Designing the Alluvial Riverbeds in Curved Paths

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Abstract. The paper presents the method of determining the shape of the riverbed in curves of the watercourse, which is based on the method of Ikeda (1975) developed for a slightly curved path in sandy riverbed. Regulated rivers have essentially slightly and smoothly curved paths; therefore, this methodology provides the appropriate basis for river restoration. Based on the research in the experimental reach of the Holeška Brook and several alluvial mountain streams the methodology was adjusted. The method also takes into account other important characteristics of bottom material - the shape and orientation of the particles, settling velocity and drag coefficients. Thus, the method is mainly meant for the natural sand-gravel material, which is heterogeneous and the particle shape of the bottom material is very different from spherical. The calculation of the river channel in the curved path provides the basis for the design of optimal habitat, but also for the design of foundations of armouring of the bankside of the channel. The input data is adapted to the conditions of design practice.

1. Introduction
The shape of the riverbed in curved path (river bend) interested many scientists since the Renaissance (da Vinci, 1503-1508, Boussinesq, 1868, Fargue, 1868; Thomson, 1876, [1-3]). Even today there are many reasons for the continuing attention given to meandering rivers by geology, geomorphology, hydrology, physics or river restoration communities. Many scientists have conducted experiments in curved paths of open channels [4-12].

Probably the most complete scientific description of the flow velocity in the river bend is reported in the work [13]. Detailed description of the instream flow based on the Navier-Stokes equations was published by Rodi [14]. Recently Rüther and Olsen [15] reported the results of modelling the curved channel using 3D model.

Ottevanger [16] developed an analytical analysis for an axially symmetrical bend of the river channel. In addition, there exists an extensive list of works that dealt with the issue of river bends. There is a lack of works that determine the shape of alluvial riverbed in inhomogeneous sand-gravel environment that would be directly applicable in the design practice. Therefore, the aim of this paper is to provide a method of determining the shape of alluvial river bed in the curved path, which can be used in design practice.
2. Material and methods

Differences between mildly and sharply curved bends were described by Ikeda [17]. He developed the method of determining the shape of alluvial river bed in a slightly curved path. Regulated river channels have essentially slightly and smoothly curved paths; therefore, this methodology is appropriate for calculation the cross-section shape of the curved paths in regulated rivers. These data provide the basis for the design of channel armoring as well as the restoration of regulated streams.

The equation was extrapolated from the conditions of steady-state river channel, [17]:

\[
\frac{h}{h_a} = e^{\frac{1}{2}A_1\left[\frac{r^2}{ra}\right]}
\]  

(1)

where:

\( h \) – Computational (actual) depth of the curved path [m]
\( h_a \) – Average depth of flow in the centre line of a straight channel [m]
\( r_a \) – Radius of the centre line of the channel [m]
\( r \) – Radius to a calculation point of a stable cross-section (variable) [m]
\( A_1 \) – Parameter dependent on the characteristics of the stream channel and the bottom material

\[
A_1 = \frac{3}{4} \left[ \frac{\mu_c \cdot c_z}{1 + \mu_c \cdot \frac{c_z}{c_x}} \right]^{\frac{1}{2}} \lambda \cdot \nu \cdot \left[ \frac{F_a(0) - 1}{\mu} \cdot \frac{\nu}{\nu} \cdot F_b(0) \right]^{\frac{1}{3}}
\]

(2)

where:

\( \mu_c \) – Coefficient of friction determined by Ikeda (1975), who recommends the value of \( \mu = 0.43 \)
\( \mu = 0.43 \)
\( c_x \) – Drag coefficient of bed particles in the flow direction
\( c_z \) – Drag coefficient of bed particles perpendicular to a water level
\( \lambda \) – Interaction factor of particles, Iwagaki recommends the value of \( \lambda = 0.592 \), [17]
\( \rho_m \) – Specific gravity of bed load sediment [kg.m\(^{-3}\)]
\( \rho \) – Specific gravity of water [kg.m\(^{-3}\)]
\( v_* \) – Friction velocity \( v_* = \sqrt{g \cdot h \cdot i} \)
\( g \) – Gravitational acceleration [m.s\(^{-2}\)]
\( h \) – Average water depth [m]
\( i_e \) – Slope of the energy grade line
\( d_e \) – Effective diameter of grains [m]
\( \nu \) – Mean profile velocity [m.s\(^{-1}\)]
\( \kappa \) – The von Kármán constant, commonly considered as \( \kappa = 0.4 \)
\( F_a(0), F_b(0) \) – Constants defined by Kikkawa [17], \( F_a(0) = -4.167 \), \( F_b(0) = -2.64 \)

