A Family of Rank Similarity Measures based on Maximized Effectiveness Difference

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Abstract—Rank similarity measures provide a method for quantifying differences between search engine results without the need for relevance judgments. For example, the providers of a search service might use such measures to estimate the impact of a proposed algorithmic change across a large number of queries — perhaps millions — identifying those queries where the impact is greatest. In this paper, we propose and validate a family of rank similarity measures, each derived from an associated effectiveness measure. Each member of the family is based on the maximization of effectiveness difference under this associated measure. Computing this maximized effectiveness difference (MED) requires the solution of an optimization problem that varies in difficulty, depending on the associated measure. We present solutions for several standard effectiveness measures, including nDCG, MAP, and ERR. Through an experimental validation, we show that MED reveals meaningful differences between retrieval runs. Mathematically, MED is a metric, regardless of the associated measure. Prior work has established a number of other desiderata for rank similarity in the context of search, and we demonstrate that MED satisfies these requirements. Unlike previous proposals, MED allows us to directly translate assumptions about user behavior from any established effectiveness measure to create a corresponding rank similarity measure. In addition, MED cleanly accommodates partial relevance judgments, and if complete relevance information is available, it reduces to a simple difference between effectiveness values.

Index Terms—Search, rank similarity, information retrieval, effectiveness measures, search engines

I. INTRODUCTION

EVEN minor modifications to ranking algorithms, training data, or document collections may cause unpredictable changes to the quality of search engine results. For any given query, documents may move up and down the ranked list. New documents may appear; others may disappear.

Traditionally, changes in search engine results are quantified by way of retrieval effectiveness measures, such as normalized discounted cumulative gain (nDCG), mean average precision (MAP), and expected reciprocal rank (ERR) [1], [2]. These measures are computed over ranked lists before and after a modification. The difference in their values — typically averaged over a number of queries — indicates the magnitude of the change.

Unfortunately, the computation of these measures depends on the existence of explicit relevance judgments. For a given query, we must know whether or not each ranked document is relevant to the query, and for some measures we must also know the degree to which each document is relevant to the query (e.g., definitive, excellent, good, fair, bad, or detrimental [3]). These judgments may be created by direct human assessment of relevance or inferred indirectly from clickthrough logs and other user interaction data [4]–[9]. Creation of these judgments involves either substantial effort on the part of assessors or large volumes of interaction data, limiting the number of queries over which the measures may be computed. Moreover, when modifications surface previously unseen and unjudged documents, these documents must be assessed immediately, or the accuracy of the effectiveness measures may be compromised.

Alternatively, changes in search results may be quantified by way of rank correlation coefficients and other rank similarity measures, avoiding the need for explicit relevance judgments. For example, the providers of a search service might use such measures to estimate the impact of a proposed change across large numbers of queries — perhaps millions of them — identifying those where the potential impact is greatest. While standard rank correlation coefficients — such as Spearman’s ρ or Kendall’s τ — might be applied for this purpose, several researchers have proposed specialized rank similarity measures that better reflect the requirements of search.

Some of this prior work focuses primarily on the problem of comparing search result lists [10]–[13], a problem which we directly address in this paper. Most notably, Webber et al. [14] carefully analyze the requirements for comparing ranked lists across a wide range of applications, including search results. Other work focuses primarily on the distinct, but related, problem of comparing system rankings under different effectiveness measures [15], [16], a problem which we do not directly address in this paper. We note, however, that progress on one problem may translate into progress on the other.

Webber et al. identify three key characteristics that particularly apply to comparisons between search results. First, users tend to focus on the top-ranked results, and more rarely view results deep in the list [9], [17]. Thus, a rank similarity measure for search results must be top-weighted, placing greater emphasis on early results, and lesser emphasis on later results. A change in the third result should have greater impact than a change in the 103rd. Second, for a given query, search engines do not typically rank more than a tiny fraction of the collection. Thus, a similarity measure for search results must handle incomplete rankings, where a document appearing in one list may not appear in the other.

Finally, since users may stop at any point in a ranked list, either because their information need is satisfied or because their patience is exhausted, a rank similarity measure for search results must appropriately handle indefinite rankings. According to Webber et al. the measure “should not arbitrarily
assign a cutoff depth, but be consistent for whatever depth is available.” Suppose the ranked lists are truncated at an arbitrary depth $K$ and we compute the measure at that depth. The value of the measure at $K$ should allow us to make predictions about the value of the measure if it were computed at depths greater than $K$, allowing us to compute bounds on the amount it can change as the depth of computation is increased. For example, perhaps the measure would only increase, or stay the same, as the depth increased (i.e., adding more documents could not make the lists less similar).

To satisfy the requirements implied by these three characteristics, Webber et al. proposed a new rank similarity measure, which they call rank biased overlap (RBO). Given two ranked lists, $A$ and $B$, let $A_{1:k}$ denote the top $k$ documents in $A$, and let $B_{1:k}$ denote the top $k$ documents in $B$. Define the overlap between $A$ and $B$ at depth $k$ as the size of the intersection between these lists at depth $k$ (i.e., $|A_{1:k} \cap B_{1:k}|$) and define the agreement between $A$ and $B$ at depth $k$ as the overlap divided by the depth. Webber et al. define RBO as a weighted average of agreement across depths, where the weights decay geometrically with depth, reflecting the requirement for top weighting:

$$\text{RBO} = (1 - \psi) \sum_{k=1}^{\infty} \psi^{k-1} \frac{|A_{1:k} \cap B_{1:k}|}{k}.$$  

In this equation, the parameter $0 < \psi < 1$ represents user persistence, with larger values representing a more patient user. When computing RBO for the comparative experiments reported in this paper, we set $\psi = 0.9$, a typical choice. The normalization factor $(1 - \psi)$ serves to map the value of RBO into the range $[0 : 1]$. In practice, RBO is computed down to some fixed depth $K$, which replaces $\infty$ in the equation, reflecting the indefinite and incomplete nature of the ranked lists.

