Influence of temperature fluctuations on continuum spectra of cosmic objects

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Abstract The presence of convective and turbulent motions, and the evolution of magnetic fields give rise to existence of temperature fluctuations in stellar atmospheres, active galactic nuclei and other cosmic objects. We observe the time and surface averaged radiation fluxes from these objects. These fluxes depend on both the mean temperature and averaged temperature fluctuations. The usual photosphere models do not take into account the temperature fluctuations and use only the distribution of the mean temperature into surface layers of stars. We investigate how the temperature fluctuations change the spectra in continuum assuming that the degree of fluctuations (the ratio of mean temperature fluctuation to the mean temperature) is small. We suggest the procedure of calculation of continuum spectra, which takes into account the temperature fluctuations. As a first step one uses the usual model of a photosphere without fluctuations. The observed spectrum is presented as a part depending on mean temperature and the additional part proportional to quadratic value of fluctuation degree. It is shown that for some forms of absorption factor the additional part in Wien’s region of spectrum can be evaluated directly from observed spectrum. This part depends on the first and second wavelength derivatives, which can be calculated numerically from the observed spectrum. Our estimates show that the temperature dependence of absorption factors is very important by calculation of continuum spectra corrections. As the examples we present the estimates for a few stars from Pulkovo spectrophotometric catalog and for the Sun. The influence of temperature fluctuations on color indices of observed cosmic objects is also investigated.

Keywords Radiative transfer · Turbulence · Temperature fluctuations · Stars: atmospheres · Color indices · Active galactic nuclei

1 Introduction

From investigation of the Sun (see Stix 1991) we know that the photosphere has chaotic changes of the temperature. They are due to convective and turbulent motions, and the existence of inhomogeneous magnetic fields, which are the specific examples of stochastic media. Recall that stochastic behavior of gas motions arises due to many types of instabilities occurring in these media. Most important is the convective instability which results in the star cells of rising and descending motions of hot and cold gas, correspondingly. For the Sun the ratio of mean temperature fluctuations \( T' \) to the mean temperature \( \langle T \rangle \equiv T_0 \) is equal to \( \eta = \sqrt{\langle T'^2 \rangle}/T_0 \approx 0.03 \). It appears this estimation is minimal because the solar spots are not taken into account. No doubt, the temperature fluctuations exist in many stars, not only in the Sun. The Sun is the only star which can be observed detailed in every point of the surface. Further we restrict ourselves by consideration of point-like cosmic objects. Apparently the most high fluctuations exist in stars with the profound convective zone and in active galactic nuclei.

In stochastic media all the values—the temperature \( T = \langle T \rangle + T' \), the radiation intensity \( I = \langle I \rangle + I' \), the absorption coefficient \( \alpha = \langle \alpha \rangle + \alpha' \) and so on, are stochastic, i.e. are characterized by its mean values, for example, \( \langle T \rangle \equiv T_0 \) and by fluctuations \( T' \). The mean value of fluctuations are equal to zero (for example, \( \langle T' \rangle = 0 \)). The mean values are determined by the average over ensemble of realizations during the time of observation. The chaotic events in space and...
time with characteristic life-time greater than the observation time are taken into account in the mean values (see, in more detail, Levshakov and Kegel 1997). From the stars and other point-like cosmic objects we observe the mean radiative flux \( \langle H \rangle \). We stress that the influence of temperature fluctuations on the spectra is purely statistical effect, depending on the specific ensemble of temperature realizations in space and time, and on the time of observation. It is known that the averaged spectra of stars, even neighboring in spectral classes, frequently differ one another (see catalog Alekseeva et al. 1996). The possible reasons may be different, and the influence of temperature fluctuations, without doubt, is one of the most important between them.

In present paper we investigate the influence of temperature fluctuations on the continuum spectra of various cosmic objects. The preliminary consideration of this subject is given in Silant’ev and Alexeeva (2008). Their paper deals only with investigation of effective thermal sources in stochastic media \( S_\lambda = \langle \alpha_\lambda(T) B_\lambda(T) \rangle \) as compared with the usual sources, \( \alpha_\lambda(T_0) B_\lambda(T_0) \), taken without the influence of temperature fluctuations.

Here \( B_\lambda(T) \) is the intensity of Planck’s radiation. It was shown that the source \( S_\lambda \) can fairly strong differ from \( \alpha_\lambda(T_0) B_\lambda(T_0) \), especially at high value of parameter \( h\nu/kT_0 \). The value \( S_\lambda \) can be both higher than \( \alpha_\lambda(T_0) B_\lambda(T_0) \) and lower one, depending on specific form of absorption coefficient \( \alpha_\lambda(T) \). Below we will obtain the formula for corrections to observed radiation flux, which arises due to existence of temperature fluctuations.

Before start a detail consideration let us show in a simple example of averaging over two temperature realizations \( (T = T_0 + T' \text{ and } T = T_0 - T') \), how arises this statistical effect. The average value of Planck’s function (in the Wien limit) is equal to:

\[
\langle B_\lambda(T) \rangle = \frac{1}{2} \left( \frac{2h\nu^2}{c^5} \right) \left[ \exp \left( -\left( \frac{h\nu}{k(T_0 + T')} \right) \right) \right]
+ \exp \left[ -\left( \frac{h\nu}{k(T_0 - T')} \right) \right] \]
\[
\simeq B_\lambda(T_0) \cosh \left( \frac{h\nu}{kT_0} \cdot \frac{T'}{T_0} \right) \geq B_\lambda(T_0).
\]

Here we used the condition \( T'/T_0 \ll 1 \).

It is seen that the observing averaged value \( \langle B_\lambda(T) \rangle \) is higher than \( B_\lambda(T_0) \). At \( T'/T_0 = 0.05 \) and \( h\nu/kT_0 = 10 \) this growing consists of 13 %. At the Rayleigh limit \( h\nu/kT \ll 1 \) the Planck function \( B_\lambda \sim T \) and \( \langle B_\lambda(T) \rangle = B_\lambda(T_0) \). For intermediate cases one has \( \langle B_\lambda(T) \rangle > B_\lambda(T_0) \).

The analogical calculations demonstrate that the mean value of absorption factor \( \langle \alpha_\lambda(T) \rangle \) can be both higher than \( \alpha_\lambda(T_0) \) and lower than this value, depending on the specific form of \( \alpha_\lambda(T) \). Joint consideration of Planck function and absorption coefficient can give rise both to positive and negative additional contribution to the usual spectra, depending on the mean temperature \( T_0 \).

Below we investigate the problem how the temperature fluctuations change the continuum spectra of cosmic objects in detail. For a number of stars and wavelengths we derived numerically the corresponding corrections to the spectra, depending on the degree of temperature fluctuations. For the cases, when the main contribution is due to bound-free transitions in negative hydrogen ions, we suggest the new technique to calculate the corrections to the spectra directly from the shape of observing spectra, without using the results of any photosphere models. Note that we consider a degree of temperature fluctuations \( \eta \) as a given parameter. The inverse problem—the estimation of \( \eta \) from observed spectra is not considered in our paper. We hope investigate this difficult problem in the future.