Ikeda (1975) verified the equations (1) and (2) on the watercourses with sandy riverbed. They showed very good agreement of the theoretical and real shape of the cross-section in maximum pool at the outer bank. The shape of the particles in alluvial sand-gravel riverbeds is substantially different from a sphere, it is therefore necessary to consider the shape and orientation of the particles.
Values of the depths in pools at the outer banks calculated in accordance with the method of Ikeda (1975) were smaller than the real ones in our study alluvial sand-gravel channels in the Slovak region of Orava and Kysuce (Table 1). The original equation considers the values of the drag coefficient of spherical sand particles deposited on the bottom. Chepil [18] provides the constant ratio of \( \frac{c_z}{c_x} = 0.85 \) for a wide range of Reynolds number, therefore, the first term of the equation (2) acquires the value as follows:

\[
0.36 = \left[ \frac{c_z \mu_c}{c_x \mu_c + 1} \right]^{\frac{1}{2}}
\]

Therefore a legitimate assumption arisen that the different hydrodynamic parameters of coarse alluvial sand-gravel material compared to sandy material create the source of the differences between the real and computational depths. That assumption raised the following questions: What is the orientation of the particles in the bottom alluvial material? How the orientation and shape of the particles impacts on the particle drag coefficient? Which accessible method can be used to determine the drag coefficient of a heterogeneous alluvial sand-gravel riverbed material for the design practice?

### Table 1. Values of the maximum depths in the curved paths of particular alternatives

| Stream name | River kilometer (rkm) | Real \( h_{max} \) in natural reach (m) | Computational \( h_{max} \) (spherical particle shape) (m) | Computational \( h_{max} \) (real particle shape) (m) |
|-------------|----------------------|----------------------------------------|----------------------------------------------------------|--------------------------------------------------------|
| Varínka     | 7.3                  | 1.6                                    | 1.3                                                      | 1.6                                                    |
| Varínka     | 12.3                 | 1.8                                    | 1.2                                                      | 1.44                                                   |
| Varínka     | 14.1                 | 2.1                                    | 1.2                                                      | 1.8                                                    |
| Kysuca      | 26.1                 | 3.7                                    | 2.7                                                      | 3.2                                                    |
| Kysuca      | 27.5                 | 3.9                                    | 3.1                                                      | 4.2                                                    |
| Čierňanka   | 9.6                  | 2.3                                    | 1.9                                                      | 2.2                                                    |
| Čierňanka   | 5.8                  | 2.1                                    | 1.9                                                      | 2.2                                                    |
| Bystrica    | 9.1                  | 2.1                                    | 1.8                                                      | 2.1                                                    |
| Zázrivka    | 3.1                  | 1.9                                    | 1.8                                                      | 2.2                                                    |

2.1. Orientation of the particles in the bottom sand-gravel material

Gilbert [19] began to study the movement of the bottom material in the laboratory channels and natural river beds already in 1917. Observation by camera was performed for example in this work [20]. As observed by these authors, trajectories of the floating bed load particles are very twisty in the first phase. Tortuosity is a consequence of the unstable position of the particles. After overcoming the initial stage of the path, the particles are stabilized in the position of least resistance; therefore the trajectories of the particles are also stabilized. The longest axis of the particle is oriented in the flow direction. The experiments took place at a relatively high ratio of \( v_z/v_r \) - high degree of the riverbed instability (the real mean velocity \( v_z \) substantially exceeded \( v_r \) – velocity which the riverbed material can withstand); however, it is not clear whether such significant instability of bottom material will be present in the natural sand-gravel watercourses. In case of our study watercourses in Orava and Kysuce, 74% of the particles were deposited in the flow direction by their longest axis with their largest area parallel to the water surface, either in straight channel, but also in the river bend. A similar finding is given by Romanovskij [21]. From the above it can be concluded that the trajectories of the
particles in sand-gravel riverbeds can be long enough for most of the particles to take a position of least resistance.

