Abstract Several kinds of astronomical observations, interpreted in the framework of the standard Friedmann–Robertson–Walker cosmology, have indicated that our universe is dominated by a Cosmological Constant. The dimming of distant Type Ia supernovae suggests that the expansion rate is accelerating, as if driven by vacuum energy, and this has been indirectly substantiated through studies of angular anisotropies in the cosmic microwave background (CMB) and of spatial correlations in the large-scale structure (LSS) of galaxies. However there is no compelling direct evidence yet for (the dynamical effects of) dark energy. The precision CMB data can be equally well fitted without dark energy if the spectrum of primordial density fluctuations is not quite scale-free and if the Hubble constant is lower globally than its locally measured value. The LSS data can also be satisfactorily fitted if there is a small component of hot dark matter, as would be provided by neutrinos of mass $\sim 0.5$ eV. Although such an Einstein–de Sitter model cannot explain the SNe Ia Hubble diagram or the position of the ‘baryon acoustic oscillation’ peak in the autocorrelation function of galaxies, it may be possible to do so e.g. in an inhomogeneous Lemaitre–Tolman–Bondi cosmology where we are located in a void which is expanding faster than the average. Such alternatives may seem contrived but this must be weighed against our lack of any fundamental understanding of the inferred tiny energy scale of the dark energy. It may well be an artifact of an oversimplified cosmological model, rather than having physical reality.

Keywords Cosmic Microwave Background · Dark energy · Inflation · Large-scale Structure

1 Introduction

Following his formulation of general relativity Einstein [29] boldly applied the theory to the universe as a whole. The first cosmological model was static to match the known universe, which at that time was restricted to the Milky way, and to achieve this Einstein introduced the ‘cosmological constant’ term (for a historical perspective, see [71]). Within a decade however Slipher and Hubble demonstrated that the nebulae on the sky are in fact other ‘island universes’ like the Milky Way and that they are mainly receding from us — the universe is expanding. Einstein wrote to Weyl in 1933: “If there is no quasi-static world, then away with the cosmological term”.

This however is not a matter of choice since general coordinate invariance, which Einstein’s equation is based on, permits an arbitrary constant (multiplied by the metric tensor) to be added to the lhs:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \lambda_{\text{metric}}g_{\mu\nu} = \frac{-T_{\mu\nu}}{M_P^2}. \quad (1)$$

Here we have written Newton’s constant, $G_N \equiv 1/8\pi M_P^2$ where $M_P \approx 2.4 \times 10^{18}$ GeV is the (reduced) Planck mass in natural units ($\hbar = k_B = c = 1$). With the subsequent development of quantum field
theory it became clear that the energy-momentum tensor on the rhs can also be freely scaled by another additive constant multiplying the metric tensor, which reflects the (Lorentz invariant) energy density of the vacuum:

$$\langle T_{\mu\nu} \rangle_{\text{fields}} = -\langle \rho \rangle_{\text{fields}} g_{\mu\nu}. \quad (2)$$

This contribution from the matter sector adds to the “bare” term from the background geometry, yielding an effective cosmological constant:

$$\Lambda = \lambda_{\text{metric}} + \frac{\langle \rho \rangle_{\text{fields}}}{M_p^2}, \quad (3)$$

or, correspondingly, an effective vacuum energy:

$$\rho_v \equiv \Lambda M_p^2. \quad (4)$$

Einstein assumed without any observational evidence that the universe is perfectly homogeneous. We know that the universe is quite isotropic about us so this is in fact likely if we are not in a special location — an assumption later dignified by Milne as the ‘Cosmological Principle’. Then using the maximally symmetric Robertson-Walker metric to describe space-time

$$\text{d}s^2 \equiv g_{\mu\nu} \text{d}x^\mu \text{d}x^\nu = \text{d}t^2 - a^2(t)[\text{d}r^2/(1 - kr^2) + r^2 d\Omega^2], \quad (5)$$

we obtain the Friedmann equations describing the evolution of the cosmological scale-factor $a(t)$:

$$H^2 \equiv \left( \frac{\dot{a}}{a} \right)^2 = \frac{\rho}{3M_p^2} - \frac{\kappa}{a^2} + \frac{\Lambda}{3}, \quad \frac{\ddot{a}}{a} = -\frac{1}{6M_p^2}(\rho + 3p) + \frac{\Lambda}{3}, \quad (6)$$

