A kinetic model for the stability of a collisional current sheet

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Abstract. A kinetic model addressing the destabilization of a current sheet by a microtearing mode is presented. For the first time, the magnetic drift, the electric potential fluctuation and collisions have been included together. As in reduced MHD, the evaluation of the current inside the resistive layer is obtained from a system of two equations linking the vector potential and the electric potential. When the electric field is neglected and using an effective magnetic drift, the magnetic drift is found to be destabilizing when combined with collisions. When both the electric potential and magnetic drift are kept, no analytical tractable solution has been found.

1. Introduction

In H-mode plasmas, the modelling of the pedestal dynamics is an important issue to predict temperature and density profiles in the tokamak edge and therefore in the core. The ”EPED” model, based on the stability of large and small scales MagnetoHydroDynamic (MHD) modes, is most commonly used to characterize the pedestal region. The EPED model has been quite successful until now [1]. However some recent analysis of JET plasmas [2] suggests that another class of instabilities, called microtearing modes (MTM) may be responsible for electron heat transport, in particular in the pedestal, and thereby play some role in determining the pedestal characteristics. MTMs belong to a class of instabilities where a modification of the magnetic field line topology is induced at the ion Larmor radius scale. This leads to the formation of magnetic islands, which can enhance the electron heat transport [3, 4, 5]. MTMs have been predicted to be unstable in modern tokamaks [2, 6, 7, 8] and other magnetic confinement devices [9, 10]. Although the stability of MTMs has been theoretically studied in the past [11, 12, 13, 14], the destabilization mechanism at play in the pedestal region is not yet well understood owing to the number of parameters possibly involved (electron temperature gradient, magnetic field curvature, magnetic shear, ...). This lack of understanding leads to an apparent disagreement between the linear analytical theory and gyrokinetic simulations. Indeed, past linear theories show that a slab current sheet is stable in the collisionless regime [11, 12] whereas, in the same regime, recent gyrokinetic simulations in toroidal geometry found unstable MTMs [8, 7, 10]. In [15], using a kinetic approach, we have presented a linear dispersion relation of a slab collisional microtearing mode destabilized by the electron temperature gradient only, neglecting the magnetic drift and the electric potential fluctuations. This theoretical linear result has been successfully compared to numerical simulations of a ”simple” microtearing mode model using the gyrokinetic code GKW [16]. However, the effects of the magnetic drift and the electric potential on the collisional microtearing mode destabilization mechanism still need to be understood. We present here a
new derivation of a current sheet model including the electric potential fluctuations and the magnetic drift in a kinetic framework.

The paper is organized as follows. In Section 2, we derive a full current sheet model by solving the Maxwell equations. The Fokker Planck equation, required to evaluate the current inside the resistive layer, is solved in the ballooning representation including the magnetic drift velocity. An expression for the current inside the resistive layer is found in a variational form. In Section 3, to validate the new calculation, we neglect the magnetic drift and the electric potential and compare the new dispersion relation to the linear calculations presented in [15]. In Section 4, we focus on the magnetic drift effect. In order to simplify the calculations, an effective magnetic drift is used. The main results of the paper are summarized in the last section.

2. Model description

To better understand the linear mechanisms at play in the MTM destabilization and how MTMs affect electron heat transport, a new analytical calculation including collisions, electromagnetic effects and the magnetic drift velocity has been formulated. In this model, the stability of a current sheet in the vicinity of a resonant surface is investigated in a kinetic framework. The evaluation of the current in the current sheet is obtained by solving the Maxwell equations, which are written in a variational form:

$$\mathcal{L} = -\frac{1}{\mu_0} \int d^3x \left| \nabla \phi \right|^2 + \int d^3x (J_\parallel A_\parallel^* - \rho \phi^*)$$

