Momentum multi-objective optimization algorithm based on black hole algorithm

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Abstract. In this paper, we mainly study the application of a multi-objective optimization algorithm based on the black hole algorithm in motor optimization. Using the combination of global search and local search, the continuous search area is better. The random search method is used in the global search, and a momentum gradient method is used in the local search, which makes the search results have faster convergence rates and easier convergence to the global optimal. A new file management strategy is used to make the optimization results more global and have better generalization ability. In practical application, the design of motor is often affected by manufacturing error, so random noise is added to the selection of optimization results, which can be more in line with the practical application. Finally, the actual motor model and complex function are used to test the performance of the optimization algorithm. Finally, the actual motor model and complex function are used to verify the performance of the optimization algorithm.

1. Introduction

Most problems belong to multi-objective optimization problems [1] in the areas of engineering design [2], path planning [3], motor design [4], and so forth. In solving the motor optimization problem, the multi-objective optimization problem is usually transformed into a single objective optimization problem by weighting. Because weights selection have a greater impact on the optimization results, personal factors play a vital part in results, but multi-objective optimization algorithm can offer multiple Pareto optimal solutions. At the same time, using multi-objective optimization algorithm also has some defects, such as high time and space complexity, optimization algorithm precocity. In the past few decades, bioinspired computation has been driven by natural and social behavior phenomena, starting from a set of initial variables and then evolving to discover multiple optimal solutions simultaneously [5]. Algorithms based on swarm intelligence have been introduced to solve MOPs, called evolutionary multi-objective optimization. Thus, it is suitable to solve MOPs by bioinspired computation. Algorithms based on Pareto advantage are the most popular multi-objective evolutionary algorithms [1], which include non-dominant ranking genetic algorithm II (NSGA-II) [6] and intensity Pareto evolutionary algorithm II (SPEA-II) [7]. At present, some machine learning algorithms such as clustering algorithms [9] and statistical methods [10] are gradually applied in optimization.

Many MOEAs have problems in these two field: the first one is high complexity of time and space and the second one is the way how to achieve the balance between diversity and convergence. In [11],
an adaptive multi-objective black hole algorithm is introduced, but there are still some problems in
local search, such as algorithm oscillation and high time complexity. Therefore, based on the black
hole algorithm (BH algorithm) [12], the author introduces a new momentum multi-objective
evolutionary algorithm in this paper in order to solve these problems, and applied a method of
combining global optimization with local optimization. Besides, the archive management strategy and
Shannon entropy are determined for the calculation of archive viscosity [13]. Analyzing the status of
evolution often uses Shannon entropy. The K-mean algorithm is used to select k categories in the file,
and the K points which is most suitable for local optimization are obtained for local optimization. The
momentum gradient [10] method is used to improve the population variation strategy in local search,
and the upper and lower bounds of population variation are determined according to the falling
momentum. In order to study the scalability of the momentum multi-objective evolutionary algorithm,
simulations are performed with test problems in different degree of difficulties, that is, ZDT3. We can
see from the simulation results that, when solving different multi-objective optimization problems, the
momentum multi-objective evolutionary algorithm performs well in diversity and convergence.

1.1. Black Hole algorithm

Black hole algorithm is based on the physical phenomenon of black hole absorption. Its basic idea
is that objects nearby a space area cannot escape its pall for it has so much mass in it. Everything that
falls into a black hole, including light, will disappear from our universe forever [12].

1. The black hole algorithm will start to work out the candidate solution of the initial optimization
problem, and use the objective function to calculate.
2. The best candidate star will be selected as the black hole and the rest stars as normal stars in each
iteration of the Black Hole. Then, stars will be pulled by the black hole to around it.
3. Black hole will possess nearby star if it get too close to the Hole and the star will disappear
forever. In this situation, there will be a new star (candidate solution) that is generated randomly and
placed in the search space. Also, there will be a new search.

\[ x_i(t + 1) = x_i(t) + \text{rand} \cdot (x_{bh} - x_i(t)) \]  (1)

1.2. Global and Local search based on K-mean algorithm

In the process of global search, initialization of stars will have a great influence on the search results,
so in the initialization process, we do not use random initialization but use the initialization method of
uniform distribution of search domain, and then use the calculated black hole to optimize each star.
The Initialization method as follows:

\[ S_0^i = x_{min}^i \]
\[ S_n^i = x_{min}^i + n \cdot (x_{max}^i - x_{min}^i) / k \]  (2)

