COSMOLOGICAL CONSTRAINTS AND SU(5) SUPERGRAVITY GRAND UNIFICATION

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ABSTRACT

The predictions of SU(5) supergravity models with radiative breaking constrained by experimental proton decay bounds are discussed. It is shown that cosmological constraints further restrict the parameter space but can be satisfied for a wide range of parameters. It is also shown that no serious fine tuning problems (either at $M_{SUSY}$ or $M_{GUT}$) exist.

1. INTRODUCTION

The observation last year [1] that the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge coupling constants, $\alpha_3$, $\alpha_2$ and $\alpha_1 \equiv (5/3)\alpha_Y$, meet at a common energy scale $\mu = M_G$ if extended from their measured values at $\mu = M_Z$ by the supersymmetric renormalization group equations (RGE) with one pair of Higgs doublets, has led to a number of investigations of other predictions of supersymmetric GUT models [2-9]. In supergravity grand unification [10] with radiative breaking of $SU(2) \times U(1)$ [11] (for reviews see [12]), the model depends on four parameters (aside from the as yet unknown t-quark mass $m_t$): $m_0$ (universal scalar mass), $m_{1/2}$ (universal gaugino mass), $A_0$ (cubic soft breaking mass) and
\[ \tan \beta \equiv \langle H_2 \rangle / \langle H_1 \rangle, \text{ where } \langle H_2 \rangle \text{ gives mass to the up quarks and } \langle H_1 \rangle \text{ to the down quarks and leptons. Thus with gauge group } SU(5), \text{ Refs. [5 and 3] discuss the No-scale model } A_0 = 0 = m_0, \text{ Ref. [7] the case } B_0 = A_0 - m_0 \text{ (where } B_0 \text{ is the quadratic soft breaking mass and can be expressed in terms of } \tan \beta \text{) and Refs. [2,4,6,7] examine the general parameter space. Ref. [9] is concerned with the } O(10) \text{ model.} \]

The fact that supergravity grand unification introduces only four unknown parameters (only two more than in the Standard Model itself) to account for the masses of 32 particles (31 new SUSY particles plus the light Higgs \( h \)) implies that there should be considerable correlation between the SUSY masses. Unfortunately if one imposes only the requirement that radiative breaking occur and the current experimental bounds on the SUSY masses, the allowed mass bands are still very broad [2,5,7] and it is difficult to make clear predictions that can be used to test the theory. The situation changes considerably, however, if the model possesses an \( SU(5) \)-type proton decay: \( p \to \bar{\nu}K \). Thus if one assumes no extreme fine tuning of parameters \( (m_0, m_\tilde{g} < 1 \text{ TeV where } \tilde{g} \text{ is the gluino}) \) and the superheavy Higgs color triplet, which mediates the decay obeys \( M_{H_3} < 3M_G \) (which in simple models is the bound that keeps the GUT couplings perturbative in size), then the parameter space allowed by current data is still fairly large e.g.: \( m_0 \gtrsim 550 \text{ GeV, } m_\tilde{g} \lesssim 450 \text{ GeV (i.e. } m_{1/2} \lesssim 150 \text{ GeV), } 1.1 \leq \tan \beta \leq 4.7 \) [2]. However, one finds a number of remarkable predictions for the SUSY masses [2]:

\[
\begin{align*}
2m_{\tilde{Z}_1} &\simeq m_{\tilde{Z}_2} \cong m_{\tilde{W}_1} \cong \left( \frac{1}{3} - \frac{1}{4} \right)m_\tilde{g} \\
& \quad m_{\tilde{Z}_3} \cong m_{\tilde{Z}_4} \cong m_{\tilde{W}_2} \\
\end{align*}
\]

where the charginos \( (\tilde{W}_i, i = 1, 2) \) and neutralinos \( (\tilde{Z}_i, i = 1 \ldots 4) \) are labeled such that \( m_i < m_j \text{ for } i < j \). In addition one finds \( m_h \lesssim 110 \text{ GeV and } m_t \lesssim 180 \text{ GeV. Further, for } m_t < 140 \text{ GeV, then } m_{\tilde{W}_i} \lesssim 100 \text{ GeV when } m_h \gtrsim 95 \text{ GeV making one of these particles (and possibly both) observable at LEP200.}\)

