The question of whether the universe is eternal or if it had a singular moment of creation is deeply intriguing. Although different versions of steady state and oscillatory models of eternal universe have been envisaged, empirical evidence suggests a singular moment of creation at the big bang. Here we analyze the oscillatory solutions for the universe in a modified theory of gravity THED (Torsion Hides Extra-Dimension) and evaluate them by fitting Type 1a supernovae redshift data. THED-gravity exactly mimics General Relativity at the kinematical level, while the modifications in its dynamical equations allow the universe to bounce between a minimum size and a maximum size with a zero average energy within each oscillation. The optimally fit oscillatory solutions correspond to a universe with (i) a small matter density requiring little to no dark matter, (ii) a significantly negative spatial curvature, (iii) a tiny negative dark energy, (iv) and an arbitrarily large maxima and small minima whose exact values depend on the tinyness of the dark energy, which can’t be empirically constrained. Alternatively, there exists non-oscillating solutions that appear as an ever-expanding universe from a single bounce at some minimum size preceded by a collapse from the infinite past. These ever-expanding solutions indeed fit the redshift data marginally better than the oscillating solutions and are at par with the best fit of standard big bang theory. They accommodate a range of matter densities requiring dark matter, positive dark energy and positive spatial curvature. A qualitative analysis of CMB power spectrum with the modified dynamical equations predicts a negative spatial curvature for the universe, in stark contrast to a near-zero curvature in the standard big bang theory. An independent constraint on the spatial curvature can further shed light on discriminating the ever-expanding and oscillatory universe scenarios.

I. INTRODUCTION

The standard model of cosmology has now been accepted to be ΛCDM, wherein the space-time evolves according to equations of General Relativity (GR) with approximately 73% dark energy (cosmological constant) and 27% cold matter (85% of which is invisible and possibly non-baryonic dark matter) on a more or less flat spatial geometry ($\Omega_\Lambda \approx 0.73$, $\Omega_M \approx 0.27$, $\Omega_k \approx 0$). This standard model implies that the universe has a singular point of creation, the big-bang, at about 14 billion years ago; however since classical GR breaks down at the singularity it is believed that a complete theory of quantum-gravity would resolve the singularity issues and explain the apparent creation of the universe. In this paper, we shall take the contrasting perspective that quantum-gravity is not essential to resolve the issues of singularity, rather a modified theory of gravity that is singularity-free at the classical level can describe the dynamics underlying the apparent ‘creation’ process. Here we analyze a modified theory of gravity that adapts Torsion to Hide an Extra-Dimension (THED-gravity) [1]; it exactly matches GR at the kinematical level, while the modifications in its dynamical equations leads to the possibility of averting classical singularities and yields eternally oscillating solutions for the universe.

A. Various Approaches to Eternal Universe

A beginning-less eternal universe has its charm because it obviates the question of creation. Soon after proposing GR, Einstein himself in 1917 contrived an unstable static universe model with a positive cosmological constant to balance the gravitational attraction of matter. With Hubble’s discovery of galactic redshifts in 1929, which made it clear that the universe is currently expanding, Fred Hoyle and colleagues envisaged an expanding steady state model with a magic-mechanism of continual matter creation [2]. After the discovery of cosmic microwave background radiation in 1965, the evolution of the universe from a hot dense state became indisputable and the big-bang model gained acceptance. The singularity issue (the creation-question) in this model has been regarded as a consequence of a lack of quantum-gravity theory at the Planck-scale by some researchers, while some others have explored various elegant approaches to resolve the singularity issue within classical-gravity. Notable approaches include models of eternal inflation [3-4], emergent universe from a cosmic seed [5], bounce cosmology [6], oscillatory universe [7-9], quasi-steady-state cosmology [10], bubble universes within blackholes [11-12].

In the eternal inflation scenario [3-4], the universe as a whole is always inflating (exponentially expanding) while small patches of it stop inflating and reheats to fill the observable universe with matter and radiation. The singularity theorems [13] imply the existence of a singularity in the past as long as certain energy conditions [14] (NEC-Null Energy condition for spatially open or flat universes, and Strong energy condition for spatially
closed universe) are satisfied. More generally, it has been shown that if the average expansion rate of the universe is positive, then irrespective of any energy condition or even the underlying theory of gravity, there must exist a singularity in the past (BGV theorem)\textsuperscript{15}. This has essentially shut the door for the possibility of inflationary models to resolve the big bang singularity issue at the classical level.

In the \textit{bubble universe inside black holes} scenario\textsuperscript{11}, every black hole formed out of a gravitational collapse gives birth to a baby-universe (possibly with different fundamental physical constants). For example, as matter collapses to extremely high densities inside a black hole, the torsional field coupled to the fermions\textsuperscript{10} is expected to generate a repulsive force and a bounce to form a baby universe\textsuperscript{12}. However, the BGV theorem\textsuperscript{15} appears to be applicable in this scenario because the expansion rate of any matter in the universe (before it starts collapsing to form the black hole) would vastly override its contraction rate during the collapse, leading us to expect a classical singularity in the past. This scenario can at best be considered an intriguing conjecture that needs quantum-gravity to resolve the details.

In the \textit{emergent universe} scenario, the universe is a fine-tuned cosmic seed that is static in the asymptotic past, and some \textit{ad hoc} mechanism triggers its inflation and expansion. In general, exotic matter that violate NEC is required to contrive such a scenario\textsuperscript{17}. However, in a spatially closed universe, the cosmic seed can be tailored to be an Einstein static universe without requiring exotic matter, but extreme fine tuning of a scalar field potential and its kinetic energy is required\textsuperscript{5}. Such a configuration would be susceptible to quantum fluctuations (particularly the homogenous mode) in the field and it is highly questionable if the universe can remain asymptotically static for infinite time in the past.

In the \textit{bounce cosmology} scenario\textsuperscript{6}, the present era of expansion of the universe is preceded by a contracting phase all the way from the infinite past. Although no fine-tuning is required, an \textit{ad hoc} bounce mechanism with a ghost condensate field\textsuperscript{18} that violates NEC has to be introduced to smoothly transition from the contraction to expansion phase. At a philosophical level, it is hard to motivate the conditions wherefrom an infinitely dilute universe would homogeneously collapse.\textsuperscript{1}

In the \textit{oscillatory universe} scenarios, the universe repeatedly goes through cycles of expansion and contracting phases. Tolman\textsuperscript{10} applied the second law of thermodynamics to the universe as a whole and concluded (irrespective of the underlying theory of gravity) that the entropy of the entire universe would have to increase in every successive cycle leading to a thermal-death of the universe in a finite time, else the volume of the universe should grow in every successive cycle. The Quasi-steady state model\textsuperscript{10} enforces this argument by explicitly invoking a traceless scalar field to continuously create new matter in the universe, thereby increasing the volume and entropy of the universe in each cycle. Some oscillatory models deal with the issue of entropy-buildup by requiring the expansion phase to be so large that all the matter debris are diluted to the extent that the universe comes back to a pristine vacuum state when it enters the contracting phase. For example the Baum-Frampton model\textsuperscript{9, 20} proposes that only a small patch of the universe comes back to the contracting phase after having jetisoned all the entropy it holds, without prescribing the actual mechanism underlying the bounce and turnaround of universe. In Steinhard-Turok model\textsuperscript{7}, a scalar field couples to the matter fields in a unique fashion and rolls over a tailored potential well (that acts as dark energy) to generate the expansion, turnaround and contraction of the universe. To induce a bounce from contraction back to expansion, the scalar field gets magically reflected reversing its momentum to produce some matter and radiation as byproducts for the next cycle (which is interpreted as inelastic collision between branes in higher dimensions). All these models involve some kind of unknown mechanism that is not obviously justifiable. Furthermore, since these models are in accord with the Tolman argument, the universe cannot oscillate within fixed bounds—the minimum size of the universe increases every cycle leading to a net positive expansion rate, and application of the BGV theorem\textsuperscript{15} implies an unavoidable singularity in the past.

