Dark matter scattering cross section in Yang-Mills theory

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We calculate for the first time the scattering cross section between lightest glueballs in SU(2) pure Yang-Mills theory, which are good candidates of dark matter. In the first step, we evaluate the interglueball potential on lattice using the time-dependent formalism of the HAL QCD method, with one lattice spacing. The statistical accuracy is improved by employing the cluster-decomposition error reduction technique and by using all space-time symmetries. We then derive the scattering phase shift and the scattering cross section at low energy, which is compared with the observational constraint on the dark matter self-scattering. We determine the lower bound on the scale parameter of the SU(2) Yang-Mills theory, on the scale parameter of the SU(2) Yang-Mills theory, as \( \Lambda > 60 \text{ MeV} \).

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The existence of a significant amount of dark matter (DM) in our Universe is supported by many physical data. Its presence was first suggested by the observations of the galactic rotation curve \[ \right \] and is now made firm by that of the density profile of the bullet clusters \[ \right \]. The quantity of DM is nowadays very precisely known thanks to the progress in the observation of the cosmic microwave background, with 27% of the energy composition of our Universe. Another important feature is the necessity of the DM in the formation of the large scale structure of the Universe \[ \right \]. Owing to the above strong arguments, the study of the DM is one of the most essential subjects of fundamental physics.

Thanks to the progress in the observation of the gravitational microlensing, the DM as astrophysical compact objects are excluded in a very wide region of their mass \[ \right \]. This suggests that the DM is mainly composed of particles which interact very weakly with the visible sector \[ \right \]. It is known that the standard model does not contain any fields which fulfill the properties of DM, and many candidates of new theories are under investigation \[ \right \]. Among the new physics, the most popular ones are extensions of the standard model with new sector(s), often protected by discrete symmetries. However, these theories often have problems with the results of direct \[ \right \] and indirect \[ \right \] detection experiments as well as with the search using colliders \[ \right \]. Due to the relatively large couplings with the SM required to keep consistency with the relic density (the so-called “WIMP miracle”) \[ \right \].

As opposed to the above “elementary” DM particles, we have the composite DM \[ \right \] which has several classes. Here we investigate the less discussed, but the most natural Yang-Mills theory (YMT), in which the lightest glueballs are candidate of DM \[ \right \]. The attractive features of this scenario are as follows. First, the mass scale is only controlled by the color number \( N_c \) through the dimensional transmutation, and the theory is thus almost free from the hierarchical problem. Second, it is also part of another very natural scenario where vectorlike quarks are present in the Lagrangian, but with masses heavier than the confining scale. In such a theory, the lightest particle of the spectrum is a glueball, which forms the massive DM. Finally, it may naturally be embedded in a more ultraviolet complete frameworks such as the grand unification or string theory \[ \right \].

The glueball is a gluonic bound state \[ \right \] which purely reflects the nonperturbative physics of nonabelian gauge theory, and it cannot be investigated perturbatively. Indeed, it has been an excellent target for nonperturbative methodologies such as lattice gauge theory \[ \right \] or holographic approaches \[ \right \] and its mass spectrum has extensively been evaluated. Despite these tremendous works, the properties of the glueballs are still obscure. In QCD, the observation of glueballs is not yet conclusive, although there are several candidates such as \( f_0(1500) \) and \( f_0(1710) \), and they are currently actively searched experimentally \[ \right \]. The determination of the glueball is essentially difficult due to the mixing with other hadronic states \[ \right \], although the production and decay patterns have extensively been studied \[ \right \]. The issue with the mixing is, of course, absent in the case of the dark YMT, since the glueball is the ground state hadron and there is no mixing with other hadrons at the scale considered.
In considering the particle DM scenario, an inevitable discussion is the self-interaction (or scattering) among DM. This feature is especially important since the DM self-interaction is given an upper bound from observations such as the density profile of the galactic halo or the collision between galaxies. As for the scale smaller than kpc, several phenomena, which were thought to be “problems” in relation to the density profile, are known to be explained with finite DM self-interaction, although these topics are still a matter of controversy. The two-body potential working between $SU(N_c)$ glueballs should have a finite range due to the mass gap of the YMT, so it is in principle possible to set constraints on the scale parameter from observations, but the relation between the latter and the scattering cross section is almost totally unknown, although it was challenged in model calculations recently almost the only way to quantify observables related to glueballs, could so far only calculate their masses, and the scattering among the lightest glueballs has to be elucidated. It is also theoretically interesting to discuss it in the context of the low energy effective field theory, since the lightest 0++ glueball is believed to have become massive due to the trace anomaly, the anomalous violation of the scale invariance.

