S-Duality as an Open String Gauge Symmetry

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Abstract

We demonstrate and discuss the connection between S-duality and string-theoretic picture-changing gauge transformation in the “infinite momentum frame”. The picture-changing transformation at zero momentum leads to the presence of the topological 5-form charge in the type IIB superalgebra (dimensionally reduced SUSY algebra of M-theory) which is attributed to the M5-brane. The topological charge defines the D5-brane state, which also is the analogue of the monopole part of the Olive-Witten’s result in field theory. The correlation functions involving this monopole-like state are computed. The five-brane states are associated with the ghost number cohomologies which we introduce in the paper. Keywords: S-Duality, picture-changing, M-theory. PACS: 04.50.+h; 11.25.Mj.
1. Introduction

The recent progress in the attempts to explore non-perturbative aspects of theories of extended objects, such as strings and p-branes, involves many remarkable features. One of them is the M-theory, the theory in eleven dimensions whose low-energy limit is the $D = 11$ supergravity. It was shown that various formulations of string theories (such as type I, IIA, heterotic $E_8 \times E_8$ or $SO(32)$ and, not without difficulties, the type IIB) and various duality symmetries that occur between them and between other p-branes follow from M-theory. Thus the M-theory appears to be a possible candidate for the underlying unification theory.

Another development is the idea of the “p-brane democracy” according to which all the extended p-dimensional objects propagating in a space-time appear on equal ground, playing equally important role from the point of view of non-perturbative physics and strings are no longer special, with the exception of the perturbative expansion (see, for instance, [2]). Apart from qualitative arguments, the practical realization of the idea of p-brane democracy is problematic, since no systematic quantization is known for p-branes; thus, for instance, the S-duality for branes is formulated on the level of their low-energy effective field theories (whose actions may also be rather conjectured than rigorously derived). One may consider, however, the different approach which still regards the theory of open strings as “more equal than others” [3,4,5] even from the non-perturbative point of view, i.e. the p-brane democracy is complemented by the “open-string aristocracy” (in the terminology of [4,3]). The basic idea is that the non-perturbative phenomena in the physics of branes, such as dualities may be understood from essentially perturbative computations in open string theories. Roughly speaking, it is the presence of the worldsheet boundaries that gives rise to the non-perturbative effects. D-branes are perhaps the best known example of that; there the massless excitations of an open string with the ends attached to a hyperplane become the collective coordinates for the transverse fluctuations of the brane. It was shown [3] that these objects are BPS saturated and carry the full set of the Ramond-Ramond charges. In this paper we shall consider the particular example of how the non-perturbative symmetries (dualities) between branes can be understood in terms of the perturbative open-string physics. Namely, we shall try to show that the well-known S-duality between strings and fivebranes (which also follows from the M-theory) is closely related to the degenerate case of a specific gauge symmetry in superstring theory - the picture-changing at zero momentum.
As for the M-theory, unfortunately very few quantitative facts are still known about its dynamics apart from the low-energy limit. One particularly promising approach is to try to formulate M-theory as a matrix model [7,8]. In this M(atrix) approach the crucial role is played by the eleven-dimensional SUSY algebra, extended by p-form central charges corresponding to p-brane states [9,10,8]. More precisely, p-brane solutions give rise to topological p-form charges, known as Page charges, which become generators in the new extended superalgebras, along with the supercharge and the momentum generator. For instance, the anticommutator of the supercharges becomes

\[ \{Q_\alpha, Q_\beta\} = \Gamma^\mu_{\alpha\beta} P_\mu + \sum_p \Gamma^{\mu_1...\mu_p}_{\alpha\beta} Z_{\mu_1...\mu_p}, \]

where each (super) p-form charge \( Z_{\mu_1...\mu_p} \) in the sum corresponds to (super) p-brane soliton [9]. The origin of these charges is supposedly related to Wess-Zumino terms in the corresponding super p-brane actions [11].

This may be illustrated on the example of supermembrane solution of \( D = 11 \) supergravity where the expression for \( Z_{\mu_1\mu_2} \) can be found explicitly [12]. Indeed, in the \( D = 11 \) supermembrane action the Wess-Zumino term is supersymmetric only up to a total derivative term; as a result integrating this boundary term over the surface of a membrane leads to the presence of the following following 2-form charge in the SUSY algebra:

\[ Z_{\mu_1\mu_2} = \int d^2\sigma \epsilon^{012} \partial_i X^{\mu_1} \partial_j X^{\mu_2} \]

This integral is nonzero if the membrane configuration defines a non-trivial 2-cycle in eleven dimensions.

