Kaluza-Klein decomposition and gauge coupling unification in orbifold GUTs

Masud Chaichian\textsuperscript{a} and Archil Kobakhidze\textsuperscript{a,b}

\textsuperscript{a}High Energy Physics Division, Department of Physical Sciences, University of Helsinki & Helsinki Institute of Physics, FIN-00014 Helsinki, Finland
\textsuperscript{b}Andronikashvili Institute of Physics, Georgian Academy of Sciences, GE-380077 Tbilisi, Georgia

Abstract

We discuss Kaluza-Klein (KK) decomposition in 5-dimensional (5D) field theories with orbifold compactification. Kinetic terms localized at orbifold fixed points, which are inevitably present in any realistic model, modify the standard KK mass spectrum and interactions of KK modes. This, in turn, can significantly affect phenomenology of the orbifold models. As an example, we discuss gauge coupling unification in N=1 supersymmetric 5D orbifold SU(5) model. We have found that uncertainties in the predictions of the model related to modification of the KK masses are large and essentially uncontrollable.
1 Introduction

Almost all currently available experimental data in particle physics are described by the Standard Model with an impressive accuracy. However, it is widely believed that various theoretical puzzles and problems of the Standard Model indicate that the theory in the present form is just the low energy limit of a more fundamental theory. Since their early days Grand Unified Theories (GUTs) are viewed as one of the most attractive candidates for such a theory. Unification of three gauge couplings within the Minimal Supersymmetric Standard Model strongly supports the basic GUT idea and low-energy broken supersymmetry.

Construction of realistic GUTs in 4 dimensions (4D), however, faces certain difficulties in understanding some phenomenologically important issues such as proper breaking of GUT gauge symmetry, doublet-triplet hierarchy, proton stability, correct relation between the masses of quarks and leptons, etc. Recently some of the problems of conventional 4D GUTs has been addressed in the context of higher-dimensional field theories with orbifold compactification. Particularly, GUT symmetry breaking can be realized through the orbifold compactification which could also naturally provide the doublet-triplet splitting and avoid coloured Higgsino (Higgs) mediated proton decay. Moreover, by placing matter fields in the higher-dimensional bulk one automatically obtains the theory with stable (in all orders of perturbation theory) proton and without SU(5) GUT relations among the masses of the down quarks and charged leptons which are apparently wrong for the two lightest families. Alternatively, restricting the third family of fermions to a SU(5)-invariant orbifold fixed-point, one can keep the celebrated b-τ unification. Other aspects of fermion masses and the possible origin of the observed structure of fermion families have been also widely discussed. Summarizing, it turns out that higher-dimensional orbifold GUTs could give a simple intrinsically geometric explanation of various problems of conventional 4D models and provide a phenomenologically more attractive bottom-up approach than the highly restricted top-down approach to the unification in higher-dimensions in the framework of fundamental string theory. Notice, however, that consistency of higher-dimensional field theories and particularly field theories on singular spaces, such as orbifolds, requires some ultraviolet (UV) completion. Such a completion is commonly viewed to be provided by the fundamental string theory.

An unspecified UV physics generally leads to uncertainties in the effective low energy field theories. Namely, from the field theory point of view one can expect appearance of certain operators localized on the orbifold fixed points which respect symmetries on the orbifold subspaces rather than the symmetries in the bulk. Among them naturally appear the relevant “renormalizable” operators. Particularly, one can write down kinetic terms of certain orbifold fields on the GUT symmetry breaking fixed point. Since they do not respect GUT symmetry one can worry that such operators could completely destroy higher-dimensional gauge coupling unification. The localized kinetic terms for the gauge fields have been studied and the influence of the kinetic fields has been investigated. The localized kinetic terms for the fermionic fields have been studied in and . It was pointed out in that assuming the unification happens in the strong coupling regime one can dilute the UV sensitivity and even improve
the apparently wrong prediction for the strong gauge coupling of the 4D supersymmetric SU(5) model. High-precision prediction for the strong gauge coupling constant obtained in [7] singles out the simple N=1 supersymmetric 5D SU(5) GUT.

In this paper we would like to reconsider the effects of the kinetic terms localized on the orbifold fixed points within the 5D effective field theories. We will show that local kinetic terms modify the KK mode decomposition and as a result the entire phenomenology of low energy orbifold field theories will be affected significantly. As an implication to the orbifold GUT, we have considered the influence of local kinetic terms on the gauge coupling unification. The essential point is that the localized kinetic terms for the Higgs and matter fields can not be constrained to be small by the requirement of strong gauge coupling unification. We have found that uncertainties related with local kinetic terms of Higgs and matter fields are generically large and uncontrollable within the effective field theory approach. In this way the predictions of the model are UV sensitive due to incalculable parameters related with localized kinetic terms. Within these uncertainties in the prediction of strong gauge coupling, one can therefore successfully incorporate different orbifold GUT models as well.

The effects of localized kinetic terms have already been discussed within different models. Importance of such terms for the quasilocalization of higher-dimensional gravity and gauge fields in the infinite volume extra dimensions has been pointed out in [14] (see also [15]). The case of large compact extra dimension(s) with single brane has been subsequently studied in [16]. It has been shown in [17] that local kinetic terms for bulk fields generally appear radiatively even if they are not present in the initial Lagrangian. Some phenomenological consequences have been also studied within the 5D orbifold theories in [18, 19]. Finally, we should point out that some results of the recent papers [19] and [20] partially overlap with the results of Section 3 of the present work. Other localized operators have been also discussed in the literature [21-24].

The paper is organized as follows. In Section 2 we will briefly remind the basic facts about N=1 supersymmetry in 5D and the orbifold compactification and set-up a simple realistic 5D SU(5) GUT. In Section 3 we discuss Kaluza-Klein (KK) reduction of theory with kinetic terms localized on the orbifold fixed points. In Section 4 we will discuss 1-loop contributions of KK modes to the evolution of gauge couplings and the problem of gauge coupling unification. The final Section 5 will be devoted to summary and conclusions.

2 Orbifold compactification and 5D SU(5) GUT

We begin with a brief review of the orbifold compactification in the simplest case of a only one extra dimension. The 5D space-time is a direct product of 4D Minkowski space-time \( M^4 \) and an extra dimension compactified on the orbifold \( S^1/Z_2 \), with coordinates \( x^M \), where \( M = 0, 1, 2, 3, 5 \) (\( x^5 = y \)). The \( S^1/Z_2 \) orbifold can be viewed as a circle of radius \( R \) with opposite points identified by action of \( Z_2 \) orbifold parity: \( Z_2: y \rightarrow -y \). The actual physical space therefore is the interval \( y \in [0, \pi R] \) with two orbifold fixed points at \( y = 0 \) and
$y = \pi R$. Under the $Z_2$ symmetry, a generic 5D bulk field $\phi(x^\mu, y)$ ($\mu = 0, 1, 2, 3$) has a definite transformation property

$$\phi(x^\mu, -y) = P\phi(x^\mu, y),$$

where the eigenvalues of $P$ must be $\pm 1$. Generally the field $\phi(x^\mu, y)$ also can have $U(1)$-twisted transformation under the periodic translation along the fifth coordinate: $y \to y + 2\pi R$:

$$\phi(x^\mu, y + 2\pi R) = U\phi(x^\mu, y),$$

where $U_\kappa = \exp(2i\pi\kappa)$. In fact this is the well-known Hosotani-Scherk-Schwarz boundary condition. The case $\kappa = \frac{1}{2}$ is a discrete version of more general case (2). Denoting the field with $(P, U_{\frac{1}{2}}) = (\pm 1, \pm 1)$ by  $\phi^{(\pm, \pm)}$, we obtain the following KK mode expansion:

$$\phi^{(++, \pm)}(x^\mu, y) = \sum_{n=0}^{\infty} \sqrt{\frac{2}{a}} f^{(++, \pm)}_n(x^\mu) \cos \left(\frac{n y}{R}\right),$$

$$\phi^{(+-, \pm)}(x^\mu, y) = \sum_{n=0}^{\infty} \sqrt{\frac{2}{a}} f^{(+-, \pm)}_n(x^\mu) \cos \left(\frac{(n + \frac{1}{2}) y}{R}\right),$$

$$\phi^{(-+, \pm)}(x^\mu, y) = \sum_{n=0}^{\infty} \sqrt{\frac{2}{a}} f^{(-+, \pm)}_n(x^\mu) \sin \left(\frac{(n + 1) y}{R}\right),$$

$$\phi^{(--, \pm)}(x^\mu, y) = \sum_{n=0}^{\infty} \sqrt{\frac{2}{a}} f^{(--, \pm)}_n(x^\mu) \sin \left(\frac{(n + \frac{1}{2}) y}{R}\right),$$

where $a = \pi R$. Upon compactification the 4D fields (KK modes) $f^{(++, \pm)}_n(x^\mu), f^{(+-, \pm)}_n(x^\mu), f^{(-+, \pm)}_n(x^\mu), f^{(--, \pm)}_n(x^\mu)$ acquire masses $\frac{n y}{R}, \frac{(n + \frac{1}{2}) y}{R}, \frac{(n + 1) y}{R}, \frac{(n + \frac{1}{2}) y}{R}$, respectively. Note that zero mode is contained only in $\phi^{(++, \pm)}$ field. We will call (3) the standard KK decomposition. In next Section we will argue that this decomposition is not valid in general.

Now let us describe the simplest $N=1$ supersymmetric SU(5) model in 5D. In the 5D bulk we have SU(5) gauge supermultiplet, transforming as an adjoint representation $V^A \sim 24$ ($A = 1, ..., 24$) and two Higgs hypermultiplets $\mathcal{H}$ and $\overline{\mathcal{H}}$ that transform as $5$ and $\overline{5}$, respectively. The 5D SU(5) gauge supermultiplet contains a vector boson $A^A_M$, two gauginos, $\lambda^A_L$ and $\lambda^A_R$, and a real scalar $\sigma^A$, which can be decomposed into a vector supermultiplet $V^A = (A^A_M, \lambda^A_L, \lambda^A_R)$, and a chiral supermultiplet $\Sigma^A = ((\sigma^A + i A^A_L)/\sqrt{2}, \lambda^A_R)$ under 4D $N=1$ supersymmetry. The hypermultiplet contains two complex scalars, $h$ and $h^c$, and two Weyl fermions, $\psi$ and $\psi^c$. They can be combined into two 4D $N=1$ chiral supermultiplets $H = (h, \psi)$ and $H^c = (h^c, \psi^c)$, which transform as $5$ and $\overline{5}$ under the SU(5) gauge group (similarly for $\overline{\mathcal{H}}$).

The 5D SU(5) symmetry is broken down to the to the SU(3)$\otimes$SU(2)$\otimes$U(1) Standard Model group by orbifold boundary conditions. This can be achieved in different ways. We will assume that the gauge symmetry breaking occurs because of non-trivial periodic boundary conditions. Namely, we choose: $U_1 = (-1, -1, -1, +1, +1)$ and $P = (+1, +1, +1, +1, +1)$,
Table 1: The particle content (bulk fields) of the orbifold SU(5) GUT, their transformation properties under the orbifold symmetries and KK masses according to the standard decomposition \( (3) \).

| \((P, U_4)\) | Gauge and Higgs fields | Bulk matter fields | KK masses |
|-------------|------------------------|-------------------|-----------|
| (+, +)      | \( V^a, H_D, \overline{H}_D \) | \( 10_{U,E}, 10', \overline{5}_D, \overline{5}_L \) | \( \frac{n}{R} \) |
| (+, −)      | \( V^\hat{a}, H_T, \overline{H}_T \) | \( 10_Q, 10'_{U,E}, \overline{5}_L, \overline{5}'_D \) | \( \frac{n+\frac{5}{R}}{R} \) |
| (−, +)      | \( \Sigma^a, H'_D, \overline{H}_D \) | \( 10'^c_{U,E}, 10''_Q, \overline{5}'_D, \overline{5}_L \) | \( \frac{n+1}{R} \) |
| (−, −)      | \( \Sigma^\hat{a}, H'_T, \overline{H}_T \) | \( 10'^c_Q, 10''_{U,E}, \overline{5}_L, \overline{5}'_D \) | \( \frac{n+\frac{5}{R}}{R} \) |

where \( U_4 \) and \( P \) act on the fundamental representation of SU(5). At the same time we have to assign opposite \( Z_2 \) orbifold parities to the 4D N=2 partners of the bulk fields, in order to keep only N=1 supersymmetry in 4D effective theory. The orbifold symmetries for all components of the vector and Higgs multiplets are shown in Table 1. Here, we split SU(5) index \( A = a, \hat{a} \) into the indices, \( a \) and \( \hat{a} \), which correspond to the unbroken and broken SU(5) generators, respectively. The subscripts \( T \) and \( D \) denote colour triplet and weak doublet components of the Higgs multiplets, respectively. Since only \((+, +)\) fields have zero modes, the massless sector of the model consists of \( \text{SU}(3) \otimes \text{SU}(2) \otimes \text{U}(1) \) N=1 vector multiplet \( V^0_a \) and Higgs doublet and anti-doublet chiral superfields \( H_D \) and \( \overline{H}_D \), colour triplet chiral superfields \( H_T \) and \( \overline{H}_T \), are massive, thus realizing doublet-triplet splitting in a simple geometric way. This also means that Higgsino mediated proton decay is absent in the model. Non-zero modes of at each KK level \( n \) \( V^a \) eat corresponding \( \Sigma^a \) becoming massive and similarly for the X-Y vector multiplets \( V^{\hat{a}} \) and \( \Sigma^{\hat{a}} \). Note also that locally at \( y = 0 \) fixed point SU(5) symmetry is exact, while it is broken on the \( y = \pi R \) fixed point. Thus the two fixed points are not equivalent.

Each generation of matter fields, are placed into hypermultiplets which transform, as usually, as \( \overline{5} \) and 10 representations of SU(5). There are different ways how to put matter fields in the bulk. They can be localized on one of the fixed points. In order to preserve the successful SU(5) \( b-\tau \) unification it is desirable to have third generation matter on the SU(5)-preserving fixed point at \( y = 0 \). On the other hand, one can place matter fields in the bulk as well. An important point is that, in such a case one has to double the representation \( \overline{5} + 10 \) by introducing \( \overline{5}' + 10' \) \( \overline{5}' \). One can see from Table 1, that zero-mode matter fields, i.e. ordinary quarks and leptons (subscripts \( Q, L, U, D \) and \( E \) denote quark doublet, lepton doublet, up-antiquark, down-antiquark and positron components of the corresponding SU(5) representations, respectively), come now from different representations. This means that X-Y gauge boson can not be responsible for the proton decay in case of all three generations being in the bulk. The proton decay will be significantly suppressed if only light generations (or part of them) are residing in the bulk, while third generation is localized on SU(5)-symmetric fixed point \( \overline{5}' \). A.

The model described above is indeed very attractive. One can see, that most of the
difficulties of the ordinary 4D SU(5) GUT can be resolved in a very simple way. It remains to see whether one can have in the above picture reliable predictions. But before discussing gauge coupling unification in the above model we would like to come back to the question of KK mode decomposition in orbifold field theories.