## 2 Basic formulas

The radiation flux in continuum \( H_\lambda^{(0)} \) is related with the Planck function \( B_\lambda \) as follows (see Sobolev 1969):

\[
H_\lambda^{(0)} = 2\pi \int_0^1 d\mu \mu I_\lambda(0, \mu) = 2\pi \int_0^1 d\tau_\lambda \int_0^1 d\mu e^{-\tau_\lambda/\mu} B_\lambda(T_0(\tau_\lambda)) \]
\[
= 2\pi \int_0^1 d\tau_\lambda E_2(\tau_\lambda) B_\lambda(T_0(\tau_\lambda)). \tag{1}
\]

Recall the notion of functions \( E_n(\tau) \) (see Sobolev 1969; Gray 1976):

\[
E_n(\tau) = \int_0^1 d\mu \mu^n e^{-\tau/\mu} = \int_1^\infty dx x^n e^{-\tau x}. \tag{2}
\]

The functions \( E_n(\tau) \) obey the relations:

\[
n E_{n+1}(\tau) = e^{-\tau} - \tau E_n(\tau),
\]
\[
\frac{dE_n(\tau)}{d\tau} = -E_{n-1}(\tau). \tag{3}
\]

At high \( \tau \) one exists \( E_n(\tau) \to \exp(-\tau)/\tau \). The figures \( E_1(\tau), E_2(\tau) \) and \( E_3(\tau) \), for example, are given in Gray (1976).

In Eq. (1) the mean temperature \( T_0 \) is considered as known function of the optical depth \( \tau_\lambda \). To find the relation between the mean temperature \( T_0 \) and optical depth \( \tau_\lambda \) is the main task at construction of photosphere models. Formula (1) does not take into account the influence of temperature fluctuations.

In Silant’ev (2005) the radiative transfer equation for mean intensity \( \langle I_\lambda \rangle = I_\lambda^{(0)}(\tau_\lambda, \mu) \) was derived. This equation has the effective source \( S_\lambda \), which was investigated.
by Silant’ev and Alexeeva (2008). The sign () in radiative transfer equation (4) means the averaging over ensemble of various realizations along the wave path, i.e. one takes into account the local temperature fluctuations.

In LTE approximation the transfer equation for \( I_{\lambda}^{(0)}(\tau_\lambda, \mu) \) acquires the form:

\[
(n \nabla) I_{\lambda}^{(0)} = -\langle \alpha_\lambda \rangle I_{\lambda}^{(0)} + S_\lambda(T_0, \eta), \\
S_\lambda(T_0, \eta) = [\langle \alpha_\lambda(T) \rangle B_\lambda(T)].
\]

(4)

(5)

As a result of averaging, all the values acquire the dependence on the degree of temperature fluctuations:

\[
\eta = \frac{\sqrt{T^2}}{T_0}.
\]

(6)

The fluctuations \( T' \) have the local character. Thus, in the case of the Sun they describes the difference between hot and cold parts of granula. The transfer equation (4) is stationary and differs from the usual transfer equation only by the feature that the temperature fluctuations are taken into account in absorption coefficient \( \langle \alpha_\lambda(T) \rangle = \alpha_\lambda(T_0 + T') \). Usually in photosphere models one assumes that all physical values are distributed spherically symmetric. We assume the same for temperature fluctuations, i.e. the degree of fluctuations \( \eta \) is independent of the place in visual semisphere of a star.

According to Silant’ev (2005), more exact expression takes into account the correlation length \( R \) of temperature fluctuations, i.e. instead of term \( \langle \alpha_\lambda \rangle \) one uses the expression \( \langle \alpha_\lambda \rangle \cdot (1 + \langle (\alpha')^2 \rangle/\langle \alpha \rangle^2 \cdot \tau_\lambda) \). Here \( \tau_\lambda = \langle \alpha_\lambda \rangle \cdot R \) is the mean optical depth of characteristic length of turbulence \( R \). Usually one exists \( \tau_\lambda \ll 1 \), i.e. the additional term is of the third degree of smallness and can be omitted. Such case takes place for solar granulations with \( R \approx 1000 \text{ km} \) in the region of wavelength, where the basic absorption is determined by negative hydrogen ion.

From Eqs. (4) and (5) one follows the expression for observed radiation flux \( \langle H_{\lambda} \rangle \), which takes into consideration the influence of temperature fluctuations:

\[
\langle H_{\lambda}(\eta) \rangle = 2\pi \int_0^{1} d\mu \mu I_{\lambda}^{(0)}(0, \mu)
\]

\[
= 2\pi \int_0^{\infty} d(\tau_\lambda) E_2(\tau_\lambda) \frac{\langle \alpha_\lambda(T) \rangle B_\lambda(T)}{\langle \alpha_\lambda(T) \rangle}.
\]

(7)

Mean values \( \langle \alpha_\lambda(T) \rangle \) are derived by the Gaussian formula (see Silant’ev and Alexeeva 2008):

\[
\langle \alpha_\lambda(T) \rangle = \frac{1}{\sqrt{2\pi}\eta} \int_{-\infty}^{\infty} dx \exp\left(-\frac{x^2}{2\eta^2}\right) \alpha_\lambda(T_0 \cdot (1 + x)).
\]

(8)

This expression is valid at \( \eta \leq 0.2 \).

When the fluctuations are absent, \( \eta = 0 \), formula (7) reverts to usual expression (1) for \( H_{\lambda}^{(0)} \), where \( d\tau_\lambda = \alpha_\lambda(T_0)dz \) and \( T_0 = \tau_\lambda \). For brevity we will use simply \( T_0 \). Recall that the radiation flux without consideration of fluctuations we denote as \( H_{\lambda}^{(0)} \). The value \( \langle \alpha_\lambda(T) \rangle B_\lambda(T) \) is calculated from Eq. (8) analogously.

Our next task is to relate the observing flux \( \langle H_{\lambda}(\eta) \rangle \) with the \( H_{\lambda}^{(0)} \), i.e. to show how the small temperature fluctuations change the continuum spectra. As an initial approximation we take the expression \( H_{\lambda}^{(0)} \), and then we will seek the corrections to this spectrum proportional to \( \eta^2 \). In Sect. 4 we will consider this problem in Wien’s region of spectrum, and for such temperatures and wavelengths where the main absorption is due to bound-free transitions of outer electron in negative hydrogen ion. In this case the flux \( \langle H \rangle \) can be found by numerical differentiation of observed spectra over wavelength \( \lambda \).

The values of fluctuation degree \( \eta \) are determined by physical conditions in every specific object. We assume \( \eta \) as a known parameter and derive the formula for estimates of spectra corrections due to temperature fluctuations for a number of objects, where absorption coefficient deals with bound-free transitions in the negative hydrogen ions. Of course, it would be very interesting and important to derive the technique how to estimate the parameter \( \eta \) from the analysis of observed spectra. It appears this complex task may be resolved by statistical comparison of objects with very close physical conditions. In this paper we do not consider this difficult problem.

