Prominent interference peaks in the dephasing Anderson model

Yannic Rath and Florian Mintert

Physics Department, Blackett Laboratory, Imperial College London,
Prince Consort Road, SW7 2BW, United Kingdom
(Dated: October 26, 2018)

The Anderson model with decoherence features a temporal evolution from localized eigenstates to a uniform spatial distribution bar any interference features. We discuss the growth and decay of pronounced interference peaks on transient time-scales and develop an analytic understanding for the emergence of these peaks.

Interference is the distinctive feature that separates the physics of classical particles from wave-mechanics, in particular quantum mechanics. Since interaction with an environment results in decoherence, which, in turn, limits the ability of any system to show interference effects, one can thus realize continuous transitions between quantum mechanical and classical regimes, parametrized by strength of the environment coupling or duration of exposure to the environment.

Typically the transition towards a classical regime implies that interference patterns are washed out such that maxima decrease and minima grow. This is strictly true in the textbook example of the double-slit experiment, with an interference pattern resulting from two amplitudes only. Patterns comprised of more amplitudes, however, can result in more complex transitions between quantum and classical, with structures in an interference pattern existing in intermediate regimes that exist neither in the perfectly quantum nor in the classical case.

A particularly intricate interplay of many interfering amplitudes is found in the Anderson model, where the destructive interference of many amplitudes is reflected by exponentially localized eigenstates. Decoherence will generally allow initially localized states to expand as destructive interference is lifted. The details of such dynamics and their stationary solutions depend on the specific properties of the environment coupling, as encoded, for example, in a Lindblad operator. Quite generically, decoherence rates are particularly high for states that can easily be distinguished by the environment, whereas they tend to be low for states that can hardly be distinguished by the environment. Interference, thus not necessarily just results in a pure attenuation of interference structures; rather, populations can start to propagate as the coherence length shrinks, while interference can still exist on sufficiently small length scales.

We will show here that propagating populations can become trapped due to interference on small length scales, resulting in the emergence of sharp interference structures that decay only once the coherence length is sufficiently small for the system to approach its classical limit.

The Anderson model for a single particle on a one-dimensional lattice with $N$ sites is defined in terms of the Hamiltonian

$$H = \sum_x \epsilon_x \langle x \rangle \langle x+1 \rangle + \langle x \rangle \langle x+1 \rangle + \langle x+1 \rangle \langle x \rangle,$$

where $\langle x \rangle$ denotes the occupation of site $x$. This Hamiltonian includes tunneling of a quantum mechanical particle on a lattice from sites $x$ to neighboring sites $x \pm 1$ with amplitude $\mathcal{T}$, and onsite energies $\epsilon_x$. In contrast to translationally invariant systems, with delocalized eigenstates (Bloch waves), any small amount of disorder, e.g. in the onsite energies $\epsilon_x$, causes a transition to exponentially localized eigenstates, resulting from destructive interference.

Decoherence can be described in terms of a master equation with a Lindbladian $\mathcal{L}$. We will consider Lindbladians satisfying

$$\mathcal{L}(\langle y \rangle \langle x \rangle) = -\gamma f(x, y) \langle y \rangle \langle x \rangle,$$

for all pairs of lattice sites $x$ and $y$. The fundamental decoherence rate of the system is denoted by $\gamma$, and $f(x, y)$ with $f(x, x) = 0$ is a distance-dependent factor. Any Lindbladian defined in this fashion induces only loss of phase coherence between different sites, but no dynamics of lattice populations.

The function $f(x, y)$ characterizes how well the environment coupling can resolve fine spatial structures in the system. The extreme case $f(x, y) = 1$ for $x \neq y$ describes a situation in which all spatial structures can be resolved equally well, but more realistic models would take into account a finite resolution with reduced decoherence rates for short spatial scales and a finite, maximum rate for asymptotically large separations. Since the behavior for large separations will not be relevant for our purposes, we will use $f = f_q$ with $f_q = |x-y|^q$ and focus mostly on the case $q = 1$. In order to avoid ambiguities in the definition of the distance between two sites, we employ open and not the usual periodic boundary conditions.

In all the following analysis, we will discuss a lattice of $N = 500$ sites, draw the onsite energies from a uniform distribution in the interval

\[0, 1\]
are largely independent of the system state occupation, and (b) to (d) depict the time-evolved densities for three different dephasing rates, including weak dephasing ($\gamma = 10^{-3} \gamma$) in (b), intermediate dephasing ($\gamma = 3 \cdot 10^{-3} \gamma$) in (c), and strong dephasing ($\gamma = \gamma$) in (d). The three instances in time depicted in (b) to (d) are chosen such that the central peak has decayed to $1/2$, $1/4$ and $1/8$ of its original height (depicted in black (dark), blue (medium) and red (light)). A clearly pronounced side-peak is growing and subsequently decaying in the case of weak dephasing (b), but intermediate and strong dephasing do not result in the emergence of such structures.