where $\kappa = 0, \pm 1$ is the 3-space curvature signature and for ordinary matter (‘dust’) and radiation we have used the ‘ideal gas’ form: $T^{\mu\nu} = pg^{\mu\nu} + (p + \rho)u^\mu u^\nu$, with $u^\mu \equiv \text{d}x^\mu/\text{d}s$. The conservation equation $T^{\mu\nu}_{\text{cons}} = 0$ implies $d(\rho a^3)/\text{d}a = -3pa^2$, so given the ‘equation of state parameter’ $w \equiv p/\rho$, the evolution history can now be constructed. Since the redshift is $z \equiv a/a_0 - 1$, for non-relativistic particles with $w \simeq 0$, $\rho_{\text{NR}} \propto (1 + z)^{-3}$, while for relativistic particles with $w = 1/3$, $\rho_{\text{R}} \propto (1 + z)^{-4}$, but for the cosmological constant, $w = -1$ and $\rho_v = \text{constant}$. Thus radiation was dynamically important only in the early universe (for $z \gtrsim 10^4$) and for most of the expansion history only non-relativistic matter is relevant. The Hubble equation can be rewritten with reference to the present epoch (subscript 0) as

$$H^2 = H_0^2 \left[ \Omega_m(1 + z)^3 + \Omega_{\kappa}(1 + z)^2 + \Omega_\Lambda \right], \quad (7)$$

$$\Omega_m \equiv \frac{\rho_m}{\rho_c}, \quad \Omega_{\kappa} \equiv -\frac{\kappa}{a_0^2 H_0^2}, \quad \Omega_\Lambda \equiv \frac{\Lambda}{3H_0^2}, \quad (8)$$

where $\rho_c \equiv 3H_0^2 M_p^2/8\pi \simeq (3 \times 10^{-12} \text{ GeV})^4 h^2$ is the ‘critical density’ and the present Hubble parameter is $H_0 \equiv 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$ with $h \simeq 0.7$, i.e. about $10^{-42}$ GeV. This yields the sum rule

$$\Omega_m + \Omega_{\kappa} + \Omega_\Lambda = 1, \quad (9)$$

so cosmological models can be usefully displayed on a ‘cosmic triangle’ [7].

As emphasised in an influential review [39], given that the density parameters $\Omega_m$ and $\Omega_{\kappa}$ were observationally constrained already to be not much larger than unity, the two terms in eq. (3) are required to somehow conspire to cancel each other in order to satisfy the approximate constraint

$$|\Lambda| \ll H_0^2, \quad (10)$$

thus bounding the present vacuum energy density by $\rho_v \ll 10^{-47} \text{ GeV}^4$ which is a factor of over $10^{120}$ below its “natural” value of $\sim M_p^4$ — the ‘cosmological constant problem’. Subsequently, several types of evidence have been advanced to argue that this inequality is in fact saturated with $\Omega_\Lambda \simeq 0.7$ ($\Rightarrow \Lambda \simeq 2H_0^2$), $\Omega_m \simeq 0.3$, $\Omega_{\kappa} \simeq 0$ (see [60,57]), i.e. there is non-zero vacuum energy of just the right order of magnitude so as to be detectable today.

In the Lagrangian of the Standard $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ Model (SM) of electroweak and strong interactions, the term corresponding to the cosmological constant is one of the two ‘super-renormalisable’ terms allowed by the gauge symmetries, the second one being the quadratic divergence
in the mass of fundamental scalar fields due to radiative corrections (see \[89\]). To tame the latter sufficiently in order to explain the experimental success of the SM has required the introduction of a ‘supersymmetry’ between bosonic and fermionic fields which is ‘softly broken’ at about the Fermi scale, \(M_{\text{EW}} \sim G_F^{-1/2} \simeq 246 \text{ GeV}\). Thus the cutoff scale of the SM, viewed as an effective field theory, can be lowered from \(M_P\) down to \(M_{\text{EW}}\), at the expense of introducing over 100 new parameters in the Lagrangian, as well as requiring delicate control of the non-renormalisable operators which can generate flavour-changing neutral currents, nucleon decay etc, so as not to violate experimental bounds. This implies a minimum contribution to the vacuum energy density from quantum fluctuations of \(O(M_{\text{EW}})\), i.e. halfway on a logarithmic scale down from \(M_P\) to the energy scale of \(O(M_{\text{EW}}^4/M_P)\) corresponding to the observationally indicated vacuum energy. Thus even the introduction of supersymmetry cannot eradicate a discrepancy by a factor of at least \(10^{60}\) between the natural expectation and observation.

