Perturbative unification of gauge couplings in supersymmetric $E_6$ models

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Abstract

We study gauge coupling unification in supersymmetric $E_6$ models where an additional U(1)$'$ gauge symmetry is broken near the TeV scale and a number of exotic matter fields from the 27 representations have $O$(TeV) mass. Solving the 2-loop renormalization group equations of gauge couplings and a kinetic mixing coupling between the U(1)$'$ and U(1)$_Y$ gauge fields, we find that the gauge couplings fall into the non-perturbative regime below the GUT scale. We examine threshold corrections on the running of gauge couplings from both light and heavy ($\sim$ GUT scale) particles and show constraints on the size of corrections to achieve the perturbative unification of gauge couplings.
Grand unification of electromagnetic, weak and strong interactions is an attractive feature of minimal supersymmetric standard model (MSSM). In grand unified theories (GUT), all matter fields in the MSSM are embedded in some larger representations of a certain GUT group such as SU(5). Among candidates of GUT groups, $E_6$ is known as a gauge group which is anomaly free, and each generation of quarks, leptons and Higgs superfields are embedded into one representation, i.e., $27$ (for a review, see [1]). The $E_6$ group could be decomposed as follows at the GUT scale:

$$E_6 \supset SO(10) \times U(1)_\psi \supset SU(5) \times U(1)_{\chi} \times U(1)_{\psi}. \quad (1)$$

From phenomenological point of view, it is often expected that one of two extra U(1) symmetries (hereafter we call it U(1)$'$) remains unbroken until the TeV scale. Then, many extra particles beyond the MSSM in $27$ may have the mass of order TeV.

In a certain extension of the MSSM, three gauge couplings are still unified at the unification scale $m_{\text{GUT}} \simeq 2 \times 10^{16}$ GeV in the leading order if the extra charged fields beyond the MSSM are embedded into a vector-like pair of complete multiplet of SU(5), e.g., $5 + \overline{5}$ or $10 + 10$. The number of such extra fields, however, is constrained from the perturbativity of gauge couplings [2]. The $27$ representation in $E_6$ contains one generation of quark/lepton superfields, an extra pair of $5 + \overline{5}$ and two SM singlet. The Higgs superfields whose scalar components break the electroweak symmetry should come from some other representations. The supersymmetric (SUSY) $E_6$ models, therefore, have at least three pairs of $5 + \overline{5}$ in addition to the MSSM fields as its low-energy spectrum.

In this paper, we study the gauge coupling unification in SUSY-$E_6$ models taking account of the kinetic mixing between $U(1)_Y$ and $U(1)'$ beyond the leading order of renormalization group equations (RGE) [3]. We solve the 2-loop RGE of three $(SU(3)_C, SU(2)_L, U(1)_Y)$ gauge couplings, the U(1)$'$ gauge couplings and the U(1)$_Y$-U(1)$'$ kinetic mixing couplings. Because of a number of extra particles beyond the MSSM, the running of gauge couplings in the SUSY-$E_6$ models are asymptotic non-free, and the gauge couplings at the GUT scale is no longer perturbative. We examine constraints on the threshold corrections of light ($\sim$ TeV scale) and heavy ($\sim$ GUT scale) particles from the perturbativity and experimental data.

We first briefly review the SUSY-$E_6$ models. As is already mentioned, a linear combination of $U(1)_\chi$ and $U(1)_\psi$ in [1] is assumed to remain in low-energy and the gauge boson $Z'$ is parametrized as

$$Z' = Z_\chi \cos \beta + Z_\psi \sin \beta. \quad (2)$$

As shown in Table [1], there are some variants of $E_6$ models corresponding to the value of mixing angle $\beta$. The couplings of matter fields and $Z'$ boson are trivial in both $\chi$- and

