Gas Density and the “Volume” Schmidt Law for Spiral Galaxies

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Abstract

The thickness of the equilibrium isothermal gaseous layers and their volume densities $\rho_{\text{gas}}(R)$ in the disc midplane are calculated for 7 spiral galaxies (including our Galaxy) in the frame of self-consistent axisymmetric model. Local velocity dispersions of stellar discs were assumed to be close to marginal values necessary for the discs to be in a stable equilibrium state. Under this condition the stellar discs of at least 5 of 7 galaxies reveal a flaring. Their volume densities decrease with $R$ faster than $\rho_{\text{gas}}$, and, as a result, the gas dominates by the density at the disc periphery. Comparison of the azimuthally averaged star formation rate $SFR$ with the gas density shows that there is no universal Schmidt law $SFR \sim \rho_{\text{gas}}^n$, common to all galaxies. Nevertheless, $SFR$ in different galaxies reveals better correlation with the volume gas density than with the column one. Parameter $n$ in the Schmidt law $SFR \sim \rho_{\text{gas}}^n$, formally calculated by the least square method, lies within 0.8 – 2.4 range and it’s mean value is close to 1.5. Values of $n$ calculated for molecular gas only are characterized by large dispersion, but their mean value is close to 1. Hence the smaller $\rho_{\text{gas}}$ the less is a fraction of gas actively taking part in the process of star formation.

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1 INTRODUCTION

The key question of galaxy evolution is the dependence of star formation rate $SFR$ on the gas density and other interstellar medium parameters, averaged by large enough area or volume for smoothing random fluctuations in gas and young stars distributions.

Scmidt [1] suggested a simple form of star formation rate parameterization: $SFR_v \sim \rho_{\text{gas}}^n$, usually called the Schmidt law. From the analysis of gas and young objects distribution in the solar vicinity, Schmidt [1] obtained $n \approx 2$. Later a large number of papers were published aimed to check and to interpret the Schmidt law, based on the gas and $SFR$ distributions in

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the discs of spiral galaxies. In particular, it was found that both local, azimuthally averaged SFR values as well as the total star formation rate in a galaxy correlate (although not too tightly) with the gas content in its disc (e.g. Madore et al., [2], Kennicutt, [3], Wong&Blitz, [4], Boissier et al., [5], Shuster et al., [6]). Being empirical by its sense, the Schmidt law and its modifications open a possibility to calculate evolution models of galaxies, parameterizing the star formation history. This allows us to understand better mechanisms which regulate the rate of star formation.

A situation is entangled by the fact that the dependence $SFR_v(\rho_{\text{gas}})$ (i.e. the local Schmidt law) and, in particular, the power $n$ can’t be found directly from observations of other galaxies, because in order to estimate the volume-related values of $SFR_v$ and $\rho_{\text{gas}}$ it is necessary to know the gas layer thickness, which may vary significantly both along the galaxy radius and from one galaxy to another. Therefore in practice the Schmidt law is often replaced by the other one, outwardly alike empirical law $SFR_s \sim \sigma^N_{\text{gas}}$ (sometimes it is called Kennicutt–Schmidt law), where the compared values are scaled to unit disc surface area, or by the law $SFR_t \sim M^N_{\text{gas}}$ for the total values $SFR_t$ and gas mass $M_{\text{gas}}$ (so-called the global Schmidt law). These dependencies are more complicated for interpretation, because the compared parameters are the integrals of heterogeneous functions of distribution along of the line-of-sight (in the first case) or allover a whole disc (for the global law). In general case, parameters $n$ and $N$ must not coincide (see the discussion of the question in papers Madore et al., [2], Tutukov, [7]). Only if at any distance from disc plane $n = 1$, the power $N$ is also equal to unit. In most cases values of $N$ that obtained for different galaxies lay within the limits of $1 < N < 2$ (Wong and Blitz, [4], Boissier et al., [5], Shuster et al., [6]), but for some galaxies they proves to be more steep. For example, for M 33 $N > 3$ (Heyer et al., [8]). For the case of the global Schmidt law when different galaxies are compared, $N \approx 1.4 - 1.5$ (Kennicutt, [9], Li et al., [10]). The scatter of $N$ remains large: for galaxies with similar gas masses $SFR_t$ may differ by an order of magnitude.

Apparently, it is possible to reproduce the Schmidt law for different star formation models using some simplifying assumptions on mechanisms of self-regulation of large-scale star formation, or considering conversion of neutral gas into dense molecular clouds and formation of stars (see e.g., [7, 10, 11, 12, 13]). Note however that usually variation of the thickness of gas layer along galactic radius as well as from one galaxy to another is ignored, and $\rho_{\text{gas}}$ is accepted to be much smaller than the volume stellar density, that is not always correct.

