Hawking radiation via tunnelling from higher dimensional Reissner-Nordström-de Sitter black holes

Shuang-Qing Wu\textsuperscript{a,\,*}, Qing-Quan Jiang\textsuperscript{b}

\textsuperscript{a}College of Physical Science and Technology, Central China Normal University, Wuhan, Hubei 430079, People’s Republic of China
\textsuperscript{b}Institute of Particle Physics, Central China Normal University, Wuhan, Hubei 430079, People’s Republic of China

Abstract

Recent work that treats the Hawking radiation as a semi-classical tunnelling process from the four-dimensional Schwarzschild and Reissner-Nordström black holes is extended to the case of higher dimensional Reissner-Nordström-de Sitter black holes. The result shows that the tunnelling rate is related to the change of Bekenstein-Hawking entropy and the exact radiant spectrum is no longer precisely thermal after considering the black hole background as dynamical and incorporating the self-gravitation effect of the emitted particles when the energy conservation and electric charge conservation are taken into account.

Key words: Black holes; Extra dimensions; Tunnelling radiation; Self-gravitation correction
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* Corresponding author.
Email addresses: sqwu@phy.ccnu.edu.cn (Shuang-Qing Wu), jiangqingqua@126.com (Qing-Quan Jiang).
1 Introduction

Hawking's remarkable discovery [1] in 1974 that black holes can radiate thermally reconciled a serious contradiction among General Relativity, Quantum Mechanics and Thermodynamics at that time and put the first law of black hole thermodynamics on a solid fundament. At a big cost, however, this discovery also caused another controversial problem: what happen to information during the black hole evaporation? This notable problem is now often called as the information loss paradox [2], since the radiation with a pure thermal spectrum has no way to be recovered after black holes have disappeared completely. Initially Steven Hawking and Kip Thorne thought that information is lost during the process of black hole evaporation, but John Preskill believed that information is not lost and can get out of the black hole. In 2004, Steven Hawking [2] changed his old opinion and argued that the information could come out if the outgoing radiation were not exactly thermal but had subtle corrections.

Recently Parikh and Wilczek [3] presented a greatly simplified model (based upon the previous developments by Per Kraus, et al. [4]) to implement the Hawking radiation as a semi-classical tunnelling process from the event horizon of the four-dimensional Schwarzschild and Reissner-Nordström black holes by treating the background geometries as dynamical and incorporating the self-gravitation correction of the radiation. The radiant spectra that they derived under the consideration of energy conservation give a leading-order correction to the emission rate arising from the loss of mass of black holes, which corresponds to the energy carried by the radiated quanta. Their result shows that the actual emission spectrum of black hole radiation deviates from strictly pure thermality, which might serve as a potential mechanism to resolve the information loss paradox. Following this program, a lot of recent work (see Ref. [5] and references therein) has been focused on extending this semi-classical tunnelling method to various cases of black holes such as those in de Sitter [6,7,8], anti-de Sitter [5,9,10] space-times, charged [8,11] and rotating [12] black holes as well as other cases [13]. But relatively less research [7,9,14] in higher dimensions is touched upon.

