Adiabatic quantum pumping through surface states in 3D topological insulators

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Abstract. We investigate adiabatic quantum pumping of ballistic Dirac fermions on the surface of a strong three-dimensional topological insulator. Two different geometries are studied in detail, a normal metal–ferromagnetic–normal metal (NFN) junction and a ferromagnetic–normal metal–ferromagnetic (FNF) junction. Using a scattering matrix approach, we show that each time a new resonant mode appears in the transport window the pumped current exhibits a maximum and provide a detailed analysis of the position of these maxima. We also predict a characteristic difference between the pumped current in NFN- and FNF-junctions: whereas the former vanishes for carriers at normal incidence, the latter is finite due to the different nature of wavefunction interference in the junctions. Finally, we predict an experimentally distinguishable difference between the pumped current and the conductance.

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1. Introduction

Recently, surface states in topological insulators have attracted a lot of attention in the condensed-matter community [1]. Both in two-dimensional (e.g. HgTe) and in three-dimensional (e.g. Bi$_2$Se$_3$) compounds with strong spin–orbit interaction the topological phase has been demonstrated experimentally [2–6]. Although these compounds are insulating in the bulk (since they have an energy gap between the conduction band and the valence band), their surface states support gapless topologically protected states. In the simplest case these low-energy states of a strong three-dimensional topological insulator can be described by a single Dirac cone at the center of the three-dimensional Brillouin zone ($\Gamma$ point) [2, 4, 7–10]. The corresponding Hamiltonian is given by [11]

$$
\mathcal{H}_0 = \hbar v_F \vec{\sigma} \cdot \vec{k} - \mu I.
$$

(1)

Here $\vec{\sigma}$ represents a vector whose three components are the three Pauli spin matrices, $I$ represents a $2 \times 2$ identity matrix in spin space, $v_F$ is the Fermi velocity, and $\mu$ is the chemical potential. The low-energy states of $\mathcal{H}_0$ (equation (1)) are topologically protected against perturbations [12]. This has prompted recent research on the transport properties of surface Dirac fermions. For example, the conductance and magnetotransport of Dirac fermions have been studied in normal metal–ferromagnet (NF) junctions, normal metal–ferromagnetic–normal metal (NPN) junctions and arrays of NF junctions on the surface of a topological insulator [13–15], suggesting the possibility of an engineered magnetic switch. An anomalous magnetoresistance effect has been predicted in ferromagnetic–ferromagnetic junctions [16]. Also, electron tunneling and magnetoresistance have been studied in ferromagnetic–normal metal–ferromagnetic (FNF) junctions [17, 18], for which it has been predicted that the conductance can be larger in the anti-parallel configuration of the magnetizations of the two ferromagnetic regions than in the parallel configuration. In addition, a large research effort has been devoted to studying models which predict the existence of Majorana fermion edge states at the interface between superconductors and ferromagnets deposited on a topological insulator [11, 19, 20].

In this article we investigate adiabatic quantum pumping of Dirac fermions through surface states of a strong three-dimensional topological insulator in the ballistic regime. Quantum

\footnote{Also, the possibility of realizing a topological insulator in alkali atomic gases has been proposed, see e.g. [6].}
pumping refers to a transport mechanism in meso- and nanoscale devices by which a finite dc current is generated in the absence of an applied bias by periodic modulations of at least two system parameters in the adiabatic regime (typically gate voltages or magnetic fields) [21–23]. In order for electrical transport to be adiabatic, the period $T$ of the oscillatory driving signals has to be much longer than the dwell time $\tau_{\text{dwell}}$ of the electrons in the system, $T = \frac{2\pi \omega^{-1}}{\tau_{\text{dwell}}}$. In the last decade, many different aspects of quantum pumping have been theoretically investigated in a diverse range of nanodevices, for example charge and spin pumping in quantum dots and quantum Hall systems [24–28], the role of electron–electron interactions [29–33], non-adiabatic driving pumps [34–36], quantum pumping in graphene mono- and bilayers [37–43] and, recently, in a two-dimensional topological insulator [44]. On the experimental side, Giazotto et al [45] have recently reported an experimental demonstration of charge pumping in an InAs nanowire embedded in a superconducting quantum interference device.

In this paper we study quantum pumping induced by periodic modulations of gate voltages or exchange fields (induced by a ferromagnetic strip) in two topological insulator devices: a NFN and a FNF-junction, see figures 1 and 2. Using a scattering matrix approach, we obtain analytic expressions for the angle-dependent pumped current in both types of junctions and show that the adiabatically pumped current reaches a maximum each time a new resonant mode appears in the junctions. By analyzing in detail the spectrum of resonant modes, we find an analytic explanation for the position of these maxima. We also predict a characteristic difference between the pumped currents in the NFN- and FNF-junctions: whereas the former vanishes for carriers at normal incidence, the latter is finite due to the different nature of wavefunction interference in the junctions. To end we highlight an experimentally distinguishable feature between the pumped current and the conductance.

