The $\eta NN$ coupling constant

Shi-Lin Zhu
Department of Physics, University of Connecticut, U-3046
2152 Hillside Road, Storrs, CT 06269-3046

Abstract

We derive the light cone QCD sum rule for the $\eta NN$ coupling constant $g_{\eta NN}$. The contribution from the excited states and the continuum is subtracted cleanly through the double Borel transform with respect to the two external momenta, $p_1^2, p_2^2 = (p - q)^2$. Our result is $\alpha_{\eta NN} = (0.3 \pm 0.15)$, which favors small values used in literatures.

PACS Indices: 14.40.Aq; 13.75.Gx; 13.75.Cs

I. INTRODUCTION

Nowadays quantum chromodynamics (QCD) is widely believed to be the underlying theory of the strong interaction. Yet the non-abelian nature of the gauge group makes analytical calculation extremely difficult in the low energy sector. A typical example is the various coupling constants of meson nucleon interaction. These couplings are inputs for the one boson exchange potentials for the nuclear forces and the analysis of the important pseudoscalar and vector meson photo- and electro-production experiments currently underway in MAMI (Mainz) and Spring8 (JHF) etc. For the pion nucleon sector there is enough precise data to extract these couplings. Then they are used as inputs to make predictions and analyze other experimental data. In the kaon nucleon hyperon sector the situation is not so encouraging. But there is still some data available. The worst occurs
in the $\eta NN$ and $\eta' NN$ sector, where knowledge of them is rather poor. In the present paper we shall focus on the calculation of $\eta NN$ coupling constant.

There were some theoretical papers on this issue. But the results from various approaches differed greatly. With $SU_f(3)$ symmetry it was found $\alpha_{\eta NN} = \frac{g_{\eta NN}^2}{4\pi} = 3.68$ from the analysis of the nucleon nucleon potential [1]. Similar values was obtained in the non-relativistic model [2]. From the analysis of forward nucleon nucleon scattering using the dispersion relation it was found that $\alpha_{\eta NN} < 1$, consistent to be zero [3]. In [4] the author was able to relate the proton matrix element of flavor singlet current in the large $N_c$ limit to the pseudoscalar meson nucleon coupling constants, leading to $\alpha_{\eta NN} \sim 1.3$. Typical values of $\alpha_{\eta NN}$ obtained in fits with one boson exchange potentials range from 3 to 7 since the eta meson does not contribute significantly to the $NN$ phase shifts and nuclear binding at normal densities [5]. However this coupling is smaller than 1 and can be neglected in the full Bonn potential [6]. $\alpha_{\eta NN}$ extracted from the reaction $\pi^-p \rightarrow \eta n$ lies between 0.6–1.7 [7]. An interesting indirect constraint of $\alpha_{\eta NN}$ comes from the $\pi$-$\eta$ mixing amplitude generated by $\bar{N}N$ loops and neutron proton mass difference using hadronic models. In order to let this amplitude agree with results from chiral perturbation theory, $\alpha_{\eta NN}$ is required to be in the range 0.32-0.53 [8]. Eta meson photo-production did not fix $\alpha_{\eta NN}$ either. In [9] $\alpha_{\eta NN}$ was suggested to around 1.0 or 1.4. Yet a recent analysis of more precise eta meson photo-production experiments in Mainz suggested smaller value of $\alpha_{\eta NN}$ [10]. In other words, the eta nucleon coupling constant is still very controversial. To derive it within an independent and reliable theoretical framework shall prove valuable. We shall use the now well developed light cone QCD sum rules (LCQSR) technique to calculate $\alpha_{\eta NN}$ in this work. Note our approach differs from all the above ones in that it starts microscopically from the QCD Lagrangian.