In this Letter, we shall investigate the Hawking radiation via tunnelling from higher dimensional Reissner-Nordström black holes in de Sitter space-time. The subject is mainly motivated by the fact that recent brane-world scenarios [15] predict the emergence of a TeV-scale gravity in the higher-dimensional theories, thus open the possibility to test Hawking effect and to explore extra dimensions by making tiny black holes in the high-energy colliders [16] (LHC) to be running in the next year. Within this context, recently there is much attention [17] being paid to examine the various properties, especially Hawking radiation of the higher-dimensional black holes. Our interest on this
topic is also due to the following two important aspects: (1) In the light of recent astronomical observation data for supernova of type Ia and the power spectrum of CMB fluctuations, it has been suggested that our universe is in a phase of accelerating expansion [18] and will asymptotically approach a de Sitter space. That is to say, the cosmological constant in our present universe might be a small but positive one. This realization has sparked a sense of urgency in resolving the quantum-gravitational mysteries of de Sitter space. (2) A conjectured de Sitter/conformal field theory (CFT) correspondence [19] defined in a manner analogous to the very successful AdS/CFT correspondence [20] has been proposed recently that there is a dual between quantum gravity on a de Sitter space and a Euclidean conformal field on a boundary of the de Sitter space. Much work has been done on finding a holographic description of de Sitter space and establishing an analogous duality. Although there has been considerable success along this line, the proposed duality is still marred by various ambiguities. Hence as an intermediate step, it may be appropriate to reinforce our understanding of de Sitter space at a semi-classical level by considering the de Sitter radiation as a semi-classical tunnelling process.

On the grounds of all described above, it becomes obvious that the further study of black holes in higher dimensions is of great important. Of particular interest is the case of Hawking radiation of the higher dimensional black holes. The aim of this Letter is to present a reasonable extension of the Kraus-Parikh-Wilczek’s semi-classical tunnelling framework in the four-dimensional spherically symmetric space-times to the case of higher dimensional Reissner-Nordström black holes in de Sitter space-time. Moreover, we shall investigate the tunnelling radiation of the massless uncharged particles and massive charged particles across the black hole event horizon and the cosmological horizon, respectively. In addition, their possible phenomenological implications to the brane-world emission rate is briefly outlined.

Our Letter is organized as follows. After introducing the metric of higher dimensional Reissner-Nordström-de Sitter black holes and their Painlevé-type extensions in Section 2, we discuss Hawking radiation of massless uncharged particles and massive charged particles as a semi-classical tunnelling process from the black hole event horizon and the cosmological horizon in Sections 3 and 4, respectively, where the tunnelling rate and emission spectrum are explicitly computed in each case. The Letter is ended up with a brief remark.

2 Higher dimensional Reissner-Nordström-de Sitter black holes and their Painlevé-type extensions

The line element of \((n + 2)\)-dimensional Reissner-Nordström-de Sitter black holes with a positive cosmological constant \(\Lambda = n(n+1)/(2l^2)\) and the electro-
magnetic one-form potential are given by [21,22]

\begin{align*}
\text{d}s^2 &= - f \text{d}t_R^2 + f^{-1} \text{d}r^2 + r^2 \text{d}\Omega_n^2, \\
A &= \pm \frac{Q}{(n-1)V_n r^{n-1}} \text{d}t,
\end{align*}

where

\begin{align*}
f &= f(M, Q, r) = 1 - \frac{\omega_n M}{r^{n-1}} + \frac{\omega_n Q^2}{2(n-1)V_n r^{2n-2}} - \frac{r^2}{l^2}, \\
\omega_n &= \frac{16\pi}{n V_n}.
\end{align*}

Here \( M \) and \( Q \) are the mass and electric charge of the black hole, respectively. \( l \) is the curvature radius of de Sitter space, \( V_n \) denotes the volume of a unit \( n \)-sphere \( d\Omega_n^2 \). (Units \( G_n + 2 = c = \hbar = 1 \) are adopted throughout this article.)

When \( M = Q = 0 \), the solution (1) reduces to the pure de Sitter space with a cosmological horizon \( r = l \) which may be very large according to the existing knowledge of the cosmological constant. When the positive mass \( M \) increases with the charge \( Q \) decreased or fixed, a black hole horizon occurs and increases its size, while the cosmological horizon shrinks. For a special value of \( M \) and \( Q \), the black hole horizon will touch the cosmological horizon. Besides, when the cosmological constant vanishes, the black hole has an outer/inner horizon located at

\[ r_{n-1}^\pm = \frac{\omega_n}{2} \left[ M \pm \sqrt{M^2 - \frac{nQ^2}{8\pi(n-1)}} \right]. \tag{4} \]

