Introduction

Measurements carried out in the dynamic mode are characterized by the dynamic error first of all caused by the primary measuring transducer (sensor) inertia. Therefore it is appropriate to consider the dynamic measurements problem within the class of LTI models [1]. The second component of the total dynamic measurements error is determined by the random high-frequency noise on the sensor output. These components of measurements error often happen to be much bigger than all the rest and they require correction [1].

Such scientists as G.I. Solopchenko, V.V. Leonov, V.A. Granovskiy and others made a great contribution to the formation and development of the dynamic measurements theory in our country. To date the most developed methods of the dynamically distorted signal correction are the ones based on A.N. Tikhonov’s regularization method which results in usage of the inverse Fourier transformation given, e.g. in papers [2–4] and the methods based on the computational solution to the convolution integral equation with the regularization parameter implementation [5–6].

Herewith, only one limit value of the dynamic error is given in the publications for all time function that is considered to be a very rough evaluation. There exist practically no results of the dynamic measurements error evaluation by the observable output signal of the sensor and information about its dynamic characteristics. The issues of the dynamic error effective correction with the lowered sensitivity to existence of noises at the sensor output were not specially considered. The problems of sensors and measuring systems parameters adaptation to the conditions of the carrying out experiment require further examination because the majority of the results were obtained for the systems with constant parameters. This makes the measuring systems accuracy limited to the characteristics of equipment and limits the computing capacity of these systems for the significant improvement of their metrological characteristics.

The scientific school headed by A.L. Shestakov covers the following issues: the dynamic measurements error analyses, the accuracy of dynamic measurements increasing on the basis of the automatic control theory methods [7–11]. This approach gives the possibility to obtain effective methods of the measured signal recovery, analyses and dynamic error reduction as well as time evaluations of the dynamic measurements error. All the activity of this scientific school is focused on the research and
development of the dynamic measuring systems models based on the modal control method [8], iterative principle of dynamic systems signal recovery [7], sliding mode control method [12, 13], parametric adaptation of systems [14, 15] and the neural network control method [16, 17].

1. Measuring transducer with parameters self-adjusted by dynamic error criterion and noise gain limitation

One of the advanced trends in the development of modern information-measuring engineering is its intellectualization [18]. The specific features of this trend are execution of complicated measuring procedures by special hardware tools and construction of measuring transducers capable to individualize processing algorithms by adaptive changing of their structure and parameters on the basis of accumulated aprioristic and received measuring information [19]. In real information-measuring systems the characteristics of noises are known approximately and can change during measurement. This makes the optimal processing of measurements data more complicated. Therefore, intelligent measuring systems adaptable to the noise conditions are of great practical interest.

Suppose a sensor is described by the liner transfer function (TF) as follows:

$$W_s(p) = \frac{Y(p)}{U(p)} = K_0 \prod_{i=1}^{m_2} (T_{2i}^2 p^2 + 2 \xi_{2i} T_{2i} p + 1) \prod_{i=m_2+1}^{n_2} (T_{2i} p + 1),$$

where $U(p)$ and $Y(p)$ are Laplace transformations of the sensor input and output signals; $T_{2i}$ and $T_{1j}$ are time constants for $i=1,2,...,n_2$ and $j=1,2,...,n_1$; $\xi_{2i}$ and $\xi_{1j}$ are damping coefficients for $i=1,2,...,m_2$ and $j=1,2,...,m_1$; $K_0$ is the static gain; $p$ is the complex number frequency.

The orders of numerator $m$ and denominator $n$ of the sensor TF (1) are defined as follows:

$$m = m_2 + n_2,$$

$$n = m_2 + n_1.$$  

The TF (1) can be represented in the following continuous form:

$$W_s(p) = \frac{Y(p)}{U(p)} = \frac{b_m \cdot p^m + b_{m-1} \cdot p^{m-1} + b_{m-2} \cdot p^{m-2} + \ldots + b_1 \cdot p + b_0}{p^n + a_{n-1} \cdot p^{n-1} + a_{n-2} \cdot p^{n-2} + \ldots + a_1 \cdot p + a_0},$$

where $b_i$ and $a_j$ are coefficients depended on the sensor TF (1) parameters for $i=0,1,...,m$ and $j=0,1,...,n-1$.

