Nucleon-Nucleon Correlations in Electromagnetically Induced Knockout Reactions

C. Giusti

Dipartimento di Fisica Nucleare e Teorica, Università degli Studi di Pavia, and Istituto Nazionale di Fisica Nucleare, Sezione di Pavia, Pavia, Italy

Abstract

The attempts to investigate correlations in electromagnetically induced one- and two-nucleon knockout are reviewed. The theoretical framework for cross section calculations is outlined and some results are presented for the exclusive $^{16}\text{O}(e,e'p)^{15}\text{N}$ and $^{16}\text{O}(e,e'pp)^{14}\text{C}$ reactions. For the $(e,e'p)$ reaction attention is focussed on extracting the spectroscopic factors. For the $(e,e'pp)$ reaction the possibility of obtaining direct and clear information on short-range correlations is discussed.

1 Introduction

Correlations in the nuclear wave function beyond the mean-field approximation are very important to describe the basic properties of nuclear structure. The mean-field (MF) or Hartree-Fock (HF) approximation gives a good description of the bulk properties of nuclei if phenomenological nucleon-nucleon ($NN$) forces are used, but employing realistic $NN$ interactions it fails badly. The failure is a consequence of the strong and repulsive short-range components of a realistic $NN$ interaction. Thus, a careful evaluation of the short-range correlations (SRC), produced by the short-range components of the $NN$ interaction, is needed to describe the nuclear properties in terms of a realistic $NN$ interaction and provide profound insight into the structure of the hadronic interaction in a nucleus. The same is true for the tensor correlations (TC), which are induced by the strong tensor components of the $NN$ interaction. Moreover, it is necessary to consider also those processes beyond the MF approximation falling under the generic name of long-range correlations (LRC), which are related to the coupling between the single-particle (s.p.) dynamics and the collective excitation modes of the nucleus and which mainly represent the interaction of nucleons at the nuclear surface. These processes can be very important in finite nuclear systems. A consistent evaluation of these different types of correlations is needed.

*E-mail: giusti@pv.infn.it, phone: +39 0382507454, fax: +39 0382526938.
Various theoretical methods have been developed to account for such correlation effects (see, e.g., [1, 2, 3]). Besides developing these methods, it has always been a challenge for nuclear physics to envisage experimental situations and observables, particularly sensitive to correlations, whose determination can give clear evidence for correlations, in particular for SRC. The hope is that the comparison between the predictions of different models and data can give detailed information on correlations and can allow one to distinguish the different models of the $NN$ interaction at short distance.

Electromagnetically induced one- and two-nucleon knockout reactions are powerful tools for this study (see, e.g., [4]). In one-nucleon knockout the conditions of individual nucleons in the nuclear medium can be explored. For an exclusive reaction, the coincidence cross section contains the one-hole spectral density function, i.e.

$$S(p_1, p'_1; E_m) = \langle \Psi_i \mid a_{p_1}^+ \delta(E_m - H) a_{p'_1} \mid \Psi_i \rangle,$$

which in its diagonal form ($p_1 = p'_1$) gives the joint probability of removing from the target a nucleon, with momentum $p_1$, leaving the residual nucleus in a state with energy $E_m$ with respect to the target ground state. In an inclusive reaction, integrating the spectral density over the whole energy spectrum produces the one-body density matrix (OBDM) $\rho(p_1, p'_1)$, that in its diagonal form gives the nucleon momentum distribution.

Likewise two-nucleon knockout represents the preferential tool for exploring the conditions of pairs of nucleons in the nuclear medium. For an exclusive reaction, the coincidence cross section contains the two-hole spectral density function, i.e.

$$S(p_1, p_2, p'_1, p'_2; E_m) = \langle \Psi_i \mid a_{p_2}^+ a_{p'_1}^+ \delta(E_m - H) a_{p_1} a_{p_2} \mid \Psi_i \rangle,$$

which in its diagonal form ($p_1 = p'_1$ and $p_2 = p'_2$) gives the joint probability of removing from the target two nucleons, with momenta $p_1$ and $p_2$, leaving the residual nucleus in a state with energy $E_m$ with respect to the target ground state. In an inclusive reaction, integrating the spectral density over the whole energy spectrum produces the two-body density matrix $\rho(p_1, p_2, p'_1, p'_2)$, that in its diagonal form and in the coordinate representation gives the pair correlation function, i.e. the conditional probability density of finding in the target a particle at $r_2$ if another one is known to be at $r_1$.

The cross sections of one- and two-nucleon knockout reactions contain information on $NN$ correlations in the spectral functions and in the density matrices. In order to extract this information, along with the experimental work, a reliable model for cross section calculations is needed, able to keep reasonably under control the reaction mechanism and all the theoretical ingredients contributing to the cross section.

The quasifree $(e, e'p)$ reaction has extensively been used to investigate the s.p. properties of nuclei and to point out the validity and the limits of the independent-particle shell model (IPSM) [4]. The fact that a pure MF picture is unable to give a precise quantitative description of $(e, e'p)$ data is due to correlations and the
discrepancy can give a measurement of correlation effects. More direct information on correlations, and in particular on SRC, is available from two-nucleon knockout reactions. First recent experiments have given clear evidence for SRC in the $^{16}\text{O}(e,e'pp)^{14}\text{C}$ reaction.

In this contribution the attempts to investigate $NN$ correlations in exclusive knockout reactions are reviewed. The theoretical framework for cross section calculations is outlined and some results are presented and discussed for $(e,e'p)$ in Sec. 2 and $(e,e'pp)$ in Sec. 3.