The remainder of the paper is organized as follows. In section 2, we describe the NFN and FNF-junctions and use a scattering matrix model to calculate the reflection and transmission coefficients of both junctions. In section 3, we analyze the condition for resonant transmission. In section 4, we calculate and analyze the adiabatically pumped current for the two different pumps. Finally, in section 5 we summarize our main results and propose possibilities for experimental observation of our predictions. In the appendix, we review the conductance of
the NFN junction and present a detailed analysis of the plateau-like steps that appear in the conductance. We also analyze and compare the conductance of the FNF-junction with parallel and anti-parallel configuration of the magnetization.

2. Normal metal–ferromagnetic–normal metal and ferromagnetic–normal metal–ferromagnetic junctions

We first describe the NFN-junction, see figure 1. The junction is divided into three regions: region $N_l$ (for $x < 0$), region $N_r$ (for $x > d$) and the ferromagnetic region $F$ in the middle. We assume the interface between $N$ and $F$ regions to be ideal, i.e. without a potential barrier. The left- and right-hand side of the junction represent the bare topological insulator. The charge carriers (surface Dirac fermions) in these regions are described by the Hamiltonian $H_0$ (equation (1)) whose eigenstates are given by

$$\psi_{\pm}^{\pm} = \frac{1}{\sqrt{2}} \left( 1 \pm e^{\pm i \alpha} \right) e^{\pm ik_{n}x} e^{i q y}. \quad (2)$$

Here $+(-)$ labels the wavefunctions traveling from the left (right) to the right (left) of the junction. The angle of incidence $\alpha$ and the momentum $k_{n}$ in the $x$-direction are given by

$$\sin(\alpha) = \frac{\hbar v_{F} q}{|\epsilon + \mu|}, \quad (3)$$

$$k_{n} = \sqrt{\left( \frac{\epsilon + \mu}{\hbar v_{F}} \right)^{2} - q^{2}}. \quad (4)$$

Here $\epsilon$ represents the energy of the Dirac fermions measured from the Fermi energy $\epsilon_{F}$, $q$ denotes the momentum in the $y$-direction and $v_{F}$ the Fermi velocity. In the normal (non-magnetic) regions $N_l$ and $N_r$ a dc gate voltage can be applied to tune the chemical potential $\mu$ in the left and right contacts and thereby control the number of charge carriers incident on the junction. We assume gate voltages to be small compared to the band-gap for bulk states ($\epsilon V_{i} < E_g \sim 1 \text{ eV}, i = l, r$), so that transport is well described by surface Dirac states [10]. In this case, the eigenstates are given by

$$\psi_{\pm}^{\pm} = \frac{1}{\sqrt{2}} \left( 1 \pm e^{\pm i \alpha_{l}} \right) e^{\pm ik_{n}x} e^{i q y}. \quad (5)$$

Formally, the Fermi energy $\epsilon_{F}$ denotes the chemical potential at zero temperature $\epsilon_{F} = \mu(T = 0)$. 

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$\text{Fig. 2.}$ Sketch of the $F_{l}NF_{r}$ junction. Ferromagnetic films are deposited on top of the topological insulator on the left and right providing exchange fields in these regions. The arrows indicate the direction of the corresponding magnetizations $M_{l}$ and $M_{r}$, see the text for further explanation.
\[ \psi_{N_r}^{\pm} = \frac{1}{\sqrt{2}} \left( \pm e^{\pm ik_n(x-d)} \right) e^{i q y}, \]  
(6)

\[ \sin(\alpha_i) = \frac{\hbar v_F q}{|\epsilon + \mu - eV_i|}, \]  
(7)

\[ k_{n_i} = \sqrt{\left( \frac{\epsilon + \mu - eV_i}{\hbar v_F} \right)^2 - q^2}, \]  
(8)

where the index \( i = l, r \) labels the left and right sides of the junction.

In the middle region of the junction \((0 < x < d)\), the presence of the ferromagnetic strip modifies the Hamiltonian by providing an exchange field. The Hamiltonian that describes the surface states in this region is then given by \( \mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\text{induced}} \), with the induced exchange Hamiltonian \[ \mathcal{H}_{\text{induced}} = \hbar v_F M \sigma_y, \]  
(9)

and magnetization \( \hat{M} = M \hat{\gamma} \). The magnitude \( M \) of the magnetization depends on the strength of the exchange coupling of the ferromagnetic film and can be tuned for soft ferromagnetic films by applying an external magnetic field \[ 16 \]. The eigenstates of the full Hamiltonian \( \mathcal{H} \) are then given by

\[ \psi_{F}^{\pm} = \frac{1}{\sqrt{2}} \left( \pm e^{\pm i \alpha_{m}} \right) e^{\pm i k_m x} e^{i q y}, \]  
(10)

with

\[ \sin(\alpha_{m}) = \frac{\hbar v_F (q + M)}{|\epsilon + \mu|} \]  
(11)

and

\[ k_m = \sqrt{\left( \frac{\epsilon + \mu}{\hbar v_F} \right)^2 - (q + M)^2}. \]  
(12)

From equation (12) we see that for a given energy there exists a critical magnetization

\[ M_c = \pm 2|\epsilon + \mu|/(\hbar v_F), \]  
(13)

beyond which the wavefunction changes from propagating to spatially decaying (evanescent) along the \( x \)-direction for all transverse \( (q) \) modes \[ 13, 14 \].

