Distributed Shortest Link Scheduling Algorithms with Constant Time Complexity in IoT Under Rayleigh Fading

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ABSTRACT For the shortest link scheduling (SLS), i.e., scheduling a given set of links with minimum time slots, we consider the distributed algorithm design by using the locality of the protocol model with high fidelity under the Rayleigh fading. Different from most previous works, focusing on distributed algorithm design under the deterministic SINR model, which ignores the fading effects in signal propagation, we first show that a successful link of protocol model is also feasible under the deterministic SINR model, then it can be scheduled successfully with high probability under the Rayleigh fading, by upper-bounding interference outside interference range of protocol model. Then on the basis of this key conclusion, we design LLS-SLS algorithm to solve SLS within $(2e\Delta_{max}^T+1)\delta \log_2 \Delta_{max}^T$ time slots for a constant $\delta$. Specifically, $\Delta_{max}^T$ is the number of a sender’s neighbors within some certain range, and can be upper-bounded. Next, based on the concept of random contention, we design CLLS algorithm to schedule all links after costing $4(\delta+1)\Delta_{max}^T \ln (\Delta_{max}^T + 1)$ time slots. In addition, extensive simulations evaluate the performance of two proposed algorithms.

INDEX TERMS Shortest link scheduling, Rayleigh fading, locality, distributed algorithms.

I. INTRODUCTION

The Internet of Things (IoT) is gaining ground in various emerging applications such as mobile healthcare, video transmissions and smart city etc., which require more stringent delay quality-of-service (QoS). However, it is a challenging problem to provide lower delay for wireless transmissions, since wireless signal strength usually fluctuates over time due to time-varying fading. On the other hand, the IoT devices are energy-constraint. Efficient link scheduling is crucial to combat the fading and interference effects of wireless channels, and provides satisfactory latency QoS to the users. Thus in this paper, we focus on the distributed algorithm design for the shortest link scheduling (SLS) (i.e., minimize the number of slots until all transmissions were successful) under Rayleigh fading.

Primarily, SLS was studied in a centralized setting. In many realistic scenarios, however, it is imperative that a distributed solution be found, since links may not obtain the global topology information of the network. For this problem, in [1], Kesselheim and Vocking proposed a simple and natural distributed algorithm that provides an $O(\log^2 n)$-approximation. In the following work [2], Halldorsson and Mitra showed that it achieves $O(\log n)$-approximation, an improvement of a logarithmic factor. Moreover, the choice of interference model has crucially impact on distributed algorithm designs for wireless networks. The SINR model can capture reality more precisely than the protocol models, but the interference relations defined by it are global and combinatorial. Thus, it is difficult to use SINR model to design distributed algorithms.
Unlike SINR model, although the validity of protocol model is doubtful because that the interference is regarded as existent only between nodes in a local neighborhood, the local nature is suitable for distributed algorithm design since distributed algorithms need commonly a local information rather than a global one.

As a result, most distributed algorithms work in protocol model even if there have been doubts on its validity. Media access under this model, apart from exceptions, is usually done probabilistically. A simple idea is to let nodes transmit randomly, with a probability inversely proportional to the ‘competition’. In particular, if every node transmits with some certain probability, we are sure that at least a constant fraction of the transmissions is successful. In addition, protocol model is beneficial to design distributed algorithms due to its locality, but its validity should be carefully examined before putting it to use. To enable distributed algorithm designs, predictable interference control, an open question is whether it is possible to develop a model of measuring interference, which has locality of protocol model and high fidelity, under Rayleigh fading.

In our previous work [3], we increased reliability and latency requirements of IoT under Rayleigh fading by solving link scheduling problem. Specifically, utilizing idea of localized global interference, two distributed algorithms were proposed. In this paper, we now present complementary, novel algorithms for low-latency performance. Following the previous works [3]–[5], we regard a link as being scheduled successfully under Rayleigh fading if its success probability is greater than $1 - \epsilon$, where $\epsilon$ is an acceptable failure probability, since Rayleigh-fading leads to the uncertainty of strength of received signal.

To eliminate effect of stochastic channel fading gain in Rayleigh fading on algorithmic design and analysis, we simplify it to deterministic signal-interference-ratio (SIR) model. The only problem is that we need to show that a successful link of deterministic SIR is also successful with probability $1 - \epsilon$ under Rayleigh fading. Moreover, to design distributed algorithms for latency minimization, we use protocol model to localize the global interference of deterministic SIR model, and show that a successful link of protocol model is also successful under deterministic SIR model (called as SIR-feasible simply).

