**B → K⁺ Transition Form Factors and the Semi-leptonic Decay B → K⁺μ⁺μ⁻**

Hai-Bing Fu¹, Xing-Gang Wu¹,², * and Yang Ma¹

¹ Department of Physics, Chongqing University, Chongqing 401331, P.R. China and
² State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, P.R. China

(Dated: November 25, 2014)

We present a detailed calculation on $B \rightarrow K^+$ transition form factors (TFFs), $V$, $A_{0,1,2}$ and $T_{2,3}$, within the QCD light-cone sum rules (LCSR). To suppress the contributions from high-twist LCDAs, we adopt a chiral correlator to do the LCSR calculation. The resultant LCSR for $B \rightarrow K^+$ TFFs do show slight dependence on those uncertain high-twist LCDAs. We suggest a convenient model for the transverse-leading-twist LCDA $\phi_{K^*}^T$, in which the parameter $B_{K^*}^T$ dominantly controls its longitudinal distribution. It is found that a single-peaked $\phi_{K^*}^T$ with $B_{K^*}^T \sim (0,0,0.1)$ leads to a better agreement with the lattice QCD predictions on $B \rightarrow K^+$ TFFs. As an application of those TFFs, we predict the branching fraction for the $B$-meson rare decay $B \rightarrow K^*\mu^+\mu^-$. The predicted differential branching fractions for $B_{K^*}^T \in [0.0,0.2]$ are consistent with the LHCb and Belle measurements within errors. And the integrated branchiong fraction $B(B \rightarrow K^*\mu^+\mu^-)$ decreases with the increment of $B_{K^*}^T$. If setting $B_{K^*}^T \sim 0.1$, we observe that the total and differential branching fractions show a better agreement with the LHCb measurement.

PACS numbers: 12.38.Aw, 13.25.Hw, 11.55.Hx

### I. INTRODUCTION

The $B$-meson rare decays mediated by the penguin-induced flavor-changing-neutral-current transition provide excellent platform for precision tests of standard model (SM) and for probing new physics beyond the SM. Those decays are always suppressed by the loop effects and by the Cabibbo-Kobayashi-Maskawa matrix elements, thus, contributions from new physics interactions could be significant and their effects to the decay rates could be observed. Among those decay channels, the $B$-meson semi-leptonic decay, $B \rightarrow K^*\mu^+\mu^-$ with $K^+ \rightarrow K\pi$, arouses people's great interest. Its measurable quantities include the muon forward-backward asymmetry, the longitudinal polarization fraction, the differential and total branching fractions, and etc. Theoretically, those observables have been studied by using the effective Hamiltonian with or without new physics contributions to the Wilson coefficients, cf. Refs. [1-17]. Experimentally, many measurements have been done by the CDF collaboration [18-20], the BABAR collaboration [21], the Belle collaboration [22], the LHCb collaboration [23-28], the ATLAS collaboration [29], and the CMS collaboration [30], respectively.

More specifically, the Belle collaboration gives the total branching fraction $B(B \rightarrow K^*\ell^+\ell^-) = (1.07^{+0.11}_{-0.10} \pm 0.09) \times 10^{-6}$ ($\ell = e, \mu$) [22] and the LHCb collaboration gives $B(B \rightarrow K^*\mu^+\mu^-) = (1.16 \pm 0.19) \times 10^{-6}$ [23-25].

Before introducing any new physics phenomenon, it is best to have a more precise estimation within the SM. To achieve a precise SM determination of $B \rightarrow K^*\mu^+\mu^-$, we are facing the problem of determining the non-perturbative hadronic matrix elements, or the $B \rightarrow K^*$ transition form factors (TFFs). There are totally seven transition form factors for the $B \rightarrow \text{vector meson}$ decays, which are main error sources for the SM predictions. In Table I, we present the relationships among the non-perturbative matrix elements and the TFFs. Here $V$ stands for the TFF defined via a vector current, $A_{0,1,2}$ are the TFFs defined via an axial-vector current, $T_1$ is the TFF defined via a tensor current, and $T_{2,3}$ are the TFFs defined via an axial-tensor current. Those $B \rightarrow K^*$ TFFs have been studied under various frameworks, such as the relativistic quark model [31, 32], the QCD light-cone sum rules (LCSR) [33-37], or the lattice QCD [38-41].

| Matrix element | TFFs | Relevant decay(s) |
|----------------|------|-------------------|
| $\langle V|\bar{q}\gamma^\mu b|B \rangle$ | $V$ | $B \rightarrow (\rho/\omega)\ell\nu_\ell$ |
| $\langle V|\bar{q}\gamma^\mu\gamma^5 b|B \rangle$ | $A_0, A_1, A_2$ | $B \rightarrow K^*\ell^+\ell^-$ |
| $\langle V|\bar{q}\sigma^{\mu\nu}q_b|B \rangle$ | $T_1$ | $B \rightarrow K^*\gamma$ |
| $\langle V|\bar{q}\sigma^{\mu\nu}\gamma^5 q_b|B \rangle$ | $T_{2,3}$ | $B \rightarrow K^*\ell^+\ell^-$ |

**TABLE I.** The relations among the $B \rightarrow \text{vector meson}$ TFFs to the matrix elements and their typical applications for the $B$-meson semileptonic and radiative decays, where $\ell$ stands for the light lepton $e$ or $\mu$. 

The QCD LCSR is applicable for low and intermediate $q^2$ region, which can be conveniently extended to all allowable $q^2$ region. Thus a more precise LCSR prediction with less theoretical uncertainties shall be helpful for a better understanding of those TFFs and the $B \rightarrow K^*\mu^+\mu^-$ decay. The LCSR is based on the operator product expansion (OPE) near the light cone, its non-perturbative dynamics can be parameterized into the light-cone distribution amplitudes (LCDAs) with increasing twists. Several approaches have been adopted to study the vector meson LCDAs, such as the $\rho$- and...
where \( e^{(\lambda)} \) stands for \( K^* \)-meson polarization vector with \( \lambda \) being its transverse (\( \perp \)) or longitudinal (\( \parallel \)) component, respectively. \( p \) is the \( K^* \)-meson momentum and \( q = p_B - p_{K^*} \) is the momentum transfer between \( B \)-meson and \( K^* \)-meson. There are some relations among those TFFs, thus not all of them are independent, i.e.,

\[
A_3(q^2) = \frac{m_B + m_{K^*}}{2m_{K^*}} A_1(q^2) - \frac{m_B - m_{K^*}}{2m_{K^*}} A_2(q^2),
\]

\[
T_3(q^2) = \frac{m_B^2 - m_{K^*}^2}{q^2} [\bar{T}_3(q^2) - T_2(q^2)].
\]

And at the large recoil region \( q^2 = 0 \), we have

\[
A_0(0) = A_3(0)
\]

\[
T_1(0) = T_2(0) = \bar{T}_3(0) = T(0).
\]

To derive the LCSRs for those TFFs, we need to deal with the following two correlators:

\[
\Pi_{\mu}(p, q) = \frac{1}{i} \int d^4 x e^{i q \cdot x} \langle K^*(p, \lambda)| T \{ \bar{s}(x) \gamma_{\mu} (1 - \gamma_5) b(x) \rangle \langle 0 | b(0)(1 + \gamma_5) q_1(0) \rangle | 0 \rangle,
\]

\[
\Pi_{1\mu}(p, q) = -i \int d^4 x e^{i q \cdot x} \langle K^*(p, \lambda)| T \{ \bar{s}(x) \sigma_{\mu\nu} q^\nu (1 + \gamma_5) b(x) \rangle \langle 0 | b(0)(1 + \gamma_5) q_1(0) \rangle | 0 \rangle,
\]

where \( q_1 \) stands for \( u \) or \( d \) quark. The correlators (7,8) are analytic \( q^2 \)-functions defined at both space-like and time-like \( q^2 \)-region.