We now describe the FNF-junction, see figure 2. Region \( F_l \) \((x < 0)\) and region \( F_r \) \((x > d)\) are ferromagnetic with different magnetizations \( M_l, M_r \) along the \( y \)-axis and corresponding wavefunctions \( \psi_{F_l} \) and \( \psi_{F_r} \) (equation (10)). The Dirac fermions in the middle region \( N \) \((0 < x < d)\) are described by the wavefunctions \( \psi_N \) (equation (2)). When calculating transport properties of the FNF-junction below, we focus on two different alignments of the magnetizations of the ferromagnetic regions: the parallel configuration, in which the magnetizations in the ferromagnetic regions point in the same direction \((M_l \parallel M_r)\), and the anti-parallel configuration, in which the magnetizations are in opposite directions \((M_l \parallel -M_r)\).

Using equations (2)–(12) we now calculate for both the NFN- and the FNF-junctions the reflection and transmission coefficients for a Dirac fermion with energy \( \epsilon \) and transverse...
momentum $q$ incident from the left. To this end, we consider a general F$_F$F$_m$F$_r$ junction, where the wavefunctions in each of the three regions left (l), middle (m) and right (r) are given by:

$$
\psi_l = \psi_l^+ + r_l \psi_l^- ,
\psi_m = p \psi_m^+ + q \psi_m^- ,
\psi_r = t_r \psi_r^+ .
$$

Here $\psi_j^\pm$ $(j = l, m, r)$ are the wavefunctions (5), (6) or (10) (depending on the junction considered) and $r_l$ and $t_l$ denote the corresponding reflection and transmission coefficients. By requiring continuity of the wavefunction at the interfaces $x = 0$ and $d$, such that $\psi_l|_{x=0} = \psi_m|_{x=0}$ and $\psi_m|_{x=d} = \psi_l|_{x=d}$, we obtain the reflection and transmission coefficients:

$$
r_l = e^{\alpha_l d} \frac{e^{2i k_{ld}}(1 + e^{i(\alpha_m + \alpha_l)}) (e^{i \alpha_m} - e^{i \alpha_l}) + (e^{i \alpha_l} - e^{i \alpha_m}) (1 + e^{i(\alpha_m + \alpha_l)})}{e^{2i k_{ld}} (e^{i \alpha_m} - e^{i \alpha_l}) (e^{i \alpha_m} - e^{i \alpha_l}) + (1 + e^{i(\alpha_m + \alpha_l)}) (1 + e^{i(\alpha_m + \alpha_l)})} ,
$$

and

$$
t_l = e^{2i k_{ld}} \frac{(1 + e^{2i \alpha_m}) (1 + e^{2i \alpha_l})}{e^{2i k_{ld}} (e^{i \alpha_m} - e^{i \alpha_l}) (e^{i \alpha_m} - e^{i \alpha_l}) + (1 + e^{i(\alpha_m + \alpha_l)}) (1 + e^{i(\alpha_m + \alpha_l)})} .
$$

Here $\alpha_j$ denotes the polar angle of the wavevector in region $j = l, m, r$ (equations (7) and (11)). The reflection and transmission coefficients $r_{tr}$ and $t_{tr}$ for Dirac fermions incident from the right are obtained in an analogous way. The expressions (15) and (16) for the reflection and transmission coefficients form the basis of our calculations of the pumped current in section 4.

3. Conditions for resonant transmission

Before calculating the pumped current, it is useful to first analyze the resonance conditions for reflection and transmission in the junctions. After setting $\alpha_l = \alpha_r = \alpha$ in equation (15), we begin by finding the conditions when the reflection coefficient is zero, i.e. $r_l = 0$. The first, trivial, condition $\alpha = \alpha_m + 2 \pi n$ corresponds to the situation of an entirely normal junction (i.e. no ferromagnetic region). The second and more interesting condition is $\sin(k_{md}) = 0$. This is the case when transmission occurs via a resonant mode of the junction and can be written as (using equations (3) and (12)):

$$
k_{md} = \frac{\epsilon + \mu}{\hbar v_F} d \sqrt{1 - \left( \sin(\alpha) + \frac{\hbar v_F M}{\epsilon + \mu} \right)^2} = n \pi .
$$

Equation (17) indicates that for a given $M$ and $\epsilon$ there are certain privileged angles $\alpha_c$ for which the barrier becomes transparent:

$$
\sin(\alpha_c) = \pm \sqrt{1 - \left( \frac{n \pi}{d (1 + \frac{\epsilon}{\mu})} \right)^2 - \frac{\tilde{M}}{|1 + \frac{\epsilon}{\mu}|}} ,
$$

with $\tilde{d} = d \mu / (\hbar v_F)$ the dimensionless barrier length and $\tilde{M} = \hbar v_F M / \mu$ the effective magnetization. We refer to the modes satisfying equation (18) as resonant modes.

