A new class of compact stars: pion stars

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Compact stellar objects offer deep insight into the physics of elementary particles in dense environments through the imprint left by merger events on the electromagnetic and gravitational wave spectra. The theoretical description of compact star interiors requires full knowledge of the equation of state (EoS) of nuclear matter and involves the non-perturbative solution of quantum chromodynamics (QCD), the theory of strongly interacting quarks and gluons. However, first-principle methods (most notably, lattice QCD simulations) are not available for high neutron densities – consequently, the EoS of neutron stars necessarily relies on a modeling of the nuclear force. Here we propose a different scenario, where the neutron density vanishes and a Bose-Einstein condensate of charged pions (the lightest excitations in QCD) plays the central role instead. This setting can be approached by first-principle methods and leads to a new class of compact stars: pion stars. As we demonstrate, pion star matter exhibits gravitationally bound configurations and is metastable against electroweak decays. If pion stars indeed exist in our Universe, this result constitutes the first occasion that the EoS and the mass-radius relation of a compact stellar object is determined from first principles within the Standard Model.

The most prominent representatives of compact stellar objects are neutron stars. The prediction of their existence and their association to the relics of core-collapse supernovae anticipated their serendipitous discovery by more than three decades. Today, more than 2600 pulsars, rotation-powered neutron stars, are known and listed in the ATNF pulsar database. However, the known pulsars are only the tip of the iceberg, as approximately a billion neutron stars are likely to exist within our galaxy. Together with other compact objects, they can be exposed by signatures from their companion stars or by gravitational wave emission, revealing information on their structure and composition. While neutron star matter consists of neutrons and protons (baryons) and, thus, features high baryon density, the proposed pion stars are substantially different. Their strongly interacting component is characterized by zero baryon density and high isospin charge (see below). Unlike neutron star matter, this system is amenable to lattice QCD simulations using standard Monte-Carlo algorithms, giving direct access to the EoS – i.e., the relation between the pressure $p$ and the energy density $\epsilon$.

Pion stars can be placed in the larger class of boson stars. Throughout their long history, boson stars were assumed to contain hypothetical elementary particles that would be either free or weakly interacting scalars. Typically being much heavier and more extended than other compact objects, it was expected that boson stars might mimic black holes or serve as candidates for dark matter within galaxies. Unlike boson stars considered previously, pion stars have no need for any beyond Standard Model constituents. According to our results, typical pion star masses range up to $M \sim 250 M_\odot$, where $M_\odot$ is the solar mass, making them two orders of magnitude heavier than neutron stars or white dwarfs and comparable to intermediate-mass black holes.

The Bose-Einstein condensation of charged pions involves the accumulation of isospin charge at zero baryon density and zero strangeness. In QCD, isospin is conserved such that the pion condensate can be triggered by the partition function $Z$ via standard thermodynamic relations (see Methods). The low-energy effective theory of this system is chiral perturbation theory ($\chi$PT), which operates with pionic degrees of freedom. According to $\chi$PT, pions condense at zero temperature if $\mu_I \geq m_\pi/2$, where $m_\pi$ is the pion mass in the vacuum. Beyond this threshold the $U(1)_I$ chiral symmetry of the light quark action is broken spontaneously by the pion condensed ground state. The corresponding phase transition is of second order and manifests itself in a pronounced rise of the isospin density $n_I$ beyond the critical point. Besides nonzero isospin, the ground state also carries nonzero electric charge density $n_Q = n_u \cdot q_u/e + n_d \cdot q_d/e = n_I/2$, where the fractional electric charges of the quarks $q_u = -2q_d = 2e/3$ enter, with $e > 0$ being the elementary charge. To relate the charge density to the isospin density, we assume that the only charged states that contribute to the pressure have zero baryon number and zero strangeness. This is indeed the case in the $T \to 0$ limit if the isospin chemical potentials $\mu_u = -\mu_d$ that couple oppositely to the up and down quark flavors, and induces opposite quark densities $n_I = 2n_u = -2n_d$. At zero temperature, the EoS of this system only depends on $\mu_I$ and follows from the partition function $Z$ via standard thermodynamic relations (see Methods).
potential is sufficiently small so that heavier charged hadrons are not excited. The strongest constraint is given by $\mu_I < m_K/2 \approx 1.8 \, m_\pi$, where $m_K$ is the kaon mass, and is fulfilled in the following calculations.