## 3 KK decomposition in 5D with orbifold compactification

Once again, we are considering 5D space-time with the fifth extra dimension $y$ being an $S^1/Z_2$ orbifold, i.e. a line of length $a = \pi R$ with two fixed points at $y = 0$ and $y = a$. As we have mentioned in the Introduction, generally one is allowed to add to the 5D bulk Lagrangian $L_5$ Lagrangians $L_0$ and $L_\pi$ localized at $y = 0$ and $y = a$ 4D subspaces. So the total Lagrangian is:

$$L = L_5 + \delta(y)L_0 + \delta(y-a)L_\pi.$$  \hfill (4)

$L_0$ and $L_\pi$ could contain various operators which respect the symmetries on the fixed points but not in the full 5D bulk. In certain cases of interest, however, they might be avoided by some symmetries (supersymmetry). In realistic models this is not the case for the kinetic terms. Indeed, the localized kinetic terms for certain 5D bulk fields inevitably appear (unless one is considering phenomenologically unacceptable theory with conformal invariance) as a result of radiative corrections. Contributions to the radiative induced local kinetic terms come not only from the fields localized at the fixed points but from the bulk fields as well [17]. These contributions are logarithmically divergent and consistency of the theory requires introduction of the corresponding local counter-terms. Residual finite parts (after cancellation of divergences) are not calculable within the effective orbifold field theory and thus should be treated as new free parameters. Basically, they can be calculated only within a more general theory which substitutes the low energy effective orbifold theory at some ultraviolet scale $\Lambda$. Thus, the free parameters of the local kinetic terms parametrize our ignorance of the fundamental UV physics.

In turn, the local kinetic terms could significantly affect the entire phenomenology of the effective low energy orbifold theories. Namely the KK mode decomposition discussed in Section 2 is not valid anymore. To see this, let us consider the simple case of 5D massless scalar field:

$$L_5 = \partial_M\Phi^+\partial^M\Phi,$$

$$L_0 = r_0\partial_\mu\Phi^+\partial^\mu\Phi,$$

$$L_\pi = r_\pi\partial_\mu\Phi^+\partial^\mu\Phi,$$ \hfill (5)

where $r_0$ and $r_\pi$ are new scale parameters associated with local kinetic terms at $y = 0$ and $y = a$, respectively. The parameters $r_0$ and/or $r_\pi$ are vanishing for the field $\Phi$ which vanishes at $y = 0$ and/or $y = a$ fixed points. The behaviour of $\Phi$ at fixed points in turn is defined by the orbifold symmetries at hand. We consider general case where $\Phi$ is either even or odd.
under the orbifold parity $Z_2$ \(1\) and it is $U(1)$-twisted under the periodic translation \(2\). The equation of motion followed from \(1\) and \(3\) is:

\[
(1 + r_0 \delta(y) + r_\pi \delta(y - a)) \partial_\mu \partial^\mu - \partial_y^2 \Phi(x^\mu, y) = 0. \tag{6}
\]

It is supplemented by the boundary conditions \(1\) and \(2\). Performing the KK decomposition of $\Phi$:

\[
\Phi(x^\mu, y) = \sum_{m_n} \phi_{m_n}(x^\mu) f_{m_n}(y), \quad -\infty \leq m_n \leq +\infty
\]

the Eq. \(6\) is split into the Klein-Gordon equation

\[
(\partial_\mu \partial^\mu + m_n^2) \phi_{m_n}(x^\mu) = 0 \tag{8}
\]

for the modes $\phi_n(x^\mu)$ with KK masses $m_n$ and the equation which determines the $y$ dependence of the modes:

\[
(\partial_y^2 + m_n^2 [1 + r_0 \delta(y) + r_\pi \delta(y - a)]) f_{m_n}(y) = 0. \tag{9}
\]

Eq. \(9\) can be solved by the method of images. The problem is actually reduced to a simple one-dimensional quantum-mechanical problem of a particle moving in a periodic potential formed by a sequence of Dirac $\delta$-functions at $y = 0 + 2an$ and $y = a + 2an$. Since the equation of motion and the boundary conditions are periodic, we can apply Floquet’s theorem and write down a general solution in different regions as:

\[
f_{m_n}(y) = \begin{cases}
A_{m_n} e^{i m_n y} + B_{m_n} e^{-i m_n y}, & 0 < y < a \\
C_{m_n} e^{i m_n y} + D_{m_n} e^{-i m_n y}, & a < y < 2a \\
e^{2iak} [A_{m_n} e^{i m_n y} + B_{m_n} e^{-i m_n y}], & -a < y < 0 \\
e^{2iak} [C_{m_n} e^{i m_n y} + D_{m_n} e^{-i m_n y}], & -2a < y < -a
\end{cases}, \tag{10}
\]

where $A_{m_n}, B_{m_n}, C_{m_n}, D_{m_n}$ are constants to be determined by matching the function $f_n(y)$ and its first derivatives at fixed points. Assuming that $f_n(y)$ is continuous across the fixed points, we get the following system of homogeneous algebraic equations for $A_{m_n}, B_{m_n}, C_{m_n}, D_{m_n}$:

\[
\tilde{\mathcal{M}}_n \begin{bmatrix} A_{m_n} \\ B_{m_n} \\ C_{m_n} \\ D_{m_n} \end{bmatrix} = 0, \tag{11}
\]

where

\[
\tilde{\mathcal{M}}_n = \begin{bmatrix}
1, & 1, & -e^{-i 2a(\kappa + m_n)}, & -e^{-i 2a(\kappa - m_n)} \\
1, & e^{-i m_n}, & -e^{-i m_n}, & -e^{-i m_n} \\
(r_0 m_n^2 + 2i m_n), & r_0 m_n^2, & -2i m_n e^{i 2a(\kappa + m_n)}, & 0 \\
(r_{\pi} m_n^2 - 2i m_n) e^{i m_n}, & r_{\pi} m_n^2 e^{-i m_n}, & 2i m_n e^{i m_n}, & 0
\end{bmatrix}. \tag{12}
\]
The necessary and sufficient condition for the existence of a nontrivial solution of (11),
\[ \det \hat{M}_n = 0, \]  
(13)
is actually nothing but the KK mass quantization condition.

Solving the system (11) one determines three out of four constants \( A_{m_n}, B_{m_n}, C_{m_n}, D_{m_n} \). The remaining one is determined from the normalization of \( f_{m_n}(y) \). In the presence of local kinetic terms the wave functions \( f_{m_n}(y) \) are generally not orthogonal, \( \int_0^a dy \ f_{m_n}^*(y) f_{m_k}(y) \neq \delta_{m_n m_k} \). This would imply that in the interactions involving these modes the KK number is not conserved, as it was obviously expected from the beginning, since the translational invariance along the fifth dimension is explicitly broken. In order to get the canonical kinetic terms for the KK modes upon the integration over the extra dimension, we imply the following orthonormality condition:
\[ \int_0^a dy \left[ 1 + r_0 \delta(y) + r_\pi \delta(y - a) \right] f_{m_n}^*(y) f_{m_k}(y) = \delta_{m_n m_k}. \]  
(14)
Note that the \( \left[ 1 + r_0 \delta(y) + r_\pi \delta(y - a) \right] \) acts as a nontrivial volume-factor in the integral over \( y \). Obviously, if the field \( \Phi(x^\mu, y) \) is real, i.e. \( \phi_{m_n}(x^\mu) = \phi_{m_n}^+(x^\mu), f_{m_n}^*(y) = f_{-m_n}(y) \), one can recast the decomposition (7) into the form:
\[ \Phi(x^\mu, y) = \sum_{m_n \geq 0} \phi_{m_n}(x^\mu) \left( f_{m_n}(y) + f_{-m_n}(y) \right). \]  
(15)
Then the normalization constants get multiplied by a factor of 2 relative to the case of complex field.