It is likely that a satisfactory resolution of the cosmological constant problem can be achieved only in a quantum theory of gravity. Recent developments in string theory and the possibility that there exist new dimensions in Nature have generated many interesting ideas concerning possible values of \(\Lambda\) (see e.g. \[86, 55, 54\]). Nevertheless it is the case that there is no accepted solution to the enormous discrepancy discussed above. The problem is not new but there has always been the hope that some day we would understand why \(\Lambda\) is exactly zero, perhaps due to a new symmetry principle. However if it is in fact non-zero and dynamically important today, the crisis is even more severe since this also raises a cosmic ‘coincidence’ problem, viz. why is the present epoch special? It has been suggested that the ‘dark energy’ may not be a cosmological constant but rather the slowly evolving potential energy \(V(\phi)\) of a hypothetical scalar field \(\phi\) named ‘quintessence’ which can track the matter energy density (see \[22\]). However this is equally fine-tuned since we need \(V^{1/4} \sim 10^{-13} \text{ GeV}\) but \(\sqrt{d^2V/d\phi^2} \sim H_0 \sim 10^{-42} \text{ GeV}\) (in order that the evolution of \(\phi\) be sufficiently slowed by the Hubble expansion), and moreover does not address the fundamental issue\(^2\) namely why are all the other possible contributions to the vacuum energy absent? Given the no-go theorem against any dynamical cancellation mechanism in eq.\((3)\) in the framework of general relativity \[83\], it might appear that solving the problem will necessarily require modification of our understanding of gravity. Interesting suggestions have been made in this context e.g. the DGP model in which gravity alone propagates in a new dimension that opens up on distance scales of \(O(H_0^{-1})\) (see \[22\]). However since this is an unnaturally large scale for a fundamental theory, this model is clearly just as fine tuned as the cosmological constant and moreover suffers from intrinsic theoretical difficulties such as violation of unitarity due to ‘ghosts’ (see \[48\]).

The situation is so desperate that ‘anthropic’ arguments have been advanced to explain why the cosmological constant is of just the right order of magnitude to allow of our existence today — if it was much higher then galaxies would never have have formed (see \[84\]). However a recent analysis \[73\] shows that the most likely value from such Bayesian arguments is in fact 20–30 times bigger than the observationally suggested value. Moreover, one cannot claim to have a rigorous understanding of the prior probability distribution, lacking a theory of the cosmological constant itself. It is commonly assumed that the prior is flat in \(\Lambda\), but if it is instead flat in say \(\log(\Lambda)\), the most likely value of \(\Lambda\) would be zero. A flat prior in \(\Lambda\) is indeed expected in quintessence-like models with a very flat potential \[85\], but such models are highly fine-tuned and have little physical basis. Whereas an uniform distribution of \(\Lambda\) does arise in the ‘landscape’ of the large number (\(\sim 10^{500}\)) of possible vacua in string theory (see \[25\]), there is no accepted measure for how these vacua may come to be populated through cosmological evolution.

Given this situation, we can ask whether the observations really require dark energy or whether this is just an artifact of interpreting the data using an over-idealised description of the universe. For the FRW model we see from eq.\((6)\) that deducing \(\Lambda\) to be of \(O(H_0^2)\) from observations should not be unexpected, this being its natural scale in such a model universe (what seems quite unnatural is the implied fundamental energy scale of \(\sqrt{H_0 M_P}\) since \(H_0\) is so much smaller than \(M_P\)). From this perspective, it is easy to see why there have been recurring claims for \(\Lambda \sim H_0^2\) — this is effectively forced upon us by the theoretical framework in which the data is interpreted.

\(^1\) While \(\Lambda \sim H_0^2\) today, we cannot have \(\Lambda \sim H^2\) always since this would amount to a substantial renormalisation of \(M_P\) in eq.\((1)\) (taking \(\alpha = 0\) as suggested by inflation); this is inconsistent with primordial nucleosynthesis which requires the ‘cosmic’ Newton’s Constant to be within a few \% of its laboratory value \[24\]. Thus arguably the most natural solution to the ‘coincidence problem’ is ruled out empirically.

