Non-Equilibrium Critical Relaxation of the 3D Heisenberg Magnets with Long-Range Correlated Disorder

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Monte Carlo simulations of the short-time dynamic behavior are reported for three-dimensional Heisenberg model with long-range correlated disorder at criticality, in the case corresponding to linear defects. The static and dynamic critical exponents are determined for systems starting from an ordered initial state. The obtained values of the exponents are in good agreement with results of the field-theoretic description of the critical behavior of this model in the two-loop approximation.

Subject Index: 022, 040, 056

§ 1. Introduction

Critical properties of disordered systems with short-range (SR) and long-range (LR) correlated randomness have been studied extensively. One important question to address is whether the introduction of weak randomness changes the universality class of phase transition. According to the well-known Harris criterion, disorder with SR correlations is relevant if \( 2 - d \nu_0 = \alpha_0 > 0 \), where \( d \) is the spatial dimension, and \( \nu_0 \) and \( \alpha_0 \) are the correlation-length and the specific-heat exponents of the pure system. This criterion is modified in the presence of LR correlations in the disorder. A special type of such a disorder has been considered by Weinrib and Halperin (WH). They showed that the disorder with power law correlation \( g(x) \sim x^{-a} \) for large separations \( x \) is relevant if \( 2 - a \nu_0 > 0 \) for \( a < d \), whereas the usual SR Harris criterion recovers for \( a \geq d \). As a result, the existence of LR correlations in the disorder gives a significant effect and a wider class of disordered systems, not only the three-dimensional diluted Ising model with point-like uncorrelated defects, can be characterized by a new universality class of critical behavior.

The power law decay for the impurity-impurity pair correlation function \( g(x) \) allows a direct geometrical interpretation. So, for integer \( a \) it corresponds to the lines (at \( a = d - 1 \)) or the planes (at \( a = d - 2 \)) of impurities of random orientation. Moreover, non-integer \( a \) is sometimes treated in terms of impurities fractal dimension. Therefore, the models with LR-correlated quenched defects have both theoretical interest due to the possibility of predicting new types of critical behavior in disordered systems and experimental interest due to the possibility of realizing LR-correlated defects in the \(^4\)He in aerogels, polymers, and disordered solids containing fractal-like defects or dislocations near the sample surface.

The critical exponents were calculated in Ref. 7) in the one-loop approximation.
using a double expansion in $\varepsilon = 4 - d \ll 1$ and $\delta = 4 - a \ll 1$. The correlation-length exponent was evaluated in this linear approximation as $\nu = 2/a$ and it was argued that this scaling relation is exact and also holds in higher order approximation. In paper\(^\text{16)}\) a renormalization analysis of WH model was carried out directly for the 3D systems in the next two-loop approximation with the values of $a$ in the range $2 \leq a \leq 3$. The static and dynamic critical exponents were calculated with the use of the Padé-Borel summation technique. The results obtained in 16) essentially differ from the results evaluated by a double $\varepsilon, \delta$-expansion in Ref. 7. The comparison of the calculated exponent $\nu$ values and ratio $2/a$ showed the violation of the relation $\nu = 2/a$, supposed in 7) as exact.

Ballesteros and Parisi\(^\text{17)}\) have studied by Monte Carlo means the critical behavior in equilibrium of the 3D site-diluted Ising model with LR spatially correlated disorder, in the $a = 2$ case corresponding to linear defects. They have computed the critical exponents of these systems with the use of the finite-size scaling techniques and found that a $\nu$ value is compatible with the analytical predictions $\nu = 2/a$.

