Impact drag force exerted on a projectile penetrating into a hierarchical granular bed

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ABSTRACT

Context. The impact of a solid object onto a small-body surface can be modeled by the solid impact onto a hierarchically structured granular target.

Aims. We develop an impact drag force model for the hierarchically structured granular target based on the experiment.

Methods. We performed a set of granular impact experiments in which the mechanical strength and porosity of target grains were systematically varied. Tiny glass beads (5 μm in diameter) were agglomerated to form porous grains of 2–4 mm in diameter. Then, the grains were sintered to control their strength. A polyethylene sphere (12.7 mm in diameter) was dropped onto a hierarchical granular target consisting of these porous grains. Motion of the penetrating sphere was captured by a high-speed camera and analyzed.

Results. We find that the impact drag force produced by the hierarchically structured granular target can be modeled by the sum of inertial drag and depth-proportional drag. The depth-proportional drag in a hierarchical granular impact is much greater than that of the usual granular target consisting of rigid grains. The ratio between the grain strength and the impact dynamic pressure is a key dimensionless parameter for characterizing this extraordinary large depth-proportional drag.

Conclusions. Grain fracturing plays an important role in the impact dynamics when the impact dynamic pressure is sufficiently larger than the grain strength. This implies that the effect of grain fracturing should be considered also for the impact on a small body. It may be that the effective strength of the surface grains can be estimated based on kinematic observations of the intrusion or touchdown of a planetary explorer.

Key words. minor planets, asteroids: general – planets and satellites: surfaces – methods: laboratory: solid state

1. Introduction

When a solid projectile impacts on a granular bed, the projectile experiences a drag force, which eventually halts the projectile motion. Low-speed granular-impact drag force and associated crater formation have been extensively studied in recent decades (Ruiz-Suárez 2013; Katsuragi 2016; Meer 2017). First, scaling relations for the penetration depth and the crater diameter were studied (Walsh et al. 2003; Uehara et al. 2003; Ambroso et al. 2005b). Then, various related studies relating to cavity formation (Lohse et al. 2004a), numerical modeling to compute stopping time (Seguin et al. 2009), splashing dynamics (Boudet et al. 2006), and crater formation by a soft projectile (Jong et al. 2017) were carried out.

The impact drag force equation has been developed based on the numerical and experimental results (Tsimring & Volfson 2005; Ambroso et al. 2005a; Katsuragi & Durian 2007; Goldman & Umbanhowar 2008). Although the proposed drag force model can reproduce the granular impact dynamics very well, there are some limitations regarding its applicability. For example, the packing fraction of the granular bed significantly affects the impact drag (Umbanhowar & Goldman 2010). Other factors such as the container wall (Seguin et al. 2008; vo Kann et al. 2010), interstitial air (Royer et al. 2011), gravity (Altschuler et al. 2014; Murdoch et al. 2017), motion history (Seguin 2019), and the wetness of grains (Marston et al. 2012) affect the granular impact dynamics. However, a simple granular impact drag model,

\[
d\frac{d^2z}{dt^2} = mg - \frac{mv^2}{d_i} - kz,
\]

has been used as a starting point for detailed modeling. Here, \(m\), \(z\), \(t\), \(g\), \(d_i\), and \(k\) are the mass of the projectile, its instantaneous penetration depth, time, gravitational acceleration, and two parameters characterizing the drag force, respectively. In this model, the vertical free-fall impact of a solid projectile onto a granular bed consisting of rigid particles is assumed. The second term in the right-hand side of Eq. (1) corresponds to the inertial drag, and the third term denotes the depth-proportional drag. The inertial drag usually results from the momentum transfer between the projectile and the target. This type of inertial drag can be observed even in a usual fluid drag. However, the origin of the depth-proportional drag is not very clear. Whereas a simple linear form \(kz\) was clearly confirmed in the experiment (Lohse et al. 2004b; Katsuragi & Durian 2007), the scaling of \(k\) showed a nontrivial form (Katsuragi & Durian 2013). Recently, granular Archimedes’ law has been considered to explain the depth-proportional drag (Kang et al. 2018). In addition, Roth et al. (2021) revealed that even the well-understood inertial drag cannot be kept constant in the steady deep penetration. However, in the free-fall impact (nonsteady) drag, the inertial drag plays a crucial role. In this study, we consider the extension of Eq. (1) to the case of hierarchical (porous and fragile) grains to build a firm basis of granular impact dynamics and its planetary application.
The advantage of using Eq. (1) is that it has an analytic solution and scaling laws for material-property dependences of the parameters \( d_1 \) and \( k \). According to Ambroso et al. (2005a) and Clark & Behringer (2013), this type of equation of motion can be solved in \( e-z \) space. Specifically, Eq. (1) has been solved by Katsuragi & Durian (2013) as

\[
\frac{v^2}{v_0^2} = e^{-\frac{z}{2}} - \frac{kd_1^2}{2mv_0^2} + \left( \frac{gd_1}{v_0} + \frac{kd_1^2}{2mv_0^2} \right) \left( 1 - e^{-\frac{z}{2}} \right),
\]