Specially, we first prove that the cumulative interference outside interference range of the protocol model can be upper-bounded under deterministic SIR model; then we show that a link can be scheduled successfully with high probability under the Rayleigh fading by ensuring that above interference has no effects on its success probability. Specifically, we bound the Rayleigh success probability of this transmission is at least $1 - \epsilon$ by using distance constraint $\Delta_{\text{max}}$ as interference range of protocol model, where $\alpha$ is the path-loss exponent, $\beta$ is the decoding threshold, $l_{\text{max}}$ is the maximum link length, $c$ is a constant [6]. Consequently, we present two distributed algorithms for SLS by using the local characteristic of protocol model with high reliability under Rayleigh fading. For settings of $\alpha = 5$, $\beta = 1$, $l_{\text{max}} = 10m$, $\epsilon = 0.1$ and $c = 0.1$, $\Delta$ is 1.8559. In summary, the main contributions of this paper are given as follows:

- Setting uniform transmission probability for each sender. All senders execute local broadcast algorithm to assign transmitting time to themselves, we design a distributed link scheduling algorithm for SLS (denoted by LLS-SLS). After achieving local broadcast, the number of senders whose transmission times are same is upper bounded and all senders can transmit successfully with high probability after $12\delta(\Delta^2 + 1)\ln\Delta^T_{\text{max}}$ time slots, where $\Delta^T_{\text{max}}$ is maximum size of neighbors of a sender.
- Based on assumption that the distance between any two nodes is at least 1m, parameter $\Delta^T_{\text{max}}$ can be upper-bounded by $\frac{1}{\pi^2} \cdot [c(\Delta + 2l_{\text{max}} + 2)]^2$, independent of network size. Specifically, LLS-SLS gives a $1 + \delta(\Delta^2 + 1)$-approximation factor of the optimum.
- Finally, utilizing localization characteristic of protocol model, we propose a random contention algorithm for SLS (denoted by CLLS) based on local information. The idea is that each sender accesses the channel in any time slot with probability $\frac{1}{\Delta^T_{\text{max}}}$ until its first success. In case of a collision, none of the involved senders is successful in this time slot.

The rest of the paper is organized as follows. Section II provides the related work. In Section III, we present network model and the overview of our design. In Section IV, we describe the model transformation from protocol model to SIR model. In Section V, we present two distributed algorithms for SLS problem, and simulations are shown in Section VI. Finally, we summarize our work and conclude the paper in Section VII.

II. RELATED WORK

In the centralized settings, SLS is closely related to capacity maximization (i.e., maximize the number of successful transmissions in a single slot), since we can execute algorithms for capacity maximization repeatedly to solve SLS. However, those works about capacity maximization were centralized and lead to equivalent bounds for SLS with $O(\log n)$-approximation (e.g., [4], [6]–[23]).

The interference localization method offers important insights of designing distributed scheduling algorithms (e.g., [24]–[27]). The main idea is to assume the aggregated interference from the senders beyond certain distance can be upper-bounded by a threshold, since the interference power level decreases over distance due to the free-space path-loss. But how to determine this key parameter may be a difficult problem.
Distributed algorithms for greedy maximal scheduling by using interference localization were proposed in [26] and [27]. They introduced the concept of interference neighborhood of a link and constrained the suffered interference inside and outside interference neighborhood being bounded (even the latter can be negligible), respectively. Then, global characteristic of the SINR model is successfully localized, links only need to perform scheduling coordination inside its interference neighborhood. However, the procedure for determining the interference neighborhood is centralized.

By combining the partition and shifting strategies with a pick-and-compare scheme [25], Zhou et al. presented a class of localized scheduling algorithms with provable throughput guarantee subject to physical interference constraints. On observing that distance dominates the interference, they partitioned the links into disjoint local link sets with a certain distance away from each other, and using this implemented localized scheduling.

Similarly, the main idea of plane-partition-based algorithms also used interference localization, i.e., deriving an upper bound of the interference received outside neighboring cell (e.g., [4], [28]–[30]).

The analysis of connectivity’s scheduling complexity, i.e., the minimal amount of time slots required until a connected structure can be scheduled, in the SINR model that deals with arbitrarily networks has been studied by Moscibroda and Wattenhofer [23]. However, they did not solve the problem of SLS but a similar one. Instead, they proved that scheduling a set of pairs of senders and receivers that all problem of SLS but a similar one. Instead, they proved that scheduling a set of pairs of senders and receivers that all

interference outside neighboring cell has been studied by Moscibroda and Wattenhofer [23]. However, they did not solve the problem of SLS but a similar one. Instead, they proved that scheduling a set of pairs of senders and receivers that all under Rayleigh fading model for the first problem, because of the uncertainty of strength of received signal under Rayleigh fading model.

B. INTERFERENCE MODEL TRANSFORMATION

Different from the determining SIR model, the strength of received signal under the Rayleigh fading model varies from time slot to time slot, which is described by channel fading gain h, as shown in Eq. (1). In addition, the success probability that a link is scheduled under Rayleigh fading model cannot be up to 1, since inherent nature of fading makes the strength of received signal uncertain. However, the existence of fading gain brings a big challenge on algorithm design and analyses. Therefore, we need to remove the effect of fading gain. Based on the works in [3] and [4], if above success probability exceeds 1 – ϵ, a link can be regarded as being scheduled successfully under Rayleigh fading model. Combining with the SIR model, if we can prove that the success probability of a link determined by SIR model is 1 – ϵ at least, the effect of fading gain can be removed. It is reasonable because of the uncertainty of strength of received signal under Rayleigh fading model.

For distributed algorithms of the SLS, Kesselheim and Vöcking introduced a measure called maximum average affectance to analyze random contention-resolution algorithms, in which each packet is transmitted with a fixed probability depending on the maximum average affectance, with O(\log^2 n)-approximation [1]. Subsequently, approximation factor was improved to O(\log n) by ensuring each link transmitting with some certain probability within logarithmic time slots [2]. Moreover, they concluded that the best possible absolute performance guarantee is logarithmic. Similar results can be obtained in [6], [27].

