The $\pi^2$ terms in the $s$–channel QCD observables

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Abstract

We analyze the effect of $\pi^2$–terms in the QCD perturbative expansions for the $s$–channel effective coupling and observables, the effect known from the 80s. We remind that these terms can be collected into specific functions — strong $s$–channel coupling $\tilde{\alpha}(s)$ and its effective powers $A_k(s)$ free of ghost singularities. Further on, we study the structure of perturbation theory for observables and its reformulation in terms of nonpower perturbation expansion over the set $\{A_k(s)\}$.

Then we discuss the influence of this effect on the numerical values of $\bar{\alpha}_s$ as extracted from experiments. The main result is that the common two-loop (NLO, NLLA) approximation widely used in the five-quark ($10 \text{ GeV} \lesssim \sqrt{s} \lesssim 170 \text{ GeV}$) region for a shape analysis contains a systematic negative error of a 1–2 per cent order of magnitude for the extracted $\bar{\alpha}_s^{(2)}$. Our physical conclusion is that the $\bar{\alpha}_s(M_Z^2)$ value averaged over the $f=5$ data

$$< \bar{\alpha}_s(M_Z^2) >_{f=5} \simeq 0.124$$

appreciably differs from the currently accepted “world average” ($= 0.118$).

1 Preamble

Usually, physical quantities in the time-like channel, like the cross-section ratio of the inclusive $e^+e^- \rightarrow$ hadron annihilation or the $\tau$–decay process, are presented in the form of two- or three-term perturbation expansion

$$\frac{R(s)}{R_0} = 1 + r(s); \quad r(s) = c_1 \bar{\alpha}_s(s) + c_2 \bar{\alpha}_s^2 + c_3 \bar{\alpha}_s^3 + \ldots$$

(our coefficients $c_k = C_k \pi^{-k}$ are normalized differently from the commonly adopted, like in Refs. [1, 2, 3]) over powers of effective QCD coupling $\bar{\alpha}_s$ which is supposed ad hoc to be of the same form as in the Euclidean domain, e.g.,

$$\bar{\alpha}_s^{(3)}(s) = \frac{1}{\beta_0 L} - \frac{b_1 \ln L}{\beta_0^2 L^2} + \frac{1}{\beta_0^3 L^3} \left[ b_1^2 \left( \ln^2 L - \ln L - 1 \right) + b_2 \right] + \frac{1}{\beta_0^4 L^4} \left[ b_1^3 \left( - \ln^3 L + \frac{5}{2} \ln^2 L + 2 \ln L - \frac{1}{2} \right) - 3 b_1 b_2 \ln L + \frac{b_3}{2} \right].$$

Here, $L = \ln(s/\Lambda^2)$ and for the beta-function we use normalization

$$\beta(\alpha) = -\beta_0 \alpha^2 - \beta_1 \alpha^3 - \beta_2 \alpha^4 + \ldots = -\beta_0 \alpha^2 \left( 1 + b_1 \alpha + b_2 \alpha^2 + \ldots \right),$$
that is also free of $\pi$ powers. Numerically,
\[ \beta_0(f) = \frac{33 - 2f}{12\pi}; \quad b_1(f) = \frac{153 - 19f}{2\pi(33 - 2f)}; \quad b_1(4 \pm 1) = 0.490_{-0.089}^{+0.076}. \]

Coefficients $c_{k \geq 3} = d_k - \delta_k$ include "$\pi^2$ structures" $\delta_k$ proportional to lower $c_k$:
\[ \delta_3 = \frac{(\pi\beta_0(f))^2}{3} c_1, \quad \delta_4 = (\pi\beta_0)^2 (c_2 + \frac{5}{6} b_1 c_1); \quad \pi^2 \beta_0^2(4 \pm 1) = 4.340_{-0.666}^{+0.723}. \quad (2) \]

These structures $\delta_k$ arise\[4, 5, 6, 7\] in the course of analytic continuation from the Euclidean to Minkowskian region. Coefficients $d_k$ should be treated as a genuine $k$th–order ones. Just they have to be calculated with the help of relevant Feynman diagrams.

