A multipurpose information engine that can go beyond the Carnot limit

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Abstract. Motivated by the recent work of Mandal and Jarzynski on an autonomous Maxwell demon information engine, we have extended their model by introducing two different heat baths. The system (demon) is coupled to a memory register (tape) and a work source. The performance of the system depends on the interplay between the two sources along with the heat baths. We have found that the system can act as an engine, refrigerator or an eraser. Even the combination of any two is possible in some parameter space. We have achieved an efficiency of the engine greater than the Carnot limit. The coefficient of performance of the refrigerator is also larger than the Carnot limit.

Keywords: numerical simulations, stationary states, transport processes/heat transfer
1. Introduction

Since the advent of the Maxwell demon 150 years ago, the study of the relationship between thermodynamics and information has attracted much attention because it concerns the foundation of the second law of thermodynamics [1–20]. The Maxwell demon was formulated as an information processing device that performs measurement including memory register and feedback at the level of thermal fluctuations. This demon refers to any general situation in which rectification of microscopic fluctuations decrease thermodynamic entropy. To preserve the second law from violating, information to memory register plays an essential role. Information entropy of the memory register compensates for the decrease of the thermodynamic entropy of the device [20–22]. Recent studies have revealed that information content and thermodynamic variables should be treated on equal footings (information thermodynamics) [9, 20, 22]. This relationship has attracted much interest recently due to advances of nano-technology which have enabled access to atomic-scale objects in a controllable manner. Experimentally, the Maxwell demon and Szilard engine have been realized in the laboratory [23–25]. This has allowed verification of several fundamental issues regarding thermodynamics of information and the physical nature of information.

Recently Mandal and Jarzynski (MJ) have introduced an autonomous Maxwell demon model [10] for a heat engine. Apart from the heat reservoir, the model consists of three parts such as the work source, the device and the information reservoir. A mass that can be raised or lowered against gravity is used as a work source, while the device operates in cycle and affects the transfer of energy among other subsystems. An information reservoir is a system that exchanges information but not energy with the device. A frictionless tape acts as an information reservoir that preserves the information. The reading and writing of information does not require any energy while passing through the demon. Depending upon the parameter, they have showed that the...
system sometimes functions as an engine or erasure or neither of them (called a dud state (MJ)).

In this paper, we have extended their model by introducing two reservoirs coupled to the system. The main motivation of our study is to find the effect of thermal bias along with the information reservoir and the work reservoir in the performance of the demon. The system reaches a unique steady state for any given set of parameters (which will be described in detail below). The dynamics are autonomous. The performance of our model basically depends on the interplay between different forces: the gravitational pull on the mass and the changing of the information content on the stream of bits written on the tape. The system can act as an engine by extracting work on an average; an erasure by removing the information from the memory register (tape), a refrigerator by absorbing heat from cold bath. Our system also exhibits a combination of any of the two in its modes of operation. For example it simultaneously acts as (i) an engine and a refrigerator, (ii) an engine and an eraser, (iii) a refrigerator and an eraser. Our results are consistent with the generalized second law of thermodynamics. We will discuss the model and findings in detail below.

2. The model

We have extended the autonomous Maxwell demon model by MJ. To this end, we have considered similar three state systems (A, B, C) but these states have different energy levels (unlike MJ), i.e. they are not degenerate (see figures 1 and 2). For simplicity we have considered the difference between the two successive energy levels to be same ($E_1$). Note that a minimum of three states are needed to observe any directed rotation. The transition can occur between A to B, B to C and vice versa, spontaneously by exchanging heat from the cold bath with temperature $T_c$ and internal energy of the system is changed. However, the transition between A to C and vice versa is restricted and depends on the value of the interacting bit written on the tape. The bit has two states 0 and 1. Hence the combined system has six states. When the transition occurs from C0 to A1, the bit state is changed from 0 to 1 and vice versa. However, the transition between C1 and A0 is not allowed. Note that, during the transition between A and B, B and C, the bit state is not changed. On the other hand, during the transition from C0 to A1, energy is absorbed from the hot bath at temperature $T_h$ by an amount $E$ while the system performs $w$ amount of work by pulling a mass $m$ to a height $\Delta h$ by a frictionless pulley in a gravitational force field $g$ ($w = mg\Delta h$). During this transition, the internal energy of the demon is increased by $2E_1$. Using energy balance one can get

$$E = w + 2E_1. \quad (1)$$

The rate of transition between C0 and A1 is related by

$$\frac{R_{C0\rightarrow A1}}{R_{A1\rightarrow C0}} = e^{-E/T_c}. \quad (2)$$

We set the Boltzmann constant $k_B = 1$. If we take $R_{C0\rightarrow A1} = 1 + \varepsilon$ and $R_{A1\rightarrow C0} = 1 + \varepsilon$, we get
A multipurpose information engine that can go beyond the Carnot limit