3 Relation of flux \( \langle H_{\lambda}(\eta) \rangle \) with \( H_{\lambda}^{(0)} \)

Assuming that the temperature fluctuations are low \( \langle \eta^2 \rangle \ll 1 \), let us use in Eq. (8) the expansion series:

\[
\alpha_\lambda(T_0 \cdot (1 + x)) = \alpha_\lambda(T_0) + b_\lambda(T_0) x + a_\lambda(T_0) x^2 + \cdots.
\]

(9)

and take into mind that \( \langle x \rangle = 0 \) and \( \langle x^2 \rangle = \eta^2 \). Dimensionless parameters \( b_\lambda(T_0) \) and \( a_\lambda(T_0) \) have the forms:

\[
b_\lambda(T_0) = \frac{T_0}{\alpha_\lambda(T_0)} \frac{\partial \alpha_\lambda(T_0)}{\partial T_0},
\]

\[
a_\lambda(T_0) = \frac{T_0^2}{2\alpha_\lambda(T_0)} \frac{\partial^2 \alpha_\lambda(T_0)}{\partial T_0^2}.
\]

(10)

Thus, the mean absorption factor acquires the form:

\[
\langle \alpha_\lambda \rangle = \alpha_\lambda(T_0) \left(1 + \frac{\alpha_\lambda(T_0) \eta^2}{2}\right).
\]

(11)
The optical depth \(\tau_\lambda\) in expression (7) is calculated according to formula:

\[
\langle \tau_\lambda \rangle = \int_0^\infty dz \langle a_\lambda(z) \rangle = \tau_\lambda + \eta^2 \int_0^{\tau_\lambda} d\tau'_\lambda \langle a_\lambda(T_0) \rangle,
\]

where we use the same notion \(d\tau_\lambda = a_\lambda(T_0(z)) dz\), as in calculation of the flux \(H_\lambda^{(0)}\) (see Eq. (1)).

Calculating by analogy the value \(\langle a_\lambda(T)B_\lambda(T) \rangle\), we obtain from Eq. (7) the following expression for observed flux \(\langle H_\lambda(\eta) \rangle\):

\[
\langle H_\lambda(\eta) \rangle = H_\lambda^{(0)} + 2\pi \eta^2 \int_0^\infty d\tau_\lambda E_2(\tau_\lambda) \left[b_\lambda(T_0) \frac{\partial B_\lambda(T_0)}{\partial T_0} \right.
\]

\[
\left. + \frac{1}{2} T_0^2 \frac{\partial^2 B_\lambda(T_0)}{\partial T_0^2} \right] + A_\lambda,
\]

\[\lambda_\alpha = 2\pi \eta^2 \int_0^\infty d\tau_\lambda B_\lambda(T_0) \left[E_2(\tau_\lambda) a_\lambda(T_0) \right.
\]

\[- E_1(\tau_\lambda) \int_0^{\tau_\lambda} d\tau'_\lambda a_\lambda(T_0) \right].
\]

The value \(T_0^\alpha \equiv T_0(\tau_0')\). The negative term with \(a_\lambda(T_0')\) in Eq. (14) arises from the expansion of the exponent \(\exp(-\langle \tau_\lambda \rangle)\) (see formula (12)) in power series with parameter \(\eta^2\).

Using the known photosphere models, i.e. the dependence between \(T_0\) and optical depth \(\tau_\lambda\), one can calculate the right part of Eq. (13) and obtain the flux \(\langle H_\lambda(\eta) \rangle\), corresponding to level of temperature fluctuation \(\eta\). The second and third terms in Eq. (13) characterize correction to model flux \(H_\lambda^{(0)}\) due to temperature fluctuations. In principle, the condition of the best coincidence of expression \(\langle H_\lambda(\eta) \rangle\) with observed flux allows us to estimate parameter \(\eta\) in the atmosphere of a cosmic object. The influence of fluctuations is most strong in the region of short waves. It appears we have to become the best coincidence just in this wave region. Taking different values for parameter \(\eta\), one can obtain more satisfactory coincidence of flux \(\langle H_\lambda(\eta) \rangle\) with the observed spectrum than that for model case \(H_\lambda^{(0)}\).

The authors are not specialists in the model construction activity. Therefore we restrict ourselves by the estimate of difference \(\langle H_\lambda(\eta) \rangle - H_\lambda^{(0)} \rangle / H_\lambda^{(0)}\) only for a number of Sun-like stars and the Sun for wavelengths where main contribution to opacity gives the bound-free electron transitions in negative hydrogen ion \(H^-\). These estimates follow directly from observed spectra (see catalog Alekseeva et al. 1996). For these wavelengths we can substitute the temperature derivatives by numerical derivatives over wavelengths in observed spectra (the results are presented in Tables 2 and 3).

First we yield this substitution for Plank’s function \(B_\lambda(T_0)\). Then such substitution will be made for some types of absorption coefficients. Note that not all types of \(\alpha(T_0)\) allow us such substitution. Finally, we will consider the spectrum in Wien’s region, simultaneously using the specific form of bound-free transitions in negative hydrogen ion \(H^-\) for those cases, where this absorption plays the dominant role.

3.1 Substitution temperature derivatives by wavelength derivatives

The Planck function has the following form:

\[
B_\lambda(T) = \frac{2hc^2}{\lambda^5} f(\lambda T), \quad f(\lambda T) = \frac{1}{e^{\frac{h\nu}{kT}} - 1},
\]

\[
\frac{\nu}{\lambda T} \equiv \frac{h\nu}{kT} = \frac{g_0}{\lambda T}.
\]

Here and what follows the wavelengths always are taken in microns. The function \(f(\lambda T)\), depending on the product \(\lambda T\) and \(T\), obeys the relations:

\[
\lambda \frac{df}{d\lambda} = T \frac{df}{dT}, \quad \lambda^2 \frac{d^2 f}{d\lambda^2} = T^2 \frac{d^2 f}{dT^2}.
\]

These relations allow us to substitute the derivatives over temperature by the derivatives over wavelength. The corresponding formulas are the followings:

\[
T \frac{d B_\lambda}{dT} = \frac{1}{T} \left(5B_\lambda + \lambda \frac{d B_\lambda}{d\lambda} \right),
\]

\[
\frac{d^2 B_\lambda}{dT^2} = \frac{1}{T^2} \left(20B_\lambda + 10 \lambda \frac{d B_\lambda}{d\lambda} + \lambda^2 \frac{d^2 B_\lambda}{d\lambda^2} \right).
\]

Absorption coefficients \(\alpha_\lambda\) in continuum spectra often have the vast regions of smooth dependence on the wavelength (see Gray 1976). In these regions they can be approximated by power law \(\alpha_\lambda \sim \alpha(T)\lambda^{n}\). Then the values \(b_\lambda(T_0)\) and \(a_\lambda(T_0)\) (see Eq. (10)) are independent of \(\lambda\): \(b_\lambda(T_0) = b(T_0)\) and \(a_\lambda(T_0) = a(T_0)\). In some cases the absorption coefficient has the form of product \(\alpha_0(\lambda)\alpha_1(T)\), i.e. \(\alpha_\lambda(T) = \alpha_0(\lambda)\alpha_1(T)\). Very important example of such dependence is the case of bound-free transitions in negative hydrogen ion \(H^-\) (see in more detail below). In such cases the values \(a_\lambda(T)\) and \(b_\lambda(T)\) also are independent of wavelength.