\(^2\) Admittedly this criticism applies also to attempts to do away with dark energy by interpreting the data in terms of modified cosmological models, but less so since these do not invoke gravitating vacuum energy at all.
2 The observational situation

2.1 The age of the universe and the Hubble constant

A recurring argument for a cosmological constant has come from consideration of the age and present expansion rate of a FRW cosmology. For a spatially flat universe, the present age is \( t_0 = 2/3H_0 \) for \( \Omega_m = 1 \) and it has often been suggested that the observed age is too long to be consistent with the observed Hubble parameter for such an universe which has always been decelerating. By contrast, an universe with \( \Omega_m = 0.3, \Omega_A = 0.7 \) would be older with \( t_0 \approx 1/H_0 \) (see \[15\]), as is also the case for a ‘Milne universe’ which expands at a constant rate.

Advances in astronomical techniques now enable direct radioactive dating using stellar spectra, e.g. the detection of singly ionized \(^{238}\)U in the extremely metal-poor star CS31802-001 in the Galactic halo implies an age of 12.5 ± 3 Gyr \[10\]. This is consistent with the (95% c.l.) range of 10.4 − 16 Gyr for the age of (the oldest stars in) globular clusters which is indirectly inferred from the observed Hertzprung-Russell diagram using stellar evolution modelling, after adding on ∼ 1 Gyr, the estimated epoch of star formation \[50\].

With regard to the present expansion rate, the \textit{Hubble Key Project} \[34\] has provided direct measurements of the distances to 18 nearby spiral galaxies (using Cepheid variables) and these have been used to calibrate five secondary methods (such as Type Ia supernovae) which probe to deeper distances and yield:

\[
H_0 = 72 \pm 3 \pm 7 \text{ km s}^{-1} \text{ Mpc}^{-1}. \tag{11}
\]

This has come to be the generally accepted value e.g. by the Particle Data Group \[87\]. It has been argued however that the \textit{HKP} data need to be corrected for local peculiar motions using a more sophisticated flow model than was actually used, and also for metallicity (heavy element abundance) effects on the Cepheid calibration — this would lower \( H_0 \) to 63 ± 6 km s\(^{-1}\) Mpc\(^{-1}\) \[62, 63\]. The sensitivity of the Cepheid calibration to metallicity is also emphasised in a reanalysis of the \textit{HKP} data which finds larger distances to the 6 SNe Ia-calibrating galaxies used by \[34\], thus obtaining \( H_0 = 62.3 \pm 1.3 \pm 5 \text{ km s}^{-1} \text{ Mpc}^{-1} \) \[67\].

Even smaller values of \( H_0 \) have been deduced using physical methods such as measurements of time delays in gravitationally lensed systems, which bypasses the traditional ‘distance ladder’ calibration and probes to far deeper distances. Using the well-measured time delays of ten multiply-imaged quasars and taking the lenses to be isothermal dark matter halos yields \( H_0 = 48 \pm 3 \text{ km s}^{-1} \text{ Mpc}^{-1} \) \[47\], however the observations are consistent with the \textit{HKP} value \[11\] if there are varying amounts of dark matter in the lensing galaxies, consistent with the mass profiles obtained for them from stellar population evolution models \[55\]. Measurements of the Sunyaev-Zeldovich effect in 41 X-ray emitting galaxy clusters also indicated a low value of \( H_0 \sim 61 \pm 3 \pm 18 \text{ km s}^{-1} \text{ Mpc}^{-1} \) for a \( \Omega_m = 0.3, \Omega_A = 0.7 \) universe, dropping further to \( H_0 \sim 54 \text{ km s}^{-1} \text{ Mpc}^{-1} \) for \( \Omega_m = 1 \) (see \[59\]). However, a recent analysis of 38 clusters finds higher values of, respectively, 76.9 ± 3.4 km s\(^{-1}\) Mpc\(^{-1}\) and 67 ± 4.5 km s\(^{-1}\) Mpc\(^{-1}\) for the two cases, with an estimated systematic uncertainty of about ±10 km s\(^{-1}\) Mpc\(^{-1}\) \[12\]. As seen in Figure 1 the systematic uncertainties presently preclude a definitive conclusion using either method.

