TESTS AT A TAU/CHARM FACTORY WITH
LONGITUDINALLY POLARIZED BEAMS

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Abstract

In a recent paper, eight semileptonic parameters were defined to specify the most general
Lorentz-invariant spin correlation functions for tau semileptonic decays. The parameters were
physically defined in terms of tau-decay partial-width intensities for polarized final states.
This paper studies how these parameters can be simply measured at a tau/charm factory
with longitudinally polarized beams without using spin-correlation techniques. Thereby the
parameters can also be used to bound the effective-mass scales $\Lambda$ for “new physics” such
as arising from lepton compositeness, leptonic CP violation, leptonic T violation, tau weak
magnetism, weak electricity, and/or second-class currents.

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Currently, the bounds are very weak for possible “new physics” in tau lepton phenomena. One of the ways in which more significant constraints could be obtained, or “new physics” be discovered, would be through experiments at a tau/charm factory \([1, 2]\) which study the structure of the charged lepton current.

In this paper we compare two simple methods for model-independent determinations of the complete Lorentz structure of tau semileptonic decays \(\tau^− \rightarrow \rho^- \nu, a_1^- \nu\). The first method is to make use of longitudinally polarized electron and positron beams (cf. Ref. 2) to study the decay chain \(\tau^- \rightarrow \rho^- \nu\) followed by \(\rho^{ch} \rightarrow \pi^{ch} \pi^o\). The second method is to make use of spin-correlations \([3, 4, 5, 6, 7]\) to study the decay sequence \(\gamma^* \rightarrow \tau^- \tau^+ \rightarrow (\rho^- \nu)(\rho^+ \bar{\nu})\) followed by \(\rho^{ch} \rightarrow \pi^{ch} \pi^o\). In both methods we include both \(\nu_L, \nu_R\) helicities and both \(\bar{\nu}_R, \bar{\nu}_L\) helicities. Similarly, we study \([8, 6, 7]\) the decay chain \(\tau^- \rightarrow a_1^- \nu\) followed by \(a_1 \rightarrow (3\pi)^-\).

In each case, we concentrate on a single distribution function (in this paper) and determine the associated “ideal statistical errors”.

**\(\mathcal{P}_L\) method** For the case of longitudinally polarized beams, we assume 100 % polarization and study the 3-variable distribution \(I_3^{\mathcal{P}}(\theta_{beam}, E_{\rho^-}, \hat{\theta}_{\pi^-})\). In the center-of-mass frame, \(\theta_{beam}\) is the angle between the final tau momentum and the initial e\(^-\) beam, and \(E_{\rho^-}\) is the energy of the final \(\rho^-\). The angle \(\hat{\theta}_{\pi^-}\) is the direction of the final \(\pi^-\) momentum in the \(\rho^-\) rest frame [when boost is directly from the center-of-mass frame].

**Stage-two Spin-Correlation method** We use the simple 4-variable distribution \([4, 6, 7]\)
which includes information on $\rho^{ch} \rightarrow \pi^{ch} \pi^{0}$. It doesn’t include information on the initial $e^{-}$ direction. This distribution is $I_4(E_{\rho^{-}}, E_{\rho^{+}}, \tilde{\theta}_{\pi^{-}}, \tilde{\theta}_{\pi^{+}})$.

**Remarks:** (1) The associated ideal statistical errors are given in the tables at the end of this paper.

(2) While more complicated distributions can be used, for a preliminary comparison of the two methods these are particularly simple distributions which, nevertheless, provide significant “analyzing powers”.

(3) In the present paper, as previously [4, 6, 7], we assume a $10^7$ ($\tau^{-}\tau^{+}$) pair data sample at 4GeV. We use branching ratios of 24.6 % for the $\rho$ mode, and 18 % for the sum of the neutral and charged $a_1$ modes, with an $a_1$ mass of 1.275 GeV.

(4) Instead of $\theta_{\text{beam}}$ which is the angle between the $e^{-}$ and $\tau^{-}$ momenta, one could use the angle between the $e^{-}$ and the final $\rho^{-}$ momenta. This would not require knowledge of the final tau direction. Work on this alternative 3-variable distribution is in progress [9].

