### TABLE 1
Details of Computational Runs

| Number of runs | Neptune | Uranus | Saturn | Jupiter | Four-planet approximation |
|----------------|---------|--------|--------|---------|---------------------------|
| Number of runs | 8       | 3      | 26     | 40      | 36                        |
| Initial conditions\(^{(1)}\): | | | | | |
| \(a_0\)        | 33 AU   | 21 AU  | 10.5 AU| 5.7 AU  | 39.27 ≤ \(a_0\) ≤ 40.12 AU| 47.37 AU |
| \(e_0\)        | 0.11-0.14 | 0.11-0.14 | 0.11-0.14 | 0.11-0.14 | 0.24 ≤ \(e_0\) ≤ 0.34 | 0.38 |
| \(i_0\)        | 5°      | 5°     | 5°     | 5°      | 1.4° ≤ \(i_0\) ≤ 12.1° | 12.4° |
| Number of computed positions | \(1.05 \times 10^6\) | \(0.91 \times 10^6\) | \(0.37 \times 10^6\) | \(0.77 \times 10^5\) | \(2.57 \times 10^7\) |
| Time step (in planet’s revolutions) | \(9.995\) | \(9.995\) | \(9.995\) | \(9.995\) | \(0.955\) |

\(^{(1)}\) taken so that the test bodies are not captured into resonances from the very beginning of the run. In the four-planet approximation, the initial conditions are taken, in accordance with observational data, near the resonances 3:2 and 2:1 with Neptune. The final distribution is not sensitive to initial conditions, because the particle’s motion becomes chaotic very soon.

\(^{(2)}\) taken to be different from 10 or 1 in order to avoid the stroboscopic effect.
FOUR COMETARY BELTS
ASSOCIATED WITH THE ORBITS OF GIANT PLANETS:
A New View of the Outer Solar System’s Structure Emerges
From Numerical Simulations

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ABSTRACT

Using numerical simulations, we examine the structure of a cometary population near a massive planet, such as a giant planet of the Solar system, starting with one-planet approximation (the Sun plus one planet). By studying the distributions of comets in semimajor axis, eccentricity, pericenter, and apocenter distances, we have revealed several interesting features in these distributions. The most remarkable ones include (i) spatial accumulation of comets near the planetary orbit (which we call the ‘cometary belt’) and (ii) avoidance of resonant orbits by comets. Then we abandon one-planet approximation and examine as to how a cometary belt is modified when the influence of all four giant planets is taken into consideration. To this end, we simulate a stationary distribution of comets, which results from the gravitational scattering of the Kuiper belt objects on the four giant planets and accounts for the effects of mean motion resonances. In our simulations, we deal with the stationary distributions computed, at different initial conditions, as 36 runs for the dynamical evolution of comets, which start from the Kuiper belt and are typically traced until the comets are ejected from the Solar system. Accounting for the influence of four giant planets makes the cometary belts overlapping, but nevertheless keeping almost all their basic features found in one-planet approximation. In particular, the belts maintain the gaps in the \((a, e)\)- and \((a, i)\)-space similar to the Kirkwood gaps in the main asteroid belt. We conclude that the large-scale structure of the Solar system is featured by the four cometary belts expected to contain 20-30 millions of scattered comets, and only a tiny fraction of them is currently visible as Jupiter-, Saturn-, etc. family comets.
1. Introduction

The outer Solar system beyond the four giant planets includes the Kuiper belt and the Oort cloud, which contain raw material left since the formation of the system. The Kuiper belt objects are thought to be responsible for progressive replenishment of the observable cometary populations, and gravitational scattering of these objects on the four giant planets could provide their transport from the trans-Neptunian region all the way inward, down to Jupiter (Fernández & Ip, 1983; Torbett 1989; Levison & Duncan, 1997; for a review, see Malhotra, Duncan & Levison, 1999 and references therein). The present paper, which continues and develops an approach started by Ozernoy, Gorkavyi, & Taidakova 2000 (≡ OGT), makes the emphasis on the structure of cometary populations between Neptune and Jupiter, both in phase space, i.e. in the space of orbital elements \( \{a, e, i\} \), and in real space. We argue that there are spatial accumulations of comets near the orbits of all four giant planets, which we name \textit{cometary belts}. These populations have the dynamical nature, because the comets belonging to the given planet’s belt are either in a resonance with the host or are gravitationally scattered predominantly on this planet.

