Traversable wormholes in $f(R, T)$ gravity

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Abstract In the present article, models of traversable wormholes within the $f(R, T)$ modified gravity theory, where $R$ is the Ricci scalar and $T$ is the trace of the energy-momentum tensor, are investigated. We have presented some wormhole models, which are formulated from various hypothesis for their matter content, i.e. various relations for their lateral and radial pressure components. The solutions found for the shape functions of the wormholes produced complies with the required metric conditions. The validity of solution is examined by exploring null, strong and dominant energy conditions. It is concluded that the normal matter in the throat may pursue the energy conditions, and it is the higher-order curvature terms, termed as the gravitational field, that supports the non-standard geometries of wormholes in the context of modified gravity.

Keywords Traversable wormholes · $f(R, T)$ gravity · Energy conditions · Shape functions

1 Introduction

The exact solutions for wormholes (WH) were obtained connecting two distinct regions of asymptotically flat space-time in general relativity (Morris and Thorne 1988; Visser 1996), also as two distinct anti-de Sitter or asymptotically de-Sitter regions (Lemos et al. 2003). Although, wormholes still have not been observed (Jordan 1959) but attempts to achieve has been proposed (Tsukamoto 2016, 2017; Zhou et al. 2016; Rahaman et al. 2014; Kuhfittig 2014; Bambi 2013; Li and Bambi 2014; Nandi et al. 2006; Harko et al. 2009). Theoretically, wormholes were predicted a long time ago. Wormholes solutions were first proposed by Einstein and Rosen (1935) with the event horizon. Many decades later, Morris and Thorne have suggested that wormholes can be traversable, which violates the energy conditions if filled with exotic matter (Morris et al. 1988).

The concept of wormhole is the most popular and intensively studied one in General Relativity research, and in modified theories of gravity, too. The minimal surface area of the attachment connecting the two regions is known as the throat of the wormhole. In wormholes Physics, the presence of throat is the fundamental feature which satisfies flaring-out conditions (Rosa et al. 2018). In the literature, wormholes are usually classified into two separate groups, namely of static and of dynamic wormholes, respectively. Pioneering work on static wormholes is due to Morris and Thorne, who demonstrated that matter inside them has negative energy, thus violating the null energy condition (NEC) (Morris and Thorne 1988). Trying to develop exact wormhole models, with the possibility either to minimize or even to completely cure this violation, has been one of the most active objects of study in the field, in the last few years.

Such aim has been fulfilled, in particular, in modified gravity theories (Agnese and La Camera 1995; Nandi et al. 1998; Bronnikov and Kim 2003; La Camera 2003; Lobo 2008a,b; Garattini and Lobo 2007, 2009, 2012; Lobo and Oliveira 2010; Singh et al. 2020; Jawad and Rani 2016; Zubair et al. 2019; Moraes et al. 2019b; Bertolami and Zum-
Primordial wormholes might be present at the very early Universe, where quantum effects play an essential role (Nojiri et al. 1999). On the other hand, if we introduce a scale factor into the original Morris-Thorne metric, this gives rise to an evolving relativistic wormhole model (dynamical wormhole model) (Bhattacharya and Chakraborty 2017). In the recent literature, there is an intensive activity related to objects of this kind, as speculating on their existence, in special, with matter satisfying the weak energy condition (WEC) or the dominant energy condition (DEC). Moreover, recently too, dynamical wormholes have been treated in terms of a two-fluid system. Other interesting studies of traversable wormholes have been carried out in the context of non-equilibrium thermodynamics in the presence of adiabatic particle creation (Pan and Chakraborty 2015).

The main challenges about the existence of wormhole are the traversable wormhole concept and the energy conditions of it. With an idea of utilizing a wormhole for time travel or interstellar travel, the concept of the traversable wormhole was suggested by Morris et al. (1988). It is notable that the concept of the traversable wormhole is completely different from the Einstein-Rosen bridge concept, (Einstein and Rosen 1935) and the work which is done by Wheeler (1962) about the charge-carrying microscopic wormhole. For example, Romero and Bellini (1999) have investigated a traversable wormhole which is created due to a magnetic monopole. The stability of traversable wormholes have been investigated for minimizing the violation of null energy condition, which is a common feature for a static wormhole in general relativity (Shinkai and Hayward 2002). The existence of static traversable wormholes has been investigated without exotic matter in the framework of Einstein-Cartan theory (Bronnikov and Galiakhmetov 2015). The traversable wormholes have been studied in both Lyra geometry and general relativity (Moradpour and Jahromi 2017). It was shown that the geometry which is obtained in the isotropic version of stress-energy tensor of dark energy can not describe a traversable wormhole in teleparallel gravity formalism (Saaidi and Nazavari 2020). Lobo and Oliveira (2009) investigated traversable wormholes using the framework of \( f(R) \) gravity and obtained the factors responsible for the violation of null energy condition and supporting the existence of wormholes. In Lobo and Oliveira (2009), they also obtained wormhole solutions for different shape functions. In fact, the wormhole throat can be threaded by the normal matter in the framework of modified gravity theories and these nonstandard wormhole geometries are supported by the higher order curvature term (Rosa et al. 2018), which can be termed as gravitational fluid. Theoretically, wormholes throat can be formulated in the absence of exotic matter (Lobo and Oliveira 2009). Wormholes were constructed by non-minimal coupling in Garcia and Lobo (2010), Montelongo Garcia and Lobo (2011). In modified gravity alone, the fundamental fields sustained the wormhole throats using generic modified gravities (Harko et al. 2013). Such solutions are also obtained in Bhawal and Kar (1992), Dotti et al. (2007), Mehdizadeh et al. (2015), Lobo (2007), Sahu et al. (2018), Anchordoqui et al. (1997), Capozziello et al. (2012), Mehdizadeh and Ziae (2017a), Golchin and Mehdizadeh (2019), Mehdizadeh and Ziae (2017b). Rosa et al. (2018) found wormholes solutions in which the null energy conditions (NEC) were obeyed by the matter not only at the throat but everywhere in a hybrid generalized metric-Palatini gravity, with action \( f(R, \lambda) \).

Indeed, examination of wormhole solutions in various modified theories is a noteworthy and important point in Theoretical Physics. The General Relativity (GR) could be adjusted in various perspectives. By the introduction of gravitational action of the \( f(R, T) \) gravity in Ricci scalar \( R \) and the trace of energy momentum tensor \( T \), T. Harko and fellow researchers (Harko et al. 2011) streamlined the \( f(R) \) theories (Starobinsky 1980; Bertolami et al. 2007, 2010; Nojiri et al. 2004, 2007; Bamba et al. 2012; Allemandi et al. 2005; Nojiri et al. 2006, 2008) by replacing the function \( f(R) \) with a random function \( f(R, T) \). This \( f(R, T) \) theory tested in Astrophysics of compact objects (Moraes et al. 2016a; Zubair et al. 2016c; Shamir 2015; Zubair and Nojiri 2015), Thermodynamics (Momeni et al. 2016; Harko 2014) and cosmology (Moraes et al. 2016b; Moraes and Correa 2016; Correa and Moraes 2016; Moraes and Santos 2016; Moraes 2015; Alvarenga et al. 2013a; Myrzakulov 2012; Sharma and Pradhan 2017; Nagpal et al. 2018; Debnath 2019; Ahmed and Alamri 2018; Sharma et al. 2019).

