System Transfer Function of Incoherent Light Illumination System

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Abstract—Generally, most imaging cases are incoherent light field illumination, and incoherent light illumination system has the advantages of simple installation and no coherent noise, therefore it has received extensive attention. The optical transfer function of illumination system is an effective method to evaluate the imaging quality of such optical system. In order to evaluate the imaging quality of an optical system, the specific form of the system transfer function should be known. However, due to the incoherent nature of the optical field, both the input function and the impulse response can only be non-negative real functions, it is difficult to deal with lots of bipolar inputs and impulse responses. In this paper, two methods for calculating the system transfer function of incoherent light illumination system under quasi-monochromatic light illumination are given. Method 1: the coherence of incoherent light is defined, and the coherent system transfer function based on the characteristics of the system transfer itself is deduced. Method 2: based on the theory of partially coherent light, started from Kirchoff's diffraction integral, the quasi-wavefront aberration is introduced, and the spatial coherence of the light field is described by the mutual intensity instead of the mutual coherent function, finally the system transfer function of incoherent illumination is derived. The calculation formula is simple and the transfer function of incoherent light system can be calculated accurately and easily. The work in this paper is helpful to the study of the new phenomena related to incoherent optical imaging.

1. Introduction
Generally, most imaging situations are incoherent light field illumination. Due to the incoherent nature of the optical field, both the input function and the impulse response only is non-negative real functions [1]. It is difficult to deal with a large number of bipolar inputs and impulse responses. With the advent of lasers and the promotion of holography [2], studies on coherent optical processing have become so active that incoherent lighting technology has once almost been overshadowed. However, the practice during many years shows that the prominent problem of coherent lighting system is the serious coherent noise, which leads to higher requirements for system components. So incoherent lighting systems have attracted extensive attention again due to their simple installation and absence of coherent noise [3].

The optical transfer function is an effective method to evaluate the imaging quality of such optical system. In 1973, Abe conjectured the relationship between the imaging system under incoherent light illumination and the light intensity, that is, the incoherent imaging system is linear to the physical
quantity of intensity. In 1971, Abbe put forward some effective measures to imaging system under incoherent light illumination and light intensity, and the relationship between the intensity of incoherent imaging system for the physical quantities is linear. Moreover, the impulse response of intensity transformation is proportional to the distribution of light intensity generated by the point source in the image plane, which is proportional to the square of the impulse response of the coherent system. At present, there are rather mature methods to calculate the optical transfer function of coherent and partially coherent light illumination systems, but there is no complete theory to calculate the system transfer function of incoherent light illumination. In this paper, firstly the statistical hypothesis of incoherent light source and the Fourier transform of lens imaging system are briefly introduced, and then the system transfer function of incoherent light illumination system is derived by two methods.

2. STATISTICAL HYPOTHESIS OF LIGHT SOURCE
In the case of incoherent illumination, the light vibration of each point is uncorrelated, and the light sources are completely independent. On the object surface, the amplitude and initial phase of all points change randomly at any time, and the amplitude and phase vary with time in such a way that they are independent and statistically independent of each other [3]. The phase amplitude vector of the spatial distribution of the light source field follows the circular Gaussian distribution, which the complex amplitude of the light field generated by the plane light source at a given point \((x, y)\) and a given time \(t\) is:

\[
U(x, y, t) = A(x, y, t) \exp[i\phi(x, y, t)]
\]

(1)

The following assumptions are satisfied: The amplitude \(A\) satisfies the Rayleigh distribution in spatial statistics:

\[
p(A) = \begin{cases} 
\frac{A}{\sigma^2} \exp\left(-\frac{A^2}{2\sigma^2}\right), & A \geq 0 \\
0, & \text{others}
\end{cases}
\]

(2)

Where \(p(A)\) is the probability density function of amplitude in (1). The phase \(\phi\) satisfied the uniform distribution on \([0,2\pi]\) in spatial statistics:

\[
p(\phi) = \begin{cases} 
\frac{1}{2\pi}, & 0 \leq \phi \leq 2\pi \\
0, & \text{others}
\end{cases}
\]

