On radiation by a heavy quark in $\mathcal{N} = 4$ SYM

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A short note on radiation by a moving classical particle in $\mathcal{N} = 4$ supersymmetric Yang-Mills theory.

I. INTRODUCTION

In the papers [1–3] radiation by a pointlike quark in $\mathcal{N} = 4$ supersymmetric Yang-Mills theory at strong coupling is investigated using the AdS/CFT correspondence in the supergravity approximation [4–6]. In this note modifications of the published radiation pattern are suggested, which are consistent with the results in [7]. This analysis is motivated by the description of electrodynamic radiation in classical electrodynamics [8, 9].

The important result in the context of radiation by an accelerated charge $e$ is given by the Abraham-Lorentz four-vector force [8, 10, 11] in classical relativistic electrodynamics [12],

$$f^\mu = \frac{2e^2}{3}(a^\nu a^\mu v^\nu + \dot{a}^\mu),$$  \hspace{1cm} (1)

where the particle velocity is $\dot{v}^\mu = v^\mu \equiv \frac{dx^\mu}{d\tau}$ and the acceleration $\ddot{v}^\mu = a^\mu \equiv \frac{dv^\mu}{d\tau}$ with the proper time $\tau$ for the particle [13] [The signature used here is $(+−−−)$]. It is important to note the orthogonality of the force to the velocity,

$$v_\mu f^\mu = 0.$$  \hspace{1cm} (2)

This force vanishes for uniformly accelerated motion, $f^\mu = 0$ [8].

II. CLASSICAL RADIATION OF ACCELERATED ELECTRONS

In order to set the framework, it is helpful to discuss radiation in classical electrodynamics. A useful approach is found in the 1949 paper by Schwinger [9].

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Assume sources, restricted to a finite domain, which emit radiation. The four-momentum of the classical electromagnetic field is given in terms of the energy-momentum tensor by integrating on a hyper-surface

\[ P_\nu = \int T^{\mu \nu} d\sigma_\mu . \]  

(3)

Gauß and Maxwell allow us to rewrite it as

\[ P_\nu = \int \partial_\mu T^{\mu \nu} d^4x = \int j_\mu F^{\mu \nu} d^4x , \]  

(4)

in terms of the current \( j_\mu = (\rho, \vec{j}) \) and the field-strength \( F^{\mu \nu} \).

In the following it is important in order to determine the radiation force that the radiation field tensor and the vector potential are introduced in terms of the retarded and advanced fields \([10, 11]\):

\[ F^{\mu \nu}_{rad} = \frac{1}{2} (F^{\mu \nu}_{ret} - F^{\mu \nu}_{adv}) . \]  

(5)

Replacing in (4) \( F^{\mu \nu} \) by \( F^{\mu \nu}_{rad} \) gives \( P^\nu_{rad} \), which for point-like charges satisfies (compare to (2))

\[ \nu_\nu \frac{dP^\nu_{rad}}{d\tau} \propto \nu_\mu F^{\mu \nu}_{rad} \nu_\nu = 0 . \]  

(6)

Using current conservation \( \partial_\mu j^\mu = 0 \) and introducing the vector potential

\[ A^\mu_{rad} = \frac{1}{2} (A^\mu_{ret} - A^\mu_{adv}) \equiv (\phi, \vec{A}) , \]  

(7)

one obtains the power

\[ \frac{dP^0_{rad}}{dt} = \int \left[ \vec{j} \cdot \vec{A} - \rho \frac{\partial \phi}{\partial t} \right] d^3x + \frac{d}{dt} \int \rho \phi d^3x . \]  

(8)

In [9], eq. (I.17), Schwinger discards the second term of this formula, which has the form of a total time derivative.

The radiation vector potential \([9]\) is expressed by

\[ A^\mu_{rad}(t, \vec{x}) = \frac{i}{2\pi} \int \exp \left[ i\omega (\vec{n} \cdot \vec{x} - \vec{x}') - (t - t') \right] j^\mu(t', \vec{x}') d^3x' dt' \omega d\omega d\Omega \frac{d\Omega}{4\pi} . \]  

(9)

A point-particle current is assumed

\[ j^\mu = e (1, v(t) = \frac{dx}{dt} ) \delta(\vec{x} - \vec{R}(t)) . \]  

