Angular distributions in $J/\psi \to p\bar{p}\pi^0(\eta)$ decays

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Abstract

The differential decay rates of the processes $J/\psi \to p\bar{p}\pi^0$ and $J/\psi \to p\bar{p}\eta$ close to the $p\bar{p}$ threshold are calculated with the help of the $N\bar{N}$ optical potential. The same calculations are made for the decays of $\psi(2S)$. We use the potential which has been suggested to fit the cross sections of $N\bar{N}$ scattering together with $N\bar{N}$ and six pion production in $e^+e^-$ annihilation close to the $p\bar{p}$ threshold. The $p\bar{p}$ invariant mass spectra is in agreement with the available experimental data. The anisotropy of the angular distributions, which appears due to the tensor forces in the $N\bar{N}$ interaction, is predicted close to the $p\bar{p}$ threshold. This anisotropy is large enough to be investigated experimentally. Such measurements would allow one to check the accuracy of the model of $N\bar{N}$ interaction.

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I. INTRODUCTION

The cross section of the process $e^+e^- \rightarrow p\bar{p}$ reveals an enhancement near the threshold [1–4]. The enhancement near the $p\bar{p}$ threshold has been also observed in the decays $J/\psi \rightarrow \gamma p\bar{p}$, $B^+ \rightarrow K^+ p\bar{p}$, and $B^0 \rightarrow D^0 p\bar{p}$ [5–7]. These observations led to numerous speculations about a new resonance [5], $p\bar{p}$ bound state [8–10] or even a glueball state [11–13] with the mass near two proton mass. This enhancement could appear due to the nucleon-antinucleon final-state interaction. It has been shown that the behavior of the cross sections of $NN$ production in $e^+e^-$ annihilation can be explained with the help of Jülich model [14, 15] or slightly modified Paris model [16, 17]. These models also describe the energy dependence of the proton electromagnetic form factors ratio $|G_p^E/G_p^M|$. A strong dependence of the ratio on the energy close to the $p\bar{p}$ threshold is a consequence of the tensor part of the $NN$ interaction.

Another phenomenon has been observed in the process of $e^+e^-$ annihilation to mesons. A sharp dip in the cross section of the process $e^+e^- \rightarrow 6\pi$ has been found in the vicinity of the $NN$ threshold [18–22]. This feature is related to the virtual $NN$ pair production with subsequent annihilation to mesons [23, 24]. In Ref. [24] a potential model has been proposed to fit simultaneously the cross sections of $NN$ scattering and $NN$ production in $e^+e^-$ annihilation. This model describes the cross section of the process $e^+e^- \rightarrow 6\pi$ near the $NN$ threshold as well. A qualitative description of this process was also achieved using the Jülich model [23].

In this paper we investigate the decays $J/\psi \rightarrow p\bar{p}\pi^0$ and $J/\psi \rightarrow p\bar{p}\eta$ taking the $p\bar{p}$ final-state interaction into account. Investigation of these processes has been performed in Ref. [25] using the chiral model. However, the tensor part of the $p\bar{p}$ interaction was neglected in that paper. To describe the $p\bar{p}$ interaction we use the potential model proposed in Ref. [24], where the tensor forces play an important role. The account for the tensor interaction allows us to analyze the angular distributions in the decays of $J/\psi$ and $\psi(2S)$ to $p\bar{p}\pi^0(\eta)$ near the $p\bar{p}$ threshold. The parameter of anisotropy is large enough to be studied in the experiments.
II. DECAY AMPLITUDE

Possible states for a $p\bar{p}$ pair in the decays $J/\psi \rightarrow p\bar{p}\pi^0$ and $J/\psi \rightarrow p\bar{p}\eta$ have quantum numbers $J^{PC} = 1^{--}$ and $J^{PC} = 1^{+-}$. The dominating mechanism of the $p\bar{p}$ pair creation is the following. The $p\bar{p}$ pair is created at small distances in the $^3S_1$ state and acquires an admixture of $^3D_1$ partial wave at large distances due to the tensor forces in the nucleon-antinucleon interaction. The $p\bar{p}$ pairs have different isopsins for the two final states under consideration ($I = 1$ for the $p\bar{p}\pi^0$ state, and $I = 0$ for the $p\bar{p}\eta$ state), that allows one to analyze two isospin states independently. Therefore, these decays are easier to investigate theoretically than the process $e^+e^- \rightarrow p\bar{p}$, where the $p\bar{p}$ pair is a mixture of different isospin states.

