Thin-Wall Aqueduct Cyclic Loading Experiment and Finite-Element Analysis

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Abstract
This paper presents aqueduct and bracket experiments under the action of low frequency cyclic pseudo static loading. A three-dimensional FE model is developed to carry out the nonlinear elastic-plastic model which is combined the elastic model and the nonlinear multi-spring model. In this model the nonlinear hysteretic characteristic of plastic hinge is simulated by the equivalent multi-springs and the biaxial moment in the cross section of the structure and axial deformation are all considered. The hysteretic characteristics of the relationship between deformation and loading of aqueducts and brackets under the action of low frequency cyclic pseudo static loading are studied by model tests. Good correlation is observed between the experimental and analysis hysteretic curves.

Keywords: concrete aqueduct; nonlinear analysis model; hysteretic characteristics of test model; low frequency cyclic pseudo static loading

1. Introduction
The project of transporting water from south to north is a big engineering in China. The aqueduct is one of the important structure of the project. In order to insure the security of aqueduct, the seismic analysis and experiment have important value¹).

In recent years, along with the design and construction of the project of transporting water from south to north, more and more designers concentrate on researching the static and dynamic analyze of the aqueduct structure. The research works mainly consist of the fluid-structure coupling, dynamic modeling, isolation of aqueduct bearing and seismic analysis of aqueduct. The previous model is simplified the aqueduct body as a solid bar, without considering the torsion-bending deformation of thin-wall structure and fluid-structure coupling. In document²) the nonlinear fluid shaking, the horizontal force of the body and overturned moment under earthquake are all calculated by the boundary element method, it implies that there would be very big horizontal force under strong earthquake. In document³) the applicability of Housner model is indicated by comparing Housner model with the analysis method of fluid-structure coupling kinetic theory. In document⁴) Arbitrary Lagrangian-Eulerian (ALE) method is put into dynamic characteristics analysis of three-dimensional girder bent aqueduct, the calculated results showed that water plays a role of frequency modulation damper. In documents⁵, ⁶) the aqueduct beam F-E model which considering the torsion-bending and restrained-torsion deformation is established and the response with multi-site earthquake input is calculated. In document⁷) the isolation mechanism of aqueduct is discussed and the isolation analysis method is put forward. In document⁸) the influence of the LRB bearing bilinear characteristic is discussed by use of the fluid-structure coupling model, the results show the effect of LRB bearing. In document⁹) a comprehensive evaluation of the static and dynamic characteristics of the aqueduct is put forward by using a U-shaped aqueduct of transferring water from south to north project.

The structures would enter nonlinear phases under large earthquakes. So the structural nonlinear model test and the structural elastic-plastic model analysis are all needed to study.

At present, most aqueduct structure model test are static or dynamic test of isolation bearing of aqueduct, the seismic test for aqueduct thin-wall structure under strong earthquake is not carry out and the elastic-plastic dynamic analytical model has not been established.⁴-⁶) Aqueduct is the thin-wall structure and has its own characteristics, the researches of the elastic-plastic seismic response analysis of the thin-wall structure is insufficient ¹⁰-¹⁴). Articles with the elastic-plastic analysis model of thin-wall aqueduct can’t be found. The method based on the linear elastic theory of aqueduct seismic response can only be used
to calculate the seismic response at elastic stage, but the seismic response at nonlinear elastic-plastic stage can not be obtained. In order to evaluate the aqueduct seismic behavior at the elastic-plastic stage, it is necessary to study the elastic-plastic seismic response of aqueduct.

Based on the characteristics of pre-stressed concrete thin-wall structure, a plastic-elastic dynamic analysis model of thin-wall concrete aqueduct is presented in this article. The nonlinear hysteretic characteristics of the aqueduct body and bracket is studied according to the model test of low frequency cyclic pseudo static loading of the pre-stressed reinforce concrete aqueduct and high strength reinforce concrete bracket. The numerical simulative P-δ hysteretic loop curve is obtained by use of the elastic-plastic analysis model of thin-wall aqueduct.

