NUCLEUS-NUCLEUS BREMSSTRAHLUNG FROM ULTRARELATIVISTIC COLLISIONS

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Abstract

The bremsstrahlung produced when heavy nuclei collide is estimated for central collisions at the Relativistic Heavy Ion Collider. Soft photons can be used to infer the rapidity distribution of the outgoing charge. An experimental design is outlined.

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I. INTRODUCTION

The Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory (BNL) will have its first beam available for experiments by the end of 1999. A basic issue is how transparent the nuclei are to each other. This issue is important because it determines the initial energy density of hot matter whose study is, after all, the ultimate physics goal. The definition of transparency is not unique; transparency may be realized as the deceleration of baryons, or of electric charge, or the number of hadrons produced. The fact that these different realizations are not tightly coupled may be illustrated by the recognition that in a cascade picture the number of produced particles decreases each time a baryon collides because of the decreased energy available in comparison with the previous collision. Hence the first baryon collision is the most significant from the particle production point of view and the last is the least significant. In contrast, each collision slows the baryon and each is equally significant because the rapidity loss per collision is practically independent of beam energy. In this paper we shall focus on the deceleration of charge as a global indicator of transparency. The advantage of charge over baryon number has to do with experimental detection. A significant fraction of baryon number may emerge in the form of neutrons and hyperons whose detection is complicated. Global stopping of electric charge is reflected in the amount of bremsstrahlung which can be readily computed and detected; this is the focus of the paper. Of course, the net charge as a function of rapidity can also be measured directly, but this requires identification of all charged particles over all of momentum space, and is much more difficult than the experiment to be discussed here.

The first theoretical study of nucleus-nucleus bremsstrahlung was done by one of the authors [1] and intended for application at the Bevalac where the beam energy was in the range of 250 to 2100 MeV per nucleon in the laboratory frame. A later study was done by Bjorken and McLerran [2] which was aimed at much higher energies. At the time of that study it was anticipated that the central rapidity region would be relatively free of baryons because of high transparency [3]. Recent experimental studies at the SPS at CERN [4] are
suggesting that many more baryons than anticipated will emerge at central rapidities. The baryon and charge rapidity distributions at RHIC may even be flat! Therefore it makes sense to reinvestigate the bremsstrahlung to be expected from central collisions of the heaviest beam nuclei at RHIC and a potential experimental design to measure it.

Emission of soft photons was also explored theoretically by Dumitru, McLerran, Stocker and Greiner in the context of the RQMD (Relativistic Quantum Molecular Dynamics) model. They point out that $\pi^0$ decay photons contribute a negligible amount to the soft photon spectrum at forward angles, suggesting the feasibility of the bremsstrahlung measurement. They propose to study charge stopping by measuring the total energy radiated as soft photons. Since the spectrum peaks at a very small angle, the measurement of total energy is actually impossible at RHIC because the detector would have to be placed within the beam pipe and would interfere with the operation of the collider. We will show that measurement of the spectrum at several angles in the range from about 2 to 10 degrees is sufficient to discriminate different stopping scenarios, and outline a detector design to do so.

II. CALCULATION OF BREMSSTRAHLUNG

Consider a central collision of two equal mass nuclei of charge $Z$ in their center of momentum frame. Denote the speed of each beam by $v_0$ and the corresponding rapidity by $y_0$, so that the rapidity gap between projectile and target nuclei is $2y_0$. Velocity and rapidity are related by $v = \tanh y$. Before collision each nucleus may be viewed as a flattened pancake with negligible thickness. The current carried by the projectile is:

\[ 1 \text{The study by Bjorken and McLerran was directed towards fixed target experiments. As an aside they boosted the bremsstrahlung distribution to the center of momentum frame, as befits a collider, but made a sign error in the Lorentz transformation. This led to the expectation that the bremsstrahlung would be in the GeV range, whereas it ought to be in the MeV range, as shown below.} \]
and by the target is:

\[ \mathbf{J}_T(x, t) = -v_0 \sigma(r_{\perp}) \delta(z + v_0 t) \hat{\mathbf{z}}, \]  

(2)

where \( \sigma(r_{\perp}) \) is the charge per unit area at a distance \( r_{\perp} \) from the central beam axis. For low frequency photons the nucleus-nucleus collision appears almost instantaneous. To the extent that the transverse rapidities of the outgoing charged particles are small compared to the beam rapidity one may then approximate the current after the collision as:

\[ \mathbf{J}_F(x, t) = \hat{\mathbf{z}} \sigma(r_{\perp}) \theta(t) \int_{-\infty}^{\infty} dy \rho(r_{\perp}, y) v(y) \delta(z - v(y) t). \]  

(3)

Here \( \rho \) represents the charge rapidity distribution and is normalized as:

\[ \int_{-\infty}^{\infty} dy \rho(r_{\perp}, y) = 2, \]  

(4)

the 2 arising because the total charge is \( 2Z \).