Using the relations (17), Eq. (13) can be written in the form:

\[
\langle H_\lambda(\eta) \rangle = H_\lambda^{(0)} + 2\pi \eta^2 \int_0^\infty d\tau_\lambda E_2(\tau_\lambda)
\]

\[
\times \left(10B_\lambda + 5\lambda \frac{d B_\lambda}{d\lambda} + \frac{\lambda^2}{2} \frac{d^2 B_\lambda}{d\lambda^2} \right)
\]

\[
+ 2\pi \eta^2 \int_0^\infty d\tau_\lambda E_2(\tau_\lambda) b(T_0)
\]

\[
\times \left(5B_\lambda(T_0) + \lambda \frac{d B_\lambda}{d\lambda} \right) + A_\lambda.
\]
The second term in the right part of this expression describes the contribution of temperature fluctuation to the spectrum due to influence of Planck’s function, i.e. this is the contribution of value \( \langle B_\lambda \rangle \). The estimate of this contribution from model spectra, for example, from models of Kurucz et al. (1974), is of interest for knowledge of influence of temperature fluctuations on continuum spectra. Beforehand, note that the contribution of \( \langle B_\lambda \rangle \) is smaller than that of term with \( b(T_0) \).

Because \( \tau_\lambda \) and \( E_2(\tau_\lambda) \) depend on wavelength \( \lambda \), the \( \lambda \)-differentiation cannot be taken out the integral in Eq. (18). But for absorption coefficients, mentioned above, exist the relations:

\[
\frac{\partial \alpha_\lambda}{\partial \lambda} = \beta(\lambda)\alpha_\lambda, \quad \frac{\partial \tau_\lambda}{\partial \lambda} = \beta(\lambda),
\]

\[
\beta(\lambda) = \frac{1}{\alpha_0(\lambda)} \frac{\partial \alpha_0(\lambda)}{\partial \lambda},
\]

which lead to the following formulas:

\[
2\pi \lambda \int_0^\infty d\tau_\lambda E_2(\tau_\lambda) \frac{\partial B_\lambda}{\partial \lambda} = \lambda \frac{\partial H_\lambda^{(0)}}{\partial \lambda} - \lambda \beta(\lambda) T_\lambda,
\]

\[
T_\lambda = \lambda H_\lambda^{(0)} - 2\pi I_\lambda^{(0)}(0, 1) = 4\pi \int_0^1 d\mu \mu [I_\lambda^{(0)}(0, \mu) - I_\lambda^{(0)}(0, 1)],
\]

\[
2\pi \lambda \int_0^\infty d\tau_\lambda E_2(\tau_\lambda) \frac{\partial^2 B_\lambda}{\partial \lambda^2} = \frac{\lambda^2}{2} \frac{\partial^2 H_\lambda^{(0)}}{\partial \lambda^2} - \lambda^2 \beta(\lambda) T_\lambda
\]

\[
- 2\lambda^2 \beta(\lambda) \frac{\partial T_\lambda}{\partial \lambda} + \lambda^2 \beta^2(\lambda) M_\lambda.
\]

When deriving these formulas, we used Eqs. (1) and (3). The values \( \beta(\lambda) \) and \( \partial \beta / \partial \lambda \) for negative hydrogen ion are presented in Table 1. They were calculated numerically from figure for \( \alpha_\lambda \) given in Gray (1976).

\[
T_\lambda = \int \frac{d\tau_\lambda}{E_2(\tau_\lambda)} = \int \frac{d\tau_\lambda}{e^{\tau_\lambda} - 1}.
\]

Table 1 Functions \( \beta(\lambda) \) and \( \partial \beta(\lambda) / \partial \lambda \) for absorption factor \( \alpha_\lambda(H^-) \)

| \( \lambda \), \( \mu \mbox{m} \) | 0.2 | 0.25 | 0.3 | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 | 0.6 | 0.65 | 0.7 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| \( \beta \) | 3.5 | 2.4 | 1.7 | 1.28 | 1 | 0.76 | 0.58 | 0.41 | 0.3 | 0.22 | 0.15 |
| \( \partial \beta / \partial \lambda \) | -28 | -18 | -11 | -6.8 | -5.3 | -4.25 | -3.5 | -2.7 | -2 | -1.5 | -1.25 |

The model is presented in a number of books (see, for example, Sobolev 1969).

Below we restrict ourselves by the Wien limit of a spectrum, when the parameter \( g_0/\lambda T \gg 1 \). In this case simple model, mentioned above, gives rise to the relations:

\[
T_\lambda \simeq -H_\lambda^{(0)},
\]

\[
M_\lambda = 2T_\lambda + 2\pi \int_0^\infty d\tau_\lambda B_\lambda(T_0)(\tau_\lambda - 1) \exp(-\tau_\lambda) \simeq H_\lambda^{(0)}.
\]

According to formulas (19) and (21), contribution of \( \langle B_\lambda(\tau_\lambda) \rangle \) into \( \langle H_\lambda(\eta) \rangle \) acquires the form:

\[
\langle H_\lambda(\eta) \rangle = H_\lambda^{(0)} + \eta^2 \left[ 10H_\lambda^{(0)} + 5\lambda \frac{\partial H_\lambda^{(0)}}{\partial \lambda} + \frac{1}{2} \lambda^2 \frac{\partial^2 H_\lambda^{(0)}}{\partial \lambda^2} \right]
\]

\[
- \lambda^2 \beta(\lambda) \frac{\partial T_\lambda}{\partial \lambda} - \left( 5\lambda \beta(\lambda) + \frac{1}{2} \lambda^2 \frac{\partial \beta(\lambda)}{\partial \lambda} \right) T_\lambda
\]

\[
+ \frac{1}{2} \lambda^2 \beta^2(\lambda) M_\lambda.
\]

This formula depends on the specific form of absorption coefficient. Because of the difference \( \langle B_\lambda(T_0, \eta) \rangle - B_\lambda(T_0) \) is positive and slowly decreases with grow of wavelength, then the term \( \langle B_\lambda(\tau_\lambda, \eta) \rangle \) gives the positive contribution to observed spectrum \( \langle H_\lambda(\eta) \rangle \). Yet the contribution of term \( b(T_0) \), in principle, can exceed the contribution from \( \langle B_\lambda(T_0) \rangle \), and, in particular, gives rise to negative correction to \( \langle H_\lambda(\eta) \rangle \).

3.2 Estimate of expressions with \( \alpha_\lambda(T_0) \)

The term \( A_\lambda \) in formula (14) describes the change of spectrum as a result of difference between \( \langle \alpha_\lambda \rangle \) and \( \alpha_\lambda(T_0) \) (see formula (11)). Negative part of \( A_\lambda \) presents the decrease of flux due to additional term with \( \eta^2 \) in \( \exp(-\tau_\lambda) \) (see (12)). The positive part describes the increasing of flux due to effective source \( \langle \alpha_2 B_\lambda \rangle \). The difference of these terms is lower than every separate term.

