$BR(b \rightarrow s\gamma)$ vs $m_{\chi^+}$
$BR(b \to s\gamma)_{1000}$
Yukawa Unification in Moduli-dominant SUSY Breaking

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ABSTRACT

We study Yukawa unification in string models with moduli-dominant SUSY breaking. This type of SUSY breaking in general leads to non-universal soft masses, \textit{i.e.} soft scalar masses and gaugino masses. Such non-universality is important for phenomenological aspects of Yukawa unification, \textit{i.e.}, successful electroweak breaking, SUSY corrections to the bottom mass and the branching ratio of $b \to s\gamma$. We show three regions in the whole parameter space which lead to successful electroweak breaking and allow small SUSY corrections to the bottom mass. For these three regions we investigate the $b \to s\gamma$ decay and mass spectra.
1 Introduction

The origin of fermion masses is one of the most important problems in particle physics. Yukawa unification is an attractive idea for fermion masses. Yukawa unification can be realized by several approaches, i.e. grand unified theories (GUTs), superstring theory and coupling reduction theories \[1\]. Within the framework of GUT different types of quarks and/or leptons are unified into one representation of a GUT group. On the other hand, the origin of couplings is unique in superstring theory including gauge couplings. Thus superstring theory can realize Yukawa unification especially for strong Yukawa couplings without a unified gauge group, although weak Yukawa couplings could be realized in terms of higher dimensional couplings. Coupling reduction theories can also realize Yukawa unification without unified symmetries, although different types of symmetries might be hidden behind coupling reduction theories \[2\].

Top-bottom-tau Yukawa unification has been studied in GUTs. That requires a large value of \( \tan \beta \simeq m_t/m_b \), where \( m_t \) and \( m_b \) are the top and bottom quark masses respectively. Phenomenological aspects of such models are very different from those with small \( \tan \beta \). In particular radiative electroweak symmetry breaking is an important issue and actually it has been discussed under assumption of universal soft supersymmetry (SUSY) breaking parameters \[3, 4\]. In the large \( \tan \beta \) case the mass parameter for the down sector Higgs field \( H_1 \) as well as the up sector Higgs field \( H_2 \) has a large and negative renormalization group equation (RGE) effects due to Yukawa couplings. Hence both of the Higgs mass parameters run from higher energy scale to the weak scale in a similar way if these masses are universal at the Planck scale. That is unfavorable for successful symmetry breaking. Thus non-universality such as \( m^2_{H_1} > m^2_{H_2} \) at the Planck scale is favorable for successful symmetry breaking with Yukawa unification \[1, 4\]. Further non-universality of squark and slepton masses affects symmetry breaking \[5, 6\] as well as other phenomenological aspects \[4\]. In general such non-universality can be derived from supergravity models.

Gaugino masses also play a role in phenomenological aspects of Yukawa unification. RGE effects due to gaugino masses generate a significant difference between \( m_{H_1} \) and \( m_{H_2} \) \[4, 5\]. Thus large gaugino masses are favorable for successful electroweak symmetry breaking. On the other hand, one of SUSY corrections to the bottom mass is proportional to the gluino mass \[8\]. Hence a small gluino mass is favorable not to obtain a large SUSY correction. In the universal scenario such SUSY corrections to the bottom mass are not suppressed \[4, 8\]. Moreover the minimal supersymmetric models with electroweak radiative breaking and universal soft mass terms at the GUT scale is ruled out due to
the constraints from the $b \rightarrow s\gamma$ decay and the condition $\Omega h^2 < 1$. It was shown in Ref. [4] that non-universality seems to be mandatory to satisfy these constraints with small SUSY corrections to the bottom quark.

SUSY breaking mechanism is a unsolved and important problem in SUSY models including superstring models. However, we can obtain generic formulae for soft SUSY breaking parameters assuming which types of fields contribute to SUSY breaking [10]. Further soft SUSY breaking terms can be parameterized simply by the gravitino mass $m_{3/2}$ and goldstino angles in the case where the dilaton and moduli fields contribute to SUSY breaking in superstring models [11, 12]. Phenomenological aspects have been discussed in several parts of the whole parameter space [13, 14]. In such framework moduli dominant SUSY breaking leads to non-universality among soft SUSY breaking terms, which could give phenomenologically interesting aspects. In particular multi-moduli cases can provide strong non-universality [12]. Recently a typical string model with multi-moduli dominant SUSY breaking is discussed in Ref. [15], showing several interesting aspects of the small $\tan \beta$ case [1]. Such non-universality between $m_{H_1}^2$ and $m_{H_2}^2$ derived from moduli-dominant SUSY breaking is useful also for successful electroweak symmetry breaking with Yukawa unification. Therefore, in this paper we study Yukawa unification of the model with non-universal Higgs masses as well as non-universal squark and slepton masses within the framework of the moduli-dominant SUSY breaking.