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1 The running of gauge couplings with the kinetic mixing in SUSY-$E_6$ models in the 1-loop level has been studied in refs. [3,4,5].
The fundamental representation $27$ in $E_6$ can be decomposed into representations in $SO(10)$ and $SU(5)$ as follows:

$$27 = \{ 16 + 10 + 1 \}_{SO(10)} = \{ (10 + 5 + 1) + (5 + 5) + 1 \}_{SU(5)}. \quad (3)$$

The $U(1)'$ charge $Q'$ of all the matter fields in a $27$ representation for the $\eta$-model is summarized in Table 2. The normalization of $U(1)'$ charge follows that of the hypercharge. The $U(1)'$ symmetry breaking could occur at near the weak scale radiatively. Discussions on this issue can be found, e.g., in refs. [10, 11].

The Lagrangian of neutral gauge bosons ($A^0$ which is a photon in the SM, $Z$ and $Z'$) is given by

$$\mathcal{L} = -\frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} - \frac{1}{4} Z'_{\mu\nu} Z'^{\mu\nu} - \frac{\sin \chi}{2} B_{\mu\nu} Z'^{\mu\nu} - \frac{1}{4} A^0_{\mu\nu} A^{0,\mu\nu} + m^2_{ZZ'} Z_{\mu} Z'^{\mu} + \frac{1}{2} m^2_{Z'} Z'_{\mu} Z'^{\mu} \quad , \quad (4)$$

where $V_{\mu\nu} = \partial_{\mu} V_{\nu} - \partial_{\nu} V_{\mu}$ for a gauge boson $V$. The parameter $\chi$ denotes the kinetic mixing angle between the hypercharge gauge boson $B$ and the $U(1)'$ gauge boson $Z'$. The mass eigenstates ($Z_1, Z_2, A$) are obtained from the gauge eigenstates ($Z, Z', A^0$) via the mass and kinetic mixing angles $\xi$ and $\chi$, respectively (see, ref. [11]). In the limit of $\xi = 0$, a shift of the

\begin{table} 
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
model & $\chi$ & $\psi$ & $\eta$ & $\nu$ \\
\hline
$\beta$ & 0 & $\pi/2$ & $\tan^{-1}(-\sqrt{5}/3)$ & $\tan^{-1}(\sqrt{15})$ \\
\hline
\end{tabular}
\caption{various $E_6$ models versus the mixing angle $\beta$}
\end{table}
U(1)′ charge to the fermions is found as
\[ L = \bar{\psi} \gamma^\mu \left( eQA_\mu + g_Z (I_3 - Q) \sin^2 \theta_W \right) Z_{1\mu} + \frac{g_E}{\cos \chi} \left( Q' - Y \frac{g_Z}{g_E} \sin \theta_W \sin \chi \right) Z_{2\mu} \psi. \] (5)
The “off-diagonal” gauge coupling \( g_{1E} \) and the kinetic mixing parameter \( \delta \) are defined as
\[ g_{1E} \equiv -g_Z \sin \theta_W \sin \chi, \] (6)
\[ \delta \equiv \frac{g_{1E}}{g_E}. \] (7)
Then, in the limit of \( \xi = 0 \), the coupling of \( Z_2 \) boson to the SM fermions are simply given by
\[ \tilde{Q}' = Q' + Y\delta. \] (8)
Note that, in this limit, the couplings of \( Z_2 \) to leptons in the \( \eta \)-model vanish when \( \delta = 1/3 \).

