A DYNAMICAL METHOD FOR MEASURING THE MASSES OF STARS WITH TRANSITING PLANETS

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ABSTRACT

As a planet transits the face of a star, it accelerates along the line of sight. The changing delay in the propagation of photons produces an apparent deceleration of the planet across the sky throughout the transit. This persistent transverse deceleration breaks the time-reversal symmetry in the transit light curve of a spherical planet in a circular orbit around a perfectly symmetric star. For “hot Jupiter” systems, ingress advances at a higher rate than egress by a factor of \( \sim 10^{-3} \). Forthcoming space telescopes such as Kepler or COROT will reach the sensitivity required to detect this asymmetry. The scaling of the fractional asymmetry with stellar mass \( M_* \) and planetary orbital radius \( a \), as \( (M_*/a^3)^{1/2} \), is different from that of the orbital period, which scales as \( (M_*/a^3)^{-1/2} \). Therefore, this effect constitutes a new method for a purely dynamical determination of the mass of the star. Radial velocity data for the reflex motion of the star can then be used to determine the planet’s mass. Although orbital eccentricity could introduce a larger asymmetry than the light-propagation delay, the eccentricity is expected to decay by tidal dissipation to negligible values for a close-in planet with no perturbing third body. Future detection of the eclipse of a planet’s emission by its star could be used to measure the light-propagation delay across the orbital diameter, \( 46.7(a/7 \times 10^{11} \text{ cm}) \), and also determine the stellar mass from the orbital period.

\textbf{Subject headings:} planetary systems — techniques: photometric

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1. INTRODUCTION

The population of known extrasolar planets that transit the faces of their parent stars has been growing steadily in recent years. It currently includes HD 209458b (Charbonneau et al. 2000), OGLE-TR-56b (Torres et al. 2004), OGLE-TR-113b (Konacki et al. 2004), OGLE-TR-132b (Moutou et al. 2004), TRES-1 (Alonso et al. 2004), OGLE-TR-111b (Pont et al. 2004), and OGLE-TR-10b (Konacki et al. 2005). For HD 209458b the transit light curve was observed with the Hubble Space Telescope (HST) to be symmetric around its centroid to an exquisite photometric precision of \( \sim 10^{-4} \) mag per data point for a few hundred data points tracing both ingress and egress (Brown et al. 2001).

In this Letter, we show that transit light curves must have a time-reversal asymmetry even if the star and the planet are perfectly symmetric and the orbit is circular. The asymmetry originates from the persistent acceleration of the planet toward the star during the transit. For a circular orbit, the planet moves toward the observer at the beginning of the transit (ingress) and away from the observer at its end (egress). This net acceleration introduces a change in the relative rate by which ingress and egress advance in the observer’s frame of reference. The effect simply follows from the unsteady change in the propagation delay of photons, which distorts the transformation of time between the planet and the observer. The variation rate of the delay changes most rapidly at the middle of the transit, when the full Newtonian acceleration vector points straight along the line of sight.

2. APPARENT TRANSVERSE DECELERATION DUE TO PROPAGATION DELAY OF PHOTONS

The observed arrival time of photons \( t_{\text{obs}} \) is given by the time they left the planet \( t \) plus the propagation delay over the distance of the planet \( D \),

\[ t_{\text{obs}} = t + D/c. \tag{1} \]

Taking the \( t \)-derivative of equation (1) gives

\[ dt_{\text{obs}} = dt(1 + v_i/c), \tag{2} \]

where \( v_i(t) = dD/dt \) is the velocity of the planet along the line of sight. Since \( v_i \) changes from negative to positive during the transit of a planet in a circular orbit, it is obvious that ingress will advance at a faster rate (i.e., a shorter \( dt_{\text{obs}} \) per \( dt \) interval) than egress. Figure 1 illustrates schematically this generic behavior. Note that equation (2) is accurate to all orders in \( |v_i/c| \), but we will keep only first-order terms in subsequent derivations.\(^1\)

\[ \underline{v}_{\text{obs}} = \frac{d\underline{x}_i}{dt_{\text{obs}}} = \frac{dt}{dt_{\text{obs}}} \frac{d\underline{x}_i}{dt} \approx \left(1 - \frac{v_i}{c}\right) \underline{v}_\perp \] \tag{3}

(Rybicki & Lightman 1979; see problem 4.7a on p. 151), where the transverse position vector \( \underline{x}_i \) corresponds to angular coordinates on the sky times the distance to the planetary system. Throughout our discussion, the terms parallel (\( \parallel \)) and transverse (\( \perp \)) are relative to the line-of-sight axis that starts at the observer and goes through the center of the stellar image. We focus on the time when the occulting planet crosses this spatial point of symmetry.

