The Slip Behavior and Source Parameters for Spontaneous Slip Events on Rough Faults Subjected to Slow Tectonic Loading

Yuval Tal¹ and Bradford H. Hager¹

¹Earth Resources Laboratory, Department of Earth, Atmospheric and Planetary Sciences, Massachusetts Institute of Technology, Cambridge, MA, USA

Abstract We study the response to slow tectonic loading of rough faults governed by velocity weakening rate and state friction, using a 2-D plane strain model. Our numerical approach accounts for all stages in the seismic cycle, and in each simulation we model a sequence of two earthquakes or more. We focus on the global behavior of the faults and find that as the roughness amplitude, \( b_r \), increases and the minimum wavelength of roughness decreases, there is a transition from seismic slip to aseismic slip, in which the load on the fault is released by more slip events but with lower slip rate, lower seismic moment per unit length, \( M_{\text{br}} \), and lower average static stress drop on the fault, \( \Delta \tau_f \). Even larger decreases with roughness are observed when these source parameters are estimated only for the dynamic stage of the rupture. For \( b_r \leq 0.002 \), the source parameters \( M_{\text{br}} \) and \( \Delta \tau_f \) decrease mutually and the relationship between \( \Delta \tau_f \) and the average fault strain is similar to that of a smooth fault. For faults with larger values of \( b_r \), that are completely ruptured during the slip events, the average fault strain generally decreases more rapidly with roughness than \( \Delta \tau_f \).

1. Introduction

The deviation of natural faults from planarity (Bistacchi et al., 2011; Brodsky et al., 2011; Brown & Scholz, 1985; Candela et al., 2009, 2012; Power et al., 1987; Power & Tullis, 1991; Renard et al., 2006; Sagy et al., 2007) results in geometric asperities and a locally heterogeneous stress field, which affect the nucleation and propagation of shear rupture and consequently earthquake source parameters, such as the maximum slip rate, seismic moment, and static stress drop. However, the exact effect of roughness on these parameters is not yet clear.

Numerical static analyses of the response of rough faults to an increase in shear load (Dieterich & Smith, 2009) or a prescribed uniform shear stress reduction on the fault (Zielke et al., 2017) show that the slip on the fault (as well as the area of the rupture in the latter study) decreases with increasing roughness amplitude. Using a quasi-static cascade rupture model for an earthquake, Candela et al. (2011) suggest that the stress drop decreases with increasing fault dimension because of the self-affine fractal geometry of faults. The quasi-static analysis of Bailey and Ben-Zion (2009) shows that frictional heterogeneities on the fault, which can represent geometrical variations to some extent, result in decreases in the average stress drop on the fault. However, although these studies provide important first-order information regarding the effects of roughness on the source parameters, they do not take into account the dynamics of the rupture and the effects of friction laws. Moreover, they do not provide any information regarding the slip rates on the fault and whether the load is released seismically or aseismically.

Several numerical studies of dynamic rupture on rough faults governed by frictional laws have been performed, all of them with strongly rate-weakening friction laws and viscoplasticity for the bulk material (Bruhat et al., 2016; Dunham et al., 2011a; Fang & Dunham, 2013; Shi & Day, 2013). These studies focus on the rupture process rather than the macroscopic source parameters and show that roughness promotes the development of self-healing rupture pulses, substantial fluctuations in rupture velocity, heterogeneous slip distribution, inelastic deformation, and diverse rupture styles, such as rupture arrests, secondary slip pulses that rerupture previously slipped fault sections, and supershear transitions. In the context of the size of earthquakes, Fang and Dunham (2013) point out that most ruptures in their simulations stop naturally before reaching the ends of the computational domain, usually when they encounter unfavorable stress conditions on compressional bends. However, these simulations adopt an artificial nucleation procedure and it is not clear whether the modeled ruptures would nucleate spontaneously into fast seismic events, if
the faults were loaded only by tectonic stress. The models are also limited to a single rupture and assume a spatially uniform initial stress field and friction parameters. It is important to examine how slip and stress heterogeneities at the end of slip events affect the rupture process at subsequent events. In addition, the contact formulation assumes that the grid points are collocated on either side of the fault during all stages of the simulation; thus, the range of roughness wavelengths is limited and the variations of the normal tractions on the fault during slip may be underestimated.

