Role of the Electromagnetic Vacuum in the Transition from Classical to Quantum Mechanics

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Abstract
We revisit the nonrelativistic problem of a bound, charged particle subject to the random zero-point radiation field (ZPF), with the purpose of revealing the mechanism that takes it from the initially classical description to the final quantum-mechanical one. The combined effect of the ZPF and the radiation reaction force results, after a characteristic time lapse, in the loss of the initial conditions and the concomitant irreversible transition of the dynamics to a stationary regime controlled by the field. In this regime, the canonical variables \(x, p\) become expressed in terms of the dipolar response functions to a set of field modes. A proper ordering of the response coefficients leads to the matrix representation of quantum mechanics, as was proposed in the early days of the theory, and to the basic commutator \([\hat{x}, \hat{p}] = i\hbar\). Further, the connection with the corresponding Fokker–Planck equation valid in the Markov approximation, allows one to obtain the (nonrelativistic) radiative corrections of QED. These results reaffirm the essentially electrodynamic and stochastic nature of the quantum phenomenon, as proposed by stochastic electrodynamics.

Keywords Zero-point radiation field · Stochastic electrodynamics · Origin of quantum operators · Elucidation of the classical-to-quantum transition

1 Introduction

This paper is devoted to an analysis of the process that takes the system described by stochastic electrodynamics (SED)—the combined system composed typically of a charged subatomic particle subject to an external potential plus the zero-point radiation field (ZPF)—from an initially classical deterministic behavior to its final behavior, described by quantum mechanics (QM).
The analysis presented is in line with the notion that has guided the efforts of a number of authors working on SED, namely that this theory can serve as a legitimate underpinning for QM. This implies that SED and QM can be simultaneously valid only by accepting the transition from classical to quantum as a real phenomenon. Up to now the existence of such process has been intuitively anticipated (although not always recognized), but no detailed satisfactory description of the transition exists so far.

One widespread line of thought within the SED community has been to consider the action of the ZPF as a mere perturbation of an otherwise classical motion, with the expectation that the quantum properties emerge as a consequence. Although this perturbative approach has led to a series of positive results ([1–10] and further references therein), it has also met with apparently unsurmountable difficulties, such as self-ionization of the H atom [11–14], and more generally it has not been able to reproduce the quantum formalism, even as an approximation [15–17]. That the ZPF cannot be treated as a mere perturbation of a classical mechanical system, is confirmed by the fact that this field changes fundamentally the behavior of the system, eventually taking control of its response, as discussed in the present paper.

In this connection it is worth recalling Nelson’s stochastic mechanics [18–20] (see also [10, 21, 22] Ch. 2 for an alternative version; we refer to either of these theories as stochastic quantum mechanics, SQM), which is a phenomenological stochastic formulation leading successfully to QM. It may be easily realized that SQM is not a classical theory, since its Newton’s law of motion violates the classical deterministic canons by the addition of a stochastic acceleration as part of its mechanical description, and by the appearance of a diffusive velocity $u(x, t)$, from which derives the quantum potential $V_Q = \frac{1}{2}(mu^2 + h\nabla \cdot u)$. Thus, SQM does not imply a transition from classical to quantum: it is already by construction a non-classical theory.

As extensively discussed in [23] and references therein, a non-perturbative statistical treatment of the SED problem reproduces in the time-asymptotic limit the quantum formalism; we therefore refer to the regime attained in this limit as the quantum regime. Further, it has been demonstrated more recently [24] that in the quantum regime, the SED mean equations of motion coincide in every detail with those of SQM, which indicates that in this regime the two descriptions are equivalent in their dynamical content. Both include the pair of mean local velocities, the flow velocity $v$ and the diffusive velocity $u$, which are also contained—even if not so conspicuously—in QM. Although one might therefore consider that the three are basically the same non-classical theory, their principles differ substantially. To arrive at QM from a classical starting point—i.e., without introducing quantum postulates—an ontological element must be added to resolve the underdetermination of QM [25, 26], one that accounts for both the indeterministic and the electromagnetic nature of the quantum phenomenon, as will become clear below. SED posits the random ZPF as this ontological element, an ingredient that is conspicuously absent in classical physics and alas hidden in QM. But as it turns out, the SED approach leading consistently to QM is not merely perturbative; it must take into account the essential role of the ZPF.

The motivation of the present paper is precisely to discuss the transition to QM based on the observation that the ZPF, in combination with the radiation reaction
force, plays a crucial role in irreversibly modifying the dynamical behavior of an otherwise classical particle. As an end effect, the variables required for the (apparently) mechanical description in the quantum regime carry with them an indelible imprint of the background field, which becomes concealed in the quantum formalism.