In the first part of this paper, the gas volume density in the plane of an equilibrium disc is calculated as a function of $R$ for several nearby galaxies. In the second part, the relation between the gas density and star formation rate is analyzed. Chosen galaxies strongly differs by their properties. They include: M 33 and M 101 — multiarmed late-type galaxies (Sc), the first of them is rather small; interacting galaxy M 51 and galaxy M 100 which are distinguished by high molecular gas content; Seyfert galaxy M 106 where the ejections from nucleus and star formation burst in the inner region are observed; massive early-type Sab galaxy M 81 which possesses a large bulge
and regular spiral structure, and, finally, our Galaxy. The main simplifications we use are the following. The gaseous layers in galaxies are assumed to be axisymmetric and being in hydrostatic equilibrium. The pressure of the gas is determined by its turbulent motion: $P_{\text{gas}} = \rho_{\text{gas}} C_z^2$, where $C_z$ is one-dimensional velocity dispersion, which is assumed to be constant (although different for atomic and molecular gas). These simplifying suggestions are definitely too tough for regions enveloped by the intensive star formation, for the inner discs deep inside dense bulges or in the neighborhood of active nucleus, and also for the far periphery of discs. Note however that gas velocity dispersion, although may slowly vary with the distance from the galaxy center, remains high enough even at large distances (see the discussion in paper Dib et al., [14]). Magnetic field pressure gradient and thermal gas pressure play significantly lesser role in the formation of gas layer thickness, at least for the case of our Galaxy (see discussion of this question in Cox review, [15]). It is essential that within the limits of approximations mentioned above the observed distribution of atomic (HI) and molecular (H$_2$) hydrogen disc thicknesses along the Galaxy’s radius can be sufficiently explained (Narayan and Jog, [16]).

2 Stellar and Gaseous Discs Thicknesses

The gaseous disc thickness depends on the turbulent gas velocity and gradient of gravitational potential along the $z$-coordinate, perpendicular to disc plane. The latter is a sum of gravitational potentials of all galaxy components. Therefore for the gas density estimation one have to know not only individual component masses, but also the vertical profile of stellar disc density. Both stellar and gaseous disc scale heights are much less than their radial scales. It allows to ignore the influence of the radial heterogeneity of discs on the vertical gradient of potential.

Our approach to the estimation of the vertical gaseous and stellar density distributions is similar to that described by Narayan and Jog, [16]. Hydrostatic equilibrium equation can be written as

$$\frac{\rho_i}{\rho_i} \frac{d\rho_i}{dz} = (K_z)_s + (K_z)_{\text{HI}} + (K_z)_{\text{H2}} + (K_z)_{\text{DM}}.$$ \hspace{1cm} (1)

Here $\rho_i$ is the volume density, index $i$ (asterisk, HI, H$_2$) corresponds to the stellar, atomic, or molecular discs, and $(C_z)_i$ is vertical dispersion of velocities.

Under assumptions already mentioned, the stellar, atomic, and molecular volume density distributions are described by the equation that follows from condition of the hydrostatic equilibrium [11] (equation (3) in paper [16]):

$$\frac{d^2 \rho_i}{dz^2} = \frac{1}{\rho_i} \frac{d\rho_i}{dz} \left[ -4\pi G (\rho_s + \rho_{\text{HI}} + \rho_{\text{H2}}) + \frac{d(K_z)_{\text{DM}}}{dz} \right] + \frac{1}{\rho_i} \left( \frac{d\rho_i}{dz} \right)^2.$$ \hspace{1cm} (2)

Here $d(K_z)_{\text{DM}}/dz = \partial^2 \psi_{\text{DM}}/\partial z^2$ describes the input of dark halo, where

$$\frac{\partial^2 \psi_{\text{DM}}}{\partial z^2} = \frac{\psi_{\text{rod}} R_C}{(R^2 + z^2)^{3/2}} \arctan \left( \frac{\sqrt{R^2 + z^2}}{R_C} \right) \left[ 1 - \frac{3z^2}{R^2 + z^2} \right] +$$
\[
\frac{z^2 R_C^2 v_{rot}^2}{(R^2 + z^2)^2} + \frac{v_{rot}^2}{(R^2 + z^2)} \left[ \frac{2z^2}{(R^2 + z^2)} - 1 \right] - \\
\]

is the \( z \)-component of the second derivative of the halo potential in cylindrical coordinates ([16]). The density distribution in the halo is accepted to be quasi-isothermal:

\[
\rho_{DM}(R) = \frac{v_{rot}^2}{4\pi G} \frac{1}{(R_C^2 + R^2)} .
\]

Here \( R \) is the distance from the center of galaxy, and \( v_{rot} \) and \( R_C \) are the circular velocity and nucleus radius of the halo, respectively.

Equations (2), applied to each galaxy component, can be reduced to the first order equations. Their self-consistent solutions are found by fourth-order Runge–Kutta method. Two boundary conditions required for the mid-plane, \( z = 0 \) are:

\[
\rho_i = (\rho_0)_i , \quad \frac{d\rho_i}{dz} = 0 .
\]

Values of the central densities \((\rho_0)_i\) can be determined from the following evident condition

\[
2 \int_0^\infty \rho_i(R, z)dz = \sigma_i(R) ,
\]

(here \( \sigma_i(R) \) is the observed radial distribution of surface, or column, densities of corresponding components) and are found by the bisection algorithm. Initially, equations (2) are solved for stellar disc assuming \( \rho_{HI, H_2} = 0 \). After that, using the obtained solution, equation for HI is solved with \( \rho_{H_2} = 0 \). Finally, equation for molecular hydrogen is solved using values obtained earlier for HI and stars. This procedure is iterated using \( \rho_i \) obtained in previous iterations, until the solution converges. Thickness of discs is calculated as doubled half-width-half-maximum (HWHM) value of corresponding density distribution.

