Enforcing Operational Properties including Blockeness for Deterministic Pushdown Automata

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Abstract: We present an algorithm which modifies a deterministic pushdown automaton (DPDA) such that (i) the marked language is preserved, (ii) lifelocks are removed, (iii) deadlocks are removed, (iv) all states and edges are accessible, and (v) operational blockness is established (i.e., coaccessibility in the sense that every initial derivation can be continued to a marking configuration). This problem can be trivially solved for deterministic finite automata (DFA) but is not solvable for standard petri net classes. The algorithm is required for an operational extension of the supervisory control problem (SCP) to the situation where the specification in modeled by a DPDA.

Keywords: Deterministic Pushdown Automata, DPDA, Blockingness, Deadlocks, Lifelocks, Accessibility, Coaccessibility, Supervisory Control

We are introducing an algorithm to transform a DPDA such that its observable operational behavior is restricted to its desired fragment. The algorithm decomposes the problem into three steps: transformation of the DPDA into a Context Free Grammar (CFG) while preserving the operational behavior, restricting the CFG to enforce operational blockiness, and the transformation of the resulting CFG via Parsers to DPDA while preserving and establishing the relevant criteria on the operational behavior. The algorithm presented here is an essential part for the effective solution of the supervisory control problem for DFA plants and DPDA specifications which is reduced (in the companion paper by Schneider, Schmuck, Raisch, and Nestmann (2014)) to the effective implementability of ensuring blockiness (solved in this paper) and ensuring controllability (solved in the companion paper by Schneider, Schmuck, Raisch, and Nestmann (2014)).

In Section 1 we define abstract transition systems (ATS) as a basis for the systems involved in the algorithm and give a formal problem statement to be solved for DPDA. In Section 2 we define the concrete transition systems appearing in the algorithm as instantiations of ATS. In Section 3 we present the extensive algorithm due to space restrictions mostly informally using a running example before we discuss the formal verification and possible improvements of the approach. The formal constructions of the algorithm are contained in Schneider and Schmuck (2013). We summarize our results in Section 4 and outline our next steps in Section 5.

1. ABSTRACT TRANSITION SYSTEMS

The concrete systems used in this paper (including DPDA, CFG, and Parsers) are instantiations of the subsequently defined class of Abstract Transition Systems (ATS). Thus, they will inherit the uniform definitions of derivations, languages, and the problem to be solved from the ATS definitions.

Throughout the paper we use the following notations.

Notation 1. Let $A$ be an alphabet and let $B$ be a set. Then (i) $A^*$ denotes the set of all finite words over $A$, (ii) $A^C = A \cup \{\lambda\}$, (iii) $A^\omega$ denotes the set of all finite and infinite words over $A$, (iv) $\cdot$ is the (sometimes omitted) concatenation operation on words (and languages), (v) $\subseteq$ is the prefix relation, (vi) $A^\omega$ is the prefix-closure of $A$, (vii) $\supseteq$ is the suffix relation, and (viii) $k:w$ denotes the $k$-Prefix of $w \in A^*$ defined by $(if w = \alpha \omega' \land k \geq 0 then \alpha \cdot ((k-1):\omega') \text{ else } \lambda)$, and (ix) $\text{dom}(A, B)$ denotes $(A \cup \{\bot\}) \times B$ where $\bot$ represents undefinedness.

Definition 1. (Abstract Transition System). $S = (E, C, S, \pi_S, R, c_0, O, o_m, o_m) \in \text{ATS}$ iff (i) $E$ is a set of step-edges, (ii) $C$ is a set of configurations, (iii) $S$ is a set of states, (iv) $\pi_S$ maps each configuration to at most one state, (v) $R$ is a binary step-relation on $\text{dom}(E, C)$, (vi) $c_0 \in C$ is the initial configuration, (vii) $O$ is the marking subset of $C$, (viii) $O$ is the set of outputs, and (ix) $o_m : C \rightarrow 2^O$ and $o_m : A \rightarrow 2^O$ define the unmarked and marked outputs for configurations.

For these ATS we define their derivations, generated languages, and subsequently the properties to be enforced.

Definition 2. (Semantics of ATS). (i) the set of derivations $D(S)$ contains all elements from $\text{dom}(E, C)^\omega$ starting in a configuration of the form $(<_e, e)$ where all adjacent $(c_1, c_1), (c_2, c_2) \in \text{dom}(E, C)$ satisfy $(c_1, c_1)R(c_2, c_2)$, (ii) the set of initial derivations $D_I(S)$ contains all elements of $D(S)$ starting with $(<_e, c_0)$, (iii) the reachable configurations $C_{\text{reach}}(S)$ are defined by $\{c \in C \mid \exists d \in D_I(S) \cdot d(n) = (e, c)\}$, (iv) the marked language $L_m(S)$ is defined by $\cup o_m(C_{\text{reach}}(S))$, and (v) the unmarked language $L_{um}(S)$ is defined by $\cup o_m(C_{\text{reach}}(S))$.