### B. Bounded Oscillating universe

For models wherein oscillations happen within fixed bounds, eternal oscillations are admissible because the BGV theorem\textsuperscript{15} does not apply; however the Tolman argument\textsuperscript{10} has to be discarded. From a fundamental statistical-mechanics perspective, there does not exist an unambiguous concept of \textit{equilibrium} or \textit{entropy} for an unconstrained isolated system. The universe is an isolated system, its volume is dynamically unconstrained, and it is in no sense equilibrating with an external thermal bath. It may be acceptable to consider a small patch of the universe as being immersed in a thermal bath constituted by the rest of the universe (and thereby allude to thermodynamic concepts of temperature and entropy), but there is no justification to impose the second law of thermodynamics to the universe as a whole when gravitational dynamics is involved. Hence in this paper we shall not consider the Tolman argument as a theoretical hindrance to prevent the universe from oscillating between fixed bounds.

In General Relativity the singularity theorems prohibit a spatially flat or open universe from smoothly oscillating.

---

\textsuperscript{1} In both \textit{emergent} and \textit{bounce} scenarios, the universe would be in a life-friendly state (ability to energetically host life) very briefly when compared to its eternal lifetime, while in the \textit{oscillatory} scenario the universe would repeatedly spend significant fraction of time in life-friendly state.
between a fixed bounds when the NEC is satisfied. The NEC is considered sacrosanct because it is satisfied by all known macroscopic matter in the universe \[21\]. However a closed universe can exhibit smooth stable oscillations within fixed bounds without violating NEC \[8\], but it requires abnormal-matter negative pressure. This implies normal-matter observed in present state of universe cannot be accounted for in an oscillating universe scenario within General Relativity. Contrastingly, in modified theories of gravity like THED \[1\], bounded oscillating universe with normal-matter (like dust) is permissible with any spatial topology.

In the oscillatory universe scenario, the minimum size at which the universe bounces back from contraction phase to expansion phase is very significant. The smaller and hotter the universe gets, the hot plasma soup could better dissolve the inhomogeneities aggregated from gravitational lumping in the prior cycle. Black holes formed in prior cycles can be particularly resistant to such dissolution into plasma soup. THED-gravity presents a simple solution by preventing the extreme inhomogeneity-buildup into black holes. It has been shown that there can exist extremely compact objects that does not violate the NEC \[22\], which would otherwise be expected to collapse into black holes in the context of GR. That is, finite pressure can restrain the gravitational collapse of a massive object even after it has shrunk to an arbitrarily small size, thus preventing the formation of singularity while leaving the exterior geometry virtually indistinguishable from that of a black hole. Hence in THED-gravity it is possible to interpret the black holes as extremely compact nonsingular objects that satisfy NEC. And we can expect such dense compact objects to dissolve into the plasma soup if the universe oscillates to a sufficiently small size, thereby not worry about accruing the inhomogeneity-buildup over cycles.

C. Organization of the paper

In section 2, we start with a review of THED-gravity and then present the cosmological equations guiding the evolution of the universe. In section 3, the oscillatory universe solutions are analyzed and the parameter space demarcating the ever-expanding and oscillatory solutions is discussed. Importantly, it is shown that the overall energy of the universe within each oscillation is precisely zero. In section 4, we analyze the implication of the cosmic microwave background (CMB) anisotropies \[24\] and infer that it suggests a negative spatial curvature for the universe (but its value cannot be constrained)—this is in stark contrast with the ΛCDM wherein the CMB data clearly points to a spatially flat universe. In section 5, we estimate the energy densities of the universe by fitting it to the Type Ia supernovae redshift data from an open source catalog \[24\] (https://sne.space/). In section 6, we discuss the implications of the results and the amicability of THED as a theory of modified gravity.

II. REVIEWING THED GRAVITY

Torsion is the natural mathematical ingredient of differential geometry that can be incorporated into the spacetime manifold (in addition to its metric structure) in order to include the gravitational effects of matter with spin \[10\]. However, torsion plays a completely different role in THED gravity \[11\]—it is not coupled to the spin of the matter fields, but it serves to hide an extra-dimension in space. A brief technical review of this theory follows. The reader can skip this review and move to the cosmological equations without loosing coherence.

We start by visualizing the 5D space-time as foliated 4D hypersurfaces (with coordinates \(x^\mu\)) along the fifth dimension \(x^5\).

\[
ds_5^2 = g_{\mu\nu}dx^\mu dx^\nu + g_{\mu 5}dx^\mu dx^5 + \Phi^2 [dx^5]^2 + g_{\mu 5}g_{\nu 5} \Phi^{-2} dx^\mu dx^\nu .
\]  

The line element is expressed in the above form for convenience because once we impose the physical constraint that the extra-dimension remains hidden, it turns out that the 4D components \(g_{\mu\nu}\) can be exactly identified as the metric tensor in torsion-free GR at a kinematical level.

The metric-compatible connection \(\tilde{\Gamma}\) in this geometry is prescribed to ensure that any motion along the fifth dimension is unobservable and its 4D components are chosen to be torsionless as in GR. This is achieved by imposing the algebraic constraints \(\tilde{\Gamma}^{\mu}_{\nu 5} = \tilde{\Gamma}^{\mu}_{5 5} = 0\) and \(\tilde{\Gamma}^{\mu}_{\nu [\alpha]} = 0\). A vierbein formulation of these constraints is elaborated in \[25\]. By analyzing the geodesic equations, we can note that any motion along the fifth dimension does not affect the 4D components of the geodesic equations. These constraints along with the metric-compatibility condition completely determine all the components of torsion in terms of the metric.

