Cartan’s Supersymmetry and
Weak and Electromagnetic Interactions

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Abstract

We apply the Cartan’s supersymmetric model to the weak interaction of hadrons. The electromagnetic currents are transformed by \( G_{12}, G_{13}, G_{132} \) and the factor \((1-\gamma_5)\) is inserted between \( \bar{l} \nu \) or \( \bar{l} \nu \) when the photon is replaced by \( W^\pm \), and between \( l\bar{l} \) or \( \nu\bar{\nu} \) when the photon is replaced by \( Z \).

Electromagnetic currents in the Higgs boson \( H^0 \) decay into \( 2\gamma \) and \( D(0^+) \), decay into \( D(0^-)\pi \) and \( D(0^-)\gamma \) in which leptons are replaced by quarks are also studied.

A possibility that the boson near the \( B\bar{B} \) thershold \( \chi_b(3P, 10.53 \text{ GeV}) \) is the Higgs boson partner \( h^0 \) is discussed.

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1 Introduction

Cartan[1] formulated the coupling of 4-dimensional spinors $A, B, C, D$ and 4-dimensional vectors $E, E'$ using the Clifford algebra, which is a generalization of quaternions and octonions. In this model there appears a triality symmetry, and one can imagine a presence of sectors of $E$ and $E'$ which cannot be detected by fermions in our detectors, in other words, fermions in our universe are transformed by $G_{12}, G_{13}, G_{123}$ and $G_{132}$ to vectors, but the vectors produced by these transformations cannot be detected by our electromagnetic probes.

We applied the Cartan’s supersymmetry to our physical system and applied to the decay of $\pi^0, \eta, \eta'$ to $\gamma\gamma$ [2, 3, 4, 5, 6]. The Pauli spinor was treated as a quaternion and the Dirac spinor was treated as an octonion. In the $\pi^0$ decay, the two final vector fields belong to the same group $(EE)$ or $(E'E')$, and we call the diagram rescattering diagram. In the decay of $\eta$, and $\eta'$, final vector fields belong to different groups $(EE')$, which we called twisted diagrams. Qualitative difference of the $\eta$ decay or $\eta'$ decay, and $\pi^0$ decay can be explained by symmetry of Cartan’s spinor.

The Clifford algebraic spinor of Cartan $\psi \in (C \otimes C^{\ell_{1,3}}) f$ is associated with the Dirac spinor

$$\psi = \xi_1 \hat{i} + \xi_2 \hat{j} + \xi_3 \hat{k} + \xi_4 = \begin{pmatrix} \xi_4 + i\xi_3 & i\xi_1 - \xi_2 \\ i\xi_1 + \xi_2 & \xi_4 - i\xi_3 \end{pmatrix}$$

$$C\psi = -\xi_{234}\hat{i} - \xi_{314}\hat{j} - \xi_{124}\hat{k} + \xi_{123} = \begin{pmatrix} \xi_{123} - i\xi_{124} & -i\xi_{234} + \xi_{314} \\ -i\xi_{234} - \xi_{314} & \xi_{123} + i\xi_{124} \end{pmatrix}$$

and the spinor operator

$$\phi = \xi_{14}\hat{i} + \xi_{24}\hat{j} + \xi_{34}\hat{k} + \xi_0 = \begin{pmatrix} \xi_0 + i\xi_{34} & i\xi_{14} - \xi_{24} \\ i\xi_{14} + \xi_{24} & \xi_0 - i\xi_{34} \end{pmatrix}$$

$$C\phi = -\xi_{23}\hat{i} - \xi_{31}\hat{j} - \xi_{12}\hat{k} + \xi_{1234} = \begin{pmatrix} \xi_{1234} - i\xi_{12} & -i\xi_{23} + \xi_{31} \\ -i\xi_{23} - \xi_{31} & \xi_{1234} + i\xi_{12} \end{pmatrix}$$

(1)