It is of interest to estimate the contribution of term \( A_\lambda \) to expressions (13) and (18). Let us replace the terms \( \alpha_\lambda(T_0) \) and \( \alpha_\lambda(T_0) \) by effective mean value \( \bar{\alpha}_\lambda \), taken at the level \( \tau_\lambda \approx 2/3 \), where is located the main source of radiation. As a result, one can obtain the expression:
\[ A_\lambda \simeq 2\pi \eta^2 \bar{\sigma}_\lambda \int_0^\infty d\tau_\lambda (2E_2(\tau_\lambda) - e^{-\tau_\lambda}) B_\lambda(T_0) \]
\[ = -2\pi \eta^2 \bar{\sigma}_\lambda \int_0^\infty d\tau_\lambda E_2(\tau_\lambda) \frac{\partial B_\lambda(T_0)}{\partial \tau_\lambda} \]
\[ \equiv \eta^2 \bar{\sigma}_\lambda T_\lambda \simeq -\eta^2 \bar{\sigma}_\lambda \delta_\lambda H_\lambda^{(0)} \quad (24) \]

This expression is less than the term \( 10H_\lambda^{(0)} \) in Eq. (23) if the value \( \bar{\sigma}_\lambda \) is less or of the order of unity. In these cases the term \( A_\lambda \) can be neglected. The coefficient \( \delta_\lambda \), according to approximate Sobolev’s (1969) formula, is estimated as the ratio of the mean absorption factor over all spectrum to the local coefficient \( \alpha_\lambda \). In the case of absorption by negative hydrogen ions \( H^- \) the Chandrasekhar’s estimation (see Chandrasekhar 1950) gives the value \( \delta_\lambda \simeq 0.26 \). This gives the estimation \( A_\lambda \simeq (1 \pm 2)H_\lambda^{(0)} \), which is less than the term \( 10H_\lambda^{(0)} \) in formula (23).

### 4 Estimates in Wien’s limit

The case \( g_0/(\lambda T_0) = 14388/(\lambda T_0) \gg 1 \) corresponds to Wien’s limit of thermal emission. Recall that at the maximum of Planck’s emission \( \lambda_{\text{max}} T = 2900 \). This gives \( g_0/(\lambda_{\text{max}} T) \simeq 5 \), i.e. the Wien approximation \( B_\lambda(T) = (2\pi^2 c^3/\lambda^5) \exp(-g_0/\lambda T) \) exists at least up to maximum of spectrum. The relative deviation from Planck’s formula at maximum of emission consists of 0.7 %. At \( g_0/\lambda T = 3.6 \) this deviation is \( \approx 2.8 \% \), that corresponds to the value \( \lambda(\mu \text{m}) T \simeq 4000 \).

This means that the Wien limit occurs practically up to \( \lambda \simeq 0.5 \mu \text{m} \) at \( T = 8000 \text{ K} \), and up to \( \lambda \simeq 1 \mu \text{m} \) at \( T \simeq 4000 \text{ K} \). Note that in this temperature interval the absorption coefficient in continuum, basically, is due to bound-free transitions of outer electron in negative hydrogen ion \( H^- \) (see Chandrasekhar 1950). Below we will give the formulas of this case, in detail. In Wien’s limit the Einstein-Milne correction due to stimulated emission (the factor \( 1 - \exp(-g_0/\lambda T) \)) is low and the formulas for absorption coefficient (see Gray 1976) become simpler.

For the absorption factors of the type

\[ a_\lambda = a_\lambda(\lambda) T^2 e^{-g_1/\lambda} \quad (25) \]

dimensionless coefficients \( b(T) \) and \( a(T) \), according to Eq. (10), acquire the form:

\[ b(T) = \gamma + \frac{g_1}{T}, \]
\[ a(T) = \frac{\gamma(\gamma - 1)}{2} + \frac{(\gamma - 1)g_1}{T} + \frac{g_1^2}{2T^2}. \quad (26) \]

Formula (25) is valid for the description bound-free transitions in \( H^- \), where \( \gamma = -1.5 \) and \( g_1 = -8700 \) (if the wavelengths are in microns).

The coefficients \( b(T) \) and \( a(T) \) obey power-law dependence on the temperature. Therefore in Wien’s approximation Eq. (18) (without the term \( A_\lambda \)) can be simplified if we transform the terms of type \( B_\lambda(T_0)/T^n \) to expressions with the differentiation over wavelength. For this aim we can use the following equalities (introducing the notion \( f_1(\lambda T) = \exp(-g_1/\lambda T) \)):

\[ \frac{f_1(\lambda T)}{T} = \frac{\lambda^2 \partial f_1}{\partial \lambda}, \quad \frac{f_1(\lambda T)}{T^2} = \left( \frac{2}{g_1} \right)^2 \frac{\lambda^2 \partial f_1 + a \partial^2 f_1}{\partial \lambda^2}. \quad (27) \]

Taking into account, that in Wien’s approximation \( B_\lambda \sim \lambda^{-5} \exp(-g_0/\lambda T) \), we derive the relations:

\[ \frac{T \partial B_\lambda}{\partial T} = \frac{g_0}{T \lambda^2} B_\lambda, \quad \frac{B_\lambda}{T} = \frac{\lambda}{g_0} \left( 5B_\lambda + \lambda \frac{\partial B_\lambda}{\partial \lambda} \right), \]
\[ \frac{B_\lambda}{T^2} = \frac{\lambda^2}{g_0^2} \left[ 30B_\lambda + 12\lambda \frac{\partial B_\lambda}{\partial \lambda} + \lambda^2 \frac{\partial^2 B_\lambda}{\partial \lambda^2} \right]. \quad (28) \]

Using these relations, and Eqs. (19) and (21), we derive Eq. (18) (without the term proportional to \( a(T) \)) in Wien’s approximation:

\[ \langle H_\lambda(\eta) \rangle = H_\lambda^{(0)} + \eta^2 \left[ C_0(\lambda) H_\lambda^{(0)} + D_0(\lambda) T_\lambda + G_0(\lambda) M_\lambda \right. \]
\[ + C_1(\lambda) \frac{\partial H_\lambda^{(0)}}{\partial \lambda} + D_1(\lambda) \frac{\partial T_\lambda}{\partial \lambda} \]
\[ \left. + C_2(\lambda) \lambda^2 \frac{\partial^2 H_\lambda^{(0)}}{\partial \lambda^2} \right]. \quad (29) \]

\[ C_0(\lambda) = 10 + 5\gamma + 30\xi, \quad C_1(\lambda) = 5 + \gamma + 12\xi, \]
\[ C_2(\lambda) = (0.5 + \xi). \quad (30) \]

\[ D_0(\lambda) = -\lambda \beta(\lambda) C_1 - \lambda^2 \frac{\partial \beta(\lambda)}{\partial \lambda} C_2, \]
\[ G_0(\lambda) = \lambda^2 \beta(\lambda) C_2, \quad D_1(\lambda) = -2\lambda \beta(\lambda) C_2. \]

Here we use the dimensionless values \( \xi = g_1 \lambda/g_0 \) with \( g_0 = 14388 \). For the case \( a_\lambda(H^-) \) one has \( \gamma = -3/2, g_1 = -8700, g_1/g_0 = -0.6 \) and \( \xi = -0.6 \). As it was mentioned earlier, we can neglect the term \( A_\lambda \) if the absorption in negative hydrogen ions are considered. According to Chandrasekhar (1950) and Sobolev (1969) this case is valid for the solar type stars with \( T_e < 7 \cdot 10^3 \text{ K} \).

In Table 2 we present the corrections to \( \langle H_\lambda(\eta) \rangle \) for a number of solar type stars (\( \beta, \text{Hyi} (G2IV), \mu, \text{Vel} (G5III), \), \( \tau \text{Per} (G0V) \)) and the Sun, using Alekseeva et al. (1996). The corrections were calculated for \( \lambda = 0.55 \mu \text{m} \) and \( 0.6 \mu \text{m} \). For these wavelengths one can neglect the contribution of free-free transitions in \( H^- \). Note that the columns with positive
numbers in Table 2 refer to $Φ_λ^{(0)}$, and the columns with negative numbers present the values $Φ_λ$. The notions of these values are given below.

