High-fidelity preparation, gates, memory and readout of a trapped-ion quantum bit

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The great potential of quantum computing requires two essential ingredients for its realization: high-fidelity quantum logic operations and a physical implementation which can be scaled up to large numbers of quantum bits [1]. We introduce a trapped-ion qubit stored in ultrastable “atomic clock” states of $^{43}\text{Ca}^+$, in which we implement all single-qubit operations with fidelities significantly above the minimum threshold required for fault-tolerant quantum computing. We measure a combined qubit state preparation and single-shot readout fidelity of 99.93%, a memory coherence time of $T_2^\ast = 50$ seconds, and an average single-qubit gate fidelity of 99.9999%. These results are achieved in a room-temperature device without the use of magnetic field shielding or dynamic decoupling techniques to overcome technical noise. The surface-electrode ion trap chip incorporates integrated resonators and waveguides for coherent manipulation of the qubit using near-field microwaves [2]. Two-qubit gates [3] and individual qubit addressing [4] have already been demonstrated using this approach, which is scalable for a many-qubit architecture.

Amongst the candidate technologies for implementing quantum information processing, individual trapped ions were early recognized as a very promising system [5–7]: the qubits are stored in internal atomic energy levels of the ions, which can be extremely stable and well isolated from the environment, and the strong Coulomb interaction between neighbouring ions can be used to mediate qubit-qubit logic. Since the first proposals, multiple-qubit algorithms have been demonstrated [8], and there has been significant progress in developing scalable ion trap technologies [9]. Long qubit memory coherence time [10], high-fidelity state preparation and readout [11], and single-qubit gates with fault-tolerant error rates [12] have all been demonstrated, in a variety of different trapped ions and experiments. In this Letter, we demonstrate all single-qubit operations (preparation, memory, gates and readout) with performances comparable to or better than previous work, and all in the same system. All errors are more than an order of magnitude below the $\approx 1\%$ fault-tolerant thresholds emerging from recent numerical calculations using surface-code error correction [13]; this is critical for the practical implementa-
lifetime, which is typically several hours in this trap under ultra-high vacuum conditions, \(< 10^{-11}\) torr). \(T_2\) coherence times are limited by the frequency stability of the qubit transition. The state energies depend on the static magnetic field \(B\) through the Zeeman effect and ambient magnetic field noise would normally limit the coherence time to a few ms. However, certain transition energies become independent of magnetic field to first order at particular values of the field, due to the non-linear dependence arising from hyperfine state mixing, and these permit particularly stable qubits \([10]\). We choose one of these so-called “atomic clock” transitions, \(S_{1/2}^0 \leftrightarrow S_{1/2}^{+1}\) (where the superscripts denote angular momentum quantum numbers \(F, M\)), which in \(^{43}\text{Ca}^+\) is field-independent at \(B_0 \approx 146\) G (figure 2b).

The relatively large magnetic field leads to a complex atomic level structure, with Zeeman splittings spanning \(\sim 500\) MHz, and because of the low-lying D levels in \(\text{Ca}^+\) there is no closed cycling transition for laser cooling. We have nevertheless identified a simple Doppler cooling method which requires only two 397 nm frequencies, a single 866 nm frequency, moderate laser powers (\(\sim 100\) \(\mu\)W) and a single beam direction. We obtain a fluorescence count rate comparable to that from a single, saturated, \(^{40}\text{Ca}^+\) ion, at \(50\ 000\) s\(^{-1}\) with a net photon detection efficiency of \(0.3\%\), which is sufficient for high-fidelity fluorescence detection.

To measure the combined state preparation and measurement (SPAM) error we repeatedly prepare the same qubit state, and read it out, averaging over preparations of the \(|\downarrow\rangle\) and \(|\uparrow\rangle\) states (see Methods). For 150,000 preparations of each qubit state, we measure the combined SPAM error to be \(6.8(5) \times 10^{-4}\) (figure 3). As the qubit readout method is not a quantum non-demolition measurement, we cannot repeat it many times to separate the preparation and readout errors, but from estimates of the various contributions to the combined error (table 1) we assign errors of \(\approx 2 \times 10^{-4}\) to the state preparation and \(\approx 5 \times 10^{-4}\) to the readout. The error contributions could all be reduced by technical improvements (such as increasing the photon detection efficiency \([11]\)), except for the optical pumping transfer to \(D_{5/2}\) which is limited to a minimum error \([10]\) of \(\approx 1 \times 10^{-4}\) (at \(B_0 = 146\) G) by the atomic structure of \(^{43}\text{Ca}^+\).
FIG. 2: (a) Microwave spectroscopy of the qubit transition, varying the static magnetic field $B$ through the field-independent point $B_0 = 146.094 \text{G}$. At each field value, the qubit transition frequency $f$ was measured by Ramsey spectroscopy, to a precision $\approx 0.1 \text{Hz}$. The field-independent qubit transition is at $f_0 = 3.199941077 \text{Hz}$ after adjusting for a $-5 \text{Hz}$ a.c. Zeeman shift due to r.f. currents in the trap electrodes. The solid line shows the expected frequency calculated using the Breit-Rabi formula assuming the known zero-field hyperfine splitting [30] and a nuclear magnetic moment [19] of $\mu_I = -1.31535 \mu_N$. (b) Qubit coherence time measurements. At each value of the Ramsey free precession time $t_R$ the phase of the second $\pi/2$ pulse was varied to produce a set of Ramsey fringes. The contrast of the fringes is fitted with an exponential decay, giving a coherence time $T_2^* = 50(10) \text{s}$.

