interaction potentials as well as the scattering lengths *a* and *a* of the binary atom boson-boson and boson-fermion interactions. The temperature is sufficiently low to validate the mean-field description of the BEC and the use of the collisionless approximation in treating the fermion dynamics, but is higher than the critical temperature for fermion pairing. The fermion and boson quantum liquids are coupled by the mean-field interaction with the interaction strengths *λ* = (2πℏ^2*a* ) (1/(*m* + 1/(*m*))). The interaction energy of fermion or boson is proportional to the local density of the other liquid: *λ* = *ρ* *ρ* for fermions and *λ* = *ρ* *ρ* for bosons.

The collective modes are small amplitude oscillations of the coupled BEC and Fermi-liquid that propagate as a plane wave with a wavevector *k*. We assume *k* is significantly shorter than the Fermi wavenumber. The phase *θ* and density *ρ* of the BEC field vary as the real parts of *θ* exp(i*θ* · *r*) and *η* exp(i*η* · *r*), with fluctuation amplitudes *θ* ≪ 1 and *η* ≪ 1. The Fermi-liquid oscillation can be described by a fluctuation of a distribution function *n*(*r*, *p*; *t*), with *r* the position and *p* the momentum. The distribution function evolves according
to the collisionless transport equation [10],
\[
\frac{\partial n}{\partial t} + \frac{\mathbf{p}}{m_{F}} \cdot \frac{\partial n}{\partial \mathbf{r}} - \nabla (\lambda_{FB}\rho_{B}) \cdot \frac{\partial n}{\partial \mathbf{p}} = 0, \tag{1}
\]
where we have approximated the velocity of the dilute fermion gas as \( \mathbf{v} \approx \mathbf{p}/m_{F} \) and the force to be the BEC-induced mean-field gradient, \( F = -\nabla [\lambda_{FB}\rho_{B}(\mathbf{r}, t)] \). Assuming \( n \) fluctuates around the equilibrium distribution \( n^{0}(\mathbf{p}) \) by deforming the local Fermi-surface, \( n \) is linearized as
\[
n(\mathbf{r}, \mathbf{p}, t) = n^{0}(\mathbf{p}) + u_{\mathbf{p}}(t) p_{F} \delta(p - p_{F}) e^{i \mathbf{k} \cdot \mathbf{r}}, \tag{2}
\]
where the second term is the fluctuation part \( \delta n_{\mathbf{p}}(\mathbf{r}, t) \), \( p_{F} \) is the equilibrium Fermi-momentum, and \( u_{\mathbf{p}}(t) \) is the amplitude of deformation. Then Eq. (1) becomes
\[
\frac{\partial}{\partial t} u_{\mathbf{p}}(t) + i \frac{\mathbf{k} \cdot \mathbf{p}}{m_{F}} u_{\mathbf{p}}(t) = -i \frac{\mathbf{k} \cdot \mathbf{p}}{p_{F}} (\lambda_{FB}\rho_{B}^{0}) \eta(t). \tag{3}
\]
Next, we describe the BEC dynamics. The hydrodynamic description of BEC equations are [11]
