One-Bit Spectrum Sensing in Cognitive Radio Sensor Networks

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Abstract
This paper proposes a spectrum sensing algorithm from one-bit measurements in a cognitive radio sensor network. A likelihood ratio test (LRT) for the one-bit spectrum sensing problem is derived. Different from the one-bit spectrum sensing research work in the literature, the signal is assumed to be a discrete random correlated Gaussian process, where the correlation is only available within immediate successive samples of the received signal. The employed model facilitates the design of a powerful detection criteria with measurable analytical performance. One-bit spectrum sensing criterion is derived for one sensor which is then generalized to multiple sensors. Performance of the detector is analyzed by obtaining closed-form formulas for the probability of false alarm and the probability of detection. The proposed one-bit LRT detector exhibits comparable performance to that of non-one-bit detectors (i.e., quadratic and energy detectors) with the lower computational complexity. Simulation results corroborate the theoretical findings and confirm the efficacy of the proposed detector in the context of highly correlated signals.

Keywords Cognitive radio · Spectrum sensing · One-bit measurements · Detection · Sensor network

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1 Introduction

Cognitive radio (CR) [12] is an emerging technology to improve the spectrum access in wireless sensor networks. It allows unlicensed or secondary users (SUs) to detect and access any available radio spectrum unused by licensed or primary users (PUs) without causing harmful interference to PUs. Hence, spectrum sensing [1–4,6,9,11, 13,15,16,18–20,24] is a vital component of a CR system to identify the state of the PUs in the network.

Spectrum sensing techniques can be categorized into narrowband spectrum sensing and wideband spectrum sensing techniques [2]. Narrowband spectrum sensing [10] investigates the problem of identifying whether a particular slice of the spectrum is idle or not. In contrast, wideband spectrum sensing [20] aims to classify individual slices of a wide frequency range, i.e., megahertz (MHz) to gigahertz (GHz) range, to be either vacant or occupied. Therefore, in majority of existing wideband spectrum sensing techniques, a simple approach is to acquire the wideband signal samples using a standard analog-to-digital converter (ADC) and then utilize appropriate signal processing techniques to identify spectral opportunities. In these techniques, however, the samples of the signal should follow Shannon’s theorem: the sampling rate must be at least twice the maximum available frequency in the signal, i.e., Nyquist rate, to avoid spectral aliasing. Hence, employing these wideband spectrum sensing techniques results in long sensing delays or leads to higher computational complexities and hardware costs. As a result, these techniques are inappropriate for a cognitive wireless sensor network (CWSN) [21,24] with simple and affordable sensors.

A number of techniques have been proposed in the literature to address the challenges, including multiband sensing (FFT-based sensing), wavelet-based sensing, and filter-bank sensing [2]. However, these approaches still suffer from the practical issues such as power consumption, feasibility of ultra-high sampling ADCs, sensing time and complexity. To avoid the high sampling rate or high implementation complexity in Nyquist systems, sub-Nyquist approaches have gained more attention, in which the sampling rates lower than Nyquist rate are employed to detect spectral opportunities. One of these sub-Nyquist approaches is compressive sensing (CS) [5,7] for detection of sparse signals [23] or spectrum sensing in cognitive radio framework [17]. However, there are some limitations on CS techniques. For instance, the sensing matrix should be properly selected to satisfy some constraints (e.g., nearly orthonormal matrices). Further, the spectrum reconstruction part of CS approach is challenging [2].

To simplify the implementation of high sampling ADCs, it is preferred to use low precision ADCs. The extreme case is to use one-bit ADCs utilizing sign measurement by a simple comparator [3,4,19]. In [3], an ultra-low power wideband spectrum sensing architecture is suggested by utilizing a one-bit quantization at the cognitive radio receiver. In [4], the same authors used a window-based autocorrelation to provide the power spectral density of the quantized signal. Recently, the authors in [19] considered the problem of detecting the presence or absence of a random wireless source with minimum latency utilizing an array of radio one-bit sensors.
1.1 Contribution

In this paper, a likelihood ratio test (LRT) detector is derived for detection of a random source with one-bit measurements. Unlike the above-mentioned research work on one-bit spectrum sensing, to reduce the complexity of the one-bit model likelihood, we employ a correlated Gaussian random process for the received signal model, where the correlation is only available within the immediate successive samples of the received signal. The employed model enables the user to design a powerful detection criteria with measurable analytical performance.

