Cosmological issues are examined when gravitino is the lightest superparticle (LSP) and R-parity is broken. Decays of the next lightest superparticles occur rapidly via R-parity violating interaction, and thus they do not upset the big-bang nucleosynthesis, unlike the R-parity conserving case. The gravitino LSP becomes unstable, but its lifetime is typically much longer than the age of the Universe. It turns out that observations of diffuse photon background coming from radiative decays of the gravitino do not severely constrain the gravitino abundance, and thus the gravitino weighing less than around 1 GeV can be dark matter of the Universe when bilinear R-parity violation generates a neutrino mass which accounts for the atmospheric neutrino anomaly.
When one considers supersymmetric extension of the Standard Model [1], one often assumes R-parity conservation. The R-parity [2] is a $Z_2$ parity which distinguishes superparticles from ordinary particles. It is motivated to prevent too fast proton decay mediated by dimension four operators [3], but it additionally results in a very interesting consequence in cosmology. Namely, thanks to the R-parity conservation, the lightest superparticle (LSP) which carries odd R-parity is stable and thus can be considered a candidate for dark matter of the Universe [4]. In fact following the standard thermal history of the Universe, the lightest neutralino (a combination of neutral gauginos and higgsinos) which is often assumed to be the LSP tends to have abundance comparable to the total mass density of the Universe for reasonable choices of supersymmetry breaking parameters [5].

The R-parity conservation is, however, not the only way to forbid the dangerous proton decays caused by the dimension four operators. One can consider R-parity violation [6] which violates, for example, only the lepton number as well. In fact one of the motivations for the R-parity violation is neutrino masses and mixing which are strongly indicated by the atmospheric neutrino anomaly [7] and the solar neutrino problem [8]. When the R-parity breaks via lepton number non-conserving interaction, a novel mechanism [6] of generating the neutrino masses and mixing takes place, which has been extensively studied in the literature (for recent works, see [9,10]). Note that R-parity itself cannot forbid dimension five proton decay operators which might be dangerous when compared with severe experimental bounds. There may be an alternative symmetry to the R-parity, which manages to sufficiently suppress the nucleon decay [11].

When the R-parity is broken, the LSP is no longer stable, but decays to ordinary particles. When the LSP is the superpartner of a standard model particle such as a neutralino or a slepton, the decay of the LSP takes place very rapidly, whose typical lifetime is much shorter than 1 sec unless the R-parity violation is extremely small. Then the LSP cannot be the dark matter of the Universe. In this case the gravitino decay occurs through usual supercurrent interaction, and thus its lifetime is rather long so that it may decay during or after the big-bang nucleosynthesis, leaving the conventional gravitino problem unchanged [12].
In this paper, we would like to consider a different case where the gravitino is the LSP under the assumption of the R-parity violation. The gravitino LSP can be realized in various scenarios of supersymmetry breaking mediation, including gauge mediation \[13\] and scenarios of low fundamental energy scale \[14\]. Even within the conventional gravity mediation, the gravitino can be the lightest among the superparticles for certain choices of supersymmetry breaking parameters.

A characteristic of this case is that the lifetime of the gravitino is very long, typically much longer than the age of the Universe, as we will see later. Therefore the gravitino LSP may be considered to be a dark matter of the Universe if its decay will not cause any problems. In fact if the decay of the gravitino LSP contains photon, which is often the case, there comes a nontrivial constraint from the diffuse photon background observation. We will show that the constraint is not very severe, allowing the gravitino to be a dark matter candidate.

To be specific, we will mainly consider the case where R-parity is violated in bilinear terms in superpotential as well as in soft supersymmetry breaking terms \[6,10\]. In this case the superpotential contains the following mass terms

\[ W = \mu H_1 H_2 + \sum_{i=1}^{3} \mu_i L_i H_2, \]

where \(H_1, H_2\) are two Higgs doublets, \(L_i (i = 1, 2, 3)\) are \(SU(2)_L\) doublet leptons, and \(\mu\) is a higgsino mass parameter and \(\mu_i\) are R-parity violating higgsino-lepton mixing masses. The soft SUSY breaking terms in the scalar potential are taken to be of the form

\[ V_{\text{soft}} = B H_1 H_2 + B_i \tilde{L}_i H_2 + m_{H_1}^2 H_1 H_1^\dagger + m_{H_2}^2 H_2 H_2^\dagger + m_{H_L}^2 \tilde{L}_i H_1^\dagger + m_{L_i}^2 \tilde{L}_i \tilde{L}_j^\dagger + \cdots, \]

where we have written only bilinear terms explicitly (in a self explanatory notation). Here \(B_i\) and \(m_{H_{L_i}}^2\) break the R-parity. The R-parity violating terms induce non-vanishing vacuum expectation values for sneutrinos, and the resulting neutrino-neutralino mixing yields a non-zero neutrino mass due to a weak-scale seesaw mechanism. Neutrino masses are also generated by loop diagrams as well, and as a consequence one can explain both the atmospheric neutrino anomaly and the solar neutrino problem in this framework. When the
supersymmetry breaking scale is around 100 GeV, the ratio $\sqrt{\mu^2_1 + \mu^2_2 + \mu^2_3}/\mu$ should be $\sim 10^{-4} - 10^{-6}$ to give the neutrino mass which explains the atmospheric neutrino anomaly \[10\].

