2DOF multi-objective optimal tuning of disturbance reject fractional order PIDA controllers according to improved consensus oriented random search method

Necati Ozbey, Celaleddin Yeroglu, Baris Baykant Alagoz, Norbert Herencsar, Aslihan Kartci, Roman Sotner

a Inonu University, Faculty of Engineering, Department of Computer Engineering, Malatya, Turkey
b Brno University of Technology, Faculty of Electrical Engineering and Communication, Department of Telecommunications, Brno, Czech Republic

GRAPHICAL ABSTRACT

The consensus curve $M(E)$ states a dynamic boundary that governs optimization process depending on the value of $E$. As $E$ decreases, it implies that set point control performance is getting better, the value of consensus curve $M(E)$ increases to meet higher disturbance rejection expectation. The logarithmic consensus coefficient $x$ is used for scaling of dynamic boundary of RDR objective. As the parameter $x$ increases and dynamic boundary $M(E)$ increases for higher disturbance rejection performance. This leads a mechanism that increase of set point performance imposes the increase of disturbance rejection performance. The logarithmic consensus coefficient can be expressed as $x = \frac{E_{\text{opt}}}{E_{\text{min}}}$ where $E_{\text{opt}}$ is a desired optimal value of $\min(E)$ and $RDR_{\text{opt}}$ is a desired optimal value for $\min(RDR_{\text{dB}}(\omega))$. Determination of the logarithmic consensus coefficient $x$ defines a consensus curve for optimal search of multi objective optimization method. The following figure illustrates a consensus curvature for the logarithmic consensus coefficient $x = 2$.

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ABSTRACT

This study presents a Fractional Order Proportional Integral Derivative Acceleration (FOPIDA) controller design methodology to improve set point and disturbance reject control performance. The proposed controller tuning method performs a multi-objective optimal fine-tuning strategy that implements a Consensus Oriented Random Search (CORS) algorithm to evaluate transient simulation results of a set point filter type Two Degree of Freedom (2DOF) FOPIDA control system. Contributions of this study have three folds: Firstly, it addresses tuning problem of FOPIDA controllers for first order time delay systems. Secondly, the study aims fine-tuning of 2DOF FOPIDA control structure for improved set point and disturbance rejection control according to transient simulations of implementation models. This enhances practical performance of theoretical tuning method according to implementation requirements.
Introduction

Several research works have been highlighted merits of fractional order dynamical system modeling for more realistic representation of real world systems when compared to integer order dynamical modeling [1–4]. Hence, fractional order dynamics and fractional order control have been turned into a major topic of control system research studies in last two decades [5]. In order to utilize advantages of fractional order dynamics in closed loop control systems, Fractional Order PID (FOPID) controllers, which allow tuning of non-integer order integral and derivative elements, have been considered as a substitute of conventional PID controllers in the field of classical control. A central motivation in the research works of FOPID controllers was to harness infinite tuning options of fractional orders dynamics to obtain more control performance merits in control laws.

Utilization of fractional order dynamics in control field have been particularly focused on enhancement of robust control performance, which is so called “fractal robustness” in the field [6,7]. Many studies revealed benefits of fractional order controllers relative to their integer order counterparts and these findings have initiated discussions on industrial use of FOPID controllers, namely industrialization of FOPID controllers [8]. In general, robustness associated with fractional order controllers has been addressed in two folds: (i) improvements of the control performance robustness against parametric perturbations of control systems [9,10], (ii) enhancement of the disturbance rejection control performance against environmental disturbances [11–16]. These two major controller design objectives have been widely considered in control system researches to improve real world control performance.

Bounds of inherent disturbance rejection capacity of negative feedback loops were discussed for unknown additive input disturbance models, and RDR measurement was proposed to express disturbance rejection capacity of closed loop FOPID control systems [13,14,16]. This was a useful step to figure out bounds of disturbance rejection capacity of closed loop systems [16]. Formulation of RDR index was derived by assuming a closed loop control system as a communication channel and RDR spectrum of the control system was expressed as the ratio of power density of reference signal relative to the power density of disturbance signal at the plant output. It resembles Signal to Noise Ratio (SNR) that was defined to evaluate signal transmission capacity of a noisy communication channel. Alagoz et al. showed that RDR performance of closed loop control systems depends on spectral power density of controllers, and practical RDR performance is bounded by stability of control systems [16].

Although increase in spectral power density of the controller function contributes to RDR index and improves disturbance rejection performance of negative feedback control loops, it deteriorates step response performance because the increasing output power of controllers causes higher overshoots and ripples that appear while settling to a set point. Further increase of RDR values finally leads to instability of closed loop control systems. Therefore, boundary of controller coefficients becomes a natural boundary for RDR performance, namely an inherent limitation for disturbance rejection capacity of closed loop systems [16]. Consequently, there exists a design tradeoff between set point performance and disturbance rejection control performance. This tradeoff brings out an essential problem for disturbance rejection controller tuning approaches. A feasible solution to this problem was to use a set point filter type 2DOF control systems. These systems perform a reference input shaping strategy by using a pre-filter function at reference input [16–18]. This pre-filter function is also known as the set point filter.