Next we discuss the gauge coupling unification in SUSY-\( E_6 \) models. It should be noted that three gauge couplings are not unified in the SUSY-\( E_6 \) models with three generations of \( 27 \) at the TeV scale. This is because it does not much the unification condition on extension of the MSSM by introducing couples of \( 5 + \overline{5} \). For the gauge coupling unification, at least a pair of SU(2) doublet (\( 2 + \overline{2} \)) should be added to the particle spectrum \(^3\). The origin of the additional pair of SU(2) doublet could be either \( 27 + \overline{27} \) or \( 78 \) in \( E_6 \). It should be explained why \( 2 + \overline{2} \) remains massless while the others decouple in a large representation. We will return this point and mention some possibilities later. In the following study, we take the U(1)′ charge of the additional SU(2) doublet to be \( \pm \frac{1}{6} \), i.e., same with \( L \) of \( 27 \) and its counter partner of \( \overline{27} \). We also study the model of Babu et al. \(^{12}\) where two pairs of \( 2 + \overline{2} \) from \( 78 \) and a pair of \( 3 + \overline{3} \) from \( 27 + \overline{27} \) are added to the \( \eta \)-model in order to achieve the quasi-leptophobicity (\( \delta \sim \frac{1}{3} \)) through the 1-loop RGE. We refer this model as the \( \eta_{\text{BKM}} \)-model in the rest of this paper. A similar study on gauge coupling unification in SUSY-\( E_6 \) models has been presented in ref. \(^{13}\) focusing on the \( \chi \)-model. The authors in ref. \(^{13}\) neglected the kinetic mixing parameter \( \delta \) in their analysis because \( \delta \) generated radiatively through the 1-loop RGE is quite small. On the other hand, we investigate the \( \eta \)- and \( \eta_{\text{BKM}} \)-models taking account of the 2-loop RGE of kinetic mixing parameters as well as gauge couplings since, as mentioned above, \( \delta \) could become sizable in these models and a \( Z' \) boson could be leptophobic which is phenomenologically attractive.

The RGE of gauge couplings \( g_i \) (\( i = 1 \sim 5 \)) is given by
\[ \frac{dg_i}{dt} = \beta_i^{(1)} + \beta_i^{(2)}, \] (9)
\[ t \equiv \ln \mu, \] (10)
\(^3\) The other choices are \( H_d \) of \( 27 \) or \( H_u \) of \( \overline{27} \).
where \( \mu \) stands for the renormalization scale, and \( \beta^{(1)}_i \) and \( \beta^{(2)}_i \) denotes the \( \beta \)-function in 1- and 2-loop levels, respectively. We adopt the SU(5) normalization for the U(1) couplings and charges, i.e.,

\[
\begin{align*}
g_1 &= \sqrt{\frac{5}{3}} g_Y, \quad g_4 = \sqrt{\frac{5}{3}} g_E, \quad g_5 = \sqrt{\frac{5}{3}} g_{1E}, \\
Q_1 &= \sqrt{\frac{3}{5}} Y, \quad Q_E = \sqrt{\frac{3}{5}} Q'.
\end{align*}
\]

The 1-loop part of the \( \beta \) functions is given by

\[
\begin{align*}
\beta^{(1)}_1 &= \frac{1}{16\pi^2} b_1 g_1^3, \\
\beta^{(1)}_4 &= \frac{1}{16\pi^2} \left\{ g_4 \left( b_E g_4^2 + b_1 g_5^2 + 2b_{1E} g_4 g_5 \right) \right\}, \\
\beta^{(1)}_5 &= \frac{1}{16\pi^2} \left( b_E g_5 g_4^2 + b_1 g_3^2 + 2b_1 g_5^2 g_5 + 2b_{1E} g_4^2 g_4 + 2b_1 g_5^2 g_4 \right), \\
\beta^{(1)}_N &= \frac{1}{16\pi^2} b_N g_5^3 \quad (\text{for } N=2, 3).
\end{align*}
\]

