Dynamical phonon-induced dephasing of an optically controlled single spin in a quantum dot

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Abstract. We study the dephasing mechanism due to carrier-phonon interaction of an operation on a single spin in a quantum dot performed by optical means. We calculate the total error of the spin-based quantum gate taking into account the influence of the carrier coupling to the phonon and photon fields as well as the imperfections of the unitary evolution such as off-resonant excitations and the nonadiabaticity of the driving. We give quantitative estimation for a $\pi$ rotation of a spin in two types of quantum dots: self-assembled InAs/GaAs and fluctuation GaAs quantum dots, and discuss the different interplay of the constituent sources of the errors.

1. Introduction

The spin of an electron confined in a quantum dot (QD) has attracted great interest due to its long coherence time [1] and has been proposed as a qubit for implementing quantum information processing schemes [2, 3]. Spin dependent optically induced charge dynamics can be realized in QD systems by means of Pauli exclusion principle and optical selection rules [4, 5], which allows one to coherently control the spin. The advantage of using optical control methods are the short switching times in comparison with nanosecond magnetic control of the spin. The recent experimental demonstration of the generation [6] and optical control [7] of a spin coherence, as well as possible read-out of the state of a single confined spin in a QD system [8] has shown, that the implementation of a quantum computer can be feasible. It has been recently proposed [9] to use the two Zeeman-split spin states of a single electron in a doped QD for storage of quantum information. Optical coherent control of a spin is possible via adiabatic Raman transitions and off-resonant coupling to a trion state, which can lead to an arbitrary spin rotation.

However, QDs are solid state structures and their confined carriers interact with the phonon environment, which leads to loss of coherence. Since the spin rotation is achieved by charge evolution the lattice response to the evolving charge density [10, 11] results in the decoherence processes. During the procedure of the rotation of a spin in a doped QD [9] a trion occupation is also induced. In consequence, radiative decay of the trion can also contribute to decoherence [12]. In addition, the ideal operation should be performed adiabatically and imperfect adiabaticity of the real evolution will also contribute to the total error. Moreover, the rotation is performed by means of frequency selective excitations (the two Zeeman-split spin states are coupled to one
trion state by $\sigma_+$-polarized laser pulses), and the off-resonant excitations can lead to an unwanted leakage to the trion state and thus to large discrepancies from the desired spin rotation.

2. Results

In this contribution, we study the influence of the phonon- and photon-induced decoherence mechanisms and of the imperfections of the evolution on a spin rotation in a QD system performed by optical means [9]. We give quantitative estimations of the total error of a $\pi$ rotation about the $z$ axis (growth direction) for two kinds of QDs: self-assembled InAs/GaAs and fluctuation GaAs systems. We indicate the dominant source of error for each QD type and study the interplay of the different contributions. Optimal conditions of the spin rotation are indicated and possible optimization is discussed.

We study a single semiconductor QD doped with one additional electron. A magnetic field is applied in the $x$ direction (Voigt configuration) and causes a Zeeman splitting between the electron states with spin-up and spin-down (see Fig. 1a). These two spin states define the logical qubit states $|0\rangle$ and $|1\rangle$ and are off-resonantly coupled to a trion state $|2\rangle$ by $\sigma_+$-polarized laser pulses with frequencies $\omega_0$ and $\omega_1$, respectively, that differ by the Zeeman splitting $\Delta_B$, thus satisfying the Raman resonance condition (see Fig. 1b). As was shown in Refs. [9] and [13], the system evolution is realized by the change of the pulse amplitudes and leads to an arbitrary rotation of the spin. The angle of rotation is determined by the pulse amplitudes and the detuning $\Delta$ from the corresponding transition energies. The axis of rotation depends on the ratio of the pulse envelopes and on the relative phase of the pulses $\gamma$. In the following discussion, we fix the pulse duration and treat the detuning as a tunable parameter while the pulse amplitudes are adjusted to achieve the desired rotation of the qubit. A detailed description of the model and the spin rotation procedure can be found in Ref. [13].

