Thermodynamical instability of self-gravitational heavy neutrino matter

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Abstract

It is shown, in the framework of the Thomas-Fermi model at finite temperature, that a cooling non-degenerate gas of massive neutrinos will, at a certain temperature, become unstable and undergo a first-order phase transition in which quasi-degenerate supermassive neutrino stars are formed through gravitational collapse. For neutrinos in the mass range of 10 to 25 keV/c², these compact dark objects could mimic the role of supermassive black holes that are reported to exist at the centres of galaxies and quasi-stellar objects.

1 Introduction

A gas of massive fermions, interacting only gravitationally, has interesting thermal properties that may have important consequences in astrophysics and cosmology. The canonical and grand-canonical ensembles for such a system have been shown to have a non-trivial thermodynamical limit [1 2]. Under certain conditions, these systems will become unstable and undergo a phase transition that is accompanied by gravitational collapse [3]. It is interesting to note that this phase transition occurs only in the case of the attractive gravitational interaction of neutral particles obeying Fermi-Dirac statistics: it neither happens in the case of the repulsive Coulomb interaction of charged fermions [4], nor does it in the case of the attractive gravitational interaction, when the particles obey Bose-Einstein or Maxwell-Boltzmann statistics. To be specific, we henceforth assume that this neutral fermion is the heaviest neutrino, which is presumably the $\nu_\tau$, although this is not essential for most of the

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subsequent discussion, as this neutral fermion could also be replaced with a light gravitino in supergravity models with low supersymmetry breaking scales [37]. We will, moreover, assume that the weak interaction can be neglected.

The ground state of a gravitationally condensed neutrino cloud, with mass below the Oppenheimer-Volkoff (OV) limit [31], where it is close to being a black hole, is a cold neutrino star [5, 6, 7, 8], in which the degeneracy pressure balances the gravitational attraction of the neutrinos. Degenerate stars of neutrinos in the mass range between $m_\nu = 10$ and 25 keV/c$^2$ are particularly interesting [4, 8, 31], as they could explain, without resorting to the black hole hypothesis, at least some of the features observed around the supermassive compact dark objects, which are reported to exist at the centres of a number of galaxies [4, 10, 12, 13, 14] and quasi-stellar objects (QSO) [17, 18, 19, 20, 28]. In fact, the upper bound for the mass of a neutrino star is given by the OV limit $M_{OV} = 0.54195 m_{Pl}^3 m_\nu^{-2} g_\nu^{-1/2}$, where $m_{Pl} = (\hbar c/G)^{1/2} = 1.22205 \cdot 10^{19}$ GeV/c$^2$ is the Planck mass and $g_\nu$ the spin degeneracy factor of the neutrinos and antineutrinos (i.e. $g_\nu = 2$ for Majorana or $g_\nu = 4$ for Dirac neutrinos and antineutrinos). The radius of such a compact dark object would be $R_{OV} = 4.4466 R_{s,OV}$, where $R_{s,OV} = 2G M_{OV}/c^2$ is the Schwarzschild radius of the mass $M_{OV}$ [31]. There is thus little difference between a neutrino star at the OV limit and a supermassive black hole of the same mass, a few Schwarzschild radii away from the object, as the last stable orbit around a black hole has a radius of 3 Schwarzschild radii anyway. For non-relativistic neutrino stars that are well below the OV limit, mass and radius scale as $MR^3 = 91.869 h^6 G^{-3} m_\nu^{-8} (2/g_\nu)^2$ [8]. For instance, the purported supermassive black hole Sgr A* at the centre of our galaxy could well be a neutrino star with $2.6 \times 10^6$ solar masses and a few tens of light-days radius [28, 31, 32, 35].

The existence of a quasi-stable neutrino in this mass range is ruled out neither by particle and nuclear physics experiments nor by direct astrophysical observations [8]. On the contrary, if the LSND experiment is actually observing $\nu_\mu \rightarrow \nu_e$ oscillations [22], and the quadratic see-saw mechanism involving the up, charm and top quarks, is the correct explanation for the smallness of the neutrino masses [23, 24], we should expect the $\nu_\tau$ mass between 6 keV/c$^2$ and 32 keV/c$^2$. In such a scenario, the $\nu_e$ and $\nu_\mu$ masses would be about $10^{-5}$ and 1eV, respectively, and the solar and atmospheric neutrino anomalies would have to be interpreted as vacuum oscillations into two different sterile neutrinos, $\nu_e^S$ and $\nu_\mu^S$, with masses slightly different from those of $\nu_e$ and $\nu_\mu$, consistent with the measured solar and atmospheric neutrino mass squared differences. Large neutrino-antineutrino asymmetries could suppress potentially disastrous active-sterile neutrino oscillations prior to nucleosynthesis [36]. The mixing angles for active-sterile neutrino oscillations involving the same flavor would be maximal, while the mixing matrix elements corresponding to flavor oscillations would be very small and exhibit a hierarchical pattern similar to that of the CKM matrix of the quark sector. The smallness of the masses of the sterile neutrinos $\nu_e^S$, $\nu_\mu^S$ and $\nu_\tau^S$ could be guaranteed by an additional see-saw mechanism in the sterile sector [38].