As is well-known, the D=11 supergravity, the low-energy limit of the M-theory, has two non-trivial solutions known as M-branes: one is the electric-type (the membrane) and another is the magnetic type (five-brane). These solutions are shown to be related by S-duality. The \( M \)-algebra therefore includes the following generators: \( \{Q^\alpha, P_\mu, Z^{A_1A_2}, Z^{A_1...A_5}\} \), where \( A = (\mu, \alpha) \). There is also a super one-form \( Z^A \), which presence in the \( M \)-algebra is quite puzzling. Its fermionic part must be related to the \( \kappa \)-symmetry; in the context of Green-Schwarz superstring theory it has been discussed in [13]. In the \( D = 10 \) context, the presence of the bosonic component is probably related to the subtleties of picture-changing at zero momentum (which is a degenerate case of BRST gauge transformation). Note that, while locally the bosonic operator \( Z^\mu \) can be absorbed into redefinition of the momentum operator, it is not always possible globally; and in string theory the operator \( Z^\mu \) has
been shown to play an interesting role in describing the string winding states \[\text{[14]}\]. The (anti)commutation relations of the M-algebra and proof that its generators satisfy Jacobi identities have been given in \[\text{[10]}\]. For future reference, some of these (anti)commutators are given by:

\[
\{Q_\alpha, Q_\beta\} = \Gamma_{\alpha\beta}^\mu (P^\mu + Z^\mu) + \Gamma_{\alpha\beta}^{\mu_1,\mu_2} Z_{\mu_1,\mu_2} + \Gamma_{\alpha\beta}^{\mu_1,\ldots,\mu_5} Z_{\mu_1,\ldots,\mu_5}
\]

\[
[P_\mu, Q_\alpha] = \Gamma_{\alpha\beta}^\nu Z^\beta - \Gamma_{\alpha\beta}^{\mu\nu} Z^\beta - \Gamma_{\alpha\beta}^{\mu_1,\ldots,\mu_4} Z_{\mu_1,\ldots,\mu_4}
\]

\[
[P_{\mu_1}, P_{\mu_2}] = \Gamma_{\alpha\beta}^{\mu_1,\mu_2} Z^\alpha Z^\beta + \Gamma_{\alpha\beta}^{\mu_1,\ldots,\mu_5} Z_{\mu_3,\ldots,\mu_5}
\]

\[
[Q_\alpha Z^\mu] = \Gamma_{\alpha\beta}^\mu Z^\beta,
\]

\[
\ldots\text{etc.}
\]

The covariant $\kappa$-symmetric action for super five-brane has been recently constructed \[\text{[15,16,17,18,19]}\], yet its relation to the 5-form of (3) is yet to be pointed out. In general, the explicit expressions for p-form charges are not known, as even the classical actions are not yet known for some super p-branes, let alone more complicated cases of intersecting branes. At the same time, it is expected that the M-algebra shall play an important role in the M-theory and in describing the non-perturbative physics of extended objects \[\text{[10]}\]. One particular question of interest is what are T- and S-dualities in the context of extended Poincare superalgebras. It shall also be of importance to understand the relation between the M-algebra and D-branes, especially given the role that D0 - branes are conjectured to play in describing the degrees of freedom in the M-theory apart from the low-energy limit \[\text{[7]}\]. Recently it has been shown that the SUSY algebra of the (M)atrix model \[\text{[8]}\] does not contain the transverse fivebrane charge, which creates problems with Lorentz invariance. It is therefore of importance to understand the role that the fivebrane plays in the (M)atrix theory. In this paper we shall discuss these questions by considering chiral reductions of the M-algebra to $D = 10$. The analysis will show a surprising relation between picture-changing formalism in string theory and S-duality; namely we will see that the picture changing transformation of the D=10 IIB Poincare superalgebra gives rise to the fivebrane state of the M-theory and will compute the S-matrix elements involving this solitonic state. S-duality shall be interpreted then as a non-perturbative counterpart of the gauge symmetry defined by picture-changing transformations.

2.Duality and Picture - Changing
Consider the chiral reduction of the $M$-algebra (3) to $D = 10$ which yields the $M$-extension of the usual (1,0) Poincare superalgebra [20]:

\[
\{Q_{\alpha}, Q_{\beta}\} = \Gamma_{\alpha\beta}^{\mu}(P_{\mu} + Z_{\mu}) + \Gamma_{\alpha\beta}^{\mu_1...\mu_5} Z_{\mu_1...\mu_5}
\]

\[
[P^\mu, Q_{\alpha}] = \Gamma_{\alpha\beta}^{\mu} Z_{\beta} - \Gamma_{\alpha\beta}^{\mu_1...\mu_4} Z_{\mu_1...\mu_4}
\]

\[
[P^{\mu_1}, P^{\mu_2}] = \Gamma_{\alpha\beta}^{\mu_1...\mu_5} Z_{\mu_3...\mu_5}
\]

\[
[Q_{\alpha}, Z^{\mu}] = -\Gamma_{\alpha\beta}^{\mu} Z_{\beta}
\]

\[
[Q_{\alpha}, Z^{\mu_1...\mu_5}] = -\Gamma_{\alpha\beta}^{\mu_1...\mu_5} Z_{\beta} + \Gamma_{\alpha\beta}^{\mu_5} Z_{\mu_1...\mu_4\beta}
\]

...etc. (4)

Compared with the M-algebra (3), the two-form is now gone and only one- and five-form charges are left, in agreement with the S-duality between strings and fivebranes in the $D = 10$ supergravity which is the low-energy limit of Green-Schwarz superstring theory. We are going to show that:

1) the appearance of the five-form in the superalgebra (14) follows from picture-changing transformation of supercharges;