Indeed, to find the gas density by the method described above, it is necessary to calculate masses of each component of a galaxy as well as to find stellar disc velocity dispersion, which varies significantly with \( R \) even for a single galaxy unlike the gas velocity dispersion.

For all galaxies from our sample, except our Galaxy, stellar disc surface densities were estimated from the available data on the radial surface brightness distribution in disc and disc's integral color index corrected for galactic absorption and inclination (based on the HYPERLEDA [17] database or original photometry taken from the literature). We ignored the central parts of galaxies where the bulge dominates and/or observed circular rotation curve is uncertain. The largest values of \( R \) we considered were limited by extention of measured rotation curve or (M 101, M 106) gas surface density distribution. To convert the surface brightness to stellar disc surface density, we used the mass-to-luminosity ratios corresponding to color index of discs (Table A3 in Bell &de Jong [18]). These ratios were used as the first approximation to decompose the rotation curve in the frames of three-component model of galaxies which consist of the bulge, exponential disc, and quasi-isothermal halo.

The accepted distances to the galaxies, disc inclinations, and the annuli, for which we compared \( SFR \) and gas density are presented in Table 1.
Table 1: Properties of the galaxies. *Columns:* (1) — galaxy, (2) — distance, (3) — inclination, (4) — major axis diameter at the B = 25 m/□′′ isophote, (5) — radial interval, for which parameter $n$ in Schmidt law is derived.

| Galaxy | $D$ Mpc | $i,$° | $D_{25}/2$ | $\Delta R_{\text{Schmidt}}$ kpc |
|--------|---------|-------|------------|------------------|
| M 33   | 0.70    | 55    | 35.4'      | 1.0 – 6.1        |
| M 51   | 8.4     | 20    | 5.6'       | 2.0 – 8.3        |
| M 81   | 3.63    | 59    | 13.45'     | 4.0 – 11.6       |
| M 100  | 17.00   | 27    | 3.7'       | 2.0 – 16.3       |
| M 101  | 7.48    | 21    | 14.4'      | 2.0 – 10.9       |
| M 106  | 7.98    | 63    | 9.3'       | 2.1 – 9.5        |
| Galaxy | —       | —     | —          | 3.0 – 13.5       |

demonstrates the color index (which was used for the mass-to-luminosity ratio), radial brightness scale length and corresponding references, references to the sources of used rotation curves. The resulting model parameters are shown in Table 3. They include the disc and halo parameters, and the star formation rates, integrated over azimuth within the distance intervals $\Delta R$. For our Galaxy, the halo and stellar disc parameters and stellar disc HI and H$_2$ dispersions are assumed the same as in the Narayan and Jog’s paper [16], where the velocity dispersion supposed to be constant for the gas, and exponentially decreasing with $R$ for stars.

To estimate stellar disc thickness, two independent methods are employed in this paper. In the first method, the stellar velocity dispersion $C_z$ is considered to be proportional to the minimum value of the radial velocity dispersion $C_r$, which provides gravitational disc with stability to perturbations in its plane. Direct measurements of the velocity dispersion of old stellar discs support the suggestion that local values of the velocity dispersion in spiral galaxies are usually close to the minimal ones necessary to provide the dynamical stability to the disc (Bottema, [19], Zasov et al., [20]). The expression for stellar velocity dispersion along the $z$-coordinate can be written as:

$$C_{z*}(R) = K \cdot 3.36 \frac{G \cdot \sigma(R) \cdot Q(R)}{\kappa(R)},$$

(6)
Table 2: Input data related to galaxy discs. *Columns: (1) — galaxy, (2) — Color index, used in the disc mass calculations and reference to original source, (3) – radial scale length and reference to original source, (4) — reference to original source of rotation curve, (5) — reference to the HI distribution source, (6) — reference to the H$_2$ distribution source.*

| Galaxy          | Color index | Brightness scale | Rotation curves | Atomic hydrogen | Molecular hydrogen |
|-----------------|-------------|------------------|----------------|-----------------|-------------------|
| M 33            | (V − I) [40] | 5.8' [41]        | [42]           | [42]            | [42]              |
| M 51            | (B − V) [17] | 87, 4'' [43]     | [44]           | [27]            | [27]              |
| M 81            | (B − V) [17] | 158'' [43]       | [44]           | [45]            | [27]              |
| M 100           | (V − I) [46] | 48.5'' [48]      | [44]           | [27]            | [27]              |
| M 101           | (B − V) [17] | 128'' [49]       | [44]           | [4]             | [4]               |
| M 106           | (B − V) [17] | 163'' *          | [47]           | [27]            | [27]              |

*In [50] the disc scale in the optical range $b_{opt}$ is given as 5.22 kpc or 163'' for the accepted distance to M 106. From the brightness distribution [51, 52], it follows that radial disc scales in different wavebands for this galaxy are practically similar, so in this paper we used $r_0 = 163''$. 

