SKEWED PARTON DISTRIBUTIONS AND REAL AND VIRTUAL COMPTON SCATTERING

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The soft physics approach to Compton scattering at moderately large momentum transfer is reviewed. It will be argued that in that approach the Compton cross section as well as other exclusive observables exhibit approximate scaling in a limited range of momentum transfer.

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1 Introduction

QCD provides three valence Fock state contributions to nucleon form factors, real (RCS) and virtual (VCS) Compton scattering at large momentum transfer: a soft overlap term with an active quark and two spectators (see Fig. 1), the asymptotically dominant perturbative contribution where by means of the exchange of two hard gluons the quarks are kept collinear with respect to their parent nucleons (see Fig. 1) and a third contribution that is intermediate between the soft and the perturbative contribution where only one hard gluon is exchanged and one of the three quarks acts as a spectator. Both the soft and the intermediate terms represent power corrections to the perturbative contribution. Higher Fock state contributions are suppressed. The crucial question is what is the relative strengths of the three contributions at experimentally accessible values of momentum transfer, i.e. at $-t$ of the order of 10 MeV$^2$? The pQCD followers (cf. Ref. 1 for recent calculations of RCS) assume the dominance of the perturbative contribution and neglect the other two contributions while the soft physics community presumes the dominance of the overlap contribution. Which group is right is not yet fully decided although comparison with the pion case seems to favour a strong overlap contribution.

In this talk I am going to report on results for Compton scattering obtained within the soft physics approach. It has been shown recently that at moderately large momentum transfer Compton scattering off protons approximately factorises into a hard photon-parton subprocess and soft proton matrix elements described by new form factors specific to Compton scattering.
These form factors, as the ordinary electromagnetic ones, represent moments of skewed parton distributions (SPD). The soft physics approach bears resemblance to the fixed-pole model advocated for in long time ago.

2 The soft physics approach to Compton scattering

For Mandelstam variables, $s$, $t$ and $u$, that are large on a hadronic scale the handbag diagram shown in Fig. 1 describes RCS and VCS. To see this it is of advantage to choose a symmetric frame of reference where the plus and minus light-cone components of $\Delta$ are zero. This implies $t = -\Delta_{+}^{2}$ as well as a vanishing skewness parameter $\zeta = -\Delta_{+}/p_{+}$. To evaluate the SPD appearing in the handbag diagram one may use a Fock state decomposition of the proton and sum over all possible spectator configurations. The crucial assumption is then that the soft hadron wave functions are dominated by virtualities in the range $|k_{i}^{2}| \lesssim \Lambda^{2}$, where $\Lambda$ is a hadronic scale of the order of 1 GeV, and by intrinsic transverse parton momenta, $k_{i}^{\perp}$, defined with respect to their parent hadron’s momentum, that satisfy $k_{i}^{\perp}/x_{i} \lesssim \Lambda^{2}$. Under this assumption factorisation of the Compton amplitude in a hard photon-parton amplitude and $1/x$-moments of SPDs is achieved.

As a consequence of this result the Compton amplitudes conserving the proton helicity are given by

$$\mathcal{M}_{\mu', \mu} = 2\pi\alpha_{em} [\mathcal{H}_{\mu', \mu} (R_{V} + R_{A}) + \mathcal{H}_{\mu', -\mu} (R_{V} - R_{A})].$$

(1)

Proton helicity flip is neglected. $\mu$ and $\mu'$ are the helicities of the incoming and outgoing photon in the photon-proton cms, respectively. The photon-quark subprocess amplitudes, $\mathcal{H}$, are calculated for massless quarks in lowest order QED. The form factors in Eq. (1), $R_{V}$ and $R_{A}$, represent $1/x$-moments.
of SPDs at zero skewedness parameter. $R_V$ is defined by

$$
\sum_a e_a^2 \int_0^1 \frac{dx}{x} \int \frac{dz}{2\pi} e^{i z x + z} \langle p' | \bar{\psi}_a(0) \gamma^+ \psi_a(z^-) - \bar{\psi}_a(z^-) \gamma^+ \psi_a(0) | p \rangle
$$

