The End for Extended Inflation?

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INTRODUCTION

The most dramatic addition to the inflationary zoo in recent years has been the extended inflation model\textsuperscript{1,2}. At a time when viable models were dominated by the notion of scalar fields slow-rolling in almost flat potentials, extended inflation reintroduced the concept of inflation occurring via a first order phase transition. In such models, inflation is driven by a scalar field which gets ‘hung up’ in a metastable vacuum state of energy density \( M^4 \), with inflation ending via the nucleation of bubbles of true vacuum, via quantum tunnelling, which can later grow and collide to reheat the universe into an isotropic Friedmann stage.

There are several reasons why this can be an interesting thing to achieve. First of all, the possibility arises that inflation can be reintroduced to realistic particle physics by identifying the inflaton field as a GUT Higgs particle or equivalent. Secondly, one has obviated the need for a flat potential (usually a requirement from a combination of sufficient inflation and small density fluctuations), and so might hope to avoid the fine-tuning problems that inflation is so commonly accused of possessing\textsuperscript{3}. And thirdly, there was the hope (which alas has not proven realisable\textsuperscript{4}), that thermalisation might involve sufficient numbers of astrophysically large bubbles as to contribute to the ‘bubbly’ structure seen in galaxy redshift surveys.

The key ingredient of extended inflation is the introduction of a means which allows the first order phase transition to complete in an inflationary universe. It has long been known that this is hard to achieve in general relativity, where Guth’s original model\textsuperscript{5} was implemented, due to a conflict between requiring a low enough bubble nucleation rate to obtain sufficient inflation and a high enough one to enable the bubbles to meet and bring inflation to an end. This is most effectively characterised by defining a nucleation parameter \( \epsilon = \Gamma / H^4 \) where \( \Gamma \) is the nucleation rate per unit volume per unit time (a constant barring very unusual and hard-to-realise field interactions\textsuperscript{6}) and \( H \) is the Hubble parameter. Hence \( \epsilon \) is the number of bubbles nucleated per Hubble volume per Hubble time. One has competing requirements

\[
\begin{align*}
\text{Sufficient inflation} & \implies \epsilon \ll 1 \\
\text{Completion of phase transition} & \implies \epsilon \approx \mathcal{O}(1)
\end{align*}
\]

In general relativity, one has the unfortunate circumstance that both \( \Gamma \) and \( H \) are constant, and so these requirements cannot both be satisfied.
What La and Steinhardt realised\textsuperscript{1} was that one requires a theory in which $H$ varies during inflation. This can be achieved by going to an ‘extended’ gravitational theory. Their original model was based on the Jordan–Brans–Dicke (JBD) theory, which gives $a \propto t^{\omega+1/2}$ and hence $\epsilon \propto t^4$. With $\epsilon$ growing with time, one can arrange an early phase in which bubble nucleation is suppressed, allowing sufficient inflation to occur before $\epsilon$ grows to unity to bring the phase transition to an end. Note that although $\epsilon$ increases quite rapidly with time, its increase with comoving scale is rather weak, especially for large $\omega$.

**Definition**: In this article, ‘extended inflation’ normally refers to all of a wide class of models which modify gravity to permit a first order phase transition to an end. Often models with dynamics mimicking the JBD theory will be discussed illustratively, and occasionally cold dark matter (CDM) will be chosen as an illustrative choice of matter content for the universe.

**CONSTRAINTS ON EXTENDED INFLATION**

Extended inflation is subject to two main constraints.

- Bubbles nucleated early in inflation are caught up in the subsequent expansion and swept up to astrophysical sizes. The bubble distribution is constrained by the isotropy of the microwave background.

- As in any inflation model, there are density perturbations generated by quantum fluctuations, in this case predominantly in the Brans–Dicke field. In extended inflation the predicted spectrum is ‘tilted’ from flat, and there are also significant long wavelength gravitational waves generated. These two perturbation spectra lead to constraints both from the microwave background and from large scale structure data.

Before proceeding onto an up-to-date account of these constraints, it is remarking on the classes of extended inflation model which exist. Soon after the original model was devised, the bubble spectrum was derived and subjected to a heuristic constraint\textsuperscript{7,8} that no more than $10^{-4}$ of the universe by volume should end up in bubbles larger than the horizon size at decoupling. Although an incredibly conservative constraint, with no particular reference to any experiment, this produced the strong limit $\omega \leq 30 + \log_{10} \frac{M}{m_{Pl}}$ where $M$ is, once again, the inflaton mass scale. Combined with present day solar system experiments\textsuperscript{9} requiring $\omega$ in excess of 500, the original model is clearly excluded.

