Inflation as a spontaneous symmetry breaking of Weyl symmetry

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Abstract

In this paper we study a novel realization of inflation, based on Weyl invariant gravity with torsion. We show that requiring the classical action for the scalar field to be Weyl invariant introduces a dilaton which induces a non trivial modification of the field space geometry of the scalar sector, which allows for inflationary phase that begins at the conformal point of the inflaton $\psi$, i.e. $\langle \psi \rangle = 0$. Since the model is Weyl invariant, the inflaton condensation models a process of spontaneous Weyl symmetry breaking. For a wide range of parameters the spectral observables of the model are in good agreement with the CMB measurements, such that the scalar spectral index and the tensor-to-scalar ratio approximately agree with those of Starobinsky’s inflation, i.e. $n_s \approx 0.96 – 0.97$ and $r \approx 3 \times 10^{-3}$. The simplest version of our model contains two scalar degrees of freedom, one of them being an exactly flat direction. If that degree is excited early on in inflation and if inflation lasts for about 60 e-foldings, we find that the Universe undergoes a short period of kination that predates inflation. Such a period strongly suppresses the amplitude of large scale CMB temperature fluctuations providing thus an elegant explanation for the lack of power in the lowest CMB multipoles.

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I. INTRODUCTION

The phenomena known as cosmic inflation [1, 2] is one of the most studied in high energy in the modern days [3]. The leading paradigm to study it is to construct a so-called effective field theory based on the breaking of time translation symmetry induced by the expansion of the universe [4].

While this point of view provides a self consistent phenomenological description, it does not shed any light on the physical nature of the phenomena, nor it reveals any of the basic principles that rule its dynamics. A possibility to gain understanding is to realize cosmic inflation in theories with enhanced symmetry, which are broken today. This would then lead to a phase transition of sort, in which the rolling inflaton field acts as the order parameter in the symmetry breaking process.

A popular choice in this direction is to invoke scale or conformal symmetry as the one realized in the initial state of the universe, and broken during the inflationary epoch [5–26]. This is also the approach advocated in this paper, however with an important twist.

The reasons why one expects conformal symmetry to be restored at very early times of the cosmic history are multiple. A compelling argument is given by the idea that quantum field theories at very short distances become approximately conformal and flow towards conformally invariant theories, the so-called fixed points of the renormalization group flow. Such a quantum theory can be, at least in principle, rigorously defined, which is what makes it so appealing. Another theoretical argument is provided by the simplicity that scale invariant models enjoy. Namely, the symmetry permits a handful of (local) operators that can be included, such that these theories differ from one another only by the number of degrees of freedom they possess and by their spin.

In this paper we consider a generic choice for the scale invariant theory, containing $N$ scalar degrees of freedom ($\in O(N)$), and in which the global scale symmetry is promoted to a local (gauged Weyl) invariance. Our conclusion are general in the sense that they do not depend on the specific of the theory, but only upon considering a Weyl invariant theory. In order to obtain a gauge invariant action, a compensating Weyl one form is introduced in the theory, which we interpret as the torsion tensor trace (as was argued in [27]).

The longitudinal component of the compensating one form acts as the Goldstone mode of the broken symmetry, and effectively behaves as a scalar field that kinetically mixes with the other scalars in the theory. The Weyl invariance of the underlying theory enlarges the configuration space of scalar fields from the original $O(N)$ invariant scalars to a negatively curved, $N + 1$ di-
imensional field space, the hyperbolic space $H^{N+1}$. This is similar to the hyperbolic field geometry studied in [9], where the authors consider it motivated by supersymmetry. That has also been discussed in [22], where the authors prove that a maximally symmetric field space leads to universal predictions, which might be responsible for the universality properties discussed in [10], thus justifying the name $\alpha$-attractors. The physical consequence of the negative curvature of the configuration space is to stretch the potential at the boundary of the field space allowing for a plateau on which a slow roll inflation is possible. In the realization of [9] the potential is stretched at large field values, $\phi \to \infty$, while in our realization the stretching occurs near the origin of the inflaton direction, i.e. $\phi \sim 0$. Another interesting effect a negative configuration space curvature can have was discussed in Ref. [28], where it was pointed out that a negative curvature can reduce the effective mass of fields during inflation, making them even negative, and thus qualitatively change the inflationary dynamics. This mechanism was dubbed geometric destabilization.

The additional scalar direction given by the Goldstone mode is flat in the sense that the Goldstone mode is only derivatively coupled to the $O(N)$ scalars, as we would expect from a Goldstone mode. If its energy density is initially big, it can dominate the preinflationary epoch with possible observable consequences. However, it does not play a dominant role in the inflationary dynamics and can thus be neglected at any later time.

Finally we briefly study quantum corrections from the matter sector, by computing the one loop quantum effective action, and conclude that the hierarchy required to obtain inflationary observables compatible with the most recent cosmological data [29] is stable. For the same reason a late time small dark energy can be expected and maintained small, thus making the tiny observed cosmological constant in this model technically natural [30].

The paper is organized as follows: we review the Weyl invariant theory of gravity in section II and then show how the negatively curved field space geometry is induced from the requirement that the theory is Weyl invariant. In section III we study the inflationary dynamics and explain the dynamics of the flat direction $\chi$, which constitutes the Goldstone mode of the broken symmetry. In section IV we discuss the model predictions, and discuss how we could detect the Goldstone mode $\chi$ in the CMB spectrum. Finally in section V we briefly discuss the quantum corrections to the model. In the Appendix one can find a comparison between this model and the Abelian symmetry breaking phenomenon, where we highlight the differences among the two.
II. WEYL INVARIANT GRAVITY WITH TORSION

Defining the torsion tensor as the antisymmetric part of the connection,

$$T^A_{\mu\nu} = \Gamma^A_{[\mu\nu]},$$

one finds an exact gauge symmetry of curvature and the geodesic equation, if the metric and the torsion are transformed according to [27],

$$T^A_{\mu\nu} \rightarrow T^A_{\mu\nu} + \delta^A_{\mu} \partial_\nu \theta, \quad g_{\mu\nu} \rightarrow e^{2\theta} g_{\mu\nu}. \quad (2)$$

That is, defining the torsion trace as,

$$\mathcal{T}_\mu \equiv -\frac{2}{D-1} T^A_{\mu A}, \quad (3)$$

one finds it transforms locally under (2) as a $U(1)$ gauge field,

$$\mathcal{T} \rightarrow \mathcal{T} + d\theta. \quad (4)$$

The transformation (2) also realises a reparametrization of proper time, $d\tau \rightarrow e^\theta d\tau$ which leaves the geodesic equation invariant [27]. Globally, however, the group of Weyl transformations is inequivalent to that of an Abelian gauge group as the corresponding group space can be 1-to-1 mapped onto the set of real numbers and it is thus non-compact.

In Ref. [27] we also showed how to straightforwardly extend such a symmetry to the classical standard model Lagrangian, which modifies scalar kinetic terms according to the prescription $^1$,

$$\partial_\mu \phi \rightarrow (\partial_\mu - \Delta_\phi \mathcal{T}_\mu) \phi \equiv \nabla_\mu \phi, \quad (5)$$

where the $\Delta_\phi$ is the scaling dimension of the field $\phi$. For canonically normalized scalars, it evaluates to $\Delta_\phi = -(D-2)/2$.

