Phase Noise Compensation with Limited Reference Symbols
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Abstract—Phase noise (PN) can cause severe link performance degradation in orthogonal frequency division multiplexing (OFDM) systems. In this work, we propose an efficient PN mitigation algorithm where the PN statistics are used to obtain more accurate channel estimates. As opposed to prior PN mitigation schemes that assume perfect channel information and/or require abundant reference symbols, the proposed PN compensation technique applies to practical systems with a limited number of reference symbols such as the 3GPP-LTE systems. Numerical results corroborate the effectiveness of the proposed algorithm.

Index Terms—Phase Noise, minimum mean square error (MMSE), multiple-input multiple-output (MIMO), OFDM, LTE.

I. INTRODUCTION

MULTIPLE-INPUT multiple-output (MIMO) and orthogonal frequency division multiplexing (OFDM) are widely used in broadband data communications. The capability of OFDM together with MIMO to achieve high-rate transmission by eliminating inter-symbol interference renders its adoption in several standards, including IEEE 802.11 Wireless Local Area Network (WLAN) and 3rd Generation Partnership Project (3GPP) Long-Term Evolution (LTE).

The phase noise (PN) introduced by the local oscillator degrades the high throughput performance of OFDM systems [1]. For this reason, extensive research efforts have been devoted to PN compensation in a variety of communication systems. The works in [2]–[4] use WLAN as the underlying system model that incorporates plentiful of reference symbols. The authors in [3] presented a two-stage training-based algorithm, while the authors in [4] approximated the PN waveform using Fourier series. However, both [2] and [4] heavily rely on the impractical assumption of ideal channel state information (CSI) at receiver. A few works [5], [6] on LTE-like systems have recently emerged in the literature as well. Nonetheless, the work [5] assumes pilot symbols in every OFDM symbol, which is not the case for real LTE systems, and the work [6] again assumes ideal CSI at the receiver. In a recent work [7], joint estimation of channel and phase noise was proposed by using a decision-directed extended Kalman filter, but abundant training sequences and pilot symbols are needed in the system model presented. More recently, several works are put forward for PN compensation in mmWave systems [8], [9].

Unlike previous works that either assume perfect CSI at receiver or rely on plentiful training/pilot signals, our contribution in this work is a pragmatic scheme for PN compensation and data detection using only a limited number of reference signals which is applicable to the LTE (and possibly future 5G) downlink subframe structure. Specifically, our novelty lies in the aspect that we view the PN contaminated channel as a whole and take the PN statistics into consideration in the 2-dimensional (2D) time-frequency minimum mean square error (MMSE) channel estimation, rather than trying to split PN from channel estimates as in the works [3], [5]. In particular, the proposed algorithm iteratively updates MIMO detection, phase noise estimation and channel estimation under a single objective. Numerical results show the substantial gain in the link performance due to the iterative updates with the improvement in channel estimation by exploiting PN statistics.

II. MIMO-OFDM SYSTEM MODEL WITH PHASE NOISE

We consider a MIMO-OFDM communication system with \( N_t \) transmit antennas, \( N_r \) receive antennas and \( N_c \) OFDM tones for transmission in a multi-path fading channel. In this MIMO-OFDM system, carrier frequency offset is ignored due to the fact that PN is shown to be more dominant in the performance degradation [5], [10]. Also, we restrict our attention to the PN at receiver only since transmitter PN can be approximated by an effective receiver PN [11].

Random PN generated by free-running oscillator is modeled as a Wiener process that exhibits Lorentzian-shape power spectrum density (PSD) [1]. Let \( \phi[n] \) denote the discrete Wiener process, then \( \phi[n] = \phi[n-1] + \epsilon[n] \), where the increment \( \epsilon[n] \) is Gaussian distributed with zero mean and variance \( \sigma^2_\epsilon = 4 \pi \beta T_s \). Here, \( \beta \) is the single-sided 3 dB bandwidth of the Lorentzian spectrum for the carrier process \( e^{j\phi[n]} \) and \( T_s \) is the sampling period. Note that, the received signals on all antennas undergo the same phase noise process, since a common local oscillator is used for all receive chains in the direct down-conversion to base band. For the sake of notational simplicity, define \( N_t' \triangleq \{1,2,\ldots,N_t\} \), \( N_r' \triangleq \{1,2,\ldots,N_r\} \) and \( N_c' \triangleq \{0,1,\ldots,N_c-1\} \). Then, the PN contaminated frequency-domain signal at the \( j \)-th receive antenna on the \( k \)-th tone can be written as