The approach followed here is not entirely new; a first proposal along this direction was advanced in [27] and a more elaborate version was presented in [10], Ch. 10. More recent work that serves as background to the present paper is contained in Refs. [23, 28]. In the interest of clarity, some previously published results are included here; however, we omit in general the detailed derivations and focus the discussion on the key elements that, taken together, help shape a coherent picture of the process leading to quantum mechanics. Herein lies the originality of the present paper: it offers a more integrated causal explanation of the emergence of quantum mechanics in its matrix formulation, from the perspective of \textsc{sed}. The present approach proves to be sufficient to arrive at a convincing picture at the level of nonrelativistic quantum mechanics (including radiative corrections); further matters, such as the quantization of the field and the relativistic formulation, obviously require additional work.

The structure of the paper is as follows. Section 2 contains a brief qualitative description of the most relevant features of the evolution of the \textsc{sed} system towards the quantum regime, with the purpose of providing an introduction to the quantitative discussion of Sect. 3. Subsection 3.1 contains an analysis of the influence of the \textsc{zpf} on the kinematics of the particle, carried out within the framework of Hamiltonian dynamics. This analysis shows that, as a result of the decay of the deterministic motion, the \textsc{zpf} takes control over the particle response, so that the description in terms of the initial canonical variables $x$, $p$, changes to one that depends on the canonical variables of the field. In Subsect. 3.2 we identify the response coefficients with the matrix elements of the operators $\hat{x}$, $\hat{p}$ that describe dipolar transitions. In Subsect. 3.3 we follow a statistical approach and use the results of the previous section to determine the diffusion coefficients appearing in the ensuing Fokker–Planck equation, valid in the quantum regime. In Subsect. 3.4 we describe the time evolution of the average energy in this regime and derive the \textsc{qed} formula for the spontaneous emission coefficient. The paper ends with a brief discussion of the most important implications of the results obtained, for a better understanding of the physical process leading to the quantum formalism.

\section{The \textsc{sed} Process Towards Equilibrium}

Let us consider an \textsc{sed} system consisting of a charged particle (typically an electron), which is subject to the action of a conservative binding force, as described by Eq. (26), and at some instant (say, at time $t_0$) gets connected to the random \textsc{zpf}, which has an energy $\hbar c^2/2$ per mode. In practice because of the stochastic nature of the problem, when speaking of one particle one has in mind an ensemble of similar systems. At the outset the dynamics is essentially that of a classical electrodynamic system; in particular, the equations of motion are satisfied with the initial values of
the mechanical variables, which are freely specified. On the other hand, the vari-
ables describing the instantaneous state of the $\text{ZPF}$ are determined by purely random
circumstances, beyond our control and even our knowledge. Therefore, the initial
dynamics follows the classical deterministic rules of electrodynamics; however, this
picture holds for a short time, as the random $\text{ZPF}$ compels the particle to start moving
unpredictably. The system then begins to accumulate information about the random
motions impressed upon it, by constructing the diffusion tensors step by step (see
Eqs. (38) below). At the same time, the radiation reaction force due to the acceler-
ations forced upon the particle by the external force and the $\text{ZPF}$, has a dissipative
effect on the particle dynamics.

In the end, a remarkable process has taken place: the accumulated effects of the
random wave field—which conveys memory through its correlations—have generated an important diffusive component of the velocity, associated with the inho-
mogeneities in the distribution density of particles and giving rise in its turn to the
quantum potential ([23] and references therein). (Analogous phenomena of density
accumulation due to a random force are discussed in detail in Ref. [29].) Eventually
the diffusion tensors reach their final form, to serve as a statistical inventory of the
fluctuations taking place in the system. Simultaneously—and most importantly—the
radiation reaction force leads to the loss of memory of the initial conditions. In the
absence of external excitations, the originally deterministic motion governed by the
external force decays and only the (stable) fluctuating motions maintained by the $\text{ZPF}$
survive; this is characteristic of the ground state. More generally, in the presence of
an external field, with the concurrent evolution of the radiation reaction and the dif-
fusion tensors the system may be taken to an excited state, which is stationary in the
radiationless approximation. This is shown in the energy-balance equation obtained
for asymptotic times, Eq. (48), where the $e^2$-dependent term on the right-hand side
is zero in the radiationless limit [23]. The classical constants of motion, in corre-
spondence with the new situation, acquire their—stationary and quantized—final
value. Indeed, the system has entered the quantum regime, and henceforth follows
the quantum rules.