Webber et al. surveyed a large number of rank similarity measures, and argued that only RBO fully satisfies the requirements of ranked search results. In creating RBO, Webber et al. drew inspiration from the user model incorporated into the rank-bias precision (RBP) effectiveness measure proposed by Moffat and Zobel [18]. This model imagines a user scanning a ranked list, starting at the top. After scanning a result, the user proceeds to the next result with probability $\psi$ and stops with probability $1 - \psi$. This simple user model provides the basis for the weights appearing in both RBP and RBO.

This close connection with the RBP effectiveness measure helps to justify the application of RBO to search results. In general, given the enormous effort made by the search community to develop and validate effectiveness measures, it seems reasonable to exploit this existing work to guide the creation of rank similarity measures targeted at search results. Taking this observation a step further, we propose a family of distance measures, each directly derivable from an associated effectiveness measure. The core idea is straightforward [19–21]:

Given two ranked lists, $A$ and $B$, what is the maximum difference in their effectiveness scores possible under a specified effectiveness measure? We call this family of distance measures Maximized Effectiveness Difference (MED). Since we typically normalize MED into the range $[0 : 1]$, the value $1 - \text{MED}$ provides an corresponding family of rank similarity measures.

Like RBO, MED is a metric in the strict mathematical sense of defining a distance between two ranked lists (see Section II). The value of this distance varies with the associated effectiveness measure. Unlike most similarity measures, MED is not a dimensionless quantity, since its value can be interpreted in terms of the associated effectiveness measure.

Like RBO, MED transfers understanding and assumptions about user behavior from an existing effectiveness measure to measure rank similarity. Unlike RBO, MED provides a method for transforming any effectiveness measure into a rank similarity measure. Both MED and RBO derive their approach to top weighting from their associated effectiveness measure, along with an ability to handle incomplete rankings, satisfying the requirements of search. MED is monotonic with increasing depth of evaluation, satisfying the requirements for indefinite rankings. In addition, MED can take appropriate advantage of known relevance information (see Section II). It is straightforward to show, from its definition, that introducing known relevance information always decreases the MED distance between two ranked lists. Moreover, if complete relevance information is known, MED reduces to a difference in effectiveness scores.

For an effectiveness measure normalized to the range $[0 : 1]$, the value of MED will also fall into the range $[0 : 1]$. As we discuss later in the paper, for some effectiveness measures this normalization requires knowledge of collection parameters, e.g., the total number of relevant documents. Since such information cannot be known without complete relevance information, we choose values for these parameters that normalize MED into the range $[0 : 1]$.

To compute MED, we must determine an assignment of relevance values to the documents appearing in the two lists that maximizes the difference in their scores under the specified effectiveness measure, an optimization process that will vary with the effectiveness measure. For some measures, such as nDCG and RBP, the computation of MED is straightforward (see Section III). For other measures, including MAP and ERR, the computation of MED requires the solution of more difficult optimization problems (see Sections VI and VII). Either way, MED reflects meaningful differences between ranked lists (see Section VII) and provides substantial advantages over other rank similarity measures on search results (see Section VIII). Finally, we consider how to extend MED beyond ranked lists, to more other effectiveness measures (see Section VIII).

II. Basic Notation and Properties

We compute MED by maximizing the effectiveness difference between two ranked lists, $A$ and $B$. If $S(A)$ is the score for list $A$ under some effectiveness measure, and $S(B)$ is the score of list $B$ under the same measure, then we seek to assign relevance values to the documents in $A$ and $B$ to maximize

$$\text{MED}(A, B) = |S(A) - S(B)|.$$  

(2)
In the remainder of the paper, we assume that scores produced by effectiveness measures are always non-negative (and we are not aware of any established effectiveness measure that can produce negative values). Thus, without loss of generality, we present algorithms that maximize
\[ S(A) - S(B). \]  
(3)
The overall maximum can be computed by swapping the lists and reapplying the algorithm.

For optimization purposes, we represent \( A \) as a vector of variables
\[ A = \langle a_1, a_2, ..., a_K \rangle, \]  
(4)
where \( a_i, 1 \leq i \leq K \), represents the relevance value assigned to the document at rank \( i \) in list \( A \). Similarly, we represent \( B \) as a vector of variables
\[ B = \langle b_1, b_2, ..., b_K \rangle, \]  
(5)
where \( b_j, 1 \leq j \leq K \), represents the relevance value assigned to the document at rank \( j \) in list \( B \). Since a document may appear in both lists, we also have a set of constraints in the form
\[ a_n = b_m, \]  
(6)
indicating that the same document appears at rank \( n \) in list \( A \) and at rank \( m \) in list \( B \), so that the same relevance value must be assigned to both variables. Since a document will appear at most once in each list, a given variable will appear in at most one constraint.

We refer to a variable that appears in a constraint as a bound variable. If a variable does not appear in any constraint — corresponding to a document that appears in only one list — we refer to it as a free variable. In the case that relevance information for a document is known from existing judgments, we assign this known value to its corresponding variable(s), which remain unchanged during maximization. We refer to these variables as predetermined variables.

For all effectiveness measures we consider in this paper, a relevance value is a number in the range \([0 : 1]\). For some measures — such as MAP — relevance is a binary property of a document, and relevance values are either 0 or 1. For graded relevance measures — such as nD\textsubscript{CG} and ERR — relevance can be one of several values from 0 up to a maximum grade \( r_G \) (i.e., the possible relevance grades are \( 0 = r_0 < r_1 < r_2 < ... < r_G \leq 1 \)). These grades indicate the level to which a document is judged relevant to the query (e.g., definitive, excellent, good, fair, etc.). For simplicity, we treat binary relevance as a special case of graded relevance, with two grades: \( r_0 = 0 \) and \( r_G = 1 \). To aid understanding, a relevance value may be interpreted as the probability that a user viewing the corresponding document will consider it to be relevant \([2, 22, 23]\), but this interpretation is not explicitly required in this paper.