\[
y_j[k] = a[k] \ast \sum_{i=1}^{N_t} (H_{ji}[k] \cdot x_i[k]) + w_j[k], \quad k \in N_c, \tag{1}
\]

where \( \ast \) denotes circular convolution, \( x_i[k] \) is the transmit signal at antenna \( i \) on the \( k \)-th tone and \( w_j[k] \sim CN(0,\sigma^2_w) \) is the additive white Gaussian noise at the \( j \)-th receive antenna. Moreover, \( a[k] \) is the frequency-domain phase noise, given by

\[
a[k] = \frac{1}{N_c} \sum_{n=0}^{N_c-1} e^{j\phi[n]} e^{-j \frac{2\pi kn}{N_c}}, \quad k \in N_c, \tag{2}
\]

and \( H_{ji}[k] \), the frequency-domain channel between transmitter \( i \) and receiver \( j \), is the DFT of time-domain channel impulse response \( \{h_{ji}[n]\}_{n=0}^{L-1} \) that is

\[
H_{ji}[k] = \sum_{n=0}^{L-1} h_{ji}[n] e^{-j \frac{2\pi kn}{N_c}}, \quad k \in N_c. \tag{3}
\]
Then, the overall MIMO-OFDM transmission with phase noise contamination can be expressed as

\[ y = (A \otimes I_{N_c})Hx + w, \]

where \( \otimes \) represents the Kronecker product and \( I_{N_c} \) is the \( N_c \)-by-\( N_c \) identity matrix. In Eq. (4), the received signal \( y = [y_0^T \; y_1^T \; \ldots \; y_{N_c-1}^T]^T \) in which the signal vector \( y[k] = [y_0[k] \; y_2[k] \; \ldots \; y_{N_c}[k]]^T \). The vectors \( x \) and \( w \) are defined similarly as \( y \). The circulant matrix \( A = \text{cir} \( a \) \) is the cyclic shift of the PN vector \( a = [a[0] \; a[1] \; \ldots \; a[N_c-1]]^T \)

The channel matrix \( H = \text{blkdiag}[H[k]]_{k \in N_c} \) with \( H[k] = [H_{f,ℓ}[k]]_{ℓ \in N_c,j \in N_c} \) and \( \text{blkdiag} \) represents block diagonal. For ease of subsequent derivations, Eq. (4) is split for each tone, shown as follows

\[ y[k] = a[0]H[k]x[k] + \sum_{r=0,r\neq k}^{N_c-1} a[k-r]H[r]x[r] + w[k], \]

in which the first term \( a[0] \) is referred to as common phase error (CPE), and the second term \( \sum_{r=0,r\neq k}^{N_c-1} a[k-r]H[r]x[r] \) is called inter-carrier interference (ICI), each element of which is zero-mean with variance \( \sigma^2_{IC1} = 2\pi\beta T_o N_t/3 \) for large \( N_c \) [2], [4].

### III. 2D MMSE Channel Estimation with CPE Term

In LTE downlink the reference symbols (or pilots) are orthogonal in both time and frequency as illustrated in Fig. 1.

Thus, 2D MMSE channel estimation for single-input single-output (SISO) system is applicable to each transmit-receive antenna pair [12]. Without loss of generality, we will present the MMSE channel estimation procedure for a SISO system, thus dropping the spatial indices \( i \) and \( j \).

Fig. 1: LTE downlink (frequency domain) reference symbol layout of antenna port 0 in a two-antenna system.