(2)
The trilinear form in these bases is

\[
\mathcal{F} = t^\phi C X \psi = t^\phi \gamma_0 x^\mu \gamma_\mu \psi \\
= x^1(\xi_{12}\xi_{314} - \xi_{31}\xi_{124} - \xi_{14}\xi_{123} + \xi_{1234}\xi_1) \\
+ x^2(\xi_{23}\xi_{124} - \xi_{12}\xi_{234} - \xi_{24}\xi_{123} + \xi_{1234}\xi_2) \\
+ x^3(\xi_{31}\xi_{234} - \xi_{23}\xi_{314} - \xi_{34}\xi_{123} + \xi_{1234}\xi_3) \\
+ x^4(-\xi_{14}\xi_{234} - \xi_{24}\xi_{314} - \xi_{34}\xi_{124} + \xi_{1234}\xi_4) \\
+ x'^1(-\xi_0\xi_{234} + \xi_{23}\xi_4 - \xi_{24}\xi_3 + \xi_{34}\xi_2) \\
+ x'^2(-\xi_0\xi_{314} + \xi_{31}\xi_4 - \xi_{34}\xi_1 + \xi_{14}\xi_3) \\
+ x'^3(-\xi_0\xi_{124} + \xi_{12}\xi_4 - \xi_{14}\xi_2 + \xi_{24}\xi_1) \\
+ x'^4(\xi_{31}\xi_{123} - \xi_{23}\xi_1 - \xi_{31}\xi_2 - \xi_{12}\xi_3)
\]

(3)

In the case of weak current, we replace the coupling \( \gamma_0 x^\mu \gamma_\mu \) to \( \gamma_0 x^\mu \gamma_\mu (1 - \gamma_5) \) and try to make the couplings between fermions and vector particles become unified in the form

\[
\sum_{i=1}^{4} (x^i C \phi C \psi + x'^i C \phi \psi)
\]

by suitable choice of 1 or \(-\gamma_5\). Except the term \( x'^i \xi_0 \xi_{123} \), which is \( x'^i \phi C \psi \) type, it is possible by the following choice

\[
\mathcal{G} = x^1(\xi_{12}\xi_{314} - \xi_{31}\xi_{124} + \langle \xi_{14}\gamma_5 \rangle \xi_{123} - \xi_{1234}(\gamma_5\xi_1)) \\
+ x^2(\xi_{23}\xi_{124} - \xi_{12}\xi_{234} + \langle \xi_{24}\gamma_5 \rangle \xi_{123} - \xi_{1234}(\gamma_5\xi_2)) \\
+ x^3(\xi_{31}\xi_{234} - \xi_{23}\xi_{314} + \langle \xi_{34}\gamma_5 \rangle \xi_{123} - \xi_{1234}(\gamma_5\xi_3)) \\
+ x^4(\langle \xi_{14}\gamma_5 \rangle \xi_{234} + \langle \xi_{24}\gamma_5 \rangle \xi_{314} + \langle \xi_{34}\gamma_5 \rangle \xi_{124} - \xi_{1234}(\gamma_5\xi_4)) \\
+ x'^1(\langle \xi_0\gamma_5 \rangle \xi_{234} + \xi_{23}\xi_4 - \xi_{24}\xi_3 + \langle -\xi_{34}\gamma_5 \rangle \xi_2) \\
+ x'^2(\langle \xi_0\gamma_5 \rangle \xi_{314} + \xi_{31}\xi_4 - \xi_{34}\xi_1 + \langle -\xi_{14}\gamma_5 \rangle \xi_3) \\
+ x'^3(\langle \xi_0\gamma_5 \rangle \xi_{124} + \xi_{12}\xi_4 - \xi_{14}\xi_2 + \langle -\xi_{24}\gamma_5 \rangle \xi_1) \\
+ x'^4(\xi_{31}\xi_{123} - \xi_{23}\xi_1 - \xi_{31}\xi_2 - \xi_{12}\xi_3).
\]

