Characterization of Lifshitz transitions in topological nodal line semimetals

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Abstract. We introduce a two-band model of three-dimensional nodal line semimetals (NLSMs), the Fermi surface of which at half-filling may form various one-dimensional configurations of different topology. We study the symmetries and “drumhead” surface states of the model, and find that the transitions between different configurations, namely, the Lifshitz transitions, can be identified solely by the number of gap-closing points on some high-symmetry planes in the Brillouin zone. A global phase diagram of this model is also obtained accordingly. We then investigate the effect of some extra terms analogous to a two-dimensional Rashba-type spin–orbit coupling. The introduced extra terms open a gap for the NLSMs and can be useful in engineering different topological insulating phases. We demonstrate that the behavior of surface Dirac cones in the resulting insulating system has a clear correspondence with the different configurations of the original nodal lines in the absence of the gap terms.

1 Introduction

Exploring novel topological phases has been one of the most fruitful avenues in condensed matter physics during the past decade. As a new type of topological phases of matter, nodal line semimetals (NLSMs) have attracted great attention in both theoretical \cite{1–9} and experimental \cite{10–13} studies during the past years. Compared with the well-known Dirac and Weyl semimetals \cite{14–17} characterized by discrete zero-dimensional (0D) band-crossing points in momentum space, NLSMs have one-dimensional (1D) band-crossing lines of the conduction and valence bands, which possess much richer configurations of Fermi surface, e.g. from the simplest case with a single loop, to the more complex linked loops \cite{18–22}, knots \cite{21,23} and crossing lines \cite{13,24–26}.

One topological invariant characterizing a nodal line of NLSMs is the Berry phase, defined along a trajectory either enclosing a 1D band-crossing line, or with one momentum varying over a period while the others being fixed \cite{1,5,9}. However, this invariant alone does not suffice to describe NLSMs because it only unveils the local topological properties in momentum space, whereas the topology of the whole Fermi surface is related to the global information of the whole Brillouin zone (BZ). Because the gap-closing lines are in general related to certain symmetries, NLSMs may be classified by the behavior of the lines on a symmetry-related plane in the BZ \cite{27}. More generally, different configurations of nodal lines are associated with the topology of Fermi surface, and the transitions between them can be understood as the Lifshitz transitions \cite{28}, which have been considered for NLSMs very recently \cite{29}. This way, a sudden change of the topology of the Fermi surface between a variety of topologically inequivalent configurations can be captured by the Lifshitz transitions.

In some recent studies, NLSMs have been taken as bases from which numerous topological phases can be generated by adding some extra terms, e.g. a 2D Chern insulator \cite{30}, a 3D Chiral insulator \cite{8}, a nodal torus semimetal \cite{31} and a 2nd order topological insulator \cite{32}. Moreover, it has been shown that the topological properties of the resulting system can be characterized by the behavior of extra terms along the nodal lines in the original NLSM, which means the nodal lines can serve as indicators of topological properties of the system. Inspired by this fact, an interesting question to ask is whether we can do it the other way around, i.e. to distinguish between different configurations of nodal lines by considering extra terms added to the system, and observing the consequences?

In this paper, we consider a simple 2-band NLSM, which exhibits Lifshitz transitions between several different configurations of the Fermi surface. The gap-closing lines are protected by the time-reversal symmetry and inversion symmetry together, while extra mirror symmetries in this model ensure that the surface states can only

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be observed under open boundary conditions (OBCs) in specific directions. We then show that the geometric structure of these gapless lines can be determined by only looking at the high-symmetry planes in momentum space, as the number of gap-closing points on the planes of \( k_z = 0 \) or \( \pi \) can exactly distinguish different Lifshitz transition regimes. Finally, we induce extra terms analogous to a 2D Rashba-type spin–orbit coupling to our model, and find that the different configurations of nodal lines can be manifested by the behavior of surface Dirac cones in the resulting system.

