Grid voltage sensor fault-tolerant control for single-phase two-level PWM rectifier

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Abstract
The main purpose of this work is to address the problem of robust grid voltage sensor fault-tolerant control for a single-phase two-level rectifier. First, considering a rectifier with disturbances and grid voltage sensor fault, the standard switched system model is constructed. Based on identical transformation, the original system is converted into an augmented system. In the augmented system, the information of the grid voltage sensor fault is included in the state vector. Second, an unknown input observer is designed for the augmented system for sensor fault estimation. The observer error is totally robust against independent deterministic disturbances. In addition, the effect of the disturbance caused by unknown disturbances or a modelling error is minimised with respect to a prescribed $H_{\infty}$ performance index. Finally, the sensor fault-tolerant control is realised by fault compensation technique, which protects the rectifier from serious performance degradation in the presence of the grid voltage sensor fault. The proposed fault-tolerant control strategy requires no supplementary hardware and all the inputs involved in the algorithm are directly available. Simulation and experimental results are exhibited to illustrate the capability of the proposed method to tolerate the grid voltage sensor fault.

1 | INTRODUCTION

Due to advantages in performance, reliability, low loss etc., AC electrical drive systems have been broadly implemented, with applications in high-speed railway traction systems for decades [1–4]. A converter, as one of the core components in a traction system, is commonly used for electric energy conversion and transmission [5]. Figure 1 shows a classic topology of a converter used in electrical railway drive systems. The converter contains three parts: a catenary-side rectifier, a DC-link circuit, and a load-side inverter. As the most front-end device of the converter, the rectifier guarantees unity power factor operation [6]. Consequently, the reliability of the rectifier thoroughly affects the performance of the whole electrical traction system [7]. Since sensors’ and insulated-gate bipolar transistors’ failure can lead to severe performance deterioration in the rectifier, great efforts have been made for the problem of fault detection and fault tolerant control (FTC) [8–11].

For the issue of the rectifier sensor FTC, many approaches have been investigated in recent years, most of which are based on fault detection and isolation (FDI) [12–17]. For instance, the authors have addressed DC-link voltage sensor FTC via the Luenberger observer-based FDI technique [13]. Moreover, the Luenberger observer is adopted in [12] to realise FDI for the rectifier with grid-side current sensor fault as well as DC-link voltage sensor fault. A sliding mode observer-based FDI approach with a novel description for switching variables has been investigated to solve the sensor FTC problem in [14]. The problem of simultaneous detection and isolation of the DC-link voltage sensor fault and transistors’ fault is successfully addressed in [15]. It is worth pointing out that the key idea in the methods presented in [12–15] is the residuals’ generation and evaluation. The residuals are commonly achieved by calculating the difference between observer outputs and rectifier outputs. The control loop reconfiguration is implemented when those residuals exceed the corresponding pre-
selected thresholds to realise the sensor FTC. However, considering that the observer is a closed-loop system, that is, the observer outputs will be inevitably affected by the rectifier outputs when some sensors are faulty, the system may experience performance degradation after the control loop reconfiguration. Moreover, the pre-selected fixed thresholds may result in detection delay and missed diagnosis for incipient sensor faults. To overcome such disadvantages, the authors have developed an incipient fault isolation strategy for the three-level rectifier with DC-link voltage sensor fault, which is based on the synthesis of sliding surface design and non-linear parameter estimation [16]. In addition, a fault estimation (FE)-based sensor FTC approach is developed in [17], which successfully compensates the effect of sensor faults by using the faults’ information, and thus, achieves good FTC performance.

In practice, it is noticed that typically three sensors are installed to guarantee the normal operation of the traction rectifier, including grid voltage sensor, catenary current sensor and DC-link voltage sensor [18]. However, the existing methods mainly focus on the catenary current sensor faults and the DC-link voltage sensor faults. The design for traction rectifiers to tolerate grid voltage sensor fault has not received much research attention yet, not to mention the case where robustness is also taken into consideration in the FTC design. Therefore, a robust FTC method that is capable of tolerating grid voltage sensor faults is of great significance, which considerably contributes to an increase in the operation safety of traction systems.

On another theoretical research forefront, it has been known that FE is an attractive alternative to traditional FDI, since the fault information is utilised to reject the fault effect directly [19]. Moreover, FE also leads to considerable simplification in the corresponding FTC design, since the detection and isolation roles are essentially embodied in FE [20]. Thus complex FDI roles and control reconfiguration mechanisms are obviated, as shown in Figure 2. Recently, FE has received considerable attention due to the potential applications in the industry. Fruitful observer-based FE methods have been investigated for different kinds of systems, for example, the unknown input observer (UIO) [21–23], sliding mode observer [24, 25], adaptive observer [26, 27], and the high gain observer [28]. Among these FE approaches, the UIO-based FE technique is proven to be an effective strategy to eliminate the disturbances via decoupling [29]. In addition, it has been widely implemented, such as in autonomous spacecraft [30], wind turbines [31], and anaerobic bioreactors [32].

Motivated by the above discussion, a robust grid voltage sensor FTC approach for the rectifier with application in railway traction systems is developed in this work. The proposed method is based on the UIO-based FE and fault compensation. The observer error is designed to be exponentially stable with a prescribed $H_{\infty}$ performance index. Based on the fast and robust FE, the sensor fault can be well compensated through the fault compensation. Thus, the high performance of the rectifier can be achieved after the proposed grid voltage sensor FTC approach is applied.

