Distinguishing Primordial Black Holes from Astrophysical Black Holes by Einstein Telescope and Cosmic Explorer

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Abstract

We investigate how the next generation gravitational-wave (GW) detectors, such as Einstein Telescope (ET) and Cosmic Explorer (CE), can be used to distinguish primordial black holes (PBHs) from astrophysical black holes (ABHs). Since a direct detection of sub-solar mass black holes can be taken as the smoking gun for PBHs, we study the detectable limits on the abundance of sub-solar mass PBHs through sub-solar mass PBH binaries in cold dark matter, and the detectable limits on the abundance of super-solar mass PBHs from mergers of sub-solar mass PBHs with super-solar mass PBHs.

1. INTRODUCTION

Ten binary black hole (BBH) mergers were detected during LIGO/Virgo O1 and O2 observing runs [1–7]. Understanding the origin of these BBHs is an essential scientific goal, which is still under intensively investigation (see e.g. [8–27]). The fact that the component masses of these BBHs observed by gravitational waves (GWs) exhibit a much heavier mass distribution than the one inferred from X-ray observations [28–31] has triggered the interest in community to speculate that the observed mergers might be due to the stellar mass primordial black holes (PBHs) [8–10].

PBHs are the black holes (BHs) that form in the early universe by the gravitational collapse of the primordial density perturbations [32, 33], and undergo quite different evolutionary histories than the astrophysical black holes (ABHs), which originate from the demise of massive stars. PBHs may contribute to a fraction of cold dark matter (CDM), and the abundance of PBHs in CDM has been constrained by a variety of experiments, e.g. extra-galactic gamma-ray [34], femtolensing of gamma-ray bursts [35], existence of white dwarfs in our local galaxy [36], Subaru/HSC microlensing [37], Kepler milli/microlensing [38], OGLE microlensing [39], EROS/MACHO microlensing [40], dynamical heating of ultra-faint dwarf galaxies [41], X-ray/radio constraints [42], cosmic microwave background (CMB) spectrum [43–46] and GWs [47–50].

An alternative way to explain LIGO/Virgo BBHs is through ABH models, whose formation and merger are guided by the evolutionary environments. There are three main channels exist in the literature. The first one is the “dynamical formation” channel, in which BHs are formed through the evolution of massive stars, then accumulated to the clusters core to pair as BBHs [18–20]. The second one is the “classical isolated binary evolution” channel, in which BBHs are formed through highly non-conservative mass transfer or common envelope ejection [21–25]. The third one is the “chemically homogeneous evolution”, in which stars evolve almost chemically homogeneously to form BHs because of the mixing of helium produced in the center throughout the envelope [26, 27]. Properties of BBHs, such as the spin [51–58], redshift [59–61], and eccentricity distributions [62], have been proposed to discriminate different channels of astrophysical origin BBH (AOBBH) models.

In this paper, we will explore and forecast the possibility of distinguishing PBHs from ABHs by using GW observations, especially by the third generation ground-based GW detectors like Einstein Telescope (ET) [63] and Cosmic Explorer (CE) [64], which are expected to detect many more BBHs than current LIGO/Virgo, at an order of O(105) events per year [65, 66]. The rest of this paper is organized as follows. In Sec. II, we focus on the sub-solar mass (∼1M⊙) BBHs. Because ABHs are expected to be heavier than the Chandrasekhar mass limit ∼1.4M⊙ [67, 68], a direct detection of sub-solar mass BHs can be the evidence of PBHs. In Sec. II A, assuming PBHs have a monochromatic mass distribution, we estimate the detectable limit on the abundance of PBHs from the targeted search by ET and CE, respectively. In Sec. II B, considering that PBHs have a broad mass distribution and all the black hole merger events detected by LIGO/Virgo are originated from PBHs, we adopt a model independent approach to constrain the abundance of PBHs with super-solar mass.
(≥ 1M⊙) from LIGO/Virgo events, and then explore the detectable limit on the abundance of sub-solar mass PBHs by searching for the BBHs containing a sub-solar mass PBH and a super-solar mass PBH. Sec. III is dedicated to the super-solar mass BBHs. The redshift evolution of the merger rate for POBBHs and AOBBHs can be quite different, which results in different redshift distributions of the expected number of observable BBHs. We estimate and forecast the event number distributions of the PBH and ABH models for ET and CE respectively, which can serve as a complementary tool to distinguish PBHs from ABHs. Finally, we summarize and discuss our results in Sec. IV.

