Exciting gauge field and gravitons in a brane-anti-brane annihilation

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In this paper we point out the inevitability of an explosive production of gauge field and gravity wave during an open string tachyon condensation in a cosmological setting, in an effective field theory model. We will be particularly studying a toy model of brane-anti-brane inflation in a warped throat where inflation ends via tachyon condensation. We point out that a tachyonic instability helps fragmenting the homogeneous tachyon and excites gauge field and contributes to the stress energy tensor which also feeds into the gravity waves.

There are many cosmological sources of gravity wave generation, for a review see [1]. In this paper we will show how an interesting scenario emerges in string cosmology where there exists plethora of examples of open string tachyon condensation [2]. We will show how gravity waves and gauge field are excited in a field theory model of brane-anti-brane annihilation particularly in a cosmological setting.

The tachyon plays an important role in terminating brane-anti-brane inflation [3], for a review see [4], when a pair comes close to each other, less than a critical distance. When the branes are far apart the tachyon field does not play any role, while the brane-anti-brane separation plays the role of an inflaton and the brane tension generate the vacuum energy density. The slow attraction of the branes towards each other gives rise to a primordial inflation and also a graceful exit. In this process the so-called tachyon field acts almost like a waterwall field like in the case of a hybrid inflation [3], where the (mass)$^2$ of the waterfall field changes from being positive-to-zero-to-negative. In all these examples the waterfall field and the tachyon in string theory examples are considered to be a homogeneous field evolving with time.

In this paper we discuss couple of important consequences of a brane-anti-brane inflation in the field theory model. Note that field theoretical treatment is an oversimplification of the brane annihilation process, but we will be content with it as a toy model to investigate excitation of gravity wave and gauge field. It would be interesting to see how stringy effects will modify our findings.

First, in a cosmological context the tachyon will never keep the homogeneity, the tachyon will be fragmented by virtue of inhomogeneity created by the quantum fluctuations, quite similar to the tachyonic preheating [8]. Second, the tachyon in string theory couples bi-linearly to the gauge fields residing on the worldvolume of the branes. It can be shown by field redefinitions that in an Einstein frame the gauge fields couple minimally, therefore, the gauge fields are Higgsed by the tachyon, i.e. the gauge fields obtain v.e.v. dependent masses. This is an ideal case when a time dependent v.e.v. can excite gauge fields at sub-and super-Hubble scales.

Third, the fragmentation of the tachyon and its inhomogeneous perturbations seeds anisotropic stress tensor which leads to gravity wave generation. The frequency of these gravity waves depends on the string scale which we take it to be close to $M_s \sim 10^{14}$ GeV (below the scale of grand unified theory). Since the scale is quite high therefore the frequency will not be detectable by future gravity wave experiments.

In particularly, we will be studying inflation in a warped throat, while the SM or the MSSM fields are located in a different throat with a different warping. For simplicity we will consider a simple $D3 - D\bar{3}$ system, where the modulus determining the inter brane separation drives inflation and inflation ends via tachyon condensation [2]. However note that the branes do carry gauge fields in their worldvolume, the tachyon in a brane-anti-brane system is a bi-fundamental field that couples only to a linear combination of the two gauge fields. These gauge fields drop out of the dynamics once the pair has annihilated [5].

When the brane and the anti-brane are coincident the action for the tachyon field responsible for the annihilation

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1 Although end of inflation via brane-anti-brane lends the greatest support for studying the tachyons, but open string tachyons could also give rise to primordial inflation [4], and furthermore tachyons can possibly generate large non-Gaussianity [6].

2 Inflation in such a setup will not directly reheat the MSSM throat [6], reheating happens indirectly via moduli belonging to the MSSM [10], see also [11]. One plausible solution is to inflate the universe within the MSSM throat [12], which would naturally reheat the universe with the MSSM degrees of freedom.

