Effectiveness of Landmark and Continuous Registrations in Reducing Inter- and Intrasubject Phase Variability

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ABSTRACT This study aimed to compare the effectiveness of applying landmark and continuous registrations in functional data analysis in reducing inter- and intrasubject phase variability of kinematic data, particularly lower extremity joint angles during the American kettlebell swing (AKS). Twenty healthy male subjects volunteered to perform the AKS test. Three different registration approaches; landmark registration, continuous registration used as an additional method to landmark registration, and continuous registration used as an alternative method to landmark registration were applied, and their effectiveness in aligning the curves was analyzed using functional permutation \(t\)-tests. All registration methods showed an improved mean curve than that of the method without registration. The root mean square error (RMSE) values between the mean unregistered curves and the individual unregistered curves were significantly higher than those of the landmark-registered curves \((p < 0.001)\) and the continuously registered curves \((p < 0.001)\). Continuous registration (as additional method) is the best registration method than landmark and continuous registrations (as alternative method), as it produced the highest mean percent decrease in the RMSE difference between the unregistered and continuously registered (as additional method) curves. Continuous registration (as additional method) appears to be the best method in reducing inter- and intrasubject phase variability than landmark and continuous registrations (as alternative method). Thus, it is recommended to implement continuous registration as an additional method to landmark registration prior to any AKS analysis. Both landmark and continuous registrations (as additional and alternative methods) enhance the likelihood of identifying significant differences between the right and left joint angles.

INDEX TERMS American kettlebell swing, continuous registration, data reduction, functional data analysis, landmark registration, time-warping function.

I. INTRODUCTION Traditionally, biomechanical data are analyzed by examining discrete measurements, such as maximum and minimum peak values of curves or waveforms of several trials from different subjects. However, these traditional methods truncate prominent features in the curves that need to be considered for more accurate analyses \([1],[2]\). Therefore, functional data analysis (FDA) is used to overcome this problem; with this method, all curves are treated as sets of functions rather than discrete measurements, and how the behaviors of all the curves change continuously over time can thus be determined \([3]\).

The most common procedures in FDA include the transformation of discrete data into functional forms, the smoothing of functional data, registration, functional principal component analysis (FPCA), and the analysis of results \([3]\). However, this research only focused on the role of registration in FDA as a curve alignment tool.

The timing of the prominent feature of the curve, which is the maximum flexion angle during the American kettlebell swing (AKS) varied from one subject to another, indicating phase variation. Meanwhile, the magnitude of the angle also varied from one subject to another, indicating amplitude variation. Averaging curves that have different phase variation causes an incorrect interpretation of the actual curves’ characteristics and features. Different subjects depicted different
strengths and abilities, causing some phase variation. Additionally, trials by a subject also vary. Therefore, inter- and intrasubject variability can be reduced by curves alignment using registration procedure in FDA. Generally, there are two types of registration, namely, landmark registration and continuous registration.

Landmark registration is the simplest curve alignment method that aligns distinct features or landmarks in such a way that their locations are similar for all curves. These prominent features or landmarks, mainly maxima, minima, or zero-crossings of curves, are aligned by transforming all curves’ selected landmarks at various time points to the same time point, enabling a representative comparison among the curves.

Nevertheless, landmark registration might not always be suitable for all types of data, as some curves do not have similar curve features or landmarks and the landmarks are ambiguous for some curves. There might be some variations that are neglected when the curve alignment is made on a single landmark point. Computationally, landmark registration is less demanding than continuous registration but, it requires an extensive data processing cost to determine the landmark locations for every curve [4]. For these reasons, continuous registration is considered an alternative method.

On the contrary, continuous registration involves entire curves for a more enhanced curve alignment rather than identifying values of specific features or landmarks. Regardless of whether the task of locating landmarks is easy or difficult, determining the timings of selected landmarks involves monotonous graphical processes [5]; thus, a fully automatic method of continuous registration needs to be used. However, to our best knowledge, none of the previous studies implement continuous registration as an alternative method to landmark registration.

Over the past years, most of the previous studies implement landmark registration to align the prominent features of kinematic or kinetic data of human movement and sports performance, such as knee joint angle during vertical jumping [6], lumbar moment during box lifting [7], displacement and velocity during gum chewing [8], ground reaction force (GRF) trajectories during walking with military-grade load carriage [9]–[11], juggling trajectories [12], [13], three-dimensional knee kinematic during treadmill walking [14], lower extremity joints kinematic during walking [15], lower extremity joints kinematic and kinetic during jump landing/cutting task [16], vertical GRF during unilateral and bilateral countermovement jumps (CMJ) [17], vertical GRF during maximal CMJ [18], and vertical GRF of stepping down during ongoing walking [19].

Landmark registration has been an advantageous curve alignment method as it made the trends of developmental stages of knee joint kinematic during vertical jump more obvious compared with the absence of registration [6] and made the differences between every increase in load by 10% of the armies’ body weight during walking with military-grade load carriage be more distinguishable [9], [11]. Landmark registration also reduces maximal CMJ waveform variability by aligning phases, although the number of landmarks to be registered should be carefully chosen to avoid over-registration [18].

