Abstract

New physics contributions to the $Z$ penguin are revisited in the light of the recently-reported discrepancy of the direct CP violation in $K \to \pi\pi$. Interference effects between the standard model and new physics contributions to $\Delta S = 2$ observables are taken into account. Although the effects are overlooked in the literature, they make experimental bounds significantly severer. It is shown that the new physics contributions must be tuned to enhance $B(K_L \to \pi^0\nu\bar{\nu})$, if the discrepancy of the direct CP violation is explained with satisfying the experimental constraints. The branching ratio can be as large as $9 \times 10^{-10}$ when the contributions are tuned at the 10% level.
1 Introduction

A deviation of the standard model (SM) prediction from the experimental result is recently reported in the direct CP violation of the $K \to \pi\pi$ decays, which is called $\epsilon'$. The latest lattice calculations of the hadron matrix elements significantly reduced the theoretical uncertainty [1–4] and yield [5,6]

$$\left( \frac{\epsilon'}{\epsilon} \right)_{\text{SM}} = \begin{cases} (1.38 \pm 6.90) \times 10^{-4}, & \text{[RBC-UKQCD]} \\ (1.9 \pm 4.5) \times 10^{-4}, & \text{[Buras et al.]} \\ (1.06 \pm 5.07) \times 10^{-4}, & \text{[Kitahara et al.]} \end{cases}$$

They are lower than the experimental result [7–10],

$$\left( \frac{\epsilon'}{\epsilon} \right)_{\text{exp}} = (16.6 \pm 2.3) \times 10^{-4}. \quad (1.2)$$

The deviations correspond to the 2.8–2.9σ level.

Several new physics (NP) models have been explored to explain the discrepancy [11–21]. In the literature, electroweak penguin contributions to $\epsilon'/\epsilon$ have been studied. In particular, the $Z$ penguin contributions have been studied in detail [11, 13, 15, 22]. The decay, $s \to dq\bar{q}$ ($q = u, d$), proceeds by intermediating the $Z$ boson, and its flavor-changing ($s$–$d$) interaction is enhanced by NP. Then, the branching ratios of $K \to \pi\nu\bar{\nu}$ are likely to be deviated from the SM predictions once the $\epsilon'/\epsilon$ discrepancy is explained. This is because the $Z$ boson couples to the neutrinos as well as the up and down quarks. They could be a signal to test the scenario.

Such a signal is constrained by the indirect CP violation of the $K$ mesons. The flavor-changing $Z$ couplings affect the indirect CP violation via so-called the double penguin diagrams; the $Z$ boson intermediates the transition, both of whose couplings are provided by the flavor-changing $Z$ couplings. Such a contribution is enhanced when there are both the left- and right-handed couplings because of the chiral enhancement of the hadron matrix elements. This is stressed by Ref. [15] in the context of the $Z'$-exchange scenario. In the $Z'$-boson case, since the left-handed coupling is installed by the SM, the right-handed coupling must be constrained even without NP contributions to the left-handed one. Such interference contributions between the NP and the SM are overlooked in Refs. [11, 13, 15, 22]. Therefore, the parameter regions allowed by the indirect CP violation will change significantly. In this letter, we revisit the $Z$-boson scenario. It will be shown that the NP contributions to the right-handed $s$–$d$ coupling are tightly constrained due to the interference, and thus, the branching ratio of $K_L \to \pi^0\nu\bar{\nu}$ is likely to be smaller than the SM predictions if the $\epsilon'/\epsilon$ discrepancy is explained. We will discuss that NP parameters are necessarily tuned to enhance the ratio. A degree of the parameter tuning will be investigated to estimate how large $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$ and $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ can become.

#1 QCD penguin contributions, e.g., through Kaluza-Klein gluons, have also been considered [11].

#2 In this letter, we focus on the $s$–$d$ transitions. The $\Delta F = 2$ transitions such as $\Delta m_B$ generally involve the interference contributions.


2 \textit{Z-penguin observables}

In this section, we briefly review the Z-penguin contributions to $\Delta S = 2$ and $\Delta S = 1$ processes in the general Z scenario. It is assumed that the left- and/or right-handed flavor-changing ($s-d$) Z couplings are involved. The effective interaction is defined as

\[ L_{\text{eff}}^Z = \left[ (\Delta_{\text{SM}}^L + \Delta_{\text{NP}}^L) (\bar{s}\gamma^\mu P_L d) + \Delta_{\text{NP}}^R (\bar{s}\gamma^\mu P_R d) \right] Z_\mu + \text{H.c.}, \]  

(2.1)

where $\Delta_L$ and $\Delta_R$ are dimensionless complex parameters at the Z-boson mass scale. The SM contribution is generated by radiative corrections. At the one-loop level, it is calculated as (c.f., Ref. [22])

\[ \Delta_{\text{SM}}^L = \frac{g^3 \lambda t}{8\pi^2 c_W} C\left(\frac{m_t^2}{m_W^2}\right), \quad \Delta_{\text{SM}}^R = 0, \]  

(2.2)

where $c_W = \cos \theta_W$ and $\lambda_i \equiv V_{id}^* V_{ij}$ with the CKM matrix $V_{ij}$. In this letter, the CKMFitter result [23] is used for the CKM elements, unless otherwise mentioned.