In control practice, the RDR spectrum analysis was used for evaluation of disturbance rejection performance of a closed loop FOPID control of magnetic levitation system, and an experimental validation of disturbance rejection performance improvements was illustrated in [15]. On the other hand, minimum RDR constraint has been used as a disturbance rejection objective in multi-objective tuning problems of PID and FOPID controllers [19–21]. However, the design tradeoff between disturbance rejection control and set point control reduces effectiveness of controller tuning methods in practice. To address this design tradeoff, a set point filter type 2DOF FOPID controller structure was implemented to enhance step response performance in case of disturbance rejection control [21]. This study also demonstrated a multi-objective pareto optimal tuning of FOPID controllers by introducing CORS algorithm. The CORS algorithm implements a consensus curve to deal with the design tradeoff that appears between set point and RDR performances [21]. Findings of this study become a motivation for the current study that extends this approach to optimal fine-tuning of 2DOF FOPID control systems according to transient control simulation results.

An accelerator term (second derivative term) was firstly adapted to PID controllers. This controller can respond the second order dynamical changes in control error and thus PIDA benefits from accelerator term to respond higher order dynamical changes in control error of closed loop control systems. This property can be expected to improve disturbance rejection control performance so that disturbance can be considered as an intermittent, higher order external dynamics that temporarily affect plant function dynamic response. In literature, tuning problem and application of PIDA controllers has been studied at a limited extent [22–24]. PIDA controllers are not highly complicated controller structures however tuning of this controller can be performed by using metaheuristic search algorithms such as particle swarm optimization, artificial bee colony etc [24]. Due to their higher computational complexity, these search algorithms may not be feasible for implementation on low cost control cards for onsite auto tuning control applications. Since possible advantages of FOPIDA controller to deal with high order dynamics, Puangdownreong have suggested tuning of FOPIDA controllers [25]. Particle swarm optimization algorithm was implemented for tuning FOPIDA controllers and control performance improvements were illustrated in [26,27]. To the best of our knowledge, tuning problem for FOPIDA controllers in order
to obtain improved disturbance rejection control performance for large time delay systems has not been a solved problem. In the current study, we address a straightforward solution for tuning problem of 2DOF FOPIDA and aim a feasible solution for the disturbance rejection control problem of large time delay control systems. For this purpose, in addition to a set point performance objective, an RDR performance objective is utilized in optimal tuning of FOPIDA controllers. Accordingly, the RDR spectrum formulation is derived for closed loop FOPIDA controllers in the following section. In further sections, the CORS tuning method is improved by initializing controller coefficients according to results of an analytical tuning method. The well known Zeigler Nichols tuning method is utilized for initial configuration of the CORS tuning method. Thus, analytical Zeigler Nichols tuning method provides a stable solution to perform a Random Search (RS) for fine-tuning controller coefficients. This fine-tuning scheme starts with results of Zeigler Nichols method, and continues searching for controller coefficients that provide a better set point and RDR performances according to transient simulation results of control systems.

The RS algorithm is a fundamental, low computational complexity and straightforward stochastic search method to find local minimum points according to random walk type strategy [28–32]. To employ this algorithm in a multi-objective controller tuning problems, RS algorithm was modified by adopting a consensus curve for pareto optimal search of solutions in case of conflicting multi-objectives [21]. One control objective requires minimization of set point error for improved step response and stability. The other objective maximizes RDR index to increase disturbance rejection capacity of resulting control systems. As a consequence, CORS algorithm can search in a guidance of a consensus curve that enforces search direction towards higher RDR values while keeping the set point control errors at low levels [21]. A major complication of metaheuristic algorithms is the finding a stable initial configuration of controller coefficients to progressively improve them according to simulation results. This problem is also solved in the current study by devising a hybrid algorithm that combines an analytical tuning method to obtain a stable initial solution, and a random search algorithm to improve this solution according to implementation requirements.

**RDR analysis of FOPIDA and theoretical background**

RDR spectrum was proposed for quantitative assessment of input disturbance rejection capacity of closed loop control systems. It resembles SNR index, which is a fundamental measure for evaluation of signal transmission performance in communication channels. The RDR analysis was carried out for closed loop control systems by considering additive input disturbance model [13,14,16] and expressed in the form of

\[
\text{RDR}(\omega) = |C(j\omega)|^2, \quad (1)
\]

where \(C(j\omega)\) stands for frequency response of controller transfer functions \(C(s)\). The \(C(j\omega)\) can be obtained by using \(s = j\omega\) in the controller transfer functions \(C(s)\). RDR is expressed in decibel (dB) \[14,16\],

\[
\text{RDR}_{\text{ad}}(\omega) = 10\log|C(j\omega)|^2. \quad (2)
\]

For more theoretical details on the formulation of RDR index, one can consider references [14] and [16]. RDR spectrum, defined by Eq. (2), provides a useful measure to assess disturbance rejection rates of control systems for each frequency components. It is noteworthy to state that Eq. (2) allows spectral assessment of additive input disturbance rejection capacity of the closed loop control systems depending on only controller parameters. In general, practical control systems work in low frequency region and higher RDR values at low frequency region is prominent to obtain satisfactory disturbance rejection control against environmental disturbances. Environmental disturbances such as alterations in operating conditions change slowly relative to controller output. Hence, a higher RDR value at low frequency region is prominent for rejection of slowly developing environmental disturbances. Integral element of controller function particularly enhances the low frequency part of RDR spectrum as shown in Fig. 1(a). (RDR spectrum of integral term \(\frac{1}{s}\) is \(10\log(k_i^2 / \omega^2)\)) Increasing RDR spectrum at higher frequency region makes control system more robust against high frequency disturbances, for instance system noises (e.g. quantization noise, sensor noises etc.) or white noises. White noise signals are random and its spectral power density spreads to the whole spectrum. Derivative element of controller function particularly enhances the high frequency part of RDR spectrum as shown in Fig. 1(a). (RDR spectrum of derivative term \(ks\) is \(10\log(k^2_s\omega^2)\)). High RDR at higher frequencies is preferable for rejection of system noise or white noises. For a fair comparison, controller coefficients are taken equal to 1 in the figure.