2 Model Hamiltonian and the geometry of lines

To model a NLSM, it is necessary for the system to have at least two bands, thus the Hamiltonian of which can be written as

\[ H(k) = \mathbf{h}(k) \cdot \sigma, \]

with \( \sigma \) the Pauli matrices acting on a pseudospin-1/2 (e.g. orbitals) space. In this work we consider a NLSM given by

\[ H_{\text{NL}}(k) = (\cos k_x + \cos k_y - m)\sigma_x + (\cos k_z + b \cos k_y)\sigma_z, \]

with \( m \) and \( b \) the parameters to control the configuration of the nodal lines. This model satisfies both the time-reversal symmetry represented by a complex conjugation operation for spinless systems, and an inversion symmetry given by \( H_{\text{NL}}(k) = H_{\text{NL}}(-k) \). The combining \( PT \) symmetry \( H_{\text{NL}}(k) = H_{\text{NL}}(-k) \) ensures the absence of \( \sigma_y \) and the existence of gapless nodal lines. Furthermore, this model also has a mirror symmetry about each of the three \( k_i - k_j \) plane, i.e. \( H_{\text{NL}}(k_i) = H_{\text{NL}}(-k_i) \) with \( i = x, y, z \). These symmetries give some restrictions to the behavior of the nodal lines, and make it possible to characterize the Lifshitz transitions by only looking at some high-symmetry planes, as discussed later in the paper.

Before further analysis of our model, here we give some general discussion about the conditions for nodal lines to exist. Essentially, 1D nodal lines in a 3D system are protected by the absence of one Pauli matrix, which is not necessarily guaranteed by the \( PT \) symmetry we specify above. This \( PT \) symmetry assumes that the inversion operation do not exchange the two components of the pseudospin. However, if the pseudospin is given by two staggered sublattices and the inversion center is between two lattice sites, the \( PT \) symmetry shall be rewritten as \( \sigma_z H_{\text{NL}}(k) \sigma_z = H_{\text{NL}}(k) \), which leads to the absence of the third Pauli matrix \( \sigma_z \).

Similarly, the absence of one Pauli matrix can also be protected by a chiral symmetry \( \sigma_i H(k) \sigma_i = -H(k) \), with \( \sigma_i \) the missing Pauli matrix in the Hamiltonian. Note that such a confinement is only for 3D NLSMs, as the momentum vector has different numbers of orthogonal components in other dimensions. For instance, in 2D systems, it requires only a single term for forming 1D gapless region in the BZ, therefore some extra symmetries are required to ensure the absence of two Pauli matrices in a 2-band system. However, a nodal line in 2D has no corresponding “drumhead” surface states (as there is not a third dimension to open), so 2D systems with nodal loops are not topological in this sense. On the other hand, in 4D systems, it requires all the three Pauli matrices to give a 1D gapless region in the BZ, so that no specific symmetry is required. Nevertheless, some extra symmetries may help identifying the loops with some high-symmetry planes, as the mirror symmetry discussed later for our model.

In our specific 3D model, the energy dispersion of the model is given by

\[ E_{\text{NL}} = \pm \sqrt{B_x^2 + B_z^2}, \]

and the nodal lines consist of points where \( E_{\text{NL}} = 0 \), the conditions of which are given by:

\[ B_x = \cos k_x + \cos k_y - m = 0, \quad (4) \]
\[ B_z = \cos k_z + b \cos k_y = 0. \quad (5) \]