QCD sum rules (QSR) [11] are successful when applied to the low-lying hadron masses and couplings. In this approach the nonperturbative effects are introduced via various condensates in the vacuum. The light cone QCD sum rule differs from the conventional short-distance QSR in that it is based on the expansion over the twists of the operators. The main contribution comes from the lowest twist operator. Matrix elements of nonlocal operators sandwiched between a hadronic state and the vacuum defines the hadron wave
functions. When the LCQSR is used to calculate the coupling constant, the double Borel transformation is always invoked so that the excited states and the continuum contribution can be subtracted quite cleanly. Moreover, the final sum rule depends only on the value of the hadron wave function at the middle point $u_0 = 1/2$ for the diagonal case, which is much better known than the whole wave function $[12]$. In the present case our sum rules involve with the eta wave function (EWF) $\varphi_\eta(u_0 = \frac{1}{2})$ etc. These parameters are universal in all processes at a given scale.

We have used QCD sum rules to study the meson nucleon strong interactions. In $[13]$ the pion is treated as the external field to analyze the possible isospin symmetry violations of the pion nucleon coupling constant. Later the light cone QSR (LCQSR) was employed to extract the $\pi NN(1535)$ coupling constant, which was found to be strongly suppressed $[14]$. The same formalism was extended to the case of vector meson nucleon interaction $[15]$. The values of the vector and tensor coupling constants and their ratios of $\rho NN$ and $\omega NN$ interaction from the LCQSR agree well with the ones from the experimental data and the dispersion relation analysis. With the advent of the eta meson distribution amplitudes up to twist four $[16]$, we are now able to calculate the $\eta NN$ coupling constant with a theoretically well developed formalism. Although $\eta$ meson is a Goldstone boson, its mass is not small in the real world and comparable with the typical hadronic scale due to the explicit breaking of $SU_f(3)$ flavor symmetry. We have included the eta mass correction in our calculation. Moreover the eta meson is an isoscalar, which leads to the big difference of the LCQSR for the $\eta NN$ coupling constant from that for the $\pi^0 NN$ coupling. We arrive at $\alpha_{\eta NN} = \frac{\varphi_\eta^2}{4\pi} = (0.3 \pm 0.15)$. The numerically small value is due to the cancellation between the leading term and mass correction terms. This point can be seen clearly in later sections.

Our paper is organized as follows: Section I is an introduction. We introduce the two point function for the $\eta NN$ vertex and saturate it with nucleon intermediate states in section II. The definitions of the eta wave functions (EWF) are also presented. Numerical analysis and a short summary is given in the last section.
II. THE LCQSR FOR THE $\eta NN$ COUPLING

We start with the two point function

$$\Pi(p_1, p_2, q) = i \int d^4x e^{ipx} \langle 0| \mathcal{T} \eta_p(x) \bar{\eta}_p(0) |\eta(q)\rangle$$

with $p_1 = p$, $p_2 = p - q$ and the Ioffe’s nucleon interpolating field \[17\]

$$\eta_p(x) = \epsilon_{abc} [u^a(x) C \gamma^\mu u^b(x)] \gamma^\mu d^c(x) ,$$

$$\bar{\eta}_p(y) = \epsilon_{abc} [\bar{u}^b(y) C \bar{u}^{aT}(y)] \bar{d}^c(y) \gamma^\nu \gamma^5 ,$$

where $a, b, c$ is the color indices and $C = i\gamma_2 \gamma_0$ is the charge conjugation matrix. For the neutron interpolating field, $u \leftrightarrow d$.

$$\Pi(p_1, p_2, q)$$

has the general form

$$\Pi(p_1, p_2, q) = F(p_1^2, p_2^2, q^2) \hat{q} \gamma_5 + F_1(p_1^2, p_2^2, q^2) \hat{p} \gamma_5 + F_2(p_1^2, p_2^2, q^2) \sigma_{\mu\nu} \gamma_5 p^\mu q^\nu$$

The sum rules derived from the chiral even tensor structure yield better results than those from the chiral even ones in the QSR analysis of the nucleon mass \[17\]. We shall focus on the tensor structure $\hat{q} \gamma_5$ and study the function $F(p_1^2, p_2^2, q^2)$ as in the QSR analysis of the pion nucleon coupling constant.

