Early Gedanken Experiments of Quantum Mechanics Revisited

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Abstract

The famous gedanken experiments of quantum mechanics have played crucial roles in developing the Copenhagen interpretation. They are studied here from the perspective of standard quantum mechanics, with no ontological interpretation involved. Bohr’s investigation of these gedanken experiments, based on the uncertainty relation with his interpretation, was the origin of the Copenhagen interpretation and is still widely adopted, but is shown to be not consistent with the quantum mechanical view. We point out that in most of these gedanken experiments, entanglement plays a crucial role, while its buildup does not change the uncertainty of the concerned quantity in the way thought by Bohr. Especially, in the gamma ray microscope and recoiling double-slit gedanken experiments, we expose the entanglement based on momentum exchange. It is shown that even in such cases, the loss of interference is only due to the entanglement with other degrees of freedom, while the uncertainty relation argument, which has not been questioned up to now, is not right.

I. INTRODUCTION

From 1927 to 1935, shortly after the construction of the basic formalism of quantum mechanics, there appeared several ingenious gedanken experiments, culminating in Einstein, Podolsky and Rosen (EPR) experiment [1,2], which has since been a focus of attention in researches on foundations of quantum mechanics, with entanglement [3] appreciated as the key notion. As we shall show here, entanglement also plays a crucial role in the other famous gedanken experiments, such as Heisenberg’s gamma ray microscope [4,5], Einstein’s single-particle diffraction and recoiling double-slit [6,7], and Feynman’s electron-light scattering scheme for double-slit [8]. To present, for the first time, a systematic analysis on these famous gedanken experiments from fully quantum mechanical point of view is one of the purposes of this paper.

Seventy years’ practice tells us to distinguish quantum mechanics itself, as a self-consistent mathematical structure, from any ontological interpretation. This does not include the sole probability interpretation, which is directly connected to observation, for instance, the probability distribution is given by the frequencies of different results of an ensemble of identically prepared systems. Whether there should be more to say about an underlying ontology, or what it is, is another issue. Furthermore, to recognize any ontological element that was mixed with quantum mechanics itself for historic reasons is the first
step in looking for the right one. Unfortunately, the Copenhagen school’s ontological version of the uncertainty relation (UR) is still adopted in many contemporary books and by many contemporary physicists, and Bohr’s analyses of the famous gedanken experiments, which are based on his interpretation of the UR, are still widely accepted. We shall comment on these analyses one by one. It turns out that both his interpretation of the uncertainty relation and his analyses of gedanken experiments based on the former are not consistent with what quantum mechanics says. It should be noted that the Copenhagen interpretation (CI) of quantum mechanics just originated in those analyses of gedanken experiments. Thus the second purpose of this paper is to point out that CI originated in a misconception of physics.

The third purpose of this paper lies in the current research frontiers. Recently there were studies on the so-called which-way experiments [9,10], which resemble the recoiling double-slit gedanken experiment in many aspects. In these experiments, the interference loss is clearly due to entanglement, rather than uncertainty relation, because no momentum exchange is involved. On the other hand, these results are regarded as another way of enforcing the so-called “complementarity principle”, coexisting with Bohr’s uncertainty relation arguments for the original gedanken experiments [9,10]. Here we expose the entanglement based on the momentum exchange and show that in such a case, which does not seem to have been investigated in laboratories, the interference loss is also only due to entanglement, while the uncertainty relation arguments are wrong. Interestingly, it will be seen that in the recoiling double-slit experiment, the interference is not entirely lost in Einstein’s proposal of measuring which-slit the particle passes.

