3D numerical simulation of gaseous flows structure in semidetached binaries

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ABSTRACT
The results of numerical simulation of mass transfer in semidetached non-magnetic binaries are presented. We investigate the morphology of gaseous flows on the base of three-dimensional hydrodynamic calculations in interacting binaries of different types (cataclysmic variables and low-mass X-ray binaries).

We find that taking into account of a circumbinary envelope leads to significant changes in the stream-disc morphology. In particular, the obtained steady-state self-consistent solutions show an absence of impact between gas stream from the inner Lagrangian point $L_1$ and forming accretion disc. The stream deviates under the action of gas of circumbinary envelope, and does not cause the shock perturbation of the disc boundary (traditional ‘hotspot’). At the same time, the gas of circumbinary envelope interacts with the stream and causes the formation of an extended shock wave, located on the stream edge. We discuss the implication of this model without ‘hotspot’ (but with a shock wave located outside the disc) for interpretation of observations. The comparison of synthetic light curves with observations proves the validity of the discussed hydrodynamic model without ‘hotspot’.

We also consider the influence of a circumbinary envelope on the mass transfer rate in semidetached binaries. The obtained features of flow structure in the vicinity of $L_1$ show that the gas of circumbinary envelope plays an important role in the flow dynamics, and that it leads to significant (in order of magnitude) increasing of the mass transfer rate. The most important contribution to this increase is due to stripping of mass-losing star atmosphere by interstellar gas flows.

The parameters of the formed accretion disc are also given in the paper.

We discuss the details of the obtained gaseous flows structure for different boundary conditions on the surface of mass-losing star, and show that the main features of this structure in semidetached binaries are the same for different cases.

The comparison of gaseous flows structure obtained in 2D and 3D approaches is presented. We discuss the common features of the flow structures and the possible reasons of revealed differences.

Key words: accretion, accretion discs – binaries: close – hydrodynamics – methods: numerical – shock waves

1 INTRODUCTION
The semidetached binaries belong to the class of interacting stars, where one component fills its critical surface that causes the mass transfer between components of the system. In general, the form of critical surface may be complex (Kruszewski 1963) and special mathematical models are required to describe the process of mass transfer in such a system (see review of these models in Lubow 1993). However, in the standard treatment, semidetached binaries are considered under the assumption that orbits of components are circular and their rotation is synchronous with the orbital movement. In this case the critical surface can be identified with internal surface (Roche surface) in the restricted
three-body problem and it is assumed, that the mass trans-
fer between components of the system occurs through the
vicinity of inner Lagrangian point $L_1$, where pressure gradi-
ent is not balanced by gravitational force.

The hydrodynamics of mass transfer through the in-
ner Lagrangian point $L_1$ has been investigated by many
authors. The detailed analysis of matter flow in the vicin-
ity of $L_1$ was carried out by Lubow & Shu (1975). Using
a perturbation method they evaluated main characteristics
of the flow. In another approach, based on the analysis of
Bernoulli integral, the stream parameters were specified as
well and the dependence of the mass transfer rate upon the
degree of Roche lobe overfilling was obtained (Paczynski
& Sienkiewicz 1972; Savonije 1978).

For adequate description of the mass transfer process in
the binary system besides determination of stream param-
eters it is also necessary to consider the further behavior of
flowlines during movement of matter from $L_1$. It is the
process of mass transfer produces the general flow structure and,
accordingly, determines basic observation evidences, there-
fore the main attention was paid to study of this question.
For the first time movement of particles leaving $L_1$ and mov-
ing in the gravitational field of binary system was consid-
ered by Warner & Peters (1972), Lubow & Shu (1975) and Flan-
nery (1975). These results were obtained using a simplified
ballistic approach for analysis of the gas movement without
taking into account of hydrodynamic effects. To study the in-
fluence of the circumbinary envelope on gas movement and,
accordingly, for a correct description of the flow, the solving
of full system of hydrodynamic equations is required. This
is possible only in the framework of rather complex mathe-
matical models.

The use of numerical methods for investigation of hy-
drodynamics of mass transfer in semidetached binaries was
limited by the computer power for a long time so 2D mod-
els were used for the analysis of the flow structure. De-
spite the restrictions of 2D approach, it allowed to consider
some details of the flow structure correctly and to obtain a
set of interesting results (see, e.g., Sawada, Matsuda & Hachisu
1986; Sawada et al. 1987; Taam, Fu & Fryxell 1991; Blondin, Richards & Malinowski 1995; Murray 1996).