2.2. Effect of the particle shape and orientation on the drag coefficient of the particles in a bottom material

If the shape of the particles in the bottom material is spherical, values of the drag coefficient of the particles are not influenced by their orientation. In case of the non-spherical particle shape the drag coefficient becomes a function of their orientation and shape.

2.3. The particle shape of the bottom material.

To evaluate the shape characteristics of the bottom material the equation may be used, [21]:

\[ \Theta = \left( \frac{a + b + c}{a \cdot b} \right)^2 \]

Where:
- \( \Theta \) – Shape factor
- \( a \) – Length of the particle - the particle largest size
- \( b \) – Particle width
- \( c \) – Particle thickness

**Table 2.** Shape characteristics of particles of the bottom material in sand-gravel river beds of watercourses

| Watercourse  | b/a | c/a | \( \Theta \) according to (3) |
|--------------|-----|-----|-------------------------------|
| Zářivka      | 0.67| 0.46| 0.630                         |
| Bystrica     | 0.67| 0.46| 0.673                         |
| Varínka      | 0.72| 0.41| 0.630                         |
| Kysuca       | 0.64| 0.62| 0.624                         |
| Čierňanka    | 0.68| 0.49| 0.620                         |
| Dunaj        | 0.71| 0.50| 0.681                         |

From Table 2 it follows that the particles of the bottom material in sand-gravel river beds of watercourses have a flat shape, therefore, their hydrodynamic parameters are clearly different from the ones of the spherical particles.

2.4. Determination of the drag coefficient of the particles of the bottom material.

To describe the development of the deformation process it is necessary to determine the relationship of resistance of the particles to their shape and orientation. Both vertical and horizontal flow impacts the particles lying at the bottom, therefore it is important to determine the drag coefficient in both directions. Based on determined shape and orientation of the particles, the values of the drag coefficient may be determined from the total drag coefficient as shown in Figure 1, where \( \delta \) is the angle between a streamline and orientation of the particles of the bed material.

In case the \( a \) axis is parallel to a streamline (the ratio of \( \nu_z/\nu_r \) is large enough to form a trajectory which allows the particle to take a position of least resistance), the value of the coefficient of friction \( \mu_c \) for \( \delta = 0^\circ \) corresponds to the drag coefficient for horizontal flowing around the particles (\( c_x \)).
Figure 1. Relationship of the coefficient of friction $\mu_c$ with particle shape and orientation in relation to the flow direction, where $\delta$ is the angle between a streamline and orientation of the particles of the bed material.

In case the particles are sporadically deposited on the bottom and the $a$ axis is not parallel to a streamline (the ratio of $v_z/v_v$ is not sufficient to form a trajectory which allows the particle take a position of least resistance), the value of $\mu_c$ for $\delta = 45^\circ$ corresponds to $c_x$. The value of the coefficient of friction $\mu_c$ for $\delta = 90^\circ$ corresponds to the drag coefficient for vertical flowing around the particles ($c_z$).

3. Results and discussion
The results have been verified in the experimental reach of the Holeška brook, where the bypass consisting of three bends have been created. The bottom of each bend was stabilized by different fraction of sand-gravel material. Discharges in the experimental reach have been provided by the outlet from the Čerenec reservoir, located 2.4 km above the reach. The paper provides selected examples at nine reference reaches of five watercourses in the Slovak region of Orava and Kysuce (Table 1). Full profile shape of the riverbed in curved path may be calculated according to equation (1).

Radius to a calculation point of a cross-section $r$ is variable. Figure 2 compares the real and a computational shape of the cross-sections in curved paths of selected study river channels. Figure 2 shows a significant difference between the depth of the straight channel represented by the value of $h_a$ and the channel depth near the concave bank. For example, in the reach of Čierňanka (rkm 5.8) there is a depth of 0.7 m in the straight channel and 1.5 m in the river bend. Thus, the foundations of armouring in the river bend are significantly different from the straight channel, it is therefore necessary to design a concave part of the riverbed based on a real asymmetrical shape.
Figure 2. Comparison of real and a computational shape of the cross-sections in curved paths of the study river channels. a) Kysuca rkm 26.1, b) Varínka rkm 7.3, c) Kysuca rkm 27.5, Čierňanka rkm 5.8

4. Conclusions
The results of measurements in the study reaches of watercourses in the Slovak region of Orava and Kysuca confirmed the good agreement of the theoretical and the actual shape of the cross-sections at the outer banks of the river bends (maximum pool). Therefore, it can be assumed that the modified method is also applicable in alluvial sand-gravel riverbeds of watercourses, where the shape of the particles of the bottom material significantly influences the shape of the cross-section in the curved path.

