Minimizing Pumping Energy Cost in Real-time Operations of Water Distribution Systems using Economic Model Predictive Control

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ABSTRACT

Optimizing pump operations is a challenging task for real-time management of water distribution systems (WDSs). With suitable pump scheduling, pumping costs can be significantly reduced. In this research, a novel economic model predictive control (EMPC) framework for real-time management of WDSs is proposed. Optimal pump operations are selected based on predicted system behavior even in receding time horizon with the aim to minimize the total pumping energy cost. Time-varying electricity tariffs are considered while all the required water demands are satisfied. The novelty of this framework is to choose the number of pumps to operate in each pump station as decision variables in order to optimize the total pumping energy costs. By using integer programming, the proposed EMPC is applied to a benchmark case study, the Richmond Pruned network. The simulation with an EPANET hydraulic simulator is implemented. Moreover, a comparison of the results obtained from using the proposed EMPC with those obtained from using trigger-level control demonstrates significant economic benefits of the proposed EMPC.

INTRODUCTION

Water distribution systems (WDSs) are critical infrastructure for modern cities. With appropriate operational management, water companies are able to provide water with the desired quantity and quality to all the customers with a reasonable operational cost. Pumping costs constitute a significant proportion of the operational costs for WDSs, and they are projected to increase in the future (van Zyl et al. 2004). An increasingly important objective in real-time optimal pump operations of WDSs is therefore to minimize pumping energy costs. During the past three decades, a large amount of research has been carried out on optimal operations of WDSs. A comprehensive literature review including over 100 scientific publications from the 1970s until 2017 on the
optimization of WDSs was conducted by Mala-Jetmarova et al. (2017). The 2017 study revealed that pump operation optimization is one of the two major areas of optimization research related to WDSs (the other area being water quality optimization).

WDS operation optimization is a very difficult problem due to a number of challenges, which include but are not limited to: 1) the equations governing the hydraulic interactions within the network are nonlinear; 2) decision variables can be either continuous or discrete; and 3) real WDSs typically consist of a large number of branches, links, loops and nodes. In addition, there are new challenges for the operation of modern WDSs, for example, 4) electricity prices can change with time; 5) disturbances to the system may occur including degradation of the pipe and pump characteristics; and 6) presence of constraints on internal states and inputs to the WDS. Consequently, to solve a pump operation problem, a mathematical model of the system is needed taking into account the nonlinear hydraulic equations and the binary nature of the pump operational decisions. The resulting mixed-integer, dynamic and nonlinear optimization problem is numerically challenging to solve in real-time. As a result, researchers often make use of a number of simplifying assumptions to reduce the computational requirements of the problem, which has resulted in the development of a variety of problem formulations and solution methods over time.

Early studies on WDS operation optimization focused on deterministic approaches, such as dynamic programming (DP) (Sterling and Coulbeck 1975), linear programming (LP) (Giacomello et al. 2013; Kurian et al. 2018), nonlinear programming (NLP) (Brion and Mays 1991; Ormsbee and Reddy 1995), and mixed integer non-linear programming (MINLP) (Biscos et al. 2003; Bagirov et al. 2013). Recently, meta-heuristic algorithms, such as genetic algorithms (Paschke et al. 2001; Kazantzis et al. 2002; van Zyl et al. 2004; Wang et al. 2009; Wu et al. 2010; Wu et al. 2012a; Wu et al. 2012b; Blinco et al. 2016), simulated annealing (da Conceição Cunha and Sousa 1999; Goldman and Mays 1999), particle swarm optimization (Wegley et al. 2000), and ant colony optimization (Ostfeld and Tubaltzev 2008; López-Ibáñez et al. 2008) have been used for finding open-loop schedule for pump operations. However, the majority of these studies did not consider real-time pump operations, where a feedback control loop uses measurements of the current system state to make informed decisions in real-time about how to choose system inputs in the future.

The well-known advantages of closed-loop controllers include robustness to system modeling errors and noise although they require real-time measurements of system states and increased online computational resources. Whilst a wide range of feedback control algorithms are in use, model predictive control (MPC) has
gained increasing popularity over the past decades due to its ability to explicitly handle system constraints and deliver improved system performance through the use of a model of the system. It is also readily deployable to multi-input-multi-output systems.