Our approach (which has a number of common elements with the ‘particle-in-cell’ computational method) is, in brief, as follows: Let us consider, for simplicity, a stationary particle distribution in the frame co-rotating with the planet (Neptune). The locus of the given particle’s positions (taken, say, as \( 6 \cdot 10^3 \) positions every 10 yrs, i.e. every \( 6 \cdot 10^3 \) Neptune’s revolutions about the Sun) are recorded and considered as the positions of \textit{many other particles} of the same origin but \textit{at a different time}. After this particle ‘dies’ (as a result of infall on a planet/the Sun or ejection from the system by a planet-perturber), its recorded positions sampled over its lifetime form a stationary distribution as if it were produced by \textit{many} particles. Typically, each run includes \( 0.7 \cdot 10^6 \) Neptune’s revolutions (10 yrs) to give \( \sim 0.7 \cdot 10^6 \) positions of a particle, which is equivalent, for a stationary distribution, to the same number of particles.

We integrate the dynamical equations for the motion of a massless particle in the gravitational field of the Sun and the four giant planets written in the rotating
\( \{x, y, z\} \)-coordinate system with the \( x \)-axis directed along the radius and the \( y \)-axis in the direction of the orbital motion of Neptune, while the origin of coordinates is placed at the center of the Sun (thereby the secular resonances are not considered here). We use an implicit second-order integrator (Taidakova 1997) appropriately adapted to achieve our goals are described in detail in OGT (2000); as shown there, the integrator for a dissipationless system provides the necessary accuracy of computations on the time scale of \( 0.5 \cdot 10^9 \) years. A big advantage of this integrator is its stability: an error in the energy (the Tisserand parameter) does not grow as the number of time steps increases if the value of the step remains the same. The latter situation is exemplified by a resonant particle – it does not approach too close to the planet so that the same time step can be taken. In contrast to resonant particles, non-resonant ones, in due course of their gravitational scatterings, approach one or another planet from time to time, and therefore one has to change the time step near the planet. Obviously, whenever the time step diminishes near the planet, an error in the Tisserand parameter slowly grows together with an increased number of the smaller time steps. Nevertheless, in our simulations a fractional error in the Tisserand parameter typically does not exceed 0.001 during \( 3 \cdot 10^6 \) Neptune’s revolution, which amounts 0.5 Gyrs (OGT 2000). To increase accuracy of computations, we use in the present paper a second iteration. While the 1st iteration yields the gravitational field between points \( A \) and \( B \) using an approximative formula based on the particle parameters at point \( A \) (because those at point \( B \) are still unknown), the 2nd iteration enables us to compute the gravitational field between \( A \) and \( B \) using a middle position between them because the position of \( B \) is already given by the 1st iteration.

We commence this paper with examining the characteristics of cometary belts in one-planet approximation, i.e. in a 3-body problem: the Sun, a planet, and a comet (Sec. 2). In Sec. 3, we consider how these cometary belts are modified when the influence of all four giant planets is taken into account. Sec. 4 contains discussion and conclusions.
2. One-planet approximation

We consider as an example a giant planet of the Saturnian mass placed on a circular orbit of radius $R_{pl}$ having a zero inclination. It is assumed that there is an outer source of comets, which injects them into the Saturn’s strong scattering zone. The boundaries of the latter are close to the region, where heliocentric orbits of comets cross the planet’s orbit: this region, which we call hereinafter the crossing zone, is defined as $a(1 - e) \leq R_{pl} \leq a(1 + e)$.

An outer source of comets could be the Kuiper belt, a scattering zone of an outer planet, or (at a much earlier stage) the disk of planetesimals.

The dynamical evolution of comets typically includes multiple gravitational scatterings of comets on the planet with eventual ejection from the system with a hyperbolic velocity. (If neighboring planets are included, some comets may enter the scattering zones of those planets). On rare occasions, there are impacts of comets with the planet.

We have computed the dynamical evolution of test comets by making a record of their orbital parameters each revolution of the planet. Assuming that the inflow of comets into the planet’s scattering zone does not change in time, we can interpret the sample of orbital parameters of test bodies as representing a stationary distribution of a large number of comets.

We simulated 26 distributions of cometary orbits totalling $0.37 \times 10^6$ positions in the form of $(a, e, i)$-points. The initial conditions for integration of cometary orbits were taken out of resonances: $a_0 = 1.1 \ a_{planet} = 10.5$ AU (for $e_0$ and $i_0$, see Table 1, where details of this and other computational runs are given as well).

