Screwed superconducting cosmic strings

C.N. Ferreira* 1

*Centro Brasileiro de Pesquisas Físicas, Rua Dr. Xavier Sigaud 150, Urca 22.290-180, Rio de Janeiro, RJ, Brazil

Abstract

We show that it is possible to build up a consistent model describing a superconducting cosmic string (SCS) endowed with torsion. A full string solution is obtained by matching the internal and the external solutions. We derive the deficit angle, the geodesics of and the gravitational force on a test-particle moving under the action of this screwed SCS. A couple of potentially observable astrophysical phenomena are highlighted: the dynamics of compact objects orbiting torsioned SCS and the accretion of matter onto it.

1crisnfer@cbpf.br
1 Introduction

Cosmic strings are a class of solutions [1] which represent topological defects that may have been formed during phase transitions in the realm of the Early Universe [2]. The GUT defects carry a large energy density and, hence, are of interest in Cosmology, as potential sources for primordial density perturbations [3]. These fluctuations would leave their imprint in the cosmic microwave background radiation (CMBR); a prediction not fully ruled out by COBE satellite observations yet [4, 5]; so, they would act as seeds for structure formation and consequently builders of the largest-scale structures in the Universe [6], such as the very high redshift superclusters of galaxies (as for instance the great wall). They may also help to explain the most energetic events in the Universe, like the cosmological gamma-ray bursts (GRBs) [7], ultra high energy cosmic rays (UHECRs) and very high energy neutrinos [8], and gravitational-wave bursts and backgrounds [9]. All these are issues deserving continuous investigation by many physicists nowadays [10].

Witten [11] has shown that cosmic strings may possess superconducting properties and may behave like both bosonic (see Ref. [12] and references therein) and fermionic strings [13, 14]. The relevant superconductivity is generated during, or very soon after, the primary phase transition in which the string was formed.

A further question of much relevance concerns the consideration of gravitational effects on the formation of SCS’s. In this direction, it has already been discussed by several authors that torsion may have been an important element in the early Universe, when the quantum effects of gravitation were drastically important [15, 16].

Our present understanding of the early Universe and the Planck era indicates that extended objects, like strings, are the best framework to describe the physics between the GUT and the Planckian scales [17]. String corrections to Quantum Gravity [18] lead to effective gravitational models where torsion plays a very relevant rôle through $\alpha'$-corrections terms involving curvature and torsion [19, 20]. So, our viewpoint in this paper is that torsion effects are to be taken into account and are supposed to persist till the cosmic strings were formed.

The Universe in its very early era ($10^{18} GeV \geq T \geq 10^{16} GeV$, the era before the GUT phase transition ) consisted of torsion particles and GUT (SU(5)) particles (heavy fermions, gauge bosons and Higgs particles); torsion particles couple with intrinsic spins of matter. Therefore, this interaction appears in the era in which the gravitational inter-
action became important for microscopic systems, such as the very early universe\cite{21}.

In \[16\], to describe the space-time defects \cite{22,23} in the early universe in an invariant form, the authors adopt a new topological invariant, obtained by means of the torsion tensor, to measure the size of defects and interpret them as the quantized dislocation flux in internal space. Related to these arguments, it may become a relevant issue to contemplate topological defects described by Superconducting Cosmic Strings (SCS) coupled torsion; they may be present as a relic in ours days.

Cartan torsion has been previously related to spinning cosmic strings \cite{24,25} from quite distinct points of view. In particular, Soleng\cite{24} has studied the case in which the cosmic string can be identified with a cylinder filled in with an anisotropic fluid possessing an intrinsic spin polarized along the string symmetry axis. It was shown there that the spin angular momentum generates torsion as a natural consequence of extending Einstein’s theory to include Riemann-Cartan geometry. Such a spinning string exhibits interesting gravitational effects, including inertial dragging and the possible generation of closed time-like lines.\cite{24} Although that theory was consistently developed, one has not discussed neither the astrophysical nor cosmological constraints on the potential implications of its realization in Nature. Moreover, as it is easy to see from the Arkuszewski-Kopczyński-Ponomariev matching conditions (Eq.18 in Ref.\cite{24}), the spin (i.e., torsion) effects do not propagate outwards. This is, of course, due to the fact that the string is composed of an anisotropic spin-polarized fermionic fluid. Nonetheless, this property (spin effects propagation) manifests itself as a space-time metric, as stressed by the definition (coupling) introduced in the Eq.(6) of Soleng’s paper\cite{24}. Conversely, our model does exhibit propagation of torsion effects outwards, and this may be responsible for the appearance of new, unexpected phenomena, as discussed below. Far-reaching effects are possible because, to obtain a complete string solution, we need to introduce superconductivity, which allows us to relate the torsion field of our model to the vortex characterizing the defect.

Recently, Kleinert has pointed out a number of aspects and effects of torsion\cite{26,27,28}. In his work, the gravitational counterpart yields consistent results only for completely antisymmetric or gradient torsion. If one takes into account that even spinless particles experience a torsion-originated force, one might naturally expect them to be also source for torsion. In a line of works related to physical consequences of torsion, Kleinert shows that deviations from Einstein gravity’s effects may be attained if one adds to the gravity
action a gradient term for torsion. According to these works, a good consistency test for gravity with torsion is the criterium that the torsion coupling to matter must be such that spinless particles run on autoparallel, rather than geodesic, trajectories. On the other hand, it is to be noticed that massless and massive gauge vector gauge bosons couple differently to torsion. To accommodate this fact with the mass generation via a Higgs mechanism, one avoids inconsistencies if the Higgs scalar runs along autoparallel trajectories.