The classical amplitude to emit electromagnetic radiation in the direction \( \mathbf{n} \) with frequency \( \omega \) is:

\[ \mathbf{A}(\mathbf{n}, \omega) = \int dt \int d^3 x \, \mathbf{n} \times (\mathbf{n} \times \mathbf{J}(x, t)) e^{i \omega (t - \mathbf{n} \cdot x)}. \]  

(5)

Here \( \mathbf{J} = \mathbf{J}_P + \mathbf{J}_T + \mathbf{J}_F \) is the total current. It is convenient to take \( \mathbf{n} = (\sin \theta, 0, \cos \theta) \). Then the distribution in frequency and direction is:

\[ \frac{d^2 N}{d\omega d\Omega} = \frac{\alpha}{4\pi^2 \omega} \sin^2 \theta \left[ \int dx dy \sigma(r_{\perp}) e^{-i\omega \sin \theta} \left[ \int dy \frac{v(y) \rho(r_{\perp}, y)}{1 - v(y) \cos \theta} - \frac{2v_0^2 \cos \theta}{1 - v_0^2 \cos^2 \theta} \right]^2 \right]. \]  

(6)

We cannot go further without some knowledge of the distribution \( \rho(r_{\perp}, y) \).

In Fig. 1 we plot the quantity \( \rho(r_{\perp}, y) \) as computed in LEXUS for central Au+Au collisions at 100 GeV per nucleon in the cm frame. LEXUS is a linear extrapolation of nucleon-nucleon scattering to nucleus-nucleus collisions. It is based on sequential nucleon-nucleon scatterings, as in free space, with energy loss taken into account. For \( r_{\perp} = 0 \) this distribution has a broad maximum at \( y = 0 \), whereas for \( r_{\perp} \approx R \), the nuclear radius,
this distribution has a broad minimum at \( y = 0 \). Overall the charge rapidity distribution is roughly flat. We have computed the bremsstrahlung from LEXUS and will display it shortly.

It is also advantageous to have a simple analytic model with a variable charge rapidity distribution. To this end suppose that \( \rho(r_\perp, y) \) is independent of \( r_\perp \). Then

\[
\frac{d^2 N}{d\omega d\Omega} = \frac{\alpha Z^2}{4\pi^2} \sin^2 \theta |F(\omega \sin \theta)|^2 \left[ \int dy \frac{v(y)\rho(y)}{1 - v(y)\cos \theta} - \frac{2v_0^2 \cos \theta}{1 - v_0^2 \cos^2 \theta} \right]^2,
\]

where \( F \) is a nuclear form factor:

\[
F = \frac{1}{Z} \int dx dy \sigma \left( \sqrt{x^2 + y^2} \right) e^{-ix\omega \sin \theta}.
\]

A solid sphere approximation should be adequate for large nuclei, in which case

\[
F(q) = \frac{3}{q^2} \left( \frac{\sin q}{q} - \cos q \right),
\]

where \( q = \omega R \sin \theta \) and \( R \) is the nuclear radius. Actually, for the range of angles and frequencies of interest to us, the nuclear form factor is practically equal to one.

The integral over rapidity can easily be performed for a flat rapidity distribution.

\[
\rho(y) = \frac{\theta(y_0 - y)\theta(y_0 + y)}{y_0}
\]

Then the photon distribution is:

\[
\frac{d^2 N}{d\omega d\Omega} = \frac{\alpha Z^2}{4\pi^2} \sin^2 \theta |F(\omega R \sin \theta)|^2 
\times \left[ \frac{2}{\sin^2 \theta} - \frac{1}{y_0 \sin^2 \theta} \ln \left( \frac{1 + v_0 \cos \theta}{1 - v_0 \cos \theta} \right) - \frac{2v_0^2 \cos \theta}{1 - v_0^2 \cos^2 \theta} \right]^2.
\]

Because \( v_0 \) is close to 1 the distribution is highly modified from the quadrupole form, being strongly peaked near small but nonzero \( \theta \). At RHIC the peak occurs at \( \theta \approx 1^\circ \).