Further gaugino masses are non-universal in moduli-dominant SUSY breaking and their magnitudes are written by a parameter, which is independent of the goldstino angle parameterizing non-universality between $m_{H_1}^2$ and $m_{H_2}^2$. Therefore our model is useful not only to show phenomenological aspects of moduli-dominant SUSY breaking, but also to study what is in general favorable for successful electroweak symmetry breaking within Yukawa unification.

This paper is organized as follows. In section 2 we review formulae for soft SUSY breaking terms, parameterizing them. We define our model and give its soft terms. In section 3 we study radiative electroweak symmetry breaking in our model with large $\tan \beta$. In section 4 we consider constraint from SUSY corrections to the bottom mass. Such corrections are usually large. However, there are three types of parameter regions leading to small SUSY corrections. These correspond to the case with the very light gluino, the small $\mu$ case and the small gaugino mass case with large squark masses. Further we discuss constraints from the $b \rightarrow s\gamma$ decay in each of these regions. In section 5 we investigate

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1 See also Ref. [16].
mass spectra of these three regions, in particular masses of the lightest neutralino, chargino and Higgs particle. Section 6 is devoted to conclusions and discussions.

2 Moduli-dominant SUSY breaking

We assume the string model which has the same massless matter content as the minimal supersymmetric standard model (MSSM), \textit{i.e.} three families of quark doublets $Q_i$, the up-types of quark singlets $U_i$, the down-type of quark singlets $D_i$, lepton doublets $L_i$ and lepton singlets $E_i$ as well as two Higgs fields. Here we consider orbifold models with three diagonal moduli fields $T_m (m = 1, 2, 3)$ as well as the dilaton field $S$. We assume dilaton and moduli fields contribute to SUSY breaking and the vacuum energy vanishes. Corresponding $F$-terms are parameterized by the gravitino mass $m_3/2$ and goldstino angles $\theta$ and $\Theta_m$ as \cite{11, 12}

\begin{equation}
(K^S_S)^{1/2} F^S = \sqrt{3} m_{3/2} \sin \theta, \quad (K^{T_m}_{T_m})^{1/2} F^{T_m} = \sqrt{3} m_{3/2} \cos \theta \Theta_m,
\end{equation}

where $K^S_S$ and $K^{T_m}_{T_m}$ are Kähler metric and $\sum_{m=1}^3 \Theta_m^2 = 1$. In this case a soft scalar mass of a field with a modular weight $n_i^m$ is obtained as \cite{11, 12}

\begin{equation}
m_i^2 = m_{3/2}^2 (1 + 3 \cos^2 \theta \sum_m n_i^m \Theta_m^2).
\end{equation}

It is obvious that there appears stronger non-universality among soft scalar masses when two fields have nonvanishing elements of modular weights corresponding to different moduli fields. Here we assume modular weights for $H_1$ and $H_2$ as

\begin{equation}
n_{H_1} = (-1, 0, 0), \quad n_{H_2} = (0, -1, 0).
\end{equation}

Such non-universality becomes strong in the moduli-dominant SUSY breaking case, while soft SUSY breaking terms are universal in the dilaton dominant SUSY breaking. We take the limit $\sin \theta \rightarrow 0$. Further it is favorable that $m_{H_1}^2 \geq m_{H_2}^2$. Thus we take here $\Theta_1 = 0$. In this case we always have $m_{H_1}^2 \geq m_{H_2}^2$ and the other goldstino angles, $\Theta_2$ and $\Theta_3$, can be written as $\Theta_2 = \sin \theta_1$ and $\Theta_3 = \cos \theta_1$. Using this angle we can write the soft masses of $H_1$ and $H_2$ as

\begin{equation}
m_{H_1}^2 = m_{3/2}^2, \quad m_{H_2}^2 = m_{3/2}^2 (1 - 3 \sin^2 \theta_1).
\end{equation}

Thus non-universality is parameterized by $\sin \theta_1$. The soft mass of $H_2$ could in principle have a negative mass squared \textit{i.e.} $m_{H_2}^2 < 0$ with a small magnitude at high energy scale, \textit{i.e.} $\sin^2 \theta_1 \geq 1/3$. However, in such case one needs fine tuning for other parameters. Thus
we restrict ourselves to the case with \( \sin^2 \theta_1 \leq 1/3 \). As will be seen, we obtain similar results around \( \sin^2 \theta \approx 1/3 \). Hence we can expect similar results for the case where \( \sin^2 \theta_1 \) exceeds \( 1/3 \) slightly.