The concatenation of derivations $d_1, d_2 \in D(S)$ is given by $(d_1 \cdot d_2)(i) = (if i \leq n then d_1(i) else d_2[(i-n)])$.
Definition 3. (Properties of ATS). (i) $S$ has a deadlock iff for some finite $d \in D_1(S)$ of length $n \in \mathbb{N}$ which is not marking (i.e., for all $k$, $d(k) = (e, c)$ implies $c \notin A$) there is no $c'$ such that $d(n) R c'$; (ii) $S$ has a lifelock iff for some infinite $d \in D_1(S)$ there is an $N \in \mathbb{N}$ such that the unmarked language of $d$ is constant from $N$ (i.e., for all $k \geq N$, $o_{um}(d(N) = o_{um}(d(k)))$; (iii) $S$ is accessible iff for each $p \in S$ there is $e \in C_{reach}(S)$ such that $\pi_{sa}(e) = p$ and for each $e \in E$ there is $d \in D_1(S)$ such that $d(n) = (e, c)$, and (iv) $S$ is operational blockfree for any finite $d_i \in D_1(S)$ of length $n \in \mathbb{N}$ ending in $d_i(n) = (e, c)$ there is a continuation $d_i' \in D_1(S)$ such that $d_i' \wedge d_i$ is a marking derivation and $d_i$ and $d_i'$ match at the gluing point $n$ (i.e., $d_i(0) = (\lambda, c)$).

By definition, for operational blockfree ATS the absence of deadlocks is guaranteed. Finally, we present the problem of enforcing the desired properties on an ATS, which will be solved for DPDA by the algorithm presented in Section 3.

Definition 4. (Problem Statement for ATS). Let $S \in \text{ATS}$. How to find $S' \in \text{ATS}$ such that (i) $L_m(S') = L_m(S)$, (ii) $S'$ is accessible, (iii) $S'$ has no deadlocks, (iv) $S'$ has no lifelocks, and (v) $S'$ is operational blockfree?

In the DFA-setting; lifelocks can not occur and the other aspects of the problem are solved by simple and efficient graph-traversal algorithms pruning out states which are either not reachable from the initial state or from which no marking state can be reached.  

2. CONCRETE TRANSITION SYSTEMS

Every deterministic context free language can be properly represented by at least three different types of finite models: a deterministic EPDA, a context free grammar (CFG) satisfying the LR(1) determinism property, and a deterministic Parser. These three types occur at intermediate steps of our algorithm which solves the problem stated in Definition 4. Therefore, the following subsections contain their definitions as instantiations of the ATS. In each of the three cases we proceed in three steps: (1) definition of EPDA, CFG, and Parser as tuples, (2) instantiation of the ATS-scheme by defining each of the ten components, and (3) characterization of the determinism conditions.

Remark 1. We provide the slightly nonstandard branching semantics for EPDA and Parses which utilize a history variable in the configurations to greatly simplify the definition of the operational-blockfreeness from Definition 3. Furthermore, this branching semantics corresponds to the intuition that the finite state realizations are generators rather than acceptors of languages, as it is customary in the context of supervisory control theory.

2.1 EPDA and DPDA

We introduce EPDA, which are NFA enriched with a variable on which the stack-operations top, pop, and, push can be executed.

\[ \begin{array}{|c|c|c|c|c|}
\hline
\text{EPDA} & \text{PDA} & \text{DPDA} & \text{NFA} & \text{DFA} \\
\hline
\text{1-popping} & \checkmark & & \checkmark & \checkmark \\
\text{deterministic} & & \checkmark & \checkmark & \checkmark \\
\text{\lambda-step-free} & & & & \checkmark \\
\text{stack-free} & & & & \checkmark \\
\hline
\end{array} \]

Table 1. Subclasses of EPDA.

Definition 5. (Extended Pushdown Automata (EPDA)). $M = (Q, \Sigma, \Gamma, \delta, \rho_0, C, F)$ in EPDA iff (i) the states $Q$, the output alphabet $\Sigma$, the stack alphabet $\Gamma$, and the set of edges $\delta$ are finite ($Q, \Sigma, \Gamma, \delta$ range over $p, \bar{p}, \alpha, \gamma, s$, respectively), (ii) $\delta: Q \times \Sigma^* \times \Gamma^* \times \Gamma^* \times \Gamma \times Q$, (iii) the end-of-stack marker $\emptyset$ is contained in $\Gamma$, (iv) the marking states $F'$ and the initial state $p_0$ are contained in $Q$, and (v) $\emptyset$ is never removed from the stack (i.e., $(p, \sigma, s, s', p') \in \delta$ and $s \nsubseteq \emptyset$ imply $s' \nsubseteq \emptyset$).

We proceed with the ATS instantiation for EPDA.

