Is space expanding in the Friedmann universe models?

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Abstract

The interpretation of the expanding universe as an expansion of space has recently been challenged. From the geodesic equation in Friedmann universe models and the empty Milne model, we argue that a Newtonian or special relativistic analysis is not applicable on large scales, and the general relativistic interpretation in terms of expanding space has the advantage of being globally consistent. We also show that the cosmic redshift, interpreted as an expansion effect, contains both the Doppler effect and the gravitational frequency shift.
I. INTRODUCTION

Although more than 75 years have passed since Hubble’s discovery of the expanding universe, there is still some confusion about how the expansion is to be interpreted. Peacock and Whiting have recently discussed radial motion through space of a free particle in Friedmann universe models. Whiting deduced the Newtonian equation of motion for a free particle and its relativistic generalization, the geodesic equation, but gave only the general solution of the Newtonian equation. The solutions show that a particle that is initially at rest relative to an observer will move inward and pass the position of the observer before moving outward. Whiting and Peacock conclude that this behavior is not in accordance with the usual interpretation that space expands.

Several papers (see for example, Refs. 7 and 8) have pointed out that even though the standard Newtonian derivation of the Friedmann equations (which govern the expansion of the universe) gives the correct form of the equations, it rests on shaky foundations. For example, using the Newtonian gravitational force law is not warranted in an infinite, homogeneous universe. Thus we should be wary of using Newtonian physics when interpreting the expansion of the universe. The question whether it is possible to distinguish by observation between the two possibilities: galaxies moving apart or the space between them expanding has been discussed by Morgan with emphasis on the observations of the fluctuations in the cosmic microwave background radiation.

The concept of space as used in the general theory of relativity is defined as a relation between reference particles and is different from the concept of space as used in Newtonian physics and in the special theory of relativity, where space is not influenced by matter and exists independently of it. Space in Newtonian physics and special relativity therefore has a mathematical character. Space in general relativistic is more physical. Matter curves space. Space is dynamic such as when there are gravitational waves, and it may even expand.

In this paper we will discuss the arguments presented in Refs. 1, 2, and 3 using a general relativistic framework to study the motion of a free particle. We will also argue that there exists a general relativistic interpretation of the cosmic redshift according to which space expands.
II. RADIAL FREE MOTION

The line element of expanding, homogeneous and isotropic universe models is (see eg. Ref. 10)

\[ ds^2 = -c^2 dt^2 + a(t) \left[ d\chi^2 + R_0^2 S_k^2(\chi/R_0)(d\theta^2 + \sin^2 \theta d\phi^2) \right], \]

where \( a(t) \) is the scale factor describing the expansion of the universe, \( \chi \) is the standard radial coordinate, comoving with the reference particles whose motion defines the so-called Hubble flow of the Universe, \( R_0 \) is the curvature radius of 3-space, and

\[ S_k(x) = \begin{cases} 
\sin x & k > 0 \\
x & k = 0 \\
\sinh x & k < 0.
\end{cases} \]

The motion of a free particle is determined by the geodesic equation

\[ \frac{d^2 x^i}{d\tau^2} + \Gamma^i_{\alpha\beta} \frac{d x^\alpha}{d\tau} \frac{d x^\beta}{d\tau} = 0, \]

where the proper time \( \tau \) is \( d\tau^2 = -ds^2 \). Here \( i \) is the spatial index and \( \alpha, \beta \) are spacetime indices.

We now consider radial motion. The only non-vanishing Christoffel symbols that are needed are

\[ \Gamma^1_{01} = \Gamma^1_{10} = \frac{\dot{a}}{a}, \quad \Gamma^0_{11} = \frac{a\ddot{a}}{c^2}, \]

where the dots denote derivatives with respect to coordinate time \( t \). The geodesic equations then reduce to

\[ \frac{d^2 \chi}{d\tau^2} = -2\frac{\dot{a}}{a} \frac{d\chi}{d\tau} \frac{d\dot{\chi}}{dt} = -2\frac{\dot{a}}{a} \frac{\dot{\chi}^2}{\dot{\tau}^2}, \]

\[ \frac{d^2 t}{d\tau^2} = -\frac{a\ddot{a}}{c^2} \left( \frac{d\chi}{d\tau} \right)^2 = -\frac{a\ddot{a}}{c^2} \frac{\dot{\chi}^2}{\dot{\tau}^2}. \]