(10)
The integrals in (8) are as follows:

\[\int \rho \phi \, d^3x = e^2 \int \delta'(t' - t + \mathbf{n} \cdot (\mathbf{R}(t) - \mathbf{R}(t'))) \, dt'd\Omega = -e^2 \int \frac{d\Omega}{4\pi} \frac{n \cdot a}{\xi^3},\] (11)

with \(\xi = (1 - \mathbf{n} \cdot \mathbf{v}), \quad a = \frac{dv}{dt},\) and from eq. (I.41) of [9]

\[\int \left[ \mathbf{j} \cdot \frac{\partial \mathbf{A}}{\partial t} - \rho \frac{\partial \phi}{\partial t} \right] d^3x = e^2 \int \frac{d\Omega}{4\pi} \left[ \frac{a^2}{\xi^3} + 2 \frac{n \cdot a \, v \cdot a}{\xi^4} - \frac{(n \cdot a)^2}{\gamma^2 \xi^5} \right]
+ \frac{d}{dt} e^2 \int \frac{d\Omega}{4\pi} \left[ -\frac{v \cdot a}{\xi^3} + \frac{n \cdot a}{\gamma^2 \xi^4} \right],\] (12)

with \(\gamma = \frac{1}{\sqrt{1 - v^2}},\) and \(v^\mu = (\gamma, \gamma \mathbf{v}).\)

Finally, using the notion of emitted power and a Schott type term (for example, in the notation of [14]), the result of the angular radiation pattern is

\[\frac{dP_{rad0}}{dt \, d\Omega} = P_{rad}(\mathbf{n}, t) = P_{emitt}(\mathbf{n}, t) + P_{Schott}(\mathbf{n}, t),\] (13)

where

\[P_{emitt}(\mathbf{n}, t) = \frac{e^2}{4\pi} \left[ \frac{a^2}{\xi^3} + 2 \frac{n \cdot a \, v \cdot a}{\xi^4} - \frac{(n \cdot a)^2}{\gamma^2 \xi^5} \right],\] (14)

and

\[P_{Schott}(\mathbf{n}, t) = \frac{e^2}{4\pi} \frac{d}{dt} \left[ -\frac{v \cdot a + n \cdot a}{\xi^3} + \frac{n \cdot a}{\gamma^2 \xi^4} \right].\] (15)

Indeed there are two terms contributing to the radiation power. Schwinger [9] claims that only the first one \(P_{emitt}(\mathbf{n}, t),\) the one denoting the emission should be retained. It has the characteristics of an irreversible energy transfer. The second one in the form of a total time derivative is reversible in nature.

Following Jackson [15] to obtain the radiated energy density of a charged particle one starts from the large distance - 1/R contribution of the Liénard-Wiechert electric field

\[E_{rad} = \frac{e}{R} \left[ \frac{n \land [(n - v) \land a]}{(1 - n \cdot v)^3} \right]
= \frac{e}{R} \left[ -\frac{a}{(1 - n \cdot v)^2} + \frac{(n \cdot a)(n - v)}{(1 - n \cdot v)^3} \right],\] (16)

to obtain from
\[ \mathcal{E}_{\text{vector}} = \frac{1}{8\pi} \left( E^2 + B^2 \right) , \]  

(17)

with \( |B_{\text{rad}}| = |E_{\text{rad}}| \)

\[ \mathcal{E}_{\text{vector}} = \frac{e^2}{4\pi R^2} \left[ \frac{a^2}{(1-n \cdot v)^4} + 2 \frac{(v \cdot a)(n \cdot a)}{(1-n \cdot v)^5} - \frac{(n \cdot a)^2}{\gamma^2(1-n \cdot v)^6} \right] . \]

(18)

There is agreement between

\[ R^2(1-n \cdot v) \mathcal{E}_{\text{vector}} = P_{\text{emitt}}(n, t) . \]

(19)

Integrating the angular dependence (see e.g. the useful integrals in Appendix A in [2]) one obtains

\[ \gamma \frac{dP_{\text{rad}}^0}{dt} = \frac{dP_{\text{rad}}^0}{d\tau} = \frac{2e^2}{3} \gamma^4 [a^2 + \gamma^2 (v \cdot a)^2] \gamma - \frac{2e^2}{3} \frac{d}{d\tau} [\gamma^4 (v \cdot a)] , \]

