Understanding the halo-mass and galaxy-mass cross-correlation functions

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ABSTRACT

We use the Millennium Simulation (MS) to measure the cross-correlation between halo centres and mass (or equivalently the average density profiles of dark haloes) in a Lambda cold dark matter ($\Lambda$CDM) cosmology. We present results for radii in the range $10 h^{-1}$ kpc $< r < 30 h^{-1}$ Mpc and for halo masses in the range $4 \times 10^{10} < M_{200} < 4 \times 10^{14} h^{-1} M_{\odot}$. Both at $z = 0$ and at $z = 0.76$ these cross-correlations are surprisingly well fitted if the inner region is approximated by a density profile of NFW or Einasto form, the outer region by a biased version of the linear mass autocorrelation function, and the maximum of the two is adopted where they are comparable. We use a simulation of galaxy formation within the MS to explore how these results are reflected in cross-correlations between galaxies and mass. These are directly observable through galaxy–galaxy lensing. Here also we find that simple models can represent the simulation results remarkably well, typically to $\lesssim 10$ per cent. Such models can be used to extend our results to other redshifts, to cosmologies with other parameters, and to other assumptions about how galaxies populate dark haloes. Our galaxy formation simulation already reproduces current galaxy–galaxy lensing data quite well. The characteristic features predicted in the galaxy–galaxy lensing signal should provide a strong test of the $\Lambda$CDM cosmology as well as a route to understanding how galaxies form within it.

Key words: cosmology: theory – dark matter – large-scale structure of Universe.

1 INTRODUCTION

Weak gravitational lensing has opened a new window on to the large-scale distribution of matter. Gravitational lensing by foreground mass induces correlated distortions, or shear, in the observed shapes of distant galaxies. In galaxy–galaxy lensing, the signal from many galaxies is added together in order to measure the average (projected) distribution of mass around galaxies. This can be interpreted as the mass in the extended dark matter haloes which surround galaxies, or, more generally, as the cross-correlation between lens galaxies and the projected mass distribution. Several groups have successfully applied this technique to large imaging surveys to derive constraints on the mass associated with galaxies as a function of galaxy properties, such as luminosity and morphology (Brainerd, Blandford & Smail 1996; dell’Antonio & Tyson 1996; Griffiths et al. 1996; Hudson et al. 1998; Fischer et al. 2000; McKay et al. 2002; Hoekstra et al. 2003; Sheldon et al. 2004; Seljak et al. 2005).

Theoretical predictions for cross-correlations between galaxies and mass, $\xi_{gm}$, have made use both of numerical simulations (Guzik & Seljak 2001; Yang et al. 2003; Tasitsiomi et al. 2004; Weinberg et al. 2004) and of analytic halo models (Seljak 2000; Guzik & Seljak 2002). Tasitsiomi et al. (2004) show that the amplitude and shape of $\xi_{gm}$ predicted by cosmological simulations, when combined with a simple model for populating haloes with galaxies, are in good agreement with observational results based on Sloan Digital Sky Survey (SDSS) data (Sheldon et al. 2004). Mandelbaum et al. (2005) also find that these simulation results can be accurately reproduced by the halo model of Seljak (2000) and Guzik & Seljak (2002).

In this work we calculate the cross-correlation between haloes and mass, $\xi_{hm}$, and between galaxies and mass, $\xi_{gm}$, in the Millennium Simulation, a very large, high-resolution simulation of a Lambda cold dark matter ($\Lambda$CDM) cosmology. We also present simple models for $\xi_{hm}$ and $\xi_{gm}$ which can be used to interpret the shapes of these functions and to extend our results to other redshifts, cosmologies and halo population models.

The organization of this paper is as follows. In Section 2 we describe the Millennium Simulation and the halo and galaxy catalogues used in this study. In Section 3 we show $\xi_{hm}$ calculated for halo samples with a range of masses and we present a model which accurately reproduces these results. In Section 4 we show $\xi_{gm}$ for both central and satellite galaxies as a function of their luminosity, and we present models for each of these and for the combined galaxy sample. Finally, we summarize our results in Section 5.

2 SIMULATIONS

This study makes use of the Millennium Simulation (Springel et al. 2005), a large cosmological $N$-body simulation carried out by the

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Virgo Consortium. In this simulation a flat ΛCDM cosmology is adopted, with Ω_m = 0.205, Ω_b = 0.045 for the current densities in cold dark matter and baryons, h = 0.73 for the present dimensionless value of the Hubble constant, σ_8 = 0.9 for the rms linear mass fluctuation in a sphere of radius 8 h^{-1} Mpc extrapolated to z = 0, and n = 1 for the slope of the primordial fluctuation spectrum. The simulation follows 21603 dark matter particles from L within a periodic cube that encompasses a mean density 200 times the critical value, and σ_h cold dark matter and baryons, fluctuation in a sphere of radius 8 h^{-1} kpc. Initial conditions were generated using the Boltzmann code CMBFAST (Seljak & Zaldarriaga 1996) to generate a realization of the desired power spectrum which was then imposed on a glass-like uniform particle load (White 1996). A modified version of the TREE-PM N-body code GADGET2 (Springel, Yoshida & White 2001b; Springel et al. 2005) was used to carry out the simulation and full particle data are stored at 64 output times approximately equally spaced in the logarithm of the expansion factor at early times and at roughly 300 Myr intervals after z = 2.

In each output of the simulation, haloes are identified using a friends-of-friends (FOF) groupfinder with a linking length of b = 0.2 (Davis et al. 1985). Each FOF halo is decomposed into a collection of locally overdense, self-bound substructures (or subhaloes) using the SUBFIND algorithm of Springel et al. (2001a). Of these subhaloes, one is typically much larger than the others and contains most of the mass of the halo. We identify this as the main subhalo and define its centre as the position of the particle with the minimum potential. The virial radius, r_200, is defined as the radius of a sphere that encompasses a mean density 200 times the critical value, and the virial mass, M_{200}, is the mass within this radius.

