Constraining primordial black holes with relativistic degrees of freedom

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Scalar perturbations in the early Universe create over-dense regions that can collapse into primordial black holes (PBH). This process emits scalar-induced gravitational waves (SIGW) that behaves like an extra radiation component and contributes to the relativistic degrees of freedom ($N_{\text{eff}}$). We show that $N_{\text{eff}}$ limits from cosmic microwave background (CMB) give promising sensitivities on both the abundance of PBHs and the primordial curvature perturbation ($P_{\text{R}}(k)$) at small scales. We show that Planck and ACTPol data can exclude supermassive PBHs with peak mass $M_* \in [3 \times 10^5, 5 \times 10^{10}] M_\odot$ as the major component of dark matter, depending on the shape of the PBHs mass distribution. Future CMB-S4 mission is capable of broadening this limit to a vast PBH mass window of $M_* \in [8 \times 10^{-3}, 5 \times 10^{10}] M_\odot$, covering sub-stellar masses. These limits correspond to the enhanced sensitivity of $P_{\text{R}}$ on scales of $k \in [10^4, 10^{22}]$ Mpc$^{-1}$, which is much smaller than those scales probed by direct perturbation power spectra (CMB and large-scale structure).

Introduction. Large density fluctuation in the early Universe create over-dense regions that can collapse into primordial black holes (PBH). This process emits scalar-induced gravitational waves (SIGW) that behaves like an extra radiation component and contributes to the relativistic degrees of freedom ($N_{\text{eff}}$). We show that $N_{\text{eff}}$ limits from cosmic microwave background (CMB) give promising sensitivities on both the abundance of PBHs and the primordial curvature perturbation ($P_{\text{R}}(k)$) at small scales. We show that Planck and ACTPol data can exclude supermassive PBHs with peak mass $M_* \in [3 \times 10^5, 5 \times 10^{10}] M_\odot$ as the major component of dark matter, depending on the shape of the PBHs mass distribution. Future CMB-S4 mission is capable of broadening this limit to a vast PBH mass window of $M_* \in [8 \times 10^{-3}, 5 \times 10^{10}] M_\odot$, covering sub-stellar masses. These limits correspond to the enhanced sensitivity of $P_{\text{R}}$ on scales of $k \in [10^4, 10^{22}]$ Mpc$^{-1}$, which is much smaller than those scales probed by direct perturbation power spectra (CMB and large-scale structure).

PBH model. To achieve efficient PBH formation, the primordial curvature perturbation $P_{\text{R}}$ needs to be boosted to $10^{-2}$ at small scales ($k \gtrsim 1$ Mpc$^{-1}$) [26, 31, 54–57]. Such a spectrum can be realised in several inflation theories [31, 55, 58–62], here we adopt a typical log-normal $P_{\text{R}}$ parameterisation peaking at a small scale
ing the third equality in (3) we have used

\[ \beta(M) \equiv \rho_*/\rho_{cr}(z) = \frac{2}{\Delta} \int_{\Delta}^{\infty} \frac{1}{\sqrt{2\pi \sigma^2}} e^{-\left(\frac{\delta^2}{2\sigma^2}\right)} \, d\delta, \]

where \( \rho_*/ \) and \( \rho_{cr}(z) \) are the PBH density and the critical density of the Universe at the time of collapse. \( \Delta \approx 0.45 \) is the threshold density contrast for gravitational collapse during radiation dominated era [8, 68, 69]. While deriving the third equality in (3) we have used \( \delta_c > \bar{\sigma} \), which remains valid for all scenarios we explored. \( \bar{\sigma}^2 \) is the coarse-grained variance for density contrast [26, 65, 70]

where the collapsing scale \( k \) is related to its enclosed mass \( M \) via Eq. (2), \( W \) is a window function, for which we adopt a Gaussian form [26, 65], \( W(x) = \exp(-x^2/2) \).