2) the picture-changing transformation is to be considered as a “generator” of S-duality. Consider the charges $Q_{\alpha}$ of (4) which are given in the NSR superstring theory by [21]:

\[
Q_{\alpha} = \oint \frac{dz}{2i\pi} e^{-\phi} \Sigma_{\alpha}(z)
\]

(5)

Here $\phi(z)$ is a bosonized superconformal ghost, $\Sigma_{\alpha}$ is spin operator for matter fields. Recall that the O.P.E. between two $\Sigma$’s is given by

\[
: \Sigma_{\alpha}(z) : \Sigma_{\beta}(w) \sim \frac{\epsilon_{\alpha\beta}}{(z-w)^{\frac{3}{2}}} + \sum_{p} \frac{\Gamma_{\alpha\beta}^{\mu_1...\mu_p}}{(z-w)^{\frac{3}{2} - \frac{p}{2}}} : \psi_{\mu_1}...\psi_{\mu_p}(z) + \text{derivatives}...
\]

(6)

where $\psi$’s are NSR worldsheet bosons and the superconformal ghosts $\beta, \gamma$ are bosonized as $\gamma = e^{\phi - \chi}; \beta = e^{\chi - \phi} \partial \chi$; and $- < \phi(z)\phi(w) >= < \chi(z)\chi(w) >= \log(z-w)$. Evaluating the anticommutator of two $Q$’s of (5) then gives

\[
\{Q_{\alpha}, Q_{\beta}\} = \oint \frac{dw}{2i\pi} \frac{1}{z-w} \oint \frac{dz}{2i\pi} \Gamma_{\alpha\beta}^{\mu} e^{-\phi} \psi_{\mu}(z) = \Gamma_{\alpha\beta}^{\mu} P_{\mu}
\]

(7)

where $P_{\mu} = \oint \frac{dz}{2i\pi} e^{-\phi} \psi_{\mu}$ is an integral of a momentum generator in the $-1$-picture; i.e. we get the standard result for superalgebra in Green-Schwarz superstring theory in a flat space-time. The supercharge of (5)-(7) has been taken in the standard $-\frac{1}{2}$-picture. Let us
further consider the commutator 
\[ [P^\mu, Q_\alpha] = \Gamma^\mu_{\alpha\beta} T_\beta \]
Here \( T_\beta \) is a new fermionic generator \[14,10\] We related it to the NSR formulation of the \( \kappa \)-symmetry generator in \[13\]. The computation using (5) gives
\[ [P^\mu, Q_\alpha] = \Gamma^\mu_{\alpha\beta} \oint \frac{dz}{2i\pi} e^{-\frac{3}{2}\phi} \Sigma_\beta \equiv \Gamma^\mu_{\alpha\beta} T_\beta \tag{8} \]
It is the integrand of \( T_\beta \) that can be shown to generate the \( \kappa \)-symmetry transformations in Green-Schwarz superstring theory, up to picture-changing transformations. Next, consider the anticommutator \[\{T_\alpha, T_\beta\}\]. It may be regarded as the anticommutator of two \( \kappa \)-transformations but, more importantly, it also has a subtle connection directly to the anticommutator of two supercharges in the supersymmetry algebra, which we shall point out later. The computation gives:
\[ \{T_\alpha, T_\beta\} = \oint \frac{dz}{2i\pi} \left( \frac{1}{2} \Gamma^\mu_{\alpha\beta} e^{-3\phi} \psi_\mu \left( \frac{9}{8} \partial \phi \partial \phi - \frac{3}{2} \partial^2 \phi \right) - \frac{3}{2} \partial \psi_\mu \partial \phi + \frac{1}{\overline{2}} \partial^2 \psi_\mu \right) + \Gamma^\mu_{\alpha\beta \mu_2} \partial (e^{-3\phi} \psi_{\mu_1} \psi_{\mu_3}) + \Gamma^\mu_{\alpha\beta \mu_2 \mu_3} e^{-3\phi} \psi_{\mu_1} \psi_{\mu_3} \tag{9} \]
Let us analyze the r.h.s. of this anticommutator. Consider the picture-changing gauge transformation, defined by the picture-changing operator:
\[ \Gamma_1 =: \delta(\gamma) (S_{\text{matter}} + S_{\text{ghost}}) := -\frac{1}{2} e^\phi \psi_\mu \partial X^\mu + e^{2\phi - \chi} b \partial \phi + e^\chi \partial \chi \tag{10} \]
Here \( S_{\text{matter}}, S_{\text{ghost}} \) are matter and ghost worldsheet supercurrents (with the superconformal ghosts \( \beta, \gamma \) being bosonized as \( \gamma = e^{\phi - x}, \beta = e^{x - \phi} \partial X, < \chi(z) \chi(w) >= - < \phi(z) \phi(w) >= \log(z - w) \)). Then, applying it twice to the sum of the first three terms in the r.h.s. of the anticommutator, we get the result equal to \( \frac{1}{16} \Gamma^\mu_{\alpha\beta} e^{-\phi} \psi_\mu \), i.e. the expression proportional to the momentum operator in the \(-1\)-picture. In other words, the r.h.s. of the anticommutator of two fermionic generators \( T \) reproduces the anticommutator \( \{Q, Q\} \) in the non-perturbative version (4) of the superalgebra in \( D = 10 \), with the five-form (fivebrane) central term proportional to \( e^{-3\phi} \psi_{\mu_1} \psi_{\mu_3} \), with the exception of the total derivative three-form term, proportional to \( \sim \partial (e^{-3\phi} \psi_{\mu_1} \psi_{\mu_3}) \) The presence of the total derivative three-form term may be interpreted as the Guven threebrane which, in turn, is nothing but the intersection of two fivebranes. Thus, one may suspect the correspondence between total derivative central terms in the super(current) algebra and intersecting branes. From now on, we will concentrate on a fivebrane central term, which appears to be more fundamental. In order to understand its relevance to the superalgebra, it is crucial
to point out the connection between the fermionic generators $T_\alpha$ and the supercharges $Q_\alpha$ of (5), which determine the regular superalgebra without central terms. Contrary to what one might suspect, $T_\alpha$ and $Q_\alpha$ are not related by the picture-changing transformation, therefore we cannot immediately interpret $T_\alpha$ just as a supercharge in another picture. Indeed, the naive application of the picture-changing operator $\Gamma_1$ to $T_\alpha$ gives zero (this is true for the terms proportional to $S_{\text{matter}}$ and to the ghost field $b$, and the term proportional to $c$ does not apply since $T_\alpha$ is a contour integral $[21][22]$). Therefore the relation between $Q_\alpha$ and $T_\alpha$ is rather subtle. Namely, consider the vertex operator

