Finite Element Analysis of Fatigue Crack of Square Bird-Beak X-joints Under Brace Axial Loading

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Abstract. This paper presents a numerical study of the fatigue crack propagation of square bird-beak hollow section welded X-joints. A pair of axial forces were applied at the brace ends, while the chord ends were free. Finite element modelling was done in ABAQUS while FRANC3D was used for the computation of stress intensity factor and crack propagation. The initial crack inserted at the site of maximum stress concentration. Paris’ law was used to compute the fatigue life. The results obtained from the finite element analysis compared with the experimental data, and the predicted crack growth was found to be in good agreement with the test observations. For such joints, the surface cracks length vs. fatigue crack growth curve proposed to ensure practicability in real applications.

1. Introduction

The bird-beak X-joints are an innovative type of weld square hollow section joints. Under the cyclic loading, such joints are most vulnerable to fatigue failure, affecting the safety of components and structures. These members are, most of the time, subject to variable load cases, resulting in a mixed mode fatigue case. Many researchers have devoted their career to study the fatigue crack growth behavior of the welded joints. Different methods are introduced to study this problem: hot spot stress method[1]; effective notch method[2]; continuum damage model[3]; and linear elastic fracture mechanics (LEFM)[4]. Mashiri et. al. [5] tested the static behavior of welded square hollow section (SHS) T-connections. Stress intensity factor (SIF) estimation of weld toe semi-elliptical crack in offshore tubular joints was proposed by Lee & Bowness [6]. Chen [7] and Tong [8] investigated the stress concentration factors (SCFs) and fatigue behavior of Square bird beak joints using the experimental method.

Moreover, many researchers are focusing on fatigue behavior and stress concentration field and are concentrating over the application of LEFM, making this the one of the most useful assessment technique, as the initial crack in the real structures is inevitable[4], [9]–[12]. Under the cyclic load, the initial crack tends to occur in the stress concentration filed, and the crack will be lengthened over time. Such process of lengthening is the crack propagation. There have been many methods to study this behavior, one of the most adopted methods is the LEFM. Crack propagation analysis using LEFM is usually conducted with Paris law[4], with the reliable SIF(K) along the crack front points. Previous researches have shown that it is difficult to calculate SIFs from the theoretical analysis. Therefore, the numerical simulation is usually utilized to obtain the SIFs and predict the crack growth.

Various researchers have proposed different initial crack shapes and its propagation paths. Generally, semi-elliptical crack with the fixed aspect ratio is assumed throughout the crack growth process. However, the crack shapes and aspect ratios change with the crack advancement and loading cycle[9].
As a result, the FEM and BEM are the most suitable methods to predict the crack growth propagation over fatigue load. Seifi and Omidvar[10] studied the modified compact test specimen (MCT) under mixed mode I + II loading effect experimentally and computationally. Experimental and simulation results of mixed mode I +II+ III loading in the crack were discussed [13]. Fatigue crack behavior of base metal and butt weld was investigated by Zong et. al.[14]. Corbani et. al.[9] proposed a line and quarter ellipse method to predict the crack shape under pure out of plane bending. Ju, X. and Tateishi, K.[12] studied the behavior of through-thickness crack under out of plane bending experimentally and numerically. Yang et. al.[15] researched the fatigue crack growth of SHS T-joints using the LEFM theory and boundary element method (BEM), which is in accordance with the experimental results. Extended finite element method (XFEM) is used to predict the crack growth in the 3D problem and reveal the sturdiness and versatility of this approach by Pathak et. al.[16]. Branco and Antunes[17] analyzed the evolution of crack shape in mode-I fatigue growth in Middle Cracked Tension specimens that included different parameters such as specimens' thickness, Poisson's ratio, the exponent of the Paris' law and cracks closure. Evaluation of SIF values and crack growth based on SIF-based flaw propagation procedure is proposed by Carpinteri et. al.[18]. Kikuchi et. al.[19] used s-version of FEM to model Fatigue Crack Growth(FCG) process under mixed mode loading. Moreover, extensive experimental and numerical analysis of static and fatigue performance has been conducted on different type of joints, such as T-[5], [8], K-[6], X-[20]–[22], TX-[23], DX-[24], and XX-[25] joints.