II. DISTINGUISH PBHS FROM ABHS BY SUB-SOLAR MASS BHS

A direct detection of sub-solar mass BHs can be taken as a smoking gun of PBHs. Nonetheless the events rate relies both on the merger rate of POBBHs and the sensitivity of GW detectors. In the following two subsections, we will explore the abilities to detect sub-solar mass BHs for different GW detectors by considering the cases when PBHs have a monochromatic and a general mass function, respectively.

A. Monochromatic mass function

In this subsection we assume all the PBHs have the same mass and estimate the detectable limits of f_{PBH} by the targeted search of POBBHs.

The redshift z evolution of the local merger rate R(z) in comoving frame for the monochromatic mass function, which takes into account the angular momentum exerted both by all PBHs and the background inhomogeneity, is given by [10, 15]

\[ R(z) = 3.9 \times 10^6 \left( \frac{t(z)}{t_0} \right)^{-32} m^{-32/37} f^2 (f^2 + \sigma_{eq}^2)^{-21/74}, \]

in units of Gpc^{−3} yr^{−1}, where \( mM_\odot \) is the component mass of BBHs measured in source frame, and \( \sigma_{eq} \) is the variance of density perturbations of the rest DM on scale of order \( O(10^6 \sim 10^9)M_\odot \) at radiation-matter equality. Following [10, 15], we choose \( \sigma_{eq} \approx 0.005 \). Here \( f_{PBH} \equiv \Omega_{pbh}/\Omega_{cdm} \) is the energy density fraction of PBHs in CDM, and is related to the total abundance of PBHs in non-relativistic matter, \( f \), by \( f_{PBH} \approx f/0.85 \). Besides, \( t(z) \) is the cosmic time at redshift of \( z \) and \( t_0 \equiv t(0) \) is the age of our universe. Note that throughout this paper, we adopt the units in which the Newtonian constant \( G \) and the speed of light in vacuum \( c \) equal to unity.

The expected number of detections, \( N_{\text{obs}} \), then follows [10, 17]

\[ N_{\text{obs}} = \int R(z) \frac{dVT}{dz} dz, \]

where \( dVT/dz \) is the spacetime sensitivity of a GW detector as a function of redshift and accounts for the selection effects of that detector. Generally, \( dVT/dz \) depends on the properties \( \xi \) (e.g. masses and spin) of a binary, and is defined as [69, 70]

\[ \frac{dVT}{dz} = \frac{dV_c}{dz} T_{\text{obs}} f(z|\xi), \]

where \( V_c \) is the comoving volume [71], \( T_{\text{obs}} \) is the observing time, and the denominator \( 1 + z \) accounts for the converting of cosmic time from source frame to detector frame due to the cosmic redshift. Here \( 0 < f(z|\xi) < 1 \) is the probability of detecting a BBH with the given parameters \( \xi \) at redshift \( z \) [72]. The 90% confidential upper limit on the binary merger rate can then be obtained by using the loudest event statistic formalism [73],

\[ R_{90} = \frac{2.303}{VT}. \]

We adopt the semi-analytical approximation from [69, 70] to calculate \( VT \) by neglecting the effect of spins for BHs and using the “IMRPhenomPv2” waveform to simulate the BBH templates. Furthermore, we set a single-detector signal-to-noise ratio (SNR) threshold \( \rho_{th} = 8 \) as a criterion of detection, which roughly corresponds to a network threshold of 12.

