Primordial flares, flux tubes, MHD waves in the early universe and genesis of cosmic gamma ray bursts

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Abstract

It is conjectured that energy sources of the gamma ray bursts are similar to energy sources which trigger solar and stellar transient activity phenomena like flares, plasma accelerated flows in the flux tubes and, dissipation of energy and acceleration of particles by the MHD waves. Phenomenologically we examine in detail the following energy sources which may trigger gamma ray bursts: (i) cosmic primordial flares which could be solar flare like phenomena in the region of intergalactic or intergalactic cluster regions, (ii) primordial magnetic flux tubes that might have been formed from the convective collapse of the primordial magnetic flux (iii) nonlinear interaction and dissipation of MHD waves that are produced from the perturbations of large-scale intergalactic or intercluster magnetic field of primordial origin. We examine in detail each of the afore mentioned phenomena keeping in mind that whether such processes are responsible for energy sources of the gamma ray bursts.

By considering the similarity of observations and prevailing physical conditions in the cosmic environment, the following study suggests that most likely and a promising energy source for creation of the gamma ray bursts may be due to primordial flares.

1 Introduction

Since the discovery of the Gamma Ray bursts (GRB), physics of these enigmatic phenomena remain elusive till today. To date, there are more than hundred theoretical models on GRB (see the bibliography web site http://ssl.berkeley).
Based on insufficient data, in the beginning period of observations, many models on GRB favored energy source of galactic origin. However, recent observations are not consistent with these models and the models which explain that source of GRB could be of cosmic origin are most favorable (Piran 1999). Though, these models can explain some of the observed phenomena, other outstanding observed properties such as energy source and isotropic distribution of GRB in the sky remain to be explained.

It is interesting to note that most of the observed properties of the GRB are almost similar to the observed properties of the solar flares. Energetics of other phenomena such as transient flows in the sunspot and heating of the solar corona are worthwhile for comparison with the energetics of GRB phenomena. It is to be noted that most of the these solar activity phenomena are directly or indirectly involved with the the ambient magnetic field. Hence, in the following study and, in an analogy with the solar transient activity phenomena, it is supposed that large scale inter galactic or inter galactic cluster magnetic field may be of primordial origin could be responsible in creating the observed cosmic GRB.

Presently it is believed (Priest 1981; Haish and Strong 1991; Parker 1994) that the source of energy produced in the solar flares is due to phenomenon called magnetic reconnection in a very compact region wherein oppositely directed magnetic flux, in the limit of finite electric conductivity, annihilate each other and releasing required amount of flare energy with the acceleration of highly energetic particles (Lenters and Miller 1998, Tsuneta and Naito 1998). Steady Evershed flows of $\sim 10^5 \text{cmsec}^{-1}$ in sunspots is interpreted as follows: (i) for overall equilibrium of current free core and current sheath of a flux tube, a upward or downward flow in the current sheath is essential (Gokhale and Hiremath 1986) and (ii) flows along a flux tube acts like siphon which is driven by pressure differences at the end of flux tube foot points ( Montesinos and Thomas 1997) . On the other hand, transient flows in the sunspots are either due to changing conditions at the foot points (Thomas 1994) or due to convective inter change of flux tubes in the penumbra (Schlichenmaier 1997). Heating of the solar corona (Sakurai 1996) is based mainly on the notion that MHD waves interact nonlinearly producing shocks which not only heat the ambient medium but also accelerate the particles (Miller 1997).

From the similarity between solar transient activity phenomena and GRB and, inspired by the theoretical works on magnetic reconnection that explain
these phenomena, we conjecture that GRB activity could be of similar origin in the distant universe. In the following study, we investigate in detail whether any such physical phenomena in the cosmic environment are responsible for the creation of the GRB. It is found from this investigation that, primordial flares (in the regions of inter galactic or inter galactic clusters), i.e., solar flare like phenomenon, may be most promising energy source for the creation of GRB. Similar studies (Qu Qin and Wang 1977; Vahia and Rao 1988) indicate relevance of stellar flares as the energy source of GRB. However, observed spectra of GRB implies that GRB must be of cosmic origin and hence it is unlikely that stellar flares may be energy source for the cosmic GRB.

