Theories of Low Energy Nuclear Transmutations

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Employing concrete examples from nuclear physics it is shown that low energy nuclear reactions can and have been induced by all of the four fundamental interactions (i) (stellar) gravitational, (ii) strong, (iii) electromagnetic and (iv) weak. Differences are highlighted through the great diversity in the rates and similarity through the nature of the nuclear reactions initiated by each.

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I. INTRODUCTION

We show below through physical examples that low energy nuclear reactions have been induced by all of the four fundamental interactions: gravitational, strong, electromagnetic and weak.

Gravity: Gravitational interactions are well known to cause nuclear reactions and fusion in a star. Were it not for nuclear activity, a star would be dynamically unstable and undergo gravitational collapse[1, 2]. In fact, standard theory predicts the collapse of a star when the nuclear fuel is exhausted as the star can no longer counteract the inward compression due to gravitation.

Nuclear: A laboratory example of low energy strong interaction fusion is provided by a fast discharge in fine deuterated polymer fibers. In such fibers, deuterons are accelerated to speeds high enough to overcome the barrier due to mutual Coulomb repulsion, giving rise to the production of 2.5 MeV neutrons through low energy reactions such as

\[ d + d \rightarrow n + ^{3}\text{He}. \]

In the same set of experiments[3], also non deuterated fibers exhibit an “anomalous” production of neutrons, at a rate over 6 orders of magnitude larger than that expected through natural contamination of deuterons in a normal material. Such experimental evidence strongly suggests an explanation in terms of weak interaction induced nuclear reactions[4]. This will be discussed in a later Sec. III B.

Electromagnetic: Purely electromagnetically induced fusion has been observed through the Coulomb explosion of large deuterium clusters -stripped of their electrons by lasers- and their subsequent blow up resulting in nuclear fusion[5]. In more recent experiments, other charged nuclei have been exploded and the ensuing nuclear reactions have been observed.

Weak: In the corona of the Sun, magnetic fields emerge from one sunspot to dive into another. The coronal magnetic field accelerates the electrons and protons enough to cause production of neutrons through weak interactions[6]. These neutrons then cause further nuclear transmutations of the available material accounting for the presence of anomalously formed nuclei on the surface of the Sun and in the Solar corona[7–10].

Once in a while, magnetic flux tubes explode leading to spectacular solar flares releasing extremely high energy particles some of which reaching the earth. Often, the resultant electromagnetic fields are large enough to disturb terrestrial satellite communications and secondary muons produced in the upper atmosphere in sufficient numbers to have been detected in the underground CERN L3+C detector[4, 11].

In the following work, we shall analyze in some detail, various mechanisms and phenomena found both in Nature and in the laboratory related to low energy nuclear reactions. We shall make a general classification of these diverse phenomena in order to delineate common themes (such as collectivity, thermal or non-thermal nature of their induction) as well as profound differences (such as the time scale for induction by a particular interaction) between nuclear reactions caused by different interactions under diverse circumstances. We shall also illustrate the interplay between various interactions necessary for the genesis of a given physical process. For the energy production in a star, all four interactions are essential.
II. GRAVITATIONAL LENT

A. From Helmholtz to Bethe

In the standard model of star formation, collectivity plays a central role. It is asserted that undamped density fluctuations cause an astronomically large number of interstellar gas (essentially hydrogen) to clump together, whereupon gravity begins to compress them further thereby increasing the protostar’s temperature.

To appreciate the subtle interplay existing between different interactions in causing nuclear reactions, the leitmotif of our paper, it is useful to recall the paradigm shifts in the view that earlier physicists had to make regarding the main source of a star’s energy production and stability, as recounted by Hans Bethe[2], a leading architect of gravitational low energy nuclear theory (Gravitational LENT).

To estimate the stability of a star, say our Sun, Helmholtz -around the 1850’s- was the first to use the only tool available to him: Newtonian gravity. If one gram of matter falls on the Sun’s surface, it acquires a potential energy \( \Delta E_{pot} \) given by

\[
\frac{\Delta E_{pot}}{gm} = -\frac{GM}{R} = -1.91 \times 10^{15} \text{ erg gm}^{-1}
\]  

(2)

A similar energy must have been set free when the Sun was assembled. Through the virial theorem, he argued, one half of it must have been set free as kinetic energy [which allows us to estimate the temperature] and the other half must have been radiated away. At present, the outward flux of radiation from our Sun is

\[
F_{\text{radiation}} = 1.96 \text{ erg gm sec}^{-1}
\]  

(3)

Hence, it was estimated that if gravitation supplies the energy, then in \( 10^{15} \text{ sec} \approx 30 \text{ million years} \) the Sun would have radiated away all its energy and would have undergone a gravitational collapse. But Darwin, other biologists and geologists argued with Helmholtz (and Kelvin[12], who concurred with Helmholtz) that they needed a much longer life time for the Sun and that his gravitational mechanism for the source of energy production in the Sun must be in error. Of course, they were right. In fact, as we will see, gravitation supplies the necessary force to hold and compress the particles collectively and raise their temperature for positively charged nuclei to overcome the Coulomb barrier but it is the weak force which ignites the nuclear fire by providing neutrons without which no deuterons or heavier nuclei could be formed.

In 1895, radioactivity in the weak sector of the standard model of fundamental interactions would be discovered by Henri Becquerel, Pierre Curie and Marie Curie and through it the age of the Earth and later that of meteorites and other geological objects would be determined. One deduces the age of the Sun from that of the meteorites to be 4.5 billion years in close agreement with the age 4.56 billions deduced from Solar oscillations[13]. Clearly, we need some interaction other than gravitation to fuel the Sun.

Since the Sun has existed for at least 4.5 billion years, we may deduce that the gain in energy per gm to be provided by whatever agency must be at least

\[
\Delta E_{\text{gain}} \gtrsim 3 \times 10^{17} \text{ erg gm}^{-1}
\]  

(4)

If not gravitation, then which of the other three EM, weak or strong can be the source? If for simplicity we assume that initially we only had electrons and protons and we want to fuse the protons into heavier nuclei, then we need the intervention of neutrons. Electromagnetism can not do it by itself because it can not change the charge. Also, the direct strong interaction between two protons to change their charges is well nigh negligible since the temperature of the core is estimated to be about \( T_{\text{core}} \approx 17 \text{ million K} \). This corresponds to a mean thermal energy for the electrons and protons to be a mere 1.46 KeV. Hence, the only alternative left is to invoke the weak interaction. The Fermi theory does provide a possibility of neutron production through the inverse beta decay

\[
e^- + p^+ \rightarrow n + \nu_e
\]  

(5)

This reaction was first proposed by Wick and later independently by Yukawa and Sakata for the observed phenomena of electron capture from the K and L shells from a large proton rich nucleus. But notice that in an electron capture, the electron is bound near the nucleus, the proton is bound within the nucleus and the subsequent neutron remains bound to the nucleus. All that escapes is the unobserved neutrino. Of course, this is not quite exact. As the closest to nucleus electron disappears, a higher shell electron jumps down to replace it, a third to displace the second, etc., thus leading to a cascade of emitted photons. It is precisely through the emitted photons, chiefly in the X-ray region,
FIG. 1: Shown is the Feynman diagram for the reaction $p^+ + p^+ \rightarrow d^+ + e^+ + \nu_e$ as in Eq. (8).

that the process can be directly registered at all. Alvarez\cite{14} provided the first clinching experimental evidence that the $^{67}$Ga nucleus captures one of its orbital electrons and converts to the stable nucleus $^{67}$Zn.