$$v_\alpha(k,\bar{k})R_\alpha = v_\alpha(k,\bar{k})e^{-\frac{1}{2}(\Sigma_\alpha e^{ikX} + \text{ghosts})}$$

where the ghost terms (not significant for correlation functions and skipped here) are introduced to insure the BRST invariance. Here $k$ is the momentum, and the auxiliary momentum $\bar{k}$ is defined so that $(k,k) = 0, (\bar{k},\bar{k}) = 0, (k,\bar{k}) = 1$, i.e. its definition is similar to the one for the dilaton vertex operator. $u_\alpha(k)$ is some constant on-shell space-time spinor. The crucial difference, however, is that the polarization tensor $v_\alpha(k,\bar{k})$ is no longer transverse, i.e. it does not satisfy the on-shell Dirac equation. Therefore the vertex $R_\alpha$ is not BRST-invariant for arbitrary $k,\bar{k}$. It is BRST-invariant, however, in the limit $k \rightarrow 0, \bar{k} \rightarrow \infty$, and in that limit the picture-changing transformation can be applied. We will refer to it as a “picture-changing in the infinite-momentum frame”, because of the mentioned $\bar{k} \rightarrow \infty$ limit. Therefore, formally we have

$$: \Gamma_1 \int \frac{dz}{2i\pi} v_\alpha(k,\bar{k})R_\alpha := u_\alpha(k)Q_\alpha$$

Now, the relation between $T$ and $Q$ is given by

$$Q_\alpha = : N_{\alpha\beta} T_\beta :$$

where the operator $N_{\alpha\beta}$ is defined as the “infinite-momentum” generalisation of the picture-changing operator $\Gamma_1$ - that is, for a local fermionic vertex at zero momentum $V_\alpha(z,k = 0) \equiv V_\alpha(z)$ such that $: \Gamma_1 V_\alpha := 0$,

$$: N_{\alpha\beta} V_\beta : (z) = \lim_{\bar{k} \rightarrow \infty, k \rightarrow 0} (\Gamma_0)^{\alpha\beta} : \Gamma_1 V_\beta e^{ikX} : (z)$$

We see that the operator $N_{\alpha\beta}$, which essentially is the special singular limit of the picture-changing transformation, maps the perturbative SUSY algebra without central
charges to the non-perturbative one, containing the fivebrane. In the limit $k \to 0 \, N_{\alpha \beta}$ effectively replaces the picture-changing, which is not well-defined at zero momentum \cite{21}. Before going further, let us comment on what appears to be a paradox - the fact that the anticommutator of two kappa-transformations gives something like the supersymmetry algebra. It is well-known that $\kappa$-symmetry algebra is quite different from the superalgebra, i.e. the combination of two kappa-transformations is again a kappa-transformation. At first glance, it is not consistent with the relation (9) The answer, however, is that the structure constants in the $\kappa$-symmetry algebra depend on superspace coordinates, namely they are proportional to $\partial \theta_{\alpha}$ where $\theta_{\alpha}$ is a GS variable, corresponding to $\sim e^{\frac{2}{\phi}} \Sigma_{\alpha}$ in the NSR formalism. The O.P.E of this fermionic structure constant with the $\kappa$-symmetry generator must be then normally ordered, giving rise to the bosonic generator, proportional to the picture-changed momentum plus central terms. Let us now return to the five-form central term in the anticommutator (9), which defines the “fivebrane emission vertex” at zero momentum. This open-string vertex is rather peculiar since it does not seem to describe the emission of any massless particle in the theory of open strings, at the same time it is not BRST trivial. Also, the ghost terms must be added to it in order to make its BRST-invariance manifest; however, because these terms are cumbersome and because they do not affect correlators, we will skip them. There is no analogue of this vertex in a picture with ghost number zero, which is quite an unusual situation for bosonic vertex operators. In other words, the essentially non-zero ghost number is a crucial factor for such a “brane emission vertex”. It has, however, the picture-changed counterpart in the $+1$-picture. Recall that the higher picture-changing operators of ghost number $n \, \Gamma_n$ (in the perturbative string theory, $\Gamma_n =: (\Gamma_1)^n : + \{ Q_{\text{BRST}}, \ldots \}$ for physical vertex operators at non-zero momentum) are given by