The eta nucleon coupling constant $g_{\eta NN}$ is defined by the $\eta N$ interaction Lagrangian:

$$\mathcal{L}_{\eta NN} = g_{\eta NN} \bar{N} i \gamma_5 \eta N .$$

At the phenomenological level the eq.\[1\] can be expressed as:

$$\Pi(p_1, p_2, q) = i\lambda_N^2 m_N g_{\eta NN}(q^2) \frac{\gamma_5 \hat{q}}{(p_1^2 - M_N^2)(p_2^2 - M_N^2)} + \cdots$$

where we include only the tensor structure $\gamma_5 \hat{q}$ only. The ellipse denotes the continuum and the single pole excited states to nucleon transition contribution. $\lambda_N$ is the overlapping amplitude of the interpolating current $\eta_N(x)$ with the nucleon state

$$\langle 0|\eta_N(0)|N(p)\rangle = \lambda_N u_N(p)$$
Neglecting the four particle component of the eta wave function, the expression for \( F(p_1^2, p_2^2, q^2) \) with the tensor structure at the quark level reads,

\[
i \int d^4x e^{ipx} \langle 0| \tilde{\Gamma}_\mu(x) \eta_\mu(0) | \eta(q) \rangle = 
2i \int d^4x e^{ipx} \epsilon^{abc} \epsilon^{a'b'c'} \text{Tr} \left\{ \gamma_\nu C_i S^{T_{a'b'}}_u(x) C \gamma_\mu i S^{a'd'}_u(x) \right\} \gamma_5 \gamma_\mu(0) d^4x \tilde{d}^{c'}(0) | \eta(q) \rangle | \eta(q) \rangle
+ 4i \int d^4x e^{ipx} \epsilon^{abc} \epsilon^{a'b'c'} \text{Tr} \left\{ \gamma_\nu C_i S^{T_{a'b'}}_u(x) C \gamma_\mu(0) u^a(x) \tilde{u}^{a'}(0) | \eta(q) \rangle \right\} \gamma_5 \gamma_\mu i S^{c'd'}_d(x) | \eta(q) \rangle \gamma_5 \gamma_\mu
\]

where \( iS(x) \) is the full light quark propagator with both perturbative term and contribution from vacuum fields \([14]\).

By the operator expansion on the light-cone the matrix element of the nonlocal operators between the vacuum and eta state defines the two and three particle eta wave function. In order to simplify the notations we use \( q \Gamma_\mu q \) to denote \( (\tilde{u} \Gamma_\mu u + \tilde{d} \Gamma_\mu d - 2s \Gamma_\mu s)/\sqrt{6} \). We also introduce \( F_\eta = f_\eta \), where \( f_\eta \) is defined as

\[
< 0 | \tilde{q}(0) \gamma_\mu q(0) | \eta(q) >= i f_\eta q_\mu .
\]

Up to twist four the Dirac components of this wave function can be written as \([16]\):

\[
< 0 | \tilde{q}(0) \gamma_\mu \gamma_5 q(x) | \eta(q) >= i f_\eta q_\mu \int_0^1 du e^{-iuq x} \left[ \varphi_\eta(u) + \frac{1}{16} m_\eta^2 x^2 A(u) \right] + \frac{i}{2} f_\eta m_\eta^2 \frac{q_\mu}{q x} \int_0^1 du e^{-iuq x} B(u) + O(x^4) ,
\]

\[
< 0 | \tilde{q}(0) i \gamma_5 q(x) | 0 >= f_\eta \mu_\eta \int_0^1 du e^{-iuq x} \varphi_P(u) ,
\]

\[
< 0 | \tilde{q}(0) \sigma_\mu \gamma_5 q(x) | 0 >= \frac{i}{6} f_\eta \mu_\eta (q_\mu x_\nu - q_\nu x_\mu) \int_0^1 du e^{-iuq x} \varphi_\sigma(u) ,
\]

