Outage Performance of MIMO Multihop Relay Network with MRT/RAS Scheme

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SUMMARY In this Letter, we investigate the outage performance of MIMO amplify-and-forward (AF) multihop relay networks with maximum ratio transmission/receiver antenna selection (MRT/RAS) over Nakagami-m fading channels in the presence of co-channel interference (CCI) or not. In particular, the lower bounds for the outage probability of MIMO AF multihop relay networks with/without CCI are derived, which provides an efficient means to evaluate the joint effects of key system parameters, such as the number of antennas, the interfering power, and the severity of channel fading. In addition, the asymptotic behavior of the outage probability is investigated, and the results reveal that the full diversity order can be achieved regardless of CCI. In addition, simulation results are provided to show the correctness of our derived analytical results.

key words: amplify-and-forward (AF), cochannel interference, MIMO, multihop, MIMO multihop relay network

1. Introduction

The use of relaying technology is a reliable way to improve the performance of wireless communication systems. Among various relay protocols, amplify-and-forward (AF) and decode-and-forward (DF) have been broadly investigated [1]. In addition to the performance of single antenna decode-and-forward (DF) have been broadly investigated [2]–[4].

In an effort to improve the performance of the relay systems, the multiple-input multiple-output (MIMO) transmission technology has been integrated into relay systems [5]. Considering the increasing complexity of using multiple antennas, antenna selection has been proposed to reduce the cost without affecting the achievable diversity gain. For instance, the performance of joint relay and antenna selection has been analyzed in [6] and the references therein.

However, the above works assume an ideal noise-limited environment. In practical wireless communication systems, the relay transmission will inevitably be subjected to co-channel interference (CCI) due to the aggressive reuse of frequency channels for high spectrum utilization. Motivated by this important observation, several works have been devoted to investigate the impact of CCI on the performance of relay networks [7]–[9]. To the best of our knowledge, the performance of MIMO multihop relay networks with maximum ratio transmission/receiver antenna selection (MRT/RAS) scheme for Nakagami-m fading environment is unexplored from the analytical point of view regardless of CCI assumptions. As such, to fill this important gap, we investigate the outage performance of MIMO multihop relay networks employing the MRT/RAS scheme with/without CCI. Specifically, we derive tight closed-form lower bounds and asymptotical analytical expressions for the outage probability of the considered system. The finding suggests that the CCI has no impact on the diversity gain achieved by the considered system.

2. Network and Channel Models

We consider an MIMO multihop relay network, which consists of the source node $S$ ($R_0$) equipped with $L_0$ antennas, $N-1$ relays $R_n$, $n = 1, 2, \ldots, N-1$, each equipped with $L_n$ antennas, and the destination $D$ ($R_N$) equipped with $L_N$ antennas. The transmission between $R_0$ and $R_N$ is done with the help of $N-1$ half-duplex relays $R_n$. The medium-access control scheme allocates a frequency band to the source for its transmission, which is further divided into orthogonal subchannels across time using a time-division scheme to permit half-duplex operation at the relays. Only one node transmits in each time slot. In the $n$th time slot, $R_n$ receives a faded noisy signal from the immediately preceding transmitting terminal $R_{n-1}$, which is also corrupted by a finite number of external CCI $Q_{n-1}$. In the next time slot, $R_n$ processes the received signal and then forwards it to $R_{n+1}$. MRT/RAS is adopted in each hop, thus the signal received from $R_{n-1}$ at the $k$-th antenna of $R_n$ can be expressed as

$$y_{n,k} = \sqrt{P_{n-1}} h_{n,k}^\dagger w_{n-1} x_{n-1} + \sum_{q=1}^{Q_n} \sqrt{P_{n,q}} f_{n,k,q} m_{n,q} + n_k$$