To illustrate, consider the three–flavor case for $\tau$–decay, $f = 4, 5$ cases for $e^+e^- \to$ hadron annihilation and $Z_0$ decay (with $f = 5$) — see Table 1 in which we also give values for the $\pi^2$–terms.

| Process | $f$ | $c_1$ | $c_2 = d_2$ | $c_3$ | $d_3 = c_3 - \delta_3$ | $\delta_3$ | $\delta_4$ |
|---------|-----|-------|-------------|-------|-----------------------|-----------|-----------|
| $\tau$ decay | 3   | $1/\pi$ | .526    | .852  | 1.389                | 0.537     | 5.01      |
| $e^+e^-$ | 4   | .318  | .155    | -0.351| 0.111                | 0.462     | 2.451     |
| $e^+e^-$ | 5   | .318  | .143    | -0.413| -0.023               | 0.390     | 1.752     |
| $Z_0$ decay | 5   | .318  | .095    | -0.483| -0.094               | 0.390     | 1.576     |

Here, all coefficients $c_k$, $d_k$ and $\delta_k$, due to normalization (1), are of an order of unity. One can see that, in the high energy region, contribution of $\delta_3$ prevails in $c_3$.

2 Preliminary quantitative estimate

In practice, the $\pi^2$–terms often dominate in higher expansion coefficients. This effect is especially strong in the $f = 5$ region. Meanwhile, just in this region people often use the so-called NLLA approximation, that is the two-term representation
\[ O(s) = C_1(\bar{\alpha}_s/\pi) + C_2(\bar{\alpha}_s/\pi)^2 \quad (3) \]
for an observable $O(s)$ when next, the three-loop, coefficient $C_3$ is not known. This is the case, e.g., with event–shape\[8\] analysis.

On the basis of the numerical estimates of Table 1, in such a case, we recommend to use the three-term expression
\[ O_3^\Delta(s) = d_1 \left\{ \bar{\alpha}_s - \frac{\pi^2 \beta_0^2}{3} \bar{\alpha}_s^3 \right\} + d_2 \bar{\alpha}_s^2 = c_1 \bar{\alpha}_s + c_2 \bar{\alpha}_s^2 - \delta_3 \bar{\alpha}_s^3 \quad (4) \]
i.e., to take into account the known predominant $\pi^2$ part of the next coefficient $c_3$. As it follows from the comparison of the last expression with the previous, two–term one, the $\bar{\alpha}_s$ numerical value extracted from eq.(4), for the same measured value $O_{\text{obs}}$, will differ by a positive quantity (e.g., in the $f = 5$ region with $\bar{\alpha}_s \simeq 0.12 \div 0.15$)

$$ (\Delta \bar{\alpha}_s)_{3} = \left. \frac{\pi \delta_3 \bar{\alpha}_s^3}{1 + 2 \pi d_2 \bar{\alpha}_s} \right|_{f=5}^{120 \div 100 \text{GeV}} = \frac{1.225 \bar{\alpha}_s^3}{1 + 0.90 \bar{\alpha}_s} \simeq 0.002 \div 0.003 $$

that turns to be numerically important.

Moreover, in the $f = 4$ region, where the three-loop approximation is commonly used in the data analysis, the $\pi^2$ term $\delta_4$ of the next order turns out also to be essential. Hence, we propose to use the four-term expression

$$ O^\Delta_4(s) = d_1 \bar{\alpha}_s + d_2 \bar{\alpha}_s^2 + c_3 \bar{\alpha}_s^3 - \delta_4 \bar{\alpha}_s^4; \quad c_3 = d_3 - \delta_3 \quad (5) $$

(instead of the three-term one (4)) that is equivalent to

$$ O^\Delta_4(s) = d_1 \left\{ \bar{\alpha}_s - \frac{\pi^2 \beta_0^2}{3} \bar{\alpha}_s^3 - b_1 \frac{5}{6} \pi^2 \beta_0^2 \bar{\alpha}_s^4 \right\} + d_2 \left\{ \bar{\alpha}_s^2 - \pi^2 \beta_0^2 \bar{\alpha}_s^4 \right\} + d_3 \bar{\alpha}_s^3 \quad (6) $$

with $\delta_3$ and $\delta_4$ defined [4, 7] in eq.(4).

