Remarks on the Causality, Unitarity and Supersymmetric Extension of the Lorentz and CPT-Violating Maxwell-Chern-Simons Model

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The gauge-invariant Chern-Simons-type Lorentz- and CPT-breaking term is here re-assessed and issues like causality, unitarity, spontaneous gauge-symmetry breaking are investigated. Moreover, we obtain a minimal extension of such a system to a supersymmetric environment. We comment on resulting peculiar self-couplings for the gauge sector, as well as on background contribution for gaugino masses.

1. Introduction

Lorentz and CPT invariances are cornerstones in modern Quantum Field Theory, both symmetries being respected by the Standard Model for Particle Physics. Yet, nowadays one faces the possibility that this scenario is only an effective theoretical description of a low-energy regime, an assumption that leads to the idea that these fundamental symmetries could be violated when one deals with energies close to the Planck scale \([1]\). Taking this viewpoint, several approaches to analyze the violation of Lorentz symmetry have been proposed in the literature. Eventually a common feature arises: the violation is implemented by keeping either a four-vector (in a CPT-odd term \([1]\)) or a traceless symmetric tensor (CPT-preserving term \([2]\)) unchanged by particle inertial frame transformations \([3]\) which is generally called spontaneous violation. Furthermore, the issue of preserving supersymmetry (Susy) while violating Lorentz symmetry is addressed to \([4]\). This breaking of Lorentz symmetry is also phenomenologically motivated as a candidate to explain the patterns observed in the detection of ultra-high energy cosmic rays, concerning the events with energy above the GZK (\(E_{GZK} \simeq 4 \times 10^{19} \text{eV}\cdot\text{T}\)) cutoff \([6]\). Moreover, measurements of radio emission from distant galaxies and quasars verify that the polarization vectors of these radiations are not randomly oriented as naturally expected. This peculiar phenomenon suggests that the space-time intervening between the source and observer may be exhibiting some sort of optical activity, the origin of which is not known.

In this context, in Section 2, we analyze the possibility of having consistency of the quantization of an Abelian theory which incorporates the Lorentz- and CPT-violating term:

\[
\Sigma_{CS} = -\frac{1}{4} \int dx^{4} \epsilon^{\mu
olga\nu} \chi^\mu A_\nu F_{\olga\nu},
\]

whenever gauge spontaneous symmetry breaking (SSB) takes place. The analysis is carried out by pursuing the investigation of unitarity and causality as read off from the gauge-field propa-
gators. We therefore propose a discussion at treeapproximation, without going through the canonical quantization procedure for field operators. In this investigation, we concentrate on the analysis of the residue matrices at each pole of the propagators. Basically, we check the positivity of the eigenvalues of the residue matrix associated to a given simple pole in order that unitarity may be undertaken. Higher-order poles unavoidably plague the theory with ghosts; this is why our analysis of the residues is restricted to the case of the simple poles. We shall find that only for $c_{\mu}$ space-like both causality and unitarity can be ascertained. On the other hand, considering that SSB is interesting in such a situation (since the mass generation mechanism induced by the Higgs scalar presupposes that the theory is Lorentz invariant), it was showed in the work of Ref. [7] that, once Lorentz symmetry is violated, there is the possibility of evading this mechanism, such that a gauge boson mass is not generated even if SSB of the local U(1)-symmetry takes place.

In view of the interesting features of the original (bosonic) model, many interesting aspects may show up whenever supersymmetry is brought about. Especially, the fermionic (gaugino) mass generation opens up a relevant discussion in connection with the presence of background-fermion condensation. The first proposal of a supersymmetry-preserving Lorentz violation was carried out in the work of Ref. [4]. The aim of that work was to investigate whether one could maintain desired properties of supersymmetric systems, namely, cancellation of divergences and the patterns of spontaneous breaking schemes, while violating Lorentz symmetry. A Lorentz breaking tensor with constants entries has been adopted following an original suggestion given by Colladay [3]. Working upon a modified Wess-Zumino model, the authors of Ref. [4] had demonstrated that convenient changes of the Susy-algebra of fermionic charges and of Susy-covariant derivatives expressions were enough to define a Susy-like invariance for the Lorentz violating starting theory. As a matter of fact the modification of the algebra was achieved by adding a particular tensor-dependent central term, of the $k_{\mu\nu}\partial^\nu$ type, where $k_{\mu\nu}$ exhibits real symmetric traceless tensor properties. As a net result, it was shown that a model for a modified-Susy invariant but Lorentz non-invariant matter system can be built. Moved by a different perspective, we present, in Section 3, an analysis on Lorentz and Susy breakings concerning degrees of freedom in the gauge field sector. We carried out the supersymmetric minimal extension for the Chern-Simons-like term [1], preserving the usual (1 + 3)-dimensional Susy algebra [5]. The breaking of Susy will follow the very same route to Lorentz breaking: the statement that $c_{\mu}$ is a constant (in the active sense) vector triggers both Lorentz and, as we shall comment on, Susy breakings. Choosing appropriate superfield extensions for the background prevents the model from displaying higher-spin excitations, and interesting self-couplings for the gauge sector as well as background contribution for the gaugino masses come up naturally as a consequence of the (initially) supersymmetric structure. The background-fermion condensates play an interesting role for the gaugino mass generation. This shall become clear after the component-field action is written down. Our conclusions are presented in Section 4.