On the one hand, in the time-like region, the long-distance quark-gluon interactions become important and, eventually, the quarks form hadrons. In this region, one can insert a complete series of intermediate hadronic states in the correlator and obtain its hadronic representation by isolating the pole term of the lowest pseudoscalar \( B \)-meson:

\[
\Pi^{H\mu}(p, q) = \Pi^{H(1)\mu}_{1}(p, q) + \Pi^{H(2)\mu}_{2}(p, q) + \Pi^{H(3)\mu}_{3}(p, q) + \Pi^{H(4)\mu}_{4}(p, q) + \sum_{j} \langle K^*| \bar{s}\gamma_{\mu} (1 - \gamma_5) b| B \rangle \langle B | bim_{6}(1 + \gamma_5) q_1(0) | 0 \rangle \frac{m_{B_{1j}}^2 - (p + q)^2}{m_{B_{1j}}^2 - (p + q)^2}.
\]
\[ \Pi^{H(1)}_\mu(p, q) = i \Pi^{H(1)}_1 \epsilon^\mu_\nu \epsilon^{\nu}(\lambda) q_\alpha p_\beta + \Pi^{H(1)}_2 \epsilon^\mu_\nu - \Pi^{H(1)}_3 (\epsilon^{\nu}(\lambda) \cdot q)(2p + q)_{\mu} + \Pi^{H(1)}_4 (\epsilon^{\nu}(\lambda) \cdot q)_{\mu} \]
\[ = \langle K^* \bar{s} \sigma_{\mu\nu} q' (1 + \gamma_5) \gamma_5 \bar{B} | B | bim_b \gamma_5 q_1 | 0 \rangle \]
\[ = \frac{(K^* \bar{s} \sigma_{\mu\nu} q' (1 + \gamma_5) \gamma_5 \bar{B} | B | bim_b \gamma_5 q_1 | 0 \rangle}{m_\bar{B} - (p + q)^2} \]
\[ + \int \frac{m_\bar{B} m_B f_B m_B + m_K^*}{m_\bar{B} - (p + q)^2} A_1(q^2) \left[ \frac{m_\bar{B} f_B}{m_\bar{B} - (p + q)^2} \right] \]
\[ + \int_{s_0}^\infty \frac{\rho_1^{H(1)}}{s - (p + q)^2} ds + \text{subtractions}, \quad (11) \]

\[ \Pi^{H(1)}_1[q^2, (p + q)^2] = \frac{m^2_B f_B(m_B + m_K^*)}{m^2_B - (p + q)^2} A_1(q^2) \]
\[ + \int_{s_0}^\infty \frac{\rho_1^{H(1)}}{s - (p + q)^2} ds + \text{subtractions}, \quad (12) \]

\[ \Pi^{H(1)}_2[q^2, (p + q)^2] = \frac{m^2_B f_B A_2(q^2)}{(m_B + m_K^*)[m^2_B - (p + q)^2]} \]
\[ + \int_{s_0}^\infty \frac{\rho_2^{H(1)}}{s - (p + q)^2} ds + \text{subtractions}, \quad (13) \]

\[ \Pi^{H(1)}_3[q^2, (p + q)^2] = \frac{2m^2_B f_B m_K^*[A_3(q^2) - A_0(q^2)]}{q^2[m^2_B - (p + q)^2]} \]
\[ + \int_{s_0}^\infty \frac{\rho_3^{H(1)}}{s - (p + q)^2} ds + \text{subtractions}, \quad (14) \]

\[ \Pi^{H(1)}_4[q^2, (p + q)^2] = \frac{2m^2_B f_B V(q^2)}{(m_B + m_K^*)[m^2_B - (p + q)^2]} \]
\[ + \int_{s_0}^\infty \frac{\rho_4^{H(1)}}{s - (p + q)^2} ds + \text{subtractions}, \quad (15) \]

where \( (B | bim_b \gamma_5 q_1 | 0) = m^2_B f_B \) with \( f_B \) standing for the \( B \)-meson decay constant. The invariant amplitudes \( \Pi^{H(1)}_{1,2,3,4} \) and \( \Pi^{H(1)}_{1,2,3,4} \) are

\[ \Pi^{H(1)}_{2B}(q^2, (p + q)^2) = \frac{m^2_B f_B(m^2_B - m^2_K^*)}{m^2_B - (p + q)^2} T_2(q^2) \]
\[ + \int_{s_0}^\infty \frac{\rho_2^{H(1)}}{s - (p + q)^2} ds + \text{subtractions}. \quad (16) \]

For convenience, in Eq.(15), we have used \( T_i(q^2) \) \( (i = 1, 3, 4) \) to stand for \( 2T_i(q^2) \), \( \bar{T}_3(q^2) \), and \( T_3(q^2) \), respectively. Here we have replaced the contributions of higher resonances and continuum states with the dispersion integrations. The continuum threshold parameter \( s_0 \) is set as the value near the squared mass of the lowest scalar \( B \)-meson. The spectral densities \( \rho_{1,2,3,4} \) can be approximated by applying the conventional quark-hadron duality ansatz

\[ \rho_{1,2,3,4}^{H(1)} = \rho_{QCD(1)}^{H(1)} \theta(s - s_0). \quad (17) \]

On the other hand, in the space-like region, the correlator can be calculated by using the QCD OPE. In this region, we have \( (p + q)^2 - m^2_B \leq 0 \) with the momentum transfer \( q^2 \sim O(1 \text{ GeV}^2) \ll m^2_B \), which corresponds to small light-cone distance \( x^2 \sim 0 \) and ensures the validity of OPE. The full \( b \)-quark propagator within the background field approach states

\[ \langle 0 | T \{ b(x) \bar{b}(0) \} | 0 \rangle = i \int \frac{d^d k}{(2\pi)^d} e^{-ikx} \frac{k + m_b}{m_b^2 - k^2} \]
\[ -ig_s \int \frac{d^d k}{(2\pi)^d} e^{-ikx} \int_0^1 dv G_{\mu\nu}(vx) \]
\[ \times \left[ \frac{1}{2} \frac{k + m_b}{m_b^2 - k^2} \sigma_{\mu\nu} + \frac{v}{m_b^2 - k^2} x_{\mu\nu} \right], \quad (18) \]

where \( G_{\mu\nu} \) is the gluonic field strength and \( g_s \) denotes the strong coupling constant. Using this \( b \)-quark propagator and carrying out the OPE for the correlator, we obtain the QCD expansion of \( \Pi_{QCD}^{H(1)}(\mu) \) with 2-particle and 3-particle Fock states’ contributions,
![Image of a page of a document with mathematical equations and symbols. The equations are related to quantum field theory, specifically to the electromagnetic form factors of the $K^*$ meson and their correlation functions. The text includes integrals, products, and various mathematical notations. The context of the document suggests it is discussing the calculation of these form factors, possibly within the framework of perturbative QCD or a similar theoretical framework. The integral expressions involve products of matrices and functions, likely representing transition amplitudes or correlation functions in the theory. The page also contains references to specific terms and constants, such as $\bar{G}_{\mu
u}(x) = \epsilon_{\mu
u\alpha\beta}G^{\alpha\beta}(x)/2$. Up to twist-4 accuracy, the non-zero meson-to-vacuum matrix elements with various $\gamma$-structures, i.e. $\Gamma = 1, i\gamma_5$ and $\sigma_{\mu\nu}$, can be expanded as [48]:

$$\langle K^*(p, \lambda)\bar{s}(x)\sigma_{\mu\nu}q_1(0)|0\rangle = -if_{K^*}^\perp \int_0^1 du e^{iu(p\cdot x)} \left\{ \left( e^{\nu}_\mu(p) - e^{\nu}_\mu(p) \right) \left[ \phi_{2K^*}^\perp(u) + \frac{m_{K^*}^2 x^2}{16} \phi_{4K^*}^\perp(u) \right] ight. 
+ \left(p_\mu x_\nu - p_\nu x_\mu\right) e^{(\lambda), x}_{\mu}(p) \left[ \phi_{3K^*}^\perp(u) - \frac{1}{2} \phi_{2K^*}^\perp(u) - \frac{1}{2} \psi_{4K^*}^\perp(u) \right],
$$