In this work, we study the case of bosonic SCS’s in Riemann-Cartan space-time with coupling terms in the potential driving the string dynamics. We aim at dealing with more realistic models which may demand supersymmetry, an essential ingredient for Grand-Unified theories and string theories. Thus, we ought to combine both gravitational and spin degrees of freedom in the same formalism; thus, torsion is required.

The main-stream of this paper is as follows: we explore the physics of torsion coupling to cosmic strings in Section II. An external solution for the SCS metric in this scenario in presented in Section III, while in Section IV we derive the corresponding metric for the internal structure of the SCS by using the weak-field approximation. The complete solution of the SCCS is obtained in Section V, by using the joint conditions. In section VI, we obtain a neat expression for the deficit angle in this context. Two applications are provided in Section VII. It is shown that such a high intensity of the gravitational force from the screwed SCS (when compared with the one generated by a superconducting string) may have important effects on the dynamics of compact objects orbiting around it, and also on matter being accreted by the string itself.

## 2 Torsion coupling to cosmic strings

Here, we propose a consistent framework for the torsion field pervading a cosmic string and define the vortex configuration for this problem. We choose here to analyze the simplest case where torsion appears. In this line of reasoning, it is possible to describe torsion as a gradient-like field

\[ S_{\mu \nu}^\lambda = \frac{1}{2} [\delta^\lambda_\mu S_{\nu} - \delta^\lambda_\nu S_{\mu}], \quad \text{(2.1)} \]

the affine connection being written as
\[ \Gamma_{\lambda \nu}^\alpha = \{\alpha_{\lambda \nu}\} + S^\alpha g_{\lambda \nu} - S_{\lambda \delta}^\alpha \gamma^\delta_{\nu}, \]  

(2.2)

where \( S_{\alpha} = \partial_{\alpha} \Lambda \) is the only piece contributing to torsion, given in Eq. (2.1). Here, the \( \Lambda \)-field is the source of torsion in the string.

We may consider a theory of gravitation possessing torsion by writing the part of the action \( I \) stemming from the curvature scalar \( R \) as:

\[
I = \int d^4 x \sqrt{g} \left[ \frac{1}{16 \pi G} R(\{\}) - \frac{\alpha}{2} \partial_{\mu} \Lambda \partial^{\mu} \Lambda \right] + I_m, \tag{2.3}
\]

where \( R(\{\}) \) is the curvature scalar of the Riemannian theory and \( I_m \) is the matter action which describes the superconducting cosmic string. The coupling constant \( \alpha \) will be specified with the help of COBE data.

We can study the SCS considering the Abelian Higgs model with two scalar fields, \( \phi \) and \( \tilde{\Sigma} \). In this case, the action for all matter fields turns out to be:

\[
I_m = \int d^4 x \sqrt{g} \left[ -\frac{1}{2} D_\mu \phi (D^\mu \phi)^* - \frac{1}{2} D_\mu \Sigma (D^\mu \Sigma)^* - \frac{1}{4} F^{\mu \nu} F_{\mu \nu} - \frac{1}{4} H^{\mu \nu} H_{\mu \nu} - V(|\phi|, |\Sigma|, \Lambda) \right], \tag{2.4}
\]

where \( D_\mu \Sigma = (\partial_\mu + i e A_\mu) \Sigma \) and \( D_\mu \phi = (\partial_\mu + i q C_\mu) \phi \) are the covariant derivatives. The reason why the gauge fields do not minimally couple to torsion is well discussed in Refs. [29, 30]. The field strengths are defined as usually as \( F^{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \) and \( H^{\mu \nu} = \partial_\mu C_\nu - \partial_\nu C_\mu \), with \( A_\mu \) and \( C_\mu \) being the gauge fields.

The potential \( V(\varphi, \sigma, \Lambda) \) triggering the symmetry breaking can be fixed by:

\[
V(\varphi, \sigma, \Lambda) = \frac{\lambda_\varphi}{4} (\varphi^2 - \eta^2)^2 + f_{\varphi \sigma} \varphi^2 \sigma^2 + \frac{\lambda_\sigma}{4} \sigma^4 - \frac{m_\sigma^2}{2} \sigma^2 + l^2 \sigma^2 \Lambda^2, \tag{2.5}
\]

where \( \lambda_\varphi, \lambda_\sigma, f_{\varphi \sigma}, m_\sigma \) and \( l^2 \) are coupling constants. Constructed this manner, this potential possesses all the ingredients that make it viable the formation of a superconducting cosmic string, as it is well-established. In addition, it is extended to include a new term describing the interaction with the torsion field. The presence of this interaction term does not affect the appearance of the string ground states. However, it contributes a torsion density in the string core due to the coupling with the charged particle flux.

The action of eq. (2.3) has a \( U(1)' \times U(1) \) symmetry, where the \( U(1)' \) group, associated with the \( \phi \)-field, is broken by the vacuum and gives rise to vortices of the Nielsen-Olesen type [31].
\[ \phi = \varphi(r)e^{i\theta}, \]
\[ C_\mu = \frac{1}{q}[P(r) - 1]\delta_\mu^0, \]

which are here written in cylindrical coordinates \((t, r, \theta, z)\), where \(r \geq 0\) and \(0 \leq \theta < 2\pi\). The boundary conditions for the fields \(\varphi(r)\) and \(P(r)\) are the same as those of ordinary cosmic strings[{11}]:

\[ \varphi(r) = \eta \quad r \to \infty \quad P(r) = 0 \quad r \to \infty \]
\[ \varphi(r) = 0 \quad r = 0 \quad P(r) = 1 \quad r = 0. \]