With this prescription and using the fact that the Ricci scalar with torsion transforms under (2) as, $R \rightarrow e^{-2\theta} R$, the Weyl invariant operators that constitute the action in four dimensions are limited to,

$$S = \int d^4x \sqrt{-g} \left[ \alpha R^2 + \xi R_{\mu\nu} R^{\mu\nu} + \frac{\xi}{2} \phi^I \phi^J \delta_{IJ} R - \frac{\lambda}{4} \left( \phi^I \phi^J \delta_{IJ} \right)^2 \right. \quad (6)$$

$$\left. - \frac{1}{2} g^{\mu\nu} \delta_{IJ} \left( \partial_\mu \phi^I - \Delta_\phi \mathcal{T}_\mu \phi^I \right) \left( \partial_\nu \phi^J - \Delta_\phi \mathcal{T}_\nu \phi^J \right) - \frac{\sigma}{4} \mathcal{T}_{\mu\nu} \mathcal{T}^{\mu\nu} \right],$$

$$\mathcal{T}_{\mu\nu} \equiv \partial_\mu \mathcal{T}_\nu - \partial_\nu \mathcal{T}_\mu. \quad (7)$$

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$^1$ Here and thereafter we use the notation $\nabla_\mu$ to denote the covariant derivative which commutes both with diffeomorphisms and Weyl transformations (2). Its definition when acting on tensors can be found in [27], and it agrees with (5) when acts on a scalar.
where $\alpha, \zeta, \xi, \lambda$ and $\sigma$ are dimensionless couplings and we allow for $O(N)$ invariant scalars $\phi^I$, with $I = 1, \cdots, N$. \(^2\)

The study of the action (6) is best carried out in the Einstein frame, that is where gravity follows the Einsteinian dynamics. This can be achieved by setting the parameters $\zeta$ and the torsion field strength coupling $\sigma$ to zero. While the former is necessary as it renders the model classically stable, the latter is not required, but it is convenient as it removes the dynamical modes of the torsion trace vector, thus simplifying the model. With this choice the transverse components of the torsion trace become constraint fields which can be consistently set to zero. The remaining longitudinal component of $T_\mu$ can be modeled by a real scalar field $\phi^0$ as follows,

$$T_\mu = \partial_\mu \phi^0. \quad (8)$$

This choice is discussed in detail in the Appendix, in which we discuss at length the Weyl gauge fixing and explain the differences between the local Weyl symmetry and the local Abelian gauge symmetry.

The Einstein’s frame action, which is on-shell equivalent to (6), reads,

$$S_E = \int d^4x \sqrt{-g} \left[ -\left( \frac{\xi^2}{16\alpha} + \lambda \right) \left( \delta_{IJ} \phi^I \phi^J \right)^2 + \frac{\xi}{8\alpha} \omega^2 \delta_{IJ} \phi^I \phi^J + \frac{\omega^2}{2} \left( \mathcal{R} + 6 \nabla^2 \phi^0 - 6 \partial_\mu \phi^0 \partial^\mu \phi^0 \right) \right. \right.$$

$$\left. - \frac{\omega^4}{16\alpha} \delta_{IJ} g^{\mu\nu} \left( \partial_\mu + \partial_\mu \phi^0 \right) \phi^I \left( \partial_\nu + \partial_\nu \phi^0 \right) \phi^J \right], \quad \equiv \int d^4x \sqrt{-g} \left[ -\left( \frac{\xi^2}{16\alpha} + \lambda \right) \left( \delta_{IJ} \phi^I \phi^J \right)^2 + \frac{\xi}{8\alpha} \omega^2 \delta_{IJ} \phi^I \phi^J + \frac{\omega^2}{2} \left( \mathcal{R} - \frac{\omega^4}{16\alpha} + 3 \omega^2 \nabla^2 \phi^0 \right) \right. \right.$$

$$\left. - \frac{1}{2} G_{AB} g^{\mu\nu} \partial_\mu \phi^A \partial_\nu \phi^B \right], \quad A, B = 0, 1, \cdots, N, \quad (9)$$

where $\omega$ and $\theta$ are Lagrange multiplier fields, $G_{AB}$ is an extended configuration space metric which includes the longitudinal torsion direction $\phi^0$,

$$G_{00} = 6 \omega^2 + \delta_{IJ} \phi^I \phi^J, \quad G_{0I} = \delta_{IJ} \phi^I = G_{I0}, \quad G_{IJ} = \delta_{IJ}, \quad (I, J = 1, \cdots, N),$$

\(^2\) In principle, we could allow for a more generic field space metric, $G_{AB}$ with a different symmetry group. Simple considerations on scale invariance show, however, that such a metric can only depend on ratios $\phi^A/\phi^B$, thus lowering the symmetry to a subgroup of $O(N)$. Since in this paper we follow a logic of simplicity, we are going to choose the maximally symmetric option, that is $O(N)$ for which $G_{IJ} = \delta_{IJ}$. Quantum corrections will in general modify the kinetic term such to replace $\delta_{IJ}$ by a more general $G_{IJ}$ of the form $G(\phi^K \phi^K \delta_{KL} / \mu^2) \delta_{IJ}$, where $\mu$ is a renormalization scale, which still respect the $O(N)$ symmetry but mildly breaks the Weyl symmetry. Furthermore, one could imagine $G_{IJ} = \eta_{IJ}$, having one or more time-like directions. This would however introduce ghost-like directions in field space, which would destabilize the field dynamics, and for that reason we shall not allow that possibility.
and we have also substituted the expression for the Ricci scalar with torsion,

\[ R^{\lambda}_{\alpha\sigma\beta} = \partial_{\sigma} \Gamma^{\lambda}_{\alpha\beta} - \partial_{\beta} \Gamma^{\lambda}_{\alpha\sigma} + \Gamma^{\lambda}_{\rho\sigma} \Gamma^{\rho}_{\alpha\beta} - \Gamma^{\lambda}_{\rho\beta} \Gamma^{\rho}_{\alpha\sigma}, \]

(10)

\[ R = g^{\alpha\beta} R^{\lambda}_{\alpha\lambda\beta} = \tilde{R} + 6 \tilde{\nabla}^{\mu} T_{\mu} - 6 g^{\mu\nu} T_{\mu} T_{\nu}. \]

(11)

Symbols with a \( \circ \) on top in (9) are computed using the metric tensor only, e.g.,

\[ \tilde{\Gamma}^{\lambda}_{\mu\nu} = \frac{g^{\lambda\sigma}}{2} \left( g_{\sigma\mu,\nu} + g_{\sigma\nu,\mu} - g_{\mu\nu,\sigma} \right), \]

denotes the Levi-Civita connection.

Varying the action (9) with respect to \( \omega \) yields,

\[ \omega^2 \equiv 4\alpha R + \xi \phi^2, \quad \phi^2 \equiv \delta_{IJ} \phi^I \phi^J \]

which means that everywhere in (9) one can exact the replacement:

\[ \omega^2 \rightarrow 4\alpha R + \xi \phi^2, \]

(13)

which will give back the original Jordan frame action (6).

