Quantum transport of Dirac electrons in graphene in the presence of a spatially modulated magnetic field

M. Tahir

Department of Physics, University of Sargodha, Sargodha 40100, Pakistan

K. Sabeeh

Department of Physics, Quaid-i-Azam University, Islamabad 45320, Pakistan

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Abstract

We have investigated the electrical transport properties of Dirac electrons in a monolayer graphene sheet in the presence of a perpendicular magnetic field that is modulated weakly and periodically along one direction. We find that the Landau levels broaden into bands and their width oscillates as a function of the band index and the magnetic field. We determine the $\sigma_{yy}$ component of the magnetoconductivity tensor for this system which is shown to exhibit Weiss oscillations. We also determine analytically the asymptotic expressions for $\sigma_{yy}$. We compare these results with recently obtained results for electrically modulated graphene as well as those for magnetically modulated conventional two-dimensional electron gas (2DEG) system. We find that in the magnetically modulated graphene system considered in this work, Weiss oscillations in $\sigma_{yy}$ have a reduced amplitude compared to the 2DEG but are less damped by temperature while they have a higher amplitude than in the electrically modulated graphene system. We also find that these oscillations are out of phase by $\pi$ with those of the electrically modulated system while they are in phase with those in the 2DEG system.
I. INTRODUCTION

The successful preparation of monolayer graphene has allowed the possibility of studying the properties of electrons in graphene [1]. The nature of quasiparticles called Dirac electrons in these two-dimensional systems is very different from those of the conventional two-dimensional electron gas (2DEG) realized in semiconductor heterostructures. Graphene has a honeycomb lattice of carbon atoms. The quasiparticles in graphene have a band structure in which electron and hole bands touch at two points in the Brillouin zone. At these Dirac points the quasiparticles obey the massless Dirac equation. In other words, they behave as massless Dirac particles leading to a linear dispersion relation $\epsilon_k = vk$ (with the characteristic velocity $v \approx 10^6 m/s$). This difference in the nature of the quasiparticles in graphene from conventional 2DEG has given rise to a host of new and unusual phenomena such as anomalous quantum Hall effects and a $\pi$ Berry phase [1][2]. Earlier it was found that if conventional 2DEG is subjected to artificially created periodic potentials in the sub-micrometer range it leads to the appearance of Weiss oscillations in the magnetoresistance. This type of electrical modulation of the 2D system can be carried out by depositing an array of parallel metallic strips on the surface or through two interfering laser beams [3, 4, 5].

Besides the fundamental interest in understanding the electronic properties of graphene there is also serious suggestions that it can serve as the building block for nanoelectronic devices [6]. Since Dirac electrons can not be confined by electrostatic potentials due to the Klein’s paradox it was suggested that magnetic confinement be considered [7]. Technology for this already exists as the required magnetic field can be created by having ferromagnetic or superconducting layers beneath the substrate [8].

In conventional 2DEG systems, electron transport in the presence of magnetic barriers and superlattices has continued to be an active area of research [9]. Recently, electrical transport in graphene in the presence of electrical modulation was considered and theoretical predictions made [11]. Along the same lines, in this work we investigate low temperature magnetotransport of Dirac electrons in a single graphene layer subjected to a one-dimensional (1D) magnetic modulation. The perpendicular magnetic field is modulated weakly and periodically along one direction.
II. FORMULATION AND ENERGY SPECTRUM

We consider two-dimensional Dirac electrons in graphene moving in the x-y-plane. The magnetic field \((B)\) is applied along the z-direction perpendicular to the graphene plane. The perpendicular magnetic field \(B\) is modulated weakly and periodically along one direction such that \(\vec{B} = (B + B_0 \cos(Kx))\hat{z}\). Here \(B_0\) is the strength of the magnetic modulation. In this work we consider the modulation to be weak such that \(B_0 << B\). We consider the graphene layer within the single electron approximation. The low energy excitations are described by the two-dimensional (2D) Dirac like Hamiltonian \((\hbar = c = 1 \text{ here})\) \([1, 2, 11]\)

\[
H = v \vec{\sigma} \cdot (\vec{\nabla} + e \vec{A}).
\] (1)

