TELLING THREE FROM FOUR NEUTRINO SCENARIOS

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In this talk I will consider the possibility of distinguishing the usual three neutrino model from scenarios in which a light sterile neutrino is also present. I will show that the confusion with the so-called 3+1 scheme can arise in some particular region of the parameter space whereas it is essentially absent for the 2+2 scheme. Then I will discuss the ambiguities in the determination of the CP-violating phase $\delta$.

1 Introduction

1.1 Why four neutrinos?

Experimental data from atmospheric neutrino experiments can be interpreted in terms of neutrino oscillations if $\Delta m^2_{\text{atm}}$ is of order $\sim (1.6 - 4) \times 10^{-3}$ eV$^2$; solar neutrino experiments, instead, point towards a $\Delta m^2_{\odot}$ of order $\sim 10^{-4}$ eV$^2$. The LSND data, on the other hand, would indicate a $\nu_\mu \rightarrow \nu_e$ oscillation with a third, very distinct, neutrino mass difference: $\Delta m^2_{\text{LSND}} \sim 0.3 - 6$ eV$^2$. Experiments such as MiniBooNE will be able to confirm it in the near future. If we consider the usual three neutrino family, it is impossible to simultaneously explain the whole ensemble of the data since there are only two independent neutrino mass differences. We should therefore include at least one more neutrino (sterile, since it must be an electroweak singlet to comply with the strong bound on the $Z^0$ invisible decay width) with a third, independent, mass difference to fit all the experimental data.

In this framework, it seems quite relevant to understand if three- and four-family models are distinguishable and, particularly, if the effect of CP violation in a three–family world could be mimicked by introducing one sterile neutrino in the mixing matrix.

In order to answer to these questions we use a Neutrino Factory experimental set-up, which has already been shown to be the best tool to explore neutrino masses and mixing (in particular, in the four-family mixing scenario in). Without entering in the details, in a
Neutrino Factory muons are accumulated in a storage ring and then decay in a straight section of the experimental apparatus producing two different flavours of neutrinos and antineutrinos, depending on the initial muon polarity. The advantage to follow this procedure is the purity of the neutrino fluxes, not achievable with usual (super)beam. In such an experimental setup, the final muons obtained, for example, via the chain:

$$\mu^+ \rightarrow e^+ \; \bar{\nu}_\mu \; \nu_e$$

$$\downarrow$$

$$\nu_\mu \rightarrow \mu^-$$

are the signal of the oscillation and are called "wrong sign muons" ($\mu^-$ appearance in a $\mu^+$ beam).

1.2 Schemes in four neutrino scenarios

When four neutrinos are considered, two very different classes of mass spectrum are possible: three almost degenerate neutrinos, accounting for the solar and atmospheric oscillations, separated from the fourth one by the large LSND mass difference (3+1 scheme); or two almost degenerate neutrino pairs, accounting respectively for the solar and atmospheric oscillations, separated by the LSND mass gap (2+2 scheme). The recent analysis of the LSND experimental data reconciles the 3+1 scheme with exclusion bounds coming from other reactor and accelerator experiments, so that the 3+1 model is now marginally compatible with the data. Moreover the recent SNO results restricted the allowed parameter region for the 2+2 scheme giving a considerably worse fit to the experimental data with respect to the pre-SNO analysis.

In the following we will focus our attention only on the 3+1 scheme, which has the three neutrino model as a limit in the case of vanishing gap-crossing angles and therefore it can more easily be confused with it.

2 Three or four families?

In a few years from now the LSND results will be confirmed by MiniBooNE or not. In case of a non-conclusive result, the three-family mixing model will be considered the most plausible extension of the Standard Model, so that long baseline experiments will be preferred with respect to the (four-family inspired) short baseline ones. In this case, will a Neutrino Factory and corresponding detectors, designed to explore the three-family mixing model, be able to tell three neutrinos from four neutrinos?

