Analytical Study of the Robustness of the Different Variants of Fractional-Order Self-Tuned Fuzzy Logic Controllers

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Abstract. Due to the evolution in technologies, it has become necessary for the systems to work in real time and for efficient output they have to be adaptive. Since the conventional Proportional-Integral-Derivative (PID) controller lacks to control the less informative non-linear systems and has fixed parameters, therefore, artificial techniques with the inclusion of adaptability has been studied in this paper. It has been observed that by implementing an extra layer of fuzzy controller in parallel, the gains of the controller could be change in real time and can highly reduce the external effects on the system as compared to the non-adaptive controller. In addition to this, different variants of fuzzy controllers like self-tuned fractional-order fuzzy PD (STFOFPD), self-tuned fractional-order fuzzy PI (STFOFPI) and self-tuned fractional-order fuzzy PID (STFOFPID) are compared and it was found that the STFOFPID outperforms all the controllers in the servo-regulatory mechanism.

Keywords: Robotics, fractional-order, self-tuned, fuzzy logic controller, PI, PD, PID

1. Introduction

The Proportional-Integral-Derivative (PID) controller has been the prime choice of the industrialists or researchers for controlling various plants due to its simplicity, low cost and most important, the ability to reduce the steady state error [1-2]. Since the PID exhibits linear dynamics, therefore it will be best suited to the linear processes. But, as the non-linear processes possess uncertainties therefore, the conventional PID is not more effective and it underperforms. To cope up with these conditions many non-linear controllers have been made in the literature but these all were requiring the exact mathematical model of the system [3-4]. Hence, these days due to advancement in the computational abilities, the researchers are keen towards the artificial intelligence method which includes Fuzzy logic[5-6], Artificial neural networks [7-8] and optimization algorithms as its agents[9]. In 1965, Lofii. A. Zadeh introduced the concept of fuzzy sets theory and the first controller based on fuzzy logic as introduced by Prof. E.H. Mamdani in 1975 [10]. Since then the Mamdani Fuzzy model become so popular and most of the researchers have been using the model to implement an efficient and the robust controller to control the particular plant [11]. The adapting and the self-tuning capability of the fuzzy logic controllers (FLCs) have made it best suitable for the uncertain and the model-free environment. Initially, the FLC was designed either in the PI (velocity form giving the incremental output) or in the PD (Position form directly giving the controller output) form. But, the PI and PD controllers individually had some demerits like PI was responsible for the oscillatory motion of the output while PD introduced undue increased damping. To cope with this situation a fuzzy PID with three inputs was made but the difficulty was to design the three dimensional rule base which could be more tedious and complex. Thus,
the PI and PD structure were combined to compensate the demerits of each other and hence the hybrid Fuzzy PID (FPID) controller was made in [13-18], which utilized the two fuzzy controllers i.e. one for fuzzy PI and another for Fuzzy PD and their outputs was combined to form PID controller. After that, a simplified form of the hybrid fuzzy PID was presented which utilized only one two-dimensional rule base making the design much simpler and easy to determine the gains.

As the researchers were busy in exploiting the fuzzy PID controllers to control various complex and non-linear plants, a new concept of fractional order theory came which was a revolutionary growth in the control process. The researchers applied the fractional order concept to the PID controller by replacing the integer order of the integral and the derivative term with the fractional order and since it provides more flexibility to the design therefore, it was observed that the fractional order controllers outperforms the integer order controllers. Hence, these days the fractional order PID controller has been tremendously applied to the PID and the fuzzy PID controllers to enhance the capability of the controller in servo and regulatory management [19-22].

In addition to all this, it is required to find the optimal values of the scaling factors of the FLCs and this has been an area of research for so many years. In the earlier phases, the tuning methods used were dependent on the plant’s mathematical model but that was not possible with the highly complex and uncertain non-linear plants. Due to this, the Genetic Algorithm (GA) technique had become increasingly popular and applied to find out the scaling factor values but it was reported that the GA does not have the better capability to converge to an optimal solution or a complex problem. Thus, the optimization algorithm based on swarm intelligence has become a promising area for the researchers. Many algorithms were designed utilizing the swarming behavior of various creatures like ants, honey bees, fish, wolf, birds and spider monkeys [23,24].