(2)

where \( v_0 \) is the impact velocity defined at the impact moment. In addition, material-property dependences of the parameters \( d_1 \) and \( k \) have also been obtained by the systematic experiments of Katsuragi & Durian (2013) as

\[
d_1 = \frac{0.25}{D_0} \rho_p \mu, \mu = \frac{\rho_p}{\rho_g},
\]

(3)

\[
kD_1^2 = \frac{12\mu}{mg} \left( \frac{\rho_p}{\rho_g} \right)^{1/2},
\]

(4)

where \( D_0, p, p_g, \) and \( \mu \) are the diameter of projectile, the density of the projectile, the bulk density of the target granular bed, and its friction coefficient, respectively. Using these relations, we can predict the penetration dynamics of a solid projectile impacting on a granular bed.

Low-speed granular impact drag is important in planetary science. Since most of the solid bodies in the Solar System are covered with granular materials such as regolith, granular impact cratering has been studied in order to understand the cratering mechanics that occur on planetary surfaces (see e.g., Melosh 1989). Recently, asteroids have been extensively explored as representative small bodies in the Solar System (e.g., Watanabe et al. 2019 and Lauretta et al. 2019). To efficiently control the missions of asteroidal surface touchdown and/or sample return, interaction between the probe and granular-regolith surface under microgravity conditions must be properly understood. Since the typical escape velocity is on the order of \( 10^4 \) m s\(^{-1} \) for kilometer-sized asteroids, the impact dynamics in such a low-speed regime should be analyzed. In addition, typical surface gravitational acceleration of such small asteroids is about four orders of magnitude less than that on Earth. The effect of gravity might be crucial in the impact cratering dynamics, as investigated by Cintala & Hörz (1989). Numerical studies to mimic the explorator situations have been carried out recently (Ballouz et al. 2021; Sunday et al. 2021; Thuillet et al. 2021). In these studies, the discrete element method (DEM) has been utilized to simulate regolith behaviors.

Recent observations of the asteroid Ryugu suggest that grains covering the asteroid have a high porosity. This fact was predicted by thermal imaging (Okada et al. 2020) and confirmed by a returned sample (Yada et al. 2021). Such porous grains could be mechanically weak and, therefore, significantly affect the impact drag force. However, it is difficult to consider grain-level porosity and/or fracturing in DEM simulations. In other words, rigid grains are assumed in usual DEM simulations. Moreover, impact drag measurement using porous grains has not been experimented thus far.

Impact mechanics among porous dust aggregates have been studied in the context of planetesimal formation (Blum 2018). Mechanical characterization and collision outcomes of porous dust aggregates have been experimentally investigated (Blum et al. 2006; Setoh et al. 2007; Michikami et al. 2007; Gützlaff et al. 2009; Katsuragi & Blum 2017). Numerical simulations using a porous projectile have also been performed recently (Planes et al. 2017, 2019, 2020). However, it is difficult to simulate the behavior of the collection of porous grains due to the computational expense. Only a small number of porous grains (dust aggregates) can be handled in numerical simulations.

In order to discuss the impact drag produced by porous grains, their collection has to be investigated. The collection of porous grains has a hierarchical structure because each porous grain usually consists of numerous tiny monomer particles. In other words, the aggregates consisting of monomer particles hierarchically compose the macroscopic granular matter. Such a hierarchical structure is an interesting research topic both in soft matter physics and planetary science, and hierarchical granular matter is an emerging research field that bridges a gap between these. Recently, collision of such hierarchical granular clusters have been investigated under microgravity conditions (Whizin et al. 2017; Katsuragi & Blum 2018). Moreover, slow compaction of a hierarchical granular column has also been performed (Pacheco-Vázquez et al. 2021). However, impact drag force has never been measured in hierarchical granular targets consisting of porous grains, even though it is quite important in order to appropriately consider the asteroidal impact phenomena. Therefore, in this study, we perform a simple experiment measuring the impact drag force using porous (fragile) hierarchical granular beds. As a result, we find a novel scaling of the drag-force parameter \( k \) for the hierarchical granular cases when grain strength is weak. In such cases, grain fracturing is the key factor for quantitatively characterizing the drag force. In addition, the obtained results are compared with the impact drag caused by rigid grains and a bulk dust aggregate.

