I. DIFFERENCES WITH THE PUBLISHED CODE SKYRME_RPA

The present code is an extension of the Hartree-Fock plus Random Phase Approximation (HF plus RPA) code that has already been published in Ref. [1]. HF is extended to HF-Bardeen-Cooper-Schrieffer (HF-BCS) and, accordingly, RPA to Quasiparticle RPA (QRPA). The scheme remains self-consistent, once the effective pairing force is introduced.

Along the code we use \( \hbar c = 197.327053 \text{ MeV fm} \) and \( mc^2 = 938.91869 \text{ MeV} \), obtained as \( (m_p + m_n)/2 \), for both neutrons and protons.

A. HF-BCS

HF equations are solved in coordinate space, assuming spherical symmetry, as explained in [1], and employing the standard form of a Skyrme interaction\(^1\). Available Skyrme interactions are: SkM*, SkP, SLy4, SLy5, SkX, KDE0-J33 (KDE33) and SAMi. The HF equations are coupled to the standard BCS equations, that in spherical symmetry, as explained in [1], and employing zero-range, density-dependent pairing force of the type \( \Delta = 1 \text{ MeV} \)

\[ V(\mathbf{r}_1, \mathbf{r}_2) = V_0 \left[ 1 + x \left( \frac{\rho_0(\mathbf{r}_1 + \mathbf{r}_2)}{2 \rho_0} \right) \right] \delta(\mathbf{r}_1 - \mathbf{r}_2) \]

where \( E \), \( u \) and \( v \) are the usual quasi-particle energies and BCS amplitudes as defined in the literature [3], and the tilde denotes the time-reversal state. \( N \) is the particle number. Either proton pairing, or neutron pairing, or both, can be active. No neutron-proton pairing is included and, thus, the particles retain their quantum number \( t_z \).

To keep \( u \) and \( v \) positive, we use the phase convention in which the single-particle (s.p.) wave functions include a factor \( i \) [Eq. (7) of Ref. [1] should be modified accordingly]. The number and gap equations, (1) and (2), are solved in a limited window which is specified by the user if the pairing is run in manual mode. In that case, an upper limit on the corresponding s.p. energy \( \varepsilon_\alpha \) and a fixed number of maximum s.p. levels above the Fermi energy to be considered should be specified. Note that only one of those two pairing cutoffs, namely the lowest, will act as the real one. These two possibilities give more flexibility in the usage of the code. Both types of cutoffs can be different for neutrons and protons. There is no lower limit imposed for this pairing window. For those users not conversant with pairing calculations or not interested in exploring pairing effects, we strongly recommend to use pairing in automatic mode, in which case, a volume pairing force (see below) has been fitted to reproduce the experimental neutron pairing gap in \(^{120}\text{Sn} \) (\( \Delta = 1.245 \text{ MeV} \)) with a neutron pairing window fixed by taking into account only bound states above the neutron Fermi energy and to a maximum of 6 neutron s.p. levels. The default pairing window in automatic mode is set in an analogous way.

The matrix elements \( V_{\alpha\alpha'} \) are calculated using a zero-range, density-dependent pairing force of the type

\[ V(\mathbf{r}_1, \mathbf{r}_2) = V_0 \left[ 1 + x \left( \frac{\rho_0(\mathbf{r}_1 + \mathbf{r}_2)}{2 \rho_0} \right) \right] \delta(\mathbf{r}_1 - \mathbf{r}_2) \]

In the above equation, \( \rho_0 = 0.16 \text{ fm}^{-3} \) while the other two parameters \( x \) and \( V_0 \) have to be adjusted. \( x = 0 \) and 0.5 define the usual volume, surface and mixed pairing. \( V_0 \) is usually adjusted to reproduce some given pairing gap but it has to be noted that the code does not make this adjustment by itself unless automatic mode is chosen where only the use of volume pairing (\( x = 0 \)) is allowed.