As will be seen later, RGE effects due to stau masses decrease \( m^2_{H_1} - m^2_{H_2} \) through a large tau Yukawa coupling \([4, 5]\). Thus small stau masses are favorable for electroweak breaking. Further the initial condition for squark masses \( m_U > m_D \) is also favorable for electroweak breaking. This initial condition is also useful for small SUSY corrections to the bottom mass. Hence we assume the following modular weights for quark and lepton fields,

\[
\begin{align*}
 n_Q &= n_U = (-1, 0, 0), \\
 n_D &= n_L = n_E = (0, -1, 0),
\end{align*}
\]

where the family index is omitted, because we assume degeneracy among three families. Under this assumption, fields \( Q \) and \( U \) have the same soft scalar mass as \( H_1 \), while \( D \), \( L \) and \( E \) have the same scalar mass as \( H_2 \).

In the moduli-dominant SUSY breaking case, gaugino masses and A-terms corresponding to moduli-independent Yukawa couplings are obtained as \([11, 12]\)

\[
\begin{align*}
 M_a &= \frac{\sqrt{3}m_{3/2}}{RcS} \sum_m \left( \frac{b^m_a}{k_a} - \delta^m_{GS} \right) D(T_m) \Theta_m, \\
 A_{ijk} &= -\sqrt{3}m_{3/2} \sum_m (1 + n^m_i + n^m_j + n^m_k) \Theta_m,
\end{align*}
\]

where \( k_a \) is a Kac-Moody level, \( \delta^m_{GS} \) is the Green-Schwarz coefficient \([17]\) and \( D(T_m) \) is the moduli-dependent function written by the Eisenstein function \( \hat{G}(T) \) as \([13]\)

\[
D(T) = \frac{(T + T^*)}{32\pi^3} \hat{G}(T),
\]

which takes values as \( D(T) = 1.5 \times 10^{-3}, 2.7 \times 10^{-2}, 9.3 \times 10^{-2}, 0.46 \) and \( 0.66 \) for \( T = 1.2, 5.0, 15, 70 \) and \( 100 \), respectively. Further \( b^m_a \) are duality anomaly coefficients, which depend on modular weights in a model as \([18]\)

\[
b^m_a = -C(G_a) + \sum_R T(R)(1 + 2n^m_a),
\]

where \( C(G_a) \) is the casimir of the adjoint representation and \( T(R) \) is the index of the \( R \) representation. In our case we have \( b^m_3 = (-6, 0, 3), b^m_2 = (-5, 1, 5) \) and \( b^m_1 = (1, -1, 11) \).
Further we take \( k_3 = k_2 = 1 \) and \( k_1 = 5/3 \). In this case gaugino masses are obtained as

\[
M_1 = \frac{\sqrt{3}m_{3/2}}{ReS} \left[ (-3/5 - \delta_{GS}) \sin \theta_1 + (33/5 - \delta_{GS}) \cos \theta_1 \right] D(T),
\]

\[
M_2 = \frac{\sqrt{3}m_{3/2}}{ReS} \left[ (1 - \delta_{GS}) \sin \theta_1 + (5 - \delta_{GS}) \cos \theta_1 \right] D(T),
\]

\[
M_3 = \frac{\sqrt{3}m_{3/2}}{ReS} \left[ -\delta_{GS} \sin \theta_1 + (3 - \delta_{GS}) \cos \theta_1 \right] D(T).
\]

(11)

In these equations we have assumed \( T_m = T \) and \( \delta_{m GS} = \delta_{GS} \) for simplicity and we take \( ReS = 2 \). In addition the \( A \)-terms are written as

\[
A_t = A_b = -\sqrt{3}m_{3/2} \cos \theta_1, \quad A_\tau = -\sqrt{3}m_{3/2} (-\sin \theta_1 + \cos \theta_1).
\]

(12)

