The Realization of Tracking Power-line Interference Adaptive Coherent Model Based on Part FFT

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Abstract. Filtering of powerline interference is very meaningful in the measurement of biomedical events recording, particularly in the case of recording signals as weak as the ECG. The available filters for powerline interference either need a reference channel or regard the frequency as fixed 50/60Hz. Relating to these problems, a new algorithm is presented. The basic principle includes the adaptive coherent model and also tracking the actual powerline frequency. In this paper, the development of a filter is briefly outlined, the basic idea is introduced and the detailed technique is also discussed.

1. Introduction
Various artifacts often contaminate electrocardiogram (ECG) recording, of which the most common are powerline interference and baseline drift. The power-line interference, as obtained from the same electrodes as the ECG, are difficult to remove, due to the frequency of the time-varying power line signal that lies within the frequency range of the ECG signal. Therefore, how to eliminate or reduce the effect of 50/60Hz interference has been one of the most important problems in biomedical signal measurement.

During the last twenty years, the literature has proposed several solutions to remove the powerline interference from ECG signals. Most of these solutions are based on ECG signals. Digital filters for AC-noise removal fall into four common categories [1]:

1) Low pass filters that also severely attenuate important frequency components of the ECG signal above a cut off frequency which lies below 50/60 Hz;

2) General notch-rejection filters which can be grouped into two categories: IIR and FIR filters. FIR filters behave similarly to low pass filters, attenuating the entire signal content together. IIR filters instead causes undesirable distortions especially after the QRS complex;

3) Adaptive filters remove interference from ECG using reference input a pure power line noise. This filtering method leaves the source signal undistorted, but it cannot follow fast changes in the interference amplitude producing an undesired ringing effect;

4) Global filters have the main drawback of producing an inter-beat average difference and are not very suitable for real-time implementations.

Although classical adaptive filtering provides a partial solution to the problem, the problem is still considered open and research continues to find an ultimate solution. Compared with conventional analog and digital filters, the adaptive coherent model proposed by Prof. Li requiring no reference signal can simultaneously reject powerline interference and baseline wander [2]. However, the relevance of the filter’s bandwidth at low-frequency and power-line interference, limits filtering
interference effect [3]. For ameliorating its performance, a part FFT used to track powerline frequency and sample frequency is accordingly adjusted. The main advantage of the developed method in comparison with other simpler and faster approaches is the accurate interference reduction in cases when the power line frequency deviates from the nominal 50 or 60 Hz. To explain the proposed algorithm, the adaptive coherent model is briefly described as follows.

2. Adaptive coherent model [2]
Adaptive coherent model by Prof. Li based on signal coherent quality includes:

- An adaptive model is established using the original signal itself
- The interference is subtracted from the original signal to eliminate powerline interference.

To present the procedure of reduction of powerline interference in ECG sampled with a frequency of K times powerline interference (50/60 Hz), let us consider the ECG series, which can be expressed as follows (supposing there is no other interference):

\[ X(n) = S(n) + N(n) \]  

where \( X(n) \) is the digitized time series of original ECG signal, then \( S(n) \) and \( N(n) \) are, respectively, the digitized time series of the true component and the interference of it, therefore the template can be defined:

\[
M(n) = \sum_{i=1}^{M} X(n - i) = \sum_{i=1}^{M} S(n - i) + \sum_{i=1}^{M} N(n - i)
\]

(2)

where \( f_s \) is the sampling rate, \( f_g \) represents the powerline interference frequency and \( M \) equals the length of interference length. For adaptive coherent model, here \( f_s \) is required to be a multiple of \( f_g \), namely \( f_s = L \cdot f_g \) (\( L \) is a integer).

It is well known that \( N(n) \) is periodic signal and \( S(n) \) is zero-mean signal because most biomedical signal amplifiers use an R-C coupling circuit. Therefore, when \( m \) is large enough, the following equations are achieved:

\[
\frac{1}{M} \sum_{i=1}^{M} N(n - L \cdot i) = N(n) \quad \text{and} \quad \frac{1}{M} \sum_{i=1}^{M} S(n - L \cdot i) = 0
\]

(3)

Then by substitution of (3) into (2), the equation (2) can be rewritten as:

\[ M(n) = \frac{1}{M} \sum_{i=1}^{M} X(n - L \cdot i) = N(n) \]

(4)

Based on the formula (1), thus the following equation is obtained:

\[ S(n) = X(n) - N(n) = X(n) - M(n) = X(n) - \frac{1}{M} \sum_{i=1}^{M} X(n - k \cdot i) \]

(5)

The above equation shows that powerline interference can be eliminated only by subtracting the model from the original signal without a reference signal and a series of complicated computations.

