Lorentz Force and Ponderomotive Force in the Presence of a Minimal Length

B. Khosropour *

Department of Physics, Faculty of Sciences, Salman Farsi University of Kazerun, Kazerun, 73175-457, Iran

Abstract

In this work, according to the electromagnetic field tensor in the framework of generalized uncertainty principle (GUP), we obtain the Lorentz force and Faraday’s law of induction in the presence of a minimal length. Also, the ponderomotive force and ponderomotive pressure in the presence of a measurable minimal length are found. It is shown that in the limit $\beta \to 0$, the generalized Lorentz force and ponderomotive force become the usual forms. The upper bound on the isotropic minimal length is estimated.

Keywords: Phenomenology of quantum gravity; Generalized uncertainty principle; Minimal length; Lorentz force; Ponderomotive force

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*E-mail: b_khosropour@kazerunsfu.ac.ir
1 Introduction

Different theories of quantum gravity such as the loop quantum gravity, string theory and noncommutative geometry have been proposed for exploring the unification between the general theory of relativity and the standard model of particle physics [1]. Although these theories are different in concepts, all of these investigations lead to unique belief which predicts the existence of a minimal length scale. An immediate consequence of existence of a minimal length gave rise to the modification of Heisenberg uncertainty principle. Nowadays the modified uncertainty principle is known generalized uncertainty principle (GUP) [2]. The generalized uncertainty principle (GUP) can be written as follows:

$$\Delta X \Delta P \geq \frac{\hbar}{2} \left[ 1 + \beta (\Delta P)^2 \right], \quad (1)$$

where $\beta$ is a positive parameter [3,4] and it is obvious that $\Delta X$ is always greater than $(\Delta X)_{\text{min}} = \hbar \sqrt{\beta}$. In the recent years, many studies have been devoted to the gravity and reformulation of quantum field theory and electrodynamics in the presence of a minimal length scale [5-20]. Quesne and Tkachuk have introduced a Lorentz covariant deformed algebra which describes a $D+1$-dimensional space time is characterized by the following deformed commutation relations:

$$[X^\mu, P^\nu] = -i\hbar[(1 - \beta P^\rho P^\nu)g^{\mu\nu} - \beta' P^\mu P^\nu], \quad (2)$$

$$[X^\mu, X^\nu] = i\hbar \frac{2\beta - \beta' - (2\beta + \beta')\beta P^\rho P^\rho}{1 - \beta P^\rho P^\rho}(P^\mu X^\nu - P^\nu X^\mu),$$

$$[P^\mu, P^\nu] = 0,$$

where $\mu, \nu, \rho = 0, 1, 2, \cdots, D$ and $\beta, \beta'$ are two positive deformation parameters. In Eq. (2), $X^\mu$ and $P^\mu$ are position and momentum operators in the framework of GUP and $g_{\mu\nu} = g^{\mu\nu} = \text{diag}(1, -1, -1, \cdots, -1)$. An immediate consequence of Eq. (2) is the following an isotropic minimal length

$$((\Delta X_i)_{\text{min}} = \hbar \sqrt{(D\beta + \beta')[1 - \beta \langle (P^0)^2 \rangle]} \quad \forall i \in \{1, 2, \cdots, D\}. \quad (3)$$

In the present work, we study the Lorentz force and ponderomotive force in the presence of a minimal length. The paper is organized as follows: In Sec. 2, the lorentz force and Faraday’s law of induction is obtained in the presence of a minimal length. In Sec. 3, the ponderomotive force in the framework of GUP is found and the relative modification of ponderomotive pressure is obtained. Also, the upper bound on the isotropic minimal length is estimated. Our conclusions are presented in Sec. 4.
2 Lorentz Force in the Presence of a Minimal Length

The aim of this section is finding the Lorentz force and Faraday’s law in the presence of a minimal length based on the Quesene-Tkachuk algebra. Hence, we need a representation which satisfies the generalized commutation relations in Eq. (2). In Ref. [23], Tkachuk introduced the following representation which satisfies the deformed algebra up to the first order in deformation parameters $\beta$ and $\beta'$

\[
X^\mu = x^\mu - \frac{2\beta - \beta'}{4}(x^\mu p_\rho p^\rho + p_\rho p^\rho x^\mu),
\]

\[
P^\mu = (1 - \frac{\beta'}{2}p_\rho p^\rho)p^\mu,
\]

where $x^\mu$ and $p^\mu = i\hbar \frac{\partial}{\partial x^\mu} = i\hbar \partial^\mu$ are position and momentum operators in usual quantum mechanics. It is interesting to note that in the special case of $\beta' = 2\beta$, the position operators commute to the first order in deformation parameter $\beta$, i.e., $[X^\mu, X^\nu] = 0$. In this special case, the Quesene-Tkachuk algebra becomes

\[
[X^\mu, P^\nu] = -i\hbar[(1 - \beta P_\rho P^\rho)g^{\mu\nu} - 2\beta P^\mu P^\nu],
\]

\[
[X^\mu, X^\nu] = 0,
\]

\[
[P^\mu, P^\nu] = 0.
\]

