Modelling and Simulation of an Anti-Tank Guided Missile

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Abstract— In order to investigate and analyze the dynamic behavior of an Anti-tank guided missile system the aerodynamics should be studied and determined precisely as they are the main corner stone in designing a mathematical model of any even flying or ground vehicle. This paper presents a sufficient and a complete analysis of the total forces and moments affecting upon a missile body causing its spatial motion. Missile DATACOM is a software adopted to get a complete picture about the force and moment coefficients as a function of Mach number, angle of attack, side slip angle and the control fin deflections[1], then a complete nonlinear mathematical model is developed for our anti-tank guided missile. The simulated results yield from the mathematical model of the missile are validated with real shooting data, the comparison shows that the missile model is close to the real missile behavior. The simulated missile is then evaluated with different target scenarios (moving and stationary) and the missile is able to hit the target in each case. Lastly more investigation should be carried out on the control loop to enhance the missile behavior against other scenarios like uncertainty in aerodynamic calculations, unmodelled dynamics and degradation in missile main thrust.

Keywords— Nonlinear, Modeling, Simulation, 6DoF, Anti-Tank Missile, Skid to Turn Missile..

I. INTRODUCTION

The first problem faced by the designer of a missile guidance system is that of translating the missile tactical problem into specifications for the guidance system design. Simulation of the system by analog or digital computer is employed as an aid to the process of the missile design i.e. simulation is a continuing aid to designer throughout the duration of the missile program[2]. When the guidance system has been designed it can be evaluated by real flight recorded data. The evaluation process and redesign are carried out utilizing simulation on computers and in turn computers are considered to be the main component in missile design and implementation process.

Simulation is a process of imitating the behavior of the actual missile system by the behavior of a set of physical equations governing the guided missile motion which solved on computers using the available numerical method yielding the behavior of the missile. This set constitutes a set of complicated differential equations involving nonlinearities of many kinds. These nonlinearities arise from aerodynamic equations of the missile or from such mechanical effects as limiting, dead-bands, backlash, and hysteresis effects. Therefore, the solution of these equations can be carried out either through [3]:

- Reducing the complexity by considering some simplifications, taking into consideration that these simplifications must be similar to the full system.
- Utilizing a great developments in computation means to solve this kind of complexity.

Therefore, by the aid of appropriate computer the differential equations describing the guided missile system can be solved to give a vision to the behavior of the missile depending on the approximation carried on the differential equations[4]. The solution of these set of the differential equations depending on the determination of force and moment coefficient which aerodynamically control the missile[5]. This paper is divided into three main sections as follows: the first section provide the total forces and moments affecting upon the missile also the mathematical representation for each force and moment. The coordinate reference frame is illustrated as well in this section. The second section shows the application of Missile DATCOM on our missile system yielding the coefficients of both forces and moments depending not only the physical dimension of the missile but also on some other parameters like Mach number, angle of attack, side slip angle and finally the deflection of the control fin located in the aft part of the missile. The validation of the simulated missile data with real shooting data is presented in the third section, the engagement of the missile with a target at different velocities is presented in this section as well. The last section includes the conclusion of this work and the investigations that have to be carried out to enhance the missile model.

II. PROPOSED MODEL

In this section, the physical configuration of the missile and the mathematical representation of both aerodynamic forces and moments acting on missile fuselage and the equations of motions are described as follows:

A. Body Model

The missile body has a plane-symmetrical configuration with a set of four orthogonal rear fins in “X” configuration responsible for the control action on the missile body. These configurations are also useful in case of deploying guidance and control techniques to maintain stability control and enhance hitting accuracy [6]. Fig. 1(a, b) shows the airframe shape of the missile fuselage and the X-form fin configuration of the body’s tail.
B. Reference Frames

To formulate a mathematical model for a missile, the equations of motion should be defined these are associated to a set of reference frames or coordinate systems. On account of this, the following three orthogonal coordinate systems were used to formulate the mathematical model [7]:

1. OX1Y1Z1 – body-fixed reference frame with its origin at the missile’s centre of mass.
2. OXYZ – velocity reference frame.
3. OXgYgZg – Earth-referenced system with its origin at the missile’s centre of mass.