The study of wormholes is also extended using \( f(R, T) \) theory. The work has been done on the cases of \( WH \) geometry in \( f(R, T) \) gravity, where the redshift function \( \phi \) is not dependent on either time or spatial coordinate (Azizi 2013; Zubair et al. 2016a). The spherically symmetric and static wormholes are considered in \( f(R, T) = f(R) + \lambda T \) gravity with different fluids (Zubair et al. 2016b). The geometry (non-commutative) of wormholes with respect to Lorentzian and Gaussian distributions of string theory is proposed as well as found the numerical and exact solutions in modified \( f(R, T) \) gravity (Zubair et al. 2017). An analytical approach is used to get the wormhole solutions in \( f(R, T) \) gravity (Moraes et al. 2017). An exponential shape function is defined for wormholes and a comparison is made between \( f(R, T) \) and \( f(R) \) gravity for the rationality of energy conditions (Samanta et al. 2018). Wormholes are studied for two types of varying Chaplygin gas and found the violation of dominated and null energy conditions in \( f(R, T) \) gravity.
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(Elizalde and Khurshudyan 2018). In Bhatti et al. (2018), an exponential \( f(R, T) \) gravity model has been considered and obtained solutions for wormhole. The charged wormhole solutions are investigated in \( f(R, T) \) gravity satisfying the energy conditions (Moraes et al. 2019a). Using Noether symmetry approach, wormhole solutions were investigated for non dust and dust distributions in \( f(R, T) \) gravity (Sharif and Nawazish 2019). Considering the different types of energy density, traversable wormhole solutions investigated in \( f(R, T) \) gravity (Elizalde and Khurshudyan 2019a). Wormhole solutions have been studied by various researchers in modified gravity theories in various set-ups (Moraes and Sahoo 2019; Sahoo et al. 2019; Banerjee et al. 2019a,b; Amir et al. 2019). An exact wormhole solution is obtained in modified theories of gravity, with \( f(R, T) = R + \lambda T \), being \( T = \rho + P_r + 2P_l \) the trace of the energy momentum tensor (Elizalde and Khurshudyan 2019b), assuming different possibilities for their matter content. Further, traversable wormholes investigated the in \( f(R, T) \) gravity by taking a new form of \( f(R, T) = R + 2\alpha \ln T \) function to minimize the appearance of exotic matter near the throat of the wormhole (Godani and Samanta 2019).

The fact that wormholes matter content is signified by an anisotropic fluid and \( T \)-dependence of the \( f(R, T) \) theory, which may be related to imperfect fluid, forms one of the strong motivation behind working with the wormholes in \( f(R, T) \) gravity (Moraes et al. 2017; Moraes and Sahoo 2017).

Recently, traversable wormhole are investigated for two particular types of shape functions i.e. (i) \( b(r) = r_0 \left( \frac{r}{r_0} \right)^y \), \( 0 < y < 1 \) (ii) \( b(r) = \frac{r_0 \log(r+1)}{\log(r_0+1)} \) and obtained wormhole structures filled with phantom fluid in \( f(R) \) gravity, taking the form of the model \( f(R) = R + \alpha R^m - \beta R^n \) (Godani and Samanta 2018). The first shape function is proposed by Cataldo et al. (2017), for a new static traversable wormholes, dubbed Schwarzschild-like wormholes, and the second one is proposed by Godani and Samanta (2018). In Godani and Samanta (2018), for both the shape functions, all the energy conditions was found to be violated for the existence of wormholes in the universe. While, in this paper, we have investigated the traversable wormholes by assuming functional form of \( f(R, T) = R + \lambda T \) of \( f(R, T) \) gravity, originally proposed by Harko et al. (2011) considering the shape function given in Godani and Samanta (2018), which has not been studied already. We found that wormholes can exist even if NEC is not violated i.e. it is not the exotic matter that threads the wormhole, but these are the extra curvature term that sustained the wormhole in the modified gravity.

The paper is organized as follows: in Sect. 2, a brief review is presented for the \( f(R, T) \) theory. In Sect. 3, the wormhole metric and its conditions are described. In Sect. 4, we presented the field equation solution for the wormhole metric in the \( f(R, T) \) gravity. In Sect. 5, we presented models of the wormhole. Our results are discussed in Sect. 6.

2 The basic formalism of \( f(R, T) \) gravity

The theory of \( f(R, T) \) gravity is given by the following action (Harko et al. 2011)

\[
S = \frac{1}{16\pi} \int d^4x f(R, T) \sqrt{-g} + \int L_m d^4x \sqrt{-g},
\]

where \( R \) is Ricci scalar and \( T \) is trace of the energy momentum tensor (EMT) in an arbitrary function \( f(R, T) \), \( g \) is the metric determinant and matter Lagrangian density \( L_m \) is the metric determinant and matter Lagrangian density \( L_m \). These are connected to the EMT (Landau and Lifshitz 1998).

\[
T_{ij} = -\frac{2}{\sqrt{-g}} \left[ \frac{\partial(\sqrt{-g} L_m)}{\partial g^{ij}} - \frac{\partial(\sqrt{-g} L_m)}{\partial (\partial^2 g^{ij})} \right].
\]

Moreover, units is taken such that \( c = 1 = G \).

Following as Harko and Lobo (2010), Harko et al. (2011), we consider \( L_m \) relies only on \( g_{ij} \) (metric component) and not depends on its differential coefficients, such that we can get

\[
T_{ij} = -\frac{\partial L_m}{\partial g^{ij}} + g_{ij} L_m.
\]

In Eq. (1), the action \( S \) is varying with \( g_{ij} \) which provides the \( f(R, T) \) field equations (Harko et al. 2011) as

\[
f_R(R, T)(R_{ij} - \frac{1}{3} R g_{ij}) + \frac{1}{6} f(R, T) g_{ij} = 8\pi G \left( T_{ij} - \frac{1}{2} T g_{ij} \right) - f_T(R, T) \left( T_{ij} - \frac{1}{3} T g_{ij} \right)
\]

\[
- f_T(R, T) \left( \theta_{ij} - \frac{1}{3} \theta g_{ij} \right) + \nabla_i \nabla_j f_R(R, T)
\]

with \( f_R(R, T) = \frac{\partial f(R, T)}{\partial R} \), \( f_T(R, T) = \frac{\partial f(R, T)}{\partial T} \) and

\[
\theta_{ij} = g^{ij} \partial T_{ij} / \partial g^{ij}.
\]

Such theories of gravity \( f(R, T) \) (as well as \( f(R, L_m) \)) can explain in their formalism a non-minimal matter-geometry coupling. It suggests that test particles moving in the field (gravitational) will not obey geodesic paths, in such theories. The geometry and matter coupling produces an additional force acting on the particles, which is perpendicular to the four-velocity.
Interestingly, an additional force relies on the Lagrangian structure of the matter (Harko and Lobo 2014). It was found in Bertolami et al. (2008), that by taking the total pressure of \( \mathcal{L}_m = p \), with \( p \) being the total pressure, the additional force vanishes. However, more natural structures of \( \mathcal{L}_m = -\rho \), with \( \rho \), represents the energy density, are more common, in the sense that they do not imply the vanishing of the additional force.

