Interaction of two bubbles with distortion in an acoustic field

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ABSTRACT

Expression of the secondary Bjerknes force of two bubbles is obtained by considering the distortion of two bubbles. The secondary Bjerknes forces in different acoustic fields are simulated, and the influence factors are analyzed and discussed. It is shown that the distortion of a bubble has an important influence on the interaction of two bubbles. The strength and even the directions of the secondary Bjerknes force of two bubbles with distortion differ considerably from the predictions of the spherical symmetry theory. The results show that when two bubbles oscillated stably in an acoustic field, the secondary Bjerknes force of two bubbles with distortion is several times more than that of two spherical bubbles in the same condition. The secondary Bjerknes force of two bubbles with distortion has more interaction distance than that of two spherical bubbles. The secondary Bjerknes force of two bubbles with distortion depends on the distance of two bubbles, the shape mode of two bubbles, the equilibrium radii of two bubbles and the driving acoustic field. The nonspherical distortion effects of the secondary Bjerknes has an importance on understanding the structure formation of bubbles and evolution process of bubble group in an acoustic field.

1. Introduction

Interaction of two gas bubbles is a famous acoustic phenomenon. The mutual interaction force of two bubbles due to the “secondary” sound fields emitted by other bubbles is called the secondary Bjerknes force [1]. The secondary Bjerknes force has an importance on many acoustic phenomena and applications, such as acoustic cavitation, multiple bubble sonoluminescence (MBSL), specifically mutual attraction and the coalescence of bubbles [2-8]. In the recent research, there are many of detailed reviews and experimental researches of the secondary Bjerknes force [9,10], such as: Yashida and Fujikawa observed the behavior of two bubbles in an ultrasonic standing wave using imaging system with a high-speed video camera, and found the direction of the force reversed at a specific separation distance of the two bubbles [11]. The theoretical researches of the secondary Bjerknes force are summarized in the following. Crum provided a simple model for the secondary Bjerknes force of two bubbles, assuming the bubble radii linearly vibrated with small amplitude [12]. Zabolotskaya and Ida theoretically demonstrated that the direction of the force depended only on the distance of the bubbles using a linear model of the coupled vibrations of two bubbles [13,14]. Harkin researched a weak forcing model of the coupled oscillations and translations of two bubbles [15]. The above researches are about the linear theory of the secondary Bjerknes force. Oguz and Prosperetti conducted numerical simulations of a nonlinear model of two vibrating and translating bubbles and demonstrated that increasing the acoustic can even reverse the direction of the force [16]. Mettin and Akhatov investigated the mutual interaction of small oscillating cavitation bubbles in a strong acoustic field, and their results show that the strength and even the directions of the resulting secondary Bjerknes force differ considerably from the predictions of the well-known linear theory [17]. These researches belong to the nonlinear research of the secondary Bjerknes force.

Despite many theoretical reports on the interaction of bubbles by assuming two bubbles remain spherical oscillations [18-26], there were little investigations on the secondary Bjerknes force of two bubbles with distortion. The experiments of sonoluminescence showed that a gas bubble is not spherical when the bubble oscillated in an acoustic field [27-32]. The nonspherical shape distortion may have an influence on the secondary Bjerknes force.

The purpose of this paper is to study the influence of the distortion of two bubbles on the secondary Bjerknes force. We aimed to investigate the complicate bubble phenomena, which is unexplained by the classical theoretical of the secondary Bjerknes force. The results can help us to understand the structure formation of bubbles in an acoustic field.

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especially the evolution process of bubble group in acoustic cavitation.

2. Mathematical model

Two gas bubbles in a liquid driven by a stationary sound field, both bubbles experience pressure oscillations of the same amplitudes and phases. For two spherical bubbles with radii $R_1$ and $R_2$ in an acoustic field, the known formula for secondary Bjerknes force can be expressed as [17]:

$$F_{BH} = -\frac{\rho}{4\pi} \left( \frac{\dot{\nu}_1(t)}{\dot{\nu}_2(t)} \right) V_1 V_2,$$

where $< >$ denotes the time average, and the dot denotes the time derivative. If we define $c_B = V_1 V_2$, the direction of the secondary Bjerknes force depends on the sign of the $\langle c_B \rangle$. If the $\langle c_B \rangle > 0$, the secondary force is attractive force, otherwise, the secondary Bjerknes force is repulsive force.

