Mean-Field Alpha Effect impacts Memory of the Solar Cycle

Soumitra Hazra\textsuperscript{1,2}, Allan Sacha Brun\textsuperscript{1} and Dibyendu Nandy\textsuperscript{3,4}

\textsuperscript{1}AIM, CEA, CNRS, Université Paris-Saclay, Université Paris-Diderot, Sorbonne Paris Cité, F-91191 Gif-sur-Yvette, France
e-mail: soumitra.hazra@cea.fr, soumitra.hazra@gmail.com
e-mail: sacha.brun@cea.fr, allan-sacha.brun@cea.fr
\textsuperscript{2}Institut d’Astrophysique Spatiale, CNRS, Univ. Paris-Sud, Université Paris-Saclay, Bât. 121, F-91405 Orsay, France
\textsuperscript{3}Department of Physical Sciences, Indian Institute of Science Education and Research Kolkata, Mohanpur 741246, West Bengal, India
\textsuperscript{4}Center of Excellence and Space Sciences India, Indian Institute of Science Education and Research Kolkata, Mohanpur 741246, West Bengal, India
e-mail: dnandi@iiserkol.ac.in

April 7, 2020

ABSTRACT

Context. Predictions of solar cycle 24 obtained from the advection-dominated and diffusion-dominated kinematic dynamo model are different if we consider only the Babcock-Leighton mechanism as a poloidal field generation source term. Yeates et al. (2008) argue that the discrepancy between the results is due to the different memory of solar dynamo for advection- and diffusion-dominated solar convection zones.

Aims. We aim to investigate the discrepancy between the solar cycle memory obtained from advection-dominated and diffusion-dominated kinematic solar dynamo models. Specifically, we want to investigate whether another poloidal field generation mechanism, namely, Parker’s mean-field alpha effect has any impact on the memory of the solar cycle.

Methods. We used a kinematic flux transport solar dynamo model where poloidal field generation takes place due to both the Babcock-Leighton mechanism and mean-field alpha effect.

Results. The addition of a mean-field alpha effect reduces the memory of the Babcock Leighton solar dynamo to within one cycle for both advection- and diffusion-dominated dynamo regimes. Our result provides an alternative resolution to the discrepancy in memory of advection- and diffusion-dominated kinematic dynamo models. We also find a moderate decrease in the correlation between the radial flux at the cycle minima and the toroidal flux at the maxima of the next cycle with the increase of mean-field alpha amplitude.

Conclusions. We confirm the conclusions of Yeates et al. (2008) and Karak and Nandy (2012) regarding the short, one-cycle memory of the solar cycle. We establish that the mean-field alpha effect impacts the memory of the solar cycle. However, correlation coefficients are highly dependent on the assumed amplitude and fluctuation levels of the mean-field alpha.

Key words. solar cycle prediction – flux transport solar dynamo model – mean-field alpha effect

1. Introduction

The magnetic field of the Sun is responsible for most of the dynamical features in the solar atmosphere. The solar cycle is the most prominent signature of solar magnetic activity in which the number of sunspots, which are strongly magnetized regions on the solar surface, varies cyclically with a periodicity of 11 years. When the Sun reaches the peak of its activity cycle, there is a large number of flares and coronal mass ejections, which can affect vulnerable infrastructures of our modern society (Schrijver et al. 2015). These important issues highlight the need for solar activity prediction, which will enable us to mitigate the impact of our star’s active behaviour (Hathaway 2009; Petrovay 2010). In recent years, many theoretical and observational studies have been performed to predict the solar activity, however, results are diverging (Pesnell 2008).

Our current understanding of the solar cycle suggests that sunspot originates from the buoyant emergence of toroidal flux tubes which are generated via dynamo mechanism inside the solar interior. Dynamo mechanism involves the idea of joint generation and recycling of the toroidal and the poloidal components of the solar magnetic field (Parker 1955). Pre-existing poloidal magnetic field components are stretched along \( \phi \)-direction due to strong differential rotation and generates the toroidal magnetic field. It is thought that toroidal field generation takes place throughout the solar convection zone, but is amplified near the base of the convection zone. Tachocline, a region of strong radial gradient in rotation and low diffusivity, offers an ideal location for storage and amplification of the toroidal magnetic field. Sufficiently strong toroidal flux tubes become magnetically buoyant and emerges at the solar surface in the form of sunspot. However, two different proposals exist in the literature for the poloidal field generation—one is due to decay and dispersal of bipolar magnetic regions at the solar surface, termed as the Babcock-Leighton mechanism (Babcock 1961; Leighton 1969) and other is due to strong helical turbulence inside the solar convection zone, known as mean-field alpha effect (Parker 1955; Steenbeck et al. 1966). In recent years, dynamo models based on the Babcock-Leighton mechanism have been successful in explaining different observational aspects regarding the solar activity (Dikpati & Charbonneau 1999; Nandy & Choudhuri 2002; Choudhuri et al. 2004; Jouve & Brun 2007; Nandy et al. 2011; Choudhuri & Karak 2012; Bhowmik & Nandy 2018; Hazra & Nandy 2019). Recently, data-driven 2.5D kinematic dynamo models and 3D
kinematic solar dynamo models are also developed to study different observational aspects regarding solar activity (Brun 2007; Jouvet et al. 2011; Yeates & Muñoz-Jaramillo 2013; Hung et al. 2017; Hazra et al. 2017; Karak & Miesch 2017; Hazra & Miesch 2018; Kumar et al. 2019). Please see these reviews for details about the solar and stellar dynamo model (Charbonneau 2005; Brun & Browning 2017).