It is convenient to express the equation of motion in terms of derivatives with respect to coordinate time. If we use

\[ \dot{\chi} = \frac{d\chi/d\tau}{dt/d\tau}, \]

\[ \ddot{\chi} = \frac{\dot{\tau}}{\tau} \frac{d\dot{\chi}}{d\tau} = \frac{\ddot{\tau}}{\tau^2} \left( \frac{d^2 \chi}{d\tau^2} - \dot{\chi} \frac{d^2 t}{d\tau^2} \right), \]

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together with Eqs. (5) and (6), we obtain
\[ \ddot{\chi} = \frac{a\dot{a}}{c^2} \chi^3 - 2\frac{\dot{a}}{a} \dot{\chi}, \] (9)

The trivial solution \( \dot{\chi} = 0 \) represents a particle following the Hubble flow. We introduce the new dimensionless variable \( y = \frac{c^2}{\dot{\chi}^2} \) and express Eq. (9) in the form
\[ \dot{y} - (\ln a^4) y = -(a^2). \] (10)

The general solution of Eq. (10) is
\[ y = a^2 + e^K a^4, \] (11)

where \( K \) is a constant of integration. Hence \( A \equiv e^K \) is a positive constant. Inserting \( y = \frac{c^2}{\dot{\chi}^2} \) in Eq. (11) leads to
\[ \dot{\chi} = \pm c(a^2 + Aa^4)^{-1/2} \] (12)

which vanishes when \( a \to \infty \). Hence in this limit the motion of the particle approaches that of the Hubble flow. We integrate Eq. (12) and find that the radial coordinate as a function of time of the free particle is
\[ \chi = \pm \int \frac{cdt}{a \sqrt{1 + Aa^2}}. \] (13)

Davis, Lineweaver, and Webb recently considered the motion of a free particle in an expanding universe in the low-velocity regime. Our treatment generalizes their description to relativistic velocities.

If we consider flat universe models with a perfect fluid with the equation of state \( P = w\rho \) with \( w = \) constant, the scale factor is (see e.g. Ref. 15 p. 334)
\[ a = \left( \frac{t}{t_0} \right)^{\frac{2}{3(1+w)}}, \] (14)

where \( t_0 \) is the present value of the cosmic time. Hence, if we measure the time in units of \( t_0 \), and normalize the scale factor to unity at \( t = t_0 \), Eq. (13) becomes
\[ \chi = \chi_0 \pm \frac{3}{2}(1 + w)ct_0 \int_1^a \frac{a^{\frac{2}{3}}(3w-1)}{\sqrt{1 + Aa^2}} da', \] (15)

where the initial condition is \( \chi = \chi_0 \) at \( t = t_0 \). We integrate Eq. (15) and substitute Eq. (14) and obtain \( \chi \) as a function of time. The proper distance of the particle from the observer
is \( l = a\chi \). Particles with \( \dot{\chi} = 0 \) define the expanding motion of the universe, i.e. its Hubble flow. If a particle, which in this context can be a galaxy, moves relative to the reference particles defining the Hubble flow, it is said to have a peculiarity, given by \( v = a\dot{\chi} \).

Whiting\(^3\) studied the motion of a test particle of negligible mass which starts with \( \dot{l} = 0 \) at some coordinate \( \chi_0 \) at time \( t = t_0 \). This criterion requires that

\[
\dot{l}(t_0) = a(t_0)\chi_0 + a(t_0)\dot{\chi}(t_0) = 0, \tag{16}
\]

so that

\[
\dot{\chi}_0 \equiv \dot{\chi}(t_0) = -\frac{\dot{a}(t_0)}{a(t_0)}\chi_0 = -H_0\chi_0, \tag{17}
\]

where \( H_0 = \frac{\dot{a}(t_0)}{a(t_0)} \) is the present value of the Hubble parameter. Hence, \( \dot{l}(t_0) = 0 \) requires the initial coordinate velocity to be directed toward the observer at \( \chi = 0 \). From Eq. (14)

\[
H_0 = \frac{2}{3(1 + w)t_0}, \tag{18}
\]

which gives

\[
\dot{\chi}_0 = -\frac{2}{3(1 + w)}\frac{\chi_0}{t_0}. \tag{19}
\]