While some researchers have shown the benefits of landmark registration [6], [9], [11], [18], other researchers [8] show that it was unbeneﬁcial. Crane et al. [8] concluded that landmark registration after time normalization is not compulsory for cyclical chewing data. Landmark registration decreases the average root mean square error (RMSE) values of displacement and velocity curves, albeit not by a statistically signiﬁcant extent [8].

Instead of using continuous registration as an alternative method to landmark registration, Ramsay et al. [5] used it as an additional method after applying landmark registration beforehand to the growth acceleration curves of 10 girls in the Berkeley Growth Study by Tuddenham and Snyder [20]. These curves were aligned using landmark registration with the aim that all curves’ post-spurt acceleration crossed zero at a similar time. Then, continuous registration was applied to the landmark-registered growth acceleration curves to align the whole pubertal growth spurt of the girls. As a result, both registration methods produced mean curves with minimum and maximum values, as well as the width of the pubertal growth spurt that is typical to that of individual curves although, continuous registration generated a better mean curve than that of landmark registration in the middle spur period centered on five years of age [5].

The positive impacts of continuous registration as an additional method to landmark registration have on the previous study by Ramsay et al. [5] and the different outcomes from the previous studies, necessitating a need to examine the impact of three different registration approaches; landmark registration, continuous registration used as an additional method to landmark registration, and continuous registration used as an alternative method to landmark registration, have on curve alignment, as the role of these registration methods in FDA is still vague.

Although FDA was introduced more than a decade ago, the application of this method is still inadequate in sports biomechanics. Previous studies on the motion analysis of the kettlebell swing; the AKS [21] and the Russian kettlebell swing [22]–[28] used the traditional method of analyzing data, in which a discrete or a single measurement of interest, such as the maximum or minimum peak of kinematic or kinetic values in specific events of interest, were measured.

A thorough search of the relevant literature yielded only one related article on the AKS, which was conducted by Mitchell et al. [21]; however, they focused only on the kinetic analysis, in which the mechanical demands of the arms and shoulders during the AKS and did not discuss the kinematics of lower extremity joint angles. Research on the kinematics and kinetics of the AKS are still equivocal and to our best knowledge, research on the application of FDA in analyzing AKS data has not yet been reported.
Therefore, the goal of this research is to compare the effectiveness of applying landmark registration and continuous registration used as additional and alternative methods to landmark registration in FDA in reducing inter- and intra-subject phase variability of kinematic data, particularly lower extremity joint angles during the AKS.

II. MATERIALS AND METHODS

A. DATA COLLECTION

Twenty healthy recreationally active male subjects with an average (±SD) age, height, and weight of 21 ± 1 years, 1.72 ± 0.07 m, and 69 ± 7.5 kg, respectively, participated in this study. Prior to participation, the subjects provided their written consent, and this study was approved by the Research Ethics Committee at Universiti Kebangsaan Malaysia (UKM PPI/111/8/JEP-2016-612). All resistance-trained subjects were confirmed to be free from any injuries. The subjects were all right-dominant for standardization. The subjects wore tight clothing and were barefoot during testing to prevent errors in the data recording process and to capture their actual movement during the AKS rather than the movement of subjects’ loose clothing. Sixteen reflective markers were attached to the anatomical landmarks of a subject’s lower extremity which are bilaterally placed on the heel, toe, ankle, tibia, knee, thigh, posterior superior iliac, and anterior superior iliac according to the lower extremity Plug-In-Gait marker placement provided by Vicon (Vicon Motion Systems Ltd, UK), based on Davis et al. [29] marker set protocol. Three-dimensional kinematic data were captured using three Vicon Nexus (Vicon Motion Systems Ltd, UK) infrared cameras (one MXF20 model and two MX3+ models) with a sampling frequency of 100 Hz. These cameras captured the movement of the reflective markers throughout the experiment.

Before the experiment was carried out, the subjects were given five to ten minutes to warm up and become familiar with the kettlebell. After several minutes, the subjects were asked to perform two sets of AKS with eight repetitions in each set with a 16-kg kettlebell (Maxx Arc Kettlebell model) as the male beginner’s kettlebell load is 16-kg [30]. Two to five minutes of rest between sets was provided to prevent injuries caused by muscle fatigue. The experimental data were collected repetitively, as the variability of a cyclical task is low [31]–[33]. The kinematic data recorded were the lower extremity joint angles, including both the right and left hip and knee flexion/extension angles and ankle plantarflexion/dorsiflexion angles. Digitization was performed using the Vicon Nexus 1.5.2 motion analysis system, and then, the recorded data were exported to Microsoft Excel. The exported cyclical data were divided into individual cycles using the live video available on the system as a reference.

A cycle of AKS consists of two phases, the flexion and extension phases. The flexion phase starts with the subject standing while gripping the kettlebell overhead (stance event) and then the subject bending down the body while swinging the kettlebell downwards in between the thighs to a maximum bending position while maintaining a neutral spine before propelling the kettlebell (maximum bending event). The extension phase starts when the subject swings the kettlebell during the maximum bending event to propel the kettlebell overhead (top swing event).

Two sets of eight repetitions of the AKS, for a total of 16 trials, were performed by each subject. However, the trials at the beginning and the end of both sets were not analyzed. The events during which the subject grabbed the kettlebell at the beginning of the set and put down the kettlebell at the end of the set were negligible for this study. The best five trials of the remaining trials of the AKS from each subject were used for further analysis forming a sample size of n = 100 trials.