The detectable limit of \( f_{PBH} \) by the targeted search from LIGO O1 & O2, LIGO Design, ET and CE are shown in Fig. 1. The effective observing time of LIGO O1 & O2 is set to their total running time of 165.6 days [3, 74]. Meanwhile LIGO Design, ET and CE are supposed to operate at 1 year with full duty. The upper limit for \( f_{PBH} \) of sub-solar mass PBHs in mass range [0.2, 1] \( M_\odot \) has been reported in [48, 49], by the null targeted search result of BBHs in that mass range. We extrapolate the results of [48, 49] in several aspects. Firstly, we adopt the merger rate presented in [15], which takes a more careful examination on the dynamical evolution of the binary systems than the one used by [48, 49] from the results of [9]. Secondly, we also estimate the detectable limits of \( f_{PBH} \) by the proposed third generation GW detectors like CE and ET. Lastly, we do not limit to the masses range of [0.2, 1] \( M_\odot \), but to the range constrained by the detectors automatically. In particular, since whether the super-solar mass BBHs observed so far are of POBBHs is still under debate, we use the dashed to illustrate the detectable limit of \( f_{PBH} \) for the super-solar mass PBHs. Furthermore, as the masses goes down
FIG. 1. Constraints on the abundance of PBHs, $f_{\text{PBH}}$, with a monochromatic mass distribution both by the non-detection of SGWBs and the null targeted search result of BBHs. The gray vertical line at $1M_\odot$ indicates that the constraints from the targeted search are only valid for the sub-solar mass PBHs, because we yet cannot conclude that none of the ten BBHs detected by LIGO/Virgo are of POBBHs. The black, purple, magenta and orange curves are the results of targeted search from LIGO O1 & O2, LIGO Design, ET and CE, respectively. The observing times of LIGO Design, ET and CE are all assumed to be 1 year. The red curve is the updated upper bound of $f_{\text{PBH}}$ constrained by the non-detection of SGWB from both LIGO O1 and O2 searches. The results from other experiments are also shown here: extra-galactic gamma-ray (EG$\gamma$) [34], existence of white dwarfs in our local galaxy (WD) [36], Subaru HSC microlensing (HSC) [37], Kepler milli/microlensing (Kepler) [38], EROS/MACHO microlensing (EROS) [40], OGLE microlensing (OGLE) [39], dynamical heating of ultra-faint dwarf galaxies (UFD) [41], and accretion constraints by CMB [43–46].

below $0.2M_\odot$, the search difficulty arises due to the number of templates, $N_{\text{temp}}$, required in the template bank scales both as the minimum mass $M_{\text{min}}$ and the starting frequency $f_{\text{min}}$ [49]

$$N_{\text{temp}} \propto (M_{\text{min}} f_{\text{min}})^{-8/3}. \quad (6)$$

The dramatic increment of computational resource limits the current GW search pipeline that it cannot deal with the BBHs with component masses far below $0.2M_\odot$ efficiently. However, in the future this difficulty may be overcomed by the improvement of search algorithm or computational technology.

In addition, we also update the constraint on $f_{\text{PBH}}$ from null search of SGWBs from LIGO O1 in [47] to both LIGO O1 and O2 runs [75–78]. The method adopted in this paper is described in Appendix. Usually one may expect that the null detection in targeted search would give a tighter constraint on the abundance of PBHs than that from SGWBs. However, the GW signal from light BBHs is so weak to be resolved individually by GW observations, and hence the null detection of SGWBs provides a more restricted constraint on the abundance of light PBHs because those weak signals can superpose to form a detectable SGWB. See a cross of the red and black curves around 0.1$M_\odot$ in Fig. 1. Actually it is also true for other detections, such as LIGO design, ET and CE.

B. General mass function – a model independent approach

A null search result of the sub-solar mass PBHs in LIGO’s O1 data has been reported in [48, 49]. However, the search was only targeted at the POBBHs with component masses between $0.2M_\odot$ and $1M_\odot$. In this subsection, we propose to search for the POBBHs with a BH of sub-solar mass and another of super-solar mass, which is expected to give a stronger signal.

Here we extend the discussion in the former subsection to the case in which PBHs have a general mass function, and assume that all the ten BBHs observed by LIGO/Virgo so far are POBBHs. Contrary to the previous works [79–82] by choosing some specific mass functions, e.g. a power-law or a lognormal ones, which are
pertinent to some specific formation models of PBHs, we take a model independent approach by binning the mass function \( P(m) \) from 0.2\( M_\odot \) to 100\( M_\odot \) as follows

\[
P(m) = \begin{cases} 
    P_0, & 0.2 M_\odot \leq m < 1 M_\odot \\
    P_1, & 1 M_\odot \leq m < 30 M_\odot \\
    P_2, & 30 M_\odot \leq m < 60 M_\odot \\
    P_3, & 60 M_\odot \leq m \leq 100 M_\odot 
\end{cases}
\]  

(7)

where \( P_i = \{P_0, P_1, P_2, P_3\} \) are four constants satisfying the normalization condition

\[
\int P(m) \, dm = 0.8P_0 + 29P_1 + 30P_2 + 40P_3 = 1.
\]  

(8)

