Electron Acceleration at a High Beta and Low Mach Number Rippled Shock

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Abstract. Electron acceleration in a high plasma beta and low Mach number quasi-perpendicular shock is investigated by using two-dimensional full particle-in-cell simulation. Although efficient shock drift acceleration followed by reconnection was observed in the previous one-dimensional simulation, no reflected electrons are found due to the effect of shock surface rippling for the particular parameters examined here. Structure of the shock transition region is complex in spite of the high beta and low Mach number situation. In addition to the ion scale fluctuations including the ripple, electron scale fluctuations are also recognized. Among these, downstream fluctuations are dominated by Alfvén ion cyclotron instability, the fluctuations in the foot are due to modified two-stream instability. Electron distribution function in the transition region indicates non-thermal nature. The energy gained by the non-thermal electrons is not explained merely by the shock drift acceleration, implying the importance of local wave-particle interactions.

1. Introduction
Recently, high beta ($\beta > 1$) and relatively low Mach number ($M_{ms} \ll 10$) shocks have been paid special attention as the accelerator of high energy particles. Here, $\beta$ is the ratio of upstream plasma thermal pressure to magnetic pressure and $M_{ms}$ is magnetosonic Mach number.

Electron acceleration at or near the high beta and low Mach number shocks is actually observed in a variety of astrophysical environments. The heliospheric termination shock is an example. Due to the large abundance of pickup ions, which is a high temperature ion component produced mainly by charge exchange between solar wind ions and interstellar neutral atoms, effective plasma beta upstream of the termination shock is high and magnetosonic Mach number is low. Voyager 2 spacecraft observed flux increase of non-thermal electrons (as well as ions) when it crossed the termination shock [1]. Galaxy cluster mergers often show radio synchrotron emissions from relativistic electrons accelerated near the merger shocks [2, 3, 4, 5, 6]. The intracluster medium usually has rather high temperature, $\sim$ 1-10 keV, so that the corresponding plasma becomes high beta.

One of the widely accepted scenarios of particle acceleration at a collisionless shock is the diffusive shock acceleration (DSA), in which non-thermal particles scattered back and forth across the shock interacting with upstream and downstream converging turbulence gain energy statistically (e.g., [7]). Although the DSA theory assumes pre-existence of mildly accelerated non-thermal seed particles, the mechanism to produce those seed particles has been still an open
question, known as the injection problem. In particular, the injection mechanism for electrons has been poorly understood. In terms of magnetized nonrelativistic shocks, the injection model of electrons proposed up to now is only for sufficiently high Mach number shocks at which strong Buneman instability can get excited in the transition region [8, 9]. The instability prefers high Mach number and low beta condition. However, Matsukiyo et al.[10] revisited the so-called shock drift acceleration (SDA) process and showed that SDA prefers high beta and low Mach number situation. They further proposed an electron injection scenario working at high beta and low Mach number shocks based on the SDA as follows. Once some electrons are accelerated through the SDA and reflected at the shock, they form a non-equilibrium velocity distribution function in the foreshock region. The distribution function leads to some instabilities, including electron beam instability and electron temperature anisotropy instability, locally in the foreshock. Some reflected electrons are scattered back toward the shock or its downstream by interacting with the self-generated waves. Then, those electrons are involved in further acceleration processes. Matsukiyo et al.[10] indirectly confirmed the above scenario by performing a series of one-dimensional particle-in-cell (PIC) simulations. Recently, Guo et al.[11] self-consistently reproduced the above scenario by using two-dimensional PIC simulation.

With higher spatial dimensions, in general, structure of a shock changes more than a little from one-dimensional case. In particular the ripple of a shock surface, which is believed to be strongly coupled with ion dynamics in the transition region, is a fundamental structure in a multi-dimensional system. Therefore, it is important to consider how the ripple affects the above injection scenario. In Guo et al.[11] the ripple in the above sense is not seen probably because of their choice of parameters as shown later. In this study we perform a 2D PIC simulation with the same physical parameters used by Matsukiyo et al.[10] in their 1D PIC simulation to discuss the effect of one-dimensional constraint. The simulation setup is briefly represented in section 2 and the results are shown in section 3. The summary and discussions are given in section 4.