The corrector of the sensor dynamic error is designed according to the dynamic model given in the papers [20, 21]. The sensor model is the actual part of the measuring system and it is described by the same differential equation:

$$\frac{d^n y_m(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y_m(t)}{dt^{n-1}} + \ldots + a_0 y_m(t) = b_m \frac{d^m U_m(t)}{dt^m} + b_{m-1} \frac{d^{m-1} U_m(t)}{dt^{m-1}} + \ldots + b_0 U_m(t),$$

where $U_m(t)$ and $y_m(t)$ are input and output signals of the sensor model.

The block diagram of the dynamic parameters measuring transducer is shown in Fig. 1. The diagram is presented in the state-space form, where $x_{1m}(t)$, $x_{2m}(t)$, ..., $x_{nm}(t)$ are state coordinates of the sensor model.

The measuring transducer with the sensor model, which is the actual part of its structure, shown in Fig. 1 has the feedback coefficients of the model as the adjustable parameters [20, 21]. These coefficients were calculated by the required dynamic error value and they were constant during the measuring experiment data processing. However, there is a possibility, in principle, to adjust the coefficients of the model feedback during measuring experiment or measurement data processing. Thus, the criterion of the parameters adjustment can be not only the convex functional of the dynamic error, but also the specially created signal of the dynamic error evaluation, which is a continuous function of time depending on the adjusted parameters.
Consider the situation when all the feedback coefficients of the sensor model are being adjusted. As a measure of disagreement between the system movements and the model, the function of an error evaluation $e_i(p)$ is chosen in the form [22]:

$$I(t) = F(e_i(t)) = e_i^2(t).$$  \hfill (6)

The essence of the gradient method is the organization of such algorithm of the coefficients $k_i(t)$ adjustment that in each time point their change was aimed at the value $I(t)$ reduction as the function of the coefficients $k_i(t)$ [23]. This requires the following system of equations implementation:
where \( \lambda \) is the constant gain coefficient determining the speed of self-adjustment.

As a result of equations for \( I(t) \) from (6) to (7) substitution the following system of equations is obtained:

\[
\begin{align*}
\frac{dI_i(t)}{dt} &= -\lambda \cdot \frac{\partial I_i(t)}{\partial k_i(t)}, \\
\quad \vdots \\
\frac{dI_n(t)}{dt} &= -\lambda \cdot \frac{\partial I_n(t)}{\partial k_n(t)},
\end{align*}
\]

(7)

According to the shown in Fig. 1 block diagram of the measuring transducer the following equation holds:

\[
e_i(t) = y(t) + V(t) - y_M(t) + k_{n-1}(t)Z_{n-1}(t) + \ldots + k_1(t)Z_2(t) + k_0(t)Z_1(t).
\]

(9)

To take the derivatives of \( e_i(t) \) that the system (8) comprises it is necessary to obtain the derivatives of the components from equation (9). According to the measuring transducer block diagram shown in Fig. 1 the following equation holds:

\[
y_M(t) = b_0x_{1M}(t) + b_1x_{2M}(t) + \ldots + b_{m-1}x_{mM}(t) + b_mx_{mM}(t).
\]

(10)

As a result of equation (10) differentiation with respect to \( k_i(t) \) for \( i = 0, 1, \ldots, n-1 \) the following equation is obtained:

\[
\frac{\partial x_{1M}^{(n)}(t)}{\partial k_i(t)} = (k_{n-1}(t) - a_{n-1})\frac{\partial x_{1M}^{(n-1)}(t)}{\partial k_i(t)} + \ldots + (k_0(t) - a_0)\frac{\partial x_{1M}^{(0)}(t)}{\partial k_i(t)} + x_{1M}^{(i)}(t).
\]

(11)

The last equation is represented in the following form:

\[
V_{li}^{(n)}(t) = (k_{n-1}(t) - a_{n-1})V_{li}^{(n-1)}(t) + \ldots + (k_0(t) - a_0)V_{li}(t) + x_{1M}^{(i)}(t),
\]

(12)

where \( V_{li}(t) = \frac{\partial x_{1M}^{(i)}(t)}{\partial k_i(t)} \) for \( i = 0, 1, \ldots, n-1 \).