Now we address the question why a mode becomes resonant in the barrier. Figure 3 shows the angle of incidence $\alpha_c$ (equation (18)) as a function of energy $\epsilon / \mu$ for different values of $n$ and for typical values of the barrier width $d$ and magnetization $M$. For a given $n$, the resonant
mode does not contribute to transmission for energies \( \epsilon_c < \epsilon < \epsilon_{c_n}^* \), with \( \epsilon_c \equiv \hbar v_F M / 2 - \mu \), because the imaginary part of the momentum \( k_m \) is non-zero and thus the mode decays along the x-direction. The critical energy \( \epsilon_{c_n}^* \) for each mode is given by

\[
\frac{\epsilon_{c_n}^*}{\mu} = \frac{n^2 \pi^2}{2 d^2 M} + \frac{\tilde{M}}{2} - 1. \tag{19}
\]

When \( \epsilon > \epsilon_{c_n}^* \), \( k_m \) is real and the mode becomes resonant. For large energies (not shown in the figure) all the modes asymptotically reach their saturation angle \( \alpha_{sat} = \pi/2 \).

The angle-dependent transmission probability of the NFN-junction \( T_{NFN}(\alpha) \equiv |t_l(\alpha)|^2 \) is given by [13, 14]

\[
T_{NFN}(\alpha) = \frac{\cos^2(\alpha) \cos^2(\alpha_m)}{\cos^2(k_m d) \cos^2(\alpha) \cos^2(\alpha_m) + \sin^2(k_m d) (1 - \sin(\alpha) \sin(\alpha_m))^2}. \tag{20}
\]

Here \( \alpha_l = \alpha_r = \alpha \) and \( \alpha_m \) denotes the polar angle of the wavevector in the middle region as defined in equation (11). This angle can be expressed in terms of \( \alpha \), using the fact that momentum is conserved along the y-axis, as

\[
\sin(\alpha_m) = \sin(\alpha) + \frac{\tilde{M}}{1 + \frac{\epsilon}{\mu}}. \tag{21}
\]

Figure 4 shows the transmission probability (equation (20)) as a function of the angle of incidence \( \alpha \) for different values of energy \( \epsilon / \mu \). The dashed (red) vertical lines correspond to the angles satisfying equation (18) for different \( n \). For low-energy excitations only negative angles \( \alpha \) (i.e. \( q \)-momenta anti-parallel to \( M \)) contribute to the transport, see figures 4(a)–(d). As the energy increases, the resonant modes move from the left to the right and also positive angles \( \alpha \) (i.e. \( q \)-momenta parallel to \( M \)) begin to contribute, see figures 4(e) and (f). Notice that the modes are not sharply peaked, except for energies at which only one mode is present (\( n = 1 \), see figure 4(a)); in general, several angles contribute to each new mode.
Figure 4. The transmission probability $T_{N}^{N}(\alpha)$ (equation (20)) as a function of the angle of incidence $\alpha$ for different values of energy $\epsilon/\mu$, (a) $\epsilon/\mu = 0.7$, (b) $\epsilon/\mu = 0.9$, (c) $\epsilon/\mu = 1.2$, (d) $\epsilon/\mu = 1.6$, (e) $\epsilon/\mu = 2.4$, and (f) $\epsilon/\mu = 2.9$. Parameters used are $d = 5$ and $M = 3$. The behavior of the resonant modes, as shown in figures 3 and 4, forms the basis for understanding the behavior of the pumped current in the next section.