The most significant contribution of this work is the robust fault-tolerant design for traction rectifiers that are subjected to grid-side voltage sensor faults, which can be further summarised in the following aspects: (1) The standard switched-system model of the rectifier is constructed with the switching signal linked to the operation of switching devices; (2) By regarding the grid voltage sensor fault as the ‘actuator fault’ in the switched system, a UIO is developed to simultaneously achieve the accurate estimation of the state and grid voltage sensor fault; (3) In the UIO design, the robustness is also taken into consideration so that the proposed FTC method is insensitive to the parameter variation.

The rest of this work is organised as follows: The standard switched-system model of the rectifier is constructed in Section 2. The UIO-based FE and the fault-tolerant scheme of the grid voltage sensor are developed in Section 3. The simulation experiment is performed in Section 4.1. Section 5 presents the conclusions.

Notations: $n \times n$ dimensional identity matrix is denoted by $I_n$, $0_{a \times b}$ represents a $a \times b$ zero matrix; the transpose matrix and inverse matrix of $A$ are denoted by $A^T$ and $A^{-1}$, respectively; $A > 0$ means the entries of $A$ are positive.

2 | MODELING

In this section, the standard switched-system model for the rectifier subjected to the grid voltage sensor faults and disturbances is constructed, which facilitates the following FE design:
2.1 Rectifier model

The topology of the electric drive rectifier is shown in Figure 3, where $L_N$, $R_N$, $C$, $R_L$, $u_N$, $i_N$, and $U_{dc}$ are the traction winding leakage inductance, the input resistance, the DC-link capacitance, the load-equivalent resistance, the grid voltage, the catenary current, and the DC-link voltage, respectively.

According to Figure 3, the mathematical description of the rectifier is given by

$$
\begin{align*}
    u_N(t) &= L_N \frac{di_N(t)}{dt} + R_N i_N(t) + u_{ab}(t) \\
    i_{dc}(t) &= i_{load}(t) + i_C(t) \\
    i_C(t) &= C \frac{dU_{dc}(t)}{dt}
\end{align*}
$$

(1)

Furthermore, the switching functions are defined as

$$
S_A = \begin{cases}
1, & S_1 \text{ or } D_1 \text{ On} \\
0, & S_2 \text{ or } D_2 \text{ On}
\end{cases}
$$

$$
S_B = \begin{cases}
1, & S_3 \text{ or } D_3 \text{ On} \\
0, & S_4 \text{ or } D_4 \text{ On}
\end{cases}
$$

(2)

Given the complementary switching rules of the transistors in the same leg of the rectifier, one has

$$
\begin{align*}
    u_{ab} &= (S_A - S_B) u_{dc} \\
    i_{dc} &= (S_A - S_B) i_N
\end{align*}
$$

(3)

Substituting the switching functions (2) and (3) with system (1) yields

$$
\dot{x} = Ax + Bu
$$

(4)

where

$$
A = \begin{bmatrix}
    \frac{R_N}{L_N} & -\frac{S_A - S_B}{L_N} \\
    \frac{S_A - S_B}{C} & 0
\end{bmatrix}, \quad B = \begin{bmatrix}
    \frac{1}{L_N} & 0 \\
    0 & \frac{1}{C}
\end{bmatrix}
$$

$$
x = \begin{bmatrix}
    i_N \\
    U_{dc}
\end{bmatrix}, \quad u = \begin{bmatrix}
    u_N \\
    i_{load}
\end{bmatrix}
$$

Figure 3. Topology of electric drive rectifier

Note that the grid voltage $u_N$ is one of the control input signals instead of the system outputs in the rectifier model (4); thus, grid voltage sensor faults are actually "actuator faults" from the perspective of the fault diagnosis theory. However, the catenary current sensor faults and DC-link voltage sensor faults can be regarded as 'sensor faults' since the catenary current $i_N$ and the DC-link voltage $U_{dc}$ are the system outputs. Therefore, the FE design of grid voltage sensor faults is completely different from that of catenary current sensor faults and DC-link voltage sensor faults (see FE design for catenary current sensor faults and DC-link voltage sensor faults in [17]).

2.2 Switched system model

For the purpose of constructing the standard switched-system model for the rectifier, a piecewise constant function $\sigma(t) : \mathbb{R} \rightarrow \mathbb{N}$ which represents the switching signal of the switched system is introduced. $\sigma(t) = i$, which denotes the $i$th subsystem is activated. Especially according to the different values $S_A - S_B$ may take, the corresponding $\sigma(t)$ is defined in Table 1.

Given the switching signal $\sigma(t)$, the rectifier model (4) can be rewritten as

$$
\dot{x} = A_\sigma x + B_\sigma u
$$

(5)

where $x$, $u$ are given in system (4) and

$$
A_1 = \begin{bmatrix}
    \frac{R_N}{L_N} & 1 \\
    0 & -\frac{1}{C}
\end{bmatrix}, \quad A_2 = \begin{bmatrix}
    \frac{R_N}{L_N} & 1 \\
    -\frac{1}{C} & 0
\end{bmatrix},
$$

$$
A_3 = \begin{bmatrix}
    \frac{R_N}{L_N} & 0 \\
    0 & -\frac{1}{C}
\end{bmatrix}, \quad B_\sigma = \begin{bmatrix}
    0 \\
    0
\end{bmatrix}
$$

Further, since $i_N$ and $U_{dc}$ are directly measurable, the switched-system model of the rectifier is given by

$$
\begin{align*}
    \dot{x} &= A_\sigma x + B_\sigma u \\
    y &= C_\sigma x
\end{align*}
$$

(6)

where $C_\sigma = I_2$.