The detector performance is investigated in single sensor and multiple sensors scenarios. Then, theoretical analysis of the detector is performed by calculating closed-form formulas for the probability of detection and probability of false alarm. The performance of the proposed one-bit LRT detector has been compared to that of the two non-one-bit detectors: quadratic and energy detectors. The proposed one-bit LRT detector exhibits comparable performance to that of non-one-bit detectors with the lower computational complexity. Simulation results show the efficacy of the LRT detector and agreement between experimental and theoretical results. The simulation results also confirm the performance enhancement of the proposed one-bit LRT detector with the increased value of the correlation coefficient ($r$) of the signal and the number of sensors. The proposed one-bit spectrum sensing detector provides competitive capabilities to save the hardware, power, and computing resources by minimizing the ADC output resolution for a large number of sensors in multiple sensor scenarios.

The rest of the paper is organized as follows. Section 2 introduces the model, the LRT detector, and the theoretical analysis for the single sensor case. In Sect. 3, the same steps are performed in the case of multiple sensors. Simulation results are presented in Sect. 5. Finally, conclusions are drawn in Sect. 6.

2 One-Bit Spectrum Sensing: Single Sensor Case

Consider a random signal $s_i$ for $1 \leq i \leq n$ in which $n$ is the total number of samples. One-bit measurements of the single sensor are modeled as

$$H_0 : \quad y_i = \text{sgn}(w_i),$$
$$H_1 : \quad y_i = \text{sgn}(s_i + w_i), \quad i = 1, 2, \ldots, n$$

where $w_i$ is Gaussian noise with zero mean and variance $\sigma^2$, $H_0$ and $H_1$ are hypotheses of the absence and the presence of the signal, respectively, and $\text{sgn}(x)$ is the indicator function ($\text{sgn}(x) = 1$ for $x \geq 0$ and $\text{sgn}(x) = 0$ for $x < 0$). Signal is assumed to be a correlated Gaussian random process with a covariance matrix which is toeplitz and banded with bandwidth 3. This means that correlation is present only between immediate successive samples. This is the case when sampling rate is less than or equal to twice the symbol rate of a digitally modulated signal. Hence, we have $\mathbb{E}(s_i^2) = \sigma_s^2$ and $\mathbb{E}(s_i s_{i+1}) = r$ while $\mathbb{E}(s_i s_{i+k}) = 0$ for $|k| > 1$. The problem is to decide the true hypothesis (absence or presence of the signal) from one-bit measurements $y = [y_1, y_2, \ldots, y_n]^T$. 

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The Neyman–Pearson LRT detector is defined as [8]:

$$\Lambda_{LR} = \frac{P(y|H_0)}{P(y|H_1)} \gtrless \lambda$$  \hspace{1cm} (1)$$

where $P(\cdot)$ is the probability mass function or probability depending on the context and $\lambda$ is the detector’s threshold. The likelihood under hypothesis $H_0$ is equal to $P(y|H_0) = (\frac{1}{2})^n$. The likelihood $P(y|H_1)$ is equal to

$$P(y_1|H_1)P(y_2|y_1, H_1)P(y_3|y_2, H_1)\ldots P(y_n|y_{n-1}, H_1)$$

since we have only one sample dependence between measurements. Also, we have $P(y_1|H_1) = \frac{1}{2}$. In “Appendix A”, the probability $P(y_2|y_1, H_1)$ is calculated to be

$$P(y_2|y_1, H_1) = p \mathbb{1}(y_1 = y_2) (1 - p) \mathbb{1}(y_1 \neq y_2)$$  \hspace{1cm} (2)$$

where

$$\mathbb{1}(y_1 = y_2) = \begin{cases} 1 & y_1 = y_2, \\ 0 & \text{else} \end{cases}$$  \hspace{1cm} (3)$$

and

$$p := P(y_2 = 1|y_1 = 1, H_1)$$  \hspace{1cm} (4)$$

A similar approach shows that $P(y_{k+1}|y_k, H_1) = p \mathbb{1}(y_k = y_{k+1}) (1 - p) \mathbb{1}(y_k \neq y_{k+1})$ for $2 \leq k \leq n - 1$. Replacing these conditional probabilities in logarithm of (1) followed by straightforward calculations lead to the final detection criterion:

$$\sum_{i=1}^{n-1} \mathbb{1}(y_i = y_{i+1}) \gtrless \eta$$  \hspace{1cm} (5)$$

In the derivation, it is assumed that $\ln \frac{p}{1-p} > 0$ which is equivalent to $p > \frac{1}{2}$ or $r > 0$. For the case of $p < \frac{1}{2}$ or equivalently $r < 0$, the detection criterion has the reverse direction. For the case of $r = 0$ in which the source samples like the noise samples are uncorrelated Gaussian random variables, the energy detector is the sole choice [8,14].

