BOUNDARY LAYER FLOW OVER A NON-LINEARLY STRETCHING SHEET WITH ETHYLENE GLYCOL BASED NANOFLUID

Jahan Akhtar\(^1\) and A K Tiwari\(^2\)
\(^1\)Department of Mathematics, ShriVenkateshwara University, Gajraula, U.P., India
\(^2\)Department of Mathematics, Birla Institute of Technology, Mesra, Patna Campus, Bihar, India

Abstract- This paper deals with radiation consequences on stagnation point flow with a non-linearly stretching sheet in a porous media immersed in an ethylene glycol based nanofluid. We also studied the issue of magneto hydrodynamic (MHD) flow of a viscous fluid done with a non-linear stretching sheet in a porous media. Various ethylene glycol based nanoparticles Copper(Cu), Alumina(Al\(_2\)O\(_3\)), and Silver(Ag) are taken into consideration.

Keyword- nanofluid, stretching sheet, magneto hydrodynamic, porous media, concentration, bvp4c.

I. INTRODUCTION

Nield and Bejan[1] ascertained the ideas and study of thermal convection in porous media. Convection in porous media is the point of enormous concern owing to its broad area of applications in civil, chemical and mechanical engineering. Flow due to stretching surface plays a vital role in different fields of engineering and industry. Many real processes involve various types of stretching velocities such as linear power-law and exponential. Specifically, such flows are generated in fibres spinning, extrusion of polymers, hot rolling glass blowing, continuous casting and manufacturing of plastic and rubber sheets. The steady mixed convection boundary layer flow from an isothermal horizontal circular cylinder inserted in a porous media filled by a nanofluid was examined by Nazar et al[2].Yih[3-6] analyzed the boundary layer analysis of natural convection over a cone. Sanaad and Mohebujjaman [7] investigated the case along a vertical stretching sheet in presence of magnetic field and heat generation. Mohmoud[8] investigated variable viscosity effects on MHD flow in presence of radiation. Hossain and Takkar[9] studied the effect of radiation using the Rosseland diffusion approximation on mixed convection along a vertical plate with uniform free steam velocity and surface temperature. Cheng[10-11] investigated the point of issue of natural convection from a vertical cone in porous media saturated by a nanofluid with mixed thermal boundary conditions. Kumaran and Ramanaih[12] examined flow over a quadratic stretching sheet. Cortell [13] examined the flow, chemical reaction and mass transfer of steady laminar boundary flow of electrically conducting second grade fluid in porous medium over a semi-infinite impermeable stretching sheet subject to transverse uniform magnetic field. Using the homotopy perturbation method (HAM). Khan et al [14] analysed the effects of variable viscosity and thermal conductivity on the flow and heat transfer in a laminar liquid film over horizontal stretching sheet. Lin et al [15] studied MHD pseudo-plastic nanofluid unsteady flow and heat transfer in a finite thin film over stretching surface with internal heat generation and he observed that as the power law index increases, unsteadiness parameter and the critical value increase, while velocity and the temperature profiles and nanofluid films decrease.Cortell[16], Afzal[17], VajraVelu[18] analyzed the effects of various parameters governing the flow of viscous fluid over a non-linearly stretching sheet. Nadeem and Hussain[19] examined analytically the problem of MHD flow of a viscous fluid over a non-
linearly porous shrinking sheet. Hamad and Ferdows [20] analyzed the heat and mass transfer analysis for boundary layer stagnation-point flow over a stretching sheet in a porous medium filled by a nanofluid with internal heat generation/consumption and suction/fuming. Nazar et al [21] studied the problem of MHD flow of a viscous fluid on a nonlinear porous shrinking sheet and he found that the dual solutions existed for positive values of the controlling parameter only. Wang [22] studied both two-dimensional and axis-symmetric stagnation flow close to a shrinking sheet in a viscous fluid. He originated that result do not exist for larger shrinking rates and not different in the two-dimensional case. After this experimental work, the flow field over a stagnation point in the direction of a stretching/shrinking sheet has drawn major attention and a good amount of literature has been generated. Makinde and Aziz [23] analyzed numerically the boundary layer flow induced in a nanofluid due to a linearly stretching sheet. The main aim of the present paper was to study the effect of various parameters on MHD radiation effect over a nonlinearly stretching sheet with different ethylene glycol based nanofluids.