It is well known that a neutrino mass of between 10 keV/c$^2$ and 25 keV/c$^2$ which is in the so-called cosmologically forbidden region given by $93\ h^{-2}\ eV/c^2 \lesssim m_\nu \lesssim 4\ \text{GeV}/c^2$, where $0.4 \lesssim h \lesssim 1$ is the Hubble parameter, is unacceptable, as it would lead to an early
neutrino-matter dominated phase some time after nucleosynthesis and prior to recombination \[21\]. In such a universe, the microwave background temperature would be attained much too early to accommodate the oldest stars in globular clusters, nuclear cosmochronometry, and the Hubble expansion age, if the Standard Model of Cosmology is correct. However, the early universe might have evolved quite differently, in the presence of such a heavy neutrino. In particular, it is conceivable that primordial neutrino stars have been formed in local condensation processes during a gravitational phase transition that must have occurred some time after the nonrelativistic heavy neutrinos started to dominate this universe, a few weeks after the big bang. Aside from reheating the gaseous phase of the heavy neutrinos, the latent heat produced during the phase transition might have contributed partly to reheating the radiation as well. Moreover, the bulk part of the heavy neutrinos (and antineutrinos) will have annihilated efficiently into light neutrinos via the Z\(^0\) in the dense interior of these supermassive neutrino stars \[4, 5, 6, 7\]. In this context, it is interesting to note that, even within the (homogeneous) Standard Model of Cosmology, the reason why neutrino masses larger than 4 GeV/c\(^2\) are again cosmologically allowed, is precisely that the annihilation of the heavy neutrinos into light neutrinos reduces the matter content of the universe. Since both these processes, reheating and annihilation, will increase the age of the universe, or postpone the time when the universe reaches the current microwave background temperature \[8, 21\], it does not seem excluded that a quasistable massive neutrino in the mass range between 10 and 25 keV is compatible with cosmological observations \[8\].

2 The Thomas-Fermi model

The purpose of this paper is to study the formation of such a neutrino star as a consequence of a thermodynamical instability during a first-order gravitational phase transition. For simplicity, we assume that the neutrino star will be sufficiently below the OV limit, so that we can treat this process non-relativistically \[25\]. The general-relativistic extension of the Thomas-Fermi model has been discussed in \[30, 33\]. The effects of general relativity become important only if the total rest-mass of the system is close to the OV limit \[30, 31, 33\]. The gravitational potential \(V(r)\) satisfies the Poisson equation

\[
\Delta V = 4\pi G m^2_\nu n_\nu, \tag{1}
\]

where the number density of the \(\tau\) neutrinos (including antineutrinos) of mass \(m_\nu\) can be expressed in terms of the Fermi-Dirac distribution at a finite temperature \(T\) as

\[
n_{\nu}(r) = \frac{g_\nu}{4\pi^2 k^3} (2m_\nu kT)^{3/2} I_{\frac{1}{2}} \left( \frac{\mu - V(r)}{kT} \right). \tag{2}
\]

Here \(I_n(\eta)\) is the Fermi function

\[
I_n(\eta) = \int_0^\infty \frac{\xi^n d\xi}{1 + e^{\xi - \eta}}. \tag{3}
\]
and $\mu$ the chemical potential. It is convenient to introduce the normalized reduced potential

$$v = \frac{r}{m_\nu G M_\odot} (\mu - V),$$

(4)

$M_\odot$ being the solar mass, and the dimensionless variable $x = r/R_0$ with the scale factor

$$R_0 = \left( \frac{3 \pi \hbar^3}{4 \sqrt{2} m_\nu^4 g_\nu G^{3/2} M_\odot^{1/2}} \right)^{2/3} = 2.1377 \text{ lyr} \left( \frac{17.2 \text{ keV}}{m_\nu c^2} \right)^{8/3} g_\nu^{-2/3}. \quad (5)$$

Using equations (2) and (4), equation (1) takes the simple form

$$\frac{1}{x} \frac{d^2 v}{dx^2} = - \frac{3}{2} \beta^{-3/2} I_{1/2} \left( \beta \frac{v}{x} \right),$$

(6)

where we have introduced the normalized inverse temperature $\beta = T_0/T$, with $T_0 = m_\nu G M_\odot / kR_0$.