From Eq. (12) we obtain

\[
\dot{\chi}_0 = -\frac{c}{\sqrt{1 + A}}. \tag{20}
\]

Equations (19) and (20) lead to

\[
A = \frac{9(1 + w)^2c^2t_0^2}{4\chi_0^2} - 1. \tag{21}
\]

Note that \( A > 0 \) requires

\[
\chi_0 < \frac{3}{2}(1 + w)c t_0 = \frac{c}{H_0}. \tag{22}
\]

This inequality may be interpreted by noting that \( c/H_0 \) is the radius of the Hubble sphere outside which the recession velocity caused by the expansion of the universe exceeds that of the speed of light. Hence, regions outside the Hubble sphere are receding faster than the speed of light, which makes \( \dot{l} = 0 \) impossible in this region, if we accept the relativistic requirement of subluminal velocities for material particles.

A plot of the proper distance \( l \) for a particle starting at \( \chi_0 = 0.1c/H_0 \) for a dust-dominated universe with \( w = 0 \) (the Einstein-de Sitter model) is shown in Fig. II. The results in Fig. II show qualitatively the same behavior as found by Whiting\(^3\) by solving the Newtonian
equation of motion. The particle is not dragged along with the Hubble flow, but falls toward and past the origin, approaching asymptotically the velocity of the Hubble flow. As pointed out by Whiting, the position of the particle does not go asymptotically to $-a(t)\chi_0$ and in that (somewhat arbitrary) sense it never rejoins the Hubble flow. The fact that the particle never rejoins the Hubble flow is part of the reason why Peacock and Whiting reject the notion of expanding space. They claim that the notion of expanding space leads us to think that a particle dropped into the Hubble flow should, like a particle dropped into a river, start following the local flow. Clearly, a free particle in a Friedmann universe does not behave like a particle dropped into a river.

In Ref. 3 the solutions of the geodesic equation are discussed with the following comment: “The free particle accelerates toward the origin away from the Hubble flow, passes through the origin, and continues out the other side. This behavior of a free particle is not what one would expect, if the ‘expansion of space’ acts like a Newtonian force pushing the galaxies apart. It is qualitatively different, the initial velocity being in the opposite direction.” Note that Whiting must be referring to the initial acceleration, because the particle starts from rest.

We can offer an explanation for this behavior in the context of an expanding space. Differentiating $l = a\chi$ twice and using Eq. (9) leads to

$$\ddot{l} = \ddot{a}\chi + \frac{a^2\ddot{a}}{c^2}\chi^3,$$

and from Eq. (17) we then obtain

$$\ddot{l}_0 = \left(\ddot{a}_0 - \frac{H_0^4}{c^2\chi_0^2}\right)\chi_0.$$  \hspace{1cm} (24)

Equation (24) shows that the particles’ initial acceleration $\ddot{l}_0$ can be directed inward, even if the overall acceleration of the universe, $\ddot{a}_0$, is outward. A free particle will accelerate inward if

$$\chi_0^2 > \frac{\ddot{a}_0}{H_0^2}\left(\frac{c}{H_0}\right)^2.$$  \hspace{1cm} (25)

From Eq. (24) we see that if the expansion of the universe is decelerating, the particle will accelerate toward the observer for every value of $\chi_0$, even outside the Hubble radius. However, outside the Hubble radius it will first be dragged outward by the superluminal expansion of the universe. The motion of the particle slows down because of the inward directed acceleration, and the free particle eventually moves toward the observer.
The inequality (25) can be rewritten so that it shows how the behavior of the test particle depends on the type of cosmic fluid dominating the universe. If we differentiate Eq. (14), we find

\[ \ddot{a}_0 = -\frac{1}{2}(1 + 3w)H_0^2. \] (26)

We substitute Eq. (26) into Eq. (25) and find that the condition for acceleration toward the observer takes the form

\[ \chi_0^2 > -\frac{1}{2}(1 + 3w)\left(\frac{c}{H_0}\right)^2. \] (27)

For a dust dominated \((w = 0)\) or radiation dominated \((w = 1/3)\) universe, the right-hand side is negative, and Eq. (27) is satisfied for every value of \(\chi_0\). The Milne universe has \(w = -1/3\) and hence \(\ddot{a} = 0\). From Eq. (24) it is seen that a free particle in the Milne universe accelerates toward the observer for any value of \(\chi_0\), although the Hubble flow is unaccelerated.

