Multifractal analysis on age-based discrimination in X-ray images for sensing the severity of COVID-19 disease

Shaobo He\textsuperscript{1,a}, C. Thangaraj\textsuperscript{2,b}, D. Easwaramoorthy\textsuperscript{2,c} (corresponding author), and G. Muhiuddin\textsuperscript{3,d}

\textsuperscript{1} School of Physical Science and Electronics, Central South University, Changsha 410083, China
\textsuperscript{2} Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Vellore, Tamil Nadu, India
\textsuperscript{3} Department of Mathematics, Faculty of Science, University of Tabuk, Tabuk 71491, Saudi Arabia

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Abstract The coronavirus, also known as COVID-19, which has been considered one of the deadliest diseases in the world, has become highly contagious, it also implants directly in the human lungs and causes severe damage to the lungs. In such case, X-ray images are widely used to analyze, detect and treat the COVID-19 patients quickly. The X-ray images without any filtering are more complex to identify the affected areas of lungs and to estimate the level of severity of various diseases. The paper analyzes the normal and filtered X-ray images through the multifractal theory and describes the effects of the infection on COVID-19 patients at different ages are classified significantly in processed X-ray images. In this study, the mean absolute error and peak signal-to-noise ratio values are calculated for comparing the noisy and denoised X-ray images using the median filter method and analyzed for comparing the severity of lung affection in X-ray images at different noise levels. Finally, the three-dimensional visualization is constructed for representative images for analyzing and comparing the fever and oxygen levels based on the ages using the corresponding Generalized Fractal Dimensions values. It is observed that the Generalized Fractal Dimensions analyze the different sets of age people’s X-ray images and shows clearly that the older people have higher complexity and the younger people have lower complexity in the infected lungs.

1 Introduction

The fractal geometry is the most important tool and it is a useful method for describing complex objects. Mandelbrot clearly stated that the classical geometric methods were insufficient to explain the complexity of all objects [1]. The heterogeneous natural objects are generally irregular and fragmentary shapes, and it is difficult to explain their complexity with respect to traditional dimensional measures [2,3]. As a result, the fractal was introduced as a non-linear geometrical method, that explain the objects with the non-integer dimension, called the fractal dimension. Mandelbrot defined a fractal mathematically as a set with a Hausdorff dimension strictly greater than its topological dimension [1,2,4]. The fractal theory offer a wide range of applications in many areas such as physics, computer science and biology [3,5]. Recently, the fractal dimension is applied effectively in the clinical image analysis.

The fractal dimension is not enough to describe an object with complex and inhomogeneous scaling features [6,7]. The most accessible non-linear techniques for working with experimental images are monofractal and correlation dimensional measures. The image’s non-uniformity or inhomogeneity cannot be described by a single-dimensional measure. This dimensional scale is insufficient to classify the experimental image’s randomness or inconsistency [8–11]. A Multifractal is an inhomogeneous set that is defined by the Generalized Fractal Dimensions (GFD) or Renyi Fractal Dimensions. The GFD is used to analyze, characterize, and quantify the irregular structure of the signals and images [12–18].

The severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) or COVID-19 disease is one of the major diseases in the medical field today [22]. It was first detected in 2019 in Wuhan Province, China. Then it spread rapidly around the world and today there are 430,257,564 confirmed COVID-19 cases, including 5,922,049 deaths as of 25 February 2022. The World Health Organization (WHO) has warned that this number could continue. The daily lives of most people have been severely affected by the impact of COVID-19 [23]. Common symptoms of the coronavirus are fever, dry cough, sore throat, fatigue, headache, and severe shortness of breath. In case heavy shortness of breath or low oxygen level in blood which can be fatal if not properly treated with oxygen [24]. And while more people have been affected due to this type of respiratory prob-
Because not knowing the severity of the disease at the right time was a major factor [25]. Thus, the polymerase chain reaction (RT-PCR) device is considered to be an excellent tool to diagnose this disease. Although its function is very effective, this tool is not used to predict the impact of the disease and to diagnose human lung problems. Therefore, medical imaging equipment such as computed tomography (CT)-Scan and X-ray are very useful for imaging the human lung and its impact. Similarly, X-ray is more accurate and less expensive than CT scan to diagnose respiratory problems caused by COVID-19 [26–29].