4. Adiabatically pumped current

We now investigate the adiabatically pumped current through NFN and FNF topological insulator junctions. In general, a pumped current is generated by slow variations of two system parameters $X_1$ and $X_2$ in the absence of a bias voltage [21, 22]. For periodic modulations $X_1(t) = X_{1,0} + \delta X_1 \cos(\omega t)$ and $X_2(t) = X_{2,0} + \delta X_2 \cos(\omega t + \phi)$, the pumped current $I_p$ into the left lead of the junction can be expressed in terms of the area $A$ enclosed by the contour that is traced out in $(X_1, X_2)$-parameter space during one pumping cycle [22]:

$$I_p = \frac{\omega e}{2\pi} \int_A dX_1 dX_2 \sum_m \Pi(X_1, X_2)$$

$$\approx \frac{\omega e}{2\pi} \delta X_1 \delta X_2 \sin \phi \sum_m \Pi(X_1, X_2),$$

with

$$\Pi(X_1, X_2) \equiv \text{Im} \left( \frac{\partial r_{\parallel}^*}{\partial X_1} \frac{\partial r_{\parallel}}{\partial X_2} + \frac{\partial t_{\parallel}^*}{\partial X_1} \frac{\partial t_{\parallel}}{\partial X_2} \right).$$

A fundamental difference between these pumps and the ones in graphene [37–42] is the nature of the spinor in the Hamiltonian (1). Whereas in graphene the spinor represents a pseudo-spin (or the sub-lattice variable), in our case it represents a real spin due to the spin–orbit interaction.
respectively (see figure 21). Equation (22b) is valid in the bilinear response regime where $\delta X_1 \ll X_{1,0}$ and $\delta X_2 \ll X_{2,0}$, and the integral in equation (22a) becomes independent of the pumping contour.

First we analyze the NFN pump, where the pumped current is generated by adiabatic variation of the gate voltages $V_l$ and $V_r$ which change the chemical potential in the left and right leads of the junction [46], respectively (see figure 1). The periodic modulation of the gate voltages is given by $V_l(t) = V_{l,0} + \delta V_l \cos(\omega t)$ and $V_r(t) = V_{r,0} + \delta V_r \cos(\omega t + \phi)$. Calculating the derivatives of the reflection and transmission coefficients $r_l$ and $t_l$ with respect to $\alpha_l$ and $\alpha_r$, substituting these derivatives into equation (23) and using $\partial \alpha_j / (\partial V_j) = \tan(\alpha_j) / (\epsilon + \mu - eV_j)$ ($j = l, r$), the angle-dependent pumped current for $V_{l,0} = V_{r,0} \equiv V_g$ is given by

$$I_p^{\text{NFN}}(\alpha) = I_0^{\text{NFN}} \frac{\cos^3(\alpha_m) \sin^2(\alpha) \cos(\alpha) \sin(2k_m d)}{\Gamma(\cos^2(\alpha) \cos^2(\alpha_m) \cos^2(k_m d) + \sin^2(k_m d)(1 - \sin(\alpha) \sin(\alpha_m))^2)}.$$

Here $I_0^{\text{NFN}} \equiv (\omega e / (8\pi)) \sin(\phi) (e \delta V_l / \mu) (e \delta V_r / \mu)$, $\Gamma \equiv -(1 + e / \mu - eV_g / \mu)^2$ and $\alpha_m$ is given by equation (21). Figure 5 shows the angle-resolved pumped current $I_p^{\text{NFN}}(\alpha)$ (equation (24)) as a function of $\alpha$ for different values of energy $\epsilon / \mu$. By comparing figures 3 and 5 we see that whenever a new mode appears in the transport window, the pumped current exhibits a maximum and changes sign. As the energy increases, more and more angles of incidence contribute to $I^{\text{NFN}}(\alpha)$, which eventually becomes a smooth oscillatory function of $\alpha$ (see panels (e) and (f)).

For slowly-varying and symmetric lead modulations, the current generated by oscillating external potentials applied to the leads can be described by the same formula as the current generated by oscillating internal parameters of the scatterer. See [46].

Notice that averaged over a pumping cycle no net left–right voltage bias is applied.
Figure 6. The pumped current $I_{\text{p}}^{\text{NFN}}$ (equation (25)) in the NFN-junction as a function of energy $\epsilon/\mu$ for $V_g = 0$, $\tilde{d} = 5$ and $\tilde{M} = 3$.

The total pumped current $I_{\text{p}}^{\text{NFN}}$ is obtained from equation (24) by integrating over all angles of incidence $\alpha$:

$$I_{\text{p}}^{\text{NFN}} = \int_{-\pi/2}^{\pi/2} I_{\text{p}}^{\text{NFN}}(\alpha) \cos \alpha \, d\alpha.$$  \hspace{1cm} (25)

In general, this integral cannot be evaluated analytically and we have obtained our results numerically. Figure 6 shows $I_{\text{p}}^{\text{NFN}}$ (in units of $I_0^{\text{NFN}}$) as a function of energy $\epsilon/\mu$. For low energies, the pumped current $I_{\text{p}}^{\text{NFN}}$ is zero as no traveling modes are allowed in the junction. As we increase the energy, each time a resonant mode appears (see figure 3) also the total pumped current exhibits a maximum and changes sign. It is interesting to compare these features of the pumped current—vanishing current and change of sign at energies where a new resonant mode appears—to the behavior of the conductance in the same junction (see for more details the appendix). The conductance exhibits plateaus and, for large energies, oscillatory behavior as a function of $\epsilon/\mu$ (see figures A.1 and A.2). The position of the plateaus in the conductance corresponds to the position of vanishing pumped current. A characteristic difference between both currents is that while the conductance generally increases as a function of voltage, the pumped current changes sign. This difference can be used to distinguish pumped currents from conductance in these junctions.