B. DATA PROCESSING

The data were time-normalized before any FDA procedures were performed. The data were time-normalized to 100 data points or frames of a cycle of AKS using the spline interpolation method. The original pattern and amplitude of the data were maintained after the process. However, the phases of the data still varied after time-normalization. Thus, registrations were applied to reduce phase variation. A cycle of AKS consists of 100 data points and is defined in the time interval, $t \in [0, 1]$ in the FDA procedures. The normalized time is merely the proportion of the AKS cycle in this time interval. The data were transformed into a functional form by smoothing using the penalized fourth-order derivative (roughness penalty approach) and quintic spline (B-spline) basis with an optimal smoothing parameter, $\lambda$, set to a value of $1 \times 10^{-12}$ before the registration procedures. The step-by-step method of how these smoothing parameters were chosen is explained in [34].

C. REGISTRATIONS AND TIME-WARPING FUNCTIONS

The curves were registered using three registration approaches, namely, landmark registration and continuous registration used as additional and alternative methods to landmark registration, and the results were compared. The time-warping functions of these methods were estimated to eliminate phase variation. A total of 100 trials from 20 subjects were registered all at once. Both registrations were applied using the fda package in R.

Landmark registration was applied using the landmarkreg function in R. In the landmark registration process, a prominent feature or landmark $t_i$ was identified in the maximum bending event for each curve $x_i$. The flexion angle was the highest in this event. A transformation $h_i$ of $t$ was estimated for each curve $x_i$ such that the registered angle functions were

$$x_i^*(t) = x_i[h_i^{-1}(t)],$$  \hspace{1cm} (1)

where aligning function $h_i^{-1}(t)$ complied with the functional inverse of $h(t)$, which was depicted as

$$h^{-1}[h(t)] = t.$$  \hspace{1cm} (2)
The time-warping functions $h_i(t)$ were estimated in a common time interval, $t \in [0, 1]$ and complied with constraints $h(0) = 0$ and $h(1) = 1$ [5]. Each angle began at time 0 and ended at time 1. The phase variation of joint angle curves was removed by estimating the time-warping functions which transformed the joint angle time $t$ to the movement execution time for subject $i$. The subject $i$ reached the maximum bending event at time $h_i(t_0)$, which corresponded to his registered time, $h_i^{-1}(t_0) = t_0$. All subjects reached the maximum bending event at time $t_0$ in terms of registered time. Accordingly, when subject $i$ reached the maximum bending event early, i.e., when $h_i(t_0) < t_0$, the aligning function $h_i^{-1}(t)$ lengthened his movement execution time, whereas when subject $i$ reached the maximum bending event late, i.e., when $h_i(t_0) > t_0$ the aligning function $h_i^{-1}(t)$ shortened his movement execution time following the maximum bending event time. The time-warping functions $h_i$ were specified by fitting a smooth function at the points $(0,0)$, $(t_0, t_i)$, and $(1,1)$ [5]. The function fitting those three points defining each warping function did not need much flexibility. Therefore, seven spline (B-spline) quintic basis functions and a very light smoothing parameter, $\lambda = 1e - 12$, were applied.

There were two approaches used in applying continuous registration. In the first approach, continuous registration was applied as an additional method to landmark registration. Continuous registration was applied to the landmark-registered curves. This approach was proposed by Ramsay and Silverman [3] and Ramsay et al. [5]. Continuous registration was used as an additional method to enhance the curve alignment of other area of curves that are not aligned as much with the use of landmark registration. Continuous registration was applied using the register.fd function in R. The registered curve $x_i[h(t)]$ and target curve $x_0(t)$ differed only in amplitude variation after landmark registration, and their values tended to be proportional to one another within the $t$ value range. A straight line passing through the origin appeared when values of the registered curve were plotted against the target curve. Principal components analysis of the integrated products of the values in the following second-order matrix $C(h)$ should reveal one component, and the smallest eigenvalue should be close to 0 [5]:

$$
C(h) = \begin{bmatrix}
\int \frac{[x_0(t)]^2 dt}{f x_0(t)x[h(t)]dt} & \int \frac{x_0(t)x[h(t)]dt}{f[x(t)]^2 dt} \\
\int \frac{[x_0(t)x[h(t)]dt}{f[x(t)]^2 dt} & \int \frac{[x(t)]^2 dt}{f[x(t)]^2 dt}
\end{bmatrix}.
$$

The time-warping functions $h_i(t)$ were estimated to minimize the smallest eigenvalue of $C(h)$ for each curve [5]. To estimate strictly monotone functions, a high basis needs to be used [5]. Hence, 14 spline (B-spline) quintic basis functions and a large smoothing parameter $\lambda = 1$, were applied. The second approach used in applying continuous registration was by using continuous registration as an alternative method to landmark registration. Continuous registration was applied to the unregistered curves. Similar procedures and smoothing parameters were applied as the first approach.