The other \(U(1)\)-symmetry, that we associate to electromagnetism, acts on the \(\Sigma\)-field. This symmetry is not broken by the vacuum; however, it is broken in the interior of the defect. The \(\Sigma\)-field in the string core, where it acquires an expectation value, is responsible for a bosonic current being carried by the gauge field \(A_\mu\). The only non-vanishing components of the gauge fields are \(A_z(r)\) and \(A_t(r)\), and the current-carrier phase may be expressed as \(\zeta(z, t) = \omega_1 t - \omega_2 z\). Notwithstanding, we focus only on the magnetic case[{12}]. Their configurations are defined as:

\[ \Sigma = \sigma(r)e^{i\zeta(z,t)}, \]
\[ A_\mu = \frac{1}{\varepsilon}[A(r) - \frac{\partial\zeta(z,t)}{\partial z}]\delta_\mu^z, \]

because of the rotational symmetry of the string itself. The fields responsible for the cosmic string superconductivity have the following boundary conditions:

\[ \frac{d}{dr}\sigma(r) = 0 \quad r = 0 \quad A(r) \neq 0 \quad r \to \infty \]
\[ \sigma(r) = 0 \quad r \to \infty \quad A(r) = 1 \quad r = 0. \]

Let us consider a SCS in a cylindrical coordinate system \((t, r, \theta, z)\), so that \(r \geq 0\) and \(0 \leq \theta < 2\pi\) with the metric defined in these coordinates as:

\[ ds^2 = e^{2(\gamma - \psi)}(-dt^2 + dr^2) + \beta^2 e^{-2\psi} d\theta^2 + e^{2\psi}dz^2, \]

where \(\gamma, \psi\) and \(\beta\) depend only on \(r\). We can write the Einstein-Cartan equations in a quasi-Einsteinian form:

\[ G_\nu^\mu(\{\}) = 8\pi G(2\alpha g^{\alpha\beta}\partial_\alpha\Lambda\partial_\beta\Lambda - \alpha\delta_\mu^\alpha g^{\alpha\beta}\partial_\alpha\Lambda\partial_\beta\Lambda + T_\nu^\mu) = 8\pi G\tilde{T}_\nu^\mu, \]

(2.11)
where \( (\{\}) \) stands for Riemannian geometric objects, \( \delta^\mu_\nu \) and \( T^\mu_\nu \) correspond to the identity and energy-momentum tensors, respectively. The \( \tilde{T}^\mu_\nu \) tensor corresponds to an energy-momentum tensor containing the torsion field.

We have seen that the dependence upon torsion is represented, in the quasi-Einsteinian form, by the \( \Lambda \)-field that has an equation of motion given by Eq.(2.18) below, whose solution shall be presented subsequently.

The SCS energy-momentum tensor is defined by

\[
T^\mu_{(scs)\nu} = \frac{2}{\sqrt{g}} \frac{\delta S_m}{\delta g_{\mu\nu}},
\]

which yields:

\[
T^t_{scs \ t} = -\frac{1}{2} \left\{ e^{2(\varphi - \gamma)} [\varphi^2 + \sigma^2] + e^{2(\varphi - \gamma)} \phi^2 P^2 + e^{-2\varphi} \sigma^2 A^2 + \frac{e^{2(2\varphi - \gamma)} (P \phi')^2}{2} + e^{-2\gamma} \left( \frac{A'}{e} \right)^2 + 2V(\varphi, \sigma, \Lambda) \right\}
\]

\[
T^r_{scs \ r} = -\frac{1}{2} \left\{ e^{2(\varphi - \gamma)} [\varphi^2 + \sigma^2] - e^{2\varphi} \phi^2 P^2 - e^{-2\varphi} \sigma^2 A^2 + \frac{e^{2(2\varphi - \gamma)} (P \phi')^2}{2} + e^{-2\gamma} \left( \frac{A'}{e} \right)^2 - 2V(\varphi, \sigma, \Lambda) \right\}
\]

\[
T^\theta_{scs \ \theta} = -\frac{1}{2} \left\{ e^{2(\varphi - \gamma)} [\varphi^2 + \sigma^2] - e^{2\varphi} \phi^2 P^2 + e^{-2\varphi} \sigma^2 A^2 + \frac{e^{2(2\varphi - \gamma)} (P \phi')^2}{2} + e^{-2\gamma} \left( \frac{A'}{e} \right)^2 + 2V(\varphi, \sigma, \Lambda) \right\}
\]

\[
T^z_{scs \ z} = -\frac{1}{2} \left\{ e^{2(\varphi - \gamma)} [\varphi^2 + \sigma^2] + e^{2\varphi} \phi^2 P^2 - e^{-2\varphi} \sigma^2 A^2 + \frac{e^{2(2\varphi - \gamma)} (P \phi')^2}{2} - e^{-2\gamma} \left( \frac{A'}{e} \right)^2 - 2V(\varphi, \sigma, \Lambda) \right\}.
\]

In the expression of eqs.(2.13-2.16) only the usual fields of the string are present. The Euler-Lagrange equations result from the variation of the Eq.(2.3) together with the conditions for the Nielsen-Olesen [31] vortex, Eqs.(2.6-2.8), and yield:

\[
\varphi'' + \frac{1}{r} \varphi' + \frac{\varphi P^2}{r^2} - \varphi [\lambda_{\phi} (\varphi^2 - \eta^2) + 2f_{\phi\sigma}\sigma^2] = 0
\]

\[
\sigma'' + \frac{1}{r} \sigma' + \sigma [A^2 + (f_{\phi\sigma}\varphi^2 + \lambda_{\sigma}\sigma^2 - m_{\sigma}^2 + l^2 A^2)] = 0
\]

\[
P'' - \frac{1}{r} P' - q^2 \varphi^2 P = 0,
\]

\[
A'' + \frac{1}{r} A' - e^2 \sigma^2 A = 0,
\]

7
while the torsion wave equation is given by:

$$\square_g \Lambda = \frac{l^2}{\alpha} \sigma^2 \Lambda.$$  \hspace{1cm} (2.18)

In the equations above, a prime denotes differentiation with respect to the radial coordinate $r$. The general solution for the SCS will be found in the weak-field approximation together with junction conditions for the external metric.