For calculation of radial oscillation of two bubbles, we used a model of coupled oscillation with fixed separation distance of two bubbles. The equation of liquid motion is given by

$$\rho \frac{\partial u_1(t)}{\partial t} = -\nabla P_1(r,t),$$

where $u_1(t)$ is the velocity field of the first bubble. Then

$$u_1(t) = -\nabla \varphi_1,$$

where $\varphi_1$ is the velocity potential of the first bubble, and can be defined as [33]:

$$\varphi_1 = \frac{R_1 R_2}{r} + \frac{R_1^{(n+1)}}{(n+1)^{m+1}} Y_n \left( \frac{\dot{a}_1 a_1}{R_1} \right).$$

Substituting Eq. (7) into Eq. (6), and doing a derivative to time, we obtain

$$\dot{u}_1(t) = \frac{2 R_1 R_2^2 + R_1 R_2^2}{r^2} \left( \frac{n+2)R_1^{(n+1)} R_1 \left( \dot{a}_1 + 2a_1 \frac{\dot{\phi}}{\phi} \right) + R_1^{(n+1)} \left[ \dot{a}_1 + 2a_1 \frac{\dot{\phi}}{\phi} + 2a_1 \left( \frac{\ddot{R}_1 R_1^2}{R_1^2} \right) \right] \right) \nabla \varphi_1 e_r.$$

(8)

We now focus on the secondary Bjerknes force of two bubbles with a nonspherical perturbation in an acoustic field. We assume that each bubble with distortion is a single shape mode, and there is no coupling between different shape modes. Considering a small distortion of the spherical interface, the distortion of the interface from $R_1$ or $R_2$ to $r_1$ and $r_2$ can be expressed as [33]:

$$r_1(\theta, R_1, t) = R_1(t) + a_n Y_n(\cos \theta),$$

(2)

$$r_2(\theta, R_2, t) = R_2(t) + b_n Y_n(\cos \theta),$$

(3)

The second bubble in a liquid under a pressure gradient $\nabla P_1$ emitted by the first bubble experiences a force $[12]$

$$F_{12} = -V_2(t) \nabla P_1 |_{r=0}.$$

(4)

The direction of the secondary Bjerknes force of two bubbles with distortion depends on the sign of $\langle c_{BN} \rangle$. If $\langle c_{BN} \rangle > 0$, the secondary force is attractive force, otherwise, the secondary Bjerknes force is repulsive force.

Combining Eq. (8) and Eq. (5), substituting the results into Eq. (4), and integrating over a period of the volume oscillations, we obtain the secondary Bjerknes force of two bubbles with distortion

$$F_{BN} = \langle F_{12} \rangle = -\frac{\rho}{4\pi} \left( \frac{\dot{\nu}_1(t)}{\dot{\nu}_2(t)} \right) \left[ \frac{16a_1^2}{3} \right] \left( \frac{n+2)R_1^{(n+1)} R_1 \left( \dot{a}_1 + 2a_1 \frac{\dot{\phi}}{\phi} \right) + R_1^{(n+1)} \left[ \dot{a}_1 + 2a_1 \frac{\dot{\phi}}{\phi} + 2a_1 \left( \frac{\ddot{R}_1 R_1^2}{R_1^2} \right) \right] \right) \nabla \varphi_1 R_1^2 \left[ 1 + \frac{3b_n^2}{R_2^2(2m+1)} \right] e_r.$$

(9)

For presentation of the results, we define

$$c_{BN} = -\frac{16a_1^2}{3} \left( \frac{n+2)R_1^{(n+1)} R_1 \left( \dot{a}_1 + 2a_1 \frac{\dot{\phi}}{\phi} \right) + R_1^{(n+1)} \left[ \dot{a}_1 + 2a_1 \frac{\dot{\phi}}{\phi} + 2a_1 \left( \frac{\ddot{R}_1 R_1^2}{R_1^2} \right) \right] \right) \nabla \varphi_1 R_1^2 \left[ 1 + \frac{3b_n^2}{R_2^2(2m+1)} \right].$$