Now we analyze the pumped current in the FNF-junction with parallel orientation of the magnetizations. In this system, the driving parameters are the magnetizations $M_l$ and $M_r$ in the left and right contacts, respectively, see figure 2, where the periodic modulation is given by $M_l(t) = M_{l,0} + \delta M_l \cos(\omega t)$ and $M_r(t) = M_{r,0} + \delta M_r \cos(\omega t + \phi)$. After calculating the derivatives of the reflection and transmission coefficients, substituting into equation (23), and

7 This behavior (vanishing pumped current at energies where the junction is fully transmitting) has also been predicted for pumped currents in other materials, e.g. semiconductor quantum dots (see e.g. [50]) and graphene (see e.g. [38, 39]).
using $\partial \alpha_j/\partial M_j = \hbar v_F/(|\epsilon + \mu| \cos(\alpha_j))$ ($j = 1, r$), the angle-dependent pumped current $I_p^{\text{FNF}}(\alpha)$ for $M_{l,0} = M_{r,0} = M$ is found to be

$$
I_p^{\text{FNF}}(\alpha) = I_0^{\text{FNF}} \frac{\cos^3(\alpha_m) \cos(\alpha) \sin(2k_md)}{\Gamma'(\cos^2(\alpha) \cos^2(\alpha_m) \cos^2(k_md) + \sin^2(k_md)(1 - \sin(\alpha) \sin(\alpha_m)))^2}.
$$

(26)

Here $I_0^{\text{FNF}} \equiv \omega e/(8\pi) \sin(\phi)(\hbar v_F \delta M_l/\mu)(\hbar v_F \delta M_r/\mu)$, $\Gamma' \equiv -(1 + \epsilon/\mu)^2$ and the polar angle of the wavevector in the middle region $\alpha_m$ is given by $\sin(\alpha_m) = \sin(\alpha) - \hbar v_F M/(|\epsilon + \mu|)$.

The total pumped current $I_p^{\text{FNF}}$ is again obtained by integrating over all angles of incidence $\alpha$. The behavior of $I_p^{\text{FNF}}$ is similar to that of the pumped current in a NFN-junction (shown in figure 6): also $I_p^{\text{FNF}}$ exhibits maxima and subsequently changes sign at the same energies as $I_p^{\text{NFN}}$, where a new mode enters the transport window. There is, however, an interesting difference between both currents for normally incident carriers: whereas the pumped current in the NFN-junction vanishes for carriers at normal incidence, $I_p^{\text{NFN}}(\alpha = 0) = 0$, the pumped current in a FNF-junction is given by $I_p^{\text{FNF}}(\alpha = 0) = \cos^3(\alpha_m) \sin(2k_md)/(\Gamma'[1 - \sin^2(\alpha_m) \cos^2(k_md)])^2$. This difference arises because the two pumps are driven by different parameters (voltages in the NFN pump and magnetizations in the FNF pump) which leads to different interference patterns in the two junctions. In the voltage-driven NFN-pump destructive interference of normally incident Dirac fermions occurs (similar effects have also been predicted for graphene [37, 40]), while the magnetization-driven FNF-pump exhibits constructive interference of Dirac fermions at normal incidence.

Finally, we briefly analyze the behavior of the pumped current as a function of the width $d$ of the middle region in the junctions. For energies below $\epsilon_\text{c}$, the pumped current of the NFN-junction decays to zero as the width $d$ increases (there are no resonant modes in the system). For energies larger than $\epsilon > \epsilon_\text{c}$, the pumped current oscillates as a function of width $d$. Figure 7 shows $I_p^{\text{NFN}}$ as a function of $d$ for $\epsilon > \epsilon_\text{c}$. As expected, the pumped current $I_p^{\text{FNF}}$ switches
direction at the values of $\tilde{d}$ given by equation (18), where a new resonant mode enters the junction. This analysis also holds for a FNF-junction.