The overall magnitude of the gaugino masses is dominated by \( D(T) \) and their ratios depend on \( \delta_{GS} \). We have \( M_3(M_Z) > M_2(M_Z) > M_1(M_Z) \) in most of the parameter space of \( \delta_{GS} \). It is important to notice that the gaugino masses are parameterized by \( D(T) \) and \( \delta_{GS} \) independent of the other soft SUSY breaking terms parameterized by \( \theta_1 \) as well as \( m_{3/2} \). As shall be shown later, the gaugino masses are very important for phenomenological aspects of Yukawa unification, electroweak symmetry breaking and SUSY corrections to the bottom mass. A large gaugino mass is favorable for successful electroweak symmetry breaking. On the other hand, a large gluino mass leads to a large SUSY correction to the bottom mass. Thus this parameterization (12) as well as eq.(4) is quite interesting not only to study phenomenology of moduli-dominant SUSY breaking, but also to investigate which types of spectra and non-universality are favorable for generic models with Yukawa unification, in particular from viewpoints of successful electroweak symmetry breaking, small SUSY corrections to the bottom mass and the experimental bounds of the \( b \rightarrow s\gamma \) decay. Moreover, for \( \delta_{GS} \simeq 3/(1 + \tan \theta_1) \), we obtain very light gluino. The possibility of having the light gluino of order \( 1 - 4 \) GeV is not excluded experimentally. In the models with the universal gaugino mass there are difficulties to get light gluino with proper radiative breaking of the electroweak symmetry satisfying experimental constraints. We will show that in our case we can have a part of the parameter space which leads to a very light gluino and satisfies all other constraints. This possibility of the light gluino is a quite interesting solution for controlling the SUSY correction \( \delta m_b \).

For the \( B \)-term its magnitude depends on a way to generate a natural \( \mu \)-term. Therefore here we take \( \mu \) and \( B \) as free parameters and we fix them requiring successful electroweak symmetry breaking.
3 Electroweak symmetry breaking in Yukawa unification

We assume the equality of the top, bottom and tau Yukawa couplings at the string scale, i.e., $\lambda_t = \lambda_b = \lambda_\tau = \lambda_G$. The RGEs of these couplings are obtained as follow:

$$\frac{dY_t}{dt} = Y_t \left( \frac{16}{3} \tilde{\alpha}_3 + 3\tilde{\alpha}_2 + \frac{13}{9} \tilde{\alpha}_1 - 6Y_t - Y_b \right),$$

$$\frac{dY_b}{dt} = Y_b \left( \frac{16}{3} \tilde{\alpha}_3 + 3\tilde{\alpha}_2 + \frac{7}{9} \tilde{\alpha}_1 - Y_t - 6Y_b - Y_\tau \right),$$

$$\frac{dY_\tau}{dt} = Y_\tau \left( 3\tilde{\alpha}_2 + 3\tilde{\alpha}_1 - 3Y_b - 4Y_\tau \right),$$

where $Y_i = \frac{\lambda_{2i}}{(4\pi)^2}$, and $i = t, b, \tau$. The pole masses of the top quark $M_t = 175$ GeV and the tau lepton $m_\tau = 1.78$ GeV are used to determine the common value of the Yukawa coupling $\lambda_G$ as well as the corresponding values of $\tan\beta$. Hence we can estimate the tree level mass of the $b$-quark, i.e., without SUSY corrections, $m_b = \lambda_b v \cos\beta$, where $v = \sqrt{\langle H_1^0 \rangle^2 + \langle H_2^0 \rangle^2} = 174$ GeV.

The fermion masses at $M_Z$-scale are related to their Yukawa couplings and we have

$$\frac{m_t^2(M_Z)}{Y_t(M_Z)} + \frac{m_b^2(M_Z)}{Y_b(M_Z)} + \frac{m_\tau^2(M_Z)}{Y_\tau(M_Z)} = (4\pi)^2 v^2,$$

which we solve for single unknown $\lambda_G$, then we find $\lambda_G \simeq 0.32$. This value leads to $\tan\beta \simeq 50$ and $m_b(M_Z) \simeq 3.3$ GeV. The experimental value of the bottom quark mass still has some uncertainty. For instance, the analysis of the $\Upsilon$ system using QCD sum rules [20] gives $m_b(m_b) = 4.13 \pm 0.06$ GeV corresponding to $m_b(M_Z) = 2.83 \pm 0.10$ GeV. On the other hand, the last lattice result shows [21] $m_b(m_b) = 4.15 \pm 0.20$ GeV and $m_b(M_Z) = 2.84 \pm 0.21$ GeV. Moreover, as mentioned in Ref. [22], the DELPHI collaboration [23] extracted $m_b(M_Z)$ to be

$$m_b(M_Z) = 2.85 \pm 0.22 \text{ (stat)} \pm 0.20 \text{ (theo)} \pm 0.36 \text{ (fragmentation)} \text{ GeV}. \quad (17)$$

The lower bound of $m_b(M_Z)$ in this case is around 2.15 GeV. This could allow for a large negative SUSY correction to $m_b$, $\delta m_b \sim 35\%$. However, here we will be conservative and we will consider the lower bound of $m_b(M_Z)$ as 2.63 GeV. Hence SUSY corrections have to be negative and of order $\delta m_b \leq 20\%$. These SUSY corrections will be discussed in the next section.

