TRANSIT TIMING VARIATIONS FOR INCLINED AND RETROGRADE EXOPLANETARY SYSTEMS

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ABSTRACT

We perform numerical calculations of the expected transit timing variations (TTVs) induced on a hot-Jupiter by an Earth-mass perturber. Motivated by the recent discoveries of retrograde transiting planets, we concentrate on an investigation of the effect of varying relative planetary inclinations, up to and including completely retrograde systems. We find that planets in low-order (e.g., 2:1) mean-motion resonances (MMRs) retain approximately constant TTV amplitudes for $0^\circ < i < 170^\circ$, only reducing in amplitude for $i > 170^\circ$. Systems in higher order MMRs (e.g., 5:1) increase in TTV amplitude as inclinations increase toward $45^\circ$, becoming approximately constant for $45^\circ < i < 135^\circ$, and then declining for $i > 135^\circ$. Planets away from resonance slowly decrease in TTV amplitude as inclinations increase from $0^\circ$ to $180^\circ$, whereas planets adjacent to resonances can exhibit a huge range of variability in TTV amplitude as a function of both eccentricity and inclination. For highly retrograde systems ($135^\circ < i < 180^\circ$), TTV signals will be undetectable across almost the entirety of parameter space, with the exceptions occurring when the perturber has high eccentricity or is very close to an MMR. This high inclination decrease in TTV amplitude (on and away from resonance) is important for the analysis of the known retrograde and multi-planet transiting systems, as inclination effects need to be considered if TTVs are to be used to exclude the presence of any putative planetary companions: absence of evidence is not evidence of absence.

Key words: celestial mechanics – methods: numerical – planetary systems

1. INTRODUCTION

Of the known extra-solar planets, more than 60 transit the host star. Of these systems, at least four show evidence for an external companion (GJ 436: Maness et al. 2007; HAT-P-13: Bakos et al. 2009; HAT-P-7: Winn et al. 2009; CoRoT-7: Queloz et al. 2009). If the transiting planet were the only planet in the system, then the period between each successive transit would be constant (neglecting complicating effects such as general relativity, stellar oblateness, and tides). The presence of additional planets in the system (which themselves may or may not transit the star) can cause perturbations to the orbit of the transiting planet, leading to detectable transit timing variations (TTVs) of the known transiting planet (Miralda-Escudé 2002; Holman & Murray 2005; Agol et al. 2005). Other studies have extended the TTV method to demonstrate the feasibility of detecting planetary moons (Kipping 2009a, 2009b) and Trojan companions (Ford & Holman 2007) in extra-solar planetary systems.

However, practical use of TTVs as a detection tool requires the solution of the difficult “inverse problem,” i.e., given a particular TTV profile, can one reconstruct (or at least restrict) the mass and orbit of the unseen perturber? This problem is non-trivial, as numerous different perturber mass–orbit configurations can as numerous different perturber mass–orbit configurations can

Several investigations have considered (and ruled out) planets more massive than the Earth in certain orbits/regions of parameter space close to many of the known transiting planets (e.g., Steffen & Agol 2005; Agol & Steffen 2007; Alonso et al. 2008; Bean & Seifahrt 2008), while Nesvorný & Morbidelli (2008) and Nesvorný (2009) have developed an approximate analytic method to try and tackle the inverse problem in a more general manner.

However, most previous investigations have focused on analyzing the effect of prograde, coplanar companions. Even where explicit investigations of inclination effects have been conducted (e.g., Nesvorný 2009), the investigations have been restricted to prograde cases, examining only relative inclinations significantly lower than $90^\circ$.

There are several reasons to consider TTVs of inclined systems. Observationally, it is possible to measure the sky-projected angle between the spin vector of a star, and the orbital angular momentum vector of a planet transiting that star via the Rossiter–McLaughlin effect. Measurements over the past year have revealed that a number of the known transiting planets are strongly inclined, or even retrograde (e.g., HAT-P-7b, Narita et al. 2009; Winn et al. 2009, as well as WASP-17b, Anderson et al. 2010).

Although the mechanism(s) driving the creation of such retrograde planets remains unclear, models used to explain high-eccentricity exoplanets via planet–planet scattering (Rasio & Ford 1996; Weidenschilling & Marzari 1996; Lin & Ida 1997; Levison et al. 1998; Papaloizou et al. 2001; Moorhead & Adams 2005; Chatterjee et al. 2008; Jurić & Tremaine 2008; Ford & Rasio 2008) and/or Kozai oscillations (e.g., Kozai 1962; Takeda & Rasio 2005; Nagasawa et al. 2008) can naturally excite large orbital inclinations (although the quantitative details of the distributions can differ greatly). Other mechanisms such as inclination pumping during migration (e.g., Lee & Thommes 2009) may also contribute.

