Dynamic modeling and vibration control of a rotating space flexible arm with enhanced active constrained layer damping treatment

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Abstract. In this paper, an active-passive control technology called as enhanced active constrained layer damping (EACLD) is applied to structural vibration suppression of a rigid-flexible coupled hub-beam system. The longitudinal shortening of the beam caused by the transverse deformation is considered so that the high-order geometrically nonlinear terms are obtained. By using the Euler-Bernoulli beam theory and describing the VEM with a complex shear modulus, and using the assumed mode method together with Lagrange’s equations, the discrete high-order dynamic equations of the system with EACLD treatment are derived in the closed-loop case with a PD controller. Based on the new dynamic model, the dynamic behavior of the rotating EACLD beam is described in the time domain. The effects of coverage ratio of the control patch, mass of edge element, control gain and edge element location on the dynamic responses of the system are investigated. The results obtained show that the influences of edge element stiffness and mass can be helpful to designs of structural vibration control.

1. Introduction

Vibration is a common physical phenomenon that occur in elastic structures. In the field of engineering, fatigues caused by unwanted vibrations will lead structures to negative conditions. The requirements for light flexible structures have been substantially increased with the development of technologies. Such structures are easy to undergo vibrations with large deformation compared to traditional structures. Based on this, many researchers have been devoting themselves to the study of vibration control problems of flexible structures, such as wind turbine blade, space arms, solar array.

There are mainly three ways for vibration control of flexible structures: the passive damping method, the smart active damping method, and the smart active-passive hybrid control technology. The first use of active constrained layer damping (ACLD) treatment on vibration control of flexible structures was investigated by Shen [1] in 1993. A traditional system of ACLD model is integrated by a piezoelectric layer, a base beam layer, a visco-elastic material(VEM) layer sandwiched in the middle which consists a beam with perfectly bonded thin piezo-sensor. Because of the controllable induced strain, the VEM layer sandwiched between the other two layers can cause irreversible energy dissipation. In 1994, Shen [2] improved his theories of Euler-Bernoulli beam based on the ACLD...
treatment and discussed the influences of boundary. Baz et al. [3] studied the effects of VEM on vibration control by adjusting the deformation of the VEM layer actively. Badre-Alam et al. [4] designed a feedback control of self-adaption damping, and they demonstrated that the method is efficient in vibration control of helicopter rotor. Sun et al. [5] studied the vibration control of a beam based on partially covered ACLD treatment of the positive feedback control. Wong et al. [6] used experiment to study self-induction ACLD treatment. Fung et al. [7] investigated a finite element model of hub-beam based on the ACLD treatment by choosing the GHM model to describe the VEM. By using the Euler-Bernoulli theory, Navazi et al. [8] presented a dynamic equation of a rotating magnetorheological tapered sandwich beam. Madeira et al. [9] used composite plate structures as model and studied the multiobjective optimization of CLD treatments, designed the number and position of constrained layer damping patches. Datta and Ray [10] presented a 3-D nonlinear finite element model of composite shell based on distributed ACLD treatments by using fractional order derivative visco-elastic constitutive relations. Liao et al. [11] added two edge elements on the traditional ACLD patches and developed a new treatment called enhanced constrained layer damping (EACLD) treatment to improve the effects of the active action caused by inverse piezo-electric effect, and modeled the edge elements as equivalent light springs with large stiffness. In this paper, the EACLD treatment is applied to structural vibration suppression of a rotating space flexible arm. By considering the effects of both stiffness and point mass of two edge elements added on the traditional ACLD patches, a comprehensive new dynamic model for a rigid-flexible coupled hub-beam system with EACLD treatment is proposed. The model is intend to improve the previous EACLD beam model that ignores the mass effect of the edge elements in Ref. [11].

2. Dynamic modeling

2.1. Description of the new model
Figure 1 shows the geometric model of EACLD, which is simplified as a sandwich composite beam fixed to a rotating rigid hub. The sandwich beam consists of three layers: the piezoelectric layer, the visco-elastic material layer and the base beam layer. Compared with the previous model with EACLD treatment [11], the two edge elements added are modelled as equivalent spring-point mass systems.

![Figure 1. New geometric model of EACLD beam.](image)

2.2. Basic assumptions
1) The piezo-sensor is tightly bonded to the base beam. There is no slip between each layer.
2) The transverse deformation of any points on each cross section of three layers is assumed to be the same.
3) The visco-elastic damping layer is modeled with complex modulus method.
4) The shear train in the VEM layer is considered only.

2.3. The description of the deformation field
Figure 2 shows the displacement relationship of the rotating EACLD beam. The axial displacement of three layers are denoted as $u_1$, $u_2$ and $u_3$, respectively. The transverse displacement of all points is $w$. The thicknesses of the three layers are $h_1$, $h_2$ and $h_3$, respectively.
Figure 2. Illustration of displacement relationship of each layer.