Let us now study electroweak symmetry breaking. The Higgs potential is written as

$$V(H_1, H_2) = \frac{1}{2} g^2 (\frac{\tau^a}{2} H_1 H_1 + H_2 \frac{\tau^a}{2} H_2)^2 + \frac{1}{2} g^2 (| H_2 |^2 - | H_1 |^2)^2 + m_1^2 | H_1 |^2 + m_2^2 | H_2 |^2 - m_3^2 (H_1 H_2 + h.c),$$

$$+ m_3^2 (H_1 H_2 + h.c), \quad (18)$$

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where
\[ m_i^2 = m_{H_i}^2 + \mu^2, \quad i = 1, 2 \]
and
\[ m_3^2 = -B\mu. \] (19)
We take \( \mu \) and \( B \) as free parameters and these are fixed by potential minimization conditions. In the large \( \tan \beta \) case the above Higgs potential has two characteristic features.

It follows from the minimization conditions that
\[ m_2^2 \simeq -\frac{M_Z^2}{2}, \] (20)
\[ m_3^2 \simeq \frac{M_A^2}{\tan^2 \beta} \sim 0, \] (21)
with
\[ M_A^2 \simeq m_1^2 + m_2^2 > 0. \] (22)
A combination of eqs. (20) and (22) gives the following constraint on the low energy parameters
\[ m_1^2 - m_2^2 > M_Z^2 \] (23)
i.e \( m_{H_1}^2 - m_{H_2}^2 > M_Z^2 \). In order to have electroweak breaking in the large \( \tan \beta \) case, the difference between the masses of the two Higgs fields should satisfy the above inequality.

In general case of the non-universal soft SUSY breaking terms, we find that the mass difference between \( m_{H_1}^2 \) and \( m_{H_2}^2 \) at the weak scale is given by
\[ m_{H_1}^2 - m_{H_2}^2 = \Delta m^2 + \alpha_{ab}M_aM_b + \beta_km_k^2 + \gamma_iA_i^2 + \lambda_{ia}A_iM_a \] (24)
where \( \Delta m^2 \) is given by \( \Delta m^2 = m_{H_1}^2 - m_{H_2}^2 \) at the string scale. In our case we have \( \Delta m^2 = 3m_{3/2}^2 \sin^2 \theta_1 \). Here \( m_k \) refer to \( m_Q, m_U, m_D, m_L \) and \( m_E \) at the string scale and \( A_i \) are the trilinear couplings \( A_t, A_b \) and \( A_\tau \).

In the case with universal soft SUSY breaking parameters, e.g. the universal scalar mass \( m_0 \) and the universal gaugino mass \( M_{1/2} \), we find
\[ \gamma_i \simeq \lambda_{ij} \simeq 0, \] (25)
\[ \alpha_{ab}M_{ab} = \alpha M_{1/2}, \quad \beta_km_k^2 = \beta m_0 \] (26)
with \( 0.1 \leq \alpha \leq 0.2 \) and \( \beta \simeq -0.2 \). The tau Yukawa coupling contributes dominantly to the third term of R.H.S. in eq.(24). Constraints such as \( \mu \simeq (1.5 - 1.7)M_{1/2} \) and \( M_{1/2} > m_0 \) are necessary for successful electroweak symmetry breaking. A lower bound on the universal gaugino mass \( M_{1/2} \) of order 300 GeV is obtained to make \( m_{H_2}^2 \) smaller than \( m_{H_1}^2 \). Thus large gaugino masses as well as a small stau mass are favorable for
successful electroweak symmetry breaking. This statement is still true in generic models. On the top of that, the non-universality $\Delta m^2$ as well as non-universality $m^2_U - m^2_D$ is favorable for electroweak symmetry breaking.

In our model the difference $m^2_{H_1} - m^2_{H_2}$ is written in terms of $m_{3/2}$, $\sin \theta_1$, $D(T)$ and $\delta_{GS}$. When we fix values of $D(T)$ and $\delta_{GS}$, this difference is written as

$$m^2_{H_1} - m^2_{H_2} \simeq (a \sin^2 \theta_1 + b)m^2_{3/2}.$$  \hfill (27)

For example in the case with $D(T) = 0.23$ and $\delta_{GS} = -5$ we find $a \simeq 3.87$ and $b \simeq 0.4$ in eq.\,(27). It is clear that in this case we have $m^2_{H_1} - m^2_{H_2} > M_Z^2$ for all values of $\theta_1 \in [0, 0.6]$ and the constraint (23) leads to $m_{3/2} \geq 150 \text{ GeV}$. Thus the gaugino mass corresponding to $D(T) \geq O(0.1)$ is large enough to lead to successful electroweak symmetry breaking for all the range of $\theta_1$ even without non-universality $\Delta m^2$. Further this case does not require very heavy gravitino mass.