(4)
When we define $\tilde{\xi}_i = \xi_{jkl}$, $1 \leq j, k, l \leq 3$, $j, k, l \neq i$, $\tilde{\xi}_{ij} = \xi_{kl}$, $1 \leq k, l \leq 4$, $l \neq i, j$, $\tilde{\xi}_{ijk} = \xi_l$, $1 \leq l \leq 4$ and $l \neq i, j, k$, $\tilde{\xi}_0 = \xi_{1234}$ and $\tilde{\xi}_{123} = \xi_4$, the couplings that can be detected becomes

$$G = \langle i\phi C X (1 - \gamma_5) \psi \rangle = \langle i\phi \gamma_0 x^\mu \gamma_\mu (1 - \gamma_5) \psi \rangle$$

$$= \sum_{i=1}^{3} x^i (\xi_{[i+1][i+2][i+3]} - \xi_{[i+1][i]+1} \xi_{[i+2][i]+1} \xi_{[i+3][i]+1} - \tilde{\xi}_{i4} \xi_{123} + \xi_{1234} \tilde{\xi}_i)$$

$$+ x^4 (\xi_{14} \xi_{234} - \tilde{\xi}_{24} \xi_{314} - \tilde{\xi}_{34} \xi_{124} + \xi_{1234} \tilde{\xi}_i)$$

$$+ \sum_{i=1}^{3} x^{i'} (-\tilde{\xi}_{0} \xi_{[i+1][i+2][i+3]} + \xi_{[i+1][i+2][i+3]} \xi_{[i+1][i+2][i+3]} - \xi_{[i+1][i+2][i+3]} \xi_{[i+1][i+2][i+3]} + \tilde{\xi}_{[i+2][i+3]} \xi_{[i+1][i+1]} \xi_{[i+2][i+3]} \xi_{[i+1][i+1]} \xi_{[i+2][i+3]} \xi_{[i+1][i+1]}$$

$$+ x^4 (\xi_{0} \xi_{123} - \xi_{23} \xi_{1} - \xi_{31} \xi_{2} - \xi_{12} \xi_{3}).$$

Here the notation $[i + k]_3$ stands for $\text{Mod}[i + k, 3]$.

If one multiplies $-\gamma_5$ to the exceptional term $x^4 \xi_{0} \xi_{123}$, the term becomes $x^4 C\phi C\psi$ type, and since there is no difference between $x^4$ and $x^4$ in the electromagnetic interaction, the weak interaction can be characterized as

$$i\phi C X C \psi + i\phi C X \psi.$$

The states $\psi$ and $C\psi$ makes a complete set of the initial state, and final states of our weak interactions is $\phi$.

In sect.2, we present Lagrangian of the weak interaction and define the Higgs field. Electromagnetic decays of Higgs bosons are studied in sect.3. Electromagnetic decays and weak decays of $B(0^+)$ bosons and $D(0^+)$ bosons are studied in sect.4, and discussion and conclusion are given in sect.5.

2 Weak interaction of leptons and hadrons

In the case of the vector particle $x^i$ and $x^{i'}$ are $W^+$, we choose $C\phi = \nu$, $\phi = \bar{\nu}$ (neutrino and antineutrino), and $C\psi = \bar{l}$, $\psi = l$ (antilepton and lepton) or $C\psi = \bar{q}$, $\psi = q$ (antiquark and quark).
In the case of the vector particle $x^i$ and $x'^i$ are $Z$, we choose the spinor $t(\psi, C\psi)$ and $t(\phi, C\phi)$ the two quarks in one triality sector, or lepton antilepton pairs and neutrino antineutrino pairs.