For the general case in higher dimensions \( (n \geq 2) \), the horizons are determined by the equation \( f(M, Q, r_h) = 0 \) which is of \( (2n) \)-order so it in general has \( (2n) \)-roots. Typically for \( n \geq 2 \), there will be three positive (real) roots of \( f(M, Q, r_h) \), with the outermost root describing a cosmological horizon, and the remaining pair describing inner and outer black-hole horizons. Just as did in the four-dimensional case [8], here we are only interested in two distinct real roots of them: the largest one is the cosmological horizon (CH) \( r_c \), the smaller one corresponds to the event horizon (EH) \( r_+ \) of the black hole. The explicit forms of these solutions are not needed for our discussions made here and are not illuminating though a detailed analysis [23] may be of some interest.

Obviously, a key trick to apply the semi-classical tunnelling analysis is to introduce a coordinate system that is well-behaved at the horizons. Thus to remove the coordinate singularities at the horizons, one must perform a Painlevé-type coordinate transformation in arbitrary dimensions

\begin{equation}
\text{d}t_R = \text{d}t \pm \frac{\sqrt{1 - f}}{f} \text{d}r, \tag{5}
\end{equation}
under which the electric potential (2) retains its previous form modulo a
gauge transformation, and the line element (1) is transformed to the following
Painlevé-like form
\[ ds^2 = -f dt^2 \pm 2 \sqrt{1 - f dt dr + dr^2 + r^2 d\Omega^2_n}, \] (6)

where the plus (minus) sign denotes the space-time line element of the out-
going (ingoing) particles across the EH and the CH, respectively. The new
metric (6) is very advantageous for us to investigate the radiation of parti-
cles tunnelling across the horizons and to do an explicit computation of the
tunnelling probability.

First, let us work with the metric in the new form (6) and obtain the ra-
dial geodesics of the massless uncharged particles that follow the radial null
geodesics, which is given by
\[ \dot{r} = \frac{dr}{dt} = \pm \sqrt{1 - f} \approx \pm \kappa_h \left( r - r_h \right), \quad \kappa_h = \kappa_h(M, Q, r_h) = \frac{1}{2} \frac{\partial f}{\partial r} \bigg|_{r=r_h}. \] (7)

where the upper (lower) sign denotes the radial null geodesics of the outgoing
(ingoing) radiation from the EH (CH), namely, \( r_h = r_+, r_c \), respectively.

Next, it is known that different from the radial null geodesics of the massless
uncharged particles, the trajectory followed by the charged massive particles
is not light-like, it does not follow the radially light-like geodesics when it
tunnels across the horizon. By treating the charged massive particle as a sort
of de Broglie s-wave, its trajectory can be approximately determined as [8,11]

\[ \dot{r} = \frac{dr}{dt} = -\frac{g_u}{2g_tr} = \pm \frac{f}{2 \sqrt{1 - f}} \approx \pm \kappa_h \left( r - r_h \right), \] (8)

where the plus (minus) sign represents the radial geodesics of the charged
particles tunnelling across the EH (CH), respectively.

In the following, we shall investigate Hawking radiation via tunnelling across
the EH and CH, respectively, and calculate the tunnelling rate from each
horizon. The overall picture of tunnelling radiation for the metric is very com-
licated, because radiation is both propagating inwards from the CH and out-
wards from the EH. A formal, complete analysis must consider both of these
effects, and there would undoubtedly be scattering taking place between the
black hole horizon and cosmological horizons [7]. To simplify the discussion,
we will consider the outgoing radiation from the EH, and ignore the incoming
radiation from the CH when we deal with the black hole event horizon. While
dealing with the CH case, we shall only consider the incoming radiation from
the CH and ignore the outgoing radiation from the EH. This assumption is
reasonable as long as the two horizons separate away very large from each
other, since the radius of the cosmological horizon is very large due to a very small cosmological constant, and the black hole event horizon considered here is relatively very small because the Hawking radiation can take an important effect only for tiny black hole typical of $1 \sim 10$-Tev energy in the brane-world scenario. On the other hand, because Hawking radiation is a kind of quantum effect, it can be neglected and may not be observed for an astrophysical black hole with typical star mass about $10M_\odot$.

