Asymptotic quasinormal modes of a coupled scalar field in the Gibbons–Maeda dilaton spacetime

Songbai Chen and Jiliang Jing

Institute of Physics and Department of Physics, Hunan Normal University, Changsha, Hunan 410081, People’s Republic of China
E-mail: csb3752@hotmail.com and jljing@hunnu.edu.cn

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Abstract
Adopting the monodromy technique devised by Motl and Neitzke, we investigate analytically the asymptotic quasinormal frequencies of a coupled scalar field in the Gibbons–Maeda dilaton spacetime. We find that it is described by
\[
\mathbf{e}^{\beta_\omega} = -\left[1 + 2 \cos \left(\frac{\sqrt{2} \pi}{2} \xi + \frac{\pi}{2}\right)\right] - \mathbf{e}^{-\beta_\omega} \left[2 + 2 \cos \left(\frac{\sqrt{2} \pi}{2} \xi + \frac{\pi}{2}\right)\right],
\]
which depends on the structure parameters of the background spacetime and on the coupling between the scalar and gravitational fields. As the parameters \(\xi\) and \(\beta_\omega\) tend to zero, the real parts of the asymptotic quasinormal frequencies become \(T_H \ln 3\), which is consistent with Hod’s conjecture. When \(\xi = \frac{91}{18}\), the formula becomes that of the Reissner–Nordström spacetime.

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1. Introduction

The study of quasinormal modes in a black-hole background spacetime has a long history [1–19]. It is shown that their frequencies and damping times are entirely fixed by the black-hole parameters and independent of the initial perturbations. Therefore, it is certain that quasinormal modes carry the characteristic information about a black hole, and it can provide a new and direct way for astrophysicists to search for black holes in the universe.

Recently, a great deal of effort [20–31] has been devoted to the study of the asymptotic quasinormal modes because Hod’s conjecture [20] shows that there may exist a connection between the asymptotic quasinormal frequencies and quantum gravity. In terms of Bohr’s correspondence principle, the transition frequencies at large quantum numbers \((n \to \infty)\) should be equal to classical oscillation frequencies. Hod [20] generalized Bohr’s correspondence principle to the black-hole physics and regarded the real parts of the asymptotic quasinormal frequencies \((n \to \infty)\) as the characteristic transition frequency for the black hole. Moreover, Hod observed that the real parts of highly damped quasinormal frequencies in a Schwarzschild black hole can be expressed as \(\omega_R = T_H \ln 3\), which is derived numerically by Nollert [21] and later confirmed analytically by Motl [22, 23] and Andersson [24]. Together with the first law of black-hole thermodynamics, Hod obtained the value of the fundamental
area unit in the quantization of black-hole horizon area. Following Hod’s works, Dreyer [25] found that the asymptotic quasinormal modes can fix the Barbero–Immirzi parameter which is introduced as an indefinite factor by Immirzi [26] to obtain the right form of the black-hole entropy in the loop quantum gravity. Furthermore, Dreyer obtained that the basic gauge group in the loop quantum gravity should be $SO(3)$ rather than $SU(2)$. These exciting new results imply that Hod’s conjecture may create a new way to probe the quantum properties of black holes.

However, the question whether Hod’s conjecture applies to more general black holes still remains open. In their deduction, it is obvious that the factor $\ln 3$ in the quasinormal frequencies plays an essential and important role. In other words, whether Hod’s conjecture is valid depends on whether the factor $\ln 3$ appears in the asymptotic quasinormal frequencies or not. Recently, we [27] probed the asymptotic quasinormal modes of a massless scalar field in the Garfinkle–Horowitz–Strominger dilaton spacetime and found that the frequency spectra formula satisfies Hod’s conjecture. For the non-flat spacetime, Cardoso and Yoshida [28, 29] found that the asymptotic quasinormal frequencies in the Schwarzschild de Sitter and anti-de Sitter spacetimes depend on the cosmological constant. Only in the case that the cosmological constant vanishes, do the real parts of the asymptotic quasinormal frequencies return $T_H \ln 3$. For the Reissner–Nordström black hole, Motl and Neitzke [23] found that the asymptotic quasinormal frequencies satisfy

$$e^{\beta_0} + 2 + 3e^{-\beta_I} = 0,$$  \hspace{1cm} (1.1)

where $\beta$ and $\beta_I$ are the inverse black-hole Hawking temperatures of the outer and inner event horizons, respectively. It is obvious that it is relevant to the electric charge $Q$ and the real parts of the quasinormal frequencies are not equal to $T_H \ln 3$. The more perplexing is that formula (1.1) does not return the Schwarzschild limit as the black-hole charge $Q$ tends to zero.