Comparing to the simplest cases with nodal lines laid in a plane, our system supports richer configurations of gap-closing lines controlled by the value of \( m \) and \( b \), as illustrated in Figure 1. Without loss of generality, we consider only positive \( m \) and \( b \) throughout the paper, and here we fixed \( m = 0.5 \) as an example to demonstrate the different types of nodal lines. In this case equation (4) gives a cylinder along \( k_z \) in the BZ, while equation (5) gives two planes parallel to \( k_x \) axis, and the 1D nodal lines are given by the intersection of the two 2D manifolds. When \( 0 \leq b < 1 \), this system holds two loops which can be mapped to each other by the mirror symmetry about \( k_z - k_y \) plane, as shown in Figure 1a. By increasing \( b \), the two loops deform and touch each other when \( b = 1 \) (Fig. 1b), and transform into two loops mirror-symmetric with respect to the \( k_y = 0 \) (or \( \pi \)) plane when \( 1 < b < 2 \) (Fig. 1c). Further increasing \( b \), these two loops will extend along \( k_z \) and merge with themselves at the boundary of the periodic BZ when \( b = 2 \), as shown in Figure 1d. When \( b > 2 \), the two loops divide into four lines going through the BZ, which can be taken as four closed loops due to the periodic condition of the BZ, as shown in Figure 1e.

In each panel we inset a simpler sketch which is topologically equivalent to the corresponding nodal lines. The Lifshitz transitions take place at \( b = 1 \) and \( b = 2 \), where the deformations of configurations are not continuous (gluing with each other or theirselves). The configurations of Fermi surface at these transition points correspond to nodal chain semimetals [13,24].

3 Surface states and phase diagram

In general, topological phases of matter are often featured by its topologically protected surface states. Here we first briefly review the surface states and their corresponding topological invariant of NLSMs. It is known that a topological NLSM has “drumhead” surface states within the regime enclosed by the projection of the nodal lines onto
a 2D surface BZ [1–4]. These surface states can be associated with a nontrivial topological invariant of the bulk states, i.e. a Berry phase

$$\gamma_c = \oint_c (\psi|i\partial_t|\psi)dt,$$  \hspace{1cm} (6)

along a trajectory $c$ enclosing a nodal line, with $\psi$ the occupied Bloch band and $\theta$ a periodic parameter which describes the trajectory $c$. For example, $c$ can be chosen as a small circle perpendicular to the nodal line, and $\theta$ is simply the phase angle of the circle. The radius of the circle does not change the Berry phase as long as the nodal line is enclosed by $c$. Alternatively, the trajectory $c$ can be continuously deformed into two lines $l_1$ and $l_2$ passing through the BZ with two momenta being fixed, and the Berry phases $\gamma_{1,2}$ along these two lines satisfies [5]

$$\gamma_c = \gamma_{l_1} - \gamma_{l_2} \mod 2\pi.$$  \hspace{1cm} (7)

Since each $\gamma_i$ is defined with two fixed momenta, i.e. fixed $k_i = (k_{ix}, k_{iy})$ with $i, j$ taking two of $x, y$ and $z$, it describes the topology along a quasi-1D system along the third momentum $k_3$, direction for a fixed point in the 2D BZ of $k_i$. The existence of surface states suggests a $\pi$ Berry phase, which is equivalent to one of the two $\gamma_i$ given by $\pi$.

In our model, due to the existence of the mirror symmetries, a given line $l$ with fixed $k_\parallel$ is enclosed by either both of the two loops (or the two pairs of the four lines when $b > 2$) or none of them, with $k_\parallel$ contains any two of $x, y$ and $z$. Therefore, for two straight lines $l_1$ and $l_2$ at $k_{\parallel,1}$ and $k_{\parallel,2}$, they can continuously deform into either a point and disappear if they are at the same side of the two loops (or the two pairs of the four lines), or two trajectories $c_1$ and $c_2$ enclosing each of the two loops (or the two pair of the four lines) if they are at the different sides of them, which leads to

$$\gamma_{l_1} - \gamma_{l_2} = \gamma_{c_1} + \gamma_{c_2} = 0 \mod 2\pi.$$  \hspace{1cm} (8)