But in practical application, we found that when the powerline frequency deviates from normal 50/60Hz, adaptive coherent model filter can’t work well. For profound understanding of this issue, the two sides of equation (5) are transformed into z plan and then the system transfer function \( H(z) \) may be written in terms of the z-transform of the input sequence \( X(n) \) and the output sequence \( S(n) \):

\[
H(z) = \frac{S(z)}{X(z)} = 1 - \frac{1}{M} \sum_{i=1}^{M} z^{-iL} = 1 - \frac{1 - z^{-LM}}{M(1 - z^{-L})}
\]

(6)

Here \( Z \) is set to \( e^{jw} \), that is to say \( Z = e^{jw} \), thus the frequency response of the system can be obtained:
Equation (7) can be seen as a band-pass filter subtracted from a global pass filter. The frequency response of the band-pass filter is determined by \( \frac{\sin(LM\omega)}{\sin(L\omega)} \), in which the numerator with short period fluctuates quickly along with the variable \( \omega \), that qualifies the bandwidth of the filter, and, on the contrary, denominator with long cycle changes slowly that decides the position of central frequency \( f_0 \) of the band-pass filter, namely \( f_0 = f_s / L \). From the periodic property of \( H(e^{j\omega}) \), it is concluded that the bandwidth on periodic interference equals to it on low frequency. Fig.1 shows filter system frequency response versus model length \( M \). Figure1 demonstrates that the larger \( M \), the smaller the ripple of pass band and the narrower as bandwidth of band-rejection, \( B_f \). Based on the above theoretic analysis, it is conclude that: i). For the sake of good low frequency property(some involved standers require the cutoff frequency of low frequency less than 0.05Hz ), the value of model length \( M \) can be increased within the range allowed by the system resource. ii). The central frequency is only decided by the \( L \), regardless of \( M \). But in fact, the actual powerline frequency is not exactly 50/60 Hz which has a variable range, 50±0.5Hz, thus the effect of powerline interference reduction deteriorates after the \( M \) exceeds specific value. As a result, the algorithm should be improved so as to satisfy powerline interference reduction and low-frequency property.

3. An Improved method

All in all, adaptive coherent model method can not simultaneously answer effectively eliminating powerline interference and good low-frequency property with the varying frequency powerline interference. For resolution of the conflict, this paper proposes an improved adaptive coherent model method which pick up the actual frequency \( f_{ac} \) of powerline interference adjusts the sample frequency \( f_s \) according to \( f_{ac} \), that is to say \( f_s = L \cdot f_{ac} \), in order to set the central frequency of the filter as \( f_{ac} \). Therefore, we can set the \( M \) value as big as possible in order to satisfy the low-frequency property without worsening the benefit of powerline interference.

3.1. The rationale of method

All the stages involved in the method implementation is shown in figure 2: i) first, we sample some number \( N \) of digitalized ECG data. ii) secondly, the spectral line \( k \) relating to the \( f_{ac} \) (the actual powerline frequency) is acquired using discrete Fourier transform(DFT) and compared with the

Figure 1. Filter frequency response versus different \( M \) value.
predicted spectral line position \( k_2 \) to specific frequency \( f \) which can be given by the formula \( k_2 = N \times f / f_s \) to get the difference. iii) Based on the difference, the sample cycle is regulated, or rather if \( k_1 < k_2 \) which suggests the actual frequency \( f_{ac} \) less than nominal 50/60Hz, \( f_s \) should be downregulated, by contrast, if \( k_1 > k_2 \) which shows the former more than the latter, should be upregulated. Then, the similar fashion is repeated until the difference error is tolerated.

![Figure 2. The scheme of sample frequency auto-adjusting](image)

3.2. Parameter selection

As we all known, the Picket Fence Effect of the FFT spectrum which is a discrete spectrum and contains information only at the specific frequencies decided by frequency resolution \( F \), may result in the true spectrum of the signal being analyzed, that have peaks at frequencies between the lines of the FFT spectrum lost [4]. Thus, in this method appropriate \( F \) selected is critical.

Considering \( F \) given by the formula \( F = f / N \), it is concluded that increasing the number of points in the acquired time-domain signal can improve the frequency resolution for a given sample frequency, whereas, results in increased system computation burden which may limit real-time application; by contrast, the less a number of points may lead to spectral line lost because of Fence Effect, which directly results in deterioration of interference suppression, for example, for \( F = 0.1 \text{Hz} \), \( B_f = 0.1 \text{Hz} \), the actual powerline frequency \( f_{ac} \) equal to 50.17Hz, is judged by program as 50.1Hz and \( f_s \) is accordingly changed, clearly, AC interference of 50.17Hz is excluded from stopband with frequency range 50.05 to 50.15Hz and can not be well reduced. In a word, \( N \) should be fixed according to the bandwidth \( B_f \) and frequency resolution \( F \).