However, model building proved fairly easy, because the conflict originates in constraints applied at wildly differing times — the bubble constraint during inflation at perhaps $10^{-30}$ sec and the solar system limit at the present time of $10^{17}$ sec. With plenty of room for manoeuvre, two strategies developed.

1. $\omega$ really is a constant less than say 25, but is concealed from present day observation by some mechanism, *eg*

   (a) A potential for $\Phi$, \textsuperscript{8}

   (b) Altered couplings to the invisible sector.\textsuperscript{10}
(c) A scale-invariant theory.\textsuperscript{11}

2. $\omega$ is not a constant,

(a) Barrow–Maeda $\omega(\Phi)$ model.\textsuperscript{12}
(b) Steinhardt–Accetta hyperextended inflation.\textsuperscript{13}

During inflation the first class act exactly as JBD theories, and produce the same void spectrum. The second class produce a spectrum that requires case-by-case examination, usually numerically.\textsuperscript{14}

The main moral of this article is that improved large scale structure and microwave background anisotropy measurements, especially the advent of COBE, have markedly changed this situation, and there are now strong constraints from structure formation which act in the opposite direction to the bubble constraints. Because the constraints act at the same time, much of the room to manoeuvre in model building has been eliminated and many models are now excluded.\textsuperscript{15}

A heuristic way to see the potential conflict is to realise that in the GR limit both bubble and perturbation spectra are scale-invariant. The same processes invoked to suppress the bubbles growth from scale-invariance will necessarily break scale-invariance of the spectra, removing short-scale power. With the COBE measurement anchoring the amplitude on large scales, there are now tight limits as to how much power can be removed on short scales\textsuperscript{15,16}. In simplistic terms, suppressing bubble growth requires that $H$ decreases as a function of time, and hence horizon-crossing scale. As the density perturbation spectrum is proportional to $H^2$, one expects perturbations leaving the horizon at later times to be smaller.

\textit{Void Constraints}

The heuristic void constraint discussed above has been much improved by Liddle and Wands\textsuperscript{17}, and extended beyond the JBD model.\textsuperscript{14} Starting from a similar expression for the spectrum at the end of inflation, they incorporate a detailed treatment of the void evolution up to decoupling, conservatively assuming the voids fill rapidly via relativistic shocks. [This analysis also depends on the choice of dark matter.] The effect of such a spectrum is then analysed in detail with regard to various experiments, with the conclusion that the all-sky coverage provided by COBE\textsuperscript{18} offers the strongest constraints.

Typically results have to be generated numerically, as in many models even the void spectrum after inflation cannot be expressed analytically. An example constraint is in the JBD theory with cold dark matter, which gives

$$\omega_{\text{CDM}} < 20 + 0.7 \log_{10} \frac{M}{m_{Pl}}$$  \hspace{1cm} (1)$$

Although the improvement in $\omega$ appears not too great, this actually corresponds to a reduction in the volume in large voids by a factor of around 100 compared to the earlier constraint. It is also worth emphasising that even this constraint was derived by neglecting many effects which would in principle generate anisotropies, conceivably larger ones than were calculated. Thus we believe that even this void constraint is very conservative, with exclusions better than 95%.
Density Perturbations and Gravitational Waves

For convenience and clarity we restrict discussion to the JBD model here. JBD extended inflation has the convenient property\textsuperscript{19,20} of being conformally equivalent to the well-investigated power-law inflation model $a \propto t^p$, where the conformal transformation yields $2p = \omega + 3/2$. This enables known results to be used directly.

The density perturbation spectrum is tilted from scale-invariance to a power-law

$$P(k) \propto k^n ; \quad n = 1 - \frac{2}{p-1} = \frac{2\omega - 9}{2\omega - 1}$$

(2)

Power-law inflation generates substantial gravitational waves\textsuperscript{21,22} (also with a power-law spectrum), and their relative contribution $R$ to large angle microwave background anisotropies is independent of multipole and given by\textsuperscript{23,15}

$$R = \frac{\Sigma_l^2(\text{grav})}{\Sigma_l^2(\text{scalar})} = \frac{12.4}{p} = \frac{50}{2\omega + 3}$$

(3)

where $\Sigma_l^2$ is the expectation of the square of the $l$-th multipole. For the allowed $\omega$ values, gravitational waves are the dominant contributors.