The life time of K capture and hence the transformation of one atom to a different atom can be of the order of seconds to months depending upon the mass difference between the parent and the daughter atom. It is well to remember that even in the vacuum, prior to the intervention of temperature, several orders of magnitude difference in the rates can occur due to differences in phase space.

Instead, for the reaction in Eq. (5), both the electron and the proton have negligible kinetic energy on the nuclear MeV scale. Hence, the mean $Q$ of the reaction is negative: $Q \approx -0.78$ MeV. One may consider the thermally induced process

$$W_{\text{thermal}} + e^- + p^+ \rightarrow n + \nu_e,$$

where the thermal radiation $W_{\text{thermal}}$ supplies kinetic energy to the initial system. But the available thermal energy even near the core of the Sun $\sim K eV$ is miniscule in comparison with the threshold energy $\sim 0.78 MeV$ required for the reaction to proceed. Hence, the process can occur through thermal fluctuations albeit with an exceedingly small probability.

The dramatic difference between the time scales involved between this case, Bethe’s estimate for the core of the Sun is $10^{30}$ years, and the observed K captures, typically a few days, is due to the large exponential threshold barrier in the first case. This will be of crucial interest for us in what follows, where the reaction in Eq. (5) would be invoked for specially designed condensed matter systems in which the electron kinetic energy can be boosted enough or in a relativistically covariant language, the electron mass can get renormalized by external electric fields radiated from currents,

$$W_{\text{EM}} + (N + 1)e^- + p^+ \rightarrow n + Ne^- + \nu_e,$$

so as to overcome the threshold barrier $Q$ value turning positive, as in the electron capture case, with substantial rates for neutron production. In Eq. (7), anticipating effects due to electron collectivity, we have generalized the reaction in Eq. (5) to the case of $N + 1$ initial electrons which end up with $N$ final electrons after the weak destruction of a single electron. As we shall see in Sec. (VI B), the presence of a large number of electrons in dense matter systems can, through the Darwin interaction, change the energy distribution of a single electron. In such cases, the rates for the reaction in Eq. (7) can be quite different from that of Eq. (6) or for electron capture.

B. Two Proton Channel Rates

Given the depressingly low rate due to the threshold barrier for the direct production of neutrons through Eq. (6), von Weizsäcker\cite{15} suggested the exothermic reaction

$$p^+ + p^+ \rightarrow d^+ + e^+ + \nu_e,$$
with a gain of energy $\sim 0.4 \text{ MeV}$ from a transmutation of two protons. For this reaction, we can estimate the energy gain/gm to be

$$\frac{\Delta E_{\text{gain}}}{\text{gm}} \approx 1.2 \times 10^{17} \left(\frac{\text{erg}}{\text{gm}}\right),$$

(9)

a value quite close to that needed in Eq. 4 in order to keep the Sun warm.

The completion of the mechanism described through the above processes requires the reaction rates for this “$p^+p^+$” channel of energy production. It was first computed by Bethe and Critchfield 1 and it involves all three, EM, weak and strong interactions of the standard model of particle physics. At low relative kinetic energy between the two protons, we have a strong Coulomb repulsion inhibiting the entire process Eq. (8). This is a two step process: first we have the essentially “zero” range weak Fermi process $p_1^+ \rightarrow \tilde{n} + e^+ + \nu_e$ followed by the very short range strong interaction process $p_2^- + \tilde{n} \rightarrow d + e^+ + \bar{\nu}_e$,

$$p_1^+ + p_2^+ \rightarrow (\tilde{n} + e^+ + \bar{\nu}_e) + p_2^- \rightarrow d + e^+ + \bar{\nu}_e,$$

(10)

where $\tilde{n}$ is an intermediate “virtual” state neutron, which combines with the second proton to fuse into a deuteron.

With an eye towards later usage for other processes, let us decompose the computation into (i) a repulsive Coulomb interaction process $p_1^+ + p_2^+$ where

$$B \approx 2\pi e^{-2\pi\gamma} \sqrt{\frac{E_g}{E}} e^{-\sqrt{E_g/E}}$$

$$E_g = (2\pi Z_1Z_2\alpha)^2 \mu c^2$$

(13)

Since, the reactions Eq. (11) of interest are exothermic for which the cross sections $\sigma \propto 1/v$ for small $v$, it is convenient to write the effective cross section as

$$\sigma_{\text{effective}}(N_1 + N_2 \rightarrow X; E) = \frac{S(E)}{E} e^{-\sqrt{E_g/E}},$$

(15)

where $S(E)$ is a characteristic of the nuclear reaction under investigation. Bethe-Critchfield 11 obtain for deuteron production through the PP chain Eq. 8

$$S(pp \rightarrow de^+\bar{\nu}_e) = 3.36 \times 10^{-25} \text{MeV barn},$$

(16)

which is exceedingly small in comparison to purely nuclear reactions, i.e., where $X$ does not contain leptons and no weak processes are involved. In fact, the reaction Eq. (8) has never been observed in any earth laboratory. By contrast, for non-leptonic initial and final states, one finds 16

$$S_{\text{nuclear}} \sim (1 \text{KeV barn}) \div (10 \text{ MeV barn}).$$

(17)

Typically (1 KeV barn) results when the transition is radiative, i.e., whenever there is a photon in the final state whereas for purely nuclear transitions we find numbers in the (MeV barn) range.

Let us pause to understand why the reaction Eq. (16) has a more than $10^{22}$ lower probability to occur than a typical strong reaction Eq. (17). Since the former is a weak process with a cross section proportional to $(G_F E)^2$, $G_F$ being
the Fermi coupling constant and $E \sim MeV$, we expect a suppression factor of about $10^{-16}$ with respect to the strong process. The extra suppression by about $10^{-6}$ arises due to the reduced probability $\sim (E/M)^2$, with $M$ the nucleon mass, for a virtual neutron to combine with a proton to form a deuteron.

Of course, Eq. (15) is not the end of the story. It would if it were a beam experiment, that is if the incident energy $E$ were fixed, but there is a Maxwellian distribution which has to be integrated upon. Hence, Bethe and later Bahcall and others inserted it and the rate of nuclear reaction at a given temperature $T$ is proportional to

$$
\left[ \frac{8}{\pi (k_B T)^3 M} \right]^{1/2} \int dE E \sigma_{eff} (E) e^{-E/(k_B T)}. \tag{18}
$$

This integration changes the final temperature or equivalently the mean energy dependence considerably in the rate of the reaction. One further point regarding $S_{\text{nuclear}}$ needs be made. Further variations can and do occur if there are resonances. This is discussed below.