For frequency-domain channel estimation in one physical resource block (PRB), we unroll the time-frequency grid along the frequency-axis first and then along the time-axis. Thus, according to the orders indicated in Fig. 1, the transmit and receive reference symbols can be arranged in column vectors \( x_p = [x_1 \; x_2 \; \ldots \; x_L]^T \) and \( y_p = [y_1 \; y_2 \; \ldots \; y_L]^T \), respectively. Let \( h_{f,ℓ} \in \mathbb{C}^{N_y \times 1} \) denote the frequency-domain channel vector at time \( ℓ \), and \( h_f \in \mathbb{C}^{N_c \times N_y} \) denote unrolled channel vector in one subframe by stacking \( h_{f,ℓ} \)'s, i.e., \( h_f = [h_{f,1}^T \; \ldots \; h_{f,N_y}^T]^T \), where \( N_o \) is the number of OFDM symbols in one subframe. Also, let \( h_{t,ℓ} \) and \( h_t \) be the time-domain counterparts corresponding to \( h_{f,ℓ} \) and \( h_f \), respectively. Dictated by the multiplicative nature as seen in Eq. (4), the CPE term \( a[0] \) is lumped with channel coefficients to form an effective channel \( h_{f,ℓ} = a[ℓ](h_{f,ℓ} - h_f) \) and so is \( h_f \). Moreover, the ICI term is treated as part of the noise process. Therefore, the least squares (LS) channel estimates on pilot tones are \( \hat{h}_{f,p}^{LS} = y_p \times x_p \), where the MATLAB notation ./ stands for element-wise division.

The MMSE estimator that uses channel second-order statistics is given by [13],

\[ \hat{h}_{f,p}^{MMSE} = R_{h_f h_f}^{-1}R_{h_f p} + \sigma^2 I \] (6)

where \( R_{h_f h_f} \), \( R_{h_f p} \) represents correlation matrix for channels on pilot tones and \( R_{h_f h_f} \) denotes the correlation between channels on all tones and pilot tones. Here, the constant \( \mu = \mathbb{E} \{1/|x_i[k]|^2 \} \) is constellation-dependent and the lumped noise variance \( \sigma^2_w = \sigma^2_a + \sigma^2_{IC1} \) can be estimated from the null subcarriers in the system guard band [2]. Note that, different from the 2-stage approach in [13], the above correlation matrices include both frequency and time correlations. Specifically,

\[ R_{h_f h_f} = \mathbb{E} \left\{ h_f^H h_f \right\} = R_F \otimes R_T \] (7)

where \( R_F \) denotes the Kronecker product, and \( R_T \) incorporates frequency correlation while \( R_F \) incorporates time correlation. In the sequel, we will elaborate on how to obtain \( R_{h_f h_f} \) in such a form, and how to generate \( R_F \) and \( R_T \) by use of the statistics of \( a[0] \).

Now consider the \( (ℓ, m) \) subblock of \( R_{h_f h_f} \), given by

\[ R_{h_f h_f}(ℓ, m) = \mathbb{E} \left\{ a[ℓ]a_m^* (a_m^* 0) h_{f,m}^H \right\} 
\quad = \mathbb{E} \{ a[ℓ]a_m^* \} \mathbb{E} \{ h_{f,m}^H \} 
\quad = \mathbb{E} \{ a[ℓ]a_m^* \} \text{FIE} \{ h_{t,ℓ} h_{t,m}^H \} F^H, \] (8)

where \( h_{t,ℓ} = \{h_{t,0}[0] \ldots h_{t,ℓ} \}^T \) being the \( i \)-th channel tap at the \( ℓ \)-th OFDM symbol, and \( F \) is the DFT matrix of appropriate size. Since we assume wide-sense stationary uncorrelated scattering channel, the channel taps at different delays are uncorrelated. Therefore,

\[ \mathbb{E}\{h_{t,ℓ}[i]h_{t,m}^*[j]\} = \begin{cases} J_0(2\pi f_d ℓ - m|T_o)\gamma_i, & i = j \\ 0, & i \neq j \end{cases} \] (9)

where \( J_0(\cdot) \) is the zero-order Bessel function of the first kind, \( f_d \) is the Doppler frequency, \( T_o \) is the time duration of one OFDM symbol and \( \gamma_i = \mathbb{E}\{|h_{t,ℓ}[i]|^2\} \) is the power of the \( i \)-th channel tap given by power-delay profile (PDP). We note that PDP is statistical information that can be measured offline, and it is assumed to be known.

