Study of surface potentials using resonant tunnelling of cold atoms in optical lattices

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Abstract
We study the feasibility of precision measurements of surface potentials at micrometre distances using resonant tunnelling of cold atoms trapped in vertical optical lattices. A modulation of an amplitude of the lattice potential induces atomic tunnelling among the lattice sites. The resonant modulation frequency corresponds to a difference of potential energy between lattice sites, which is defined by an external force, i.e. gravity. The vicinity of the surface alters the external potentials, and hence the resonant frequency. Application of this method allows the accurate study of Casimir-type potentials and improvement of the present experimental validity of the Newtonian gravitational potential at a distance range of 5–30 μm.

(Some figures may appear in colour only in the online journal)

1. Introduction
Interest in surface potentials spans from fundamental issues to the phenomenology of surface forces. From the theoretical point of view, at length scales smaller than 100 μm QED interactions, i.e. Casimir-type potentials, such as the Casimir–Polder potential and the thermal Livshitz potential [1, 2], become important. Some recent theories, going beyond the standard model, foresee deviations of the gravity interaction from Newton’s law [3–7] that occur at spatial distances and with strength, which depend on the details of the model. Accurate measurements of surface forces have been performed using a variety of experimental techniques, e.g., micro-cantilevers [8–11], torsion pendulum [12–14], micromechanical torsional oscillators [15], atomic force microscopes [16] and torsion balances [17]. The major difficulties of such mechanical experiments are the exact knowledge of the geometry of the setup (distance, surface roughness, etc), the precise measurement of the very small forces involved and strong electrostatic forces between interacting bodies, with unknown strength and spatial dependence. The latest progresses in laser cooling allow surface potentials on short distances with a high spatial resolution to be investigated using the micrometric size of laser cooled atomic clouds [4, 18–22]. In [19, 20], the authors demonstrated a shift of trap frequency of atomic Bose–Einstein condensates (BEC) induced by temperature-dependent Livshitz potentials. Recently new proposals were presented for measurements of atom–surface forces using Bloch oscillations [18], and sub-Hz optical atomic transitions [21, 22].

In this paper we study the reachable accuracy of the measurements of surface potentials with the method based on the coherent resonant tunnelling of ultra-cold atoms trapped in modulated optical lattices. As was demonstrated, the resonant frequency of such tunnelling can be measured with a high precision; further, it has been proposed to employ this phenomenon for precision measurements, for example, of the Casimir–Polder interaction [23–25]. A shift of the resonant tunnelling frequency measured with localized atomic samples can provide direct measurements of surface potentials. This method can be applied for a search of possible deviations from the Newtonian gravitational law at short distances.

The paper is organized as follows. In section 2 we give a basic theoretical description of atomic wavefunctions in tilted periodic potentials. We briefly discuss the phenomenon of coherent tunnelling in driven lattice potentials and its possible application for the study of surface potentials. In section 3 we show calculations of the expected shift of the resonant frequency in the vicinity of a surface and discuss the accuracy of measurements of Casimir-type potentials. Then we discuss the expected improvements of experimental constraints for a Yukawa-type gravitational potential. In section 4 we discuss a practical implementation of the proposed method. We provide a concrete and feasible ‘to-do’ list for an experimental realization of the proposed method.
2. Theoretical background and methods

2.1. Atoms in driven optical lattices

We consider cold atoms trapped in optical lattices in the presence of an external force. Optical lattices create a periodic potential for atoms that originates from the ac Stark shift produced by the interference pattern of two counter propagating laser beams. If the frequency of the lattice light is red detuned from the main atomic transition, the optical lattice traps the atoms at the antinodes of the interference pattern. Transverse confinement is provided by the Gaussian profile of the lattice beam. The sinusoidal modulation of the power of the vertical optical lattice leads to a tilted periodic potential of the form

$$U(z, t) = mgz + \frac{U_0}{2} \cos[2k_Lz](1 + \kappa \cos(\omega_M t)), \quad (1)$$

where \(mgz\) is the gravity potential, \(U_0\) is the lattice depth, \(\kappa\) is the modulation amplitude, \(k_L\) is the optical-lattice wave vector and \(\omega_M\) is the modulation frequency. The recoil energy \(E_r = \hbar^2 k_L^2 / 2m\) associated with the absorption or emission of a photon of the lattice laser is the natural energy unit for the lattice trap depth.

This potential is well studied in a static case, i.e. when \(\kappa = 0\). The quasi-eigenstates in tilted periodic potentials are known as Wannier–Stark (WS) states. The wavefunction of these states is localized in wells of the lattice potential.