3 Radiation of massless uncharged particles as tunnelling

Before proceeding to discuss Hawking radiation of charged massive particles as a semi-classical tunnelling process, let us first consider the uncharged radiation outgoing from the EH. We adopt the picture of a pair of virtual particles spontaneously created just inside the horizon. The positive energy virtual particle can tunnel out and materialize as a real particle escaping classically to infinity, its negative energy partner is absorbed by the black hole, resulting in a decrease in the mass of the black hole, at the same time, the size of EH will reduce and the radius of CH will enlarge. Since the emitted particle can be treated as a shell of energy $\omega$, Eqs. (6) and (7) should be modified when the particle’s self-gravitation is incorporated. Taking into account the energy conservation only, the mass parameter in these equations will be replaced with $M \to M - \omega$ when the particle of energy $\omega$ tunnels out of the EH. So the radial null geodesics of the uncharged particles tunnelling out from the EH is

$$\dot{r} = 1 - \sqrt{1 - f(M - \omega, Q, r)} \approx k_+(M - \omega, Q, r_+)(r - r_+).$$

(9)

The imaginary part of the action of the uncharged particles can be expressed as

$$\text{Im}S = \text{Im} \int_{r_i}^{r_f} \int_0^{P_r} dP_r' dr = \text{Im} \int_{r_i}^{r_f} \int_M^{M - \omega} \frac{dr}{r} d(M - \omega'),$$

(10)

where we have used the Hamilton equation

$$\dot{r} = \frac{d(M - \omega)}{dP_r},$$

(11)

and denoted $r_i$ and $r_f$ the locations of the EH before and after the particle of energy $\omega$ tunnels out, respectively.

Substituting Eq. (9) into Eq. (10) and switching the order of integration, the imaginary part of the action can be easily evaluated by deforming the contour around the single pole $r = r'_+$ at the EH and reads
\[ \text{Im} S = \text{Im} \int_M^{M-\omega} \int_{r_i}^{r_f} \frac{dr}{1 - \sqrt{1 - f(M - \omega', Q, r)}} d(M - \omega') \]

\[ = - \int_M^{M-\omega} \frac{\pi}{\kappa_+(M - \omega', Q, r_+')} d(M - \omega') \]

\[ = - \frac{2\pi}{\omega_n} (r_n - r_i^n) = -\frac{1}{2} \Delta S_{EH}, \quad (12) \]

where \( S_{EH} = V_n r_+^{n}/4 = 4\pi r_+^{n}/(n\omega_n) \) is the Bekenstein-Hawking entropy of the EH.

Since the geometrical optical limit and the “s-wave” approximation can be used here, using the semi-classical WKB method the tunnelling probability is found to be related to the imaginary part of the action via \( \Gamma \sim e^{-2\text{Im} S} \), so the tunnelling rate at the EH is

\[ \Gamma \sim e^{-2\text{Im} S} = e^\frac{4\pi}{\omega_n} (r_n^n - r_i^n) = e^{\Delta S_{EH}}, \quad (13) \]

where \( \Delta S_{EH} = S_{EH}(M - \omega, Q) - S_{EH}(M, Q) \) is the difference of black hole entropy after and before the particle emission. Obviously, the derived emission spectrum actually deviates from pure thermality, perfectly generalizing those obtained in Refs. [3,4,7].

