Superconductivity in carbon nanotube ropes: Ginzburg-Landau approach and the role of quantum phase slips

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We derive and analyze the low-energy theory of superconductivity in carbon nanotube ropes. A rope is modeled as an array of ballistic metallic nanotubes, taking into account phonon-mediated plus Coulomb interactions, and Josephson coupling between adjacent tubes. We construct the Ginzburg-Landau action including quantum fluctuations. Quantum phase slips are shown to cause a depression of the critical temperature $T_c$ below the mean-field value, and a temperature-dependent resistance below $T_c$.

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Over the past decade, the unique mechanical, electrical, and optical properties of carbon nanotubes, including the potential for useful technological applications, have created a lot of excitement. While many of these properties are well understood by now, the experimental observation of intrinsic [1, 2] and anomalously strong proximity-induced [3] superconductivity continues to pose open questions to theoretical understanding. In this paper we analyze 1D superconductivity found in ropes of carbon nanotubes [4, 5], starting from a microscopic model of the rope in terms of an array of individual clean single-wall nanotubes (SWNTs), with attractive phonon-mediated on-tube interactions and inter-tube nearest-neighbor Josephson couplings. Then the Luttinger interaction parameter $g_c$ and the Josephson coupling $\lambda$ describing transfer of Cooper pairs are crucial microscopic parameters, which, fortunately, can be estimated rather accurately [6, 7]. The coupled-chain problem corresponding to superconducting nanotube ropes, where typically less than hundred metallic SWNTs are present [1, 3], does neither permit classical Ginzburg-Landau (GL) theory nor a standard self-consistent BCS approach, in contrast to the situation encountered in, e.g., quasi-1D organic superconductors [2]. At this time, nanotube ropes represent wires with the smallest number of transverse channels showing intrinsic superconductivity, even when compared to the amorphous MoGe wires of diameter $\approx 10$ nm studied in Ref. [10], where still several thousand channels are available. Based on a microscopic derivation of the quantum GL action, we show that quantum phase slips (QPS’s) [11, 12, 13, 14] are crucial for an understanding of experimental results [1, 3]. First, they cause a depression of the transition temperature $T_c$ below the mean-field critical temperature $T_c^0$. Furthermore, for $T < T_c$, a finite resistance $R(T)$ due to QPS’s appears, which exhibits approximate power-law scaling. Below we determine the full temperature dependence of $R(T < T_c)$ for arbitrary rope length.

We consider a rope consisting of $N$ metallic SWNTs, where disorder is assumed to be negligible [22]. The validity of modeling the rope as an array of ballistic 1D quantum wires has recently been discussed in Ref. [3]. Since the K point degeneracy is inessential here, an individual SWNT can be described as a spin-1/2 Luttinger liquid, where the combined effects of Coulomb and phonon-mediated interactions lead to an interaction parameter $g_c$ [8], where $g_c = 1$ refers to the noninteracting case and $g_c > 1$ ($g_c < 1$) signals effectively attractive (repulsive) interactions. We mention in passing that lattice commensurabilities and electron-electron backscattering can be neglected in the intrinsically doped SWNTs encountered in practice [15, 16]. In a thick rope, Coulomb interactions are expected to be largely screened off, and $g_c > 1$ due to breathing-phonon exchange [8]. The only weak screening in the thinnest ropes studied in Refs. [1, 3] is probably linked to the absence of superconductivity in these samples. To probe superconductivity in ultrathin ropes, it is necessary to externally screen Coulomb interactions. In principle, three different inter-tube coupling mechanisms should now be taken into account, namely (i) direct Coulomb interactions, (ii) Josephson coupling (Cooper pair hopping), and (iii) single-electron hopping. The last process is strongly suppressed due to the generally different chirality of adjacent tubes [17], and, in addition, for $g_c > 1$, inter-SWNT Coulomb interactions can be neglected [8]. Therefore the most relevant mechanism is Josephson coupling between adjacent SWNTs.