As there is a spatial separation between the source layers of the toroidal and poloidal field, there must be some effective communication mechanism between these layers. While magnetic buoyancy plays the primary role for transporting the toroidal flux from the base of the convection zone to the solar surface; different other flux transport mechanisms, namely, diffusion, meridional flow, and turbulent pumping, share the role of transporting the poloidal flux from the surface to the base of the convection zone. It has been shown that finite time required for the magnetic flux transport impacts the predictability of the solar cycle (Yeates et al. 2008; Jouve et al. 2010). Dikpati et al. (2006) used an advection-dominated based dynamo model (where the meridional flow is the primary flux transport mechanism) to predict the solar cycle 24 and found that cycle 24 should have been a strong one. We note that Dikpati et al. (2006) used a weak tachocline alpha effect in their model. However, Choudhuri et al. (2007) used a diffusion-dominated dynamo model (diffusion is the primary flux transport mechanism) to predict the solar cycle 24 and found that cycle 24 will be a weaker one. Yeates et al. (2008) have shown that memory of the solar cycle in the diffusion-dominated dynamo is shorter (only one cycle) while the memory of the solar cycle in advection-dominated dynamo lasts over a few solar cycles. They suggest that the difference in the memory of the solar cycle in two regimes results in different predictions of the solar cycle. Later, Karak & Nandy (2012) has shown that the introduction of turbulent pumping reduces the memory of the solar cycle into one cycle in both advection and diffusion-dominated dynamo models; this impacts the capability of these kind of models for prediction. Turbulent pumping transports the magnetic field vertically downwards; however, there is also a significant latitudinal component in the strong rotation regime (Ossendrijver et al. 2012; Käpylä et al. 2006b; Mason et al. 2008; Do Cao & Brun 2011; Hazra & Nandy 2016).

Most of the dynamo based prediction models completely ignored the contribution of distributed mean-field alpha effect. They consider the Babcock-Leighton mechanism as the only poloidal field generation mechanism for their prediction models. However, some studies indicate that the mean-field alpha effect plays an important role in solar dynamo models and is necessary to recover the solar cycle from grand-minima like episodes (Pipin & Kosovichev 2011; Pipin et al. 2013; Passos et al. 2014; Hazra et al. 2014b; Incoglu et al. 2019). Recently Bhowmik & Nandy (2018) considers both the Babcock-Leighton mechanism and mean-field alpha effect as poloidal field generation mechanism in their model to predict the strength of the solar cycle 25. Here, in this paper, we want to explore the importance of the mean-field alpha effect in the context of solar cycle memory and predictability. We find that the presence of mean-field alpha reduces the memory into one cycle for both advection and diffusion dominated regime. We provide the details about our solar dynamo model in Section 2 followed by a discussion of our results in Section 3. Finally, in the last section, we present the conclusions of our study.

2. Model

Our \((\alpha - \Omega)\) kinematic solar dynamo model solves the evolution equations for the toroidal and poloidal components of solar magnetic fields (Moffatt 1978; Charbonneau 2005):

\[
\frac{\partial A}{\partial t} + \frac{1}{s} \left[ \mathbf{v}_p \cdot \nabla (sA) \right] = \eta \left( \nabla^2 - \frac{1}{s^2} \right) A + S_p
\]

\[
\frac{\partial B}{\partial t} + s \left[ \mathbf{v}_p \cdot \nabla \left( \frac{\mathbf{B}}{s} \right) \right] + \left( \nabla \cdot \mathbf{v}_p \right) B = \eta \left( \nabla^2 - \frac{1}{s^2} \right) B + s \left( \nabla \times (\Omega \mathbf{A}) \right) \cdot \nabla \Omega + \frac{1}{s} \frac{\partial \left( s^2 B \right)}{\partial r}
\]

where, \(s = r \sin(\theta)\) and \(v_p\) is the meridional flow. We specify the differential rotation and turbulent magnetic diffusivity by \(\Omega\) and \(\eta\) respectively. \(B\) represents the toroidal magnetic field component and \(A\) represents the vector potential of the poloidal magnetic field component. \(S_p\) in the poloidal field evolution equation is the source term for the poloidal field; while the second term on the RHS of the toroidal field evolution is the source term for the toroidal field due to differential rotation.

We do not consider small scale convective flows in this model. However, we consider an effective turbulent diffusivity in our model to capture the mixing effects due to convective flows. We do not have a reliable estimate of the diffusivity value inside the convection zone at this moment. However, diffusivity value near the surface is well constrained by surface flux transport dynamo models; as well as from observations (Komm et al. 1995; Muñoz-Jaramillo et al. 2011; Lemerle et al. 2013). Diffusivity value near the surface have been found to be a few times of \(10^{15}\) \(\text{cm}^2/\text{s}\). It is still unclear how these surface values change as a function of depth in the solar convection zone. We have assumed a profile which keeps value close to the surface one except in the tachocline where it drops by several orders of magnitude due to the reduced level of turbulence there. Recent theoretical studies also suggest a diffusivity value of the order of \(10^{16}\) \(\text{cm}^2/\text{s}\) inside the convection zone (Parker 1979; Miesch et al. 2012; Cameron & Schüssler 2016). We use a two-step radial diffusion profile which has the following form:

\[
\eta(r) = \eta_{\text{broad}} + \frac{\eta_c - \eta_{\text{broad}}}{2} \left( 1 + \text{erf} \left( \frac{r - r_c}{d_c} \right) \right) + \frac{\eta_s - \eta_c}{2} \left( 1 + \text{erf} \left( \frac{r - r_s}{d_s} \right) \right),
\]

