Methodology of Assessing Strength of Metal Structure Elements with Cracks

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Abstract. Technological defects such as cracks can exist in the elements of metal structures, or cracks can appear during operation. Calculation of crack resistance, supplementing usual calculation of strength, is designed to protect such a structure from premature failure due to the unstable development of the crack. The crack resistance parameter can vary both due to increasing stress and due to increasing the length of the crack. These two factors are interrelated, therefore, usual strength calculation does not give a real estimate of strength, an actual safety factor is lower than that in usual calculation. Therefore, it is proposed alongside with the safety factor for strength to introduce a safety factor for the crack length. In the work, in general form there were obtained relations for reasonable selection of the safety factor for the crack length. For frequently used fracture criteria, dependences were derived for selecting the safety factor for the crack length. The dependences are also given for determining the breaking stress corresponding to the existing crack and the critical length of the crack under existing stresses for various fracture criteria. Based on the results obtained, there was obtained a method of analyzing strength of structural elements with cracks, which allowed determining the permissible crack length and the actual safety factor of structures. The results are of great importance with viscous and mixed fracture, where the criteria of nonlinear fracture mechanics are used.

1. Introduction
Recently, in connection with the advent of highly stressed structures operating under extreme conditions and increasing the requirements for reliability of their elements, there has been growing interest in the problem of materials and structures fracture. Attention to this problem is also caused by the fact that structural defects, such as cracks, either obviously exist in structural elements, or they arise during operation. Fracture-like defects are often found in welded joints of metal structures, pipelines and pressure vessels. The design of these structures has been given a lot of attention in publications.

The needs of engineering practice require the development of methods of calculating the structural elements with cracks strength. Calculation of crack resistance, supplementing usual calculation of strength, is intended to contribute to measures of protecting structures against premature failure and to establish tolerances for the safe dimensions of cracks and defects. The general problems of such calculations are described in monographs [1, 2].

Determining the stress-strain state of bodies with cracks is performed by analytical and numerical methods and is described in many books and scientific articles. They come down to determining the
parameter of fracture mechanics at the crack tip. The number of articles dealing with determining these parameters continues to grow [3-13].

For strength calculations of bodies with cracks, the justified selection of the fracture criterion and the justified designation of the safety factor for the crack length are very important. Selecting the fracture criterion depends on the product material, the crack length and the fracture mechanics parameter used. A lot of research is also dealing with the development of new criteria, the improvement and development of existing criteria [14-21].

From this brief review it follows that the scope of research for determining the stress state parameters at the crack tip, for developing criteria for the limiting state of bodies with cracks is growing every year, which indicates the problem urgency. At the same time, the analysis of calculations for strength of metal structures and machine parts with cracks shows that there is no sound and generally accepted methodology of such calculations. Most journal articles are private in nature and consider the features of specific structures failure. In addition, the modern concept of “fit for purpose” requires reasonable selection of the fracture criteria and the assignment of safety factors, the development of an algorithm for determining the desired parameters (critical crack length, breaking load) from the criterion relations. The justified selection of the safety factor for the crack length is difficult and very important in practical terms.

The purpose of this study is to justify selection of the safety factor for the crack length and development on this basis of a common methodology for assessing strength of bodies with crack-like defects.

The authors have been studying the problems of calculating and assessing the structural elements with cracks strength for a long time and have a good scientific basis [22, 23].

2. Methods
Methods of calculating strength with brittle fracture, based on linear fracture mechanics, in particular on the stress intensity factor (SIF), are well developed. The use of viscous materials and the size of cracks restriction on the part of flaw inspection lead to increasing the nominal breaking stresses exceeding the yield strength, as a result of which viscous failure occurs. Under these conditions, for strength calculations, it is necessary to apply the criteria of nonlinear fracture mechanics. Applying these criteria requires further research and analysis.

The criteria of nonlinear fracture mechanics are not entirely obvious, therefore, a lot of criteria are proposed. One-parameter criteria are usually expressed in terms of the J-integral, less often in terms of crack opening criteria. In the case of large local elastoplastic deformations, the deformation criterion is applied.

When metal structures are fractured, there occurs mixed fracture, which can be described using two-parameter criteria. The most common criteria are relationships expressed in terms of the SIF or the crack resistance parameter [2, 24, 25]. However, these relations give noticeable errors for large plastic strains. Under these conditions, two-parameter criteria should be applied, expressed in terms of the J-integral [2, 19] or the strain rate.