Even so this raises the question whether there may be spatial variations in the measured Hubble rate, e.g. because we are inside an underdense region (‘Hubble bubble’) that is expanding faster than the average \[58\]. Such voids are seen in all surveys of large-scale structure (see e.g. \[36\]) and are persistent indications from galaxy counts that we may be located in such a region (e.g. \[33\]). Recent work using SNe Ia detects a drop in the expansion rate of \( \delta H = 6.5 \pm 1.8 \% \) at a distance of 7400 km s\(^{-1}\) \[45\]), although this is sensitive to the precise manner in which corrections are made for reddening due to dust \[29\]. The measured CMB dipole (due to our peculiar motion with respect to the CMB rest frame) implies a general bound on the variance of such deviations, if the density fluctuations are gaussian, of \( (\delta H)^2 R < 10.5 h^{-1} (R/\text{Mpc})^{-1} \) in a sphere of radius \( R \) \[51\].

Moreover the \textit{HKP} data show statistically significant variations in \( H_0 \) of 9 km s\(^{-1}\) Mpc\(^{-1}\) across the sky \[53\]. The SNe Ia dataset also shows a significant systematic difference in the expansion rate in the North and South Galactic hemispheres \[68\].

To conclude, current estimates of the Hubble constant are typically quoted with 10% uncertainty but range between 60 and 75 km s\(^{-1}\) Mpc\(^{-1}\) (see \[13\]). Thus the age argument cannot be used to definitively exclude the Einstein–de Sitter (E-deS) model and argue for a cosmological constant. More worryingly, there are indications for local inhomogeneity, as well as anisotropy in the Hubble flow
which can significantly bias cosmological inferences drawn assuming an exactly homogeneous FRW model (e.g. [40][21]). It is essential that the fundamental assumption of homogeneity underlying the standard cosmology be rigorously tested, now that a wealth of data is available to carry out such tests.

Fig. 1 Hubble diagram for Cepheid-calibrated secondary distance indicators from the Hubble Key Project [34], along with deeper measurements using SNe Ia (filled circles), gravitational lenses (triangles) and the Sunyaev-Zeldovich effect (circles), with model predictions (from [10]).

2.2 The deceleration parameter

The most exciting observational development in studies of the Hubble expansion rate have undoubtedly been in measurements of the deceleration parameter $q \equiv \frac{dH^{-1}}{dt} - 1 = \frac{\Omega_m}{\Omega_m+\Omega_\Lambda}$. This has been found to be negative through deep studies of the Hubble diagram of SNe Ia pioneered by the Supernova Cosmology Project [58] and the High-z SN Search Team [60]. Their basic observation was that distant supernovae at $z \sim 0.5$ are $\Delta m \sim 0.25$ mag (corresponding to $10^{\Delta m/2.5} - 1 \sim 25\%$) fainter than would be expected for a decelerating universe such as the $\Omega_m = 1$ E-deS model. This has been interpreted as implying that the expansion rate has been accelerating since then, consequently the observed SNe Ia are actually further away than expected.

The measured apparent magnitude $m$ of a source of known absolute magnitude $M$ yields the ‘luminosity distance’:

$$m - M = 5 \log \left( \frac{d_L}{\text{Mpc}} \right) + 25, \quad d_L = (1 + z) \int_0^z \frac{dz'}{H(z')}.$$  \hspace{1cm} (12)

which is sensitive to the expansion history, hence the cosmological parameters. According to the second Friedmann equation [6] an accelerating expansion rate requires the dominant component of the universe to have negative pressure. The more mundane alternative possibility, namely that the SNe Ia appear fainter because of absorption by intervening dust, can be observationally constrained since this would also lead to characteristic reddening, unless the dust has unusual properties [2]. It is more difficult to rule out that the dimming is due to evolution, i.e. that the distant SNe Ia (which exploded over 5 Gyr ago) are intrinsically fainter by $\sim 25\%$ (e.g. [24]. Although SNe Ia are believed to result from the thermonuclear explosion of a white dwarf, there is no “standard model” for the progenitor(s) (see [38]), hence there may well be luminosity evolution which would complicate the use of SNe Ia as ‘standard candles’.
However it is known (using nearby SNe Ia with independently measured distances) that their time evolution is tightly correlated with their peak luminosities such that the intrinsically brighter ones fade faster. This can be used to make corrections to reduce the scatter in the Hubble diagram using various empirical methods such as a ‘stretch factor’ to normalise the observed apparent peak magnitudes [58] or the ‘Multi-colour Light Curve Shape’ method [60]. Such corrections are essential to reduce the scatter in the data sufficiently so as to allow meaningful deductions to be made about the cosmological model. It is a matter of concern that the corrections made by different methods do not always correlate with each other when applied to the same objects (see [51]), especially since there is no physical understanding of the observed correlations.