Semi-analytic techniques have been used to simulate the evolution of the galaxy population within the Millennium Simulation, as described by Springel et al. (2005) and Croton et al. (2006). In this approach the evolution of the baryonic component is followed using a set of simple prescriptions for gas cooling, star formation, supernova and active galactic nucleus feedback, chemical enrichment, galaxy merging and other relevant physical processes. These models can be applied repeatedly to the stored histories of the dark matter haloes and subhaloes with different parameter choices for the model, or indeed with different physical assumptions in the modeling. In this work we use the semi-analytic galaxy catalogue described in detail in Croton et al. (2006). We note that various aspects of the clustering properties of haloes and galaxies in the Millennium Simulation have been investigated in Springel et al. (2005), Gao, Springel & White (2005), Harker et al. (2006), Wang, Li & Kauffmann (2006), Springel, Frenk & White (2006), Croton et al. (2006), Croton, Gao & White (2007), Gao & White (2007) and Li et al. (2007). Studies of halo density profiles in the simulation have been carried out by Neto et al. (2007) and Gao et al. (2008).

3 HALO-MASS CROSS-CORRELATIONS

Given a density field, ρ(x), the density fluctuation field is defined as

\[ \delta(x) = \frac{\rho(x) - \bar{\rho}}{\bar{\rho}}. \] (1)

The two-point autocorrelation function can then be written as

\[ \xi(r) \equiv \left< \delta(x) \delta(x + r) \right>. \] (2)

Assuming isotropy reduces this to a function of the separation, ξ(r). This is interpreted as a measurement of the excess probability above random of finding a pair of objects with separation r (Peebles 1980). In the case of two different populations of objects, \( \delta_i(x) \) and \( \delta_j(x) \), the two-point cross-correlation function is given by

\[ \xi_{ij}(r) \equiv \left< \delta_i(x) \delta_j(x + r) \right>. \] (3)

If we consider the cross-correlation between halo centres and mass, the cross-correlation function \( \xi_{hm}(r) \) is simply the spherically averaged halo density profile averaged over all haloes in the sample. This can be seen by combining equations (1) and (3) to give

\[ \xi_{hm}(r) = \frac{\left< \delta_h(r) \right> - \bar{\rho}_m}{\bar{\rho}_m}, \] (4)

where \( r \) is the radial distance from the halo centre and \( \bar{\rho}_m = \rho_{crit} \Omega_m \) is the mean density of the Universe.

Fig. 1 shows halo-mass cross-correlations along with the mass autocorrelation function for the Millennium Simulation. The halo-mass cross-correlations are computed using the centres of the main subhaloes of all FOF haloes with mass \( M_{200} \geq 4 \times 10^{10} h^{-1} M_\odot \), corresponding to \( \geq 50 \) particles. Seven halo samples are shown, each spanning a factor of 2 in mass. The number of haloes in each sample is listed in Table 1. We have computed cross-correlations also for the mass ranges not included in this plot and table, and we have checked that they also fit the models we discuss below. However, for clarity we refrain from showing them or discussing them further. Our cross-correlations were calculated using multiscale Fourier techniques as discussed in Smith et al. (2003). On small scales we checked the results using direct pair counts. Because of the very large numbers of haloes which we average together, the purely statistical uncertainties in our cross-correlations (due to halo-to-halo profile variations, substructure, etc.) are extremely small. As a result, we do not put error bars on our results and when fitting them below we will weight all points equally.

The shape of the halo-mass cross-correlation is clearly defined by two parts, commonly referred to as the one- and two-halo terms since they are dominated by particles within the same halo and...
in different haloes, respectively. Fig. 1 shows that on large scales \(\xi_{\text{hm}}(r)\) follows closely the mass autocorrelation function, \(\xi_{\text{mm}}\), with a mass-dependent offset in amplitude known as the halo bias factor, \(b(M)\). The transition between the two regimes is remarkably sharp and takes place at an overdensity of approximately seven times the mean density, i.e. \(\xi_{\text{hm}} \sim 6\).

We construct a simple model for \(\xi_{\text{hm}}(r)\) as follows:

\[
\xi_{\text{model}}(r; M) = \begin{cases} 
\xi_{\text{hs}}(r) & \text{if } \xi_{\text{hm}}(r) \geq \xi_{\text{hs}}(r), \\
\xi_{\text{zs}}(r) & \text{if } \xi_{\text{hm}}(r) < \xi_{\text{hs}}(r),
\end{cases}
\]

(5)

\[
\xi_{\text{hs}}(r) = \frac{\rho_{\text{halo}}(r; M) - \bar{\rho}_m}{\bar{\rho}_m},
\]

(6)

\[
\xi_{\text{zs}}(r) = \frac{b(M)\xi_{\text{hm}}(r)}{r},
\]

(7)

where \(\xi_{\text{hm}}(r)\) is the mass autocorrelation function predicted by linear theory. The main ingredients of the model are the halo density profile, \(\rho_{\text{halo}}\), and the bias factor \(b(M)\) which we now describe in detail.