In the following, detection probability and false alarm probability are calculated for the LRT detector of (5) in the case of $p > \frac{1}{2}$. Detection statistic is defined as $Y = \sum_{i=1}^{n-1} \mathbb{1}(y_i = y_{i+1})$. The decision is

$$\hat{d} = \begin{cases} 1 & Y \geq \eta \\ 0 & Y < \eta \end{cases}$$  \hspace{1cm} (6)$$

Hence, the false alarm probability $P_{fa} = P(\hat{d} = 1|H_0) = P(Y > \eta|H_0)$ is equal to

$$P_{fa} = \mathbb{P}\left( \sum_{i=1}^{n-1} \mathbb{1}(y_i = y_{i+1}) \geq \eta \mid H_0 \right) \approx Q\left( \frac{\eta - \mu_0}{\sigma_0} \right)$$  \hspace{1cm} (7)$$
where \( Q(\cdot) \) is the Q-function, \( \mu_0 = E(Y|H_0) \), \( \sigma_0^2 \) is the variance of \( Y \) subject to hypothesis \( H_0 \) and it is assumed that the detection statistic \( Y = \sum_{i=1}^{n-1} I(y_i = y_{i+1}) \) is Gaussian due to the central limit theorem (CLT). In “Appendix B”, \( \mu_0 \) and \( \sigma_0^2 \) are calculated to be:

\[
\mu_0 = \frac{1}{2}(n - 1) \\
\sigma_0^2 = \frac{1}{4}(n - 1) 
\]  

(8)

The detection probability \( P_d = \mathbb{P}(\hat{d} = 1|H_1) = \mathbb{P}(Y \geq \eta | H_1) \); we have:

\[
P_d = \mathbb{P}
\left(
\sum_{i=1}^{n-1} I(y_i = y_{i+1}) \geq \eta \mid H_1
\right)
\approx Q\left(\frac{\eta - \mu_1}{\sigma_1}\right)
\]  

(9)

where \( \mu_1 = \mathbb{E}(Y|H_1) \), \( \sigma_1^2 \) is the variance of \( Y \) subject to hypothesis \( H_1 \). In “Appendix C”, \( \mu_1 \) and \( \sigma_1^2 \) are calculated as:

\[
\mu_1 = 2p(n - 1) \\
\sigma_1^2 = 2p(1 - 2p)(n - 1).
\]  

(10)

(11)

3 One-Bit Spectrum Sensing: Sensor Network Case

Consider a sensor network with \( N \) nodes. Each sensor performs a one-bit measurement as

\[
H_0 : \quad y_{ki} = \text{sgn}(w_{ki}), \\
H_1 : \quad y_{ki} = \text{sgn}(s_i + w_{ki})
\]

where \( 1 \leq i \leq n \) is the time index, \( 1 \leq k \leq N \) is the sensor index, \( N \) is the total number of sensors, \( w_{ki} \) is the Gaussian noise of \( k \)'th sensor, and \( s_i \) is the signal sample with the same model as assumed in Sect. 2.

The LRT detector will be [8]:

\[
A_{LR} = \frac{\mathbb{P}(\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_n \mid H_0)_{H_0} \geq \lambda}{\mathbb{P}(\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_n \mid H_1)_{H_1} \geq \lambda}
\]  

(12)

where \( \mathbf{x}_i = [y_{1i}, y_{2i}, ..., y_{Ni}]^T \) is the measurements of all sensors at \( i \)'th time instant. We will have \( \mathbb{P}(\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_n \mid H_0) = \left(\frac{1}{2}\right)^nN \). Also, \( \mathbb{P}(\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_n \mid H_1) \) is equal to:

\[
\mathbb{P}(\mathbf{x}_1 \mid H_1)\mathbb{P}(\mathbf{x}_2 \mid \mathbf{x}_1, H_1)\mathbb{P}(\mathbf{x}_3 \mid \mathbf{x}_2, H_1)...\mathbb{P}(\mathbf{x}_n \mid \mathbf{x}_{n-1}, H_1)
\]  