2. Simulation Setup
A shock is produced by the so-called injection or reflecting wall method. An upstream magnetized plasma is continuously injected from the right-hand boundary. The plasma is reflected at the left-hand boundary and mixture of the incoming and the reflected plasmas results in a downstream medium. The shock is produced at a boundary between the upstream and the downstream plasmas. Since the simulation frame is the downstream rest frame, the shock propagates in time from left to right. The injection flow is parallel to the shock normal which is along the x-axis. The upstream magnetic field is in the x – y plane. The size of a spatial grid is \( \Delta x = \Delta y \approx \lambda_{De} \) where \( \lambda_{De} \) denotes the electron Debye length, and the number of super-particles per cell for the injection plasma is \( N_p = 80 \) for both electrons and ions. The system size is \( L_x \times L_y = 10000\Delta x \times 1024\Delta y \), which corresponds to \( 67.45c/\omega_{pe} \times 6.91c/\omega_{pe} \). Here, \( c/\omega_{pe} \) denotes the ion inertial length of the upstream plasma. For the parameters below, \( L_y \) is enough to reproduce the ion scale ripple structure which may affect the dynamics of high energy electrons. However, it may not be enough to investigate behaviours of high energy ions. Time resolution is \( \Delta t = 0.0578\omega_{pe}^{-1} \), where \( \omega_{pe} \) is the electron plasma frequency. The Alfvén and the magnetosonic Mach numbers of the shock is \( M_A = 7.1 \) and \( M_{ms} = 2.6 \), respectively. The ion to electron mass ratio is realistic \((m_i/m_e = 1836)\), the magnetization parameter, or the squared ratio of electron cyclotron to plasma frequencies, is \( \sigma = \omega_{ce}^2/\omega_{pe}^2 = 1/9 \). The upstream plasma beta is \( \beta = 3(\beta_e = \beta_i = 1.5) \) and the shock angle, the angle between upstream magnetic field and the shock normal, is \( \Theta_{Bn} = 85^\circ \).
3. Results

3.1. Shock Profiles

Fig. 1 shows a snapshot of shock profile at $\Omega_i t = 6.4$. Here, $\Omega_i$ denotes ion cyclotron frequency defined by upstream magnetic field strength. The left panels indicate three components of magnetic field ($B_x, B_y, B_z$) and the middle panels show those of electric field ($E_x, E_y, E_z$). In the right panels from the top the profiles of electron and ion densities ($n_e, n_i$) and total magnetic field ($B$) are presented, respectively. The shock is around the center in $x$. In contrast to the previous one-dimensional simulation, rather complex multi-scale fluctuations are found in the transition region. In each panel $y$-averaged field profile is indicated by solid line (arbitrary vertical scale). Typical structures of a supercritical quasi-perpendicular shock, i.e., foot, ramp, overshoot and undershoot, are well defined in the right panels, two densities as well as total magnetic field strength. The ripple, the mode 2 structure in the overshoot along the shock surface, is also clearly seen in these panels. The above field structures are more or less time stationary.

In Fig. 2 ion (left panels) and electron (right panels) phase space densities along $y = L_y/2$ at $\Omega_i t = 6.4$ are shown. Ion phase space indicates characteristics of a typical supercritical quasi-perpendicular shock where most of the incident ions transmit and a fraction of them reflected at the shock denote large scale gyro motions downstream, which are seen also in the one-dimensional simulation. On the other hand, all electrons transmit and no reflected electrons are produced, which is in contrast to the one-dimensional case.
The origin of rippling in a quasi-perpendicular shock was discussed by a number of authors [12, 13, 14]. Although the issue has not been completely resolved yet, it is considered that downstream temperature anisotropy of ions may play a crucial role. In the particular case here the downstream temperature anisotropy is provided by the ions once reflected by the shock and convected back into the downstream region. We estimated that their temperature anisotropy in the undershoot is $T_{r,\perp}/T_{r,\parallel} \sim 16$ and local relative density to transmit ions is $n_r/n_{tr} \sim 0.5$. The linear dispersion analysis using these values with assuming a locally homogeneous plasma results in that the maximum growth occurs at $k_{\parallel}/\omega_{pi} \sim 1.3$ corresponding mode number in $y$ is 1 or 2. This is what is observed in the undershoot of $B_x$ and $B_z$ in Fig.1. The waves are generated through Alfvén ion cyclotron (AIC) instability [12]. The ripple observed in the overshoot appear to be related strongly with these waves.