II. UNCERTAINTY RELATION AND PHOTON BOX EXPERIMENT

We have now learned that the momentum-position UR,

\[ \Delta x \Delta p \geq \hbar / 2, \]

is an example of the general relation [13]

\[ \Delta A \Delta B \geq |\langle [A, B] \rangle| / 2, \]

where \( A \) and \( B \) are two operators, \( [A, B] = AB - BA, \Delta A = (\langle A^2 \rangle - \langle A \rangle^2)^{1/2}, \langle \ldots \rangle \) represents quantum mechanical average, which is over a same quantum state for different terms in an inequality. This is only a relation between standard deviations of the respective measurements following a same preparation procedure [14]: if the same preparation procedure is made many times, but followed by measurements of \( A \), or by measurements of \( B \), or by measurements of \( [A, B] \) (if it is not a c-number), and the results for \( A \) have standard deviation \( \Delta A \) while the results for \( B \) have standard deviation \( \Delta B \), and the results for \( [A, B] \) give an average \( \langle [A, B] \rangle \), then \( \Delta A \), \( \Delta B \) and \( \langle [A, B] \rangle \) satisfy relation (2). The UR is only a consequence of the formalism of quantum mechanics, rather than a basic postulate or principle. It has nothing to do with the accuracy of the measuring instrument, nor with the disturbance between incompatible measurements, which are performed on different, though identically prepared, systems.
The mathematical derivation in Heisenberg’s original paper, though only for a special wavefunction, is consistent with the correct meaning. But an ontological interpretation was given and was more emphasized on [4,5]. In Copenhagen school’s version [4–6], largely justified through the analyses of the famous gedanken experiments, it is vague whether an UR is an equality or an inequality, and an uncertainty is interpreted as the objective absence or the absence of exact knowledge of the value of the physical quantity, caused by measurement or “uncontrollable” interaction with another object, at a given moment. For instance, in a diffraction, the momentum uncertainty of the particle is thought to be “inseparately connected with the possibility of an exchange of momentum between the particle and the diaphragm” [12]. The UR was understood as that determination of the precise value of \( x \) causes a disturbance which destroys the precise value of \( p \), and vice versa, in a single experiment.

The notion of uncertainty at a given moment or in a single run of the experiment is beyond the formalism of quantum mechanics. Experimentally, an uncertainty or standard deviation can only be attached to an ensemble of experiments. Furthermore, the uncertainty is determined by the quantum state, and may remain unchanged after the interaction with another object. In fact, in an ideal measurement, the buildup of entanglement does not change the uncertainty. Hence the uncertainty is not caused by an interaction with a measuring agency.

Now people understand that there does not exist an “energy-time UR” in the sense of \((2)\) [14]. We make some new comments in the following. In Bohr’s derivation [5,6], one obtains an approximate equality through the relation between the frequency width and the time duration of a classical wave pulse, or transforms the momentum-position UR to an “energy-time UR” by equating an energy uncertainty with the product of the momentum uncertainty and velocity, and incorrectly identifying time interval or “uncertainty” as the ratio between the position uncertainty and the velocity. Sometimes it was a time interval, while sometimes it was a “time uncertainty”, that appeared in the “energy-time UR” [6]. Therefore, they actually provide only a dimensional relation. A later justification due to Landau and Peierls was from the transition probability induced by a perturbation [15,16].

For a system consisting of two parts, between which the interaction is treated as a time-independent perturbation, the probability of a transition of the system after time \( t \), from a state in which energies of the two parts being \( E \) and \( \epsilon \) to one with energies \( E' \) and \( \epsilon' \), is proportional to \( \sin^2((E' + \epsilon' - E - \epsilon)t/2\hbar)/(E' + \epsilon' - E - \epsilon)^2 \). Thus the most probable value of \( E' + \epsilon' \) satisfies \( |E + \epsilon - E' - \epsilon'|t \sim \hbar \). This interpretation has changed the meaning of “energy-time UR”, and has nothing to do with the general relation Eq. (2). Furthermore, this relation only applies to the specific situation it concerns, and it is inappropriate to regard it as a general relation for measurement, with the two parts interpreted as the measured system and the measuring instrument. A counter-example was given by Aharonov and Bohm [17]. In fact, the relation obtained in this way must be an equality with \( \hbar \) and cannot be an inequality, since the transition probability oscillates rapidly with \( |E + \epsilon - E' - \epsilon'|t \); if it is \( 2\hbar \), the transition probability is zero.