The total HF-BCS energy can be calculated, and is calculated by the code, in two different manners. It can be calculated from the force, or energy functional, directly. In this case

\[ E = E_{\text{kin}} + E_{\text{Skyrme}} + E_{\text{Coul}} + E_{\text{pair}}. \]

The first two terms are the kinetic and Skyrme contributions to the energy and are written as the volume integrals of the corresponding energy densities, \( E_{\text{kin}} \) and

\[ E_{\text{Skyrme}} \]

\[ E_{\text{Coul}} \]

\[ E_{\text{pair}} \]
\[ E = \frac{1}{2} \sum \alpha v_\alpha^2 \varepsilon_\alpha + \frac{1}{2} E_{\text{kin}} + E_{\text{rearr}} + E_{\text{pair}}, \]  
(5)

where the first term includes the s.p. energies \( \varepsilon_\alpha \), and \( E_{\text{rearr}} \) is the rearrangement term associated with the density-dependent term in the Skyrme force (as in Eq. (109) of [5]), as well as with the Coulomb exchange energy within the Slater approximation. The equality between the two expressions (4) and (5) for the total energy is, among the rest, a useful test for the convergence of the HF-BCS equations.

Regarding the calculation of charge radii, we take into account the electromagnetic finite size of the proton and the neutron. That is, we assume the proton and neutron electric sizes as \( \langle r_p^2 \rangle = 0.64 \text{ fm}^2 \) and \( \langle r_n^2 \rangle = -0.11 \text{ fm}^2 \), respectively, as well as the anomalous proton magnetic moment \( \kappa_p = \mu_p - 1 \) with \( \mu_p = 2.79 \) and neutron magnetic moment \( \kappa_n = \mu_n = 1.91 \). The expression for the charge radius \( \langle r^2 \rangle_{\text{ch}} \) used is derived in the non-relativistic limit and can be written as [6] (see also [7]),

\[ \langle r^2 \rangle_{\text{ch}} = \langle r_p^2 \rangle + \langle r_n^2 \rangle + \frac{N}{Z} \langle r^2 \rangle_{\text{nuc}} + \frac{1}{Z} \left( \frac{\hbar}{mc} \right)^2 \sum_{\alpha} v_{\alpha \tau}^2 (2j_\alpha + 1) \kappa_\tau \langle \sigma \cdot l \rangle_\alpha, \]  
(6)

where \( \tau = p, n \) indicates if the nucleon is a proton or a neutron, respectively, and \( \langle r_p^2 \rangle \) is the mean square radius of the density distribution of protons,

\[ \langle r^2 \rangle_p = \int dr r^2 \rho_p(r). \]  
(7)

We also evaluate the isospin mixing \( \varepsilon \) in the obtained s.p. wave-functions as discussed in Sec. IIA of Ref. [8]. Specifically,

\[ \varepsilon^2 \equiv \frac{1}{N - Z + 2} \sum_{j_p = j_n} (2j_p + 1)v^2_{\alpha \beta}u^2_{\alpha \beta} \mathcal{O}_{np}^2 \]  
(8)

where we have omitted the \( \alpha \) label (previously defined) for the sake of simplicity and clarity. The overlap factor \( \mathcal{O}_{np} \) between the neutron \( (n) \) and proton \( (p) \) radial part \( R_{n,l}(r) \) of the considered s.p. wave function is defined as

\[ \mathcal{O}_{np} \equiv \int_0^\infty dr r^2 R_{n,l,p}(r)R_{n,l,n}(r). \]  
(9)

### B. QRPA

The QRPA equations are

\[ \begin{pmatrix} A_{\alpha \beta, \gamma \delta} & B_{\alpha \beta, \gamma \delta} \\ -B_{\alpha \beta, \gamma \delta} & A_{\alpha \beta, \gamma \delta} \end{pmatrix} \begin{pmatrix} X^\nu_{\alpha \gamma} \\ Y^\nu_{\alpha \beta} \end{pmatrix} = E^\nu \begin{pmatrix} X^\nu_{\alpha \beta} \\ Y^\nu_{\alpha \beta} \end{pmatrix}, \]  
(10)

The basis on which these equations are written is made up with two quasiparticle states, like \((\alpha \beta)\) and \((\gamma \delta)\). These respect the usual selection rules in that they are coupled to good angular momentum \( J \) and parity \( \pi \). The upper limit on the two quasiparticle energy can be specified (see below). On the other hand, the code automatically discards configurations for which both quasiparticle states are fully occupied or both have an occupation probability lower than \(10^{-6}\).