(1)

where $\phi^*$ and $A_\parallel^*$ are the complex conjugate of the electric potential and the vector potential, respectively and $\rho$ is the charge density. The extremalisation of the functional with any variation of $\phi^*$ and $A_\parallel^*$ is equivalent to the linearized electro-neutrality and Ampère equations. One important property of $\mathcal{L}$ is that it vanishes when $\phi$ and $A_\parallel$ match the solution of Maxwell equations. In this linear model a simple geometry of circular concentric magnetic surfaces is considered, where $r$ is the minus radius, $\varphi$ the toroidal angle and $\theta$ the poloidal angle. To study the electrons dynamics including the magnetic drift, the ballooning representation appears as the appropriate tool. Indeed, from linear gyrokinetic simulations presented in [15], the mode structure is observed to be strongly peaked at the low field side mid-plane. The system of equations is solved in the “ballooning” space [17] using the field aligned coordinates $(r, \alpha, \eta)$. Here, $\alpha = \varphi - q(r)\theta$ is a transverse coordinate and $\eta = \theta$ plays the role of a coordinate along the unperturbed field lines. Any perturbed field, for instance the vector potential $A_\parallel$, at given toroidal wave number $n$ and complex frequency $\omega$, is written:

$$A_\parallel(r, \alpha, \eta, t) = \sum_{p=-\infty}^{\infty} \hat{A}_\parallel(\eta + 2p\pi) \exp \{i n \alpha + i n q(r) (\eta_k - 2p\pi) - i \omega t \} + c.c.$$ 

(2)

where $\eta_k$ is the ballooning angle. The perturbed current $J_\parallel$ in the current layer can be calculated analytically including collisions, electromagnetic effects, the magnetic drift velocity and using a constant $A_\parallel$ approximation [18] ($A_\parallel$ is constant in the current layer). The starting point is to determine the current in the parallel direction $J_\parallel = \sum_{species} e \int d^3v f_{n\omega}(\eta, v_\parallel, \mu)$ using the Fokker-Planck equations for each charged species. The distribution function $f_{n\omega}$ for each species at given $n\omega$ is split in an unperturbed part $F_{eq}$ taken as an unshifted Maxwellian of temperature $T_{eq}$, and a perturbed part $\hat{f}_{n\omega}(\eta, v_\parallel, \mu)$, where $v_\parallel$ is the parallel velocity and $\mu$ the magnetic moment. The perturbed distribution function $\hat{f}_{n\omega}$ is itself written as the sum of an adiabatic part and a resonant part $\hat{f}_{n\omega} = -F_{eq} \frac{h_{n\omega}}{T_{eq}} + \tilde{g}_{n\omega}$, where $h_{n\omega} = e \left( \tilde{\phi} - v_\parallel \hat{A}_\parallel \right)$
the perturbed Hamiltonian, $\phi(\eta)$ and $\hat{A}_y(\eta)$ are the $n, \omega$ Fourier components of the electric and vector potentials, $e$ the algebraic charge and $\tilde{g}_{n\omega}$ is solution of the following kinetic equation:

$$
(\omega - k_\parallel v_\parallel - \omega_d) \tilde{g}_{n\omega} = \frac{F_{eq}}{T_{eq}} (\omega - \omega^*) \mathcal{J} \hat{h}_{n\omega} + i\mathcal{C}(\tilde{g}_{n\omega}) \tag{3}
$$

where $\mathcal{J}$ is the gyro-average operator, $\mathcal{C}$ is a linearized Fokker-Planck operator (pitch-angle scattering only), $\omega^* = \omega^T(\frac{1}{\eta_e} + \frac{\zeta^2 - \frac{\zeta}{2}}{\eta_i})$ is the kinetic diamagnetic frequency with $\zeta = v/v_{Th}$ and $\omega^T_k = \frac{k_\parallel v_\parallel}{2 \sqrt{m_e v_r n_e R/L_e}}$ where $v_{Th} = \sqrt{2T_e/m_e}$ is the electron thermal velocity and $\eta_e = L_{ne}/L_{Te}$ is the ratio of the electron density scale length $L_{ne}$ to the electron temperature scale length $L_{Te}$ (all gradients are calculated at the reference magnetic surface $r = r_0$). The electron and ion mass and temperature are noted $m_e$, $m_i$, $T_e$ and $T_i$, respectively. $\rho_i = \frac{m_i}{eB_0}$ is the ion Larmor radius and $R$ is the major radius.