In most of the gedanken experiments discussed by Bohr, the “energy-time UR” was touched on only as a vague re-expression of the momentum-position UR. It was directly dealt with only in the photon box experiment, proposed by Einstein in sixth Solvay conference held in 1930 [3]. Consider a box filled with radiation and containing a clock which controls
the opening of a shutter for a time $T$ (Fig. 1). Einstein suggested that the energy escaped from the hole can be measured to an arbitrary precision by weighing the box before and after the opening of the shutter. In Bohr’s analysis, the box is weighed by a spring balance. He argued that for a determination of the displacement, there is an accuracy $\Delta q$ connected to a momentum uncertainty $\Delta p \sim h/\Delta q$. It was thought that it should be smaller than the impulse given by gravitation to a mass uncertainty $\Delta m$ during the interval $T$, i.e. $h/\Delta q < T g \Delta m$. On the other hand, $\Delta q$ is thought to be related to a $\Delta T$ through gravitational redshift, $\Delta T/T = g \Delta q/c^2$, where $g$ is the gravitational constant, $c$ is the light velocity. Consequently, $\Delta T \Delta E > h$, where $E = mc^2$. One problem in this derivation is that in the UR, the momentum uncertainty is an intrinsic property of the box, there is no reason why it should be smaller than $T g \Delta m$. Another problem is that gravitation redshift causes a change of time, hence in the above derivation, $\Delta T$ corresponding to an uncertainty $\Delta q$ has to be “an uncertainty of the change of $T$” if $g$ is taken as a constant. In contemporary established physics, both $T$ and $g$ cannot have well-defined uncertainties. To cut the mess, we simply regard Bohr’s analysis as no more than giving dimensional relations.

Although we do not need to go further, it may be noted that the state of the box plus the inside photons is entangled with the state of the photons which have leaked out.

III. GAMMA RAY MICROSCOPE EXPERIMENT.

Introduced during the discovery of UR in 1927, the gamma ray microscope experiment greatly influenced the formation of CI. The approach of Copenhagen school is as follows. Consider an electron near the focus $P$ under the lens (Fig. 2). It is observed through the scattering of a photon of wavelength $\lambda$, thus the resolving power of the microscope, obtained from Abbe’s formula in classical optics for a classical object, gives an uncertainty of any position measurement, $\Delta x \sim \lambda/(2 \sin \epsilon)$, where $x$ is parallel to the lens, $2 \epsilon$ is the angle subtended by the aperture of the lens at $P$. For the electron to be observed, the photon must be scattered into the angle $2 \epsilon$, correspondingly there is a latitude for the electron’s momentum after scattering, $\Delta p_x \sim 2h \sin \epsilon/\lambda$. Therefore $\Delta x \Delta p_x \sim h$. Heisenberg’s initial analysis, which had been acknowledged to be wrong, attributed the momentum uncertainty to the discontinuous momentum change due to scattering.

A quantum mechanical approach can be made for a general situation. The account is totally different from the views of Bohr and Heisenberg. Suppose that the state of the electron before scattering is

$$|\Phi\rangle_e = \int \psi(r)|r\rangle_e d\mathbf{r} = \int \phi(p)|p\rangle_e dp,$$

and that the state of the photon is a plane wave with a given momentum $k$,

$$|\Phi\rangle_{ph} = \frac{1}{\sqrt{2\pi}} \int e^{ik \cdot r}|r\rangle_{ph} d\mathbf{r} = |k\rangle_{ph},$$

where $\hbar$ is set to be 1. Before interaction, the state of the system is simply $|\Phi\rangle_e |\Phi\rangle_{ph}$. After interaction, the electron and the photon become entangled, until decoherence. If decoherence does not happen till the detection of the photon, the situation is similar to
EPR experiment. If decoherence occurs before the photon is detected, as in a more realistic situation, the observation is made on a mixed state. The entangled state after scattering is

\[ |\Psi\rangle = \int \int \phi(p) C(\delta p)|p + \delta p\rangle_c|k - \delta p\rangle_{ph} dp d(\delta p) \]

\[ = \int \int C(\delta p)\psi(r)e^{i\delta p \cdot r}|r\rangle_e|k - \delta p\rangle_{ph} dr d(\delta p), \]

where \(\delta p\) is the momentum exchange between the electron and the photon, subject to the constraint of energy conservation, \(C(\delta p)\) represents the probability amplitude for each possible value of \(\delta p\) and is determined by the interaction. Note that the states before and after scattering are connected by a \(S\) matrix, which depends only on the interaction Hamiltonian. This is because the interaction happens in a very local regime and very short time interval, i.e. in the time and spatial scales concerning us, we may neglect the time interval of the interaction.