Many studies show that criteria parameters are interconnected. This is explained by the fact that they are all based on the conditions of plastic flow through a single diagram of material deformation and reflect the ability of the material to deform plastically in local volumes. Therefore, these criteria are interconnected through the parameters of the deformation diagram. Obviously, a connection is possible between the parameters of the nonlinear fracture mechanics in the elastoplastic region and the SIF. The presence of such a relationship significantly expands the range of applied problems solved by linear fracture mechanics.

The crack resistance parameter in the criterion relations, regardless of the methods of their calculation and the selected criteria, depends on the current stress\(\sigma\), crack length \(l\), and characteristic part dimension \(b\). General criteria can be written as:

\[ F(\sigma, l, b) \leq F_c, \]
where the equality is achieved in the case of $\sigma=\sigma_p$, $l=l_k$ the crack critical length.

The SIF, the J-integral, the crack opening width, etc. can be used as the $F$ criterion. The right side of the inequality, which includes the limiting value of the crack resistance parameter in the general case, can depend on breaking stress $\sigma_p$. This is especially true for multi-parameter criteria. In addition, $F_c$ is usually determined on the sample that, which differs from the calculated element in shape, dimensions and loading pattern. The discrepancy between the limit parameters of the fracture mechanics of the sample and the element is taken into account by the construction factor $\psi$, by which the limit value of the crack resistance parameter is multiplied [2].

The structural factor is determined empirically and is presented in the form of an empirical dependence. So, for two-dimensional products working in bending and stretching, there is recommended

$$\psi \approx \sqrt{d_e/d_o},$$

where $d_e, d_o$ is the element and sample dimension along the crack.

Taking into account these notes, the condition of structure strength will take the form:

$$F(\sigma,l,b) \leq F_c(\sigma_p,\psi).$$

(1)

The calculated crack toughness parameter $F$ can vary due to both the load and the crack length. Therefore, it is advisable to introduce safety factors that distinguish between these two cases [26]. When conducting strength calculations, the calculated stress is taken from the strength condition, for example $\sigma_p = \sigma_b/n$, where $\sigma_b$ is tensile strength, $n$ is the safety factor. To determine the permissible crack length for given design stress, the right-hand side of (1) should be reduced, for example, by the factor of $m$. Thus, at the fracture stage, the calculated equation for strength in the presence of a crack and static loading takes the form:

$$F(\sigma_b/n,l,b) = F_c(\sigma_p,\psi)/m.$$

(2)

Here $m$ is the safety factor for this fracture toughness characteristic $F_c$ or the safety factor for the length of the crack, showing how many times it is necessary to increase $F$ by increasing the length of the crack at a given stress level so that $F$ reaches its limit value $F_c$. For $m = 1$, from equation (2) we obtain the critical crack length $l_k$ corresponding to given stress $\sigma_n$, and for a reasonably selected value of $m$, the permissible crack length $l_0$.

If the design has a permissible crack $l_0$ from equation (2) with $m = 1$, we find the nominal breaking stress $\sigma_p(l_0)$. Then the actual safety factor will become less than $n$ and will be equal to

$$n_0 = \sigma_p(l_0)/\sigma_n.$$

(3)

The actual safety factor $n_0$ shows how many times the external load must be increased so that the permissible crack becomes critical.

Equation (2) is often transcendental. In this case, for carrying out the above calculations, the graphical-analytic method with constructing a fracture diagram is effective (Figure 1).

You can enter a factor for reducing the safety factor $\sigma=\sigma_p/\sigma_p$. Then you can write down:

$$n_0 = n/\sigma_a.$$

That is, the actual safety factor is determined by means of reducing the $n$ factor. Work [2] proposes to determine the actual safety factor from the relation:

$$n_0 = n^{1-a/2},$$

where $a$ is an indicator equal to the ratio of the area under the critical fracture diagram (above the calculated stress level) to the area of the triangle $0.5(\sigma_b-\sigma_n)l_k$ (Figure 1, c).
Figure 1. Towards the graphical analytic method of strength assessment.