2 One-nucleon knockout

2.1 The Plane-Wave Impulse Approximation

In the one-photon exchange approximation, where the incident electron exchanges a photon of momentum $q$ and energy $\omega$ with the target, the most general form of the coincidence $(e,e'N)$ cross section involves the contraction between a lepton tensor $L_{\mu\nu}$ and a hadron tensor $W_{\mu\nu}^T$. The lepton tensor is produced by the matrix elements of the electron current and, neglecting the effect of the nuclear Coulomb field on electrons, contains only electron kinematics. The components of the hadron tensor are given by bilinear combinations of the Fourier transforms of the transition matrix elements of the nuclear current operator between initial and final nuclear states, i.e.

$$J^{\mu}(q) = \int \langle \Psi_f | \hat{J}^{\mu}(r) | \Psi_i \rangle e^{iq \cdot r} dr.$$  \hspace{1cm} (3)

In the plane-wave impulse approximation (PWIA), i.e. neglecting final-state interactions (FSI) of the ejected particle, the $(e,e'p)$ cross section is factorized as a product of a kinematical factor, the (off-shell) electron-proton cross section and, for an exclusive reaction, the one-hole diagonal spectral density function

$$S(p_m,E_m) = \sum_{\alpha} S_{\alpha}(E_m)|\phi_{\alpha}(p_m)|^2,$$  \hspace{1cm} (4)

where the missing momentum $p_m$ is the recoil momentum of the residual nucleus. At each value of $E_m$, the momentum dependence of the spectral function is given by the momentum distribution of the quasi-hole states $\alpha$ produced in the target nucleus at that energy and described by the normalized overlap functions (OF) $\phi_{\alpha}$ between the target ground state and the states of the residual nucleus. The (normalization) spectroscopic factor (s.f.) $S_{\alpha}$ gives the probability that the quasi-hole state $\alpha$ is a pure hole-state in the target. In an IPSM $\phi_{\alpha}$ are the s.p. states of the model and $S_{\alpha} = 1(0)$ for occupied (empty) states. In reality, the strength of a quasi-hole state is fragmented over a set of s.p. states, and $0 \leq S_{\alpha} < 1$. The fragmentation of the strength is due to correlations and the s.f. can thus give a measurement of correlation effects.

The PWIA is a simple and clear picture that is able to describe the main qualitative features of $(e,e'p)$ cross sections, but is unable to give a precise
quantitative description of data. For the analysis of data a more refined theoretical treatment is needed. The calculations for this analysis were carried out with the program DWEEPY [6], within the theoretical framework of a nonrelativistic distorted-wave impulse approximation (DWIA), where FSI and Coulomb distortion of the electron wave functions are taken into account.

2.2 The Distorted-Wave Impulse Approximation

The DWIA treatment of the matrix elements in Eq. (3) is based on the following assumptions:

i) An exclusive process is considered, where the residual nucleus is left in a discrete eigenstate $|\Psi_B^{\alpha}(E)\rangle$ of its Hamiltonian, with energy $E$ and quantum numbers $\alpha$.

ii) The final nuclear state is projected onto the channel subspace spanned by the vectors corresponding to a nucleon, at $r_1$, and the residual nucleus in the state $|\Psi_B^{\alpha}(E)\rangle$. This assumption neglects effects of coupled channels and is justified by the considered asymptotic configuration of the final state.

iii) The (one-body) nuclear-current operator does not connect different channel subspaces. Thus, also the initial state is projected onto the selected channel subspace. This assumption is the basis of the direct knockout (DKO) mechanism and is related to the IA.

The transition matrix elements in Eq. (3) can thus be written in a one-body representation as

$$J^{\mu}(q) = \int \chi_{E\alpha}^{(-)}(r_1) \hat{J}^{\mu}(r, r_1) \phi_{E\alpha}(r_1) [S_\alpha(E)]^{1/2} e^{iq \cdot r_1} dr_1 dr,$$

where

$$\chi_{E\alpha}^{(-)}(r_1) = \langle \Psi_B^{\alpha}(E)|a_{r_1}|\Psi_t^1\rangle$$

is the s.p. distorted wave function of the ejectile and the overlap function

$$[S_\alpha(E)]^{1/2} \phi_{E\alpha}(r_1) = \langle \Psi_B^{\alpha}(E)|a_{r_1}|\Psi^1\rangle$$

describes the residual nucleus as a hole state in the target. The spectroscopic strength $S_\alpha(E)$ is the norm of the overlap integral in the right-hand side of (6) and gives the probability of removing from the target a nucleon at $r_1$ leaving the residual nucleus in the state $|\Psi_B^{\alpha}(E)\rangle$.

The scattering state in Eq. (6) and the normalized bound state $\phi_{E\alpha}(r_1)$ in Eq. (7) are consistently derived in this model from an energy-dependent non-Hermitean optical model Feshbach Hamiltonian. In standard DWIA calculations, however, phenomenological ingredients are employed. The nucleon scattering state is eigenfunction of a phenomenological optical potential, determined through a fit to elastic nucleon-nucleus scattering data including cross sections and polarizations. Phenomenological bound-state wave functions are usually adopted for the OF, which thus do not include correlations. In the analysis of data these functions were calculated in a Woods-Saxon well, where the radius was determined to fit the
experimental momentum distributions and the depth was adjusted to reproduce the experimentally observed separation energy of the bound final state. The normalization of the wave function was fitted to the data. In order to reproduce the magnitude of the experimental cross sections, a reduction factor was applied to the calculated results. This factor was then identified with the s.f.

The “experimental” s.f. extracted in these DWIA analyses indicate that the removal of the s.p. strength for quasi-hole states near the Fermi energy is about 60-70%. The s.f. gives a measurement of correlation effects, but, since in the \((e,e'p)\) analyses it is obtained through a fit to the data, in practice it can include besides correlations also the effect of other contributions which are neglected or not adequately described in the model. It can be identified with the s.f. only if all the theoretical ingredients contributing to the cross section are reasonably under control. On the other hand, the fact that this model, with phenomenological ingredients, was able to give an excellent description of \((e,e'p)\) data, in a wide range of nuclei and in different kinematics (see, e.g., \([4, 7]\)), gives support and consistency to this whole picture and to the interpretation of the s.f. extracted in comparison with data.

