Method of distinction of pure d-wave state from the mixed state in HTSC

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Direct observation of the collective modes in unconventional superconductors (USC) by microwave impedance technique experiments has made the study of the collective excitations in these systems very important. One remaining question is the exact form of the order parameter of unconventional superconductors. Extended s-wave pairing, mixture of s- and d-states, as well as of different d-wave states are among possibilities. We have considered the mixtures of $d_{x^2-y^2}$ and $d_{y^x}$ states in high temperature superconductors (HTSC) and have, for the first time, derived the full set of equations for the collective mode spectrum in the mixed d-wave state with an arbitrary admixture of $d_{y^x}$ state. The results we have obtained will allow us to calculate the whole collective mode spectrum, which may then be used for interpretation of sound attenuation and microwave absorption data as well as for identification of the type of pairing and order parameter in unconventional superconductors. In particular, this will allow one to estimate the extent of admixture of $d_{y^x}$ state in the possible mixed state.

\section{1. Introduction}

One of the main problems of unconventional superconductivity is the determination of a type of pairing and an order parameter\textsuperscript{1}. While now the type of pairing is established for a lot of unconventional superconductors (s-pairing is realized in some electron-type HTSC (Nd$_2$CeCuO$_4$) and MgB$_2$, d-pairing in a hole-type HTSC and in some electron-type HTSC (NCCO and other compounds), organic superconductors, some HFSC (UPd$_2$Si$_2$, CeIn$_3$, CeNi$_2$Ge$_2$ etc.) and p-pairing in Sr$_2$RuO$_4$ (HTSC), UPt$_3$ (HFSC)) the exact form of the order parameter is still unknown for the most of USC. It is known that there is a $d$-wave pairing in a most hole-type oxide superconductors, but at the same time the different ideas concerning extended s-wave pairing, mixture of s- and d-states, as well as of different d-wave states still discuss actively. One of the cause of such a situation is the uncertainty in answer the question: do we have exact zero gap along some chosen lines in momentum space (like the case of $d_{x^2-y^2}$) or gap is anisotropic but nonzero everywhere (except maybe some points). Existing experiments (tunneling etc.) do not give the certain answer this question while the answer is quite principle. From other side there are some experiments\textsuperscript{2} which could be explained\textsuperscript{3} under suggestion about realization in HTSC of a mixed states, like $d_{x^2-y^2} + id_{y^x}$. Annett \textit{et al.}\textsuperscript{4} considered the possibility of mixture of different d-wave states in HTSC and came to conclusion that mixture of $d_{x^2-y^2}$ and $d_{y^x}$ states is the most likely state. Brusov \textit{et al.}\textsuperscript{5} have suggested one of the possible ways to distinguish the mixture of two $d$-states from pure $d$-states. For this they considered the mixed $d_{x^2-y^2} + id_{y^x}$ state and calculated the spectrum of collective modes in this state. The comparison of this spectrum with the spectrum of a pure $d$-wave states of HTSC shows that they are significantly different and could be the probe of the symmetry of the order parameter in HTSC. Thus the probe of the spectrum in ultrasound and/or microwave absorption experiments could be used to distinguish the mixture of two $d$-wave states from pure $d$-wave states.

To create the theoretical basis for investigation of possible mixed superconducting state in unconventional superconductors by sound attenuation and microwave absorption I derive for the first time a full set of equations for collective modes spectrum in $d_{x^2-y^2}$ - state with small admixture of $d_{y^x}$ state. This case is the most interesting one because we suppose that dominant state is $d_{x^2-y^2}$ state and admixture of $d_{y^x}$ state is small, around 3-10%. In this case it is possible to expand all expressions in powers of small $\epsilon$ and obtain the corrections to the spectrum of pure $d_{x^2-y^2}$ state, which has been found before. Obtained equations allow to calculate the whole collective mode spectrum in mixed $d_{x^2-y^2} + id_{y^x}$ state and distinguish this state from pure $d$-wave states (whose collective mode spectrum has been calculated earlier) by ultrasound attenuation and microwave absorption experiments.