Last years the possibility of numerical hydrodynamic simulation of
mass transfer in the framework of more realistic 3D mod-
els (Nagasawa, Matsuda & Kuwahara 1991; Hirose, Osaki
& Minishige 1991; Molteni, Belvedere & Lanzafame 1991;
Sawada & Matsuda 1992; Lanzafame, Belvedere & Molteni
1992, 1994; Belvedere, Lanzafame & Molteni 1993; Meglicki,
Wickramasinghe & Bicknell 1993; Armitage & Livio 1996)
appeared. In particular, these authors considered the forma-
tion of accreting disc in semidetached binaries (Nagasawa
et al. 1991; Sawada & Matsuda 1992) and investigated the
interaction of stream of matter leaving $L_1$ with the disc (Hi-
rose et al. 1991; Armitage & Livio 1996).

Unfortunately, many 3D investigations were carried out
during a rather small time-scales and this fact did not allow
to consider the real flow morphology, accordingly, to evalu-
ate the influence of forming circumbinary envelope on the
flow structure. Some progress in the investigation of gen-
eral flow structure in semidetached binaries was achieved in
works Molteni et al. (1991), Lanzafame et al. (1992, 1994),
and Belvedere et al. (1993), where 3D numerical simulations
were carried out on the sufficiently large time intervals. A set

of interesting results was obtained in these works, however
using of method SPH (Smoothed Particle Hydrodynamics)
did not allow to consider the influence of the circumbinary
envelope on the flow structure as far as the computational re-
strictions of SPH method did not permit to investigate flows
with large density gradients, and, accordingly, the account
of the influence of circumbinary envelope on the mass trans-
fer was not quite correct. For the first time the morphology
of gaseous flows in binaries was accurately considered by
authors in Bisikalo et al. (1997a,b).

In this work we present the results of 3D numerical
study of the flow structure in semidetached non-magnetic bi-
naries. TVD (Total Variation Diminishing) method of solv-
ing of hydrodynamic equations used in this paper has al-
lowed to investigate the morphology of gaseous flows in the
system and to consider the influence of forming circumbinary
envelope, despite the presence of significant density gradi-
ents. The numerical simulations of mass transfer in semide-
tached binaries have been conducted on large time intervals
that allowed to consider the main features of flow structure
in steady-state regime. The earlier conclusions on the flow
structure for low-mass X-ray binary (Bisikalo et al. 1997a,b)
are generalized in present work for the wider class of objects.

The paper is structured as follows. In Section 2, the
properties of used physical, mathematical and numerical
models are described. Section 3 contains the results of nu-
merical simulations. In this Section the stream-disc interac-
tion, comparison of synthetic light curves with observations,
flow structure in the vicinity of $L_1$, influence of accepted
boundary conditions, and comparison of the results obtained
in 2D and 3D models are discussed. Our conclusions follow
in Section 4.

2 THE MODEL

2.1 Physical model

The semidetached binaries such as cataclysmic variables
(CVs), low-mass X-ray binaries (LMXBs) and supersoft X-
ray sources (SSS) show a lot of interesting observation evi-
dences. The observations of CVs light curves and X-ray light
curves of LMXBs offer bright evidences of a complex flow
structure in these systems and allow to make assumptions
on the structure of gaseous flows. In particular, in some cata-
clysmic binaries, the most well studied of which is Z Cha,
the complex picture of eclipse (‘double eclipse’) is observed
(see, e.g., Hack & La Dous 1993; Cherepashchuk et al. 1996).
For its explanation the hypothesis of an ‘hotspot’ in interac-
tion zone between the stream and the disc outer edge was
suggested (Smak 1970). For a number of X-ray sources such
as X1822-371 (‘dipping’ sources), a significant decrease of
the radiation flux at orbital phase 0.8 is observed. It is ex-
plained by a bulge in the accretion disc created by impact of
the stream with the disc (White & Holt 1982; Mason 1989;
Armitage & Livio 1996).

 Doubtless, the presence of such appreciable observa-
tional evidences of the complex flow structure in semide-
tached binaries requires a detailed study of the gas flow. In
this work the investigations of the flow structure are car-
died out for two types of semidetached binaries: 1) with pa-
rameters typical for low-mass X-ray binaries, where main

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sequence dwarf fills its Roche lobe and transfers the matter to neutron star, and 2) with parameters typical for CVs, where the accreting star is a white dwarf. It is assumed in our model, that magnetic field is negligible and does not influence the gas flow in considered systems.

For the numerical simulations we use systems with parameters similar to those for X1822-371 (Armitage & Livio 1996), and for Z Cha (Goncharskij, Cherepashchuk & Yagola 1985). For the first system it is assumed, that the mass-losing component has the mass \( M_1 \) equal to 0.28\( M_\odot \); the gas temperature on the surface is \( T = 10^4 \ K \); the mass of compact star is \( M_2 = 1.4M_\odot \); the orbital period of the system is \( P_{\text{orb}} = 1^d.78 \); and the distance between centres of components is \( A = 7.35R_\odot \). For Z Cha the following parameters are adopted: the mass of mass-losing red dwarf is \( M_1 = 0.19M_\odot \); the temperature of gas on the surface is \( T = 5 \times 10^3 \ K \); the mass of the compact star – white dwarf is \( M_2 = 0.94M_\odot \); the orbital period of the system is \( P_{\text{orb}} = 0^d.674 \); and \( A = 0.78R_\odot \). We suppose that mass-losing stars in both models fill their Roche lobes. The accretor radius for Z Cha is adopted as \( R_2 = 0.009R_\odot \), that is equal to the radius of a typical white dwarf. For X1822-371 where the accretor is a neutron star, the computer facilities do not permit us to resolve the real star size, therefore we adopt the accretor radius equal to \( R_2 = 0.05R_\odot \). It should be noted that the larger accretor radius adopted for the LMXB case does not influence the flow structure at distances larger than \( R_2 \), therefore the obtained results are correct for all calculated region except the small zone around neutron star \((r \leq R_2)\).