## 2 SIMPLE 3-VARIABLE DISTRIBUTION IN CASE OF LONGITUDINALLY POLARIZED BEAMS

For the tau decay sequence $\tau^{-} \rightarrow \rho^{-} \nu$ followed by $\rho^{-} \rightarrow \pi^{-} \pi^{0}$,

$$I_3^P(\theta_B, E_{\rho^{-}}, \tilde{\theta}_{\pi^{-}}) = \sum \rho_{hh}^{\text{prod}}(e^{-} e^{+} \rightarrow \tau^{-} \tau^{+}) \rho_{hh}(\tau^{-} \rightarrow \rho^{-} \nu \rightarrow \pi^{-} \pi^{0} \nu)$$  \hspace{1cm} (1)

The summation is over the $\tau^{-}$ helicity, $h = \pm 1/2$. Since the azimuthal angle of the beam direction has been integrated over, the off-diagonal elements in the density matrices do not appear in Eq.(1).
The production density matrix for the $\tau^-$ is

$$
\rho_{\lambda_1,\lambda'_1}^{LR} = \begin{pmatrix}
\sin^4 \theta_B/2 + \frac{m^2}{s} \sin^2 \theta_B & -\frac{m}{\sqrt{s}} e^{-i\phi_B} \sin \theta_B \\
-\frac{m}{\sqrt{s}} e^{i\phi_B} \sin \theta_B & \cos^4 \theta_B/2 + \frac{m^2}{s} \sin^2 \theta_B
\end{pmatrix}
$$

(2)

$$
\rho_{\lambda_1,\lambda'_1}^{RL} = (-)^{\lambda'_1-\lambda_1} (\rho_{-\lambda_1,-\lambda'_1}^{LR})^* \tag{3}
$$

(i.e. these are used in the case when only $\tau^-$ decay products are observed). In these equations, $\theta_B = \theta_{\text{beam}}$ is the angle between the $e^-$ and the $\tau^-$ momenta (and between the $e^+$ and the $\tau^+$ momenta).

For the antiparticle leg with the $\tau^+$ but referred still to the $e^-$ beam, the production density matrix is

$$
\bar{\rho}_{\lambda_2,\lambda'_2}^{LR} = \begin{pmatrix}
\cos^4 \theta_B/2 + \frac{m^2}{s} \sin^2 \theta_B & -\frac{m}{\sqrt{s}} e^{i\phi_B} \sin \theta_B \\
-\frac{m}{\sqrt{s}} e^{-i\phi_B} \sin \theta_B & \sin^4 \theta_B/2 + \frac{m^2}{s} \sin^2 \theta_B
\end{pmatrix}
$$

(4)

$$
\bar{\rho}_{\lambda_2,\lambda'_2}^{RL} = (-)^{\lambda_2-\lambda'_2} (\bar{\rho}_{-\lambda_2,-\lambda'_2}^{LR})^* \tag{5}
$$

(i.e. these are used in the case when only $\tau^+$ decay products are observed). In Eq.(5), $\theta_B$ is still the angle between the $e^-$ and the $\tau^-$ momenta (and between the $e^+$ and the $\tau^+$ momenta).

3 STAGE-TWO SPIN-CORRELATION FUNCTION $I_4$

For comparison with the case of longitudinally-polarized beams, we use the simple 4-variable S2SC function
\begin{align}
I(E'_\rho, E'_\bar{\rho}, \bar{\theta}_1, \bar{\theta}_2) &= |T(+-)|^2 \rho_{++}\bar{\rho}_{--} + |T(-+)|^2 \rho_{--}\bar{\rho}_{++} \\
&+ |T(++)|^2 \rho_{++}\bar{\rho}_{++} + |T(--)|^2 \rho_{--}\bar{\rho}_{--}
\end{align}

(6)

In terms of probabilities, the quantum-mechanical structure of this expression is apparent, since the $T(\lambda_{\tau^-}, \lambda_{\tau^+})$ helicity amplitudes describe the production of the $(\tau^-\tau^+) \rightarrow Z^0$, or $\gamma^* \rightarrow \tau^-\tau^+$.