The spinor of a neutrino is defined as

$$\Psi_p = \begin{pmatrix} \eta_p \\ \chi_p \end{pmatrix} = \begin{pmatrix} i\sigma^2 \chi_p^T \\ \chi_p \end{pmatrix},$$

where we define

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

and

$$\sigma^\mu = (1, \sigma) \quad \bar{\sigma}^\mu = (1, -\sigma).$$

and

$$\eta_p \eta_p = m \chi_p \chi_p$$

The Lagrangian of a Majorana neutrino is given as

$$\mathcal{L}_M = \chi_p^\dagger \bar{\sigma}^\mu i\partial_\mu \chi_p - \frac{m}{2}(\chi_p \cdot \chi_p + \bar{\chi}_p \cdot \bar{\chi}_p),$$

and our neutrinos are consistent with Majorana neutrinos.

The leptons, quarks and neutrinos have triality sectors

$$b, t | \tau, \nu_\tau, \quad s, c | \mu, \nu_\mu, \quad u, d | e, \nu_e.$$  

The neutrinos $\nu_e, \nu_\mu$ and $\nu_\tau$ are described by Kobayashi-Maskawa matrix as

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ -s_1 s_2 & c_1 s_2 c_3 + s_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}.$$
The scattering amplitude of neutrinos or electrons is

\[
T(\nu_e \rightarrow \nu_e) = \frac{g^2}{8M_W^2} \left\{ [\bar{\Psi}_\nu \gamma_\mu (1 - \gamma_5) \Psi_e] [\bar{\Psi}_e \gamma^\mu (1 - \gamma_5) \Psi_\nu] - \frac{1}{2} [\bar{\Psi}_\nu \gamma_\mu (1 - \gamma_5) \Psi_e] [\bar{\Psi}_e \gamma^\mu (1 - \gamma_5) \Psi_\nu] - 4 \sin^2 \theta_W [\bar{\Psi}_e \gamma^\mu \Psi_\nu] \right\}
\]

(7)

where the second term contains the neutral current contribution.

Leptons are defined by left-chiral field

\[
\Psi_L = \begin{pmatrix} \Psi_{1L} \\ \Psi_{2L} \end{pmatrix}
\]

whose covariant derivative is

\[
D_\mu \Psi_L = (\partial_\mu + i g \frac{\tau^i}{2} W^i_\mu - i g' B_\mu) \Psi_L
\]

and right-chiral field \( \Psi_{1R} \), whose covariant derivative is

\[
D_\mu \Psi_{1R} = (\partial_\mu - i g' B_\mu) \Psi_{1R}
\]

The weak bosons \( W \) are weighted by isospin operator \( \tau^i/2 \), and the gluons \( G \) are weighted by the color operator \( \lambda^a/2 \).

The covariant derivative of left-handed quark field is

\[
D_\mu \Psi_{Q_L} = (\partial_\mu + i g_s \frac{\lambda^a}{2} G^a_\mu + i \frac{g'}{2} B_\mu) \Psi_{Q_L}
\]

The covariant derivative of \( u_R \) and \( d_R \) quarks are

\[
D_\mu \Psi_{u_R} = (\partial_\mu + i g_s \frac{\lambda^a}{2} G^a_\mu + i \frac{2g'}{3} B_\mu) \Psi_{u_R}
\]

\[
D_\mu \Psi_{d_R} = (\partial_\mu + i g_s \frac{\lambda^a}{2} G^a_\mu - i \frac{g'}{3} B_\mu) \Psi_{d_R}
\]

(8)

The Lagrangian of the \( U(1)_Y, SU(2)_L \) and \( SU(3)_C \) gauge fields is

\[
\mathcal{L}_{GB} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} Tr(G_{\mu\nu} G^{\mu\nu})
\]
where

\[ F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \]
\[ W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu - ig[W_\mu, W_\nu] \]
\[ G_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu - ig[G_\mu, G_\nu] \quad (9) \]

Higgs field \( H \) belongs to \( SU(2)_L \) doublet and can be expressed as

\[ H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}, \quad H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}, \]

The gauge-covariant derivative \( D_\mu H \) is

\[ D_\mu H = (\partial_\mu + ig \frac{\tau^i}{2} W^i_\mu + ig' B_\mu) H \]

