Determination of the fraction of electrical energy released in the oncoming supersonic airflow using long spark discharge

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Abstract. In present work an indirect measurement of the fraction of stored electrical energy, that by initiating long spark discharge of submicrosecond duration transforms into air heating, that moves near the surface of an aerodynamic model at supersonic speed with Mach number \( M = 2 \), was carried out. Using the Particle Image Velocimetry method the flow velocity behind the front of an induced quasi-cylindrical shock wave that occurs when heat is released with long spark discharge was measured. This process was modeled in a two-dimensional formulation using a numerical simulation with OpenFOAM package. By comparing the data obtained in the experiment and in the numerical simulation the value of the fraction of stored electrical energy, which goes to the local heating of air, was determined.

1. Introduction

The initiation of an electric discharge near the surface of an aerodynamic body, streamlined by the oncoming airflow, is currently being investigated in many laboratories in the world. The use of energy release in the oncoming airflow is interesting from the point of view of practical application for changing the flow pattern of an aerodynamic body, for example: changing the position of the separation point on the wing model with sub- [1] and supersonic [2, 3] free-stream velocities, increasing lift of the wing at supercritical angles of attack [4], preventing stall flow from the leading edge of the wing [5], controlling the laminar-turbulent transition [6], controlling the ignition of combustion in supersonic flows of combustible mixtures [7], control of the interaction process of a shock wave and a turbulent boundary layer [8]. The main advantages of this method of active action over mechanical or pneumatic are high peak power, low response time, the ability to mount the discharge device flush with a streamlined surface. At the same time, the use of a spark discharge as an instrument for transmitting energy to the incoming airflow has a number of advantages, among which a relatively wide range of energy released in the flow can be noted. First of all, this circumstance is due to the fact that air is predominantly involved in the electrical discharge, and not the surface of the body, which is streamlined by oncoming flow, so that this surface is not subject to the destructive action of high temperature and chemically active components of the discharge plasma. In addition, the
initiation of this type of discharge is possible in conditions of high humidity of the incoming air flow [9].

Typically, in experiments, a spark discharge is initiated in a pulsed or pulse-periodic mode, and often the discharge duration is much shorter than the characteristic gasdynamic time. Since the initiation of an electric discharge occurs with usage of a particular high-voltage electrical circuit, the discharge parameters are determined by the parameters of this circuit. In particular, the duration of a spark discharge $\tau$ is often determined by a value of order $RC$, where $R$ is the effective resistance of the discharge circuit during the existence of an electric spark discharge, and $C$ is the total capacitance of the capacitors in the circuit in which the energy is initially stored. On the other hand, the amount of heat energy $Q$, which is released into the gas in which a spark occurs, is associated with the electrical energy $E$ stored in the capacitor $C$:

$$Q = \eta E = \eta \frac{CU^2}{2},$$

where $U$ is the value of the voltage on the capacitor at the time of electrical breakdown, $\eta$ is the fraction of stored electrical energy that goes into heat during electrical discharge. It should be noted that the above formula does not take into account the residual voltage on the capacitor; for simplicity, this value is taken to be zero. From the above relations it follows that the parameters of energy release into the flow can be set by the parameters of the electrical circuit. For example, the flow velocity determines the discharge duration, thus limiting the permissible capacitance of the circuit. In other words, if the energy input should be quasi instantaneous, then the duration of the electric discharge should be set in accordance with the characteristic gasdynamic time. It is possible to formulate a qualitative rule: if the flow velocity is higher, the smaller capacitance of the discharge circuit should be used. Since the value of energy depends on this capacitance, in order to maintain its value, it is necessary to increase the voltage on the capacitor, which discharges to the discharge gap. The value of the fraction $\eta$ of electric energy $E$, which is converted into heat of the incoming air flow, remains unknown. The rest of the electrical energy can go to electromagnetic radiation, heat losses in the supply wires and other elements of the discharge circuit (for example, on switching devices), on the excitation of other degrees of freedom of the molecules of air. It should also be noted that if one initiate a spark discharge in a pulse-periodic mode, the discharge frequency will be determined by the value of $W/E$, where $W$ is the power of the high-voltage source of the discharge circuit.