\(^1\) Even though the planet is not emitting the observed photons, the timing of its occultation is dictated by the instant at which stellar photons graze its outer surface; this timing is distorted by the same propagation-delay effect that would exist if the grazing photons were emitted by the planet itself rather than the star, since the propagation history of the photons before they graze the planet’s boundary is irrelevant.
Fig. 1.—Schematic illustration of the propagation-delay effect. The transiting planet is moving toward the observer \( v_\parallel < 0 \) during ingress and away from the observer \( v_\parallel > 0 \) during egress. As a result, the observed time interval of partial eclipse is shorter at ingress relative to egress (see eq. [2]). This breaks the time-reversal symmetry of the observed light curve and introduces a fractional difference of \( \delta \) in the temporal slope of the light curve between ingress and egress, where \( \delta \) is the net gain in the planet’s line-of-sight velocity over the transit duration \( T \). [See the electronic edition of the Journal for a color version of this figure.]

values of the transverse acceleration of the planet across the sky,

\[
\dot{v}_\perp = \frac{2v_\parallel}{c} \quad \text{and} \quad \dot{v}_\parallel = \frac{v_\parallel}{c}
\]

(4)

(Loeb 2003), where \( \dot{v}_\perp \) is the observed acceleration and \( \dot{v}_\parallel \) is the actual Keplerian acceleration of the planet. The last term on the right-hand side of equation (4) implies that the apparent transverse acceleration of the planet gets a contribution from its Keplerian acceleration along the line of sight, \( v_\parallel \). This term is the source of our effect.

For simplicity, we assume that the orbital plane is viewed edge-on by the observer; this geometry is exact for transits that cross the center of the star (“central transits”) and is a very good approximation more generally as long as the orbital radius is much larger than the radius of the star. For a circular planetary orbit, Newtonian dynamics implies that there will be no transverse acceleration of the planet at the spatial center of symmetry of the orbit, where

\[
\dot{v}_\perp = 0, \quad v_\perp^2 = \frac{GM_*}{a}; \quad \dot{v}_\parallel = \frac{GM_*}{a^2}, \quad v_\parallel = 0.
\]

(5)

Here \( M_* \) is the stellar mass and \( a \) is the planet’s orbital radius. However, equation (4) implies that at the same time there would be an apparent transverse deceleration of the planet in the observer’s frame of reference,

\[
\dot{v}_\perp \text{obs} = -\frac{v_\perp}{c} \dot{v}_\parallel = -\frac{v_\perp}{v_\parallel} \left( \frac{GM_*}{a^2} \right)^{1/2},
\]

(6)

which is directed opposite to \( v_\perp \). This persistent apparent deceleration implies that the planet will cross ingress and egress at different apparent rates. Thus, even if the stellar image is perfectly symmetric, the orbit is circular, and the planet is perfectly spherical, there is an inherent asymmetry in the transit light curve due to the nearly steady value of \( \dot{v}_\parallel = GM_*/a^2 \) during the transit. Newtonian dynamics alone predicts a nonvanishing \( \dot{v}_\perp \) as soon as the planet moves away from the transit center, but for a circular orbit this deviation would maintain the time-reversal symmetry between ingress and egress.

The fractional asymmetry in the transit light curve is of order the fractional change in \( v_\perp \text{obs} \) over the transit duration, since the rates by which ingress and egress proceed are proportional to \( v_\text{obs} \). For a total transit duration \( T = 2\pi a/v_\parallel \gg T \), the slope of the initial drop and final rise in the transit light curve will differ by a fractional amplitude

\[
\delta \equiv \frac{\dot{v}_\text{obs}}{v_\text{obs}} = \frac{\dot{v}_\text{obs}^T}{v_\text{obs}} = \frac{v_\perp}{c} \frac{2\pi T}{T} = 1.078 \times 10^{-4} \left( \frac{M_*}{1.1 M_\odot} \right) \left( \frac{a}{7 \times 10^{11} \text{ cm}} \right)^{-2} \left( \frac{\tau}{3 \text{ hr}} \right).
\]

(7)

For a “hot Jupiter” such as HD 209458b, the ingress phase \( v_\parallel < 0 \) would proceed at a rate that is higher by fractional amplitude of \( \sim 10^{-4} \) than the egress phase \( v_\parallel > 0 \). Planets that are closer in by a factor of a few could produce an asymmetry of up to \( \delta \sim 10^{-3} \). OGLE-TR-56b, for which \( a = 3.5 \times 10^{11} \text{ cm} \), provides \( \delta = 3 \times 10^{-4} \) (see Gaudi et al. [2005] for a compilation of all known transit systems and the prospects for detecting others).