An accelerating universe has \(-1 < w < -1/3\). In this case the Hubble flow accelerates, \(\ddot{a}_0 > 0\). Let \(w = -1/3 - (2/3)\Delta w\) with \(0 < \Delta w < 1\). The condition for acceleration of the free particle toward the observer is then

\[ \chi_0^2 > \left(\frac{c}{H_0}\right)^2 \Delta w, \] (28)

where \(\chi_0\) is less than the Hubble radius because \(\Delta w < 1\). Even in this case the particle will accelerate inward for sufficiently large initial distance, although it may be dragged outward with a velocity after release directed away from the observer because of the cosmic expansion.

The inward directed acceleration of a free particle in a universe with accelerated Hubble flow and repulsive gravitation is somewhat surprising and needs explanation. The general relativistic explanation of this behavior is the following. Consider a particle that is not moving freely, but is initially kept at a fixed physical distance from the origin. At the position of the particle space expands so that reference points with fixed values of \(\chi\) move outward. Hence, new reference points with smaller values of \(\chi\) pass the particle as time proceeds. The velocity of the reference points is \(\dot{\chi}\) which decreases as \(\chi\) decreases. This decrease means that at a fixed distance from an arbitrarily chosen origin the expansion of space slows down. This slowing down has nothing to do with the dynamics of the Hubble flow, but is due to its inhomogeneity as observed from the origin. Regions with lower expansion velocity move outward. The particle has an initial peculiar velocity directed inward that is adjusted to...
a larger velocity of the Hubble flow than its later value. Hence, it starts moving inward
because of a retarded expansion of space at its position.

Davis et al. have also treated this problem. However, their expression for \( \ddot{r} \) (Eq. (14)
in Ref. 4) lacks the last term of our Eq. (24). The reason is that they have not used
the geodesic equation to deduce the equation of motion, but a scaling argument for the
momentum. From their equation they conclude that whether the particle approaches us or
recedes from us depends only on the sign of the acceleration of the Hubble flow \( \ddot{a}_0 \). As we
have shown, this conclusion is not correct.

When discussing the meaning of an expanding universe, we should bear in mind that
physical arguments, like the ones we have given, are usually framed within the homogeneous
Friedmann-Robertson-Walker model. To reject the concept of expanding space based on
the common-sense observation that space locally, for example, in my room, shows no sign
of expanding is not relevant, because the assumption of homogeneity breaks down on small
scales. In some idealized models there is, in principle, an effect of the expansion of space
on small scales. One case that has been studied is a spherical mass distribution embedded
in an Einstein-de Sitter universe. It is found that circular orbits around the spherical
mass will expand, although at a modest rate (roughly \( 10^{-23} \text{m/year} \) for an orbit of radius of
1 astronomical unit around a solar-mass star).

III. SUPERLUMINAL VELOCITY IN THE MILNE UNIVERSE

The Milne universe is the Minkowski spacetime described from an expanding reference
frame. The coordinate transformation between the Minkowski coordinates, \( (T, R) \) and the
Milne coordinates \( (t, \chi) \) is

\[
R = c t \sinh \left( \frac{\chi}{c t_0} \right), \quad T = t \cosh \left( \frac{\chi}{c t_0} \right),
\]

which leads to the line element

\[
ds^2 = -c^2 dt^2 + \left( \frac{t}{t_0} \right)^2 \left[ d\chi^2 + \sinh^2 \chi (d\theta^2 + \sin^2 \theta d\phi^2) \right].
\]

From the transformation (29) it follows that

\[
R = c T \tanh \left( \frac{\chi}{c t_0} \right), \quad c^2 T^2 - R^2 = c^2 t^2.
\]
The Minkowski coordinates are the coordinates of a rigid inertial reference frame of an arbitrarily chosen reference particle P in the expanding cloud of particles defining the Milne universe model. The time $T$ is the private time of P. The time $t$ is measured on clocks following all of the reference particles. As seen from Eq. (31) the space $T = \text{constant} = T_1$ has a finite extent, $cT_1$. This space is the private space of an observer following the particle P. The space $t = \text{constant} = t_1$ is represented by a hyperbola in the Minkowski diagram of the P observer (see Fig. 2). It is defined by simultaneity of the clocks carried by all the reference particles and is called the public space or only the space of the universe model. It has infinite extension in spite of the fact that the Big Bang has the character of a point event in the Milne universe model.