Here, only three out of the four \( P_i \)s are independent and we will choose \( \bar{\theta} = \{P_1, P_2, P_3\} \) as free parameters, which will be fitted by the ten BBHs from LIGO’s O1 and O2 observing runs. In this subsection, we are only interested in LIGO’s O1 search for the sub-solar mass BBHs. Here we denote the abundance of PBHs in the mass range \([m_{\text{min}}, m_{\text{max}}] = [0.2 M_\odot, 100 M_\odot] \). Hence, \( m_{\text{min}} = 0.2 M_\odot \) corresponds to the lower mass bound of LIGO’s O1 search for the sub-solar mass ultracompact binaries in [48].

The merger rate for PBHBs with a general mass function is given in [10].¹ The time dependent comoving merger rate density for a general normalized mass function, \( P(m|\bar{\theta}) \), takes the form

\[
\mathcal{R}_{12}(t|\bar{\theta}) \approx 3.9 \cdot 10^6 \times \left( \frac{t}{t_0} \right)^{-34} f^2(f^2 + v_0^2)^{-\frac{33}{4}} \times \min \left( \frac{P(m_1|\bar{\theta})}{m_1}, \frac{P(m_2|\bar{\theta})}{m_2} \right) \left( \frac{P(m_1|\bar{\theta}) + P(m_2|\bar{\theta})}{m_1 + m_2} \right) \times (m_1 m_2)^{\frac{3}{16}} (m_1 + m_2)^{-\frac{33}{16}},
\]  

(9)

in units of Gpc\(^{-3}\) yr\(^{-1}\), where the component masses \( m_1 \) and \( m_2 \) are in units of \( M_\odot \). The time (or redshift) dependent merger rate can be obtained by integrating over the component masses

\[
\mathcal{R}(t|\bar{\theta}) = \int \mathcal{R}_{12}(t|\bar{\theta}) \, dm_1 \, dm_2.
\]  

(10)

The local merger rate density distribution then reads [79]

\[
\mathcal{R}_{12}(t_0|\bar{\theta}) = R P(m_1, m_2|\bar{\theta}),
\]  

(11)

where the local merger rate \( R \equiv \mathcal{R}(t_0|\bar{\theta}) \) is chosen such that the population distribution of BBH mergers, \( P(m_1, m_2|\bar{\theta}) \), is normalized.

In order to extract the population parameters \( \{\bar{\theta}, R\} \) from the merger events observed by LIGO/Virgo, it is necessary to perform the hierarchical Bayesian inference on the BBHs’ mass distribution [3, 59, 69, 70, 85–87]. If we have the data of \( N \) BBH detections, \( d = (d_1, \ldots, d_N) \), then the likelihood for an inhomogeneous Poisson process is [59, 85–87]

\[
p(d|\bar{\theta}, R) \propto R^N e^{-R|\bar{\theta}|} \prod_i \int d\bar{\lambda} \, p(d_i|\bar{\lambda}) \, p(\bar{\lambda}|\bar{\theta}),
\]  

(12)

where \( \bar{\lambda} \equiv \{m_1, m_2\} \). The likelihood of an individual event \( p(d_i|\bar{\lambda}) \) is proportional to the posterior of that event \( p(\bar{\lambda}|d_i) \), as the standard priors on masses for each event in LIGO/Virgo analysis are taken to be uniform. We will use the public available posterior samples of ten BBHs [3, 7] from LIGO/Virgo observations to evaluate the integral in Eq. (12). Meanwhile, \( \beta(\bar{\theta}) \) is defined as

\[
\beta(\bar{\theta}) \equiv \int d\bar{\lambda} \, VT(\bar{\lambda}) \, p(\bar{\lambda}|\bar{\theta}),
\]  

(13)

in which \( VT(\bar{\lambda}) \) is given by Eq. (5). The posterior probability distribution \( p(\bar{\theta}, R|d) \) can be directly estimated by

\[
p(\bar{\theta}, R|d) \propto p(d|\bar{\theta}, R) \, p(\bar{\theta}, R),
\]  

(14)

where as usual the prior distribution \( p(\bar{\theta}, R) \) [4, 69] is chosen to be uniform for \( \bar{\theta} \) parameters and log-uniform for local merger rate \( R \),

\[
p(\bar{\theta}, R) \propto \frac{1}{R}.
\]  

(15)

Integrating over \( R \) in Eq. (14), it is then easily to obtain the marginalized posterior

\[
p(\bar{\theta}|d) \propto \left[ \beta(\bar{\theta}) \right]^{N-1} \prod_i \int d\bar{\lambda} \, p(d_i|\bar{\lambda}) \, p(\bar{\lambda}|\bar{\theta}),
\]  

(16)

which has been widely used in previous population inferences [3, 4, 69, 70, 79, 88].