\[
\frac{\partial \rho_{B}(\mathbf{r}, t)}{\partial t} = -\nabla \cdot \left( \rho_{B}(\mathbf{r}, t) \frac{\hbar}{m_{B}} \nabla \theta_{B}(\mathbf{r}, t) \right), \tag{4}
\]
\[
-h \frac{\partial \theta_{B}(\mathbf{r}, t)}{\partial t} = -\hbar^{2} \nabla^{2} \rho_{B}(\mathbf{r}, t) \tag{5}
\]
\[
\frac{1}{2m_{B}} \nabla^{2} \rho_{B}(\mathbf{r}, t) + \lambda_{BB}\rho_{B}(\mathbf{r}, t) + \lambda_{FB}\rho_{F}(\mathbf{r}, t),
\]
where \( \lambda_{BB} \) is the boson-boson interaction strength, \( \lambda_{BB} = (4\pi\hbar^{2}a_{BB})/m_{B} \). The coupling to the fermion system arises from the mean-field interaction term \( \lambda_{FB}\rho_{F}(\mathbf{r}, t) \approx \lambda_{FB}\rho_{F}^{0} + \lambda_{FB}(p_{F}^{3}/2\pi^{2}) \langle u(t) \rangle \exp(\mathbf{i} \mathbf{k} \cdot \mathbf{r}) \) with \( \langle u(t) \rangle \) the angular average, evaluated on the Fermi surface, \( \langle u(t) \rangle = (4\pi)^{-1} \int u_{\mathbf{p}}(t) d\Omega_{\mathbf{p}}, |\mathbf{p}| = p_{F} \). Linearizing the fluctuation, combining Eqs. (4) and (5), then using the Bogoliubov dispersion \( \omega_{k}^{2} = kc \sqrt{1 + (\xi)^{2}} \), where \( c \) is the single BEC-sound velocity, \( c = (\lambda_{BB}\rho_{B}(m_{B})^{1/2}) \), and \( \xi \) is the BEC coherence length, \( \xi = (16\pi\rho_{B}^{0}a_{BB})^{-1/2} \), we obtain the collective BEC-dynamics,
\[
\frac{\partial^{2} \eta(t)}{\partial t^{2}} = -\left( \omega_{k}^{2} \right)^{2} \eta(t) - 3k^{2}c^{2} \left( \lambda_{FB}\rho_{B}^{0} \right) \langle u(t) \rangle . \tag{6}
\]
Substituting Eq. (6) into Eq. (5) and imposing harmonic time dependence, \( \eta(t) = \eta \exp(-i\omega t) \), \( u_{\mathbf{p}}(t) = \mathbf{v}_{\mathbf{p}} \exp(-i\omega t) \) give the collective mode equation
\[
s^{2} (v_{F}/c)^{2} - (1 + q^{2})^{2} = \frac{1}{2} \int_{-1}^{1} \frac{x}{s - x} dx, \tag{7}
\]
where
\[
F = \frac{k_{F}a_{FB}}{2\pi} \left( \frac{a_{FB}}{a_{B}} \right) \left( 1 + \frac{m_{F}}{m_{B}} \right) \left( 1 + \frac{m_{B}}{m_{F}} \right) , \tag{8}
\]
with \( k_{F} = p_{F}/\hbar \), and where \( s \equiv \omega/(kv_{F}) \) and \( q \equiv k\xi \). The integral on the right hand side stems from the angular average (\( x \) is the cosine of the angle between \( \mathbf{k} \) and \( \mathbf{p} \)). Eq. (7) is an s-wave zero-sound dispersion with a Fermi-liquid parameter [4] that has a resonant denominator
\[
F_{0} = F/ \left[ s^{2} (v_{F}/c)^{2} - (1 + q^{2}) \right]. \tag{9}
\]

