Non–Baryonic Dark Matter

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The best particle candidates for non–baryonic cold dark matter are reviewed, namely, neutralino, axion, axino and Majoron. These particles are considered in the context of cosmological models with the restrictions given by the observed mass spectrum of large scale structures, data on clusters of galaxies, age of the Universe etc.

1. Introduction (Cosmological environment)

Presence of dark matter (DM) in the Universe is reliably established. Rotation curves in many galaxies provide evidence for large halos filled by nonluminous matter. The galaxy velocity distribution in clusters also show the presence of DM in intercluster space. IRAS and POTENT demonstrate the presence of DM on the largest scale in the Universe.

The matter density in the Universe $\rho$ is usually parametrized in terms of $\Omega = \rho / \rho_c$, where $\rho_c \approx 1.88 \cdot 10^{-29} h^2 \text{ g/cm}^3$ is the critical density and $h$ is the dimensionless Hubble constant defined as $h = H_0 / (100 \text{ km/s/Mpc})$. Different measurements suggest generally $0.4 \leq h \leq 1$. The recent measurements of extragalactic Cepheids in Virgo and Coma clusters narrowed this interval to $0.6 \leq h \leq 0.9$. However, one should be cautious about the accuracy of this interval due to uncertainties involved in these difficult measurements.

Dark Matter can be subdivided in baryonic DM, hot DM (HDM) and cold DM (CDM).

The density of baryonic matter found from nucleosynthesis is given as $\Omega_b h^2 = 0.025 \pm 0.005$.

Hot and cold DM are distinguished by velocity of particles at the moment when horizon crosses the galactic scale. If particles are relativistic they are called HDM particles, if not – CDM. The natural candidate for HDM is the heaviest neutrino, most naturally $\tau$-neutrino. Many new particles were suggested as CDM candidates.

The structure formation in Universe put strong restrictions to the properties of DM in Universe. Universe with HDM plus baryonic DM has a wrong prediction for the spectrum of fluctuations as compared with measurements of COBE, IRAS and CfA. CDM plus baryonic matter can explain the spectrum of fluctuations if total density $\Omega_0 \approx 0.3$.

There is one more form of energy density in the Universe, namely the vacuum energy described by the cosmological constant $\Lambda$. The corresponding energy density is given by $\Omega_{\Lambda} = \Lambda / (3 H_0^2)$. Quasar lensing and the COBE results restrict the vacuum energy density: in terms of $\Omega_{\Lambda}$ it is less than 0.7.

Contribution of galactic halos to the total density is estimated as $\Omega_a \sim 0.03 - 0.1$ and clusters give $\Omega_{\Lambda} \approx 0.3$. Inspired mostly by theoretical motivation (horizon problem, flatness problem and the beauty of the inflationary scenarios) $\Omega_0 = 1$ is usually assumed. This value is supported by IRAS data and POTENT analysis. No observational data significantly contradict this value.

There are several cosmological models based on the four types of DM described above (baryonic DM, HDM, CDM and vacuum energy). These models predict different spectra of fluctuations to be compared with data of COBE, IRAS, CfA etc. They also produce different effects for cluster-
cluster correlations, velocity dispersion etc. The simplest and most attractive model for a correct description of all these phenomena is the so-called mixed model or cold-hot dark matter model (CHDM). This model is characterized by following parameters:

\[ \Omega_\Lambda = 0, \Omega_0 = \Omega_\nu + \Omega_{CDM} + \Omega_{HDM} = 1, \]
\[ H_0 \approx 50 \text{ km s}^{-1} \text{Mpc}^{-1} (h \approx 0.5), \]
\[ \Omega_{CDM} : \Omega_{HDM} : \Omega_\nu \approx 0.75 : 0.20 : 0.05, \] (1)

where \( \Omega_{HDM} \approx 0.2 \) is obtained in ref. [2] from damped Ly\( \alpha \) data. Thus in the CHDM model the central value for the CDM density is given by

\[ \Omega_{CDM} h^2 = 0.19 \] (2)

with uncertainties within 0.1.

The best candidate for the HDM particle is \( \tau \)-neutrino. In the CHDM model with \( \Omega_\nu = 0.2 \) mass of \( \tau \) neutrino is \( m_\nu \approx 4.7 \text{ eV} \). This component will not be discussed further.

The most plausible candidate for the CDM particle is probably the neutralino (\( \chi \)): it is massive, stable (when the neutralino is the lightest supersymmetric particle and if R-parity is conserved) and the \( \chi\chi \)-annihilation cross-section results in \( \Omega_{\chi} h^2 \sim 0.2 \) in large areas of the neutralino parameter space.