These experiments have led recently to discovery of collective modes in UBe$_{13}$ (heavy fermion superconductor) by microwave impedance technique experiments and in Sr$_2$RuO$_4$ (high temperature superconductor) by ultrasound attenuation experiments. Feller \textit{et al.}\textsuperscript{6} have presented results of a microwave surface impedance study of the heavy fermion superconductor UBe$_{13}$. They clearly have observed an absorption peak whose frequency- and temperature-dependence scales with the BCS gap
function \( \Delta(T) \). This was the first direct observation of the resonant absorption into a collective mode, with energy approximately proportional to the superconducting gap. This discovery opens a new page in study of the collective excitations in unconventional superconductors. The significance of studying of collective modes connects to the fact that they exhibit themselves in ultrasound attenuation\(^6\) and microwave absorption\(^6\) experiments, neutron scattering, photoemission and Raman scattering\(^8\). The large peak in the dynamical spin susceptibility in HTSC arises from a weakly damped spin density-wave CM. This gives rise to a dip between the sharp low energy peak and the higher binding energy hump in the ARPES spectrum. Also, the CM of amplitude fluctuation of the d-wave gap yields a broad peak above the pair-breaking threshold in the \( B_{1g} \) Raman spectrum\(^8\).

2. Path integral model of d-wave pairing
We used Brusov’s model\(^1\) for \( d \)-pairing in superconductors, obtained by the path integral method. The model is described by the effective functional of action

\[
S_{\text{eff}} = g^{-1} \sum_{p,j,a} c_{ia}^+(p)c_{ia}(p) + \frac{1}{2} \ln \det \frac{\hat{M}(c_{ia}^+c_{ia})}{\hat{M}^{c_{ia}^+c_{ia}}}, \tag{1}
\]

where \( c_{ia}^{(0)} \) is the condensate value of Bose-fields \( c_{ia} \) and \( \hat{M}(c_{ia}^+c_{ia}) \) is the \( 4 \times 4 \) matrix depending on Bose-fields and parameters of quasi-fermions. The number of degrees of freedom in the case of \( d \)-pairing is equal to 10, i.e., we must have five complex canonical variables, which can be naturally chosen in the form

\[
c_1 = c_1 + c_{22}, \quad c_2 = c_1 - c_{22}, \quad c_3 = c_{12} + c_{21}, \quad c_4 = c_{13} + c_{31}, \quad c_5 = c_{23} + c_{32}.
\]

In the canonical variables, the effective action has the form

\[
S_{\text{eff}} = (2g)^{-1} \sum_{p,j} c_j^+(p)c_j(p)(1+2\delta_{jl}) + \frac{1}{2} \ln \det \frac{\hat{M}(c_{j}^+c_{j})}{\hat{M}^{c_{j}^+c_{j}}}, \tag{2}
\]

where

\[
\begin{align*}
M_{11} &= Z^{-1} \left[ i\omega + \xi - \mu(H\sigma) \right] \delta_{p_1p_2}, \quad M_{22} = Z^{-1} \left[ - i\omega + \xi + \mu(H\sigma) \right] \delta_{p_1p_2}, \\
M_{12} &= M_{21}^* = (\beta V)^{-1/2} \left[ \frac{15}{32\pi} \right]^{1/2} \left[ c_i \left( 1 - 3 \cos^2 \theta \right) \right. \\
&\quad + c_2 \sin^2 \theta \cos^2 \varphi + c_3 \sin^2 \theta \sin 2\varphi + c_4 \sin 2\cos \varphi + c_5 \sin 20 \sin \varphi. \tag{3}
\end{align*}
\]

Here \( p = (k, \omega) \); \( \omega = (2n+1)\pi T \) are Fermi-frequencies and \( x = (x, \tau) \), \( \xi \) is the kinetic energy with respect to Fermi-level, \( \mu \) - magnetic moment of quasifermion, \( H \)-magnetic field, \( \sigma = (\sigma_x, \sigma_y, \sigma_z) \) – Pauli-matrices. This functional determines all the properties of the model SC Fermi-system with \( d \)-pairing. We have generalized Brusov et al.\(^3\) consideration for the case of arbitrary admixture of \( d \)-state.

We consider the mixed \( d_{x^2-y^2} + i xd_{xy} \) state in high temperature superconductors (HTSC) and derive for the first time a full set of equations for collective modes spectrum in mixed \( d \)-wave state with arbitrary admixture of \( d \)-state.