(13)

where \( \mathbb{P}(\mathbf{x}_1 \mid H_1) = \left(\frac{1}{2}\right)^N \). Also, \( \mathbb{P}(\mathbf{x}_2 \mid \mathbf{x}_1, H_1) \) is equal to \( \prod_{k=1}^{N} \mathbb{P}(y_{k2} \mid y_{k1}, H_1) \) where \( \mathbb{P}(y_{k2} \mid y_{k1}, H_1) = p^{I(y_{k2} = y_{k1})}(1-p)^{I(y_{k2} \neq y_{k1})} \). We will have
\[ P(\mathbf{x}_2|\mathbf{x}_1, H_1) = (1 - p)^N \left( \frac{p}{1 - p} \right) \sum_k \mathbb{I}(y_{k2} = y_{k1}) \] 

(14)

Similar calculations lead to

\[ P(\mathbf{x}_{i+1}|\mathbf{x}_i, H_1) = (1 - p)^N \left( \frac{p}{1 - p} \right) \sum_k \mathbb{I}(y_{k,i+1} = y_{k,i}) \]

Replacing these conditional probabilities into (13) and (12) leads to the following criterion for \( p > \frac{1}{2} \):

\[ Y := \sum_{i=1}^{n-1} \sum_{k=1}^{N} \mathbb{I}(y_{k,i+1} = y_{k,i}) \overset{H_1}{\geq} \eta \]

(15)

which is a direct generalization of detection criterion in single sensor case in (5). For the case of \( p < \frac{1}{2} \), the direction of the detection criterion in (15) is reversed. The case of \( p = \frac{1}{2} \) is the same as that in the single sensor case where there is no information to detect the presence of the signal.

In the next step, detection probability and false alarm probability are calculated for the LRT detector of (15) when \( p > \frac{1}{2} \). False alarm probability \( P_{fa} = P(\hat{d} = 1|H_0) = P(Y > \eta|H_0) \) is equal to

\[ P_{fa} \approx Q \left( \frac{\eta - m_0}{s_0} \right) \]

(16)

where \( m_0 = \mathbb{E}(Y|H_0) \) and \( s_0^2 \) is the variance of \( Y \) subject to hypothesis \( H_0 \) and it is assumed that the detection statistic \( Y \) is Gaussian due to the CLT. In “Appendix C”, \( m_0 \) and \( s_0^2 \) are calculated to be:

\[ m_0 = \frac{1}{2}(n - 1)N \]
\[ s_0^2 = \frac{1}{4}(n - 1)N \]

(17)

The detection probability \( P_d = P(\hat{d} = 1|H_1) = P(Y \geq \eta|H_1) \). We have:

\[ P_d \approx Q \left( \frac{\eta - m_1}{s_1} \right) \]

(18)

where \( m_1 = \mathbb{E}(Y|H_1) \) and \( s_1^2 \) is the variance of \( Y \) subject to hypothesis \( H_1 \). In “Appendix D”, \( m_1 \) and \( s_1^2 \) are calculated as

\[ m_1 = 2p(n - 1)N \]
\[ s_1^2 = 2p(1 - 2p)(n - 1)N \]

(19) (20)

4 Complexity Analysis

This section presents the complexity analysis of the proposed one-bit LRT detector in comparison with some other detectors. Two other non-one-bit (real-valued) detectors which use the real-value of the observations are quadratic detector and energy...
Table 1  Computational complexities of various algorithms for \( N \) nodes

| Algorithms                     | Multiplications | Additions | Comparison |
|-------------------------------|-----------------|-----------|------------|
| Quadratic detector           | \( Nn^2 + Nn \)  | \( Nn^2 - 1 \) | 1          |
| Energy detector               | \( Nn \)         | \( Nn - 1 \)  | 1          |
| Proposed one-bit LRT detector | 0               | \( Nn - N - 1 \) | \( Nn - N + 1 \) |