In the ballooning representation, $k_\parallel$ must be understood as an operator $-i\frac{1}{\eta_0 R_0} \frac{\partial}{\partial \eta}$. The magnetic drift frequency is defined as $\omega_d = \frac{ne\mu B_0}{r_0 - eB_0} \left( \cos \eta + s_0 (\eta - \eta_k) \sin \eta \right)$ where $\eta_k$ is the ballooning angle and $s_0 = \frac{r_0 \partial q_{dr}}{r_{eq}}$ the magnetic shear at the reference resonant surface. The functional Eq. (1) can be written indifferently in the physical space or in the ballooning space. Note that in ballooning representation $|\nabla_\perp|^2$ should be replaced by $k_\perp^2 = k_\perp^2 [1 + s_0^2(\eta - \eta_k)^2]$, where $k_0 = \frac{ne}{\eta_0}$ is the reference poloidal wave number.

Finally, the functional can be written as:

$$
\mathcal{L} = -\frac{1}{\mu_0} \int d^3x \left| \nabla_\perp \hat{A}_\parallel \right|^2 + \sum_{\text{species}} \int d^3x \frac{N_{eq} e^2 T_{eq}}{\omega} \left| \phi \right|^2 - \sum_{\text{species}} \int d\tau \tilde{g}_{n\omega} \hat{h}_{n\omega}^* \tag{4}
$$

where $d\tau = d^3x d^3p$ is the volume element in phase space. It turns out that the Hamiltonian formulation is not the most suitable one to get close to the traditional resistive MHD formulation. Indeed one would like to manipulate the parallel electric field rather than a perturbed Hamiltonian $\hat{h}_{n\omega}$. This is done via the following transformation

$$
\frac{\hat{h}_{n\omega}}{e} = \frac{\omega - k_\parallel v_\parallel - \omega_d}{\omega} \phi + \frac{\omega_d}{\omega} \phi - v_\parallel \hat{\phi} \tag{5}
$$

where $\hat{\phi} = \hat{A}_\parallel - k_\parallel \phi$ is proportional to the parallel electric field. It is then convenient to replace the distribution function $\tilde{g}_{n\omega}$ by $\tilde{g}_{n\omega} = \tilde{g}_{n\omega} - \frac{F_{eq}}{T_{eq}} \frac{e - \omega^*}{\omega} \mathcal{J} \hat{\phi}$. The new distribution function is solution of

$$
(\omega - k_\parallel v_\parallel - \omega_d) \tilde{g}_{n\omega} = \frac{F_{eq}}{T_{eq}} (\omega - \omega^*) \mathcal{J} \left( \frac{\omega_d}{\omega} \phi - v_\parallel \hat{\phi} \right) \tag{6}
$$

The functional becomes

$$
\mathcal{L} = -\frac{1}{\mu_0} \int d^3x \left| \nabla_\perp \hat{A}_\parallel \right|^2 + \int d\tau F_{eq} e^2 \frac{\omega - \omega^*}{\omega} \frac{1 - \mathcal{J}^2}{\omega} \left| \phi \right|^2 + \sum_{\text{species}} \int d\tau \frac{F_{eq} e^2}{T_{eq}} \frac{\omega^* \omega_d}{\omega^2} \left| \mathcal{J} \hat{\phi} \right|^2 + \sum_{\text{species}} \mathcal{L}_{res}
$$

where

$$
\mathcal{L}_{res} = -e \int d\tau \hat{\tilde{g}}_{n\omega} \mathcal{J} \left( \frac{\omega_d}{\omega} \hat{\phi} - v_\parallel \hat{\phi} \right)^* \tag{7}
$$