Local search is often used to improve the performance of the algorithm. After the file is full, the
better initial solution in the file is selected. In order to reduce the complexity of the algorithm, the K-
mean algorithm is used to calculate the Euclidean distance between the solution sets. First, the
Initialize cluster centroids \( \mu_1, \mu_2, \mu_3, \ldots, \mu_k \in S^n \) randomly. Then repeat until convergence:

\{ 
   For every i, set:
   \[ c_i' := \arg \min_j \| s_i - \mu_j \|^2 \]  (3)

   For each j, set:
   \[ \mu_j := \frac{\sum_{i=1}^{n} 1 \{ c_i' = j \} s_i}{\sum_{i=1}^{n} 1 \{ c_i' = j \}} \]  (4)
\}

And the K best initial solutions are obtained. The elite mutation strategy of a genetic algorithm is
used for local search, which is helpful for the algorithm to fall into local optimization.
1.3. Momentum gradient learning rate

In the process of local search, the elite mutation strategy generates some new solutions by random oscillations on the basis of the initial solution. When the optimization algorithm converges to the local optimal, the learning rate will be adjusted accordingly, so that the algorithm can enter the local elite search to prevent the algorithm from entering the local optimal. The fixed learning rate will make the convergence too slow and the amplitude of vibration too large to get optimal. Using a momentum gradient method, through the previous search of the pheromone, the adjustment of each learning rate can be affected by the momentum many times before, and the premature information can be gradually forgotten, and the better optimization effect can be obtained by adjusting the learning rate. The fixed learning rate can make the convergence too slow and the amplitude of the oscillation too large to get the best, using a method of momentum gradient, through the pheromone of the previous search, adjust the learning rate to get better optimization effect.

\[
\begin{align*}
I_{W_0} &= 1 \\
I_{W_1} &= \beta I_{W_0} + (1 - \beta)\nabla W_1 \\
&\quad \vdots \\
I_{W_n} &= \beta I_{W_{n-1}} + (1 - \beta)\nabla W_n
\end{align*}
\]

Where \( \nabla W \) is the gradient sequence and \( \beta \) is the weighting function. In this paper, the \( \beta \) is 0.1.

When performing a local search, the formula of the algorithm is:

\[
W(t + 1) = W(t) + \nabla W(t) \cdot \text{Gaussian}(0, \sigma^2)
\]

1.4. Archive Manage Strategy

In the archives management, because of the fixed file size, the new solution and the old solution conflict, the new solution when enters the file question, here proposed the archives management strategy to deal with the following question. First, the new answer is abandoned When the old answer is good than the new answer. Then if the new answer is all better than the old answer, the old answer is removed and the new answer enters the archive. Finally, if the new answer and all the old answer are not dominant, the new answer will enter the archive when the archive is not contented. But when the archive is contented, calculating the viscosity of the archive, If the new answer is more concentrated, give up the new answer. The viscosity of the solution set \( D_n \) calculated between the characteristics of individual \( i \) and individual \( j \), as follows.

\[
D_n = \min \sqrt{\sum_{k=1}^{m} (S_i^k - S_j^k)^2}
\]

1.5. Input parameter adjustment

In the practical application, due to the factors of the manufacturing process and the production level, the actual production cannot be as accurate as the optimization result, and in order to improve the robustness of the optimization result, some disturbance is added in the parameter input, which also can avoid the early maturity of the optimization algorithm, and the 6-sigma principle in the industrial manufacturing is followed.

\[
x_y = x_y[1 + \text{Gaussian}(0, \sigma^2)]
\]

2. Simulation and optimization result verification

2.1. Evaluation Criteria for Optimization Results

In 1949, Shannon et al. in their paper [14] introduced a concept of Shannon entropy, which before the reception was presented as a mode of the amount of information which is losing. And Shannon entropy is as follows:

\[
H(T) = -K_c \sum_{r \in T} p_i(t) \log_2 p_i(t)
\]
where the $p_i(x)$ is the probability which is the Probability of occurrence of the i. Ke is the fixed value, and $T$ is the set of events $t$.