Here \(\vec{\sigma} = \{\vec{\sigma}_x, \vec{\sigma}_y\}\) are the Pauli matrices and \(v\) characterizes the electron velocity. We employ the Landau gauge and write the vector potential as \(\vec{A} = (0, Bx + (B_0/K) \sin(Kx), 0)\) where \(K = 2\pi/a\) and \(a\) is the period of the modulation. The Hamiltonian given by Eq. (1) can be expressed as

\[
H = -iv \vec{\sigma} \cdot \vec{\nabla} + ev \vec{\sigma}_y Bx + ev \vec{\sigma}_y \frac{B_0}{K} \sin(Kx).
\] (2)

The above Hamiltonian can be written as

\[
H = H_0 + H',
\] (3)

where \(H_0\) is the unmodulated Hamiltonian given as

\[
H_0 = -iv \vec{\sigma} \cdot \vec{\nabla} + ev \vec{\sigma}_y Bx
\]

and

\[
H' = ev \vec{\sigma}_y \frac{B_0}{K} \sin(Kx).
\]

The Landau level energy eigenvalues without modulation are given by

\[
\varepsilon(n) = \omega_y \sqrt{n}
\] (4)

where \(n\) is an integer and \(\omega_y = v \sqrt{2eB}\). As has been pointed out \([11]\) the Landau level spectrum for Dirac electrons is significantly different from the spectrum for electrons in conventional 2DEG which is given as \(\varepsilon(n) = \omega_c (n + 1/2)\), where \(\omega_c = eB/m\) is the cyclotron frequency.
The eigenfunctions without modulation are given by

$$\Psi_{n,k_y}(r) = e^{ik_y y} \sqrt{2L_y l} \left( \frac{-i\Phi_{n-1}[(x + x_0)/l]}{\Phi_n[(x + x_0)/l]} \right)$$

where

$$\Phi_n(x) = \frac{e^{-x^2/2}}{\sqrt{2\pi n!}} H_n(x)$$

and $l = \sqrt{1/eB}$ is the magnetic length, $x_0 = l^2 k_y$, $L_y$ is the $y$-dimension of the graphene layer and $H_n(x)$ are the Hermite polynomials. Since we are considering weak modulation $B_0 << B$, we can apply standard perturbation theory to determine the first order corrections to the unmodulated energy eigenvalues in the presence of modulation

$$\Delta \varepsilon_{n,k_y} = \int_{-\infty}^{\infty} dx \int_0^{L_y} dy \Psi^*_{n,k_y}(r) H'(x) \Psi_{n,k_y}(r)$$

with the result

$$\Delta \varepsilon_{n,k_y} = \omega_0 \cos(Kx_0) \left( 2\sqrt{n}e^{-u/2}[L_{n-1}(u) - L_n(u)] \right)$$

where $\omega_0 = \frac{evB_0}{K^2}$, $u = K^2 l^2/2$ and $L_n(u)$ are the Laguerre polynomials. Hence the energy eigenvalues in the presence of modulation are

$$\varepsilon(n, k_y) = \varepsilon(n) + \Delta \varepsilon_{n,k_y} = \omega_g \sqrt{n} + \omega_0 \cos(Kx_0) G_n$$

with $G_n(u) = 2\sqrt{n}e^{-u/2}[L_{n-1}(u) - L_n(u)]$. We observe that the degeneracy of the Landau level spectrum of the unmodulated system with respect to $k_y$ is lifted in the presence of modulation with the explicit presence of $k_y$ in $x_0$. The $n = 0$ landau level is different from the rest as the energy of this level is zero and electrons in this level do not contribute to diffusive conductivity calculated in the next section. The rest of the Landau levels broaden into bands. The Landau bandwidths $\sim G_n$ oscillates as a function of $n$ since $L_n(u)$ are oscillatory functions of the index $n$.