Using ten BBH events from LIGO’s O1 and O2 runs, we find the median value and 90\% equal-tailed credible intervals for the parameters \( \{\bar{\theta}, R\} \) to be \( P_1 = 2.1^{+0.7}_{-0.8} \times 10^{-2}, P_2 = 5.4^{+3.7}_{-3.1} \times 10^{-3}, P_3 = 5.1^{+15.2}_{-1.6} \times 10^{-4}, \) and \( R = 308^{+193}_{-132} \) Gpc\(^{-3}\) yr\(^{-1}\), from which we also infer the fraction of PBHs in CDM to be \( f_{PBH} = 3.3^{+2.4}_{-1.8} \times 10^{-3} \). Such an abundance of PBHs is consistent with previous estimations that \( 10^{-3} \lesssim f_{PBH} \lesssim 10^{-2} \), confirming that the dominant fraction of CDM should not originate from PBHs in the mass range \([0.2, 100] M_\odot \) [9, 10, 15, 79–81]. From now on we will investigate the possibility of detecting sub-solar mass BBHs. Here we denote the abundance of PBHs in the mass range \([0.2, 1] M_\odot \) as

\[
f_{PBH} \equiv f_{PBH} P_0 \, \Delta m_0.
\]  

(17)

where \( \Delta m_0 = (1 - 0.2) M_\odot = 0.8 M_\odot \). As a consequence of the above analysis, it is then straightforward to infer

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¹ In [80–84], the authors ignored the relative distribution of PBHs, and cannot guarantee that PBH binaries are formed from the closest neighboring PBHs.
the upper bound of \( f_{PBH0} \) to be \( f_{PBH0} \leq 1.8 \times 10^{-3} \) by LIGO’s O1 and O2 runs. In the future, if third generation ground based GW detectors are in operation, the detection ability will be greatly enhanced and we have more chance to detect the sub-solar mass BBHs if they do exist. In addition to search for the BBHs with two sub-solar mass components as LIGO/Virgo have done, we also propose to search for the BBHs with one sub-solar mass component (with mass lying in \([0.2, 1]M_\odot\)) and another super-solar mass one (with mass lying in \([1, 100]M_\odot\)). Using the loudest event statistic formalism [see Eq. (4)] and the values of \( \theta \) inferred from LIGO’s O1 and O2 runs, \( f_{PBH0} \) can be constrained to an unprecedented level. Assuming no such BBHs will be detected, ET implies \( f_{PBH0} \leq 4.1 \times 10^{-7} \) while CE implies \( f_{PBH0} \leq 4.5 \times 10^{-8} \). The results of the constraints on \( f_{PBH} \) (and \( f_{PBH0} \)) when PBHs have a broad mass distribution are shown in Fig. 2. Note that the red line in Fig. 2 shows the upper limit of \( f_{PBH0} \leq 1.6 \times 10^{-2} \) from the null targeted search of BBHs with two sub-solar mass components, assuming that PBHs take a flat distribution in the mass range \([0.2, 1]M_\odot\).

It is worthy to note that the targeted search of BBHs with a sub-solar mass and a super-solar mass components will improve the detectable limit of \( f_{PBH} \) by an order of \( \mathcal{O}(10^2 \sim 10^3) \), comparing to the targeted search for BBHs with two sub-solar mass BHs as shown in Fig. 1.

III. DISTINGUISH PBHS FROM ABHS BY SUPER-SOLAR MASS BHS

Besides the method of using sub-solar mass BBHs to distinguish PBHs from ABHs, there is another way by exploring the redshift evolution of the event rate of super-solar mass BBHs. In [10], we found that the merger rate of POBBHs increases as a function of redshift \( z \), namely \( \mathcal{R}(z) \propto (1+z)^{-34/37} \), which is independent on both the abundance and mass function of PBHs. However, the merger rate predicted by the AOBBHs will firstly increases with \( z \), then peaks around \( z \sim 1 \), and lastly rapidly decreases with \( z \). Fig. 3 shows the merger rate of POBBHs and AOBBHs as a function of redshift \( z \). The difference of the merger rate between these two models increases at higher redshift. Currently, LIGO can only observe BBHs at low redshifts (with \( z < 1 \)), but future GW detectors such as CE and ET will be able to probe much higher redshifts (with \( z \geq 10 \)). In this section, we will demonstrate how well the third generation ground-based detectors like CE and ET can be used to distinguish PBHs from ABHs according to the quite different event rate distributions at high redshifts.