Considering system (6) is subjected to grid voltage sensor faults and disturbances, one has

$$
\begin{align*}
    \dot{x} &= A_\sigma x + B_\sigma u + D_\sigma d + W_\sigma \omega + F_\sigma f \\
    y &= C_\sigma x
\end{align*}
$$

(7)

| $\sigma(t)$ in the rectifier model |
|-----------------------------------|
| $S_A - S_B$ | 1 | -1 | 0 |

Table 1
where $d$, $\omega$, and $f$ stand for the deterministic disturbances, the bounded unknown vector caused by either unknown disturbances or modelling errors, and the grid voltage sensor faults, respectively; $D_{1\sigma}$, $W_\sigma$, and $F_\sigma$ denote the distribution matrices with respect to $d$, $\omega$, and $f$, respectively.

**Assumption 1** $f$ is a derivable signal and the $q$-order derivative of $f$ is identically equal to zero, that is, $f^{(q)}(\cdot) \equiv 0$.

The deterministic disturbance $d$ denotes those disturbances for which the effect on the system is known, that is, the disturbance distribution matrix $D_\sigma$ can be calculated. The unknown vector $\omega$ stands for the disturbances caused by either unknown disturbances or modelling errors. In addition, it is noticed that the distribution matrix $D_\sigma$ can be set as a zero matrix with appropriate dimension if the rectifier does not experience the deterministic disturbances $d$ in practice.

## 3 | DESIGN OF UIO-BASED FE AND FTC

In this section, an augmentation transformation for the rectifier model is performed to construct the augmented system. Then, an UIO is introduced to accomplish the estimation for the grid voltage sensor fault. Finally, the fault compensation technique is implemented to achieve the grid voltage sensor FTC of the rectifier.

### 3.1 | UIO design

A preliminary step of the FE design is to introduce an identical augmentation transformation for system (7). We first denote the following augmented vectors and matrices with $q$ being set as 4:

$$
\bar{A}_\sigma = 
\begin{bmatrix}
  A_\sigma & F_\sigma & 0_{2\times1} & 0_{2\times1} \\
  0_{1\times1} & 0_{1\times1} & I_1 & 0_{1\times1} \\
  0_{1\times1} & 0_{1\times1} & 0_{1\times1} & I_1 \\
  0_{1\times1} & 0_{1\times1} & 0_{1\times1} & 0_{1\times1} \\
  0_{1\times1} & 0_{1\times1} & 0_{1\times1} & 0_{1\times1} \\
  \end{bmatrix},
$$

$$
\bar{x}(t) = \begin{bmatrix}
  x(t) \\
  \dot{x}(t) \\
  \ddot{x}(t) \\
  \vdots \\
  \dddot{x}(t) \\
  \end{bmatrix},
\tilde{B}_\sigma = 
\begin{bmatrix}
  B_\sigma \\
  0_{1\times2} \\
  0_{1\times2} \\
  \vdots \\
  0_{1\times2} \\
  \end{bmatrix},

\bar{D}_\sigma = 
\begin{bmatrix}
  D_\sigma \\
  0_{1\times1} \\
  0_{1\times1} \\
  \vdots \\
  0_{1\times1} \\
  \end{bmatrix},
$$

Then we construct the augmented system which is given by

$$
\begin{align*}
\dot{\bar{x}}(t) &= \bar{A}_\sigma \bar{x}(t) + \tilde{B}_\sigma u(t) + \bar{D}_\sigma d(t) + W_\sigma \omega(t) + IF^{(4)}(t) \\
y(t) &= C_\sigma \bar{x}(t)
\end{align*}
$$

(8)

Since the grid voltage sensor fault $f$ is included in the augmented state vector $\bar{x}(t)$, if there exists a stable UIO for the system (8), then the estimation for $f$ can be achieved.

For the augmented system (8), the following UIO is introduced to realize the state vector reconstruction:

$$
\begin{align*}
\dot{\bar{\theta}}(t) &= N_\sigma \bar{\theta}(t) + G_\sigma u(t) + (L_{1\sigma} + L_{2\sigma}) y(t) \\
\dot{\bar{y}}(t) &= \bar{\theta}(t) + H_\sigma y(t)
\end{align*}
$$

(9)

where $\bar{\theta}(t)$ is the transitional variable, $\bar{y}(t)$ is the estimation value of $\bar{x}(t)$; matrices $N_\sigma$, $G_\sigma$, $L_{1\sigma}$, $L_{2\sigma}$ and $H_\sigma$ are the observer gains to be designed.

Define the estimation error vector $e(t) = \bar{x}(t) - \bar{y}(t)$. Considering system (9), the error is derived as follows:

$$
e(t) = \bar{x}(t) - (\bar{\theta}(t) + H_\sigma y(t)) = M_\sigma \bar{x}(t) - \bar{\theta}(t)
$$

(10)

where $M_\sigma = I_6 - H_\sigma C_\sigma$.

From (10), the error dynamic can be achieved.

$$
\dot{e}(t) = M_\sigma \dot{\bar{x}}(t) - \dot{\bar{\theta}}(t)
= M_\sigma \bar{A}_\sigma \bar{x}(t) + M_\sigma \tilde{B}_\sigma u(t) + M_\sigma \bar{D}_\sigma d(t) + M_\sigma W_\sigma \omega(t) - (L_{1\sigma} + L_{2\sigma}) y(t)
= (M_\sigma \bar{A}_\sigma - L_{1\sigma} C_\sigma) e(t) + (M_\sigma \tilde{B}_\sigma - G_\sigma) u(t)
+ (M_\sigma \bar{D}_\sigma - L_{2\sigma}) y(t)
+ M_\sigma W_\sigma \omega(t)
$$

(11)