Remarkably, this theory is indistinguishable from GR at a kinematical level. First, it turns out that the 4D metric on the hypersurfaces, \(g_{\mu\nu}\), is functionally independent of \(x^5\)—making all the hypersurfaces identical; this can be viewed as a strong validation to our procedure of hiding the extra-dimension, and to set the cylindrical condition that the entire 5D metric is independent of \(x^5\). Second, the components of the connection and the curvature tensor that are tangential to any hypersurface are identical to those in GR with metric \(g_{\mu\nu}\). Mathematically, this implies that the terms contributed by the torsion exactly cancel-off the terms contributed by the effect of the extra-dimension in the computation of the 4D components of connection and curvature. Hence, hiding the extra-dimension by requiring that the physically observable 4D motion is oblivious to any extra-dimensional motion, leads to a theory where the kinematic equations are identical to those in GR. Moreover, this formalism does not require us to explicitly choose a topology for the extra dimension—it could be either a compact dimension or a large unbounded dimension.
Since torsion is not dynamically independent and is completely metric dependent, the field equations are derived from the action-principle with pure metric variations, and they take the form\(^2\)

\[
G_
u^\mu - H_
u^\mu = \Sigma_\nu^\mu, \tag{2}
\]

Here \(G^\mu_\nu\) is the standard Einstein tensor constructed from the 4D metric \(g^\mu_\nu\) as in GR, and \(R^\mu_\nu\) and \(R\) are respectively the 4D Ricci tensor and Ricci scalar constructed from the 4D metric \(g^\mu_\nu\) as in GR. The additional term \(H^\mu_\nu\) (under cylindrical condition) takes the form

\[
H^\mu_\nu = \nabla_\nu J^\mu - (\nabla J)\delta^\mu_\nu + J_\nu \delta^\mu_\nu - (J J)\delta^\mu_\nu, \tag{4}
\]

where \(J_\mu \equiv \Phi^{-1} \partial_\mu \Phi\) is a 4D vector whose indices are raised and lowered with the 4D metric \(g^\mu_\nu\) and its inverse. Similarly, the covariant derivative, \(\nabla_\nu\), is defined as in GR with Christoffel connection expressed in terms of the 4D metric \(g^\mu_\nu\). Note that the 4D components of the field equations only involve the 4D metric components \(g^\mu_\nu\), while the metric components \(g^\mu_5\) have completely decoupled out. Particularly, \(g^\mu_5 = 0\) and \(\Sigma_\nu^5 = 0\) are always consistent solutions to the field equations, but there is absolutely no necessity to impose them. Since the only 4D metric components \(g^\mu_\nu\) and the 4D components of stress tensor \(\Sigma^\mu_\nu\) are relevant for physical observations, eq. 2 is sufficient to solve for all the physical degrees of freedom including \(\Phi\). The unavoidable extra dimensional stress tensor component \(\Sigma^5_\nu\) can be defined to be \(R/2\) consistent with the field equations. Clearly, this theory reduces to GR when \(J_\mu\) vanishes. In GR, the Bianchi identity automatically implies the conservation of stress tensor, which is not true here because of the presence of the term \(H^\mu_\nu\) in eq. 2. Nevertheless, if we impose 4D matter conservation, \(\nabla_\mu \Sigma^\mu_\nu = 0\), it would imply \(\nabla_\mu H^\mu_\nu = 0\). Moreover, since this condition must hold in vacuum regions of spacetime, and since there is no reason for a purely geometry-dependent tensor \(H^\mu_\nu\) to behave differently in vacuum vs matter-filled regions, we would expect any smooth solution of field equations (eq. 2) to automatically satisfy \(\nabla_\mu H^\mu_\nu = 0\).

### A. Cosmological equations

For homogeneous and isotropic universe, the 4D line element in spherical-polar coordinates is given by

\[
\text{ds}^2 = -\text{dt}^2 + a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 \right), \tag{5}
\]

\(^2\) In the original derivation of eq. 2 presented in [1], the sign convention adopted to define the curvature and the Einstein tensor is different.

where \(a(t)\) is the scale factor, and \(k = 0\) corresponds to spatially flat universe, \(k > 0\) corresponds to spatially closed universe, and \(k < 0\) corresponds to spatially open universe. The nonvanishing components of the Einstein tensor for the above metric is given by

\[
\begin{align*}
-G_t^t &= 3(\dot{a}/a)^2 + 3k/a^2, \\
-G_r^r &= 2(\dot{a}/a) + (\ddot{a}/a)^2 + k/a^2, \\
G_\theta^\theta &= G_\phi^\phi = G_t^r,
\end{align*}
\]  

where over-dot denotes a derivative with respect to time. With the 4D metric (eq. 5) embedded in the 5D geometry (eq. 1), and acknowledging that the extra-dimensional field \(\Phi\) can functionally depend only on time \(t\), the only nonvanishing component of \(J_\mu\) is \(J_t\) and the nonvanishing components of \(H^\mu_\nu\) in eq. 4 are

\[
\begin{align*}
H^t_t &= 3J_t(\dot{a}/a), \\
H^r_r &= 2J_t(\dot{a}/a) + J_t + J_t^2, \\
H^\theta_\theta &= H^\phi_\phi = H^r_t.
\end{align*}
\]  

The conservation equation \(\nabla_\mu H^\mu_\nu = 0\) implies that either \(J_t = 0\) or \(J_t = \dot{a}/a\). That is, either \(\Phi(t)\) is a constant, which would give rise to the usual Friedman-Robertson-Walker (FRW) cosmology, or \(\Phi(t) = \dot{a}(t)\). Focusing on the latter case, \(H^\mu_\nu\) simplifies to

\[
\begin{align*}
H^t_t &= 3\ddot{a}/a, \\
H^r_r &= 2(\dot{a}/a) + (\ddot{a}/a). \tag{8}
\end{align*}
\]

and the field equations (eq. 2) reduce to

\[
\begin{align*}
3(\dot{a}/a)^2 + 3k/a^2 + 3\ddot{a}/a &= 8\pi G\rho, \tag{9} \\
4(\dot{a}/a)^2 + (\ddot{a}/a)^2 + k/a^2 &= -8\pi G P. \tag{10}
\end{align*}
\]

Here \(\rho\) and \(P\) are the density and pressure of the 4D matter, and \(G\) is the Newton’s gravitational constant. The matter conservation equation follows from the field equations, exactly resembling FRW cosmology.

\[
\dot{\rho} + 3(\rho + P)\dot{a}/a = 0. \tag{11}
\]

For the standard matter equation of state, \(P = w\rho\), it is well known that the density evolves with time as \(\rho(t) \propto a(t)^{-3(1+w)}\). The Null Energy condition (NEC) is satisfied if \(\rho + P \geq 0\), which is the kind of matter we shall restrict our attention to.

To keep the analysis simple, we shall just consider normal pressure-less matter \((w = 0)\) with density \(\rho_m\) and a cosmological constant \((w = -1)\) of density \(\Lambda\). The NEC is satisfied if \(\rho_m \geq 0\) irrespective of the sign of \(\Lambda\). To solve the cosmological equations (eq. 9-11), we shall set the initial conditions at the present moment \((t = 0)\) to be such that \(a(t = 0) = 1\) and \(\dot{a}(t = 0) = 1\), assuming that the universe is currently expanding. This sets the units of length and time scales in our numerical computations to be such that the Hubble constant \(H_0 \equiv [\dot{a}/a]_{t=0}\) and the velocity of light \(c\) are taken to be 1. This implies the physical time and physical length should be numerically interpreted in the units of \(H_0^{-1}\) and \(cH_0^{-1}\) respectively.
FIG. 1. The parameter space \( \{\Omega_k, \Omega_M, \Omega_{\Lambda}\} \) is classified into regions that accommodate solutions of universe that (i) bounces from a minimum nonzero size and (ii) turns around from an expansion phase to a contraction phase at some maximum size. The flat transparent horizontal surface marks the \( \Omega = 0 \). The green-surface (nearly horizontal) marks the boundary below which an expanding universe turns around at a maximum and starts contracting, while above the surface an expanding universe will continue to expand forever. The red-surface (nearly vertical) marks the boundary below which an expanding universe turns around into regions that accommodate solutions of universe that (i) bounces from a minimum nonzero size and (ii) turns around into the red-surface and the other close to the green-surface. Effectively, we need the parameters to be in the bottom-left quadrant of the parameter space (marked by the brown dot) supports bounded oscillating universe. For visual clarity, the values of \( \Omega_{\Lambda} \) is restricted between \( \pm 0.5 \).