Assuming the Lagrangian matter in this paper as \( \mathcal{L}_m = -\rho \). It is therefore possible to write Eq. (5) as

\[
\theta_{ij} = -2T_{ij} - \rho g_{ij}.
\]

(6)

Let us suppose the function \( f(R,T) = 2f(T) + R \), where \( f(T) \) is a random function of \( T \). The field equations (4) and (5) of \( f(R,T) \) gravity takes the form

\[
R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} + 2f'(T)T_{ij}
+ [2\rho f''(T) + f'(T)]g_{ij},
\]

(7)

where \( R_{ij} \) is Ricci tensor and \( f'(T) = \frac{df(T)}{dT} \). By taking \( f(T) = \lambda T \), with a constant \( \lambda \), we can rewrite the above as

\[
G_{ij} = (8\pi + 2\lambda)T_{ij} + \lambda(2\rho + T)g_{ij},
\]

(8)

where \( G_{ij} \) is usual Einstein tensor. In this work, we have considered a linear combination of \( R \) and \( T \) as \( f(R,T) = R + 2\lambda T \). Such a functional form was originally proposed in \( f(R,T) \) gravity (Harko et al. 2011), and in several approaches such as Moraes et al. (2018, 2019b), Moraes and Sahoo (2017), Sharma and Pradhan (2017), Sharma et al. (2019), among many others. The functional form used in this article, \( f(R,T) = R + 2\lambda T \), does not predict a non-minimal matter-geometry coupling (mixture term like \( RT \)). A density-dependent supplementary acceleration, acting on massive test particles, is induced in the presence of a non-minimal coupling between geometry and matter. The extra force on massive particles generated by the geometry-matter coupling is always present, even in the case \( \mathcal{L}_m = p \), and causes a deviation from geodesic paths (Harko and Lobo 2014; Bertolami et al. 2008; Moraes et al. 2018; Haghani et al. 2013; Harko 2014). The considered form is the simplest one, which reduces to general relativity for the choice of \( \lambda = 0 \).

### 3 Wormhole metric and its conditions

In Schwarzschild coordinate \((t,r,\theta,\phi)\), the spherical symmetric wormhole metric is (Morris and Thorne 1988; Visser 1996)

\[
ds^2 = -U(r)dt^2 + \frac{dr^2}{V} + r^2d\Omega^2,
\]

(9)

where \( \Omega^2 = d\theta^2 + \sin^2\theta d\phi^2 \) and \( V = 1 - \frac{b(r)}{r} \). The \( U(r) \) that denotes redshift function and the \( b(r) \) that denotes shape function given in Eq. (9) will follow the conditions mentioned below (Morris and Thorne 1988; Visser 1996)

1. The coordinate \( r \) named as radial co-ordinate, ranges \( r_0 \leq r \leq \infty \), where \( r_0 \) refers to the radius of the throat.
2. The shape function \( b(r) \) fulfills the condition at the throat

\[
b(r_0) = r_0,
\]

(10)

and for the out of throat i.e. for \( r > r_0 \)

\[
0 < 1 - \frac{b(r)}{r}.
\]

(11)

3. The flaring out condition must be satisfied by shape function \( b(r) \) at the throat, i.e.

\[
b'(r_0) < 1,
\]

(12)

where superscript (’) represent derivative with respect to \( r \).

4. The limit required for asymptotically flatness of the space-time geometry

\[
\frac{b(r)}{r} \rightarrow 0 \quad \text{as} \quad |r| \rightarrow \infty.
\]

(13)

5. At the throat \( r_0 \), redshift function \( U(r) \) must be non-vanishing and finite.

To achieve the asymptotically flat behavior, we can take \( U(r) = \text{constant} \). Since the constant redshift function can be absorbed in rescaled time coordinate, we take \( U(r) = 1 \), such as Cataldo et al. (2011), Rahaman et al. (2007).

#### 3.1 Energy conditions

It is important to mention that the flaring-out condition has a purely geometric nature. However, in classical general relativity, through the Einstein field equation, one can deduce that the matter threading the wormhole throat violates the NEC. Generally, the NEC arises when one refers back to the Raychaudhuri equation, given by

\[
\frac{d\theta}{d\tau} + \frac{1}{2} \dot{\theta}^2 + \sigma_{ab} \sigma^{ab} - \omega_{ab} \omega^{ab} = -R_{ab}k^ak^b.
\]

(14)

Where \( \theta, \omega_{ab} \), \( \sigma_{ab} \) and \(-R_{ab}^k k^k \) are the expansion, rotation and shear associated to the null vector field \( k^a \) and Ricci tensor, respectively. The Raychaudhuri equation is also a purely geometric statement, and as such it makes no reference to any gravitational field equations. Now, the shear is a “spatial” tensor with \( \sigma^2 = \sigma_{ab} \sigma^{ab} \geq 0 \) and \( \omega_{ab} \equiv 0 \) for any hyper-surface orthogonal congruences, so that the condition
for attractive gravity reduces to $R_{ab}k^ak^b \geq 0$. The positivity of this latter quantity ensures that geodesic congruences focus within a finite value of the parameter labeling points on the geodesics. In general relativity, contracting both sides of the Einstein field equation $G_{ab} \equiv R_{ab} - \frac{1}{2}Rg_{ab} = k^2T_{ab}$ with any null vector $k^a$, one can write the above condition in terms of the stress-energy tensor given by $T_{ab}k^ak^b \geq 0$ (Alvarenga et al. 2013b; Sharif and Zubair 2013).