(3)

Where \(p(\phi)\) is the probability density function of amplitude in (1). The intensity of the light source is uniform and does not have spatial coherence at all, that is:

\[
\langle U^*(x, y)U(x', y') \rangle = I \delta(x - x', y - y')
\]

(4)

Where \(I\) is the ensemble average of the electric field \(E\) mode squared at a certain point in the light field, and angle brackets represent the time average. The light source is a quasi-monochromatic narrow band light source with a fixed central wavelength \(\lambda\).

3. THE IMAGING PROCESS

3.1 Transfer hypothesis
Although it is assumed that the thermal source does not have spatial coherence, the instantaneous
values at various locations in the space during the propagation of the light field still have the characteristic of amplitude superposition, which the Fresnel-Kirchoff’s propagation law is still valid for the diffraction law of coherent light. Fig.1 is a schematic diagram of the propagation of light field in air. The light source is located in the $x_1$-$y_1$ plane, and $U(x_1, y_1, t)$ is the phase amplitude vector of the light source at time $t$.

$U(x_2, y_2, t)$ represents the phase amplitude vector of the light field formed by the propagation of $U(x_1, y_1, t)$ to the plane $x_2$-$y_2$ (ignoring the time experienced by the square propagation). If $U(x_1, y_1, t)$ is the electric field distribution at time $t$ of the light source $S$ in the pane $x_1$-$y_1$, then the electric field distribution when the light field generated by this light source is transmitted to the plane $x_2$-$y_2$ is [3]:

$$U(x_2, y_2, t) = \frac{1}{j2\pi} \int \int U(x_1, y_1, t) \exp \left\{ j \frac{\pi}{\lambda z} \left( (x_2 - x_1)^2 + (y_2 - y_1)^2 \right) \right\} dx_1 dy_1$$

(5)

Figure 1. The propagation of the light field.

3.2 The Fourier transform of a lens

In the case of incoherent illumination, the amplitude and phase of points on the object surface change with time in a way that is statistically irrelevant, so in the image plane, the impulse response corresponding to each point will also change irregularly with time [4]. The final image intensity distribution will depend on the statistical relationship between these impulse responses and on the amplitude and phase of the points illuminated on the surface. The imaging device is shown in Fig.2. If the light source is close to the imaging object for illumination, the light source and the plane where the object is located are $x_0$-$y_0$, and the imaging plane is $x$-$y$.

Suppose the imaging object is an amplitude type object: the transmittance function is $f(x_0, y_0)$, so the light field distribution on the rear surface of the object is:

$$U_0(x_0, y_0, t) = Af(x_0, y_0) \exp[\varphi(x_0, y_0, t)]$$

(6)
According to transmission (4), the light field distribution on the front surface of the lens is:

\[
U_i(x_i, y_i, t) = \frac{1}{\lambda d_i} \int \int U_{ii}(x_i, y_i, t) \times \exp \left\{ j \frac{\pi}{\lambda d_i} \left[ (x_i - x)^2 - (y_i - y)^2 \right] \right\} dx_i dy_i
\]  

(7)

In an optical system, lens has two functions. On the one hand, it is a pupil, which acts as a limiting wavefront, on the other hand, it acts as a transforming wavefront. So after the phase modulation of the lens, the light field transmitted to the rear focal plane of the lens is distributed as follows:

\[
U_i(x_i, y_i, t) = t(x_i, y_i)U_i(x_i, y_i, t)
\]  

(8)

Where \( t(x_i, y_i) \) is the phase transformation function of the lens. In this paper, we make it as:

\[
t_i(x_i, y_i) = p(x_i, y_i) \exp \left\{ -j \frac{x_i^2 + y_i^2}{2f} \right\}
\]  

(9)