The above proton decay constraint is sufficiently powerful to eliminate the preferred models of Ref. [7] and the No-scale \( SU(5) \) model over the entire parameter space [3]. (For the No-scale case one need not even use the fine tuning constraint if one imposes the cosmological requirement that the LSP be electrically neutral.) Thus No-scale models are
viable only if they can suppress the $p \rightarrow \bar{\nu}K$ decay mode, such as is done in the flipped $SU(5)$ supergravity model \[13\].

The purpose of this letter is to discuss the role of cosmology in a GUT theory which allows proton decay via dimension five operators. We shall show that a supergravity GUT theory which is constrained both by proton decay limit and the cosmological relic density limit (which avoids overclosing the universe) allows a wide domain of the parameter space on a reduced four dimensional manifold (more precisely a five dimensional shell) in contradiction to the conclusions of a recent analysis on this topic \[14\]. We also show that the conclusions drawn in Ref. \[14\] concerning fine tuning are inaccurate, and there are no serious fine tuning problems either at $M_{SUSY}$ or $M_G$.

2. COSMOLOGICAL CONSTRAINTS

Recently detailed analyses of the neutralino relic density in N=1 supergravity unified models have been carried out using the superparticle spectrum generated by the radiative electroweak symmetry breaking \[14-16\]. It is found that the relic density constraint $\Omega_{\tilde{Z}_1} h^2 \lesssim 1$ (where $\Omega_{\tilde{Z}_1}$ is the ratio of the lightest neutralino mass density to the critical mass density and $h$ is Hubble constant measured in units of 100 km/sec Mpc) limits additionally the allowed parameter space of the supergravity models when one imposes the naturalness condition discussed above, $m_{\tilde{g}}, m_0 \lesssim 1$ TeV. [If squark and gluino masses in excess of 5 TeV are allowed, the relic density constraint is easily satisfied \[15\] (due to the many open channels), as is the p-decay constraint (due to the suppression from the large SUSY masses in the dressing loop \[17\]).]

A detailed analysis of the allowed parameter space under the simultaneous constraints of proton stability \[17\] and neutralino relic density not overclosing the universe \[18\] will be given elsewhere \[19\]. We give here a brief discussion. For an arbitrary point in parameter space one finds $\Omega_{\tilde{Z}_1} \approx 100$. As has been pointed out \[14-16\], however, the neutralino annihilation rate is significantly enhanced when the annihilation occurs close (within a few GeV) to the $h$ boson being on shell, i.e. $2m_{\tilde{Z}_1} \approx m_h$. To calculate the $\tilde{Z}_1$ relic density near the $h$ pole, we follow the general analysis of Ref. \[18\] making use of the cross section for $\tilde{Z}_1 + \tilde{Z}_1 \rightarrow h^* \rightarrow f + \bar{f}$ where $f$ is a final state fermion \[20\]. However, near the
pole, it is necessary to take the thermal average of the rigorous cross section, \( < \sigma v > \) (\( v=\text{relative velocity} \)), as discussed in detail in Ref. [21], rather than use the expansion \( \sigma v \approx a + bv^2/6 \). The thermal average can no longer be done analytically, but must be performed numerically. (To our knowledge, previous calculations of SUSY relic densities have not included this important modification.) From Eq. (1), the condition that the intermediate \( h \) is nearly on-shell can be viewed as a constraint relating \( m_{\tilde{g}} \) to \( m_h \), and we find strong suppression of \( \Omega_{\tilde{Z}_1} h^2 \) over a range of gluino masses \( \lesssim 5\text{-}20 \text{ GeV} \) wide. Thus the inclusion of the relic density constraint reduces the five dimensional parameter space, \( m_0, m_{\tilde{g}}, A_0, \tan \beta \), and \( m_t \) to a four dimensional submanifold (actually a five dimensional shell \( \approx 5\text{-}20 \text{ GeV} \) wide) where the annihilation of the \( \tilde{Z}_1 \) is enhanced so that \( \Omega_{\tilde{Z}_1} h^2 < 1 \). If \( m_t \) is experimentally determined (as one hopes it soon will be at the Tevatron) then one will be left with a three dimensional subspace (more precisely a four dimensional shell) depending on the parameters \( m_0, A_0, \) and \( \tan \beta \).