In the inertial and rigid Minkowski coordinate system the velocity of a reference particle with comoving coordinate $\chi$ is

$$\frac{dR}{dT} = c \tanh \left( \frac{\chi}{ct_0} \right), \quad (32)$$

which is less than $c$ for all values of $\chi$. However, in the expanding cosmic frame it is different. Here the velocity of the reference particle as defined by an observer at the origin is given by Hubble’s law

$$v = Hl = \frac{\dot{a}}{a} a \chi = \dot{a} \chi = \frac{1}{t_0} \chi. \quad (33)$$

Hence the reference particles have superluminal velocity at sufficiently great distances from the observer. According to special relativistic kinematics superluminal velocity is problematic because the particles cannot move through space with a velocity greater than $c$. However, according to the general relativistic interpretation, the reference particles define the public space of the universe model, and there is no limit to how fast space itself can expand.

In the Milne universe model there exists a global inertial frame that we have called a Minkowski frame. In such a frame the finite extension of the infinite cosmic space is due to the Lorentz contraction. The distances between the reference particles are Lorentz contracted relative to the corresponding distances in the Milne frame. The Lorentz contraction makes the infinite public space look like a finite private space in the Minkowski frame, and makes the velocities of the reference particles less than the velocity of light. However, in universe models that are not empty, no global Minkowski frame exists. Globally, Minkowski coordinates, or Cartesian coordinates, can describe only the flat tangent space of a curved
space. Such coordinates are often called local inertial coordinates and can be defined only in limited neighborhoods of points or world lines.\textsuperscript{10,15} In curved spacetime there is only the cosmic frame. The only real space is the public space. Although we have considered homogeneous universe models and the special theory is valid locally, we cannot apply a special relativistic kinematics to define the cosmic kinematics globally, which is the main reason that the special relativistic conception of particles moving through space cannot be applied to define the expansion of the universe. The Hubble flow has a global cosmic character, and the general theory of relativity is needed to define it properly, which leads to the concept of an expanding space.

IV. COSMIC REDSHIFT, DOPPLER EFFECT AND GRAVITATIONAL FREQUENCY SHIFT

In Ref. \textsuperscript{3}, it is argued that the cosmic redshift should not be interpreted as an expansion effect, but as a curvature effect. The general formula for the cosmic redshift is

\[
z = \frac{1}{a(t_e)} - 1,
\]

where \(t_e\) is the emission time of a light signal, and the scale factor has been normalized so that \(a(t_0) = 1\) today.

In this section we shall accept the general relativistic interpretation of Eq. (34) according to which the wavelength of the emitted radiation has been stretched during its travel from the object to the observer by a factor equal to the ratio of the distance between two galaxy clusters at the time of observation and the time of emission. If, for example, \(z = 1\), the distances have been doubled during the time the radiation moved from the source to the observer.

We shall show that the cosmic redshift can be separated into a Doppler shift and a gravitational frequency shift if the object is close to the observer, cosmically speaking, that is, if \(H_0(t_0 - t_e) \ll 1\), where \(H_0\) is the present value of the Hubble parameter. This result was first derived by Herman Bondi.\textsuperscript{16} Alternative derivations can be found in Refs. 17 and 18.