Before we begin the calculation of electrical conductivity it is necessary to discuss the regime of validity of the perturbation theory presented above. For large $n$ the level spacing given by Eq. (4) goes as $\omega_g(\sqrt{n} - \sqrt{n-1}) \rightarrow \omega_g \frac{1}{2\sqrt{n}}$ and the width of the $n$th level given by Eq.(8) goes as $2\omega_0 n^{1/2}$. There is therefore a value of $n$ at which the width becomes equal to the spacing and the perturbation theory is no longer valid. This occurs when

$$n_{\text{max}} = \sqrt{2\pi^2 B'}$$

where $B' = \frac{1}{e a^2} = 0.0054 T$ for $a = 350nm$. For a fixed electron density and
the period of modulation this suggests the maximum value of the magnetic modulation $B_0$ above which it is necessary to carry out a more sophisticated analysis.

III. ELECTRICAL CONDUCTIVITY WITH PERIODIC MAGNETIC MODULATION

To calculate the electrical conductivity in the presence of weak magnetic modulation we use Kubo formula to calculate the linear response to an applied external field. In a magnetic field, the main contribution to Weiss oscillations comes from the scattering induced migration of the Larmor circle center. This is diffusive conductivity and we shall determine it following the approach in [10, 11] where it was shown that the diagonal component of conductivity $\sigma_{yy}$ can be calculated by the following expression in the case of quasielastic scattering of electrons

$$\sigma_{yy} = \frac{\beta e^2}{L_x L_y} \sum_\zeta f(E_\zeta) [1 - f(E_\zeta)] \tau(E_\zeta) (v_\zeta y)^2$$

(10)

$L_x, L_y,$ are the dimensions of the layer, $\beta = \frac{1}{k_B T}$ is the inverse temperature with $k_B$ the Boltzmann constant, $f(E)$ is the Fermi Dirac distribution function, $\tau(E)$ is the electron relaxation time and $\zeta$ denotes the quantum numbers of the electron eigenstate. The diagonal component of the conductivity $\sigma_{yy}$ is due to modulation induced broadening of Landau bands and hence it carries the effects of modulation in which we are primarily interested in this work. $\sigma_{xx}$ does not contribute as the component of velocity in the $x$-direction is zero here. The collisional contribution due to impurities is not taken into account in this work.

The summation in Eq.(10) over the quantum numbers $\zeta$ can be written as

$$\frac{1}{L_x L_y} \sum_\zeta = \frac{1}{2\pi L_x} \int_{-\frac{L_x}{2}}^{\frac{L_x}{2}} dk_y \sum_{n=0}^{\infty} = \frac{1}{2\pi l^2} \sum_{n=0}^{\infty}$$

(11)

The component of velocity required in Eq.(10) can be calculated from the following expression

$$v_\zeta y = \frac{\partial}{\partial k_y} \varepsilon(n, k_y).$$

(12)

Substituting the expression for $\varepsilon(n, k_y)$ obtained in Eq.(9) into Eq.(12) yields

$$v_\zeta y = \frac{2\omega_0 u}{K} \sin(Kx_0) G_n(u)$$

(13)
With the results obtained in Eqs.(11), (12) and (13) we can express the diffusive contribution to the conductivity given by Eq.(10) as

\[ \sigma_{yy} = A_0 \Phi \] (14)

where

\[ A_0 = 2\omega_0^2 e^2 \tau \beta \] (15)

and the dimensionless conductivity \( \Phi \) is given as

\[ \Phi = 4ue^{-u} \sum_{n=0}^{\infty} \frac{ng(E_n)}{[g(E_n) + 1]^2}[L_{n-1}(u) - L_n(u)]^2 \] (16)

where \( g(E) = \exp[\beta(E - E_F)] \) and \( E_F \) is the Fermi energy.