Before discussing decays of the gravitino LSP, we would like to briefly mention decays of the next lightest superparticle (NLSP) which is now the lightest superpartner among the Standard Model particles. In the R-parity conserving case, the NLSP decays into gravitino with some electromagnetic and/or hadronic activities. The decay width of the NLSP is roughly of the order

$$\Gamma_{\text{NLSP}} \sim \frac{1}{16\pi} \frac{m_{\text{NLSP}}^5}{M_{\text{pl}}^2 m_{3/2}^2}$$

assuming that the decay is a two-body decay. Here $M_{\text{pl}} \approx 2.4 \times 10^{18}$ GeV is the reduced Planck scale, $m_{3/2}$ is the gravitino mass, and $m_{\text{NLSP}}$ is the NLSP mass. If the decay occurs during or after the big-bang nucleosynthesis epoch, which is the case for a relatively heavy gravitino, the abundances of the light elements will be significantly changed. This issue was discussed in Refs. \[15\]–\[17\] and an upper bound on the gravitino mass is obtained, provided that the Universe follows the standard thermal evolution.

Now if we consider the R-parity violation, the situation changes drastically. The NLSP decays into ordinary particles (i.e. R-parity even particles) via the R-parity violating interaction, and its lifetime becomes much shorter than 1 sec. Thus the decay of the NLSP becomes harmless \[16\].

Let us next turn to the decay of the gravitino LSP. The lifetime of the gravitino is long because it experiences the gravitational interaction suppressed by $M_{\text{pl}}$ and also the small R-parity violating coupling is involved for the LSP decay. In fact the lifetime is typically much longer than the age of the Universe. To see this, let us consider the bilinear R-parity violation with the heaviest neutrino mass fixed around 0.07 eV. We assume that the lightest neutralino is bino-dominant. Then the dominant decay mode of the gravitino LSP is into a photon and a neutrino. This decay occurs through the interaction
\[ L_{\text{int}} = -\frac{i}{8M_{\text{pl}}} \bar{\psi}_\mu [\gamma^\nu, \gamma^\rho] \gamma^\mu \lambda F_{\nu\rho}, \]  

(4)

where \( \bar{\psi}_\mu \) is the gravitino field, \( F_{\nu\rho} \) is the field strength for the photon, and \( \lambda \) represents the superpartner of the photon, “photino”, which contains a neutrino component via neutralino-neutrino mixing after the sneutrino develops the vacuum expectation value. Thus we evaluate the lifetime of the gravitino as follows:

\[ \Gamma(\tilde{G} \rightarrow \gamma\nu) = \frac{1}{32\pi} |U_{\gamma\nu}|^2 \frac{m^3_{3/2}}{M_{\text{pl}}^2} \left( 1 - \frac{m^2_\nu}{m^2_{3/2}} \right)^3 \left( 1 + \frac{1}{3} \frac{m^2_\nu}{m^2_{3/2}} \right) \approx \frac{1}{32\pi} |U_{\gamma\nu}|^2 \frac{m^3_{3/2}}{M_{\text{pl}}^2}, \]  

(5)

where \( U_{\gamma\nu} \) represents the neutrino contamination into the “photino”. This is approximately related to the neutrino mass as follows

\[ |U_{\gamma\nu}|^2 \simeq \cos^2 \theta_W \frac{m_\nu}{m_\chi}, \]  

(6)

where \( m_\chi \) is the mass of the bino-dominant lightest neutralino and \( \theta_W \) denotes the Weinberg angle. Using a representative value \( |U_{\gamma\nu}|^2 \simeq 7 \times 10^{-13} \) which corresponds to \( m_\chi \simeq 80 \text{ GeV} \), we find the gravitino lifetime to be

\[ \tau_{3/2} = \Gamma^{-1}(\tilde{G} \rightarrow \gamma\nu) \simeq 8.3 \times 10^{26} \text{ sec} \times \left( \frac{m^3_{3/2}}{1\text{GeV}} \right)^{-3} \left( \frac{|U_{\gamma\nu}|^2}{7 \times 10^{-13}} \right)^{-1}. \]  

(7)

Note that the lifetime becomes even longer as the gravitino mass decreases. Thus we conclude that the gravitino is very long lived, whose lifetime is much longer than the age of the Universe.

The long-lived gravitinos are generated in the early Universe. In the standard big-bang cosmology, they were in thermal equilibrium and then frozen out while they were relativistic. In this case their abundance would be comparable to those of other light Standard Model particles. If the Universe experiences inflationary expansion, then the primordial abundance is completely diluted and the gravitinos are regenerated in the thermal bath after the reheating. The abundance of the gravitinos depends on the reheat temperature after the inflation \[ . \]  

[15]. Recently there has been claimed that a non-thermal production mechanism during inflationary epoch may work to produce more abundant gravitinos \[ IS \], though how it works
will depend very much on inflation models. Thus we conclude that one can always consider a scenario of inflation and subsequent reheating so that the gravitino abundance lies in a right range where they constitute dark matter of the Universe.