The matrix elements \( A \) and \( B \), in the angular momentum-coupled representation, written on the HF-BCS basis two-quasiparticle basis, have the form

\[ A_{\alpha \beta, \gamma \delta} = \frac{1}{\sqrt{1 + \delta_{\alpha \beta}} \sqrt{1 + \delta_{\gamma \delta}}} \times \left[ (E_\alpha + E_\beta) \delta_{\alpha \gamma} \delta_{\beta \delta} + G_{\alpha \beta, \gamma \delta}(u_\alpha v_\beta u_\gamma v_\delta + v_\alpha v_\beta v_\gamma v_\delta) + F_{\alpha \beta \gamma \delta}(u_\alpha v_\beta v_\gamma u_\delta + v_\alpha v_\beta u_\gamma v_\delta) - (1)^{j_\alpha + j_\beta - j_\gamma - j_\delta} F_{\alpha \beta \gamma \delta}(u_\alpha v_\beta v_\gamma u_\delta + v_\alpha v_\beta u_\gamma v_\delta) \right], \]  
(11)

\[ B_{\alpha \beta, \gamma \delta} = \frac{1}{\sqrt{1 + \delta_{\alpha \beta}} \sqrt{1 + \delta_{\gamma \delta}}} \times \left[ -G_{\alpha \beta \gamma \delta}(u_\alpha v_\beta v_\gamma v_\delta + v_\alpha v_\beta u_\gamma u_\delta) - (1)^{j_\alpha + j_\beta - j_\gamma - j_\delta} F_{\alpha \beta \gamma \delta}(u_\alpha v_\beta v_\gamma u_\delta + v_\alpha v_\beta u_\gamma v_\delta) \right], \]  
(12)
with

\[ G_{\alpha\beta\gamma\delta} = \sum_{m_\alpha, m_\beta, m_\gamma, m_\delta} \langle j_{\alpha} m_\alpha | j_{\beta} m_\beta | JM \rangle \langle j_{\gamma} m_\gamma j_{\delta} m_\delta | J' M' \rangle V_{\alpha\beta, \gamma\delta}^{pp}, \]

(13)

\[ F_{\alpha\beta\gamma\delta} = \sum_{m_\alpha, m_\beta, m_\gamma, m_\delta} \langle j_{\alpha} m_\alpha | j_{\beta} m_\beta | JM \rangle \langle j_{\gamma} m_\gamma j_{\delta} m_\delta | J' M' \rangle V_{\alpha\beta, \gamma\delta}^{ph}. \]

(14)

\( V_{\alpha\beta, \gamma\delta}^{ph} \) are the (uncoupled) matrix elements of the particle-hole effective interaction, and \( V_{\alpha\beta, \gamma\delta}^{pp} \) represent the (uncoupled) matrix elements of the particle-particle effective interaction. The p-h effective interaction is described in detail in Ref. [1]. In fact, one can immediately spot that the matrix element in Eq. (14) is the same that has been defined in Eq. (16) of Ref. [1]; the different notation here has the purpose of distinguishing it clearly from the matrix element \( G \) that is the one of the pairing force and reads

\[ G_{\alpha\beta\gamma\delta} = R \sum_{\lambda} \left( (-)^{J_{\beta} + J_{\gamma} + J} \left\{ \frac{J_{\alpha}}{J_{\gamma}} \frac{J_{\lambda}}{J_{\delta}} \right\} \langle \alpha || Y_{\lambda} || \beta \rangle \langle Y_{\lambda} || \delta \rangle + (-)^{J_{\beta} + J_{\gamma}} \left\{ \frac{J_{\alpha}}{J_{\gamma}} \frac{J_{\lambda}}{J_{\delta}} \right\} \langle \alpha || Y_{\lambda} || \delta \rangle \langle Y_{\lambda} || \beta \rangle \right). \]

(15)

where \( R \) is the radial integral

\[ R = \int \frac{dr}{r^2} u_\alpha (r) u_\beta (r) u_\gamma (r) u_\delta (r) V_{\text{pair}} (r). \]

(16)

The excitation operators and associated sum rules used in the code are those discussed in Ref. [1]. Regarding spurious states: the HF-BCS is known to break different symmetries present in the original Hamiltonian. In principle QRPA should restore those symmetries exactly. However, numerical implementations are not exact, and spurious states do not appear necessarily at zero energy as expected. As a rule, the spurious state should be the first excited state, close to zero energy, in the calculated spectrum. Specifically, within the present code: i) for \( 0^+ \) excitations, we have not implemented any projection technique to avoid the spurious state originated from the violation of the particle number; and ii) for \( 1^- \) excitations, we use the operators defined in [1] to approximately remove the spurious state that arise from the violation of translational invariance.