In our paper\(^\text{18)}\) the integrated Monte Carlo simulations of the short-time dynamic behavior have been carried out for 3D Ising and XY models with LR-correlated disorder at criticality, in the case corresponding to linear defects. Both static and dynamic critical exponents were determined for systems starting separately from ordered and disordered initial states. The obtained values of the exponents are in good agreement with results of the field-theoretic description of the critical behavior of these models in the two-loop approximation.\(^\text{16)}\)

Also, in Ref. 19) the authors have performed extensive Monte Carlo simulations of critical statics of 3D Ising model with LR-correlated disorder with linear defects. The Swendsen-Wang algorithm was used alongside with a histogram reweighting technique and the finite-size scaling analysis to evaluate the values of critical exponents. It was shown that obtained estimates for exponents differ from both previous numerical simulations\(^\text{17), 18)}\) and results of renormalization-group (RG) calculations.\(^\text{7), 14), 16)}\)

The present paper is devoted to numerically investigations of critical dynamics in short-time regime of 3D Heisenberg magnets with LR-correlated defects. Insertion disorder with LR correlations or extended defects must modify a conventional critical dynamics of pure Heisenberg ferro- or antiferromagnets which is described by the model J and the model G in the classification of Hohenberg and Halperin\(^\text{20)}\) and can lead to relaxational critical dynamics described by the model A with number components $n = 3$ for order parameter. The results of Monte Carlo study of the nonequilibrium behavior in the planar magnetics described by 2D XY-model with quenched structural defects can be evidence of it.\(^\text{21)}\) A significant changes in the time dependence of the autocorrelation function have been observed in the low-temperature phase due to localization of the spin excitations on structural defects.

The method of short-time critical dynamics\(^\text{22)}\) gives the possibility to determine both static and dynamic critical exponents modified by LR correlations of impurities.\(^\text{18)}\) In the following section, we introduce the 3D Heisenberg model with isotropic distributed linear defects and scaling relations for the short-time critical dynamics. In §3, we derive the critical relaxation in short-time regime for disordered Heisenberg
systems starting from an ordered initial state and cite values of static and dynamic critical exponents obtained with the use of the leading corrections to scaling. The final section contains analysis of the main results, their comparison with results of other investigations, and our conclusions.

§ 2. Description of the model and methods

We have considered the following 3D site-diluted ferromagnetic Heisenberg model Hamiltonian defined in a cubic lattice of linear size $L$ with periodic boundary conditions:

\[ H = -J \sum_{\langle i,j \rangle} p_i p_j \vec{S}_i \cdot \vec{S}_j, \]  

(2.1)

where $\vec{S}_i = (S_i^x, S_i^y, S_i^z)$, the sum is extended to the nearest neighbors, $J > 0$ is the short-range exchange interaction between spins $\vec{S}_i$, and the $p_i$ are quenched random variables ($p_i = 1$, when the site $i$ is occupied by spin, and $p_i = 0$, when the site is empty), with LR spatial correlation. An actual $p_i$ set will be called a sample from now on. We have studied the next way to introduce the correlation between the $p_i$ variables for WH model with $a = 2$, corresponding to linear defects. We start with a filled cubic lattice and remove lines of spins until we get the fixed spin concentration $p$ in the sample. We remove lines along the coordinate axes only to preserve the lattice symmetries and equalize the probability of removal for all the lattice points. This model was referred to in 17) as the model with non-Gaussian distribution noise and characterized by the isotropic impurity-impurity pair correlation function decays for large $r$ as $g(r) \sim 1/r^2$.

In this paper we have investigated disordered Heisenberg magnets with the spin concentrations $p = 0.8$. We have considered the cubic lattices with linear size $L = 128$. The Metropolis algorithm has been used in simulations. We consider only the dynamic evolution of systems described by the model A in the classification of Hohenberg and Halperin.20) The Metropolis Monte Carlo scheme of simulation with the dynamics of a single-spin flips reflects the dynamics of model A and enables us to compare obtained critical exponents $z$ to the results of RG description of the critical dynamics of this model.16)