Eq. (7) can be understood as follows. The first term is the magnetic energy, and gives rise to the field part of the Ampère equation. The second term is the polarization term (including
functions of $\bar{\nu}$. It is convenient to introduce a new variable $\bar{\nu}$, where $\bar{\nu}(v)$ is the electron-ion collisional frequency and $\nu = \frac{v}{\bar{\nu}}$ is the pitch-angle variable (which varies between $-1$ and $1$), and $v$ the velocity modulus. It is natural to choose $(v, \xi)$ as new velocity variables, the volume integration being $d\tau = d^3x 2\pi v^2 dv d\xi$. It is then convenient to expand $\hat{g}_{\nu\omega}(\eta, \xi, v)$ over a basis of Legendre polynomials $P_\ell(\xi)$.

$\hat{g}_{\nu\omega}(\eta, \xi, v) = \sum_{\ell=0}^{+\infty} \hat{g}_{\nu\omega \ell}(\eta, v) P_\ell(\xi)$

(9)

Indeed Legendre polynomials are eigenfunctions of the collision operator: $C(P_\ell) = -\ell(\ell+1)\nu_{ei} P_\ell$.

One has $P_0(\xi) = 1$, $P_1(\xi) = \xi$, and all other polynomials can be calculated via the recurrence $P_{\ell+1} = (2\ell + 1)\xi P_\ell - \ell P_{\ell-1}$. We restrict the calculation to the two first components $\hat{g}_{\nu\omega 0}$ and $\hat{g}_{\nu\omega 1}$. The closure $2P_2 = 3\xi P_1 - P_0$ is determined using the recurrence formula. The electron gyro-averager operator can be set to one because of the small electron gyroradius, and we ignore in the following the second-order compressional term $\frac{\omega_e}{\omega} \phi$, this term tends to be small. The projections onto the 2 first polynomials give the following relations

$(\omega - \omega_d) \hat{g}_{\nu\omega 0} - \frac{1}{3} k_{||} v \hat{g}_{\nu\omega 1} = 0$  (10)

$(\omega - \omega_d + i\nu_{ei}) \hat{g}_{\nu\omega 1} - k_{||} v \hat{g}_{\nu\omega 0} = -e\frac{F_{eq}}{T_{eq}} (\omega - \omega^*) v \hat{\xi}_||$  (11)

To make the calculation tractable, the dependence of $\omega_d$ on $\xi$ is ignored, i.e. $\omega_d = \frac{n_0}{r_0} \frac{2}{3} \frac{mv^2}{eB_0 R_0} (\cos \eta + s_0 (\eta - \eta_k) \sin \eta)$, corresponding to $\xi^2 = 1/3$, is assumed. Note that as the magnetic drift frequency depends on $\eta$, Eqs(10, 11) are two coupled differential equations in $\eta$. It is convenient to introduce a new variable $\bar{\eta}(v, \eta)$, instead of $\eta$, and defined by the differential equation

$\frac{d\bar{\eta}}{d\eta} = \frac{\sqrt{3} q R_0}{v} (\omega - \omega_d(v, \eta))$  (12)

which is invertible as long as the root $\eta_0(v)$ of the equation $\omega = \omega_d(v, \eta_0)$ is larger than the mode width in $\eta$. We choose by convention $\bar{\eta} = 0$ when $\eta = 0$. All functions can then be expressed as functions of $\bar{\eta}$ in place of $\eta$. We also introduce the functions $\lambda(v, \bar{\eta}) = \left(1 + i\frac{\nu_{ei}(v)}{\omega - \omega_d(v, \bar{\eta})}\right)^{1/2}$ and $Q(v, \bar{\eta}) = \frac{F_{eq}}{T_{eq}} \frac{\omega - \omega^*}{\omega - \omega_d(v, \bar{\eta})} v$. The square root is chosen such that the imaginary part of $\lambda$ is positive. The function $\hat{g}_{\nu\omega 1}(v, \bar{\eta})$ is then solution of the differential equation