Consider now the case of the massless uncharged radiation incoming from the CH, Eqs. (6) and (7) should also be modified when the particle’s self-gravitation is incorporated. After taking into account the energy conservation, the mass parameter in these equations will be replaced with \( M \rightarrow M + \omega \) when the particle of energy \( \omega \) tunnels into the CH, and the radial null geodesics of the uncharged particles tunnelling into the CH becomes

\[ \dot{r} = -1 + \sqrt{1 - f(M + \omega, Q, r)} \approx -\kappa_c (M + \omega, Q, r_c) (r - r_c). \quad (14) \]

Applying the Hamilton equation

\[ \dot{r} = -\frac{d(M + \omega)}{dP_r}, \quad (15) \]

the imaginary part of the action of the uncharged particles can be computed in a similar manner by deforming the contour around the single pole \( r = r'_c \) at the CH.
\[ \text{Im} S = -\text{Im} \int_{r_{ic}}^{r_{fc}} \int_{-M}^{-(M + \omega)} \frac{dr}{r} d(M + \omega') \]
\[= \text{Im} \int_{-M}^{-(M + \omega)} \int_{r_{ic}}^{r_{fc}} \frac{dr}{1 - \sqrt{1 - f(M + \omega', Q, r)}} d(M + \omega') \]
\[= - \int_{-M}^{-(M + \omega)} \frac{\pi}{\kappa_c(M + \omega', Q, r')} d(M + \omega') \]
\[= - \frac{2\pi}{n\omega_n} \left( r^n_{fc} - r^n_{ic} \right), \quad (16) \]

where \( r_{ic} \) and \( r_{fc} \) are the locations of the CH before and after the uncharged particle tunnels into the CH, respectively. In terms of the Bekenstein-Hawking entropy of the CH: \( S_{CH} = V_n r^n / 4 = 4\pi r^n_{CH} / (n\omega_n) \), the tunnelling rate at the CH is
\[ \Gamma \sim e^{-2\text{Im} S} = e^{\frac{4\pi}{n\omega_n} (r^n_{fc} - r^n_{ic})} = e^{\Delta S_{CH}}, \quad (17) \]

where \( \Delta S_{CH} = S_{CH}(M + \omega, Q) - S_{CH}(M, Q) \) is the change of entropy of the CH after and before the particle tunnels into the CH. Once again, the actual radiant spectrum is no longer precisely thermal.

4 Radiation of massive charged particles via tunnelling

Now, we proceed to discuss Hawking radiation of charged massive particles from the EH. Since the emitted particle can be treated as a shell of energy \( \omega \) and charge \( q \), Eqs. (6) and (8) should be modified when the particle’s self-gravitation is incorporated. Taking into account the energy conservation and the charge conservation, the mass and charge parameters in these equations will be replaced with \( M \rightarrow M - \omega \) and \( Q \rightarrow Q - q \) when the particle of energy \( \omega \) and charge \( q \) tunnels out of the EH. So the outgoing radial geodesics of the charged massive particle tunnelling out from the EH and the non-zero component of electro-magnetic potential are, respectively,
\[ \dot{r} = f(M - \omega, Q - q, r) \approx \kappa_+(M - \omega, Q - q, r_+) \left( r - r_+ \right), \quad (18) \]
\[ A_t = \frac{Q - q}{(n - 1)V_n r^{n-1}}. \quad (19) \]

When we investigate the tunnelling process of a charged massive particle, the effect of the electro-magnetic field outside the black hole should be taken into consideration. So the matter-gravity system consists of the black hole and the outside electro-magnetic field whose Lagrangian function \(-(1/4) F_{\mu\nu} F^{\mu\nu}\)
is described by the generalized coordinate \( A_\mu = (A_t, 0, 0, 0) \). As the generalized coordinate \( A_t \) is an ignorable one, to eliminate this degree of freedom completely, the imaginary part of the action should be written as

\[
\text{Im} S = \text{Im} \int_{t_i}^{t_f} \left( L - P_{A_t} \dot{A}_t \right) dt = \text{Im} \int_{r_i}^{r_f} \left[ \int_{(0, 0)}^{(P_r, P_{A_t})} \left( \dot{r} dP_r - \dot{A}_t dP_{A_t} \right) \right] \frac{dr}{r}, \tag{20}
\]

where \( r_i \) and \( r_f \) represent the locations of the EH before and after the particle of energy \( \omega \) and charge \( q \) tunnels out, and \((P_{A_t}, P_r)\) are two canonical momenta conjugate to the coordinates \((A_t, r)\), respectively.