With the rapid heat release in a gas using a spark discharge, a region with increased temperature and pressure is formed. In this case decay of discontinuity takes place, which leads to the formation of unsteady flow with the presence of a shock wave. If the discharge is pointlike, i.e. the length of the spark channel is comparable to its diameter, the front of the shock wave has a spherical shape. If the length of the spark channel is many times greater than its diameter, then a quasi-cylindrical shock wave is formed, which front shape is a cylindrical surface with an axis of symmetry corresponding to the location of the long spark discharge. It should be noted that a spark discharge can be represented by a set of pointlike discharges which occur simultaneously and are located along a straight line at a distance between adjacent ones comparable to the size of an individual spark. In this case, each discharge is the source of a separate spherical shock wave, and when they interfere one cylindrical shock wave is formed. With initiation of spark discharge on the surface of an aerodynamic model streamlined by airflow, the formation of a shock wave leads to short-term momentum transfer to the flow and to the emergence of local areas of high pressure, which can significantly change the flow pattern. This process can be modeled both experimentally and with the help of numerical simulation. In the numerical simulation it is necessary to know a number of parameters of the physical process of energy release into the incident flow, which will determine the parameters of unsteady flow that occurs, such as the fraction $\eta$ of electrical energy $E$ converting to gas heat, the duration of pulse heating $\tau$. In the experiment, when using a particular scheme of initiation of the discharge, the discharge duration $\tau$ can be measured, and by photographing the discharge the volume of the energy release region can be determined. However, $\eta$ depends on many process parameters and often cannot be measured directly, but can only be determined indirectly. This paper presents the results of
determining the fraction $\eta$ of stored electric energy, determined indirectly from the results of experiments and numerical simulation of the process described in [3]. In the work, experimentally and with the help of numerical simulation, quasi-instantaneous local heat generation using long spark discharge near the surface of the NACA airfoil streamlined by supersonic airflow with a positive angle of attack and with the presence of a separation region is simulated. The use of long spark discharge allows to assume the two-dimensional nature of the process and to carry out a two-dimensional computer simulation with possibility of fine mesh usage, which allows simulation the viscous flow in the boundary layer and its separation near the trailing edge.

2. Experimental setup
Experiments were carried out on supersonic wind tunnel ST-4 of the atmospheric-vacuum type, and a detailed description of the facility can be found in [2, 3, 10]. The description of the important aspects of the facility and some important parameters are noted below. The description of the aerodynamic model with a discharge device, high-voltage circuit for initiating long spark discharge and systems for diagnosing the flow and measuring the electrical parameters of the process are presented below.

2.1. Supersonic wind tunnel
The facility is intended for the formation of a supersonic jet of air in the working chamber. It consists of the following main parts: a vacuum system with a volume of 240 m$^3$, a working chamber with a characteristic size of 1 m and equipped with a number of transparent windows for diagnosing the process of flow around a model, de Laval nozzle with an output diameter of 110 mm, and a series of locking devices for starting and stopping supersonic flow. The principle of operation is as follows. After pumping out the gasholders using vacuum pumps to a pressure of less than 10 kPa, they are filled with atmospheric air through de Laval nozzle, which is designed for Mach number $M = 2$, and the jet velocity profile along its diameter has a rectangular shape (under fully expanded regime). The air is taken from the atmosphere, it passes the nozzle, the working chamber, then enters the gasholders and fills them during test run. In the working chamber in a submerged supersonic jet, the jet behind the exit section of the nozzle, an aerodynamic model to be tested in a supersonic airflow is placed. The resulting static pressure in the submerged jet has a value close to $p_{st} = 0.15$ bar, flow velocity $V_0 = 500$ m/s, static temperature $T_{st} = 160$ K. While supersonic flow is maintained behind the output section of de Laval nozzle, the static pressure $p_{st}$ in the jet in this section constant. This jet flows in a space where there is almost stationary air with pressure $p_a$. This pressure of the ambient air during the test run varies from within 3 kPa to 20 kPa, and the flow regime changes from under-expansion to over-expansion. At the time moment when $p_a = p_{st}$, so-called ideally expanded regime of flow occurs, which is characterized by a rectangular flow velocity profile and the absence of rarefaction waves or shock waves, that start from the nozzle lip. It is important to note that the experimental results presented in this work were obtained near the regime of ideally expanded jet.