The body coordinate system is transformed to the fixed (earth) coordinate system throughout the Euler’s angles (ψ, θ, γ) such that the transformation matrix (\( T_b^g \)) equals [8]:

\[
T_b^g = \begin{bmatrix}
C\theta C\psi & -S\theta S\psi C\gamma & S\theta C\psi + C\theta S\psi S\gamma & -C\theta S\gamma \\
C\theta S\psi & C\theta C\psi & -S\theta C\theta & -C\theta S\psi \\
S\theta S\psi & C\theta S\psi & C\theta C\psi & -S\theta S\gamma \\
C\theta C\gamma & -C\theta S\gamma & S\theta & S\theta C\gamma - C\theta S\gamma
\end{bmatrix}
\]

(1)

Furthermore the Velocity and Body coordinate systems are transformed using the Side Slip angle (β) and the Angle of Attack (α), such that the transformation matrix from Body to Velocity coordinate systems is as follow [9].

\[
T_v^b = \begin{bmatrix}
C\alpha C\beta & -C\beta S\alpha & S\beta \\
S\alpha & C\alpha & 0 \\
-C\beta C\alpha & S\beta & C\alpha
\end{bmatrix}
\]

(2)

Velocity coordinate systems are as follow [9].

Also the Velocity and ground reference frames can be related to each other by Euler’s angles (\( \phi, \theta, \chi \)) and consequently the transformation matrix from Velocity to Ground is \( T_v^g \) [3]

\[
T_v^g = \begin{bmatrix}
C\phi C\theta & S\theta S\phi C\chi - C\theta S\phi S\chi & S\theta C\phi + C\theta S\phi C\chi & -C\theta S\phi S\gamma \\
S\phi & C\phi S\theta & -C\phi C\theta & S\phi S\theta S\gamma \\
-C\theta S\phi & C\phi S\theta S\chi + C\theta S\phi C\chi & C\phi C\theta & S\phi C\theta S\gamma \\
C\phi S\gamma & -C\phi C\gamma & S\phi & C\phi C\gamma S\theta
\end{bmatrix}
\]

(3)

Where, \( C\phi, S\phi \) are \( \cos \phi, \sin \phi \) etc

C. Acting Forces and Moments

The aerodynamic forces which does not pass through the c.g. appears on the control surfaces when they are deflected and originate a moment resulting in missile rotation around c.g. this rotation of the missile frame with respect to the trajectory changes the missile angle of attack (\( \alpha \)) and the side slip angle (\( \beta \)) and in turn the missile is subjected to lifting and lateral forces [10]. So a complete investigation on the forces and moments acting and affecting the missile during its flight is represented in the upcoming subsections.

1. Thrust Force (T)

   This force is acting on the missile due to the thrust of the rocket motor even booster or sustainer and the only main component is acting along the missile longitudinal axis (\( x_1 \)) [11]

   \[ T = T_{x_1} \]

   (4)

2. Missile Weight (G)

   The missile weight is determined by the following relationship [11]

   \[ G = m_g \cdot g \]

   (5)

   Where:
   
   \( m_g \)……instantons missile mass
   
   \( g \)……the vector of gravity acceleration

3. Aerodynamic forces

   It is usually distributed into velocity coordinate system, which related to the directions of the missile. The components of aerodynamic forces are given by [12]:

   \[ X = C_x \cdot S \cdot q \]
   \[ Y = C_y \cdot S \cdot q \]
   \[ Z = C_z \cdot S \cdot q \]

   (6)

   Where:
   
   X……drag force.
   
   Y……lift force.
   
   Z……Side force.
   