Mathematically, MED is a metric, regardless of the associated effectiveness measure. Non-negativity, identity and symmetry are straightforward. To demonstrate the triangle inequality, consider three ranked lists \( A, B, \) and \( C \). Let \( A' \) and \( B' \) represent the assignment of relevance values that maximizes the effectiveness difference between \( A \) and \( B \), i.e., \( \text{MED}(A, B) = |S(A') - S(B')| \). Let \( C' \) be any assignment of relevance values to \( C \) that is consistent with \( A' \) and \( B' \), such that documents are assigned the same relevance values in all three lists. Let \( A'' \) and \( B'' \) represent the assignment of relevance values that maximizes the effectiveness difference between \( A \) and \( C \). Let \( B''' \) and \( C''' \) represent the assignment of relevance values that maximizes the effectiveness difference between \( B \) and \( C \). Now,
\[ \text{MED}(A, B) = |S(A') - S(B')| \]  
(7)
\[ = |S(A') - S(C') + S(C') - S(B')| \]
\[ \leq |S(A') - S(C')| + |S(C') - S(B')| \]
\[ \leq |S(A'') - S(C''')| + |S(C'''') - S(B''')| \]
\[ = \text{MED}(A, C) + \text{MED}(C, B). \]

The second-last step holds by the definition of MED, as maximizing effectiveness difference.

The degree to which MED satisfies our requirements with respect to top-weighting, indefinite rankings, and incomplete rankings, depends upon the associated effectiveness measure. While we are not aware of any established effectiveness measure that cannot handle incomplete rankings, a few measures — such as precision@\(k\) — are not top weighted. Moreover, several effectiveness measures — such as precision@\(k\) and nD\textsubscript{CG}@\(k\) — are parameterized by a depth \( k \). Changing the value of \( k \) effectively creates a new measure. For such effectiveness measures, extending the ranked lists beyond \( k \) does not change the value of the measure, or the value of MED. For these measures, MED appropriately supports indefinite rankings only to depths less than or equal to \( k \). When \( K < k \), we compute MED by filling ranks \( K + 1 \) to \( k \) with free variables in both lists. Other measures — such as RBP and ERR — are not parameterized by depth and are notionally computed to infinity, providing stronger support for indefinite rankings.

III. Simple Dot Product Measures

In this section, we maximize Equation\(3\) for a class of simple but widely used effectiveness measures. These measures may be expressed as a normalized dot product between a vector of relevance values and a vector of rank-based discount values. The class includes RBP and nD\textsubscript{CG}, along with other measures \([24]\).

Let \( C \) be a ranked list (which could be either \( A \) or \( B \)) represented by a vector \( C = (c_1, c_2, ...) \) of relevance values. Let \( D = (d_1, d_2, ...) \) be a vector of discount values, where the value of \( d_i \) depends only on the rank, \( i \). Effectiveness is computed as the normalized dot product of \( C \) and \( D \):
\[ S(C) = \frac{C \cdot D}{N}, \]  
(8)
where the normalization factor \( N \) is a constant, which may depend on assumptions about the user or on characteristics of the document collection, such as the number of relevant documents it contains at each relevance grade. Generally, normalization serves to map the value of the measure into the range \([0 : 1]\). To compute MED, we require \( N > 0 \) (and we
are not aware of any established effectiveness measure where this requirement does not hold).

Each \( d_i \) in the discount vector may be interpreted as the probability that a user scanning the search results will reach rank \( i \), and hence view the document at that rank \([2], [18], [23]\), but this interpretation is not explicitly required in this paper. To compute MED, we require discount values to be non-negative and to decrease monotonically with increasing rank, i.e., \( d_i \geq d_j \) if \( i < j \) (and we are not aware of any established effectiveness measures where this requirement does not hold).

Maximizing Equation \([3]\) for simple dot product measures is straightforward. First, we set all predetermined variables in both lists to their known values. Second, we set all free variables in \( A \) to \( r_G \) and all free variables in \( B \) to \( r_0 = 0 \). If \( S(A) - S(B) \) is maximized, then all free variables in \( A \) must have value \( r_G \), for otherwise we could increase \( S(A) - S(B) \) by increasing the value of these free variables. Similarly, if \( S(A) - S(B) \) is maximized, then all free variables in \( B \) must have value 0.

Finally, given a constraint \( a_n \equiv b_m \), the contribution of these variables to \( S(A) - S(B) \) is

\[
\frac{a_n d_n - b_m d_m}{N}.
\]

Since discount decreases monotonically with increasing rank, \( S(A) - S(B) \) is maximized by setting \( a_n = b_m = r_G \) if \( n < m \), or \( a_n = b_m = 0 \) if \( n > m \). If \( n = m \) then these variables contribute nothing to the value of \( S(A) - S(B) \) and can be set to any value.

For example, we may express precision@\( k \) as a simple dot product measure using a vector of binary relevance values, a normalization factor of \( k \), and a discount with the first \( k \) values set to one and the remaining values set to 0:

\[
D = \langle d_1 = 1, d_2 = 1, \ldots, d_k = 1, d_{k+1} = 0, \ldots \rangle.
\]

In the case that the ranked lists are not fully specified to depth \( k \), we complete them with arbitrarily chosen free variables. Applying the relevance assignment process above, we see that MED for the precision@\( k \) effectiveness measure (which we call MED-precision@\( k \)) is just one minus the overlap between \( A \) and \( B \) at depth \( k \):

\[
1 - \frac{|A_{1:k} \cap B_{1:k}|}{k}.
\]

If there are no predetermined variables, then for simple dot measures it is not hard to show that maximizing \( S(B) - S(A) \) produces the same value as maximizing \( S(A) - S(B) \). In this case, there is no need to swap lists and recompute.

A. Computing MED-RBP

Moffat and Zobel \([18]\) define the formula for computing rank biased precision (RBP) as:

\[
S(C) = (1 - \psi) \sum_{i=1}^{\infty} c_i \psi^{i-1},
\]

where \( C = (c_1, c_2, \ldots) \) is a vector of graded relevance values, although the definition works equally well for binary values. For the experiments reported in this paper, we assume \( r_G = 1 \).