Since the non-linear higher plants possess the nonlinearities like dead time, saturation time-delay etc., due to which the parameters of the plant may change abruptly and also the dynamics of the plant may get change due to the inclusion of the disturbance effects and the noise at the actuator’s as well the plant output. In this scenario, the FLC with fixed scaling factors, membership functions and the rule base would not be sufficient to eliminate the non-linear effect. Considering this, many researchers attempted to make an adaptive FLC by changing any one or more parameters of the FLCs like scaling factors, rule base and the membership functions either online or offline. Later, it was reported that the tuning of rule base and the membership functions was a tedious task and hence tuning of scaling factors in the real time has become increasingly popular to give an adaptive nature to the FLC by employing another FLC to give a tuning factor $\alpha$ which changes the output scaling factors of the FLC and make it more robust as compared to the non-tunable FLC.

In the present paper, the features of the self-tuning capability of the Fuzzy PID controller has been exploited and a robust controller along with the fractional-order technique (STFOFPID) has been designed whose scaling factors are determined by the meta-heuristic optimization algorithm like Cuckoo Search Algorithm (CSA). The efficacy of the controller has been explained by applying it to a highly complex multi coupled robotic manipulator system which has a wide range of applications in the real world and compared with the other variants of the fuzzy controller like self-tuned Fractional-order Fuzzy PI (STFOFPI) and the self-tuned Fractional-order Fuzzy PD (STFOFPD).

The paper has been organized in 4 sections as follows: section 1 gives the introduction and literature survey. Section 2 describes the structure of the different variants of self-tuned fuzzy logic controllers. In the third section, the simulation study of the plant along with the different controllers has been done to compare all of them and in the last section the results obtained are concluded.

2. Controller structure and description
The presented section represents the block diagram of the controllers used in this paper to control the end-effector movement of the robotic manipulator.

2.1 Self-tuned fuzzy PID controller.

Figure 1 shows the block diagram of the self-tuning fractional order fuzzy PID controller in which two FLC layer has been employed where the FLC-1 is utilized to realize the structure of a fractional order fuzzy PID controller while the FLC-2 is used to change the scaling factors by multiplying them with its output value $\alpha$ in the run time and thus, producing an output in the form of self-tuning fractional order fuzzy PID ($U_{STFOPID}$) as represented in the equation 1.

$$U_{STFOPID} = f\left(K_P e(t), K_D \left(s^\lambda e(t)\right)\right)\left(\frac{\alpha K_{PI}}{s^\mu}\right) + \alpha K_{PD}$$ (1)

Here, $f$ represents the rule base function of the fuzzy logic giving non-linear output. Where, $K_P$, $K_D$, $K_{PI}$ and $K_{PD}$ are the scaling factors of the controller and $\lambda$ and $\mu$ are the order of the integral and the derivative term respectively.

2.2 Self-tuned fuzzy PI controller

Figure 2. represents the block diagram of the self-tuning fractional order fuzzy PI controller (STFOFPI). The fuzzy PI takes the same structure as that of the fuzzy PD but the output is in the incremental form and the output of the PI controller is obtained by integrating the output of the FLC as shown in the equation 2.
Here, $K_P$, $K_D$ and $K_{PI}$ are the scaling factors of the controller and $\lambda$ and $\mu$ are the order of the integral and the derivative terms respectively.

2.3 Self-tuned fuzzy PD controller

\[ U_{STFOFPD} = f \left( K_P e(t), K_D \left( s^\lambda e(t) \right) \right) \ast \left( \frac{a K_{PD}}{s^\mu} \right) \]  \hspace{1cm} (3)

2.4 Description of FLC implementation

The FLC implementation is a three step process as shown in the figure 4. The input is fuzzified with the help of membership functions which may be of different shapes like triangular, trapezoidal, singleton,
sigmoidal and Gaussian etc. in this paper Gaussian membership functions have been used for the both FLC-1 and FLC-2 as shown in figure 5 (a) and (b). The FLC-1 is used to produce the controller output therefore, the rule base of the FLC-1 is decided to compute the values of $\Delta u$ and $u$ for PI and PD controller respectively as given in table 1. while the rule base of the FLC-2 is designed in order to improve the overall performance of the controller by computing the value of $\alpha$ as given in table 2.

For inferencing, Mamdani model have been used giving the fuzzy output as min-max composition of the inputs and the crisp output is obtained by defuzzification through the center of gravity method (COG).