where \(\eta_{\text{broad}} = 10^8 \text{ cm}^2/\text{s}\) is the diffusivity at the bottom of the computational domain, \(\eta_c = 10^{22} \text{ cm}^2/\text{s}\) is the diffusivity in the convection zone and \(\eta_s = 2 \times 10^{12} \text{ cm}^2/\text{s}\) is the near surface supergranular diffusivity. Other parameters, which characterize the transition from one value of diffusivity to another are taken as \(r_c = 0.73 R_\odot, d_c = 0.015 R_\odot, r_s = 0.95 R_\odot, \text{ and } d_s = 0.015 R_\odot\). We use an analytic fit to the observed helioseismic rotation data as our differential rotation profile (see Nandy et al. 2011, Muñoz-Jaramillo et al. 2009).

\[
\Omega(r, \theta) = 2\pi\Omega_c + \pi \left( 1 - \text{erf} \left( \frac{r - r_c}{d_c} \right) \right) \left( \Omega_r - \Omega_c + (\Omega_p - \Omega_c)\Omega_\delta(\theta) \right),
\]

\[
\Omega_\delta(\theta) = \cos^2(\theta) + (1 - a) \cos^4(\theta),
\]
where $\Omega_r$, $\Omega_\phi$, and $\Omega_\psi$ represents the rotation frequency of the core, equator, and the pole respectively. We take $\Omega_r = 432\text{ nHz}$, $\Omega_\phi = 470\text{ nHz}$, $\Omega_\psi = 330\text{ nHz}$, $r_n = 0.7R_0$, $d_\phi = 0.025R_0$ (half of the tachocline thickness), and $\alpha = 0.483$.

Recent helioseismic results have not yet converged on the accurate measure about the structure of the meridional flow (Rajaguru & Antia [2015], Jackiewicz et al. [2015], Zhao & Chen [2016]). As the information about the meridional flow structure is absent at present moment, we have used a single cell meridional circulation $(v_p)$ profile which transports the field poleward at the surface and equatorward at the base of the convection zone. One can see Jouve & Brun (2007), Hazra et al. (2014a, 2014b), Hazra & Nandy (2016) for a discussion on the role of multi cellular or shallow meridional flow profiles. We obtained the profile for the meridional circulation $(v_p)$ for a compressible flow inside the convection zone using the following equation: $\nabla(v_pv_p) = 0$ so, $\rho v_p = \nabla \times (\phi \psi_0)$ where $\psi_0$ is prescribed as:

$$\psi r \sin \theta = \phi_0 (R - R_\psi) \sin \left[ \frac{\pi (r - R_\psi)}{(R_0 - R_\psi)} \right] \left[ 1 - e^{0.1r/p_1} \right] \times \left[ 1 - e^{0.1(\theta - \pi/2)} \right] e^{-(\pi - \pi/2)/2}$$

where $\phi_0$ controls the maximum speed of the flow. We take the following parameter values to obtain the profile for meridional circulation: $\beta_1 = 1.5$, $\beta_2 = 1.8$, $\epsilon = 2.0000001$, $r_0 = (R_0 - R_\psi)/4$, $\Gamma = 3.47 \times 10^3$, $\gamma = 0.95$, $m = 3/2$. $R_0 = 0.65R_\odot$ corresponds to the penetration depth of the meridional flow. $R_\psi = 0.55R_\odot$ is the bottom boundary of our computational domain. Both observation of small scale features on the solar surface and helioseismic inversions indicate that the surface flow from equator to the pole has an average speed of 10-25 ms$^{-1}$ (Komm et al. 1993, Snodgrass & Dailey 1996, Hathaway et al. 1996). In our model, meridional flow speed at the surface lies within the range of 10-25 ms$^{-1}$ and reduces to 1 ms$^{-1}$ at the base of the convection zone.

To explore the importance of the mean-field alpha effect, we consider two distinct scenarios. In one scenario, poloidal field generation takes place only due to the Babcock-Leighton mechanism; while in the other scenario poloidal field alpha generation takes place due to the combined effect of the Babcock-Leighton mechanism and mean-field alpha effect. In the first scenario, $S_p = S_{BL}$, where $S_{BL}$ is the source term for the poloidal field due to the Babcock-Leighton mechanism. We model the poloidal field source term due to the Babcock-Leighton mechanism in their dynamo model (Nandy & Choudhuri 2001, Muñoz-Jaramillo et al. 2010, Nandy et al. 2011, Hazra & Nandy 2013, 2016). It has been shown that the double ring algorithm captures the essence of the Babcock-Leighton mechanism in a better way compared to other formalism (Muñoz-Jaramillo et al. 2010). We have provided the details of our double ring algorithm in the Appendix. Please note that when we model the Babcock-Leighton mechanism via double ring algorithm, we make the Babcock-Leighton source term in the equation (1) as zero, and we modify the poloidal field by the poloidal fields associated with the double ring (basically, $A(i,j)$ is modified by $A(i,j) + A_{\text{double-ring}}$) at regular time interval. As the double-ring algorithm works above a certain threshold, a recovery mechanism is necessary to recover an activity level of the Sun from grand minima like phases. However, some previous studies indicate that even if there is no sunspot during grand minima, there are still many ephemeral regions at the solar surface which obey the Hale’s polarity law. These ephemeral regions may contribute to the poloidal field generation mechanism during this time (Priest 2014, Svanda et al. 2016, Karak & Miesch 2018). Thus, we also added an extra Babcock-Leighton source term due to ephemeral regions which acts on the weak magnetic field regime. Please see Appendix for the details of the Babcock-Leighton source terms. In this way, we ensure the effectiveness of the Babcock-Leighton mechanism throughout our simulation.