II. BASIC EQUATIONS

Consider the two-dimensional boundary layer flow due to a stretching/shrinking sheet. The fluid is a ethylene glycol based nanofluid containing three types of nanoparticles such as copper, alumina and silver. The stretching/shrinking velocity \( U_w(x) \) and the ambient velocity \( U_\infty(x) \) are assumed to vary non-linearly from the stagnation point, i.e. \( U_w(x) = ax^n \) and \( U_\infty(x) = cx^n \), where \( a \) and \( c \) are constant with \( c > 0 \). We note that \( a < 0 \) and \( a > 0 \) correspond to stretching and shrinking sheets, respectively. It is also assumed that the base fluid and nanoparticles are in thermal equilibrium and no slip occurs between them. The thermo physical properties of regular fluid and nanoparticles are given in Table 1.

| Physical properties | Ethylene glycol | Copper Cu | Alumina Al_2O_3 | Silver Ag |
|---------------------|----------------|-----------|-----------------|-----------|
| \( c_p \) (J/kg K)  | 2415           | 385       | 765             | 235       |
| \( \rho \) (kg/m^3) | 1114.4         | 8933      | 3970            | 10500     |
| \( k \) (W/mK)      | 0.252          | 400       | 46              | 429       |
| \( \beta \times 10^5 \) (1/K) | 65         | 1.67      | 0.63            | 1.89      |

The basic steady conservation of mass, momentum and thermal energy equations for nanofluid in porous media by using boundary-layer approximations in the presence of radiation, MHD and viscous dissipation can be written as:

**Equation of mass conservation:**

\[
\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{1}
\]

**Equation of momentum:**

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2(x)}{\rho_{nf}} u + U_\infty \frac{dU_\infty}{dx} - \frac{v_{nf}}{K} u \tag{2}
\]
The associated boundary conditions are:

\[ u = u_w(x) = cx^n, \quad v = 0; \]
\[ T = T_w(x) = T_\infty + bx^m; \]
\[ u \to 0; \quad T \to T_\infty \quad \text{as} \quad y \to \infty. \]

where \( u \) and \( v \) are the velocity component along the \( x \) and \( y \) directions, \( n \) and \( m \) are the non-linear stretching parameter, surface temperature parameter and concentration parameter respectively. The temperature on the wall is \( T_w \) and the influence is retained at consistent temperature \( T_\infty \). \( \alpha_{nf} \) is the effective viscosity, \( \alpha_{nf} \) is the thermal diffusivity, \( \rho_{nf} \) is the density and \( \nu_{nf} \) is the kinematic viscosity of the nanofluid.

\[ \rho_{nf} = (1 - \phi)\rho_f + \phi \rho_s, \]
\[ \alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}, \quad \mu_{nf} = \frac{\mu_f}{(1 - \phi)^{\frac{1}{2}}}, \]
\[ (\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s, \]
\[ k_{nf} = \frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)} \]
\[ k_f = \frac{(k_s + 2k_f)}{k_s}. \]

Here, \( \phi \) is the nanoparticles volume division, \( (\rho C_p)_{nf} \) is the heat capacity and \( k_{nf} \) is the thermal conductivity of the nanofluid. \( k_s \) is the thermal conductivity of nanoparticles and \( k_f \) is the thermal conductivity of base fluid. Using Rosseland approximation[24] for radiation, the radiative heat flux is interpreted as:

\[ q_r = -\frac{4\sigma}{3k} \frac{\partial T^4}{\partial y} \]