It is noticed that in the error dynamic (11), $M_\sigma \bar{A}_\sigma$, $\bar{B}_\sigma$, $C_\sigma$, $D_\sigma$, and $W_\sigma$ are known matrices; $N_\sigma$, $G_\sigma$, $L_{1\sigma}$, $L_{2\sigma}$, and $H_\sigma$ are the parameter matrices to be designed. Further, we set

$$
\begin{align*}
M_\sigma \bar{A}_\sigma - L_{1\sigma} C_\sigma - N_\sigma &= 0 \\
(M_\sigma \bar{A}_\sigma - L_{1\sigma} C_\sigma) H_\sigma - L_{2\sigma} &= 0 \\
N_\sigma \tilde{B}_\sigma - G_\sigma &= 0 \\
M_\sigma \bar{D}_\sigma &= 0
\end{align*}
$$

(12)

Thus, the error system (10) becomes

$$
\dot{e}(t) = (M_\sigma \bar{A}_\sigma - L_{1\sigma} C_\sigma) e(t) + M_\sigma W_\sigma \omega(t)
$$

(13)

### 3.2 | Stability analysis

The stability analysis for the error system (10) is given in the following theorem.
Theorem 1. Given a decay rate $\beta > 0$, scalar $\gamma > 0$, if the matrices’ conditions in (12) are satisfied and there exist a symmetric positive definite matrix $P \in \mathbb{R}^{n \times n}$, Matrix $L \in \mathbb{R}^{n \times 2}$ such that the following condition holds:

$$
\begin{bmatrix}
\Omega

\end{bmatrix}
\begin{bmatrix}
P M_s W_s

- \gamma I
\end{bmatrix} < 0 \tag{14}
$$

where $\Omega = (M_s \bar{A}_s)^T P + PM_s \bar{A}_s - C_s^T L^T - L C_s + \beta P + I$ and $L_{1s} = P^{-1} L$, then the error system (10) is exponentially stable with the $H_{\infty}$ performance index $\gamma$.

Proof. (1) We first set $\omega(t) = 0$ and chose the Lyapunov function given by

$$
V(t) = e^T(t) P e(t).
$$

Before the stability analysis, the switching sequence is defined as $\{t_0, t_1, t_2, \ldots, t_k, \ldots\}$ where $t_k$ stands for the time when the system switches between several subsystems. Further, considering $V(t) + \beta V(t)$ when $t \in [t_k, t_{k+1})$, one has

$$
\dot{V}(t) + \beta V(t) = e^T(t) P e(t) + e^T(t) Pe(t) + \beta e^T(t) Pe(t)
= e^T(t) \left[ (M_s \bar{A}_s - L_{1s} C_s)^T P + P (M_s \bar{A}_s - L_{1s} C_s) + \beta P \right] e(t).
$$

Condition (14) guarantees

$$
(M_s \bar{A}_s - L_{1s} C_s)^T P + P (M_s \bar{A}_s - L_{1s} C_s) + \beta P < 0. \tag{16}
$$

Substituting (16) with (15), we know

$$
\dot{V}(t) + \beta V(t) < 0, \quad t \in [t_k, t_{k+1}) \tag{17}
$$

which further implies

$$
V(t_k^+ \leq V(t) \leq V(t_k^-). \tag{18}
$$

and

$$
V(t) \leq e^{-\beta(t-t_k^-)} V(t_k^+). \tag{19}
$$

Owing to the fact that $t - t_k \leq t_{k+1} - t_k$, one has

$$
V(t) \leq e^{-\beta(t_{k+1} - t_k)} V(t_{k+1}^+). \tag{20}
$$

Considering $V(t_{k+1}^+) = V(t_{k+1}^+)$, the inequality together with (20) implies

$$
V(t_{k+1}^+) \leq e^{-\beta(t_{k+1} - t_k)} V(t_k^+). \tag{21}
$$

Considering any time instant $t$, let $N$ represent the total switching number of discontinuities of $\sigma(t)$ on $[0, t]$. Clearly, when $t \in [0, t]$ $t_N$ is the last switching time. Without loss of generality, and presuming that $t \neq t_N$, one has

$$
V(t^+) = e^{-\beta(t-t_N)} V(t_N^-). \tag{22}
$$

Iterate (21) from $t = t_{N-1}$ to $t = 0$. Thus, one has

$$
V(t_N^-) \leq e^{-\beta(t_{N-1} - t_N)} V(t_{N-1}^-) \leq e^{-\beta(t_{N-2} - t_{N-1})} e^{-\beta(t_{N-1} - t_{N-2})} V(t_{N-2}^-) \leq e^{-\beta(t_{N-1} - t_{N-2})} e^{-\beta(t_{N-2} - t_{N-3})} \ldots e^{-\beta(t_0 - t_1)} V(0) = e^{-\beta N} V(0). \tag{23}
$$

Substituting inequality (23) with (22) yields

$$
V(t) \leq e^{-\beta t} V(0). \tag{24}
$$

Since $\beta > 0$, inequality (24) guarantees

$$
V(t) \to 0 \quad \text{as} \quad t \to \infty \tag{25}
$$

which further implies

$$
e(t) \to 0 \quad \text{as} \quad t \to \infty. \tag{26}
$$

Inequality (24) shows that the error $e(t)$ converges exponentially to zero when $\omega(t) = 0$.

(2) In the following, the proof of the error system with the prescribed $H_{\infty}$ performance index in the presence of $\omega(t)$ is given.