Transfer function of FOPID controller is commonly written in general form of

\[
C_{\text{FOPID}}(s) = k_p + k_i s^{-1} + k_s s^n, \quad (3)
\]

![Fig. 1](image-url)
where parameters $k_p$, $k_a$ and $k_i$ are gain coefficients and the parameters $s$ and $\mu$ are fractional orders of FOPID controllers. Design of a FOPID controller involves tuning of these five design parameters in order to obtain a desired control response. The RDR of closed loop FOPID control systems was derived as [14],

$$\text{RDR}_{\text{FOPID}}(\omega) = \left( k_p + k_a s^{-\lambda} \cos(\frac{\pi}{2} \lambda) + k_0 s^\mu \cos(\frac{\pi}{2} \mu) \right)^2 + \left( k_o s^{-\lambda} \sin(\frac{\pi}{2} \lambda) - k_0 s^\mu \sin(\frac{\pi}{2} \mu) \right)^2. \quad (4)$$

Transfer function of FOPIDA controller is written in general form by adding accelerator term $k_2 s^2$ to FOPID controller function as

$$C_{\text{FOPIDA}}(s) = k_p + \frac{k_i}{s} + k_2 s^\mu + k_a s^2, \quad (5)$$

where the additional parameter $k_a$ is the accelerator coefficient. The accelerator term $k_2 s^2$ considers changes in velocity of control error of closed loop control. Thus, the second order dynamics in control error can contributes to the control law of FOPIDA controllers. A benefit of the accelerator term appears at high frequency disturbance rejection performance because this term increases RDR performance at high frequencies more than the derivative element of controller. (RDR spectrum of accelerator element $k_2 s^2$ is $10 \log(k_2 s^2)$. FOPIDA controller design requires tuning of those six design parameters, where five of those parameters are coefficients of FOPID and an additional parameter is the accelerator coefficient. The RDR of closed loop FOPIDA control system can be derived by using $s = j\omega$ in equation (1).

$$\text{RDR}_{\text{FOPIDA}}(\omega) = \left( k_p + k_a j \omega \cos(\frac{\pi}{2} \lambda) + k_0 j \omega \cos(\frac{\pi}{2} \mu) - k_0 j \omega^2 \right)^2 + \left( k_0 j \omega \sin(\frac{\pi}{2} \mu) - k_0 j \omega^{-\lambda} \sin(\frac{\pi}{2} \lambda) \right)^2. \quad (6)$$

For disturbance reject controller design, the following RDR constraints can be used to specify a lower boundary for disturbance rejection capacity of the resulting control system at an operating frequency range of $\omega \in \left( \omega_{\text{min}}, \omega_{\text{max}} \right]$. 

$$\min_{\omega \in \left( \omega_{\text{min}}, \omega_{\text{max}} \right]} \left( \text{RDR}_{\text{FOPID}}(\omega) \right) \geq M, \quad (7)$$

where $M \in \mathbb{R}$ is a design specification. This constraint infers that the lowest RDR performance should be equal or greater than the lower boundary $M$ [16].

To investigate effects of accelerator term to disturbance rejection performance of the closed loop control system, we compare RDR spectrums of conventional FOPID controller and FOPIDA controller for equal values of coefficients $k_p = 1$, $k_a = 1$, $k_i = 1$, $\lambda = 1$ and $\mu = 1$ and $k_a = 1$. Fig. 1(b) reveals that RDR performance of FOPIDA controller is equal or greater than RDR performance of FOPID controller except RDR values around the angular frequency $\omega = 1$ rad/sec. Inset of Fig. 1(b) is a close view of this part of spectrum. This characteristic implies that a harmonic disturbance at 1 rad/sec deteriorates disturbance rejection performance of FOPIDA controller. Such performance deteriorations come out a need for special consideration of low frequency disturbance rejection performance when designing FOPIDA controllers. The integral compensators ($k_i$) are widely used for removal of steady state errors [33]. As it is shown in Fig. 1(a), the integer order integral compensator contributes RDR performance at low frequency region. (RDR spectrum of integral element $\frac{k_i}{s}$ is $10 \log(1/\omega^2)$). A future study can address enhancement of low RDR performance at the low frequency region by using an integral compensator parallel to FOPIDA controllers.