Earlier (OGT) we computed the stationary distribution of test comets in the space of orbital coordinates $(a, e)$ and $(a, i)$. As distinct from OGT (1999), where each comet was represented as a point on the phase plane $[(a, e)$ or $(a, i)]$, in the present paper we wish to compute the distribution of the ‘surface density’ (more accurately, the 2D-density) of comets on the phase plane. To this end, we make a record of coordinates of test comets each 10 revolutions of the planet. Then we sort out the computed $0.37 \times 10^6$ cometary coordinates
into two 2D data files: a $500 \times 100$ array in the $(a, e)$-plane ($a < 2.5 R_{pl}$) and a $500 \times 180$ array in the $(a, i)$-space. The following bins are used: $\Delta a = 0.005 R_{pl}$, $\Delta e = 0.01$, $\Delta i = 0.5^\circ$. Fig. 1a,b show, in the $(a, e)$ and $(a, i)$-spaces, the surface density of comets, which are gravitationally scattered on a planet of the Saturnian mass. The following features are worth mentioning:

(i) in the $(a, e)$-space, the comets are stretched along the boundaries of the planet’s strong scattering zone;

(ii) resonant gaps at the resonances $2:3$, $1:1$, $3:2$, $2:1$, $3:1$, etc., are well pronounced;

(iii) outside the scattering zone, the dynamical evolution of test bodies occurs slowly (in a diffusive way) resulting in clusterings, which could be named *diffusive accumulations*. As can be seen in Fig. 1a, these accumulations are separated from the the right boundary $a(1 - e) = R_{pl}$ of the crossing zone by a noticeable ‘trough’ of a decreased surface density of comets.

Fig. 2 a,b,c,d show distributions of comets in semimajor axis, pericenter distance, apocenter distance, and heliocentric distance, respectively. The vertical coordinate is a measure of a number of comets within the bin of 0.01 $R_{pl}$. Dash line delineates the region occupied by comets with distances of pericenter $< 0.5 R_{pl}$, i.e. those comets whose chances to be discovered are the best. Such objects could be called ‘visible comets’ (which is correct for Jupiter, although for more distant planets the visibility condition should be more stringent).

Fig. 2a reveals a rich resonant structure of the cometary belt. Arrows show positions of particular resonances. A detailed analysis (OGT) indicates that the smaller the mass of a planet (in the range $M_{\text{Uranus}} < M < M_{\text{Jupiter}}$), the more rich is its resonant structure. This is caused by the fact that, for large planetary masses, different resonances are partly overlapping.

Fig. 2b demonstrates that the pericenter distances of the scattered comets are located
close to the orbit of the planet, slightly exceeding it. This is determined by dynamics of comets and is an important feature of the cometary belt.

The distribution of comets in apocenter distance (Fig. 2c) demonstrates an appreciable concentration of comets to the planet’s orbit. If we only select comets with small distances of pericenter (as mentioned above, it is the condition to have these comets visible), it turns out that the apocenter distances of those comets are indeed rather close to the orbit of the planet (see dashed curve in Fig. 2c). As is known, it is this circumstance which has been used to define the particular ‘cometary family’.

Fig. 2d (the distribution in heliocentric distance) demonstrates that a strong concentration of comets toward the planetary orbit exists not only in phase space, but in usual space as well. The cometary belt has a pronounced maximum near the host planet’s orbit. Interestingly, the simulated distributions of surface density of both visible and all comets have two maxima. As for the curve of visible comets, this is explained by the fact that the probability to find a comet at various distances from the Sun has two maxima – at the pericenter and apocenter, and since the visible comets have quite similar orbital parameters those two maxima are clearly revealed. As for the simulated comets belonging to the same cometary belt, the right (outer) peak on the surface density curve appears because the pericenter distances of all outer (relative to the planet) comets are rather close to each other, whereas their apocenter distances differ substantially. Meanwhile the left (inner) peak on the surface density curve appears because the apocenter distances of all inner (relative to the planet) comets are rather close to each other, whereas their pericenter distances are very different. Our simulations indicate that the regions of the largest surface density of comets are located slightly outside the boundaries of the crossing zone (see Fig. 1a). Therefore pericenters of the outermost comets do not coincide with apocenters of the innermost comets. As a result, this leads to two maxima in the surface density of comets near the planet’s orbit.