In typical experimental conditions of vertical optical lattices these states can be considered as stationary states of the lattice potential. The tilted periodic potential leads to the formation of a discrete ladder of energy states, the WS ladder, spaced by the Bloch frequency [26] in energy units,

$$v_0 = \frac{mg \lambda}{2h}, \quad (2)$$

where \(m\) is the atomic mass, \(g\) is the gravity acceleration, \(\lambda = 2\pi / k_L\) is the wavelength of the lattice light and \(h\) is the Planck constant.

The modulation of the lattice potential causes a delocalization of the atomic states through coherent tunnelling of atomic wavefunctions among different states of the WS ladder. As demonstrated experimentally [23, 24], the coherent tunnelling is a resonant process where resonant frequencies match the spacing between states of the WS ladder. The tunnelling probability amplitude as a function of the modulation frequency \(\nu\) exhibits a shape proportional to \(\sin(\nu - v_0) / \nu\) in the frequency domain based on the generalized model of the two-level system. Here \(\text{sinc}(x)\) is the resonance function \(\sin(x)/x\) for a two-level transition probability, \(v_0\) is the resonant frequency, \(\Gamma\) is the linewidth of the transition and \(\nu\) is the modulation frequency. In the absence of decoherence sources, the linewidth (\(\Gamma\)) is Fourier limited; the narrow tunnelling transitions allow measuring resonant frequencies with a high precision.

2.2. Method of studying surface potentials

We discuss the possibility of using resonant tunnelling in amplitude-modulated lattice potentials as a probe for surface potentials, i.e. Casimir-type or non-Newtonian Yukawa-type potentials. The basic scheme of our method is shown in figure 1. A sample of cold atoms is positioned close to a plane horizontal surface, and loaded into a vertical far-off-resonance optical lattice. The optical lattice is formed by an incoming beam and its reflection from the horizontal surface. Then the optical power of the lattice beam is periodically modulated. If the modulation frequency is equal to an energy difference of the levels of the WS ladder, coherent tunnelling occurs between resonant lattice sites. For the atoms far from the surface, the energy difference of the levels is set by the Earth’s gravity. In the vicinity of the surface, the WS energy levels are shifted by the surface potentials. This causes a shift of the resonant frequency, which depends on distances \(z\) from the surface.

If site-to-site tunnelling occurs on distances (\(\delta\)) larger than a size of the atomic sample, the resonant tunnelling causes an appearance of a separated atomic cloud. Otherwise resonant tunnelling appears as a broadening of the initial atomic sample. Although a study of the surface potentials is also possible by measuring such broadening for the sake of simplicity we focus on the case in which the appearance of a separated atomic cloud is directly observed. Here we suggest observing a single tunnelling process in contrast to [23, 25] where multiple resonant tunnelling transitions occurs. The surface potentials are studied by measuring the shift of the tunnelling resonant frequency at different distances, and observed directly by absorption imaging.

This method can be extended to a study of possible deviations from the Newtonian gravitational law at short distances. Expected deviations from the Newtonian gravitational potential are much weaker than the Casimir-type potential. Thus differential measurements to exclude the contribution of the Casimir-type potential are desirable. A study of Yukawa-type gravitational potentials is possible using the scheme proposed in [4], where the measurements are performed above two different materials with drastically different densities that are covered with a thin layer of a good conductor (for example gold), forming the so-called Casimir shield. This provides the identical Casimir potential above all surfaces of the sample.
2.3. Sensitivity of the method

The described method relies on accurate measurements of a number of tunnelled atoms for various modulation frequencies. Fluctuations or uncertainties of these atomic numbers directly affect sensitivity. There are a few factors that might cause such fluctuations or uncertainties. The shot noise in the atomic numbers poses a fundamental limit to a signal-to-noise ratio. For example, for an atomic cloud of $10^5$ atoms and the fraction of tunnelled atoms of about 10%, it limits a signal-to-noise ratio to 100. Imperfections of the imaging system, such as limited numerical aperture of imaging lenses or limited quantum efficiency of the CCD camera, will further decrease a signal-to-noise ratio. Instabilities of a power of lattice beams cause a fluctuation in the number of tunnelled atoms. Short-term instabilities for time-scales smaller than interrogation times will be effectively averaged out during each interrogation time. A long-term instability seems more problematic. Fluctuations of the atomic numbers in a first approximation are linear to fluctuations of the lattice beam powers. Assuming long-term instability of the beam power of $10^{-3}$ and a fraction of tunnelled atoms of about 10% we obtain a signal-to-noise ratio of 500.