Thus,

$$a = S_1 / S_2 = 2 \int_0^{l_k} \frac{\sigma_p(l) dl}{(\sigma_0, \sigma_n)}$$

where $\sigma_p(l)$ is the diagram of critical stresses calculated from $(2)$ at $m=1$.

A reasonable selection of the safety factor for the crack length is very important in practical terms and there is still no consistent approach in this matter. The following approach was proposed in [2]. Let’s require that, with a permissible crack length, the nominal breaking stress is not less than yield strength. Let’s write down criterion relation $(1)$ at the boundary of the brittle and viscous states for $\sigma=\sigma_T, l=l_T$:

$$F(\sigma_T, l_T) = F(\sigma_T)$$

and calculation equation of strength $(2)$ for $\sigma=\sigma_n, l=l_T$:

$$F(\sigma_n, l_T) = F(\sigma_n) / m_0.$$ 

From here we obtain the required coefficient

$$m_0 = \frac{F(\sigma_T, l_T)}{F(\sigma_n, l_T)} / \frac{F(\sigma_T)}{F(\sigma_n)}.$$ 

(4)

Failure in the presence of a crack of permissible length will be viscous if $m > m_0$, and brittle if $m < m_0$. However, selection of $m$ according to formula (4) does not solve the problem of selecting a truly permissible crack length but only indicates the possible nature of the fracture and is suitable only
for $\sigma_n \leq \sigma_T$. To select the coefficient $m$, it is advisable to use a critical fracture diagram [2]. Let’s find the coefficient $m$ from the criterion relation for $\sigma = \sigma_0/\alpha$, $l=l_0$:

$$F(\sigma_0/\alpha, l_0) = F_c(\sigma_0/\alpha)$$

and from the calculation equation for $\sigma = \sigma_n$, $l=l_0$:

$$F(\sigma_n, l_0) = F_c(\sigma_n)/m_1.$$  

To determine the coefficient $m$, we obtain the formula

$$m_1 = \frac{F(\sigma_0/\alpha, l_0) \cdot F_c(\sigma_0)}{F_c(\sigma_0/\alpha)}.$$  

(5)

The $\alpha$ value taking into account the earlier given relations will take the form: $\alpha = n^{a/2}$. From the two values of the coefficient $m$ it is natural to accept the larger one:

$$m = \max(m_0, m_1).$$  

(6)

3. Results

Let’s obtain expressions for determining the safety factor for the crack length according to various criteria of fracture. When using the Irving force criterion, we have:

$$F_c = K_{IC}, \quad F = \sigma \sqrt{\pi l f(\lambda)}, \quad (\lambda = l/b),$$

where $K$ is the coefficient of stress intensity; $K_{IC}$ is its limiting value; $f(\lambda)$ is the correction function taking into account the element dimension finiteness.

Then by formulas (4) and (5) we obtain:

$$m_0 = \sigma_T/\sigma_n = n\sigma_T/\sigma_b, \quad m_1 = \sigma_0/\alpha \sigma_n = n\alpha = n^{1-a/2}.$$

If when determining the SIF we take into account the Irving correction, then

$$F = \sigma \sqrt{\pi l \left(1 + \sigma^2/2\sigma_T^2\right) f(\lambda)}$$

and by formulas (4) and (5) we obtain

$$m_0 = n \frac{\sigma_T}{\sigma_b} \sqrt{\frac{1.5}{1 + \left(\frac{\sigma_0}{\alpha \sigma_T}\right)^2}}, \quad m_1 = n^{1-a/2} \sqrt{\frac{1 + \sigma_0/\alpha \sigma_T^2}{1 + \left(\frac{\sigma_0}{\alpha \sigma_T}\right)^2}}.$$

If the criterion relation is expressed through the $J$–integral, then $F=J$, $F_c=Jc$. In the conditions of small-scale fluidity we can write down [1]

$$J = \frac{K^2}{E} = \sigma^2 \pi l f^2(K)/E.$$

Then by formulas (4) and (5) we obtain

$$m_0 = (n\sigma_T/\sigma_b)^2, \quad m_1 = (n/\alpha)^2 = n^{2-a}.$$

For the power law of strengthening with the $m$ parameter [1]

$$J = \begin{cases} &2K^2/(1+m)E, \quad \sigma < \sigma_T; \\ &2K^2(\sigma/\sigma_T)^{(1-m)/m}/(1+m)E, \quad \sigma > \sigma_T. \end{cases}$$