Figure 2 shows an example magnitude-redshift diagram of SNe Ia obtained by the Supernova Search Team [61] using a carefully compiled ‘gold set’ of 142 SNe Ia from ground-based surveys, together with 14 SNe Ia in the range $z \sim 1 - 1.75$ discovered with the Hubble Space Telescope. The latter are brighter than would be expected if extinction by dust or simple luminosity evolution ($\propto z$) is responsible for the observed dimming of the SNe Ia up to $z \sim 0.5$, and thus support the earlier indication of an accelerating cosmological expansion. However alternative explanations such as luminosity evolution proportional to lookback time, or extinction by dust which is maintained at a constant density are still viable. Moreover for reasons to do with how SNe Ia are detected, the dataset consists of approximately equal subsamples with redshifts above and below $z \sim 0.3$. It has been noted that this is also the redshift at which the acceleration is inferred to begin and that if these subsets are analysed separately, then the 142 ground-observed SNe Ia are consistent with deceleration; only when the 14 high-$z$ SNe Ia observed by the HST are included is there a clear indication of acceleration [58]. Clearly further observations are necessary particularly at the rather poorly sampled intermediate redshifts $z \sim 0.1 - 0.5$ — precisely the redshift range where the dark energy is supposed to have come to dominate the expansion leading to acceleration. Further observations have been made by the Supernova Legacy Survey [6] and ESSENCE [82] at the upper end of this redshift range, and these analyses have confirmed the previous indications of accelerated expansion.

![Fig. 2](image-url) The residual Hubble diagram of SNe Ia relative to the expectation for an empty universe, compared to cosmological models; the bottom panel shows weighted averages in redshift bins (from [61]).
A very different picture emerges however when the consistency of different data sets is examined more closely [68]. As mentioned earlier, the nearby \((z < 0.2)\) SNe Ia data indicates anisotropic expansion (at > 95\% c.l.), suggestive of an unidentified systematic error. While data from the North Galactic hemisphere are consistent with accelerated expansion, data from the South Galactic hemisphere are not conclusive in this regard. Even when the full-sky data is used, Figure 3 shows that \(q_0 = 0\) is still permitted at 2\(\sigma\), although \(q_0 = 0.5\) (the decelerating E-deS model) is rejected at this level. These authors conclude: “Our model independent test fails to detect acceleration of the Universe at high statistical significance” [68].

\[ H_0/\text{FractionBarExt/FractionBarExt/FractionBarExt/FractionBarExt/FractionBarExt/FractionBarExt/FractionBarExt} \]

\[ q_0 = 0.96 \quad 1 \quad 1.04 \quad 1.08 \]

Fig. 3 Results from a model-independent fit to the Hubble law for three SNe Ia data sets with \(z < 0.2\) — the horizontal axis shows the derived Hubble parameter relative to a fiducial value of \(H_0 = 65 \text{ km s}^{-1} \text{ Mpc}^{-1}\). The value of \(q_0 \approx 0.5\) deduced from WMAP data is shown for comparison (from [68]).

