Shake table testing of a passive negative stiffness device with curved leaf springs for seismic response mitigation of structures

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Summary
Past research has shown that negative stiffness may be used to good effect in mitigating the response of seismically excited structures. This has led to the emergence of several new negative stiffness devices of varying sizes, complexity, effectiveness, and practicality. In this study, a passive negative stiffness device composed of curved leaf springs using geometric nonlinearity is presented. The proposed device is designed to be simple and compact so as to be easily installed and exhibits approximately constant negative stiffness over a large range of displacements. Beyond these displacements, the negative stiffness produced by the device increases, which can help to limit the transmission of large damper forces at peak structure displacements. At displacements approaching the deformation limit of the device, the device produces positive stiffness so that stability of the structure is ensured. The objective of the research presented herein is to investigate the basic behavior of the device. Shaking table tests on specimens consisting of a vibrating part and the prototype passive negative stiffness device subjected to sinusoidal and simulated earthquake waves were conducted. The results revealed that the specimens exhibited negative restoring force characteristics, and due to the effect of the negative stiffness, an increase in the apparent period was demonstrated. Moreover, reduction of the response acceleration was shown without remarkable increase of the response displacement under the simulated earthquake inputs. Also, a numerical analysis to simulate the shake table testing and evaluate the control effect for a full-scale building model was carried out.

KEYWORDS
apparent period, geometric nonlinearity, leaf springs, negative restoring force, negative stiffness, snap-through buckling

1 | INTRODUCTION

In recent years, several technologies have been proposed to passively achieve negative stiffness as a method for controlling the earthquake response of structures. Adopting negative stiffness decreases the equivalent stiffness of structures,
thereby increasing the apparent period, and is expected to reduce the structure absolute response when subjected to earthquakes.

As a previous passive technology to generate negative stiffness, a friction damper\textsuperscript{1-6} using pendulum-type friction bearings has been proposed, whose curvature is opposite to conventional ones and exerts a negative stiffness by the weight of the structure itself. In addition, compressed coil springs\textsuperscript{7-17} have been proposed that make use of geometric nonlinearity to produce variable negative stiffness that is suitable for application in the apparent weakening seismic protection scheme. The latter has also been achieved using rotating and translating components in combination with preloaded springs.\textsuperscript{18,19} Compressed coil springs have been combined with fluid viscous dampers for negative stiffness damping of cables,\textsuperscript{20,21} with friction sliders for a nonlinear energy sink system,\textsuperscript{22} with a flexible support connected in series,\textsuperscript{23} and with a passive resettable stiffness damper for hybrid base isolation of a five-story building.\textsuperscript{24} Another passive negative stiffness device utilizes permanent magnets arranged in a conductive pipe, resulting in eddy-current negative stiffness damping.\textsuperscript{25} As previous techniques that can produce negative stiffness based on the inertial force of the mass body, those using the inertial mass of the rotating body\textsuperscript{26,27} and using the inertial mass of the fluid\textsuperscript{28} have also been studied. A displacement-dependent oil damper,\textsuperscript{29} whose damping force passively decreases with increasing displacement, has been proposed, as for past devices that exert a kind of pseudo-negative stiffness effect. In addition, numerical investigation and optimum design of a negative stiffness damper for vibration control of stay cables have been conducted.\textsuperscript{30-32}

In the present study, a new passive negative stiffness device composed of curved leaf springs is proposed. It utilizes the reversal phenomenon due to elastic snap-through buckling to produce negative restoring force characteristics. If such a device using a curved surface shape can be realized, it is expected to produce a negative stiffness in the load-deformation relationship due to the geometrical nonlinearity effect with a relatively simple and compact mechanism. In this paper, two prototype specimens of the passive negative stiffness device were manufactured and incorporated into a single degree of freedom (SDOF) system on a shaking table. The basic characteristics of the SDOF system with the device and subjected to sinusoidal waves and simulated earthquake motions were investigated. Also, a numerical analysis for simulating the shake table testing and evaluating the control effect for a full-scale building model was conducted. The present paper expands on a previous study\textsuperscript{33} by presenting new experimental and analytical results and findings.

2 | PASSIVE NEGATIVE STIFFNESS DEVICE DESCRIPTION

To produce negative restoring force characteristics, the proposed passive negative stiffness device utilizes precompressed curved leaf springs that are subject to the reversal phenomenon due to elastic snap-through buckling, such as that used in a common type of pin, as shown in Figure 1. The energy stored in the precompressed springs leads to an initial negative stiffness prior to the onset of snap-through buckling, whereas during snap-through buckling the negative stiffness increases substantially. Once the displacement limit of the device has been reached, the springs provide a large positive restoring force. Therefore, the proposed device can be described as having displacement-dependent restoring force characteristics with both negative and positive stiffness.

Figure 2 depicts a schematic diagram showing how the proposed device works. In Figure 2a, two curved leaf springs are rigidly connected to an upper and lower beam, such that their curvatures are in opposite directions. The leaf springs
are initially compressed between the beams, and the distance between beams in the Y-direction is held constant. The curvature of the leaf spring on the left in Figure 2a is concave right at the top, then concave left in the middle, and back to concave right at the bottom. Meanwhile, the curvature of the leaf spring on the right is concave left at the top, then concave right in the middle, and concave left at the bottom. Regions A, B, C, and D in Figure 2a (see dotted lines) indicate the portions of the leaf springs subject to snap-through buckling during relative motion of the upper and lower beams in the X-direction. For example, if the lower beam is fixed and the upper beam moves to the left (negative X-direction) as shown in Figure 2b, snap-through buckling will occur in Regions B and C, where a reversal in curvature of the leaf springs can be observed. Similarly, if the upper beam moves to the right (positive X-direction) as shown in Figure 2c, snap-through buckling will occur in Regions A and D. When snap-through buckling occurs, a force is exerted on the upper beam in the direction of, and increasing with, the beam displacement. This results in negative restoring force characteristics, that is, negative stiffness.

