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Relativistic effects and primordial non-Gaussianity in the galaxy bias

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Abstract. When dealing with observables, one needs to generalize the bias relation between the observed galaxy fluctuation field to the underlying matter distribution in a gauge-invariant way. We provide such relation at second-order in perturbation theory adopting the local Eulerian bias model and starting from the observationally motivated uniform-redshift gauge. Our computation includes the presence of primordial non-Gaussianity. We show that large scale-dependent relativistic effects in the Eulerian bias arise independently from the presence of some primordial non-Gaussianity. Furthermore, the Eulerian bias inherits from the primordial non-Gaussianity not only a scale-dependence, but also a modulation with the angle of observation when sources with different biases are correlated.

Keywords: dark matter theory, cosmological perturbation theory

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1 Introduction

Cosmological inflation [1] has become the dominant paradigm to understand the initial conditions for the Cosmic Microwave Background (CMB) anisotropies and Large Scale Structure (LSS) formation. This picture has recently received further spectacular confirmation by the Wilkinson Microwave Anisotropy Probe (WMAP) seven year set of data [2]. Present and future [3] data may be sensitive to the non-linearities of the cosmological perturbations at the level of second- or higher-order perturbation theory. The detection of these non-linearities through the non-Gaussianity (NG) [4] has become one of the primary experimental targets.

A possible source of NG could be primordial in origin, being specific to a particular mechanism for the generation of the cosmological perturbations. This is what makes a positive detection of NG so relevant: it might help in discriminating among competing scenarios which otherwise might be indistinguishable. Indeed, various models of inflation, firmly rooted in modern particle physics theory, predict a significant amount of primordial NG generated either during or immediately after inflation when the comoving curvature perturbation becomes constant on super-horizon scales [4]. While standard single-field [5] and two(multi)-field [6] models of inflation generically predict a tiny level of NG, ‘curvaton-type models’ [7–9], in which a significant contribution to the curvature perturbation is generated after the end of slow-roll inflation by the perturbation in a field which has a negligible effect on inflation, may predict a high level of NG [10]. Alternatives to the curvaton model are those models characterized by the curvature perturbation being generated by an inhomogeneity in the decay rate [11, 12] of the inflaton field. Other opportunities for generating the curvature perturbation occur at the end of inflation [13] and during preheating [14]. All these models generate a level of NG which is local, since the NG part of the primordial curvature perturbation is a local function of the Gaussian part generated on superhorizon scales. It has now become common to parametrize the level of NG through a dimensionless quantity $f_{NL}$ which sets the magnitude of the three-point correlation function [4]. In momentum space, the three point function (bispectrum), arising from the local NG is dominated by the so-called “squeezed” configuration, where one of the momenta is much smaller than the other two and it is parametrized by the non-linearity parameter $f_{NL}^{loc}$. Other models, such as DBI inflation [15] and ghost inflation [16], predict a different kind of primordial NG, called “equilateral”, because the three-point function for this kind of NG is peaked on equilateral configurations, in which the lengths of the three wave-vectors forming a triangle in Fourier space are equal [17]. The equilateral NG is parametrized by an amplitude $f_{NL}^{equil}$ [18].
Present limits on NG are summarized by $-10 < f_{NL}^{loc} < 74$ and $-214 < f_{NL}^{\text{equil}} < 266$ at 95\% CL [2, 19, 20].

It is clear that detecting a significant amount of NG and its shape either from the CMB or from the LSS offers the possibility of opening a window into the dynamics of the universe during the very first stages of its evolution and to understand what mechanism gave rise to the cosmological perturbations. Besides in the CMB anisotropies, NG is particularly relevant in the high-mass end of density perturbations, i.e. on the scale of galaxy clusters, since the effect of NG fluctuations becomes especially visible on the tail of the probability distribution function [21]. Furthermore, and more relevantly for us, primordial NG also alters the clustering of dark matter halos (where galaxies are supposed to reside) inducing a scale-dependent bias on large scales [22]. In the local Eulerian biasing model [23, 24] the galaxy density field at a given position is described as a local function of the dark matter density field $\delta$ at the same position. On large scales, the galaxy density field can be therefore expressed as a Taylor expansion in powers of $\delta$ whose coefficients define the bias parameters. As the primordial NG generates a cross-talk between short and long wavelengths, it alters significantly the local bias and introduces a strong scale dependence in it [25]. The same result can be also obtained through the peak-background split theory [26] where the underlying idea behind the generation of a local bias is that galaxies tend to form in regions where the dark matter density field is larger than some threshold value in Lagrangian space.