On the other hand, in the case with $D(T) = 0.027$ and $\delta_{GS} = -5$, we find $a \simeq 3.5$, $b \simeq -0.16$ in eq.\,(27). Now to require $m^2_{H_1} - m^2_{H_2} > M_Z^2$ we obtain constraints in both $m_{3/2}$ and $\theta_1$. Moreover we also require that the mass of the pseudoscalar Higgs $m_A$ satisfies the experimental constraint $i.e. m_A \geq 40 \text{ GeV}$. Combining this constraint with the above ones we find that $m_{3/2} \geq 300 \text{ GeV}$ and $\theta_1 > 0.2 \text{ rad}$. Therefore we conclude that in our model with non-universality between $m^2_{H_1}$ and $m^2_{H_2}$, the scalar masses need no longer be smaller than gaugino masses. Further, as we will show in the next section, the large hierarchy ($m_i \gg M_a$) is favoured for obtaining small SUSY correction to the bottom quark mass. This shows that the value of $D(T)$ plays an important role in studying the Yukawa unification scenario.

4 SUSY corrections to $m_b$ and the $b \to s\gamma$ decay rate

In this section we calculate SUSY corrections to the bottom mass in the model with successful electroweak symmetry breaking. We are interested in finding regions of the parameter space $m_{3/2}$, $\theta_1$, $T$ and $\delta_{GS}$ which allow small SUSY corrections to $m_b$ and at the same time have proper electroweak breaking.

The bottom mass receives SUSY corrections as $m_b = \lambda_b v_1 (1 + \delta m_b)$. Here dominant contributions to $\delta m_b$ are due to the sbottom-gluino and stop-chargino loops, given in Ref. \[8\]

$$\delta m_b = \frac{2\alpha_3}{3\pi} M_\tilde{g} \tan \beta I(m^2_{b_1}, m^2_{b_2}, M^2_\tilde{g})$$
\[ + \ Y_t A_t \tan \beta \ I(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2, M_\tilde{g}^2), \quad (28) \]

where \( M_\tilde{g}, m_{\tilde{b}_i} \) and \( m_{\tilde{t}_i} \) are the gluino, sbottom and stop eigenstate masses respectively. The integral function \( I(a, b, c) \) is given by

\[ I(a, b, c) = \frac{ab \ln(a/b) + bc \ln(b/c) + ac \ln(c/a)}{(a-b)(b-c)(a-c)}. \quad (29) \]

The function \( I(a, b, c) \) is of order \( 1/m_{\text{max}}^2 \) where \( m_{\text{max}} \) is the largest mass running in the corresponding loop. The first term of R.H.S. in eq.(28) is rather dominant. For small SUSY corrections models should satisfy at least one of the following conditions, \( M_\tilde{g} \ll m_\tilde{q} \) or \( \mu \ll m_\tilde{q} \) where \( m_\tilde{q} \) represent the heaviest mass of the third generation squark eigenstates. By scanning the parameter space of this model, we find that there are three different regions which lead to \( \delta m_b \leq 20\% \). These regions correspond to very light gluino of 1–4GeV, small \( \mu \) and small gaugino mass of \( O(100) \)GeV with large sparticle. In all these regions large non-universality between \( m^2_{H_1} \) and \( m^2_{H_2} \) is favorable. Thus \( \theta_1 \) runs from 0.3-0.6 rad. i.e. around \( \sin^2 \theta = 1/3 \) to give maximum non-universality between \( m^2_{H_1} \) and \( m^2_{H_2} \).

### 4.1 Light gluino scenario

As mentioned above, at \( \delta_{GS} \simeq 3/(\tan \theta_1 + 1) \) the value of \( M_3 \) is very close to zero while \( M_1 \) and \( M_2 \) are of order \( m_3/2 D(T) \). In this case we have strong non-universality between \( M_3 \) and other gaugino masses. For such value of \( \delta_{GS} \) the gluino mass \( M_\tilde{g} \) is of order few GeV, then the SUSY correction \( \delta m_b \) becomes small. For other values of \( \delta_{GS} \), similarly small values of \( M_3 \) can be obtained if \( D(T) \) is of order \( 10^{-3} \), i.e. \( T \simeq O(1) \). However, in this case \( M_1 \) and \( M_2 \) are also suppressed and then it is difficult to satisfy the LEPII lower bound on the chargino mass. Hence we will not consider such case, but we will concentrate the case with \( \delta_{GS} \) very close to \( 3/(\tan \theta_1 + 1) \), which leads to \( M_\tilde{g} < 10 \)GeV. In this case the value \( D(T) > 0.1 \) is required to have proper electroweak breaking. The corresponding values of \( \delta m_b \) for this region of the parameter space is found to be smaller than 10%.