Since the frequency dependence of bremsstrahlung photons is \( 1/\omega \), for low frequencies anyway, there is no loss in information in integrating over a range of frequencies. For the detector described in the next section we can consider all photons with energies between 10 keV and 3 MeV. Fig. 2 shows the spectra for central gold collisions at RHIC using both
LEXUS and a flat charge rapidity distribution which is independent of $r_\perp$. The two models give almost identical results, as may be expected based on Fig. 1.

We have also computed the spectra resulting from three different final charged particle rapidity shapes that span the possibilities at RHIC. These shapes are shown in Fig. 3. The curve $\alpha$ represents a maximal stopping scenario, $\beta$ an approximately 50% stopping case, and $\gamma$ the case of minimal stopping. The results are shown in Fig. 4. Once again we have taken only photon energies between 10 keV and 3 MeV so as to match the experimental design described in the following section. There is a very clear difference in the photon spectrum between the cases of “full stopping” and “transparency”. The cases of “50% stopping” and a flat rapidity distribution are very similar, although they are still very different from the other two cases, and even these can be distinguished with sufficient statistics and angular resolution in an experiment.

To conclude this section we note that the classical bremsstrahlung formula ought to be accurate for photon energies less than $\hbar/\tau$, where $\tau$ is the time elapsed between first nuclear contact and the last hard scattering of the charged hadrons. For $\tau$ on the order of 10 to 50 fm/c this restricts the validity to photons with energies less than 20 to 4 MeV, respectively.

III. A DETECTOR DESIGN

The bremsstrahlung yields calculated above are measurable in a relatively modest experiment. The basic features of the bremsstrahlung which allow this are the low energies, less than a few MeV, and concentration to forward angles, with almost all of the radiation within 10° of the beam line. In the following we describe a simple experiment which could detect and measure this bremsstrahlung. This is meant as an example of how it might be done. Of course, a real experiment would need to be concerned with many other details and might well choose somewhat different dimensions, number of detectors, and so on. The discussion presented below is limited in detail as appropriate for this paper. However, we believe that sufficient information is presented to show the feasibility of the measurement.
A. Direct $\pi^0$ Background

An experiment to detect and measure this radiation can be based on the detection of gamma rays with energies ranging from 10 keV to 3 MeV. At first thought, one might imagine that the most dangerous backgrounds are photons from the decay of $\pi^0$ mesons which happen to lie in this energy range. Because of the large mass of the $\pi^0$ (140 MeV) the typical $\pi^0$ decay photon has substantially greater energy. Essentially the only way a $\pi^0$ decay photon can be in this range is for the meson to be moving away from the detector and then to decay at backward angles, with just the right kinematics.

As an example, we note that calculations using HIJET [8] show that for the experiment described below, the $\pi^0$ decay background is smaller than the expected bremsstrahlung rate (for 3° and the flat rapidity spectrum) by a factor of $4 \times 10^4$. Thus, this background is negligible, as pointed out already by Dumitru et al. [3]. Incidentally, if in the future, very precise measurements of the bremsstrahlung were desired, this background could be calculated from the $\pi^0$ spectrum, which would be known by that time, and subtracted.

B. Experimental Configuration

The photon detector proposed is shown in Fig. 5. The Ge detector detects the low energy photons by photoabsorption for the low end of the spectrum and Compton scatters for the high end. Detected energy in excess of 3 MeV will not be accepted as a valid event. The role of the cylindrical scintillation counter surrounding the Ge cube is to veto higher energy gamma rays and charged particles. The aluminum cold finger is connected to a liquid nitrogen supply (not shown) and keeps the Ge detector at liquid nitrogen temperature.

The dominant gamma ray flux is from the decay of $\pi^0$ mesons and is in the several GeV range. The pairs produced by these photons will be detected and vetoed by the scintillation counter. Compton scatters with final electron energy above about 3 MeV will also leave the Ge and be vetoed by the scintillator. The precise response of the counter assembly
to the low energies of the bremsstrahlung photons can be well predicted but can also be calibrated with the use of sources. The detected energy spectrum will thus not be the true bremsstrahlung spectrum but will be calculable from the expected bremsstrahlung spectrum (properly weighting photoabsorption and Compton scattering).

Since the shape of the expected frequency spectrum does not depend on the stopping or the detection angle, the agreement between the measured and predicted spectral shapes will be an important check that the experiment is actually measuring the bremsstrahlung.

One aspect of the detection system is that the background particles not be so numerous that the apparatus has no live time to detect the real bremsstrahlung photons. With the collimation scheme proposed below, the live time is estimated to be 85%. The magnet and the collimator are necessary to reduce the dead time due to charged particles to reach this level of live time.