The charge density $n_Q$ and the pion condensate $\sigma_\pi$ are obtained as expectation values involving the Euclidean path integral over the gluon and quark fields discretized on a space-time lattice. The positivity of the measure in the path integral\textsuperscript{5} ensures that standard importance sampling methods are applicable. Since the spontaneous symmetry breaking associated to pion condensation does not occur in a finite volume, the simulations are performed by introducing a pionic source parameter $\lambda$ that breaks the symmetry explicitly.\textsuperscript{12} Physical results are obtained by extrapolating this auxiliary parameter to zero as discussed in Ref.\textsuperscript{14} The details of our lattice setup are described in Methods.

The dependence of $n_Q$ on the isospin chemical potential is shown in the left panel of Fig. 1, clearly reflecting the phase transition to the pion condensed phase at $\mu_I = m_\pi/2$. Due to effects from the finite volume and the small but nonzero temperature employed in our simulations, the density below $\mu_I = m_\pi/2$ is not exactly zero. To approach the thermodynamic and $T = 0$ limits consistently, we employ $\chi$PT. In particular, we set the density to zero below $m_\pi/2$ and fit the lattice data to the form predicted by $\chi$PT around the critical chemical potential (see Methods). Matching this fit to a spline interpolation of the lattice results at higher isospin chemical potentials gives the continuous curve shown in the left panel of Fig. 1. The resulting $n_Q(\mu_I)$ curve is used to calculate the EoS, as shown in the right panel of Fig. 1.

![Figure 1: Left panel: Phase transition between the vacuum and the pion condensed phase, as exhibited by the charge density. The lattice data are fitted using $\chi$PT (yellow curve) and matched to a spline interpolation (blue curve). The error bars indicate the standard error of the mean of $n_Q$. Right panel: Equation of state in the pion condensed phase in the QCD sector and for the electrically neutral systems also including leptons (either muons or electrons). The width of the curves represents the standard error of the mean and incorporates statistical uncertainties as well as the uncertainty in the lattice pion mass for the pion-lepton systems.](image)

In the vacuum phase, charged pions decay weakly into leptons, with a characteristic lifetime of $\tau_\pi \approx 10^{-8}$ s. However, the condensed phase carries nonzero electric charge that is stable against weak decays. To see this, consider the following argument: since the spontaneously broken symmetry group is part of the local gauge group of electromagnetism, the pion condensed phase is a superconductor, where the electric charge eigenstates $\pi^+$ and $\pi^-$ mix with each other. One linear combination ($\tilde{\pi}_1$) plays the role of the Goldstone mode, while the other one ($\tilde{\pi}_2$) is heavy, fulfilling $m_{\tilde{\pi}_2} > m_\pi$.\textsuperscript{5} In the presence of dynamical photons the Goldstone mode disappears via the Higgs mechanism,\textsuperscript{14} at the cost of a nonzero photon mass $m_\gamma \propto e|\sigma_\pi|$. Thus, only $\tilde{\pi}_2$ can decay via weak interactions. However, if the temperature is sufficiently low (i.e. $T \ll m_{\tilde{\pi}_2}$), this mode is not excited and no weak decay of electrically charged states can occur. An obvious analogue to this situation is the spontaneous symmetry breaking and the associated Higgs mechanism in the electroweak sector of the Standard Model. Below the electroweak scale the Higgs condensate is stable and carries nonzero weak hypercharge. The Higgs boson and the gauge bosons decay weakly but are irrelevant for low-temperature physics due to their large masses.

To ensure stability under electromagnetism, the pion condensate is neutralized by a gas of leptons. In the present approach we consider the leptons to be free relativistic particles. A systematic improvement over this assumption is possible by taking into account $O(\alpha^2)$ electromagnetic effects perturbatively, both in the electroweak sector and in lattice QCD simulations. The lepton density $n_l$ is controlled by a lepton chemical potential $\mu_l$, from which the leptonic contribution to the pressure $p_l$ and to the energy density $\epsilon_l$ can be obtained, similarly to the QCD sector. We require local charge neutrality to hold, $n_Q + n_l = 0$, which uniquely determines the lepton chemical potential in terms of $\mu_I$ (see Methods). We mention that a similar construction, assuming
a first-order phase transition for pions, was discussed in Ref.\textsuperscript{15}