In the light of recent measurements of the Hubble constant the CHDM model faces the age problem. The lower limit on the age of Universe \( t_9 > 13 \text{ Gyr} \) (age of globular clusters) imposes the upper limit on the Hubble constant in the CHDM model \( H_0 < 50 \text{ km s}^{-1} \text{Mpc}^{-1} \). This value is in slight contradiction with the recent observations of extragalactic Cepheids, which can be summarized as \( H_0 > 60 \text{ km s}^{-1} \text{Mpc}^{-1} \). However, it is too early to speak about a serious conflict taking into account the many uncertainties and the physical possibilities (e.g. the Universe can be locally overdense - see the discussion in ref. [3]).

The age problem, if to take it seriously, can be solved with help of another successful cosmological model \( \Lambda \text{CDM} \). This model assumes that \( \Omega_0 = 1 \) is provided by the vacuum energy (cosmological constant \( \Lambda \)) and CDM. From the limit \( \Omega_\Lambda < 0.7 \) and the age of Universe one obtains \( \Omega_{CDM} \geq 0.3 \) and \( h < 0.7 \). Thus this model also predicts \( \Omega_{CDM} h^2 \approx 0.15 \) with uncertainties 0.1. Finally, we shall mention that the CDM with \( \Omega_0 = \Omega_{CDM} = 0.3 \) and \( h = 0.8 \), which fits the observational data, also gives \( \Omega h^2 \approx 0.2 \). Therefore \( \Omega h^2 \approx 0.2 \) can be considered as the value common for most models.

In this paper we shall analyze several candidates for CDM, best motivated from the point of view of elementary particle physics. The motivations are briefly described below.

**Neutralino** is a natural lightest supersymmetric particle (LSP) in SUSY. It is stable if R-parity is conserved. \( \Omega_\chi \sim \Omega_{CDM} \) is naturally provided by annihilation cross-section in large areas of neutralino parameter space.

**Axion** gives the best known solution for strong CP-violation. \( \Omega_a \sim \Omega_{CDM} \) for natural values of parameters.

**Axino** is a supersymmetric partner of axion. It can be LSP.

**Majoron** is a Goldstone particle in spontaneously broken global \( U(1)_{B-L} \) or \( U(1)_L \). KeV mass can be naturally produced by gravitational interaction.

Apart from cosmological acceptance of DM particles, there can be observational confirmation of their existence. The DM particles can be searched for in the direct and indirect experiments. The direct search implies the interaction of DM particles occurring inside appropriate detectors. Indirect search is based on detection of the secondary particles produced by DM particles in our Galaxy or outside. As examples we can mention production of antiprotons and positrons in our Galaxy and high energy gamma and neutrino radiation due to annihilation of DM particles or due to their decays.

### 2. Axion

The axion is generically a light pseudoscalar particle which gives natural and beautiful solution to the CP violation in the strong interaction [4] (for a review and references see[3]). Spontaneous breaking of the PQ-symmetry due to VEV of the scalar field \( < \phi > = f_{PQ} \) results in the production of massless Goldstone boson. Though \( f_{PQ} \) is a free parameter, in practical applications...
it is assumed to be large, \( f_{PQ} \sim 10^{10} - 10^{12} \text{ GeV} \) and therefore the PQ-phase transition occurs in very early Universe. At low temperature \( T \sim \Lambda_{QCD} \sim 0.1 \text{ GeV} \) the chiral anomaly of QCD induces the mass of the Goldstone boson \( m_a \sim \Lambda_{QCD}^2 / f_{PQ} \). This massive Goldstone particle is the axion. The interaction of axion is basically determined by the Yukawa interactions of field(s) \( \phi \) with fermions. Triangular anomaly, which provides the axion mass, results in the coupling of the axion with two photons. Thus, the basic for cosmology and astrophysics axion interactions are those with nucleons, electrons and photons.

Numerically, axion mass is given by

\[
m_a = 1.9 \cdot 10^{-3} (N/3) (10^{10} \text{ GeV} / f_{PQ}) \text{ eV},
\]

where \( N \) is a color anomaly (number of quark doublets).

All coupling constants of the axion are inversely proportional to \( f_{PQ} \) and thus are determined by the axion mass. Therefore, the upper limits on emission of axions by stars result in upper limits for the axion mass. Of course, axion fluxes cannot be detected directly, but they produce additional cooling which is limited by some observations (e.g., age of a star, duration of neutrino pulse in case of SN etc). In Table 1 we cite the upper limits on axion mass from ref. [5], compared with revised limits, given recently by Raffelt [6].