In the second Scenario, $S_p = S_{BL} + S_{MF}$, where $S_{MF}$ is the poloidal field source term due to mean-field alpha effect. It implies that the poloidal field is generated due to both the Babcock-Leighton mechanism and mean-field alpha effect. We model the mean-field alpha effect following this equation:

$$S_{MF} = S_{\phi} \frac{\cos \theta}{4} \left[ 1 + \text{erf}\left( \frac{(r - r_1)}{d_1} \right) \right] \left[ 1 - \text{erf}\left( \frac{(r - r_2)}{d_2} \right) \right] \times \frac{1}{1 + \left( \frac{\theta}{\pi/2} \right)^2}$$

where $r_1 = 0.71R_0$, $r_2 = R_0$, $d_1 = d_2 = 0.25R_0$, and $B_{\phi p} = 10^4 G$ i.e. the upper threshold. $S_{\phi}$ controls the amplitude of the mean-field alpha effect. The function $1/(1 + \left( \frac{\theta}{\pi/2} \right)^2)$ ensures that this additional $\alpha$ effect is only effective on weak magnetic field strength (below the upper threshold $B_{\phi p}$) and the values of $r_1$ and $r_2$ ensure that this additional mechanism takes place inside the bulk of convection zone (see top panel of Fig. 1 for radial profile of the mean-field $\alpha$-coefficient). We set the critical value of $S_{\phi}$ such that our model generates periodic cycles if we consider mean-field alpha effect as the only poloidal field generation mechanism. Critical value of $S_{\phi}$ is 0.14 m s$^{-1}$ for our model. Please see the right-hand side of the upper panel for the radial profile of the mean-field $\alpha$ coefficient.

We perform all of our dynamo simulations within the meridional slab $0.55R_\odot < r < R_{\odot}$ and $0 < \theta < \pi$ with a resolution of $300 \times 300$ (i.e., $N_r = N_\theta = 300$). We set $A = 0$ and $B = \sin(2\theta) \sin(\pi(r - 0.55R_\odot)/(R_\odot - 0.55R_\odot))$ as dipolar initial conditions for our simulations. Finally, we solve the dynamo equations with proper boundary conditions suitable for the Sun. As our model is axisymmetric, we set both poloidal and toroidal fields as zero ($A = 0$ and $B = 0$) at the pole ($\theta = 0$ and $\theta = \pi$), to avoid any kind of singularity. The inner boundary condition at the bottom of the computational domain ($r = 0.55R_\odot$) is of a perfect conductor. So, at $r = 0.55R_\odot$, both the toroidal and poloidal field components vanish (i.e., $A = 0$ and $B = 0$). At the surface, we assume there is the only radial component of the solar magnetic field which is necessary for stress balance between the subsurface and coronal magnetic fields (van Ballegooijen & Mackay 2007). We set $B = 0$ and $\partial (\theta A)/\partial r = 0$ as a top boundary condition at the surface ($r = R_{\odot}$).

3. Results

To bring out the impact of mean-field alpha effect in the memory of the solar cycle, we perform kinematic solar dynamo simulations in two different regimes, advection-dominated ($\nu_\zeta = 1 \times 10^4 \text{ cm}^2 \text{ s}^{-1}$, $v_0 = 25 \text{ m s}^{-1}$) and diffusion-dominated regimes ($\nu_\zeta = 1 \times 10^2 \text{ cm}^2 \text{ s}^{-1}$, $v_0 = 15 \text{ m s}^{-1}$). Advection-dominated regimes are characterized by the dominance of meridional circulation as a major poloidal flux transport mechanism from the
surface to the base of the convection zone; while diffusion-dominated regimes are characterized by the dominance of diffusion (Yeates et al. 2008; Karak & Nandy 2012). We define the advective flux transport time scale following the suggestions of Yeates et al. (2008). Advective flux transport time scale is the time taken for the meridional circulation to transport poloidal fields from $r = 0.95 R_\odot$, $\theta = 45^\circ$ to the location at the tachocline where the strongest toroidal field is formed ($\theta = 60^\circ$). The meridional flow speed of the order of 25 m s$^{-1}$ at the surface yields an advective flux transport time scale about 9-10 years (with a flow speed of 15 m s$^{-1}$ it becomes 16 years). While turbulent diffusion of the order of $1 \times 10^{12}$ cm$^2$ s$^{-1}$ gives us the diffusion flux transport time scale $(L^2/\eta$ where $L$ is the depth of the convection zone) of 14 years. In the diffusion dominated regime, the diffusion time scale is shorter compared to the advection time scale; while in the advection dominated regime, it is the reverse.

