Noisy kink in microtubules

H.C. Rosu\textsuperscript{a,c} \textsuperscript{1}, J.A. Tuszyński\textsuperscript{b} \textsuperscript{2} and A. González\textsuperscript{a} \textsuperscript{3}

\textsuperscript{a} Instituto de Física de la Universidad de Guanajuato, Apdo Postal E-143, Léon, Gto, México
\textsuperscript{b} Department of Physics, University of Alberta, Edmonton, AB, T6G 2J1, Canada
\textsuperscript{c} Institute of Gravitation and Space Sciences, Magurele-Bucharest, Romania

ABSTRACT. We study the power spectrum of a class of noise effects generated by means of a digital-like disorder in the traveling variable of the conjectured Ginzburg-Landau-Montroll kink excitations moving along the walls of the microtubules. We have found a $1/f^\alpha$ noise with $\alpha \in (1.82-2.04)$ on the time scales we have considered.

PACS: 87.15.-v, 72.70+m, 87.10.+e

\textsuperscript{1}Electronic mail: rosu@ifug.ugto.mx
\textsuperscript{2}Electronic-mail:jtus@phys.ualberta.ca
\textsuperscript{3}Electronic mail: gonzalez@ifug.ugto.mx
One can find extensive, descriptive presentations of microtubules (MTs) in many biological papers. Here, we shall give only elementary definitions as follows. They are ubiquitous protein polymers in eukaryotic cells belonging to the category of biological filaments and making the most part of the cytoskeleton. They are hollow cylinders 25 nm in outer diameter and 17 nm in the inner one, with lengths ranging from nm to mm in some neuronal cells. The walls of the cylinders are usually made of 13 (the seventh Fibonacci number) protofilaments laterally associated. The surface structure of MT walls is very interesting [1]. Structurally, MTs are quasi one-dimensional chains of tubulin polar dimers (negative $\alpha$ and positive $\beta$ monomers, each of 4 nm in length) undergoing conformational changes induced by the guanosine triphosphate to diphosphate (GTP-GDP) hydrolysis. The whole assembly process of a MT is due to the hydrolyzation of GTP. The cation Mg$^{++}$ is essential to increase the affinity of tubulin for binding GTP and thus for generating MTs. Moreover, a unique dynamical property is the so-called dynamical instability [2] which is a random growth and shrinkage of the more active plus ends of MTs. It has raised much interest in recent years [3].

An energy-transfer mechanism in MTs by means of Ginzburg-Landau-Montroll (GLM) kink-klike protofilament excitations has been discussed by Satarić, Tuszyński and Zakula [1] in 1993. Also sine-Gordon (SG) solitons have been discussed by Chou, Zhang and Maggiora in 1994 [4]. We recall that various types of solitons have found interesting applications in biological physics (Davydov’s model [5], DNA/RNA [6]). Usually, in order to get nonlinear differential equations one performs a continuum limit for some lattice models in which discreteness effects are neglected. Some authors have shown that such effects might be important and suggested various ways of including them in continuous differential equations [8]. The digital disorder we shall comment on and use next is just a possible way to incorporate discreteness in a moving solitonic pattern.

In 1993, Rosu and Canessa [9] introduced a digital-like disorder in the Davydov $\beta$-kink leading to the $1/f^\alpha$ noise, with $\alpha \approx 1$, in the dynamics of that kink, and also commented on the multifractal features of the dynamics of the $\beta$-kink. The procedure is as follows. In the traveling variable $\xi = x - v_K t$ of the slowly moving kink (the acoustic Lorentz factor $\gamma_L \approx 1$)
one puts $x_i = x_0 + \Delta x_i$, where $x_0$ is the position of the center of mass of the kink, or its central position along the chain, and $\Delta x$ are small random displacements around $x_0$ (say, in the interval $\pm 1$). In the calculations, one can fix $x_0 = 0$. For each random position $\Delta x_i$ chosen from a uniform distribution, one calculates by means of a fast-Fourier-transform algorithm the noise power spectrum of the time series of the signal (considered to be the kink) in order to get the time correlations of the fluctuations of the signal, i.e.,

$$S_K(f) \propto \frac{1}{\tau} \left\langle \left| \int_0^\tau K(\xi) e^{2\pi i ft} dt \right|^2 \right\rangle,$$

(1)

where $0 < t < \tau \approx 1/f$ and $K(\xi)$ is the kink function, and the brackets stand for averaging over ensembles. This approach to noise effects has been taken from the literature on the self-organized criticality (SOC) paradigm, see, e.g., [10]. On the other hand, the standard treatment of noise effects when they originate in thermal fluctuations is by means of the Langevin equation method. Valls and Lust [11] have studied the effect of thermal noise on the front propagation in the GL case. They have found a crossover between constant-velocity propagation at early times and diffusive behavior at late times.