The equation to solve is then

\[
\text{THED : } \left[\frac{\dot{a}}{a}\right]^2 - \Omega_k/a^2 = \Omega_m/a^3 + \Omega_{\Lambda} \quad (12)
\]

\[
\text{FRW : } \left(\frac{\dot{a}}{a}\right)^2 - \Omega_k/a^2 = \Omega_m/a^3 + \Omega_{\Lambda} \quad (13)
\]

In effect, the Friedman equations in GR are modified in THED gravity by the inclusion of the first term in l.h.s of eq. (13), namely \( \left[\frac{\dot{a}}{a}\right]^2 \). In the FRW case, \( \Omega_m + \Omega_{\Lambda} + \Omega_k = 1 \), while in the THED \( \Omega_m + \Omega_{\Lambda} + \Omega_k = 1 - q \), where deceleration parameter \( q = -[\frac{\ddot{a}}{a}]_t \).

**Physical values of the energy densities** : The physical value of \( \Omega_k \) is given by \( \Omega_k = -k c^2/H_0^2 \). The physical density of matter is \( (3H_0^2/8\pi G)\Omega_m \) and the energy density of the cosmological constant is \( (3H_0^2/8\pi G)\Omega_{\Lambda} \). For example, if \( H_0 = 70 \text{ km/s/Mpc} \), the physical matter density turns out to be \( \Omega_m \approx 0.9 \times 10^{-28} \text{ kg/m}^3 \). The observed matter density of luminous gasses in the universe only amounts to a contribution of \( \Omega_m \approx 0.005 \); however with theoretical extrapolations from bigbang neucleosynthesis the total baryonic matter can amount to \( \Omega_m \approx 0.05 \), while the standard model \( \Lambda \text{CDM} \) requires \( \Omega_m \approx 0.27 \) including dark matter contributions.

**III. OSCILLATORY UNIVERSE SOLUTIONS**

Oscillatory solutions to eq. (13) exist when the parameters \( \{\Omega_k, \Omega_M, \Omega_{\Lambda}\} \) lie in a particular region of the parameter space, as depicted in fig. 1. For the expanding universe to reach a maximum size \( a_{\text{max}} \) and turn around to begin contracting, we require the acceleration at the maximum to be negative. That is

\[
\ddot{a}_{\text{max}} = \Omega_k/a_{\text{max}} + \Omega_\Lambda a_{\text{max}} + \Omega_M/a_{\text{max}}^2 < 0
\]

Similarly, for the universe to bounce back from a minimum size \( a_{\text{min}} \), we require the acceleration at the minima to be positive.

\[
\ddot{a}_{\text{min}} = \Omega_k/a_{\text{min}} + \Omega_\Lambda a_{\text{min}} + \Omega_M/a_{\text{min}}^2 > 0
\]

However, \( a_{\text{max}} \) and \( a_{\text{min}} \) are themselves implicitly determined by the parameters through eq. (13). It turns out that for the universe to have a nonsingular bounce, the parameters have to be to the left of the red-surface in figure 1, while for the universe to turn around from a maximum, the parameters should be below the green-surface. Effectively, we need the parameters to be in the bottom-left quadrant as indicated by a brown dot. There are two important features of the oscillatory profile that can be gleaned from the position of the brown dot, namely the minimum size of the universe \( a_{\text{min}} \) and the periodicity of oscillations \( \tau \).

Firstly, the minimum size of the universe \( a_{\text{min}} \) depends on how far away the parameters are from the red-surface. If the parameters lie very close to the red-surface, then \( a_{\text{min}} \) is very close to zero. Since observations suggest that the universe should have been very hot and dense in the past, we would require that \( a_{\text{min}} \approx 0 \), and hence we shall primarily focus on the parameter values that lie almost on the red-surface in the bottom-left quadrant of figure 1. Empirically, it turns out that the left side of the red-surface in figure 1 is given by

\[
\Omega_\Lambda \geq \Omega_\Lambda^R \equiv 2(-\Omega_k + 1) - 4\Omega_M \quad (14)
\]

where \( \Omega_\Lambda^R \) is the value of \( \Omega_\Lambda \) at the red-surface.

Secondly, the periodicity of oscillations \( \tau \) is determined by how far away the parameters are from the green-surface. The closer the parameters get to the green-surface, \( \tau \) increases without bounds. Thus the parameter values close to the intersection of the red and green surfaces will correspond to universes that have \( a_{\text{min}} \to 0 \) and \( \tau \to \infty \).

For \( \Omega_k \geq 0 \) (spatially flat/open universe), the green-surface is given by \( \Omega_\Lambda = 0 \), while for \( \Omega_k < 0 \) (spatially closed universe), the green-surface has \( \Omega_\Lambda > 0 \) as can be seen in figure 1. Moreover for \( \Omega_k \geq 0 \), as the parameters get close to the green surface the value of \( a_{\text{max}} \) grows large towards \( \infty \), while for \( \Omega_k < 0 \), the value of \( a_{\text{max}} \) remains bounded while the periodicity alone grows unbounded.

Figure 2 illustrates the influence of the parameters on the oscillatory profile, namely \( a_{\text{min}}, a_{\text{max}} \) and \( \tau \). For simplicity we fix \( \Omega_M = 1 \) for three different values of \( \Omega_k = 0, \pm 1 \). We shall pick two values of \( \Omega_\Lambda \), one close to the red-surface and the other close to the green-surface. Imagine the brown dot in fig. 1 lying close to the red-surface, and then imagine it to be vertically transported.
FIG. 2. Illustration of oscillatory solutions of \( a(t) \) for three pairs of \( \{\Omega_k, \Omega_M\} \). The top row corresponds to parameters lying close to the red-surface in fig. 1 and the bottom row corresponds to the parameters lying close to the green-surface in fig. 1.

- From the top row, note that \( a_{\min} \simeq 0 \), but the value of \( a_{\max} \) and \( \tau \) depend on the parameters.

- As we move away from the red-surface towards the green-surface (bottom row), note that \( a_{\min} \), \( a_{\max} \) and \( \tau \) all increase. It is particularly crucial to note that this happens even with \( \Omega_k \leq 0 \) (wherein the green-surface lies at a positive \( \Omega_A \)), as seen in the bottom left panel (although it is not visually discernible that \( a_{\min} \) has moved away from 0).