(20)

which, with \( a^\mu = (\gamma^4 v \cdot a, \gamma^2 a + \gamma^4 (v \cdot a)v) \), can be written as

\[ \frac{dP_{\text{rad}}^0}{d\tau} = -f^0 = -\frac{2e^2}{3} \left[ a^\mu a_\mu v^0 + \dot{a}^0 \right] , \]

(21)

i.e. the zero component of the (negative) relativistic Abraham-Lorentz vector force \( f^\mu = f_{\text{emitt}}^\mu + f_{\text{Schott}}^\mu \)

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\[ = \frac{2e^2}{3} \left[ (\frac{d^2 x}{d\tau^2})^2 \frac{dx^\mu}{d\tau} + \frac{d^3 x^\mu}{d\tau^3} \right] . \]

(22)

The first term represents an irretrievable loss of energy, the second, the Schott contribution, is a total time differential, which contributes nothing to an integral by \( d\tau \), when the initial value of \( a^\mu \) is returned to at the end \[12\].

In summary, as Schwinger stated already in 1949, only the spectrum for the irreversible transfer \( P_{\text{emitt}}(n, t) \) is the relevant one for discussing the emitted radiation, and therefore should be retained. It is consistent with the derivation via the radiative electric field \( 16 \) as given in \[15\].

III. CLASSICAL \( \mathcal{N} = 4 \) SYM RADIATION

In order to calculate the radiation power one has to add to the vector part a contribution due to a massless scalar field \( \chi \)

\[ \partial_\mu T_{\text{scalar}}^{\mu\nu} = j_\chi \partial^\nu \chi , \]

(23)
with the current
\[ j_\chi = \rho_\chi = e_{\text{eff}} \sqrt{1 - v^2} \delta(x - R(t)) \].

This scalar contribution leads to
\[
P_{\text{scalar}}(n, t) = \frac{e_{\text{eff}}^2}{4\pi} \left[ \frac{\gamma^2 (v \cdot a)^2}{\xi^3} - 2 \frac{(v \cdot a)(n \cdot a)}{\xi^4} + \frac{(n \cdot a)^2}{\gamma^2 \xi^5} \right]
\[ + \frac{e_{\text{eff}}^2}{4\pi} \frac{d}{dt} \left[ \frac{v \cdot a}{\xi^3} - \frac{n \cdot a}{\gamma^2 \xi^4} \right], \]
(25)

(see also [1, 2]). Adding the vector parts (14) and (15) from the previous section the weak coupling angular spectrum is given by
\[
P_{\text{rad}}(n, t) = \frac{\lambda}{32\pi^2} \frac{a^2 + \gamma^2 (v \cdot a)^2}{\xi^3} - \frac{\lambda}{32\pi^2} \frac{d}{dt} \left[ \frac{n \cdot a}{\xi^3} \right].
\]
(26)
The term for \( P_{\text{emitt}}(n, t) \) may also be expressed as
\[
P_{\text{emitt}}(n, t) = \frac{\lambda}{32\pi^2} \frac{\gamma^2 [a^2 - (v \wedge a)^2]}{(1 - n \cdot v)^3}.
\]
(27)

Performing the angular integration gives
\[
\int \gamma P_{\text{rad}}(n, t) \, d\Omega = \frac{\lambda}{8\pi} \frac{\gamma^4 [a^2 + \gamma^2 (v \cdot a)]}{\gamma} - \frac{\lambda}{8\pi} \frac{d}{d\tau} \left[ \gamma^4 (v \cdot a) \right].
\]
(28)

Up to the coupling this expression agrees with the one from classical electrodynamics, given by (20). The \( \mathcal{N} = 4 \) SYM Abraham-Lorentz force [16, 17] in the weak coupling limit reads
\[
f_{\text{SYM,weak}}^\mu = \frac{\lambda}{8\pi} \left[ a^\nu \epsilon_{\nu\mu} + \dot{a}^\mu \right],
\]
(29)
allowing the same interpretation as in classical electrodynamics given above. \( f_{\text{SYM,weak}}^\mu \) satisfies the constraint (2).