We derive the formulas for the decay rate of the process $J/\psi \rightarrow p\bar{p}x$, where $x$ is one of the pseudoscalar mesons $\pi^0$ or $\eta$. The following kinematics is considered: $k$ and $\epsilon_k$ are the momentum and the energy of the $x$ meson in the $J/\psi$ rest frame, $p$ is the proton momentum in the $p\bar{p}$ center-of-mass frame, $M$ is the invariant mass of the $p\bar{p}$ system. The following relations hold:

$$p = |p| = \sqrt{\frac{M^2}{4} - m_p^2}, \quad k = |k| = \sqrt{\epsilon_k^2 - m^2}, \quad \epsilon_k = \frac{m_{J/\psi}^2 + m_p^2 - M^2}{2m_{J/\psi}},$$  \hspace{1cm} (1)

where $m$ is the mass of the $x$ meson, $m_{J/\psi}$ and $m_p$ are the masses of a $J/\psi$ meson and a proton, respectively, and $\hbar = c = 1$. Since we consider the $p\bar{p}$ invariant mass region $M - 2m_p \ll m_p$, the proton and antiproton are nonrelativistic in their center-of-mass frame, while $\epsilon_k$ is about 1 GeV.

The spin-1 wave function of the $p\bar{p}$ pair in the center-of-mass frame has the form [17]

$$\psi^j_\lambda = e_\lambda u^j_1(0) + \frac{u^j_2(0)}{\sqrt{2}} [e_\lambda - 3\hat{p}(e_\lambda \cdot \hat{p})],$$ \hspace{1cm} (2)

where $\hat{p} = p/p$, $e_\lambda$ is the polarization vector of the spin-1 $p\bar{p}$ pair,

$$\sum_{\lambda=1}^{3} e^i_\lambda e^j_\lambda = \delta_{ij},$$ \hspace{1cm} (3)

$u^j_1(r)$ and $u^j_2(r)$ are the components of two independent solutions of the coupled-channels
radial Schrödinger equations

\[ \frac{p_n^2}{m_p} \chi_n + V \chi_n = 2E \chi_n, \]
\[ V = \begin{pmatrix} V^I_S & -2\sqrt{2} V^I_T \\ -2\sqrt{2} V^I_T & V^I_D - 2V^I_T + \frac{6}{m_pr^2} \end{pmatrix}, \quad \chi_n = \begin{pmatrix} u_n^I \\ w_n^I \end{pmatrix}. \tag{4} \]

Here \( E = \frac{p^2}{2m_p} \), \( V^I_S \) and \( V^I_D \) are the \( N\bar{N} \) potentials in \( S \)- and \( D \)-wave channels, and \( V^I_T \) is the tensor potential. Two independent regular solutions of these equations are determined by their asymptotic forms at large distances \[ \begin{align*}
    u_1^I(r) &= \frac{1}{2ipr} [S_{11}^I e^{ipr} - e^{-ipr}], \\
    u_2^I(r) &= \frac{1}{2ipr} S_{21}^I e^{ipr}, \\
    w_1^I(r) &= -\frac{1}{2ipr} S_{12}^I e^{ipr}, \\
    w_2^I(r) &= \frac{1}{2ipr} [-S_{22}^I e^{ipr} + e^{-ipr}],
\end{align*} \tag{5} \]

where \( S_{ij}^I \) are some functions of energy.

The Lorentz transformation for the spin-1 wave function of the \( p\bar{p} \) pair can be written as

\[ \tilde{\psi}_\lambda^I = \psi_\lambda^I + (\gamma - 1) \hat{k}(\psi_\lambda^I \cdot \hat{k}), \tag{6} \]

where \( \tilde{\psi}_\lambda^I \) is the wave function in the \( J/\psi \) rest frame, \( \hat{k} = k/k \), and \( \gamma \) is the \( \gamma \)-factor of the \( p\bar{p} \) center-of-mass frame. The component collinear to \( k \) does not contribute to the amplitude of the decay under consideration because the amplitude is transverse to \( k \). As a result, the dimensionless amplitude of the decay with the corresponding isospin of the \( p\bar{p} \) pair can be written as

\[ T_{\lambda\lambda'}^I = \frac{G_I}{m_{J/\psi}} \psi_\lambda^I [k \times \epsilon_{\lambda'}], \tag{7} \]

Here \( G_I \) is an energy-independent dimensionless constant, \( \epsilon_{\lambda'} \) is the polarization vector of \( J/\psi \),

\[ \sum_{\lambda'=1}^2 \epsilon^I_{\lambda'} \epsilon'^I_{\lambda'} = \delta_{ij} - n^i n^j, \tag{8} \]

where \( n \) is the unit vector collinear to the momentum of electrons in the beam.