The lattice gauge theory simulation, which is currently almost the only way to quantify observables related to glueballs, could so far only calculate their masses in a finite volume lattice was first conceived by Lüscher [492]. This method was then successfully implemented in numerical calculations, and results for several systems are currently available [494, 501]. As an alternative approach, we also have the HAL QCD method in which the phase shift is indirectly extracted via the potential. This approach is known to be able to extract the interhadron potential without waiting for the formation of the plateau, which greatly reduces the computational cost. After intense discussions on its formalism, the HAL QCD method is now well established.

In this letter, we discuss the self-interaction of the DM in the YMT by calculating the interglueball scattering cross section in $SU(2)$ lattice gauge theory employing the HAL QCD method. Our goal is to constrain the scale parameter of the $SU(2)$ YMT from observations. We also extrapolate our results to $N_c \geq 3$ using the large $N_c$ argument.

To proceed, we simulate $N_c = 2$ pure YMT on lattice, with $\beta = 2.5$. The scale of the lattice is expressed in units of the scale parameter $\Lambda$ which is unknown, since the mass and other dimensionful quantities related to the DM are not known. The result of our work will actually set a constraint on $\Lambda$. The relation between the lattice spacing and $\Lambda$ was fitted through the calculation of the string tension. For the general $N_c$, we have

$$\frac{\Lambda}{\sqrt{\sigma}} = 0.503(2)(40) + \frac{0.33(3)(3)}{N_c^2}. \quad (1)$$

By using the string tension $a\sqrt{\sigma} = 0.1834(26)$ calculated on $16^4$ lattice [525], the lattice spacing of the $SU(2)$ YMT with $\beta = 2.5$ is $a = 0.107(8)\Lambda^{-1}$. We used the pseudo-heat bath method to generate 2 million configurations with the volume $16^3 \times 24$. In this study, we only use one lattice spacing since the formalism does not depend on the renormalization scale (see later).

We now define the 0++ glueball operator:

$$\phi(t, \vec{x}) = \text{Re}[P_{12}(t, \vec{x}) + P_{12}(t, \vec{x} + a\vec{c}_3) + P_{23}(t, \vec{x}) + P_{23}(t, \vec{x} + a\vec{c}_2)],$$

where $P_{ij}$ are the plaquette operator in $i-j$ direction, and $a\vec{c}_{1,2,3}$ are the unit vector. Since the 0++ glueball has the same quantum number as the vacuum, we have to subtract its expectation value as $\langle \phi(t, \vec{x}) \rangle = \langle \phi(t, \vec{x}) \rangle - \langle \phi(t, \vec{x}) \rangle$ in order to calculate the physical glueball correlators. We may also improve the glueball operator using the APE smearing [330, 341, 342]. With this optimization, we obtained the glueball mass $m_\phi = 0.6857(28)$ (lattice unit) in our setup.

The physical information of the scattering between two hadrons can be extracted from the Nambu-Bethe-Salpeter (NBS) amplitude. For the glueball two-body scattering, it is defined as follows:

$$\Psi_{\phi\phi}(t, \vec{x} - \vec{y}) \equiv \frac{1}{V} \sum_{\vec{r}} \langle 0 | T[\phi(t, \vec{x} + \vec{r})\phi(t, \vec{y} + \vec{r})] \mathcal{J}(0) | 0 \rangle,$$  \quad (3)

where $\mathcal{J}$ is the source operator with the same quantum number as the two-glueball state. Here two important features have to be mentioned. First, in the case of the glueball, $\mathcal{J}$ may be chosen as an $n$-body operator $(\phi^n)$ with arbitrary positive-definite integer $n$, due to the 0++ quantum number. Second, the multi-glueball operators also have expectation values which have to be subtracted. This subtraction has to be operated for both the source and the sink, but it can easily be shown that the removal of one side will automatically do for the other one. For computational convenience, we will remove the expectation value of $\mathcal{J}$. The correlator is purely gluonic, and the statistical error is significant in the lattice calculation. To improve the signal, we use all space-time symmetries (space-time translations and cubic rotations) to effectively increase the statistics. We also employ the cluster-decomposition error reduction technique (CDERT) to remove the fluctuation of the vacuum insertion. In our work, we used the cut $r \leq 8$ (lattice unit) for which the...
Let us compare the efficiency of the signal thanks to the CDERT with the cutoff $r \leq 8$.