Figure 3 shows a conceptual diagram of the load–displacement relationships when the proposed passive negative stiffness device is applied to a vibration control structure with a conventional friction damper. Due to the negative stiffness effect, vibration systems incorporating the proposed device are expected to have the following advantages:

- The apparent natural period of structures can be increased.
- The absolute acceleration, shear force, and overturning moment response of structures when subjected to earthquakes would be reduced.
- When installing energy dissipation devices in buildings, it is possible to mitigate the increase in the axial force of the columns and the total bending deformation due to the addition of the dampers.
• Even with the same amount of energy absorption by installing dampers, a larger damping factor can be obtained.
• The range of application is expected to include the vibration control structures in which optimal tuning is required, such as mass damper systems.
• The increased burden on the foundation due to the installation of energy dissipation devices would be mitigated in the case of seismic control retrofitting for existing structures.

3 | EXPERIMENTAL METHODS

3.1 | Test specimens

Vibration system specimens with and without passive negative stiffness devices were manufactured and mounted on a shaking table in the present study. A photograph of the specimen with the negative stiffness device and the shaking table is shown in Figure 4.

The vibrating part of the specimen consisted of a moving part, a main bearing, and positive stiffness restoring members and was designed and manufactured to move in one horizontal direction (i.e., shaking table excitation direction). The mass of the moving part was 0.509 kg. As the main bearing, a linear guide was installed in the direction of shaking. As positive stiffness restoring force members, multiple tension coil springs were used in both series and parallel. For each spring on each side, an initial tensile deformation was applied so the springs could work in both directions (i.e., plus and minus directions) without looseness.

3.2 | Passive negative stiffness device

Photographs of the passive negative stiffness device are shown in Figure 5. It was intended to exhibit negative stiffness through geometric nonlinearity and consisted of leaf springs with a curved shape. Two curved leaf springs were...
fabricated and placed bilaterally symmetrical to each other, and both ends of the springs were fixed to rigid beams to make a single negative stiffness device.

A total of two negative stiffness device specimens, named PNSD-1 and PNSD-2, were manufactured and used in the shaking table test. Table 1 gives the specifications of the specimens. Figure 6 shows the production drawings of the leaf springs for the negative stiffness devices before the bending process. Each leaf spring is composed of three plates—a center plate and two outer plates. The outer plates have bolt holes at the ends, whereas the center plate has a bolt hole in the middle. The initial curvature of the leaf springs was achieved by bolting the ends of the outer plates to the middle of the center plate. In doing so, a three-dimensional curved surface was constructed. PNSD-2 was designed with a larger plate width with the aim of obtaining a higher negative stiffness.

The passive negative stiffness device was installed with one side fixed to the vibration part and the other side to the shaking table via a sublinear guide. The sublinear guide was set in the direction orthogonal to the shaking direction and was used to adjust the internal span of the negative stiffness device during installation, such that each device was initially compressed. The sublinear guide was positioned so that the inner span of the negative stiffness device was held at 73 mm (see Table 1) during the shaking experiments. In order to alleviate the occurrence of shock and the remarkable increase in response acceleration when the negative stiffness device reaches its deformation limit, a cushioning material made of sponge was installed just before the deformation limit.

### 3.3 Input motions

Two kinds of input motions, sinusoidal waves and simulated earthquake waves, were used in the shaking table tests. Examples of the input motions are shown in Figure 7.
The sinusoidal waves all had a frequency of 1.3 Hz with a wave number of five cycles but had displacement amplitudes varying from 8 to 12 mm. The frequency of the sinusoidal waves was set to 1.3 Hz to match the natural frequency of the vibrating part of the specimen without the passive negative stiffness device installed. That natural frequency was obtained by free vibration measurements as described in Section 4.2. Figure 7a shows an example of the time history of the sinusoidal wave (input amplitude 10 mm).

Two simulated earthquake waves, named Wave-S and Wave-L, were prepared. Each seismic wave was given the characteristic that the target velocity response spectrum (damping factor 0.05) had a constant range in the period exceeding approximately 0.77 s and the periodic component longer than 2.0 s was cut off in consideration of the capacity of the shaking table. A shorter and a longer envelope function in the time domain was given for Wave-S and Wave-L, respectively, and the phase characteristic in the time domain was given by random numbers for each seismic wave. The input magnifications of the seismic waves were 0.30, 0.40, and 0.48 times for Wave-S and 0.30, 0.40, and 0.42 times for Wave-L, respectively. Examples of time history acceleration waveforms and velocity response spectra of the simulated earthquake waves are shown in Figure 7b–e.
3.4 | Shaking table and vibration direction

A uniaxial shaking table with a maximum acceleration of 1.0 G, maximum displacement of 100 mm, and maximum payload of 1000 N was used for the shaking table tests. The specimen was mounted on the shaking table, and the shaking tests were carried out in the horizontal direction as shown in Figure 4.

3.5 | Measurement

The absolute accelerations on the top of the shaking table (excitation direction) and on the vibrating part of the specimen (excitation direction) were measured using strain gauge-type accelerometers. A laser displacement meter was installed on the shaking table to measure the relative displacement of the vibrating part of the specimen. The measurement sampling rate was 200 Hz. The acceleration and displacement data obtained from the shaking table tests were subjected to a smoothing process by taking a moving average with a rectangular window of 0.035 s.

3.6 | Experiment parameters and shaking conditions

As previously discussed, experiments were performed for the vibrating part of the specimen with and without the two types of passive negative stiffness devices (PNSD-1 and PNSD-2). Furthermore, two types of input motions, sinusoidal waves and simulated earthquake waves with varying input amplitudes, were used. When a residual displacement occurred in the specimen after the end of each shaking, it was returned to the neutral position, and then the next shaking was performed.