The decay rate of the process \( J/\psi \rightarrow p\bar{p}x \) can be written in terms of the dimensionless amplitude \( T_{\lambda\lambda'}^I \) as (see, e.g., [26])

\[ \frac{d\Gamma}{dM d\Omega_p d\Omega_k} = \frac{p_k}{2^9 \pi^5 m_{J/\psi}^2} |T_{\lambda\lambda'}^I|^2, \tag{9} \]
where $\Omega_p$ is the proton solid angle in the $p\bar{p}$ center-of-mass frame and $\Omega_k$ is the solid angle of the $x$ meson in the $J/\psi$ rest frame.

Substituting the amplitude (7) in Eq. (9) and averaging over the spin states, we obtain the $p\bar{p}$ invariant mass and angular distribution for the decay rate

$$
\frac{d\Gamma}{dM d\Omega_p d\Omega_k} = \frac{G_f^2 p k^3}{2^{11} \pi^8 m_{J/\psi}^4} \left\{ \left| u_1'(0) \right|^2 + \frac{1}{\sqrt{2}} \left| u_2'(0) \right|^2 \right\} + \frac{3}{2} \left[ \left| u_2'(0) \right|^2 - 2 \sqrt{2} \text{Re} \left( u_1'(0) u_2'^* (0) \right) \right] \left[ \left| \hat{n} \cdot \hat{k} \right|^2 - 2 (\hat{n} \cdot \hat{p}) (\hat{n} \cdot \hat{\bar{p}}) (\hat{p} \cdot \hat{k}) \right].
$$

The invariant mass distribution can be obtained by integrating Eq. (10) over the solid angles $\Omega_p$ and $\Omega_k$:

$$
\frac{d\Gamma}{dM} = \frac{G_f^2 p k^3}{2^{11} \pi^8 m_{J/\psi}^4} \left( \left| u_1'(0) \right|^2 + \left| u_2'(0) \right|^2 \right) \left[ 1 + \cos^2 \vartheta_k \right],
$$

where $\vartheta_k$ is the angle between $\hat{n}$ and $\hat{k}$. However, the angular part of this distribution does not depend on the features of the $p\bar{p}$ interaction. The proton angular distribution in the $p\bar{p}$ center-of-mass frame is more interesting. To obtain this distribution we integrate Eq. (10) over the solid angle $\Omega_k$:

$$
\frac{d\Gamma}{dM d\Omega_p} = \frac{G_f^2 p k^3}{2^{11} \pi^8 m_{J/\psi}^4} \left( \left| u_1'(0) \right|^2 + \left| u_2'(0) \right|^2 \right) \left[ 1 + \gamma' P_2 (\cos \vartheta_p) \right],
$$

where $\vartheta_p$ is the angle between $\hat{n}$ and $\hat{p}$, $P_2 (x) = \frac{3x^2 - 1}{2}$ is the Legendre polynomial, and $\gamma'$ is the parameter of anisotropy:

$$
\gamma' = \frac{1}{4} \frac{\left| u_2'(0) \right|^2 - 2 \sqrt{2} \text{Re} \left( u_1'(0) u_2'^* (0) \right)}{\left| u_1'(0) \right|^2 + \left| u_2'(0) \right|^2}.
$$

Averaging (10) over the direction of $\hat{n}$ gives the distribution over the angle $\vartheta_{pk}$ between $\hat{p}$ and $\hat{k}$:

$$
\frac{d\Gamma}{dM d\Omega_{pk}} = \frac{G_f^2 p k^3}{2^{11} \pi^8 m_{J/\psi}^4} \left( \left| u_1'(0) \right|^2 + \left| u_2'(0) \right|^2 \right) \left[ 1 - 2 \gamma' P_2 (\cos \vartheta_{pk}) \right].
$$

Note that this distribution can be written in terms of the same anisotropy parameter (14).

The mass spectrum (11) and the anisotropy parameter (14) are sensitive to the tensor part of the $N\bar{N}$ potential and, therefore, gives the possibility to verify the potential model.
III. RESULTS AND DISCUSSION

