Intertwined spin, charge and pair correlations in the two-dimensional Hubbard model in the thermodynamic limit

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This manuscript was compiled on February 11, 2022

The high-temperature superconducting cuprates are governed by intertwined spin, charge, and superconducting orders. While various state-of-the-art numerical methods have demonstrated that these phases also manifest themselves in doped Hubbard models, they differ on which is the actual ground state. Finite cluster methods typically indicate that stripe order dominates while embedded quantum cluster methods, which access the thermodynamic limit by treating long-range correlations with a dynamical mean field, conclude that superconductivity does. Here, we report the observation of fluctuating spin and charge stripes in the doped single-band Hubbard model using a quantum Monte Carlo dynamical cluster approximation (DCA) method. By resolving both the fluctuating spin and charge orders using DCA, we demonstrate that they survive in the doped Hubbard model in the thermodynamic limit. This discovery also provides a new opportunity to study the influence of fluctuating stripe correlations on the model’s pairing correlations within a unified numerical framework. Using this approach, we also find evidence for pair-density-wave correlations whose strength is correlated with that of the stripes.

Hubbard model | stripe | dynamical cluster approximation | ...

A common element of strongly correlated materials is the existence of several nearly degenerate states, which compete or cooperate to produce novel phases of matter \cite{1}. For example, in the high-temperature (high-$T_c$) superconducting cuprates, multiple theoretical studies and experiments point to intertwined orders of spin and charge stripes, charge- and pair-density-waves, and unconventional superconductivity \cite{1–7}. Understanding the relationships among these orders and how they shape the cuprate phase diagram is a central problem in condensed matter physics.

Addressing this question using nonperturbative methods remains challenging as even the simplest correlated electron models also contain near-degenerate orders, which can be difficult to discern from one another. For example, state-of-the-art numerical studies have identified a plethora of low-energy states in the single-band Hubbard and $t$-$J$ models that contend for the ground state \cite{5, 6, 8–25}. Methods like Hartree-Fock mean-field theory \cite{10–12}, density matrix renormalization group (DMRG) \cite{5, 13–16}, density matrix embedding theory \cite{13, 17}, variational Monte Carlo \cite{18}, auxiliary field quantum Monte Carlo \cite{5, 6, 13, 19}, and infinite projected entangled-pair states \cite{13, 20} tend to find static stripe order, i.e. unidirectional spin and charge density waves, as the ground state, and recent determinant quantum Monte Carlo (DQMC) calculations \cite{8, 9} and minimally-entangled typical thermal states (METTS) \cite{25} have found evidence for spin stripe correlations at finite temperatures. In contrast, quantum cluster methods like cellular dynamical mean-field theory \cite{24, 26, 27} and the dynamical cluster approximation (DCA) \cite{28, 29}, which, unlike finite size cluster techniques, directly access the thermodynamic limit, typically find superconducting solutions with a $d$-wave symmetry \cite{21–24}. While approximate calculations of this sort with large unit cells but correlations restricted to single sites or small clusters have found evidence of stripes \cite{30–33}, more reliable calculations with clusters large enough to accommodate the stripe periodicity have yet to find any indication of stripe-like solutions. It is crucial that we understand and resolve these differences to properly identify the properties of the Hubbard model in the thermodynamic limit. Moreover, this dichotomy has made it difficult to understand the relationships between the relevant orders since each method has its approximations, which introduce systematic errors that can bias towards particular solutions. To overcome this issue, it is desirable to identify a single framework capable of identifying the relevant states to avoid compounding systematic biases.

Here, we demonstrate that quantum Monte Carlo DCA \cite{28} methods can detect short-range stripe correlations in the two-dimensional single-band Hubbard model. The QMC-based impurity solver captures the intra-cluster correlations exactly, DCA Simulations: PM; DQMC Simulations: PM; Code Development: PM, SK, GB, SJ, and TAM; Supervision: SJ and TAM; Funding acquisition: SJ and TAM; Writing original draft: PM, SJ, and TAM; Writing review & editing: All authors.

The authors declare no competing interests.