The three– and two–term structures in curly brackets are related to specific expansion functions $\tilde{\alpha}$ and $\mathfrak{A}$ defined below (10) and entering into the non-power expansion (11).

To estimate roughly the numerical effect of using this last modified expression (4), we take the case of $e^+ e^-$ inclusive annihilation. For $\sqrt{s} \simeq 3 \div 5 \text{ GeV}$ with $\bar{\alpha}_s \simeq 0.28 \div 0.22$ one has

$$ (\Delta \bar{\alpha}_s)_{4} = \left. \frac{\pi \delta_4 \bar{\alpha}_s^4}{1 + 2 \pi d_2 \bar{\alpha}_s} \right|_{f=4}^{3 \div 5 \text{ GeV}} = \frac{1.07 \bar{\alpha}_s^4}{1 + 0.974 \bar{\alpha}_s} \simeq 0.005 \div 0.002 $$

— an important effect on the level of ca $1 \div 2\%$.

Moreover, the $(\Delta \bar{\alpha}_s)_4$ correction turns out to be noticeable even in the lower part of the $f = 5$ region! Indeed, at $\sqrt{s} \simeq 10 \div 40 \text{ GeV}$ with $\bar{\alpha}_s \simeq 0.20 \div 0.15$ we have

$$ (\Delta \bar{\alpha}_s)_{4} \bigg|_{10 \div 40 \text{ GeV}} \simeq 0.71 \bar{\alpha}_s^4 \simeq (1.1 \div 0.3) \cdot 10^{-3} \quad (\lesssim 0.5\%) $$

3 Non-power expansion in the Minkowskian region

The so–called $\pi^2$ terms in the $s$–channel perturbative expansions for the invariant coupling and observables have a simple origin.
As it is well known, the usual invariant coupling originally defined in terms of real constants $z_i$, counter-terms of finite Dyson renormalization transformation, can be expressed via a product of dressed symmetric vertex and propagator amplitudes taken at space-like values of their arguments.

$$\bar{\alpha}(Q^2, \alpha) = \alpha \Gamma^2(Q^2, \alpha) \prod_i d_i(Q^2, \alpha).$$

Hence, by construction, it is a real function defined in the Euclidean region.

Transition to the time-like region, with logs branching $\ln Q^2 \to \ln s - i\pi$ transforms all relevant amplitudes into complex functions $\Gamma(s, \alpha), d_i(s, \alpha)$. Here, the problem of appropriate defining of effective coupling in the time-like domain arises.

For this goal, we shall follow the idea devised in the early 80s by Radyushkin [4] and Krasnikov–Pivovarov [5]. There, an integral transformation $R$ reverse to the dipole representation for the Adler function has been used.

We propose to treat this representation as an integral operation

$$R(s) \to D(z) = Q^2 \int_0^\infty \frac{ds}{(s+z)^2} R(s) \equiv D \{R(s)\}$$

transforming a function $R(s)$ of a real positive (time-like) argument into a function $D(z)$ given in the cut complex plane with analytic properties equivalent to those following from the Källen–Lehmann integral representation. In particular, the function $D(Q^2)$ is real on the positive (space-like) real axis at $z = Q^2 + i0; Q^2 \geq 0$.

