Spin excitations in fermion condensates

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We investigate collective spin excitations in two-component fermion condensates with special consideration of unequal populations of the two components. The frequencies of monopole and dipole modes are calculated using Thomas-Fermi theory and the scaling approximation. We demonstrate that spin oscillations have more sensitivity to the interaction and the properties of the condensates than the density oscillations.

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I. INTRODUCTION

Since the realization of the Bose-Einstein condensed (BEC) atomic gases\cite{1,2}, there has been much interest in ultracold trapped atomic systems to study quantum many-body phenomena. Besides the Bose-Einstein condensates (BEC)\cite{1,2,3}, one can now study degenerate atomic fermi gases\cite{4} and bose-fermi mixtures\cite{5}. These systems offer great promise to exhibit new and interesting phenomena of quantum many-particle physics. An important diagnostic signal for these systems is the spectrum of collective excitations. Such oscillations are common to a variety of many-particle systems and are often sensitive to the interaction and the structure of the ground state.

However, for harmonically trapped gases of either bosonic or fermionic atoms, the frequencies of the density oscillations are quite insensitive to the strength of the interaction. In particular, the dipole oscillation frequency is completely independent of the interaction\cite{10}. As we will see, excitations where the two components move out of phase (spin excitations) behave quite differently.

The underlying theoretical tool to treat dynamic problems of dilute quantum gases is time-dependent mean field theory. This reduces to the random-phase approximation (RPA) for small amplitudes, and the theory in this form has been applied to density oscillations of these systems\cite{7,8}. When the single-particle spectrum is regular, the long-wavelength excitations are collective and simpler methods can be used to calculate the frequencies, in particular with sum rules\cite{6} or the scaling approximation\cite{12,13,14}. In this work we will follow the last approach, which we believe is justified by the extreme regularity of the harmonic trapping potential. The scaling method is physically quite transparent, but it is in fact equivalent to the theory based on energy-weighted sum rules\cite{13}, as used for example in ref.\cite{6}.

In the dilute limit, the interaction in a two-component fermion condensate is characterized by a single number, the scattering length $a$. We will only consider here the case of positive scattering lengths which
correspond to a repulsive interaction. When the interaction is attractive, the system is superfluid in its ground state and the excitation properties are controlled by the energy gap.

The ground state properties of the two-component fermion system with a repulsive interaction was treated by Sogo and Yabu [11], who showed that there are three regimes, depending on interaction strength. For small interaction strengths, the ground state has equal densities of the two components, which we call the "paramagnetic" regime. Beyond a certain threshold in interaction strength, the ground state becomes "ferromagnetic", that is with unequal densities of the two components. At very large interaction strengths, the minority component vanishes. We shall call this the single-component phase. In this work, we will treat systems in the presence of an external magnetic field, which will produce a net spin in the ground state even in the paramagnetic regime.

We now briefly describe the spin modes that could be excited by a time-dependent external magnetic field. The simplest field to consider is one that is spatially uniform. However, such fields cannot change the spatial part of the single-particle wave functions of the condensate and do not induce internal excitations. The response to a uniform field is identical to that of a noninteracting ensemble of \( N_1 - N_2 \) atoms, where \( N_i \) is the number of atoms in component \( i \). The next field to consider has a dipole spatial dependence. Due to the constraints of Maxwell’s equations, the overall multipolarity of such a field is \( J = 2 \), but we shall follow the common terminology calling it dipole. The spin-dipole mode has been discussed previously in in refs. [6] and [7]. Ref. [6] treats the the spin dipole mode as overdamped and does not discuss its frequency. In ref. [7], the spin dipole response was calculated in RPA for a particular value of the coupling strength, and it was found to be rather narrow and close to the unperturbed oscillation frequency. In our work here, we will calculate the frequency under a variety of conditions: different coupling strengths and in the presence of an external static magnetic field. The damping of the mode is beyond the scope of this study.