(10)
radial oscillation of the first bubble is described as [22]:

\[
\left(1 - \frac{\dot{R}_1}{c}\right) \ddot{R}_1 = 2m \ddot{R}_2 - \frac{1}{\rho c^2} \left(1 + \frac{\dot{R}_1}{c}\right) P_{\text{atm}} \frac{R_1}{R_2} \frac{d}{dt} P_{\text{atm}} - \frac{1}{d} \left(2\dot{R}_1^2 \dot{R}_2^2 \ddot{R}_2\right),
\]

and

\[
P_{\text{atm}} = \left(P_0 + \frac{2\sigma}{R_{\text{eq}}^2} \left(\frac{R_{\text{eq}}}{R_1}\right)^{3n} - \frac{2\sigma}{R_1} \frac{4\dot{R}_1}{R_1} - P_0 - P_0\right)
\]

Exchanging the subscripts 1 and 2 yields the dynamical equation for the second bubble.

Following previous authors, the dynamics for the distortion amplitude of the first bubble is given by [28]

\[
\ddot{a}_1 + B_1(t)\dot{a}_1 - A_1(t)a_1 = 0.
\]

Then

\[
A_1(t) = (n - 1) \frac{\ddot{R}_1}{R_1} - (n - 1)(n + 1)(n + 2) \frac{\sigma}{\rho R_1^3} - \left[(n - 1)(n + 2)\right] \frac{\delta_1}{R_2^3} \frac{2\dot{R}_2}{R_2^2},
\]

and

\[
B_1(t) = \frac{3\ddot{R}_1}{R_1^2} + \left[(n + 2)(2n + 1) - 2n(n + 2)\right] \frac{\delta_1}{R_2^3} \frac{2\dot{R}_2}{R_2^2}.
\]

If we assume fluid flow only in a thin layer around the bubble, the thin layer of thickness \(\delta_1\) can be expressed as [30]:

\[
\delta_1 = \frac{1}{m} \sqrt{\frac{c}{\Delta n}} |\dot{R}_1|.
\]

the dynamics for the distortion amplitude of the second bubble can be expressed as:

\[
\ddot{a}_2 + A_2(t)\dot{a}_2 - B_2(t)a_2 = 0.
\]

3. Numerical results

We now focus on the influence of the nonspherical distortion on the secondary Bjerknes force of two bubbles. When the nonspherical distortion appears, the shape of a gas bubble in an acoustic field will recover spherical shape or deviate spherical shape more and more. For
the second situation above, the nonpherical distortion will lead the bubble to breakup within a few oscillation cycles. The second case is not in the scope of this paper.

The secondary Bjerknes force of two bubbles with distortion was calculated. Almost all choices of \( R_1, R_2, a_n \) and \( b_m \) in this paper only consider the first driving period.

3.1. The secondary Bjerknes force in different acoustic fields

Fig. 1 is a phase diagram of stability about a bubble with distortion. The black region indicates shape stable region which shows a bubble can oscillate stably, and a bubble with distortion does not collapse because of the nonpherical distortion. The white region indicates shape instable region, which is not in the scope of this paper, so we only discuss the secondary Bjerknes force in a stable region in Fig. 1.