### D. DECOMPOSITION INTO AMPLITUDE AND PHASE SUM OF SQUARES

A method that can measure the effect of registration towards amplitude and phase variations has been outlined by Kneip and Ramsay [35]. With this method, the amplitude and phase variations were decomposed, enabling the effect of before and after registration of $N$ functional groups to be compared, where $x_t$ represents the unregistered functions, $y_t$ represents the registered functions, and $h_t$ represents the time-warping functions. The $\bar{x}$ and $\bar{y}$ denoted as the mean unregistered and registered functional groups, respectively. The total mean square error is denoted as:

$$
MSE_{total} = N^{-1} \sum_{i=1}^{N} \int [x_i(t) - \bar{x}(t)]^2 dt. \quad (4)
$$

A constant $C_R$ is defined as follows:

$$
C_R = 1 + \frac{N^{-1} \sum_{i=1}^{N} \left[ Dh_i(t) - N^{-1} \sum_{i=1}^{N} Dh_i(t) \right] }{N^{-1} \sum_{i=1}^{N} \int y_i^2(t) dt}. \quad (5)
$$

where $C_R \geq 1$ associated with the covariation between the deformation functions $Dh_i$ and the squared registered functions $y_i^2$. Thus, the mean square errors of amplitude and phase variations are decomposed respectively, as follows:

$$
MSE_{amp} = C_R N^{-1} \sum_{i=1}^{N} \int \left[ y_i(t) - \bar{y}(t) \right]^2 dt, \quad (6)
$$

$$
MSE_{phase} = C_R \int \bar{x}^2(t) dt - \int x^2(t) dt, \quad (7)
$$

whereby the total mean square error (4) can be defined as

$$
MSE_{total} = MSE_{amp} + MSE_{phase}. \quad (8)
$$

By using this decomposition method, a squared multiple correlation index of the proportion of the total variation due to phase as denoted below

$$
R^2 = \frac{MSE_{phase}}{MSE_{total}}. \quad (9)
$$

### E. ROOT MEAN SQUARE ERROR

Several measures were evaluated to examine the effectiveness of landmark and continuous registrations (as additional and alternative methods) in reducing the inter- and intrasubject phase variability. The time-warping functions of the landmark-registered and continuously registered (as additional and alternative methods) curves were compared. The effects of these registrations on the mean curves were assessed by comparing the peak curves and the peak time of the mean unregistered, landmark-registered, and continuously registered (as additional and alternative methods) curves of lower extremity joint angles during the AKS. The RMSE values between the unregistered and
landmark-registered curves, as well as the RMSE values between the unregistered and continuously registered (as additional and alternative methods) curves, were compared using the Wilcoxon signed-rank statistical test. The RMSE values were calculated between the mean curve and the individual curves for each group of curves, as conducted by Sadeghi et al. [36], [37] and Crane et al. [8]. The statistical analysis was performed using IBM SPSS version 22. The percent decrease in the RMSE difference between the unregistered and landmark-registered curves, and between the unregistered and continuously registered (as additional method) curves, and then, the percent decrease was averaged.

\[
(a - b)/a \times 100\% \tag{10}
\]

where \(a\) represents the RMSE values between the mean unregistered and the individual unregistered curves and \(b\) represents the RMSE values between the mean registered and the individual registered curves and then, the percent decrease was averaged.

**F. FUNCTIONAL PERMUTATION T-TEST**

The effects of both registrations between the two groups of functional curves were assessed by using functional permutation \(t\)-tests. The functional permutation \(t\)-tests were used to assess the significant differences between the unregistered and landmark-registered curves, between the unregistered and continuously registered (as additional method) curves, and between the unregistered and continuously registered (as alternative method) curves. Correspondingly, the difference between the right and left hip, knee, and ankle joint angles of the unregistered, landmark-registered, and continuously registered (as additional and alternative methods) curves were compared with functional permutation \(t\)-tests. The \(t\)-statistic for the functional \(t\)-test was given as

\[
T(t) = \frac{|\bar{x}_1(t) - \bar{x}_2(t)|}{\sqrt{1/n_1 \text{Var}[x_1(t)] + 1/n_2 \text{Var}[x_2(t)]}} \tag{11}
\]
where $\bar{x}_1$ and $\bar{x}_2$ are sample means and $n_1$ and $n_2$ are the sample sizes. Permutation tests shuffle the curves’ labels arbitrarily and reevaluate the maximum of $T(t)$ with new labels, repetitively, forming a null distribution [5]. Null distributions were created between the two groups of functional curves tested, including between the unregistered and landmark-registered curves, between the unregistered and continuously registered (as additional method) curves, and between the unregistered and continuously registered (as alternative method) curves. Null distributions were also created between the right and left joint angles of the unregistered, landmark-registered, and continuously registered (as additional and alternative methods) curves. The default number of permutations, i.e., 200, was used to create a null distribution, while the default critical upper quantile of the null distribution, i.e., 0.95 (95% confidence interval), was compared with the observed $t$-statistic. The null hypothesis testing results indicated there were no significant differences between the two mean curves, whereas the alternative hypothesis testing results showed that the two mean curves were significantly different as follows:

$$H_0 : \mu_1 = \mu_2$$

$$H_a : \mu_1 \neq \mu_2$$

(12)

III. RESULTS

A. APPLICATION OF LANDMARK AND CONTINUOUS REGISTRATIONS

After smoothing, landmark registration was applied to the lower extremity joint angles, as shown in Fig. 1. The unregistered curves were plotted side by side with the corresponding landmark-registered curves and time-warping functions results for comparison. In this process, for all joint angles that were studied, a prominent landmark at the maximum bending event was used for alignment. As a result, all maximum points were aligned approximately at the same time with landmark registration, thus decreasing the phase variation. Moreover, the continuously registered joint angle curves were aligned
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FIGURE 3. Graphs of (a) the unregistered and (b) continuously registered (as alternative method) curves, as well as the respective (c) time-warping functions of lower extremity joint angles studied during the AKS.

over a cycle of AKS relative to the landmark-registered curves, although they were not as precisely aligned as the landmark-registered curves at the landmark point, as shown in Fig. 2. Fig. 2 shows the results of applying continuous registration as an additional method to landmark registration whereas Fig. 3 shows the results of applying continuous registration as an alternative method to landmark registration.