This weight parameter \( \varepsilon \) is bounded by \(-1 < \varepsilon < 1\). The other transitions occur due to the connection to the cold bath \( T_c \) and the rates are related by

\[
\frac{R_{B0 \to A0}}{R_{A0 \to B0}} = \frac{R_{C0 \to B0}}{R_{B0 \to C0}} = e^{-E_1/T_c} = \frac{R_{B1 \to A1}}{R_{A1 \to B1}} = \frac{R_{C1 \to B1}}{R_{B1 \to C1}}.
\]

All the allowed transitions are shown in figure 2(b) and they form a linear chain (figure 3). The model cannot exhibit a directed rotation, only back and forth transitions are possible. To get any possible rotation we need to introduce the change of bit externally. We consider the tape as moving with a rate \( \tau^{-1} \) from left to right. The system interacts only with the nearest bit (one bit at a time). Each bit is allowed to interact with the system with time \( \tau \) before the next bit takes its place. The earlier bit with new

Figure 1. A schematic diagram of our model: the demon is a three state system that is coupled with a external load and two thermal reservoirs (not shown). A sequence of bits (tape) passes from left to right at a constant speed. The nearest bit interact with the demon. For positive load i.e, \( w > 0 \), the mass is lifted at an amount \( \Delta h \) for every transition \( C \to A \), while for every transition \( A \to C \) the mass is lowered by the same amount. However \( w < 0 \) the mass is connected to the right side of the small circle, so the transition \( C \to A \) lowers the mass and the transition \( A \to C \) lifts it up.

Figure 2. Six possible states depending on demon and the bit. (a) The difference between the energy levels between A and B is \( E_1 \), similarly the difference between B and C is also \( E_1 \). The energy absorbed/released during any transition between \( C_0 \) and \( A_1 \) is denoted as \( E \) such that \( E = w + 2E_1 \). (b) All the allowed transitions and corresponding bath where energy is exchanged.

\[
\varepsilon = \tanh \left( \frac{E}{2T_h} \right).
\]

This weight parameter \( \varepsilon \) is bounded by \(-1 < \varepsilon < 1\).
information content moves forward along with the tape. If $\tau$ is very small the system hardly evolves during this time. If $\tau$ is large, the system gets enough opportunity to evolve along with the bit. After time $\tau$, the joint state may change and depends on the incoming bit. During this transition, the actual state of the system (A, B or C) is not changed and hence it does not involve any energy cost. Take an example: if the demon state is B and the outgoing bit in state 1 then the joint state will be B1. Now if the incoming bit is 0, then the joint state at the beginning of the new cycle will change to simply B0. Note that during this transition from B1 to B0 the internal energy of the demon is not changed since the demons actual state is fixed at B.

In our present study we simulate this problem and analyze our findings.

3. Degenerate energy levels with a single thermal reservoir

First we set $E_1 = 0$ ($E = w$) and $T_h = T_c$. This case correspond to the original problem of MJ. Now all the levels are degenerate. Heat is exchanged only during the transition between A1 and C0 from a single bath with temperature $T$. Let $\delta$ denote excess number of 0 in the incoming bit stream,

$$\delta = p(0) - p(1).$$

(5)

Here $p(0)$ and $p(1)$ represent probability of 0 and 1, respectively, in the incoming bit stream and they are normalized i.e. $p(0) + p(1) = 1$. The Shannon entropy of the incoming bit stream (written on the memory register or tape) is given by

$$S = -\sum_i p_i \ln p_i = -p(0) \ln p(0) - p(1) \ln p(1).$$

(6)