\(|\Psi\rangle\) may be simply re-written as

\[ |\Psi\rangle = \int \psi(r)|r\rangle_e|r\rangle_s dr, \]

where \(|r\rangle_s = \int C(\delta p)e^{i\delta p \cdot r}|k - \delta p\rangle_{ph} dp d(\delta p)\) represents that a scattering takes place at \(r\).

More generally, if the incipient photon is also a wave-packet, i.e., \(|\Phi\rangle_{ph} = \int \phi_{ph}(k)|k\rangle_{ph} dk = \int \psi_{ph}(r)|r\rangle_{ph} dr\), then \(|\Psi\rangle = \int \int \phi(p) \phi_{ph}(k) C(\delta p)|p + \delta p\rangle_e|k - \delta p\rangle_{ph} dp d\delta p\)

It is clear that an uncertainty is an intrinsic property determined by the quantum state, its meaning is totally different from that considered by Bohr from the perspective of the optical properties of the microscope such as the resolution power as given by classical optics. How an uncertainty changes depends on the states before and after the interaction, and it may remain unchanged.

In an ideal measurement as discussed by von Neumann [34], if the system’s state is \(\sum k \psi_k \sigma_k\), then after interaction with the apparatus, the system-plus-apparatus has the entangled state \(\sum k \psi_k \sigma_k \alpha_k\). \(\sigma_k\) and \(\alpha_k\) are orthonormal sets of the system and apparatus, respectively. In such a case, the expectation value of any observable of the system and thus its uncertainty is not changed by the interaction with the apparatus.

IV. DETECTION OF A DIFFRACTED PARTICLE

At the Fifth Solvay Conference held in 1927, Einstein considered the diffraction of a particle from a slit to a hemispheric screen [7] (Fig. 3). He declared that if the wavefunction represents an ensemble of particles distributed in space rather than one particle, then \(|\psi(r)|^2\) expresses the percentage of particles presenting at \(r\). But if quantum mechanics describes individual processes, \(|\psi(r)|^2\) represents the probability that at a given moment a same particle shows its presence at \(r\). Then as long as no localization is effected, the particle has the possibility over the whole area of the screen; but as soon as it is localized,
a peculiar instantaneous action at a distance must be assumed to take place which prevents
the continuously distributed wave from producing an effect at two places on the screen.

By definition, the concept of probability implies the concept of ensemble, which means the
repeat of identically prepared processes. Therefore as far as \( \psi(r) \) represents a probability
wave rather than a physical wave in the real space, it makes no difference whether it is
understood in terms of an ensemble or in terms of a single particle. What Einstein referred
to here by ensemble is effectively the classical ensemble, i.e. only the probability plays
a role while \( \psi(r) \) does not matter directly. The essential problem of Einstein is how a
classical event originates from the quantum state, which was unsolved in the early years and
remains, of course, an important and active subject. In the fully quantum mechanical view,
the diffraction is not essential for Einstein’s problem.

Here comes the entanglement between the detector and the particle. The state of the
combined system evolves from the product of those of the particle and the screen into an
entangled state

\[
|\Psi\rangle = \int \psi(r)|r\rangle_p|r\rangle_d dr,
\]

where \( |r\rangle_p \) is the position eigenstate of the particle, \( |r\rangle_d \) represents that a particle is detected
at \( r \). The measurement result of the particle position is described by a classical ensemble,
with the diagonal density matrix

\[
\rho_p = \int |\psi(r)|^2 |r\rangle_p\langle r|dr.
\]

von Neumann formulated this as a postulate that in addition to the unitary evolution,
there is a nonunitary, discontinuous “process of the first kind” \[18\], which cancels the off-
diagonal terms in the pure-state density matrix \( |\Psi\rangle\langle\Psi| \), leaving a diagonal reduced density
matrix \( \rho_r = \int |\psi(r)|^2 |r\rangle_p\langle r|d|r\rangle_d\langle r|dr \), which implies \( \rho_p \). Equivalently the projection postu-
late may also apply directly to the particle state to obtain (8). There are various alternative
approaches to this problem. Nevertheless, the projection postulate should effectively valid
in most situations.