In Figure 1a we demonstrate an example of the integral paths of these Berry phases with red lines and circles. In either case, we can see that there is no topological difference between the quasi-1D systems at $k_{i,1}$ and $k_{i,2}$. Hence the nontrivial topologies of the two nodal loops (or the two pairs of the four lines) always cancel out each other and make no difference when considering the existence of surface states at a given $k_\parallel$. Consistent with this symmetry-based perspective, numerical results have shown no surface state when OBC is taken in $x, y$ or $z$ direction. We would also like to point out that in some cases, a system with Berry phase $\gamma = 0$ ($\mod 2\pi$) may also have fourfold surface states, which correspond to a winding number of 2 along $l$ [9]. In our model, however, the Berry phases $\gamma_{c_{1,2}}$ of the two loops (or the two pairs of the four lines) always correspond to opposite windings of the pseudospin texture, hence a straight line $l$ enclosed by both of them can only have a total winding of zero, and as such this still suggests no surface state.

Nevertheless, we find that in our model the nontrivial topology of the nodal lines can be manifested by surface states when OBC is taken in a direction without mirror symmetry. Specifically, we consider a rotated axis with $k_\pm = 1/2(k_x \pm k_y)$, as the model does not possess a mirror symmetry about either of $k_\pm - k_z$ planes. As a consequence, “drumhead” surface states emerge when OBC is taken in either $k_+$ or $k_-$ direction. In order to illustrate the “drumhead” states, we numerically diagonalize the Hamiltonian with OBC along $k_+$ (i.e. $x + y$) direction, while $k_z$ and $k_-$ are taken as fixed parameters. In Figure 2 we show the areas (marked in yellow (shadow)) with degenerate zero-energy surface states, which are enclosed by the projections of nodal lines (blue (dark) lines) in the 2D surface BZ of $k_2$ and $k_\parallel$.

Next we focus on the high-symmetry planes of $k_2 = 0$ and $\pi$, as the mirror symmetry about $k_x - k_y$ plane is still preserved in the new coordinates. We find that the noncontinuous deformation in our model only occurs at these planes, therefore we can characterize the Lifshitz transitions by focusing on the gapless points and surface states with $k_z$ fixed at these two values. In Figure 3 we demonstrate the spectra as a function of $k_\parallel$, with OBC along $k_\parallel$, direction and $k_z = 0$ and $\pi$. The different configurations in Figure 1 clearly show different features of surface states on the high-symmetry planes. When the two loops are connected by the mirror symmetry about $k_x - k_y$ plane (Fig. 1a), there is no gapless point or surface state in either of the two planes, as shown in Figures 3a and 3b. When $1 < b < 2$, the two loops are connected by the mirror symmetry about $k_x - k_z$ plane (Fig. 1c), and four gapless points with surface states connecting them emerge only for $k_z = \pi$ as shown in Figures 3c and 3d. Finally, when the two loops divide into four lines (Fig. 1e), gapless
Let $m$ and $b$, our results can be extended to arbitrary $m$ and $b$ by considering certain transformations of the Hamiltonian. Explicitly, the Hamiltonian is invariant under the operations $C_1$: $\sigma_y H(k_x, k_y, k_z, m, b) \sigma_y = H(k_x + \pi, k_y + \pi, k_z + \pi, -m, b)$, and $C_2$: $\sigma_x H^\dagger(k_x, k_y, k_z, m, b) \sigma_x = H(k_x, k_y, k_z + \pi, m, -b)$. These two operations map the positive $m$ and $b$ to negative ones, respectively, and the combination of $C_1$ and $C_2$ reverses signs of both the two parameters.

### 4 Effect of extra gap terms

It has been shown that the a NLSM can be turned to an insulator by introducing some extra gap terms, and the behavior of surface states of the resulting insulator has a correspondence with the original gap-closing lines of the NLSM [8,30,32]. In other words, one can obtain the information of Fermi surface by adding some extra physical effects to open a band gap, and investigating the surface states of the resulting insulator. More precisely, suppose the zeros of extra terms form one or several 1D lines (referred as zero-lines here after), the surface states shall have a Dirac cone whenever such a line is enclosed by a gap-closing loop in the original NLSM [8]. In Figure 5 we show several examples of geometric relations between zero-lines and the original nodal loops, which correspond to different number of surface Dirac cones.

