Quantum vortex creep
: Hall and dissipative tunneling

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Abstract

Within the framework of the path-integral approach we study the quantum vortex creep for the situation where both the Hall and the dissipative dynamics are simultaneously present. We calculate the relaxation rate and the crossover temperature separating the thermal activation and the quantum tunneling processes for anisotropic or multilayer superconductors. The results are compared with the available experimental data.

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After the discovery of high temperature superconductors (HTSC) the relaxation of magnetization has been the subject of intensive studies because of the possibility of quantum tunneling of vortices in the system. According to the Anderson-Kim model, the vortices can move out of the pinning sites by the thermal activation process, which proceeds at a rate proportional to \( \exp(-U_0/k_B T) \), where \( U_0 \) is the height of the energy barrier generated by the pinning mechanism. This process induces a redistribution of vortices, hence of the current loops related with the vortices, causing a change in the magnetization logarithmically with time

\[
M(t) = M_0[1 - Q(T) \ln(t/t_0)],
\]

where \( Q(T) = k_B T/U_0 \) is the so called magnetic viscosity. Since the rate is expected to vanish at zero temperature, the magnetization tends to be unchanged as the temperature \( T \) approaches zero. However, it has been realized that the thermal activation theory fails to explain experimental observations of temperature independent magnetic relaxation in HTSC at low temperatures. This has led to introduce the concept of quantum tunneling of vortices trapped in the pinning potential with the collective pinning length \( L_c \). Since a pinned vortex is in some metastable state, the quantum tunneling of the vortex is governed by the frequency of the pinning barrier hindering the tunneling process. The rate of the quantum tunneling can be thus obtained with the relevant dynamics, whereas the rate of the thermal activation process does not depend on particular dynamics.

Recently, Blatter et al. studied the quantum vortex tunneling for the case where the dissipative term is dominant in the dynamical equation of motion of vortex, and many experimental results have been interpreted within this limit. On the one hand, Feigel'man et al. estimated the low-lying level spacing (\(~ \hbar \omega_0 \simeq \Delta^2/\epsilon_F \) in the vortex core and the transport relaxation time of the charge carriers \( \tau_r \) at \( T = 0 \) by using a linear extrapolation of the normal state resistivity \( \rho_n \), and obtained \( \omega_0 \tau_r \gg 1 \) in HTSC at low temperature which is the superclean limit. The criteria distinguishing the dissipative and the Hall vortex motion is the
magnitude of the quantity $\omega_0 \tau_r$, therefore they proposed that the Hall motion should be dominant in clean HTSC. Most recently, however, van Dalen et al. [9]-[11] observed experimentally that in YBCO or BiSCCO systems, the vortex tunneling at low temperatures may occur in an intermediate regime between the purely dissipative and the superclean Hall tunneling. Prior to the experiments, Stephen [12] studied the quantum tunneling of vortex with the dissipative and the Hall terms by using the approximate form for the mean-square quantum fluctuation with a simple harmonic pinning potential up to a cutoff. As is noted in Ref. [13], since the cutoff model potential can be considered to be of the form $(q/q_0)^2 - (q/q_0)^n$ with $n \to \infty$ where $q$’s are coordinates of the potential, the steep part of the model potential underestimates the suppression factor, i.e., the effective action in the case of damped systems. So they remarked that a special care should be taken in replacing the smooth potential by one which has a discontinuity.

In this paper, we study the quantum vortex tunneling for the case where the dissipative and the Hall dynamics are simultaneously present, considering the pinning potential barrier generated by disorder with a cubic term [2, 7, 14] along the $x$ axis near criticality, which should better model the real pinning potential. Since the tunneling rate takes the form $\Gamma_Q \propto \exp(-S_{cl}/\hbar)$, where $S_{cl}$ is the least action obtained from the classical trajectory, we calculate the classical equations of motion in the imaginary time path-integral formalism. When both the dynamics are simultaneously considered for the equation of motion, the resultant integral equation becomes complicated by the presence of the cubic term in the pinning potential, which compels us to rely on numerical methods to obtain the solutions.