We define

$$
\Gamma(t) = \int_0^T \left( e^T(t) y(t) - \gamma^2 \omega^T(t) \omega(t) + \beta V(t) \right) dt. \tag{27}
$$

Based on (27), one has

$$
\Gamma(t) = \int_0^T \left( e^T(t) y(t) - \gamma^2 \omega^T(t) \omega(t) + \beta V(t) \right) \left( + \dot{V}(t) \right) dt - \int_0^T V(t) \dot{V}(t) dt
= \int_0^T \left( e^T(t) \dot{\omega}(t) + 2 e^T(t) P N_s \bar{D_s} \omega(t) - \gamma^2 \omega^T(t) \omega(t) \right) dt - \int_0^T V(t) \dot{V}(t) dt. \tag{28}
$$

Given the initial condition $e(0) = 0$, it yields that

$$
\int_0^T \dot{V}(t) dt = e^T(t) P e(t) - e^T(0) P e(0) = V(t) > 0. \tag{29}
$$
The condition (14) together with inequalities (28) and (29) implies
\[ e^T(t)e(t) - \gamma^2 \omega^T(t)\omega(t) + \beta V(t) + \dot{V}(t) < 0. \] (30)

Note that \( \dot{V}(t) + \beta V(t) > 0 \) has been proved in (17). So, we know
\[ e^T(t)e(t) - \gamma^2 \omega^T(t)\omega(t) < 0. \] (31)

From (31), it is clear that the error system (10) has \( H_\infty \) performance index \( \gamma \). Thus, the proof is completed.

### 3.3 FTC design

Different from the control loop reconfiguration involved in FDI-based methods, the FTC system based on FE is realised through fault compensation, since the fault \( f \) has been successfully estimated. Through the proposed UIO, we can reconstruct the grid voltage sensor fault \( \hat{f} \) as
\[ \hat{F}(t) = [0_{1 \times 2} \ 0_{1 \times 1} \ 0_{1 \times 1} \ 0_{1 \times 1}] \bar{x}(t) \] (32)
where \( \hat{F}(t) \) is the estimation of \( f \).

Further, subtracting \( \hat{F} \) from the measured grid voltage \( u_N \) yields
\[ u_{comp} = u_N - \hat{F} \] (33)
where \( u_{comp} \) is the compensated grid voltage.

The compensated grid voltage \( u_{comp} \) is regarded as the new input to the control algorithm, instead of the measured grid voltage \( u_N \), the grid voltage sensor FTC design is thus completed. The diagram of the fault-tolerant system is shown in Figure 4.

**Remark 1** Clearly, the pre-design of the control algorithm that guarantees the normal rectifier operation is out of the scope of this work. Nonetheless, it should be pointed out that the proposed fault tolerant method can be implemented in any kind of controller as long as \( u_N \) is one of the control inputs.

**Remark 2** Note that \( i_{load} \) is one of the control input signals of the proposed UIO (9). However, \( i_{load} \) is typically unmeasurable in the power electric traction system. In fact, \( i_{load} \) can be calculated using the parameters in the inverter or the traction motor (e.g., [5, 16, 33]). Moreover, note that the performance of the proposed UIO is independent of load properties. Thus, in this work, \( i_{load} \) is assumed to be known and the resistor is adopted as the load instead of the inverter-fed motor for simplicity.

### 4 VERIFICATION

The proposed robust grid voltage sensor fault-tolerant control method is verified in this section. The classic dq axis decouple-control algorithm is used to guarantee the unity power factor.
operation as well as the stability of the DC-link voltage. The parameters related to rectifier and control algorithm are given in Table 2. Moreover, the solutions for condition (14) and the gain matrices in (9) are given in Appendix.

In the following section, two cases are considered in order to assess the performance of the proposed fault-tolerant strategy.

**Case 1:** The performance of the proposed grid voltage sensor FTC without considering parameter variation is verified in this case. The grid voltage sensor fault \( f \) is injected, whose mathematical expression is given in (34). In addition, the load variation given in Table 3 is presented to simulate the lightly loaded condition.

\[
f(t) = \begin{cases} 
0, & 0 \leq t < 5s \\
-6000(t - 5.1) + 300, & 5s \leq t < 5.1s \\
37500(t - 5.2) - 300, & 5.1s \leq t < 5.2s \\
0, & 5.2s \leq t < 5.4s \\
300, & 5.4s \leq t < 8s \\
0, & 8s \leq t < 5s 
\end{cases} \quad (34)
\]

It is worth pointing out that the fault signal \( f \) is designed to be more complex than the common sensor faults that occur in the traction system, reported in [18]. The purpose is to illustrate that the proposed FTC is able to tolerate any kind of fault satisfying \( f(t) = 0 \). In addition, the drifting of the sensor’s measurement value is typically a kind of fault that changes slowly in the time domain. However, in this work the changing rate of the grid voltage sensor fault \( f \) is artificially set as high in order to achieve an obvious fault-tolerant control performance in a short time.

**Case 2:** Subsequently, the robustness of the proposed grid voltage sensor FTC is verified in this case. The grid voltage sensor fault, inductance variation, and the load variation are taken into consideration simultaneously. The variation of \( L_N \) is given in Table 4 while the grid voltage sensor fault and the load variation have already been given in (34) and (3), respectively.

Given that the grid side inductance \( L_N \) is the parameter with the greatest impact on the control performance among all the parameters in Table 2 [6], the variation of \( L_N \) is considered to test the robustness of the proposed FTC method.

Before the FTC performance of the proposed method is verified, the control performance of the rectifier subjected to the grid-side voltage sensor fault \( f \) without the FTC unit involved is tested. As shown in Figure 5, after the occurrence of the sensor fault \( f(t = 5s) \), the measured grid voltage \( u_N \) changes accordingly. Since the abnormal measured grid voltage is used as the controller input, the control performance, consequently, is seriously degraded. As a result, both \( i_N \) and \( U_{dc} \) are out of the acceptable range, which may lead to subsequent catastrophic failure of the whole system.