$$
= R_V(t) \bar{u}(p') \gamma^+ u(p) + R_T(t) \frac{i}{2m} \bar{u}(p') \sigma^{+\rho} \Delta_{\rho} u(p),
$$

\[ (2) \]

where the sum runs over quark flavours $a$ ($u, d, \ldots$), $e_a$ being the electric charge of quark $a$ in units of the positron charge. $R_T$ being related to nucleon helicity flips, is neglected in [3]. There is an analogous equation for the axial vector nucleon matrix element, which defines the form factor $R_A$. Due to time reversal invariance the form factors $R_V, R_A$ etc. are real functions.

As shown in [3] form factors can be represented as generalized Drell-Yan light-cone wave function overlaps. Assuming a plausible Gaussian $k_{\perp}$-dependence of the soft Fock state wave functions, one can explicitly carry out the momentum integrations in the Drell-Yan formula. For simplicity one may further assume a common transverse size parameter, $\bar{a}$, for all Fock states. This immediately allows one to sum over them, without specifying the $x_i$-dependence of the wave functions. One then arrives at

$$
F_1(t) = \sum_a e_a \int dx \exp \left[ \frac{1}{2} \bar{a}^2 t \frac{1-x}{x} \right] \{ q_a(x) - \bar{q}_a(x) \},
$$

$$
R_V(t) = \sum_a e_a^2 \int dx \exp \left[ \frac{1}{2} \bar{a}^2 t \frac{1-x}{x} \right] \{ q_a(x) + \bar{q}_a(x) \},
$$

\[ (3) \]

and the analogue for $R_A$ with $q_a + \bar{q}_a$ replaced by $\Delta q_a + \Delta \bar{q}_a$. $q_a$ and $\Delta q_a$ are the usual unpolarized and polarized parton distributions, respectively. The result for $F_1$ can also been found in [3].

The only parameter appearing in (3) is the effective transverse size parameter $\bar{a}$; it is known to be about 1 GeV$^{-1}$ with an uncertainty of about 20%.

Thus, this parameter only allows some fine tuning of the results for the form factors. Evaluating, for instance, the form factors from the parton distributions derived by Glück et al. (GRV) [3] with $\bar{a} = 1$ GeV$^{-1}$, one already finds good results. Improvements are obtained by treating the lowest three Fock states explicitly with specified $x$-dependencies. For the valence Fock distribution amplitude, for instance, a form proposed in Ref. [9] was used

$$
\Phi_{123}^{BK} = 60 x_1 x_2 x_3 (1 + 3x_1),
$$

\[ (4) \]

valid at a factorisation scale of 1 GeV. All higher Fock states were still treated in the global way, using the parton distributions of Ref. [3] as input. Results for
Figure 2: The Dirac (left) and the vector Compton (right) form factor of the proton as predicted by the soft physics approach. Data are taken from [4]. The data on the magnetic form factor, $G_M$, are shown in order to demonstrate the size of spin-flip effects.

$t^2 F_1$ and $t^2 R_V$ obtained that way are displayed in Fig. 2. Both the scaled form factors, as well as $t^2 R_A$, exhibit broad maxima and, hence, mimic dimensional counting rule behaviour in the $t$-range from about 5 to 15 GeV$^2$. For very large momentum transfer the form factors turn gradually into the soft physics asymptotics $\sim 1/t^4$. This is the region where the perturbative contribution ($\sim 1/t^2$) takes the lead.

It is to be stressed that the perturbative contribution to the proton form factor evaluated from the distribution amplitude [9] within the modified perturbative approach [11] is as small as only a few $\%$ of the experimental value. A similarly small value is expected for Compton scattering. Therefore, the analysis performed in Ref. [3] is to be regarded as a consistent calculation in which the soft contribution clearly dominates for experimentally accessible values of momentum transfer.