Table 1
Astrophysical upper limits on axion mass

|            | 1990   | 1995   |
|------------|--------|--------|
| sun        | 1 eV   | 1 eV   |
| red giants | 1 - 10^{-2} eV | very uncertain |
| hor.-branch stars | not considered | 0.4 eV |
| SN 1987A  | 1 - 10^{-3} eV | 1 - 10^{-2} eV |

As one can see from the Table the strong upper limit, given in 1990 from red giants, is replaced by the weaker limit due to the horizontal-branch stars. The upper limit from SN 1987A was reconsidered taking into account the nucleon spin fluctuation in \( N + N \rightarrow N + N + a \) axion emission.

There are three known mechanisms of cosmological production of axions. They are (i) thermal production, (i) misalignment production and (iii) radiation from axionic strings.

The relic density of thermally produced axions is about the same as for light neutrinos and thus for the mass of axion \( m_a \sim 10^{-2} \text{ eV} \) this component is not important as DM.

The misalignment production is clearly explained in ref. [5].

At very low temperature \( T \ll \Lambda_{QCD} \) the massive axion provides the minimum of the potential at \( \theta = 0 \), which corresponds to conservation of CP. At very high temperatures \( T \gg \Lambda_{QCD} \) the axion is massless and the potential does not depend on \( \theta \). At these temperatures there is no reason for \( \theta \) to be zero: its values are different in various casually disconnected regions of the Universe. When \( T \rightarrow \Lambda_{QCD} \) the system tends to go to potential minimum (at \( \theta = 0 \)) and as a result oscillates around this position. The energy of these coherent oscillations is the axion energy density in the Universe. From cosmological point of view axions in this regime are equivalent to CDM. The energy density of this component is approximately

\[
\Omega_a h^2 \approx 2 \cdot (m_a / 10^{-5} \text{ eV})^{-1.18}.
\]

Uncertainties of the calculations can be estimated as \( 10^{\pm 0.5} \).

Axions can be also produced by radiation of axionic strings [6, 7]. Axionic string is a one-dimension vacuum defect \( \phi_{PQ} > 0 \), i.e. a line of old vacuum embedded into the new one. The string network includes the long strings and closed loops which radiate axions due to oscillation. There were many uncertainties in the axion radiation by axionic strings (see ref. [5] for a review). Recently more detailed and accurate calculations were performed by Battye and Schellard [8]. They obtained for the density of axions

\[
\Omega_a h^2 \approx A (m_a / 10^{-5} \text{ eV})^{-1.18}
\]

with \( A \) limited between 2.7 and 15.2 and with uncertainties of the order \( 10^{\pm 0.6} \). The overpro...
duction condition $\Omega_\alpha h^2 > 1$ imposes lower limit on axion mass $m_\alpha > 2.3 \cdot 10^{-5}$ eV.

Fig. 1 shows the density of axions $\Omega_\alpha h^2$ as a function of the axion mass $m_\alpha$. The upper limits on axion mass from Table I are shown above the upper absciss (limits of 1990) and below lower absciss (limits of 1995). The overproduction region $\Omega_\alpha h^2 > 1$ and the regions excluded by astrophysical observations are shown as the dotted areas.

The axion window of 1995 (shown as undotted region) became wider and moved to the right as compared with window 1990. The horizontal strip shows $\Omega_{CDM} = 0.2 \pm 0.1$ as it was discussed in Introduction. One can see from Fig. 1 that string and misalignment mechanisms provide the axion density as required by cosmological CDM model, if axion mass is limited between $7 \cdot 10^{-5}$ eV and $7 \cdot 10^{-4}$ eV. However, in the light of uncertainties, mostly in the calculations of axion production, one can expect that this ”best calculated” window is between $3 \cdot 10^{-5}$ and $10^{-3}$ eV. This region is partly overlapped with a possible direct search for the axion in nearest-future experiments (see Fig. 1 and refs. [1]).

3. Axino

In supersymmetric theory the PQ-solution for strong CP-violation should be generalized. Within this theory the PQ symmetry breaking results in the production of the Goldstone chiral supermultiplet which contains two scalar fields and their fermionic partner – axino ($\tilde{a}$). The scalar fields enter the supermultiplet in the combination $(f_{PQ} + s) \exp(a/f_{PQ})$, where $s$ is a scalar field, saxino, which describes the oscillations of the initial field $\phi$ around its VEV value $< \phi > = f_{PQ}$, and $a$ is the axion field. This phase transition in the Universe occurs at temperature $T \sim f_{PQ}$. As we saw in the previous section the axion is massless at this temperature and since supersymmetry is not broken yet, the axino and saxino are massless, too. The axion acquires the mass in the usual way due to chiral anomaly at $T \sim \Lambda_{QCD}$, while saxino and axino obtain the masses due to global supersymmetry breaking.