where $f_{K^*}^\perp$ represents the $K^*$-meson decay constant,

$$\langle K^*(p, \lambda)\bar{s}(x)\sigma_{\alpha\beta}g_\mu G^{\mu\nu}(x)\bar{q}_1(0)|0\rangle = m_{K^*}^2 f_{K^*}^\perp \left[ p_\mu \left( p_\alpha g_{\beta\nu} - p_\beta g_{\alpha\nu} \right) - p_\nu \left( p_\alpha g_{\beta\mu} - p_\beta g_{\alpha\mu} \right) \right] \times \Phi_{3K^*}^\perp, (v, p, x),$$

and have set

$$g_{\mu\nu}^\perp = g_{\mu\nu} - \frac{p_\mu x_\nu + p_\nu x_\mu}{p \cdot x},$$

$$e^\mu = e^\mu_p \frac{x_\mu}{p \cdot x} \left( p_\mu - \frac{m_{K^*}^2}{2(p \cdot x)} x_\mu \right) + e^\mu_{1\mu},$$

$$K(v, p, x) = \int D\alpha e^{ipx(\alpha_1 + \alpha_3)} K(\alpha).$$

Here $D\alpha = d\alpha_1 d\alpha_2 d\alpha_3 (1 - \alpha_1 - \alpha_2 - \alpha_3)$ and $K(\alpha)$ stands for the twist-3 or twist-4 DA $\Phi_{3K^*}^\perp, (\alpha), \Psi_{4K^*}^\perp, (\alpha)$ or $\Psi_{4K^*}^\perp, (\alpha)$, in which $\alpha = \{\alpha_1, \alpha_2, \alpha_3\}$ corresponds to the momentum fractions carried by the antiquark, quark and gluon, respectively.

Then, by equating the correlators within different $q^2$-regions and by applying the conventional Borel transformation, we obtain the required LCSRs for the $B \to K^*$ TFFs, which are listed in the following equations:

$$A_1(q^2) = \frac{m_B m_{K^*} f_{K^*}^\perp}{f_B m_B^2 (m_B + m_{K^*})} \left\{ \int_0^{1} du e^{\frac{m_B^2}{2M^2} u} \left[ \Theta(c(u, s_0)) \phi_{2K^*}^\perp(u) + \Theta(c(u, s_0)) \psi_{3K^*}^\perp(u) - \frac{1}{4} \right] \right. 
\times \left[ \frac{C - 2m_B^2}{u^2 M^2} \Theta(c(u, s_0)) - \frac{1}{u} \Theta(c(u, s_0)) \right] \phi_{4K^*}^\perp(u) - 2 \left[ \frac{C}{u^2 M^2} \Theta(c(u, s_0)) - \frac{1}{u} \right] \psi_{4K^*}^\perp(u) 
\times \Theta(c(u, s_0)) \right\} I_2(u) = \left[ \frac{2m_B^2}{u^2 M^2} \Theta(c(u, s_0)) + \Theta(c(u, s_0)) \right] H_3(u) + \int D\alpha e^{ipx(\alpha_1 + \alpha_3)} K(\alpha).$$

$$- \frac{1}{2X^2} \Theta(c(X, s_0)) \left(4v - 1\right) \Psi_{4K^*}^\perp, (\alpha) - \Psi_{4K^*}^\perp, (\alpha) \right\}.$$
\[ A_2(q^2) = \frac{m_b(m_B + m_{K^*})m_{K^*}^2 f_{K^*}}{f_{B} m_B^2} \left\{ \int_0^1 \frac{du}{u} e^{\frac{m_B^2 - s(u)}{m_B}} \left[ \frac{1}{m_{K^*}} \Theta(c(u, s_0)) \phi_{2,K^*}^1(u, \mu) - \frac{1}{M^2} \bar{\Theta}(c(u, s_0)) \psi_{3,K^*}^2(u) \right] \\
- \frac{1}{4} \left[ \frac{m_B^2}{u^2 M^4} \bar{\Theta}(c(u, s_0)) + \frac{1}{u M^2} \bar{\Theta}(c(u, s_0)) \phi_{2,K^*}^1(u) + 2 \left[ \frac{C - 2m_B^2}{u^2 M^4} \bar{\Theta}(c(u, s_0)) - \frac{1}{u M^2} \bar{\Theta}(c(u, s_0)) \right] \times I_L(u) - \frac{1}{M^2} \bar{\Theta}(c(u, s_0)) H_3(u) \right\} + \int D\alpha_i \int_0^1 dv e^{\frac{m_B^2 - s(u)}{M^2}} \theta(c(X, s_0)) \left[ (4\nu - 1) \Psi_{4,K^*}^2(\alpha) - \bar{\Psi}_{4,K^*}^2(\alpha) + 4\nu \Phi_{4,K^*}^2(\alpha) \right], \]

\[ V(q^2) = \frac{m_b(m_B + m_{K^*})f_{K^*}}{f_B m_B^2} \left\{ \int_0^1 \frac{du}{u} e^{\frac{m_B^2 - s(u)}{M^2}} \left[ \Theta(c(u, s_0)) \phi_{2,K^*}^1(u, \mu) - \frac{m_B^2}{4u^2 M^4} \bar{\Theta}(c(u, s_0)) + \frac{1}{u M^2} \right] \times \bar{\Theta}(c(u, s_0)) \phi_{2,K^*}^1(u) \right\}, \]

\[ A_3(q^2) - A_0(q^2) = \frac{m_b m_{K^*} f_{K^*}}{2f_B m_B^2} \left\{ \int_0^1 \frac{du}{u} e^{\frac{m_B^2 - s(u)}{M^2}} \left\{ - \frac{1}{m_{K^*}} \Theta(c(u, s_0)) \phi_{2,K^*}^1(u, \mu) - \frac{m_B^2}{4u^2 M^4} \Theta(c(u, s_0)) \phi_{2,K^*}^1(u) - \frac{2}{u M^2} \bar{\Theta}(c(u, s_0)) \phi_{2,K^*}^1(u) \right\} \times \bar{\Theta}(c(u, s_0)) I_L(u) - \frac{1}{M^2} \bar{\Theta}(c(u, s_0)) H_3(u) + \int D\alpha_i \int_0^1 dv e^{\frac{m_B^2 - s(u)}{M^2}} \left[ \frac{5}{4X^2 M^2} \bar{\Theta}(c(X, s_0)) \Psi_{4,K^*}^2(\alpha) \right], \]

\[ T_1(q^2) = \frac{m_B^2 m_{K^*}^2 f_{K^*}}{m_B^2 f_B} \left\{ \int_0^1 \frac{du}{u} e^{\frac{m_B^2 - s(u)}{M^2}} \left\{ \frac{1}{m_{K^*}} \Theta(c(u, s_0)) \phi_{2,K^*}^1(u, \mu) - \frac{m_B^2}{4u^2 M^4} \Theta(c(u, s_0)) \phi_{2,K^*}^1(u) - \frac{2}{u M^2} \bar{\Theta}(c(u, s_0)) \phi_{2,K^*}^1(u) \right\} \times \bar{\Theta}(c(u, s_0)) I_L(u) - \frac{1}{M^2} \bar{\Theta}(c(u, s_0)) H_3(u) \right\}, \]