2.3 Overlap Functions and Correlations

Explicit calculations of the hole spectral function and the associated fully correlated OF for complex nuclei are very difficult. Only recently the first successful parameter-free comparison of experiment and theory including the absolute normalization in \(p\)-shell nuclei has been performed for the \(^7\)Li\((e,e'p)\) reaction \([8]\). For heavier nuclei a calculation able to account for the effects due to all types of correlations appears extremely difficult, since it requires excessively large model space. The effects of a spectral function containing only SRC and TC \([3]\) and only LRC \([10]\) have been investigated in the \(^{16}\)O\((e,e'p)\) reaction. A method to deal with SRC and LRC consistently has been proposed and applied in ref. \([11]\) to \(^{16}\)O. In this application, however, only the s.f. and not the OF have been calculated.

Recently, a general procedure has been adopted \([12]\) to extract the OF and the associated s.f. on the base of the OBDM. The advantage of this procedure is that it avoids the complicated task of calculating the nuclear spectral function, but its success depends on the availability of realistic calculations of the OBDM.

This procedure has been applied \([13, 14, 15, 16, 17]\) to OBDM of \(^{16}\)O and \(^{40}\)Ca constructed within different correlation methods, such as the Jastrow Correlation Method (JCM) \([13]\), the Correlated Basis Function (CBF) theory \([18, 19]\), the Green’s Function Method (GFM) \([20]\), and the Generator Coordinate Method (GCM) \([21, 22]\). The OF and the s.f. have then been used to calculate the cross section of one-nucleon removal reactions \([15, 16, 17]\).

An example is displayed in Fig. \([1]\) for the \(^{16}\)O\((e,e'p)\) reaction \([16]\). Calculations have been done with same code DWEEPY \([8]\) used in the original analysis of the NIKHEF data \([23]\), in the same conditions and with the same optical potential \([24]\), but the phenomenological s.p. bound state wave functions have been replaced.
by the theoretically calculated OF. The results are presented in terms of the reduced cross section \( \sigma \), defined as the cross section divided by a kinematical factor and the elementary off-shell electron-proton scattering cross section, which is the quantity that in PWIA gives the momentum distribution of the quasi-hole state. The experimental data were taken in the so-called parallel kinematics, where the momentum of the outgoing nucleon is fixed and is taken parallel or antiparallel to the momentum transfer. Different values of \( p_m \) are obtained by varying the electron scattering angle and therefore the magnitude of the momentum transfer. In Fig. 1 the calculated reduced cross sections are able to reproduce with a fair agreement the experimental distributions. They are anyhow sensitive to the shape of the various functions. The differences are larger at large values of \( p_m \), where correlation effects are more sizable. The best agreement with data, for both transitions, is obtained with the OF emerging from the OBDM calculated within the GFM \( \text{[20]} \) and corresponding to the most refined calculation of the OBDM.

The results obtained with the different OF are compared in the figure with those given by the HF wave function, which is calculated in a self-consistent way using the Skyrme-III interaction. Besides the HF wave function, whose norm is equal to one, all the OF contain a s.f. These factors are listed in Table I (column I). They account for the contribution of correlations included in the OBDM, which cause a depletion of the quasi-hole states. Only SRC are included in the OBDM of refs. \( \text{[13, 19, 21]} \), whereas also TC are taken into account in refs. \( \text{[13, 20]} \). Indeed the s.f. are lower for the functions including also TC. These OF, however, do not include LRC, which should produce further depletion of the quasi-hole states \( \text{[11]} \). In order to reproduce the size of the experimental cross section, a reduction factor has been applied in Fig. 1 to the calculated results. These factors are also listed in Table I (column II). They can be considered as additional s.f. reflecting the depletion of the quasi-hole state produced by the correlations not included in the OBDM, namely LRC for the OBDM of refs. \( \text{[13, 20]} \), LRC and TC for those of refs. \( \text{[13, 19, 21]} \), and all the correlations for the HF wave function. The product of the two factors, in column III, can thus be considered as the total s.f. accounting for the combined effect of all the correlations. Indeed for \( 1p_{1/2} \) these factors are in reasonable agreement with the s.f. (0.77) obtained in the calculation of ref. \( \text{[11]} \), where both SRC and LRC are consistently included. The fact that for \( 1p_{3/2} \) these factors are lower than the one found in \( \text{[11]} \) (0.76) is presumably due to the approximations used in that calculation, which is unable to reproduce the experimentally observed splitting of the \( 3/2^- \) state. Further work is currently in progress to account for the complexity of the low-energy structure of \( ^{16}\text{O} \).