It represents the amount of disorder present per bit. We have ignored the correlations between the successive bits. $S$ quantifies the information content in the incoming bit stream. Consider the case when all incoming bits are 0; which implies $\delta = 1$ and $S = 0$. After the interaction time $\tau$, if the joint state ends up at any of the following states A1, B1 or C1, the outgoing bit is changed to 1. This implies during this time $\tau$, there is one excess number of upward transitions compared to the downward transitions. If the final state is in A0, B0 or C0 the outgoing bit will be in 0 state. For this situation,
the number of upward transitions is balanced by the downward transitions. In the long
time limit \( t \gg \tau \), the system will attain the steady state along with the tape. The out-
go\ing bit stream will be a mixture of 0 and 1 with probability \( p'(0) \) and \( p'(1) \), respectively. Then the Shannon entropy of the outgoing bit stream
\[
S' = -p'(0) \ln p'(0) - p'(1) \ln p'(1).
\]
will be always positive. This means some information has been written on the tape
\( \Delta S = S' - S > 0 \). Every 1 in the outgoing bit stream represents \( w \) work is extracted by
pulling the mass at an amount \( \Delta h \), taking the heat from the bath. Hence for \( \delta = 0 \), on
an average we have extracted work by rectifying the noise from the single heat bath.
This is possible because the entropy of the memory register (tape) increases. Now if we
consider entropy of the memory register and bath, it together follows the second law
\[
\Delta S + \Delta S_B \geq 0.
\]
\( \Delta S_B \) represents entropy change of the bath. It may be noted that in the steady
state, the entropy change of the demon is zero on average.

For \(-1 < \delta < 1\) the evolution of the combined system will depend on two forces:
gravitational pull on the mass (depends on weight parameter \( \varepsilon \)) and the randomiza-
tions of bits (\( \delta \)) along with the duration of the contact time of each bit \( \tau \). The average
number of clockwise (CW) rotation is given by
\[
\phi = p'(1) - p(1) = \frac{1}{2}(\delta - \delta')
\]
where \( \delta' = p'(0) - p'(1) \). Note that during each upward transition (CW rotation) \( E \)
energy is absorbed from the heat bath and the same amount of work extracted (\( w = E \)).
Then on average
\[
Q = -E\phi
\]
heat will be dissipated into the bath. As all the states of the demon are degenerate, one
can easily find
\[
W = -w\phi = Q
\]
work will be done on the system on average. For \( \phi > 0 \), work is extracted on average
\( (W < 0) \) and the system acts as an engine.

For \( \delta = 0 \) the incoming bit stream contains maximum information and \( S(0) = \ln 2 \).
Before the beginning of each interval, the probability of finding the demon in the upper
state or lower state is equal \( (p(0) = p(1) = \frac{1}{2}) \). However after the interaction, the system
will relax and the lower state will have larger probability \( (p'(0) > \frac{1}{2} > p'(1)) \). This leads
to the information content in the outgoing bits being less than that of the incoming
bits \( (\Delta S < 0) \). Since \( \phi < 0 \), we have \( W > 0 \) (equation (10)). This means work is done on
the system to erase the information contained in the tape and the system acts as an
erasure.

If all the incoming bits are 1 (\( \delta = -1 \)), after the interaction we obtain a mixture of
0 and 1 in the outgoing bits. Every 0 in the outgoing bit stream represents a counter
clockwise (CCW) rotation. Hence on average \( W > 0 \) and \( \Delta S > 0 \). Here the system
neither behaves as an engine nor an erasure and we call it dud.
Till now we have considered \( w > 0 \). The negative \( w \) represents the load is connected to the right side of the pulley. It makes for every transition \( C \rightarrow A \) the mass is lowered, while for every transition from \( A \rightarrow C \) the mass is lifted up. As a result for \( \delta = 1 \) the system acts as a dud, while for \( \delta = -1 \) the system performs as an engine.

In figure 4(A) we have plotted our numerically obtained phase diagram for \( \tau = 1 \). We find depending on the parameter \( \delta \) and \( \varepsilon \) the system can exhibit engine (red region), erasure (green region) or dud (blue region). This phase diagram exactly matches with the theoretical plots given by MJ. This also verifies the correctness of our simulation.

In the phase diagram, we have found there are three boundary lines that separate the different operating regions. At \( \varepsilon = 0 \), we have set \( E = 0 \) or \( w = 0 \). That means during the CW or CCW rotation no work is done on the system and heat absorption is zero and thus \( W = 0 \) and \( Q = 0 \). The second law becomes \( \Delta S > 0 \) since \( \Delta S_B = \frac{Q}{T} = 0 \). We define this phase boundary where \( W = 0 \) as line 1 (shown as black line in figure 4) and it separates the engine and the dud regions.