Since the decay products contain photons, we have to next consider constraints on the abundance of the photons produced. The photon number flux induced by the gravitino decay has a peak at the maximum photon energy \( E_\gamma = \frac{m_{3/2}}{2} \) and the maximum flux there is estimated as

\[
F_{\gamma,\text{max}} = E_\gamma \frac{dF}{dE_\gamma d\Omega}|_{E_\gamma = \frac{m_{3/2}}{2}} \\
\simeq \frac{n_{3/2,0}}{4\pi \tau_{3/2} H_0} \left( \frac{2E_\gamma}{m_{3/2}} \right)^{3/2} \left| \frac{U_{\chi\nu}}{\Omega_{3/2}} \right|^2 \left( \frac{\Omega_{3/2}}{0.3} \right) \left( \frac{h}{0.7} \right) \left( \frac{17 \times 10^{-13}}{E_{\gamma} \text{GeV}} \right),
\]

where \( n_{3/2,0} = 10.54 \text{(cm}^{-3}) (m_{3/2}/\text{keV}^{-1}) (\Omega_{3/2} h^2) \) is the gravitino number density at present, \( \Omega_{3/2} \) stands for the gravitino mass density normalized by the critical density of the Universe, and \( H_0 = 100 h \text{ km/sec/Mpc} \) is the Hubble constant. We can obtain a constraint on the abundance \( \Omega_{3/2} \) by requiring that the flux obtained in Eq. (8) does not exceed the observed diffuse photon background, which is fitted as

\[
F_{\gamma,\text{obs}}(E_\gamma) \simeq (1.5 \pm 0.3) \times 10^{-6} \text{(cm}^2 \cdot \text{str} \cdot \text{sec})^{-1} (E_\gamma/\text{GeV})
\]

for \( 20 \text{ MeV} < E_\gamma < 10 \text{ GeV} \). We show our result in fig.1. In the \( m_{3/2} - \Omega_{3/2} \) plane, the region excluded by the consideration of the diffuse photon background is shown. We find that only a tiny region in the upper-right side is excluded. Here we conservatively took a 2 \( \sigma \) error and imposed the constraint

\[
F_{\gamma,\text{max}} \leq 2.1 \times 10^{-6} \text{(cm}^2 \cdot \text{str} \cdot \text{sec})^{-1} (E_\gamma/\text{GeV}).
\]

The result implies that, as far as the gravitino mass is less than about 1 GeV, \( \Omega_{3/2} \sim 0.1 - 1 \) is allowed and thus the gravitino can constitute dark matter. Note that actual limits on the gravitino mass depend on the magnitude of the R-parity violating couplings, as we can see from Eq. (8).
Here we would like to briefly discuss another case where trilinear Yukawa couplings are dominant sources of R-parity violation and neutrino masses. In this case the neutrino masses are induced at one-loop level and the gravitino decays to a neutrino and a photon also at the one-loop level. Therefore when we relate the decay width with the neutrino mass as we did in Eqs. (3) and (4), the decay width contains additional loop factor $\alpha/4\pi \sim 10^{-3}$ (with $\alpha$ the fine structure constant) compared to the previous case, and the lifetime will be longer by the inverse of the one loop factor. Since the photon flux coming from the gravitino decay is inversely proportional to the lifetime, the constraint from the photon background becomes even weaker than the bilinear case.

In this paper, we have considered cosmology of the light gravitino scenario when the R-parity is violated. Unlike the R-parity conserving case, the decay of the NLSP becomes harmless because it rapidly decays to ordinary particles through the R-parity violating interaction and its lifetime is much shorter than 1 sec. On the other hand, we showed that the lifetime of the gravitino LSP is very long, typically by several orders of magnitude longer than the age of the Universe. The decay products of the gravitino generically contain photons, and thus the abundance may be constrained by the observations of the diffuse photon background. Our analysis showed, when the bilinear R-parity violation induces the neutrino mass which gives the neutrino oscillation solution of the atmospheric neutrino anomaly, that the long-lived gravitino can constitute the dark matter of the Universe without conflicting the observations of the diffuse photon background. As far as the gravitino is heavier than $\sim 1$ keV, it behaves as a cold dark matter, preferable from the arguments on the structure formation, while for the mass of order 100 eV or less it becomes a warm dark matter which is apparently ruled out [20]. To summarize, we conclude that even when the R-parity is not conserved and thus the LSP is unstable, it can be the dark matter, if it is the gravitino.
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FIG. 1. Excluded region from the consideration of the diffuse photon background in $m_{3/2}$-$\Omega_{3/2}$ plane. The shaded region is excluded. We fix the neutrino-neutralino mixing $|U_{\gamma\nu}|^2 = 7 \times 10^{-13}$ and take 2 $\sigma$ error for the observed flux.