According to the argument of Janssen et al.23) obtained with the RG method and $\varepsilon$-expansion, one may expect a generalized scaling relation for the $k$th moment the magnetization

\[ m^{(k)}(t, \tau, L, m_0) = b^{-k\beta/\nu} \times m^{(k)}(b^{-z}t, b^{1/\nu} \tau, b^{-1}L, b^{x_0}m_0) \]  

(2.2)

is realized after a time scale $t_{\text{mic}}$ which is large enough in a microscopic sense but still very small in a macroscopic sense. In Eq. (2.2), $b$ is a spatial rescaling factor, $\beta$ and $\nu$ are the well-known static critical exponents, and $z$ is the dynamic exponent, while the new independent exponent $x_0$ is the scaling dimension of the initial magnetization $m_0$ and $\tau = (T - T_c)/T_c$ is the reduced temperature.
The short-time dynamic method in part of critical evolution description of system starting from the ordered initial state is essentially the same as the non-equilibrium relaxation method proposed by Ito in Refs. 24) and 25) for non-equilibrium critical behavior study.

Since the system is in the early stage of the evolution the correlation length is still small and finite size problems are nearly absent. Therefore, we generally consider $L$ large enough ($L = 128$) and skip this argument. We measured the time evolution of the magnetization determined as follows:

$$m(t) = \left[ \frac{1}{N_s} \left( \left( \sum_i^{N_s} p_i S_i^x \right)^2 + \left( \sum_i^{N_s} p_i S_i^y \right)^2 + \left( \sum_i^{N_s} p_i S_i^z \right)^2 \right)^{1/2} \right], \quad (2.3)$$

where angle brackets denote the statistical averaging, the square brackets are for averaging over the different impurity configurations, and $N_s = pL^3$ is a number of spins in the lattice.

The question arises how a completely ordered initial state with $m_0 = 1$ evolves, when heated up suddenly to the critical temperature. In the scaling form (2.2), one can skip besides $L$, also the argument $m_0 = 1$,

$$m^{(k)}(t, \tau) = b^{-k\beta/\nu} m^{(k)}(b^{-z} t, b^{1/\nu} \tau). \quad (2.4)$$

The system is simulated numerically by starting with a completely ordered state, whose evaluation is measured at or near the critical temperature. The quantities measured are $m(t)$ and $m^{(2)}(t)$. With $b = t^{1/z}$, one avoids the main $t$ dependence in $m^{(k)}(t)$ and for $k = 1$ one has

$$m(t, \tau) = t^{-\beta/\nu z} m(1, t^{1/\nu z} \tau) = t^{-\beta/\nu z} \left( 1 + a t^{1/\nu z} \tau + O(\tau^2) \right). \quad (2.5)$$

For $\tau = 0$, the magnetization decays by a power law $m(t) \sim t^{-\beta/\nu z}$. If $\tau \neq 0$, the power law behavior is modified by the scaling function $m(1, t^{1/\nu z} \tau)$. From this fact, the critical temperature $T_c$ and the critical exponent $\beta/\nu z$ can be determined.

The scaling form of magnetization in Eq. (2.5) is presented as follows:

$$\ln m(t, \tau) = (-\beta/\nu z) \ln t + \ln m(1, t^{1/\nu z} \tau) \quad (2.6)$$

after differentiation with respect to $\tau$ gives the power law of time dependence for the logarithmic derivative of the magnetization in the following form:

$$\partial_\tau \ln m(t, \tau)|_{\tau=0} \sim t^{1/\nu z}, \quad (2.7)$$

which allows to determine the ratio $1/\nu z$. On the basis of the magnetization and its second moment, the cumulant

$$U_2(t) = \frac{m^{(2)}(t)}{(m)^2} - 1 \sim t^{d/z} \quad (2.8)$$
Fig. 1. Time evolution of the magnetization $m(t)$ for different values of the temperature $T$ (a) and for critical temperature $T = T_c = 1.197$ (b).

is defined. From its slope, one can directly measure the dynamic exponent $z$. Consequently, from an investigation of the system relaxation from ordered initial state with $m_0 = 1$, the dynamic exponent $z$ and the static exponents $\beta$ and $\nu$ can be determined.