MPC was initially developed for the chemical processing industry in the late 1970s (García et al. 1989). It relies on an internal model of the process dynamics to predict system states and outputs given a sequence of control actions over a finite receding horizon. An optimization problem is solved at each sampling instant to determine an optimal control sequence that minimizes deviations of the process states from some predetermined operating setpoints. MPC utilizing a quadratic cost on state deviations and control actions has been applied to many water applications, such as river management and operation of open channel systems (Nasir et al. 2018; Nasir et al. 2019a; Nasir et al. 2019b).

Economic MPC (EMPC) is a relatively new extension of MPC, where the cost function is generally formulated using an arbitrary objective function that captures some economic aspect of process systems to be controlled (Rawlings et al. 2012; Ellis et al. 2016). Given its superior "economic" performance, EMPC has found recent applications in various areas, including building energy systems (Ma et al. 2012), gas pipeline networks (Gopalakrishnan and Biegler 2013), electric vehicle charge planning (Halvgaard et al. 2012), inventory management (Subramanian et al. 2014), and wastewater treatment processes (Zeng and Liu 2015). EMPC has been found to be a suitable control strategy for operational management of WDSs. An earlier deployment of EMPC for the water distribution problem was proposed by Cembrano et al. (2000), where the simulations only considered a two pump system model and locally optimal solutions were found. More recently, this approach has been revisited with using flow-based and pressured-based models of WDSs (Wang et al. 2017). The works by Wang et al. (2017) and Salomons and Housh (2020) have suggested the use of EMPC for WDSs but the authors have made simplifications including approximating the ON/OFF behavior of pumps, as well as using simplified plant and economic/energy models.

This study proposes a novel EMPC framework for real-time management of WDSs, where the full nonlinear pumping models and accurate energy pricing are taken into account to investigate the benefits of EMPC relative to the trigger-level control. Within this EMPC framework, a modeling methodology for WDSs is introduced, which includes the number of pumps in each pump station as integer decision variables. The advantage of this framework is to make sure that accurate economic pumping costs are included in the control system. Based on the objectives of the operational management of WDSs, economic cost functions and constraints
have been formulated as part of the EMPC optimization problem. By using integer programming to solve the corresponding EMPC optimization problem online with updated feedback information, optimal pump operations can be obtained while minimizing the pumping costs. A case study for the Richmond Pruned network is presented here based on the original Richmond network introduced in the study by van Zyl et al. (2004). The closed-loop online simulation results with EMPC connected to the Richmond Pruned EPANET model, and a comparison of the results with trigger-level control demonstrate the effectiveness of EMPC.

The remainder of this paper is organized as follows. First, the EMPC framework for real-time pump operations is proposed. Second, the Richmond Pruned case study is described in detail. Then, the closed-loop online simulation results are discussed for six different demand loading cases and a comparison of the results with those obtained by trigger-level control is presented. Finally, conclusions are drawn and future research directions are suggested.

ECONOMIC MODEL PREDICTIVE CONTROL FOR OPTIMAL PUMP OPERATIONS

In this section, a novel EMPC framework is systematically described for real-time operational management of WDSs. A mathematical modeling methodology is first introduced to find the flow-based model of a WDS. Then, the cost functions and constraints are formulated. Together with the above setup, the EMPC optimization problem is presented.

WDS Model

To design an EMPC controller, a lower-order prediction model of a WDS is required to describe the system dynamics. In this research, a flow-based model of WDSs is considered, which consists of $T$ water storage tanks, $P$ pump stations with a total of $n_j$ parallel pumps located in the $j$-th pump station, and $D$ water demands. The corresponding variable assignments are shown in Table 1.

For each water storage tank $i = 1, \ldots, T$, the variable $x_i$ represents the water depth in the tank. The volume balance for the storage tanks can be written as

$$x_i(k+1) = x_i(k) + \frac{\Delta t}{S_i} \left( q_i^{(in)}(k) - q_i^{(out)}(k) \right), \quad i = 1, \ldots, T, \quad k = 0, 1, \ldots, N,$$

where $q_i^{(in)}$ and $q_i^{(out)}$ represent the inflow and outflow for the $i$-th water tank, $k$ refers to the $k$-th time step from a total of $N$ time steps, $\Delta t$ is the sampling period and $S_i$ is the area (plan view) of the $i$-th tank. The inflow to a tank is usually determined by the outflows from pump stations while the outflow from a tank is usually
determined by the water demands \( d_s, s = 1, \ldots, D \) from each water demand node.