The DWIA calculations with the different OF for the \( ^{16}\text{O}(\gamma,p)^{15}\text{N}_{g.s.} \) reaction at \( E_\gamma = 60 \text{ MeV} \) are displayed in Fig. 2. \( \text{[10]} \). The same theoretical ingredients, i.e. OF, s.f., and consistent optical potentials, have been adopted as in \( (e,e')p \). Moreover, the reduction factor determined in comparison with the \( (e,e')p \) data has been applied, in order to allow a consistent comparison of \( (e,e')p \) and \( (\gamma,p) \) results. In photon-induced reactions a different kinematics is explored. In fact in this case the energy and momentum transfer cannot be independently varied.
They are constrained by the condition $\omega = |\mathbf{q}| = E_\gamma$, and only the high-momentum components of the nuclear wave function are probed, higher values than in the usual kinematics of $(e,e'p)$ experiments. Thus, it is not strange that the differences given by the different OF are so large, in particular at backward angles, which correspond to higher values of $p_m$. The agreement of the DWIA calculations with data is poor. For the $(\gamma,p)$ reaction the validity of the DKO mechanism related to the DWIA, which is clearly stated for $(e,e'p)$, is much more questionable and large contributions are expected also by two-nucleon processes, such as those involving two-body meson-exchange currents (MEC). Indeed a better agreement with data is obtained in Fig. 2 when MEC are added. Here MEC have been added to the DKO mechanism within the theoretical framework of ref. [29], in a microscopic and unfactorized calculation. In order to reduce the complexity of the calculation, however, only the contribution due to the seagull diagrams with one-pion exchange has been included. This is certainly an approximation, but the seagull current here considered should give the main contribution of the two-body current in the photon-energy range above the giant resonance and below the pion production threshold. MEC give an important contribution and bring the results closer to data. The differences for the various OF are however still large and for some functions the agreement with data is still poor.

The best agreement and a fair description of data is given by the OF from GFM [20], which is able to give also the best description of $(e,e'p)$ data and with the same s.f. This result, that has been confirmed also in different situations [16], is a strong indication in favour of a consistent description of the two reactions and gives further support to our results for $(e,e'p)$, where the contribution to the depletion of the quasi-hole states produced by the different types of correlations has been established. A part of this contribution, however, is still obtained as a reduction factor in comparison with data. Thus, also in this analysis the s.f. can be affected by other effects not included or not adequately described in the theoretical model.

### 2.4 Spectroscopic Factors and Relativistic Effects

Various contributions and their effect on the extracted s.f. have been studied in recent years. Only a small contribution is expected from two-body currents in $(e,e'p)$ [30, 31, 32]. A proper treatment of the c.m. motion leads to an enhancement of the extracted s.f. by about 7% [33]. A further enhancement is obtained in relativistic DWIA (RDWIA) analyses with relativistic optical potentials [34, 35].

Fully relativistic models based on the RDWIA have been developed by different groups [36, 37]. In these approaches the bound nucleons are described by s.p. Dirac wave functions in the presence of scalar and vector potentials fitted to the ground-state properties of the nucleus, and the scattering wave function is solution of the Dirac equation with relativistic optical potentials obtained by fitting elastic proton-nucleus scattering data. Also RDWIA analyses are able to give a good description of $(e,e'p)$ data. RDWIA calculations are necessary for the analyses of the new $(e,e'p)$ data from Jlab [37] in kinematic conditions inaccessible in previous experiments, where the four-momentum transfer squared $Q^2$ was less than 0.4 (GeV/c)$^2$ and
the outgoing proton energy generally around 100 MeV. It is anyhow important to check the relevance of relativistic effects also in the kinematics at lower energies of the previous experiments, whose data were analyzed with a nonrelativistic DWIA treatment.

Relativistic effects as well as the differences between relativistic and nonrelativistic calculations have been investigated in different papers where RDWIA treatments have been developed. The differences, however, are usually evaluated starting from the basis of a relativistic model where terms corresponding to relativistic effects are cancelled or where nonrelativistic approximations are included. Although very interesting, these investigations do not correspond to the result of a comparison between RDWIA and the DWIA calculations carried out with the program DWEEPY. In fact, DWEEPY is based on a nonrelativistic treatment where some relativistic corrections are introduced in the kinematics and in the nuclear current operator. On the other hand, only indirect comparison between relativistic and nonrelativistic calculations can be obtained from the available data analyses carried out with DWEEPY and in RDWIA. In fact, the two types of calculations make generally use of different optical potentials and bound state wave functions, and the difference due to the different theoretical ingredients cannot be attributed to relativity.

In order to investigate the relevance of genuine relativistic effects through a direct comparison between RDWIA calculations and the results of DWEEPY, a fully relativistic RDWIA model for the \((e,e'p)\) reaction has been developed and its numerical results have been compared with the corresponding results given by DWEEPY. In order to make the comparison as consistent as possible, in the nonrelativistic calculations the bound state is the normalized upper component of the Dirac spinor and the scattering state is the solution of the same Schrödinger-equivalent optical potential of the relativistic calculation. This is not the best choice for DWEEPY, but the same theoretical ingredients are to be used for a clear comparison between the two approaches.

An example is shown in Fig. 3, for the \(^{16}\text{O}(e,e'p)\) reaction in comparison with the NIKHEF data. Only small differences are found between the two calculations in this kinematics. The reduction (spectroscopic) factor applied to the calculated reduced cross sections in order to reproduce the size of the experimental results is 0.7 for RDWIA and 0.65 for DWIA, for both the transitions, which confirms that somewhat higher spectroscopic factors are obtained in RDWIA.

The systematic investigation carried out in ref. [36] indicates that relativistic effects increase with the energy and in particular with the energy of the outgoing proton. The DWIA approach can be used with enough confidence at the energies around 100 MeV of previous \((e,e'p)\) experiments, and, with some caution, up to about 200 MeV. This confirms the validity of the analyses carried out with DWEEPY at lower energies. A fully relativistic calculation is anyhow convenient at 200 MeV and necessary above 300 MeV. Thus, RDWIA must be used in comparison with the recent data from JLab, at \(Q^2 = 0.8 \,(\text{GeV}/c)^2\) and \(T_1' = 433 \text{ MeV}\). Here the RDWIA model gives an excellent description of data keeping the same spectroscopic factor (0.7) extracted in the comparison with the NIKHEF data of
2.5 Conclusions

The results of the study of $NN$ correlations in the $(e,e'p)$ reaction on complex nuclei can be summarized as follows. The s.f. account for the depletion of the quasi-hole states produced by $NN$ correlations. The depletion found in the DWIA analyses is $\sim 30 - 40\%$. The s.f. are usually extracted from the comparison between data and DWIA results and can be affected by all the uncertainties of the model. Theoretical investigations within different correlation methods indicate that only a few percent of the depletion is due to SRC (see also [38, 41]). When TC are added to SRC the depletion amounts to $\sim 10\%$, at most $\sim 15\%$ in heavy nuclei. Further depletion is given by LRC. A full and consistent calculation of the OF and of the s.f. including SRC, TC and LRC is still unavailable.