$$\Gamma_n = e^{n \phi} S \partial S \ldots \partial^{n-1} S$$

(15)

where $S = S_{\text{matter}} + S_{\text{ghost}}$ is the sum of matter and ghost worldsheet supercurrents. Then

$$: \Gamma_4 e^{-3 \phi} \psi_{\mu_1} \ldots \psi_{\mu_5} := e^{\phi} \psi_{\mu_1} \ldots \psi_{\mu_5}$$

(16)

Of course, this alternative formulation of the five-form term has no analogue in the 0-picture as well. Analogously, the counterpart of the three-brane total derivative central term in the $+1$-picture is given by $\partial (e^{\phi} \psi_{\mu_1} \ldots \psi_{\mu_3})$. One may check that

$$\{ : \Gamma_2 T_{\alpha} :, : \Gamma_2 T_{\beta} \} = \Gamma^\mu_{\alpha \beta} \, : \Gamma_2 e^{- \phi} \psi_{m \mu} : + \Gamma_{\alpha \beta}^{\mu_1 \ldots \mu_3} \partial (e^{\phi} \psi_{\mu_1} \ldots \psi_{\mu_3}) + \Gamma_{\alpha \beta}^{\mu_1 \ldots \mu_5} e^{\phi} \psi_{\mu_1} \ldots \psi_{\mu_5}$$

(17)
For the sake of convenience, from now on we will be using the version of the five-form term in the +1-picture in our analysis, since its properties are quite equivalent to its −3-picture counterpart. In general, any brane emission vertex in a negative picture appears to have the equivalent counterpart in the non-negative picture, even though picture-changing for such non-perturbative vertices involves many subtleties. It is therefore enough to consider non-negative pictures to study the properties of such brane vertex operators. As we have already mentioned, the remarkable property of the five-form operator is that it cannot be connected with any operator of ghost number zero by means of picture-changing. Namely, the following formal consideration may be given. Let \( \{ V_n \} \) be the set of physical states (or the set of vertex operators) having ghost number \( n > 0 \). Let us further define \( \{ \tilde{V}_n \} \) as the set of those vertices of ghost number \( n \) which can be obtained as a result of picture-changing transformation of vertices of lower non-negative ghost numbers, i.e.

\[
\{ \tilde{V}_n \} \supset \Gamma_1 \{ V_{n-1} \} \cap \Gamma_1 \{ V_{n-2} \} \cap \Gamma_1 \{ V_{n-1} \} \cap \ldots \cap \Gamma_1 \{ V_0 \} \cap \ldots \cap \Gamma_1 \{ V_{n-1} \}
\]

where \( \Gamma_n \) : are picture-changing operators of higher ghost numbers.

Consider the cohomology classes :

\[
H_n = \frac{\{ V_n \} \setminus \{ \tilde{V}_n \} \setminus \{ \tilde{V}_0 \}}{\{ \tilde{V}_n \}} \quad (18)
\]

The vertex \( V(z, k) \) of (16) is then the element of \( H_1 \). In general, we expect all the perturbative string states to belong to the “elementary” ghost number cohomology \( H_0 \), while cohomologies of non-zero ghost numbers account for the non-perturbative physics, i.e. branes. Note that, while it is of a conformal dimension 1, the charge \( Z_{\mu_1 \ldots \mu_5} \) should, in principle, be an integral over the volume of a fivebrane of some expression of a dimension 5, in connection with Wess-Zumino term for a fivebrane. We conjecture therefore that the five-form \( e^{-3\phi} \psi_{\mu_1} \cdots \psi_{\mu_5} \) in the algebra (9), which is expressed in terms of open string variables, is essentially of a D-brane origin. The state \( V(z, k = 0)|0> \) should be related to the boundary state of a D5-brane, where \( |0> \) is an open string vacuum. Indeed, computing various S-matrix elements of the vertex operator \( V(z, k) = e^{\phi} \psi_{\mu_1} \cdots \psi_{\mu_5} e^{ikx} \) in open string theory one may check that they all vanish, which is not surprising since this operator does not describe the emission of any massless particle in the perturbative string spectrum. In the presence of D-branes, however, the S-matrix elements of \( V(z, k) \) are nonzero, due to its interaction with Ramond-Ramond charges. In fact, \( Z_{\mu_1 \ldots \mu_5} \) defines a solitonic-type state which interacts with Ramond-Ramond (RR) charges of a D-brane. The computation gives the nonzero S-matrix elements \( < V(z_1)V^{RR}(z_2, k_1)V^{RR}(z_3, k_2) > \)
and $< V(z_1, k_1) V^{RR}(z_2, k_2) V^{NS-NS}(z_3, k_3) >$ on a disk with mixed Dirichlet - Neumann boundary conditions which determine the scattering off D-brane with the inserted boundary state (16) (in the first of them the vertex $V(z_1)$ must be multiplied by an anti-holomorphic part). The first one is just the analogue of the p-brane gravitational lensing of [23]; while the second describes an interesting physical process - the Ramond-Ramond particle becoming the NS-NS particle due to interaction with the monopole-like state (16). In this paper we shall present the computation of the matrix element corresponding to this process. The vertex operators in the appropriate pictures must have a total left + right ghost number $-2$ in order to to compensate the ghost number anomaly:

$$V^{RR}(z, \bar{z}, k) = e^{-\frac{i}{2}(\phi + \bar{\phi})} \Sigma_\alpha \bar{\Sigma}_\beta \Gamma_{\nu_1 \ldots \nu_p} \psi_{\mu_1 \ldots \mu_p}(k) e^{ikX},$$

$$V^{NS-NS}(z, \bar{z}, k) = e^{-\phi - \bar{\phi}} \psi_{\mu_1 \ldots \mu_2} \bar{Z}^{\mu_1 \ldots \mu_5}.$$

The “tilde” in the polarization tensor $\tilde{Z}$ of $V(z, k)$ is used to avoid a confusion with the $Z$ of (16). Note that, as it has been explained in [23], we may consider the correlation function on the half-plane first and then, if needed, map it conformally to the disk. Then, extending the (anti)holomorphic fields to the whole complex plane, we get the following relations between holomorphic and antiholomorphic parts reproducing all the O.P.E.’s on the half-plane:

$$\bar{\psi}_\mu(\bar{z}) = \pm \psi_\mu(z)$$

$$X_\mu(z) = \pm X_\mu(\bar{z})$$

(20)

The sign depends on whether there are Neumann or Dirichlet boundary conditions for particular $\mu$’s; and

$$\bar{\Sigma}_\alpha(\bar{z}) = M_{\alpha\beta} \Sigma_\beta(\bar{z})$$

(21)

where $M_{\alpha\beta} = (\Gamma^0 \ldots \Gamma^p)_{\alpha\beta}$. We now have to consider:

$$< V^{RR}(z, \bar{z}, k_1) V^{NS-NS}(w, \bar{w}, k_2) V(0, 0) > = (F^{(p)} M)^{\alpha\beta}(k_1) B^{\mu\nu}(k_2) \bar{Z}^{\mu_1 \ldots \mu_5} \times$$

$$\times < e^{-\frac{i}{2} \bar{\phi} \Sigma_\alpha(z) e^{-\frac{i}{2} \bar{\phi} \Sigma_\beta(\bar{z}) e^{-\phi} \psi_{\mu}(w) e^{-\bar{\phi} \bar{\psi}_\nu(\bar{w}) e^{\phi} \psi_{\mu_1} \ldots \psi_{\mu_5}(0) > \times$$

$$\times < e^{i(k_1 X)(z_1, \bar{z}_1) e^{i(k_2 X)(z_2, \bar{z}_2)} >$$

(22)

Here $F^{(p)}(k) = \Gamma_{\nu_1 \ldots \nu_p} F_{\mu_1 \ldots \mu_p}(k)$ is the Ramond-Ramond field strength contracted with the antisymmetrized product of gamma-matrices, $B^{\mu\nu}(k)$ is the axion’s polarization tensor and $\bar{Z}^{\mu_1 \ldots \mu_5}$ is the topological charge associated with the boundary vertex $e^{\phi} \psi_{\mu_1} \ldots \psi_{\mu_5}$.
While the correlator of two exponents of $X^\mu$’s in (22) (the four-point function) is easy to compute, some work is needed to calculate the five-point function involving the fermions, ghosts and spin fields. This five-point correlator is computed by applying the O.P.E. rules involving spin fields and worldsheet fermions and using the theorem about the poles of meromorphic function. The result of the computation is given by:

$$\langle e^{-\frac{\phi}{2} \sum_{\alpha}(z)e^{-\frac{\phi}{2} \sum_{\beta}(\bar{z})} e^{-\phi \psi^\mu(w)} e^{-\phi \bar{\psi}^\nu(\bar{w})} e^{\phi \psi^{\mu_1} \ldots \psi^{\mu_5}(0)} \rangle = \frac{2(\Gamma^\mu_{\nu} \Gamma^{\mu_1 \ldots \mu_5})_{\alpha\beta}}{(z-w)(w-\bar{w})^2\bar{z}^2} + \frac{(\Gamma^\mu_{\nu} \Gamma^{\mu_1 \ldots \mu_3})_{\alpha\beta} \gamma^{\mu_4 \nu} \gamma^{\mu_5 \rho} + \text{perm.}(\mu, \nu; \mu_1 \ldots \mu_5)}{(z-w)(w-\bar{w})(\bar{z}-\bar{w})\bar{z}\bar{w}} + \frac{(\Gamma^\mu_{\nu} \Gamma^{\mu_1 \ldots \mu_3} \Gamma^\mu_{\nu})_{\alpha\beta} + \text{perm.}(\mu, \nu; \mu_1 \ldots \mu_5)}{(z-w)(w-\bar{w})(\bar{z}-\bar{w})\bar{z}\bar{w}} + \frac{(\Gamma^\mu_{\nu} \Gamma^{\mu_1 \ldots \mu_5})_{\alpha\beta} + \text{perm.}(\mu, \nu; \mu_1 \ldots \mu_5)}{(z-w)(w-\bar{w})(\bar{z}-\bar{w})\bar{z}\bar{w}} + \frac{(\Gamma^\mu_{\nu} \Gamma^{\mu_1 \ldots \mu_5} \Gamma^\mu_{\nu})_{\alpha\beta}}{(z-w)(w-\bar{w})^2\bar{z}^2} \right)$$