The action (9) still contains a Weyl gauge redundancy. A convenient gauge choice is the Weyl gauge which amounts to fixing \( \omega \) to some (constant) physical scale. From (9) we see that \( \omega \) defines the Planck scale, i.e.

\[ \omega^2 \equiv 4\alpha R + \xi \phi^2 \rightarrow M_{P}^2, \]

(14)

completely fixing the Weyl gauge. Since now we have defined a reference scale, any other scale in the model can be defined with respect to that reference scale. This accords with the general notion that all physical quantities (or, equivalently, measurements) can be represented as dimensionless ratios.

Note that, because the gauge fixing condition (14) can be imposed only if either \( \phi \) or \( R \) (or both) does not vanish, we have to take the initial conditions such that at least one condensate does not vanish. In other words, the conformal point at which all scalar condensates vanish is singular, and a proper discussion of its significance is beyond the scope of this work. In the construction of the mechanism analyzed in this paper, we assume that the scalar (inflaton) field begins at its conformal point, \( \langle \phi \rangle = 0 \). This choice is motivated by the conformal symmetry of the UV theory. However, the initial value for the Ricci scalar is such to satisfy (14), which necessarily leads to \( R \neq 0 \). The dynamics that follow are governed by a transfer of energy (and entropy) between the space-time...
fluctuations and the scalar field, as it can be best understood by considering the scalar potential and noticing that the potential energy of the scalar field drops from its value at the beginning of inflation to a lower value when the field reaches the global minimum. In this sense, the symmetry breaking described in this paper is due to the initial conditions, which are taken at the point of enhanced symmetry. Then inflation happens as a consequence of energy exchange between the gravitational field and the scalar field. While the Ricci curvature breaks conformal invariance even near $\phi = 0$, it is still conceivable that conformal symmetry is realized in the far UV, *i.e.* at scales much greater than $\omega = M_P$. A proper understanding of the UV conformal fixed point is, however, beyond the scope of this work. \(^3\)

Coming back to our inflationary model, the extended *configuration space metric* in (9) reads,

$$G_{AB} d\phi^A d\phi^B = \left(6 M_P^2 + \rho^2 \delta_{IJ} \right) (d\phi^0)^2 + 6 \rho d\rho d\phi^0 + \delta_{IJ} d\phi^I d\phi^J ,$$

and has a negative configuration space Ricci curvature,

$$R = - \frac{N(N+1)}{6 M_P^2} ,$$

which identifies it as a hyperbolic geometry $\mathbb{H}^{N+1}$.

To see this more explicitly, consider the following coordinate transformations in field space. First, let us define polar coordinates for the $O(N)$ scalars,

$$\begin{align*}
\phi^1 &= \rho \cos \theta_1 , \\
\phi^2 &= \rho \sin \theta_1 \cos \theta_2 , \\
\phi^3 &= \rho \sin \theta_1 \sin \theta_2 \cos \theta_3 , \\
\vdots \\
\phi^{N-1} &= \rho \sin \theta_1 \sin \theta_2 \cdots \sin \theta_{N-2} \cos \theta_{N-1} , \\
\phi^N &= \rho \sin \theta_1 \sin \theta_2 \cdots \sin \theta_{N-2} \sin \theta_{N-1} , \\
\phi' \phi'' &= \rho^2
\end{align*}$$

$$\implies \phi' d\phi'' = \rho d\rho ,$$

$$G_{AB} d\phi^A d\phi^B = \left(6 M_P^2 + \rho^2 \right) (d\phi^0)^2 + 2 \rho d\rho d\phi^0 + \left(d\rho^2 + \rho^2 d\Omega_{N-1} \right) ,$$

where $d\Omega_{N-1}$ is the metric on the $(N-1)$-sphere, $S^{N-1}$. Finally, defining,

$$\phi^0 = \chi - \frac{1}{2} \log \left(1 + \frac{\rho^2}{6 M_P^2} \right) ,$$

brings the metric into a diagonal form,

$$G_{AB} d\phi^A d\phi^B = \left(6 M_P^2 + \rho^2 \right) d\chi^2 + \frac{6 M_P^2 d\rho^2}{6 M_P^2 + \rho^2} + \rho^2 d\Omega_{N-1} ,$$

which can be further simplified by redefining

$$\frac{d\rho}{\sqrt{6 M_P^2 + \rho^2}} = d\psi \implies \rho = \sqrt{6 M_P} \sinh(\psi) ,$$

$$\implies G_{AB} d\phi^A d\phi^B = 6 M_P^2 \left[ d\psi^2 + \left(\cosh^2(\psi) d\chi^2 + \sinh^2(\psi) d\Omega_{N-1} \right) \right] .$$

\(^3\) Curiously, if initial conditions support cosmic inflation, then its geometry can be approximated by that of de Sitter space, which exhibits a global conformal symmetry SO(2,4).
This form of the metric makes it explicit what components of the fields $\phi^I$ are the Goldstone modes and which are the directions acquiring a vev, the latter being identified with the direction $\psi$ which, as we show in the next section, is the inflaton direction.

III. INFLATIONARY DYNAMICS

In this section we study inflation in the model presented in the previous section and show that one can obtain an inflationary model that conforms with all existing data.

In the diagonal field coordinates (18) the gauge fixed action (9) specialized to $\mathcal{N} = 1$ case 4, reads,

$$S = \int d^4x \sqrt{-g} \left[ -\left( \frac{9\xi^2}{4\alpha} + 36\lambda \right) M_p^4 \sinh^4(\psi) + \frac{3\xi}{4\alpha} M_p^4 \sinh^2(\psi) - \frac{M_p^4}{16\alpha} \right. \\
\left. + \frac{M_P^2}{2} R - \frac{6M_P^2}{2} g^{\mu
u} \left[ (\partial_\mu \psi)(\partial_\nu \psi) + \cosh^2(\psi)(\partial_\mu \chi)(\partial_\nu \chi) \right] \right].$$

Let us begin the analysis by recalling the background cosmological metric,

$$ds^2 = -dt^2 + a(t)^2 \delta_{ij} dx^i dx^j,$$

where $a = a(t)$ denotes the scale factor. For simplicity the spatial sections in (21) are assumed to be flat, i.e. $dx^i \cdot dx^j = \delta_{ij} dx^i dx^j$ $(i, j = 1, 2, 3)$.

The equations of motion for the background (homogeneous) fields, $\psi = \psi(t)$ and $\chi = \chi(t)$, are best analyzed in terms of $e$-folding time, $dN = H dt$, where $H = \dot{a}/a$ is the Hubble parameter and $\dot{a} \equiv da/dt$. In $e$-folding time the scalar equation of motion and the Friedman equations decouple, that is defining the principal (first) slow roll parameter as $\epsilon_1 \equiv \epsilon = -\frac{\dot{H}}{H} = 3 \left[ (\dot{\psi})^2 + \cosh^2(\psi)(\dot{\chi})^2 \right]$, $H^2 = \frac{V(\psi)}{(3-\epsilon)M_P^2}$, (22)

$$\frac{\psi''}{3-\epsilon} + \psi' + \frac{M_P^2}{6} \frac{\partial \log V(\psi)}{\partial \psi} = 0,$$

$$\left[ H e^{3N} \cosh^2(\psi) \dot{\chi} \right]' = 0,$$

(23)

a prime ('') denotes a derivative with respect to $N$ and

$$V(\psi) = M_P^4 \left[ \left( \frac{9\xi^2}{4\alpha} + 36\lambda \right) \sinh^4(\psi) - \frac{3\xi}{4\alpha} \sinh^2(\psi) + \frac{1}{16\alpha} \right].$$

4In this work we focus to study only the simplest O(1) case of a real scalar field. The more general $O(N)$ case contains $N - 1$ Goldstones, and we postpone the study of their effect onto the inflationary dynamics to future publication. It is well known that the dynamics of multi-field Goldstone-like inflationary fields can produce interesting effects, see e.g. Ref. [31]. For now it suffices to note that, when the angular velocities are small, $|\theta_I| \ll M_P$ ($I = 1, \cdots, N - 1$), we expect the effect of Goldstones to be unimportant during inflation.
FIG. 1: The figure illustrates how the curved geometry of field space allows for a large flat plateau near the origin, where inflation may take place. The flat region is a consequence of the negatively curved geometry of field space.