2. The Gauge-Higgs Model

We propose to carry out our analysis by starting off from the action:

$$
\Sigma = \int d^4x \left\{ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\mu}{4} \eta_{\nu\mu} A_\nu F_{\rho\lambda} \varepsilon^{\mu\nu\rho\lambda} + \frac{M^2}{2} A_\mu A^\mu \right\},
$$

where $\mu \nu \mu = c_\mu$ and the mass term, $M^2$, comes from the spontaneous symmetry breaking mechanism. The propagator may be obtained after a lengthy algebraic manipulation. Its explicit form in momentum space is

$$
\langle A_\mu A_\nu \rangle = \frac{i}{D} \left\{ -(k^2 - M^2) \left( \eta_{\mu\nu} - \frac{k_{\mu}k_{\nu}}{k^2} \right) + \left( \frac{D}{M^2} - \frac{\mu^2 (v \cdot k)^2}{(k^2 - M^2)} \right) \frac{k_{\mu}k_{\nu}}{k^2} + -i \mu \varepsilon_{\mu\nu\rho\lambda} v^\rho k^\lambda \right\} +
$$
and we find only a single non-vanish eigenvalue for the residue matrix: \( \lambda = \frac{1}{M^2} (M^2 + 2k_3^2) > 0 \). This opens up a very interesting conclusion: the \( M^2 \)-pole, appearing in the longitudinal sector, describes a physically realizable scalar mode. We are left with a very peculiar result: The vector potential accommodates 3 physical excitations, each of them carrying a single degree of freedom; so, the external background influences the gauge field by drastically changing its physical content: instead of describing a 3–degree of freedom massive excitation, it rather describes 3 different massive excitations, each carrying one physical degree of freedom.

3. The Supersymmetric Extension of the Maxwell-Chern-Simons Model

Adopting covariant superspace-superfield formulation, we propose the following minimal extension for (1):

\[
A = \int d^4x d^2\theta d^2\bar{\theta} \{ W^a(D_a V) S + c.c. \}
\]

where the superfields \( W_a, V, S \) and the SUSY-covariant derivatives \( D_a, \overline{D}_a \) hold the definitions:

\[
D_a = + \frac{\partial}{\partial \theta^a} + i \sigma^\mu_{\dot{a}a} \partial_\mu \partial^a,
\]

\[
\overline{D}_a = - \frac{\partial}{\partial \theta^a} - i \theta^a \sigma^\mu_{\dot{a}a} \partial_\mu;
\]

from \( \overline{D}_b W_a (x, \theta, \bar{\theta}) = 0 \) and \( D^a W_a (x, \theta, \bar{\theta}) = \overline{D}_a W^a (x, \theta, \bar{\theta}) \), it follows that

\[
W_a (x, \theta, \bar{\theta}) = -\frac{1}{4} D^2 D_a V.
\]

Its \( \theta \)-expansion reads as below:

\[
W_a (x, \theta, \bar{\theta}) = \lambda_a (x) + i \theta^b \sigma^\mu_{\dot{b}a} \partial^\mu \lambda_a (x) + \frac{1}{4} \bar{\theta}^2 \theta^2 \partial^\mu \partial_\mu \lambda_a (x) + 2 \theta a D (x) + \bar{\theta}^2 \theta^2 \sigma^\mu_{\dot{a}a} \partial_\mu D (x) + \sigma^\mu_{\dot{a}a} b \theta b F_{\mu \nu} (x) + \frac{i}{2} \sigma^\mu_{\dot{a}a} b \sigma^\nu_{\dot{b}a} \partial^\nu \partial_\mu \lambda^a (x) \theta^2
\]

and \( V = V^\dagger \). The Wess-Zumino gauge choice is taken for \( V \) with no loss of generality as far as the action (5) is gauge-invariant. The background
superfield is so chosen to be a chiral one. Such a constraint restricts the maximum spin component of the background to be an $s = \frac{1}{2}$ component-field, showing up as a Susy-partner for a spinless dimensionless scalar field. Also, one should notice that $S$ happens to be dimensionless. Taking $D_\sigma S(x) = 0$ the superfield expansion for $S$ then reads:

$$S(x) = s(x) + i\theta \sigma^\mu \partial_\mu s(x) - \frac{1}{4} \partial^2 \theta^\mu \partial_\mu s(x) + \sqrt{2} \theta \psi(x) + \frac{i}{\sqrt{2}} \theta^\mu \partial_\mu \psi(x) + \theta^2 F(x).$$