5. Summary and discussion

To summarize, we have analyzed quantum pumping by Dirac fermions in NFN- and FNF-junctions on top of a three-dimensional topological insulator. For low energies, the pumped current exhibits a maximum and subsequently changes sign when a new resonant mode appears in the transport window. This is our key result, and provides an experimentally distinguishable signature between the pumped current and the conductance (which increases in plateau-like steps as a function of the bias voltage and does not change sign). In addition, we predict an interesting difference between pumping in NFN- and FNF-junctions for normally incident Dirac fermions: whereas the pumped current in NFN-junctions vanishes at normal incidence, the pumped current in FNF-junctions is finite, due to the different nature of interference of Dirac fermions in both junctions. Experimentally, the NFN-pump could be realized by depositing a ferromagnetic film on top of a topological insulator, which induces an exchange field of 5–50 meV [47, 48]. For typical parameters $\omega/(2\pi) = 5$ GHz, barrier width $d = 10–20$ nm and gate voltages on the order of 10 meV, the predicted pumped current is on the order of 10 fA far from the resonant tunneling condition, going up to $0.1–1$ pA or higher close to resonance. This is well within reach of experimental observation. The FNF-pump may be more difficult to realize in practice, since it requires oscillating magnetizations. Possible ways to realize this pump could be by moving the two ferromagnetic layers coherently using a nanomechanical oscillator [49] or by using an inverse spin-Galvanic effect [51].

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Appendix. Conductance

The conductance $G_{\text{NFN}}$ of a topological insulator NFN-junction has been studied in earlier work by Mondal et al [13, 14], who predicted oscillatory behavior of $G_{\text{NFN}}$ as a function of the applied bias voltage (see also figure A.1). In this appendix we briefly review their results and then add a quantitative explanation for the oscillations of the conductance in terms of the resonant modes (discussed in section 3). This explanation is crucial for understanding both the plateau-like behavior of the conductance and for comparing this with the behavior of the pumped current in section 4. We also calculate and analyze the conductance in topological insulator FNF-junctions.

The general expression for the conductance $G_{\text{NFN}}$ across the NFN-junction in terms of the transmission probability $T_{\text{NFN}}(\alpha)$ (see equation (20)) is given by

$$G_{\text{NFN}} = \left( G_0 / 2 \right) \int_{-\pi/2}^{\pi/2} T_{\text{NFN}}(\alpha) \cos \alpha \, d\alpha. \quad (A.1)$$

Here $G_0 \equiv 2e^2 / \pi \rho(eV)\hbar v_F W$, $\rho(eV) = |\mu + eV|/(2\pi(\hbar v_F)^2)$ denotes the density of states, $W$ is the sample width, and the integration is over the angles of incidence $\alpha$. 

New Journal of Physics 14 (2012) 113003 (http://www.njp.org/)
Figure A.1. The conductance $G_{NFLN}$ of the NFLN junction (equation (A.1)) as a function of the applied bias voltage $eV/\mu$ for $V_l = V_r = 0$ and for different values of the effective magnetization $\tilde{M} \equiv \hbar v_F M/\mu = 3$ (solid blue line), 3.5 (dashed green line), 4 (dot-dashed red line) and 4.5 (dotted light-blue line). The effective junction width $d \equiv \mu d/(\hbar v_F) = 5$.

Figure A.1 shows the conductance of the NFLN junction as a function of the applied bias voltage $eV/\mu$ (which plays the role of the energy $\epsilon$ here) for different values of the effective magnetization $\tilde{M} \equiv \hbar v_F M/\mu$. For a given magnetization $M$, the conductance is zero for $eV < \epsilon_c$, where the critical energy $\epsilon_c \equiv \hbar v_F M/2 - \mu$. Below this energy there are no traveling modes inside the barrier. Our results agree with previous results in the literature [13, 14]. We see from figure A.1 that the conductance changes from plateau-like to oscillatory as $eV/\mu$ increases. In order to provide an explanation for this behavior we first analyze the plateau-like regime in detail. The first plateau corresponds to the situation in which the first transmission mode appears at $\alpha = -\pi/2$ (see also figure 3 in the main text). As the energy increases, subsequent resonant modes appear and the conductance increases in a step-like manner. The plateaus are not sharp, since the modes are not sharply peaked, but have a distribution around a particular angle of incidence, see figure 4. Once the bias voltage is large enough for there to be contributions from both positive and negative angles of incidence, the conductance becomes oscillatory. For very large bias voltages ($eV \gg \epsilon_c$, not shown in the figure), the effect of the magnetic barrier disappears and the conductance $G_{NFLN}/G_0 \rightarrow 1$.

Figure A.2 shows the conductance as a function of $eV$ for several values of applied gate voltages $V_l = V_r \equiv V_g$. As expected, the features of the conductance remain the same as $V_g$ increases, but the critical energy $\epsilon_c = \hbar v_F M/2 + eV_g/2 - \mu$ for the onset of the conductance increases and the spacing between consecutive resonant modes decreases. As a result the plateaus become narrower.