II. GROUND STATE

We begin with the expression for the energy of a dilute trapped condensate in the mean-field approximation:

\[
E_T = \int d^3 r \left[ -\frac{\hbar^2}{2m} \sum_{n, s=1,2} \sum_{s=1,2} \psi_{ns}^* \nabla^2 \psi_{ns} + \frac{m}{2} (\Omega_L^2 r_1^2 + \Omega_T^2 r_2^2 + \Omega_L^2 r_3^2) (\rho_1 + \rho_2) \\
+ g \rho_1 \rho_2 - B (\mu \rho_1 - \mu \rho_2) \right].
\]

Here \( \psi_{ns} \) are orbital wave functions indexed by spin \( s = 1, 2 \), \( \Omega_L, T \) are the longitudinal and transverse frequencies of the trapping field, \( g \) is the coupling strength of a contact interaction, \( \mu_s \) are the magnetic moments of the atoms, and \( B \) is an external magnetic field. The densities \( \rho_s \) are given by the usual sum over occupied orbitals: \( \rho_s = \sum_n^{occ} |\psi_{n,s}|^2 \).

We shall use the Thomas-Fermi approximation to evaluate the first term, the kinetic energy. We can
then make a change of variables to simplify the appearance of the Thomas-Fermi equations, similar to the scaling defined in ref. [11]. With the scaled variables and the Thomas-Fermi approximation, the expression for the energy is, up to an additive constant,

$$\tilde{E}_T = \int d^3x \left[ \sum_{i=1,2} \left\{ \frac{3}{5} n_i^{5/3} + x^2 n_i \right\} + g n_1 n_2 - \tilde{B}(n_1 - n_2) \right].$$  \hspace{1cm} (2)

Here the variables are defined

$$x_j = \left( \frac{m^2 \Omega_j}{3 \pi^2 \hbar^3} \right)^{1/3} r_j \quad (j = 1 \sim 3),$$

$$n_i = \left( \frac{m}{\hbar^2} \right)^3 \frac{2}{9 \pi^2} \rho_i \quad (i = 1, 2),$$

$$\tilde{B} = \left( \frac{m}{\hbar^2} \right)^3 \frac{2}{9 \pi^2} B \mu,$$

$$\tilde{E}_T = \frac{4 m_{12}^{12} \Omega_L \Omega_2^2}{(3 \pi^2)^{2/3} \hbar^{12}} E_T.$$  \hspace{1cm} (3)

The TF equations for the densities $n_{1,2}$ are derived by variation of the energy (3) with a constraint on the total number of particles. This yields

$$n_1^{2/3} + gn_2 = e_f - x^2 + \tilde{B}$$  \hspace{1cm} (4)

$$n_2^{2/3} + gn_1 = e_f - x^2 - \tilde{B}$$  \hspace{1cm} (5)

In this equation the Lagrange multiplier $e_f$ has the meaning of the Fermi energy. The solution of these equations in the absence of a magnetic field is discussed in detail in ref. [11]. These authors make an additional rescaling to eliminate $g$, but we do not do that here. For any positive $g$, there is a value of $e_f$ above which the system becomes ferromagnetic. At $g = 1$, the critical point is at $e_f = 20/27$. Below that value, the minimum energy is obtained for the paramagnetic phase, having $n_1 = n_2$. Just above that value, both components are present but the densities are unequal. When $e_f$ is increased past $e_f = 1$, the system goes into a single-component phase in the center of the trap, at $x = 0$. Away from the center of the trap, the Fermi energy is effectively reduced, allowing the paramagnetic phase to persist in the outer part of the condensate cloud. Thus the minority component will form a hollow sphere, which we will call the hollow spin phase.

Now consider the effect of a magnetic field. If $\tilde{B} \neq 0$, the densities $n_{1,2}$ will unequal no matter what the value of $g$. Still, one can distinguish two kinds of behavior near the center of the trap. If the densities of both spins decrease as one moves away from the center, we shall call it paramagnetic. However, for a certain range of parameters, the density of the minority component may increase initially, moving away from the center. This is the ferromagnetic phase. The system can also form the hollow spin phases in the presence of a magnetic field. When the external magnetic field $\tilde{B}$ increases more, the minority component disappears completely, giving what we call the single-spin phase.