The component-wise counterpart for the action equation is as follows:

$$A_{\text{comp.}} = \int d^4x \left\{ -\frac{1}{2} (s + s^*) F_{\mu\nu} F^{\mu\nu} + i\partial_\mu (s - s^*) \epsilon^{\mu\alpha\beta\nu} F_{\alpha\beta} + 4D^2 (s + s^*) + 2i \bar{\lambda} \sigma^\mu \partial_\mu \lambda - 2i s^* \bar{\lambda} \sigma^\mu \partial_\mu \lambda + \bar{\lambda} \lambda F^* - 2\sqrt{2} \bar{\lambda} \psi D + -2\sqrt{2} \bar{\lambda} \psi D \right\}$$ (6)

As one can easily recognize, the first two lines display the 4D Chern-Simons-like term \(\mathfrak{L}\), where the vector $c_\mu$ is expressed as the gradient of a real background scalar: $c_\mu = \partial_\mu \sigma$, for $s = \xi + i\sigma$. Such a reduction of the vector into a gradient of a scalar field stems directly from the simultaneous requirements of both gauge\(^2\) and supersymmetry invariances.

Another interesting feature of this model concerns the presence of self-couplings for the gauge sector: the fermionic background field, $\psi$, triggers the coupling of the gauge boson (through the field-strength) to the gaugino. Moreover, using the field equation for the gauge auxiliary field $D$ one arrives at a quartic fermionic fields coupling - $\lambda \psi \psi$ -, and the background nature of $\psi$ indicates a background contribution for the gaugino mass\(^3\).

Concerning the breaking of Lorentz symmetry, realized by assuming $c_\mu = \partial_\mu \sigma$ to be constant under the action of particle inertial frame transformations, one should observe that such an assumption implies that the scalar component-field $\sigma$ must be linear in the coordinates, $\sigma = c_\mu x^\mu$. As a matter of fact, a linear dependence on $x^\mu$ cannot be implemented by means of a Susy-covariant constraint (i.e., Susy-covariant derivatives acting on $S$), and, in that sense, the choice of a rigid $\partial_\mu \sigma$ breaks Susy in exact analogy to the Lorentz breaking scheme adopted. To better establish such a correspondence, one can consider the choice for constant $\partial_\mu \sigma$ to be accompanied by a constant $\psi$ requirement (and a constant auxiliary field, $F$, as well\(^4\)). In this context, a (passive) Susy-transformation keeps the status of all component-fields unchanged.

As a first step towards generalizations of the presented minimal Susy-extension, we have also obtained the following non-polynomial formulation:

$$A_{n.p} = \frac{1}{4} \int d^4x \left\{ d^2\theta \left[ W^a W_a e^{(hS)} \right] + \text{c.c.} \right\}, \ (7)$$

whose full component-field expression may be found in the work [5]. One should realize, from the expression above, that a quartic fermion-field coupling is already present at order $h^2$, even if the field equation for the auxiliary field $D$ is not used to eliminate it.

4. Concluding Comments

Working on the gauge-field sector of a system with a Lorentz breaking 4D-Chern-Simons-like term, we have been able to derive its minimal supersymmetric extension. One should already realize the presence of new couplings induced by the background (passive-)superfield components. The assumption that the Lorentz break-

\(^2\)The gauge invariance of action equation will become clearly manifest in the next section, where we rephrase the supersymmetrization of the 4D Chern-Simons-like term in a formulation restricted to the chiral (anti-chiral for the h.c. counterpart) sector of superspace.

\(^3\)We shall analyze the propagator structure for the gauge component-fields in a forthcoming communication. We anticipate that a constant $\psi$ component-field configuration is compatible with the supersymmetry algebra.

\(^4\)In fact, a constant auxiliary field $F$ is singled out as a susy-invariant parameter, as far as one deals with a constant $\psi$. 

ing is implemented by means of a constant vector, regarded as a background input, finds its Susy-counterpart in a set of requirements on the space-time dependence of each component-field of the background superfield, $S$. A scalar field, $s$, linearly dependent on $x^\mu$, as well as a constant spinor field, $\psi$, arise as a consequence of gauge invariance, and these results impose that, eventually, coupling terms are to be regarded as mass terms. A complete analysis of the propagator structure for the gauge supermultiplet, both in superspace and in component-fields, is mandatory, including an interesting study of the gaugino (background-)induced mass. In terms of components, the explicit breaking of the Lorentz symmetry becomes manifest through the appearance of a gauge field-gaugino mixed propagator induced by the action term that involves the gauge potential, the gaugino and the background fermion. This is a rather peculiar point and, in deriving the full set of propagators, it will become clear that the gauge field and its fermionic partner will share a common dispersion relation, for which the background-fermion condensate interferes in competition with the external vector responsible for the Lorentz breaking. We shall very soon report our efforts in this matter elsewhere.

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