\[ T_2(q^2) = \frac{m_B^2 f_{K^*}^2}{m_B^2 f_B} \int_0^1 \frac{du}{u} e^{\frac{m_B^2 - s(u)}{M^2}} \left\{ \frac{1}{m_{K^*}} \Theta(c(u, s_0)) \phi_{2,K^*}^1(u, \mu) - \frac{m_B^2}{4u^2 M^4} \Theta(c(u, s_0)) \phi_{2,K^*}^1(u) - \frac{2}{u M^2} \bar{\Theta}(c(u, s_0)) \phi_{2,K^*}^1(u) \right\} \times \bar{\Theta}(c(u, s_0)) + \frac{2q^2}{u M^2} \bar{\Theta}(c(u, s_0)) I_L(u) - \frac{1}{M^2} \bar{\Theta}(c(u, s_0)) H_3(u) \right\}, \]

\[ T_3(q^2) = \frac{m_B^2 f_{K^*}^2}{m_B^2 f_B} \int_0^1 \frac{du}{u} e^{\frac{m_B^2 - s(u)}{M^2}} \left\{ \frac{1}{m_{K^*}} \Theta(c(u, s_0)) \phi_{2,K^*}^1(u, \mu) - \frac{m_B^2}{4u^2 M^4} \Theta(c(u, s_0)) \phi_{2,K^*}^1(u) - \frac{2}{u M^2} \bar{\Theta}(c(u, s_0)) \phi_{2,K^*}^1(u) \right\} \times \bar{\Theta}(c(u, s_0)) + \frac{4}{u M^2} \bar{\Theta}(c(u, s_0)) (m_B^2 - m_{K^*}^2) I_L(u) + \frac{2}{u M^2} \bar{\Theta}(c(u, s_0)) H_3(u) \right\}, \]

\[ \text{where } H = q^2/(m_B^2 - m_{K^*}^2) \text{ and } C = m_B^2 + u^2 m_{K^*}^2 - q^2. \]

\[ s(\varrho) = \sqrt{m_B^2 - \varrho(q^2 - m_{K^*}^2)}/\varrho \text{ with } \varrho = 1 - q, \]

\[ X = a_1 + a_3, \quad c(\varrho, s_0) = \varrho s_0 - m_B^2 + \varrho q^2 - \varrho m_{K^*}^2. \]

\[ \bar{\Theta}(c(\varrho, s_0)) \text{ is the usual step function, } \bar{\Theta}(c(\varrho, s_0)) \text{ and } \end{equation}}
\[
\int_0^1 du \frac{du}{2u^3 M^4} e^{-s(u)/M^2} \Theta(c(u, s_0)) f(u) \\
= \int_0^1 du \frac{du}{2u^3 M^4} e^{-s(u)/M^2} f(u) + \Delta(c(u_0, s_0)). (35)
\]

The surface terms \(\delta(c(u_0, s_0))\) and \(\Delta(c(u_0, s_0))\) for the 2-particle DAs are
\[
\delta(c(u, s_0)) = e^{-s_0/M^2} \frac{f(u_0)}{C_0},
\]
\[
\Delta(c(u, s_0)) = e^{-s_0/M^2} \left[ \frac{1}{2u_0 M^2} \frac{f(u_0)}{C_0} - \frac{\bar{u}_0}{2C_0} \frac{d}{du} \left( \frac{f(u)}{uC} \right) \bigg|_{u=u_0} \right],
\]
where \(C_0 = m_0^2 + u_0^2 m_K^2 - q^2\) and \(u_0\) is the solution of \(c(u_0, s_0) = 0\) with \(0 \leq u_0 \leq 1\). We do not present the surface terms for the 3-particle DAs, whose contributions are quite small and can be safely neglected. The simplified functions \(I_3(u)\) and \(H_3(u)\) are defined as
\[
I_3(u) = \int_0^u dv \int_0^v \frac{dv}{w} \frac{\phi_{2,K}^+(w)}{w} \left( - \frac{1}{2} \phi_{2,K}^+(w, \mu) \right),
\]
\[
H_3(u) = \int_0^u dv \left[ \psi_{2,K}^+(v) - \phi_{2,K}^+(v, \mu) \right]. (36)
\]

### III. NUMERICAL RESULTS AND DISCUSSION

#### A. Basic input

In doing the numerical calculations, we take the \(K^+\) decay constant \(f_{K^+} = 0.185(9)\) GeV [48], the \(b\)-quark pole mass \(m_b = 4.80 \pm 0.05\) GeV, the \(K^+\)-meson mass \(m_{K^+} = 0.892\) GeV, the \(B\)-meson mass \(m_B = 5.279\) GeV [49], and \(f_B = 0.160 \pm 0.019\) GeV [42].

As shall be shown later, the dominant contributions to LCSRs (26–33) are from the leading-twist LCDA \(\phi_{2,K^+}^\perp\). The chiral-even DAs \(\phi_{2,K^+}^\parallel, \phi_{2,K^+}^\perp, \psi_{2,K^+}^\parallel, \Phi_{2,K^+}^\parallel, \Phi_{2,K^+}^\perp\) that are at the \(\delta^1\)-order provide zero contributions; and all non-zero twist-3 and twist-4 LCDAs, which are at the \(\delta^2\)-order, can only provide less than 10% contribution to the LCSRs. Thus, theoretical uncertainties from the choices of high-twist LCDAs themselves are highly suppressed. For clarity, we take those high-twist LCDAs to be the ones suggested by Ref. [48]. The \(K^+\)-meson transverse leading-twist DA \(\phi_{2,K^+}^\perp\) can be derived from its light-cone wavefunction (LCWF). They are related via the relation
\[
\phi_{2,K^+}^\perp(x, \mu_0) = \frac{2\sqrt{3}}{f_{K^+}^\perp/C_{K^+}^\perp} \int_{|k^\perp|^2 \leq \mu_0} \frac{d|k^\perp|^2}{16\pi^3} \psi_{2,K^+}^\perp(x, k^\perp), (37)
\]
where \(f_{K^+}^\perp/C_{K^+}^\perp\) is the reduced vector decay constant with \(C_{K^+}^\perp = \sqrt{3}\).

Following the idea of Ref. [50], one can separate the \(K^\ast\)-meson LCWF into radial part and spin-space part (we call it as the WH model), whose radial part \(\psi_{R, K^\ast}^\perp\) is constructed from the BHL-prescription [51] and the spin-space part \(\chi_{K^\ast}^h h_2(x, k^\perp)\) is from the Wigner-Melosh rotation [52]. More specifically, the WH LCWF states
\[
\psi_{2,K^\ast}^\perp(x, k^\perp) = \sum_{h_1 h_2} \chi_{K^\ast}^{h_1} h_2(x, k^\perp) \psi_{R, K^\ast}^\perp(x, k^\perp), (38)
\]
where
\[
\psi_{R, K^\ast}^\perp \propto [1 + B_{2,K^\ast}^\perp \cdot C_2^\perp (\xi)] \\
\times \exp \left[ -b_{2,K^\ast}^\perp \left( \frac{k^2 + m_q^2}{x} + \frac{k^2 + m_q^2}{\bar{x}} \right) \right]. (39)
\]

The \(s\)-quark with mass \(m_s\) carries a fraction \(x\) of the meson light-cone momentum and the light-quark with mass \(m_q\) carries \(\bar{x}\) or \((1 - x)\) of the meson light-cone momentum. The spin-space wavefunction
\[
\chi_{K^\ast}^{h_1 h_2}(x, k^\perp) = \frac{\bar{x}m_s + xm_q}{\sqrt{k^2_\perp + (\bar{x}m_s + xm_q)^2}}. (40)
\]

Then, one can get the WH-DA
\[
\phi_{2,K^\ast}^\perp(x, \mu_0) = \frac{A_{2,K^\ast}^\perp \cdot \sqrt{3} x Y}{8 \pi^3/2} \left[ 1 + B_{2,K^\ast}^\perp \cdot C_2^\perp (\xi) \right] \\
\times \exp \left[ -b_{2,K^\ast}^\perp \left( \frac{\bar{x}m_s + xm_q}{\sqrt{k^2_\perp + (\bar{x}m_s + xm_q)^2}} \right) \right] \\
\times \left[ \text{Erfc} \left( b_{2,K^\ast}^\perp \cdot \sqrt{\frac{m_q^2 + Y^2}{x \bar{x}}} \right) - \text{Erf} \left( b_{2,K^\ast}^\perp \cdot \sqrt{\frac{Y^2}{x \bar{x}}} \right) \right]. (41)
\]

where \(\text{Erfc}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt\), \(Y = \bar{x}m_s + xm_q\) and the constitute quark mass \(m_q \approx 300\) MeV and \(m_s \approx 450\) MeV. In addition to the normalization condition, the average value of the squared transverse momentum can be regarded as another constraint, which is defined as
\[
\langle k_{T, K^\ast}^\perp \rangle = \frac{\int dxdz k_{T, K^\ast}^\perp |\psi_{2,K^\ast}^\perp(x, k^\perp)|^2}{\int dxdz |\psi_{2,K^\ast}^\perp(x, k^\perp)|^2}. (42)
\]

We take its value to be 0.37 GeV, which is consistent with that of Ref. [50] for the light-mesons.