(7)

Then by formulas (4) and (5) we obtain

$$m_0 = (\sigma_T/\sigma_n)^{(1+m)/m} = (n\sigma_T/\sigma_b)^{(1+m)/m},$$

$$m_1 = (\sigma_0/\sigma_n)^{(1+m)/m} = (n/\alpha)^{(1+m)/m} = n(1-a/2)^{(1+m)/m}.$$

If we use the deformation criterion, then
where $\hat{e}$ is relative deformation; $\hat{e}_c$ its limiting value determined according recommendations [26].

The local deformation intensity (at the crack tip) [26]

$$\bar{e}_c = K_c \hat{e}_c \left( \frac{K_T}{\sigma_T} \right)^{n/2},$$  \hspace{1cm} (8)

where $K_c$ is the strain concentration factor (SCF) at the crack tip;

$K_T = K_T/\sigma_T$ is a relative value of the SIF determined by the linear mechanics of fracture;

$p = \frac{2-0.5(1-m)(1-n)}{m+1}$, $q = 0$ at $\bar{e} < 1$, $q = (1-m)/(m+1)$ at $\bar{e} \geq 1$. Now by formula (4) we obtain

$m_0 = \frac{p_l^{m+1} q^{(m+1)/m}}{\sigma_0^{m+1}},$ where $p_2 = \frac{2-0.5(1-m)(1-n_0/\sigma_0)}{m+1}$.

Since $\bar{e}_c < 1$, $q = 0$, and $\bar{e}_c = 1$. Then

$$m_0 = \frac{(\sigma_T/\sigma_0)^{p_2}}{m_0} = (n_0^{1/2}/\sigma_0)^{p_2},$$ \hspace{1cm} (9)

In the same conditions by formula (5) we obtain

$$m_1 = \left( \frac{\sigma_0}{\sigma_n} \right)^{q} \left( \frac{\sigma_0}{\sigma_n} \right)^{p_1} = n_0^{1/2} q \left( \frac{\sigma_0}{\sigma_n} \right)^{p_2},$$ \hspace{1cm} (10)

where $p_l = \frac{2-0.5(1-m)}{m+1}$, $q = (1-m)/(m+1)$.

Let’s consider two-parameter criteria of fracture. Such a criterion expressed in the SIF terms can be presented in the form [2, 14]:

$$J = J_c \left[ 1 - \frac{\sigma}{\sigma_c} \right]^{1/m} = F_c(\sigma),$$

Then, taking into account expression (7)

$$F = \frac{2\sigma^2 \hat{e}^2}{(1+m)/E} \left( \frac{\sigma}{\sigma_T} \right)^{1-m} \left( \frac{\sigma}{\sigma_T} \right)^{1-m} = \frac{2\sigma^2 \hat{e}^2}{(1+m)/E} \left( \frac{\sigma}{\sigma_T} \right)^{1-m}.$$

Now by formulas (4) and (5) we obtain

$$m_0 = \left( \frac{\sigma_n}{\sigma_0} \right)^{1-m} \left[ 1 - \left( \frac{\sigma_n}{\sigma_0} \right)^{1/m} \right]^{m+1} = \left( \frac{\sigma_0}{\sigma_n} \right)^{m+1} \left[ 1 - \left( \frac{\sigma_0}{\sigma_n} \right)^{1/m} \right]^{m+1},$$

$$m_1 = \left( \frac{\sigma_n}{\sigma_0} \right)^{1-m} \left[ 1 - \left( \frac{\sigma_n}{\sigma_0} \right)^{1/m} \right]^{m+1} = n \left( 1-\frac{2m}{1-m} \right) \left( \frac{\sigma_0}{\sigma_n} \right)^{m+1}.$$

For large elastic-plastic deformations based on the Neuber solutions, the authors proposed a two-parameter criterion of fracture that can be written down in the form:
The ultimate SCF value is \( K_{lc} = K_{lc} \sqrt{2\pi l} \), and according to (8), \( K_{lc} = F = K_0 \sigma_l \).