Next we introduce this scheme to our model to distinguish between different configurations of nodal lines. In order to do so, the zero-lines of extra terms need to have different geometric relations with each configuration. Here we consider the following extra terms added to the model:

$$H_{\text{gap}} = \sin k_z \tau_z \sigma_y + \sin k_y \tau_y \sigma_y, \quad (9)$$

with $\tau$ the Pauli matrices acting on another (pseudo)spin-1/2 space. These gap terms can be viewed as a 2D Rashba type (pseudo)spin–orbit coupling along $k_y$ direction, which also couples the two components of $\sigma$. The zeros of these terms form four straight zero-lines through the BZ along $k_y$, with $k_x$ and $k_z$ equal to either 0 or $\pi$. Due to the anticommuting relation between $H_{\text{gap}}$ and the Hamiltonian (2), the eigen-energies of the resulting system are doubly degenerate, and take the form of

$$E = \pm \sqrt{E_{\text{NL}} + E_{\text{gap}}}, \quad (10)$$

with $E_{\text{gap}} = \pm \sqrt{\sin^2 k_y + \sin^2 k_z}$.

In Figure 6 we illustrate the two doublet bands nearest to $E = 0$ under OBC along $x$, with the same parameters as in Figure 1. The different configurations of gap-closing lines in the original NLSM can be directly distinguished by the surface Dirac cones in the spectrum. We note that with the parameter we choose, the zero-lines never cross the nodal lines except for $m = 0.5, b = 2$ in Figure 6d, hence the system is an insulator in the bulk with the parameters in Figures 6a–6c and 6e. Therefore the gapless dispersion in corresponding panels must exist on the surface of the system.
Fig. 3. The spectra as a function of $k_-$. $k_z$ is fixed at 0 or $\pi$, and OBC is taken along $k_+$ direction with 100 lattice sites. The parameters are $m = 0.5$ and $b = 0.5$, 1.5 and 2.5 from left to right.

Fig. 4. Phase diagram of Hamiltonian (2) versus $m$ and $b$, insets are graphs geometrically equivalent to corresponding gapless regimes in the BZ. $v_1$ and $v_2$ are the number of gapless points on planes of $k_z = 0$ and $= \pi$, respectively.

When $b < 1$, the two nodal loops do not enclose any of the zero-lines, and there is no any gapless states in the spectrum (Fig. 6a). When $b = 1$, the two loops deform into a pair of crossing loops which encloses the zero-line at $k_z = k_x = 0$, and surface gapless states with quadratic dispersion emerge at $k_y = k_z = 0$ when OBC is taken along $x$ (Fig. 6b). In the regime with $1 < b < 2$, the crossing loops divide into two separate loops, and the quadratic states also divide into two surface Dirac cones in Figure 6c, as a zero-line is enclosed by both loops. Keep increasing $b$, we can see that the bulk gap closes at $b = 2$ (Fig. 6d) and reopens after this point, and leave another two surface Dirac cones (Fig. 6e). This pair of Dirac cones corresponds to the zero-line at $k_z = 0$ and $k_z = \pi$, which now is also enclosed by the two “effective loops”, each of them constructed by a pair of the four nodal lines in Figure 1e.