We define the growth $G = \max_{i,j} \mathcal{P}(t_f) - \mathcal{P}(0)$ of a peak with ground states that have their center of mass in the interval $[249.5, 250.5]$ around the center of the chain. The resulting ensemble average $\overline{\mathcal{P}}(x)$ for 1000 of such disorder realizations is depicted in Fig. 2 after a propagation time chosen for the three dephasing regimes such that the peak height of the ensemble average has reached half of its initial value. As one can see, strong dephasing results in a rather broad peak but negligible tails, whereas weak dephasing yields more narrow peaks but pronounced tails. Close to the center of the chain, i.e. close to the initial peak, and far out in the tails, the ensemble average for intermediate dephasing lies between the corresponding data for weak and strong dephasing; between those regimes (in this case between approx. $x = 270$ and $x = 300$), however, the intermediate dephasing results in an average population $\overline{\mathcal{P}}(x)$ that is larger than in the two extreme regimes.

The existence of a pronounced side-peak as visible in Fig. 1 in the weak dephasing regime, is clearly consistent with the narrow peak and pronounced tails in the ensemble average shown in Fig. 2 but since the distance between side-peak and main peak depends on the realization of disorder, such side-peaks can not be unambiguously verified in the ensemble average. We will therefore characterize peaks in interference patterns in terms of their topographic prominence $\overline{\mathcal{P}}$, which characterizes to what extent a peak stands out in front of the background. In the present case of one-dimensional structures, the side prominence of a peak with respect to its left/right side can be defined as the difference between its height and the height of the lowest point between itself and the next higher peak to the left/right. The overall prominence $\mathcal{P}$ for any peak is then given by the smaller value of the two side prominences. We define the growth $G = \max_{i,j} \mathcal{P}(t_f) - \mathcal{P}(0)$ of a
peak as the maximum of the difference of prominence at different times and the prominence $\Delta$ for a given system (characterized by realization of disorder and dephasing rate) as the maximal growth with the maximum taken over all side-peaks.

Fig. 2 shows two ensemble averages of this prominence as a function of the dephasing rate $\gamma$, extracted from an ensemble of 5000 disorder realizations. The blue (light) curve corresponds to the half of the realizations resulting in lower prominence for very weak dephasing with $\gamma = 10^{-9}T$, whereas the black (dark) curve represents the ensemble average over the other half of realizations. Both sub-ensembles share several features: the overall prominence drops to zero for fast dephasing ($\gamma \geq T$), confirming the absence of side-peaks in this limit. The growth of the average prominence with decreasing dephasing rate up to $\gamma \simeq 10^{-2}T$ is essentially the same in both sub-ensembles, and the prominence is independent of the dephasing rate $\gamma$ in the regime $\gamma < 10^{-4}T$. The central difference between both sub-ensembles lies in the maximum of prominence at around $\gamma \simeq 3 \cdot 10^{-3}T$ in the ensemble with lower prominence, as opposed to the monotonic decrease in prominence with increasing dephasing rate $\gamma$ for the ensemble with higher prominence.

In the following, we will present a mostly analytic description for the emergence of side-peaks in the slow dephasing regime. Based on this description we can then develop the physical understanding of the enhanced prominence for intermediate dephasing ($\gamma \simeq 3 \cdot 10^{-3}T$) shown in Fig. 2 and the enhanced average population for intermediate dephasing at intermediate distances from the original peak shown in Fig. 2.

In the regime of slow dephasing, the system state will be an incoherent mixture of energy eigenstates for all times, if it is initially prepared in such a state. The dephasing (in the site basis) only results in incoherent transitions between energy eigenstates, as described by the rate equation

$$\langle \psi_i \mid \dot{\rho} \mid \psi_i \rangle = -\gamma \sum_j \eta_{ij} \langle \psi_j \mid \rho \mid \psi_j \rangle,$$

with the coupling elements

$$\eta_{ij} = \sum_{x,y} f(x, y) \langle x \mid \psi_i \rangle \langle y \mid \psi_j \rangle \langle \psi_j \mid x \rangle,$$

given in terms of overlaps between energy eigenstates $|\psi_{i,j}\rangle$ and site eigenstates $|x\rangle$ and $|y\rangle$, and if the function $f(x, y)$ for those two sites is sufficiently large, which means that coherent superpositions of $|x\rangle$ and $|y\rangle$ decay quickly.

