Experimental Highlight of Hysteresis Phenomenon in Rolling Contact

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Abstract. In literature, the hysteresis phenomenon in rolling contacts is studied considering both rolling friction and sliding friction. Removal of sliding friction in experimental tests from a concentrated contact is a serious challenge. The paper proposes a method and presents a device ensuring pure rolling between two identical discs, normally loaded. Using photoelastic material for the two rolling discs, by means of photoelastic method, the hysteresis phenomenon due to rolling friction is qualitatively confirmed.

1. Theoretical aspects

Mechanics of rigid body theory is based on the hypothesis of undeformable bodies that is mathematically expressed by the fact that the distance between any two points of the body does not change in time, regardless of the magnitude of applied loads. However, when explaining the rolling friction phenomenon, this assumption must be overlooked. Voinea, [1], considers a disc of radius \( r \), resting on a rigid horizontal plane and loaded by a horizontal force \( F \) and its own weight \( G \), as in figure 1.

**Figure 1.** Contact between a rigid cylinder and a rigid half-space.

**Figure 2.** Contact between an elastic cylinder and an elastic half-space.
The reaction force from the plane has two components: the normal reaction, \( N \) and the tangential reaction \( T \) (or the friction force). The equilibrium equation for moments written with respect to the contact point leads to:

\[ F = 0 \]  

(1)

According to equation (1), but contrary to practice, the body could be brought to rolling motion by any small force. In order to explain the plane resistance to rolling, the hypothesis of rigid plane must be disregarded and therefore, a deformable plane should be considered. Under this assumption, the contact between the cylinder and the plane doesn’t occur along a generatrix but on a surface on which a pressure distribution arises, asymmetrical with respect to the vertical line passing through the center of the cylinder, as seen in figure 2. The pressure distribution ought to be asymmetrical as the force torsor reduced to the initial contact point should contain the normal force \( N \) and the moment \( M_r \) with respect to the initial contact point. The point of reduced force torsor can be chosen to be situated at a distance \( e \) from the theoretical contact point where the moment is zero, the condition being:

\[ M_r < eN \]  

(2)

The maximum value of eccentricity \( e \) for which the disc still rests immobile is named rolling coefficient and is denoted by \( s \). Since \( e \leq s \), by multiplying by \( N \) this inequality, one obtains:

\[ M_r = eN \]  

(3)

As long as the next condition is fulfilled, the body will not effectuate rolling motion:

\[ M_r = eN \leq sN, \]  

(4)

The maximum value of eccentricity \( e \) is named coefficient of rolling friction.

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**Figure 3.** Case of pure sliding occurring between two bodies.

**Figure 4.** Case of general plane motion when both sliding and rolling friction exist.

There are practical situations when between two bodies only relative translation motion occurs, as represented in figure 3, and just the sliding friction as friction type. For most of the cases, [2,3,4,5], the relative motion between two bodies is a general spatial motion. For the situation of a contacting cylinder 1 and a prismatic part 2, the upper body 1 is in relative plane motion with respect to the body 2 and this relative motion is characterized by the velocity \( v_{12} \). In figure 4, the relative motion between the bodies is characterized by the relative angular velocity vector \( \omega_{12} \), parallel to the axis of the cylinder and by the relative velocity between the theoretical points of contact, \( v_{12} \). At first sight, if the following condition is fulfilled:
and, for a value of sliding friction coefficient large enough (where $R$ is the radius of the cylinder), it could appear that the pure rolling condition is satisfied. Actually, due to deformability, [6], the instantaneous axis of relative rotation doesn’t coincide with the initial contact generatrix and therefore the pure rolling condition is violated, [7].

2. Constructive requirements

According to the above observation, pure rolling will occur between the two bodies only in the case in which the instantaneous axis of relative motion between the two bodies coincides to the initial contact straight line and this conducts to the idea of using two deformable cylinders of precise equal radii, of similar materials, [6], pressed one against the other. The contact surface is a plane one and the axis of cylinders are symmetrically placed about it for any load. The geometrical and loading symmetry ensures the plane shape of contact surface and will contain the axode of relative motion. Pure rolling occurrence requires, besides the geometrical condition of equal cylinders radii, precise equality of angular velocities of cylinders.

3. Photoelasticity considerations

The photoelastic method was chosen to study the stress field produced between the two discs. The main advantages of the method consist in the option of global stress field analysis from the tested parts and in the fact that the stress field structure remains unaltered as the probes are not supposed to direct contact. The main disadvantage of the method is the limitation of applying the method only for plane models. For spatial models, a special study method – freezing stresses method, must be applied, [8]. The basis for the photoelastic studies is the birefringence phenomenon. Since the proposed testing device scheme assumes a plane model loaded by a plane force system, the usual photoelastic methodology is applied.