For instance, in the 1st term, the factor $|T(+,-)|^2 \equiv \text{“Probability to produce a } \tau^- \text{ with } \lambda_{\tau^-} = \frac{1}{2}$ and a $\tau^+$ with $\lambda_{\tau^+} = -\frac{1}{2} \text{”}$ is multiplied by the product of the decay probability, $\rho_{++}$, for the positive helicity $\tau^- \rightarrow \rho^-\nu \rightarrow (\pi^-\pi^0)\nu$ times the decay probability, $\bar{\rho}_{--}$, for the negative helicity $\tau^+ \rightarrow \rho^+\bar{\nu} \rightarrow (\pi^+\pi^0)\bar{\nu}$.

4 COMPARISON OF TWO METHODS

Both of the above methods involve the same composite decay density matrices for $\tau^- \rightarrow \rho^-\nu \rightarrow (\pi^-\pi^0)\nu$, ..., and similarly for the $a_1$ decay mode. So when defining the parametrization of these decay matrices, it is convenient to simultaneously report the associated ”ideal statistical errors” .

I: Measurement of general semileptonic parameters:

The 8 tau semi-leptonic decay parameters \[\] for $\tau^- \rightarrow \rho^-\nu, ...$, are defined for the four polarized $\rho_{L,T}\nu_{L,R}$ final states: The first parameter is simply $\Gamma \equiv \Gamma_L^+ + \Gamma_T^+$, i.e. the partial width for $\tau^- \rightarrow \rho^-\nu$. The second is the chirality parameter $\xi \equiv \frac{1}{\Gamma}(\Gamma_L^- + \Gamma_T^-)$. Equivalently,

$\xi \equiv (\text{Prob } \nu_{\tau} \text{ is } \nu_L) - (\text{Prob } \nu_{\tau} \text{ is } \nu_R)$, or

$$\xi \equiv |<\nu_L|\nu_{\tau}>|^2 - |<\nu_R|\nu_{\tau}>|^2$$

(7)

So a value $\xi = 1$ means the coupled $\nu_{\tau}$ is pure $\nu_L$. $\nu_L$ ($\nu_R$) means the emitted neutrino has L-handed (R-handed) polarization. For the special case of a mixture of only $V & A$ couplings and
\( m_{\nu_r} = 0, \xi \rightarrow |g_L|^2 - |g_R|^2 \) and the “stage-one spin correlation” parameter \( \zeta \rightarrow \xi \). The subscripts on the \( \Gamma \)’s denote the polarization of the final \( \rho^- \), either “L=longitudinal” or “T=transverse”; superscripts denote “\( \pm \)” for sum/difference of the \( \nu_L \) versus \( \nu_R \) contributions.

The remaining partial-width parameters are defined by

\[
\zeta \equiv (\Gamma_L^- - \Gamma_T^-) / (\mathcal{S}_\rho \Gamma), \quad \sigma \equiv (\Gamma_L^+ - \Gamma_T^+) / (\mathcal{S}_\rho \Gamma). \tag{8}
\]

To describe the interference between the \( \rho_L \) and \( \rho_R \) amplitudes, we define

\[
\begin{align*}
\omega & \equiv \frac{I_R^-}{(\mathcal{R}_\rho \Gamma)}, \quad \eta \equiv \frac{I_R^+}{(\mathcal{R}_\rho \Gamma)} \\
\omega' & \equiv \frac{I_L^-}{(\mathcal{R}_\rho \Gamma)}, \quad \eta' \equiv \frac{I_L^+}{(\mathcal{R}_\rho \Gamma)} \tag{9}
\end{align*}
\]

where the measureable \( LT \)-interference intensities are

\[
\begin{align*}
I_R^\pm & = \left| A(0, -\frac{1}{2}) \right| \left| A(-1, 1) \right| \cos \beta_a \pm \left| A(0, 1) \right| \left| A(1, 1) \right| \cos \beta_a R \\
I_L^\pm & = \left| A(0, -\frac{1}{2}) \right| \left| A(-1, 1) \right| \sin \beta_a \pm \left| A(0, 1) \right| \left| A(1, 1) \right| \sin \beta_a R \tag{10}
\end{align*}
\]