3 However, more realistic models would include a stack of $N$ (anti)branes located at the bottom of the throat and a number of $M$ mobile branes moving towards them. After annihilation the remaining stack of branes supports an unbroken $U(N-M)$ [13, 14, 15]. The original $U(N) \times U(M)$ is higgsed down to $U(N-M)$ when the tachyon develops a v.e.v. at the end of inflation.
of the pair is described by \cite{16,17}:

\[
S = -2T_{D3} \int \sqrt{-g} dx^4 e^{-2\pi \alpha'T} \left[ 1 + 8\pi (\alpha')^2 \ln (2) D^\mu T \partial_\mu T + \frac{\gamma \alpha'^2}{8} (F_\mu^+ - F_\mu^-)^2 \right] \tag{1}
\]

Here \( F^+ \) and \( F^- \) are the gauge fields that live in the worldvolume of the brane and anti-brane respectively. The tachyon is a bi-fundamental field that couples only to a linear combination of the two gauge fields \( D_\mu T = \partial_\mu T - (A_\mu^+ - A^-_\mu) T \). If we expand the exponential term as: \( e^{-2\pi \alpha'T} = 1 - 2\pi \alpha'T \) and write the \( D3 \) brane tension as: \( T_{D3} = 1/(2\pi)^3 (\alpha')^2 g_s \). The numerical pre-factor \( 8\pi \ln (2) \) can be absorbed in a field re-definition and the action with \( \chi = (\sqrt{2\ln 2} r/\pi R) T \) can be brought to the form (the scalar part only) \(^4\):

\[
S = \int d^4x \left( \frac{2\pi^4}{R^4} T_{D3} + \frac{1}{2} D_\mu \chi D^\mu \chi - \frac{\gamma}{2} M_4^2 \left( \frac{\Lambda_4}{R^2} \right)^2 \chi^2 \right) = \int d^4x \left( \Lambda_4 + \frac{1}{2} D_\mu \chi D^\mu \chi - \frac{1}{2} m_T^2 \chi^2 \right) \tag{2}
\]

Denoting \( r/R = w \), the warp factor, we see that the cosmological constant given by the brane tension and the tachyon mass depend only on the warped string scale \( m = wM_s \), and therefore the Hubble constant during inflation and the tachyon mass during reheating are related \( \Lambda_4 = 2w^4 T_{D3} = w^4 2M_s^4/(2\pi)^3 g_s = 2m^4/(2\pi)^3 g_s \).

![FIG. 1: The polynomial approximation we use has the advantage of allowing us to modify the tachyon mass at \( \phi = 0 \) independently of the quartic term. The downside is that the value of the potential at its minimum is negative.](image)

In order to understand the aftermath of inflation, we need to include the gauge fields and we also need to model the scalar potential in the tachyonic phase by a polynomial function of the fields.

\[
\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} D_\mu \chi D^\mu \chi^* + \frac{1}{2} m^2_{\text{inflaton}} \phi^2 + \frac{1}{2} m_T^2 |\chi|^2 \left( \phi^2 - \phi_0^2 \right) + \frac{\lambda}{4} |\chi|^4 - \frac{1}{4} F^2 \tag{3}
\]

\(^4\) To find the warped tachyon mass we write the metric in the warped throat takes the form (up to corrections in a KS type throat): \( ds^2_{10} = \frac{r^2}{R^2} ds^2_4 + \frac{R^2}{r^2} \left( dr^2 + r^2 d\Omega^2 \right) \), see Ref. \cite{18}.

The value of \( \phi_0 \) represents the critical distance at which the tachyon field becomes unstable. We had to add a quartic term for the tachyon potential, so that the overall potential is bounded from below. Otherwise the simulation cannot be performed. We are interested in the dynamics of the tachyon at the top of its potential, \( \chi \approx 0 \), where the quartic term is negligible.

We have verified that the quartic term does not change the results of our simulations as far as the tachyonic phase is concerned. Making the quartic term smaller will result in a potential with deeper minima, and therefore a longer tachyonic phase. However, since the VEV of the field increases exponentially with time, the duration of the tachyonic phase depends only logarithmically in the coefficient of the quartic term. The quartic term has a large influence during the turbulent phase after the tachyonic amplification has ended. Since the minimum is deeper, the field will reach this minimum with higher kinetic energy, and therefore more energy is transferred to the gravity waves in a given time interval.

