Difference frequencies of nonradial pulsation modes and repetitive outbursts in classical Be stars

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Abstract. Space photometry has revealed rich pulsation spectra in classical Be stars. Often frequency pairs can be found that differ by nearly the frequency of a third genuine stellar variability. The lowest currently known of these so-called difference (or \( \Delta \)) frequencies reach 0.03 c/d, which is the upper limit of outburst frequencies. The potential of such slow variations for pulsation-assisted mass loss is discussed.

1. Introduction

Classical Be stars pose two core problems: (i) How do they eject matter? (ii) How can ejecta form a Keplerian disk? As Ghoreyshi et al., Klement et al., and Vieira et al. illustrate in these proceedings, viscosity is the key process that drives the exchange of angular momentum between gas parcels so that \(~1\%\) of the matter reaches an orbit. By contrast, the first Be-star challenge has not yet found a well-documented general solution. The only case of clear causality was presented by Rivinius et al. (1998), who observed that outbursts of \( \mu \) Cen occur when two of six spectroscopic nonradial pulsation (NRP) modes are in phase at the stellar surface. When ranked by velocity amplitude, both the first and the second as well as the first and the third mode combine in this fashion. Very plausibly, it was interpreted as a beat phenomenon although direct evidence of a typical beat pattern was not produced. Rivinius et al. (2001) identified all three as \( \ell = m = 2 \) g-modes which prevail in most Be stars (Rivinius et al. 2003).

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This relatively thin factbase was widely adopted and modified to the more general notion that during outbursts of Be stars an unspecified number of NRP modes somehow combine to liberate the energy needed to eject significant amounts of matter (e.g., Huat et al. 2009; Kurtz et al. 2015). This conclusion supposedly received support from photometric observations of Be stars in outburst when many spikes in the power spectra change their strength, with a large fraction of them becoming detectable only around the outbursts. To this end, the spikes were interpreted as NRP modes. However, Baade et al. (2016) and Rivinius et al. (2016) questioned this identification and showed that much of the variability power involved arises from circumstellar processes.

2. Space photometry

Spectroscopic monitoring of broad-lined stars pulsating in multiple modes and undergoing additional outbursts is a very resource-demanding undertaking. However, not only is the technique expensive, but in about 20 years it has found only that one case of \( \mu \) Cen. 28 Cyg and \( \eta \) Cen were eventually confirmed by photometry (cf. Sect. 3).

The advent of photometry with tiny to medium-sized satellites and corresponding costs has opened up new opportunities with the amenities of space observatories: SMEI (Howard et al. 2013), MOST (Walker et al. 2003), CoRoT (Baglin et al. 2006), Kepler (Gilliland et al. 2010), and BRITE (Pablo et al. 2016). All five missions have observed Be stars. Examples (except for MOST) will be discussed in the next section.

Even precision photometry is not perfect: In rapid rotators such as Be stars, frequencies are subject to large rotational modification and can be treacherous indicators of mode types. Moreover, the latitudinal component of low-order \( g \)-mode velocity fields is a strong identifying symptom in spectra of pole-on stars whereas the photometric signal is largest in the equatorial plane but not visible in true pole-on stars. In Be stars, there is an additional photometric pitfall: Light from the central star is reprocessed in the disk. Therefore, variable amounts of circumstellar matter contribute an additional variability by this kind of light echo, which is weakest at inclination angles around 70 degrees (Haubois et al. 2012). A strong example is \( \mu \) Cen, where this effect precluded the detection by BRITE of the spectroscopic modes (Baade et al. 2016), the photometric signature of which is low at the given low inclination.

3. Difference (or \( \Delta \)) frequencies

In the following, frequency differences and difference (\( = \Delta \)) frequencies need to be carefully distinguished: The former can be calculated between any two frequencies. A \( \Delta \) frequency is something much more specific because, in addition to being a simple frequency difference, it plays its own role as a third variability. This is not the case for arbitrary frequency differencies, which do not normally have any physical meaning; in power spectra of synthetic data with two sinusoidal variations, difference (and sum) frequencies do not occur. \( \Delta \) frequencies were found in the following Be stars:

\( \eta \) Centauri (B2): Two retrograde \( \ell = 2 \) \( g \)-modes with 1.732 and 1.764 c/d are known from spectra (Rivinius et al. 2003). In agreement with the nearly equator-on orientation and the low H\( \alpha \) line-emission activity, BRITE detected both with relatively large amplitudes slightly above 2 mmag (Baade et al. 2016). A frequency of 0.034 c/d roughly equal to the difference between the two NRP frequencies was also discovered. It has
two unexpected properties: At 15 mmag its amplitude is 2.5 times as large as the sum of the two NRP modes, perhaps indicative of nonlinear mode coupling. And it is not a beat frequency but an approximately sinusoidal variability in its own right. Increased scatter around the extrema of the associated light curve and modifications of the amplitude and phase of the circumstellar Štefl frequency (Baade et al. 2016) with the stellar $\Delta$ frequency imply that the latter modulates and perhaps even drives mass loss.

**28 Cygni** (B2.5): Two (numerically imperfect) $\Delta$ frequencies occur in BRITE data (Baade et al., in prep.): $0.051 \text{ c/d} \approx 1.598 \text{ c/d} - 1.544 \text{ c/d}$ and $0.217 \text{ c/d} \approx 1.598 \text{ c/d} - 1.380 \text{ c/d}$. The amplitude (8.5 mmag) of the slower variability exceeds those ($\leq 5.5$ mmag) of the two retrograde quadrupole $g$-modes (Tubbesing et al. 2000) while in the second tuple the amplitudes are less different. Indications of a link to mass loss exist as in $\eta$ Cen but are much weaker. Like in $\mu$ Cen (provided this star does possess genuine $\Delta$ frequencies), the two $\Delta$ frequencies have one parent NRP frequency in common so that only three NRP modes seem involved in their formation.

**HD 50209** (B9): From CoRoT observations, Diago et al. (2009) suggest the presence of rotational frequency splitting although the implied frequency of the zonal mode ($m=0$) would be unusually low ($0.108 \text{ c/d}$). Indications of rotational splitting are not commonly found in Be stars. If HD 50209 is such a case, it illustrates that a frequency difference is not a difference ($\Delta$) frequency: The frequency difference is a quantity with high diagnostic potential but does not correspond to a separate stand-alone variability.

**HD 49330** (B1): In the last third of the 137-d monitoring period with CoRoT, Huat et al. (2009) detected a 0.03-mag outburst, during which the two base frequencies, $11.86 \text{ c/d}$ and $16.89 \text{ c/d}$, lost part of their power. The difference between them appears as a genuine $\Delta$ frequency at $5.03 \text{ c/d}$ (confirmed by Rivinius et al., in prep.). In contemporary echelle spectra, Floquet et al. (2009) determined an azimuthal mode order $\ell \approx 4$ for the $11.86-\text{c/d}$ variability and $\ell \approx 6$ for the higher-frequency mode.

**HD 186567**: The Kepler data were analyzed by Kurtz et al. (2015) and Rivinius et al. (2016). The results based on conventional Lomb-Scargle power spectra agree fairly perfectly. In particular, the two investigations found a $\Delta$ frequency at $0.276 \text{ c/d}$ corresponding to $4.010 \text{ c/d} - 3.734 \text{ c/d}$. Both of the latter frequencies are the strongest peaks in a group. Rivinius et al. also conducted a wavelet transform. It revealed the two amplitudes as time dependent on a scale of $\sim 20$ days with variations being antiphased. In stark contrast, the variability with the $\Delta$ frequency only showed a slow and perfectly smooth trend over the four years of observations; however, this is also favoured by the wavelet method.

**ALS 10705**: Rivinius et al. (2016) analyzed Kepler data and found that the frequency differences $4.77 \text{ c/d} - 4.41 \text{ c/d}$ and $5.13 \text{ c/d} - 4.77 \text{ c/d}$ are about equal, namely $0.36 \text{ c/d}$. There is a genuine frequency at $0.35 \text{ c/d}$ so that the two pairs may not only be a case of frequency splitting but give rise to genuine $\Delta$ frequencies. In addition, the difference between $1.58 \text{ c/d}$ and $0.59 \text{ c/d}$ ($0.99 \text{ c/d}$) is nearly coincident with the largest-amplitude variability at $0.98 \text{ c/d}$. This could constitute a third $\Delta$ frequency.