Our considerations will begin by discussing the pancake vortex in the $xy$ plane with the length $L_c$ along the $z$ axis, where $L_c$ is the collective pinning length which can be expressed in terms of the mass anisotropy parameter $\varepsilon_a^2 = m/M < 1$, the coherence length $\xi$, and the depairing and critical current densities $j_0$ and $j_c$: $L_c \simeq \varepsilon_a \xi (j_0/j_c)^{1/2}$ within the weak collective pinning theory. [2] The supercurrent
characterized by the parameter \( \epsilon = (1 - j/j_c)^{1/2} \) is along the \( y \) direction. Since the length scale in this study is much larger than the size of a vortex core, a vortex can be regarded as a point-like object. Neglecting the inertial mass term in the equation of motion, Kopnin et al.\[8\] derived the following equation of motion:

\[
\eta \dot{\mathbf{u}} + \alpha \mathbf{u} \times \dot{\mathbf{z}} = \frac{\phi_0}{c} \mathbf{j} \times \dot{\mathbf{z}} + \mathbf{F}_p, \tag{2}
\]

where \( \phi_0 = hc/2e \) is the flux quantum, \( \mathbf{u} \) the distortion of the vortex from the equilibrium position in the \( xy \) plane, \( \mathbf{F}_p \) the pinning force, and \( \eta \) and \( \alpha \) are the viscous and Hall drag coefficients given by\[2, 8\]

\[
\alpha = \alpha_0 \frac{(\omega_0 \tau_r)^2}{1 + (\omega_0 \tau_r)^2}, \quad \eta = \alpha_0 \frac{\omega_0 \tau_r}{1 + (\omega_0 \tau_r)^2}. \tag{3}
\]

Here \( \alpha_0 = \pi \hbar n_s \), \( \tau_r = m/n e^2 \rho_n \), \( \Delta = \hbar v_F/\pi \xi \), and \( n_s \) is the density of charge carriers, \( m \) its effective mass, and \( v_F \) the Fermi velocity.

To study the tunneling process at zero temperature, we need to consider the imaginary time path integral\[15\]

\[
\int D\mathbf{u}_x \int D\mathbf{u}_y \exp(-S_E/\hbar). \tag{4}
\]

Considering the effect of dissipation on the tunneling, we use the model introduced by Caldeira and Leggett\[13\] and others\[2, 7, 12\] in which dissipation is modeled by coupling the vortex degree of freedom to a bath of harmonic oscillators. By combining the pinning potential with a cubic term with the Lorentz term in Eq. (2) the model potential for the pancake vortex is taken as\[2, 7, 14\]

\[
V_p(u_x, u_y) - \frac{\phi_0 ju_x}{c} \equiv \frac{V_0}{2} \left[ (\frac{u_y}{R})^2 + c_1 \epsilon (\frac{u_x}{R})^2 - \frac{2}{3} c_2 (\frac{u_x}{R})^3 \right], \tag{5}
\]

where \( \epsilon = \sqrt{1 - j/j_c} \), \( c_{1,2} \sim 1 \), \( V_0 \sim (\phi_0/4\pi \lambda_{xy})^2 \), and \( \lambda_{xy} \) is the bulk planar penetration depth.\[16\]

Now, the Euclidean action of this system takes the form\[17\]

\[
S_E = L_c \int_{-\infty}^{\infty} d\tau \left\{ -i\alpha \frac{du_x}{d\tau} u_y + \frac{V_0}{2} \left[ (\frac{u_y}{R})^2 + c_1 \epsilon (\frac{u_x}{R})^2 - \frac{2}{3} c_2 (\frac{u_x}{R})^3 \right] \right. \\
+ \left. \frac{1}{2} \int_{-\infty}^{\infty} d\tau_1 K_0(\tau - \tau_1) [\mathbf{u}(\tau) - \mathbf{u}(\tau_1)]^2 \right\}, \tag{6}
\]
where the so called nonlocal influential function becomes \( K_0(\tau) = \frac{1}{2\pi} \int_0^\infty d\omega J(\omega) \exp(-\omega |\tau|) \) where \( J(\omega) \) is the spectral density whose form will be discussed later.

Let us now introduce \( u_{x0}, u_{y0}, \) and \( \tau_0 \) as the length scales for \( u_x, u_y, \) and \( \tau, \) respectively, which are to be set by the following scaling analysis for Eq. (6).