This, in a nutshell, we have learned along the years from the development of SED. All in all it implies an extraordinary transition from the initially deterministic
dynamics to a non-deterministic one controlled by the $\text{ZPF}$, which has however not
been analyzed in detail so far and we set out to address here. In particular, as antici-
pated in Ref. [28], and further elaborated in what follows, the permanent action of
the $\text{ZPF}$ turns out to induce a gradual and irreversible change in the nature of the
variables that describe the particle dynamics: they eventually become the particle
variables controlled by the field. This feature is seen to be at the core of the operator
representation of quantum mechanics.
3 Quantitative Approach

3.1 Kinematics of the SED System; Algebraic Structure

In a classical Hamiltonian system, consisting of a particle subject to a conservative force \( f(x) \), the (phase-space) algebraic structure is determined by the Poisson brackets in terms of the full set of the particle’s canonical variables \( \{x_i; p_i\} \) (we use a semicolon for the set of canonical variables, to distinguish it from the Poisson bracket),

\[
\{x_i(t), p_j(t)\}_{xp} = \delta_{ij},
\]  

with \( i, j = 1, 2, 3 \). However, the complete SED system, which is also a Hamiltonian system, consists of the particle subject to the conservative force plus the radiation field; therefore the full set of canonical variables must include those of the (infinite number of) field modes,

\[
\{q;p\} = \{x_i, q_a; p_i, p_a\},
\]  

with \( q_a, p_a \) the canonical variables corresponding to the mode of the radiation field of (circular) frequency \( \omega_a \). A discrete set of frequencies is considered here, for reasons that will become clear later.

At the initial time \( t_0 \), when particle and field start to interact, the full set of canonical variables is given by

\[
\{q_o,p_o\} = \{x_{io}, q_{ao}; p_{io}, p_{ao}\},
\]  

with \( x_{io}, p_{io} \) the initial values of the particle’s coordinate and momentum components and \( q_{ao}, p_{ao} \) the canonical variables corresponding to the original field modes, which are those of the zpf alone. Because the system is Hamiltonian, the variables at time \( t \), \( q = q(t), p = p(t) \), are related to \( q_o, p_o \) via a canonical transformation, and therefore the Poisson bracket of any two functions \( f, g \) can be taken equivalently with respect to the variables at time \( t \) or at time \( t_0 \),

\[
\{f, g\}_{qp} = \{f, g\}_{q_o,p_o}.
\]  

Since we are interested in the description of the dynamics of the particle, our focus will be on the Poisson bracket of particle variables \( x_i, p_j \), which at any time \( t \) satisfies of course

\[
\{x_i(t), p_j(t)\}_{qp} = \delta_{ij}.
\]  

From Eq. (4) it follows that this is equal to the Poisson bracket with respect to the full set of initial variables,

\[
\{x_i(t), p_j(t)\}_{q_o,p_o} = \{x_i(t), p_j(t)\}_{x_o,p_o} + \{x_i(t), p_j(t)\}_{q_{ao},p_{ao}}.
\]  

As discussed in Ref. [28] (and considered again in Subsect. 3.3), a result of the dissipative effect upon the systematic motion due to the radiation reaction force is that
the particle eventually loses memory of its initial conditions \( x_o, p_o \). Therefore, for times larger than a characteristic dissipation time \( \tau_d \), Eq. (6) reduces to

\[
\{ x_i(t), p_j(t) \}_{q_o p_o} \to_{t > \tau_d} \{ x_i(t), p_j(t) \}_{q_o p_o},
\]

and by combining Eqs. (5–7) we get the condition

\[
\{ x_i(t), p_j(t) \}_{q_o p_o} = \delta_{ij} \quad (t > \tau_d).
\]

An order-of-magnitude calculation of a typical dissipation time gives [30]

\[
\tau_d \approx 10^{-11} \text{ s},
\]

taken as the time needed for a “classical” radiating electron moving initially in an orbit of Bohr radius to collide with the nucleus [31].

Let us introduce normal field variables \( a_a, a_a^* \), such that

\[
a_a = e^{i \phi_a}, \quad a_a^* = e^{-i \phi_a},
\]

with \( \phi_a \) statistically independent, random phases in \((-\pi, \pi)\), as corresponds to the ZPF. Since the total energy per frequency of the mode \( a \) of the ZPF, taking into account the two polarizations, is \( \delta(\omega_a) = \hbar \omega_a \), the transformation between normal field variables and field quadratures is given by

\[
\omega_a q_a^0 = \sqrt{\hbar \omega_a / 2(a_a + a_a^*)}, \quad p_a^0 = -i \sqrt{\hbar \omega_a / 2(a_a - a_a^*)}.
\]

These transformation rules applied to the Poisson bracket with respect to the field variables give

\[
i \hbar \{ x_i, p_j \}_{q_o p_o} = \{ x_i, p_j \}_{a_a a_a^*} = [x_i, p_j],
\]

with the bilinear form \([f, g]\) defined in general as the transformed Poisson bracket [10, 28].

\[
[f, g] \equiv \sum_a \left( \frac{\partial f}{\partial a_a} \frac{\partial g}{\partial a_a^*} - \frac{\partial g}{\partial a_a} \frac{\partial f}{\partial a_a^*} \right).
\]

From Eqs. (8) and (11) we conclude that for times \( t > \tau_d \) the bilinear form \([x_i, p_j]\) must satisfy the condition

\[
[x_i, p_j] = i \hbar \delta_{ij}. \quad (t > \tau_d)
\]