IV. RADIATION IN \( \mathcal{N} = 4 \) SYM AT STRONG COUPLING

Based on the work by [1] Hatta et al. [2] performed a detailed and transparent calculation of the radiation pattern by a heavy quark in \( \mathcal{N} = 4 \) SYM at strong coupling, to be followed rather closely. The result consists of two parts for the energy density, to be identified as
\[
P_{\text{emitt}}(n, t) = \frac{\sqrt{\lambda} \gamma^2 [a^2 - (v \wedge a)^2]}{8\pi^2 (1 - n \cdot v)^3},
\]
(30)
and a term in form of a total time derivative

\[ P_{tt}(n, t) = \frac{\sqrt{\lambda}}{24\pi^2} \frac{d}{dt} \left[ \frac{v \cdot a}{\xi^3} - \frac{n \cdot a}{\gamma^2 \xi^4} \right], \tag{31} \]

which is - up to the couplings - the same given by (12) in classical electrodynamics [9].

In the notation of \[2\] \( P_{\text{emit}}(n, t) = R^2 \xi E^{(1)}_{\text{rad}}(t, r) \) and \( P_{tt}(n, t) = R^2 \xi E^{(2)}_{\text{rad}}(t, r) \). It is noted in [2] that integrating the sum of these two terms with respect to \( d\Omega \) does not give a proper Abraham-Lorentz force [16, 17], and the constraint (2) is not satisfied, as it is the case in the weak coupling limit, when compared with (29).

A possible source of this deficiency may be found that only retarded contributions for the radiation are taken into account, instead of following the prescription given e.g. by (5) and (7) in the previous sections.

There is no need to repeat the derivations given in [2], but instead relying on the expressions of the energy density in the gauge theory, i.e. on the Minkowski boundary given therein.

First consider the quantity

\[ E_A = \frac{\sqrt{\lambda}}{4\pi^2} \int dt_q \delta(W_q) \left( \frac{A_1}{\gamma^2 \Xi^2} + \frac{\partial}{\partial t_q} \frac{A_0}{\gamma \Xi} \right), \tag{32} \]

with the definitions

\[ W_q \equiv -(t - t_q)^2 + |r - r_q|^2, \quad \Xi \equiv (t - t_q) - v_q \cdot (r - r_q) = \frac{1}{2} \frac{dW_q}{dt_q}. \tag{33} \]

In [2] the integral is evaluated by the retarded condition: \( t_r = t_r(t, r) \) denotes the value of \( t_q \) for which \( W_q(t_q) = 0 \), with

\[ t - t_r = |r - r_q(t_r)| = R. \tag{34} \]

Writing \( \delta(W_q) = \delta(t_q - t_r)/2|\Xi| \) the result in the large \( R \)-limit taken from [2] is

\[ E_A^{\text{ret}} = \frac{\sqrt{\lambda}}{8\pi^2 R^2 |\Xi|} \left( \frac{\gamma^4 [a^2 - (v \wedge a)^2]}{\xi^2} (2 - \xi) \right) + \frac{\sqrt{\lambda}}{8\pi^2 R^2 |\Xi|} \frac{\partial}{\partial t_r} \left( \frac{n \cdot a + \gamma^2 (v \cdot a)(2 - \xi)}{\xi^2} \right). \tag{35} \]