In our model, the Higgs potential is written as

\[ V(H) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2 \]

and the vacuum expectation value of \( H \) is chosen to be

\[ \langle H_u \rangle_{\text{min}} = \begin{pmatrix} 0 \\ \nu_u \end{pmatrix}, \quad \langle H_d \rangle_{\text{min}} = \begin{pmatrix} \nu_d \\ 0 \end{pmatrix}, \]

where \( \nu_u = \frac{\mu_u}{\sqrt{2}\lambda}, \quad \nu_d = \frac{\mu_d}{\sqrt{2}\lambda}, \quad \mu_u^2 + \mu_d^2 = 2\mu^2 \) and define

\[ H_{0u}' = H_0 - \nu_u \quad \text{and} \quad H_{0d}' = H_0 - \nu_d. \]

The covariant derivative of \( H_u \) becomes

\[ D_\mu H_u = D_\mu \begin{pmatrix} H_u^+ \\ H_{0u}' \end{pmatrix} + D_\mu \begin{pmatrix} 0 \\ \nu_u \end{pmatrix} \]

where

\[ D_\mu \begin{pmatrix} 0 \\ \nu_u \end{pmatrix} = \frac{i\nu_u}{2} \begin{pmatrix} gW_{1\mu} - igW_{2\mu} \\ -gW_{3\mu} + g'B_\mu \end{pmatrix}. \]
Similarly
\[ D_\mu H_d = D_\mu \begin{pmatrix} H_{0d}^t \\ H_d \end{pmatrix} + D_\mu \begin{pmatrix} \nu_d \\ 0 \end{pmatrix} \]
where
\[ D_\mu \begin{pmatrix} \nu_d \\ 0 \end{pmatrix} = i \nu_d \begin{pmatrix} gW_{3\mu} \\ gW_{1\mu} + igW_{2\mu} - g'B_\mu \end{pmatrix}. \]

The masses of the gauge bosons are
\[ \mathcal{L}_{MGB} = \frac{\nu^2}{4} g^2 (W_1^2 + W_2^2) + \frac{\nu^2}{4} (gW_3 - g'B)^2 \]
\[ = m_{W}^2 W^{+\mu} W^{-\mu} + \frac{1}{2} m_{Z}^2 Z_{\mu} Z^{\mu}, \tag{10} \]

where \( \nu^2 = \nu_u^2 + \nu_d^2. \)

The possible masses of the Higgs particles are
\[ m_{H^\pm} = m_{W}^2 + m_{A^0}^2 \]
for charged massive states, and
\[ m_{h^0}^2 = \frac{m_{A^0}^2 + m_{Z}^2}{2} - \frac{1}{2} \sqrt{(m_{A^0}^2 + m_{Z}^2)^2 - 4m_{A^0}^2 m_{Z}^2 \cos^2 2\beta} \]
\[ m_{H^0}^2 = \frac{m_{A^0}^2 + m_{Z}^2}{2} + \frac{1}{2} \sqrt{(m_{A^0}^2 + m_{Z}^2)^2 - 4m_{A^0}^2 m_{Z}^2 \cos^2 2\beta}, \tag{11} \]

where \( \tan \beta = \nu_u / \nu_d, \) for neutral massive states.

When \( \cos 2\beta = 0, m_{h^0}^2 = 0, m_{H^0}^2 = m_{A^0}^2 + m_{Z}^2, \) and \( m_{Z} = 91.2 \text{GeV}, m_{H^0} = 125 \text{GeV} \)\[13\] gives
\[ m_{A^0} = 85.5 \text{GeV}. \]

and \( m_{W} = 80.4 \text{ GeV} \) yields \( m_{H^\pm} = 117 \text{ GeV}. \)