This most important result indicates that the symplectic relation between the particle variables \( x_i \) and \( p_j \) becomes determined (for \( t > \tau_d \)) by their functional dependence on the normal ZPF variables as is expressed in Eq. (12), with the scale given by Planck’s constant.
3.2 Stationary States and Transitions; Operators

From Eqs. (12) and (13) it is clear that the variables \( x_i, p_j \) are now driven by a set of field modes, denoted by \( \{a\} \). In order to further analyze the implications of this result, let us for the moment consider that sufficient time has elapsed for the system to have reached the quantum regime, with stationary states of motion resulting from the combined effect of the radiation reaction force and the driving force of the (stationary) zpf on the particle [23]. In Subsect. 3.3 we come back to the discussion of how this regime is attained.

In the absence of external radiation, the field is in its ground state, namely the zpf, and correspondingly also the particle is in its ground state, characterized by its energy \( E_o \). In the presence of external excitations the particle may reach an excited state \( n \), with energy \( E_n = E_o + \omega_n \). Of importance in the present context is that the symplectic structure of the problem is already defined, which means that (13) can be applied to any stationary state, be it the ground state or an excited state \( n \). Let us tag the corresponding variables \( x_i, p_j \) with the subindex \( n \), and write, using (12),

\[
[x_i, p_j]_{nn} = \sum_a \left( \frac{\partial x_n}{\partial a_a} \frac{\partial p_n}{\partial a^*_a} - \frac{\partial p_n}{\partial a_a} \frac{\partial x_n}{\partial a^*_a} \right) = i\hbar \delta_{ij}. \tag{14}
\]

For ease of notation we refer in what follows to the one-dimensional case. Equation (14) implies necessarily that the variables \( x_n, p_n = m\dot{x}_n \) are linear functions of \( \{a_a, a^*_a\} \). The field modes involved are those to which the particle responds, i.e., those that can take the particle from state \( n \) to another state, say \( k \). We therefore write ([28]; see also [10, 27])

\[
x_n(t) = \sum_k x_{nk} a_n e^{-i\omega_{nk}t} + \text{c.c.}, \quad p_n(t) = \sum_k p_{nk} a_n e^{-i\omega_{nk}t} + \text{c.c.},
\]

where \( a_{nk} \) is the normal variable associated with the field mode that connects state \( n \) with state \( k \), and \( x_{nk}, p_{nk} = -im\omega_{nk} x_{nk} \) are the corresponding response coefficients [28]. Introduction of these expressions into the bilinear form Eq. (14) gives

\[
[x, p]_{nn} = 2im \sum_k \omega_{kn} |x_{nk}|^2 = i\hbar,
\]

whence

\[
\sum_k \omega_{kn} |x_{nk}|^2 = \hbar/2m. \tag{16}
\]

Further, since the field variables \( a_{nk}, a_{n'k} \) connecting different states \( n, n' \) with state \( k \) are independent random variables, by combining Eqs. (12), (15) and (16) one gets

\[
[x, p]_{nn'} = i\hbar \delta_{nn'}.
\]

The coefficients \( x_{nk} \) and the normal variables \( a_{nk} \) refer to the transition \( n \rightarrow k \) involving the frequency \( \omega_{kn} \), whilst \( x_{kn} \) and \( a_{kn} \) refer to the inverse transition, with \( \omega_{nk} = -\omega_{kn} \); therefore, from (15),
whence Eq. (17) takes the form

\[ \sum_k (x_{nk}p_{kn'} - p_{n'k}x_{kn}) = i\hbar \delta_{nn'} . \]

The coefficients \( x_{nk} \) and \( p_{nk} \) can therefore be organized as the elements of matrices \( \hat{x} \) and \( \hat{p} \), respectively, with as many rows and columns as there are different states, and such that Eq. (19) becomes

\[ [\hat{x}, \hat{p}]_{nn'} = i\hbar \delta_{nn'} , \]

which is precisely the matrix formula for the quantum commutator

\[ [\hat{x}, \hat{p}] = i\hbar . \]

The (basic) quantum commutator is thus identified with the Poisson bracket of the system’s response variables \( x, p \) with respect to the normal field variables \( \{a, a^*\} \). The matrix coefficient \( x_{nk} \) represents the (dipolar) response amplitude of the system in state \( n \) to the field mode \( (nk) \). It is clear that if the particle responds to a given mode \( (nk) \) of the zpf, it also has the capacity to respond to the mode \( (nk) \) of an external field. Whenever \( x_{nk} \neq 0 \), the system can thus make a transition from state \( n \) to state \( k \) and vice versa—as expressed in the well-known selection rules for dipolar transitions.

From this point on, using the fact that \([\hat{x}, \hat{p}]\) defines a Lie algebra that satisfies the Leibniz rule and the Jacobi identity, it is a direct matter to derive the expressions that complete the quantum matrix formalism [28].