As a conjecture let us consider

\[ E_A^{\text{rad}} = \frac{1}{2} (E_A^{\text{ret}} - E_A^{\text{adv}}) \tag{36} \]
by performing the integral for $E_{\text{rad}}^A$ starting from (32) but using the advanced condition with
\begin{equation}
  t - t_r = -|\mathbf{r} - \mathbf{r}_q(t_r)| = -R.
\end{equation}
This amounts to the substitutions, when on top $\mathbf{n} \rightarrow -\mathbf{n}$, which does not affect the force,
\begin{equation}
  \Xi \rightarrow -R(1 - \mathbf{n} \cdot \mathbf{v}),
\end{equation}
i.e.
\begin{equation}
  \xi \rightarrow -\xi,
\end{equation}
whereas $\frac{\partial}{\partial t_r} \xi$ remains unchanged.
From (35) one obtains
\begin{equation}
  E_{\text{adv}}^A = \frac{\sqrt{\lambda}}{8\pi^2 R^2|\xi|} \left( \frac{\gamma^4[a^2 - (\mathbf{v} \wedge \mathbf{a})^2](2 + \xi)}{\xi^2} - \frac{n \cdot \mathbf{a} + \gamma^2(\mathbf{v} \cdot \mathbf{a})(2 + \xi)}{\xi^2} \right),
\end{equation}
and
\begin{equation}
  E_{\text{rad}}^A = -\frac{\sqrt{\lambda}}{8\pi^2} \frac{\gamma^4[a^2 - (\mathbf{v} \wedge \mathbf{a})^2]}{R^2\xi^2}
  + \frac{\sqrt{\lambda}}{8\pi^2 R^2 \xi} \frac{\partial}{\partial t_r} \left( \frac{n \cdot \mathbf{a} - \gamma^2(\mathbf{v} \cdot \mathbf{a})}{\xi} \right).
\end{equation}
In an analogous way the contribution $E_{\text{rad}}^B$ is evaluated, starting from
\begin{equation}
  E_{\text{ret}}^B = -\frac{\sqrt{\lambda}}{8\pi^2 R^2|\xi|} \left( \frac{\gamma^4[a^2 - (\mathbf{v} \wedge \mathbf{a})^2](-\frac{1}{\gamma^2} + 2\xi - \xi^2)}{\xi^3} \right)
  - \frac{\sqrt{\lambda}}{8\pi^2 R^2 |\xi|} \frac{\partial}{\partial t_r} \left( \frac{n \cdot \mathbf{a}(\xi - 1)}{\xi^3} + \frac{n \cdot \mathbf{a} - \gamma^2(\mathbf{v} \cdot \mathbf{a})(2 - \xi)}{\xi^2} + \frac{1}{|\xi|} \frac{\partial}{\partial t_r} \left[ \frac{1}{6(2\xi^2 + 1)} \mathbf{v} \cdot \mathbf{a} \right] \right).
\end{equation}
After the substitution (39) $E_{\text{adv}}^B$ and then $E_{\text{rad}}^B$ is obtained,
\begin{equation}
  E_{\text{rad}}^B = -\frac{\sqrt{\lambda}}{8\pi^2 R^2} \left( \frac{\gamma^4[a^2 - (\mathbf{v} \wedge \mathbf{a})^2](-\frac{1}{\gamma^2} - \xi^2)}{\xi^4} \right)
  - \frac{\sqrt{\lambda}}{8\pi^2 R^2 \xi} \frac{\partial}{\partial t_r} \left( \frac{n \cdot \mathbf{a}}{\xi^2} - \frac{n \cdot \mathbf{a}}{\xi} \right) + \frac{1}{|\xi|} \frac{\partial}{\partial t_r} \left[ \frac{1}{6(2\xi^2 + 1)} \right].
\end{equation}
Finally, adding $E_{\text{rad}}^A$ and $E_{\text{rad}}^B$ the angular radiation power in the strong coupling limit is obtained
\begin{equation}
  P_{\text{rad}}^\text{strong}(\mathbf{n}, t) = P_{\text{emitt}}(\mathbf{n}, t) + P_{\text{Schott}}(\mathbf{n}, t)
  = \frac{\sqrt{\lambda}}{8\pi^2} \frac{\gamma^2[a^2 - (\mathbf{v} \wedge \mathbf{a})^2]}{\xi^3} - \frac{\sqrt{\lambda}}{8\pi^2 dt} \frac{n \cdot \mathbf{a}}{\xi^3}.
\end{equation}
The total time derivative term $P_{\text{Schott}}(n, t)$ differs from $P_{tt}(n, t)$ in (31).

Up to the dependence on the coupling $\lambda$ the same angular radiative spectrum is found in the weak as well as in the strong coupling limit of the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory, i.e. $\frac{1}{4} \rightarrow \sqrt{\lambda}$. As in electrodynamics [9] it is suggestive that for strong coupling as well only the emission spectrum $P_{\text{emitt}}(n, t) = \sqrt{\lambda} \, \pi / 2 \, \gamma^{2} \left[ a_{\mu} a_{\nu} v^{\mu} + \dot{a}^{\mu} \right]$ is the relevant one for radiation, i.e. for the irreversible energy transfer [7].

Furthermore the force [16, 17] is

$$ f_{\mu}^{\text{SYM, strong}} = \frac{\sqrt{\lambda}}{2\pi} \left[ a_{\nu} a_{\nu} v^{\mu} + \dot{a}^{\mu} \right]. $$ (45)

Up to the coupling dependence the Abraham-Lorentz forces in classical electrodynamics as well as in the Yang-Mills theory have the same dependence on the acceleration $a^{\mu}$ and the velocity $v^{\mu}$, when comparing (1), (29) and (45). All do satisfy the constraint (2).

