Solar active regions: a nonparametric statistical analysis

J. Pelt\textsuperscript{1}, M. J. Korpi\textsuperscript{2}, and I. Tuominen\textsuperscript{2}

\textsuperscript{1} Tartu Observatory, 61602 Tõravere, Estonia
\textsuperscript{2} Observatory, PO Box 14, FI-00014 University of Helsinki, Finland

Received —; accepted —

ABSTRACT

Context. The sunspots and other solar activity indicators tend to cluster on the surface of the Sun. These clusters very often occur at certain longitudes that persist in time. It is of general interest to find new and simple ways to characterize the observed distributions of different indicators and their behaviour in time.

Aims. In the present work we use Greenwich sunspot data to evaluate statistical but not totally coherent stability of sunspot distribution along latitudes as well as longitudes. The aim was to obtain information on the longitudinal distribution of the underlying spot-generating mechanism rather than on the distribution and migration of sunspots or sunspot groups on the solar surface. Therefore only sunspot groups were included in the analysis, and only the time of their first appearance was used.

Methods. We use simple nonparametric approach to reveal sunspot migration patterns and their persistency.

Results. Our analysis shows that regions where spots are generated tend to rotate differentially as the spots and spot groups themselves do. The activity areas, however, tend to break down relatively fast, during 7-15 solar rotations.

Conclusions. This study provides a challenge for solar dynamo models, as our results are consistent with the presence of a non-axisymmetric spot-generating mechanism experiencing differential rotation (known as phase mixing in dynamo theory). The new nonparametric method introduced here, completely independent of the choice of the longitudinal distribution of sunspots, was found to be a very powerful tool for spatio-temporal analysis of surface features.

Key words. Sun: activity – Sun: magnetic fields – sunspots – methods: statistical

1. Introduction

Modern observations of the Sun are so rich in detail that astronomers are eventually struck by “embarrassment of riches”. When spatio-temporal properties of the smaller features - say spots, flares \textit{etc} - are treated with well-established vigour, the analysis of spatially larger or temporarily longer patterns is very complicated. Even the nomenclature of the phenomena is not well-established - for instance the time-space clusters of the local phenomena can be called as “active longitude” (Losh 1938, Vitinskij 1969), “Sonnenfleckenherd” (Becker 1955), “active region” (Bumba & Howard 1965), “sunspot nest” (Castenmiller et al. 1986) , “complex of activity” (Gaizauskas et al. 1983) or “hot spot” (Bai 1988). There is, in addition, a problem with proper definition of such extended patterns.

It is generally thought that the tracers of solar activity - sunspots, flares \textit{etc} - are randomly generated manifestations of the larger scale mean magnetic field of the Sun generated by a hydromagnetic dynamo process. An analogy with a submerged animal blowing out bubbles is quite appropriate in this context (see Bai 2003). What can we tell about the swimming speed and size of the animal, if only random bubbles are observable? How deep in water is the animal?

The answers to these kind of questions depend very much on the method of analysis used. Very often subjective judgement is involved, either through steps of visual processing or through involvement of freely chosen procedure parameters (bin sizes, zone widths, detection limits, preselection criteria \textit{etc}).

From the statistical analysis point of view we can divide the previously used methods along two lines: how the input data is transformed before computing final statistics and what kind of statistics are used. Some typical but random examples:

- Aggregated data (daily Wolf numbers) and correlation analysis (Bogart 1982),
- Raw heliographic longitudes and longitude-wise binning (Trotter & Billings 1962, Warwick 1965),
- Transformed (using trial rotation velocity) longitudes and $\chi^2$ statistic (Bai 1987),
Transformed (using latitude-dependent rotation velocities) longitudes and pattern matching (Usoskin et al. 2003; Pelt et al. 2006 hereafter PBKT),

Spherical harmonic decomposition and time series analysis of mode amplitudes, phases and phase-walks (Juckett 2003).

