1 Introduction

Current and future experimental studies of exclusive and semi-exclusive processes require accurate calculations of relevant hadronic matrix elements for exclusive processes. I present recent applications of this method to the pion electromagnetic form factor and to the form factors of $\gamma^*\rho \to \pi$ and $\gamma^*\gamma \to \pi^0$ transitions.

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1 Pion Form Factors from QCD Light-Cone Sum Rules

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Light-cone sum rules have proved to be very useful in calculating hadronic matrix elements for exclusive processes. I present recent applications of this method to the pion electromagnetic form factor and to the form factors of $\gamma^*\rho \to \pi$ and $\gamma^*\gamma \to \pi^0$ transitions.

\[ F_\pi(Q^2) = \frac{8\pi\alpha_s f_\pi^2}{9Q^2} \int_0^1 du \frac{\varphi_\pi(u)}{\bar{u}}^2, \]

\((\bar{u} \equiv 1 - u)\) obtained by the convolution of twist-2 distribution amplitudes $\varphi_\pi(u)$ of the initial and final pion with the $O(\alpha_s)$ quark hard-scattering kernel. The major unsolved problem is to estimate the so-called soft, or end-point contributions to this form factor. These contributions are expected to be important at intermediate momentum transfers $Q^2 \sim 1 \div 10 \text{ GeV}^2$. Obviously,
a consistent solution of this problem requires a calculation of both soft and hard contributions in one and the same framework beyond perturbation theory.

A promising and largely universal method of calculating hadronic matrix elements is provided by QCD light-cone sum rules (LCSR). This approach combines the light-cone operator-product expansion (OPE) with the conventional SVZ sum rule technique. Proceeding from the basis of QCD perturbation theory, LCSR incorporate elements of nonperturbative long-distance dynamics parametrized in terms of hadronic distribution amplitudes with different twist and multiplicity.

In this talk, I describe recent applications of LCSR to various pion form factors, including the pion electromagnetic form factor and the form factors of $\gamma^*\rho(\omega) \to \pi$ and $\gamma^*\gamma \to \pi^0$ transitions.

2 Pion electromagnetic form factor

I begin with explaining the general idea of the LCSR approach using an important example of the pion electromagnetic form factor. More detailed derivation can be found in [8]. The starting object is the vacuum-pion correlation function

$$ T_{\mu\nu}(p,q) = i \int d^4x e^{iqx} \langle 0| T\{j_{em}^\mu(0)j_{em}^\nu(x)\}|\pi^+(p)\rangle, \tag{2} $$

where $j_{em}^\mu = e_u \bar{u}\gamma_\mu u + e_d \bar{d}\gamma_\mu d$ is the quark electromagnetic current. One of the pions is put on-shell and the second one is replaced by the generating current $j_5^\mu = \bar{d}\gamma_\mu \gamma_5 u$. For the on-shell pions, $p^2 = m_\pi^2$ vanishes in the chiral limit adopted here.

At fixed large $Q^2 = -q^2 >> \Lambda^2_{QCD}$ the correlation function is a function of a single invariant $(p - q)^2$. Depending on the value of this variable the amplitude $T_{\mu\nu}$ corresponds to different physical pictures. At large spacelike $(p - q)^2$, $(p - q)^2 >> \Lambda^2_{QCD}$, the interval $x^2 \to 0$ and one is able to employ the light-cone operator expansion for the product of two currents in (2). The amplitude can then be factorized in a convolution of the pion distribution amplitude and the hard scattering amplitude yielding

$$ T_{\mu\nu} = 2if_{\pi\rho}\frac{1}{16Q^2} \int_0^1 du \frac{u\varphi_\pi(u)}{uQ^2 - u(p - q)^2} + \ldots. \tag{3} $$

In the above, only the leading order, twist 2 contribution is shown, the ellipses denoting $O(\alpha_s)$ corrections and higher twist terms.

Decreasing $|(p - q)^2|$, one gradually approaches the physical region in the channel of the pion current. The interval $x^2$ deviates from zero, the light-cone
OPE diverges and factorization is lost. Finally, at \((p-q)^2 = m^2\) one deals with the emission and propagation of an on-shell pion which is then elastically scattered by the electromagnetic current. In this region, the amplitude \(T_{\mu\nu}\) is dominated by the one-pion contribution to its hadronic representation (dispersion relation)

\[
T_{\mu\nu}(p, q) = 2if_{\pi}(p-q)_{\mu}p_{\nu}F_\pi(Q^2)\frac{1}{m^2 - (p-q)^2} + \int ds \frac{\rho_{\mu\nu}(s)}{s - (p-q)^2}. \tag{4}
\]

The one-pion term is proportional to the pion decay constant \(f_\pi\) and to the desired form factor \(F_\pi(Q^2)\). The integral over \(\rho_{\mu\nu}\) in (4) contains contributions of higher mass intermediate states with the pion quantum numbers.