The order parameter in \( d_{x^2-y^2} + i xd_{xy} \) state takes the following form

\[
\Delta_0(T) \left[ \left( \delta_{i1} \delta_{a1} - \delta_{i2} \delta_{a2} \right) + i\epsilon \left( \delta_{i1} \delta_{a2} - \delta_{i2} \delta_{a1} \right) \right], \tag{4}
\]

or in canonical variables \( \Delta_0(T)(0; \sin^2 \theta \cos 2\varphi; i\epsilon \sin^2 \theta \sin 2\varphi; 0; 0) \).

The gap equation has the following form

\[
g^{-1} + \frac{\alpha Z^2}{2\beta V} \sum_p \frac{\sin^4 \Theta[\cos^2 2\phi + \epsilon^2 \sin^2 2\phi]}{\omega^2 + \xi^2 + \Delta_0^2 \sin^4 \Theta[\cos^2 2\phi + \epsilon^2 \sin^2 2\phi]} = 0, \tag{5}
\]

where

\[
\Delta_0 = 2cZ\alpha, \quad \alpha = (15/32\pi)^{1/2} \text{and gap } \Delta^2(T) = \Delta_0^2 \sin^4 \Theta[\cos^2 2\phi + \epsilon^2 \sin^2 2\phi]. \tag{6}
\]

For limited case \( \epsilon = 0 \) one gets \( d_{x^2-y^2} \) state with order parameter \( (0; \sin^2 \theta \cos 2\varphi; 0; 0) \).

The gap equation in this case has the following form
The coefficients of the quadratic form are proportional to the sums of the products of Green's functions of
\[ (21) \]

and gap \( \Delta'(T) = \Delta_0 \sin^2 \Theta \cos^2 2\phi \).

For the case \( \epsilon=1 \) one gets Brusov et al.'s case of equal admixtures of \( d_{z^2,r^2} \) and \( d_{xy} \) states.

3. Equations for spectrum of collective modes in \( d_{z^2-r^2} + i\epsilon d_{xy} \) state with arbitrary admixture of \( d_{xy} \) state

The spectrum of collective excitations in the first approximations is determined by the quadratic part of
\( S_{rf} \), obtained after shift \( c_j \to c_j + c_0 \) where \( c_j \) are the condensate values of \( c_j \), which take the following form,
\[ c_j(\rho) = (\beta \rho)^{1/2} c_\delta \rho \delta_j \] and \( b_2 = 2, \quad b_3 = 2i\epsilon \) with all remaining components of \( b_0 \) equal to zero.

Excluding terms involving \( g^{-1} \) by gap equation, one obtains the following form for the quadratic part of \( S_{r} \)
\[ S_{H} = \frac{\alpha^2 Z^2}{8\beta V} \sum_p \frac{[c_0^0 Y^*][c_0^0 Y]}{\omega^2 + \xi^2} \sum_j \left[ (1 + 2\delta l) c_j(p) c_j(p) + Z^2/4 \beta V \sum_{\rho_1,\rho_2} \frac{1}{M_1 M_2} \right] \]
\[ \int [c(p)Y^*(p_1)]^2 + \int [c(p)Y^*(p_2)]^2 - \Delta_1^2 [c(p)Y^*(p_2)]^2 \]

(21)

Here \( [Y^*] = c_1 \left( 1 - 3\cos^2 \theta \right) + c_2 \sin^2 \cos 2\varphi + c_3 \sin^2 \sin 2\varphi + c_4 \sin 2\varphi + c_5 \sin 2\varphi + c_6 \sin 2\varphi \).

The coefficients of the quadratic form are proportional to the sums of the products of Green's functions of quasifermions. At low temperatures \( T < T \) we can go from a summation to an integration and evaluate these integrals by using the Feynman equality. It is easy to evaluate the integrals with respect to variables \( \omega \) and \( \xi \) and then with respect to parameter \( \alpha \) and the angular variables.