In the case of uniform dephasing (i.e. $f(x, y) = 1$ for $x \neq y$), the latter condition is obsolete, such that generally many transitions between energy eigenstates are possible. Any eigenstate will therefore decay into a mixture of many eigenstates, and fine structures that are contained in each eigenstate tend to average out in this mixture. One would thus expect not to observe any pronounced side-peaks for this decoherence model, and we verified in numerical simulations that this is indeed the case. If, on the other hand, the environment is limited in its resolution of small spatial structures, such that the function $f(x, y)$ is negligible for small separations $|x - y|$, then only contributions with sufficiently large separation between $|x\rangle$ and $|y\rangle$ will remain.
and $|y\rangle$ can contribute substantially to the sum in Eq. (3). Since, in addition, the transition amplitudes $\langle x|\psi_i\rangle\langle\psi_i|y\rangle$ become exponentially small with growing separation between $|x\rangle$ and $|y\rangle$, only a very small number of terms contributes substantially to the sum in Eq. (3). As statistics with low numbers is prone to ‘rare’ events with large deviations from the average, this implies a substantially larger likelihood of particularly strong coupling between one pair or few pairs of eigenstates, whereas a summation over many terms would more generically result in many coupling constants of comparable magnitude. The system state is thus typically given by a mixture of only few energy eigenstates, and because of this restriction to a small number of eigenstates, spiky structures do not necessarily average out, but side-peaks can arise.

The dependence on the transition amplitude $\langle x|\psi_i\rangle\langle\psi_i|y\rangle$ asserts that side-peaks will arise predominantly in the vicinity of the tails of the exponentially localized initial state, which is exactly what can be observed in Fig. 1. The dephasing rate $\gamma$ enters the rate equations (Eq. (2)) only as a global prefactor and has no further impact on the coupling structure, which explains why the prominence becomes independent of $\gamma$ for sufficiently weak dephasing in Fig. 3.

Drawing further conclusions from Eq. (3) requires additional knowledge of specific properties on the energy eigenstates. This seems to be hopeless at the first sight since the most basic assertion of Anderson localization is that the character of eigenstates changes dramatically even in the presence of only weak disorder [2]. Despite the dramatic change from delocalized to exponentially localized states, however, we found that the localized states do inherit properties like sign changes surprisingly well from their delocalized counterparts. Estimating the coupling constants $\eta_{ij}$ in terms of eigenstates $\langle\phi_i\rangle$ of the disorderless system thus gives additional insight into the coupling between energy eigenstates. The sinusoidal solutions $\langle x|\phi_i\rangle = \sqrt{2/(N+1)}\sin(k_i x)$ with wave number $k_i = \pi r/(N + 1) + \pi$ and corresponding eigenvalue $2\gamma \cos k_i$, thus leads to an estimate of the coupling matrix $\eta$ that allows to draw qualitative conclusions. Any two sinusoidal solutions $\langle x|\phi_i\rangle$ and $\langle x|\phi_j\rangle$ with substantially different wave numbers result in negligible coupling $\eta_{ij}$ since the oscillatory character tends to result in many cancellations in the summation of Eq. (3), and only coupling between spectrally close eigenstates is sizeable. This property can also be observed very clearly for disordered systems and, together with the above findings, defines a rather stringent condition:

*a prominent side-peak can only occur if there is a strongly peaked eigenstate that is spectrally close to the initial state, and both of these states have large transition amplitudes $\langle x|\phi\rangle\langle\phi|y\rangle$ for pairs of sites $x$ and $y$ with sizeable decoherence rate.*

For sufficiently large dephasing rates (roughly $\gamma \gtrsim 10^{-6} \Gamma$), the system state can no longer be described as a purely incoherent mixture of energy eigenstates, but coherences between energy eigenstates can build up. The first observable consequence of this is that the overall dynamics slows down $[13]$, such that it takes longer for the initial peak to dissolve into the flat steady state distribution. This resembles the onset of a quantum Zeno effect $[14]$, even though decoherence is still too weak for complete suppression of population dynamics. Populations can therefore still propagate through the chain, but, as shown in Fig. 2, the propagation over large distances is suppressed as compared to the weak dephasing regime. Since dynamics on shorter scales usually has a more coherent character than dynamics on larger scales, populations can thus propagate so that an initially localized peak can start to dissolve, resulting in the fine interference structures that are apparent in Fig. 1. The onset of quantum Zeno dynamics then tends to prevent such a structure to expand over a larger spatial region. This suppression, in turn, implies an enhanced probability to remain in the vicinity of the initial distribution, which is reflected by the increased probability $\tilde{P}$ for sites $270 \lesssim x \lesssim 290$ for intermediate dephasing in Fig. 2.