Using the numerical approach developed in Tal (2017) and Tal and Hager (2017), we study the response of rough faults governed by rate and state friction (Dieterich, 1979; Ruina, 1983) to slow tectonic loading over time equivalent to about two seismic cycles, if a smooth fault were considered, and examine the effects of roughness on the stress drop, seismic moment, and whether the fault slips seismically or aseismically. The method enables the modeling of the whole seismic cycle, including the slow aseismic nucleation, the dynamic propagation, and the arrest of the rupture. We focus on the scale of small earthquakes and choose the minimum roughness wavelength to be at the size close to the lab samples (20 cm) and thus use the observed lab-scale rate and state friction laws without upscaling the constitutive parameters. We provide some of the quantities in dimensionless form and believe that trends obtained here are also valid for larger earthquakes, but it is important to note that the complexity and heterogeneity of the medium surrounding large faults are expected to be larger and that the friction laws should be modified to include the large reduction in the friction coefficient observed in large slip experiments (e.g., Di Toro et al., 2011).

2. Model Description

We study the behavior of rough finite faults with length, $L_f$, embedded in a 2-D elastic medium with dimensions $3L_f \times 2L_f$ (Figure 1a). We assume a plane-strain model and apply the following boundary conditions: (1) a prescribed slow horizontal velocity of $V_{bx} = \pm \frac{L_f}{20} \times 10^{-10}$ m/s at the top and bottom boundaries of the medium, (2) zero vertical velocities on all the boundaries; initial stresses $\sigma_{xx0}$, $\sigma_{yy0}$, and $\sigma_{xy0}$, and horizontal normal tractions on the left and right boundaries $\sigma_{bxx} = \sigma_{bx0}$. During the dynamic stages, the model includes an absorbing layer with gradual Rayleigh damping. (b) The mesh structure around the left side portion of a rough fault with length of 30 m. The mesh is refined with hanging nodes to give elements on the fault with dimensions of about 1.56 × 1.56 cm.
The roughness of faults can be measured by the average deviation of the profile from planarity (root mean square height), which for a profile with length $L_f$ is found to satisfy the following scaling:

$$h(L_f) = b_r L_f^H,$$

where $b_r$ is the root mean square prefactor and $H$ is the Hurst exponent. Candela et al. (2012) compiled earthquake rupture trace data (Klinger, 2010) with profilometers and light detection and ranging scan measurements from exhumed faults to generate a description of roughness over nine decades of length scales. They suggested that natural faults have a self-affine geometry with $H = 0.6$ and $b_r$ ranging from 0.001 to 0.01 in the slip direction. However, while measurements at a particular wavelength bandwidth seem to fit this estimation, the data as a whole may be better fit with larger value of $H$ (Shi & Day, 2013). In order to study the effects of the geometry of the fault on the rupture process, we perform 72 simulations of self-affine faults with $H = 0.8$, in which we vary $L_f, b_r$, and the minimum wavelength of roughness, $\lambda_{\text{min}}$ (Figure 2). We consider fault lengths of $L_f = 20$ m, 30 m, and 40 m and three different geometries for each length. For each geometry we generate eight profiles, in which $b_r$ ranges between $b_r = 0.001, 0.002, 0.005$, and 0.01 and $\lambda_{\text{min}}$ ranges between 0.2 m and 1 m. For reference, we also run a simulation with a smooth fault for each length.

Figure 2. The fault profiles examined in this study: We consider a total of nine different general geometries for fault lengths of $L_f = 20$ m (a), 30 m (b), and 40 m (c). For each geometry eight profiles are generated with roughness prefactor values of $b_r = 0.001, 0.002, 0.005$, and 0.01 and minimum wavelengths of $\lambda_{\text{min}} = 0.2$ m and 1 m. For reference, we also run a simulation with a smooth fault for each length.
The fault is governed by rate and state friction, which is given in the standard aging formulation (Dieterich, 1979; Ruina, 1983) by

$$\mu = \mu^* + a \ln \left( \frac{v + v_{th}}{v^*} \right) + b \ln \left( \frac{v^* \theta}{L^*} \right),$$  \hspace{1cm} (2)

$$\dot{\theta} = 1 - \frac{\theta(v + v_{th})}{L^*},$$  \hspace{1cm} (3)

where $a$ and $b$ are rate and state constitutive parameters, $v$ is the slip rate, $v^*$ is a reference slip rate, $\mu^*$ the steady state friction at $v = v^*$, $\theta$ is a state variable, and $L^*$ is the characteristic sliding distance. We add a threshold velocity term, $v_{th} = 10^{-13}$ m/s to avoid singularity at $v = 0$. We do not consider the effect of normal tractions variations on $\dot{\theta}$. The mechanical properties of the medium, friction law parameters, and initial stresses are given in Table 1.