The loop function is

\[ C(x) = \frac{x}{8} \left[ \frac{x - 6}{x - 1} + \frac{3x + 2}{(x - 1)^2} \ln x \right]. \]  

(2.3)

In the following, we omit the subscript “NP” in $\Delta_{\text{NP}}^L$ and $\Delta_{\text{NP}}^R$.

2.1 $\epsilon_K$ and $\Delta m_K$

The $\Delta S = 2$ observables involve the indirect CP violation $\epsilon_K$ and the mass difference $\Delta m_K$ in the $K^0 - \bar{K}^0$ mixing. In particular, since $\epsilon_K$ has been measured precisely, and the SM prediction has been estimated accurately, it provides a severe constraint. The SM and NP contribute as

\[ \epsilon_K = e^{i\varphi} \left( \epsilon_{K}^{\text{SM}} + \epsilon_{K}^{\text{NP}} \right), \]  

(2.4)

where $\varphi = (43.51 \pm 0.05)^\circ$. The NP contribution is given by the double penguin diagrams with the Z boson exchange,

\[ \epsilon_{K}^{\text{NP}} = \sum_{i=1}^{8} (\epsilon_{K})_i^Z, \]  

(2.5)

where the right-hand side is

\[ (\epsilon_{K})_1^Z = -4.26 \times 10^7 \text{ Im } \Delta_L \text{ Re } \Delta_L, \quad (\epsilon_{K})_2^Z = -4.26 \times 10^7 \text{ Im } \Delta_R \text{ Re } \Delta_R, \]

\[ (\epsilon_{K})_3^Z = 2.07 \times 10^9 \text{ Im } \Delta_L \text{ Re } \Delta_R, \quad (\epsilon_{K})_4^Z = 2.07 \times 10^9 \text{ Im } \Delta_R \text{ Re } \Delta_L. \]  

(2.6)
In addition, the interference terms between the SM and NP contributions are

\[
\begin{align*}
(\epsilon_K)_5^Z &= -4.26 \times 10^7 \text{Im } \Delta_L^\text{SM} \text{ Re } \Delta_L, \\
(\epsilon_K)_6^Z &= -4.26 \times 10^7 \text{Im } \Delta_R^\text{SM} \text{ Re } \Delta_L, \\
(\epsilon_K)_7^Z &= 2.07 \times 10^9 \text{Im } \Delta_L^\text{SM} \text{ Re } \Delta_R, \\
(\epsilon_K)_8^Z &= 2.07 \times 10^9 \text{Im } \Delta_R \text{ Re } \Delta_L^\text{SM}.
\end{align*}
\] (2.7)

The numerical factors are found in Ref. [15], where renormalization group corrections and long-distance contributions are included [24]. These interference terms between the SM and NP contributions have been overlooked in Refs. [11,13,15,22]. They will be shown to be significant in the next section.*3

The latest estimation of the SM value is [25]

\[
\epsilon_K^\text{SM} = (2.24 \pm 0.19) \times 10^{-3}.
\] (2.8)

On the other hand, the experimental result is [10]

\[
|\epsilon_K^\text{exp}| = (2.228 \pm 0.011) \times 10^{-3}.
\] (2.9)

They are well consistent with each other, and \(\epsilon_K^\text{NP}\) must satisfy

\[
-0.39 \times 10^{-3} < \epsilon_K^\text{NP} < 0.37 \times 10^{-3},
\] (2.10)

at the 2\(\sigma\) level.#4

The kaon mass difference \(\Delta m_K\) consists of the SM and NP contributions:

\[
\Delta m_K = \Delta m_K^\text{SM} + \Delta m_K^\text{NP}.
\] (2.11)