For the adequate description of the flow structure it is necessary to take into account the influence of radiative processes on hydrodynamics. The accurate consideration of non-adiabatic processes in the numerical model increases the time of calculations significantly. Therefore, taking into account our computing environment and the necessity of carrying out calculations at large time-scales for getting of the steady-state solution, we simplify the model by using the adiabatic hydrodynamics and an ideal gas equation of state

\[
P = (\gamma - 1)\rho E \quad (P - \text{pressure}, \rho - \text{density}, \varepsilon - \text{specific internal energy}).
\]

To consider the energy losses the ratio of specific heats is assumed to be close to unity: \( \gamma = 1.01 \), that corresponds to the near-isothermal case (Landau & Lifshiz 1959). Such a technique for taking into account the radiative losses is well known and is used in practice repeatedly (see, e.g., Sawada et al. 1986, 1987; Spruit et al. 1987; Molteni et al. 1991; Matsuda et al. 1992; Bisikalo et al. 1995).

2.2 Mathematical model

To describe the gas flow, the system of 3D hydrodynamic equations in integrated form and an ideal gas equation of state with the ratio of specific heats \( \gamma = 1.01 \) is used. The system of equations for the fluid element with volume \( V \) and surface \( \Sigma \) has the following form:

\[
\frac{\partial}{\partial t} \int_V \rho \, dV + \int_V \rho (v \cdot n) \, d\Sigma = 0
\]

\[
\frac{\partial}{\partial t} \int_V \rho v \, dV + \int_V \rho (v \cdot n) \, d\Sigma + \int_{\Sigma} P \, n \, d\Sigma = \int_V \rho F \, dV
\]

The specific external force \( F \) includes the Coriolis force, gravitational forces of two point masses, and centrifugal force, and has the form:

\[
F = - \nabla \Phi + 2 \cdot (v \times \Omega)
\]

where the Roche potential \( \Phi \) can be written in the form:

\[
\Phi = \frac{GM_1}{r - r_1} - \frac{GM_2}{r - r_2} - \frac{1}{2} \Omega^2 (r - r_c)^2.
\]

Here \( v \) – velocity; \( h \) – specific full enthalpy: \( h = \varepsilon + |v|^2/2 + P/\rho \); \( E \) – specific full energy: \( E = \varepsilon + |v|^2/2 \); \( \Omega = (0, 0, \Omega) \); \( \Omega = 2\pi/P_{\text{orb}} \); \( r_1, r_2 \) – the centres of components of the system; and \( r_c \) – the centre of mass of the system.

The boundary conditions on the surface of mass-losing star are determined under the assumption that this star fills its Roche lobe. To build the boundary conditions on the Roche lobe we use a standard procedure of solving of Riemann problem between two regions – one on the surface of the star and other corresponding to the nearest computational cell (see, e.g., Sawada et al. 1986; Sawada & Matsuda, 1992). On the surface of mass-losing star we choose the value of density \( \rho = \rho_0 \). We also guess that everywhere on the surface of this component the gas velocity is directed along the surface normal, and its value is fixed to be equal to the local sonic velocity \( v = c_s \). For Z Cha we also consider the case, when the gas velocity is fixed to be zero for all the surface of mass-losing star.

It should be noted that the boundary value of density on the surface of mass-losing star has no influence on the solution, due to scaling of the system of equations with respect to \( \rho \) (with simultaneous scaling of \( P \)). So an arbitrary value of \( \rho_0 \) can be accepted in calculations, however, when considering the particular system with known mass-loss rate, to determine the real values it is necessary to increase the calculated values of density in accordance with the scale, defined by ratio of real value of density on the surface of mass-losing star to the model one.

On the surface of the compact star and the outer numerical boundary the free outflow conditions are used.

As initial conditions the low-density ambient matter \( \rho \sim 10^{-6} \rho_0 \) in rest in the rotational frame is accepted. Subsequently, this matter is forced out from the system by the gas injecting from mass-losing star.

2.3 Numerical model

The main question for numerical simulation of the hydrodynamic models is the choosing of solving method and appropriate scheme for the system of equations. Among the large variety of finite-difference schemes the so-called Godunov-type schemes (Godunov 1959) are considered to be the most exact ones.