where $K \approx 0.4−0.7$ is the velocity dispersion ratio $C_z/C_r$, determined by the stability disc condition against the bending modes, $\sigma(R)$ designates the surface disc density, $\kappa(R) = \sqrt{2 \frac{\nu(R)}{R} \left( \frac{\nu(R)}{R} + \frac{d\nu}{dR} \right)}$ — epicyclic frequency for the rotation curve $\nu_{rot}(R)$. Strictly speaking, stability parameter Toomre $Q(R)$ is equal to unit only for the idealized case of thin homogeneous disc exposed to radial perturbations. N-body modeling of marginally stable disc with various input data (see e.g. [21] and references in this paper) demonstrates that the $Q$-parameter varies along the radius in the range of 1.2–3 for a large part of disc within several radial scale lengths. In this paper we used the approximation formula $Q(R)$ from [21]:

$$Q_T^* = A_0 + A_1 \left( \frac{R}{R_0} \right) + A_2 \left( \frac{R}{R_0} \right)^2,$$

where

$A_0 = 1, 46, \ A_1 = -0, 19, \ A_2 = 0, 134$; where $R_0$ is radial disc scalelength.

The epicyclic frequency is calculated for M 33 and M 100 using the observed rotation curve. After that, the resulting radial dependence of disc thickness was smoothed. For the galaxies M 51, M 81, M 101 and M 106,
Table 3: Model parameters of galaxies discs and halos and star formation rates. *Columns*: (1) — galaxies, (2) — linear radial scale length, (3) central surface density of a disc in a model, (4) — asymptotical velocity, (5) — halo nucleus radius, (6) — star formation rates $SFR_{int}$ (7) — radial intervals, for which $SFR_{int}$ is related.

| Galaxy  | Disc parameters | Halo parameters | $SFR_{int}$ | $\Delta R$ |
|---------|----------------|----------------|--------------|------------|
|         | Scale length, kpc $\sigma_0$, $M_\odot/pc^2$ $v_\infty$,km/sec $R_e$, kpc | $M_\odot$/year | kpc |
| M 33    | 1,2 657,5       | 109,8 1,41     | 0,4          | 0 – 6,5    |
| M 51    | 3,6 1238        | 120 3,25       | 5,9          | 0,5 – 17,4 |
| M 81    | 2,8 1710        | 88 4,6        | 1,3          | 0,1 – 11,6 |
| M 100   | 4,0 1175        | 295,7 5,3     | 6,5          | 0 – 21,8   |
| M 101   | 4,6 629         | 236 5,2       | 5,3          | 0,7 – 27,9 |
| M 106   | 6,3 934         | 157 8        | 3,3          | 0,7 – 20,2 |
| Galaxy  | 3,2 641         | 220 5        | 3,7          | 3 – 13,5   |

where the irregularities of rotation curves $v_{rot}(R)$ may lead to considerable errors in $\kappa$, we use the initially “smoothed” rotation curves, calculated for three-component model galaxies (best fit models). For all galaxies (except the Galaxy) the values of $C_{zs}(R)$, calculated from the equations above, are used to estimate the half-thickness of stellar disc $h_*(R)$ and volume densities of stellar and gaseous discs in the galactic plane. Parameter $K$ in equation (6) is assumed to be 0.5.

The results of calculation of the half-thickness (defined as HWHM) of stellar disc $h_*(R)$, and HI and $H_2$ layers are shown in figures 1a – 7a (upper panel). Radial dependence $h_*(R)$ for our Galaxy practically coincides with Narayan&Jog results because the same equations and input data are used (though mathematical method of solution of the equations is different). Note, however, that $h_*(R)$ for the Galaxy in our paper is extended farther from the center. The results for molecular gas are also noticeably different from those derived by Narayan and Jog because the input data were adopted from another source.
Figure 1: Model radial dependencies for M 33. Above: a half thicknesses of the stellar disc and gas layers, below — their volume densities in the midplane. Black thick continuous lines correspond to stellar disc, green lines — to HI layer, red lines — to H\textsubscript{2} layer; blue dash thin lines are fit the model of constant stellar disc thickness: a half thicknesses of the stellar disc in the figure above and the total gas density in the figure below.

Our results show that only for our Galaxy and maybe M 33 it can be concluded that the stellar disc thickness weakly changes with galactocentric distance within a large range of \( R \). In M 33, disc thickness at the periphery falls down. However, this result is not reliable and needs to be confirmed, because for this galaxy it may be caused by the smoothing of rather irregular curve. For other our galaxies the distribution of \( h_*(R) \) is determined more reliably and disc’s thicknesses tends to grow with \( R \) as it can be seen from figures 2–7 (top panels).