The third important aspect of the analysis is time coverage of the observations. It is quite easy to find recurrent patterns in short time series, but coherence tends to break down very fast for longer datasets.

In this paper we in a certain sense try to return to the square one, back to the very basics. Using very simple considerations and avoiding all freely chosen parameters we try to get answers to the questions:

- Is there a tendency for surface elements to occur at certain longitudes that persist over time?
- How this persistence and differential rotation of the surface elements are connected?
- How long typical correlations in activity persist?

Our aim is not so much to perform another statistical analysis of the well-known and already extensively analysed data, but to introduce a new nonparametric method of analysis involving no physical, geometrical or statistical prior assumptions. In Section 2 we introduce our method of analysis, in Section 3 we present the results obtained for the Greenwich sunspot data set, and finally in Sections 4 and 5 we discuss our results in the light of previous statistical analyses.

2. Method of analysis

2.1. Nonparametric method

Let us assume that we have two sets of longitudes: \( \lambda_i^{(1)}, i = 1, \ldots, N \) and \( \lambda_j^{(2)}, j = 1, \ldots, M \). Their values belong to the interval \( 0^\circ \leq \lambda \leq 360^\circ \) and we assume that \( N \leq M \) (if otherwise, we can always swap the sets). We want to characterize the similarity or the difference between the two longitude distributions somehow. The general theory of directional measurements is considered in mathematical statistics (see for instance the latest monograph by Mardia & Jupp 2000 and references therein), but here we need a more specific method, namely one without any underlying statistical assumptions or parametric models for the distributions involved.

We propose the following very simple nonparametric method: The circular distance between two longitudes \( \lambda_k \) and \( \lambda_l \) we define as usually done

\[
\Delta \lambda_{k,l} = \min(|\lambda_k - \lambda_l|, 360^\circ - |\lambda_k - \lambda_l|).
\]  

Let us take a particular longitude \( \lambda_k^{(1)} \) from the first set.

Among the longitudes of the second set there is always a value whose circular distance from the selected value is the smallest, let us denote this distance as \( \Delta \lambda_k \). All together we can compute \( N \) such values - for each longitude in the first set. Now we select the longest distance among them and denote it simply as \( \Delta \). It is quite clear that, in the particular case when the first set is just a subset of the second one, \( \Delta = 0 \). If the sets differ, then \( \Delta > 0 \). In principle such max-min distance between two longitude sets is already a useful statistics; its full power, however, is revealed if we properly normalize it.

For a particular set sizes \( N \) and \( M \) we can compute the mathematical expectation of \( \Delta \) for completely random distributions of longitudes in both sets. Let us denote this expectation as \( \bar{\Delta} \). Our final statistics which measures statistical distance between the two sets of longitudes is then:

\[
D = \frac{\Delta}{\bar{\Delta}}.
\]  

If we want to stress that the distance \( D \) is computed for two particular indexed longitude sets, say for index \( n \) (\( N \) longitudes) and index \( m \) (\( M \) longitudes) then we use notation

\[
D(n, m) = \frac{\Delta_{n,m}}{\bar{\Delta}_{N,M}}.
\]

The mathematical expectations \( \bar{\Delta}_{N,M} \) depend only on the integers \( N \) and \( M \), and can be pretabulated. In our calculations we used approximations obtained from randomly generated longitudes for 10000 statistically independent runs.

It is quite obvious that for absolutely random pairs of longitude sets our distance will have a value around 1. For weakly correlated sets values are less than one, and values higher than 1 can occur when the longitude sets involved are constrained in a certain way due to which they cannot form all the patterns which occur for randomly generated sets. In the case of sunspot groups, for instance, the distributions are constrained by group sizes. Randomly generated points can fall arbitrarily close, which is not true for sunspot longitudes, because for them the group centres are separated by definition.