Density profiles illustrating these three regimes are shown in Fig. 1. The panels show the densities as a function of distance $x$ in a weak magnetic field ($g^2 \tilde{B} = 1.0 \times 10^{-4}$) and at three different interaction
strengths: \( g = 0.95(a), 1.05(b) \) and 1.15(c). From top to bottom, the panels show paramagnetic (a), ferromagnetic (b), and the hollow spin phases (c). The solid and dashed lines represent the scaled density distribution of major and minor components of fermions.

The phase boundaries as function of \( \tilde{B} \) and \( g \) are shown in Fig. 2. The upper and lower columns show same results, but in the lower column the vertical line is rescaled by factor \( g^2 \).

The dashed lines represents the border between the paramagnetic and ferromagnetic phases, which is given by \( \partial^2 n_2(x)/\partial x^2 = 0 \) at \( x = 0 \).

The solid line represents the results solved by \( n_2(0) = 0 \) in eq. (5). This line crosses the dashed line at \( g = \sqrt{2} \) and \( \tilde{B} = 1/27 \). The curvature of the minority component density, \( \partial^2 n_2(x)/\partial x^2 \) at \( x = 0 \), is negative when \( g < 1/\sqrt{2} \) and positive when \( g > 1/\sqrt{2} \) This solid line shows the border between the single and paramagnetic spin phases when \( g < 1/\sqrt{2} \), and the border between the ferromagnetic and hollow spin phases when \( g < 1/\sqrt{2} \).

In the hollow phases there are two boundaries for the minority fermion density where \( n_2 = 0 \) and \( n_1 \neq 0 \). On this boundary the density of majority fermion must satisfy

\[
n_1^{\frac{2}{3}} - gn_1 = 2\tilde{B}.
\]

Here we should consider the following function:

\[
f(n) = n^{\frac{2}{3}} - gn - 2\tilde{B}.
\]

The derivative of the above equation is

\[
\frac{d}{dn}f(n) = f'(n) = \frac{2}{3}n^{-\frac{1}{3}} - g.
\]

Since \( f'(8/27g^3) = 0 \), \( f(0) = f(1/g^3) = -2B < 0 \) and \( f'(1/g^3) < 0 \), \( f(n) \) becomes maximum at \( n = 8/27g^3 \). Thus \( f(8/27g^3) > 0 \) is required, otherwise \( f(n) \leq 0 \), and there is no boundary of \( n_2 \), which means \( n_2 = 0 \) in all regions. From that we can obtain the condition for \( \tilde{B} \) as

\[
\tilde{B} < \tilde{B}_{max} = \frac{1}{2} \left( \frac{4}{9g^2} - \frac{8}{27g^2} \right) = \frac{2}{27g^2} \approx 0.074g^{-2}.
\]

The long-dashed represents \( \tilde{B} = 2/27g^{-2} \approx 0.074g^{-2} \) for \( g > 1/\sqrt{2} \), which means the border between the hollow and single spin phases.

In Fig. 3 we plot the number asymmetry between two components \((N_1 - N_2)/N_T\), where \( N_T = N_1 + N_2 \). When the external magnetic field is small, the asymmetry of the fermion number is very small and almost independent of the coupling in the paramagnetic phase (\( g \lesssim 1.0 \)) except when the coupling is very small, where the system is close to the border between the paramagnetic and single spin phases. As the coupling increases further and exceeds about \( g \approx 1.0 \), the system is changed from the paramagnetic spin phase to the ferromagnetic one. The number asymmetry of the fermion number also increases As the external magnetic field becomes larger, the change of the number asymmetry is not drastic, though the rapid change of the dipole frequency still remains.
III. DIPOLE EXCITATIONS