The following representations satisfy Eq. (5), in the first order in $\beta$:

\[
X^\mu = x^\mu,
\]

\[
P^\mu = (1 - \beta p_\rho p^\rho)p^\mu.
\]

2.1 The Modified Lorentz Force

The Lorentz force is the combination of electric and magnetic force on a charged particle due to electromagnetic fields. If we consider a particle of charge $q$ moves with velocity $\mathbf{v}$ in the presence of an electric field $\mathbf{E}$ and a magnetic field $\mathbf{B}$, then the Lorentz force is given by [24]

\[
\frac{dp}{dt} = q(\mathbf{E} + \mathbf{V} \times \mathbf{B}).
\]

We know that $p$ transforms as the space part of the momentum four-vector,

\[
p^\mu = (p^0, \mathbf{p}) = (\frac{E}{c}, \mathbf{p}) = m(U^0, \mathbf{U}),
\]
and $U^\alpha$ is the velocity four-vector. The covariant form of the Lorentz force in a 3+1-dimensional space time can be written as follows[24]

$$\frac{dp^\alpha}{d\tau} = qU_\rho F^{\alpha \rho},$$

(10)

where $F^{\alpha \rho}$ is the electromagnetic field tensor and $\tau$ is the proper time which are the following definition

$$F^{\alpha \rho} = \partial^\alpha A^\rho - \partial^\rho A^\alpha,$$

(11)

$$\tau = \frac{t}{\gamma} = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}. $$

(12)

In a 3+1 dimensional space time, the components of the electromagnetic field tensor $F^{\alpha \rho}$ can be written as

$$F^{\alpha \rho} = \begin{pmatrix}
0 & -E_x/c & -E_y/c & -E_z/c \\
E_x/c & 0 & -B_z & B_y \\
E_y/c & B_z & 0 & -B_x \\
E_z/c & -B_y & B_x & 0
\end{pmatrix}. $$

(13)

Now, we obtain the electromagnetic field tensor in the presence of a minimal length based on the Quesne-Tkachuk algebra. For this purpose, let us write the electromagnetic field tensor in Eq. (11) by using the representations Eqs. (6) and (7), that is

$$x^\mu \rightarrow X^\mu = x^\mu, $$

$$\partial^\mu \rightarrow D^\mu := (1 + \beta \bar{h}^2 \Box) \partial^\mu, $$

(14)

where $\Box := \partial_\nu \partial^\nu$ is the d’Alembertian operator. If we substitute Eq. (14) into Eq. (11), we will obtain the following modified electromagnetic field tensor

$$F^{\alpha \rho} = D^\alpha A^\rho - D^\rho A^\alpha = (1 + \beta \bar{h}^2 \Box) \partial^\alpha A^\rho - (1 + \beta \bar{h}^2 \Box) \partial^\rho A^\alpha $$

(15)

$$= F^{\alpha \rho} + \beta \bar{h}^2 \Box F^{\alpha \rho} = (1 + \beta \bar{h}^2 \Box) F^{\alpha \rho}. $$

The term $\beta \bar{h}^2 \Box F^{\alpha \rho}$ in Eq. (15) can be considered as a minimal length effect. By inserting the modified electromagnetic field tensor into Eq. (10), the modified Lorentz force can be obtained as follows

$$\frac{dp^\alpha}{d\tau} = qU_\rho F^{\alpha \rho} = qU_\rho (1 + \beta \bar{h}^2 \Box) F^{\alpha \rho}. $$

(16)

According to Eq. (13), Eq. (15) can be written in the vector form as follows

$$\frac{dp}{dt} = q[E(x,t) + v \times B(x,t)] $$

(17)

$$+ q\beta \bar{h}^2 (\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2)[E(x,t) + v \times B(x,t)]. $$