\[
< 0 | \tilde{q}(0) \sigma_{\alpha\beta} \gamma_5 q S G_{\mu\nu}(ux) q(x) | \eta(q) >=
\]

\[
if_\eta \mu_\eta \eta_3 [ (q_\mu q_\alpha q_{\alpha\beta} - q_\beta q_\alpha q_{\alpha\beta}) - (q_\mu q_\beta q_{\alpha\nu} - q_\alpha q_{\beta\nu} q_{\mu\nu}) ] \int D\alpha_i \varphi_\eta(\alpha_i) e^{-iux(\alpha_i + v_\alpha)} ,
\]

\[
< 0 | \tilde{q}(0) \gamma_\mu \gamma_5 q S G_{\alpha\beta}(ux) q(x) | \eta(q) >=
\]

\[
f_\eta m_\eta^2 \left[ q_\beta (q_{\alpha\mu} - \frac{x_\alpha q_\mu}{q \cdot x}) - q_\alpha (q_{\beta\mu} - \frac{x_\beta q_\mu}{q \cdot x}) \right] \int D\alpha_i \varphi_\perp(\alpha_i) e^{-iuq x(\alpha_i + v_\alpha)}
\]

\[
+ f_\eta m_\eta^2 \frac{q_\mu}{q \cdot x} (q_\alpha x_\beta - q_\beta x_\alpha) \int D\alpha_i \varphi_\parallel(\alpha_i) e^{-iuq x(\alpha_i + v_\alpha)}
\]
and

\[
<0|\bar{q}(0)\gamma_\mu g_\mu \tilde{G}_{\alpha\beta}(vx)q(x)|\eta(q)> = \\
-ig_\eta m_\eta^2 \left[ q_\beta \left( g_{\alpha\mu} - \frac{x_\alpha q_\mu}{q \cdot x} \right) - q_\alpha \left( g_{\beta\mu} - \frac{x_\beta q_\mu}{q \cdot x} \right) \right] \int D\alpha_i\bar{\phi}_\perp(\alpha_i)e^{-iqx(\alpha_1 + \nu_\alpha)} \\
-ig_\eta m_\eta^2 \frac{q_\mu}{q \cdot x} (q_\alpha x_\beta - q_\beta x_\alpha) \int D\alpha_i\bar{\phi}_{\parallel}(\alpha_i)e^{-iqx(\alpha_1 + \nu_\alpha)}. \tag{15}
\]

The operator \( \tilde{G}_{\alpha\beta} \) is the dual of \( G_{\alpha\beta} \): \( \tilde{G}_{\alpha\beta} = \frac{1}{2} \epsilon_{\alpha\beta\delta\rho} G^{\delta\rho} \); \( D\alpha_i \) is defined as \( D\alpha_i = d\alpha_1d\alpha_2d\alpha_3(1 - \alpha_1 - \alpha_2 - \alpha_3) \). Due to the choice of the gauge \( x^\mu A_\mu(x) = 0 \), the path-ordered gauge factor \( P \exp(ig_\eta \int_0^1 dx^\mu A_\mu(x)) \) has been omitted.

The EWF \( \varphi_\eta(u) \) is of twist two, \( \varphi_F(u) \), \( \varphi_\sigma(u) \), and \( \varphi_{3\eta} \) are of twist three, while \( A(u) \), part of \( B(u) \) and all the EWFs appearing in eqs. (14), (15) are of twist four. The EWFs \( \varphi(x_i, \mu) \) (\( \mu \) is the renormalization point) describe the distribution in longitudinal momenta inside the eta meson, the parameters \( x_i (\sum_i x_i = 1) \) representing the fractions of the longitudinal momentum carried by the quark, the antiquark and gluon.

The normalization and definitions of the various constants can be found in [16]. Some of them are \( \int_0^1 du \varphi_\eta(u) = \int_0^1 du \varphi_\sigma(u) = 1 \), \( \int D\alpha_i\varphi_\perp(\alpha_i) = \int D\alpha_i\varphi_{\parallel}(\alpha_i) = 0 \) etc.