In modified theories of gravity the gravitational field equations can be rewritten as an effective Einstein equation, given by $G_{ab} = k^2T_{ab}^{eff}$, where $T_{ab}^{eff}$ is an effective stress-energy tensor containing the matter stress-energy tensor $T_{ab}$ and the curvature quantities, arising from the specific modified theory of gravity considered. Now, the positivity condition $R_{ab}k^ak^b \geq 0$ in the Raychaudhuri equation provides the generalized NEC, $T_{ab}^{eff}k^ak^b \geq 0$, through the modified gravitational field equation (Harko et al. 2013). Thus, NEC for any null vector is defined as $T_{ab}k^ak^b \geq 0 \Leftrightarrow$ NEC. Alternatively, NEC can also be defined in terms of principal pressures as $\forall i$, $\rho + p_i \geq 0 \Leftrightarrow$ NEC. The physical interpretation of the NEC is that the energy density as measured by any observer is non-negative. WEC for a time like vector is defined as $T_{ab}k^ak^b \geq 0 \Leftrightarrow$ WEC. This WEC in terms of principal pressures can also be defined as $\rho \geq 0$ and $\forall i$, $\rho + p_i \geq 0 \Leftrightarrow$ WEC. The physical significance of this condition is that it claims the local energy density must be positive as measured by any time-like observer. For a time-like vector, SEC is defined as $(T_{ab} - \frac{1}{2}g_{ab})k^ak^b \geq 0 \Leftrightarrow$ SEC, where $T$ denotes trace of energy momentum tensor. SEC in terms of principal pressures is defined as $T = -\rho + \sum_j p_j$ and $\forall j$, $\rho + p_j \geq 0$, $\rho + \sum_j p_j \geq 0 \Leftrightarrow$ SEC. The violation of SEC is a necessary condition to have a wormhole geometry. DEC for a time-like vector is defined as $DEC \Leftrightarrow T_{ab}k^ak^b \geq 0$ and $T_{ab}k^a$ is not space like. Alternately, DEC is also defined in terms of principal pressures $\rho \geq 0$ and $\forall i$, $p_i \in [-\rho, +\rho] \Leftrightarrow$ DEC. The physical significance of this energy condition says that the energy density will be positive locally and that the energy flux is time-like or null.

4 Solutions of field equations for wormholes in $f(R, T)$ gravity

Considering that the description of matter is described by an anisotropic fluid of the form

$$T^i_j = \text{diag}\{-\rho, p_r, p_t, p_t\},$$

(15)

where $\rho = \rho(r)$, $p_r = p_r(r)$, $p_t = p_t(r)$ are energy density, radial pressure and transverse (measured orthogonal to radial directions) pressure, respectively and the trace $T$ of the Eq. (15) can be obtained as $T = -\rho + p_r + 2p_t$.

The field equation components from Eq. (8) for the metric given by Eq. (9) with Eq. (15) are obtained as:

$$\frac{b'}{r^2} = (8\pi + \lambda)\rho - \lambda(p_r + 2p_t),$$

(16)

$$\frac{b}{r^3} = \lambda\rho + (8\pi + 3\lambda)p_r + 2\lambda p_t,$$

(17)

$$\frac{b-b'r}{r^3} = \lambda\rho + \lambda p_r + (8\pi + 4\lambda)p_t,$$

(18)

the field equations given by Eqs. (16), (17) and (18), yields the following solutions

$$\rho = \frac{b'}{r^2(8\pi + 2\lambda)},$$

(19)

$$p_r = -\frac{b}{r^3(8\pi + 2\lambda)},$$

(20)

$$p_t = \frac{b - b'r}{2r^3(8\pi + 2\lambda)}.$$  

(21)

5 Models of wormholes

We shall investigate models of wormholes for two different shape functions in this section.

5.1 Shape function $b(r) = \frac{r_0 \log(r+1)}{\log(r_0+1)}$

For our first model we take radial pressure $p_r$ and energy density $\rho$ related as

$$p_r = \omega\rho,$$  

(22)

where equation of state parameter $\omega$ in terms of $p_r$ is called radial state parameter. The $f(R, T)$ gravity model is considered with the shape function (Alvarenga et al. 2013b)

$$b(r) = \frac{r_0 \log(r+1)}{\log(r_0+1)}.$$  

(23)

For this shape function, wormholes solutions are obtained. The energy density, radial pressure, tangential pressure can be obtained by using field equations (19)–(21) as

$$\rho = \frac{r_0}{(1+r)\log(1+r)\log(1+r_0)},$$

(24)

$$p_r = \frac{r_0 \ln(1+r)}{\ln(1+r)\ln(r_0+1)},$$

(25)

$$p_t = \frac{1}{2}\left(\frac{r_0 \ln(1+r)}{\ln(1+r)} - \frac{r_0 r}{\ln(1+r_0 + 1)}\right) \times r^{-3}(8\pi + 2\lambda)^{-1},$$

(26)

$$\rho + p_r = \frac{1}{2}\left(\frac{r_0 (r + r \ln(1+r) + \ln(1+r))}{(1+r)\ln(1+r_0 + 1)r^3(4\pi + \lambda)}\right),$$

(27)
\[ \rho + p_r = \frac{1}{4} \frac{r_0 (r + r \ln(1 + r) + \ln(r + 1))}{(1 + r) \ln(r_0 + 1) r^3 (4\pi + \lambda)} , \]
\[ \rho + p_r + 2p_l = \frac{r_0 (1 + r)}{\ln(r_0 + 1) r^3 (4\pi + \lambda)} , \]
\[ \rho - |p_r| = \frac{r_0}{(1 + r) \ln(r_0 + 1) r^3 (8\pi + 2\lambda) - \frac{1}{2} \left( \frac{r_0 (1 + r)}{\ln(r_0 + 1)} - (1 + r) \ln(r_0 + 1) \right) r r_0}{(1 + r) \ln(r_0 + 1) r^3 (8\pi + 2\lambda)} \times r^{-3} (8\pi + 2\lambda)^{-1} , \]
\[ p_l - p_r = -\frac{1}{4} \frac{r_0 (r + r \ln(1 + r) + \ln(1 + r))}{(1 + r) \ln(r_0 + 1) r^3 (4\pi + \lambda)} . \]

We can see directly from Fig. 1, that all the conditions i.e. fundamental needs of shape functions, such as throat condition, flaring out condition and typical asymptotically flatness condition are satisfied for WH geometry. It can be observed from Eqs. (27, 28) and Figs. 2(a) and 2(b) the validation region of null energy condition (NEC) i.e. \( \rho + p_r \geq 0 \) and \( \rho + p_l \geq 0 \). This is the weakest restriction and just represents the attractive nature of gravity. In all the appropriate plots, the range of the parameter \( \lambda \) is \(-4 < \lambda < 4\) which is \(-4\pi\).

The strong energy condition (SEC) derives from the attractive nature of gravity and its shape arises directly from the analysis of a spherically symmetrical metric in the GR system. From (29) the SEC is plotted in Fig. 3(a). The SEC \( \rho + p_r + 2p_l \) is found to be positive, decreasing and tending towards zero with the increment of \( r \).

The dominant energy condition (DEC) limited the rate of the transfer of energy to the speed of light. DECs are obtained in Eqs. (30, 31). DEC is plotted in Figs. 3(b) and 4(a).
one can observe that violation of \( \rho \geq |p_r| \) for present model. We also observe from Fig. 4(b), that anisotropy parameter is negative with the change in \( r \), which signifies that the geometry is attractive nature.