Now let us consider some particular examples of fields with different orbifold symmetries.

**Z\(_2\)-even, periodic field, \( \Phi^{(+,+)}(x^\mu, y) \).** Since the \( Z_2 \)-even field is nonvanishing at both boundaries, \( r_0 \) and \( r_\pi \) are in general non-zero. Taking \( \kappa = 0 \) (periodic boundary condition), we obtain the following solution of the Eqs. (11):
\[ f_{m_n}(y) = A_{m_n} \left( e^{im_n y} + \frac{2im_n + r_0 m_n^2}{2im_n - r_0 m_n^2} e^{-im_n y} \right), \quad 0 \leq y \leq a. \]  
(16)
Then the KK mass quantization condition (13) becomes:
\[ \tan (m_n a) = -\frac{m_n (r_0 + r_\pi)}{2 \left( 1 - \frac{r_0 r_\pi m_n^2}{4} \right)}, \]  
(17)
and the normalization constants \( A_{m_n} \) are:
\[ A_0 = \frac{1}{\sqrt{2} \left( 2a + r_0 + r_\pi \right)} \]  
(18)
for the massless zero mode \((m_0 = 0)\) and

\[
A_{m_n} = \frac{1}{\sqrt{2a + \frac{r_0}{a} + \frac{r_0 + r_\pi}{a} + \frac{r_0 r_\pi m_n^2}{(1 + \frac{r_0^2 m_n^2}{4}) (1 + \frac{r_\pi^2 m_n^2}{4})}}} \tag{19}
\]

for non-zero modes \((m_n \neq 0)\).

The Eq. (17) for KK masses \(m_n\) can be solved numerically. Here we consider some analytic approximations assuming first that all \(r\)-factors are positive. If \(r\)-factors are small enough, \(\frac{r_0}{a} \ll 1\), KK mode decomposition is reduced to the standard one (see (3)) with KK masses \(m_n \approx \frac{n}{R}, n = 0, 1, \ldots\) However, for large \(r\)-factors, \(\frac{r_0}{a} \gg 1\), KK modes substantially deviate from those discussed in the previous Section. As an extreme case consider \(\frac{r_0}{a} \gg 1\) \((r_0 \approx r_\pi \equiv r, \xi = \frac{r}{a})\). Then KK modes at each level \(n \geq 2\) once again approach their standard values

\[
m_n \approx \frac{n}{R} - \frac{4}{\xi^{2a}} n, \quad n = 2, 3, \ldots \tag{20}
\]

while the first excited mode is extremely light:

\[
m_1 \approx \frac{2}{a} \sqrt{1 - \frac{1}{\xi}} \approx \frac{2}{\pi R \sqrt{\xi}} < \frac{1}{R}. \tag{21}
\]

The appearance of the light mode in this limit is a peculiar property of the presence of two opaque boundaries \([19]\). Nothing similar happens if one sets one of the two \(r\)-factors to zero. In the case of graviton (instead of the scalar field) with such sufficiently light mode one could have, e.g., an interesting new bigravity model \([25]\).

**Z\(_2\)-even, anti-periodic field, \(\Phi^{(+,-)}(x^\mu, y)\).** In this case the field vanishes on the boundary \(y = a\). Thus taking \(r_\pi = 0\) and \(\kappa = \frac{1}{2}\) (anti-periodic boundary condition), we obtain the solution

\[
f_{m_n}(y) = A_{m_n} \left(e^{im_n y} - e^{-im_n(y-2a)}\right), \quad 0 \leq y \leq a, \tag{22}
\]

which is amended by the quantization condition

\[
cot (m_n a) = \frac{r_0 m_n}{2}. \tag{23}
\]

The normalization constants in (22) are

\[
A_{m_n} = \frac{1}{\sqrt{2a + \frac{r_0}{a} + \frac{r_0}{1 + \frac{r_0^2 m_n^2}{4} (1 + \frac{r_\pi^2 m_n^2}{4})}}} \tag{24}
\]

Once more one can discuss analytically some limiting approximations. For small \(r_0, \frac{r_\pi}{a} \equiv \xi \ll 1\) we essentially reproduce the standard KK modes and their masses (3),

\[
m_n \approx \frac{(n + \frac{1}{2})}{R \left(1 + \frac{\xi}{2}\right)}. \tag{25}
\]
An opposite limit of large $\xi \gg 1$ gives KK masses substantially lighter compared to the standard case:

$$m_n \approx \frac{n}{R} + \frac{2}{\xi a \pi n} < \frac{(n + \frac{1}{2})}{R}, \quad (26)$$

for $n = 2, 3, \ldots$ and a lighter first excited mode:

$$m_1 \approx \sqrt{\frac{2}{\xi \pi R}} << \frac{1}{2R}. \quad (27)$$

**$Z_2$–odd, anti-periodic field, $\Phi^{(-,-)}(x^\mu, y)$.** In this case the boundary at $y = 0$ is transparent for the KK modes. Thus taking $r_0 = 0$ and $\kappa = \frac{1}{2}$ we obtain

$$f_{mn}(y) = A_{mn} \left( e^{imny} + e^{-imn(y-2a)} \right), \quad 0 \leq y \leq a. \quad (28)$$

Mass quantization condition and normalization constants can be obtained changing $r_0$ by $r_\pi$ in the corresponding formulas (23) and (24). Thus in the case of equivalent fixed points ($r_0 = r_\pi$) KK masses of $(+, -)$ and $(-, -)$ modes are degenerate. The approximations of small and large $r_\pi$ are evidently given by the same formulas (25) and (26), (27) (with $\xi = \frac{r_\pi}{a}$), respectively.

**$Z_2$–odd, periodic field, $\Phi^{(-,+)}(x^\mu, y)$, ** is unaffected by the presence of boundaries, since orbifold symmetries force the field to be vanishing at both boundaries. Hence the standard KK mode decomposition discussed in Section 2 remains unchanged in this case.

Before going further, few remarks concerning $r$-factors are in order. So far we have assumed that $r_0$ and $r_\pi$ are positive. Treating them as free parameters one can ask whether both or one of them can be negative. That is to say, whether the local kinetic terms on the fixed points can have wrong sign. An obvious constraint in such cases comes from the appearance of negative norm states. One can see, however, that in certain cases the ghost KK modes can be avoided. Consider for example $(+, +)$ modes and assume $r_0 = -r_\pi$. Then, from (17), (18) and (19), it is clear that the KK modes behave as in the standard case without local kinetic terms, whenever $r$-factors are large or small. The contributions from the localized kinetic terms are cancelled out and boundaries become transparent for the corresponding modes. For the $(+, -)$ ($(-, -)$) modes and negative $r_0$ ($r_\pi$), ghost-free effective theory would imply $2a \left( 1 + \frac{r_0 m_n^2}{4} \right) + r_0 > 0 \left( 2a \left( 1 + \frac{r_\pi m_n^2}{4} \right) + r_\pi > 0 \right)$ for all $n$. Obviously, the above ghost-free condition can be satisfied, for instance, when $r_0 > -2a$ ($r_\pi > -2a$). Many other possibilities can be found as well. The overall message here is that the KK mass spectrum and wave functions crucially depend not only on the size of $r$-factors but on their signs as well.