Since the steps to derive LCQSRs are very similar to those in [14, 15], we present final sum rule directly. Interested readers may consult the above papers for details.

\[
m_N \lambda_N^2 g_{\eta NN} e^{-\frac{M_2^2}{M^2}} = \\
-\frac{u_0}{2\pi^2} \varphi_\eta(u_0) M^2 f_2(\frac{s_0}{M^2}) + \frac{m_\eta^2}{4\pi^2} 2u_0 \left[ F^u_\eta A(u_0) + F^d_\eta B(u_0) \right] M^4 f_1(\frac{s_0}{M^2}) \\
-\frac{F^u_\eta}{9\pi^2} a_\mu \left[ \varphi_\sigma(u_0) + \frac{u_0}{2} \varphi'_\sigma(u_0) \right] M^2 f_0(\frac{s_0}{M^2}) \\
+ \frac{1}{12\pi^2} F^u_\eta \mu_\eta \eta_3 a_\mu m_\eta^2 I_1[\varphi_3] - \frac{1}{6\pi^2} F^u_\eta \mu_\eta \eta_3 a I_2[\varphi_3] M^2 f_0(\frac{s_0}{M^2}) \\
+ \frac{1}{2\pi^2} \left( F^u_\eta + F^d_\eta \left\{ \frac{1}{4} I_2[\varphi_\parallel] - \frac{1}{4} I_2[\varphi_\perp] + I_1[\varphi_\perp] - I_4[\varphi_\parallel] - I_4[\varphi_\perp] \right\} \right) M^4 f_1(\frac{s_0}{M^2}) \\
+ \frac{1}{2\pi^2} m_\eta^4 (F^u_\eta + F^d_\eta \left\{ -I_3[\varphi_\parallel] - I_3[\varphi_\perp] - I_5[\varphi_\parallel] - I_5[\varphi_\perp] + I_5[\varphi_\perp] \right\} M^2 f_0(\frac{s_0}{M^2}) \right), \tag{16}
\]

where \( f_n(x) = 1 - e^{-x} \sum_{k=0}^{n} \frac{x^k}{k!} \) is the factor used to subtract the continuum, \( s_0 \) is the continuum threshold, \( u_0 = \frac{M_1^2}{M_1^2 + M_2^2} \), \( M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2} \), \( M_1^2, M_2^2 \) are the Borel parameters, and \( \varphi'_\sigma(u_0) = \frac{d\varphi_\sigma(u)}{du} |_{u=u_0} \). In order to make comparison with the sum rule for \( n^0 NN \) coupling constant \( g_{\eta NN} \), we have labeled the eta meson decay constant \( F_\eta \) with the flavor index.
The functions \( I_i[\varphi_{3\eta}] \) etc are defined as:

\[
\phi_B(u_0) = -\int_0^{u_0} duB(u),
\]

\[
I_1[F] = 2u_0 \int_0^{u_0} d\alpha_1 \int_0^{1-u_0} d\alpha_2 \frac{F(\alpha_1, \alpha_2, 1-\alpha_1-\alpha_2)}{(1-\alpha_1-\alpha_2)^2} (1 - 2u_0 + \alpha_1 - \alpha_2),
\]

\[
I_2[F] = \int_0^{u_0} d\alpha_1 \frac{F(\alpha_1, 1 - u_0, u_0 - \alpha_1)}{u_0 - \alpha_1} + \int_0^{1-u_0} d\alpha_2 \frac{F(u_0, \alpha_2, 1 - u_0 - \alpha_2)}{1 - u_0 - \alpha_2} - 2 \int_0^{u_0} d\alpha_1 \int_0^{1-u_0} d\alpha_2 \frac{F(\alpha_1, \alpha_2, 1 - \alpha_1 - \alpha_2)}{(1 - \alpha_1 - \alpha_2)^2},
\]