There is a report of the search of \( H^+ \)\[19\] using the \( t \to H^+ b \) decay and \( H^+ \to \tau\nu_\tau \) which yields \( m_{H^+} = 120 \text{GeV}, \) but in this analysis, the branching fraction \( B(H^+ \to \tau\nu_\tau) \) could not be well determined, and it was assumed to be equal to 1. We expect that it is due to
instability of the $H^+$ state. The requirement that $m_{H^\pm}^2 = m_W^2 + m_{A_0}^2 = (120 \text{ GeV})^2$ gives $m_{A_0} = 78.0 \text{ GeV}$ and $m_{H^0}$ becomes 125 GeV, with

$$m_{A_0} = 78.0 \text{ GeV} \quad \text{and} \quad \cos 2\beta = \pm 0.1878.$$ 

These parameters give $m_{H^0} = 125 \text{ GeV}$, with $m_{A_0} = 78.0 \text{ GeV}$ and $\cos 2\beta = \pm 0.1878$.

The mass of $\chi_b(3P)$ is slightly below the $B\bar{B}$ threshold and there remains a possibility that the SUSY-breaking potential\[7\],

$$V_{SSB} = v(H_u^+ H_u^0) i\tau_2 \left( \begin{array}{c} H_d^0 \\ H_d^* \end{array} \right) + v^*(H_d^0 H_d^+)(-i\tau_2) \left( \begin{array}{c} H_u^+ \\ H_u^0 \end{array} \right)$$

$$= v(H_u^+ H_d^- - H_u^0 H_d^0) + h.c. \quad (12)$$

where $v > 0$, makes $H^\pm$ unstable, and the $h^0$ appears as the $\chi_b(3P)$.

Detailed study of the structure of $\chi_b(3P)$ is necessary to clarify the Higgs meson physics.

3 \hspace{1cm} $H(0^+) \to \ell\bar{\ell}\ell\bar{\ell} \to 2\gamma$

As a model of $H(0^+)$, we use spinor fields in Clifford algebra\[11, 12\], and study its decay into $2\gamma$. In the lepton-antilepton annihilation and quark-antiquark annihilation to $\gamma$, the helicity of the lepton and the antilepton are assumed to be parallel. Typical diagrams of $\phi - C\phi$ or $\psi - C\psi$ decays into a $\gamma$ in the standard model are shown in Figure 11.

In the octonion bases, in addition to the lepton-antilepton pair $\phi - C\phi$ or $\psi - C\psi$, the pair $\phi - \psi$ or $C\phi - C\psi$ decays into a $\gamma$.

A scalar boson $\Psi(0^+)$ or $\Phi(0^+)$ decays into $\gamma(\ell\ell)\gamma(\ell\ell)$, where $\ell$ stands here for $e$ or $\mu$.

There are 8 diagrams each\[10\].

In the Figure 3 $x_s$ and $x'_s$ stand for leptons or antileptons that decay into $\gamma$s and $\xi_s$s stand for photons or gauge bosons. We studied decay modes of $H_0$ in \[10\].
Figure 1: Typical diagrams of lepton-antilepton $\phi - C\phi$ or $\psi - C\psi$ decay into a $\gamma$.

Figure 2: Typical diagrams of lepton-antilepton $\phi - \psi$ or $C\phi - C\psi$ decay into a $\gamma$.

4 $B(0^+) \to B(0^-)\pi$ and $B(0^+) \to D_s(0^+)\mu^+\nu_\mu$ 

Experimentally, presence of charmed strange meson $D_s(0^+)$ was exciting and the presence of bosonized strange meson $B_s(0^+)$ and hadronic decays of a $B(0^+)$ meson was studied in lattice simulation\[16, 17]. Experimentally the $B(0^+)$ decay into $B(0^-)\pi^+$ is not observed. However, $D_s(0^+)^+\pi^0$ and $D_s(0^-)^+\pi^0$ are observed\[18]

\[
D(0^-)^+(1968\text{MeV})\pi^0 \quad 5.8 \pm 0.7\% \\
D(0^-)^+(1968\text{MeV})\gamma \quad 94.2 \pm 0.7\%.
\]

Quarks and anti quarks are expressed by $\phi, C\phi, \psi$ and $C\psi$. In the analysis of Higgs boson decay into $2\gamma s, \phi, \psi$ and $C\phi, C\psi$ pair creation/annihilation was consistent with $\gamma$ creation/annihilation\[10].