4 | EXPERIMENTAL RESULTS

4.1 | Quasi-static loading tests

As an independent test from the dynamic shake table tests, a quasi-static loading was carried out using PNSD-1 or PNSD-2 alone for each of the horizontal (shear) and vertical (compression) directions. The horizontal loading test was
conducted on PNSD-1 or PNSD-2 installed in the testing system for the shake table tests (i.e., the movement part, main linear guide, and sublinear guide in Figure 4), excluding the positive stiffness restoring members and the cushioning material. In the horizontal loading, the movement part of the testing system was displaced via a loadcell in the shear direction of installed PNSD-1 or PNSD-2. The horizontal reaction force and displacement were measured by the loadcell and laser displacement meter, respectively. In the vertical loading, PNSD-1 or PNSD-2 was compressed via the loadcell without being installed into the testing system, and the reaction force was measured by the loadcell. The measurement sampling rate was 200 Hz. The displacement and load data of the loading tests were smoothed by taking a moving average with a rectangular window of 0.035 and 1.0 s, respectively.

The hysteresis loops of PNSD-1 and PNSD-2 obtained by the quasi-static loading for the horizontal (shear) direction are shown in Figure 8 with solid lines. Overall, the results exhibited an increase in negative stiffness as the displacement increased. Specifically, the hysteresis loops for both PNSD-1 and PNSD-2 showed that the devices produced approximately constant negative stiffness over a wide displacement range due to precompression of the curved leaf springs, followed by a sharp increase in negative stiffness due to the occurrence of snap-through buckling. The hysteretic component (i.e., the thickness of the loops) appearing in Figure 8 was due to the resistance of the main linear guide when sliding.

From the vertical (compression) direction loading, where each device was displaced from the inner span of around 82 (corresponding to before installation into the testing system) to 73 mm (corresponding to after installed into the testing system) for the compressive direction, a vertical stiffness of 0.165 and 0.254 N/mm was obtained for PNSD-1 with the 5-mm main plate width and PNSD-2 with the 7 mm width, respectively. The ratio of the obtained vertical stiffness of the two devices (= 0.254/0.165 = 1.54) roughly corresponded to the ratio of the main plate width of the two devices (= 7/5 = 1.4).

Based on the obtained vertical (compressive) stiffness of the devices with an assumption that it has a linear characteristic, its contribution to a horizontal (shear) force was theoretically calculated. As shown in Figure 9, the approximated horizontal force component due to the vertical stiffness can be expressed as follows:

\[
F_x = -F_a \sin \theta 
\]  
\[
F_a = K_a (L_y + D_y - L_a) 
\]  
\[
L_a = (L_y^2 + D_x^2)^{0.5} 
\]  
\[
\theta = \arctan \left( \frac{D_x}{L_y} \right) 
\]

**Figure 8** Load–displacement relationships of passive negative stiffness devices PNSD-1 or PNSD-2 alone obtained from quasi-static loading for horizontal direction (along with theoretical calculation curve due to vertical compression stiffness)
The obtained relationships of the horizontal force $F_x$ and displacement $D_x$ based on Equations 1–4 for PNSD-1 and PNSD-2 are depicted in Figure 8 with dashed lines. The magnitude of the theoretically calculated initial negative stiffness in the small range for PNSD-2 was larger than that for PNSD-1. On the other hand, in the loading tests, almost the same initial negative stiffness was shown for PNSD-1 and PNSD-2. The discrepancy between the theoretical and experimental negative stiffness in the small range may be due to the offset effect by positive stiffness exerted by the curved shape of the leaf springs adopted in this study. Moreover, when the displacement exceeded around ±20 mm, the difference between the calculation and experiment significantly increased. The difference between the results obtained by the quasi-static loading tests and the theoretical calculation based on the vertical stiffness at displacements greater than ±20 mm can be attributed to the effect of snap-through buckling in the test specimen, which was not accounted for in the calculation. A theoretical derivation for the horizontal force considering both the offset effect by positive stiffness of the curved leaf springs and the snap-through buckling effect is the subject of ongoing research by the authors.

4.2 Free vibration measurement

Prior to the shaking table tests, a free vibration measurement was conducted. The free vibration measurement was performed for the specimen without the negative stiffness device (i.e., with the positive stiffness restoring members alone). The free vibration was generated by releasing the vibrating part of the specimen after giving it an initial displacement of 40 mm. The natural frequency and the damping factor of the specimen were obtained from the measured time history displacement waveform. The natural frequency was determined based on the time difference between the peaks of the waveform, whereas the damping factor was taken as the logarithmic damping rate calculated from the amplitude.
ratio of the peaks of the waveform. Natural frequencies and damping factors were averaged for three free vibration measurements.

The average natural frequency and damping factor of the specimen without the negative stiffness device (with the positive stiffness restoring members alone) was determined to be \( f = 1.30 \text{ Hz} \) (\( T = 0.77 \text{ s} \)) and \( h = 0.097 \), respectively. As described in Section 3.3, the frequency of the sinusoidal wave used in the shaking table tests was set to correspond to this natural frequency \( f \). The relatively large value of the damping factor \( h \) may be attributed to the resistance force of the main linear guide used as the horizontal bearing in the shaking direction.

### 4.3 Results of sinusoidal inputs

Figure 10 shows the response obtained from the sinusoidal wave (displacement amplitude 10 mm) input without the passive negative stiffness device (i.e., with positive stiffness restoration members alone). Figure 10a shows the time history response acceleration of the vibrating part of the specimen, where gradual amplification of the acceleration due to resonance is observed up to around 7 s. After 7 s, the amplitude of the input acceleration becomes nearly 0 (see Figure 7a), and damped free vibration of the acceleration response is observed in Figure 10a. Figure 10b shows the hysteresis loops of the inertial force–displacement relationship. In addition to the positive stiffness, a bilinear-type hysteresis loop shape was observed due to the resistance of the main linear guide that was used as a horizontal bearing (as described in Section 3.1). Regarding this hysteresis loop, a linear function approximated by the least-squares method is also shown in Figure 10b, along with the decision coefficient \( R^2 \). From this result, the horizontal stiffness for the positive stiffness restoring members alone was determined to be \( k_{rs} = 0.034 \text{ N/mm} \). Using the horizontal stiffness of the restoring members and the system mass, the natural frequency of the vibrating system without the negative stiffness device was calculated to be 1.30 Hz, which agrees well with the value determined through free vibration measurements in Section 4.2. Figure 10c shows a hysteresis loop whose vertical axis is the force obtained by subtracting the contribution of the horizontal stiffness \( k_{rs} \) from the inertial force on the vertical axis in Figure 10b. The loop after subtraction exhibited a shape similar to rigid plasticity, and its maximum section force (i.e., force at zero displacement) was about 0.25 N.

