Constraints on $H_0$ from WMAP and baryon acoustic oscillation measurements

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We report the constraints of $H_0$ obtained from Wilkinson Microwave Anisotropy Probe (WMAP) combined with the latest baryonic acoustic oscillations (BAO) measurements. We use the BAO measurements from the 6dF Galaxy Survey (6dFGS), the SDSS DR7 main galaxies sample (MGS), the BOSS DR12 galaxies and the eBOSS DR14 quasars. Adding the recent BAO measurements to the cosmic microwave background (CMB) data from WMAP, we constrain cosmological parameters \( \Omega_m = 0.298 \pm 0.005 \), \( H_0 = 68.36^{+0.53}_{-0.52} \text{ km s}^{-1} \text{ Mpc}^{-1} \), \( \sigma_8 = 0.8170^{+0.0175}_{-0.0179} \) in a spatially flat \( \Lambda \) cold dark matter (\( \Lambda \)CDM) model, and \( \Omega_m = 0.302 \pm 0.008 \), \( H_0 = 67.63 \pm 1.30 \text{ km s}^{-1} \text{ Mpc}^{-1} \), \( \sigma_8 = 0.7988^{+0.0345}_{-0.0338} \) in a spatially flat \( \text{wCDM} \) model, respectively. The combined constraint on \( w \) from CMB and BAO in a spatially flat \( \text{wCDM} \) model is \( w = -0.96 \pm 0.07 \). Our measured $H_0$ results prefer a value lower than \( 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \), consistent with the recent data on CMB constraints from Planck (2018), but in 3.1 \( \sigma \) tension with local measurements of Riess et al. (2018) in \( \Lambda \)CDM and \( \text{wCDM} \) framework, respectively. Compared with the WMAP alone analysis, the WMPA+BAO analysis reduces the error bar by 75.4\% in \( \Lambda \)CDM model and 95.3\% in \( \text{wCDM} \) model.

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I. INTRODUCTION

As we know, the Hubble constant $H_0$ are in tension between the Cosmic Microwave Background (CMB) measurements from Planck [1, 2] and the type Ia supernova measurements from SH0ES [3, 4] (SNe, $H_0$, for the Equation of State of dark energy). The value of $H_0$ can be directly obtained by the Hubble Space Telescope (HST) and the CMB measurements (see [5] for review on determining the Hubble constant). Riess et al. (2016) [3] reported $H_0 = 73.24 \pm 1.74 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (2.4\% precision) from Cepheids in the hosts of Type Ia supernovae (SNIa). Recently Riess et al. (2018) [4] improves the precision to 2.3\%, yielding 73.48 \pm 1.66 \text{ km s}^{-1} \text{ Mpc}^{-1}. On the other hand, the Planck survey reported $H_0 = 67.27 \pm 0.66 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (0.98\% precision; TT,TE,EE+lowE) in 2015 [1] and 67.27 \pm 0.60 \text{ km s}^{-1} \text{ Mpc}^{-1} (0.89\% precision; TT,TE,EE+lowE) in 2018 [2]. There exist a 3.7\( \sigma \) tension between the new results of Planck and SH0ES. Addison et al. (2016) [6] have discussed the internal tension inferred from the Planck data itself. They have analyzed the Planck TT power spectra in detail and found that the Hubble constant $H_0 = 69.7 \pm 1.7 \text{ km s}^{-1} \text{ Mpc}^{-1}$ at the lower multipoles ($\ell < 1000$) and $H_0 = 64.1 \pm 1.7 \text{ km s}^{-1} \text{ Mpc}^{-1}$ at the higher multipoles ($\ell \geq 1000$). The measured value of $H_0$ is much lower in the case of $\ell \geq 1000$.