As a result, measuring the clustering properties of haloes is a sensitive probe of primordial NG which could be detected or significantly constrained by the various planned large-scale galaxy surveys, both ground based (such as DES, PanSTARRS and LSST) and in space (such as EUCLID and ADEPT) [27].

When analyzing the impact of NG onto the bias of galaxies various points should be addressed. As the NG effects show up at second-order in perturbation theory, a full consistent second-order computation should be performed. Furthermore, since the primordial NG manifests itself on large cosmological scales one should treat carefully the second-order relativistic effects. This automatically calls for a gauge-invariant formulation of the observables at hand, while so far in the literature only the Newtonian and post-Newtonian contributions have been included for the dark matter contrast (both in the synchronous-comoving and Poisson gauge, see [28, 29]). A gauge-invariant formulation is also needed when dealing with real observables. For instance, the observed redshift and position of galaxies are affected by the matter fluctuations between the source galaxies and the observer. Moreover, one needs to generalize the bias relation between the observed galaxy fluctuation field to the underlying matter distribution in a gauge-invariant way. This latter step has been taken at the linear level in ref. [30] by defining a gauge-invariant dark matter density contrast in the uniform redshift gauge. The goal of this paper is to generalize such bias relation to second-order. We clarify from the very beginning that we will not include other effects such as the redshift-space distortion and the magnification effect discussed at the linear level by [30].

In this paper we adopt the local Eulerian bias model [23, 24] and show that a refined gauge-invariant treatment of the Eulerian bias at second-order in perturbation theory leads to the prediction that the bias is scale-dependent on large scales even in the absence of primordial NG and that the latter generates an angular modulation if sources with different biases are cross-correlated.

The paper is organized as follows. In section 2 we provide the tools for the second-order gauge-invariant formulation of the Eulerian bias parameters. In section 3 we proceed with the explicit computation of the gauge-invariant Eulerian bias at second-order in perturbation theory. Finally, section 4 contains our conclusions.
2 Local gauge-invariant Eulerian description of the galaxy bias

In this section we describe how to obtain a physically motivated gauge-invariant expression for the Eulerian galaxy bias. We will adopt the bias model described in refs. [23, 24]. In this approach the (smoothed) galaxy number density field at a given position \( x \) and time \( \tau \) is assumed to be a local function of the (smoothed) underlying CDM mass density at the same location and instant

\[
\delta_g = b^E_1 \delta + \frac{1}{2} b^E_2 \delta^2 + \cdots ,
\]

(2.1)

where \( \delta_g \) is the number overdensity of galaxies and \( \delta \) is the cold dark matter (CDM) overdensity. This approach is essentially phenomenological, with the bias coefficients \( b^E_1 \) and \( b^E_2 \) to be determined from observations. Notice that in eq. (2.1) the bias parameters are not gauge-invariant. While comparing the theoretical predictions (the matter power spectrum) with observations (the galaxy power spectrum) does not represent a problem on sub-horizon scales where the matter density perturbations computed in the different gauges all coincide, it is a delicate operation on scales comparable with the horizon where different gauges provide different results even at the linear level (see, e.g., [30]).

Truly gauge-independent perturbations must be exactly constant in the background spacetime. This apparently limits one’s ability to make a gauge-invariant study of quantities that evolve in the background spacetime, e.g. density perturbations in an expanding cosmology. In practice one can construct gauge-invariant definitions of unambiguous, that is physically defined, perturbations (see, e.g., the discussion of ref. [31]). These are not unique gauge-independent perturbations, but are gauge-invariant in the sense commonly used by cosmologists to define a physical perturbation. There is a distinction between quantities that are automatically gauge-independent, i.e., those that have no gauge dependence (such as perturbations about a constant scalar field), and quantities that are in general gauge-dependent (such as the curvature perturbation) but can have a gauge-invariant definition once their gauge-dependence is fixed (such as the curvature perturbation on uniform-density hypersurfaces). In other words, one can define gauge-invariant quantities which are simply a coordinate independent definition of the perturbations in the given gauge. This can be often achieved by defining unambiguously a specific slicing into spatial hypersurfaces. In this sense it should be clear that one may define an infinite number of, e.g., gauge-invariant density contrasts. Which one to use is a matter that can be decided only considering how the determination of a given observable is performed. We will come back to this point later. For the time being it suffices to say that the local prescription of the Eulerian bias can be expressed in terms of gauge-independent (in the sense just described) CDM and galaxy density contrasts

\[
\delta_{g_i} = b^E_1 \delta_{g_i} + \frac{1}{2} b^E_2 (\delta_{g_i})^2 + \cdots .
\]

(2.2)

One of our goals is to account for the non-linear (second-order) contribution of the CDM matter density field to the bias parameters. In fact one can split the CDM density contrast into a linear and a second-order contribution

\[
\delta_{g_i} = \delta_{g_i(1)} + \frac{1}{2} \delta_{g_i(2)} + \cdots .
\]

(2.3)

Since the second-order part \( \delta_{g_i(2)} \) will be expressed as a convolution in terms of \( (\delta_{g_i})^2 \), this will produce an additional contribution to the bias parameter \( b^E_2 \) in eq. (2.2). To compute this contribution though we must come back to the issue of which gauge-invariant contrasts
we should take. In fact with the notation “gi” we will intend in the following that the density contrasts will be evaluated in a gauge-invariant way starting from the uniform-redshift gauge. Notice that in this way the bias parameters are automatically gauge-invariant.