Now by formula (4) taking into account transformations resulting in expression (9), we have:

\[
m_0 = \left( \frac{n^m \sigma}{\sigma_l} \right)^p - \frac{1-n^{1/m}}{1-(\sigma_l/\sigma)^{1/m}}.
\]

Then by formula (5) taking into account transformations resulting in expression (10), we obtain:

\[
m_1 = n^{(1-a)q} \frac{(\sigma_l/\sigma)^p}{(\sigma_l/\sigma)^q} - \frac{1-n^{1/m}}{1-n^{a/m}}.
\]

Reasonable selection of the safety factor for the crack length allows determining the permissible crack length \( l_k \) from equation (2). The expressions for determining the critical crack length \( l_c \) (and consequently \( l_k \)), as well as the breaking stresses \( \sigma_p \), can be obtained from the corresponding criteria relations given above. If in condition of strength (1) we take a strict equality, then we’ll find out the breaking stress corresponding to the existing crack length. If in this equality the stress is equated to the limiting value \( \sigma_l \) (\( \sigma_l = \sigma_T \) for the yield zone and \( \sigma_l = \sigma_p \) for the strengthening zone), then we’ll find the critical crack length for the acting stress. So from the force criterion expressed through the SIF, taking into account the Irving correction, we obtain:

\[
\sigma_p = \sqrt{1 + 2(K_{lc}/\sigma_l)^2 \pi l - 1}} = \frac{K_{lc}^2}{\sigma_l}.
\]

For the energy criterion expressed through the \( J \) – integral we obtain the following expressions:

– for small-scale fluidity

\[
\sigma_p = \sqrt{EJ/(\pi l}}; \quad l_k = \sqrt{EJ/(\pi l}}.
\]

– with developed plastic deformations

\[
\sigma_p = \sqrt{\frac{J_1 E (1+m)}{2\pi l^2}(\lambda)} \sigma_l^{(1-m)/(1+m)}; \quad l_k = \frac{J_1 E (1+m)}{2\pi l^2(\lambda)} (\sigma_l/\sigma_l)^{(1+m)/m}.
\]

For the two-parameter criterion in the SIF terms we have

\[
l_k = \frac{K_{lc}^2 (1-\sigma_0 q)}{\sigma_0^q (\lambda)},
\]

and for determining breaking stresses it is needed to find the roots of the following transcendental equation:

\[
\left( \frac{\sigma_p}{\sigma_0} \right)^q + \left( \frac{\sigma_p(\lambda)}{K_{lc}} \right)^2 = 1.
\]

4. Discussion

Based on the results obtained, an improved methodology of calculating the structural elements with cracks strength has been developed. When developing the methods of strength calculations, two types of calculations should be distinguished: design and verification calculations. Let us first consider the design calculation, which consists in determining the critical and permissible crack length and the actual safety factor of the product with a permissible crack.

1. Selecting the material and standard safety factor for tensile strength \( n \).
2. Selecting the design scheme and determining the dimensions of the defect-free element from the strength conditions.
3. Analyzing the structure for the presence or possibility of cracking during operation.
4. Selecting the proposed type of fracture and the corresponding criterion of fracture mechanics.
5. Determining the limiting value of the crack resistance parameter $F_c$ ($K_{IC}, I_{IC}, \delta_c, \varepsilon_c$) and the designation of the safety factor $n_b(n_c, or n_v)$, the loading rigidity coefficient $\eta$, and other design factors $\psi$. The safety factor for brittle $n_c$ and viscous failure $n_v$ takes into account the validity of selecting the limiting value of the crack resistance parameter and the accuracy of its calculation determining.

The coefficient $\eta$ takes into account the difference between the stress-strain state of the element and plane strain, for which the limiting parameters are determined. Theoretical calculations using the Broeck model lead to the relation that can be reduced to the form:

$$\eta = \sqrt{t/m}$$

where $t_m = 2.5 (K_{IC}/\sigma_T)^2$; $t$ is the element width.

The experimental data provide the basis for the next assessment of this coefficient:

$$\eta = 1 \text{ at } t \geq t_m; \quad \eta = \sqrt{t/m} \text{ at } t < t_m.$$

6. Calculating the crack resistance parameter. A crack is supposed to be emerging from a dangerous point in the most probable direction. Sometimes, several such directions are selected, and after calculating the crack resistance parameter, the direction that corresponds to the highest value of the crack resistance parameter is accepted. This parameter must be presented analytically or graphically.

$$(K, J, \delta, \alpha) = F(\sigma, l, b).$$

When calculating by the finite element method such presentations give regression dependences.