Definition 6. (PDA—ATS Instantiation). An EPDA $M = (Q, \Sigma, \Gamma, \delta, \rho_0, C, F)$ instantiates the ATS scheme $(E, C, S, \pi_{sa}, R, c_0, A, o_m, o_{um})$ via: (i) $E = \delta$ (ii) $\bigcup C(M) \triangleq \bigcup (Q \times \Sigma^* \times \Gamma^*)$ where $(p, w, s) \in C(M)$ consists of a state $p$, a history variable $w$ (storing the symbols generated), and the stack-variable $s$ (iii) $Q \cup \{\emptyset\}$ (iv) $\bigcup \{p, w, s\} \in \delta \rightarrow M: \bigcup (\delta, C(M))^2 \text{ defined by } (e, (p, w, s')) \rightarrow_M ((p, \sigma, s', s''', p'), w', \sigma, s', s''') \text{ (vi) } \bigcup \{p_0, \emptyset, \emptyset\} \in (p, w, s) \in C(M) \text{ } p \in F \text{ (vii) } (\bigcup \Sigma^* \text{ (ix) } o_m(p, w, s), o_{um}(p, w, s) \{w\})$

The well known sub-classes of EPDA having one or more of the properties below are defined in Table 1.  

Definition 7. (Sub-classes of EPDA). An EPDA is 1-popping iff every edge pops precisely one element from $\Gamma$ from the stack. An EPDA is deterministic iff for every reachable configuration all two distinct steps append distinct elements of $\Sigma$ to the history variable. An EPDA is $\lambda$-step-free iff no edge is of the form $(p, \lambda, s', s', p')$. An EPDA is stack-free iff every edge is of the form $(p, \alpha, \emptyset, \emptyset, p')$.

2.2 CFG and LR(1)

A CFG (e.g., defined by Ginsburg and Greibach (1966)) is a term-replacement system replacing a nonterminal with a word over output symbols and nonterminals.

Definition 8. (Context-Free Grammars (CFG)). $G = (N, \Sigma, P, S) \in \text{CFG}$ iff (i) the nonterminals $N$ (ranging over $A, B)$, the output alphabet $\Sigma$, and the productions $P$ are finite (ii) $P: N \times (N \cup \Sigma)^*$, and (iii) the initial nonterminal $S$ is contained in $N, N \cup \Sigma$ and $(N \cup \Sigma)^*$ range over $\kappa$ and $\nu$, respectively. Productions $(A, v)$ are written $A \rightarrow v$.

Definition 9. (CFG—ATS Instantiation). A CFG $G = (N, \Sigma, P, S)$ instantiates the ATS scheme $(E, C, S, \pi_{sa}, R, c_0, A, o_m, o_{um})$ via: (i) $E = \delta$ (ii) $\bigcup C(G) = (N \cup \Sigma)^*$ (iii) $\{\Sigma \cup N \} \{\emptyset\}$ take the first nonterminal (if present) of the configuration $(v) \rightarrow (v) \in C(P, C(G))^2$ given by $(e, (v_1, A : v_2)) \rightarrow_C ((A, v), (v_1, \cdot, v_2)) (v) \rightarrow (v) \in S (vii) \{\emptyset\} \Sigma^*$ (viii) $\{\emptyset\} \Sigma^*$ (ix) $o_m(v) \{v\} (x) \{o_{um}(v) \{v\})$

Thus, $\lambda$ steps may not be enabled simultaneously with other steps.  

The output symbols of a CFG are usually called terminals.
The LR(1)-condition below, which corresponds to the determinism property of EPDA, depends on the restriction of the step-relation to the replacement of the right-most nonterminal which will be denoted by the index rm.

**Definition 10. (LR(1)-Condition).** According to Sippu and Soisalon-Soininen (1990) (page 52)\(^5\), LR(1) is the set of all CFG for which (assuming \( x \in \Sigma^* \))

(i) \((\langle S, F \rangle \vdash_{rm}^{\tau} (v_1, v_1, v_1, w_1)) \vdash_{rm}^{\tau} (A_1, v_1, v_1, w_1)\),

(ii) \((\langle S, F \rangle \vdash_{rm}^{\tau} (v_2, v_2, A_2, w_2)) \vdash_{rm}^{\tau} (A_2, v_2, v_2, w_2)\),

(iii) \(v_2 = v_2 - v_1 = x_1\), and

(iv) \(1:v_1 = 1:|x-w_1|\), imply

(v) \(v_1' = v_2\), \(A_1 = A_2\), and \(v_1 = v_2\).

Intuitively, if a parser for a CFG has generated the shorter prefix \(v_1'\), it must be decide by fixing the next symbol \((1:v_1 = 1:|x-w_1|)\) whether \((A_1, v_1)\) is to be applied backwards or whether for \( x \neq \lambda \) another symbol of \( x \) should be generated or for \( x = \lambda \) the production \((A_2, v_2)\) is to applied backwards.\(^6\)

\[\begin{align*}
\text{Step 6:} & \quad (G_0, \text{DPDA}) \\
\text{Step 7:} & \quad (G_0, \text{EDPDA}) \\
\text{Step 8:} & \quad (G_0, \text{EDPDA}) \\
\text{Step 9:} & \quad (G_0, \text{EDPDA}) \\
\text{Step 10:} & \quad (G_0, \text{DPDA}) \\
\text{Step 11:} & \quad (G_0, \text{DPDA}) \\
\text{Step 12:} & \quad (G_0, \text{DPDA})
\end{align*}\]

