Abstract

The nonet symmetry scheme seems to describe rather well the masses and $\eta - \eta'$ mixing angle of the ground state pseudo-scalar mesons and is thus expected to be also a good approximation for the matrix elements of the pseudo-scalar density operators which play an important role in charmless two-body $B$ decays with $\eta$ or $\eta'$ in the final state. In this talk, I would like to report on a recent work on the $B^- \to K^- \eta, K^- \eta'$ decay using nonet symmetry for the matrix elements of pseudo-scalar density operators. We find that the branching ratio $B \to PP$, with an $\eta$ meson in the final state agrees well with data, while those with an $\eta'$ meson are underestimated by $20-30\%$. This could be considered as a more or less successful prediction for QCDF, considering the theoretical uncertainties involved. This could also indicate that an additional power-suppressed terms could bring the branching ratio close to experiment, as with the $B \to K^*\pi$ and $B \to K^*\eta$ decay for which the measured branching ratios are much bigger than the QCDF predictions.

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I. INTRODUCTION

The $B \to K\eta, K\eta'$ decays have been analysed in recent papers [1, 2, 3] in QCD Factorization (QCDF), in perturbative QCD (pQCD) [4, 5] and in soft collinear effective theory (SCET) [6]. In QCDF the $B \to K\pi$ branching ratio could be understood with a moderate contribution from annihilation terms [7, 8]. Similarly, without fine tuning, the $B \to K\eta'$ branching ratio is predicted to be larger than that of $B \to K\pi$ in qualitative agreement with experiment, but is still underestimated by $20-30\%$ compared with the measured value.

Apart from the power-suppressed $O(1/m_b)$ annihilation terms, the main theoretical uncertainties are the $B \to \eta'$ transition form factor and the pseudo-scalar density matrix elements for $\eta'$. Historically, there is an approximate $SU(3)$ relation between the octet pseudo-scalar density [9] but there is no known explicit expression for the singlet pseudo-scalar density in the nonet symmetry scheme. In this talk I would like to discuss a recent work [10] in which we show that nonet symmetry for the quark mass term in $\eta - \eta'$ implies nonet symmetry for the pseudo-scalar density matrix elements. With the nonet symmetry expression for the pseudo-scalar density matrix elements in $\eta - \eta'$, we obtain in QCDF a $B \to K\eta'$ branching ratio, though sufficiently large, is still below the measured value by $20-30\%$, but a large $B \to \eta'$ form factor or additional power-suppressed terms could bring the predicted value closer to experiment.

II. NONET SYMMETRY IN THE $\eta - \eta'$ SYSTEM

Since QCD interactions through the exchange of gluons are flavor-independent, the wave function for the pseudo-scalar meson nonet is also expected to be flavor-independent in the limit of vanishing current quark mass. The quark mass term is the leading term in the large $N_c$ expansion while higher order terms in the chiral Lagrangian [11] is $O(1/N_c)$ and is thus suppressed in the large $N_c$ limit. This justifies the nonet symmetry for the pseudo-scalar meson mass matrix, the off-diagonal quark mass term $< \eta_0 | H_{SB} | \eta_8 >$ then gives an $\eta - \eta'$ mixing angle $\theta = -18^\circ$ in good agreement with the value determined from the two-photon decay width of $\eta$ and $\eta'$ as mentioned in [10]. However, from the Gell-Mann-Okubo (GMO) mass formula, we would have

$$m_{\eta}^2 = m_{8}^2 - \tan^2 \theta (m_{\eta'}^2 - m_{8}^2)$$

(1)

which gives, for $\theta = -18^\circ$, $m_\eta = 483$ MeV, about 60 MeV below experiment. This indicates that chiral logarithms and chiral Lagrangian higher order terms [11, 12] which are second order in $SU(3)$ breaking as the $(\sin \theta)^2$ term in Eq.(1) could contribute to $m_8$ and shift $m_\eta$ upward by a similar
amount with the result that the \( \eta \) mass is very close to the GMO value and a large \( \eta - \eta' \) mixing angle is obtained, rather than the small value of \(-10^\circ\) given by the GMO formula for \( m_s \). We now use the nonet symmetry mass term to derive the pseudo-scalar density matrix element for \( \eta, \eta' \) which allows a calculation of \( B \rightarrow K\eta' \) as shown in the following sections.