In the (idealized) rope crystal, (metallic) SWNTs of radius $R$ are arranged on a trigonal lattice with lattice constant $a = 2R + b$, where $b = 0.34$ nm [17]. Choosing the $x$-axis parallel to the rope, and numbering the SWNTs by $j = 1, \ldots, N$, with center at $\vec{r}_j = (y_j, z_j) = n_1 \vec{a}_1 + n_2 \vec{a}_2$, where $\vec{a}_1 = a(1, 0)$ and $\vec{a}_2 = a(1/2, \sqrt{3}/2)$ span the trigonal lattice, allowed indices $(n_1, n_2)$ corresponding to $j$ follow from the condition $|\vec{r}_j| \leq R_{\text{rope}}$. A given rope radius $R_{\text{rope}}$ then fixes the number of tubes $N$. The Josephson coupling matrix $\Lambda_{ij}$ is nonzero only for nearest-neighbor pairs $(i, j)$, where $\Lambda_{ij} = \lambda$. Only singlet pairing of electrons on the same tube is important [8], leading to the on-tube order parameter $\mathcal{O}_j(x, \tau) = \sum_{\sigma} a_{\sigma}^\dagger \psi_{\sigma,j}^\dagger \psi_{\sigma,-j}$, where $\psi_{\sigma,j}(x, \tau)$ is the electron operator for a right- or left-moving electron $(r = \pm)$ with spin $\sigma = \pm$ on the $j$th SWNT, and $0 \leq \tau < 1/T$ is imaginary time (we put $\hbar = k_B = 1$). In bosonized language [15],

$$\mathcal{O}_j = (\pi a_0)^{-1} \exp \left[ i \sqrt{2} \pi \theta_{c,j} \right] \cos \left[ \sqrt{2} \pi \varphi_{s,j} \right],$$  \hspace{1cm} (1)
where $a_0 = 0.246$ nm is the SWNT lattice spacing, $\varphi_{c/s,j}(x, \tau)$ denotes the charge/spin boson field on the $j$th SWNT, and $\theta_{c/s,j}$ is the dual field to $\varphi_{c/s,j}$. The Euclidean action is

$$S = \sum_{j=1}^{N} S_{LL}[\theta_{c,j}, \varphi_{s,j}] - \sum_{jk} A_{jk} \int dx d\tau \mathcal{O}_j \mathcal{O}_k,$$  

(2)

where the on-tube fluctuations are governed by an effective Luttinger liquid action $\mathcal{L}$, 

$$S_{LL}[\theta_{c}, \varphi_{s}] = \int dx d\tau \left\{ \frac{v_c}{2} \left[ \left( \partial_x \theta_c / v_c \right)^2 + \left( \partial_\tau \theta_c \right)^2 \right] + \frac{v_s}{2g_s} \left[ \left( \partial_x \varphi_s / v_s \right)^2 + \left( \partial_\tau \varphi_s \right)^2 \right] \right\},$$

with $v_{c/s} = v_F / g_{c/s}$ for Fermi velocity $v_F = 8 \times 10^6$ m/sec, and $g_s = 1$ due to spin $SU(2)$ invariance. Note that the model (2) and our results below apply beyond the specific system under study here, see also Refs. [9, 18].

Next we employ a Hubbard-Stratonovich transformation in order to decouple the Josephson term in Eq. (2), using the complex-valued order parameter field $\Delta_j(x, \tau)$. This allows to write the partition function as

$$Z = \int \mathcal{D} \Delta \exp \left\{ - \sum_j S_0[\Delta_j] - \int dx d\tau \sum_{jk} \Delta^*_j D_{jk} \Delta_k \right\},$$

(3)

where $\Delta^*$ is the complex conjugate field. The $N \times N$ matrix $D$ denotes the positive definite part of $\Delta^{-1}$, since order parameter modes corresponding to negative eigenvalues of $\Delta$ can never become critical. Integration over the on-tube boson field fluctuations then leads to

$$S_0[\Delta] = - \ln \int \mathcal{D} \theta_c \mathcal{D} \varphi_s e^{-S_{LL}[\theta_{c}, \varphi_{s}] - \int dx d\tau (\Delta^* \cdot C \cdot \Delta + \Delta^* \cdot C \cdot \Delta)}.$$

(4)

The expectation value of Eq. (4) can be computed as $\langle O_j \rangle = \sum_k D_{jk}(\Delta_k)$, where $\langle \Delta_j \rangle$ is averaged using the action corresponding to Eq. (4).