3. APPROACH

**Motivation:** For example, the DPDA \( G_0 \) in Figure 1 exhibits a lifelock generating the output \( b \) reaching \( p_1, p_3 \) arbitrarily often, a lifelock (and blocking situations) generating the output \( ab \) reaching \( p_2 \) arbitrarily often, a non-accessible state \( p_4 \) (along with the edge leading to it), but no deadlock. Observe that the cause \((G_0)\) does not properly distinguish between an even or odd number of generated states is structurally separated from the lifelock at \( p_2 \). Thus, the intuitive solution \( G_{int} \) (see Figure 1) is obtained by splitting the state \( p_0 \) and by removing junk. Any formal construction must detect the states with a deadlock, a lifelock, or a blocking situation, determine the cause of that problem, and make a decision on how to fix the problem.

**Solution:** In Figure 2 we have depicted our approach in the subsequently explained steps. The basic idea is to (Steps 1–3) transform the DPDA \( G_0 \) into a CFG \( G_3 \), (Step 4)

\[\begin{align*}
\text{Step 0:} & \quad p_0 \rightarrow p_1 & \text{Step 9:} & \quad (G_0, \text{DPDA}) \\
\text{Step 1:} & \quad (G_1, \text{DPDA}) \\
\text{Step 2:} & \quad (G_2, \text{DPDA}) \\
\text{Step 3:} & \quad (G_3, \text{DPDA}) \\
\text{Step 4:} & \quad (G_4, \text{DPDA}) \\
\text{Step 5:} & \quad (G_5, \text{DPDA}) \\
\text{Step 6:} & \quad (G_6, \text{DPDA}) \\
\text{Step 7:} & \quad (G_7, \text{DPDA}) \\
\text{Step 8:} & \quad (G_8, \text{DPDA}) \\
\text{Step 9:} & \quad (G_9, \text{DPDA}) \\
\text{Step 10:} & \quad (G_{10}, \text{DPDA}) \\
\text{Step 11:} & \quad (G_{11}, \text{DPDA}) \\
\text{Step 12:} & \quad (G_{12}, \text{DPDA})
\end{align*}\]

\[\begin{align*}
\text{Figure 1. DPDA } G_0 \text{ and } G_{int} \text{ generating } \{a^2b^m|b(bd)^n | n \in N\}.
\end{align*}\]
obtain an LR(1) grammar $G_4$ by restricting $G_3$ to establish operational blockfreeness and absence of lifelocks, (Steps 5–11) transform the LR(1) grammar into a DPDA $G_{11}$ preserving the desired properties, and finally (Step 12) remove all inaccessible states and edges.

Steps 1–4 and 7–12 preserve the marked language. Steps 1–3 and 7–12 preserve the unmarked language while step 4 restricts the unmarked language to the prefix closure of the marked language. Steps 5 and 6 are not meant to preserve the (un)marked language as they are only intermediate results of the translation in Step 7.

**Approximating Accessibility:** Throughout the following presentation we omit states and edges which are obviously inaccessible: such states and edges are detected by over-approximating the possible ≤$k$-length prefixes of stacks in reachable configurations. The $k$-overapproximation $R : Q → Q → 2^{$p \leq k$}$ is the least function satisfying the following rules: (i) initial configuration: $k:\square \in R(p_0,p_0)$, (ii) closure under steps: if $γs ∈ R(p,p)$ and $(p,α,γ,s,p') ∈ δ$ then $k:(s'\cdot s) ∈ R(p,p')$ and $k:(s'\cdot s') ∈ R(p',p')$, and (iii) transitivity: if $s ∈ R(p,p')$ and $s' ∈ R(p',p'')$ then $s' ∈ R(p,p'')$.

For example, in $G_0$ the state $p_4$ is obviously inaccessible because the set of all ≤1-length prefixes of stacks of reachable configurations with state $p_4$ is empty. However, we would obtain $λ$ to be a ≤0-length prefix of a reachable configuration with state $p_4$; i.e., by increasing the parameter for the length of the calculated prefixes a better result may be obtained. For DFA and $k=0$ the standard DFA-accessibility-operation is obtained. For arbitrary DPDA step 12 alone enforces accessibility.

Applying this approximation implicitly in the running example, we now describe the steps of the algorithm solving the problem stated in Definition 4.