![Figure 4. Block diagram of Fuzzy Logic Controller](image)

![Figure 5. (a) The membership functions for the inputs of FLC -1 and FLC-2 and the output of the FLC-1 (b) The membership functions for the output of the FLC-2.](image)

| E   | NL | NM | N  | Z  | P  | PM | PL |
|-----|----|----|----|----|----|----|----|
| NL  | NL | NL | NL | N  | N  | N  | Z  |
| NM  | NL | NM | NM | N  | Z  | P  |
| N   | NL | NM | N  | Z  | P  | PM |
| Z   | NL | NM | N  | Z  | P  | PM |
| P   | NM | N  | Z  | P  | PM | PL |
| PM  | N  | Z  | P  | PM | PM | PL |
| PL  | Z  | P  | P  | PM | PL | PL |

| E   | NL | NM | N  | Z  | P  | PM | PL |
|-----|----|----|----|----|----|----|----|
| NL  | VB | BG | MB | MD | VS | ZE | ZE |
| NM  | BG | BM | SM | SM | VS | ZE |
| N   | BM | BM | MD | VS | MD | SM | VS |
| Z   | MD | SM | VS | ZE | VS | MS | MD |
| P   | VS | SM | MD | VS | MD | MB | MB |
| PM  | ZE | VS | SM | SB | MB | BG | BG |
| PL  | ZE | ZE | VS | SM | MB | BG | VB |
2.5 Implementation of the Fractional order

Although, there are various techniques like G-L, R-L, Caputo are available which are basically a finite difference approximation of fractional derivative with long memory behavior and suitable for the fractional order implementation, but the Oustaloup’s band limited frequency domain rational approximation method is best suitable in the real hardware implementation using higher order Infinite Impulse Response (IIR) type Analog and Digital filters. The Oustaloup’s recursive approximation in frequency domain is given by equation (4), representing a higher order analog filter:

\[ S^m \cong K \prod_{k=-N}^{N} \left( \frac{s^{k+N+1}}{s^{m} + \omega_k^h} \right) \]

where the poles, \( \omega_k = \omega_b \left( \frac{\omega_h}{\omega_b} \right)^{\frac{k+N}{2N+1}} \)

the zeros, \( \omega_k' = \omega_b \left( \frac{\omega_h}{\omega_b} \right)^{\frac{k+N}{2N+1}} \)

and the gain \( K = \omega_b^m \)

In this paper, the Oustaloup approximation method have been used for fractional order implementation with the order of filter as \( N=3 \), and the frequency band \( (\omega_b, \omega_h) \) as (10-2, 102).

3. Simulation Study

This section presents the performance of the controller by applying it to a non-linear multi-coupled two-link manipulator with payload (TLMP) system. The figure 6 shows the arrangement of the TLMP which looks like a human hand to grasp an object (Payload) and move it along the desired trajectory.

![Figure 6. TLPM structure](image_url)

The system is described by equation (5) given below [25].

\[
\begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix}
\begin{bmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2
\end{bmatrix} +
\begin{bmatrix}
P_{11} \\
P_{21}
\end{bmatrix} +
\begin{bmatrix}
f_{r1} \\
f_{r2}
\end{bmatrix} +
\begin{bmatrix}
f_{n1p} \\
f_{n2p}
\end{bmatrix} =
\begin{bmatrix}
\tau_1 \\
\tau_2
\end{bmatrix}
\]

The table 3 depicts the parameters values for the plant.
Table 3. link 1 and link 2 parameters for the system

| Parameter for Link 1 | Values       | Parameter for Link 2 | Values       |
|----------------------|--------------|----------------------|--------------|
| \(m_1\)              | 0.392924 kg  | \(m_2\)              | 0.094403 kg  |
| \(l_1\)              | 0.2032 m     | \(l_2\)              | 0.1524 m     |
| \(l_{c1}\)           | 0.104648 m   | \(l_{c2}\)           | 0.081788 m   |
| \(b_{1p}\)           | 0.141231 m/radian/s | \(b_{2p}\) | 0.3530776 m/radian/s |
| \(I_{1p}\)           | 0.0011411 kg\cdot m^2 | \(I_{2p}\) | 0.0020247 kg\cdot m^2 |
| \(m_p = 0.566699 kg\) |              |                      | g=9.81 m/s^2 |

The control set up of the plant has been implemented on the MATLAB/ Simulink and the parameters of a particular controller found out by applying the Cuckoo search Algorithm (CSA) which finds out the best possible parameters of the controller by minimizing the value of the cost function \(J\) which is taken as the weighted function of the Integral of the absolute error (IAE) and the integral of the absolute value of the Change in the controller output (IACCO) as given in the equation (6) as follows

\[
J = w_1 f_1 + w_2 f_2
\]  
(6)

