Controller Based Observer in Switched System with Norm Bounded Uncertainty

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Abstract: Problem statement: This study discusses the robust stabilization of norm bounded discrete switched systems. Approach: The proposed method is using the second Lyapunov approach and the poly-quadratic function concept. The stabilization conditions are written through linear matrix inequality relations. The control law is based on a static output feedback with the use of a switched observer. The synthesis conditions of the controller are written in the form of linear matrix inequalities difficult to resolve by current numerical solvers. That’s why relaxations are proposed to mitigate the pessimism of LMI conditions obtained. Results: The poly-quadratic Lyapunov approach provides a constructive way to tackle uncertainty in the switched framework. The feasibility is illustrated by the example of discrete uncertain switched systems. Conclusion: With these results, the study of stability can be achieved for arbitrary switching laws, state-dependent, time dependent or generated by a controller. However, the implementation of the control law is possible only if the switching status is well known in real time.

Key words: Matrix inequalities, robust stabilization, arbitrary switching laws, inequality relations, systems combine continuous, stability analysis

INTRODUCTION

Many natural and artificial systems work in different operating modes, each with its own dynamic. The car changes from one dynamic mode to another with every change of speed, the human heart switches between different modes depending on the emotional state of the person. These systems concoct continuous dynamics with both synchronous or asynchronous discrete events. Such class of systems is called hybrid systems (Liberzon, 2003)

The literature has shown a growing interest in switched systems. Switched systems are hybrid systems defined by a set whose elements are dynamic continuous and/or discrete time models with commutation law which define, in time, the jumps between the elements, leading to a non stationary dynamic system. Some recent results are given in (Daafouz et al., 2001; 2002a) where a sufficient (but relatively non restrictive compared to the quadratic approach) stability condition for discrete switched systems is provided using the poly-quadratic approach recently proposed by (Daafouz and Bernussou, 2001) to analyze stability and stabilization control of Linear “Time Varying systems”.

Other approaches are concerned with the determination of an act of switching between several controllers guaranteeing the stability of systems (Skafidas et al., 1999; Liberzon, 2003; Hespanha and Morse, 2002) which operating constraints require switching between multiple controllers.

This study proposes an extension of this works in the case when the switches are made between uncertain LTI systems. The control investigated is of state feedback control, observer based and dynamic controller.

MATERIALS AND METHODS

Let us consider a discrete autonomous switched system (1) where each subsystem is vitiated by a norm bounded uncertainty (Maherzi et al., 2007; Zhou and Khargonekar, 1987; 1988), this system can be described by the following equalities (1):

\[
x(k+1) = \sum_{i=1}^{M} E_i (A_i + \Delta A_i)x(k)
\]

(1)

With Eq. 2 and 3:

\[
\Delta A_i = D_i F_i E_i
\]

(2)

And:

\[
F_i^T F_i \leq \gamma_i I
\]

(3)
Where:
\( \ell = \) Switching index
\( M = \) Number of subsystems Eq. 4 and 5:
\[
\xi'_M = \begin{cases} 1 & \text{if the state matrix } A \in (A_i + \Delta A_i) \\ 0 & \text{if not} \end{cases}
\]
\( \xi = \sum_{i=1}^{M} \xi'_i = 1; \xi = [\xi'_1, \ldots, \xi'_N]^{T} \)

**Analysis of stability:** By using Theorem developed by (Daafouz and Bernussou, 2001) the system (1) is poly-quadratically stable if and only if there are \( N \) symmetric positive definite matrices \( S_1 \ldots S_N \) and \( N \) symmetric positive definite matrices \( G_1 \ldots G_N \) matrices of appropriate dimensions satisfying Eq. 6:
\[
G_i + G_i^{T} - S_i G_i^{T} (A_i + \Delta A_i) G_i^{T} \geq 0 \quad \forall (i,j) \in (e \times e)
\]
\[
(A_i + \Delta A_i)G_i \quad S_j
\]
\[
G_i^{T} \Delta A_i^{T} \geq 0
\]
(6)
(7)
\[ \Delta A_i = D_i (-F_i) E_i \]
(8)