**Step 1:** We transform the DPDA into a simple DPDA (called SDPDA subsequently) such that every edge is of one of three forms: a generating edge $(p,α,γ,γ,p')$ or a push edge $(p,λ,γ,s,p')$, or a pop edge $(p,λ,γ,γ,p)$. The operation is based on four steps: (i) split every edge of the form $(p,α,γ,s,p')$ into $(p,α,γ,s,p')$ and $(p',λ,γ,s,p')$, (ii) split every neutral edge of the form $(p,λ,γ,s,p')$ into $(p,λ,γ,s',p')$ and $(p',λ,γ,s,p')$ for every $s' \in S'$ and (iii) split every rule of the form $(p,α,γ,s,p')$ into $(p,α,γ,s',p')$ and $(p',λ,γ,γ,p)$ for every $s' ∈ S$. Note that the fresh states to be used in each of the four steps contain the edge for which they have been constructed (i.e., $p''$ in the first step is $(p,σ,γ,s,p')$). The operation has been adapted from Knuth (1965) (1) correcting the handling of neutral edges involving the $\square$ symbol (for example, the self loop at $p_2$ in $G_0$ would have been handled incorrectly).

*Without using the transition rule we obtain the 0- and 1-overapproximations $R_0$ and $R_1$ of $G_0$ (where we omit empty sets): $R_0 = \{ (p_0 \rightarrow \{ p_1 \rightarrow \{(λ, \rightarrow \{(λ), p_1 \rightarrow \{(λ, p_2 \rightarrow \{(λ, p_3 \rightarrow \{(λ))\}, p_2 \rightarrow \{(λ, p_3 \rightarrow \{(λ), p_3 \rightarrow \{(λ, p_4 \rightarrow \{(λ, p_5 \rightarrow \{(λ, p_6 \rightarrow \{(λ))\}, p_7 \rightarrow \{(λ\rightarrow \{(λ))\})\)\}$ $R_1 = \{ (p_0 \rightarrow \{(λ, \rightarrow \{(λ, p_0 \rightarrow \{(λ, \rightarrow \{(λ), p_1 \rightarrow \{(λ, p_1 \rightarrow \{(λ, p_2 \rightarrow \{(λ, p_1 \rightarrow \{(λ, p_1 \rightarrow \{(λ, p_2 \rightarrow \{(λ, p_3 \rightarrow \{(λ, p_3 \rightarrow \{(λ, p_3 \rightarrow \{(λ, p_4 \rightarrow \{(λ, p_5 \rightarrow \{(λ, p_6 \rightarrow \{(λ))\), p_7 \rightarrow \{(λ\rightarrow \{(λ))\})\} \}$

*Figure 3.* The simple DPDA $G_1$, the simple DPDA $G_2$ not exhibiting double marking, and the LR(1)-grammar $G_4$.

and by (2) logging the involved edges in the fresh states as explained before. For the DPDA $G_0$ from Figure 1 the SDPDA $G_1$ in Figure 3 results (up to renaming of the states).

**Step 2:** We transform the SDPDA $G_1$ into a SDPDA $G_2$ such that once the SDPDA $G_2$ has generated an output, it has to generate another symbol before entering a marking state again. For the example automaton $G_1$ this means that the lifelock at $p_2$, $p_3$ is problematic. We are reusing the construction from Knuth (1965): Every state is duplicated (the duplicated states are neither initial nor marking). Then, the edges are defined such that the automaton $G_2$ operates on the original states until it reaches a marking state. Once this happens, the automaton either remains in the original states by using a generating edge or it switches to the duplicated states. The automaton remains in the duplicated states until switching to the original states using any generating edge. For the SDPDA $G_1$ from Figure 3 the SDPDA $G_2$ in the same figure results. Note, the lifelock in $p_1$, $p_3$ has been removed by the cost of another lifelock in $p_1$, $p_3$ generating the same output $b$.

**Step 3 & Step 4:** We transform the SDPDA $G_2$ in step 3 into the CFG $G_3$ using a construction from Knuth (1965). We restrict the CFG $G_3$ in step 4 to the LR(1) grammar $G_4$ (see Figure 3) by removing all productions from $G_3$ which do not appear in any marking derivation of $G_3$. That is, the accessible and coaccessible part is constructed using
a fixed-point algorithm in each case. For the accessible part: the accessible nonterminals are the least set of nonterminals \( \mathcal{A} \) such that the initial nonterminal is contained in \( \mathcal{A} \) and for any production \( A \rightarrow v \): if \( A \in \mathcal{A} \), then the nonterminals of \( v \) are contained in \( \mathcal{A} \). For the coaccessible part: the coaccessible nonterminals are the least set of nonterminals \( \mathcal{A} \) such that for any production \( A \rightarrow v \): if the nonterminals of \( v \) are contained in \( \mathcal{A} \) then \( A \in \mathcal{A} \). The equivalence of \( G_2 \) and \( G_4 \) w.r.t. the marked language can best be understood by comparing the derivations in Figure 4.

The following three properties explain the correctness of the construction:

1. The nonterminals of the form \( L_{p,A} \) (for example \( L_{p_1,a} \)) guarantee a marking derivation of the SDPDA starting in \( p \) not modifying the stack starting with \( A \).
2. The nonterminals of the form \( L_{p,A,p'} \) (for example \( L_{p_2,p_3} \)) guarantee a derivation of the SDPDA starting in \( p \) not modifying the stack starting with \( A \) and reaching a configuration in which the \( A \) is removed and the state \( p' \) is reached.
3. For any configuration \( (p_1,w_1 \ldots w_n) \) there are \( p_2 \ldots p_n \) such that \( (p,w,\gamma_1 \ldots \gamma_n) \) is reachable by \( G_2 \) iff \( w \in L_{p_1,g_1} \ldots L_{p_{n-1},\gamma_{n-1}} \) is reachable by \( G_4 \).