Where \( \sigma \) is the Stefan-Boltzmann constant and \( k \) is the mean absorption coefficient. We consider that the temperature difference within the flow is very small as \( T^4 \) can be expressed as linear function of temperature. Accordingly, expanding \( T^4 \) in a Taylor series about \( T_\infty \) and neglecting higher order terms, we get:

\[ T^4 \approx 4T_\infty^4 - 3T_\infty^4 \quad \text{(7)} \]

Using equations (7) and (8), equation (3) reduces to:

\[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \frac{\partial^2 T}{\partial y^2} + \frac{\nu_{nf}}{k_0} \left( \frac{\partial u}{\partial y} \right)^2 \]

Where \( k_0 = \frac{3N_R}{3N_R + 4} \). It is worth quoting that the ideal solution for energy equation, Equation (9), without thermal radiation effect can be obtained from equation (8), which reduces to
\[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \frac{\partial^2 T}{\partial y^2} \]

as \( N_R \to \infty (i.e., k_o \to 1) \) and viscous dissipation vanishes.

We consider that the extraneous electric field and polarization effects in equation (2) are negligible, as the magnetic field \( B(x) \) is in the form

\[ B(x) = B_0 x^{(n-1)/2} \]

we considered that the magnetic Reynolds number is insufficient, so that the induced magnetic field is very small.

\[ u = cx^n f'(\eta); \]

The similarity transformations are:

\[ v = -\sqrt{\frac{c v(n+1)}{2}} x^{n-\frac{1}{2}} \left\{ f(\eta) + \frac{n-1}{n+1} \eta f'(\eta) \right\}; \]

\[ \eta = \sqrt{\frac{c(n+1)}{2}\nu} x^{n-\frac{1}{2}} y; \theta(\eta) = \frac{T-T_w}{T_w-T_w}; \]

The equations (1), (2) and (7) became:

\[ f''' - (1-\phi) \frac{2e}{2} \left\{ 1 - \phi + \frac{\rho(t)}{\rho(t)} \right\} \times \]

\[ \times \frac{2n}{n+1} \left( f'^2 - \frac{n+1}{2n} \eta f'' + Rf' \right) = 0 \]

\[ \frac{1}{Pr} \left( \frac{k_{nf}}{k_f} \right) \left( \frac{\theta''}{\theta_0} + \frac{Ec}{1-\phi} x^{2n-1} f'' \right) \]

\[ + \left( 1 - \phi + \frac{(\rho(Cp)_{nf}}{(\rho(Cp)_{f})} \right) \left( f\theta - \frac{2m}{n+1} f' \right) \theta = 0 \]

So that all akin solutions put \( m = 2n \) in equation (13), which reduces to

\[ \frac{1}{Pr} \left( \frac{k_{nf}}{k_f} \right) \left( \frac{\theta''}{\theta_0} + \frac{Ec}{1-\phi} x^{2n-1} f'' \right) \]

\[ + \left( 1 - \phi + \frac{(\rho(Cp)_{nf}}{(\rho(Cp)_{f})} \right) \left( f\theta - \frac{4n}{n+1} f' \right) \theta = 0 \]

Subjected to the boundary conditions (5) which becomes

\[ f(0) = 0, f'(0) = 1, \theta(0) = 0, h(0) = 0, \]

\[ f'(\infty) = 1, \theta(\infty) = 0, h(\infty) = 0 \text{ as } \eta \to \infty \]

where \( Pr = \frac{\nu f}{\alpha f} \) is the Prandtl number, \( Ec = \frac{u_w^2}{(Cp)_{f}(T_w-T_w)} \) is the Eckert number, \( R = \frac{\nu_{nf}}{K_{nf}} + \frac{\sigma B_0^2}{\rho_{nf} \nu} \) is the combined magnetic and porosity parameter.