The relatively small scale fluctuations seen in the foot is due to the modified two-stream instability [15, 16, 17, 18]. The local distribution function of ions are again divided into two populations, transmitted core ions and reflected ions. The reflected ions are non-gyrotropic so that they are regarded as a cross-field ion beam. Dispersion analysis (not shown) reveals that the generated instability is the modified two-stream instability [19, 16, 17]. Smaller wavelength fluctuations are also present in the foot, ramp, and overshoot, although they are not so clear from Fig.1. They propagate along the local magnetic field so that they appear to be electron temperature anisotropy driven whistler waves.

![Figure 2](image1.png)  
**Figure 2.** (left) Ion and (right) electron phase space densities along $y = L_y/2$ at $\Omega_i t = 6.4$.

![Figure 3](image2.png)  
**Figure 3.** Orbits of two well accelerated electrons. The top three panels show time evolution of energy, momentum (parallel and perpendicular), fraction of energy gained through the shock drift process. The bottom panel denotes the corresponding orbit in $P_\parallel - P_\perp$ momentum space.

3.2. Electron Acceleration

Fig.3 represents the trajectories of two electrons which are well energized in the system. When they encounter the overshoot, they pass through the region where the magnetic field is locally
weak. This weak magnetic field region in the overshoot appears due to the rippling. The local loss-cone angle in such a region is larger than that defined by average magnetic field in the overshoot. Therefore, if an electron in the process of SDA comes to such a region, it easily enters inside of the loss-cone so that transmits. In the one-dimensional model typical time for an electron to spend in the transition region during the SDA process is roughly about $\sim \Omega_i^{-1}$ [20]. All of the electrons involved in the process of SDA encounter the weak magnetic field region within the above time period by moving along the local magnetic field. As a result, there are no reflected electrons in this particular case.

However, some electrons are locally accelerated to non-thermal energy in the transition region. Fig.4 denotes the local electron energy distribution function obtained in the overshoot. The non-thermal population is clearly seen. The above discussed electrons are the two of them. In Fig.3 it is recognized that there are several acceleration phases during the passage of the transition region. In most of the acceleration phases perpendicular momentum increases. This is in contrast to the SDA in which parallel momentum mainly increases. The trajectories in the $p_\perp - p_\parallel$ space show a number of arcs implying that wave-particle interactions play a role. The energy gained through the drift motion along the motional electric field is roughly estimated in Fig.3. In the third panels from the top $\Delta \gamma_{drift} = -eE_z \Delta z/m_e c^2$ is calculated, where $E_z$ denotes the upstream motional electric field and $\Delta z$ is the distance of drift in the $z$-direction (anti-)parallel to the motional electric field. Since the motional electric field decreases in the transition region, the actual field at the position of each electron may be weaker than $E_z$. Therefore, the values of $\Delta \gamma_{drift}$ may be overestimated than the actual energies gained through the shock drift process. Nevertheless, the corresponding energy gain is limited. Hence, at least a part of energy is gained through the process different from the SDA.