3.1 The one-halo term

The density profiles of CDM haloes have been studied extensively with high-resolution N-body simulations over the past decade. Early results indicated that the density increases steeply towards the centres of haloes (Frenk et al. 1985; Quinn, Salmon & Zurek 1986; Dubinski & Carlberg 1991). Navarro, Frenk & White (1996, 1997, hereafter NFW) suggested the following simple fitting formula to describe the density profile of simulated haloes:

\[
\rho_{\text{NFW}}(r) = \rho_{200}\left(\frac{r}{r_{200}}\right)^{-
2} e^{-r/r_{200}},
\]

(8)

where \(\rho_{200}\) is the critical density. Note that the slope of the NFW profile is shallower (steeper) than the isothermal profile inside (outside) the characteristic scale radius, \(r_s\). Integrating this density profile out to the virial radius, \(r_{200}\), gives the following relation for the dimensionless density parameter

\[
\delta_0 = 200 \frac{\rho_{200}}{3} - \frac{1}{1 + \frac{c_{200}}{r_{200}}},
\]

(9)

where \(c_{200} = r_{200}/r_s\). Since the halo mass and virial radius are related through \(M_{200} = 200 \rho_{200}(4\pi/3)_{200}\), the independent parameters in the NFW profile are effectively the halo mass and concentration. Furthermore, these properties are known to be correlated in simulated haloes, in the sense that low-mass haloes are more central concentrated than high-mass haloes. This is generally interpreted in terms of the mean density of the Universe at the time of formation of a halo. Since low-mass systems typically collapse at higher redshift, the characteristic density and concentration of such systems is larger with respect to high-mass systems. The concentration–mass relation has been studied extensively with cosmological simulations and several authors have proposed models for predicting the average value of \(c_{200}\) as a function of halo mass and redshift (NFW; Bullock et al. 2001; Eke, Navarro & Steinmetz 2001; Macciò et al. 2007; Neto et al. 2007; Gao et al. 2008). Adopting such a model therefore fully specifies the density profile of a typical halo of a given mass \(M_{200}\) at a given redshift.

The most recent high-resolution N-body simulations have revealed small but significant deviations from the NFW formula. Navarro et al. (2004, hereafter N04) show that the density profiles become shallower towards the halo centre more gradually than the NFW formula predicts, causing NFW fits to underestimate the density in the inner regions. These authors propose an improved fitting formula with the following form:

\[
\ln \frac{\rho_{200}}{\rho_{r_2}} = -\frac{2}{\alpha} \left[ \frac{r_{r_2}}{r_{r_2}} \right]^\alpha - 1,
\]

(10)

where \(\rho_{r_2}\) and \(r_{r_2}\) are the density and radius at which the logarithmic slope of the density profile \(d \ln \rho_{200}/d \ln r = -2\), and the parameter \(\alpha\) controls the rate of change of the logarithmic slope with radius. Higher values of \(\alpha\) cause the profile to become shallower more quickly towards the centre of the halo. Unlike the NFW profile, equation (10) does not converge to a power law at small radii, instead reaching a finite density at the centre. N04 found a relatively small range of values, 0.12 \(\leq \alpha \leq 0.22\), to fit the density profiles of simulated haloes over a wide range in mass (8 \(\times 10^9 \lesssim M_{200} \lesssim 8 \times 10^{14} h^{-1} M_\odot\)). A spatial density profile of this form was first proposed in a different context by Einasto (1965) so we will hereafter refer to it as the Einasto profile. Prada et al. (2006) recently used the NFW and Einasto formulae to fit halo density profiles out to radii far beyond the virial radius. These authors find the mean density to give a significant contribution to the density profile at large distances, i.e. the halo profile is better fitted by

\[
\rho_{\text{halo}}(r) = \rho_{200}(r) + \bar{\rho}_m.
\]

(11)

With this modification, the one-halo term becomes

\[
\xi_{\text{hs}}(r) = \frac{\rho_{200}(r)}{\bar{\rho}_m}.
\]

(12)

In Sections 3.3 and 3.5 we investigate the accuracy of our model for \(\xi_{\text{hm}}\) using the NFW and Einasto fitting formulae in the one-halo term of the model.
3.2 The two-halo term

The two-halo term of the model is specified by the bias factor, \(b(M)\), and the mass autocorrelation function calculated from linear theory, \(\xi_{\text{lin}}(r)\). Halo bias has been studied extensively in the context of hierarchical structure formation scenarios. Assuming a Gaussian distribution of initial density fluctuations specified by \(\sigma(M)\), the \(r_{\text{ms}}\) linear mass fluctuation (extrapolated to \(z = 0\)) within spheres that on average contain mass \(M\), Mo & White (1996) derive an analytic model for halo bias, \(b(v, z)\), where

\[
v = \left[ \frac{\delta_i(z)}{\sigma(M)} \right]
\]

(13)

is the dimensionless amplitude of fluctuations, or peak height, that produces haloes of mass \(M\) at redshift \(z\) and \(\delta(z)\) is the linear overdensity (again extrapolated to \(z = 0\)) for which a spherical perturbation would collapse at redshift \(z\). The characteristic halo mass for clustering \(M_c(z)\) is defined by \(\sigma(M_c) = \delta(z)\) and haloes more (less) massive than \(M_c\) are more (less) strongly clustered than the underlying mass density field. For the Millennium Simulation cosmology, \(M_c(z = 0) = 6.15 \times 10^{12} h^{-1} M_\odot\).

Mo & White (1996) showed that their model accurately describes the bias in the autocorrelation function of dark matter haloes with respect to that of the mass in cosmological \(N\)-body simulations. Further testing against higher resolution simulations has led to modifications and improvements in the model (Jing 1998; Governato et al. 1999; Kravtsov & Klypin 1999; Sheth & Tormen 1999; Colberg et al. 2000; Sheth, Mo & Tormen 2001; Seljak & Warren 2004; Mandelbaum et al. 2005). Most recently, Gao et al. (2005) compared these models with halo bias in the Millennium Run. Fig. 1 of Gao et al. (2005) shows that the Millennium Run results are reasonably well matched by the bias model of Sheth & Tormen (1999)

\[
b(v) = 1 + \frac{av^2 - 1}{\delta_1} + \frac{2p}{\delta_1[1 + (av^2)^p]},
\]

(14)

with the parameter values \(a = 0.73\) and \(p = 0.15\) of Mandelbaum et al. (2005) based on fits to the simulations of Seljak & Warren (2004). We therefore adopt equation (14) with these parameter values as the bias formula in the two-halo term of our model for \(\xi_{\text{lin}}\).