II. INPUT OF THE CODE

In this section we briefly describe the input file \texttt{hfbcs-qrpa.in} needed to run the code. The format of the input file should be as follows:

| Line | Input | Explanation |
|------|-------|-------------|
| 1    | FORCE | Name of the Skyrme interaction to be used [options: SkM*, SkP, SLy4, SLy5, SkX, KDE33, SAMi] |
| 2    | MESH STEP | Number of mesh points and step (fm) |
| 3    | A Z | Mass number (A) and proton number (Z) |
| 4    | EPS | Required accuracy for the HFBCS equations. Specifically, it corresponds to the maximum accepted difference between consecutive iterations and it is used for: single-particle energy, gap and chemical potential. |
| 5    | FLAG BCSP BCSN | Automatic (0) or manual(1) BCS calculations. Active (1) non-active (0) for protons. Active (1) non-active (0) for neutrons. Only if manual mode has been chosen next three lines (6-8) should be added: |
| 6    | LEVP LEVN | Number of levels taken above the Fermi level in the BCS calculation for protons and neutrons. |
| 7    | EMAXP EMAXN | Maximum proton and neutron energies for the BCS calculation. |
| 8    | \(-V_0 \) x | Parameters of the pairing interaction to be used defined in the previous section. |
| 9    | CALEX | Type of calculations [options: |
- UNP: stands for unperturbed calculation, residual interaction set to zero
- TDA: Tamm-Dancoff Approximation is used
- RPA: Random Phase Approximation is used.

10 J P
Total angular momentum and pairity (+1 or -1). At the moment only natural parity is considered.

11 ECP ECN
Minimum and maximum energy for the considered two body neutron-neutron and proton-proton basis states.

### III. OUTPUT OF THE CODE

The code produces different output files. Definitions of the different quantities such as operators or transition densities are as in skyrme_rpa [1]. Below a brief description:

| File name                  | Description                                                                 |
|---------------------------|-----------------------------------------------------------------------------|
| hfbcs-qrpa.out            | main output of the code                                                      |
| quasiparticle_states.out  | full list of quasi-particle states used in the calculation.                  |
| basis_states.out          | full list of two-body neutron-neutron and proton-proton basis states used in the calculation. |
| d.out                    | proton and neutron densities                                                 |
| td.out                   | proton and neutron transition densities for all excited states              |
| strength_*.out           | three files contain isoscalar, isovector and electromagnetic strength functions (reduced transition probabilities convoluted by a Lorenzian function with a energy step of 0.1 MeV and width of 1 MeV) |

In the main output file (hfbcs-qrpa.out) details on the parameters of the used functional as well as active terms are printed. One-body center of mass correction is active for all the Skyrme interaction assuming the same prescription as in skyrme_rpa. Convergence information and a list of quasi-particle levels is given. Total binding energy as well as its different contributions are printed in two different forms (see the discussion above). Firstly, the energy is calculated as the integral of the energy density. Then, the energy is calculated via the sum of the eigenvalues. In this latter case the rearrangement energy is needed. The calculation of the binding energy in two different ways serves as a strong test for the convergence the code. Neutron and proton root mean square radii are also printed and the root mean square radius is estimated. Average pairing gaps, and isospin mixing in the ground state wave function are also given in the output. Finally, different sum rules and isoscalar, isovector and electromagnetic reduced transition probabilities are listed.

### IV. EXAMPLE

In this section, we show an input (hfbcs-qrpa.in) example of the code which uses the manual pairing option (FLAG=1):

```
SLy5
200 0.1
120 50
1.d-6
1 0 1
0 6
0. 0.
870.6 1.
RPA
2 +1
0. 100.
```

The main output (hfbcs-qrpa.out) of the code corresponding to this input is given below.
READING SKYRME INTERACTION TO BE USED

NAME: SLy5

SKYRME FORCE PARAMETERS IN STANDARD FORM:

\[
\begin{align*}
t_0 &= -2484.88000 \\
x_0 &= 0.77800 \\
t_1 &= 483.13000 \\
x_1 &= -0.32800 \\
t_2 &= -549.40000 \\
x_2 &= -1.00000 \\
t_3 &= 13763.00000 \\
x_3 &= 1.26700 \\
g &= 0.16667 \\
W_0 &= 126.00000 \\
W_0p &= 126.00000
\end{align*}
\]