For each pump station \( j = 1, \ldots, P \), the variable \( n_j \) represents the number of parallel pumps that are currently operating, and the variable \( q_j \) represents the total outflow from the \( j \)-th pump station. The relationship between \( n_j \) and \( q_j \) can be formulated as

\[
q_j(k) = \phi_j(n_j(k)), \quad j = 1, \ldots, P, \quad k = 0, 1, \ldots, N,
\]  

(2)

where \( \phi_j(n_j) \) is a static function that can be obtained from experimental or simulated (e.g. EPANET) data, and \( n_j \) takes integer values from 0 to \( \bar{n}_j \). Note that the outflow also depends on water depths in tanks and the approximation in Eq. (2) ignores this dependence due to that the variations in water depths are usually relatively small.

In general, based on Eqs. (1) and (2), the flow-based prediction model for a WDS can be summarized as follows:

\[
x(k + 1) = f(x(k), u(k), d(k)),
\]

(3a)

\[
0 = g(u(k), d(k)), \quad k = 0, 1, \ldots, N,
\]

(3b)

where \( x = [x_1, \ldots, x_T]^T \), \( u = [n_1, \ldots, n_P]^T \), \( d = [d_1, \ldots, d_D]^T \) denote the vectors of tank water depths, the number of pumps operating in each pump station, and the water demands across the whole water network. Note that for complex WDS, the model in Eq. (3) includes the static equations in Eq. (3b) obtained from using the Kirchhoff’s law for the nodes without storage capability. In this research, the water demands are assumed to be known along a prediction horizon as considered in EMPC by using demand forecasting methods, such as the ones introduced in the studies by Wang et al. (2016) and Salomons and Housh (2020).

**Cost Functions and Constraints Setup**

In the following, cost functions and constraints are defined based on the objectives for real-time operational management of WDSs.
Cost Functions

The main objective is to minimize the pumping energy cost of a WDS. The power consumption $\tilde{P}$ for an operating pump is modeled as

$$\tilde{P} = \frac{\gamma Q H_p}{\eta_s \eta_m}, \quad (4)$$

where $\gamma$ is specific weight of water in units of [N/m$^3$], $Q$ is the pump flow [m$^3$/s], $H_p$ is the pump head [m], $\eta_s$ is the pump shaft efficiency and $\eta_m$ is the motor efficiency. It is assumed that $\eta_s$ and $\eta_m$ are both constants.

From Eq. (4), the energy consumed by a pump operating for a total time interval of $\Delta t$ is

$$E = \tilde{P} \Delta t. \quad (5)$$

From Eqs. (4) and (5), the pump head $H_p$ is used, however, the flow-based model (3) assumes pump energy usage is independent of static head (which is a simplification as it ignores any relationship to the state $x$). In the economic cost function, we use an approximation of the power consumption and the consumed energy. Taking into account the time-varying electricity prices from the tariff database, the pumping energy cost can be computed at each sampling interval for the different prices. For the $j$-th pump station, given a time-varying electricity price at time step $k$ as $\alpha_j(k)$, the pumping energy cost can be expressed as

$$\ell_j(n_j(k), \alpha_j(k)) = \alpha_j(k) g_j(n_j(k)), \quad j = 1, \ldots, P, \ k = 0, 1, \ldots, N, \quad (6)$$

where $g_j(n_j)$ is a function that estimates the energy consumed by $n_j$ parallel pumps at the $j$-th pump station (assuming that all the pumps in a pump station are of the same type).

The total economic cost function for the $P$ pump stations as a function of the vector $u(k)$ of the actual number of pumps operating in each pump station can be stated as

$$\ell_e(u(k), \alpha(k)) = \sum_{j=1}^{P} \ell_j(n_j(k), \alpha_j(k)), \quad k = 0, 1, \ldots, N, \quad (7)$$

where the subscript $e$ refers to the "economic" cost function associated with pumps that are operating and $\alpha(k) = [\alpha_1(k), \ldots, \alpha_P(k)]^T$.

While it cannot be directly attributed to the short term operating costs of the network, excessive pump
switching is considered undesirable as it can lead to mechanical degradation and early replacement (with a consequent economic impact on the long term operation of the network). As a result, an implicit economic cost that is associated with turning pumps on and off is introduced using the following economic penalty

\[
\ell_p(u(k)) = \| \Delta u(k) \|_R^2 = \Delta u(k) R \Delta u(k), \quad k = 0, 1, \ldots, N, \quad (8)
\]

where \( \Delta u(k) = u(k) - u(k-1) \), and \( \| \cdot \|_R \) refers to the weighted 2-norm by a positive-definite matrix \( R \). The matrix \( R \) may be tuned to account for different types pumps in different pump stations, or the different network architecture implicitly introducing more switching behavior in some links.