The physical quantities of interest are the skin friction coefficient and the local Nusselt number are defined as
\[
c_f = \frac{2\mu_{nf}}{\rho_f (u_* (x))^2 \left( \frac{\partial u}{\partial y} \right)_{y=0}} \\
- k_{nf} \left( \frac{\partial T}{\partial y} \right)_{y=0} \\
Nu_x = \frac{k_f (T_w - T_\infty)}{k_f (T_w - T_\infty)} 
\]
(16)

with \( \mu \) and \( k \) are the dynamic viscosity and thermal conductivity, respectively. Using non-dimensional variables, we have

\[
\sqrt{\frac{c}{2\nu_f}} c_f = \frac{\sqrt{(n+1)x^{\frac{n+1}{2}}}}{(1-\phi)^{2.5}} f''(0) \\
\sqrt{\frac{2\nu_f}{c}} Nu_x = -\frac{k_{nf} (n+1)x^{\frac{n+1}{2}}}{k_f} \theta'(0) 
\]
(18)

### III. METHOD OF SOLUTION

The sets of nonlinear governing differential Equations. (12)-(15) with boundary conditions (16) constitute a two-point boundary value problem. These equations are solved using “bvp4c” function of MATLAB software package. The function has three input variables namely: M-file enumerating an ordinary differential equation system of the design, M-file enumerating the boundary values, and an initial approximation of the result prepared with the MATLAB function “bvpinit”. The output variable of “bvp4c” answers the solution in the form \([f(\eta), f'(\eta), f''(\eta), f'''(\eta)]\), which can be deliberated at any given framework with the MATLAB function “deval”.

### IV. RESULTS AND DISCUSSION

In order to bring out prime characteristic of the flow over a non-linearly stretching sheet in a porous media with Copper (Cu), Alumina (Al\(_2\)O\(_3\)), and Silver (Ag) nanoparticles are depicted in Fig 1-6 for variant values of volume fraction \( \phi \), combined porosity parameter and magnetic parameter R, Eckert number Ec, Prandtl number Pr and thermal radiation parameter \( N_R \). Table 2 shows computational values \(- f''(0)\) and \(- \theta'(0)\) for Copper, Alumina and Titanium nanoparticles with different values \( \phi \). From Table 2, it is clear that the Nusselt number is minimum at \( \phi = 0.5 \) for all the three nanofluids.

**Table 2. Values related to the skin friction and Nusselt number for different values of \( \phi \) when \( Pr=6.25, n=0.8, Ec=0.1, R=0.5 \)**

| \( \phi \) | \( Cu \) | \( Al_2O_3 \) | \( Ag \) | \( Cu \) | \( Al_2O_3 \) | \( Ag \) |
| --- | --- | --- | --- | --- | --- | --- |
| 0.0 | 0.3489 | 0.3489 | 0.3489 | 2.2807 | 2.2807 | 2.2807 |
| 0.1 | 0.4113 | 0.3484 | 0.4293 | 1.1914 | 1.9423 | 1.8821 |
| 0.2 | 0.4267 | 0.3331 | 0.4523 | 1.6240 | 1.6584 | 1.5728 |
| 0.3 | 0.4126 | 0.3036 | 0.4409 | 1.3912 | 1.4281 | 1.3195 |
Figure 1: $\phi=0.1, 0.2, 0.3, n=0.5, R=0.3$. Velocity profile for different values of solid volume fraction $\phi$.

Figure 2: $\phi=0.1, 0.2, 0.3, Pr=6.2, Ec=0.1, n=0.4, N_R=1$. Temperature profile for different values of solid volume fraction $\phi$. 

| $\phi$ | $f' (\eta)$ | $\phi=0.1, 0.2, 0.3$ |
|--------|-------------|----------------------|
| 0.4    | 0.3775      | 0.2706               |
| 0.5    | 0.3267      | 0.2279               |
|        | 0.4055      | 0.3523               |
|        | 1.1754      | 0.9972               |
|        | 1.2311      | 1.0172               |
|        | 1.1007      | 0.9096               |
Figure 3: \( R=0.1, 0.2, 0.3, \phi=0.1, n=0.8 \). Velocity profile for different values of combined magnetic and porosity parameter \( R \)

Figure 4: \( Ec=0.0, 0.5, 1, \phi=0.1, n=0.4, Pr=6.2, N_R=1 \). Temperature profile for different values of Eckert number

Figure 5: \( Pr=0.72, 1, 3, \phi=0.1, Ec=0.5, n=0.4, N_R=2 \). Temperature profile for different values of Prandtl number
Figure 6: \( N_R = 1, 2, 3, \phi = 0.1, \text{Pr}=3, \text{Ec}=0.5, n=0.4 \). Temperature profile for different values of thermal radiation \( N_R \).