Our numerical approach (Tal, 2017; Tal & Hager, 2017) is based on the mortar finite element formulation (Bernardi et al., 1993), in which nonmatching meshes are allowed across the fault and the contacts are continuously updated; thus, it enables slip that is large relative to the size of the elements near the fault, as well as an accurate modeling of the variations of normal tractions during slip. We extend the 2-D large sliding mortar formulation of Popp et al. (2009) and Gitterle et al. (2010) to dynamic problems and consistently implement the rate and state friction law into the method. The method uses Lagrange multipliers with dual spaces discretization (Wohlmuth, 2000) to enforce the continuity of stress, nonpenetration condition, and frictional resistance on the fault in a weak integral sense. This concept is combined with the primal-dual active set strategy (Brunssen et al., 2007; Hüeber et al., 2008; Hüeber & Wohlmuth, 2005) to enable an efficient local elimination of the discrete Lagrange multipliers by static condensation and an efficient semismooth Newton algorithm for the solution of the nonlinear system of equations. Moreover, the discretization of the friction law involves a procedure to condense out the state variables, thus eliminating the addition of another set of unknowns into the system.

Because we use a finite element-based method, the meshing structure is quite flexible. In this study, we refine the mesh near the fault using a hanging nodes technique, thereby enabling accurate representation of the fault geometry. The resulting elements on the fault have dimensions of about $1.56 \times 1.56$ cm (Figure 1b), which is equal to $l_{min}/13$ and $l_{max}/64$ for $l_{min} = 0.2$ m and 1 m, respectively. Under plane-strain conditions, the smallest length scale to be resolved numerically for a growing nucleation zone on a fault governed by rate and state friction with the aging law scales roughly as $L_b = L_{th} \ln(1/s_0)$ (Ampuero & Rubin, 2008; Perfettini & Ampuero, 2008). Here $s_0$ is the normal traction and $G$ and $v$ are the shear modulus and Poisson’s ratio of the medium, respectively. With the parameters adopted in this study (Table 1), the average $L_b \approx 53.3$ cm and is resolved with about 34 elements. We show that problem is well resolved with the chosen grid spacing in the supporting information.

The method uses a quasi-static backward Euler time discretization scheme when inertial effects are negligible and implicit Newmark scheme (Newmark, 1959) for dynamic analysis. Because both schemes are implicit, the implementation of variable time stepping is straightforward, and the whole seismic cycle can be modeled, including a completely spontaneous nucleation process. Based on the current values of slip rates, we estimate the time step size at the next time step such that the average incremental slip of the 40 fastest nodes along the fault will not be larger than 0.4 $L$. This procedure results in a time step size that represents the evolution of the friction coefficient well without reducing the time step size to values that lead to simulations with an excessive number of time steps. We switch between quasi-static and dynamic time integration schemes when the average slip rate at the 40 fastest nodes on the fault is larger than $5 \times 10^{-5}$ m/s. We use the Newmark time integration scheme with small algorithmic damping ($\beta = 0.35$ and $\gamma = 0.7$) for the dynamic stage and also add an absorbing layer with gradual Raleigh damping near the boundaries of the model.

Currently, the numerical method adopted in this study accounts only for an elastic rheology of the medium surrounding the fault. This approximation limits the amount of deformation that the medium can experience.

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### Table 1: Model Parameter Values

| Parameter                        | Value |
|----------------------------------|-------|
| **Frictional properties**        |       |
| Direct-effect parameter          | $a$   | 0.01 |
| Evolution-effect parameter       | $b$   | 0.012 |
| Reference velocity               | $v$   | $10^{-6}$ m/s |
| Reference friction               | $\mu$ | 0.6  |
| Characteristic sliding distance  | $L$   | 20 $\mu$m |
| Initial friction                 | $\mu_0$ | 0.57 |
| Initial state variable           | $\theta_0$ | 1 s |
| **Bulk properties**              |       |
| Young’s modulus                  | $E$   | 60 GPa |
| Poisson’s ratio                   | $\nu$ | 0.25 |
| Density                          | $\rho$ | 2,700 kg/m$^3$ |
| **Initial remote stresses**      |       |
| Horizontal stress                | $\sigma_{xx0}$ | 100 MPa |
| Vertical stress                  | $\sigma_{yy0}$ | 100 MPa |
| Shear stress                     | $\sigma_{xy0}$ | 57 MPa |
before unrealistic stresses larger than the Coulomb failure criterion are accumulated around the fault. Therefore, we set the total time of simulation to the time during which two large events would occur, if a smooth fault were considered. Because we do not model a long sequence of earthquakes, the initial conditions have a significant effect on the results, despite the spontaneous nucleation. We conceptually begin the simulation at the end of an earthquake that ruptured the whole fault and choose the initial friction parameters accordingly. We assume that the earthquake approached slip rate on the order of 1 m/s during the rupture and that the state variable and the friction coefficient had no time to evolve and are equal to their steady state values at this slip rate, \( \theta_0 = \frac{L}{1} \) and \( \mu_0 = \mu^* + (a - b) \ln \left( \frac{1}{\sqrt{\gamma}} \right) \approx 0.57 \), respectively. For faults with small value of \( b_r \), the initial shear stress in the medium is chosen such that \( \sigma_{xy0} = \mu_0 \sigma_{xy0} \). As the roughness of the fault increases, the ratio of shear traction to normal traction on some segments along the fault increases and they begin to slip under smaller shear stress. In these cases, we prescribe \( \sigma_{xy} < \mu_0 \sigma_{xy0} \) but in order to maintain similar initial conditions in all simulations, we do not allow the state variable to evolve until the remote shear stress exceeds the initial shear stress calculated for a smooth fault.