- The values of \( \Omega_A \) are chosen in order to get close to the red and green surfaces (but not exactly on the surfaces), we can nevertheless get arbitrarily close to the surfaces by fine-tuning the chosen \( \Omega_A \). If we fine-tuned \( \Omega_A \) much closer to the respective surfaces, the oscillations in the top row of fig. 2 would not show any visually discernible change (because \( a_{\min} \) is already very close to zero), but the scale of oscillations in the bottom row would be drastically affected.

- For \( \Omega_k = 0, +1 \) in the bottom row, \( a_{\max} \) is finite because the choice \( \Omega_A = -0.1 \) is not on the green-surface. However, we could move much closer to the green-surface by pushing \( \Omega_A \) to 0, and that would push \( a_{\max} \rightarrow \infty \) along with \( \tau \rightarrow \infty \), as shown in fig. 3. But for \( \Omega_k = -1 \) (bottom-left panel), moving arbitrarily close to the green-surface will only push \( \tau \rightarrow \infty \) but \( a_{\max} \) will saturate and remain bounded, as hinted by the extremely flat plateau (rather than a peak)—a very lengthy static-phase for the universe at the crest of the oscillation.

- Deceleration Parameter: For the oscillatory solutions with very small \( a_{\min} \), the universe must necessarily be decelerating at the present epoch. Note that with \( \Omega_k > 0 \), we need \( \Omega_A \) to be negative and also satisfy eq. (14). The deceleration parameter given by \( q = 1 - \left[ \Omega_A + \Omega_k + \Omega_M \right] \) then simplifies to \( q \simeq \Omega_M - \Omega_A/2 \) which has to be positive. The value of \( q \) can be negative only for parameters that lie far away from the red surface or lie above the green surface in fig. 1.

The parameters thus drastically affect the oscillatory profile, however there is a constant integral of motion that evaluates to zero for any parameter values in the oscillatory regime. This constant of motion can be interpreted as the average energy of the universe within each cycle.

### A. Zero Energy Oscillations

The equation determining the dynamics of \( a(t) \), namely eq. (13), can be rewritten as

\[
\frac{d}{dt}(a\dot{a}) = [\Omega_M/a^3 + \Omega_k/a^2 + \Omega_A]a^2 \equiv E a^2
\]  

(15)
where \( E \equiv [\Omega_M/a^3 + \Omega_k/a^2 + \Omega_\Lambda] \) is the total energy density of the universe at any moment. Equivalently,

\[
\frac{1}{2} d(a\dot{a})^2 = Ea^3 \dot{a} \, dt
\]  

(16)

Since the RHS is an exact differential and since \( \dot{a} \) vanishes at both the minima and maxima within an oscillation, the RHS must vanish when integrated from the minima to the maxima within a cycle.

\[
\int_{t_{\text{min}}}^{t_{\text{max}}} E a^3 \dot{a} \, dt = \int_{a_{\text{min}}}^{a_{\text{max}}} E a^3 \, da = 0
\]  

(17)

Thus the average energy of the universe within each half-cycle is by itself zero. This is a constant of motion, and is particularly useful to analytically evaluate \( a_{\text{max}} \). In the limit \( a_{\text{max}} \gg a_{\text{min}} \), the above integral reduces to

\[
\Omega_M + \frac{1}{2} \Omega_k a_{\text{max}} + \frac{1}{4} \Omega_\Lambda a_{\text{max}}^3 = 0,
\]  

(18)

from which \( a_{\text{max}} \) can be extracted. For \( \Omega_k > 0 \), as we approach the green surface \((-\Omega_\Lambda \rightarrow 0)\), the value of \( a_{\text{max}} \) and \( \tau \) are proportional to \( 1/\sqrt{-\Omega_\Lambda} \). This is well illustrated in fig. 3 as the log-log plot of \( a_{\text{max}} \) and \( \tau \) vs \(-\Omega_\Lambda\) turns out to be a straight line with a slope \(-1/2\). We can then deduce from the above equation that

\[
a_{\text{max}} \simeq \sqrt{2\Omega_k \Omega_\Lambda} \quad \text{as} \quad \{-\Omega_\Lambda \rightarrow 0\}
\]  

(19)

It should be noted that \( \Omega_M \) is implicitly fixed by eq. 14 so that the parameters lie close to the red-surface of fig. 1.

It is important to note that Eq. 17 does not denote the conventional sense of time-averaging over energy, rather the averaging is done over a cosmic time unit that flows in lieu with the expansion rate of the universe. This is however not the only constant of motion. We can derive other constants of motion, for example a time-averaged surface-energy in each half-cycle must also be zero.

\[
\int_{t_{\text{min}}}^{t_{\text{max}}} E a^2 \, dt = 0
\]  

(20)

This can be interpreted as the time-averaged energy over any 2D-comoving surface in the universe to be zero.

The existence of these constants of motion is a very unique property of the dynamical equations because it can be written in the form of first order exact differential (eq. 15) with a term \( \dot{a} \) inside the differential. This is not possible in the simpler FRW equation (eq. 9) where there are no second derivative terms of \( a(t) \).

**IV. SPATIAL FLATNESS AND CMB DATA**

It is often claimed that the cosmic microwave background (CMB) provides an independent geometric confirmation that the universe is almost spatially flat. The power spectrum of the CMB radiation shows the first acoustic peak at a multipole moment of \( \ell \simeq 200 \), and this feature is interpreted as a strong evidence for a flat universe with \( \Omega_k \approx 0 \). This is a very model-dependent interpretation from the \( \Lambda \)CDM perspective, and no such inference can be deduced for the oscillatory universe scenario in THED gravity.

Two quantities need to be computed from the cosmological model in order to predict the position of the first acoustic peak of CMB power spectrum. (i) The size of the sound-horizon at the time of photon-decoupling \( h_d \), that happens in the past when the universe was very hot at a redshift \( z_d \approx 1100 \). This gives an estimate of the size of the region that could have physically interacted to homogenize before the photons decoupled from the plasma soup to start free streaming. (ii) The radial coordinate \( r_d \) from which the decoupled photons at time \( t_d \) free stream to reach us at the present epoch. If \( a_d = 1/(1 + z_d) \) is the scale factor at the time of decoupling, then the ratio \( (h_d/a_d r_d) \) gives the angular width in the sky that corresponds to a typical hot patch in the microwave background anisotropies, and we would expect the first peak of the CMB power spectrum at a multipole moment \( \ell \approx a_d r_d/h_d \).

\[
h_d = \int_{t_0}^{t_d} \frac{(c_s/c) \, dt}{\sqrt{\Omega_M} \int_{z_d}^{\infty} (1 + z)^{-5/2} \, dz}
\]  

(21)

where \( c_s = c/\sqrt{3} \) is the sound velocity in plasma, \( t_d \) is the time of photon-decoupling, and \( t_o \) is the time before which there was no causal contact between any two spatial points. \( t_o \) would be the big bang instant in the context of \( \Lambda \)CDM at \( z \rightarrow \infty \). Since \( (1 + z) = 1/a(t) \), the integral can be rewritten in terms of \( z \). For \( \Lambda \)CDM, the Friedman equations (eq. 13) express \( (\dot{a}/a) \) as a simple function of \( (1 + z) \). Because \( z_d \gg 1 \), only the \( \Omega_M \) term is relevant leading to the above expression for \( h_d \). The radial coordinate \( r_d \) from which the just-decoupled pho-
tons (usually referred to as the surface of last scattering) would reach us at the present epoch can be computed from

$$\int_0^{r_d} \frac{dr}{\sqrt{1 + \Omega_k r^2}} = \int_0^0 \frac{dt}{a(t)} = \int_0^{r_d} \frac{dz}{(\dot{a}/a)} \quad (22)$$

From a given coordinate position $r_d$, the sound-horizon of a given size $h_d$ would appear to have a smaller angular width in a spatially open universe than in a flat universe, while it would appear to have a larger angular width in a spatially closed universe. Calculations show that in flat space $\Lambda$CDM predicts an angular width corresponding to a power spectrum peak at $\ell \sim 200$, in concordance with what is observed from CMB power spectrum. Hence for $\Lambda$CDM, it is inferred that $\Omega_k \approx 0$.