If we want to study more specifically SRC, we have seen that they account for only a small part of the depletion of the quasi-hole states. This depletion is compensated by the admixture of high-momentum components in the nuclear wave function. Thus, one might think to investigate SRC studying the high-momentum components of the s.p. wave functions in exclusive one nucleon knockout experiments. Indeed we have found large differences for the cross sections calculated with the different OF at high values of the missing momentum. It is not clear, however, if these differences are due to correlations or to the different methods used in the calculations of the OBDM. Microscopic calculations of the momentum distribution [3] give indeed a strong enhancement of the high-momentum components due to SRC, but this enhancement shows up at large values of $E_m$. In exclusive $(e,e'p)$ experiments one does not measure the whole momentum distribution, but the spectral function at the energy corresponding to the specific final state that is considered. In general low-lying discrete states of the residual nucleus are considered, corresponding to low values of the energy, while the missing strength due to SRC is found at high values of the momentum but also at large values of the excitation energy, well above the continuum threshold, where other competing processes are present. This makes a clear-cut identification of SRC in $(e,e'p)$ very difficult.

This identification appears possible in two-nucleon knockout reactions. Here particular situations can be envisaged where the knockout of the two nucleons is entirely due to correlations. These situations appear very well suited to study SRC.

3 Two-Nucleon Knockout

Since a long time electromagnetically induced two-nucleon knockout reactions have been devised as the preferential tool for investigating SRC. In fact, direct insight into SRC can be obtained from the situation where the electromagnetic probe hits, through a one-body current, either nucleon of a correlated pair and both nucleons are then ejected from the nucleus. This process is entirely due to correlations.
But two nucleons can also and naturally be ejected by two-body currents due to meson exchanges and $\Delta$ isobar excitations. These two competing processes, both produced by the exchange of mesons between nucleons, require a careful and consistent treatment. Their role and relevance, however, is different in different reactions and kinematics. It is thus possible, with the help of theoretical predictions, to envisage appropriate situations where various specific effects can be disentangled and separately investigated.

Interesting and complementary information is available from electron and photon-induced reactions, but the electron probe is preferable to study SRC. In fact, two-body currents predominantly contribute to the transverse components of the nuclear response. Only these components are present in photon-induced reactions that appear thus generally dominated by two-body currents. Also the longitudinal component, dominated by correlations, is present in electron-induced reactions. The possibility of independently varying the energy and momentum transfer of the exchanged virtual photon allows one to select kinematics where the longitudinal response and thus SRC are dominant.

A combined study of $p\bar{p}$ and $n\bar{p}$ knockout is needed for a complete information. Correlations are different in $p\bar{p}$ and $n\bar{p}$ pairs. They are stronger in $n\bar{p}$ pairs and thus in $np$ knockout due to the tensor force, that is predominantly present in the wave function of a $np$ pair. But also two-body currents are much more important in $np$ knockout, while they are strongly suppressed in $p\bar{p}$ knockout, where the charge-exchange terms of the two-body current do not contribute. Therefore, the $(e,e'p\bar{p})$ reaction was devised as the preferential process for studying SRC in nuclei. It is however clear that, since different effects can be emphasized in suitable conditions for different reactions, a combined study of $p\bar{p}$ and $n\bar{p}$ knockout induced by real and virtual photons is needed to unravel the different contributions and obtain clear and complete information on $NN$ correlations.

Exclusive reactions, for transitions to specific discrete eigenstates of the residual nucleus, are of particular interest for this study. One of the main results of the theoretical investigation is the selectivity of exclusive reactions involving different final states that can be differently affected by one-body and two-body currents [42, 43]. Thus, the experimental resolution of specific final states may act as a filter to disentangle the two reaction processes. $^{16}$O is a suitable target for this study, due to the presence of discrete low-lying states in the experimental spectrum of $^{14}$C and $^{14}$N well separated in energy. From this point of view, $^{16}$O is better than a light nucleus, which lacks specific final states.

### 3.1 The Theoretical Framework

The theoretical framework for two-nucleon knockout is formally similar to the model for one-nucleon knockout outlined in Sec. II. The transition matrix elements in Eq. (3), whose bilinear combinations give the components of the hadron tensor $W^{\mu\nu}$, represent the basic ingredients of the calculation. The model [4, 44] is still based on the two assumptions of an exclusive process, for the transition to a discrete
eigenstate of the residual nucleus, and of the DKO mechanism, but a different
final nuclear state must be considered, with two outgoing nucleons and the residual
nucleus, and the nuclear-current operator is the sum of a one-body and a two-body
part, corresponding to the two competing reaction processes already mentioned.
The two-body current includes terms due to the lowest order diagrams with one-
pion exchange, namely seagull, pion-in-flight and diagrams with intermediate ∆
isobar configurations [45]. All these terms contribute to pn knockout while only the
non charge-exchange terms in the ∆ current operator contribute to pp knockout.

The matrix elements of Eq. (3) can be written as

\[ J^\mu(q) = \int \psi_i^*(r_1, r_2) J^\mu(r, r_1, r_2) \psi_f(r_1, r_2) e^{iq \cdot r} dr_1 dr_2, \quad (8) \]

where the two-nucleon overlap integral \( \psi_i \) and the two-nucleon scattering state \( \psi_f \)
are consistently derived from an energy-dependent non-Hermitian Feshbach-type
Hamiltonian for the considered final state of the residual nucleus. In practice, since
it would be extremely difficult to achieve this consistency, the treatment of initial
and final states proceeds separately with different approximations.