As it does in RBO, the parameter \( 0 < \psi < 1 \) represents user persistence, with larger values representing a more patient user. We may easily express this formula in the form of Equation \([8]\) making it straightforward to compute MED for the RBP measure (MED-RBP). When computing MED-RBP for the experiments reported in this paper, we set \( \psi = 0.9 \), a typical choice.

MED-RBP provides strong properties in support of indefinite rankings. For ranked lists specified only to depth \( K \), we compute MED-RBP down to infinite depth by assuming arbitrary free variables in both lists below \( K \). To maximize \( S(A) - S(B) \), in list \( A \) we set these free variables to 1, and in list \( B \) we set these free variable to 0, so that we maximize:

\[
S(A) - S(B) = (1 - \psi) \left( \sum_{i=1}^{K} (a_i - b_i) \psi^{i-1} + \sum_{i=K+1}^{\infty} \psi^{i-1} \right)
\]

\[
= (1 - \psi) \left( \sum_{i=1}^{K} (a_i - b_i) \psi^{i-1} \right) + \psi^K. \tag{13}
\]

If the ranked lists are later specified to greater depth, increasing \( K \) and potentially introducing new bound and predetermined variables, MED cannot increase. Moreover, the MED-RBP distance cannot decrease by more than \( 2\psi^K \) as the depth goes to infinity.

B. Computing MED-nDCG

At the time Järvelin and Kekäläinen \([1]\) created normalized discounted cumulative gain (nDCG) no other established effectiveness measure accommodated graded relevance values. Since then, nDCG has become widely used for Web-related research. In this paper, we work with a version of nDCG that has become standard in the research literature and through industry practice \([2], [9]\):

\[
S(C) = \left( \frac{1}{\text{ideal DCG}} \right) \sum_{i=1}^{k} \frac{c_i}{\log (i+1)}. \tag{14}
\]

\( C = (c_1, c_2, \ldots) \) is a vector of relevance values, where each \( c_i = 1 \) if \( r_0 \ldots r_G \). When computing nDCG for our experiments, we set \( k = 20 \), a typical choice.

For nDCG@\( k \), relevance values are computed using the formula \([2]\):

\[
r_j = \frac{2^j - 1}{2G}, \quad j = 0, \ldots, G. \tag{15}
\]

For our experiments, the test collection employs three relevance grades, so that \( r_0 = 0, r_1 = 1/4 \), and \( r_2 = r_G = 3/4 \). These grades indicate documents that are judged to be non-relevant, relevant, and highly relevant, respectively.

We may express Equation \([14]\) in the form of Equation \([8]\) with a discount vector of

\[
D = (1, \ldots, 1/\log (i+1), \ldots). \tag{16}
\]

Järvelin and Kekäläinen call the dot product \( C \cdot D \) discounted cumulative gain (DCG). DCG is then normalized by ideal DCG to give nDCG.

Ideal DCG is a constant, defined as the maximum DCG achievable over the collection, which can be determined by
ranking all the most relevant documents first, followed by the next most relevant, and so on. In theory, determining ideal DCG may require exhaustive judging, which is rarely feasible on realistically sized collections. In practice, ideal DCG is usually estimated from the known relevant documents surfaced during an retrieval experiment. If changes to ranking algorithms surface new relevant documents, the estimate of ideal DCG may grow and the value of nDCG may drop, even when the changes improve the algorithm. When applying nDCG as an effectiveness measure, care must be taken to account for this potential growth in ideal DCG.

When computing MED for the nDCG measure (MED-nDCG), we may have no judgments at all. Fortunately this can be avoided to some extent. The definition of MED

\[ \text{理想 nDCG} = \mathcal{N} = \sum_{i=1}^{k} \frac{r_{C_i}}{\log (i + 1)} \]  

Using this normalization factor, the value of MED-nDCG will fall in the range \([0 : 1]\).

Admittedly, we do lose one property of MED with this approach. The calculation of nDCG requires the estimation of a constant that depends solely on the collection, not on the relevant documents being measured. With no relevance judgments available, we have no estimate for the value of this constant. By normalizing it away, as we do above, we lose the property that MED can be interpreted as an actual maximum difference in effectiveness values. MED-nDCG continues to be a metric in the mathematical sense, able to measure distances between rankings, but these distances are scaled by an unknown constant. In this regard, MED-nDCG is no different than existing correlation coefficients, including RBO, which are also dimensionless quantities.

Alternatively, for some queries we may know, or be able to reasonably guess, properties of the collection that would impact normalization. For example, we may know through the application of a query type classifier that a given query is navigational, and we may know from experience that a navigational query typically has one highly relevant document and a small number — no more than a dozen, say — of marginally relevant documents. Under these assumptions the computation of MED-nDCG would require a different normalization constant and additional constraints on the optimization, limiting the values of variables. We leave the exploration of this idea for future work.

IV. COMPUTING MED-MAP

Mean average precision (MAP) is defined over a vector \( C = (c_1, c_2, ...) \) of binary relevance values as:

\[ S(C) = \frac{1}{R} \sum_{i=1}^{k} c_i \cdot \text{precision}@i = \frac{1}{R} \sum_{i=1}^{k} \sum_{j=1}^{c_i} c_j, \]  

where \( R \) indicates the number of relevant documents in the collection, and \( k \) an arbitrary maximum depth for computation. Although it is rarely made explicit in the research literature as MAP@\( k \), this maximum depth is as important for MAP as it is for precision@\( k \) and nDCG@\( k \). Changing \( k \) effectively creates a different measure. When computing MED for our experiments, we set \( k = 100 \), a typical choice.