The reverse operation is expressible in the form of a contour integral

$$R(s) = \frac{i}{2\pi} \int_{s-i\varepsilon}^{s+i\varepsilon} \frac{dz}{z} D_{pt}(-z) \equiv R[D(Q^2)].$$

With the help of the latter, one can define [4, 5] an effective invariant time-like coupling $\tilde{\alpha}(s) = R[\bar{\alpha}_s(Q^2)]$. Omitting some technical details, we give a few resulting expressions.

E.g., starting with one–loop $\bar{\alpha}_s^{(1)} = [\beta_0 \ln(Q^2/\Lambda^2)]^{-1}$ one has

$$\tilde{\alpha}_s^{(1)}(s) = \frac{1}{\beta_0} \left[ \frac{1}{2} - \frac{1}{\pi} \arctan \frac{L}{\pi} \right]_{L>0} = \frac{1}{\beta_0 \pi} \arctan \frac{\pi}{L}; \quad L = \ln \frac{s}{\Lambda^2}. \quad (8)$$

At the same time, to $(\bar{\alpha}_s^{(1)}(Q^2))^2$ and $(\bar{\alpha}_s^{(1)}(Q^2))^3$ there correspond

$$\mathcal{A}_2^{(1)}(s) \equiv R[(\bar{\alpha}_s^{(1)}(Q^2))^2] = \frac{1}{\beta_0^2 [L^2 + \pi^2]^2}$$

and

$$\mathcal{A}_3^{(1)}(s) = \frac{L}{\beta_0^3 [L^2 + \pi^2]^2}.$$
In the two–loop case, for a “popular” expression

\[ \beta_0 \tilde{\alpha}^{(2)}_{s,\text{pop}}(Q^2) = \frac{1}{l} - b_1(f) \frac{\ln l}{T^2}; \quad l = \ln \frac{Q^2}{\Lambda^2} \]

one obtains\[4\] the two-loop “pop” effective s–channel coupling

\[ \tilde{\alpha}^{(2)}_{\text{pop}}(s) = \left( 1 + \frac{b_1 L}{L^2 + \pi^2} \right) \tilde{\alpha}^{(1)}(s) - \frac{b_1}{\beta_0} \ln \frac{\sqrt{L^2 + \pi^2} - 1}{L^2 + \pi^2}. \quad (9) \]

Both the expressions (8) and (9) are monotonically decreasing with a finite IR \( \tilde{\alpha}(0) = 1/\beta_0(f = 3) \simeq 1.4 \) value. Meanwhile, higher functions go to the zero \( \tilde{\alpha}_k(0) = 0 \) at the IR limit.

In the case \( L \gg \pi \), it is possible to expand \( \tilde{\alpha} \) and \( \tilde{\alpha}_k \) in powers of \( \pi^2/L^2 \). Then functions \( \tilde{\alpha} \) and \( \tilde{\alpha}_2 \) can be presented as expansions in powers of common \( \tilde{\alpha}_s \simeq 1/L \). They correspond to curly brackets in (9).

In [4, 5], as a starting point for observables in the Euclidean, i.e., space–like domain \( Q^2 > 0 \), the perturbation series

\[ D_{pt}(Q^2) = 1 + \sum_{k \geq 1} d_k \tilde{\alpha}_s^k(Q^2) \]

has been assumed. It contains powers of usual, RG summed, invariant coupling \( \tilde{\alpha}_s(Q^2) \) that obeys unphysical singularities in the infrared (IR) region around \( Q^2 \simeq \Lambda_3^2 \).

By using the R transformation, we obtain in the Minkowskian region the “transformed” expansion over a non-power set of functions

\[ R_{\tilde{\alpha}}(s) \equiv R \left[ D_{pt}(Q^2) \right] = 1 + \sum_{k \geq 1} d_k \tilde{\alpha}_k(s); \quad \tilde{\alpha}_k(s) = R \left[ \tilde{\alpha}_s^k(Q^2) \right] \quad (10) \]

free of the mentioned singularities. Properties of these functions have been analyzed in detail in our previous paper[13] — see also Ref. [14]. For a more detailed numerical information on the functions \( \tilde{\alpha}, \tilde{\alpha}_2 \) and \( \tilde{\alpha}_3 \) see Ref. [15].