In section 2, we briefly describe the theory of magnetic reconnection. Application of magnetic reconnection for the explanation of GRB is presented in section 2.1. In section 3, we present plasma flows from the primordial flux tubes. We present physics of the MHD waves and their dissipation in the GRB environment in section 4. Results and conclusions are presented in the last section. Preliminary results of this study have been presented at the IAU symposium 195 (Hiremath 1999) and details are presented in the following paper.

2 Theory of magnetic reconnection

Let oppositely directed magnetic flux of large length scale \( L \) are merged with inflow velocity \( v_i \). This merging of flux will form a current sheath of thickness \( \delta \). Then the law of magnetic induction according to MHD description is

\[
\frac{\partial B}{\partial t} = \text{curl}(v \times B) + \eta \nabla^2 B,
\]

where \( B \) is strength of the magnetic field, \( v \) is the velocity and \( \eta \) is the magnetic diffusivity of the plasma. The first term in the above equation is due to convective flow and second term represents the magnetic diffusion of the plasma. Outside the region of reconnection, magnetic diffusivity is very low and magnetic field is glued to the plasma and moves along with the plasma. That is magnetic field is frozen to the plasma. This condition of infinite electric conductivity fails in the region of magnetic field reconnection by producing very high gradients of currents and electric fields. Dissipation of these strong currents leads to annihilation of magnetic field in the region
of magnetic reconnection where steady state exists so that convective and resistive terms in equation (1) are equal. The amount of energy released by the annihilation of magnetic field $B$ and cube of length $L$ is estimated to be $\sim L^3B^2$. The ratio of convective to resistive term called "magnetic Reynolds number" is given as follows

$$R_m = \frac{v_{in}\delta}{\eta},$$

where $\delta$ is the thickness of reconnection region. The assumption of incompressibility and conservation of flux yields

$$v_{out} = \frac{L}{\delta}v_{in} = v_a,$$

where $v_{out}$ is out flow velocity, $v_{in}$ is the inflow velocity and, $v_a$ is the ambient Alfven wave velocity whose perturbations are perpendicular to the field line and travel along it. Outside the region of magnetic reconnection, the convective term in equation (1) dominates over the resistive term. That is $R_m >> 1$ and hence the electric field is non-dissipative. However, inside the layer of thickness $\delta$, $R_m << 1$ and, the electric field $E = \eta J$ is dissipative which leads to domination of kinetic effects. For example, two dimensional simulations (Brown 1999 and references there in) indicate that magnetic flux and electron flow decouple in the accelerating region resulting in acceleration of electron beams by ejecting with super-Alfvenic velocity.

## 2.1 Primordial flares

When I say primordial flares, it means that flares of cosmic origin in the optically thin medium situated either in the regions of inter galactic or inter galactic clusters. The length scales are of cosmic dimension and, cosmic flares are produced by the oppositely directed magnetic flux due to peculiar motion of cosmic bodies such as galaxies or cluster of galaxies or near the sites of supernova ejections from the galaxy. The region of magnetic reconnection is assumed to be formed by merging of oppositely directed magnetic field which is of primordial origin. The typical observed peculiar velocity fields of galaxies are $\sim 10^7$ cm/sec and length scales of magnetic elements $\sim 10^{24}$cms. It is expected that in order to reproduce the observed magnetic fields $\sim \mu$ G in galaxies, galaxy clusters and inter galactic clusters (Kroneberg 1994;
Vallee et al. 1987; Lesch and Chiba 1997; Bagchi et al. 1998; Taylor and Perely 1993), a primordial field of \( \sim 10^{-9} \) G is required.