2.1 On the gauge-invariant formulation: uniform-redshift gauge based perturbations

In the following we consider a spatially flat Universe filled with a cosmological constant \( \Lambda \) and a non-relativistic pressureless fluid of Cold Dark Matter (CDM), whose energy-momentum tensor reads \( T_{\mu\nu} = pu_{\mu}u_{\nu} \) where \( u^\mu \) \( (u_\mu u^\mu = -1) \) is the comoving four-velocity. Following the notations of ref. [32], the perturbed line element around a spatially flat FRW background reads

\[
\text{d} s^2 = a^2(\tau)\{-(1+2\phi)d\tau^2 + 2\dot{\omega}_i d\tau dx^i + [(1-2\psi)\delta_{ij} + \hat{\chi}_{ij}][dx^i dx^j]\}, \tag{2.4}
\]

where \( a(\tau) \) is the scale factor as a function of conformal time \( \tau \). Here each perturbation quantity can be expanded into a first-order (linear) part and a second-order contribution, as for example, the gravitational potential \( \phi = \phi^{(1)} + \phi^{(2)}/2 \). Up to now we have not chosen any particular gauge. We can employ the standard split of the perturbations into the so-called scalar, vector and tensor parts, according to their transformation properties with respect to particular gauge. We can employ the standard split of the perturbations into the so-called scalar, vector and tensor parts, according to their transformation properties with respect to the 3-dimensional space with metric \( \delta_{ij} \), where scalar parts are related to a scalar potential, vector parts to transverse (divergence-free) vectors and tensor parts to transverse trace-free tensors. Thus \( \phi \) and \( \psi \), the gravitational potentials, are scalar perturbations, and for instance, \( \dot{\omega}^{(r)}_i = \partial_i \omega^{(r)} + \omega^{(r)}_i \), where \( \omega^{(r)} \) is the scalar part and \( \omega^{(r)}_i \) is a transverse vector, i.e. \( \partial^i \omega^{(r)}_i = 0 \) \((r) = (1,2) \) stand for the \( r \)th-order of the perturbations). The symmetric traceless tensor \( \hat{\chi}_{ij} \) generally contains a scalar, a vector and a tensor contribution, namely \( \hat{\chi}_{ij} = D_{ij} \chi + \partial_i \chi_j + \partial_j \chi_i + \chi_{ij} \), where \( D_{ij} = \partial_i \partial_j - (1/3)\nabla^2 \delta_{ij} \), \( \chi_i \) is a solenoidal vector \( (\partial^i \chi_i = 0) \) and \( \chi_{ij} \) represents a traceless and transverse (i.e. \( \partial^i \chi_{ij} = 0 \) tensor mode.\(^1\)

As we pointed out before, there is an infinite number of ways to define gauge-invariant density contrasts which differ by other gauge-invariant combinations. Since one observes galaxies rather than the underlying matter distribution and the latter at the source galaxy position is related to the mean matter density at the observed redshift \( z \), a good choice to define gauge-invariant density constraints related to each other by a bias factor seems the one involving the observed redshift \( z \) [30]. At first order a coordinate transformation reads \( x^\mu \to x'^\mu - \xi^{\mu}_{(1)} \) where \( \xi^{\mu}_{(1)} = (\alpha^{(1)}, \xi^{(1)i}) \). The matter density contrast transforms as \( \delta^{(1)} \to \delta^{(1)} + \hat{\rho}/\tilde{\rho} \alpha^{(1)} \), where now dot stands for differentiation with respect to the conformal time and \( \tilde{\rho} \sim (1+z)^3 \) is the background matter energy density; similarly the first-order perturbation of the observed redshift transforms as \( z^{(1)} \to z^{(1)} + \dot{z} \alpha^{(1)} \) (here \( z \) is the unperturbed redshift).