The photons in the $D(0^+)_s \to D(0^-)_s\gamma$ appear from the interactions

\[
C\phi^*\gamma_0\gamma_\mu\phi, \quad C\psi^*\gamma_0\gamma_\mu\psi \\
\phi^*\gamma_0\gamma_\mu\psi + h.c. \quad C\phi C\psi^* + h.c.
\]

The pion is a quark-antiquark system coupled to angular momentum 0, and in the $D(0^+_s)^* \to D(0^-_s)\pi$, they appear from the divergence of an axial current.

\[
C\phi^*\gamma_0\gamma_5\phi.
\]
Figure 3: Typical diagrams of $H(0^+)$ decay into $\ell\ell\ell\ell$ which reduce to $2\gamma$s.

In order that quarks and antiquarks in $D(0^+)_s$ mesons are expressed by $\xi_*, i, \xi_* j$, or $\xi_* k$, the polarization of quark-antiquarks in pions or $D(0^-)$ becomes $q(I)\bar{q}(I)$, as shown in Figures 6 and 7. We take interactions between quarks in $D(0^+)$ to be Coulomb type, in order that the relative p-wave becomes well determined, and distinguish them from that in $D(0^-)$.

Figure 4: A quark-antiquark $\phi - \psi$ or $C\phi - C\psi$ decay into $2\gamma$ via $\pi^0$.

When the helicities of the quark-antiquark are not parallel, decay into $2\gamma$ via $a_2(2^+)$ may occur.

In the Figures 6 and 7, $x_*$s and $x'_*$s stand for gluons. The lines $\xi_{12} k$ and $\xi_{31} j$ in the Figure 6 and the lines $\xi_{23} i$ and $\xi_{12} k$ in the Figure 7 are s-quarks.

Corresponding to $D(0^-)_s\pi^0 \rightarrow D(0^-)_s2\gamma$ decay, in $D(0^-)_s\gamma$ decay, $q(\phi)\bar{q}(\psi) \rightarrow \gamma$ and $q(C\phi)\bar{q}(C\psi) \rightarrow \gamma$ occur, as shown in Figure 11 i.e. the number of quarks per one $\gamma$ that contribute to $D(0^-)_s\gamma$ decay is 4 times larger than that contribute to $D(0^-)_s\pi$ decay. The
absence of $B_s(0^+) \rightarrow B_s(0^-) \pi^+$ decay is expected to be due to the presence of triality sector $(s, c|\mu, \nu_\mu)$, which makes the weak decay of $B(0^+) \rightarrow D(0^+)s^{\mu} + \nu_\mu$ stronger than the strong decay of $B(0^+) \rightarrow B(0^-)\pi^+$. A $D(0^+)_s$ decays to $D(0^-)_s + \pi$, and a $D(0^-)_s$ decays to $K + \text{anything}$ by about 57% via strong interactions.

5 Discussion and conclusion

We showed that Cartan’s supersymmetry can be applied to weak interactions of leptons and hadrons. Consistency with the electromagnetic interaction was also confirmed.

The model of Higgs boson predicts presence of two neutral scalar bosons of masses $m_{H^0}$ and $m_{h^0}$ and charged scalar boson of mass $m_{H^\pm}$. An adjustment of $m_{H^0} = 125$ GeV and
Figure 7: Typical diagrams of $D(0^+)^* \rightarrow D(0^-) + \pi(q(\bar{q}(\xi^{**})\bar{q}(\xi^{***})))$.