Distributions shown in Figs. 2a to 2d make it quite convincing that there is a spatial accumulation of comets near the planetary orbit. We name such an accumulation the
cometary belt. This population has the dynamical nature, because the comets belonging to the given planet’s belt are either in a resonance with the host or are gravitationally scattered predominantly on this planet.

Summarizing similarities and dissimilarities between the cometary belt and cometary family, we can conclude that the distribution in apocenter distance is the only one which looks alike for both, whereas all other distributions are different due to observational selection to which the cometary family is highly sensitive.

The above material concerns the cometary belt around a planet of the Saturnian mass. We have computed the surface density distribution of comets around a planet of the Jovian, Uranian, and Neptunian mass as well. The details of our computational runs are given in Table 1. The data on the surface density of comets for all for giant planets, in one-planet approximation, are shown in Fig. 3. As can be seen, the smaller the planet’s mass, the larger is the surface density contrast.

3. Four-planet approximation

It would be important to see which features of cometary belts are kept invariable when we abandon one-planet approximation and take into account gravitational fields of all four giant planets. However, to save the computational time, we continue to neglect eccentricities and inclinations of the planets (as shown in OGT, accounting for non-zero planetary eccentricities does not lead to any oversimplifications) and we also neglect secular resonances. We have simulated 36 distributions of cometary orbits totalling \(25.7 \times 10^6\) positions, or \((a, e, i)\)-points. The details of our computational runs are summarized in Table 1. The initial conditions for orbit integrations are taken in the Kuiper belt: we consider those objects whose orbits intersect the Neptune’s orbit. They belong to the resonances 3:2 and 2:1, but the angle ‘the test body – the Sun – Neptune’ was taken in such a way that the test body rapidly leaves the resonance owing to a close encounter with the planet. Available numerical computations (Malhotra et al. 1999 and refs. therein) confirm
that the above resonances are temporary and their de-population might indeed explain the origin of the so-called scattered disk objects.

Fig. 4a,b show the ‘surface density’ (more accurately, the 2D-density) of comets in the $(a, e)$- and $(a, i)$-space governed by gravitational scatterings on all four giant planets.

We use, as we did before in Fig. 1, the logarithmic grey scale, with the only difference that each shade differs 100-fold from the neighboring one. The basic time step used is 0.001 (in units of one Neptune’s revolution, taken to be $2\pi$). The time step was taken smaller as the test comet approaches the planet. These improved simulations provide a 4-fold better accuracy compared to our earlier approach (OGT), where we used a larger basic time step (0.002). The results of both simulations turn out to be close to each other.

As can be seen from Fig. 4a, the resonant gaps in Neptune’s zone as well as gaps in the resonances 1:1 near Uranus, Saturn and Jupiter are well pronounced (at not too large eccentricities).

Our simulations indicate a progressive, sharp decrease in surface density of comets between orbits of Neptune and Jupiter (see Fig. 5d). This decrease is characterized by the transfer functions computed in OGT. We notice that a substantial decrease in surface density of comets between orbits of Neptune and Jupiter is consistent with the fact that the number of the known Centaur objects at different heliocentric distances has been found not to change substantially with the distance. Bearing in mind that the observational selection is the larger, the bigger is the distance of an object both from the Sun and the observer, the above fact implies that the number of Centaurs should sharply increase toward the Kuiper belt.

As can be seen from Figs. 4a and 4b, our simulations apparently do not explain the known comets with the largest eccentricities $e > 0.9$ or inclinations $i > 40^\circ$. Such objects, which mostly belong to the Saturn-, Uranus-, and Neptune-family comets, appear to be very rare in the dynamical evolution of short-period comets. A similar conclusion was made by Levison & Duncan (1997) who argue that the majority of such comets (of Halley type)
could be produced by a journey from the Oort cloud, and not the Kuiper belt.

Fig. 5a to 5d show the distributions of the cometary populations governed by all four giant planets in semimajor axis, distance of pericenter, distance of apocenter, and heliocentric distance, respectively.

Fig. 5a demonstrates that the resonant structure in the entire cometary population is preserved despite the gravitational influence of all four giant planets. The resonant structure is especially rich in the outer part of the Neptunian cometary belt, where the influence of the other giant planets is somewhat weakened. On the other hand, the resonant structure in the distribution of comets with $a < 30$ AU is much more smoothed, which can be explained by a strong interaction with all giant planets, including the most massive ones.