III. NETWORK MODEL AND DEFINITIONS

A. NETWORK MODEL

Considering a set of wireless communications, denoted by \( L = \{ l_1, l_2, \ldots, l_n \} \), where \( l_i \) represents a transmission from sender \( s_i \) to receiver \( r_i \). The Euclidean distance between node \( i \) and node \( j \) is denoted by \( d_{ij} \) and assume that it is at least 1. Let \( d_{\text{max}} \) be the maximum link length. The distance between link \( l_i \) and link \( l_j \) is the distance from the former sender to the latter receiver. Transmission power is set to \( P \).

Under Rayleigh-fading, the strength of received signal from sender \( s_i \) to receiver \( r_j \) is a random variable, denoted by \( P h_{ij} / d_{ij}^\alpha \), and is exponentially distributed with mean \( P d_{ij}^\alpha / \bar{h} \), where \( \bar{h} \) is the channel fading gain between nodes and \( \alpha \) is the path-loss exponent. As the noise power has negligible effect on the results, we then ignore its influence on transmission of a link [11]. [11]. Then, the signal-interference-ratio (SIR) of a link \( l_i = (s_i, r_i) \) with respect to a scheduling set \( S \) is the ratio of the signal strength received at \( r_i \) from \( s_i \) to the signal strength received at \( r_i \) from other non-intended senders in \( S \). Mathematically, the received SIR of link \( l_i = (s_i, r_i) \) in the presence of \( S \) is given by

\[
\gamma_i = \frac{h_{ii}/d_{ii}^\alpha}{\sum_{j \neq i, \forall j \in S} h_{ji}/d_{ji}^\alpha},
\]

the receiver \( r_i \) can successfully decode the signal transmitted by its sender \( s_i \), if the SIR is above a certain threshold \( \beta \), this is \( \gamma_i \geq \beta \).

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Random fading gain and global interference are two challenging problems for designing and analyzing distributed algorithms under Rayleigh fading. To solve those two problems, we first make a transformation from deterministic SIR model to Rayleigh fading model for the first problem, by proving that a \textit{SIR-feasible} link (i.e., this link can be scheduled successfully in the SIR model) is also scheduled successfully with high probability under Rayleigh fading. Next, for the second problem, we apply the protocol model to localize global interference of deterministic SIR and show that a successful link of protocol model also is \textit{SIR-feasible}.
For protocol model, a link is successful when corresponding intended receiver falls outside the interference ranges of other non-intended senders, which can be simply abstracted to a disc of the radius $\Delta_{l_{\text{max}}}$, and settings of $\Delta$ will be given in Formula (2). To localize the global interference and design distributed algorithms, we need prove that the interference outside some range can be bounded, and has no effect on the successful transmission of a link. In the following section, we bound the interference outside $\Delta_{l_{\text{max}}}$ by defining the neighbor of a sender, and show that success probabilities of a link are 1 and $1 - \epsilon$ under deterministic SIR and Rayleigh fading, respectively.

Definition 1: Neighbors of a sender $v$ are a set of other senders whose distance satisfies $d_{vw} \leq (\Delta + 1)l_{\text{max}}$ for $w$, denoted by a set $N_v$.

Define $\Delta_{l_{\text{max}}}$ as the maximum neighbor size of a sender in the network. In the following analysis, we prove that $\Delta_{l_{\text{max}}}$ can be upper-bounded by a constant.

Following the existing researches [31]–[33], we employ Aloha as the medium access control (MAC) protocol, in which each sender $v$ simply transmits with probability $q$ and keeps silent with probability $1 - q$. Then, Our goal is to show that, although protocol model is intrinsically local, it is available to guarantee efficient medium access using this simple and completely distributed manner.

Definition 2: [34] For a receiver $x$, the probabilistic interference at $x$ under deterministic SIR, denoted by $\psi_x$, is defined as the expected value of interference experienced by $x$ in a certain time slot, this is

$$\psi_x = \sum_{v \in S, v \neq x} q_v \frac{p}{d_{vx}},$$

where $q_v$ is the transmitting probability of sender $v$.

At the end of this section, we present Lemmas 1 and 2 that give sufficient conditions that applying protocol model measures interference under Rayleigh fading from point of success probability of a link, which are the basis of further designing distributed algorithms. The proof of Lemma 1 is based on a technique of bounding the size of disjoint discs within some certain range. The proof of Lemma 2 is based on the characteristic of Rayleigh fading and settings of parameter $\Delta$. The detailed proof is given in the following section.

Lemma 1: For a sender $v$, if the sum of all transmission probabilities of its neighbors starting to transmit can be upper-bounded, denoted by $q_{\text{sum}}$, then $v$ can transmit successfully with some certain probability.

In the following section, we will prove $q_{\text{sum}}$ is a constant.

Lemma 2: A successful link of protocol model is also successful with high probability under Rayleigh fading, when $\Delta$ is defined as a function of $\alpha$, $\beta$, $l_{\text{max}}$, $c$ and $\epsilon$:

$$\Delta = \max \left\{ \left[ \frac{g_8}{a^2c^2l_{\text{max}}} \right] \frac{\pi^2}{\ln \left( \frac{1}{1 - \epsilon} \right)} \right\},$$

where $c = \frac{1}{2 + \max(2, (2^{1/9} + 1/9)g_8^{-1/9})^{1/9}}$ [6].