3.3 Model fitting results

Fig. 2 shows the accuracy of our model when we choose the halo density profile to be of NFW form. The downward pointing arrows indicate the transition scale \(r_{\text{trans}}\) between the one- and two-halo regimes of the model, also listed in Table 1. We take the halo mass in the model to be the geometric mean of the upper and lower halo mass limits in each halo sample, i.e. \(M_{\text{model}} = (M_{\text{lo}}M_{\text{hi}})^{1/2}\). Here we take the concentration to be a free parameter and compare our best-fitting values to the concentration–mass relations proposed by various authors. The best-fitting value is obtained by minimizing the root mean square of \((\xi_{\text{lin}} - \xi_{\text{model}})/\xi_{\text{lin}}\).

Several trends are apparent in the deviations from our model fits. In the one-halo regime, the NFW profile fits the halo density profile to within about 10 per cent, but there is a clear indication of the systematic ‘U-shaped’ residuals found by N04 for fits to individual halo density profiles with the NFW formula. The measured profiles are lower than the fits at intermediate radii but higher at the largest and smallest radii. This indicates that the shape of the NFW profile does not perfectly capture the simulation results. We return to this issue later, when we examine the accuracy of fits using equation (10) to model the halo density profile.

Figure 2. Deviations between our measured halo mass cross-correlations and the simple model given by equations (5)-(7) assuming the NFW profile in the one-halo term. On large scales, \(r \gtrsim 3 h^{-1} \text{Mpc}\), the deviations are dominated by a quasi-linear distortion that is apparent in the ratio of the mass autocorrelation function to the linear theory prediction, \(\xi_{\text{lin}}/\xi_{\text{lin}}\) (solid line). For low-mass haloes, \(M_{\text{200}} \lesssim 3.2 \times 10^{11}\), the model also fails on intermediate scales suggesting a cut-off in power should be applied to the linear prediction on these scales. Otherwise the accuracy of the model is better than \(\pm 10\) per cent.

In the two-halo regime, the deviations in the model at large separations, \(r \gtrsim 3 h^{-1} \text{Mpc}\), are dominated by the quasi-linear distortion due to the large-scale movement of haloes with respect to each other. This is illustrated by the solid line in Fig. 2 which shows the ratio of the mass autocorrelation function to the linear theory prediction, \(\xi_{\text{lin}}/\xi_{\text{lin}}\). This distortion has been investigated in numerous studies of the matter power spectrum (Ma & Fry 2000; Seljak 2000; Scoccimarro et al. 2001; Smith et al. 2003; Cole et al. 2005) and various correction factors have been proposed. In this work we prefer to neglect the distortion and adopt the simpler assumption of pure linear theory (Peacock & Smith 2000). This approach simplifies the calculation of the model and avoids the introduction of the additional parameters that are present in correction factors proposed by Smith et al. (2003) and Cole et al. (2005). The cost is a systematic residual in our model fits of the order of 25 per cent at \(\sim 10 h^{-1} \text{Mpc}\).

In the two lowest mass halo samples shown in Fig. 2, the two-halo term in the model appears to overpredict \(\xi_{\text{lin}}\) at overdensities corresponding to \(\xi_{\text{lin}} \gtrsim 7\). This suggests that the linear theory prediction should also be modified by a cut-off in power at small scales, as suggested by Smith et al. (2003). We investigate this in further detail in Fig. 3, where we examine the linear and quasi-linear terms predicted by the Smith et al. (2003) HALOFIT model for the Millennium Simulation cosmology. Using the notation of Smith et al. (2003), the correlation function is related to the power spectrum by the integral relation

\[
\xi(r) = \int \Delta^2(k) \frac{\sin kr dk}{kr} \frac{dk}{k},
\]

(15)

and the HALOFIT model power spectrum is given by the sum of the quasi-linear and one-halo terms,

\[
\Delta^2(k) = \Delta^2_{\text{QL}}(k) + \Delta^2_{\text{1H}}(k),
\]

(16)

where the quasi-linear term \(\Delta^2_{\text{QL}}(k)\) includes an exponential cut-off at the non-linear wavenumber \(k_0 \simeq 0.3 h \text{Mpc}^{-1}\) for \(\Lambda\text{CDM}\).
Figure 3. Top panel: Mass autocorrelation function, $\xi_{\text{nn}}$, and halo-mass cross-correlation functions, $\xi_{\text{hm}}$, for low-mass haloes ($6.4 \times 10^{11} < M_{200} < 12.8 \times 10^{11}$). Also shown are the predictions of linear theory $\xi_{\text{lin}}$ and of the Smith et al. (2003) HALOFIT model. Neither $\xi_{\text{lin}}$ nor the HALOFIT quasi-linear model $Q$ provide a good match to the shape of $\xi_{\text{hm}}$ on large scales, though the corresponding HALOFIT one-halo term $Q_H$ (see equation 16) is a good match to our data. Bottom panel: The ratio between $\xi_{\text{lin}}$ and the linear and quasi-linear predictions shows that the shape of $\xi_{\text{hm}}$ is marginally better fitted by $\xi_{\text{lin}}$ than by $Q$. The top panel of Fig. 3 shows that the correlation function calculated from the HALOFIT model power spectrum provides a good fit to the mass autocorrelation function $\xi_{\text{mm}}$. The cut-off in power at high $k$ built into the HALOFIT model corresponds to a flattening of the corresponding correlation function, $Q$, with respect to the linear theory prediction, $\xi_{\text{lin}}$ at an overdensity $Q \simeq 4$. Smith et al. (2003) provide fitting formulae for $\Delta_H^2(k)$ and $\Delta_Q^2(k)$, each with numerous parameters tuned to give a good match to the matter power spectrum in various CDM-based cosmological $N$-body simulations. However, Fig. 3 shows that $Q$ cannot be used to improve the model for $\xi_{\text{hm}}$ of low-mass haloes. This is illustrated in the bottom panel, where we show the ratios $\xi_{\text{lin}}/Q$ and $\xi_{\text{lin}}/\xi_{\text{lin}}$ for haloes with mass $6.4 \times 10^{11} < M_{200} < 12.8 \times 10^{11} h^{-1} M_{\odot}$. We find that using the quasi-linear term from the HALOFIT model actually produces a marginally poorer fit to the measured shape of halo-mass cross-correlations. The HALOFIT model imposes a cut-off in power at the non-linear wavenumber $k_s$. Our results suggest that the Smith et al. (2003) prescription adopts a value of $k_s$ that is too low in the present context. Modifying their prescription for the non-linear scale would require the recalibration of all of the fitting formulae in the HALOFIT model in order to recover the fit to the mass autocorrelation function. This is beyond the scope of the present paper.