A practical and general solution of the low RDR problems is to perform fine-tuning of FOPIDA controller implementations according to the minimum RDR constraint (Eq. (7)). To address the low RDR problems, the current study implements this fine-tuning option by using the CORS algorithm. To verify validity of fine-tuning options for RDR enhancement process, one should theoretically demonstrate the existence of FOPIDA controller coefficient configurations that can surpass RDR of FOPID controllers. For this reason, a sufficient condition is figured out to validate improvement of RDR performance of FOPIDA controllers relative to RDR performance of FOPID controllers. This sufficient condition can be expressed as $\text{RDR}_{\text{FOPIDA}}(\omega) - \text{RDR}_{\text{FOPID}}(\omega) > 0$. By using equations (6) and (4), this condition can be obtained as

$$2k_p + 2k_0 j \omega \cos(\frac{\pi}{2} \lambda) + 2k_0 j \omega \cos(\frac{\pi}{2} \mu) < k_0 j \omega^2. \quad (8)$$

This sufficient condition verifies the existence of an infinite set of FOPIDA controller coefficients that can surpass RDR performance of FOPID controller at any desired frequency component. (See appendix section for the derivation of the sufficient condition) This theoretical consideration validates the fine-tuning option of FOPIDA controllers.

**FOPIDA controller design by consensus curve oriented RS algorithm**

Fig. 2 shows a block diagram of set point filter type 2DOF closed loop control structure that can be a preferable solution for enhancement of set point performance in case of disturbance rejection control [16]. In this control structure, a set point filter $F(s)$ is employed to smooth reference input signal $r(t)$ via filtering out high frequency components from the reference input $r(t)$. In case of a powerful controller, which is also an indication of high RDR performance, high frequency components of $r(t)$ leads to fast alterations (e.g. high overshoots, multiple ripples) at the system output during settling period. High overshoots, ripples (also known as ringing effect in electronics) and longer settling periods are not desirable for sensitive set point control applications such as level control applications. The set point filter $F(s)$ can smooth the reference input signal $r(t)$ and this allows more consistent and asymptotical settling effect. This type smooth settling reduces unnecessary ripples that can cause longer settling periods and more energy consumption in control actions. To allow none-overshoot smooth settling characteristic in control system response, a first order pre-filter function [16] is implemented as

$$F(s) = \frac{a}{s + a}. \quad (9)$$

where constant $a = 1/\tau_f$ and the parameter $\tau_f$ is the time constant of the filter. Step response of this filter function yields a first order dynamic response that settles to its input value without producing any overshoot. Such a filter describes preferable step response characteristics, which can be particularly desirable for precise level or alignment control applications for instance temperature control, liquid level control or control of smoothly alignment tasks of an equipped heads or vehicles. Previously, utilization of this type set point pre-filters as a reference model was shown for shaping the reference input in adaptive control [34]. Essentially, the function $F(s)$ is employed to describe a desired trajectory of step response, of which the closed loop control systems can track. As shown in the block diagram in Fig. 2, closed loop control system tracks the filter output $r_f$, and this pre-filter acts as a reference model. On the
other hand, the 2DOF closed loop control structure is used to deal with design tradeoff appearing between set point control and disturbance rejection control performances [16]. This effect can be explained as following:

High disturbance rejection requires strong or aggressive controllers, which is possible by using control laws with high power density. Such a high power density control law can easily deteriorate set point control performance because of forming high overshoots and ripples while settling to the set point [16]. To reduce those overshoots and ripples in settling, a first order pre-filter \( F(s) \) is used to eliminate very high frequency components in step waveform and to smooth reference input signal before applying to the closed loop control system [16,21]. Thus avoids excitation of high frequency components at the controller output and consequently, diminishes high overshoots and ripples at the output of control system while settling to set points [16]. We assumed an additive input disturbance model to represent impacts of environmental disturbance on the control system.

The mean squared control error (MSCE) from transient simulation [36] is used to measure set point control performance that is given by

\[
E = \frac{1}{T} \int_{0}^{T} e(t)^2 dt. \tag{10}
\]

The primary control objective of the controller tuning is commonly the minimization of MSCE, which is written by \( \min(E) \) [36]. The parameter \( T \) is the observation time for MSCE calculations. We performed transient simulation of the control system and obtained instant control errors \( e(t) \) in order to calculate \( E \). The observation time \( T \) is configured to the total simulation time in these simulations. The minimization of \( E \) leads to decrease the magnitude of control errors signal that is written by \( e(t) = r(t) - y(t) \). This enforces \( e(t) \) to approximate to zero, which implies settling of the plant output \( y \) to the desired reference input \( r \). This primary objective assures set point tracking and stability of the closed loop control system.

The secondary control objective is to increase disturbance rejection performance without degrading set point control performance. To perform the disturbance rejection control objective for closed loop control systems, the minimum RDR constrains, given by Eq. (7), is utilized as a secondary objective of multi-objective optimal tuning problem. A consensus curve, which describes a dynamic boundary for acceptable RDR performance depending on \( E \), is defined as

\[
M(E) = -\log E. \tag{11}
\]

Then, the minimum RDR value in the RDR spectrum is limited by the consensus curve \( M(E) \). This condition is expressed as

\[
\min_{\omega \in [\omega_{\text{min}}, \omega_{\text{max}}]} \{ \text{RDR}_{\text{dB}}(\omega) \} \geq M(E). \tag{12}
\]