![Figure 4. Energy distribution function of electrons in the overshoot.](image)

![Figure 5. Feasibility of SDA in one-dimensional shocks. The colored region denotes $M_A - \Theta_{Bn}$ parameter space in which some electrons on a velocity shell center at upstream plasma flow velocity are adiabatically reflected. The gray region corresponds to the shocks with $\beta_e(= \beta_i) = 1.5, \sigma = 1/9, \eta = 3$, while the black region to the shocks with $\beta_e(= \beta_i) = 10, \sigma = 1/29, \eta = 1$, where $\eta$ is the shell radius in the unit of upstream electron thermal velocity. The solid circle (square) indicates the parameter used in this study and [10] ([11]).](image)
4. Summary and Discussions

In the simulation presented here no incoming electrons are reflected, although the reflection was evident in the previous one-dimensional simulation with the same upstream physical parameters [10]. The reason is the difference of microstructure in the shock transition region between the one and the two-dimensional cases. In contrast to the one-dimensional case multi-scale fluctuations in electromagnetic fields are clearly seen in the two-dimensional simulation (Fig.1). These fluctuations do not alter the $y$-averaged structures from the one-dimensional simulation. However, the motion of energetic electrons is strongly modified. The ripple plays a crucial role. During its interaction with the shock, a high energy electron encounters the region where the local magnetic field is weak and slips through the region before reflected. This weak magnetic field region is produced as a result of the rippling. In this particular case we believe that the ripple is caused by AIC instability driven by the temperature anisotropy of reflected ions. As McKean et al. [13] pointed out, the local non-gyrotropic reflected ions may drive the instability, although the linear analysis in this region is not straightforward. In the undershoot, however, the local ion distribution function is more gyrotropic and the waves observed in this region may be related with or affected by the ripple in the overshoot. The standard linear analysis assuming the temperature anisotropy of gyrotropic reflected ions gives the maximum growth at the mode number in $y$ between 1 and 2, which is consistent with the simulation result.

When the upstream plasma beta becomes high, the temperature anisotropy becomes small [12]. In Guo et al.[11] using $\beta = 20$ the maximum temperature anisotropy of the whole ions in the transition region is roughly estimated as $\sim 3$ from their Fig.2. If the reflection ratio is same as our case and the temperature anisotropy is dominantly provided by the reflected ions, the temperature anisotropy of the reflected ions may be roughly half of our case. The maximum growth of the AIC instability in the undershoot in such a case occurs at $k_{\parallel}c/\omega_{pi} < 0.1$. The corresponding waves can not be actually reproduced in the simulation, because their wavelengths in $y$ are much larger than the system size.

Fig.5 indicates a feasibility diagram of SDA. The colored regions denote $M_A - \Theta_{Bn}$ parameter space where some electrons on the velocity shell having a radius of $\eta$ times the upstream electron thermal velocity centered at the upstream plasma bulk velocity can be reflected at a shock when two dimensional effects are not taken into account (see details in Matsukiyo et al.[10]). The gray region corresponds to such a parameter space for $\beta_e(=\beta_i) = 1.5$, $\sigma = 1/9$, and $\eta = 3$. Our present simulation is done with the parameters marked by an open circle just below the upper boundary. The black region is similar one but for $\beta_e(=\beta_i) = 10$, $\sigma = 1/29$, and $\eta = 1$. The former two values are used in Guo et al.[11] and the open square denotes the parameters they chose. In contrast to the gray region, $\eta = 1$ is assumed here, because the velocity for $\eta = 3$ exceeds the speed of light in this particular case. For comparison, the boundary for $\eta = 1$ in the first case is indicated by the dashed line. From the figure, it is found that the effect of rippling may push down the boundary. Although we believe that there is a parameter region where the SDA based injection scenario can work, the effect of the ripple should be taken into account.