**Constraints**

In a WDS, physical limitations of water depths in the tanks and the availability of pump stations must be taken into account. For the \( T \) water tanks in the network, the constraint describing the physical limitations or operational limits for water depths can be formulated by

\[
x(k) \leq x(k) \leq \bar{x}, \quad k = 0, 1, \ldots, N, \quad (9)
\]

where \( x \) and \( \bar{x} \) denote the vectors of lower and upper bounds for tank water depths, respectively.

For the \( P \) pump stations in the network, the constraints on the number of available pumps can be described by

\[
0 \leq u(k) \leq \bar{u}, \quad k = 0, 1, \ldots, N, \quad (10)
\]

where \( \bar{u} = [\bar{n}_1, \ldots, \bar{n}_P]^T \) denotes the vector of the available number of parallel pumps in each pump station. Note that the elements of the vector \( \bar{u} \) take on integer values.

**Optimization Problem Formulation**

According to the discussion above, the EMPC controller for the operational management of WDSs can be implemented by solving a finite-horizon optimization problem considering a prediction horizon \( H_p > 0 \), that is, at each time \( k \geq 0 \), solve
\[
\text{minimize } \sum_{t=k}^{k+H_p-1} \left( \ell_e(\tilde{u}(t)) + \ell_p(\tilde{u}(t)) \right),
\]

subject to
\[
\begin{align*}
\tilde{x}(t + 1) &= f(\tilde{x}(t), \tilde{u}(t), \tilde{d}(t)), & t = k, \ldots, k + H_p - 1, \\
\underline{x} &\leq \tilde{x}(t + 1) \leq \bar{x}, & t = k, \ldots, k + H_p - 1, \\
0 &\leq \tilde{u}(t) \leq \bar{u}, & t = k, \ldots, k + H_p - 1, \\
\tilde{x}(k) &= x(k), \\
[\tilde{d}(k), \ldots, \tilde{d}(k + H_p - 1)]^T &= [d(k), \ldots, d(k + H_p - 1)]^T,
\end{align*}
\]

where the tilde \(\sim\) refers to the predicted variables. Eq. (11a) is the total cost function, Eq. (11b) is a constraint that the future states must obey the plant dynamics of a WDS based on the volume balance model in Eq. (3), Eq. (11c) constrains the water depth in each tank, Eq. (11d) constrains the number of parallel pumps that can be operated in each pump station, Eq. (11e) is the initialization constraint to feed back the current measured water depth \(x(k)\) as the first predicted state \(\tilde{x}(k)\), and Eq. (11f) are the demand forecasts over the prediction horizon \(H_p\) under the assumption of no uncertainty for demand forecasts.

From the solutions of the optimization problem in Eq. (11) at any time \(k\), the optimal control action at sampling time \(k\) is chosen by using a receding horizon strategy as
\[
u(k) = \tilde{u}^*(k),
\]

where \(u^*(k)\) is the first value of the optimal control sequence as determined from solving Eq. (11). Thus, at each simulation time step \(k\), a new optimization in Eq. (11) is carried out for a prediction horizon \(H_p\) but only the first control input is implemented. At the next simulation time step \(k + 1\), all the system states are updated using the measured data from the WDS, and the optimization in Eq. (11) is solved again with new updated system states.

The EMPC controller in Eq. (11) provides real-time optimal control action for the management of WDSs. Assuming all demands are periodic, the closed-loop systems obtained using the EMPC controller converges to
optimal periodic trajectories, as studied in the related research work in the study by Wang et al. (2018). However, the closed-loop stability of the proposed EMPC can be further enhanced by using alternative approaches, such as terminal costs/constraints, or the average performance constraint, see the study by Angeli et al. (2012).