In the limit $\beta \rightarrow 0$, the generalized Lorentz force in Eq. (16) becomes the usual Lorentz force.
2.2 Faraday’s Law of Induction in the Presence of a Minimal Length

If we consider a circuit wire in the presence of a magnetic field, we find the induced electromotive force in the wire. The induced electromotive force around the circuit wire is equal to Faraday’s law of induction, that is

$$\epsilon = -\frac{d\phi_B}{dt},$$

(18)

where $\phi_B$ is the magnetic flux through the wire. It is easy to derive the Faraday’s law from the Lorentz force and the Maxwell equations. As we know that the electric field and the induced electromotive force are defined by

$$E(x, t) = \frac{F(x, t)}{q},$$

(19)

$$\epsilon = \int_c E \cdot dL.$$

(20)

If we substitute Eq. (19) into Eq. (20) and considering Lorentz force in Eq. (8), we have

$$\epsilon = \int_c E \cdot dL = \int_c \frac{F(x, t)}{q}$$

(21)

$$= \int_c E \cdot dL + \int_c (v \times B(x, t)) \cdot dL.$$

Hence, if we use the Kelvin-Stokes theorem and considering the Maxwell-Faraday’s equation ($\nabla \times E = -\frac{\partial B}{\partial t}$), we will obtain

$$\epsilon = -\int \int_S ds \cdot \frac{\partial B(x, t)}{\partial t} + \int_c (v \times B(x, t)) \cdot dL.$$

(22)

Now, using the Leibniz integral rule and the $\nabla \cdot B = 0$, we can find the following Faraday’s law of induction

$$\epsilon = -\frac{d}{dt} \int \int_S ds \cdot B(x, t) = -\frac{d\phi_B}{dt}.$$

(23)

Let us find the Faraday’s law in the presence of a minimal length. If we use Eqs. (17) and (21), we can write the modified induced electromotive force as follows

$$\epsilon_{ML} = \int_c \frac{F_{ML}}{q} = \left[\int_c E \cdot dL + \int_c (v \times B) \cdot dL\right]$$

(24)

$$+ \beta\hbar^2 \left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2\right) \left[\int_c E \cdot dL + \int_c (v \times B) \cdot dL\right].$$
According to our previous work[11], we have found that the homogeneous Maxwell’s equations \((\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \nabla \cdot \mathbf{B} = 0)\) were not modified by the effect of minimal length. Therefore by using the Kelvin-Stokes theorem and Maxwell-Faraday, Eq. (24) will be became

\[
\epsilon_{ML} = \left[ -\int \int_S ds \frac{\partial \mathbf{B}(x, t)}{\partial t} + \int_c (\mathbf{v} \times \mathbf{B}(x, t)) \cdot d\mathbf{L} \right] \tag{25}
\]

\[+ \beta \hbar^2 \left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) \left[ -\int \int_S ds \frac{\partial \mathbf{B}(x, t)}{\partial t} + \int_c (\mathbf{v} \times \mathbf{B}(x, t)) \cdot d\mathbf{L} \right].\]

Now, by using the Leibniz integral rule and \(\nabla \cdot \mathbf{B} = 0\), the Faraday’s law of induction in the presence of a minimal length can be obtained as follows

\[
\epsilon_{ML} = \left[ -\frac{d}{dt} \int \int_S ds \cdot \mathbf{B}(x, t) \right] + (\hbar \sqrt{\beta})^2 \left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) \left[ -\frac{d}{dt} \int \int_S ds \cdot \mathbf{B}(x, t) \right] \tag{26}
\]

\[= \left[ -\frac{d\phi_B}{dt} \right] + (\hbar \sqrt{\beta})^2 \left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) \left[ -\frac{d\phi_B}{dt} \right].\]

3 Ponderomotive Force in the Presence of a Minimal Length

Light waves exert a radiation pressure which is usually very weak and hard to detect. When high-powered microwaves or laser beams are used to confine plasmas, however, the radiation pressure reaches to very high value[25]. When high-powered microwaves applied to a plasma, this force is coupled to the particles in a somewhat subtle way and is known the ponderomotive force. Many nonlinear phenomena have a simple explanation in terms of the ponderomotive force. For deriving this nonlinear force is to consider the motion of an electron in the oscillating \(\mathbf{E}\) and \(\mathbf{B}\) fields of a wave. By vanishing dc \(\mathbf{E}_0\) and \(\mathbf{B}_0\) fields, the electron equation of motion is [25]

\[m \frac{d\mathbf{v}}{dt} = -e(\mathbf{E}(r) + \mathbf{v} \times \mathbf{B}(r)). \tag{27}\]

