Investigation on large responses of the streamwise velocity with the effect of spanwise Lorentz force

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Abstract: The flow of a weakly conductive fluid (i.e. seawater) can be controlled by Lorentz forces generated by the suitably chosen magnetic and electric fields, which has significant effects for applications in the drag reduction and oscillatory suppression. However, the control efficiency is very low due to the application of large amplitude for Lorentz force. Therefore, the large response, induced by a small Lorentz force, is the key to enhance the flow control efficiency. In this paper, the amplification mechanism of the velocity response with the effect of Lorentz force in weakly conductive fluids is investigated, where the Lorentz force is applied on the lower wall in the channel. The analytic solutions of the velocity responses in linear stage are derived with linear stability theory, when the amplitude of Lorentz force is far less than 1. From the discussions of the analytic solutions, the mechanism is revealed on large responses which is induced by the small Lorentz force in the flow field. The results show that the flow along the spanwise direction is induced by Lorentz force, which leads to the momentum exchange of fluids in the wall-normal direction. Therefore, the large responses of velocity are generated due to the high-speed fluids transferring to the wall. Moreover, the responses depend on the parameters of Lorentz force, i.e. wave number $K_z$, effective penetration $\Delta$, including proportional to amplitude $A$ and square of Reynolds number $Re^2$. With the increase of $K_z$, the response decreases monotonously. However, with the increase of $\Delta$, the response increases first ($\Delta<0.4$), reaches the maximum at $\Delta=0.4$, and then decreases ($\Delta>0.4$). Finally, the maximum response is obtained and the corresponding amplification is 480, when the two parameters are optimized, i.e. $K_z=2, \Delta=0.4$.

1. Introduction

The viscous flow passing a moving body may induce deceleration, vibration and instability
which is usually undesired, which can be suppressed by active flow control. Therefore, active flow control has a wide applied prospect and great practical value in the drag reduction and vibration suppression to improve the performance of planes, ships, missiles and rockets[1-4]. However, these active approaches consume much energy, which is even an order of magnitude larger than the energy saved from drag reduction and vibration suppression. It is important to improve efficiencies for applications of active flow control.

As one of the active flow control methods, flow control by Lorentz force has many advantages, such as fast responses, great control effects, flexibility and so on. It is widely used in the laminar flow control and the turbulence flow control[5,6]. As early as the 1960s, Lorentz force, which can be generated with electromagnetic actuators, was used to control flow by Gailitis[7]. With the development of Lorentz force control, more and more researchers focused on its efficiency. Berger[8] investigated a channel turbulent flow with Lorentz force control at low Reynolds number and discussed the control efficiency of Lorentz force for drag reduction. The control efficiency was defined as the ratio of the energy saved for drag reduction to the energy used for Lorentz force. It was found that skin-friction drag can be reduced approximately 40% with the application of Lorentz force. However, the energy used was an order of magnitude larger than that saved. The control efficiency decreased further with the increase of Reynolds number. In similar studies, O’Sullivan[9] found that it was difficult to have significant effects with a small Lorentz force in a flow field. With the increase of the Lorentz force, the energy used was always larger than that saved. Shatrov[10] numerically investigated the drag reduction in a turbulent channel flow with time-dependent and constant Lorentz forces, respectively. The results indicated that the energy consumed was larger than the energy saved for all the cases. Gad-el-Hak[11] investigated a closed-loop feedback control for the similar studies, and found that the control efficiency was not still improved obviously. In addition, Rogers[12] and Sankar[13] studied the efficiencies of the lift increase and drag reduction based on the flow past hydrofoil with the application of Lorentz force and analyzed the control processes and efficiencies under different conditions. Chen[14] also investigated the flow past hydrofoil with the effect of Lorentz force. It was found that the control effect was improved significantly with the increase of Lorentz force whereas the control efficiency was not improved. Chen[15] numerically studied the cylinder wake with the entire and local Lorentz forces, respectively. The results show that the effects of flow control were similar with entire Lorentz force and with local Lorentz force near the separation points. However, the energy consumed was larger than the energy saved even for the local Lorentz force.

The above-mentioned studies indicate that the significant effects of flow control by Lorentz force can be obtained, whereas the control efficiency is very low. Our research group has also done many studies on the optimal flow control[5, 6]. However, most of the approaches, such as parameter optimization, location optimization, feedback control and so on, cannot improve the control efficiency significantly. The key problem is that a large response can only be induced by a large force in previous studies. Therefore, it is possible to significantly improve the efficiency of control if a large response can be induced by a small force.