Having now the statistic to measure distances between different distributions of longitudes we can go further. For a sequence of longitude sets we can compute a mean distance between neighbouring sets:

\[
\bar{D} = \frac{1}{K - 1} \sum_{k=1}^{K-1} D(k, k + 1),
\]

where \( K \) is the number of the sets. We can also investigate how the distance depends on the mutual positions of particular sets:

\[
\bar{C}(l) = \frac{1}{K - l} \sum_{k=1}^{K - l} D(k, k + l).
\]

Eventually \( \bar{D} = \bar{C}(1) \). The statistic \( \bar{D} \) allows us to investigate rotational properties of the sunspot groups and statistic \( \bar{C}(l) \) will be used to estimate how persistent the longitudinal correlations are.
2.2. Rotation and frames

Heliographic longitudes are defined using the so-called Carrington frame, which rotates against fixed stars with the exact period of $P_C = 25.38$ days. The mean rotation period if observed from the Earth is $P_0 = 27.2753$ days. The Carrington frame is a formal construct and real features on the Sun need not to follow it exactly.

Let us fix a certain longitude $\lambda^{(C)}$ of a particular persistent feature on the Sun rotating with the Carrington angular velocity. Then its longitude for different Carrington rotations $i$ will be fixed: $\lambda^{(C)} = \left[ \lambda^{(C)} + i \times 360^\circ \right] = \lambda^{(C)}$, angular brackets denoting here and below reduction to the interval $(-360^\circ, 360^\circ)$. Because the angular velocity of the Carrington frame is $\Omega_C = \frac{360^\circ}{P_C}$ degrees per day we can rewrite cycle dependent sequence of longitudes as

$$\lambda^{(C)}_i = \left[ \lambda^{(C)} + i \times \Omega_C P_C \right], \ i = 0, 1, \ldots$$

(6)

The actual angular velocity of an arbitrary feature on the Sun need not to be exactly $\Omega_C$. Let longitude of the first occurrence of such feature be $\lambda$. Then cyclic reoccurrences of it can be described using a correcting term $\Delta \Omega$:

$$\lambda_l = \left[ \lambda + i \times (\Omega_C + \Delta \Omega) P_C \right], \ i = 0, 1, \ldots$$

(7)

The corrected frame rotates against Carrington frame with angular velocity $\Delta \Omega$ degrees per day. For convenience we introduce also notion for sidereal angular velocity of the accelerated or decelerated frames $\Omega = \Omega_C + \Delta \Omega$.

In what follows, we measure the angular velocity in degrees per day, latitude in degrees, and periods in days, and give the values in these units.

2.3. Algorithms

All ingredients of the method of analysis described, we can now formulate our basic algorithms.

As an input data we use a set of time tagged longitudes $t_i, \lambda_i, l = 1, \ldots, L$, amounting to $L$ pairs of data. Using the time points $t_i$ we divide records into subintervals with the length $27.2753$ (Carrington rotations). This procedure is not absolutely exact because the observation timing depends on the somewhat eccentric orbit of the Earth. Fortunately the errors involved are small and we can ignore them. From the point of generality and objectivity our choice is quite natural. Historical observations are all done from Earth and consequently the features can be observed only half a time. However, during the Carrington rotation we can record what happens at all longitudes. As far as timing is considered, due to the rotation some processes can actually start earlier than observed. This excludes short-living processes (shorter than Carrington rotation) from our analysis.

It is also possible to divide observations into longer subintervals. Then we increase statistical stability of our estimates (more observations in subsets) but loose resolution in time. We consider time step with the length of one Carrington rotation to be optimal.

We assume that the features on the surface of the Sun rotate with angular velocity which is different from the Carrington velocity $\Omega_C$. For a certain trial angular velocity $\Omega$ and for each Carrington cycle $i$ we can compute longitude corrections:

$$\lambda_i = i \times \Delta \Omega P_C = i \times (\Omega - \Omega_C) P_C, \ i = 0, 1, \ldots$$

(8)

By subtracting rotation number dependent corrections from measured longitudes and properly reducing results to interval $(0, 360)$ we build transformed longitudes:

$$\lambda^{(T)} = [\lambda_i - \lambda_i].$$

(9)

They can be analysed using the statistics introduced above. We can also say that we transform longitudes in the Carrington frame into longitudes in the comoving frame. The frame rotation velocity $\Omega$ is a free parameter of the procedure. We expect that the distributions of the transformed longitudes depend on $\Omega$ and the highest level of correlation in the longitude distribution will show up as a minimum of the distance statistic $D$.