2.2. Aerodynamic model with discharger
An aerodynamic model, which was a wing model (model of the NACA64a212 profile) with a chord length of 98 mm and a span of 150 mm, was placed in the submerged supersonic jet. Due to the presence of tilt mechanism inside the working chamber, the model was placed at a certain angle of attack to the oncoming air flow and motionlessly relative to the working chamber, with the leading edge parallel to the nozzle’s exit cross-section. At the top surface of the model, at a distance of 38 mm from the trailing edge, a dovetail shaped groove (figure 1a) is milled to install conductive segments of the discharger into this groove. Figure 1b shows a photograph (top view) of an aerodynamic model with a discharger on the surface. It is important to note that the material from which the model is made is fused silica, which is resistant to high temperatures and to chemically active components of discharge plasma. The use of such material allows one to be sure that during the experiments the surface of the aerodynamic model is not subject to destruction (geometry changes), which is important when comparing the results with numerical simulation.
An electric spark discharge on the surface of the model was created using segmented discharger. It consists of a set of trapezoidal segments with cross-section corresponding to a slot in the model. Segments have a width of 3.5 mm and are made from a Kovar, which is characterized by a relatively low value of thermal expansion that is close to this value for fused silica. The trapezoid-shaped dielectric layers, made of 0.05 mm thick Teflon film, are placed between the segments. The resistance between the outer segments of the discharger is very large, but when high voltage is applied to them, electrical spark discharge occurs in the air between outer segments. Due to this design of the discharge device, it is possible to significantly reduce the voltage required for gap breakdown. In addition, due to the fact that the discharge occurs in a supersonic airflow, the use of a segmented discharger provides a fixed place for the release of energy, i.e. near the surface of the model above the discharger. The experiments were carried out at angle of attack to the oncoming flow equal to 14 degrees, as in experiment [3].

2.3. Controlled high voltage circuit
To apply high voltage to the discharger gap (i.e. to the outer segments of the discharger), an electrical circuit was used, that is shown on figure 2. The circuit is a Marx generator consisting of two stages of $NC_1$ capacitance, where $N$ is the number of $C_1$ capacitors in each stage. The number of capacitors $N$ and the capacitance of each capacitor $C_1$ can be varied to change the amount of electrical energy $E$ stored in the high-voltage circuit. A controlled switch $D_1$ (a three-electrode air gap) is connected between the stages, which is designed to start a high-voltage generator through a high-voltage pulse applied to the control electrode of the switch $D_1$. This pulse is formed using a high-voltage transformer (ignition coil) with an output voltage of about 20 kV, to the primary coil of which a low-voltage pulse is fed, but with large current amplitude. Additionally, the circuit contains an uncontrolled switch $D_2$ (two-electrode air gap), which is triggered immediately after $D_1$. Switch $D_2$ is necessary to prevent current leakage from the circuit while charging its capacitors.

A distinctive feature of this generator circuit is that none of the two outer segments of the discharger is grounded. If the generator is connected to a high voltage source $U$, when the switches $D_1$
and $D_2$ are triggered, the potential of one of these outer segments becomes close to $-U$ (relative to the ground), and for the other one it becomes $+U$, so the resulting potential difference will be $2U$. In this case, the absolute value of the potential difference between any segments of the discharge device and grounded parts of the working chamber will not exceed the $U$ value. Therefore, the selected high-voltage generator circuit ensures the position of the spark discharge strictly above the discharger surface.

It should be noted that another circuit was used in experiments earlier where the spark discharge occurred spontaneously when the voltage in the outer segments reached the breakdown value [2, 3]. The need to use a controlled high-voltage circuit is due to the use of the Particle Image Velocimetry technique, in which there is a significant value of delay relative to the input trigger that associated with the large value of pumping time of the working body in the laser cavity [11]. In this experiment this delay is greater than characteristic gasdynamic time.