In addition, previous dynamical studies have suggested that systems of prograde and retrograde planets are more stable than standard prograde–prograde cases (Gayon & Bois 2008; Gayon et al. 2009; Smith & Lissauer 2009), essentially because the planets spend less time “close together” and thus perturbations are smaller.

Given the observational evidence for highly inclined and retrograde systems, and the suggestion of enhanced stability in such systems, we investigate and quantify the hypothesis that retrograde systems will have a significantly reduced TTV profile compared to a standard prograde case.

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the mid-time of the transit, \( \delta t \). Observations of transit times are perturbed by the light travel time, \( \delta t_{ltt} \), which is the speed of light. The observable transit time variations are calculated as \( \delta t(i) = t_i + \delta t_{ltt}(i) - i \times \hat{P} - \hat{t}_0 \), where the constant \( \hat{P} \) and \( \hat{t}_0 \) are determined by linear least-squares minimization of \( \sum (\delta t(i))^2 \).

We neglect any motion of the stellar center between the time of light emission and the time of transit.

### 2.2. TTV Maps

In Sections 3.2 and 3.3, we produce TTV contour maps in \((i_2, e_2)\) and \((\frac{a_2}{a_1}, e_2)\) parameter space. These plots are produced by fixing the mass and the initial orbit of the transiting hot-Jupiter \((e_1 = 0 \text{ and } a_1 = 0.05)\), and then varying the orbit of the Earth-mass perturber.

For each set of configurations—i.e., semimajor axis, \(a_2\), eccentricity, \(e_2\), and inclination, \(i_2\), of the perturbing planet—we conduct five simulations randomizing the other orbital elements (argument of pericenter, \(\omega\), longitude of ascending node, \(\Omega\), and initial mean anomaly, \(M\)).

We then calculate the rms of \(\delta t(i)\) for each \(a, e, i\) configuration. The required contour plots are then plotted using an approximately logarithmic color scale (shown in Figures 2 and 3).

We restrict our simulations to the region defined by the prograde coplanar three-body Hill-stability criterion of Gladman (1993). This ensures that all of the \(i = 0\) simulations are stable, and ensures for the \(i > 0\) systems that we have covered all of the guaranteed stability zone and more (see Veras & Armitage (2004) for a discussion of the stability limit as a function of inclination). As an additional check on systems with extremely large TTVs close to the Hill-stability criterion boundary, we conducted 1000 longer-term \(\text{MERCURY}\) simulations. In most cases, the Earth-mass planet experiences significant changes in semimajor axis within 1000 years, due to the “kicks” at pericenter passage. Thus, although these planets do not collide, the less massive planet can experience some orbit variation.

### 3. RESULTS

#### 3.1. Example TTVs for a Hot-Jupiter System

In Figure 1(a), we show a circular hot-Jupiter at 0.05 au being perturbed by an Earth-mass companion with \(e_2 = 0.02\) located on the external 2:1 mean-motion resonance (MMR). We find that although the prograde systems \((0^\circ \leq i \leq 90^\circ)\) have TTV profiles that differ significantly, their overall amplitudes...
are similar. In contrast, the amplitude of the $i = 180^\circ$ signal is very low ($\sim 1$ s). The results for the 3:1 case in Figure 1(b) are qualitatively similar.

In contrast, for the 5:1 MMR system (Figure 1(c)), while the amplitude of the $i = 180^\circ$ signal is extremely low, there are now significant differences in the behavior at smaller angles: the $i = 0^\circ$ and $i = 180^\circ$ now have similarly low amplitudes, with an approximate maximum occurring at $i = 90^\circ$.

Away from resonance (Figures 1(d)–(f)), we again find that the $i = 180^\circ$ signal is generally significantly lower than that observed for cases with $i < 90^\circ$, but that the detailed dependence on inclination is far from obvious, with some cases (Figures 1(d) and (f)) having the highest amplitude at $i = 0^\circ$, while the low-eccentricity plot of Figure 1(e) has a particularly low-amplitude signal at $i = 0^\circ$.

We note that there are a number of frequencies evident in Figure 1, with periods from a few months to $\gg 10$ years. In general, the short-term oscillations tend to be driven by variations in $e_2$, leading to variations in the distance of closest approach between the two planets. The longer term quadratic trends are primarily due to outward drifts in $a_2/a_1$ as a result of kicks at close approach.

The large number of different frequencies evident in the plots also serves to highlight the importance of considering the sampling period over which observations and simulations are to be conducted and compared.