In Fig.(1), we plot the dimensionless conductivity given by Eq.(16) as a function of inverse magnetic field at temperature \( T = 6K \) and electron density \( n_e = 3 \times 10^{11} cm^{-2} \). The dimensionless magnetic field is introduced given as \( b = \frac{B}{B'} \) with \( B' = \frac{1}{ea} \). In the region of high magnetic field we can see SdH oscillations superimposed on the Weiss oscillations.

IV. ASYMPPTOTIC EXPRESSIONS

To get a better understanding of the results of the previous section we will consider the asymptotic expression of conductivity where analytic results in terms of elementary functions can be obtained following [11]. We shall compare the asymptotic results for the dimensionless conductivity obtained in this section with the results obtained for a magnetically modulated conventional 2DEG system. We shall also compare these results with those of graphene that is subjected to only the electric modulation.

The asymptotic expression of dimensionless conductivity can be obtained by using the following asymptotic expression for the Laguerre polynomials

\[ \exp^{-u/2} L_n(u) \rightarrow \frac{1}{\sqrt{\pi \sqrt{nu}}} \cos(2\sqrt{nu} - \frac{\pi}{4}) \]. (17)

Note that the asymptotic results are valid when many Landau Levels are filled. We now take the continuum limit:

\[ n \rightarrow \frac{1}{2} \left( \frac{lE}{v} \right)^2, \sum_{n=0}^{\infty} \rightarrow \left( \frac{l}{v} \right)^2 \int_0^{\infty} EdE \] (18)
to express the dimensionless conductivity in Eq.(16) as the following integral

$$
\Phi = \frac{8\sqrt{u}}{\sqrt{2\pi}} \left( \frac{l}{v} \right)^3 \int_0^\infty dE \frac{E^2 g(E)}{[g(E) + 1]^2} \sin^2(1/2\sqrt{u/n}) \sin^2(2\sqrt{n}u - \pi/4)
$$

(19)

where $u = 2\pi^2/b$.

Now assuming that the temperature is low such that $\beta^{-1} \ll E_F$ and replacing $E = E_F + s\beta^{-1}$, we rewrite the above integral as

$$
\Phi = \frac{8p^2a}{vb^2\beta} \sin^2 \left( \frac{\pi}{p} \right) \int_{-\infty}^\infty ds \frac{\cosh^2(s/2)}{(e^s + 1)^2} \sin^2 \left( \frac{2\pi p}{b} - \frac{\pi}{4} + \frac{2\pi a}{vb\beta} s \right)
$$

(20)

where $p = \frac{E_{Fv}}{a} = k_Fa = \sqrt{2\pi n_e}a$ is the dimensionless Fermi momentum of the electron. To obtain an analytic solution we have also replaced $E$ by $E_F$ in the above integral except in the sine term in the integrand.

The above expression can be expressed as

$$
\Phi = \frac{8p^2a}{vb^2\beta} \sin^2 \left( \frac{\pi}{p} \right) \int_{-\infty}^\infty ds \cosh^2(s/2) \sin^2 \left( \frac{2\pi p}{b} - \frac{\pi}{4} + \frac{2\pi a}{vb\beta} s \right)
$$

(21)

The above integration can be performed by using the following identity [12]:

$$
\int_0^\infty dx \frac{\cos ax}{\cosh^2 \beta x} = \frac{a\pi}{2\beta^2 \sinh(a\pi/2\beta)}
$$

(22)

with the result

$$
\Phi = \frac{2p^2T}{\pi^2 bT_D} \sin^2 \left( \frac{\pi}{p} \right) \left[ 1 - A \left( \frac{T}{T_D} \right) + 2A \left( \frac{T}{T_D} \right) \sin^2 \left[ 2\pi \left( \frac{p}{b} - \frac{1}{8} \right) \right] \right]
$$

(23)

where $k_BT_D = \frac{b_v}{4\pi^2a}, \frac{T}{T_D} = \frac{4\pi^2a}{vb\beta}$ and $A(x) = \frac{x}{\sinh(x)} - (x-\to\infty) - \to 2xe^{-x}$.