3 Results on Compton scattering

The amplitude (1) leads to the RCS cross section

$$\frac{d\sigma}{dt} = \frac{d\hat{\sigma}}{dt} \left[ \frac{1}{2} (R_V^2(t) + R_A^2(t)) - \frac{us}{s^2 + u^2} (R_V^2(t) - R_A^2(t)) \right]. \quad (5)$$

It is given by the Klein-Nishina cross section

$$\frac{d\hat{\sigma}}{dt} = \frac{2\pi\alpha_s^2}{s^2 + u^2} \frac{s^2 + u^2}{-us}, \quad (6)$$

multiplied by a factor that describes the structure of the proton in terms of two form factors. Evidently, if the form factors scale as $1/t^2$, the Compton
cross section would scale as $s^{-6}$ at fixed cm scattering angle $\theta$. In view of the above discussion (see also Fig. 2), one therefore infers that approximate dimensional counting rule behaviour holds in a limited range of energy. The magnitude of the Compton cross section is quite well predicted as is revealed by comparison with the admittedly old data measured at rather low values of $s$, $-t$ and $-u$ (see Fig. 3). Cross sections of similar magnitude have been obtained within the perturbative approach (evaluated from very asymmetric distribution amplitudes) and within the diquark model. The latter model is a variant of the standard perturbative approach in which diquarks are considered as quasi-elementary constituents of the proton.

The soft physics approach also predicts characteristic spin dependencies of the Compton process. Of particular interest is the initial state helicity correlation

$$A_{LL} \frac{d\sigma}{dt} = \frac{2\pi\alpha^2_{em}}{s^2} R_V(t) R_A(t) \left(\frac{u}{s} - \frac{s}{u}\right).$$

(7)

Approximately, $A_{LL}$ is given by the corresponding subprocess helicity correlation $A_{LL} = (s^2 - u^2)/(s^2 + u^2)$ multiplied by the dilution factor $R_A(t)/R_V(t)$. Thus, measurements of both the cross section and the initial state helicity correlation allows one to isolate the two form factors $R_V$ and $R_A$ experimentally. In Fig. 3 predictions for $A_{LL}$ are shown. Interestingly, the diquark model predicts the opposite sign for $A_{LL}$. For the final state helicity correlation, $C_{LL}$, and for the helicity transfer from the incoming photon to the outgoing proton, $K_{LL}$, one finds

$$C_{LL} = K_{LL} = A_{LL},$$

(8)

while the helicity transfer from the initial to the final state photon, $D_{LL}$,
is unity since the photon helicity is conserved. Since in the soft physics approach photon helicity flips are zero and proton helicity flips are neglected many other polarization observables are zero, e.g. all single spin asymmetries or correlations between longitudinal and sideways polarizations (normal to the particle’s momentum and in the scattering plane). In the diquark model\cite{13}, on the other hand, the photon and proton helicity flip amplitudes are non-zero although suppressed by either $1/\sqrt{s}$ or $1/s$; there are even perturbatively generated phase differences between the helicity amplitudes. Therefore, the diquark model predicts small deviations from zero for the latter polarization observables.

The VCS contribution to the unpolarized $ep \to ep\gamma$ cross section is decomposed as (see e.g.\cite{17})

$$\frac{d\sigma}{dsdQ^2d\varphi dt} \sim \frac{d\sigma_T}{dt} + \varepsilon \frac{d\sigma_L}{dt} + \varepsilon \cos 2\varphi \frac{d\sigma_{TT}}{dt} + \sqrt{2\varepsilon(1+\varepsilon)} \cos \varphi \frac{d\sigma_{LT}}{dt},$$