As in electrodynamics [8, 18] the forces $f_{\mu}^{\text{SYM, weak}}$ and $f_{\mu}^{\text{SYM, strong}}$ vanish in weak and strong coupling $\mathcal{N} = 4$ SYM, respectively, for uniformly accelerated motion [2, 19], e.g. along the $x$ direction, $x^{\mu} = (\frac{1}{g} \sinh(g\tau), \frac{1}{g} \cosh(g\tau) = \sqrt{t^{2} + \frac{1}{g^{2}}}, 0, 0)$, i.e. $a_{\mu} a^{\mu} = -v_{\mu} \dot{a}^{\mu} = -g^{2}$, although radiation is emitted.

In the spirit of [20] a phenomenological derivation of $P_{\text{rad}}^{\text{strong}}$ (44) could be done as follows: retain only $P_{\text{emitt}}(n, t)$ of (30) as derived in [2], calculate after the angular integration the force $f_{\text{emitt}} = \frac{\sqrt{\lambda}}{2\pi} a_{\nu} a^{\nu} v^{\mu}$. Enforce the orthogonality (2) to $v^{\mu}$, together with $f_{\mu}^{\text{SYM, strong}} = 0$ for uniformly accelerated motion, by adding the Schott-type term $\frac{\sqrt{\lambda}}{2\pi} \dot{a}^{\mu}$. This one is consistently obtained after integrating $P_{\text{Schott}}(n, t)$ assumed to be of the same form in strong as well as in weak coupling (compare with (26)).

In [3] the time averaged energy density of an oscillating quark with small linear oscillations is derived, $v_{q}(t) = \epsilon \Omega \cos \Omega T$, $a_{q} = -\epsilon \Omega^{2} \sin \Omega T$ and $\epsilon << 1$. It is asymptotically isotropic and - after correcting the numerical coefficient by a factor 6 - given by

$$ \int_{-\infty}^{+\infty} dt < T_{00}(t, \vec{x}) > = \frac{\epsilon^{2} \Omega^{4} \sqrt{\lambda}}{16\pi^{2} R^{2}} \int_{-\infty}^{+\infty} dt, $$ (46)

which is consistent with the result by A. Mikhailov [7], namely for

$$ P_{\text{emitt}} = \frac{\sqrt{\lambda}}{2\pi} a_{q}^{2}(t). $$ (47)
V. CONCLUSION

The essence of this note is based on the structure of the Abraham-Lorentz force $f^\mu$ \cite{1}, which holds even for strong coupling, with the properties of the orthogonality to the velocity and its vanishing for uniformly accelerated motion. It is surprising that the force - up to its strength - is the same in relativistic electrodynamics, as well as in weak and strong coupling $\mathcal{N} = 4$ SYM, although the underlying angular distributions $P(n, t)$ are different.

For the $\mathcal{N} = 4$ SYM model in the strong coupling limit the special case of synchrotron radiation with frequency $\omega_0$ is considered in \cite{1}. An independent derivation of the synchrotron radiation in this model is given in \cite{21}.

In this case with $\mathbf{v} \cdot \mathbf{a} = 0$, $\mathbf{a}^2 = v^2 \omega_0^2$ the energy density \cite{14} reads

$$P_{\text{rad strong}}^n(n, t) = \frac{\sqrt{\lambda} \omega_0^2}{8\pi^2 \xi^4} \left[3 - (4 + \gamma^{-2})\xi - 3v^2 \sin^2 \Theta + 2\xi^2\right], \quad (48)$$

which differs from the one using $P_{tt}$ of \cite{31} (compare with eq. (3.71) in \cite{1} and with eq. (6.5) in \cite{2}).

A quantity of interest considered in \cite{1} is the time-averaged angular distribution of power, given by

$$\frac{dP_{\text{emitt}}(n)}{d\Omega} = \frac{\omega_0}{2\pi} \int_0^{2\pi/\omega_0} dt \left[ \frac{\sqrt{\lambda}}{8\pi^2} \frac{a^2}{(1 - n \cdot v)^3} \right], \quad (49)$$

with $1 - n \cdot v = 1 - v \sin \Theta \sin (\phi - \omega_0 t)$.

For this periodic motion for synchrotron radiation the contributions of the total time derivatives $P_{\text{Schott}}$, as well as $P_{tt}$, vanish in the time-averaged distribution. In any case, when emitted radiation is considered total time derivative terms should not be retained. They do not represent irreversible loss of energy in contrast to $P_{\text{emitt}}$ \cite{9, 12}.