### 3 The external solution

Now, we go on solving the previous set of equations for an observer outside the SCS stressed by torsion focusing on the external metric which satisfies the constraint $r_0 \leq r \leq \infty$. The external contribution to the energy-momentum of the string reads

$$\mathcal{T}_\mu^\nu = \frac{1}{4} g^{\mu\alpha} g^{\beta\rho} F_{\alpha\beta} F_{\nu\rho} - \delta_\nu^\mu g^{\sigma\alpha} g^{\beta\rho} F_{\sigma\beta} F_{\alpha\rho}.$$  \hspace{1cm} (3.1)

This tensor is the external energy-momentum tensor of a SCS with no torsion. If we consider the asymptotic conditions, Eq.(2.7) and Eq.(2.9), we see that the only field that does not vanish is the $A_\mu$-field. This field is responsible for the conduction of the string current. The torsion contribution to the external energy-momentum tensor is given by

$$\mathcal{T}_\mu^{\nu_{tor}} = 2\alpha g^{\mu\alpha} \partial_\alpha A_\nu \Lambda - \alpha \delta_\nu^\mu g^{\sigma\alpha} \partial_\alpha A_\beta \Lambda.$$  \hspace{1cm} (3.2)

For this configuration, the energy-momentum tensor displays the following symmetry properties:

$$\mathcal{T}_t^t = -\mathcal{T}_r^r = \mathcal{T}_\theta^\theta = -\mathcal{T}_z^z.$$  \hspace{1cm} (3.3)

Then, the only one component of $\Lambda$ in Eq.(2.18) to be solved is the $r$—dependent function $\Lambda(r)$. The solution reads:

$$\Lambda(r) = \lambda \ln(r/r_0).$$  \hspace{1cm} (3.4)

The vacuum solution of Eqs.(2.11) are found from the symmetries Eq.(3.3). Hence the solutions of $\beta(r)$ and $\gamma(r)$ are given by
\[ \beta = Br, \quad \gamma = m^2 \ln r/r_0. \] (3.5)

To find the \( \psi \)-solution, we can use the condition:

\[ R = 2\Lambda^2 e^{2(\psi - \gamma)}. \] (3.6)

This condition is different from the usual one\[12\] because the scalar curvature \( R \) does not vanish, and opposely it is linked to the torsion-field \( \Lambda \). Then, this condition has the same form as the one for a SCS in a scalar-tensor theory \[32\]. By making use of solutions (3.4), (3.5), we find:

\[ \psi = n \ln (r/r_0) - \ln \frac{(r/r_0)^{2n} + k}{1 + k}. \] (3.7)

Thus we see from the solutions of the SCCS Eqs. (3.5, 3.7), that there exists a relationship between the parameters \( n, \lambda \) and \( m \) given by \( n^2 = \lambda^2 + m^2 \).

With the above results, we find that the external metric for the SCS takes the form:

\[ ds^2 = \left( \frac{r}{r_0} \right)^{-2n} W^2(r) \left[ \left( \frac{r}{r_0} \right)^{2m^2} (-dt^2 + dr^2) + B^2 r^2 d\theta^2 \right] + \left( \frac{r}{r_0} \right)^{2n} \frac{1}{W^2(r)} dz^2, \] (3.8)

with \( W(r) = [(r/r_0)^{2n} + k]/[1 + k] \).

As it is clear, the external solution alone does not provide a complete description of the physical situation. We proceed hereafter to find the junction conditions to the internal metric, in order to obtain an appropriate accounting for the nature of the source and its effects on the surrounding space-time.

### 4 SCS solution: The weak-field approximation

Now, let us find the Einstein-Cartan solutions for a SCCS by considering the weak-field approximation. Thus, the space-time metric may be expanded in terms of a small parameter \( \varepsilon \) about the values \( g_{(0)\mu\nu} = diag(-1, 1, 1, 1) \), then:

\[ g_{\mu\nu} = g_{(0)\mu\nu} + \varepsilon h_{\mu\nu}, \]
\[ \tilde{T}_{\mu\nu} = \tilde{T}_{(0)\mu\nu} + \varepsilon \tilde{T}_{(1)\mu\nu}. \] (4.1)
The $\tilde{T}_{(0)\mu\nu}$ tensor corresponds to the energy-momentum tensor in a space-time with no curvature. However, torsion is embedded. $\tilde{T}_{(1)\mu\nu}$ represents the part of the energy-momentum tensor containing curvature and torsion. Next we proceed to define some important quantities useful for the analysis to come.

The energy-momentum density and tension of the thin SCCS are given by:

$$U = -2\pi \int_0^{r_0} \tilde{T}^t_{(0)t}rdr;$$

$$T = -2\pi \int_0^{r_0} \tilde{T}^r_{(0)r}rdr.$$  \(4.2\)

The remaining components follow as

$$X = -2\pi \int_0^{r_0} \tilde{T}^r_{(0)r}rdr;$$

$$Y = -2\pi \int_0^{r_0} \tilde{T}^\theta_{(0)\theta}rdr.$$  \(4.3\)

The energy conservation in the weak-field approximation reduces to

$$r \frac{d\tilde{T}^r_{(0)r}}{dr} = (\tilde{T}^\theta_{(0)\theta} - \tilde{T}^r_{(0)r}),$$  \(4.4\)

where $\tilde{T}_{(0)\mu\nu}$ represents the trace of the energy-momentum tensor with torsion.