In the present work, we use the modification of explicit TVD Roe scheme (Roe 1986) for numerical solving of the system of hydrodynamic equations. The original scheme (first order of spatial approximation) is modified by monotonic flux limiters in the Osher’s form (Chakravarthy & Osher 1985) that makes the scheme of third order of approximation. The special model simulations show, that the given
scheme permits to describe adequately the flow structure including shock waves and tangential discontinuities and does not result in artificial fluctuations and smearing of features of flow. Moreover the used scheme permits to consider the flows with large density gradients, that of special importance for consideration of the influence of circumbinary envelope on the flow structure.

The system of hydrodynamic equations is solved in Cartesian coordinate system, which is predetermined as follows:

- the zero of coordinate system is located in the centre of mass-losing star;
- $X$ axis is directed from the centre of mass-losing star to the accretor;
- $Z$ axis is directed along the axis of orbital rotation;
- $Y$ is determined to get right-handed coordinate system.

The computation region is a parallelepiped $[-A..2A]\times[-A..A]\times[0..A]$ (due to symmetry about the equatorial plane calculations were conducted only in the top half-space). Non-uniform difference grids (more fine near the accretor) containing $78\times60\times35$ gridpoints for the system X1822-371 and $84\times65\times33$ gridpoints for the system Z Cha is used.

Solving of the system of equations has been carried out from initial conditions up to the steady-state regime. To check the establishment of the steady-state regime we have numerically monitored flow parameters (density and pressure) as a function of time. We have checked these parameters inside the spheres around accretor (with different radii) and inside the sphere close to the outer boundary. When the flow patterns do not depend on the time we suppose that steady-state is reached. To assure that the obtained solution is steady we have continued the calculations additionally during 3–5 orbital periods. The runs have been stopped at 12 orbital period for X1822-371, and at 20 orbital periods for Z Cha. Characteristic time step in both runs is approximately $10^{-4}$ orbital period, so the total number of steps

Figure 1. 3D view of density isosurface at the level \( \rho = 0.005\rho_0 \). The values of coordinates $X$, $Y$ and $Z$ are in units $R_\odot$. Accretor is marked by the filled circle. Cross-sections of density isosurface by planes $XZ$ and $YZ$ passing through the accretor are also shown.
3 RESULTS AND DISCUSSION

Let us consider the characteristic features of the flow structure in semidetached binaries, obtained in the framework of 3D hydrodynamic model described in Section 2. As it was mentioned above, the calculations have been carried out for typical representatives LMXBs and CVs. The obtained results testify qualitatively similar nature of the flow in considered systems, that, in turn, permits to establish the general character of the steady flow structures for semidetached non-magnetic binaries.

Taking into account the qualitative similarity of results, the general properties of flow structure will be described below for the X1822-371 system. The results obtained for the system Z Cha will be used to discuss the quantitative characteristics.

3.1 Stream-disc interactions

The general structure of the gaseous flows, illustrating the morphology of mass transfer in the system X1822-371, is presented in Fig. 1, where 3D view of density isosurface at the level $0.005\rho_0$ is shown. The cross-sections of density isosurface by planes $XZ$ and $YZ$ passing through the accretor are also shown. The flow structure presented in Fig. 1 is steady-state and corresponds to a time exceeding 10 orbital periods. The analysis of presented results allows to reveal the following features of the flow structure:

i) the matter of the stream is redistributed into three parts: the first part forms a quasi-elliptic accretion disc; the second part moves around the accretor beyond the disc; the third part of the stream moves towards the external Lagrangian point $L_2$, then a fraction of this matter leaves the system, while a considerable amount of the gas changes the direction of motion due to Coriolis force and comes back to the system;

ii) the interaction between the stream and the disc is shock-free;

iii) the stream of matter moving from the vicinity of $L_1$ changes the sizes as it spreads towards the accretor; the thickness of the stream decreases, and its width in the orbital plane increases;

iv) the thickness of the accretion disc is smaller then the stream thickness.

A more detailed analysis of the structure of gaseous flows in the system and evaluation of linear sizes of the disc can be carried out for the flow structure in the equatorial plane. In Fig. 2 density isolines and velocity vectors in this plane for the region with dimensions from 2 to $10R_\odot$ on axis $X$ and from $-3$ to $3R_\odot$ on axis $Y$ are presented. In Fig. 2 four flowlines are shown as well, labelled by markers ‘a’, ‘b’, ‘c’ and ‘d’. The accretor is marked by the filled circle. Vector in the upper right corner corresponds to the value of velocity of 800 km s$^{-1}$. 