CASE STUDY: THE RICHMOND PRUNED NETWORK

Case Study Description

The Richmond water network was taken from a part of the Yorkshire water supply area in the UK described in the study by van Zyl et al. (2004). Its EPANET simulation model can be found at the University of Exeter website. To test the proposed EMPC framework, a portion of the original Richmond network, called the Richmond Pruned network, was developed based on the Richmond skeleton model. The layout of this network is shown in Fig. 1. This network consists of one water storage tank, two pump stations (one that contains two parallel pumps 1A and 2A; a booster pump station that contains a single pump 3A), one demand sector located at node 10. The demand multipliers $d_m(k)$ within 24 hours are given in Fig. 2, in which the average demand multiplier is 1.0. It is assumed that the demand multipliers are repeated every 24 hours. The actual daily demand flow at node 10, $d_{10}(k)$, is given by the product of a given base demand $\bar{d}_{10}$ and the demand multiplier $d_m(k)$, that is, $d_{10}(k) = \bar{d}_{10}d_m(k)$.

Mathematical Model of the Richmond Pruned Network

The mathematical model of the Richmond Pruned network is introduced in this section. Based on the EPANET results of this network, the data used to fit relationships for EMPC are reported in Table 2, where the total inflow $q_A$ to the storage tank $A$ and power of the individual pumps for pump station 1 (PS1) and pump station 2 (PS2) are shown with different combinations of pump operations defined by the numbers of operating pumps $n_1$ and $n_2$.

Based on the topology of this network as shown in Fig. 1, the function in Eq. (1) can be explicitly written as

$$x(k + 1) = x(k) + \frac{\Delta t}{S_A} (q_A(k) - d_{10}(k)),$$

where $S_A$ is the plan area of Tank A and $q_A$ is the inflow to Tank A. In Eq. (1), the units for water depth are meters ([m]) and the units for all the flow variables (inflows, outflows and water demands) are cubic meters per second ([m$^3$/s]).

1. https://emps.exeter.ac.uk/engineering/research/cws/resources/benchmarks/operation/richmond.php
To find the total inflow $q_A$ to Tank $A$, a polynomial equation can be used in Eq. (2)

$$q_A(k) = c_1 n_1(k) + c_2 n_1(k)^2 + c_3 n_2(k) + c_4 n_1(k)n_2(k),$$

(14)

where $c_1 = 25.215$, $c_2 = 30.84$, $c_3 = 43.23$ and $c_4 = 57.89$. These parameters are fitted so that Eq. (14) accurately reproduces the flow values obtained from the EPANET as shown in Table 2.

**Cost Functions and Constraints**

According to Table 2, for the Richmond Pruned network, the economic cost in Eq. (6) can be approximated by

$$\ell_e(u(k), \alpha(k)) = \alpha(k) p(n_1(k) + n_2(k)) \Delta t, \quad k = 0, 1, \ldots, N,$$

(15)

where $\alpha(k)$ is time-varying price at time $k$, $\Delta t$ is the sampling time, and $p$ is an approximation of total power consumed at both pump stations, which is assumed to be a constant. For the Richmond Pruned network, we assume an identical electricity cost price variation (i.e. the tariff of PS1, $\alpha = 2.41$ pence/kWh (UK) in the off-peak period and $\alpha = 6.79$ pence/kWh in the peak period - the values for PS1 in the original Richmond network) is used for both pump stations. Based on the EPANET results in Table 2, it is obvious that the pump selection of $n_1 = 2$ and $n_2 = 0$ is not a good option since the energy consumed is relatively high but only provides a small pumping flow. So this selection is excluded in the search for optimal pump operations. For the remaining four pump selections, the constant power was set to be $p = 40.21$ obtained from the power values for four selections of $n_1 + n_2$ in Table 2.

The weighting matrix $R$ in Eq. (8) is chosen to be $R = \text{diag}(100, 50)$. The order of the terms is chosen to provide a reasonable balance between economic cost of operation, $\ell_e$, and the desire to reduce switching of the pumps to a level commensurate with typical behavior. The relative weighting between the two terms is an artefact of the asymmetric network used in this case study. The weighting term for PS1 is relatively larger than the one for PS2. This is due to the fact that when Pump 3A in PS2 is turned on, one or two pumps may operate in PS2.

Based on the original Richmond network, the water depth constraint for Tank A (i.e. the capacity of this tank) can be found in the EPANET file as

$$x \leq x(k) \leq 3.37,$$

(16)
where the minimum water depth was chosen to be $x = 1.4m$ for the simulation.