Figure 7: \( \text{Pr}=6.25, n=0.8, \text{Ec}=0.1, R=0.5 \). Skin friction for variation in \( \phi \).

Figure 8: \( \text{Pr}=6.25, n=0.8, \text{Ec}=0.1, R=0.5 \) Nusselt number for variation in \( \phi \).
The effect of the nanoparticles volume fraction $\phi$ on the velocity and temperature profile is shown in fig 1 and 2. In Fig 1, the velocity profile decreases with the increase in value of $\phi$ to a certain level and again it increases to 1.

Fig 2 exhibits the effect of nanoparticles volume fraction $\phi$ on temperature profile. The temperature profile decreases as the volume fraction $\phi$ increases and there is an increase in thermal conductivity and thermal boundary layer thickness. This happens due to the presence of solid nanoparticles which leads to further thinning of the velocity boundary layer thickness.

Fig 3 exhibits for combined magnetic and porosity parameter $R$ for different values of $n$. It is obvious from the figure that the velocity profile decreases with the increase in $R$ to a fixed level and again it increases to 1 and it is minimum for $R=3$ for all the three nanoparticles.

Fig 4 is for variation in Eckert number with $\phi = 0.1$ and $N_R = 1$. Eckert number expresses the relationship between a flow's kinetic energy and the boundary layer enthalpy difference, and is used to characterize heat dissipation. With increase in Eckert number there is a decrease in temperature profile. The thermal boundary layer thickness increase with increasing Ec.

Fig 5 is for variation in Prandtl number $Pr$ on temperature profile. The Prandtl number is a dimensionless number, entitled after the German physicist Ludwig Prandtl, defined as the ratio of momentum diffusivity to thermal diffusivity. It is apparent from the figure that the temperature profile decreases with increasing Prandtl number and the thermal boundary layer thickness decreases.

The effect of variation in thermal radiation $N_R$ on temperature profile is depicted in Fig 6. It is obvious from the figure, that the temperature profile decreases with the increase in value of $N_R$ and the thermal boundary layer thickness decreases. This reality disclosed the result that the decrease in value of $N_R$ for given $k$ and $T_w$ means an enhancement in Rosseland absorptivity $k_1$. According to equations (3) and (5), the divergence of the radiative heat flux decreases as $k_1$ increases which in turn decrease the rate of heat transferred to the fluid and hence the fluid temperature decreases and thermal layer also decreases with increase in $N_R$.

Fig 7 and Fig 8 is for variation in $-f''(0)$ and $-\theta(0)$ for variant thermo-physical properties of ethylene glycol and nanoparticles.

V. CONCLUSION

- The local Nusselt number is maximum at $\phi = 0.0$.
- There is an increment in thermal boundary layer thickness with increase in Eckert number.
- An increase in thermal radiation $N_R$ yields a decrease in the nanofluids temperature.
- The increase in solid volume fraction $\phi$ shows an increment in skin friction to a certain level.

REFERENCES

[1] Nield DA, Bejan A. Convection in porous media, 3rd edn. Springer, New York (2006)
[2] Nazar R, Tham L, PopI, Ingham DB. Mixed convection boundary layer flow from a horizontal circular cylinder embedded in a porous medium filled with a nanofluid. TranspPorousMed. 2011;86:517–536
[3] Yih KA. The effect of uniform lateral mass flux effect on free convection about a vertical cone embedded in a saturated porous medium. Int. Comm. Heat Mass Transfer. 1997;24(8):1195–1205
[4] Yih KA. Uniform lateral mass flux effect on natural convection of non-Newtonian fluids over a cone in porous media. Int. Comm. Heat Mass Transfer.1998. 25(7):959–968
[5] Yih KA. Effect of radiation on natural convection about a truncated cone. Int. J. Heat Mass Transfer.1999. 42; 4299–4305
[6] Yih KA. Coupled heat and mass transfer by free convection over a truncated cone in porous media: VWT/VWC or VHF/VMF. Acta Mech. 1999. 137; 83–97