In the scattering state the mutual interaction between the two outgoing nucleons
is neglected and only the interaction of each of the outgoing nucleons with the
residual nucleus is considered by means of a phenomenological optical potential.

For the \(^{16}\text{O}(e,e'pp)^{14}\text{C}\) reaction the two-nucleon overlap functions are taken
from the calculation of the spectral function [42, 46], where both LRC and SRC
are included. A two-step procedure has been applied in this calculation where LRC
and SRC are treated in a separate but consistent way. The calculation of LRC is
performed in a SM space large enough to incorporate the corresponding collective
features which influence the pair removal amplitudes. The s.p. propagators used for
this dressed Random Phase Approximation (RPA) description of the two-particle
propagator also include the effect of both LRC and SRC. This yields s.f. for low-
lying states of \(^{15}\text{N}\) which represent the closest agreement with \((e,e'p)\) data to date [11]. In the second step that part of the pair removal amplitudes which describes the
relative motion of the pair is supplemented by defect functions obtained from the
same G-matrix which is also used as the effective interaction in the RPA calculation.

The two-nucleon OF for a discrete final state of \(^{14}\text{C}\), with angular momentum
quantum numbers \( JM \), is expressed in terms of a combination of relative and c.m.
wave functions [42]. The combination coefficients contain contributions from a SM
space which includes the 0s up to the 1p0f shells. The c.m. radial wave function is
that of a harmonic oscillator (h.o.). SRC are included in the radial wave function \( \phi \)
of relative motion through a defect function defined by the difference between \( \phi \) and
the uncorrelated relative h.o. wave function. These defect wave functions depend
on the quantum numbers of the relative motion. Thus, SRC depend on the relative
state and, since different components of relative and c.m. motion contribute to each
transition, the role of SRC can be different for different final states.
3.2 Evidence for SRC in the $^{16}\text{O}(e,e'pp)^{14}\text{C}$ Reaction

A numerical example is shown in Fig. 4, where the cross sections of the $^{16}\text{O}(e,e'pp)^{14}\text{C}$ reaction are displayed for the transitions to the $0^+$ ground state and to the $1^+$ state at 11.31 MeV. Results are shown for two kinematical settings considered in the experiments performed at NIKHEF [5, 47] and MAMI [48].

Different components of relative and c.m. motion contribute to the two final states [12]: $^1S_0$ and $^3P_1$ relative waves (the notation $^{2S+1}l_j$, for $l = S, P, D$, is used here for the relative states), which are combined with a c.m. orbital angular momentum $L = 0$ and 1, respectively, for the $0^+$ state, and $^3P_0$, $^3P_1$, $^3P_2$, all combined with $L = 1$, for the $1^+$ state. The value of $L$ determines the shape of the recoil-momentum distribution. Indeed in Fig. 4 for the $1^+$ state, where only components with $L = 1$ are present, the momentum distributions have a typical $p$-wave shape, while the $s$-wave shape obtained for the $0^+$ state indicates that in the two considered kinematics the cross section is dominated by the component with $L = 0$ and thus by $^1S_0$ pp knockout. The component with $L = 1$, due to $^3P_1$, becomes meaningful only at large values of $p_m$, where the contribution of the $s$ wave gets lower.

The comparison between correlated and uncorrelated relative wave functions [42, 46] indicates that SRC play a different role in different relative states: they are quite strong for the $^1S_0$ state and much weaker for $^3P$ states. Moreover, also the role of the isobar current is strongly reduced for $^1S_0$ pp knockout, since there the generally dominant contribution of that current, due to the magnetic dipole $NN \leftrightarrow N\Delta$ transition, is suppressed [15, 19]. Thus, the role of SRC is emphasized in $^1S_0$ knockout, while the role of the $\Delta$ current is emphasized in $^3P$ knockout. This explains the different role of the two reaction processes for the two final states in Fig. 4: the transition to the $1^+$ state is dominated by the two-body current, while for the $0^+$ state, where $^1S_0$ pp knockout plays the main role, the cross section is dominated by the one-body current and thus my SRC.

The final result is determined by all the ingredients of the model, but it is clear that the two reaction processes play a different role for the two final states. Thus, the experimental resolution of different states may act as a filter to disentangle and separately investigate the contributions due to SRC and two-body currents.

Data have confirmed the predictions of this model. A reasonable and in some cases an excellent agreement with the available data [5, 47, 48] has been obtained. The comparison has clearly shown the validity of the DKO mechanism for transitions leading to the lowest-lying states of $^{14}\text{C}$ and has confirmed the predicted selectivity of the exclusive reaction involving discrete final states, which are differently affected by SRC and two-body currents. In particular, clear evidence for SRC has been obtained for the transition to the ground state [5].

This important result means that two-nucleon knockout reactions can be used to study and hopefully determine SRC. More theoretical and experimental work is however needed for this study. More data are expected from MAMI for all the exclusive $^{16}\text{O}(e,e'pp)^{14}\text{C}$, $^{16}\text{O}(e,e'np)^{14}\text{N}$, $^{16}\text{O}(\gamma,pp)^{14}\text{C}$, and $^{16}\text{O}(\gamma,pn)^{14}\text{N}$
knockout reactions [50]. A combined study of different reactions is needed for a complete information on $NN$ correlations.

Good opportunities to increase the richness of information available from two-nucleon knockout reactions are also offered by polarization measurements [51]. Reactions with polarized particles give access to a larger number of observables, hidden in the unpolarized case, whose determination can impose more severe constraints on theoretical models. Thus, a combined analysis of cross sections and polarization observables would make possible it to disentangle the different contributions and shed light on the genuine nature of correlations in nuclei.