PARAMETERS OF THE SKYRME ENERGY DENSITY FUNCTIONAL:

as defined in J. Dobaczewski and J. Dudek, Phys. Rev. C52, 1827 (1995)

\[
\begin{align*}
C_{0}\rho &= -931.83000 + 860.18750 *\rho^{0.16667} \\
C_{1}\rho &= 793.91916 - 1013.30087 *\rho^{0.16667} \\
C_{0}\delta\rho &= -76.52453 \\
C_{1}\delta\rho &= 16.37485 \\
C_{0}\tau &= 56.24937 \\
C_{1}\tau &= 23.95020 \\
C_{0}s &= -172.69916 + 439.84254 *\rho^{0.16667} \\
C_{1}s &= 310.61000 - 286.72917 *\rho^{0.16667} \\
C_{0}\nabla J &= -94.50000 \\
C_{1}\nabla J &= -31.50000 \\
C_{0}\Delta s &= 46.08734 \\
C_{1}\Delta s &= 14.06234 \\
C_{0}T &= -15.66645 \\
C_{1}T &= -64.53312
\end{align*}
\]

HF+BCS CALCULATION IN A BOX

NUCLEUS:
- Mass number 120.
- Proton number 50.

BOX PARAMETERS:
- N. of points 200
- Step size 0.10E+00

CONVERGENCE PARAMETERS:
- Error spe 0.10E-05
- Error gap eq. 0.10E-05
- Error num. eq. 0.10E-05
POTENTIAL INCLUDED IN THE CALCULATIONS:
- SKYRME STANDARD WITH
  | ___ J2 TERMS... YES
  | ___ TENSOR..... NO
- COULOMB DIRECT..... YES
- COULOMB EXCHANGE..... YES
- SPIN-ORBIT........... YES
- DEFAULT PAIRING...... NO
- PROT. PAIRING (BCS) . NO
- NEUT. PAIRING (BCS) . YES

ENERGY CUT..... 0.000000000000000E+00
LEV. ABOVE F... 6

Type: delta force
Vp= V0(1-x(rho/rho0))^g
V0 = 0.87060E+03
x = 0.10000E+01
rho0= 0.16000E+00
G = 0.10000E+01

-CENTER OF MASS CORR.. YES

CONVERGENCE:
- At iteration number: 77
- Maximum error in spe: 0.99830E-05

SINGLE PARTICLE ENERGIES, OCCUP., V^2 AND GAPS

PROTON STATES

1S1/2  E=-0.4664E+02  DEG= 0.2000E+01  V^2= 0.1000E+01  D= 0.0000
1P3/2  E=-0.3873E+02  DEG= 0.4000E+01  V^2= 0.1000E+01  D= 0.0000
1P1/2  E=-0.3778E+02  DEG= 0.2000E+01  V^2= 0.1000E+01  D= 0.0000
1D5/2  E=-0.2980E+02  DEG= 0.6000E+01  V^2= 0.1000E+01  D= 0.0000
1D3/2  E=-0.2756E+02  DEG= 0.4000E+01  V^2= 0.1000E+01  D= 0.0000
2S1/2  E=-0.2559E+02  DEG= 0.2000E+01  V^2= 0.1000E+01  D= 0.0000
1F7/2  E=-0.2040E+02  DEG= 0.8000E+01  V^2= 0.1000E+01  D= 0.0000
1F5/2  E=-0.1634E+02  DEG= 0.6000E+01  V^2= 0.1000E+01  D= 0.0000
2P3/2  E=-0.1522E+02  DEG= 0.4000E+01  V^2= 0.1000E+01  D= 0.0000
2P1/2  E=-0.1361E+02  DEG= 0.2000E+01  V^2= 0.1000E+01  D= 0.0000
1G9/2  E=-0.1083E+02  DEG= 0.2000E+01  V^2= 0.1000E+01  D= 0.0000