The critical dynamic exponent $z$ of the model can be obtained also from the time evolution of the ratio

$$F_2(t) = \frac{m^{(2)}(t)|_{m_0=0}}{[m(t)]^2|_{m_0=1}} \sim \frac{t(d-2\beta/\nu)^{z}}{t^{-2\beta/\nu^z}} = t^{d/z}. \quad (2.9)$$

§3. Measurements of the critical exponents for 3D Heisenberg model with linear defects

We have performed simulations on three-dimensional cubic lattices with linear size $L = 128$, starting from an ordered initial state. We would like to mention that measurements starting from a completely ordered state with the spins oriented in the same direction ($m_0 = 1$) are more favorable, since they are much less affected by fluctuations, because the quantities measured are rather big in contrast to those from a random start with zero or small initial magnetization ($m_0 \ll 1$). Therefore, for careful determination of the critical temperature and critical exponents for 3D Heisenberg model with linear defects, we investigate the relaxation of this model from a completely ordered initial state.

Initial configurations for systems with the spin concentration $p = 0.8$ and randomly distributed quenched linear defects were generated numerically. Starting from those initial configurations, the system was updated with Metropolis algorithm. Simulation have been performed up to $t = 1000$ Monte Carlo steps per spin (MCS/s).

We measured the time evolution of the magnetization $m(t)$ and the second moment $m^{(2)}(t)$, which also allow to calculate the time-dependent cumulant $U_2(t)$ in Eq. (2.8).

In Fig. 1(a) the magnetization $m(t)$ for samples with linear size $L = 128$ at $T = 1.191, 1.194, 1.197, 1.20$, and $1.203$ is plotted in log-log scale. The resulting
curves in Fig. 1 have been obtained by averaging over 2800 samples with different linear defects configurations with 25 runs for each sample. We have determined the critical temperature $T_c = 1.197(2)$ from best fitting of these curves by power law. The magnetization $m(t)$ at the critical temperature $T = T_c$ is plotted in Fig. 1(b).

In order to check up the critical temperature value independently, we have carried out in equilibrium the calculation of cumulant $U_4$, defined as

$$U_4 = \frac{1}{2} \left( 3 - \frac{\langle m^4 \rangle}{\langle m^2 \rangle^2} \right),$$  

and the correlation length\(^{27)\)

$$\xi = \frac{1}{2 \sin (\pi / L)} \sqrt{\frac{\chi}{F} - 1},$$  

$$\chi = \frac{1}{N_s} \langle M^2 \rangle,$$  

$$F = \frac{1}{N_s} \langle \Phi \rangle,$$  

$$\Phi = \frac{1}{3} \sum_{k=\{x,y,z\}} \sum_{n=1}^{3} \left( \left| \sum_j p_j S_j^k \exp \left( \frac{2\pi i x_{n,j}}{L} \right) \right|^2 \right).$$

where $(x_{1,j}, x_{2,j}, x_{3,j})$ are coordinates of $j$-th site of lattice.

The cumulant $U_4(L, T)$ has a scaling form

$$U_4(L, T) = u \left( L^{1/\nu} (T - T_c) \right).$$

The scaling dependence of the cumulant makes it possible to determine the critical temperature $T_c$ from the coordinate of the points of intersections of the curves specifying the temperature dependence $U_4(L, T)$ for different $L$. In Fig. 2(a) the computed curves of $U_4(L, T)$ are presented for lattices with sizes $L$ from 32 to 128.
As a result it was determined that the critical temperature is $T_c = 1.197(2)$. In this case for simulations we have used the Wolff single-cluster algorithm with elementary MCS/s step as 5 cluster flips. We discard 256 MCS/s for equilibration and then measure after every MCS with averaging over 2048 MCS/s. The results have been averaged over 1000 different samples with 25 runs for each sample.