Like ideal nDCG, determining \( R \) theoretically requires exhaustive judging. However, as we did for ideal nDCG, when computing MED for the MAP measure (MED-MAP) we assume that the collection contains an arbitrary number of relevant documents. To compute MED-MAP, we replace \( R \) with \( k \) in Equation [18], which guarantees that the value of MED-MAP will fall in the range \([0 : 1]\):

\[ S(C) = \frac{1}{k} \sum_{i=1}^{k} c_i \sum_{j=1}^{c_i} c_j. \]  

Unfortunately, Equation [19] is quadratic in its relevance values and does not fit the simple dot product form assumed in Section [III]. Maximizing \( S(A) - S(B) \) requires a little more effort than it does for those measures. Fortunately, this maximization problem can be re-expressed as a quadratic 0-1 optimization problem, a heavily researched and well understood class of problems [25], [26]. Our goal is to assign relevance values to the documents in lists \( A \) and \( B \) that maximizes

\[ S(A) - S(B) = \frac{1}{k} \left( \sum_{i=1}^{k} a_i \sum_{j=1}^{i} a_j - \sum_{i=1}^{k} b_i \sum_{j=1}^{i} b_j \right). \]  

If \( S(A) - S(B) \) is maximized, it is not hard to show that the free variables in \( A \) must be set to one, and the free variables in \( B \) must be set to zero. Of course, predetermined variables must be set to their known values.

After setting the values for free and predetermined variables, our next step is to replace each pair of variables appearing in a constraint with a single variable. To do this, we create a combined variable vector \( Z = (z_1, z_2, ..., z_k) \), where \( k \leq k' \) is the number of constraints. Each document appearing in both \( A \) and \( B \) corresponds to a single variable in \( Z \). The ordering of \( Z \) is arbitrary. Equation [20] can now be re-written in the form

\[ Z^T Q Z + L^T Z + F, \]  

where \( Q \) is a matrix of order \( k' \), \( L \) is a vector of dimension \( k' \), and \( F \) is a constant. Equation [21] is the standard form for quadratic 0-1 optimization, a heavily studied NP-complete problem. Quadratic 0-1 optimization is equivalent to the weighted max-cut problem, one of Karp’s original 21 NP-complete problems.

To approximate MED-MAP, we implemented a version of tabu search [26], a standard local search method that creates and maintains a set of disallowed (or tabu) moves to avoid repeated visits to suboptimal solutions. Unfortunately — while the computation of MED for dot product measures is essentially instantaneous — the computation of MED-MAP may require a second or so on a typical desktop machine.

V. COMPUTING MED-ERR

Expected reciprocal rank (ERR) is based on the cascade model of user browsing behavior over search results [2], [22].
The model implicitly assumes that the user is seeking a single relevant document. After entering a search query and receiving a result list, the user scans the list, starting at the first result. With probability \( c_1 \), the user finds the information she seeks and stops browsing. Otherwise, with probability \( 1 - c_1 \), she continues to the second result, and so on. In general, if the user reaches the result at rank \( i \), she finds the information she seeks with probability \( c_i \), or continues on to the result at rank \( i + 1 \) with probability \( 1 - c_i \). Thus, the probability that the user reaches rank \( i \) is

\[
\prod_{j=1}^{i-1} (1 - c_j).
\] (22)

ERR is then defined as the expected reciprocal rank where the user’s information need is satisfied:

\[
S(C) = \sum_{i=1}^{\infty} c_i \prod_{j=1}^{i-1} (1 - c_j).
\] (23)

ERR uses the same relevance grades as nDCG, as defined by Equation [15]. ERR is not normalized; its value naturally falls into the range \([0 : 1]\) (with a maximum value \(< 1.0\)).

The cascade model is closely related to the user model incorporated into RBO and RBP, as described in the introduction. For RBO and RBP, the probability that the user will move from rank \( i \) to \( i + 1 \) is constant (\( \psi \)). Under the cascade model, this inter-rank transition probability depends on the relevance of the document at rank \( i \), and the probability that the user will reach rank \( i \) depends on the relevance of all the documents appearing before it. If a few highly relevant documents appear above rank \( i \), the probability of reaching that rank becomes relatively small. For example, if \( r_G = 3/4 \), it takes only four such documents for the probability in Equation [22] to drop below 1%. After viewing five such documents, less than one in a thousand users will continue.

Under ERR, if \( S(A) - S(B) \) is maximized, it is possible to show that free variables in \( A \) must be set to \( r_G \) and free variables in \( B \) must be set to \( r_0 = 0 \). Moreover, if \( S(A) - S(B) \) is maximized, it is possible to show that bound variables must either be set to \( r_G \) or 0, apart from cases where the value of the variable does not change the value of the difference. In these cases, we can arbitrarily set the variables to \( r_G \) or 0. We omit proofs for these claims, which are straightforward but tedious.

Thus, in maximizing \( S(A) - S(B) \) we set free and bound variables to either \( r_G \) or 0, allowing us to ignore intermediate relevance grades. Unfortunately, maximizing \( S(A) - S(B) \) still requires us to solve a highly non-linear optimization problem, with what are effectively 0-1 constraints. Fortunately, we can take advantage of properties of ERR to efficiently approximate the solution. In particular, as noted previously, the value of ERR largely depends on the position of the first few relevant documents. As the user views more and more relevant documents, the probability she will continue to lower ranks drops exponentially.

Consider the calculation of \( S(A) \) for some assignment to the variables in \( A \). We start calculating the summation at rank \( i = 1 \). Suppose at rank \( k \) we have encountered \( p \) variables with value \( r_G \). The sum over the remaining ranks is bounded by

\[
\epsilon = \sum_{i=k+1}^{\infty} \frac{c_i}{i} \prod_{j=1}^{i-1} (1 - c_j)
\] (24)

\[
\leq \sum_{i=p+1}^{\infty} r_G^{i-1} \prod_{j=1}^{i-1} (1 - r_G)
\]

\[
= (1 - r_G)^p \sum_{i=p+1}^{\infty} \frac{r_G}{i} \prod_{j=p+1}^{i-1} (1 - r_G)
\]

\[
< \frac{r_G (1 - r_G)^p}{p+1} \sum_{i=0}^{\infty} (1 - r_G)^i
\]

\[
= \frac{(1 - r_G)^p}{p+1}.
\]

If \( p = 5 \) and \( r_G = 3/4 \) then \( \epsilon < 0.0002 \). This bound on \( S(A) \) is also a bound on \( S(A) - S(B) \), since setting variables in \( B \) to \( r_G \) increases \( S(B) \) and decreases the difference.