The coefficients \( b_1, b_E \) and \( b_{1E} \) are given by

\[
b_1 = \text{Tr}(Q_1^2), \quad b_E = \text{Tr}(Q_E^2), \quad b_{1E} = \text{Tr}(Q_1 Q_E),
\]

while the coefficient of r.h.s. in eq. (16) is given by

\[
b_N = \sum T(N) - 3C_2(G),
\]

where

\[
T(N) = \frac{1}{2}, \quad C_2(N) = \frac{N^2 - 1}{2N}, \quad C_2(G) = N,
\]

for a fundamental representation \( N \) and an adjoint representation \( G \) of SU(\( N \)). It should be summed over all charged fields under SU(\( N \)). The coefficients \( b_i \) (\( i = 1, 2, 3 \)), \( b_E \) and \( b_{1E} \) in the \( \eta \)- and \( \eta_{BKM} \)-model are summarized in Table 3, where the factor \( a \) denotes the U(1)' charge of additional SU(2) doublets. The 1-loop RGE can be solved easily by assuming \( g_1 = g_2 = g_3 = g_4 \) and \( \delta = 0 \) (\( g_5 = 0 \)) at the GUT scale. The magnitude of extra U(1) coupling \( g_E \) and the kinetic mixing parameter \( \delta (= g_{1E}/g_E) \) at the \( m_Z \) scale are \( g_E/g_Y = 1.03 \) and \( \delta = 0.018 \) for the \( \eta \)-model with \( a = 1/6 \) while \( g_E/g_Y = 0.86 \) and \( \delta = 0.29 \) for the \( \eta_{BKM} \)-model, where the U(1)_Y gauge coupling \( g_Y \) is fixed at \( g_Y = 0.36 \) \cite{4}. We note that the kinetic mixing parameter \( \delta \) in the \( \eta_{BKM} \)-model is close to the leptophobicity condition, \( \delta = 1/3 \). The coefficients of 1-loop \( \beta \)-functions summarized in Table 3 tell us that gauge couplings \( g_1 \sim g_4 \) are asymptotically non-free and the running of gauge couplings is expected to be affected by taking account of the 2-loop contributions.

\footnote{The RGE given in this paper is based on the interactions in eq. (5). We note here that the RGE with the kinetic mixing between two U(1) in a most general ("symmetric") basis has been given in ref. [14].}
We take \( a = \frac{1}{6} \) in our analysis.

The 2-loop contributions to the RGE for \( g_1, g_4 \) and \( g_5 \) are summarized as

\[
(16\pi^2)\beta_1^{(2)} = 4Q_1^{4}g_1^5 + 4Q_1^{3}g_1^3g_5^2 + 8Q_1^{3}Q_Eg_1^3g_4 + 4Q_1^{2}Q_E^2g_1^3g_4^2 + \sum_{N=2,3} 4C_2(N)Q_1^{3}g_1^3g_N^2, \tag{20}
\]

\[
(16\pi^2)\beta_4^{(2)} = 4Q_E^{4}g_4^5 + 16Q_4^{3}Q_Eg_5g_4^4 + 4Q_4^{2}Q_E^2g_1^3g_4^2 + 24Q_4^{2}Q_E^2g_5^3g_4^2 + 8Q_4^{1}Q_E^3g_1^3g_4^2 + 16Q_4^{3}Q_Eg_5g_4^2 + 4Q_4^{2}g_1^3g_4^2 + 4Q_4^{2}g_5^2g_4 + \sum_{N=2,3} C_2(N) \left( 4Q_E^{4}g_4^3 + 8Q_1Q_Eg_5g_4^2 + 4Q_1^3g_5^2g_4 \right) g_N^2, \tag{21}
\]

\[
(16\pi^2)\beta_5^{(2)} = 4Q_1^{4}g_5^5 + 16Q_4^{3}Q_Eg_5g_4^4 + 12Q_1^{4}g_5^2g_3^2 + 24Q_1^{2}Q_5^{2}g_1^2g_4^3 + 32Q_1^{3}Q_E^2g_1^2g_5^2 + 16Q_4^{3}Q_Eg_5g_4^2 + 8Q_1^{2}Q_E^3g_1^2g_4^2 + 28Q_4^{2}Q_E^2g_1^2g_4^2 + 4Q_4^{2}Q_E^2g_5^2g_4 + 8Q_1Q_4^3Q_E^2g_1^2g_4 + 8Q_1Q_4^3Q_E^3g_1^2g_4 + \sum_{N=2,3} C_2(N)g_N \left( 8Q_4^{2}g_5^2g_4 + 4Q_4^{2}g_5^3 + 8Q_1Q_Eg_5^2g_4 \right) + 8Q_1Q_Eg_5^2g_4 + 4Q_1^3g_5^2g_4^3, \tag{22}
\]