In this paper, the viscosity of archives based on the calculation of entropy:

$$H(t) = -N(t) \sum_{n=1}^{N(t)} \sum_{m=1}^{M} p_{n,m}(t) \log_2 p_{n,m}(t);$$

$$p_{n,m}(t) = \frac{D_{n,m}(t)}{N(t)M}$$

(10)

### 2.2. complex function optimization

In order to test the ability of the algorithm, the ZDT3 function is used to verify the momentum multi-objective optimization algorithm. The function can verify the performance of the algorithm in terms of concaveness, convexity, dispersion, nonuniformity, multimodality and so on. Using momentum multi-objective optimization algorithm optimizes this function, and the iterations $T = 500$, the number of stars $K = 300$, and archive size $N = 50$. The optimization function is as follows:

$$\min f_1(x) = x;$$

$$\min f_2(x) = g(x)[1 - \sqrt{\frac{x_1}{g(x)}} - \frac{x_1 \sin(10\pi x_1)}{g(x)}]$$

$$s.t. \quad g(x) = 1 + 9 \sum_{i=2}^{n} x_i, x \in [0, 1], i = 1, 2, ..., n, n = 30.$$ (11)

The optimization results are shown in the following figure:

*Figure 1. Algorithm optimization result and Shannon entropy*

Because of the complexity and nonlinear of ZDT3, the optimal solution curve of the function is ladder. From the optimization results, the obtained pareto is distributed on the optimal path of the function, and the distribution is uniform, and the Shannon entropy curve is also very stable.

### 2.3. Motor model optimization

Based on the analysis of the electromagnetic field model in [16], the thrust model and copper loss model of The eddy-losses of Linear PM synchronous machine are obtained. The optimization objectives are motor thrust, permanent magnet volume, and copper loss. The optimized parameters are as follows:
Table 1. Optimization design variables of the Linear PM synchronous machine

| Variable                  | Symb | Unit   | Range                       |
|---------------------------|------|--------|-----------------------------|
| Outer Radius of Coil      | Rs   | mm     | \( R_{i1} < R_s \leq 34 \) |
| Outer Radius of PM        | Rm   | mm     | \( R_{i2} + 5 < R_m < R_s \) |
| Inner Radius of PM        | Rb   | mm     | \( 5 < R_b < 28 \)           |
| Width of PM               | 2b   | mm     | \( 0.5 \tau_p < 2b < 0.9 \tau_p \) |
| Coils Number              | N    | --     | --                          |
| Air Gap                   | \( \sigma_s \) | mm     | 1                           |
| Inner Radius of Coil      | Ri1  | mm     | \( R_m + \sigma_s \)         |
| Current Density           | \( J_m \) | A/mm² | 4                           |
| Rated Speed               | \( v_N \) | m/s   | 9                           |

The optimization function is:

\[
\begin{align*}
\text{min} & : V = 2\pi \left( R_s^2 - (R_s - D_z)^2 \right) b \\
\text{min} & : 1/F = \frac{1}{\sqrt{2j_m \sum_{n=1}^{\infty} K_T n \left[ \cos \left( m \left( z - \tau_p \right) \right) \sin \left( \omega t \right) + \cos \left( \frac{7\tau_p}{6} \right) \sin \left( \omega t - \frac{2\pi}{3} \right) \right]}} \\
\text{min} & : P_{vis} = 3 \left( J_m \sigma_s \right)^2 R \\
\text{s.t.} & : 180 \leq f_1(x) = E_m \leq 400
\end{align*}
\]

(12)

The optimization results are shown in the following figure:

Figure 2. Algorithm optimization result and The Shannon Entropy Evolutionary

From the result of the optimization, the optimization curve is smooth and the pareto is uniform, and the Shannon entropy based on the viscosity of the file is gradually raised and stabalized at a higher level with the optimization iteration times.

3. Conclusion

Based on the improvement of black hole algorithm, this paper proposes a more efficient momentum multi-objective optimization algorithm which applies the mutation in genetic algorithm to local search. In this algorithm, the momentum gradient method is used to set the learning rate to avoid local optimization and accelerate the convergence speed of the algorithm. In the Momentum multi-objective optimization algorithm, Greater entropy deputies better consistency and variety. The K-mean algorithm in machine learning is used to select the initial value of local search. Calculate the viscosity between the solution sets in the file management, so that the viscosity of the archive is minimized. The optimization performance of the actual motor and the Momentum multi-objective optimization algorithm is tested. Momentum multi-objective optimization algorithm has a good balance of variety and convergence in the validation function, although its convergence rate will have a great impact on the performance of some multimodal problems. In terms of time complexity, Momentum multi-objective optimization algorithm is also very excellent. Therefore, Momentum multi-objective optimization algorithm is a new and powerful multi-objective optimization algorithm.
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