For room temperature non-thermal induction of neutron production such as electron capture in a plasmon polariton set up, there is no cross section only a capture rate. To calculate a cross section from a rate one requires two beams so that one may define a cross section as a rate divided by a flux. In the context of computing rates when the beams are in reality oscillating plasma oscillations, one is perhaps better off simply quoting the rates of processes. In the Fermi theory of point interaction, the electron has to be at the top of the nucleus for the reaction to go. The method suggested\[17] employs the process of radiation induced electron capture by protons or deuterons to produce ultra low momentum neutrons and neutrinos. In the neighborhood of metallic hydride cathodes, protons or deuterons can do such inverse beta decay conversions through the collective plasma modes of the charged particles. Hence, its proper description is through Eq. (7) and not through a two body cross section since electrons and protons or deuterons oscillate back and forth at a frequency $\Omega$ and have a root mean square lateral displacement $\sqrt{<u^2>}$ and when a renormalized invariant mass $m^* > \gamma_o m$ with $\gamma_o > 2.5$ is reached for an initial electron, it can cause the Fermi inverse beta reaction. This is strikingly different from a beam situation. Failure to do so and analyze this reaction using two body cross section at a fixed velocity would lead to quite erroneous results.

C. Production of $\alpha$ Particles via Four Protons

The deuterons produced from the PP reaction can in turn produce $^3He$ and an $\alpha$:

$$
d+d \to ^4He + \gamma \tag{19}
$$

$$
d+d \to n + ^3He \tag{20}
$$

$$
p+d \to ^3He + \gamma \tag{21}
$$

As discussed in the Sec [VII C] barring selection rules, $S$ for reactions Eq. (19) and Eq. (21) should be $\sim KeV$ barn, whereas for Eq. (20) it should be $\sim MeV$ barn. Also, as mentioned in Eq. (1), the neutron in Eq. (20) has an energy of 2.5 $MeV$ which has been measured in exploding wire experiments\[3] as well as in the Coulomb explosion experiments\[5]. Schwinger\[18] considered the radiative reaction Eq. (21) and observed that at low energies, if one includes just the $S$-wave, the dipole radiation would not contribute because of the positive intrinsic parities of the $p$, $d$ and $^3He$ and negative parity of $\gamma$ and hence it should be highly suppressed. The same argument should also apply to the radiative decay Eq. (19). On the other hand, all the intrinsic parities are positive in Eq. (20) and hence this process can proceed via $S$-wave. It is thus encouraging that both in exploding wire and Coulomb explosion experiments, reaction Eq. (20) has been observed and to our knowledge reaction Eq. (19) not. It is suppressed at the level of $10^{-7}$ in the vacuum.

Proceeding on with the PP chain, it has become customary in the astrophysics literature to invoke the Bethe-von Weizsäcker weak process twice and write it as a four initial proton process

$$
p+p+p+p \to ^4He + e^+ + e^+ + \nu_e + \nu_e. \tag{22}
$$

Needless to say that there is no possibility to ever measure this reaction directly on earth.

On the other hand, for a partial verification of the PP chain in the Sun, the detection of neutrinos arising from the fast part of the cycle was undertaken. As discussed in the next section, nuclei between helium and carbon are unstable. One such unstable process which concerns bound and free electron capture from a $^7Be$ nucleus leading to $^7Li$ nucleus and emission of an electron neutrino has been widely investigated\[19]. The electron capture reaction

$$
e^- + ^7Be \to ^7Li + \nu_e, \tag{23}
$$
half-life has been measured in the laboratory\textsuperscript{[20]}
\[
\tau[^7\text{Be}(e^-,\nu_e)^7\text{Li}] = (4.61 \pm 0.001) \times 10^6 \text{ sec.,}
\] (24)
leading to a rate \((2.169 \pm 0.005) \times 10^{-7} \text{ Hz}\). This implies, that after 1 sec., we should find about \(2.17 \times 10^{14}\) \(^7\text{Li}\) atoms and the same number of neutrinos if we had an initial supply of say \(10^{22}\) \(^7\text{Be}\) atoms. Historically, the inverse electron capture reaction
\[
^{37}\text{Cl}(\nu_e,e^-)^{37}\text{Ar}
\] (25)
was employed for the Solar neutrino detection on earth by Davis and others.

D. CNO cycle and beyond

We mention the CNO cycle employed in massive stars simply to direct attention to the role of resonances. Early workers \textsuperscript{[21]} \textsuperscript{[22]} had difficulty in bridging the gap between \(^4\text{He}\) and \(^{12}\text{C}\) in the stars since the elements in this gap are unstable. Given the great abundance of helium, the process of fusing 3 alpha particles was invoked
\[
^{4}\text{He} + ^4\text{He} + ^4\text{He} \rightarrow ^{12}\text{C} + \gamma.
\] (26)
Since it requires the simultaneous presence of three particles in the initial state, the probability should be extremely small except that this process is blessed with a double resonance. Salpeter\textsuperscript{[23]} pointed the first resonance which is the unstable nucleus \(^8\text{Be}\) with approximately the same mass as two alpha particles and Hoyle\textsuperscript{[24]} noted the second i.e., \(^8\text{Be}\) and \(^4\text{He}\) have the same mass as an excited state of \(^{12}\text{C}\). Of course, the reaction Eq.(26) can not be observed in the laboratory, but the resonances can and have been observed.

Bethe first emphasized the great importance of the above process which should be the main reaction producing stable Carbon, active when the temperatures reach around \(1.1 \times 10^8\) Kelvin. Carbon is of course considered crucial for the buildup of elements. Around the same temperature, another capture of an alpha particle produces oxygen
\[
^{12}\text{C} + ^4\text{He} \rightarrow ^{16}\text{O} + \gamma.
\] (27)
We shall not follow the astrophysical consequences any further, except to remind the reader of the intricate interplay between all the four interactions in producing various low energy reactions leading to very different rates.

III. STRONG LEPT

We now turn our attention to laboratory examples where strong interaction fusion reactions, such as
\[
d + d \rightarrow n + ^3\text{He},
\] (28)
have been directly observed through clean signals of threshold 2.5 MeV neutrons. As discussed in the last section, the Coulomb barrier is quite large at low energies, see Eq.\textsuperscript{(15)}, and thus it is well known that in the vacuum this process would have a vanishingly small probability. In the core of the stars, when the temperature becomes sufficiently high to overcome the barrier, the process is supposed to occur. In reality, the results of such experiments do not yet have practical importance. An alternative method using fast electrical discharge through thin deuterated polymer fibers has been very successful in causing fusion in many laboratories. This method is not directly thermal but uses an electrical impulse to accelerate the deuterons enough to overcome the Coulomb barrier. We shall discuss it in the next subsection using an example of a particular experiment.