If we parameterize the NP contribution as

\[
\frac{\Delta m_K^\text{NP}}{\Delta m_K^\text{exp}} = \sum_{i=1}^{8} R_i^Z,
\] (2.12)

the right-hand side is estimated as

\[
\begin{align*}
R_1^Z &= 6.43 \times 10^7 \left[ (\text{Re } \Delta_L)^2 - (\text{Im } \Delta_L)^2 \right], \\
R_2^Z &= 6.43 \times 10^7 \left[ (\text{Re } \Delta_R)^2 - (\text{Im } \Delta_R)^2 \right], \\
R_3^Z &= -6.21 \times 10^9 \text{Re } \Delta_L \text{ Re } \Delta_R, \\
R_4^Z &= 6.21 \times 10^9 \text{Im } \Delta_L \text{ Im } \Delta_R.
\end{align*}
\] (2.13)

*3 Charm-quark loops also contribute to \(\Delta_L^\text{SM}\) in Eq. (2.7). In reliable estimations, long-distance effects and dimension-eight operators need to be taken into account [24]. However, such a work is beyond the scope of this letter. Instead, if we estimate the charm-quark loop at the Z-boson mass scale, the real component of \(\Delta_L^\text{SM}\) is suppressed by about 10%.

#4 The SM estimation \(\epsilon_K^\text{SM}\) is sensitive to the CKM elements. If one uses \(V_{cb}\) that is determined by the exclusive \(B \to D^{(*)} \ell\nu\) decays [26], \(\epsilon_K^\text{SM} = (1.73 \pm 0.18) \times 10^{-3}\) is obtained [27]. Then, \(\epsilon_K^\text{NP} = (0.50 \pm 0.18) \times 10^{-3}\) are required at the 1\(\sigma\) level.
The interference terms between the SM and NP contributions are

\[ R^Z_5 = 12.9 \times 10^7 \, \text{Re} \, \Delta^\text{SM}_L \, \text{Re} \, \Delta_L, \quad R^Z_6 = -12.9 \times 10^7 \, \text{Im} \, \Delta^\text{SM}_L \, \text{Im} \, \Delta_L, \]
\[ R^Z_7 = -6.21 \times 10^9 \, \text{Re} \, \Delta^\text{SM}_L \, \text{Re} \, \Delta_R, \quad R^Z_8 = 6.21 \times 10^9 \, \text{Im} \, \Delta^\text{SM}_L \, \text{Im} \, \Delta_R. \quad (2.14) \]

The numerical factors are found in Ref. [15]. The interference terms have been overlooked in the literature.

The experimental result is [10]

\[ \Delta m^\text{exp}_K = (3.484 \pm 0.006) \times 10^{-15} \, \text{GeV}. \quad (2.15) \]

The SM prediction involves a sizable contribution from long-distance effects, and the uncertainty is large. Hence, we simply require that the NP contribution does not exceed the experimental value with allowing the 2\(\sigma\) uncertainty:

\[ |\Delta m^\text{NP}_K| < 3.496 \times 10^{-15} \, \text{GeV}. \quad (2.16) \]

This constraint will turn out to be much weaker than \(\epsilon^\text{NP}_K\).

### 2.2 \(\epsilon'/\epsilon\)

The flavor-changing Z interaction also contributes to \(\Delta S = 1\) observables. The direct CP violation \(\epsilon'/\epsilon\) is shown as

\[ \frac{\epsilon'}{\epsilon} = \left( \frac{\epsilon'}{\epsilon} \right)^\text{SM} + \left( \frac{\epsilon'}{\epsilon} \right)^\text{NP}. \quad (2.17) \]

The NP contribution is estimated as

\[ \left( \frac{\epsilon'}{\epsilon} \right)^\text{NP} = -2.64 \times 10^3 \, B^{(3/2)}_8 \left( \text{Im} \, \Delta_L + \frac{c_W^2}{s_W^2} \text{Im} \, \Delta_R \right). \quad (2.18) \]

The lattice calculation yields \(B^{(3/2)}_8 = 0.76 \pm 0.05\). This formula is provided in Ref. [15], where the terms which are not proportional to \(B^{(3/2)}_8\) are omitted. This approximate result is valid at the 10\% accuracy. The coefficient in the parenthesis is \(c_W^2/s_W^2 \simeq 3.33\). Thus, \(\epsilon'/\epsilon\) can be enhanced easily by \(\Delta^\text{NP}_R\).

As mentioned in Sec. 1, the SM prediction deviates from the experimental result at the 2.8–2.9\(\sigma\) level. In this letter, we require that the discrepancy of \(\epsilon'/\epsilon\) is explained at the 1\(\sigma\) level, and thus, \((\epsilon'/\epsilon)^\text{NP}\) is required to satisfy

\[ 10.0 \times 10^{-4} < \left( \frac{\epsilon'}{\epsilon} \right)^\text{NP} < 21.1 \times 10^{-4}, \quad (2.19) \]

where Ref. [6] is used for the SM prediction.