In this paper, the amplification mechanism of the velocity response with the effect of Lorentz force in weakly conductive fluids is investigated, where the Lorentz force is applied on the lower wall in the channel. The analytic solutions of the responses in linear stage are derived with linear stability theory, when the amplitude of Lorentz force is far less than 1. From the
discussions of the analytic solutions, the mechanism is revealed on large responses which is induced by the small Lorentz force in the flow field. Moreover, the variations of responses with the different parameters are also discussed.

2. Governing equations

The laminar flow of a weakly conductive fluid with the effect of Lorentz force, which is only employed on the lower wall of the channel, is shown in figure 1. The original point of rectangular coordinate system is located on the lower wall, where \( x, y \) and \( z \) represent the streamwise, normal and spanwise directions, respectively. The size of the channel domain in these three directions is \( L_x \times L_y \times L_z = (4\pi / 3) \times 2 \times (2\pi / 3) \).

![Figure 1. Diagrammatic sketch of channel flow.](image)

The incompressible three-dimension (3D) channel flow can be described with the non-dimensional Navier-Stokes equations with Lorentz force imposed as a source term, which are written as

\[
\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{\text{Re}} \nabla^2 \mathbf{u} + \mathbf{f},
\]

\[
\nabla \cdot \mathbf{u} = 0,
\]

(1)

(2)

In equations (1) and (2), all variables are nondimensionalized with respect to the center line velocity \( \mathbf{u} \) and the channel half width \( u \) [6]. \( \mathbf{u} \) is the velocity vector, \( \mathbf{f} \) is the pressure, \( \text{f} \) is Reynolds number, where \( \text{f} \) is the kinematic viscosity. The source term \( \mathbf{f} \) is the Lorentz force, which can be described as the following

\[
\mathbf{f} = (0,0,f_z),
\]

(3)

where

\[
f_z = Ae^{-y/\Delta} \cos(kz),
\]

(4)

Here, \( f_z \) represents the Lorentz force along the spanwise direction, \( A \) is the dimensionless amplitude, \( \Delta \) is the effective penetration, and \( k = \frac{2\pi}{L_z} \cdot K_z = 3K_z \), where \( K_z \) is the wave number along the spanwise direction. The distribution of Lorentz force only depends on space, i.e., cosine and exponential decay in the spanwise and normal directions, respectively.
Considering the characteristics of channel flow[6], the no-slip boundary condition is used in the normal direction of the wall, and the periodic boundary condition is used in the streamwise and spanwise directions. Moreover, the initial condition is the basic flow

\[ U_b(y) = 1 - (1 - y)^2 = 2y - y^2, \quad (0 \leq y \leq 2). \]

3. Analytic solutions in linear stage

Equations (1) and (2) in the rectangular coordinate system is written as follows

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = -\frac{\partial p}{\partial x} + \frac{1}{Re} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right),
\]

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -\frac{\partial p}{\partial y} + \frac{1}{Re} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right),
\]

\[
\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{Re} \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) + f_z,
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0,
\]

where

\[
\begin{pmatrix}
  u(y, z) \\
  v(y, z) \\
  w(y, z) \\
  p(x, y, z)
\end{pmatrix} = \begin{pmatrix}
  U_b(y) \\
  0 \\
  0 \\
  P_b(x)
\end{pmatrix} + \begin{pmatrix}
  \hat{u}(y, z) \\
  \hat{v}(y, z) \\
  \hat{w}(y, z) \\
  \hat{p}(y, z)
\end{pmatrix},
\]

\[ U_b(y) \] and \[ P_b(x) \] are the basic flow and pressure. \[ \hat{u}, \hat{v}, \hat{w} \] and \[ \hat{p} \] are the responses induced by Lorentz force \[ f_z \] when the flow field is steady.

Substituting equation (9) into equations (5)–(8), we have the equations (10)–(13) due to

\[
\frac{dP_b(x)}{dx} = \frac{1}{Re} \frac{d^2U_b(y)}{dy^2} \text{ in channel flow},
\]

\[
\hat{v} \frac{\partial U_b}{\partial y} + \hat{v} \frac{\partial \hat{u}}{\partial y} + \hat{w} \frac{\partial \hat{u}}{\partial z} = \frac{1}{Re} \left( \frac{\partial^2 \hat{u}}{\partial y^2} + \frac{\partial^2 \hat{u}}{\partial z^2} \right),
\]