In order to calculate the observable events rate, we first need to know the merger rate of AOBBHs, which is dependent on the star formation rate (SFR) and the time delay between the formation and merger of AOBBHs. In this paper, we consider the “WWp” model [89] of ABH formation and “Fiducial+PopIII” model [90] of SFR. A well summary of this channel can be found in [79] (see the references therein). For PBHs, we assume they have a broad mass distribution of Eq. (7) and adopt the best-fits from Sec. II B.
FIG. 4. Redshift distribution of the expected number of observable BBHs, $dN_{\text{obs}}/dz$, for CE (top panel) and ET (bottom panel), respectively. The blue and red lines are for the POBBHs and AOBBHs, respectively. For both the POBBHs and AOBBHs, we only count the BBHs with masses in the range of $5M_\odot \leq m_2 \leq m_1 \leq 95M_\odot$.

The redshift dependent observable events number density of a GW detector can be calculated by

$$dN_{\text{obs}} = \int \int dm_1 dm_2 R_{12}(z) \frac{dT}{dz},$$

(18)

Integrating over the redshift $z$ results in the total number of observable events, $N_{\text{obs}}$,

$$N_{\text{obs}} = \int dz \frac{dN_{\text{obs}}}{dz},$$

(19)

Note that Eq. (2) is a special case of Eq. (19) when $m_1 = m_2 = m$. Fig. 4 shows the expected number of observable BBHs as a function of redshift for CE and ET, respectively. The third generation GW detectors like CE and ET are expected to detect $O(10^5)$ BBH mergers each year and dig much deeper at redshifts, and the fact that the redshift distribution of $dN_{\text{obs}}/dz$ for POBBHs and AOBBHs are quite different from each other, can be taken as a complementary tool to distinguish these two formation models of BBHs.

IV. SUMMARY AND DISCUSSION

Even though LIGO/Virgo have detected GW events from the coalescences of BBHs, the origin of these black holes is still unknown. In this paper, we explore how well the next generation detectors, such as ET and CT, can be used to distinguish PBHs from ABHs.

Firstly, we investigate the possibility of direct detection of sub-solar mass BBHs, hence validating the existence of PBHs. For PBHs with a monochromatic mass function, we estimate and forecast the detectable limit of $f_{\text{PBH}}$ from the targeted search of BBHs by LIGO, ET and CE, respectively. Furthermore, in order to get a better sensitivity, we propose to search for the BBHs containing a sub-solar mass PBH and a super-solar mass PBH. We predict that the abundance of PBHs in the mass range $[0.2, 1]M_\odot$ can be constrained to an order of $O(10^{-7})$ and $O(10^{-8})$ if no such BBHs are to be detected by ET and CE, respectively.

Secondly, we explore the possibility of utilizing the redshift evolution of merger rate of super-solar mass BBHs to distinguish PBHs from ABHs. We estimate and forecast the redshift distribution of the expected number of observable BBHs for the PBH and ABH models, respectively. When the third generation ground-based GW detectors like CE and ET are in operation, it is expected to detect $O(10^5)$ BBH mergers each year and reach much deeper redshift ($z \gtrsim 10$), and the redshift distribution of detectable BBH events can serve as an alternative means to distinguish PBHs from ABHs.

Throughout this paper we assume that all the LIGO/Virgo BBHs originate from the same formation channel. However, this assumption can be too oversimplified because the observed BBHs might be a mixing of POBBHs and AOBBHs. To identify each BBH as a POBBH or AOBBH will be quite difficult in this scenario. In addition to the mass and redshift distribution of BBHs, other informations, e.g. spin distribution, will also be invaluable in order to find out the progenitors of the LIGO/Virgo BBHs. For instance, it is expected that PBHs formed in the early universe have negligible spins [91, 92], while the ABHs, which originated from the Population III star binaries, favor a relative high spin distribution [93].