In this equation, \( p(x_i, y_i) \) is the optical pupil function of lens \( l \). By substituting (5), (6) and (7) into (4), the light field distribution on the image surface can be obtained as follows:

\[
U(x, y, t) = \frac{1}{\lambda d_i} \int \int \int \int U_i(x_i, y_i, t) \times \exp \left\{ \frac{\pi}{\lambda d_i} \left[ (x - x_i)^2 - (y - y_i)^2 \right] \right\} dx_i dy_i \times
\]

\[
t(x, y) = \exp \left\{ -j \frac{x^2 + y^2}{2f} \right\}
\]  

(10)

Further arrangement:

\[
U(x, y, t) = \frac{1}{\lambda d_i} \exp \left\{ \frac{\pi}{\lambda d_i} (x^2 + y^2) \right\}
\]

(11)

Since \( U_0(x_0, y_0, t) \) has a random phase and does not affect the intensity distribution of the final detection, therefore, \( \exp \left\{ \frac{2\pi}{\lambda d_i} (x^2 + y^2) \right\} \) can be ignored, and the distribution of image surface field is:

\[
U(x, y, t) = \frac{1}{\lambda d_i} \exp \left\{ \frac{\pi}{\lambda d_i} (x^2 + y^2) \right\}
\]

(12)

According to the definition of the Fourier transform:
If the focal length of the lens is $f$, according to the geometric optical imaging theory:

$$\frac{1}{d_x} \frac{1}{d_y} = \frac{1}{f}$$  \hfill (14)

By substituting $t_i(x_i, y_i)$ and (13) into (12), and ignoring the constant terms independent of integration, the distribution of the image surface light field is:

$$U(x, y, t) = \int \frac{p(x, y)}{\delta(x-x_i, y-y_i)} \exp \left(-j \frac{2\pi}{\lambda d_x} (x-x_i, y-y_i) \right) dx_i dy_i$$

$$= F \left[ p(x, y) \right]_{x_i, y_i} * U_i \left( \frac{d_x}{d_i x}, \frac{d_y}{d_i y}, t \right)$$ \hfill (15)

In the equation, $F \left[ p(x, y) \right]$ is the system impulse response function. The above is the derivation process of the optical field transfer function. The conclusion is the same as the coherent optical imaging theory. According to the definition of the system itself, the impulse response function of the system is only related to the imaging system, and has nothing to do with the input of the system.

### 4. ACCURATE DERIVATION OF LIGHT INTENSITY TRANSFER FUNCTION IN INCOHERENT LIGHTING SYSTEM

Based on the above content, two derivation ideas of light intensity transfer function in incoherent lighting system are introduced: 1) define the coherence of incoherent light, and deduce the transfer function based on the characteristics of system transfer; 2) according to the theory of partially coherent light, the system transfer function of partially coherent lighting is directly derived.

#### 4.1 Define the Coherence of Incoherent Light

According to the coherence assumption of the light source in (4), the intensity of the light field can be obtained [5]:

$$I(x, y) = \left| U^* (x, y, t) U(x, y, t) \right|^2$$ \hfill (16)

Add (14) into (15) to get:

$$I(x, y) = \left| \left[ p(x, y) \right]_{x_i, y_i} * U_i \left( \frac{d_x}{d_i x}, \frac{d_y}{d_i y}, t \right) \right|^2$$ \hfill (17)

Since any function can be understood as the sum of countless delta functions, make the impulse response function of the system as the sum of three delta functions to get:

$$p(x, y) = \delta(x, y) + \frac{1}{2} \delta(x-\Delta, y) + \frac{1}{2} \delta(x+\Delta, y)$$ \hfill (18)