where the trace over all charged fields under the gauge groups are understood. The explicit values of \((Q_1^1, Q_4^3Q_E, Q_1^2Q_E, Q_1^4Q_E)\) in each model are summarized in Table 4.

| \( \eta \) | \( b_1 \) | \( b_2 \) | \( b_3 \) | \( b_E \) | \( b_{1E} \) |
|----------------|--------|--------|--------|--------|--------|
| \( \bar{\eta}_{\text{BKM}} \) | \( \frac{28}{5} \) | 4 | 0 | \( 9 + \frac{12}{a} \) | \( -\frac{9}{a} \) |
| \( \eta_{\text{BKM}} \) | \( \frac{23}{5} \) | 5 | 1 | \( \frac{77}{5} \) | \( -\frac{16}{5} \) |

Table 4: Explicit values of factors in eqs. (20), (21) and (22). We take \( a = \frac{1}{6} \) in our study.

The 2-loop contributions to the RGE of non-abelian couplings \( g_N (N = 2, 3) \) is given as
Figure 1: Predictions on $\alpha(m_Z)$ (black), $\alpha_s(m_Z)$ (red) and $\sin^2 \theta_W(m_Z)$ (blue) in the $\eta$-model with $a = \frac{1}{6}$. Each band stands for the region on $(m_{\text{GUT}}, r_{\text{GUT}})$ plane which satisfies $\Delta \chi^2 < 4$.

follows

$$(16\pi^2)^2 \beta_N^{(2)} = g_N^5 \left[ \{2C_2(G) + 4C_2(N)\} T(N)d(N') - 6 \{C_2(G)\}^2 \right]$$

$$+ g_N^2 g_N^2 \{4C_2(N')T(N)d(N')\}$$

$$+ g_N^3 \left( 2Q_1^2 g_1^2 + 2Q_1^2 g_5^2 + 4Q_1 Q_E g_5 g_4 + 2Q_E^2 g_1^2 \right) d(N'), \quad (23)$$

where $g_{N'}$ and $d(N')$ denote the gauge coupling and the dimension of charged fields of another non-abelian gauge group $\text{SU}(N')$, respectively. For example, $d(N') = 3$ (1) for a triplet (singlet) in $\text{SU}(3)$ for $N = 2$. In RGE of $g_i$ ($i = 1-5$), contributions from the Yukawa couplings of fermions at the 2-loop level are not included. Since the Yukawa couplings of exotic fermions and the flavor mixings are model dependent, including their effects increases the model parameters and makes the analysis complicated. The impacts of the Yukawa couplings to our results will be discussed later.

Next we test the gauge coupling unification in SUSY-$E_6$ models numerically. We solve the 2-loop RGE from the GUT scale taking the unification scale $m_{\text{GUT}}$ and the unified gauge coupling $\alpha_{\text{GUT}}$ as inputs. In the analysis, we introduce a ratio of $\alpha_{\text{GUT}}$ in 1- and 2-loop levels as;