The crossing of $\xi/L$ was introduced as a convenient method for calculating of $T_c$ in $28)$. In Fig. 2(b) the computed curves of temperature dependence of ratio $\xi/L$ are presented for lattices with the same sizes, the coordinate of the points of intersections of which also gives the critical temperature $T_c = 1.198(5)$. The value of $T_c = 1.197(2)$ we selected as the best for subsequent investigations of the Heisenberg model with linear defects and with spin concentration $p = 0.8$. In Fig. 3 the cumulant $F_2(t)$ and in Fig. 4, the logarithmic derivative of the magnetization $\partial_\tau \ln m(t, \tau)|_{\tau=0}$ with respect to $\tau$ are plotted on a log-log scale at $T = T_c(p)$. The $\partial_\tau \ln m(t, \tau)|_{\tau=0}$ have been obtained from a quadratic interpolation between the three curves of time evolution of the magnetization in Fig. 1 for the temperatures $T = T_c$, $T = T_c \mp 0.003$ and taken at the critical temperature $T_c = 1.197(2)$. The resulting curves in Figs. 1 and 3 have been obtained at the critical temperature by averaging over 3800 samples with 25 runs for each sample.

In contrast to short-time dynamics of the pure systems, we can observe the crossover from dynamics of the pure system on early times of the magnetization evolution from $t \approx 15$ up to $t \approx 35$ MCS/s to dynamics of the disordered system with the influence of long-range correlated defects for $t > 80$ MCS/s. The same crossover phenomena were observed in the Monte Carlo simulated critical short-time dynamic behavior of diluted 3D Ising systems with point-like defects and in behavior of 3D Ising and XY systems with linear defects.

The existence of different regimes in short-time dynamic evolution of disordered system can be demonstrated clearly on critical time behavior of the ratio $F_2(t)$ (Fig. 3). We have analyzed the time dependence of the $F_2(t)$ in the time interval

Fig. 3. Time evolution of the cumulant $F_2(t)$ is plotted on a log-log scale at $T = T_c(p)$.

Fig. 4. Time evolution of the logarithmic derivative of the magnetization $\partial_\tau \ln m(t, \tau)|_{\tau=0}$ with respect to $\tau$ is plotted on a log-log scale.
Fig. 5. Dependence of the mean square error \( \sigma_z \) as a function of the right border of the time interval \( t \in [80, t_{\text{right}}] \).

t \in [15, 35] \text{ MCS/s} where the \( F_2(t) \) is best fitted by power law with the exponent \( d/z = 1.464(22) \). This value of \( d/z \) gives the dynamic exponent \( z = 2.049(31) \), corresponding to the pure \( O(n = 3) \) Heisenberg model.\(^{31}\) An analysis of the \( F_2(t) \) slope measured in the interval \( t \in [80, 300] \text{ MCS/s} \) shows that the exponent \( d/z = 1.217(3) \) which gives \( z = 2.465(6) \). The dependence of the mean square error \( \sigma_z \) as a function of the right border of the time interval \( t \in [80, t_{\text{right}}] \) is presented in Fig. 5. The time interval \([80, t_{\text{right}}]\) for \( t_{\text{right}} = 300 \) gives the minimum of errors for exponent \( z \) in comparison with value \( z = 2.591(10) \), which is evaluated in the time interval \( t \in [80, 1000] \).

The same analysis of time dependences of the magnetization \( m(t) \) and the logarithmic derivative of the magnetization \( \partial_{\tau} \ln m(t, \tau)|_{\tau=0} \) in the initial time interval gives the exponent values \( \beta/\nu z = 0.249(1) \) and \( 1/\nu z = 0.692(15) \) with the use of \( z = 2.049(31) \) we obtain the values of exponents \( \nu = 0.705(26) \), \( \beta = 0.360(9) \) and \( \beta/\nu = 0.510(10) \). These values are in good agreement with exponents \( \nu = 0.7048(30) \), \( \beta = 0.3636(45) \) and \( \beta/\nu = 0.5158(102) \) obtained in Ref. 33) on the basis of high resolution Monte Carlo study of pure 3D Heisenberg model critical behavior.