The consensus curve \( M(E) \) states a dynamic RDR boundary that governs optimization process depending on the value of \( E \). As \( E \) decreases, it implies that the set point control performance is getting better, the value of consensus curve \( M(E) \) increases to meet higher disturbance rejection expectations. This property leads a mechanism such that an improvement in set point performance imposes the increase in disturbance rejection performance. The logarithmic consensus coefficient \( \alpha \) is used for scaling of dynamic boundary of RDR objective. When the parameter \( \alpha \) is set to higher values, the dynamic boundary \( M(E) \) increases to provide higher disturbance rejection performance. A suitable logarithmic consensus coefficient can be found by

\[
\alpha = -\frac{\text{RDR}_{\text{dB}}}{\log_{\text{min}}} \tag{13}
\]

where \( E_{\text{min}} \) is a desired optimal value of \( \min(E) \), and \( \text{RDR}_{\text{dB}} \) is a desired optimal value for \( \min_{\omega \in [\omega_{\text{min}}, \omega_{\text{max}}]} \{ \text{RDR}_{\text{dB}}(\omega) \} \). Determination of the logarithmic consensus coefficient \( \alpha \) configures a consensus curve for optimal search of multi-objective optimization method. Fig. 3 illustrates a consensus curve for the logarithmic consensus coefficient, \( \alpha = 2 \). The update condition in step 5 allows optimization of controller coefficients in the allowed design region, which is above the consensus curve in Fig. 3. This region represents a set of acceptable solutions to deal with tradeoff between opposing design objectives. The low performance region, which is below the consensus curve, is forbidden because designs in this region are not acceptable in term of multi-objective design performance.

In this study, the disturbance reject control problem of the large time delay systems is considered. These systems can be represented by a first order time delay transfer function

\[
G(s) = \frac{K_{dc}}{Ts + 1} e^{-\tau s}, \tag{14}
\]

where the parameter \( K_{dc} \) is static gain of plant function, \( \tau \) is the time constant of dominating first order dynamics of systems, and \( L \) is the time delay, which is also known as dead time or apparent time delay of the system. Due to large time delay, optimal tuning of integral component of FOPID controllers yields very low values for coefficient of integrator element \( (k_i) \) relative to other gain coefficients. Such a low integrator gain causes weak integral operation and it may leave steady state errors in set point control applications [33]. Therefore, practical controller design task for a large time delay plant needs a special concern for set point control [37].

In the previous study, CORS algorithm was proposed by modifying a classical RS algorithm in order to perform optimization in guidance of a consensus curve [21]. In the current study, a significant modification to improve design performance of CORS algorithm is that initial values of design coefficient are configured according to results of an optimal tuning method. This provides a good initial design point to further optimize control systems for improved disturbance rejection control performance. Therefore, to implement analytical tuning, we configure initial coefficients of FOPIDA controller designs according to Zeigler Nichols method in the current study. Zeigler Nichols method is a well-known and widely accepted analytical tuning method. Coefficients of Zeigler Nichols method is fine-tuned by the CORS algorithm. Since, there is no suggestion of Zeigler Nichols method for the accelerator.
coefficient and fractional orders, the initial value for accelerator coefficient \((k_{a0})\) is set to zero, and the initial values for fractional orders \((\lambda_a \text{ and } \mu_a)\) are set to one. Consequently, PID design of Zeigler-Nichols method is progressively evolved to a FOPIDA controller design. For 2DOF design of FOPIDA controller, the pre-filter parameter \(a\) is determined regarding the time delay and time constant of plant functions. Thus, FOPIDA designs can track the first order dynamics of the pre-filter, properly. A feasible time constant for the pre-filter function was empirically found as the time delay plus a fraction of the time constant of plant function \((\tau_f = L + \frac{\lambda_o}{\mu_o})\). Typical value of the \(\gamma > 0\) is around \(1-5\). Then, a relevant pre-filter coefficient \(a\) is written by

\[
a = \frac{\gamma}{\tau_f + \tau}.
\]  

Steps of the improved CORS algorithm to fine-tune 2DOF design of FOPIDA controller are as follows:

**Step 1 (Initial Configuration):** Set initial values \(k_p = k_{io}, k_d = k_{do}, k_1 = k_{o1}, k_2 = k_{o2}, \lambda = \lambda_o, \mu = \mu_o\) according to an optimal controller design method. (Use Zeigler Nichols method for initial values of \(k_p, k_o, k_{io}, k_{do}, k_{o1}, k_{o2}\) and set \(k_o = 0, \lambda_o = 1 \text{ and } \mu_o = 1\) ) Set pre-filter parameter \(a\) according to Eq. (15) and set \(E_{min}\) to a large value (typically \(1000\)). Configure random search lengths \(c_p, c_d, c_i, c_c, c_p\) and \(c_d\).

**Step 2 (Random Search):** Generate new candidate for controller coefficients by using random walks in parameter search space as follows

\[
k_{pm} = k_p + (\text{rand} - 0.5)c_p,
\]

\[
k_{dm} = k_d + (\text{rand} - 0.5)c_d,
\]

\[
k_{im} = k_i + (\text{rand} - 0.5)c_i,
\]

\[
\lambda_n = \lambda + (\text{rand} - 0.5)c_i,
\]

\[
\mu_n = \mu + (\text{rand} - 0.5)c_i,
\]

\[
k_{an} = k_a + (\text{rand} - 0.5)c_a.
\]

**Step 3 (Performance Evaluation):** Perform transient simulation for these candidate coefficients and calculate the error function \(E\) for a step response with a \(T\) simulation time. Then, calculate \(\text{min}(RDR_{db})\) for the operating frequency range of \(\omega \in \left(\omega_{min}, \omega_{max}\right)\).