The usual way to derive the pseudo-scalar density matrix elements between the vacuum and the pseudo-scalar meson nonet is to consider the matrix elements of the divergence of the axial vector currents between the vacuum and pseudo-scalar meson nonet. For \( \pi, K \) meson, we have [10]:

\[
f_\pi B_0(m_u + m_d) = (m_u + m_d)(0|\bar{u} i\gamma_5 d|ud),
\]

\[
f_K B_0(m_u + m_s) = (m_u + m_s)(0|\bar{u} i\gamma_5 s|us).
\]

and for \( \pi^0 \)

\[
f_u B_0(m_u + m_d) = (m_u + m_d)(0|\bar{u} i\gamma_5 u|uu).
\]

with the \( \pi \) and \( K \) meson masses the usual expressions in terms of \( B_0 \) and the current quark mass [11, 13]. To first order in \( SU(3) \) breaking quark mass term, the decay constants \( f_q \) (\( q = u, d, s \)) is (putting \( f_{q\bar{q}} = f_q \)),

\[
f_\pi = f_{ud} \approx f_u, \quad f_K = f_{us} = (1 + \epsilon) f_{ud},
\]

\[
f_s = (1 + 2 \epsilon) f_u \approx (1 + \epsilon) f_K.
\]

Consider now the divergence of the \( I = 0 \) axial vector current:

\[
A_{\mu} = (\bar{u} \gamma_\mu \gamma_5 u + \bar{d} \gamma_\mu \gamma_5 d), \quad A_{s\mu} = \bar{s} \gamma_\mu \gamma_5 s.
\]

we have:

\[
\partial A_n = 2(m_u \bar{u} i\gamma_5 u + m_d \bar{d} i\gamma_5 d) + 2 \frac{\alpha_s}{4\pi} G \tilde{G}.
\]

\[
\partial A_s = 2m_s \bar{s} i\gamma_5 s + \frac{\alpha_s}{4\pi} G \tilde{G}.
\]

Taking the matrix elements of \( \partial A_n \) and \( \partial A_s \) between the vacuum and \( \eta_{0,8} \), we obtain:

\[
f_u \frac{1}{\sqrt{3}}(m_0^2 + B_0 \frac{2}{3}(m_s + 2\hat{m})) = f_u \frac{1}{\sqrt{3}}m_0^2 - f_u \frac{1}{\sqrt{6}} B_0 \frac{2\sqrt{2}}{3}(\hat{m} - m_s) + 2 \frac{1}{\sqrt{3}} \hat{m}(0|\bar{u} i\gamma_5 u|uu),
\]

\[
f_s \frac{1}{\sqrt{3}}(m_0^2 + B_0 \frac{2}{3}(m_s + 2\hat{m})) = f_s \frac{1}{\sqrt{3}}m_0^2 - f_s \frac{2}{\sqrt{6}} B_0 \frac{2\sqrt{2}}{3}(\hat{m} - m_s) + 2 \frac{1}{\sqrt{3}} m_s(0|\bar{s} i\gamma_5 s|ss).
\]

Similarly, for \( \eta_8 \):

\[
f_u \frac{1}{\sqrt{3}} B_0 \frac{2}{3}(2m_s + \hat{m}) = -f_u \frac{1}{\sqrt{3}} B_0 \frac{2\sqrt{2}}{3}(\hat{m} - m_s) + 2 \frac{1}{\sqrt{6}} \hat{m}(0|\bar{u} i\gamma_5 u|uu),
\]

\[
-f_s \frac{2}{\sqrt{6}} B_0 \frac{2}{3}(2m_s + \hat{m}) = -f_s \frac{1}{\sqrt{3}} B_0 \frac{2\sqrt{2}}{3}(\hat{m} - m_s) - 2 \frac{2}{\sqrt{6}} m_s(0|\bar{s} i\gamma_5 s|ss).
\]
In deriving the above expressions, we have used the nonet symmetry mass formula, i.e \( m_\eta^2 = B_0 \frac{2}{3} (2m_s + \hat{m}) \), \( m_0^2 = \bar{m}_0^2 + B_0 \frac{2}{3}(m_s + 2\hat{m}) \) and \( m_{08}^2 = B_0 \frac{2}{3}\sqrt{2}(-m_s + \hat{m}) \) where \( \hat{m} = (m_u + m_d)/2 \) and \( \bar{m}_0 \) is the anomaly contribution to \( m_0 \), the singlet \( \eta_0 \) mass. The second term on the r.h.s of Eqs. \((8,9)\) and the first term on the r.h.s of Eqs. \((10,11)\) are the pole terms due to \( \eta - \eta' \) mixing. In the limit \( m_u = m_d = 0 \), the l.h.s and r.h.s of Eq. \((10)\) become \( f_\eta m_8^2/\sqrt{2} \) in agreement with the divergence equation Eq. \((5)\). Because of cancellation between the pole contribution and other quark mass terms, Eqs. \((8,9)\) are reduced to the simplified form of Eq. \((2)\) or Eq. \((3)\). The pseudo-scalar density matrix elements in \( \eta_0 \) are then given by:

\[
\langle 0 | \bar{u} i \gamma_5 u | u \bar{u} \rangle = B_0 f_\eta, \quad \langle 0 | \bar{s} i \gamma_5 s | s \bar{s} \rangle = B_0 f_s. \tag{12}
\]