Here, we give condensed information that will be enough for a few illustrations.

**Table 2**

| \( \sqrt{s}/\text{GeV} \) | 5  | 10 | 15 | 20 | 30 | 50 | 60 | 90 | 150 |
|--------------------------|----|----|----|----|----|----|----|----|----|
| \( \tilde{\alpha}_s(s) \) | .235 | .195 | .177 | .165 | .153 | .137 | .133 | .125 | .115 |
| \( \tilde{\alpha}(s) \)   | .221 | .186 | .170 | .160 | .148 | .136 | .132 | .123 | .114 |
| 10\( \tilde{\alpha}_2 \)   | .456 | .330 | .275 | .246 | .214 | .180 | .169 | .149 | .129 |
| 100\( \tilde{\alpha}_3 \)  | .871 | .555 | .436 | .357 | .299 | .232 | .213 | .177 | .143 |
Both in the Figure 1 and in Table 2, we give 3-loop solutions for $\bar{\alpha}_s$ as well as for the modified, so-called global (for detail, see paper [13]) functions $\tilde{\alpha} = \mathcal{A}_1$, $\mathcal{A}_2$ and $\mathcal{A}_3$ calculated within the MS scheme for the cases $\Lambda(5) = 215$ GeV, $\bar{\alpha}_s(M_Z^2) = 0.118$ and $\Lambda(5) = 290$ GeV, $\bar{\alpha}_s(M_Z^2) = 0.125$.

Figure 1: Effective global Minkowskian, $\tilde{\alpha}$, and Euclidean, $\alpha_{an}$ expansion functions, as compared with the standard one $\bar{\alpha}_s$ (at $\Lambda(5) = 350$ MeV and $\bar{\alpha}_s(M_Z^2) = 0.118$).

We have chosen these two cases as limiting ones as far as in many practical cases real figures lie between these limits.

In the first figure we give three curves $\bar{\alpha}_s$, $\tilde{\alpha}$ and $\alpha_{an}$ related to the same physical case for $\Lambda_3 = 350$ MeV and $\bar{\alpha}_s(M_Z^2) = 0.118$. The curves $\tilde{\alpha}$ and $\alpha_{an}$ on the figure go a bit slanting than usual, the $\bar{\alpha}_s$, dotted curve. This is quite natural, as they both are regular in the vicinity of the $\Lambda$ singularity.

Meanwhile, only two first, $\tilde{\alpha}$ and $\alpha_{an}$ have direct physical meaning (compare with conclusion of [13]). Just their values have to be determined from any given experiment. Nevertheless, in the four- and five–flavour regions one can still refer to $\bar{\alpha}_s$ and $\bar{\alpha}_s(M_Z^2)$ as to traditional theoretical objects.

Now, instead of (1), with due account to (10), we have

$$r(s) = \frac{\tilde{\alpha}(s)}{\pi} + d_2 \mathcal{A}_2(s) + d_3 \mathcal{A}_3(s)$$

(11)
with beautifully decreasing coefficients $d_k$. Just this nonpower expansion, strictly speaking, should be used instead of its approximations, eqs. (11) and (13), for data analysis in the time-like region.

At the same time, in the Euclidean, we have also non-power expansion

$$d(Q^2) = \frac{\alpha_{\text{an}}(Q^2)}{\pi} + d_2 A_2(Q^2) + d_3 A_3(Q^2)$$

(12)

that can be related to (11) by transformation (7) in the framework of Invariant Analytic Approach (refs.[16, 17]).

These non-power expansions, free of unphysical singularities, jointly form a correlated system. The latter has been studied in detail in Refs.[13] and [18]. We call it Analytic Perturbation Theory (APT).