The nonlinearity comes partly from the \(\mathbf{v} \times \mathbf{B}\) term, hence the term is no larger than \(\mathbf{v}_1 \times \mathbf{B}_1\), where \(\mathbf{v}_1\) and \(\mathbf{B}_1\) are the linear-theory values. Another part of the nonlinearity, comes from evaluating \(\mathbf{E}\) at the actual position of the particle rather than its initial position. By considering a wave electric field of the form

\[E = E_s(r) \cos(\omega t), \tag{28}\]

where \(E_s(r)\) contains the spatial dependence and also in first order, we may vanish the \(\mathbf{v} \times \mathbf{B}\) term in Eq. (27), therefore we have

\[m \frac{d\mathbf{v}_1}{dt} = -e\mathbf{E}(r_0).\]
\[ v_1 = \frac{-e}{m\omega} E_s \sin(\omega t), \]  
(29)

\[ \delta(r_1) = \frac{e}{m\omega^2} E_s \cos(\omega t). \]  
(30)

For finding the second order, we expand \( E(r) \) about \( r_0 \)

\[ E(r) = E(r_0) + (\delta r_1 \cdot \nabla) E|_{r=r_0} + \ldots, \]  
(31)

and from Maxwell’s equation, we can obtain \( B_1 \) as follows

\[ B_1 = -\frac{1}{\omega} \nabla \times E_s|_{r=r_0} \sin(\omega t). \]  
(32)

Now, we are ready to study the second order part of Eq. (27), hence we have

\[ m\frac{dv_2}{dt} = -e[(\delta r_1 \cdot \nabla) E + v_1 \times B_1]. \]  
(33)

Inserting Eqs. (29), (30) and (32) into Eq. (33) and averaging time, we can find the ponderomotive force on a single electron as follows

\[ f = -\frac{1}{4} \frac{e^2}{m\omega^2} \nabla |E_s|^2. \]  
(34)

Here we used \( \langle \sin^2(\omega t) \rangle = \langle \cos^2(\omega t) \rangle = \frac{1}{2}. \)

Now, let us obtain the ponderomotive force in the presence of a minimal length. According to Eq. (17), the electron equation of motion in the presence of a minimal length is given by

\[ m\frac{dv}{dt} = -e[E(r) + v \times B(r)] \]  
(35)

\[ - e\beta \hbar^2 \left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) [E(r) + v \times B(r)]. \]

From Eq. (33), the modified second order part of Eq. (35) can be written as follows

\[ m\frac{dv_2}{dt} = -e[(\delta r_1 \cdot \nabla) E + v_1 \times B_1] \]  
(36)

\[ - e\beta \hbar^2 \left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) [(\delta r_1 \cdot \nabla) E + v_1 \times B_1]. \]

By inserting Eqs. (29), (30) and (32) into Eq. (36), we will obtain

\[ m\frac{dv_2}{dt} = -\frac{e^2}{m\omega^2} [(E_s \cdot \nabla) E_s (\cos^2(\omega t)) + (E_s \times \nabla \times E_s) \sin^2(\omega t)] \]  
(37)

\[ - \frac{e^2}{m\omega^2} \beta \hbar^2 \left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) [(E_s \cdot \nabla) E_s (\cos^2(\omega t)) + (E_s \times \nabla \times E_s) \sin^2(\omega t)]. \]
After simplifying and averaging time, we can obtain the following generalized ponderomotive force on a single electron

$$\langle f \rangle_{ML} = -\frac{1}{4} \frac{e^2}{m_\omega^2} \left[ \nabla |E_s^2| - \left( (\hbar \sqrt{\beta})^2 \nabla^2 \right) \nabla |E_s^2| \right].$$ \hspace{1cm} (38)

It should be noted that in the limit $\beta \to 0$, the generalized ponderomotive force in Eq. (38) becomes the usual ponderomotive force. Another hand, if we substitute $\beta' = 2\beta$ into Eq. (3), we will find the following isotropic minimal length up to the first order over the deformation parameter $\beta$

$$(\Delta X_i)_{min} = \hbar \sqrt{(D + 2)\beta}, \quad \forall i \in \{1, 2, \cdots, D\}.$$ \hspace{1cm} (39)

The isotropic minimal length in three spatial dimensions is given by

$$(\Delta X)_{min} = \hbar \sqrt{5\beta}.$$ \hspace{1cm} (40)