The saxino is not of great interest for cosmology: it is heavy and it decays fast (mostly into two gluons). The axino can be the lightest supersymmetric particle and thus another candidate for DM.

How heavy the axino can be? The mass of axino has a very model dependent value. In the phenomenological approach, using the global supersymmetry breaking parameter $M_{SUSY}$ one typically obtains (e.g. [11, 12])

$$m_\alpha \sim M_{SUSY}^2 / f_{PQ}$$

(6)

For example, if global SUSY breaking occurs due to VEV of auxiliary field of the goldstino supermultiplet $< F > = F_g$, then the axino mass appears due to interaction term $(g/f_{PQ})\tilde{a}\tilde{a}F$ (F has a dimension $M^2$), and using $< F > = F_g = M_{SUSY}^2$ one arrives at the value (1).

The situation is different in supergravity. In ref. [3] the general analysis of the axino mass is given in the framework of local supersymmetry. It was found that generically the mass of axino in these theories is $m_\alpha \sim m_{3/2} \sim 100$ GeV. Even in case when axino mass is small at tree level, the radiative corrections raise this mass to the value $\sim m_{3/2}$. This result holds for the most general form of superpotential. The global SUSY result, $m_\alpha \sim m_{3/2}^2 / f_{PQ}$, can be reproduced in the local SUSY only if one of the superpotential coupling constants is very small, $\lambda < 10^{-4}$, which implies fine-tuning. Thus, the axino is too heavy to be a CDM particle.

The only exceptional case was found by Goto and Yamaguchi [14]. They demonstrated that in case of no-scale superpotential the axino mass vanishes and the radiative corrections in some specific models can result in the axino mass $10 - 100$ keV, cosmologically interesting. This beautiful case gives essentially the main foundation for axino as CDM particle.

The cosmological production of axinos can occur through thermal production or due to decays of the neutralinos $[15, 16]$. The axion chiral supermultiplet contains two particles which can be CDM particles, namely axion and axino. In this section we are interested in the case when axino gives the dominant contribution. In particular this can take place in the range $2 \cdot 10^9$ GeV $< f_{PQ} < 2.7 \cdot 10^{10}$ GeV where axions are cosmologically unimportant.
Since axino interacts with matter very weakly, the decoupling temperature for the thermal production is very high \[ T_d \approx 10^9 \text{GeV} \left( f_{PQ}/10^{11} \text{GeV} \right) \].

Therefore, axinos are produced thermally and thermally at the reheating phase after inflation. The relic concentration of axinos can be easily evaluated for the reheating temperature \( T_R \) as

\[
\Omega_{\tilde{a}} h^2 \approx 0.6 \frac{m_{\tilde{a}}}{100 \text{ keV}} \left( \frac{3 \cdot 10^{10} \text{ GeV}}{f_{PQ}} \right)^2 \frac{T_R}{10^9 \text{ GeV}} \]

Reheating temperature \( T_R \leq 10^9 \text{ GeV} \) gives no problem with the gravitino production. The relic density (8) provides \( \Omega_{\text{CDM}} h^2 \approx 0.2 \) for a reasonable set of parameters \( m_{\tilde{a}}, f_{PQ} \) and \( T_R \). One can easily incorporate in these calculations the additional entropy production if it occurs at EW scale (17).

If the axino is LSP and the neutralino is the second lightest supersymmetric particle, the axinos can also be produced by neutralino decays (15, 16, 17). According to estimates of ref. (17) the axinos are produced due to \( \chi \to \tilde{a} + \gamma \) decays at the epoch with red-shift \( z_{\text{dec}} \approx 10^8 \). Axinos are produced in these decays as ultrarelativistic particles and the free-streaming prevents the growth of fluctuations on the horizon scale and less. At red-shift \( z_{nr} \approx 10^4 \) axinos become non-relativistic due to adiabatic expansion (red shift). From this moment on the axinos behave as the usual CDM and the fluctuations on the scales \( \lambda \geq (1 + z_{nr}) c t_{nr} \) (which correspond to a mass larger than \( 10^{15} M_\odot \)) grow as in the case of standard CDM. For smaller scales the fluctuations, as was explained above, grow less than in CDM model. Therefore, as was observed in ref. (17), the
axinos produced by neutralino decay behave like HDM. It means that axinos can provide generically both components, CDM and HDM, needed for description of observed spectrum of fluctuations.

Unfortunately stable axino is unobservable. In case of very weak R-parity violation, decay of axinos can produce a diffuse X-ray radiation, with practically no signature of the axino.