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#5 The latest lattice simulation, which includes the long-distance contributions, provides \(\Delta m^\text{SM}_K = (3.19 \pm 1.04) \times 10^{-15} \, \text{GeV}\) [28]. However, it is performed on masses of unphysical pion, kaon and charmed quark.
2.3 $K^+ \to \pi^+ \nu \bar{\nu}$ and $K_L \to \pi^0 \nu \bar{\nu}$

The (ultra-) rare kaon decay channels, $K^+ \to \pi^+ \nu \bar{\nu}$ and $K_L \to \pi^0 \nu \bar{\nu}$, are correlated with $\epsilon'/\epsilon$ as well as $\epsilon_K$ and $\Delta m_K$ in the general $Z$ scenario.\[^6\] They are represented as $[15,30]$

$$
\mathcal{B}(K^+ \to \pi^+ \nu \bar{\nu}) = \kappa_+ \left( \frac{\text{Im} X_{\text{eff}}}{\lambda^5} \right)^2 + \left( \frac{\text{Re} \lambda}{\lambda} P_c(X) + \frac{\text{Re} X_{\text{eff}}}{\lambda^5} \right)^2, \quad (2.20)
$$

$$
\mathcal{B}(K_L \to \pi^0 \nu \bar{\nu}) = \kappa_L \left( \frac{\text{Im} X_{\text{eff}}}{\lambda^5} \right)^2. \quad (2.21)
$$

Here, $X_{\text{eff}}$ satisfies

$$
X_{\text{eff}} = \lambda_t (1.48 \pm 0.01) + 2.51 \times 10^2 (\Delta_L + \Delta_R), \quad (2.22)
$$

where the first term in the right-hand side is the SM contribution. Also, $\lambda = |V_{us}|$, $\kappa_+ = (5.157 \pm 0.025) \times 10^{-11} (\lambda/0.225)^5$, and $\kappa_L = (2.231 \pm 0.013) \times 10^{-10} (\lambda/0.225)^8$. The charm-quark contribution is $P_c(X) = (9.39 \pm 0.31) \times 10^{-4}/\lambda^4 + (0.04 \pm 0.02)$. Using the CKMfitter result for the CKM elements, we obtain

$$
\text{Re} X_{\text{eff}} = -4.83 \times 10^{-4} + 2.51 \times 10^2 (\text{Re} \Delta_L + \text{Re} \Delta_R), \quad (2.23)
$$

$$
\text{Im} X_{\text{eff}} = 2.12 \times 10^{-4} + 2.51 \times 10^2 (\text{Im} \Delta_L + \text{Im} \Delta_R). \quad (2.24)
$$

Also, the SM predictions are

$$
\mathcal{B}(K^+ \to \pi^+ \nu \bar{\nu})_{\text{SM}} = (8.5 \pm 0.5) \times 10^{-11}, \quad (2.25)
$$

$$
\mathcal{B}(K_L \to \pi^0 \nu \bar{\nu})_{\text{SM}} = (3.0 \pm 0.2) \times 10^{-11}. \quad (2.26)
$$

On the other hand, the experimental results are $[31,32]$

$$
\mathcal{B}(K^+ \to \pi^+ \nu \bar{\nu})_{\text{exp}} = (17.3^{+11.5}_{-10.5}) \times 10^{-11}, \quad (2.27)
$$

$$
\mathcal{B}(K_L \to \pi^0 \nu \bar{\nu})_{\text{exp}} \leq 2.6 \times 10^{-8}. \quad [90\% \text{ C.L.}] \quad (2.28)
$$

Although the current constraints on the NP contributions are very weak, their measurements will be improved significantly in the near future. The NA62 experiment at CERN, which already started the physics run at low beam intensity in 2015, has a potential to measure $\mathcal{B}(K^+ \to \pi^+ \nu \bar{\nu})$ at the 10% precision by 2018 $[33]$. The KOTO experiment at J-PARC is designed to improve the sensitivity for $\mathcal{B}(K_L \to \pi^0 \nu \bar{\nu})$, which enables us to measure it at the 10% level of the SM value $[34,35]$. As one can see from Eqs. (2.18) and (2.22), the NP contributions to $\mathcal{B}(K \to \pi \nu \bar{\nu})$ are correlated with those to $\epsilon'/\epsilon$ in the general $Z$ scenario. Thus, if the $\epsilon'/\epsilon$ discrepancy is a signal of the scenario, these experiments would detect NP effects.