Using Eq. (34) and making a Taylor expansion of \(a(t)\) to second order in \(H_0(t_0 - t_e)\) one
obtains (see Ref. 10 p. 781)

\[ z = H_0(t_0 - t_e) + \left(1 + \frac{q_0}{2}\right)H_0^2(t_0 - t_e)^2, \]  

(35)

where \( q_0 = -(a\ddot{a}/\dot{a}^2)_{t=t_0} \) is the deceleration parameter. The special relativistic formula for the Doppler shift is

\[ z_D = \sqrt{1 + v_e^2} - 1, \]  

(36)

where \( v_e \) (in units where \( c = 1 \)) is the velocity of the source when it emitted the light. To second order in \( v_e \) Eq. (36) gives

\[ z_D = v_e + \frac{1}{2}v_e^2. \]  

(37)

The second-order expansion in Eq. (35) is valid for objects with small values of \( z \) having an expansion velocity, \( v_e = \dot{a}_e\chi_e \) which is much less than the velocity of light. Hence, we can use the velocity from the Hubble law in the approximate relation (37) for the Doppler shift and obtain

\[ z_D = \dot{a}_e\chi_e + \frac{1}{2}\dot{a}_e^2\chi_e. \]  

(38)

A Taylor expansion of \( \dot{a} \) gives to first order

\[ \dot{a}_e = \dot{a}_0 - \ddot{a}_0(t_0 - t_e) = H_0 + q_0H_0^2(t_0 - t_e). \]  

(39)

To second order in \( H_0(t_0 - t_e) \) the radial coordinate of the source is\textsuperscript{10}

\[ \chi_e = t_0 - t_e + \frac{1}{2}H_0(t_0 - t_e)^2. \]  

(40)

Note that since \( \chi_e \) contains no zero order term, it is sufficient to calculate \( \dot{a}_e \) to first order in \( H_0(t_0 - t_e) \) as in Eq. (39), in order to obtain an expression for \( \dot{a}_e\chi_e \) which is correct to second order. We substitute Eqs. (39) and (40) into Eq. (38) and obtain the redshift due to the Doppler effect

\[ z_D = H_0(t_0 - t_e) + (1 + q_0)H_0^2(t_0 - t_e)^2. \]  

(41)

If gravity is attractive so that the cosmic expansion decelerates, the photons fall downward from the source to the observer in the gravitational field. The gravitational frequency shift is then a blueshift. If gravity is repulsive and the cosmic expansion accelerates, the gravitational frequency shift is a redshift. In both cases it is given by

\[ z_G = -\dot{\phi}_e, \]  

(42)
where \( \phi_e \) is the gravitational potential at the emitter, and the potential has been defined so that \( \phi_0 = 0 \) at the observer.

To simplify the calculation we now assume that the universe is dominated by dust. Hence the gravitational potential at the emitter is

\[
\phi_e = \int_0^{\chi_e} \frac{GM}{\chi^2} d\chi = \frac{4\pi G}{3} \rho_0 \int_0^{\chi_e} \chi^2 d\chi = \frac{2\pi G}{3} \rho_0 \chi_e^2.
\]

(43)

The Friedmann equations give

\[
\frac{4\pi G}{3} \rho_0 = q_0 H_0^2,
\]

(44)

so the gravitational potential at the emitter is

\[
\phi_e = \frac{1}{2} q_0 H_0^2 \chi_e^2.
\]

(45)

By using Eq. (40) we obtain to second order in \( H_0(t_0 - t_e) \)

\[
\phi_e = \frac{1}{2} q_0 H_0^2 (t_0 - t_e)^2.
\]

(46)

Hence the gravitational blueshift is

\[
z_G = -\frac{1}{2} q_0 H_0^2 (t_0 - t_e)^2,
\]

(47)

From Eqs. (35), (41), and (47) we have

\[
z = z_D + z_G.
\]

(48)

We have shown that for small values of \( z \) the cosmic redshift can be separated into a Doppler effect and a gravitational frequency shift. This derivation shows that it is not correct to invoke the Doppler effect as an explanation for the cosmic redshift. It is already contained in the redshift interpreted as an expansion effect.

Chodorowski\(^{19}\) has recently claimed that the cosmological redshift is a relativistic Doppler shift by considering the Milne universe model. This interpretation is possible in this model, because \( z_G = 0 \) for the Milne universe model. However, as we have shown, it is not a correct interpretation of the cosmological redshift for arbitrary universe models. Note also that the explanation of the redshift as some sort of curvature effect fails for the Milne universe which has a flat spacetime.