The variables \( V_{li}(t) \) are obtained by the solution to equation (12). Time derivatives of these coordinates are obtained as the state coordinates by the solution to these equations. If the coefficients \( k_i(t) \) change slowly, then the variables \( V_{li}(t) \) are output coordinates of LTI systems, where the signal \( x_{1M}^{(i)}(t) \) is active at the input. Thus, the derivative of the first component in equation (6) is written as follows:

\[
\frac{\partial y_M(t)}{\partial k_i(t)} = b_0V_{li}(t) + b_1V_{li}(t) + \ldots + b_mV_{li}(t).
\]

(13)

Determine the derivatives \( \frac{\partial Z_i(t)}{\partial k_i(t)} \) from equation (9). According to the measuring transducer block diagram shown in Fig. 1 the following equation holds:

\[
Z_i^{(n)}(t) = y(t) + V(t) - y_M(t) + (k_{n-1}(t) - a_{n-1})Z_{n-1}(t) + \ldots + (k_0(t) - a_0)Z_1(t).
\]

(14)
As a result of equation (14) differentiation with respect to \( k_i(t) \) for \( i = 0, 1, ..., n-1 \) the following equation is obtained:

\[
\frac{\partial Z_{i}^{(n)}(t)}{\partial k_{i}(t)} = -\frac{\partial y_{M}(t)}{\partial k_{i}(t)} + (k_{n-1}(t) - a_{n-1}) \frac{\partial Z_{i}^{(n-1)}(t)}{\partial k_{i}(t)} + \ldots + (k_{0}(t) - a_{0}) \frac{\partial Z_{i}(t)}{\partial k_{i}(t)} + Z_{i}^{(1)}(t).
\] (15)

The last equation is represented in the following form:

\[
q_{ii}^{(n)}(t) = -\frac{\partial y_{M}(t)}{\partial k_{i}(t)} + (k_{n-1}(t) - a_{n-1}) q_{ii}^{(n-1)}(t) + \ldots + (k_{0}(t) - a_{0}) q_{ii}(t) + Z_{i}^{(1)}(t),
\] (16)

where \( q_{ii}(t) = \frac{\partial Z_{i}(t)}{\partial k_{i}(t)} \) for \( i = 0, 1, ..., n-1 \).

The variables \( q_{ii}(t) \) are determined similarly to \( V_{ii}(t) \) as the solution to differential equation (16) with the input signal \( \frac{\partial y_{M}(t)}{\partial k_{i}(t)} \), and their derivatives as output coordinates of LTI systems.

\textbf{Fig. 2. Block diagram of the measuring transducer with self-adjusting according to the dynamic error parameters}

Designed on the basis of equations (12)–(16) shown in Fig. 2 the block diagram of the measuring transducer with self-adjusting according to the dynamic error parameters demonstrates all important
relations, when the measuring transducer is implemented in the analog form. Moreover, it can be considered as the structural representation of the differential equations which are necessary to be numerically integrated, when the measuring transducer is implemented as a program of the sensor signal digital processing.

2. Dynamic model of measuring system with sensor model in sliding mode

In A.L. Shestakov’s papers [7–11], to provide the proximity of the sensor described by the TF (1) and its model output signals, the feedbacks were introduced. It is also possible to provide the proximity of these signals by introduction a sliding mode into the system. To provide the sliding mode a nonlinear element (relay with the gain coefficient $K$) is used. In general, the sensor has the measured state vector with the measured state coordinates $x_1, x_2, \ldots, x_m$.

The sensor described by the TF (1) is presented by the following differential equations:

$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = x_3,$$
$$\vdots$$
$$\dot{x}_n = U - a_0x_1 - a_1x_2 - \ldots - a_mx_m - \ldots - a_{n-1}x_{n-1},$$

$$\bar{Y} = C \cdot \vec{X} = \begin{bmatrix} c_1 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 \\ 0 & 0 & c_{m-1} & 0 \\ 0 & 0 & 0 & c_m \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_m \\ \vdots \\ x_{n-1} \\ x_{n-1} \end{bmatrix},$$

where $U$ is the sensor input, $\bar{Y}$ is the sensor outputs vector of dimension $m \times n$; $a_0, a_1, \ldots, a_m, \ldots, a_{n-1}, b_0$ and $c_1, \ldots, c_m$ are constant coefficients; $\vec{X}$ is the state coordinates vector of the sensor; $x_1, x_2, \ldots, x_m, \ldots, x_n$ are sensor state variables; $C$ is the diagonal matrix of the sensor output signals coefficients.

In some cases there exists a possibility of the data obtaining about some part of the sensor state variables. Then, for the measurable state coordinates the coefficients are $c_i \neq 0$ for $i = 1, \ldots, m$ and $m < n$, and for unmeasurable state coordinates the coefficients are $c_i = 0$ for $i = m + 1, \ldots, n$. In general, the possibility to measure the full vector of the sensor states parameters ($\vec{X}$, i.e. coefficients $c_i \neq 0$ for $i = 1, \ldots, n$) is meant.