To get a coherent picture of the physics behind the formalism it is important to bear in mind at all times the meaning just disclosed of the operators as response functions acting on the (state) vectors of a Hilbert space. Thus for instance, by introducing the vectors (summation over the repeated index \( k \) is understood)

\[ u_n = x_{nk}\varepsilon_k, \quad v_n = p_{nk}\varepsilon_k, \]

with \( \varepsilon_k \) unit orthogonal vectors, and applying the Cauchy–Schwarz inequality to their scalar products,

\[ |u_n \cdot u_n^*| |v_n \cdot v_n^*| \geq |u_n \cdot v_n^*|^2 , \]

we obtain, with \( p_{nk} = -im\omega_{kn}x_{nk}, \)

\[ \sum_{k \neq n} |x_{nk}|^2 \sum_{k' \neq n} |p_{nk'}|^2 \geq |m \sum_{k \neq n} \omega_{kn} |x_{nk}|^2 |^2 = \left( \frac{\hbar}{2} \right)^2 . \]

In writing the last equality we have used the Thomas–Reiche–Kuhn sum rule, Eq. (16). This result may be written in the more familiar form, using (21),
\[(\Delta \hat{x})^2 (\Delta \hat{p})^2 \geq \left| [\hat{x}, \hat{p}] \right|^2 = \left( \frac{\hbar}{2} \right)^2, \quad (25)\]

valid for any \( n \). The Heisenberg inequality represents therefore, from this perspective, a restriction on the minimum value of the product of the entire set of response coefficients, with Planck’s constant representing the scale of the fluctuations of the zpf. Further, from the derivation just presented it is clear that the operators \( \hat{x}, \hat{p} \) do not represent trajectories and there need not be a phase-space description associated with them.

### 3.3 Statistical Dynamic Description in the Markov Approximation

Let us now turn to the (initial) dynamics of the sED particle, which is usually described by the (non relativistic) equation of motion, known as Braffort–Marshall equation

\[ m\ddot{x} = f(x) + m\tau \dot{x} + eE(t), \quad (26) \]

where \( f(x) \) is the external binding force and \( m\tau \dot{x} \) stands for the radiation reaction force, with \( \tau = 2e^2/3mc^3 \) \[10\]. For an electron, \( \tau \approx 10^{-23} \) s. \( E(t) \) represents the electric component of the zpf taken in the long-wavelength approximation, with time correlation given in the continuum limit by \((j, k = 1, 2, 3)\)

\[ \langle E_k(s)E_j(t) \rangle = \delta_{kj}\varphi(t - s), \quad (27a) \]

where the spectral function

\[ \varphi(t - s) = \frac{2\hbar}{3\pi c^3} \int_0^\infty d\omega \omega^3 \cos\omega(t - s), \quad (27b) \]

corresponding to an energy \( \hbar\omega/2 \) per mode, represents a highly colored noise. In order to analyze the effect of the different forces on the particle dynamics, we introduce an expansion of \( x(t) \) in terms of powers of \( e \)

\[ x = x^{(0)} + x^{(1)} + x^{(2)} + \ldots = x^{(0)} + \sum_{s=1}^\infty x^{(s)}, \quad (28) \]

where \( x^{(s)} \) stands for the contribution of order \( e^s \), \( e \) being here the coupling factor of the particle to the field. (For neutral electromagnetic particles a different coupling needs to be considered, which would depend on the charge distribution). From Eq. (26) follows the hierarchy (summation over repeated indices is understood)

\[ m\ddot{x}_i^{(0)} = f_i(x^{(0)}) + m\tau \dot{x}_i^{(0)}, \quad (29) \]

\[ m\ddot{x}_i^{(1)} = \frac{\partial f_i}{\partial x_j} \bigg|_{x^{(0)}} x_j^{(1)} + eE_i(t), \quad (30) \]
By replacing, as is customary, the third-order time derivative by its approximate first-order expression $\tau \left( \frac{df_i}{dt} \right)$ in Eq. (29), one may readily see that the deterministic solution of the homogeneous part of (26) (i.e., in the absence of the zpf) decays within a time lapse of the order of the lifetime $\tau_d$, determined by the value of $\frac{df_i}{dt}$, as was mentioned above. Further, from Eq. (30) it follows that $x_i^{(1)}$ is a purely stochastic variable, describing a non-decaying motion driven by the electric component of the zpf, which may be written in the form

$$x_i^{(1)} = e \int_{-\infty}^{t'} ds G_{ik}(t,s) E_k(s), \quad (32)$$

where the Green function $G_{ij}(t,s)$ is a solution of the equation

$$m \frac{\partial^2}{\partial t^2} G_{ik}(t,s) = \left. \frac{\partial f_i}{\partial x_l} \right|_{x(0)} G_{ik}(t,s), \quad (33)$$

with initial conditions

$$G_{ik}(t,t) = 0, \quad \lim_{s \to t} \frac{\partial}{\partial t} G_{ik}(t,s) = \frac{1}{m} \delta_{ik}. \quad (34)$$