The control input can be set as $u(k) = [n_1(k), n_2(k)]^T$ with the constraints

$$[0, 0]^T \leq u(k) \leq [2, 1]^T,$$  \hspace{1cm} (17a)

$$n_1(k) \geq n_2(k),$$  \hspace{1cm} (17b)

$$n_1(k) - n_2(k) \leq 1,$$  \hspace{1cm} (17c)

where Eq. (17a) indicates the available pumps in pump station 1 and 2. Eqs. (17c) ensure that the pump selection of $n_1 = 2$ and $n_2 = 0$ is not used.

**RESULTS**

The optimization problem was solved by integer programming implemented with the Yalmip toolbox (Löfberg 2004) and the Artelys Knitro solver (Artelys 2020). The EMPC controller was connected to the EPANET hydraulic simulator (Rossman 2000). The optimal control action from the EMPC controller was sent to this simulator via the EPANET-Matlab toolkit (Eliades et al. 2016). The EMPC controller was implemented with the sampling time $\Delta t = 1$ hour = 3600 seconds and the prediction horizon was chosen to be $H_p = 24$ hours. With different base demands at node 10, the closed-loop simulation results for 4 days (96 hours) are shown in the following sections.

**Simulation Results of EMPC with EPANET**

To assess the performance of the proposed EMPC, the closed-loop simulations with the Richmond Pruned network in EPANET have been carried out from the same initial water depth $x(0) = 3.12m$ in Tank A. In PS1, pumps 1A and 2A are of the same type. In simulations, we arbitrarily choose to use Pump 2A when $n_1 = 1$ and Pumps 1A and 2A when $n_1 = 2$. For the demand sector at Node 10, six different water demand loading cases were chosen by setting the base demands as $\bar{d}_{10} = 5, 15, 25, 35, 45, 55$ L/s, respectively. In simulations, it was verified that with the proposed EMPC controller, this network can be operated to satisfy all the demands up to $\bar{d}_{10} = 57.9$ L/s but fails above this level (demands exceed capacity of the pumps and Tank A empties). For the demand loading larger than $\bar{d}_{10} = 57.9$ L/s, no controller or operational strategy can handle it since the average demand exceeds maximum pumping capacity. For the cases when $\bar{d}_{10} > 5$ L/s, for the step $t \geq 8$ in the prediction horizon $H_p$ of (11), the integer decision variables $\tilde{u}(t)$ were relaxed and continuous decision variables were used instead in order to reduce the computational burden required to find the solution.
For these six cases, the water depth variations of Tank A are shown in Fig. 3. From the same initial water depth, the EMPC optimization results show Tank A fills up and then decreases before arriving at the predefined minimum depth of $\bar{x} = 1.4$ m. In general, the tank fills up when the electricity tariff is low and empties when it is high. Since the demand multipliers follow a periodic pattern over 24 hours, the water depth variations also follow a periodic behavior for each of these six cases after the effect of the initial conditions has subsided.

For $\bar{d}_{10} = 5, 25, 45$ L/s, the resulting pump operations are shown in Fig. 4 to Fig. 6. As shown in Fig. 4, since the base demand is small, only one pump in PS1 is required, and pumping only takes place in the off-peak tariff period as shown in Fig. 4(a). When the demand is increased to $\bar{d}_{10} = 25$ L/s, operation of one pump in PS1 is not enough to provide the required flows and pumping in PS2 is required, but only in the off-peak tariff period as shown in Fig. 5(b) while some pumping in PS1 is necessary in the peak tariff period. However, only one pump (out of 2 pumps) in PS1 is operated and this pump is switched off for some of the time during the peak tariff period. When the demand is increased to $\bar{d}_{10} = 45$ L/s, both pump stations are required to be operated in order to provide enough water. From the optimal solution of EMPC, a single pump in PS1 is operated during the peak tariff period and two pumps in PS1 are operated in the off-peak tariff period as shown in Fig. 6(a). For PS2, the single pump is needed for most of the time and is only switched off for some of the time during the peak tariff period.

In the cases in Figs. 4 and 5, most of the pumping takes place in the off-peak tariff period and no unexpected pump switches happened. For the case when $\bar{d}_{10} = 45$ L/s, the optimal pump operations are shown in Figs. 6(a)-6(b) and the water depth variation of Tank A is shown in Fig. 6(c). From the beginning, Tank A drains since the demand is high and Pump 1A in PS1 is not turned on since it is in the peak tariff period. As the water depth in Tank A gets close to the lower limit, both Pumps 1A and 2A in PS1 are eventually turned on in order to avoid the water depth going below 1.4 m but Pump 1A is only used for a short period of time in the peak tariff period but used most of time in the off-peak tariff period. Pump 3A in PS2 still operate until around $k = 40$. Since it is still in the peak tariff period and there is enough water in Tank A, Pump 3A is turned off for some time and turned on in the off-peak period.