Suppose, as in the measuring transducer considered above, differential equations of the sensor model are identical to differential equations of the sensor:

$$\dot{x}_{1M} = x_{2M},$$
$$\dot{x}_{2M} = x_{3M},$$
$$\vdots$$
$$\dot{x}_{nM} = U_M - a_0x_{1M} - a_1x_{2M} - \ldots - a_mx_{mM} - \ldots - a_{n-1}x_{nM},$$

$$\bar{Y}_M = C \cdot \vec{X}_M = \begin{bmatrix} c_1 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 \\ 0 & 0 & c_{m-1} & 0 \\ 0 & 0 & 0 & c_m \end{bmatrix} \begin{bmatrix} x_{1M} \\ \vdots \\ x_{mM} \\ \vdots \\ x_{nM} \end{bmatrix},$$

where $U_M$ is the sensor model input; $\bar{Y}_M$ is the sensor model outputs vector of dimension $m \times n$; $\vec{X}_M$ is the state coordinates vector of the sensor model; $x_{1M}, x_{2M}, \ldots, x_{mM}, \ldots, x_{nM}$ are the sensor model state variables.

For the sliding mode implementation the following sliding surfaces are chosen:

$$\bar{S} = C \cdot \vec{X} - C \cdot \vec{X}_M = C \cdot (\vec{X} - \vec{X}_M).$$
While moving in the sliding mode \( \tilde{S} = 0 \) the sensor outputs vector \( \tilde{Y} \) and the sensor model outputs vector \( \tilde{Y}_M \) are equal. As long as the sensor and its model are described by the identical differential equations, their input signals will be equal too \( (U = U_M) \). This makes it possible to evaluate the measured signal \( U \) by the sensor model signal \( U_M \).

The block diagram of the measuring transducer is shown in Fig. 3.

![Block diagram of the measuring transducer](image)

If the sliding surfaces are \([24]\) \( \tilde{S} = C \cdot (\bar{X} - \bar{X}_M) = 0 \), then the following equality holds:

\[
\bar{X} = \bar{X}_M. \tag{20}
\]

Thus, state coordinates of the sensor and the sensor model are equal correspondingly \( x_1 = x_{1M}, \)
\( x_2 = x_{2M}, \ldots, x_n = x_{nM}. \)

On the basis of the following difference derived from equations (17) and (18):

\[
\dot{x}_n - \dot{x}_{nM} = U - U_M + a_0 (x_1 - x_{1M}) + a_1 (x_2 - x_{2M}) + \ldots + a_m (x_m - x_{mM}) + \ldots + a_{n-1} (x_n - x_{nM}),
\]

and the equality of state coordinates, the following equation is obtained:

\[
U - U_M = 0. \tag{21}
\]

Thus, the usage of the sliding surface vector and achievement of the sliding mode appearance in the system provide the equality of output and input signals of the measuring transducer. However, in real measuring systems there exists no possibility to observe the majority of the sensor state coordinates. Besides, even there exists the information about the majority of the state coordinates, it is impossible to achieve the sliding mode on all the sliding surfaces as long as a large number of the closed nonlinear contours occur in the system, which self-oscillations can cause difficulties in the sliding mode imple-
mentation and they can even lead to the exit from this mode. Therefore, it is appropriate to use only the dominant of the measurable sensor state coordinates (having the greatest value \( i \), for which \( c_i \neq 0 \)).

Consider a special case of the measuring transducer synthesis with the usage of the dominant of the measurable sensor state coordinates. Suppose the dominant measurable sensor state coordinate is \( x_m \), for which \( c_m \neq 0 \) and \( c_{m+1} = 0 \).

The sensor is described similar to equation (27) except for measured output signals:
\[
\dot{x}_1 = x_2, \\
\dot{x}_2 = x_3, \\
\vdots \\
\dot{x}_n = U - a_0 x_1 - a_1 x_2 - \ldots - a_m x_m - \ldots - a_{n-1} x_n, \\
Y = [x_1 c_1 \ldots x_{m-1} c_{m-1} x_m c_m]_T.
\]

Then, the equations for the sensor model will take the following form:
\[
\dot{x}_{1M} = x_{2M}, \\
\dot{x}_{2M} = x_{3M}, \\
\vdots \\
\dot{x}_{nM} = U_M - a_0 x_{1M} - a_1 x_{2M} - \ldots - a_m x_{mM} - \ldots - a_{n-1} x_{nM}, \\
\bar{Y}_M = [x_{1M} c_1 \ldots x_{(m-1)M} c_{m-1} x_{mM} c_m]_T.
\]

For the sliding mode implementation the following sliding surfaces are chosen:
\[
S = (x_m c_m - x_{mM} c_m) = (x_m - x_{mM}) c_m.
\]  

The block diagram of the measuring transducer with the measurable coordinate \( x_m \) is shown in Fig. 4.