While the relic density constraint reduces the allowed range of the remaining parameters somewhat, significantly it still leaves available a wide range of these parameters. Fig. 1 shows the allowed region in the parameters \( m_{\tilde{g}}, \alpha_H \) (\( \alpha_H \equiv 1/\tan \alpha_H \)) for a characteristic example \( m_0 = 600 \text{ GeV}, A_t = 0, \mu > 0 \) where \( A_t \) is the t-quark \( A \) parameter at the electroweak scale. (In Figs. 1 and 2, we have allowed the more conservative bound of \( M_{H_3} < 6 M_G \).) We see that \( m_{\tilde{g}} \) ranges from 200 GeV to 450 GeV and \( 22^\circ \leq \alpha_H < 41^\circ \), much as when only the p-decay constraint was imposed [2]. (The allowed \( m_{\tilde{g}} \) band is \( \approx 10 \text{ GeV} \) wide.) Note also that allowed ranges of parameters satisfying both relic density and proton decay constraints exist for a full range of values of \( m_t \) and not for only “special” values of \( m_t \) as stated in Ref. [14]. Fig. 2 shows the dependence of the allowed region on \( m_{\tilde{g}} \) and \( A_t \) for \( m_0 = 600 \text{ GeV} \) and \( \alpha_H = 30^\circ \) (\( \tan \beta = 1.73 \)). The \( m_{\tilde{g}} \) band is again 10-20 GeV wide and the range of \( A_t \) is similar to that obtained before [2]. Again, allowed regions in parameter space exist for a range of values of \( m_t \). Fig. [3] shows the importance of using the rigorous analysis of Ref. [21] rather than the approximation \( < \sigma v > = a + bx \) (\( x = kT/m_{\tilde{Z}_1} \)). As one can see from comparison of the exact and the approximate analysis, the approximate analysis would introduce large errors.

The above analysis shows that the relic density constraint combined with the proton
decay constraint for arbitrary \( m_t \) leads to a wide range of allowed parameters, in contrast to the analysis in Ref. [14]. Further, the model has many experimentally testable predictions. There is, however, an alternate framework which retains all the predictions of the standard \( SU(5) \) model in terrestrial experiments, and eliminates the constraint of the neutralino relic density altogether by a sufficiently rapid decay of the neutralinos. We begin by recalling that we are dealing with an \( N = 1 \) supergravity theory which is an effective remnant theory at the scale \( M_G \) of some more unified structure. As such, it can possess at the scale \( M_G \) operators with \( \text{dim} > 3 \) in the superpotential scaled by \( M_P \) or the compactification mass \( M_C \). While the effect of these operators on the computation of the superparticle spectrum via radiative breaking [2-9] would be negligible, they can significantly affect the cosmology resulting from the model.