\[
\hat{v} \frac{\partial \hat{v}}{\partial y} + \hat{w} \frac{\partial \hat{v}}{\partial z} = -\frac{\partial \hat{p}}{\partial y} + \frac{1}{Re} \left( \frac{\partial^2 \hat{v}}{\partial y^2} + \frac{\partial^2 \hat{v}}{\partial z^2} \right),
\]

\[
\hat{v} \frac{\partial \hat{w}}{\partial y} + \hat{w} \frac{\partial \hat{w}}{\partial z} = -\frac{\partial \hat{p}}{\partial z} + \frac{1}{Re} \left( \frac{\partial^2 \hat{w}}{\partial y^2} + \frac{\partial^2 \hat{w}}{\partial z^2} \right) + f_z,
\]
\[
\frac{\partial \tilde{v}}{\partial y} + \frac{\partial \tilde{w}}{\partial z} = 0 , \quad (13)
\]

For the amplitude of Lorentz force \(0 < A \ll 1\), the values of \(\tilde{u}, \tilde{v}, \tilde{w}\) and \(\tilde{p}\) are very small. Therefore, equations (14)–(17) can be obtained with linear stability theory[16]

\[
\tilde{v} \frac{\partial U_y}{\partial y} = \frac{1}{\text{Re}} \left( \frac{\partial^2 \tilde{u}}{\partial y^2} - \frac{\partial^3 \tilde{u}}{\partial z^2} \right) , \quad (14)
\]

\[
\frac{1}{\text{Re}} \left( \frac{\partial^2 \tilde{v}}{\partial y^2} + \frac{\partial^3 \tilde{v}}{\partial z^2} \right) = \frac{\partial \tilde{p}}{\partial y} , \quad (15)
\]

\[
\frac{1}{\text{Re}} \left( \frac{\partial^2 \tilde{w}}{\partial y^2} + \frac{\partial^3 \tilde{w}}{\partial z^2} \right) = \frac{\partial \tilde{p}}{\partial z} - f_z , \quad (16)
\]

\[
\frac{\partial \tilde{v}}{\partial y} + \frac{\partial \tilde{w}}{\partial z} = 0 , \quad (17)
\]

The form of analytic solution can be described with equation (18) due to the periodic boundary condition in \(z\) direction in channel flow

\[
\begin{aligned}
\tilde{u}(y, z) &= \tilde{u}(y)e^{ikz} / 2 + \tilde{u}^*(y)e^{-ikz} / 2 \\
\tilde{v}(y, z) &= \tilde{v}(y)e^{ikz} / 2 + \tilde{v}^*(y)e^{-ikz} / 2 \\
\tilde{w}(y, z) &= \tilde{w}(y)e^{ikz} / 2 + \tilde{w}^*(y)e^{-ikz} / 2 \\
\tilde{p}(y, z) &= \tilde{p}(y)e^{ikz} / 2 + \tilde{p}^*(y)e^{-ikz} / 2
\end{aligned} \quad (18)
\]

The form of Lorentz force can also be written as the following

\[
f_z = Ae^{kz} \cos(\kappa z) = Ae^{kz} / 2 + Ae^{-kz} / 2 \quad , \quad (19)
\]

Substituting the first items of equations (18) and (19) into equations (14)–(17), we have

\[
\tilde{v}(y) \frac{\partial U_y}{\partial y} = \frac{1}{\text{Re}} \left( \frac{\partial^2 \tilde{u}}{\partial y^2} + k^2 \tilde{u}(y) \right) , \quad (20)
\]

\[
\frac{1}{\text{Re}} \left( \frac{\partial^2 \tilde{v}}{\partial y^2} + k^2 \tilde{v}(y) \right) = \frac{\partial \tilde{p}(y)}{\partial y} , \quad (21)
\]

\[
\frac{1}{\text{Re}} \left( \frac{\partial^2 \tilde{w}}{\partial y^2} + k^2 \tilde{w}(y) \right) = i\kappa \tilde{\nu}(y) - Ae^{2kz} \quad , \quad (22)
\]

\[
\frac{\partial \tilde{\nu}(y)}{\partial y} + i\kappa \tilde{\nu}(y) = 0 , \quad (23)
\]