![Fig. 4. Block diagram of the measuring transducer with the measurable coordinate \( x_m \)](image)
If the sliding surface is \( S = (x_m - x_{mM})c_m = 0 \), then the coordinates equality \( x_m = x_{mM} \) holds.

On the basis of the following difference derived from equations (22) and (23):
\[
\dot{x}_n - \dot{x}_{nM} = U - U_M + a_0(x_1 - x_{1M}) + a_1(x_2 - x_{2M}) + \ldots + a_m(x_m - x_{mM}) + \ldots + a_{n-1}(x_{n-1} - x_{nM}),
\]  
and the equality of state coordinates \( x_m = x_{mM} \), the following equation is obtained:
\[
U - U_M = 0.
\]  
Thus, the usage of the sliding surface (24) and achievement of the sliding mode appearance in the system provide the equality of output and input signals of the measuring transducer.

The sliding mode is accompanied by high-frequency noises; therefore it is necessary to filter the wanted signal. The issues of the filtration will be considered in subsequent papers.

In practice there exists no possibility to get the additional state coordinates from the majority of the sensors. The measured signal in such cases is the sensor output signal \( Y \).

Suppose the sensor is described by the following differential equations:
\[
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= x_3, \\
&\vdots \\
\dot{x}_n &= U - a_0x_1 - a_1x_2 - \ldots - a_{n-1}x_{n-1}, \\
Y &= x_1b_0 + x_2b_1 + \ldots + x_nb_n,
\end{align*}
\]  
where \( U \) and \( Y \) are the sensor input and output signals; \( a_0, a_1, \ldots, a_{n-1} \) and \( b_0, b_1, \ldots, b_m \) are constant coefficients; \( x_1, x_2, \ldots, x_n \) are sensor state variables.

Similarly to the previous cases the differential equations of the sensor model are identical to the differential equations of the sensor (27):
\[
\begin{align*}
\dot{x}_{1M} &= x_{2M}, \\
\dot{x}_{2M} &= x_{3M}, \\
&\vdots \\
\dot{x}_{nM} &= U_M - a_0x_{1M} - a_1x_{2M} - \ldots - a_{n-1}x_{nM}, \\
Y_M &= x_{1M}b_0 + x_{2M}b_1 + \ldots + x_{nM}b_{n-1},
\end{align*}
\]  
where \( U_M \) and \( Y_M \) are input and output signals of the sensor model; \( a_0, a_1, \ldots, a_{n-1} \) and \( b_0, b_1, \ldots, b_m \) are constant coefficients; \( x_{1M}, x_{2M}, \ldots, x_{nM} \) are sensor model state variables.

For the sliding mode implementation the following sliding surface is chosen:
\[
S = Y - Y_M.
\]  
When moving in the sliding mode (\( S = 0 \)) the sensor output signal \( Y \) and the sensor model output signal \( Y_M \) are equal. As long as the sensor and its model are described with the identical differential equations, input and output signals are also equal (\( U = U_M \)). This makes it possible to evaluate the measured signal \( U \) by the sensor model signal \( U_M \).

The sliding mode is implemented by meeting the following conditions [24]: when \( S > 0 \), the derivative \( \dot{S} \) must be negative (\( \dot{S} < 0 \)) and when \( S < 0 \), the derivative \( \dot{S} \) must be positive. According to the value of \( S \):
\[
\dot{S} = Y - Y_M.
\]  
As a result of the derivatives \( \dot{Y} \) and \( \dot{Y}_M \) from (27) and (28) to (30) substitution the following equation holds:
\[
\dot{S} = \dot{x}_1b_0 + \dot{x}_2b_1 + \ldots + \dot{x}_{n-1}b_{n-1} - \dot{x}_{1M}b_0 - \dot{x}_{2M}b_1 - \ldots - \dot{x}_{nM}b_n.
\]  
On the basis of the suggested sliding surface a measuring transducer is developed. The block diagram of the measuring transducer is shown in Fig. 5. To provide the sliding surface (\( S = 0 \)) the sensor and the corrector with the sensor model in its structure are used in the block diagram proposed.
The corrector in this case serves as a tracing system which when \( S = Y - Y_M < 0 \) reduces \( Y_M \) ensuring \( \dot{S} > 0 \) and when \( S = Y - Y_M > 0 \) increases \( Y_M \), approximating the difference to zero, ensuring \( \dot{S} < 0 \). As a result \( Y_M \) tends to \( Y \) and \( U_M \) tends to \( U \). Terms of regularization are included in the structure of the measuring system. This is the main idea of the measuring transducer in the sliding mode construction.