Using the resulting EoS, the mass $M$ and radius $R$ of pion stars can be computed by solving the Tolman-Oppenheimer-Volkov (TOV) equations,\textsuperscript{16,17} which describe hydrostatic equilibrium in general relativity, assuming spherical symmetry. Further stability analyses are performed by requiring the star to be robust against density perturbations\textsuperscript{18} and radial oscillations.\textsuperscript{19} Our main result is the mass-radius relation for gravitationally stable pion stars, composed of a pion condensate and electrons, see the left panel of Fig. 2. The results for an electrically charged pure pion star (our preliminary results for this case were presented in Ref.\textsuperscript{20}) are also included in the figure. In addition, we considered a pion-muon system (muons decay weakly into electrons) and a mixture of electrons and muons in chemical equilibrium by setting their respective chemical potentials equal.

We observed that the gravitationally stable configurations for the latter setup cannot maintain a muonic component and are thus identical to those for the pion-electron system. Fig. 2 reflects the $R \sim \text{constant}$ behavior for pure pion stars (with masses below $7 \, M_\odot$) – a telltale sign for an interaction-dominated EoS. The slope changes by the addition of leptons, scaling as $MR^3 \sim \text{constant}$, similarly to stars made of fermions. Moreover, one can see that pion stars are considerably heavier and larger when compared to other branches of compact stars, attaining masses and radii up to $M \sim 250 \, M_\odot$ and $R \gtrsim 30,000$ km. This comparison is more strikingly illustrated in the right panel of Fig. 2, where neutron stars and white dwarfs, together with regular stars, are also included.\textsuperscript{21,22}

![Figure 2: Left panel: Mass-radius relations of pion stars, composed of a pion condensate and electrons ($\pi + e$). For comparison, a pure pion star ($\pi$) with net electric charge and a pion-muon system ($\pi + \mu$) is also exhibited. The colored (gray) segments mark the gravitationally stable (unstable) solutions. Right panel: Mass-radius relations of regular stars,\textsuperscript{22} white dwarfs and neutron stars\textsuperscript{21} and pion stars. The dark blue area marks the region excluded by causality in both panels and the background color represents the compactness $\beta \propto M/R$ of the objects (darker colors indicate compacter stars). The width of the curves for pion stars indicates the standard error of the mean, incorporating statistical uncertainties and the uncertainty in the lattice pion mass.](image)

We note that about $10^{-4}$ s after the Big Bang, the Universe consisted mainly of a pion gas with electrons, muons and neutrinos in weak equilibrium. It is therefore conceivable that a seed of charged pion condensed matter, neutralized by leptons, could have been created at that time. Assuming that a pion star can indeed be formed, we now consider its stability in free (cold) space. While the Bose-Einstein condensate in the bulk of the star behaves coherently and does not decay, the situation is different near the surface of the star, where $n_Q$ approaches zero and temperature effects become important. Provided that in this dilute environment, pions are well described by a free complex scalar field theory, the critical temperature for condensation can be found semi-analytically to be $T_c(n_Q) \sim 2\pi/m_\pi \left(n_Q/\zeta(3/2)\right)^{2/3}$. Thus, for cosmic temperatures around $T \approx 2.7$ K, the pion star boundary is marked by a critical charge density $n_Q \approx 10^{-19}$ $\text{fm}^{-3}$, corresponding to an energy density $\epsilon_c$. The nonzero pressure in this region results in an outward flow of pions subject to weak decays and a subsequent rearrangement of the star matter characterized by the speed of sound $v_s$. The latter follows from the EoS as $v_s^2 = \partial p/\partial \epsilon$, giving $v_s \approx 5$ km/s at the star boundary. Assuming that the corresponding mass loss proceeds along the mass-radius relation $M(R)$, it is described by the differential equation $dM/dt = -4\pi R^2(M) v_s \epsilon_c$. This results in typical lifetimes of the order of a million years for the heaviest stars. The recombination of pions, together with the capture and conversion of further charged particles into the condensate counteracts this process, making the above estimate a lower bound for the lifetime. Moreover, we stress that through $v_s$ the mass loss is very sensitive to the precise EoS for low density and might be subject to large corrections from perturbative QED effects that we neglected here.
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mass used in the simulations originates primarily from the uncertainty of the lattice scale and amounts to 2%.