Fig. 5b (distribution of simulated comets in pericenter distance) reveals four major maxima indicating the existence of the four cometary belts associated with the giant planet orbits. The separation into four belts becomes more evident for comets with large $a$ (say, with $39 < a < 75$ AU, as can be seen in Fig. 4a), because such comets are dynamically governed by the planet whose orbit turns out to be located nearby the comet’s pericenter.

The general distribution of comets in distance of apocenter (Fig. 5c) does not reveal appreciable concentrations to any planet’s orbit, except that of Jupiter. The other, less contrast peaks are hard to be seen, because their apocenter maxima are easily destroyed by the influence of the planets (even the apocenter branch of the Neptunian belt is somewhat dissolved by the three innermost giant planets).

Distribution of comets in heliocentric distance (Fig. 5d) reveals a density peak near Neptune and another one near Jupiter, i.e. around the boundaries where those planets are the only hosts. Absence of noticeable density peaks associated with the orbits of Uranus and Saturn is not surprising, because those peaks are overlapped by a vast number of comets belonging to the Neptunian cometary belt. To illustrate this, we show in Fig. 5d, in one-planet approximation, the distribution of comets near each giant planet orbit. We assume that the density maxima in each belt are proportional to the transfer functions
found by OGT (obviously, we need this assumption only to illustrate how the different belts could be populated relative each other).

Our 36 runs performed in the four-planet approximation allow to construct a steady-state distribution consisting of $25.7 \times 10^6$ positions of test comets. Of this number of comets, only 815 penetrated into the zone of visible comets, with distances of pericenter less than 2.5 AU. Interestingly, this number turns out not to differ much from the total number of Jupiter family comets estimated, with accounting for observational selection, to be $800 \pm 300$ (Fernández et al. 1999). This implies that our simulations indicate the total number of comets in the Solar system, with the size of several km (typical for Jupiter family comets), to be as large as 20-30 millions.

According to our simulations, the number of gravitationally scattered comets of the Neptunian belt is as large as $(10-20) \cdot 10^6$. Using recent observations (Marsden 1999), which indicate that the number of kuiperoids exceeds the number of scattered Neptunian comets by a factor of 50, we estimate the total number of kuiperoids to be $5 \cdot 10^8 - 10^9$ bodies, which is fairly consistent with $8 \cdot 10^8$ inferred by Jewitt (1999) from available observations.

4. Discussion and Conclusions

One-planet approximation (the Sun plus one planet) employed in Sec. 2 suggests that each giant planet can host a cometary belt – an accumulation of comets associated with the planet’s orbit. This is a non-trivial result: in principle, the distribution of comets governed by the gravitational fields of the Sun and the planet could be alike the main asteroidal belt, i.e. not to have any concentration toward the planet’s orbit. The above accumulation has the dynamical nature implying that each comet in the belt is either gravitationally scattered predominantly on this planet or is in a resonance with it. The major problem would be to verify whether this accumulation found in one-planet approximation is survived when the influence of all giant planets is taken into account. As shown in Sec. 3, this is
indeed the case for a cometary population originating in the Kuiper belt and eventually distributed in the steady-state between the orbits of Neptune and Jupiter. Although the cometary belts of all four giant planets can be traced using the distributions in semimajor axis, distance of pericenter, and heliocentric distance, the belts are overlapped. The four-planet approximation indicates that only a tiny fraction of comets is able to penetrate from an outer planet’s zone into the zone of the nearest inner neighbor. As a result, different cometary constituents are seen in the superposition of the belts with a different confidence: the Uranian and Saturnian belts are barely seen having as a background the copious Neptunian belt, and only the latter (plus, in part, the Jovian belt) appear to be well pronounced. Nevertheless, as shown in Sec. 2, the comets of each belt, regardless of its richness, are concentrated to the host’s orbit. Despite the destructive influence of the four giant planets, the cometary belts maintain their major features found in one-planet approximation: (i) the belts are the more clear separated, the larger are semimajor axes of comets, and (ii) the resonant accumulations and gaps, although lose a little, are still well delineated.