Thus, to approximate MED-ERR, we adopt a brute force approach, trying all combinations of up to \( p = 5 \) bound variables in \( A \) down to depth 30. We set each combination to the value \( r_G \), compute the value of ERR, and take the maximum across the combinations. This brute force approximation requires a dozen milliseconds or so on a typical desktop machine.

VI. VALIDATION

To validate MED, we employ a set of experimental runs submitted to the TREC 2005 Robust Retrieval Track [27]. As our primary goal, we hope to demonstrate that MED reflects meaningful differences between retrieval runs. The TREC Robust Track provides a particularly appropriate dataset for this purpose because its retrieval topics were chosen for their anticipated difficulty, with many standard retrieval algorithms performing poorly on them. As a result, track participants applied an unusually wide variety of retrieval methods, including some entirely novel methods, particularly in the area of query expansion.

A total of 17 groups participated in the track submitting a total of 74 runs. We start by examining differences between runs submitted by the same group. If we assume that each group used a core retrieval approach across all its runs, we hope to interpret MED values in terms of meaningful changes between these runs. Among the groups who submitted multiple runs, we selected the eight groups with the best overall performance. While we now focus our attention on these eight groups, we note that results for the excluded groups exhibit consistent behavior, with the general observations we make below applying to these groups as well.

We computed MED-nDCG between all pairs of runs within each group, averaging across the 50 topics used in the track. Figure [1] shows the results, with one chart for each group, appearing in alphabetical order. Within each chart, pairs are ordered by increasing MED distance. Each bar corresponds to
Fig. 1. Intra-group MED-nDCG@20 values for selected TREC 2005 Robust Retrieval Track participants.
a single pair of runs, with the value of MED-nDCG given on the y-axis.

Each run is identified by a code indicating the type of information used to automatically formulate the query. Like many TREC tasks, retrieval topics for the Robust Track provide multiple expressions of the associated information need, with each topic including:

- a title field (T), providing one to three keywords expressing the information need,
- a description field (D), providing a single sentence expressing the information need, and
- a narrative (N), providing a longer expression of the information need.

Run codes indicate the set of fields used for query formulation. For example, the codes TD1 and TD2 in the Arizona State University chart both indicate runs where the title and description were used for query formulation. Within each set of codes, numbers are assigned arbitrarily. In addition, IBM submitted a single manual run (M1) in which the query formulation process included human assistance.

We indicate the types of queries in each pair using unique colors and shading. For example, title vs. title runs are colored in strawberry, description vs. description runs are colored in lime, and title vs. description runs are colored in tangerine. The charts for the Chinese Academy of Science, Hummingbird, Illinois, and Massachusetts contain only pairs of these types. An interesting trend is apparent in these four charts, with title vs. title pairs and description vs. description pairs being noticeably closer than title vs. description pairs.

Digging deeper, we consider the runs from Arizona State University. From the chart, we see that TD1, TD2, and D1 are all several times closer to each other than they are to the other two runs. These two runs, T1 and TD3, are closer to each other than they are to any of the other runs. The group’s workshop report provides an explanation [28]. Reading their report, we learn that T1 and TD3 used the same query expansion model, which differed from the expansion model used by the other runs. Digging to the other groups reveals similar relationships reflected in MED-nDCG. For example, as shown in the chart from IBM, manual runs tend to be farther from automatic runs than automatic runs are from each other.

The computation of MED does not depend on relevance judgments. However, we would hope that MED distance provides some indication about actual effectiveness differences. Figure 2(a) plots (the absolute value of) actual nDCG@20 differences against MED-nDCG@20 distances across all pairs for the groups listed in Figure 1. The plot includes both intra-group pairs and inter-group pairs, 703 pairs in total. The plot
shows a clear correlation, although the MED-nDCG distances have much higher values.

As relevance judgments become available, creating predetermined variables, MED-nDCG distances become increasingly correlated with actual nDCG differences, moving closer and closer to the actual differences. Figure 2(b) plots actual nDCG differences against MED-nDCG after we set variables to predetermined values from the using 25% of available TREC relevance judgments, randomly selected. Figure 2(c) provides an equivalent plot for 75% of available TREC judgments, and Figure 2(d) provides an equivalent plot for 100% of available TREC judgments. In this last plot, all of the MED-nDCG distances do not match actual differences because TREC 2005 Robust Track runs were not fully judged down to depth 20, and TREC assumes unjudged documents to be non-relevant.

Figure 3 provides comparisons between RBO, MED-nDCG, MED-RBP, MED-MAP and MED-ERR. In all plots, most of the points fall toward the upper right, indicating larger differences in the pairs. The pairs appearing towards the lower left are generally from the same groups, using the similar retrieval techniques.

Given that MED-RBP and RBO share a common user model, we would expect a high correlation between them. Figure 3(a) shows the correlation. The strength of this correlation suggests that other variants of MED may similarly reflect the user models underlying those measures. Interestingly, MED-RBP is even more highly correlated with MED-nDCG@20, as shown in Figure 3(b). This correlation is related to the choice of persistence parameter $\psi = 0.9$. Lower and higher values for $\psi$ produce weaker correlations. Figures 3(c) and 3(f) show how values of MED-nDCG change for different values of $k$ (5, 20, and 100).

### VII. Comparison with Prior Work

As discussed in the introduction, the RBO similarity measure of Webber et al. [14] directly inspired our efforts. As part of that paper, Webber et al. provide a substantial review of related research up to early 2010, when the final version of their paper was submitted and accepted. We encourage readers who are interested in a thorough analysis of this prior work to consult that paper. In this section, we touch only on the prior work that is most closely connected with our efforts, including papers appearing since early 2010.