Some authors suggested [30] that Hod’s conjecture about $T_H \ln 3$ is valid only in the pure gravitational perturbations case. Formula (1.1) does not return the Schwarzschild limit as the black-hole charge $Q$ tends to zero because it describes a complicated effect of the material–gravitational coupling. After considering the interaction between the matter and gravitational fields, Hod’s conjecture may possess a more general form. Thus, it is necessary to study the contribution of the interaction between the matter and gravitational fields to the asymptotic quasinormal modes in the more general background spacetimes. Recently, we [31] studied the asymptotic quasinormal modes of a coupled scalar field in the Garfinkle–Horowitz–Strominger dilaton spacetime. We found that the asymptotic quasinormal frequencies depend both on the structure parameters of the background spacetime and on the coupling between the matter and gravitational fields. Moreover, we noted that only in the minimal coupled case, are the real parts of the asymptotic quasinormal frequencies consistent with Hod’s conjecture about $T_H \ln 3$.

In this paper, our main purpose is to investigate the asymptotic quasinormal modes of a coupled scalar field in the Gibbons–Maeda dilaton black-hole background. Our plan for the paper is as follows. In section 2, we derive analytically the asymptotic quasinormal frequency formula of a coupled scalar field in the Gibbons–Maeda dilaton spacetime by making use of the monodromy method. Finally, a summary and some discussions are presented.

2. Asymptotic quasinormal frequencies formula of a coupled scalar field in the Gibbons–Maeda dilaton spacetime

The metric for the Gibbons–Maeda dilaton spacetime is [32]

$$ds^2 = \frac{(r^2_1 - r^2_2)(r - r_2)}{r^2 - D^2} dr^2 + \frac{r^2 - D^2}{(r - r^2_1)(r - r^2_2)} dr^2 + (r^2 - D^2)(d\theta^2 + \sin^2 \theta \, d\phi^2),$$  \hspace{1cm} (2.1)
Asymptotic quasinormal modes of a coupled scalar field

where

\[ D = (P^2 - Q^2)/2M \quad \text{and} \quad r'_\pm = M \pm \sqrt{M^2 + D^2 - P^2 - Q^2}. \]

The parameters \( P \) and \( Q \) represent the black-hole magnetic and electric charges. When \( P = 0 \) and \( D = 0 \), the metric returns the Garfinkle–Horowitz–Strominger dilaton spacetime and the Reissner–Nordström spacetime, respectively.

We introduce a coordinate change

\[ r^2 = r'^2 - D^2, \] (2.2)

and metric (2.1) can be rewritten as

\[
\begin{align*}
\text{d}s^2 &= -(\sqrt{r^2 + D^2 - r'_-})(\sqrt{r^2 + D^2 - r'_+}) \frac{r^2}{r^2} \text{dr}^2 + \frac{r^2}{(r^2 + D^2)} \text{dr}_+^2 + r^2(\text{d}\theta^2 + \sin^2 \theta \text{d}\phi^2),
\end{align*}
\]

(2.3)

The outer and inner horizons lie in \( r'_\pm = \sqrt{r^2_{\pm} - D^2} \).

The general perturbation equation for a coupled massless scalar field in the dilaton spacetime is

\[
\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu
u} \partial_\nu) \psi - \xi R \psi = 0, \] (2.4)

where \( \psi \) is the scalar field and \( R \) is the Ricci scalar curvature. The coupling between the scalar and gravitational fields is represented by the term \( \xi R \psi \), where \( \xi \) is a numerical coupling factor.

After adopting the WKB approximation \( \psi = e^{-i \omega t} Y(\theta, \phi) \), introducing the ‘tortoise coordinate’ change

\[
x = \sqrt{r^2 + D^2} - D + \frac{r^2 - D^2}{r'_+ - r'_-} \ln \left[ \frac{\sqrt{r^2 + D^2 - r'_+}}{r'_+ - D} \right] - \frac{r^2 - D^2}{r'_+ - r'_-} \ln \left[ \frac{\sqrt{r^2 + D^2 - r'_-}}{r'_- - D} \right]
\]

(2.5)

and substituting equations (2.3) and (2.5) into equation (2.4), we can obtain the radial perturbation equation for a coupled scalar field in the Gibbons–Maeda dilaton spacetime as

\[
\frac{d^2 \phi}{dx^2} + (\omega^2 - V[r(x)]) \phi = 0,
\]