   S……characteristic area
   
   q……Dynamic pressure

   The aerodynamic force coefficients can be written as

   \[ C_x = C_x(\alpha, \beta, M, \delta_e, \delta_r) \]
   \[ C_y = C_y(\alpha, \beta, M, \delta_e, \delta_r) \]
   \[ C_z = C_z(\alpha, \beta, M, \delta_e, \delta_r) \]

   (7)

   Where:
   
   M………Mach number
   
   \( V_m \)……..missile velocity
   
   \( V_a \)……..sound velocity at the missile position
   
   \( \delta_e \)……rudder fin deflection
   
   \( \delta_a \)……elevation fin deflection

4. Acting Moments on the Missile Body

   The main affecting moment on the missile body (X form missile type) is the aerodynamic moment. The aerodynamic moment originates from the aerodynamic forces that don’t pass through the missile c.g. This moment has three components; pitch, yaw and roll moments. Usually the aerodynamic moment is given by
its components along the axes of board coordinate system as follows[12]:

\[ M_{x1} = m_{x1} \cdot I_{xx} \cdot S \cdot q \]
\[ M_{y1} = m_{y1} \cdot I_{yy} \cdot S \cdot q \]
\[ M_{z1} = m_{z1} \cdot I_{zz} \cdot S \cdot q \]  \hspace{1cm} (8)

Where:
\[ I_{xx}, I_{yy}, I_{zz} \]  \hspace{1cm} Characteristic linear dimensions of the missile
\[ m_{x1}, m_{y1}, m_{z1} \]  \hspace{1cm} dimensionless aerodynamic moment coefficient

The aerodynamic moments’ coefficients are dependent upon the body aerodynamic shape; for the underlying missile, the aerodynamic moments’ coefficients are given as follows:

\[ m_{x1} = m_{x1} \delta_r + m_{x1} w_{x1} \frac{L_{x1}}{V_m} \]
\[ m_{y1} = m_{y1} \delta_t + m_{y1} w_{y1} \frac{L_{y1}}{V_m} + m_{y1} \beta \]
\[ m_{z1} = m_{z1} \delta_r + m_{z1} w_{z1} \frac{L_{z1}}{V_m} + m_{z1} \alpha \]  \hspace{1cm} (9)

Where:
\[ m_{x1,n}, m_{y1,n}, m_{z1,n} \]  \hspace{1cm} is the aerodynamic derivative due to fin deflections
\[ w_{x1}, w_{y1}, w_{z1} \]  \hspace{1cm} is the angular velocity components.
\[ m_{x1,n}, m_{y1,n}, m_{z1,n} \]  \hspace{1cm} is the aerodynamic derivative due to angular velocity

Lastly the total force and moments acting on the missile body can be illustrated on Fig. (2).

Fig. 2. Acting forces and moments on the missile fuselage

D. Equation of the missile motion

The equations describing the missile motion in the space can be derived easily from Newton’s second law of motion which states that the summation of all external forces acting on a body must be equal to the time rate of change of its momentum and the summation of all external moments acting on a body must be equal to the time rate of change of its momentum[13]:

\[ \sum F = \frac{d(m \cdot v)}{dt} \]
\[ \sum M = \frac{dt}{dt} \]  \hspace{1cm} (10)

Where:
\[ m \]  \hspace{1cm} is the instantaneous missile mass
\[ v \]  \hspace{1cm} is the missile velocity
\[ H \]  \hspace{1cm} is the angular momentum

The following assumptions are considered during derivation of the equations of missile motion:

i- The mass of the missile is remains constant during any particular dynamic analysis.

ii- The missile is assumed to be a rigid body which means that any two points on or within the airframe remains fixed w.r.t each other.

iii- The earth is an inertial reference frames

Then

\[ \sum F = \frac{d(m \cdot v)}{dt} = (m \cdot v) \ddot{v} + m \cdot (w \cdot V) \]  \hspace{1cm} (11)

So, the linearized equations of missile motion are obtained as follows

\[ \sum \Delta F_{x1} = m(V_{x1} + V_{x1} \omega_{x1} - V_{y1} \omega_{y1}) \]
\[ \sum \Delta F_{y1} = m(V_{y1} + V_{y1} \omega_{y1} - V_{z1} \omega_{z1}) \]
\[ \sum \Delta F_{z1} = m(V_{z1} + V_{z1} \omega_{z1} - V_{x1} \omega_{x1}) \]  \hspace{1cm} (12)

Also

\[ F_{x1} = m_{g1} + T_x + X \]
\[ F_{y1} = m_{g2} + T_y + Y \]
\[ F_{z1} = m_{g3} + T_z + Z \]  \hspace{1cm} (13)