V. COMPARISON WITH MAGNETICALLY MODULATED CONVENTIONAL TWO-DIMENSIONAL ELECTRON GAS SYSTEM

We start by comparing the energy spectrum and velocity expression obtained in Eq.(9) and Eq.(13) with similar expressions for the conventional 2DEG where the electron spectrum is parabolic [9]. For the energy spectrum, we find that the Landau level spectrum is significantly different from that of standard electrons in conventional 2DEG. The first term
\( \omega \sqrt{n} \) with \( \omega_g = v \sqrt{2eB} \) in Eq.(9) has to be compared with \( \omega_c(n + 1/2) \) with \( \omega_c = eB/m \) for standard electrons. The modulation effects are in the second term where the essential difference is in the structure of the function \( G_n(u) = 2 \sqrt{n} e^{-u/2} \left[ L_{n-1}(u) - L_n(u) \right] \). We find that there are essentially two basic differences: Firstly, for Dirac electrons we have a difference of two successive Laguerre polynomials whereas we had the sum of the Laguerre polynomials in the corresponding term for standard electrons in 2DEG. Secondly, the expression for Dirac electrons is multiplied by the square root of the Landau band index \( \sqrt{n} \) that was absent in the expression for standard electrons. The above mentioned differences in the \( G_n(u) \) function cause the velocity expression for the Dirac electrons given by Eq.(13) to be different from that of the standard electrons.

As expected, these differences in the energy spectra and velocities lead to different results for the diffusive conductivity in the two cases. We now compare the results obtained for the asymptotic expression of the diffusive conductivity \( \sigma_{yy} \). To make this comparison possible we first express \( \Phi \) given by Eq.(23) as

\[
\Phi = \frac{4k_F^2 k_B T}{v e^2 B^2 \alpha} \sin^2 \left( \frac{\pi}{p} \right) F
\]

where \( F = \left[ 1 - A \left( \frac{\pi}{T_D} \right) + 2A \left( \frac{\pi}{T_D} \right) \sin^2 \left[ 2\pi \left( \frac{p}{b} - \frac{1}{8} \right) \right] \right] \). We now compare the results for dimensionless conductivity obtained in Eq.(24) with those presented in Eq. (18) of [9](a). We find that the result for graphene (Dirac electrons) system differs from that of the conventional 2DEG (standard electrons) system by a factor \( \frac{2\pi^2}{\pi^2 b} \sin^2 \left( \frac{\pi}{p} \right) \). For the system under consideration \( p \sim 50 \) and in this limit if we take \( \sin^2 \left( \frac{\pi}{p} \right) \rightarrow \left( \frac{\pi}{p} \right)^2 \) it yields the factor \( \frac{2}{b} \).

Hence we conclude that the amplitude of the oscillations in the conductivity will be reduced by this factor in the magnetically modulated graphene system compared to the conventional 2DEG under the same conditions. For the parameters considered in this work, the conductivity is larger by a factor of \( \approx 44 \) (at magnetic field 0.5T) in the 2DEG system compared to graphene. In Fig.(2) we plot the dimensionless conductivity versus inverse magnetic field for magnetically modulated graphene and 2DEG. Note that in Fig.(2) the dimensionless conductivity for conventional 2DEG is rescaled by a factor .023.

The temperature scale for damping of Weiss oscillations in graphene can be obtained from Eq.(23) and is characterized by \( T_D \) given above while the characteristic tempererature for 2DEG is given in [9](a) as \( k_B T_a = (\hbar \omega_c / 4\pi^2) ak_F \). Comparing \( T_D \) and \( T_a \) we obtain \( \frac{T_D}{T_a} = \frac{v_F}{v} \) where \( v_F \) is the Fermi velocity in 2DEG. This shows that Weiss oscillations in graphene are
less damped with temperature compared to 2DEG due to the difference in Fermi velocities in the two systems.