(2.6)

where

\[ V[r(x)] = \frac{(\sqrt{r^2 + D^2} - r'_+)(\sqrt{r^2 + D^2} - r'_-)}{r^2} \left[ l(l + 1) \right] - \frac{3D^2(\sqrt{r^2 + D^2} - r'_+)(\sqrt{r^2 + D^2} - r'_-)}{r^6} \left[ \sqrt{r^2 + D^2}(r'_+ + r'_-) - 2r'_+ r'_- \right] + \xi R, \] (2.7)

and

\[ R = \frac{2D^2[D^2 + r^2 - \sqrt{D^2 + r'_+ r'_-} + r'_+ r'_-]}{r^6}. \] (2.8)

As in [23, 27], extending equation (2.6) analytically to the whole complex \( r \)-plane and comparing the local with global monodromy of \( \phi(r) \) along the selected contour \( L \), we can obtain the asymptotic quasinormal frequency spectra in the Gibbons–Maeda dilaton.
Figure 1. The complex $r$-plane and the contour $L$. The shaded regions denote the area $\text{Re}(x) < 0$.

black-hole spacetime. Because the regions for the different signs of $\text{Re}(x)$ (as shown in figure 1) are clearly different from those in the Garfinkle–Horowitz–Strominger dilaton spacetime case, the contour $L$ we selected is more complicated than the former. Here, to avoid the inner horizon at $r = \sqrt{r'' - D^2}$, the selected contour only goes around the outer horizon at $r = \sqrt{r'' + D^2}$, where the boundary conditions and the monodromy are well defined. In order to compute conveniently, we introduce the variable $z = x - 2\pi i (r'' - D) + 2\pi i (r'' + D)$. For $r = 0$, we have $z = 0$. In the vicinity of the point $r = 0$, we have

$$z \sim \frac{r^4}{4D(r'' - D)(r'' - D)},$$  \hspace{1cm} (2.9)

and the behaviour of the Ricci scalar curvature and the potential is

$$R \sim \frac{2D^2(r'' - D)(r'' - D)}{r^6},$$  \hspace{1cm} (2.10)

and

$$V[r(z)] \sim -\frac{3 - 2\xi}{16\xi^2}.$$  \hspace{1cm} (2.11)

We make the identification $j = \frac{\sqrt{2\xi + 1}}{2}$, and then the perturbation equation (2.6) can be rewritten as

$$\left(\frac{d^2}{dz^2} + \omega^2 + \frac{1 - j^2}{4z^2}\right)\phi(z) = 0.$$  \hspace{1cm} (2.12)

As before, from the boundary condition at point $A$,

$$A_+ e^{-i\omega} + A_- e^{-i\omega} = 0,$$  \hspace{1cm} (2.13)

we easily obtain that the asymptotic form of $\phi(z)$ at the point marked $A$ is

$$\phi(z) \sim (A_+ e^{i\omega} + A_- e^{-i\omega}) e^{-i\omega z},$$  \hspace{1cm} (2.14)

To follow the contour $L$ and approach point $B$, we first turn an angle $\frac{\pi}{2}$ around the origin point, corresponding to $2\pi$ around $z = 0$. From the Bessel function behaviour near the point of origin, we find that after the $2\pi$ rotation the asymptotics are

$$\phi(z) \sim (A_+ e^{i\omega z} + A_- e^{-i\omega z}) e^{-i\omega z} + (A_+ e^{3i\omega z} + A_- e^{-3i\omega z}) e^{i\omega z}.$$  \hspace{1cm} (2.15)
Then we must go out along the lobe where the behaviour of the wavefunction can be approximated as purely oscillatory. After going around the lobe, we return to the Bessel region a second time. However, this time, we must add an additional distance to \( z \), namely,

\[
\delta = -2\pi i \left( \frac{r'^2 - D^2}{r'_+ - r'_-} \right).
\] (2.16)

Thus, the general solution to equation (2.12) is

\[
\phi(z) = B_+ e^{3i\alpha} + B_- e^{-3i\alpha} + c_+ \sqrt{\omega(z - \delta)} J_{-j/2}(\omega(z - \delta)) + c_- \sqrt{\omega(z - \delta)} J_{j/2}(\omega(z - \delta)).
\] (2.17)

In terms of the continuity and asymptotic behaviour of the wavefunction, we have

\[
A_+ e^{3i\alpha} + A_- e^{-3i\alpha} = (B_+ e^{i\alpha} + B_- e^{-i\alpha}) e^{-i\omega\delta},
\] (2.18)