The time-warping functions of the continuously registered (as additional method) curves converged toward the middle part of the curves, which were the landmark-registered time, while that of the landmark-registered curves and continuously registered (as alternative method) curves did not converge. The landmark-registered curves were aligned at selected landmarks only while the other regions of the curves were left unregistered. Conversely, the continuously registered (as additional and alternative methods) curves were aligned at the entire curves as shown in Fig. 2 and 3.

Table 1 shows the decomposition of lower extremity joint angles into amplitude and phase sum of squares with the application of landmark registration, and continuous registration (as additional and alternative methods). The decomposition was applied to compare the unregistered curves with their landmark-registered counterparts, the landmark-registered curves with their continuously registered (as additional method) counterparts, and the unregistered curves with their continuously registered (as alternative method) counterparts. For all joint angles studied, the amplitude mean square error (MSE\textsuperscript{amp}) for landmark-registered curves (except right and left knee joint angles) and continuously registered (as additional method) curves was higher than the phase mean square error (MSE\textsuperscript{phase}) while the MSE\textsuperscript{phase} for continuously registered (as alternative method) curves was higher than the MSE\textsuperscript{amp}. Additionally, for all joint angles studied, the MSE\textsuperscript{phase} of continuously registered (as alternative method) curves was higher than that of landmark-registered curves. However, the MSE\textsuperscript{amp} of continuously registered (as alternative method) curves was lower than that of landmark-registered curves. Based on Table 1, the proportion of total variation explained by phase $R^2$ is 0.376 after landmark registration of the right hip angle curves. This result indicates that...
TABLE 1. Amplitude, phase and total mean square error (MSE), proportion of total variation explained by phase $R^2$ and constant $C$ for landmark-registered, and continuously registered curves (as additional and alternative) of lower extremity joint angles studied during AKS.

| Joint Angle | Measure | Landmark Registration | Continuous Registration (as additional method) | Continuous Registration (as alternative method) |
|-------------|---------|-----------------------|-----------------------------------------------|-----------------------------------------------|
| Left Hip    | $MSE_{\text{unreg}}$ | 149.63                | 73.887                                        | 79.446                                        |
|             | $MSE_{\text{landmark}}$ | 86.89                | 58.094                                        | 204.396                                       |
|             | $R^2$    | 0.367                 | 0.440                                         | 0.720                                         |
|             | $C$      | 1.004                 | 1.014                                         | 1.010                                         |

37.6% of the variation in right hip angle curves is associated to phase. The percentage phase variation after continuous registration (as alternative method) was the highest recorded, followed by that of after continuous registration (as additional method) and landmark registration as the least for both right and left hip and ankle joint angles. Meanwhile, for right and left knee joint angles, the percentage phase variation after continuous registration (as alternative method) was also the highest recorded, followed by that of after landmark registration and continuous registration (as additional method) as the least.

B. MEAN UNREGISTERED, LANDMARK-REGISTERED AND CONTINUOUSLY REGISTERED CURVES

After registration, the unregistered, landmark-registered, and continuously registered (as additional and alternative methods) curves were averaged to produce their respected mean curves, as illustrated in Fig. 4. There were slight differences in the peak time among the mean unregistered curves, mean landmark-registered curves, mean continuously registered (as additional and alternative method) curves for all joint angles, as illustrated in Table 2. Table 2 shows the maximum peak curves and the peak time of the mean unregistered, mean landmark-registered, and mean continuously registered (as additional and alternative methods) curves of lower extremity joint angles during the AKS. For the right hip angle, the peak time of the mean unregistered curve, 43% of AKS cycle, was slightly earlier than that of the mean landmark-registered curve and the mean continuously registered (as additional method) curve, both at 44% of AKS cycle. Similar trends were found for the other joint angles that were studied. However, the trend was inconsistent for the left ankle angle, as the peak times for the mean unregistered, and mean landmark-registered curves were the same, i.e., 46% of AKS cycle and a bit later than that of the mean continuously registered (as additional method) curve, i.e., 45% of AKS cycle. Almost all joint angles’ peak times of the mean continuously registered (as alternative method) curves were slightly earlier than those of the mean landmark-registered and mean continuously registered (as additional method) curves.