In the first scenario, we consider the Babcock-Leighton mechanism as the only poloidal field generation mechanism. We first perform the simulation without any fluctuation. We are able to reproduce a solar-like cycle with an 11-year periodicity. We also confirm that the periodicity of the solar cycle decreases with the speed of the meridional flow (Dikpati & Charbonneau 1999). Periodicity lies within the range of 7 years to 18 years depending on the speed of the meridional flow at the surface varies from 25 m s$^{-1}$ to 10 m s$^{-1}$. However, in reality, the Babcock-Leighton mechanism is a random process. The stochastic nature of the Babcock-Leighton mechanism arises from the random buffeting of flux tubes during their rise through the turbulent convection zone, yielding a significant scatter in tilt angles of the active region (Longcope & Choudhuri 2002; McCintock & Norton 2016). Motivated by these facts, we introduce stochastic fluctuation in the poloidal field generation source term by setting $K_1 = K_\text{base} + K_\text{fluc}$corr$(t, \tau_{corr})$ with $K_\text{base} = 100$ in the double ring algorithm. Here $\sigma$, the uniform random number, lies between -1 and +1. We run all our next simulations with a 60% fluctuation in the Babcock-Leighton mechanism. So in our simulation $K_\text{fluc} = 0.6K_\text{base}$ (see Appendix). Our choice of fluctuation levels is inspired by observations as well as the eddy velocity distributions present in 3D turbulent convection simulations (Miesch et al. 2008; Racine et al., 2011; Passos et al., 2012).

The middle panel of Fig. 1 shows the butterfly diagram at the base of the convection zone from a simulation in the diffusion-dominated regime and the bottom panel shows the variation of $B_{\phi}^2$ (a proxy of sunspot number) with time at the base of the convection zone. We note that the sensitivity of the peak amplitude of the $B_{\phi}^2$ is due to the choice of fluctuation levels. We calculate the polar radial flux $\Phi_r$ and toroidal flux $\Phi_\phi$ using the prescription suggested by Karak & Nandy (2012) and Yeates et al. (2008). The toroidal flux $\Phi_\phi$ is calculated by integrating $B_\phi^2(r, \theta)$ within a layer of $r = 0.677 R_\odot - 0.726 R_\odot$, and within the latitude $10^\circ - 45^\circ$; while the radial flux $\Phi_r$ is calculated by integrating $B_r^2(r, \theta)$ at the solar surface within the latitude $70^\circ - 89^\circ$. We note that there is a 90° phase difference between the radial flux and the toroidal flux. Radial flux is maximum at the minima of the solar cycle. We find the peak value of $\Phi_r$ and $\Phi_\phi$ for each cycle and study the cross-correlation between the surface radial flux $\Phi_r$ of cycle n and the toroidal flux of cycle n, n+1, n+2 and n+3. We perform the same study for both advection-dominated and diffusion-dominated dynamo simulations. Cycle to cycle correlation gives us the extent of correlation between the radial and toroidal flux. As per suggestions of Yeates et al. (2008) and Karak & Nandy (2012), the extent of the correlation is the indicator of the memory of the solar cycle. We run our stochastically forced dynamo model for a total of 250 solar cycles to generate the correlation statistics.

Figure 2 shows that in the advection-dominated regime, surface radial flux $\Phi_r$ correlates with the toroidal flux $\Phi_\phi$ff of cycles $(n+1)$ and $(n+2)$ with Spearman correlation coefficients 0.90 and 0.49 respectively. On the other hand, Figure 3 indicates that in the diffusion-dominated regime, surface radial flux $\Phi_r$ only correlates with the toroidal flux $\Phi_\phi$ff of the next cycle (correlation coefficient 0.95). Yeates et al. (2008) studied the memory of the solar cycle and found that in the diffusion-dominated regime surface radial flux $\Phi_r$ only correlates with the next cycle toroidal flux $\Phi_\phi$ff. While, in the advection-dominated regime, surface radial flux $\Phi_r$ correlates with the toroidal flux $\Phi_\phi$ff of subsequent few cycles $(n+1)$, $(n+2)$ and $(n+3)$. Yeates et al. (2008) and Karak & Nandy (2012) found higher correlation coefficients in the advection dominated regimes compared to us. This is probably due to our choice of modelling magnetic buoyancy by the double-ring algorithm. In summary, our result agrees with the results of Yeates et al. (2008) when we consider the Babcock-Leighton mechanism as the only poloidal field generation process.

In the second scenario, we consider both the Babcock-Leighton mechanism and mean-field alpha effect as a poloidal field generation mechanism. However, the mean-field alpha effect is also a random process, not a deterministic one. As the mean-field alpha effect arises due to helical turbulence inside the turbulent convection zone; thus the mean-field alpha effect is also a stochastic process. Motivated by these facts, we introduce randomness in the mean-field alpha by setting $S_\odot = S_{\text{base}} + S_{\text{fluc}}\sigma(t, \tau_{corr})$. Here $\sigma$ is a uniform random number lies between -1 and +1. We set the correlation time $\tau_{corr}$, in a way such that at least 10 fluctuations are there within a single solar cycle. We run our simulations with a 60% fluctuation in the Babcock-Leighton mechanism and different level of fluctuations in the mean-field alpha effect. Figures 4 and 5 show that in the case of both advection and diffusion dominated regime, surface radial flux $\Phi_r$ only correlates with the toroidal flux of the next cycle with Spearman correlation coefficients 0.85 and 0.89 respectively. Please note that we use the results of the model with a 60% fluctuation in the Babcock-Leighton mechanism and 50% fluctuation in the mean-field alpha effect to generate Figure 4 and 5. In our model, the presence of the mean-field alpha effect reduces the memory of the solar cycle. Table 1 shows that the correlation strength decreases with the increase of mean-field alpha strength. We note that peak toroidal flux of n-th cycle is weakly correlated with the peak radial flux of n-th cycle in the advection dominated regime when we choose $S_\odot = 0.20$ ms$^{-1}$. We also notice that our model gives an unstable solution in the advection dominated regime if we increase the value of mean-field alpha ($S_\odot$) more than 0.20 ms$^{-1}$. Table 2 confirms the validity of our result for different levels of fluctuations in the mean-field alpha. More importantly, we get one cycle memory even for steady mean-field alpha (no fluctuation in mean-field alpha) for both advection and diffusion dominated region.