### Algorithm 1 Local Broadcast within $2l_{\text{max}}$

1: $\theta = 12(\Delta^2 + 1)$, $q = \frac{1}{2\Delta_{l_{\text{max}}}}$, slot = $2\theta \ln \Delta_{l_{\text{max}}}$
2: $\text{slot}_1 = \delta \log_2 \Delta_{l_{\text{max}}}$, start = 0
3: while slot do
4: if $v$ starts to transmit then
5: Update start$_t = \text{start}_v + \text{slot}_1$
6: Send message rej$_m$ to neighbors within $2l_{\text{max}}$
7: else
8: Listen
9: end if
10: if $v$ receives rej$_m$ and does not broadcast it before then
11: Update start$_v$ with max{start$_t$} + slot, $i$ is a neighbor of sender $v$ within $2l_{\text{max}}$
12: end if
13: end while
14: Senders whose starts are same start to execute Algorithm 2

### V. DISTRIBUTED ALGORITHMS FOR SLS

#### A. LOCAL BROADCAST BASED SLS

In this section, we present a distributed algorithm for latency minimization. Each sender starts to broadcast a message with probability $q$, such that after achieving local broadcast, the number of senders simultaneously transmitting in a certain area of the network is bounded to permit each sender to perform a successful communication within logarithmic time slots with high probability.

We apply the communication model in [38] and [39] and as follows. The pseudo-code of local broadcast is shown in Algorithm 1 with synchronous message passing model, in which each node sends/receives to/from all of its neighbors in each communication round, and the communication rounds are synchronized. If we assume that each round takes one time unit, then the time complexity of the algorithm is the number of time units taken from start to completion. Also, we can set a maximum number of rounds for the algorithm execution according to its time complexity.

Note that, there may exist two colliding cases, one is that a sender receives at least two messages rej$_m$ concurrently from different neighbors, which is called as the hidden terminal problem. Another is that at least two senders receive message rej$_m$ from each other in a time slot. Next, we show that those two case can be solved with high probability by settings of transmission probability and transmission time to make local broadcast.

In the first case, the probability that a sender receives at least two messages rej$_m$ concurrently, denoted by $p_{\text{con}}^1$, is

$$p_{\text{con}}^1 = (1 - q) \left( q^2 + q^3 + \cdots + q^{\Delta_{l_{\text{max}}}} \right) \leq \left( 1 - \frac{1}{2\Delta_{l_{\text{max}}}} \right)^{\frac{\Delta_{l_{\text{max}}}}{\sum_{j=2}^{\Delta_{l_{\text{max}}}} \frac{1}{2j}}}.$$
be the smallest number of discs \( D \)
given slot is \( R \)
slot \( i \), i.e., ensuring they can transmit successfully within
senders whose \( q \) transmitting node in that slot. Assume that
each other can have different
its new
start
have known their transmission times by updating their
start
rej
. Then, senders receiving \( rej_{m} \)
with high probability, and above two colliding problems can not
exist with high probability. After \( slot_{1} \) time slots, all senders
have known their transmission times by updating their \( start \)
with the maximum value of \( start \) recorded by their neighbors
inside \( 2l_{\max} \), respectively.

Specifically, if a sender starts to transmit, then it increases
its \( start \) to \( start + slot_{1} \) and notifies its neighbors within \( 2l_{\max} \)
its new \( start \). In this way, senders which are neighbors from
each other can have different \( start \) and execute Algorithm 2 in
different slots. The purpose of \( slot \) is to ensure those senders
whose \( start \) is the minimum can execute Algorithm 2, i.e., ensuring they can transmit successfully within \( slot \) slots.

Fact 1: \([34]\) Consider two discs \( D_{1} \) and \( D_{2} \) of radii \( R_{1} \) and \( R_{2} \), respectively, assume that \( R_{1} > R_{2} \), we define \( \chi_{R_{1},R_{2}} \) to be the smallest number of discs \( D_{2} \) needed to cover the larger disc \( D_{1} \). It holds that
\[
\chi_{R_{1},R_{2}} \leq \frac{2\pi}{3\sqrt{3}} \cdot \frac{(R_{1} + 2R_{2})^2}{R_{2}^2}.
\]

First, based on the assumption in Section III that the time complexity of Algorithm 1 is a constant.

Theorem 1: Local broadcast of Algorithm 1 can be achieved within \( O(1) \).

Proof: In each slot, a sender \( v \) independently transmits a
\( rej_{m} \) with probability \( q \) and sends \( rej_{m} \) to its neighbors inside \( 2l_{\max} \). Then, senders receiving \( rej_{m} \) change their corresponding \( start \) and then transmit again with probability \( q \). The purpose of \( slot_{1} \) is to ensure the process of message exchange in local broadcast can be achieved with high probability, and above two colliding problems can not exist with high probability. After \( slot_{1} \) time slots, all senders
have known their transmission times by updating their \( start \)
with the maximum value of \( start \) recorded by their neighbors
inside \( 2l_{\max} \), respectively.