I want to thank all the colleagues who contributed to this work, in particular A.N. Antonov, M.K. Gaidarov, A. Meucci and F.D. Pacati.

References

[1] A.N. Antonov, P.E. Hodgson and I.Zh. Petkov, *Nucleon Momentum and Density Distributions in Nuclei* (Clarendon Press, Oxford, 1988).

[2] A.N. Antonov, P.E. Hodgson and I.Zh. Petkov, *Nucleon Correlations in Nuclei* (Springer–Verlag, Berlin, 1993).

[3] H. M"uther and A. Polls, Prog. Part. Nucl. Phys. 45, 243 (2000).

[4] S. Boffi, C. Giusti, F.D. Pacati and M. Radici, *Electromagnetic Response of Atomic Nuclei* (Clarendon Press, Oxford, 1996).

[5] R. Starink et al., Phys. Lett. B 474, 33 (2000).

[6] C. Giusti and F.D. Pacati, Nucl. Phys. A473, 717 (1987); Nucl. Phys. A485, 461 (1988).

[7] L. Lapikás, Nucl. Phys. A553, 297c (1993).

[8] L. Lapikás, J. Wesseling, and R.B. Wiringa, Phys. Rev. Lett. 82, 4404 (1999).

[9] A. Polls et al., Phys. Rev. C 55, 810 (1997).

[10] K. Amir-Azimi-Nili, H. M"uther, L.D. Skouras, and A. Polls, Nucl. Phys. A604, 245 (1996); K. Amir-Azimi-Nili et al., Nucl. Phys. A625, 633 (1997).

[11] W.J.W. Geurts, K. Allaart, W.H. Dickhoff, and H. M"uther, Phys. Rev. C 53, 2207 (1996).

[12] D. Van Neck, M. Waroquier, and K. Heyde, Phys. Lett. B 314, 255 (1993).
[13] M.V. Stoitsov, S.S. Dimitrova, and A.N. Antonov, Phys. Rev. C 53, 1254 (1996).

[14] S.S. Dimitrova et al., J. Phys. G 23, 1685 (1997).

[15] M.K. Gaidarov et al., Phys. Rev. C. 60, 024312 (1999).

[16] M.K. Gaidarov et al., Phys. Rev. C 61, 014306 (2000).

[17] M.V. Ivanov, M.K. Gaidarov, A.N. Antonov, and C. Giusti, Phys. Rev. C 64, 014605 (2001).

[18] D. Van Neck, L. Van Daele, Y. Dewulf, and M. Waroquier, Phys. Rev. C 56, 1398 (1997).

[19] F. Arias de Saavedra, G. Co’, A. Fabrocini, and S. Fantoni, Nucl. Phys. A 605, 359 (1996).

[20] A. Polls, H. Müther, and W.H. Dickhoff, Proceedings of the Conference on Perspectives in Nuclear Physics at Intermediate Energies, Trieste, 1995, edited by S. Boffi, C. Ciofi degli Atti, and M.M. Giannini, (World Scientific, Singapore, 1996), p.308.

[21] A.N. Antonov, Chr.V. Christov, and I.Zh. Petkov, Nuovo Cim. 91A, 119 (1986); A.N. Antonov, I.S. Bonev, Chr.V. Christov, and I.Zh. Petkov, Nuovo Cim. 100A, 779 (1988); A.N. Antonov, I.S. Bonev, and I.Zh. Petkov, Bulg. J. Phys. 18, 169 (1991); A.N. Antonov, I.S. Bonev, Chr.V. Christov, and I.Zh. Petkov, Nuovo Cim. 103A, 1287 (1990).

[22] M.V. Ivanov, A.N. Antonov, and M.K. Gaidarov, Int. J. Mod. Phys. E 9, No.4, 339 (2000).

[23] M. Leuschner et al., Phys. Rev. C 49, 955 (1994).

[24] P. Schwandt et al., Phys. Rev. C 26, 55 (1982).

[25] C. Barbieri and W.H Dickhoff, Phys. Rev. C 63, 034313 (2001); nucl-th/0108023.

[26] D.J.S. Findlay and R.O. Owens, Nucl. Phys. A 279, 389 (1977).

[27] F. de Smet et al., Phys. Rev. C 47, 652 (1993).

[28] G.J. Miller et al., Nucl. Phys. A 586, 125 (1995).

[29] G. Benenti, C. Giusti, and F.D. Pacati, Nucl. Phys. A 574, 716 (1994).

[30] S. Boffi and M. Radici, Nucl. Phys. A 526, 602 (1991).