$\frac{\partial^2 \hat{g}_{\nu\omega 1}}{\partial \bar{\eta}^2} + \lambda^2 \hat{g}_{\nu\omega 1} = -Qe\hat{\xi}_||$  (13)

while $\hat{g}_{\nu\omega 0} = -i \frac{1}{\sqrt{3}} \frac{\partial \hat{g}_{\nu\omega 1}}{\partial \bar{\eta}}$. We note that only $\hat{g}_{\nu\omega 1}$ is needed since the resonant functional can be expressed as

$L_{res} = e \int d\tau \hat{g}_{\nu\omega 1} \frac{v}{3} \hat{\xi}_||^*$  (14)
where the volume element in the ballooning space is \( d\tau = \frac{d\eta}{2\pi^2} 4\pi v^2 dv \). Let us introduce the function \( \Lambda(v, \bar{\eta}) = \int_0^\infty d\eta' \lambda(v, \bar{\eta}) \). If \( \frac{\partial \Lambda}{\partial \bar{\eta}} \ll \frac{1}{\bar{\eta}} \frac{\partial g_{\omega \Lambda}}{\partial \eta} \) (WKB approximation [19]), then \( e^{\pm \Lambda} \) are solutions of the homogeneous equation, and a formal solution of Eq. (13) is

\[
\hat{g}_{\omega \Lambda} = -e \int_{-\infty}^{\infty} \frac{d\eta'}{2\lambda(v, \bar{\eta}')} \exp \left\{ i \left[ \Lambda(v, \bar{\eta}) - \Lambda(v, \bar{\eta}') \right] \right\} Q(v, \bar{\eta}') \hat{\Theta} (\bar{\eta}') \\
+ e \int_{+\infty}^{\infty} \frac{d\eta'}{2\lambda(v, \bar{\eta}')} \exp \left\{ i \left[ \Lambda(v, \bar{\eta}) - \Lambda(v, \bar{\eta}') \right] \right\} Q(v, \bar{\eta}') \hat{\Theta} (\bar{\eta}')
\]

The WKB approximation is justified in the weakly collisional regime and when \( \omega \gg \omega_d \), so that the variations \( \lambda(v, \bar{\eta}) = \left(1 + i \frac{\nu_d(v)}{\omega_d(v, \bar{\eta})}\right)^{1/2} \) with \( \bar{\eta} \) are slow compared to the variation of \( \hat{g}_{\omega \Lambda} \). Plugging the formal solution Eq. (15) into the functional Eq. (14) provides formally the required functional

\[
\mathcal{L}_{\text{res}} = -\frac{1}{3} \int_0^{+\infty} dv 4\pi v^4 F_{eq} e^2 \frac{T_{e,eq}}{e}\int_{-\infty}^{+\infty} \frac{d\eta d\eta'}{2\pi} \frac{\omega - \omega_e^*}{\omega - \omega_d(v, \bar{\eta})} \frac{1}{2i\lambda(v, \bar{\eta})} \left\{ \Theta (\bar{\eta} - \bar{\eta}') \exp \left\{ i \left[ \Lambda(v, \bar{\eta}) - \Lambda(v, \bar{\eta}') \right] \right\} + \Theta (\bar{\eta}' - \bar{\eta}) \exp \left\{ i \left[ \Lambda(v, \bar{\eta}') - \Lambda(v, \bar{\eta}) \right] \right\} \right\} \hat{\Theta} (\bar{\eta}') \hat{\Theta}^* (\bar{\eta})
\]

where \( \Theta \) is an Heaviside function. We consider the long wavelength limit, where \( 1 - \mathcal{J}^2 = \frac{1}{4} k_\perp^2 \rho_i^2 \). The ion inertia functional then becomes