For the case of plane stress state in a body, this stress state will be completely characterized in a Cartesian coordinate system by a second order tensor, $T$:

$$
T = \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix}
$$

(6)

where $\sigma_x$, $\sigma_y$ are the normal stresses and $\tau_{xy} = \tau_{yx}$ are the shear stresses. The eigenvalues of this tensor are the values of principal normal stresses:

$$
\lambda_1 = \sigma_{\text{max}}, \quad \lambda_2 = \sigma_{\text{min}}
$$

(7)

The principal directions of the tensor corresponding to the principal normal stresses are characterized by the orthogonal versors $e_1$ and $e_2$, and between the versors $i$ and $e_1$ the angle $\varphi$ appear, figure 5.

![Figure 5. Cartesian coordinate system and the directions of principal stresses.](image_url)

Three families of curves are used in photoelastic studies:
Isochromatics, defined as geometrical locus of points for which the maximum shear stress is the same

$$\tau_{\text{max}} = (\sigma_{\text{max}} - \sigma_{\text{min}})/2 = \text{const.}$$  \(\text{(8)}\)

Isoclinics, defined as the locus of the points in the specimen along which the principal stresses are in the same direction.

$$\varphi = \tan^{-1}\left[\frac{2\tau_{xy}}{(\sigma_x - \sigma_y)}\right] = \text{constant.}$$  \(\text{(9)}\)

Isoclinics and isochromatics are overlapped and cannot be separated.

The isopachs are the curves along which the sum of normal principal stresses is constant or the mean normal stress is constant.

$$\sigma_1 + \sigma_2 = \sigma_x + \sigma_y = \text{tr}(\mathbf{T}) = \text{const.}$$  \(\text{(10)}\)

where \(\text{tr}(\mathbf{T})\) represents the trace of stress tensor \(\mathbf{T}\).

For a model subjected to a plane stress state, the complete stress tensor in a point is found by knowing the characteristic parameters of the three curves passing through the considered point.

From the three families of curves, the most used are the isoclinics. In white light, the isoclinics appear as a succession of colored curves. In polarized light, these curves become more clear, the image of isochromatics field is a succession of dark and bright curves in white and black. As the load of a region increases, the isochromatics gather tighter and the brightness of the region increases. For the next considerations, is important the observation that an unloaded contour appears dark. In the case of loaded contour, it becomes bright due only to principal normal hoop stress.

4. Design principles for the device

As previously précised, for the study of rolling friction effect, between the two cylinders, rolling motion without sliding must exist. The presence of sliding friction alters the symmetry of isochromatics field. In the isochromatics field, the presence of a concentrated force on the contour is evidenced by the occurrence of a pattern named „peacock eye”, \([9]\), figure 6. The level curves are circles tangent in the loading point.

![Figure 6](image)

Figure 6. Isochromatic field near a concentrated normal load on boundary.

In order to emphasize the effect of friction force upon the isochromatics field, a dry friction contact was modeled using finite element method, for friction coefficient \(\mu = 0\) and \(\mu = 0.05\) respectively. The results are presented in figure 7.
The isochromatics field from an elastic half-plane normally loaded, without friction (left) and with friction (right).

The presence of friction leads to asymmetrical loading and therefore to the occurrence of asymmetry in isochromatics field.

With the above considerations, the following device scheme is proposed, Fig. 8. Two identical discs, with parallel axis, made of photoelastic material, are brought into contact and rotated in opposite ways with strictly equal angular velocities. The prefect equality of the angular velocities of the wheels is accomplished by mounting them on two rotating shafts, the kinematical linkage between the shafts consisting in a gear train. The contact region between the cylinders must be free to allow the polarized light waves to pass.

To this end, the gears from the shafts must have smaller diameters compared to the photoelastic discs and the shafts must be actuated by two gears ensuring opposite rotations direction of the discs, figure 9.

In order to load the discs radial, the distance between the gears 1 and 4 must be adjustable. The condition of equality for angular velocities of gears 1 and 4 plus the possibility of adaptable axis distance lead to the conclusion that one of the gears 1 or 4 must be a sun gear. The driving motion is made using a d.c. motor, because it allows adjusting the angular velocity via the voltage. The reduced available space on the polariscope table for attaching the testing device imposed the use of a fifth gear on which shaft a belt wheel was placed.
The mechanism from figure 9, ensures the conditions that the rotations of the shafts 1 and the one on which the photoelastic discs will be fitted should be rigorously equal, due to equal number of teeth of the gears 2 and 3, and 1 and 4, respectively. The inconvenience of this mechanism is that, due to fixed positions of shafts, cannot ensure radial loading of photoelastic discs. To respond to this requirement, the final version of the mechanism was proposed, as presented in figure 10. Thus, the photoelastic disc united with gear 1 has a fixed center while the second disc has the center on the lever P of a differential mechanism.