Similarly, substituting the second items into equations (14)–(17), we have

\[
\tilde{v}^*(y) \frac{\partial U_y}{\partial y} = \frac{1}{\text{Re}} \left( \frac{\partial^2 \tilde{u}^*}{\partial y^2} + k^2 \tilde{u}^*(y) \right) , \quad (24)
\]
\[
\frac{1}{\text{Re}} \left( \frac{\partial^2}{\partial y^2} - k^2 \right) \psi^*(y) = \frac{\partial \hat{p}^*(y)}{\partial y}, \tag{25}
\]

\[
\frac{1}{\text{Re}} \left( \frac{\partial^2}{\partial y^2} - k^2 \right) \hat{w}^*(y) = -ik \hat{p}^*(y) - Ae^{\frac{y}{\Delta}}, \tag{26}
\]

\[
\frac{\partial \hat{v}^*(y)}{\partial y} - ik \hat{w}^*(y) = 0, \tag{27}
\]

Then, one obtains

\[
\hat{u}(y) = iA \text{Re}^2 \hat{f}(k, \Delta, y), \tag{28}
\]

\[
\hat{u}^*(y) = -iA \text{Re}^2 \hat{f}(k, \Delta, y)
\]

\[
\hat{v}(y) = \frac{iA \text{Re}}{2} \left[ \frac{\partial^2 f(k, \Delta, y)}{\partial y^2} - k^2 f(k, \Delta, y) \right] \frac{1}{(1 - y)}, \tag{29}
\]

\[
\hat{v}^*(y) = -\frac{iA \text{Re}}{2} \left[ \frac{\partial^2 f(k, \Delta, y)}{\partial y^2} - k^2 f(k, \Delta, y) \right] \frac{1}{(1 - y)}
\]

\[
\hat{w}(y) = -\frac{A \text{Re}}{2} \left[ \frac{\partial^3 f(k, \Delta, y)}{\partial y^3} - k^2 \frac{\partial f(k, \Delta, y)}{\partial y} \frac{1}{(1 - y)k} - \frac{\partial^2 f(k, \Delta, y)}{\partial y^2} - k^2 f(k, \Delta, y) \frac{1}{(1 - y)^2 k} \right], \tag{30}
\]

\[
\hat{w}^*(y) = -\frac{A \text{Re}}{2} \left[ \frac{\partial^3 f(k, \Delta, y)}{\partial y^3} - k^2 \frac{\partial f(k, \Delta, y)}{\partial y} \frac{1}{(1 - y)k} - \frac{\partial^2 f(k, \Delta, y)}{\partial y^2} - k^2 f(k, \Delta, y) \frac{1}{(1 - y)^2 k} \right]
\]
Finally, the analytic solutions of the responses in linear stage can be obtained as shown in equations (32)–(35).

\[ \ddot{u}(y,z) = -\text{ARe}^2 \ast f(k,\Delta,y) \ast \sin(kz), \]  

\[ \ddot{v}(y,z) = -\text{ARe}^2 \frac{\partial^3 f(k,\Delta,y)}{\partial y^3} \frac{-k^2 f(k,\Delta,y)}{(1-y)} \ast \sin(kz), \]  

\[ \ddot{w}(y,z) = -\text{ARe}^2 \left[ \frac{\partial^3 f(k,\Delta,y)}{\partial y^3} \frac{-k^2 f(k,\Delta,y)}{(1-y)k} \right. \right. \left. \left. - \frac{\partial^2 f(k,\Delta,y)}{\partial y^2} \frac{-k^2 f(k,\Delta,y)}{(1-y)^2 k} \right] \ast \cos(kz), \]
\[
\begin{align*}
\tilde{p}(y,z) &= \frac{A}{2} \frac{1}{k} \left\{ \frac{1}{(1-y)} \left[ \frac{\partial^3 f(k,\Delta,y)}{\partial y^3} - 2k^2 \frac{\partial^3 f(k,\Delta,y)}{\partial y^2} + k^4 \frac{\partial f(k,\Delta,y)}{\partial y} \right] \\
&\quad + \frac{6}{(1-y)^3} \left[ \frac{\partial^3 f(k,\Delta,y)}{\partial y^3} - k^2 \frac{\partial f(k,\Delta,y)}{\partial y} \right] \\
&\quad + \frac{6}{(1-y)^3} \left[ \frac{\partial^3 f(k,\Delta,y)}{\partial y^3} - 2 \frac{\partial f(k,\Delta,y)}{\partial y} \right] - 2e^{-\frac{z}{\Delta}} \right\} \sin(kz) \\
\end{align*}
\]

As we can see from equation (32), the velocity response \( \tilde{u} \) along the streamwise direction induced by the Lorentz force is proportional to \( \text{Re}^2 \) in linear stage. Therefore, it is possible to generate a large response in the flow field with the application of a small Lorentz force.