Formula (29), with taking into account Eq. (22), acquires the form:

$$
\langle H_λ(η) \rangle \equiv H_λ^{(0)}(1 + η^2Φ_λ)
$$

$$
= H_λ^{(0)} + η^2 \left\{ Φ(0)H_λ^{(0)} + A_0H_λ^{(0)} + A_1λ \frac{∂H_λ^{(0)}}{∂λ} \right. \\
+ \left. ξλ^2 \frac{∂^2H_λ^{(0)}}{∂λ^2} \right\},
$$

(31)

where we introduce the notion $β' = \frac{∂β}{∂λ}$, and the factors $A_0$ and $A_1$ have the following form:

$$
A_0 = (5 + λβ)γ + ξ(30 + 12λβ + λ^2(β^2 + β')), \\
A_1 = γ + ξ(12 + 2λβ).
$$

(32)

The term $Φ(0)H_λ^{(0)}$ coincides with that in square brackets in Eq. (23), if the relations (22) are used. This term has the form:

$$
Φ_λ^{(0)}H_λ^{(0)} = \left[ 10 + 5λβ + \frac{λ^2}{2}(β^2 + β') \right] H_λ^{(0)} \\
+ (5 + λβ)λ \frac{∂H_λ^{(0)}}{∂λ} + \frac{λ^2}{2} \frac{∂^2H_λ^{(0)}}{∂λ^2}.
$$

(33)

Recall that this term presents contribution of $⟨B_λ⟩$.

4.1 Results of calculations and the discussions

Usually the star fluxes are presented in logarithmic scale $lg H_λ$ or in star magnitudes $m_λ = -2.5l g H_λ$. Our formulas have the derivatives $∂H_λ/∂λ$ and $∂^2H_λ/∂λ^2$. Differentiation of the relation $ln H_λ = ln 10 l g H_λ$ gives rise to expressions:

$$
\frac{∂H_λ}{∂λ} = H_λ ln 10 \frac{∂ lg H_λ}{∂λ} = -H_λ ln 10 \frac{∂ m_λ}{2.5} \frac{∂λ}{∂λ},
$$

(34)

$$
\frac{∂^2H_λ}{∂λ^2} = H_λ \left[ ln 10 \frac{∂^2 lg H_λ}{∂λ^2} + \left( ln 10 \frac{∂ lg H_λ}{∂λ} \right)^2 \right]
$$

$$
= H_λ \left[ \frac{(ln 10 \frac{∂ m_λ}{2.5})^2}{2.5} - \frac{ln 10 \frac{∂^2 m_λ}{∂λ^2}}{2.5} \right].
$$

(35)

These formulas show that the derivatives are proportional to $H_λ$. It means that the expression (31) may be presented in the form:

$$
\frac{(H_λ(η)) - H_λ^{(0)}}{H_λ^{(0)}} \equiv \frac{ΔH_λ}{H_λ^{(0)}} = η^2Φ_λ.
$$

(36)

The value $ΔH_λ$ can be presented as a corresponding change $ΔT$ (see Sobolev 1969). As a result, we derive the estimate:

$$
ΔT \approx (2.8/4) \left( \frac{g_0}{λT_0} \right)^{-1} η^2Φ_λ T_0.
$$

(37)

Instead of $T_0$ in such estimates one may be taken the effective temperature $T_e$ in the regions, where $B_λ(T_e)$ satisfactory describes the spectrum. Recall that in Wien’s region even low temperature changes $ΔT$ give rise to fairly high changes of fluxes.

Let us present the explicit formula for function $Φ_λ$, if the absorption coefficient has form (25). This formula follows directly from Eqs. (31)–(35):

$$
Φ_λ = 10 + 5λβ + \frac{λ^2}{2}(β^2 + β') + A_0
$$

$$
+ (5 + λβ + A_1)λ ln 10 \frac{∂ lg H_λ}{∂λ} + \left( ln 10 \frac{∂ lg H_λ}{∂λ} \right)^2.
$$

(38)

First of all, we obtain the part of expression $Φ_λ$, which is due to contribution of mean Planck’s function $⟨B_λ⟩$. This expression can be obtained from Eq. (38), where terms with $γ$ and $ξ$ have to be omitted. Let us denote this expression as $Φ_λ^{(0)}$. To calculate this term one can use the model spectra, or use the observed spectra $⟨H_λ(η)⟩$, assuming that at low values $η$ exists approximate equality $⟨H_λ(η)⟩ \simeq H_λ^{(0)}$.

As it was mentioned earlier, the spectra depend strongly on the contribution of the coefficient $b_λ(T_0)$ (see Eq. (10)), which describe the change of effective source $⟨α_λ B_λ⟩$ due to fluctuations of first temperature derivatives in absorption factor $α(λ) T$ and in Planck’s function $B_λ(T)$. As a role, the first derivatives characterize the changes most sensitively than the second ones. The contribution of second order derivatives give rise to less contribution to changes in spectra. For this reason, the calculation of $Φ_λ^{(0)}$ is useful for comparison with basic terms proportional to mean value of the product of first order derivatives.
Table 3 Functions $\Phi^{(0)}_\lambda$ and $\Phi_\lambda$ for spectra of $\alpha$ PsA and $\beta$ Ari

| $\lambda$, $\mu m$ | 0.45 | 0.47 | 0.50 | 0.55 | 0.60 |
|-------------------|------|------|------|------|------|
| $\alpha$ PsA, $\Phi^{(0)}_\lambda$ | 1.7  | 1.4  | 1.0  | 0.4  | 0.0  |
| $\alpha$ PsA, $\Phi_\lambda$       | −10.2| −11.3| −12.2| −13.5| −13.4|
| $\beta$ Ari, $\Phi^{(0)}_\lambda$  | 3.1  | 2.8  | 1.9  | 1.4  | 0.9  |
| $\beta$ Ari, $\Phi_\lambda$       | −8.6 | −9.2 | −10.8| −11.0| −11.1|

It is of interest, that calculations, presented in Table 2, show that the terms with $\beta$ and $\beta'$ give comparatively low contribution to $\Phi_\lambda$, less than $(2\div5)$ % compared with other terms. Note, that this low contribution is not the consequence of smallness of $\beta$ and $\beta'$ (see Table 1), and basically are due to powers of small wavelengths ($\lambda \sim 0.5\, \mu m$) at coefficients $\beta$ and $\beta'$. Therefore for estimations of $\langle B_\lambda \rangle$ (term $\Phi^{(0)}_\lambda$) can be used the simple formula:

$$
\phi^{(0)}_\lambda H^{(0)}_\lambda \simeq 10 H^{(0)}_\lambda + 5\lambda \frac{\partial H^{(0)}_\lambda}{\partial \lambda} + \lambda^2 \frac{\partial^2 H^{(0)}_\lambda}{\partial \lambda^2}.
$$

The term with $b_\lambda$ can be simplified accordingly. The sense of these simplifications consists of conclusion, that to obtain the estimates in Eq. (18), one can take out of the integrals the differentiations over wavelength. Apparently, this can be used in other cases, if the absorption coefficients change slowly in the range of considering spectrum, and if the spectrum itself has fairly smooth form. In this case one can introduce the large-scale differentiation over wavelengths for spectrum itself, and use the averaged absorption factor.