Finally, we show that \( \Delta_{\max}^{T} \) can be regarded as a constant. Recall that the distance between any two nodes is at least 1. For each sender \( v \), by using Fact 1, all possible neighbors in an extended ring of radius \((\Delta + 1)l_{\max} + l_{\max} \) around sender \( v \) are at most
\[
\Delta_{\max}^{T} \leq \chi^{(\Delta + 1)l_{\max} + l_{\max}, 1} - 1
\]
\[
< \frac{2\pi}{3\sqrt{3}} \cdot (2\Delta + 2)l_{\max}^2, \tag{4}
\]
and are at least
\[
\Delta_{\max}^{T} \geq \chi^{(\Delta + 1)l_{\max} + l_{\max}, 1} - 1
\]
\[
\geq \frac{2\pi}{3\sqrt{3}} (\Delta + 4)^2. \tag{5}
\]
Along with time slots in first case, the total time slots are at most \( 2(\Delta l_{\max} + 1)\delta \log_{2} \Delta_{\max}^{T} \) and can be regarded as a constant.

Theorem 2: The message complexity of Algorithm 1 is \( O(n) \).

Proof: In local broadcast, each sender sends \( rej_{m} \) to its neighbors within \( 2l_{\max} \). By using Fact 1, the number of neighbors within \( 2l_{\max} \) is at most
\[
\chi^{2, l_{\max} + l_{\max}, 1} - 1 < \frac{2\pi}{3\sqrt{3}} (3l_{\max} + 2)^2.
\]
Thus, the number of all messages is at most \( n(3l_{\max} + 2)^2 \in O(n) \).

Lemma 3: For each active sender \( v \), the sum of transmission probabilities for its active neighbors is bounded, by the execution of the algorithm, this is,
\[
\sum_{v \in N_{v}} q \leq \frac{9\sqrt{3}}{4\pi}.
\]

Proof: For active sender \( v \), the distance between any one of active neighbors and node \( v \), by local broadcasting with \( rej_{m} \), is at least \( 2l_{\max} \). Then, the distance between the receiver of \( v \) and any active neighbors of \( v \) is at least \( l_{\max} \). Thus, all discs of radius \( l_{\max} \) centered at those neighbors are disjoint, and the number of neighbors being active in an extended ring of area \( \pi [(\Delta + 2)^2 l_{\max}^2 - l_{\max}^2] \) is at most
\[
\frac{\pi [(\Delta + 2)^2 l_{\max}^2 - l_{\max}^2]}{\pi l_{\max}^2} \leq 3(\Delta + 1),
\]
then, the summed transmission probabilities are at most
\[
\sum_{v \in N_{v}} q \leq 3(\Delta + 1) \cdot \frac{1}{2\Delta l_{\max}}
\]
\[
\leq 3(\Delta + 1) \cdot \frac{1}{4\pi} \leq \frac{9\sqrt{3}}{4\pi}.
\]

Lemma 3 now yields the claim.

Next, we prove that interference measured by a receiver \( x \) from active senders which are away the sender of \( x \) is at least \((\Delta + 1)l_{\max} \) can be bounded.

Lemma 4: Consider a sender \( v \) and interference range \( \Delta_{\max} \). In a schedule, the sum of transmission probabilities of all active senders outside this range can be bounded by a constant, i.e., if \( \sum_{v \in N_{v}} q \leq c \), then the probabilistic interference
experienced by $x$ (v’s receiver) under deterministic SIR model, caused by active nodes outside this range can be bounded by

$$\psi_{x}^{w \notin N_v} \leq 3 \frac{a - 1}{a - 2} \cdot \frac{P}{\Delta^{a - 2} c l_{\text{max}}^a}. \quad (7)$$

Proof: Consider rings $\text{Ring}_k$ of width $(\Delta + 1) l_{\text{max}}$ around $v$, containing all senders $w$ whose distance satisfies $k(\Delta + 1) l_{\text{max}} \leq d_{\text{wv}} \leq (k+1) \Delta l_{\text{max}}$, here $k \geq 1$ is a constant. In each $\text{Ring}_k$, after achieving local broadcast, all discs of radius $l_{\text{max}}$ centered at those senders which have same value of $\text{start}$ must be located entirely in an extended ring $\text{Ring}_k^+$ of area

$$\text{Area}(\text{Ring}_k^+) < \pi \left[ (k+1) \Delta l_{\text{max}} + l_{\text{max}}^2 - (k \Delta l_{\text{max}} - l_{\text{max}})^2 \right]$$

$$= \pi (2k + 1) \left( \Delta^2 + 2 \Delta \right) l_{\text{max}}$$

$$< \pi 2(2k + 1) \Delta^2 l_{\text{max}}, \quad (8)$$

and any two discs are disjoint since the distance between them are $2 \Delta l_{\text{max}}$.

By local broadcast, the distance from each active sender $w$ in $\text{Ring}_k$ $(k \geq 1)$ to $v$’s receiver, denoted by $x$, is at least $k \Delta l_{\text{max}}$. By applying a standard geometric area argument, we can bound the probabilistic interference $\psi_{x}^{\text{Ring}_k}$ incurred by active nodes located in ring $\text{Ring}_k$ ($k \geq 1$) as

$$\psi_{x}^{\text{Ring}_k} = \sum_{v \in \text{Ring}_k} \psi_{v}^{v}$$

$$\leq \frac{\text{Area}(\text{Ring}_k^+)}{\text{Area}(l_{\text{max}})} \sum_{w \in \text{Ring}_k} q \cdot \frac{P}{(k \Delta l_{\text{max}})^a}$$

$$\leq \frac{2k + 1}{k a} \cdot \frac{1}{\Delta^{a - 2} c l_{\text{max}}^a} \cdot \frac{P}{l_{\text{max}}^a}$$

$$\leq \frac{3}{k a - 1} \cdot \frac{1}{\Delta^{a - 2} c l_{\text{max}}^a} \cdot \frac{P}{l_{\text{max}}^a}. \quad (9)$$