4 Numerical illustrations

To illustrate, let us start with a few cases in the $f = 5$ region.

To begin with, consider the decay. According to the Particle Data Group (PDG) overview (see their Fig.9.1 on page 88 of Ref.[1]), this is (with $\bar{\alpha}_s(M_\Upsilon) \simeq 0.170$ and $\bar{\alpha}_s(M_Z^2) = 0.114$) one of the most “annoying” points of their summary of $\bar{\alpha}_s(M_Z^2)$ values. It is also singled out theoretically. The expression for the ratio of decay widths starts with the cubic term

$$R(\Upsilon) = R_0 \bar{\alpha}_s^3(M_\Upsilon)(1 + e_1 \bar{\alpha}_s) \quad \text{with} \quad e_1 \simeq 1.$$  

(13)

Due to this, the $\pi^2$ correction is rather big here

$$A_3 \simeq \bar{\alpha}_s^3 \left( 1 - 2(\pi \beta_0)^2 \bar{\alpha}_s^2 \right).$$

(14)

Accordingly,

$$\Delta \bar{\alpha}_s(M_\Upsilon) = \frac{2}{3} (\pi \beta_0)^2 \bar{\alpha}_s^3(M_\Upsilon) \simeq 0.0123,$$

that corresponds to

$$\Delta \bar{\alpha}_s(M_Z) = 0.006 \quad \text{with} \quad \bar{\alpha}_s(M_Z) = 0.120.$$  

(15)

Now, let us turn to a few cases analyzed by the three-term expansion formula (11). For the first example, take $e^+e^- \text{ hadron annihilation}$ at $\sqrt{s} = 42 \text{ GeV}$ and $11 \text{ GeV}$.

A common form (see, e.g., Eq.(15) in Ref.[2]) of theoretical presenting of the QCD correction in our normalization looks like

$$r_{e^+e^-}(s) = 0.318 \bar{\alpha}_s(s) + 0.143 \bar{\alpha}_s^2 - 0.413 \bar{\alpha}_s^3.$$  

(16)

\footnote{First proposal of taking into account this effect in the $\Upsilon$ decay was discussed more than a quarter of century ago. Nevertheless, in current practice it is neglected.}
Starting with \( r_{e^+e^-}(42) \simeq 0.0476 \), one has \( \bar{\alpha}_s(42) = 0.144 \). Along with our new philosophy, one should use instead

\[
r_{e^+e^-}(s) = 0.318 \bar{\alpha}(s) + 0.143 \mathcal{A}_2(s) - 0.023 \mathcal{A}_3(s)
\]

(17)

that yields \( \bar{\alpha}(42) = 0.142 \) with \( \bar{\alpha}_s(42) = 0.145 \) and \( \bar{\alpha}_s(M^2_Z) = 0.127 \) to be compared with \( \bar{\alpha}_s(M^2_Z) = 0.129 \) under a usual analysis.

Quite analogously, for \( r_{e^+e^-}(11) \simeq 0.0661 ; \bar{\alpha}_s(11) = 0.200 \), we obtain \( \bar{\alpha}(10) = 0.190 \) that corresponds to \( \bar{\alpha}_s(M^2_Z) = 0.124 \) instead of 0.130.

For the next example, we take the \( Z_0 \) inclusive decay. Experimental ratio \( R_{Z} = \Gamma(Z_0 \to \text{hadrons})/\Gamma(Z_0 \to \text{leptons}) = 20.783 \pm .029 \) is usually presented as follows: \( R_Z = R_0(1 + r_Z(M^2_Z)) \) with \( R_0 = 19.93 \). A common form (see, e.g., Eq.(15) in Ref.[4]) of presenting of the QCD correction in our normalization looks like

\[
r_Z(M^2_Z) = 0.3326 \bar{\alpha}_s + 0.0952 \bar{\alpha}_s^2 - 0.483 \bar{\alpha}_s^3.
\]