The displacements of the top and bottom of the VEM layer in the $x$ direction are

$$ u_a = u_3 - \frac{h_1}{2} \frac{\partial w}{\partial x} $$

$$ u_b = u_5 - \frac{h_1}{2} \frac{\partial w}{\partial x} $$

(2.1)

The angle of VEM layer around the $Y$-axis is $\phi_y = \frac{u_b - u_a}{h_2}$. The shear strain of the VEM layer is expressed as

$$ \gamma = \frac{\partial w}{\partial x} + \phi_y = \frac{u_b - u_a + d}{h_2} \frac{\partial w}{\partial x} $$

(2.2)

where $d = \frac{h_1 + 2h_2 + h_3}{2}$.

The axial displacement of the VEM layer is given by

$$ u_z = \frac{1}{2} (u_a + u_b) = \frac{1}{2} \left[ (u_3 + u_5) + d \frac{\partial w}{\partial x} \right] $$

(2.3)

where $d = \frac{1}{2} (h_1 - h_2)$.

The position vector of any points on the $i$-th ($i = 1, 2, 3$) layer in the $XOZ$ coordinate system is

$$ r_i = \Theta (r_0 + u_i) $$

(2.4)

where $\Theta$ is the coordinate transformation matrix

$$ \Theta = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} $$

(2.5)

$r_0 = [R + x \ 0]^T$ and $u_i = [u_i \ w_i]^T$ are the position vector of any points on the $i$-th ($i = 1, 2, 3$) layer in the $xoz$ coordinate system. The axial displacements of piezoelectric and base beam layer are

$$ u_1(x,t) = w_1(x,t) + w_c(x,t) = w_1(x,t) - \frac{1}{2} \int_0^1 \left( \frac{\partial w(z,t)}{\partial z} \right)^2 dz $$

$$ u_3(x,t) = w_3(x,t) + w_c(x,t) = w_3(x,t) - \frac{1}{2} \int_0^1 \left( \frac{\partial w(z,t)}{\partial z} \right)^2 dz $$

(2.6)

where $w_1$ and $w_3$ are the longitudinal extension quantities. $w_c$ is the coupling term.

The position vectors of the two edge elements in the $XOZ$ coordinate system are
\[
\begin{align*}
\mathbf{r}_l &= \Theta ( \mathbf{r}_l + \mathbf{u}_l ) \\
&= (R + x_l + w_l(x,t))x + w(x,t)y + \mathbf{r}_l \\
\mathbf{r}_u &= \Theta ( \mathbf{r}_u + \mathbf{u}_u ) \\
&= (R + x_u + w_u(x,t))x + w(x,t)y
\end{align*}
\]
(2.8)

\[
\begin{align*}
\mathbf{r}_l &= [R + x_l \ 0]^T, \mathbf{u}_l = [u_l \ w_l]^T, \mathbf{r}_u &= [R + x_u \ 0]^T, \mathbf{u}_u = [u_u \ w_u]^T.
\end{align*}
\]
(2.9)

2.4. Kinetic energies

The kinetic energy of the system is composed of six parts and can be given by

\[
T = T_1 + T_2 + T_3 + T_4 + T_5 + T_6
\]

\[
\begin{align*}
&= \frac{1}{2} \int \dot{\mathbf{r}}^2 + \frac{1}{2} \sum_{i=1}^3 \rho_i A_i \dot{r}_i^2 \mathbf{r}_i dx + \frac{1}{2} \int \mathbf{r}_i \mathbf{A}_i \ddot{r}_i \mathbf{r}_i dx + \frac{1}{2} m_i \dot{r}_i^2 + \frac{1}{2} \int m_i \ddot{r}_i \mathbf{r}_i dx \\
&= \frac{1}{2} \int \dot{\mathbf{r}}^2 + \frac{1}{2} \rho_i A_i \sum_{i=1}^3 \left[ \frac{\mathbf{r}_i \mathbf{A}_i \ddot{\mathbf{r}}_i \mathbf{r}_i}{m_i} + \mathbf{r}_i \mathbf{A}_i \dot{\mathbf{r}}_i \mathbf{r}_i + \mathbf{r}_i \mathbf{A}_i \dot{\mathbf{r}}_i \mathbf{r}_i + \mathbf{r}_i \mathbf{A}_i \dot{\mathbf{r}}_i \mathbf{r}_i \right] dx \\
&= \frac{1}{2} \int \left[ \frac{\mathbf{r}_i \mathbf{A}_i \ddot{\mathbf{r}}_i \mathbf{r}_i}{m_i} + \dot{\mathbf{r}}_i^2 + \mathbf{r}_i \mathbf{A}_i \dot{\mathbf{r}}_i \mathbf{r}_i + \mathbf{r}_i \mathbf{A}_i \dot{\mathbf{r}}_i \mathbf{r}_i + \mathbf{r}_i \mathbf{A}_i \dot{\mathbf{r}}_i \mathbf{r}_i \right] dx
\end{align*}
\]
(2.10)