\[^6\] The branching ratios of $K \to \pi^\pm \ell^- \ (\ell = e, \mu)$ are also affected in the general $Z$ scenario. However, $K^+ \to \pi^+ \ell^+ \ell^-$ and $K_S \to \pi^0 \ell^+ \ell^-$ are dominated by a long-distance contribution through $K \to \pi \gamma^* \to \pi \ell^+ \ell^- \ [29]$. On the other hand, such a contribution to $K_L \to \pi^0 \ell^+ \ell^-$ is forbidden by the CP symmetry, but is dominated by an indirect CP-violating contribution, $K_L \to K_S \to \pi^0 \ell^+ \ell^- \ [29]$. Therefore, it is challenging to discuss short-distance NP contributions in these channels.
2.4 $K_L \to \mu^+ \mu^-$

The branching ratio of $K_L \to \mu^+ \mu^-$ is also sensitive to the NP contributions to the flavor-changing $Z$ couplings. Theoretically, only the short-distance (SD) contributions can be calculated reliably. They are shown as [15,36,37]

$$B(K_L \to \mu^+ \mu^-)_{SD} = \kappa_{\mu} \left( \frac{\text{Re} \lambda c P_c(Y)}{\lambda} + \frac{\text{Re} Y_{eff}^\lambda}{\lambda^5} \right)^2,$$

(2.29)

where $\kappa_{\mu} = (2.01 \pm 0.02) \cdot 10^{-9} (\lambda/0.225)^8$. The charm-quark contribution is $P_c(Y) = (0.115 \pm 0.018) \cdot (0.225/\lambda)^4$. Using the CKMFITTER result, we obtain

$$\text{Re} Y_{eff} = -3.07 \times 10^{-4} + 2.51 \times 10^2 \left( \text{Re} \Delta_L - \text{Re} \Delta_R \right),$$

(2.30)

where the first term in the right-hand side is the SM contribution, and the minus sign between $\Delta_L$ and $\Delta_R$ is due to the axial-vector current. The SM value is obtained as

$$B(K_L \to \mu^+ \mu^-)_{SD, SM} = (0.83 \pm 0.10) \times 10^{-9}. \tag{2.31}$$

On the other hand, it is challenging to extract a short-distance part in the experimental data $B(K_L \to \mu^+ \mu^-)_{exp} = (6.84 \pm 0.11) \cdot 10^{-9}$ [10], because of huge long-distance contributions through $K_L \to \gamma^* \gamma^* \to \mu^+ \mu^-$ [38]. An upper bound on the short-distance contribution is [38]

$$B(K_L \to \mu^+ \mu^-)_{SD} < 2.5 \times 10^{-9}. \tag{2.32}$$

Since the constraint is much weaker than the SM uncertainties, we ignore them for simplicity and impose a bound on the $Z$ couplings,

$$-1.08 \times 10^{-6} < \text{Re} \Delta_L - \text{Re} \Delta_R < 4.05 \times 10^{-6}. \tag{2.33}$$

The real parts of the NP contributions are constrained by $B(K_L \to \mu^+ \mu^-)$.

3 Analysis

In this section, we examine the general $Z$ scenario quantitatively. Although the discrepancy of $\epsilon'/\epsilon$ could be explained by the scenario, the parameter regions would be constrained by $\epsilon_K$, $\Delta m_K$ and $K_L \to \mu^+ \mu^-$. In particular, the interference between the SM and NP contributions, Eq. (2.7), enhances $\epsilon_K$ significantly. Consequently, wide parameter regions will be excluded. Therefore, the discrepancy of $\epsilon'/\epsilon$ will be explained by tuning the model parameters such that the limit of $\epsilon_K$ is satisfied. Let us introduce a quantity which parameterizes the tuning:

$$\xi = \max(\xi_1, \xi_2, \ldots, \xi_8), \quad \text{with} \quad \xi_i = \left| \frac{(\epsilon_K)^Z_i}{\epsilon_K^{NP}} \right|.$$

(3.1)
For instance, if $\epsilon_K^\text{NP}$ is dominated by a single term, one obtains $\xi \simeq 1$ and there is no tuning in the model parameters. If the maximal value of $(\epsilon_K)_i^Z$ is about ten times larger than $\epsilon_K^\text{NP}$, $\xi \sim 10$ is obtained; the model parameters are tuned such that there is a cancellation among $(\epsilon_K)_i^Z$ at the 10% level.

### 3.1 Simplified scenarios

First, we consider the following simplified scenarios (c.f., Ref. [39]),

- left-handed scenario (LHS): $\Delta_R = 0$,
- right-handed scenario (RHS): $\Delta_L = 0$,
- pure imaginary scenario (ImZS): $\text{Re} \Delta_L = \text{Re} \Delta_R = 0$,
- left-right symmetric scenario (LRS): $\Delta_L = \Delta_R$.