Based on the simple flare model described in section 2 and available observed informations on GRB, I estimate the approximate size of the region involved and the required amount of energy released in GRB. I also estimate the strength of the magnetic field and thickness of the reconnection layer at the site of GRB formation. Using this information and conservation of mass and flux, I estimate strength of the magnetic field of region before the formation of the GRB. Then I compare estimated strength of magnetic field and velocity of outflow \( v_{out} \) from the site of flare region with the strength of the observed large-scale magnetic field and peculiar velocity of the galaxies.

We know that the amount of energy released by annihilation of magnetic field of strength \( B \) and cube of length scale \( L \) is \( \sim L^3 B^2 \). Observations (Meegan et al. 1997) show that total amount of energy released by GRB activity is \( \sim 10^{53} \) to \( 10^{56} \) ergs if the GRB sources are at cosmic distance. In order to produce maximum energy of \( \sim 10^{56} \), the length \( L \) of the cube should be \( \sim 10^{12} \) cms and field strength in the reconnection region to be \( \sim 10^{10} \) G. In fact, observations (Piran 1999) of GRB spectrum in absorption lines does indicate that field of \( 10^{12} \) G is required for explanation of the GRB spectrum of non thermal origin.

If we assume \( L = \delta \), then from equation (3), this implies that \( v_{in} = v_{out} = v_a \). Alfven velocity \( v_a \) can be calculated from the inferred magnetic field (\( \sim 10^{10} G \)) and observed maximum time scale (\( \sim 1000 \) sec) of the GRB. This gives the Alfven velocity of \( \sim 10^9 \) cm/sec and density of the reconnection region \( \sim 10 g/cm^3 \). Note that \( v_{in} = v_a \) is 100 times more than the observed velocity (\( \sim 10^7 \) cm/sec) of the galaxies. This calculation is also in contradiction with the conventional flare mechanisms (Petschek 1964) wherein we have \( v_{out} = v_a = 0.1 v_{in} \). This indicates that thickness of the reconnecting layer must be less than length scale \( L \). If we accept that \( v_{out} = 0.1 v_{in} \), then law of conservation of mass yields density in the vicinity of the in flowing region to be \( \sim 1 gm cm^{-3} \).

Let us now reverse back, using inferred length scale and strength of magnetic field in region of reconnection, and ask what could be the strength of initial magnetic field that might have been distributed over large length scales. If so, does it match with strength of the observed large-scale magnetic
fields in the universe. Conservation of flux leads to a relation

$$B_{\text{initial}} = B_{\text{present}} \frac{L^2}{R_{\text{initial}}^2},$$

(4)

where $B_{\text{initial}}$ is the initial strength of the magnetic field, $B_{\text{present}}$ is strength of magnetic field in the reconnection region and $R_{\text{initial}}$ is initial length scale of the magnetic field. If we take the observed large-scale magnetic field then for typical length scale $R_{\text{in}} \sim 10 \text{kpc} \sim 10^{22} \text{cm}$, we get $B_{\text{initial}} \sim 10^{-10} \text{G}$ which is almost similar to the expected field strength of primordial origin from the observations. Hence the typical rate of GRB is expected to be $\sim R_{\text{initial}}/v_{\text{in}} \sim 10^5$ years which is very close to the observed rate of GRB $\sim 10^6$ years.

3 Primordial flux tubes

Let us assume that magnetic flux tubes, similar to solar flux tubes, were formed by the convective collapse of the magnetic flux in the early history of the universe when the matter was dominated over the radiation. If those flux tubes of primordial origin survive upto the present age, then length scale of such a flux tube is given as follows

$$L = (\tau \eta)^{1/2},$$

(5)

where $\tau$ is survival life time of the flux tube and $\eta$ is the magnetic diffusivity. This equation is derived from the equation of magnetic induction in which it is assumed that the directions of velocity of the flux tube and the direction of ambient magnetic fields are parallel to each other. By taking survival life time equivalent to age of the universe ($\sim 10 \text{billion yrs} \sim 10^{17} \text{sec}$) and turbulent magnetic diffusivity $\sim 10^{32} \text{cm}^2 \text{sec}^{-1}$ (eg. equation 3.29 from Lesch and Chiba 1997), we get length scale $L$ of the primordial flux tube is $\sim 10^{25}$ cms. In the calculation of turbulent magnetic diffusivity we take plasma velocity to be the observed peculiar velocity of galaxies ($\sim 10^7 \text{cms sec}^{-1}$) and typical intergalactic length scales of 1kpc.