(18)

To \( [r_Z]_{\text{obs}} = 0.04184 \) there corresponds \( \bar{\alpha}_s(M^2_Z) = 0.1241 \) with \( \Lambda^{(5)}_{\text{MS}} = 292 \text{ MeV} \). In the APT case, from

\[
r_Z(M^2_Z) = 0.3326 \bar{\alpha}(M^2_Z) + 0.0952 \mathcal{A}_2(M^2_Z) - 0.094 \mathcal{A}_3(M^2_Z)
\]

(18)

we obtain \( \bar{\alpha}(M^2_Z) = 0.122 \) and \( \bar{\alpha}_s(M^2_Z) = 0.124 \) that relates to \( \Lambda^{(5)} = 290 \text{ MeV} \). Note that here the three-term approximation of (3) gives the same relation between the \( \bar{\alpha}_s(M^2_Z) \) and \( \bar{\alpha}(M^2_Z) \) values.

Nevertheless, in accordance with our preliminary estimate for the \( (\Delta \bar{\alpha}_s)_4 \) role, even the so-called NNLO theory needs some \( \pi^2 \) correction in the \( W = \sqrt{s} \lesssim 50 \text{ GeV} \) region.

Now, turn to the experiments in the HE Minkowskian (mainly with a shape analysis) that usually are confronted with two-term expression (3). As it has been shown below, the main theoretical error in the \( f = 5 \) region can be expressed in the form

\[
(\Delta \bar{\alpha}_s(s)|_{f=5})_{20 \pm 100 \text{ GeV}} \simeq 1.225 \bar{\alpha}_s^3(s) \simeq 0.002 \div 0.003 .
\]

(19)

An adequate expression for the shift of an equivalent \( \bar{\alpha}_s(M^2_Z) \) value is

\[
[\Delta \bar{\alpha}_s(M^2_Z)]_3 = 1.225 \bar{\alpha}_s(s)\bar{\alpha}_s(M^2_Z)^2.
\]

(20)

We give results of our approximate APT calculations, mainly by Eqs.(19) and (20), in the form of Table 3 and Figure 2. At the last column of the Table 3 in brackets we indicate difference between the APT and usual analysis. By bold figures the results of the three–loop analysis are singled out.
Table 3
The APT revised part \((f = 5)\) of Bethke’s Table 6

| Process          | √s  | loops | \(\bar{\alpha}_s\) (s) | \(\bar{\alpha}_s(m_Z^2)\) | \(\bar{\alpha}_s\) (s) | \(\bar{\alpha}_s(m_Z^2)\) |
|------------------|-----|-------|-------------------------|---------------------------|-------------------------|---------------------------|
| \(\Upsilon\)-decay| 9.5 | 2     | ..170                   | ..114                     | ..182                   | ..120 (+6)                |
| \(e^+e^-[\sigma_{had}]\) | 10.5| 3     | ..200                   | ..130                     | ..198                   | ..129(-1)                 |
| \(e^+e^-[j & sh]\) | 22.0| 2     | ..161                   | ..124                     | ..166                   | ..127(+3)                 |
| \(e^+e^-[j & sh]\) | 35.0| 2     | ..145                   | ..123                     | ..149                   | ..126(+3)                 |
| \(e^+e^-[\sigma_{had}]\) | 42.4| 3     | ..144                   | ..126                     | ..145                   | ..127(+1)                 |
| \(e^+e^-[j & sh]\) | 44.0| 2     | ..139                   | ..123                     | ..142                   | ..126(+3)                 |
| \(e^+e^-[j & sh]\) | 58  | 2     | ..132                   | ..123                     | ..135                   | ..125(+2)                 |
| \(Z_0 \rightarrow \text{had.}\) | 91.2| 3     | ..124                   | ..124                     | ..124                   | ..124 (0)                 |
| \(e^+e^-[j & sh]\) | 91.2| 2     | ..121                   | ..121                     | ..123                   | ..123(+2)                 |
| \(e^+e^-[j & sh]\) | 133 | 2     | ..113                   | ..120                     | ..115                   | ..122(+2)                 |
| \(e^+e^-[j & sh]\) | 161 | 2     | ..109                   | ..118                     | ..111                   | ..120(+2)                 |
| \(e^+e^-[j & sh]\) | 172 | 2     | ..104                   | ..114                     | ..105                   | ..116(+2)                 |
| \(e^+e^-[j & sh]\) | 183 | 2     | ..109                   | ..121                     | ..111                   | ..123(+2)                 |
| \(e^+e^-[j & sh]\) | 189 | 2     | ..110                   | ..123                     | ..112                   | ..125(+2)                 |