2. Experiment

2.1. Free-fall impact setup

The experimental system we used in this study was a simple free-fall setup (Fig. 1a). A polyethylene sphere of diameter \( D_p = 12.7 \) mm and density \( \rho_p = 1050 \) kg m\(^{-3} \) was held by a pull-type electromagnet. This projectile-release unit was mounted on a tall height gauge (Mitsutoyo, HW-100) to control the free-fall height \( h \). A granular bed consisting of various kinds of grains (explained in the next subsection) was prepared by simply pouring grains in a cylindrical container with an inner radius of 80 mm and a depth of 50 mm. Any disturbance such as tapping and/or shaking was not applied during the target preparation. By retracting the movable part of the pull-type electromagnet system, the projectile commenced a free fall with zero initial velocity. Then, the dropped projectile impacted on the granular bed. The motion of the projectile was filmed by a high-speed camera (Photon, SA-5). The image-acquiring conditions were as follows: the frame rate was 12,000 fps, the size of the image was 896 × 704 pixels, and spatial resolution was 30 μm pixel\(^{-1} \). The range of free-fall height of the projectile was varied from \( h = 10 \) to 320 mm. Thus, the impact velocity range was \( v_0 \approx 0.44–2.5 \) m s\(^{-1} \). The identical sphere projectile was used in all experiments. We performed five trials for each experimental condition to confirm the reproducibility. In the following, average data are analyzed unless otherwise noted.

To mimic actual asteroidal situations, microgravity and vacuum environments should be reproduced. However, to reveal the fundamental physical aspect with a simple setup, we performed all the experiments under atmospheric conditions and under \( 1g = 9.8 \) m s\(^{-2} \) gravitational acceleration condition. In this
experiment, only the range of impact speed ($10^{-1}–10^{0}$ m s$^{-1}$) was close to the asteroidal landing condition.

2.2. Grain preparation and characterization

Hierarchical granular beds consisting of porous and fragile grains were prepared by agglomeration and sintering. The first step was to agglomerate monomer grains (tiny glass beads with a typical diameter of 5 μm (2–10 μm), Potters Ballotini, EMB-10) using a pan-type granulator (AS-ONE, PZ-01R). Monomer grains and small amount of water (2% of the monomers’ mass) were mixed in the rotating pan. Then, the agglomerates were formed due to the effects of capillary bridges and van der Waals forces. Subsequently, most of the agglomerates were dried at 105°C for 24 h by using a drying oven (Yamato Scientific, DVS402). After that, agglomerates were sieved to collect the grains of desired size range, $d = 2–4$ mm. To increase the strength of the agglomerated grains, these grains were sintered. The sintering temperature (650, 750, or 850°C) and duration (1, 2, or 64 h) were controlled by using an electric furnace (AS-ONE, SMF-2). To characterize the grains, we measured the friction coefficient $\mu = \tan \theta_r$ ($\theta_r$ is the angle of repose), the bulk density $\rho_g$, the bulk (macroscopic) packing fraction $\phi$, and the compression strength $Y_g$.

To measure $Y_g$, uniaxial compression tests were performed. A grain was sandwiched by stainless steel plates and vertically compressed by using a universal testing machine (Shimadzu, AG-X). During the compression, applied force and vertical displacement were recorded. A typical result of the compression test is shown in Fig. 1b. In the early stage, compression force increases linearly with the displacement. The force curve in this stage is similar to elastic one. When the compression force reaches a certain point, it suddenly drops due to the fracturing. The peak compression force at this point $F_{peak}$ is divided by the grains’ cross-sectional area $A = \pi (d/2)^2 = 7.1 \times 10^{-6}$ m$^2$ ($d = 3$ mm) to estimate the strength, $Y_g = F_{peak}/A$. In the compression test, the compression rate was fixed at 5 mm min$^{-1}$. Although this compression rate was much smaller than the free-fall impact speed, we have confirmed that the rate-dependence of the measured strength is limited to several factors over a wide dynamic range of compression rate (although in the slow regime). Due to the technical limitation, we employed $Y_g$ (measured with slow compression rate) as a typical strength value.

The images and physical properties of the grains used in this experiment are presented in Fig. 2 and Table 1, respectively. The six types of grains (Figs. 2a–f and Tables 1a–f) had hierarchical structure. Among them, four types of grains (Figs. 2c–f and Tables 1c–f) were sintered. The grains labeled “S650C02h” were sintered at 650°C for 2 h. Other labels similarly indicate the sintering temperature and duration. Wet and dry grains were produced only by agglomeration (without sintering). The wet grains were not dried at all, while dry grains experienced 105°C drying. For comparison, spherical glass beads of 2 mm in diameter were also used as nondeformable (rigid) grains (Fig. 2g and Table 1g). These granular materials possessed the following physical properties: the static friction coefficient $\mu = \tan \theta_r$ ($\theta_r$ is the angle of repose) ranging from 0.45–0.88, the bulk density $\rho_g$ ranging from 600–1500 kg m$^{-3}$, the bulk packing fraction $\phi$ ranging from 0.25–0.63, and the strength of each grain $Y_g$ varying from $5.5 \times 10^{3}$–$1.5 \times 10^{5}$ kPa. In general, friction coefficients of hierarchical grains were higher than that of rigid glass beads because of their surface roughness. Moreover, except for [S850C01h], the grains possessed quite a low bulk density and packing fraction, consistent with Katsuagri & Blum (2017, 2018). However, the physical properties of [S850C01h] were almost similar to those of rigid glass beads. This was probably due to the elimination of pores in the agglomerated grains as a consequence of the intense sintering. In any case, as seen in Table 1, we successfully varied $Y_g$ over four orders of magnitude.