The distribution of PBH abundance in different masses can be calculated via [26, 65, 70, 71]

\[ \Phi = \sum f_{bh} \frac{1}{\sqrt{2\pi \sigma^2}} \exp \left[ -\frac{\ln(M/M_*)^2}{2\sigma^2} \right], \]
After neutrinos decouple, the cosmic radiation energy density \( \rho_t \) is a sum of contributions from CMB photon (\( \gamma \)), neutrino (\( \nu \)) and GW,

\[
\rho_t = \rho_\gamma + \rho_\nu + \rho_{gw}
\]

with

\[
\rho_\gamma = \frac{\pi^2}{15} T_\gamma^4, \quad \rho_{gw} = \frac{7\pi^2}{120} N_{\text{eff}} T_\gamma^4, \quad \rho_\nu = \frac{\pi}{16} T_\nu^4,
\]

here \( T_\gamma = 2.728(1+z) \) K and \( T_\nu = (4/11)^{1/3} T_\gamma \) are temperatures of CMB and neutrino respectively. Since the behavior of GW density mimics that of neutrino, in Eq. (11) we model their total energy density by the effective number of neutrino species,

\[
N_{\text{eff}} \equiv N_{\text{SM}}^{\text{eff}} + \Delta N_{\text{eff}}, \quad (12)
\]

where \( N_{\text{SM}}^{\text{eff}} = 3.046 \) represents the contribution from SM neutrino [42–44], \( \Delta N_{\text{eff}} \) parameterises the GW density. Comparing \( \rho_{gw} = \Omega_{gw} \rho_c (1+z)^4 \) with Eq. (11), one can easily show that

\[
\Delta N_{\text{eff}} = 8.3 \times 10^4 \Omega_{gw}(\bar{\theta}),
\]

where \( \bar{\theta} \) indicates the model parameters. Depending on the choice of parameterisation, \( \bar{\theta} \) can be either the \( \mathcal{P}_3 \) parameters \( (A, \sigma, k_*) \) defined in Eq. (1), or the PBH parameters \( (f_{bh}, \sigma_*, M_*) \) defined in Eq. (6). For a given set of \( \mathcal{P}_\mathcal{R} \) parameters, the value of \( \Omega_{gw} \) is directly determined by integrating Eq.(7). To obtain the relation between \( \Omega_{gw} \) and PBH parameters, we constructed a three dimensional grid in \( \mathcal{P}_\mathcal{R} \) parameter space and calculated the corresponding \( \Omega_{gw} \) and \( (f_{bh}, \sigma_*, M_*) \) for each point in the grid, using the large discrete sample of \( \Omega_{gw}(f_{bh}, \sigma_*, M_*) \) obtained in the process, we then built an interpolation function to calculate \( \Omega_{gw} \).

We combine Planck [78, 79] and ACTPol [80, 81] data and use the Markov Chain Monte Carlo (MCMC) chain in the CosmoMC package [82–85] to constrain cosmological parameters, and obtain \( N_{\text{eff}} < 3.213 \) for 95% C.L.

\(^2\) For future CMB observations, we use the benchmark value of \( \Delta N_{\text{eff}} < 0.027 \) (95% C.L.) from the CMB Stage IV experiment [47, 51–53]. Combining both current and future constraints, we have

\[
\Delta N_{\text{eff}} < \begin{cases} 
0.167 & \text{Planck+ACTPol (PA)} \\
0.027 & \text{CMB Stage IV (S4)} 
\end{cases}
\]

(14)

Using Eq. (13), Eq. (14) is inverted to the upper bounds on SIGW density at 95% C.L.

\[
\Omega_{gw} < \begin{cases} 
2.01 \times 10^{-6} & \text{PA} \\
3.25 \times 10^{-7} & \text{S4} 
\end{cases}
\]

(15)

In the following, we will substitute Eq. (1) into Eq. (7) to calculate \( \Omega_{gw} \) or numerically using the \( \Omega_{gw}(f_{bh}, \sigma_*, M_*) \) interpolation function, and then use the upper bound in Eq. (15) to constrain the model parameters.