The same expression for \( \eta_8 \) is obtained similarly from Eqs. \((10,11)\). Thus, \( \langle 0 | \bar{u} i \gamma_5 d | \pi^+ \rangle \), \( \langle 0 | \bar{u} i \gamma_5 u | \pi^0 \rangle \) and \( \langle 0 | \bar{s} i \gamma_5 s | K^+ \rangle \), and the matrix element \( \langle 0 | \bar{u} i \gamma_5 u | u \bar{u} \rangle \) and \( \langle 0 | \bar{s} i \gamma_5 s | s \bar{s} \rangle \) in \( \eta_{0,8} \) are, apart from the decay constant \( f_q \), essentially the same, given by the parameter \( B_0 \) and are consistent with nonet symmetry.

Since experimentally, \( m_{08}^2 = -(0.81 \pm 0.05) m_K^2 \) is rather close to the nonet symmetry value of \( m_{08}^2 \approx -0.90 m_K^2 \), we expect nonet symmetry for the pseudo-scalar density matrix elements in \( \eta - \eta' \) would be valid to this accuracy. Since \( m_8^2 \) gets about 15% increase from higher order terms \( L_4, L_5, L_6, L_8 \) and chiral logarithms, Eqs. \((10,11)\) show that \( \langle 0 | \bar{s} i \gamma_5 s | s \bar{s} \rangle \) in \( \eta \) will be increased by a similar amount. A possible similar 15% increase for \( m_8^2 \) would also increase \( \langle 0 | \bar{s} i \gamma_5 s | s \bar{s} \rangle \) in \( \eta_0 \) by a similar amount and would be additional source of enhancement for the \( B \to K \eta' \) branching ratio.

### III. THE \( B^- \to K^-(\eta, \eta') \) AND \( B^- \to \pi^-(\eta, \eta') \) DECAYS

The \( B \to M_1 M_2 \) decay amplitude in QCD Factorization(QCDF) is given by \((2,8)\):

\[
A(B \to M_1 M_2) = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pb} V_{ps}^* \times \left( \sum_{i=1}^{10} a_i^p \langle M_1 M_2 | O_i | B \rangle_H + \sum_{i} f_B f_{M_1} f_{M_2} b_i \right), \tag{13}
\]

where the QCD coefficients \( a_i^p \) contain vertex corrections, penguin corrections, and hard spectator scattering contributions. The hadronic matrix elements \( \langle M_1 M_2 | O_i | B \rangle_H \) of the tree and penguin operators \( O_i \) are given by factorization model \((2,14)\) and \( b_i \) are annihilation terms. The values for \( a_i^p, p = u, c \) and \( b_i \) computed from the expressions in \((4,8)\) at the renormalization scale \( \mu = m_b \) and with \( m_b = 4.2 \text{GeV} \) are given in the published work \((10)\). We estimate the CKM matrix element
$V_{ub}$ and the CKM angle $\gamma$ from the $(db)$ unitarity triangle \cite{15}:

$$|V_{ub}| = \left| \frac{V_{cb}^* V_{cd}}{V_{td}^* V_{ts}} \right| \sin \beta \sqrt{1 + \frac{\cos^2 \alpha}{\sin^2 \alpha}} \sin \gamma \sqrt{1 + \cos^2 \alpha \sin^2 \alpha}.$$ \hspace{1cm} (14)

With $\alpha = (99^{+13}_{-9})^\circ$ \cite{16} and $|V_{cb}| = (41.78 \pm 0.30 \pm 0.08) \times 10^{-3}$ \cite{17}, we find

$$|V_{ub}| = 3.60 \times 10^{-3}.$$ \hspace{1cm} (15)

which is quite close to the exclusive data \cite{17} $|V_{ub}| = (3.33 - 3.51) \times 10^{-3}$.

