Influence of the flywheel profile on the energy storage capacity of the kers system

L Milica and G Andrei

1Mechanical Engineering Department, “Dunarea de Jos University” of Galati, Galati, Romania

E-mail: milica.lucian@ugal.ro

Abstract. When speaking about other energy storage devices, flywheels can be viable alternatives due to the low degradation process during the entire operating life, and its numerous energy discharge rates. In this paper, the problem of the flywheel shape, present in the KERS system, is approached analytically. The analysis was performed on three series of rotors and the results were validated using a CAD program. The linear relationship between the mass of the flywheel and its rotation axis was emphasized by Pappus-Guldin theorem. Based on the results it was shown that the value of kinetic energy is higher when the mass of the flywheel is concentrated closer to the axis of rotation.

1. Introduction

The KERS mechanical system has been developed since 2009 by the Williams F1 team for the purpose of energy storage and recovery, being a viable alternative to the hybrid system due to the high reliability and low wear and tear during the charge-discharge energy cycles throughout its lifetime [1]. Unlike the hybrid system whose energy efficiency is reduced because of multiple energy conversions, (mechanical energy - electricity, electricity - chemical energy) the KERS system increases the fuel efficiency of the fuel by approximately 20% [2]. Moreover, the production of the KERS system that provides 60 kW and 15 kW is 3-4 times respectively and 4-5 times, respectively more economical than the hybrid engines [3].

Figure 1. Price variation per kW depending on how the energy is produced [3].
The KERS system is a device for storing mechanical energy as rotating kinetic energy. This system is composed of a flywheel powered by an electric motor-generator (EMG), magnetic bearings, an electronic interface that contains an inverter for converting the DC current supplied by the flywheel into AC current to the EMG, a vacuum insulation housing, a continuous variable transmission system (CVTS) and a clutch between the flywheel and the CVTS that connects this system to the main transmission shaft.

The EMG practically has a common body with the flywheel having the stator mounted on the outer walls of the housing. The permanent magnets of the engine are incorporated into the composite structure of the flywheel thus forming a compact structure. The EMG performs mechanical-electrical energy conversion and vice versa [4, 5] while permanently controlling the steering wheel speed [6-8].

The flywheel is the major component of the KERS system. It is generally built of composite materials (carbon filament wrapped around a steel hub) whose characteristics (lightweight, high strength and rigidity, anisotropy) allow it to operate at high speeds [9-12].

The literature advances various solutions for the problem of the amount of energy that the flywheel can provide. Many of these works focus on the material from which it is built and also on the arrangement of the different layers of material that compose it [13-15]. In this context, the present paper aims to determine how the flywheel profile shape influences the value of the energy produced by the flywheel when this is made from a single material.

2. The characteristics of the flywheel
Let a flywheel like the one in Figure 2 of mass m and volume V, having the length l and the outer radius R and the inner radius r.

We have the following relationships:
\[ \rho = \frac{dm}{dV} \rightarrow dm = \rho \cdot dV \]  
(1)

\[ dV = 2\pi R \cdot dr \cdot l \]  
(2)

From relationships (1) and (2) it resulted:
\[ dm = \rho \cdot 2\pi R \cdot dr \cdot l \]  
(3)

From the expression of the moment of inertia I we have:
\[ I = \int_{r}^{R} R^2 \cdot dm = \int_{r}^{R} \rho \cdot 2\pi R^3 \cdot dr \cdot l = 2\pi \rho l \cdot \int_{r}^{R} R^3 \cdot dr = \frac{\pi \rho l}{2} \cdot (R^4 - r^4) = \frac{\pi \rho l}{2} \cdot (R^2 - r^2) \cdot (R^2 + r^2) \]  

(4)

But \( m = \rho \cdot V \) and \( V = (R^2 - r^2) \cdot \pi l \) it resulted:

\[ m = (R^2 - r^2) \cdot \rho \pi l \]

(5)

Based on relations (4) and (5) we have the final expression of the moment of inertia:

\[ I = \frac{m}{2} \cdot (R^2 + r^2) \]

(6)

The kinetic energy \( E \) produced by the flywheel has the expression:

\[ E = \frac{(I \cdot \omega^2)}{2} = \frac{[m \omega^2 \cdot (R^2 + r^2)]}{4 \cdot I} \]

(7)

The amount of energy that the flywheel can store depends on the three major elements [16]:
- the state of maximum tension of the materials the flywheel is made of;
- the geometric profile of the flywheel that influences its moment of inertia and the K-shaped factor;
- the angular velocity \( \omega \).

Based on the relation (7) of the three elements mentioned above, two others are added:
- the flywheel mass;
- the thickness of the flywheel wall.