By substituting Eq. (9) into Eq. (8), we get

\[ R_{h_f h_f}(ℓ, m) = J_0(2\pi f_d ℓ - m|T_o)\text{FIE} \{ a[ℓ]a_m^* \} \text{FIF}^H \] (10)

where \( \Gamma = \text{diag}(\gamma_0, \ldots, \gamma_{L-1}) \) is a diagonal matrix. We recognize that the frequency auto-correlation matrix \( R_F \) is given by

\[ R_F = \mathbb{E} \{ h_f h_f^H \} = \text{IFE} \{ h_{t,ℓ} h_{t,ℓ}^H \} F^H = \text{FIE} \{ h_f h_f^H \} \] (11)

and we define the \( (ℓ, m) \) entry of the temporal auto-correlation matrix \( R_T \) by

\[ R_T(ℓ, m) = J_0(2\pi f_d ℓ - m|T_o)\text{FIE} \{ a[ℓ]a_m^* \} \] (12)
where the computation of $\mathbb{E}\{a_{\ell}[0]a_{m}^{*}[0]\}$ is shown in Appendix A. When $\ell = m$, $\mathbb{E}\{a_{\ell}[0]^{2}\} \approx 1 - 2\pi\beta T_{s} N_{c}/3$ \cite{1}, and for $\ell \neq m$,
\[
\mathbb{E}\{a_{\ell}[0]a_{m}^{*}[0]\} = \frac{e^{-2\pi\beta T_{s} N_{c} |m-\ell|}}{N_{c}^{2}} \cdot \frac{1 - \cos(2\pi\beta T_{s} N_{c})}{1 - \cos(2\pi\beta T_{s})}.
\]
Finally, $R_{h/k'}(\ell, m) = R_{T}(\ell, m)R_{F}$ and thus $R_{h/k'} = R_{T} \otimes R_{F}$. Then, the correlation matrices $R_{h/k'}$ and $R_{h/2}$ are generated by selecting corresponding time-frequency slots from $R_{h/k'}$.

IV. ITERATIVE DETECTION AND ESTIMATION ALGORITHM

In this section, we will present a PN mitigation algorithm that iteratively updates the estimates of channel, phase noise and data symbols. The primary objective of interest is to minimize the squared-error $\|y - (A \otimes I_{N_{r}})Hx\|^2$. Due to the unknown phase noise matrix $A$ and channel matrix $H$, $x$ cannot be recovered directly via this objective function. Instead, we modify the objective function as follows
\[
\|y - (A \otimes I_{N_{r}})Hx\|^2 = \|y - (A/[a[0] \otimes I_{N_{r}}])a[0]Hx\|^2,
\]
where $H' = a[0]H$ is available through the 2D MMSE channel estimation in Section III, and the modified phase noise matrix $A' = A/[a[0]$ is initialized to $I_{N_{r}}$, based on which we can perform CPE compensation.

However, the initial estimate of $\hat{x}$ is fairly coarse, and the elements on the diagonal band of matrix $A$ are not negligible, though $a[0]$ is dominating. Also, the effective channel $a[0]H$ is estimated in presence of high-level noise (lumped ICI and channel noise). In consideration of these facts, we propose to interatively update the three quantities, namely, phase noise matrix $A'$, channel matrix $H'$ and signal vector $x$, one after another following the LS principle.

For evaluating frequency-domain phase noise, we rewrite the transmit-receive relation in Eq. (4) as follows
\[
y = (A' \otimes I_{N_{r}})H'x + w = \text{circ}(H'x)N_{r}a' + w,
\]
where $\text{circ}(H'x)N_{r}$ means a cyclic shift of the vector $H'x$ by $N_{r}$ elements vertically, and the resulting matrix is of size $N_{c}N_{r} \times N_{c}$ Here, we use the fact that $A' = \text{circ}(a')$, where $a' = a/[a[0]$. Now, we obtain an over-determined system of $N_{c}N_{r}$ equations to estimate parameters $a'$ of dimension $N_{c}$. For better estimation, we adopt the method in \cite{3} to further reduce the number of unknown parameters. Concretely, we estimate time-domain decimated phase noise samples $e'$ instead of $a'$, and they are related by
\[
a' \approx \frac{1}{N_{c}}F_{a}P_{c}e',
\]
where $F_{a}$ is an $N_{c} \times N_{c}$ DFT matrix and $P_{c}$ is a linear interpolation matrix of size $N_{c} \times M(M \ll N_{c})$ as specified by Eq. (15) in \cite{3}.