To approach the point \( B \), we must finally turn the second \( 2\pi \) rotation around the point \( z = \delta \). The asymptotic forms of the wavefunction \( \phi(z) \) near the point \( B \) become

\[
\phi(z) \sim (B_+ e^{3i\alpha} + B_- e^{-3i\alpha}) e^{-i\omega\delta} e^{-i\omega z} + (B_+ e^{5i\alpha} + B_- e^{-5i\alpha}) e^{-i\omega\delta} e^{i\omega z}.
\] (2.19)

Running over the large semicircle to come back to the point \( A \), we find that the wavefunction \( e^{-i\omega z} \) must multiply the factor

\[
\frac{(B_+ e^{3i\alpha} + B_- e^{-3i\alpha}) e^{-i\omega\delta}}{A_+ e^{i\alpha} + A_- e^{-i\alpha}} = -(1 + 2 \cos \pi j) e^{-i\omega z} (2 + 2 \cos \pi j).
\] (2.20)

In the globe monodromy analysis of \( \phi(z) \) around the point \( r = r_+ \), we find that the coefficient of \( e^{-i\omega z} \) must be multiplied by \( e^{i\beta_1} \), where \( \beta_1 = 4\pi \left( \frac{r'^2 - D^2}{r'_+ - r'_-} \right) \) is the inverse black-hole Hawking temperature of the outer event horizon. Then matching the local and globe monodromies of the function \( \phi(z) \), we find that the asymptotic quasinormal frequency spectra formula of a coupled scalar field in the Gibbons–Maeda spacetime is

\[
e^{i\beta_1} = - \left[ 1 + 2 \cos \left( \frac{\sqrt{2\xi + 1}}{2} \right) \right] e^{-i\beta_1/2} \left[ 2 + 2 \cos \left( \frac{\sqrt{2\xi + 1}}{2} \right) \right],
\] (2.21)

where \( \beta_1 = 4\pi \left( \frac{r'^2 - D^2}{r'_+ - r'_-} \right) \) is the inverse black-hole Hawking temperature of the inner event horizon. As our expectation, the asymptotic quasinormal frequency spectra formula of a coupled scalar field in the Gibbons–Maeda dilaton spacetime also depends on the coupling between the scalar and the gravitational fields. Moreover, we find that as both \( P = 0 \) and \( \xi = 0 \), the real part of quasinormal frequency becomes \( T_H \ln 3 \), which is consistent with Hod’s conjecture.

3. Summary and discussion

We have investigated the analytical forms of the asymptotic quasinormal frequencies for a coupled scalar field in the Gibbons–Maeda dilaton spacetime by adopting the monodromy technique. As in the Garfinkle–Horowitz–Strominger dilaton spacetime, the asymptotic quasinormal frequencies depend not only on the structure parameters of the background spacetime, but also on a couple constant \( \xi \). The fact again tells us that the interaction between the matter and gravitational fields affects the frequency spectra formula of the asymptotic quasinormal modes. Moreover, formulae (2.21) and (1.1) look like very similar. Both formulae have a correction term which comes from the phase shift \( \delta \). As formula (1.1) cannot yield the Schwarzschild limit when the electric charge \( Q \) tends to zero, the frequency
spectra formula (2.21) still does not return to the formula of a coupled scalar field in the Garfinkle–Horowitz–Strominger dilaton black hole as the magnetic charge $P$ approaches zero. Similarly, formula (2.21) also cannot return that of in the Reissner–Nordström spacetime as $D$ approaches zero. The mathematical reason for this behaviour is that the variable $z$ and the effective potential $V$ depend heavily on the parameters $D$ and $P$. In the cases $D = 0$ and $P = 0$, the forms of the variable $z$ and the effective potential $V$ near the point $z = 0$ are clearly different from equations (2.9) and (2.11). However, when both $\xi$ and $P$ vanish, we find that the real part of quasinormal frequency in the Gibbons–Maeda dilaton spacetime can become $T_H \ln 3$, which agrees with that of a minimal coupled scalar field in the Garfinkle–Horowitz–Strominger dilaton spacetime. More interestingly, when $\xi = \frac{91}{18}$, formula (2.21) becomes $e^{\beta \omega} + 2 + 3e^{-\beta \omega} = 0$, which is entirely consistent with that of the Reissner–Nordström black-hole spacetime. This may imply that the coupling between the matter and gravitational field plays an important role in the asymptotic quasinormal modes. The relation between the couple factor $\xi$ and the asymptotic quasinormal modes in more general spacetime needs to be studied more deeply in future.

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