To compute the overall metric, we use the Einstein-Cartan equations in the quasi-Einsteinian Eq.(2.11), where it gets the form $G^{\mu\nu}(\{\}) = 8\pi G \tilde{T}^{\mu\nu}$ in the weak-field approximation, with the tensor $\tilde{T}^{\mu\nu}$ (being first order in $G$) containing torsion. After integration, we have:

$$\int_0^{r_0} rdr(\tilde{T}^\theta_{(0)\theta} + \tilde{T}^r_{(0)r}) = r_0^2 \tilde{T}^r_{(0)r}(r_0) = r_0^2 \left[ \frac{A'^2(r_0)}{2e^2} + \frac{\alpha}{2} \Lambda^2(r_0) \right].$$  \(4.5\)

To find the internal energy-momentum tensor, it is more convenient to use Cartesian coordinates\[12\]. For this purpose, we calculate the cross-section integrals of $\tilde{T}^x_{(0)x}$ and $\tilde{T}^y_{(0)y}$; in Cartesian coordinates, they read as below:

$$\tilde{T}^x_{(0)x} = c[\sigma'^2 + \sigma^2 + \alpha A'^2] + s \sqrt{2q} \left( \frac{P'}{qr} \right)^2 + \frac{1}{2} \left( \frac{P'}{q} \right)^2 - 2V - 1 \frac{\sigma^2 A^2}{2} - 2V;$$  \(4.6\)

where $c = \cos^2 \theta - \frac{1}{2}$ and $s = \sin^2 \theta - \frac{1}{2}$. We then find:

$$\int r dr d\theta \tilde{T}^x_{(0)x} = \int r dr d\theta \tilde{T}^y_{(0)y} = \pi \int r dr \left[ \left( \frac{P'}{qr} \right)^2 - \sigma^2 A^2 - V \right] = -W.$$  \(4.7\)
Using the fact that $\tilde{T}^r_r + \tilde{T}^\theta_\theta = \tilde{T}^x_x + \tilde{T}^y_y$, then we have:

$$X + Y = 2W = -2\pi r_0^2 \left[ \frac{A'(r_0)}{e^2} + \alpha \Lambda'^2(r_0) \right], \quad (4.8)$$

which can be computed by integration of Eq.(2.17) so as to give

$$A'(r) = \frac{eJ}{\sqrt{2\pi r}}, \quad J = \sqrt{2}\pi e \int_0^{r_0} r dr \sigma^2 A, \quad (4.9)$$

where $J$ is the current density. Thus, the torsion density can be computed by integration of Eq.(2.18)

$$\Lambda' = \frac{S}{\sqrt{2\pi \alpha r}}, \quad S = \sqrt{2}\pi l^2 \int_0^{r_0} r dr \sigma^2 \Lambda, \quad (4.10)$$

where $S$ is the torsion density. With these considerations, we find the string structure. Then, we obtain

$$W = -\frac{1}{2\pi} (J^2 + \nu S^2), \quad (4.11)$$

with $\nu = 1/\alpha$.

In addition, we can assume that the string is infinitely thin so that its stress-energy tensor is given by

$$\tilde{T}^{\mu\nu}_{\text{string}} = \text{diag}[U, -W, -W, -T] \delta(x) \delta(y). \quad (4.12)$$

Is worth noticing that definitions for both string energy $U$ and tension $T$, as in equations [12, 32], already incorporate information about the torsion.

By virtue of the presence of the external current, we use the form Eq.(1.12) for the string energy-momentum tensor as well as Eq.(3.1) and Eq.(3.2) for the external energy-momentum tensor in linearized solution to zeroth order in $G$. In the sense of distributions we have,

$$\nabla^2 \ln(r/r_0) = 2\pi \delta(x) \delta(y)$$

$$\nabla^2 (\ln(r/r_0))^2 = 2/r^2$$

$$\nabla^2 (r^2 \partial_i \partial_j \ln(r/r_0)) = 4 \partial_i \partial_j \ln(r/r_0). \quad (4.13)$$

The energy-momentum tensor of the string source $\tilde{T}^{(0)\mu\nu}$ (Cartesian coordinates) possesses no curvature, which is the well-known result [12, 32], but does have torsion which produces the following energy-momentum tensor
\[ \tilde{T}_{(0)tt} = U \delta(x) \delta(y) + \frac{(J^2 + \nu S^2)}{4\pi} \nabla^2 \left( \ln \frac{r}{r_0} \right)^2, \]
\[ \tilde{T}_{(0)zz} = -T \delta(x) \delta(y) + \frac{(J^2 - \nu S^2)}{4\pi} \nabla^2 \left( \ln \frac{r}{r_0} \right)^2, \]
\[ \tilde{T}_{(0)ij} = (J^2 + \nu S^2) \delta_{ij} \delta(x) \delta(y) - \frac{(J^2 + \nu S^2)}{2\pi} \partial_i \partial_j \ln(r/r_0), \quad (4.14) \]

where the trace is given by \( \tilde{T}_{(0)} = -(U + T - J^2 - \nu S^2) \delta(x) \delta(y) - \frac{\nu S^2}{2\pi} \nabla^2 \left( \ln \frac{r}{r_0} \right)^2. \)

Now, let us find the matching conditions to the external solution. For this purpose, we shall use the linearized Einstein-Cartan equation in the form
\[ \nabla^2 h_{\mu\nu} = -16\pi G (\tilde{T}_{(0)\mu\nu} - \frac{1}{2} g_{(0)\mu\nu} \tilde{T}_{(0)}). \quad (4.15) \]

The internal solution to Eq. (4.15) with time-independent source yields:
\[ h_{tt} = -4G[J^2(\ln(r/r_0))^2 + (U - T + J^2 + \nu S^2) \ln(r/r_0)] \]
\[ h_{zz} = -4G[J^2(\ln(r/r_0))^2 + (U - T - J^2 - \nu S^2) \ln(r/r_0)] \]
\[ h_{ij} = -[2G(J^2 + \nu S^2)r^2 \partial_i \partial_j - 4G \delta_{ij}(U + T + J^2 + \nu S^2) \ln(r/r_0)] + S^2 \left( \ln \frac{r}{r_0} \right)^2. \quad (4.16) \]

This corresponds to the solution found in Cartesian coordinates. We note that the torsion appears explicitly in the transverse components of the metric. To analyze the solution for the with junction condition to the external metric, let us transform it back into cylindrical coordinates.