4. **Be stars without (firmly) detected $\Delta$ frequencies**

BRITE observations of several other Be stars were searched for $\Delta$ frequencies. For most of them some excuses can be construed as to why none was found:

- **Late-type Be Stars**: In agreement with most other cool Be stars, no periodic short-term variability with semi-amplitude above 1 mmag was seen in $\kappa^1$ Lup
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(B9.5) and μ Lup (B8). The 5.9-mmag amplitude (0.6586 c/d) in ν Pup (B8) is, therefore, remarkable but the only variability above the detection threshold.

• Intermediate-type Be stars: Short-periodic variability was also not detected in 17 Tau (B6), 23 Tau (B6), and η Tau (B7). At blue wavelengths, ψ Per (B5) exhibits a large-amplitude (10.3 mmag) variability with 0.0355 c/d, which numerically matches the difference between 0.7182 c/d (1.8 mmag) and 0.6829 c/d (1.9 mmag). Blue and red data are of different quality and quantity and do not give the same results. In fairly limited datasets, φ And (B5; only blue data available) and HR 1113 (B7; only red data available) appear single-periodic with 0.6500 c/d or 1.3085 c/d (1.1 mmag) and 1.488 c/d (2.7 mmag), respectively.

• In limited data, ω Ori (B3) does not present a significant isolated frequency but a frequency group (Baade et al. 2016) around 1.05 c/d, which apparently changed from 2014 to 2015 and perhaps explains the difficulty of establishing a stable frequency (Neiner et al. 2002). In even sparser data of 25 Cyg (B3), only one possible frequency was found (1.86 c/d, 1.5 mmag). Well-observed other earlier-type Be stars have much richer power spectra as illustrated in the following.

• 48 Per (B3): Three large-amplitude variations with widely spaced frequencies were detected in this earlier-type shell star (viewed equator-on): 0.0311 c/d (5.4 mmag), 1.074 c/d (4.3 mmag), and 2.2594 c/d (4.2 mmag). The slow variability is not in any obvious way related to more rapid variations.

• 60 Cyg (B1): Among the many significant variabilities, one stands out because both its frequency (3.363 c/d) and amplitude (6.4 mmag) are large.

• 25 Ori (B1): There are two candidate Δ frequencies with large amplitudes: 0.0268 c/d (11.3 mmag) ≈ 2.9530 c/d (2.0 mmag) - 2.9889 c/d (4.8 mmag) and 0.1897 c/d (15.9 mmag) ≈ 1.6782 c/d (6.9 mmag) - 1.4888 c/d (9.3 mmag). Separated by three months, phases of slow, roughly sinusoidal variations occurred with full amplitudes between 200 and 300 mmag.

• γ Cas (B0): In the combined blue and red data (to extend the time span) several frequencies were found: 0.2734 c/d (1.1 mmag), 0.9728 c/d (2.2 mmag), 2.1838 or 2.1950 c/d (0.9 mmag), and 2.4797 c/d (2.7 mmag). The latter is ~3 times the frequency of 0.8225 c/d derived by Henry & Smith (2012) from single-site ground-based photometry, which BRITE does not detect at the 0.6 mmag level. If the true frequency is the higher one, its interpretation as rotational modulation would put other parameters under considerable stress. However, the only frequency seen with SMEI is 0.821895 c/d. For a Be star, 0.27 c/d is an unusual frequency. But it is very close to the difference between 2.4797 c/d and 2.1950 c/d. A possible very low frequency at 0.0204 c/d (2.9 mmag) requires confirmation.

• δ Sco (B0): In red-light observations, this single-lined binary showed several slow variations: 0.0937 c/d (0.8 mmag), 0.0305 c/d (1.9 mmag), and 0.01402 c/d (1.5 mmag). The latter one could be the difference between 1.0316 c/d (0.8 mmag) and 1.0176 c/d (0.9 mmag). Unfortunately, the blue data are of lower quality and not sufficient to support or question the red data.

• The BRITE observations of 28 CMa (B2) became available only after this workshop. The SMEI data are noisy, and the star is seen at low inclination.

• 27 CMa (B3) is a strong shell star, i.e. viewed through the disk. At V=4.7 mag it is very faint for SMEI; the delivery of BRITE data took place after this workshop. In summary, this list is inconclusive. (It is also very inhomogeneous because the number of data, the photon noise, and systematic errors vary widely.) Very slow variations (0.02 to 0.05 c/d) without obvious instrumental or other explanations as artifacts, are
wide-spread but cannot always be related to more rapid variations. Some time scales reach 30-40% of the time span of the data and need confirmation.