**TABLE 2** Parameters related to the rectifier and the control algorithm

| Description                        | Symbol | Value   |
|------------------------------------|--------|---------|
| RMS of grid voltage               | \( u_N \) | 1500 V  |
| Traction winding leakage inductance | \( L_N \) | 0.22 mH |
| Input resistance                   | \( R_N \) | 0.1 Ω   |
| Reference DC-link voltage          | \( U_{dc} \) | 3000 V  |
| DC-link capacitance                | \( C \) | 2 mF    |
| Rated load equivalent resistance   | \( R_L \) | 10 Ω    |
| Switching frequency                | \( f_s \) | 1.25 kHz |

**TABLE 3** Variation of \( R_L \)

| Time(s) | \([0, 5.3)\) | \([5.3, \infty)\) |
|---------|--------------|------------------|
| \( R_L \) | 10 Ω         | 20 Ω             |

**TABLE 4** Variation of \( L_N \)

| Time(s) | \([0, 5.1)\) | \([5.1, 5.15)\) | \([5.15, 5.2)\) | \([5.2, 5.25)\) | \([5.25, \infty)\) |
|---------|--------------|------------------|-----------------|-----------------|-------------------|
| \( L_N \) | 0.22mH       | 0.25mH           | 0.22mH          | 0.21mH          | 0.22mH            |

**Figure 5** Control performance of rectifier subjected to grid voltage sensor fault \( f \) without fault tolerant control unit involved.
4.1 Simulation

In practice, the estimated grid voltage sensor fault $\hat{F}$ (32) and the compensated grid voltage $u_{comp}$ (33) are calculated in a digital signal processor (DSP). Neither of them is measurable and their visualisation is unnecessary and not intuitive. However, the trajectory of $\hat{F}$ and $u_{comp}$ will be helpful in understanding how the proposed FTC system works. Therefore, in order to present $\hat{F}$ and $u_{comp}$ intuitively, a simulation based on MATLAB/Simulink is performed.

To avoid confusion, a diagram shown in Figure 6 is used to illustrate the relationship between three voltage-related terms.

In the first case, only the grid voltage sensor fault is considered and the simulation results are shown in Figure 7. As can be seen from Figure (7a), the estimated sensor fault $\hat{F}$ precisely tracks the actual sensor fault $f$ which is injected in the grid voltage sensor, that is, the FE is accurate in the absence of the parameter variation. Specifically, when tracking the slope fault and the fault described by cubic function, the tracking error converges very quickly (about 0.02 s). Due to the characteristic of discontinuity, error convergence takes a relatively long time (about 0.05 s) when tracking the constant fault. Therefore, based on the accurate real-time FE, the compensated grid voltage $u_{comp}$ is still sound after the sensor fault occurs, as can be seen from Figure (7b), Figure (7c), and Figure (7d). As a result, the performance of the real grid voltage $u_{real}$ is not degraded by the grid fault sensor fault $f$.

In the second case, the sensor fault and inductance variation are taken into consideration simultaneously. The corresponding simulation results are shown in Figure 8. By comparing Figure (8a) with Figure (7a), we know that the FE accuracy will be inevitably affected by the parameter variation. However, since the robustness is taken into account in the UIO design, the maximum FE error is about 50V when the inductance $L_N$ changes from 0.22 to 0.25mH and about 25V when $L_N$ changes from 0.22 to 0.21mH. Compared with the grid voltage, which is 1500V RMS, the FE error is small. Therefore, the compensated grid voltage $u_{comp}$ is only slightly affected, as can be seen from Figure (8b), Figure (8c), and Figure (8d). Subsequently, the performance of the real grid voltage $u_{real}$ is totally acceptable.

![FIGURE 6 Simulation results of Case 1: fault estimation accuracy (a) and fault tolerant control performance after constant fault (b), slope fault (c), and fault described by cubic function (d)](image)

![FIGURE 7 Simulation results of Case 2: fault estimation accuracy (a) and fault tolerant control performance after constant fault (b), slope fault (c), and fault described by cubic function (d)](image)
4.2 Hardware-in-the-loop test

The hardware-in-the-loop (HIL) experimental setup consists of a DSP, a dSPACE real-time simulator, an interface board, and a host PC, which is shown in Figure 9. The controller that guarantees the normal rectifier operation and the proposed FTC algorithm are implemented in the DSP whose control chip is TMS320F28335. Further, the dSPACE simulator embedded the DS1006 board is used to perform the rectifier model and the sensor fault injection. The interface board is used for connecting the signals between the DSP and the dSPACE simulator. The host PC is used to monitor the data and debug the programs. The control frequency of the DSP is set to 25 kHz.

FIGURE 8 Hardware-in-the-loop test results of Case 1: overview (a) and transient fault tolerant control performance after constant fault (b), slope fault (c), fault described by cubic function (d), and load variation (e).

FIGURE 9 Hardware-in-the-loop test results of Case 2: overview (a) and transient fault tolerant control performance after parameter variation (b), (c).
As mentioned earlier, both the estimated grid voltage sensor fault $\tilde{F}$ and the compensated grid voltage $u_{\text{comp}}$ are calculated in the DSP and are not measurable. Further, note that the grid voltage has a great influence on the rectifier's performance since it is one of the control inputs of the rectifier controller. Thus, the real grid voltage, the catenary current, and the DC-link voltage are presented to demonstrate the performance of the proposed FTC method in the HIL test.

The HIL test results of Case 1 are shown in Figure 10. It can be seen from Figure 10 that the real grid voltage is hardly affected by the grid voltage sensor fault, which further indicates that the proposed FTC method is of good performance. Specifically, due to the FE tracking error during the adjustment time when tracking the voltage sensor fault, both the catenary current $i_N$ and the DC-link voltage $U_{dc}$ experience slight deterioration before the FE error convergence, which are shown in Figure (10b), Figure (10c), and Figure (10d), respectively. In addition, from Figure (10c) we can see that the real grid voltage remains unchanged, and $i_N$ and $U_{dc}$ change as the load changes, which implies that the proposed FTC performance is not affected by load variation.