Here \( \beta_a \equiv \phi_a^a - \phi_a^0 \), and \( \beta_a^R \equiv \phi_a^a - \phi_a^0 R \) are the measurable phase differences of of the associated helicity amplitudes \( A(\lambda_\rho, \lambda_\nu) = |A| \exp i \phi \).

Four of these parameters \( (\xi, \zeta, \sigma, \omega) \) appear in the \( \rho_{hh} \) density matrix which occurs in the above distribution functions, \( I_3^P \) and \( I_4 \).

**Formulas for** \( \tau \rightarrow \rho \nu : \)

The composite decay density matrix elements are simply the decay probability for a \( \tau_1^- \) with helicity \( \frac{1}{2} \) to decay \( \tau^- \rightarrow \rho^- \nu \rightarrow (\pi^- \pi^0) \nu \) since

\[
\frac{1}{\Gamma} \frac{dN}{d(\cos \theta_1^\tau) d(\cos \bar{\theta}_1)} = \rho_{hh} \left( \theta_1^\tau, \bar{\theta}_1 \right) \tag{11}
\]

and for the decay of the \( \tau_2^+ \),

\[
\bar{\rho}_{hh} = \rho_{-h,-h} \ (\text{subscripts} \ 1 \rightarrow 2, a \rightarrow b) \tag{12}
\]

6
For a $\tau^-_{1}$ with helicity $\frac{h}{2}$ to decay $\tau^- \to \rho^- \nu \to (\pi^-\pi^0)\nu$,

$$\rho_{hh} = \frac{1}{8}(3 + \cos 2\theta_1)S + \frac{1}{32}(1 + 3 \cos 2\theta_1)D$$ \hspace{1cm} (13)

where

$$S = 1 + h\zeta S_{\rho} \cos \theta_{1}$$ \hspace{1cm} (14)

$$D = -S(1 - \cos 2\omega_1) + (\sigma S_{\rho} + h\zeta \cos \theta_{1})(1 + 3 \cos 2\omega_1) + h\omega R_{\rho} A \sqrt{2} \sin 2\omega_1 \sin \theta_{1}.$$ \hspace{1cm} (15)

with the Wigner rotation angle $\omega_1 = \omega_{1}(E_{\rho}), \ [4]$.

**Formulas for $\tau \to a_1 \nu$**:

For $\tau^- \to a_1^- \nu \to (3\pi)^-\nu$, with $\tau^-$ helicity $\lambda_1 = h/2$ where

$$\rho_{hh} = \frac{1}{8}(3 + \cos 2\theta_1)S_{a_1} - \frac{1}{32}(1 + 3 \cos 2\theta_1)D_{a_1}$$ \hspace{1cm} (16)

$$S_{a_1} = 1 + h\zeta S_{a_1} \cos \theta_{1}$$ \hspace{1cm} (17)

$$D_{a_1} = S_{a_1}(3 + \cos 2\omega_1) + (\sigma S_{a_1} + h\zeta \cos \theta_{1})(1 + 3 \cos 2\omega_1) + h\omega R_{a_1} A \sqrt{2} \sin 2\omega_1 \sin \theta_{1}.$$ \hspace{1cm} (18)

For the CP conjugate process, $\tau^+ \to a_1^+ \bar{\nu} \to (3\pi)^+\bar{\nu}$, with $\tau^+$ helicity $\lambda_2 = h/2$,

$$\bar{\rho}_{hh} = \rho_{-h,-h} \text{ (subscripts} \ 1 \rightarrow 2, a \rightarrow b)$$ \hspace{1cm} (19)

**Ideal statistical errors for measurement of $\xi, \zeta, \sigma, \text{ and } \omega$**:

For the $10^7 (\tau^-, \tau^+)$'s at 4 GeV, we determine the ideal statistical errors in the same manner as in our earlier papers, see Ref. 4.