Once step 4 has been completed, for the given DPDA a marked language equivalent CFG has been constructed which is leftfreeckfree, accessible, and operational blockfree (and by that deadfreeckfree).

**Step 5 & Step 6 & Step 7:** In these steps we are following, with some modifications, the constructions in Sippu and Soisalon-Soininen (1990).

In step 5 we are constructing the \( \sigma \)-augmented version \( G_5 \) of \( G_4 \): A new initial nonterminal \( S' \) and the production \( S' \rightarrow \sigma S \) are added where \( S \) is the old nonterminal. This modification allows for a simpler construction procedure of the LR(1)-machine and the LR(1)-parser in steps 6 and 7.

In step 6 we are constructing the LR(1)-machine \( G_6 \) (depicted in Figure 5) for the LR(1)-grammar \( G_5 \). The output alphabet of the DFA \( G_6 \) is the union of the output alphabet and the nonterminals of \( G_5 \). The steps of the parser (between two states \( p,p' \) of \( G_6 \)) will depend on the elements of \( p \): these elements are called items which are formally four-tuples containing a production with a marker splitting the right hand side of the production and a lookahead symbol. The DFA \( G_6 \) has two kinds of edges: the edges labeled with an output symbol \( \alpha \) represent the action where the parser generates \( \alpha \), the edges labeled

### Figure 4
Corresponding initial derivations of the SDPDA \( G_2 \) and the LR(1) grammar \( G_4 \).

| SDPDA \( G_2 \) | LR(1) \( G_4 \) |
|-----------------|-----------------|
| \( G_2(p_0, \emptyset) \) | \( L_{p_0, \emptyset} \) |
| \( G_2(p_0, a) \) | \( G_4(a \Rightarrow L_{p_0, a}) \) |
| \( G_2(p_0, a \cdot a) \) | \( G_4(a \Rightarrow L_{p_0, a} \cdot p_1 \cdot L_{p_1, a}) \) |
| \( G_2(p_0, <a>) \) | \( G_4(a \Rightarrow L_{p_0, a} \cdot L_{p_1, a}) \) |
| \( G_2(p_0, a <a>) \) | \( G_4(a \Rightarrow a \cdot L_{p_0, a} \cdot L_{p_1, a}) \) |

### Figure 6
The rules of the LR(1)-parser \( G_7 \) with initial state 1 and marking set \( \{6\} \).

| Reduce Rules | Shift Rules |
|--------------|-------------|
| 1-3-5|\( \sigma \) → 1-6| 1| a → 1-2 |
| 2-8-10|\( \sigma \) → 2-7| 1| b → 1-3 |
| 3-4|\( \sigma \) → 3-5| 2| a → 2-9 |
| 8-4|\( \sigma \) → 8-10| 8| a → 8-4 |
| 9-13-16|\( \sigma \) → 9-12| 9| a → 9-15 |
| 17|\( \sigma \) → 17-18| 17| d → 13-17 |
| 9-15-22|\( \sigma \) → 9-13| 14-26-27| d → 14-19 | 14-26-27| d → 14-19 |
| 15-23-25|\( \sigma \) → 15-22| 15| a → 15-24 |
| 23-26-27|\( \sigma \) → 23-25| 23| b → 23-26 |
| 24-29-30|\( \sigma \) → 24-28| 24| a → 24-15 |
| 24-14-19|\( \sigma \) → 24-29| 24| b → 24-14 |
| 29-33|\( \sigma \) → 29-30| 31| b → 31-32 |
| 29| b → 29-31 |

Remark 2. According to Sippu and Soisalon-Soininen (1990), the parser \( G_7 \) is a correct prefix parser. However, that is a too weak assertion: their definition of the unmarked language considers a symbol the parser has fixed but not generated not to be part of the generated.

11 While in Sippu and Soisalon-Soininen (1990) no effective algorithm is presented for this operation we have been able to verify such a construction.

12 The state with no items has been removed from the visualization in Figure 5.
Figure 5. The LR(1)-machine $G_6$. Edges generating terminals (relevant for shift-rules) and items with marker $>$ at the beginning of the right hand side (relevant for reduce rules) are printed in red.

unmarked word. Since the mode of operation we are interested (control of (embedded) discrete event systems), we had to find new proofs to verify that our stronger condition is also satisfied by the generated parser $G_7$.

**Step 8:** Since DPDA are not capable of terminating the generation by fixing an end-of-output marker, we are modifying the parser $G_7$ by removing all rules involving the end-of-output marker $\lambda$ and by changing the set of marking states such as $G_8$ (depicted in Figure 7) marks in $(s,p,w,f)$ iff some edge $s'p'\lor\to s''\lor$ has been removed. While it is not mentioned in Sippu and Soisalon-Soininen (1990), we discovered that this fragment of removal rules preserves the (un)marked language because the parser reaches a configuration in which such an edge is enabled if and only if the stack can be entirely reduced by subsequently executed reduce rules. This optimization also speeds up the parsing process using the presented construction in any other context (e.g., parsing of programming languages for which it has originally been designed).