Figure 6 shows an elevation view of the experiment. (The small counter system which determines the vertex position is not shown.) The apparatus has four photon detectors of the type shown in Fig. 5 at polar angles of 3°, 5°, 7°, and 9°. As can be seen from Figs. 2 and 4, this permits a characterization of the bremsstrahlung.

A view of the proposed experiment looking back towards the interaction point from the detector side is shown in Fig. 7. The collimator slit is narrow and matched to the detector width. The magnet sweeps charged particles to the left and right, as seen in Fig. 7. Most of the charged particle flux then misses the narrow slit in the collimator and does not reach the detector. As mentioned above, this feature is necessary to achieve a sufficiently large live time for the experiment.

C. Backgrounds to the Measurement and Their Solution

The studies and simulations which have been done indicate that the only background in the experiment arises from interactions which occur in the beam pipe, the collimator, the ancillary material, etc. which produce π⁰ mesons which in turn produce gamma rays
which in turn shower in the various materials. The gamma ray showers have a non-negligible probability of producing a low energy gamma in the bremsstrahlung detection range. The location of these showers is widely spread out physically and essentially occurs where the particles produced in the gold-gold collision interact with the surrounding material.

This background is considerable and is comparable to the bremsstrahlung signal in rate. Thus, the experiment must measure this background since the calculations are not precise enough to allow a theoretical subtraction. It is also essential to use a Be, rather than Al, beam pipe to keep this background at a level which can be measured in a reasonable time.

There is a simple way to measure this background. Consider one photon detector. If a plug, small in area - matched to the small size of the detector - is placed behind the collimator, between the primary collision and the detector, it will attenuate the bremsstrahlung flux by a large amount which can be reliably calculated, and calibrated with sources. However, it will have a negligible effect on the background sources which are present during the runs without the plug. This is because of the wide distribution of these sources and the small volume of the plug. The plug has a big effect on the true signal (attenuates it) because it completely blocks the path of direct photons from the interaction to the detector. The plug can be small because the detector is small, less than 1 cm$^3$ at 4 meters from the interaction.

The situation is somewhat complicated by the fact that the plug creates new background via the showering of $\pi^0$ gammas in it (which would not have showered in the no plug run). This means that we have to do the plug experiment twice, with two different thickness plugs. With the three measurements, one can solve for the signal rate (and incidentally, of course, the background rates).

An evaluation of the systematics of such a measurement indicates that a significant measurement could be accomplished in 300 hours of running even at the initially anticipated RHIC luminosity of 1 central event per second.
IV. CONCLUSION

We have shown that the bremsstrahlung emitted during a high energy heavy ion collision is a sensitive indicator of the degree of stopping of the positive charges and presumably, therefore, of the transparency of the collisions. The bremsstrahlung is emitted at relatively low photon energies, typically less than a few MeV and is concentrated at small angles to the line of the colliding beams.

An experimental design has been presented which illustrates that the bremsstrahlung is measureable at RHIC with a modest apparatus and in a reasonable time.

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FIGURE CAPTIONS

Figure 1: The distribution $\rho$ as a function of transverse position and rapidity from LEXUS.

Figure 2: The number of bremsstrahlung photons with energies between 10 keV and 3 MeV as a function of angle for central collisions of gold at RHIC. The two curves correspond to LEXUS and a flat charged particle rapidity distribution.

Figure 3: Curves $\alpha$, $\beta$, $\gamma$ represent potential charged particle rapidity spectra for central collisions of gold at RHIC. They categorize “full stopping”, “50% stopping”, and “near transparency”, respectively. Only the $y > 0$ curves are shown: the spectra are symmetric about $y=0$. 
Figure 4: The number of bremsstrahlung photons with energies between 10 keV and 3 MeV as a function of angle for central collisions of gold at RHIC. The three curves correspond to those labelled $\alpha$, $\beta$, $\gamma$ in the previous figure.

Figure 5: Sketch of the bremsstrahlung photon detector. The small Ge block ($0.8 \times 0.8 \times 0.5$ cm$^3$) detects the low energy radiation. The cylindrical scintillator is operated in anticoincidence to veto pairs and energetic Compton scatters.

Figure 6: Elevation view of the bremsstrahlung experiment. The magnetic field is along the $y$ axis (vertical) and of strength 20 kG.

Figure 7: View of the experiment along the beam, looking back from the detector side.
$\rho(r_\perp, y)$