This is equivalent to the following inequality Eq. 7 and 8:
\[
\begin{bmatrix}
G_i + G_i^{T} - S_i G_i^{T} (A_i + \Delta A_i) G_i^{T} \\
(A_i + \Delta A_i)G_i \\
S_j
\end{bmatrix} 
\begin{bmatrix}
0 \\
G_i^{T} \Delta A_i^{T} \\
\Delta A_i G_i \\
0
\end{bmatrix} > 0
\]
(7)
(8)

Knowing that Eq. 9:
\[
\begin{bmatrix}
0 \\
G_i^{T} \Delta A_i^{T} \\
\Delta A_i G_i \\
0
\end{bmatrix} \leq \begin{bmatrix}
G_i^{T} E_i E_i G_i \\
0 \\
0 \\
\gamma^{2} D_i D_i^{T}
\end{bmatrix}
\]
(9)

So we can say that if Eq. 10:
\[
\begin{bmatrix}
G_i + G_i^{T} - S_i G_i^{T} (A_i + \Delta A_i) G_i^{T} \\
(A_i + \Delta A_i)G_i \\
S_j
\end{bmatrix} 
\begin{bmatrix}
0 \\
G_i^{T} \Delta A_i^{T} \\
\Delta A_i G_i \\
0
\end{bmatrix} > 0
\]
(10)

Then inequality (7) is true. Thereby lead to the following conditions Eq. 11:
\[
\begin{bmatrix}
G_i + G_i^{T} - S_i G_i^{T} (A_i + \Delta A_i) G_i^{T} \\
(A_i + \Delta A_i)G_i \\
S_j
\end{bmatrix} 
\begin{bmatrix}
G_i^{T} E_i E_i G_i \\
0 \\
0 \\
\gamma^{2} D_i D_i^{T}
\end{bmatrix} > 0
\]
(11)

Applying the Schur complement leads to the following proposal.

**Proposition 1:** By using Theorem developed by (Daafouz et al., 2002b) the system described by Eq. 1 is a poly-quadratically stable “if and only if there are \( N \) symmetric positive definite matrices \( S_1 \ldots S_N \) and \( N \) symmetric positive definite matrices \( G_1 \ldots G_N \) matrices” of appropriate dimensions satisfying Eq. 12:
\[
\begin{bmatrix}
G_i + G_i^{T} - S_i G_i^{T} (A_i + \Delta A_i) G_i^{T} \\
(A_i + \Delta A_i)G_i \\
S_j
\end{bmatrix} 
\begin{bmatrix}
0 \\
G_i^{T} \Delta A_i^{T} \\
\Delta A_i G_i \\
0
\end{bmatrix} > 0
\]
(12)

**Synthesis of dynamic control based on a switched observer:** This part aims to design switched observer to reconstruct the unknown states of the system (1), these states will then be used to formulate a stabilizing control law. The ultimate goal would be to stabilize the observation error and system states (Liu and Duan, 2005).

**Form of switched observer:** Let us consider an observer of the form Eq. 13:
\[
\dot{x}(k+1) = \sum_{i=1}^{M} \xi_i(k) \left( \dot{A}_i \dot{x}(k) + B_i u(k) \right) + L_i (y(k) - \dot{y}(k))
\]
(13)

The gains observation have to be calculated to guarantee the convergence to zero of the observation error Eq. 14:
\[
\varepsilon(k) = x(k) - \dot{x}(k)
\]
(14)

The evolution of the observation error is described by the following Eq. 15:
\[
\varepsilon(k+1) = \sum_{i=1}^{M} \xi_i(k) \left( A_i + \Delta A_i - \dot{A}_i \right) x(k) + L_i (y(k) - \dot{y}(k))
\]
(15)

We see here that the problem of reconstruction does not make much sense, since the observation error term depends on a term such like an open loop gain on which \( L \) has no effect. In this term we chose to consider \( A_i \neq \dot{A}_i \) as well and does not constrain the \( \dot{A}_i \) value to have an additional degree of freedom (Vidal et al., 2002; Richard, 2003).