In THED gravity, the term $(\dot{a}/a)$ in equations 21 and 22 cannot be expressed as an explicit function of $(1 + z)$ because the dynamics is guided by eq. 13 with the presence of the acceleration term $\ddot{a}$. The integrals in equations 21 and 22 have to be computed in the time domain after explicitly identifying the time $t_d$ from the numerical solution to $a(t)$. Moreover, the acceleration becomes very large as $a_{min} \rightarrow 0$, hence the calculations strongly depend on the value of $a_{min}$. By ignoring the acceleration term, we can obtain an upper bound for $(\dot{a}/a)$ and compute the integral in eq. 21 (analogous to the computation in $\Lambda$CDM) to obtain a lower bound for the horizon size. So, for a given set of parameters, the horizon-size should be significantly larger than that computed from $\Lambda$CDM. If we assume that the post-decoupling evolution of the universe is similar to the $\Lambda$CDM scenario, then we are witnessing a significantly bigger hot spot in the CMB anisotropy at a smaller angular width, indicating a spatially open universe. On the flip side, the acceleration term could significantly affect the calculation of eq. 22 and increase the value of $r_d$, which would sway our inference towards a positively curved universe (this is however very unlikely because the acceleration term would play more significant role in computing $h_d$ at extremely small values of $a_{min}$).

In an eternally oscillating universe, it is inappropriate to assume that there was no causal contact between spatially separated points when the universe is at its minimum size. In fact the remnant anisotropies from the previous cycle of the universe would be expanded out in the early phase of the current cycle and appear as the CMB anisotropy now. Hence if the universe is spatially flat, we should expect the hot spots in the CMB to be significantly bigger than its appearance. This again indicates that the universe is spatially open. Since we cannot place a numerical value on the size of $\Omega_k$ due to the lack of knowledge of $a_{min}$ and the size of remnant anisotropies from the prior cycle, we shall consider all possibilities equally likely while fitting the model to the Supernova data.

V. FITTING SUPERNOVA REDSHIFT DATA

Type Ia supernovae are uniquely qualified to serve as ‘standard candles’ to estimate the distance of the parent galaxy to which they belong. This is because each supernova explosion is always expected to occur with a fixed peak luminosity as a white dwarf accretes material from a companion star to reach the Chandrasekhar limit of 1.44 solar masses. Theoretically the absolute magnitude $M$ of a Type Ia supernova should be $-19.5$, however there are some variations of the range $\pm 2$ which can be corrected for by analyzing the shape of the supernova’s light curve in the days following the explosion. The absolute magnitude of a supernova and its apparent magnitude as observed from Earth would together give a distance estimate, which when plotted against the redshift is known as the Hubble diagram.

From the cosmological equations, a Hubble diagram for the model can be generated by following these steps:

1. Numerically integrate the cosmological equation with given initial conditions at $t = 0$ to obtain $a(t)$ for all $t$ beyond the most recent minima.

2. The wavelength of a photon traveling through the universe linearly scales with the scale factor $a(t)$, and the red shift $z$ of the light emitted by a supernova at some time $t$ in the past as observed at $t = 0$ is

$$1 + z = 1/a(t)$$

3. The comoving distance travelled by light from a supernova at time $t$ to Earth at $t = 0$ is

$$\chi = \int_t^0 \frac{dt'}{a(t')} = \int_0^r \frac{dr'}{\sqrt{1 + \Omega_k r'^2}}$$

and the radial coordinate $r$ of that supernova is $S(\chi) = \{\chi, [\Omega_k]^{-1/2} \sin(\sqrt{[\Omega_k]} \chi), [\Omega_k]^{-1/2} \sinh(\sqrt{[\Omega_k]} \chi)\}$ for $\Omega_k = \{0, -+, +\}$ respectively.

4. The luminosity distance $d_L$ is defined in terms of the source luminosity $L$ and the observed energy flux $F$ such that $4\pi d_L^2 F = L$, which can be computed for the expanding universe in terms of the redshift to be

$$d_L = S(\chi)(1 + z)$$

5. The magnitude modulus of a supernova is defined as $\mu \equiv$ absolute magnitude - apparent magnitude. From the definition of luminosity magnitude, the relationship between the magnitude modulus and the luminosity distance is given by

$$\mu = 5 \log_{10}(d_L) + 25 \quad (23)$$
To generate the Hubble diagram for the model, we need to first back-calculate the value of \( t \) for any given \( z \) from the numerical solution of \( a(t) \), and then evaluate \( \chi, d_L \) and \( \mu \) from the above steps to plot \( \mu \) vs \( z \).

Plotting the Hubble diagram of the observed data is more involved, because the observed magnitude of supernova has to be corrected for absorption effects of interstellar gas on a case to case basis. We shall however not be concerned of those details, and just use the reported post-processed magnitude from a large data set. The open supernova catalog [24] (https://sne.space/) contains data from over 8000 Type Ia supernovae providing the details of each individual measurement and an estimate of \( d_L \) after such corrections are applied. The list of supernovae names along with their redshift and magnitude modulus is given in the supplementary information, and the associated Hubble diagram is plotted in the top left panel of fig. 4.

The binning-choice essentially determines the way we

\[ \text{where } d_L \text{ is measured in Megaparsecs (Mpc).} \]

\[ \text{The Hubble constant is generally measured in units of km/s/Mpc. The } d_L \text{ obtained from the model should be multiplied by a factor } cH_0^{-1} \text{ to make it physically dimension-full.} \]

\[ \text{A. Binned analysis on log } z \text{ - axis} \]

The Supernova dataset has been steadily growing to include larger values of redshift data, and the parameters of the standard model of cosmology ΛCDM has been fitted with increasing precision on this dataset [27]. Now, with the availability of a very large data set containing more than 8000 data points with over 100 data points with \( z > 1 \) [24], we can examine the model-accuracy corresponding to different scales of the universe separately.

To estimate the model parameters that best fit the data, the standard strategy is to take into account the uncertainty in the measurement of \( z \) and \( \mu \) for each data point, and compute the parameters that maximizes the likelihood of the dataset. But here we shall take a simpler approach by ignoring the error-bars in the data, and segregating the data into appropriately sized bins with sufficiently many data points within each bin. The range of \( z \)-values within each bin gives the scale of the universe represented by that bin. Any model-data discrepancy within a bin is a measure of the deviation of the model from the data at that scale of the universe. The model-data discrepancies from all bins should be equally weighted to obtain a measure of overall model accuracy. Just because more data is available at some scale, that specific scale should not over-represented in evaluating the model accuracy.