That the cosmic redshift cannot be interpreted as a Doppler effect may be seen by considering an emitter at rest relative to an observer at the origin. Because such an emitter has
a peculiar velocity, we shall first deduce the general formula for the cosmic redshift from an 
object with an arbitrary peculiar velocity, $v_p$. This velocity is that of the emitter relative 
to an observer at the position of the emitter, comoving with the Hubble flow. Assume the 
source emits radiation with wavelength $\lambda_e$. Then the observer at the emitter comoving with 
the Hubble flow measures a wavelength 

$$\lambda = \sqrt{\frac{1+v_p/c}{1-v_p/c}} \lambda_e, \quad (49)$$

because of the Doppler effect. The wavelength of the radiation received by an observer at 
rest at the origin at a time $t_0$ when $a(t_0) = 1$ is 

$$\lambda_0 = \frac{\lambda}{a_e} = \frac{1}{a_e} \sqrt{\frac{1+v_p/c}{1-v_p/c}} \lambda_e, \quad (50)$$

where $a_e = a(t_e)$ is the value of the scale factor at the time of emission $t_e$. Hence, the 
redshift is in general given by 

$$z = \frac{\lambda_0}{\lambda_e} - 1 = \frac{1}{a_e} \sqrt{\frac{1+v_p/c}{1-v_p/c}} - 1. \quad (51)$$

An emitter at rest relative to the observer has a peculiar velocity $v_p = -H_e \chi_e$. Thus the 
redshift of this emitter is 

$$z = \frac{1}{a_e} \sqrt{\frac{1-H_e \chi_e/c}{1+H_e \chi_e/c}} - 1, \quad (52)$$

which in general does not vanish. It has hence been shown that the radiation from an object 
at a cosmological distance at rest relative to the observer has a non-vanishing redshift, which 
shows that it is not natural to interpret the redshift as a Doppler effect. A similar result 
was obtained in Ref. 4.

V. CONCLUSION

Einstein once said to Heisenberg, “It is the theory that tells what we observe.” Heisen- 
berg said that this idea helped him in arriving at the uncertainty relation when he struggled 
to find the physical significance of quantum mechanics (see Ref. 20). It is no less true in 
cosmology. To be able to construct a picture of the world that can be expressed by our 
ordinary language, we must interpret the observational data within a theory that gives us
the concepts to be used in forming this picture. Describing the expansion of the universe we have either a Newtonian or special relativistic picture with an absolute space or a general relativistic picture with a dynamical space. According to the Newtonian picture the galaxies move through space, but according to the general theory of relativity space expands and the galaxies follow the expanding space. Which picture is most natural – the special relativistic or the general relativistic? Let us consider Hubble’s law, which says that the velocity of the galaxies away from us is proportional to their distance from us. It implies that at sufficiently great distances the velocities become superluminal. Think of the Milne universe, the Minkowski space described from an expanding reference frame. Because spacetime is flat, special relativity is sufficient to describe the kinematics in this spacetime, and special relativity says that the velocities of the galaxies have to be less than the velocity of light. It seems that there is an inconsistency here. The solution is that Hubble’s law refers to the general relativistic space defined by simultaneity on the clocks following the reference particles. It is valid in the public space of the universe model, not the private space of a particular observer.

Peacock and Whiting have pointed out an unexpected behavior of free particles in Friedmann universe models. If a distant particle at rest relative to an observer is let free, it is not dragged along with the Hubble flow of the universe. Even in a universe with accelerated expansion, the particle accelerates towards the observer. They claim that such motion is not what one expects if the expansion of the universe is an expansion of space.

We have shown, however, that the general relativistic conception of an expanding space does in fact offer a natural, although not immediately obvious, explanation of the motion of a free particle. Introducing an observer into the homogeneous universe breaks the homogeneity. There is an inhomogeneous Hubble flow relative to the observer, where the velocity increases with the distance from the observer. Also the region with a given velocity expands, implying that the velocity of the Hubble flow decreases with time at a fixed distance from the observer. Hence, the peculiar velocity towards the observer of a particle initially at rest relative to the observer, is adjusted to a larger expansion velocity of space than the expansion velocity at later times. In other words, the acceleration of the particle is directed towards the observer because of the retarded expansion of space at its position.