**Step 4 (Consensus and Coefficient Update):** If the update condition \((E < E_{min}\) and \(\text{min}(RDR_{db}) \geq M(E_{min})\)) is satisfied, then update the current controller coefficients by using candidate coefficients; \(k_p \rightarrow k_{pm}, k_d \rightarrow k_{dm}, k_i \rightarrow k_{im}, \lambda \rightarrow \lambda_n, \mu \rightarrow \mu_n, k_a \rightarrow k_{an}\). Then, update the minimum error \(E_{min} = E\).

**Step 5 (Update of Dynamic Lower Boundary):** Calculate the dynamic RDR boundary \(M(E_{min}) = -2\log\left[E_{min}\right]\) for the current minimum error \(E_{min}\).

**Step 6 (Stopping Criteria):** If \(E_{min}\) is adequately small or a maximum iteration count is exceeded, end the optimization. Otherwise go to step 2.

The constants \(c_p, c_d, c_i\) and \(c_a\) are RS lengths for each gain coefficients and, \(c_c\) and \(c_p\) are random search lengths for fractional orders. These RS lengths specify a maximum bouncing range for each coefficient. During optimizations, the minimum value of \(E\) is stored in \(E_{min}\) parameter. Therefore, \(E_{min}\) should be set very high values at initialization of optimization.

**Illustrative design examples**

This section presents three design examples to demonstrate applications of proposed design method. Fig. 4 illustrates a flow chart that depicts incorporation of improved CORS algorithm and transient control simulations of 2DOF FOPIDA control systems. The CORS algorithm sends candidate controller coefficients to MATLAB Simulink (MS) simulation environment in order to carry out transient control simulations. MSCE of each candidate solution is calculated according to simulation results. Fractional order derivative and integral elements were implemented in these simulations according to Oustaloup’s method by using FOTF Matlab toolbox [38].

**Example 1 (Large Time Delay Systems):** Let’s design a set point filter type 2DOF FOPIDA control system for a large time delay plant model

\[
G(s) = \frac{3.13}{433.33s + 1}e^{-50\tau}
\]

for a logarithmic consensus coefficient \(\alpha = 1\). This plant function represents a linear model of the experimental platform Basic Process Rig 38-100 Feedback Unit, which was used by Monje et al. to demonstrate performance of fractional order controllers in industrial applications [11]. According to model parameters of this plant function, the experimental system presents 50 sec time delay in responding to a change in the reference input. After this apparent time delay, the system settles according to a dominating first order system pole with 433.33 sec time constant and a DC gain of 3.13. Such a large time delay plant complicates the closed loop controller design due to the requirement of very small integrator coefficients, which make it very sensitive to realization issues. Non-ideal realization of fractional order elements may fail results of analytical tuning methods in real control applications because analytical optimal tuning models rely on an ideal and theoretical model of fractional order elements. Therefore, a fine-tuning with respect to practical realization model of optimal controllers improves real world performance of control system implementations in the case of analytical optimal tuning.

By using parameters of Rig 38-100 feedback unit, which are \(K_o = 3.13, \tau = 433.33\) and \(L = 50\), initial values of controller design coefficients were obtained \(K_p = 3.3227, K_o = 78.25, K_{io} = 0.0313, \lambda_o = 1, \mu_o = 1\) according to Ziegler-Nichols tuning method and \(a = 0.0041\) according to Eq. (15). These values were configured as initial value of coefficients in the CORS algorithm. The proposed CORS algorithm was performed for 10 iterations. For a fast response of control system, \(\gamma\) parameter of pre-filter was set to 5. The MS simulations of proposed 2DOF FOPIDA control system were run 5000 sec. Set point of basic process rig 38-100 feedback unit was 0.47 [11]. Hence, a step input with the amplitude of 0.47 was applied to reference input in the simulations. At the simulation time 2500 sec, a step disturbance with amplitude of 0.3 was applied to the input of plant model. Based on MS simulation results, MSCEs for each candidate design was calculated and sent back to the CORS algorithm at each iteration of optimization process. When the optimization was completed, a fine-tuned FOPIDA controller function was obtained as

\[
C_{FOPIDA}(s) = \frac{3.3817 + 0.0283s}{s^{3.0283} + 80.0205s^{1.0162} - 0.0108s^2}
\]

Parameters of controller functions, which were used for performance comparisons, are listed in Table 1. Fig. 5 shows performance of controllers. Table 2 summarizes set point and disturbance rejection control performances of these controllers. The 2DOF FOPIDA controller settles in 663 sec, which is the shortest settling time without any overshoot and ripples. A step disturbance was applied after settling, the 2DOF FOPIDA control system was settled back to the set point 0.47 in 300 sec with 21% overshoot and 3 slight ripples. The 2DOF FOPIDA control was the fastest in resetting and the shortest in overshoots in disturbance simulations. These performance analyses indicate that 2DOF FOPIDA controller can present much better set point control and disturbance rejection
Fig. 4. A flow chart that depicts incorporation of the CORS algorithm and the transient control simulations.