7. Determining the critical crack length $l_k$ from the condition

$$F(\sigma_n, l_k) = \eta F_c(\sigma_n)/n_b \quad (\sigma_n = \sigma_b/n).$$

8. Selecting the safety factor for the crack length by formulas (4), (5) and (6).

9. Determining the permissible crack length $l_0$ from the condition

$$F(\sigma_n, l_0) = \eta F_c(\sigma_n)/n_b m.$$  

10. Determining critical stress $\sigma_p$ corresponding to the permissible crack length

$$F(\sigma_p, l_0) = \eta F_c(\sigma_p)/n_b.$$  

11. Determining the actual safety factor by formula (3).

Verification calculations consist in checking the condition that the dimension of the existing defect $l$ is smaller than the permissible crack length, as well as in determining the actual safety factor according to the selected criterion.

1. Selecting the design scheme and determining the stress-strain state of the element (rated design stress $\sigma_d$).

2. Selecting the proposed type of fracture and the corresponding criterion of fracture mechanics.

3. Determining the limit value of the crack resistance parameter $F_c$ and assigning the safety factor $n_b$, the coefficient of loading rigidity $\eta$, and other design factors $\psi$.

4. Calculating the crack resistance parameter and determining dependence (11).

5. Determining $l_k$ from equation (12) and checking the condition

$$l + r_f < l_k,$$

where the correction for the plastic zone

$$r_f = (K_f/\chi\sigma_T)^2/2\pi.$$
Here $\chi = 1$ for the plane stress state and $\chi = \sqrt{3}$ for plane deformation. If this condition is not met, then it is necessary to reduce the load or to stop further use of the product.

6. Assigning the rated safety factor for ultimate strength $n$ and reasonable selecting the safety factor $m$ for the crack length according to formulas (4), (5) and (6).

7. From equation (13) determining the permissible crack length and checking the condition

$$1 + r \leq l_0.$$ 

8. From equation (14) determining $\sigma_p(l_0)$ and checking the condition $\sigma_p \leq \sigma_n$.

9. Determining the actual safety factor by formula (3).

Based on the results of calculations, draw conclusions or develop recommendations for the further operation of structures.

This work difference from similar works consists in separating the safety factor by strength and by crack length in strength calculations. The need for this is caused by the fact that the criterion parameter (for example, SIF) can vary both due to increasing stress and due to increasing the crack length. These two factors are interrelated, especially in viscous and mixed fracture, where criteria of nonlinear fracture mechanics are used. Therefore, the usual strength calculation does not give a real estimate of strength, since it does not take into account slow subcritical growth of the crack length. The actual safety factor is lower than that when calculating the permissible stresses or the adopted safety factor for strength.

In order to implement this approach, it is necessary to make reasonable selection of the safety factor for the crack length. Solving this problem was one of the goals of the work. The goal has been completely achieved, the problem has been solved in general terms, regardless of the specific type of the fracture criterion. The safety factor for the crack length depends on the safety factor for tensile strength, the type of critical fracture diagram, reflecting the properties of the material, and the specific value of the crack resistance parameter. Selecting this coefficient allows determining the permissible crack length, which is important when operating structures with cracks.

The practical value of the work consists in the development of methods for strength analysis of structural elements with cracks. This is one more goal of the work that has also been achieved.

5. Conclusions
Based on the results of the work, the following conclusions can be drawn.

1. In general terms, the relations have been obtained for reasonable selection of the safety factor for the crack length.

2. For frequently used one and two parametric criteria, dependencies have been derived for selecting the safety factor for the crack length.

3. Explicit expressions have been obtained for determining the breaking stress corresponding to the existing crack and the critical length of the crack under the existing stress for various fracture criteria.

4. A technique has been developed for strength analysis of structural elements with cracks.

Theoretical results are applicable for any type of fracture, regardless of the fracture criterion used. The developed methodology for strength calculation allows determining the critical and permissible crack length, the actual safety factor of structures with a permissible or existing crack. The results of the work can be used in designing metal structures with cracks in construction, in lifting and transporting machines, in pipelines for oil and gas, in pressure vessels, as well as machine parts.

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