The sensitivity is directly affected by the broadening of tunnelling resonances. Broadening might be caused by various sources of decoherences, similar to decoherences that lead to dephasing of Bloch oscillations in [30]. The atomic collisions, or the mean-field interaction in the case of BECs, appear to be the main candidate. As was demonstrated in [30], for high atomic densities a scattering length $a_0$ needs to be $\leq 0.5 a_B$ (where $a_B$ is the Bohr radius) for preservation of coherent Bloch oscillations on the time-scale of $\sim 10$ s. For thermal clouds this constraint can be far more relaxed due to lower atomic densities. A typical time between atomic collisions should be longer than an interrogation time. For example, for $^{174}$Yb this implies densities of the order of $10^{10}$ cm$^{-3}$.

Fluctuations of the lattice wavelength shift resonant frequencies, which after averaging over many tunnelling profiles appears as broadening. Such fluctuations of about 100 MHz are reported to be the main source of uncertainty in [25] for the gravity measurement, limiting the relative uncertainty to $2 \times 10^{-7}$. Locking the lattice laser to an atomic transition or ultra-stable cavity can decrease the uncertainty to at least a factor of 100. Trapped atoms move in a transverse direction, and thus experience different potential barriers; however, this affects the magnitude of the tunneling rate, but does not affect its resonant frequency, and hence does not decrease the sensitivity [23].

A modulation time is a trade-off: a longer modulation time potentially can lead to narrower resonances; however, various decoherence sources tend to limit resonance linewidth, and further, atoms will be lost due to collisions with background gas. Based on recent experiments [24, 25], a modulation time of the order of 10 s appears optimum. For example, assuming a signal-to-noise ratio of 10, a modulation time of 10 s and an average over the 100 resonance spectra, one can expect sensitivity of the order of 0.3 mHz. Assuming tunnelling over the eight lattice sites corresponds to the sensitivity of measurements of a local gravity acceleration of $\sim 70 \times 10^{-9}$ for $^{88}$Sr and $\sim 35 \times 10^{-9}$ for $^{174}$Yb. Such parameters are rather modest and easily reachable with present experimental techniques. More ambitiously one can assume a signal-to-noise ratio of 50, an interrogation time of 30 s and that an average over the 1000 resonance spectra sensitivity is expected of the order of $\sim 7 \mu$Hz. Although such parameters seem reachable with present experimental techniques, they require a combination of state-of-the-art imaging and stabilization of lattice beams and a large sample of laser-cooled atoms.

3. Projected precision of surface potential measurements

Generally Casimir-type potentials can be written as the sum of two parts, where the first $U_{\text{CP}}(z)$ comes from zero-point electromagnetic field fluctuations and the second $U_{\text{th}}(z)$ from the thermal fluctuations of the electromagnetic field,

$$U_{\text{CP}}(z) = U_0(z) + U_{\text{th}}(z).$$  (3)

For distances much larger than $\lambda/2\pi$, where $\lambda$ is the wavelength of the strongest optical transition, one can apply the so-called static approximation to estimate the atom–surface interaction in the case of atomic polarizability $\alpha$ that can be substituted by its static polarizability [28]. The first term gives rise to the Casimir–Polder asymptotic behaviour [1]

$$U_0(z) = -\frac{a_0}{4\pi \varepsilon_0} \frac{3\hbar c}{8\pi^2} \frac{1}{z^3} \left(\frac{1}{\varepsilon(\alpha)} - 1\right).$$  (4)

Here $\varepsilon_0$ is the vacuum permittivity, $\hbar$ is the reduced Plank constant, $c$ is the speed of light, $a_0$ is the static polarizability, $\varepsilon$ is the static dielectric function of the substrate and $z$ is the distance from the surface. The function $\phi(\varepsilon)$ is defined as in [27].

The second contribution was first considered by Lifshitz [2]. At large distances the thermal contribution approaches the so-called Lifshitz law

$$U_{\text{th}}(z) = -\frac{a_0}{4\pi \varepsilon_0} \frac{kb T}{4z^3} \left(\frac{1}{\varepsilon(\alpha)} - 1\right),$$  (5)

where $k_B$ is the Boltzmann constant and $T$ is the temperature of an environment. For material with high conductivity, such as gold or copper, $\varepsilon \gg 1$. This simplifies equation (3),

$$U_{\text{CP}}(z) = -\frac{a_0}{4\pi \varepsilon_0} \frac{kb T}{4z^3} + \frac{3\hbar c}{8\pi^2} \left(\frac{1}{\varepsilon(\alpha)} - 1\right).$$  (6)