(9)

where $\varepsilon$ denotes the ratio of longitudinal and transverse photon flux in the Compton process and $\varphi$ is the azimuthal angle between the electron and hadron planes. The partial cross sections in (9) refer to the scattering of transverse and longitudinal photons and to the transverse-transverse and longitudinal-transverse interference terms. The VCS partial cross sections have also been calculated in Ref.\cite{3}. Comparing with the only other available results, namely those from the diquark model\cite{17}, one observes that the transverse cross section in both approaches comes out rather similar, while the other three cross sections are generally larger and with a smoother $Q^2$-dependence in the soft physics approach than in the diquark model. In contrast to the diquark model the transverse-transverse interference term, $d\sigma_{TT}/dt$ is strictly zero in the limit $Q^2 = 0$. In addition to VCS the full $ep \to ep\gamma$ cross section receives substantial contributions from the Bethe-Heitler process, in which the final state photon is radiated by the electron. Dominance of the VCS contribution requires high energies, small values of $|\cos \theta|$ and an out-of-plane experiment, i.e. an azimuthal angle larger than about 60°. The relative importance of the VCS and of the Bethe-Heitler contributions is rather similar in both the approaches the soft physics approach\cite{3} and the diquark model\cite{17}.

In Ref.\cite{17} the relevance of the beam asymmetry for $ep \to ep\gamma$

$$A_L = \frac{d\sigma(+)-d\sigma(-)}{d\sigma(+)+d\sigma(-)},$$

(10)

where the labels + and − denote the lepton beam helicity, has been pointed out. It is sensitive to the imaginary part of the longitudinal-transverse interference in the Compton process, while $d\sigma_{LT}/dt$ measures its real part. In the soft
physics approach, \( A_L \) is zero since all amplitudes are real within the accuracy of the calculation. In the diquark model\(^\text{17}\), on the other hand, \( A_L \) is non-zero due to the perturbatively generated phases of the VCS amplitudes. In regions of strong interference between the Compton and the Bethe-Heitler amplitudes the beam asymmetry is even spectacularly enhanced. In the standard perturbative approach\(^\text{14}\) a non-zero value of \( A_L \) is also to be expected.

### 4 Scaling - evidence for pQCD?

It is particularly interesting that the soft physics approach can account for the experimentally observed approximate scaling, i.e dimensional counting rule behaviour, at least for Compton scattering and for form factors. One may object that the perturbative explanation (leaving aside the logarithms from the running of \( \alpha_s \) and from the evolution) works for many exclusive reactions, while in the soft physics approach the approximate counting rule behaviour is accidental, depending on specific properties of a given reaction. It however seems that the approximate counting rule behaviour is an unavoidable feature of the soft physics approach. As discussed in Ref.\(^\text{3}\) similar arguments as for Compton scattering lead to a factorisation of baryon-baryon and meson-baryon amplitudes into hard scatterings of spin 1/2 partons and soft hadronic matrix elements which are represented by form factors analogue to the electromagnetic or Compton ones. In elastic pp scattering, for instance, the form factor

\[
F_{pp}^V(t) = \sum_a \int dx \exp \left[ \frac{1}{2} \hat{a}^2 t \frac{1-x}{x} \right] \{ q_a(x) + \bar{q}_a(x) \} 
\]

appears. All these form factors are smooth functions of the momentum transfer and, if scaled by \( t^2 \), exhibit a broad maximum in the \(-t\)-range from about 5 to 15 GeV\(^2\), set by the transverse hadron size, i.e. by a scale of order 1 GeV\(^{-1}\). The position, \( t_0 \), of the maximum of \( t^2 F_i \), where \( F_i \) is any of the soft form factors, is determined by the solution of the implicit equation

\[
- t = 4 \hat{a}^{-2} \left( \frac{1-x}{x} \right)^{-1}_{F_i,t}.
\]

The mean value \( \langle \frac{1-x}{x} \rangle \) comes out around 0.5 at \( t = t_0 \), hence, \( t_0 \approx 8 \hat{a}^2 \). Since both sides of Eq. \(\text{12}\) increase with \(-t\) the maximum of the scaled form factor, \( F_i \), is quite broad. The scaling behaviour of the form factors lead to approximative \( s^{-10} \) \((s^{-8})\) scaling of \( d\sigma/dt \) in baryon (meson)-baryon scattering around \( s = 10 \text{ GeV}^2 \). For elastic proton-proton scattering, shown in Fig. \(\text{1}\) fair agreement with experiment is obtained. The proton-proton data\(^\text{18}\) show
fluctuations superimposed to the $s^{-10}$ behaviour. These fluctuations, if a real dynamical feature, tell us that there still is another momentum scale relevant in that kinematical region, contradicting the very idea of dimensional scaling. Theoretical interpretations of these fluctuations have been attempted in\textsuperscript{19}.