In this section we derive an expression for the dipole frequency using the scaling method. Without loss of generality, we take the direction of motion along the $z$-axis. The time-varying density will be parameterized with a collective coordinate $\lambda_i$ as

$$\rho_i(r) = \rho_i^{(0)}(r_1, r_2, r_3 - \lambda_i),$$  \hspace{1cm} (10)

where $\rho_i^{(0)}(r)$ is the density distribution of the ground state. Substituting eq.(10) into eq. (1), we can obtain the variation of the total energy up to the order of $O(\lambda^2)$ as

$$\Delta E_T = E_T - E_T^{(0)} \approx \frac{1}{2} m \Omega_L^2 (\lambda_1^2 N_1 + \lambda_2^2 N_2) + A(2\lambda_1 \lambda_2 - \lambda_1^2 - \lambda_2^2),$$  \hspace{1cm} (11)

where $E_T^{(0)}$ is the ground state energy ($\lambda_i = 0$), and

$$A = \frac{g}{2} \int d^3r \frac{\partial \rho_i^{(0)}}{\partial z} \frac{\partial \rho_j^{(0)}}{\partial z}.$$  \hspace{1cm} (12)

When we consider the time-dependence of $\lambda_i$, the mass parameter with respect to $\lambda_i$ is obtained as $mN_i$. Then the classical equation of motion for $\lambda$ is harmonic, giving rise to the following eigenvalue equation for the oscillation frequencies,

$$\begin{pmatrix}
  m(\Omega_L^2 - \omega^2)N_1 - 2A & 2A \\
  2A & m(\Omega_L^2 - \omega^2)N_2 - 2A
\end{pmatrix}
\begin{pmatrix}
  \lambda_1 \\
  \lambda_2
\end{pmatrix} = 0.$$  \hspace{1cm} (13)

The eigenvalues of this equation are given

$$\omega^2 = \Omega_L^2 - (A/mN_1 + A/mN_2) \pm (A/mN_1 + A/mN_2).$$  \hspace{1cm} (14)

One eigenvalue is $\omega = \Omega_L$ with the eigenvector $\lambda_1 = \lambda_2$. This is the in-phase oscillation of the system which follows Kohn’s theorem.

The other eigenvalue is

$$\omega = \frac{\omega_D}{\Omega_L} \equiv \{\Omega_L^2 - (2A/mN_1 + 2A/N_2)\}^{\frac{1}{2}}.$$  \hspace{1cm} (15)

The eigenvector is given by $\lambda_1/N_1 = -\lambda_2/N_2$. In this mode, the two components move in opposite directions keeping their center of mass at rest.

Here we should give a comment. If we consider the dipole oscillation in the transverse relation, the frequency normalized by the transverse trapped frequency $\omega_D/\Omega_T$ is also equivalent to the right-hand-side of eq. (15).
Using the variable transformation explained in the previous sections, the dipole frequency is written as
\[
\omega_D/\Omega_L(T) = \left\{1 - \tilde{A} \left( \frac{1}{N_1} + \frac{1}{N_2} \right) \right\}^{1/2},
\]
where
\[
\tilde{A} = \frac{g}{6} \int d^3x \frac{\partial n_1}{\partial x} \frac{\partial n_2}{\partial x},
\]
\[
\tilde{N}_i = \int d^3x n_i = \frac{2m^9}{3^5 \pi^{10} \hbar^{15}} N_i.
\]

In Fig. 4 we show the frequency of the dipole oscillation in the out-of-phase oscillation as a function of the coupling constant, \(g\), with various external magnetic fields, \(g^2 \tilde{B} = 1.0 \times 10^{-4}\) (dashed), \(1.0 \times 10^{-3}\) (solid), \(1.0 \times 10^{-2}\) (long-dashed), and \(5.0 \times 10^{-2}\) (chain-dotted), respectively. For comparison we also show the frequencies in the symmetric system in zero magnetic field (dotted line).

As the coupling becomes larger, the dipole frequency decreases monotonically until \(g \approx 1.0\), and sharply increases above that. For \(g \lesssim 1.0\), where the number asymmetry is small, the frequency \(\omega_D\) does not strongly depend on the external magnetic field. In the paramagnetic spin phase \((g \lesssim 1)\), the density distribution of the minor component of fermion \(n_2\) is similar to that of the major component \(n_1\), and the integrand in \(\tilde{A} > 0\) is positive in all regions. As the coupling increases, the size of the fermi gas becomes larger, and then the frequency \(\omega_D\) monotonously decreases.

