Segregation and Phase Inversion in a Simple Granular System.

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Abstract

The segregation and the phase inversion are investigated through a simple granular system which consists of only two inelastic hard spheres in a square box with an energy source. With the variation of the coefficient of restitution, the mass ratio between two spheres or the box size, we show that two types of segregated states and crossover between them are realized in such a small simple system. PACS number(s):

Granular materials exhibit various complex phenomena\(^1,2\). Examples are segregations of particles mixtures with different properties which appear by shaking or stirring them, or confined them in a horizontally rotating drum, and so on\(^3-18\). Recently, many studies for several segregated patterns and the crossover between them are reported\(^10-18\).

In this paper, instead of carrying out simulations with many particles, we choose a simple system consisting of inelastic hard spheres with different masses. Although the system is so simple, it will be shown to exhibit segregation of particles. This system is expected as a simple model of local processes of highly excited granular mixtures.

Such small systems with elastic hard discs are recently investigated, and are found to show a prototype of solid-liquid phase transition, glass transition, and the transition between the static and dynamic friction\(^19-21\). In the following, we show that this simple system realizes

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the crossover between two segregated states like the change between different segregated patterns obtained in recent studies\textsuperscript{10–18}.

The system under consideration consists of two inelastic hard sphere particles with unit radius and different masses which are confined in a two-dimensional square box (Fig. 1). The left-hand wall is set at the origin of the horizontal axis and is in contact with the energy source. All walls are rigid and have a length larger than 4. The interaction between a particle and the walls without an energy source occurs only through hard-core collisions. We give the position of a light and a heavy particle in the horizontal direction $x_L$ and $x_H$ as the distance from the left wall, and the mass $m_L = 1$ and $m_H = M > 1$. The interaction between two particles occur through inelastic hard-core collisions with the the coefficient of restitution $e$. A particle hitting the left-hand wall in contact with the energy source with the velocity $(v_h, v_v)$ bounces back with the velocity $(V_h, V_v)$ ($V_h > 0$). Here, subscripts $h$ and $v$ indicate, respectively, the horizontal and vertical directions. In this paper, we employ a heat bath of the temperature $T$ as the energy source because of the simplicity. Then, the velocity $(V_h, V_v)$ is chosen randomly from the probability distributions $P_h(V_h)$ and $P_v(V_v)^{22}$:

\begin{align}
P_h(V_h) &= \frac{m_i V_h}{T} \exp\left(-\frac{m_i V_h^2}{2T}\right) \\
P_v(V_v) &= \left(\frac{m_i}{2\pi T}\right)^{\frac{3}{2}} \exp\left(-\frac{m_i V_v^2}{2T}\right),
\end{align}

where $T$ is the temperature of the heat bath fixed as unity. (We give the Boltzmann constant as $1$.)

In the following, we perform the simulation of the above system with the length of walls $S$ is $= 4.05$, $= 4.2$, and $= 4.5$. For this range of $S$, particles are densely packed and the collision between particles occur frequently. Now, in order to characterize the distribution of two particles, we define the segregation parameter $\delta \equiv < x_L - x_H >_t$ where $< >_t$ means the time average. If $\delta \sim 0$, the particles are not segregated while the sign of $\delta$ gives the average configuration of the two particles.

Now, we draw the phase diagram of the model against $\{M, e\}$ ($M > 1$ and $e < 1$), according to the size of $S$. Figure 2 shows the diagram for (a) $S = 4.05$, (b) $S = 4.2$ and (c)
$S = 4.5$. Here, $\delta > 0$ holds in the region with $+$, $\delta < 0$ holds in the region with $-$, and $\delta \sim 0$ holds in the shadow area drawn by multiple of points. (In this paper, we regard the case with $-(S-d) \times 10^{-3} < \delta < (S-d) \times 10^{-3}$ as $\delta \sim 0$ where $d$ is the diameter of the particle.) Independently of $S$, each phase diagram has following characteristics. For small $M$, $\delta \sim 0$ or $\delta > 0$ holds over all the range of $e < 1$. When $M$ is increased, the system realizes two states with $\delta > 0$ and $\delta < 0$ and the crossover between them depends on $e$. In particular, critical value $e$ to realize the crossover between states with $\delta < 0$ and $\delta > 0$ is independent of $M$ for large limit of $M$. However, crossover points shift to larger $e$ and larger $M$ with the increase of $S$. This means the change of the packing fraction of particles is also relevant to the crossover between $\delta < 0$ and $\delta > 0$ states. Moreover, the crossover points for $S = 4.05$ form a curve with 'N' shape, while the curve becomes smooth with the increase of $S$ like in Fig. 2 (b) and (c).