Based on Equation (7) we can have an image of how each parameter in its composition influences the amount of energy that the flywheel can store. Thus it can be observed that adopting solutions that allow higher angular speeds to be obtained is more advantageous than increasing the mass of the flywheel. On the other hand, although we would be tempted to increase the outer or the inner radius of the steering wheel, this choice would lead to an increase over the permissible limit of the tangential stresses while it is desired to obtain higher angular speeds.

The main problem in choosing the size of the flywheel, besides the concern for amount of energy it can store is that of internal tensions. The values of the tangential stresses \( \sigma_t \) and radial stresses \( \sigma_r \) is presented in Figure 3.

![Figure 3. The propagation directions for the stresses inside the flywheel.](image-url)
The tangential and radial stresses depending on the radial distance \( r \) from the axis of rotation are given by the relations [17]:

\[
\sigma_t = \frac{3 + \nu}{8} \left( R^2 + r^2 - \frac{1 + 3\nu}{3 + \nu} \cdot \frac{R^2 \cdot r^2}{r_o^2} \right) \cdot \rho \omega^2
\]

\[
\sigma_r = \frac{3 + \nu}{8} \left( R^2 + r^2 - \frac{1}{3 + \nu} \cdot R^2 \cdot \frac{r^2}{r_o^2} \right) \cdot \rho \omega^2
\]

In which \( \nu \) represents Poisson's coefficient and \( \rho \) is the density of the material.

For any angular velocity \( \omega \) of the flywheel, the tangential and radial stresses will be maximum when \( r = r_o \). The expressions of the stresses are given by the relations (10) and (11):

\[
\sigma_{t,\text{max}} = \frac{3 + \nu}{8} \left( 2R^2 + r^2 - \frac{1 + 3\nu}{3 + \nu} \cdot r^2 \right) \cdot \rho \omega^2
\]

\[
\sigma_{r,\text{max}} = \frac{3 + \nu}{8} \left( R^2 - r^2 \right) \cdot \rho \omega^2
\]

Another important indicator used to determine the stress states that determine volumetrically the shape deformations is the von Mises stress \( \sigma_{v,M} \), whose expression is [18]:

\[
\sigma_{v,M} = \sqrt{\frac{1}{2} \left\{ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right\}}
\]

where the \( \sigma_1, \sigma_2, \sigma_3 \) are the main stresses on the three spatial directions.

Considering that in the case of the flywheel these stresses are present only on two of the three directions, the expression (12) becomes:

\[
\sigma_{v,M} = \sqrt{\frac{1}{2} \left\{ (\sigma_t - \sigma_r)^2 + \sigma_r^2 + \sigma_t^2 \right\}}
\]

Also, a parameter to be followed in determining the shape and dimensions of the flywheel is the radial displacement \( \Delta_r \). The expression of this parameter is given by the relation:

\[
\Delta_r = \frac{(3 + \nu) \cdot (1 - \nu)}{8 \cdot E} \left( R^2 + r^2 - \frac{1 + \nu}{3 + \nu} \cdot r^2 - \frac{1 + \nu}{1 - \nu} \cdot R^2 \right) \cdot \rho \omega^2
\]

where \( E \) represents the elasticity modulus of the material the flywheel is made of.

As shown in [19], there is a close connection between the maximum energy \( E_{\text{max}} \) that the flywheel can store, its maximum angular velocity \( \omega_{\text{max}} \) and the maximum internal stress \( \sigma_{\text{max}} \). Thus:

\[
E_{\text{max}} \rightarrow \omega_{\text{max}}
\]

\[
\omega_{\text{max}} \rightarrow \sigma_{\text{max}}
\]

Based on the maximum energy \( E_{\text{max}} \), the specific energy \( e_s \) is obtained depending on the material from which the flywheel is made. The expression of the specific energy is given by the relation:

\[
e_s = \frac{E_{\text{max}}}{m}
\]
3. The conditions that determine the maximum kinetic energy

We generate three series of rotors that have the following parameters in common (Figure 4): \( r, h, V, \rho \) and \( \sigma \) (the stress accepted by the designer).