$$r_{\text{GUT}} \equiv \frac{\alpha_{\text{GUT}}^{2-\text{loop}}}{\alpha_{\text{GUT}}^{1-\text{loop}}}, \quad (24)$$
where $\alpha_{\text{GUT}}$ is found by solving the 1-loop RGE, i.e., $1/\alpha_{\text{GUT}}^{1-\text{loop}} = 3.17$. Then we obtain the input $\alpha_{\text{GUT}} \equiv \alpha_{\text{GUT}}^{2-\text{loop}}$ by varying $r_{\text{GUT}}$. In general, the unification scale $m_{\text{GUT}}$ is understood as a scale where the SU(5) symmetry is broken to the SM gauge group, and the $E_6$ symmetry breaking scale may be higher than $m_{\text{GUT}}$. In our study, however, we assume that the $E_6$ symmetry is broken at $m_{\text{GUT}}$ since the $\eta$-model is obtained when $E_6$ is directly broken to a rank 5 group as is already mentioned above. We also introduce the intermediate scale $m_I$ in which the threshold corrections by exotic particles beyond the MSSM in SUSY-$E_6$ models are switched on, i.e., the $\beta$-functions in SUSY-$E_6$ model change to MSSM at $m_I$. The mass scale of all MSSM particle are assumed to be 1 TeV. Solving the 2-loop RGE with these input parameters, we compare the gauge couplings at the $m_Z$ scale with the experimental values \[1\]

\[
1/\alpha(m_Z) = 127.944 \pm 0.014, \tag{25}
\]

\[
\alpha_s(m_Z) = 0.1185 \pm 0.0006, \tag{26}
\]

\[
sin^2 \theta_W(m_Z) = 0.23116 \pm 0.00012. \tag{27}
\]

We give our results for $m_I = 4, 5, 6$ and 7 TeV in Figs. 1 ($\eta$-model with $a = 1/6$) and 2 ($\eta_{\text{BKM}}$-model). These figures show the regions which satisfy $\Delta \chi^2 < 4$ for $\alpha(m_Z)$, $\alpha_s(m_Z)$ and $\sin^2 \theta_W(m_Z)$ on the $(m_{\text{GUT}}, r_{\text{GUT}})$ plane by blue, red and black bands, respectively. Three gauge couplings ($g_1, g_2, g_3$) are successfully unified when three bands cross each other on the $(m_{\text{GUT}}, r_{\text{GUT}})$ plane. No such a crossing of three bands, however, is found in the figures. Discrepancies of three bands decreases as $m_I$ increases, since the running of gauge couplings coincides with that in the MSSM in the limit of $m_I \to m_{\text{GUT}}$. The intermediate scale $m_I$ is, therefore, required to be high for the coupling unification of SUSY-$E_6$ models. We note that the kinetic mixing parameter $\delta$ in eq. (7) is highly suppressed as $O(10^{-2} - 10^{-3})$ in a wide range of parameter space.

Finally we discuss threshold corrections from the heavy particles whose mass scale is around the GUT scale. We have mentioned particle spectrum which is charged under the U(1)$'$ gauge symmetry i.e., three 27 representations and extra matters for the coupling unification. We have so far not discussed, however, the heavy particles such as the Higgs fields to break the $E_6$ symmetry. Although such fields decouple from the light spectrum by getting the GUT scale mass, these fields may contribute to the running of gauge couplings near the GUT scale. Such corrections are called the heavy particle threshold corrections which are proportional to $c \times \ln M/m_{\text{GUT}}$ where $M$ is a mass of the field and the coefficient $c$ is determined by the charge of the field under $E_6$. Since we have not considered concrete heavy spectrum of SUSY-$E_6$ models which will decouple after the $E_6$ breaking, we estimate the magnitude of threshold corrections from the heavy particles required for the successful coupling unification.

Introducing a parameter $\Delta$, which accounts for the threshold corrections from the heavy
particles, the gauge couplings at the $m_Z$ scale can be expressed as

$$\frac{1}{\alpha_i(m_Z)} = \frac{1}{\alpha_i(m_Z)_{1+2\text{loops}}} + \Delta_i.$$  

We perform the $\chi^2$-fit of $\Delta_i$ to the data of $\alpha(m_Z), \alpha_s(m_Z)$ and $\sin^2 \theta_W(m_Z)$ in eqs. (25)-(27) for the intermediate scale $m_I = 5$ and 10 TeV, and results are summarized in Table 5. As expected, $\Delta_i$ is required to be sizable for the coupling unification when the intermediate scale $m_I$ is smaller.