However, we need to point out that it is not sufficient to distinguish different configurations of the original nodal lines by only looking at the number of surface Dirac cones in a system with fixed parameters. For example, the two different cases in Figures 5b and 5c both have two surface Dirac cones. To see the difference, one need to observe how the surface Dirac cones behave when the parameters are tuned continuously. By fixing the extra terms and tuning the parameters in the nodal line Hamiltonian (2), the two nodal loops in Figure 5b can merge and reshape into two loops, none of them encloses the zero-line. In this procedure the nodal loops never cross the zero-line, hence the bulk gap is always opened, and one shall observe the two Dirac cones (Fig. 6b) merge into one quadratic gapless point (Fig. 6b), and eventually annihilate (Fig. 6b). Whereas in Figure 5c, the two Dirac cones are related to two zero-lines within the same
loop, hence they cannot disappear continuously by tuning only the nodal loop enclosing them, unless the nodal loop crosses the zero-lines and the system become gapless at the crossing point. On the other hand, if we fix the parameters in the nodal line Hamiltonian and tuning the extra terms, the two zero-lines in Figure 5c may merge and disappear, thus their two corresponding surface Dirac cones may merge and annihilate with each other, while the ones related to Figure 5b cannot disappear in this way.

5 Summary

In summary, we have studied a simple 2-band model of NLSMs with $PT$ symmetry, which exhibits various configurations of nodal lines in momentum space, including two loops related to each other by mirror symmetries about $k_x - k_y$ or $k_x - k_z$ planes, and four nodal lines extending through the BZ. The transitions between these configurations, namely the Lifshitz transitions, correspond to noncontinuous transformations of the nodal lines, which occur only at the high-symmetry planes with $k_z = 0$ and $\pi$ in the BZ. The existence of “drumhead” surfaces of the NLSM, on the other hand, requires OBC in a direction where the system is not mirror-symmetric to itself, and their behavior in the two high-symmetry planes also reflects the Lifshitz transitions in our model. According to these features, we obtain a phase diagram regarding different configurations of nodal lines. Finally, we introduce some extra terms analogous to a 2D Rashba-type spin–orbit coupling, which open a gap in the system. The behavior of surface Dirac cones in the resulting insulating system under OBCs can manifest the nodal lines in the original semimetallic system.

Finally, we would also like to point out that although it may not be an easy task to find a realization of real materials, it is possible to simulate our model and its Lifshitz transition using some artificial systems. One promising experimental setup is the circuit systems, which may not be an easy task to find a realization of real materials, it is possible to simulate our model and its Lifshitz transition using some artificial systems. One promising experimental setup is the circuit systems, which may not be an easy task to find a realization of real materials, it is possible to simulate our model and its Lifshitz transition using some artificial systems.