where the first four parts \( T_1, T_2, T_3, \) and \( T_4 \) are the kinetic energies of the hub, the piezoelectric layer, the VEM layer, and the base beam layer, respectively; \( T_5, T_6 \) are the kinetic energies of the two edge elements, respectively. \( J \) is the moment of inertia of the hub. \( A_i \) and \( \rho_i \) are the cross section area and mass density of \( i-th \) layer.

2.5. Potential energies

The potential energies of the closed-loop system are given by

\[
\begin{align*}
U &= U_1 + U_2 + U_3 + U_4 \\
&= \frac{1}{2} \int \left[ E_i A_i (\ddot{w}_i)^2 + \frac{1}{2} k (\Delta u_s)^2 \right] dx \\
&= \frac{1}{2} \int \left[ E_i A_i (\dot{w}_i)^2 + \frac{1}{2} k (\Delta u_s)^2 \right] dx
\end{align*}
\]
(2.11)

where \( U_1, U_2, U_3, \) and \( U_4 \) are the potential energies of the piezo-electric, the VEM, the base beam layer and spring, respectively. \( G^* \) is the complex shear modulus. \( I_i, E_i \) are the area moments of inertia and Young’s moduli of the \( i \) layer.

2.6. Derivation of dynamic equation

By using the assumed mode method, the axial deformation \( w_i \) and transverse deformation \( w \) can be written as
where \( H(x) = \int^1_0 \phi^T(\eta) \phi' d\eta \) is the shape function.

For closed-loop system, the actuator voltage by using a proportional and derivative(PD) controller is

\[
\phi_c = -K_p \phi - K_d \dot{\phi}
\]  \hspace{1cm} (2.15)

where \( K_p \) is the proportional control gain and \( K_d \) is derivative control gain of the PD control. \( \phi_c \) is the output sensing voltage and can be written as

\[
\phi = -\frac{k_{31} D_b}{g_{31} C} \int^2_0 \varepsilon^2 d\varepsilon \text{d}x
\]  \hspace{1cm} (2.16)

where \( C \) is the sensor capacitance, \( D_a = h_t/2 \), \( k_{31} \) is the electromechanical coupling factor, \( g_{31} \) is piezoelectric voltage constant.

The virtual work done by the piezoelectric force and moment is expressed as

\[
\delta W_p = Q^T_p \delta q = E_t A \int^v_0 (\varepsilon_p \delta u_1 + d \varepsilon_p^f \delta w') \text{d}x
\]  \hspace{1cm} (2.17)

where \( \varepsilon_p = d \varepsilon / h_t \) is the strain induced by piezoelectric effect, \( d_{31} \) is the strain constant.

The generalized piezoelectric force vector can be expressed as

\[
Q_p = \begin{pmatrix} 0 & Q_{p}^T & 0 \end{pmatrix} \text{T}
\]  \hspace{1cm} (2.18)

where \( Q_p \) are vectors of the generalized piezoelectric force and moment, respectively.

\[
Q_{p} = g(K_p + K_d p) \int^v_0 \phi^T \left( \begin{array}{c} \delta u_1 \\ \delta q_2 \\ \delta q_3 \end{array} \right) \text{d}x
\]  \hspace{1cm} (2.19)

\[
Q_{m} = d_{31} g(K_p + K_d p) \int^v_0 \phi^T \left( \begin{array}{c} \delta q_2 \\ \delta q_3 \\ \delta q_3 \end{array} \right) \text{d}x
\]  \hspace{1cm} (2.20)

where \( p = \frac{d}{dt} \).

The generalized external force associated with the external torque \( Q_e \) can be obtained by

\[
\delta W_e = Q^T_e \delta q
\]  \hspace{1cm} (2.21)

where the external torque can be expressed as

\[
Q_e = \begin{pmatrix} F_e & 0 & 0 \end{pmatrix} \text{T}
\]  \hspace{1cm} (2.22)