Assuming a spin gap, taking $T = 0$, and allowing only static homogeneous configurations $\Delta_j(x, \tau) = \Delta_0$, Eq. (4) can be evaluated explicitly [18]. Saddle-point analysis then yields a relation between the mean-field critical temperature and the $T = 0$ superconducting gap, see Ref. [18]. For general order parameter $\Delta_j(x, \tau)$ or arbitrary temperature, however, integration over the Luttinger phase fields in Eq. (4) is impossible. To make progress, it is instructive to construct the GL action [13, 20], where it is crucial to include quantum fluctuations. A systematic approach proceeds via cumulant expansion of Eq. (4) up to quartic order in the expansion parameter $|\Delta| / 2\pi T$ [20]. We stress that this expansion is carried out for the single-chain problem, and is not restricted to $N \gg 1$. Assuming slowly varying configurations $\Delta_j(x, \tau)$, gradient expansion yields the Lagrangian in Eqs. (3) and (4) as

$$L = \sum_{jk} V_{jk} \Delta^*_j \Delta_k + \sum_j \left[ W_1^{-1} - A \right] \Delta_j \Delta^*_j$$

$$+ B |\Delta_j|^4 + \frac{c_s^2 |\partial_\tau \Delta_j|^2}{2},$$

(5)

with the Mooij-Schön plasma velocity [21], $c_s = v_c \sqrt{\tilde{C} / D}$, and $V_{jk} = \sum_{\alpha} \left( W_1^{-1} - W_1^{-1} \right) (\alpha/j) (\alpha/k)$, where $W_\alpha$ denote the eigenvalues of $A$ in descending order ($\alpha = 1, \cdots, N$), with eigenvectors $|\alpha\rangle$, and the $\alpha$-summation extends over $W_\alpha > 0$ only. Furthermore, the positive coefficients $A, B, C$ are with $\gamma = v_c / v_s$ given by

$$A = \frac{\gamma g_c}{2\pi^2 v_c} (\pi a_0 T / v_c) g_c^{-1} + g_s^{-2} \tilde{A},$$

$$B = \frac{\gamma g_c}{32\pi^2 v_c} (\pi a_0 T / v_c) g_c^{-1} + 2g_s^{-6} \tilde{B},$$

$$C = \frac{\gamma g_c}{4\pi^2 v_c} (\pi a_0 T / v_c) g_c^{-1} + g_s^{-4} \tilde{C}.$$  

(6)

Putting $g_s = 1$, dimensionless $g_c$-dependent numbers are defined as

$$\tilde{C} = \int dz \frac{w^2}{f_c(z) f_s(z)},$$

(7)

where we use the notations $z = (w, u)$, $f_c(z) = |\sinh(w + iu)|^{1/g_c}$, $f_s(z) = |\sinh(\gamma w + iu)|$, and $\int dz = \int_{-\infty}^{\infty} du$. Here $\tilde{D}$ is given by Eq. (4) with $w^2 \to 1 \to w^2$, and

$$\tilde{B} = \int dz_1 dz_2 dz_3 \left[ \frac{4}{f_c(z_2) f_c(z_1)} - \frac{f_c(z_1) f_c(z_2)}{f_c(z_2) f_c(z_3) f_s(z_1) f_s(z_3) f_s(z_2)} \right] \times \left[ \frac{f_s(z_1) f_s(z_2)}{f_s(z_2) f_s(z_3) f_s(z_1) f_s(z_3) f_s(z_2)} (1 \leftrightarrow 2) + (1 \leftrightarrow 3) \right],$$

(8)

with $z_{ij} = (w_i - w_j, u_i - u_j)$. The quantity $\tilde{B}$ is evaluated using the Monte Carlo method, see also Ref. [18]. For $g_c = 1$, we first numerically reproduced the exact result $\tilde{B} = 8\pi^2 \tilde{C}$ with $\tilde{C} = 7\pi^2 (3/4)$. Numerical values can then be obtained for arbitrary $g_c$. From Eq. (5), previous GL results for the infinite 2D array of coupled 1D chains are recovered [8, 15]. In that case, the $V_{ij}$ term in Eq. (5) leads to transverse gradients because $\alpha$ corresponds to transverse momentum $k_\perp$, with $W_\alpha^{-1} - W_\alpha^{-1} \propto k_\perp^2$. Note that Eq. (5) additionally includes quantum fluctuations and allows to describe the case of arbitrary $N$.

