Size Segregation and Convection of Granular Mixtures Almost Completely Packed in the Rotating Thin Box.

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Abstract

Size segregation of granular mixtures which are almost completely packed in a rotating drum is discussed with an effective simulation and a brief analysis. Instead of a 3D drum, we simulate 2D rotating thin box which is almost completely packed with granular mixtures. The phase inversion of radially segregated pattern which was found in a 3D experiment are qualitatively reproduced with this simulation, and a brief analysis is followed. Moreover in our simulation, a global convection appears after radial segregation pattern is formed, and this convection induces axially segregated pattern.

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Granular materials exhibit some complex phenomena. One example is size segregation which occurs by shaking or stirring them. Mixtures of granular materials which are partially-filled in horizontally rotating drum also segregate by size. Recent experiments of such rotating drum have shown two types of segregation patterns. One of them is radial segregation where large materials accumulate near the walls of drum and small materials

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accumulate to the central region around the rotating axis. The other is axial segregation where the system evolves to form alternating bands one of which is rich in small material and the other is rich in large material. These phenomena have been explained by recent numerical and analytical studies. Most of these studies explained such segregation considering the difference of dynamic angle of reposes between large and small materials at flowing surface of granular bed.

Recently, radial and axial segregation were observed in an experiment of a horizontally rotating drum which is almost completely packed with granular mixtures. Moreover, as explained in the following, phase inversion between two types of radial segregation patterns takes place when $w$, the angular velocity of the rotating drum, passes through a critical value. When $Aw^2 < g$, large particles accumulate near the wall of the drum and small particles accumulate to the central region. On the contrary, small particles accumulate near the wall of drum and large materials accumulate to the inner region of the drum when $Aw^2 > g$. Here, $A$ is the radius of the drum, and $g$ is the acceleration of gravity. In this system, there are little surface flow because the drum is almost completely packed with granular mixtures. Thus, previous numerical and analytical studies which have taken into account the difference of dynamic angle of reposes between large and small materials cannot explain these phenomena. In this paper, we discuss the mechanism of such segregation and phase inversion phenomena in a rotating drum almost filled with granular mixtures. First, we reproduce experimental results with a simple simulation. Second, we make an analytical study of the phase inversion phenomena between two types of radial segregation.

In order to simplify the simulation, we set the following situation. Instead of a 3D drum the radius and the width of which are respectively $A$ and $B$, we use a 2D box which is rotating along a horizontal axis with the length $2A$ and the width $B$. Here, the length of 2D box corresponds to the diameter of 3D drum. The rotation axis of this box is the line of half length $A$, and this box rotates with angular velocity $w$ (Fig.1). We employ the following particle model which is one of the simplest model of granular dynamics. The equation of the motion of the $i$th particle is
\[ \dot{x}_i = -\sum_{j=1}^{N} \theta(r_i + r_j - |x_i - x_j|)\{\nabla V(r_i + r_j - |x_i - x_j|) + \eta(v_i - v_j)\} + F_{i}^{ex} \] (1)

\[ V(r_i + r_j - |x_i - x_j|) = \frac{k}{2}(r_i + r_j - |x_i - x_j|)^2 \] (2)

Here, \(\theta\) is the Heaviside function, \(N\) is the total number of particles, \(k\) and \(\eta\) are respectively the elastic constant and the viscosity coefficient, and \(x_i(x_i, y_i), v_i(v_{x_i}, v_{y_i})\) and \(r_i\) are, respectively, the position, the velocity and the radius of \(i\)th particles. The elastic constant \(k\) and the viscosity coefficient \(\eta\) are related to the coefficient of restitution \(e\) and the collision time \(t_{col}\), time period during collision. In this model, the effect of particles rotation is neglected.