### 5.2 Shape function \( b(r) = r_0(\frac{r}{r_0})^\gamma, 0 < \gamma < 1 \)

In this model, the wormholes solutions are obtained by using the shape function \( b(r) = r_0(\frac{r}{r_0})^\gamma, 0 < \gamma < 1 \) (see in Godani and Samanta 2018; Cataldo et al. 2017). In this case energy density \( \rho \), radial pressure \( p_r \), tangential pressure \( p_t \) are obtained by using field equations (19)–(21)

\[
\rho = r_0 \left( \frac{r}{r_0} \right)^\gamma r^{-3}(8\pi + 2\lambda)^{-1}
\]

\[
p_r = r_0 \left( \frac{r}{r_0} \right)^\gamma r^{-3}(8\pi + 2\lambda)^{-1}
\]

\[
p_t = \frac{1}{2} r_0 \left( \frac{r}{r_0} \right)^\gamma - r_0 \left( \frac{r}{r_0} \right)^\gamma r^{-3}(8\pi + 2\lambda)^{-1}
\]

\[
\rho + p_r = r_0 \left( \frac{r}{r_0} \right)^\gamma r^{-3}(8\pi + 2\lambda)^{-1} + r_0 \left( \frac{r}{r_0} \right)^\gamma r^{-3}(8\pi + 2\lambda)^{-1}
\]

\[
\rho - p_t = r_0 \left( \frac{r}{r_0} \right)^\gamma r^{-3}(8\pi + 2\lambda)^{-1} + 1/2 \left( r_0 \left( \frac{r}{r_0} \right)^\gamma - r_0 \left( \frac{r}{r_0} \right)^\gamma r^{-3}(8\pi + 2\lambda)^{-1}
\]

\[
\rho + p_r + 2p_t
\]

\[
= r_0 \left( \frac{r}{r_0} \right)^\gamma r^{-3}(8\pi + 2\lambda)^{-1} + r_0 \left( \frac{r}{r_0} \right)^\gamma r^{-3}(8\pi + 2\lambda)^{-1}
\]

\[
+ \left( r_0 \left( \frac{r}{r_0} \right)^\gamma - r_0 \left( \frac{r}{r_0} \right)^\gamma \right) r^{-3}(8\pi + 2\lambda)^{-1}
\]

We can also see from Fig. 5 that all the conditions mentioned earlier for WH geometry are satisfied for the second model. Further, in Figs. 6(a) and 6(b), we plot the validity region of NEC. The validity of the SEC can also be seen in Fig. 7(a). One can observe from Fig. 7(b) that violation of DEC (\( \rho > |p_r| \)), \( \rho - p_t \) is positive as \( r \) increases shown in
Fig. 6 (a) NEC, $\rho + p_r$, (b) NEC, $\rho + p_t$

Fig. 7 (a) SEC, $\rho + p_r + 2p_t$, (b) DEC, $\rho - |p_r|$

Fig. 8 (a) DEC, $\rho - |p_t|$, (b) DEC, $p_t - p_r$

It can be seen from Fig. 8(b), that anisotropy parameter is negative with the change in $r$, which signifies that the geometry is of attractive nature.

6 Discussion

Modified gravitational theories have received considerable attention over the last few decades as a possible alternative to GR. The paramount feature of wormholes geometries is non-vacuum solutions of Einstein's field equations. According to this theory, they are filled with a matter which is different from the normal matter and is known as exotic matter. Several researchers found exotic matter to be very useful in examining whether different modified gravity models were responsible for violating energy conditions via the effective energy-momentum tensors while the usual matter obeys these conditions (Morris and Thorne 1988; Visser 1996). In the present paper we explored different models of static wormhole solutions with two different shape functions $b(r) = r_0 \log(r+1) \log(r_0+1)$ and $b(r) = r_0(\frac{r}{r_0})^\gamma$, $0 < \gamma < 1$ in the theory of $f(R,T)$ gravity. One can observe from Fig. 1 and Fig. 5 that both the shape functions satisfy throat condition, flaring out condition and typical asymptotically flatness condition for WH geometry.

Furthermore, redshift function $U(r)$ is assumed to be constant, it indicates that a theoretical traveller’s tidal gravitational force is null. In both cases, we found that the geometries of the wormhole can exist even if the NEC is not violated by the usual matter i.e. it is not the exotic matter that...
threads the WH, but these are the extra curvature ingredients that sustained the wormhole in a modified gravity context (Visser 1996). We analyzed the nature of SEC and WEC in the context of $f(R,T)$ gravity with two different types of shape functions to investigate this study. We emphasized the details of wormhole geometry with variable shape function and constant redshift.

In both the models, it is observed that, for appropriate values of the parameters, we have obtained a wormhole solution with valid NEC and SEC, while DEC is also satisfied in terms of $p_t$ and only DEC is violated in terms of $p_r$. The presence of these solutions complies with the understanding that traversable wormholes enabled by additional fundamental gravitational fields can occur without the need for exotic matter. On the other hand, the rather realistic construction needed to build a complete space-time wormhole solution may suggest that in this class of theories there are not many such solutions around.

The solutions found for the shape functions of the wormholes would decide the metric conditions. The non-standard geometries of wormholes originate from the normal matter in the throat. Although plenty of wormholes solutions have been explored in the literature, it is useful to find geometries that minimize the usage of exotic matter. In the framework of modified gravity, it has also been shown that the normal matter can be imposed to satisfy the null energy condition, and it is the higher order curvature terms, interpreted as a gravitational fluid, that sustain these non-standard wormhole geometries, fundamentally different from their counterparts in general relativity (Azizi 2013; Lobo and Oliveira 2009).

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References

Agnese, A.G., La Camera, M.: Wormholes in the Brans-Dicke theory of gravitation. Phys. Rev. D 51, 2011 (1995)

Ahmed, N., Alamri, S.Z.: A stable flat universe with variable cosmological constant in $f(R,T)$ gravity. Res. Astron. Astrophys. 18(10), 123 (2018)

Allemandi, G., Borowiec, A., Francaviglia, M., Odintsov, S.D.: Dark energy dominance and cosmic acceleration in first order formalism. Phys. Rev. D 72, 063505 (2005)

Alvarenga, F.G., de la Cruz-Dombriz, A., Houndjo, M.J.S., Rodrigues, M.E., Sáez-Gómez, D.: Dynamics of scalar perturbations in $f(R,T)$ gravity. Phys. Rev. D 87(10), 103526 (2013a)

Alvarenga, F.G., Houndjo, M.J.S., Monwanou, A.V., Orou, J.B.C.: Testing some $f(R,T)$ gravity models from energy conditions. J. Mod. Phys. 4, 130 (2013b)

Amir, M., Banerjee, A., Maharaj, S.D.: Shadow of charged wormholes in Einstein–Maxwell-dilaton theory. Ann. Phys. 400, 198–207 (2019)

Anchordoqui, L.A., Perez Bergliaffa, S.E., Torres, D.F.: Brans-Dicke wormholes in nonvacuum space-time. Phys. Rev. D 55, 5226 (1997)