- This scenario is realized by chargino contributions to the $Z$ penguin in the supersymmetric model [17, 19, 40–42].
- Such a setup is provided by Randall-Sundrum models with custodial protection [43].
- In axial-symmetric scenarios, $\Delta_L = -\Delta_R$, there are no NP contributions to $K \to \pi \nu \bar{\nu}$. 

Figure 1. The $Z$-penguin observables are displayed in LHS (left panel) and RHS (right). In the green (light green) regions, the $\epsilon'/\epsilon$ discrepancy is explained at 1 (2) $\sigma$. The blue and the orange shaded regions are excluded by $\epsilon_K$ and $\mathcal{B}(K_L \to \mu^+ \mu^-)$, respectively. The ratios of $\mathcal{B}(K_L \to \pi^0 \nu \bar{\nu})/\mathcal{B}(K_L \to \pi^0 \nu \bar{\nu})_{\text{SM}}$ and $\mathcal{B}(K^+ \to \pi^+ \nu \bar{\nu})/\mathcal{B}(K^+ \to \pi^+ \nu \bar{\nu})_{\text{SM}}$ are shown by the red and black dashed contours, respectively.
As shown below, these scenarios do not require large parameter tuning in $\epsilon_{K}^{\text{NP}}$. However, $\mathcal{B}(K \to \pi \nu \bar{\nu})$ will turn out to be small.

In Fig. 1, the $Z$-penguin observables are shown as functions of $\Delta_{L,R}$ for LHS and RHS. In the green (light green) regions, the $\epsilon'/\epsilon$ discrepancy is explained at 1 (2)$\sigma$. They depend only on the imaginary component of $\Delta_{L,R}$. One can see that $\epsilon'/\epsilon$ is enhanced by the right-handed $Z$ coupling, $\Delta_{R}$, more than $\Delta_{L}$.

The blue regions are excluded by the $\epsilon_{K}$, and the orange regions by the $\mathcal{B}(K_{L} \to \mu^{+}\mu^{-})$. It is clear that the constraint from $\epsilon_{K}$ is much severer in RHS than LHS due to the interference contributions, Eq. (2.7). There is no constraint from $\Delta m_{K}$ in the parameter regions of the plots.

The red and black dashed contours represent $\mathcal{B}(K_{L} \to \pi^{0}\nu \bar{\nu})/\mathcal{B}(K_{L} \to \pi^{0}\nu \bar{\nu})_{\text{SM}}$ and $\mathcal{B}(K^{+} \to \pi^{+}\nu \bar{\nu})/\mathcal{B}(K^{+} \to \pi^{+}\nu \bar{\nu})_{\text{SM}}$, respectively. Here and hereafter, $\mathcal{B}(K_{L} \to \pi^{0}\nu \bar{\nu})_{\text{SM}}$ and $\mathcal{B}(K^{+} \to \pi^{+}\nu \bar{\nu})_{\text{SM}}$ denote the central values of the SM predictions, Eqs. (2.25) and (2.26). It is found that $\mathcal{B}(K_{L} \to \pi^{0}\nu \bar{\nu})$ cannot be as large as the SM value as long as $\epsilon'/\epsilon$ is explained in LHS or RHS. On the other hand, if the $\epsilon'/\epsilon$ discrepancy is explained by LHS, the NP contribution to $\mathcal{B}(K^{+} \to \pi^{+}\nu \bar{\nu})$ is limited by $\mathcal{B}(K_{L} \to \mu^{+}\mu^{-})$. In contrast, $\epsilon_{K}$ constrains RHS.

Next, we consider $\text{Im} Z S$. The situation is often considered to amplify $(\epsilon'/\epsilon)_{\text{NP}}$ but suppress $\epsilon_{K}^{\text{NP}}$. In Fig. 2, the $Z$-penguin observables are shown as functions of $\text{Im} \Delta_{L,R}$. The most severe constraint is from $\epsilon_{K}$ due to the interference between the SM and NP contributions, while the other bounds are weak and absent in the plot. Since there are no real components of $\Delta_{L,R}$, $\mathcal{B}(K^{+} \to \pi^{+}\nu \bar{\nu})$ is correlated with $\mathcal{B}(K_{L} \to \pi^{0}\nu \bar{\nu})$. 

Figure 2. The $Z$-penguin observables are displayed in $\text{Im} Z S$. Notations of the lines and shaded regions are the same as in Fig. 1.
Figure 3. The $Z$-penguin observables are displayed in LRS. Notations of the lines and shaded regions are the same as in Fig. 1.