3 The spatial curvature and the matter density

Although the first indications for an accelerating universe from SNe Ia were rather tentative, the notion that dark energy dominates the universe became widely accepted rather quickly (e.g. [7]). This was because of two independent lines of evidence which also suggested that there is a substantial cosmological constant. The first was that contemporaneous measurements of degree-scale angular fluctuations in the CMB by the Boomerang [24] and MAXIMA [37] experiments provided a measurement of the sound horizon (a ‘standard ruler’) at recombination (see [39] and thereby indicated that the curvature term \(\kappa \approx 0\), i.e. the universe is spatially flat. The second was that, as had been recognised for some time, several types of (mainly local) observations indicate that the amount of matter which participates in gravitational clustering is much less than the critical density, \(\Omega_m \approx 0.3\) (see [56]). The cosmic sum rule [9] then requires that there be some form of ‘dark energy’, unclustered on the largest spatial scales probed in the measurements of \(\Omega_m\), with an energy density of \(1 - \Omega_m \approx 0.7\). This was indeed consistent with the value of \(\Omega_m \approx 0.7\) suggested by the SNe Ia data [58, 60] leading to the widespread identification of the dark energy with a cosmological constant. In fact all data to date are consistent with \(w = -1\) and this ‘concordance model’ is termed \(\Lambda\)CDM since the matter content must mostly be cold dark matter (CDM) given the constraint from primordial nucleosynthesis on the baryonic component \(\Omega_B h^2 \approx 0.02 \pm 0.002\) (see [33]).
Subsequently a major advance has come about with precision measurements of the CMB anisotropy by the Wilkinson Microwave Anisotropy Probe [69,70], and of the power spectrum of galaxy clustering by the 2 degree Field Galaxy Redshift Survey [19] and the Sloan Digital Sky Survey [72]. The paradigm which these measurements test is that the early universe underwent a period of inflation which generated a gaussian random field of small density fluctuations ($\delta \rho / \rho \sim 10^{-5}$) with a nearly scale-invariant ‘Harrison-Zeldovich’ spectrum: $P(k) \propto k^n, n \simeq 1$), and that these grew by gravitational instability in the sea of (dark) matter to create the large-scale structure, as well as leaving a characteristic anisotropy imprint on the ‘last scattering surface’ of the CMB. The latter is a snapshot of the oscillations in the coupled baryon and photon fluids as the plasma (re)combines suddenly and the universe becomes transparent at $z \sim 1000$ (see [39]). The amplitudes and positions of the resulting ‘acoustic peaks’ in the angular power spectrum of the CMB are sensitive to the cosmological parameters and it was recognised that precision measurements of CMB anisotropy can thus be used to determine these accurately (e.g. [13,46]). However in practice there are many ‘degeneracies’ in this exercise hence prior assumptions have to be made concerning some of the parameters (e.g. [14,28]). An useful analogy is to see the generation of CMB anisotropy and the formation of LSS as a sort of cosmic scattering experiment, in which the primordial density perturbation is the “beam”, the universe itself is the “detector” and its matter content is the “target”. In contrast to the situation in the laboratory, neither the properties of the beam, nor the parameters of the target or even of the detector are known — only the actual “interaction” may be taken to be gravity. In practice therefore assumptions have to be made about the nature of the dark matter (e.g. ‘cold’ non-relativistic or ‘hot’ relativistic?) and about the nature of the primordial perturbation (e.g. adiabatic or isocurvature?) as well as its spectrum, together with further ‘priors’ (e.g. on the curvature parameter $\kappa$ or the Hubble constant $h$) before the cosmological density parameters can be inferred from the data. Note in particular that $\Lambda$ is dynamically quite negligible at such large redshifts, so its only effect is to change the distance to the last scattering surface, hence the angular scale of the observed CMB anisotropies (in particular the position of the first acoustic peak).

Nevertheless as seen in Figure 4, the angular spectrum of the CMB measured by WMAP is in impressive agreement with the expectation for a flat $\Lambda$CDM model, assuming a power-law spectrum for the primordial (adiabatic) perturbation [69,70]. The fitted parameters for the 3-year data are $\Omega_B h^2 = 0.02229 \pm 0.00073$, $\Omega_m h^2 = 0.1277 \pm 0.008$, and $h = 0.732 \pm 0.031$ consistent with the HKP value [11]. The spectral index is obtained to be $n = 0.958 \pm 0.016$ which is as expected in simple models of slow-roll inflation. It is particularly impressive that the prediction for the matter power spectrum (obtained by convoluting the primordial perturbation with the CDM ‘transfer function’) is in excellent agreement with the power spectrum of galaxy clustering measured by both 2dFGRS and SDSS. The power spectrum from spectral observations of the ‘Lyman-α forest’ (intergalactic gas clouds) is also concordant (e.g. [44]). Having established the consistency of the $\Lambda$CDM model, this is used to draw tight constraints e.g. on a ‘hot dark matter’ (HDM) component which translates into a (95% c.l.) bound on the summed neutrino masses of $\sum m_\nu < 0.66$ eV [70].