The binning-choice essentially determines the way we
evaluate the model. We could simply choose bins of constant width along the $z$-axis, but note that the number of data points in the bins with low $z$ values is much higher than those with large $z$ (see bottom-left panel of figure 4). Alternatively, we could choose bins of variable sizes along the $z$ axis such that each bin has a fixed number of data points. We should however not segregate the scales of the universe based on the existing number of data points at various scales; rather the segregation of scales (or bins) should be based on some theoretically appealing feature of the data.

By definition, the difference in magnitude of two light sources is explicitly a measure of the perceptual difference in brightness between them. So any uncertainty in the measurement of magnitude can be expected to be a constant (or at least uncorrelated to the magnitude itself)—this is indeed well established in psychophysics as the Weber-Fechner law. This suggests a natural way to segregate the data into equidistant bins along the measured luminosity magnitude, so that the uncertainty in all the bins will be the same. However, since the magnitude modulus $\mu$ is dependent on the redshift $z$, we can rescale the $z$-axis so that $\mu$ appears linear along the rescaled $z$-axis, and the data can be binned along the rescaled $z$-axis. The Hubble diagram from the dataset is plotted in the top-left panel of figure 4 and the top-right panel shows the same Hubble diagram on a logarithmic-$z$-axis.

Notice that on a logarithmic-$z$-axis, the dependent variable $\mu$ is almost linear. So, the natural binning-choice is to segregate the data into equidistant bins along log $z$ axis.

Every bin is considered equal in evaluating the model accuracy irrespective of the number of data points it contains (as long as there exists at least one data point). The error in $\mu$ for the $i$-th data point $\{z_i, \mu_i\}$ is computed as

$$\langle (\mu_{\text{model}} - \mu_{\text{data}})^2 \rangle_{\text{bin}}$$

at various scales of the universe (quantified as bins) is plotted for FRW model with some illustrative parameter values. The bins are laid out on log $z$ axis with a width of $\Delta \log z = 0.05$ and the data is distributed across the bins as shown in the bottom-left panel of fig. 4. The best fit parameters are $\Omega_M = 0.27$, $\Omega_k = 0$, $\Omega_\Lambda = 0.73$, $H_0 = 70$ km/s/Mpc.
model-data discrepancy $\Delta \mu_i = |\mu_{\text{model}}(z_i) - \mu_i|^2$, which is then averaged within each bin, and finally aggregated over all the bins to obtain a measure of model-data discrepancy $e$.

$$e = \sum_{\text{bin}} \langle |\mu_{\text{model}} - \mu_{\text{data}}|^2 \rangle_{\text{bin}}$$  \hspace{1cm} (24)

We could view the above described measure of model-data discrepancy with skepticism because the data within each bin is significantly scattered, especially in the very low $z$ range (see top-right panel of fig. 4). Nevertheless, since the intrinsic variability in the data within each bin would affect all models (irrespective of the choice of model parameters) in the same way, we do not expect it to have significant effect on estimating the best fit model parameters. This concern can be alleviated by evaluating the measure of model accuracy of FRW cosmology for various model parameters, see figure 5.

The bins are laid out on the log $z$ axis with a width of $\Delta \log z = 0.05$ as shown in bottom-left panel of fig. 4. However, due to sparsity of data at the extremes, we can only populate a total of 133 bins and the model accuracy is evaluated only in the range of $z \in (0.001, 1.7)$, Figure 5 shows the error within each bin $\langle |\mu_{\text{model}} - \mu_{\text{data}}|^2 \rangle_{\text{bin}}$ for different sets of parameters in $\Lambda$CDM. The minimal value of $e$ is attained for $\Omega_{\Lambda} = 0.73$, $\Omega_M = 0.27$, $\Omega_k = 0$, $H_0 = 70$ km/s/Mpc conforming with the standard consensus. This validates eq. (24) as a measure of model accuracy and can hence be trusted to evaluate and estimate parameters in THED gravity.

**B. Parameter Estimation**

To estimate the parameters in THED gravity, let us keep note of the following. (i) The parameters are restricted to lie very close to the red surface in fig. 1 so that the universe was extremely hot and dense in the past, implying that $\Omega_{\Lambda}$ is determined by the choice of $\Omega_k$ and $\Omega_M$ through eq. (14). (ii) The observed density of luminous matter in the universe only amounts to $\Omega_M = 0.005$, and the total baryonic matter density can be estimated to be at most $\Omega_M = 0.05$. (iii) The CMB power spectrum analyzed with the modified dynamical equations suggests that the universe might be negatively curved ($k < 0$, $\Omega_k > 0$).

Figure 6 shows the model-data discrepancy as a function of $\Omega_k$ for certain chosen values of $\Omega_M$. The green line corresponds to the intersection of the green and red surfaces in fig. 1. The oscillating solutions lie on the left side of the green line and the ever-expanding solutions lie on the right side of the green line of fig. 6. For any given $\Omega_M$ there exists $\Omega_k$ that minimizes the model-data discrepancy, but it lies on the left side of the green line wherein the universe bounces off from $a_{min}$ to expand forever. For any $\Omega_M < 0.3$, the best fit values have
Model accuracy for THED parameters

![Graphs showing model accuracy for THED parameters](image)

FIG. 7. The model-data discrepancy $\langle |\mu_{\text{model}} - \mu_{\text{data}}|^2 \rangle_{\text{bin}}$ at various scales of the universe (quantified as bins) is plotted for THED gravity with some illustrative parameter values. The bins are laid out on log $z$ axis with a width of $\Delta \log z = 0.05$ and the data is distributed across the bins as shown in the bottom-left panel of fig. [4]. In the left column, the parameters are very close to the green surface with $\Omega_\Lambda$ being a very small negative number, and these correspond to oscillatory solutions with very large periodicity. The middle column shows the parameters that best fit the data for chosen values of $\Omega_M$, but these correspond to ever-expanding solutions because these parameters lie above the green surface in fig. [4].

$e \simeq 11$, comparable to the best fit value from $\Lambda$CDM. For larger $\Omega_M$, the best fit value of $e$ grows larger, however that is irrelevant given the observed matter density in the universe.

Figure [7] illustrates the model accuracy across the bins for three values of $\Omega_M$, corresponding to luminous matter ($\Omega_M = 0.005$), baryonic matter ($\Omega_M = 0.05$), and dark matter in $\Lambda$CDM ($\Omega_M = 0.25$). All panels in the left column have a tiny negative $\Omega_\Lambda$, so that the parameters lie at the intersection of the green and red surfaces of fig. [1]. Among the oscillatory solutions, these are the solutions that best fit the data for any given $\Omega_M$. The smaller the choice of $|\Omega_\Lambda|$, the larger will be $a_{\text{max}}$ and $\tau$; however they cannot be determined by fitting the parameters to high precision with supernova data because no significant change occurs in the solutions for $a(t)$ at $t < 0$ (past) for $|\Omega_\Lambda| \leq 0.01$. So, based on the fits we could for instance place an upper bound on $|\Omega_\Lambda|$ and infer from fig. [6] that for a universe with $\Omega_M = 0.25$, $a_{\text{max}}$ is at least 10 times the current size and the periodicity $\tau$ is at least 50 times the current Hubble time $\simeq 1$ trillion years.