Summing up the interference over all rings yields

$$\psi_{x}^{w \notin N_v} \leq \sum_{k=1}^{\infty} \psi_{x}^{\text{Ring}_k} \leq \frac{3}{k a - 1} \cdot \frac{1}{\Delta^{a - 2} c l_{\text{max}}^a} \cdot \frac{P}{l_{\text{max}}^a}$$

$$\leq \frac{3}{a - 1} \cdot \frac{1}{\Delta^{a - 2} c l_{\text{max}}^a} \cdot \frac{P}{l_{\text{max}}^a},$$

which concludes the proof of the lemma.

Moreover, Lemma 1 now yields the claim according to conclusions of Lemmas 3 and 4.

Corollary 1: A successful transmission of protocol model is SIR-feasible.

Proof: If a link is successful in protocol model, there is no active non-intended senders within $\Delta l_{\text{max}}$ around the receiver of this link. Based on Equation (7), received SIR at this receiver under deterministic SINR is

$$\frac{P d_{\text{wi}}}{P d_{\text{wi}}^2} \geq \frac{1}{\ln \left( \frac{1}{1 - \epsilon} \right)} \beta \geq \beta.$$ 

Therefore, it is successful under deterministic SIR model.

Next, we give another method to measure the upper bound of interference received outside interference range of protocol model under deterministic SIR model. From proof process of Rayleigh success probability in [31] and [35], the summed interference beyond $(\Delta + 1) l_{\text{max}}$ can be expressed as

$$\psi_{x} = \sum_{w \notin \text{Ring}_v \in S} q \cdot \frac{P}{d_{\text{wv}}^a}.$$ 

Note that $d_{wx} \geq \Delta l_{\text{max}}$. By regarding $d_{wx}$ as a continuous random variable, we get

$$\psi_{x} = \sum_{w \notin \text{Ring}_v \in S} q \cdot \frac{P}{d_{\text{wv}}^a}$$

$$< \alpha c \int_{\Delta l_{\text{max}}}^{+\infty} \frac{P}{\alpha^a} dx = \frac{acP}{\alpha - 1} \frac{1}{(\Delta l_{\text{max}})^{a - 1}}. \quad (9)$$

For maximum link length, its Rayleigh success probability can be expressed as

$$\exp \left( - \frac{\beta P d_{\text{max}}}{P} \cdot \psi_{x} \right).$$

Corollary 2: Setting $\Delta$ based on Equation (2), a transmission is successful under Rayleigh fading.

We consider the worst case, i.e., the success probability of a link whose length is maximum. Combining with Formulas (7) and (9) and using $\Delta$ in Equation (2), Rayleigh success probability of this link can be ensured with $1 - \epsilon$. Moreover, this link is also SIR-feasible under deterministic SIR from Corollary 4 in [31].

In this way, we can design distributed algorithms that rely strictly on local information by using local characteristic of protocol model under deterministic SIR model, which ensures each selected link has Rayleigh success probability of $1 - \epsilon$.

So far, we have shown that neighbors and active neighbors before and after executing Algorithm 1 for each sender $v$ and can be upper-bounded respectively using Formulas (4) and (6), and a transmission is successful if there is no neighbors of its sender transmitting (Corollary 2).

Finally, we propose Algorithm 2 to schedule all active senders with high probability by costing $\delta \beta \log \Delta T_{\text{max}}$ time slots at most. The pseudo-code is given in Algorithm 2.

Algorithm 2: Local Link Scheduling for SLS (LLS-SLS)

1: $q = \frac{1}{(\Delta + 1)^2}$
2: while $\delta \beta \log \Delta T_{\text{max}}$ time slots do
3: transmit() with probability $q$
4: if $v$ successfully transmits then
5: Stop
6: end if
7: end while
8: Return the maximum $\text{start}$ for all nodes

Fact 2: [37] Given a set of probabilities $p_1, \ldots, p_n$, with $\forall i : p_i \in [0, \frac{1}{2}]$, the following inequalities hold:

$$\left( \frac{1}{4} \right)^{\sum_{k=1}^{n} p_k} \leq \prod_{k=1}^{n} (1 - p_k) \leq \left( \frac{1}{e} \right)^{\sum_{k=1}^{n} p_k}. \quad (10)$$
Theorem 3: All active senders can be scheduled successfully with $O(1)$ whp.