Figure 11 shows the inertial force–displacement relationship obtained by the sinusoidal wave inputs when the passive negative stiffness device PNSD-1 was installed. Figure 11a–d shows the results when the input displacement amplitude of the sinusoidal wave was 8, 10, 11, and 12 mm, respectively. These hysteresis loops demonstrated a clear
tendency of decreasing the inertial force when the response displacement exceeded around 20 mm. From this, the effect of reducing the load by installing the negative stiffness device was confirmed. However, in Figure 11c,d, the inertial force started to increase rapidly around the point where the response displacement reached roughly 30 mm or more. This was because the vibration part contacted the cushioning material (Section 3.2).

Hysteresis loops (sinusoidal wave input) corresponding to the force produced by the passive negative stiffness device PNSD-1 are shown in Figure 12. The force produced by the negative stiffness device PNSD-1 was obtained by subtracting the contribution due to the horizontal stiffness $k_{rs}$ of the positive stiffness restoring members from the inertial force of the vertical axis in Figure 11. As a result, the resistance force from the main linear guide used as the horizontal bearing (Section 4.2) is still included in the forces shown in Figure 12, and the thickness of the hysteresis loops (i.e., energy absorption) is attributed to the linear guide rather than the negative stiffness device. The input amplitudes of the sinusoidal waves were 8 mm for Figure 12a, 10 mm for Figure 12b, 11 mm for Figure 12c, and 12 mm for Figure 12d. From these subtracted hysteresis loops, it was determined that the negative stiffness device PNSD-1 exhibited approximately constant negative equivalent stiffness over the range of displacements $-25$–$20$ mm. Beyond this range of displacements, the negative stiffness increased substantially, before changing to positive stiffness around $-34$ and $31$ mm, as the vibration component of the specimen contacted the cushioning material.

The initial negative stiffness of PNSD-1 over the range of displacements $-25$–$20$ mm can be attributed to the precompression of the leaf springs during installation of the device (Section 4.1). To determine the initial negative stiffness $k_{ns1,i}$ of PNSD-1, the stiffness $k_{rs}$ of the system without the negative stiffness device was subtracted from the initial stiffness $k_{net1,i}$ of the vibrating system with the negative stiffness device ($k_{ns1,i} = k_{net1,i} - k_{rs}$). The initial stiffness $k_{net1,i}$ of the vibrating system with the negative stiffness device was determined by applying a least-squares approximation to the force–displacement data between displacements $±20$ mm for the four different inputs. The initial stiffness $k_{net1,i}$ corresponding to the input amplitudes of 8, 10, 11, and 12 mm were found to be 0.023, 0.023, 0.022, and 0.023 N/mm, respectively, leading to an average initial stiffness of $k_{net1,i} = 0.023$ N/mm. The initial negative stiffness of PNSD-1 was then calculated using $k_{rs}$ to be $k_{ns1,i} = -0.011$ N/mm. Using the average initial stiffness $k_{net1,i} = 0.023$ N/mm and the system mass, the initial natural frequency of the vibrating system with PNSD-1 was calculated to be 1.07 Hz.
result, the initial natural frequency of the vibrating system was reduced by around 18% (i.e., 22% increase in initial natural period) due to the addition of the negative stiffness device.

As observed from Figure 12, there is an increase in the negative stiffness produced by PNSD-1 beyond the range of displacements $-25\text{–}20 \text{ mm}$. This increase in negative stiffness may be attributed to the occurrence of snap-through buckling in the leaf springs of the device. In order to determine the snap-through buckling negative stiffness $k_{nsl,s}$ of PNSD-1, a least-squares approximation was first used to determine the snap-through buckling stiffness $k_{net,1,s}$ of the vibrating system with the negative stiffness device. The approximation was applied to the force–displacement results for the 12-mm input, where full snap-through buckling was observed for both negative and positive displacements (see Figure 12). The approximation was performed for data within a 3-mm range of displacements in each direction: $-32 \text{ to } -29 \text{ mm}$ in the negative direction and $27 \text{–}30 \text{ mm}$ in the positive direction. The snap-through buckling stiffness $k_{net,1,s}$ in the negative and positive directions were approximated to be $-0.024$ and $-0.043$ N/mm, respectively, leading to an average snap-through buckling stiffness for the vibrating system with the negative stiffness device of $k_{net,1,s} = -0.033$ N/mm. The snap-through buckling negative stiffness $k_{nsl,s}$ of PNSD-1 was then calculated as the difference between the snap-through buckling stiffness $k_{net,1,s}$ of the vibrating system with the negative stiffness device and the stiffness of the system $k_{rs}$ without the device ($k_{nsl,s} = k_{net,1,s} - k_{rs}$). The snap-through buckling negative stiffness was found to be $k_{nsl,s} = -0.067$ N/mm, or around six times the initial negative stiffness of PNSD-1.

It was also observed from Figure 12 that for the negative stiffness device PNSD-1, the maximum section force at zero response displacement was roughly 0.25 N, and thus, no significant increase in the section force occurred by adding the negative stiffness device to the vibrating component of the specimen. This means that energy consumption by the negative stiffness device PNSD-1, such as that due to friction, was small.