Going to the uniform redshift gauge where the linear perturbation of redshift vanishes relates \( \alpha^{(1)} \) to the linear perturbation of redshift in the old gauge, \( \alpha^{(1)} = -z^{(1)}/\tilde{z} \). Therefore the gauge invariant definition of the matter density contrast (and similarly for the halo one) is [30]

\[
\delta^{\text{gi}(1)} = \delta^{(1)} - \frac{z^{(1)}}{1+z}. \tag{2.5}
\]

At second-order the procedure is more involved, but straightforward. The coordinate transformation reads \( x^\mu \to x^\mu + \xi^{\mu}_{(1)} + \frac{1}{2}(\xi^{\mu}_{(2)}, \xi^{\mu}_{(2)i}) \) where \( \xi^{\mu}_{(2)} = (\alpha^{(2)}, \xi^{(2)i}) \). Under this

\(^1\)In what follows, for our purposes we will neglect linear vector modes since they are not produced in standard mechanisms for the generation of cosmological perturbations (as inflation), and we also neglect tensor modes at linear order, since they give a negligible contribution to LSS formation.
coordinate transformation the density matter contrast and the redshift perturbation transform as

\[ \delta^{(2)} \rightarrow \delta^{(2)} + \frac{\dot{\rho}}{\rho} \alpha^{(2)} + \alpha^{(1)} \left( \frac{\ddot{\rho}}{\rho} \alpha^{(1)} + \frac{\dot{\rho}}{\rho} \dot{\alpha}^{(1)} + 2 \frac{\delta^{(1)} \dot{\rho}}{\rho} \right) + \xi_{i}^{(1)} \left( \frac{\dot{\rho}}{\rho} \partial_{i} \alpha^{(1)} + \frac{3}{2} \frac{\delta^{(1)} \rho}{\rho} \partial_{i} \right), \]

\[ z^{(2)} \rightarrow z^{(2)} + \dot{z} \alpha^{(2)} + \alpha^{(1)} \left( \dot{\alpha}^{(1)} + \ddot{z} + 2 \dot{z} \right) + \xi_{i}^{(1)} \left( \dot{z} \partial_{i} \alpha^{(1)} + 2 \partial_{i} \dot{z} \right). \] (2.6)

Going to the uniform redshift gauge where second-order perturbation of the redshift vanishes gives

\[ \alpha^{(2)} = -z^{(2)} z^{-2} - \frac{\xi_{i}^{(1)}}{z} \partial_{i} z^{(1)}. \] (2.7)

To completely solve the uniform redshift gauge at second-order we must also specify the first-order spatial gauge shift \( \xi_{i}^{(1)} \). A natural choice is to pick worldlines comoving with the fluid. The (scalar) velocity transforms as \( v^{i} = e^{i} v_{\mu} u^{\mu} \). Thus from an arbitrary spatial gauge we can transform to the comoving gauge by the spatial gauge transformation \( \xi_{i}^{(1)} = \int v_{i}^{(1)} d\tau \). Such a choice leads to the second-order gauge invariant matter density contrast (and similarly for the halo one)

\[ \frac{1}{2} \delta^{i(2)} = \frac{1}{2} \left( \frac{z^{(2)} - 3}{1 + z} \right) + \frac{z^{(1)} z^{(1)}}{2(1 + z)} + \frac{z^{(1)^{2}}}{(1 + z)^{2}} - \frac{z^{(1)}}{z} \delta^{i(1)} - \frac{3}{1 + z} \left( \partial_{i} \delta^{i(1)} - 3 \frac{\partial_{i} z^{(1)}}{1 + z} \right) \int d\tau v_{i}^{(1)}. \] (2.8)

The next step amounts to determining the expression of the redshift perturbation in terms of the perturbations of the metric (2.4) and other quantities. Photons suffer a redshift \( z \) during their travel from the emitter \( E \) to the observer \( O \); the emitted frequency \( \omega_{E} \) and the observed one \( \omega_{O} \) are related by \( \omega_{O} = \omega_{E} / (1 + z) \). Here \( \omega = -g_{\mu \nu} u^{\mu} k^{\nu} \), where \( u^{\mu} \) is the four-velocity of the observer or emitter and \( k^{\nu} = dx^{\nu} / d\lambda \) is the wave vector of the photon in the conformal metric, tangent to the null geodesic \( x^{\nu}(\lambda) \) (\( \lambda \) is the affine parameter) followed by the photon from the emission to the observation point. We do not report the full calculation of the redshift perturbation which basically amounts to solving for the photon trajectory. The computation can be found in ref. [33]. Expanding the frequency as \( \omega = \bar{\omega}(1 + \omega^{(1)} + \frac{1}{2} \omega^{(2)}) \), at first-order one obtains