From Equations (12) and (13), we can get that:

\[ T_{x1} = C_s \cdot S_{q1} + m_{g1} = m(V_{x1} + V_{x1} \omega_{x1} - V_{y1} \omega_{y1}) \]
\[ T_{y1} = C_s \cdot S_{q1} + m_{g2} = m(V_{y1} + V_{y1} \omega_{y1} - V_{z1} \omega_{z1}) \]
\[ T_{z1} = C_s \cdot S_{q1} + m_{g3} = m(V_{z1} + V_{z1} \omega_{z1} - V_{x1} \omega_{x1}) \]  \hspace{1cm} (14)

but

\[ T_{x1} = T_x, T_{y1} = T_y, T_{z1} = T_z \]
\[ g_{x1} = -g \sin \theta \cos \gamma \]
\[ g_{y1} = -g \cos \theta \cos \gamma \]
\[ g_{z1} = g \sin \gamma \cos \theta \]

From the previous equation it can be considered that:

\[ V_{x1} = \frac{C_s \cdot S_{q1}}{m} \cdot V_{x1} - g \sin \theta \omega_{x1} - V_{y1} \omega_{y1} \]
\[ V_{y1} = \frac{C_s \cdot S_{q1}}{m} \cdot V_{y1} + g \cos \theta \cos \gamma \omega_{y1} - V_{z1} \omega_{z1} \]
\[ V_{z1} = \frac{C_s \cdot S_{q1}}{m} \cdot V_{z1} - g \cos \theta \sin \gamma \omega_{z1} - V_{x1} \omega_{x1} \]  \hspace{1cm} (15)

The linear displacement in GCS (kinematic translation of missile c.g. motion) is as follows:

\[ X = V_{x1} \cdot (C \cdot \theta) - (C \cdot \theta \cdot S \cdot \phi) \cdot Y \]
\[ Y = V_{x1} \cdot (S \cdot \phi) - (S \cdot \phi \cdot S \cdot \phi \cdot Y \cdot Z) \cdot V_{x1} \]
\[ Z = V_{x1} \cdot (S \cdot \phi \cdot \theta) + (S \cdot \phi \cdot S \cdot \theta \cdot Y) \cdot V_{y1} \]
\[ - (S \cdot \phi \cdot S \cdot \theta \cdot Y) \cdot V_{x1} \]  \hspace{1cm} (16)
The aerodynamic force and moments coefficients are mainly a function of angle of attack \( \alpha \) and side slip angle \( \beta \). Thus it is necessary to show the dependence of these angles upon the velocity components as follows:

\[
\alpha = \tan^{-1}\left(\frac{V_y}{V_x}\right) \quad (17)
\]

\[
\beta = \sin^{-1}\left(\frac{V_z}{V_x}\right)
\]

The guided missile is assumed to be a massless body that is firmly attached to the board coordinate system. The position of the missile’s centre of gravity, the location of wing or tail, etc.), the flight conditions used in this study are included in the input file of Missile DATCOM and are given in Table II. The Missile DATCOM accepts an input text file including the missile physical parameters and geometric characteristics (the shape and type of the wing/tail aerofoil, the position of the centre of gravity, the location of wing or tail, etc.), the flight conditions, the mass properties, and so on [1]. The main parameters of the physical configuration of the missile and the flight conditions used in this study are included in the input file of Missile DATCOM and are given in Table II. The missile’s body was divided into stations as to supply detailed information of the physical geometry of the missile.