VI. COMPARISON WITH ELECTRICALLY MODULATED GRAPHENE SYSTEM

We will now compare the results obtained in this work with results obtained in [11] for the case of electrically modulated graphene system. We will first compare the energy spectrum in the two cases. The difference in the energy spectrum due to modulation effects was obtained in Eq.(8). If we compare this result with the corresponding expression for the electrically modulated case, we find the following differences: Firstly, in the magnetic modulation case we have the difference of two successive Laguerre polynomials whereas we had the average of two successive Laguerre polynomials in the electric case. Secondly, in the magnetic modulation case the energy eigenvalues are multiplied by the square root of the Landau band index $\sqrt{n}$ that was absent in the expression for the electric case. These differences cause the velocity expression for the Dirac electrons given by Eq.(13) to be different from that of electrons in the electrically modulated system.

We now compare the expressions for dimensionless conductivity $\Phi$ given by Eq. (23) with the electrically modulated case (Eq.(22) in [11]). We find that in the magnetically modulated case we have $\sin^2 x$ functions in place of $\cos^2 x$ functions for the electric case which results in oscillations being out of phase in the two cases. We also find that the amplitude of the oscillations in the magnetic case are larger by a factor of $\frac{8\pi^2}{b}$ compared to the electrically modulated case. For the parameters considered in this work, the conductivity is larger by a factor of $\approx 1.2$ (at magnetic field $0.5T$) in the magnetically modulated graphene system compared to the electrically modulated one.

Exact expression of the dimensionless conductivity $\Phi$ for the electric and magnetic modulated graphene system is shown in Fig.(3) as a function of the inverse magnetic field at temperature $T = 6K$, electron density $n_e = 3 \times 10^{11}cm^{-2}$ and period of modulation $a = 350nm$. In Fig.(3) we can clearly see that the Weiss oscillations in the dimensionless conductivity are enhanced, they have a larger amplitude, in the magnetically modulated case compared to the electrically modulated case for the same parameter values. Furthermore, we note that the oscillations in the magnetic and electric modulated cases have a $\pi$
phase shift. We also observe that in the region of high magnetic field SdH oscillations are
superimposed on the Weiss oscillations. The oscillations are periodic in $1/B$ and the period
depends on electron density as $\sqrt{n_e}$ in both the magnetic and electric modulated cases. The
characteristic damping temperature is the same for both the systems.

To better understand the increase in amplitude of Weiss oscillations in the magnetically
modulated graphene system compared to the electrically modulated one we consider the
difference in bandwidths in the two cases [13]. Important feature is the additional $\sqrt{n}$ factor
in the perturbed energy eigenvalues for the magnetically modulated case which is absent
in the electrically modulated case. The result is that the bandwidth in the magnetically
modulated case is approximately greater by a factor of $4\sqrt{n}$ compared to the electrically
modulated graphene system.

VII. CONCLUSIONS

In this work we have investigated the electrical transport properties of Dirac electrons
in a monolayer graphene sheet in the presence of a perpendicular magnetic field that is
modulated weakly and periodically along one direction. Our primary focus has been the
study of Weiss oscillations in the diffusive magnetoconductivity $\sigma_{yy}$ of this system. We have
compared the results obtained with those obtained for magnetically modulated conventional
2DEG system and with those of the graphene system subjected to only the electric modula-
tion. We find that in the magnetically modulated graphene system Weiss oscillations in the
magnetoconductivity have a reduced amplitude compared to the conventional 2DEG but are
more robust with respect to temperature. In comparison with the electrically modulated
graphene case, we find that the conductivity is larger in amplitude. We also find that the
oscillations in the magnetoconductivity in graphene are $\pi$ phase shifted with respect to the
electrically modulated case whereas they are in phase with the conventional 2DEG subjected
to magnetic modulation.

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∗Present address: Department of Physics, Blackett Laboratory, Imperial College London, London SW7 2AZ, United Kingdom.

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Fig. (1): The dimensionless conductivity versus inverse magnetic field.

$T = 6K$

$a = 350nm$

$n_e = 3 \times 10^{11} \text{ cm}^2$