2.1 Experimental set-up and strategy adopted

We consider the following “reference set-up”: neutrino beams resulting from the decay of $2 \times 10^{20} \mu^+$'s and $\mu^-$'s per year in a straight section of an $E_\mu = 50$ GeV muon accumulator. A realistic 40 Kton detector of magnetized iron is used and five years of data taking for each polarity is envisaged. Detailed estimates of the corresponding expected backgrounds ($b$) and efficiencies ($\epsilon$) have been included in the analysis. We follow the analysis in energy bins as made in. The $\nu_e \rightarrow \nu_\mu$ channel, the so-called golden channel, will be the main subject of our investigations.

Let $N_{i\nu}$ be the total number of wrong-sign muons in a four neutrino theory detected when the factory runs in polarity $\mu^+$ or $\mu^-$, grouped in energy bins specified by the index $i$, and three possible distances, corresponding to $L = 732$ Km, $L = 3500$ Km and $L = 7332$ Km. In order to
simulate a typical experimental situation we generate a set of “data” $n^i$ by smearing the number of wrong sign muons:

$$n^i = \frac{\text{Smear}(N^i_{4\nu}, \epsilon^i + b^i)}{\epsilon^i} - b^i.$$  \hfill (1)

Finally, the ”data” are fitted to the theoretical expectation in the three-neutrino model as a function of the neutrino parameters under study, using a $\chi^2$ minimization:

$$\chi^2 = \sum_i \left( \frac{n^i - N^i_{3\nu}}{\delta n^i} \right)^2,$$  \hfill (2)

where $\delta n^i$ is the statistical error for $n^i$ (errors on background and efficiencies are neglected). The output of interest of this procedure are the values of the $\chi^2 \leq \chi^2_{68\%, n\,dof}$, for which the hypothesis of confusion can be considered accepted.

In order to get results that can be easily understood, we need to restrict the 4-$\nu'$s parameter space in our numerical simulations: the $4 \times 4$ unitary mixing matrix has six independent angles and three CP-violating phases! We only considered CP-conserving four neutrino schemes. We fixed $\theta_{12} = \theta_\odot = 22.5^\circ$ and $\theta_{23} = \theta_{\text{atm}} = 45^\circ$. Then we allowed a variation for $\theta_{14} = \theta_{24} = \epsilon = 2^\circ$, $5^\circ$ and $10^\circ$ (taken to be equal for simplicity) and the two free parameters we considered are $\theta_{13}$ in the interval $[1^\circ, 10^\circ]$ and $\theta_{34}$ in $[0^\circ, 50^\circ]$. For the three family case, we still fixed $\theta_{12} = \theta_\odot$ and $\theta_{23} = \theta_{\text{atm}}$ since the differences from the two models depend on the small $\Delta_\odot$ and on the angles $\theta_{14}$, $\theta_{24}$ and $\theta_{13}$ that we took small. In this case the parameter space consists of the angle $\theta_{13}$ (varying in the same interval as in the 3+1 scheme) and the CP-violating phase $\delta$ in the interval $[-180^\circ, 180^\circ]$.

The results of the fits depend heavily on the values of the small gap-crossing angles $\theta_{14}$ and $\theta_{24}$. If their value is small ($2^\circ$), since the 3+1 model has a smooth limit to the three-neutrino theory, the fit is possible for almost every value of the other parameters. This is shown in fig. 2, where in the dark regions of the “dalmatian dog hair” plot the three–neutrino model is able to fit at 68% c.l. the data generated with those parameter values.

![Figure 1: Plots at 68 % in the four–family plane for different baselines for $\theta_{14} = \theta_{24} = 2^\circ$. From left to right: L = 732 Km; L = 3500 Km; L = 7332 Km.](image)

Increasing the value of $\theta_{14}$ and $\theta_{24}$, the extension of blotted regions decreases. This is shown for $\theta_{14} = \theta_{24} = 5^\circ$ in fig. 2.