**Sensitivity.** For given \( \sigma \) and \( k \), \( \mathcal{P}_R \) is determined by \( k_* \) and \( A \) parameters. Once we set \( k_* = k \exp[(1-n_c)\sigma^2] \), which gives a \( \mathcal{P}_R \) spectra that peaks at \( k \) according to Eq. (4), the upper bound on \( \mathcal{P}_R(k) \) can be obtained by fixing \( A \) to its upper limit derived by iteratively solving Eq. (15). To avoid overlapping with existing large scales constraints [29], we restrict the solution to the range of \( k > 10 \) Mpc\(^{-1} \), which corresponds to \( M_* < 5 \times 10^{10} M_{\odot} \) via Eq. (2).

In the majority of parameter space we explored, our results show that PA constrains \( \mathcal{P}_R \) to be \( \sim \mathcal{O}(10^{-1}) \) over a wide range of \( k \), and the constraint becomes more stringent for a wider spectral width \( \sigma \), which is mainly due to the fact that the peak amplitude of \( \mathcal{P}_R \) is roughly inversely proportional to \( \sigma \). For a sharp spectra with \( \sigma = 0.1 \), our results yields \( \mathcal{P}_R \lesssim 1 \), whereas the limit tightens to \( \mathcal{P}_R \lesssim 0.1 \) for \( \sigma = 2 \). In most cases, the linear part in Eq. (1) can be safely ignored, so that \( \mathcal{P}_R \propto A \) and \( \Omega_{gw} \propto A^2 \), thus our \( \mathcal{P}_R \) upper limit is proportional to the maximally allowed \( \sqrt{\Omega_{gw}} \). Therefore, compared to PA, the CMB-S4 experiment almost uniformly improves

![Comparison of \( \mathcal{P}_R \) constraints. The shaded regions indicate the excluded parameter space. We show limits derived in this work for \( \sigma = 0.5 \) in blue and red regions, corresponding to the current excluded space from PA and the forecasted exclusion from S4 respectively, a wider spectra with larger \( \sigma \) would yield more stringent constraints. Regions in cyan and yellow colors indicate the constraints from CMB and LSS [29], and the non-detection of \( \gamma \)-rays from Ultracompact minihalos (UCMHs) [56]. Regions in magenta, green, and brown colors indicate the exclusion of \( \mathcal{P}_R \) with width of \( \sigma = 0.5 \), from Big-Bang Nucleosynthesis (BBN) [25, 76], European Pulsar Timing Array (EPTA) and advanced LIGO (aLIGO) [25].](image)
its $\mathcal{P}_R$ constraint by 60% on all scales.

Figure 1 compares our PA and S4 results with other leading $\mathcal{P}_R$ constraints taken from Refs. [25, 29, 56]. At $k < 4\,\text{Mpc}^{-1}$, $\mathcal{P}_R$ is well measured by CMB anisotropy and LSS to $\sim 2 \times 10^{-9}$ [29]. Between $k = [4.3 \times 10^7, 10^9] \,\text{Mpc}^{-1}$, the non-detection of gamma rays from Ultra-compact minihalos (UCMHs) [56] constrains $\mathcal{P}_R \lesssim 10^{-6}$. All other limits shown in Fig. 1 are for $\mathcal{P}_R$ with a log-normal width of $\sigma = 0.5$, set by the Big-Bang Nucleosynthesis (BBN) and GW observations, e.g. EPTA (European Pulsar Timing Array) and aLIGO (advanced LIGO) [25]. Our PA data constrains $\mathcal{P}_R \lesssim 0.28$, which is the strongest $\mathcal{P}_R$ limit for $k \in [10^5, 10^{10}] \,\text{Mpc}^{-1}$ up-to-date. Projected limits from S4 further tighten to $\mathcal{P}_R \lesssim 0.11$. Future GW detectors such as Taiji [35], Tian-Qin [86], LISA [36] and SKA [87–89] can probe $k$ scales from $10^5$ to $10^{14} \,\text{Mpc}^{-1}$ [26] and potentially improve our constraints further.