We do not consider turbulent pumping in our simulations. However, Karak & Nandy (2012) indicate that turbulent pumping can impact the memory of the solar cycle. Karak & Nandy (2012) found that in the case of both advection and diffusion dominated regimes, surface radial flux $\Phi_r$ only correlates with the next cycle toroidal flux if they introduce turbulent pumping in their model. We also find that surface radial flux $\Phi_r$ correlates only with the toroidal flux $\Phi_\phi$ff of the next cycle when we also consider turbulent pumping in our simulation. Thus, turbulent
pumping also has an impact on the memory of the solar cycle. However, Karak & Nandy (2012) did not consider the mean-field alpha effect in their dynamo simulation with turbulent pumping.

### 4. Summary

In summary, we have demonstrated the importance of the mean-field alpha effect on the memory of the solar cycle. We find that solar cycle memory is only limited to one cycle. This result supports the earlier suggestions (Schatten et al. 1978; Solanki et al. 2002; Yeates et al. 2008; Karak & Nandy 2012; Muñoz-Jaramillo et al. 2013; Sanchez et al. 2014). Our analysis shows that the presence of a mean-field alpha effect in the dynamo model can shed light on the existing discrepancy between the memory obtained from advection and diffusion-dominated Babcock-Leighton solar dynamo models.

Previously, it has been shown that the relative efficiency between different flux transport mechanisms governs the memory of the solar cycle. In the model, which considers the Babcock-Leighton mechanism as the only poloidal field generation mechanism, the time scale of transporting the poloidal flux from surface to the base of the convection zone basically governs the memory of the solar cycle. Even for a modest radial turbulent pumping speed of 2 m s\(^{-1}\), the time scale for transporting the poloidal flux from surface to the base of the convection zone is only 3.4 year. The introduction of turbulent pumping in the flux transport dynamo model makes the model pumping dominated in both advection and diffusion dominated scenarios, eventually impacting the memory of the solar cycle. In the situation where turbulent pumping dominates the vertical flux transport mechanism, the relative efficiency of other flux transport mechanisms namely, meridional circulation and turbulent diffusion is less significant.

In our model, the poloidal field generation takes place due to the combined effect of both the Babcock-Leighton mechanism and the mean-field alpha effect. As poloidal field generation takes place throughout the convection zone due to mean-field alpha effect, this poloidal flux can amplify and become magnetically buoyant at the base of the convection zone quickly. Thus, the presence of the mean-field alpha effect reduces solar cycle memory into one cycle in both advection-dominated and diffusion-dominated dynamo models. We also find a moderate decrease in the correlation between the radial flux at the cycle minima and the toroidal flux at the maxima of the next cycle when we consider mean-field alpha effect in our model.

### 5. Appendix: Modeling the Babcock-Leighton mechanism

Poloidal field generation at the surface takes place due to the decay and dispersal of the bipolar sunspot regions as well as ephemeral regions. We model the poloidal field generation mechanism at the surface due to ephemeral regions following...
where we take $r_1 = 0.95 R_0$, $r_2 = R_0$, $d_1 = d_2 = 0.15 R_0$, and $B_{dp} = 10^7 G$. $S_1$ controls the amplitude of the poloidal field generation mechanism due to ephemeral regions. We take $S_1 = 0.13$ in our model.

In our model, we follow the prescription of the double-ring algorithm proposed by Durney (1997) to model the active region. In this algorithm, we define the $\phi$ component of potential vector $A$ associated with the active region as:

$$A_{\omega}(r, \theta) = K_1 A(\Phi) F(r) G(\theta),$$

(8)

where constant $K_1$ ensures the super-critical dynamo solution and $A(\Phi)$ defines the strength of the ring doublet. $F(r)$ is defined as:

$$F(r) = \begin{cases} 0 & r \leq R_0 \quad \text{or} \quad r \geq R_0 - R_{ar}, \\ \frac{1}{2} \sin^2 \left[ \frac{1}{2 \pi} \int_0^\theta \left( \sigma_1 - \frac{\chi}{2} \right) d\theta \right] & R_0 < r < R_0 - R_{ar}, \end{cases}$$

(9)

where $R_0$ is the solar radius and penetration depth of the active region is $R_{ar} = 0.85 R_0$. $G(\theta)$ in the integral form is defined as:

$$G(\theta) = \frac{1}{\sin \theta} \int_0^\theta [B_+ \left( \sigma_1 - \frac{\chi}{2} \right) \sin(\theta) \sin(\theta) d\theta],$$

(10)

where $B_+$ represents the strength of positive (negative) ring:

$$B_+(\theta) = \begin{cases} 0 & \theta < \theta_{ar} + \frac{\chi}{2} - \frac{\Lambda}{2}, \\ \frac{1}{2} \left( 1 + \cos \left( \frac{2 \chi}{\Lambda} (\theta - \theta_{ar} + \frac{\chi}{2}) \right) \right) & \theta_{ar} + \frac{\chi}{2} - \frac{\Lambda}{2} \leq \theta < \theta_{ar} + \frac{\chi}{2} + \frac{\Lambda}{2}, \\ 0 & \theta \geq \theta_{ar} + \frac{\chi}{2} + \frac{\Lambda}{2}. \end{cases}$$

where $\theta_{ar}$ is colatitude of the double ring emergence and the diameter of each polarity of the double ring is $\Lambda$. We take the latitudinal distance between the centers of the double ring as $\chi = \arcsin[\sin(\beta) \sin(\Delta_\theta)]$, where $\Delta_\theta$ is the angular distance between polarity centers and $\beta$ is the active region tilt angle. We take $\Lambda$ and $\Delta_\theta$, as 6° for our model.