Azizi, T.: Wormhole geometries in $f(R,T)$ gravity. Int. J. Theor. Phys. 52, 3486 (2013)

Bamba, K., Capozziello, S., Nojiri, S., Odintsov, S.D.: Dark energy cosmology: the equivalent description via different theoretical models and cosmography tests. Astrophys. Space Sci. 342, 155 (2012)

Bambi, C.: Can the supermassive objects at the centers of galaxies be traversable wormholes? The first test of strong gravity for mm/sub-mm very long baseline interferometry facilities. Phys. Rev. D 87, 107501 (2013)

Banerjee, A., Moraes, P.H.R.S., Correa, R.A.C., Ribeiro, G.: Wormholes in Randall-Sundrum braneworld (2019a). arXiv preprint arXiv:1904.10510

Banerjee, A., Singh, K., Jasim, M.K., Rahaman, F.: Traversable wormholes in $f(R,T)$ gravity with conformal motions (2019b). arXiv preprint arXiv:1908.04754

Bertolami, O., Zambuial Ferreira, R.: Traversable wormholes and time machines in non-minimally coupled curvature-matter $f(R)$ theories. Phys. Rev. D 85, 104050 (2012)

Bertolami, O., Boehmer, C.G., Harko, T., Lobo, F.S.N.: Extra force in $f(R)$ modified theories of gravity. Phys. Rev. D 75, 104016 (2007)

Bertolami, O., Lobo, F.S.N., Paramos, J.: Non-minimum coupling of perfect fluids to curvature. Phys. Rev. D 78, 064036 (2008)

Bertolami, O., Frazao, P., Paramos, J.: Accelerated expansion from a non-minimal gravitational coupling to matter. Phys. Rev. D 81, 104046 (2010)

Bhattacharya, S., Chakraborty, S.: $f(R)$ gravity solutions for evolving wormholes. Eur. Phys. J. C 77(8), 558 (2017)

Bhatti, M.Z., Yousaf, Z., Illyas, M.: Existence of wormhole solutions and energy conditions in $f(R,T)$ gravity. J. Astrophys. Astron. 39, 69 (2018)

Bhawal, B., Kar, S.: Lorentzian wormholes in Einstein-Gauss-Bonnet theory. Phys. Rev. D 46, 2464 (1992)

Bronnikov, K.A., Galiazhmetov, A.M.: Wormholes without exotic matter in Einstein–Cartan theory. Gravit. Cosmol. 21(4), 283 (2015)

Bronnikov, K.A., Kim, S.W.: Possible wormholes in a brane world. Phys. Rev. D 67, 064027 (2003)

Capozziello, S., Harko, T., Koivisto, T.S., Lobo, F.S.N., Olmo, G.J.: Wormholes supported by hybrid metric-Palatini gravity. Phys. Rev. D 86, 127504 (2012)

Cataldo, M., Meza, P., Minning, P.: N-dimensional static and evolving Lorentzian wormholes with cosmological constant. Phys. Rev. D 83, 044050 (2011)

Cataldo, M., Liempi, L., Rodrigues, P.: Traversable Schwarzschild-like wormholes. Eur. Phys. J. C 77(11), 748 (2017)

Correa, R.A.C., Moraes, P.H.R.S.: Configurational entropy in $f(R,T)$ brane models. Eur. Phys. J. C 76(2), 100 (2016)

Debnath, P.S.: Bulk viscous cosmological model in $f(R,T)$ theory of gravity (2019). arXiv preprint arXiv:1907.02238

Dehghani, M.H., Hendi, S.H.: Wormhole solutions in Gauss-Bonnet-Born-Infeld gravity. Gen. Relativ. Gravit. 41, 1853 (2009)

Dotti, G., Oliva, J., Troncoso, R.: Static wormhole solution for higher-dimensional gravity in vacuum. Phys. Rev. D 75, 024002 (2007)

Einstein, A., Rosen, N.: The particle problem in the general theory of relativity. Phys. Rev. 48, 73 (1935)
Elizalde, E., Khurshudyan, M.: Wormhole formation in $f(R,T)$ gravity: varying Chaplygin gas and barotropic fluid. Phys. Rev. D 98, 123525 (2018)

Elizalde, E., Khurshudyan, M.: Wormholes with $\rho(R,R')$ matter in $f(R,T)$ gravity. Phys. Rev. D 99(2), 024051 (2019a)

Elizalde, E., Khurshudyan, M.: Wormhole models in $f(R,T)$ gravity. Int. J. Mod. Phys. D 28(15), 1950172 (2019b). arXiv:1909.11037

Garattini, R., Lobo, F.S.N.: Self sustained phantom wormholes in semi-classical gravity. Class. Quantum Gravity 24, 2401 (2007)

Garattini, R., Lobo, F.S.N.: Self-sustained traversable wormholes in noncommutative geometry. Phys. Lett. B 671, 146 (2009)

Garattini, R., Lobo, F.S.N.: Self-sustained wormholes in modified dispersion relations. Phys. Rev. D 85, 024043 (2012)

Garcia, N.M., Lobo, F.S.N.: Wormhole geometries supported by a nonminimal curvature-matter coupling. Phys. Rev. D 82, 104018 (2010)

Godani, N., Samanta, G.C.: Traversable wormholes and energy conditions with two different shape functions in $f(R)$ gravity. Int. J. Mod. Phys. D 28(02), 1950039 (2018)

Godani, N., Samanta, G.C.: Static traversable wormholes in $f(R,T) = R + 2\alpha \ln T$ gravity. Chin. J. Phys. 62, 161–171 (2019)

Golchin, H., Mehdizadeh, M.R.: Quasi-cosmological traversable wormholes in $f(R)$ gravity. Eur. Phys. J. C 79(9), 777 (2019)

Haghani, Z., Harko, T., Lobo, F.S.N., Sepangi, H.R., Shahidi, S.: Further matters in space-time geometry: $f(R,T,R_{\mu\nu}, T_{\mu\nu})$ gravity. Phys. Rev. D 88(4), 044023 (2013)

Harko, T.: Thermodynamic interpretation of the generalized gravity models with geometry—matter coupling. Phys. Rev. D 90(4), 044067 (2014)

Harko, T., Lobo, F.S.N.: $f(R,L_m)$ gravity. Eur. Phys. J. C 70, 373 (2010)

Harko, T., Lobo, F.S.N.: Generalized curvature-matter couplings in modified gravity. Galaxies 3(3), 410 (2014)

Harko, T., Kovacs, Z., Lobo, F.S.N.: Thin accretion disks in stationary axisymmetric wormhole spacetimes. Phys. Rev. D 79, 064001 (2009)

Harko, T., Lobo, F.S.N., Nojiri, S., Odintsov, S.D.: $f(R,T)$ gravity. Phys. Rev. D 84, 024020 (2011). arXiv:1104.2669 [gr-qc]