A. Deuterated wires and neutron production

Let us focus on the exploding wire experiment by Stephanakis et al\textsuperscript{[3]}. Their experimental setup is summarized below:

Fibers used: Polyethylene and Polypropylene
Density of deuterium \(> 10^{19}/cm^3\)
High power discharge \(\geq 10^{12}\) Watts
Typical discharge currents $\sim 1.2$ MegaAmperes
Peak Voltage $\sim 600$ KiloVolts
Duration of the discharges $\sim 50$ nanoseconds
Length of the fibers $\sim 3.5$ cm
Diameter of the fibers 10 to 190 microns
Mean kinetic energy of the electrons: (1 to 10) KeV
Mean electron speed $v/c \approx (0.06$ to $0.2)$

Through the equipartition theorem, it was deduced that when the plasma contained deuterium ions, it had also a temperature equivalent to kinetic energies in the range $(1 \div 10)$ KeV, sufficient to overcome the Coulomb barrier and cause fusion. Reading from the graph of their integrated yield $Y$ of 2.5 MeV neutrons, the yield seems to scale initially with the diameter $D$ according to $Y \propto 1/D^2$ and then it seems to saturate for higher diameters. The authors state, without proof, that the variation in the yield with fiber diameter can be understood on the basis of pressure and energetics. Their integrated yield $Y \sim 10^{10}$ neutrons of 2.5 MeV for thin deuterated fibers of diameter $D \sim 20$ micron. It is indeed a hefty rate giving ample proof that fusion had occurred. But they were puzzled by an anomalously high production of neutrons by natural i.e., non deuterated fibers as discussed below.

### B. High neutron rates from non deuterated exploding wires

While the production of 2.5 MeV neutrons from deuterated fibers appeared to satisfy expectations from the standard strong interaction branch Eq. (28), an apparent paradox was generated when yields from normal fibers were found to be considerably higher than that expected. The isotopic abundance of deuterium in normal matter is about $1.5 \times 10^{-4}$. The fusion yield being proportional to the square of the density of deuterium, the expectations were that natural fiber yield should be about $5 \times 10^{-7}$ lower than the deuterated fiber yield, for the same fiber geometry. But measurements proved otherwise. The normal yields are about 4 to 5 orders of magnitude higher than that expected. It is natural to conclude that some mechanism other than the strong interaction fusion is responsible for this effect.

There are also other natural phenomena where neutrons have been observed in normal matter with virtually nil deuteron contamination. Examples are lightning[25] and thunderstorms[26]. In both cases, an anomalous production of neutrons has been reported. For a normal fiber, if all we have are electrons and protons then the only way to produce neutrons is via weak interactions.

### C. Normal Fibers and Weak Interactions

We shall discuss at some length the dynamics of weak neutron production in a later section. Here we shall just anticipate some results. As discussed earlier, for an electron-proton system, we are not afflicted by the Coulomb barrier, as a matter of fact the process is helped by Coulomb attraction. But to produce a neutron, about 0.78 MeV needs to be put in the system to reach threshold for this reaction. To thermally excite an electron to such an energy is not possible even at the core of most stars. However, under special circumstances there are electromagnetic means e.g., high electric and magnetic fields[4], or “smart” materials[27] which can directly convert elastic or other mechanical energy into EM signals, to supply such an energy.

For a thin fiber, it is useful to consider an initial state of a large number of $(N+1)$ interacting electrons undergoing a weak process with a proton

$$W + (N+1)e^- + p \rightarrow Ne^- + n + \bar{\nu}_e.$$ \hspace{1cm} (29)

We shall see later that the importance of having a large number of “spectator” electrons, even though only one is destroyed at a time, is due to the induction of a coherent Darwin interaction[28] between the electrons which leads to a high collective contribution to the reaction energy thereby enhancing the nuclear activity. It produces a strong flux of neutrons in normal fibers, lightning and in thunderstorms.

### IV. ELECTRIC LENT

In recent experiments, exploding molecular clusters of deuterium atoms have been produced. First, a weak laser pulse hits the cluster internally ionizing the atoms within the cluster and it is then followed by a strong laser pulse photo-ejecting a large number of electrons completely out of molecular cluster. This leaves the cluster with a large positive charge ($Q = Ne$) in a sphere of small radius $R$. Clusters with large $Q$ in a small $R$ explode. Ejected
deuterons from different clusters then collide with sufficient kinetic energy to overcome the Coulomb barrier and lead to observed fusion events. Such experiments have now been repeated in several laboratories around the world and results bear out their theoretical analysis.

Let us estimate what magnitudes for the fields are required for a Coulomb explosion. Let \( V, E \) and \( P \) denote respectively the voltage, electric field and the stress on the system of charges.

\[
\begin{align*}
V &= \frac{Q}{R} = \frac{Ne}{R}, \\
E &= \frac{Q}{R^2} = \frac{Ne}{R^2}, \\
P &= \frac{E^2}{8\pi} = \frac{N^2e^2}{8\pi R^4}. 
\end{align*}
\]

The tensile strength of a material \( P_c \) is defined as the maximum allowed stress before the material disintegrates.

\[
E_c = \sqrt{8\pi P_c},
\]

for \( E < E_c \), material \( \Rightarrow \) stable, for \( E > E_c \), material \( \Rightarrow \) unstable.

Typical explosion fields are of the order of

\[
E_c \sim 10^{11} \text{Volts/m}. \]

We shall see later that similar fields would be required for inducing weak processes through surface plasmon polaritons on metallic hydride surfaces.

We can deduce a hot disintegration temperature \( T_c \) for Coulomb explosion devices, using Eqs. (30-31):

\[
eV_c = k_B T_c \sim \left( \frac{R}{100 \text{Å}} \right) \text{KeV} \quad T_c \sim 10^7 K,
\]

of the order of the temperature near the core of the Sun.

A. Coulomb explosions of deuterium clusters

Through the collision between two deuterons from different clusters, Zweiback et al [5] and Buersgens et al [29], observed the production of neutrons via the reaction Eq. (28):

\[
d + d \rightarrow ^3\text{He}(0.82\text{MeV}) + n(2.45\text{MeV}),
\]

the peak of their signal showing up at the expected characteristic neutron energy of \( (2.45 \pm 0.2) \text{ MeV} \).