The solution of Eq. (33) satisfying these conditions is

$$G_{ik}(t,s) = \left. \frac{\partial x_i(t)}{\partial p_k(s)} \right|_{x(0)}, \quad (35)$$

whence (32) becomes

$$x_i^{(1)} = e \int_{-\infty}^{t} ds \left. \frac{\partial x_i(t)}{\partial p_k(s)} \right|_{x(0)} E_k(s), \quad (36a)$$

and its time derivative gives

$$p_i^{(1)} = e \int_{-\infty}^{t} ds \left. \frac{\partial p_i(t)}{\partial p_k(s)} \right|_{x(0)} E_k(s). \quad (36b)$$

Notice that in these expressions, the derivatives $\partial x_i(t)/\partial p_k(s)$, $\partial p_i(t)/\partial p_k(s)$, which are functions of the exact solution of Eq. (26), are to be calculated to zero order in $e$, i.e., at $x(0)$. As is clear from Eq. (31) and the following equations of the hierarchy, the higher-order solutions $x_i^{(r)}$ with $r > 1$ are all determined (although indirectly) by the field.

These results describe with revealing detail several important aspects of the dynamics. Equations (36) show how $x_i^{(1)}(t)$ and $p_i^{(1)}(t)$ are constructed by the zpf starting from zero, at specific rates determined by the external force. We observe
that along the evolution of the system, the dynamics undergoes a qualitative change, the \(zpf\) moving from being merely the source of some noise impressed on the particle motion, to injecting upon it indeterministic properties and gaining control of the response, after a period of the order of \(\tau_d\).

Alternatively, a statistical treatment of the problem, following an approach that is standard in the theory of stochastic processes, starts from Eq. (26) and leads by means of a smoothing process to a generalized Fokker–Planck equation (GFPE) for the probability density \(Q(x, p, t)\) ([32, 33] p. 209). The GFPE is an integro-differential equation, or equivalently, a differential equation containing an infinite number of time-dependent terms, which express the memory-laden buildup of the diffusion. At the initial time the system is far from equilibrium and there is no diffusion; once the Markov approximation applies, the FPE is reduced to a true Fokker–Planck equation [34], i.e., to a second-order differential equation,

\[
\frac{\partial Q}{\partial t} + \frac{1}{m} \frac{\partial}{\partial x_i} p_i Q + \frac{\partial}{\partial p_i} f_i Q + m \tau \frac{\partial}{\partial p_i} \dot{x}_i Q = D_{ij}^{px} \frac{\partial^2 Q}{\partial p_i \partial x_j} + D_{ij}^{pp} \frac{\partial^2 Q}{\partial p_i \partial p_j},
\]

with diffusion tensors given by ([23] Ch.4 and refs. therein)

\[
D_{ij}^{px}(t) = e \left\{ x_i E_j \right\} = e^2 \int_{-\infty}^{t} ds \frac{\partial x_i(t)}{\partial p_k(s)} \left|_{x(0)} \right. \left\{ E_k(s)E_j(t) \right\},
\]

\[
D_{ij}^{pp}(t) = e \left\{ p_i E_j \right\} = e^2 \int_{-\infty}^{t} ds \frac{\partial p_i(t)}{\partial p_k(s)} \left|_{x(0)} \right. \left\{ E_k(s)E_j(t) \right\},
\]

where the brackets denote averaging over the realizations of the stochastic field, and the derivatives are calculated at \(x^{(0)}\).

Notice that by taking the product of \(E_j(t)\) with \(x_i^{(1)}\) given by Eq. (36a) and averaging, one obtains precisely Eq. (38a), and similarly, the averaged product of \(E_j(t)\) with \(p_i^{(1)}\) given by Eq. (36b) gives Eq. (38b). This shows that the Markov approximation is consistent with the notion that the field has taken control of the variables \(x, p\). In the radiationless approximation, the FPE (37) leads to the Schrödinger equation in configuration space ([23] Ch. 4). Further, since \(e \langle x \cdot E \rangle\) is the mean work realized by the field on the particles, \(\text{Tr}D^{px}\) represents a radiative contribution to the mean energy of the particles. Indeed, this has been shown to correspond to the nonrelativistic formula for the Lamb shift [10, 30, 35], which assigns a clear meaning to the origin of the Lamb shift, well in accord with the intuitive interpretation suggested originally by Welton [36] (detailed discussions of this point can be seen in [23] Ch. 6, [35], and [37] Ch. 7). Similarly, \(\text{Tr}D^{pp}\) obtained from Eq. (38b) is related with the mean power absorbed from the field along the orbital motion; we shall deal with this term in Subsect. 3.4.