The question whether the stellar discs thickness in real galaxies changes with \( R \) seems to be more simply resolved for edge-on galaxies. Nevertheless, the available estimates obtained from photometric data are rather discrepant. There are arguments both for the thickness constancy (see for example van der Kruit, [22]), and for the disc flaring — especially for early-type disc galaxies, where the disc thickness increases approximately 1.5 times within one radial brightness scalelength (de Grijs and Peletier, [23]). The latter conclusion is in a good agreement with our calculations, at least for Sab—
Sb galaxies. Our estimations are based on an assumption of marginal disc stability. Hence, if the discs actually keep their thicknesses constant, being nevertheless in a dynamical stable condition, it would mean that our model underestimates the discs thickness at least in their inner regions, and, as a sequence, the discs are dynamically overheated there.

In the second method of calculation of the gas densities, we propose that the half thicknesses of the isothermal stellar disc \( h_* \) is independent of \( R \) and is equal to \( 0.2 R_0 \), where \( R_0 \) is the photometric disc scalelength. Note that the change of the accepted value of \( h_*/R_0 \) influences the resulting gas density, but weakly affects the shape of the radial density dependencies in general.

In the model of fixed stellar disc thickness a calculating procedure should be different because the solution may not be self-consistent in this case. If the input of gas into the general surface disc density is small, the vertical density distribution in the isothermal stellar disc can be described by the well-known equation:

\[
\rho_* = \rho_{0*} \left( \text{sech} \left( \frac{|z|}{z_0} \right) \right)^2
\]  

(8)

where

\[
 z_0 = \frac{C_{z*}^2}{\pi G \sigma_{tot}}
\]  

(9)

Figure 2: Model radial dependences for M 51. Designations are the same as on the Figure 1.
and $\sigma_{\text{tot}}$ — the total surface disc density at given $R$.

Gravitational field of the stellar disc creates the vertical acceleration

$$K_z = -k^2 \cdot th(|z|/z_0),$$

where $k^2 = 4\pi G \rho_0 z_0$. Gas density equilibrium distribution (HI or H$_2$) in such a field is described by the equation [24]:

$$\rho_i(z) = \rho_{0i} \left( \text{sech} \left( \frac{|z|}{z_0} \right) \right)^{\alpha},$$

where $\alpha = k^2 z_0 / C_{zi}^2$.

Using (9) and taking into account that the stellar disc surface density is $\sigma_* = 2\rho_0 z_0$, we get:

$$\alpha = 2 \left( \frac{C_{z_*}}{C_{zi}} \right)^2 = \frac{2\pi G \sigma_* z_0}{C_{zi}^2}.$$

Joint solution of equations (5, 11, 12) enables us to estimate the gas density $\rho_{0i}(R)$ in the disc plane for a given scaleheight $z_0$. For the case of our Galaxy, the radial gas distribution and velocity dispersions in HI, H$_2$ and stellar discs are taken the same as in Narayan and Jog’s paper [16]: $C_z(\text{HI}) = 8 \text{ km/sec},$
Figure 4: Model radial dependences for M 100. Designations are the same as on the Figure 1.

\[ C_z(H_2) = 5 \text{ km/sec}, \]  and for stars an exponential function of \( R \) is accepted following \[25\]):

\[ C_{z*}(R) [\text{km/sec}] \approx 50 \cdot \exp \left\{ -\frac{R [\text{kpc}]}{8,74} \right\}. \]  (13)

The velocity dispersion \( C_{z*}(R) \) for \( R > 3 \text{ kpc} \) is defined to be close to the values obtained in N-body simulations of marginally stable discs (see Figure 8 in Khoperskov and Tyurina paper \[26\]). For six remaining galaxies we assumed the gas velocity dispersions along the \( z \)-axis to be \( C_z(HI) = 9 \text{ km/sec} \) for \( HI \) and \( C_z(H_2) = 6 \text{ km/sec} \) for \( H_2 \).

Results of calculation of the volume density in stellar discs, and corresponding values of densities in the midplane are illustrated in figures 1–7 (lower pannels). Dashed line shows distribution of the total gas density \( \rho_{gas} \) in the model with fixed stellar disc thickness. Note that both approaches give qualitatively similar results, although the volume gas density decreases slower along the radius in the discs with constant thickness.

It is obvious that the gas volume density falls along the radius faster than the surface density because of gas layer flaring at large \( R \). The density of stellar discs decreases even faster than gaseous, so these densities become nearly equal at the discs periphery. For example, in M 33 the gas density
prevails over the stellar one starting with $R \approx 5\, \text{kpc}$. In our Galaxy it is observed starting with $R \approx 12\, \text{kpc}$ . Nevertheless, the stellar disc remains dominated by the surface density over gaseous one even at this distance because of the larger half thickness.