Substituting the Hamilton’s equations of motion

\[
\dot{r} = \frac{dH}{dP_r}(r, A_t, P_{A_t}), \quad dH(r, A_t, P_{A_t}) = d\left(M - \omega \right), \tag{21}
\]

\[
\dot{A}_t = \frac{dH}{dP_{A_t}}(A_t, r, P_r), \quad dH(A_t, r, P_r) = A_t d(Q - q), \tag{22}
\]

into Eq. (20), and switching the order of integration yield the imaginary part of the action

\[
\text{Im} S = \text{Im} \int_{r_i}^{r_f} \int_{(M, Q)}^{(M - \omega, Q - q)} \left[ d(M - \omega') - \frac{Q - q'}{(n - 1)V_n r^{n-1}} d(Q - q') \right] \frac{dr}{r}
\]

\[
= \text{Im} \int_{(M, Q)}^{(M - \omega, Q - q)} \int_{r_i}^{r_f} 2\sqrt{1 - f(M - \omega', Q - q', r)} \frac{d(M - \omega')}{f(M - \omega', Q - q', r)} \left[ d(M - \omega') - \frac{Q - q'}{(n - 1)V_n r^{n-1}} d(Q - q') \right] dr. \tag{23}
\]

The above integral can be evaluated by deforming the contour around the single pole \( r = r_+ \) at the EH. Doing the \( r \) integral first, we find

\[
\text{Im} S = - \int_{(M, Q)}^{(M - \omega, Q - q)} \frac{\pi}{\kappa_+(M - \omega', Q - q', r_+)} \left[ d(M - \omega') - \frac{Q - q'}{(n - 1)V_n r_+^{n-1}} d(Q - q') \right] . \tag{24}
\]

Using the area-entropy formulae of the EH: \( S_{EH} = V_n r_+^n / 4 = 4\pi r_+^n / (n\omega_n) \), one can prove an identity which is essentially equivalent to the differential form of the first law of black hole thermodynamics

\[
\frac{\pi}{\kappa_+(M, Q, r_+)} \left[ dM - \frac{Q}{(n - 1)V_n r_+^{n-1}} dQ \right] = \frac{2\pi}{\omega_n} r_+^{n-1} dr_+ = \frac{1}{2} dS_{EH}. \tag{25}
\]
By means of this identity, we can easily finish the integration and obtain

\[
\text{Im} S = - \int_{r_i}^{r_f} \frac{2\pi r_i^{n-1} dr_i}{\omega_n} = - \frac{2\pi}{n\omega_n} (r_f^n - r_i^n) \\
\equiv - \frac{1}{2} \int_{(M, Q)}^{(M-\omega, Q-q)} dS' = - \frac{1}{2} \Delta S_{EH}. \tag{26}
\]

The tunnelling probability for the outgoing radiation at the EH is still represented by Eq. (13) with \( \Delta S_{EH} = S_{EH}(M-\omega, Q-q) - S_{EH}(M, Q) \) being the difference of black hole entropy after and before the particle emission. Obviously, the derived emission spectrum actually deviates from pure thermality.