4. Majoron

The Majoron is a Goldstone particle associated with spontaneously broken global $U(1)_{B-L}$ or $U(1)_Y$ symmetry. The symmetry breaking occurs due to VEV of scalar field, $<\sigma>$, and $\sigma$ splits into two fields, $\rho$ and $J$:

$$\sigma \rightarrow (v_\sigma + \rho) \exp(iJ/v_\sigma).$$

The field $J$ is the Majoron. A mass of $\sim$ keV can be obtained due to gravitational interaction [13, 19]. The keV Majoron has the great cosmological interest since the Jeans mass associated with this particle, $m_{\text{Jeans}} \sim m_{\chi}^2/v_\sigma^2$, gives the galactic scale $M \sim 10^{12} M_\odot$. In all other respects the keV Majoron plays the role of a CDM particle. It is assumed usually that the Majoron interacts directly only with some very heavy particles (e.g. with the right-handed neutrino $\nu_R$). It results in very weak interaction of the Majoron with the ordinary particles (leptons, quarks etc) and thus makes the Majoron ”invisible” in the accelerator experiments.

The cosmological production of the Majoron occurs through thermal production [19, 20] and radiation by the strings [19] as in case of the axion. However, under imposed observational constraints, the Majoron in models [13, 19] has to be in general unstable with lifetime much shorter than the age of Universe. A successful model was developed in ref.[21], where the Majoron was assumed to interact with the ordinary particles through new heavy particles. For the cosmological production it was considered the phase transition associated with the global $U(1)_{B-L}$ symmetry breaking, when the Majorons were produced both directly and through $\rho \rightarrow J + J$ decays.

Another interesting possibility was considered recently in ref.22. In this model the Majoron is rather strongly coupled with $\nu_\mu$ and $\nu_\tau$ neutrinos ($J\nu_\mu, \nu_\tau$ coupling). The Majorons are produced through $\nu_\tau \rightarrow J + \nu_\mu$ decays. The strong interactions between Majorons reduces the relic abundance of the Majorons to the cosmologically required value.

The Majoron signature in all these models is given by $J \rightarrow \gamma + \gamma$ decays, which result in the production of keV X-ray line in the X-ray background radiation.

5. Neutralino

The neutralino is a superposition of four spin 1/2 neutral fields: the wino $\tilde{W}_3$, bino $\tilde{B}$ and two Higgsinos $\tilde{H}_1$ and $\tilde{H}_2$:

$$\chi = C_1 \tilde{W}_3 + C_2 \tilde{B} + C_3 \tilde{H}_1 + C_4 \tilde{H}_2$$

The neutralino is a Majorana particle. With a unitary relation between the coefficients $C_i$ the parameter space of neutralino states is described by three independent parameters, e.g. mass of wino $M_2$, mixing parameter of two Higgsinos $\mu$, and the ratio of two vacuum expectation values $\tan \beta = v_2/v_1$.

In literature one can find two extreme approaches describing the neutralino as a DM particle.

(i) **Phenomenological approach.** The allowed neutralino parameter space is restricted by the LEP and CDF data. In particular these data put a lower limit to the neutralino mass, $m_\chi > 20$ GeV. In this approach the usual GUT relation between gaugino masses, $M_1 : M_2 : M_3 = \alpha_1 : \alpha_2 : \alpha_3$, is used as an additional assumption, where $\alpha_i$ are the gauge coupling constants. All other SUSY masses which are needed for the calculations are treated as free parameters, limited from below by accelerator data.

One can find the relevant calculations within this approach in refs.23, 24 and in the review 25 (see also the references therein). There are large areas in neutralino parameter space where the neutralino relic density satisfies the relation (2). This is especially true for heavy neutralinos with $m_\chi > 100 - 1000$ GeV, ref.26. In these areas
there are good prospects for indirect detection of neutralinos, due to high energy neutrino radiation from Earth and Sun (see [24] and references therein) as well as due to production of antiprotons and positrons in our Galaxy. The direct detection of neutralinos is possible too, though in more restricted parameter space areas of light neutralinos (see review [25]).

This model-independent approach is very interesting as an extreme case: in the absence of an experimentally confirmed SUSY model it gives the results obtained within most general framework of supersymmetric theory.

(ii) Strongly constrained models. This approach is based on the remarkable observation that in the minimal SUSY SU(5) model with fixed particle content, the three running coupling constants are based on the remarkable observation that in the minimal SUSY SU(5) model with fixed particle content, the three running coupling constants meet at one point corresponding to the GUT mass $M_{GUT}$. Because of the fixed particle content of the model, its predictions are rigid and they strongly restrict the neutralino parameter space. This is especially true for the limits due to proton decay $p \rightarrow K^+ \nu$. As a result very little space is left for neutralino as DM particle. Normally neutralinos overclose the Universe ($\Omega_\chi > 1$). The relic density decreases to the allowed values in very restricted areas where $\chi \chi$-annihilation is accidentally large (e.g., due to the $Z^0$ exchange term - see ref. [29]). Thus, this approach looks rather pessimistic for neutralino as DM particle.

