Vortex-forced-oscillations of thin flexible plates

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I. INTRODUCTION

Fluid-structure interactions of a slender flexible cantilevered-element and vortices in an otherwise steady flow is considered here by investigating the dynamics of thin low-density polyethylene sheets subject to periodic forcing due to Bénard-Kármán vortices in a 2-meter long narrow water channel. The vortex shedding frequency $f_s$ is varied via the mean flow speed $U_0$ and the cylinder diameter $d_0 = 10, 20$ and 40 mm, while the structures’ bending resistance is properly controlled via its Young’s modulus $E$, thickness $e_0$ and length $L_0$. Thereby, it is first shown that the non-dimensional time-averaged sheet deflection, namely, the sheet reconfiguration $h_0/L_0 \sim C_u^{\nu/2}$ and also, the time-averaged drag force $F_d \propto U_0^{2+V}$, where $V \leq 0$ is the well-known Vogel number for flexible structures in a steady flow and $C_u = 12\left(\frac{E}{\rho U_0^2} \frac{d_0}{h_0}\right)$ is the Cauchy number comparing the relative magnitude of the profile drag force over a typical elastic restoring force, if the sheet were rigid. Measurements and a simple model based on torsional-spring-mounted flat plate illustrate that the tip amplitude $\delta_0$ is not only directly proportional to the characteristic size of the eddies, say $d_e$, but also to the sheet mechanical properties and the vortex flow characteristics such that $\delta_0/d_e \sim C_\nu^{(1+V)/2} \sqrt{U_0/f_d d_e}$. Furthermore, a rich phenomenology of structural dynamics including vortex-forced-vibration, lock-in with the sheet natural frequency, flow-induced vibration due to the sheet wake, multiple-frequency and modal response is reported.

A rigid circular cylinder, when exposed to a fluid flow, expresses a wide variety of dynamics depending on not only its mechanical properties and the flow characteristics, but also its boundary conditions. For example, Strouhal [1] and Rayleigh [2] illustrated that the singing notes of thin wires and strings, subject to an air-stream, are a function of the relative velocity and Rayleigh [2] illustrated that the singing notes of thin wires and strings, subject to an air-stream, are a function of its mechanical properties and the flow characteristics, but also its boundary conditions. For example, Strouhal [1]

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Therefore, it is first shown that the non-dimensional time-averaged sheet deflection, namely, the sheet reconfiguration $h_0/L_0 \sim C_u^{\nu/2}$ and also, the time-averaged drag force $F_d \propto U_0^{2+V}$, where $V \leq 0$ is the well-known Vogel number for flexible structures in a steady flow and $C_u = 12\left(\frac{E}{\rho U_0^2} \frac{d_0}{h_0}\right)$ is the Cauchy number comparing the relative magnitude of the profile drag force over a typical elastic restoring force, if the sheet were rigid. Measurements and a simple model based on torsional-spring-mounted flat plate illustrate that the tip amplitude $\delta_0$ is not only directly proportional to the characteristic size of the eddies, say $d_e$, but also to the sheet mechanical properties and the vortex flow characteristics such that $\delta_0/d_e \sim C_\nu^{(1+V)/2} \sqrt{U_0/f_d d_e}$. Furthermore, a rich phenomenology of structural dynamics including vortex-forced-vibration, lock-in with the sheet natural frequency, flow-induced vibration due to the sheet wake, multiple-frequency and modal response is reported.

In often applications and in nature, mechanical structures exposed to a fluid flow could be very flexible in order to alleviate flow-generated forces. For example, a flexible structure exposed to a steady flow undergoes static reconfiguration whereby the profile drag experienced by the former is reduced, in comparison with that of its rigid counterpart. In turn, the resulting internal bending stresses and furthermore, the modified flow angle of attack might alter both the structural dynamics and the wake characteristics. Such effects are only recently considered for flow-bent cylinders to explore associated FSI, namely, reduction in oscillation amplitude, multi-frequency response and lock-in mode, to name a few, and references therein. Also, a related topic, namely, the flutter of a flat-plate, be it rigid or flexible, has received a large number of attention during the past two decades. Whereas rare are the studies on VIV of a single cantilevered slender blades whose one-end is anchored to the flow bed.

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In this context, using single specimens of four different freshwater plants in laboratory flumes whose floor is covered with uniform density of shorter artificial grass, Siniscalchi and Nikora \cite{30} correlated individual plant movement and its drag force fluctuations with respect to upstream turbulence. Also, they remarked spatial flapping-like movement in all plant species, with the propagation velocity of perturbations being comparable to the approach flow velocity. More recently, a series of works by Jin et al. \cite{37,38,39,40} put light upon a rich phenomenology of Fluid-Structure Interactions for dynamically reconfigured flexible blades. Firstly, if $U_0$ is the average flow speed, $w_b$ is the base width of flexible plates facing a channel flow and $\nu$ the liquid's kinematic viscosity, for various Reynolds numbers $Re_b = \frac{U_0w_b}{\nu}$ ranging from 3000 to $3 \times 10^4$ \cite{57} point out that moderately-long flexible structures (aspect ratio $L_b/w_b \in (2, 3)$) vibrate in the stream-wise direction at their natural frequency while the wake fluctuations beat at a frequency of vortex shedding. This implies that energy induced from the fluid-induced motion of flexible blades \cite{11} might be controlled by properly tuning the structures natural frequencies. On the other hand, Jin et al. \cite{38} by analysing the role of tip geometry on moderately-short flexible plates (aspect ratio $L_b/w_b \in (2, 3)$) illustrated that the structural dynamics are governed by both wake-fluctuations and non-linear modulations of structural bending. Furthermore, these cantilevered-structures presented a maximum tip oscillation intensity at some critical Cauchy number which compares the profile drag force experienced by the structure if it were rigid and the characteristic internal restoring force generated due to an external force. Later, Jin et al. \cite{40} used a flexible blade of aspect ratio 4 at different inclination angles to the incoming flow and highlighted the presence of three modes of tip-oscillations, namely, fluttering, tip-twisting and orbital modes which occur respectively at increasing Cauchy number $C_y$. Orbital modes are characterized by large-amplitude coupled twisting and bending deformations, and they occur for sufficiently large inclination angles. Much is to follow in the perspective of these studies, for instance, the influence of mass ratio.

Furthermore, long flexible structures in nature and in applications occur rarely as an isolated object. And so, artificial canopies of flexible blades are pertinent to study the effect of wind on trees, terrestrial plant fields and aquatic vegetation \cite{27,28,42}. Indeed, a wide variety of mechanically activated phenomena through FSI in plants, or artificial canopies are crucial for sediment transport, water quality, biodiversity of aquatic species. Among the well-known examples, honami (Japanese: ho = crops and nam = wave) \cite{30,43,44} and monami (Japanese: mo = aquatic plant) \cite{45,46,47,48}, respectively, represent coherent motion of crops and aquatic canopies when the flow resistance is sufficiently high. In these cases, the proposed mechanistic views generally involve the two-way coupling between flow vortices and the flexible canopy of plants \cite{27,42,40}. Furthermore, velocity spectrum and eddies in the incoming flow modulate the motion of flexible structures \cite{50}. Also there is now some evidence \cite[Chap. 5]{51} that the mechanical response of flexible blades in a channel flow might depend strongly on the Cauchy number $C_y$. It seems, therefore, important to know how vortices of different sizes interact with cantilevered flexible blades at various flow velocities and blade physical characteristics.

In the wake of these previous investigations, we study the motion of a thin flexible sheet when it encounters a regular array of vortices generated by Bénard-Kármán vortex shedding behind a cylinder. Thereby, we seek to provide few insights into the dynamic reconfiguration (IV), tip amplitude (V), tip frequency (VI) and modes of oscillation (VII), for a good range of Cauchy numbers. Such results may not only contribute to the physics of the structural dynamics of plant canopies that exhibit coherent motion such as honami and monami but also, to a novel kind of FSI which involve slender flexible objects exposed to coherent vortical structures.