### 3.4 Halo concentrations

Fig. 4 shows the best-fitting NFW concentration parameter values versus halo mass for our model fits. Also shown are the predictions of the concentration–mass models proposed by Eke et al. (2001, hereafter ENS), Bullock et al. (2001, hereafter B01), and the power-law fit of Macciò et al. (2007, hereafter M07) which is also a good representation of the results of Neto et al. (2007). In each case we have used the parameters recommended by the authors to draw the relation. Our best-fitting concentrations appear to follow the B01 and M07 results for halo masses $M_{\text{vir}} \lesssim 5 \times 10^{12}$, but at higher masses the results are better described by the ENS model. We find that a power-law provides a reasonable fit to the data, with the same normalization, but a slightly shallower slope ($c \propto M_{\odot}^{-0.1}$) than found by M07 and Neto et al. (2007) ($c \propto M_{\odot}^{-0.1}$). We note that the difference between all these models is relatively small with respect to the 1σ scatter, $\Delta \log(c_{\text{vir}}) = 0.14$, found for individual haloes in B01. (This scatter is represented by the offset dashed lines in Fig. 4.) Since the ENS model provides a reasonably close match to our best-fitting concentration values and has been calibrated for redshifts $z > 0$ and for different cosmologies, we adopt it as our halo concentration model hereafter. We note, however, that Gao et al. (2008) find that the ENS model significantly underestimates the concentration of massive haloes at high redshift (for which ENS had no data) so we recommend using the ‘revised NFW’ model of Gao et al. (2008) in applications which require concentrations for such haloes.

Figure 4. NFW concentration versus halo mass from fits of our model to $\xi_{\text{hm}}$ are compared with published concentration–mass relations as discussed in the text. The black solid line shows a power-law fit to our results, $c \propto M_{\odot}^{-0.1}$. The vertically offset versions of the B01 model (the dashed curves) enclose the 1σ scatter in concentration found by B01 for haloes of given mass.
3.5 Halo density profile

In this subsection we investigate whether our model is improved if we change the choice of halo density profile. We here use the Einasto profile (equation 10), the improved fitting formula proposed by N04. This contains an additional shape parameter, $\alpha$, which controls the rate at which the slope of the density profile changes with radius, with typical values lying in the range $[0.12, 0.22]$ (N04; Prada et al. 2006).

Fig. 5 shows the deviations of the measured halo-mass cross-correlations from our best fits using the Einasto formula. Here we focus on halo samples with $M_{200} > 6.4 \times 10^{11} h^{-1} M_\odot$, since deviations from the fits are then not dominated by the inadequacy of the two-halo term on intermediate scales (see Section 3.3). The deviations are reduced significantly by using the Einasto model, $\lesssim 5$ per cent compared to $\lesssim 10$ per cent for fits using the NFW profile. The best-fitting values of $\alpha$ are shown as a function of halo mass in Fig. 6. We find that $\alpha$ tends to increase with halo mass, ranging from $\alpha \simeq 0.18$ for $M_{200} \simeq 3 \times 10^{11} h^{-1} M_\odot$ to $\alpha \simeq 0.22$ for $M_{200} \simeq 3 \times 10^{14} h^{-1} M_\odot$.

Figure 5. Deviations between our measured halo-mass cross-correlations and the simple model given by equations (5)–(7) with the Einasto profile in the one-halo term. On small scales the Einasto profile provides a better fit than the NFW profile, with deviations $\lesssim 5$ per cent.

3.6 Higher redshift results

Since galaxy–galaxy lensing studies are sensitive to lenses at redshifts well above zero, it is interesting to check our models against cross-correlations at $z > 0$. In the COSMOS survey, for example, the galaxy–galaxy lensing signal is significant for lenses over the redshift range $0.2 < z < 1.2$ with the main contribution from $z \simeq 0.5–0.9$ (A. Leauthaud, private communication).

Fig. 7 shows $\xi_{hm}$ for the $z = 0.755$ output of the Millennium Simulation. The results are qualitatively similar to those at $z = 0$, and the bottom panel of the figure shows the deviations from the best fits using our (Einasto) model. We find that the deviations are of the order of $\pm 10$ per cent for the higher redshift $\xi_{hm}$. Fig. 6 also shows the best-fitting values of $\alpha$ for this redshift. As in the $z = 0$ case, we find that $\alpha$ tends to increase with mass for $M_{200} \gtrsim 3 \times 10^{11} h^{-1} M_\odot$. In the right-hand panel of this figure we plot the best-fitting $\alpha$ values against the peak height defined by equation (13). Plotted in this way, we find that the $z = 0$ and 0.755 results are in good agreement with each other. The similar analysis of Gao et al. (2008) suggests that the upturn in the $\alpha$–$v$ relation at large $v$ is real, and that the disagreement between the data for the two redshifts at small

Figure 6. Left-hand panel: Best-fitting value of $\alpha$ from the Einasto fitting formula versus halo mass. Higher values of $\alpha$ correspond to density profiles that curve more strongly in a log–log plot. The best-fitting value of $\alpha$ tends to increase with halo mass, and with redshift at given halo mass. Right-hand panel: The same data for $\alpha$ are plotted against the peak-height parameter $v(M, z)$ defined in the text. Comparison with results from the resolution study of Gao et al. (2008) suggests that the upturn at the highest $v$ may be real, but that the disagreement at small $v$ is likely a result of inadequate resolution for haloes with $M_{200} < 10^{12} h^{-1} M_\odot$.