First we compute how the mean distance between neighbouring rotations $D$ depends on angular velocity $\Omega$. Then we can use the best value (producing the highest level of correlation) for angular velocity to compute how distances depend on the interval between rotations (using the statistic $C(t)$).

3. Data analysis

Here we describe how we apply the presented statistical method to study the particular case of sunspots.

The most comprehensive (in time) compilation of sunspot data was downloaded from the Science at NASA web site\textsuperscript{1}. The same minor corrections as in PBKT were introduced. In this paper we used the full data set covering years 1874-2008, or in terms of Carrington rotations, the rotations 275-2074. From all the data base records we chose only sunspot groups, leaving out single spots. In this way all the entries in the final set have equal statistical weight. For each sunspot group we selected only the record of its first occurrence. This is important aspect of our analysis. We do not track sunspots as they rotate, but are interested in the movement of the underlying spot-generating structures. The final compiled data sets cover rotations 275-2074 with 16053 records for the Northern hemisphere and 275-2071 rotations with 15858 records for the Southern hemisphere; the compiled data sets are available on the web\textsuperscript{2}.

3.1. Mean angular velocity

For the first approximation we can assume that the mechanism generating the sunspots rotates as a rigid body. Then we can measure its angular velocity using $D$ statistic by comparing different longitude correction schemes.

\textsuperscript{1} http://solarscience.msfc.nasa.gov/greenwch.shtml
\textsuperscript{2} http://www.aai.ee/~pelt/soft.htm
and choosing the one that produces the lowest value of the statistic. The results can be best illustrated by displaying \( \bar{D} \) as a function of \( \Omega \) - the actual sidereal angular velocity of the frame. In Fig. 1 such functions are displayed for both solar hemispheres. As we see both curves show very clear and indicative minima. The absolute minimum for the Northern data is positioned at 14.348 and for Southern data at 14.403. The curves themselves are somewhat fluctuating so that we found useful to estimate the minima using local fits of the fifth-degree polynomials also. The resulting values from the fitting procedure are 14.365 ± 0.001 and 14.409 ± 0.002 for North and South, deviating from the absolute minimum values by 0.017 and 0.006, respectively.

From these results we can see that the Southern part of the mechanism tends to rotate slightly faster. However the difference is too small, especially if to take into account the roughness of our method, to be conclusive.

### 3.2. Differential rotation

Sunspots and other activity indicators rotate with different angular velocities at different latitudes. By tracking particular objects in time it is possible to build a smooth curve to reveal the overall pattern of such differential, latitude dependent, rotation. Our statistic \( \bar{D}(\Omega) \) does not track single sunspots or the actual movement of sunspot groups, as we include only the first appearance of the sunspot groups. This way we can check whether the spot-generating mechanism itself rotates differentially or not. For that purpose we divided the observed groups into four subsets along latitudes (per hemisphere) and computed \( \bar{D}(\Omega) \) for every group. The latitude limits for the subsets where chosen to make them as equal in size as possible. The typical curves are shown in Fig. 2. The exact determination of the minima for the curves is somewhat complicated. If we locally fit polynomials into the curves as we did above, we can get estimates with high formal precision (0.001-0.002). The differences between the absolute numerical minima and the fitted minima, however, can be quite large (up to 0.045). Therefore we can claim that the probable statistical errors of the obtained minima are around 0.02 degrees per day.

The full set of the absolute minima for all the eight curves is given in Table 1. In Fig. 3 the traditional least squares fit of the obtained angular velocities \( \Omega_i \) is presented.