2. Thin-Wall Aqueduct and Bracket Experiment

2.1 Specimen Model Design of Aqueduct Body

(1) The specimen model of aqueduct body use channel section (the ratio of prototype and model is 7:1), the specimen total length is 4.4m, its net length is 4m, the constraint condition is simple supported. The height of the specimen cross section is 400mm, the bottom is 140mm wide, the slab is 140mm thick, the protective layer is 20mm thick, there are class I longitudinal bar $6\phi 6$ along the two side webs, longitudinal bar $12\phi 6$ and pre-stressed reinforcement $3\times(7\phi 5)$ along slab, the reinforcement ratio $\rho$ is 0.98%. Class hoop reinforcement $\phi 6@20$, reinforcement ratio per unit volume $\rho_{SV}$ is 0.98%.

(2) The concrete strength grade of specimen is C30 and the specimen is made of pea gravel concrete.

(3) According to original design requirements, considering pre-stress loss, the stress of the pre-stressed steel of aqueduct is 820MPa. With the principle that the strains at upper and lower edge of prototype and model are the same, the initial curvature and initial compressive strain between prototype and the model could be calculated and compared using the gradient method. The calculation results show the pretension force of the model needed 200MPa.

2.2 Design of Aqueduct Bracket Model

(1) The bracket with circular section is 1.6m high, the diameter is 200mm. There are class II longitudinal bar $8\phi 12$, the reinforcement ratio $\rho$ is 2.88%; Class hoop reinforcement $\phi 8@80$, reinforcement ratio per unit volume $\rho_{SV}$ is 1.26%; (2) The cross section of bracket tie beam is rectangular with $b\times h=300\times 200$mm. There are class II longitudinal bar $4\phi 12$, the reinforcement ratio $\rho$ is 0.84%; Class hoop reinforcement $\phi 8@80$, reinforcement ratio per unit volume $\rho_{SV}$ is 0.42%. The concrete strength grade of specimen is C50 and the specimen is made of pea gravel concrete. The loading system is determined according to document 19).

2.3 Loading System

(1) According to Technical Specification for Seismic Experiment Method (JGJ101-96), the loading system is carried out.

(2) The mixed loading system is adopted, before sample yield, loading control system is used, after sample yield, displacement control system is used.

2.4 Description of Failure Procedure

2.4.1 Description of aqueduct failure

(1) The earliest crack appears in the middle of simple supported span. As the loading increased, many smaller cracks appear in the middle of span. (2) After the specimen yield, the new cracks will not appear any more, but the original cracks would be longer and wider quickly. (3) The final failure mode of horizontal or vertical loading specimen is bending failure with concrete crushing and scaling in middle span, cracks cross section of specimen and the longitudinal steel could be seen at the cracks.

In the whole process, the specimen remains higher bearing capacity. Its ductility is good and the ability of energy dissipation is also good. In the unloading stage, some cracks at the lower part of pre-stressed concrete specimen are shut. These cracks are invisible and their residual deformations are all very small. It shows that pre-stressed concrete aqueduct model has a good crack closure properties and deformation recovery capabilities.
2.4.2 Description of bracket failure

(1) The vertical bending cracks first appear at the top and bottom of the bracket tie beam; (2) As sequencing cyclic loading, horizontal bending cracks appear at both inside and outside bottom of the column; (3) With the increasing of horizontal loading and cyclic times, the bending cracks develop at the tie beam and the bottom of the bracket. With the protective layer of control section scaling, based on cracks as center the plastic hinge extend outside gradually. As sequencing cyclic loading, high strengthen concrete bracket specimen can form plastic hinge where the structure failure mainly concentrate on it. During small amplitude value of displacement cyclic loading, the strength and the stiffness of specimen both are not degenerated. But with the amplitude value of displacement increase, the strength, both stiffness and deformation recovery ability decrease greatly. The strength and stiffness of specimen are decreasing as the number of cyclic loading increased. Either the amplitude value of displacement cyclic loading is small or big, the specimen has good seismic performance.

2.5 Stiffness Degeneration

The stiffness degeneration of aqueduct can be seen in Fig.5.

(1) According to calculation, the vertical original stiffness is about 43.66 percent bigger than the transverse stiffness. It is obviously that the stiffness degenerates during the whole loading stage and mainly takes place from crack stage to plastic stage. In this stage the curve is nearly linear. Along with displacement increases, the stiffness degenerates and approach steady in the end. (2) During loading prophase, the change of stiffness degeneration under vertical loading is notable than that of under transverse loading. After the plastic stage, the transversal stiffness is nearly the same as the vertical stiffness. (3) During the loading anaphase, the stiffness of reverse loading is a little smaller than that of positive loading.