**Control and stabilization of the system:** The new augmented system is Eq. 16-22:
\[(x(k+1)) = \sum_{i=0}^{n} s_i(k) \phi \begin{pmatrix} x(k) \\ e(k) \end{pmatrix}\]  
\(\phi = \begin{pmatrix} A_i + \Delta A_i + B_i k_i \\ \Delta_i + \Delta A_i \end{pmatrix}
\]
\(\Delta_i = A_i - \hat{A}_i\)

Let:
\[\Phi^I = \Phi + \Delta F \hat{E}_i \]
\[\hat{E}_i = \begin{bmatrix} D_i \\ D_i^T \end{bmatrix}\]

Using results of last proposition, system (16) is poly-quadratically stable “if and only if there are N symmetric positive definite matrices \(S_1...S_N\) and N symmetric positive definite matrices \(G_1...G_N\) matrices” of appropriate dimensions satisfying Eq. 23:

\[
\begin{bmatrix}
G_i + G_i^T - S_i & G_i \Phi_i^T & G_i \hat{E}_i^T \\
\Phi_i G_i & S_i & 0 \\
\hat{E}_i G_i & 0 & 1 \\
0 & \hat{D}_i & 0 & 0
\end{bmatrix}
> 0 \forall (i,j) \in (e \times e)
\]

By rewriting \(G_i\) as follows (24) and after the change of variables we arrive at the following stability condition (25) Eq. 24:

\[G_i = \begin{bmatrix} G_{i,1} & G_{i,2} \\
G_{i,3} & G_{i,4}\end{bmatrix}\]

**Proposition 2:** The system is poly-quadratically stable “if and only if there are N symmetric positive definite matrices \(S_1...S_N\) and N symmetric positive definite matrices \(G_1...G_N\) matrices” of appropriate dimensions satisfying Eq. 25-29:

\[
\begin{bmatrix}
G_i + G_i^T - S_i & \Gamma \psi_3 & 0 \\
\cdot & S_i & 0 \\
\cdot & 0 & 1 \\
0 & \cdot & 0 & \gamma_i^T
\end{bmatrix}
> 0
\]

\[\Gamma = \begin{bmatrix}
G_{i,j}^T(A_i + B_i k_i) + R_i - G_{i,j}^T B_i k_i + G_{i,j}^T A_i - R_i - U_i C_i \\
G_{i,j}^T(A_i + B_i k_i) + F_i - G_{i,j}^T B_i k_i + G_{i,j}^T A_i - F_i - V_i C_i
\end{bmatrix}
\]

\[\psi = \begin{bmatrix}
G_{i,j}^T D_i \\
G_{i,j}^T D_i + G_{i,j}^T D_i
\end{bmatrix}
\]

\[\Delta_i = A_i - \hat{A}_i\]

\[\hat{A}_i = A_i - \Delta_i\]

\[L_i = G_{i,j}^T V_i\]
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$G_{i,j} = G_{i,s}$, the condition (25) becomes Eq. 37-42:

$$
G_i + G_j^T - S_j \Gamma_{si} \psi_{si} 0 \\
(\cdot)^T S_j 0 E_j \\
(\cdot)^T 0 I 0 \\
0 (\cdot)^T 0 \gamma^{-1} 
$$

(37)

$$
\Phi_{si} = \\
G_i^T(D_j + G_i^T D_j) \\
G_j^T (A_j + B_j k_j) + F_j - G_j^T B_j k_j + G_j^T A_j - F_j - V_j C_j
$$

(38)

$$
\psi_{si} = \\
G_i^T D_j + G_j^T D_j
$$

(39)

With:

$$
\hat{A}_i = A_i - \Delta_i
$$

(40)

$$
\Delta_i = (G_{i,s})^{-1} F_j
$$

(41)

$$
L_j = G_j^T V_j
$$

(42)

RESULTS

It is clear that these methods are conservative, because the control gains and observers are calculated separately and we imposed a special structure in the matrix variable $G$. However these methods are interesting because they allow a relaxation of conditions LMIs to be solved by the freedom granted to the matrices $\hat{A}_j$, required for the construction of the observers.