Figure 13 shows the inertial force–displacement relationship obtained from the sinusoidal wave input when the passive negative stiffness device PNSD-2 was installed in the test specimen. Figure 14 shows hysteresis loops whose vertical axes are the corresponding forces produced by the passive negative stiffness device PNSD-2, which were once again obtained by subtracting the contribution of the horizontal stiffness $k_{rs}$ by the positive stiffness restoring members from
the inertial force in Figure 13. The input amplitude of the sinusoidal wave is 8 mm for (a), 10 mm for (b), 11 mm for (c), and 12 mm for (d). From Figures 13 and 14, a similar tendency as in the case of the device PNSD-1 (Figures 11 and 12) was generally shown for the device PNSD-2. For PNSD-2, the range of displacements over which the initial negative stiffness was approximately constant was observed to be ±25 mm, after which there was an increase in the negative stiffness due to snap-through buckling and then a rapid transition to positive stiffness at −34 and 31 mm as the specimen contacted the cushioning material. Using the same method as with PNSD-1, the initial negative stiffness of PNSD-2 was approximated to be $k_{n2,i} = -0.010 \text{ N/mm}$, and it was determined that the addition of the negative stiffness device to the vibrating system resulted in a 16% reduction in the initial natural frequency (i.e., 19% increase in initial natural period). Meanwhile, the snap-through buckling negative stiffness was approximated to be $k_{n2,s} = -0.086 \text{ N/mm}$ ($k_{n2,s} = 8.6k_{n1,s}$).

Comparison of the results for PNSD-1 and PNSD-2 reveals that there is no significant difference between the initial negative stiffness of the devices but that the snap-through negative stiffness of PNSD-2 is around 1.3 times that of PNSD-1 ($k_{n2,s} = 1.3k_{n1,s}$). The ratio of the snap-through negative stiffness of PNSD-2 to PNSD-1 is comparable with the ratio of their plate widths, which was determined to be 1.4.

Figure 15 shows a comparison of the time history response displacement waveforms for the vibrating part of the specimen without the passive negative stiffness device (i.e., with the positive stiffness restoring members alone) and with the passive negative stiffness device PNSD-2 and subjected to the sinusoidal wave with a 10-mm amplitude. Compared with the waveform without the passive negative stiffness device (thick line), the waveform with the negative stiffness device PNSD-2 (thin line) showed a delay in the time at which the peaks occurred. For the case without the negative stiffness device, the maximum response displacement became large because the vibration system, whose natural period was 1.3 Hz (Section 4.2), resonated with the 1.3-Hz sinusoidal wave input. On the other hand, when the negative stiffness device was installed, the apparent period of the vibration system was elongated and deviated from the resonance, and thus, the maximum response displacement was reduced. Specifically, elongation of the period is apparent after the response displacement amplitude increased beyond 25 mm (around $t = 4 \text{ s}$), when a reduction in apparent stiffness of the vibrating system was caused by snap-through buckling in PNSD-2. The peak shear force ($Q_{\max}$), response

![Figure 13](image_url)
acceleration ($A_{\text{max}}$), and response displacement ($D_{\text{max}}$) of the vibrating specimen with and without PNSD-2 for the sinusoidal wave with a 10-mm amplitude are provided in Table 2. The results show that the peak displacement of the specimen with PNSD-2 exceeded its snap-through buckling displacement of ±25 mm, leading to a deviation from resonance and smaller peak responses compared with the specimen without the negative stiffness device.

The peak responses corresponding to PNSD-2 at input amplitudes of 8, 11, and 12 mm, as well as the peak responses for PNSD-1 at all input amplitudes, are also included in Table 2. Comparison of the peak responses for the vibrating specimen with PNSD-1 and PNSD-2 with the specimen without the negative stiffness devices shows a similar trend to that described above, that is, a reduction in the peak responses when snap-through buckling occurs. However, it is worth pointing out that the peak responses for PNSD-1 at an input amplitude of 11 mm, and PNSD-1 and PNSD-2 at an input amplitude of 12 mm, were influenced by contact between the vibrating specimen and the cushioning material as
the specimen reached the deformation limit. As a result, comparison of these peak responses with those from the other cases is difficult.

### 4.4 Results of simulated earthquake inputs

Figure 16 shows the inertial force–displacement relationship obtained from the simulated earthquake Wave-S (input magnification ×0.48) input for the case without the passive negative stiffness device (i.e., with the positive stiffness restoring members alone). As a result, a bilinear loop similar to the sinusoidal wave input (Figure 10b) was observed.

Figure 17a shows the inertial force–displacement relationship obtained by Wave-S (×0.48) input when the passive negative stiffness device PNSD-2 was installed. Figure 17b shows the hysteresis loop of the corresponding force produced by the negative stiffness device PNSD-2 obtained by subtracting the contribution of the horizontal stiffness $k_{rs}$ of the positive stiffness restoring members. It can be seen that the negative stiffness device exhibited a negative restoring force even in the random-like earthquake excitation. Figure 17 also shows that the peak response displacement for the vibrating specimen with PNSD-2 is around 34 mm in the negative direction, which is just short of the location of the cushioning material. As a result, the response of the specimen is unaffected by impact with the material.

Figure 18a shows the comparison of the time history response acceleration waveform in the case of Wave-S (×0.48) input without the passive negative stiffness device and with the negative stiffness device PNSD-2. Compared with the case without the negative stiffness device (thick line), the maximum response acceleration was clearly reduced for the case with the negative stiffness device PNSD-2 (thin line). This is due to the installation of the negative stiffness device lengthening the apparent period of the vibration system, and thus, the response acceleration decreased because of the

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**TABLE 2 Maximum response for sinusoidal wave inputs**

| Negative stiffness device | Input amplitude (mm) | $Q_{\text{max}}$ (N) | $A_{\text{max}}$ (m/s²) | $D_{\text{max}}$ (mm) |
|---------------------------|----------------------|----------------------|----------------------|----------------------|
| No device                 | 10                   | 2.23                 | 4.38                 | 64.0                 |
| PNSD-1                    | 8                    | 0.77                 | 1.51                 | 26.5                 |
|                           | 10                   | 0.79                 | 1.55                 | 31.6                 |
|                           | 11                   | 1.02                 | 2.00                 | 34.1                 |
|                           | 12                   | 2.21                 | 4.34                 | 38.9                 |
| PNSD-2                    | 8                    | 0.76                 | 1.49                 | 25.1                 |
|                           | 10                   | 0.88                 | 1.72                 | 30.4                 |
|                           | 11                   | 0.80                 | 1.57                 | 33.4                 |
|                           | 12                   | 2.30                 | 4.53                 | 36.0                 |

Notes: $A_{\text{max}}$: peak response acceleration. $D_{\text{max}}$: peak response displacement. $Q_{\text{max}}$: peak response shear force (inertial force).
effect of the constant velocity response spectrum band (i.e., acceleration response monotonously decreasing band) of the input motion shown in Figure 7c.