3. Results

Figure 3 shows typical example images of the projectile penetration and associated surface deformation. The impact moment ($t = 0$) at which the projectile bottom reaches the target surface is identified by the video images. Figures 3a–f shows the results of hierarchical grains (wet type) and Figs. 3g–l represents the results of rigid glass beads. These examples show the representative behaviors of fragile- and rigid-grain cases, respectively. In both cases, the projectile was dropped from $h = 80$ mm. However, penetration behaviors varied depending on the type of
In both situations, the projectile was dropped from the impinged zone of the hierarchical granular target are significantly damaged and compressed, whereas the glass beads are not broken at all.

The temporal variation of the projectile position was measured from the raw video data and the example results computed from the data shown in Fig. 3 are plotted in Figs. 4a, b. The level $z = 0$ corresponds to the $z$ at the impact moment ($t = 0$), and the vertically downward direction corresponds to the positive direction of $z$. Furthermore, the instantaneous velocity of the projectile $v(t) = dz/dt$ can easily be computed, as shown in Figs. 4c, d. To reduce the noise level of velocity data, three successive data points are averaged in the following analysis. As a corollary, the projectile velocity at $t < 0$ agrees with the constant acceleration with $g = 9.8 \text{ m s}^{-2}$ (dashed red lines in Figs. 4c, d). The impact velocity is simply defined by $v(0)$. After the impact, the projectile impacting the hierarchical granular target (wet grains) decelerates in a relatively short time. For the rigid glass beads target, on the other hand, while the early-stage deceleration is significant, the late-stage deceleration is weaker than that in the hierarchical granular target case. As a consequence, the maximum penetration depth $z_{\text{max}}$ becomes larger in the case of glass beads. This is a counterintuitive result because the soft fragile grains causes larger deceleration. These two cases represent the extreme situations of fragile and rigid grains. Thus, we must check the universality of this behavior.

To check the generality of this tendency, both the projectile impacting a hierarchical granular bed (wet grains), and the panels (f)–(k) present the impact onto a rigid glass beads bed. In both situations, the projectile was dropped from $h = 80 \text{ mm}$ ($v_0 = 1.2$ and $1.4 \text{ m s}^{-1}$ for wet-grains and glass-beads cases, respectively). In the right panels (f, l), topview images of the impacted surfaces of (f) the hierarchical-granular-bed (wet grains) and (l) the rigid-glass-beads case are displayed. The impact points are marked with red laser points.

Table 1. Physical properties of target grains.

| Name | $\mu$ | $\rho_g$ (kg m$^{-3}$) | $\phi$ | $Y_g$ (kPa) |
|------|-------|------------------------|-------|-------------|
| (a) Wet | 0.60 ± 0.02 | 600 ± 30 | 0.25 ± 0.01 | 5.5 ± 1.5 |
| (b) Dry | 0.67 ± 0.03 | 630 ± 30 | 0.26 ± 0.01 | 18 ± 7 |
| (c) S650C02h | 0.53 ± 0.02 | 660 ± 30 | 0.28 ± 0.01 | 92 ± 21 |
| (d) S650C64h | 0.73 ± 0.03 | 650 ± 60 | 0.27 ± 0.02 | (6.0 ± 3.7) × 10$^2$ |
| (e) S750C01h | 0.87 ± 0.03 | 700 ± 10 | 0.29 ± 0.01 | (3.5 ± 1.5) × 10$^3$ |
| (f) S850C01h | 0.88 ± 0.03 | 1390 ± 80 | 0.58 ± 0.03 | (3.9 ± 1.7) × 10$^4$ |
| (g) Glass beads | 0.45 ± 0.02 | 1500 ± 60 | 0.63 ± 0.03 | (1.5 ± 10.2) × 10$^5$ |

Notes. Wet and Dry correspond to grains without sintering. Wet grains are not dried at all. The label $TTTXXh$ indicates sintered grains at $TTT^\circ \text{C}$ for $XX$ hours. $\mu$, $\rho_g$, and $\phi$ are the friction coefficient measured by the angle of repose, the bulk density, and the packing fraction of the target granular layer, respectively. $Y_g$ denotes the strength of the agglomerated grains. Errors of $\mu$ indicate the measurement uncertainty and other errors indicate the standard deviation of multiple measurements.