See Table 1 for the errors for $(\xi, \zeta, \sigma, \omega)$ based on $I^P_3$ and on $I_4$. In general, by using longitudinally-polarized beams the errors for the $\rho^-$ mode are slightly less than 0.4% and about a factor of 7 better than by using the S2SC function $I_4$. The CP tests for these semileptonic parameters are
worse by the $P_L$ method, and about the same by the S2SC method. Typically the $a_1$ values are 2-4 times worse than the $\rho$ values. However, for $\xi$, the error for the $a_1$ mode by the $P_L$ method is about 3 times better than that for the $\rho$ mode.

II: Two tests for non-CKM-type leptonic $CP$ violation if only $\nu_L$ and $\bar{\nu}_R$ couplings:

Here we use a different parametrization of the composite decay density matrix since we assume only $\nu_L$ couplings. For the $\rho$ mode we use

\begin{equation}
\rho_{hh} = (1 + h \cos \theta^2_1) \left[ \cos^2 \omega_1 \cos^2 \tilde{\theta}_1 + \frac{1}{2} \sin^2 \omega_1 \sin^2 \tilde{\theta}_1 \right] \\
+ \frac{r_a^2}{2} (1 - h \cos \theta^2_1) \left[ \sin^2 \omega_1 \cos^2 \tilde{\theta}_1 + \frac{1}{2} \left( 1 + \cos^2 \omega_1 \right) \sin^2 \tilde{\theta}_1 \right] \\
+ h \frac{r_a}{\sqrt{2}} \cos \beta_a \sin \tilde{\theta}_1 \sin 2\omega_1 \left[ \cos^2 \tilde{\theta}_1 - \frac{1}{2} \sin^2 \tilde{\theta}_1 \right] 
\end{equation}

The dynamical parameters to be experimentally measured are the polar parameters $\beta_a = \phi_{a,1} - \phi_0^a$, $\beta_b = \phi_{b,1} - \phi_0^b$, and $r_a = |A(-1,-\frac{1}{2})|/|A(0,-\frac{1}{2})|$, $r_b = |B(1,\frac{1}{2})|/|B(0,\frac{1}{2})|$. In the standard lepton model with a pure $(V-A)$ coupling, the predicted values are $\beta_{a,b} = 0$, $r_{a,b} = \sqrt{2m_\rho/\mu} \simeq \sqrt{2}m_\rho/m_\tau \simeq 0.613$.

For the $\tau^- \to a_1^- \nu \to (\pi^- \pi^- \pi^+)\nu, (\pi^0 \pi^0 \pi^-)\nu$ modes,

\begin{equation}
\rho_{hh} = (1 + h \cos \theta^2_1) \left[ \sin^2 \omega_1 \cos^2 \tilde{\theta}_1 + \left( 1 - \frac{1}{2} \sin^2 \omega_1 \right) \sin^2 \tilde{\theta}_1 \right] \\
+ \frac{r_a^2}{2} (1 - h \cos \theta^2_1) \left[ (1 + \cos^2 \omega_1) \cos^2 \tilde{\theta}_1 + \left( 1 + \frac{1}{2} \sin^2 \omega_1 \right) \sin^2 \tilde{\theta}_1 \right] \\
- h \frac{r_a}{\sqrt{2}} \cos \beta_a \sin \tilde{\theta}_1 \sin 2\omega_1 \left[ \cos^2 \tilde{\theta}_1 - \frac{1}{2} \sin^2 \tilde{\theta}_1 \right] 
\end{equation}

Here $\tilde{\theta}_1$ specifies the normal to the $(\pi^- \pi^- \pi^+)$ decay triangle, instead of the $\pi^-$ momentum direction used for $\tau^- \to \rho^- \nu$. The Dalitz plot for $(\pi^- \pi^- \pi^+)$ has been integrated over so that it is not necessary to separate the form-factors for $a_1^- \to (\pi^- \pi^- \pi^+)$. In the standard lepton model with a pure $(V-A)$ coupling, for the $a_1$ mode $r_{a,b} = 1.01$ for $m_{a_1} = 1.275GeV$. 