The potential (24) possesses a nearly flat region near the point of enhanced symmetry, \( \psi = 0 \) (see figure 1a), which is instrumental for prolongation of the inflationary phase. An analogous potential that is not however motivated by conformal symmetry is plotted on figure 1b for the same choice of the couplings. Note that the minimum of the potential in figure 1a is at a lower value of the field, implying that the models with a negative configuration space curvature roll typically over smaller distances. This feature is due to the sudden curving of the potential that can be seen in figure 1a induced by the negative configuration space curvature. The main consequence of such a sharp turn of the potential is, as we shall demonstrate later, a lower value of the slow roll parameters in the window \( N = 50 - 60 \) e-foldings before the end of inflation. This translates into a lower value of the tensor to scalar ratio, \( r \simeq 16 \epsilon_1 \), which renders our model viable for a wider range of the parameter \( \xi \) as compared to the flat geometry case. In addition, due to the smaller field excursion in the hyperbolic geometry case \( (O(1)m_p = O(1) \sqrt{8\pi M_p}) \), it may be therefore easier to tame the Planck scale operators and the infamous \( \eta \) problem may be less severe, or even absent, in models with a negative configuration space curvature.

The potential (24) has a maximum at \( \psi = 0 \), at which the potential energy equals,

\[
V(0) = \frac{M_P^4}{16\alpha},
\]

and two symmetric minima,

\[
\pm \psi_m = \pm \ln \left[ \mu + \sqrt{\mu^2 + 1} \right], \quad \text{with} \quad \mu = \sqrt{\frac{\xi}{6(\xi^2 + 16\alpha \lambda)}},
\]

where \( \xi = 10^{-3} \times 10^{-10} \times 10^{-9} \times 10^{-8} \).
at which the potential minimizes at a value,

\[ V_m = V(\pm \psi_m) = \frac{\lambda}{\xi^2 + 16\alpha\lambda} M_p^4, \]  

(27)
such that, as the field rolls down from its maximum at \( \psi \approx 0 \) to its minimum, the potential energy density changes by, \( \Delta V = -M_p^4/[(\lambda/\xi^2) + (16\alpha)^{-1}] \). From (27) we see that the potential energy at the end of inflation is positive and potentially large. Indeed, unless the coupling constant \( \lambda \) is extremely small (and/or \( \xi, \alpha \) extremely large), the energy density (27) will be much larger than the corresponding density in dark energy. Therefore, the amount in (27) ought to be compensated by an almost equal, negative contribution. In fact, such a negative contribution exists in the standard model. Indeed, it was pointed out in Ref. [30] that such a fine tuning can be exacted by adding both the negative contributions generated at the electroweak scale by the Higgs field condensate and the chiral condensate generated at the quantum chromodynamic (QCD) transition. In the same reference it was observed that, once this fine tuning is done at one renormalization scale, it will remain stable under an arbitrary change of the renormalization scale, which is due to the technical naturalness that arises from the enhanced symmetry at the point of vanishing vacuum energy.

A nontrivial consequence of Weyl symmetry, is the absence of any potential for the \( \chi \) field. Since this is a remnant of a broken local symmetry, when quantum effects are included the \( \chi \)-flatness must be preserved to all orders in perturbation theory. Namely, quantum effects can (and will) modify the term multiplying \((\partial \chi)^2\) and they can generate non-local terms, but they can never generate a potential (zero derivative) term.

An important consequence of the \( \chi \)-flatness is the conservation of its canonical momentum, \( \pi_\chi = H e^{3N} \cosh^2(\psi) \chi' \) implied by the equation of motion for \( \chi \) in (22). \(^5\) This then means that,

\[ \chi' = \frac{c}{H e^{3N} \cosh^2(\psi)}, \]

(28)

where \( c \) is a constant with the meaning of field space angular momentum. If \( c \) is very large, it can change the initial dynamics, producing a period of kination – defined as the cosmological epoch during which kinetic energy of a scalar field dominates the Universe’s dynamics \(^{32, 33}\) – during which the matter fluid is characterized by the equation of state, \( P = \rho \). Since \( \chi' \) scales as \( 1/a^3 \) and moreover it is proportional to \( 1/\cosh^2 \psi \), which also decays, any \( c \) contribution to the Universe’s energy density will rapidly dilute and the Universe will quickly enter a slow roll

\(^5\) An analogous conservation of the angular momentum of the Goldstone modes governs their dynamics, as each of the Goldstones exhibits a flat direction as well.
inflationary regime governed by the evolution of $\psi$. Nevertheless, kination should not be readily dismissed. Indeed, as we argue at the end of section IV, a pre-inflationary period of kination may produce interesting observable effects in the cosmic microwave background on very large angular scales, at least if inflation does not last for too long.

IV. RESULTS

In this section we show the results that a numerical analysis of Eqs. (22–23) yields. There are 4 free parameters that control the dynamics of this theory - 3 coupling constants $\alpha$, $\xi$, $\lambda$ and the field-space angular momentum $c$. In this section we will see what role each of them plays in the predictions of the model.

We start with $\alpha$, as its role is simplest to understand. If we extract $\alpha$ out of all terms in the potential (24) we get,

$$V(\psi) = M_P^4 \left( \frac{9\xi^2}{4\alpha} + 36\lambda \right) \sinh^4(\psi) - \frac{3\xi}{4\alpha} \sinh^2(\psi) + \frac{1}{16\alpha},$$

and looking at (23), we see that – for fixed $\xi$ and $\tilde{\lambda} \equiv \alpha \lambda$ – the coupling $\alpha$ controls the size of the potential, cf. also (25), and thus the Hubble parameter at the beginning of inflation, as well as the scalar and tensor spectra of cosmological perturbations (recall that the corresponding amplitudes are, $\Delta_s^2 \approx H^2/(8\pi^2\epsilon M_P^2)$ and $\Delta_t^2 \approx 2H^2/(\pi^2 M_P^2) \approx 16\epsilon \Delta_s^2$, respectively) and thus can be fixed by requiring the COBE normalization of the scalar power spectrum, $\ln(10^{10} \Delta_s^2) = 3.089 \pm 0.036$.