Finally, LRS is shown in Fig. 3. Similarly to the cases of RHS and ImZS, most of the parameter regions are excluded by $\epsilon_K$. It is noticed that both $\epsilon^'/\epsilon$ and $\epsilon_K$ are (accidentally) consistent with the experimental results even when $\text{Re } \Delta_{L,R}$ is large. The NP contributions to $\mathcal{B}(K_L \to \mu^+\mu^-)$ vanish because the process is the axial-vector current. Then, $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ can be as large as the experimental limit.

In Fig. 4, contours of the tuning parameter $\xi$ are shown for LHS, RHS and ImZS on the plane of the branching ratios of $K \to \pi\nu\bar{\nu}$. We scanned the whole parameter space of $\Delta_{L,R}$ in each scenario and selected the parameters where $\epsilon^'/\epsilon$ is explained at the 1$\sigma$ level, and the experimental bounds from $\epsilon_K$, $\Delta m_K$, and $\mathcal{B}(K_L \to \mu^+\mu^-)$ are satisfied (see the previous section for the experimental constraints). Then, $\xi$ was estimated at each point. Several parameter sets predict the same $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ and $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$. Among them, the smallest $\xi$ is chosen in Fig. 4 for given $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ and $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$. Hence, one can read at least how large tuning is required. In the allowed parameter space, $\xi = O(1)$. Thus, there is no tight tuning in these scenarios.

From the figure, one finds that $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$ is smaller than the SM value by more than 10%. Hence, the scenarios could be tested by the KOTO experiment. On the other hand, $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ depends on the scenarios. In LHS, we obtain $0 < \mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})/\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})_{\text{SM}} < 2$, and tighter tunings are required for smaller branching ratios. In RHS, $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ is comparable to or larger than the SM value, but cannot be twice as large. In ImZS, the branching ratios are perfectly correlated and displayed by a line in Fig. 4. Then, $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ is not far away from the SM one.
Figure 4. Contours of the tuning parameter $\xi$ are shown in LHS, RHS and ImZS. Here, “SM” in the axis labels denotes the central values of $B(K_L \to \pi^0 \nu \bar{\nu})_{SM}$ and $B(K^+ \to \pi^+ \nu \bar{\nu})_{SM}$, Eqs. (2.25) and (2.26). In the colored regions, $\epsilon'/\epsilon$ is explained at 1$\sigma$, and the experimental bounds of $\epsilon_K$, $\Delta m_K$, and $B(K_L \to \mu^+ \mu^-)$ are satisfied. The right region of the blue dashed line is allowed by the measurement of $B(K^+ \to \pi^+ \nu \bar{\nu})$ at 1$\sigma$.

Fig. 5 is a result of the tuning parameter $\xi$ in LRS. The whole parameter region of the model is scanned in a similar way as Fig. 4. It is found that $B(K_L \to \pi^0 \nu \bar{\nu})$ does not exceed about a half of the SM value, while $B(K^+ \to \pi^+ \nu \bar{\nu})$ can be large without introducing a tight tuning. Thus, the scenario could be tested by the NA62 experiment easily.

### 3.2 General scenario

Let us consider the full parameter space in the general $Z$ scenario. Both $\Delta_L$ and $\Delta_R$ are turned on. Then, $B(K^+ \to \pi^+ \nu \bar{\nu})$ and/or $B(K_L \to \pi^0 \nu \bar{\nu})$ can be enhanced by allowing the tuning for $\epsilon_K^{NP}$.

In Fig. 6, the branching ratios of $K \to \pi \nu \bar{\nu}$ and the tuning parameter are shown for $(\epsilon'/\epsilon)^{NP} = 15.5 \cdot 10^{-4}$ and $\epsilon_K^{NP} = 0.37 \cdot 10^{-3}$. It is found that $B(K_L \to \mu^+ \mu^-)$ and the tuning parameter restrict the flavor-changing $Z$ couplings, and thus, the NP contributions to $K \to \pi \nu \bar{\nu}$.

In Fig. 7, contours of the tuning parameter $\xi$ are shown. The whole parameter space of the general $Z$ scenario is scanned. In the colored regions, $\epsilon'/\epsilon$ is explained
at the 1σ level, and the experimental bounds of \( \epsilon_K, \Delta m_K \), and \( \mathcal{B}(K_L \to \mu^+\mu^-) \) are satisfied (see the previous section for the experimental constraints). For given \( \mathcal{B}(K^+ \to \pi^+\nu\bar{\nu}) \) and \( \mathcal{B}(K_L \to \pi^0\nu\bar{\nu}) \), the smallest \( \xi \) is chosen among the parameter sets which predict the same branching ratios.