At present it is difficult to explain the $H_0$ disagreement in the standard cosmological model. The tensions among datasets could be due to some underestimated systematic error associated with the experiments. Of course, we cannot exclude the possibility of new physics beyond the \( \Lambda \)CDM cosmology [7,13], so the additional crosschecks are expected. In this paper we again call for another independent precise CMB measurements, namely Wilkinson Microwave Anisotropy Probe (WMAP). The 9-year WMAP reported a 3\% precision determination of $H_0 = 70.0^{+2.2}_{-2.0} \text{ km s}^{-1} \text{ Mpc}^{-1}$ in a spatially flat \( \Lambda \)CDM model [14]. On the other hand, Cheng et al. [15] combined various BAO data sets to get relatively tight constraints on $H_0 = 68.17^{+1.55}_{-1.56} \text{ km s}^{-1} \text{ Mpc}^{-1}$. Combining the recent BAO measurements, Wang et al. [16] reported $H_0 = 69.13 \pm 2.34 \text{ km s}^{-1}$ (3.38\% precision). In addition, The Advanced LIGO and Virgo [17] reported the a strong signal of GW170817 from the merger of a binary neutron-star system and determined the Hubble constant $H_0 = 70.0^{+12.0}_{-8.0} \text{ km s}^{-1} \text{ Mpc}^{-1}$. Addison et al. (2018) [18] show that WMAP, Atacama Cosmology Telescope (ACT), South Pole Telescope (SPT) surveys, and primordial deuterium abundance constraints can be used

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together with some BAO data to provide the values of $H_0$, which are $2.4 \sim 3.1\sigma$ lower than SH0ES, independent of Planck. Combining galaxy and Ly$\alpha$ BAO observations with the primordial deuterium abundance, a value of $H_0 = 66.98 \pm 1.18$ km s$^{-1}$ Mpc$^{-1}$ has been estimated. This value also have 3$\sigma$ tension with local $H_0$ measurement. These measurements, independent of SH0ES and Planck constraints, seem to favor a lower $H_0$ value that is more consistent with Planck result. See [19–30] for more literature on $H_0$.

Beyond the spatially flat standard cosmological model and without SH0ES and Planck measurements, it would be interesting to study the Hubble constant constraints. So, in this work we combine the BAO with WMAP measurements to place constraints on $H_0$ in $\Lambda$CDM and $\omega$CDM cosmology. The paper is organized as follows. In section II we will introduce the model and data sets used in this work. In section III we present our main results. We conclude in section IV.

II. MODEL AND DATA

In this paper we discuss the spatially flat $\Lambda$CDM and $\omega$CDM model, first using the nine-year WMAP data only, then combined with the additional BAO data sets. We use the BAO measurements from the 6dFGS survey [31], the SDSS DR7 MGS [32], the BOSS DR12 (9-zbin) [33], and the eBOSS DR14 measurement [34]. Their effective redshifts and constraints are listed in Table I.

| Experiment      | z_{eff} | Measurement | Constraint                        |
|-----------------|---------|-------------|-----------------------------------|
| 6dFGS           | 0.106   | $r_d/D_V$   | $(664 \pm 25)(r_d/r_{d, fid})$ Mpc |
| SDSS DR7 MGS    | 0.15    | $D_V$       | $(664 \pm 25)(r_d/r_{d, fid})$ Mpc |
| BOSS DR12 (9zbin) | 0.31   | $D_A/r_d; H \ast r_d$ | $(6.29 \pm 0.14)$ Mpc; $(11.55 \pm 0.70) \times 10^3$ km/s |
|                 | 0.36    | $D_A/r_d; H \ast r_d$ | $(7.09 \pm 0.16)$ Mpc; $(11.81 \pm 0.50) \times 10^3$ km/s |
|                 | 0.40    | $D_A/r_d; H \ast r_d$ | $(7.70 \pm 0.16)$ Mpc; $(12.12 \pm 0.30) \times 10^3$ km/s |
|                 | 0.44    | $D_A/r_d; H \ast r_d$ | $(8.20 \pm 0.13)$ Mpc; $(12.53 \pm 0.27) \times 10^3$ km/s |
|                 | 0.48    | $D_A/r_d; H \ast r_d$ | $(8.64 \pm 0.11)$ Mpc; $(12.97 \pm 0.30) \times 10^3$ km/s |
|                 | 0.52    | $D_A/r_d; H \ast r_d$ | $(8.90 \pm 0.12)$ Mpc; $(13.94 \pm 0.39) \times 10^3$ km/s |
|                 | 0.56    | $D_A/r_d; H \ast r_d$ | $(9.16 \pm 0.14)$ Mpc; $(13.79 \pm 0.34) \times 10^3$ km/s |
|                 | 0.59    | $D_A/r_d; H \ast r_d$ | $(9.45 \pm 0.17)$ Mpc; $(14.55 \pm 0.47) \times 10^3$ km/s |
|                 | 0.64    | $D_A/r_d; H \ast r_d$ | $(9.62 \pm 0.22)$ Mpc; $(14.60 \pm 0.44) \times 10^3$ km/s |
| eBOSS DR14      | 1.52    | $D_V$       | $(3843 \pm 147)(r_d/r_{d, fid})$ Mpc |