A comparison of the time history response displacement waveforms without the passive negative stiffness device and with the negative stiffness device PNSD-2 subjected to Wave-S (×0.48) input is shown in Figure 18b. In comparison with the case without the negative stiffness device (thick line), the maximum response displacement increased in the minus side for the case with the negative stiffness device PNSD-2 (thin line), but the increase in the maximum displacement was relatively small.

Closer examination of Figure 18a,b reveals an interesting feature of the vibrating system with the negative stiffness device. Figure 18a shows that leading up to 13.5 s, the acceleration response of the system with and without the negative stiffness device was similar in period, but that the amplitude of the response for the system with the negative stiffness device was slightly smaller than that for the system without the device. Figure 18b shows that during this same time period, the response displacement for the system with the negative stiffness device is slightly larger than that for the system without the device, but that the response displacement is within the range ±25 mm (indicated by horizontal dotted lines). This is the range of displacements over which the initial stiffness of the vibrating system is approximately constant and has been reduced due to the addition of the negative stiffness device. As expected, the reduced initial stiffness for the system with the negative stiffness device results in a reduction in acceleration, but an increase in displacement, compared with the system without the device. Just after 13.5 s, Figure 18a reveals an obvious lengthening of the period, along with a substantial reduction in amplitude, in the acceleration response of the system with the negative stiffness device compared with the system without the device. The cause for the shift in period is revealed in the displacement response shown in Figure 18b, where an increase in displacement in the negative direction beyond −25 mm

**Figure 17**  Hysteresis loops with passive negative stiffness device (PNSD-2) for simulated earthquake input (Wave-S × 0.48)

**Figure 18**  Comparison of time history response between with or without passive negative stiffness device (PNSD-2) for simulated earthquake input (Wave-S × 0.48)
may be observed at around 13.5 s. As a result, snap-through buckling occurred, causing a substantial reduction in stiffness and lengthening of the period for the system with the negative stiffness device, thereby limiting its response compared with the system without the device.

Figure 19 shows the response hysteresis loops obtained from Wave-L (×0.42) input for the case without the passive negative stiffness device. Figure 20a,b shows the loops when PNSD-2 was installed and the loops of the corresponding force by subtracting the contribution of $k_{rs}$ of the positive stiffness restoring members under Wave-L (×0.42) input. Figure 20 reveals that the peak response displacement for the vibrating specimen with PNSD-2 is around 31 mm in the negative direction, and therefore, the response of the specimen is unaffected by impact with the cushioning material.

Figure 21a,b shows the time history acceleration and displacement waveforms, respectively, in the case of Wave-L (×0.42) input without the passive negative stiffness device and with the negative stiffness device PNSD-2. Figure 21a shows that once again, the system with the negative stiffness device exhibited a smaller peak acceleration and smaller overall accelerations throughout the duration of the event. However, Figure 21b shows that the reduced accelerations were achieved at the cost of increased displacements, with both the peak displacement and overall displacements being larger for the system with the negative stiffness device. Figure 21b also shows a lengthening of the vibration period just prior to around 55 s, which can be attributed to the occurrence of snap-through buckling when the response displacement exceeded 25 mm in the negative direction. The effect of the snap-through buckling on the acceleration response is observed in Figure 21a at around 55 s, where the acceleration of the system with the negative stiffness device is considerably smaller than that of the system without.

Table 3 summarizes the obtained maximum responses from the experiments under simulated earthquake inputs, where $W$, $Q_{\text{max}}/W$, $R_{A\text{max}}$, and $R_{D\text{max}}$, respectively, denote weight of the mass point of the specimen, the peak response

**Figure 19** Hysteresis loop without passive negative stiffness device for simulated earthquake input (Wave-L × 0.42)

**Figure 20** Hysteresis loops with passive negative stiffness device (PNSD-2) for simulated earthquake input (Wave-L × 0.42)
TABLE 3 Maximum response for simulated earthquake inputs

| Input wave (magnification) | Negative stiffness device | \( Q_{\text{max}} \) (N) | \( Q_{\text{max}}/W \) | \( A_{\text{max}} \) (m/s²) | \( D_{\text{max}} \) (mm) | \( R_{A_{\text{max}}} \) | \( R_{D_{\text{max}}} \) |
|---------------------------|--------------------------|-----------------------|----------------------|----------------------|----------------------|------------------|------------------|
| Wave-S (×0.30)            | No device                | 0.43                  | 0.085                | 0.84                 | 9.7                  | –                | –                |
|                           | PNSD-2                   | 0.40                  | 0.080                | 0.78                 | 10.3                 | 0.93             | 1.07             |
| Wave-S (×0.40)            | No device                | 0.94                  | 0.189                | 1.86                 | 23.5                 | –                | –                |
|                           | PNSD-2                   | 0.61                  | 0.122                | 1.20                 | 20.4                 | 0.65             | 0.87             |
| Wave-S (×0.48)            | No device                | 1.27                  | 0.255                | 2.50                 | 33.8                 | –                | –                |
|                           | PNSD-2                   | 0.71                  | 0.142                | 1.39                 | 34.2                 | 0.56             | 1.01             |
| Wave-L (×0.30)            | No device                | 0.52                  | 0.105                | 1.03                 | 12.2                 | –                | –                |
|                           | PNSD-2                   | 0.45                  | 0.090                | 0.88                 | 14.8                 | 0.85             | 1.22             |
| Wave-L (×0.40)            | No device                | 0.84                  | 0.168                | 1.65                 | 20.6                 | –                | –                |
|                           | PNSD-2                   | 0.67                  | 0.135                | 1.32                 | 26.6                 | 0.80             | 1.29             |
| Wave-L (×0.42)            | No device                | 0.87                  | 0.174                | 1.70                 | 22.3                 | –                | –                |
|                           | PNSD-2                   | 0.64                  | 0.129                | 1.26                 | 31.2                 | 0.74             | 1.40             |
| Mean                      |                          | 0.76                  | 1.14                 |                      |                      |                  |                  |