The amplitude variation between the mean unregistered, mean landmark-registered, and mean continuously registered (as additional and alternative methods) curves for all joint angles can be identified as shown in Fig. 4. The mean landmark-registered and mean continuously registered (as additional and alternative methods) curves appeared to be higher in amplitude compared to the mean unregistered curve. Based on Table 2, for all joint angles studied, the maximum peak of the mean landmark-registered curve was the largest recorded, followed by that of the mean continuously registered (as additional and alternative methods), and mean unregistered curves were the lowest. For both legs, the maximum peak of the mean unregistered, mean landmark-registered, and mean continuously registered (as additional and alternative methods) knee angles were the highest, followed by those of the hip angles, and those of the ankle angles were the lowest.

When comparing between the right and left legs, the maximum peaks of the mean unregistered, mean landmark-registered, and mean continuously registered (as additional and alternative methods) curves of the right hip and knee angles were higher than those of the left hip and knee angles. In contrast, the peaks of the mean unregistered, mean landmark-registered, and mean continuously registered curves of the right ankle angles were lower than those of the left ankle angles.

C. MEAN PERCENT DECREASE IN ROOT MEAN SQUARE ERROR

The RMSE values between the mean unregistered curve and the individual unregistered curves, between the mean landmark-registered curve and the individual landmark-registered curves, and between the mean continuously registered (as additional and alternative methods) curve and the individual continuously registered (as additional and alternative methods) curves for all joint angles studied, were computed and averaged as shown in Table 3. The percent decrease in the RMSE difference between unregistered
and landmark-registered curves, as well as the RMSE difference between unregistered and continuously registered (as additional and alternative methods) curves, were computed and averaged for all joint angles studied, as shown in Table 3.

The RMSE values between the mean unregistered curves and the individual unregistered curves were significantly higher than those of the landmark-registered curves ($p < 0.001$) for all joint angles studied. Following the same trend, the RMSE values between the mean unregistered curves and
TABLE 3. RMSE values between the mean unregistered, mean landmark-registered, and mean continuously registered (as additional and alternative methods) curves and their corresponding unregistered, landmark-registered, and continuously registered (as additional and alternative methods) individual curves of lower extremity joint angles studied during the AKS.

| Joint Angle | Unregistered Mean (SD) | Landmark-registered Mean (SD) | Continuous registered (additional) Mean (SD) | p-value | Decrease (%) |
|-------------|------------------------|-------------------------------|-----------------------------------------------|---------|--------------|
| Right Hip   | 15.01 (6.0)            | 12.09 (5.0)                  | <0.001*                                       | 11.9    |              |
| Left Hip    | 13.43 (6.0)            | 10.75 (5.4)                  | <0.001*                                       | 14.4    |              |
| Right Knee  | 14.27 (5.8)            | 9.83 (2.6)                   | <0.001*                                       | 24.8    |              |
| Left Knee   | 14.99 (5.4)            | 10.45 (3.2)                  | <0.001*                                       | 24.9    |              |
| Right Ankle | 6.44 (2.4)             | 5.62 (2.2)                   | <0.001*                                       | 10.1    |              |
| Left Ankle  | 7.48 (3.1)             | 6.25 (3.1)                   | <0.001*                                       | 15.1    |              |

Note: *Statistically significant difference (p<0.05)

D. DIFFERENCES BETWEEN UNREGISTERED AND LANDMARK-REGISTERED CURVES, AND BETWEEN UNREGISTERED AND CONTINUOUSLY REGISTERED CURVES

Another test to prove the relevancy of implementing registration in FDA of the kinematic data of lower extremity joint angles during the AKS was performed by using functional permutation $t$-tests. Functional permutation $t$-tests were applied to determine the significant differences between the unregistered and landmark-registered curves,
Out of the three joint angles for which the unregistered curves were tested, the only joint angle that showed a significant difference between the right and left legs was the ankle angle. For two out of the three joint angles for which the landmark-registered curves were tested, there were significant differences between the right and left joint angles. Concurrently, for the continuously registered (as additional and alternative methods) curves, all joint angles showed significant differences between the right and left legs.

IV. DISCUSSION
This study aimed to compare the effectiveness of applying landmark registration and continuous registration used as additional and alternative methods to landmark registration in FDA in reducing inter- and intrasubject phase variability of kinematic data, particularly lower extremity joint angles during the AKS.

Continuous registration was implemented using two different approaches which were as additional and alternative methods to landmark registration. All joint angles curves were ensured to fulfill two basic requirements for curve registration which are; all curves must be of the same pattern from one another and only differ in terms of intensity and dynamics [38]. The effectiveness of landmark registration depends on the curves’ maximum, minimum, inflection points, or zero-crossings visibility and must be ensured to be of the same homogeneous group [8]. The joint angles curves of AKS have a similar structural pattern and trait with only a single identifiable landmark (i.e., the maximum bending event).

The time-warping functions act as the indicator of how the timing of the continuously registered (as additional method) curves was transformed to fit the selected landmarks of landmark-registered curves. These might be the reason for the convergence of time-warping functions of continuously registered (as additional method) curves at the landmark-registered time. The time-warping functions of the landmark-registered and continuously registered (as alternative method) curves did not converge at any time region as they both were applied to transform the unregistered curves.