Fig. 3. Typical penetration snapshots of the projectile impacting onto granular beds and the resultant surface deformation. The panels (a)–(e) show the projectile impacting a hierarchical granular bed (wet grains), and the panels (g)–(k) present the impact onto a rigid glass beads bed. In both situations, the projectile was dropped from $h = 80 \text{ mm}$ ($v_0 = 1.2$ and $1.4 \text{ m s}^{-1}$ for wet-grains and glass-beads cases, respectively). In the right panels (f, l), topview images of the impacted surfaces of (f) the hierarchical-granular-bed (wet grains) and (l) the rigid-glass-beads case (after removing the projectile) are displayed. The impact points are marked with red laser points.

target grains. For example, we can observe ejector splashing only in the case of rigid grains (glass beads). In addition, by comparing the surface deformation (Figs. 3f, l), we find that grains at the impinged zone of the hierarchical granular target are significantly damaged and compressed, whereas the glass beads are not broken at all.

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In Fig. 6, experimentally obtained impact drag force is necessary. Thus, we consider the application in the hierarchical granular targets, quantitative analysis of the gravitational acceleration before the impact can also be confirmed (the slope of \( g = 9.8 \text{ m s}^{-2} \) is indicated by dashed red lines).

projectile and the target becomes quite small in this regime. Due to these effects, the very shallow data deviate from the scaling of Eqs. (3) and (4), as in the previous studies (Katsuragi & Durian 2013; Katsuragi & Blum 2017). Thus, the data of \( z_{\text{max}} < 2 \text{ mm} \) are not used in the following analysis. The black dashed curve in Fig. 5 shows the scaling, \( z_{\text{max}} \propto v_{0}^{2/3} \) which corresponds to \( z_{\text{max}} \propto E_{k}^{1/3} \) (\( E_{k} \) is impact kinetic energy). This relation has been confirmed in previous similar experiments (Uehara et al. 2003; Katsuragi & Blum 2017) and also qualitatively captures the data trend observed in this study as well, namely that the current experimental result is consistent with the conventional scaling.

In Fig. 5, the difference in \( z_{\text{max}} \) among fragile grains cannot be clearly observed. The error bars indicating the standard deviation of five experimental runs are too large to clearly distinguish the variation depending on \( Y_{g} \), particularly in the case of the agglomerated fragile grains. This implies that the static quantity such as \( z_{\text{max}} \) is insufficient to characterize the strength-dependent behavior. Therefore, we have to analyze the time-resolved dynamics to quantitatively discuss the penetration dynamics.

4. Analysis

To understand the physical mechanism causing shallow \( z_{\text{max}} \) in the hierarchical granular targets, quantitative analysis of the impact drag force is necessary. Thus, we consider the applicability of the model of Eqs. (1)–(4) to the hierarchical granular target cases. In Fig. 6, experimentally obtained \( v(z) \) data (colored curves) and the corresponding fittings to Eq. (2) (dashed black curves) are presented. For the hierarchical granular targets consisting of low-strength grains (Figs. 6a, b), gentle deceleration after the impact can be observed in the convex shape of the \( v(z) \) curves. As the grain strength \( Y_{g} \) increases, however, the shape of the \( v(z) \) curves becomes similar to that of rigid glass beads (Figs. 6c–g). The fitting results demonstrate that all the experimental data can be well fitted by the model. This implies that the drag force of hierarchical granular targets can also be expressed by the combination of the inertial drag and the depth-proportional drag, as written in Eq. (1). Meanwhile, variation in the shape of \( v(z) \) curves should be explained by the difference in the two fitting parameter values, \( d_{1} \) and \( k \).

The estimated values of \( d_{1} \) and \( k \) are plotted in Figs. 7a and b, respectively. The corresponding data are also listed in Table 2. Horizontal dashed lines in Fig. 7 indicate the expected values computed from the scaling relations for the rigid grains case (Eqs. (3) and (4)). In the following, we refer to the parameter values expected from the scaling of \( d_{1} \) and \( k \) as \( d_{1s} \) and \( k_{s} \), to avoid confusion. Because \( \mu \) and \( \rho_{s} \) slightly depend on the target materials (Table 1), the expected values of \( d_{1s} \) and \( k_{s} \) distribute around the typical values, \( d_{1s} \approx 5 \text{ mm} \) and \( k_{s} \approx 5 \text{ kg s}^{-2} \). Although the range of \( d_{1} \) spans about one order of magnitude (from 1 to 10 mm), the ratio between experimental results and scaling expectations \( d_{1}/d_{1s} \) seems to be almost constant (see also Fig. 8a). This systematic deviation between the scaling expectation \( d_{1s} \) and the measured \( d_{1} \) probably comes from the difference in the experimental setup, such as the system size. For instance, the size of container used in this study is not very large compared to those used in previous studies (Katsuragi & Durian 2007, 2013). The narrow container might slightly increase the drag force due to the wall effect. Then, the value of \( d_{1} \) becomes small. Moreover, the velocity dependence of \( d_{1} \) probably results from the very shallow penetration, which reduces the actual contact area between the projectile and the target. Since these tendencies can be similarly confirmed for all targets, they presumably result from such boundary effects and are not the material-dependent inherent behaviors. Namely, we consider that \( d_{1} \) value is roughly predictable by the scaling of Eq. (3). Its value is independent of the target grain rigidity. However, the value of \( k \) significantly deviates from the expected values of \( k_{s} \), particularly in the small \( Y_{g} \) regime. The difference exceeds one order of magnitude in the cases of wet and dry grains. It can be noticed that the hierarchical grains whose \( Y_{g} \) values are relatively low possess an extremely large \( k \) value compared with \( k_{s} \). The hierarchical structure of the grains is likely responsible for the large value of \( k \). In fact, similarly large \( k \) values were also confirmed in the impact experiment onto a bulk dust aggregate target (Katsuragi & Blum 2017).
Furthermore, the $k$ value also depends on $v_0$ (Fig. 7b). In the hierarchical granular target cases, $k$ shows an increasing trend as $v_0$ increases. In rigid grain cases, an opposite trend can be confirmed. This means that the effect of $k$ dominates the drag force when $Y_g$ is small and $v_0$ is large. In other words, the drag force depends not only on the material properties, but also on the impact inertia.