What did Bohr say? Instead of addressing the event on the screen, Bohr discussed the
correlation between the particle and the diaphragm using his version of UR \[6\] . Einstein’s
problem was conceived as to what extent a control of the momentum and energy transfer can
be used for a specification of the state of the particle after passing through the hole. This is
a misunderstanding: even if its position on the diaphragm is specified, the particle is still not
in a momentum eigenstate, moreover, the particle still has nonzero position wavefunction at
every point on the screen after arrival; what is lost is the interference effect.

V. RECOILING DOUBLE-SLIT AND FEYNMAN’S ELECTRON-LIGHT
SCATTERING

After Bohr’s argument of the single-slit diffraction, Einstein proposed the recoiling
double-slit arrangement \[6,7\]. Consider identically prepared particles which, one by one,
are incident on a diaphragm with two slits, and then arrive at a screen. Einstein argued
against UR as follows.
As shown in Fig. 4, midway between a stationary diaphragm $D_1$ with a single slit as the particle source and a screen $P$, a movable diaphragm $D_2$ is suspended by a weak spring $Sp$. The two slits $S'_2$ and $S''_2$ are separated by a distance $a$, much smaller than $d$, the distance between $D_1$ and $D_2$. Since the momentum imparted to $D_2$ depends on whether the particle passes through $S'_2$ or $S''_2$, hence the position in passing through the slits can be determined. On the other hand, the momentum of the particle can be measured from the interference pattern.

Bohr pointed out that the two paths’ difference of momentum transfer is $\Delta p = p\omega$ (this is a fault, in this setup, it should be $2p\omega$), where $\omega$ is the angle subtended by the two slits at the single slit in $D_1$. He argued that any measurement of momentum with an accuracy sufficient to decide $\Delta p$ must involve a position uncertainty of at least $\Delta x = h/\Delta p = \lambda/\omega$, which equals the width of the interference fringe. Therefore the momentum determination of $D_2$ for the decision of which path the particle passes involves a position uncertainty which destroys the interference. This was regarded as a typical “complementary phenomenon” [3].

In Feynman’s light-electron scattering scheme, which slit the electron passes is observed by the scattering with a photon. One usually adopts Bohr’s analysis above by replacing the momentum of the diaphragm as that of the photon.

Bohr’s argument means that in determination of which slit the particle passes, its momentum uncertainty becomes smaller enough. Clearly this may not happen. For instance, as we have seen in our analysis on the gamma ray microscope, scattering with a plane wave increases the momentum uncertainty on the contrary. This is an indication that Bohr’s analysis is not correct.

Bohr’s reasoning is avoided in a proposal using two laser cavities as the which-slit tag for an atomic beam [9], and in a recent experiment where the internal electronic state of the atom acts as the which-slit tag [10,11]. However, as a showcase of the contemporary influence of CI, there was no doubt on Bohr’s argument in the original gedanken experiment, and the current experimental results were framed in terms of Copenhagen ideology, even the debate on whether UR plays a role in such which-way experiments was titled as “Is complementarity more fundamental than the uncertainty principle?” [10]. We shall show that it is a universal mechanism that interference is destroyed by entanglement with another degree of freedom. Here the momentum exchange is just the basis for the entanglement. Bohr’s analysis is not consistent with the fully quantum mechanical account.