The decomposition method developed by Kneip and Ramsey [35] able to compute the amount of amplitude and phase variations on the landmark-registered and continuously registered curves (as additional and alternative methods). Therefore, it can be used as an indicator of determining the suitable type of registration rather than depending on visual interpretation only. Based on the results from Table 1, it can be deduced that landmark registration emphasizes more on the amplitude alignment than phase alignment whereas...
The amplitude variation between the mean unregistered, mean landmark-registered, and mean continuously registered (as additional and alternative methods) curves for all joint angles can be identified as shown in Fig. 4. The registered curves can be inferred as well-registered curves if they have continuous registration (as alternative method) performs the opposite to landmark registration. These findings were corresponding to the fact that continuous registration (as alternative method) is applied to the unregistered curves. Landmark registration focuses on aligning the specific maximum landmark point rather than the other regions of the curves. The purpose of applying continuous registration to the landmark-registered curves is to reduce phase variation on the other region of curves that is disregarded after landmark registration. The proportion of total variation explained by phase $R^2$ reported having positive value for all registration methods studied on all joint angles as shown in Table 1.

These results indicate that none of the registration methods applied have over-registered the joint angles curves. In any event of the negative value of $R^2$ the registration process can be inferred to have over-registered the curves [5]. These outcomes suggest that all registration methods studied can be used to align the curves. It all depends on the main focus of the study. If the alignment of the landmark point is of interest, then landmark registration is the best option, provided that continuous registration is applied afterward to compensate for phase variation of the other unregistered regions of the curves.

The $p$-value of functional permutation t-tests between the unregistered right and left curves, between the landmarked-registered right and left curves, between the continuously registered (as additional method) right and left curves, and between the continuously registered (as alternative method) right and left curves of lower extremity joint angles studied during the AKS.

**FIGURE 6.** Functional permutation t-tests between (a) the unregistered right and left curves, between (b) the landmarked-registered right and left curves, between (c) the continuously registered (as additional method) right and left curves, and between (d) the continuously registered (as alternative method) right and left curves of lower extremity joint angles studied during the AKS.

**TABLE 5.** The $p$-value of functional permutation t-tests between the unregistered right and left curves, between the landmarked-registered right and left curves, between the continuously registered (as additional method) right and left curves, and between the continuously registered (as alternative method) right and left curves of lower extremity joint angles studied during the AKS.

| Joint Angle | Unregistered right & left | Landmarked-registered right & left | Continuously registered (as additional method) right & left | Continuously registered (as alternative method) right & left |
|-------------|--------------------------|-----------------------------------|----------------------------------------------------------|----------------------------------------------------------|
| Hip         | 0.085                    | 0.025*                            | 0.01*                                                    | <0.001*                                                 |
| Knee        | 0.26                     | 0.2                               | <0.001*                                                  | <0.001*                                                  |
| Ankle       | <0.001*                  | <0.001*                           | <0.001*                                                  | <0.001*                                                  |

Note: *Statistically significant difference ($p<0.05$)
higher and sharper peaks and valleys than the unregistered curves because the reason amplitude and phase variations are mixed is to smear variation over a wider range of \( t \) values [5]. It can be seen that the mean landmark-registered and mean continuously registered (as additional and alternative methods) curves have a higher and sharper peak than the mean unregistered curves. The higher amplitude of the curves indicating a better curves alignment and thus, a more accurate peak magnitude average of all subjects can be calculated as inter- and intra-subject variability are reduced. These results imply that the decision of whether to use registration for the smoothed, time-normalized functional data affects the amplitude variations of the mean joint angles curves. Landmark and continuous registrations (as additional and alternative methods) tend to slightly increase the peak lower extremity joint angles of AKS. These outcomes corresponding to the findings of Sadeghi et al. [36] and Sadeghi et al. [37], as curve and continuous curve registrations, respectively, are the cause of a slight increase in peak muscle powers in gait curve. Nevertheless, curve and continuous curve registrations implemented by these researchers [36], [37], respectively were based on the technique developed by Ramsay and Li [39]. The registration approach by Silverman [40] is the pioneer of the method used by Ramsay and Li [39]. Based on the registration approach by Silverman [40], the unregistered curves were registered using time-shifting by reducing the mean squared distance between each curve and a target function. Subsequently, Ramsay and Li [39] extended the work of Silverman [40] by applying Procrustes fitting method that estimates the transformation by defining the target function with a suitable degree of smoothing. This approach is also known as continuous monotone registration and it is very effective but overly relies on the target function [41]. Despite the different approaches used, the findings from both studies [36], [37], are indeed beneficial for comparison to the registration approach used, which is by Ramsay et al. [5].

The peak mean joint angles results in Table 2 portray the role of each lower extremity joint studied during the maximum bending event of AKS. Executing the hip hinging movement of the AKS leads to more bending at the hip joint, the knee joint is the primary contributor when the subject propels the kettlebell overhead, and the ankle joint stabilizes the body throughout the whole process. Since all subjects were right-dominant, the peaks of the right side of the joints angles, particularly hip and knee joint angles tend to be higher than that of the left side of the joints angles, except for the ankle angles. These outcomes might indicate that the left ankle joint plays a slightly more crucial role than the right ankle joint in stabilizing the body throughout the AKS, although the subjects are all right-dominant and performed symmetrical and bilateral movements such as the AKS.