Generally, the internal appearances and the infected areas of the lungs cannot be analyzed, examined and segmented by the physicians clearly. It is very difficult to describe the internal appearances, patterns and affected portions of patients’ lung as it has different lobe systems with detailed structures. Thus, X-ray images are highly complex grayscale images for examining the human lungs. There are certain challenges mainly for physicians to analyze the X-ray images. GFD can be utilized for identifying the nature and severity of the disease through the proper techniques in image analysis. GFD’s role in image analysis will be extremely useful in detecting noise levels of complex images [30–32].

The primary uses and significance of this study are listed below. It is expected to demonstrate the disease severity at different age groups of COVID-19 patients through the graphical comparison using GFD method. Also, the comparison is performed between the different age groups of noisy images with their GFD spectra of COVID-19 patients and then the different age groups of denoised images with their GFD spectra of COVID-19 patients. Furthermore, the denoising performance of the tested X-ray grayscale images is compared by analyzing the quality of the images using the performance metrics such as mean absolute error (MAE) and peak signal noise ratio (PSNR) for X-ray images of COVID-19 patients. The three-dimensional plots with fever level, oxygen values and its GFD range are graphically illustrated about the age-wise variations.

The remaining sections are structured as follows: Section 2 presents the Renyi Entropy, generalized fractal dimensions, median filter and qualitative measures. Section 3 describes the experimental data description. Section 4 details about the results and discussion of the research study. Section 5 concludes the study.

## 2 Methods

Some of the basic ideas and methods are discussed in this section elaborately.

### 2.1 Renyi entropy

Renyi entropy plays a very important role in information theory. Renyi Entropy generalizes generally Hartley entropy, Shannon entropy and Collision entropy. Also, the Renyi entropy was introduced by a Hungarian mathematician named Alfred Renyi. The generalized entropy of a given probability distribution is called

| Patient Age | Gender | SpO2 (%) | Fever (°C) |
|-------------|--------|----------|------------|
| 65 Years    | F      | 84       | 37.5       |
| 70 Years    | M      | 97       | 37.8       |
| 71 Years    | F      | 93       | 39         |
| 77 Years    | M      | 96       | 39         |
| 78 Years    | M      | 92       | 38.2       |
| 83 Years    | M      | 92       | 38.9       |
| 83 Years    | M      | 92       | 38.9       |
| 88 Years    | M      | 96       | 37.1       |
The Renyi Entropy is defined as
\[
S_q = \frac{1}{1-q} \log_2 \left( \sum_{i=1}^{N} p_i^q \right),
\]
where \( q\neq 1 \) is the probability distribution and order. Also, the given \( p_i \in [0,1] \) is the probability of \( x_i \), \( i \in \{1,2,\ldots,N\} \).

### 2.2 Generalized fractal dimensions for grayscale images

The non-linearity occurs generally in all real-time image acquisition through the experimental noises and errors. In a typical pipeline for real-time medical image processing, it is common to have many filters included to form, shape, detect, and manipulate the image information. Furthermore, each of these filter types can be parametrized to work one way under certain circumstances and another way under a different set of circumstances using filter rule generation. The goals vary from noise removal to feature abstraction. Filtering image data are a standard process used in all image processing systems as a preprocessing technique. In this context, the non-linear measure is suitable to analyze the clinical images at different levels such as original, noisy and denoised categories.

The Renyi entropies are crucial in non-linear analysis and statistics as indices of uncertainty or randomness. They also lead to a spectrum of indices of Fractal Dimension, so called as Renyi fractal dimensions or generalized fractal dimensions. In this section, we describe to determine the noise level of grayscale images using generalized fractal dimensions (GFD) [8–10].

The following procedure is constructed to form the suitable probability distribution and to define GFD for a grayscale image through the Renyi entropy.

Let \( N \) be the number of boxes required to cover the grayscale image being evaluated with box size \( \varepsilon \). The probability \( p_i \) for the the tested grayscale image for \( i^{th} \) box of size \( \varepsilon \) is defined as
\[
p_i = \frac{M_i}{M},
\]
where \( M_i \) is the mass of the tested grayscale image included in the corresponding \( i^{th} \) box of size \( \varepsilon \) and \( M \) is the total mass of the tested grayscale image.

The Renyi Fractal Dimensions or GFD of order \( q \in (-\infty, \infty) \) such that \( q \neq 1 \). For the known probability distribution of the given grayscale image can be constructed as
\[
D_q = \lim_{\varepsilon \to 0} \frac{1}{q-1} \log_2 \left( \frac{\sum_{i=1}^{N} p_i^q}{\log_2 \varepsilon} \right). \tag{2}
\]
Here \( D_q \) is called the generalized Renyi Entropy.