If such an isolated and untwisted flux tube form with a current free core bounded by thin current sheath, is in an overall equilibrium with stratified atmosphere, then there is a dominant flow along the thin current sheath and a net downward force on the core or vice-versa (Gokhale and Hiremath,
1986). The total amount of energy released in time $t$ of such a plasma flow in the current sheath of a flux tube is given as follows

$$E \sim (\frac{B^2}{4\pi})(\Delta x)t,$$

(6)

where $B$ is strength of the magnetic field in the flux tube, $\Delta x$ is the total thickness of the current sheath. In the case of sun, typical sunspots have total area of 100 millionths of hemisphere and typical size of umbra is $\sim 20\%$ of size of the total area. If we take length scale of the solar flux to be size of the convective envelope which is $\sim 10^{10}$ cms, then the sunspots radii which consist of umbra and penumbra must be $\sim 1\%$ of the length of the flux tube. We copy similar picture and physical properties of the solar flux tube to cosmic flux tube. Assuming that current sheath is wholly concentrated in penumbral region, simple calculations yield radius of cosmic flux tube be $\sim 10^{23}$ cms and total thickness of current sheath which is situated between umbra and penumbral boundary to be $\sim 10^{56}$ cm$^2$. After substituting these values in equation (6), we have $E \sim 10^{56}B^2t$. That means in order to get required energy for GRB phenomena in a time scale of 1 sec, we require magnetic field strength $B$ of the flux tube of $\sim 1$ G.

4 MHD waves in the cosmic environment

One more expected source of energy for the production of GRB is the dissipation of MHD waves which travel along and perpendicular to the large-scale cosmic magnetic field of primordial origin. Some of these MHD waves are believed to be responsible for heating of the solar corona (Kuperus, et al. 1981, Sakurai 1996).

For uniform magnetic field and for incompressibility, we have Alfven waves whose perturbations are perpendicular to the magnetic field and travel along the magnetic filed lines. First order perturbations show that amplitudes of perturbations never become nonlinear since there are no density or pressure perturbations involved with these waves and hence never dissipate. They simply escape to the space of inter galactic or inter cluster medium. However when oppositely traveling Alfven waves meet each other resulting beat period between two waves (Wentzel 1976) convert their density fluctuations into slow mode MHD waves (explained in the following text) and then dissipate their energy. Following Wentzel (1976) we compute in the following energy
flux released by the dissipation and length scales of dissipation of the MHD waves. The flux $F$ due to dissipation is given as follows

$$F = \frac{B^2}{8\pi} \frac{v_a^2}{A^2},$$

(7)

where $A$ is amplitude of the waves and $v_a$ is Alfvén velocity. By taking the typical observed peak flux of GRB980425 which is of $\sim 10^{-7} \text{erg cm}^{-2} \text{sec}^{-1}$, we require the amplitude of waves to be $\sim 0.01 \text{cm sec}^{-1}$. In this calculation, ambient density is taken from the observed density $\sim 10^{-31} \text{gm cm}^{-3}$ and strength of uniform magnetic field is taken to be $\sim 10^{-10} \text{G}$ (strength of large scale magnetic field of primordial origin) and Alfvén velocity turns out to be $\sim 10^6 \text{cm sec}^{-1}$. The length scale over which these Alfvén waves are dissipated is $\sim 10^{-18} \text{cm}$. Thus it is natural to expect that in the cosmic environment pure Alfvén waves can not be the candidates for source of the GRB.