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References
[1] L. da Vinci, “Codex Arundel 263”, pp. 216, 1503–1508.
[2] J. Boussinesq, “Mémoire sur l'influence de frottement dans les mouvements réguliers des fluides; xii — essay sur le mouvement permanent d'un liquide dans un canal horizontal axe circulaire,” Journal de Mathématiques Pures et Appliquées, vol. 13, pp. 377-424, 1868. (in French)
[3] L. Fargue, “Étude sur la corrélation entre la configuration du lit et la profondeur d'eau dans les rivières à fond mobile,” Annales Des Ponts Et Chaussées 1868. (in French)
[4] H. J. de Vriend, “Steady flow in shallow channel bends,” (Technical Report) Fluid Mech. Lab., Dep. of Civ. Eng., Delft University of Technology, 489 pp., 1981a.
[5] H. J. de Vriend, “Velocity redistribution in curved rectangular channels,” J. Fluid Mech. 107, pp. 423–439, 1981b.
[6] T. Blokland, “Turbulence measurements in a curved flume,” MSc thesis, Delft University of Technology, 1985. (in Dutch)
A. J. Odgaard, and M. A. Bergs, “Flow processes in a curved alluvial channel,” *Wat. Resour. Res.*, vol. 24, pp. 45–56, 1988.

F. E. Hicks, Y. C. Jin, and P. M. Steffler, “Flow near sloped bank in curved channel,” *J. Hydr. Eng.*, vol. 116, pp. 55–70, 1990.

P. J. Whiting, and W. E. Dietrich, “Experimental studies of bed topography and flow patterns in large amplitude meanders: 2. Mechanisms,” *Wat. Resour. Res.*, vol. 29, pp. 3615–3622, 1993.

A. Tominaga, M. Nagao, and I. Nezu, “Flow structure and momentum transport processes in curved open-channels with vegetation,” *Proc. 28th IAHR Congress*, Graz, Austria. Institute for Hydraulics and Hydrology, 1999.

R. Booij, “Measurements and large-eddy simulations of the flows in some curved flumes,” *J. Turbulence*, vol. 4, pp. 1–17, 2003.

K. Blanckaert, and H. J. de Vriend, “Secondary flow in sharp open channel bends,” *J. Fluid Mech.*, vol. 498, pp. 353–380, 2004.

I. L. Rozovskij, “Movement of water at curves of the open channel (Dvizheniye vody na povorote otkrytogo rusa),” Akademia nauk, Kyjev, Ukraina 185 p. 1957. (in Russian)

W. Rodi, “Turbulence Models and Their Application in Hydraulics,” International Association for Hydraulic Research, Delft. 1980.

N. Rüther, and N. R. B. Olsen, “Modelling free-forming meander evolution in a laboratory channel using three-dimensional computational fluid dynamics,” *Geomorphology*, vol. 89, pp. 308–319, 2007.

W. Ottevanger, K. Blanckaert, and W.S.J. Uijttewaal, “Processes governing the flow redistribution in sharp river bends,” *Geomorphology*, vol. 163–164, pp. 45–55, 2012.

S. Ikeda, “On secondary flow and bed profile in alluvial curved open channel,” *Proc. XVth IAHR Congress*, Sao Paulo (Paper B-14), 1975.

W. S. Chepil, “The use of evenly spaced hemispheres to evaluate aerodynamic forces on a soil surface,” *Eos Trans. AGU*, Vol. 39, pp. 397-404, 1958.

G. K. Gilbert, “Hydraulic-mining debris in the Sierra Nevada” *Professional Paper 105*, 188 p., Government Printing Office Washington, 1917.

K. I. Rossinskij, and K. S. Ljubomirova, “Hopping movement of river sediments (Skachkoobraznoye dvizheniye rechnykh nanosov),” *Dinamika i termika rečných potokov*. Moskva, 1972. (in Russian)

V. V. Romanovskij, “Research and calculation of hydraulic parameters of sediments (Issledovaniya i raschet gidravlicheskih parametrovnanosov),” Leningrad, 1973. (in Russian)