A comparison of the quadratic potential term with the cubic one in the \( x \) direction determines the relevant length scale along \( x; \) \( u_{x0} \sim \epsilon Rc_1/c_2. \) Equating the quadratic potential terms in the \( x \) and \( y \) directions, we obtain the length scale for the displacement in the \( y \) direction; \( u_{y0} \sim \epsilon^{3/2} Rc_1^{3/2}/c_2. \) The time scale for the imaginary time, \( \tau_0 \sim \alpha R^2/(V_0 \sqrt{c_1}) \), is set by comparing the Hall term with the quadratic potential in the \( y \) direction. Keeping these points in mind, it is convenient to use the dimensionless variables

\[
u_x = (\frac{c_1 \epsilon R}{2c_2}) \bar{u}_x, \quad \nu_y = (\frac{c_1^{3/2} \epsilon^{3/2} R}{2c_2}) \bar{u}_y, \quad \tau = (\sqrt{2R^2 \alpha_0}/\sqrt{c_1 \epsilon V_0}) \tilde{\tau}.
\]

The Ohmic dissipation\[2, 7, 12, 18\] in Eq. (2), which is obtained if \( J(\omega) = \eta \omega \) after elimination of the oscillators, leads to the influential function

\[
K_0(\tau) = \eta/2\pi |\tau|^2.
\]

Then, the Euclidean action is simplified as

\[
S_E = (\frac{\sqrt{2}c_1^{5/2}}{4c_2^2})(L_c \alpha_0 R^2)\epsilon^{5/2} I,
\]

\[
I = \int_{-\infty}^{\infty} d\tau \{ \epsilon \alpha_1 \frac{d}{d\tau} \bar{u}_x - \frac{1}{2} \bar{u}_x^2 + \frac{1}{2} \bar{u}_y^2 - \frac{1}{6} \bar{u}_x^3 + \frac{1}{4} \eta_1 \int_{-\infty}^{\infty} d\tilde{\tau}_1 \frac{[\bar{u}_x(\tilde{\tau}) - \bar{u}_x(\tilde{\tau}_1)]^2 + c_1 \epsilon [\bar{u}_y(\tilde{\tau}) - \bar{u}_y(\tilde{\tau}_1)]^2}{|\tau - \tau_1|^2} \},
\]

where the dimensionless Hall and dissipation coefficient are given by

\[
\alpha_1 = \frac{1}{\sqrt{2 \epsilon} (\omega_0 \tau_r)^2}, \quad \eta_1 = \frac{\sqrt{2 \epsilon}}{2\pi \sqrt{c_1} \epsilon (\omega_0 \tau_r)^2}.
\]

Let us briefly consider the case of a pure Hall type which is relevant in super-clean superconductors.\[7\] In this limit the integral (14) does not include \( \eta_1 \)-term. Noting that \( I \sim \alpha_1 \int_{-\infty}^{\infty} d\tau \bar{u}_y^{cl} d\bar{u}_x^{cl} / d\tau = \alpha_1 \times (\text{numerical quantity}) \) where \( \bar{u}_x^{cl} \) and
$\bar{u}^c_y$ are classical trajectories in the Hall limit, the approximate least action takes the simple form
\[
S_{cl} \sim (L_c \alpha_0 R^2) \epsilon^{5/2} \alpha_1 \sim (L_c \alpha_0 R^2) \epsilon^{5/2},
\]
which agrees up to a numerical factor with the result in Table 1 obtained by the explicit instanton solution. In the case where the dissipation term is dominant in the vortex dynamics, $\alpha_1$-term is not included in the integral (10). Noting that $I \sim \eta_1 \times \text{(numerical quantity)}$ and $\eta_1 \sim \omega_0 \tau_r / \sqrt{\epsilon}$ in this limit, $S_{cl}$ is approximately
\[
S_{cl} \sim (L_c \alpha_0 R^2) \epsilon^{5/2} \eta_1 \sim (L_c \alpha_0 R^2) \epsilon^2 (\omega_0 \tau_r),
\]
which is also consistent with the result in Table 1 up to a numerical factor.