We regard the rotating axis as \(x\) axis \((y = 0)\), and the length direction (radius direction for 3D drum) as \(y\) direction in this simulation. \(F_{i}^{ex}\) is the external force which directly acts on \(i\)th particle not by collision. Since gravitation and centrifugal force work on each particle, \(F_{i}^{ex}(F_{x_i}^{ex}, F_{y_i}^{ex})\) is given as following.

\[ F_{x_i}^{ex} = 0 \] (3)

\[ F_{y_i}^{ex} = y_i w^2 - g \sin(wt) \] (4)

The above equations are calculated with the Euler’s scheme. The time step \(\delta t\) is set enough small such that \(\delta x\), the displacement of the \(i\)th particle during \(\delta t\), does not exceed a given value. We set \(2A = 7.43, B = 24.0, t_{col} = 0.05\), \(e\) between a pair of particles is 0.99, and \(g = 3.0\). Moreover, boundaries of box \(|x| = A\) or \(|y| = B\) are given as the visco-elastic walls with \(e = 0.95\). Total number of particles is \(N = 750\), the ratio of particle numbers between large and small particles is 1:4, the ratio of average radius between them is 2:1, and 10% polydispersity for large and small particles’ radius is given. The packing density which is defined as \([\text{the area of region occupied by particles}] / [\text{the area of 2D box}]\) is estimated about 84%. It means this system is not completely packed. In practice, the movement of the center of the mass of particles appears through the rotation process in our simulation. However, the distance of this movement from the average position is almost same as the...
average radius of small particles which is enough small compared to $A$. Hence, this system is regarded as an almost completely packed system. Because of such polydispersity and the movement of the center of mass of particles, each particle in the system barely moves.

We pack particles at random at the initial condition and simulate with several values of $w$. Figure 2 (a) and (b) are typical patterns of radial segregation for (a)$w = 0.5$ ($Aw^2 < g$), and (b)$w = 1.5$ ($Aw^2 > g$). Figure 2 (a) indicate that large particles accumulate near the wall of drum ($|y| = A$) and small particles gather around the central region when $Aw^2 < g$. Figure 2 (b) indicate small particles accumulate near the wall of drum and positions of large particles accumulate to the central region when $Aw^2 > g$. These results qualitatively correspond to the experimental results of rotating drum which is almost completely packed with granular mixtures. The pattern illustrated in Fig.2 (b) is stable whereas the pattern like Fig.2 (a) evolves to the pattern like Fig.3 (c). This is because the fluctuation of large particles concentration in axial direction grows up slowly like followings. Now, we consider the case that the direction of gravity is the negative direction in $y$. It means that $y = -A$ corresponds to the bottom of box. Near the bottom of box, the region in which large particles are packed exists. Above this region, the region in which small particle is rich exists. In convenience, we name boundaries between these two regions S-L-boundary. Because of the fluctuation of large particles concentration near the bottom, S-L-boundary has finite inclination. Small particles on this slope cannot invade to the region in which large particle is rich because large particles are packed densely. However, along this boundary, small particles can flow down. Then, the amount of the small particles flow in $y < 0$ region is very small compared to that in the $y > 0$ region because particles in $y < 0$ region are packed more densely than in $y > 0$ region. Moreover, above the region rich in small particles, the region rich in large particle exists again. We name the boundary between these two regions L-S-boundary. Large particles in this region cannot invade into the region in which small particle is rich because they cannot go through small voids which appear in the region rich in small particles. Because of friction working at L-S-boundary, however, some parts of large particles in this region near L-S-boundary are dragged by the flow of small particles, which flow down along
the slope of S-L-boundary. Thus, some of large particles around L-S-boundary flow down
along the boundary. Hence, the axial concentration of large particles in this region fluctuates
and this fluctuation grows up along S-L-boundary (Fig. 3 (a)→(b)→(c)). The direction of
gravity periodically changes between the negative direction and the positive direction in
y with the rotation of the box. Thus, above mentioned flow of particles in all around the
system forms global convection along S-L-boundaries like Fig. 4. Furthermore, the convection
makes the fluctuation of large particles concentration grow up, and changes the segregation
pattern from the radial to the axial. Moreover, the axial segregation pattern is kept stable
by the convection.