\section{II. MATERIALS, SET-UP AND METHODS}

Five different cases of low-density polyethylene (density $\rho_b = 920 \text{ kg m}^{-3}$ and mass ratio 0.93) sheets are considered (see Table I) by changing the sheet length $L_b$ and the thickness $e_b$. The latter is chosen to be always very small compared to all other dimensions. Here, it is pointed out that cutting extra-thin polyethylene sheets to prepare long blades is a delicate task, as the process often leads to some local plastic deformation. For the sake of simplicity, the sheet width $w_b = 15 \text{ mm}$ and sheet material are kept identical for all experiments discussed here and it is expected that the results are qualitatively similar for other materials since the Cauchy number

$$C_y = \frac{1}{2} \frac{\rho U_0^2}{E} \left( \frac{L_b^3 w_b}{I} \right) = 12 \left( \frac{C_d \rho U^2}{E} \right) \left( \frac{L_b^3}{e_b^3} \right),$$

(1)

which expresses the ratio of the flow drag and the elastic restoring forces, varies in wide a range, namely, between $O(1)$ and $10^3$. Here, $U_0$ is the depth-averaged flow speed, $E$ is the Young’s modulus, and the second moment of inertia for a thin flexible sheet is taken as $I = w_b e_b^3/12$ to obtain the second formula on the right-hand-side. And throughout the present work, the profile drag coefficient $C_d$ of laterally-confined flexible sheets of width $w_b = 15 \text{ mm}$ is taken as $C_d = 6$, based on measurements in Barsu et al. \cite{32} [see fig. 6a]. Furthermore, an estimate for the natural
frequency of a fixed-free cantilever beam in a fluid is

\[
f_{ni} = a_i \sqrt{\frac{EI/w_bL_b^4}{\rho_b c_b + \pi C_M \rho w_b L_b/4}} = a_i \sqrt{\frac{E (c_b/L_b)^3}{12\rho_b c_b L_b + 3\pi C_M \rho w_b L_b}},
\]

where \(a_i\) is a non-dimensional constant with \(a_1 = 0.56\) and \(a_2 = 3.5\) for the first-order and second-order natural frequency, resp.) \(^{20,52}\) and \(C_M\) is the added mass coefficient. This is taken as the undamped bending frequency of all sheets here despite the fact that this natural frequency is usually attributed to beams in fluids that undergo small-amplitude vibrations about an equilibrium position based on the assumption that a beam presents only small deflections compared to their length. Note that such a frequency might not be relevant to long flexible sheets considered in this study since they exhibit large deflections when subject to the flow drag in experiments presented here. And also, these sheets are, in fact, pre-tensioned due to the presence of a mean flow drag.

All experiments are performed in a long-narrow water channel as schematized in figure 1(a). Water from the pump (700 – 2800 lit/h) passes through a fine grid in the inlet and flows out in to a narrow 2-meter long channel of width 4 cm and height 25 cm. By properly adjusting the outlet gate and the horizontal slope of the channel, it is possible to maintain a free-surface flow of uniform water height across the entire channel. Sufficiently far from the inlet, at a

FIG. 1: Schematic view of (a) the 2-meter long water channel along with (b) top and (c) front view showing how the cylindrical obstacle is set-up in order to impose a vortex street forcing on a flexible sheet downstream.

(a) Water channel set-up

(b) Front (camera) view

(c) Side view

(d) Sheet coordinates
little more than 1-meter, a circular cylinder of diameter \( d_0 \) is fixed with its axis parallel to the floor, but perpendicular to the flow. Behind the cylinder (downstream), one of the thin, flexible and lighter-than-water rectangular sheet is exposed to an uniform channel flow directly orthogonal to the sheet’s section.

The shedding frequency were obtained using Ultrasonic Doppler Velocimetry (UDV). As compared to the cylinder Reynolds number \( Re_c = U_0 d_0 / \nu \), the hydraulic Reynolds number \( Re_h = U_0 D_h / \nu \) varies from 1730 to 6920, wherein the hydraulic diameter is \( D_h = 4 w_b h_0 / (2 w_b + h_0) \) and the water height \( h_w = 22.1 \) cm is kept constant for all experiments.

Indeed, a first series of experiments consist of measuring the sheet deflection \( h_0 \) (not presented here) in a fully-developed uniform water flow without vortices. These measurements are then used to fix the cylinder vertical location for each run in the vortex-forced-vibration experiments. Thus, for each set of sheet physical properties and flow conditions, namely, the water speed \( U_0 \) and the cylinder diameter \( d_0 \), the latter’s bottom is placed at a distance \( h_0 \) from channel floor such that \( h_0 \) is equal to the deflected height of a sheet in an equivalent steady, uniform flow of same flow rate and water height. In the range of depth-averaged flow speed \( U_0 = 2.2 – 8.8 \text{ cm s}^{-1} \) and diameters \( d_0 = 10, 20, 40 \text{ mm} \) used here, the cylinder Reynolds number \( Re_d = U_0 d_0 / \nu \) varies over a decade, between 236 and 3773.

An Ultrasonic Doppler Velocimetry (UDV) is used to obtain vortex street characteristics at the center plane of the channel and at a fixed position downstream, equal to 3 times the cylinder diameter. The probe measures the

| ID     | \( L_b \) (mm) | \( e_b \) (mm) | \( E \) (\( \times 10^3 \) Pa) | \( C_y = \frac{C_{y0} U^2}{E - \left( \frac{d_0^2}{4} \right)} \) | \( B_\alpha = \frac{\Delta p g e_b}{E - \left( \frac{d_0^2}{4} \right)} \) | \( f_{n1} \) (Hz) | Symbol |
|--------|----------------|---------------|-----------------------------|-----------------------------------|-------------------------------|----------------|-------|
| S442   | 84             | 0.19          | 210                         | 5 – 88                             | 0.55                          | 0.286          | <     |
| S1263  | 240            | 0.19          | 210                         | 130 – 2000                        | 12.8                          | 0.035          | □     |
| S1400  | 84             | 0.06          | 230                         | 908 – 4200                        | 8.3                           | 0.042          | ○     |
| S2000  | 200            | 0.10          | 250                         | 2100 – 9800                       | 32.1                          | 0.018          | ★     |
| S4000  | 240            | 0.06          | 230                         | 12500 – 98200                     | 192.4                         | 0.005          | ⋄     |

**TABLE I:** Geometric and mechanical properties of thin low-density polyethylene sheets along with the range of related Cauchy numbers attained in this work. All sheets are less denser than water (density \( \rho_s = 920 \text{ kg m}^{-3} \)) and their width \( (w_b = 15 \text{ mm}) \) is kept constant throughout this work. The sheet ID also indicates their stiffness ratio given by \( L_b/e_b \).

**TABLE II:** Characteristics of Bénard-Kármán vortices for a range of water speed \( U_0 = 2.2 – 8.8 \text{ cm s}^{-1} \). The shedding frequency were obtained using Ultrasonic Doppler Velocimetry (UDV). As compared to the cylinder Reynolds number \( Re_c = U_0 d_0 / \nu \), the hydraulic Reynolds number \( Re_h = U_0 D_h / \nu \) varies from 1730 to 6920, wherein the hydraulic diameter is \( D_h = 4 w_b h_0 / (2 w_b + h_0) \) and the water height \( h_w = 22.1 \) cm is kept constant for all experiments. The reduced frequency \( u = f_c / f_n \geq 1 \) for all cases considered here and also, it increases almost as \( C_y^{0.15} \) (not shown here).
FIG. 2: Bénard-Kármán vortex shedding frequency and the related Strouhal number (a) $\text{St}_0 = f_v d_0 / U_0$ based on the depth-averaged water speed $U_0$ (b) $\text{St}_{h0} = f_v d_0 / U_{h0}$ based on cylinder center velocity $U_{h0}$ as computed from the classical Coles law [54] for fully-developed channel flows.