3. Results

3.1. The Effect of Roughness on the Earthquake Sequence

Figure 3 shows the evolutions of the maximum slip rate and average shear traction on the fault for different fault geometries. For brevity we show only the results of Geo-1 (20 m), Geo-4 (30 m), and Geo-7 (40 m) here. Similar trends are observed for the other six geometries. We determine the initial and final stages of the slip events from curves of the evolution of the average shear traction on the fault, where the initial stages correspond to peaks in the curve and the final stages correspond to the stages in which the average traction begins to increase linearly with time. Note that for slow slip events, the final stages coincide with the minima of the curves, while in fast slip events, those stages are slightly after the minima because of dynamic effects.

In general, as \( b_r \) increases, faults experience more slip events but with lower slip rates and lower stress drops. A more rigorous estimation of the stress drops is provided in section 3.3, but the major trends are clearly observed in Figure 3. There is no clear relationship between \( \lambda_{\text{min}} \) and \( L_f \) and the number of events (Figure 4), but the slip rates and stress drops generally increase with increasing \( \lambda_{\text{min}} \), mostly for large values of \( b_r \). The effect of \( L_f \) seems to be significant only between \( L_f = 20 \) m and \( L_f = 30 \) m. Faults with \( b_r \leq 0.001 \) slip only seismically. They experience two fast slip events with slip rates of few meters per second. Faults with \( b_r = 0.002 \) experience two to three slip events, most of them fast. Faults with \( b_r = 0.005 \) experience both slow and fast slip events. The roughest faults (\( b_r = 0.01 \)) experience fast slip events only for \( \lambda_{\text{min}} = 1 \) m. For faults with \( b_r = 0.01 \) and \( \lambda_{\text{min}} = 0.2 \) m, the maximum slip rates are generally between \( 10^{-8} \) and \( 10^{-4} \) m/s. In two slip events on fault with Geo-8 (40), the maximum slip rates approach larger values, but these values are observed only on a small portion of about 4 m along the fault.

Under the same initial and loading conditions, rougher faults include segments with favorite orientation that start to slip earlier and partially release the shear load. In the case of \( b_r \geq 0.005 \), some of the load is initially released by a few localized slow slip events. As the roughness decreases, the fault accumulates more shear load before the first slip events; thus, the peak stress, stress drop, and the maximum slip rate increase with decreasing roughness. Later slip events depend on the final loading and frictional conditions of prior events, and these parameters do not always decrease with increasing roughness. For example, in the case of a fault with \( L_f = 20 \) m and \( \lambda_{\text{min}} = 0.2 \) m (Figure 3b), a fault with \( b_r = 0.002 \) experiences slightly larger peak stress and stress drop during the third slip event than those observed during the second slip event on a fault with \( b_r = 0.001 \). For faults with \( b_r = 0.002 \) and 0.005, these parameters are usually larger as the slip event is later in the sequence, while for smoother faults there is no clear trend.

3.2. Seismic Moment

In this section, we analyze each slip event in the 75 earthquake sequences mentioned above and examine the effect of roughness on the seismic moment. Because we use a 2-D plane strain model, we examine the moment per unit length, \( M_{0,1d}(t) \), which is given by

\[
M_{0,1d}(t) = G |u(x, t)| dx = G L \overline{D},
\]

where \( G \) is the shear modulus, \( L_f \) is the fault length, \( \overline{D} \) is the average slip on the fault, and \( u(x, t) \) is the slip
accumulated from the beginning of the slip event. The seismic moment defined above may involve also the slip accumulated over time scales of hours and days, and, for fast slip events, it is important to estimate how much of the total seismic moment is released during the dynamic stage of the rupture. To estimate the moment release during the dynamic stage of the rupture, $M_{0,1d}(t)$, we examine the evolution of the moment rate, which is given by

$$M_{0,1d}(t) = G I(x, t)dx,$$

where $v(x, t)$ is the slip rate. Figure 5a shows the evolution of $M_{0,1d}$, $M_{0,dyn,1d}$, and $M_{0,1d}$ during the first slip event on a fault with Geo-4 (30 m), $b_r = 0.001$, and $\lambda_{min} = 0.2$ m. We define the beginning of the dynamic stage of the rupture at the time when the moment rate exceeds a threshold value of $M_{th,ini} = 5 \times 10^9$ N/s and

Figure 3. The evolution of maximum slip rate (a, c, and e) and average shear traction (b, d, and f) obtained for the fault geometries Geo-1 (20 m), Geo-4 (30 m), and Geo-7 (40 m) shown in Figure 2, as well as for smooth faults.
To provide insight into the rupture process, as well as its state at these threshold values, Figures 5c and 5d show the evolution of slip rate and slip along the fault, respectively. The nucleation stage, $M_{th,ini}$, corresponding to a stage in which the whole rupture is about 9 m, and the slip rates along a 4–5 m portion of the rupture are on the order of a few cm/s.