In the intermediate regime, i.e. when the Hall and the dissipative dynamics are simultaneously present, the classical trajectories of $\bar{u}_x$ and $\bar{u}_y$ should be determined by the Euler-Lagrange equation derived from Eq. (10):
\[
i \alpha_1 \frac{d\bar{u}_y}{d\tau} + \bar{u}_x - \frac{\bar{u}_x^2}{2} = \eta_1 \int_{-\infty}^{\infty} d\tau_1 \left( \frac{d\bar{u}_x}{d\tau_1} \right) \frac{1}{\tau_1 - \tau} = 0,
\]
\[
-i \alpha_1 \frac{d\bar{u}_x}{d\tau} + \bar{u}_y - \eta_1 c_1 \epsilon \int_{-\infty}^{\infty} d\tau_1 \left( \frac{d\bar{u}_y}{d\tau_1} \right) \frac{1}{\tau_1 - \tau} = 0,
\]
where $\bar{u}_x(-\tau) = \bar{u}_x(\tau)$ and $\bar{u}_y(-\tau) = -\bar{u}_y(\tau)$. As in the dissipative case, the equations (14) and (15) are nonlocal in time, which necessitates the Fourier transformation as follows:
\[
\left[ \frac{(\alpha_1 \omega)^2}{1 + \pi \eta_1 c_1 \epsilon |\omega|} \right] + 1 + \pi \eta_1 |\omega|) \bar{u}_x(\omega) = \frac{1}{\sqrt{8\pi}} \int_{-\infty}^{\infty} d\omega_1 \bar{u}_x(\omega - \omega_1) \bar{u}_x(\omega_1),
\]
and
\[
\bar{u}_y(\omega) = \frac{-\alpha_1 \omega}{1 + \pi \eta_1 c_1 \epsilon |\omega|} \bar{u}_x(\omega)
\]
where $\bar{u}_x(-\omega) = \bar{u}_x(\omega)$ and $\bar{u}_y(-\omega) = -\bar{u}_y(\omega)$. Also, we note that Eq. (10) is reduced to
\[
I = \int_{-\infty}^{\infty} d\omega \left\{ \frac{1}{2} \left[ \frac{1}{1 + \pi \eta_1 c_1 \epsilon |\omega|} \bar{u}_x(\omega) \bar{u}_x(-\omega) \right. \right.
\]
\[
- \frac{1}{6} \int_{-\infty}^{\infty} \frac{d\omega_1}{\sqrt{2\pi}} \bar{u}_x(\omega) \bar{u}_x(\omega_1) \bar{u}_x(\omega + \omega_1) \},
\]

after Fourier transform and integrating over the \( \bar{u}_y(\omega) \) variable.

Even though Eq. (14) is a one-dimensional problem with respect to \( \bar{u}_x \), its analytic solution becomes complex by the presence of the nonlocal term arising from the cubic potential (5). We have therefore numerically solved the above integral equations as follows.

Let us introduce, for a given function \( F = F(\omega) \),

\[
\mathcal{L}_F(\omega) \equiv \left\{ \frac{(\alpha_1 \omega)^2}{1 + \pi \eta_1 \epsilon_1 |\omega|} + 1 + \pi \eta_1 |\omega| \right\} F(\omega),
\]

\[
\mathcal{R}_F(\omega) \equiv \frac{1}{\sqrt{8\pi}} \int_{-\infty}^{\infty} d\omega_1 F(\omega - \omega_1) F(\omega_1).
\]

(19)

With the function \( F(\omega) \) in the above definitions replaced by the Fourier transformed classical path \( \bar{u}_x(\omega) \), the equation (16) corresponds to

\[
\mathcal{L}_F(\omega) - \mathcal{R}_F(\omega) = 0.
\]

(21)

Let us now define, for a given function \( F \),

\[
\delta_F \equiv \lim_{\Omega \to \infty} \frac{1}{\Omega} \int_0^\Omega d\omega |\mathcal{L}_F(\omega) - \mathcal{R}_F(\omega)|.
\]

(22)

If a trial function \( F \) has a set of adjustable parameters \( \{ p_i \} \), \( \delta_F \) becomes a function of \( \{ p_i \} \) and if the set \( \{ p_i^* \} \) which minimizes \( \delta \) can be found, the function \( F \) with the parameter set \( \{ p_i^* \} \) will be the desired solution of the integral equation. As shown in Table 1, a natural choice for the trial function \( F \) for general values of \( \alpha_1 \) and \( \eta_1 \) would be to linearly combine the two extreme solutions such as

\[
F(\omega) = p_1 \omega / \sinh(p_2 \omega) + p_3 \exp(p_4 |\omega|),
\]

(23)

where \( p_1, p_2, p_3, \) and \( p_4 \) are parameters to be controlled in the variational procedure.