Experimental Strouhal number $\text{St}_0 = f_v d_0 / U_0$ data based on the depth-averaged water speed $U_0$ are presented in figure 2(a). Here, the results are given in terms of the gap-to-diameter ratio ($h_0 / d_0$), as it is conventional in previous works which consider the influence of a planar wall on the vortex dynamics [55, 56, 57, 58]. Although the data does not display any general trend, it is observed that for the smallest diameter ($d_0 = 10$ mm) and $h_0 / d_0 > 2$, the Strouhal number is about approximately 0.22. This value corresponds to the classical Strouhal number measurements in the absence of the channel floor for moderate cylinder Reynolds numbers [4, 59]. For larger diameters, namely $d_0 = 20, 40$ mm, there is quite a scatter in the Strouhal number $\text{St}_0$ between 0.25 – 0.4. Nonetheless, this dispersion can be understood if the local velocity $U_{h0}$ at the cylinder center is used. For this purpose, via the depth-averaged water speed $U_0$ and the water height $h_w$, the channel flow profile $U(y)$ was computed using the so-called Coles law [54] by taking commonly used empirical constants as in, for instance, Kirkgöz and Ardiçlioğlu [60]. And so, by taking $U_{h0} = U(y = h_0)$, the new rescaled Strouhal number $\text{St}_{h0} = f_v d_0 / U_{h0}$, shown in figure 2(b), presents much smaller dispersion in the range of gap-to-diameter ratio studied here. Furthermore, the globally decreasing trend of the rescaled Strouhal number $\text{St}_{h0}$ data in this work is similar to previous experimental [56, 61, 62] and 3-D LES numerical investigations [62, *]. As discussed in the introductory section of Sarkar and Sarkar [62], the trend at small and moderate gap-to-diameter ratio ($h_0 / d_0 \lesssim 4$) is either a growing or a decreasing function of $h_0 / d_0$ depending on the cylinder Reynolds number, the boundary layer thickness at the channel floor and the presence of a free-surface. In conclusion, UDV-based results in the present study are consistent with previous works at moderate Reynolds numbers [56, 61, 62], if a proper velocity scale is taken for the Strouhal number.

III. SHEET-TIP DYNAMICS DUE TO BÉNARD-KÀRMÀN VORTICES: GENERAL REMARKS

As discussed before, each experiment is set-up by placing the cylinder base at a known distance $h_0$ equal to the static deflection height. Note that the cylinder is located at about 1-meter from the channel entrance. The cylinder then sheds a Bénard-Kármán vortex street of known shedding frequency (see VII) which encounters a flow-bent flexible sheet downstream. A high-resolution, full-frame digital camera (Sony α7) is used to image the Vortex-Forced-Oscillations (VFO) of the flexible sheet at a rate of 25 images per second for a time period of 7 to 8 minutes (see supplementary videos). The resulting images are analysed using the open-source freeware *ImageJ* [63] and algorithms therein for
brightness thresholding \[64, 65\], edge detection, etc. Sample images and the corresponding edge detection (dots, pink) are shown in figures 3(a) and (b). Clearly, the contour of the sheet is well-detected. The sheet “tip” is taken as the center of the last identified sheet edge and thereby, a robust sheet tip detection can be obtained by these techniques. Such an identified sheet’s tip (⋆), along with the manually demarcated sheet’s foot (×) are also displayed in figures 3(a) and (b). This allows for a well-resolved tip detection amplitudes in the order of a few 10-th of a millimetre.

\[3(a) \text{ S442: } C_y = 11.3, \quad Re_b = 2.5 \times 10^3, \quad Re_d = 1.4 \times 10^3 \quad \text{and} \quad (b) \text{ S400: } C_y = 7.83 \times 10^3, \quad Re_b = 6.2 \times 10^3, \quad Re_d = 3.4 \times 10^3.\]

Here, dots (pink) show the detected sheet edge while an asterisk (blue) and a cross (red) each indicate the sheet’s tip and foot, respectively.

If \(x_b(t)\) and \(y_b(t)\) are respectively the horizontal and vertical position of the sheet tip, the corresponding horizontal and vertical fluctuations are then defined as

\[
\tilde{x}_b(t) = x_b(t) - \bar{x}_b, \quad \tilde{y}_b(t) = y_b(t) - \bar{y}_b, \tag{3}
\]

where an over-bar indicates a time-averaged variable. Now consider the sheet-tip fluctuations for a few typical cases provided in figures 4. Time evolution of the fluctuations (\(\tilde{x}_b, \tilde{y}_b\)) in millimetres are shown on the left while the corresponding peak-normalised spectra for the tip’s vertical position are provided on the right. For the sake of clarity, only a minute-long evolution is given. Note that these are raw data, by eliminating a few outliers beyond 4 times the standard deviations but without any prior moving-average.

Figures 3(a) & (b) display the temporal tip response of the sheet S442 to Bénard-Kármán vortices shed by the same cylinder (\(d_0 = 40\) mm) at the lowest and highest average water speed, respectively. In the former case, at \(U_0 = 3.1\) cm s\(^{-1}\), both vertical and horizontal oscillations are of the same order of magnitude and also, they are synchronous. As the speed increases to \(U_0 = 7.9\) cm s\(^{-1}\), \(y\)-fluctuations present an almost proportionally-increased back-and-forth amplitude while \(x\)-fluctuations are much smaller in magnitude. Clearly, \(x\)-tip detection seems to be not so robust for this case. Nonetheless, vertical sheet tip position exhibits a reasonably continuous evolution wherein its frequency of oscillation \(f_x\) seems to be greater than the case with smaller water speed \(U_0 = 3.1\) cm s\(^{-1}\). This is readily visible in the corresponding power spectral density shown on the immediate right of the same figures. Here, arrows are used to indicate the forcing frequency (vortex shedding) \(f_v = 0.56\) Hz and the first natural frequency \(f_{n1} = 0.29\) Hz, respectively: the sheet tip oscillates at the vortex shedding frequency for these two cases. Furthermore, both these examples correspond to the sheet for which the relevant Cauchy numbers \(C_y = 9 \times 10^4\) are the smallest. In comparison, figure 4(c) gives the tip fluctuations for S400, at one of the largest Cauchy numbers \(C_y = 9.29 \times 10^4\). This sheet exhibits large amplitude oscillations in the vertical direction as big as the cylinder diameter \(d_0 = 40\) mm while the horizontal oscillations remain small (\(\lesssim 4\) mm) as before. In particular, the back-and-forth \(y\)-fluctuation amplitude is about 4 times larger than the case S442 at the same average water speed. The corresponding spectra of vertical tip position does not present a peak at the sheet natural frequency \(f_v = 5 \times 10^{-3}\) Hz. However, a large low-frequency peak (0.15 Hz), along with a smaller second peak at the vortex shedding frequency \(f_v = 0.55\) Hz, are visible.
Qualitatively similar temporal characteristics are observed for the sheet tip when the diameter $d_0$ is decreased. For example, figure 4(d) provides a typical data at $d_0 = 10$ mm to be compared with its equivalent case at the same water speed and sheet physical properties given in figure 4(c). Firstly, the vertical oscillations are diminished almost in proportions to the diameter-ratio. Secondly, the power spectral density presents a peak neither at the vortex shedding frequency $f_v = 0.55$ Hz, nor at the sheet natural frequency $f_v = 5 \times 10^{-3}$ Hz, but instead at almost the same frequency ($0.16 \text{ Hz} < f_v$) as for the case with the larger diameter $d_0 = 40$ mm.

In summary, at smaller average water speeds and stiffness ratios $L_b/e_b$, flexible sheets seem to exhibit small-amplitude flutter, i.e., $x$, $y$ oscillations are comparable, and are also much smaller than the cylinder diameter $d_0$. Tip fluctuations are larger at higher speeds and bigger stiffness ratio $L_b/e_b$, leading to oscillations comparable to $d_0$. The peak-normalised power spectra suggests that the tip motion in the $y$ direction oscillates either at the vortex shedding frequency $f_v$ or at a different frequency lesser than $f_v$ as and when $d_0$ decreases, or $L_b/e_b$ increases. Moreover, as seen in figures 3 a given sheet’s local curvature could be either single-signed, or multiple-signed, depending on the sheet stiffness and vortex street’s width. In the following, these general remarks are further analyzed.