The cluster size can be measured through light scattering. When the wave length \( \lambda \) of the incident light in a Raleigh light scattering experiment is large on the scale of the cluster radius \( R \), then the elastic scattering amplitude is determined by the polarizability \( \alpha \).

\[
F_{\omega i} = \left( \frac{\omega}{c} \right)^2 \alpha(\omega) \epsilon^*_f \cdot \epsilon_i,
\]

so the elastic differential cross-section reads

\[
\left( \frac{d\sigma_{el}}{d\Omega} \right)_{i \rightarrow f} = |F_{\omega i}(\omega)|^2 = \left( \frac{\omega}{c} \right)^4 |\alpha(\omega)|^2 |\epsilon^*_f \cdot \epsilon_i|^2
\]

and the integrated elastic cross-section reads

\[
\sigma_{el} = \frac{8\pi}{3} \left( \frac{\omega}{c} \right)^4 |\alpha(\omega)|^2
\]

The total cross-section is given by

\[
\sigma_{tot} = \frac{4\pi c}{\omega} \sum m F_{ii} = \frac{4\pi \omega}{c} \sum m |\alpha(\omega)|
\]
The radius $R$ of the clusters has been determined empirically via light scattering cross sections as a function of the temperature $T$ of the gas jet producing the clusters. For a gas pressure of 55 atmospheres, as the temperature was increased from 85K to 120K, the measured radii of the clusters decreased from about 55 Å to about 12 Å.

The temperature $T$ of the gas jet producing the clusters also determines the absorption cross section for the light. Thus, the cluster radius can be determined by the absorption of light. Such measurements have also been made. In addition, the yield as a function of the radius of the clusters have been measured. The experimental results are consistent with theoretical computations described above.

**B. Coulomb breakup of molecules**

The success achieved with deuteron fusion led to a series of experiments using other molecular clusters[30, 31] through which detailed dynamical mechanisms for the explosions could be understood along with the structure of the molecules themselves. It has become an imaging tool as well, see for instance[32]

**V. ELECTRO-WEAK LENT**

A detailed review of electro-weak interaction induced lent [EW LENT] can be found[4], hence we shall be very brief here. For systems containing only electrons and protons, as amply discussed in the preceding sections, weak interactions between electrons and protons are essential in producing neutrons. Once there is a supply of low energy neutrons, subsequent nuclear transmutations can occur at a high rate. It is crucial for the production of initial neutrons from low energy $e^-p^+$ in a condensed matter environment, that an input energy $W \geq 0.78 \text{ MeV}$ be supplied somehow to overcome the threshold barrier [see Eq.(6)]. To accelerate the electrons thermally to MeV's in a condensed matter system is impracticable and one is left to search for EM means for the needed level of electron acceleration. There are essentially three options for electron acceleration which have already produced good results

1. **Fast Lasers:** Very fast femtosecond lasers can and have been employed successfully to create mono-energetic electron beams from 100 MeV[33–35] up to 1 GeV[36] and table top electron accelerators have been constructed. While such table top accelerators are extremely useful devices for all sorts of studies, they are not quite suitable for inducing electroweak LENT.

2. **Magnetic means:** As mentioned in the Introduction, an excellent example of EW LENT in nature is provided through the magnetic field acceleration of electrons and protons via the betatron mechanism in the Solar corona. We shall discuss it in the next section (VI).

3. **Electric means:** A laboratory example is an electric field acceleration of electrons when piezoelectric rocks are crushed. This would be discussed in Sec(VII) devoted to LENT induced by ‘smart’ materials.

A very promising avenue for EW LENT yet not completely demonstrated in the laboratory is through plasmon polaritons on the surface of metallic hydrides. Under suitable conditions in particular, $|ecE/\Omega > \gamma_0(mc^2)|$, where $E$ is the mean electric field, $\Omega$ is the resonant frequency, $m$ is the electron mass and $\gamma_0 \sim 2.5$, weak production of neutrons can occur. In fact, experimentally the Naples group has observed neutrons generated through plasma in a Mizuno type battery. The detection of neutrons were made through nuclear CR-39 detectors and these results are repeatable[37]. Same as above also holds for piezo electrics. The difference is that the resonant frequency for metallic hydrides is in the infrared whereas acoustic frequencies are in the microwave range. Thus, considerably higher electric fields are needed for LENT to occur on metallic surfaces.

**VI. SOLAR CORONA**

Galileo first saw solar sunspots through his optical telescope and to this day the physical origin of these dark sunspots is unclear. Early in the last century, Hale and others[38, 39] discovered that magnetic flux tubes exit out of one sunspot to then dive into another sunspot. Such an uninterrupted magnetic flux tube has a part of it submerged below the solar surface where the electronic density is high and a part which is out in the solar corona where the density of electrons is low. A schematic picture is shown in FIG. 2. Such a “floating” flux tube is held up by the magnetic buoyancy[40]. There are oppositely directed vortex currents of electrons and protons which loop around the outer magnetic flux tubes.
FIG. 2: Shown schematically is a magnetic flux tube which exits the solar photosphere (shaded region) and enters the solar corona (clear region) through a sunspot on the chromosphere boundary. The magnetic flux tube then exits the solar corona and enters back into the photosphere through a second sunspot on the chromosphere boundary. The walls of magnetic flux tube are a vortex of circulating electric currents.

A. Solar carpet

An intricate pattern of stable steady state magnetic flux tubes form a “magnetic carpet” and they store considerable magnetic energy. This collective magnetic energy $W_{\text{mag}}$ can be estimated to be high enough to cause inverse beta decay similar to that discussed previously

$$W_{\text{magnetic}} + (N + 1)e^- + p \rightarrow Ne^- + n + \nu_e$$  \hspace{1cm} (39)

Let $\delta I$ denote a small change in the current going around the vortex circumference. It would lead to a small change in the magnetic field energy $\delta \mathcal{E}$

$$\delta \mathcal{E} = \Phi \delta I.$$  \hspace{1cm} (40)

If $L$ denotes the length of the vortex circumference of the magnetic flux tube, then the destruction of one electron through the weak reaction Eq.\,(39) implies

$$\delta I = -\frac{ev}{L},$$  \hspace{1cm} (41)

with $v$ denoting the component of the relative velocity between an electron and a proton which is tangential to the circumference. Letting $\Phi = B\Delta S$ and $\delta \mathcal{E} = -W_{\text{magnetic}}$, we have

$$W_{\text{magnetic}} = (ecB) \left[ \frac{\Delta S}{L} \right] \left[ \frac{v}{c} \right].$$  \hspace{1cm} (42)

For a cylindrical flux tube,

$$\frac{\Delta S}{L} = \left[ \frac{\pi R^2}{2\pi R} \right] = \frac{R}{2},$$  \hspace{1cm} (43)

yielding

$$W_{\text{magnetic}} = 15 \text{ GeV} \times \left[ \frac{R}{\text{Kilometer}} \right] \left[ \frac{B}{\text{KiloGauss}} \right] \left[ \frac{v}{c} \right].$$  \hspace{1cm} (44)

Employing the estimates

$$R \sim 100 \text{ Km}; B \sim 1 \text{ K Gauss}; \frac{v}{c} \sim 10^{-2},$$  \hspace{1cm} (45)
that is quite large enough for an appreciable production of neutrons from the protons. These neutrons thereby give rise to further nuclear transmutations into higher mass nuclei via neutron capture and subsequent beta decays.

B. Betatron mechanism

Let us discuss here a non-thermal acceleration process which leads to a generation of quite energetic particles in the solar corona. As described in the last section, a mechanism is required to induce nuclear reactions well outside the solar photosphere.