Another important point to stress is that $r$-factors (in general case when interactions are included) are actually scale dependent and undergo renormalization. Thus the physical KK masses should be determined using the corresponding renormalization group equations. Note, however, that in certain cases of interest this effects might be negligible.
Having determined the KK masses and wave functions from the free (linearized) theory one can discuss now the interactions. As a representative example let us add to the free Lagrangian (4), (5), a $\phi^4$-potential $V$:

$$V = [\lambda_5 + \lambda_0 \delta(y) + \lambda_{\pi} \delta(y - a)] (\Phi^+ \Phi)^2, \quad (29)$$

where $\lambda_5$ is a 5D coupling, while $\lambda_0$ and $\lambda_{\pi}$ are couplings of the interaction terms localized at boundaries $y = 0$ and $y = a$, respectively. The effective 4D couplings can be derived upon KK mode expansion. They are given as:

$$\lambda_{ijkl} = \lambda_5 \int_0^a dy \left[ 1 + \frac{\lambda_0}{\lambda_5} \delta(y) + \frac{\lambda_{\pi}}{\lambda_5} \delta(y - a) \right] f_{m_i}^* f_{m_j} f_{m_k}^* f_{m_l}. \quad (30)$$

Note, that the KK number is violated in these interactions in general even if the localized potentials are absent, $\lambda_0, \lambda_{\pi} = 0$. Only in the standard case when the local kinetic terms are also absent the KK-number (i.e. the fifth component of the momentum) is conserved.

The effective 4D couplings (30) are determined through the rather complicated integrals of the product of wave functions $f_{m_a}$. However, one can readily say something about interactions involving zero modes. Consider for instance $\lambda_{00kl}$. Once again in general KK-number is still not conserved. Suppose now that $\frac{\lambda_0}{\lambda_5} = r_0$ and $\frac{\lambda_{\pi}}{\lambda_5} = r_{\pi}$. Then (30) is considerably simplified and one gets (using orthonormality condition (14)):

$$\lambda_{00kl} = |A_0|^2 \lambda_5 \delta_{kl}. \quad (31)$$

These interactions conserve the KK-number and are universal for all KK modes. The effective 4D coupling is given by:

$$\lambda_4 = \frac{\lambda_5}{2 (2a + r_0 + r_{\pi}). \quad (32)}$$

This is the familiar relation between 4D and 5D couplings except the fact that the ”volume” of extra space, $a$, is replaced by the effective volume, $a + r_0/2 + r_{\pi}/2$. So, if the bulk and localized couplings are aligned with kinetic $r$-factors in the way assumed above, the interaction of zero modes with those of non-zero KK modes are universal and given by (32). This is obviously true also when zero modes interact with different bulk fields with different orbifold symmetries, provided, of course, that the interactions respect these symmetries as it is required by the consistency of the theory.

In fact the above situation is exactly what happens in the case of bulk gauge theories. Indeed, consider some non-Abelian gauge field in the bulk with local kinetic terms on the boundaries:

$$\mathcal{L} = -\frac{1}{4} F_{MN}^a F^{aMN} - \frac{1}{4} g_0^2 \delta(y) F_{\mu\nu}^a F^{a\mu\nu} - \frac{1}{4} \frac{g_{\pi}^2}{g_0^2} \delta(y - a) F_{\mu\nu}^a F^{a\mu\nu}, \quad (33)$$

where $F_{MN}^a = \partial_M A_N^a - \partial_N A_M^a + g_5 f^{abc} A_M^b A_N^c$ is the gauge field strength, $g_5$ bulk gauge coupling and $g_0$ and $g_\pi$ couplings for the localized terms. In this case $r$-factors are given by
r_0 = \frac{g_2^2}{g_0} \text{ and } r_\pi = \frac{g_2^2}{g_\pi} \text{. Linearized equations for the wave functions } f_{m_n} \text{ (they are real in this case) have exactly the same form (in the unitary gauge, } A_5 = 0 \text{) as in the case of scalar field discussed above (see Eq. (6)). The massless zero modes of gauge field } (A^a_\mu \text{ assumed to be } Z_2\text{-even and periodic}) \text{ have the following self-interactions:}

\begin{align*}
g_{0kl} &= \frac{g_5}{\sqrt{a + \frac{r_\mu}{2} + \frac{r_\pi}{2}}} \delta_{kl} \equiv g_4, \\
g_{00kl} &= \frac{g_5^2}{a + \frac{r_\mu}{2} + \frac{r_\pi}{2}} \delta_{kl} \equiv g_4^2. \quad (34)
\end{align*}

The zero mode also universally interacts with bulk matter fields irrespective of the } r\text{-factors of the local kinetic terms of the matter fields (which might be different from those for the gauge field) and matter fields localized on the orbifold fixed points. Clearly, all these features are dictated by the gauge invariance. Note, however, that the above statement is not true for the KK modes of the gauge fields. For instance, fields localized on different fixed points would interact with the KK mode of the same gauge boson differently, if the fixed points are not equivalent.

From the above discussion it is clear that the localized kinetic terms will have profound consequences for the entire phenomenology of higher-dimensional theories. Modification of collider phenomenology in some limited cases has been recently studied in [16, 18, 19]. In the following Section we will discuss the effects of localized kinetic terms on the gauge coupling unification in the framework of 5D GUT model described in Section 2.

### 4 Gauge coupling unification in realistic SU(5) orbifold GUT

An important consistency check of any GUT model is its prediction for the standard gauge couplings } \alpha_1(M_Z), \alpha_2(M_Z), \alpha_3(M_Z) \text{ at low energies, namely at } M_Z \text{ (Z-boson mass). They are related to the unified gauge coupling } \alpha_{GUT}(M_{\text{GUT}}) \equiv \alpha_1(M_{\text{GUT}}) = \alpha_2(M_{\text{GUT}}) = \alpha_3(M_{\text{GUT}}) \text{ at the unification scale } M_{\text{GUT}} \text{ through the renormalization group equations. At two-loop level the minimal 4D supersymmetric SU(5) model gives the following predictions for the strong gauge coupling } \alpha_3(M_Z):

\begin{equation}
\alpha_3^{SU(5)}(M_Z) \simeq 0.130 \pm 0.004 \pm \Delta^{SU(5)} \quad (35)
\end{equation}

and for the unification scale:

\begin{equation}
M_{\text{GUT}}^{SU(5)} \simeq 2 \cdot 10^{16} \text{ GeV}, \quad (36)
\end{equation}

where the experimental values for } \alpha_1(M_Z) \text{ and } \alpha_3(M_Z) \text{ couplings are taken as input parameters. The prediction (35) should be compared with the high-precision experimental value [21]:

\begin{equation}
\alpha_3^{\text{exp}}(M_{\text{GUT}}) = 0.119 \pm 0.002. \quad (37)
\end{equation}
The main uncertainty in (35), $\Delta^{SU(5)}$, originates from the uncertainties in masses of GUT particles, while the first uncertainty is related to the variations in the mass spectrum of the electroweak Higgs bosons and superparticles at the TeV-scale. In the minimal supersymmetric SU(5), $\Delta^{SU(5)}$ is parametrized through the coloured Higgs (Higgsino) mass $M_T$, $\Delta^{SU(5)} = \frac{3\alpha_{GUT}}{10\pi} \ln \left( \frac{M_T}{M_{GUT}} \right)$. Thus, large negative contribution $\Delta^{SU(5)} \approx -0.01$ needed to fit the experimental value (35) can be obtained by lowering the mass $M_T$, $M_T \sim 0.08M_{GUT}$. However, this possibility is excluded from the unacceptably fast proton decay induced by a low mass coloured Higgsino[28]. Thus the minimal 4D SU(5) GUT indeed contradicts the data. In more involved GUT models, the GUT threshold corrections depend on a larger number of unknown parameters which can neither be constrained from independent data, nor unambiguously predicted. In this situation we essentially loose the predictivity.