Add (17) into (16) to get:
The final result is:

$$I(x,y) = \left\{ \left[ \delta(x,y) + \frac{1}{2} \delta(x-\Delta_x,y) + \frac{1}{2} \delta(x+\Delta_x,y) \right] * U \left( \frac{d_{x_i}}{d_{x_o}}, \frac{d_{y_i}}{d_{y_o}}, t \right) \right\}$$

$$= I_i \left( \frac{d_{x_i}}{d_{x_o}}, \frac{d_{y_i}}{d_{y_o}} \right) + \frac{1}{4} I_i \left( \frac{d_{x_i}-\Delta_x}{d_{x_o}}, \frac{d_{y_i}}{d_{y_o}} \right) + \frac{1}{4} I_i \left( \frac{d_{x_i}+\Delta_x}{d_{x_o}}, \frac{d_{y_i}}{d_{y_o}} \right)$$

(19)

The final result is:

$$I(x,y) = \left\| \rho(x,y) \right\|_{\alpha} * I_i \left( \frac{d_{x_i}}{d_{x_o}}, \frac{d_{y_i}}{d_{y_o}} \right)$$

(20)

Above all is the definition of the coherence of incoherent light. Based on the characteristics of the system transfer itself, the light intensity transfer function is derived.

### 4.2 Theory of Partially Coherent Light

The actual light field is essentially a kind of fluctuation phenomenon [6]. The classical theory of partially coherent light is that the light emitted by two sub-light sources in the field of light causes the vibration at another point in space to be equal to, and the two points respectively cause the superposition of the vibration at this point. The cross-correlation function is used to represent partially coherent light, and the cross-correlation function describes the correlation between light fields located at two spatial points with a certain relative time delay [7].

#### 4.2.1 Transmission of mutual intensity under quasi-monochromatic conditions

Mutual coherence function on a certain surface $\Sigma_1$ in the space is known and mutual coherence function on another surface $\Sigma_2$ in the space is solved, which is known as the spread of mutual coherence function. The actual light field signal is a function, and the Fourier transform of the real function results in an extra negative frequency. In information optics, information is mainly processed from the perspective of frequency domain. The extra negative frequency brings a lot of inconvenience to the processing of information. In order to facilitate the processing of signals in frequency domain, a complex function corresponding to a real function (its Fourier transform has only one positive frequency) is constructed to replace the original real function [9].

Figure 3. Amplitude distribution of electrical image

Considering that two points light sources $P_1$ and $P_2$ pass through the optical system $h$, and superposition diffraction of $S_1$ and $S_2$ on the same actual image plane is formed at the exit pupil. Because heterogeneous field is considered as a linear combination of the disturbance of monochromatic. According to the huygens-fresnel principle, for each monochromatic light of frequency $\nu$, The complex amplitudes of $Q_1$ and $Q_2$ at any two points on the image plane can be expressed:
\[ U(Q_1, \nu) = \frac{1}{j \lambda} \int_{s_1} U(P_1, \nu) \frac{K(\theta_1)}{r_1} \exp[jkr_1] ds_1 \]  
(21)

\[ U(Q_2, \nu) = \frac{1}{j \lambda} \int_{s_2} U(P_2, \nu) \frac{K(\theta_2)}{r_2} \exp[jkr_2] ds_2 \]  
(22)

\[ U(P_1, \nu) \text{ and } U(P_2, \nu) \text{ respectively represent the complex amplitude at } P_1 \text{ and } P_2 \text{ of the two point sources; } \theta_1 \text{ and } \theta_2 \text{ respectively represent the Angle between the line and the horizontal direction between } P_1 \text{ and } Q_1 \text{ and the Angle between the line and the horizontal direction between } P_2 \text{ and } Q_2; K(\theta_1) \text{ and } K(\theta_2) \text{ respectively represent two constants about } \theta_1 \text{ and } \theta_2 \text{ respectively; } r_1 \text{ and } r_2 \text{ are the distance between } P_1 \text{ and } Q_1 \text{ and } P_2 \text{ and } Q_2. \]