The Lagrangian in Eq.\(^{11}\) describes thebrane-anti-brane system only when the brane and the anti-brane are coincident. In our approximate Lagrangian, Eq.\(^{13}\), we include the dependence of the tachyon mass on the inflaton v.e.v., that is the brane-anti-brane separation.

To study the gravity waves, we follow the approach of \cite{19} were the perturbations of the Einstein equations are studied. We start by considering the metric fluctuations: \( ds^2 = -dt^2 + a^2(t) (\delta_{ij} + h_{ij}) dx^i dx^j \). The corresponding equation for the fluctuations of the metric reads: \( \Box h_{ij} = 16\pi G \delta T_{ij} \), where \( T_{ij} \) is the anisotropic stress-energy tensor. Just as in \cite{19} we follow the evolution of only the traceless part of the tensor fluctuations: \( h_{ij} - \nabla^2 h_{ij} = 16\pi G \Pi_{ij} \), where \( \Pi_{ij} = T_{ij} - \frac{1}{3} \delta_{ij} T \). We may neglect the expansion of the universe since \( M_s \gg H \) at the time when tachyon is rolling after inflation.

The stress-energy tensor gets contributions from both the charged tachyon and uncharged inflaton, as well as the gauge fields. Details of the calculation of the stress-energy tensor will be presented in a separate paper. Here we simply quote the results:

\[
\Pi_{ij} = F_{iC} F_{jC}^* - \frac{1}{3} \delta_{ij} F_{kC} F^{kC} - D_i \chi D_j \chi^* + \frac{1}{3} \delta_{ij} D_k \chi D_k^* \chi^* - \partial_i \phi \partial_j \phi^* + \frac{1}{3} \delta_{ij} \partial_k \phi \partial_k \phi^* \tag{4}
\]

When studying the effects of the gauge fields we look at both the energy the gauge fields pick up during the evolution as well as their contribution as a source for the gravitational waves. The energy density of the gravitational waves is simply the \( t_{00} \) component of the energy-momentum tensor of the gravitational waves: \( t_{00} = \frac{1}{32\pi G} \left( \partial_t h_{TT} \partial_0 h_{ij}^{TT} \right) \) where \( TT \) stands for the transverse-traceless part of the tensor fluctuations. The energy density calculated simply for \( h_{ij} \) is related to that for \( h_{ij}^{TT} \) by a numerical factor given by the angular integral of the projection operator that transforms \( h_{ij} \) into \( h_{ij}^{TT} \). We also use the synchronous gauge for the metric, so the \( h_{00} \)
and the $h_{0i}$ components of the metric fluctuations are set to zero.

![Graph](image)

**FIG. 2:** Gravity wave energy with varying initial amplitudes. Note that different initial amplitudes saturate at a similar value for a fixed tachyon mass $\sim M_s/2$.

The parameters of the model are set to reflect the typical values for an inflationary model. We set the charge coupling the gauge field to the tachyon to 1, and the same for the quartic self-coupling of the tachyon, $\lambda = 1$. The mass of the inflaton is taken to be $m_\phi^2 = 0.01$, much smaller than the tachyon mass. We set the Newton constant to a value corresponding to the typical string mass scale of the tachyon to the inflaton field, is chosen to take 4 $\times 10^{−6}$ and the quartic self-coupling of the tachyon is taken to have a value for a fixed tachyon mass

$\chi(k) = c_k/k^{3/2}$. The Fourier coefficients for the tachyon, thus the real and imaginary parts are being independent random numbers.

![Graph](image)

**FIG. 3:** The energy components of the tachyon and gauge field for the tachyon mass $M_s/2$. Note that the total energy remains conserved.