In several recent works [30]-[35] less restricted SUSY models were considered. In particular the limits due to proton decay were lifted. A GUT model was not specified or less restrictive SO(10) model was used [35]. Although, the neutralino can be heavy in these models, the prospects for indirect detection, including the detection of high energy neutrinos from the Sun and Earth, are rather pessimistic [32]. The direct detection is possible in many cases [33, 34, 35].

(iii) Relaxed restrictions. In section 5.3, following refs. [36], [37] we shall analyze the restrictions to neutralino as DM particle, imposed by basic properties of SUSY theory. As in many previous works, a fundamental element of analysis is the radiatively induced EW symmetry breaking (EWSB) [38]. However, some mass unification conditions at the GUT scale are relaxed (see ref. [39]). The powerful restriction from the no-fine-tuning condition is added.

5.1. SUSY theoretical framework

The basic element which should be used in the analysis is a supersymmetry breaking and induced by it (through radiative corrections) electroweak symmetry breaking [38]. We shall refer to this restriction as to the EWSB restriction.

One starts with unbroken supersymmetric model described by some superpotential. It is assumed that local supersymmetry is broken by supergravity in the hidden sector, which communicates with the visible sector only gravitationally. This symmetry breaking penetrates into the visible sector in the form of global supersymmetry breaking. More specifically it is assumed that the symmetry breaking terms in the visible sector are the soft breaking terms given at the GUT scale $Q^2 \sim M^2_{GUT}$ by the following expression:

$$L_{sb} = m_0^2 \sum_a |\phi_a|^2 + m_1/2 \sum_a \lambda_a \lambda_a +$$

$$+ A m_0 f_Y + B m_0 \mu H_1 H_2$$

where $\phi_a$ are scalar fields of the model (sfermions and two Higgses $H_1$ and $H_2$), $\lambda_a$ are gaugino fields, $f_Y$ are trilinear Yukawa couplings of fermions and Higgses and the last term is an additional (relative to the superpotential term $\mu H_1 H_2$) soft breaking mixing of two Higgses. Here and everywhere below we specify the scale at which an expression and parameters are defined.

The soft breaking terms (11) are described by 5 free parameters: $m_0, m_1/2, A, B$ and $\mu$. This implies the strong assumption that all scalars $\phi_a$ and all gauginos $\lambda_a$ at the GUT scale have the common masses $m_0$ and $m_1/2$, respectively. This assumption can be relaxed, as we shall discuss later.

The soft breaking terms (11) together with supersymmetric mixing give the following potential defined at the EW scale at the tree level:

$$V = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 -$$

$$- B m_0 (H_1 H_2 + h c) +$$

$$+ g_1^2 + g_2^2 (|H_1|^2 - |H_2|^2)^2,$$
The mass parameters \( m_1 \) and \( m_2 \) at the GUT scale are equal to

\[
m_1^2(GUT) = m_2^2(GUT) = \mu^2 + m_0^2, \tag{13}
\]

with \( \mu \) defined at the GUT scale, too. The term \( \mu^2 \) in Eqs.\((12),(13)\) appears due to the mixing term \( \mu H_1 H_2 \) in the superpotential of unbroken SUSY.

The radiative EWSB occurs due to evolution of \( m_{H_2} \), the mass of \( H_2 \), which is connected with the upper components of the fermion doublets and in particular with t-quark. Because of the large mass of t-quark and consequently the large Yukawa coupling \( Y_{tH_2} \), \( m_{H_2}^2 \) evolves from \( m_0^2 \) at the GUT scale to the negative value \( m_{H_2}^2 < 0 \) at the EW scale. At this value the potential \( (12) \) acquires its minimum and the system undergoes the EW phase transition coming to the minimum of the potential. At EW scale in the tree approximation the conditions of the potential minimum (vanishing of the derivatives) give:

\[
\nu^2 = \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{M_Z^2}{2} \tag{14}
\]

\[
\sin 2\beta = \frac{-2B\mu}{m_{H_1}^2 + m_{H_2}^2 + 2\mu^2} \tag{14}
\]

With these equations we obtain one connection between five free parameters describing the soft-breaking terms \( (11) \). Thus the number of independent parameters is reduced to four, e.g.\( m_0, m_{1/2}, A, \mu \) (or \( \tan \beta \)).