We obtain constraints by utilising the COBE 10\textsuperscript{0} result\textsuperscript{17} with error bars doubled to give something like a 2-sigma result: $\sigma_{10}\textsuperscript{2} = (1.1 \pm 0.4) \times 10^{-5}$. We include the gravitational wave contribution. The spectral slopes and amplitudes are uniquely determined by the parameters $M$ and $\omega$.

**COMBINED CONSTRAINTS**

The constraints on the inflation parameters $M$ and $\omega$ are plotted in figure 1. All lines are 95% exclusions or better. The microwave anisotropy line applies regardless of the choice of dark matter. The void constraint line does depend on this choice as it governs the efficiency of void filling, and we plot both cold and hot dark matter results to indicate the spread. One immediately sees that regardless of $M$, $\omega$ is constrained to be less than about 17, and hence $n$ can be no larger than 0.76. The inflaton mass scale is also strongly constrained.

Can such a tilt in the spectrum, coupled with the dominant gravitational contribution to the COBE result, be compatible with small-scale measurements? In these models the answer is a clear no. Figure 2 considers the specific choice of standard CDM, and plots constraints on the amplitude and tilt of the perturbation spectrum. The COBE range implies a very low small-scale amplitude ($\sigma_{8,\text{CDM}}$ is the variance of the mass in $8h^{-1}$ Mpc spheres, often indicated as the inverse of the bias parameter) at small $n$. One can compare this with several experiments. As an example, we choose constraints on the amplitude from the QDOT survey\textsuperscript{24}, following a procedure of Efstathiou, Bond and White\textsuperscript{25}, which indicate quite a high short-scale amplitude in order to explain peculiar velocities. Recent results\textsuperscript{26} from comparison of POTENT with IRAS are similar. One can see immediately that the region $n < 0.84$ is excluded. In fact, things are probably much worse even than that, because more recent microwave results\textsuperscript{27} make it extremely unlikely that the true
value is towards the top of the COBE range. And one can see that if the central COBE value is instead taken as a limit, the constraint tightens by a considerable amount.

With an admixture of hot dark matter or a choice \( \Omega < 1 \) (with a cosmological constant to retain spatial flatness) things again get worse, because yet more short-scale power is removed (remember our normalisation to COBE is dark matter independent). The slight weakening of the void constraint is unable to compensate for this worsening state of affairs. One can thus say that any model of extended inflation which shares the dynamics of the JBD theory during inflation is convincingly ruled out.

**CONCLUSIONS**

- The combination of big bubble and perturbation constraints is severe.
- The suppression of big bubbles is inextricably linked to the breaking of scale-invariance of the perturbation spectra.
- Models sharing the JBD dynamics are excluded. In addition it is easy to show the Barrow–Maeda \( \omega(\Phi) \) model, which has a more stringent bubble constraint than the JBD model, is also excluded.

To our knowledge, this rules out all extended inflation models bar two.

1. The Steinhardt–Accetta hyperextended inflation model, utilising a means of ending inflation dubbed 'mode 2' by Crittenden and Steinhardt. In this model, inflation ends not by bubble collisions but instead by the complex dynamics ending inflation while the bulk of the universe is still hung up in false vacuum. The phase transition later completes in the post-inflationary universe. There are no significant large bubbles and hence no bubble constraint. However, obtaining working models is tricky and probably requires the introduction of a mass for the gravitational scalar.

2. The 'plausible' double inflation model. This relies on a subsequent phase of slow-roll inflation following the original first-order inflation, in order to erase some of the effects of the bubbles. This can weaken the constraints sufficiently to allow a working model.

It is well worth remarking that neither of these models end inflation at the bubble collision phase. It is fair to say then that there appears no known working model in which inflation ends via bubble collisions.

To end, we should mention the escape clause. While the bubble constraints always apply, the perturbation constraints assume that the inflationary perturbations are responsible for large scale structure. If one can suppress these further and invoke another means of forming structure, then they can be evaded. One such option would be to form cosmic strings after or near the end of inflation. However, the constraint on \( M \) implies a reheat temperature of no more than a few times \( 10^{14} \) GeV. Perhaps this can be achieved with global strings.
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