We present non-Newtonian correction to the gravity potential in a conventional form of the Yukawa-type potential (equation (7)). These hypothetical corrections are usually parameterized by a Yukawa-type term which adds to the Newtonian potential [3, 4]. The modified potential for two masses $m_1$ and $m_2$ has the form

$$V_{\text{Y}}(r) = -G\frac{m_1 m_2}{r} \left(1 + \alpha e^{-r/\lambda_{\text{Y}}}ight),$$  (7)

where $G$ is Newton’s gravitational constant, $r$ is the distance between masses, $\alpha$ is the relative magnitude of the Yukawa correction and $\lambda_{\text{Y}}$ its spatial range. Assuming that the distance between the atomic cloud and the surface is much smaller than all other linear sizes (i.e. an approximation of an infinite plane), one obtains the Yukawa-type potential in the form

$$U_{\text{Y}}(z) = 2\pi G\rho_0 m \lambda_{\text{Y}}^2 e^{-2\lambda_{\text{Y}}^2 z},$$  (8)

where $\rho_0$ is the bulk density of the material and $m$ is the mass of the atom. In the approximation of an infinite plane the
Newtonian gravity causes an overall shift of the potential, but it does not introduce a potential gradient.

3.1. Study of the Casimir-type potentials

First we consider the effect of the Casimir-type potentials’ atom–surface interaction based on equation (6), we discuss the feasibility of probing non-Newtonian gravitational correction later. Two-electron atoms such as strontium or ytterbium are considered to be good candidates for precision measurements due to their insensitivity to the stray magnetic field. $^{88}$Sr is proven to be a promising candidate also because of its large mass.

Table 1 shows the shift of the optical lattice potential due to the contribution of the Casimir potential for $^{88}$Sr atoms in different wells of the optical lattice potential.

| Lattice site | 2 | 5 | 10 | 20 | 40 |
|--------------|---|---|----|----|----|
| $z$ (μm)     | 0.40 | 1.20 | 2.53 | 5.19 | 10.5 |
| $U/L(2\pi\hbar)$ (Hz) | 7.9×10$^2$ | 102 | 6.5 | 0.52 | 0.050 |
| $F_c/(\hbar\omega_0)$ | 31.2 | 0.15 | 4×10$^{-3}$ | 160×10$^{-6}$ | 7.2×10$^{-6}$ |

Figure 2. Potential energy of an atom trapped in the vertical optical lattice (equation (1)) in the vicinity of a conducting surface at the temperature 300 K (red solid line). The potential is calculated for $^{88}$Sr near a conducting surface in the region from 0 to 2.5 μm. For comparison we show the lattice potential without any surface potentials (dashed green line), with a Casimir-type potential in the form of equation (4) (blue dotted line).

Figure 3. Shift of the resonant frequency of coherent tunnelling due to the Casimir-type potentials. The $^{88}$Sr atoms tunnel towards the surface. The red solid and green dashed curves are calculated for the tunnelling distances $\delta$ of eight and four lattice sites respectively. The horizontal blue dotted line is the level of the sensitivity of 0.3 mHz.

The expected frequency shift $\Delta \nu$ can be compared with the expected sensitivity of 0.3 mHz. All our calculations are performed for a lattice constant of 266 nm, which corresponds to a wavelength of the lattice laser of 532 nm. We compute the shift of the resonant frequency due to the Casimir potential (figure 3). The shift of the resonance depends on the direction of the tunnelling, i.e. towards or away from the surface. In figure 3 and further in the paper we consider only the case of tunnelling towards the surface. The precision of the method is sensitive enough to detect the change of the temperature of environment in resolution of a few kelvins.

The result of resonant tunnelling through many lattice sites can be observed directly by absorption of fluorescence imaging. For illustration we demonstrate a calculated profile of the column density versus the distance from the surface in figure 4. For these calculations we assume a small tunnelling flux, i.e. a fraction of atoms which have tunnelled twice is negligible. The typical depth of the lattice potential is much larger than the recoil energy $E_r$. In this case WS wavefunctions are well localized in each potential well.

For small distances the non-zero size of atomic samples yields a noticeable effect. Namely atoms sitting in different lattice sites experience different potential shifts and hence have different resonant frequencies. Such differences have to be compared with the linewidth of the transitions. For distances of 10 μm or smaller a relative shift of potential energy for atoms sitting in two neighbouring sites are larger than the Fourier limited linewidth of 32 mHz for the modulation time of 10 s. Basically in the frequency space strongly non-linear, curved Casimir-type potentials cause inhomogeneous broadening of tunnelling profiles. Although such broadening can be computed, the sensitivity of this method relies on a small linewidth of resonances and, hence, is directly affected. Loading the majority of atoms in one lattice site would be ideal; otherwise such broadening can be a major limit of reaching precision for distances smaller than 10 μm.