5. Discussion

5.1. Commonalities of variabilities with Δ frequencies

With such a small number of examples, which may not all belong to the group to be defined, and incomplete information about most of them, the following compilation is not likely to fully survive critical scrutiny for long but may serve as a first guidance:

- In most stars with spectroscopic mode identifications (η Cen, µ Cen, 28 Cyg), the parent variations are retrograde quadrupole g-modes. The only counter example is HD 49330 with ℓ ≈ 4 and ℓ ≈ 6.
- In no star is the structure of the surface velocity field observationally constrained. For the understanding of the nature of Δ frequencies it is of high interest to compare it to those of the parent variabilities (which may also reside in the circumstellar disk).
- In some stars, there is a mismatch at the ≥1% level between inferred and calculated frequency differences. Its significance is not clear, especially since some of the parent frequencies are members of broad frequency groups.
- Most Δ frequencies correspond to periods ranging from days to weeks. With 0.2 d, HD 49330 is again an outlier. Perhaps, this frequency triplet is better described in terms of a sum frequency.
- Amplitudes of Δ frequencies can be both smaller or larger than those of their parent variabilities. Only in η Cen is the amplitude of one Δ frequency more than twice as large as the amplitude sum of the parent frequencies.
- Stars can exhibit more than one Δ frequency.
- One parent frequency can be involved in at least two Δ frequencies.
- In some stars (µ Cen [if it does possess genuine Δ frequencies], η Cen and, perhaps, 28 Cyg) there is evidence of the active involvement of difference frequencies in mass loss.
- With the exception of 25 Ori, amplitude variations during outbursts are not well documented for Δ frequencies. In wavelet transforms (which are technically biased to such results), Δ frequencies may exhibit substantially less amplitude and phase wobble than other variations (HD 186567).

5.2. What is the physical significance of Δ frequencies?

With mismatches in frequency of up to a few percent, such coincidences may in stars with several base frequencies arise by chance in ~ 10% of all cases. However, if the amplitude is larger than those of the parent frequencies (η Cen, µ Cen) or there are links to a variable star-to-disk mass-transfer rate (µ Cen, η Cen, 28 Cyg), the false-alarm probability becomes negligible. That is, frequency differences as characterized above are real as a general phenomenon but individual examples may be spurious.

In pulsating stars, difference (Δ) frequencies are a subset of combination frequencies, which comprise differences, sums, and mixed combinations of eigenfrequencies. They are known from white dwarfs (e.g., Winget et al. 1994), RR Lyrae stars (e.g., Moskalik & Poretti 2003), and δ Scuti stars (e.g., Breger et al. 1998). In the rapidly rotating hybrid β Cep / SPB star β Cen Aa, the Δ frequency 0.86 c/d has the highest
amplitude of all detected variations (Pigulski et al. 2016); for early-type stars in general see Kurtz et al. (2015). The theoretical foundation for such nonlinear mode coupling was laid by Dziembowski (1982). However, in Be stars circumstellar processes may be involved, which would render the two phenomena incomparable.

As discussed by Rivinius et al. (2016), rapid rotation adds an important extra twist: Numerical relations between frequencies in the observer’s frame do not necessarily hold in the corotating frame, especially considering the change of the latter implied by differential rotation. That is, either such numerical relations need to be looked at with much skepticism or they carry valuable diagnostic information. For instance, differential rotation may be one way to understand the observed imperfect frequency matches. Perhaps, the nature of \( \Delta \) frequencies may also shed some light on the unexplained strong spectroscopic preponderance of retrograde modes in Be stars.

In the context of possible pulsation-assisted mass loss, \( \Delta \) frequencies may be more relevant than sum frequencies because the much increased time scales may enable stronger deviations from adiabaticity. Unfortunately, the repetition time scales of outbursts of most Be stars are much longer (Mennickent et al. 2002, \( \mu \) Cen is a rare exception) than the lengths of photometric data strings that can be safely assumed to be free of long-term instrumental or other artifacts. More importantly, for a general explanation of outbursts by \( \Delta \) frequencies, they would need to be present in all Be stars concerned. As an observational goal, this is still very far away.

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