To summarize, we have studied the gauge coupling unification in the SUSY-$E_6$ model taking account of both the $U_Y$-$U(1)'$ mixing and the 2-loop contributions to the RGE. The minimal model which maintains the coupling unification consists of three generation of $27$ and a pair of $2 + \bar{2}$. As an example, we focused on the $\eta$-model which breaks the $E_6$ symmetry directly into the SM gauge group. We also studied the $\eta_{\text{BKM}}$-model where two $2 + \bar{2}$ and one $3 + \bar{3}$ are added to three generations of $27$. We found that results in the 1-loop RGE are significantly affected by 2-loop corrections, and constraints on experimental measurements of $\alpha, \alpha_s$ and $\sin^2 \theta_W$ at the $m_Z$ scale require the intermediate scale $m_I$ to be much higher than $O(1 \text{ TeV})$. We also obtained constraints on the size of threshold corrections from heavy particles $\Delta_i$ to achieve the coupling unification.

A few comments are in order. Throughout our analysis, we neglected contributions from the Yukawa couplings of fermions for simplicity. Since, in general, the Yukawa couplings negatively contribute to the running of gauge couplings, they might affect the results if they
are not negligible. In our analysis, we have so far expressed the threshold corrections from heavy particles by model independent parameters $\Delta_i$ in eq. (28). The parameters $\Delta_i$, however, can be understood to represent a sum of contributions from the Yukawa couplings and the heavy threshold corrections, and the combinations of two contributions are constrained by experimental data as shown in Table 5.

We also comment on how additional massless $2 + \bar{2}$ at the GUT scale originated from $E_6$ multiplets. One way is the sliding singlet mechanism [16] or the missing partner mechanism [17], which have been known as solutions to the doublet-triplet splitting problem in the SU(5) GUT. Another way is the mechanism in extra dimensional models [18]. Let us suppose a five dimensional model compactified on an orbifold $S^1/Z_2$ and the compactification scale is the GUT scale. We consider the case where the SM fields except for Higgs doublets are localized on the fixed points while the gravity and the $E_6$ multiplets including Higgs fields propagate in the bulk. The $E_6$ gauge symmetry is broken by the boundary conditions ($Z_2$ parity). If only the Higgs doublet components $2 + \bar{2}$ are assigned to be $Z_2$ even and the others are $Z_2$ odd, then only $2 + \bar{2}$ remains to be massless at the GUT scale and the others have at least the GUT scale masses.

Table 5: Constraints on the heavy particle threshold corrections $\Delta_i$ for $m_I = 5$ and 10 TeV. The correlation $\rho$ between the error in $\Delta_1$ and that in $\Delta_2$ is given as $\rho = -0.68$ in each case.

| $m_I = 5$ TeV | $\eta$-model | $\eta_{\text{BKM}}$-model |
|--------------|--------------|--------------------------|
| $\Delta_1$  | $-1.013 \pm 0.011$ | $-0.990 \pm 0.011$ |
| $\Delta_2$  | $-1.006 \pm 0.016$ | $-0.875 \pm 0.016$ |
| $\Delta_3$  | $0.011 \pm 0.043$ | $0.494 \pm 0.043$ |

| $m_I = 10$ TeV | $\eta$-model | $\eta_{\text{BKM}}$-model |
|----------------|--------------|--------------------------|
| $\Delta_1$    | $-0.677 \pm 0.011$ | $-0.542 \pm 0.011$ |
| $\Delta_2$    | $-0.672 \pm 0.016$ | $-0.429 \pm 0.016$ |
| $\Delta_3$    | $0.344 \pm 0.043$ | $0.948 \pm 0.043$ |

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