The role of $\xi$ can be understood from the requirement that the potential should have a sufficiently large flat region around the origin. From Eqs. (26) and (27) we see that the minima will be at a large (super-Planckian) value if $\iota \gg 1$. In this case, inflation will be long (the total number of e-foldings $N_{\text{tot}}$ will be much larger than the required 60) and these models are large field inflationary models. In the opposite limit when $\iota \ll 1$, $\psi_m \ll 1$, inflation will typically be short ($N_{\text{tot}} \ll 60$) and one gets a small field inflation. Obviously, viable inflationary models must satisfy $N_{\text{tot}} \geq 60$ and belong to large field models, for which $\iota \gg 1$. As we will see below, it is also typically the case that $16\alpha \lambda \ll \xi^2$, such that $\iota \approx 1/\sqrt{6\xi}$, and (26) yields $\psi_m \approx \ln(2\iota) \approx \frac{1}{2} \ln(2/(3\xi))$. In figure 2 we illustrate the two relevant cases, large field (left panel) and small field (right panel) inflationary potentials. Upon solving the background equations of motion, one can confirm that increasing $\xi$ (for fixed $\lambda$ and $\alpha$), decreases the duration of inflation, as can be seen in figure 3.

To summarize, the following picture has emerged:
FIG. 2: The hierarchy effect between $\tilde{\lambda} = \alpha \lambda$ and $\xi^2$. Left panel. There are deep minima at $\psi = \pm \psi_m$, $\psi_m \gg 1$ when the hierarchy $1 > \xi^2 > \tilde{\lambda}$ is observed. Right panel. When the hierarchy $\xi^2 > \tilde{\lambda}$ is not observed, the minima get close to the origin and become very shallow, almost unobservable by eye, although they are still present.

FIG. 3: Increasing $\xi$ (for fixed $\alpha$ and $\lambda$) decreases the duration of inflation.

1) The parameter $\lambda \ll 1$ controls the vacuum energy density at the end of inflation. More precisely, from Eq. (27) it follows that $\lambda/(\xi^2 + 16\alpha \lambda) = V_m/M_p^4$ controls the post-inflationary vacuum energy density, which in the limit when $\xi^2 \gg 16\alpha \lambda$ reduces to $\lambda/\xi^2$. This vacuum energy should be of the order of the electroweak energy density, $\rho_{EW} \sim 10^{-66} M_p^4$, implying that $\lambda$ ought to be extremely small, i.e. $\lambda \sim 10^{-66}\xi^2$.

2) The parameter $\alpha \gg 1$ controls the amplitude of the scalar and tensor power spectra, as
it fixes the value of $H$ at the beginning of inflation. It is therefore fixed by the COBE normalization of the amplitude of scalar cosmological perturbations to be about, $\alpha \sim 10^9$, where to get the estimate we took, $r = 16 \epsilon \simeq 0.003$ of the Starobinsky model.

3) The parameter $\xi \ll 1$ controls the duration of inflation, such that a small $\xi$ implies a long inflation (large field model); a large $\xi$ implies short inflation (small field model), see figure 4. More precisely, it is actually $\tau^2 = 6(\xi^2 + 16\alpha \lambda)/\xi$ that controls the duration of inflation, which in the limit when $\xi^2 \gg 16\alpha \lambda$ reduces to $6\xi$.

Finally, one can argue that $16\alpha \lambda \ll \xi^2$ as follows. If this condition were not met, would imply (from (27)) a vacuum energy density at the end of inflation, $V_m \simeq (M_{Pl}^4/16\alpha)\left[1 - \xi^2/(16\alpha \lambda)\right]$, which is comparable to the vacuum energy at the beginning of inflation given by (25). If this energy is to be almost compensated by the vacuum energy from the electroweak symmetry breaking, this would mean that inflation would have to happen at the electroweak scale. Inflation at the electroweak scale is possible, but it is much more fine tuned than inflation close to the grand unified scale, at which $H \sim 10^{13}$ GeV, and thus theoretically disfavored.

A. Model predictions for $n_s$ and $r$

In what follows, we present inflationary predictions of our model in slow roll approximation. There are essentially four observables from the Gaussian cosmological perturbations - from which two have been observed and for the other two there are limits and there are only limits on non-Gaussianities. The scalar and tensor spectrum can be written as,

$$\Delta_s^2(k) = \Delta_{s^*}^2 (k/k_s)^{n_s-1}, \quad \Delta_t^2(k) = \Delta_{t^*}^2 (k/k_t)^{n_t}$$

(30)

where $\Delta_{s^*}$ is the amplitude of the scalar spectrum, which is fixed by the COBE normalization,

$$A_s \equiv \Delta_{s^*}^2 = (2.105 \pm 0.030) \times 10^{-9}$$

(31)

and $n_s$ is the spectral slope defined such that, when $n_s = 1$, the scalar spectrum is scale invariant. Planck (and other available) data constrain $n_s$ at $k_s = 0.05$ Mpc$^{-1}$ as,

$$n_s = 0.9665 \pm 0.0038 \quad (68\% \text{ CL})$$

(32)

---

6 By this we mean that the formulas for $n_s$, $\alpha_s$ and $r$ that we use are the leading order in slow roll. The background evolution is solved however exactly.

7 If one includes the running of the scalar and tensor spectral index then there are six observables.
In slow roll approximation, \( n_s \) can be expressed in terms of slow roll parameters \( \epsilon_1 \) and \( \epsilon_2 = (d/dN) \ln(\epsilon_1) \) as,

\[
n_s = 1 - 2\epsilon_1 - \epsilon_2, \tag{33}
\]

where higher order (quadratic, etc.) corrections in slow roll parameters have been neglected. On the other hand, we have no measurements of the tensor spectrum. The current upper bound is expressed in terms of the scalar-to-tensor ratio, defined as,

\[
r = \frac{\Delta_s^2}{\Delta_t^2}, \tag{34}
\]

which to leading order in slow roll parameters reads, \( r = 16\epsilon_1 \), and it is constrained by the data at \( k_* = 0.002 \text{ Mpc}^{-1} \) as,

\[
r < 0.065 \quad (95\% \text{ CL}) \tag{35}
\]

Because \( r \) has not yet been measured, there is no meaningful constraint on the tensor spectral index, \( n_t = -2\epsilon_1 = -r/8 \). Sometimes one also quotes limits on the running of the scalar spectral index, defined as \( \alpha_s = dn_s/d\ln(k) \), which makes up the fifth Gaussian observable. The current constraints on \( \alpha_s \) are quite modest, \(-0.013 < \alpha_s < 0.002\), and they are not strong enough to significantly constrain our model, in which \( \alpha_s \) is second order in slow roll parameters and thus quite small.

In what follows, we investigate how the model predictions depend on the parameters of the model \( \xi, \lambda \) and \( \alpha \). The dependence on \( \alpha \) is the simplest, as it is completely fixed by the COBE normalization and by the scalar-to-tensor ratio \( r \) as,

\[
\alpha = \frac{1}{24\pi^2 r A_{s*}}. \tag{36}
\]

Now, as we shall see below, \( r \) can be well approximated by, \( r \approx 0.003 \) (with \( \sim 30\% \) accuracy), implying that \( \alpha \approx 7 \times 10^8 \).