IV. TIME-AVERAGED SHEET RECONFIGURATION

Related to the sheet tip dynamics is the mean sheet tip position. In the absence of a vortex street, at any chosen flow rate and constant water height, a sheet bends in the flow direction and exhibits a deflected height $h_b < L_b$. This leads to the well-known profile drag reduction since the drag force experienced by a flexible sheet $F_d = C_d \frac{1}{2} \rho U^2 h_b w_b$ is smaller compared to the profile drag $C_d \frac{1}{2} \rho U^2 L_b w_b$, if the same sheet was rigid (here, $C_d$ is the sheet drag coefficient).
This drag reduction is often quantified using the so-called static reconfiguration number $\mathcal{R} = h_b/L_b$ as a function of the flow speed $U_0$. It is now well-established that the drag reduction can be expressed as a power law, namely

$$F_d \propto \frac{1}{2} \rho U_0^{2+\mathcal{V}} A_f,$$

where $A_f = L_b w_b$ is the undeformed sheet frontal area and $\mathcal{V} < 0$ is known as the Vogel number \[24\] such that $h_b/L_b \sim U_0^{\mathcal{V}}$. For long thin blades that are anchored to the flow bed at its one end, the Vogel number $\mathcal{V} = -2/3$ \[27\].

![Diagram](image)

**FIG. 5:** [TOP] Time-averaged sheet reconfiguration $\bar{R}_b = \bar{h}_b/L_b$ in the presence of Bénard-Kármán vortices as a function of the average water speed across the sheet height ($U_h$). It illustrates that the average Vogel number $\mathcal{V} \approx -0.6 \pm 0.1$ for all cases, except for sheets S442 (◁). [BOTTOM] Same data expressed in terms of the local Cauchy number

$$C_y h_b^2 = 12 \left( C_d \frac{\rho U_h^2}{E} \right) \left( \frac{L_b}{e_b} \right)$$

and compared with continuous lines (green) as obtained from the bending beam model (eqn. 6). The inset compares the average sheet deflection against the corresponding cylinder’s vertical position which is equal to the sheet height ($\pm 2$ mm) in an uniform flow in the absence of Bénard-Kármán vortices.

Since the time averaged-reconfiguration of a sheet should also provide a measure of the average drag-reduction, if any, during the vortex-forced-motion of thin flexible sheets, it is reasonable to define a dynamic reconfiguration number $\bar{R} = \bar{h}_b/L_b$, analogous to the static reconfiguration number $\mathcal{R}$. In this context, figure 6 (top) presents the non-dimensional time-averaged deflected height $\bar{h}_b$ for each of the five sheets. Here, instead of the channel-depth-averaged water speed ($U_0 = 22 - 88$ mm s$^{-1}$), we present our experimental results in terms of the average water speed across the time-averaged deflected sheet height $\bar{h}_b$ based on the classical Coles law for the channel velocity profile $U(y)$, so that $U_h = \int_0^{h_b} U(y) dy/\bar{h}_b$. Each symbols, namely, $\triangle$, $\square$, $\bigcirc$, $\star$, and $\diamond$, represent different sheets of increasing stiffness ratio $(L_b/e_b)$, respectively, provided in Table I. Also, each column of figures correspond to data for a particular cylinder diameter $d_0$. In all case corresponding to a fixed $L_b/e_b$, we observe that the average reconfiguration number decreases when the flow speed increases. Furthermore, it is possible to associate a power law and hence, a Vogel number $\mathcal{V}$ for each stiffness ratio $(L_b/e_b)$. Expect for the sheet with the smallest stiffness, all
the other sheets present a Vogel number $V \approx -0.6 \pm 0.1$. Similar values were previously obtained in the same water channel, but for submerged artificial canopies of kevlar sheets undergoing static reconfiguration [32, see fig 4(a)]. Finally, we observe a small decrease in Vogel number as well, in the case of vortices shed by cylinders of increasing diameter. Furthermore, the reconfigured sheet height, say $R = h_b/L_b$ is usually given as a function of the Cauchy number $C_y = C_d \frac{1}{2} \rho U_0^2 L_b^3 w_b / EI \approx \frac{C_y}{b} \rho g w_b \Delta y / L_b^3$. Indeed, a low Cauchy number $C_y \ll 1$ represents the case of a sheet that undergoes very little deflection in the flow direction ($R = h_b/L_b \approx 1$) since the drag force experienced by the sheet is sufficiently small with respect to the internal restoring force. Whereas the case of $C_y \gg 1$ indicates a flexible sheet with large deflection (or equivalently, a small blade reconfiguration number $R_0 \ll 1$) and hence, a reduced overall sheet drag force compared to its rigid counterpart.

It is now well-established that a bending beam under large deflection, accounting for the local flow drag and the Archimedes force due to buoyancy provides a satisfactory mechanical model for static reconfiguration of flexible sheets [20, 31, 32, 66]. Since a single rectangular sheet resembles a bluff body, as for the above-mentioned works, the sheet’s skin friction is assumed to be negligible here for the sake of simplicity. 1. For a sheet due to a steady flow, an expression for the restoring bending moment is simply given by the bending beam model for thin flexible sheets [68]. If $s$ is the curvilinear coordinate along the sheet and $\theta (s)$ the local sheet deflection as represented in fig. 1(d), the restoring bending moment $M(s)$ at any arbitrary distance $s$ from the sheet’s foot should be given by

$$EI \frac{d\theta}{ds} = \int_s^{L_b} (x(\xi) - x(s)) dF_A - \int_s^{L_b} (\xi - s) dF_D, \tag{5}$$

where $dF_A = \Delta \rho g (e_b w_0 d\xi)$ is the local Archimedes force and $dF_D = C_d \frac{1}{2} \rho U_0^2 \sin^2 (\theta(\xi)) dA_f$ is the normal component of the local profile drag, with $\Delta \rho = \rho - \rho_w$ the density difference between the fluid and the sheet, and $dA_f = w_d d\xi$ the local frontal area of the reconfigured sheet 2. The above equation can be further simplified by taking both $U(s) \equiv U_0$ and $EI$ to be invariant across the sheet. Therefore

$$\frac{d^3 \theta}{ds^3} = B_a \left( \sin \theta (1 - \tilde{s}) \frac{d\theta}{ds} \cos \theta - C_y \sin^2 \theta, \tag{6} \right)$$

where $\tilde{s} = s/L_b \in [0, 1]$ is the non-dimensional curvilinear coordinate, $B_a = \Delta \rho g w_e b L_b^3 / EI$ is the so-called Buoyancy number and $C_y = C_d \rho w_b L_b^3 U_0^2 / 2EI$ is the Cauchy number. The typical values for these non-dimensional numbers are also provided in Table I. This model equation can be readily solved by applying the boundary conditions at the sheet extremities $\theta = \pi/2$ and $d\theta/ds = d^2 \theta/ds^2 = 0$, for $\tilde{s} = 0$ and $\tilde{s} = 1$, respectively.

Let us now investigate the dynamic reconfiguration number as a function of Cauchy number, as provided in figure 5 (bottom). As before, we use the average water speed $U_0$ across the time-averaged deflected sheet height $h_b$ to express data in terms of the local Cauchy number $C_y = 12 C_d \frac{1}{2} \rho U_0^2 / E (L_b / e_b)^3$. Symbols represent experiments and continuous lines are computed from the bending beam model in eqn. 6 at each various Cauchy and Buoyancy number. For a given blade, say for instance S442 as represented by $\triangle$, as $C_y$ increases, the sheet reconfiguration decreases monotonically. However, at a fixed Cauchy number $C_y$, reconfiguration data from all sheets do not collapse on a single master curve. A closer observation reveals that this is due to buoyancy effects which tends to increase the reconfiguration for sheets with larger Buoyancy number $B_a = 12 \Delta \rho g c_b / E (L_b / e_b)^3$. Despite the fact that the above-mentioned bending beam model is only valid for the case of a static reconfiguration under a steady uniform flow, it predicts the trend with the Cauchy number and also, the buoyancy number for all cases. It displays a reasonable agreement for $C_y > \mathcal{C}(1)$. However, differences with experimental data are visible as the diameter of the upstream cylinder increases (see also, figures 6 for comparison between sheet shape and the ones computed from the expression 6).