References

1. A.A. Burkov, M.D. Hook, L. Balents, Phys. Rev. B 84, 235126 (2010)
2. H. Weng, Y. Liang, Q. Xu, R. Yu, Z. Fang, X. Dai, Y. Kawazoe, Phys. Rev. B 92, 045108 (2015)
3. Y. Kim, B.J. Wieder, C. Kane, A.M. Rappe, Phys. Rev. Lett. 115, 036806 (2015)
4. R. Yu, H. Weng, Z. Fang, X. Dai, X. Hu, Phys. Rev. Lett. 115, 036807 (2015)
5. D.-W. Zhang, Y.X. Zhao, R.-B. Liu, Z.-Y. Xue, S.-L. Zhu, Z.D. Wang, Phys. Rev. A 93, 043617 (2016)
6. Z. Yan, Z. Wang, Phys. Rev. Lett. 117, 087402 (2016)
7. L.-K. Lim, R. Moessner, Phys. Rev. Lett. 118, 016401 (2017)
8. L. Li, C. Yin, S. Chen, M.A.N. Araujo, Phys. Rev. B 95, 121107 (2017)
9. L. Li, S. Chesi, C. Yin, S. Chen, Phys. Rev. B 96, 081116 (2017)
10. Y. Wu, L.-L. Wang, E. Mun, D.D. Johnson, D. Mou, L. Huang, Y. Lee, S.L. Bud’ko, P.C. Canfield, A. Kaminski, Nat. Phys. 7, 667 (2016)
11. G. Bian, T.-R. Chang, R. Sankar, S.-Y. Xu, H. Zheng, T. Neupert, C.-K. Chiu, S.-M. Huang, G. Chang, I. Belopolski, D.S. Sanchez, M. Neupane, N. Alidoust, C. Liu, B. Wang, C.-C. Lee, H.-T. Jeng, C. Zhang, Z. Yuan, S. Jia, A. Bansil, F. Chou, H. Lin, M.Z. Hasan, Nat. Commun. 7, 10556 (2016)
12. J. Hu, Z. Tang, J. Liu, X. Liu, Y. Zhu, D. Graf, K. Myhro, S. Tran, C.N. Lau, J. Wei, Z. Mao, Phys. Rev. Lett. 117, 016602 (2016)
13. Q. Yan, R. Liu, Z. Yan, B. Liu, H. Chen, Z. Wang, L. Lu, Nat. Phys., DOI:10.1038/s41567-017-0041-4
14. X. Wan, A.M. Turner, A. Vishwanath, S.Y. Savrasov, Phys. Rev. B 83, 205101 (2011)
15. S.M. Young, S. Zaheer, J.C.Y. Teo, C.L. Kane, E.J. Mele, A.M. Rappe, Phys. Rev. Lett. 108, 140405 (2012)
16. T. Morimoto, A. Furusaki, Phys. Rev. B 89, 235127 (2014)
17. B.-J. Yang, N. Nagaosa, Nat. Commun. 5, 4898 (2014)
18. C. Zhong, Y. Chen, Z.-M. Yu, Y. Xie, H. Wang, S.A. Yang, S. Zhang, Nat. Commun. 8, 15641 (2017)
19. W. Chen, H.-Z. Lu, J.-M. Hou, Phys. Rev. B 96, 041102 (2017)
20. Z. Yan, R. Bi, H. Shen, L. Lu, S.-C. Zhang, Z. Wang, Phys. Rev. B 96, 041103 (2017)
21. M. Ezawa, Phys. Rev. B 96, 041202 (2017)
22. P.-Y. Chang, C.-H. Yee, Phys. Rev. B 96, 081114 (2017)
23. R. Bi, Z. Yan, L. Lu, Z. Wang, Phys. Rev. B 96, 201305 (2017)
24. T. Bzduek, Q. Wu, A. Regg, M. Sigrist, A.A. Soluyanov, Nature 538, 75 (2016)
25. Z. Yan, Z. Wang, Phys. Rev. B 96, 041206 (2017)
26. M. Ezawa, Phys. Rev. B 96, 041205 (2017)
27. R. Okugawa, S. Murakami, Phys. Rev. B 96, 115201 (2017)
28. I.M. Lifshitz, Sov. Phys. JETP 11, 1130 (1960)
29. A. Bouhou, A.M. Black-Schaffer, arXiv:1710.04871v1 (2017)
30. M. Li, M.A.N. Araújo, Phys. Rev. B 94, 165117 (2016)
31. N. Trker, S. Moroz, Phys. Rev. B 97, 075120 (2018)
32. L. Li, H.H. Yap, M.A.N. Araújo, J. Gong, Phys. Rev. B 96, 235424 (2017)
33. V.V. Albert, L.I. Glazman, L. Jiang, Phys. Rev. Lett. 114, 173902 (2015)
34. J. Ningyuan, C. Glazman, A. Sommer, D. Schuster, J. Simon, Phys. Rev. X 5, 021031 (2015)
35. C.H. Lee, S. Imhofs, C. Berger, F. Bayer, J. Brehm, L.W. Molenkamp, T. Kiessling, R. Thomale, arXiv:1705.01077v3 (2017)
36. S. Imhof, C. Berger, F. Bayer, J. Brehm, L. Molenkamp, T. Kiessling, F. Schindler, C.H. Lee, M. Greiter, T. Neupert, R. Thomale, arXiv:1708.03647v1 (2017)