TABLE I: BAO distance measurements used in this work.

The angular diameter distance takes the form of

$$D_A(z) = \frac{1}{1 + z} \int_0^z \frac{dz'}{H(z')}.$$  \hspace{1cm} (1)

For the comoving sound horizon at the end of the baryon drag epoch $z_d$, we take $r_d \equiv r_s(z_d)$. The volume-averaged effective distance $D_V$ is a combination of the angular diameter distance $D_A(z)$ and Hubble parameter $H(z)$,

$$D_V(z) = \left[ (1 + z)^2 D_A^2(z) \frac{cz}{H(z)} \right]^{1/3}. \hspace{1cm} (2)$$

We use the CosmoMC package [35] to sample the parameter space. Our analysis employs the same Markov Chain Monte Carlo (MCMC) formalism used in previous analyses [36–41]. We explore the WMAP-only and WMAP+BAO likelihood with MCMC simulations of the posterior distribution for the six base parameters as given in Planck Collaboration [1] [2]. This approach naturally generates the likelihoods of parameters, which are marginalized over all other fitting parameters. The six basic parameters are the baryon density today, $\Omega_b h^2$, the cold dark matter density today, $\Omega_c h^2$, 100 $\times$ approximation to $r_s/D_A$, 100$\theta_{MC}$, the reionization optical depth, $\tau$, the log power of the primordial curvature perturbations, $\ln(10^{10}A_s)$, and the scalar spectrum power-law index, $n_s$. 


III. RESULT

FIG. 1 and FIG. 2 shows 2-dimensional marginalized constraints on the six MCMC sampling parameters of the ΛCDM model and wCDM model used to explore the posterior of parameters, and plotted against the following derived parameters (the Hubble constant $H_0$, matter density parameter $Ω_m$, and late-time clustering amplitude $σ_8$). Our results are based on 9-year WMAP (TT,TE,EE+lensing). Here we plot the results using the combined datasets of 6dF+MGS+DR12(9-zbin)+DR14 (labeled by BAO). The blue contours show the constraints using 9-year WMAP data alone, and the red contours include BAO data sets (WMAP+BAO). It is easy to see the BAO method is sensitive to the change of $H_0$, $Ω_m$ and $σ_8$, so it can be effectively improve the constraint of WMAP-only.

**FIG. 1:** Likelihood contours (68% and 95%) of cosmological parameters in a flat ΛCDM model derived from WMAP and WMAP+BAO respectively.

**FIG. 2:** Likelihood contours (68% and 95%) of cosmological parameters in a flat wCDM model derived from WMAP and WMAP+BAO respectively.
FIG. 3: Confidence contours for $\Omega_m$-$H_0$ in $\Lambda$CDM and $w$CDM model using WMAP+BAO data sets.

FIG. 3 presents 68% and 95% likelihood contours for the $\Omega_m$-$H_0$ plane for the WMAP+BAO data sets. The red contours correspond to a flat $\Lambda$CDM model and the blue contours correspond to the $w$CDM model. FIG. 4 shows the marginalized likelihood distribution of $H_0$ and summarized the $H_0$ measurements from other two methods. Blue line and red lines show constraints in $\Lambda$CDM and $w$CDM model respectively using 9-year WMAP data and BAO data. Clearly, adding the recent BAO as a complementary to WMAP, our measured $H_0$ results consistent with the recent data on CMB constraints from Planck (2018), which prefer a value lower than 70 km s$^{-1}$ Mpc$^{-1}$. These shows in 3.1 and 3.5$\sigma$ tension with local measurements of Riess et al. (2018) in $\Lambda$CDM and $w$CDM framework, respectively.