As the coupling becomes \(g \gtrsim 1\) one component of the fermions is partially converted into the other component, and ferromagnetism appears in the central region. In Fig. 4, \(\partial n_2/\partial x > 0\) for \(x < x_c\) and \(\partial n_2/\partial x < 0\) for \(x > x_c\), where \(x_c \approx 0.32\); namely there is a ferromagnetic region for \(x < x_c\). The contribution from this ferromagnetic region to \(\tilde{A}\) is negative. As the coupling increases, the critical position \(x_c\) moves to the surface, \(\tilde{A}\) becomes smaller, and then \(\omega_D\) increases.

In the case of a strong magnetic field, the qualitative behavior is similar. In strong coupling \(g \gtrsim 1\) the slope of the density function for the minor fermion \(n_2\) is smaller in the strong magnetic field than that in the weak magnetic field. As the external magnetic field increases, the dipole frequency becomes smaller.
In this section we study the monopole oscillation in spherical trap, $Ω_T = Ω_L = Ω_M$. Generally the monopole and quadrupole oscillations are coupled, but qualitative properties are not so significant difference between the two collective oscillations. In order to study the monopole oscillation, we introduce the following scaling:

$$\rho_i(r) = e^{3\lambda_i} \rho_i^{(0)}(e^{\lambda_i} r).$$  \hspace{1cm} (19)$$

Under this scaling the total energy becomes

$$E_T = \sum_{i=1,2} \{e^{2\lambda_i} T_i + e^{-2\lambda_i} U_i\} + V_{12},$$  \hspace{1cm} (20)$$

where the $T_i$ and $U_i$ are the kinetic and harmonic oscillator energy parts in the ground state, respectively. The interaction energy $V_{12}$ appears

$$V_{12} = ge^{3\lambda_1+3\lambda_2} \int d^3 r \rho_1^{(0)} (e^{\lambda_1} r) \rho_2^{(0)} (e^{\lambda_2} r)$$

$$\approx -g \int d^3 r \{\lambda_1 \rho_1^{(0)} r \frac{\partial}{\partial r} \rho_2^{(0)} + \lambda_2 r \frac{\partial}{\partial r} \rho_1^{(0)} \rho_2^{(0)}\} + K_{11} \lambda_1^2 + K_{22} \lambda_2^2 + 2K_{12} \lambda_1 \lambda_2$$  \hspace{1cm} (21)$$

with

$$K_{11} = \frac{g}{2} \int d^3 r \{(3+r \frac{\partial}{\partial r}) \rho_1^{(0)} \{r \frac{\partial}{\partial r} \rho_2^{(0)}\},$$  \hspace{1cm} (22)$$

$$K_{22} = -\frac{g}{2} \int d^3 r \{r \frac{\partial}{\partial r} \rho_1^{(0)} \{(3+r \frac{\partial}{\partial r}) \rho_2^{(0)}\},$$  \hspace{1cm} (23)$$

$$K_{12} = \frac{g}{2} \int d^3 r \{(3+r \frac{\partial}{\partial r}) \rho_1^{(0)} \{(3+r \frac{\partial}{\partial r}) \rho_2^{(0)}\}.$$  \hspace{1cm} (24)$$

The mass parameter of the monopole oscillation is given by the mass times the mean square radius. Then the monopole oscillation mode and frequencies $\omega_M$ are obtained by

$$[B_M \omega^2 - C_M] \lambda = 0.$$  \hspace{1cm} (25)$$

with

$$B_M = \frac{m}{2} \begin{pmatrix} X_1 & 0 \\ 0 & X_2 \end{pmatrix},$$  \hspace{1cm} (26)$$

$$C_M = \begin{pmatrix} C_{11} & K_{12} \\ K_{12} & C_{22} \end{pmatrix} = \begin{pmatrix} 2T_1 + 2U_1 + K_{11} & K_{12} \\ K_{12} & 2T_2 + 2U_2 + K_{22} \end{pmatrix},$$  \hspace{1cm} (27)$$

where

$$X_i = \int d^3 r \rho_i(r) r^2.$$  \hspace{1cm} (28)$$
Using the variable transformation explained in the Sec. II,