Averaged \(<\bar{\alpha}_s(M_Z^2)>_{f=5}\) values \(0.121; 0.124; \)

\(^a\)“j & sh” = jets and shapes; Figures in brackets in the last column give the difference \(\Delta\bar{\alpha}_s(M_Z^2)\) between common and APT values.

\(^b\)Taken from Ref.

Let us note that our average over events from Table 6 of Bethke’s review \cite{2} nicely correlates with recent data of the same author (see Summary of Ref.\cite{19}). The best \(\chi^2\) fit yields \(\bar{\alpha}_s(M_Z^2)_{[2]} = 0.1214\) and \(\bar{\alpha}_s(M_Z^2)_{\text{APT}} = 0.1235\). This gives minimum \(\chi^2_{[2]} = 0.197\) and \(\chi^2_{\text{APT}} = 0.144\) with impressive ratio \((\simeq 0.73)\) illustrating the effectiveness of the APT procedure.

On the Fig.2 by open circles and bullets (◦, •) we give two– and three–loops data mainly from Fig.10 of paper \cite{2}. The only exclusion is the \(\Upsilon\) decay taken from the Table 6 of the same paper. By crosses we marked the new “APT values” calculated approximately mainly with help of Eq.\cite{19}.

For clearness of the \(\pi^2\) effect, we skipped the error bars. They are the same as in the mentioned Bethke’s figure and we used them for calculating \(\chi^2\).
Figure 2: The new APT analysis for $\bar{\alpha}_s$ in the five-flavour time-like region. Crosses (+) differ from circles (○, ⋄) by $\pi^2$ correction (19). Solid APT curve relates to $\Lambda_{\text{MS}}^{(5)} = 270$ MeV and $\bar{\alpha}_s(M_Z^2) = 0.124$. To compare, we give also the standard (dot-and-dash curve) $\bar{\alpha}_s$ (at $\Lambda^{(5)} = 213$ MeV and $\bar{\alpha}_s(M_Z^2) = 0.118$) taken from Fig.10 of paper [2].

5 Conclusion

We have established a few qualitative effects:

1. Effective positive shift $\Delta \bar{\alpha}_s = +0.002$ in the upper half ($\geq 50$ GeV) of the $f = 5$ region for all time-like events that have been analyzed up to now in the NLO mode.

2. Effective shift $\Delta \bar{\alpha}_s \simeq +0.003$ in the lower half ($10 \div 50$ GeV) of the $f = 5$ region for all time-like events that have been analyzed in the NLO modes.

3. The new value

$$\bar{\alpha}_s(M_Z^2) = 0.124$$

by averaging over the $f = 5$ region.

These results are based on a plausible hypothesis on the “$\pi^2$ – terms” prevalence in expansion coefficients for observable in the Minkowskian do-
main. The hypothesis has some preliminary support but needs to be checked in a more detail.

Nevertheless, our result (21) being taken as granted, rises two physical questions:

– The issue of self-consistency of QCD invariant coupling behavior between the “medium (f = 3, 4)” and “high (f = 5, 6)” regions.
– The new “enlarged value” (21) can influence various physical speculations in the several hundred GeV region.

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