For uniform magnetic field and compressive plasma, the medium of magnetic plasma has following three types of waves. The \textit{shear Alfvén waves} which are similar to the Alfvén waves as discussed above.

The \textit{fast MHD waves} in which the thermal and the magnetic compressions are in phase. And finally, the \textit{slow MHD waves} whose thermal and magnetic perturbations are in out of phase. For very weak magnetic field, $v_s >> v_a$ and $v_f = v_s$, while for a strong magnetic field $v_s << v_a$ and $v_f = v_a$. Here, $v_s$ is the sound velocity if there is no magnetic field, $v_f$ represents velocity of the fast MHD wave and $v_a$ is the Alfvén wave velocity. The fast waves move perpendicular to magnetic field lines. In the cosmic environment, assuming that the ambient medium consists of hydrogen plasma only, we get the ambient sound speed for the ambient temperature $\sim 10^3 \text{K}$, to be $\sim 10^4 \text{cm sec}^{-1}$ which is smaller by factor of 100 compared to the Alfvén wave velocity. In this context, the maximum energy flux that can be dissipated from the fast MHD waves is $\sim \rho A^2 v_a$, where $A$ is amplitude of the wave. Hence, in order to get GRB flux, the required amplitude of waves should be $\sim 10^9 \text{cm sec}^{-1}$. The length scale of dissipation of this flux is $\sim 1.7 \text{cms}$.

In case of \textit{slow MHD waves}, thermal and magnetic perturbations are out of phase. These waves can travel only in direction close to the direction of the magnetic field. For very weak fields, $v_s >> v_a$ and for strong fields $v_s << v_a$, then waves travel along the direction of field lines only. In case of intermediate field, $v_{slow} = v_a$, where $v_{slow}$ is the velocity of slow MHD
wave, then the allowed direction of travel lies between a cone with half an angle $\theta = 27^\circ$. The wave energy flux of the slow modes is $\sim \rho A^2 v_{slow}$. Then the required amplitudes of the waves should be $\sim 10^{10} cm sec^{-1}$. The length scales over which these slow waves dissipate their energy is $\sim 10^3 cms$.

5 Conclusions and discussion

Our conclusions of this study are as follows:

(i) Assuming that energy sources of the gamma ray bursts are similar to the energy sources which trigger solar and stellar transient activity phenomena like flares, plasma accelerated flows in the flux tubes and, dissipation of energy by the MHD waves, we estimated physical parameters in the cosmic environment and compared with the observations.

(ii) If primordial flares are responsible for the creation of GRB, the following physical parameters are needed in order to explain the observed maximum amount of energy released by the GRB. The source region requires length scale of $\sim 10^{12} cms$, strength of the annihilating magnetic field $\sim 10^{10} G$ and the inflow velocity of the plasma may be $\sim 10^9 cm/sec^{-1}$ if we assume that thickness is equal to length scale of the reconnection region. By the theory of conventional flare mechanism and conservation of mass, we also estimated density of the reconnection region to be $\sim 10 g/cm^3$. Finally, by taking the observed typical length scale of the cosmic environment and conservation of magnetic flux, we estimated the strength of the primordial magnetic field which is almost similar to expected strength of the magnetic field of primordial origin. Lastly, we estimated typical rate of GRB per galaxy to be $\sim 10^5 yrs$.

(ii) If plasma flows (that dissipate and release required amount of energy) in the primordial flux tubes are responsible for the creation of GRB, the required length of the flux tube may be $\sim 10^{25} cms$. Assuming that total amount of energy released from the current sheath of the primordial flow is same as the amount of energy released in GRB, it is estimated that the magnetic field strength of the flux tube may be $\sim 1 G$.