**Step 9:** Since DPDA are not capable of fixing output symbols without generating them, we add the fixed output component of a configuration into the state of the configuration. For every shift rule of the form $p(\alpha \rightarrow p\lor'\lor')\alpha$ the rules $(\lambda, \alpha) \rightarrow p(\lor, \lor')\lambda$ and $(\lambda, \alpha) \rightarrow p(\lor', \lor')\lambda$ are used. For every reduce rule of the form $s(\alpha \rightarrow s'\lor')\alpha$ the rules $s(\lambda, \alpha) \rightarrow s'(\lor, \lor')\lambda$ and $s(\lambda, \alpha) \rightarrow s'(\lor', \lor')\lambda$ are used. The resulting parser $G_9$ is depicted in Figure 8.

It is then possible to verify, that all reachable configurations of the resulting parser $G_9$ have an empty fixed output component. We call the parser $G_9$ essentially EDPDA because it uses none of the extra capabilities of the parser formalism. **Step 10:** The essentially EDPDA parser $G_9$ can be translated into the EDPDA $G_{90}$ (depicted in Figure 9) by using for every rule of the form $s(\alpha \rightarrow s'\lor')\lambda$ the edge $(p, \sigma, s^{-1}, s'^{-1}, p')$. Marking and initial states of $G_{90}$ are taken from $G_9$. 

![Figure 5](image-url)
Step 11: Since DPDA are not capable of popping strictly more than one symbol from the stack, we split such edges into multiple edges to obtain the DPDA $G_{11}$. To preserve determinism, the splitting of edges with the same source entails the merging of partially identical edges until the recursive split identifies their distinctness. For example, the edges $(p, σ, s′, s_1, p_1)$ and $(p, σ, s, s′, s_2, p_2)$ share a common prefix $s$ on the popping component.

Since DPDA are not capable of popping strictly less than one symbol from the stack, we modify the automaton by replacing any edge $(p, σ, λ, s, p')$ with $(p, σ, γ, s, γ, p)$ for any $γ$ of the stack alphabet of $G_{10}$. For soundness, recall that the stack-bottom-marker can never be removed from the stack.

Step 12: Finally, accessibility of states and edges can be enforced by reusing the presented steps 1–4. For a DPDA we are executing steps 1–4. From the productions obtained by step 4 we can determine by executing the steps 1–3 backwards (which are by our construction injective in the sense that for each constructed production/edge $x$ a unique edge $e$ can be determined for which $x$ has been constructed). Using this backwards computation, we are able to determine the accessible edges of a DPDA. The accessible states are the sources and edges of any of the accessible edges. The inaccessible states and edges are then removed to obtain the DPDA $G_{12}$ from Figure 10.

We are not aware of comparable constructions ensuring accessibility of DPDA, however, using the decidability of emptiness from Hopcroft and Ullman (1979) it is possible to test a single (and by that every) edge for accessibility; this approach has been used in Griffin (2006). Our approach is superior as we are executing a single test on all edges simultaneously.

Verification: The soundness of the presented algorithm (w.r.t. the problem Definition 4) has been verified in the interactive theorem prover Isabelle/HOL (Paulson et al., 2011) apart from the following steps for which only pen-and-paper proofs exist yet and which are to be completed

Testing: The presented algorithm has been implemented in Java for rapid prototyping and in C++ as a plugin to the libFAUDES (2006-2013) tool. The implementations have been used successfully for many examples including the running example of this paper.

Optimizations: The algorithm can be optimized in different ways. (i) The runtime of the algorithm depends primarily on the steps 3 and 4 because $G_3$ would have an enormous amount of productions. We can greatly restrict the set of productions to be generated by exploiting the structure of the input DPDA using the reachability overapproximation presented on page 4. (ii) Furthermore, steps 3 and 4 can be merged such that only productions are generated which are coaccessible. This alternative trades runtime for space-requirements (the size of $G_4$ is usually not much greater than $G_1$ but the runtime is increased by the length of the longest derivation necessary in $G_2$ to reach all states). (iii) Another optimization merges adjacent edges in EDPDA which are intermediate results. This optimization decreases the runtime of the subsequently executed operations. The formal definition and verification in Isabelle/HOL of such intermediate operations is left for future work.

4. CONCLUSION

The algorithm presented in this paper optimizes the behavior of a DPDA whilst preserving its marked language by first translating the DPDA into another model (LR(1) grammars) in which the desired properties can be enforced using simple constructions and by translating the obtained solution back into DPDA while preserving the desired properties.