In the proposed block diagram a non-linear element such as a relay with the input signal \( S = Y - Y_M \) is used for the sliding mode occurrence. The gain coefficient \( K \) that affects both the signal amplitude at the relay output and the relay switching frequency was introduced after the non-linear element. With the increase of the coefficient \( K \) the switching frequency of the relay element also increases. Therefore, it is necessary to choose the coefficient \( K \) so that the high-frequency oscillations range lies outside the main signal spectrum. Moreover, at the relay element output high-frequency components emerge. This distorts the measured signal. Therefore, for the efficient recovery of the sensor input signal, it is necessary to set the low-pass filter after the relay element and the gain coefficient \( K \).

It should be noted that in a closed circuit with the sensor model and the nonlinear element self-oscillations can occur. As a result there exists a mismatch at the relay element input that leads to the exit from the sliding mode.

3. Neural network dynamic model of sensor

The discrete analog of the continuous TF (4) is represented in the following general form:

\[
W_i(z) = \frac{Y(z)}{U(z)} = \frac{\beta_0 + \beta_1 \cdot z^{-1} + \beta_2 \cdot z^{-2} + \ldots + \beta_{n-1} \cdot z^{-n+1} + \beta_n \cdot z^{-n}}{1 - \alpha_1 \cdot z^{-1} - \alpha_2 \cdot z^{-2} - \ldots - \alpha_{n-1} \cdot z^{-n+1} - \alpha_n \cdot z^{-n}},
\]

where \( U(z) \) and \( Y(z) \) are z-transformations of the sensor input and output signals; \( \beta_i \) and \( \alpha_j \) are coefficients depending on the sensor TF (4) parameters and the sampling period \( T \) for \( i = 0, 1, \ldots, n \) and \( j = 1, 2, \ldots, n \).
The difference equation corresponding to the discrete TF (4) of the sensor is as follows:

\[
y(k) - \sum_{j=1}^{n} \alpha_j \cdot y(k-j) = \sum_{i=0}^{n} \beta_i \cdot u(k-i),
\]

where \( u(k) \) and \( y(k) \) are samples of the sensor input and output signals at discrete times \( t_k = k \cdot T \) for \( k = 0, 1, 2, ... \).

The relationship between the output and input of the discrete sensor model is described by the following recurrence equation derived from the last one:

\[
y(k) = \sum_{i=0}^{n} \beta_i \cdot u(k-i) + \sum_{j=1}^{n} \alpha_j \cdot y(k-j).
\]

The parameters of the discrete model (32) is determined on the basis of the linear artificial neural network (ANN) sensor model. The block diagram of the general ANN sensor model is shown in Fig. 6. The model specified is the ANN that in its turn is the recurrent dynamic perceptron with the linear activation function \( f_a \) and the zero bias.

![Fig. 6. Block diagram of the general ANN sensor model](image)
The recurrence equation that determines the relationship between input and output of the ANN sensor model is as follows

\[ y^*(k) = \sum_{i=0}^{n} w_i \cdot u(k-i) + \sum_{j=1}^{n} v_j \cdot y^*(k-j), \quad (35) \]

where \( u(k) \) and \( y^*(k) \) are samples of the sensor input signal and the output signal of the ANN sensor model at discrete times \( t_k = k \cdot T \) for \( k = 0, 1, 2, \ldots \); \( w_i \) and \( v_j \) are adjustable coefficients (weights) of the ANN sensor model for \( i = 0, 1, \ldots, n \) and \( j = 1, 2, \ldots, n \).