Let us now extract the scattering phase shift. The direct way to calculate it is to Fourier transform the NBS amplitude and inspect the momentum modulation of the energy (so-called Lücker’s method) \[503\]. This approach was successful in the mesonic sector, but in the case of the glueball, we encounter a problem, due to the necessity of taking the plateau of the NBS amplitude. Indeed, the NBS amplitude mixes with the single glueball (2-point) correlator so that taking the plateau will always show the single glueball mass as the energy of the system. We might, of course, remove the one-glueball state by diagonalizing the NBS amplitude, but the glueball spectrum has other resonances close to the two-body threshold, so the extraction of the two-glueball scattering in this approach is highly challenging. An alternative approach to calculating the scattering phase shift is to indirectly extract it via the potential (HAL QCD method) \[504\]. This method has the crucial advantage that we do not need the ground state saturation for obtaining the potential \[504\]. In particular, the glueball correlators are in general very noisy, so the use of this method is almost mandatory if one wants to keep good statistical accuracy. In addition, the potential handled in the HAL QCD method does not depend on the renormalization scale \[502\]. We however have to keep in mind that the potential is not an observable, and it may depend on the choice of the operators.

Let us now describe the formalism of the calculation of the interglueball potential on lattice. The nonlocal potential $U(\vec{r}, \vec{r}')$ is extracted according to the following time-dependent Schrödinger-like equation \[509\]

$$ \left[ \frac{1}{4m_{\phi}} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{1}{m_{\phi}} \nabla^2 \right] R(t, \vec{r}) = \int d^3r' U(\vec{r}, \vec{r}') R(t, \vec{r}'), $$

(4) where $R(t, \vec{r}) \equiv \frac{\psi_{\phi}(t, \vec{r})}{e^{-\frac{1}{2m_{\phi}} \phi(t, \vec{r})}}$. Here $t$ should be chosen so that $1/t$ is less than the inelastic threshold $m_{\phi}(=3m_{\phi} - 2m_{\phi})$. In our calculation with $\beta = 2.5$, it is enough to take the data from $t = 2$. The physics addressed is then nonrelativistic, and the potential should be to a good approximation local and central $U(\vec{r}, \vec{r}') \approx V_{\phi\phi}(\vec{r}) \delta(\vec{r} - \vec{r}')$. Nonlocal and angular momentum dependent terms appear in the derivative expansion but are neglected in this work.

The result of our work is plotted in Fig. 2. The potential at $r = 0$ and $r = 1$ (in lattice unit) looks attractive, but these data points have to be considered as contact terms (note that the glueball operator has a spread of one lattice unit). In the fit of the potential, we then remove the above points. The important feature of our result is that the interglueball potential is repulsive at short range ($r \sim 0.2 \Lambda^{-1}$). Here the simplest interglueball process that we would come up with in the effective field theory is the scalar glueball exchange, which however induces an attractive force. Our result is therefore suggesting that other more important processes are relevant. Another remarkable point is that the potential becomes very noisy in the long range. This is because the potential is obtained by dividing by the NBS amplitude which damps the error bar by more than twice, which demonstrates its efficacy.

![FIG. 1. Glueball NBS amplitude with 1-body source ($J = \phi$) measured at $t = 2$. We compare the improvement of the signal thanks to the CDERT with the cutoff $r \leq 8$.](image)

![FIG. 2. Interglueball potential calculated on lattice in the $SU(2)$ YMT. The fits with two fitting forms are also displayed. The grey band shows the uncertainty.](image)
\(\chi^2/\text{d.o.f.} = 0.8\).

Now that we have the analytic form of the potential, we calculate the scattering phase shift and the cross section. The scattering phase shift is obtained by simply solving the following (s-wave) Schrödinger equation:

\[
\frac{\partial^2 \phi}{\partial r^2} + k^2 - m_\phi V(r) \phi(r) = 0. \quad (5)
\]

The wave function asymptotically behaves as \(\phi(r) \propto \sin[|k| r + \delta(k)]\), where \(\delta(k)\) is the scattering phase shift.