The quadratic detector is the optimal LRT detector for the case of a random Gaussian noise in a random white Gaussian noise [14]. It calculates \( t = z^T Q z \) where \( z \) is the real-valued observation vector and compares it with a threshold where \( Q = \frac{1}{\sigma_s^2} C_s (\sigma_s^2 I + C_s)^{-1} \) and \( C_s \) is the covariance matrix of the random signal. The energy detector simply calculates the energy of the signal \( E = \sum_{i=1}^{n} z_i^2 \) and compares it with a threshold to detect the presence or absence of the signal. For the case of multiple sensors, we simply add the test statistics for all sensors and then compare that with a threshold. For calculating the computational complexity of the detectors, we enumerate the number of multiplications, additions, and comparisons of the test statistics of one-bit LRT detector and these two detectors. The computational complexity of the three detectors is shown in Table 1. It is seen from Table 1 that the proposed one-bit LRT detector does not need any multiplications. Further, the number of additions required in the proposed one-bit LRT detector is less than those of in the quadratic and energy detectors. Although the number of comparisons in the proposed one-bit LRT detector is greater than that of in the quadratic and energy detectors, the overall computational complexity of the proposed one-bit LRT detector is lower than the quadratic and energy detectors when all the mathematical operations are taken into account. This is because the ADC output resolution is minimized by considering the extreme scenario of the quantization, i.e., one-bit quantization. Even for large number of sensors \( N \) (as it can be implied from Table 1), the proposed one-bit LRT detector exhibits lower computational complexity compared to the non-one-bit detectors as the required number of multiplications and additions are less than that of the non-one-bit (real-valued) quadratic and energy detectors.

5 Simulation Results

This section presents the simulation results. Correlated random signal is generated as described in Sect. 2, with parameters \( r \) and \( \sigma_s = 1 \). Noise is generated as a Gaussian uncorrelated random process with zero mean and variance \( \sigma = \frac{\alpha_s}{10 \text{SNR}} \), where SNR is the signal-to-noise ratio. In all simulations, number of time samples are assumed to be \( n = 40 \). For comparison of the detectors, the detection probability versus false alarm probability known as receiver operating characteristic (ROC) is depicted. For the Monte Carlo simulation, the experiments are repeated 5000 times and detection probability and false alarm probability are averaged over all the trials. Moreover, to verify the theoretical analysis, we compare the experimental results with the theoretical
results. Four experiments are presented to study the performance of the proposed detector in the single sensor and multiple sensor cases.

In the first experiment, a single sensor is used for spectrum sensing. Four signals are examined with correlation coefficients \( r = 0.1, 0.3, \) and \( 0.5 \). A good agreement between experimental and theoretical ROC curves is shown in Fig. 1. Also, it shows that by increasing the correlation coefficient, the performance of the detector improves.

In the second experiment, we utilize a cognitive sensor network with 1, 5, and 20 sensors. The correlation coefficient of signal is \( r = 0.5 \). ROC curves for \( \text{SNR} = 0 \text{dB} \) and \( \text{SNR} = -5 \text{dB} \) are sketched in Figs. 2 and 3, respectively. It shows that by increasing the number of sensors, the detector performance improves. It also demonstrates a reasonable agreement between theoretical and experimental ROC curves. From Figs. 2 and 3, it is seen that the theoretical curve slightly deviates from the simulation curve when the number of sensors increases, however, the simulated result follows the same trend as the theoretical result. This slight deviation is due to the approximation of the detection probability \( P_d \) with Gaussian Q-function in (9).

In the third experiment, we consider a cognitive wireless sensor network with 20 sensors and with \( \text{SNR} = -5 \text{dB} \). In this experiment, the effect of correlation coefficient between successive samples of random signal on the performance of the proposed one-bit LRT detector is investigated. We considered three cases of \( r = 0.3, r = 0.5, \) and \( r = 0.7 \). The ROC curves of these three cases are shown in Fig. 4. It shows that by increasing the correlation coefficient, the performance of the one-bit LRT detector improves. In fact, in the one-bit case, there is no information of amplitude of the observations and we utilize the correlation information of successive samples to detect the absence or presence of the signal.

In the last experiment, we utilize a cognitive sensor network with 5 sensors. The correlation coefficient of signal is \( r = 0.7 \) and the SNR is equal to \(-5 \text{dB}\). In this experiment, the performance of our proposed one-bit LRT detector is compared to
two other real-valued detectors. The first detector is the quadratic detector which is an optimum LRT detector for the case of a random Gaussian signal in a random Gaussian noise [14]. The second detector is the energy detector [14]. The ROC curves for the proposed one-bit detector and these two well-known detectors in the real-valued case are depicted in Fig. 5. This figure shows that the performance of the one-bit LRT detector is comparable to two other real-valued detector with lower computational complexity as discussed in Sect. 4.