3 Star formation and the Schmidt law

The star formation rate $SFR_s$ per unit of disc area, and its variation along the radius of spiral galaxies, as well as the relationship between $SFR_s$ and other galactic parameters, were examined by many authors (see Introduction for some references). In this paper we take radial distributions $SFR_s(R)$ obtained from the combined UV and far IR brightness data following the papers by Boissier et al. [27], Hirashita et al. [28], Buat et al. [29]. We use the data from Boissier et al. [27] where UV absorption profiles smoothed over $\sim 100''$ are presented. This approach supposes that all the energy absorbed in UV range reradiates in FIR, so that brightness ratio UV/FIR is the quantitative criterion of this absorption. The resulting values of UV brightness corrected for the absorption weakly depend upon the optical properties of dust and the spatial distribution of dust and luminous material, i.e. they are stable.
against the model choice $[30]$. Following $[28, 29]$, star formation rate may be written as:

$$SFR = \frac{SFR(UV)}{1 - \varepsilon}. \quad (14)$$

Here $1 - \varepsilon$ is a fraction of not absorbed UV quanta, that is estimated by ratio FIR/UV, and $SFR(UV) = C_{2000} \cdot L_{2000}$ is star formation rate, calculated by UV radiation intensity, $C_{2000}$ is a coefficient derived in stellar population modeling, and $L_{2000} [\text{erg sec}^{-1} \text{A}^{-1} \text{pc}^2]$ designates the monochromatic luminosity at 2000Å. For stellar population model with solar abundance and Salpeter initial mass function (IMF) of stars in the interval $0,1 – 100 \, M_\odot$ coefficient $C_{2000}$ is taken as $2.03 \cdot 10^{-40} \left[\frac{M_\odot}{\text{yr-erg/(sec-A)}}\right]$ $[29]$. Note that the estimates of $SFR_s$, obtained by the same way, we used in our previous paper $[31]$.

A different way is needed to estimate the star formation rate for our Galaxy. We tentatively admit that the $SFR$ changes along $R$ parallel to the azimuthally averaged FIR brightness distribution provided by the radiation of several hundred star forming regions studied by Bronfman et al. $[32]$. The radial distribution curve (see Figure 10 in that paper) was calibrated in such a way that for the solar vicinity $SFR_s$ equals to $4 \cdot 10^{-9} \, M_\odot/(\text{yr} \cdot \text{pc}^2)$. Being

Figure 6: Model radial dependences for M 106. Designations are the same as on the Figure 1.
integrated all over the galactic disc (within the limits considering in this paper), it gives the total $SFR_t \approx 3.6 \, M_\odot/\text{yr}$, which is in a good agreement with the available integral estimations of $SFR_t$\cite{33}.

Radial dependencies $SFR_s(R)$ for our set of galaxies are shown in Figure 8. In Figure 9, the star formation rate compares with surface gas density (Kennicutt–Schmidt relationship). For the combined data taken for all galaxies, the relationship really exists, although the individual curves form a broad bundle. Similar behavior of graphs can be found in other papers (see e.g. Figure 10 in paper Boissier et al.\cite{5}). Note that for M 81 the dependence is practically absent, and for M 106 it is ambiguous: in the inner part of the galaxy $SFR_s$ decreases with growing $\sigma_{\text{gas}}$. Note that Boissier et al. found the same behavior of $SFR_s$ for M 31 (see Figure 6 in \cite{34}).

It is essential that in the case when the surface gas density is replaced by the volume density in the disc plane, the scatter of curves in the diagram for individual galaxies reduces and they become considerably better expressed (Figure 9b). Position of our Galaxy on the diagram is not distinguish significantly from the other galaxies if not to take into account that in the inner part where gaseous density is higher the star formation rate remains rather moderate. This leads to the non-monotonic character of curve. Boissier et al.\cite{5} also noted that the Schmidt law does not fulfill in the inner part of

Figure 7: Model radial dependences for our Galaxy. Designations are the same as on the Figure 1 (except of the absence of thin dashed lines).
If we want to consider the Schmidt law in its classical form $SFR_v \sim \rho_{gas}^n$, the star formation rate must also be related to the unit of volume. Since star formation is tightly connected with molecular gas, we assume $SFR_v = SFR/2h_{H_2}$ as the mean volume density in regions of star formation, where $h_{H_2}$ is the half thickness of molecular gas layer. The star formation rate per unit volume $SFR_v$ is compared with the total gas density in the disc plane in Figure 10. The diagram for marginally stable stellar discs whose thickness changes with $R$ (case a), and the diagram for discs with a fixed thickness (case b) are shown separately. In the latter case, the curve for the Galaxy is not shown because we do not need to make the assumption of the disc thickness constancy in its case. As it can be seen from Figure 10b, transition from surface to volume gas and $SFR$ densitied decreases the difference between the galaxies. For the constant thickness disc model (Figure 10b) the dependencies for different galaxies are not in too good concordance. However, even in this case, as in the case of changing disc thickness, all galaxies except M 51 and M 101 indicate the Schmidt law parameter $n > 1$ (see Table 4).

The gas volume density in the inner parts of M 51 and M 100 is the
highest and, as it is expected, the most high star formation rate per volume unit is observed there (Figure 10a). In spite of general similarity between the curves for individual galaxies, the range of $SFR_v$ remains rather high: a typical deviation from the mean linear dependence (dashed line) corresponds to a factor of about 3. However, it is worth to mention that accuracies of estimation of the gas density and star formation rate are of the same magnitude.

Note that M 33 whose dependence $SFR_s(\rho_{gas})$ is especially steep (parameter $N$ in Kennicutt–Schmidt law exceeds 3) does not outlies considerably from the other galaxies in Figure 10. The high value of $N$ for this galaxy may be naturally explained as the result of a strong flaring of gas layer along the radius.