Now, we focus on the constraints from \( b \to s\gamma \) decay [24]. In SUSY models, there are additional contributions to the decay besides the SM one. For large \( \tan \beta \), the dominant supersymmetric contribution comes from the charged Higgs and chargino exchanges. As explained in Ref.[14] the chargino exchange contribution is enhanced for large \( \tan \beta \), and it becomes sizable. The chargino contribution could give rise to a substantial destructive interference with SM and Higgs amplitudes, depending on the sign of \( \mu \). We follow the
Before discussing the constraints of $b \to s\gamma$ on the parameter space, it is worthwhile to show the correlation between the masses of the lightest chargino and charged Higgs. This is helpful in understanding our estimation to the branching ratio (BR) of $b \to s\gamma$. In Fig.1, we plot the charged Higgs mass versus the lightest chargino mass for $\mu < 0$.

![Figure 1: The mass of charged Higgs versus the mass of lightest chargino, and $\mu < 0$.](image)

We notice from this figure that the charged Higgs tends to be heavier than the lightest chargino. This feature is very special for this region while in most of the parameter space the charged Higgs is lighter. In Figs.2 we show the values of the $BR(b \to s\gamma)$ corresponding to the values of $D(T)$ in the light gluino region of the parameter space we have determined above for $\mu < 0$. In this case we find the chargino contribution gives destructive interference with the charged Higgs amplitude. For $\mu > 0$ the chargino contribution is additive to charged Higgs and the SM ones, so that the total branching ratio becomes larger than the experimental limit $4 \times 10^{-4}$.

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3According to our convention, the chargino is heavier in the case of $\mu > 0$. 

Figure 2: $BR(b \to s\gamma)$ as a function of $D(T)$, and $\mu < 0$.

It is remarkable that the experimental limits $1 \times 10^{-4} \leq BR(b \to s\gamma) \leq 4 \times 10^{-4}$ impose an upper bound on the value of $D(T)$, $D(T) \leq 0.45$. The value of $\mu$ is decreasing as $D(T)$ increases. Such behavior is shown in Fig. 3 for $m_{3/2} = 1$ TeV, $\sin^2 \theta_1 \simeq 1/3$ and $\delta_{GS} \simeq 2.1$.

For $D(T) \geq 0.45$ we find that the angle $\phi$, where

$$
\tan 2\phi = 2\sqrt{2}m_W\frac{-\mu \sin \beta + M_2 \cos \beta}{M_2^2 - \mu^2 - 2M_W^2 \cos 2\beta},
$$

(30)

Figure 3: The value of $|\mu|$ as a function of $D(T)$. 
changes its sign. Note that $M_2$ is increasing as $D(T)$ increases. This angle determines one of the unitary matrices that diagonalise the chargino mass matrix and it has contribution to the chargino amplitude 14. Then the sign of the chargino contribution is also changed and it becomes additive. Hence we find that the value of the $BR(b \to s\gamma)$ becomes larger than the experimental upper bound for $D(T) \geq 0.45$ as Fig.2 shows. The same is happening for $\mu > 0$, and due to this sign changing the $BR(b \to s\gamma)$ jumps from values larger than the upper bound $4 \times 10^{-4}$ to values lower than the lower bound $1 \times 10^{-4}$. Actually there is the region which satisfies the $b \to s\gamma$ constraint for $\mu > 0$, but it is very narrow. Moreover for each value of $0.1 \leq D(T) \leq 0.45$ it is observed from Fig.2 that the experimental limits impose another constraints on $m_{3/2}$. For instance, for $D(T) = 0.1$, the gravitino mass should be larger than 600 GeV.

4.2 Small $\mu$ scenario

There appears a parameter region with small $\mu$ enough to lead to $\delta m_b \leq 20\%$. In our model such small $\mu$ is obtained as $m_{3/2} > 200$GeV, large $D(T)$ such as $[0.6, 1]$ and $\delta_{GS} \simeq 3/(1 + \tan \theta_1)$, say 1.5–1.9.\footnote{Further a value of $\delta_{GS} < 1.5$ makes the gluino very heavy increasing $\delta m_b$.} Here the value $D(T) = 0.6$ corresponds to $T \sim 90$. For this region Fig.4 shows values of $\mu$ against the gluino mass $M_{\tilde{g}}$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.png}
\caption{The value of $|\mu|$ versus the gluino mass.}
\end{figure}

This figure shows the ratio of $|\mu|/M_{\tilde{g}}$ is less than 1/5 for very heavy gluino while $\mu$ is about half of $M_{\tilde{g}}$ for smaller gluino mass. For the latter case suppression of $\delta m_b$ is due to
smallness of the product $\mu M_3$ compared with $\tilde{m}_g^2$. Further Fig.5 shows $BR(b \to s\gamma)$ for this region with $\mu < 0$.