The observed time \( t_\text{obs} \) is a slightly distorted version of the time axis \( t \), along which the light curve is symmetric. We may write \( t_\text{obs} = t(1 + \epsilon) \), where \( \epsilon(t) = (t/2\pi) \delta \) and we have shifted \( t = 0 \) to be at the transit centroid. The observed photon flux, \( F(t_\text{obs}) \), corresponds to the flux at an undistorted time \( t \approx t_\text{obs}[1 - \epsilon(t)] \), while \( F(-t_\text{obs}) \) corresponds to \( t \approx -t_\text{obs}[1 - \epsilon(-t)] \) \( = -t_\text{obs}[1 + \epsilon(t)] \). For small deviations, we may expand the photon flux as a function of time to leading order, \( F(t + \Delta t) \approx F(t) + dF/dt|_t \Delta t \). Since \( \epsilon(t) = -\epsilon(-t) \), \( F(t) \approx F(-t) \), and \( dF/dt|_t \approx -dF/dt|_{-t} \), the time-dependent photometric asymmetry of the light curve is given by

\[
\Delta F(t_\text{obs}) = F(t_\text{obs}) - F(-t_\text{obs}) = -\frac{d \ln F}{d \ln t_\text{obs}} \frac{t_\text{obs}}{\tau} \delta,
\]

(8)

where we have set \( t_\text{obs} = 0 \) at the transit centroid (for which \( dF/dt_\text{obs} = 0 \) and kept terms to leading order in \( \epsilon \ll 1 \). Given a preliminary transit light curve, \( F(t_\text{obs}) \), and an approximate knowledge of the system parameters, it is possible to predict the light-curve asymmetry under the assumption of a circular orbit. Figure 2 shows the expected \( \Delta F(t_\text{obs}) \) for HD 209458b based on the light-curve data from Brown et al. (2001). For a transit depth of \( 2 \times 10^{-3} \) mag produced by a close-in planet, the net photometric asymmetry would typically amount to a few percent of \( \delta \) or, equivalently, to a peak value of \( \Delta F \) in the range \((0.1-1) \times 10^{-5}\). The photometric sensitivity of \( \sim 10^{-4} \) mag per data point achieved with \( HST \) for HD 209458b (Brown et al. 2001) provides a net sensitivity to a time-reversal asymmetry of \( \sim 10^{-7}(N/10^3)^{-1/2} \), where \( N \approx 10^3 \) is the total number of independent data points during ingress and egress. Our effect appears to be on the borderline of being detectable with existing techniques. A statistically significant (\( \sim 3 \sigma \)) detection of the asymmetry in equation (7) for HD 209458b...
the eccentricity effect is smaller than the propagation-delay effect in equation (6). In the absence of perturbers, any initial orbital eccentricity is expected to decay exponentially through tidal dissipation on an e-folding timescale

\[ t_{\text{circ}} = \frac{e}{\dot{e}} = \left( \frac{4TQ_p}{63 \times 2\pi} \right) \left( \frac{M_p}{M_*} \right) \left( \frac{a}{R_p} \right)^5 \]

\[ = 1.4 \times 10^{-2} \left( \frac{Q_p}{10^5} \right) \left( \frac{M_p}{6 \times 10^{-5}M_*} \right) \left( \frac{a}{75R_p} \right)^5 \text{ Gyr} \]  

\[ \quad \text{(10)} \]

(Goldreich & Soter 1966), where the quality parameter \(Q_p\) (∼10^1 for Jupiter) is inversely proportional to the dissipation rate in the planet’s interior (Iannaou & Lindzen 1993; Ogilvie & Lin 2004) and \(R_p\) is the planet’s radius. Aside from the unknown \(Q_p\), the parameter values in equations (7) and (10) were chosen to match HD 209458b. Since the orbital circularization timescale is shorter by up to 2 orders of magnitude than the age of planetary systems such as HD 209458b (∼5 Gyr; see Mazeh et al. 2000; Cody & Sasselov 2002), the eccentricity-driven asymmetry is expected to diminish for a close-in planet unless a third body pumps its orbital eccentricity (Bodenheimer et al. 2003).