Next we look at the dependence on \( \xi \). In figure 4 we illustrate how the duration of inflation depends on \( \xi \). The case \( \xi \ll 1 \) falls into the class of \textit{large field models} (left panel) and one gets \( N_{\text{tot}} \gg 60 \). On the other hand, the case \( \xi \leq 1 \) is a \textit{small field model} and yields only a few e-folds, \textit{i.e.} \( N_{\text{tot}} \ll 60 \) (right panel), making it not viable for inflationary model building. We note that one gets enough e-foldings of inflation (\( N_{\text{tot}} \sim 60 \)) when \( \psi_m \sim 3 - 4 \), which belongs to the class of \textit{intermediate field models}, in which during inflation the inflaton \( \psi \) rolls over approximately one (unreduced) Planck mass, \( \Delta\psi \approx \psi_m \sim m_P = \sqrt{8\pi}M_P \).
(a) For $\xi = 1.5 \cdot 10^{-3}$ inflation lasts for long time, much more than the required 60 e-foldings.

(b) For a large $\xi$ ($\xi = 0.07$) inflation does not any more last long enough as $N_{\text{tot}} \approx 10 \ll 60$.

FIG. 4: The duration of inflation in a large (left) and small (right) field inflationary model.

FIG. 5: $r$ vs $n_s$ for varying $\xi$. As $\xi$ decreases, $r$ and $n_s$ approach those of Starobinsky’s inflation. As $\xi$ increases, $n_s$ and $r$ mildly decrease.

In figure 5 we show how $n_s$ and $r$ change as $\xi$ varies for a fixed $\tilde{\lambda} \equiv \alpha \lambda$. The figure shows that, in the limit when $\xi$ is very small, one reproduces the $n_s$ and $r$ of Starobinsky’s model; increasing $\xi$ introduces a deviation from Starobinsky’s model such that both $n_s$ and $r$ decrease. Note that $\xi$ cannot be too large, since $n_s$ decreases as $\xi$ increases, eventually dropping outside the region of validity of figure 5. In figure 7 we plot the running of the scalar spectral index, $\alpha_s$, and the corresponding observational $2\sigma$ contours. As one can see, our model predicts a rather small $\alpha_s$, which is well within the experimental limits when $\xi \ll 1$.

In figure 6 we show the effect of the hyperbolic field space geometry on the tensor to scalar ratio.
FIG. 6: $r$ vs $n_s$ for $\xi = 0.25 \times 10^{-3}$ in the two cases of hyperbolic and flat field space geometry. Here the box indicates the 2$\sigma$ contour as given in [29]. As we can see the flat geometry exceeds the 95% confidence level contours for $r$ for this particular value of $\xi$, while the hyperbolic geometry sits comfortably at a low value of $r$.

Compared to the case of flat geometry, the slow roll parameters stay smaller for a longer time (in this sense the hyperbolic potential is more “flat”), which eventually translates into a suppression in the tensor to scalar ratio. For small values of $\xi \approx 10^{-4}$ the flat space model is excluded, while the hyperbolic space model gives reasonable predictions as long as the condition $\xi^2 \gg 16\alpha\lambda$, as described in section III, is satisfied.

Finally, let us discuss the dependence on $\lambda$. As explained above, $\lambda$ primarily controls the vacuum energy at the end of inflation, and for that reason it ought to be small enough. When $\alpha\lambda \ll \xi^2/16$ the inflationary observables depend only very weakly on $\lambda$, and the dependence on $\lambda$ starts becoming significant as $\alpha\lambda \sim \xi^2/16$ or larger. However, when the latter condition is satisfied, the vacuum energy left when $\psi$ settles on its minimum is big enough to quickly dominate the universe and drive a second phase of accelerated expansion. In this case, the model would predict eternal inflation and would be as such ruled out. Hence we must require $\lambda \ll \xi^2/16$, in which limit any dependence on $\lambda$ essentially drops out.
FIG. 7: $\alpha_s$ vs $n_s$ for varying $\xi$. As $\xi$ decreases, $\alpha_s$ does not vary much, while $n_s$ approaches the value of Starobinsky’s inflation. The rectangle limits this time denote the $1\sigma$ contour as given in [29].

**B. The role of the flat direction**

So far we have not yet discussed how the model predictions depend on the dynamics of the flat direction $\chi$. Unless the initial kinetic energy stored in $\chi$ is large, it will not affect the above analysis in any significant manner. Consider however the case when the initial kinetic energy in $\chi$ is large. This is equivalent to taking the parameter $c$ defined in Eq. (28) to be initially large. One can easily convince oneself that in this case the energy density of the Universe will early on scale as, $\propto c^2/[a^6 \cosh^2(\psi)]$, which corresponds approximately to a period of kination, during which kinetic energy of a scalar field dominates and $\epsilon_1 \approx 3$. A brief period of kination followed by slow roll inflation is clearly visible in figure 8. From the value of the Hubble parameter at the beginning of inflation, $H_I \approx 10^{13}$ GeV, we easily get an estimate for the maximum number of e-foldings in kination,

$$ (N_{\text{kin}})_{\text{max}} \lesssim \frac{1}{6} \ln \left( \frac{M_P^2}{H_I^2} \right) \approx 4, $$

(37)

where the estimate is obtained by assuming that the initial energy density in kination is at most Planckian.

A proper analysis of the spectrum of scalar cosmological perturbations requires solving for small perturbations of two fields, where the adiabatic mode is a linear combination of the two
fields, fluctuations of $\chi$ and of $\psi$. During kination mostly $\chi$ will source scalar cosmological perturbations, during inflation it will be $\psi$, while at the transition from kination to inflation it will be a linear combination of the two. Therefore as a rough approximation one can assume: the fluctuations of the $\chi$ field source the adiabatic mode during kination, the fluctuations of the $\psi$ field source it during inflation, and the transition period is instantaneous. An inspection of figure 8 suggests that the transition lasts for about one e-folding, of equivalently about one Hubble time, $\Delta t_{\text{transition}} \sim 1/H$. This then means that sub-Hubble modes ($k/a \gg H$) behave adiabatically, i.e. their matching leads to an exponentially suppressed mixing between positive and negative frequency modes, and thus to an exponentially suppressed particle production and the amplification for these modes can be neglected. On other other hand, super-Hubble modes ($k/a \ll H$) can be treated in the sudden matching approximation, such that these modes inherit the highly blue spectrum from kination, $\Delta_s \propto (k/k_*)^3$, for which $n_s \approx 4$. The resulting power spectrum is shown in figure 9.

As it was originally pointed out by Starobinsky [34] (see also [35]), apart from a break in its slope, the scalar power spectrum also exhibits a memory effect, manifested as damped oscillations which propagate into quite large momenta. These oscillations can be clearly seen in figure 9 where for definiteness we have assumed that the Hubble parameter at the matching equals to $k \sim 3H_0$ (which roughly corresponds to the CMB multipole $l \sim 3$). The scale at which the spectrum breaks can be shifted left or right by a suitable change in the Hubble parameter at the matching, or

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8 This approximation neglects the effect of turning of the trajectory in field space [31], and a proper understanding of this effect we postpone for a future publication.
equivalently by changing the duration of the inflationary phase. Even though the oscillations are
generated by the matching at $k \sim 3H_0$, they are visible all the way to $k \sim 30H_0$, or equivalently
$l \sim 30$. In order to better understand whether these oscillations are strong enough to be detectable
we show the upper and lower contours obtained by adding and subtracting the cosmic variance
(shown as green lines). We emphasize that, even though the size of the oscillations is smaller than
the cosmic variance, their cumulative effect might be statistically significant and one should look
for their effect in the data.