We make a general account applicable to both single-slit diffraction and many-slit interference. Let us assume that just before diffraction by the slit(s), the state of the particle is

$$|\Phi(t_0 - 0)\rangle_p = \int \psi(r_0, t_0)|r_0\rangle_p dr_0 = \int \phi(p, t_0)|p\rangle_p dp$$

(10)

where $r_0$ belong to the slit(s). After diffraction, the state is

$$|\Phi(t > t_0)\rangle_p = \int \psi(r, t)|r\rangle_p dr,$$

(11)

with

$$\psi(r, t) = \int \psi(r_0, t_0)G(r, t; r_0, t_0)dr_0.$$  

(12)
where $G(r, t; r_0, t_0)$ is a propagator. The interference appears since the probability that the diffracted particle is at $r$ is $\int |\psi(r_0, t_0)G(r, t; r_0, t_0)d\mathbf{r}_0|^2$, instead of $\int |\psi(r_0, t_0)G(r, t; r_0, t_0)|^2d\mathbf{r}_0$. The diffraction does not change the uncertainties of position and momentum, as seen from $|\Phi(t \to t_0)| \to |\Phi(t_0 - 0)|$.

Before interacting with the photon ($t < t_0$), the diaphragm has a definite position $r_i$. This is a gedanken experiment, in which the relevant degrees of freedom are well isolated. Hence the diaphragm is described by the quantum state

$$|\Psi(t < t_0)\rangle_d = |r_i\rangle_d = \int \delta(r - r_i)|r\rangle_d d\mathbf{r} = \frac{1}{\sqrt{2\pi}} \int e^{-i\mathbf{k} \cdot \mathbf{r}_i}|\mathbf{k}\rangle_d d\mathbf{k},$$

and the state of the combined system of particle-plus-diaphragm is $|\Phi(t < t_0)\rangle_p |r_i\rangle_d$. If the diaphragm is fixed, the state of whole system after diffraction is the product of $|r_i\rangle_d$ and the state of the particle as given by Eq. (11).

If the diaphragm is moveable, after the interaction, the state of the combined system is an entangled one. Right after the scattering,

$$|\Psi(t + 0)\rangle = \frac{1}{\sqrt{2\pi}} \int \int C(\delta\mathbf{p})\phi(\mathbf{p}, t_0)e^{-i\mathbf{k} \cdot \mathbf{r}_i}|\mathbf{p} + \delta\mathbf{p}\rangle_p|\mathbf{k} - \delta\mathbf{p}\rangle_d d(\delta\mathbf{p}) d\mathbf{p} d\mathbf{k}$$

$$= \int \int C(\delta\mathbf{p})\psi(r_0, t_0)e^{i\delta\mathbf{p} \cdot \mathbf{r}_0}e^{-i\delta\mathbf{p} \cdot \mathbf{r}_i}|r_i\rangle_d d(\delta\mathbf{p}) dr_0.$$}

Then,

$$|\Psi(t > t_0)\rangle = \int \int \int C(\delta\mathbf{p})\psi(r_0, t_0)G(r, t; r_0, t_0)e^{i\delta\mathbf{p} \cdot \mathbf{r}_0}U(t, t_0)e^{-i\delta\mathbf{p} \cdot \mathbf{r}_i}|r_i\rangle_d d(\delta\mathbf{p}) dr_0 dr_0$$

$$= \int \psi(r_0, t_0)G(r, t; r_0, t_0)|r_0\rangle_p |r_0\rangle_s(t) dr_0 dr,$$

where $U(t, t_0)$ represents the evolution of the diaphragm state, and

$$|r_0\rangle_s(t) = \int C(\delta\mathbf{p})e^{i\delta\mathbf{p} \cdot \mathbf{r}_0}U(t, t_0)e^{-i\delta\mathbf{p} \cdot \mathbf{r}_i}|r_i\rangle_d d(\delta\mathbf{p}).$$

Generally speaking, this entanglement is not maximal, hence the interference is not completely destroyed.

In Feynman’s electron-light scattering scheme, we may suppose that the scattering takes place at the slit(s). In general, the photon is a wave packet $|\Phi\rangle_{ph} = \int \phi_{ph}(\mathbf{k})|\mathbf{k}\rangle_{ph} d\mathbf{k}$, then $|\Psi(t > t_0)\rangle = \int \int \int C(\delta\mathbf{p})\psi(r_0)G(r, t; r_0, t_0)e^{i\delta\mathbf{p} \cdot \mathbf{r}_0}U(t, t_0)e^{-i\delta\mathbf{p} \cdot \mathbf{r}_i}\psi(r_1)|r_1\rangle_{ph} d(\delta\mathbf{p}) dr_0 dr_1 dr.$

Again, the interference is not completely destroyed. If the photon is a plane wave, as we have known in the above discussions on the gamma ray microscope, the position uncertainty remains unchanged while the momentum uncertainty increases, contrary to Bohr’s claim.