6 Conclusion

In this paper, we have derived the LRT detector for one-bit spectrum sensing problem in single sensor and multiple sensor cases for a correlated Gaussian signal model. The detectors utilize correlation available within successive samples of the received signal to obtain the detection criteria. Closed-form detection and false alarm probabilities are
Fig. 4 ROC curve of the LRT detector for multiple sensor case ($N = 20$) and for SNR $= -5$ dB for different values of correlation coefficient ($r$)

Fig. 5 ROC curve of the one-bit LRT detector for multiple sensor case ($N = 5$) and for SNR $= -5$ dB in comparison with Energy detector and Quadratic detector in real-valued case

derived in single and multiple sensor scenarios. Simulation results show the efficacy of the detector specially when the correlation coefficient is large or the number of sensors increases. The performance of the proposed one-bit LRT detector has been compared to that of the two non-one-bit detectors: quadratic and energy detectors. The proposed one-bit LRT detector exhibits comparable performance to that of non-one-bit detectors with the lower computational complexity. Moreover, the simulations results corroborate the theoretical analysis. The simulation results also confirm the performance enhancement of the proposed one-bit LRT detector with the increased value of the correlation coefficient ($r$) of the signal and the number of sensors. The proposed one-bit spectrum sensing detector provides competitive capabilities to save the hard-
ware, power, and computing resources by minimizing the ADC output resolution for a large number of sensors in multiple sensor scenarios.

**Appendix A: Calculating** $\mathbb{P}(y_2|y_1, H_1)$

We first calculate the four probabilities $\mathbb{P}(y_2 = 1|y_1 = 1, H_1) = p$, $\mathbb{P}(y_2 = 0|y_1 = 1, H_1) = 1 - p$, $\mathbb{P}(y_2 = 1|y_1 = 0, H_1) = p'$, and $\mathbb{P}(y_2 = 0|y_1 = 0, H_1) = 1 - p'$. The probability $p = \mathbb{P}(y_2 = 1|y_1 = 1, H_1)$ is equal to

$$p = \frac{\mathbb{P}(z_1 + s_2 \geq 0|w_1 + s_1 \geq 0)}{\mathbb{P}(z_1 \geq 0)} = 2 \mathbb{P}(z_1 \geq 0, z_2 \geq 0)$$

(21)

where $z_1 = s_1 + w_1$, $z_2 = s_2 + w_2$, and $p(z_1 \geq 0) = \frac{1}{2}$. To calculate $p(z_1 \geq 0, z_2 \geq 0)$, note that $z_1$ and $z_2$ are correlated Gaussian random variables with covariance matrix $C$ with elements $C_{11} = E(z_1^2) = \sigma_1^2 + \sigma_2^2$, $C_{12} = C_{21} = E(z_1 z_2) = r$ and $C_{22} = E(z_2^2) = \sigma_2^2 + \sigma_1^2$. Therefore, joint probability density function (pdf) is $f(z_1, z_2) = \frac{1}{2\pi\sqrt{\text{det}(C)}} \exp(-\frac{1}{2}z C^{-1} z^T)$ where $z = [z_1, z_2]$. Hence, we have $p = 2 \int_0^{+\infty} \int_0^{+\infty} f(z_1, z_2) dz_1 dz_2$. To calculate the other probability $p'$, we consider that $p' = \mathbb{P}(y_2 = 1|y_1 = 0, H_1) = \frac{\mathbb{P}(y_2 = 1, y_1 = 0|H_1)}{\mathbb{P}(y_2 = 1|H_1)} = 2 \mathbb{P}(y_2 = 1, y_1 = 0|H_1) = 2(\frac{1}{2} - \frac{p}{2}) = 1 - p$ which leads to (2).

**Appendix B: Calculating $\mu_0$ and $\sigma_0^2$**

We have $\mu_0 = \sum_{i=1}^{n-1} \mathbb{E}[y_i = y_{i+1}|H_0] = \frac{1}{2}(n-1)$. Also, we have $\sigma_0^2 = \mathbb{E}(Y^2|H_0) - \mathbb{E}^2(Y|H_0)$ in which $\mathbb{E}(Y|H_0) = \frac{1}{2}(n-1)$ and

$$\mathbb{E}(Y^2|H_0) = \sum_{i,i'} \mathbb{E}(I(y_i = y_{i+1}) I(y_{i'} = y_{i'+1})|H_0)$$

(22)

where the expectation is equal to

$$\mathbb{P}(y_i = y_{i+1} \wedge y_{i'} = y_{i'+1}|H_0) = \begin{cases} \frac{1}{4} & i = i' \\ \frac{1}{4} & i \neq i' \end{cases} (23)$$

Replacing (23) into (22) results in (8).

