Top quark threshold production in $\gamma\gamma$ collisions: current theoretical issues

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Abstract

The top quark-antiquark pair threshold production at future High Energy Photon Colliders is considered in view of the recent advances in the theoretical description of the nonrelativistic heavy quark dynamics.

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The future Photon Colliders provide an opportunity of experimental study of high energy $\gamma\gamma$ interactions which can be used for the top quark-antiquark pair production. Using the laser backscattering method one can obtain $\gamma\gamma$ colliding beams with the energy and luminosity comparable to those in $e^+e^-$ annihilation [1]. Theoretical study of the top quark-antiquark pair production near the two-particle threshold [2] is based on the key observation that the relatively large electroweak top quark width $\Gamma_t$ and characteristic scale of the nonrelativistic Coulomb dynamics $\alpha_s^2 m_t$ are considerably larger than $\Lambda_{QCD}$ and serve as an effective infrared cutoff for long distance nonperturbative strong interaction effects. This makes perturbative QCD applicable for the theoretical description of the threshold top quark physics. At the same time numerically $\Gamma_t \sim \alpha_s^2 m_t$ and the Coulomb effects are not completely dumped by the non-zero top quark width that should be properly taken into account.

High quality experimental data that can be obtained in the experiments along with a very accurate theoretical description of them make the processes of top-antitop pair threshold production a promising place for investigating quark-

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gluon interactions. This investigation concerns both general features of interaction and precise quantitative properties such as the determination of numerical values of the strong coupling constant $\alpha_s$, the top quark mass, and the top quark width. The main features of $\gamma\gamma \rightarrow t\bar{t}$ threshold production are rather similar to the properties of the $e^+e^- \rightarrow t\bar{t}$ process. However, the strong interaction and relativistic corrections are different for them. Therefore a simultaneous analysis of these two processes extends possibilities of studying fine details of the top quark threshold dynamics. Moreover, the $S$ and $P$ partial waves of the final state top quark-antiquark pair produced in $\gamma\gamma$ collisions can be separated by choosing the same or opposite helicities of the colliding photons. This gives an opportunity of direct measurement of the $P$ wave amplitude which is strongly suppressed in the threshold region in comparison to the $S$ wave one and provides us with an additional independent probe of the top quark interactions.

Recently an essential progress has been achieved in study of the top quark-antiquark pair production in $e^+e^-$ annihilation reviewed in [3]. This analysis was extended to $\gamma\gamma$ collisions in the next-to-leading order (NLO) [4] and next-to-next-to-leading order (NNLO) [5] of the perturbative and relativistic expansion. Below we outline the approach used in these papers and summarize the obtained results. We consider the normalized total cross section

$$ R(s) = \frac{\sigma(\gamma\gamma \rightarrow t\bar{t})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}, \quad (1) $$

which can be decomposed into the sum $(R^{++} + R^{+-})/2$ of the cross sections $R^{++}$ and $R^{+-}$ for the colliding photons of the same and opposite helicity respectively. Near the threshold the heavy quarks are nonrelativistic so that one may consider the quark velocity $v$ as a small parameter. An expansion in $v$ may be performed directly in the Lagrangian of QCD by using the effective theory framework of nonrelativistic QCD (NRQCD) [6]. In the effective theory the dynamics of the heavy quarks is governed by the nonrelativistic Schrödinger equation and by their multipole interaction to the dynamical ultrasoft gluons. The corrections from harder scales are contained in the Wilson coefficients leading to an expansion the $\alpha_s$ as well as in the higher-dimensional operators corresponding to the expansion in $v$. In the threshold region the cross sections are determined by the imaginary part of the correlators of the nonrelativistic vector/axial quark currents which can be related to the nonrelativistic Green function $G(x,y,E)$ and its derivatives at the origin

$$ R^{++}(s) = \frac{24\pi q^4 N_c}{m_t^2} \left( C^{++}(\alpha_s) - \frac{E}{3 m_t} + \ldots \right) \text{Im}G(0,0,E), $$

$$ R^{+-}(s) = \frac{32\pi q^4 N_c}{m_t^2} \left( C^{+-}(\alpha_s) + \ldots \right) \partial^2_{xy} \text{Im}G(x,y,E)|_{x,y=0}, \quad (2) $$
where \( N_c = 3 \), \( q_t = 2/3 \), \( E = \sqrt{s} - 2m_t \) is the energy of a quark pair counted from the threshold and ellipsis stand for the high order relativistic corrections. A symbolic notation \( \partial_{xy}^2 \) is used for the operator that singles out the \( P \) partial wave of \( G(x, y, E) \). Note that the cross section \( R^{++} \) is suppressed by the factor \( v^2 \) in comparison to \( R^{++} \). Therefore only the NLO corrections to \( R^{+-} \) cross section is of practical interest. The nonrelativistic Green function satisfies the Schrödinger equation

\[
(H_C + \Delta H - E) G(x, y, E) = \delta^{(3)}(x - y),
\]