The qubit coherence time was measured by performing Ramsey experiments (without any dynamic decoupling pulses [20]) on the $S_{1/2}^0 \leftrightarrow S_{1/2}^\pm$ qubit transition at $f_0 = 3.200 \text{GHz}$ (see Methods). Ramsey delays up to $t_R = 16 \text{sec}$ were used, with results shown in figure 2b. An exponential decay $\exp(-t_R/T_2^*)$ fitted to the data gives a coherence time $T_2^* = 50(10) \text{sec}$. The coherence time may be limited by residual magnetic field drift (the qubit’s second-order field dependence is $d^2 f/dB^2 = 2.4 \text{mHz/mG}^2$), instability of the local oscillator, and fluctuations in the amplitude of the trap r.f. voltage (we measure an a.c. Zeeman shift of $-5 \text{Hz}$ in the qubit frequency due to the effect of r.f. currents in the trap electrodes [21]). The reduction in fringe contrast could also be due to effects unrelated to the qubit coherence, for example heating of the ion during $t_R$ which increases readout error due to Doppler-broadening of the $393 \text{nm}$ shelving transition. We note that longer coherence times have been measured in large ensembles, using trapped ions [22] and nuclear spins [23] (in the latter case, only with multiple dynamical decoupling pulses).

The fidelity of single-qubit gates driven by one of the near-field integrated microwave electrodes was measured by the technique of randomized benchmarking [24], which yields an average gate error appropriate to a computational context. We use the same method as ref. [12], which reports the previous lowest single-qubit gate error. Having prepared the qubit in $|\uparrow\rangle$, we apply a pre-programmed pseudo-random sequence of logical gates, where each logical gate comprises a Pauli gate followed by a Clifford gate. The sequence terminates by rotating the qubit into either $|\downarrow\rangle$ or $|\uparrow\rangle$, chosen with equal probability. Clifford gates are randomly chosen to rotate the qubit about the $\pm x$, $\pm y$, or $\pm z$ axes on the Bloch sphere; Pauli gates are randomly chosen to rotate about the $\pm x$, $\pm y$, or $\pm z$ axes, or to be a $\pm I$ identity gate. In the experiment, each Clifford gate is performed by a microwave $\pi/2$-pulse and each Pauli gate by a pair of $\pi/2$-pulses. Identity gates are implemented using delays of the same duration ($12 \mu\text{s}$) as the $\pi/2$-pulses, $\pm z$ rotations as an identity followed by a rotation of the logical frame of the qubit for subsequent pulses. The microwaves are generated by a frequency-octupled 400 MHz direct digital synthesis (DDS) source, fed via a switch to one of the m.w. electrodes (figure 1b); the enhancement provided by the integrated m.w. resonator and the proximity of the ion to the electrode means that a low m.w. power ($0.1 \text{mW}$) is sufficient and a power amplifier is not necessary. The m.w. power was periodically calibrated during the experiments using a sequence of 751 $\pi/2$-pulses. The qubit was kept at the field-independent point by servoing the magnetic field as in the coherence time measurements (see Methods).

Each pseudo-random sequence is applied many times,
and we compare the measured final qubit state with the expected outcome for that sequence. We apply sequences of various lengths, up to 2000 computational gates, and use 32 distinct sequences at each length (to average over systematic variations, since some sequences are more susceptible to errors than others). Results are shown in figure 4, where we deduce an average error per gate of 1.0(3) × 10^{-6}, while the intercept agrees with the independently measured SPAM error of 6.8(5) × 10^{-4} (dashed line). The duration of the longest sequences was 160 ms. Error bars represent statistical uncertainty, assuming binomial statistics.