Notes: \( Q_{\text{max}} \): peak response shear force (inertial force). \( A_{\text{max}} \): peak response acceleration. \( D_{\text{max}} \): peak response displacement. \( W \): weight of mass point of specimen. \( Q_{\text{max}}/W \): peak response shear coefficient. \( R_{A_{\text{max}}} \): ratio of peak response acceleration (PNSD-2/no device). \( R_{D_{\text{max}}} \): ratio of peak response displacement (PNSD-2/no device).
shear coefficient, ratio of the peak response acceleration of PNSD-2 to that of no device, and ratio of the peak response displacement of PNSD-2 to that of no device. In addition, the mean for each \( R_{A_{\text{max}}} \) and \( R_{P_{\text{max}}} \) is listed in Table 3. From this table, it can be seen that the response acceleration was evidently reduced and the response displacement did not increase significantly on the average by installing the passive negative stiffness device. Furthermore, it should be noted that the peak responses shown in Table 3 were obtained for vibrating specimens without supplemental energy dissipation. Past research has shown that adding viscous damping in combination with a passive negative stiffness device can simultaneously reduce the peak displacements and accelerations of a seismically excited SDOF elastic system.11

5 | NUMERICAL SIMULATION

5.1 | Response analysis of the shake table tests

To simulate the response behavior observed in the shake table testing, a nonlinear time history response analysis was conducted. Structural analysis software SNAP (Version 7)34 was used for the analysis. A numerical SDOF system model was prepared. The model properties were given based on the specimen and results of the shake table tests of PNSD-2 (in Sections 3 and 4). The mass of the SDOF model was set to 0.509 kg.

For the restoring force characteristics of the model, a parallel combination of three nonlinear spring elements was adopted, as shown in Figure 22. The first was a bi- or trilinear elastic element with stiffness softening (Type EL3 in the software34) as shown in Figure 22c. For this spring, the initial stiffness of 0.024 N/mm \((= k_{rs} + k_{ns,1})\), the softening point force of 0.55 N, and the negative stiffness after softening of \(-0.052\) N/mm \((= k_{rs} + k_{ns,2})\) were given. The second was a bilinear elastic element with stiffness hardening (Type EL2 in the software34) as shown in Figure 22d. For this spring, the initial stiffness of 0.0001 N/mm, the hardening displacement of 31 mm for positive and \(-34\) mm for negative sides, and the stiffness after hardening of 1.02 N/mm \((= 30k_{rs})\) were given. The third was an elastic–plastic normal bilinear hysteretic element (Type BL2 in the software34) as shown in Figure 22e. For this spring, the initial stiffness of 1.7 N/mm \((= 50k_{rs})\), the yield force of 0.18 N, and the second stiffness ratio of 0 were given. This spring element was incorporated to represent the damping effect \((h = 0.097)\) of the main linear guide that was observed during testing. The integrated (net) restoring force of the SDOF system is shown as Figure 22f.

Because the energy absorption of the passive negative stiffness device was regarded as not being large, an initial stiffness proportional viscous damping with damping factor of 0.02 was assumed for the natural period at the state of the restoring members with positive stiffness \((k_{rs})\) alone. For numerical integration, Newmark-\(\beta\) method with \(\beta = 0.25\) and a sampling frequency of 1.0 kHz were used. For input motions, the observed acceleration measured on the shake table at the cases of sinusoidal wave (amplitude 12 mm) and Wave-L \((x \times 0.42)\) inputs of PNSD-2 was used.

The obtained shear force–displacement relationships of the numerical response are shown in Figure 23 for each input case. The shear force was calculated by the sum of the restoring forces by the three parallel spring elements and the assumed viscous damping force. The hysteresis loops including negative stiffness behavior of the analytical results
roughly agreed with those observed in the shake table tests, from a comparison with Figure 13d for sinusoidal wave with 12-mm amplitude and Figure 20a for Wave-L × 0.42 input.

### 5.2 Numerical evaluation of control effect for full-scale structural system

As mentioned in Section 2, the advantages of the proposed device include reduction of the response acceleration, shear force, and overturning moment. In this section, a trial seismic response analysis was carried out using a hypothetical model of a full-scale three-story building with/without a large capacity passive negative stiffness device. The target of the analysis was to understand the response control effect for the other stories when the negative stiffness device is incorporated into only a limited story of a multistory building by taking advantage of the shear force reduction performance to the story where the device is installed.