Figure 3 (a) shows $\delta$ as a function of $e$ for $M = 1.5$, $M = 2.0$, $M = 3.65$ and $M = 6.0$ with $S = 4.05$. For large $M$, $\delta$ takes a minimum value at about $e \sim 0.825$ and the minimum value decreases with the increase of $M$. In Fig. 3 (b), $\delta$ is plotted as a function of $M$ for $e = 0.825$, $e = 0.625$ and $e = 0.55$. Here, $\delta$ has a peak at $M \sim 1.3$. With the decrease of $e$, $\delta$ for larger $M$ becomes larger than that for $M$ at the peak.

To explain the existence of the two segregated states with $\delta > 0$ and $\delta < 0$, we consider two effects determining the sign of $\delta$, respectively; One effect contributes to the increase of $\delta$ and the other contributes to the decrease of $\delta$ with the decrease of $e$.

First, we study the former effect. The light particle’s velocity just after the contact with the energy source on the left-hand wall tends to be faster than the heavy particle’s because these velocities are a decrease function of the mass of each particle. On the other hand, by iterations of the collision between two particles, their velocities approach with each other because $e < 1$. This implies that the light particle tends to be located farther from the left-hand wall than to the heavy particle. Thus, $\delta$ is expected to increase with the decrease of $e$. The contribution of this effect is expected to increase with $1 - e$, and as a rough approximation, it is assumed to be proportional to $1 - e$. 

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Second, we study the effect by which $\delta$ is decreased with the decrease of $e$. If $e$ is given a value close to 1, the approach of two particles' velocities is slow. Then if $M >> 1$, the light particle moves much faster than the heavy one for most of the time and the collision between two particles occurs frequently. Then, the light particle behaves like a potential barrier for the motion of the heavy particle. In this case, the heavy particle’s motion to go across this potential barrier is important to determine the particle distribution. The kinetic energy of the heavy particle with the case of $x_L < x_H$ is smaller than that of $x_L > x_H$ because, in the former case, the heavy particle cannot make a contact with the heat bath directly and the energy is supplied only by collisions with the light particle. This means that the mean velocity of the heavy particle of the case $x_L < x_H$ is smaller than that of $x_L > x_H$, while the ratio between velocities is roughly estimated to be $e : 1$. Then, the ratio between the time required to switch from $x_L < x_H$ to $x_L > x_H$ and that to switch from $x_L > x_H$ to $x_L < x_H$ is given as $1 : e$. Here, this effect on $\delta$ is prominent only for the case with the large collision frequency which is almost proportional to $e$. In addition, this collision frequency increases with the decrease of $S$. Then, the contribution from the above effect is approximately estimated as $\delta_1 \sim (e - 1)/e/C(S)$ (C is an increase function of $S$).

By the combination of these effects, $\delta$ for large $M$ is given as $\delta \sim A\delta_0 + B\delta_1 = (1 - e)(A - \frac{B}{C(S)}e)$. Here, $A$ and $B$ are given as positive constant values. With adequate $A$, $B$, and $C$ holding $0 < AC(S)/B < 1$, $\delta$ takes a negative value for large $e$ ($1 > e > AC(S)/B$) and a positive value for small $e$ ($0 < e < AC(S)/B$). This result also gives that the crossover value of $e$ between the state with $\delta < 0$ and that with $\delta > 0$ becomes larger with the increase of $S$. If $M$ is small, the contribution of $\delta_1$ is expected little. Thus, $\delta > 0$ is realized for small $M$.

In this paper, the mass segregation and the phase inversion are investigated through a system which consists of only two inelastic hard spheres in a square box with heat bath. With the variation of the coefficient of restitution, the mass ratio or the box size, two types of states with different particle distributions and crossover between them are observed for the case with a large mass ratio between two particles. The system we studied here may
look too small and simple. However, we expect that this system can describe local dynamics of highly excited granular systems, and provide a basis for the understanding of particle segregations in granular system consisting of many particles.

The effects of the gravity, the size differences and the friction should be considered in order to investigate the generality of obtained phenomena like the crossover between different segregated states\textsuperscript{10–18}. Further analytical study of this system to clarify the presented behavior as well as the study of systems with three or more particles are necessary in future.

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FIGURES

Fig. 1. Illustration of the system with two inelastic hard spheres and energy source (left).

Fig. 2. Phase diagrams of the system for each set \((M,e)\) with (a) \(S = 4.05\), (b) \(S = 4.2\) and (c) \(S = 4.2\). + and − mean the sign of \(\delta\), and \(\delta \sim 0\) in the shadow area.

Fig. 3. (a) \(\delta\) as the function of \(e\) for \(M = 1.5, M = 2.0, M = 3.65\) and \(M = 6.0\), and (b) \(\delta\) as the function of \(M\) for \(e = 0.825, e = 0.625\) and \(e = 0.55\) with \(S = 4.05\).
A. Awazu Figure 1
A. Awazu Figure 3

(a) 

(b)