By means of an appropriate procedure of the input and target training sets formation that reflects the relationship between the input and output of the discrete sensor model, the parameters (weights) of the ANN sensor model can be adjusted during the training process so that the samples of the ANN sensor model output will be equal to the corresponding discrete samples of the sensor output signal for a given level of the accuracy (that does not exceed the machine accuracy of calculations and rounding of the intermediate results). Meanwhile, the indicated possibility follows from the linearity and the compliance of the discrete and the ANN models of the sensor. Indeed, if \( y^*(k) = y(k) \) for \( k = 0, 1, 2, \ldots \), then from equations (34) and (35) the following equality holds:

\[ \sum_{i=0}^{n} \beta_i \cdot u(k-i) + \sum_{j=1}^{n} \alpha_j \cdot y(k-j) = \sum_{i=0}^{n} w_i \cdot u(k-i) + \sum_{j=1}^{n} v_j \cdot y(k-j). \quad (36) \]

As a result of the last expression transformation, the following equality holds:

\[ \sum_{i=0}^{n} (\beta_i - w_i) \cdot u(k-i) + \sum_{j=1}^{n} (\alpha_j - v_j) \cdot y(k-j) = 0. \quad (37) \]

Provided the sensor input is nonzero, the last equality becomes the identity only when \( \beta_i = w_i \) for \( i = 0, 1, \ldots, n \) and \( \alpha_j = v_j \) for \( j = 1, 2, \ldots, n \).

Thus, if as a result of the ANN sensor model training, samples of its output signal are equal to the corresponding samples of the output signal of the sensor with the TF (1), then the values of adjusted parameters of the ANN model will be the values of the sensor discrete model (32) parameters. Therefore, it is appropriate to choose the error function between the target and the actual output of the ANN sensor model as the criterion of the model under consideration training.

The discussed above approach to the ANN sensor model creation can be used for the solution to the problem of the dynamic measurement error correction caused by the sensor described by the TF (1) inertia. Then, this problem is formulated as the problem of the dynamic distorted sensor input signal recovery on the basis of the respective samples of its output signal.

Taking into account this statement it is necessary on the basis of the ANN sensor model to create the ANN inverse sensor model. This ANN inverse sensor model should provide the recovery of the dynamically distorted sensor input signal, i.e. implement the inverse relationship between the sensor input and output.

Consider the discrete model described by the TF (32) in order to obtain the structure of the generalized inverse ANN sensor model. This TF can be represented in the inverse form as follows:

\[ w^{-1}_s(z) = \frac{U(z)}{Y(z)} = \left( \frac{1}{\beta_0} - \frac{\alpha_1}{\beta_0} \cdot z^{-1} - \frac{\alpha_2}{\beta_0} \cdot z^{-2} - \cdots - \frac{\alpha_n}{\beta_0} \cdot z^{-n} \right)^{-1} = \frac{1}{\beta_0} + \frac{\alpha_1}{\beta_0} \cdot z^{-1} + \frac{\alpha_2}{\beta_0} \cdot z^{-2} + \cdots + \frac{\alpha_n}{\beta_0} \cdot z^{-n} = 1 - \frac{\alpha_1}{\beta_0} \cdot z^{-1} - \frac{\alpha_2}{\beta_0} \cdot z^{-2} - \cdots - \frac{\alpha_n}{\beta_0} \cdot z^{-n} = \frac{1}{\beta_0} + \frac{\beta_1}{\beta_0} \cdot z^{-1} + \frac{\beta_2}{\beta_0} \cdot z^{-2} + \cdots + \frac{\beta_n}{\beta_0} \cdot z^{-n} = \frac{1}{\beta_0} + \frac{\beta_1}{\beta_0} \cdot z^{-1} + \frac{\beta_2}{\beta_0} \cdot z^{-2} + \cdots + \frac{\beta_n}{\beta_0} \cdot z^{-n} = \frac{1}{\beta_0} - \frac{\beta_1}{\beta_0} \cdot z^{-1} - \frac{\beta_2}{\beta_0} \cdot z^{-2} - \cdots - \frac{\beta_n}{\beta_0} \cdot z^{-n} = \frac{1}{\beta_0} + \frac{\beta_1}{\beta_0} \cdot z^{-1} + \frac{\beta_2}{\beta_0} \cdot z^{-2} + \cdots + \frac{\beta_n}{\beta_0} \cdot z^{-n} = \frac{1}{\beta_0} - \frac{\beta_1}{\beta_0} \cdot z^{-1} - \frac{\beta_2}{\beta_0} \cdot z^{-2} - \cdots - \frac{\beta_n}{\beta_0} \cdot z^{-n}. \quad (38) \]
After introduction of the following notation: \( \mu_0 = \frac{1}{\beta_0}, \mu_i = -\frac{\alpha_i}{\beta_0} \) and \( \lambda_i = -\frac{\beta_i}{\beta_0} \) for \( i = 1, 2, \ldots, n \), the last equation is represented as follows:

\[
W_s^{-1}(z) = \frac{U(z)}{Y(z)} = \frac{\mu_0 + \mu_1 \cdot z^{-1} + \mu_2 \cdot z^{-2} + \ldots + \mu_{n-1} \cdot z^{-n+1} + \mu_n \cdot z^{-n}}{1 - \lambda_1 \cdot z^{-1} - \lambda_2 \cdot z^{-2} - \ldots - \lambda_{n-1} \cdot z^{-n+1} - \lambda_n \cdot z^{-n}}. \tag{39}
\]