Table 1
Coefficients of controllers designed for $G(s)$.

| Tuning Method                  | $k_0$ | $k_d$ | $K_i$   | $k_a$ | $\lambda_a$ | $\mu_a$ |
|--------------------------------|-------|-------|---------|-------|--------------|---------|
| FOPID (Monje et al. [11])     | 0.61  | 4.38  | $10^{-2}$ | 0     | 0.8968       | 0.4773  |
| Optimal PID (Matlab)          | 0.55  | -57.69| $10^{-3}$ | 0     | 1            | 1       |
| 2DOF FOPIDA                   | 3.38  | 80.02 | $10^{-2}$ | $-10^{-2}$ | 0.9676       | 1.0162  |
control performances than those of other controllers in this exam-
ple. Fig. 5(a) compares RDR performances of controllers to validate
disturbance rejection improvements via RDR spectrum. Fig. 5(b)
shows step and disturbance responses of the proposed 2DOF FOPIDA
control system for 5000 sec. One can observe in Fig. 5(b)
that the proposed control system settles without any overshoot
in a satisfactory period. For disturbance rejection simulation, the
control systems were disturbed at 2500 sec by a step disturbance
and, disturbance rejection performance of the proposed FOPIDA
control is more satisfactory than those of other control systems.
These simulation results clearly demonstrate that the proposed
2DOF FOPIDA control system can improve both set point control
performance and disturbance rejection control performance. The
results in figure also confirm the data in Table 2. The figure also
indicates the disturbance rejection performance improvement of
the FOPIDA controller compared to the optimal FOPID controller
designed by Monje et al. in [11]. Fig. 5(c) shows evolution of con-
trol errors and results reveals performance improvements of the
2DOF FOPIDA control system in term of robust control perform-
ance. This observation indicates that both set point control and
disturbance rejection performance can be further enhanced by
the improved CORS algorithm.

To test controller in more realistic simulations, an additive type
white noise with power of $5 \times 10^{-6}$ was inserted to feedback loop
to mimic sensor measurement noise. Fig. 6 shows responses of each
control system under a step disturbance and sensor noise condi-
tions. Variances of system outputs are computed for comparison
of overall set point control performances as;

$$r^2 = 0.0053 \text{ for FOPID (Monje et al. [11]),}$$

$$r^2 = 0.0157 \text{ for Optimal PID (Matlab) and}$$

$$r^2 = 0.0044 \text{ for 2DOF FOPIDA. The variance of 2DOF FOPIDA con-
trol system output is measured lower than variance of other con-
troller's outputs, and it is an indication of robust control
performance improvement.}$$

Example 2 (TRMS Nonlinear Model): This example demonstrates
performance of 2DOF FOPIDA control for a nonlinear model of
TRMS experimental setup. This nonlinear model of the main rotor
was provided by producer of TRMS experimental setup [35,36].In
this control problem, the vertical angle of the main rotor is con-
trolled by regulating terminal voltage of the DC electric motor. This
control action adjusts rotational velocity of propeller to hover the
main rotor at the desired angle. Due to nonlinear aerodynamics
of propeller blades, this example introduces a nonlinear set point
control problem. In this example, we tested performance of three
controllers. These are a classical PID controller, a conventional
FOPID controller and the proposed 2DOF FOPIDA controller. The
optimal PID controller for the main rotor control of TRMS setup
is provided by Feedback Inc as [35,36]

$$C_{PID}(s) = \frac{5}{s} + \frac{8}{s^2} + 10s.$$  \hspace{1cm} (24)

The FOPID controller was tuned according by the CORS algo-
rithm as

$$C_{FOPID}(s) = 5.04 + \frac{7.96}{s_{\text{imp}}} + 10.022s^{1.13}. \hspace{1cm} (25)$$

The 2DOF FOPIDA controller was designed by the fine-tuning of
improved CORS algorithm as

$$C_{FOPIDA}(s) = 9.87 + \frac{7.12}{s_{\text{imp}}} + 11.78s^{1.10} - 0.95s^2. \hspace{1cm} (26)$$

The CORS algorithm was initialized by using the parameters of
optimal PID controller. When the optimization is completed,
$\min[RDR_{dB}]$ was obtained 21.68 dB for a minimum MSCE
$E_{\text{min}} = 3.81 \times 10^{-1}$. Fig. 7(a) shows step and disturbance responses
of these controllers. The 2DOF FOPIDA controller can enhances
set point and disturbance rejection control performances
compared to responses of other controllers. Fig. 7(b) reveals
improvements of disturbance rejection control via 2DOF FOPIDA
controller. Fig. 7(c) shows changes of control errors and confirms
improvement in disturbance rejection control.