**Regenerating the Poloidal Field:**

To recreate the poloidal field in the solar surface, first, we choose randomly a latitude from both the hemispheres where the toroidal field exceeds the buoyancy threshold at the bottom of the convection zone. Then, we use a non-uniform probability distribution function to ensure that randomly chosen latitude always remains within the observed active region belt. Next, we calculate the tilt of the corresponding active region following the expression prescribed in [Fan et al. (1994)].

$$\beta \propto Q_0^{1/4} B_0^{-5/4} \sin(\lambda),$$

(12)

where $Q_0$ is the toroidal field associated flux, $B_0$ is the local field strength and $J$ is the chosen latitude for the ring emergence. The constant that appear in equation 12 is fixed in a way such that the tilt angle lies between $3^\circ$ and $12^\circ$. Next, we remove a part of the magnetic field with the same angular size of the emerging active region from this toroidal ring. We have reduced the magnetic field strength of the toroidal ring from which active region erupts. We set the toroidal field strength in a way such that the energy of the full toroidal ring with the new magnetic field is equal to the energy of the partial toroidal ring with the old magnetic field (after removing a chunk of the magnetic field). Finally, we place the ring doublets with these calculated properties at the near-surface layer at the chosen erupted latitude. Figure 1 (right side-top panel) shows the poloidal field line contours associated with the double ring in both hemispheres for one particular time step.

Acknowledgements. We thank Antione Strugarek and Eric Buchlin for reading this manuscript and providing useful suggestions. We thank the Université Paris-Saclay (IRS SPACEOBS grant), ERC Synergy grant #810218 (Whole Sun project), INSU/PSMN and CNES Solar Orbiter for supporting this research. The Center of Excellence in Space Sciences India (CESSI) is supported by the Ministry of Human Resource Development, Government of India.

**References**

Balogh, H. W. 1961, ApJ, 133, 572

Blouwink, P. & Nandy, D. 2018, Nature Communications, 9, 5209

Brun, A. S. 2007, Astronomische Nachrichten, 328, 1137

Brun, A. S. & Browning, M. K. 2017, Living Reviews in Solar Physics, 14, 4

Cameron, R. H. & Schüssler, M. 2016, A&A, 591, A46

Charbonneau, P. 2005, Living Reviews in Solar Physics, 2, 2

Choudhuri, A. R., Chatterjee, P., & Jiang, J. 2007, Physical Review Letters, 98, 131103

Choudhuri, A. R., Chatterjee, P., & Nandy, D. 2004, ApJ, 615, L57

Choudhuri, A. R. & Karak, B. B. 2012, Physical Review Letters, 109, 171103

Dikpati, M. & Charbonneau, P. 1999, ApJ, 518, 508

Dikpati, M., de Toma, G., & Gilman, P. A. 2006, Geophys. Res. Lett., 33, L05102

Do Cao, O. & Brun, A. S. 2011, Astronomische Nachrichten, 332, 907

Durney, B. R. 1997, ApJ, 486, 1065

Fan, Y., Fisher, G. H., & McClymont, A. N. 1994, ApJ, 436, 907

Hathaway, D. H. 2009, Space Sci. Rev., 144, 401

Hathaway, D. H., Gilman, P. A., Harvey, J. W., et al. 1996, Science, 272, 1306

Haza, G., Choudhuri, A. R., & Miesch, M. S. 2017, ApJ, 835, 39

Haza, G., Karak, B. B., & Choudhuri, A. R. 2014a, ApJ, 782, 93

Haza, G. & Miesch, M. S. 2018, ApJ, 864, 110

Haza, S. & Nandy, D. 2013, in Astronomical Society of India Conference Series, Vol. 10, Astronomical Society of India Conference Series 327, 884

Käpylä, P. J., Korpi, M. J., Essendriev, M., & Stix, M. 2006a, A&A, 455, 401

Käpylä, P. J., Korpi, M. J., & Tuominen, I. 2006b, Astronomische Nachrichten, 327, 884

Karak, B. B. & Miesch, M. 2017, ApJ, 847, 69

Karak, B. B. & Miesch, M. 2018, ApJL, 860, L26

Karak, B. B. & Nandy, D. 2012, ApJ, 761, L13

Komm, R. W., Howard, R. F., & Harvey, J. W. 1993, Sol. Phys., 147, 207

Komm, R. W., Howard, R. F., & Harvey, J. W. 1995, Sol. Phys., 158, 213

Kumar, R., Jouve, L., & Nandy, D. 2019, A&A, 623, A54

Kömür, R. W., Howard, R. F., & Harvey, J. W. 1993, Sol. Phys., 147, 207

Leighton, R. B. 1969, ApJ, 156, 1

Lembrer, A., Charbonneau, P., & Carignan-Dugas, A. 2015, ApJ, 810, 78

Longcope, D. C. & Choudhuri, A. R. 2002, Sol. Phys., 205, 63

Loureiro, E. H., Hughes, D. W., & Tobias, S. M. 2008, MNRAS, 391, 467

Mason, J., Hughes, D. W., & Tobias, S. M. 2008, MNRAS, 391, 467

McClintock, B. H. & Norton, A. A. 2016, ApJ, 818, 7

Miesch, M. S., Brun, A. S., DeRosa, M. L., & Toomre, J. 2008, ApJ, 673, 557

Miesch, M. S., Featherstone, N. A., Rempel, M., & Trampedach, R. 2012, ApJ, 757, 128