3.2. Study of the Yukawa-type potentials

We perform similar calculations for the frequency shift of the Yukawa-type gravitational potential. $^{174}$Yb is almost...
twice as heavy as $^{88}$Sr, which improves the projected accuracy for measurements of gravitational potentials. In figure 5 we plot the differential shift of the resonant frequency due to the Yukawa-type gravitational potential, based on equation (8), i.e. we plot the difference of the frequency shift above two materials with different densities for a range of the $\lambda$ parameters. One can evaluate reachable constraints on $\alpha$, within given sensitivity. To compare the expected constraints of our method on $\alpha$ with the present experimental constraints we use the conventional way of presentation of the experimental data, such as the $\alpha$–$\lambda$ plane shown in figure 6. The left-lower corner of the plot presents the region of short $\lambda_{gr}$ and small $\alpha$, which needs to be investigated experimentally. Using the value of the precision 0.1 mHz we predict the experimental constraints that can be achieved with our scheme. The high spatial resolution allows a substantial improvement of the present experimental constraints for the short wavelengths ($\lambda_{gr} \leq 10 \mu m$). For instance for $\lambda_{gr} = 1 \mu m$ an improvement on $\alpha$ of a factor of $\sim 10^3$ is reachable for a distance of 5 $\mu m$ between the atomic sample and the surface.

### 4. Experimental realization

In order to perform the described measurements several experimental steps need to be performed.

- An atomic probe, i.e. a sample of cold atoms, has to be prepared.
- The atomic sample has to be accurately positioned at the needed distance from the studied surface.
- After modulation of the optical lattice one has to do readout by absorption or fluorescence imaging.

Here, we briefly discuss the most important experimental details that might affect the accuracy of the measurements.

The presented scheme, potentially, can be performed with any laser cooled atomic species. However, atoms with small elastic cross section and low density atomic samples are preferable. Zero-spin atoms have a clear advantage, since they are insensitive to magnetic fields.

Stray electric and magnetic fields originating from contaminations of the surface were reported to be significant possible error sources [19, 32, 33]. Typical local electric field strengths caused by adsorbates are reported to be 100 V cm$^{-1}$.
or less [31–33] for distances of 10 μm. The strength of electric fields caused by the patch potential is expected to be of the same order of magnitude or smaller [34]. For example, for strontium a fractional shift of resonant frequency caused by the electric field equals Δf/ν = αSrE/(2mSr), where αSr is the electrostatic polarizability of Sr and E is the electric field strength, and it is of the order of 10−11, i.e. well below the resolution of the described experiment. Further a surface might be cleaned by heating as it was done in the Cornell group [32].

The focus of the lattice beam does not necessarily coincide with a position of the atoms, which causes an additional gradient of the potential that shifts the resonant frequency. However for the study of the surface potentials in the range of 5–30 μm the change of the gradient is negligible for the lattice beam waist of 100 μm or larger.

Imaging poses an important problem. The accuracy of the measurements strongly relies on a precise determination of the distance between atoms and a surface. In the recent experiment [19] at distances of 6–10 μm, the uncertainties on z are around 0.2 μm; this implies a limit of about 10% on the measurement of Ucp. The imaging resolution of about ~1 μm was achieved recently [36, 37]; further the short wavelengths used for imaging of Sr and Yb (461 and 399 nm respectively) are advantageous. Alternatively atom–surface distances can be accurately calibrated using an optical elevator as demonstrated in [35]. The use of the retro-reflected lattice is a substantial advantage for determination of the distance from the surface. Since the lattice phase is defined on the conducting surface by the boundary condition, the resolution of the imaging system should be just sufficient to distinct positions of the different lattice sites. Then the exact position can be inferred.

5. Conclusion

In this paper we have studied the feasibility of precision measurements of the surface potentials with a high spatial resolution at distances in the range of 5–30 μm using coherent resonant tunnelling of atoms trapped in optical lattices. Simple calculations show that this method can lead to more accurate measurements of Casimir-type potentials. The search for possible deviations from the Newtonian gravitational potential is also possible. The scheme of differential measurements should allow substantial improvements of constraints on possible non-Newtonian gravity for λg smaller than 10 μm.

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