With increasing $b_r$, the complexity of the rupture increases and may include multiple transitions between the “slow” and dynamic stages. For example, Figure 6a shows the evolution of $M_{0,1,d}$, $M_{odyn,1,d}$ and $M_{0,1}$ during the third slip event of the sequence obtained for fault with Geo-4 (30 m), $b_r = 0.01$, and $\lambda_{min} = 1$ m. Most of the event is aseismic, but the figure represents a time range of 85 s, which includes three short subevents where the rupture propagates dynamically ($M_{0,1,d} > 5 \times 10^9$). Between the subevents, $M_{0,1,d}$ decreases by a few orders of magnitude, and there is an additional slow subevent. The slip contours along the fault (Figure 6b) show a very complex behavior. The dynamic subevents are localized to small portions of the fault, and the slip accumulated during these subevents is much smaller than the total slip of the event. The propagations of the ruptures during the dynamic subevents occur mostly on portions of the fault preferably oriented for slip (Figure 6c) and usually while rerupturing a portion of the fault that slipped previously during the event. For example, during the first dynamic subevent, the rupture is locked on a barrier on the right and propagates...
dynamically only to the left, rerupturing a portion of the fault with a negative slope. It stops propagating dynamically at a local peak of the fault topography.

Figure 7a shows the total seismic moment of all the slip events in the 75 earthquake sequences, as well as the average value of the moment for each combination of $b_r$, $\lambda_{\text{min}}$, and $L_f$. It is important to note that in the case of $b_r \geq 0.005$, there might be slip events that are not included here because they are too small to affect the behavior of the average shear traction on the fault. In general, the average and the maximum values of the seismic moment decrease with increasing $b_r$ and decreasing $L_f$ and $\lambda_{\text{min}}$, where the effect of $\lambda_{\text{min}}$ is mostly observed for $b_r \geq 0.005$. For $\lambda_{\text{min}} = 1$ m, the decrease in the moment of the largest event with $b_r$ is quite small, where the
maximum moment on a fault with \( b_r = 0.01 \) is about 60–70% of that on a smooth fault. A larger decrease is observed for \( \lambda_{\text{min}} = 0.2 \) m, with about 1 order of magnitude difference between a smooth fault and a fault with \( b_r = 0.01 \). The average moment decreases more rapidly than the maximum moment, with a significant decrease already for smaller values of \( b_r \). In the case of smooth faults, \( M_{0,1,0} \) and \( M_{0,1,0,d} \) of the second event in the sequence are smaller than those of the first one. The finiteness of the fault leads to stress concentrations near the tips of the fault at the end of the first event; thus, the second event nucleates near the ends of the fault and rupture process is different.

Figure 7b shows the effect of roughness on the moment ratio \( M_{0,1,0,d}/M_{0,1,0} \) for the fast slip events. In the few cases where the fast slip events include several dynamic subevents (see, e.g., Figure 6), \( M_{0,1,0,d} \) is calculated as the sum of the dynamic moment release in all the subevents. Similar to the total seismic moment, the average and the maximum values of the moment ratio generally decrease with increasing roughness prefactor and decreasing minimum wavelength. The effect of the length of the fault is less consistent. The moment ratio generally increases between \( L_f = 20 \) m and \( L_f = 30 \) m (Note that in the case of \( L_f = 20 \) m and \( \lambda_{\text{min}} = 1 \) m there is only one fast event for \( b_r = 0.01 \)), but moment ratios of faults with \( L_f = 40 \) m are not always higher than those of faults with \( L_f = 30 \) m. However, it is important to note that the dynamic moment itself, which can be obtained by multiplying the data in Figure 7b with the corresponding data in Figure 7a, clearly increases with increasing length of the fault.