Here, by substituting Eq. (40) into Eq. (38), we have

$$f_{ML} = -\frac{1}{4} \frac{e^2}{m_\omega^2} \left[ \nabla |E_s^2| - \left( \frac{(\Delta X)^2_{min}}{5} \nabla^2 \right) \nabla |E_s^2| \right].$$ \hspace{1cm} (41)

The force per $m^3$ is $f$ times the electron density $n_0$, which can be defined in terms of plasma frequency ($\omega_p$) as follows

$$\omega_p = \sqrt{\frac{4\pi n_0 e^2}{m_e}}.$$ \hspace{1cm} (42)

From Eqs. (41) and (42) and since $E_s^2 = 2\langle E^2 \rangle$, we finally have for the generalized ponderomotive force the formula

$$F_{ML} = -\frac{\omega_p^2}{\omega^2 8\pi} \left[ \nabla \langle E^2 \rangle - \left( \frac{(\Delta X)^2_{min}}{5} \nabla^2 \right) \langle E^2 \rangle \right].$$ \hspace{1cm} (43)

Also, as we know the ponderomotive pressure has the following definition

$$F = -\nabla P_{pond}.$$ \hspace{1cm} (44)

According to Eq. (44), we can obtain the generalized ponderomotive pressure as follows

$$P_{pond} = \frac{\omega_p^2}{\omega^2 8\pi} \left[ \langle E^2 \rangle - \left( \frac{(\Delta X)^2_{min}}{5} \nabla^2 \langle E^2 \rangle \right) \right].$$ \hspace{1cm} (45)
If we consider the first term \((P_{pond})_0\), the usual ponderomotive pressure and the second term is its correction due to the considered minimal length effect \((P_{pond})_{ML}\), the relative modification of ponderomotive pressure can be found as

\[
\Delta P_{pond} = \frac{\omega_p^2}{\omega^2 40\pi} \left[ ((\Delta X)^2_{\text{min}} \nabla^2) \langle E^2 \rangle \right].
\] (46)

Now we can estimate the upper bound on the isotropic minimal length in the modified ponderomotive pressure. If we consider the value of ponderomotive pressure is about \(10^6 Pa\) and also assuming \(\omega_p \approx 10^{28} Hz\), \(\omega \approx 10^{13} Hz\). Therefore from Eq. (46), we can estimate the following upper bound on the the isotropic minimal length

\[
10^6 \approx 10^{30} \left[ ((\Delta X)^2_{\text{min}} \nabla^2) \langle E^2 \rangle \right],
\] (47)

\((\Delta X)_{\text{min}} \leq 10^{-12} m\).

It is interesting to note that the estimation on the isotropic minimal length is near to the minimal observable distance which was proposed by Heisenberg \((L \sim 10^{-15} m)\)\([26,27]\). On the other hand, Nouicer has investigated the Casimir effect in the framework of GUP and obtains \((\Delta X^i)_{\text{min}} \leq 15 \times 10^{-9} m\). The upper bound on the isotropic minimal length in Eq. (47) is compatible with the results of Refs. [30] and [31].

Although the above value for the minimal length \(((\Delta X^i)_{\text{min}} \approx 10^{-12} m)\) is about six orders of magnitude larger than the electroweak length scale \((10^{-18} m)\), this value is near to the reduced Compton wave length of electron \((\lambda_c = 3.86 \times 10^{-13} m)\).

4 Conclusions

In the last decade, many investigations have been done to compute the corrections of different phenomena of electrodynamics and quantum mechanics in the framework of GUP. According to these studies, we can find that the implications of GUP can be sought at atomic scales \([28,29]\). In this study, first we have obtained the Lorentz force in the presence of a minimal length based on the Quesne-Tkachuk algebra. Then from the generalized Lorentz force we have found the Faraday’s law of induction in the framework of GUP. As many nonlinear phenomena have a explanation in terms of the ponderomotive force, we have investigated the ponderomotive force in the presence of a minimal length scale. Also, the generalized ponderomotive pressure has been obtained. It should be emphasized that in the limit \(\beta \to 0\), all of the generalized lorentz force and Faraday’s law and ponderomotive force become the ordinary forms. Finally, the upper bound on the isotropic minimal length scale has been estimated by using the experimental value of ponderomotive pressure. It is interesting to note that the estimation on the isotropic minimal length was near to the minimal observable distance which was proposed by Heisenberg \((L \sim 10^{-15} m)\).
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