![Figure 4. The flywheel parameters.](image)

The flywheel series will have different \( x \) and \( y \) dimensions. Also, the distance \( c \) (the radius of the circle described by the center of gravity of the area of section \( A \) with the center on the axis of rotation) will be different. From the condition \( V = \text{ct.} \) (constant) a relationship between \( x \) and \( y \) will result. It is known that the volume \( V \) of a rotating body is given by the relation (18) based on the Pappus-Guldin theorem:

\[
V = 2 \cdot \pi \cdot c \cdot A
\]  

(18)

The expressions for \( c \) and \( A \) are known:

\[
c = r + \frac{h}{3} \cdot \frac{x + 2y}{x + y}
\]

(19)

\[
A = \frac{h}{2} (x + y)
\]

(20)

The condition \( V = \text{ct.} \) becomes:

\[
2 \cdot \pi \cdot \left( r + \frac{h}{3} \cdot \frac{x + 2y}{x + y} \right) \cdot \frac{h}{2} (x + y) - V = 0
\]

(21)

Expression (21) represents a linear relationship between \( x \) and \( y \), the graph of which is shown in Figure 5 below:

![Figure 5. The line determined by the equation (21).](image)

The parametric equations of the line \( x_0 \) and \( y_0 \) are:

\[
x = x_0 \cdot \lambda
\]

(22)
The condition that the allowable stress \( \sigma_a \) appears on rotation is:

\[
\sigma_a = \rho V \omega^2 / c \overset{4}{\cdot} A
\]

(24)

From the relation (24) it results \( \omega^2 \) and hence the kinetic energy of the flywheel \( E_{\text{max}} \), depending on the \( \lambda \). In Excel, 21 flywheel \( \lambda = (0, 0.05, 0.1, ... 1) \) are generated for three data sets. For the three data sets we have \( y_n < y_n < y_n \) and \( c_n < c_n < c_n \).

Figure 6. Angular speeds for the three series of flywheels.

Figure 7. Kinetic energy for the three series of flywheels.
The diagrams show that the kinetic energy is maximum when $x = x_0$ and $y = 0$, in other words when the mass is closer to the axis of rotation (Figure 6-7).

4. Validation of the results using a CAD program
Taking into account the notations in the previous paragraph, a series of flywheels have been generated, having in common the dimensions $r + h = 78\text{mm}$ and $y = 190\text{mm}$.

![Figure 8. Flywheel type $F_1$.](image)

![Figure 9. Flywheel type $F_2$.](image)

![Figure 10. Flywheel type $F_3$.](image)

![Figure 11. Flywheel type $F_4$.](image)

![Figure 12. Flywheel type $F_5$.](image)

![Figure 13. Flywheel type $F_6$.](image)

![Figure 14. Flywheel type $F_7$.](image)

![Figure 15. Flywheel type $F_8$.](image)
To validate the results presented above, we used the capabilities of the CATIA program, which generated flywheels of different sections.

In Figures 8-15 the flywheels generated are denoted by \( F_1 - F_8 \). The flywheels are considered to be made of the same material. The values \( V, \rho, c, A, I \) were returned by the program and based on the relation (24) it resulted the expression of the angular speed, imposing \( \sigma_u = 200\text{MPa} \). Based on the maximum kinetic energy \( E_{\text{max}} \) given by the relation (7) we obtained the specific energy \( e_s \) based on the relation (17). Table 1 shows the values for \( V, m, c, A, I \).

**Table 1.** Values for \( V, m, c, A, I \) for the \( F_1 - F_8 \) flywheels.

| Flywheel type | \( A \left[ m^2 \right] \) | \( V \left[ \frac{m^3}{2} \right] \) | \( c \left[ m \right] \) | \( I \left[ \text{kg} \times m^2 \right] \) | \( m \left[ \text{Kg} \right] \) |
|---------------|----------------|-----------------|---------|-----------------|---------|
| \( F_1 \)     | 0.027          | 0.0007          | 0.027   | 0.0110          | 11.00   |
| \( F_2 \)     | 0.004          | 0.0005          | 0.031   | 0.0100          | 7.86    |
| \( F_3 \)     | 0.004          | 0.0009          | 0.043   | 0.0380          | 14.35   |
| \( F_4 \)     | 0.006          | 0.0100          | 0.037   | 0.0280          | 15.72   |
| \( F_5 \)     | 0.007          | 0.0005          | 0.024   | 0.0070          | 7.86    |
| \( F_6 \)     | 0.006          | 0.0005          | 0.025   | 0.0050          | 7.07    |
| \( F_7 \)     | 0.010          | 0.0008          | 0.027   | 0.0160          | 12.58   |
| \( F_8 \)     | 0.005          | 0.0006          | 0.041   | 0.0130          | 9.43    |

Figure 16 shows the variation of the specific energy \( e_s \) and confirms that this value is higher when the body mass is concentrated closer to the axis of rotation.
5. Conclusion
The global tendency in the automotive industry in the context of the current climate change situation is represented by the need to produce vehicles with the lowest carbon dioxide emissions. This has led to the development of alternative solutions to the expensive electrical batteries, thus, reducing the car-manufacturing cost.