Using the renormalization group equations (RGE) one can follow the evolution of the scalar particles (Higgses and sfermions) and spin 1/2 particles (gauginos) from the masses \( m_0 \) and \( m_{1/2} \) at the GUT scale to the masses at the EW scale. Analogously, the evolution of the coupling constants can be calculated. In particular the masses of Higgses at EW scale are given by

\[
m_{H_i}^2 = a_i m_0^2 + b_i m_{1/2}^2 + c_i A^2 m_0^2 + d_i A m_0 m_{1/2}, \tag{15}
\]

where \( a_i, b_i, c_i \) and \( d_i \) are numerical coefficients, which depend on \( \tan \beta \).

Equivalently, using Eqs.\((14)\) one finds

\[
M_Z^2 = J_1 m_{1/2}^2 + J_2 m_0^2 + J_3 A^2 m_0^2 + J_4 A m_0 m_{1/2} - \mu^2, \tag{16}
\]

where \( J_i \) are also numerical coefficients.

Eq.\((16)\) allows to impose the no-fine-tuning condition in the neutralino parameter space. Indeed, one can keep large values of masses in the rhs of Eq.\((14)\) only by the price of accidental compensation between the different terms. It is unnatural to expect an accidental compensation to a value less than 1% from the initial values. This is the no-fine-tuning condition.

Naturally this condition is just the same as the one due to the radiative corrections to the Higgs mass.

### 5.2. Restrictions: the price list

Within the theoretical framework outlined above one can choose the restrictions from the following price list:

- Soft breaking terms \( (11) \) and EWSB conditions \( (12) \),
- No-fine-tuning condition,
- Particle phenomenology (constraints from accelerator experiments and the condition that the neutralino is the LSP),
- Restrictions due to \( b \to s\gamma \) decay,
- Meeting of coupling constants at \( M_{GUT} \),
- \( b - \tau \) and \( b - \tau - t \) unification,
- Restrictions due to \( p \to K\nu \) decay.

Using some (or all) restrictions listed above one can start calculations for the neutralino as DM particle. The regions where the neutralinos are overproduced (\( \Omega_\chi h^2 > 1 \)) must be excluded from consideration and the allowed region should be determined according to the chosen cosmological model (e.g. \( \Omega_\chi h^2 = 0.2 \pm 0.1 \) for the CHDM model). For the allowed regions the signal for direct and indirect detection can be calculated.

### 5.3. SUSY models with basic restrictions

Accepting all restrictions listed above one arrives at a rigid SUSY model, with the neutralino parameter space being too strongly constrained. In ref.\[37\] the SUSY models with basic restrictions were considered. These restrictions are as
follows:
(i) Radiative EWSB, (ii) No fine-tuning stronger than 1%, (iii) RGE and particle phenomenology (accelerator limits on the calculated masses and the condition that neutralino is LSP), (iv) Limits from $b \to s\gamma$ decay taken with the uncertainties in the calculations of the decay rate and (v) $0.01 < \Omega_\chi h^2 < 1$ as the allowed relic density for neutralinos. Rather strong restrictions are imposed by the condition (ii); in particular it limits the mass of neutralino as $m_\chi < 200$ GeV.

At the same time some restrictions are lifted as being too model-dependent: (i) No restrictions are imposed due to $p \to K\nu$ decay, (ii) Unification of coupling constants at the GUT point is allowed to be not exact (it is assumed that new very heavy particles can restore the unification), (iii) unification in the soft breaking terms is relaxed. Following ref.[36] it is assumed that masses of Higgses at the GUT scale can deviate from the universal value $m^2_{H_i}(\text{GUT}) = m^2_0(1 + \delta_i)$ ($i = 1, 2$).

This non-universality affects rather strongly the properties of neutralino as DM particle: the allowed parameter space regions become larger and neutralino is allowed to be Higgsino-dominated, which is favorable for detection.

Some results obtained in ref.[37] are illustrated by Figs. 2 - 6. The regions allowed for the neutralino as CDM particle are shown everywhere by small boxes.

In Fig. 2 the regions excluded by the LEP and CDF data are shown by dots and labelled as LEP. The regions labelled "fine tuning" have an accidental compensation stronger than 1% and thus are excluded. No-fine-tuning region inside the broken-line box corresponds to a neutralino mass $m_\chi \leq 200$ GeV. The region "EWSB+particle phenom." is excluded by the EWSB condition combined with particle phenomenology (neutralino as LSP, limits on the masses of SUSY particles etc). In the region marked by rarified dotted lines neutralinos overclose the Universe ($\Omega_\chi h^2 > 1$). The solid line corresponds to $m_0 = 0$. The regions allowed for neutralino as CDM particle ($0.01 < \Omega_\chi h^2 < 1$) are shown by small boxes. As one can see in most regions the neutralinos are overproduced. The allowed regions correspond to large $\chi\chi$ annihilation cross-section (e.g. due to $Z^0$-pole).