Working independently of Webber et al., and publishing at roughly the same time, Sun et al. [11] identified a similar set of desiderata for rank similarity in the context of search. These desiderata essentially include a requirement for top-weighting, and an ability to handle indefinite and incomplete results. In addition, they suggest that measures should be symmetric, should be computationally efficient, and should allow meaningful aggregation over multiple queries. To address these desiderata, Sun et al. defined a similarity measure based on a weighted version of the Hoeffding distance. They then applied this measure to visualize differences in search engines through multidimensional scaling.

MED satisfies the additional desiderata of Sun et al. Symmetry is straightforward from the definition of MED. The versions of MED detailed in this paper satisfy the efficiency requirement to varying degrees, and all are reasonably efficient. Computation of MED for the dot product measures is essentially instantaneous, with run times dominated by I/O and data conversion. For members of the MED family not covered in this paper, computational efficiency will depend on the details of the associated effectiveness measure. MED also derives its approach to aggregation from the associated effectiveness measure, where an arithmetic mean is typical, but other approaches are possible [29].

Kumar and Vassilvitskii [13] proposed various extensions to the Kendall’s $\tau$ and Spearman’s footrule correlation coefficients intended to address the requirements of search. These extensions are intended to handle top weighting, known document relevance, and the similarity between documents. They demonstrated that these extensions maintain the Diaconis-Graham inequality, which guarantees that Kendall’s $\tau$ and Spearman’s footrule differ by at most a constant factor.

Like Webber et al. [14] and Sun et al. [11], Kumar and Vassilvitskii suggest a list of desiderata for rank similarity in the context of search. These desiderata include support for top weighting and the triangle inequality. While all their requirements are not precisely defined, MED appears to satisfy them with one interesting exception. The family members of MED defined in this paper do not provide support for inter-document similarity. When comparing two rankings, replacements or swaps of documents with similar content may not greatly impact the user, and a similarity measure might reasonably reflect this consideration. From the perspective of MED, we can trace this requirement back to the associated effectiveness measures, which also do not appropriately handle documents with similar or duplicate content. As researchers begin to consider these issues in the design of effectiveness measures [30], solutions will transfer naturally to MED.

Several other researchers have also adapted existing rank similarity measures to the requirements of search, particularly the Kendall’s $\tau$ correlation coefficient [12], [15], [16]. These efforts primarily address the issue of top weighting, paying less attention to incomplete and indefinite rankings. In addition, rather than measuring differences between result lists, much of this work focuses on the problem of comparing system rankings for the purpose of validating newly proposed effectiveness measures, and other evaluation methodologies. In our work, rather than adapting existing rank similarity measures to the requirements of search, we adapt existing search effectiveness measures to the computation of rank similarity, inheriting important properties of these effectiveness measures and cleanly accommodating known relevance information.

Rank similarity measures provide a method for comparing search results without the need for relevance information. Numerous researchers have examined the related problem of estimating effectiveness measures and comparing systems using limited relevance information. Most notably, Carterette et al. [19]–[21] define algorithms for selecting minimal sets of documents for judging, with the aim of ordering a group of retrieval systems at a given confidence level. Similar in spirit to MED, and using equivalent methods, documents are selected to maximize differences in retrieval effectiveness scores.
(a) MED-RBP vs. RBO
(b) MED-RBP vs. MED-nDCG@20
(c) MED-MAP vs. MED-nDCG@20
(d) MED-ERR vs. MED-nDCG@20
(e) MED-nDCG@5 vs. MED-nDCG@20
(f) MED-nDCG@100 vs. MED-nDCG@20

Fig. 3. Comparisons between RBO, MED-nDCG, MED-RBP, MED-MAP and MED-ERR for all runs from the TREC 2005 Robust Retrieval Track.

Interestingly, in his doctoral thesis Carterette [21, pages 156–157] notes the need for rank similarity measures that incorporate top-weighting and an ability to handle incomplete rankings. To address these shortcomings of traditional correlation coefficients, such as Kendall’s $\tau$ and Spearman’s $\rho$, he proposes his own rank similarity measure based on differences in reciprocal rank. He employs this measure to study properties of his judging algorithms and TREC runs.

We take his work a step further, recognizing and demonstrating that maximized effectiveness difference itself can form the basis for measuring rank similarity. In addition, we extend the idea of maximized effectiveness difference to effectiveness measures that did not exist at the time of his thesis, including ERR and RBP (and in the next section, measures beyond the ranked list). Some of our ideas could also be applied back to his work, including our generalization for the dot-product measures, our formulation for MAP, and our formulation for ERR.

Apart from Webber et al. [14] and Carterette et al. [19–21], the work closest to ours is the AnchorMap measure proposed by Buckley [10]. AnchorMap compares two ranked lists by assuming that the top $k$ documents from one list are relevant, and then computing MAP for the other list using this relevance information. While AnchorMap is not symmetric, and has other limitations [14], the idea captures the essence of our proposal.

Several researchers have used rank similarity measures to monitor and compare commercial search engines [11], [13], [14], [31]. Cardoso and Magalhães [31] applied RBO to compare the behavior of two of the most heavily used commercial search engines. In addition, they applied Jensen-Shannon divergence to measure the similarity between search results based on the contents of the top documents returned. They argue that this second approach provides deeper insights into the differences between the search engines, a view that reflects some concerns of Kumar and Vassilvitskii [13].

In other related work, Büttcher et al. [32] trained a classifier to predict the relevance of unjudged documents. Jensen et al. [33] combined limited manual judgments with automatically generated pseudo-judgments to evaluate search results in dynamic environments. Yilmaz et al. [34] developed sampling methods for estimating standard effectiveness measures. Sakai and Kando [35] explored the impact of missing relevance judgments on standard effectiveness measures across four large test collections.