The purpose of the present work is to investigate the noise produced by the same type of disorder as in [9] in the case of the GLM kink conjectured in MTs.

The main assumption in [4] is that the assembly of tubulin dimers/dipoles ($D_n$) form a quasi one-dimensional ferrodistortive system for which the double-well on-site potential model is a standard framework

$$V(D_n) = -\frac{1}{2}AD_n^2 + \frac{1}{4}BD_n^4 .$$

(2)

The variable $D$ has been identified in [4] with the amount of $\beta$-state distortion vertically projected (the $\beta$-state is defined as having the mobile electron within the $\beta$-monomer). Moreover, in [4] a GL hamiltonian/free-energy with intrinsic electric field and dissipation effects included led to the dimer Euler-Lagrange dimensionless equation of motion (EOM) in the traveling coordinate of the anharmonic oscillator form (with linear friction)

$$|A|D'' - \gamma_0 v_K D' - F(D) = 0 ,$$

(3)

where $\gamma$ is the friction coefficient, $\alpha = |A|\gamma_0^2/Mv_{\text{sound}}^2$ and $F(D) = AD - BD^3 + qE$, with $q$
denoting the effective charge of a single dimer of mass M, and $E$ the magnitude of the intrinsic electric field. This EOM is known to have a unique kink solution given by the formula

$$K(\xi) = a + \frac{b - a}{1 + \exp(\beta \xi)} \equiv a + \frac{\beta}{\sqrt{2}} (1 - \tanh(\beta \xi/2)) ,$$

(4)

where $K = D/\sqrt{|A|/B}$ is a rescaled dipole variable, $\beta = (b - a)/\sqrt{2}$, whereas $a$ and $b$ are two of the solutions of the cubic equation

$$F(K) \equiv (K - a)(K - b)(K - c) \equiv K^3 - K - \sigma = 0 ,$$

(5)

where $\sigma = q\sqrt{B/|A|}E$ and $u_0$ units are used, where $u_0 = \sqrt{|A|/E} \approx 1.4 \cdot 10^{-11}$ m is the amplitude of the dimer displacement (shift of the double-well potential). Notice that the GLM kink is thin. Its width is $w_K = \frac{1}{\beta} \approx 0.7u_0$.

As we said, the type of digital disorder we consider here is very close to the ideas of the SOC paradigm that we understand in the broad sense of both spatial and temporal scaling of the dynamical state of the system [13]. The spatial scaling (self-similarity) is of the (multi)fractal type while the temporal scaling leads to $1/f^{\alpha}$ noises.

A strong motivation for dealing with digital dynamics is the possibility of generating broken symmetries and therefore of having various types of dynamical phase transitions [14]. Thus, digital disorder, though might look a rather ad-hoc approach, focuses on both self-organized properties of MTs, i.e., to driven steady states with long-range spatio-temporal correlations, and to (dynamical) phase transitions, since digital dynamics allows for symmetry breaking.

Our results are displayed in Figures 1 and 2 and show that the noise introduced by the digital disorder in the GLM kink variable is practically of the $1/f^2$ (Brownian) type on the time scales we have considered. At low frequencies there is the known cross-over to a white noise due to the finite system size, which is moving to lower frequencies as the size of the system is increased [15].

On the other hand, the deviation from the power law at high frequencies is an artifact due to the so-called aliasing [16]. The Brownian noise we have obtained is not unexpected since it is a common occurrence in SOC models in any finite dimension [17], and only mean-field calculations reproduced the $1/f$ noise [18].
Finally, we would like to mention that if polarization does not exactly follow the displacement one needs a system of two coupled partial differential equations leading to two coupled traveling kink waves. Of course, the nonlinear models seem to be too simple-minded for the MT complexity. Nevertheless they provide guides for further insight and perhaps some partial answers.