$m_{H^+} = 120$ GeV predicts $m_{h^0} \simeq 11.2$ GeV. There are possibility that $H^\pm$ is unstable and hard to detect, and the boson $\chi_b(3P, 10.53$ GeV) [14, 15] near the $B\bar{B}$ threshold is the Higgs boson partner $h^0$.

Detailed study of $\chi_b(3P, 10.53$ GeV) decay may be helpful for clarifying whether the $\chi_b(3P)$ can be understood as an $h_0$.

The world of matters transformed by $G_{23}$ can be understood through our detectors, and the world of matters transformed by $G_{12}, G_{13}, G_{123}$ and $G_{132}$ would be understood through studies of neutrino-hadron interactions and $H^0$ and $h^0$ decay patterns.

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References

[1] Cartan É. (1966), *The theory of Spinors*, Dover Pub..

[2] Furui S. (2012a), Fermion Flavors in Quaternion Basis and Infrared QCD, Few Body Syst. 52, 171-187.
[3] Furui S. (2012b), The Magnetic Mass of Transverse Gluon, the B-Meson Weak Decay Vertex and the Triality Symmetry of Octonion, Few Body Syst. 53, 343.

[4] Furui S. (2012c), The flavor symmetry in the standard model and the triality symmetry, Int. J. Mod. Phys. A27 1250158, arXiv:1203.5213.

[5] Furui S. (2013), Axial anomaly and triality symmetry of octonion, Few Body Syst. DOI 10.1007/s0061-013-0719-9, arXiv:1301.2095 [hep-ph].

[6] Furui S. (2014a), Axial anomaly and triality symmetry of leptons and hadrons, Few Body Syst. 55, 1083, arXiv:1304.3776 [hep-ph].

[7] Labelle P. (2010) Supersymmetry Demystified, McGraw Hill.

[8] Kobayashi M. and Maskawa M. (1973), CP-Violation in the Renormalizable Theory of Weak Interaction, Prog. Theor. Phys. 49, 652.

[9] Lounesto P. (1993), Clifford algebras and Hestenes spinors, Foundation of Physics, 23, 1203-1237.

[10] Furui S. (2014b), Triality selection rules of Octonion and Quantum Mechanics, arXiv:1409.3761 [hep-ph].

[11] Lounesto P. (2001), in Clifford Algebras and Spinors 2nd ed. , Cambridge University Press.

[12] Hestenes D. (1986), Clifford Algebras and their Applications in Mathematical Physics, Reidel, Dordrecht/Boston, 321.

[13] Aad G. et al., (ATLAS Collaboration) (2012), Combined search for the Standard Model Higgs boson in $pp$ collisions at $\sqrt{s}=7$ TeV with the ATLAS detector, Phys. Rev. D86, 032003.
[14] Aad G. et al., (ATLAS Collaboration) (2012), Observation of a New \( \chi_b \) State in Radiative Transitions to \( \Upsilon(1S) \) and \( \Upsilon(2S) \) at ATLAS, Phys. Rev. Lett. 108, 152001.

[15] Abazov V.M. et al., (D0 Collaboration) (2012), Observation of a narrow mass state decaying into \( \Upsilon(1S)+\gamma \) in \( p\bar{p} \) collisions at \( \sqrt{s} = 1.96 \) TeV, Phys. Rev. D 86, 031103(R).

[16] Green A.M., Koponen J., McNeile C., Michael C. and Thompson G. (UKQCD Collaboration) (2003) Excited B mesons from lattice, hep-lat/0312007.

[17] McNeile C., Michael C., Thompson G. (UKQCD Collaboration) [2004], Hadronic decay of a scalar B meson from the lattice, hep-lat/0404010.

[18] Olive, K.A. et al, (Particle Data Group) (2014), Review of Particle Physics, Chinese Physics C 38, 090001.

[19] The CMS Collaboration (2012), Search for a light Charged Higgs boson in Top quark decays in pp collision at \( \sqrt{s} = 7 \) TeV, hep-ex/1205.5736 v3.