(iii) Finally, we considered nonlinear interaction and dissipation of MHD waves in the cosmic environment and estimated flux of the energy released. It is expected that, in addition to thermal sound waves, cosmic environment with magnetic field can accommodate Alfven waves, shear Alfven waves, fast
and slow MHD waves. In case of Alfven waves and in order to get the observed GRB flux, it is estimated that the amplitudes of the waves that dissipate into shocks may be $\sim 0.01 \text{ cm sec}^{-1}$. Similarly, in case of fast and slow MHD waves the required amplitudes of the waves may be $\sim 10^9 \text{ cm sec}^{-1}$ and $\sim 10^{10} \text{ cm sec}^{-1}$.

Most of the phenomena explained in this study appear to be promising sources of energy for the creation of GRB. However, depending upon the observational constraints, one can accept or reject these phenomena. For example, among the MHD waves, if we assume that Alfven waves are responsible for the GRB phenomena, then estimated dissipation lengths over which shock be formed is very small. For example, observations of GRB bursts temporal variability on a time scale $\Delta t \sim 10 \text{ m sec}$ implies that source size should be less than 3000 kms (Piran 1999). Though, dissipation length is within the observational limit, it is very difficult to believe that such low amplitude Alfven waves create GRB.

In case of either fast or slow modes though both the waves form the shock, dissipate the required amount of GRB flux, magnitude of the amplitude of the waves involved in dissipation is enormous and author is not aware as to whether observations do indicate any such amplitudes of either thermal or magnetic perturbations. Since, density perturbations are involved in fast and slow MHD waves, it will be interesting if such signatures are detected in future observations.

Primordial flux tubes are also good candidates for the acceleration of matter to the required energy of GRB. The difficulty in this case is how to dissipate the accelerated plasma matter and moreover stability of such flux tubes (in cosmic environment) should be examined carefully. Future observations of Faraday rotation measurements of distant cosmic radio sources may reveal the nature and dimension of such primordial flux tubes if they were created in the early universe.

Among all the physical processes considered for the investigation of the energy sources of GRB, the most promising candidate appears to be primordial flares. This can be gauged by the following similarities: (i) observations of solar flares and observations of GRB and, (ii) inferred physical parameters from model of the primordial flare with the observed parameters of the cosmic environment. These similarities can also be found in the earlier work (Vahia and Rao 1988).
The GRB are similar to solar flares in the following important observed properties: (i) the source size is compact, (ii) time scales is \( \sim \) seconds to minutes, (iii) radiation is observed in most of the electromagnetic spectrum, (iv) non-thermal energy spectrum and (v) polarization < 30%.

In this study, we inferred from flare theory that in order to get the required observed GRB energy, length scale of source region may be \( \sim 10^{12} \text{cms} \) and strength of the magnetic field \( \sim 10^{10} \text{G} \). The difficulty here is how to annihilate such a large scale region within few seconds. The answer lies in the instabilities (Hood 1986) created by attaining such a structure of the magnetic fields. Once instability starts, reconnection starts with in few seconds, accelerate the particles very close to velocity of light and shock structures would be formed which follows the creation of non-thermal spectrum. It is also estimated from this flare study that primordial magnetic field may be \( \sim 10^{-10} \text{G} \) which is very close to the strength of the magnetic field expected from the observations. It is known from solar flare observations that before eruption of the flare, gradients of magnetic fields occur over the solar surface. It will be interesting to get observational information regarding development of any such strong gradients of magnetic fields (inferred from the Faraday polarization data in radio domain), at the site of GRB flare, before it’s eruption. In the present study we phenomenologically modeled GRB as a primordial flare phenomenon. However, detailed solution of MHD equations is essential in order understand the GRB phenomenon completely.

In summary, this study indicates that solar like transient MHD phenomena, especially primordial flares in the inter galactic or inter galaxy cluster region, may be most promising energy source for creation of GRB phenomena. Though most of the observed properties of GRB phenomena and solar flare phenomena are similar, additional observational informations such as signature of the gradients in the cosmic magnetic fields at the site of production of GRB is required in order to prove our proposed conjectures in this study.

Acknowledgements

This paper is dedicated to my beloved parents who constantly encouraged my research carrier when they were alive.

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