![Fig. 1. Statistic \( \bar{D}(\Omega) \) for the full dataset of the Northern hemisphere (thick line) and for the full dataset of the Southern hemisphere (thin line). Minima indicate the best fitting comoving velocities.](image)

![Fig. 2. Statistic \( \bar{D}(\Omega) \) for the latitude strip 0.0 – 9.8. Thick line - Northern hemisphere, thin line Southern hemisphere. Minima indicate the best fitting comoving velocities.](image)

![Fig. 3. Differential rotation curve. Thick curve - fit for Southern data, thin curve - full fit.](image)
sented. We estimated parameters $A$ and $B$ for a simple model:

$$\Omega_i = A + B \sin^2(\theta_i),$$

(10)

where $\theta_i$-s are mid-latitudes of the eight belts. The estimated parameters were $A = 14.438 \pm 0.035$ and $B = -1.13 \pm 0.17$ correspondingly. For the Southern hemisphere the latitude dependence of the angular velocity is monotonically decreasing polewards and resembles quite well the curves obtained from sunspot tracking. For the Northern hemisphere the latitudinal behaviour is not monotonous, and the two latitude bands nearest to the equator are somewhat slower than expected. If we used only Southern velocities for the fitting procedure, the resulting fit was better with parameters $A = 14.490 \pm 0.001$ and $B = -1.276 \pm 0.044$. At the moment we do not try to find any statistical or physical interpretation of this result; more importantly to us, the results obtained in this section clearly demonstrate that the simple nonparametric method can be successfullly used to study the differential rotation of the solar activity tracers.

### 3.3. Break down times

The results of the previous section clearly show that the longitudinally concentrated spot-generating mechanism is subject to differential rotation. Kinematic mean-field dynamo theory predicts (e.g. Krauskopf & Rädler 1980) that in the parameter regime where nonaxisymmetric dynamo modes can be excited, the nonaxisymmetric modes are non-oscillatory and rotate rigidly with angular velocity different from the overall rotation period. The phenomenon of phase-mixing, i.e. the nonaxisymmetric modes becoming affected by differential rotation, is against these predictions; our results, however, are consistent with the phase-mixing effect.

Let us now try to quantify the effect of the differential rotation on the nonaxisymmetric structures by calculating the characteristic time needed to break down a longitudinally elongated structure for different latitude strips. Using the estimated $B$ values from Eq. (10) we can define a breakdown time for the strip of latitudes $(\theta_1, \theta_2)$:

$$T_{bd} = \frac{\Delta \phi}{B \sin^2 \theta_1 - \sin^2 \theta_2},$$

(11)

where $\Delta \phi$ is the phase distance over which the hypothetical longitudinal pattern can be regarded to be destroyed. A reasonable value for the parameter $\Delta \phi$ comes from a following simple observation. Let us assume that at the latitudes $\theta_1$ and $\theta_2$ we have $K$ observations. Let the longitudes coincide for the starting point in time. The statistics introduced above are based on finding the nearest “neighbours”. In our case for every observation at one latitude there is exactly one “neighbour” at the other. To break the ties between the neighbours we need to rotate the observation at the other latitude. That means we need to have relative shifts which are longer than half the distance between two consecutive observations or formally $\Delta \phi = 0.5 \times 360^\circ/\bar{K}$, where $\bar{K}$ is a mean number of observations per Carrington rotation. Applying all this to the same latitude intervals as in Sect. 3.2 we obtain the break up times listed in Table 3.2.

### 3.4. Decorrelation time

So far we have demonstrated that the sunspot group distributions along longitudes for sequential Carrington rotations are correlated. We also computed the best fitting mean angular velocity $\bar{\Omega}$ for several latitude strips, and as a result found out a clear differential rotation pattern. Next we are interested in estimating the approximate life-times of the correlated features found from the sunspot data. For that purpose we use the obtained mean angular velocities for each latitude strip and compute $\bar{C}(l)$ curves to learn how fast the correlation between appropriately rotated longitude sets fades off.