VIII. BEYOND THE RANKED LIST

MED may be extended to measure distances between search results beyond the ranked list. As a simple example, we
consider the \textit{U-measure} proposed by Sakai et al. \cite{Sakai_2005}. This measure assumes that a retrieval result is not expressed as a ranked list of documents, but rather as a \textit{trailtext}, a concatenation of all text presented to (or seen by) the user. Given a trailtext of length \( l \), the \textit{U-measure} is computed as

\[
U = \frac{1}{N} \sum_{i=1}^{l} c_i d_i.
\]

Just as Equation 1 sums over documents in the ranked list, the \textit{U-measure} sums over character offsets in the trailtext. The value \( c_i \) represents the graded relevance value of the character at offset \( k \), and the value \( d_i \) represents a position-oriented discount. For their experiments, Sakai and Dou use no normalization (\( N = 1 \)) and a linear discount (\( d_i = 1 - i/l \)) but clearly the measure can be generalized to other discounts and normalization values.

The \textit{U-measure} provides one method for evaluating retrieval systems that attempt to return sub-document components, such as text passages, to the user. For example, a system might respond to a query by extracting relevant passages from books, and other longer documents, placing them in order for reading by the user. Essentially this problem was addressed by a task included as a part of the TREC 2004 HARD Track \cite{Ridenour_2005}.

The 2004 HARD Track included 25 topics for which relevance judging was performed at the sub-document level, by identifying offsets and lengths of relevant passages within documents. The corpus for the track comprised more than a half-a-million news articles from the year 2003, drawn from a variety of sources, including the Associated Press, the New York Times, and the Washington Post. Participating systems returned a ranked list of passages for each of the passage-level topics, where each passage consisted of a starting byte offset within a document and a length in bytes.

Since characters within passages cannot be arbitrarily reordered, standard rank correlation coefficients are not appropriate to compute similarities between these passage-oriented results. However, we may apply MED for the \textit{U-measure} (MED-U) for this purpose. Although we have switched from documents to characters, the properties of MED from Section \[ III \] continue to hold; nothing in that section requires variables to represent documents.

The \textit{U-measure} may be viewed as a type of dot-product measure, as discussed in Section \[ III \]. We apply the methods of that section to compute MED-U over passage-oriented retrieval runs submitted to the track. We compute the measure down to depth \( l = 12,000 \), consistent with HARD Track practice, and use binary values for \( c_i \), i.e., a character is either relevant or not. Results are shown in Figure 4.

Figure 4 plots actual \textit{U-measure} differences vs. U-MED across pairs of passage-oriented topics taken from passage-oriented runs. For visual clarity, we plot a 20\% sample, selected randomly. Figure 4(a) plots intra-group pairs; Figure 4(b) plots inter-group pairs. In contrast to Figure 2(a) retrieval results tend to be very different from one another, although runs from the same group tend to be closer than runs from different groups. Many pairs from different groups are completely different (with the maximum possible MED-U@12000 value of 2999.75). These large differences may reflect the unusual nature of the task, as well as unfamiliarity with passage retrieval. If passage retrieval was a better understood task, we might much expect closer results, particularly for intra-group runs.

In other work, Smucker and Clarke \cite{Smucker_2005}, \cite{Smucker_2006}, present a general effectiveness measure, called \textit{time-biased gain}, which may be applied to measure search results in many forms. For example, retrieval results might be specified as summaries, snippets, or video clips. As the user interacts with the results, the total benefit to the user is expressed as a function \( G(t) \), the \textit{cumulative gain} at time \( t \geq 0 \). This gain is realized as relevant material is encountered by the user, perhaps by reading relevant text or viewing relevant video. For a retrieval result \( X \), with associated cumulative gain function \( G_X(t) \), time-biased gain is

\[
S(C) = \int_0^\infty \frac{dG_X}{dt} D(t) dt.
\]

\( D(t) \) is a decay function, typically exponential, indicating the probability that the user interacts with the material until time \( t \). Given two retrieval results, \( A \) and \( B \), we treat the material shared by these results as \textit{bound material}, material
appearing in only one result as free material, and material with known relevance as predetermined material. To compute MED(A,B), we must assign relevance to bound and free material to maximize

\[ S(A) - S(B) = \int_0^\infty \frac{d(G_A - G_B)}{dt} D(t) dt \] (27)

MED-RBP, MED-nDCG, MED-ERR, and even MED-MAP are all specific examples of this general equation, under appropriate list-oriented definitions for gain and decay [30], [39]. We leave to future work the application of this equation to measure search-result distances beyond the ranked list.

IX. CONCLUDING DISCUSSION

Rank similarity measures cannot replace traditional measures effectiveness for determining the absolute performance of search engines. However, they are more easily applied over larger numbers of queries, without the need for relevance judgements, providing an additional tool for assessing the scope of a search engine change. The MED family of rank similarity measures, presented in this paper, satisfies various desiderata suggested in prior work for rank similarity measures in the context of search. Unlike the rank similarity measures presented in this prior work, the MED family allows us to translate our understanding and assumptions about user behavior from an existing effectiveness measure to create a rank similarity measure. In addition, MED cleanly accommodates partial relevance judgments, when this information is available. If complete relevance information is available, MED reduces to a simple difference between effectiveness values. Software to compute MED for various effectiveness measures is available at \texttt{plg.uwaterloo.ca/~claclark/med}.

Differences between retrieval results might also be studied under conditions other than the “worst case” assumption of MED. For example, we might assume that an unjudged document is equally likely to be relevant or non-relevant. Under this assumption, we can compute a distribution of differences, perhaps treating the mean difference as a similarity measure, although it may not be a metric in the mathematical sense. The probability of relevance for an unjudged document could be conditioned on rank, or computed by a classifier, with the goal of estimating actual differences. Some of these ideas have been partially explored in related work, as discussed. We leave additional exploration as future work.

Some newer proposals for effectiveness measures incorporate more complex user models, creating interesting and challenging optimization problems, which we hope to examine in the future. Several research groups have suggested measures that reward novelty and diversity in search results [2], [23], [40]. Computing MED for these effectiveness measures may require the explicit assignment of query interpretations to documents in order to maximize effectiveness difference. Other proposals use time as the primary indicator of user effort, creating measures that reflect the impact of snippets and other user interface features [40], [41]. Some of these proposals employed simulation as a method for determining effectiveness [38], [39], [41], and computing MED for these effectiveness measures may also require extensive simulation.

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