We have carried out minimization of \( \delta \) of Eq. (22) with the trial function of Eq. (23) by the conjugate gradient method. It turns out that the variational method is very successful, covering all ranges of \( \omega_0 \tau_r \) for arbitrary values of \( \epsilon \). In principle, a perfect minimization of \( \delta \) should make it to be zero, but due to the
numerical accuracy in carrying out integrations involved in the procedure (or it may be possible that the trial function of the form in Eq. (23) cannot perfectly minimize \( \delta \)), our numerical calculation typically gave \( \delta / \bar{L} \leq 10^{-3} \), where \( \bar{L} \) is the average value of \( L_{\omega}(\omega) \) in the integrand of Eq. (22). Nevertheless, the accuracy is far good enough for our purpose of evaluation of the action integral in later stages.

The accuracy of the variational method has been also checked by comparing the solutions by this method with those by the direct iterations of differential equations (14) and (15) in the range of \( \omega_0 \tau_r \) where the solutions by the iteration method are available. The two solutions always match excellently, which confirms that our variational method produces true instanton solutions.

Typical instanton solutions thus obtained are illustrated in Fig. 1. The classical Euclidean action \( S_{\text{cl}} \) is given by a numerical integration of Eq. (10) with the instanton solutions. We then have the relaxation rate or the magnetic viscosity \( Q(T = 0) \) which equals \( \hbar / S_{\text{cl}} \), in other words, \( Q(0)/Q_0 = 2\sqrt{2}/I \) where \( Q(0) \equiv Q(T = 0) \) and \( Q_0^{-1} = \epsilon^{5/2}(\pi n_s L_c \xi^2) \). Here we have set \( c_{1,2} \) to be 1 and replaced \( R \) by the coherence length \( \xi \). The results of evaluation of \( Q(0)/Q_0 \) versus \( \omega_0 \tau_r \) for \( \epsilon = 1, 0.1, 0.01, \) and 0.001 are shown in Fig. 2. We first point out that our \( Q(0)/Q_0 \) is different from the result in Ref. [12], which reflects the fact that the infinitely steep potential which underestimates the suppression factor is carefully treated in our model potential. As is noted in Table 1, the approximate value of \( \epsilon^{5/2}Q(0) \) in units of \( \pi n_s L_c \xi^2 \) is 15/18 in the Hall limit and \( 2\epsilon_{1/2}/(\pi \omega_0 \tau_r) \) in the dissipative limit, which agree with the limiting values of the figure. Even though \( Q(0)/Q_0(= \hbar / S_{\text{cl}}) \) looks going toward infinity as \( \omega_0 \tau_r \to 0 \), it actually does not diverge in that limit. In the pure dissipative case, the classical action \( S_{\text{cl}} \) vanishes as \( \omega_0 \tau_r \to 0 \) in our consideration, which is expected from the limiting behavior of Eq. (13). However, in this situation the inertia term not included in this work influences the results. By including the massive term, in this limit the approximate form of the classical action becomes \( S_{\text{cl}} / \hbar \sim L_c \sqrt{m_v \bar{V}_0} \xi \epsilon^{5/2} \) which is independent
of $\omega_0 \tau_r$, where $m_v$ is the inertia mass of a vortex. In general the massive term is small in the region except for the limit mentioned before and usually can be neglected as compared to either the dissipative or the Hall term. In the intermediate tunneling regime, our curves display an interesting feature which is the clear existence of minimum of $Q(0)/Q_0$ for each $\epsilon$. Even at $\epsilon = 1$, there exists a shallow yet clear minimum near $\omega_0 \tau_r \sim 3.5$ (see the inset). As $\epsilon$ becomes smaller, i.e., as $j \rightarrow j_c$, the minimum becomes much more pronounced with its location moving toward $\omega_0 \tau_r \approx 1$ at the same time. The existence of the minimum of $Q$ can be understood by noting the following qualitative features of the relaxation rate based on the dimensional estimation. Noting that for the pure Hall motion the classical trajectory is dominant in the frequency scale $|\omega| \sim 