Equating (3) and (4) one extracts the form factor \(F_\pi(Q^2)\) applying the standard elements of the QCD sum rule technique. Specifically, the quark-hadron duality is used: \(\rho_{\mu\nu}(s) = 1/\pi\text{Im}T_{\mu\nu}(s)\Theta(s - s^2_0)\), where \(\text{Im}T_{\mu\nu}(s)\) is calculated from (4) and \(s^2_0 \approx 0.7\ \text{GeV}^2\) is the effective threshold for the pion channel determined from the two-point QCD sum rule. Furthermore, the Borel transformation in the variable \((p-q)^2\) is performed. The resulting LCSR, in the zeroth order in \(\alpha_s\) and in the twist 2 approximation, reads

\[
F_\pi(Q^2) = \int_{u_0}^{1} du \varphi_\pi(u, \mu_u) \exp\left(-\frac{uQ^2}{\mu^2_u}\right) \lim_{Q^2 \to \infty} \frac{\varphi'_\pi(0)}{Q^2} \int_{0}^{\infty} ds \frac{s e^{-s/M^2}}{Q^4}, \tag{5}
\]

where \(\varphi'_\pi(0) = -\varphi'_\pi(1)\), \(M^2\) is the Borel parameter, \(u_0 = Q^2/(s^2_0 + Q^2)\). The factorization scale \(\mu^2_u = \bar{u}Q^2 + uM^2\) corresponds to the quark virtuality in the correlation function. This sum rule perfectly behaves at \(Q^2 \to \infty\), in contrast to the conventional QCD sum rule for \(F_\pi(Q^2)\) based on the local OPE. The \(1/Q^4\) behaviour of (5) corresponds to the soft end-point mechanism, provided that in \(Q^2 \to \infty\) limit the integration region in (5) shrinks to a point \(u = 1\).

Perturbative \(O(\alpha_s)\) corrections to the correlation function (5) considerably improve the accuracy of the soft contribution. More importantly, in \(O(\alpha_s)\) one recovers the \(\sim 1/Q^2\) asymptotic behaviour corresponding to the hard perturbative mechanism. The \(O(\alpha_s)\) twist 2 term was calculated in \cite{8}. Including this contribution in the LCSR and retaining the first two terms of the sum rule expansion in powers of \(1/Q^2\) one obtains:

\[
F_\pi(Q^2) = \frac{\alpha_s}{2\pi} C_F \int_{0}^{s_0} ds \frac{e^{-s/M^2}}{Q^2} \int_{0}^{1} du \frac{\varphi_\pi(u)}{\bar{u}} + \varphi'_\pi(0) \int_{0}^{s_0} ds \frac{s e^{-s/M^2}}{Q^4}
\]
Figure 1: The light-cone sum rule predictions for the pion electromagnetic form factor using asymptotic distribution amplitude (dashed), CZ distribution (dotted) and fit to the data (solid).

\[ + \frac{\alpha_s}{4\pi} F_0 \int_0^{s_0} \frac{ds \, s \, e^{-s/M^2}}{Q^4} \left\{ \varphi'_\pi(0) \left[ -9 + \frac{1}{3} \pi^2 + \ln s \mu^2 - \ln^2 \frac{s}{Q^2} \right] 
\right. \\
+ (2 \ln \frac{s}{\mu^2} - 3) \int_0^1 du \left[ \frac{\varphi_\pi(u) - \bar{u} \varphi'(0)}{u^2} \right] + (2 \ln \frac{s}{\mu^2} - 8) \int_0^1 du \frac{\varphi_\pi(u)}{\bar{u}} \right\} \]. (6)

It is remarkable that the leading asymptotic $O(1/Q^2)$ term in (6) coincides with provided that the two-point sum rule for $f_\pi$ yields $\int_0^{s_0} ds \, e^{-s/M^2} = 4\pi^2 f_\pi^2$, and $\int_0^1 du \varphi_\pi(u) / \bar{u} = 3$ for the asymptotic $\varphi_\pi(u) = 6u\bar{u}$. Furthermore, it is instructive to split the $0 < u < 1$ integration region in LCSR into “hard” ($u < u_0$) and “soft” ($u > u_0$) parts. This separation reveals that the $O(\alpha_s/Q^4)$ hard contribution is large and negative while the soft $O(1/Q^4)$ and $O(\alpha_s/Q^4)$ contributions are positive yielding considerable cancellations in the sum rule.