Fig. 4 Angular power spectrum of the CMB measured by WMAP (black points – 3 year data, gray points – 1 year data) and the fit to the $\Lambda$CDM model (black line – 3 year data, red line – 1 year data); the right panel shows the predicted matter power spectrum compared with the SDSS data (from [70]).
It must be pointed out however that cosmological models without any dark energy can fit exactly the same data by making different assumptions for the ‘priors’. For example, an E-deS model is still allowed if the Hubble parameter is as low as $h \simeq 0.46$ and the primordial spectrum is not scale-free but has a change in its slope at a wavenumber $k \simeq 0.01 \text{ Mpc}^{-1}$ [11]. This is a toy model but an even better fit is obtained as shown in Figure 5 with a physically motivated ‘bump’ in the range $k \sim (0.01-0.1) \text{ Mpc}^{-1}$ [11]. Such a feature is plausible in ‘multiple inflation’ based on supergravity [1] wherein spontaneous symmetry breaking phase transitions occurring during inflation create sharp changes in the mass of the inflaton field. To satisfactorily fit the LSS power spectrum also requires that the matter not be pure CDM but have a ‘hot’ component, e.g. of neutrinos with (approximately degenerate) mass 0.5 eV (i.e. $\sum \nu = 1.5 \text{ eV}$) which contribute $\Omega_\nu \simeq 0.1$. This had been noted independently for the 2dFGRS data [31] and such a mass is within the sensitivity reach of the forthcoming KATRIN $\beta$-decay experiment [27]. It is important to note that this ‘cold + hot’ dark matter (CHDM) model has $\Omega_B h^2 \simeq 0.02$, just as required by primordial nucleosynthesis, but because of the low Hubble parameter the corresponding baryon density is about 10%. This is quite consistent with the observed baryon fraction in X-ray clusters, which is often used (e.g. 3) to deduce $\Omega_m \sim 0.3$, hence the presence of dark energy, adopting the HKP prior on $h$. Moreover the value of $\sigma_8$, the variance of mass fluctuations on cluster scales, is about 0.7 in this model, which is marginally consistent with measurements of cosmic shear due to weak gravitational lensing by large-scale structure (e.g. 30).

However such an E-deS cosmology fails to match the ‘baryon acoustic oscillation’ (BAO) peak observed in the galaxy autocorrelation function using luminous red galaxies at $z \sim 0.35$ in SDSS extending over a large survey volume. Although the measured amplitude is below the expectation of the $\Lambda$CDM model, the position is just as expected in this model [30]. The physical scale of the BAO peak is independent of the cosmological model, but its observed position (in redshift space) is sensitive to the Hubble parameter [11], hence the E-deS cosmology with $h \sim 0.5$ cannot match the data as seen in Figure 5.
4 Conclusions

We now have a ‘cosmic concordance’ model with $\Omega_m \sim 0.3$, $\Omega_L \sim 0.7$ which is supposedly consistent with all astronomical data but has no satisfactory explanation in terms of fundamental physics. One might hope to eventually find explanations for the dark matter (and baryonic) content of the universe in the context of physics beyond the Standard Model but there appears to be little prospect of doing so for the apparently dominant component of the universe — the dark energy which behaves as a cosmological constant. Cosmology has in the past been a data-starved science so it has been appropriate to consider the simplest possible cosmological models in the framework of general relativity. However now that we are faced with this serious confrontation between fundamental physics and cosmology, it would seem prudent to reconsider the underlying assumptions, especially that of exact homogeneity.

The observed isotropy of the CMB along with a belief in the Cosmological Principle has generally been taken to imply homogeneity but this has come to be questioned of late. There is growing interest in the inhomogeneous but isotropic Lemaître-Tolman-Bondi model of a local void (see [19]). It has been shown [18, 19] that a LTB model can in principle give an explanation of the oddities in the local Hubble flow as well as account for the SNe Ia Hubble diagram without invoking acceleration (see also [20]). Recently it has been claimed [21] that with a large local void of size $\sim 450h^{-1}$ Mpc the angular diameter distance of the BAO peak can also be matched in this model. Such local inhomogeneity may be responsible for generating the mysteriously aligned low multipoles in the WMAP sky [22]. A void of this size does seem extremely unlikely, but one has recently been actually detected at $z \sim 1$ and appears to be responsible for the anomalously ‘cold spot’ seen by WMAP [23].

Landau famously said “Cosmologists are often wrong, but never in doubt”. The situation today is perhaps better captured by Pauli’s enigmatic remark — the present interpretation of the data may be “...not even wrong”. However we are certainly not without doubt! The crisis posed by the recent astronomical observations is not one that confronts cosmology alone; it is the spectre that haunts any attempt to unite two of the most successful creations of 20th century physics — quantum field theory and general relativity. It would be fitting if the cosmological constant which Einstein allegedly called his “biggest blunder” proves to be the catalyst for triggering a new revolution in physics in this century.

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