The complex amplitude of any point \( Q_1 \) on the image plane is removed from the negative frequency part, and the negative Fourier transform is carried out on the positive frequency and construct a complex function corresponding to the real function (the Fourier transform has only one positive frequency) to replace the original real function. Then the distribution of the image plane field is:

\[ U(Q_1, t) = \int_{\nu} U(Q_1, \nu) \exp[-j2\pi \nu t] d\nu \]

\[ = \int_{\nu} \frac{K(\theta)}{jcr_1} U(P_1, \nu) \exp[-j2\pi \nu (t - \frac{r_1}{c})] d\nu ds_1 \]

\[ = \int_{\nu} \frac{\pi}{jcr_1} U(P_1, \nu) \exp[-j2\pi \nu (t - \frac{r_1}{c})] d\nu K(\theta) ds_1 \]  
(23)

In the quasi-monochromatic field approximation, the effective spectral width is much less than the average frequency, then the above formula can be changed into:

\[ U(Q_1, t) = \frac{1}{j \lambda} \int_{s_1} U(P_1, t - \frac{r_1}{c}) \frac{K(\theta)}{r_1} ds_1 \]  
(24)

In the same way:

\[ U(Q_2, t) = \frac{1}{j \lambda} \int_{s_2} U(P_2, t - \frac{r_2}{c}) \frac{K(\theta)}{r_2} ds_2 \]  
(25)

According to the classical theory of partially coherent light, the mutual coherence function is used to represent partially coherent light, so:

\[ \Gamma(Q_1, Q_2, \tau) = \left\langle U(Q_1, t + \tau) U^*(Q_2, t) \right\rangle \]  
(26)

\( \Gamma(Q_1, Q_2, \tau) \) is correlation between the image of time interval \( \tau \) at \( Q_1 \) and \( Q_2 \).

Add (23) and (24) to (25):

\[ \Gamma(Q_1, Q_2, \tau) = \int_{\nu} \left\langle \int_{s_1} U(P_1, t + \tau - \frac{r_1}{c}) U^*(P_2, t - \frac{r_2}{c}) \frac{K(\theta)}{r_1} \frac{K(\theta)}{r_2} ds_1 ds_2 \right\rangle \]

\[ = \int_{\nu} \int_{s_1} \int_{s_2} \Gamma(P_1, P_2, \tau + \frac{r_2 - r_1}{c}) \frac{K(\theta)}{r_1} \frac{K(\theta)}{r_2} ds_1 ds_2 \]  
(27)

\( \frac{r_2 - r_1}{c} \) is the time difference required for the transmission of light from \( P_1 \) and \( Q_1 \) to \( P_2 \) and \( Q_2 \).
Under quasi-monochromatic field approximation, the spectral lines of light is very narrow, the optical path difference is far less than the coherence length, mutual intensity $J(P_1, P_2)$ can substitute mutual coherence intensity $\Gamma(P_1, P_2, 0)$ to describe the spatial coherence of light field:

$$
\Gamma(P_1, P_2, \frac{r_2 - r_1}{c}) = \Gamma(P_1, P_2, 0) \exp\left(-j \frac{2\pi}{\lambda} (r_2 - r_1)\right)
$$

(28)

Add $\Gamma(Q_1, Q_2, 0) = J(Q_1, Q_2)$ and (27) to (26):

$$
J(Q_1, Q_2) = \int \int J(P_1, P_2) \exp\left[-j \frac{2\pi}{\lambda} (r_2 - r_1)\right] \frac{K(\theta_1)}{\Delta r_1} \frac{K(\theta_2)}{\Delta r_2} \, ds_1 \, ds_2
$$

(29)

This is the propagation theorem of mutual intensity.