$$\left\{ \frac{\omega_M}{\Omega_M} B_M - \tilde{C}_M \right\} \lambda = 0 \quad (29)$$

with

$$\tilde{B}_M = \begin{pmatrix} \tilde{X}_1 & 0 \\ 0 & \tilde{X}_2 \end{pmatrix}, \quad (30)$$

$$\tilde{C}_M = \begin{pmatrix} 2 \tilde{T}_1 + 2 \tilde{X}_1 + \tilde{K}_{11} & \tilde{K}_{12} \\ \tilde{K}_{12} & 2 \tilde{T}_2 + 2 \tilde{X}_2 + \tilde{K}_{22} \end{pmatrix}, \quad (31)$$

where

$$\tilde{X}_i = \int d^3 x \ n_i(x) x^2 \quad (32)$$

$$\tilde{T}_i = \frac{3}{5} \int d^3 x \ [n_i(x)]^{\frac{5}{2}}, \quad (33)$$

$$\tilde{K}_{11} = -\frac{g}{2} \int d^3 x \ (3n_1 + x \frac{\partial n_1}{\partial x})(x \frac{\partial n_2}{\partial x}), \quad (34)$$

$$\tilde{K}_{22} = -\frac{g}{2} \int d^3 x \ (x \frac{\partial n_1}{\partial x})(3n_2 + x \frac{\partial n_2}{\partial x}), \quad (35)$$

$$\tilde{K}_{12} = \frac{g}{2} \int d^3 x \ (3n_1 + x \frac{\partial n_1}{\partial x})(3n_2 + x \frac{\partial n_2}{\partial x}) = \frac{g}{2} \int d^3 x \ \left( x^2 \frac{\partial n_1}{\partial x} \frac{\partial n_2}{\partial x} \right). \quad (36)$$

In the symmetric system $\rho_1 = \rho_2$ the monopole frequencies are given by $\omega_M^2 = (C_{11} + K_{12})/mX_1$ and $(C_{11} - K_{12})/mX_1$. The eigenvector becomes $\lambda_1 = \lambda_2$ (in-phase) in the respect to the former frequency, and $\lambda_1 = -\lambda_2$ (out-of-phase) in the latter frequency.

In Fig. 5 we show the frequencies of monopole oscillations as functions of the coupling constant $g$ with $g^2 \tilde{B} = 1.0 \times 10^{-4}$ (a) and $1.0 \times 10^{-2}$ (b). The solid and dashed lines represent the smaller and larger frequencies of two modes, which we call mode-1 and mode-2, respectively.

In Fig. 5a we also plot the frequencies of out-of-phase and in-phase modes in the symmetric system without external magnetic field with the thin long-dashed and dotted lines, respectively. The frequencies of the in-phase oscillation in the symmetric system is about $\omega_M \approx 2\Omega_M$, which is the frequency in a non-interacting trapped ideal gas, for any asymmetry. As the coupling constant increases, on the other hand, the frequency of the out-of-phase mode monotonously decreases.

In the weak coupling $g \lesssim 1$, the frequencies of the mode-1 and mode-2 are almost same as those of the in-phase and out-of-phase modes in the symmetric system. As the coupling constant increases, on the other hand, the frequency of the mode-2 rapidly increases above $g \approx 1$. When the coupling constant increases further, the frequency of the out-of-phase mode approaches to that of the in-phase mode, and the level mixing between the two modes occurs. Even after the level mixing, the frequency of the mode-1 increases while the frequency of the mode-2 rarely vary. From those we can suppose that the modes-1 and -2 are almost out-of-phase and in-phase modes, though the two modes are mixed and exchange their roles in strong coupling.
In order to confirm that, here, we calculate the mixing angle \( \theta_m \) defined as

\[
\lambda = \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \cos \theta_m + \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \end{pmatrix} \sin \theta_m,
\]

(37)

where \( \lambda \) is an eigenvector obtained from eq. (29). When \( \theta_m = 0, \pi \) and \( \theta_m = \pi/2 \), the modes exhibit the pure in-phase and out-of-phase monopole oscillation of two-component fermions.

