Abstract

We study the radiative decay $Z \to \nu \bar{\nu} \gamma$ within an effective Lagrangian approach. Using the search for energetic single–photon events in the data collected by the L3 Collaboration, we get direct bounds on dimension–six and dimension–eight operators associated with the $\tau$–neutrino magnetic moment and the anomalous electromagnetic properties of the $Z$ boson. As a by-product of our calculation, we reproduce the L3 result for the bound on $\mu_{\nu\tau}$.

The effective Lagrangian approach concerning the local $SU(2)_L \times U(1)_Y$ symmetry linearly realized [1] has been used recently to explore the consequences of physics beyond the Standard Model (SM) at lepton [2], hadron [3] and $\gamma\gamma$ colliders [4]. Also, this approach has been used to constrain the anomalous electromagnetic couplings of the $W$ boson, the $t$ quark [5] and the neutrinos [6] from the known experimental bounds on the rare decays $b \to s \gamma$ [7] and $\mu \to e \gamma$ [8]. In the present letter we point out that the recent measurement of energetic single–photons at LEP arising from the radiative decay $Z \to \nu \bar{\nu} \gamma$ [9] leads to direct constraints on dimension–eight and dimension–six operators associated with the anomalous electromagnetic properties of the $Z$ vector boson and the $\tau$–neutrino magnetic moment, respectively.

The radiative decay $Z \to \nu \bar{\nu} \gamma$ can not be induced at the tree level in the SM. In the effective Lagrangian approach this decay could proceed through

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Fig. 1. Feynman diagrams contributing to the decay $Z \to \nu\bar{\nu}\gamma$ in the effective Lagrangian approach. The heavy dots denote effective vertices.

The Feynman diagrams shown in Fig. 1, where the dots indicate effective vertices induced by dimension–six or dimension–eight operators which modify the SM weak sector. The anomalous $\nu\bar{\nu}\gamma$ vertex arises from the dimension–six operators [6]:

$$O_{\nu W}^{ab} = \bar{\ell}_a \sigma^{\mu
u} W_{\mu
u}^i \gamma^i \bar{\nu}_R^b,$$

$$O_{\nu B}^{ab} = \bar{\ell}_a \sigma^{\mu
u} B_{\mu\nu} \bar{\nu}_R^b,$$  \hspace{1cm} (1)

while the four–point vertex $Z\nu\bar{\nu}\gamma$ arises from the dimension–eight operators

$$O_1^8 = i (\phi_1^\dagger \phi) \bar{\ell}_a \gamma^\mu D^\nu \ell_L^a W_{\mu\nu}^i,$$  \hspace{1cm} (3)
\[ O_2^8 = i(\phi^\dagger \phi) \bar{\ell}_L^a \gamma^\mu D^\nu \ell_L^a B_{\mu\nu}, \quad (4) \]

\[ O_3^8 = i(\phi^\dagger D^\mu \phi) \bar{\ell}_L^a \gamma^\nu \tau_i \ell_L^a W_{\mu\nu}^i, \quad (5) \]

\[ O_4^8 = i(\phi^\dagger D^\mu \phi) \bar{\ell}_L^a \gamma^\nu \ell_L^a B_{\mu\nu}. \quad (6) \]

All these operators preserve the SU(2)_L \times U(1)_Y SM gauge symmetry. We have denoted with standard notation the SU(2)_L and U(1)_Y tensor field strength tensors \( W_{\mu\nu} \) and \( B_{\mu\nu} \), respectively, as well as the SU(2)_L left–handed lepton doublet \( \ell_L^a \), the respective right–handed neutrinos \( \nu_R^a \), the Pauli matrices \( \tau_i \), the Higgs field \( \tilde{\phi} = i\tau^2 \phi^* \) and the respective covariant derivative \( D_\mu \) [10].

Previous studies on the anomalous ZZ\( \gamma \) coupling shown in Fig. 1(c) have used an U(1)_{em} gauge invariant parametrization [9,11]. However, it is important to notice that in the effective Lagrangian approach there are no effective operators of dimension lower than eight, which are SU(2)_L \times U(1)_Y gauge invariant and could lead to an anomalous ZZ\( \gamma \) vertex. As a consequence, it is expected that this vertex is highly suppressed in the SM. This situation has been confirmed by an U(1)_{em} gauge invariant calculation of the CP-conserving and CP-violating off-shell ZZ\( \gamma \) vertex [12]. Since in the effective Lagrangian approach the non–standard ZZ\( \gamma \) vertex can not be generated at this level, we will not consider its effect on the \( Z \rightarrow \nu \bar{\nu} \gamma \) radiative decay.

It is possible to establish in the effective Lagrangian approach the order of perturbation theory in which SM gauge invariant non–renormalizable operators may be generated in the underlying theory[13]. In particular, loop generated operators appear with a characteristic suppression factor \( \sim 1/(4\pi)^2 \) which significantly decreases the magnitude of their effects. For example, in the case of the radiative decay of the SM Higgs boson into two photons, it was found that some tree level generated operators of dimension–eight may compete with dimension–six operators which are generated at the one–loop level[14]. In our case, the dimension–six operators (1)–(2) are induced at the one–loop level in the underlying theory, whereas the dimension–eight operators (3)-(6) are induced at the tree level. As a consequence, we expect that all these operators give similar contributions to the \( Z \rightarrow \nu \bar{\nu} \gamma \) decay through the anomalous \( \nu \bar{\nu} \gamma \) and \( Z\nu \bar{\nu} \gamma \) vertices. We will ignore CP–violating effects in this decay relying on general expectations that the scale of CP–violation is greater than \( \Lambda \), the scale used in the effective Lagrangian approach to denote the characteristic energy in which non–standard effects are expected to become directly observable.