Add $Q_1 = Q_2 = Q$ to (28), light intensity distribution on plane $\Sigma_2$ is:

$$
I(Q) = \int \int J(P_1, P_2) \exp\left[-j \frac{2\pi}{\lambda} (r_2 - r_1)\right] \frac{K(\theta_1) K(\theta_2)}{\Delta r_1 \Delta r_2} \, ds_1 \, ds_2
$$

(30)

In order to simplify the above equation, the near-axis approximation can be adopted, so it is assumed that the size of the light source and the observation area are much smaller than the distance between the light source and the observation plane. In this case, the following approximate relationship is established:

$$
\frac{1}{r_1} \frac{1}{r_2} = \frac{1}{z^2}
$$

$$
K(\theta) = K(\theta_1) K(\theta_2) = 1
$$

$$
r_1 = z + \frac{(x_1 - \xi)^2 + (y_1 - \eta)^2}{2z}
$$

$$
r_2 = z + \frac{(x_2 - \xi)^2 + (y_2 - \eta)^2}{2z}
$$

(31)

Obviously, outside the light source $I(P_1) = 0$, therefore the integral limit of (29) can be set to $(-\infty, +\infty)$, and let $\Delta x = x_2 - x_1, \Delta y = y_2 - y_1$. In the near-axial approximation, (29) is:

$$
I(Q) = \int \int J(P_1, P_2) \exp\left[-j \frac{2\pi}{\lambda} (x_2 + y_2) - (x_1 + y_1)\right] \times
$$

$$
\exp\left[-j \frac{2\pi}{\lambda} (\Delta x \xi + + \Delta y \eta)\right] \, d\xi \, d\eta \, d\chi
$$

(32)

If it is completely irrelevant then $I(Q) = 0$. In this case light cannot propagate, which means that the illumination light cannot be completely irrelevant. If it can be replaced by the function sinc, then the intensity of the light field will be:
The above formula is the expression of the intensity of the light field at any point after the incoherent light is modulated by an optical system under the condition of quasi-monochromatic light, and then the intensity transfer function of the incoherent light illuminating lens imaging system is calculated.

4.2.2 Imaging system

After the modulation of the lens imaging system in Fig. 2, the expression for the intensity of the light field is:

\[
I(Q) = \frac{1}{d_x d_y} \iint J(x, y) \exp \left\{ \frac{j \pi}{Z} ((x^2 + y^2) - (x_1^2 + \Delta x^2 + (y_1 + \Delta y)^2)) \right\} \times \exp \left\{ -\frac{2\pi}{Z} (\Delta x + \Delta y) \right\} d\Delta x d\Delta y \times \frac{1}{Z^2} \Delta^2
\]

(33)

\[
= \iint J(P_1, P_2) \exp \left\{ -\frac{2\pi}{Z} (x + y, \Delta x + \Delta y) \right\} \exp \left\{ -\frac{j \pi}{Z} (\Delta x^2 + \Delta y^2) \right\} \times \exp \left\{ -\frac{2\pi}{Z} (\Delta x + \Delta y) \right\} d\Delta x d\Delta y \times \frac{1}{Z^2} \Delta^2
\]

The above formula is the expression of the intensity of the light field at any point after the incoherent light is modulated by an optical system under the condition of quasi-monochromatic light, and then the intensity transfer function of the incoherent light illuminating lens imaging system is calculated.

Simplify it a little bit to get:

\[
I(Q) = \frac{1}{d_x d_y} \iint J(x, y) \times \exp \left\{ -\frac{2\pi}{d_x} (x, \Delta x + y_0) \right\} \times \exp \left\{ \frac{j \pi}{d_y} (\Delta x + \Delta y) \right\} \times \exp \left\{ -\frac{j \pi}{d_x} (x, \Delta x + y) \right\} \times \exp \left\{ \frac{j \pi}{d_y} (\Delta x^2 + \Delta y^2) \right\} \times \exp \left\{ -\frac{j \pi}{d_x} (x, \Delta x + \Delta y) \right\} \times \exp \left\{ \frac{j \pi}{d_y} (\Delta x^2 + \Delta y^2) \right\} \times \exp \left\{ -\frac{j \pi}{d_x} (x, \Delta x + y) \right\} \times \exp \left\{ \frac{j \pi}{d_y} (\Delta x^2 + \Delta y^2) \right\} \times \exp \left\{ -\frac{j \pi}{d_x} (x, \Delta x + \Delta y) \right\} \times \exp \left\{ \frac{j \pi}{d_y} (\Delta x^2 + \Delta y^2) \right\}
\]