As an example, consider the second generation of quarks and leptons and supplement the particle content of the model by an \( SU(5) \), singlet field \( \nu^c \) and an \( SO(10) \) singlet field \( N \) so that they can be grouped into nonets of \( SU(3)_C \times SU(3)_L \times SU(3)_R \). We denote these nonets by the representation \( L(1, 3, \bar{3}), Q(3, \bar{3}, 1) \) and \( Q^c(\bar{3}, 1, 3) \) of \([SU(3)]^3\). Next we extend the superpotential of the theory by adding the minimal terms \( W_s + \lambda[(Tr\Sigma^2)/M_G^2]TrQLQ^c \), where \( W_s \) contains the singlet fields \( \nu^c \) and \( N \) and generates superheavy VEVs for them, and \( \Sigma \) is the 24 of \( SU(5) \) whose VEV, \( \text{diag} \Sigma = M(2, 2, 2, -3, -3) \) breaks \( SU(5) \) to \( SU(3)_C \times SU(2)_L \times U(1)_Y \) at \( M_G \). The second term contains the factor

\[
TrQLQ^c = -DND^c + D\ell^c\ell^c - D\nu^c\nu^c - q\ell D^c + qHu^c + qH'd^c
\]

(2)

where \( D, D^c \) are the superheavy Higgs color triplets, \( \ell \) and \( q \) are lepton and quark fields, and \( H \) and \( H' \) are the two light Higgs doublets. We note that both \( SU(5) \) and \([SU(3)]^3\) can be embedded into \( E_6 \), pointing to a possible common origin of the normal and “Planck slop” terms from a more unified structure.

After spontaneous breaking, \( W_s \) generates a VEV for \( \nu^c \) which spontaneously produces a violation of R-parity as can be seen from Eq. (2). This also gives superheavy masses to the \( N \) and \( \nu^c \) fields. The \( \nu^c \) VEV growth generates a \( D - d \) and \( D^c - d^c \) mixing. Diagonalization of the \( D - d \), etc. mass terms [22] gives an effective R-parity violating interaction in the quark-lepton sector which can decay the lightest neutralino. Thus after
diagonalization, the R-parity violating interaction determining this decay is

\[ \mathcal{L}_{\text{int}} = g(\tilde{Z}_1\mu_L)(s^a c_L^a) ; \quad g = \frac{2e}{m_{\tilde{\mu}}^2} f_{22} U s_1 \]  

(3)

where \( \mu(x) \) is the muon field, \( s^a(x), c^a(x) \) (\( a = \) color index) the strange and charm quark fields, \( f_{22} \) the \( SU(5) \) coupling in \( f_{ij}\bar{H}_x\bar{M}_{iY}M_j^{XY} \) (\( \bar{H}_x = 5 \) Higgs field, \( \bar{M}_Y = 5, M^{XY} = 10 \) matter fields, \( i, j = \) generation indices), \( U \) is the projection of the \( \tilde{Z}_1 \) state onto the photino state, \( s_1 \) is the \( D - d \) mixing parameter given by

\[ s_1 \approx -y/(m_D^2 + y^2)^{1/2} ; \quad y = \frac{<\Sigma^2>}{M_C^2} <\nu^c> \lambda \]  

(4)

and \( m_D \equiv M_{H_3} \) is the superheavy Higgs color triplet mass. The partial lifetime of the \( \tilde{Z}_1 \) is then

\[ \tau(\tilde{Z}_1 \rightarrow \bar{c}s\mu^+) \simeq (1 \times 10^{-19}\text{sec}) (f_{22}s_1 U)^{-2} \left(\frac{m_{\tilde{\mu}}}{m_{\tilde{Z}_1}}\right)^4 \left(\frac{1\text{GeV}}{m_{\tilde{Z}_1}}\right) \]  

(5)

with \( f_{22} = (m_s/M_Z)(e/\sin\alpha_H \sin 2\theta_W) \). For \( <\Sigma> = <\nu^c> = M_G \approx 10^{16} \text{ GeV}, M_C = 5 \times 10^{17} \text{ GeV}, M_D = 3M_G, \lambda = 1, U = 1, m_{\tilde{\mu}} = 500 \text{ GeV}, m_{\tilde{Z}_1} = 50 \text{ GeV}, \) one finds \( \tau \approx 10^{-4}\text{sec} \). Thus typically the neutralino is very short lived so that it will not leave any significant cosmological trace. However, the lifetime of the neutralino is still large enough that it will decay well outside the detection chamber in collider experiments, so that all of the characteristic missing \( E_T \) signals of supersymmetric particles will be maintained.