NEUTRON STATES

1S1/2  E=-0.5616E+02  DEG= 0.2000E+01  V^2= 0.1000E+01  D= 0.2017
1P3/2  E=-0.4748E+02  DEG= 0.4000E+01  V^2= 0.1000E+01  D= 0.4678
1P1/2  E=-0.4615E+02  DEG= 0.2000E+01  V^2= 0.1000E+01  D= 0.4191
1D5/2  E=-0.3787E+02  DEG= 0.5999E+01  V^2= 0.9998E+00  D= 0.7963
1D3/2  E=-0.3481E+02  DEG= 0.3999E+01  V^2= 0.9998E+00  D= 0.7045
2S1/2  E=-0.3334E+02  DEG= 0.1999E+01  V^2= 0.9997E+00  D= 0.8519
1F7/2  E=-0.2775E+02  DEG= 0.7993E+01  V^2= 0.9991E+00  D= 1.1555
1F5/2  E=-0.2250E+02  DEG= 0.5992E+01  V^2= 0.9987E+00  D= 1.0469
2P3/2  E=-0.2194E+02  DEG= 0.3993E+01  V^2= 0.9982E+00  D= 1.1819
2P1/2  E=-0.2013E+02  DEG= 0.1995E+01  V^2= 0.9975E+00  D= 1.1866
1G9/2  E=-0.1747E+02  DEG= 0.9937E+01  V^2= 0.9937E+00  D= 1.4813
2D5/2  E=-0.1137E+02  DEG= 0.5787E+01  V^2= 0.9644E+00  D= 1.2543
1G7/2  E=-0.9976E+01  DEG= 0.7133E+01  V^2= 0.8916E+00  D= 1.3924
3S1/2  E=-0.9077E+01  DEG= 0.1618E+01  V^2= 0.8089E+00  D= 1.0877
\[ \text{2D3/2} \quad E=-0.8573E+01 \quad \text{DEG}= 0.2530E+01 \quad V^2= 0.6325E+00 \quad D= 1.2771 \]
\[ \text{1H11/2} \quad E=-0.7227E+01 \quad \text{DEG}= 0.2978E+01 \quad V^2= 0.2481E+00 \quad D= 1.7057 \]
\[ \text{2F7/2} \quad E=-0.1526E+01 \quad \text{DEG}= 0.5666E-02 \quad V^2= 0.6894E-03 \quad D= 0.4313 \]
\[ \text{2F5/2} \quad E= 0.1169E+01 \quad \text{DEG}= 0.0000E+00 \quad V^2= 0.0000E+00 \quad D= 0.0000 \]
\[ \text{1H9/2} \quad E= 0.1856E+01 \quad \text{DEG}= 0.0000E+00 \quad V^2= 0.0000E+00 \quad D= 0.0000 \]
\[ \text{1I13/2} \quad E= 0.2809E+01 \quad \text{DEG}= 0.0000E+00 \quad V^2= 0.0000E+00 \quad D= 0.0000 \]

\section*{Contributions to the Total Energy (MeV):}

\section*{Integral of the Energy Density:}
\[ E(\text{KIN}) = 0.21769E+04 \]
\[ E(\text{SKYRME}) = -0.34793E+04 \]
\[ E(\text{t0}) = -0.12698E+05 \]
\[ E(\text{t3}) = 0.82849E+04 \]
\[ E(\text{t1,t2}) = 0.93408E+03 \]
\[ E(\text{SO}) = -0.54885E+02 \]
\[ E(\text{CD}) = 0.36706E+03 \]
\[ E(\text{CE}) = -0.19131E+02 \]
\[ E(\text{pair}) = -0.99067E+01 \]
\[ E(\text{TOT}) = -0.10193E+04 \]

\section*{HF Energy + \text{E}_\text{REA}:}
\[ E(\text{KIN}) = 0.10885E+04 \]
\[ E(\text{SPE}) = -0.14010E+04 \]
\[ E(\text{REA}) = -0.69678E+03 \]
\[ E(\text{pair}) = -0.99067E+01 \]
\[ E(\text{TOT}) = -0.10193E+04 \]

\section*{Numerical Checks:}
\[ \frac{E_{\text{INT}}}{E_{\text{HF}}}-1 \]
\[ \text{REL. ERR. (\%)} = 0.36779E-03 \]

\section*{Root Mean Square Radii}
\[ R(\text{N}) = 0.47278E+01 \]
\[ R(\text{P}) = 0.45865E+01 \]
\[ R(\text{CH}) = 0.46418E+01 \]
\[ R^2(\text{CH}) = 0.21546E+02 \]
\[ R^2(\text{p}) = 0.64000E+00 \]
\[ R^2(\text{n})/Z = -0.15400E+00 \]
\[ R^2(\text{SO-n}) = -0.38991E-01 \]
\[ R^2(\text{SO-p}) = 0.63250E-01 \]

\section*{Average Pairing Gaps (MeV):}
\[ \text{GAP(P)} = 0.00000E+00 \]
\[ \text{GAP(n)} = 0.14505E+01 \]