2.4. Diagnostics and equipment

Particle Image Velocimetry (PIV) was used for visualization and to measure velocity of unsteady flow caused by long spark discharge initiation near the trailing edge of the wing model. A more detailed description of the methodology and a description of its application under the experimental conditions are given in [3]. It is important to note that although the discharge must obviously be initiated before the flow pattern was recorder, the PIV shooting was a triggering event for the reason given above. The visible area for measuring the flow velocity was 36x48 mm$^2$. The PIV video camera was positioned in such a way relative to the wing model, that in the field of its visibility there were the place of initiation of the discharge, the trailing edge and the region of airflow above the model surface. In the experiment, an oil aerosol was used for seeding a supersonic flow, obtained using generator based on Laskin nozzle [11]. The average diameter of a droplet in an aerosol does not exceed 1 μm, and the particle relaxation time in the flow $\tau_p$ is near 4 μs [11]. The size of the visible region and the speed of the incoming air flow determined the magnitude of the delay between the laser pulses, which was 0.8 μs. The recording was carried out at times that correspond to the fully expanded regime in the facility $(p_{at} = p_{at})$ with a maximum possible frequency of 14 Hz and the number of frames equal to 139, i.e. about 10 seconds (the maximum possible operational time of the supersonic wind tunnel is 1 minute).

To register the electrical and temporal properties of each discharge pulse, a high-resolution 12-bit LeCroy WaveRunner 66 Zi digital oscilloscope was used, one of the channels of which received a signal from a high-voltage probe (1:1000 probe LeCroy PPE20KV), connected to high voltage generator. The TTL signal from the PIV system, corresponding to the moment when the flow pattern was recorded, was fed to another channel. The oscilloscope recording was also carried out for 10 seconds with a time discretization of 100 ns. Thus, the voltage at the time of the breakdown $U$, the discharge duration $\tau$, the residual voltage on the capacitors of the high-voltage generator $U_{max}$, and the time of recording of each frame of PIV, i.e. the value of the delay $\Delta t$ between the beginning of the discharge and the moment of recording of each frame. Since in each test run of such frames there were 139, it is possible for each test run to build statistics on the distribution of discharge parameters during the launch time, which will reflect the stability of the parameters of the selected discharge initiation scheme. Value of electrical energy, stored in capacitors of high voltage generator at the moment of discharge, can be calculated as:

$$E = NG_1(U^2 - U_{min}^2),$$

and the duration of discharge $\tau$ was measured by oscillograms as time period, during which voltage on capacitors falls from 0.9$U$ to 0.1$U$ $(U_{min} < 0.1U)$. It should be noted that in presented work measured value of $\tau$ was 0.2−0.6 μs, which is much less than characteristic gasdynamic time, with energy $E$ range from 0.55−6.9 J.

To control the flow regime, synchronously with the PIV-recording, the $p_{at}$ and $p_{at}$ values were measured in the working chamber. Therefore, after processing the data, it was possible to assign to each frame of the PIV record the flow regime and also to find the frame number that most closely
matches the fully expended flow regime. These measurements were carried out using Honeywell vacuum pressure gauges and 16-bit National Instruments NI-6363 ADC.

To determine the place of initiation of discharge on the upper surface of the wing model, synchronous video recording was additionally performed using a RedLake Motion Pro X3 color digital video camera. The frames were taken synchronously with the initiation of the discharge, so that when viewing the resulting recording, it was possible to determine the fact of successful initiation of the discharge and confirm the place of initiation near the discharger surface. The obtained images were used to determine the length of the energy release region and its transverse size.

Synchronization of all recording equipment with discharge initiation was performed using Berkeley Nucleonics BNC-575 pulse generator.

3. Numerical simulation

Computer simulation of the process of rapid heat release on the surface of the wing model was carried out in the OpenFOAM v.5 package, using sonicFoam solver [12]. The calculation was performed on a quad-core processor with a peak computing power of about 100 Gflops. The supersonic flow of a viscous heat-conducting gas (air) was simulated without taking into account turbulence. The computational domain had a characteristic size of 1 m; it contained a wing profile model with geometrical dimensions corresponding to the wing model used in the experiments. In the computational domain a grid was constructed, and it consists of hexagonal computational cells. The grid is thickened when approaching the surface of the wing profile, i.e. the height of the computational cell decreases when approaching a streamlined surface. This is done in order to correctly calculate the flow in the boundary layer near the surface of the model and to obtain the separation flow observed in the experiment. Thus, the computational domain is actually divided into two, one of which surrounds the surface of the profile streamlined by supersonic airflow, and corresponds to the flow in the boundary layer, and inviscid flow is simulated in the rest of the computational domain.