For the fermion matrices we employ stout smearing of the gauge fields. The determinants of $M$ where $M = \det M$ and the temperature $T$ approximate $Z = \int DA \prod [\det M_1^{1/4} \det M_3^{1/4}] e^{-S_\pi}$, where $S_\pi$ is the gluonic part of the QCD action, for which we use the tree level-improved Symanzik discretization. The quark masses $m_{ud}$ and $m_s$ are tuned to their physical values so that, in particular, $m_{ud} = m_s$. We discretize $M_{ud}$ and $M_s$ using the rooted staggered formulation, so that the Euclidean path integral over the gauge field $A_\mu$ becomes

$$ n_I = \frac{1}{V_4} \frac{\partial \log Z}{\partial \mu_I} = \frac{1}{4V_4} \langle \text{tr} M^{-1}_{ud} \gamma_0 \tau_3 \rangle, \quad \sigma_\pi = \frac{1}{V_4} \frac{\partial \log Z}{\partial \lambda} = \frac{i}{4V_4} \langle \text{tr} M^{-1}_{ud} \gamma_5 \tau_2 \rangle, $$

where $V_4 = V/T$ is the four-dimensional volume of the system that includes the spatial volume $V = (N_4 a)^3$ and the temperature $T = (N_4 a)^{-1}$ in terms of the lattice spacing $a$ and the lattice geometry $N_4^3 \times N_4$. The extrapolation of the isospin density to $\lambda = 0$ is performed using the singular value representation of the massive Dirac operator.\textsuperscript{13}

Here we perform simulations on a $24^3 \times 32$ lattice ensemble with a lattice spacing of $a \approx 0.29$ fm, a wide range of chemical potentials $0 < \mu_I/m_\pi \leq 1$ and three pionic source parameters $0.17 \leq \lambda/m_{ud} \leq 0.88$. The systematic uncertainties originating from lattice artefacts and from neglecting $O(\epsilon^2)$ electromagnetic effects will be investigated in a future publication. The volume of our system is around $7$ fm\(^3\), sufficiently large so that finite size effects are under control. The temperature is well below the relevant QCD scales so that it well approximates $T = 0$.

Equation of state and the TOV equations

In $\chi$PT the isospin density reads,\textsuperscript{5}

$$ n_I^{\chi PT} = 2\mu f_\pi^2 \left[ 1 - \left( \frac{m_\pi}{2\mu_I} \right)^4 \right] \cdot \Theta(\mu_I - m_\pi/2), $$

where $f_\pi$ is the chiral limit of the pion decay constant, which is the only free parameter for the $\chi$PT fit depicted in the left panel of Fig. 1. For free relativistic leptons, the density is

$$ n_I(\mu) = \frac{1}{3\pi^2} (\mu^2 - m_l^2)^{3/2} \cdot \Theta(\mu - m_l), $$

where $m_l$ is the lepton mass. The pionic pressure and energy density is calculated from $n_I(\mu_I)$ at zero temperature via

$$ p = \frac{\log Z}{V_4} = \int_0^{\mu_I} d\mu'_I n_I(\mu'_I), \quad \epsilon = -p + \mu_I n_I, $$
and very similarly for the leptons, using $n_i(\mu_l)$. The impact of a temperature of $T \approx 2.7$ K on the pionic and leptonic EoS is of the order of $e^{-m_\pi/T}$ and $e^{-m_l/T}$, respectively and can be safely neglected.

After requiring local charge neutrality $n_i = n_Q = n_l/2$, the pion-lepton system is unambiguously characterized by the lepton chemical potential $\mu_l$. The total pressure $p$ and energy density $\epsilon$ enter the TOV equations,\textsuperscript{16,17} which can be rewritten in terms of the chemical potentials as

$$\frac{d\mu_l}{dr} = -G\mu_l \frac{M + 4\pi r^3 p}{r^2 - 2rGM} \left[ 1 + 2 \frac{\mu_l}{\mu_l} \right] \left[ 1 + 4 \frac{n_l'}{n_l} \right]^{-1}, \quad M(r) = 4\pi \int_0^r r' r'^2 \epsilon(r'), \quad (7)$$

where $G$ is Newton’s constant, the primes denote derivatives with respect to the corresponding chemical potentials and we used natural units $c = \hbar = 1$. Eq. (7) is integrated numerically up to the star boundary $r = R$, where the pressure vanishes and the total mass $M = M(R)$ is attained. The points of the mass-radius curves in Fig. 2 correspond to different values of the central chemical potential. The data that support the findings of this study are available from the authors on request.