In Fig. 6 we show the results with the external magnetic fields \( g^2 \tilde{B} = 1.0 \times 10^{-4} \) (a) and \( g^2 \tilde{B} = 1.0 \times 10^{-2} \) (b). In the case of the small magnetic field (Fig. 6a), the two modes, the mode-1 and the mode-2, are almost clear out-of-phase and in-phase oscillations before the level mixing. Around \( g \approx 1.2 \), the modes-1 and -2 are mixed, and the two collective modes exchange their roles in the strong interaction limit. This feature is less clear, but also seen in the case of the strong magnetic field (Fig. 6b).

V. SUMMARY

In this paper we study the spin excitation on the dipole and monopole oscillations in the asymmetric two-component fermion condensed system using the scaling method. As for the dipole oscillations the in-phase and the out-of-phase modes are completely decoupled even in the asymmetric system. The two modes can be coupled in the monopole oscillation, but these two modes are rarely mixed except when their frequencies are close.

In all kinds of oscillations the in-phase motions do not have any special behavior even if the two components are largely asymmetric. The frequencies of the in-phase modes are not so different from those of the non-interacting system.

On the other hand the frequencies of these out-of-phase oscillations monotonously decreases as the repulsive force between the two kinds of fermions becomes larger. As the coupling further increases, the system changes to ferromagnetic and, the frequencies of the out-of-phase oscillations very suddenly increase, thought those of the in-phase oscillations do not vary even if the ferromagnetism appears. In addition the frequencies and M1 excitation strength show differences between different number asymmetry for \( g \gtrsim 1 \), where the ferromagnetism appears.

Thus significant information of many body systems can be gotten only from the out-of-phase oscillations. In this work we describe the two components of fermions with different two spin states. It may be thought to be difficult to establish such experiments because we need two kinds of external magnetic fields to control the interaction and the asymmetry. However a similar work can be done in systems including two different kinds of fermi gases, where we do not need an external magnetic field to construct the asymmetry. If we use two kinds of fermions with different masses, we can study a variety of systems.

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FIG. 1: The scaled density distribution of two components fermion at $g = 0.95$ (a), $g = 1.05$ (b) and $g = 1.15$ (c) with $e_f = 20/27$ and $g^2 \tilde{B} = 1.0 \times 10^{-4}$. The solid and dashed lines represents the distribution of the major and minor components, respectively.
FIG. 2: The phase diagram of the two components of fermion. The symbols "S", "P", "F" and "H" denote the region of the single-component, paramagnetic, ferromagnetic, single and hollow phases.
FIG. 3: The asymmetry of the two components as a function of the coupling constant $g$, at $e_f = 20/27$. The dashed, sold, long-dashed and dotted lines represent the results with the external magnetic fields $g^2 \tilde{B} = 1.0 \times 10^{-4}$, $1.0 \times 10^{-3}$, $1.0 \times 10^{-2}$ and $5.0 \times 10^{-2}$, respectively.
FIG. 4: The frequencies of the dipole oscillations as a function of the coupling constant $g$. The dashed, sold, long-dashed and dotted lines represent the results with the external magnetic fields $g^2 \tilde{B} = 1.0 \times 10^{-4}$, $1.0 \times 10^{-3}$, $1.0 \times 10^{-2}$ and $5.0 \times 10^{-2}$, respectively. The dotted line denote the result of the symmetric system.
FIG. 5: The frequency of the monopole oscillations with the external magnetic fields with $g^2 \tilde{B} = 1.0 \times 10^{-4}$ (a) and $1.0 \times 10^{-2}$ (b). The thick solid and dashed lines represent the results of the out-of-phase and in-phase oscillation modes, respectively. The thin solid and dashed lines denote the result the out-of-phase and in-phase oscillation modes of the symmetric system, respectively.
FIG. 6: The mixing angle of the two modes of the monopole oscillation with $g^2 \tilde{B} = 1.0 \times 10^{-4}$ (a) and $1.0 \times 10^{-2}$ (b). The thick solid and dashed lines represent the results of the mode-1 and mode-2, respectively.