Based on the results shown in Fig. 4, and Table 2, it can be deduced that landmark and continuous registrations (as additional and alternative methods) produce an improved mean curve than that of the method without registration (time-normalization). This conclusion is supported by the later analysis of the RMSE value. The larger RMSE value between the mean unregistered curve and their individual unregistered curves compared to those of the mean landmark-registered and continuously registered (as additional and alternative methods) curves indicate that landmark and continuous registrations (as additional and alternative methods) reduce inter- and intra-subject phase variability. The results of the comparison of the mean decrease between all registration approaches imply that continuous registration (as additional method) is the best method for reducing inter- and intra-subject phase variability. Continuous registration (as additional method) appears to be a better method of reducing inter- and intra-subject phase variability than landmark and continuous registrations (as alternative method). Thus, it is recommended to implement continuous registration (as additional method) prior to any AKS analysis, provided that landmark registration is applied to the unregistered curve beforehand. Nevertheless, the implementation of continuous registration (as alternative method) gives a satisfactory result. Therefore, it is recommended to apply continuous registration (as alternative method) for a large amount of data to reduce the processing time of locating each prominent landmark of the curves.

However, our findings are inconsistent with those reported by Crane et al. [8]. Despite the mean decrease by 18.6% and 20.3% in the RMSE difference between unregistered and landmark-registered displacement and velocity curves, respectively, the \( t \)-test performed on both mean RMSE values showed the differences were not statistically different (displacement \( p = 0.2249 \), velocity \( p = 0.0726 \)) [8]. On the other hand, our findings are homogenous with those reported by Sadeghi et al. [36] and Sadeghi et al. [37]. The mean decrease in the mean RMSE difference between the unregistered and curve-registered muscle power curves of able-bodied subjects during walking was 13.6% and the highest statistically significant reductions in the RMSE values were found at the hip, 12.5% and knee joints, 23.0%, in the sagittal plane [36]. These results are supported by the results of a later study performed by Sadeghi et al. [37], as there was a mean decrease of 25% in the RMSE difference between the unregistered and continuously curve-registered hip angular displacement curves in the sagittal plane. The results of these studies by Sadeghi et al. [36], and Sadeghi et al. [37] show that phase variability was reduced by curve and continuous curve registrations, respectively.

The results of the comparison between the unregistered and the landmark-registered curves, as well as between the unregistered and the continuously registered curves (as additional and alternative methods) using functional permutation \( t \)-tests clearly show that the phase alignments by these registration methods generate sets of registered curves that were significantly different from the sets of unregistered curves. These findings suggest that landmark and continu-
ous registrations (as additional and alternative methods) play important roles in FDA since time-normalization alone is insufficient to reduce phase variation. Landmark registration only aligned the selected landmark points which cause phase variation in other regions of the curves. Consequently, the difference between the unregistered and landmark-registered curves could be less notable. There are times when landmark registration may not be appropriate and can produce unintended effects [42]. For this reason, landmark registration is applied if the unnecessary temporal or spatial variation can be removed [43].

The results of the comparison between the unregistered, landmark-registered, and continuously registered (as additional and alternative methods) right and left lower extremity joint angles highlight on the importance of implementing registration in FDA. Both landmark and continuous registrations (as additional and alternative methods) enhance the likelihood of identifying significant differences between the right and left joint angles. These results emphasize that symmetrical and bilateral movements, such as the AKS, impact the two sides of joint angles, differently in terms of the lower extremity joint angles. Continuous registration, either used as additional or alternative methods has proven to be a comprehensive curve registration method compared to landmark registration and the method without registration (time-normalization). The consideration of the whole-curve alignment using continuous registration allows thorough data analyses to be made, as the difference between the two sets of curves can be identified using functional permutation t-tests.

However, this study has a limitation. Landmark and continuous registrations (as additional and alternative methods) remove the ability to examine the temporal differences. This limitation is compensated by the decomposition of amplitude and phase variation sum of square of each registration applied. In this way, the amplitude and phase variation can be measured.

V. CONCLUSION
Landmark and continuous registrations (as additional and alternative methods) tend to slightly increase the peak lower extremity joint angles of AKS. Landmark and continuous registrations (as additional and alternative methods) produce an improved mean curve than that of the method without registration (time-normalization). Landmark and continuous registrations (as additional and alternative methods) reduce inter- and intra-subject phase variability as they produced larger RMSE value than that of the method without registration. Continuous registration (as additional method) is the best registration method than landmark and continuous registrations (as alternative method) as it produced the highest mean percent decrease in the RMSE difference between the unregistered and continuously registered (as additional method) curves. It is suggested that the mean continuously registered (as additional method) curves to be used as a reference to compare any AKS patterns. Continuous registration (as additional method) appears to be the best method of reducing inter- and intra-subject phase variability than landmark and continuous registrations (as alternative method). Thus, it is recommended to implement continuous registration (as additional method) prior to any AKS analysis, provided that landmark registration is applied to the unregistered (time-normalized) curve beforehand. Nevertheless, the implementation of continuous registration (as alternative method) gives a satisfactory result. Therefore, it is recommended to apply continuous registration (as alternative method) for a large amount of data to reduce the processing time of locating each prominent landmark of the curves. Both landmark and continuous registrations (as additional and alternative methods) enhance the likelihood of identifying significant differences between the right and left joint angles.

CONFLICT OF INTEREST STATEMENT
The authors declare no conflicts of interest with any person, people, or organizations that might influence the findings.

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