Here, it is inferred that the dynamic reconfiguration curve is strikingly similar to that of the quasi-static regime. This is further elucidated in the inset of figures 5 (bottom) wherein the sheet deflection height $h_b$ in the absence of vortices is compared with $R = h_b/L_b$. Firstly, the mean blade tip position is situated just above the value obtained for the quasi-static case. In all the cases studied here, the sheet deflects a little lesser in the presence of Bénard-Kármán vortices. In other words, the vortices slightly “lift-up” the sheet’s tip, up to approximatively 5 – 10% of the blade length. Note that this “lift-up” effect is less visible for large dynamic reconfiguration $R_b \ll 1$ and small vortices.

1 Although a recent work by Bhati et al. [67] suggests that this is not always the case

2 Note that the effect of the sheet’s curvature on the bending moment due to the drag force and the tensile stress along the length of the sheet are neglected for simplicity.
FIG. 6: Typical time-averaged sheet shape as compared to bending beam model (eqn. [7]), allowing for the effect of buoyancy in a steady, uniform channel flow.
(\(d_0 = 10\) mm) as inferred for the case S4000, denoted by ◊ in figure 5(b). Secondly, it suggests that the dynamic sheet reconfiguration is only slightly modified, notwithstanding that the relatively wide variation of the tip oscillation amplitude observed in this study, as described in the following section. Finally, figure 6 compares the time-averaged sheet shape with that computed via the bending beam model (eqn. 6). It indicates that the model provides a satisfactory estimate. This might not only be due to the restriction in the model that the average-water speed is uniform across the sheet but also due to the fact that thin polyethylene sheets used in this study present local plastic deformation, as already explained in section III. Note that the latter fact can also be inferred in the first column of figure 6 which provides comparisons for the case with smallest tip oscillations, namely, the one at the slowest speed (\(U_0 = 22\) mm \(s^{-1}\)) and the smallest cylinder diameter (\(d_0 = 10\) mm).

V. SHEET OSCILLATION AMPLITUDE

FIG. 7: Tip oscillation amplitude \(\delta_b\) as a function of the water speed \(U_{b0}\) at the cylinder center (a) \(d_0 = 40\) mm, (b) \(d_0 = 20\) mm and (c) \(d_0 = 10\) mm for five different sheet physical properties (see also, Table I). (d) All data from above but, here, given with respect to an equivalent vortex circulation \(U_{b0}d_0\) and (e) Normalised amplitude \(\delta_b/d_0\) as a function of Cauchy number. Note that the \(U_{b0}d_0\) is equivalent to the local cylinder Reynolds number \(Re_b = U_{b0}d_0/\nu\).

The general observations evoked at the end of the section III can be quantified by defining a proper expression for the oscillation amplitude. While it is possible to simply define the amplitude as the maximum excursion of the observed tip motion, a more robust definition is to define it as a statistical measure. For this purpose, it is useful to
define the tip amplitude based on the standard deviation as in

\[ \delta_b \approx 2\sqrt{\frac{x_b^2}{\bar{R}^2} + \frac{y_b^2}{\bar{R}^2}} = 2\sqrt{(x_b(t) - x_0)^2 + (y_b(t) - y_0)^2}. \]  

Figures 7(a) – (c) then display such an amplitude as a function of the local water speed \( U_{h0} = U(y = h_0) \) computed from the Coles law channel velocity profile, as before in section IV for various cylinder diameters. Here, each symbol corresponds to a stiffness ratio as given in Table I. As already remarked, figure 7(a) at a given \( d_0 = 40 \) mm confirms that tip fluctuation amplitude increases proportionally with the water speed \( U_{h0} \) for a given sheet. In addition, sheets with larger stiffness ratio \( L_b/e_b \) show increasingly bigger amplitudes. In particular, note that the data for sheets (○, S1263 and ○, S1400) with approximately same stiffness ratio fall almost on the same linear trend line. Now, as the cylinder diameter is decreased, as in figures 7(b) and (c), similar behaviors are again observed but the tip amplitudes \( \delta_b \) are smaller, as well. When all data are put together with respect to \( U_{h0}d_0 \) which is proportional to the local cylinder Reynolds number, as in figure 7(d), it is clear that the tip amplitude not only increases with the sheet stiffness ratio \( L_b/e_b \), but also with \( U_{h0}d_0 \). Finally, we present in figure 7(e) the non-dimensional tip oscillation amplitude \( \bar{\delta}_b/d_0 \) as a function of the sheet Cauchy number \( C_y \) given in eqn. 4. Here, almost always, all data corresponding to a given cylinder diameter increase monotonically with the Cauchy number. It can, therefore, be inferred from these observations that the relevant first-order magnitude of \( \bar{\delta}_b/d_0 \) might be captured by the Cauchy number and the details of the vortex-laden flow influences the rest.

Note that results in section IV suggest that the time-averaged reconfiguration of all sheets here is essentially similar to the sheet reconfiguration in an steady, uniform flow. So, it is first assumed that \( i \) the mean flow in the channel provides the average sheet deflection along the flow direction, and also, it is expected that \( ii \) the periodic excitation by Bénard-Kármán vortices then provides the necessary vibrational energy for the flow-bent flexible sheet. In order to illustrate this effect, let us consider a toy-model shown in figure 5(b). It consists of rigid flat plate supported by a torsional spring of stiffness, say \( K \), and exposed to a steady, uniform flow containing a regular array of Bénard-Kármán vortices, each moving at some characteristic velocity in the direction of the steady flow. A simple model can be derived if we decompose the total work done by the vortex-laden flow on the flexible sheet into two distinct parts: \( i \) the steady component of the flow leads to the average angular position, say \( \bar{\xi} \), and \( ii \) the periodic interaction between the vortices and spring-supported flat plate results in torsional vibrations of the spring, and hence the plate’s angular position \( \xi(t) \). Then, locally, the average profile drag-induced moment should be balanced by the average restoring moment in the deflected sheet \( EI\text{d}\theta/\text{ds} \), where \( d\theta/\text{ds} \) is the sheet’s local curvature \( (1/R_c) \). As already observed in section IV at very large Cauchy number which compares drag force against the elastic restoring force, \( \bar{\delta}_b \ll L_b \) and hence, it can be safely assumed that \( EI\text{d}\theta/\text{ds} \approx EI/\bar{h}_b \), since \( R_c \) is approximately equal to the time-averaged sheet deflection \( \bar{h}_b \) (see figure 5). And so, we obtain

\[
\frac{EI}{\bar{h}_b} \sim \left( C_y^{1/3} \bar{\rho} U_0^2 w_b \bar{h}_b \right) \times \bar{h}_b, \\
\Rightarrow \frac{\bar{h}_b}{L_b} \sim C_y^{1/3},
\]

a result analogous to the well-known scaling for the static reconfiguration number \( \chi_2, \chi_3 \) that leads to drag reduction in flexible plates. When \( C_y \ll 1 \), on the other hand, \( \bar{h}_b \approx L_b \). Note that the experimental data provided in the previous section, as in figure 5 match fairly well with the large-Cauchy number scaling law, irrespective of Buoyancy number. In general, the above result could also be expressed as \( \bar{h}_b/L_b \sim C_y^{1/2} \), where the Vogel number \( \mathcal{V} = -2/3 \) at \( C_y \gg 1 \) \( \chi_2, \chi_3 \).

Furthermore, we propose that the vibrational energy of the sheet is solely taken from Bénard-Kármán vortices at some rate depending on the characteristics of the incoming unsteady flow, and during some time scale proportional to the shedding period \( 1/f_c \). Therefore, for the toy-model, we have

\[
\frac{1}{2} \xi \left( \frac{\delta_t}{L_b} \right)^2 \approx \left( \frac{1}{2} \bar{\rho} U_0^3 w_b d_v \right) \frac{1}{f_c},
\]

where the left-hand-side is the vibrational energy of the torsional spring and the right-hand-side is the product of local kinetic energy transfer rate, taken here as proportional to \( 1/2 \bar{\rho} U_0^3 w_b d_v \), and the typical timescale during which the transfer takes place periodically, i.e., \( 1/f_c \). Here, \( d_v \) is some typical lengthscale of Bénard-Karman vortices. Also, in the above expression, the stiffness \( K \xi \) of the pre-tensioned torsional spring can be ascertained from the equilibrium condition that \( K \xi \equiv EI\text{d}\theta/\text{ds} \approx EI/\bar{h}_b \), with \( \xi \equiv h_0/L_b \) when \( Ca \gg 1 \) since \( h_b \ll L_b \). Now, in terms of the Vogel number \( \mathcal{V} = -2/3 \) at \( C_y \gg 1 \) (or \( \mathcal{V} = 0 \) at \( C_y \ll 1 \), the dynamic reconfiguration number is given by \( \bar{h}_b/L_b \sim C_y^{1/2} \).
FIG. 8: Schematic of the flow and sheet parameters along with that of the torsional spring model.