8
Ideal statistical errors for two tests for "non-CKM-type" leptonic CP violation:

Tables 2 & 3 show respectively the sensitivities of the $\rho$ and $a_1$ modes for measurements by the two tau-polarization methods. By either polarization technique, the moduli ratio $r_a$ versus $r_b$ can be measured to better than 0.1%. The phase differences $\beta_a, \beta_b$ can be measured to about $7^\circ$ by these techniques; however, in the S2SC case the $I_7$ distribution is about 2 times as sensitive so inclusion of more variables to describe the final state may also give significant improvement in the case of longitudinally-polarized beams.

III: Measurement of effective-mass scales $\Lambda$ for additional “Chiral Couplings”:

In Ref. 7, the above semileptonic parameters have been expressed in terms of additional “chiral couplings” in the charged-current which could arise due to “new physics”.

The most general Lorentz coupling for $\tau^- \rightarrow \rho^- \nu_{L,R}$ is

$$\rho^*_\mu \bar{u}_{\nu_{\tau}} (p) \Gamma^\mu_{\tau} (k)$$

(22)

where $k_{\tau} = q_\rho + p_\nu$. It is convenient to treat the vector and axial vector matrix elements separately. In Eq.(22)

$$\Gamma^\mu_V = g_V \gamma^\mu + \frac{f_M}{2\Lambda} \tau \sigma^{\mu\nu}(k-p)_\nu + \frac{g_{S-}}{2\Lambda} (k-p)^\mu + \frac{g_{S+}}{2\Lambda} (k+p)^\mu + \frac{g_{T_+}}{2\Lambda} \tau \sigma^{\mu\nu}(k+p)_\nu$$

$$\Gamma^\mu_A = g_A \gamma^\mu \gamma_5 + \frac{f_E}{2\Lambda} \tau \sigma^{\mu\nu}(k-p)_\nu \gamma_5 + \frac{g_{P-}}{2\Lambda} (k-p)^\mu \gamma_5 + \frac{g_{P+}}{2\Lambda} (k+p)^\mu \gamma_5 + \frac{g_{T_5}}{2\Lambda} \tau \sigma^{\mu\nu}(k+p)_\nu \gamma_5$$

(23)

The parameter $\Lambda = \text{"the effective-mass scale of New Physics"}$. In effective field theory this is the scale at which new particle thresholds are expected to occur or where the theory becomes non-perturbatively strongly-interacting so as to overcome perturbative inconsistencies. It can also be interpreted as a measure of a new compositeness scale. In old-fashioned renormalization theory $\Lambda$ is the scale at which the calculational methods and/or the principles of “renormalization”
breakdown. Without additional theoretical or experimental inputs, it is not possible to select what is the "best" minimal set of couplings for analyzing the structure of the tau’s charged current. For instance, by Lorentz invariance, there are the equivalence theorems that for the vector current

\[ S \approx V + f_M, \quad T^+ \approx -V + S^- \]  

(24)

and for the axial-vector current

\[ P \approx -A + f_E, \quad T_5^+ \approx A + P^- \]  

(25)

On the other hand, dynamical considerations such as lepton compositeness would suggest searching for an additional tensorial \( g_+ = f_M + f_E \) coupling which would preserve \( \xi = 1 \) but otherwise give non-(\( V - A \))-values to the semi-leptonic parameters. For instance, \( \sigma = \zeta \neq 1 \) and \( \eta = \omega \neq 1 \).

**Effective-mass scale bounds for additional “chiral couplings”:**

Tables 4 & 5 respectively give the limits \[ F \] on \( \Lambda \) in the case of purely real and imaginary coupling constants for additional “chiral couplings”. Scales of the order of \( 1 TeV \) can be probed for some real coupling constants.