For majority of PBH formation theories [72–74, 90–93], our PBHs follow an extended distribution that can be well described by the log-normal parameterisation in Eq. (6). Using the $\Omega_{\text{GW}}(f_{\text{bh}}, \sigma_{\text{bh}}, M_{\text{bh}})$ interpolation function described in previous section, it can be shown numerically that $\Omega_{\text{GW}}$ increases with $f_{\text{bh}}$, and here we derive our $f_{\text{bh}}$ upper bounds by iteratively solving Eq. (15). In Fig. 2 we show the $f_{\text{bh}}$ limits for PBHs with distribution widths of $\sigma = 1$ and $\sigma = 2$, combined with existing bounds of monochromatic PBH limits (assuming all PBHs having same mass), summarised in Refs. [8, 77]. For constraint from PA with $\sigma = 1$, the blue (filled) region in Fig. 2 shows that it excludes supermassive PBHs with $M_{\*} \in [3 \times 10^5, 5 \times 10^{10}] \,\text{M}_\odot$ as the dominant DM component ($f_{\text{bh}} < 1$), spanning more than 5 orders of magnitude. The limit covers the range set by X-ray binary (light blue) [94] and LSS (yellow) [8]. For $\sigma = 2$, the exclusion window expands to $[33, 5 \times 10^{10}] \,\text{M}_\odot$, which constitutes the widest constraints of PBH up-to-date. The red region and dashed line show the sensitivity of projected CMB-S4 experiment on PBH abundance, which can exclude PBHs in $[8 \times 10^{-5}, 5 \times 10^{10}] \,\text{M}_\odot$ while setting the most stringent PBH constraints in a large mass range ($[7 \times 10^{-3}, 5 \times 10^{10}] \,\text{M}_\odot$). Compared to other leading constraints in the supermassive PBH window around $[3 \times 10^{4}, 5 \times 10^{10}] \,\text{M}_\odot$, the limits from projected S4 is stronger by more than 10 orders of magnitude. Fig. 3 shows the complete constraints for a range of $\sigma$ values. For fixed $f_{\text{bh}}$, we find that $\Omega_{\text{GW}}$ increases with both $\sigma$ and $M_{\*}$, therefore our $f_{\text{bh}}$ limit tightens as we increase either $\sigma$ or $M_{\*}$.

**Summary.** Scalar-perturbation induces PBH formation which emits SIGWs that behave like a relativistic species, the CMB constrained $\Delta N_{\text{eff}}$ can yield stringent limits on the scalar power spectra $\mathcal{P}_R$ at small scales $10 < k < 10^{22} \,\text{Mpc}^{-1}$, much smaller than the direct CMB power spectra measurement. Using a log-normal parameterization for $\mathcal{P}_R$, which naturally arises in e.g Hornedskel gravity theory [26] and serves as a good approxi-
mation for a wide class of perturbation theories, we show that Planck and ACTPol experiments give the currently most stringent $P_\mathcal{R}$ constraints in $k \in [10^8, 10^{22}] \text{ Mpc}^{-1}$. For PBHs with a log-normal width of $\sigma_\epsilon = 1$, Planck and ACTPol exclude supermassive PBHs with peak mass $M_\bullet \in [4 \times 10^5, 5 \times 10^{10}] \text{ M}_\odot$ as the dominant DM component, while future CMB-S4 experiment can improve the constraints by more than 10 orders of magnitudes and potentially exclude $M_\bullet \in [8 \times 10^{-5}, 5 \times 10^{10}] \text{ M}_\odot$ mass window, which constitutes a sensitive test on small-scale primordial perturbations.

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