![Figure 2. Density isolines and velocity vectors in the equatorial plane of the system. Roche equipotentials are shown by dashed lines. Four flowlines, labelled by markers ‘a’, ‘b’, ‘c’ and ‘d’ are also presented. The accretor is marked by the filled circle. Vector in the upper right corner corresponds to the value of velocity of 800 km s$^{-1}$.](image-url)
`c` and `d`. These flowlines illustrate the directions of matter flows in the system.

The analysis of results presented in Fig. 2 verifies the above conclusion that the part of matter of the stream falls in the disc at once (flowline `d`), and then loses the angular momentum under the action of numerical viscosity and accretes. The obtained quantitative evaluations show, that in steady-state regime the fraction of accreted matter, for the given semidetached binary, is approximately equal to 75 per cents of the total amount of matter injected into the system.

The part of matter, that remains in the system and influences the flow structure (flowlines `a`, `b`, and `c` in Fig. 2) is of special interest. Hereafter we shall name this part of matter by a circumbinary envelope. It should be noted, that a significant part of the gas of circumbinary envelope (see flowlines `a` and `b`) interacts with the matter ejected from the surface of mass-losing star. The influence of this part of circumbinary envelope on the flow structure results in considerable change of the mass transfer regime. The detailed analysis of this effect will be presented in subsection 3.3 of this paper. Another part of circumbinary envelope (see flowline `c`) makes a revolution around the accretor and shocks the stream edge, facing the orbital movement. This interaction results in a significant change of the general flow structure in the system and, in particular, in absence of ‘hotspot’ in the disc, as well as to the formation of an extended shock wave, located along the stream edge. Detailed description of the influence of this part of circumbinary envelope on the morphology of gas flows in the system is presented below.

To estimate linear sizes of the disc it is necessary to find the marginal (‘last’) flowline along which the matter falls directly in the disc. Flowline `d` in Fig. 2 is the marginal one and it is easy to determine sizes of the calculated quasi-elliptic accretion disc: $2.3 \times 2.0 R_\odot$ ($0.31 \times 0.27 A_\odot$). The thickness of the disc increases with the distance from the accretor and changes from $\sim 0.05$ to $\sim 0.27 R_\odot$ (or from 0.9 to 5.2 accretor radii). Similar evaluations of the parameters for quasi-elliptical disc using the marginal flowline is obtained for Z Cha system as well. For this system the forming steady-state disc has the size $0.25 \times 0.22 R_\odot$ ($0.32 \times 0.28 A_\odot$), and its thickness varies in the range from $\sim 0.006$ to $\sim 0.04 R_\odot$ (or from 0.7 to 4.5 accretor radii). It should be noted that for considered binary systems with approximately equal components mass ratios, but considerably different various characteristic parameters (separation $A$ and orbital period), radii and disc thickness (dimensionalized using $A$) are approximately identical. Moreover, locations of discs in the relation to the accretor Roche lobe also coincide for different systems.

The conducted analysis shows, that in all flowlines, belonging to the disc, up to the marginal flowline `d` the flow is smooth one. The absence of breaks indicates the shock-free interaction between the stream and matter of the disc. There is no a ‘hotspot’ on the disc edge.

The presented flow structure shows that the stream deflects under the action of the gas of circumbinary envelope (flowline `c` in Fig. 2), approaches the disc along a tangent line and does not cause any shock perturbation of the disc edge.

At the same time the analysis of results shows that the interaction between stream and circumbinary envelope results in formation of an extended shock wave, located along the stream edge turned towards orbital movement. The parameters of this shock wave, as well as the total energy production in it can be evaluated from Fig. 3. Here the normalized distribution of the energy production specific rate $\delta E$ (erg s$^{-1}$ cm$^{-2}$) along the shock wave in the equatorial plane of the system. The boundaries of the shock wave are marked by dashed lines.

$$\delta E \propto \frac{1}{4} \frac{GM_2 \dot{M}}{\Delta R_{\text{out}}} \text{ (erg s}^{-1}),$$

where $\Delta R_{\text{out}}$ is the distance of mass loss by mass-losing star and $R_{\text{out}}$ is the distance between the hypothetical ‘hotspot’ and the accretor. In the standard model the mass transfer rate $\dot{M}$ is defined as:

$$\dot{M} = \rho c a S,$$

where $S$ is the stream cross-section in the vicinity of $L_1$

Figure 3. Normalized to unit distribution of the energy release specific rate $\delta E$ (erg s$^{-1}$ cm$^{-2}$) along the shock wave in the equatorial plane of the system. The boundaries of the shock wave are marked by dashed lines.
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3.2 Comparison of synthetic light curves with observations

At the present time the maximum information on the flow structure can be obtained from the analysis of light curves of cataclysmic variables. The well-known humps on the light curves are observed for cataclysmic variables in quiescent state (see, e.g., Hack & La Dous 1993). These humps repeat regular on the orbital period and have an amplitude up to 1 magnitude (Wood et al. 1986). Moreover, for five cataclysmic binaries in quiescent state (Z Cha, OY Car, V2051 Oph, HT Cas, IP Peg) the so-called double eclipse is observed.