An analysis of the \( U_2(t) \) slope measured in the interval \( t \in [80, 300] \text{ MCS/s} \) shows that the exponent \( d/z = 1.170(136) \) which gives \( z = 2.564(368) \). The value of the dynamic exponent \( z \) obtained from the ratio \( F_2 \) in Eq. (2.9) is more preferred because evolution of \( F_2(t) \) is less fluctuated than \( U_2(t) \), and, therefore, a greatly large statistics is necessary for obtaining the same quality results from measurements of \( U_2(t) \) as is in the case with \( F_2(t) \). So, the slope of magnetization \( m(t) \) and its derivative \( \partial \ln m(t)|_{\tau=0} \) measured in the interval \( t \in [80, 300] \text{ MCS/s} \) provides the ratio of exponents \( \beta/\nu z = 0.150(1) \) and \( 1/\nu z = 0.483(22) \) which give \( \beta = 0.311(16) \) and \( \nu = 0.840(47) \).

These values of exponents can be compared with results of the field-theoretic description of the critical behavior of Heisenberg model with LR-correlated defects in the two-loop approximation \( z = 2.264, \nu = 0.798, \) and \( \beta = 0.384 \) calculated in Ref. 16) for the case of Heisenberg system with linear defects when the correlation parameter \( a = 2 \). Some numerical differences of these values exceeding the limits
Fig. 6. Dependence of the mean-square errors $\sigma$ of the fits for (a) the cumulant $F_2$, (b) magnetization $m$, and (c) the logarithmic derivative of the magnetization $\partial_{\tau}\ln m(t, \tau)$ as a function of the exponents $d/z$, $\beta/\nu z$ and $1/\nu z$ for $\omega/z = (\omega/z)_{\text{min}}$ and time interval $[80, 300]$.

Fig. 7. Dependence of global mean-square error (a) $\Delta d/z$, (b) $\Delta \beta/\nu z$ and (c) $\Delta 1/\nu z$ for all time intervals as a function of the exponent $\omega/z$.

of statistical errors of simulation and numerical approximations should not be discouraged since the obtained effective values of exponents cannot be considered as final.

In the next stage in order to obtain accurate values of the critical exponents, we have considered the influence of a leading corrections to the scaling on asymptotic values of exponents. We have applied the following expression for the observable $X(t)$:

$$X(t) = A_x t^\delta (1 + B_x t^{-\omega/z}),$$

where $\omega$ is an exponent of the leading corrections to scaling, $A_x$ and $B_x$ are fitting parameters, and an exponent $\delta = -\beta/\nu z$ when $X \equiv m(t)$, $\delta = d/z$ when $X \equiv F_2(t)$ or $X \equiv U_2(t)$, and $\delta = 1/\nu z$ when $X \equiv \partial_{\tau}\ln m(t, \tau)|_{\tau=0}$. The expression in Eq. (3.7) reflects the scaling transformation in the critical range of time-dependent corrections to scaling in the form of $t^{-\omega/z}$ to the usual form of corrections to scaling $t^{\nu\omega}$ in equilibrium state for time $t$ comparable to the order parameter relaxation time $t_r \sim \xi^2 \Omega(k \xi)$.\textsuperscript{20}

