Neutrino-electron scattering with a new source of CP violation

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Abstract. According to previous works, there is a possibility for increasing the difference between Dirac and Majorana cross section for a neutrino-electron scattering process if we take into account that the neutrino longitudinal polarization can be different from minus one. In this work, we study the difference between Dirac and Majorana scattering process but we introduce an additional effective interaction that depends on complex coupling constants for the neutrinos. Thus, in this more general case, we have two additional parameters, the phase of the neutrino couplings and one parameter \( \epsilon \) related to the effective coupling of the new interaction.

1. Introduction
One of the open questions in particle physics about neutrinos is referred to the nature of this particle, that is, if the neutrino is a Dirac or a Majorana particle. We can remember now that in Dirac case the particle is different than its associated antiparticle, otherwise, if we have a Majorana particle then it is identical to its own antiparticle [1]. That is one crucial difference between them. It is important to establish a way to determine to which of the both categories the neutrino belongs, although we know that there is a clear difference in both cases, it is very difficult to determine via a neutrino-electron scattering process if the neutrino belongs to one or other kind, because Dirac process differs from Majorana by terms that are proportional to the neutrino mass, which is usually small. Then, in order to increase such difference other authors have explored in the past the possibility of including the neutrino parallel polarization term in the calculation of the cross section, that is, they take this value different from minus one [2].

In this work, additionally to this approach we propose and consider a new effective interaction similar in its functional form to Standard Model neutral current interaction when the polarization term is considered again. With all these ingredients the cross section for the neutrino-electron scattering process is calculated but not only we explore the possibilities to increase the difference between Dirac and Majorana processes but actually we are able to define a region in which a Dirac neutrino is indistinguishable from a Majorana neutrino, for a certain neutrino energy and polarization values.
2. Neutrino-electron scattering in the Standard Model

In the Standard Model the amplitudes for the neutrino-electron scattering process are given by

\[ M_D = -i \frac{G_F}{\sqrt{2}} \left[ \bar{u}^f_{\nu} \gamma^\mu \left( 1 - \gamma^5 \right) u^{i}_{\nu} \right] \left[ \bar{u}^{\ell}_{\nu} \gamma^\mu \left( g^{\ell}_V - g^{\ell}_A \gamma^5 \right) u^{i}_{\nu} \right], \tag{1} \]

in the Dirac case, and by

\[ M_M = -i \frac{G_F}{\sqrt{2}} \left[ \bar{\nu}^f_{\nu} \gamma^\mu \left( g^{\ell}_V - g^{\ell}_A \gamma^5 \right) u^{i}_{\nu} \right] \left[ \bar{u}^{\ell}_{\nu} \gamma^\mu \left( 1 - \gamma^5 \right) v^{i}_{\nu} \right], \tag{2} \]

for the Majorana case. The effective coupling constants are defined as follows,

\[ g^{\ell}_V = -\frac{1}{2} + 2 \sin^2 \theta_W + \delta^{\ell}_{\nu e}, \quad g^{\ell}_A = -\frac{1}{2} + \delta^{\ell}_{\nu e}, \tag{3} \]

with \( \ell = e, \mu, \tau \).

In the last expression the second term, the neutrino block, includes the contribution from the anti-neutrino-electron scattering because in Majorana case the particle is identical to its own anti-particle, then if we pretend to obtain the completed amplitude for the scattering process this contribution cannot be despised. Related to the last equation, if we consider that the neutrino is a Majorana particle, then the following identity is valid \[ \bar{\nu}^{f}_{\nu} \gamma^\mu \left( 1 - \gamma^5 \right) v^{i}_{\nu} = \bar{u}^{f}_{\nu} \gamma^\mu \left( 1 + \gamma^5 \right) u^{i}_{\nu}, \tag{4} \]

and the amplitude that appears in equation (2) can be rewritten as

\[ M_M = i \frac{2G_F}{\sqrt{2}} \left[ \bar{u}^{f}_{\nu} \gamma^\mu \left( g^{\ell}_V - g^{\ell}_A \gamma^5 \right) u^{i}_{\nu} \right] \left[ \bar{\nu}^{f}_{\nu} \gamma^\mu \gamma^5 u^{i}_{\nu} \right]. \tag{5} \]

Now it is possible to see that Majorana amplitude, as opposed to Dirac amplitude, does not have a vector term in the neutrino block, thus identity (4) allows cancellation of the vectorial part in the amplitude.

As we can see from equations (5) and (1), their functional forms are different, for this reason it is reasonable to expect that after making all calculations needed to find the cross section for the scattering process, these cross sections do not look identical therefore we would expect to see a difference, but experimentally it does not happen because of the very small neutrino mass.