Figure 11 exhibits the HIL test results of Case 2. It is clear that the real grid voltage is still good after the sensor fault and parameter variation, as can be seen from Figure (11a). By comparing the details of Figure (10c) and Figure (11b), Figure (10d) and Figure (11c), we know that the inductance variation deteriorates the grid voltage sensor FTC. However, owing to the robust design of the proposed UIO, the effect of the parameter variation on the compensated grid voltage is minimised. Thus, the power factor is not seriously degraded and
the stability of the DC-link voltage is not destroyed. The rectifier system is able to continue to function normally in the presence of the grid voltage sensor fault and parameter variation.

5 | CONCLUSION

A robust FE-based FTC approach for an electrical traction rectifier subjected to disturbances and grid voltage sensor faults is proposed in this work. By establishing the augmented model for the rectifier with the fault and its derivatives being augmented state vectors, the FE design is converted to the observer design. Through the proposed UIO, accurate estimation of the grid voltage sensor fault can thus be accomplished. In addition, the observer error is robust against the unknown disturbances. The proposed FE algorithm is applicable to any kind of faults satisfying that $F(q)(\cdot) \equiv 0, q \in \mathbb{N}$. The FTC system is realised through the synthesis of FE and fault compensation. Finally, both simulation and HIL test confirm the effectiveness of the proposed FTC method.

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REFERENCES

1. Hill, R.J.: Electric railway traction. part 2: traction drives with three-phase induction motors. Power Eng. J. 8(3), 143–152 (1994)
2. Cheok, A.D., et al.: High power ac/dc converter and dc/ac inverter for high speed train applications. Tencen Proc. 1, 423–428 (2000)
3. Bertini, S., Ghari, S.: Ac/dc/ac high voltage traction drives with quasi-zero reactive power demand. IEEE Trans. Power Electron. 8(4), 632–639 (1993)
4. Ronanki, D., Singh, S.A., Williamson, S.S.: Comprehensive topological overview of rolling stock architectures and recent trends in electric railway traction systems. IEEE Trans. Transport. Electrific. 3(3), 724–738 (2017)
5. Yang, C., et al.: Voltage difference residual-based open-circuit fault diagnosis approach for three-level converters in electric traction systems. IEEE Trans. Power Electron. 35(3), 3012–3028 (2020)
6. Song, W., et al.: Deadbeat predictive power control of single-phase three-level neutral-point-clamped converters using space-vector modulation for electric railway traction. IEEE Trans. Power Electron. 31(1), 721–732 (2016)
7. Wang, H., Liserre, M., Blaabjerg, F.: Towards reliable power electronics: challenges, design tools, and opportunities. Ind. Electron. Mag. IEEE. 7(2), 17–26 (2013)
8. Shi, L.: Online diagnostic method of open-switch faults in pwm voltage source rectifier based on instantaneous ac current distortion. IET Electr. Power Appl. 12(3), 447–454 (2017)
9. Lu, H., et al.: Fault-tolerant predictive current control with two-vector modulation for six-phase permanent magnet synchronous machine drives. IET Electr. Power Appl. 12(2), 169–178 (2018)

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10. Sobanski, P., Kaminiski, M.: Application of artificial neural networks for transistor open-circuit fault diagnosis in three-phase rectifiers. IET Power Electron. 12(9), 2189–2200 (2019)

11. Caseiro, L.M.A., Mendes, A.M.S.: Real-time ight open-circuit fault diagnosis in three-level neutral-point-clamped voltage-source rectifiers based on instant voltage error. IEEE Trans. Ind. Electron. 62(3), 1669–1678 (2015)

12. Youssef, A.B., Khil, S.K.E., Slama-Belkhodja, I.: State observer-based sensor fault detection and isolation, and fault tolerant control of a single-phase pwm rectifier for electric railway traction. IEEE Trans. Power Electron. 28(12), 5842–5853 (2013)

13. Kim, S.C., et al.: Fault tolerant control of de-link voltage sensor for three-phase ac/dc/ac pwm converters. J. Power Electr. 14(4), 659–703 (2014)

14. Xia, J., et al.: Sensor fault diagnosis and system reconfiguration approach for electric traction pwm rectifier based on sliding mode observer. IEEE Trans. Ind. Appl. 53(5), 4768–4778 (2017)

15. Youssef, A.B., Khil, S.K.E., Slama-Belkhodja, I.: Open-circuit fault diagnosis and voltage sensor fault tolerant control of a single phase pulsed width modulated rectifier. Math. Comput. Simulat. 131, 234–252 (2017)

16. Zhang, K., et al.: Incipient voltage sensor fault isolation for rectifier in railway electrical traction systems. IEEE Trans. Ind. Electron. 64(8), 6763–6774 (2017)

17. Gong, Z., et al.: Sensor-fault-estimation-based tolerant control for single-phase two-level pwm rectifier in electric traction system. IEEE Trans. Power Electron. 35(11), 12274–12284 (2020)

18. Yang, C., et al.: A fault-injection strategy for traction drive control systems. IEEE Trans. Ind. Electron. 64(7), 5719–5727 (2017)

19. Jian, H., et al.: Fault estimation and fault-tolerant control for switched fuzzy stochastic systems. IEEE Trans. Fuzzy Syst. 26(5), 2993–3003 (2018)

20. Lan, J., Patton, R.J.: A new strategy for integration of fault estimation within fault-tolerant control. Automatica. 69, 48–59 (2016)