To explain these light curves the hypothesis of ‘hotspot’ is widely used. According to this hypothesis the ‘hotspot’ is formed as a result of the shock interaction of stream of matter leaving \( L_1 \) with the outer edge of the accretion disc (see, e.g., Smak 1970; Hack & La Dous 1993; Shore, Livio & van den Heuvel 1994). However, as it follows from the numerical studies of steady-state self-consistent gaseous flows structure the stream of matter from \( L_1 \) does not cause the shock perturbation of the disc. In the hydrodynamic model presented above the zone of energy release is located outside the disc. This zone is formed due to the shock interaction between the gas of circumbinary envelope revolving accretor and the stream of matter from \( L_1 \).

To be convinced of the adequacy of the considered model we have built the synthetic light curve for cataclysmic variable Z Cha and have compared it with observations. To obtain the synthetic light curves the techniques described in papers by Khruzina (1992) and by Khruzina & Cherepashchuk (1994) is used. The detailed description of this method adapted for the considered hydrodynamic model is given in Bisikalo et al. (1998).

Observable and synthetic light curves are presented on Fig. 4. The comparison of these curves shows the good qualitative agreement. Practically all characteristic features of the observable light curve are repeated on the theoretical one. It should be also noted that we have built synthetic light curves for the different types of cataclysmic variables. The comparison of obtained curves (Bisikalo et al. 1998) with observation shows that in the framework of considered model with energy release zone located outside the disc it is possible to explain practically all types of observable light curves.

Figure 4. a) Optical light curve of cataclysmic variable Z Cha (Wood et al. 1986); b) synthetic light curve of Z Cha obtained for the hydrodynamic model without ‘hotspot’ (Bisikalo et al. 1998).

3.3 Flow structure in the vicinity of \( L_1 \)

The stream parameters in the vicinity of \( L_1 \) were specified both in analytical (Paczyński & Sienkiewicz 1972; Lubow & Shu 1975; Savonije 1978) and numerical (see, e.g., Nazarenko 1993) works. The main characteristics of matter stream obtained in various works differ only in details, and at present time, these expressions are widely used as standard for the mass transfer analysis in semidetached binaries (see, e.g., Pringle & Wade 1985; Shore et al. 1994). Unfortunately, in all these works the influence of forming circumbinary envelope on the flow structure in the vicinity of mass-losing star was not taken into account. In this work we can consider the contribution of circumbinary envelope on the basis of 3D numerical simulations.

The general flow structure in the equatorial plane of considered system is presented on Fig. 5, where density isoflows and velocity vectors are presented. This figure is similar to Fig. 2 but the results are presented for larger region with sizes from \(-5\) to \(13R_\odot\) on \( X \) axis and from \(-6\) to \(6R_\odot\) on \( Y \) axis. On Fig. 5 four flowlines (labelled by markers ‘\( a\)’, ‘\( b\)’, ‘\( c\)’, ‘\( d\)’), illustrating the directions of flows in the system are shown as well.

The analysis of results presented on Fig. 5 shows, that the significant part of the gas of circumbinary envelope (flowlines ‘\( a\)’ and ‘\( b\)’) reaches the surface of mass-losing star (Roche lobe) and prevents the gas to escape from the star surface. A part of matter of the envelope (flowlines ‘\( c\)’ and

Supposing that \( R_{out} \) is equal to the radius of the accretion disc obtained from the hydrodynamic calculations presented above it is possible to estimate \( \Delta E_{spot} \). The comparison of the energy release rate in the shock wave \( \Delta E_{shock} \) with \( \Delta E_{spot} \) shows, that \( \Delta E_{shock} \) exceeds \( \Delta E_{spot} \) by factor 2. The obtained evaluation of energy release in cataclysmic binary Z Cha indicates that for this type of semidetached binaries \( \Delta E_{shock} \) is approximately equal to \( \Delta E_{spot} \) and the main part of energy is also released in a part of the shock close to the disc. It means that in the considered model without ‘hotspot’ the value of additional energy release is enough to explain the observable excess of luminosity. Of course, it is not sufficient proof of the adequacy of the considered model. The real proof of the model adequacy can be obtained only from the direct comparison of the synthetic light curves with observations.
Figure 5. The same as Fig. 2 for extended region of the system. Four flowlines labelled by markers ‘a’, ‘b’, ‘c’ and ‘d’ show the directions of gas movement near the mass-losing star. Vector in the upper right corner corresponds to the value of velocity of 800 km s$^{-1}$.

‘d’) blows away the gas from the surface of mass-losing star and becomes involved into the process of stream formation.