(a) Flow and sheet parameters

(b) Torsional spring model

Hence, it can be deduced that $K \sim \left( \frac{EI}{L_b} \right) C_y^{-\nu}$, and thereafter the expression leads to

$$
\left( \frac{\delta_b}{L_b} \right)^2 \sim \left( \frac{\rho U_0^2 w_b L_h}{EI} \right) \left( \frac{U_0 d_v}{f_v} \right) C_y^{\nu},
$$

$$
\delta_b \sim C_n^{\nu} St_v^{-1/2}, \quad (11)
$$

where $St_v = f_v d_v / U_0$ is the Strouhal number based on a typical size of the eddies, and $n \equiv (1 + \nu) / 2 = 1/6$, or respectively $1/2$, for sufficiently large Cauchy numbers $C_y \gg 1$, or otherwise.

In fact, figure (e) could be seen as an illustration of the above result, if one allows a constant Strouhal number $St_h$, an uniform velocity profile $U_h = U_0$ and the typical length scale $d_v$ of the vortices to be equal to the cylinder diameter $d_0$. However, as inferred in section II, $St_h$ varies between $0.42$ and $0.22$ depending on the cylinder diameter and this also, implies that the typical vortex size and strength varies with the ratio $h_0 / d_0$. Also, the channel flow is not uniform. Thus, it is expected that the amplitude data should show lesser dispersion if some details of the Bénard-Kármán vortices shed by the cylinders are known.

A proper Particle Image Velocimetry (PIV) measurements in the flow domain which includes both the cylinder and the entire deflected sheet, notwithstanding the recirculation zone behind the sheet, is a huge task and is beyond the scope of the present work. For the purpose of this work, a set of PIV measurements in the region immediately downstream of the cylinder are undertaken. Not all configurations were considered but, in this study, only six pairs of $(h_0, U_0)$ were chosen for each cylinder diameter $d_0$. Nonetheless, these parameters cover the wide range of values for $h_0$, $U_0$ and $d_0$ presented in figures. A high-speed camera with a resolution of 1024 px × 1024 px is used to capture images of a particle-seeded flow at a frame rate of 125 fps. For the measurement, tracer particles with density $1005 \, \text{kg m}^{-3}$ and a diameter of $50 \, \mu\text{m}$ and $80 \, \mu\text{m}$ are added to the flow. A system of mirrors scatters a LASER beam into a thin LASER sheet which, depending on the cylinder diameter $d_0$, covered from 6 to 10 times $d_0$. Standard recommendations were followed for seeding, lighting and the relevant post-processing using DaVis Lavision software. Finally, to obtain the frequency associated with the Bénard-Kármán vortex street, a Fast Fourier Transformation is performed on the instantaneous vertical velocity given by PIV measurements. The frequency associated with the maximum spectral density is considered to be the vortex shedding frequency. To decrease the error, this process is repeated for each streamwise location on the centreline behind the cylinder to compute an average shedding frequency $f_{PIV}$.

Such PIV measurements are compared with the UDV-measured shedding frequency $f_{UDV} = f_c$ in figure (a). The scatter plot also provides colored data points from black to bright-yellow which represent the ratio between $h_0 / d_0 = 6$ and $h_0 / d_0 = 0.25$, respectively. Clearly all data fall between the trend line $f_{PIV} = f_{UDV}$ and $f_{PIV} = 0.8 f_{UDV}$. For larger frequencies, irrespective of $h_0 / d_0$, the equality is less pronounced. Furthermore, the instantaneous vorticity field can be computed from the measured velocity field at $x = 3d_0$ from the cylinder center. The absolute value of instantaneous vorticity profile presents a maximum at a vertical position corresponding to either a counter-clockwise, or clockwise, rotating vortex. Now, at each time step, the absolute maximum vorticity including the sign is counted...
FIG. 9: (a) Comparison between shedding frequency measured using Particle Image Velocimetry ($f_{PIV}$) and UDV ($f_{UDV}$) and (b) The product $\omega m d_0$ against the cylinder centreline velocity $U_{h0}$ where average maximum vorticity $\omega_m$ is obtained from the histogram of absolute maximum in the instantaneous vorticity profile at a fixed stream-wise location $x = 3d_0$.

FIG. 10: All data from figure 7(e) are expressed in terms of the rescaled oscillation amplitude of the sheet’s free-end $\delta_b/d_0$ as given by the eqn. 11 versus the Cauchy number $C_y = 12 \left( C_d \frac{\rho U_0^2}{E} \right) \left( L_b^3 / e_b^3 \right)$. Two distinct regimes are visible here.
FIG. 11: Power spectral density for all experimental cases as a function of the forcing frequency $f_v$, corresponding to imposed periodic shedding of Bénard-Kármán vortices. The frequency content of the [TOP] vertical velocity behind the cylinder, as measured using an UDV and [BOTTOM] vertical fluctuations of the sheet’s free-end. Dashed line is given simply to show the trend while symbols are provided for the sake of reference only (see figure 10 for the corresponding sheet physical properties).

in order to build-up an histogram (not provided here). In general, such an histogram displays two peaks, each representing the most-likely maximum vorticity of the clockwise and counter-clockwise vortices. The half-distance between these peaks is then referred to be the average maximum vorticity $\omega_m$ contained in shed vortices for a given set of experimental flow conditions, namely, $h_0$, $d_0$ and $U_0$. Figure 9(b) displays the product $\omega_m d_0$ as a function of the water speed at the cylinder centreline $U_{h0}$. Again, despite the variations in $h_0/d_0$, it is observed that for a given cylinder diameter, $\omega_m d_0 \propto U_{h0}$. So, by assuming that the maximum vorticity in Bénard-Kármán vortices is $U_{h0}/d_v$ where $d_v$ is some typical size of the vortex core, it is then possible to note from figure 9(b), the ratio $d_v/d_0$ for different cylinder sizes $d_0$.

Thereby, all the ingredients necessary to test the proposed scaling in eqn. 11 is now obtained. Figure 10 presents the same tip amplitude data given in figure 7(e) but instead, they are rescaled in terms of the experimentally obtained values of the Strouhal number $St_h$, cylinder centreline velocity $U_{h0}$ and the characteristic vortex length scale $d_v/d_0$, from figure 9(b). Clearly, experimental data ranging over all Cauchy numbers investigated here are regrouped around two distinct trend lines, namely $C_y^{1/2}$ and $C_y^{1/6}$, each corresponding to the case of moderately small and large Cauchy numbers, respectively, as expressed by 11. In the former, the sheet reconfiguration is sufficiently small and so, it represents vibration of a rigid sheet forced by a vortex street. Whereas, in the latter case, sheets can be considered to be flexible. These results strongly suggest that the torsional spring model contains the essential mechanism to explain the observed vortex-forced-vibration of thin sheets. They also imply that, analogous to drag-reduction via reconfiguration under an external flow, a flexible sheet experiences a smaller vibration amplitude compared to that of a rigid sheet when excited by a Bénard-Kármán vortex street.

VI. SHEET BEATING FREQUENCY

Figure 11 (a) displays a color plot of the power spectral density of the UDV-measured, instantaneous vertical velocity at the cylinder mid-span $z = d_0/2$ and at a distance $3d_0$ downstream the cylinder wake. As already mentioned, UDV measurements were acquired at 2 MHz during 3 minutes. Colors, from bright yellow to black, represent the
normalised spectra of the y-component velocity field across the cylinder diameter. Here, the spectra at each ycoordinate \((y \in [h_0, h_0 + d_0])\) is first computed and then an average in the discrete Fourier space is taken to obtain the normalised spectra. The latter presents a maximum (bright yellow, in figure 11 (a)) at a frequency \(f\) which is, by definition, equal to the vortex shedding frequency \(f_v\).