Observational data should be interpreted in as unified a way as possible. If we interpret the cosmic redshift as a Doppler effect, then the emitters are thought of as moving through
space. However, the Doppler effect interpretation of the cosmic redshift is valid only in flat spacetime, that is, in the empty Milne universe model. Generally, it does not give the full frequency shift. In universe models with matter and energy, spacetime is curved, and there is a cosmic gravitational field. The effect of this field on the observed frequency of light from distant sources is not included in the Doppler effect. As we have shown, both the Doppler effect and the gravitational frequency shift are included in the interpretation of the redshift due to the expansion of space during the time the light from a galaxy travels toward us.

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1 J. A. Peacock, “An introduction to the physics of cosmology,” in Modern Cosmology, Proceedings of the Como School 2000, edited by S. Bonometto, V. Gorini, and U. Moschella (IOP, Bristol, 2002), p. 9.

2 J. A. Peacock, <www.roe.ac.uk/~jap/book/additions.html>.

3 A. B. Whiting, “The expansion of space: Free particle motion and the cosmological redshift,” The Observatory 124, 174–189 (2004).

4 T. M. Davis, C. H. Lineweaver, and J. K. Webb, “Solutions to the tethered galaxy problem in an expanding universe and the observation of receding blueshifted objects,” Am. J. Phys. 71, 358–364 (2003).

5 T. M. Davis and C. H. Lineweaver, “Expanding confusion: Common misconceptions of cosmological horizons and the superluminal expansion of the universe,” Publications Astro. Soc. Australia 21, 97–109 (2004).
C. H. Lineweaver and T. M. Davis, “Misconceptions about the big bang,” Sci. Am. 292 [xx need issue # xx], 24–33 (2005).

F. J. Tipler, “Newtonian cosmology revisited,” MNRAS 282, 206–210 (1996).

F. J. Tipler, “Rigorous Newtonian cosmology,” Am. J. Phys. 64, 1311–1315 (1996).

J. A. Morgan, “Are galaxies receding or is space expanding,” Am. J. Phys. 56 777 (1988).

C. W. Misner, K. S. Thorne, and J. A. Wheeler, Gravitation (Freeman, 1973), p. 284.

R. Gautreau, “Imbedding a Schwarzschild mass into cosmology,” Phys. Rev. D 29, 198–206 (1984).

Olaf Gron and P. D. Rippins, “Singular shell embedded into a cosmological model,” Gen. Re. Grav. 35, 2189–2215 (2003).

R. H. Price, “In an expanding universe, what doesn’t expand?,” gr-qc/0508052.

H. P. Robertson and T. W. Noonan, Relativity and Cosmology (W. B. Saunders, 1968), Sec. 16.1.

S. Carroll, Spacetime and Geometry: An Introduction to General Relativity (Addison-Wesley, 2004), p. 93.

H. Bondi, “Spherically symmetrical models in general relativity,” MNRAS 107, 410–425 (1947).

J. A. Peacock, Cosmological Physics (Cambridge, Cambridge University Press, 1998), Exer. 3.4.

Y. B. Zeldovich and I. D. Novikov, Relativistic Astrophysics (Chicago, University of Chicago Press, 1983).

M. J. Chodorowski, “Is space really expanding? A counterexample,” astro-ph/0601171.

W. Heisenberg, Physics and beyond (New York, Harper & Row, 1971).

Figure Captions
FIG. 1: Proper distance (in units of $c/H_0$) of a free particle from an observer at the origin as a function of time (in units of $t_0$) (full line) and trajectories of particles comoving with the Hubble flow (dashed lines) for $w = 0$ (Einstein-de Sitter).
FIG. 2: Minkowski diagram with reference to the comoving inertial reference frame of an arbitrary particle in the Milne universe. All the clocks at rest in this frame show the same time $T$ as the clock at the spatial origin of the reference frame. The coordinate $R$ is the distance from the origin particle measured at $T = \text{constant}$. The vertical time axis is the world line of the origin particle. The fan of lines is the set of world lines of the reference particles defining the Milne universe. The horizontal line at $T = a$ represents the simultaneity space, called the private space of the origin particle, at an arbitrary point of time. The cosmic time $t$ is the time measured by clocks following all of the test particles. This time is the time since the Big Bang measured on these clocks. The hyperbola $t = a$ represents the space at a fixed cosmic time, called the public space of the Milne universe. The two lines from a point on the time axis down to the Big Bang light cone represent a backward light cone of an observer on the origin particle.