At the phase boundary between the engine and the erasure region we have \( W = 0 \) and \( \Delta S = 0 \). We denote this boundary line as line 2 (as denoted by the blue line in figure 4). It implies \( Q = 0 \) and total entropy production is also zero. This can only happen when the probability distribution of bits does not change during the interactions: \( p'(0) = p(0) \) and \( p'(1) = p(1) \). Which implies the initial bit stream is such that the demon is already in equilibrium with the bath. \( p(0) = \frac{e^{E/T}}{1 + e^{E/T}} \) and \( p(1) = \frac{1}{1 + e^{E/T}} \); hence line 2 occurs at \( \delta = p(0) - p(1) = \varepsilon \). Note that here the demon operates reversibly and total entropy production is zero.

The third boundary line which separates the erasure and the dud region is called line 3 (as shown by the brown line in figure 4). Here, \( \Delta S = 0 \) but we have observed from the phase diagram \( W > 0 \). The second law reduces to \( \Delta S_B > 0 \). This is only possible if the probability distribution of bits gets altered after the evolution along with the
A multipurpose information engine that can go beyond the Carnot limit

demon: \( p'(0) = p(1) \) and \( p'(1) = p(0) \). Line 2 and line 3 must cross each other at \( \delta = 0 \) because for this case the two conditions together hold \( (p(0) = p(1) = \frac{1}{2}) \). Putting \( \delta = 0 \) one obtains \( \epsilon = 0 \) (as this is the condition of line 2). Hence this crossing point must be on line 1 and three lines will meet at this point.

For \( \delta < 0 \) and \( \epsilon \to 1 \) the upper states are initially more populated \( (p(1) > p(0)) \). During the evolution the system will relax. The final state will depend on the relaxation time \( \tau \). If \( \tau \) is large it is possible to reach even \( p'(0) > p(1) \). Then the system will act as an erasure. In figure 4(B) we have plotted the same phase diagram for \( \tau = 10 \). It is observed that as \( \tau \) increases, the erasure region covers more area.

4. Non-degenerate energy levels with a single thermal reservoir

Now we set \( E_1 > 0 \) but keep the system connected with a single bath. The states are non degenerate. Hence for any allowed transition, heat is exchanged with this bath. For transitions between A and B or B and C, \( E_1 \) energy is exchanged with the bath. This changes the internal energy of the demon by the same amount. For transition from C0 to A1 (CW rotation), the system absorbs \( E \) amount of heat and pulls the mass by doing \( w \) amount of work. During this transition the internal energy of the system is increased by \( 2E_1 \). The corresponding energy balance equation is

\[
E = w + 2E_1.
\]

In figure 5(A) we have plotted the numerically obtained phase diagram for temperature \( T = 1.0 \) and \( E_1 = 0.5 \), the interaction time is set at \( \tau = 1.0 \). Line 1 is now shifted because if we set \( w = 0 \), it makes \( E = 2E_1 = 1.0 \) which eventually leads to \( \epsilon = 0.46 \). At line 2 \( (\Delta S = 0 \text{ and } W = 0) \) from the earlier condition we get \( p(0) = \frac{e^{-2E_1/\tau}}{1 + e^{-2E_1/\tau}} \) and \( p(1) = \frac{1}{1 + e^{-2E_1/\tau}} \). It leads to \( \delta = \tanh((E - 2E_1)/2T) < \epsilon \). As before, line 3 \( (\Delta S = 0 \text{ and } W > 0) \) meets line 2 at \( \delta = 0 \), which implies \( E = 2E_1 \) and hence these two lines meet with line 1 at a single point as shown in the figure 5(A). As we increase the cycle time \( (\tau = 10) \), line 3 is again shifted and the erasure region increases (shown in figure 5(B)).

5. Non-degenerate energy levels with two thermal reservoirs

Let us consider the case when the bath temperatures are different. For this case we set \( T_c = 0.5 \) and \( T_h = 1.0 \). Note that during each transition the internal energy of the system will change. For each transition couple to the cold bath, the internal energy of the demon is changed by \( E_1 \); while the internal energy is changed by an amount \( 2E_1 \) during each transition along with the hot bath. However, in steady state the average internal energy does not change. Hence the demon transfers heat from one bath to another during its operation. The transfer of energy is only possible if we take non-degenerate levels \( (E_1 \neq 0) \). The main objective of our study is to find the effect of thermal bias along with information reservoir and the work reservoir in the performance of the demon. Since
φ represents the average CW rotation, the total work done on the system is
\[ W = -\phi w \]
while the heat dissipated to hot bath is given by
\[ Q_E = -\phi E. \]  
(13)
As the demon does not accumulate energy on average, using the first law, the heat
dissipated to the cold bath is given by
\[ Q_c = \phi 2E_1. \]  
(14)