From the above observations and analyses, we introduce a dimensionless number $Y_{g}/p_0 v_0^2$ to characterize the degree of grain rigidity. When $Y_{g}/p_0 v_0^2 \gg 1$, the grain strength is much greater than the impact inertia (dynamic pressure). Thus, fracturing of grains cannot be induced in this regime. As a result, impact kinetic energy is dissipated by the friction and gravitational potential energy to splash ejector grains. However, when $Y_{g}/p_0 v_0^2 \ll 1$, the impact inertia can cause grain fracturing and compression. In this regime, impact kinetic energy is mainly dissipated by grain fracturing.

Therefore, the value of $k$ could be related to $Y_{g}/p_0 v_0^2$. In Fig. 8, nondimensionalized parameters, $d_1/d_{ls}$ and $k/k_s$, are plotted as functions of $Y_{g}/p_0 v_0^2$. As seen in Fig. 8a, $d_1/d_{ls}$ is roughly independent of $Y_{g}/p_0 v_0^2$. That is why we can consider $d_1$ to be almost independent of the grain strength. The weak $v_0$
Fig. 6. Instantaneous relations between projectile velocity $v$ and penetration depth $z$. The solid colored curves indicate the experimental results. The dashed black curves are the fitting by Eq. (2). Although the qualitative behavior of $v(z)$ curves depends on the experimental conditions, all the experimental data can be fitted by the model.

Fig. 7. The fitting parameter values: (a) $d_1$ characterizing the inertial drag force and (b) $k$ characterizing the depth-proportional drag force. The colored dashed lines in both plots represent the scaling expectations $d_1s$ and $k_s$ estimated by Eqs. (3) and (4). The deviation of $d_1$ from $d_1s$ is less than one order of magnitude and $d_1$ values tend to approach $d_1s$ in the large $v_0$ limit. However, $k$ becomes much greater than the scaling expectations particularly in soft (porous and fragile) hierarchical targets. The error bars indicating the standard deviation of five experimental runs.

Fig. 8. Normalized parameter values vs. the nondimensionalized grain strength: (a) $d_1/d_{1s}$ vs. $Y_g/\rho_p v_0^2$ and (b) $k/k_s$ vs. $Y_g/\rho_p v_0^2$. The normalized $d_1$ values distribute around $d_1/d_{1s} \approx 0.5$. In the regime of $Y_g/\rho_p v_0^2 \ll 1$, $k/k_s$ can be scaled as $k/k_s = C(Y_g/\rho_p v_0^2)^{-\alpha}$, with $C = 2.4 \pm 0.3$ and $\alpha = 0.32 \pm 0.03$. In the regime of $Y_g/\rho_p v_0^2 \gg 1$, $k/k_s$ is close to unity. The inset of (b) shows the log–log plot of the identical data. The error bars indicate the standard deviation of five experimental runs.
dependence of $d_1/d_3$, probably originates from the variation in the contact area between the projectile and the target. When $n_0$ is large ($Y_g/p_0v_0^2$ is small), the measured $d_1$ value approaches $d_3$ because of the sufficient contact area. On the other hand, $k/k_s$ shows nontrivial dependence on $Y_g/p_0v_0^2$. A clear decreasing trend of $k/k_s$ can be confirmed in the regime of $Y_g/p_0v_0^2 \ll 1$. To quantitatively evaluate this trend, data in this regime are fitted to the power-law form:

$$k/k_s = C \left( \frac{Y_g}{p_0v_0^2} \right)^{-a},$$  

(5)