\section{The Matching Conditions}

It is possible to find the junction conditions for the external solution \[33\]. In the case of a space-time with torsion, we may find the matching conditions using the fact that \([\{a_{\mu\nu}\}]^{(+)\prime} = \{a_{\mu\nu}\}]^{(-)\prime}, \) and the metricity constraint \([\nabla_\rho g_{\mu\nu}]^{(+)} = [\nabla_\rho g_{\mu\nu}]^{(-)} = 0, \) to get the continuity conditions
\[ [g_{\mu\nu}]_{r=r_0}^{(-)} = [g_{\mu\nu}]_{r=r_0}^{(+)}, \]
\[ [\partial g_{\mu\nu}^\rho / \partial x^\nu]^{(+)} + 2[g_{\alpha\rho} K_{(\mu\nu)}]^{(+)} = [\partial g_{\mu\nu}^\rho / \partial x^\nu]^{(-)} + 2[g_{\alpha\rho} K_{(\mu\nu)}]^{(-)}, \quad (5.1) \]

where \((-)\) represents the internal region and \((+)\) corresponds the external region around \( r = r_0. \) In analyzing the junction conditions we notice that the contortion contributions do not appear neither in the internal nor in the external regions \[33, 34\].
To match our solution with the external metric, we used the metric in cylindrical coordinates, which is obtained from the coordinate transformations:

\[ r^2 \partial_i \partial_j \ln(r/r_0) dx^i dx^j = r^2 d\theta^2 - dr^2, \]  

Unfortunately, for this goal we cannot use the metric the way it stands. Therefore, we have to change the radial coordinate to \( \rho \), using the constraint (symmetry) \( g_{\rho \rho} = -g_{tt} \), to have, to first order in \( G \),

\[ \rho = r[1 + a_1 - a_2 \ln(r/r_0) - a_3 (\ln(r/r_0)^2)]. \]  

In this case, we have \( a_1 = G(4U + J^2 + \nu S^2) \), \( a_2 = 4GU \) and \( a_3 = -2G(J^2 + \nu S^2) \), which corresponds to the magnetic configuration of the string fields \( [12] \). The transformed metric yields:

\[ g_{tt} = -\left\{1 + 4G[J^2(\ln(\rho/r_0))^2 + (U - T + J^2 + \nu S^2) \ln(\rho/r_0)]\right\} = -g_{\rho \rho} \]

\[ g_{zz} = \left\{1 - 4G[(J^2 + \nu S^2)(\ln(\rho/r_0))^2 + (U - T + J^2 - \nu S^2) \ln(\rho/r_0)]\right\} \]

\[ g_{\theta \theta} = \rho^2 \left\{1 - 8G(U + \frac{(J^2 + \nu S^2)}{2}) + 4G(U - T - J^2 - \nu S^2) \ln(\rho/r_0)] + 4GJ^2(\ln(\rho/r_0))^2\right\}. \]

(5.4)

Now, we are ready to find the external parameters \( B \), \( n \) and \( m \) as functions of the source structure. If we consider the junction of the Eq.(5.1), after the linearization, and using the limit \( |n \ln(\rho/r_0)| << 1 \), we have:

\[ n \left(\frac{1-k}{1+k}\right) = 2G(U - T - J^2 - \nu S^2) \]

\[ B^2 = 1 - 8G \left(U + \frac{(J^2 + \nu S^2)}{2}\right) \]

\[ m^2 = 4G(J^2 + \nu S^2). \]

(5.5)

Using the derivative of the expression Eq.(3.4) and the Eq.(4.10), we arrive at

\[ \lambda = \tilde{G} S. \]

(5.6)

where \( \tilde{G} = \frac{\nu}{\sqrt{2\pi}} \). This expression completes the derivation of the full metric components.

In analyzing the metric of the SCS with torsion, we note that the contribution of torsion appears in the \( \theta \theta \)-metric component, which is important for astrophysical applications such as gravitational lensing studies, because this component is linked to the deficit angle.
6 Screw effect on the propagation of particles and light

In this section, we analyze the metric around a screwed superconducting cosmic string. Let us investigate the deficit angle. If we consider the metric Eqs. (5.4), projected into the space-time perpendicular to the string, i.e., \( dz = 0 \), then we have:

\[
ds^2_{\perp} = (1 - h_{tt})[-dt^2 + dr^2 + (1 - b)r^2d\theta^2],
\]
with \( h_{tt} \) given by

\[
h_{tt} = -4G(J^2(\ln(\rho/r_0))^2 + (U - T + J^2 + \nu S^2) \ln(\rho/r_0)) \]
(6.2)

Then, to first order in \( G \), the deficit angle gets:

\[
\delta \theta = b\pi = 8\pi G\{U + J^2(1 + \ln(\rho/r_0)) + \frac{1}{2}\nu S^2(1 - 2\ln(\rho/r_0))\}.
\]
(6.3)