As before, line 1 \((W = 0)\) is shifted up to \(\epsilon = 0.46\). But for this case three lines do not meet at a point (figure 6(A)). It opens up a triangular area (cyan full box region) in the middle. Because from the earlier condition line 2 occurs when
\[ p(0) = \frac{e^{\epsilon/T_h} - 2E_1/T_h}{1 + e^{\epsilon/T_h} - 2E_1/T_h}, \]
and
\[ p(1) = \frac{1}{1 + e^{\epsilon/T_h} - 2E_1/T_h}. \]  
Now analytical treatment \((\delta = 0)\) reveals that the crossing point of
line 2 and line 3 occurs at \(\epsilon = 0.76\), which is consistent with our simulation. Line 1 and
line 2 confines the engine region \((W < 0\) as denoted by red, cyan, black color); while
line 2 and line 3 confines the erasure region \((\Delta S < 0\) as denoted by green, cyan, yellow
regions) as before. Apart from this, we get a new refrigerator region. The system acts as
a refrigerator when heat is absorbed from the cold bath, i.e. \(Q_c < 0\). At line 2, not only
\(W = 0\), \(\Delta S = 0\) and \(\Delta S_{\text{tot}} = 0\) but also individually \(Q_h = 0\) and \(Q_c = 0\) (since \(\phi = 0\)).
The left side of line 2 (yellow, pink, black regions), \(Q_c\) turns out to be negative and
the system works as a refrigerator. Hence, apart from four separate regions of engine
(red), erasure(green), refrigerator(pink) and dud(blue), there appear three more new and
interesting regions in the phase diagram where the system performs simultaneously
as an (i) erasure and refrigerator \((\Delta S < 0\) and \(Q_c < 0\) yellow region), (ii) refrigerator and
engine \((Q_c < 0\) and \(W < 0\) black region) even (iii) erasure and engine \((\Delta S < 0\) and \(W < 0\)
cyan region). For the same parameters in figure 6(B) we have plotted the phase dia-
gram by increasing the interaction time of the demon with each bit to \(\tau = 10\). As before
the erasure region covers more space. The figures are self explanatory.
5.1. Discussions

For a given point in the phase diagram the demon operates in a steady state. The change in entropy of the demon is zero on average. The bath entropy production is given by 
\[ \Delta S = Q_c/T_c + Q_h/T_h. \]
The total entropy production, \( \Delta S_{\text{tot}} \) will be the sum of the entropy change of the memory register \( \Delta S \) and the bath entropy production \( \Delta S_B \):

\[ \Delta S_{\text{tot}} = \Delta S + \Delta S_B. \]  \hspace{1cm} (15)

It is important to mention that apart from the earlier two forces (randomizations of bits and pull of gravity), now the demon experiences a temperature difference between the baths. The phase diagram is a result of interplay of all these three forces. It is observed that \( \Delta S_{\text{tot}} \) remains always greater than zero except at line 2 where it reaches zero (figure 7). On this line the demon operates in a thermodynamically reversible mode. Hence the appearance of all the phases are consistent with the generalized second law of thermodynamics, namely \( \Delta S_{\text{tot}} \geq 0 \). In the reversible mode, the distributions of bits in the incoming and the outgoing bit stream are not altered, which results in all the quantities \( W, Q_h, Q_c, \Delta S, \Delta S_B \) individually being zero at line 2.

In between the line 2 and line 3, \( \Delta S < 0 \) and the system behaves as an erasure. However, from the phase diagram (figures 6(A) and (B)) we observe that this region is divided into three separate parts. At high \( \varepsilon \), in between line 2 and 3 (yellow region), the demon acts as an erasure as well as a refrigerator. Here \( W > 0 \) and \( \Delta S_B > 0 \). Therefore work is done on the demon to absorb heat from the cold bath, besides, it erases some information written on the tape. In the triangular area in the phase diagram enclosed by the three lines (cyan region), we find \( W < 0 \), furthermore it acts as an erasure. In this regime even \( \Delta S < 0 \), the thermal bias plays a dominant role and makes \( \Delta S_B \) more positive so that \( \Delta S_{\text{tot}} > 0 \). Note that \( Q_h < 0 \) and work is extracted on average.
A multipurpose information engine that can go beyond the Carnot limit

In the remaining part (green region) it performs as a conventional erasure where work is done on the system to erase the information.