(1)

where $P_{n-1}$ is the transmit power of $R_{n-1}$, $m_{n,q}$ is the $q$-th interfering signal with an average power $P_{n,q}$ at $R_n$, and $Q_n$ is the number of interferers at $R_n$. $h_{n,k}$ is the $L_{n-1} \times 1$ channel link vector between $R_{n-1}$ and the $k$-th receiver antenna of $R_n$, and its entries follow independent and identically distributed (i.i.d.) Nakagami-$m$ distribution with parameters $(m_{h_n}, \Omega_{h_n})$. $w_{n-1} = \frac{h_{n,k}}{\|h_{n,k}\|}$ is the $L_{n-1} \times 1$ weight vector.
and $f_{n,k,q}$ is the channel coefficient between the $q$-th interfering and the $k$-th receiver antenna of $R_n$, which follows Nakagami-$m$ distribution with parameters $(m_n, \Omega_n)$. $n_k$ is the AWGN at $R_n$ with $E[|n_k|^2] = \sigma^2$, and $x_{n-1} = G_{n-1} y_{n-1,k}$ is the signal relayed from $R_{n-1}$, $G_{n-1}$ is given based on AF relay protocol [8]

$$G_{n-1} = \left( \frac{P_{n-2} \|h_{n-1,k}\|^2_f + \sum_{q=1}^{Q_n} P_{n-1,q} |f_{n-1,k,q}|^2 + \sigma^2}{\gamma_{\min}} \right)^{-1}$$  \hspace{1cm} (2)

Based on the above model, the best antenna can be selected based on the following equation

$$k^* = \arg \max_{k=1,2,...,L_n} \gamma_{n,k}$$ \hspace{1cm} (3)

where $\gamma_{n,k} = P_{n-1} \|h_{n,k}\|^2_f / \sigma^2$.

According to Eqs. (1)–(3), the end-to-end (e2e) SINR of MIMO multihop AF relay networks with MRT/RAS is

$$\gamma_{\text{end}} = \left[ \prod_{n=1}^{N} \left( 1 + \frac{\sum_{q=1}^{Q_n} \gamma_{n,k}}{\gamma_n} \right) - 1 \right]^{-1}$$ \hspace{1cm} (4)

where $\gamma_n = \gamma_{n,k^*}$, and $\gamma_{n,k} = P_{n,q} |f_{n,k,q}|^2 / \sigma^2$ is the $q$-th interference-to-noise ratio (INR) at the selected $R_n$ antenna.

In general, an exact analysis of the statistics of e2e SINR in Eq. (4) is very challenging. Hence, to circumvent this difficulty, we adopt the following tight upper bound [8]

$$\gamma_{\text{end}} \leq \gamma_{\text{up}} = \min(\gamma_{1,\text{eff}}, \gamma_{2,\text{eff}}, \ldots, \gamma_{N,\text{eff}})$$ \hspace{1cm} (5)

where $\gamma_{n,\text{eff}} = \gamma_n / (\gamma_{\text{up}} + 1)$ and $\gamma_{\text{up}} = \sum_{q=1}^{Q_n} \gamma_{n,k}$.

3. Outage Performance Analysis

As an important performance indicator for wireless communication systems, the outage probability can be expressed as

$$P_{\text{out}}(\gamma) = P(\gamma_{\text{end}} \leq \gamma) = F_{\gamma_{\text{end}}}(\gamma)$$ \hspace{1cm} (6)

Based on Eq. (5), it is lower bounded by

$$F_{\gamma_{\text{up}}}(\gamma) = 1 - \prod_{n=1}^{N} \left[ 1 - F_{\gamma_{n,\text{eff}}}(\gamma) \right]$$ \hspace{1cm} (7)

To evaluate the impact of CCI on the performance of MIMO multihop AF relay systems with MRT/RAS, the following two cases of interest are considered, i.e. MIMO multihop AF relay networks without CCI and MIMO multihop AF relay networks with CCI.

3.1 MIMO Multihop AF Relay Systems without CCI

If CCI signals do not exist in the system, we have the following important results.

**Theorem 1:** The outage probability of MIMO multihop AF relay networks without CCI is lower bounded by

$$P(\gamma_{\text{up}} \leq \gamma) = 1 - \prod_{n=1}^{N} \left[ 1 - F_{\gamma_{n,\text{eff}}}(\gamma) \right]$$ \hspace{1cm} (8)