As noted in the paper, the cutoff frequency of the filter required by involved standards should be no more than 0.05Hz, thus the bandwidth \( B_f \) on powerline and its harmonic is less than 0.1Hz. As \( B_f \) can be calculated by \( 2 \times 50/(L \times M) \) and the given sampling frequency is 400Hz, the length M of 128 can satisfy the requirement. Considering picket fence effect, the resolution bias error should be no more than 0.05Hz, that is to say \( f_{ac} = 0.05 \text{Hz} \), in order to well eliminate the powerline interference, for example, to the last example, if \( f_{ac} = 50.17 \text{Hz} \) and \( f = 0.05 \text{Hz} \), the frequency judged by program only can be either 50.15Hz or 50.20Hz and the corresponding stopband frequency range are, respectively, 50.10–50.20 Hz or 50.15–50.25 Hz which apparently ensure \( f_{ac} \) fall into either of them. For obtaining this \( F \), the number \( N \) of points acquired in the time domain with the sampling frequency equal to 400Hz calculated from the expression \( N = f_s / F \) is 8000. Commonly, \( N \) is set as \( 2^{13} \) for radix-2 FFT algorithm.

3.3. Partial FFT for tracking powerline frequency

Given N-point sequence \( x(n) \), its discrete-time Fourier transform is given by the expression

\[
X(k) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi nk/N} = \sum_{n=0}^{N-1} x(n)W_N^{nk}, \quad k=0,1,\ldots,N-1.
\]

Commonly, both \( x(n) \) and \( W_N^{nk} \) are complex, and \( X(k) \) is also complex. Thus each DFT sample \( X(k) \) requires \( N \) complex multiplications and \( N-1 \) complex additions, and there are \( N \) samples to compute, so direct DFT computation would require (at most) \( N^2 \) complex multiplications and additions. FFT is the fast algorithm of its computation, which is based on a sum of butterfly computations, each of which requires one complex multiplication and two complex additions. To see this, a diagram of 8-point decimation-in-time FFT, with inputs in bit-reversed order is shown in figure 3. With the FFT algorithm, on the other hand, there are \( r = \log_2 N \) stages, each of which consists of \( N/2 \) butterfly
computations and requires up to N/2 complex multiplications by twiddle factors, N/2 complex additions and N/2 complex subtractions. Hence, the computations of r stages include:

- complex multiplications \( N/2 \log_2 N \)
- complex additions \( N \log_2 N \)

Although the computations of FFT are much less than that of direct DFT, the computations of 2^13-points of FFT on DSP (Digital Signal Processor) are significant and thus the method should be further optimized.

From the above theoretical analysis, we can know that for the given frequency resolution \( F \) is lower as sampling frequency, fewer as sampling points. Since the computations of FFT are in proportion to \( N \), the time-saving can be achieved through reducing the sampling frequency. Therefore, the paper presents a method of omitting points for simulating a sample with 200Hz, thus, for \( F=0.05 \), the number of sampling points only needs \( 200/0.05=4096 \).

In addition, it is known that the spectral lines contributing to the algorithm are only some of them in each stage of FFT. For example, if \( F=0.05 \), \( Fs=200Hz \) and the frequency of powerline ranges from 50.05 to 50.15Hz, only 21 lines of the resulting spectral lines sequence \( X(k) \) are beneficial to tracking the frequency. Hereby, the portion of each stage of FFT needed can be deduced.

To say how to simplify the FFT, the computation procedure is showed on 8-points FFT. If the final portion \( X(1) \) is interesting, From fig.3, there are \( y_2(1) \) and \( y_2(5) \) in third stage, \( y_1(1) \) and \( y_1(3) \), \( y_1(5) \) and \( y_1(7) \) in the second stage, and all lines in the first stage. That is to say, the computation of FFT of the given points can be reduced by means of only calculating the specific spectral lines. Fig.4 shows calculation quantity vs the number of point.

4. Algorithm comparison

To compare the effect of filter before and after amended ECG, with three types of powerline deviation.
state as input signal is used to test this algorithm, the results is showed in figure 5, figure 6, figure 7. Among each of them, the above trace is for the original filter and the below one is for filter amended.

![Figure 5](image1.png)  
Figure 5. The result of ECG with standard 50Hz interference

![Figure 6](image2.png)  
Figure 6. The result of ECG with 50.5Hz interference

![Figure 7](image3.png)  
Figure 7. The result of ECG with interference frequency deviation form 50Hz to 50.5Hz

Figure 5 demonstrates that ECG with standard 50Hz powerline interference can be well suppressed with both. But figure 6 shows that to powerline interference with 50.5Hz, the effect of filtering is worsening for the narrowness of the stopband width of the original filter, while, the effect of the improved filter is still good since the central frequency of stopband can be changed along with frequency fluctuation. Figure 7 shows the state of powerline frequency deviates from 50Hz to 50.5Hz.

5. Conclusions
The Adaptive coherent model method is easily implemented and suppresses the interference well, but at the same time is limited in practical application area, such as ECG, because of failing to satisfy powerline interference suppression and low-frequency properties, which results from its primary rationale. This paper further investigates the algorithm and presents an improved method which tracks the actual powerline frequency using partial FFT and then accordingly adjusts the sampling frequency to ameliorate the filter. From the above experimental results, it is known that powerline interference with frequency deviating slowly can still be well reduced.

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