We know that, when the string possesses current, there appear gravitational forces. We shall consider the effect that torsion plays on the gravitational force generated by SCS on a particle moving around the defect, assumed here the has particle no charge. We consider the particle speed \( |v| \leq 1 \), condition under which the geodesic equation becomes:

\[
d^2x^i/d\tau^2 + \Gamma^i_{tt} = 0,
\]
where \( i \) is the spatial coordinate, and the connection can be written as in Eq.(2.2). The gravitational acceleration around the string gets the form

\[
a = \frac{1}{2}(\nabla h_{tt} - \tilde{G}\frac{S}{\rho}),
\]
(6.5)

with \( g_{tt} = -1 - h_{tt} \) in Eq.(5.4). We also note that the gravitational pull is related to the \( h_{tt} \) component that has explicit dependence on the torsion, as shown in Eq.(6.2). From the last equation, the force the SCCS exerts on a test particle can be explicitly written as

\[
f = -\frac{m}{\rho}\left[2GJ^2\left(1 + \frac{(U - T + \nu S^2)}{J^2} + 2\ln(\rho/r_0)\right) + \tilde{G}S\right].
\]
(6.6)

where \( m \) is the mass of the particle.
Let us consider the deflection of particles moving in the geometry of the string. For that, we work with the metric in Cartesian coordinates,

\[ ds^2 = (1 - h_{tt})[dt^2 - dx^2 - dy^2], \]  

(6.7)

with \( h_{tt} \) given by (7.2), where for simplicity, we consider \( d_z = 0 \). The linearized geodesic equations in this metric takes the form

\[ 2\ddot{x} = -(1 - \dot{x}^2 - \dot{y}^2)\partial_x h_{tt} + (1 - \dot{y}^2)\partial_y \phi \]  

(6.8)

\[ 2\ddot{y} = -(1 - \dot{x}^2 - \dot{y}^2)\partial_y h_{tt} + (1 - \dot{x}^2)\partial_x \phi, \]  

(6.9)

with dots referring to derivatives with respect to \( t \). To calculate the transverse velocity the particles acquire after passing by the string, we consider that the particles are flowing with initial velocity, \( v \), and we can integrate over the unperturbed trajectory, \( x = vt \) and \( y = y_0 \)

The result is the velocity impulse in the \( y \) direction after the string has passed. Then, particles enter the wake with a transverse velocity:

\[ v_t = 4\pi G(U + T + I^2 + \nu S^2)v_\gamma + \frac{4\pi \tilde{G}S}{v_\gamma}. \]  

(6.10)

The first term is the usual velocity impulse of the particles due to the deficit angle; the additional term is nothing but the torsion contribution.

A quick glance at the last equation may illustrate the essential role torsion eventually plays in the context of cosmic string. If torsion is present a new attractive gravitational force acts on test particles. According to Pogosian and Vachaspati[5], it is possible that wiggles may impart significant longitudinal velocities to string segments; a characteristic torsion also imprints the same effect, as we can verify with the help of (6.10). In this statement, we consider \( \nu \) arbitrary, but, in the case \( \nu \) is of the order of \( G \), the contribution of the angular deficit is of second order in \( G \), hence negligible; the second contribution is of order of \( G \), and may therefore have non-negligible effects.
7 Conclusions and Remarks

It is possible that torsion may have had a physically relevant role during the early stages of the Universe’s evolution. Along these lines, torsion fields may be regarded as potential sources for dynamical stresses which, when coupled to other fundamental fields (the gravitational field, for instance), might have had a significant contribution during the phase transitions leading to formation of topological defects, such as the SCSs we focused on here. It therefore seems a crucial issue to investigate basic models and scenarios involving cosmic defects within a context where torsion degrees of freedom are suitably accounted for. We showed that torsion has a small, but non-negligible, contribution to the geodesic equation obtained from the contortion term.

From a physical point of view, this contribution is responsible for the appearance of an additional attractive force acting on a massive test-particle, as explained in Section 6; this attractive force is important to the accretion of matter by the wake; in such a situation, the screwed cosmic string behaves very much like wiggly cosmic strings, assuming, of course, the validity of the Pogosian and Vachaspati conclusion [5].

To incorporate the Pogosian and Vachaspati statement[5], we propose that their straight strings with small-scale structures (wiggles) may resemble the strings endowed with torsion in our picture (screwed strings). In so doing, we postulate that the small-scale structures existing in wiggly strings can be approximately scaled to the geometrical deformation torsion produces on ordinary strings. Admitting this premise leads us to the idea that the primordial spectrum of perturbations in the CMBR, as observed by COBE, may reasonably be reproduced if one uses the freedom in the parameter-space of numerical models of structure formation based on wiggly strings.

The shape of the matter (radiation) power spectrum can be obtained by following the evolution of a network of long ordinary straight strings interacting with the universe matter (radiation) content. A string evolves in such a way that its characteristic curvature radius at time \( t \) is \( \sim t \) (the reader is addressed to Ref.[35], and references therein, for an exhaustive discussion on this subject). Each string moves with typical speed \( v \) through matter. The translational motion of a string creates a wedge-shaped wake. Once the wake forms, particles fall into it with transversal velocity, \( v_t \sim 8\pi G \mu \gamma \) (where \( \mu \) is the linear mass density of the string, \( \gamma = (1 - v^2)^{-1/2} \)), towards the plane behind the string [5]. Test particles that do not collide travel a distance shorter than the wake width:
Now that we have already studied the wake formation, we are in a position to analyze the wake generating by screwed superconducting cosmic string. The study of the effects of a cosmic string passing through matter is of great importance to understand the current organization of matter in the Universe and in this context many authors have already considered this problem in General Relativity.[36]

Massless particles (such as photons) will be deflected by an angle \( \delta \theta = 4 \pi G (U + T + I^2) \) in the case where \( \tilde{G} \sim G \). From the observational point of view, it would be impossible to distinguish a screwed string from its General Relativity partner, just by considering effects based on deflection of light (i.e., double image effect, for instance). On the other hand, trajectories of massive particles will be affected by the torsion coupling (which generates by torsioned space-time). These results are compatible with the Kleinart conclusions already shown in the Introduction.