However, the high cost of producing these batteries has led to the design of systems capable of utilizing the energy recovered from braking which will significantly contribute to reducing carbon dioxide emissions.

This paper presents an analytical approach to the problem of the flywheel shape existent in the KERS system. The results were compared with a CAD program. It was pointed out that there is a linear relationship between the mass of the flywheel and its axis of rotation based on the Pappus-Guldin theorem.

It has been shown that the value of the kinetic energy is higher when the mass of the flywheel is concentrated closer to the axis of rotation. The results were validated based on the flywheels generated through the CATIA software. The level of kinetic energy stored by the profiled flywheel can be determined using the graphs of the specific energy’s variation, as shown above.

6. References
[1] Krack M, Secanell M, Mertiny P 2011 Rotor design for high-speed flywheel energy storage systems, Carbon Rosario, editor, Energy Storage in the Emerging Era of Smart Grids, In Tech, pp 41–68
[2] Burn J 2014, Auto EXPRESS. Volvo flywheel KERS tech revealed, https://www.autoexpress.co.uk/Volvo/s60/86320/volvo-flywheel-kers-tech-revealed
[3] Jörgensson M, Eriksson S, Sjögren R, Dzafic A, Arvidsson R, Verbakel M, Holweg E 2012 F-KERS – Kinetic Energy Recovery System with mechanical flywheel sub 80g CO2 2020
[4] Parfomak P.W 2012 Energy Storage for Power Grids and Electric Transportation: A Technology Assessment, Congressional Research Services: Washington, DC, USA
[5] Akhil A, Huff G, Currier A. B, Kaun B. C, Rastler D. M, Chen S. B, Cotter A. L, Bradshaw D. T, Gauntlett W. D 2013 DOE/EPRI 2013 Electricity Storage Handbook in Collaboration with NRECA, U.S. Department of Energy: Oak Ridge, TN, USA
[6] Su W, Jin T, Wang S 2010 Modeling and Simulation of Short-term Energy Storage: Flywheel. In Proceedings of the 2010 International Conference on Advances in Energy Engineering Modeling, Beijing, China, pp 9-12
[7] Meng Y. M, Li T.C, Wang L 2008 Simulation of controlling methods to flywheel energy storage on charge section, In Proceedings of the Third International Conference on Electric Utility Deregulation and Restructuring and Power Technologies, Nanjing, China, pp 2598-2602
[8] Zhang X, Yang J 2014 An improved discharge control strategy with load current and rotor speed compensation for high-speed flywheel energy storage system, In Proceedings of the 17th International Conference on Electrical Machines and Systems, Hangzhou, China, pp 318–324
[9] Xingjian D, Kai Z, Xiao-zhang Z 2011 Design and test of a 300Wh composites flywheel energy storage prototype with active magnetic bearings, Proceedings of ICREPQ-11 Conference, Spain, 1, pp 380-385
[10] Sung H. K, Myung K. H, Sang H. C, Tae-Hyun S 2006 Design and spin test of a hybrid composite flywheel rotor with a split type hub, IEEE J Compos Mater, 40, pp 2113-2130
[11] Christopher D. A, Beach R 1998 Flywheel technology development program for aerospace applications, IEEE AES System Mag, 13, pp 9–14
[12] Genta G 1985 Kinetic energy storage. 1st Edition. Theory and practice of advanced flywheel systems, Elsevier, pp 70-150
[13] Ha S. K, Han H. H and Han Y. H 2008 Design and manufacture of a composite flywheel press-fit multi-rim rotor, *Journal of Reinforced Plastics and Composites*, 27, pp 953-965
[14] Krack M, Secanell M and Mertiny P 2010 Cost optimization of hybrid composite flywheel rotors for energy storage, *Structural & Multidisciplinary Optimization*, 41, pp 779-795
[15] Arvin A. C. and Bakis C. E 2006 Optimal design of press-fitted filament wound composite flywheel rotors, *Composite Structures*, 72, pp 47-57
[16] Arslan M. A 2008 Flywheel geometry design for improved energy storage using finite element analysis, *Material Design*, 29, pp 514-518
[17] Norton R. L 1998 Machine design: An integrated approach, *Prentice Hall (Upper Saddle River, N.J.)*
[18] Dahlberg T 2001 Teknisk hållfasthetslära, *Lund: Studentlitteratur AB*
[19] Pena-Alzola R, Sebastian R, Quesada J, and Colmenar A 2011 Review of flywheel based energy storage systems, *International Conference of Power Engineering, Energy and Electrical Drives*, pp 1-6