4. Results and discussions

From equation (32), the analytic solution \( \tilde{u} \) is a function with respect to the parameters \( A; \text{Re}; k(K_r); \Delta \), where \( \tilde{u} \) is proportional to \( A \) and \( \text{Re}^2 \). However, the relation between \( \tilde{u} \) and \( k(K_r); \Delta \) is complex, which is related to the distribution of Lorentz force. Therefore, the response mechanism induced by Lorentz force can be revealed from this relation.

The responses of velocities induced by Lorentz force at \( A=10^{-4}; K_r = 2; \Delta = 0.4 \) for \( \text{Re}=2000 \) are shown in figure 2, where (a), (b), (c) and (d) correspond to the distribution of Lorentz force, spanwise, normal and streamwise velocities in the \( z-y \) plane, respectively. In the figure, the background colors describe the values, where the red and blue refer to positive and negative values, respectively. The vectors show velocities \( (\tilde{w},\tilde{v})^T \) in the \( y-z \) plane. Based on equation (4), the distribution of Lorentz force is cosine in the spanwise direction and exponential decay in the normal direction as shown in figure 2(a). With the effect of the Lorentz force, the velocity \( \tilde{w} \) is induced near the lower wall as shown in figure 2(b), which has the same period and phase with the Lorentz force. Then, an upward/downward normal velocity of fluid, i.e., the velocity \( \tilde{v} \) in figure 2(c), is generated with pressure and suction effects of fluid motion along the spanwise direction. The distributions of \( \tilde{v} \) and \( \tilde{w} \) have the same period and 1/4 phase-difference. Therefore, the absolute value of \( \tilde{v} \) is large while that of \( \tilde{w} \) is small near the lower wall, and vice versa. Finally, the effect of \( \tilde{v} \) leads to momentum transfer, i.e., the high-speed fluid moving downward and the low-speed fluid moving upward, which leads to a response of streamwise velocity \( \tilde{u} \) as shown in figure 2(d). The red and blue alternate with representatives high and low speed streaks, which are near the lower wall.
From the discussion in figure 2, a response of streamwise velocity $\bar{u}$ can be induced by Lorentz force. However, the intensity of the responses is distinct with the different parameter values. The variations of the response $\text{max} \bar{u}$ with effective penetration $\Delta$ and wave number $K_z$ are shown in figure 3, where $\text{max} \bar{u}$ is defined as the maximum value of $\bar{u}(y,z)$. For different $\Delta$, each $\text{max} \bar{u}$ increases rapidly at first and then decreases slowly with the increase of $\Delta$ and the corresponding optimal value of $\Delta$ has a slight decrease with the increase of $K_z$ when $\text{max} \bar{u}$ reaches the maximum. For different $K_z$, the $\text{max} \bar{u}$ monotonously decreases with the increase of $K_z$. In particular, the maximum response reaches $4.8 \times 10^{-3}$, which means the amplification effect $\left(4.9 \times 10^{-3}\right)/\left(10^{-3}\right) = 490$ for $K_z = 2; \Delta = 0.4$. 

Figure 2. The responses of velocities induced by Lorentz force at $A = 10^{-3}; K_z = 2; \Delta = 0.4$ for Re=2000.
Figure 3. The variations of the response $\max \dot{u}$ with effective penetration $\Delta$ and wave number $K_z$.

In order to reveal the mechanism of the large responses at different values of parameters, the distributions of $\dot{u}$ induced by Lorentz force with different $K_z$ are shown in figure 4, where figure 4(a), (b), (c) and (d) correspond to the cases of $K_z = 2, 3, 4$ and $5$ in the $z$-$y$ plane, respectively. In the figure, the background colors describe the values of $\dot{u}$ and the vectors show velocities $(\dot{w}, \dot{v})$ in the $z$-$y$ plane. Two pairs of streaks are generated near the lower wall for $K_z = 2$ in figure 4(a). With the increase of $K_z$, the numbers of streaks increase, which leads to the flow passage narrowed. Therefore, the velocities $\dot{v}$ and $\dot{w}$ decrease, which means the momentum exchange suppressed. Therefore, the amplitude of $\dot{u}$ is decreased and the distribution of that moves downward.