Now, considering that incident neutrino has a longitudinal polarization different than minus one, that is, they are not completely left-handed particles, it is possible to gain a certain degree of dissimilarity between Dirac and Majorana processes. To quantify how dissimilar the cross sections can be, a function is defined \[ D_{DM} \left( E_{\nu_{\ell}}^{Lab}, s_{||} \right) = \left| \frac{\frac{d\sigma^D_{E_{\nu_{\ell}}^{Lab}}}{dE_{\nu_{\ell}}^{Lab}} - \frac{d\sigma^M_{E_{\nu_{\ell}}^{Lab}}}{dE_{\nu_{\ell}}^{Lab}}}{\frac{d\sigma^D_{E_{\nu_{\ell}}^{Lab}}}{dE_{\nu_{\ell}}^{Lab}}} \right|, \tag{6} \]

which measures the percentage of difference between Dirac and Majorana as a function of neutrino energy and polarization. Of course, all variables involved are expressed in the laboratory reference frame. Figure 1 shows the difference between Dirac and Majorana for a set of distinct polarization values.
3. A new effective interaction

A straightforward way to introduce a more general interaction is to suppose an additional neutral current interaction with some special properties. Thus, we consider that its functional form remains equal to Standard Model neutral current interaction, but neutrino coupling constants for this case are the complex numbers $g'_{\nu \ell A} = e^{i\alpha}$ and $g'_{\nu \ell V} = e^{-i\beta}$, and not the typical values for this scattering process [5]. Thus, in the general case of complex coupling constants there are additional parameters, the phases of these constants just as reference [6] reports. Moreover, the electron axial and vector coupling constants do not change and have the same ordinary values.

3.1. Dirac case

We are interested again in the total amplitude for the neutrino-electron scattering process, but considering now that there is an additional contribution to amplitude (1) that comes from the new interaction described above. In Dirac case we have

$$
M'_{D} = -\frac{\sqrt{2}}{G} \left[ \bar{u}_f \gamma^\mu (\tilde{g}'_{\nu \ell V} - \tilde{g}'_{\nu \ell A} \gamma^5) u_\nu \right] \left[ \bar{u}_e \gamma^\mu \left( g'_{\ell V} - g'_{\ell A} \gamma^5 \right) u_e \right],
$$

where

$$
\tilde{g}'_{\nu \ell V} = 1 + \epsilon g'_{\nu \ell V} \quad \text{and} \quad \tilde{g}'_{\nu \ell A} = 1 + \epsilon g'_{\nu \ell A}.
$$

The small dimensionless parameter $\epsilon$ is defined as follows

$$
\epsilon = \frac{G'_F}{G_F}.
$$

Here, $G'_F$ corresponds to the effective coupling for the new physics interaction and it is according with $G'_F \ll G_F$ and consequently implies that $\epsilon \ll 1$. When $\epsilon = 0$, the new interaction is totally suppressed and we will recover the standard amplitudes in both cases, Majorana and Dirac.

3.2. Majorana case

On the other hand, if the neutrino is a Majorana particle, then it is necessary to incorporate the contribution that comes from new interaction for the neutrino and anti-neutrino to the total

\[ Figure 1. \text{ Difference between the Majorana and Dirac neutrino-electron elastic scattering cross section as a function of the incident neutrino energy and } m_\nu = 1eV. \text{ The black solid line corresponds to a polarization } s_\parallel = -0.9, \text{ dotted red line for } s_\parallel = -0.85, \text{ broken green line for } s_\parallel = -0.82 \text{ and chain blue line for } s_\parallel = -0.78. \]
amplitude in (2), that is
\[ M_{M'} = -i \frac{G_F}{\sqrt{2}} \left[ \bar{u}^f_{\ell} \gamma_\mu \left( g^V_{\nu} - g^A_{\nu} \gamma^5 \right) u^i_{\nu} \right] \left[ \bar{u}^f_{\nu} \gamma^\mu \left( \tilde{g}^V_{\nu} - g^A_{\nu} \gamma^5 \right) u^i_{\nu} \right]. \] (10)

In the same way in which the identity (4) was obtained by applying the fact that the neutrino is a Majorana particle, we can construct an equivalent expression for the anti-neutrino term in (10) but considering that neutrino coupling constants are complex numbers, so the following identity is valid
\[ \bar{u}^f_{\nu} \gamma^\mu \left( \tilde{g}^V_{\nu} - g^A_{\nu} \gamma^5 \right) v^i_{\nu} = \bar{u}^f_{\nu} \gamma^\mu \left( \tilde{g}^V_{\nu} + g^A_{\nu} \gamma^5 \right) u^i_{\nu}. \] (11)