1/\alpha_1$ in Eq. (16) which is also understood by the solution $\bar{u}_x(\omega)$ in Table I, we find that from Eqs. (12) and (13) the correction to the pure Hall classical action $S_0^H$ by the small dissipation $\eta_1$ is of the order $\eta_1/\alpha_1$, that is, the corrected classical action becomes $(1 + \eta_1/\alpha_1)S_0^H \sim (1 + 1/\omega_0 \tau_r)S_0^H$. The relaxation rate is then $Q \sim \omega_0 \tau_r/(1 + \omega_0 \tau_r)$, which increases monotonically as a function of $\omega_0 \tau_r$ in the large $\omega_0 \tau_r$ regime. The fact that the classical action increases (or the corresponding relaxation rate decreases) by inclusion of the dissipation is well known in the macroscopic quantum tunneling problems, which is also physically clear in the sense that the dominant (Hall) motion is hindered by the frictional force, resulting in decreasing the relaxation rate. In the opposite limit, on the other hand, the correction to the pure dissipative action $S_0^D$ by the small Hall contribution $\alpha_1$ is of the order $(\alpha_1/\eta_1)^2$. The corrected classical action is then $(1 + (\alpha_1/\eta_1)^2)S_0^D \sim \omega_0 \tau_r(1 + (\omega_0 \tau_r)^2)$, leading to the relaxation rate $Q \sim 1/[\omega_0 \tau_r(1 + (\omega_0 \tau_r)^2)]$. $Q$ now decreases monotonically as a function of $\omega_0 \tau_r$ in the small $\omega_0 \tau_r$ regime. Therefore, it is expected that there exists a minimum of $Q$ in the intermediate range of $\omega_0 \tau_r$ because the relaxation rate decreases monotonically for small $\omega_0 \tau_r$ but increases monotonically for large $\omega_0 \tau_r$. This suggests us to predict observation of minimum in an experiment.
As is shown in Fig. 2, our results are valid in any value of $\epsilon \leq 1$. It is well known that the tunneling rates get larger for smaller $\epsilon$ since the height and the width of the barrier are proportional to the parameter $\epsilon$. However, there might exist the quantum tunneling even in the absence of the external current, i.e., $\epsilon = 1$. Actually the existence of the quantum tunneling is determined by estimating the magnitude of the quantum tunneling rate $\Gamma_Q$. If we apply our result of $\epsilon = 1$ to the existing experimental data,[16] where $Q(0)/Q_0 \approx 2.3$ and $Q_0^{-1} \approx 78$ in the YBa$_2$Cu$_3$O$_7$/PrBa$_2$Cu$_3$O$_7$ multilayer systems[1] and $Q(0)/Q_0 \approx 2.0$ and $Q_0^{-1} \approx 66$ in the BiSr$_2$CaCu$_2$O$_8$ single crystal,[10] we obtain $S_{cl}/\hbar \approx 33$ for both systems. This gives $\Gamma_Q \approx \omega \exp(-S_{cl}/\hbar) \approx 4.66 \times 10^{-3} \ (4.66 \times 10^{-4})$sec$^{-1}$ for the attempt frequency $\omega(\approx V_0\epsilon^{1/2}/(\xi^2 n_s))$ of order $10^{12} \ (10^{11})$sec$^{-1}$ for YBCO/PBCO (BiSCCO), which is a resonable value to ensure the observable rate of tunneling. Also, using the value of $Q(0)/Q_0$ for both systems, we get $\omega_0 \tau_r \approx 0.29$, which corresponds to the Hall angle $\Theta_H = \arctan(\alpha_1/\eta_1) = \arctan(\omega_0 \tau_r) \approx 16^\circ$ for the former, and $\omega_0 \tau_r \approx 0.37 \ (\Theta_H \approx 20^\circ)$ for the latter. This implies that for those systems the vortex tunneling at low temperatures occurs in an intermediate regime between the purely dissipative tunneling and the superclean Hall tunneling. It should be also noted that the quantum creep in the oxygen deficient YBa$_2$Cu$_3$O$_x$ films[11] occurs in the intermediate regime as well, where the Hall angle depends on the oxygen content.

In the thermal activation regime, the classical solutions $\bar{u}_x(\bar{\tau})$ and $\bar{u}_y(\bar{\tau})$ do not depend on $\bar{\tau}$[24] and the integration range in Eq. (10) reduces to between $-\bar{\beta}/2$ and $\bar{\beta}/2$ where $\bar{\beta} = \beta \hbar(\sqrt{\pi} e V_0/\sqrt{2} R^2 \alpha_0)$. Then we obtain the classical action $S_{cl} = S_T = \beta \hbar U_0$, where $U_0 = L_c V_0 \epsilon^3/6$, and the corresponding escape rate