To further assess the performance of the proposed EMPC, the total pumped water volume, the total consumed energy, and the total economic cost are calculated for six water demand loading cases and the computational results are reported in Table 3. Note that the results of the economic costs are actual pumping energy costs by implementing the chosen pump operations. As the volume of pumped water increases, the consumed energy
also increases as well as the total cost. The total cost per unit volume is also computed in Table 3. Since more water is pumped in the peak tariff period, the total cost per unit volume also increases as the water demand increases. Furthermore, the average pump efficiencies for pumps 1A, 2A and 3A were obtained from the EPANET simulator and reported in Table 3. It can be seen that with the proposed EMPC controller, all the average pump efficiencies are around 70%.

**Comparison with Trigger-Level Control**

To compare the performance of EMPC with a traditional pump operational control strategy, the traditional trigger-level control is also applied for the Richmond Pruned network. The trigger levels for three pumps are shown in Table 4. With \( \bar{d}_{10} = 5 \) L/s, the results are compared in Fig. 7. Since trigger-level control does not take into account the time-varying electricity price, the pumping flows will occur at any time based on the water depth in Tank A without regard for the peak and off-peak tariff periods as shown in Fig. 7(a). In contrast, for EMPC, the pumping flows are only operated in the off-peak tariff period for \( \bar{d}_{10} = 5 \) L/s. Consequently, as shown in Fig. 7(b) and 7(c), the total pumped water volume and the total energy consumed are similar for both EMPC and trigger-level control while total costs are very different. The cost from the operation optimized with the EMPC is significantly lower than that obtained from trigger-level control.

The results obtained from trigger-level control with six different demand loading cases are reported in Table 5. Compared to the results obtained from the EMPC reported in Table 3, the total pumped water volume, the total energy consumed and the total pumping energy costs are calculated as well as the total cost per unit volume of water pumped. Based on the results of the total cost per unit volume, these costs are similar for the three larger water demand loading cases. Moreover, as also shown in Table 5, the cost ratio between trigger-level control and EMPC is computed by

\[
\text{Cost Ratio} = \frac{\text{Trigger-level Cost per m}^3}{\text{EMPC Cost per m}^3}. \tag{18}
\]

According to the cost ratio results in Table 5, the optimal pump operations with the EMPC has gained an economic benefit between 3% and 250% compared to trigger-level control. The relative benefits are more significant at lower demand flows, as there are more opportunities for intelligent scheduling to increase economic efficiency. On the other hand, at the higher demands, the utilization of the pumps is very high and must be operated during the high tariff period in order to satisfy demands. In this case, there is little opportunity for
improving the economic performance and the benefits of the proposed approach are reduced.

CONCLUSIONS

In this study, a novel EMPC framework has been proposed for real-time operational management of WDSs. The optimal pump operations are chosen by minimizing pumping energy costs approximated with a flow-based model of WDSs. To demonstrate the utility and advantage of this proposed EMPC framework, the Richmond Pruned case study has been used. The closed-loop simulation with an EMPC controller and an EPANET hydraulic simulator has shown that the EMPC framework is effective and efficient in finding a set of optimal pump operations with minimum pumping energy costs taking into account time-varying electricity prices. Less pumping occurs during the peak tariff period and only when it is necessary while more pumping is operated in the off-peak period. The performance of the EMPC has also been compared to the tradition operational control based on trigger level values. From this comparison, under the smallest demand loading case, the energy consumption obtained with the EMPC and the trigger-level control is similar at the end of the simulation time but the pumping energy cost is quite different. It has been shown that for a small demand, the cost for trigger-level control is 2.5 times more expensive than that obtained with the EMPC. So the significantly lower cost obtained from the EMPC compared to the trigger-level control is due to the fact that the EMPC directly takes into account the time-varying electricity prices.

Uncertainties exit in both the mathematical model of the WDS and the forecasts of demands and electricity prices. These uncertainties may lead to solution that cannot be feasibly applied in real-world applications, or solutions with operational performance degradation. Therefore, robustness considering these uncertainties need to be incorporate in WDS operational control in the future.