The results of this kind of analysis are displayed on Figs. 4 to 11. On these plots the thick lines indicate mean level over the last 25 points, which gives an asymptotic level of convergence. As can well be seen from the plots,
the inherent scatter of the $\bar{C}(l)$ curves is quite large; this is due to the physical variability of the activity level and the roughness of the statistic. Therefore it is hard to fix the point where the asymptotic level is achieved. Some aspects of the curves, however, are quite indicative. First – the shortest decorrelation time is obtained for the highest latitudes. This is obviously because the width (in degrees) of the high altitude strips is wider and covers very differently rotating spot groups. Secondly – the strips nearest to the equator show the longest correlations. We can quite safely claim that certain level of correlation is visible up to time span of 15 rotations.

Comparing the estimated decorrelation times for different latitude strips with the break-down times from Table 3.2 we can see that for lower latitudes the decorrelation times obtained from our analysis are much shorter.
4. Discussion

To put our results into a general context we will compare them with a sample of previous analyses.

Large amount of the solar variability research is not based on a full set of sunspot observations but on some aggregated form of data. Most typically the daily Wolf sunspot numbers are used. For instance Bogart [1982] analysed these numbers using autocorrelation functions and power spectra. The major results were quite similar to ours - the rotation period around 27 days was detected and persistence of activity zones was claimed to be of the order of 10 solar rotations. In principle correlation functions and power spectra can be considered to be parameter-free statistics. The aggregated nature of the Wolf numbers, however, does not allow to analyse latitude dependence of the active clusters.

There is a number of analyses which use longitudinal phase binning of the surface features. For instance in a series of papers Bai [1987,1988] used comoving frames (as in our work) to seek rotation velocities which enhance statistical contrast of longitudinal distribution of solar flares. The transformed longitudes for each trial rotation velocity were binned into 12 bins and variance of obtained distributions was computed. The possibility of differential rotation was not taken into account. To study the persistence of particular active regions were visually tracked and displayed as “family trees”. In describing his results the author proposed a general scheme to characterize hierarchical patterns of solar activity:

- single events (sunspots, flares),
- active regions,
- activity complexes,
- active zones.

The lifetimes of the activity centres increase with hierarchy - from days to several years. In a later work Bai [2003] used Rayleigh-type statistics to analyse transformed longitudes. He computed standard spectra which are sensitive to unimodal distributions and spectra which are sensitive to bimodal distributions. As a result he found that some characteristics of the longitude distributions are rather persistent in time, even up to decades. Our results describe average behaviour of the solar activity and consequently some long-lived elements do not have strong influence, as they are mixed with other elements whose lifetimes are shorter. It should be also stressed that $D$ and $\tilde{C}(l)$ statistics do not depend on any assumption about modality of the underlying variability. All the “modes” are automatically accounted for.

Probably the most popular method to study the kinematics of the solar surface features is a standard power spectrum analysis and its variants (just an example - Temmer et al. 2004, Giordano 2008). This kind of analysis can be applied to latitude strips and in this way the differential rotation can be taken into account. From the first sight Fourier analysis seems to be essentially nonparametric. However, the fact that it uses single harmonics as base functions prescribes certain form of preferred activity distributions. The results of the Fourier methods are often given as a list of certain periods which show up in power spectra or on wavelet plots. The periodicity claim itself is quite a strong statement, as it is often very hard to find physically solid timing mechanisms for periods which strongly differ from the obvious one - that of solar rotation.

We want to stress here that in the proposed statistical method no assumptions about the particular form of the activity indicator distributions are made. Even more - the statistics $D$ and $\tilde{C}(l)$ are not seeking certain clusters or other kind of patterns, they are just used to check whether the “birth places” of surface elements are correlated or not. This makes the new method somewhat similar to the method of “family trees” (Bai 2003) or longitude-time diagrams (Brouwer & Zwaan 1990).