Proof: From Lemmas 1 and 3 and Corollary 2, we can see that a sender $v$ only competes with its neighbors. For receiver $x$, defining $P_1$ as the probability that $v$ is the only active sender centered at $v$ within $(\Delta + 1)l_{max}$. Then

$$P_1 = q_v \prod_{u \in N_v} (1 - q_u)$$

$$\geq q_v \prod_{u \in N_v} (1 - q_u)$$

$$\geq q_v \cdot \left(\frac{1}{4}\right) \sum_{u \in N_v} q_u.$$

(11)

The last inequality is by Fact 2. By Inequality (6), we have

$$\sum_{u \in N_v} q_u \leq \frac{1}{6(\Delta^2 + 1)} 3(\Delta^2 + 1) = \frac{1}{2}.$$

Thus $P_1 \geq 1 - \frac{1}{12(\Delta^2 + 1)}$. After $\delta \theta \ln \Delta_T^{max}$ time slots, the probability that transmission from $v$ to $x$ is successful at least once is at least

$$1 - \left(1 - \frac{1}{12(\Delta^2 + 1)}\right)^{\delta \theta \ln \Delta_T^{max}} \geq 1 - \frac{1}{\Delta_T^{max}} \delta.$$

Next, we give a worst instance where the last one link can be scheduled after some certain time slots.

In the worst case, all senders within $(\Delta + 1)l_{max}$ are neighbors with each other. Expectantly, only one sender starts to transmit in a time slot. Without loss of generality, assume that the order of transmission starting to transmit is from the left node to the right node, as shown in Fig. 1. When sender $v$ starts to transmit, it notifies its neighbors starting to transmit after $slot_1 + slot$ time slots. Then, Right nodes of sender $v$ start to transmit one by one. Finally, sender $w$ will transmit after $slot_1 + \Delta_T^{max}slot$ time slots.

Theorem 4: In a time slot, Algorithm 2 can give a scheduling set, which consists of a fraction of the optimal one, denoted by $\frac{3(\Delta^2 + 1)}{4(\Delta^2 + 2)}$. The approximation ratio of Algorithm 2 is a constant: $\frac{3\Delta^2 + 4}{\Delta^2 + 2}$.

Proof: In a time slot, denote the number of senders that execute $transmit()$ concurrently selected by Algorithm 2 and the optimal schedule by $U_{lls}$ and $U_{opt}$, respectively. In Algorithm 2, for sender $v$, if no active neighbors obtained by Algorithm 1 are executing $transmit()$ with probability $q$, it will transmit successfully. Expectantly, at most one active sender within $(\Delta + 1)l_{max}$ executes $transmit()$ by settings of $q = \frac{1}{6(\Delta^2 + 1)}$, and it is selected into $U_{lls}$ according to Corollary 2. For a optimal schedule, we regard those active neighbors can be scheduled concurrently with sender $v$ in the same slot. Then, according to Inequality (6),

$$|U_{opt} \setminus U_{lls}| \leq 3(\Delta^2 + 1).$$

Thus, we get

$$\frac{U_{lls}}{U_{opt}} = \frac{|U_{opt} \setminus U_{lls}|}{U_{lls}} + 1 \leq 3\Delta^2 + 4.$$

From above description, we know the last active senders can be scheduled by costing $slot_1 + \Delta_T^{max}slot$ time slots. Denote the number of slots until all active senders are scheduled successfully by Algorithm 2 and the optimal solution by $U_{lls}$ and $U_{opt}$, respectively. Thus, approximation factor of SLS problem is given by

$$\frac{U_{lls}}{U_{opt}} = \frac{slot_1 + \Delta_T^{max}slot}{slot_1 + slot}$$

$$= \frac{e\Delta_T^{max} + 6(\Delta^2 + 1)\Delta_T^{max}}{e\Delta_T^{max} + 6(\Delta^2 + 1)}$$

$$< 1 + \frac{6}{e} (\Delta^2 + 1).$$

B. Random Contention Based Algorithm for SLS

Random contention-resolution algorithms are probably the most intuitive way to share limited resources among several agents in a distributed fashion. The idea is that each node accesses the resource in any time slot with a certain probability $q$ until its first success [36]. The algorithm from [2] is a natural backoff scheme, denoted by Distributed, in the tradition of ALOHA. It is run synchronously, but independently, on each sender of a link. However, their algorithm needs global information for setting transmission probability for each sender and calculating interference at each receiver.

In this section, we present an improved algorithm of random contention-resolution for SLS problem. This algorithm takes advantages of the local decision of the protocol model and the high feasibility of the SIR model. We start by giving a conclusion that if $q$ is chosen small enough, a fraction of the transmissions is successful.

Intuitively, when the probability $q_v$ is set right for each sender, a large fraction of the links will transmit successfully, in expectation. This is argued in the following Lemma. What remains is proving that the last one link can be scheduled after some certain time slots.
Similar to Formula (11), sender $v$ transmits successfully if there is no active senders within $(\Delta + 1)l_{\text{max}}$, corresponding success probability is at least

$$P_1 \geq \frac{1}{\Delta l_{\text{max}}} \cdot \frac{1}{4}. $$

Due to senders only compete with their neighbors from Corollary 2, there are at most $(\Delta l_{\text{max}} + 1)$ senders within $(\Delta + 1)l_{\text{max}}$. The expected number of remaining unsuccessful links in first slot is at most

$$(\Delta l_{\text{max}} + 1) \left( 1 - \frac{1}{4\Delta l_{\text{max}}} \right).$$

After consuming $4(\delta + 1)\Delta l_{\text{max}} \ln(\Delta l_{\text{max}} + 1)$ time slots, the expected number of remaining unsuccessful transmissions is at most $\frac{1}{(\Delta l_{\text{max}} + 1)^2}$. This is

$$(\Delta l_{\text{max}} + 1) \left( 1 - \frac{1}{4\Delta l_{\text{max}}} \right)^4(\delta + 1)\Delta l_{\text{max}} \ln(\Delta l_{\text{max}} + 1) \geq \frac{1}{(\Delta l_{\text{max}} + 1)^2}. $$

### VI. SIMULATIONS

In this section, we evaluate how the distributed algorithms performs with uniform power assignment by simulator MATLAB. Simulations are carried out on random networks constructed by randomly placing senders on a 1000 x 1000m$^2$ plane and are done over 200 different networks. Each corresponding receiver is placed by choosing the angle and the distance to the sender uniformly at random from a fixed interval. In this way, a minimal and a maximal distance between sender and its receiver can be specified. The related parameters are given in Table 1.