The present simulation implies that local wave-particle interactions in the transition region appear to play some roles in the injection process. Although there are no reflected electrons in the present case, high energy portion of the electrons in the transition region is clearly non-thermal as seen in Fig.4. The increase of their energy is not provided merely by the drift motion along the shock surface anti-parallel to the motional electric field. Particle trajectories imply the local wave-particle interactions in the transition region. In the simulation at least two electron scale waves are confirmed to be present. One is the oblique whistler waves generated in the foot as a result of modified two-stream instability, which are visible in $E_x$ and all components of $B$ at $X \sim 15$ in Fig.1. Another one is also whistler waves but propagating almost parallel to the local magnetic field with smaller wavelengths (although not so clear in Fig.1), which appear to be driven by local electron temperature anisotropy. The detailed interaction and energization
mechanisms need to be examined elsewhere. Here, we infer that at least in some parameter region injection is achieved by the combined effects of the wave-particle interactions and the SDA. Probably for smaller $B_n$, some electrons locally energized and pitch angle scattered through the wave-particle interactions during the SDA may be reflected and escape from the shock toward upstream to be involved into further acceleration process. The scenario will be examined in the near future.

In the end we comment on the effect of dimensionality of the simulation. Thomas[21] reported that although there is essentially no qualitative difference in shock structure between two and three dimensional simulations, there are some quantitative differences between them. The amplitude of ripple is smaller in three dimensional case than in two dimensional case. Also the average values at the overshoot of both magnetic field and plasma density are lower in three dimensional simulation. These effects may affect the actual reflection ratio of electrons.

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References
[1] Decker, R. B., Krimigis, S. M., Roelof, E. C., Hill, M. E., Armstrong, T. P., Gloeckler, G., Hamilton, D. C., and Lanzerotti, L. J., 2008, *Nature*, **454**, 67
[2] Willson, M. A. G., 1970, *MNRAS*, **151**, 1
[3] Fujita, Y., and Sarazin, C. L., 2001, *Astrophys. J.*, **563**, 660
[4] Govoni, F., and Feretti, L., 2004, *Int'l J Modern Phys. D*, **13**, 1549
[5] van Weeren, R. J., Röttgering, H. J. A., Brüggen, M., Hoeft, M., 2010, *Science*, **330**, 347
[6] Lindner R. R., Baker A. J., Hughes J. P., Batta`glia N., Gupta N., Knowles K., Marriage T. A., Menanteau F., Moodley K., Reese E. D., and Srianand R., 2014, *Astrophys. J.*, **786**, 49
[7] Blandford, R. D., and Eichler, D. 1987, *Phys. Rep.*, **154**, 1
[8] Amano, T., and Hoshino, M., 2007, *Astrophys. J.*, **661**, 190
[9] Matsumoto, Y., Amano, T., and Hoshino, M., 2013, *Phys. Rev. Lett.*, **111**, 215003
[10] Matsukiyo, S., Ohira, Y., Yamazaki, R., and Umeda, T., *Astrophys. J.*, **742**, 47
[11] Guo, X., Sironi, L, and Narayan, R., *Astrophys. J.*, **794**, 153
[12] Winske, D., and Quest, K. B., 1988, *J. Geophys. Res.*, **93**, 9681
[13] McKeen, M. E., Omid, N., and Krauss-Varban, D., 1995, *J. Geophys. Res.*, **100**, 3427
[14] Lowe, R. E., and Burgess, D., 2003, *Ann. Geophys.*, **21**, 671
[15] Matsukiyo, S., and Scholer, M., 2006, *J. Geophys. Res.*, **111**, A06104
[16] Umeda, T., Kidani, Y., Matsukiyo, S., and Yamazaki, R., 2012, *J. Geophys. Res.*, **117**, A03206
[17] Umeda, T., Kidani, Y., Matsukiyo, S., and Yamazaki, R., 2012, *Phys. Plasmas*, **19**, 042109
[18] Umeda, T., Kidani, Y., Matsukiyo, S., and Yamazaki, R., 2014, *Phys. Plasmas*, **21**, 022102
[19] Matsukiyo, S., and Scholer, M., 2003, *J. Geophys. Res.*, **108**, 1459
[20] Krauss-Varban, D., Burgess, D., Wu, C., S., 1989, *J. Geophys. Res.*, **94**, 15089
[21] Thomas V. A., 1989, *J. Geophys. Res.*, **94**, 12009