Next, the transmit-receive equation in Eq. (4) is rearranged to apply LS channel estimation
\[
y = (A' \otimes I_{N_{r}})H'x + w = (A' \otimes I_{N_{r}})Xh' + w,
\]
where $h' = [(h'_{1})^{T} (h'_{2})^{T} \ldots (h'_{i})^{T} \ldots (h'_{N_{r}N_{c}})^{T}]^{T}$ and $h'_{ji} = [H_{ji}[0]H_{ji}[1] \ldots H_{ji}[N_{c} - 1]]^T/a[0]$. In Eq. (17), the $N_{c}N_{r}$-by-$N_{r}N_{r}$ matrix $X$, consisting of elements from vector $x$, can be concatenated as $X = [X_{1} X_{2} \ldots X_{j} \ldots X_{N_{r}}]$, where the submatrix $X_{j}$ is specified as
\[
X_{j}(m, n) = \begin{cases}
    x_{i}[k], & \text{if } m = kN_{r} + j,
    n = (\nu - 1)N_{c} + k + 1, \\
    0, & \text{otherwise}.
\end{cases}
\]

Algorithm Iterative Detection and Estimation (IDE)

1: Obtain initial $\hat{H}'$ by 2D MMSE channel estimation, and initialize $A' = I_{N_{r}}$.
2: repeat
3: MIMO detection by LS estimation
\[
\hat{x} = \arg\min_{\hat{x}} \|y - (\hat{A}' \otimes I_{N_{r}})\hat{H}'x\|^{2},
\]
and then slice $\hat{x}$ to the nearest constellation point.
4: Time-domain phase noise estimation is given by
\[
\hat{e}' = \arg\min_{\hat{e}'} \|y - \text{circ}(\hat{H}'\hat{x})N_{r} \cdot \frac{1}{N_{c}}F_{a}\hat{P}_{c}\hat{e}'\|^{2}.
\]
Transform $\hat{e}'$ back to frequency domain $\hat{a}' = F_{a}\hat{P}_{c}\hat{e}'/N_{c}$, and normalize $\hat{a}' = \hat{a}'/[a[0]$. Then obtain $A' = \text{circ}(\hat{a}')$.
5: Update channel estimation in time domain
\[
\hat{h}'_{ji} = \arg\min_{\hat{h}'} \|y - (\hat{A}' \otimes I_{N_{r}})X\hat{F}_{H}\hat{h}'\|^{2}
\]
and then $\hat{h}' = F_{H}\hat{h}'$. Finally, reshape $\hat{h}'$ to $\hat{H}'$.
6: until Reach maximum iterations or no significant improvement of the objective function.

The above procedures are summarized in the iterative detection and estimation (IDE) algorithm. We notice that each update step involves large-scale matrix multiplications and LS computations. In practical LTE systems, $N_{r}, N_{t} \ll N_{c}$, and also $M, L \ll N_{c}$ by algorithm construction. By fixing $N_{r}, N_{t}, M$ and $L$ as constant parameters, the complexity of the matrix multiplication and the LS computation using QR factorization \cite{14} are both $O(N_{c}^{3})$ per iteration. Therefore, the overall complexity is $O(N_{c}^{3})$ with finite maximum iterations.

V. NUMERICAL RESULTS

This section provides simulation results under LTE structure with $2 \times 2$ spatially independent MIMO in a 3 MHz channel bandwidth, in which 180 out of 256 subcarriers are occupied.
and the rest are guard tones. 16-QAM is used for modulating the data symbols, and reference symbols are generated from the 4 outermost corner of the 16-QAM constellation. The tone spacing is 15 KHz, and thus the sampling period \( T_s = 1/(15000 \times 256) \). Further, one subframe in LTE system, containing 14 OFDM symbols, lasts for 1 millisecond, thus the OFDM symbol period \( T_o = 1/14 \) ms. The multipath fading channel used in the simulation is the ITU Pedestrian B (PedB) model, in which the tap delays are at \([0.0 \ 0.2 \ 0.8 \ 1.2 \ 2.3 \ 3.7]\) micro-second with power delay profile \([0.0 \ -0.9 \ -4.9 \ -8.0 \ -7.8 \ -23.9]\) measured in dB. We assume pedestrian velocity at \( v = 5 \) km/h (= 5/3.6 m/s) and a carrier frequency \( f_c = 2 \) GHz. Moreover, we set the number of decimated time-domain phase noise elements \( M = 50 \), and the maximum number of iterations for IDE algorithm is 5.