**Proof:** When the considered system is free of CCI, we have $\gamma_{n,\text{eff}} = \gamma_n$. Hence, to obtain the CDF of $\gamma_{\text{end}}$, we first give the CDF of $\gamma_{n,\text{eff}}$ as follows

$$F_{\gamma_{n,\text{eff}}}(\gamma) = \frac{\Gamma(m_n L_n - 1, \frac{\gamma}{\gamma_{\text{up}}})}{\Gamma(m_n L_n)}$$ \hspace{1cm} (9)

where $\gamma_{\text{up}} = P_{n-1} \bar{\Omega}_n / \sigma^2$, $\Gamma(\cdots)$ is the lower incomplete Gamma function [10, Eq. (8.350.1)], and $\Gamma(\cdots)$ is the Gamma function [10, Eq. (8.310.1)]. According to Eq. (3) with [11, Eq. (9.9)], the CDF of $\gamma_n$ can be derived as

$$F_{\gamma_n}(\gamma) = \prod_{k=1}^{L_n} \frac{\gamma(m_n L_n - 1, \frac{\gamma}{\gamma_{\text{up}}})}{\Gamma(m_n L_n)} = 1 - \sum_{l=1}^{L_n} \beta_{\text{Lmm}} \left( \frac{\gamma}{\gamma_{\text{up}}} \right)^{\alpha_{\text{Lmm}}} \exp \left( - \frac{\alpha_{\text{Lmm}} \gamma}{\gamma_{\text{up}}} \right)$$ \hspace{1cm} (10)

Finally, substituting Eq. (10) into Eq. (7), the exact analytical expression for the lower outage bound of the considered system without CCI can be obtained as Eq. (8) after some simple mathematical manipulations.

To gain further insights, we now look into the high SNR regime, and derive the asymptotic expression for the outage probability.

**Corollary 1:** When the considered system is free of CCI, the asymptotic outage probability is given by

$$P(\gamma_{\text{up}} \leq \gamma) \approx 1 - \prod_{n=1}^{N} \left[ 1 - \left( \frac{m_n L_n - 1}{(m_n L_n - 1)!} \right)^{\gamma_{\min}} \right]$$ \hspace{1cm} (11)

where $n^* = \min_{n=1,2,...,N} \{m_n L_n - 1\}$, the outage diversity gain $G_{\text{anc}} = m_{n^*} L_{n^* - 1} L_n$, and the outage array gain $G_{\text{anc}} = \left( \sum_{n=1}^{N} \{m_n L_n - 1\}^{L_n - 1} \right)^{\gamma_{\min}} \gamma_{\text{up}}$. 
Proof 2: To derive the asymptotical CDF of \( F_{\gamma_n}(\gamma) \), we first express the incomplete Gamma function using the series expansion as \([10, Eq. (8.354.1)]\)

\[
\Gamma(m, \theta x) = (\theta x)^m \sum_{n=0}^{\infty} \frac{(-1)^n \theta^n x^n}{n! (m+n)} \approx \frac{1}{m}.
\]

Then, substituting Eq. (12) into Eq. (10), the CDF of \( F_{\gamma_n}(\gamma) \) can be approximated as

\[
F_{\gamma_n}(\gamma) = \left( \frac{y}{\gamma_n} \right)^{\alpha_{\text{mnw}}} \frac{1}{(\alpha_{\text{mnw}})!!} \sum_{n=0}^{\infty} \frac{(-1)^n \gamma_n^n}{n! (\alpha_{\text{mnw}}+n)} x^n.
\]

To this end, substituting Eq. (13) into Eq. (7), we have the asymptotic outage probability as shown in Eq. (11).

3.2 MIMO Multihop AF Relay Systems with CCI

If CCI signals are considered, we have the following key result.

Theorem 2: The outage probability of MIMO multihop AF relay networks with MRT/RS in the presence of CCI is lower bounded by

\[
\Pr(\gamma_{up} \leq \gamma) \leq 1 - \sum_{\ell_1=0}^{m_{\text{h},1-L_{\ell_1}}} \prod_{s=1}^{\ell_1} \sum_{\gamma_{\ell_s}=1}^{\theta_{\ell_s}} \prod_{\gamma_{\ell_s}=1}^{\theta_{\ell_s}} \left\{ \prod_{n=1}^{N} \left( \frac{\eta_{\text{mnw}} \gamma_{\ell_s} \Gamma(n \gamma_{\ell_s} + m_{\ell_s} Q_{\ell_s})}{\gamma_{\ell_s}^{\theta_{\ell_s}} \gamma_{\ell_s}} \right) \exp\left( -\frac{n \gamma_{\ell_s}}{\gamma_{\ell_s}} \right) \right\} \gamma_{\ell_s}^{-n}
\]

where \( \gamma_{\ell_s} = P_{\ell_s} \Omega_{f_s}/\sigma_{f_s}^2 \)