21. Zhang, H., et al.: Sensor fault estimation of switched fuzzy systems with unknown input. IEEE Trans. Fuzzy Syst. 26(3), 1114–1124 (2018)

22. Cristofaro, A., Johansen, T.A.: Fault tolerant control allocation using unknown input observers. Automatica. 50(7), 1891–1897 (2014)

23. Gao, Z., Liu, X., Chen, M.Z.Q.: Unknown input observer-based robust fault estimation for systems corrupted by partially decoupled disturbances. IEEE Trans. Ind Electron. 63(4), 2537–2547 (2016)

24. Shen, Y., et al.: Descriptor reduced-order sliding mode observers design for switched systems with sensor and actuator faults. Automatica. 76, 282–292 (2017)

25. Abi, H., Edwards, C.: Robust fault reconstruction for linear parameter varying systems using sliding mode observers. Int. J. Robust Nonlinear Control. 24(14), 1947–1968 (2015)

26. Cristofaro, A., Johansen, T.A.: Fault tolerant control allocation using unknown input observers. Automatica. 50(7), 1891–1897 (2014)

27. Yang, H., Jiang, B., Staroswiecki, M.: Observer-based fault-tolerant control for a class of switched nonlinear systems. IET Control Theory & Appl. 1(5), 1523–1532 (2007)

28. Gao, Z., Breikin, T., Wang, H.: High-gain estimator and fault-tolerant design with application to a gas turbine dynamic system. IEEE Trans. Contr. Syst. Technol. 15(4), 740–753 (2007)

29. Chen, J., Patton, R.J., Zhang, H.Y.: Design of unknown input observers and robust fault detection filters. Int. J. Contr. 63(1), 85–105 (1996)

30. Fonol, R., et al.: A class of nonlinear unknown input observer for fault diagnosis: application to fault tolerant control of an autonomous spacecraft. In: 10th UKACC International Conference on Control (2014)

31. Sun, X., Patton, R.J.: Robust actuator multiplicative fault estimation with unknown input decoupling for a wind turbine system 2013 Conference on. In: Control and Fault-Tolerant Systems (SysTol) (2013)

32. Lopez-Estrada, F.R., et al.: Robust sensor fault estimation for descriptor-lpv systems with unmeasurable gain scheduling functions: application to an anaerobic bioreactor. Int. J. Appl. Math. Comput. Sci. 25(2), 233–244

33. Garramiola, F., et al.: De-link voltage and catenary current sensors fault reconstruction for railway traction drives. Sensors. 18(7) (1998–2018)

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APPENDIX

In this work, $D_{\sigma}$, $W_{\sigma}$, and $F_{\sigma}$ are set as

$$D_{\sigma} = \begin{bmatrix} 10 \\ 10 \end{bmatrix}, \quad W_{\sigma} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad F_{\sigma} = \begin{bmatrix} \frac{1}{L_N} \end{bmatrix},$$

The condition (14) is solved by setting $\beta = 30$ and $\gamma = 0.01$ in advance. Thus, the matrices are obtained, which are given by

$$N_1 = \begin{bmatrix} -5.10e3 & -8.77e1 & -2.27e2 & 0 & 0 & 0 \\ 1.18e3 & -3.58e3 & -2.27e2 & 0 & 0 & 0 \\ -4.42e3 & 2.82e3 & 0 & 1e0 & 0 & 0 \\ -3.21e5 & 2.15e5 & 0 & 0 & 1e0 & 0 \\ -8.93e6 & 5.98e6 & 0 & 0 & 0 & 1e0 \\ -8.58e7 & 5.74e7 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$N_2 = \begin{bmatrix} 1.23e6 & -8.4e5 & -2.38e6 \\ -8.4e5 & 1.02e6 & 1.06e6 \\ -2.38e6 & 1.06e6 & 2.84e7 \\ 2.38e1 & 2.36e1 & -1.06e3 \\ 2.81e4 & -7.38e2 & 2.38e1 \\ 1.48e4 & -6.78e2 & 2.36e1 \\ 6.31e4 & -3.16e3 & 1.40e2 \\ -3.16e3 & 2.09e2 & -1.16e1 \\ 1.40e2 & -1.16e1 & 7.383e-1 \end{bmatrix},$$

$$N_3 = \begin{bmatrix} -4.60e3 & 3.67e2 & 2.27e2 & 0 & 0 & 0 \\ 6.80e2 & -4.03e3 & -2.27e2 & 0 & 0 & 0 \\ -4.42e3 & 2.82e3 & 0 & 1e0 & 0 & 0 \\ -3.21e5 & 2.15e5 & 0 & 0 & 1e0 & 0 \\ -8.93e6 & 5.98e6 & 0 & 0 & 0 & 1e0 \\ -8.58e7 & 5.74e7 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$P = \begin{bmatrix} 1.23e6 & -8.4e5 & -2.38e6 \\ -8.4e5 & 1.02e6 & 1.06e6 \\ -2.38e6 & 1.06e6 & 2.84e7 \\ 2.38e1 & 2.36e1 & -1.06e3 \\ 2.81e4 & -7.38e2 & 2.38e1 \\ 1.48e4 & -6.78e2 & 2.36e1 \\ 6.31e4 & -3.16e3 & 1.40e2 \\ -3.16e3 & 2.09e2 & -1.16e1 \\ 1.40e2 & -1.16e1 & 7.383e-1 \end{bmatrix},$$

$$G_{\sigma} = \begin{bmatrix} 5e-1 & 5e-1 \\ 5e-1 & 5e-1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$H_{\sigma} = \begin{bmatrix} 5e-1 & 5e-1 \\ 5e-1 & 5e-1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}.$$