For illustration, let us consider a section of Fig. 2, \( P_a = 1.34 \times 10^6 \text{Pa} \), \( f = 20 \text{kHz} \), \( R_{10} = 1.8 \mu \text{m} \) and \( R_{20} = 2 \mu \text{m} \), for a constant distance of two bubbles of \( d = 200 \mu \text{m} \). The radial oscillations of two bubbles are shown in Fig. 2 (a), the bubble of \( R_{20} = 2 \mu \text{m} \) oscillates violently in the above acoustic field. The bubble of \( R_{20} = 2 \mu \text{m} \) expands to the max radius at 33μs s, and the maximum radius is 12.45 times more than equilibrium radius during a strong nonlinear oscillation. The bubble collapses at 36.1 μs, then the bubble oscillates around the equilibrium radius. For the bubble of \( R_{10} = 1.8 \mu \text{m} \), the radial oscillation is violent in the same driving field. The maximum radius is 7.05 times more than the equilibrium radius in one driving period, and collapses to the equilibrium radius. Then, the bubble grows up and shrinks once again, and oscillates around the equilibrium radius. The distortions of two bubbles occur in the initial stage of oscillation in the above driving condition, which are shown in Fig. 2 (b) and (c). From Fig. 2 (b) and (c) can also be seen that the secondary shape amplitudes of two bubbles are very small, and the distortion do not lead the bubble to breakup. With increasing of the time, the distortions of two bubbles disappear, and two bubbles return to spherical symmetrical oscillations.

We numerical simulate \( c_{\text{BN}} \) with time in one driving period, and compare it with the \( c_{\text{B}} \) of two spherical bubbles, which is shown as Fig. 3. The results show that averaging of \( c_{\text{BN}} \) yields a negative Bjerknes force, and \( \langle c_{\text{BN}} \rangle = -3.2208 \times 10^{-18} \text{m}^4/\text{s}^2 \). Thus two bubbles with distortion repulse each other. The function \( c_{\text{B}} \) of two spherical bubbles is altered according to Fig. 3 (a), and \( \langle c_{\text{B}} \rangle = 1.4176 \times 10^{-18} \text{m}^4/\text{s}^2 \). In this case, two spherical bubbles attract each other. It is found that the distortion of two bubbles can increase the interaction of two bubbles by contrasting \( \langle c_{\text{BN}} \rangle \) with \( \langle c_{\text{B}} \rangle \), and even change the directions of the secondary Bjerknes force. Although the distortion disappears with times, the sound effects of shape oscillation of bubbles can increase obviously the secondary Bjerknes force. This may be the reason of the formation of the structure of bubbles in a strong acoustic field.

According to the classical theory of the secondary Bjerknes force, the secondary Bjerknes force is very small in a weak driving field. Therefore the secondary Bjerknes force is omitted in lots of researches. But the experiments of bubbles showed that bubbles can form some stable bubble structure in a weak acoustic field [34]. These experimental results showed that the interaction of bubbles is big enough in a weak acoustic field. We also want to research the secondary Bjerknes force of two bubbles with the distortion, and try to give the explanation for the formation of the bubble structure in a weak acoustic field.

According to the phase diagram of stability in Fig. 1, the calculations are carried out with driving pressure \( P_a = 0.9 \times 10^6 \text{Pa} \), driving frequency \( f = 20 \text{kHz} \), \( m = n = 2 \) and a constant second bubble size of \( R_{20} = 2 \mu \text{m} \). The main results are presented in Fig. 4 (a) and (b). The normalized maximum radius of the first bubble, \( R_{1_{\text{max}}}/R_{10} \), is plotted with its equilibrium radius \( R_{10} \) in Fig. 4 (a). The normalized maximum shape amplitude \( (n = 2) \) of the first bubble \( a_{2_{\text{max}}}/R_{10} \) with the change of
the equilibrium radius of the first bubble is shown in Fig. 4 (b). The results show that the bubbles of 1–10 μm oscillate periodically, and appear a weak bouncing behavior without collapse in the above driving acoustic field. Due to the surface tension, bubbles which are smaller than 2 μm oscillate with very low amplitudes and nearly linearly. The amplitudes of bubbles are bigger than 2 μm increase with the radii of bubbles, but the maximum radii of bubbles (2–10 μm) are only 1.05–1.65 times more than their equilibrium radii in the weak acoustic field. The maximum shape amplitudes of bubbles (1–10 μm) are only 0.01–0.48 times of their equilibrium radii, which show the nonspherical distortions are weak and bubbles could not breakup because of nonspherical distortion. The distortions of bubbles (1–10 μm) will disappear and the oscillations of bubbles will return to spherical symmetry after several driving periods.