Finally, we turn to calculate the tunnelling rate at the CH. Different from the tunnelling behavior across the EH, the charged particle is found to tunnel into the CH. Ignoring the effect of the EH and taking into account the energy conservation and electric charge conservation, the mass and charge parameters in Eqs. (6) and (8) will be replaced with \( M \rightarrow M + \omega \) and \( Q \rightarrow Q + q \), and generally speaking, the size of EH will expand and the radius of CH will shrink when the charged particle tunnels into the CH. At this moment, the radial geodesics of the charged massive particle tunnelling into the CH and the non-vanishing component of electro-magnetic potential are

\[
\dot{r} = - \frac{f(M + \omega, Q + q, r)}{2\sqrt{1 - f(M + \omega, Q + q, r)}} \approx - \kappa_c(M + \omega, Q + q, r_c)(r - r_c), \tag{27}
\]

\[
A_t = - \frac{Q + q}{(n-1)V_n r_{n-1}}. \tag{28}
\]

It should be stressed that the first law of thermodynamics is satisfied both at the EH and CH, respectively, but one must note that the definitions of energy, temperature, and electric potential receive an opposite sign in both cases [21]. With this issue in mind, the calculation of the tunnelling rate at the CH is almost similar to those did in the EH case. In this way, one can get

\[
\text{Im} S = - \text{Im} \int_{r_{ic}}^{r_{fc}} \int_{(M-\omega, Q-q)}^{(M, Q)} \left[ d(M + \omega') - \frac{Q + q'}{(n-1)V_n r_{n-1}} d(Q + q') \right] dr \\
= \text{Im} \int_{(M-\omega, Q-q)}^{(M-\omega, Q-q)} \int_{r_{ic}}^{r_{fc}} 2\sqrt{1 - f(M + \omega', Q + q', r)} \left[ d(M + \omega') \\
- \frac{Q + q'}{(n-1)V_n r_{n-1}} d(Q + q') \right] dr, \tag{29}
\]

where \( r_{ic} \) and \( r_{fc} \) are the locations of the CH before and after the charged particle tunnels into the CH, respectively. Carrying out the integration, we
have

\[
\text{Im} S = - \int_{(-M, -Q)}^{(-M-\omega, -Q-q)} \frac{\pi}{\kappa_c(M + \omega', Q + q', r'_c)} \left[ d(M + \omega') - \frac{Q + q'}{(n-1)V_n r'_c^{(n-1)}} d(Q + q') \right]
\]

\[
= -\frac{1}{2} \int_{(M, Q)}^{(M+\omega, Q+q)} d\mathcal{S}' = -\frac{1}{2} \Delta S_{CH} = -\frac{2\pi}{n\omega_n} \left( r_{fc}^n - r_{ic}^n \right),
\]

in which we have used an identity similar to Eq. (25) that holds at the CH. Thus the tunnelling rate is still given by Eq. (17) where \( \Delta S_{CH} = S_{CH}(M + \omega, Q + q) - S_{CH}(M, Q) \) is the change of entropy of the CH after and before the particle tunnels into the CH. Once again, the actual radiant spectrum is no longer precisely thermal and fits well to the universal relation presented in the standard semi-classical tunnelling formalism [3,4].

5 Concluding remarks

In summary, we have investigated Hawking radiation of the massless uncharged particles and massive charged particles as a semi-classical tunnelling process from the black hole event horizon and the cosmological horizon of higher dimensional Reissner-Nordström black holes in de Sitter space-time, respectively. Our results show that the tunnelling rate is related to the change of Bekenstein-Hawking entropy and the exact radiant spectrum is no longer precisely thermal after considering the black hole background as dynamical and incorporating the self-gravitation effect of the emitted particles when the energy conservation and electric charge conservation are taken into account, thus perfectly extending the Kraus-Parikh-Wilczek’s semi-classical tunnelling framework in the four-dimensional case to the higher dimensional spherically symmetric case.

The leading-order, energy-independent term in the emission spectrum is the well-known pure thermal spectrum, but the energy-dependent terms when computed to any desired order in \( \omega \), are indicative of a “greybody” factor in the emission spectrum. This means that a deviation from pure thermality probably have an important effect on the number of the brane localized particles emitted per unit time. This issue deserves further research in the future.
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