We put the \(K^\ast\)-meson leading-twist LCDA parameters for some typical choices of \(B_{2,K^\ast}^\perp\) in Table II, where we have taken \(B_{2,K^\ast}^\perp \in [-0.2, 0.2]\). By varying \(B_{2,K^\ast}^\perp\) from \(-0.2\) to \(0.2\), the \(\phi_{2,K^\ast}^\perp\) changes from the single-peaked shape with highly suppressed end-point behavior to the double-humped shape. Conventionally, one can expand the light meson’s LCDA as a Gegenbauer expansion. As a comparison, we also present the first and second Gegenbauer moments \(a_n^\parallel\) and \(a_n^\perp\) in Table II, which are calculated via the formula
\[
a_n^\parallel(\mu_0) = \frac{\int_0^1 dx \phi_{2,K^\ast}^\perp(x, \mu_0) C_n^\perp(\xi)}{\int_0^1 dx \bar{x} |C_n^\perp(\xi)|^2}. (43)
\]
$$\gamma$$ the one-loop anomalous dimension obtained from QCD evolution equation \[ a \]

The resultant first and second Gegenbauer moment $a_1^\perp$, $a_2^\perp$ at $\mu_0 = 1.0 \text{GeV}$ are also presented.

| $B_{2,K^*}$ | $A_{2,K^*}$ | $b_{2,K^*}$ | $a_1^\perp$ | $a_2^\perp$ |
|-------------|-------------|-------------|-------------|-------------|
| $-0.2$      | $36.233$    | $0.696$     | $0.015$     | $-0.206$    |
| $-0.1$      | $35.370$    | $0.683$     | $0.023$     | $-0.123$    |
| $0.0$       | $33.802$    | $0.663$     | $0.030$     | $-0.034$    |
| $+0.1$      | $31.555$    | $0.636$     | $0.035$     | $0.064$     |
| $+0.2$      | $28.889$    | $0.604$     | $0.038$     | $0.170$     |

TABLE II. The $K^*$-meson transverse leading-twist LCDA parameters $A_{2,K^*}$ and $b_{2,K^*}$ under some typical choices of $B_{2,K^*}$. 

FIG. 1. A comparison of $\phi_{2,K^*}^\perp(x, \mu_0 = 1\text{GeV})$ under various models. As for the WH-DA model, which are shown by shaded band, we adopt $B_{2,K^*} = [-0.2, 0.2]$. The Gegenbauer moments $a_1^\perp$ at any other scale can be obtained from QCD evolution equation \[ a \], $a_1^\perp(\mu) = a_1^\perp(\mu_0)(\alpha_s(\mu)/\alpha_s(\mu_0))^{\gamma_1^\perp/\beta_0}$, where $\beta_0 = 11 - 2n_f/3$ and the one-loop anomalous dimension $\gamma_1^\perp = 4C_F[\psi(n+2)+\gamma_E-1]$ with $\psi(n+1) = \sum_{k=1}^{n} 1/k - \gamma_E$.

In addition to the WH-DA model, other $\phi_{2,K^*}^\perp$ models have also been suggested in the literature. Following the standard Gegenbauer expansion, it is suggested by Ball and Braun (we call it as the BB-DA model) \[ b \], $a_1^\perp(1\text{GeV}) = 0.04(3)$ and $a_2^\perp(1\text{GeV}) = 0.10(8)$. Another typical model based on the AdS/QCD theory has been suggested in Refs.\[ c \], we call it as the AdS/QCD-DA model. We put the twist-2 LCDAs $\phi_{2,K^*}^\perp(x, \mu_0)$ in Fig. 1, in which the WH-DA, BB-DA and AdS/QCD-DA models are presented as a comparison. The small asymmetries of those LCDAs indicate small $K^*$-meson SU(3)-breading effects. Fig. 1 shows that the WH-DA model provides a convenient form to mimic the behavior of various DA models: when taking $B_{2,K^*}^\perp \approx 0.03$, the WH-DA behaves closely to the AdS/QCD-DA; when taking $B_{2,K^*}^\perp \approx 0.08$, the WH-DA behaves closely to the BB-DA. In the following, we shall take the WH-DA model with $B_{2,K^*}^\perp \in [0.0, 0.2]$ to do our discussions. If we have precise measurements for the processes involving $K^*$-meson, we can fix $B_{2,K^*}^\perp$ and get the determined $\phi_{2,K^*}^\perp$ behavior by comparing theoretical predictions with the data.

B. $B \to K^*$ transition form factors

We adopt the following criteria to set the LCSR parameters, such as the Borel window and $s_0$, for the $B \to K^*$ TFFs. First, we require the continuum contribution to be less than 30% of the total LCSR. Second, we require all high-twist DAs’ contributions to be less than 15% of the total LCSR. Third, the derivatives of the LCSR \[ d \] with respect to $-1/M^2$ provide the LCSR for $m_B$. And, to be self-consistent, we require all the predicted $B$-meson masses from those LCPRs to be full-filled in comparing with the experimental one, e.g. $|m_B^\text{LCSR} - m_B^\text{exp}|/m_B^\text{exp} \leq 0.1\%$. The determined $s_0$ and $M^2$ for $B \to K^*$ TFFs at the large recoil region $q^2 = 0$ are presented in Table III. As a minor point, the flatness of those TFFs over the Borel parameter $M^2$ \[ e \], as can be indicated by small $M^2$-uncertainties shown in Table IV, also ensure the reliability of our present calculation. As for $T_1$, $T_2$ and $\bar{T}_3$, their $s_0$ and $M^2$ are to be the same, which are consistent with the relations $T_1(0) = T_2(0) = \bar{T}_3(0)$.

| $B_{2,K^*}^\perp$ | $s_0^{A_1}$ | $M_2^{-1}$ | $s_0^{A_2}$ | $M_2^{A_2}$ | $s_0^{A_1 - a}$ | $M_2^{A_1 - a}$ | $\gamma$ | $s_0$ | $M_2^{0}$ | $s_0^{T_1}$ | $M_2^{T_1}$ | $s_0^{T_3}$ | $M_2^{T_3}$ |
|------------|-------------|-------------|-------------|-------------|-------------|-------------|---------|--------|-------------|-------------|-------------|-------------|-------------|-------------|
| $0.0$      | $38.3(5)$   | $9.2(5)$    | $36.1(5)$   | $9.2(5)$    | $37.5(5)$   | $10.0(5)$   | $39.3(5)$ | $12.9(5)$| $39.7(5)$   | $13.5(5)$   | $36.8(5)$   | $9.8(5)$    | $38.7(5)$   | $13.5(5)$   |
| $0.1$      | $36.1(5)$   | $8.6(5)$    | $33.6(5)$   | $8.1(5)$    | $34.8(5)$   | $9.0(5)$    | $37.0(5)$ | $12.0(5)$| $37.4(5)$   | $12.5(5)$   | $34.2(5)$   | $8.7(5)$    | $34.5(5)$   | $11.0(5)$   |
| $0.2$      | $33.7(5)$   | $7.9(5)$    | $31.2(5)$   | $6.9(5)$    | $32.2(5)$   | $8.0(5)$    | $34.5(5)$ | $11.0(5)$| $34.8(5)$   | $11.4(5)$   | $31.7(5)$   | $7.5(5)$    | $34.5(5)$   | $11.4(5)$   |