Similarly, we use the current determination $|V_{td}/V_{ts}| = (0.208^{+0.008}_{-0.006})$ from the $B_s^0 - \bar{B_s}^0$ mixing measurements \cite{18} to obtain the angle $\gamma$:

$$|V_{td}| = \left| \frac{V_{cb}^* V_{cd}}{V_{td}^* V_{ts}} \right| \sin \gamma \sqrt{1 + \frac{\cos^2 \alpha}{\sin^2 \alpha}}.$$ \hspace{1cm} (16)

which gives $\gamma = 66^\circ$ ($|V_{ts}| = 1$) and $\alpha = 91.8^\circ$, in good agreement with the value found in the current UT-fit value of $(88 \pm 16)^\circ$. For other parameters, we use $m_s(2 \text{ GeV}) = 80 \text{ MeV}$, $f_u = f_\pi$, $f_s = f_\pi \left(1 + 2\left(\frac{f_K}{f_\pi} - 1\right)\right)$, the current theoretical values \cite{19}:

$$F^{B\pi}_0(0) = 0.258, \quad F^{BK}_0(0) = 0.33,$$ \hspace{1cm} (17)

$F^{B\eta, B\eta'}$ from the $u$ quark content in $\eta$ and $\eta'$:

$$F^{B\eta}(0) = 0.58 F^{B\pi}_0(0), \quad F^{B\eta'}(0) = 0.40 F^{B\pi}_0(0).$$ \hspace{1cm} (18)

and the pseudo-scalar density matrix elements:

$$\langle 0|\bar{s} i\gamma_5 s|\eta \rangle = C_\eta B_0 f_s, \quad \langle 0|\bar{s} i\gamma_5 s|\eta' \rangle = C_{\eta'} B_0 f_s,$$ \hspace{1cm} (19)

obtained with the $s$ quark content $C_\eta = -0.57$, $C_{\eta'} = 0.82$ and an $\eta - \eta'$ mixing angle $(-22 \pm 3)^\circ$ \cite{13}.

### TABLE I: The branching ratio $\mathcal{B}(B \to P\eta, P\eta')$ in QCDF

| Decay Modes | QCDF BR ($\times 10^{-6}$) | Exp. \cite{20} |
|-------------|-----------------------------|----------------|
| $B^- \to \pi^- \pi^0$ | 5.05 | 5.7 ± 0.4 |
| $\bar{B}^0 \to K^- \pi^+$ | 18.25 | 19.04 ± 0.6 |
| $B^- \to \pi^- \eta$ | 3.39 | 4.4 ± 0.4 |
| $B^- \to \pi^- \eta'$ | 1.91 | 2.6^{+0.6}_{-0.5} |
| $B^- \to K^- \eta$ | 0.43 | 2.2 ± 0.3 |
| $B^- \to K^- \eta'$ | 48.26 | 69.7^{+2.8}_{-2.7} |
As shown in Table I with a moderate annihilation term ($\rho_A = 0.6$) and the current theoretical value for the $F^{B\pi}$ and $F^{BK}$ form factors, QCDF predictions are in reasonable agreement with experiment, except for the $B \to K\eta'$ branching ratio which is underestimated by $20 - 30\%$. One could increase the $F^{B\eta'}$ form factor to produce better agreement with experiment for $B^- \to \pi^- \eta'$ for which the prediction in Table I is below the Babar value of $(4.0 \pm 0.8 \pm 0.4) \times 10^{-6}$ and to bring the predicted $B \to K\eta'$ branching ratio closer to experiment, but so far there seems to be no evidence for a large $B \to \eta'$ form factor compared with the nonet symmetry value as seen from the new Babar upper limit $B(B^+ \to \eta'\ell^+\nu)/B(B^+ \to \eta\ell^+\nu) < 0.57$ which is consistent with nonet symmetry for the $B \to \eta, \eta'$ form factors given in Eq. (18).

IV. CONCLUSION

We have shown that nonet symmetry for the pseudo-scalar meson mass term implies nonet symmetry for the pseudo-scalar density matrix elements. With this approximate relation, we obtained an improved estimate for the $B \to P\eta' (P = K, \pi)$ branching ratios. With a moderate annihilation contribution consistent with the measured $B \to K\pi$ branching ratio, we find that a major part of the $B \to K\eta'$ branching ratio could be obtained by QCDF. Without fine tuning or a large $F^{B\to \eta'}$ form factor, we find that the $B \to K\eta'$ branching ratio is underestimated by $20 - 30\%$. This could be considered as a more or less successful prediction for QCDF, considering the theoretical uncertainties involved. This could also indicate that an additional power-suppressed terms could bring the branching ratio close to experiment, as with the $B \to K^*\pi$ and $B \to K^*\eta$ decay for which the measured branching ratios are much bigger than the QCDF prediction.

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