Since, as said above, the variables \(x, p\) have become controlled by the field, we may introduce into the above expressions the results obtained for the kinematics in Subsect. 3.1. We observe that in Eqs. (36) and (38), the integrands contain factors of the form \(\partial x_i(t) / \partial p_j(s), \partial p_i(t) / \partial p_j(s)\), which may be written in terms of Poisson brackets.
calculated at $x^{(0)}$, i.e., to zero order in $e$. According to what we have learned in Subsect. 3.2, for times $s, t$ larger than $\tau_d$ these Poisson brackets should be replaced by the corresponding quantum commutators, to be calculated in a given stationary state, say $n$ (for ease of notation we omit the subindex $n$ when possible),

$$\{x_k(s), x_i(t)\}_{xp} \to \frac{1}{i\hbar} [\hat{x}_k(s), \hat{x}_i(t)], \quad \{x_k(s), p_i(t)\}_{xp} \to \frac{1}{i\hbar} [\hat{x}_k(s), \hat{p}_i(t)].$$

Therefore, by setting $t_o = -\infty$ we may safely neglect the initial contribution to the integral for $t_o \leq s \leq t_o + \tau_d$ and write Eqs. (36) in the form

$$\chi^{(1)}_i = \frac{e}{i\hbar} \int_{-\infty}^{t} ds \, [\hat{x}_k(s), \hat{x}_i(t)] E_k(s),$$

$$p^{(1)}_i = \frac{e}{i\hbar} \int_{-\infty}^{t} ds \, [\hat{x}_k(s), \hat{p}_i(t)] E_k(s).$$

Similarly, from Eqs. (38) we obtain for the diffusion tensors

$$D^{px}_{ij}(t) = e \langle x_i E_j \rangle = \frac{e^2}{i\hbar} \int_{-\infty}^{t} ds \, [\hat{x}_k(s), \hat{x}_j(t)] \langle E_k(s) E_j(t) \rangle,$$

$$D^{pp}_{ij}(t) = e \langle p_i E_j \rangle = \frac{e^2}{i\hbar} \int_{-\infty}^{t} ds \, [\hat{x}_k(s), \hat{p}_j(t)] \langle E_k(s) E_j(t) \rangle.$$

Using Eqs. (27) for the time correlation of the zpf, we get

$$\text{Tr} D^{px}(t) = e \langle x \cdot E \rangle = -\frac{2ie^2}{3\pi c^3} \int_{-\infty}^{t} ds \, [\hat{x}_i(s), \hat{x}_j(t)] \int_0^\infty d\omega \, \omega^3 \cos \omega (t - s),$$

$$\text{Tr} D^{pp}(t) = e \langle p \cdot E \rangle = -\frac{2ie^2}{3\pi c^3} \int_{-\infty}^{t} ds \, [\hat{x}_i(s), \hat{p}_j(t)] \int_0^\infty d\omega \, \omega^3 \cos \omega (t - s).$$

### 3.4 Energy Balance

We now use the above result, Eq. (43b), to analyze the energy balance for the system in a state $n$. The equation of evolution for the average energy is obtained by multiplying the dynamical equation (26) with $p$ and averaging over the ensemble,
The second term on the rhs represents the average power absorbed from the field. Writing the commutator in (43b) in terms of the matrix elements corresponding to state $n$ with the help of Eqs. (15) and (18) gives

$$[\dot{x}_i(s), \hat{p}_j(t)]_{nm} = 2im \sum_i \sum_k |x_{ink}|^2 \omega_{ikn} \cos \omega_{ikn}(t-s), \quad (45)$$

whence

$$(\text{Tr} \mathcal{D}^{pp})_n = e(p \cdot E)_n = \frac{4me^2}{3\pi c^3} \sum_i \sum_k |x_{ink}|^2 \omega_{ikn} \int_0^\infty d\omega \alpha^3 \int_-\infty^t ds \cos \omega(t-s) \cos \omega_{ikn}(t-s)$$

$$= \frac{2me^2}{3c^3} \sum_i \sum_k |x_{ink}|^2 \omega_{ikn}^4 \left[ \delta(\omega - \omega_{ikn}) - \delta(\omega + \omega_{ikn}) \right]. \quad (46)$$

In its turn, the first term on the rhs of (44) represents the average power lost by radiation reaction, which is readily calculated using Eqs. (15),

$$\tau(p \cdot \dot{x})_n = -\frac{2e^2}{3c^3} \sum_i \sum_k |x_{ink}|^2 \omega_{ikn}^4. \quad (47)$$

Introducing Eqs. (46) and (47) in (44) one gets

$$\frac{d}{dt} \langle H \rangle_n = -\frac{4e^2}{3c^3} \sum_i \sum_{\omega_{ikn} > 0} |x_{ink}|^2 \omega_{ikn}^4. \quad (48)$$