Acknowledgments

This work was partially supported by CONACyT (Mexico) and by NSERC (Canada).

References

[1] E.-M. Mandelkow et al., J. Cell Biology 102 (1986) 1067; E. Mandelkow et al., Neurobiology of Aging 16 (1995) 347.

[2] T. Mitchison and M. Kirschner, Nature 312 (1984) 232, 237.

[3] D. Kuchnir Fygenson, E. Braun, and A. Libchaber, Phys. Rev. E 50 (1994) 1579; H. Flyvbjerg, T.E. Holy and S. Leibler, Phys. Rev. Lett. 73 (1994) 2372; P.M. Bayley, M.J. Schilstra, and S.R. Martin, J. Cell Sci. 95 (1990) 33; D.J. Odde, L. Cassimeris, and H.M. Buettner, Biophys. J. 69 (1995) 796.

[4] M.V. Satarić, J.A. Tuszyński, R.B. Žakula, Phys. Rev. E 48 (1993) 589. See also: J.A. Tuszyński et al., J. Theor. Biol. 174 (1995) 371.

[5] K.C. Chou, C.T. Zhang, G.M. Maggiora, Biopolymers 34 (1994) 143.

[6] A.S. Davydov, Solitons in Molecular Systems (Reidel, Dordrecht, 1985).

[7] G. Gaeta, C. Reiss, M. Peyrard, T. Dauxois, Riv. Nuovo Cimento 4 (1994).

[8] J.C. Kimball, Phys. Rev. B 21 (1980) 2104; A. Sánchez and L. Vázquez, Phys. Lett. A 152 (1991) 184; J.A. Combs and S. Yip, Phys. Rev. B 28 (1983) 6873; M. Otwinowski, J.A. Tuszyński, J.M. Dixon, Phys. Rev. A 45 (1992) 7263.

[9] H. Rosu and E. Canessa, Phys. Rev. E 47 (1993) R3818.
[10] J. Vitting Andersen and O.G. Mouritsen, Phys. Rev. A 45 (1992) R5331.

[11] O.T. Valls and L.M. Lust, Phys. Rev. B 44 (1991) 4326.

[12] M.A. Collins, A. Blumen, J.F. Currie, and J. Ross, Phys. Rev. B 19 (1979) 3630.

[13] P. Bak, C. Tang and K. Wiesenfeld, Phys. Rev. Lett. 59 (1987) 381; Phys. Rev. A 38 (1988) 364.

[14] M.Y. Choi and B.A. Huberman, Phys. Rev. B 28 (1983) 2547.

[15] T. Fiig and H.J. Jensen, J. Stat. Phys. 71 (1993) 653; N.C. Pesheva, J.G. Brankov, and E. Canessa, Phys. Rev. E 53 (1996) 2099.

[16] H.J. Jensen, Mod. Phys. Lett. B 5 (1991) 625.

[17] H.J. Jensen, K. Christensen, and H.C. Fogedby, Phys. Rev. B 40 (1989) 7425; J. Kertész and L.B. Kiss, J. Phys. A 23 (1990) L433; H.F. Chau and K.S. Cheng, Phys. Rev. A 46 (1992) R2981.

[18] C. Tang and P. Bak, J. Stat. Phys. 51 (1988) 797.
Fig. 1

Double logarithmic plot of the power spectrum of the digital noise perturbing the motion of the GLM kink moving along MTs at constant $v_K = 2\text{m/s}$. The fitted slopes are as follows: (a) -2.032 (b) -2.0175 (c) -2.0164 for the time scales corresponding to that of the dimer, ten times bigger, and hundred times bigger, respectively. The errors in the slopes are at the level of 0.0003 for each case.
The same plot as in Fig. 1 for $v_K = 100\text{m/s}$. The fitted slopes are (a) -2.0083 (b) -1.8641 (c) -1.8205 (d) -2.0000 for temporal scales of $10^{-2}$, $10^{-1}$, $10^{0}$, and $10^{1}$ times that of the tubulin dimer, respectively.

The level of the errors is the same as in Fig. 1.