Three full-scale 3DOF numerical models were prepared. One was a mainframe (MF) model without any dampers nor negative stiffness devices. Another was a friction damped (FD) model with a damper installed at the third story of the MF model. The last was a negative stiffness (NS) model with a passive negative stiffness device incorporated into the third story of the FD model. The MF model was a linear equivalent shear 3DOF system. Properties including stiffness and viscous damping of the MF model are shown in Table 4. The natural period of the MF model was 0.39, 0.16, and 0.10 s for the first, second, and third modes, respectively. The viscous damping coefficient in each story (Table 4) was set such that initial stiffness proportional damping with damping factor of 0.02 for the first mode of the MF model was obtained. For the FD model, a normal bilinear hysteretic element with an elastic–plastic restoring force characteristic was used for the friction damper, which was parallely set at only the third story of the MF model. The initial stiffness, stiffness after yielding, and yielding force of the bilinear element were 800 MN/m, 0, and 2.5 MN, respectively. For the NS model with a passive negative stiffness device, two elastic bilinear restoring force with softening or hardening were parallely added at only the third story of the FD model. For the softening restoring force, a change of stiffness to the second negative stiffness was set to occur at a displacement of ±6 mm; and the initial and second negative stiffness were set to be −45 and −360 MN/m, respectively. For the hardening restoring force, the hardening displacement, initial stiffness, and second stiffness were set to ±40 mm, 0.01 MN/m, and 400 MN/m, respectively. It should be noted that such a large capacity negative stiffness device has not yet been designed and tested, but to grasp the control effect to a full-scale structure, this analysis using the hypothetical modeling was done.

| Story | Story height [m] | Mass (t) | Stiffness (MN/m) | Viscous damping coefficient (MNs/m) |
|-------|------------------|----------|------------------|-------------------------------------|
| 3     | 4                | 500      | 400              | 1.0                                 |
| 2     | 4                | 500      | 600              | 1.5                                 |
| 1     | 4                | 500      | 800              | 2.0                                 |

![Response shear force versus displacement obtained from analysis results for SDOF system with PNSD-2](image)

**FIGURE 23** Response shear force versus displacement obtained from analysis results for SDOF system with PNSD-2

**TABLE 4** Property of main frames of full-scale analytical models
For input motions, 10 observed earthquakes, which consisted of 1940 El Centro, 1952 Taft, 1968 Hachinohe, 1978 Tohoku, and 1995 Kobe earthquakes for each NS and EW directions, were used. Each input motion was normalized such that the peak ground velocity (PGV) was equal to 0.35 and 0.5 m/s. Velocity response spectra of the input motions can be found in a previous work. For numerical integration, Newmark- method with and a sampling frequency of 1.0 kHz were used. The same software from Section 5.1 was used for the analysis.

Comparisons of response hysteresis loops (story shear force vs. story drift) at the third story for each full-scale numerical model are shown in Figure 24 (PGV = 0.35 m/s, Taft NS) and Figure 25 (PGV = 0.5 m/s, Kobe EW). Owing to the effect of the negative stiffness device, the NS model exhibited a decreased stiffness and a smaller response shear force than those of the MF and FD models. For all 10 PGV = 0.35 m/s input motions, no peak response at the third story of the NS model reached the hardening displacement (i.e., ±40 mm). However, for the PGV = 0.5 m/s inputs, two (Tohoku EW and Kobe EW) of the 10 exceeded the hardening displacement at the third story of the NS model.

The peak responses obtained for each model were averaged for each PGV group (0.35 and 0.5 m/s) of the input motions. The mean values of the peak responses are summarized in Table 5. For the NS model, the shear force at the third story was significantly decreased because of the addition of negative stiffness, and as a result, the base shear and the overturning moment at the first story were clearly reduced in comparison with the MF and FD models. Moreover, the averaged peak story drift for the first and second stories of the NS model was reduced compared with those of the MF and FD models. From these results, if a negative stiffness device is installed at the third story only, the response drift, shear force, and overturning moment of the lower stories would be mitigated. However, an adequate lateral deformation capacity needs to be ensured at the story where the negative stiffness device is incorporated to accommodate the occurrence of a larger story drift.

**FIGURE 24** Response shear force versus story drift at the third story of 3DOF full-scale models under Taft NS (PGV = 0.35 m/s) input

**FIGURE 25** Response shear force versus story drift at the third story of 3DOF full-scale models under Kobe EW (PGV = 0.5 m/s) input
SUMMARY AND CONCLUSIONS

In the present study, a passive negative stiffness device composed of curved leaf springs using geometric nonlinearity was proposed. Shaking table tests were conducted on vibration specimens with and without the prototyped negative stiffness device and subject to both sinusoidal waveforms and simulated earthquake waves. A nonlinear time history response analysis using a SDOF system with negative stiffness was conducted to simulate the shake table testing. Also, the seismic response control effect using a hypothetical full-scale 3DOF building model was numerically evaluated. From the results of the study, the following may be concluded:

1. The negative stiffness device is capable of producing approximately constant negative stiffness over a large range of displacements, with a substantial increase in negative stiffness due to snap-through buckling prior to reaching the deformation limit of the device.

2. The width of the plates that are used to construct the curved leaf springs has little effect on the initial negative stiffness of the device while having a more significant effect on the snap-through negative stiffness.

3. As a result of the sinusoidal wave input and simulated earthquake wave input, the negative stiffness effect due to installing the passive negative stiffness device was demonstrated, and the apparent natural period of the vibration system was elongated. The elongation of the apparent natural period was most significant with the onset of snap-through buckling.

4. As a result of the simulated earthquake inputs, by installing the passive negative stiffness device, the response acceleration was reduced without significantly increasing the response displacement on average. The main reason for this was that the natural period of the vibration system without the negative stiffness device was within the constant velocity response spectrum band of the simulated earthquake wave and the apparent period increased by the installation of the negative stiffness device.

5. The passive negative stiffness device used in this study had a deformation limit, and the inertial force increased when the vibration part of the specimen contacted the cushioning material. In the absence of the cushioning material, the deformation limit of the device is expected to act as a brace for ensuring the stability of the structure by producing positive stiffness.

6. The numerical response hysteresis loops using the SDOF model with negative stiffness roughly agreed with the results obtained from the shake table tests. For the full-scale 3DOF model with a negative stiffness device installed at the third story, the base shear and overturning moment at the first story and the drift for the first and second stories were reduced compared with those for the models without negative stiffness device.

Future research tasks include establishing a theoretical model of the proposed device, achieving a high-capacity passive negative stiffness device, improvement of load–deformation relationship, and controlling of the deformation limit.

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