Proof 3: According to [7], the PDF of \( \gamma_{\ell_s} \) is expressed as

\[
f_{\gamma_{\ell_s}}(z) = \frac{z^{m_{\ell_s} Q_{\ell_s} - 1}}{\gamma_{\ell_s}^{m_{\ell_s} Q_{\ell_s}} (m_{\ell_s} Q_{\ell_s} - 1)!} \exp\left( -\frac{z}{\gamma_{\ell_s}} \right)
\]

By invoking the concepts of probability theory, the CDF of \( F_{\gamma_{\ell_s}}(\gamma) \) can be written as

\[
F_{\gamma_{\ell_s}}(\gamma) = \Pr\left( \frac{\gamma_{\ell_s}}{\gamma_{\ell_s} + 1} < \gamma \right) = 1 - \int_0^{\gamma_{\ell_s}} \sum_{\ell_1=0}^{m_{\text{h},1-L_{\ell_1}}} \prod_{s=1}^{\ell_1} \sum_{\gamma_{\ell_s}=1}^{\theta_{\ell_s}} \prod_{\gamma_{\ell_s}=1}^{\theta_{\ell_s}} \left\{ \frac{z^{m_{\ell_s} Q_{\ell_s} - 1}}{\gamma_{\ell_s}^{m_{\ell_s} Q_{\ell_s}} (m_{\ell_s} Q_{\ell_s} - 1)!} \exp\left( -\frac{z}{\gamma_{\ell_s}} \right) \right\} dz
\]

4. Numerical Results and Discussion

In this Section, the Monte Carlo simulations are provided to validate the derived analytical expressions. Without loss of generality, the transmit power of all nodes \( R_s \) is assumed to be equal, i.e., \( P_s \), the channel mean powers are given by \( \Omega_{f_s} = 1 \) and \( \Omega_{f_s} = 1 \), and the SNR threshold \( \gamma \) is 5dB. With the aim to highlight the joint effect of the number of antennas and the channel fading severity, CCI only has impact on the outage array gain.


\[ S_{12} = \{1, 1, 1, 1, 2, 3, 3\}, S_{13} = \{2, 2, 2, 2, 1, 1, 1\}. \]

Figure 1 plots the outage probability versus \( P_s \) for three hop MIMO relay networks without CCI. We can see that, the SNR upper bound in Eq. (5) is tight when SNR increases. The analytical results are in exact agreement with the simulation curves, and the asymptotic curves tight converge to the exact curves in the high SNR regime, which verifies the accuracy of our analytical derivation. In addition, as can be expected, the outage probability decreases as the number of transmitter and receiver antennas increases, e.g., \( S_{13} \) can achieve better performance than \( S_{11} \). Furthermore, as the diversity order is governed by the minimum of \( m_{bh} \) \( L_{m-1} L_n \) for each hops, the higher the value of \( \min \{m_{bh} L_{m-1} L_n\} \), the better the outage performance. Figure 2 plots the outage probability versus \( P_s \) for three hop MIMO relay networks in the presence of two i.i.d interferers. The channel fading severity parameters between CCI and \( R_n \) is \( m_{fa} = 2 \). Two kinds of different interfering power are considered, i.e., \( P_{nm} = 1 \) dB and \( P_{nm} = 5 \) dB. As can be expected, the outage performance can be significantly improved when the fading severity \( m_{bh} \) increases, which indicates the outage probability is closely related to the channel quality of each link. However, increasing the interfering power will decrease the outage performance, while we can improve the outage probability of the system by increasing the number of antennas at each nodes, since the extra number of antennas provide the additional outage array gain to the system.

5. Conclusion

This Letter has studied the outage performance of MIMO multihop AF relay networks with MRT/RAS scheme. The closed-form analytical expressions for the outage probability of the considered system with/without CCI were derived, they provide a fast and efficient means of evaluating system performance. Moreover, simple and informative high SNR approximations for the outage probability were provided, they enable us to investigate the impact of the number of antennas at each node, channel fading severity parameters, and the interfering power on the outage performance. Our finding reveals that CCI has no impact on the outage diversity gain, and it only affects the outage array gain achieved by the considered system.

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