**Appendix C: Calculating $\mu_1$ and $\sigma_1^2$**

We have $\mu_1 = \sum_{i=1}^{n-1} \mathbb{E}[y_i = y_{i+1}|H_1] = \sum_{i=1}^{n-1} \mathbb{P}(y_i = y_{i+1}|H_1) = 2p(n-1)$. Also, we have $\sigma_1^2 = \mathbb{E}(Y^2|H_1) - \mathbb{E}^2(Y|H_1)$ in which $\mathbb{E}(Y|H_1) = 2p(n-1)$ and

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\[ E(Y^2|H_1) = \sum_{i, i'} \mathbb{E}(\mathbb{I}(y_i = y_{i+1}) \mathbb{I}(y_{i'} = y_{i'+1})|H_1) \] (24)

where the expectation is equal to

\[
\mathbb{P}(y_i = y_{i+1} \land y_{i'} = y_{i'+1})|H_1)
= \begin{cases} 
\mathbb{P}(y_i = y_{i+1}|H_1) = 2p : & i = i' \\
\mathbb{P}(y_i = y_{i+1}|H_1)\mathbb{P}(y_{i'} = y_{i'+1}|H_1) = 4p^2 : & i \neq i'
\end{cases}
\] (25)

Replacing (25) into (24) results in (11).

**Appendix D: Calculating \(m_0\) and \(s_0^2\)**

We have \(m_0 = \sum_{i=1}^{n-1} \sum_{k=1}^{N} \mathbb{E}(y_{ki} = y_{k,i+1}|H_0) = \frac{1}{2}(n-1)N\). Also, we have \(s_0^2 = \mathbb{E}(Y^2|H_0) - \mathbb{E}^2(Y|H_0)\) in which \(\mathbb{E}(Y|H_0) = \frac{1}{2}(n-1)N\) and

\[
\mathbb{E}(Y^2|H_0) = \sum_{i,k,i',k'} \mathbb{E}(\mathbb{I}(y_{ki} = y_{k,i+1}) \mathbb{I}(y_{k'i'} = y_{k',i'+1})|H_0) \] (26)

where the expectation is equal to

\[
\mathbb{P}(y_{ki} = y_{k,i+1} \land y_{k'i'} = y_{k',i'+1})|H_0)
= \begin{cases} 
\frac{1}{2} : & i = i' \land k = k' \\
\frac{1}{4} : & i \neq i' \lor k \neq k'
\end{cases}
\] (27)

Replacing (27) into (26) results in (17).

**Appendix E: Calculating \(m_1\) and \(s_1^2\)**

We have \(m_1 = \sum_{i=1}^{n-1} \sum_{k=1}^{N} \mathbb{E}(y_{ki} = y_{k,i+1}|H_1) = \sum_{i,k} \mathbb{P}(y_{ki} = y_{k,i+1}|H_1) = 2p(n-1)N\). Also, we have \(s_1^2 = \mathbb{E}(Y^2|H_1) - \mathbb{E}^2(Y|H_1)\) in which \(\mathbb{E}(Y|H_1) = 2p(n-1)N\) and

\[
\mathbb{E}(Y^2|H_1) = \sum_{i,k,i',k'} \mathbb{E}(\mathbb{I}(y_{ki} = y_{k,i+1}) \mathbb{I}(y_{k'i'} = y_{k',i'+1})|H_1) \] (28)

where the expectation is equal to

\[
\mathbb{P}(y_{ki} = y_{k,i+1} \land y_{k'i'} = y_{k',i'+1})|H_1)
= \mathbb{I}(i = i' \land k = k')\mathbb{P}(y_{ki} = y_{k,i+1}|H_1) + \\
\mathbb{I}(i \neq i' \lor k \neq k')\mathbb{P}(y_{ki} = y_{k,i+1}|H_1)\mathbb{P}(y_{k'i'} = y_{k',i'+1}|H_1) \]
which is

\[
\mathbb{P}( (y_{ki} = y_{k,i+1}) \land (y_{k'i'} = y_{k',i'+1}) | H_1) = \begin{cases} 
2p : & i = i' \land k = k' \\
4p^2 & i \neq i' \lor k \neq k' 
\end{cases} \quad (29)
\]

Replacing (1) into (28) results in (20).

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