Illustration example: It is a benchmark example, commonly used in the literature, which we added an uncertainty of norm bounded type.

$$
\begin{bmatrix}
0.2113 & 0.0087 & 0.4524 \\
0.0824 & 0.8096 & 0.8075 \\
0.7599 & 0.8474 & 0.4832
\end{bmatrix}
$$

$$
A = (I + aI) \\
\hat{A}_i = \\
0.0876 & 0.9036 & 0.8694
$$

$$
B = \\
0.2749 & 0.4899 \\
0.8807 & 0.7741
$$

One of the two actuators can fail, in this case the matrix $B$ may take three different values:

$$
B_1 = \\
0.6135 & 0.6538 \\
0.2749 & 0.4899 \\
0.8807 & 0.7741
$$

$$
B_2 = \\
0.6135 & 0.0000 \\
0.2749 & 0.0000 \\
0.8807 & 0.0000
$$

$$
B_3 = \\
0.000 & 0.6538 \\
0.000 & 0.4899 \\
0.000 & 0.7741
$$

A representation norm bounded uncertainty in the dynamical matrix can be given by:

$$
\Delta A_i = DFE; D_i = A_i; F^T F \leq \alpha^2 I; E = I
$$

For a maximum value of $= 0.18$, the proposition 1, allows us to obtain the state feedback gains by replacing $A_i$ by:

$$
A_i + B_i K_i :
$$

$$
\begin{bmatrix}
-1.4882 & 0.7063 & 1.7174 \\
0.7982 & -1.6053 & -2.5123
\end{bmatrix}
$$

$$
\begin{bmatrix}
-0.6604 & -0.9712 & -2.5123 \\
0.0000 & 0.0000 & 0.0000
\end{bmatrix}
$$

Based on the condition (5.47) we obtain the following observers:

$$
\begin{bmatrix}
0.2203 & 0.0207 & 0.4472 \\
0.1076 & 0.8584 & 0.8031 \\
0.7844 & 0.9036 & 0.8694
\end{bmatrix}
$$

$$
\begin{bmatrix}
0.0876 & 0.9036 & 0.8694 \\
0.7692 & 0.9396 & 0.5361
\end{bmatrix}
$$

1011
The conditions obtained from the formulation norm bounded uncertainties have the advantage of reducing the number of LMIs to be solved compared to the polytopical formulation of uncertainty which, admittedly, is less conservative but generates much of LMIs to solve.

Figure 1 and 2 shows the evolution of controls $u_1$ and $u_2$ for the uncertainty range with a loss of control $u_1$ between times $[15s 20s]$ and loss of control $u_2$ between times $[0s 10s]$.

**DISCUSSION**

The poly-quadratic Lyapunov approach provides a constructive way to tackle bounded uncertainty in the switched framework. The controller synthesis conditions are dependent on information provided by observers and the feasibility of LMIs generated by the calculations. We believe that the proposed conditions are not pessimistic and against this approach may offer alternatives to conventional methods of synthesis of current regulator for this particular type of system.

**CONCLUSION**

With these results, the study of stability can be achieved for arbitrary switching laws, state-dependent, time dependent or generated by a controller. However, the implementation of the control law is possible only if switching law status is known in real time (Gao *et al.*, 2004; Hetel *et al.*, 2008).
In terms of outlook, an important point, to seek, is to develop conditions based even on partial knowledge of the switching law without necessarily having a thorough knowledge of real-time switching.

Information obtained in advance of the switching law can afford to give less restrictive conditions, not having to account for any switching law possible. Indeed, it is possible in some practical applications, such as control systems via computer networks, to estimate an interval containing the delay as a function of various parameters of the network. It will be interesting, in this case, to define conditions of stabilization not only taking into account the parameter uncertainties but also uncertainties about the delay.

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