\[
\int d\tau \frac{F_{eq} e^2 (\omega - \omega_e^*)}{\omega} (1 - \mathcal{J}^2) \left| \hat{\phi} \right|^2 = \int d^3x \frac{N_{eq} m_i}{B_0^2} \frac{\omega - \omega_{pi}^*}{\omega} \left| \nabla_\perp \hat{\phi} \right|^2
\]

where \( \omega_{pi}^* = -\frac{k_\parallel T_{i,eq}}{eB_0 L_{pi}} \) and \( L_{pi} \) is the ion pressure gradient length. The interchange drive in the functional becomes:

\[
\sum_{\text{species}} \int d\tau \frac{F_{eq} e^2 \omega^* \omega_d}{\omega^2} \left| \mathcal{J} \hat{\phi} \right|^2 = \int d\tau \frac{F_{eq} e^2 \omega^* \omega_d}{\omega^2} \left| \hat{\phi} \right|^2
\]

The final form of the functional then becomes:

\[
\mathcal{L} = -\frac{1}{\mu_0} \int d^3x \left| \nabla_\perp \hat{A} \right|^2 + \int d^3x \frac{N_{eq} m_i}{B_0^2} \frac{\omega - \omega_{pi}^*}{\omega} \left| \nabla_\perp \hat{\phi} \right|^2 + \int d\tau \frac{F_{eq} e^2 \omega^* \omega_d}{\omega^2} \left| \hat{\phi} \right|^2 + \mathcal{L}_{\text{res}}
\]

where

\[
\mathcal{L}_{\text{res}} = -\frac{q R_0}{\sqrt{3}} \int_0^{+\infty} dv 4\pi v^3 \frac{F_{eq} e^2}{T_{e,eq}} (\omega - \omega_e^*) \int_{-\infty}^{+\infty} \frac{d\eta}{2\pi} \frac{d\eta'}{2\lambda(v, \bar{\eta})} \left\{ \Theta (\bar{\eta} - \bar{\eta}') \exp \left\{ i \left[ \Lambda(v, \bar{\eta}) - \Lambda(v, \bar{\eta}') \right] \right\} \right\} \hat{\Theta} (\bar{\eta}') \hat{\Theta}^* (\bar{\eta})
\]

and \( \hat{\Theta} (\bar{\eta}) = \hat{A}_\parallel (\bar{\eta}) - \frac{\omega - \omega_{\omega \Lambda}}{\omega} \frac{1}{\partial \eta} \hat{\phi} \). The complex form of the functional can be simplified by assuming an effective magnetic drift term.
3. Magnetic microtearing modes

In the classical theory of tearing/microtearing mode, the solution is split into an external currentless solution where \( \nabla^2 A_{||} = 0 \), and a layer where the current allows a matching across the resonant surface. The external solution is \( A_{||}(0)e^{-|k_0 x|} \) in the physical space. The “A-constant” approximation consists in assuming that the vector potential varies slowly within the current layer \( A_{||}(x) \approx A_{||}(0) \). The matching between the two solutions is represented by the usual tearing index \( \Delta' = \left[ \frac{1}{A_{||}} \frac{dA_{||}}{dx} \right]_{L} \). For a microtearing mode \( \Delta' = -2 |k_0| \).

The same procedure can be applied in the ballooning space: the internal solution \( \hat{A}_{||}(x) \) which is constant in the physical space becomes a function \( \hat{A}_{||}(\eta) \) in the ballooning space. Microtearing modes are very localized around the resonant surface. Using an effective magnetic drift frequency \( \omega_d(v, \eta) = \omega_d(v, 0) \), consistent with a ‘strong ballooning approximation’ combined with the constant \( A_{||} \) approximation, a residue integration method in the variable \( x \), noticing that \( \frac{d\zeta}{d\tau} = \frac{1}{|A_{||}|} \) and using the WKB approximation the total functional for a pure magnetic microtearing mode (\( \phi = 0 \)) becomes:

\[
\mathcal{L} = -\frac{2 |k_0|}{\mu_0 |d|} |A_{||}(0)|^2 + i \frac{\pi}{\sqrt{3}} qR_0 |A_{||}(0)|^2 \int_0^{+\infty} dv 4\pi \nu_{\text{eq}}^2 \frac{F_{\text{eq}}e^2}{T_{\text{eq}}} \frac{\omega - \omega^*}{\left(1 + i \frac{\nu_{\text{eq}(v)}}{\omega - \omega_d(v, 0)} \right)^{1/2}} \tag{21}
\]

where \( d \) is the distance to the resonance surface and \( N_{\text{eq}} \) is the local density. Let us now introduce a few normalizations. All frequencies are normalized to the electron temperature diamagnetic frequency \( \omega^*_T = \frac{\omega_T}{\omega^*_e} \), i.e. \( \Omega = \frac{\omega}{\omega^*_e} \), \( \nu_{\text{eq}} = \frac{\nu_{\text{e,th}}}{\nu_{\text{eq}}} \), and \( \Omega_d(v, \eta) = \frac{\omega_d(v, \eta)}{\omega^*_e} \), where \( \nu_{\text{e,th}} \) is the thermal collision frequency. All velocities are normalized to the electron thermal velocity \( v_{\text{Te}} = \sqrt{\frac{2T_{\text{e,eq}}}{m_e}} \). The condition \( \mathcal{L} = 0 \) then reads

\[
\frac{1}{\beta^*} = 8i \frac{\sqrt{\pi}}{3} \int_0^{+\infty} d\zeta \zeta^2 e^{-\zeta^2} \frac{\Omega - \Omega^*(\zeta)}{\left[\zeta^3 + i \frac{\nu_{\text{eq}}}{\Omega - \Omega^*(\zeta)} \right]^{1/2}} \tag{22}
\]

where \( \beta^* = \frac{2v_{\text{Te}}N_{\text{eq}}T_{\text{e,eq}} q_0 R_0}{B^2} \), \( \rho_e = \frac{m_v v_{\text{Te}}}{e R_0} \), \( \Omega^*(\zeta) = \frac{1}{\rho_e} + \zeta^2 - \frac{3}{2} \), \( \eta_e = \frac{m_v v_{\text{Te}}}{e R_0} \), and \( \Omega_d = \frac{4 T_{\text{e,eq}}}{3 m_e} \).

For zero magnetic drift \( \Omega_d = 0 \) and electric potential, the previous result obtained for the slab collisional tearing mode is recovered [15]. The electron temperature gradient is at the origin of the collisional microtearing mode destabilization.

4. Effect of an effective magnetic drift on microtearing modes

The form Eq. (1) at marginal stability (real \( \omega \)) provides a way to understand the mechanisms that underlie the instability, based on the sign of the current. The first term, always negative, represents the magnetic energy needed to bend field lines. It is thus stabilizing. The second term in absence of electric potential is destabilizing when the normalised current density \( j_{||}(x)/A_{||}(0) \) is positive. From Eq. (21) and Eq. (22), the current density expressed in the variable \( \rho \) such that \( \rho = \frac{1}{\sqrt{3}} \frac{v_{\text{Te}}}{\omega - \omega_d} \) is given by the relation

\[
\frac{j_{||}(\rho)}{\mu_0 A_{||}(0)} = -\frac{8}{\sqrt{3} \pi} q_0 R_0 \frac{\rho_e}{s_0 L_{\text{Te}}} \int_0^{+\infty} d\zeta \zeta^4 e^{-\zeta^2} \frac{\Omega - \Omega^*(\zeta)}{1 + i \frac{\nu_{\text{eq}}}{\Omega - \Omega^*(\zeta)} - \zeta^2 \rho^2} \tag{23}
\]