2.6 Skeleton Curve

The aqueduct model skeleton curve can be seen in Fig.6. According to calculation, the ultimate displacement under transversal loading is obviously bigger than that of under vertical loading. In the reverse loading stage, the ultimate displacement under transversal loading is 70.62 percent bigger than that of under vertical loading. In positive loading stage, the ultimate displacement under transversal loading is 70.09 percent bigger than that of under vertical loading. In the positive loading stage, the different of ultimate loading between transversal loading and vertical loading is not big. But in reverse loading stage, the ultimate loading under transversal loading is 35.66
percent smaller than that of under vertical loading.

3. Nonlinear Finite-Element Analysis Model for Thin-Wall Aqueduct

The establishment of the nonlinear Finite-Element analysis model of thin-wall aqueduct is the basis of the nonlinear time-procedure analysis of seismic responses of aqueduct. Based on the characteristics of pre-stressed concrete thin-wall structure, a multi-spring model of thin walled concrete aqueduct is proposed. According to the mechanical equivalent relationship from section material to nonlinear spring \(^{15,16}\), the load-displacement hysteretic curve of the nonlinear spring is determined. The model combined the linear elastic model with nonlinear multi-spring model for analyzing the dynamic characteristics of thin-wall concrete aqueduct, the influence of the biaxial moment in the cross section of the structure and axial deformation are considered, the nonlinear hysteretic characteristic of the plastic hinge is simulated by using the multi-spring equivalent relationship and the influences of the concrete, reinforcing bar and pre-stressed reinforcement are all taken into consideration.

3.1 Basic Assumption

The elastic-plastic properties of the thin wall space aqueduct are simulated by elastic-plastic element of the thin-wall aqueduct. Using multiple nonlinear springs to simulate the plastic deformation properties of plastic-hinge region, and make some assumptions as follows:(1) According to the components of the structure, the elastic-plastic element of the thin-wall aqueduct can be classified into beam elements and nonlinear spring elements, the elastic deformation occurs mainly in the internal of the elastic elements, the plastic deformation occurs mainly in nonlinear spring area. (2) Practically, both the transverse shear rigidity and the axial torsion rigidity of engineering structure are large, so the transverse shear deformation and the torsion deformation are assumed to be linear and the nonlinear deformations only axial deformation and bending deformation. (3) The element deformation in elastic-plastic area is conformed with the plane section assumption, and the cross section is typically divided into series finite fiber elements parallel to the axis with a uniaxial stress-strain relationship assigned to each fiber material. By using the principles of force equivalent, several fiber elements can be equivalent substituted by a nonlinear spring. (4) The bond-slip relationship among concrete, reinforcing bar and pre-stressed reinforcement is not considered. (5) The mass of the nonlinear spring is neglected.

3.2 Nonlinear Element Characteristic Matrix of Thin-Wall Aqueduct

The nonlinear element of thin-wall aqueduct is shown in Fig.7 and its dynamic equilibrium equation can be written as follow

\[
[M][\dot{\mathbf{u}}] + [C][\dot{\mathbf{u}}] + [K][\mathbf{u}] = \mathbf{f}
\]

(1)

According to the Guyan static condensation method, the freedoms of node \(j\) can be condensed and the multi-spring elastic-plastic element of thin wall aqueduct only has the freedoms of node \(i\) and node \(j\).

The static equilibrium equation can be written as follow

\[
\begin{bmatrix}
\mathbf{K}_{11} & \mathbf{K}_{1j} \\
\mathbf{K}_{j1} & \mathbf{K}_{jj}
\end{bmatrix}
\begin{bmatrix}
\Delta_i \\
\Delta_j
\end{bmatrix}
=
\begin{bmatrix}
\mathbf{F}_i \\
\mathbf{F}_j
\end{bmatrix}
\]

(2)

\(\Delta_i, \Delta_j, \Delta_j'\) are the displacements of the nodes. After static condensation

\[
\Delta_j = -\mathbf{K}_{jj}^{-1}\mathbf{K}_{ji}\Delta_i - \mathbf{K}_{jj}^{-1}\mathbf{K}_{jj'}\Delta_j'
\]

(3)

Substitute Equation (3) in (2) gives

\[
\begin{bmatrix}
\mathbf{K}_{11} & \mathbf{K}_{1j} \\
\mathbf{K}_{j1} & \mathbf{K}_{jj}
\end{bmatrix}
\begin{bmatrix}
\Delta_i \\
\Delta_j
\end{bmatrix}
=
\begin{bmatrix}
\mathbf{F}_i \\
-\mathbf{K}_{jj}^{-1}\mathbf{K}_{ji}\mathbf{F}_i - \mathbf{K}_{jj}^{-1}\mathbf{K}_{jj'}\mathbf{F}_j
\end{bmatrix}
\]