In order to study the decoherence mechanism due to the carrier-phonon interaction of a spin rotation in a QD, we use a perturbative theory describing the open system evolution under arbitrary driving [10, 11, 13]. The effect of the carrier interaction with the phonon reservoir is calculated using the second-order Born expansion of the evolution equation for the density matrix. We include the effect of the driving field exactly and treat the carrier-phonon interaction within a perturbation theory (see Ref. [13] for details). The carrier-phonon Hamiltonian reads $H_{c-\text{ph}} = |2\rangle\langle 2| \sum_k F_{22}(k) \left( b_k + b_k^\dagger \right)$, where $b_k$ and $b_k^\dagger$ are phonon annihilation and creation operators, respectively, and $F_{22}(k)$ is the coupling constant. We calculate the total error of a spin rotation through an angle of $\pi$ about the $z$ axis (growth direction), performed by means of the Gaussian pulses. We consider two types of QDs: a small self-assembled InAs/GaAs QD with the wave function width in-plane $l = 5$ nm and in the $z$ direction $l_z = 1$ nm, and large fluctuation GaAs QD with $l = 15$ nm and $l_z = 2$ nm.

The error due to carrier-phonon interaction in the considered system has two origins. One source is the pure dephasing effects [14, 15] and can be decreased by means of large detunings and long pulse durations. The second contribution is the phonon-assisted transition to the trion state, thus cannot be decreased by slowing down the evolution and becomes the dominant source for the total error.
The phonon-induced errors as a function of detuning for a fixed pulse duration $\tau_p = 30$ ps for the small self-assembled (solid lines) and large fluctuation (dashed lines) QDs. The total error $\delta_{\text{tot}}$ as a function of detuning for self-assembled QDs together with the constituent sources: the phonon-induced error $\delta_{\text{ph}}$, the photon-induced error $\delta_{\text{rad}}$, and the error due to the unitary corrections $\delta_u$. (c) As in (b) but for fluctuation QDs.

The carrier-phonon interaction strongly depends on the confinement size: the influence of the phonon environment is stronger for smaller QDs (see Fig. 2b,c). For small detunings, the difference between the error for two QD types is small, but it becomes significant for larger detunings. At higher temperatures and for detunings of several ps$^{-1}$, the error initially increases with growing detuning and then decreases for large detunings of several ps$^{-1}$. At low temperature, the error is almost constant for small detunings and decreases for detunings larger than 0.2 ps$^{-1}$. The maximum values of the error correspond to high probabilities of the phonon absorption by the carriers.

In the considered procedure of spin rotation, not only coupling of the carriers to the phonon environment contributes to the total error. The spin rotation requires a certain occupation of the trion, which leads to an additional decoherence mechanism resulting from the nonzero probability of the radiative decay of the trion. The effect of the photon interaction is calculated using the perturbation theory as for the phonons. The carrier-phonon interaction reads $H_{c-\text{rad}} = \frac{i}{\sqrt{2}} \sum_{q,\lambda} g_{q,\lambda} c_{q,\lambda}^\dagger \left( e^{i\omega t} |0\rangle \langle 2| + e^{(i\omega t - \gamma)} |1\rangle \langle 2| \right) + \text{H.c.}$ with coupling constants $g_{q,\lambda}$ and photon creation operators $c_{q,\lambda}^\dagger$. The radiative error $\delta_{\text{rad}}$ is proportional to the trion lifetime $\tau_{\lambda}$ which depends on the type and size of the QD, and decreases with the growing size of the system. We assume that the trion lifetime is $\tau_{\lambda} = 1$ ns in a self-assembled QD [16]. In fluctuation QDs, the radiative lifetime is much shorter [17]; we will assume $\tau = 200$ ps. The contribution to the error due to the trion radiative decay decreases with growing detuning (see Fig. 2b,c), since the trion occupation is then reduced. For small detunings, the system is excited almost resonantly and the trion occupation is large, which leads to large radiative errors, which in this regime are linear in the pulse duration $\tau_p$, $\delta_{\text{rad}} \approx \tau_p / \tau_{\lambda}$. In order to suppress the influence of the photon environment, one needs large detunings of tens of ps$^{-1}$.

Apart from the influence of the carrier coupling to the phonon and photon environments, the imperfections of the unitary evolution also contribute to the total error of the operation on a spin-based qubit. The requirement of adiabatic evolution as well as the effect of the off-resonant
excitations impose limitations on the coherent control of the spin. In realistic experiments, it is not possible to change the pulse amplitudes infinitely slowly, which leads to additional errors. Moreover, if the values of the detuning approach the Zeeman splitting $\Delta_B = 1.5$ ps$^{-1}$, the error is very large, since one spin state is then almost resonantly coupled to the trion state, which induces a large trion occupation inhibiting the coherent control of the spin. This error due to these two limitations, $\delta_u$, is independent of the size of the QD (see Fig. 2b,c), since it results from the conditions of the driving and the resulting evolution. This error is large around the resonance at $\Delta = \Delta_B$ and also for small detunings, when the adiabatic condition is not met. In the latter case it is relatively small because relatively long pulse duration $\tau_p = 30$ ps is assumed.