After putting together all the factors in (22) we obtain the final expression for the matrix element corresponding to the process of the NS-NS particle producing the Ramond-Ramond state due to the interaction with the Olive-Witten monopole (16):

$$V^{RR}(\tilde{z}, \tilde{k}_1) V^{NS-NS}(w, \bar{w}, k_2) V(0) = \frac{(z-\bar{z})^{2k_1^\parallel} (w-\bar{w})^{2k_2^\parallel} (z-w)(k_1 k_2)}{(z-\bar{w})^{k_1^\perp + k_2^\perp - k_1^\parallel - k_2^\parallel}} \times \left[ \text{Tr}(F(k_1) MB(k_2) \tilde{Z}) G_1(z, \tilde{z}, w, \bar{w}) + \text{Tr}(F(k_1) M \tilde{Z} B(k_2)) G_2(z, \tilde{z}, w, \bar{w}) \right] + \text{Tr}(F(k_1) MT^\mu \tilde{Z} T^\nu) B_{\mu\nu}(k_2) G_3(z, \tilde{z}, w, \bar{w}) + \text{Tr}(F(k_1) MT^\mu \Gamma^{\mu_1 \ldots \mu_4} \tilde{Z})_{\mu_1 \ldots \mu_5} B_{\mu\nu}(k_2) G_4(z, \tilde{z}, w, \bar{w}) \right] \delta(k_1^\parallel + k_2^\parallel)$$

Here $k_1^\parallel$ and $k_1^\perp$ are the longitudinal and transverse components of the $k^\mu$’s with respect to the D p-brane; while the matrices $F(k_1) = \Gamma^{\mu_1 \ldots \mu_5} F_{\mu_1 \ldots \mu_5}(k_1)$, $B(k_2) =$
\[ \Gamma^\mu \nu B_{\mu \nu}(k_2) \text{ and } \tilde{Z} = \Gamma^{\mu_1 \ldots \mu_5} \tilde{Z}_{\mu_1 \ldots \mu_5} \text{ are the polarization tensors contracted with anti-symmetrized } \Gamma\text{-matrices.} \]

The functions \[ G_i(z, \bar{z}, w, \bar{w}), i = 1, \ldots, 4 \text{ are given by:} \]

\[ G_1(z, \bar{z}, w, \bar{w}) = \frac{2}{(z - w)(w - \bar{w})^2 \bar{z}^2} + \frac{2}{(z - \bar{w})(w - \bar{w})^2 \bar{z}^2} + \frac{1}{(w - \bar{w})(z - \bar{w})^2 \bar{z}^2} - \frac{1}{2(w - \bar{w})(z - \bar{w})\bar{z} \bar{w}} + (z \leftrightarrow \bar{z}); \]

\[ G_2(z, \bar{z}, w, \bar{w}) = \frac{1}{z^2(w - \bar{w})} \left( \frac{1}{\bar{z} - w} - \frac{1}{\bar{z} - \bar{w}} \right) + \frac{1}{\bar{z} w \bar{w} (w - \bar{w})} \left( \frac{1}{z - w} - \frac{1}{z - \bar{w}} \right) + (z \leftrightarrow \bar{z}); \]

\[ G_3(z, \bar{z}, w, \bar{w}) = \left( \frac{1}{(z - \bar{w})(\bar{z} - w)w \bar{w}} \right) + \frac{1}{(z - w)(\bar{z} - \bar{w})w \bar{w}} \left( \frac{1}{\bar{z}} + \frac{1}{w - \bar{w}} \right) + (z \leftrightarrow \bar{z}) \]

\[ G_4(z, \bar{z}, w, \bar{w}) = \frac{1}{(w - z)(w - \bar{w}) \bar{z} \bar{w}} \left( \frac{1}{\bar{z} - w} + \frac{1}{w} \right) + \frac{1}{(z - \bar{w})(w - \bar{w}) \bar{z}} \left( \frac{1}{z(z - w)} - \frac{1}{w(\bar{z} - w)} \right) + (z \leftrightarrow \bar{z}) \]

This concludes the computation of the correlation function (22).