![Power Spectrum](attachment:image.png)

**FIG. 9:** Power Spectrum in the sudden matching approximation. Notice the sudden drop in power
on the largest scales. The upper and lower green lines represent the cosmic variance contours.

As it is known from the Planck Collaboration [29] (see also the earlier observations of COBE
and WMAP [36, 37]), the temperature anisotropy spectrum of CMB has a dip in the low $l$ part
of the spectrum which has statistical significance of $2 \sim 3\sigma$ which could be explained by a pre-
inflationary period of kination. Such an explanation is viable only if inflation lasts for about 60
e-foldings. While this may seem like a tuning of inflation, we point out that 60 e-foldings are
obtained in our model if the field rolls during inflation by about 1 (unreduced) Planck scale, which
might be the natural scale of inflation. Namely, large field inflationary models might be hampered
by quantum operators of large dimensions. In our model however, such operators may be forbidden
by conformal symmetry; for a recent discussion of the role of such operators for inflation see [38].
V. QUANTUM CORRECTIONS AT 1 LOOP

In this section we investigate the quantum corrections to the action (6) and show that the model, and in particular the hierarchy required for the inflationary observables to match the Planck data, i.e. \( \lambda \ll \xi \ll \alpha \), is preserved by quantum corrections.

So far we have learned that the theory (9) contains one massive scalar field \( \phi \) (defined in Eq. (12)), \( N-1 \) massless Goldstone bosons, one flat direction (denoted by \( \chi \)), one conformal gauge degree of freedom (denotes by \( \omega \)), one vector degree of freedom (which can be removed by setting \( \sigma = 0 \)) and one massless, spin two degree of freedom (the graviton). For simplicity here we shall consider the O(1) case, in which there are no Golstone bosons. Furthermore, the contribution of the massless scalar \( \chi \) to the (infrared) effective action is suppressed when compared to that of the massive scalar and therefore can be neglected.

Let us now consider the one loop contribution of the massive scalar \( \phi \). As it is well known, the quantum effective action that describes the quantum theory is given, at one loop, by the classical action plus the term \((i/2)\text{Tr} \log(i\Delta)^{-1}\), where \( i\Delta \) is the scalar propagator of the theory, which for the action (6) is given by the solution of,

\[
\left[ \nabla_\mu \nabla^\mu + \xi R - 3\lambda \phi^2 \right]i\Delta(x; y) = i\delta^D(x-y) \sqrt{-g(x)},
\]

where \( \phi \) is the value of the (possibly space-time dependent) scalar field condensate, i.e. \( \phi = \langle \hat{\phi} \rangle \) and \( D \) is the dimension of space-time.

While the effective action is highly non-local \([39]\), its infrared limit (in which \( \xi R, \lambda \phi^2 \gg \|\Box\| \)) is quite simple. Indeed, the unregularized effective action is given by,

\[
\Gamma = \int \frac{d^Dx}{(4\pi)^{D/2}} \sqrt{-g} \left\{ \left( -\left( \xi - \frac{1}{6} \right) R + \lambda \phi^2 \right)^{\frac{D}{2}} \Gamma \left( \frac{D}{2} \right) - \frac{1}{3} \left( \frac{1}{180} R_{\alpha(\beta\gamma)\delta} R^{\alpha(\beta\gamma)\delta} - \frac{1}{180} R_{(\alpha\beta)} R^{(\alpha\beta)} + \frac{(D-2)^2}{48} T_{\alpha\beta} T^{\alpha\beta} \right) \right. \left[ -\left( \xi - \frac{1}{6} \right) R + \lambda \phi^2 \right]^{D-4} \Gamma \left( 2 - \frac{D}{2} \right) \right\}. \tag{39}
\]
From this action we can read off the quantum corrections to the interaction vertices. We find,

\[ \delta \alpha = \frac{(\xi - \frac{1}{6})^2}{16\pi^2} \log \left[ \frac{-\left(\xi - \frac{1}{6}\right) R + 3\lambda \phi^2}{\mu^2} \right], \]

\[ \delta \xi = -\frac{(\xi - \frac{1}{6}) \lambda}{8\pi^2} \log \left[ \frac{-\left(\xi - \frac{1}{6}\right) R + 3\lambda \phi^2}{\mu^2} \right], \]

\[ \delta \lambda = -\frac{\lambda^2}{16\pi^2} \log \left[ \frac{-\left(\xi - \frac{1}{6}\right) R + 3\lambda \phi^2}{\mu^2} \right], \]

where we assumed that, \(-\left(\xi - \frac{1}{6}\right) R + 3\lambda \phi^2 > 0\). This shows that the quantum corrections at one loop can maintain a specific hierarchy between the coupling constants, namely,

\[ \lambda \ll \xi \ll \alpha, \quad \Rightarrow \quad \delta \lambda \ll \delta \xi \ll \delta \alpha, \]

which also happens to be the required hierarchy to obtain convincing inflationary predictions, as we showed in the previous section.

This discussion, however, does not take into account possible quantum gravitational corrections. While to reliably estimate their contribution one should set up a perturbative quantum gravity calculation, such as the one that has performed in [18], we also expect these contribution to yield corrections that are suppressed at energy scales below the Planck energy. As long as the space-time curvature remains within such a bound, therefore, we do not expect quantum gravitational correction to substantially change the conclusions of this section.

VI. CONCLUSIONS

We investigate a simple inflationary model (6) which exhibits local Weyl (or conformal) symmetry at the classical level which is realized by the space-time torsion. We show that in its simplest realization the model contains two scalar fields and one massless spin two field. One of the scalars corresponds to a flat direction \( \chi \), which is a remnant of conformal symmetry and therefore the flatness is protected from quantum corrections. The other scalar (\( \psi \)) is massive and can support inflation. The noteworthy property of the model is a negative configuration space curvature\(^9\) which is responsible for the flattening of the effective potential for \( \psi \) which is crucial for obtaining a long

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\(^9\) A similar feature flanks the so-called \( \alpha \)-attractor models [9] constructed from a supergravity model.
lasting inflation. While this feature can be present in other realisation of the spontaneous symmetry breaking pattern such as in the model of [15], it appears that the maximal symmetry is related to the enhanced, local, Weyl symmetry. The predictions of low tensor to scalar ratio, which seems typical for Weyl invariant models, seems to be in line with the results found in [11]. Our analysis shows that, for a typical choice of parameters of the model, the model approximately reproduces the results of Starobinsky inflation, which again seem to appear as a ‘universal attractor’. However, variation of the coupling constants yields significant variation of in the predictions of the model as regards the scalar spectral index \( n_s \) and the tensor-to-scalar ratio \( r \), which can be used to test the model by the next generation of CMB satellite missions such as COrE [40]. Therefore, for quite a large range of couplings (namely \( \xi < 0.01, \lambda \ll \xi \)), our model is viable in light of the currently available data. We also point out that, if a lot of energy is initially stored in the flat direction, the universe will undergo a short period of kination, followed by quasi-de Sitter inflation. Such a sequence is characterized by a lack of power in low momentum modes, a break in the power spectrum and a memory manifested as damped oscillations in the power spectrum.