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Significance Statement

The high-temperature superconducting cuprates are governed by intertwined striped magnetic and charge orders in addition to superconductivity. Remarkably similar behavior has also been seen in numerical calculations for the Hubbard model describing the copper-oxygen layers in these materials. Finite cluster methods typically find that spin and charge stripe order dominates, while embedded quantum cluster methods, which access the thermodynamic limit, often conclude that superconductivity does. Here, we report the observation of fluctuating spin and charge stripes in an embedded cluster calculation for the Hubbard model. This discovery demonstrates that striped states survive in the thermodynamic limit and allows us to study their influence on the model’s superconducting properties, where we find evidence for pair-density-wave correlations intertwined with the stripe correlations.
while longer-range correlations are treated in a mean-field that approximates the infinite system. The observation of fluctuating stripes with this method therefore provides crucial confirmation that such correlations persist in the thermodynamic limit. It also fixes the discrepancy between finite-cluster and quantum cluster methods. Our results allow us to examine the relevant periodicity such that the spatial modulations are not averaged out by the DCA mean field. Using this unified framework, we find evidence for short-range pair-density-wave (PDW) correlations, whose strength is correlated with the strength of the spin and charge stripe correlations.

**Model**

We consider the two-dimensional single-band Hubbard Hamiltonian defined on a rectangular lattice $N = N_x \times N_y$

$$H = - \sum_{i,j,\sigma} t_{ij} \left( c_{i \sigma}^\dagger c_{j \sigma} + \text{h.c.} \right) - \mu \sum_{i,\sigma} n_{i \sigma} + U \sum_{i} n_{i \uparrow} n_{i \downarrow}. \quad [1]$$

Here, $c_{i \sigma}^\dagger$ ($c_{i \sigma}$) creates (annihilates) a spin-$\sigma$ electron on site $i$; $n_{i \sigma} = c_{i \sigma}^\dagger c_{i \sigma}$ is the number operator; $t_{ij}$ is the hopping integral between sites $i$ and $j$; $\mu$ is the chemical potential; and $U$ is the on-site Hubbard repulsion. Throughout, we restrict $t_{ij}$ to nearest- ($t$) and next-nearest-neighbor ($t'$) hopping only and set $U = 6t$ to facilitate comparisons to Ref. [8]. We then solved Eq. (1) using DCA (28) and with a continuous-time QMC impurity solver (23), as implemented in the DCA++ code (34), and complementary DQMC calculations (35, 36).

Previous DQMC calculations (8, 9) have demonstrated that the high-temperature spin-stripe correlations in the single- and multi-band Hubbard model are fluctuating in nature, where short-range spatial correlations appear over several unit cells and are fluctuating in time. In general, static ($\omega = 0$) correlation functions, which integrate over the imaginary time dynamics, are expected to be more sensitive to fluctuating short-range order (37) compared to the corresponding equal-time ($\tau = 0$) correlation functions (8). We have found that this is indeed the case for the spin, charge, and PDW-like correlations reported here (see SI Appendix), supporting the interpretation that the observed stripe correlations are fluctuating in nature.

To study the fluctuating spin stripes, we measured the static staggered spin-correlation function $S(t,\omega = 0) = (-1)^{x+y+\sigma} \frac{1}{T} \int_0^T \sum_{\tau} \langle \xi_{i}^{(\sigma)}(\tau) \xi_{j}^{(\sigma)}(0) \rangle d\tau$, where $\xi_{i}^{(\sigma)} = \frac{1}{2} (c_{i \uparrow}^{\dagger} c_{i \downarrow} - c_{i \downarrow}^{\dagger} c_{i \uparrow})$ is the $z$-component of the local spin operator at site $i$. The fluctuating charge stripe correlations are assessed by measuring the static density-density correlation function $N(r,\omega = 0)^{\dagger} = \frac{1}{T} \sum_{\tau} \langle n_{i+x}(\tau) n_{i+y}(0) - \langle n_{i+x}(\tau) \rangle \langle n_{i+y}(0) \rangle \rangle d\tau$, where $n_{i} = \sum_{\sigma} n_{i \sigma}$ is the local density operator. The pairing tendencies are accessed by measuring the static pairing correlation function in the $d$-wave channel $P_{d}(r,\omega = 0) = \frac{1}{T} \sum_{\tau} \langle \Delta_{i}^{\dagger}(\tau) \Delta_{j}^{\dagger}(0) \rangle d\tau$, where $\Delta_{i} = c_{i \uparrow}^{\dagger} (c_{i \downarrow}^{\dagger} + c_{i \downarrow} - c_{i \downarrow}^{\dagger} c_{i \downarrow} - c_{i \downarrow}^{\dagger} c_{i \downarrow}^{\dagger} + c_{i \uparrow}^{\dagger} c_{i \downarrow} - c_{i \uparrow}^{\dagger} c_{i \uparrow} - c_{i \downarrow}^{\dagger} c_{i \uparrow})$ destroys a singlet pair of electrons with $d$-wave symmetry. We also determined the structure of the pairing interaction by explicitly solving the Bethe-Salpeter equation (BSE) in the particle-particle singlet channel to obtain its leading eigenvalues and eigenvectors (22, 38). Due to the large cluster sizes and the self-consistency loop, our DCA calculations are significantly more expensive than the corresponding DQMC calculations. For this reason, we focus on an average density $\langle n \rangle = 0.8$, where we have observed strong stripe correlations.
We have observed similar patterns for different cluster sizes and geometries, including staggered spin-spin correlation functions, the zero-frequency (Fig. 2) and short-range charge stripe-like pattern laying on both sides. As a comparison, the stripe-like pattern in the thermodynamic limit. Nevertheless, the observation of stripes with DCA provides crucial evidence that they persist in the thermodynamic limit.