Furthermore, TTV profiles may change in amplitude and frequency by several orders of magnitude simply due to a variation in the initial values of the mean anomalies (Veras & Ford 2009). This means that while approximate expressions for TTV amplitude dependences on mass, semimajor axis separation, etc., can serve as a useful guide, individual system variations can be far removed from the mean. We do not intend this statement as a criticism of previous work, but rather as a cautionary note against over-reliance on approximations when analyzing individual systems which can exhibit great sensitivity to initial conditions. We do not seek in this Letter to address in detail all such issues, but instead defer a more detailed consideration to a companion paper (D. Veras et al. 2010, in preparation), where a more thorough examination of the coplanar prograde case will be presented, allowing a much more detailed exposition of the huge number of variables which can affect the TTV signal.

### 3.2. Eccentricity–Inclination Contour Plots

To better understand the TTV amplitudes as a function of inclination, for each of the semimajor axis plots in Figure 1 we perform a search of the $(e_2, i_2)$ parameter space, producing contour plots in Figure 2 showing the rms TTV amplitude variation at that semimajor axis separation as the eccentricity and inclination of the outer planet is varied.

Figure 2. Plots at fixed $(a_2/a_1)$ showing the median rms TTV amplitude as a function of $e_2$ and $i_2$. The contour key is displayed in the bottom right-hand corner. The semimajor axes selected for plots (a)–(e) are the same as those of Figures 1(a)–(e). We can see significant differences in behavior as a function of inclination depending on (1) position on/off resonance, (2) the type of any resonance, and (3) the eccentricity of the perturber.
Figure 3. Median rms TTV amplitudes in the \((a_2/a_1, e_2)\) plane for various \(i_2\). The contour key is shown at the top left in plot (a). In each \((a_2/a_1, e_2)\) plot, we see a strong increase in amplitude (1) at high eccentricity and (2) close to MMRs. As the inclination is increased there is an obvious general decrease in TTV amplitude, particularly pronounced for the retrograde orbits.

In the 2:1 case of Figure 2(a) the resulting contour plot is rather uniform, with the only area of significant reduction being in the \(e_2 < 0.1, i \sim 180^\circ\) region. An approximately similar result can be seen for the 3:1 case.

For the 5:1 case (Figure 2(c)), the situation changes: the region of low amplitude close to \(i = 180^\circ\) significantly expands, while another low-amplitude region emerges at \(e_2 < 0.1\) and \(i < 45^\circ\).

For Figure 2(d) the perturber is located very close to (but not on) the 2:1 MMR. The plotted behavior is particularly rich, showing huge variations in the predicted TTV amplitude as functions of both eccentricity and inclination.

Finally, the non-resonant case in Figure 2(e) with \(a_2/a_1 = 3.6\) shows a fairly simple decline in TTV amplitude as a function of inclination.

3.3. TTV Maps for Varying \(a_2/a_1\)

We now look at a different projection of the data, looking in the \((a_2/a_1, e_2)\) plane at the TTV structure as a function of inclination.

In Figure 3(a) we reproduce the coplanar prograde results of Agol et al. (2005), showing the pronounced “flames of resonance” (Veras & Ford 2009). The majority of the non-resonant parameter space has TTV signals of comparatively low amplitude (<30 s). It is only when eccentricities become very high, or the planets are in/near MMRs that the TTV signal amplitudes become large.

As the relative inclination of the planetary orbits is increased, for low inclinations changes in TTV amplitude are relatively subtle; for the \(i = 30^\circ\) and \(i = 45^\circ\) cases (Figures 3(c) and (d)) the rms TTV signal is reduced in some high-eccentricity locations, corresponding to the reduction previously seen in the close-to-resonance map of Figure 2(d). In addition, we can also see that spikes around higher resonances start to drop lower down, i.e., the low-eccentricity amplitude increases as the inclination decreases, an effect noted in Section 3.2 for the 5:1 resonance plot of Figure 2(c).

For larger relative inclinations, the expected TTV signal amplitude starts to drop across much of the \((a_2/a_1, e_2)\) parameter space (see the \(i = 90^\circ\) plot in Figure 3(d)). The reduction in high-amplitude signals at high eccentricities is the most marked decrease, but at any given eccentricity one tends to find a reduction in amplitude (shifting of the contours) compared to the \(i = 0^\circ\) plot.

When the systems are pushed into retrograde orbits, the difference becomes more pronounced (e.g., \(i = 135^\circ\) in Figure 3(e) and \(i = 180^\circ\) in Figure 3(f)). For these inclinations, we find a reduction in amplitude across large swathes of the
plot, with significant amplitude remaining only near MMRs. In addition, the TTV amplitude and the width of regions with large TTVs (near MMRs) are markedly reduced.