After spontaneous symmetry breaking, the operators (1)-(6) induce the following parametrization for the \( \nu \bar{\nu} \gamma \) and \( Z\nu \bar{\nu} \gamma \) effective couplings,

\[ M_{\mu\nu}^{(a)} = \frac{\epsilon}{v} \bar{u}(p_2) \gamma_\mu (g_\gamma - g_\Lambda \gamma_5) \gamma^{-1}_\nu \sigma_{\nu\alpha} k^\alpha v(p_1), \quad (7) \]
\[ M^{(b)}_{\mu\nu} = \frac{\epsilon_8}{v^2} \bar{u}(p_2)(1 - \gamma_5)(k_\mu \gamma_\nu - \bar{k} g_{\mu\nu})v(p_1), \]  

(8)

where we have used the kinematic variables shown in Fig. 1, \( q = k + p_2 \), \( g_{V,A} = g/4c_w \) are the couplings of \( Z \) to neutrinos in the SM and \( v \) is the SM vacuum expectation value. The coefficients \( \epsilon_{6,8} \) summarize all the information that can be gathered from the heavy degrees of freedom associated with new physics effects and are expressed in terms of dimensionless coupling constants \( \alpha_i \) and the scale \( \Lambda: \epsilon_6 = \epsilon_6^W + \epsilon_6^B \) and \( \epsilon_8 = \epsilon_8^2 + \epsilon_8^3 + \epsilon_8^4; \) with \( \epsilon_6^i = \alpha_6^i (v/\Lambda)^2 \) and \( \epsilon_8^i = \alpha_8^i (v/\Lambda)^4 \).

It is easy to see that contributions (7) and (8) do not interfere. Furthermore, assuming the simple situation that cancellation among different operators does not take place, we get the following expressions for the distribution of the photon energy \( (x = E_k/M_Z) \) arising from diagrams 1(a) and 1(b),

\[ \frac{d\Gamma^{(a)}}{dx} = \frac{\epsilon_6^2 (g_V^2 + g_A^2) M_Z^3}{72\pi^3 v^2} x(3(1 - 2x) + x^2), \]  

(9)

\[ \frac{d\Gamma^{(b)}}{dx} = \frac{\epsilon_8^2 M_Z^5}{18\pi^3 v^4} x^3(1 - x). \]  

(10)

The measurement of energetic single–photons at LEP arising from the decay \( Z \to \nu \bar{\nu} \gamma \) has been used to set a direct limit on the \( ZZ \gamma \ U(1)_{em} \)–gauge invariant coupling and the magnetic moment of the \( \tau \) neutrino. For the purposes of the present analysis in the framework of the effective Lagrangian approach, the search for the energetic single–photons events on the data collected by the L3 collaboration may be translated easily into bounds on the coefficients \( \epsilon_6 \) and \( \epsilon_8 \) contained in the energy distributions (9) and (10). In order to reduce backgrounds, the L3 collaboration required the photon energy to be greater than one half the \( e^+e^- \) beam energy. The L3 collaboration obtained a branching ratio limit of one part in a million when the energy of the photon in \( Z \to \nu \bar{\nu} \gamma \) is above 30 Gev. Integrating (9) and (10) over the relevant range of energy we obtain the following bounds on the \( \epsilon_{6,8} \) coefficients which correspond to the two events selected from the L3 data

\[ \epsilon_6 < 0.192, \]  

(11)

\[ \epsilon_8 < 0.165. \]  

(12)

The constraint (11) can be translated into an upper limit on the \( \tau \)–neutrino magnetic moment in units of Bohr magnetons \( \mu_B \),

\[ \mu_{\nu_\tau} < 2.62 \times 10^{-6} \mu_B. \]  

(13)
Our bound (13) is consistent with the L3 limit $\mu_{\nu_\tau} < 3 \times 10^{-6}$ [9]. This means that the determination of this quantity is independent of the scale involved in the effective vertex (7): the dimension–six operators (1) and (2) induce in our case a scale given by $v$, while the L3 limit used the traditional scale given in terms of the electron mass. Maltoni and Vysotski [15] reproduced our calculation for $\Gamma^{(a)}$ and $\Gamma^{(b)}$ recently. Our bounds given in (11)-(13) agree with their results on the coefficients $\epsilon_{6,8}$ and $\mu_{\nu_\tau}$. There is a small difference among their bounds and ours due to the fact that we are considering the L3 branching ratio and the known experimental value for the full $Z$ width decay [8], while they considered directly the L3 value for $N_{Z \rightarrow \text{had}}$. In particular, our bound on $\mu_{\nu_\tau}$ compares favourably with the bound $\mu_{\nu_\tau} < 4 \times 10^{-6} \mu_B$ obtained from low–energy experiments [16] and $\mu_{\nu_\tau} < 2.7 \times 10^{-6} \mu_B$ obtained from the invisible width of the $Z$ boson [17], and it is close to the one derived from a beam–dump experiment [18]. It is interesting to notice that these bounds on the $\tau$–neutrino magnetic moment are still weaker than the known bounds on the magnetic moments of electron and muon neutrinos [19] and the transition magnetic moments $\nu_\tau \rightarrow \nu_i \gamma$ obtained from the experimental bound on $\mu \rightarrow e\gamma$ also within the effective Lagrangian approach [6].

In conclusion, we have obtained direct bounds on the $\tau$–neutrino magnetic moment and the dimension–eight operators (3)-(6). These bounds reflect the natural consequence that non–standard effects may become enhanced when SM contributions are highly suppressed. We know that this is the situation with the magnetic moments of the neutrinos and the $ZZ\gamma$ vertex. In the case of the $Z\nu\bar{\nu}\gamma$ effective vertex, the SM calculation for the respective box diagrams is not available yet, but we expect to have a similar situation in this case [20].

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