\section*{Isospin Mixing in the HF G.S.
\[ |HF> = \sqrt{1-e^{-2}}|T_0, T_0> + e|T_0+1, T_0> \]

\[ e^{-2} (%) = 0.31506E+00 \]

CONSIDERED TRANSITIONS WITH:
\[ J = 2 \]
\[ \text{PARITY} = 1 \]

MOMENTS OF THE STRENGTH FUNCTION

- **NON-ENERGY WEIGHTED SUM RULE:**
  \[ m(0)_{IS} = 0.26417E+05 \]
  \[ m(0)_{IV} = 0.11253E+05 \]

- **INVERSE ENERGY WEIGHTED SUM RULE:**
  \[ m(-1)_{IS} = 0.10068E+05 \]
  \[ m(-1)_{IV} = 0.14632E+04 \]

- **ENERGY WEIGHTED SUM RULE:**
  \[ m(1)_{IS} = 0.20875E+06 \]
  \[ m(1)_{IV} = 0.23841E+06 \]
  \[ m(1)_{[D.C.]IS} = 0.21407E+06 \]
  \[ \text{Exhaus. IS(%)} = 97.51375 \]
  IV DOUBLE COMMUTATOR EWSR NOT AVAILABLE

- **ENERGY CUBE WEIGHTED SUM RULE:**
  \[ m(3)_{IS} = 0.42103E+08 \]
  \[ m(3)_{IV} = 0.16041E+09 \]

AVERAGE EXCITATION ENERGIES

\[ \text{E}_{\text{CONSTR.}} (IS) = 0.45533E+01 \]
\[ \text{E}_{\text{CENTR.}} (IS) = 0.79021E+01 \]
\[ \text{E}_{\text{SCAL.}} (IS) = 0.14202E+02 \]
\[ \text{E}_{\text{CONSTR.}} (IV) = 0.12765E+02 \]
\[ \text{E}_{\text{CENTR.}} (IV) = 0.21187E+02 \]
\[ \text{E}_{\text{SCAL.}} (IV) = 0.25939E+02 \]

REDUCED TRANSITION PROBABILITIES

| E   | BEL_is | FRAC_NEWISR | BEL_em | BEL_iv | FRAC_NEWISR |
|-----|--------|-------------|--------|--------|-------------|
| 1   | 0.13051E+01 | 0.11155E+05 | 0.42229E+02 | 0.12264E+04 | 0.11249E+02 |
| 2   | 0.27263E+01 | 0.29313E+02 | 0.11096E+00 | 0.41912E+00 | 0.15080E+00 |
| 3   | 0.30799E+01 | 0.57219E+03 | 0.21660E+01 | 0.87101E+02 | 0.24539E+00 |
Finally, for this example, we show in Fig. 1 the reduced transition probabilities contained in hfbcs-qrpa.out and their convolution with a Lorenzian function with a width of 1 MeV. The latter information is also given by the code and can be found in the file strength_is.out for the isoscalar response and strength_iv.out for the isovector response.

![Reduced transition probabilities](image)

**FIG. 1.** Reduced transition probabilities (bars) for the text example contained in hfbcs-qrpa.out and their convolution with a Lorenzian function (solid lines) with a width of 1 MeV. The latter information is also given by the code and can be found in the file strength_is.out for the isoscalar response (upper panel) and strength_iv.out for the isovector response (lower panel).
[1] G. Colò, L. Cao, N. Van Giai, and L. Capelli, Computer Physics Communications 184, 142 (2013).
[2] F. Stancu, D. Brink, and H. Flocard, Physics Letters B 68, 108 (1977).
[3] P. Ring and P. Schuck, “The nuclear many-body problem,” (Springer, 1980).
[4] E. Chabanat, P. Bonche, P. Haensel, J. Meyer, and R. Schaeffer, Nuclear Physics A 635, 231 (1998).
[5] W. Ryssens, V. Hellemans, M. Bender, and P.-H. Heenen, Computer Physics Communications 187, 175 (2015).
[6] C. J. Horowitz and J. Piekarewicz, Phys. Rev. C 86, 045503 (2012).
[7] E. Chabanat, P. Bonche, P. Haensel, J. Meyer, and R. Schaeffer, Nuclear Physics A 627, 710 (1997).
[8] X. Roca-Maza, H. Sagawa, and G. Colò, Phys. Rev. C 102, 064303 (2020).