[31] V. Van der Sluys, J. Ryckebusch, and M. Waroquier, Phys. Rev. C 49, 2695 (1994).
[32] J.E. Amaro, A.M. Lallena, and J. A. Caballero, Phys. Rev. C 60, 014603 (1999).
[33] D. Van Neck, et al., Phys. Rev. C 57, 2308 (1998).
[34] Y. Jin, D.S. Onley, and L.E. Wright, Phys. Rev. C 45, 1311 (1992); Y. Jin and D.S. Onley, Phys. Rev. C 50, 377 (1994); J.M. Udías et al., Phys. Rev. C 48, 2731 (1993).
[35] S. Boffi, C. Giusti, F.D. Pacati, and F. Cannata, Nuovo Cim. 98, 291 (1987).
[36] A. Meucci, C. Giusti and F.D. Pacati, Phys. Rev. C 64, 014604 (2001).
[37] J. Gao et al., Phys. Rev. Lett. 84, 3265 (2000).
[38] A. Fabrocini and G. Co', Phys. Rev. C 63, 044319 (2001).
[39] E.D. Cooper, S. Hama, B.C. Clark, and R.L. Mercer, Phys. Rev. C 47, 297 (1993).
[40] W. Pöschl, D. Vretenar, and P. Ring, Comput. Phys. Commun. 103, 217 (1997).
[41] S.R. Mokhtar, M. Anguiano, G. Co' and A.M. Lallena, Annals Phys. 293, 67 (2001).
[42] C. Giusti et al., Phys. Rev. C 57 1691 (1998).
[43] C. Giusti, H. Müther, F.D. Pacati and M. Stauf, Phys. Rev. C 60, 054608 (1999).
[44] C. Giusti and F.D. Pacati, Nucl. Phys. A535, 573 (1991).
[45] C. Giusti and F.D. Pacati, Nucl. Phys. A641, 297 (1998).
[46] W.J.W. Geurts et al., Phys. Rev. C 54 1144 (1996).
[47] C.J.G. Onderwater et al., Phys. Rev. Lett. 81, 2213 (1998).
[48] G. Rosner, Prog. Part. Nucl. Phys. 44, 99 (2000).
[49] P. Wilhelm, J.A. Niskanen and H. Arenhövel, Nucl. Phys. A597, 613 (1996).
[50] J.R.M. Annand et al., MAMI proposal Nr: A1/5-98; J. Ahrens et al., MAMI proposal Nr: A2/4-97.
[51] C. Giusti and F.D. Pacati, Phys. Rev. C 61, 054617 (2000); Eur. Phys. Journ. A 12, 69 (2001).
Table 1: Spectroscopic factors for the $^{16}\text{O}(e,e'p)$ knockout reaction leading to the $1/2^-$ ground state and to the $3/2^-$ excited state of $^{15}\text{N}$. Column I gives the s.f. deduced from the calculations with different OBDM of $^{16}\text{O}$; II gives the additional reduction factors determined through a comparison between the $(e,e'p)$ data of ref. [23] and the reduced cross sections calculated in DWIA with the different overlap functions; III gives the total s.f. obtained from the product of the factors in columns I and II.

| OBDM   | $^{1p}_{1/2}$ |   |   | $^{1p}_{3/2}$ |   |   |
|--------|---------------|---|---|---------------|---|---|
|        | I  | II | III | I  | II | III |
| HF     | 1.000 | 0.750 | 0.750 | 1.000 | 0.550 | 0.550 |
| JCM [13] | 0.953 | 0.825 | 0.786 | 0.953 | 0.600 | 0.572 |
| CBF [18] | 0.912 | 0.850 | 0.775 | 0.909 | 0.780 | 0.709 |
| CBF [19] | 0.981 | 0.900 | 0.883 | 0.981 | 0.600 | 0.589 |
| GFM [20] | 0.905 | 0.800 | 0.724 | 0.915 | 0.625 | 0.572 |
| GCM [21] | 0.988 | 0.700 | 0.692 | 0.988 | 0.500 | 0.494 |
Figure 1: Reduced cross sections of the $^{16}$O($e,e'p$) reaction as a function of the missing momentum $p_m$ for the transitions to the $1/2^-$ ground state and to the first $3/2^-$ excited state of $^{15}$N in parallel kinematics, with an incident electron energy $E_0 = 520.6$ MeV and an outgoing proton energy $T_1' = 90$ MeV. The optical potential is from ref. [24]. The OF are derived from the OBDM of GFM [20] (solid line), CBF [18] (long-dashed line), CBF [19] (long-dot-dashed line), JCM [13] (short-dot-dashed line), and GCM [21] (short-dashed line). The dotted line is calculated with the HF wave function. The positive (negative) values of $p_m$ refer to situations where $|q| < |p'|$ ($|q| > |p'|$). The experimental data are taken from ref. [23]. The theoretical results have been multiplied by the reduction factor given in column II of Table I (from ref. [16]).
Figure 2: Angular distribution of the cross section of the $^{16}\text{O}(\gamma,p)$ reaction for the transition to the $1/2^-$ ground state of $^{15}\text{N}$ at $E_\gamma = 60$ MeV. The separate contribution given by the one-body current (DWIA) and the final result given by the sum of the one-body and the two-body seagull current (DWIA+MEC) are shown. Line convention as in Fig. 1. The optical potential is from ref. [24]. The experimental data are taken from refs. [26] (black circles), [27] (open circles) and [28] (triangles). The theoretical results have been multiplied by the reduction factors listed in column II of Table I, consistently with the analysis of $(e,e'p)$ data (from ref. [16]).
Figure 3: Reduced cross sections of the $^{16}\text{O}(e,e'p)$ reaction as a function of the missing momentum $p_m$ for the transitions to the $1/2^-$ ground state and to the first $3/2^-$ excited state of $^{15}\text{N}$ in the same kinematics as in Fig. 1. The solid lines give the RDWIA result [36] the dotted lines the nonrelativistic result. The optical potential is from ref. [39] and the bound state wave functions from ref. [40] (from ref. [36]).
Figure 4: The differential cross section of the reaction $^{16}\text{O}(e,e'pp)^{14}\text{C}$ for the transitions to the $0^+$ ground state and to the $1^+$ state at 11.31 MeV. In the left panels a super-parallel kinematics is considered with $E_0 = 855$ MeV, $\omega = 215$ MeV and $q = 316$ MeV/c. Positive (negative) values of $p_m$ refer to situations where $p_m$ is parallel (anti-parallel) to $q$. In the right panels $E_0 = 584$ MeV, $\omega = 212$ MeV, $q = 300$ MeV/c, $T'_1 = 137$ MeV and $\gamma_1 = -30^\circ$, on the opposite side of the outgoing electron with respect to the momentum transfer. The defect functions for the Bonn-A $NN$ potential and the optical potential of ref. [24] are used. Separate contributions of the one-body and the two-body $\Delta$ current are shown by the dotted and dashed lines, respectively. The solid curves give the final result.