Figure 7. Top panel: Halo-mass cross-correlations calculated at $z = 0.755$. Bottom panel: Deviations between $\xi_{hm}$ and our model assuming the ENS halo concentration relation and the Einasto density profile. Deviations from the model are of the order of 10–20 per cent.
4.1 Cross-correlations between central galaxies and mass

We first consider cross-correlations between central galaxies and mass. Since central galaxies are placed on the most bound particle of their host haloes, \( \xi_{gm} \) for central galaxies is equivalent to \( \xi_{hm} \) averaged over the corresponding sample of FOF haloes, and so should be similar to the \( \xi_{hm} \) presented in Section 3 provided that central galaxies are selected according to a property that correlates well with host halo mass. Fig. 9 shows \( \xi_{gm} \) for central galaxies selected according to \( r \)-band absolute magnitude in the range \(-20 \leq M_r \leq -24 \). These cross-correlations indeed appear very similar in shape to those of Fig. 1.

We apply the simple model presented in Section 3 based on the Einasto profile of equation (10). We take the mass of the host halo as a free parameter which determines the bias factor according to equation (14) and the radius and concentration of the mean halo density profile according to the ENS model. The best-fitting value of the halo density profile parameter \( \alpha \) is about 0.15 for the central galaxy \( \xi_{gm} \). This is quite similar to the values found above for haloes in the relevant mass ranges.

The distribution of host halo masses for central galaxies in the luminosity range \(-21 < M_r < -20 \) is shown in Fig. 10. This distribution is highly asymmetric and ranges over four orders of magnitude of their host haloes, \( \xi_{gm} \) for central galaxies is equivalent to \( \xi_{hm} \) averaged over the corresponding sample of FOF haloes, and so should be similar to the \( \xi_{hm} \) presented in Section 3 provided that central galaxies are selected according to a property that correlates well with host halo mass. Fig. 9 shows \( \xi_{gm} \) for central galaxies selected according to \( r \)-band absolute magnitude in the range \(-20 < M_r < -24 \). These cross-correlations indeed appear very similar in shape to those of Fig. 1.

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4.2 Cross-correlations between satellite galaxies and mass

We now consider cross-correlations between satellite galaxies and the mass. These galaxies were once associated with the dominant subhalo of a FOF group but are now centred on a secondary subhalo within a larger FOF group. They may have experienced significant mass-loss due to tidal stripping. Indeed, as noted above, a significant fraction are ‘orphans’ and have lost their subhalo entirely, remaining associated with the particle which lay at subhalo centre when it was last identified. Observation shows that the baryonic components of galaxies are substantially overdense with respect to the dark matter, and simulations suggest that they are therefore more resistant to tidal disruption (Katz, Hernquist & Weinberg 1992; Katz, Weinberg & Hernquist 1996). The semi-analytic galaxy formation model takes this into account by allowing orphans to survive for a dynamical friction time before merging them with the central galaxies of their haloes.

Table 2 shows the numbers of central and satellite galaxies in the Millennium Simulation model of Croton et al. (2006) as a function of $M_r$. Although there are fewer satellites than central galaxies for all $M_r \leq -17$, the fraction of satellites increases with decreasing luminosity, and is about 40 per cent of the galaxy population for $-19 < M_r < -17$. The fraction of orphans also increases with decreasing luminosity, and is more than half of the total satellite population in the $-19 < M_r < -17$ magnitude range.

Fig. 11 shows $\xi_{gm}$ for ‘subhalo satellites’, i.e. satellite galaxies hosted by intact subhaloes (top panel) and for orphan satellites (bottom panel). For both types of satellite, the cross-correlation function is positively biased with respect to $\xi_{mm}$ on large scales, $r \gtrsim 1\, h^{-1}\, \text{Mpc}$. This reflects the fact that relatively bright satellite galaxies almost all reside in haloes more massive than $M_\ast$, the
Satellites, the mean mass of the haloes hosting the satellites, and the mean distances of the subhalo and orphan satellites from the centres of their haloes.

\[ \xi \] (top panel) and without (bottom panel) associated subhaloes. In both cases, \( \xi \) for subhalo satellites shows an upturn due to the mass bound to the innermost region of the host halo, which is likely due to the enhanced density of subhalos within the host halo.

- **Table 2.** Galaxy samples from the Millennium Simulation: columns give the mass and satellite galaxies with cross-correlation between the mass and satellite galaxies with

- **Figure 11.** Cross-correlation between the mass and satellite galaxies with (top panel) and without (bottom panel) associated subhaloes. In both cases, \( \xi_{gm} \) is positively biased on large scales, \( r \gtrsim 1 \, h^{-1} \) Mpc indicating that satellites preferentially reside in high-mass haloes. On small scales \( \xi_{gm} \) for subhalo satellites shows an upturn due to the mass bound to the individual subhaloes, whereas the orphan satellite \( \xi_{gm} \) reflects the density profile of the host halo convolved with the radial distribution of the orphan satellites.

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Fig. 12 shows the deviations of this model from the satellite $\xi_{gm}$ measured in the simulation. Since a non-linear model for $\xi_{mm}$ is used as the basis of the satellite model on large scales, the systematic residuals in the fits to the halo and central galaxy cross-correlation functions are not present in the satellite model fits. The main deficiency in the satellite model is in reproducing the scale-dependent bias. The fits shown in Fig. 12 were obtained using single values of $r_\beta = 2 h^{-1} \text{Mpc}$ and $\beta = 0.5$. They could be improved somewhat by varying these values for the different satellite galaxy luminosity bins, but systematic residuals still remain because the exponential function is only a crude match to the shape of the scale-dependent bias.