The central feature of our solar accelerator [6] centers around the Faraday’s law and is closely analogous to a betatron[41]. Conceptually, the betatron is a step up transformer whose secondary coil is a toroidal ring of accelerating charged particles circulating about a time varying magnetic flux tube. The solar magnetic flux tube acts as a step up transformer mechanism in transferring particle kinetic energy outward from the inner photosphere to circulating charged particles located in the corona outside.

Circulating currents located deep in the photosphere can be viewed conceptually as a net current $I_{primary}$ circulating around a primary coil. Circulating currents found high in the corona can be viewed as a net current $I_{secondary}$ circulating around a secondary coil. If $K_{primary}$ and $K_{secondary}$ represent, respectively, the charged particle kinetic energies in the primary and secondary coils, then one finds the step up transformer power equation

$$\dot{K}_{primary} = I_{primary} V_{primary}$$

$$= K_{secondary} = I_{secondary} V_{secondary}$$

(47)

where $V_{primary,secondary}$ represent the voltages across the primary, secondary coils respectively. The total kinetic energy transfer

$$\Delta K_{primary} = \int I_{primary} V_{primary} (dt)$$

$$= \Delta K_{secondary} = \int I_{secondary} V_{secondary} (dt).$$

(48)

The essence of the step up transformer mechanism is that the kinetic energy distributed among a very large number of charged particles in the photosphere can be transferred via the magnetic flux tube to a distributed kinetic energy shared among a distant much smaller number of charged particles located in the corona, i.e. a small accelerating voltage in the primary coil produces a large accelerating voltage in the secondary coil. The resulting transfer of kinetic energy is collective from a large group of charged particles to a smaller group of charged particles. The kinetic energy per charged particle of the dilute gas in the corona may then become much higher than the kinetic energy per charged particle of the more dense fluid in the photosphere. In terms of the connection between temperature and kinetic energy, the temperature of the dilute gas in the corona will be much higher than the temperature of the more dense fluid photosphere.

It is clear that for the betatron, or equivalently the transformer mechanism, to be efficient in sustaining large electrical fields, the coronal electrical conductivity must not be too large. For an estimation of the conductivity, we need to know the density of the electrons in the corona. Fortunately, we have a quite reliable experimental measurement of the electronic density through its effect on the gravitational bending of light in the radio wave band near the Sun. Given its importance, we shall sketch the methodology of this measurement.

Bending of an EM beam near the Sun may be looked upon as a change in the refractive index, from unity for the vacuum. For frequencies $\omega >> \omega_P$, where $\omega_P$ is the plasma frequency, the effective refractive index $\tilde{n}(r, \omega)$ at a position $r$, where the charged particle density is $n(r)$, reads [42 [43]

$$\tilde{n}(r, \omega) = 1 + \frac{2GM_{Sun}}{c^2 |r|} - \frac{2\pi n(r)c^2}{m\omega^2}.$$  

(49)

Successful measurements of the gravitational bending of electromagnetic waves with frequencies in the visible and all the way down to the high end of the radio spectrum are legion. These experiments provide a direct proof that any coronal conductivity disturbance on the expected gravitational bending of electromagnetic waves for frequencies down to 12.5 GigaHertz must be negligible. Deviations from the general relativistic effect have allowed quite accurate measurements of the electronic density in the Solar corona.
The estimates from even lower frequency radio wave probes used for gravitational bending [43] put the coronal conductivity in the megahertz range. For comparison, we note that the typical conductivity of a good metal would be more than ten orders of magnitude higher [44]. Hence, these data lead us to conclude that the Solar corona is close to being an insulator and eons away from being a metal and there are no impediments toward sustaining electrical fields within it [45–47]. Thus, our proposed transformer mechanism and its subsequent predictions for the corona remain intact.

C. Solar flares and Solar explosions

When sufficiently high kinetic energy is reached in the circulating currents around the flux tubes floating in the corona, the flux tubes may explode violently into a solar flare. The loss of magnetic energy during the flux tube explosion is rapidly converted into charged particle kinetic energy. The relativistic high energy products of the explosion yield both nuclear and elementary particle interactions. The extraordinary explosion which occurred on July 4, 2000 was strong enough to have generated particles, for example an excess muon flux, some of which were observed experimentally at the CERN L3 + C detector [11]. It offered a wonderful opportunity to infer that protons with an average energy of 40 GeV were produced in the Solar corona. Independently, the BAKSAN underground muon measurement [48] provided experimental evidence for protons of energy in excess of 500 GeV in the Solar flare of September 29, 1989. We show below some estimates [4].

For a large Solar flare exploding in a time ($\Delta t$), the break up of a magnetic tube would yield, through the Faraday law, a mean acceleration voltage $\bar{V}$ around the walls given by

$$\bar{V} = \frac{\Delta \Phi}{\Delta t} \quad (50)$$

Inserting as before, $\Delta \Phi = B \Delta S$, where $B$ denotes the mean magnetic field before the explosion and $\Delta S$ the inner cross-sectional area, we have for the mean acceleration energy

$$e\bar{V} = ecB \frac{\Delta S}{\Lambda} \quad \text{where} \quad \Lambda = c\Delta t. \quad (51)$$

For a cylindrical geometry, we may again rewrite it in a useful form

$$e\bar{V} \approx 30 \text{ GeV} \times \left[ \frac{B}{\text{KiloGauss}} \right] \left[ \frac{\pi R^2}{\Lambda \times \text{Kilometer}} \right]. \quad (52)$$

For a coronal mass ejecting flux tube exploding in a time $\Delta t = 10^2$ seconds, we use the estimated values to derive

$$B \approx 1 \text{ KiloGauss};$$
$$R \approx 10^4 \text{ Kilometers}$$
$$\Lambda \approx 3 \times 10^7 \text{ Kilometers}$$
$$e\bar{V} \approx 300 \text{ GeV}$$

(53)

Physically, it corresponds to a colliding beam of electrons and protons with a beam energy of 300 GeV. Experimental measurements by L3 and Baksan confirm the very high energy acceleration of electrons and protons and our proposed mechanism is consistent with the data. In [4] we have also made predictions for the flux of the positron from such flares.

Needless to say, that the measurements already made belie the notion that all nuclear reactions in the Sun occur in the core of the Sun. Similar conclusions can be drawn for any star with a high enough magnetic field.

D. Fusors

It is often overlooked that in a D-D or a D-T system [49] -driven by modest static electric fields- inertial electrostatic confinement has been achieved leading to fusion. Such devices known as fusors have existed since the 1960’s. The basic idea is to use a hollow transparent (perforated) spherical cathode enclosed by a concentric spherical anode, and the device filled with deuterium or deuterium and tritium at low pressure. Ionization of the gas occurs via corona discharge. A potential difference of around 80 KV in commercial devices accelerates deuterons radially, and despite collisional losses, substantial number of collisions take place with energies over 15 KeV and fusion takes place with
the release of neutrons. Much lower voltages can be used. In fact, the voltage threshold for D-T fusion (the lowest) is very modest, comparable to that used in many photocopiers, well below the 10 KV of old black and white television sets or furnace ignition transformers, and also well below the typical 15 KV for neon signs or the 25 KV used in old-style colour TV sets. For D-D fusion, the threshold voltage is again quite modest around 20 KV, and corresponds to an energy that can and indeed has been reached by pyroelectric devices as discussed in section VII A.