The secondary Bjerknes force of two bubbles with distortion is shown as Fig. 4 (d). The secondary Bjerknes forces of different bubble pairs are different in direction and magnitude. But there is a big change by comparing with the secondary Bjerknes force of two spherical bubbles in the same driving condition (Fig. 4 (c)). The secondary Bjerknes force considered distortion can be 10^1 times more than the secondary Bjerknes force of two spherical bubbles in a certain parameters of bubbles and a weak acoustic field. This result shows that the secondary Bjerknes force of two bubbles with distortion is big enough and is not omitted in a weak acoustic field, although the oscillation is not violent enough. Two bubbles oscillate in an acoustic field with distortion, which contain radial oscillate and shape oscillate. The radiation sound fields emitted by the shape oscillations of two bubbles overlap with each other, and the secondary Bjerknes force enlarges several times. So the interaction of two bubbles with distortion has two parts: the force caused by radial oscillations and the force caused by shape oscillations, Therefore the interaction of two bubbles with distortion is more bigger than that of two bubbles without distortion. This force is big enough to form some stable bubble structure in a certain condition. These results can also explain some bubble phenomenon in experiments.

Fig. 4. Resonancelike response and the secondary Bjerknes force (a) Normalized maximum radius \( \frac{R_{1\text{max}}}{R_0} \) vs the equilibrium radius of the first bubble (b) Normalized maximum shape amplitude \( \frac{a_{1\text{max}}}{R_0} \) vs the equilibrium radius of the first bubble (c) Secondary Bjerknes force of two spherical bubbles vs the equilibrium radius of the first bubble (d) Secondary Bjerknes force of two bubbles with distortion vs the equilibrium radius of the first bubble.

Fig. 5. The secondary Bjerknes force and the distance of bubbles. (a) \( F_B \) vs \( R_{10} \) for \( d = 500 \mu m \) (a) \( F_{BN} \) vs \( R_{10} \) for \( d = 500 \mu m \) (c) \( F_B \) vs \( R_{10} \) for \( d = 1000 \mu m \) (d) \( F_{BN} \) vs \( R_{10} \) for \( d = 1000 \mu m \).
3.2. Influence of the distance of two bubbles on the secondary Bjerknes force

The distance of two bubbles has an influence on the secondary Bjerknes force, the change of the secondary Bjerknes force of two bubbles with distortion in different distances is researched. For illustration, let us consider a section of Fig. 5, \( P_a = 0.9 \times 10^5 \text{Pa} \), \( m = n = 2 \), for a constant second bubble size of \( R_{20} = 2 \mu m \). The secondary Bjerknes force of two bubble with distortion is plotted vs the equilibrium radius of the first bubble in Fig. 5 (b) and (d). The curves in Fig. 5 (b) and Fig. 5 (d) are results for the distance of 500 \( \mu m \) and 1000 \( \mu m \). The results also were compared with the secondary Bjerknes forces of two spherical bubbles in the same condition, which are shown in Fig. 5 (a) and (c).

The results show that the secondary Bjerknes force of two bubbles with distortion also decreases with the increasing of distance of two bubbles. The interaction of bubbles has a certain interactional distance. With the increase of the distance, the interaction of bubbles decreases. If the distance is big enough, two bubbles are uncoupled and the secondary Bjerknes force is near to 0, which is shown in Fig. 5 (c) and Fig. 5 (d). But for two bubbles with distortion, the results show that the secondary Bjerknes force of two bubbles with distortion has a farther interactional distance compared with spherical bubbles. A gas bubble with distortion has an acoustic effect on the other bubbles in a farther distance.

3.3. The influence of the shape modes on the secondary Bjerknes force

The shape mode of a gas bubble with distortion has an important influence on the secondary Bjerknes force. For illustration, let us consider a section of Fig. 6, \( P_a = 0.9 \times 10^5 \text{Pa} \), for a constant second bubble size of \( R_{20} = 2 \mu m \) and a constant shape mode of the second bubble \( m = 2 \). The curves in Fig. 6 (a), (b) and (c) are results for different shape modes of the first bubble.