As in Figure 9, M 106 does not follow the general dependency: its gas density very slowly change along the radius, whereas star formation rate decreases apart from the centre. However, the situation for this galaxy becomes different if the molecular gas density $\rho_{H_2}$ (Figure 11a, b) is considered rather than the total gas density. As one can see, for all galaxies including M 106 star formation rates increases with the increasing of the molecular gas density $H_2$. In this case, the correlation also becomes more tight if we consider $SFR$ per unit volume rather than per unit area. Note however that for the whole
Figure 10: Star formation rate over the unit of volume $SFR_v$ against the volume gas density (classical Schmidt law). Left — model with changing stellar disc thickness, right — model where stellar disc thickness is fixed. Designations are the same as on the Figure 8. In the panel (b) the Galaxy is not shown.

Complex of galaxies $SFR_v$ better correlates with total gas density, than with molecular gas density (Figure 10).

In Table 4, the coefficients of inclination are presented for the straight lines shown in Figures 9b, 10a and 11b and obtained by the least square method for each galaxy (i.e. parameters $n$ in the Schmidt law)

$$\log SFR = n \log \rho + \text{const.}$$  \hspace{1cm} (15)

Accuracy of the data for individual galaxies is different, so there is no sense to mix up all the data for all galaxies to calculate a common correlation coefficient or the mean value of $n$. Instead, we calculate the values of $n$ for each galaxy separately, and then find their mean value. The r.m.s. of $n$ that illustrates the scatter of these coefficients is calculated in a simple way:

$$D = \frac{1}{7} \sum_{i=1}^{7} (n_i - < n >)^2.$$  \hspace{1cm} (16)

Values of $n_i$, $< n >$, and $D$ for the relations considered above are given in Table 4.
4 Discussion and conclusions

As it follows from this paper, the assumption of marginally stable stellar discs leads to conclusion that their thickness, at least in some galaxies, changes significantly (usually increases) along the radius. The thickness of equilibrium gas layer increases in all cases — either nearly linearly within a wide range of $R$ (M 33, M 51, M 81 and M 101), or nonlinearly, with a positive second derivative (M 100 and the Galaxy). Volume gas density in the midplane in all cases decreases within the considered range of $R$ down to several units of $10^{-25}$ g/cm$^3$. In all cases the stellar density decreases steeper, than the gas density, so at the peripheries of galaxies the gas mid-
Figure 11: “Star formation rate over molecular gas volume density” diagram. Left — SFR is related to the unit of surface area, right — to the unit of volume. Designations are the same as in Figure 8.

plane density becomes comparable with the stellar density or even exceeds it.

To compare the half thicknesses (HWHM) with the densities of stellar discs and gaseous layers in the galaxies it is convenient to consider their parameters at a fixed radius, say, \( R \approx 2R_0 \) (see Table 5). As it follows from the Table, the relative thickness of stellar disc \( h_*/R_0 \) at \( R = 2R_0 \) lies within the range of 0.1–0.3. Our Galaxy is the most thin \( (h_*/R_0 \approx 0.1) \) in our sample, whereas the stellar discs of two giant early type spiral galaxies (M 81 and M 106) are almost three times thicker. The thicknesses of gas layers differ less than stellar ones in the galaxies. The most thin gas layer (in M 33) is less than twice thinner than the most thick one (in M 101). The gas density (at distance \( 2R_0 \)) in all cases is equal to several units of \( 10^{-24} \) g/cm\(^3\) with no obvious dependence on the morphological type: in M 101 (Sc) it is approximately the same as in M 81 (Sab), and in our Galaxy (Sb–Sbc) it is close to that in M 33.

The comparison of the gaseous density with the star formation rates (figures 9b, 9d, and 10b) gives an evidence that the replacement of surface gas density (Figure 9b) by the gas volume density leads to more tight dependencies both for the “surface” and “volume” star formation rates. The reason
Table 5: Stellar disc and gas layers thicknesses and the densities of gas at $R \approx 2R_0$. Columns: (1) — a galaxy, (2) — stellar disc thickness to radial scalelength ratio, (3) — atomic gas disc thickness, (4) — molecular gas disc thickness, (5) — volume gas density (including helium).

| Galaxy | $h_*(2R_0)/R_0$ | $h_{\text{HI}}(2R_0)$, pc | $h_{\text{H}_2}(2R_0)$, pc | $\rho_{\text{gas}}(2R_0)$, g/cm$^3$ |
|--------|-----------------|-----------------|-----------------|------------------|
| M 33   | 0.18            | 87.8            | 57.6            | 6.3 \times 10^{-24} |
| M 51   | 0.23            | 123.6           | 81.3            | 3.5 \times 10^{-24} |
| M 81   | 0.34            | 129.2           | 85.7            | 1.1 \times 10^{-24} |
| M 100  | 0.20            | 108.8           | 71.0            | 6.1 \times 10^{-24} |
| M 101  | 0.19            | 173.8           | 114.8           | 1.5 \times 10^{-24} |
| M 106* | 0.27            | 150.0           | 99.2            | 2.2 \times 10^{-24} |
| Galaxy | 0.11            | 100.3           | 61.4            | 4.4 \times 10^{-24} |

*For M 106 all the values are given for $R = 1.5R_0$.