$$\Gamma_T \propto \exp(-S_{cl}/\hbar) = \exp(-U_0/k_B T),$$

which is the Boltzmann formula representing a pure thermal activation. Comparing $\Gamma_T$ with $\Gamma_Q(\propto \exp(-S_{cl}/\hbar))$, we have the crossover temperature $k_B T_0 \sim A\epsilon^{1/2} Q(0)/Q_0$ from the thermal to quantum regime where $A = \phi_0^2/(96\pi^3 \lambda^2 \xi^2 n_s)$
and find that the quantum process dominates at $T < T_0$. Using $\xi = 1.5\text{nm (2.0nm)}$, $\lambda = 150\text{nm (250nm)}$, and $n_s = 5 \times 10^{27}/\text{m}^3 (3.5 \times 10^{27}/\text{m}^3)$ for YBCO multilayer systems (BiSCCO single crystal), for $\epsilon = 1$ we get $T_0 \sim 9.48\text{ K (2.36 K)}$ which agrees reasonably with the experimental results.

In conclusion, we have studied the quantum vortex creep for the case where the Hall and the dissipative dynamics are simultaneously present by using the standard instanton method. We have introduced the cubic potential term to better model the pinning potential and solved the resultant integral equations numerically. In a comparison with available experimental data, our results indicate that the quantum vortex creep may occur in an intermediate regime between the Hall and the dissipative regimes for highly anisotropic and multilayer superconductors. It has been also found that there exists a minimum in a scan of the relaxation rate by $\omega_0\tau_r$ for every $\epsilon$ we have studied, whose observation we predict in an experiment with nonzero external currents. The temperature is set to zero in this study, but an extension to finite temperatures will be discussed elsewhere. It will be also interesting to apply our approach to systems with columnar defects produced by irradiation with heavy ions.

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\( \omega_0 \tau_r \pi \sqrt{\epsilon_f} \) becomes smaller than \( \sim 0.3 \), iterations converge very slowly, and in worse cases, even if we choose the initial trial functions well approximating the actual solutions, slight numerical errors can drive iterations away from convergence so as to make the iteration scheme to break down.

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Table 1: Instantons in the $\bar{\tau}$ space and in the $\omega$ space, and the corresponding actions in the Hall limit ($\omega_0\tau_r \gg 1$) and the dissipative limits ($\omega_0\tau_r \ll 1$). Here $c_H = 18c_1^{5/2}/15c_2^2$ and $c_D = \pi c_1^2/2c_2^2$.

|                  | Hall $[7]$                       | dissipative $[3, 20]$               |
|------------------|----------------------------------|-------------------------------------|
| $\bar{u}_x(\bar{\tau})$ | $3/\cosh^2(\bar{\tau}/2\alpha_1)$ | $4/[1 + (\bar{\tau}/\pi\eta_1)^2]$ |
| $\bar{u}_x(\omega)$   | $\frac{12\alpha_1}{\sqrt{2\pi}} \frac{(\alpha_1 \pi \omega)}{\sinh(\alpha_1 \pi \omega)}$ | $(2\pi)^{3/2} \eta_1 \exp(-\pi\eta_1 |\omega|)$ |
| $S_{cl}/(Lc_0R^2)$    | $c_H \epsilon^{5/2} \frac{(\omega_0 \tau_r)^2}{1+(\omega_0 \tau_r)^2}$ | $c_D \epsilon^2 \frac{\omega_0 \tau_r}{1+(\omega_0 \tau_r)^2}$ |
Figure 1: Typical instanton solutions: $\bar{u}_x(\bar{\tau})$ (top) and $-i\bar{u}_y(\bar{\tau})$ (bottom) for $\epsilon = 1$ when $\omega_0 \tau_r \gg 1$ (a), $\omega_0 \tau_r = 1$ (b), and $\omega_0 \tau_r = 0.35$ (c).

Figure 2: Dependence of the quantum vortex creep relaxation rate $Q(0)/Q_0$ on $\omega_0 \tau_r$ at zero temperature, for $\epsilon = 0.001$ (a), 0.01 (b), 0.1 (c), and 1 (d). Also is shown, to logarithmic accuracy, the relaxation curve (e) from Ref. [12] (Eqs. (16) and (20)). Note that the approximate value of $Q(0)/Q_0$ becomes 15/18 in the Hall limit ($\omega_0 \tau_r \gg 1$) and $2\epsilon^{1/2}/(\pi \omega_0 \tau_r)$ in the dissipative limit ($\omega_0 \tau_r \ll 1$). In the inset the plot (a) for $\epsilon = 1$ is magnified.