In this study, an un-cracked finite element model was built by ABAQUS, and then the initial crack is inserted in FE model. The fatigue crack growth of square bird beak joints (SBBJ-X) at the weld toe under brace axial loading is investigated. Paris’ Law and maximum tensile stress criteria are used to predict the crack growth, and the results are compared with the experimental observations. Moreover, the variation of SIF on the initial crack front is also analyzed.

2. Fatigue test specimens and test setup

Two square bird-beak joints (SBBJ-X) are formed. These specimens are designed based on the three non-dimensional parameters, brace to chord width ratio (β = b1/b0), chord-wall slenderness ratio(2γ = b0/t0), brace to chord wall thickness ratio(τ = t1/t0). b1 and b0 represent the sectional width of chord and brace respectively. Similarly, t0 and t1 correspond to the thickness of the chord and brace respectively. Chords of length L0 and braces of the length Lj are used. The geometric configuration of the specimen is shown in figure. 1 and the geometric dimensions of the joints are enlisted in Table 1.

Chord and brace members are connected by 80% partial joint penetration (PJP) groove-fillet welds. The cold-formed steel tubes, which are of Q420 grade, are used to fabricate the joints which strictly conforms to Chinese standard GB/T 6728-2002[26]. Standard coupons were tested for the uniaxial test. Measured mechanical properties are 494 MPa, 597 MPa and 202 GPa for yield stress (Sy), ultimate tensile stress (Su) and Young's modulus (E) respectively.

Before the test, to conform to Chinese standard GB/T 6728-2002[26] specimens were prepared for the test as (1) rigid steel platform was fixed onto the ground for fix support, (2) lower brace end and rigid steel platform was connected via a pin connection, (3) fatigue actuator and upper brace end was connected by a pin connection. (4) Then, a pair of axial tension force was applied at the brace ends in

| Specimen | Chord | Brace | Non-dimensional Parameters | Axial-Load (kN)(P) |
|----------|-------|-------|-----------------------------|-------------------|
|          | b0 (mm) | t0 (mm) | L0 (mm) | b1 (mm) | t1 (mm) | L1 (mm) | β = (b1/b0) | 2γ = (b0/t0) | τ = (t1/t0) |                |
| SBBJ-X-1 | 200   | 12    | 1200   | 90    | 8     | 500    | 0.45    | 16.67   | 0.67   | 63               |
| SBBJ-X-2 | 200   | 9.5   | 1200   | 120   | 4.5   | 500    | 0.6     | 21.05   | 0.47   | 63               |
the direction of brace axis. Load within the stress range is implemented by using the actuator of maximum capacity 200 kN with the load ratio of 0.1.

Fatigue load cycle was recorded on loading machine itself. Crack initiation was determined by human eye aided with the magnifying glass. Thin layer of white paint was sprayed at the potential initial crack location (i.e., at the junction surface) to assist the crack detection. During the fatigue tests, human inspection had performed to record surface crack length in every 2h until the end of the test. Two 12 megapixels HD cameras were always focused on the potential crack initiation location to avoid human error during the test.

3. Fatigue Life Prediction

FRANC3D software is used for modeling and re-meshing of the specimen, and also for the prediction of crack growth. This software was developed in the Cornell University by Fracture Analysis Group (FAG) [27]. Initially, Boundary Element Method (BEM) was used to predict the fatigue crack growth. After reformation and redesign of the software replacing BEM with finite element method to work in conjunction with FE solver were developed by fracture analysis consultant [28]. This software can calculate the new crack front based on displacement provided by the commercially available FE solver. Then the crack is inserted in the un-cracked FE model, and the model is re-meshed with advancing front meshing algorithm to simulate the further crack growth. Kink angle, which can be determined by four criteria: maximum strain energy release rate criterion (SERR); generalized stress criterion (GEN); maximum shear stress criterion (MSS); maximum tensile stress criterion (MTS), is used to determine the direction of crack growth in this model. The maximum tensile stress criterion is used to determine the kink angle[28]. Similarly, constant amplitude model, one of the models in FRANC3D, is used for the crack growth analysis. Subsequently, the Paris’ crack growth model, which can predict the crack propagation behavior accurately, is used to calculate the extension of crack front points[4]. And the SIF and Paris’s constant are of importance to the crack propagation behavior. Extended crack front location can be calculated based on the equivalent stress intensity factor (Keq), which is determined by equation (1) in mixed mode loading[29]:

\[ K_{eff} = \sqrt{K_{I}^2 + (\alpha K_{II})^2 + (\beta K_{III})^2} \] (1)
Where \( \alpha = \frac{K_{Ic}}{K_{IIc}} = 1.155 \) and \( \beta = \frac{K_{Ic}}{K_{IIc}} = 1.0 \), which are material independent parameters\(^{[10]}\) \(^{[12]}\). Fracture toughness and threshold fracture toughness of the Q420 grade steel is taken as \( K_c = 1900 \text{ MPa mm}^{1/2} \) and \( K_{th} = 148 \text{ MPa mm}^{1/2} \) respectively as the recommendation from BS7910\(^{[30]}\). The governing universal fatigue crack threshold for common steel is considered in the equation (2) which is as follows.

\[
K_{th} = 170 - 214 \times R \text{ (Unit: MPa mm}^{1/2} \text{)}
\]  

(2)

An increment in each step of crack extension (\( \delta_m \)) is manually provided at the location of maximum equivalent SIF. Among both available methods (manual and automatic), for the consideration of the crack growths, maximum crack growth in each step should not be more than 30% of the cumulative crack length. And the propagation at the discrete points along the crack front is approximated using the Paris’ equation, in which the maximum crack extension was manually keyed by equation (3).

\[
\delta = \delta_m \left( \frac{K_{eq}}{K_{eq,m}} \right)^m
\]  

(3)

The cubic spline curve fitting approach is implemented for fitting the crack front points using one of the curve fitting tools in FRANC3D. It should be noted that FRANC3D smoothens the entire crack front, making simulated crack front different from the experimental observations \(^{[9]}\). Meanwhile, the simulated crack front leaves some error from the theoretical statement which states crack growth direction is always perpendicular to the crack front \(^{[9]}\).

The number of fatigue cycle is determined by the Paris’ Law as:

\[
da/dN = C \left( \Delta K_{eff} \right)^m
\]  

(4)

Where \( a \) is the crack length, \( N \) is the number of cycle of propagation life, \( C = 2.18 \times 10^{-13} \) and \( m = 3 \) respectively for the Q420 steel are material constants. \( da/dN \) is expressed in unit mm/cycle and \( \Delta K_{eff} \) in Nmm\(^{3/2}\).

Thus, Fatigue life in each step is calculated as follows \(^{[4]}\):

\[
\Delta N = \frac{D_m}{C \Delta K_{eff}^m}
\]  

(5)

Since the crack front is tested under the brace axial loading, there exists Mode II, and Mode III loading of SIF at crack presents at the weld toe. These are considered as the effective SIF \(^{[31]}\) in the simulation process as:

\[
\Delta K_{eff} = \sqrt{\Delta K_I^4 + 8 \Delta K_{II}^4 + \frac{(8 \Delta K_{III})^4}{1 - v}}
\]  

(6)