Harko, T., Lobo, F.S.N., Mak, M.K., Sushkov, S.V.: Modified-gravity wormholes without exotic matter. Phys. Rev. D 87(6), 067504 (2013)

Jawad, A., Rani, S.: Non-minimal coupling of torsion—metry satisfying null energy condition for wormhole solutions. Eur. Phys. J. C 76(12), 704 (2016)

Jordan, P.: The present state of Dirac’s cosmological hypothesis. Z. Phys. 157, 112 (1959)

Kuhfittig, P.K.F.: Gravitational lensing of wormholes in the galactic halo region. Eur. Phys. J. C 74(99), 2818 (2014)

La Camera, M.: Wormhole solutions in the Randall-Sundrum scenario. Phys. Lett. B 573, 27 (2003)

Landau, L.D., Lifshitz, E.M.: The Classical Theory of Fields. Pergamon, Oxford (1998)

Lemos, J.P.S., Lobo, F.S.N., Quinet de Oliveira, S.: Morris-Thorne wormholes with a cosmological constant. Phys. Rev. D 68, 064004 (2003)

Li, Z., Bambi, C.: Distinguishing black holes and wormholes with orbiting hot spots. Phys. Rev. D 90, 024071 (2014)

Lobo, F.S.N.: A general class of braneworld wormholes. Phys. Rev. D 75, 064027 (2007)

Lobo, F.S.N.: Exotic solutions in general relativity: traversable wormholes and ‘worm drive’ spacetimes. In: Classical and Quantum Gravity Research, pp. 1–78. Nova Science Publishers, New York (2008a). ISBN 978-1-60456-366-5. arXiv:0710.4474 [gr-qc]

Lobo, F.S.N.: General class of wormhole geometries in conformal Weyl gravity. Class. Quantum Gravity 25, 175006 (2008b)

Lobo, F.S.N. (ed.): Wormholes, Warp Drives and Energy Conditions. Fundam. Theor. Phys., vol. 189. Springer, Berlin (2017)

Lobo, F.S.N., Oliveira, M.A.: Wormhole geometries in $f(R)$ modified theories of gravity. Phys. Rev. D 80, 104012 (2009)

Lobo, F.S.N., Oliveira, M.A.: General class of vacuum Brans-Dicke wormholes. Phys. Rev. D 81, 067501 (2010)

Mandal, S., Sahoo, P., Sahoo, P.K.: Wormhole model with a hybrid shape function in $f(R,T)$ gravity (2019). arXiv:1911.13247 [gr-qc]

Mehdizadeh, M.R., Ziaie, A.H.: Einstein-Cartan wormhole solutions. Phys. Rev. D 95(6), 064049 (2017a)

Mehdizadeh, M.R., Ziaie, A.H.: Dynamic wormhole solutions in Einstein-Cartan gravity. Phys. Rev. D 96(12), 124017 (2017b)

Mehdizadeh, M.R., Kord Zangeneh, M., Lobo, F.S.N.: Einstein-Gauss-Bonnet traversable wormholes satisfying the weak energy condition. Phys. Rev. D 91(8), 084004 (2015)

Momeni, D., Moraes, P.H.R.S., Myrzakulov, R.: Generalized second law of thermodynamics in $f(R,T)$ theory of gravity. Astrophys. Space Sci. 361(7), 228 (2016)

Montelongo Garcia, N., Lobo, F.S.N.: Nonminimal curvature-matter coupled wormholes with matter satisfying the null energy condition. Class. Quantum Gravity 28, 085018 (2011)

Moradpour, H., Jahromi, A.S.: Static traversable wormholes in Lyra manifold. Int. J. Mod. Phys. D 27(03), 1850024 (2017)

Moraes, P.H.R.S.: Cosmological solutions from induced matter model applied to $5D f(R,T)$ gravity and the shrinking of the extra coordinate. Eur. Phys. J. C 75(4), 168 (2015)

Moraes, P.H.R.S., Correa, R.A.C.: Braneworld cosmology in $f(R,T)$ gravity. Astrophys. Space Sci. 361(3), 91 (2016)

Moraes, P.H.R.S., Sahoo, P.K.: Modeling wormholes in $f(R,T)$ gravity. Phys. Rev. D 96(4), 044038 (2017)

Moraes, P.H.R.S., Sahoo, P.K.: Wormholes in exponential $f(R,T)$ gravity (2019). arXiv preprint arXiv:1903.03421

Moraes, P.H.R.S., Santos, J.R.L.: A complete cosmological scenario from $f(R,T^a)$ gravity theory. Eur. Phys. J. C 76, 60 (2016)

Moraes, P.H.R.S., Arbáñil, J.D.V., Malheiro, M.: Stellar equilibrium configurations of compact stars in $f(R,T)$ gravity. J. Cosmol. Astropart. Phys. 1606, 005 (2016a)

Moraes, P.H.R.S., Ribeiro, G., Correa, R.A.C.: A transition from a decelerated to an accelerated phase of the universe expansion from the simplest non-trivial polynomial function of $T$ in the $f(R,T)$ formalism. Astrophys. Space Sci. 361(7), 227 (2016b)

Moraes, P.H.R.S., Correa, R.A.C., Lobato, R.V.: Analytical general solutions for static wormholes in $f(R,T)$ gravity. J. Cosmol. Astropart. Phys. 2017, 029 (2017)

Moraes, P.H.R.S., Correa, R.A.C., Ribeiro, G.: Evading the non-continuity equation in the $f(R,T)$ cosmology. Eur. Phys. J. C 78(3), 192 (2018)

Moraes, P.H.R.S., de Paula, W., Correa, R.A.C.: Charged wormholes in $f(R,T)$ extended theory of gravity. Int. J. Mod. Phys. D 28(08), 1950098 (2019a)

Moraes, P.H.R.S., Sahoo, P.K., Kulkarni, S.S., Agarwal, S.: An exponential shape function for wormholes in modified gravity. Chin. Phys. Lett. 36, 120401 (2019b)

Morris, M.S., Thorne, K.S.: Wormholes in space-time and their use for interstellar travel: a tool for teaching general relativity. Am. J. Phys. 56, 395 (1988)

Morris, M.S., Thorne, K.S., Yurtsever, U.: Wormholes, time machines, and the weak energy condition. Phys. Rev. Lett. 61(1446), 1446 (1988)

Myrzakulov, R.: FRW cosmology in $f(R,T)$ gravity. Eur. Phys. J. C 72, 2203 (2012)

Nagpal, R., Pacif, S.K.J., Singh, J.K., Bamba, K., Beesham, A.: Analysis with observational constraints in $\Lambda$-cosmology in $f(R,T)$ gravity. Eur. Phys. J. C 78(11), 946 (2018)
Traversable wormholes in $f(R,T)$ gravity.

