The method of the calculation of the average contact pressure in
the interference joints of the details of different length

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Abstract. The publication describes a universal calculation method of the average contact pressure
in the interference joint with different lengths of fitting parts. The formulas given in the article
were obtained under the condition of uniform radial displacement of the mating surface of that of
the joined parts, the deformation of which causes the concentration of contact pressure at the
mating boundaries after assembly.

The method provides sufficient accuracy for engineering calculations. It is distinguished by physic
clarity, visibility and the simplicity of usage. It can be used for design and verification of
calculations of smooth cylindrical joints with a long shaft and a long external member.

Key-words: contact pressure, interference joints, deformation, deformation wave

1. Introduction
Interference joints are used to connect parts of machines and mechanisms which work in different
conditions and for different purposes of loading and usage. The reason of that is their properties such as
simplicity of construction, compactness, the ability to transfer circular, axial forces, or the combination of
them.

The strength and bearing capacity of interference joints is determined by the value of contact pressure (q)
that occurs at the joint after assembly and depends on many factors, including the interference (δ), the
size and shape of the fitting parts to be joined, their relative position [1, 2, 8], the state of the contacting
surfaces [1, 3, 5, 6], the physicomechanical properties of the materials of the parts [1, 2, 8, 9, 10], etc.
Thereby, the improvement of the known methods and the development of new ones are important and
relevant for calculating contact pressure and estimating the bearing capacity of the considered joints.

2. Problem statement
The new universal engineering method is shown of determination of average of contact pressure in
interference joints with different lengths of fitting parts to be joined.

3. Theory
The peculiarities of macro- and microgeometry of the mating surfaces of the parts to be joined by fitting
with the interference of details lead to the fact that in real joints the contact has a discrete character [1, 4,
8]. Due to the complexity of determining the actual contact area and the actual contact pressure in
engineering practice, the calculation method based on the Lame formulas [1, 2, 7, 8] is widely used. In
accordance with this method, the contact pressure \( q_0 \) arising in the joint after assembly of cylindrical parts
of identical length is determined by the formula:
\[ q_0 = \frac{\delta}{d(C_1/E_1 + C_2/E_2)} \]

where \( C_1 = \frac{1+(d_1/d)^2}{1-(d_1/d)^2} \mu_1 \); \( C_2 = \frac{1+(d_2/d)^2}{1-(d_2/d)^2} \mu_2 \); \( E_1, E_2, \mu_1, \mu_2 \) are the modulus of elasticity and Poisson's ratio materials of the shaft and sleeve, respectively; \( d, d_1, d_2 \) are the sizes of parts to be joined (figure 1).

**Figure 1.** Interference joint: 1 is the shaft; 2 is the external member; 3 is the cantilever; 4 is the internal surface cantilever.

In real mechanisms, the parts to be joined are usually of different lengths and complex geometries. For example, at the junction of a gear or pulley with a shaft which has a greater length (figure 1, a), and at the joints of a wheel pair of the tyre and wheel center the size \( l_2 \) the external member of the size \( l_1 \) is larger the internal member of the size \( l_1 \) (figure 1, c).

The average value of contact pressure \( (q_{av}) \) in the joint with a longer shaft (figure 1, a) is calculated by the formula:

\[ q_{av} = \frac{\delta}{d(C_1/E_1 + C_2/E_2)} \]

where \( \chi \) is the parameter, influence the effect on the value of \( q \) protruding shaft ends. The value of \( \chi \) is determined from the graphs constructed for individual specific ratios of size \( d_1 \) and \( d \) of the shaft in [1, 7, 8].