It is rather evident: star formation takes place in a narrow layer near the midplane, hence this process is sensitive namely to the volume gas density, whereas the directly measured surface gas density depends, in addition, to the gas layer thickness (more precisely — to the vertical density profile), which differs from galaxy to galaxy.

It is worth to remind that the regions that are very close to the center (in all galaxies but M 33), or are located at the disc peripheries (beyond the well defined spiral arms) need a special study and were not considered in this paper. Indeed, here we ignored the influence of the bulge on the vertical density profiles. The accepted value of the turbulent velocity may also be not suitable for all radii. A formal inclusion of the most inner and outer regions into our diagrams leads to the resulting dependencies “gas density – SFR” which are less regular and more different between galaxies.

Of the galaxies we consider, M 106 stands out by almost constant density $\rho_{\text{gas}}$ between 4 and 10kpc. The increasing of stellar disc thickness along the radius in this galaxy accompanies by the increasing of the surface gas density out of the centre within the wide range of $R$. By this reason this galaxy differs from the others in the diagrams “SFR – gas density”. The similar abnormal
behavior of this galaxy on the Kennicutt-Schmidt diagram was also revealed by Boissier et al. [5]. However the molecular gas density in M 106 behaves in the common manner i.e. steeply decreases with \( R \), that explains its “normal” position at the diagram \( SFR_v - \rho_{H2} \). Note, that this galaxy is not typical in other aspects: it differs from the others by the presence of the active nucleus, as well as by the very active star formation in the central part of the disc.

The values of \( n \) in the Schmidt law \( SFR_v \sim \rho_{gas}^n \) for the galaxies we consider are rather different (see Table 4). For all galaxies except M 101 the exponent \( n > 1 \), and its mean value is close to “standard” value \( \approx 1.4 \) for “global” Kennicutt-Schmidt law (column (3) of the Table). However, the mean value of \( n \) approaches unit, if to compare the \( SFR_v \) with the molecular gas density (column (4) in the Table).

The most “steep” relationships \( SFR(\rho_{gas}) \) are obtained for M 33, M 81 and the Galaxy. In the case of M 33 it is caused by the steep decreasing of the \( SFR \) with the galactocentric distance (see Figure 8). In the case of M 81, it is the consequence of the atypically slow decrease of the gas volume density along \( R \) (see Figure 3) when the radial gradient of \( SFR \) is moderate. For these three galaxies, the exponent \( n \) is close to the “classical” value \( n = 2 \), suggested earlier by Schmidt.

In conclusion, parameter \( n \) by no means can be considered as a universal one. The Schmidt law has a very approximate nature, and it fulfills much better when the volume instead of the surface gas density is used. In this case the exponent \( n < 2 \) (with some possible exceptions). It is important that in spite of a large dispersion its value becomes closer to unit in the mean if to replace the total gas density \( \rho_{gas} \) with the molecular gas density \( \rho_{H2} \).

It is evident that relationship between the volume gas density and \( SFR \) for galaxies should not be too tight because the process of star formation depends on a number of parameters besides the mean gas density. The latter seems to be a crucial factor only for the most dense gas such as the gas in the nuclei of molecular clouds where the HCN radiation comes from. Indeed, as observations show, the star formation rate depends linearly on the gas mass determined by the HCN line intensity, i.e. for the most dense molecular gas \( n \approx 1 \) (Gao and Solomon, [35, 36]). Although the number of galaxies we considered is small, the results allow to propose that the value of \( n \) is on average close to unit even for the less dense molecular gas, that reveals itself in CO line. Since \( n > 1 \) for the total \((HI + H2)\) gas density, one may conclude that not only star formation rate, but also the efficiency of star formation \((SFR \) per gas mass unit\) decreases along with \( \rho_{gas} \). In other words, the less is the gas density, the longer time the gas remains in the rarified atomic state (that is a time scale of gas consumption is larger). It agrees with general conception, according to which the fraction of the interstellar gas which takes active part in the process of a star formation, decreases when the mean total density of gas becomes lower (see the discussion in [4, 37]).

The other important factor, besides the gas density, that determines the star formation rate at a given radius \( R \) is the surface density of the old stellar disc \( \sigma_* \) (see papers [31, 38, 39]). As it was shown by Zasov and Abramova [31] on the example of four well-studied galaxies, local star formation efficiency defined as \( SFE = SFR/\sigma_{gas} \) changes approximately as \( \sigma_*^{0.7} \) in a wide inter-
val of $R$. Partially this relationship may be explained by higher volume gas density (for a given surface density) in those regions, where $\sigma_*$ is higher (that is closer to galaxy center). Nevertheless this relationship cannot be reduced to the simple exponential Schmidt law (neither “volume” nor “surface” one) because the clearly defined relationship between the gas density and surface disc density is absent. It evidences the existence of more deep connection between the present-day star formation on the one hand and already formed stellar disc and the gas density on the other hand, which cannot be described by simple empiric laws.

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