A further increase of the gap-crossing angles makes the distinction of the two models possible for all values of the other, variable parameters.
Assuming a detector with better resolution, (i.e. increasing the number of energy bins from five to ten) we have not observed a sensible reduction of the blotted regions for $\epsilon = 2^\circ$, due to the extremely poor statistics per energy bin. On the other hand, for $\epsilon = 5^\circ$ we observe a sensitive reduction of the confusion regions.

To explain all the previous results, we can consider the following plots in which we represented the transition probabilities for the two models for the oscillation parameters fixed to some representative value (fig. 3):

For $\epsilon = 2^\circ$ (plot on the left) the four–neutrino probability is very similar to the three–neutrino result and confusion is always possible. At low and intermediate distances ($L \sim 3000$ Km) the oscillation probabilities in the two models are quite similar: at this distance we expect therefore that it will be difficult to tell three from four neutrinos. At larger distances ($L > 5000$ Km) the distinction will in general be possible for $\theta_{34}$ large enough.

For $\epsilon = 5^\circ$ (plot on the right), at the shortest distance, we can observe that the LSND oscillation dominates in the oscillation probability: the 3+1 model gives a probability larger than the three–neutrino theory and confusion is not possible. The oscillation probabilities at the intermediate distance are very similar in the two models, and the confusion is therefore maximal in this case. At larger distances the situation remains essentially unchanged.
3 CP violation vs. more neutrinos

So far, we have not presented in detail the results of a fit of the four–family “data” in the parameter space of the three-family model. If a non-vanishing CP violating phase is found when fitting with the three–family model the 3+1 (CP conserving) “data”, its values are generally not too large. To illustrate this conclusion, we show the situation in the three family parameter space for a typical point in the dark region (fig. 4), where we present the 68%, 90% and 99% confidence level contours for typical three–neutrino fits (one for each distance) to 3+1 neutrino “data” generated with \( \theta_{14} = \theta_{24} = 2^\circ \) and five energy bins.

![Confidence level contours for typical points](image1)

Figure 4: Confidence level contours for typical points where the three–neutrino theory can well reproduce (3+1)–neutrino “data” at the three distances studied: from left to right, \( L = 732, 3500 \) and 7332 Km.

The representativeness of the values chosen for the plots in fig. 4 can be understood noting that, out of about 6000 successful fits, 31% (50%, 37%) for \( L = 732 \) (3500, 7332) Km give for the CP violating phase a value \(-15^\circ < \delta < 15^\circ\), that is the amount of the leptonic CP-violation is not large.

Suppose now to follow an inverse procedure with respect to that described previously, that is to generate ”data” in the usual three-neutrino model; can we miss a large CP-violating phase in the three-family model by fitting in a CP-conserving 3+1 theory? For this purpose we choose to vary \( 1^\circ \leq \theta_{13} \leq 10^\circ \) and \( 60^\circ \leq \delta \leq 120^\circ \). The answer to the question depends on the baseline, as we can see in fig. 5.

![Examples of regions in the plane](image2)

Figure 5: Examples of regions in the \((\theta_{13}, \delta)\) plane where the (3+1)–neutrino theory with vanishing CP violating phases can reproduce at 90% c.l. three–neutrino “data”. From left to right: \( L = 732 \); \( L = 3500 \); \( L=7332 \).

The confusion is possible in many a case for \( L = 732 \) Km, it is very difficult to obtain for the intermediate distance, \( L = 3500 \) Km, and at the largest \( L \) an intermediate situation holds. Fitting simultaneously data at two different distances the possibility of confusion is strongly
reduced and in fact it vanishes if the data at intermediate distance are used in any combination with the others. We also observed that no confusion would be possible assuming the larger values $\epsilon = 5^\circ$ or $10^\circ$.

Our result seems to imply that if the data would point to a maximal CP violating phase in the three–neutrino theory, it would be very difficult to describe them in a theory without CP violation, even if with more neutrinos.

Eventually in the 2+2 scheme the ambiguity with a three neutrino theory is essentially absent due to the impossibility to recover the three family formulae in any limit.

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