Gerrymandering under Uncertain Preferences (Student Abstract)

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Abstract
Gerrymandering is the manipulating of redistricting for political gain. While many attempts to formalize and model gerrymandering have been made, the assumption of known voter preference, or perfect information, limits the applicability of these works to model real world scenarios. To more accurately reason about gerrymandering we investigate how to adapt existing models of the problem to work with imperfect information. In our work, we formalize a definition of the gerrymandering problem under probabilistic voter preferences, reason about its complexity compared to the deterministic version, and propose a greedy algorithm to approximate the problem in polynomial time under certain conditions.

Introduction
Gerrymandering refers to manipulating district assignments such that one candidate or political party gains some kind of advantage. This advantage manifests as a substantial difference in the proportional of seats won by a candidate and the proportion of votes won. Gerrymandering is a commonly used tactic in many countries around the world including the United States where a recent example includes North Carolina, where a state court ruled the district map unconstitutional under the North Carolina State constitution in 2019 (Wines 2019). Some court cases have restricted gerrymandering, but it still remains widely used and so developing models with which we are able to reason about the effect and potential of gerrymandering remains critically important. While existing work has been done into formalizing a definition of gerrymandering, much of the work makes the assumption of constant and known voter preferences. This assumption allows models to be simpler and reflects how many people will vote, but we believe it also limits the applicability of these existing models to real world scenarios.

In particular, we use a definition of gerrymandering as a decision problem from (Cohen-Zemach, Lewenberg, and Rosenschein 2018), $A_{GM}$, and extend it to include voters with non-constant preferences. $A_{GM}$, shown to be NP-Complete, was originally defined to be whether a graph of voters could be partitioned to create connected components (districts) such that a given candidate would win a specific number of districts in the election.

Problem Definition
We define $PROB-GERRY$ as follows: given an undirected graph of voters, each with a probability distribution over the linear orders of candidates, can we partition the graph to create a set of connected components corresponding to a valid district assignment? A valid district assignment must ensure that a target candidate has at least a given probability to win a specified number of districts, as well at most another probability to lose a specified number of districts under a given voting rule. Lastly, there is a restriction placed on the ratio of the size from the largest district to the smallest district. We include two bounds, a lower bound on number of districts won, and an upper bound on number of districts lost. Adding both of these conditions ensures that the candidate has sufficient odds to both win by a given margin, but also never to lose by a given margin as well. A potentially gerrymandered map and district assignment is given for both the deterministic and non-deterministic version of the problem are given in 2. Other assignments of the probabilistic map give the same chance for green to win 3+ districts, but also increase the chance green loses 4+ districts.

Problem Complexity

Theorem 1 $PROB-GERRY$ is NP-Hard
To prove this, we show that there exists a reduction from $A_{GM}$ to $PROB-GERRY$ by creating an instance of $PROB-GERRY$ where voters are given constant preferences. Since...
Algorithm 1: Greedy Algorithm for PROB-GERRY

Initialize a new voter graph $G'$ with no edges

while Number of districts is more than desired amount do

    $e = \arg\max_{e \in E(G) \setminus E(G')} \Pr(\text{candidate wins } \frac{1}{k} \text{ proportion of current districts}) \text{ given that } \text{RatioCap}(G' \cup \{e\}) > r$

    if $e = \text{None}$ then

        $e = \arg\min_{e \in E(G) \setminus E(G')} \text{RatioCap}(G' \cup \{e\})$

        $E(G') = E(G') \cup \{e\}$

end while

Testing and Results

We tested our greedy algorithm by generating graphs of voters. Each voter was assigned a position in $[0, 1]^2$ from a normal distribution, and two candidates were created at $(0.75, 0.75)$ and $(0.25, 0.25)$. We generated graphs using the procedure described in (Cohen-Zemach, Lewenberg, and Rosenschein 2018). We then specified our problem to draw 5 districts and see if there is a way that candidate 1 can win 3+ districts with 70% probability and lose 3+ districts with 25% probability. We used the Plackett-Luce model to assign voters a probability to each linear order of candidates using the distance from each candidate to a voter. We analyzed the success rate of finding an assignment on 1000 random graphs of 100 voters while changing the ratio cap (figure 3). We conclude that the ratio cap strongly determined the chance of success and that improvements in how to handle this cap is important in developing a more applicable algorithm. We also analyzed running time based on number of voters and candidates to confirm that our algorithm scaled in polynomial and exponential time respectively. These results indicated that even with ideal conditions, large numbers of voters or even just a few candidates causes even this greedy algorithm to take large amounts of time.

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