### 3.3. Static Stress Drop

Another important earthquake source quantity is the static stress drop, \( \Delta \tau_{\text{st}} \), which is defined as the difference between the average shear stress on the fault before and after the slip event. Because the local stress change varies over the fault, the averaging methodology may affect the estimated \( \Delta \tau_{\text{st}} \). Noda et al. (2013) compared three different measures for averaging heterogeneous stress drop distributions: the moment-based, the rupture area-based, and the energy-based stress drop measures. For small levels of heterogeneity, the three measures of the average stress drop were similar, but with increasing heterogeneity significant differences were observed. Similarly to Bailey and Ben-Zion (2009) and Cocco et al. (2016), we use a rupture area-based measure, which averages the local stress drops over the portions of the fault that slipped more than a threshold value, \( u_{\text{th}} \). For natural earthquakes the actual constitutive law that governs the fault is unknown and \( u_{\text{th}} \) is defined as a certain percentage of the average or maximum slip on the fault (e.g., Cocco et al., 2016). In this numerical study the fault is governed by rate and state friction with a known characteristic sliding distance; thus, we simply define the threshold slip as \( u_{\text{th}} = 5 \) L.

Figure 8a shows the evolution of shear traction, \( \tau_{\text{s}} \), with the normalized slip \( u/L \) at six locations on the fault during the first slip event in the sequence obtained for fault with \( L_f = 30 \) m, \( b_r = 0.001 \), and \( \lambda_{\text{min}} = 1 \) m (Geo-4). Because in this case the whole fault is ruptured, the static stress drop, and the static stress of the dynamic stage, \( \Delta \tau_{\text{dyn},\text{th}} \), are directly obtained from the plot showing the average behavior (Figure 8b). Note not to confuse the latter with the dynamic stress drop, which is defined by conventions as the difference between the initial average shear traction and the average residual sliding resistance (Kanamori & Rivera, 2006; Madariaga, 1976) and, in this case, is smaller than \( \Delta \tau_{\text{st}} \) by about 18% because of dynamic overshooting. The slip and the shear traction are plotted from the beginning of the simulation, and on each curve three circles denote different stages of the event. The black circle denotes the beginning of the slip event, which corresponds to the maximum of the average shear traction, \( \tau_{\text{p,av}} \). The blue circle denotes the beginning of the dynamic stage of the rupture. The red circle denotes the end of the slip event, which coincides with the end of the dynamic stage for this case. At the beginning of the event, the node located at 18.5 m along the fault already experienced slip of \( u/L \approx 15 \) and reduction of about 2.5 MPa in \( \tau_{\text{st}} \). Before the beginning of the dynamic stage, \( \tau_{\text{st}} \) slightly increases but then decreases further and approaches the steady state value. The node located at 22.5 m also experiences slip and reduction in \( \tau_{\text{st}} \) before the beginning of the dynamic stage.

The spatial distributions of shear traction along the fault at the stages described above are shown in Figure 8c. At the stage corresponding to the beginning of the event, the rupture nucleates at a region on the fault between 17.5 m and 20.5 m with a reduction in \( \tau_{\text{st}} \) and the development of stress concentrations at the tips. At the stage corresponding to the transition to the dynamic stage of the rupture, the rupture is 9.5 m long with a single pulse propagating to the left and two pulses propagating to the right. The region that already slips shows small spikes in \( \tau_{\text{st}} \) at intervals of \( \lambda_{\text{min}}/2 \). These spikes are due to changes in the normal traction on
the fault as it slips. At the end of the slip event, the rupture extends over the whole fault and the magnitudes of the spikes increase.

Figure 9 summarizes the values of $\Delta \tau_t$ and $\Delta \tau_t(\text{dyn})$ in all of the slip events, as well as the average value for each combination of $b_r$, $\lambda_{\text{min}}$, and $L_f$. In general, $\Delta \tau_t$ and $\Delta \tau_t(\text{dyn})$ decrease with increasing $b_r$ with a large decrease already between a smooth fault and a fault with $b_r = 0.001$. While the average values show a completely consistent decrease, the maximum values show an increase between $b_r = 0.001$ and 0.002 in some cases. The effect of $\lambda_{\text{min}}$ is significant for $b_r \geq 0.005$, where $\Delta \tau_t$ and $\Delta \tau_t(\text{dyn})$ decrease with decreasing $\lambda_{\text{min}}$. On the contrary to the seismic moment, the effect of $L_f$ is not consistent, except for smooth faults, which show an increase in $\Delta \tau_t$ and $\Delta \tau_t(\text{dyn})$ with increasing $L_f$.

4. Discussion

4.1. Scaling Relations

While both the moment and the stress drop clearly decrease with increasing roughness, it is important to examine the role of roughness in the scaling of the two. In static theories (e.g., Kanamori & Anderson, 1975), the constant stress change $\Delta \sigma$ on a fault with characteristic dimension $L = A^{1/2}$ is given by

$$\Delta \sigma = C G \frac{\bar{D}}{L}$$  \hspace{1cm} (6)

and is related to the seismic moment $M_0 = GAD$ via

$$\Delta \sigma = C \frac{M_0}{L^{1/2}}$$  \hspace{1cm} (7)

where $C$ is a shape factor and $A$ is the area of the fault. These relations, together with the observational data, which shows that the stress drop is independent of the seismic moment (Abercrombie, 1995; Abercrombie & Leary, 1993; Aki, 1967; Allmann & Shearer, 2009; Cocco et al., 2016; Kanamori & Anderson, 1975 and references therein), suggest that the rupture process is self-similar with a constant fault strain $\bar{D}/L$ for small and large earthquakes.