Eq. (7) also reveals that the mode structure depends on two relevant ratios: the velocity ratio \( c/v_{F} \) and the interaction parameter, \( F \). The velocity ratio \( c/v_{F} \) provides the ratio of time scales. The oscillation periods of a long wavelength BEC and a collective fermion mode of wavenumber \( k \) are \( \sim (kc)^{-1} \) and \( \sim (kv_{F})^{-1} \), respectively. They are also the times that take the quantum liquids to respond to a density perturbation of spatial variation on the length scale \( k^{-1} \).

FIG. 1: The scaled velocity \( s \) (in units of the Fermi-velocity) for the collective damped and undamped modes as a function of \( c/v_{F} \) for \( q = 0 \). solid : undamped mode for \( F = 0.01 \), dash-dotted: undamped mode for \( F = 0.5 \), dashed: damped mode for \( F = 0.01 \), and dotted: damped mode for \( F = 0.5 \).

In the long wavelength limit (i.e., \( k \to 0 \)), if the BEC-response is significantly slower than the fermion frequency, \( c/v_{F} \ll 1 \), the BEC fluctuation fails to follow the fermion fluctuation and oscillates out-of-phase, giving an effective repulsive fermion-fermion interaction as opposed to the attractive one in the static limit. In this regime, the undamped solution to Eq. (7) describes a collective excitation with the characteristics of the zero-sound mode of a pure fermion gas with weak repulsive interactions. Its velocity, \( v \) is nearly equal to \( v_{F} \) (see Fig. 1). On the other hand, in the limit, \( c/v_{F} \gg 1 \), the BEC is ‘fast’ enough to be able to follow fermion oscillations adiabatically, thus validating the static description of BEC-mediated fermion-fermion interactions. Since the effective interaction is attractive, the mode is not a pure Fermi-liquid form of zero-sound, but the one that resembles a pure BEC with \( v \approx c \).
As a function of $c/v_F$, the undamped collective excitation crosses over from pure fermion-like zero sound to pure BEC-like sound around $c/v_F = 1$, but the width of the transition regime depends on $F$. Setting $s = 1 + \epsilon$, $\epsilon \ll 1$, the beginning of the transition can be estimated as $(1 + F [-1 + (1/2 \ln (\epsilon/2))]^{-1/2}$. A crossover from pure fermion-like zero-sound to a pure BEC-like sound mode is also evident from the collective mode dispersion if $c/v_F \leq 1$ [19]. Figure 2 shows the avoided cross-over from a linear to a Bogoliubov dispersion. In the crossover region centered around $k \simeq \xi^{-1}(v_F/c)^2 - 1$. For mixtures within this transition regime, the undamped collective mode is a strong admixture with boson and fermion density fluctuations of comparable amplitude. In the language of optics, the mixture changes its index of refraction in the spatial region in which the trapped mixture has a local velocity ratio within the transition range. This shell can act as a boundary region from which a propagating wavepacket of density fluctuations is reflected.

In a mixture with $c/v_F < 1$, the mediated interactions remain weak so that the Bogoliubov mode of the single BEC appears as a second mode with a dispersion that is slightly modified by the fermion mediated boson-boson interactions. The zero-sound deformation of the local Fermi surface creates a cloud of fermion density fluctuations that accompanies (‘dresses’) the propagating BEC-excitation. This collective excitation damps by Landau damping and the complex solution $s = r - i\gamma$ with $0 < r < 1$, $\gamma > 0$. Finding this mode is rather delicate: A damped excitation, $\exp [-i (\omega - i\Gamma)t]$ where $\Gamma = k v_F \gamma$, requires $r - i\gamma$ in the left-hand side of Eq. 4, while the integral on the right-hand side, which describes a retarded mediated interaction, requires the replacement $s \to r + i\gamma$. The numerical evaluation shows that, for $c/v_F < 1$ and $F \ll 1$, the damping rate can be well-approximated by the Fermi golden rule for phonon annihilation accompanied by a fermion particle-hole excitation: $\Gamma = ck (\pi F/4) (c/v_F) = \gamma \tau_0^{-1}$, where $\tau_0 = (m_F/2m_B)(ck^2)^{-1}$ is the lifetime of the mode. In atom trap experiments, $\tau_0$ can be order of milliseconds – long enough to measure the damping but short enough to keep a propagating wavepacket within the spatial region where the system is approximately homogeneous. Even for a rather large value of $F$, such as $F = 0.5$, $\Gamma$ obtained from the numerical evaluations agree with the one using Fermi Golden rule with 30% error at $c/v_F \sim 0.8$, where the damped zero-sound mode vanishes.

The damped mode dispersion relation in Fig. 2 terminates at $k$ before entering the crossover region. This termination point is reminiscent of the $^4$He-mode for which the termination is connected with the decay into quasiparticle pairs [12].

While the imaginary part of the dispersion relation merely describes the damping of the collective mode and not the instability of the homogeneous mixture as claimed by Ref. 6, Eq. 4 does give insight into the stability issue. When the interaction parameter $F$ exceeds unity, the damped mode $s$ takes on a purely imaginary value, $s = i\gamma$, on account of which the right-hand side of Eq. 7 reduces to $\gamma \arctan (\gamma^{-1}) - 1$ and which does signal the instability of the homogeneous mixture. A degenerate fermion system of $\rho^F > \rho^F_{\text{crit}}$.

$$\rho^F_{\text{crit}} = \frac{4\pi}{3a^2_{BB}} \left[ \frac{a^B_F}{a^F_B} \frac{m_B}{m_F} \right]^3, \quad (10)$$

is immiscible to a BEC and an initially homogeneous mixture spontaneously separates into either regions of pure fermion and pure BEC systems or regions of pure fermion and mixed fermion-BEC phases [14]. The experimentalist can realize the separation by tuning an external magnetic field near the zero-point of the boson-boson scattering length $a_{BB}$ in a Feshbach resonance [15]. Eq. 7 also provides the dynamics of the onset of the instability. In response to a sudden change of $a_{BB}$ in an initially homogeneous gas mixture, the quantum gases gather into ‘clumps’ of single phase matter before the clumps congregate into larger regions of single phase matter.