where $C = 2.4 \pm 0.3$ and $a = 0.32 \pm 0.03$ are obtained by the fitting. The dotted black curve in Fig. 8b shows the result of this fitting. Ideally, impact-induced grain fracturing should be negligible in the regime of $Y_g/p_0v_0^2 \gg 1$ and the value of $k/k_s$ would converge to $C$, whose value is on the order of unity. Indeed, the $k/k_s$ value in $Y_g/p_0v_0^2 \gg 1$ is close to unity. In other words, $k$ recovers the conventional scaling value $k = k_s$ when $Y_g$ is sufficiently greater than $p_0v_0^2$. In summary, by introducing a dimensionless number $Y_g/p_0v_0^2$, two limiting behaviors can be seamlessly connected and systematically analyzed by the dimensionless number over a range of six orders of magnitude.

5. Discussion

The form of the dimensionless number $Y_g/p_0v_0^2$ is identical to the dimensionless number $\pi_3 = Y_g/p_0v_0^2$ defined in the II group scaling for impact cratering (e.g., Melosh 1989; Katsuragi 2016). However, they are not identical in terms of quantity. In the conventional II group scaling, $Y$ denotes the bulk strength of the target material, whereas $Y_g$ used in this study is the strength of constituent grains. In general, bulk strength and grain strength are different quantities. In this study, we find that the strength of constituent grains is useful to characterize the impact drag force. In the II group scaling for impact cratering, $\pi_3$ becomes relevant when the target strength is sufficiently large or when the resultant crater size is small enough. That situation corresponds to the typical strength-dominant cratering regime. However, in this study, $Y_g/p_0v_0^2$ becomes a relevant scaling parameter when $Y_g$ is small or when the impact inertia is large. When $Y_g$ is sufficiently large, we can neglect the effect of grain fracturing and the behavior is dominated by gravity. This tendency is opposite to the conventional crossover between gravity-dominant and strength-dominant regimes in impact-cratering dynamics. Thus, the scaling found in this study is relevant only when the target material has a hierarchical granular structure and the grains’ internal strength is sufficiently small.

Substituting the relation of Eq. (5) into Eq. (4), a simple form can be obtained:

$$k = 8C \mu g \sqrt{p_0v_0^2} D_p^2 \left( \frac{Y_g}{p_0v_0^2} \right)^{-\alpha}.$$  

(6)

It is important to note that this relation is applicable only in the regime of $Y_g/p_0v_0^2 \ll 1$. The scaling of the original form (Eq. (4)) is recovered in $Y_g/p_0v_0^2 \gg 1$. Here, we consider the physical meaning of the scaling of Eq. (6). This simple scaling can be explained by energy conservation, $E_k = Y_g V$, where $V$ is the volume of grains fractured by the impact. By carefully observing the impacted hierarchical granular bed, we realize that a certain amount of porous grains are compressed or fractured in the vicinity of the impact point. By introducing a characteristic length scale of the impact-induced damage zone $D_4$, we simply assume a relation $V \sim D_4^3$. By using the projectile density and diameter, the impact kinetic energy is expressed as $E_k \sim \rho_0 D_p^4 v_0^2$. Taking them into account, energy conservation can be written as $Y_g D_4^3 \sim \rho_0 D_p^4 v_0^2$. This relation is rewritten as

$$\left( \frac{Y_g}{p_0v_0^2} \right)^{1/3} \sim \frac{D_4}{D_p}.$$  

(7)

This power-law form is equivalent to Eq. (5), with $a = 1/3$. Substituting Eq. (7) into Eq. (6), the scaling form of $k$ in the regime of $Y_g/p_0v_0^2 \ll 1$ is expressed by the damaged length scale $D_4$ as follows:

$$k \sim \mu \sqrt{p_0v_0^2} g D_p D_4.$$  

(8)

This relation is dimensionally sound and includes physically relevant quantities. Based on the actual observation of the impacted targets, we consider that $D_4$ is larger than the size of crater produced by the impact. Indeed, wider damage zone was confirmed in the previous impact experiment using a bulk dust aggregate (Güttler et al. 2009). However, Eq. (8) suggests a simple proportionality between $k$ and $D_4$. This relation might imply the unreasonably large $D_4$ at the small $Y_g/p_0v_0^2$ limit. In the above discussion, we simply assumed homogeneous energy distribution in the damaged volume $V \sim D_4^3$. However, the degree of damage must be more or less inhomogeneous and localized. To quantitatively evaluate the validity of Eq. (8), we have to measure the compaction and its localization induced by the impact. In other words, Eq. (8) is the first-order approximation and should be improved based on the characterization of the damage zone. The detailed observation of the damage zone by measuring spatial distribution of the fragmented particles and internal porosity is the most important future problem.