where \( H_C \) is the Coulomb Hamiltonian and \( \Delta H \) stands for the high order corrections in \( \alpha_s \) and \( v \). The leading order approximation for the nonrelativistic Green function is obtained with the Coulomb Hamiltonian and sums up the singular \( (\alpha_s/v)^n \) Coulomb terms to all orders. The higher order terms in the effective Hamiltonian are known up to NNLO including \( O(\alpha_s^2) \) perturbative corrections [7]. Corresponding corrections to the Coulomb Green function have been obtained in [8–11] and to its derivative in [4]. The ultrasoft gluons do not contribute in NNLO. The perturbative Wilson coefficients \( C^{++} \) and \( C^{+-} \) in Eq. (2) are known in NLO. To complete the NNLO analysis of the \( R^{++} \) cross section the NNLO contribution to the coefficient \( C^{++} \) has to be computed. Now only the \( O(\alpha_s^2) \) anomalous dimension [5] and the Abelian part [12] of this coefficient are available. Because the threshold dynamics is nonrelativistic and is rather insensitive to the hard momentum details of top quark decays the instability of the top quark can be implemented simply by the complex energy shift \( E \to E + i\Gamma_t \) in Eq. (3). This accounts for the leading imaginary electroweak contribution to the leading order NRQCD Lagrangian and the most essential features of the physical situation are caught within this approximation. However, in the case of \( P \) wave production the above prescription is not sufficient for a proper description of the entire effect of the non-zero top quark width and more thorough analysis is necessary (see [4] for detailed discussion).

To summarize, the complete NLO analytical expressions for the \( R^{++} \) and \( R^{+-} \) cross sections are known and a bulk of NNLO corrections to the total cross section of unpolarized or equally polarized photons is available. The cross sections \( R \) in NNLO and \( R^{+-} \) in NLO are plotted on Fig. 1. and Fig. 2. respectively as the functions of energy for several values of the “soft” normalization point \( \mu \) of the strong coupling constant of the nonrelativistic Coulomb problem. In the numerical analysis we neglect the unknown non-logarithmic \( O(\alpha_s^2) \) part of the Wilson coefficient \( C^{++} \). Taking into account the result for the similar coefficient in the analysis of the photon mediated \( t\bar{t} \) production in \( e^+e^- \) annihilation obtained in [13] we suppose this contribution to lead only to a few percent change of the overall normalization of the cross section. The main conclusions we draw from the results of the numerical analysis are: (i) the ground state Coulomb resonance is distinguishable in the cross section \( R \)
while in $R^{+−}$ the resonances are completely smeared out by the top quark width; (ii) the total cross section of unpolarized or equally polarized photons suffers from the large corrections and is quite sensitive to the normalization point of the strong coupling constant in NNLO.

The importance of the high order corrections to the cross section has been proved by explicit calculation of certain classes of the next-to-next-to-next-to-leading corrections (see [17] as a review). In [18] the retardation effects caused by the ultrasoft gluons were analysed. The leading logarithmically enhanced $O(\alpha_s^3)$ corrections to the resonance energy and normalization have been computed in [19]. The Abelian part of the subleading logarithmic corrections was obtained in [20] in the context of positronium lifetime calculation. On the basis of this analysis and from the normalization scale dependence of the NNLO result we estimate the high order corrections to the cross section to be at least 10% in magnitude.

In the NNLO analysis the convergence of the series for the resonance energy can be essentially improved by employing an infrared safe mass parameter instead of the top quark pole mass [11,14–16]. As a consequence, using the value of $\alpha_s$ with 2% uncertainty as an input the $\overline{MS}$ top quark mass can be obtained from the resonance peak with the accuracy $\sim 100$ MeV. On the contrary, the behavior of the perturbative series for the cross section normalization cannot be improved in this way that makes the high precision determination of $\alpha_s$ and $\Gamma_t$ to be problematic. At the same time the perturbation theory behavior of the total cross sections of $tt$ production in $e^+e^-$ annihilation and $\gamma\gamma$ collision is very similar. Thus the ratio of the cross sections taken, for example, at the resonance energies is expected to have very nice perturbative properties such as fast convergence and stability against changing the normalization scale. This quantity can probably be used to extract the value of $\alpha_s$ with rather high accuracy. To exploit this property for the precision determination of the strong coupling constant one has to complete the calculation of the NNLO correction to the coefficient $C^{++}$ that now is the main challenge for the theoretical study of the $\gamma\gamma \rightarrow tt$ threshold production.

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Fig. 1. The normalized cross section $R(E)$ in the leading order (solid lines), NLO (bold dotted lines) and NNLO (bold solid lines) for $m_t = 175$ GeV, $\Gamma_t = 1.43$ GeV, $\alpha_s(M_Z) = 0.118$ and $\mu = 50$ GeV, 75 GeV and 100 GeV. The dotted line corresponds to the result in Born approximation.

Fig. 2. The normalized cross section $R^{++}(E)$ in the leading order (dotted lines) and NLO (bold solid lines) for $m_t = 175$ GeV, $\Gamma_t = 1.43$ GeV, $\alpha_s(M_Z) = 0.118$ and $\mu = 50$ GeV, 75 GeV and 100 GeV. The solid line corresponds to the result in Born approximation.