In this respect, the situation in orbifold GUT, where GUT symmetry breaking is achieved by orbifold compactification, seems more promising. Firstly, ignoring for a moment possible operators localized on the orbifold fixed points, the mass spectrum of GUT particles and their KK modes are essentially determined by few parameters, namely in 5D by a single compactification radius $R$. Secondly, one can avoid the proton decay constraints by intrinsically geometric mechanism without extension of particle content of the model. Thirdly, although each gauge coupling above the compactification scale receives radiative corrections which depend on a certain power of the cut-off scale, the relative slope of the gauge couplings and thus the low energy predictions will have the usual logarithmic scale dependence. This is because of the underlying bulk GUT symmetry. However, as we have seen in the previous Section the local kinetic terms might substantially disturb the KK mass spectrum and one may worry whether one can actually retain the predictivity power.

In fact the problem appears already at tree level. Consider for definiteness a realistic SU(5) GUT in 5D described in Section 2. Since on the orbifold fixed point $y = a$ SU(5) symmetry is broken down to the SU(3)$\otimes$SU(2)$\otimes$U(1), one is allowed to write down only a SU(3)$\otimes$SU(2)$\otimes$U(1)-symmetric (rather than SU(5)-symmetric) gauge kinetic terms with different $r$-factors, $r^i_\pi$ ($i = 1, 2, 3$ correspond to U(1), SU(2) and SU(3), respectively). Then the 4D effective gauge coupling (see Eq. (34)) at some unification scale will be

$$g^4_i(M_{GUT}) = \frac{g_5(M_{GUT})}{\sqrt{a + \frac{r_0(M_{GUT})}{2} + \frac{r^i_\pi(M_{GUT})}{2}}}.$$  

(38)

If $r^i_\pi(M_{GUT})$ are large and non-universal, $r^i_\pi(M_{GUT}) \gtrsim a$, then the gauge coupling unification is ruined already at the tree level. Some extra assumption beyond the SU(5) framework is evidently needed to suppress the localized gauge kinetic terms at SU(5)-breaking orbifold fixed point, i.e. $\frac{r^i_\pi}{a}$ should be $<< 1$.

The explanation to why the $r$-factors of gauge localized kinetic terms might be small has been proposed by Hall and Nomura[6, 7]. The key assumption is that the theory enters

\[\text{Several mechanisms to suppress Higgsino mediated proton decay is known (see e.g. [27]). However, all of them require introduction of new particles and/or interactions, and hence new parameters, beyond the minimal SU(5). This, in turn, significantly reduces the predictive power of such models.}\]
into the strong gauge coupling regime at $M_{GUT}$. This could naturally happen because above the compactification scale the asymptotic freedom is lost and couplings grow rapidly due to the power-low running of couplings in higher dimensions. Naive dimensional analysis gives the following estimates for the strong bulk gauge coupling,

$$g_5(M_{GUT}) \simeq \sqrt{\frac{16\pi^2}{C_M GUT}}$$

and for strong gauge couplings for the local kinetic terms one has:

$$g_0(M_{GUT}) \simeq \sqrt{\frac{16\pi^2}{C}}$$

$$g^i_\pi(M_{GUT}) \simeq \sqrt{\frac{16\pi^2}{C_i}}$$

(here $C$'s are group theoretic factors, $C \simeq 5$ for SU(5) and $C_i \simeq (1, 2, 3)$) [7]. Within this assumption $r$-factors are indeed small, $r_0(M_{GUT}) = g_2^2 g_0^2 \ll 1$, $r_\pi(M_{GUT}) = g_2^2 g_\pi^2 \ll 1$, providing that extra dimension is large, $a M_{GUT} >> 1$ ($a = \pi R$). Since the low energy values for the effective 4D couplings $g_4^i$ are known to be of the order $\sim 0.7$, from the above assumption one can estimate the energy gap between the compactification scale $\frac{1}{R}$ and strong coupling unification scale $M_{GUT}$: $M_{GUT} R \simeq 60$. That is to say, there are approximately $N_{KK} \simeq 60$ KK modes in the energy region between the compactification scale $\frac{1}{R}$ and the unification scale $M_{GUT}$.

To see how this works, let us calculate the KK mode corrections in the model described in the Section 2 with standard KK masses, see Table 1. One obtains at one loop leading-log approximation:

$$\alpha^{-1}_3(M_Z) = \alpha^{-1}_{SU(5)}(M_Z) + \frac{6}{7\pi} \sum_{n=0}^{N_{KK}} \ln \left( \frac{n + 1}{n + \frac{1}{2}} \right),$$

(39)

(here $\alpha^{-1}_{SU(5)}(M_Z)$ is the prediction of the 4D supersymmetric SU(5) for (35) without GUT thresholds, $\Delta_{SU(5)} = 0$) for the strong gauge coupling and

$$\ln \left( \frac{M_c}{M_Z} \right) = \ln \left( \frac{M_{GUT}^{SU(5)}}{M_Z} \right) + \frac{4}{7} \sum_{n=0}^{N_{KK}} \ln \left( \frac{n + 1}{n + \frac{1}{2}} \right) - \ln N_{KK}.$$ (40)

Note that the KK mode correction to $\alpha_3(M_Z)$ (the last term in Eq. (39)) works in the right direction, lowering $\alpha_3^{SU(5)}(M_Z)$ (35). Taking $N_{KK} = 60$, one obtains correction from KK modes $\delta \alpha^{-1}_3(M_Z) \approx 0.715$ which to the brings prediction for the strong gauge coupling,

$$\alpha_3(M_Z) \approx 0.119 \pm 0.004,$$ (41)

in remarkable agreement with the experimental value (37). At the same time, the compactification scale $M_c$ is lower,

$$M_c \approx 1.5 \cdot 10^{15} \text{ GeV},$$ (42)

than the typical 4D unification scale (36). This means that the first KK modes of X-Y intermediate vector bosons might induce unacceptably fast decay of proton. This inevitably forces us to put all or part of light generations into the 5D bulk. Then the fast proton decay can be avoided [4, 5, 6].

Let us now calculate possible corrections to the above result (41), (42) due to the modification of the KK mass spectrum by the local kinetic terms. Consider first gauge vector multiplets $V^a$ and $\bar{V}^\alpha$. The $r$-factors for these superfields are controlled by the strong coupling
unification assumption and has been estimated above. Thus, the corresponding leading-log correction can be unambiguously computed. We find:

$$\delta \alpha_3^{-1}(M_Z) \approx -\frac{N_{KK}}{2\pi} \times \left[ \frac{72}{7} \ln \left(1 - \frac{7}{10N_{KK}}\right) - 9 \ln \left(1 - \frac{4}{5N_{KK}}\right) + \frac{9}{7} \ln \left(1 + \frac{1}{2N_{KK}}\right) \right], \quad (43)$$

where we have used the approximations for the KK masses discussed in the previous Section. With $N_{KK} = 60$ we have $\delta \alpha_3^{-1}(M_Z) \approx -0.103$, which is indeed a small correction (as it was expected) and it lies in the range of theoretical uncertainties estimated in [4].

However, large uncertainties in the prediction of strong gauge coupling $\alpha_3(M_Z)$ might originate from the contributions of the Higgs and matter KK modes. The point is that, unlike gauge fields, their local kinetic terms are not controlled by the strong coupling unification assumption. The corresponding $r$-factors are in general arbitrary parameters and thus the KK mode spectrum cannot be reliably computed. A general treatment of possible uncertainties is rather difficult since we have no any independent experimental data to constrain the corresponding KK masses and as well we do not know any principle how to estimate possible size of the corresponding $r$-factors. Instead, just to stress the importance of possible corrections, we give here some numerical examples assuming certain hierarchies for the $r$-factors of different fields.