In equation (6) we recover, at zero temperature, the well-known Lane-Emden differential equation \[6, 8\]

$$\frac{d^2 v}{dx^2} = - \frac{v^{3/2}}{\sqrt{x}}. \quad (7)$$

The solution of the differential equation (6) requires boundary conditions. We assume here that the neutrino gas is enclosed in a spherical cavity of radius $R$ corresponding to $x_1 = R/R_0$. We further require the total neutrino mass to be $M_\nu$, and we allow for the possibility of a pointlike mass $M_B$ at the origin, which could be, e.g., a compact seed of baryonic matter. $v(x)$ is then related to its derivative at $x = x_1$ by

$$v'(x_1) = \frac{1}{x_1} \left( v(x_1) - \frac{M_B + M_\nu}{M_\odot} \right),$$

(8)

which, in turn, is related to the chemical potential by $\mu = kT_0 v'(x_1)$. $v(x)$ at $x = 0$ is given by the point mass at the origin, i.e. $M_B/M_\odot = v(0)$.

Similarly to the case of the Lane-Emden equation, it is easy to show that equation (6) has a scaling property: if $v(x)$ is a solution of equation (6) at a temperature $T$ and a cavity radius $R$, then $\tilde{v}(x) = A^3 v(Ax)$, with $A > 0$, is also a solution at the temperature $\tilde{T} = A^4 T$ and the cavity radius $\tilde{R} = R/A$.

It is important to note that only those solutions that minimize the free energy are physical. The free-energy functional is defined as \[2\],

$$F[n_\nu] = \mu[n_\nu] N_\nu - W[n_\nu] - kT g_\nu \int \frac{d^3 r d^3 p}{(2\pi \hbar)^3} \times$$
\[
\ln \left[ 1 + \exp \left( -\frac{p^2}{2m\nu kT} - \frac{V[n\nu]}{kT} + \frac{\mu[n\nu]}{kT} \right) \right],
\]

where
\[
V[n\nu] = -Gm^2_\nu \int d^3 r' \frac{n\nu(r')}{|\vec{r} - \vec{r}'|},
\]

and
\[
W[n\nu] = -\frac{1}{2} Gm^2_\nu \int d^3 r d^3 r' n\nu(r)n\nu(r')\frac{1}{|\vec{r} - \vec{r}'|}.
\]

The chemical potential in equation (9) varies with density, so that the number of neutrinos \( N_\nu = M_\nu/m_\nu \) is kept fixed.

All the relevant thermodynamical quantities, such as number density, pressure, free energy, energy, and entropy, can be expressed in terms of \( v/x \), i.e.

\[
n_\nu(x) = \frac{M_\odot}{m_\nu R_0^3} \frac{3}{8\pi} \beta^{-3/2} I_\frac{3}{2} \left( \frac{\beta v}{x} \right),
\]

\[
P_\nu(x) = \frac{M_\odot T_0}{m_\nu R_0^3} \beta^{-5/2} I_\frac{3}{2} \left( \frac{\beta v}{x} \right) = \frac{2}{3} \epsilon_{\text{kin}}(x),
\]

\[
F = \frac{1}{2} \mu N_\nu + \frac{1}{2} kT_0 R_0^3 \int d^3 x n_\nu(x) \frac{v(x) - v(0)}{x} - R_0^3 \int d^3 x P_\nu(x),
\]

\[
E = \frac{1}{2} \mu N_\nu - \frac{1}{2} kT_0 R_0^3 \int d^3 x n_\nu(x) \frac{v(x) + v(0)}{x} + R_0^3 \int d^3 x \epsilon_{\text{kin}}(x),
\]

\[
S = \frac{1}{T}(E - F).
\]

3 Numerical results

We now turn to the numerical study of a system of self-gravitating massive neutrinos with an arbitrarily chosen total mass \( M = 10M_\odot \), varying the cavity radius \( R \). Owing to the scaling properties, the system may be rescaled to any physically interesting mass. For definiteness, the \( \nu_\tau \) mass is chosen as \( m_\nu = 17.2 \text{ keV}/c^2 \) which is about the central value of the mass region between 10 and 25 keV/c^2 [8], that is interesting for our scenario. In Figure 1, our results for a gas of neutrinos in a cavity of radius \( R = 100R_0 \) are presented. We find three distinct solutions in the temperature interval \( T = (0.049 \div 0.311)T_0 \); of these only two are
physical solutions, namely, those for which the free energy assumes a minimum. The density distributions corresponding to such two solutions are shown in the first plot in Figure 1.

The solution that can be continuously extended to any temperature above the mentioned interval is referred to as “gas”, whereas the solution that continues to exist at low temperatures and eventually becomes a degenerate Fermi gas at $T = 0$ is referred to as “condensate”. In Figure 1, we also plot various extensive thermodynamical quantities (per neutrino) as functions of the neutrino temperature. The phase transition takes place at a temperature $T_t$, where the free energy of the gas and of the condensate become equal. The transition temperature $T_t = 0.19442T_0$ is indicated by the dotted line in the free-energy plot. The top dashed curve in the same plot corresponds to the unphysical solution. At $T = T_t$ the energy and the entropy exhibit a discontinuity, and thus there will be a substantial release of latent heat during the phase transition. An important and currently still open question is, how and to which type of matter or radiation this latent heat, which can be interpreted as the binding energy of the neutrino stars, will be transferred.

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