Table II gives marginalized parameter constraints from the WMAP CMB spectra with and without BAO. Parameter 68% intervals in the $\Lambda$CDM model and $w$CDM model from WMAP CMB power spectra in combination with BAO. The first group is the base six parameters in $\Lambda$CDM model, which are sampled in the MCMC analysis. The second group lists the representative derived parameters ($H_0$, $\Omega_m$ and $\sigma_8$). The third group shows the $\chi^2$ of WMAP and each BAO data sets. The column labeled ‘WMAP’ is 9-year WMAP only. The first two columns give results of six parameter $\Lambda$CDM from 9-year WMAP data, with and without BAO measurements. The last two columns give results in $w$CDM framework from WMAP data only and when BAO are added. Adding BAO measurements to WMAP, we constrain cosmological parameters $\Omega_m = 0.298 \pm 0.005$, $H_0 = 68.36^{+0.53}_{-0.52}$ km s$^{-1}$ Mpc$^{-1}$ (0.78 % precision), $\sigma_8 = 0.8170^{+0.0159}_{-0.0175}$ for a flat $\Lambda$CDM model, and $\Omega_m = 0.302 \pm 0.008$, $H_0 = 67.63 \pm 1.30$ km s$^{-1}$ Mpc$^{-1}$ (1.93 % precision), $\sigma_8 = 0.7988^{+0.0345}_{-0.0338}$ for a flat $w$CDM model. The combined constraint on $w$ from WMAP+BAO in a flat $w$CDM model is $w = -0.96 \pm 0.07$. Compared with the WMAP alone analysis, the WMAP+BAO analysis reduces the error bar by 75.4% in $\Lambda$CDM model and 95.3% in $w$CDM model.
IV. SUMMARY AND DISCUSSION

In this paper, we determine the Hubble constant $H_0$ using the cosmic microwave background (CMB) data from WMAP and the latest baryon acoustic oscillation (BAO) measurements in a spatially flat $\Lambda$CDM and wCDM cosmology. Adding BAO measurements to WMAP, we constrain cosmological parameters $\Omega_m = 0.298 \pm 0.005$, $H_0 = 68.36^{+0.53}_{-0.52}$ km s$^{-1}$ Mpc$^{-1}$ (0.78 % precision), $\sigma_8 = 0.8170^{+0.0159}_{-0.0175}$ in a flat $\Lambda$ cold dark matter (CDM) model, and $\Omega_m = 0.302 \pm 0.008$, $H_0 = 67.63 \pm 1.30$ km s$^{-1}$ Mpc$^{-1}$ (1.93 % precision), $\sigma_8 = 0.7988^{+0.0345}_{-0.0338}$ in a flat wCDM model. The combined constraint on $w$ from CMB and BAO for a flat wCDM model is $w = -0.96 \pm 0.07$. By adding the recent BAO as a complement to WMAP, our measured $H_0$ results consistent with the recent data on CMB constraints from Planck (2018), which prefer a value lower than 70 km s$^{-1}$ Mpc$^{-1}$. These shows in 3.1 and 3.5σ tension with local measurements of Riess et al. (2018) in $\Lambda$CDM and wCDM framework, respectively. Compared with the WMAP alone analysis, the WMPA+BAO analysis reduces the error bar by 75.4% in $\Lambda$CDM model and 95.3% in wCDM model.

Our results indicate that the combination of WMAP and BAO datasets gives a tight constraint on the Hubble constant comparable to that adopting Planck data. In order to soften the model-dependent constraint using CMB and BAO data, we also extend our analysis to use general dark energy model (wCDM cosmology), but there is still a significant tension between the global fitting CMB and BAO datasets and local determination.

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