### 3.3.1. External 2:1 MMR

Since highly inclined systems only have large rms TTVs in/near MMRs (Figure 3(f)), we examine in more detail one such resonant region. The external 2:1 MMR is known to be of importance for exoplanet systems from both theoretical (e.g., Sándor et al. 2007) and observational (e.g., Tinney et al. 2006) studies.

On and adjacent to the 2:1 MMR region, the TTV signal is extremely sensitive to changes in semimajor axis and/or eccentricity. In the coplanar prograde case (Figure 4(a)), a fractional increase of just a few percent in either $a_2$ or $e_2$ causes the TTV amplitude to jump by more than an order of magnitude.

As the inclination increases to 30° and 45°, we find that the central region of the MMR maintains an approximately constant TTV amplitude. The regions just outside the resonance feel the strongest effects. For example, regions near $a_2/a_1 = 1.54$ show TTV amplitudes reduced by an order of magnitude for many eccentricities (compare Figure 4(a) and Figure 4(c)). This decreases the width of the region of parameter space around the 2:1 MMR resonance with TTV signals $>30$ s. In addition, the general reduction in “background amplitude” causes the minor resonances to become more prominent, e.g., $a_2/a_1 \sim 1.69$.

As the inclination increases to 90° and 135°, TTV signals $>30$ s are restricted purely to the resonance regions, with regions away from resonance having signals $<30$ s almost everywhere, even at high eccentricity. As the inclination reaches 180°, and the planets are now again coplanar but with completely counter-rotating orbits, we find TTV amplitude over 100 s only in very narrow regions around the first- and second-order MMRs.

Finally, we note that as we move from Figures 4(e) and (f), the amplitude in regions slightly offset from resonance appears to increase slightly at high eccentricities. However, this only occurs at low-order resonances (small semimajor axis separations—see Figures 3(e) and (f)) as well as Figure 2. More generally, away from such regions (see Figure 3(f)), retrograde orbits tend to produce the smallest TTVs.

### 3.3.2. Internal Perturbers

Finally, for completeness we consider an Earth-mass planet on an orbit interior to the Jupiter-mass planet, keeping the more massive planet at $a_1 = 0.05$ AU and $e_1 = 0$, and then monitor the TTVs of the massive planet due to the interior perturber.

From Figure 5 it seems that the background (non-resonant) regions now retain an approximately constant TTV signal irrespective of inclination, while there is a tendency for the TTV signal to become stronger around the MMR regions as the inclination is increased toward 90°, but then drop away to almost nothing as the inclination further increases toward $i = 180°$. 
4. CONCLUSIONS

We have investigated the TTV signals for systems of highly inclined and retrograde planets. We find the following.

1. In the vicinity of exterior MMRs the inclination dependence is complex: low-order resonances maintain a high TTV amplitude for all regions \( i < 170^\circ \), declining in amplitude only for low-eccentricity cases close to \( i = 180^\circ \), whereas higher order resonances display an increase in TTV amplitude as inclinations rise from \( 0^\circ \) to \( 45^\circ \). Moreover, the regions immediately adjacent to MMRs show extreme sensitivity to changes in perturber \( a, e, \) and \( i \).

2. Exterior perturbers away from resonances tend to show a slow decrease in TTV amplitude with increasing inclination, although regions adjacent to resonances can show remarkably complex behavior.

3. Perturbing planets on interior orbits display a slightly different behavior: away from resonance the amplitude remains approximately constant with inclination, but around MMRs the perturbations become stronger as the inclination increases toward \( 90^\circ \) before decreasing again beyond \( 90^\circ \).

We note the following.

1. Absence of evidence is not evidence of absence: planets in retrograde orbits should be expected to produce markedly reduced TTV signals as compared to the standard prograde case. For an Earth-mass perturber in an anti-aligned orbit, almost the entirety of the sample parameter space would result in a very small or undetectable TTV signal (unless the planet happened to be fortuitously located precisely on a strong MMR).

2. Retrograde orbits may be a natural way to explain transiting systems in which little or no TTV signal is observed, but in which the radial velocity observations point toward the existence of an additional planetary companion (e.g., GJ 436, HAT-P-13, etc.).

3. In addition to the TTV considerations in this work, it is important to acknowledge that inclined orbits in multi-planet systems will precess, leading to variations in transit duration. More work is required to try and understand whether a combination of TTV signals with transit duration variation signals could remove some of the degeneracies inherent in this problem.

4. Many additional dependences (e.g., sampling period, perturber mass, relative mean anomaly, etc.) can significantly alter the expected TTV signal for a particular system. We defer the provision of a detailed investigation of such matters to a companion paper (D. Veras et al. 2010, in preparation).

5. Future work could also investigate in more detail the regions above the Hill-stability curve, seeking to identify the (likely large) TTVs for any stable systems which exist in that region.

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