### 2.3 Median filter

Image noise is a random variation of brightness or color that is usually caused by electronic noise, such as noise from a scanner and a digital camera. Noise also refers to unwanted, spurious, and extraneous information contained in the image capture. Median filtering technique
is one of the best methods for removing noise in the image and signal. It is a nonlinear image processing method that reduces “salt and pepper” noise. This type of noise reduction is a common pre-processing technique used to improve the outcomes of post-processing (such as edge detection on an image). Median filtering is commonly applied in digital image processing because it preserves edges while reducing noise under specific conditions [19–21]. In this study, the salt and pepper noise to make the original X-ray grayscale images as noisy images and denoised the corrupted original X-ray grayscale images by the median filter. We examined noise levels in the images to analyze and diagnose standard X-ray grayscale images of COVID-19 patients of different ages.

Figure 1 contains sample images of male and female patients of different ages affected by COVID-19. They are 27, 40, 49, 60, 70, 65, 71, and 78 years old, respectively.

### 2.4 Qualitative measures

The mean absolute error (MAE) and peak signal-to-noise ratio (PSNR) have been used to evaluate the performance of the median filter’s denoising technique in
The age-based X-ray images of COVID-19 patients are considered in this research study. The experimental X-ray imaging repository are openly available in Societa Italiana di Radiologia Medica e Interventistica (SIRM), Milano, Italy [33]. The detailed information about the patients considered in the aforementioned database is as follows.

The first patient 27-year-old man. General malaise and fever from 5 days, from 2 days cough and difficulty breathing Bilateral parenchymal thickenings, more evident on the right, of a phlogistic nature. The next patient is a 40-year-old male patient, he was
Table 2  Comparison of denoising performances of tested X-ray grayscale images

| COVID-19 Patient Age | Noisy Image Corrupted by Salt and Pepper Noise with Density 0.05 | Denoised Image using Median Filter |
|----------------------|---------------------------------------------------------------|-----------------------------------|
|                      | MAE    | PSNR  | MAE    | PSNR  |
| 27 Years Old Male    | 6.3699 | 17.8152 | 0.8840 | 41.9659 |
| 30 Years Old Male    | 6.4798 | 18.2442 | 0.9388 | 43.7544 |
| 40 Years Old Male    | 6.2239 | 18.0928 | 0.7669 | 35.5903 |
| 45 Years Old Male    | 6.4025 | 18.3157 | 1.3986 | 37.0159 |
| 49 Years Old Female  | 6.4296 | 17.8203 | 0.8848 | 43.5873 |
| 52 Years Old Female  | 6.3041 | 18.1260 | 1.2519 | 35.0325 |
| 57 Years Old Male    | 6.3537 | 17.8005 | 1.4707 | 34.2238 |
| 60 Years Old Male    | 6.3408 | 18.5699 | 0.9080 | 41.4729 |
| 60 Years Old Male    | 6.3710 | 18.4533 | 1.4823 | 36.9525 |
| 65 Years Old Female  | 6.3735 | 18.1572 | 1.4629 | 27.4058 |
| 70 Years Old Male    | 6.3565 | 18.0595 | 0.6548 | 38.9232 |
| 71 Years Old Female  | 6.4090 | 18.2582 | 0.5993 | 40.7105 |
| 77 Years Old Male    | 6.4415 | 18.5167 | 1.4131 | 37.6146 |
| 78 Years Old Male    | 6.3836 | 18.1306 | 1.6168 | 36.7110 |
| 82 Years Old Female  | 6.4022 | 17.8379 | 1.3475 | 27.7178 |
| 83 Years Old Male    | 6.4011 | 18.3155 | 1.5512 | 31.1375 |
| 88 Years Old Male    | 6.3744 | 18.0381 | 1.7273 | 35.4226 |

4 Results and discussion

Here we examine X-ray images of COVID-19 disease patients of different ages. First we randomly selected the patients’ ages 40, 49, 60, 70, 57, 83, 27, 65, 71 and 78, respectively. We also transformed the X-ray images of the patients so taken into size. Then, the original X-ray images are modified and examined to attain the age-wise classification. The images taken for the first test were subjected to a salt and pepper noise of 0.05 and the contaminated image was then given in Fig. 2. The distorted images are then denoised by a median filtering technique. Denoised images are shown in Fig. 3.