The middle column in figure [7] shows the best fit solutions that correspond to the various minimas in fig. [6]. These solutions lie to the left of the green line in figure [4] and are ever-expanding single bounce solutions. Their $e$ values are at par with that of the best fits from $\Lambda$CDM (compare with fig. [5]). They all have $\Omega_\Lambda > 0$, and $\Omega_k < 0$ for larger $\Omega_M$. For a spatially flat universe the best fit
solution has \( \{ \Omega_M = 0.15, \Omega_k = 0, \Omega_\Lambda = 1.4, e \simeq 10.95 \} \), which is at par with best fit of ΛCDM \( \{ \Omega_M = 0.27, \Omega_k = 0, \Omega_\Lambda = 0.73, e \simeq 10.95 \} \).

VI. DISCUSSION

The modified dynamical equations of THED gravity (eq. 13) allows for a variety of scenarios. The hidden torsion in the theory allows the universe to bounce from an arbitrarily minimum size. The energy densities of various components in the universe, namely matter \( \Omega_M \), curvature \( \Omega_k \) and dark energy \( \Omega_\Lambda \), play a succinct role in distinguishing the various scenarios of whether the universe is ever-expanding from a bounce or if it is oscillating? if oscillating, what is its minimum and maximum size and the periodicity of its cycle? The minimum size of the universe \( a_{\text{min}} \) depends on the fine balance between the energy densities and how close they lie to the red-surface of fig. 1. At its minimum, we could expect the universe to be at least as hot as 8.7 MeV (Binding energy of Iron) or even 150 MeV (QCD phase transition temperature) so that nucleosynthesis could occur every cycle afresh, requiring an \( a_{\text{min}} \simeq 10^{-12} \). But we should keep in mind that this is an external imposition we need to lay on the parameters of the model, and it cannot be constrained by fitting the supernovae data. On the other hand, the maximum size of the universe and its periodicity in the oscillatory scenario depends on the tinyness of the negative dark energy (eq. 19), which also cannot be constrained by the supernovae data. With a heuristic numerical bound of \( |\Omega_\Lambda| < 0.01 \), we can expect the oscillation cycle to be at least 1 trillion years long. Most intriguingly, irrespective of the size and periodicity of the oscillations, the average energy within each oscillation is precisely zero, which is a unique consequence of the form of the dynamical equations.

The binned analysis approach that we have adopted to fit the supernovae data is simplistic and powerful enough to evaluate the model-data discrepancies at different scales independently, because the dataset we use is large enough to well-populate the bins at all scales. The error measure \( e \) (eq. 24) certainly depends on the choice of the binwidths, and we had chosen the binwidths so that there are around 100 data points within each bin on average (see bottom right panel of figure 4), with an optimistic assumption that the uncertainties within each bin would wash away. This evaluation criteria can be made more sophisticated to extract the best-fit parameters, but the analysis used here is sufficient to catch a birds-eye view on how the parameters affect the various aspects of the oscillatory profile of the solutions.

A crucial development in this analysis could come from an independent constraint on the curvature of the universe. The analysis of CMB power spectrum suggests a plausible negative curvature on the universe, but no concrete constraint on the curvature can be deciphered. The lack of such a constraint leaves the field open for a range of admissible matter densities and curvature. However, weighing-in the theoretical elegance of having a zero-energy oscillatory universe, we prefer a negatively curved universe with a small matter density and small negative dark energy.

[1] K. H. Shankar, A. Balaraman, and K. C. Wali, Physical Review D 86, 024007 (2012).
[2] F. Hoyle, Monthly Notices of the Royal Astronomical Society 108, 372 (1948).
[3] A. D. Linde, Physics Letters B 108, 389 (1982).
[4] A. Albrecht and P. J. Steinhardt, Physical Review Letters 48, 1220 (1982).
[5] G. F. Ellis and R. Maartens, Classical and Quantum Gravity 21, 223 (2003).
[6] P. Creminelli and L. Senatore, Journal of Cosmology and Astroparticle Physics 2007, 010 (2007).
[7] P. J. Steinhardt and N. Turok, Physical Review D 65, 126003 (2002).
[8] P. W. Graham, B. Horn, S. Rajendran, and G. Torroba, Journal of High Energy Physics 2014, 163 (2014).
[9] P. H. Frampton, International Journal of Modern Physics A 30, 1550129 (2015).
[10] F. Hoyle, G. Burbidge, and J. V. Narlikar, The Astrophysical Journal 410, 437 (1993).
[11] L. Smolin, The life of the cosmos (Oxford University Press, 1999).
[12] N. J. Poplawski, arXiv preprint arXiv:1410.3881 (2014).
[13] S. W. Hawking and G. F. R. Ellis, The large scale structure of space-time, Vol. 1 (Cambridge university press, 1973).
[14] M. Visser and C. Barcelo, arXiv preprint gr-qc/0001099 (1999).
[15] A. Borde, A. H. Guth, and A. Vilenkin, Physical review letters 90, 151301 (2003).
[16] F. W. Hehl, P. Von der Heyde, G. D. Kerlick, and J. M. Nester, Reviews of Modern Physics 48, 393 (1976).
[17] S. Mukherjee, B. Paul, N. Dadhich, S. Maharaj, and A. Beesham, Classical and Quantum Gravity 23, 6927 (2006).
[18] N. Arkani-Hamed, H.-C. Cheng, M. A. Luty, and S. Mukhopadhyay, Journal of High Energy Physics 2004, 074 (2004).
[19] R. C. Tolman, Physical Review 38, 1758 (1931).
[20] L. Baum and P. H. Frampton, Physical review letters 98, 071301 (2007).
[21] E. Curiel, arXiv preprint arXiv:1405.0403 (2014).
[22] K. H. Shankar, General Relativity and Gravitation 49, 33 (2017).
[23] D. N. Spergel, L. Verde, H. V. Peiris, E. Komatsu, M. Nolta, C. Bennett, M. Halpern, G. Hinshaw, N. Jarosik, A. Kogut, et al., The Astrophysical Journal Supplement Series 148, 175 (2003).
[24] J. Guillochon, J. Parrent, L. Z. Kelley, and R. Margutti,
[25] K. H. Shankar and K. C. Wali, Modern Physics Letters A 25, 2121 (2010).
[26] A. G. Riess, W. H. Press, and R. P. Kirshner, The Astrophysical Journal 473, 88 (1996).
[27] A. G. Riess, L.-G. Strolger, J. Tonry, S. Casertano, H. C. Ferguson, B. Mobasher, P. Challis, A. V. Filippenko, S. Jha, W. Li, et al., The Astrophysical Journal 607, 665 (2004).
[28] G. T. Fechner, Elements of psychophysics, 1860. (Appleton-Century-Crofts, 1948).