To further improve the accuracy of the LCSR, one also has to include the higher twist corrections. Physically, these corrections take into account both the transverse momentum of the quark-antiquark state and the contributions of higher Fock states in the pion wave function. In addition to the twist 4 terms, in the factorizable twist 6 contributions proportional to the square of the quark condensate have been calculated. The latter corrections turn out to be comfortably small. Adding twist 4.6 terms to the twist 2 (leading and $O(\alpha_s)$) parts yields the LCSR prediction for $F_\pi(Q^2)$ shown in Fig. 1 for the pion distribution amplitude $\varphi_\pi(u, \mu) = 6u\bar{u} \left[ 1 + a_2(\mu) C_2^{3/2}(u - \bar{u}) \right]$ with
two choices: \( a_2 = 0 \) (asymptotic) and \( a_2(1 \text{ GeV}) = 2/3 \) (CZ). The available experimental data seem to rule out the CZ option. The fit of the LCSR to these data yields \( a_2(1 \text{ GeV}) = 0.12 \pm 0.07^{+0.05}_{-0.07} \) where the first (second) uncertainty is experimental (theoretical).

Another way to employ the LCSR prediction is to subtract the leading asymptotic \( 1/Q^2 \) term (the first term in (6)) from the sum rule. The remaining power suppressed part can be called “nonperturbative” contribution \( F_{\text{nonp}}(Q^2) \). One then obtains a quantitative prediction for the full pion form factor by adding \( F_{\text{nonp}}(Q^2) \) to the perturbative QCD result which is currently known with the NLO accuracy. The LCSR prediction for \( F_{\text{nonp}}(Q^2) \) is plotted in Fig. 2. It turns out to be numerically moderate due to abovementioned cancellations between soft and hard \( O(1/Q^4) \) contributions in the twist 2 part.

### 3 \( \gamma^* \rho \to \pi \) and \( \gamma^* \gamma \to \pi \) transition form factors

Similar to the pion electromagnetic form factor, the \( \gamma^* \rho \to \pi \) or \( \gamma^* \omega \to \pi \) transition form factors can be measured by extracting the one-pion exchange in the electroproduction of \( \rho \) or \( \omega \). The form factors are defined as \( \frac{1}{p^0(p)} \langle \pi^0(p) \mid j_{\mu}^{em} \mid \omega(p - q) \rangle \simeq \langle \pi^0(p) \mid j_{\mu}^{em} \mid \rho^0(p - q) \rangle = \frac{F^{\rho \pi}(Q^2)m_{\rho}^{-1}e_{\mu}e_{\nu}e_{\alpha}e_{\beta}p^\beta}{e_{\nu}} \), \( e_{\nu} \) being the polarization vector of \( \rho \) or \( \omega \). The derivation of LCSR for \( F^{\rho \pi}(Q^2) \) basically follows the procedure described in the previous section. The
underlying correlation function is

$$\int d^4x e^{-iqx} \langle \pi^0(p) | T \{j^c_\mu(x) j^c_\nu(0) \} | 0 \rangle = i\epsilon_{\mu\nu\alpha\beta} q^\alpha p^\beta F_{\gamma^*\pi}(Q^2, (p-q)^2). \quad (7)$$

If both $Q^2$ and $|(p-q)^2|$ are sufficiently large, the light-cone OPE is valid for the amplitude $F_{\gamma^*\pi}(Q^2, (p-q)^2)$ starting from the leading twist 2 term:

$$F_{\gamma^*\pi}(Q^2, (p-q)^2) = \sqrt{2} f_\pi \int_0^1 \frac{du \varphi_\pi(u)}{uQ^2 - u(p-q)^2} + \ldots \quad (8)$$