Consider for example bulk matter fields. Each type of matter fields (with the same orbifold symmetries) at each KK level $n$ are arranged in the full SU(5) multiplets, $\overline{5} + 10$, see Table 1. Thus if their masses are degenerate, they will not give any correction at one loop level as it is the case for the zero modes, i.e. ordinary quarks and leptons in the limit of vanishing masses. This is indeed the case for the standard KK decomposition as well. In the general case, it is clear that $(-, +)$ KK matter fields as well as $(+, -)$ KK matter fields also do not give any extra correction. In the case of $(-, +)$ the KK mass spectrum is standard since for these fields both boundaries at $y = 0$ and $y = a$ are transparent. Also $(+, -)$ fields feel only SU(5)-invariant fixed point at $y = 0$ and thus KK mass spectrum is SU(5)-invariant. However, different KK modes collected in $(+, +)$ and $(-, -)$ fields might have SU(5) non-universal masses at each KK level due to the non-universal local kinetic terms at SU(5)-violating fixed point $y = a$. Once again, $r$-factors for these fields cannot be computed within the effective orbifold field theory and should be treated as independent free parameters. As a representative example of possible uncertainties in the calculation of $\alpha_3(M_Z)$, consider $(-, -)$ matter fields. Assume that $r_\pi$-factors for the SU(2)-doublet fields, $10^e_Q$ and $\overline{10}^e_L$ are approximately the same and large, $r_{(10^e_Q, \overline{10}^e_L)}^{(10^e_Q, \overline{10}^e_L)} \equiv \xi_1 \gg 1$, so that the KK masses can be approximated according to (26), (27), while SU(2)-singlet fields have small $r_\pi$-factors, $r_{(16^e_{Q, E}, \overline{16}^e_{D, D})}^{(16^e_{Q, E}, \overline{16}^e_{D, D})} \equiv \xi_2 << 1$, so that their KK masses are given by (25). In this case we obtain the following correction to $\alpha_3(M_Z)$:

$$\delta \alpha_3^{-1}(M_Z) \approx -\frac{15\eta}{14\pi} \times$$
\[
\ln \left( \pi N_{KK} \sqrt{\frac{\xi_1}{2}} \right) - N_{KK}^{-1} \sum_{n=0}^{N_{KK}-1} \ln \left( \frac{n+1}{n+\frac{1}{2}} \right) + N_{KK} \ln \left( 1 + \frac{\xi_2}{2} \right) \] , \quad (44)
\]

where \( \eta \) is the number of matter generations propagating in the bulk. Taking now, for instance, \( \xi_1 = \frac{1}{\xi_2} = 50 \) and \( N_{KK} = 60 \) as before, we obtain a large correction, \( \delta \alpha_3^{-1}(M_Z) \approx -1.645 \eta \), which pushes up the strong gauge coupling constant to an unacceptable value:

\[
\alpha_3(M_Z) \approx 0.148, \text{ for } \eta = 1, \\
\alpha_3(M_Z) \approx 0.196, \text{ for } \eta = 2. \quad (45)
\]

The corresponding correction to the compactification scale \( M_c \) of (40) is: \( \delta \ln \left( \frac{M_c}{M_Z} \right) \approx 1.38 \eta \), i.e.

\[
M_c \approx 6 \cdot 10^{15} \text{ GeV for } \eta = 1, \\
M_c \approx 2.4 \cdot 10^{16} \text{ GeV for } \eta = 2. \quad (46)
\]

If only the 10-plets are residing in the bulk, as it is favoured by some phenomenological considerations [7], then the correction (44) is reduced by only a factor 0.7 which is still too large. For example, for \( \eta = 2 \) and for the above values of \( x_1 \) and \( x_2 \) one obtains \( \alpha_3(M_Z) \approx 0.162 \). Alternatively, assuming that \( \xi_1 \ll 1 \) and \( \xi_2 \gg 1 \), the rhs of Eq. (44) changes the sign (with \( \xi_1 \leftrightarrow \xi_2 \)) and thus \( \alpha_3(M_Z) \) gets dramatically lowered. We have found large enough corrections for moderate values of \( \eta \)-factors as well.

Certainly, many different numerical examples can be presented which stress the importance of the correct treatment of KK mass spectrum in the presence of local kinetic terms. We are not aiming here to give a more general analysis of possible uncertainties caused by the \( \eta \)-factors of different fields. However, already the above representative numerical examples are clearly warning us that uncertainties in the calculation of the strong gauge coupling are actually large and essentially uncontrollable. Strictly speaking, \( \eta \)-factors (especially for the Higgs and matter fields) can neither be computed or estimated nor can be constrained by the available experimental data. Thus we are lead to the conclusion that one cannot obtain reliable predictions in the framework of orbifold GUTs. Nevertheless, orbifold GUT models could still be viewed as interesting theoretical schemes where many attractive features, such as GUT symmetry breaking, doublet-triplet splitting, suppression of the proton decay etc., could have their counterparts in a more fundamental theory, where, hopefully, the ambiguity related with local kinetic terms can be also resolved. In this respect, it seems that many other GUT models which are excluded in the standard KK decomposition approach might still accommodate the experimental data within large uncertainties expected in general. A particularly interesting question is whether one could have low (intermediate) energy unification in simple orbifold GUTs assuming some particular set of \( \eta \)-factors, which, in turn, can be verified in high energy experiments.
5 Summary and outlook

In this paper we have discussed KK decomposition in higher-dimensional theories with orbifold compactification. We have shown that the standard KK decomposition is not valid due to the presence of kinetic terms localized at the orbifold fixed points. We have also found that the KK mass spectrum and interactions of KK modes are significantly modified and the phenomenology of various orbifold models must be reconsidered. As an illustrative example, we have considered gauge coupling unification in recently proposed realistic 5D orbifold SU(5) GUT. We have shown that large uncertainties in the low-energy predictions of the model appear once the local kinetic terms for Higgs and matter fields are included into consideration and that the predictivity of the model is essentially lost.

Clearly, our analysis of KK decomposition is relevant as well for a wide class of other models extensively discussed in the literature. It will be interesting to see how the supersymmetry and electroweak symmetry breaking as well as collider phenomenology of particular models with large compactification radius \[29\] is affected when the correct KK mode decomposition we have presented in this work is adopted.

Although we have discussed here the case of a single flat extra dimension, local kinetic terms are also generally expected when the extra space-time is not flat, for example, in the case of warped geometry. Thus recently discussed unification in AdS$_5$ \[30\], as well as other phenomenological aspects of the scenario in \[31\], might be significantly modified as well. Finally, it will be certainly interesting to extend this work to the case of general higher dimensions, where some non-trivial features might emerge.

Acknowledgments. We thank Z. Berezhiani, H.P. Nilles and Z. Tavartkiladze for discussions. A.K. would like also to gratefully acknowledge stimulating research atmosphere at Gran Sasso Summer Institute ”New Dimensions in Astroparticle Physics” where this work was finalized.

This work was done by partial financial support of the Academy of Finland under the Project No. 54023.
References

[1] H. Georgi, S. Glashow, *Phys. Rev. Lett.* **32** (1974) 438.

[2] See e.g., P. Langacker, N. Polonsky, *Phys. Rev.* **D47** (1993) 4028;
   J. Bagger, K. T. Matchev, D. Pierce, *Phys. Lett.* **B348** (1995) 443 and references therein.

[3] Y. Kawamura, *Prog. Theor. Phys.* **103** (2000) 613; *Prog. Theor. Phys.* **105** (2001) 999.

[4] G. Altarelli, F. Feruglio, *Phys. Lett.* **B511** (2001) 257.

[5] A. B. Kobakhidze, *Phys. Lett.* **B514** (2001) 131.