It can be seen that the secondary Bjerknes force is biggest when the shape mode of the first bubble is equal to 2 (\( n = 2 \)) in the above driving acoustic field in Fig. 6 (a). For \( n = 3 \), the secondary Bjerknes force is smaller than that \( n = 2 \), which is shown as Fig. (b). For \( n = 4 \), the secondary Bjerknes force is smallest, which is shown as Fig. (c). These results show that the shape mode of a gas bubble with distortion has an important contribution for the secondary Bjerknes force. According to the reference [31], the amplitude of the secondary shape mode of a gas bubble is bigger than that other shape order in the same driving acoustic, so the radiation field of a bubble with secondary shape mode is biggest. Therefore, the secondary Bjerknes force of two bubbles with shape mode \( n = 2 \) and \( m = 2 \) is biggest. The secondary Bjerknes force of two bubbles whose shape mode orders are bigger than 2 is smaller.

Fig. 6. The secondary Bjerknes force of two bubbles with distortion for different shape modes of the first bubble (a) \( F_{BN} \) vs \( R_{10} \) for \( n = 2 \), \( m = 2 \) (b) \( F_{BN} \) vs \( R_{10} \) for \( n = 3 \), \( m = 2 \) (c) \( F_{BN} \) vs \( R_{10} \) for \( n = 4 \), \( m = 2 \).

Fig. 7. The secondary Bjerknes force of two bubbles with distortion vs \( R_{10} \) for different radii of the second bubble (a) \( F_{BN} \) vs \( R_{10} \) for \( R_{20} = 2 \mu m \) (b) \( F_{BN} \) vs \( R_{10} \) for \( R_{20} = 3 \mu m \) (c) \( F_{BN} \) vs \( R_{10} \) for \( R_{20} = 4 \mu m \).

3.3. The influence of the shape modes on the secondary Bjerknes force

The shape mode of a gas bubble with distortion has an important influence on the secondary Bjerknes force. For illustration, let us consider a section of Fig. 6, \( P_a = 0.9 \times 10^5 \text{Pa} \), for a constant second bubble size of \( R_{20} = 2 \mu m \) and a constant shape mode of the second bubble \( m = 2 \). The curves in Fig. 6 (a), (b) and (c) are results for different shape modes of the first bubble.

It can be seen that the secondary Bjerknes force is biggest when the shape mode of the first bubble is equal to 2 (\( n = 2 \)) in the above driving acoustic field in Fig. 6 (a). For \( n = 3 \), the secondary Bjerknes force is smaller than that \( n = 2 \), which is shown as Fig. (b). For \( n = 4 \), the secondary Bjerknes force is smallest, which is shown as Fig. (c). These results show that the shape mode of a gas bubble with distortion has an important contribution for the secondary Bjerknes force. According to the reference [31], the amplitude of the secondary shape mode of a gas bubble is bigger than that other shape order in the same driving acoustic, so the radiation field of a bubble with secondary shape mode is biggest. Therefore, the secondary Bjerknes force of two bubbles with shape mode \( n = 2 \) and \( m = 2 \) is biggest. The secondary Bjerknes force of two bubbles whose shape mode orders are bigger than 2 is smaller.

Fig. 8. The secondary Bjerknes force of two bubbles with distortion for driving frequencies (a) \( F_{BN} \) vs \( f \) for \( R_{10} = 2 \mu m \) and \( R_{20} = 2 \mu m \) (b) \( F_{BN} \) vs \( f \) for \( R_{10} = 2 \mu m \) and \( R_{20} = 3 \mu m \).
3.4. The influence of the equilibrium radii of two bubbles on the secondary Bjerknes force