TABLE III. The determined $B \to K^*$ continuum threshold $s_0$ and the Borel parameter $M^2$ at the large recoil region for the WH-DA model with $B_{2,K^*}^\perp = 0.0, 0.10, 0.20$, respectively. The central values are for $m_0 = 4.80 \text{GeV}$.  

\[ f \] The flatness has sometimes been adopted as a criteria to set the Borel window, cf. a review on the QCD sum rules \[ g \].
TABLE IV. The $B \to K^+$ TFFs at the large recoil region, where the errors are squared average of all mentioned error sources. $\mu = 2.2\text{GeV}$. The Ball and Zwicky (BZ) [34], the AdS/QCD [46] predictions are presented as a comparison.

| $B_{K^+}^\perp$ | $A_1(0)$ | $A_2(0)$ | $V(0)$ | $A_0(0)$ | $T_1(0)$ | $T_2(0)$ | $T_3(0)$ |
|-----------------|-----------|-----------|--------|-----------|-----------|-----------|-----------|
| 0.0             | 0.305^{+0.10}_{-0.18} | 0.274^{+0.011}_{-0.04} | 0.424^{+0.021}_{-0.024} | 0.389^{+0.02}_{-0.03} | 0.343^{+0.019}_{-0.023} | 0.216^{+0.012}_{-0.014} |
| 0.1             | 0.294^{+0.013}_{-0.016} | 0.259^{+0.012}_{-0.02} | 0.411^{+0.015}_{-0.026} | 0.389^{+0.015}_{-0.028} | 0.331^{+0.022}_{-0.018} | 0.202^{+0.011}_{-0.011} |
| 0.2             | 0.283^{+0.012}_{-0.016} | 0.244^{+0.016}_{-0.020} | 0.398^{+0.015}_{-0.020} | 0.375^{+0.005}_{-0.005} | 0.319^{+0.020}_{-0.016} | 0.189^{+0.011}_{-0.011} |
| BZ [34]         | 0.292 | 0.259 | 0.411 | 0.374 | 0.333 | 0.202 |
| AdS/QCD [46]    | 0.249 | 0.235 | 0.277 | 0.285 | 0.255 | 0.155 |

TABLE V. The fitted parameters $a_i$ and $b_i$ for the $B \to K^+$ TFFs. All LCSR parameters are set to be their central values. $\Delta$ is the quality of fit.

We present the $B \to K^+$ TFFs at the large recoil region $q^2 = 0$ in Table IV, in which $B_{K^+}^\perp = 0.0, 0.1$ and 0.2 are adopted. In Table IV, we also present the available predictions by Ball and Zwicky (BZ) [34] and the AdS/QCD predictions by Ref.[46]. During the calculation, the factorization scale $\mu$ is set as the usual typical momentum transfer of $B \to K^*$, i.e. $\mu \sim (m_B^2 - m_K^2)^{1/2} \sim 2.2\text{GeV}$. It is noted that even though, we have taken different correlators to do the LCSR calculation in comparison to that of Ref.[34], the central values of our present LCSRs agree with those of Ref.[34] under the similar twist-2 LCDA with $B_{K^+}^\perp \sim 0.1$ or $a_2^\perp \sim 0.1$. This agreement could be treated as a cross check of different LCSR calculations.

We present the $B \to K^*$ TFFs at the large recoil region $q^2 = 0$ in Table IV, in which $B_{K^+}^\perp = 0.0, 0.1$ and 0.2 are adopted. In Table IV, we also present the available predictions by Ball and Zwicky (BZ) [34] and the AdS/QCD predictions by Ref.[46]. During the calculation, the factorization scale $\mu$ is set as the usual typical momentum transfer of $B \to K^*$, i.e. $\mu \sim (m_B^2 - m_K^2)^{1/2} \sim 2.2\text{GeV}$. It is noted that even though, we have taken different correlators to do the LCSR calculation in comparison to that of Ref.[34], the central values of our present LCSRs agree with those of Ref.[34] under the similar twist-2 LCDA with $B_{K^+}^\perp \sim 0.1$ or $a_2^\perp \sim 0.1$. This agreement could be treated as a cross check of different LCSR calculations.

The high-twist LCDA$s$ are uncertain and their uncertainties are highly suppressed. Because of the suppression of the most uncertain high-twist contributions, our present LCSRs provide a good platform for testing the behaviors of the leading-twist $a_2^\perp$. Moreover, Table IV shows that all $B \to K^*$ TFFs decrease with the increment of $B_{K^+}^\perp$, i.e. smaller TFFs can be achieved for a bigger value of $B_{K^+}^\perp$.

The LCSRs for the $B \to K^*$ TFFs are applicable in the region $0 \leq q^2 \leq 14\text{GeV}^2$. They can be extrapolated into the physical allowable region, $0 \leq q^2 \leq (m_B - m_{K^*})^2 \sim 19\text{GeV}^2$ via the formulae

$$F_i(q^2) = F_i(0) \frac{1 - a_i q^2/m_B^2 + b_i (q^2/m_B^2)^2}{1 - a_i q^2/m_B^2 + b_i (q^2/m_B^2)^2}.$$

where $F_i$ stands for the TFFs $A_0, A_1, V$ or $T_1, T_2, T_3$, respectively. The parameters $a_i$ and $b_i$ can be determined by requiring the “quality” of fit ($\Delta$) to be less than one [34, 42]. Here $\Delta$ is defined as

$$\Delta = \frac{\sum (F_i(t) - F_i^{\text{fit}}(t))^2}{\sum (F_i(t))^2} \times 100,$$

where $t \in [0, 0.5, \ldots, 0.5, 1] 14\text{GeV}^2$. The derived fitting parameters are put in Table V.

We put the extrapolated $B \to K^*$ TFFs $A_0, A_1, V(q^2)$, $V(q^2)$ and $T_1, T_2, T_3(q^2)$ in Figs. 2 and Figs. 3, in which the available lattice QCD predictions [41] and the AdS/QCD predictions [46] are also included as a comparison. The $B \to K^*$ TFFs become smaller with the increment of $B_{K^+}^\perp$, especially in large $q^2$-region. To compare of the lattice QCD predictions, a much larger $B_{K^+}^\perp$ or a too smaller $B_{K^+}^\perp$ is not allowable. For example, a more broader leading-twist LCDA with $B_{K^+}^\perp \geq 0.2$ could be excluded by the lattice QCD calculation; a single-peak transverse leading-twist LCDA with $B_{K^+}^\perp \sim 0.0, 0.1$ shows a better agreement of the lattice QCD predictions over all TFFs. It is noted that the AdS/QCD predictions are consistent with the lattice QCD predictions on $A_2$, $V$, $T_1, T_2$ but not $A_0$ and $T_2$, which are always smaller than the lattice QCD predictions. Thus a larger differential branching fraction for $B \to K^*\mu^+\mu^-$ at large $q^2$ region, $q^2 > 12\text{GeV}^2$, are achieved [46].