In Katsuragi & Blum (2017), the unexpectedly large $k$ value was also reported in the impact onto a bulk dust aggregate. In that study, the effective strength of the target material was estimated as $k/\pi D_p = 200$ kPa. Corresponding effective strength for the hierarchical granular target used in this study is $k/\pi D_p = 1$ kPa (a typical value of $k = 40$ kg s$^{-2}$ is used). In other words, the effective strength of a bulk dust aggregate is 10$^2$ times greater than that of a hierarchical granular bed. This tendency is qualitatively reasonable because the hierarchical granular matter seems to be weaker than the bulk dust aggregate. According to Blum et al. (2006) and Katsuragi & Blum (2017), the stress required to open a crack in a bulk dust aggregate (tensile strength) can be estimated as 19 kPa. This value is close to the strength of the soft hierarchical grains produced in this study ($Y_g$ value without sintering). The difference in the penetration strength between a hierarchical granular bed and a bulk dust aggregate could result from the effect of the hierarchical grains’ size. Moreover, the actual regolith grains covering planetary bodies have various sizes and strengths. Size and strength distributions might also affect the impact drag force. Such higher-order effects are future issues to be solved in order to properly consider planetary application. The effect of environmental conditions like atmospheric pressure and gravity should also be investigated to discuss practical planetary application.

In this study, thermal effect is completely neglected. This could be justified only in the low-speed regime. When the impact velocity is very large, a high temperature produced by the impact could melt the target material. In such a situation, the scaling we obtained in this study (Eq. (5)) cannot be applicable. When $Y_g$ is
large, $\rho g Y_{\text{g}}^2$ must also be very large to satisfy $Y_g/\rho g Y_{\text{g}}^2 \ll 1$. Thus, there must be a certain upper limit of $Y_g$ above which the completely different physical process dominates the phenomenon. Based on this study, the upper limit of $Y_g$ is at least greater than $10^3$ kPa because the clear fracturing effect can be confirmed in this ($Y_g \leq 10^3$ kPa) regime. In other words, the impact drag force is dominated by the strength $Y_g$ only when $Y_g$ is small. As mentioned above, this is contrastive with the definition of “strength-dominant impact cratering”, in which the strength $Y_g$ should be greater than the gravitational pressure $\rho g D_0$ (see e.g., Melosh 1989 or Katsuragi 2016). Much more systematic studies are required in order to conclude the scaling form and take into account the various effects. The parameter variations are still limited in this study. However, we believe that this study opens a new direction of study of granular impact, which is necessary for planetary science.

Finally, we briefly discuss the possible application potential of this study. For example, if we can record the kinematic information of the penetrator, rober, or any kind of solid object impacting onto a surface of a planetary body, we can estimate the values of $d_1$ and $k$. Then, if we can independently estimate the density and friction coefficient of the regolith layer, the strength of regolith grains can be computed based on the model developed in this study. In general, strength and density might have a certain relation, as seen in Table 1. Once we develop a calibration for that relation, we might be able to estimate various properties of the regolith grains simultaneously. It should be noted that we do not have to perform additional free-fall experiments particularly for this analysis. The only thing we have to do is record the motion of the instrument which touches down on the surface of planetary bodies. To properly derive the physical characteristics of the regolith layer, much more systematic experiments that measure, for example, the damage zone, are required, as already discussed above. However, this study proposes a new methodology for a simple exploration of the fragile (porous) regolith layer by low-speed impact.

6. Conclusions

In this study, we experimentally investigated the drag force exerted on a projectile that impacts onto a bed formed by grains with a hierarchical structure. Sintering was used to create hierarchical granular beds with various grain strengths. By the impact, target grains were broken and compressed if target grains were sufficiently weak. From the fittings of $d(z)$ curves of the projectile motion, we found that the impact drag in the hierarchical granular targets can be explained by the drag force model developed for the impact onto a granular bed consisting of rigid grains. However, the contribution of the depth-proportional drag force $k z$ increased significantly in the hierarchical granular target case. In this study, we experimentally found a novel scaling relation, $k/k_{\text{z}} \sim \left(Y_g/\rho g Y_{\text{g}}^2\right)^{-1/3}$ at $Y_g/\rho g Y_{\text{g}}^2 \ll 1$. This implies that the grain fracturing effect dominates the drag force when $Y_g$ is small compared to $\rho g Y_{\text{g}}^2$. This tendency is in contrast with the conventional $\Pi$ group scaling for impact cratering analysis, in which strength dominates the mechanics when the bulk target strength is relatively large. According to a simple energy conservation assumption, $k$ might relate to the size of the damage zone, $D_0$. In addition, we evaluated the penetration strength $k/\pi D_0$ of the target bed directly from the impact drag measurement. As a result, we found that the hierarchical granular bed has an intermediate penetration strength in between a granular bed consisting of rigid grains and a bulk dust aggregate. To obtain the general impact drag-force model that is applicable to various granular targets, we have to consider the effects of other parameters such as the thermal effect and the grain size in the hierarchical structure.

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