Physical states in the $(p-q)^2$-channel include vector mesons $\rho, \omega, \rho', \omega', \ldots$ and a continuum of hadronic states with the same quantum numbers. The hadronic dispersion relation can be written as

$$F_{\gamma^*\pi}(Q^2, (p-q)^2) = \frac{\sqrt{2} f_\pi}{m_\rho^2 - (p-q)^2} + \frac{1}{\pi} \int_{s_0^\rho}^\infty ds \frac{\text{Im} F_{\gamma^*\pi}(Q^2, s)}{s - (p-q)^2}. \quad (9)$$

Here, the $\rho$ and $\omega$ contributions are combined in one ground-state resonance term using $m_\rho \simeq m_\omega$ and $3\langle \rho | j^c_\mu | 0 \rangle \simeq \langle \rho' | j^c_\mu | 0 \rangle = (f_\rho/\sqrt{2}) m_\rho e_\rho^*$. For the higher states above the effective threshold $s_0^\rho$, the quark-hadron duality is used with the spectral density $1/\pi \text{Im} F_{\gamma^*\pi}(Q^2, s)$ calculated from (8). The resulting LCSR

$$F_{\rho\pi}(Q^2) = \frac{f_\pi}{3f_\rho} \int_0^1 \frac{du \varphi_\pi(u)}{u} \exp \left( -\frac{Q^2(1-u)}{uM^2} + \frac{m_\rho^2}{M^2} \right) + O(\text{twist 4}) \quad (10)$$

was obtained in [10] where one can find additional details and numerical predictions. Note that the form factor (10) corresponds to the soft mechanism with $1/Q^4$ behaviour at large $Q^2$. Due to helicity suppression, the perturbative $O(\alpha_s)$ correction to $F_{\rho\pi}(Q^2)$ should have the same $1/Q^4$ behaviour. This and twist 6 corrections still have to be included in (10).

Finally, let me focus on the $\gamma^*\gamma \rightarrow \pi$ transition. Since the real photon is a large-distance object, the transition form factor $F_{\gamma^*\pi}(Q^2)$ contains nonperturbative contributions beyond the light-cone expansion of two electromagnetic quark currents. The leading $1/Q^2$ asymptotics of $F_{\gamma^*\pi}(Q^2)$ is well known [3] and given by (8) at $(p-q)^2 \to 0$. The calculation of the contributions suppressed by powers of $1/Q^2$ is a nontrivial task. Within LCSR approach, one has to invoke the photon distribution amplitudes. A reliable estimate is nevertheless
Figure 3: Form factor of the $\gamma^*\gamma \rightarrow \pi^0$ transition calculated with the asymptotic (solid), CZ (long dashed) and BF (short-dashed) wave function of the pion in comparison with the experimental data points and with the interpolation formula from (dash-dotted).

possible using the hadronic dispersion relation (where the resonance term is determined by the LCSR and the integral over higher states is estimated using duality. One can analytically continue this relation to $(p-q)^2 \rightarrow 0$ provided that it does not contain subtraction terms (a similar approach was used in for the structure function of the real photon). The result shown in Fig. 3 again has a better agreement with the experimental data in the case of the asymptotic pion distribution amplitude. The recent update of $F_{\gamma\pi}(Q^2)$ includes the $O(\alpha_s)$ correction decreasing the leading order results in Fig. 3 by 15-20%.

4 Conclusions

Light-cone sum rules provide a powerful tool for calculating various form factors at intermediate momentum transfers. In this framework, both soft end-point and hard perturbative mechanisms can be incorporated. The LCSR results for the pion electromagnetic form factor indicate that the soft contribution is indeed large. However, an essential part of it is cancelled by the $O(\alpha_s/Q^4)$ corrections originating from the hard region. As a result, the overall power suppressed correction to the perturbative QCD factorization turns out to be relatively moderate. Comparison of LCSR predictions for the pion form factors with the available data indicates an almost asymptotic pion distribution amplitude $\varphi_\pi(u)$. More precise measurements of the form factors are needed to tightly constrain the nonasymptotic parts of $\varphi_\pi(u)$.

The LCSR approach is extremely useful also for heavy flavour exclusive decays, in particular, for the form factors of $B \rightarrow \pi, K, \rho, K^*$ transitions. The sum rules for heavy-to-light and light-to-light transition form factors share
a common nonperturbative input, that is the set of light hadron distribution amplitudes. Moreover, the structure of perturbative corrections to the pion form factor turns out to be very similar to the heavy quark limit of the LCSR for $B \to \pi$ in $O(\alpha_s)$. One may conclude that within a universal approach of QCD light-cone sum rules, both fields, the hard exclusive processes and the heavy flavour decays can mutually benefit.

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