\[
I_3[F] = 2u_0 \int_0^{u_0} d\alpha_1 \int_0^{1-u_0} d\alpha_2 F(\alpha_1, \alpha_2, 1 - \alpha_1 - \alpha_2) \frac{(u_0 - \alpha_1)(1 - u_0 - \alpha_2)}{(1 - \alpha_1 - \alpha_2)^2},
\]

\[
I_4[F] = 2u_0 \int_0^{u_0} d\alpha_1 \int_0^{1-u_0} d\alpha_2 \frac{F(\alpha_1, \alpha_2, 1 - \alpha_1 - \alpha_2)}{1 - \alpha_1 - \alpha_2},
\]

\[
I_5[F] = 2u_0 \int_0^{u_0} d\alpha_1 \int_0^{1-u_0} d\alpha_2 F(\alpha_1, \alpha_2, 1 - \alpha_1 - \alpha_2) \frac{u_0 - \alpha_1}{1 - \alpha_1 - \alpha_2},
\]

\[
I_6[F] = 2u_0 \int_0^{u_0} d\alpha_1 \int_0^{u_0-\alpha_1} d\alpha_3 F(\alpha_1, 1 - \alpha_1 - \alpha_3, \alpha_3),
\]

\[
I_7[F] = 2u_0 \left\{ \int_0^{u_0} d\alpha_1 \frac{F(\alpha_1, 1 - u_0, u_0 - \alpha_1)}{u_0 - \alpha_1} - \int_0^{1-u_0} d\alpha_2 \frac{F(u_0, \alpha_2, 1 - u_0 - \alpha_2)}{1 - u_0 - \alpha_2} \right\},
\]

where \( F = \varphi_{3\eta}, \varphi_\parallel, \varphi_\perp, \tilde{\varphi}_\parallel, \tilde{\varphi}_\perp \).

III. DISCUSSION

Since eta meson is an isoscalar, we have \( F_\eta^u = F_\eta^d = F_\eta = \frac{f_\eta}{\sqrt{2}} \). Replacing the \( \eta \) index with \( \pi \) and \( F_\eta \) by \( f_\pi \) in (14), we recover the sum rule for \( g_{\pi NN} \) (14). Note \( f_\pi^u = -f_\pi^d = f_\pi \). In other words, the twist four terms involved with three particle pion wave functions vanish due to isospin symmetry. The first term in (16) is the leading twist two term. The third term is of twist three and of the same sign as the leading term. The second term comes from two particle EWF and the remaining terms all come from three particle EWFs. Although they are of twist four except the fourth term, their contribution is greatly

7
enhanced by the factor $m_\eta^2$ in contrast with $m_\pi^2$ in the $\pi NN$ coupling case. Moreover they are of the opposite sign as the leading twist two and three terms, which leads to strong cancellation. In other words, large mass and isoscalar structure of eta meson causes $g_{\eta NN}$ to be much smaller than $g_{\pi NN}$.

The sum rule (16) is symmetric and diagonal, which requires the Borel parameters $M_1^2 = M_2^2$, i.e., $u_0 = \frac{1}{2}$. The working interval for analyzing the QCD sum rule (16) is $0.9\text{GeV}^2 \leq M_B^2 \leq 1.8\text{GeV}^2$, a standard choice for analyzing the various QCD sum rules associated with the nucleon. In order to diminish the uncertainty due to $\lambda_N$, we shall divide (16) by the Ioffe’s mass sum rule for the nucleon:

$$32\eta^4 \lambda_N^2 e^{-\frac{M_B^2}{m_N^2}} = M^6 f_2 \left( \frac{s_0}{M^2} \right) + \frac{b}{4} M^2 f_0 \left( \frac{s_0}{M^2} \right) + \frac{4}{3} a^2 - \frac{a^2 m_0^2}{3 M^2}. \quad (25)$$

The various parameters which we adopt are $f_\eta = (0.133\pm0.01) \text{GeV}$ [18], $\eta_3 = 0.013$, $a = -4\pi^2 < 0 |\bar{q}q|0 > = 0.67 \text{GeV}^3$, $\mu_\eta = 2.13 \text{GeV}$ [16] at the scale $\mu = 1 \text{GeV}$, $s_0 = 2.25 \text{GeV}^2$, $m_N = 0.938 \text{GeV}$, $\lambda_N = 0.026 \text{GeV}^3$ [17].