The difference equation corresponding to the inverse discrete TF (39) of the sensor is as follows:

\[
u(k) - \sum_{j=1}^{n} \lambda_j \cdot u(k-j) = \sum_{i=0}^{n} \mu_i \cdot y(k-i). \tag{40}
\]

The relationship between the input and output of the inverse discrete sensor model is as the following recurrence equation derived from the last one:

\[
u(k) = \sum_{i=0}^{n} \mu_i \cdot y(k-i) + \sum_{j=1}^{n} \lambda_j \cdot u(k-j). \tag{41}
\]

The structure of equation (41) is similar to the structure of equation (34) for the direct discrete sensor model. Therefore the structure of the ANN inverse sensor model will also be the same as the structure of the ANN direct sensor model.

Fig. 7. Block diagram of the general ANN inverse sensor model
The block diagram of the general ANN inverse sensor model is shown in Fig. 7. This model is the ANN that is the recurrent dynamic perceptron with the linear activation function $f_a$ and the zero bias. The structure of this model completely corresponds to recurrence equation (41). The recurrence equation for the ANN inverse sensor model is as follows:

$$u^*(k) = \sum_{i=0}^{n} w'_i \cdot y(k-i) + \sum_{j=1}^{n} v'_j \cdot u^*(k-j),$$

(42)

where $u^*(k)$ are samples of the sensor output signal and the output signal of the ANN inverse sensor model at discrete times $t_k = k \cdot T$ for $k = 0,1,2,...$; $w'_i$ and $v'_j$ are weights of the generalized ANN sensor model for $i = 0,1,...,n$ and $j = 1,2,...,n$.

The criterion for the ANN inverse sensor model training as in the case of the ANN sensor model training is the minimum of the training error represented as the standard deviation between the target and actual outputs of the ANN inverse sensor model. Obviously, both in the criterion and in the training procedure of the ANN inverse sensor model it is necessary to swap the input and target training sets towards the criterion and the training procedure of the ANN sensor model.

**Conclusion**

Three general approaches to the dynamic measurement error correction based on the sensor model are described. The first approach implements the adaptive control method, the second one is based on the method of the sliding mode control, and the third one uses the neural network control method.

These approaches provide correction of the dynamic measurement error component caused by the inertia of the primary measuring transducer.

In addition, the vital issue in the problem of the total dynamic measurement error correction is the cancellation of random high-frequency noises at the sensor output during the process of its input signal recovery, as well as the issues of the considered models sustainability in case of the high-ordered transfer function of the primary measuring transducer. These problems are solved by the additional filtration of the signal recovered and the reduction of the sensor model order.

Different ways of solution to problems indicated on the basis of proposed three general approaches to the dynamic measurement error correction will be considered in subsequent papers.

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Предметом исследования являются измерительные системы и первичные измерительные преобразователи, функционирующие в динамическом режиме. Целью исследования является уменьшение динамической погрешности измерений, обусловленной инерционностью первичного измерительного преобразователя и случайными высокочастотными шумами и помехами, присутствующими на его выходе. Рассмотрены три обобщенных способа коррекции динамической погрешности измерений, базирующихся на структуре динамической измерительной системы с динамической моделью первичного измерительного преобразователя (датчика). Первый из них реализует метод адаптивного управления параметрами измерительной системы, второй – метод скользящих режимов в динамических измерительных системах и третий – нейросетевой метод. Указанные подходы базируются на принципах классической и современной теории автоматического управления и обеспечивают коррекцию динамической погрешности измерений путем восстановления входного сигнала первичного измерительного преобразователя, что ранее в классической постановке требовало решения интегрального уравнения типа свертки и использования обратного преобразования Фурье.

Ключевые слова: теория автоматического управления, динамические измерения, динамическая погрешность измерений, динамическая модель датчика, адаптивное управление, скользящий режим, нейросетевая модель.

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