To describe the spatial distribution of comets in the Solar system, astronomers traditionally use such a term as ‘cometary family’ (e.g. Jupiter family comets). This term, although introduced on a purely observational basis, turned out to be very valuable as it helped to reveal the first structures in the cometary population. However, since it has been found from numerical simulations that the cometary population in the zone of giant planets is very populous (Levison & Duncan 1997; OGT), it becomes more and more clear that the part of this population observed in the form of the above cometary families is no more than the ‘tip of the iceberg’. In this paper, we give new evidence in favor of this. Moreover, the distribution of the cometary populations between Jupiter and Neptune simulated in OGT and the present paper is important to compute the distribution of dust in the outer Solar system (GOTM 1999).

Each cometary family is characterized by the same apocenters of comets as the orbit of its host planet. Therefore, each family of comets turns out to be a part of a more general
dynamical substance described in this paper – the cometary belt. Such belts, as shown above, should exist near each giant planet’s orbit. The basic features of each cometary belt are determined by its dynamical interaction with the gravitational field of the host planet, and these features, as distinct from those in cometary families, do not depend upon observational selection. The dynamical term ‘cometary belt’ seems to be more justifiable and helpful than the observational term ‘cometary family’. The latter is meaningful to characterize the visible part of a cometary belt, which steadily grows as soon as more and more faint objects are registered with improving techniques. Further simulations would be highly desirable to separate possible contributions to the cometary belts from the Kuiper belt and Oort cloud.

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Figure Captions

Figure 1.

a. 2D density of a cometary belt in coordinates ‘eccentricity – semimajor axis’. To represent the number of comets in each cell, a logarithmic grey scale is used, i.e. each shade differs 10-fold from the neighboring one. Heavy curves represent the boundaries of the crossing zone, and the region above the dashed line is the zone of visible comets ($q < 0.5 R_{pl}$). Numerous resonant gaps are seen.

b. 2D density of a cometary belt in coordinates ‘inclination angle – semimajor axis’. The same logarithmic grey scale as in a is used. Numerous resonant gaps are clearly seen at all inclinations.

Figure 2.

a. Distribution of comets in semimajor axis, with a bin size $\Delta a = 0.005 R_{planet}$. Various resonant gaps are indicated by arrows. The region shown by dashed lines is occupied by visible comets, whose perihelion distances are the smallest.

b. Distribution of comets in the distance of pericenter. Dashed line indicates the region of visible comets concentrated at the left edge of the distribution.

c. Distribution of comets in the distance of apocenter. Visible comets form a peak near the planet’s orbit.

d. Surface density of the cometary population as a function of heliocentric distance. Visible comets are concentrated near or inside the host planet’s orbit.

Figure 3.

Surface density of cometary populations (logarithmic scale) as a function of the heliocentric distance shown for all four giant planets (in one-planet approximation).
Figure 4.

a. 2D density of the simulated cometary population of the Solar system in coordinates ‘eccentricity – semimajor axis’ (the four-planet approximation). Four cometary belts of the giant planets can be seen. The boundaries of the crossing zones are shown by heavy lines, and the region occupied by visible comets ($q < 2$ AU) is located above the dashed line. Crosses stand for asteroids of the main belt (the first 100 objects of the list), small triangles stand for short-periodic comets (112 objects), large triangles stand for Centaurs (15 objects), and diamonds stand for the Kuiper belt objects (191).

b. 2D density of the four cometary belts in coordinates ‘inclination angle – semimajor axis’ (the four-planet approximation). Designations are the same as in a.

Figure 5.

a. Distribution of simulated comets of the Solar system in semimajor axis (the four-planet approximation). Arrows indicate various Neptunian resonances. A region inside Neptune’s orbit, where the strong scattering zones of different planets are overlapped, looks more uniform than a well structured outermost zone.

b. Distribution of comets in the distance of pericenter (the four-planet approximation). Arrows indicate the four well pronounced peaks which correspond to the four cometary belts. Dashed line is for comets with $39 < a < 75$ AU.

c. Distribution of comets in the distance of apocenter (the four-planet approximation). There is a noticeable peak only around Jupiter’s orbit, the other peaks in this distribution are associated with various local resonances and diffusion accumulations of comets.

d. Surface density of the cometary population as a function of heliocentric distance (the four-planet approximation). Curve 1 is the four-planet approximation, and curve 2 is the sum of 4 one-planet approximations (dotted line shows the Neptunian belt, dashed line shows the Uranian belt, dash-dotted line stands for the Saturnian belt, and least-populated belt is that of Jupiter).