From Eq. (5), we obtain the mean-field critical temperature

$$T^0_c = \frac{v_c}{\pi a_0} \left( \frac{\tilde{A} W_1}{2\pi^2 v_F} \right) g_c / (g_c - 1).$$

(9)
Assuming sufficiently thick ropes such that Coulomb interactions can be neglected, in concrete estimates we shall put $q_e = 1.3$ \cite{18}, with Josephson coupling $\lambda / v_F = 0.02$ \cite{18}. Numerical evaluation yields

$$\hat{A} \simeq 30.72, \quad \hat{B} \simeq 293.1, \quad \hat{C} \simeq 12.12, \quad \hat{D} \simeq 7.78. \quad (9)$$

Equation \ref{18} then predicts, e.g., $T^0_c = 2.3$ K for $N = 31$, which is slightly above reported experimental values \cite{12,18}. In what follows, we focus on temperatures below $T^0_c$. Writing $\Delta_j = |\Delta_j| \exp \{i \phi_j(x, \tau)\}$, the amplitudes $|\Delta_j|$ are finite, with a gap for fluctuations around the mean-field value. At not too low temperatures, they are found from the saddle point equation

$$\sum_j V_{ij} |\Delta_j| + (W_{1}^{-1} - A)|\Delta_j| + 2B|\Delta_j|^3 = 0, \quad (10)$$

whose numerical solution (via a Newton-Raphson root finding scheme) yields the transverse order parameter profile and $\Delta_0 = \sum_j |\Delta_j| / N$. Typical results for $\Delta_0 / 2\pi T$ are shown in Fig. \ref{18} which demonstrates that GL theory quantitatively holds down to $T \approx T^0_c/2$. In our discussion below, it is useful even down to $T = 0$. Fixing the amplitudes $|\Delta_j|$ at their mean-field values, the resulting Lagrangian governing the massless phase fluctuations (Goldstone modes) is

$$L = \sum_j \frac{\mu_j}{2 \pi} |c_s(\partial_x \phi_j)^2 + c_s^{-1}(\partial_\tau \phi_j)^2|$$

$$+ \sum_{i>j} 2V_{ij} |\Delta_i||\Delta_j| \cos(\phi_i - \phi_j), \quad (11)$$

with dimensionless quantities $\mu_j = 2\pi C |\Delta_j|^2 / c_s$. Electromagnetic potentials can then be coupled in by standard Peierls substitution rules \cite{24}, e.g., allowing to describe the Meissner effect. Furthermore, dissipative effects can be included following Ref. \cite{14}.

In the 1D situation encountered here, superconductivity can be destroyed by thermally activated or quantum phase slips \cite{13}. Following arguments similar to the ones of Ref. \cite{18}, we find that only QPS’s play a role. Numerical evaluation of Eq. \ref{18} shows that well below $T^0_c$, transverse fluctuations are heavily suppressed \cite{22}, and therefore QPS’s can be described using the action

$$S = \frac{\mu}{2 \pi} \int dx d\tau \left[ c_\sigma^{-1}(\partial_x \phi)^2 + c_\sigma(\partial_\tau \phi)^2 \right], \quad (12)$$

where $\mu = \sum_j \mu_j$ is a dimensionless rigidity. For not too low temperature, and neglecting transverse fluctuations,

$$\mu(T) = a_0 N \left[ 1 - (T/T^0_c)^{(n_c-1)/n_c} \right], \quad (13)$$

where $a_0 \simeq 4\pi \bar{\Delta}(\bar{C}\bar{D})^{1/2} / \bar{B}$, resulting in $a_0 \simeq 12.7$ for $g_e = 1.3$. Remarkably, at $T = 0$, Eq. \ref{18} coincides up to a prefactor of order one with the rigidity $\mu$ obtained from standard mean-field relations \cite{20}, $\mu = \pi^2 n_e R^2 / 2 m^* c_s = a_0 N$, where $n_e$ is the density of condensed electrons. At $T = 0$, this implies $a_0 \simeq v_F / c_s$. We conclude that the GL prediction \ref{18} for $\mu(T)$ is robust and useful even outside its validity regime.

QPS’s are topological vortex-like excitations of the superconducting phase field $\phi(x, \tau)$. For rope length $L \rightarrow \infty$ and thermal length $L_T = c_s / \pi T \rightarrow \infty$, a QPS with core at $(x_1, \tau_1)$ and winding number $n_1 = \pm 1$ (higher winding numbers are irrelevant) is $\phi(x, \tau) = n_1 \tan^{-1}((x-x_1) / c_s(\tau-\tau_1))$ \cite{22}, where the finite $L, L_T$ solution follows by conformal transformation. This form solves the equation of motion for Eq. \ref{18} with a singularity at the core, where superconducting order is locally destroyed. The local loss of condensation energy density $E_c$ (this may also contain other energy costs \cite{11}) leads to the core action $S_c = \kappa^2 E_c / c_s$, with core radius $\kappa$ as a variational parameter. The optimal value of the core radius is $\kappa = (c_s \mu / 2E_c)^{1/2}$, where $S_c \approx \mu / 2$, and $\kappa$ now serves as UV cutoff length of the field theory. To leading order in $\kappa / L, \kappa / L_T$, the hydrodynamic action \ref{18} of a vortex is $S_{\text{vd}} = \mu \ln [\min (L, L_T) / 2 \kappa] + S'(L / L_T)$, where $L / L_T$ measures the anisotropy of this finite-size 2D Kosterlitz-Thouless problem. In particular, we find $S'(L / L_T \approx L) \simeq 0.11 \mu$, while in the opposite limits, $S' \simeq 2 \mu / \pi$. Below $S'$ is taken into account as renormalization of $S_c$.