[6] L. Hall, Y. Nomura, *Phys. Rev.* **D64** (2001) 055003
   Y. Nomura, *Phys. Rev.* **D65** (2002) 085036.

[7] L. J. Hall, Y. Nomura, *Phys. Rev.* **D65** (2002) 125012; [hep-ph/0205067](http://arxiv.org/abs/hep-ph/0205067).

[8] R. Contino, L. Pilo, R. Rattazzi, E. Trincherini, *Nucl. Phys.* **B622** (2002) 227.

[9] A. Hebecker and J. March-Russell, *Nucl. Phys.* **B625** (2002) 128.

[10] A. Hebecker and J. March-Russell, *Nucl. Phys.* **B613** (2001) 3;
    R. Barbieri, L. Hall, Y. Nomura, [hep-ph/0106190](http://arxiv.org/abs/hep-ph/0106190);
    J. Bagger, F. Feruglio, F. Zwirner, [hep-th/0107128](http://arxiv.org/abs/hep-th/0107128);
    T. Li, *Phys. Lett.* **B520** (2001) 377; *Nucl. Phys.* **B619** (2001) 75;
    T. Asaka, W. Buchmuller, L. Covi, *Phys. Lett.* **B523** (2001) 199.
    L. Hall, H. Murayama, Y. Nomura, [hep-th/0107245](http://arxiv.org/abs/hep-th/0107245);
    L. J. Hall, Y. Nomura, T. Okui, D. R. Smith, *Phys. Rev.* **D65** (2002) 035008.

[11] L. Hall, J. March-Russell, T. Okui, D. R. Smith, [hep-ph/0108161](http://arxiv.org/abs/hep-ph/0108161).
    N. Haba, Y. Shimizu, T. Suzuki, K. Ukai, *Prog. Theor. Phys.* **107** (2002) 151;
    N. Haba, T. Kondo, Y. Shimizu, *Phys. Lett.* **B535** (2002) 271;
    A. Hebecker, J. March-Russell, [hep-ph/0205143](http://arxiv.org/abs/hep-ph/0205143).

[12] M. Chaichian, J. L. Chkareuli, A. Kobakhidze, [hep-ph/0108131](http://arxiv.org/abs/hep-ph/0108131).

[13] T. Watari, T. Yanagida, *Phys. Lett.* **B532** (2002) 252;
    K. S. Babu, S. M. Barr, B. Kyae, *Phys. Rev.* **D65** (2002) 115008;
    K. Hwang, J. E. Kim, *Phys. Lett.* **B540** (2002) 289.

[14] G. R. Dvali, G. Gabadadze, M. Porrati, *Phys. Lett.* **B485** (2000) 208;
    G. R. Dvali, G. Gabadadze, *Phys. Rev.* **D63** (2001) 065007.
[15] G. R. Dvali, G. Gabadadze, M. A. Shifman, *Phys. Lett.* **B497** (2001) 271; S. L. Dubovsky, V. A. Rubakov, *Int. J. Mod. Phys.* **A16** (2001) 4331; M. Chaichian, A. B. Kobakhidze, *Phys. Rev. Lett.* **87** (2001) 171601.

[16] M. Carena, A. Delgado, J. Lykken, S. Pokorski, M. Quiros, C. E. Wagner, *Nucl. Phys. B**609** (2001) 499; G. R. Dvali, G. Gabadadze, M. Kolanovic, F. Nitti, *Phys. Rev.* **D64** (2001) 084004.

[17] H. Georgi, A.K. Grant, G. Hailu, *Phys. Lett.* **B506** (2001) 207.

[18] H. C. Cheng, K. T. Matchev, M. Schmaltz, hep-ph/0204342, hep-ph/0205314.

[19] M. Carena, T. M. Tait, C. E. Wagner, hep-ph/0207056.

[20] B. Kyae, hep-th/0207272.

[21] M. Chaichian, A. B. Kobakhidze, M. Tsulaia, *Phys. Lett.* **B505** (2001) 222.

[22] Y. Nomura, D. R. Smith, N. Weiner, *Nucl. Phys.* **B613** (2001) 147.

[23] S. Groot Nibbelink, H. P. Nilles, M. Olechowski, *Phys. Lett.* **B536** (2002) 270; hep-th/0205012.
D. Marti, A. Pomarol, hep-ph/0205034.
K. A. Meissner, H. P. Nilles, M. Olechowski, hep-th/0205166.

[24] A. Hebecker, *Nucl. Phys.* **B632** (2002) 101; A. Hebecker, J. March-Russell, *Phys. Lett.* **B539** (2002) 119; F. Paccetti Correia, M. G. Schmidt, Z. Tavartkiladze, hep-ph/0204080.

[25] For recent model of bigravity within the brane world scenario, see I. I. Kogan, S. Mouslopoulos, A. Papazoglou, *Phys. Lett.* **B501** (2001) 140.

[26] Review of Particle Physics, Particle Data Group, *Euro. Phys. J.* **C15** (2000) 1.

[27] G. R. Dvali, *Phys. Lett.* **B287** (1992) 101; K. S. Babu, S. M. Barr, *Phys. Rev.* **D48** (1993) 5354; J. Hisano, T. Moroi, K. Tobe, T. Yanagida, *Phys. Lett.* **B342** (1995) 138; I. Gogoladze, A. Kobakhidze, *Phys. Atom. Nucl.* **60** (1997) 126; Z. Berezhiani, Z. Tavartkiladze, M. Vysotsky, hep-ph/9809301; Z. Chacko, R. N. Mohapatra, *Phys. Rev.* **D59** (1999) 011702; Z. Berezhiani, I. Gogoladze, A. Kobakhidze, *Phys. Lett.* **B522** (2001) 107; Y. Achiman, M. Richter, *Phys. Lett.* **B523** (2001) 304; K. S. Babu, S. M. Barr, *Phys. Rev.* **D65** (2002) 095009.
[28] For a recent discussion see e.g., H. Murayama, A. Pierce, *Phys. Rev.* **D65** (2002) 055009.

[29] I. Antoniadis, *Phys. Lett.* **B246** (1990) 377;
A. Pomarol, M. Quirós, *Phys. Lett.* **B438** (1998) 255;
I. Antoniadis, S. Dimopoulos, A. Pomarol, M. Quirós, *Nucl. Phys.* **B544** (1999) 503;
R. Barbieri, L.J. Hall, Y. Nomura, *Phys. Rev.* **D63** (2001) 105007;
T. Appelquist, H. C. Cheng, B. A. Dobrescu, *Phys. Rev.* **D64** (2001) 035002;
T. G. Rizzo, *Phys. Rev.* **D64** (2001) 095010;
C. Macesanu, C. D. McMullen, S. Nandi, *Phys. Rev.* **D66** (2002) 015009.

[30] A. Pomarol, *Phys. Rev. Lett.* **85** (2000) 4004;
L. Randall, M. D. Schwartz, *JHEP* **0111** (2001) 003; *Phys. Rev. Lett.* **88** (2002) 081801;
K. Choi, H. D. Kim and I. W. Kim, [hep-ph/0202257](http://arxiv.org/abs/hep-ph/0202257), [hep-ph/0207013](http://arxiv.org/abs/hep-ph/0207013).
W. D. Goldberger, I. Z. Rothstein, [hep-th/0204156](http://arxiv.org/abs/hep-th/0204156), [hep-th/0208060](http://arxiv.org/abs/hep-th/0208060).
K. Agashe, A. Delgado, R. Sundrum, [hep-ph/0206099](http://arxiv.org/abs/hep-ph/0206099).

[31] L. Randall, R. Sundrum, *Phys. Rev. Lett.* **83** (1999) 3370; *Phys. Rev. Lett.* **83** (1999) 4690.