Standard Temperature and Pressure Superconductivity

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Abstract Superconductivity at standard temperature and pressure is far from the extreme conditions where new fundamental laws of physics are expected to arise [1]. Yet it is widely believed that the Landau-Ginzburg-Wilson-Fisher paradigm of broken symmetry and renormalization does not give a satisfactory account of the phenomenon [2]. Almost a decade ago, we used the Bardeen-Cooper-Shrieffer wavefunction to show that superconductors have topological order [3]. Here we report progress using topological order to look for standard temperature and pressure superconductivity.

Keywords superconductivity · hydrides · entanglement · topological

1 Introduction

The recent discovery of conventional superconductivity at 203 K at high pressures in the sulfur hydride system [4] raises the possibility of observing quantum effects on a truly macroscopic scale [5] at standard temperature (273.15K) and pressure (1 bar). Perhaps the most remarkable of these effects are those which show the significance of the electromagnetic potential $A$ in quantum theory [6]. The electric current density $j = -A/\mu_0\lambda_L^2$ in a superconductor, where, $\lambda_L$ is the penetration depth and $\mu_0$ the magnetic permeability of free space, is such an effect [7].

Conventional superconductors with transition temperatures close to standard temperature have Fermi pressure in the normal state far above standard pressure [5] [8]. This follows from the hierarchy $T_F \approx 3.5 \times 10^4 T_c$ between the Fermi temperature of the normal state $T_F$ and the critical temperature $T_c$ for the transition into the superconducting state for conventional superconductors. Taking the transition temperature close to standard, we find Fermi pressure $P_F = 100$ Mbar on the scale of the high pressures used by Drozdov et al. [4] to stabilize the superconducting sulfur hydride system. Further, the Fermi wavelength $\lambda_F = 2\pi/k_F \approx 0.06$ nm is roughly the size of the hydrogen atom in its ground state. This result supports the suggestion [10] that hydrogen dominant metallic alloys are a good place to look for conventional superconductivity at temperatures close to standard. From these estimates, we conclude that superconductivity at standard temperature and pressure cannot be conventional.

Conventional or not, all superconductors have topological order [3] [11] [12] [13] [14]. In a simply connected system, that is, one without holes or handles, the ground state wave function responds rigidly to transverse electromagnetic potentials $\nabla \cdot A = 0$ with irrotational, dissipationless flow of electric current [5] [7]. For a multiply connected system, such as ring, we have more than one superconducting ground state and the number of such states does not change when we stretch or bend the system while leaving unchanged the number of holes and handles [15] [16].

2 Method

We look for topological order using quantum entanglement [3] [17]. First, we glue together the two ends of a pair of superconducting cylinders $A$ and $B$ to make a hollow ring. Next we do a singular value (Schmidt) decomposition

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of the superconducting quantum ground state on the ring $|\psi\rangle$:

$$|\psi\rangle = \sum_i \sqrt{\lambda_i} |\phi_i^A\rangle|\phi_i^B\rangle,$$

(1)

Here, the singular values $\lambda_1, \lambda_2, \ldots, \lambda_r$ are real, non-negative numbers where $r$ is the rank of the quantum state. The states $\{|\phi_i^A\rangle\}$ form an orthonormal basis spanning the sub-space of quantum states of $A$ which fuse to form $|\psi\rangle$. Similarly $\{|\phi_i^B\rangle\}$ span the states in $B$ which participate in the fusion. Finally, we analyze the entanglement spectrum $\{\lambda_i\}$ for topological order. In particular, we look for spectral flow of the singular values $\lambda_i$ as we change the momentum around the compact direction of the cylinder in the state $|\phi_i^A\rangle$, or equivalently, $|\phi_i^B\rangle$.

For simplicity, we take the superconductor to have $p_x + ip_y$ pairing between electrons on the square lattice, motivated by the observation of such pairing in superfluid helium-three, superconducting strontium ruthenate, and the quantum Hall liquid at filling fraction $\nu = 5/2$[3]. The pairing comes with two distinct topological orders: weak-pairing, called BCS after Bardeen-Cooper-Schrieffer, and strong-pairing, called BEC after Bose-Einstein-Condensate. We use the BCS wave function to describe both phases[3]. The long-range entanglement of Cooper pairs causes flow in the entanglement spectrum of the weak-pairing (BCS) phase but not in the strong-pairing phase (BEC). The strong-pairing phase has Abelian topological order which requires more subtle techniques to detect[3].