In figure 8(A) we have plotted $W$, $Q_h$, $Q_c$ as a function of $\varepsilon$ for $\delta = 1$. As every incoming bits are 0, here the demon always write some information into the tape and results in $\Delta S > 0$. We find work can be extracted when $1 > \varepsilon > 0.46$ and heat is transfered from the hot to cold bath. Thus the demon acts as an engine while writing information on the tape. In the other region ($\varepsilon < 0.46$) work is done on the system as well as information is written on the tape and making it dud. Further if we plot the efficiency of the engine defined as $\eta = \frac{W}{Q_h}$, we find $\eta$ can exceed even the Carnot bound $\eta_c = 1 - \frac{T_c}{T_h} = 0.5$ for our case(figure 8(B)). Moreover, in the reversible limit at $\varepsilon = 1$, $\eta$ tends to a limiting point 1, while both $W$ and $Q_h$ become zero!

In figure 9(A) we have fixed $\delta = -1$ and plotted $W$, $Q_h$ and $Q_c$ versus $\varepsilon$. It is observed that $Q_c$ always remains negative and act as a refrigerator irrespective of the sign of $W$. For $\varepsilon < 0.46$ the demon takes heat from the cold bath as well as work is extracted on average. It simultaneously acts as a refrigerator and as an engine. This is quite possible because for $\delta = -1$ all the incommoding bits are 1, the outgoing bits will contain a mixture of 0 and 1. The system always writes information on the tape and

Figure 7. Histogram of $\Delta S_{tot}$ for different values of $\delta$ and $\varepsilon$ at $T_h = 1.0$; $T_c = 0.5$, $E_1 = 0.5$, $\tau = 1.0$.

Figure 8. (A) Variation of $Q_h$, $Q_c$, $W$ with $\varepsilon$ for $\delta = 1.0$, $T_h = 1.0$, $T_c = 0.5$, $E_1 = 0.5$ and $\tau = 1.0$. (B) Variation of efficiency of engine $\eta$ (red line) with $\varepsilon$ while the Carnot limit $\eta_c = 0.5$ (blue dashed line).
A multipurpose information engine that can go beyond the Carnot limit

\[ \Delta S > 0. \] Moreover it is observed from figure 9(B) that the coefficient of performance of the refrigerator \( \sigma = \frac{Q_h}{W} \) is greater than the Carnot limit \( \sigma_c = \frac{T_c}{T_c - T_h} = 1 \) in our case and it even takes a negative value (\( \varepsilon < 0.46 \)) as work is extracted while heat is absorbed from the cold bath. In this respect it can be mentioned that \( \phi \) represents the average CW rotation. The total work done on the system is \( W = -\phi w \) while the heat dissipated to the hot and cold bath is given by \( Q_h = -\phi E \) and \( Q_c = \phi 2E_1 \), respectively. Then \( \eta, \sigma \) becomes independent of \( \phi \). As we have already fixed \( E_1 = 0.5 \) for given \( \varepsilon(E) \), both \( \eta \) and \( \sigma \) become constant and they do not depend on \( \delta \). Although \( W, Q_h \) and \( Q_c \) will depend on \( \delta \) because \( \phi \) is a function of \( \delta \), but, their ratio becomes independent of \( \delta \) for given \( \varepsilon \).

6. Conclusion

We have constructed a simple model of the autonomous Maxwell demon, which is not manipulated by an external force but requires an information reservoir (tape) to which it can write information. The demon is also connected to two thermal baths. The system reaches a unique steady state depending on the model parameters, where it exchanges energy from the thermal reservoirs, performs work and writes information on the tape at a constant rate. The phase diagram shows that the demon can work as an engine by lifting the mass, an eraser by removing the information written on the tape, or a refrigerator by transferring heat from cold bath. Apart from these, we have also found the demon acts simultaneously as an (i) erasure and refrigerator, (ii) refrigerator and engine, and (iii) erasure and engine. We have shown that all this modes of operation are thermodynamically possible since they do not violate the generalized second law of thermodynamics. The efficiency of an engine and coefficient of performance of refrigerator can exceed the Carnot limit.

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