The next step is to analyze a QPS gas, where textbook analysis \cite{22} leads to the picture of an interacting Coulomb gas of charges $n_q = \pm 1$, fugacity $y = e^{-S_c}$, and total charge zero. For $\mu > \mu^* \approx 2$, QPS’s are confined into neutral pairs, quasi-long range superconductivity is present, but QPS’s cause a finite resistance below $T^0_c$ \cite{11}. For $\mu = \mu^*$, QPS proliferation leads to a Kosterlitz-Thouless transition to the normal metallic state. (Of course, here “normal” does not imply Fermi-liquid behavior.) The transition temperature $T_c$ is therefore not $T^0_c$ but follows from the condition $\mu(T_c) = \mu^*$. Equation \ref{18} then yields

$$T_c / T^0_c = (1 - \mu^* / a_0 N)^{q_c / (g_e - 1)}. \quad (14)$$
This $T_c$ depression is normally rather weak, e.g. for $N = 31$, we obtain $T_c/T^0 = 0.97$, but for small $N$, the effect can be large. Furthermore, other mechanisms not included in our model could act to effectively reduce $\alpha_0$ and hence $T_c$, e.g. disorder and heating effects [11], or the electromagnetic environment [14].

The temperature dependence of the linear resistance $R(T) = V/I$ for $T < T_c$ can be obtained by computing the voltage drop $V$ for applied current $I$. Expanding the vortex partition function up to order $y^2$ and extracting the imaginary part of the free energy $F(I)$ using the Langer approach [24], $\Gamma(\pm I) = -2i\text{Im}F(\pm I)$ can be interpreted as the rate for a phase slip by $\pm 2\pi$ [11]. The average change in phase is then $\langle \dot{\phi} \rangle = 2\pi[\Gamma(I) - \Gamma(-I)]$, which from the Josephson relation implies a voltage drop $V = \pi[\Gamma(I) - \Gamma(-I)]/e$. Using $\epsilon = \pi h I/e$, the rate $\Gamma(\epsilon)$ follows for $L, L_T \gg \kappa$ but arbitrary $L/L_T$ in the form

$$\Gamma(\epsilon) = \frac{1}{\kappa e^4} \int_{-L/2}^{L/2} dx \int_{-\infty}^{\infty} dt e^{i\epsilon t - \pi(x/c_s) + G(t-x/c_s)},$$

where $G(t) = \ln[(L_T/\kappa)\sinh(\pi T|t|)] + i(\pi/2)\text{sgn}(t)$. To evaluate the rate $\Gamma(\epsilon)$ for arbitrary $L/L_T$, we replace the boundaries for the $x$-integral by a soft exponential cutoff, switch to integration variables $t' = t - x/c_s$ and $t'' = t + x/c_s$, and use the auxiliary relation

$$e^{-c_s|t''-t'|/L} = \frac{c_s}{\pi L} \int_{-\infty}^{\infty} ds e^{-is(t''-t')}. $$

It is now straightforward to carry out the $t', t''$ time integrations, and some algebra yields the linear resistance

$$\frac{R}{R_q} = \left( \frac{\pi y \Gamma(\mu/2)}{\Gamma(\mu/2 + 1/2)} \right)^2 \frac{2\pi L}{\kappa} \left( \frac{L_T}{\kappa} \right)^{3-2\mu} \times \int_0^\infty du \frac{2\pi}{1 + u^2} \left[ \frac{\Gamma(\mu/2 + iu L_T/2L)}{\Gamma(\mu/2)} \right]^4 (15)$$

in units of the resistance quantum $R_q = \pi h/2e^2$. Equation (15) leads to good agreement with experimental data [13]; a detailed comparison will be given in Ref. [22]. For $L/L_T \gg 1$, the $u$-integral approaches unity, and hence $R \propto T^{2u-3}$, while for $L/L_T \ll 1$, dimensional scaling arguments give $R \propto T^{2u-2}$. The exponents are determined by the temperature-dependent stiffness [13]. While both power-law behaviors have been reported in Ref. [11], Eq. (15) describes the full crossover for arbitrary $L/L_T$. In Refs. [1, 2], typical lengths were $L \approx 1\mu m$, which indeed puts one into the crossover regime $L_T \approx L$.