In the present work we use the potential model suggested in Ref. [24]. The parameters of this model have been fitted using the \( p\bar{p} \) scattering data, the cross section of \( N\bar{N} \) pair production in \( e^+e^- \) annihilation near the threshold, and the ratio of the electromagnetic form factors of the proton in the timelike region. By means of this model and Eq. (11), we predict the \( p\bar{p} \) invariant mass spectra in the processes \( J/\psi \to p\bar{p}\pi^0 \) and \( J/\psi \to p\bar{p}\eta \). The isospin of the \( p\bar{p} \) pair is \( I = 1 \) and \( I = 0 \) for, respectively, a pion and \( \eta \) meson in the final state. The model [24] predicts the enhancement of the decay rates of both processes near the threshold of \( p\bar{p} \) pair production (see the red band in Fig. 1). The invariant mass spectra predicted by our model are similar to those predicted in Ref. [25] with the use of the chiral model. Very close to the threshold the enhancement factor turned out to be slightly overestimated in comparison with the experimental data, as it is seen from Fig. 1. We have tried to refit the parameters of our model in order to achieve a better description of the invariant mass spectra of the decays considered. The predictions of the refitted model are shown in Fig. 1 with the green band. It is seen that the refitted model fits better the invariant mass spectra of \( J/\psi \) decays. However, the discrepancy in the cross sections of \( n\bar{n} \) production in \( e^+e^- \) annihilation and the charge-exchange process \( p\bar{p} \to n\bar{n} \) have slightly increased after refitting.

![Fig. 1](image1.png)  
![Fig. 1](image2.png)

Fig. 1. The invariant mass spectra of \( J/\psi \) decays to \( p\bar{p}\pi^0 \) (left) and \( p\bar{p}\eta \) (right). The red/dark band corresponds to the model [24] and the green/light band corresponds to the refitted model. The phase space behavior is shown by the dashed curve. The experimental data are taken from Refs. [5, 27, 28]. The measurement of Ref. [5] is adopted for the scale of the left plot.
An important prediction of our model is the angular anisotropy of the $J/\psi$ decays. This anisotropy is the result of $D$-wave admixture due to the tensor forces in $N\bar{N}$ interaction. The anisotropy (see Eqs. (13) and (15)) is characterized by the parameters $\gamma^I$ and $\gamma^0$ (14) for the $p\bar{p}\pi^0$ and $p\bar{p}\eta$ final states, respectively. The dependence of the parameters $\gamma^I$ on the invariant mass of the $p\bar{p}$ pair is shown in the left side of Fig. 2. For $pp$ invariant mass about $100 - 200$ MeV above the threshold, significant anisotropy of the angular distributions is predicted. The distributions over the angle between the proton momentum and the momentum of the electrons in the beam are shown in the right side of Fig. 2. Note that the anisotropy in the distribution over the angle $\vartheta_{pk}$ is expected to be two times larger than in the distribution over the angle $\vartheta_p$ (compare Eqs. (13) and (15)).

There are some data on the angular distributions in the decays $J/\psi \to p\bar{p}\pi^0$ [27] and $J/\psi \to p\bar{p}\eta$ [28]. However, these distributions are obtained by integration over the whole
Fig. 3. The invariant mass spectra for the decays $\psi(2S) \to p\bar{p}\pi^0$ (left) and $\psi(2S) \to p\bar{p}\eta$ (right). The red/dark band corresponds to the model [24] and the green/light band corresponds to the refitted model. The phase space behavior is shown by the dashed curve. The experimental data are taken from Refs. [29–31]. The measurement of Ref. [29] is adopted for the scale of both plots.

$p\bar{p}$ invariant mass region. Unfortunately, our predictions are valid only in the narrow energy region above the $p\bar{p}$ threshold. Therefore, we cannot compare the predictions with the available experimental data. The measurements of the angular distributions at $p\bar{p}$ invariant mass close to the $p\bar{p}$ threshold would be very helpful. Such measurements would provide another possibility to verify the available models of $N\bar{N}$ interaction in the low-energy region.

The formulas written above are also valid for the decays $\psi(2S) \to p\bar{p}\pi^0$ and $\psi(2S) \to p\bar{p}\eta$ with the replacement of $m_{J/\psi}$ by the mass of $\psi(2S)$. The invariant mass spectra for these decays are shown in Fig. 3. The angular distributions for these processes are the same as for the decays of $J/\psi$ because they depend only on the invariant mass of the $p\bar{p}$ pair.

IV. CONCLUSIONS

Using the model proposed in Ref. [24], we have calculated the effects of $p\bar{p}$ final-state interaction in the decays $J/\psi \to p\bar{p}\pi^0(\eta)$ and $\psi(2S) \to p\bar{p}\pi^0(\eta)$. Our results for the $p\bar{p}$ invariant mass spectra close to the $p\bar{p}$ threshold are in agreement with the available experimental data. The tensor forces in the $p\bar{p}$ interaction result in the anisotropy of the angular distributions. The anisotropy in the decay $J/\psi \to p\bar{p}\pi^0$ and especially in the $J/\psi \to p\bar{p}\eta$ decay are large enough to be measured. The observation of such anisotropy close to the $p\bar{p}$ threshold would
allow one to refine the model of $N\bar{N}$ interaction.

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