We stress that this is not a speculative idea, but rather a mature technology, and tabletop devices can be purchased commercially[50] which produce \((10^6 \div 10^{11})\) neutrons per second. Indeed, a quick internet search will readily reveal that fusors are popular science fair projects built by students.

VII. LENT IN SMART MATERIALS

Amongst a large class of “smart” or more appropriately, specifically designed materials with special properties, pyro and piezo electric materials have been demonstrated both theoretically as well as experimentally to lead to low energy nuclear reactions. A pyroelectric crystal develops an electric field due to (adiabatic) changes in its temperature and its opposite, that is an applied electric field causing an adiabatic heating or cooling of the system is called the electrocaloric effect. In a piezo electric crystal, mechanical stress is directly converted to the development of electric fields and vice versa. Under suitable conditions, sufficient electric fields [voltages] have been generated in both cases to lead to neutron production. We shall discuss each of them briefly.

A. Pyroelectric LENT

In [51, 52], it was experimentally shown that through two pyroelectric crystals when heated or cooled produced nuclear \(dd\) fusion evidenced by the observation of the characteristic signal of 2.5 MeV neutrons. The system was used to ionize the gas and accelerate the ions up to 200 KeV. As we have seen in the previous sections, e.g. Eq.(28), this acceleration is much more than sufficient to cause \(dd\) fusion. The measured yields are in good agreement with the calculated yield.

The formal definition of a pyroelectric crystal is that within a single domain, the electric induction \(\mathbf{D}_o\) ≠ 0. When such a pyroelectric crystal is heated or cooled, it gets spontaneously polarized under equilibrium conditions. The often quoted phenomenological expression presumably valid in the linear regime gives the effective electric field \(E_{eff}\) generated inside a pyroelectric crystal due to a temperature change \(\Delta T\)

\[
E_{eff} = \phi(\Delta T),
\]

where \(\phi\) is the pyroelectric coefficient. Pyroelectric crystals are modeled as parallel plate capacitors with the accelerating voltage \(V\) given by

\[
V = \frac{Q}{C}, \quad Q = E_{eff}A, \quad C = \frac{A \epsilon}{4 \pi t},
\]

where \(A\) is the area, \(\epsilon\) is the dielectric constant and \(t\) the thickness of the crystal. Hence, in this simple model, the energy imparted to an electron or an ion of charge \(|e|\) reads

\[
|e|V = \frac{4 \pi e \phi(\Delta T)}{\epsilon}.
\]

If two crystals are used face to face, then the energy above is doubled.

For definiteness, we shall discuss briefly the experiment described in[52], where lithium tantalate \([LiTaO_3]\) crystals were employed since they have a particularly large value for this coefficient,

\[
\phi \approx 190 \frac{\mu C}{m^2 K} \approx 57 \frac{Gauss}{K}, \quad \phi \approx 1.71 \times 10^4 \frac{Volts}{cm K}.
\]

At atmospheric pressure, the change in polarization charge is quickly masked by the accumulation of charges from the air. Hence, it is in the vacuum that we see the surface charge change and the resulting electric field.

For, \(t = 1\ cm\) and \(\epsilon = 46\) for a two crystal set up[52], the increase in energy should be

\[
2|e|V = 933\ KeV,
\]
whereas the maximum electron energy gain measured is only about 200 $K_eV$. For ions, it is even less.

The deuterium fill gas in a vacuum chamber was ionized by the strong electric field between two polarized crystals and the deuterons were accelerated into a deuterated polystyrene target at energies above the threshold for fusion. To obtain the maximum accelerating voltage, two crystals were employed to double the voltage and a catwhisker tip was employed to increase the electric field. The crystals were heated up to 130$^\circ C$ and then cooled to room temperature. Very clear signals for neutrons with energies $(2.5 \pm 0.1) MeV$ were observed by the usual time of flight method.

B. Piezoelectric LENT

Over six decades, considerable experimental evidence has been gathered for high energy particle production during the fracture of certain kinds of crystals [53–57]. Fracture induced nuclear transmutations and the production of neutrons have been clearly observed[58–63]. The production of neutrons appears greatly enhanced if the solids being fractured are piezoelectric materials. Common examples of a piezoelectric are: hair, bone, quartz.

In a recent paper[27], we have theoretically analyzed the problem and shown the manner in which the mechanical pressure in a piezoelectric stressed solid about to fracture can organize the energy so that neutrons can be produced. Since some of the arguments may not be familiar to many readers, we shall repeat below some of the relevant theory.

The energy per unit volume of a piezoelectric material may be written as

$$du = Tds + \sigma_{ij}dw_{ij} - P \cdot dE,$$  \hspace{1cm} (59)

where $s$ is the entropy per unit volume; $\sigma_{ij}$ and $w_{ij}$ are the stress and strain tensors, $P$ and $E$ are the polarization and electric field vectors. The adiabatic piezoelectric tensor is defined as[64]

$$\beta_{i,jk} = \left( \frac{\partial P_i}{\partial w_{jk}} \right)_{s,E} = - \left( \frac{\partial \sigma_{jk}}{\partial E_i} \right)_{s,w}$$ \hspace{1cm} (60)

The effective mechanical electric field interaction energy density, up to quadratic order, reads

$$\beta_{i,jk} = - \left( \frac{\partial^2 u}{\partial E_i \partial w_{jk}} \right) = - \left( \frac{\partial^2 u}{\partial w_{jk} \partial E_i} \right);$$ \hspace{1cm} (61)

$$u = - \beta_{i,jk} E_i w_{jk},$$ \hspace{1cm} (62)

leading to the piezoelectric Hamiltonain

$$\mathcal{H}_{\text{piezo}} = - \int (d^3x) \beta_{i,jk} E_i w_{jk}.$$ \hspace{1cm} (63)

Eq.\,(63) provides a direct link between a large strain (say a fracture caused by stress) causing a large electric field. If one uses this interaction twice, then to second order in $\beta$, the EM photon propagator would receive a contribution from the mechanical acoustic phonon propagator in the following way.