Similarly, figure 11 (b) presents the power spectral density for the sheet-tip \(y_b(t)\). For the sake of comparison, the x-axis is kept the same as just before. Therefore, this figure illustrates the energy content of the sheet-tip oscillation for each forcing frequency, equal to the shedding frequency of the Bénard-Kármán vortices \(f_v\). Note that the sheet tip position spectra is not quite the same as the y-component velocity power spectra discussed just before. Clearly, there are many cases where the power spectrum is wide for a fixed \(f_v\) : sheet’s vibrational energy is distributed across different frequencies and the dominant frequency \(f_h\) varies from case to case. Nonetheless, the dominant frequency of sheet tip fluctuations is observed, in general, to be lower than the vortex shedding frequency \(f_v\), as already briefly inferred in section III. Also, a second peak is often visible for some cases in figure 11 (b).

To further elucidate the distribution of vibrational energy in the sheet-tip vertical motion, figure 12 provides a few dominant tip frequencies \(f_h\), i.e., those frequencies corresponding to the prominent peaks in the corresponding power spectral density, for various imposed vortex shedding frequency. Size of the data points are proportional to the normalized power spectra. In this figure, data in each row corresponds to a single sheet characteristics, in the ascending order of the sheet stiffness ratio \(L_0/e_b\) while each column denotes data from experiments with the same cylinder diameter \(d_0\). The cylinder diameter \(d_0\) increases from left to right. Here, the continuous line (green) provided to indicate the cases when \(f_h = f_v\), i.e., the cases where the observed beating frequency of the sheet is equal to the forcing frequency due to Bénard-Kármán vortices. Now it can be inferred from the data displayed in the last column, for experiments with the largest cylinder (\(d_0 = 40\) mm), that the dominant frequency in the sheet-tip fluctuations is simply equal to the forcing frequency \(f_v\), irrespective of the Cauchy number \(C_y = 12\left(C_d \rho U^2 / E\right)\left(L_0^3 / e_b^3\right)\). This is in contrast with the case when \(d_0 = 10\) mm wherein \(f_h < f_v\) for all sheets and multiple prominent peaks in the power spectra of the tip’s vertical fluctuation \(y_b(t)\). However, for some case, namely, S442 and S1263, the prominent frequencies are about the first, or second natural sheet frequency, respectively, as given by eqn. 2. Whereas, for the other cases, the data is scatter around the dashed-line (blue) which denotes the expression \(f_h = 0.145U_0/h_b\), the flat-plate shedding frequency based on the sheet deflected height \(h_b\), as known from the dynamic sheet reconfiguration. We also observe similar dynamical regimes for the intermediate cylinder size \(d_0 = 20\) mm : (i) For the most rigid sheet S442 and for \(C_y < 20\), the dominant blade frequency is seen to be the forcing frequency \(f_v\), with a few dominant peaks either at the sheet’s first natural frequency \(f_{n1}\), (ii) For the most flexible sheets, namely, S2000 and S4000 and for \(C_y > 10^4\) the sheets display oscillations at the vortex shedding frequency of an inclined flat plate given by \(f_h = 0.145U_0/h_b\), and (iii) For moderate stiffness ratio and Cauchy numbers, the sheet oscillates either at one of its natural frequency which is close to the forcing frequency \(f_v\) or at the inclined plate shedding frequency \(f_h\). Finally, it is pointed out here that no conclusive evidence for a critical Cauchy number, nor a critical length scale for the vortices, is observed in these data. Nonetheless these observations strongly suggest that, in general, there is a transition from the forced-vortex-synchronous sheet-tip oscillation regime \(f_h = f_v\) to either a regime wherein the sheet-tip oscillations resemble the classical lock-in mode or a regime wherein the tip vibrations are induced by its wake characteristics. And the transition between these dynamical oscillation modes should depend on the relative size of the Bénard-Kármán vortices and the stiffness ratio.

VII. MODAL OSCILLATIONS : FLUTTER AND TRAVELING WAVE MODES

In the section III on the general remarks on sheet-tip dynamics, it was also pointed out that certain sheet-tips exhibit mild flutter-like back-and-forth motion while others present strong vertical oscillations (see figure 2). Indeed, the instantaneous shape of the most stiff sheet S442 and the least stiff sheet S4000 in figure 2 are distinctly different. In the former case, the local curvature does not change its sign throughout the length of the sheet while the latter presents a wave-like shape. This is the subject matter of this section.

Figure 13 (a) displays a color plot of the normalized local vertical oscillation amplitude \(\delta\) at different fixed points on the sheet as indicated by the curvilinear coordinate \(s\) so that \(\delta(s, t) = 2\sqrt{\tilde{y}(s, t)^2}\) at different fixed points on the sheet as indicated by the curvilinear coordinate \(s\) so that \(\delta(s, t) = 2\sqrt{\tilde{y}(s, t)^2}\) at different fixed points on the sheet as indicated by the curvilinear coordinate \(s\) so that \(\delta(s, t) = 2\sqrt{\tilde{y}(s, t)^2}\) at different fixed points on the sheet as indicated by the curvilinear coordinate \(s\) so that \(\delta(s, t) = 2\sqrt{\tilde{y}(s, t)^2}\). Data presented in figure 13 (a) corresponds to the entire observation period (7 mins) but data close to the sheet foot are not presented as the image detection near the channel bottom is poor, except for the sheet anchoring point. Also, bright yellow represents the upward motion \((\tilde{y}(s, t) > 0)\) of the blade with respect to its time-averaged position, and vice versa \((\tilde{y}(s, t) < 0)\) for the darker colors. A careful observation then indicates that there is a lag in upward fluctuations at the sheet-tip \((s = L_b)\) and the points located at \(s < L_b\). The same is true for downward fluctuations as well. This lag is readily visible in figure 13 (b) which compares the \(y\)-fluctuations at \(s = 0.9L_b\) and \(s = 0.3L_b\) (see also, supplementary video III). It is seen from the video that, in general, two oscillation
FIG. 12: A detailed view of the measured blade peak frequency as a function of the vortex shedding frequency $f_v$ for each case. Symbols denote the same cases as in figure 7. Symbol size at each forcing frequency $f_v$ is proportional to the normalized power spectral density. Here, the continuous line (——) represents $f_b = f_v$ while the dot-dash line (− ⋅ −) and the dotted line (⋯) indicate the sheet’s first and second natural frequency $f_{n1}$ and $f_{n2}$, respectively, and (− − −) is the flat-plate shedding frequency based on the sheet deflected height $f_h = 0.145U_b/h_b$. Also, in each case, pink bands indicate the forcing frequency range.
FIG. 13: (a) Spatio-temporal evolution of the normalized vertical displacement \( \tilde{y}(s, t)/2\sqrt{\tilde{y}(s, t)^2} \) for S2000 at \( U_0 = 8.8 \text{ cm s}^{-1} \), (b) Comparison between vertical blade displacement at two different points on the sheet, namely, \( \tilde{y}(s = 0.9L_b, t) \) and \( \tilde{y}(s = 0.3L_b, t) \) illustrates a time lag \( \Delta T_\delta > 0 \) and (c) Evolution of the time lag \( \Delta T_\delta \) as given by the cross-correlation between \( \tilde{y}(s = 0.9L_b, t) \) and \( \tilde{y}(s, t) \) for various values of \( s \ll L_b \), \( U_w = 10.8 \text{ cm s}^{-1} \) is the speed at which the vertical fluctuations propagate towards the sheet-tip is obtained by a linear data fit (---).