The distribution of host halo masses for satellite galaxies in the luminosity range $-21 < M_r < -20$ is shown in Fig. 13. The distribution spans five orders of magnitude in mass. Nevertheless, the best-fitting model recovers the mean halo mass to within about 30 per cent. The bottom panel shows mean and best-fitting halo mass values as a function of $M_r$. The values obtained from fitting galaxies and the mass:

$$\xi_{gm, sat}(r) = \frac{\rho_{halo}(r;c,M)}{\rho_m} + b(M_{\text{host}})\xi_{mm}(r) \left[ 1 + \beta \exp \left(-\frac{r}{r_\beta}\right) \right]. \quad (20)$$

On scales $r \gg r_\beta$, $\xi_{sat}$ is equal to the product of the mass autocorrelation function and the bias factor $b(M_{\text{host}})$, where $M_{\text{host}}$ is the average mass of haloes which host satellite galaxies of a given luminosity. On scales $r \lesssim 2 h^{-1} \text{Mpc}$, a variable bias is apparent in the satellite $\xi_{gm}$. We attempt to model this with an exponential function. In equation (20), the parameters $r_\beta$ and $\beta$ control the characteristic scale and amplitude of this scale-dependent bias. On small scales, $r \lesssim 0.1 h^{-1} \text{Mpc}$, the satellite model is dominated by the first term due to the subhalo density profile, which, as discussed above, we take to be identical to that of a central galaxy of the same luminosity.

Fig. 13. Top panel: Distribution of host halo $M_{200}$ for satellite galaxies with $-21 < M_r < -20$. Bottom panel: Mean host halo $M_{200}$ (open diamonds) with 20th and 80th percentile values (dotted lines) are compared with best-fitting values recovered by modelling the satellite-mass cross-correlations (solid circles).
the cross-correlations are accurate for host halo masses in the range $-22 < M_r < -19$, but can differ from the true values by as much as 50 per cent for host haloes outside this range.

The model parameters entering the first term in equation (20) are the concentration, $c$, and the mass, $M_{200}$, of the halo density profile fitting formula. However, the best-fitting parameter values (shown in Fig. 13; the concentration is taken as the typical value for haloes of each mass) are not easily interpreted.

### 4.3 Cross-correlations with mass for all galaxies

In Fig. 14 we present cross-correlations of all galaxies with mass. This is simply the linear combination of the cross-correlations for central and satellite galaxies, weighted by the relative fractions of each type, i.e.

$$\xi_{gm, all} = (1 - f_{sat})\xi_{gm, central} + f_{sat}\xi_{gm, sat},$$

where the fraction of satellite galaxies is defined as $f_{sat} = N_{sat}/(N_{central} + N_{sat})$. The satellite fraction in each luminosity range strongly affects the shape of $\xi_{gm}$ for galaxies of that luminosity. Table 2 shows that $f_{sat}$ ranges from 40 per cent in the lowest luminosity bins, $M_r > -19$, to 5 per cent for $M_r < -22$.

Since equation (21) is a linear combination of two functions which can differ in value by several orders of magnitude and since $f_{sat}$ is of the order of unity, $\xi_{gm, all}$ is dominated by the larger of $\xi_{gm, central}$ and $\xi_{gm, sat}$ at most radii. For instance, for $-24 < M_r < -23$, $\xi_{gm, central}/\xi_{gm, sat} \gg 1$ at all radii, thus equation (21) gives $\xi_{gm, all} \approx (1 - f_{sat})\xi_{gm, central}$ and the cross-correlation function is very similar to $\xi_{gm, central}$. At lower luminosities, $\xi_{gm, sat}/\xi_{gm, central} \gg 1$ at intermediate radii, $0.1 h^{-1} \text{Mpc} < r < 1 h^{-1} \text{Mpc}$, so $\xi_{gm, all} \approx f_{sat}\xi_{gm, sat}$ and in this case the cross-correlation function resembles that of the satellite galaxies.

The combined $\xi_{gm}$ model for central and satellite galaxies contains the following three parameters: $M_{central}$, $M_{host}$ and $f_{sat}$. However, in fitting this composite model to the measured cross-correlations, we obtained best-fitting parameter values which were not in good agreement with the true values of $M_{central}$, $M_{host}$ and $f_{sat}$. This suggests that the shape of $\xi_{gm}$ is degenerate, with various combinations of the free parameters giving almost equivalent fits.

It is interesting to note that our cross-correlation model is based on components which deviate strongly from power laws, i.e. $\xi_{gm}$, $\xi_{mm}$ and $\xi_{halo}$. Nevertheless, at intermediate luminosities $\xi_{gm, all}$ is reasonably well described by a power-law over a wide range in radius. In order to illustrate this point, we plot in Fig. 15 the deviations from power-law fits to $\xi_{gm, all}$. In particular, we find that in the range $-21 < M_r < -20$, $\xi_{gm, all}$ is well fitted by a power law with slope $r^{-1.8}$ and the deviations from the fit are $<10$ per cent at all radii!

For galaxies more luminous than $M_r < -21$, the deviations from best-fitting power laws become $>50$ per cent at some radii. Galaxies in this luminosity range are dominated by central galaxies, so our model for $\xi_{gm, central}$ can be used to fit observations and provide detailed checks on various aspects of the cosmological and galaxy formation models. At lower luminosities, however, we conclude that some independent information regarding the satellite fraction must be included in order to extract useful information about the average mass of the haloes which host satellites. Alternatively, shear data can be stacked around galaxies that are observed to be brighter than all their neighbours, and thus are very likely central galaxies.