C. The branching fraction of $B \to K^*\mu^+\mu^-$

As an application, we apply the extrapolated $B \to K^*$ TFFs to study the branching fraction of the dileptonic decay $B \to K^*\mu^+\mu^-$. This decay is very useful for precise tests of the standard model and for probing new physics beyond standard model. Its differential branching fraction can be written as [56].
FIG. 2. The extrapolated $B \to K^*$ axial-vector and vector TFFs $A_{0,1,2}(q^2)$ and $V(q^2)$. As a comparison, the lattice QCD [41] and AdS/QCD [46] predictions are also presented.

\[
\frac{d\mathcal{B}}{dq^2} = \tau_B \frac{G_F^2 \alpha^2 |V_{tb}V_{ts}|^2 \sqrt{\lambda v}}{24 \pi^5} \left\{ (2m_{\mu}^2 + m_B^2) s \left[ 16(|A|^2 + |C|^2)m_B^4 \lambda + 2(|B_1|^2 + |D_1|^2) \right. \right.
\]
\[
\left. \times \frac{\lambda + 12 rs}{rs} + 2(|B_2|^2 + |D_2|^2)m_B^4 \lambda^2 \left. \right] - 4 \text{Re}(B_1 B_2^*) + 4 \text{Re}(D_1 D_2^*) \right\} \frac{m_B^2}{r_s} (1 - r - s)
\]
\[
+ 6m_B^2 \left[ -16|C|^2 m_B^4 \lambda + 4 \text{Re}(D_1 D_2^*)m_B^2 \lambda \frac{m_B^2}{r} - 4 \text{Re}(D_2 D_3^*)m_B^2 (1 - r) \lambda \frac{m_B^2}{r} + 2|D_3|^2 m_B^2 s \frac{m_B^2}{r} \right]
\]
\[
- 4 \text{Re}(D_1 D_2^*) m_B^2 \left( 24|D_1|^2 + 2|D_2|^2 \frac{m_B^2}{r} (2 + 2r - s) \right) \left\} \right\}
\]

(46)

where $r = m_{K^*}^2/m_B^2$, $s = q^2/m_B^2$, and the phase-space factor $\lambda = 1 + r^2 + s^2 - 2r - 2s - 2rs$. The muon velocity $v = (1 - 4m_{\mu}^2/q^2)^{1/2}$, where $m_{\mu}$ is muon mass. $\tau_B$ is the average lifetime of those of $B^0$ and $B^+$-mesons [49]. The coefficients $A, B_{1,2}, C$ and $D_{1,2,3}$ are functions of $B \to K^*$ TFFs and the Wilson coefficients, whose explicit forms can be read from Ref. [56].

Our predictions for the differential branching fraction of $B \to K^* \mu^+ \mu^-$ are presented in Fig. 4, where the LHCb data [23–25] and the AdS/QCD prediction [46] are included as a comparison. For the LHCb data, we have adopted the weighted average of the branching fractions for $B^+ \to K^{*0} \mu^+ \mu^-$ [23–24] and $B^0 \to K^{*0} \mu^+ \mu^-$ [25].

We take three typical values for the WH-DA model, i.e. $B_{2,K^*} = 0.0, 0.1$ and 0.2, respectively. In lower $q^2$ region, the branching fractions for $B_{2,K^*} \in [0.0, 0.2]$ are almost coincide with each other; in higher $q^2$ region, the branching fractions decrease with the increment of $B_{2,K^*}$. A single peaked behavior for $\phi_{2,K^*}$ leads to a larger branching fraction. All the curves for $B_{2,K^*} \in [0.0, 0.2]$ are consistent with the LHCb data within errors, especially for larger $q^2$ region, $q^2 > 2\text{GeV}^2$. This indicates that even though the $B \to K^*$ TFFs behave quite differently under various choices of $B_{2,K^*}$, their differences to the branch-
The differential branching fraction are suppressed due to correlations among different TFFs.

By integrating the differential branching fraction (46) over all allowable $q^2$ region, we obtain

$$B(B_{2;K^*} = 0.0) = (1.347^{+0.642}_{-0.479}) \times 10^{-6};$$
$$B(B_{2;K^*} = 0.1) = (1.113^{+0.626}_{-0.540}) \times 10^{-6};$$
$$B(B_{2;K^*} = 0.2) = (0.911^{+0.505}_{-0.334}) \times 10^{-6};$$

where the errors are squared averages of the errors caused by varying $s_0$, $M^2$ and $m_b$ within the determined/adopted ranges shown in Sec.III.A and by taking the $B$-meson lifetimes $\tau(B^0) = 1.519 \pm 0.005$ ps and $\tau(B^+) = 1.638 \pm 0.004$ ps [49]. Those results indicate that the total branching fraction for $B \to K^* \mu^+ \mu^-$ decrease with the increment of $B_{2;K^*}$. It is found that when taking $B_{2;K^*} \sim 0.1$, the predicted total branching fraction shows a better agreement with the LHCb measurements [23–25] within errors, which is also agrees with a pQCD prediction by including the $\mathcal{O}(\alpha_s)$ and $\Delta_{QCD}/m_b$

2 Here, we adopt the central values for all the Wilson coefficients within the SM [12] to do the calculation, and the uncertainties of them have not been taken into consideration.
corrections, i.e. \( B(B \to K^*\mu^+\mu^-) = (1.19 \pm 0.39) \times 10^{-6} \) [57]. When the precision of experimental measurements has been improved in the future, we may get a definite conclusion on the \( K^* \)-meson transverse leading-twist LCDA \( \phi_{2,K^*}^{||} \).

IV. SUMMARY

We have recalculated the \( B \to K^* \) TFFs by using the LCSR approach and by taking the \( SU_f(3) \)-breaking effect into consideration. To constrain the pollution from the uncertain high-twist LCDAs to the LCSRs for those TFFs, we have adopted the chiral correlator to do the LCSR calculation.

The LCSRs (26–33) show that the chiral-even DAs \( \phi_{2,K^*}^{||}, \phi_{3,K^*}^{||}, \phi_{3,K^*}^{+}, \phi_{3,K^*}^{-} \), \( \phi_{3,K^*}^{\perp} \), \( \Phi_{3,K^*}^{||}, \Phi_{3,K^*}^{+}, \Phi_{3,K^*}^{-}, \Phi_{3,K^*}^{\perp} \) provide zero contributions to the LCSRs and all the remaining non-zero twist-3 and twist-4 LCDAs can only provide less than 10\% total contributions to the LCSRs. Thus the resultant LCSRs for the \( B \to K^* \) TFFs show slight dependence on the high-twist LCDAs, which inversely provide us a good chance for testing the properties of the transverse leading-twist LCDA \( \phi_{2,K^*}^{||} \).

To compare with the lattice QCD predictions on the \( B \to K^* \) TFFs, we suggest a convenient model (41) for \( \phi_{2,K^*}^{||} \), in which a single parameter \( B_{2,K^*}^{||} \) dominantly controls its longitudinal behavior. By comparing with the lattice QCD predictions, we observe that a too large \( B_{2,K^*}^{||} \) or a too small \( B_{2,K^*}^{||} \) is not allowable. For example, a more broader double-humped transverse leading-twist LCDA with \( B_{2,K^*}^{||} > 0.2 \) could be excluded by the lattice QCD calculation; while, a single-peaked transverse leading-twist LCDA with \( B_{2,K^*}^{||} \sim (0.0, 0.1) \) shows a better agreement of the lattice QCD predictions over all TFFs. At the present, the lattice QCD predictions are with large errors, a more accurate lattice prediction shall lead to a better constrain on \( \phi_{2,K^*}^{||} \).

As an application of the obtained \( B \to K^* \) TFFs, we have further predicted the differential branching fraction of \( B \to K^*\mu^+\mu^- \). The predicted branching fractions for \( B_{2,K^*}^{||} \in [0.0, 0.2] \) are consistent with the LHCb and the Belle measurements within errors, especially for the intermediate and large \( q^2 \) region, \( q^2 > 2\text{GeV}^2 \). In those \( q^2 \) region, the differential branching fraction decreases with the increment of \( B_{2,K^*}^{||} \), which indicates that a single peaked behavior for \( \phi_{2,K^*}^{||} \), leads to a larger differential branching fraction. The integrated branching fraction \( B(B \to K^*\mu^+\mu^-) \) also decreases with the increment of \( B_{2,K^*}^{||} \). When taking \( B_{2,K^*}^{||} \sim 0.1, \) both the differential and integrated branching fractions show a better agreement with the LHCb measurements [23–25] within errors.