At $u_0 = \frac{1}{2}$ the values of various eta meson wave functions are: $\varphi_\eta(u_0) = 1.05$, $A(u_0) = 4.14$, $\phi_B(u_0) = 0$, $\varphi_\sigma(u_0) = 1.44$, $\varphi'_\sigma(u_0) = 0$, $I_1[\varphi_{3\eta}] = 0$, $I_1[\varphi_\parallel] = 0.026$, $I_2[\varphi_{3\eta}] = -0.9375$, $I_2[\varphi_\parallel] = 0$, $I_3[\varphi_{\perp}] = 0$, $I_4[\varphi_\perp] = 0$, $I_4[\varphi_{3\eta}] = -0.313$, $I_5[\varphi_\parallel] = -0.032$, $I_5[\varphi_{\perp}] = 0.0396$, $I_6[\varphi_\perp] = -0.052$, $I_6[\varphi_{3\eta}] = 0.044$, $I_7[\varphi_{\perp}] = 0$ at $u_0 = \frac{1}{2}$ and $\mu = 1 \text{GeV}$.

The dependence on the Borel parameter $M^2$ of $g_{\eta NN}$ are shown in FIG 1 with $s_0 = 2.35, 2.25, 2.15 \text{ GeV}^2$. The final sum rule is stable in the working region of the Borel parameter $M^2$. We obtain:

$$g_{\eta NN} = (1.7 \pm 0.3). \quad (26)$$

In the above numerical analysis we have used relatively large quark condensate value $< \bar{q}q > = - (240 \pm 10)^3 \text{MeV}^3$, which corresponds to $a = 0.67 \text{GeV}^3$. In the literatures another value $< \bar{q}q > = - (225 \pm 10)^3 \text{MeV}^3$ and $a = 0.55 \text{GeV}^3$ is also used. Since we are not able to know very precisely the quark condensate value, we also present the variation of $g_{\eta NN}$ with $M^2$, $s_0$ with $a = 0.55 \text{GeV}^3$ in FIG 2. In this case we have,

$$g_{\eta NN} = (2.1 \pm 0.3). \quad (27)$$

We have included the uncertainty due to the variation of the continuum threshold.
and the Borel parameter $M^2$ in (26) and (27). In other words, only the errors arising from numerical analysis of the sum rule (16) are considered. Other sources of uncertainty include: (1) the truncation of OPE on the light cone at the twist four operators. For example, the four particle component of EWF is discarded explicitly; (2) the EWFs are estimated with QCD sum rule, which also induces some errors; (3) the continuum model used in the subtraction of contribution from the higher resonances and continuum spectrum; (4) errors in $f_\eta$ etc.

With all these uncertainties we arrive at

$$\alpha_{\eta NN} = (0.3 \pm 0.15) .$$

For the $\eta, \eta'$ sector instanton effects might be important. It's well known that a large part of $\eta'$ mass comes from the $U_A(1)$ anomaly. Through $\eta - \eta'$ mixing instantons also affect eta meson mass and decay constant $f_\eta$. Fortunately, we know from phenomenological analysis that the $\eta - \eta'$ mixing angle is about $-20$ degrees [18]. So for eta meson such effects may be not so large as in the $\eta'$ channel. Direct instantons favor strongly the scalar and pseudoscalar channel and might affect the mass sum rules for the mesons in these channels. In our QCD sum rule analysis of eta NN coupling constant we have chosen the tensor structure $\hat{q}\gamma_5$. Moreover, we have used the experimental values for $m_\eta, f_\eta$ as inputs instead of invoking the eta meson mass sum rules to extract them. Hence the possible correction from instantons is expected to be relatively small.