![Graph](image)

**FIG. 4:** The energy pumped into the gauge field and the gravity waves.

![Graph](image)

**FIG. 5:** Snap-shots of iso-surface of the energy density for the gauge field, gravity waves, tachyon and the inflaton (from left-to-right) at $t = 300$.

The inflaton starts with a v.e.v. of $|\phi| = \phi_0$, corresponding the the onset of the tachyonic instability. On top of this v.e.v. we superimpose fluctuations corresponding to the deSitter phase, just like for the tachyon. The gauge fields have only the fluctuations corresponding to the deSitter phase. We take the gravitational waves to start at varying amplitude, see Fig. (2), all of them saturate to a similar amplitude. The time is measured in a string unit.

In Fig. (3) we have shown various energy components of the tachyon. The potential energy of the tachyon is negative given our choice of a potential. We observe that the production of gravitational waves does not start until the gradient energy (dark blue curve) of the tachyon starts to pick up, see Figs. (4, 5). The first ”spike” in
the energy graph, see Fig. (4), corresponds to the first oscillation of the tachyon, but the field stays almost homogeneous during this first oscillation. When the gradient energy picks up and the field condensate fragments, then only the gravity waves are produced and also the gauge field is excited. The energy in the $\phi$ field remains negligible.

The energy stored in the gauge field also get exponentially amplified due to the fragmentation of the tachyon field. Although they carry most of the energy, see Fig. (4), compared to the gravity waves, their amplification happens at a slightly later stage, i.e. compare the two in Fig. (4) for tachyon mass $M_\chi/2$.

In Figs. (5) we have shown 4 snap-shots of the iso-surface of the constant energy density for the gauge field, the gravity waves, the tachyon and the inflaton. Couple of points to note here, all the fields except the inflaton shows a remarkable departure in the homogeneity. Except the inflaton all fields undergo long wavelength excitations (they all look relatively smooth on small scales), while the inflaton obtains the largest inhomogeneity on the smaller scales. This is due to the fact that there is no long wavelength amplification for the inflaton, rather the inflaton starts oscillating rapidly with an enhanced frequency by virtue of the coupling $\sim m_\chi^2 |\chi|^2$. However as the tachyon fragments, the $\chi$ field no longer retains the homogeneity, therefore the inflaton obtained space dependent mass which fragments the inflaton on sub-Hubble scales.

Let us now comment on the gravity waves, its amplitude and frequency. Due to the tachyonic growth the amplitude of the frequency is given by: $\rho_{GW}/\rho_0 \sim t_{00}/M_\chi^3 \sim \langle h_{ij} h^{ij} \rangle_V$, where $V$ stands for the volume average. From Fig. (11) it is clear that the factor $\rho_{GW}/\rho_0 \sim 10^{-6}$ while its frequency depends on the details of the expansion history of the universe, however no matter what happens lately, the frequency will be well beyond the reach of the future gravity wave detectors.

To summarize, we have shown a brane-anti-brane annihilation inevitably ends up in fragmentation of a tachyon which excites gauge field and gravity waves simultaneously. We also note that the excitations are generically long wavelength in nature, except that of the inflaton. However our lattice box size is such that we can only explore sub-Hubble processes. The amplification of gauge and gravity waves depends on the mass of a tachyon. A larger mass tachyon leads to a quicker growth as compared to a smaller tachyon mass. The amplitude of the gravity waves could be quite large, i.e. $\rho_{GW}/\rho_0 \sim 10^{-6}$ but their frequency will be towards the high end of the spectrum where future detection will not be foreseeable. Our simulation also unveil the first study of exciting gravity waves where the gauge field has been taken into account. We found that the initial burst of gravity waves arises from the fragmentation of the tachyon field, at only later stages the gauge field feeds into the production of gravity waves. One point to note here that we have not studied turbulence of the gauge field and the gravity waves, which will be considered in a separate publication.

Acknowledgements:- The numerical simulations have been done on the Imperial College HPC cluster. AM is partly supported by grant (MRTN-CT-2006-035863).

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