Acknowledgments: This work was supported in part by Natural Science Foundation of China under Grant No.11275280, by the Fundamental Research Funds for the Central Universities under Grant No.07XKJ03, and by the Open Project Program of State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences under Grant No.Y3KF311CJ1.

[1] S. Descotes-Genon, J. Matias, and J. Virto, Phys. Rev. D 88, 074002 (2013).
[2] C. Bobeth, G. Hiller, D. van Dyk, J. High Energy Phys. 1007, 098 (2010).
[3] A.K. Alok, A. Datta, A. Dighe, M. Durrnissay, D. Ghosh et al., J. High Energy Phys. 1111, 121 (2011).
[4] A.K. Alok, A. Datta, A. Dighe, M. Durrnissay, D. Ghosh et al., J. High Energy Phys. 1111, 122 (2011).
[5] S. Descotes-Genon, D. Ghosh, J. Matias, M. Ramon, J. High Energy Phys. 1106, 099 (2011).
[6] C. Bobeth, G. Hiller, and D. van Dyk, J. High Energy Phys. 1107, 067 (2011).
[7] D. Becirevic and E. Schneider, Nucl. Phys. B 854, 321 (2012).
[8] W. Altmannshofer, P. Paradisi, and D. M. Straub, J. High Energy Phys. 1204, 008 (2012).
[9] C. Bobeth, G. Hiller, D. van Dyk, and C. Wacker, J. High Energy Phys. 1201, 107 (2012).
[10] J. Matias, F. Mescia, M. Ramon, and J. Virto, J. High Energy Phys. 1204, 102 (2012).
[11] F. Beujoan, C. Bobeth, D. van Dyk, and C. Wacker, J. High Energy Phys. 1208, 030 (2012).
[12] W. Altmannshofer, P. Ball, A. Bharucha, A.J. Buras, D.M. Straub, M. Wick, J. High Energy Phys. 0901, 019 (2009).
[13] W. Altmannshofer and D. M. Straub, J. High Energy Phys. 1208, 121 (2012).
[14] D. Becirevic, E. Kou, A. Le Yaouanc, and A. Tayduganov, J. High Energy Phys. 1208, 099 (2012).
[15] S. Descotes-Genon, J. Matias, M. Ramon, and J. Virto, J. High Energy Phys. 1301, 048 (2013).
[16] C. Bobeth, G. Hiller, and D. van Dyk, Phys. Rev. D 87, 034016 (2013).
[17] S. Descotes-Genon, T. Hurth, J. Matias, and J. Virto, J. High Energy Phys. 1305, 137 (2013).
[18] T. Aaltonen et al. (CDF Collaboration), Phys. Rev. Lett. 108, 081807 (2012).
[19] T. Aaltonen et al. (CDF Collaboration), Phys. Rev. Lett. 107, 201802 (2011).
[20] CDF Collaboration, CDF public note: 10894.
[21] J. Lees et al. (BABAR Collaboration), Phys. Rev. D 86, 032012 (2012).
[22] J.T. Wei et al. (BELLE Collaboration), Phys. Rev. Lett. 103, 171801 (2009).
[23] R. Aaij et al. (LHCb Collaboration), J. High Energy Phys. 1207, 133 (2012).
[24] R. Aaij et al. (LHCb Collaboration), Phys. Rev. Lett. 111, 191801 (2013).
[25] T. Blake and N. Serra (LHCb Collaboration) Report No. CERN-LHCb-CONF-2012-008 (2012).
[26] R. Aaij et al. (LHCb Collaboration), J. High Energy Phys. 1308, 131 (2013).
[27] R. Aaij et al. (LHCb Collaboration), J. High Energy Phys. 1409, 177 (2014).
[28] R. Aaij et al. (LHCb Collaboration), J. High Energy Phys. 1406, 133 (2014).
[29] ATLAS Collaboration, ATLAS-CONF-2013-038, ATLAS-COM-CONF-2013-043 (2013).
[30] CMS Collaboration, CMS-PAS-BPH-11-009.
[31] A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Korner, and V. E. Lyubovitskij, Eur. Phys. J. direct C 4, 18 (2002).
[32] D. Ebert, R. N. Faustov, and V. O. Galkin, Phys. Rev. D 82, 034032 (2010).
[33] P. Ball and V. M. Braun, Phys. Rev. D 58, 094016 (1998).
[34] P. Ball and R. Zwicky, Phys. Rev. D 71, 014029 (2005).
[35] A. Khodjamirian, T. Mannel, and N. Offen, Phys. Rev. D 75, 054013 (2007).
[36] P. Ball and R. Zwicky, Phys. Lett. B 633, 289 (2006).
[37] A. Ali, P. Ball, L.T. Handoko, G. Hiller, Phys. Rev. D 61, 074024 (2000).
[38] D. Becirevic, V. Lubicz, and F. Mescia, Nucl. Phys. B 769, 31 (2007).
[39] Z. Liu, S. Meinel, A. Hart, R. R. Horgan, E. H. Muller, and M. Wingate, arXiv: 1101.2726.
[40] R.R. Horgan, Z. Liu, S. Meinel, and M. Wingate, arXiv: 1310.3722.
[41] R.R. Horgan, Z. Liu, S. Meinel and M. Wingate, Phys. Rev. D 89, 094501 (2014).
[42] H.B. Fu, X.G. Wu, H.Y. Han and Y. Ma, arXiv: 1406.3892.
[43] H.B. Fu, X.G. Wu, H.Y. Han, Y. Ma and H.Y. Bi, Phys. Lett. B 738, 228 (2014).
[44] P. Ball and V.M. Braun, Phys. Rev. D 58, 094016 (1998).
[45] H.M. Choi, C.R. Ji, Phys. Rev. D 75, 034019 (2007).
[46] M. Ahmady, R. Campbell, S. Lord and R. Sandapen, Phys. Rev. D 89, 074021 (2014).
[47] T. Huang, Z.H. Li, X.G. Wu and F. Zuo, Int. J. Mod. Phys. A 23, 3237 (2008).
[48] P. Ball, V.M. Braun, A. Lenz, J. High Energy Phys. 0708, 090 (2007).
[49] K.A. Olive, et. al. (Particle Data Group), Chin. Phys. C 38, 090001 (2014).
[50] X.G. Wu and T. Huang, Phys. Rev. D 82, 034024 (2010); Phys. Rev. D 84, 074011 (2011); T. Huang, T. Zhong, and X.G. Wu, Phys. Rev. D 88, 034013 (2013).
[51] S.J. Brodsky, T. Huang and G.P. Lepage, in Particles and Fields-2, edited by A.Z. Capri and A.N. Kamal (Plenum, New York, 1983), p.143; S.J. Brodsky, G.P. Lepage, T. Huang and P.B. MacKenzie, in Particles and Fieds 2, edited by A.Z. Capri and A.N. Kamal (Plenum, New York, 1983), p.83.
[52] F.G. Cao and T. Huang, Phys. Rev. D 59, 093004 (1999).
[53] G.F. de Teramond and S.J. Brodsky, arXiv: 1203.4025; arXiv: 1208.3021.
[54] S.J. Brodsky, G.F. de Teramond, and H.G. Dosch, arXiv: 1302.5399; arXiv: 1302.4105.
[55] P. Colangelo and A. Khodjamirian, QCD sum rules, a modern perspective, arXiv:0010175 (hep-ph).
[56] T.M. Aliev, M. Savci, and A. Ozpineci, Phys. Rev. D 56, 4260 (1997).
[57] A. Ali, E. Lunghi, C. Greub, and G. Hiller, Phys. Rev. D 66, 034002 (2002).