\[ \frac{z^{(1)}}{1 + z} = \omega^{(1)}_{\bar{\omega}} - \omega^{(1)}_{\bar{\omega}} = \phi^{(1)}_{\bar{\omega}} - \phi^{(1)}_{\bar{\omega}} + v_{i}^{(1)} e_{i} - v_{i}^{(1)} e_{i} + I_{1}(\lambda_{\bar{\omega}}), \]

\[ I_{1}(\lambda_{\bar{\omega}}) = \int_{\lambda_{\bar{\omega}}}^{\lambda_{\bar{\omega}}} d\lambda \dot{A}^{(1)} , \]

\[ A^{(1)} = \psi^{(1)} + \phi^{(1)} + \dot{\psi} e^{i} - \frac{1}{2} \chi^{(1)} e^{i} e^{j} , \] (2.9)

where \( e^{i} \) indicates the zero-th order three-dimensional vector indicating the photon direction from which they arrive at the observer \( O \). In the expression above one recognizes the Sachs-Wolfe effect due to the change in the gravitational potential at the source’ and observer’s points, the Doppler contribution due to the peculiar velocities of the emitter and the observer and the integrated Sachs-Wolfe effect along the photon trajectory.
At second-order expressions are more involved and the perturbation of the redshift may be written as [33]

\[
\frac{1}{2} \frac{z^{(2)}}{1 + z} = \frac{1}{2} \left[ \omega^{(2)}_\epsilon - \omega^{(2)}_\lambda - 2 \omega^{(1)}_{\epsilon \lambda} \omega^{(1)}_\lambda + 2 \left( \omega^{(1)}_\lambda \right)^2 \right]
\]

\[
= \frac{1}{2} \phi^{(2)}_\epsilon - \frac{1}{2} \phi^{(2)}_\lambda + \frac{1}{2} v^{(2)j}_\epsilon e_i - \frac{1}{2} v^{(2)j}_\lambda e_i + \frac{3}{2} \left( \phi^{(1)}_\epsilon \right)^2 - \frac{1}{2} \left( \phi^{(1)}_\lambda \right)^2 - \phi^{(1)}_\lambda \phi^{(1)}_\epsilon + I_2(\lambda) + v^{(1)i}_\epsilon e_i \phi^{(1)}_\lambda
\]

\[- \left( I_1(\lambda) + v^{(1)i}_\lambda e_i \right) \left( 2 \phi^{(1)}_\lambda - \omega^{(1)}_\lambda + \frac{1}{2} \chi^{(1)ij}_\lambda e_i e_j - \omega^{(1)}_\epsilon e_i + \phi^{(1)}_\lambda + v^{(1)i}_\lambda e_i \right)
\]

\[-x^{(1)}_\epsilon \lambda - x^{(1)i}_\lambda + x^{(0)i} e_i \left( \phi^{(1)}_\lambda - \omega^{(1)}_\lambda \right), e_i \omega^{(1)}_\epsilon - \phi^{(1)}_\lambda \left( \frac{1}{2} \omega^{(1)}_\lambda - 2 \omega^{(1)}_\epsilon \right) e_i + \lambda^{(1)ij}_\epsilon e_i
\]

\[
+ \frac{1}{2} I^{(1)i}_\epsilon e_i + v^{(1)i}_\lambda e_i \left( \phi^{(1)}_\lambda - \omega^{(1)}_\lambda + \frac{1}{2} \chi^{(1)ij}_\lambda e_i e_j - \phi^{(1)}_\lambda \right)
\]

\[-v^{(1)i}_\epsilon \left( -\omega^{(1)}_\lambda + \omega^{(1)}_\lambda + 2 \phi^{(1)}_\lambda e_i - \chi^{(1)ij}_\lambda e_j - I^{(1)}_i(\lambda) \right)
\]

\[+ \left( v^{(1)i}_\epsilon e_i - v^{(1)i}_\lambda e_i + I_1(\lambda) \right)^2 \left( \phi^{(1)}_\gamma - \phi^{(1)}_\lambda \right) \left( v^{(1)i}_\epsilon e_i - v^{(1)i}_\lambda e_i + I_1(\lambda) \right), \quad (2.10)
\]

where

\[
I^{(1)}_1(\lambda) = \int_{\lambda_0}^{\lambda} d\lambda A^{(1)i},
\]

\[
I_2(\lambda) = \int_{\lambda_0}^{\lambda} d\lambda \left[ \frac{1}{2} A^{(2)} - (\omega^{(1)}_i - \chi^{(1)}_{ij} e^j) (k^{(1)i} e^j + e^i k^{(1)0}) + 2 k^{(1)0} A^{(1)} \right]
\]

\[+ 2 \omega^{(1)} e^i A^{(1)} + x^{(0)i} A^{(1)} + x^{(0)i} A^{(1)}, \quad (2.11)
\]