Take \( q = (\theta, q_1^T, q_2^T, q_3^T) \) as the generalized coordinate vector, and employ the second kind Lagrange’s equations. The dynamic equation of the closed-loop system can be written as

\[
\begin{aligned}
\dot{q} &= \frac{1}{2} \int^x_0 \phi^T \left( \begin{array}{c} \dot{u}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{array} \right) \text{d}x \\
0 &= \frac{1}{2} \int^x_0 \phi^T \left( \begin{array}{c} \delta u_1 \\ \delta q_2 \\ \delta q_3 \end{array} \right) \text{d}x \\
0 &= \frac{1}{2} \int^x_0 \phi^T \left( \begin{array}{c} \delta q_2 \\ \delta q_3 \\ \delta q_3 \end{array} \right) \text{d}x
\end{aligned}
\]
$$
\begin{bmatrix}
M_{11} & M_{12} & M_{13} & M_{14} \\
M_{21} & M_{22} & M_{23} & M_{24} \\
M_{31} & M_{32} & M_{33} & M_{34} \\
M_{41} & M_{42} & M_{43} & M_{44}
\end{bmatrix}
\begin{bmatrix}
\dot{\theta} \\
\dot{q}_{a1} \\
\dot{q}_{a3} \\
\dot{q}_{w}
\end{bmatrix}
=
\begin{bmatrix}
Q_{\theta} \\
Q_{a1} \\
Q_{a3} \\
Q_{w}
\end{bmatrix}
$$

\text{(2.23)}

3. Numerical simulations and analysis

The properties of the rotating beam system based on EACLD treatment are expressed in Table 1. The sandwiched beam moves in \(X-Y\) plane. Driving torque is \(F_r\) at the root. \(F_r=\tau \exp(-120\tau)\).

| \(R\)     | 0 | \(\rho_1\) | 7600 Kg/m³ |
|----------|---|------------|-------------|
| \(l\)    | 300 mm | \(\rho_2\) | 1250 Kg/m³ |
| \(b\)    | 12.7 mm | \(\rho_3\) | 2700 Kg/m³ |
| \(h_1\)  | 0.762 mm | \(G^*\) | 0.2615 Mpa |
| \(h_2\)  | 0.5 mm | \(\eta\) | 0.38 |
| \(h_3\)  | 2.286 mm | \(d_{31}\) | \(2.3 \times 10^{11}\) m/V |
| \(E_1\)  | 64.9 Gpa | \(g_{31}\) | 0.216 Vm/N |
| \(E_2\)  | 29.8 Mpa | \(k_{31}\) | 0.12 |
| \(E_3\)  | 71 Gpa | \(k_{3w}\) | 12 |
| \(\tau\) | 0.5 N·m | \(k\) | \(10^6\) N/m |

Table 1 The properties of EACLD treated beam

Figure 3(a) and (b) show the effects of the mass of two edge elements on the EACLD beam system. The PD control gains \(K_p\), \(K_d\) are chosen as 1, -0.005; the coverage ratio is 70% and the position of the left edge element \(x_1 = 0.045\) m; the length of control layer is 0.21 m. When the mass is zero (\(m=0\)), the EACLD beam system becomes the previous system proposed in Ref. [11]. It is found that the amplitude and frequency with mass being 0.05 Kg become smaller and lower, respectively in the vibration curves.

![Figure 3](image-url)  
Figure 3. Effect of edge element mass on EACLD beam
Figure 4 shows the effects of the control gains on the EACLID beam system. The coverage ratio is 60% and the position of the left edge element $x_i=0.06\,\text{m}$; the length of control layer is 0.18\,m; the mass of edge element is selected as 0.005\,Kg. It is clear that the tip transverse deformation decreases with the increase of control gains.

![Figure 4. Effect of PD control gains on EACLID beam](image)

Figure 5(a), (b) and (c) show comparisons of the responses of the EACLID treated beam under different coverage ratios, respectively. The mass of each edge element is chosen to be 0.003\,Kg; the PD control gains $K_p, K_d$ are chosen as 1, -0.005; the control layer is placed in the middle of the base beam. By changing the coverage from 50\% to 80\%, it can be found that when the coverage is about 60\%, the rotating beam performs better.

![Figure 5](image)
Figures 6(a), (b), (c) and (d) show comparisons of the responses of the EACL D beam under different locations of edge element. The mass of each edge element is chosen to be 0.003 Kg; the PD control gains $K_p, K_d$ are chosen as 1, -0.005; the coverage ratio is selected as 60%; the length of control layer is 0.18m. In the case of distance being around 0.08 m, it is obvious that this case is superior to other three cases.
4. Conclusions
In this paper, a high order dynamic equation of a space arm with EACLD treatment is developed. By numerical simulations, the EACLD treatment considering both effects of the stiffness and the mass of two edge elements can be applied to the vibration control of the flexible sandwiched beam as the most effective method by chosen appropriate parameters. The results also show that the vibrations are greatly suppressed choosing proper values of PD control gains. It is found that the damping performance of vibration control of the system are not proportional with the selections of parameters such as coverage ratios, locations of edge element. Based on this, we should focus on the parameter optimization of the system to achieve an economical and effective vibration control goal.

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