For comparison, we use centralized single slot scheduling algorithm by Goussevskaia, Halldörsson and Wattenhofer (GHW) [6] to solve SLS problem. Their algorithm is a simple greedy algorithm, where the links are processed in a non-decreasing order of length, and each link is included in the set of active senders if the affectance of the link, caused by the current set of active links is less than or equals to a constant $c$, where

$$c = \frac{1}{2 + \max(2, ((2^3 \cdot 9 + 1)\beta^2 - 1)\frac{1}{2 - 2})}. $$

Even though GHW algorithm is an $O(1)$-approximation algorithm, we realized that for the algorithm to be competitive on our random instances, the constant was too low, resulting in very small sets of active senders.

| Symbol | Meanings | Value |
|--------|----------|-------|
| $n$ | The number of links | 200 |
| $l_{\text{min}}$ | The minimum link length | 1m |
| $l_{\text{max}}$ | The maximum link length | 100m |
| $\alpha$ | The path-loss exponent | 5 |
| $\beta$ | Deciding threshold | 1 |
| $\epsilon$ | Acceptable failure probability | 0.1 |
| $P$ | Transmission power | 6mW |

First, we consider the impacts of the number of links on time complexity and schedule length, as shown in Figs. 2 and 3, respectively. The SIR parameters were set to $\alpha = 5$, $\beta = 1$, $P = 6\text{mW}$, maximum link length is $l_{\text{max}} = 10\text{m}$ and $\epsilon = 0.1$. Over $n$ increasing, the number of time slots needed increases for algorithms LLS-SLS, CLLS and Distributed [2]. This is because that the probability of nodes starting to transmit decreases for greater $n$, thus they need more time slots to start to transmit. As could be expected, GHW algorithm can not compare with LLS-SLS, CLLS and Distributed. Specifically, GHW computes 1.14, 1.36, and 1.87 times longer schedules than CLLS and Distributed, respectively. Algorithm Distributed computes shorter schedules than LLS-SLS and CLLS since it uses the dynamic transmission probability, namely the schedule length is increased by 32.8% for LLS-SLS and 28.3% CLLS. More important, CLLS only needs local information but without considering the global interference.

Then, setting $n = 200$ and other settings keep the same as before. The influences of the path-loss exponent on the time complexity and schedule length are shown in Figs. 4 and 5, respectively. As $\alpha$ increases, smaller path-loss exponent means there are less neighbors for a sender since $(\Delta + 1)l_{\text{max}}$. 

![FIGURE 2. The schedule length vs. $n$.](image1)

![FIGURE 3. The number of time slots vs. $n$.](image2)
decreases, i.e., the numbers of time slots needed for LLS-SLS and CLLS decrease. Compared with Distributed, the number of time slots needed for CLLS decreases 38.88%, and its schedule length increases 37.68% on average.

Next, in Figs. 6 and 7, we analyze the influence of the decoding threshold. Greater decoding threshold means there are more neighbors for a sender since $(\Delta + 1)_{l_{\text{max}}}$ gets greater, and then the numbers of time slots needed for LLS-SLS and CLLS increase. On average, Distributed needs 1.262 times the number of time slots than CLLS, but the latter computes a 1.745 times schedule length than the former. A sender will compete with more neighbors to try to transmit successfully with a smaller $\frac{1}{l_{\text{max}}}$, causing more schedules.
The influence of the link length is given in Figs. 8 and 9. We can see that Distributed needs more time slots than CLLLS for smaller link length, but over $l_{\text{max}}$ increasing, a sender will have more neighbors and compete with them with a smaller transmission probability, resulting in more time slots and schedules needed. CLLLS computes a 1.853 times schedules than Distributed on average. Moreover, GHW computes 1.311 and 2.43 times schedules than CLLLS and Distributed, respectively.

To sum up, the simulations show that although Distributed computes a shorter schedule for different link size, path-loss exponent and decoding threshold, CLLLS needs more less time slots and has localized characteristics, which has an advantage in realistic environment.

**VII. CONCLUSION**

In this paper, by utilizing the protocol model to localize global interference of SIR model, we first show that a successful link under protocol model is also SIR-feasible. Furthermore, we prove that the probability of a SIR-feasible link is at least $1 - \epsilon$ under Rayleigh fading. Based on the conclusion, we can design localized distributed link scheduling algorithms of SIR model under Rayleigh fading. Furthermore, combing random contention with localization characteristic of protocol model, SLS can be solved within logarithmic time complexity. Finally, the simulations verifies our theoretical analysis of the designed distributed algorithms.

Only uniform power assignment is considered, hence one future direction is to consider other methods of power control. Another promising research direction is the dynamic network scenarios where nodes move randomly to broadcast or collect data.

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