where \( d_e = \frac{\omega_T}{\omega^*_e} \) is the electron skin depth (\( \omega^2_{\text{pe}} = N_{\text{eq}}e^2/m_e c_0, \omega_{\text{pe}} \) the electron plasma frequency). The main role of the magnetic drift appears to be a modification of the collisional frequency,
whose dependence on velocity is now modified, i.e. \( \nu_e \) is replaced by \( \frac{1}{c^2} \frac{\nu_e}{\Omega - \Omega_d} \). One key point however is that the current far from the resonant surface \( \rho \to \infty \) is not affected, i.e. 
\[
\frac{j_i(\rho)}{\mu_0 A_i(0)} \simeq \frac{2}{\sqrt{3} \pi} \left( \Omega - \frac{1}{\eta_e} \right) \frac{1}{\rho^2}.
\]
The dispersion relation of microtearing modes is generically of the form \( \Omega = \frac{1}{\eta_e} + \alpha_e \), with \( \alpha_e > 0 \) (e.g. \( \alpha_e = \frac{1}{2} \) in the collisionless regime, \( \alpha_e = \frac{5}{4} \) in the collisional regime). Hence the current far from the resonant surface is always destabilizing. Since it does not depend on the collision frequency, it is not affected by the magnetic drift. The situation close to the resonant surface has to be different. It is known that microtearing modes are marginally stable without magnetic drift and no collisions, which implies that the radial integral of the current vanishes. Since the current is positive (destabilizing) far from the resonant surface, it must be negative (stabilizing) in some regions closer to the resonant surface. Intuition would tell us that collisions should decrease this current, and therefore destabilize the microtearing mode. Moreover, the magnetic drift should increase this effect by enhancing the effect of collisions. This can be seen in a better way by looking at the stability criterion Eq. (22) at weak collisionality and weak magnetic drift frequency, i.e. 
\[
\frac{1}{\beta^*} = 8i \sqrt{\frac{\pi}{3}} \int_0^{+\infty} d\zeta \zeta^3 e^{-\zeta^2} \left( \alpha_e + \frac{3}{2} - \zeta^2 \right) \left[ 1 - i \frac{2}{\sqrt{3} \pi} \frac{\nu_e \zeta}{\Omega} \left( 1 + \frac{\Omega_d}{\Omega} \zeta^2 \right) \right]
\] (24)
The imaginary part cancels when \( \alpha_e = \frac{1}{2} \), as expected. The real part provides the relation 
\[
\frac{1}{\beta^*} = 2 \sqrt{3 \pi} \nu_e \Omega \left( 1 + \frac{1}{6} \frac{\Omega_d}{\Omega} \right).
\]
The magnetic drift helps, but only in conjunction with collisions. In other words, the effect of the magnetic drift disappears when \( \nu_e = 0 \). In summary the magnetic drift has a destabilizing effect, in synergy with collisionality.

5. Conclusion

In this paper, we have presented a derivation of a current sheet model for microtearing modes in a kinetic framework. For the first time, the magnetic drift, the electric potential and collisions have been included consistently. Previous gyrokinetic simulations have shown that microtearing modes have a poloidally localised mode structure and as a consequence the ballooning representation has been used to evaluate the current inside the resistive layer. Moreover, the full expression of the current inside the resistive layer being rather complex, an effective magnetic drift has been employed. This model recovers a previous expression where the magnetic drift and the electric potential were neglected. The magnetic drift is found to be destabilising, but only in conjunction with a finite collisionality. Finally, as in reduced MHD, the evaluation of the current inside the resistive layer is obtained from a system of two equations linking the vector potential (and as a consequence the current) and the electric potential. Keeping both electric potential and magnetic drift no analytical tractable solution has been found.

In future work, in order to compute the current inside the resistive layer, the system of two equations should be solved numerically using an eigenvalue code. The dispersion relation of a microtearing mode destabilizing a current sheet will be obtained matching the current inside the resistive layer to the external current. Then, this dispersion relation could be compared linear gyrokinetic simulations in order to "reconcile" linear theory and simulations and to improve the understanding of microtearing mode destabilization mechanism.
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