The secondary Bjerknes force is sensitive to the equilibrium radii of bubbles in a weak acoustic field, and the secondary Bjerknes forces are different with different equilibrium radii of bubbles. If we choose \( P_0 = 0.9 \times 10^5 \text{Pa} \), \( f = 20 \text{ kHz} \), \( d = 200 \mu \text{m} \), the shape mode of first bubble \( \eta = 2 \) and the shape mode of the second bubble \( \eta = 2 \), the secondary Bjerknes force of two bubbles with distortion is simulated. The curves in Fig. 7(a), (b) and (c) are results for different equilibrium radii of two bubbles. It can be seen that the secondary Bjerknes forces of different bubble pairs have a big difference in the magnitude and direction in the Fig. 7(a), (b) and (c). When the bubble radial oscillations are weak, the shape oscillations have an important influence on the radiation acoustic field. The radial oscillations and shape oscillations of different bubble pairs are different, so the interaction of two different bubble pairs is different. The secondary Bjerknes forces of bubble pairs with a smaller bubble and a bigger bubble are bigger than that of bubble pairs with two smaller bubbles. Besides, for bubble pairs with one bubble constant, the radius of other bubble is bigger and the interaction is stronger in a weak acoustic field. Such as Fig. 7(a), (b) and (c), for a constant bubble size of the first bubble, the equilibrium radius of the second bubble increases from \( 2 \mu \text{m} \) to \( 3 \mu \text{m} \) and \( 4 \mu \text{m} \), the secondary Bjerknes force can increase 2 and 10 times. These results may be the reason of clusters of large bubbles in a weak acoustic field [34].

3.5. The influence of the driving frequency on the secondary Bjerknes force

The driving frequency has an important influence on the secondary Bjerknes force of two bubbles with distortion in a weak acoustic field. For illustration, let us consider a section of Fig. 8, \( P_0 = 0.9 \times 10^5 \text{Pa} \), for a constant shape mode of first bubble \( \eta = 2 \) and a constant shape mode of the second bubble \( \eta = 2 \). The second Bjerknes force of two bubbles with distortion for different driving frequencies is shown in Fig. 8. The curves in Fig. 8(a) and (b) are results for \( R_0 = 2 \mu \text{m} \), \( R_0 = 2 \mu \text{m} \) and \( R_0 = 2 \mu \text{m} \), \( R_0 = 3 \mu \text{m} \). The results show that the secondary Bjerknes force of two bubbles with distortion is related to the driving frequency in a weak acoustic field. If the driving frequency is close or equal to the resonance frequency of the first bubble or the second bubble, the secondary Bjerknes force increase obviously in a weak acoustic field. When the driving frequency is much smaller or even larger than the resonance frequency of the first bubble or the second bubble, the secondary Bjerknes force is smaller in a weak acoustic field.

4. Conclusion

The secondary Bjerknes force of two gas bubbles with distortion is obtained based on the nonspherical perturbations. The mutual interaction forces of two coupled bubbles with distortion in a strong (weak) acoustic field have been investigated. The results also compare with the secondary Bjerknes force of two spherical bubbles in the same condition. For a strong low-frequency acoustic field, if the distortions of two bubbles are weak and bubbles do not breakup, the distortions may make the interaction force increase several times. When two bubbles oscillate in a weak acoustic field, the distortions of two bubbles also make the interaction force increase more than \( 10^3 \) times. The results also show that the secondary Bjerknes force of two bubbles with distortion has more interactional distance than that of two spherical bubbles. The distance of two bubbles, shape mode, the radii of two bubbles and acoustic parameter have an important influence on the secondary Bjerknes force. Our findings are important for stable bubble structures in an acoustic field, especially for collective bubble phenomena in a weak acoustic field. According to the classical theory of the secondary Bjerknes force, the secondary Bjerknes force is very small in a linear modeling and omitted in most researches. The classical theory do not explain the collective bubble phenomena in a weak acoustic field, but we find the secondary Bjerknes force of two bubbles is big enough by considering the nonspherical distortion in a weak acoustic field. Although we only consider the Bjerknes force of two bubbles with distortion during the first driving period, the interaction of two non-spherical bubbles also has an important influence on the bubble motion. The interaction of bubbles can change the motion trend of small bubbles, which make the phenomenon of bubble coalescence or separation occur. These may be the reason of the formation of bubble structure in an acoustic field.

Future investigations will address to further details of bubble structures in a weak acoustic field, and provide a theory for the researches of bubble groups in an acoustic field.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Author statement

Yan Ma performed Conceptualization, Data curation, Formal analysis, Methodology and Writing-original draft. Guoqian Zhang performed Validation and Visualization. Tao Ma performed Conceptualization and Software. All authors read and contributed to the manuscript.

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