(4)

The element stiffness matrix of node \(j\) is as follow

\[
\mathbf{\bar{K}} = \begin{bmatrix}
\mathbf{K}_{11} - \mathbf{K}_{1j}\mathbf{K}_{jj}^{-1}\mathbf{K}_{ji}^{-1}\mathbf{K}_{1j} & \mathbf{K}_{1j}\mathbf{K}_{jj}^{-1}\mathbf{K}_{jj'}
\end{bmatrix}
\]

(5)

The transition matrix of the freedoms of node \(i\) and node \(j\) can be written as

\[
\begin{bmatrix}
\mathbf{\bar{F}}_i \\
\mathbf{\bar{F}}_j
\end{bmatrix}
=
\begin{bmatrix}
\mathbf{K}_{ji} & \mathbf{K}_{jj}
\end{bmatrix}
\begin{bmatrix}
\Delta_i \\
\Delta_j
\end{bmatrix}
=
\begin{bmatrix}
-\mathbf{K}_{ji}^{-1}\mathbf{K}_{ji}\mathbf{F}_i - \mathbf{K}_{ji}^{-1}\mathbf{K}_{jj'}\mathbf{F}_j
\end{bmatrix}
\]

(6)

The element mass matrix and damping matrix after freedom condensation

\[
\begin{bmatrix}
\mathbf{\bar{M}}_i \\
\mathbf{\bar{C}}_i
\end{bmatrix}
=
\begin{bmatrix}
\mathbf{T}^T & 0
\end{bmatrix}
\begin{bmatrix}
\mathbf{M} & \mathbf{C} & \mathbf{T}^T
\end{bmatrix}
\begin{bmatrix}
\mathbf{T} & \mathbf{C} & \mathbf{T}
\end{bmatrix}
\]

(7)

The matrix \([\mathbf{M}], [\mathbf{C}]\) are element mass matrix and damping matrix before freedom condensation that can be seen in documents \(^{6,16}\).

The elastic-plastic dynamic model of the beam element can be established by simulating the aqueduct model.

3.3 Nonlinear Spring Hysteretic Characteristics of Thin-Wall Aqueduct Nonlinear Element

When the nonlinear earthquake responses of the
aqueduct is analyzing, the nonlinear spring hysteretic characteristic parameters of plastic area should be determined at first. The method is as follows: when the bending moment-curvature (M–φ) hysteretic curve of the cross section is calculated by gradient method[5, 16], the P–δ hysteretic curve of the equivalent nonlinear spring at correspond position is calculated simultaneously. The substitution principle is that the effect of nonlinear spring's force should be the same as that of the concrete and reinforcing bar in the substituted area. During the bending deformation of the section under biaxial bending moment, the neutral axis doesn't parallel with the inertia principal axes of cross section, so it requires that, when dividing elements and solving section internal force, the bidirectional dividing should along the principal axis. The strain calculation is accorded to plane section assumption and the internal force and axial deformation of equivalent nonlinear spring in a certain position can be solved as follow

\[
\begin{align*}
\delta_x &= \sum_{j=1}^{n_s} \sigma_s y_j A_{s,j} + \sum_{j=1}^{n_i} \sigma_i y_j A_{i,j} + \sum_{k=1}^{n_p} \sigma_{p,k} A_{p,k} x_k \delta_y = \sum_{j=1}^{n_s} \sigma_s x_j A_{s,j} + \sum_{j=1}^{n_i} \sigma_i x_j A_{i,j} + \sum_{k=1}^{n_p} \sigma_{p,k} A_{p,k} y_k
\end{align*}
\]

\[
P_s = \sum_{j=1}^{n_s} \sigma_{e,j} A_{e,j} + \sum_{j=1}^{n_i} \sigma_{s,j} A_{s,j} + \sum_{k=1}^{n_p} \sigma_{p,k} A_{p,k}
\]

X_s = \frac{P_s}{\sum_{j=1}^{n_s} \sigma_{e,j} A_{e,j} + \sum_{j=1}^{n_i} \sigma_{s,j} A_{s,j} + \sum_{k=1}^{n_p} \sigma_{p,k} A_{p,k}}