Now, we discuss the mechanism of the phase inversion of the radial segregation pattern.
In order to discuss this phenomenon, we consider following assumptions hold according to
previous studies\cite{3, 4}. Compared to large particles, small particles can move more easily in
granular bed because they can move through smaller voids. It means that small particles
are more directly drifted by external force than large particles. Then, we assume that small
particles, compared to large particles, tend to move to the direction in which external force
works. Large particles can also move through voids if sizes of them are larger than that
of large particles. Then, large particles tend to move to the low particles density region in
which such large voids tend to appear. By use of these assumptions, the mechanism of the
phase inversion of radial segregation pattern is discussed. The force which works on each
particle is given by eq.(3) and eq.(4). We put \((X, Y) = (ycos(\omega t), ysin(\omega t))\). Here, the
direction of gravitation is from \(Y > 0\) to \(Y < 0\). Now, the region in drum is given by circle
\(X^2 + Y^2 \leq A^2\), and the region in which \(F_i^{ex} < 0\) for \(y > 0\) and \(F_i^{ex} > 0\) for \(y < 0\) are satisfied
is given by the circle
\[
X^2 + (Y - \frac{g}{2\omega^2})^2 < \left(\frac{g}{2\omega^2}\right)^2. \tag{5}
\]
In convenience, we name the former circle ‘circle 1’ and the latter ‘circle 2’. The external
forces work toward the center of circle 1 on each particle in circle 2, and toward the cir-
cumference of circle 1 in another region in circle 1. Now, the movement of particles in the
$Y < 0$ region seems quite small compared to that in the $Y > 0$ region because the packing of particles in the $Y < 0$ region is more dense than that in the $Y > 0$ region. Hence, we need to consider the movement of particles only in the $Y > 0$ region. When $Aw^2 < g$, circle 2 covers up to the circumference of circle 1 in $Y > 0$ (Fig. 5). It means that in most of $Y > 0$ region, the external force works toward the center of drum, so that small particles move toward the center of drum. Because of the movement of small particles, low density region appears near the upper most position of drum and large particles accumulate around there. On the contrary, when $Aw^2 > g$, circle 1 completely covers circle 2 (Fig. 5). It means that small particles in circle 2 move toward the center of drum, and small particles out of circle 2 move toward the wall of drum. Then, since low particle density space appears around the circumference of circle 2, large particles accumulate there. The length between the center of circle 1 and the most far point on the circumference of circle 2 from the center of circle 1 is given by $\frac{g}{w^2}$. Hence, When $Aw^2 > g$, annular pattern the radius of which is estimated about $\frac{g}{w^2}$ is formed by large particles. Thus, the relation between the angular velocity of drum $w$ and the radius $\bar{y}$ where large particles aggregate is given by

$$\bar{y} = A \cdot \cdots (w < \left( \frac{g}{A} \right)^{\frac{1}{2}})$$

(6)

$$\bar{y} \sim \frac{g}{w^2} \cdots (w > \left( \frac{g}{A} \right)^{\frac{1}{2}}).$$

(7)

The phase inversion angular velocity $w_c$ is given by $w_c = (\frac{g}{A})^{\frac{1}{2}}$.

In this paper, by use of simulations and brief analysis, we discussed the radial segregation and the axial segregation of granular mixtures which are almost completely packed in a horizontally rotating drum. By simulating a 2D horizontally rotating box, we reproduced two types of radial segregation patterns and the phase inversion between them, which have been found in previous experiments. Furthermore, in this simulation, global convection was observed to appear after the radial segregation pattern is formed, and this convection caused the axial segregation pattern. Moreover, by use of the competition between gravitation and centrifugal force which depends on the angular velocity of drum, we explained the phase
inversion and found the critical angular velocity at which the phase inversion takes place. In order to explain the phase inversion, we assumed that small particles, relatively, tend to move in the direction of external force and large particles move to the lower particle density region. The justification of this assumption remains to be made.

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21. \( t_{col} = (2k - \eta^2)^{\frac{1}{2}} \), \( e = \exp\left(\frac{-\eta r}{t_{col}}\right) \)
A. Awazu  Figure 1
A. Awazu  Figure 4
\[ X + \frac{Y-g}{2w^2} < \frac{g}{2w^2} \]
\[ (y - g \sin(wt) < 0) \]

Circle 1
Circle 2

Direction of External Force

A. Awazu  Figure 5