Moffatt, H. K. 1978, Magnetic field generation in electrically conducting fluids

Moore, R. L., & Marsch, K. 2000, J. Geophys. Res., 105, 11

Nandy, D. & Choudhuri, A. R. 2002, Science, 296, 1671

Nandy, D., Blouwink, P., & Miesch, M. S. 2011, Nature, 471, 80
Ossendrijver, M., Stix, M., Brandenburg, A., & Rüdiger, G. 2002, A&A, 394, 735
Parker, E. N. 1955, ApJ, 121, 491
Parker, E. N. 1979, Cosmical magnetic fields. Their origin and their activity
Passos, D., Charbonneau, P., & Beaudoin, P. 2012, Sol. Phys., 279, 1
Passos, D., Nandy, D., Hazra, S., & Lopes, I. 2014, A&A, 563, A18
Pesnell, W. D. 2008, Sol. Phys., 252, 209
Petrovay, K. 2010, Living Reviews in Solar Physics, 7, 6
Pipin, V. V. & Kosovichev, A. G. 2011, ApJ, 741, 1
Pipin, V. V., Zhang, H., Sokoloff, D. D., Kuzanyan, K. M., & Gao, Y. 2013, MNRAS, 435, 2581
Priest, E. 2014, Magnetohydrodynamics of the Sun
Racine, É., Charbonneau, P., Ghizaru, M., Bouchat, A., & Smolarkiewicz, P. K. 2011, ApJ, 735, 46
Rajaguru, S. P. & Antia, H. M. 2015, ApJ, 813, 114
Sanchez, S., Fourmer, A., & Aubert, J. 2014, ApJ, 781, 8
Schatten, K. H., Scherrer, P. H., Svalgaard, L., & Wilcox, J. M. 1978, Geophys. Res. Lett., 5, 411
Schrijver, C. J., Kauristie, K., Aylward, A. D., et al. 2015, Advances in Space Research, 55, 2745
Snodgrass, H. B. & Dailey, S. B. 1996, Sol. Phys., 163, 21
Solanki, S. K., Krivova, N. A., Schüssler, M., & Fligge, M. 2002, A&A, 396, 1029
Steenbeck, M., Krause, F., & Rädler, K. H. 1966, Zeitschrift Naturforschung Teil A, 21, 369
van Ballegooijen, A. A. & Mackay, D. H. 2007, ApJ, 659, 1713
Švanda, M., Brun, A. S., Roudier, T., & Jouve, L. 2016, A&A, 586, A123
Yeates, A. R. & Muñoz-Jaramillo, A. 2013, MNRAS, 436, 3366
Yeates, A. R., Nandy, D., & Mackay, D. H. 2008, ApJ, 673, 544
Zhao, J. & Chen, R. 2016, Asian Journal of Physics, 25, 325
Fig. 1. Top Panel: The left side shows the radial profile of mean-field $\alpha$-coefficient and the Babcock-Leighton source term due to ephemeral regions and the right-hand side shows the poloidal field line contours obtained from the double ring algorithm in the northern and southern hemispheres respectively. Middle Panel: Butterfly diagram generated from our simulation in diffusion dominated region. Bottom Panel: Typical variation of $B^2_\odot$ at the base of the solar convection zone with time.
Fig. 2. Cross-correlation between the radial flux ($\Phi_r$) of cycle $n$ and the toroidal flux ($\Phi_{\text{tor}}$) of cycle $n$, $n+1$, $n+2$, $n+3$ in the advection-dominated region. The poloidal field is generated only due to the Babcock-Leighton mechanism. Spearman correlation coefficients with significance level are inscribed inside the plots. Here, we consider 60% fluctuations in the Babcock-Leighton mechanism.
Fig. 3. Cross-correlation between the radial flux ($\Phi_r$) of cycle $n$ and the toroidal flux ($\Phi_{tor}$) of cycle $n$, $n+1$, $n+2$, $n+3$ in the diffusion-dominated region. The poloidal field is generated only due to the Babcock-Leighton mechanism. Spearman correlation coefficients with significance level are inscribed inside the plots. Here, we consider 60% fluctuations in the Babcock-Leighton mechanism.
Fig. 4. Cross-correlation between the radial flux ($\Phi_r$) of cycle $n$ and the toroidal flux ($\Phi_{tor}$) of cycle $n$, $n+1$, $n+2$, $n+3$ in the advection-dominated region. The poloidal field is generated due to both the Babcock-Leighton mechanism and mean-field alpha effect. Spearman correlation coefficients with significance level are given inside the plots. Here, we consider 60% fluctuations in the Babcock-Leighton mechanism and 50% fluctuation in the mean field alpha. Value of mean-field alpha constant factor $S_{\odot}$ is 0.14.
**Fig. 5.** Cross-correlation between the radial flux ($\Phi_r$) of cycle $n$ and the toroidal flux ($\Phi_{tor}$) of cycle $n$, $n+1$, $n+2$, $n+3$ in the diffusion-dominated region. Poloidal field is generated due to both the Babcock-Leighton mechanism and mean-field alpha effect. Spearman correlation coefficients with significance level are given inside the plots. Here, we consider 60% fluctuations in the Babcock-Leighton mechanism and 50% fluctuation in the mean field alpha. Value of mean-field alpha constant factor $S_\alpha$ is 0.14.