We have used the least-squares method for the best approximation of the simulation data $X(t)$ by the expression in Eq. (3.7). Minimum of the mean square errors $\sigma_\delta$ of this fitting procedure determines the exponents $\delta$ and $\omega/z$. As an example, we plot in Fig. 6 for time interval $[80, 300]$ the $\sigma_{d/z}$ (a) for the cumulant $F_2$ as a function of the exponent $d/z$ for $\omega/z = 0.40$, the $\sigma_{\beta/\nu z}$ (b) for the magnetization $m$ as a function of the exponent $\beta/\nu z$ for $\omega/z = 0.24$ and the $\sigma_{1/\nu z}$ (c) for the logarithmic derivative of the magnetization as a function of the exponent $1/\nu z$ for $\omega/z = 0.41$. All obtained minimal values of $\delta$ are averaged for all time intervals. The distribution
Fig. 8. Distribution function $\rho(d/z)$ for different time intervals. The solid line corresponds to calculated averaged value $d/z = 1.329$.

Fig. 9. Dependence of global mean-square error $\Delta \beta/\nu z$ as a function of the exponent $\omega/z$ for different values of $T$.

function $\rho(\delta)$ for different time intervals is presented in Fig. 8 for instance $\delta = d/z$. The global mean-square error

$$\Delta_\delta = \sqrt{\frac{\sum_{i=1}^{N} (\delta_i - \overline{\delta})^2 \Delta t_i}{\sum_{i=1}^{N} \Delta t_i}}$$

is calculated on the basis of averaged values $\overline{\delta}$, where $N$ is the number of time intervals, which demonstrates minimum of errors. The dependence of global mean-square error $\Delta_{d/z}$ (a) for the cumulant $F_2$, $\Delta_{\beta/\nu z}$ (b) for the magnetization $m$ and $\Delta_{1/\nu z}$ (c) for the logarithmic derivative of the magnetization as a function of the exponent $\omega/z$ is presented in Fig. 7. The critical exponents are calculated for $\omega/z$
Table I. Values of the obtained critical exponents and comparison with results of renormalization group (RG) and Monte Carlo (MC) calculations.

|                  | $z$     | $\beta/\nu$ | $\nu$   | $\beta$  | $\omega$   |
|------------------|---------|--------------|---------|----------|------------|
| $m_0 = 1$, LR system stage | 2.257(61) | 0.510(78)  | 0.770(74) | 0.393(77) | 0.786(45)  |
| $m_0 = 1$, pure system stage  | 2.049(31) | 0.510(10)  | 0.705(26) | 0.360(9)  |            |
| LR system $(a = 2)$        | 2.264   | 0.482       | 0.798    | 0.384     |            |
| Prudnikov et al., 2000 (Ref. 16)) RG $d = 3$ | 2.291(29) | 0.490(5)   | 0.766(17) | 0.375(5)  |            |
| Prudnikov et al., 2010 (Ref. 34)) RG $d = 3$ | 0.88     |              |          |           |            |
| Blavats'ka et al., 2001 (Ref. 35)) RG $d = 3$ | 0.5178(13) | 0.7048(30) | 0.3636(45) |            |            |
| Pure system           | 2.020(7) |              |          |           |            |
| Guida et al., 1998 (Ref. 32)) RG $\varepsilon$-exp. | 0.5188(23) | 0.7045(55) | 0.3655(45) |            |            |
| Chen et al., 1993 (Ref. 33)) MC | 0.5158(102) | 0.7048(30) | 0.3636(45) |            |            |

which corresponds to minimum of $\Delta_\delta$.

The value of exponent $d/z = 1.336(35)$ was computed with $\omega/z = 0.40(1)$, value $\beta/\nu z = 0.228(7)$ was computed with $\omega/z = 0.24(1)$ and value $1/\nu z = 0.589(4)$ was computed with $\omega/z = 0.41(1)$. It can be determined the values of the critical exponents $z = 2.245(60)$, $\nu = 0.757(26)$, $\beta = 0.388(15)$. For the averaged value $\omega/z = 0.36(7)$ it was computed values of the exponents $d/z = 1.329(36)$, $\beta/\nu z = 0.226(29)$, and $1/\nu z = 0.575(40)$. On the basis of these values, we determine the final values of the critical exponents $z = 2.257(61)$, $\nu = 0.770(74)$, $\beta = 0.393(77)$, and $\omega = 0.786(45)$. The statistical errors for exponents are estimated by dividing all data into five sets.