In the remaining part of this appendix we study the conductance in a topological insulator FNF-junction, as shown in figure 2. We consider both the junction with parallel and with anti-parallel magnetizations in the ferromagnetic regions. In the parallel configuration (using $\alpha_l = \alpha_r \equiv \alpha = \sin^{-1}(\hbar v_F (q + M)/|\epsilon + \mu|)$ and $\alpha_m = \sin^{-1}(\hbar v_F q/|\epsilon + \mu|)$ in equation (16)), we
Figure A.2. The conductance $G_{\text{NFN}}$ of the NFN junction as a function of $eV/\mu$ for different values of gate voltages: $eV_{g}/\mu = 0$ (solid blue line), 1 (dashed green line), 2 (dot-dashed red line) and 3 (dotted light-blue line). As before, $\tilde{M} = 3$ and $d = 5$.

find that the conductance is similar to the conductance of a NFN-junction, as displayed in figure A.1. A minor difference between the two is that in the FNF-junction the first resonant mode occurs for positive $\alpha$ (i.e. transverse momentum parallel to $M$) and as the energy increases, the resonances move towards negative values of the angle $\alpha$. This, however, does not affect the total conductance as we sum over all possible angles of incidence, and the same analysis as presented above for the NFN-junction can be applied to understand the FNF-junction with parallel magnetization.

In the case of anti-parallel alignment of the magnetizations in the two ferromagnetic regions, the transmission probability $T_{\text{FNF},\text{AP}}(\alpha_l, \alpha_r) \equiv \left| t_{rl}(\alpha_l, \alpha_r) \right|^2$ is given by [17, 18]

$$T_{\text{FNF},\text{AP}}(\alpha_l, \alpha_r) = \cos^2(\alpha_l) \cos^2(\alpha_m) / \left( \cos^2(k_md) \cos^2 \left( \frac{\alpha_l + \alpha_r}{2} \right) \cos^2(\alpha_m) \right. + \sin^2(k_md) \left[ \cos \left( \frac{\alpha_l - \alpha_r}{2} \right) - \sin \left( \frac{\alpha_l + \alpha_r}{2} \right) \sin(\alpha_m) \right]^2 \right).$$

(A.2)

The conductance of the FNF-junction in the anti-parallel configuration is obtained from equation (A.2) by multiplying $T_{\text{FNF},\text{AP}}(\alpha_l, \alpha_r)$ with $\cos(\alpha_l)/\cos(\alpha_l)$ and integrating over the allowed angles of incidence, i.e. from $\alpha_{c1} = \sin^{-1}(2\hbar v_F M/(|\epsilon + \mu|) - 1)$ to $\alpha_{c2} = \sin^{-1}(2\hbar v_F M/(|\epsilon + \mu|) + 1)$:

$$G_{\text{FNF},\text{AP}} = G_0/2 \int_{\alpha_{c1}}^{\alpha_{c2}} T_{\text{FNF},\text{AP}}(\alpha_l, \alpha_r) \cos(\alpha_l) \, d\alpha_l.$$  

(A.3)

8 This restriction of the angles of incidence comes from the fact that the minimum and the maximum value of the angle $\alpha_r$ for the transmitted wavefunction is $-\pi/2$ and $\pi/2$ respectively.

New Journal of Physics 14 (2012) 113003 (http://www.njp.org/)
Figure A.3. The conductance $G^{\text{FNF},\text{AP}}$ of the FNF junction in the anti-parallel configuration as a function of $eV/\mu$, for $\tilde{M} = 3$ (blue solid line), 3.5 (green dashed line) and 4 (red dot-dashed line). The parameter $\tilde{d} = 5$.

Figure A.4. The angle-dependent total transmission $G^{\text{FNF},\text{AP}}(\alpha_l, \alpha_r)$ (equation (A.4)) for the anti-parallel configuration of the FNF-junction as a function of the angle of incidence $\alpha_l$ for (a) $eV/\mu = 2.47$, (b) $eV/\mu = 2.57$, (c) $eV/\mu = 3.10$ and (d) $eV/\mu = 3.40$. Parameters used are $\tilde{M} = 3$ and $\tilde{d} = 5$.

with

$$G^{\text{FNF},\text{AP}}(\alpha_l, \alpha_r) = \frac{\cos(\alpha_r)}{\cos(\alpha_l)} T^{\text{FNF},\text{AP}}(\alpha_l, \alpha_r).$$

(A.4)

Figure A.3 shows the conductance $G^{\text{FNF},\text{AP}}$ of the FNF-junction in the anti-parallel configuration (equation (A.3)). From the horizontal axis we see that the critical energy $\epsilon_c$ for the onset of the conductance is larger than in the corresponding parallel configuration (figure A.1). Moreover, as the bias voltage increases the conductance does not exhibit plateaus, but instead increases in an oscillatory manner. This oscillatory behavior can be understood from figure A.4, which shows $G^{\text{FNF},\text{AP}}(\alpha_l, \alpha_r)$ for four different values of $eV$ (note that all the angles ($\alpha_l$, $\alpha_m$ and $\alpha_r$) can be expressed in terms of one angle, which we choose to be $\alpha_l$). From figure A.4 we observe that the area under the curve oscillates as a function of the applied bias voltage, which leads to oscillations in the conductance.
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New Journal of Physics 14 (2012) 113003 (http://www.njp.org/)
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