FIG. 14: (a) The speed \( U_w \) at which \( y \)-fluctuations travel towards the sheet-tip as a function of the time-averaged reconfiguration number \( \langle R \rangle = \langle h \rangle_b / L_b \) and (b) the normalized wave speed \( U_w/U_{h0} \) versus the Cauchy number \( C_y \), where \( U_{h0} \) is water speed at the cylinder center.
states are observed. In a first regime which resembles a low-amplitude flutter, the blade moves forth-and-back about a mean reconfigured position and in the second regime, transverse waves originate at some point \( s < L_0 \) and move along the sheet’s length towards its free-end. Figure 14 (c) presents measured time lag \( \Delta t(s) \) between the sheet y-fluctuations at some arbitrary point \( s \in [0, L_b] \), i.e., \( \bar{y}(s, t) \), with respect to sheet tip motion \( \bar{y}(s = L_0, t) \). The time lag \( \Delta t_b \) at some point on this sheet increases linearly with its distance from the sheet-tip. Indeed, this should correspond to a constant speed at which the transverse waves progress towards the sheet-tip which in this case is \( U_w = 3.8 \text{ cm s}^{-1} \), a little greater than the depth-averaged water speed \( U_0 = 2.2 \text{ cm s}^{-1} \).

Indeed, it is possible to systematically compute this lag for each of the case studied here. Figure 14 displays such measurements of the transverse wave speed \( U_w \). Data corresponding to \( U_w = 0 \) show no time lags i.e., \( \Delta t_b = 0 \). Clearly, almost all data points corresponding to the most stiff sheet S442 present measurements of the transverse wave speed \( U_w \) when the Cauchy number \( C_y \geq 10^2 \), the transverse wave speed \( U_w > 0 \). This implies that all other sheets display wavy modal oscillations with transverse waves which advance in the flow direction. Note that these observations are valid for all cylinder diameters \( d_0 \) in our experiments. Note that shortest sheets in this study are about 84 mm long and the biggest cylinder is about 40 mm. When \( C_y \) is sufficiently large, \( \bar{h}_b \leq 0.5L_b \) (see figure 9). For these cases, the most of the bending stress in the sheet is concentrated at the foot where the local sheet curvature is approximately \( \bar{h}_b \). Thereby, an effective free-end of length say \( l \sim L_b - \bar{h}_b \) is available for wave-like sheet motion if this length scale is at least comparable with, or greater than, the typical size of a vortex shed from the cylinder. This suggests that transverse waves appear only when the Cauchy number is substantially large so that the typical vortex size is comparably smaller than the length of the “stress-free” end of a flexible sheet. Furthermore, these two regimes can be identified with those found in the case of the non-dimensional oscillation amplitude \( \delta_b / d_0 \). The flutter mode is seen to occur when \( \delta_b / d_0 \propto C_y^{1/2} \) whereas the flag-like oscillations occur in the regime when \( \delta_b / d_0 \propto C_y^{1/6} \). Finally, in figure 13 the observed wave speed \( U_w \) varies between 1 to 3 times the local water speed at the cylinder centre \( U_{h0} \). And as the Cauchy number increases, it decreases and tends towards \( U_w / U_{h0} \sim 1 \).

VIII. CONCLUSIONS

The motion of an isolated quasi-2D artificial sheet subject to a transverse water flow that advect a periodic array Bénard-Kármán vortices shed by a cylinder upstream is experimentally investigated. Thin polyethylene sheets of varying lengths and thicknesses (\( L_b = 84 - 240 \text{ mm}; e_b = 0.06 - 0.19 \text{ mm} \)) and three different cylinder diameters \( d_0 = 10, 20 \) and 40 mm are used for this purpose in a long-narrow water channel. Each experiment consist of rigidly anchoring the sheet to the channel bottom and then systematically exciting its free-end by vortices shed by a cylinder upstream. The forcing frequency is \( f_s = 0.14 - 2.1 \text{ Hz} \) for different depth-averaged water speed \( U_0 = 22 - 88 \text{ mm s}^{-1} \), so that the cylinder Reynolds number varies between \( Re_d = \rho U_d d_0 / \mu = 240 - 3800 \).

Our experiments show that the time-averaged reconfiguration of a thin sheet follows qualitatively the same scaling with the Cauchy number \( C_y = 12 (C_a^2 \rho U_0^2 / E) (L_b^3 / e_b^3) \) as in the case of a thin sheet in an uniform, steady flow bends, so that its time-averaged profile drag is reduced. A simple bending beam model for steady flow which takes into account the drag force, and also the buoyancy force, as in [31], provides a reasonably good match with observations, if the flow speed \( U_0 \) is replaced by that of a local speed based on depth-averaged steady velocity profile given by Coles law [54]. Hence, the mean sheet dynamic reconfiguration number is very much similar to that of its static counterpart, as if the flow is steady. In addition, this suggests that the average drag force \( F_d = C_d 1/2 \rho U_0^2 \bar{h}_b w_b \propto U_0^{2+V} \). Here, \( \bar{h}_b \propto L_b U_0^3 \) is the time-averaged sheet deflection and \( V < 0 \) for drag reduction is the so-called Vogel number and it is observed to be \( V \approx -0.6 \pm 0.1 \) in the present work.

For a given blade thickness \( (e_b) \) and length \( (l_b) \), the oscillation amplitude \( (\delta_b) \) of the sheet tip increases with the Reynolds number \( Re_d \). It is also demonstrated that the Cauchy number is the appropriate parameter to scale all data in terms of the non-dimensional amplitude \( \delta_s / d_0 \). The underlying mechanism that controls sheet-tip oscillations is then analyzed via a toy-model. It consists of torsional spring-mounted rigid flat plate subject to an external flow which is disposed into a uniform steady flow and a regular array of vortices. If the former is taken to control the average sheet reconfiguration and the latter is assumed to provide the necessary work for the forced vibration the sheet, it is then shown that the rescaled oscillation amplitude \( \delta_s / d_s \sim C_y^{(1+V)/2} St_{s}^{-1/2} \), where \( d_s \) is the typical length scale of the vortex core such that the modified Strouhal number \( St_s = f_s d_s / U_0 \). In particular, for a relatively rigid sheet wherein the average drag force provides only a very little sheet deflection i.e., when \( V = 0, \delta_s / d_s \sim C_y^{1/2} \). This corresponds to the cases where the Cauchy number is moderate. Also, in this case, the sheet vibrates like a rigid curved plate as its local curvature does not change sign over the sheet’s entire length. On the other hand, at large Cauchy number \( C_y > 10^2 \), for a relatively flexible sheet wherein the average drag force results in a strong sheet reconfiguration i.e., when \( V \approx -2/3 \) [27], the non-dimensional vibration amplitude of the sheet free-end \( \delta_b / d_0 \sim C_y^{1/6} \). And in this case,
the sheet exhibits modal oscillations like a flapping flag. Here, transverse waves which travel forward towards the sheet free-end appear. The forward speed of such waves can attain as high as three times the local flow speed at the cylinder centreline $U_{b0}$, depending on the Cauchy number.

In regards to the beating frequency $(f_b)$ of the sheet free-end, three dynamical regimes are observed in this study: (i) Tip oscillations follow the forcing frequency $f_0$, corresponding to vortex shedding from the upstream cylinder, (ii) Tip oscillation frequency is related to vortex shedding behind a free inclined rigid plate of frontal height equal to the average sheet deflection $h_b$ such that $f_b \sim 0.145 U_{f0}/h_b$, and (iii) Sheet vibrations occur at one of its natural frequency $f_n$, near the forcing frequency. When the cylinder diameter $d_0 = 40$ mm, sheet tip oscillates at the forcing frequency $f_n$, irrespective of the Cauchy number studied here ($C_y < 10^4$). For smaller cylinders, the sheet displays \textit{flow-induced vibration} controlled by its wake characteristics as in (ii) or a \textit{lock-in} motion at its natural frequency as in (iii). Furthermore, this transition is possibly dependent on the average sheet reconfiguration and sheet thickness as well.

As mentioned in the classical book on Flow-Induced-Vibration by Blevins [18], many studies on flow-induced oscillations can often be classified into two general categories depending on the incoming flow, namely, a steady or an unsteady flow. Our work is a sub-category of the latter case wherein we provide a case study for interactions between eddies and a flexible sheet. Indeed, more work is necessary to understand and hence predict the above-mentioned dynamical regimes via flow visualization, PIV measurements of the flow around the sheet and in its wake. Moreover, the pertinence of these results for a canopy of flexible structures like plants (artifical or natural), and for different mass ratio as well, is left for future investigations.

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