Although we observe similar fluctuating spin stripes in both zero-frequency (Fig. 1) and equal-time (SI Appendix) spin-spin correlation functions, the zero-frequency (Fig. 2) and equal-time (SI Appendix) density-density correlation functions show qualitatively different behaviors. The density-density correlations have much stronger imaginary time dependence. A detailed imaginary time analysis can be found in SI Appendix. As a result, the fluctuating charge stripe pattern is only observed in the static ($\omega = 0$) correlation functions. Fig. 2 plots $N(r, \omega = 0)$ for the same $16 \times 4$ DCA and DQMC simulations shown in Fig. 1. We observe the central blue region surrounded by two red regions on both sides, signaling short-range charge stripe fluctuations but with a shorter period. As $t'/t$ increases from negative to positive, unlike the spin case where the blue region extends to weaken the stripe, the charge blue region does not extend. In the electron-doped case ($t' = 0.2t$), however, the central region is dominated by a staggered $(\pi, \pi)$ CDW-like correlation, with the sub-dominant stripe-like pattern laying on both sides. As a comparison, the DQMC result in Fig. 2c shows a stronger stripe pattern with a shorter period than the corresponding DCA result in Fig. 2b.

The presence of the spin and charge stripes is more readily observed by examining the dynamical spin $S(Q, \omega)$ and charge $N(Q, \omega)$ susceptibilities (SI Appendix), which measure the collective fluctuations. Here, we consider the static limit ($\omega = 0$), which can be obtained by Fourier transforming the corresponding static real-space correlation functions. Fig. 3 summarizes $S(Q, \omega = 0)$ along $Q = (Q_x, \pi)$ (top row) and $N(Q, \omega = 0)$ approaches employing extended clusters. The origin of the reduced correlations is unclear at this time. One possibility is that the correlations observed by DQMC would weaken as the cluster size increases. Another is that the mean field reduces the effective correlations in the DCA treatment of the problem or tends to favor uniform states and restore $C_4$ symmetry in the cluster. Nevertheless, the observation of stripes with DCA provides crucial evidence that they persist in the thermodynamic limit.

Results and Discussion

Figure 1 plots $S_{\text{stag}}^{\text{ag}}(r)$ for several values of the next-nearest neighbor hopping $t'$, where we see clear evidence for spin stripe correlations in our DCA calculations (Panels a, b, d-f). Here, we employ large $16 \times 4$ clusters embedded in the DCA self-consistent mean-field, where we access temperatures as low as $T = 0.167t$ (inverse temperature $\beta = 6/t$). Since the staggered spin-spin correlation function imposes a sign flip on every other site, the positive blue regions in the middle of each panel represent short-range antiferromagnetic (AFM) correlations. In contrast, the negative red regions represent AFM regions but with a $\pi$ phase shift. As $t'/t$ decreases from positive to negative, red negative regions form more prominently on both sides of the central blue region, signaling the formation and growth of AFM stripe fluctuations, similar to those observed in finite size DQMC calculations (8, 9). In general, we find that the boundary between the red and blue regions mixes, suggesting that the stripes are incommensurate. We have observed similar patterns for different cluster sizes and geometries, including $8 \times 8, 8 \times 6$ and $8 \times 4$ clusters, and in the spin-\textit{xx} correlations (see SI Appendix).