The details of the flow structure near the inner Lagrangian point are shown in Fig. 6, where the same flow parameters as in Fig. 5 are presented in small region of the equatorial plane with sizes from 1 to 4$R_\odot$ for $X$-direction and from $-1.5$ to $1.5R_\odot$ for $Y$-direction. Fig. 6 shows that the gas of circumbinary envelope considerably changes the flow structure near $L_1$ and, in particular, strips off part of star atmosphere. In Fig. 6 we can see also the asymmetry of the influence of circumbinary envelope on the stream. The gas moving from above – on the way of orbital movement is accreted by mass-losing star and only in a small area in the proximity of $L_1$ it blows out the matter from the surface and transfers it into the stream. The gas of the envelope, coming to $L_1$ from below, strips off the matter from significant part of the surface.

The considered effect of ‘stripping off’ the matter from the surface of mass-losing star considerably changes the usual point of view on the mechanism of stream formation and on the mass transfer parameters. According to the standard model, the atmosphere structure near the surface of mass-losing star is determined by the equation of hydrostatic equilibrium (see, e.g., Lubow & Shu 1975; Pringle & Wade 1985). For the adopted parameters of the system and the temperature (sonic velocity) of the gas on the mass-losing star surface, the energy of the gas is not sufficient for the direct escape from the star surface. Therefore the matter flow connected with thermal escape will be negligible in comparison with the flow through the vicinity of $L_1$. The gas of surface layer can flow along the surface of the star, however in this case the total mass flow to the system increases slightly (Lubow & Shu 1975). The situation changes drastically in the case of taking into account of a circumbinary envelope, as far as in this case the gas of surface layer can be ‘stripped off’ and blown away into the system.

On the significant part of the star surface the rarefied gas of circumbinary envelope has sufficiently large momentum for capturing the matter from the surface layer. Subsequently this matter forms the stream and determines the value of the total mass transfer rate together with the gas leaving the vicinity of $L_1$. The analysis of results shows, that in the considered systems X1822-371 and Z Cha the mass...
transfer rate is one order of magnitude higher than one predicted by theoretical estimations, calculated without taking into account the influence of circumbinary envelope for the same values of gas parameters on the surface of mass-losing star.

### 3.4 Influence of the accepted boundary conditions on the flow structure

The results of calculations for semidetached binaries of various types show the qualitative similarity of the flow structure. Formally, this fact allows to assert, that the change of parameters of the system does not result in considerable changes of the flow structure. However, before making a conclusion about the universal character of the proposed model, let us consider possible variations of the flow structure due to changes of boundary parameters of the gas, injected into the system.

The study of this problem is stimulated first of all by the fact that the adopted boundary conditions on the mass-losing star surface determine the flow structure. Moreover, in view of limited clarification of the behavior of the surface layers of star when it fills Roche lobe, there is some arbitrariness in specifying of the appropriate boundary conditions.

As it was mentioned above, to specify the boundary conditions on the surface of mass-losing star it is necessary to estimate the temperature, the density and gas velocity. The first parameter – temperature is determined from the observations and its value is known with the satisfactory accuracy. Second parameter – density does not influence the solution due to scaling of the system of equations upon \( \rho \). Therefore, the only boundary condition that is not well defined and that can influence the flow structure is the gas velocity.

The results of calculations for low-mass X-ray binary X1822-371 and for cataclysmic binary Z Cha have been obtained under the assumption, that the gas on the mass-losing star surface has velocity equal to the local sonic velocity, i.e. the Mach number \( M \) is equal to 1 and it is the limiting case of maximum possible value of the boundary gas velocity. Another limiting case corresponds to zero gas velocity. So, calculations for Z Cha have been reconducted under the assumption that the boundary gas velocity is equal to zero, in order to study the influence of the boundary conditions on the flow structure.

The results obtained under this assumption show that there are some insignificant quantitative changes in the flow structure. In particular: i) the mass transfer rate, in comparison with the first model (\( M=1 \)), has decreased now by 25 per cents; ii) the total energy production in the shock wave has decreased by 15 per cents. At the same time, the common flow structure has preserved all characteristic features.

In runs conducted with different boundary parameters we have obtained the formation of circumbinary envelope which deflects the stream and leads to the shock-free stream-disc interaction. This fact allows to conclude that the considered
model of the flow structure for semidetached non-magnetic binaries is an universal one.

3.5 Comparison of results obtained in 3D and 2D models

For a long time the 2D approach was the main one in numerical simulations of mass transfer in binaries. Increasing of computational facilities allows to include the third dimension into the hydrodynamic models and as a consequence to make them more realistic. Unfortunately the 3D models are very time-consuming and accordingly are not very refined, so the question of 2D approach applicability is not scholastic one.

To evaluate the validity of 2D approach we have made the 2D simulation of the flow structure for Z Cha binary system. To make the comparison accurately we have used the same parameters and boundary conditions as in 3D model described above. The results of calculations for 2D and 3D models are presented on Figs 7a and 7b accordingly. On these figures density isolines and velocity vectors in region of the equatorial plane with sizes from 0.0 to 1.5$R_\odot$ for $X$-direction and from $-0.75$ to $0.75R_\odot$ for $Y$-direction are shown.