and

\[
k^{(1)0}(\lambda) = \phi^{(1)}_\lambda - \phi^{(1)}_\epsilon + \frac{1}{2} \chi^{(1)ij}_\lambda e_i e_j - 2 \phi^{(1)}_\lambda - \omega^{(1)}_\epsilon e_i + I_1(\lambda),
\]

\[
k^{(1)i}(\lambda) = 2 \phi^{(1)}_\lambda e^i + \omega^{(1)}_\lambda - \chi^{(1)}_{ij} e_j - 2 \phi^{(1)}_\lambda e^i - \omega^{(1)}_\epsilon e_i + \chi^{(1)}_{ij} e_j - I^{(1)}_i(\lambda),
\]

\[
x^{(1)}_\lambda(\lambda) = (\lambda - \lambda_0) \left[ \phi^{(1)}_\lambda - \phi^{(1)}_\epsilon + \frac{1}{2} \chi^{(1)ij}_\lambda e_i e_j \right] + \int_{\lambda_0}^{\lambda} d\lambda \left[ -2 \phi^{(1)}_\lambda - \omega^{(1)}_\lambda e^i + (\lambda - \lambda_0) A^{(1)} \right],
\]

\[
x^{(1)i}_\lambda(\lambda) = (\lambda - \lambda_0) \left[ 2 \phi^{(1)}_\lambda e^i + \omega^{(1)}_\lambda - \chi^{(1)}_{ij} e_j \right]
\]

\[+ \int_{\lambda_0}^{\lambda} d\lambda \left[ 2 \phi^{(1)}_\lambda e^i + \omega^{(1)}_\lambda - \chi^{(1)}_{ij} e_j + (\lambda - \lambda_0) A^{(1)i} \right]. \quad (2.12)
\]

These long expressions allow the determination of the gauge-independent density contrasts. In practice, one evaluates them adopting the most appropriate gauge.

3 The computation of the gauge-invariant Eulerian bias

In this section we proceed with the computation of the contribution to the second-order local Eulerian bias $b^2$ coming from the gauge-invariant matter density contrast defined in eq. (2.8), including primordial non-Gaussianity.
We find it convenient to perform such a computation in the comoving-orthogonal gauge which, as we restrict ourselves to the case of irrotational dust plus \( \Lambda \), is also synchronous. Indeed, the fluid four-velocity field can be written as \( u^{\mu} = (1/a, 0, 0, 0) \). The possibility of making the synchronous, time-orthogonal gauge choice and comoving gauge choice simultaneously is a peculiarity of fluids with vanishing spatial pressure gradients, i.e. vanishing acceleration, which holds at any time, i.e. also beyond the linear regime.\(^{2}\)

In the synchronous and comoving gauge the line element reads

\[
ds^2 = a^2(\tau) \left[ -d^2\tau + h_{ij}(x, \tau)dx^idx^j \right]. \tag{3.1}
\]

Working in the synchronous comoving gauge, the spatial coordinate does not evolve with time, \( x(\tau) = x(\tau = \tau_0) \equiv q (q \text{ would correspond to the comoving Lagrangian coordinate for the fluid element}) \). To simplify our expressions we limit ourselves to the case of a pure CDM-gauge-invariant density contrast can be written as

\[
\delta^{g(1)}(q, \tau) = \frac{2}{3H^2\Omega_m} \nabla^2 \varphi(q) + \varphi(q) + \tau \epsilon^i \varphi_i(q), \tag{3.2}
\]

where \( \tau \) is linked to \( \lambda_c \) and hence to the emission redshift, \( \varphi(q) \) is the peculiar scalar gravitational potential and we have got rid of the terms defined at the observer’s point which may be absorbed in the monopole term. Notice that in Fourier space the time evolution of the gauge-invariant density contrast can be written as

\[
\delta^{g(1)}(k, \tau) = f^{g(1)}(k, \tau) \delta^{g(1)}(k, \tau),
\]

\[
f^{g(1)}(k, \tau) = 1 - \frac{6}{k^2\tau^2} - 6i \hat{k} \cdot \hat{n}, \tag{3.3}
\]

where \( \delta^{g(1)}(k, \tau) = -\frac{k^2}{6} \varphi(k) \) is the matter density contrast in the synchronous gauge.