5 Summary

The soft physics approach leads to detailed predictions for RCS and VCS as well as for form factors. The predictions exhibit interesting features and characteristic spin dependences with marked differences to other approaches. Dimensional counting rule behaviour for form factors, Compton scattering and perhaps for other exclusive observables is mimicked in a limited range of momentum transfer. This tells us that it is premature to infer the dominance of perturbative physics from the observed scaling behaviour. The soft contributions although formally representing power corrections to the asymptotically leading perturbative ones, seem to dominate form factors and Compton scattering for momentum transfers around 10 GeV\textsuperscript{2}. However, a severe confrontation of this approach with accurate large momentum transfer data on RCS and VCS is still pending.

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References

1. A. Kronfeld and B. Nižić, Phys. Rev. D44, 3445 (1991); Erratum ibid. D46, 2272 (1992); M. Vanderhaeghen, P.A.M. Guichon and J. Van de Wiele, Nucl. Phys. A622, 144c (1997).
2. R. Jakob and P. Kroll, Phys. Lett. B315, 463 (1993), Erratum ibid. B319, 545 (1993); P. Kroll and M. Raulfs, Phys. Lett. B387, 848 (1996); V. Braun and I. Halperin, Phys. Lett. B328, 457 (1994); L.S. Kisslinger and S.W. Wang, Nucl. Phys. B399, 63 (1993).
3. M. Diehl, T. Feldmann, R. Jakob and P. Kroll, Eur. Phys. J. C8, 409 (1999) and hep-ph/9903268, to be published in Phys. Lett. B.
4. A.V. Radyushkin, Phys. Rev. D58, 114008 (1998).
5. D. Müller et al., Fortschr. Physik 42, 101 (1994), hep-ph/9812448; X. Ji, Phys. Rev. Lett. 78, 610 (1997); Phys. Rev. D55, 7114 (1997); A.V. Radyushkin, Phys. Rev. D56, 5524 (1997).
6. S.J. Brodsky, F.E. Close and J.F. Gunion, Phys. Rev. D6, 177 (1972).
7. V. Barone et al., Z. Phys. C58, 541 (1993); A.V. Afanasev, hep-ph/9808291.
8. M. Glück, E. Reya and A. Vogt, Z. Phys. C67, 433 (1995); Eur. Phys. J. C5, 461 (1998); M. Glück, E. Reya, M. Stratmann and W. Vogelsang, Phys. Rev. D53, 4775 (1996).
9. J. Bolz and P. Kroll, Z. Phys. A356, 327 (1996).
10. A.F. Sill et al., Phys. Rev. D48, 29 (1993).
11. J. Bolz, R. Jakob, P. Kroll, M. Bergmann and N.G. Stefanis, Z. Phys. C66, 267 (1995).
12. M.A. Shupe et al., Phys. Rev. D19, 1921 (1979).
13. P. Kroll, M. Schürmann and W. Schweiger, Intern. J. Mod. Phys. A6, 4107 (1991).
14. G.P. Lepage and S.J. Brodsky, Phys. Rev. D22, 2157 (1980).
15. M. Anselmino, P. Kroll and B. Pire, Z. Phys. C36, 89 (1987).
16. A.M. Nathan, hep-ph/9807397 and these proceedings.
17. P. Kroll, M. Schürmann and P.A.M. Guichon, Nucl. Phys. A598, 435 (1996).
18. C.W. Akerlof et al., Phys. Rev. 159, 1138 (1967); J.V. Allaby et al., Phys. Lett. 25B, 156 (1967).
19. B. Schrempp and F. Schrempp, Phys. Lett. 55B, 303 (1975); B. Pire and J.P. Ralston, Phys. Lett. 117B, 233 (1982).