The initial conditions in the whole region were set by the quantities corresponding to the experimental conditions (parameters of the incoming airflow): static pressure $p_{st}$=0.15 bar, static temperature $T_{st}$=160 K, speed $V_0$=500 m/s. Air viscosity was calculated using the Sutherland formula. A zero velocity of gas motion was set on the profile surface, and the surface itself was adiabatic.

As a result of the calculation, the position of the separation point $X_{sp}$ relative to the trailing edge of the wing model, the aerodynamic coefficients of the lift $C_l$ and the drag $C_d$ were determined. It is shown that at angle of attack of 0 and 10 degrees, these parameters $X_{sp}$, $C_l$, $C_d$ change by no more than 1% with an increase in the number of computational cells from 230 000 to 300 000. It was considered grids with the number of computational cells from 80 000 to 300 000. Further calculations were carried out with the number of cells 230 000 and with a time step of 10 ns. A comparison was made of the stationary flow patterns obtained in the numerical simulation and measured in the experiment (without initiating heat release) by the position $X_{sp}$ of the separation point and the velocity of the air flow near the heat release place, and this comparing has showed good agreement between experiment and simulation.

Simulation of unsteady process of heat release using long spark discharge was carried out by applying an additional field of increased temperature and pressure to the previously calculated stationary flow pattern. This increase in temperature $\Delta T$ and pressure $\Delta p$ was taken to be equal to [13]:

$$\Delta p = (\gamma - 1) \frac{2E}{\nu} \cdot \Delta T = T_{st} \frac{\Delta p}{p_{st}},$$

(2)

where $\nu$ is the volume in which heat is released by the discharge, $\gamma$ is the heat capacity ratio for air, is assumed to be a constant value of 1.4, $E$ is the electrical energy stored in the capacitor, $\eta$ is the fraction of this energy that goes into heat. The change in density inside the discharge region was neglected, i.e. a quasi-instantaneous isochoric process of air heating was simulated. The volume of the discharge region $\nu$ was calculated as the product of the cross section of the discharge channel $S$ by its length $L$. The length $L$ was specified by the number of discharger segments (in the experiment it was 9 cm), and
was determined from the discharge photos (the resulting width of the discharge channel was equal to 2 mm).

4. Obtained results

Both in the simulation and in the experiment, the obtained unsteady flow patterns with fast heat release show that the after spark discharge in the oncoming supersonic airflow near the surface of the wing model a cylindrical shock wave forms. Its center of symmetry shifts relative to the surface of the wing model downstream due to the presence of an external supersonic flow. The front of a cylindrical shock wave moves relative to the main supersonic flow, and behind it the air gains some additional impulse due to the expansion of the gas region heated by an electric discharge. The gas flow speed $V_2$ relative to the speed of the incident flow depends on the time after the discharge and on its energy. On the figure 3 unsteady flow patterns (contours of $y$-component of velocity vector field), obtained in experiment and in numerical simulation, is presented at site near the trailing edge of wing model. On figure 3a the averaged PIV-data is presented for angle of attack 14 degrees, stored electrical energy $E = 3.0$ J, delay after discharge $\Delta t = 48 \mu s$. On the figure 3b unsteady flow pattern, obtained in simulation, is presented for the same angle of attack, released energy is $0.2$ J, delay after energy release is $\Delta t = 50 \mu s$. On both patterns dashed curve is visible part of shock wave front, red circle is place of energy release, open circle is the place, where $V_2$ is measured. Area near the trailing edge with warm colour is the flow separation region.

![Figure 3](image_url)

**Figure 3.** Unsteady flow patterns obtained in experiment (a) and in numerical simulation (b). External airflow with $M=2$ is from left to right.

The velocity of the front of a cylindrical shock wave decreases with time, and at the same time moment after discharge the magnitude of this velocity and velocity $V_2$ will be greater if the value of the stored electrical energy $E$ is greater. It is important to note that in the experiment the duration of the electric discharge was measured, and in the energy range $E = 0.55$–$6.9$ J the duration $\tau$ was from $0.2 \mu s$ to $0.6 \mu s$, that is much less than characteristic gasdynamic time, and this circumstance makes it possible to consider the process of heat release to be almost instantaneous (as it is in computer simulation). It should be noted that the measured value of $V_2$ did not exceed 300 m/s in the indicated energy range $E = 0.55$–$6.9$ J. Considering the value of the tracer relaxation time $\tau_p = 4 \mu s$ mentioned in subsection 2.4, gives a distance less than 2 mm. When considering the distribution of the velocity $V_2$, measured with PIV, from the distance $r$ from the center of symmetry of a cylindrical shock wave, it is clear that the width of the region where the tracers lag behind the flow does not exceed 2 mm (see figure 3).