At second order one first writes the matter density contrast in Fourier space as

\[
\frac{1}{2} \delta^{g(2)}(k, \tau) = \int \frac{d^3k_1d^3k_2}{(2\pi)^3} k_\delta^{g(1)}(k_1, k_2; \tau) \delta^{g(1)}(k_1, \tau)\delta^{g(1)}(k_2, \tau) \delta_D(k_1 + k_2 - k). \tag{3.4}
\]

With this position one finds that, in Fourier space, the relation between the galaxy number density and the CDM density contrasts following the Eulerian prescription (3.2) is given by

\[
\delta^{g(1)}(k, \tau) = b^P \delta^{g(1)}(k, \tau) + \frac{1}{2} \int \frac{d^3k_1d^3k_2}{(2\pi)^3} \frac{b_{\delta}^P}{b_{\delta}^P}(k_1, k_2; \tau) \delta^{g(1)}(k_1, \tau)\delta^{g(1)}(k_2, \tau) \delta_D(k_1 + k_2 - k), \tag{3.5}
\]

\(^2\)The choice of the comoving-orthogonal gauge to evaluate the gauge-invariant Eulerian bias might be motivated by other reasons. In particular, if one does not treat the bias parameters as phenomenological coefficients, but desires to have a theoretical prediction for them starting from the Lagrangian bias (see, e.g., [34]). A simple analytic model for the gravitational clustering of dark matter haloes to understand how their spatial distribution is biased relative to that of the mass was developed in ref. [35]. The statistical distribution of dark haloes within the initial density field (assumed Gaussian) is determined by an extension of the Press-Schechter formalism and is done therefore at the Lagrangian level. One then expects that the gauge-invariant Eulerian description to be therefore simpler to formulate through the synchronous gauge. Furthermore, the non-linear spherical collapse description necessary to compute the halo mass function through the (extended) Press-Schechter approach, requires the choice of the comoving-orthogonal gauge.

\(^3\)To solve for the integral appearing in eq. (2.9) one can use, e.g., the results contained in the appendix of ref. [36]. One can also readily check that the same expression is obtained, e.g., in the Poisson gauge.
be found in ref. [7]. Indeed, the latter gets a correction arising from the last term of the expression \( (2) \) (corresponding in Fourier space to \( 7 \Psi^2 \)).

The kernel \( K_\delta^2 \) is conveniently computed in the Poisson gauge (rather than in the synchronous gauge). In the presence of large local non-Gaussianities, we can introduce a gravitational potential which, at some initial epoch and deep in matter domination, reads

\[
\phi_{in} = \phi_{in}^{(1)} + f_{NL}^{loc} (\phi_{in}^{(1)})^2 - \langle \phi_{in}^{(1)} \rangle,
\]

with the dimensionless non-linearity parameter \( f_{NL}^{loc} \) setting the level of quadratic local NG. In the case of large local non-Gaussianities, \( f_{NL}^{loc} \gg 1 \), one finds [28, 29]

\[
\begin{align*}
\frac{1}{2} \delta^{(2)} &= \frac{\tau^4}{252} \left[ 5 \left( \nabla^2 \varphi \right)^2 + 2 \varphi^{i j} \varphi_{, i j} + 7 \varphi^i \nabla^2 \varphi_{, i} \right] - f_{NL}^{loc} \frac{\tau^2}{6} \nabla^2 \varphi, \\
\frac{1}{2} \psi^{(2) i} &= \frac{\tau^3}{18} \left( - \varphi^{i j} \varphi_{, j} + 6 \Psi_{, i} \right) + f_{NL}^{loc} \frac{\tau}{3} \phi_{, i}^2,
\end{align*}
\]

where \( \nabla^2 \Psi \equiv -\frac{1}{2} \left( (\nabla^2 \varphi)^2 - \varphi_{, i k} \varphi_{, i k} \right) \). Inserting these expressions into eq. (2.8), we obtain

\[
\begin{align*}
K_\delta^2 (k_1, k_2; \tau) &= \frac{5}{7} + \frac{2 (k_1 \cdot k_2)^2}{k_1^2 k_2^2} - \frac{18 i}{7 k_1^2 k_2^2} \left[ (k_1 \cdot n) + (k_2 \cdot n) \right] \left[ 1 - \frac{(k_1 \cdot k_2)^2}{k_1^2 k_2^2} \right] \\
&+ 6 i \left[ (k_1 \cdot n) + (k_2 \cdot n) \right] \left[ \frac{8}{21} + \frac{2 (k_1 \cdot k_2)^2}{k_1^2 k_2^2} \right] + 6 f_{NL}^{loc} \frac{k^2}{k_1^2 k_2^2 \tau^2} \\
&- 18 i f_{NL}^{loc} \frac{\left[ (k_1 \cdot n) + (k_2 \cdot n) \right]}{k_1^2 k_2^2 \tau^3},
\end{align*}
\]