Integration in (49) leads to

$$\frac{dP_{\text{emitt}}(n)}{d\Omega} = \frac{\sqrt{\lambda}}{8\pi^2} a^2 \gamma^5 \frac{1 + \frac{v^2}{2} \sin^2 \Theta}{(\gamma^2 \cos^2 \Theta + \sin^2 \Theta)^{5/2}}, \quad (50)$$

which agrees with the result eq. (3.72) derived in \cite{1}. The total power emitted,

$$P_{\text{emitt}} = \frac{\sqrt{\lambda}}{2\pi} [\gamma^2 t \omega_0]^2, \quad (51)$$

is the same as the result obtained in \cite{1, 2, 7}.
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[1] C. Athanasiou, P. M. Chesler, H. Liu, D. Nickel, K. Rajagopal, “Synchrotron radiation in strongly coupled conformal field theories,” Phys. Rev. D81 (2010) 126001. arXiv:1001.3880 [hep-th].

[2] Y. Hatta, E. Iancu, A. H. Mueller, D. N. Triantafyllopoulos, “Radiation by a heavy quark in N=4 SYM at strong coupling,” Nucl. Phys. B850 (2011) 31-52. arXiv:1102.0232 [hep-th].

[3] K. Maeda and T. Okamura, “Radiation from an accelerated quark in AdS/CFT,” Phys. Rev. D77 (2008) 126002 [arXiv:0712.4120 [hep-th]].

[4] J. M. Maldacena, ”The large N limit of superconformal field theories and supergravity,” Adv. Theor. Math. Phys. 2 (1998) 231–252, [hep-th/9711200].

[5] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, ”Gauge theory correlators from non-critical string theory,” Phys. Lett. B428 (1998) 105–114, [hep-th/9802109].

[6] E. Witten, ”Anti-de Sitter space and holography,” Adv. Theor. Math. Phys. 2 (1998) 253–291, [hep-th/9802150].

[7] A. Mikhailov, “Nonlinear waves in AdS / CFT correspondence,” hep-th/0305196.

[8] F. Rohrlich, Classical Charged Particles, Third Edition, World Scientific Publ., Singapore (2007), Sect. 6.

[9] J. Schwinger, “On the Classical Radiation of Accelerated Electrons,” Phys. Rev. 75 (1949) 1912.

[10] P. A. M. Dirac, ”Classical theory of radiating electrons,” Proc. Roy. Soc. A167 (1938) 148.

[11] F. Rohrlich, ”The definition of electromagnetic radiation,” Il Nuovo Cimento 21 (1961) 811.

[12] W. Thirring, A Course in Mathematical Physics 2: Classical Field Theory, Sect. 2.4, Second Edition, Springer Verlag New York Wien (1986).

[13] E. Eriksen and Ø. Gron, “Electrodynamics of hyperbolically accelerated Charges, IV. Energy-momentum conservation of radiating charged particles,” Ann. Phys. 297 (2002) 243-294.

[14] see, for example, D. V. Gal’tsov and P. Spirin, “Radiation reaction reexamined: Bound mo-
mentum and Schott term,” Grav. Cosmol. 12 (2006) 1-10. [hep-th/0405121].

[15] J. D. Jackson, Classical Electrodynamics, Second Edition, Wiley, New York (1975), Sect. 14.

[16] M. Chernicoff, J. A. Garcia, and A. Guijosa, “Generalized Lorentz-Dirac Equation for a Strongly-Coupled Gauge Theory,” Phys. Rev. Lett. 102 (2009) 241601, [arXiv:0903.2047].

[17] M. Chernicoff, J. A. Garcia, and A. Guijosa, “A Tail of a Quark in N=4 SYM,” JHEP 09 (2009) 080, [arXiv:0906.1592].

[18] T. Fulton and F. Rohrlich, ”Classical radiation from a uniformly accelerated charge,” Ann. Phys. (N.Y.) 9 (1960) 499-517.

[19] B. -W. Xiao, “On the exact solution of the accelerating string in AdS(5) space,” Phys. Lett. B665 (2008) 173-177. [arXiv:0804.1343 [hep-th]].

[20] L. D. Landau and E. M. Lifshitz, The Classical Theory of Fields, Fourth Revised English Edition, Elsevier, Amsterdam (2010), § 76.

[21] V. E. Hubeny, “Holographic dual of collimated radiation,” [arXiv:1012.3561].