We have divided the original images of the given COVID-19 patients into three age-wise groups respectively. The first classification was 40-year-old male, 49-year-old female, a 60-year-old male and 70-year-old male, respectively. The second classification is 40-year-old male, 49-year-old female, 57-year-old male and 83-year-old male. The third classification system is 27 years old, 65 years old, 71 years old and 78 years old. The original images of these classified COVID-19 patients are compared with their respective denoised images using a median filtering technique. The MAEs and PSNRs of each age group are presented in Table 2.
patients are first illustrated in Fig. 4 with the GFD Spectra.

Figure 4 represents the $D_q$ values for original X-ray images depicted in Fig. 1, which were taken from COVID-19 patients with different ages. Consequently, Fig. 5 reveals the GFD spectra calculated for X-ray images corrupted by salt and pepper noise with the noise density 0.05 as in Fig. 2. Besides, the X-ray images from COVID-19 patients with different age groups are depicted in Fig. 3 after denoising process using the median filter and the GFD spectra for those images are estimated and explored in Fig. 6.

Figure 4a illustrates the first age-wise GFD Spectra. Furthermore, GFD Spectra accurately illustrates the severity of age-related COVID-19 in Fig. 4a. The COVID-19 patient’s X-ray GFD curve of a 40-year-old male has lower complexity than the GFD curve of other ages and the GFD curve will be increases rapidly in age based. From this, it is clear that younger people are less susceptible to COVID-19 than older people. Similarly, Fig. 4b, c clearly shows that the GFD curve of older adults is higher than the GFD curve of younger adults compared to the X-ray images of COVID-19 patients on an age-related basis, respectively.

The GFD curve of the Noisy X-ray images of the COVID-19 patients is compared in Fig. 5a–c. The complexity of the 49-year-old patient is slightly higher than that of the 60-year-old patient, as illustrated by this. In Fig. 5b, a 57-year-old patient’s GFD curve is seen to be higher than that of an 83-year-old patient with
greater complexity. When comparing the GFD curves of patients aged 27 and 78, it is obvious that Fig. 5c has more GFD variations than the original grayscale images.

The GFD of denoised X-ray images is shown in Fig. 6a–c. Figure 6 also clearly demonstrates the severity of the most prominent COVID-19 infection in terms of age. The GFD curve correctly demonstrates that it has a minor impact on the younger and a massive impact on the older.

Finally, using quality measures such as MAE and PSNR, the performance of the denoising procedure for all evaluated grayscale X-ray images is examined and illustrated in Table 2. Table 2 quality measures further revealed that the denoising procedure is effective for all types of grayscale X-ray images tested. From this comparison, the MAE and PSNR values of all the COVID-19 patient X-ray images are clearly every image can be denoised effectively using the GFD.

Figures 7, 8 and 9 are shows the different q values in the GFD compare to data on fever and oxygen levels based on the COVID-19 patients’ ages. When older adults are exposed to COVID-19, the $D_q$ value rises dramatically.

When comparing GFD values of X-ray images from COVID-19 patients with different ages, the severity of the disease can be graphically classified in denoised cases. Furthermore, the values of the mean absolute error rate and the peak signal noise ratio are evidently supporting the age discrimination with respect to GFD spectrum in denoised X-ray images, as it was reduced with its noise level. Based on this, it is confirmed that the complexity is easily detected when analyzing images of COVID-19 patients using GFD after denoising process to perform the age-wise discrimination. Thus, the rate of lung affection is comparatively higher in aged patients than the younger patients affected by the coronavirus.

5 Conclusion

In this context, we are able to significantly predict the age-related vulnerability rate of COVID-19 patients by calculating the multifractal dimensions for X-ray grayscale images. The multifractal GFD Spectra for the original, noisy, and denoised images of COVID-19 diseases are computed and observed that there is less complex in GFD spectra for younger patients and more complex in GFD spectra for older patients. It is also noticed that the deadly virus is more dangerous for the elderly people than in the young persons. In the denoising process, MAE and PSNR values clearly show the obtained GFD Spectra discriminates the patients in denoised X-ray images with respect to age variations. The depicted 3-D visualizations are constructed based on age, SpO2 and Fever levels and also projected the same as less complexity for younger than the older people. It is ultimately concluded that, multifractal analysis is performed well for X-ray images in the analysis of age classification for COVID-19 patients and estimated their level of severity in the infected lungs.

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Data Availability Statement This manuscript has associated data in a data repository [33].

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