The clumping is initiated by the exponential growth of the unstable collective mode eigenvectors of wavenumber $k = q\xi^{-1}$ and grows at a rate $\gamma \tau_0^{-1}$. The fastest growing eigenvectors of wavenumber $q^*$ dominate the dynamics and determine the size of the clumps as well as the rate of formation [16]. By phase separating a BEC-fermion mixture in a cigar-shaped trap, the experimentalist can prevent the single phase clumps to move past each other and measure the size of the clumps frozen in place, as demonstrated in the phase separation of BEC’s [17]. If the instability is sufficiently weak,
\(F - 1 \ll 1\), the right-hand side of Eq. (7), is well approximated by \(-\gamma(\pi/2) - 1\) and the unstable eigenvectors \(q \in (0, \sqrt{F - 1})\) grow at a rate \(\gamma q \tau_0^{-1}\) with 
\[
\gamma(q) = \left(\pi F/4 + \sqrt{(\pi F/4)^2 - (c/v_F)^2[1 + q^2 - F]}\right) \text{ giving a dominant wavenumber } q\xi^{-1} \text{ with }
q^2 = \frac{1}{128} \left[64(F - 1) + F\pi(c/v_F)^2 \times \left[3F\pi - \sqrt{(3F\pi)^2 + 128(c/v_F)^2(F - 1)}\right] \right].
\]

We expect the average clump size to be \(\sim \xi(2\pi/q')\) and the clumps to form on a time \(\sim \tau_0(\gamma q')^{-1}\).

Finally, we remark that previously developed cold atom BEC-techniques are ideally suited to probe the collective modes of fermion-boson mixtures. A focused laser beam or crossed beams in a two-photon Bragg-scattering setup can create a density variation near the middle of the trap, the subsequent propagation of which can be imaged optically or by a time-of-flight measurement. The dual mode structure of well separated group velocities would manifest itself by spatially separating the initial wave packet into two wavepackets with the faster one moving at a speed \(c/v_F\) instead of the speed of ordinary sound (\(c/v_F/\sqrt{3}\) in dilute fermions). The 40\% difference in velocity, in stark contrast to the few percent difference in the strongly interacting \(^3\)He-fluid, can be easily measured. The slower wavepacket moves at a speed closer to \(c\) and is damped. Furthermore, if the spatially varying equilibrium fermion and BEC-densities in the atom trap reach values where the local Fermi and BEC sound velocities are equal, we expect that this region can act as a boundary from which the faster wavepacket is reflected. The low energy collective modes of the finite size system can similarly be located inside or outside the boundary.

In conclusion, we have shown that a quantum degenerate gas mixture of a BEC and single component fermion gas exhibits a surprisingly rich collective mode structure that is closely related to the phenomenon of mediated interactions. Even though the boson mediated fermion-fermion interactions are attractive in the static limit, the mixture can support a long lived zero-sound mode that resembles that of a pure Fermi-liquid in the resonant dynamics of the phonon-mediation if \(F < 1\), since the effective interaction is repulsive. As a function of the velocity ratio \(c/v_F\), the undamped mode undergoes an avoided crossing near \(c \sim v_F\) and then, becomes a single BEC-sound like mode as \(c/v_F\) increases. A second zero-sound mode appears in the slow BEC regime \(c/v_F < 1\). This mode undergoes Landau damping and terminates. If \(F > 1\), the homogeneous mixture becomes unstable and undergoes phase separation for which we derive the relevant time and length scales.

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[19] From \(\xi k_F = (m_F/2m_B)(v_F/c)\), we find that even if \(c\) and \(v_F\) are comparable, the inverse BEC coherence length can remain significantly smaller than \(v_F\) provided the mixture consists of light boson atoms immersed in a Fermi-sea of much heavier fermion atoms (e.g hydrogen immersed in \(^{40}\)K, or \(^{7}\)Li mixed with \(^{85}\)Rb). In that case, Eq. (7) can describe excitations of wavelength comparable to \(\xi^{-1}\).