where \( k = |k_1 + k_2| \) and we have performed an expansion in \( (k_1 \tau)^{-1} \ll 1 \) (\( i = 1, 2 \)). Notice that in the kernel the primordial non-Gaussian piece coming from the second-order density contrast is post-Newtonian and is damped by two powers of \( (k_1 \tau) \) with respect to the Newtonian leading terms. It is important to stress that the Newtonian part of the kernel does not coincide with the one of the matter density contrast in the Poisson gauge which can be found in ref. [28]. Indeed, the latter gets a correction arising from the last term of the expression (2.8): passing from the Poisson gauge to the gauge-invariant density contrast the piece \( [7 \tau^4 \varphi^i \nabla^2 \varphi_{, i}/252] \) (corresponding in Fourier space to \( k_1 \cdot k_2 (k_1^2 + k_2^2)/(2k_1^2 k_2^2) \) gets cancelled. Furthermore, there are terms which are damped by only one power of \( (k_1 \tau) \): they originate from the velocity contributions in gauge-invariant definition of the matter density contrast and they are absent if a gauge-dependent definition of the matter density contrast is adopted. The same holds for the last contribution, damped by three powers of \( (k_1 \tau) \). This term comes from the primordial NG term in \( \delta^{(2)} \) (which gives the dominant contribution to the second-order redshift perturbation (2.10)). All other relativistic effects have been neglected.

Notice that for high values of \( f_{NL}^{loc} \), the new relativistic terms in eq. (3.9) do not alter the effect of primordial non-Gaussianity in the galaxy bias because in the limit \( f_{NL}^{loc} \gg 1 \) the effect on the bias is practically appearing at the linear level. It is only when \( f_{NL} \sim O(1) \) that the second-order relativistic effects start to be important, see also [37].

Nevertheless, adopting a gauge-invariant expression for the density contrast brings two new and interesting features in eq. (3.6) for the Eulerian bias \( b_E^2 \). First, it acquires a large-scale dependence from the kernel \( K_\delta^2 \) even when there is no primordial NG. Secondly, the
primordial NG, if present, introduces a dependence on the line of sight \( \mathbf{n} \) which comes from terms like \( \mathbf{v} \cdot \mathbf{n} \), which are necessary to realize the gauge-invariant definition of the matter density contrast. Therefore in the Eulerian bias there appears scale-dependent contributions which get also modulated. Notice that such \( \mathbf{v} \cdot \mathbf{n} \) contributions do not represent the usual redshift-space distortion effects, but rather they appear as effective corrections to the Eulerian bias. Obviously, when computing the power spectrum of two objects with the same bias on large scales, this modulation disappears being the power spectrum real. Nevertheless, this does not happen when computing the power spectrum of two objects with different bias. In such a case, the resulting bias in the presence of some primordial NG is not only scale-dependent \([22]\), but also depending on the angles \( \cos \theta = \hat{\mathbf{k}} \cdot \hat{\mathbf{n}} \) between the vector \( \mathbf{k} \) and the vector indicating the line-of-sight. A computation similar to the one leading to a scale-dependent bias when some large local NG is included leads to a correction to the bias on large scales for two objects with different bias b_E^a(k) and b_E^b(k):

\[
\Delta b_E^1(k, R_1, R_2) = 9 i f_{NL} \delta_c(z) \frac{\Omega_m H_0^2}{k^2 T(k)} H(z) f(\Omega_m) \left( b_E^a(k, R_1) \frac{k^2 \sigma^2_c(R_1)}{\sigma^2(R_1)} - b_E^b(k, R_2) \frac{k^2 \sigma^2_c(R_2)}{\sigma^2(R_2)} \right) \cos \theta ,
\]

where we have generalized our computation to a ΛCDM model, \( \Omega_m \) is the dark matter critical density, \( T(k) \) is the linear transfer function, \( f(\Omega_m) \approx \Omega_0^{0.6} \) and \( \sigma^2(R) \) and \( \sigma_c^2(R) \) are the variance of the density contrast and of the velocity at a radius \( R \), respectively.

4 Conclusions

In this paper we have described the computation of the Eulerian bias at second-order in perturbation theory adopting the local bias model. Paying attention to the gauge-invariant issues which necessarily arise when dealing with relativistic effects on large scales and with real observables, we have shown that some interesting effects show up. First of all, the Eulerian bias acquires a scale-dependence on large scales even if the primordial NG is totally negligible. Secondly, the primordial non-Gaussianity induces in the bias a modulation with the line of observation when sources with different biases are observed. Of course, we are well aware of the fact that our results are not complete in the sense that not all effects have been included. In this paper we have restricted ourselves to that part of the observed galaxy density contrast which is directly proportional to the dark matter density contrast through the bias. On the contrary, we have not discussed, for instance, the redshift-space distortion and the magnification effects. We leave it for future work.

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