Considering the effect of roughness on $M_{0.1d}$ (see Figure 7), the fault strain $\bar{D}/L = M_{0.1d}/G L^2$ clearly decreases with increasing roughness for a given $L_f$, where $L$ here is equal to the actual length of the rupture, $L_{\text{rup}}$, which may be smaller than the length of the fault. This was also shown by Dieterich and Smith (2009), who performed static analysis with the boundary element method. However, the break of self-similarity with roughness does not suggest that $\bar{D}/L_{\text{rup}}$ changes with the size of the fault. Moreover, it is important to understand the relationship with $\Delta \tau_t$, which is a measured quantity in the spontaneous dynamic rupture simulations performed in this study.

Figure 10a summarizes the values of $\Delta \tau_t$ versus $\bar{D}/L_{\text{rup}}$ for all the slip events in the 75 earthquake sequences simulated in this study. As a reference, the figure also shows the static relationship in equation (6) for a smooth 1-D finite fault subjected to different values of uniform shear traction reduction, which is given by $\Delta \tau_t = 1.16 \frac{4G}{\pi(1-v)} \frac{\bar{D}}{L_{\text{rup}}}$. Because of the finite dimensions of the model, this solution deviates from that of Starr (1928) by 16%. For $b_r \leq 0.002$, $\Delta \tau_t$ and $\bar{D}/L_{\text{rup}}$ decrease mutually and the static relationship is generally preserved. For fast slip events on faults with $b_r \geq 0.005$, the relationship between $\Delta \tau_t$ and $\bar{D}/L_{\text{rup}}$ diverges from the static solution, with generally smaller reduction of $\Delta \tau_t$ with roughness than that of $\bar{D}/L_{\text{rup}}$. For the slow slip events the data are very scattered, but many of those slip events do not rupture the whole faults. The values
of Δτ, in the events that do rupture the whole fault are generally larger than those expected by the static relationship. Similar trends are also observed for the parameters Δτ_{(t,dyn)} and D_{dyn} = L_{rup} of the dynamic stages of the fast slip events (Figure 10b), where D_{dyn} is the average slip accumulated during the dynamic stage of the rupture.

Dieterich and Smith (2009) suggested that the relationship between D and Δσ can be approximated by an effective stiffness as (K_{s} + K_{r}) = Δσ, where K_{s} is the stiffness of a smooth fault and K_{r} is for the additional resistance from the roughness. Although our simulations suggest that the stress drop itself decreases with roughness, we do observe an increase in the effective stiffness of the faults with roughness. For faults with b_{r} ≥ 0.005 that are completely ruptured during the slip events, the ratio Δτ/L_{rup} increases with increasing b_{r} and decreasing λ_{min} (Figure 10). As the effective stiffness increases, the faults become more stable, with a transition to aseismic deformation, as observed in the simulations (Figure 3).

The effect of the length of the faults on Δτ, D/L_{rup}, Δτ_{(t,dyn)}, and D_{dyn}/L_{rup} is consistent only for smooth faults (See also Figure 9). For dynamic ruptures on smooth faults, as L_{r} increases, the rupture propagates to a larger distance with larger slip rates behind the fronts. Thus, the average slip rate on the fault is larger and consequently also the values D/L_{rup} and Δτ_{r}. Roughness introduces more complexity into the rupture process, which decreases this effect and may partially explain the seismological observations of self-similar behavior of D/L and the independence of stress drop on the moment. It is important to note that plasticity or damage on smooth faults also decreases the growth of slip rate with distance (e.g., Dunham et al., 2011b). Moreover, in this study, we consider only faults with L_{r} ≤ 40. With increasing L_{r}, the complexity and heterogeneity of the medium surrounding the fault are expected to be larger and the friction laws should be modified to include the large reduction in the friction coefficient observed at large slip experiments (e.g., Di Toro et al., 2011). We do not observe the decrease in stress drop with increasing fault dimension suggested by Candela et al. (2011), but, as mentioned above, the range of L_{r} in our study is quite small.
4.2. Dimensionality