As long as the discs are not in contact, the disc 4 presents its own rotation around the center \( \omega_4 \), that is composing with the velocity of differential and planet wheel carrier according to Willis formula, [10].

\[
\frac{\omega_4 - \omega_p}{\omega_3 - \omega_p} = \frac{z_4}{z_3}
\]  

(11)
When the contact between the photoelastic discs happens, the planet wheel carrier becomes immobile, \( \omega_p = 0 \), the relation 10 becomes:

\[
\frac{\omega_4}{\omega_3} = \frac{z_4}{z_3} = \frac{z_1}{z_2} = \frac{\omega_1}{\omega_2}.
\]  

(12)

In addition, the angular velocities \( \omega_2 \) and \( \omega_4 \) are equal in magnitude but of opposite directions and therefore, finally \( \omega_4 = -\omega_1 \). The experimental testing device is presented in Figure 11, made based on the kinematical scheme from Figure 10. The driving motion is made using a d.c. motor and a belt transmission.

![Experimental device](image)

**Figure 11.** Detail and general view of the experimental device.

5. Experimental results

The experimental device was attached to the stage of a VISHAY P500 Series Polariscope. The rotative speed is measured using a non contact tachometer. The experimental test aimed observing the isochromatics field obtained for different loads and angular velocities. For very small loads, one can notice that the contact between the two discs appears faintly lighted due to first isocromatics occurrence. During increasing load period, the next isochromatics appear, situated in the next vicinity of contact point. With further increased load, the lighted region dimensions increase and the distinction between isochromatics is more difficult to be made due to brightness of the loaded region. For a strong load rise, the brightness increases further but the angular velocity of the discs decreases considerable due to augmented internal friction from the material of the two discs.

In figure 12 the experimental isochromatics of the two discs are shown, correlated to the rotating directions. The most interesting aspect appears in figure 12 b), where in the region of contact exit from the contours of the discs, a brightness gradually decreasing can be observed. This brightness can be caused by:

- Plastic deformations;
- Friction forces presence;
- Hysteresis phenomenon.

Within the assumption of plastic deformations, the entire periphery of the discs should be lighted on the entire contour.

The hypothesis that sliding friction occurs between the two discs could be caused by different values of the diameters of the two photoelastic discs and possible errors from the condition of equal angular velocities of the shafts of the photoelastic discs, due to execution errors and device fitting.
errors. If the assumption of friction is considered, than in the contact point the friction forces are equal in magnitude but with opposite directions and the isochromatics fields from the two discs should be anti-symmetrical.

The only feasible remaining hypothesis is the one considering that the brightness is produced by the hysteresis phenomenon. The material from the vicinity of contact point enters the contact and is rapidly deformed. After leaving the contact, the material remains loaded for a while and this fact determines the occurrence of brightness on the discs contours in the contact exit region.

![Figure 12](image-url)

**Figure 12.** Experimental isochromatics; increasing loads from a) to d).

For increased values of load F, the isochromatics field is symmetrical in the absence of motion. Under rolling conditions, for increased normal loads, the hysteresis phenomenon cannot be marked because with increasing load, the intensity and extent of brightened area increases too. This is due to the fact that the stresses (and their optical effect, the brightness) due to hysteresis become insignificant compared to the stress field produced by the Hertzian contact.

### 6. Conclusions

The paper presents a device designed for the study of rolling contacts based on the photoelastic method. Most of the photoelastic researches are made on probes in static conditions.
The test rig shows a silent running, confirmed by the isochromatics shape from the two discs as during running at a rotating speed of 2400 rpm the aspect is quasi static. The contact hysteresis phenomenon was revealed.

During experimental tests, it was noticed that for increasing load, the angular velocity of the motor drive decreases. In order to maintain a constant rotating speed, a new motor drive is required, having greater power or a stiffer characteristic.

Another solution for obtaining uniform motion is mounting a disc flywheel on one of the shafts. The device can also be used for the study of rolling friction with sliding, by modifying the discs, having different radii but with fixed distance between axes.

For further studies, a proposed improvement of the testing device consists in using photoelastic discs of equal diameters but employing pairs of exchangeable wheels used in accomplishing a controlled sliding.

Acknowledgement:
This paper was supported by the project "Progress and development through post-doctoral research and innovation in engineering and applied sciences – PRiDE - Contract no. POSDRU/89/1.5/S/57083", project co-funded from European Social Fund through Sectorial Operational Program Human Resources 2007-2013.

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