First element yields \( \gamma \) and by multiplying 2nd element by \( \sin \gamma \) and 3rd element by \( \cos \gamma \) and add to yield \( \dot{\theta} \), then multiply 2nd element by \( \cos \gamma \) and 3rd by \( \sin \gamma \) and subtract to yield \( \dot{\psi} \).

\[
\begin{bmatrix}
\omega_x \\
\omega_y \\
\omega_z
\end{bmatrix} = 
\begin{bmatrix}
\cos \theta \cos \gamma & \sin \theta & 0 \\
-\cos \theta \sin \gamma & \cos \theta \cos \gamma & \sin \gamma \\
\sin \theta & -\cos \theta \cos \gamma & \cos \theta \sin \gamma
\end{bmatrix}
\begin{bmatrix}
\dot{\psi} \\
\dot{\theta} \\
\dot{\gamma}
\end{bmatrix}
\quad (18)
\]

\[
\begin{align*}
\dot{\psi} &= (\omega_y \sin \gamma - \omega_z \sin \gamma) \cos \theta \cos \gamma \\
\dot{\theta} &= \omega_y \sin \gamma + \omega_z \cos \gamma \\
\dot{\gamma} &= \omega_y - (\omega_y \cos \gamma - \omega_z \sin \gamma) \tan \theta
\end{align*}
\]

From Newton’s second law of rotary motion which states that the summation of all external moments acting on a body must be equal to the time rate of change of its moment of momentum i.e.

\[
\sum M = \frac{d(H)}{dt} \quad (20)
\]

\[
\sum M = \frac{dH}{dt} = H \dot{H} + \dot{\omega} \notimes \dot{H} \quad (21)
\]

For two plane of symmetry, like the underlying missile

\[
\begin{align*}
\dot{H}_z &= I_{zz} \omega_{z1} \\
\dot{H}_y &= I_{yy} \omega_{y1} \\
\dot{H}_x &= I_{xx} \omega_{x1}
\end{align*}
\]

And

\[
\begin{align*}
\omega_x &= \sin \theta \omega_y \cos \gamma - \cos \theta \omega_z \sin \gamma \\
\omega_y &= \cos \theta \omega_y \cos \gamma + \sin \theta \omega_z \sin \gamma \\
\omega_z &= \cos \theta \omega_y - \sin \theta \omega_z \\
\end{align*}
\]

The output file of Missile DATCOM gives the total aerodynamic forces and moments coefficients as multidimensional pages. Each page corresponds to a selected value of the Mach number, the sideslip angle, the altitude and the rear fin deflection.
A. Lift Force Aerodynamic Coefficient ($C_{y1}$)

For efficient flight at subsonic speeds, it is necessary to use wings that have as large a span as possible to reduce the drag. Additionally, it is advantageous to use aerofoil section that are slightly curved. The missile is symmetrical in pitch and yaw planes. The coefficient of lifting force is a function of the angle of attack ($\alpha$), rear fin deflection ($\delta$) according to the following relations:

$$C_{y1} = C_{y10} + C_{y1}^\alpha \sin \alpha + C_{y1}^\delta \delta$$  (25)

Where $C_{y10}$ is the coefficient of lifting force when $\alpha=\delta=0$. For the underlying system it is equal to zero because the missile is symmetrical about its longitudinal axis. From Missile DATCOM output file we can get the lift force coefficient at every rear fin deflection as shown Fig.3.

B. Drag Force Coefficient ($C_{x1}$)

Trajectory calculations and autopilot design require the missile configuration and a method by which the drag coefficient can be efficiently calculated by responsible accuracy. Essentially, there are two separate contributions to the drag force on the missile through its flight, one due to the pressure and viscous forces on the missile ($C_{x0}$) while the other is associated with the production of lift ($C_{x1}$) such that:

$$C_{x1} = C_{x0} + C_{x1}$$  (26)

So, the total drag force coefficient can be also yielded from DATCOM as follows in Fig.4.

C. Side Force Coefficient ($C_l$)

The side force originates from the deflection of the rudder fins as shown in Fig.5.

D. Pitching Moment Coefficient ($m_{z1}$)

The moment arises from non-zero angle of attack ($\alpha$) and when the aerodynamic force is related to the missile c.g. it causes the missile to rotate in the pitch plane and consequently it is called pitch moment and can be expressed as follows:

$$M_{z1} = m_{z1} \cdot I_{zz} \cdot S \cdot q$$  (27)

The DATCOM result for pitching moment coefficient due to elevator fin deflections is shown in Fig.6.