In Table 3 we presented the values of functions $\Phi^{(0)}_\lambda$ and $\Phi_\lambda$ for stars $\alpha$ PsA and $\beta$ Ari, calculated from homogeneous catalog (Alekseeva et al. 1996). The tables of star magnitudes $m_\lambda$ in this catalog are presented with the interval $\Delta \lambda = 0.0025\, \mu m$. The first star is of the class A3V with $T_e \simeq 8300\, K$, and the second one is of the class A5V with $T_e \simeq 5800\, K$. Spectra of these stars does not determined by absorption in negative ion $H^-$. To receive the estimates of temperature fluctuations influences we used the absorption coefficients in continuum given in Allen (1973). We used the parabolic interpolation of data from this book for intervals $\lambda = 0.4, 0.5, 0.8\, \mu m$ and for $\theta = 5040/\lambda = 0.4, 0.5, 0.6$ at the value of electron pressure $\lg P_e = 1$. As a result, we derived the formula:

$$
\alpha_\lambda(\theta) = 10 C(\lambda, \theta),
$$

$$
C(\lambda, \theta) = \frac{25}{3} \left[ \lambda^2 (20 \theta^2 - 21 \theta + 5.3) + \lambda (-24 \theta^2 + 25.5 \theta - 6.33) + (5.8 \theta^2 - 6.18 \theta + 1.468) \right].
$$

Taking into account that $\lambda$-dependence in this expression is fairly smooth, we used for estimations the mentioned above (see Eq. (39) and so on) simple expressions.

The negative corrections $\Phi_\lambda$ denote that the temperature (see the notions (36) and (37)) is less compared with the temperature obtained from photosphere models, which do not take into account the temperature fluctuations. Especially important conclusion from our estimates is that most considerable corrections to $\langle H_\lambda(\eta) \rangle$ are due to terms with $b_\lambda(\eta)$ (see Eq. (10)). These terms give rise to negative corrections whereas the contribution of $\langle B_\lambda(\eta) \rangle$ gives positive corrections. The contribution of terms with $b_\lambda$ is determined by the average of product of first temperature derivative from absorption factor and from the Planck function, i.e. most important linear expansions over $T'$ are averaged.

It is of interest to note that observed Sun’s spectrum in ultraviolet region is lower than that determined by accepted effective temperature (Stix 1991; Chance and Kurucz 2010). Usually this effect is related with influence of absorption of metals. However, the possible contribution of temperature fluctuations is also not eliminated. Though this mechanism is not studied in detail, but preliminary we can note that the calculations of source function of new transfer equation (4) in ultraviolet region demonstrate the strong difference from the usual Planck’s function.

The estimates (37) for the Sun at $\lambda = 0.55\, \mu m$ and $T_e \simeq 5770\, K$ give $\Delta T \simeq -(2.8 \div 4) \eta^2 \cdot 9.8 T_e \simeq -(2.8 \div 4) \cdot 3.2470 \eta^2$, i.e. $\Delta T \simeq -(31 \div 45)\, K$ for $\eta = 0.03$, and $\Delta T \simeq -(87 \div 125)\, K$ for $\eta = 0.05$. In percents these estimates are $(0.5 \div 0.8)\, %$ and $(1.5 \div 2.1)\, %$ of $T_e$, correspondingly.

These estimates are given for $\lambda \sim 0.55\, \mu m$ and practically do not depend on low changes in taking $T_0 \sim T_e$. Note, that to estimate the change in effective temperature of a star, one has calculate the change of all the photosphere spectrum in the range of all wavelengths.

5 Influence of temperature fluctuations on color indices of radiating objects

The color index of observing objects is characterized by the difference of radiation fluxes between standard wavelength intervals. Usually the observed fluxes $F_\lambda$ correspond to standard wavelength intervals near the central wavelengths $(\lambda_U = 0.36\, \mu m, \lambda_B = 0.44\, \mu m, \lambda_V = 0.55\, \mu m, \lambda_R = 0.7\, \mu m)$. For example, the color index $U - B$ is determined according to formula:

$$
U^{(0)} - B^{(0)} = -2.5 \lg \frac{F_U^{(0)}}{F_B^{(0)}} + Const. \quad (41)
$$

The value $Const$ is taken from the reasons of convenience. Frequently one takes $Const = 0$ for stars of A0V-type.
Usually one considers that fluxes $F_{\lambda}^{(0)} = B_{\lambda}(T_0)$, where temperature $T_0$ on an average describes the observed part of spectrum. This relation formally follows from usual transfer equation at supposition that the source of radiation $B_{\lambda}(T_0)$ is independent of optical depth in radiating region. The radiating objects can have the fluctuating temperature. In this case the source is described by the formula $\langle \alpha_\lambda(T) B_{\lambda}(T)/B_{\lambda}(T) \rangle ≡ S_{\lambda}(T_0, \eta)/(B_{\lambda}(T))$ (see Eqs. (4) and (5)).

As we know, in this case the flux $\langle H_{\lambda}(\eta) \rangle \equiv \langle F_{\lambda} \rangle$ depends on the degree of temperature fluctuations $\eta$. Assuming that this effective source is independent of optical depth, we obtain the expression $\langle F_{\lambda} \rangle = S_{\lambda}(T_0, \eta)/(B_{\lambda}(T))$. This means that the color indices $\langle U \rangle - \langle B \rangle$ and so on depend on degree of fluctuations $\eta$. As usually, we restrict ourselves to low fluctuations $\eta^2 \ll 1$ and instead of Eq. (41) obtain the relation:

$$\langle U \rangle - \langle B \rangle = U^{(0)} - B^{(0)} + \eta^2 \Delta(U - B),$$

where $\Delta(U - B)$ is expressed by the formula:

$$\Delta(U - B) = -2.5(K_{\lambda{g}} - K_{\lambda{b}}),$$

$$K_{\lambda} = \frac{T_0^2}{2B_{\lambda}(T_0)} \frac{\partial^2 B_{\lambda}(T)}{\partial T_0^2} + \frac{T_0}{B_{\lambda}(T_0)} \frac{\partial B_{\lambda}(T)}{\partial T_0} \cdot B_{\lambda}(T_0)$$

$$\equiv K_{\lambda}^{(0)} + K_{\lambda}^{(1)} b_{\lambda}(T_0).$$

Analogous formulas characterize the color indices $B - V$ and $V - R$. The numerical values for coefficients $K_{\lambda}^{(0)}(T_0)$ and $K_{\lambda}^{(1)}(T_0)$ are presented in Table 4 for wavelengths $\lambda = 0.36; 0.44; 0.55$ and $0.7$ µm. Note, that these factors are positive. At growing of wavelengths these coefficients diminish.