3. FINE TUNING AT \( M_{\text{SUSY}} \)

The problem of fine tuning first arose in non-SUSY GUTs due to the quadratic divergence of \( m_H \), the Higgs mass i.e. \( m_H^2 = m_0^2 - b\tilde{\alpha}\Lambda^2 \), where \( m_0 \) is the bare mass, \( \tilde{\alpha} \) is a coupling constant, \( \Lambda \) is the cutoff and \( b \) is a constant. Thus if \( m_H = O(M_Z) \), then one must fine tune \( m_0^2 \) to 24 decimal places when \( \Lambda = M_G \). One may formalize this argument [23,14] by defining the parameter \( c = (m_0^2/m_H^2)(\partial m_H^2/\partial m_0^2) \). Then \( c = m_0^2/m_H^2 \approx c\tilde{\alpha}\Lambda^2/m_H^2 \approx 10^{24} \), i.e. \( \log c \) is the number of fine tuning decimal places. In supersymmetry the fine tuning problem resurfaces in the radiative breaking equation [11]
\[
\frac{1}{2} M_Z^2 = (m_{H_1}^2 - m_{H_2}^2 \tan^2 \beta)/(\tan^2 \beta - 1) - \mu^2 .
\] (6)

Here \( \mu \) is the \( H_1 - H_2 \) mixing mass, \( m_{H_1,2} \) are the Higgs masses which can be expressed in terms of the GUT scale parameters \( m_0, m_{1/2}, A_0, B_0, \mu_0 \) by the renormalization group equations. The situation here is more complex as there are many parameters \( a_i = m_{0,1/2}^2, \mu_0^2, m_{1/2}^2 \) etc. One may define \( c_i \equiv (a_i/M_Z^2)(\partial M_Z^2/\partial a_i) \) and require \( c_i < \Delta_i \) with, say \( \Delta_i = 10^2 \) as in Ref. [14]. There are however, a number of ambiguities that need to be addressed. Thus one can always make transformations on the parameters, \( a_i' = f_i(a_i) \) sending \( c_i \) to \( c_i' \), increasing or decreasing the value of a given \( c_i \) in this way. (The \( a_i \) in general are complicated functions of the hidden sector of the theory [12]. Thus which set of functions of \( a_i \) are “fundamental” and hence to be preferred is unknown at present.) Also rescaling all parameters to a single one e.g. \( m_{1/2} \) by writing \( \xi_0 = m_0/m_{1/2}, \xi_A = A_0/m_{1/2} \) etc. (as done in Ref. [14]) artificially increases the remaining c-parameter as \( c_{1/2} \) then equals \( \Sigma c_i \).

In Ref. [14], the choice \( a_i = \mu^2 \) and \( m_t^2 \) was made. The authors then found only \( c_t \) “too high” (and then only by a factor of 2-3). We believe it is incorrect to use \( m_t \) as a fine tuning parameter as (presumably) the top will shortly be discovered, and then the only correct thing would be to insert its experimental value. But even allowing the choice \( a = m_t^2 \), one could reduce the value of \( c_t \) by a factor of two merely by replacing \( M_Z^2 \) by \( M_Z \) in the definition of \( c_t \), which would then satisfy the criteria of Ref. [14].

The above discussion shows that the fine tuning criteria used in Ref. [14] is ambiguous up to factors of 2-10. When one fine tunes to 24 decimal places as in non-SUSY GUTs, these ambiguities are unimportant. But if one is talking about conditions such as \( \Delta < 10^2 \), no clear conclusions can be drawn. The only reasonable constraint is the qualitative one used in Ref. [2] that squark and gluino masses be less than 1 TeV (which is also approximately the detection upper bound at the LHC and SSC).