If the string is moving with normal velocity, \( v \), through matter, a transversal velocity appears that is given by Eq.(6.10). We can see that there exists a new contribution to the transversal velocity \( v_t = \frac{4 \pi G S}{\gamma} \) given by torsion. We saw that the propagation of photons is unaffected by a screwed superconducting cosmic string, and is only affected by the angular deficit. This result shows us that the effect of torsion on massive particles is qualitatively different from its effect on radiation; this aspect becomes especially relevant when calculating CMBR-anisotropy and the power spectrum as wiggly cosmic strings. One expects that this feature could help to partially by-pass the current difficulties in reconciling the COBE normalized matter power spectrum with the observational data in the cosmic string model.

Acknowledgments: The author is indebted to J. H. Mosquera-Cuesta and L.C.Garcia-de-Andrade for participation at an early stage of this work. Thanks are also due to J. A. Helayel-Neto and V. B. Bezerra for long discussions, suggestions and careful criticisms on the original manuscript of this paper. C.N.Pq.- Brazil is acknowledge for the Graduate fellowship.

References

[1] A.Vilenkin, Phys. Rev. D 23, 852, (1981); W.A.Hiscock, Phys. Rev. D 31, 3288, (1985); J.R.GottIII, Astrophys. J. 288, 422, (1985); D. Garfinkel, Phys. Rev. D 32
1323, (1985).

[2] T.W. Kibble, J. of Phys. A9, 1387 (1976).

[3] P. P. Avelino et al., Phys. Rev. Letts. 81, 2008 (1998).

[4] G. F. Smoot, Lectures at D.Chalonge School on Large-Scale Structure of the Universe, Erice, Dic 12-17 (1999).

[5] L. Pogosian and T. Vachaspati, Phys. Rev. D 60, 083504 (1999).

[6] J. Magueijo and R.H. Brandenberger, astro-ph/0002030, (2000); T.W.B. Kibble, Phys. Rep 67, 183, (1980).

[7] R. H. Brandenberger, A. T. Sornborger and M. Trodden, Phys. Rev. D. 48, 940 (1993); V. Berezinshy, B. Hnatyk and A. Vilenkin, astro-ph/0001213 (2000).

[8] H. J. Mosquera Cuesta and D. Moréjon González, to appear in Phys. Letts. B. (2000).

[9] B.Allen and P.Casper, Phys. Rev. D 51, 1546 (1995), gr-qc/9407023; Phys. Rev. D 50, 2496 (1994), gr-qc/9405005.

[10] S.W.Hawking and S.F.Ross, Phys.Rev.Lett. 75, 3382 (1995), gr-qc/9506020 (1995); Audretsch and A.Economou, Phys. Rev. D 44,3774 (1991).

[11] E. Witten, Nucl.Phys. B249, 557 (1985).

[12] P. Peter and D. Puy, Phys. Rev. D 48, 5546 (1993).

[13] R. Jackiw and P. Rossi, Nucl. Phys B 190, 681, (1981).

[14] S.C.Davis, Int. J. Theor. Phys. 38, 2889 (1999), hep-ph/9901417 (1999); S.C.Davis et al., Phys. Rev. D 62, 043503, (2000), hep-ph/9912356 (1999).

[15] V. De Sabbata, IL Nuovo Cimento, 107 A, 363, (1994).

[16] Y.Duan, G. Yang and Y. Jiang, Helv. Phys. Acta 70, 565, (1997)

[17] S. Deser and A.N. Redlich, Phys. Lett. 176B (1986) 350;

[18] D.G. Boulware and S. Dese, Phys. Rev. Lett. 55 (1986) 2656 and Phys. Lett. 175B (1986) 409;
[19] S. Deser, "Gravity from Strings", in Unification of Fundamental Interactions Proc. of Nobel Symposium 67, ed. by L. Brink, R. Marnelius, J.S. Nilsson, P. Salomonson and B.-S. Skagerstam, Marstrand - Sweden, June 1986.

[20] I. L. Shapiro, hep-th/0103093 to appear in Phys. Rept.

[21] S. Ogino, Prog. Theor. Phys. 73, 84, (1985).

[22] D. K. Ross, Int. J. Theor. Phys. 28, 1333, (1989)

[23] V. De Sabbata, IL Nuovo Cimento, A107 363, (1994)

[24] H.H. Soleng, Gen. Rel. Grav. 24, 111, (1992).

[25] P.S.Letelier, Class. Quant. Grav. 12, 471 (1995); P.S.Letelier, Class. Quant. Grav. 12, 2224, (1995).

[26] H.Kleinert, Phys.Lett. B440 283 (1998).

[27] H.Kleinert, Gen. Rel. Grav. 32 769 (2000).

[28] H.Kleinert, Gen. Rel. Grav. 32 1271 (2000).

[29] V. de Sabbata and M.Gasperini, “Introduction to Gravitation ”, World Scientific Publishing (1985)

[30] F.W.Hehl et al., Rev. Mod. Phys. 48, 393 (1976).

[31] H.B Nielsen and P. Olesen, Nucl.Phys. 61, 45, (1973).

[32] C.N.Ferreira, M.E.X. Guimarães and J.A.Helayel-Neto, Nucl.Phys. B 581, 165, (2000).

[33] W.Arkuszewski et al., Commun.Math.Phys. 45, 183 (1975).

[34] R.A.Puntigam and H.H.Soleng, Class. Quantum Grav. 14, 1129 (1997).

[35] J. Silk and A. Vilenkin, Phys. Rev. Letts. 53, 1700 (1984).

[36] T. Vachaspati, Phys. Rev. Lett. 57, 1655, (1986); Phys. Rev. D 45, 3487, (1992); A. Stebbins , S. Veraraghavan, R. H. Brandenberger, J. Silk and N. Turok, Astrophys. J. 1, 322 (1987).