Example 3 (Automatic Voltage Regulator (AVR) Model): This
example illustrates control of an AVR model by using the proposed
2DOF FOPIDA control scheme. The AVR systems are important
components of power systems that contribute to power quality

![Fig. 5.](image-url)
of an electricity grid by stabilizing terminal voltage of generators [39]. However, a number of factors such as load variability or demand fluctuation in power systems can disturb AVR terminal voltage. The preservation of voltage stability is important for a reliable power generation. The objective of AVR system control is keeping the terminal voltage of a generator at a desired set point level [39]. Ramezanian et al. used Particle Swarm Optimization (PSO) and chaotic ant swarm (CAS) optimization methods to design an optimal FOPID controller for the linear AVR model in [39].

$$C_{FOPID_{PSO}}(s) = 1.26 + \frac{0.55}{s^{1.18}} + 0.23s^{1.25}$$

(27)

$$C_{FOPID_{CAS}}(s) = 1.05 + \frac{0.44}{s^{1.06}} + 0.25s^{1.11}$$

(28)

The 2DOF FOPIDA controller is retuned by using improved CORS algorithm.

$$C_{FOPID}(s) = 1.50 + \frac{0.65}{s^{1.79}} + 0.27s^{1.25} - 0.000287s^2$$

(29)

The step and disturbance responses of these controllers are illustrated in Fig. 8. Figure reveals set point and disturbance rejection control performance improvements by using 2DOF FOPIDA control. Main reason of these improvements is the fine-tuning of optimal FOPID controller to obtain better disturbance rejection according to transient simulation of the AVR model.

These illustrative examples reveal that practical control performance of optimal tuning methods can be further enhanced by performing fine-tuning according to the transient simulation of control systems. A major complication in this type of metaheuristic optimization problems is interruption of transient simulations due to unstable design points. The unstable design points mainly result in overflow of simulation parameters, and it leads to interruption of metaheuristic optimization tasks before a successful completion. Such interruptions in transient control simulation can be a serious concern for implementation of metaheuristic optimization methods in the optimal tuning of control systems. To address this complication in the current study, a hybrid tuning approach is implemented, which combines stable solutions of analytical optimal tuning method with flexibility of the stochastic search: Analytical tuning methods provide a stable design point, and the proposed CORS algorithm performs fine-tuning of the design point by considering transient simulations of implementation models of control systems. This strategy allows fine-tuning of control systems around the optimal design points and contributes to practical performance of optimal controller design methods.

**Conclusions**

This study introduced a computer-aided controller design methodology for improvement of disturbance reject control performance of control systems. The CORS tuning algorithm becomes more effective by cooperation of optimal tuning methods. The improved CORS algorithm starts with controller coefficients of optimal tuning methods and further optimizes controller coefficients to increase disturbance rejection performance according to the consensus curve. The consensus curve is proposed to govern the optimization process towards controller solutions that results in higher disturbance rejection performance and lower set point error. To measure disturbance rejection performance of control loops, RDR spectrum for FOPIDA controllers was obtained. Then, contributions of FOPIDA to disturbance rejection control performance were investigated.

Simulation results indicate that the proposed CORS algorithm can deal with two short-coming of analytical optimal tuning methods:

(i) Due to increasing complexity and difficulties in finding analytical solutions of complicated equation systems, analytical tuning methods do not consider sophisticated design specification and constraints. The CORS algorithm can fine-tune...
results of analytical tuning methods to meet additional design specifications and requirements such as disturbance rejection control.

(ii) Analytical tuning methods indeed assume an ideal and continuous realization of controller functions. In practice, realization of FOPID controller functions is based on approximate equivalent models [38,40–45]. The fine-tuning efforts according to transient responses of realization models are required to validate optimal controller coefficients. Such a fine-tuning based on realization models can reduce the gap between results of theoretical solutions and their practical realizations.

Compliance with ethics requirements

This article does not contain any studies with human or animal subjects.
Declarations of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A

A sufficient condition to improve RDR of FOPIDA controllers:

Property. A sufficient condition, which increases RDR values of FOPIDA controllers relative to RDR values of FOPID controllers, is expressed at any given frequency as

\[2k_{p} + 2k_{\omega}^{-1}\cos\left(\frac{\omega}{2}\right) + 2k_{\omega}^{2}\cos\left(\frac{\pi}{2} \mu\right) < k_{o}^{2}\]

Proof. The inequality, \(RDR_{\text{fopida}}(\omega) - RDR_{\text{fopid}}(\omega) > 0\), suggests a sufficient condition that infers the case that RDR values of FOPID controllers is greater than RDR performance of FOPID controllers at a given any angular frequency \(\omega\). Then, by using \(RDR_{\text{fopida}}(\omega)\) (equations (6)) and \(RDR_{\text{fopid}}(\omega)\) (equation (4)), this condition can be written by

\[(k_{p} + k_{\omega}^{-1}\cos(\frac{\omega}{2}) + k_{\omega}^{2}\cos(\frac{\pi}{2} \mu)) + (k_{\omega}^{-1}\sin(\frac{\omega}{2}) - k_{\omega}^{-1}\sin(\frac{\pi}{2} \mu)) > 0\]

This inequality can be reorganized as

\[2k_{p} + 2k_{\omega}^{-1}\cos\left(\frac{\omega}{2}\right) + 2k_{\omega}^{2}\cos\left(\frac{\pi}{2} \mu\right) - k_{o}^{2} > 0\]

Then, one can express the sufficient conditions as

\[2k_{p} + 2k_{\omega}^{-1}\cos\left(\frac{\omega}{2}\right) + 2k_{\omega}^{2}\cos\left(\frac{\pi}{2} \mu\right) < k_{o}^{2}\]

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