E. Yawing Moment Coefficient ($m_{y1}$)

Like pitching moment; yawing moment is also arisen from the non-zero side slip angle ($\beta$) and consequently the side force causes the missile to rotate around its yaw axis ($Y_1$) and can be expressed as follows:

$$M_{y1} = m_{y1} \cdot I_{yy} \cdot S \cdot q$$  (28)

The DATCOM result for yawing moment coefficient due to rudder fin deflections is shown in Fig.7.

These coefficients are then implemented in the full mathematical model of the missile as well as the equations of motion to get the missile trajectory.

E. Flow Chart of the missile Simulation model

The set of differential equations governing the missile spatial motion is solved using Rung Kutta-4 method. First the initial target and missile parameters are manually entered to the program and depending on the pitch and yaw loops illustrated in Fig.8 the error is filtered, compensated and the steering commands are generating causing the missile rear fins to deflect with ($\delta_\alpha$) in pitch plane and ($\delta_\gamma$) in yaw plane.
According to the missile fin deflections ($\delta$), Mach number (M), angle of attack ($\alpha$) and side slip angle ($\beta$) the corresponding force and moment coefficients are obtained using lookup table yielding the new missile position and new angular position in yaw plane ($\psi$) and pitch plane ($\vartheta$), these calculations are carried out till the missile interception yielding the miss distance in pitch and yaw planes or missile ground hit occurs.

**F. Validation of the Missile Model**

The missile model is validated by comparing its output flight path trajectory with real recorded shootings results. Fig.9 and Fig.10 show the radial velocities and the trajectories between real and simulated missile data. Fig.11 (a,b,c,d) show the engagement of the missile with a target in different scenarios.

**IV. CONCLUSIONS AND FUTURE WORK**

This paper has presented the total forces and moments that affecting on the missile body and also the coordinate reference frames are then defined. Taking into consideration the forces and moments affecting on the missile fuselage, the equations of motion that governing the missile spatial motion are derived. A special software technique (Missile DATCOM) is adopted to calculate the coefficients of forces and moments as a function of Mach number, angle of attack, side slip angle and control fin deflections. The complete 6 DOF is created based on the
DATCOM results such that the simulated results show a satisfactory behaviour when validated with real shooting results. However the simulated missile success to attack the missile at different scenarios but it fails to hit it with thrust degradation so more investigation on the controller loop will be carried out in the future work.

REFERENCES

[1] USAF, "Missile DATCOM " USAF, ed., 1998.
[2] Islam H. Elnady, Ahmed M. Kamel, Yehia Z. Elhalwag, “Modeling and Simulation of an Aerial Gliding Body in Free-Fall,” the International Journal of Engineering Research and Technology vol. 7, no. 8, 2018.
[3] G. El-Sheikh, Guidance System and Theory, Cairo, 2010.
[4] L. S. a. Collaborators, Principles of Guided Missile Design, 1955.
[5] A.N.Ouda, “Adaptive Autopilot Design for Flying Vehicle,” the International journal of dynamic and control (IJDAC), 2017.
[6] A. M. Kamal, “Modeling, Analysis and Identification of Airplane Flight Dynamics,” Military Technical College, Cairo, 2014.
[7] J. D. Anderson, FUNDAMENTALS OF AERODYNAMICS, 1991.
[8] A. H. A. Elsayed Ahmed, A. N. Ouda, H. Eldin Hussein Ahmed, H. Mohamed Abd-Elkader, “Modeling of a Small Unmanned Aerial Vehicle,” the International Journal of advanced robotic and automation vol. 4, no. 126, 2015.
[9] A. E. E.-H. Shady A. Kader, A. N. Ouda, “Quadcopter System Modeling and Autopilot Synthesis,” the International Journal of Engineering Research and Technology, vol. 3, no. 11, 2014.
[10] P. Zarcan, “Tactical and Strategic Missile Guidance,” Progress in Astronautics and Aeronautic. AIAA, 1994.
[11] R.M.Goreckt, A baseline 6 degree of freedom mathematical model of generic missile, DSTO Systems Sciences Laboratory Australia 2003.
[12] N. Jack N, Missle Aerodynamics, Mountain View, USA, 1988.
[13] G. M. Siouris, Missile Guidance and Control Systems, Ohio,USA, 2003.