The determination of the fraction $\eta$ of the electric energy $E$, which goes to gas heating, was carried out by comparing data from flow patterns obtained in experiment and in computer simulation. The values of the velocity $V_2$ behind the shock wave front (normalized to the magnitude of the velocity of the oncoming flow $V_0$) were compared at different time moments after discharge initiation and at
different values of the energy $E$. Both experiment and simulation show that over time after the initiation of the discharge in the range of stored electrical energy $E = 0.1 - 10\, J$, the dependence $V_2(t)$, divided by the oncoming flow velocity $v_0$, has the form:

$$\frac{V_2(t)}{v_0} = kt^n,$$

(3)

where $k$ increases with increasing of energy $E$, and $n$ in the specified range $E$ remains almost constant and equal to $-1$ in the range of time $t$ from $10\, \mu s$ to $60\, \mu s$. The indicated time range is limited by possibilities of measuring the velocity $V_2$ in the experiment: when $t < 10\, \mu s$ such measurements cannot be carried out due to the small size of the gasdynamic perturbation region, and at $t > 60\, \mu s$, the shock wave front leaves the region of visualization. Experimental data and computer simulation results with different values of $E$ show that the dependence $k(E)$ can be represented as:

$$k(E) = cE^b,$$

(4)

where $c$ and $b$ are constants independent of $E$, they are determined by approximation of the obtained dependences $k(E)$. Approximation of the data obtained in computer simulation gives the value $b = 0.26 \pm 5\%$, which corresponds to the results of the analytical model in the theory of a very intense explosion in the two-dimensional case [14]:

$$V_2(t) = \frac{4}{(d+2)(\gamma+1)} \left( \frac{E_s}{\rho_s} \right)^{\frac{1}{d+\gamma}} t - \frac{d}{\alpha s^2},$$

(5)

where $E_s$ is amount of released energy, $\rho_s$ is density of undisturbed air, $d$ is number of space dimensions (in this case $d = 2$). But this correspondence is only partial, because power $n = -1$, obtained from experimental data and numerical simulation (see equation (3)), does not correspond to theoretical value (see equation (5)). This is due the fact that in this case rapid heat release cannot be described as very intense explosion, and the pressure behind cylindrical shock wave is not much greater then pressure of undisturbed air (Mach number of front of cylindrical shock wave is less than 2). When comparing the dependencies $k(E)$ obtained in the experiment and in the simulation (see figure 4), it turns out that the value of $\eta$ is $0.07 \pm 20\%$, where this error is an approximation error of the experimental data.

**Figure 4.** $k$ dependence on $E$: 1—numerical simulation, 2—approximation by equation (4), 3—experimental data. Plotted vertical error corresponds to error of approximation by equation (3).

5. Conclusions

The experimental study of the process of rapid heat release using long spark discharge on the surface of a wing model streamlined by supersonic airflow with Mach number $M = 2$ and attack angle of 14 degrees and with various discharge energy values was carried out, and a two-dimensional simulation of this process was performed. By comparing the measured and calculated values of the additional velocity that air gains behind the front of a cylindrical shock wave, it was found that the fraction of electrical energy that goes to local gas heating is about 7%. This value turns out to be less
than in other works ([13, 15]), and this circumstance suggests that in computer modeling of such phenomena it is necessary to take into account specific conditions and method of discharge initiation. As shown by numerical simulation, additional velocity gained by air after expansion of a small volume of air participates in electric discharge depends on stored electrical energy according to a power law, and under the conditions of described experiment power of this dependence is 1/4, which corresponds to the result in the theory of a very intensive explosion in the two-dimensional case. Obtained result can be used for computer simulation of unsteady process of rapid heat release in supersonic airflow at experimental conditions, described in this paper and in [2, 3].

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