4. FINE TUNING AT \( M_C \)

We mention briefly some additional points concerning the problem of fine tuning at the GUT scale. There are three theoretically satisfactory methods of breaking \( SU(5) \) to the Standard Model group while maintaining light Higgs doublets and superheavy Higgs
triplets. The first is that originally proposed in global SUSY GUT models [24], which requires a fine tuning that, however, is natural due to the SUSY no renormalization theorems. These models use a VEV growth of a 24 representation to break $SU(5)$. The second is the missing partner models [25] using 50, $\overline{50}$ and 75 representations of $SU(5)$ where the VEV growth in the 75 breaks $SU(5)$. The third method makes use of a global $SU(6)$ symmetry in the GUT sector [26]. The breaking of the GUT group then makes the Higgs doublets pseudo Goldstone bosons and hence automatically light. We view this last method as being more elegant than the missing partner models in either flipped $SU(5)$ or normal $SU(5)$.

The third method illustrates the fact that fine tuning in the GUT sector may not be as invidious as other fine tunings. Thus the physics of the GUT sector is at present unknown, and some higher symmetry (perhaps from string theory) may naturally force two coupling constants to be equal, thus keeping the Higgs doublets light: this is precisely what happens in the $SU(6)$ model above.

5. DISCUSSION

At present there is no acceptable string model, Calabi-Yau, orbifold or 4-D construction, which is consistent with the coupling constant unification analysis [1]. Thus the fact that a GUT model may possibly have string antecedents appears irrelevant at this point, as it may have to be significantly modified if and when a viable string model is constructed. More relevant is the “possibility” that the No-scale model could determine the soft breaking parameters dynamically. For the complete No-scale model, where only the $m_{1/2}$ soft breaking mass is non-zero at the GUT scale, this would lead to a unique SUSY mass spectrum at a fixed value of $m_t$. Implementing this is the most interesting question facing the No-scale model, since then very likely it could be experimentally determined whether it is right or wrong. Barring this theoretical development, one should look for experimental differences between different supergravity models. One of these involves proton decay, where the flipped $SU(5)$ model suppresses the $p \to \bar{\nu}K$ decay [13], while $SU(5)$ supergravity expects it to be seen at the Super Kamiokande experiment.

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Note added: After completing this work we received the preprint Ref. [27]. These authors also find a significant region of parameter space simultaneously satisfying the $SU(5)$ proton decay and cosmological constraints. They also reconfirm the mass relations of Eq. (1).

FIGURE CAPTIONS

Fig. 1. Region in $m_{\tilde{g}} - \alpha_H$ parameter space allowed by the combined proton decay and $\tilde{Z}_1$ relic density constraints for $m_0 = 600$ GeV, $A_t = 0.0$, $\mu > 0$ ($\tan \beta \equiv 1/\tan \alpha_H$). $A_t$ is the t-quark $A$ parameter at the electroweak scale. The dashed curve is for $m_t = 110$ GeV, solid curve for $m_t = 125$ GeV, dot-dash for $m_t = 140$ GeV.

Fig. 2. Region in $m_{\tilde{g}} - A_t$ parameter space allowed by the combined proton decay and $\tilde{Z}_1$ relic density constraints for $m_0 = 600$ GeV, $\alpha_H = 30^\circ$ ($\tan \beta = 1.73$), $\mu > 0$. Different curves as in Fig. 1.

Fig. 3. Contribution to $\Omega_{\tilde{Z}_1} h^2$ from the h pole for $m_t = 125$ GeV, $m_0 = 600$ GeV, $A_t/m_0 = 0.5$, $\tan \beta = 1.73, \mu > 0$. The solid line is the exact result from thermal averaging over the Higgs pole. The dashed line is the approximate result when one expands $\sigma v \approx a + b v^2 / 6$.

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