For comparison, Fig. 1c shows $S_{\text{stag}}^{\text{ag}}(r, \omega = 0)$ obtained from a DQMC calculation at $T = 0.22t$ ($\beta = 4.5/t$), $t' = -0.25t$. The DQMC results are consistent with the corresponding DCA results (Fig. 1b). (A systematic comparison between DCA and an earlier DQMC study (8) focusing on the real-space equal-time spin-spin correlation function at the density $n = 0.875$ is given in SI Appendix.) Since DQMC treats the system exactly on an extended but finite cluster, one must perform a finite-size scaling analysis to access the thermodynamic limit. On the other hand, DCA accesses the thermodynamic limit by embedding its clusters in a dynamical mean field that approximates the rest of the system. Comparing Figs. 1b and 1c, we find that DCA predicts weaker stripe correlations compared to DQMC for the same $t'$, despite the lower temperature. This observation may help explain why stripes have previously gone unobserved in quantum cluster

Fig. 2. The real-space static density-density correlation function of the single-band Hubbard model, obtained from DCA and DQMC simulations. Results are shown for parameters in a one-to-one correspondence with those shown in Fig. 1.
along $Q = (Q_x, 0)$ (bottom row) for different values of the next-nearest-neighbor hopping $t'$. When the spin stripe correlations are strong, they should manifest as incommensurate peaks in $S(Q, 0)$ centered at $(2\pi/a)(0.5 \pm \delta_s, 0.5)$ while the charge stripes should manifest as incommensurate peaks in $N(Q, 0)$ centered at $(2\pi/a)(0.5 \pm \delta_c, 0)$, where $\delta_s = 2\delta_c$. To check this, we fit the spectra with pairs of Lorentzian functions plus a constant background and extracted the corresponding values of $\delta_s, \delta_c$. In all cases, the susceptibilities are well represented by the fits, and the resulting ratio $r = \delta_s / \delta_c$ is given in the top panels of Fig. 3, where we find $0.66 > r > 0.3$. Especially in the cases with $t' < 0$, these values are close to the expected $r = 0.5$ result, for which the periodicity of the charge fluctuations is twice that of the spin fluctuations.

To examine how the spin and charge stripes form, we also extracted the values of $\delta_s, \delta_c$, and $r = \delta_s / \delta_c$ as a function of temperature for $t' = -0.3t$, as shown in Fig. 3k. (The corresponding susceptibility data is provided in SI Appendix.) We find that $\delta_s$ remains relatively fixed as a function of temperature, while $\delta_c$ appears to lock in to its value of $\delta_c \approx 2\delta_s$ as the temperature is lowered. From Figs. 1-3, we can see that the incommensurability $\delta_s$ (or period) of the fluctuating spin stripes depends weakly on temperature but strongly on $t'$, while the incommensurability $\delta_c$ (or period) of the fluctuating charge stripes depends strongly on temperature but weakly on $t'$. Fig. 3l and Fig. 3m show the correlation lengths $\xi_s$ and $\xi_c$ for $t' = -0.3t$, for the fluctuating spin and charge stripes, respectively. These values are determined from the inverse of the half-width-half-maximum of the Lorentzian fits, but we obtain very similar estimates from exponential fits of the real-space correlations. We find that both the spin and charge correlation lengths extracted this way are relatively short at the temperatures we can access but grow with decreasing temperature. We also find that the charge correlation length is substantially shorter than the spin correlation length. These results suggest that in the hole-doped case, the charge stripes emerge at lower energy scales than spin stripes do in the Hubbard model, and that the periodicity of the former locks into the value set by the latter. This finding is counter to the idea that the charge order forms prior to the spin order (39, 40).

In the electron-doped case, the comparison between Figs. 1f and 2f shows that the charge stripe appears at higher energy scale instead.