Where,

\[
f_1 = \int |e_1(t)|dt + \int |e_2(t)|dt
\]  
(7)

And,

\[
f_2 = \int |\Delta u_1(t)|dt + \int |\Delta u_2(t)|dt
\]  
(8)

Where, \(e_1(t)\) and \(e_2(t)\) are the instantaneous error for the link 1 and link 2 respectively as shown by equations (9) and (10) and \(\Delta u_1(t)\) and \(\Delta u_2(t)\) are the instantaneous change in the controller output for the link 1 and link 2 respectively. The weight \(w_1\) and \(w_2\) are taken as 0.9 and 0.1 respectively.

\[
e_1(t) = r_1(t) - \theta_1(t)
\]  
(9)

\[
e_2(t) = r_2(t) - \theta_2(t)
\]  
(10)

Here, \(r_1(t) = -2\sin(\pi/3)\) and \(r_2(t) = 2\sin(\pi/2)\) are the reference signals and \(\theta_1(t)\) and \(\theta_2(t)\) are the outputs of the link 1 and link 2 respectively.

After successful completion of the optimization algorithm, the parameters of the controllers obtained are given in the table 4, and with the graph of cost function V/s iteration curve as shown in the figure 7, it is clear that the STFOFPID controller achieves the less value of the cost function as compared to others indicating to reduce the error upto the high extent as compared to all other controllers. Figure 8, shows the comparison of the magnitude of the cost function of the controllers and authorized the results obtained by the curve shown in the figure 7. To justify the accuracy of the controllers, output curves of the link 1 and link 2 are shown in figure 9 which shows that the system controlled by the STFOFPID is more near to the reference signal as compared to other controllers.
Table 4. Parameters of the controllers obtained after the optimization

| Controllers | STFOFPID | STFOFPI | STFOFPD | FOFPD |
|-------------|----------|---------|---------|-------|
| Parameters  | Link1    | Link1   | Link2   | Link2 |
| $K_P$       | 1        | 410.77  | 3.7912  | 97.535|
| $K_D$       | 0.0186   | 0.027   | 0.0594  | 0.1   |
| $K_{PI}$    | 500      | 142.65  | 300     | 50    |
| $K_{PD}$    | 467.84   | 5.73    | 494.48  | 13.11 |
| $\lambda$  | 0.8554   | 0.1959  | 0.9     | 0.773 |
| $\mu$      | 0.9      | 0.8930  | 0.1457  | 0.1273|

**Figure 7.** Minimization of the cost function values of the system with respect to the no. of iterations.

**Figure 8.** Comparison of the magnitude of the cost function of the different controllers
3.1 Study of performance of the controllers under external effects

To verify the efficacy of the self-tuning property of the controller a disturbance signal of magnitude 1N-m is applied to the controller output for the time interval of 5 to 6 secs. After the simulation, the obtained values of the cost function for different controllers is shown in the figure 10. It shows that still the STFOFPID has less value of the cost function having the better capability to reject the effect of disturbances as the gains of the controller are adjusted to nullify the effect of the disturbance. The figure 11 shows the error curve of the system for the controllers during the duration and it is observed that the variation in the STFOFPID is lowest while the variation in the FOFPD is the highest. Thus we can say that the self-tuning capability of the device increased the robustness of the device while since PID has the advantage over PD and PI therefore the STFOFPID performs in the best manner.

**Figure 9.** Output curves of the system controlled by the different controllers (a) Link 1 (b) Link 2

**Figure 10.** Comparison of the magnitude of the cost function of the different controllers under the effect of disturbance
Figure 11. Error curves of the system controlled by the different controllers under the effect of disturbance

4. Conclusion

From the simulated results obtained in the section 3, it is concluded that the PID controller is having better capability to reduce the error as compared to the PI and PD as the steady state error is reduced by the integral term and the stability is provided by the derivative term. Also, by the inclusion of the another FLC layer the parameters of the controller could be change online and make the system adaptive in nature. Thus on comparison of FOFPD with the STFOFPD, it is found that the performance of the later is better for nominal tracking as well as for the disturbance rejection. Further the STFOFPD was compared with the STFOFPI and STFOFPID and it was observed that the STFOFPID outperforms all other in nominal tracking and the rejection of the external effects.

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