ACKNOWLEDGMENTS

We would like to thank Lu Chen, Yun Fang, Fan Huang, Jun Li, Lang Liu, Shi Pi, You Wu, Yu Sang, Sai Wang, Hao Wei, Chen Yuan, and Xue Zhang for useful conversations. We acknowledge the use of HPC Cluster of
ITP-CAS. This work is supported by grants from NSFC (grant No. 11690021, 11575271, 11747601), the Strategic Priority Research Program of Chinese Academy of Sciences (Grant No. XDB23000000, XDA15020701), and Top-Notch Young Talents Program of China. This research has made use of data, software and/or web tools obtained from the Gravitational Wave Open Science Center [94] (https://www.gw-openscience.org), a service of LIGO Laboratory, the LIGO Scientific Collaboration and the Virgo Collaboration. LIGO is funded by the U.S. National Science Foundation. Virgo is funded by the French Centre National de Recherche Scientifique (CNRS), the Italian Istituto Nazionale della Fisica Nucleare (INFN) and the Dutch Nikhef, with contributions by Polish and Hungarian institutes.

Appendix: Constrain \( f_{\text{PBH}} \) by SGWB

Another way to constrain \( f_{\text{PBH}} \) is through SGWB [47], which is a superposition of the energy spectra emitted by the BBHs that are unlikely to be resolved individually. This method differs from the one by targeted search mainly due to it also utilize the redshift dependence of the merger rate (see Eq. (A.2) below), as the targeted search is more sensitive to the local merger rate. The energy-density spectrum of a SGWB is characterized by \( \Omega_{\text{GW}}(\nu) \) in denominator of Eq. (A.2) converts the merger rate, \( R(z) \), from source frame to detector frame. We adopt the best-fit results from Planck [96] that \( \Omega_{k} = 9.15 \times 10^{-5}, \Omega_{\Lambda} = 0.3089, \) and \( \Omega_{\Lambda} = 1 - \Omega_{m} - \Omega_{r} \). The cutoff redshift is chosen to be \( z_{\text{max}} = \nu_{3}/\nu - 1 \) [47], in which \( \nu_{3} \) is given by Eq. (A.3) below. Furthermore, the energy spectrum \( dE_{\text{GW}}/d\nu_{s} \), emitted by an individual BBH with equal component masses \( m_{1} = m_{2} = m \), is approximated by [99, 101, 102]

\[
\frac{dE_{\text{GW}}}{d\nu_{s}} = \frac{\pi^{2/3} M^{5/3}}{3} \nu_{s}^{-1/3}, \quad \nu_{s} < \nu_{1}, \quad \nu_{1} \leq \nu_{s} < \nu_{2}, \quad \nu_{2} \leq \nu_{s} < \nu_{3},
\]

where \( M = m_{1} + m_{2} = 2m \) is the total mass of the BBH, \( \eta = m_{1}m_{2}/M^{2} = 1/4 \), and \( \nu_{i} = (a_{i}n^{2} + b_{i}n + c_{i})/\pi M \) with \( i = \{1, 2, 3\} \). The coefficients \( a_{i}, b_{i} \) and \( c_{i} \) are presented in Table II of [103]. Again, all masses are measured in the source frame and in units of \( M_{\odot} \).

For a network of \( n \) individual detectors, the SNR \( \rho \) for measuring the SGWB with an observation time \( T_{\text{obs}} \) is given by [95, 104]

\[
\rho = \sqrt{2T_{\text{obs}} \int d\nu \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\Gamma_{I,I}(\nu) S_{b}(\nu)}{P_{n,n}(\nu)}}^{1/2}, \quad \text{(A.4)}
\]

where \( P_{n,n}(\nu) \) is the auto power spectral density for the noise in detector \( I \) and \( \Gamma_{I,I}(\nu) \) is the overlap reduction function [105, 106]. In Eq. (A.4), \( S_{b} \) is the strain power density spectrum of a SGWB, which is related to \( \Omega_{\text{GW}} \) through [104]

\[
S_{b}(\nu) = \frac{3H_{0}^{2}}{2\pi^{2}} \frac{\Omega_{\text{GW}}(\nu)}{\nu^{3}}. \quad \text{(A.5)}
\]

Here we set \( \rho = 1 \), which corresponds to 1\( \sigma \) confidential level, as the criterion for the detection of SGWBs.

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