In general, in both the moveable diaphragm and the photon scattering schemes, the change of the uncertainty is dependent of the states before and after the interaction. It is not right to simply say that the momentum uncertainty becomes smaller, as thought by Bohr. One should also note that there are various possible momentum exchanges $\delta\mathbf{p}$, subject to energy conservation, and this is independent of the position on the diaphragm, or which slit the particle passes in the double-slit experiment. In general, both before and after
the interaction with the diaphragm, the particle is in a superposition of different momentum eigenstates. Even after the detection of the particle on the screen, the states of the diaphragm and particle still do not reduce to those with a definite momentum exchange. This was not appreciated by either Bohr or Einstein, and is another point inconsistent with Bohr’s analysis, which is based on a classical picture supplemented by an uncertainty relation.

VI. CONCLUDING REMARKS

Regarding Copenhagen school’s view of uncertainty, one cannot directly prove or disprove the notion of uncertainty at a given moment in a single run of experiment, which is beyond the standard formalism of quantum mechanics. However, it is clearly wrong to attribute the uncertainty to the interaction with a “measuring agency”. It is also wrong to regard the uncertainty as a bound for the accuracy of the measuring instrument, given by classical physics, as done in Bohr’s analyses of gamma ray microscope and recoiling double-slit. On the other hand, it is inappropriate to regard the consequence of the interaction simply as causing the uncertainty, while neglect the buildup of the entanglement, which may not change the uncertainty, or may change it but not in the way thought by Bohr.

We have seen that Bohr’s analyses of the gedanken experiments are not consistent with the quantum mechanical accounts. This indicates that the essence of quantum mechanics cannot be simply reduced to a classical picture supplemented by an uncertainty relation. More weirdness of quantum phenomena comes from the superposition, especially the entanglement. The crucial importance of entanglement in quantum mechanics was not well appreciated in the early gedanken experiments, with the attention focused on uncertainty relation in Copenhagen school’s version. However, it was finally exposed in EPR experiment, and had been noted in an earlier paper [19].

Had Bohr been aware of this, he might say that the entanglement versus interference implements the “complementary principle”, and might be happy that more commonness exists between the early gedanken experiments and EPR-experiment than discussed in his reply to EPR paper [12], where most of the discussions deal with the “mechanical disturbance” in the diffraction experiment by using uncertainty relation argument; regarding EPR, it is only said that “even at this stage there is essentially the question of an influence on the very conditions which define the possible types of predictions regarding the future behavior of the system”. Note that if the “very condition” refers to the quantum state, it is just the basis of the discussions of EPR, and that Einstein did consider it as possible to relinquish the assertion that “the states of spatially separated objects are independent on each other”, but with wavefunction regarded as a description of an ensemble [20]. The wording of the Copenhagen interpretation is so vague and flexible that one can easily embed new meanings, as people have been doing in the past many years. However, no matter how to refine its meaning, the “complementary principle” does not provide any better understanding than that provided by quantum mechanics itself. Afterall, the language of physics is mathematics rather than philosophy. In fact, decoherence based on entanglement could have been studied in the early days, had not there been the advent of the Copenhagen interpretation [21], which originated in the misconception on the early gedanken experiments.

I thank Claus Kiefer for useful discussions.
Figures
Figure 1. Photon box. Copied from Ref. [6].
Figure 2. Gamma ray microscope. Copied from Ref. [7].
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Fig. 1. Photon box. Copied from Ref. [6].

Fig. 2. Gaussian ray beam. Copied from Ref. [7].

Fig. 3. Detection of different panels. Copied from Ref. [7].

Fig. 4. Recording of ultrasonic. Copied from Ref. [7].
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