For close-in planets such as HD 209458b, our effect is larger than a different source of light-curve asymmetry that has already been discussed in the literature, namely, the planet’s obliquity (Seager & Hui 2002). This is because close-in planets are expected to rotate slowly and possess a small projected obliquity as a result of strong tidal locking with their orbital revolution. Their rotation axis is expected to be normal to their orbital plane, which is viewed nearly edge-on (in the case of HD 209458b, the orbital inclination angle is \(i = 86^\circ\) ± 16°; Mazeh et al. 2000). Another source of asymmetry is rotation of the star. Typical projected rotation speeds of \(v_\text{rot}\sin i \sim 4 \text{ km s}^{-1}\) (Mazeh et al. 2000; Queloz et al. 2000) would produce a Doppler offset (Loeb & Gaudi 2003) between the flux emitted by the approaching and receding sides of the star of order \(v_\text{rot}/c \sim 1.3 \times 10^{-5}\), which is much smaller than the effect considered here. If the stellar rotation axis is significantly misaligned with the normal of the orbital plane and the transit is not central, then the rotation-induced oblateness of the stellar image could generate a fractional (δ-equivalent) asymmetry of less than \(\frac{1}{2}v_\text{rot}R_*/GM_* \sim 5 \times 10^{-7}\), which is again well below our effect for HD 209458b, where the misalignment angle must be small (Queloz et al. 2000).

The propagation-delay relation between \(t_{\text{obs}}\) and \(i\) in equation (1) can easily be incorporated into a computer program that searches for the best-fit Keplerian orbit under the constraints of a given data set. The delay-corrected Keplerian fit would provide new dynamical constraints on the planetary system. Such a fit would involve the same number of free parameters as the standard Keplerian fit.

3. DISCUSSION

We have shown that the propagation delay of light introduces an apparent transverse deceleration of a planet on the sky during its transit across the face of its parent star (eq. [6]). This persistent deceleration breaks the time-reversal symmetry of the transit light curve for a spherical planet in a circular orbit around a spherical star. Throughout the transit, the planet’s velocity along the line of sight, \(v_\text{s}\), changes at a nearly steady rate, \(GM_p/a^2\). This produces a steady change in the transfor-
Additional special relativistic or general relativistic effects are involving asymmetries from the oblateness of the planet (Hui face of the star and would compete against other small effects would be contaminated by noise from inhomogeneities on the 2002; Agol et al. 2004; Holman & Murray 2005), our effect perturbation of another planet.

as HD 209458b, unless it is being pumped by the gravitational ligible levels through tidal dissipation for close-in planets such as COROT or Kepler (Borucki et al. 2003), which are scheduled for launch within 2–3 years. Because this asymmetry has a unique scaling with stellar mass and orbital radius (\(\propto M_*/a^2\)), its detection, together with the reflex motion of the star, will allow determination of the star and planet masses, as well as the orbital radius, using purely dynamical data.

Orbital eccentricity could induce a stronger asymmetry in the light curve but is expected to decay exponentially to negligible levels through tidal dissipation for close-in planets such as HD 209458b, unless it is being pumped by the gravitational perturbation of another planet.

Similarly to other transit timing residuals (Miralda-Escude 2002; Agol et al. 2004; Holman & Murray 2005), our effect would be contaminated by noise from inhomogeneities on the face of the star and would compete against other small effects involving asymmetries from the oblateness of the planet (Hui & Seager 2002; Seager & Hui 2002) or the rotation of the star. Additional special relativistic or general relativistic effects are of order \(~(v/c)^2\) or \(~\phi/c^2\), or smaller, where \(\phi\) is the gravitational potential produced by the star (\(\phi \sim v^2\)); these corrections are orders of magnitude smaller than the propagation-delay effect discussed here.

Finally, we note that a change of the opposite sign in \(v_i\) occurs when the planet goes behind the star. In this case, ingress would be slower than egress. There is no net change in the orbital period over a full closed orbit. However, when the planet enters its own (secondary) eclipse by the star, the photons it emits will be delayed relative to primary eclipse (the transit) by the difference in emission times plus the light-travel time across the orbital diameter, as implied by equation (1). For a circular orbit, the time interval between the centroids of the primary and secondary eclipses would be longer by \(\delta T_{1/2} = 2a/c\) than half the full orbital period. Future detection of a planet’s infrared emission would then allow one to determine the orbital radius from the light-propagation delay of \(\delta T_{1/2} = 4.67(a/7 \times 10^7 \text{ cm})/c\). The full orbital period \(T\) would then yield the stellar mass through \(M_\star = a^3/[2(G/72\pi^3)]\). An eccentricity could change the light-curve history of the illuminated planet and, in particular, make the time from primary eclipse to secondary eclipse different from the time back to primary eclipse.

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