We believe that the 2-D plane strain simulations in this study provide important information regarding the role of roughness in the rupture process, especially given that the amplitude of roughness in natural faults is significantly smaller in the direction of slip than in the perpendicular direction (e.g., Candela et al., 2012; Sagy et al., 2007). However, it is important to note that there are fundamental differences between 2-D and 3-D modeling of rupture dynamics on rough faults. In 3-D fault models, the rupture growth has mixed modes II and III and the effect of fault irregularities is different. While the additional mode tends to limit the growth of the rupture, the effect of the irregularities is weaker in 3-D than in 2-D. On a 3-D fault surface, the rupture can surround and bypass the irregularities, while, in 2-D models, the rupture can overcome barriers only by jumping over them. To our knowledge, there are no direct comparisons between simulations of dynamic ruptures on 2-D and 3-D rough faults. The 2-D study of Dunham et al. (2011a) and the 3-D study of Shi and Day (2013) show similar trends such as fluctuations in rupture velocity, heterogeneous slip distribution, and the excitation of high-frequency accelerations. Moreover, using a 2-D model, Bruhat et al. (2016) show that roughness promotes supershear transitions. Three-dimensional simulations of strike-slip faults (Yao, 2017) suggest that supershear transitions that are not affected by free surface interaction are favored by rougher faults, as suggested by the 2-D model. However, supershear episodes induced by free surface interactions are favored by smoother faults.

We focus here on the global behavior of the earthquakes, and because we obtain 1-D slip distributions, we study the effects of the roughness on the moment per unit length rather than extrapolating the 1-D slip distribution in the z direction and calculating the 2-D moment (e.g., Lapusta & Rice, 2003). However, we believe that the trends obtained here with plane strain modeling, which is equivalent to a fault with a very large dimension in the z direction, are also adequate for faults with dimension in the z direction comparable to \( L_f \) and that the obtained 1-D slip distribution could be extrapolated in the z direction to give a 2-D seismic moment.

To examine this assumption, we consider the static problem of a square fault with dimensions \( L_f \times L_f \) under constant reduction of shear stress. Using 3-D numerical models, Parsons et al. (1988) and Noda et al. (2013) found that the shape factor for this geometry is \( C = 2.53 \). We calculate an equivalent shape factor \( C_{eq} \) by extrapolating the 1-D slip distribution \( u(x) \) obtained by the plane strain solution (Starr, 1928) to a 2-D slip distribution \( u(x, z) \) on a square fault. Assuming an elliptic slip distribution in the z direction, with \( u(x, 0) = u(x) \) and \( u(x, \pm L_f/2) = 0 \), the average slip on the fault is given by

\[
\bar{D}(x, z) = \frac{L_f \Delta \sigma}{3G} \int_{-L_f/2}^{L_f/2} \left[ 1 - \frac{(x - L_f/2)^2}{(L_f/2)^2} \right] \left[ 1 - \frac{z^2}{(L_f/2)^2} \right] dx dz/L_f, \quad \frac{L_f}{2} \leq z \leq L_f/2
\]  

and \( C_{eq} = 2.16 \), which is only 15\% smaller than that obtained with the 3-D models. This simple analysis may suggest that the trends obtained by the 2-D model (1-D fault) can be extrapolated to a 2-D fault with similar dimension in the z direction but with small overestimation of the slip obtained for a given stress drop and vice versa. It is important to note that the analysis does not take into account the dynamics of the rupture.

5. Conclusions

We study the response of rough faults governed by velocity weakening rate and state friction to slow tectonic loading, using the mortar-based method developed in Tal (2017) and Tal and Hager (2017). The method enables the modeling of the whole seismic cycle, including the slow aseismic nucleation, the dynamic propagation, and the arrest of the rupture.
We focus on the global behavior of the faults and find that as the roughness prefactor $b_r$ increases, there is a transition from seismic slip behavior to aseismic slip behavior, in which the load on the fault is released by more slip events but with lower slip rate, seismic moment $M_{0.1,i}$ and the average static stress drop $\Delta \tau_i$. These parameters also decrease with decreasing minimum wavelength of roughness, especially for large values of $b_r$.

Because the deformation in the slip events may occur over time scales of hours and days and involves also aseismic slip, for the fast slip events we also estimate the seismic moment and stress drop during only the dynamic stage of the ruptures. We find even larger decreases of these parameters with increasing roughness compared to their corresponding total values.

For $b_r \leq 0.002$, the source parameters $M_{0.1,i}$ and $\Delta \tau_i$ decrease mutually and the relationship between $\Delta \tau_i$ and the average fault strain is similar to that of a smooth fault. For faults with larger amplitude of roughness that are completely ruptured during the slip events, the strain decreases more rapidly with roughness than $\Delta \tau_i$.

A consistent effect of the length of the fault on $\Delta \tau_i$ and $M_{0.1,i}$ is observed only for smooth faults; thus, we speculate that roughness may be one of the reasons for the stress drop being independent of magnitude.

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