In short summary we have calculated the eta nucleon coupling constant with the light cone QCD sum rules. The continuum and the excited states contribution is subtracted rather cleanly through the double Borel transformation. Our approach differs from all the available methods in the extraction of $g_{\eta NN}$ and starts from the quark gluon level. So it is independent and more reliable to some extent. Our result of $\alpha_{\eta NN}$ favors the small value. Except the nonrelativistic quark model and fits with one boson exchange potentials, other approaches tend to yield small values for $\alpha_{\eta NN}$. However, in such potentials, the eta meson was treated as some effective degree of freedom to model other multi-meson correlations. Hence the eta meson in these potentials can not be related to the real eta meson seen in the photo- or electro-production experiments in a simple way. In other words, the $\alpha_{\eta NN}$
in these potentials may be not the same quantity as the coupling we have calculated. We hope our extraction of $\alpha_{NN}$ can be used to analyze future eta meson photo- and electro-production experiments.

[1] O. Dumbrajs et al., Nucl. Phys. B 216, 277 (1983).
[2] Y.-W. Yu and Z.-Y. Zhang, Nucl. Phys. A 426, 557 (1984).
[3] W. Grein and P. Kroll, Nucl. Phys. A 338, 332 (1980).
[4] T. Hatsuda, Nucl. Phys. B 329, 376 (1990).
[5] R. Brockmann and R. Machleidt, Phys. Rev. C 42, 1965 (1990).
[6] R. Machleidt, Adv. Nucl. Phys. 19, 189 (1989); K. Holinde, Nucl. Phys. A 543, 143c (1992).
[7] J. C. Peng, Proc. LAMPF Workshop on photon and neutral meson physics at intermediate energies-LA-11177-C, eds. H. W. Baer et al. (1987).
[8] J. Piekarewicz, Phys. Rev. C 48, 1555 (1993).
[9] M. Benmerrouche and N. C. Mukhopadhay, Phys. Rev. Lett. 67, 1070 (1991).
[10] L. Tiator, C. Bennhold, and S.S. Kamalov, Nucl. Phys. A 580, 455 (1994).
[11] M.A. Shifman, A.I. Vainshtein, and V.I. Zakharov, Nucl. Phys. B 147, 385, 448, 519 (1979).
[12] I. I. Balitsky, V. M. Braun and A. V. Kolesnichenko, Nucl. Phys. B 312, 509 (1989).
[13] W-Y. P. Hwang, Ze-sen Yang, Y.S. Zhong, Z.N. Zhou and Shi-Lin Zhu, Phys. Rev. C57, 61 (1998).
[14] Shi-Lin Zhu, W.-Y. P. Hwang, and Y.-B. Dai, Phys. Rev. C 59, 442 (1999).
[15] Shi-Lin Zhu, Phys. Rev. C 59, 435 (1999); ibid. 59, 3455 (1999).
[16] P. Ball, JHEP 9901, 010 (1999).
[17] B.L. Ioffe, Nucl. Phys. B188, 317 (1981); [E] B191, 591 (1981).
[18] Particle Data Group, C. Caso et al., Euro. Phys. J. C 3, 1 (1998).
Figure Captions

FIG 1. The sum rule for $g_{qNN}$ as a function of the Borel parameter $M^2$ with $a = 0.67\text{GeV}^3$ and the continuum threshold $s_0 = 2.35, 2.25, 2.15\text{GeV}^2$.

FIG 2. The same notations as in FIG 1 except $a = 0.55\text{GeV}^3$. 
\[ \frac{\tilde{s}_{\eta NN}}{M^2} \]

- \( s_0 = 2.35 \text{GeV}^2 \)
- \( s_0 = 2.25 \text{GeV}^2 \)
- \( s_0 = 2.15 \text{GeV}^2 \)