Previous DCA studies have observed a finite-temperature transition to the $d$-wave superconducting state (21, 38). Now that DCA also finds evidence for fluctuating stripes, both in the spin and charge sector, it is natural to ask how they affect the formation of Cooper pairs. To answer this question, we examine the static $d$-wave pairfield susceptibility $P_d(r, \omega = 0)$ calculated with DCA on a $16 \times 4$ cluster as shown in Fig. 4. Interestingly, as $t'$ is reduced and varied to more negative values, the static pairfield correlations develop a modulated striped pattern with a sign change ($\pi$ phase shift) suggestive of a pair-density-wave (3). This trend is similar to that found for the spin stripe fluctuations in Fig. 1, except that for $t' = -0.25t$ the modulation is less visible possibly due to a change in the Fermi surface topology from electron- to hole-like (41). Fig. 5 shows the corresponding plots of the Fourier-transformed static $d$-wave pairfield susceptibility $P_d(Q, \omega = 0)$ fitted with a pair of Lorentzian functions. For $t' = 0.2t$, we observe a single peak at $Q = (0, 0)$. But as $t'$ is lowered and varied to more negative values, one sees that the peak flattens out. In this case, the best fit is obtained with two separate Lorentzians centered at $(2\pi/a)(0.5 \pm \delta_p, 0)$, consistent with the periodic pair-
density-wave like modulation observed in Fig. 4.

These observations suggest that striped modulations in the pairfield correlations develop together with the spin and charge stripe correlations. At the temperatures we have studied, it is clear from Fig. 5 that their signature in the momentum structure of the pairfield susceptibility is not as strong as that of the spin and charge stripes. Nevertheless, the striped modulation is clearly visible in the real space structure in Fig. 4. Whether the PDW-like modulations become stronger at lower temperatures and potentially lead to a superconducting instability to a PDW state, however, remains an open question that we are unable to address because of the Fermion sign problem.

**Conclusion**

Our results demonstrate that fluctuating spin and charge stripe orders are a property of the doped single-band Hubbard model in the thermodynamic limit. Moreover, by accessing these phases using an embedded cluster technique, we are able to examine the ways in which the stripe fluctuations couple to superconducting correlations in the model. Concomitant with the spin stripes, we find that the d-wave pairing correlations develop a similar periodic stripe modulation, indicative of a pair-density wave.

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**Materials and methods**

**Dynamical cluster approximation.** To study the single-band Hubbard model, we use the dynamical cluster approximation (DCA) (21, 28, 29, 34). Complete details of the DCA algorithm can be found in Ref. (29). The DCA coarse-grains momentum space to represent the bulk lattice in the thermodynamic limit by a finite size cluster that contains \(N_e\) sites and is embedded in a self-consistent mean-field. This mean-field represents the remaining degrees of freedom and is determined self-consistently from the solution of the cluster problem.

With the assumption of short-ranged correlations, the self-energy \(\Sigma(k, i\omega_n)\) is well approximated by the cluster self-energy \(\Sigma(K, i\omega_n)\), where \(K\) are the cluster momenta. The coarse-grained single-particle Green’s function

\[
\tilde{G}(K, i\omega_n) = \frac{N_c}{N} \sum_{k'} G(K + k', i\omega_n)
\]

is then obtained by averaging the lattice Green’s function \(G(k, i\omega_n)\) over the \(N/N_c\) momenta \(k'\) in a square patch about the cluster momentum \(K\) that has an area of size \(1/N_c\) of that of the first Brillouin zone. This reduces the bulk problem to a finite size cluster, which we solve using the continuous-
time, auxiliary-field, quantum Monte-Carlo algorithm (CT-AUX) (23).

In our DCA++ simulations, the expansion order of the CT-AUX QMC is typically in the range of 100 - 3000, depending on temperature and $t'$. Depending on the average fermion sign for a given parameter set, we have obtained 10 million - 2 billion samples for the correlation functions. Usually 6 - 8 iterations were needed to obtain good convergence for the DCA mean-field.

**Determinant quantum Monte Carlo.** We also perform DQMC(35, 36) simulation on the single-band Hubbard model to obtain the spin, charge and pairing correlation functions for finite-size clusters. We divide the imaginary time interval $[0, \beta]$ into $L$ discrete steps with step size fixed at $\Delta \tau = 0.1$, and rewrite the partition function using the Trotter formula neglecting terms of order $O(\Delta \tau^2)$.

We perform equal-time measurements every other full space-time sweep and unequal-time measurements every 4th sweep. We use 5000 independently seeded Markov chains and for each chain, we use 50,000 warmup sweeps and 400,000 measurement sweeps. This large amount of data set leads to reliable statistics despite the severe Fermion sign problem.