Y_s = \frac{P_s}{\sum_{j=1}^{n_s} \sigma_{e,j} A_{e,j} + \sum_{j=1}^{n_i} \sigma_{s,j} A_{s,j} + \sum_{k=1}^{n_p} \sigma_{p,k} A_{p,k}}


Fig.8. Transverse Moment-curvature Hysteretic Curve of Aqueduct Specimen

P_s and δ_s are the nonlinear spring internal force and unit linear axial deformation. x_s, y_s are the position coordinates of the nonlinear spring. φ_x, φ_y are the rotational curvature of section. n_c, n_s, n_p are the micro element number of concrete, reinforcement and pre-stressed reinforcement in the cross section which nonlinear spring substituted.

Fig.8. is the moment-curvature curve of the aqueduct body calculated with gradient method. Fig.9. is the axial force-displacement curve of the nonlinear spring in the cross section of the bracket column.

3.4 Numerical Simulation of Load-Displacement Hysteretic Curve of the Model

3.4.1 Numerical simulation of load-displacement hysteretic curve of the aqueduct body

From Fig.1. of the sketch map of loading, the simply supported aqueduct body is considered as the simply supported thin-wall aqueduct at which mid span act concentrated cyclic loading. Elastic-plastic element is set in mid span because the plastic hinge would appear in mid span and elastic elements are used in other positions. The global stiffness matrix is adjusted during numerical simulation. The calculation methods including Gauss-Seidel iteration method or Wilson-θ incremental method.

3.4.2 Numerical simulation of load-displacement hysteretic curve of the brackets

Brackets are fixed at the foundation, 450KN vertical dead load act on the top of two columns, and the left side of the cross beam is under horizontal cyclic loading. Elastic-plastic elements are set in these four positions because of the plastic hinge, and elastic elements are used in other positions.

4. Aqueduct Body and Brackets Pseudo Static Experiment and Model Verification

4.1 Loading-displacement Hysteretic Curve of Aqueduct

Fig.10. and Fig.12. are the horizontal and vertical loading-displacement hysteretic curves of aqueduct body. From figures when loading is small, the hysteretic curve is linear before cracking and the displacement in the middle span is small. Before the loading get limiting load, with the loading increasing, the rate of displacement increase is more than that of loading, and the hysteretic curve is noticeably bending. The area of hysteretic loop increase gradually. At this time, the consume energy mainly depend on plastic hinge mechanism. Because of damage accumulation, the loading stiffness and unloading stiffness are decreased gradually and showing obvious non-elastic properties. With the loading cycle times increasing, it shows the rigidity degeneration. Fig.11. and Fig.13. are the horizontal and vertical loading-displacement numerical simulating hysteretic curves of aqueduct body which compared with Fig.10. and Fig.12., the two curves accord well. It shows that the experimental results well accord with the calculation results by applying the aqueduct elastic-plastic dynamic model. So the basic assumption and the elastic-plastic dynamic
model of the aqueduct could be used to analyze the nonlinear earthquake responses of reinforced concrete aqueduct structure.

4.2 Hysteretic Curve of Loading-displacement of Bracket

Fig.14. is the hysteretic curve of loading-displacement of aqueduct bracket. When the horizontal load is small, which in the initial loading cycle, the bracket specimen is in the elastic stage and the hysteretic curve is linear. The residual deformation is less after unloading. The hysteretic curve shows nonlinear characteristic under yield loading. According to the loading method of displacement control, with the loading cycle time increased, the loading-unloading strength and rigidity degradation would take place continuously. Fig.15. is numerical simulating curve. Comparing Fig.14. and Fig.16., the two figures accord well.

5. Conclusions

On the basis of features of pre-stressed concrete thin-wall structure, the analysis model of the thin-wall space aqueduct is established. In the model the nonlinear hysteretic characteristic of the plastic hinge is simulated by using the multi-spring equivalent relationship and the influences of the biaxial moment in the cross section of the structure and axial deformation are considered. The hysteretic characteristics of the relationship between deformation and brackets under the action of low frequency cyclic pseudo static loading are studied by model test. The results show that test hysteretic curve accord with numerical simulated hysteretic curve well. So the Elastic-plastic aqueduct model and the basic assumption are both right, furthermore, the model can be used to analyzing the nonlinear earthquake response of aqueduct, predicting the strong earthquake response, the internal force and the failure characteristics during nonlinear stage.
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