The literature about the longitudinal distribution of solar activity indicators is so wide that it is not reasonable to compare our results with all of them. It suffices to state that the general patterns revealed so far are quite similar to those described above. The major shortcomings of the previously used methods include the dependence of the results on some prefixed parameters or on the choice of a particular distribution model.

5. Conclusions

When introducing a new method to analyse solar activity patterns we started from certain methodological principles:

- Input data must be homogeneous, comprehensive and cover as long time base as possible.
- The analysis method must be free from any prefixed constants.
- The method must not depend on the model of the activity indicator distributions (unimodal, bimodal etc.).
- The computations must be as simple as possible.

The results obtained using the new method can be ranked using these underlying principles. We start from the most evident and methodologically “clean” facts and proceed towards the statements which can be doubted or refined using additional devices.

- The distribution of sunspots is determined by the underlying large-scale mechanism which is more persis-
tent than sunspots themselves. This shows up as a tendency of new sunspots to occur near the places where the previous sunspots were observed.

- The mean rotation velocity of the large-scale features for the Northern hemisphere is 14.35 ± 0.02 and for the Southern hemisphere 14.40 ± 0.02.
- The rotation velocities for Northern and Southern hemispheres differ slightly. Consequently both velocities manifest certain statistical averages and do not suggest a strong meridional coupling.
- The large-scale patterns of activity take part in differential rotation. The differential rotation curve is somewhat shallower if to compare with curves obtained from sunspot tracking (see Zapallà & Zuccarello [1991]).
- The differential rotation for the Southern hemisphere is more similar to that obtained from sunspot analysis. Differential rotation curve for the Northern hemisphere deviates from the general rotation law especially near the equator.
- The strong tendency for the spot groups to cluster on certain longitude dies off with time. The longest observable correlations can reach 15 Carrington rotations.
- The correlations between rotations are more pronounced for lower latitudes.
- The observation of the spot-generating mechanism being affected by differential rotation is suggestive of phase mixing occurring in the solar convection zone; such a phenomenon is not predicted by conventional mean-field dynamo theory.

The set of formulated results is impressive if to take into account the simplicity of the analysis method used.

Acknowledgements. Part of this work was supported by the Estonian Science Foundation grant No. 6813 and Academy of Finland grant No. 112020.

References
Bai T., 1987, ApJ, 314, 795
Bai T., 1988, ApJ, 328, 860
Bai T., 2003, ApJ, 585, 1114
Becker U., 1955, Z. Astrophys., 37, 47
Bogart R.S., 1982, Solar Physics, 76, 155
Brouwer M.P., Zwaan C., 1990, Solar Physics, 129, 221
Bumba V., Howard R., 1965, ApJ, 141, 1502
Castenmiller M.J.M, Zwaan C., van der Zalm E.B.J., 1986, Solar Physics, 105, 237
Gaizauskas V., Harvey K.L., Harvey J.W., Zwaan C., 1983, ApJ, 265, 1056
Giordano S., Mancuso S., 2008, ApJ, 688, 656
Juckett D., 2003, A&A, 399, 731
Krause F., Rädler K.-H., 1980, Mean-field magnetohydrodynamics and dynamo theory, Akademie-Verlag Berlin
Losh N.M., 1938, Publ. Observ. Michigan, 7, 127
Mardia K.V., Jupp P., 2000, Directional Statistics (2nd edition), John Wiley and Sons Ltd.
Pelt J., Brooke J., Korpi M., Tuominen I., 2006, A&A, 460, 875 (PBKT)
Tempel M., Veronig A., Rybák J., Brajša R., Hanslmeier A., 2004, Solar Physics, 221, 325
Trotter D.E., Billings D.E., 1962, ApJ, 136, 1140
Usoskin I.G., Berdyugina S.V., Poutanen J., 2005, A&A, 441, 347
Vitinskij Ju.I., 1969, Solar Physics, 7, 210
Warwick C.S., 1965, ApJ, 141, 500
Zapallà R.A., Zuccarello F., 1991, A&A, 242, 480