**ACKNOWLEDGMENTS.** The authors would like to thank T. P. Devereaux, E. Huang, B. Moritz, and D. J. Scalapino for useful discussions. This work was supported by the Scientific Discovery through Advanced Computing (SciDAC) program funded by the U.S. Department of Energy, Office of Science, Advanced Scientific Computing Research and Basic Energy Sciences, Division of Materials Sciences and Engineering. This research used resources of the Oak Ridge Leadership Computing Facility, which is a DOE Office of Science User Facility supported under Contract DE-AC05-00OR22725. A part of the analysis of the results performed by T. A. M. was supported by the U.S. Department of Energy, Office of Basic Energy Sciences, Materials Sciences and Engineering Division. This manuscript has been authored by UT-Battelle, LLC under Contract No. DE-AC05-00OR22725 with the U.S. Department of Energy. The United States Government retains and the publisher, by accepting the article for publication, acknowledges that the United States Government retains a non-exclusive, paid-up, irrevocable, world-wide license to publish or reproduce the published form of this manuscript, or allow others to do so, for United States Government purposes. The Department of Energy will provide public access to these results of federally sponsored research in accordance with the DOE Public Access Plan (http://energy.gov/downloads/doe-public-access-plan).

References.

1. E Fradkin, SA Kivelson, JM Tranquada, Colloquium: Theory of interlaced orders in high temperature superconductors. Rev. Mod. Phys. 87, 457–482 (2015).
2. JM Tranquada, BJ Sternlieb, JD Axe, Y Nakamura, S Uchida, Evidence for stripe correlations of spins and holes in copper oxide superconductors. Nature 375, 561–563 (1995).
3. DF Agterberg, et al., The physics of pair-density waves: Cuprate superconductors and beyond. Annu. Rev. Condens. Matter Phys. 11, 231–270 (2020).
4. JM Tranquada, Cuprate superconductors as viewed through a striped lens. Adv. Phys. 69 (2021).
5. M Qin, et al., Absence of superconductivity in the pure two-dimensional Hubbard model. Phys. Rev. X 10, 031016 (2020).
6. M Qin, T Schäfer, S Andergassen, P Corboz, E Hombach, The Hubbard model: A computational perspective. arXiv/2104.00064 (2021).
7. R Arpaia, G Ghifinhilli, Charge order at high temperature in cuprate superconductors. J. Phys. Soc. Jpn. 90, 111005 (2021).
8. EW Huang, CB Mendel, HC Jiang, B Moritz, TP Devereaux, Stripe order from the perspective of the Hubbard model. npj Quantum Mater. 3, 22 (2018).
9. EW Huang, et al., Numerical evidence of fluctuating stripes in the normal state of high-Tc cuprate superconductors. Science 358, 1161–1164 (2017).
10. J Zaaren, O Gunnarsson, Charged magnetic domain lines and the magnetism of high-Tc oxides. Phys. Rev. B 40, 7391–7394 (1989).
11. K Machida, Magnetism in La$_2$CuO$_4$ based compounds. Phys. C 158, 192–196 (1989).
12. M Kato, K Machida, H Nakashima, M Fujita, Soliton lattice modulation of incommensurate spin density wave in two dimensional Hubbard model -a mean field study-. J. Phys. Soc. Jpn. 59, 1047–1058 (1990).
13. BX Zheng, et al., Stripe order in the underdoped region of the two-dimensional Hubbard model. Science 358, 1155–1160 (2017).
14. SR White, DJ Scalapino, Stripes on a 6-leg Hubbard ladder. Phys. Rev. Lett. 91, 136403 (2003).
15. HC Jiang, TP Devereaux, Superconductivity in the doped Hubbard model and its interplay with next-nearest hopping $t'$. Science 365, 1424–1428 (2019).
16. YF Jiang, J Zaaren, TP Devereaux, HC Jiang, Ground state phase diagram of the doped Hubbard model on the four-leg cylinder. Phys. Rev. Res. 2, 030373 (2020).