Comparison of Figs 7a and 7b shows that steady-state flow structures obtained in 2D and 3D models have a set of common features:

i) the accretion disc is formed;

ii) the circumbinary envelope plays an important role in the formation of the gaseous flows structure;

iii) the stream-disc interaction is shock-free, and the ‘hotspot’ does not exist in both models;

iv) part of the stream revolves around the accretor and interacts with itself causing the formation of shock wave ‘I’ on the outer edge of the stream;

v) the interaction of gas of the stream with circumbinary envelope causes the formation of the shock waves labelled by marker ‘II’.

Resuming the above points we may conclude that the qualitative characteristics of the flow structure in the inner region for the considered binary are similar. In turn, it means that 2D model give a rather correct qualitative description of the gas flow structure in this region.

As it follows from a comparison of results presented in Figs 7a and 7b there are also some quantitative differences of the flow structure in the vicinity of the accretor. In particular, in 3D case the accretion disc has an elliptic form and the stream of the matter leaving $L_1$ goes close to the accretor. In 2D case the disc is more circular and the angle of deviation of the stream is greater than in 3D case, therefore the stream goes far from the accretor. This fact may be confirmed by the results presented in Fig. 8, where the density distributions (normalized on unity) along the line passing through the accretor parallel to $Y$ axis are shown.
From physical point of view the close moving of the stream in 3D case is rather evident, because in this case we consider all three dimensions and gas of the disc can expand on $Z$-direction, that, in turn, allows the stream to go closer to the accretor. The distance between the stream and the accretor defines the ratio between the gas flow leaving the system through the vicinity of $L_2$, and the gas flow revolving around accretor. This difference in the gas fluxes leads to the flow structure changes in the outer regions of the system. In 3D case (where the stream moves close to accretor) the most part of the stream is involved into movement around accretor, while in 2D case the gas outflow through the vicinity of $L_2$ is more effective. In particular, for 2D case we see that the dominant gas flux leaving the system through $L_2$ moves around the centre of mass of the binary system in clock-wise direction and causes the formation of typical bow shock labelled by ‘III’, while in 3D case this bow shock does not arise.

It should be noted that the obtained 3D solution was a steady-state one. The run have been conducted up to 20 orbital period and we have not found changes in the flow structure. For 2D case the solution is quasi steady-state and even for large evolution time (few orbital periods) quasi periodic changes of the flow structure is observed.

Resuming the comparison between obtained 2D and 3D solutions, we may say that for the considered case of $\gamma = 1.01$:

i) the 2D model gives a rather good qualitative description of the flow structure in the inner regions of the binary, while at the outer regions the 2D results are not reliable.

ii) the 2D model does not give an adequate quantitative description.

4 CONCLUSION

This work deals with the results of 3D numerical simulation of the gaseous flows structure in semidetached binaries of various types. The analysis of results shows the significant influence of rarefied gas of circumbinary envelope on the flow patterns in these systems. The gas of circumbinary envelope interacts with the stream of matter and deflects it. This leads, in particular, to the shock-free (tangential) interaction between the stream and the outer edge of forming accretion disc, and, as the consequence, to the absence of ‘hotspot’ in the disc.

At the same time it is shown, that the interaction of the gas of circumbinary envelope with the stream results in the formation of an extended shock wave located along the stream edge. The comparison of synthetic light curves with observations proves the validity of discussed hydrodynamic model without ‘hotspot’.

It should be mentioned that taking into account of circumbinary envelope also leads to a drastic change of the mass transfer parameters in the system. The calculated mass transfer rate increases in order of magnitude as compared with values got from the standard model. Moreover, the gas of circumbinary envelope changes the flow structure near the surface of mass-losing component that eventually influences the common structure of gas flows in the system and consequently affects the interpretation of observational data.

The qualitative similarity of the obtained solutions for
the various types of semidetached systems permits to speak about some universal character of the considered hydrodynamic model for non-magnetic semidetached binaries. The main features of the obtained flow structure are summarized on Fig. 9, where gaseous flows in systems, location of the shock wave, as well as forming quasi-elliptical accretion disc are presented schematically.

It should be noted that the presented results are obtained for the steady-state case. For the non-stationary (transient) case when morphology of the flow is determined by external factors and is not self-consistent, more features of the flow may appear, in particular, arising of the zone of the disc-stream shock interaction is possible. For example, if the disc was formed before the filling by the mass-losing star its Roche lobe than, after the beginning of mass exchange, the occurrence of ‘hotspot’ would be possible. Nevertheless, the disc-stream shock interaction is possible. For example, if the characteristic lifetime of ‘hotspot’ it is naturally to take the interval, during which the quantity of matter, transferred to the system by the shock wave, as well as forming quasi-elliptical accretion disc are presented schematically.

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