[7] Samad, M.A, and Mohebujjaman, M, (2009), “MHD Heat and Mass Transfer Free Convection Flow Along a Vertical Stretching Sheet in Presence of Magnetic Field With Heat Generation,” Res. J. Appl. Sci. Eng. Technol. , 1(3), pp. 98-106

[8] Mahmoud, M. A. A., (2007), “Variable Viscosity Effects of Hydromagnetic Boundary Layer Flow Along a Continuously Moving Vertical Plate in the Presence of Radiation,” Appl. Math. Sci., 1(17), pp. 779-814.

[9] Hossain, M. A., and Takher, H.S., (1996), “Radiation Effect on Mixed Convection Along a Vertical Plate With Uniform Surface Temperature,” Heat Mass Transfer, 31(4), pp. 243-248

[10] Cheng CY. Soret and Dufour effects on heat and mass transfer by natural convection from a vertical truncated cone in a fluid-saturated porous medium with variable wall temperature and concentration. Int. Comm. Heat Mass Transfer. 2010.37; 1031–1035

[11] Cheng CY. Nonsimilar boundary layer analysis of double-diffusive convection from a vertical truncated cone in a porous medium with variable viscosity. Int. Comm. Heat Mass Transfer. 2010. 37; 1031–1035

[12] Kumaran V, RamanaihG. A note on the flow over stretching sheet. Arch Mech. 2010. 116; 229–233

[13] R. Cortell, MHD flow and mass transfer of an electrically conducting fluid of second grade in a porous medium over a stretching sheet with chemically reactive species, Chem. Eng. Process. 46 (2007) 721-728

[14] Y. Khan, Q.B. Wu, N.Faraz, A. Yildirim, The effect of variable viscosity and thermal conductivity on a thin film flow over a shrinking/stretching sheet, Comput, Math. Appl, 61 (2011) 3391-3399

[15] Y. Lin, L. Zheng, X. Zhang, L. Ma, G. Chen, MHD pseudo-plastic nanofluid unsteady flow and heat transfer in a finite thin film over stretching surface with internal heat generation, International Journal of Heat and Mass Transfer 84 (2015) 903-911

[16] Cortell R. Heat and fluid flow due to non-linearly stretching surfaces. Appl Math Comput. 2011. 217;7564–7572

[17] Afzal N. Momentum and thermal boundary layers over a two-dimensional or axisymmetric non-linear stretching surface in a stationary fluid. Int J Heat Mass Transf. 2010.53;540–547

[18] Vajravelu K. Viscous flow over a nonlinearly stretching sheet. A ppl Math Comput. 2001. 124;281–288

[19] NadeemS. HussainA. MHD flow of a viscous fluid on a nonlinear porous shrinking sheet with homotopy analysis method. Appl. Math. Mech. 2009. 30(12);1569-1578

[20] Hamad M, FerdowsM. Similarity solution of boundary layer stagnation-point flow towards a heated porous stretching sheet saturated with a nanofluid with heat absorption/generation and suction/blowing: a lie group analysis. Commune Nonlinear SciNumerSimulat. 2011. 17(1);132-140

[21] Nazar R, PopI, Arifin,NM, AliFM.Dual solution in MHD flow on a nonlinear porous shrinking sheet in a viscous fluid. [J].Boundary Value Problems. 2013. 32

[22] Wang CY. Stagnation flow towards a shrinking sheet. Int J Non Lin Mech. 2008.43;377-382

[23] MakindeOD, Aziz A. Boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition. Int J Therm Sci. 2011; 50, 1326–1332

[24] Brewster MQ. Thermal Radiative Transfer Properties, Wiley, Canada 1992