### 3. Conclusion

We have shown that the 5-form topological term, whose presence in the non-perturbative Poincare superalgebra for type IIB strings must follow from the M-theory, is obtained as a result of a special gauge transformation of the perturbative open string superalgebra, which the picture-changing at zero momentum is. This is consistent with the idea of “open string aristocracy” which implies that many aspects of the non-perturbative physics of branes, such as S-duality in our case, in fact may be hidden in the perturbative structure of open strings. In the example that we have presented in this paper, the string-theoretic analogue of the monopole part in the Olive-Witten’s result for field theory has appeared as a consequence of the singularity of picture-changing at \( k = 0 \) (I am grateful to Augusto Sagnotti for pointing out to me this connection). Alternatively, one may think of the picture-changing operator at zero momentum (or, equivalently, in the infinite momentum frame) as effectively creating boundaries on the worldsheet which, in turn, give rise to various non-perturbative effects. This is in accordance with the apparently D-brane nature of the 5-form charge in the superalgebra (9). It would certainly
be interesting to explore the connection between these two viewpoints, as well as to understand in more details how the presence of the boundaries modifies current algebras in general. This may help us to understand better the still obscure relation between p-brane solutions of low-energy effective theories and p-brane dynamics in general, by using the interplay between the physics in the target space and on the worldsheet (worldvolume). For instance, due to the relation between target space and worldsheet metrics:

$$\gamma_{ij} = \partial_i X^\mu \partial_j X^\nu G_{\mu\nu}(X)$$ (29)

a singularity in the metric of a given p-brane solution shall induce a singularity on the worldsheet which, in turn, one may try to represent as an insertion of some boundary vertex operator. P-brane dynamics shall be explored then by studying the properties of the insertion. We hope to address this question in future papers; it would be especially interesting to apply this program to the case of intersecting branes since their classical action is yet to be constructed. Next, recall that the monopole-like state (16) which we have also associated with the boundary state of a D5-brane appears to be an element of a “ghost-number” cohomology class $H_1$ defined in (18). One may expect that BRST non-trivial elements of higher cohomology classes $H_n$’s should somehow contain the non-perturbative part of an open-string spectrum (D-branes) and dualities shall then appear as mappings between these cohomology groups. Studying the properties of these cohomologies shall involve many subtleties connected with the picture-changing and it requires certain accuracy.

We conclude by giving some intuitive arguments that emphasize the connections between picture-changing, ghost number cohomologies and S and T-dualities. As is well known, the total ghost number $n$ of a correlation function (corresponding to some term in the low-energy effective action) and the genus $g$ of the worldsheet (the expectation value of the dilaton) are related by the constraint

$$n = 2(1 - g).$$ (30)

At the same time, the picture-changing at zero momentum essentially generates the map between the ghost number cohomologies $H_n$. Due to (30), a map between the $H_n$’s should correspond to the transformation of the dilaton field, and this is where the relation with the S-duality shall appear. One may also anticipate an important role of the Ramond-Ramond vertices in the relation between S-duality and ghost number cohomologies due
to their property to effectively change the genus of the worldsheet \[23,24\]. As to the connection with T-duality, consider the space of pseudodifferential superforms \(\Lambda^{r|s}\) with \(r\) and \(s\) being even and odd degrees respectively \[27\] (these forms may also be understood as the extended objects of the dimension \(r|s\) propagating in a target space \[28\]). While the usual differentiation changes the even degree \(r\) only, the odd degree \(s\) is shifted by the operation analogous to the picture-changing, i.e. the dimension of the extended object is shifted by the transformation of the picture-changing:

\[
\Gamma_1 : \Lambda^{r|s} \rightarrow \Lambda^{r|s-1}
\]

On the other hand, T-duality, which exchanges the Dirichlet and Neumann boundary conditions on the ends of an open string transforms a D-brane into either \((D+1)\)- or \((D-1)\)-brane, depending on whether the T-duality is performed in transverse or tangent directions with respect to the D-brane. To make this argument precise, we need to understand the relation between the string-theoretic picture-changing formalism and the one involving the superforms, in more accurate terms.

**Erratum**

The previous version of this paper contained the incorrect computation of the anti-commutator of two supercharges in the \(+\frac{1}{2}\) – *picture*. We claimed to have obtained the 5-form central term (16) in the anticommutator. However, the accurate calculation shows that this term appears to be proportional to the matrix factor \(\Gamma_\mu \Gamma^{\mu_1 \cdots \mu_5} \Gamma^\mu\), which vanishes in \(D = 10\) - and this factor was missing in the previous version. Therefore, the central terms in the superalgebra do not appear under the naive picture-changing, contrary to what was previously suggested. The mistake in the computation led to the erroneous assumption that the non-perturbative central terms in the superalgebra may be obtained just by applying the usual picture-changing transformation to the perturbative superalgebra (7). However, as it was shown here, the relation between perturbative superalgebra (7) and the one containing the fivebrane requires the serious modification of the picture-changing transformation. This modification, described in (14) is referred to as “picture-changing in the infinite momentum frame”. This name suggests certain relevance to the matrix theory, which we hope to discuss in the upcoming paper. The author is very thankful to N.Berkovits and E.Witten, whose kind remarks have pointed out the computational error contained in the previous version of this paper, leading to incorrect physical interpretations.

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