This onset of quantum Zeno dynamics also explains the maximum of prominence for intermediate dephasing rates in Fig. 3. In the sub-ensemble with less prominence, there tend to be several side peaks with similar prominence in the weak dephasing regime, and the growth of several peaks imposes limits on the growth of each individual peak. For intermediate dephasing, however, the growth of peaks further away from the initial peak is suppressed by the onset of quantum Zeno dynamics. Since dephasing is not strong enough to suppress the growth of all side-peaks, this results in a more focused flow of population to selected peaks.

The rise and decay of interference structures indicates neatly the interplay of interference and decoherence. For the sake of specificity most of the explicit situations shown here correspond to the function $f(x, y) = |x - y|$, but none of the results discussed here are specific to this particular model. We found qualitatively the same results for $f_q(|x - y|) = |x - y|^q$ with $q = 2, 3$ and 4, with the only difference that side-peaks get more pronounced with increasing $q$. In light of the above discussion, this is as expected, since a large value of $q$ tends to restrict the summation in Eq. (3) to fewer terms so that the system state is a mixture of fewer eigenstates.

The unavoidable overhead required for the sim-
ulation of open quantum systems makes generalizations to two- or three-dimensional systems [15] prohibitively expensive for simulations on classical computers. Quantum simulators [16] – be it analog or digital – on the other hand can be viable options for such endeavors. Analog quantum simulators, e.g. with atomic gases in optical lattices [17, 18], realize predominantly coherent dynamics, but temporally modulated site energies can introduce dephasing in a highly controlled fashion. Digital quantum simulations with currently existing hardware [19] comprised of several tens of qubits suffer inherently from decoherence, and the quantum simulation of the dephasing Anderson model would be perfectly suited to turn such a limitation into an asset.

Acknowledgement This work benefited greatly from stimulating discussions with Yoshitaka Tanimura, Rob Nyman and Johannes Knolle. Financial support by JSPS in terms of the fellowship S16025 and hospitality by Kyoto University are gratefully acknowledged.

[1] Y.-S. Ra, M. C. Tichy, H.-T. Lim, O. Kwon, F. Mintert, A. Buchleitner, and Y.-H. Kim, Proc. Natl. Acad. Sci. U.S.A. 110, 1227 (2013)
[2] P. W. Anderson, Phys. Rev. 109, 1492 (1958)
[3] B. Kramer and A. MacKinnon, Rep. Prog. Phys. 56, 1469 (1993)
[4] S. A. Gurvitz, Phys. Rev. Lett. 85, 812 (2000)
[5] K. Fujii and K. Yamamoto, Phys. Rev. A 82, 042109 (2010)
[6] S. Genway, I. Lesanovsky, and J. P. Garrahan, Phys. Rev. E 89, 042129 (2014)
[7] I. Yusipov, T. Laptveva, S. Denisov, and M. Ivanchenko, Phys. Rev. Lett. 118, 070402 (2017)
[8] O. S. Vershiman, I. T. Yusipov, S. Denisov, M. V. Ivanchenko, and T. V. Laptveva, EPL 119, 56001 (2017)
[9] K. Hornberger and J. E. Sipe, Phys. Rev. A 68, 012105 (2003)
[10] M. Llobera, J. Archaeol. Sci. 28, 1005 (2001)
[11] For weak disorder only states at the edge of the spectrum are exponentially localized, but since the system is initialized in its ground state, this statement holds true for all relevant transitions.
[12] L. Banchi and R. Vaia, Journal of Mathematical Physics 54, 043501 (2013)
[13] D. A. Huse, R. Nandkishore, F. Pietracaprina, V. Ros, and A. Scardicchio, Phys. Rev. B 92, 014203 (2015)
[14] B. Misra and E. C. G. Sudarshan, J. Math. Phys. 18, 756 (1977)
[15] A. Rodriguez, L. J. Vasquez, and R. A. Römer, Phys. Rev. Lett. 102, 106406 (2009)
[16] I. M. Georgescu, S. Ashhab, and F. Nori, Rev. Mod. Phys. 86, 153 (2014)
[17] J. Billy, V. Josse, Z. Zuo, A. Bernard, B. Hambrecht, P. Lugar, D. Clément, L. Sanchez-Palencia, P. Bouyer, and A. Aspect, Nature 453, 891 (2008)
[18] G. Roati, C. Deerrico, L. Fallani, M. Fattori, C. Fort, M. Zaccanti, G. Modugno, M. Modugno, and M. Inguscio, Nature 453, 895 (2008)
[19] N. M. Linke, D. Maslov, M. Roetteler, S. Debnath, C. Figgatt, K. A. Landsman, K. Wright, and C. Monroe, Proc. Natl. Acad. Sci. U.S.A. 114, 3305 (2017)