When designing the interference joints, it is often necessary to know not only the average value of the contact pressure, but also to understand how a particular structural element affects its value, where the concentration zones \( q \) are located, what is the maximum value \( q \) in these zones. This publication is devoted of the solution of some of these problems.
Consider the joint shown in figure 1, a. As the results of the research in [7, 8] show, after assembly shaft 1 is deformed after assembly not only within the conjugation $AB$, but also beyond its boundaries in zones $AC$ and $BF$, the width ($L_{ac}$) of which is $(0.25 - 0.3)d$. In section $C$, the radial displacement of the outer surface of the shaft is zero, and in section $D$, it has a maximum value $U_1$, calculated using the Lame formula. When $d_i=0$ the formula for calculating $U_1$ takes the form:

$$ U_1 = \frac{q_1}{2E} (1 - \mu_t) \cdot d. \quad (2) $$

At constant pressure [7], the radial displacement of the surface of the part 1 in section $A$ is $0.5U_1$. The approximate schedule of displacement of its points on the area $CD$ is shown in figure 1, b. A similar picture takes place at the border $B$ of the joint. In the zones $AD$ and $BE$ of the joint, the concentration of contact pressure is observed, as a result its average value increases.

To deform the shaft in the zone of the $AC$, it is necessary to do work ($A_{ad}$), the value of which during the elastic deformation of the material is calculated by the formula:

$$ A_{ad} = 0.5U_{1ac} q_{aw} L_{ac} \pi d, $$

where $U_{1ac}$ is the average value of the radial displacement of the shaft surface in the deformation zone of $AC$; $q_{aw}$ is the average pressure that must be applied to this surface to ensure its radial displacement $U_{1ac}$. Accept $L_{ac} = 0.25d$. If use a linear model, then $U_{1ac} = 0.25U_1$ and $q_{aw} = 0.25q_0$.

For the possibility of deformation of the shaft in the joint in the zone of $AC$, it is necessary to create an additional force action equal to $\Delta q_1$ and uniformly distributed over the contact area. Then the work ($A_{ad}$), performed by $\Delta q_1$, can be calculated by the formula:

$$ A_{ad} = 0.5\Delta q_1 U_1 L \pi d. $$

In this formula, the conjugation size $L$ is equal to $l_2$. The value of the additional contact pressure $\Delta q_1$ is determined by the condition of equality of works $A_{ad}$ and $A_{ad}$. After the conversion, the formula for calculating $\Delta q_1$ takes the form:

$$ \Delta q_1 = \frac{q_0 d}{64L}, $$

The average value of contact pressure in the joint with a long shaft:

$$ q_{aw} = q_0 + K \cdot \Delta q_1, \quad (3) $$

where $K$ is the number of protruding shaft ends. $K = 1/2$.

After assembly, the cantilever element 3 is deformed in joint with a long external member (figure 1, c), forming a deformation half-wave with a height of $\Delta U_2$ (figure 1, d). At the same time, the concentration of contact pressure is observed at the conjugation boundary in the zone $AD$, which leads to an increase in its average value. In accordance with [8, 9]

$$ \Delta U_2 = \frac{2q_0 d L}{E (d_0^2 - d^2)}, $$

Necessary to do the work of $Ac$ to ensure the radial displacement of the surface 4 of the cantilever 3. At the elastic deformation of the material parts 2

$$ A_c = 0.5\Delta q_c U_{2ac} \pi d l_c. $$

In this formula $q_c - q_0 (1 - 0.67\Delta U_2/U_2), U_{2ac} = U_2 (1 - 0.67\Delta U_2/U_2)$. In reality, a force on the details is created only in the joint. Consequently, the additional contact pressure $\Delta q_2$ is necessary to create for to deformation the cantilever 3. At a uniform distribution of pressure over the matting surface $AB$, it the additional work $A_{ad2}$ is calculated by the formula:

$$ A_{ad2} = 0.5\Delta q_2 U_2 \pi d L. $$

In the joint under consideration, $L = l_1$. The value of $\Delta q_2$ is determined by the condition $A_c = A_{ad2}$. After conversion obtain

$$ \Delta q_2 = q_0 l_c \frac{L}{(1 - 0.67\Delta U_2/U_2)^2}. $$

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Where \( U_2 = \frac{q_0 d}{2E_s(d_2^2 - d^2)} \left[ (1 - \mu_2)h_2^2 + (1 + \mu_2)d_2^2 \right] \)

The average value of the contact pressure in the joint with a long external member:

\[ q_{av} = q_0 + K \cdot \Delta q_2, \]

where \( K \) - the number of cantilever elements in the external member.