It is found that the regions with \( \xi \gtrsim 10 \) are almost disfavored by the current measurement of \( \mathcal{B}(K^+ \to \pi^+\nu\bar{\nu}) \) except for those close to the Grossman-Nir bound \([44]\). Compared to the simplified cases in Fig. 4, \( \mathcal{B}(K_L \to \pi^0\nu\bar{\nu}) \) can be enhanced. It can be as large as \( \mathcal{B}(K_L \to \pi^0\nu\bar{\nu}) \approx 6 \times 10^{-11} \) and \( 9 \times 10^{-10} \) for \( \xi \approx 2 \) and 10, respectively. In other words, \( \mathcal{O}(10)\% \) tunings are required to enhance \( \mathcal{B}(K_L \to \pi^0\nu\bar{\nu}) \) by an order of magnitudes compared the SM prediction, as long as \( \epsilon'/\epsilon \) is explained and the bound of \( \epsilon_K \) is satisfied in the general \( Z \) scenario. The KOTO experiment can probe such large branching ratios in the near future.

4 Conclusion

The recent discrepancy of \( \epsilon'/\epsilon \) may be a sign of the NP contribution to the flavor-changing \( Z \) coupling. In this letter, we revisited the scenario with appropriately taking account of the interference effects between the SM and NP contributions to the \( \Delta S = 2 \) observables. It was shown that the effects tend to enhance \( \epsilon_K \) when the

\[
\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu}) \quad \text{at 1σ level, and the experimental bounds of } \epsilon_K, \Delta m_K, \text{ and } \mathcal{B}(K_L \to \mu^+\mu^-) \text{ are satisfied (see the previous section for the experimental constraints). For given } \mathcal{B}(K^+ \to \pi^+\nu\bar{\nu}) \text{ and } \mathcal{B}(K_L \to \pi^0\nu\bar{\nu}), \text{ the smallest } \xi \text{ is chosen among the parameter sets which predict the same branching ratios.}
\]

It is found that the regions with \( \xi \gtrsim 10 \) are almost disfavored by the current measurement of \( \mathcal{B}(K^+ \to \pi^+\nu\bar{\nu}) \) except for those close to the Grossman-Nir bound \([44]\). Compared to the simplified cases in Fig. 4, \( \mathcal{B}(K_L \to \pi^0\nu\bar{\nu}) \) can be enhanced. It can be as large as \( \mathcal{B}(K_L \to \pi^0\nu\bar{\nu}) \approx 6 \times 10^{-11} \) and \( 9 \times 10^{-10} \) for \( \xi \approx 2 \) and 10, respectively. In other words, \( \mathcal{O}(10)\% \) tunings are required to enhance \( \mathcal{B}(K_L \to \pi^0\nu\bar{\nu}) \) by an order of magnitudes compared the SM prediction, as long as \( \epsilon'/\epsilon \) is explained and the bound of \( \epsilon_K \) is satisfied in the general \( Z \) scenario. The KOTO experiment can probe such large branching ratios in the near future.

4 Conclusion

The recent discrepancy of \( \epsilon'/\epsilon \) may be a sign of the NP contribution to the flavor-changing \( Z \) coupling. In this letter, we revisited the scenario with appropriately taking account of the interference effects between the SM and NP contributions to the \( \Delta S = 2 \) observables. It was shown that the effects tend to enhance \( \epsilon_K \) when the
right-handed coupling is turned on. Consequently, $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$ is smaller than the SM prediction in the simplified scenarios as long as $\epsilon'/\epsilon$ is explained.

In the general $Z$ scenario, $\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$ can be larger by allowing parameter tunings. It is found that we require tunings at the $\mathcal{O}(10)$% level to enhance the branching ratio by an order of magnitudes compared to the SM prediction. In fact, it can be as large as $6 \times 10^{-11}$ and $9 \times 10^{-10}$ for $\xi \simeq 2$ and 10, respectively. This implies that the NP contributions to $\epsilon_K$ are tuned at the 50% and 10% levels, respectively. The KOTO experiment could probe such large branching ratios in the near future. Then, the signal would provide fruitful information for the UV completion.

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Figure 7. Contours of the tuning parameter $\xi$ are shown in the general $Z$ scenario. In the colored regions, $\epsilon'/\epsilon$ is explained at $1\sigma$, and the experimental bounds of $\epsilon_K$, $\Delta m_K$, and $B(K_L \to \mu^+\mu^-)$ are satisfied. The region between the blue dashed lines is allowed by $B(K^+ \to \pi^+\nu\bar{\nu})$ at $1\sigma$. There are no available model parameters above the Grossman-Nir bound, and the rightmost uncolored region is excluded by $B(K_L \to \mu^+\mu^-)$.

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