Nandi, K.K., Bhattacharjee, B., Alam, S.M.K., Evans, J.: Traversable wormholes in the Jordan and Einstein frames. Phys. Rev. D 57, 823 (1998)

Nandi, K.K., Zhang, Y.Z., Zakharov, A.V.: Gravitational lensing by wormholes. Phys. Rev. D 74, 024020 (2006)

Nojiri, S., Odintsov, S.D.: Gravity assisted dark energy dominance and cosmic acceleration. Phys. Lett. B 599, 137 (2004)

Nojiri, S., Odintsov, S.D.: Introduction to modified gravity and gravitational alternative for dark energy. Int. J. Geom. Methods Mod. Phys. 4, 115 (2007)

Nojiri, S., Obregon, O., Odintsov, S.D., Osetrin, K.E.: Can primordial wormholes be induced by GUTs at the early universe? Phys. Lett. B 458, 19 (1999)

Nojiri, S., Odintsov, S.D., Sami, M.: Dark energy cosmology from higher-order, string-inspired gravity and its reconstruction. Phys. Rev. D 74, 046004 (2006)

Nojiri, S., Odintsov, S.D., Tretyakov, P.V.: From inflation to dark energy in the non-minimal modified gravity. Prog. Theor. Phys. Suppl. 172, 81 (2008)

Noureen, I., Zubair, M.: Dynamical instability and expansion-free condition in $f(R,T)$ gravity. Eur. Phys. J. C 75(99), 62 (2015)

Noureen, I., Zubair, M., Bhatti, A.A., Abbas, G.: Shear-free condition and dynamical instability in $f(R,T)$ gravity. Eur. Phys. J. C 75(7), 323 (2015)

Pan, S., Chakraborty, S.: Dynamic wormholes with particle creation mechanism. Eur. Phys. J. C 75(1), 21 (2015)

Rahaman, F., Kalam, M., Sarker, M., Ghosh, A., Raychaudhuri, B.: Wormhole with varying cosmological constant. Gen. Relativ. Gravit. 39, 145 (2007)

Rahaman, F., Kuhfittig, P.K.F., Ray, S., Islam, N.: Possible existence of wormholes in the galactic halo region. Eur. Phys. J. C 74, 2750 (2014)

Romero, J.M., Bellini, M.: Traversable wormhole magnetic monopoles from Dymnikova metric. Eur. Phys. J. Plus 134(11), 579 (2019)

Rosa, J.L., Lemos, J.P.S., Lobo, F.S.N.: Wormholes in generalized hybrid metric-Palatini gravity obeying the matter null energy condition everywhere. Phys. Rev. D 98(6), 064054 (2018)

Saaidi, K., Nazavari, N.: Traversable wormhole solutions in Rastall teleparallel gravity. Phys. Dark Universe 28, 100464 (2020)

Sahoo, P., Kirchner, A., Sahoo, P.K.: Phantom fluid wormhole in $f(R,T)$ gravity (2019). arXiv preprint arXiv:1906.04048

Sahu, S.K., Ganebo, S.G., Wemdemariam, G.G.: Kaluza-Klein tilted cosmological model in Lyra geometry. Iran. J. Sci. Technol. A 42(3), 1451 (2018)

Samanta, G.C., Godani, N., Bamba, K.: Traversable Wormholes with Exponential Shape Function in Modified Gravity and in General Relativity: A Comparative Study (2018). arXiv:1811.06834v1 [gr-qc]

Shamir, M.F.: Locally rotationally symmetric Bianchi type I cosmology in $f(R,T)$ gravity. Eur. Phys. J. C 75(8), 354 (2015)

Sharif, M., Nawazish, I.: Viable wormhole solutions and Noether symmetry in $f(R,T)$ gravity. Ann. Phys. 400, 37 (2019)

Sharif, M., Zubair, M.: Energy conditions constraints and stability of power law solutions in $f(R,T)$ gravity. J. Phys. Soc. Jpn. 82, 014002 (2013)

Sharma, U.K., Pradhan, A.: Cosmology in modified $f(R,T)$-gravity theory in a variant $\Lambda(T)$ scenario-revisited. Int. J. Geom. Methods Mod. Phys. 15(01), 1850014 (2017)

Sharma, U.K., Zia, R., Pradhan, A., Beesham, A.: Stability of LRS Bianchi type-I cosmological models in $f(R,T)$-gravity. Res. Astron. Astrophys. 19(4), 055 (2019)

Shinkai, H.A., Hayward, S.A.: Fate of the first traversible wormhole: black hole collapse or inflationary expansion. Phys. Rev. D 66, 044005 (2002)

Singh, K.N., Banerjee, A., Rahaman, F., Jasim, M.K.: Conformally symmetric traversable wormholes in modified teleparallel gravity (2020). arXiv:2001.00816 [gr-qc]

Starobinsky, A.A.: A new type of isotropic cosmological models without singularity. Phys. Lett. B 91, 99 (1980)

Tsukamoto, N.: Strong deflection limit analysis and gravitational lensing of an Ellis wormhole. Phys. Rev. D 94(12), 124001 (2016)

Tsukamoto, N.: Retrolensing by a wormhole at deflection angles $\pi$ and $3\pi$. Phys. Rev. D 95(8), 084021 (2017)

Visser, M.: Lorentzian Wormholes: From Einstein to Hawking. Springer, New York (1996)

Wheeler, J.A.: Geometrodynamics. Academic Press, San Diego (1962)

Yousaf, Z., Ilyas, M., Zaeem-ul-Haq Bhatti, M.: Static spherical wormhole models in $f(R,T)$ gravity. Eur. Phys. J. Plus 132(6), 268 (2017)

Zhou, M., Cardenas-Avendano, A., Bambi, C., Kleihaus, B., Kunz, J.: Search for astrophysical rotating Ellis wormholes with X-ray reflection spectroscopy. Phys. Rev. D 94(2), 024036 (2016)

Zubair, M., Noureen, I.: Evolution of axially symmetric anisotropic sources in $f(R,T)$ gravity. Eur. Phys. J. C 75(6), 265 (2015)

Zubair, M., Waheed, S., Ahmad, Y.: Static spherically symmetric wormholes in $f(R,T)$ gravity. Eur. Phys. J. C 76(8), 444 (2016a)

Zubair, M., Waheed, S., Ahmad, Y.: Static spherically symmetric wormholes in $f(R,T)$ gravity. Eur. Phys. J. C 76(8), 444 (2016b)

Zubair, M., Abbas, G., Noureen, I.: Possible formation of compact stars in $f(R,T)$ gravity. Astrophys. Space Sci. 361(1), 8 (2016c)

Zubair, M., Mustafa, G., Waheed, S., Abbas, G.: Existence of stable wormholes on a non-commutative-geometric background in modified gravity. Eur. Phys. J. C 77(10), 680 (2017)

Zubair, M., Saleem, R., Ahmad, Y., Abbas, G.: Exact wormholes solutions without exotic matter in $f(R,T)$ gravity. Int. J. Geom. Methods Mod. Phys. 16, 1950046 (2019)