![Figure 4](image)

**FIG. 4:** Randomized benchmarking of single-qubit gates. The start of an example pseudo-random sequence is shown: each computational gate is comprised of three physical π/2 pulses (absent in the case of identity I operations or z rotations). An additional final π/2 pulse rotates the qubit into the (|↓⟩, |↑⟩) basis for measurement. If the measured state disagrees with that expected, an error is recorded. For each sequence length, 32 distinct sequences are used, each one being repeated ∼100 times to measure the total error. Results are shown in the plot, where repeated runs have been offset horizontally for clarity. The gradient of the fitted straight line gives an average error per gate of 1.0(3) × 10^{-6}, while the intercept agrees with the independently measured SPAM error of 6.8(5) × 10^{-4} (dashed line). The duration of the longest sequences was 160 ms. Error bars represent statistical uncertainty, assuming binomial statistics.

| preparation/readout operation          | error          |
|----------------------------------------|----------------|
| stretch state S_{1/2}^{i+} preparation | < 1 × 10^{-4}  |
| transfer to qubit (3 or 4 m.w. π-pulses) | 1.8 × 10^{-4}  |
| transfer from qubit (4 m.w. π-pulses)   | 1.8 × 10^{-4}  |
| optical pumping transfer S_{1/2}^{i+} → D_{3/2} | 1.7 × 10^{-4}  |
| time-resolved fluorescence detection    | 1.5 × 10^{-4}  |

**TABLE I:** Error contributions: (top) state preparation and readout experiment; (bottom) single-qubit randomized benchmarking experiment (EPG: error per gate). The error contributions are estimates based on auxiliary experiments, experimentally measured parameters, and theoretical models of the various processes [19].

**METHODS**

**State preparation:** The qubit is initialized as follows. We first optically pump the ion to the S_{1/2}^{i+} state using circularly σ^+ polarized 397 nm light close to resonance with the S_{1/2}^\leftrightarrow P_{1/2}^{i+} and S_{1/2}^\leftrightarrow P_{1/2}^{i+} transitions. The fidelity of the optical pumping process is limited by imperfect polarization of the 397 nm beam, leaving population in (predominantly) ground level M = +3 and M = +2 states. We use a microwave technique to eliminate this error: after clearing out the S_{1/2}^3 states with a pulse of 397 nm σ^+ light on the S_{1/2}^3 ↔ P_{1/2}^3 transition, we drive microwave π-pulses on the S_{1/2}^{i+} → S_{1/2}^{i+} and S_{1/2}^{i+} → S_{1/2}^{i+} transitions, followed by another 397 nm σ^+ pulse to clear out S_{1/2}^3. The sequence of microwave and clear-out pulses can be repeated as often as necessary. We estimate that this sequence prepares the S_{1/2}^{i+} state with < 1 × 10^{-4} error. A series of three (or four) microwave π-pulses on the transitions indicated in figure 1, then transfers the ion to the |↑⟩ (or |↓⟩) qubit state, as
State readout: To read out the qubit state, a state-selective optical pumping method is used to implement the transfer \((|\downarrow\rangle, |\uparrow\rangle) \rightarrow (4S_{1/2}^{3/2}, 3D_{5/2})\), followed by fluorescence detection. First, three microwave π-pulse transfers transfer population in \(|\uparrow\rangle\) to \(S_{1/2}^{+4}\), and a fourth π-pulse transfers \(|\downarrow\rangle\) to \(S_{1/2}^{3/2}\). Population in \(S_{1/2}^{+4}\) is then "shelved" in the metastable \(3D_{5/2}\) level by a repeated sequence of \((393\text{ nm } \sigma^+, 850\text{ nm } \sigma^+, 850\text{ nm } \pi)\) pulses, as described in [11]; this shelving method is more involved than readout in ions without low-lying D levels, but more robust to imperfections in laser polarizations. Finally the 397 nm and 866 nm Doppler-cooling lasers are applied again and we detect whether or not the ion was shelved by the absence or presence of 397 nm fluorescence. Time-resolved photon counting is used to discriminate against spontaneous decays from the \(D_{5/2}\) level during the detection period [11].

Coherence time measurements: The microwave \(\pi/2\)-pulses are derived from a local oscillator referenced to an atomic clock with a nominal stability of \(< 10^{-11}\) (accuracy and long-term stability were verified by comparison with GPS [19]). We measure the contrast of Ramsey interference fringes as the Ramsey delay time \(t_R\) is increased, for delays up to 16 sec. These measurements are demanding on the stability of experimental conditions since, with a single qubit, each data point requires several hundred Ramsey experiments. To ensure that the applied magnetic field remains close to the field-independent point, the frequency of the field-dependent \(S_{1/2}^{+4} \leftrightarrow S_{1/2}^{3/2}\) transition is periodically measured by the computer controlling the experiment, and an appropriate correction is applied to the magnetic field coil current.

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AUTHOR CONTRIBUTIONS

T.P.H., D.T.C.A., C.J.B. and D.M.L. built the apparatus, designed and performed experiments, analysed data and wrote the paper. L.G. assisted with building apparatus and preliminary experiments. H.A.J. and D.N.S. simulated the atomic system. N.M.L. analysed data and produced the figures. All authors discussed the results and the text of the manuscript.

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