4. Model Generation

The loading, boundary conditions, construction and geometric dimension of these type bird-beak X-joints are all of the symmetry. Thus, one-eighth of SBBJ-X specimen is modeled in ABAQUS, as shown in figure 2. Sub-model is defined as shown in the enlarged view in figure 2. The brace axial force is applied in the FE model, and the stress distribution is analyzed in ABAQUS to locate the node with maximum stress from where the crack initiates being, as shown in figure 3. Through the static analysis, the location of the initial crack for FRANC3D can be manually determined, while the initial crack length for FRANC3D can be obtained from the experimental results. Moreover, the initial crack is aligned parallel to the welding direction and is perpendicular to the chord surface.
The flaws presented in the typical steel structures are of a semi-elliptical shape which is used for the simulation of the initial shape of the crack using a special module in FRANC3D[28]. Crack front template around the crack tip is defined with three rings and eight circumferential elements. Quarter-point Wedge elements are used around the crack front followed by twenty-node hexahedral elements, while the remaining portion is re-meshed with tetrahedron element by using front advancing re-meshing algorithm technique. The crack is then inserted into the un-cracked FE model as shown in figure 4.

![Figure 2. Meshing and Boundary Conditions](image)

After the insertion of crack into FRANC3D, load ratio of 0.1 is taken into account in the simulation. The input file created by FRANC3D is used for stress analysis in ABAQUS. The obtained displacements are used to calculate the SIF at corresponding crack front locations. The value of the crack extension is manual where the maximum crack occurs at the location of maximum SIF, but is as explained in equation (3) in other location. The newly formed cracks obtained from the crack extension is subjected to un-cracked model and the process repeats until the crack propagates to reach the symmetrical boundary condition. This crack propagation of the specimen is shown in figure 5.

![Figure 3. Stress Analysis of Un-Cracked Model](image)
5. Results and Discussion
The stress intensity factors are calculated in SBBJ-X joints with different initial crack lengths under brace axial loading, as shown in figure 6. It shows that with the initial crack length in SBBJ-X-1 specimen being placed parallel with the welding, average SIF in the Mode I is increased by 12% with the crack length increasing from 4 mm to 8 mm, while it is increased to 22% with the crack length getting to 18 mm. Similarly, in case of SBBJ-X-2 specimen, the initial crack is placed parallel with the welding, average SIF in the Mode I is increased by 23% with the increase of crack length from 4 mm to 12 mm, while it is increased to 32% with the crack length getting to 19 mm. Based on the SIF curve, it is observed that the SIF near the crack front ends have the higher values and middle has lower values, which means there is comparatively more growth in the ends than that of through-thickness direction. This growth continues until the symmetrical end of the specimen is reached. Once the crack reach the symmetrical end, small portion of crack undergoes compression resulting in the crack closure which, to date, cannot be further accounted by FRANC3D. In order to address such limitations, Corbani et al. [9] proposed a new method of eliminating this shortcoming by combining the crack front as a straight line and quarter ellipse. However, this has not been performed in this paper and can be a matter of new research.
length. As in the figure 7, for SBBJ-X-1 specimen, with the increase in the initial crack length from 4 mm to 8 mm, the fatigue life decreases by 19%, while it decreases by 44% with the initial crack length increasing from 4 mm to 18 mm. Similarly, in case of SBBJ-X-2, there is also a decrease in fatigue life by 27% and 39% for an increase in the initial crack length from 4 to 12 mm and 4 to 19 mm respectively, as shown in figure 7.

The effect of the initial crack length over two specimens are examined for the fatigue crack growth which illustrates the effect of an initial crack length over the fatigue life and cracks growth as shown in figure 7. As seen in figure 6, the stress intensity factor is higher in the longer length of the initial crack length, and it is lower in the smaller initial crack length. This behavior of SIF implies that shorter initial crack length will have a longer fatigue life than that of longer initial crack length. The numerical simulation of crack complies with the experimental results.

6. Conclusion

In this paper, the effects of brace axial loading on the crack propagation of SBBJ-X specimens are investigated using a FEM approach. It demonstrates that SIF has evident effects on the fatigue behavior of the joints. Fatigue life is obtained by using the linear elastic fracture mechanics. The initial crack length has an eminent effect on fatigue life. Having control over the initial flaw increases the fatigue life, thus, will enhance the life of the structure. These numerical results are in good agreement with the available test results. This analysis demonstrates the effectiveness of finite element analysis in such steel joints in evaluating the fatigue life for the future works.
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