**Appendix: A comparison between the breaking of Weyl symmetry and Abelian gauge symmetry**

It is important to understand that there are differences between the breaking of local Weyl symmetry and that of local Abelian gauge symmetry. The purpose of this Appendix is to underpin the similarities and differences between the two. Our starting point is the Einstein frame action (9) and the action for the Abelian-Higgs model (also known as scalar quantum electrodynamics, or short SQED),

\[
S_{\text{SQED}} = \int d^4x \sqrt{-g} \left[ - \frac{1}{4} g^{\mu\nu} g^{\rho\sigma} F_{\mu\nu} F_{\rho\sigma} - g^{\mu\nu} \left\{ (\partial_{\mu} - ieA_{\mu})\varphi^* (\partial_{\nu} + ieA_{\nu})\varphi \right\} - V(\varphi) \right],
\]

(46)

where \( e \) is the gauge coupling (electric charge), \( F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu} \) is the field strength associated to the Abelian gauge field \( A_{\mu} \), \( \varphi \) is a complex scalar and

\[
V(\varphi) = -\mu^2 \varphi \varphi^* + \lambda (\varphi \varphi^*)^2
\]

(47)

is the potential, which for \( \mu^2 > 0 \) exhibits spontaneous symmetry breaking that ‘breaks’ the local gauge symmetry.

Note first that the action (9) is invariant (i.e. it transforms into itself) under local Weyl transformations,

\[
\omega \to \omega e^{-\xi(x)}, \varphi^I \to e^{-\xi(x)} \hat{\varphi}^I, T_{\mu} \to T_{\mu} + \partial_{\mu} \xi(x), g_{\mu\nu} \to e^{2\xi(x)} g_{\mu\nu},
\]

(48)
where \( \zeta(x) \) is an arbitrary (regular) function of space and time. This means that a suitable choice of \( \zeta \) can fix \( \omega \) to a nonvanishing constant, which defines the Planck scale \( M_P \). This completely fixes Weyl symmetry.

Analogously, the Abelian-Higgs action (46) is invariant under local gauge transformations,

\[
A_\mu \rightarrow A_\mu + \partial_\mu \tilde{\zeta}, \quad \varphi \rightarrow \exp[-ie\tilde{\zeta}(x)]\varphi,
\]

where \( \tilde{\zeta} = \tilde{\zeta}(x) \) is an arbitrary scalar gauge function.

The equation of motion for the torsion trace is obtained by varying the action (9),

\[
\nabla_\mu \nabla_\nu T^{\nu} - \nabla_\mu \nabla_\nu T_\mu - (6\omega^2 + \phi^2) \mathcal{T} = \frac{1}{2} \partial_\nu (\phi^2 + 6\omega^2) \equiv \mathcal{J}_\nu, \quad \mathcal{J}_{[\nu,\mu]} = 0.
\]

Notice that the source current \( \mathcal{J}_\nu \) is purely longitudinal, which is opposite to what happens in gauge theories. Indeed, the equation of motion of the gauge field implied by the action (46) reads,

\[
\nabla_\mu \nabla_\nu A_\nu - \nabla_\mu \nabla_\nu A_\mu - 2e^2\varphi\varphi^* A_\nu = ie [\varphi \partial_\nu \varphi^* - \varphi^* \partial_\nu \varphi] \equiv J_\nu, \quad \nabla_\mu J_\mu = 0,
\]

where the scalar electromagnetic current \( J_\mu \) is purely transverse. (The current transversality condition is a consistency condition that can be traced back to the gauge symmetry: since a massive gauge field contains at most 3 physical degrees of freedom, the current \( J_\mu \) can have at most 3 independent components.)

The first difference to notice in Eq. (50) and Eq. (51) is that, in (50), the effective mass does not vanish at zero scalar field condensate, \( \phi \rightarrow 0 \). This is due to the gauge fixing condition, \( \omega^2 = 4\alpha R + \xi\phi^2 \rightarrow M_P^2 \), which guarantees that the curvature condensate does not vanish when \( \phi \) vanishes.

In order to reduce it to the backbones, let us recast the gauge field equation (51) in flat space (Minkowski) limit \( g_{\mu\nu} = \eta_{\mu\nu} \). It is instructive to study (51) in the flat space-time limit, and assume that the scalar condensate is constant, such that (51) reduces to a Proca theory with a mass term given by,

\[
M_A^2 \equiv 2e^2\langle \varphi \varphi^* \rangle.
\]

Acting suitable derivative operators on the Proca equation (51) separates it into transverse and longitudinal equations as follows,

\[
(\partial^2 - M_A^2)\partial_\nu A_\nu = \partial_\mu J_\nu, \quad J_\nu = ie \langle \varphi \partial_\nu \varphi^* - \varphi^* \partial_\nu \varphi \rangle \equiv J_\nu, \quad M_A^2 \partial^\nu J_\nu = 0, \quad (\partial_\nu M_A^2 = 0),
\]

\[
(\partial_\nu M_A^2)\partial_\nu J_\nu = 0, \quad (\partial_\nu M_A^2 = 0),
\]
which tell us that (if $\partial_\nu M_A^2 = 0$) the three propagating degree of freedom are transverse, in the 
Lorenz sense, and massive. Notice that the Lorenz condition, $\partial^\nu A_\nu = 0$, is exact as long as $M_A^2 \neq 0$ and $\partial_\nu M_A^2 = 0$.

In the case of Weyl symmetry breaking, upon following an analogous procedure, one obtains

$$ (\partial^2 - M_T^2)\partial_{[\mu}T_{\nu]} = 2(\partial^2 T_{[\mu} - \partial_{[\mu}\partial^\lambda T_{\lambda]}T_{\nu]}, \quad (55) $$

$$ -M_T^2\partial^\nu T_{\nu} = \frac{1}{2}\partial^2\langle \phi^2 \rangle + (\partial^\nu\langle \phi^2 \rangle)T_{\nu}, \quad M_T^2 = \frac{6}{2}M_P^2 + \langle \phi^2 \rangle, \quad (56) $$

which tell us that the transverse modes are only sourced by higher order interactions with the 
longitudinal mode (i.e. they are not sourced at linear order), while the longitudinal degree of 
freedom is sourced at linear order. This is to be contrasted with the Abelian gauge theory, in 
which the transverse modes are sourced in the linearised theory, while the longitudinal mode is 
source-free at leading order.

This analysis gives a more formal justification of the Ansatz we used in section [11] $T_\mu = \partial_\mu \phi^0$, 
and justifies our proposition to take the longitudinal component of the torsion trace as an effective 
scalar degree of freedom. Indeed, Eq. (56) tells us that the scalar $\phi^0$ mixes with the other scalar 
degree of freedom, and does not lead to a Lorenz condition as it is the case in the massive Proca 
theory. This scalar is the Goldstone mode of the broken local dilatation symmetry, and mixes non- 
linearly with all the scalars of the original theory according to (17), which is the field redefinition 
that diagonalizes the field space metric.

While instructive, the above analysis is not rigorous. A rigorous analysis would entail a proper 
(Dirac) analysis of constraints and dynamical degrees of freedom, and we leave it for future work.

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