In the context of the DM, we are interested in the (s-wave) low energy limit of the cross section \(\sigma_{\phi\phi} = \lim_{k \to 0} \frac{4\pi}{k^2} \sin^2[\delta(k)]\). From the two fitting forms, we obtain \(\sigma_{\phi\phi} = (3.5 - 3.8)\Lambda^{-2}\) (Yukawa), and \(\sigma_{\phi\phi} = (7.5 - 8.0)\Lambda^{-2}\) (Yukawa+Gaussian), with the band denoting the statistical error. By considering the difference between them as the systematic error, the interglueball scattering cross section for the \(SU(2)\) YMT is

\[
\sigma_{\phi\phi} = (3.5 - 8.0)\Lambda^{-2} \quad \text{(stat. + sys.)}. \quad (6)
\]

Let us now try to derive the constraint on \(\Lambda\) from observational data. The most robust bound on the DM cross section is given by the observation of the shape of the galactic halo \(454, 456\) and galactic collisions \(528–532\). Here we adopt that of Ref. \(\text{[532]}\): \(\sigma_{\text{DM}/m_{\text{DM}}} < 0.47 \text{cm}^2/\text{g}\). By substituting our result \(6\), we obtain

\[
\Lambda > 60 \text{ MeV}. \quad (7)
\]

This is the first constraint on the YMT as the theory of DM derived from first principle.

Let us mention future improvements that would be in order to further quantify the results. Our calculation assumed the local potential, but it is not entirely certain that the nonlocality is negligible, since the glueball operator has a finite spread. The quantification of the nonlocal physics has recently been developed by HAL QCD Group, with an involved discussion in the operator dependence \(\text{[521, 532, 533]}\). Through this discussion, we are expecting to unveil the artifact encountered in the potential at lattice unit zero and one (see Fig. \(2\)).

Let us add several comments on these results. If we also consider the discussion of Spergel and Steinhardt which derived a lower limit on the DM cross section \(\sigma_{\text{DM}/m_{\text{DM}}} > 0.45 \text{cm}^2/\text{g}\) \(\text{[440]}\), we can almost determine the scale parameter of the YMT. As seen in the beginning, this bound could be set by inspecting that the so-called core-vs-cusp, too-big-to-fail, and missing satellite problems could be resolved by assuming a finite DM scattering cross section. This argument has however recently been revised \(\text{[467, 471, 473, 478, 481, 483]}\), and there are now alternative possibilities to resolve the above-mentioned problems \(\text{[467, 470, 474, 477, 482, 484]}\). Therefore, we leave the lower limit to the scale parameter open and wait for a conclusive resolution.

Our discussion of the constraint on the scale parameter of the YMT can be extended to larger \(N_c\)’s. The YMT’s with \(N_c \geq 3\) are indeed even more interesting as a candidate of dark sector since the deconfinement transition at finite temperature is of the first order \(\text{[350, 535, 541]}\), and they may be probed with the background gravitational waves \(\text{[542, 555]}\). Although we do not have simulated them, we can qualitatively estimate the limits on their scale parameters according to the large \(N_c\) argument. Since the cross section scales as \(1/N_c^4\), we can derive the lower limits on \(\Lambda\) as

\[
\Lambda_{N_c} > 60 \left(\frac{2}{N_c}\right)^{\frac{3}{4}} \text{MeV}. \quad (8)
\]

Of course, the large \(N_c\) corrections are not small for \(N_c = 2\), so the first principle calculations on lattice definitely have to be done for small \(N_c\). Our future work will be to calculate the interglueball cross section for larger \(N_c\) until the large \(N_c\) corrections be small. The completion of this project means that we can handle the DM in the YMT with all \(N_c\) to good accuracy.

It would also be interesting to compare the interglueball cross section at low energy with that derived from the glueball effective field theory which can be constructed according to the conformal Ward identity \(\text{[265, 266, 273, 278, 279, 489, 491, 492]}\). Determining the glueball EFT and its low energy constants may give us an important insight into other fields such as the conformal field theory or hadron physics. If again the large \(N_c\) expansion holds, the determination of the cross section at a sufficiently large \(N_c\) would probably mean the quantification of the conformal physics.

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