3 Results

The spectral flow for $p_x + ip_y$ superconductor on the square lattice in both weak-pairing and strong-pairing phases is shown in Fig. 1. We plot the single-particle eigenvalues $f(E_{\text{cut}}(q))$ as a function of the momentum along the compact direction of the cylinder $q$. Here, the momentum around the cylinder $q$ are measured relative to the Brillouin zone edge $\pi/a$ with $a$ the lattice constant. We set the Planck constant $\hbar = 1$ and note that $f(E_{\text{cut}}) = (\exp(E_{\text{cut}}) + 1)^{-1}$, is the Fermi function.

In particular, the singular vectors are product states of the form $\prod_q |n_1(q), n_2(q), \ldots n_L(q)\rangle$, where, $L$ is the length of the cylinder, and the occupation numbers $n_i(q)$ take the values 0 (do not occupy mode $i$) or 1 (occupy mode $i$), with $i = 0, 1, \ldots, L - 1$. The singular values in (1) may then be written as products over momenta and mode number, and labelled by the occupation numbers $\{n_i(q)\}$:

$$\lambda(\{n_i(q)\}) = \prod_{q,i} (n_i(q)f(-E_i(q))+(1-n_i(q))f(E_i(q))).$$

(2)

where, the single-particle eigenvalues $f(E_i(q))$ for each $i = 0, 1, \ldots, L - 1$ are plotted in Fig. 1. Notice that the single-particle singular values fall into bands labelled by the index $i = 0, 1, \ldots, L - 1$, where, $L$ is the length of the cylinder. In particular, in the weak-pairing phase but not the strong pairing phase, we find a mode with $E(q) \to 0$ as $q \to 0$. This indicates a spectral flow and a strong entanglement between the two halves of the ring in the weak-pairing phase shown in Fig. 1(a) but not in the strong-pairing phase shown in Fig. 1(b).

We find that the dispersion of this mode is robust to changes of the ground-state wavefunction which leave the $p_x + ip_y$ superconductor in the weak pairing phase. For example, we may increase the size of the superconducting energy gap $\Delta$ and decrease the Fermi temperature $T_F$ without leaving the weak-pairing phase[3]. It follows that the dispersing mode in Fig. 1(a) remains as a signature of the weak-pairing phase even for superconductors where $T_c/T_F$ is larger than the conventional value.

4 Conclusion

We reviewed recent progress in using quantum entanglement to detect topological order in superconducting quantum states. While topological order is a generic property of all superconductors, we found a subtle distinction between weak-pairing (BCS) and strong-pairing (BEC) in the case of $p_x + ip_y$ pairing. In particular, the entanglement spectrum exhibits spectral flow for the non-Abelian topological ordered case of weak-pairing (BCS). For strong-pairing (BEC) another method is needed to detect its Abelian topological order.

We imagine it may be interesting to turn the calculation around and use the dispersing zero mode in the entanglement spectrum of the weak-pairing phase as a sensitive test for superconductivity with long-range quantum entanglement of the Cooper pairs. In particular, we find that the dispersing zero mode is robust to changes in the superconducting wavefunction which take it out of the conventional regime of small superconducting gap and large normal state Fermi temperature. We envision using long-range quantum entanglement of Cooper pairs as a tool to search through the vastness of Hilbert space and find promising superconducting wavefunctions whose superconductivity will persist at standard temperature and pressure.

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Fig. 1 Entanglement spectrum (singular value decomposition) $f(E_{ent})$ of $p_x + ip_y$ superconductor on a hollow ring in (a) the weak pairing (BCS) and (b) strong pairing (BEC) phase as a function of momentum $qa/\pi$ along the compact direction of the cylinder formed when we cut the ring into two cylinders by making cuts at opposite sides of the ring. Notice the lowest singular value at each momentum $q$ disperses towards $E_{ent} \rightarrow 0$ as $q \rightarrow 0$ in (a) but not in (b). In both phases, we find $L$ modes at each momentum $q$, where $L$ is the length of each cylinder. The momenta $q$ are quantized by the circumference of the cylinder but form continuous bands labelled by the mode number. The lowest mode in (a) has wavefunction localized near the edge of the cylinder. The modes at higher $f(E_{ent})$ are spread through the bulk of the cylinder away from the cuts which divide the original ring into two cylinders.

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