In the joint in (Figure 1, e), the magnitude of the contact pressure is determined by the presence of a cantilever element at the external member and the protruding end of the internal member. The average value of contact pressure in this connection:

\[ q_{av} = q_0 + \Delta q_1 + \Delta q_2. \]

4. Experimental results

The correctness of the proposed solution was tested using formula (1) and the finite element method (FEM). The (Figure 2 and 3) shows the graphs of the dependence of the average value of the contact pressure on the size \( L \) of the matting surface. The main parameters of the investigated joints is \( d_1 = 0; d = 40 \text{ mm}; d_2 = 60 \text{ mm}; \delta = 0.06 \text{ mm}. \) Mechanical characteristics of materials is \( E_1 = E_2 = 2 \cdot 10^5 \text{ MPa}; \mu_1 = \mu_2 = 0.28. \) In joints with a long shaft (Figure 1, a) is \( l_1 = l_2 + 40 \text{ mm}. \) The size \( L \) of the mating surface, equal to the length \( l_2, \) varied in the interval \((1 - 0.1)d\).

\[ q_{av} \]

Figure 2. The dependence graphic of \( q_{av} \) from size \( L \) of a mating surface in the joint of a long shaft and a short external member.

In connections with a long spanning part (Figure 1, c), the size \( L, \) equal to the length \( l_1 \) of the shaft, varied in the interval \((1 - 0.25)d\), and the size \( l_2 \) of the cantilever had values of \( 0.2d \) (Figure 3, a), \( 0.15d \) (Figure 3, b), \( 0.05d \) (Figure 3, c).

On all graphs, curve 1 is plotted based on the results of calculation using the proposed method, curve 2 using formula (1) (Figure 2) and FEM (Figure 3). In addition, for comparison, the values of \( q_0 \) are given (line 3).

5. Results discussion

Graphs in figure 2 show that with a joint length \( L \geq 0.3d, \) the traditional method according to formula (1) and the method of calculating the average value \( q \) proposed by formula (3) give very close results. Their divergence increases with decreasing size \( L. \) So at \( L = 0.2d, \) it is 6\%, and at \( L = 0.1d \) it increases to 16\%.
Figure 3. The dependence graphic of $q_{ag}$ from size $L$ of a mating surface in the shaft joint with longer external member: a – at $l_c=0.2d$; b – at $l_c=0.15d$; c – at $l_c=0.05d$. 
This indicates that the adopted linear dependence of the radial displacement of the shaft surface on the position of the cross section in the zone of deformation $AC$ at a small length $L$ of conjugation gives a rough approximation. If the graph of radial displacement of the shaft surface in the zone of deformation of the $AC$ is approximated by a polygonal line $AMC$ (Figure 1, b), then the formula for calculating $\Delta q_1$ at $d_1 = 0$ takes the form

$$\Delta q_1 = \frac{0.009q_0d_0}{L}.$$  

The results of the calculation of the $q_{av}$ value by the refined formula (5) are shown in figure 2 (curve 4). Graphs in figure 3 show that with the length $l_c$ of the cantilever commensurate with the parameter $L$ of the geometric mating surface, the divergence between the calculation results obtained by different methods can be significant. So at $l_c/L = 0.8$ it was 20%. In real construction, this ratio sizes is unlikely. In the remaining variants, the discrepancy was (0.45 ... 7.6)%, which is acceptable for engineering calculations.

**6. Conclusions**

The results of the study indicate the reliability of the proposed method of calculating the average value of contact pressure in the interference joints of parts of different lengths. The proposed method is simple and visibility. It allows to quantify the influence of different the constructive features of the interference joints of the parts on the value of contact pressure and the carrying capacity of the joint. Within the framework of the adopted calculated model, the proposed method makes it possible to identify concentration $q$ and determine the type of its distribution in these zones.

The method is universal. It can be used in calculating both smooth and interference joints, as well as joints with a modified mating surface.

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