Using last equation, the Majorana amplitude in (10) is now given by
\[ M_{M'} = -i \frac{G_F}{\sqrt{2}} \left[ \bar{u}^f_{\nu} \gamma_\mu \left( \xi_V - \xi_A \gamma^5 \right) u^i_{\nu} \right] \left[ \bar{u}^f_{\nu} \gamma^\mu \left( g^V_{\nu} - g^A_{\nu} \gamma^5 \right) u^i_{\nu} \right], \] (12)
where the new effective coupling constants are \( \xi_V = 2i \text{Im} \left\{ g^V_{\nu} \right\} \) and \( \xi_A = 2 \text{Re} \left\{ g^A_{\nu} \right\}, \) or equivalently
\[ \xi_V = -2\epsilon \sin (\beta) \quad \text{and} \quad \xi_A = 2 \left[ 1 + \epsilon \cos (\alpha) \right]. \] (13)

On the one hand, equation (7) looks similar to (1), actually we only rewrote that equation in a more general expression, where axial and vector coupling constants are complex and they depend on \( \epsilon \) and phases \( \alpha \) and \( \beta, \) but on the other hand the amplitude for a scattering process in Majorana case with that additional interaction, does not look like (5), because of their functional forms are disparate, that is, while in the Dirac case the amplitudes have axial and vector terms in the neutrino block, but different values, in Majorana case when a new interaction is introduced in the model, with the properties described above, the resulting amplitude has, not only a vector term as in (5), but an axial and a vector terms. Thus, the use of identity (11) does not cancel the vector part in the Majorana amplitude for the scattering process with a new interaction.

We will assume for simplicity that the value of the phases of neutrino complex coupling constants are identical, that is \( \alpha = \beta = \delta. \) Thus, for a given value of incoming neutrino energy \( E_{\nu_{\ell}}^{\text{Lab}} \) the total cross sections will be three-parametric families that depend on the dimensionless parameter \( \epsilon, \) the phase of vector and axial couplings \( \delta \) and the neutrino longitudinal polarization \( s_||. \) Figure 2 shows the function defined in (6) but changing \( d\sigma^M / dE_{\nu_{\ell}}^{\text{Lab}} \) by \( d\sigma^M / dE_{\nu_{\ell}}^{\text{Lab}} \), where \( d\sigma^M / dE_{\nu_{\ell}}^{\text{Lab}} \) is the differential cross section referred to the lab frame for a neutrino-electron scattering process with the new interaction, and \( \ell \neq e \) (left) and \( \ell = e \) (right) in (5). The dotted blue lines and the dashed red lines in the plots correspond to \( D_{DM'} \left( E_{\nu_{\ell}}^{\text{Lab}}, s_\parallel = -0.9, \epsilon = 0.01, \delta \right) \) for \( \delta = 0 \) (up) and \( \delta = \pi \) (down), respectively, while green dash-dotted line is the difference between \( d\sigma^D / dE_{\nu_{\ell}}^{\text{Lab}} \) and \( d\sigma^D / dE_{\nu_{\ell}}^{\text{Lab}} \) when \( \delta = \pi. \) The solid line corresponds to the difference function \( D_{DM'} \left( E_{\nu_{\ell}}^{\text{Lab}}, s_\parallel = -0.9, \epsilon = 0 \right) \).

4. Conclusion
For some appropriate values for parameters \( \epsilon, s_\parallel \) and \( \delta \) it is possible to obtain an overlap region, just as we can see from plots in Figure 2. Once we take all contributions for neutrino-electron scattering process into account, this overlap region suggests that for a certain energy \( E_{\nu_{\ell}}^{\text{Lab}} \) of the incident neutrino there is confusion between Dirac and Majorana neutrinos if we include an additional interaction with vector and axial complex coupling constants for the neutrino, then it is not possible to distinguish with this model if the neutrino is a Dirac or Majorana particle taken specific values for the involved parameters, even if the longitudinal polarization \( s_\parallel \) is different from minus one.
Figure 2. The blue dotted line and the red dashed line are the differences between $d\sigma^D/dE^L_{\text{Lab}}$ and $d\sigma^M/dE^L_{\text{Lab}}$ for $\delta = 0$ and $\delta = \pi$, respectively; dash-dotted line is the plot for $D_{DD'}$ with $\delta = \pi$; the solid line corresponds to equation (6). The parameter values for these plots are $s_{||} = -0.9$, $\epsilon = 0.01$ and a neutrino mass $m_\nu = 1$ eV.

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