The dependences of global mean-square error $\Delta_{\beta/\nu z}$ as a function of the exponent $\omega/z$ for different values of $T$ are presented in Fig. 9 and they demonstrate that temperature $T = 1.197$, which was chosen as critical, gives the minimal value of fitting errors.

§4. Analysis of results and conclusions

The present results of Monte Carlo investigations allow us to recognize that the short-time dynamics method is reliable for the study of the critical behavior of the systems with LR-correlated disorder and is the alternative to traditional Monte Carlo methods. But in contrast to studies of the critical behavior of the pure systems by the short-time dynamics method, in case of the systems with quenched disorder corresponding to randomly distributed linear defects after the microscopic time $t_{mic} \approx 10$ there exist three stages of dynamic evolution. In the time interval of $15 - 35$ MCS/s, the power-law dependences are observed in the critical point for the magnetization $m(t)$, the logarithmic derivative of the magnetization $\partial_t \ln m(t, \tau)$, and the cumulant $F_2(t)$, which are similar to that in the pure system. In the time interval $[80, 300]$, the power-law dependences are observed in the critical point which are determined by the influence of disorder. In the intermediate time interval the crossover behavior is observed in the dynamic evolution of the system. However,
The cumulant $U_4$ as a function of the scaling variable $(T - T_c)L^{1/\nu}(1 + bL^{-\omega})$ with using correction to scaling procedure for calculated values $\nu = 0.770$ and $\omega = 0.786$ is shown in Fig. 10. Approximation of function $f(U_4, 1/L) = [\ln U_4/\ln(T - T_c)]^{-1}$ (Fig. 11) gives a possibility to estimate the exponent $\nu = 0.758(10)$, which is in good agreement with final value $\nu = 0.770(74)$ (Table I).

The dynamic and static critical exponents were computed with the use of the leading corrections to scaling for the 3D Heisenberg model with linear defects and their values $z = 2.257(61)$, $\nu = 0.770(74)$, $\beta = 0.393(77)$, and $\omega = 0.786(45)$ can be considered as final. In a summary Table I, we present the values of critical exponents $z$, $\beta/\nu$, $\nu$, $\beta$, and $\omega$ obtained in this paper by comprehensive Monte Carlo simulations of the short-time critical evolution of the diluted 3D Heisenberg model with linear defects from an ordered initial state with $m_0 = 1$. For comparison, we give in Table I the results of renormalization group (RG) and Monte Carlo (MC) calculations of these exponents for pure 3D Heisenberg model\(^{31-33}\) and diluted 3D Heisenberg model with linear defects.\(^{16,34,35}\)

The values of exponents obtained in this paper demonstrate very good agreement in the limits of statistical errors of simulation and numerical approximations with results of the RG field-theoretic description from Ref. 16), calculated with the use of the Padé-Borel (PB) summation technique to $d = 3$ expansion series, and particularly with results from Ref. 34), where the Padé-Borel-Leroy (PBL) and the self-similar approximation (SSA)\(^{36}\) resummation methods were also applied to series from Ref. 16). The obtained value of the correction-to-scaling exponent $\omega = 0.78(31)$ demonstrates a sufficiently good agreement with value of $\omega = 0.88$, obtained in Ref. 35) by the RG field-theoretical method with fixed dimension ($d = 3$) for Heisenberg model with isotropic distributed linear defects ($a = 2$).
The obtained results confirm the strong influence of LR-correlated quenched defects on the critical behavior of the systems described by the many-component order parameter. As a result, a wider class of disordered systems, not only the three-dimensional diluted Ising model, can be characterized by a new type of critical behavior induced by randomly distributed quenched defects and effects of their spatial correlations.

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