The proposed IDE scheme is compared with the algorithm presented for an LTE-like system in [5]. However, the algorithm in [5] requires pilot symbols in every OFDM symbol, which is not the case for real LTE subframe structure. Thus, besides the frequency interpolation that the authors proposed, we perform another interpolation along the time axis to obtain CSI for OFDM symbols without training signals. Moreover, we compare the IDE algorithm with CPE compensation, which is described in Eq. (19) (essentially the first iteration of IDE algorithm) based on the MMSE channel estimates of Eq. (6). Particularly, two channel estimates are considered for CPE compensation – with and without the statistics of \( a[0] \) in MMSE channel estimation. Last but not least, two benchmarks, the cases of no compensation and no PN, are also plotted for comparison purpose.

Fig. 2 shows the bit error rate (BER) performance against signal-to-noise ratio (SNR), while Fig. 3 demonstrates BER versus \( \beta \). It is clearly seen in Fig. 2 and Fig. 3 that the IDE algorithm outperforms all other algorithms in between the two benchmarks, and it is also observed that the CPE compensation with channel estimates considering \( a[0] \) is superior to that without taking \( a[0] \) into account in the channel estimation. Further, it is interesting to note that the compared algorithm in [5] performs close to the inferior CPE compensation in Fig. 2 though it outperforms the latter in Fig. 3 when \( \beta \) is large. The poor performance of the algorithm in [5] is possibly due to the sparser reference symbols (based on LTE setup) than those considered by the authors of [5] and also caused by the less accurate channel estimates by interpolation. Moreover, since our proposed IDE algorithm is iterative, we show its BER performance as iterations increase. It is clear from Fig. 4 that 5 iterations will suffice.

VI. CONCLUSION

In this correspondence, we proposed an iterative phase noise and channel estimation and compensation algorithm that is well suited to the wireless systems with sparse pilots arrangement. This algorithm starts with a fairly accurate channel estimate, and then follows the LS principle to perform iterative updates. Its effectiveness has been shown via computer simulations. To combat imperfections, joint detection-decoding schemes have been proposed recently [15]–[20]. Specially, the iterative methods in [21] may be used for more effective phase noise compensation. Moreover, the proposed IDE algorithm is heuristic in nature, so future works may consider a better termination criterion with provable performance bounds.
The carrier process autocorrelation follows \[ R_{aa} = \mathbb{E} \left[ e^{j\phi[n+k]}(e^{j\phi[n]}e^{j\phi[k]}\right] = \exp(-2\pi\beta T_s|k|). \] (22)

Given \( R_{aa} \), the correlation \( \mathbb{E} \{ a_l[0]a_{m}^*[0] \} \) for \( m \neq \ell \) is

\[
\begin{align*}
E \{ a_l[0]a_{m}^*[0] \} & = \frac{1}{N_c^2} \sum_{p=0}^{N_c-1} \sum_{q=0}^{N_c-1} \sum_{N_c-1}^{N_c-1} e^{j\phi[N_c,\ell+p]} e^{-j\phi[N_c,m+q]} e^{-2\pi\beta T_s[N_c,m+q-N_c,\ell-p]} \\
& = \frac{e^{-2\pi\beta T_s N_c(m-\ell)}}{N_c^2} \cdot \sum_{p=0}^{N_c-1} e^{2\pi\beta T_s p} \sum_{q=0}^{N_c-1} e^{-2\pi\beta T_s q}
\end{align*}
\]

(23)

\[
= \frac{e^{-2\pi\beta T_s N_c(m-\ell)}}{N_c^2} \cdot \frac{1 - \cos(2\pi\beta T_s N_c)}{1 - \cos(2\beta T_s)}. \]

Considering the symmetric case for \( m < \ell \), we obtain

\[
E \{ a_l[0]a_{m}^*[0] \} = \frac{e^{-2\pi\beta T_s N_c(m-\ell)}}{N_c^2} \cdot \frac{1 - \cos(2\pi\beta T_s N_c)}{1 - \cos(2\beta T_s)},
\]

for any \( \ell \neq m \).

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