17. BX Zheng, GKL Chan, Ground-state phase diagram of the square lattice Hubbard model from density matrix embedding theory. Phys. Rev. B 93, 035126 (2016).
18. K Ido, TH Ohshe, M Imada, Competition among various charge-inhomogeneous states and d-wave superconducting state in Hubbard models on square lattices. Phys. Rev. B 97, 045138 (2018).
19. S Sorella, The phase diagram of the Hubbard model by variational auxiliary field quantum Monte Carlo (2021).
20. P Corboz, TM Rice, M Troyer, Competing states in the $t$-$t'$ model: Uniform $d$-wave state versus stripe state. Phys. Rev. Lett. 113, 046402 (2014).
21. TA Maier, M Jarrell, TC Schulthus, PRC Kent, JB White, Systematic study of $d$-wave superconductivity in the 2D repulsive Hubbard model. Phys. Rev. Lett. 95, 237001 (2005).
22. TA Maier, MS Jarrell, DJ Scalapino, Structure of the pairing interaction in the two-dimensional Hubbard model. Phys. Rev. Lett. 96, 047005 (2006).
23. E Gull, P Werner, O Parcollet, M Troyer. Continuous-time auxiliary-field Monte Carlo for quan- tum impurity models. Eur. Lett. 82, 57003 (2008).
24. G Sordi, P Sérim, K Haule, AMS Tremblay, Strong coupling superconductivity, pseudogap, and melt transition. Phys. Rev. Lett. 108, 216401 (2012).
25. A Wietek, YY Ha, SR White, A Georges, EM Stoudenmire, Antiferromagnetism, and the pseudogap in the doped Hubbard model at finite temperature. Phys. Rev. X 11, 031007 (2021).
26. A Foley, SV Verret, AMS Tremblay, D Sénéchal, Coexistence of superconductivity and antiferromagnetism in the Hubbard model for cuprates. Phys. Rev. B 99, 184510 (2019).
27. G Kollath, S Savasov, G Palsson, G Biroli, Cellualr Dynamical Mean Field Approach to Strongly Correlated Systems. Phys. Rev. Lett. 87, 186401 (2001).
28. M Jarrell, T Maier, C Huistros, S Mouloudi, Quantum Monte Carlo algorithm for nonlocal corrections to the dynamical mean-field approximation. Phys. Rev. B 64, 195130 (2001).
29. T Maier, M Jarrell, T Pruschke, MH Hettler, Quantum cluster theories. Rev. Mod. Phys. 77, 1027–1080 (2005).
30. R Peters, N Kawakami, Spin density waves in the Hubbard model: A DMFT approach. Phys. Rev. B 89, 151134 (2014).
31. M Fleck, A Lichtenstein, E Pavarini, One-Dimensional Metallic Behavior of the Stripe Phase in La$_2$-Sr$_2$CuO$_4$. Phys. Rev. Lett. 84, 4962–4965 (2000).
32. TI Vanhala, P Törnä, Dynamical mean-field theory study of stripe order and $d$-wave supercon- ductivity in the two-dimensional Hubbard model. Phys. Rev. B 97, 1–10 (2018).
33. SS Dash, D Sénéchal, Charge- And pair-density-wave orders in the one-band Hubbard model from dynamical mean field theory. Phys. Rev. B 103, 1–10 (2021).
34. UR Hämmerl, et al., DCA++: A software framework to solve correlated electron problems with modern quantum cluster methods. Comput. Phys. Commun. 246, 106709 (2020).
35. R Blankenbecler, DJ Scalapino, RL Sugar, Monte Carlo calculations of coupled boson-fermion systems. I. Phys. Rev. D 24, 2278–2286 (1981).
36. SR White, et al., Numerical study of the two-dimensional Hubbard model. Phys. Rev. B 40, 506–516 (1989).
37. SA Kivelson, et al., How to detect fluctuating stripes in the high-temperature superconductors. Rev. Mod. Phys. 75, 1201–1241 (2003).
38. A Guldbrand, S Johnston, T Maier, Orbital structure of the effective pairing interaction in the high-temperature superconducting cuprates. npj Quantum Mater. 6, 26 (2021).
39. O Zachar, SA Kivelson, VJ Emery, Landau theory of stripe phases in cuprates and nickelates. Phys. Rev. B 57, 1442 (1998).
40. E Berg, et al., Dynamical Layer Decoupling in a Stripe-Ordered High-Tc Superconductor. Phys. Rev. Lett. 99, 127003 (2007).
41. W Wu, et al., Pseudogap and Fermi-Surface Topology in the Two-Dimensional Hubbard Model. Phys. Rev. X 8, 21048 (2018).