The algorithm guarantees accessibility (every state and every edge is required for some marking derivation), life-lockfreeness (there is no initial derivation executing infinitely many steps without generating an output symbol), deadlockfreeness (non-extendable initial derivations are ending in marking states), and finally the operational blockfreeness (every initial derivation can be extended into a marking derivation).
The operational blockfreeness is sufficient to conclude that the unmarked language is the prefix closure of the marked language of the resulting DPDA.

The algorithm does not minimize the size of the automaton, in fact, the size of the resulting DPDA is usually increased and is growing according to Geller et al. (1975) in some cases exponentially.

The algorithm presented here is a crucial part of the presented solution of the supervisory control problem for DFA plants and DPDA specifications which is reduced (in the companion paper by Schneider, Schmuck, Raisch, and Nestmann (2014)) to the effective implementability of ensuring blockfreeness (solved in this paper) and ensuring controllability (solved in the companion paper by Schmuck, Schneider, Raisch, and Nestmann (2014)).

5. FUTURE WORK

- **Petri nets**: Since the problem of establishing blockfreeness is unsolvable for standard Petri net classes (Giua and DiCesare, 1994, 1995), we intend to determine Petri net classes $P$ that can be translated (preserving the marked language) into a DPDA $G$ such that the DPDA generated by our algorithm $G'$ can be translated back into a Petri net from $P$ to solve the problem for such a Petri net class.

- **Visibly Pushdown Tree Automata (VPTA)**: VPTA introduced by Chabin and Réty (2007) are the greatest known subclass of DPDA which are closed under intersection. For the context of the Supervisory Control Theory we intend to determine an algorithm which solves the problem from Definition 4 for VPTA because (i) plant and controller can then be generated by VPTA, while this decreases the expressiveness for the controller language it also increases the expressiveness for the plant language, and (ii) the closed loop is again a VPTA, which allows for the iterative restriction of a plant language by horizontal composition of controllers. The algorithm presented here may be reusable: the output of the algorithm, when executed on a VPTA, may be (convertible) into a VPTA. Therefore, when using VPTA for plants, specifications, and controllers, the supervisory controller synthesis can be extended to yet another domain.

- **Nondeterminism**: For the context of the Supervisory Control Theory there is no reason to restrict oneself to deterministic controllers. However, for these systems the desired property of operational blockfreeness is not guaranteed for language blockfree controllers. Therefore, when extending the domain of the algorithm to PDA the proofs will become more complex as the preservation of marked and unmarked language is no longer sufficient for the preservation of the operational blockfreeness as discussed in Schneider et al. (2014).

REFERENCES

Aho, A.V. and Ullman, J.D. (1972). *The theory of parsing, translation, and compiling*. Prentice-Hall, Inc., Upper Saddle River, NJ, USA.

Chabin, J. and Réty, P. (2007). Visibly pushdown languages and term rewriting. In B. Konev and F. Wolter (eds.), *FroCoS*, volume 4720 of *Lecture Notes in Computer Science*, 252–266. Springer.

Geller, M.M., III, H.B.H., Szymanski, T.G., and Ullman, J.D. (1975). Economy of descriptions by parsers, dpda’s, and pda’s. In *FOCS*, 122–127. IEEE Computer Society.

Ginsburg, S. and Greibach, S.A. (1966). Deterministic context free languages. *Information and Control*, 9(6), 620–648.

Giua, A. and DiCesare, F. (1994). Blocking and controllability of petri nets in supervisory control. *IEEE Transactions on Automatic Control*, 39(4), 818–823. doi: 10.1109/9.386260.

Giua, A. and DiCesare, F. (1995). Decidability and closure properties of weak petri net languages in supervisory control. *IEEE Transactions on Automatic Control*, 40(5), 906–910. doi:10.1109/9.384227.

Griffin, C. (2006). A note on deciding controllability in pushdown systems. *IEEE Transactions on Automatic Control*, 51(2), 334–337.

Hopcroft, J.E. and Ullman, J.D. (1979). *Introduction to Automata Theory, languages and computation*. Addison-Wesley Publishing company.

Knuth, D.E. (1965). On the translation of languages from left to right. *Information and Control*, 8(6), 607–639.

libFAUDES (2006-2013). Software library for discrete event systems. URL http://www.rt.uni-erlangen.de/libfaudes.

Paulson, L., Nipkow, T., and Wenzel, M. (2011). Isabelle/HOL. URL http://isabelle.in.tum.de.

Schmuck, A.-K. and Schneider, S. (2013). Supervisory controller synthesis for deterministic pushdown automata—enforcing controllability least restrictively. *WODES’14*.

Schneider, S. and Schmuck, A.-K. (2014). Supervisory controller synthesis for deterministic pushdown automata specifications. Technical report, Technical University of Berlin, URL http://www.tu-berlin.de/?25631.

Schneider, S., Schmuck, A.-K., Raisch, J., and Nestmann, U. (2014). Reducing an operational supervisory control problem by decomposition for deterministic pushdown automata. *WODES’14*.

Sippu, S. and Soisalon-Soininen, E. (1990). *Parsing Theory*, volume II: LR(k) and LL(k) Parsing of EATCS Monographs on Theoretical Computer Science. Springer-Verlag.