It is seen from Eq. (43) that the color indices strongly depend on specific form of absorption coefficients. This is seen clearly from the example, when we use the absorption factor (25) for bound-free transitions in negative hydrogen ions. In this case $b_{\lambda}(T_0)$ has the form (26) with $\gamma = -1.5$ and $g_1 = -8700$. The term $K_{\lambda}^{(0)}$ describes the influence of temperature fluctuations without contribution of absorption factor. This term is positive, as it was mentioned earlier. The expressions $\Delta(U - B)$, $\Delta(B - V)$, and $\Delta(V - R)$ with allowance for only $K_{\lambda}^{(0)}$-term are negative for all mean temperatures $T_0$. The consideration of absorption coefficients gives rise to the result that the total coefficient $K_{\lambda}^{(0)} + K_{\lambda}^{(1)} b_{\lambda}(T_0)$ strongly differs from the term $K_{\lambda}^{(0)}$ (see Table 5). In Table 5 we also present the values of $\Delta(U - B)$ and so on, which take into account both coefficients—$K_{\lambda}^{(0)}$ and $K_{\lambda}^{(1)}$.

At comparatively low temperatures the correction to color indices is negative. Then, with the grow of the temperature, this correction acquires positive sign. Most high the influence of temperature fluctuation is in color index $U - B$, i.e. for small wavelengths. It seems, that in active

| $T_0, K$ | $4 \cdot 10^3$ | $5 \cdot 10^3$ | $6 \cdot 10^3$ | $7 \cdot 10^3$ | $8 \cdot 10^3$ | $9 \cdot 10^3$ | $10 \cdot 10^3$ | $11 \cdot 10^3$ | $12 \cdot 10^3$ | $13 \cdot 10^3$ | $14 \cdot 10^3$ | $15 \cdot 10^3$ | $16 \cdot 10^3$ | $17 \cdot 10^3$ |
|----------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| $K_{\lambda}^{(0)}$ | 39.93 | 23.98 | 15.60 | 10.73 | 7.71 | 5.72 | 4.37 | 2.73 | 1.82 | 1.29 | 0.95 |
| $K_{\lambda}^{(1)}$ | 9.99 | 8.00 | 6.67 | 5.73 | 5.03 | 4.49 | 4.07 | 3.45 | 3.03 | 2.72 | 2.49 |
| $K_{\lambda{g}}$ | 25.27 | 14.93 | 9.57 | 6.51 | 4.63 | 3.42 | 2.60 | 1.61 | 1.08 | 0.77 | 0.57 |
| $K_{\lambda{b}}$ | 8.18 | 6.55 | 5.47 | 4.72 | 4.16 | 3.73 | 3.40 | 2.92 | 2.59 | 2.35 | 2.17 |
| $K_{\lambda{g}}$ | 14.93 | 8.65 | 5.46 | 3.67 | 2.60 | 1.91 | 1.45 | 0.90 | 0.61 | 0.43 | 0.32 |
| $K_{\lambda{b}}$ | 6.55 | 5.26 | 4.42 | 3.83 | 3.40 | 3.07 | 2.82 | 2.46 | 2.21 | 2.03 | 1.90 |
| $K_{\lambda}^{(0)}$ | 8.27 | 4.70 | 2.93 | 1.96 | 1.38 | 1.02 | 0.78 | 0.49 | 0.33 | 0.24 | 0.18 |
| $K_{\lambda}^{(1)}$ | 5.17 | 4.18 | 3.54 | 3.10 | 2.78 | 2.54 | 2.36 | 2.09 | 1.91 | 1.78 | 1.68 |

| $T_0, K$ | $4 \cdot 10^3$ | $5 \cdot 10^3$ | $6 \cdot 10^3$ | $7 \cdot 10^3$ | $8 \cdot 10^3$ | $9 \cdot 10^3$ | $10 \cdot 10^3$ | $11 \cdot 10^3$ | $12 \cdot 10^3$ |
|----------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| $K_{\lambda{g}}$ | 3.21 | -1.92 | -4.08 | -4.98 | -5.31 | -5.36 | -5.28 | -4.96 |
| $K_{\lambda{b}}$ | -4.79 | -6.29 | -6.57 | -6.43 | -6.12 | -5.79 | -5.46 | -4.87 |
| $K_{\lambda{g}}$ | -9.14 | -8.40 | -7.57 | -6.83 | -6.20 | -5.67 | -5.24 | -4.56 |
| $K_{\lambda{b}}$ | -10.73 | -8.84 | -7.51 | -6.54 | -5.81 | -5.25 | -4.81 | -4.16 |
| $\Delta(U - B)$ | -20.00 | -10.94 | -6.24 | -3.62 | -2.04 | -1.06 | -0.45 | 0.21 |
| $\Delta(B - V)$ | -10.88 | -5.26 | -2.49 | -1.00 | -0.18 | 0.29 | 0.55 | 0.77 |
| $\Delta(V - R)$ | -3.97 | -1.12 | 0.14 | 0.71 | 0.96 | 1.05 | 1.07 | 1.01 |
galactic nuclei the temperature fluctuations may be high. In these cases the consideration of fluctuations can change the color indices considerably. Besides, at introduction of color indices would be useful, though roughly, take into account the temperature gradient near the surface of emitting object, for example, using the mentioned approximate formula in Sobolev (1969). It appears the inverse problem of estimation of degree of temperature fluctuations \( \eta \) is simpler from consideration of color indices of cosmic objects with like physical conditions.

6 Conclusion

Let us present short discussion of obtained results. The existence of stochastic processes in photospheres of stars, active galactic nuclei etc., which arises due to convective and turbulent motions, generation and evolution of magnetic fields, give rise to stochastic behavior of temperature, intensity of radiation and absorption coefficients. These stochastic processes determine the mean radiation flux \( \langle H_\lambda \rangle \). This flux differs from the flux \( H_\lambda^{(0)} \), which takes into account only dependence of all values from mean temperature \( T_0 \) and is independent of temperature fluctuations \( T' \). The numerous models of photospheres determine the mean temperature as a function of distance from the photosphere’s surface.

Assuming the degree of temperature fluctuations \( \eta = \sqrt{\langle T'^2 \rangle / T_0} \) as a low value, we obtain the relation between the observed flux \( \langle H_\lambda \rangle \) and model flux \( H_\lambda^{(0)} \). The difference between these fluxes is proportional to \( \eta^2 \) and can assume both positive and negative values. This is determined by the specific form of absorption coefficient. In Wien’s limit this difference can be estimated from the observed spectrum, because it depends of the first and the second derivatives over wavelength, which can be calculated numerically directly from the observed spectrum and is independent of specific choice of photosphere model.

We calculated this difference of fluxes for the Sun, and solar type stars (\( \beta \) Hyi, \( \mu \) Vela and \( \iota \) Per) for \( \lambda = 0.55 \) \( \mu \)m and \( \lambda = 0.6 \) \( \mu \)m, using the homogeneous Pulkovo spectrophotometric catalog (see Alekseeva et al. 1996). Also we obtain the corresponding estimates for stars \( \alpha \) PsA and \( \beta \) Ari.

These estimates demonstrate that the most high correction is due to mean value of product of first temperature derivatives of the absorption coefficient and the Planck function. The estimations show that the difference of spectra \( \langle H_\lambda \rangle \) and \( H_\lambda^{(0)} \) is fairly high and can be used for correction of model results, especially in ultraviolet region of spectra. We also described the influence of temperature fluctuations on the color indices. It appears these corrections can be high, especially for active galactic nuclei.

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