In order to choose the proper conditions for the spin rotation leading to a high fidelity, one has to take into account all the above discussed contributions to the total error. Let us discuss the interplay of the constituent errors for the two types of QDs. For both quantum dot types, one has to avoid the detunings with values close to the Zeeman splitting. For self-assembled QDs (see Fig. 2b), the phonon-induced error is dominant, which results from the strong confinement. In this system, the trion decay rate is slow, which leads to the smaller radiative error in comparison with the phonon-induced one (for detunings smaller than $15$ ps$^{-1}$). In the case of the large fluctuation QDs, the strength of the confinement is smaller and the trion lifetime is much shorter. The phonon-induced error is now dominant only in the range of the detuning between 0.2 and 2 ps$^{-1}$. For other detunings, the dominant source of error is the carrier coupling to the photon environment. For shorter trion lifetimes, this error would linearly decrease with the detuning (apart from the vicinity of the Zeeman splitting). The influence of the contributing errors, which have many different origins and depend differently on the detuning, leads to a nontrivial total error $\delta_{tot}$. To achieve small errors of the spin rotation, large detunings are needed. For coherent quantum control, errors with values below $10^{-4}$ are required, which can be achieved only for small QDs with detunings of tens of ps$^{-1}$ at $T = 10$ K. In the case of fluctuation QDs, for large detunings the error is at least one order of magnitude larger than in the case of self-assembled QDs.

3. Conclusion

To summarize, we have studied the different sources of the error of an optically controlled single spin in a QD. We have considered two types of QDs and discussed the different influence of the contributions to the total error indicating the dominant source. This has allowed us to identify self-assembled QDs as the most favorable structures and to find the optimal driving conditions for the coherent quantum control.

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References

[1] Hanson R, Witkamp B, Vandersypen L M K, Willems van Beveren I H, Elzerman J M and Kouwenhoven L P 2003 Phys. Rev. Lett 91 196802
[2] Calarco T, Datta A, Fedichev P, Pazy E and Zoller P 2003 Phys. Rev. A 68 123310
[3] Feng M, D’Amico I, Zanardi P and Rossi F 2003 Phys. Rev. A 67 014306
[4] Pazy E, Bialatti E, Calarco T, D’Amico I, Zanardi P, Rossi F and Zoller P 2003 Europhys. Lett. 62 175
[5] Emary C and Sham L J 2007 J. Phys. Cond. Matt. 19 056203
[6] Dutt M V G, Cheng J, Li B, Xu X, Li X, Berman P R, Steel D G, Bracker A S, Gammon D, Economou S E, Liu R-B and Sham L J 2005 Phys. Rev. Lett. 94 227403
[7] Greilich A, Oulton R, Zhukov E A, Yugova I A, Yakovlev D R, Bayer M, Shabaev A, Efros A L, Merkulov I A, Stavarache V, Reuter D and Wieck A 2006 Phys. Rev. Lett. 96 227401
[8] Atatüre M, Dreiser J, Badolato A and Imamoglu A 2007 Nature Phys. 3 101
[9] Chen P, Piemarchocci C, Sham L J, Gammon D and Steel D G 2004 Phys. Rev. B 69 075320
[10] Roszak K, Grodecka A, Machnikowski P and Kuhn T 2005 Phys. Rev. B 71 195333
[11] Grodecka A, Jacak L, Machnikowski P and Roszak K. 2005 in: Ling P. A, editor, *Quantum Dots: Research Developments* (Nova Science Publisher, NY) p 47 (Preprint cond-mat/0404364)

[12] Caillet X and Simon C 2007 *Eur. Phys. J. D* **42** 341

[13] Grodecka A, Weber C, Machnikowski P and Knorr A 2007 *Phys. Rev. B* in press (Preprint arXiv.org:0706.1989)

[14] Förstner J, Weber C, Danckwerts J and Knorr A 2003 *Phys. Rev. Lett.* **91** 127401

[15] Alicki R, Horodecki M, Horodecki P, Horodecki R, Jacak L and Machnikowski P 2004 *Phys. Rev. A* **70** 010501(R)

[16] Langbein W, Borri P, Woggon U, Stavarache V, Reuter D and Wieck A D 2004 *Phys. Rev. B* **70** 033301

[17] Li X, Wu Y, Steel D, Gammon D, Stievater T, Katzer D, Park D, Piermarocchi C and Sham L J 2003 *Science* **301** 809