To conclude, we have studied superconductivity in carbon nanotube ropes, starting from a model of ballistic SWNTs with attractive intra-tube interactions and inter-tube Josephson coupling. We have constructed the Ginzburg-Landau theory including quantum fluctuations. This allows for detailed predictions about the critical temperature $T_c$ and the QPS-induced resistance below $T_c$. If repulsive Coulomb interactions can be screened off efficiently, our theory suggests that superconductivity may survive down to only a few transverse channels in clean nanotube ropes. — We acknowledge useful discussions with A. Altland, H. Bouchiat, F. Essler, and A. Tsvelik. This work has been supported by the EU network DIENOW and by the SFB-TR 12 of the DFG.

[1] M. Kociak, A. Y. Kasumov, S. Gueron, B. Reulet, I. I. Khodos, Y. B. Gorbatov, V. T. Volkov, L. Vaccarini, and H. Bouchiat, Phys. Rev. Lett. 86, 2416 (2001).
[2] Z. K. Tang, L. Zhang, N. Wang, X. X. Zhang, G. H. Wen, G. D. Li, J. N. Wang, C. T. Chan, and P. Sheng, Science 292, 2462 (2001).
[3] A. Kasumov, M. Kociak, M. Ferrier, R. Deblock, S. Gueron, B. Reulet, I. Khodos, O. Stephan, and H. Bouchiat (2003), cond-mat/0307260.
[4] A. Y. Kasumov, R. Deblock, M. Kociak, B. Reulet, H. Bouchiat, I. I. Khodos, Y. B. Gorbatov, V. T. Volkov, C. Journet, and M. Burghard, Science 284, 1508 (1999).
[5] A. F. Morpurgo, J. Kong, C. M. Marcus, and H. Dai, Science 286, 263 (1999).
[6] J. González, Phys. Rev. Lett. 88, 076403 (2002).
[7] J. González, Phys. Rev. B 67, 014528 (2003).
[8] A. De Martino and R. Egger, Phys. Rev. B 67, 235418 (2003).
[9] H. J. Schulz and C. Bourbonnais, Phys. Rev. B 27, 5856 (1983).
[10] C. N. Lau, N. Markovic, M. Bockrath, A. Bezryadin, and M. Tinkham, Phys. Rev. Lett. 87, 217003 (2001).
[11] A. D. Zaikin, D. S. Golubev, A. van Otterlo, and G. T. Zimanyi, Phys. Rev. Lett. 78, 1552 (1997).
[12] D. S. Golubev and A. D. Zaikin, Phys. Rev. B 64, 014504 (2001).
[13] M. Tinkham, Introduction to Superconductivity, 2nd Edition (McGraw-Hill, Inc., 1996).
[14] H. P. Büchner, V. B. Geshkenbein, and G. Blatter (2003), cond-mat/0306617.
[15] R. Egger and A. O. Gogolin, Phys. Rev. Lett. 79, 5082 (1997).
[16] C. Kane, L. Balents, and M. P. A. Fisher, Phys. Rev. Lett. 79, 5086 (1997).
[17] A. A. Maarouf, C. L. Kane, and E. J. Mele, Phys. Rev. B 61, 11156 (2000).
[18] S. T. Carr and A. M. Tsvelik, Phys. Rev. B 65, 195121 (2002).
[19] S. Lukyanov and A. B. Zamolodchikov, Nucl. Phys. B 493, 571 (1997).
[20] N. Nagaosa, Quantum Field Theory in Condensed Matter Physics (Springer Verlag, 1999).
[21] J. E. Mooij and G. Schön, Phys. Rev. Lett. 55, 114 (1985).
[22] A. De Martino and R. Egger (2003), in preparation.

[23] P. M. Chaikin and T. Lubensky, Principles of Condensed Matter Physics (Cambridge University Press, 2000).
[24] J. S. Langer, Ann. Phys. (N.Y.) 41, 108 (1967).
[25] Since about 2/3 of all SWNTs are semiconducting, the rope contains $\approx 3N$ SWNTs. In our estimates, to take this into account, below we consider a reduced Josephson coupling.