DATA AVAILABILITY

Some or all data, models, or code generated or used during the study are available in a repository online in accordance with funder data retention policies.

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### TABLE 1. Variable assignment for modeling.

| Item                          | Variable | Minimum Value | Maximum Value |
|-------------------------------|----------|---------------|---------------|
| Tank water depth              | $x_i$    | 0             | $\bar{x}_i$   |
| Inflow to the tank            | $q_i^{(in)}$ | 0             | -             |
| Outflow from the tank         | $q_i^{(out)}$ | 0             | -             |
| Number of pumps operating     | $n_j$    | 0             | $\bar{n}_j$   |
| Outflow from pump station     | $q_j$    | 0             | -             |
| Water demand                  | $d_s$    | 0             | -             |
| Subscript indexing the tanks  | $i$      | 1             | $T$           |
| Subscript indexing the pump stations | $j$ | 1             | $P$           |
| Subscript indexing the water demands | $s$ | 1             | $D$           |
TABLE 2. Fitting data for pumping flows and powers (values from EPANET).

| $n_1$ | $n_2$ | $q_A$ [L/s] | PS1 Power [kW] | PS2 Power [kW] |
|-------|-------|-------------|----------------|----------------|
| 0     | 0     | 0           | 0              | 0              |
| 1     | 0     | 25.21       | 46.32          | 0              |
| 2     | 0     | 30.82       | 87.03          | 0              |
| 1     | 1     | 43.23       | 59.52          | 21.41          |
| 2     | 1     | 57.88       | 98.45          | 22.19          |

$n_1$ - the number of pumps operating in PS1;
$n_2$ - the number of pumps operating in PS2.
### TABLE 3. EMPC results for 4 days.

| $\dot{d}_{10}$ [L/s] | Tot. Vol. Pumped [m³] | Tot. Energy [kWh] | Tot. Cost (£) | Cost per m³ [£/m³] | $\bar{\eta}_1$ [%] | $\bar{\eta}_2$ [%] | $\bar{\eta}_3$ [%] |
|-----------------------|------------------------|-------------------|----------------|-------------------|----------------|----------------|----------------|
| 5                     | 1282                   | 712               | 17.16          | 0.0134            | 0              | 66.67          | 0              |
| 15                    | 4836                   | 2712              | 96.69          | 0.0200            | 0              | 70.17          | 60.24          |
| 25                    | 7969                   | 4493              | 207.28         | 0.0260            | 0              | 68.74          | 60.25          |
| 35                    | 11148                  | 6427              | 319.24         | 0.0286            | 69.71          | 68.83          | 64.04          |
| 45                    | 14282                  | 8327              | 444.01         | 0.0311            | 69.79          | 72.97          | 62.83          |
| 55                    | 17425                  | 10795             | 595.67         | 0.0342            | 69.75          | 70.88          | 68.37          |
**TABLE 4.** The trigger-level controller setup.

| Pump | Trigger Level* - ON [m] | Trigger Level - OFF [m] |
|------|--------------------------|-------------------------|
| 1A   | \( \leq 2.37 \)         | \( \geq 2.98 \)        |
| 2A   | \( \leq 1.40 \)         | \( \geq 3.25 \)        |
| 3A   | \( \leq 1.90 \)         | \( \geq 3.11 \)        |

* Tank A; Lower trigger level values for 2A and 3A were reduced compared to the original Richmond EPANET input file.
**TABLE 5.** Trigger-level control results for 4 days.

| $d_{10}$ [L/s] | Tot. Vol. [m$^3$] | Tot. Energy [kWh] | Tot. Cost [£] | Cost per m$^3$ [£/m$^3$] | Cost Ratio |
|----------------|-------------------|-------------------|----------------|--------------------------|------------|
| 5              | 1311              | 753               | 42.96          | 0.0328                   | 2.50       |
| 15             | 4664              | 2540              | 150.17         | 0.0322                   | 1.55       |
| 25             | 7823              | 4385              | 242.34         | 0.0310                   | 1.16       |
| 35             | 10862             | 6477              | 410.32         | 0.0378                   | 1.28       |
| 45             | 14448             | 8746              | 515.52         | 0.0357                   | 1.16       |
| 55             | 17502             | 10899             | 618.74         | 0.0354                   | 1.03       |

Cost Ratio = Trigger-level Cost per m$^3$/EMPC Cost per m$^3$. 
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