When the system is in its ground state there is no contribution to the sum in Eq. (48), which confirms that the energy lost by radiation is compensated in the mean by the energy extracted from the zpf and detailed energy balance holds. By contrast, when the system is in an excited state, Eq. (48) gives for the rate of change of the energy ([23, 35] Ch. 6)

$$\frac{d}{dt} \langle H \rangle_n = -\sum_i \sum_{\omega_{ikn} > 0} \hbar \omega_{ikn} A_{ink}, \quad (49a)$$

with

$$A_{ink} = \frac{4e^2}{3\hbar c^3} \sum_i |x_{ink}|^2 \omega_{ikn}^3. \quad (49b)$$

which coincides with the QED formula for the Einstein spontaneous emission coefficient and serves to demonstrate that the radiation reaction and the zpf contribute...
equal parts to the spontaneous emission rate (see [37] for a discussion of this point in the context of QED).

4 Final Comments and Conclusions

To recap, firstly we have introduced the electromagnetic ZPF as part of the quantum ontology with the purpose of addressing several longstanding issues of quantum mechanics. In particular, its presence assigns a physical cause to the quantum fluctuations and thus accounts for the indeterministic nature of the quantum phenomenon. By introducing the ZPF we are considering that atomic matter is made of electric charges (or electromagnetic particles, more generally) that fill the space with radiation as a result of their permanent wiggling. This does not preclude the possibility that other fields should be required at some point to complete the quantum ontology; to arrive at quantum mechanics, however, the electromagnetic vacuum proves to be sufficient. The image of an isolated atom or a quantum particle in empty space appears thus as an idealization with no real counterpart. If it were experimentally feasible to introduce an atom or any quantum system in a volume completely free of radiation, including the ZPF, this would provide a valuable possibility to subject the theory to testing.

Secondly, the introduction of the ZPF is seen to represent more than the mere addition of a new element to the ontology. It changes the (apparently) mechanical nature of the quantum problem to an electrodynamic one. In addition to ensuring (and explaining) the atomic stability by compensating for the energy lost by radiation [23], it offers a possible explanation for the so-called quantum jumps [38], along with an understanding of the mechanism of entanglement [39, 40] and of the electron spin as an emergent property [41], as well as the (nonrelativistic) radiative corrections proper of QED [23, 35].

Thirdly, with the present work we have completed the picture by showing that as a consequence of dissipation, after a time lapse of order $\tau_d$ the ZPF becomes the driving force for the particle. We embody this central role of the ZPF in a kinematic condition, which explains what represents perhaps the most obscure of all quantum features: the representation of dynamical variables by operators in a Hilbert space, the elements of which are the response functions associated with the (dipolar) transitions induced by the field. This, in its turn, helps clarify another intriguing question of QM that sounds as an oxymoron: how is it possible that the description provided by the quantum formalism of a stationary quantum state is to be made in terms of a collection of transition amplitudes between states? The suggested functionality of the transition coefficients as the building blocks of the operator representation has moreover an historical value, since in Born’s hands (and as was involuntarily suggested by Heisenberg) they came to be the omnipresent elements of the quantum description.

A point of contention is whether the resulting theory is still “classical”. The inclusion of the ZPF can be seen as either a relinquishment of classical physics (by the introduction of a foreign element), an addition, or even a correction to classical
physics (by recognizing the reality of this field). These diverse points of view are present in the literature; see in particular Ref. [42]. That the inclusion of the zpf represents a rupture with classical physics is confirmed, some would say, by the fact that it eventually leads to quantum mechanics, which is no longer classical physics. Interestingly, hardly anyone would argue that Brownian motion is no longer “classical physics”—even though the chaotic and diffusive behavior of the molecules cannot be explained without consideration of the embedding medium. In fact, classical physics began to consider stochastic processes as part of it with the advent of Brownian motion in the hands of Einstein, followed by Smoluchowski. Thus from the point of view of sed, one might consider Einstein the grandfather of qm—and one of its (pre-)founders. There are of course important elements that make the quantum stochastic process differ essentially from Brownian motion, namely the colored noise spectrum and the temporal and spatial correlations of the stationary zpf as opposed to a white noise [24]. The commutator $[\hat{x}, \hat{p}]$ has a universal (state-independent) form thanks to the functional dependence on $\omega$ in the transformation equations (10). Further, the memory associated with the colored noise spectrum is essential for the buildup of diffusion during the transition to the quantum regime. The question then arises of whether in other cases where there is a correlated, wave-like, stationary background noise, one should expect quantization to emerge; this is an issue that has prompted a whole new branch of research under the name of hydrodynamic quantum analogies [43].

From the perspective gained with the present work, one may say that quantum dynamics is a brand-new variant and extension of classical physics into the stochastic domain, which finds a place of its own, both because of the distinctive behavior of quantum systems and for the wealth of phenomena and applications to which it gives rise.

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Declarations

Conflict of interest The authors declare no conflict of interest.

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