Our models can be used to predict $\Delta \Sigma(R)$ for direct comparison with galaxy–galaxy lensing measurements. Examples are shown in the lower panel of Fig. 14. It is notable that these curves show a variety of scales and features which should be directly observable. A proper comparison with observation needs to account for (i) the sensitivity of our predictions to cosmological parameters; (ii) their sensitivity to the more uncertain parts of the galaxy formation modelling (e.g. the treatment of orphan satellites); (iii) the systematic effects caused by the gravity of the baryons; (iv) the projection effects (e.g. light-cone averaging and source redshift

![Figure 14](image1.png)

**Figure 14.** Top panel: Cross-correlations between all galaxies and the mass at $z = 0$. High-luminosity galaxy samples are dominated by central galaxies, therefore $\xi_{gm}$ resembles our simple model for $\xi_{mm}$. Low-luminosity galaxy samples contain a significant fraction of satellite galaxies and $\xi_{gm}$ follows the shape of $\xi_{mm}$. Bottom panel: $\Delta \Sigma(R)$ corresponding to $\xi_{gm}$.

![Figure 15](image2.png)

**Figure 15.** Deviations from the best-fitting power laws of our measured cross-correlations between all galaxies and the mass.
averaging) expected in real data and (v) random and systematic noise due to observational uncertainties. This is beyond the scope of this paper. It is nevertheless interesting to compare the lower panel of Fig. 14 to fig. 1 of Seljak et al. (2005) which gives observational results from the SDSS in the same format and for the same bins of absolute magnitude. The agreement is very encouraging. At $r \sim 1.0 \, h^{-1} \, $Mpc both the observations and the model give $\Delta \Sigma(R) \sim 4 h M_{\odot}/$pc$^{-2}$ for all $M_r > -22$, but a $-4$ times larger value for $-22 > M_r > -23$. At $r \sim 0.1 h^{-1} \, $Mpc, the values of $\Delta \Sigma(R)$ for both the model and the SDSS data decrease smoothly with luminosity from $\sim 120 h M_{\odot}/$pc$^{-2}$ for $-22 > M_r > -23$ to $\sim 10 h M_{\odot}/$pc$^{-2}$ for $-18 > M_r > -19$. The amplitudes agree to within the (relatively large) observational errors over this full range of luminosity and radius. With improved observational data it will clearly become possible both to estimate cosmological parameters and to test the various assumptions underlying our model.

Related observational results have been published very recently by Sheldon et al. (2007) and Johnston et al. (2007) who analysed the weak lensing signal for stacks of large numbers of clusters selected from the SDSS. Although these stacks assumed clusters to be centred on their identified brightest members, the stacks were carried out for bins of cluster richness or total luminosity rather than for bins of central galaxy luminosity. They are thus similar but not identical to the kind of stacks analysed here. Johnston et al. (2007) analysed these data with a simplified LCDM model with parameters very similar to those of the Millennium Simulation, finding good agreement. Thus we would expect to find similarly good agreement when their data are compared directly with the simulation. A fully quantitative comparison will, however, require modelling of the Sheldon et al. (2007) cluster selection, binning, and stacking procedures, and so is beyond the scope of this paper.

5 SUMMARY

We have calculated cross-correlations between halo centres and the mass, and between galaxies and the mass in the Millennium Simulation of a LCDM cosmology. The shape of the halo-mass cross-correlation function $\xi_{hm}$ is well fitted by a simple two-part model. On small scales, $\xi_{hm}$ is specified by the halo density profile which is accurately described by the Einasto model advocated by N04. On large scales, $\xi_{hm}$ is specified by the mass autocorrelation function predicted by linear theory and the halo bias model of Sheth & Tormen (1999). The deviations between $\xi_{hm}$ and the best-fitting model are dominated by the quasi-linear distortion in $\xi_{mm}$, but are otherwise $\lesssim 5$ per cent.

The cross-correlation functions of central galaxies, $\xi_{gm,central}$, are reasonably well fitted by our model for $\xi_{hm}$. The best-fitting models recover the mean halo mass of central galaxies to within 30 per cent. The cross-correlations of satellite galaxies, $\xi_{gm,sub}$ appear qualitatively different from those of haloes or central galaxies. Their shape is similar to that of $\xi_{mm}$, with an upturn at small scales due to the mass associated with the individual subhaloes in which most satellites reside. A model for $\xi_{gm,sub}$ based on these features reproduces the cross-correlation functions to within 10 per cent and recovers the mean host halo mass to within 50 per cent.

The cross-correlation function of all galaxies, $\xi_{gm,all}$ is simply a linear combination of $\xi_{gm,central}$ and $\xi_{gm,sub}$ weighted by the relative fractions of central and satellite galaxies. For very luminous galaxies, the satellite fraction $f_{sat} \lesssim 10$ per cent, and $\xi_{gm,all}$ is dominated by the contribution of $\xi_{gm,central}$. At intermediate luminosities, $-22 < M_r < -20$, $\xi_{gm,all}$ is reasonably well fitted by a power law. At lower luminosities the cross-correlation is dominated by the satellite galaxy contribution.

The conversion from the three-dimensional cross-correlations to the directly observable mean tangential shear [which is proportional to $\Delta \Sigma(R)$, the difference between the mean enclosed surface density and the local surface density at each projected radius $R$] accentuates features in the cross-correlations. Galaxy–galaxy lensing surveys typically contain enough information to separate the lenses into likely satellites and likely central systems. As a result, the features seen in our predictions should provide information on cosmological parameters, tidal stripping processes and the exact way in which galaxies trace the dark matter. Our current models already agree quite well with the SDSS data presented by Seljak et al. (2005). If the sharp features in $\xi_{hm}(r)$ are clearly detected in future data at the positions where they are expected, this will provide a major challenge to theories which try to replace dark matter by a modification of Einsteinian gravity.

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