Let $\chi_{ij}$ and $\tilde{\chi}_{ij}$ be the adiabatic electric susceptibilities at constant strain and at constant stress respectively

$$\chi_{ij} = \left( \frac{\partial P_i}{\partial E_j} \right)_{s,w}, \hspace{1cm} \tilde{\chi}_{ij} = \left( \frac{\partial P_i}{\partial E_j} \right)_{s,\sigma}.$$ \hspace{1cm} (64)

These two are related through the elastic response tensor

$$D_{ijkl} = \left( \frac{\partial w_{ij}}{\partial \sigma_{kl}} \right)_{s,E},$$ \hspace{1cm} (65)

via the thermodynamic identity

$$\tilde{\chi}_{ij} = \chi_{ij} + \beta_{i,lm} D_{lm,nq} \beta_{j,nq}.$$ \hspace{1cm} (66)

The above static equations are easily generalized to dynamical frequency dependent susceptibilities $\chi_{ij}(\omega)$ and $\tilde{\chi}_{ij}(\omega)$ and the phonon propagator $D_{lm,nq}(\omega)$. Phonon modes described by the phonon propagator affect the dynamical susceptibilities through the relations

$$D = E + 4\pi P$$ \hspace{1cm} (67)

$$\epsilon_{ij}(\omega) = \delta_{ij} + 4\pi \tilde{\chi}_{ij}(\omega)$$ \hspace{1cm} (68)

$$\tilde{\chi}_{ij}(\omega) = \chi_{ij}(\omega) + \beta_{i,lm} D_{lm,nq}(\omega) \beta_{j,nq}.$$ \hspace{1cm} (69)
Clearly then, the electric susceptibility of a piezoelectric material would be affected by the frequencies associated with the mechanical acoustic frequencies occurring in fractures of such materials. The velocity of sound \( v_s \) compared to the velocity of light \( c \) obeys \( v_s/c \approx 10^{-5} \) and hence for similar sized cavities

\[
\left( \frac{\omega_{\text{phonon}}}{\omega_{\text{photon}}} \right) \approx 10^{-5} \text{ for similar sized cavities.} \tag{70}
\]

These phonon modes will then appear in \( \epsilon(\omega) \).

The nuclear reactions are supposed to proceed through the weak capture of high electric field accelerated electrons by protons as described in Section (V). We need to calculate the mean energy of the electrons in condensed matter when accelerated by an electric field

\[
\frac{dp}{dt} = eE, \tag{71}
\]

and the electron energy is estimated to be

\[
W = \sqrt{(mc^2)^2 + \langle |p|^2 \rangle e^2}, \tag{72}
\]

If \( P_E(\omega)(d\omega) \) denotes the mean squared electric field strength in a band width \((d\omega)\), then Eqs. (71,72) imply

\[
E^2 = \int_0^\infty d\omega P_E(\omega) \tag{73}
\]

\[
\langle |p|^2 \rangle = e^2 \int_0^\infty d\omega \omega^2 P_E(\omega). \tag{74}
\]

If \( \Omega \) is a dominant frequency, then the above equation may be written as

\[
\langle |p|^2 \rangle = \frac{e^2 E^2}{\Omega^2}. \tag{75}
\]

Let \( m^* \) be the (electrically) renormalized mass. Then, in terms of the vacuum electron mass \( m \), we may write

\[
m^* = \gamma m \text{ where } \gamma = \sqrt{1 + \left( \frac{eE}{mc\Omega} \right)^2} \tag{76}
\]

For electron capture to occur, \( \gamma \geq \gamma_0 \approx 2.5 \). For a rough estimate of the electric field, we may assume that most of the stress at a fracture is electric, in which case

\[
\sigma_{\text{fracture}} \approx \frac{E^2}{8\pi} \Rightarrow E \approx 10^5 \text{ Gauss}, \tag{77}
\]

where we have used Griffith’s rule that \( \sigma_{\text{fracture}} \approx 10^{-3} \sigma_{\text{bond}} \), where \( \sigma_{\text{bond}} \) is the stress necessary to break all the chemical bonds. For a relevant frequency in the microwave range \( \Omega \approx 6 \times 10^{10} / \text{sec} \) we have \( \gamma \approx 30 \). We can convert this into the neutron production rate through the Fermi interaction

\[
\Gamma[\bar{e} + p \rightarrow n + \nu_e] \approx \left( \frac{G_F m^2}{\hbar c} \right)^2 \left( \frac{mc^2}{\hbar} \right) \gamma^2 \tag{78}
\]

\[
\sim 0.6 \text{ Hz for } \gamma = 30. \tag{79}
\]

The transition rate per unit time per unit area of micro-crack surface may be written as

\[
\nu_2 = n_2 \Gamma[\bar{e} + p \rightarrow n + \nu_e], \tag{80}
\]

where \( n_2 \) is the number of protons per unit area in the first few layers of the quartz granite. Typical values are

\[
n_2 \approx \frac{10^{14}}{\text{cm}^2} \Rightarrow \nu_2 \approx \frac{10^{15} \text{ Hz}}{\text{cm}^2}, \tag{81}
\]

a neutron production rate sufficiently large to be measured and to cause even other nuclear transformations. Incidentally, in the piezoelectric case, the neutrons are not ultra cold.
C. Novel uses for the Tokamak

Traditional “hot fusion” programs have been able to generate neutrons at total rate of about $10^{17}$/sec. in the $(dd)$ and $(dt)$ modes, unfortunately lasting only for about 2 seconds [65] [66]. Thus, great improvements are still required for a steady state running of the Tokamaks along with the curtailment of the generated radioactivity. As emphasized by Byrne [68], fusion reactors would require a “biological shield”, as their outermost shell made of some heavy element such as lead, whose purpose is to remove the surviving 14 $MeV$ neutrons and to attenuate the $\gamma$ ray flux. But, this would generate a substantial amount of undesirable radioactivity [67]. In fact, Byrne concludes that “It is not immediately obvious that the problems of radioactive waste disposal, which have plagued fission power programmes across the world, will be entirely absent in the thermonuclear power plants of the future.”

In view of the formidable scientific and technical problems with hot fusion programs yet to be resolved, it is pertinent to ask whether Tokomaks, machines constructed at great public cost, may be put to a different but important practical use. Given the extremely large electromagnetic fields created with great ingenuity within these devices, we propose to use it as a Proof of Concept machine for electro-weak LENT by running electrons and protons rather than just deuterons and tritons as has been done so far. It may be difficult at the beginning to adapt a Tokomak for this purpose but our proposal seems feasible.

VIII. CONCLUSIONS AND FUTURE PROSPECTS

The leitmotif of this paper has been to provide direct evidence that low energy nuclear transmutations [LENT] can and have been induced by gravitational, weak, EM and strong interactions both in Nature as well as in the laboratory. Also, that quite often three or four of the fundamental interactions are needed to lead to the final LENT. Due to this interplay of different interactions each with its own widely different coupling strength, one finds a great diversity in the rates of the reactions.

Hence, we may conclude that the reality of LENT is no longer a matter of debate because it is soundly based on the Standard Model. The focus must now be on the employment of technology to realize electroweak LENT and thereby provide sorely needed practical applications of the Standard Model of particle physics. Extraordinary advances already made in realizing up to 1 GeV table top electron accelerators and high gain factors in generating electric fields in surface plasmon polariton resonances on nano particles bode well for the future of electroweak LENT.

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