After calculating all integrals except over the angular variables and equating the determinant of the resulting quadratic form to zero one gets the following set of ten equations, which determine the whole spectrum of the collective modes for \( d_{z^2-r^2} + i\epsilon d_{xy} \) state at arbitrary \( \epsilon \):

\begin{align*}
\int_0^1 dx \int d\phi \left\{ \frac{\omega^2 + 4(1-x^2)^2[\cos^2 2\phi + \epsilon^2 \sin^2 2\phi]}{\omega^2 + 4(1-x^2)^2[\cos^2 2\phi + \epsilon^2 \sin^2 2\phi]} \right\} & = 0 \\
\int_0^1 dx \int d\phi \left\{ \frac{\omega^2 + 4(1-x^2)^2[\cos^2 2\phi + \epsilon^2 \sin^2 2\phi]}{\omega^2 + 4(1-x^2)^2[\cos^2 2\phi + \epsilon^2 \sin^2 2\phi]} \right\} & = 0 \\
\int_0^1 dx \int d\phi \left\{ \frac{\omega^2 + 4(1-x^2)^2[\cos^2 2\phi + \epsilon^2 \sin^2 2\phi]}{\omega^2 + 4(1-x^2)^2[\cos^2 2\phi + \epsilon^2 \sin^2 2\phi]} \right\} & = 0 \\
\int_0^1 dx \int d\phi \left\{ \frac{\omega^2 + 4(1-x^2)^2[\cos^2 2\phi + \epsilon^2 \sin^2 2\phi]}{\omega^2 + 4(1-x^2)^2[\cos^2 2\phi + \epsilon^2 \sin^2 2\phi]} \right\} & = 0 \\
\int_0^1 dx \int d\phi \left\{ \frac{\omega^2 + 4(1-x^2)^2[\cos^2 2\phi + \epsilon^2 \sin^2 2\phi]}{\omega^2 + 4(1-x^2)^2[\cos^2 2\phi + \epsilon^2 \sin^2 2\phi]} \right\} & = 0 \\
\int_0^1 dx \int d\phi \left\{ \frac{\omega^2 + 4(1-x^2)^2[\cos^2 2\phi + \epsilon^2 \sin^2 2\phi]}{\omega^2 + 4(1-x^2)^2[\cos^2 2\phi + \epsilon^2 \sin^2 2\phi]} \right\} & = 0 \\
\int_0^1 dx \int d\phi \left\{ \frac{\omega^2 + 4(1-x^2)^2[\cos^2 2\phi + \epsilon^2 \sin^2 2\phi]}{\omega^2 + 4(1-x^2)^2[\cos^2 2\phi + \epsilon^2 \sin^2 2\phi]} \right\} & = 0 \\
\int_0^1 dx \int d\phi \left\{ \frac{\omega^2 + 4(1-x^2)^2[\cos^2 2\phi + \epsilon^2 \sin^2 2\phi]}{\omega^2 + 4(1-x^2)^2[\cos^2 2\phi + \epsilon^2 \sin^2 2\phi]} \right\} & = 0 \\
\int_0^1 dx \int d\phi \left\{ \frac{\omega^2 + 4(1-x^2)^2[\cos^2 2\phi + \epsilon^2 \sin^2 2\phi]}{\omega^2 + 4(1-x^2)^2[\cos^2 2\phi + \epsilon^2 \sin^2 2\phi]} \right\} & = 0 \\
\int_0^1 dx \int d\phi \left\{ \frac{\omega^2 + 4(1-x^2)^2[\cos^2 2\phi + \epsilon^2 \sin^2 2\phi]}{\omega^2 + 4(1-x^2)^2[\cos^2 2\phi + \epsilon^2 \sin^2 2\phi]} \right\} & = 0 \end{align*}
These equations determine the whole spectrum of collective modes in mixed $d^{x^2-y^2}$ state of high temperature superconductors (HTSC) with arbitrary admixture of $d_{xy}$ state. Knowledge of the collective mode spectrum could be used for interpretation of the sound attenuation and microwave absorption data as well as for identification of the type of pairing and order parameter in unconventional superconductors.

In particular, they allow to estimate the extent of admixture of a small $\varepsilon$ and obtain the corrections to the equations for spectrum of pure $d^{x^2-y^2}$ state, which have been found before. New developments see in.

The following substitutions have been used: $\cos\theta = x$, $\omega = \omega/\Delta_d$.

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