We list the ideal statistical error for the presence of an additional \( V + A \) coupling as an error \( \delta(\xi_A) \) on the chirality parameter \( \xi_A \) for \( \tau^- \rightarrow A^-\nu \). Equivalently, if one ignores possible different L and R leptonic CKM factors, the effective lower bound on an additional \( W^\pm_R \) boson (which couples only to right-handed currents) is

\[ M_R = \left\{ \delta(\xi_A)/2 \right\}^{-1/4} M_L \]  

(26)

So \( \delta(\xi) = 0.0012(0.0018) \) respectively correspond to \( M_R > 514 GeV(464 GeV) \).

In some cases for real coupling constants, the S2SC method gives a bound about a factor of 2 better than that for the \( I_P^F \) method. Here also it is important to extend the present analysis
in the case of longitudinally-polarized beams to see what occurs when additional variables are included in the description of the final state.

5 CONCLUSIONS

In the present paper two simple tau-polarization techniques have been compared for possible use at a tau/charm factory to study the $J_{\text{charged lepton}}^{\text{charged lepton}}$ current. For measurement of the semileptonic parameters, $\xi, \zeta, \sigma, \omega$, the $P_L$ method using $I_3^P$ is about 7 times better than the S2SC($I_4$) method. Both methods are comparable for the two tests for non-CKM-type leptonic $CP$ violation. In some cases the S2SC($I_4$) method gives about a 2 times stronger bound on additional chiral couplings.

In the case of the S2SC method, additional kinematic variables have been shown to be important to include in describing the final state. Thereby additional semileptonic parameters can be measured and significantly greater analyzing powers can be achieved. More analysis is needed to see if the same is true when additional variables are included in the case of the $P_L$ method which exploits longitudinally-polarized beams. This should be true because the 3 variables so-far included in $I_3^P$ do not fully exploit the special kinematics of the tau threshold region.

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Table Captions

Table 1: Ideal statistical errors for measurements at 4 GeV of the fundamental parameters $\xi, \zeta, \sigma$, and $\omega$ by either (i) the simple $I^P_3$ distribution function for $\tau^- \rightarrow \rho^- \nu$ using longitudinally-polarized $e^-e^+$ beams, or by (ii) the stage-two spin-correlation function $I_4$ for the sequential decay of an off-mass-shell photon $\gamma^* \rightarrow \tau^- \tau^+$ with $\tau^- \rightarrow \rho^- \nu$ and $\tau^+ \rightarrow \rho^+ \bar{\nu}$, etc. For each parameter, the first row assumes CP-invariance, for instance $\xi = \bar{\xi}$; then the following row contains the corresponding the statistical errors for measurement of the same parameter not assuming CP-invariance. We use $10^7 \gamma^* \rightarrow \tau^- \tau^+$ events.

Table 2: Ideal statistical errors for two tests for CP violation in $\tau \rightarrow \rho \nu$ by (i) the S2SC function, $I_4$, or by (ii) the longitudinally-polarized beam distribution function, $I^P_3$.

Table 3: Ideal statistical errors for two tests for CP violation in $\tau \rightarrow a_1 \nu$ by (i) the S2SC function, $I_4$, or by (ii) the longitudinally-polarized beam distribution function, $I^P_3$. 

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Table 4: “Chiral Couplings”: Limits on $\Lambda$ in GeV for real coupling constants. For the $\rho$ and $a_1$ modes, the $T^+ + T_5^+$ coupling is equivalent to the $V - A$ coupling; and $T^+ - T_5^+$ is equivalent to $V + A$. For $V + A$ only, the entry is for $\xi_A$; by Eq.(26) these values can be converted to a bound on the $M_R$ mass of a R-handed $W^\pm$.

Table 5: “Chiral Couplings”: Limits on $\Lambda$ in GeV for pure imaginary coupling constants. For the $\rho$ and $a_1$ modes, the $T^+ + T_5^+$ coupling is equivalent to the $V - A$ coupling; and $T^+ - T_5^+$ is equivalent to the $V + A$.

Tables are available by airmail or FAX—contact author by email.