Figure 5: $BR(b \to s\gamma)$ as a function of $m_{\chi^+}$, and $\mu < 0$.

As a result, the $b \to s\gamma$ decay requires no more constraints on this parameter region.

4.3 Small gaugino mass scenario with large sparticle

An alternative scenario for making $\delta m_b \leq 20\%$ is to have small gaugino masses of $O(100)$ GeV with large sparticle of order TeV. For instance, we find that for $m_{3/2} \simeq 2$ TeV, and $D(T) \simeq 0.04 - 0.2$ we can obtain small SUSY correction to $m_b$.

The computation of the $b \to s\gamma$ shows that the value of the $BR(b \to s\gamma)$ is decreased by increasing the value of $D(T)$. In this region we find that the values of the branching ration are within the experimental limits and no further constraints are obtained.

5 SUSY spectrum in Yukawa unification

In this section, we investigate the SUSY spectrum in the three regions of the parameter space which lead to small SUSY correction to $m_b$.

In the region corresponds to very light gluino, the lightest neutralino is found to be $O(100)$ GeV. It is not the lightest supersymmetric particle (LSP), because obviously the gluino is much lighter. Hence the lightest neutralino is unlike a dark matter candidate in
this region. However, in string models there are many other candidates for dark matter like for example the axion or other singlet fields. The lightest chargino in this region is found to be of order 100 GeV and we have imposed the experimental constraint $m_{\chi^+} > 84$ GeV in determining this region. The mass of the lightest Higgs in this region lies between 80-125 GeV.

On the other hand, in the small $\mu$ region, we find that the lightest neutralino is the LSP and its mass is given in Fig.6 as a function of the gravitino mass.

![Figure 6: The mass of the LSP in the region of small $\mu$ as a function of $m_{3/2}$.

In this region a small mass of $m_\chi$ is allowed. Thus this LSP becomes a dark matter candidate. Further we have $M_2 > \mu$, hence the lightest chargino mass is of order $\mu$. The lightest Higgs mass $m_h$ in this region satisfies $100 \text{ GeV} < m_h < 130 \text{ GeV}$.

Finally for the region with small gaugino masses of $O(100)$GeV and the scalar masses of order TeV we find that the lightest neutralino is the LSP and its mass and the mass of the lightest chargino are of $O(100)$ GeV. Further the mass of the lightest Higgs is about 125 GeV.

6 Conclusions

We have studied Yukawa unification within the framework of superstring models with moduli-dominant SUSY breaking. Large non-universality between $m_{H_1}$ and $m_{H_2}$ as well as non-universality $m_U^2 - m_D^2$ and small stau masses is favorable for successful electroweak
symmetry breaking. Large gaugino masses $M_a$ are also suitable. On the other hand, SUSY corrections to the bottom mass are in general large. To obtain sufficiently small SUSY corrections gives strong constraints on allowed parameter regions. There appear three types of regions leading to small SUSY corrections. These correspond to the very light gluino scenario with the mass of order $1 - 4\text{GeV}$, the small $\mu$ scenario and the small gaugino mass scenario with heavy squark masses. In the very light gluino scenario the $b \to s\gamma$ decay constrains gaugino masses as $D(T) \leq 0.45$, i.e. $M_a/m_{3/2} \leq O(0.5)$ for $a = 1, 2$. For the other two scenarios, we have no further constraint from $BS(b \to s\gamma)$.

There also appears the region where the lightest neutralino is light enough for a dark matter candidate in the small $\mu$ scenario. This scenario requires a large value of $T \sim O(100)$ corresponding to $D(T) > 0.6$. Such large value for $T$ might not be natural. The value of $T$ is determined by a nonperturbative mechanism. It is not clear that such large value could be realized by any mechanism. However, in the whole parameter space one could find out regions with small $\sin \theta$, but not exactly $\sin \theta = 0$ which lead to spectra similar to the case with $D(T) > 0.6$ and $\sin \theta = 0$.

Gauge symmetry breaking can lead to another source of non-universality of soft scalar masses, i.e. $D$-term contributions \[2\]. Such non-universality could also lead to interesting aspects to be studied \[2\].

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