(a). $K_z = 2$  

(b). $K_z = 3$
To compare the values of $\hat{u}$ with different $K_z$, the profiles of $\hat{u}$ for the maximum value in spanwise direction, i.e. $\max_{z} \hat{u}$, at $A=10^{-5}; \Delta = 0.4; \text{Re} = 2000$ are shown in figure 5. From this figure, the response $\max_{z} \hat{u}$ decreases dramatically with the increase of $K_z$. Particularly, the value of $\max_{z} \hat{u}$ is negative near the upper wall, which means the opposite direction with that near the lower wall. The reason is that the low-speed fluid near the upper wall is shifted downward with the effect of Lorentz force.

Figure 4. The distributions of $\hat{u}$ induced by Lorentz force in the y-z plane at $A=10^{-5}; \Delta = 0.4; \text{Re} = 2000$ for different $K_z$.

Figure 5. The profiles of $\hat{u}$ for the maximum value in spanwise direction at $A=10^{-5}; \Delta = 0.4; \text{Re} = 2000$ for different $K_z$. 
Moreover, the distributions of \( \mathbf{u} \) induced by Lorentz force for \( K_z = 2 \) with different \( \Delta \) are shown in figure 6, where figure 6(a), (b), (c) and (d) correspond to the cases of \( \Delta = 0.1, 0.4, 0.9 \) and 1.5 in the \( z-y \) plane, respectively. Similarly with figure 4, the background colors describe the values of \( \mathbf{u} \) and the vectors show velocities \( (\mathbf{w}, \mathbf{v})^T \) in the \( z-y \) plane. From figure 6(a), two pairs of streaks are generated near the lower wall for \( \Delta = 0.1 \). When \( \Delta \) is increased from 0.1 to 0.4, the numbers of streaks are the same while the locations of the streaks move upward as shown in figure 6(b). When \( \Delta \) is increased from 0.4 to 0.9, two pairs of streaks with opposite direction are generated near the upper wall, while the intensity of the streaks near the lower wall is decreased as shown in figure 6(c). With the further increase of \( \Delta \) in figure 6(d), the intensity of the streaks near the upper wall is further increased whereas those near the lower wall are decreased. Comparing the velocity vectors in figures 6(a)–6(d), it can be seen that the convergent effect of velocity vectors grows firstly and then decays with the increase of \( \Delta \), which is related to the elongate passages in the wall-normal (y) direction. Therefore, the response \( \mathbf{u} \) increases firstly and then decreases with the increase of \( \Delta \).

**Figure 6.** The distributions of \( \mathbf{u} \) induced by Lorentz force in the \( z-y \) plane at \( \Delta = 10^{-5}; K_z = 2; \text{Re} = 2000 \) for different \( \Delta \).
To compare the values of $\tilde{u}$ with different $\Delta$, the profiles of $\max_{z} \tilde{u}$ in figure 6 are shown in figure 7. With the increase of $\Delta$, the value of $\max_{z} \tilde{u}$ near the lower wall increases first ($\Delta < 0.4$), reaches the maximum at $\Delta = 0.4$, and then decreases ($\Delta > 0.4$). However, the direction of $\max_{z} \tilde{u}$ near the upper wall is opposite with that near the lower wall. Moreover, the absolute value of $\max_{z} \tilde{u}$ near the upper wall increases with the increase of $\Delta$, which is related to the convergent effect and direction of the velocity vectors as mentioned in figure 6.

**Figure 7.** The profiles of $\max_{z} \tilde{u}$ at $A = 10^{-5}; K_z = 2; \text{Re} = 2000$ for different $\Delta$.

5. Conclusion
In this paper, the amplification mechanism of the velocity response with the effect of Lorentz force in channel flow is investigated. The analytic solutions of the responses in linear stage can be obtained with linear stability theory. The results show that the flow along the spanwise direction is induced by Lorentz force, which leads to the momentum exchange of fluids in the wall-normal direction. Therefore, the large responses of velocity are generated due to the high-speed fluids transferring to the wall. Moreover, the responses depend on the parameters of Lorentz force, i.e. wave number $K_z$, effective penetration $\Delta$, including proportional to $A$ and $\text{Re}^2$. With the increase of $K_z$, the numbers of streaks increase, which leads to the flow passage narrowed and the momentum exchange suppressed. Therefore, the response $\tilde{u}$ decreases monotonously. With the increase of $\Delta$, the convergent effect of velocity vectors grows firstly and then decays. Therefore, the response $\tilde{u}$ increases first ($\Delta < 0.4$), reaches the maximum at $\Delta = 0.4$, and then decreases ($\Delta > 0.4$). Finally, the maximum response is obtained and the corresponding amplification is 480, when the two parameters are optimized, i.e. $K_z = 2; \Delta = 0.4$. 
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