(34)

In the small range of observation points \(Q_1\) and \(Q_2\), \(R(P_1)\) and \(R(P_2)\) are equal each other, which is equivalent to the background. So two constant terms of the mutual coherent function can be ignored, and the cross-term is used to represent the intensity. Then, the intensity expression of the light field is:
\[
I(Q) = \frac{1}{d^2} \frac{1}{d^2} \int \int \int \int J(x_{0}, y_{0}) \times \\
\exp \left[-j \frac{2\pi}{d^2} (x_{0} \Delta x_{0} + y_{0} \Delta y_{0}) \right] dx_{0} dy_{0} dx_{1} dy_{1} \cdot P^{*}(\xi_{0} + \Delta \xi, \eta_{0} + \Delta \eta) \times \\
P(\xi_{0}, \eta_{0}) \exp \left[-j \frac{2\pi}{d^2} (x_{0} \Delta x_{0} + y_{0} \Delta y_{0}) \right] dx_{0} dy_{0} dx_{1} dy_{1} 
\]

If \( P(\xi_{0}, \eta_{0}) \) is symmetric, the intensity expression of the light field is:

\[
I(Q) = \frac{d^2}{2} \int \int \int \int J(x_{0}, y_{0}) \exp \left[-j \frac{2\pi}{d^2} (x_{0} \Delta x_{0} + y_{0} \Delta y_{0}) \right] dx_{0} dy_{0} dx_{1} dy_{1} \times \\
P^{*}(\xi_{0}, \eta_{0}) \exp \left[-j \frac{2\pi}{d^2} (x_{0} \Delta x_{0} + y_{0} \Delta y_{0}) \right] dx_{0} dy_{0} dx_{1} dy_{1} 
\]

5. Conclusion

This paper deduces the intensity transfer function of incoherent light illumination system from two angles, and proves that the imaging system performs linear transformation of intensity when incoherent light illumination is used. Furthermore, the impulse response of this intensity transformation is simply proportional to the square of the module of the impulse response under coherent light illumination. This point is also easy to understand in physics. When incoherent light is used for illumination, the phase difference between the points on the object is changing with time. When the observed light intensity is an average, the change of the interference effect at any time will be averaged. So the impulse response in the image plane must be superimposed by intensity.

References
[1] Jiang H Z, Zhao J L, Di J L, et al. Correction of Nonparaxial and Misfocus Aberrations in Digital Lensless Fourier Transform Holography[J]. Acta Optica Sinica, 2008, 28(8): 1457-1462.
[2] He Y L, Chen Z Y, Wu F T. Effects of Coherent and Light Sources on Resolution of Bessel Beam Imaging System[J]. Laser and Optoelectronics Progress, 2016, 53(09):090301.
[3] Joseph W G. Introduction to Fourier optics[M]. New York: Roberts and Company Publisher, 2005 :35-66.
[4] Born, M. Wolf, E. Principles of optics[J]. Beijing: Science Press, 1981.
[5] Song Q H, Li J C, Liu Z Q, et al. Defocusing Image Field Calculation of Imaging System Illuminated by Coherent Light[J]. Acta Optica Sinica, 2017, 37(12):1207001.
[6] Zhang S, Gao L, Xiong J, et al. Spatial Interference: From Coherent to Incoherent[J]. Physical Review Letters, 2009, 102(7) :073904.
[7] Zhang M, Wei Q, Shen X, et al. Lensless Fourier—Transform Ghost Imaging with Classical Incoherent Light[J]. Physical Review A, 2007, 366(6) :569-574.
[8] Hu X Y, Tian A L, Wei W. Polarization Imaging of An Edge Object with Partially Coherent Light[J]. Physical Review A,2007, 366(6) :569-574.