Fig. 3 and Fig. 2 differ only by universality: in Fig. 3 $\delta_1 = \delta_2 = 0$ (mass–unification), while in Fig. 4 $\delta_1 = -0.2$ and $\delta_2 = 0.4$. The allowed region in Fig. 3 becomes much larger and is shifted into the Higgsino dominated region. Figs. 4 and 5 are given for $\tan\beta = 53$. This large value of $\tan\beta$ correspond to $b-\tau-t$ unification of the Yukawa coupling constants. Again one can notice that in the mass–unification case ($\delta_1 = \delta_2 = 0$) only a small area is allowed for neutralino as CDM particle, while in non-universal case ($\delta_1 = 0, \delta_2 = -0.2$) the allowed area becomes larger and shifts into the gaugino dominated region.

In Fig. 6 the scatter plot for the rate of direct detection with the Ge detector is given for the non-universal case ($\delta_1 = 0, \delta_2 = -0.2$) and $\tan\beta = 53$. We notice that, for some configurations, the experimental sensitivity is already at...
Figure 3. Case $\delta_1 = -0.2, \delta_2 = 0.4$ and $\tan \beta = 8$.

the level of the predicted rate.

6. Conclusions

1. The density of CDM needed for most cosmological models is given by $\Omega_{CDM} h^2 = 0.2 \pm 0.1$. There are four candidates for CDM, best motivated from point of view of elementary particle physics: neutralino, axion, axino and majoron.

2. There are different approaches to study the neutralino as DM particle in SUSY models with R-parity conservation.

   In the phenomenological approach, apart from the LEP-CDF limits, very few other constraints are imposed. In the Minimal Supersymmetric Model only one GUT relation between gauginos masses, $M_1 : M_2 : M_3 = \alpha_1 : \alpha_2 : \alpha_3$, is used. While the coupling constants are known, the mass parameters needed for calculations are taken as the free parameters. Many allowed configurations in the parameter space give the neutralino as DM particle with observable signals for direct and indirect detection (antiprotons and positrons in our Galaxy and high energy neutrinos from the Sun and Earth).

   The other extreme case, the complete SUSY SU(5) model with fixed particle content, with meeting of coupling constants at the GUT point and with the constraints due to proton decay, leaves very little space for the neutralino as DM particle.

   In the third option the SUSY soft breaking terms (11) and induced EW symmetry breaking are used as the general theoretical framework. Combined with a no-fine-tuning condition this framework already gives essential restrictions. In particular, fine tuning allowed at the level larger than 1% results in the neutralino being lighter than 200 GeV. Here the neutralino is gaugino dominated, which is unfavorable for direct detection. If we employ further other restrictions, such as exact unification of gauge coupling constants and soft-breaking parameters at the GUT scale, $b \rightarrow s \gamma$ limit and $b - \tau$ unification, the model becomes as rigid as the one considered above.

   If, on the other hand, we relax some con-
Figure 5. Case $\delta_1 = 0, \delta_2 = -0.2$ and $\tan \beta = 53$.

Figure 6. Rate for direct detection ($\delta_1 = 0, \delta_2 = -0.2$ and $\tan \beta = 53$).

3. In the framework of supersymmetric theory, the PQ-mechanism for solving the problem of strong CP violation, results in a Goldstone supermultiplet which contains axion and its fermionic superpartner, axino. Both of them can be CDM particles. The most important parameter here is the scale of PQ symmetry breaking $f_{PQ}$ which is observationally constrained as $2 \cdot 10^9$ GeV $< f_{PQ} < 8 \cdot 10^{11}$ GeV. The axion can be the CDM particle, if its mass is $10^{-5} - 10^{-3}$ eV (the corresponding values of $f_{PQ}$ are between $2 \cdot 10^9$ GeV and $6 \cdot 10^{10}$ GeV). For larger values of $f_{PQ}$ the axino can be CDM particle if a mechanism of SUSY breaking provides its mass within the interval $10 - 100$ keV. The axino can provide both CDM and HDM components needed to fit the cosmological observations. The axion can be directly observed (e.g. in microwave cavity experiments) while the axino dark matter is practically unobservable.

4. The Majoron with keV mass can be the warm DM particle which explains the galactic scale ($M \sim 10^{12} M_\odot$) in the structure formation problem. The decay of the Majoron to two photons can produce an observable X-ray line in the cosmic background radiation.

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