Analysis of Transforming dq Impedances of Different Converters to A Common Reference Frame in Complex Converter Networks

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Abstract—dq impedance-based method has been widely used to study the stability of three-phase converter systems. As the dq impedance model of each converter depends on its local dq reference frame, the dq impedance modeling of complex converter networks gets complicated. Because the reference frames of different converters might not fully align, depending on the structure. Thus, in order to find an accurate impedance model of a complex network for stability analysis, converting the impedances of different converters into a common reference frame is required. This paper presents a comprehensive investigation on the transformation of dq impedances to a common reference frame in complex converter networks. Four different methods are introduced and analyzed in a systematic way. Moreover, a rigorous comparison among these approaches is carried out, where the method with the simplest transformation procedure is finally suggested for the modeling of complex converter networks. The performed analysis is verified by injecting two independent small-signal perturbations into the d and the q axis, and doing a point-by-point impedance measurement.

Index Terms—Complex converter networks, impedance transformation, synchronous rotating dq frame, stability analysis.

I. INTRODUCTION

NOWAYS, power electronic converters are widely used in three-phase ac systems to interface renewable energy sources to the power grid [1]. Despite the stable design of the control loops at each converter, when connecting it to the grid, some dynamic interactions may appear, which will cause oscillations and even lead to stability concerns. In this context, stability analysis of power converter systems is a critical aspect [2], [3]. Among different approaches for small-signal stability analysis, the impedance-based method is validated to be an effective tool, since it features the property of modularity by dividing the whole interconnected system into source and load subsystems. Besides, both subsystems can also be regarded as “black boxes” without the prior knowledge of their internal structures and parameters [4].

Impedance-based method was firstly proposed in dc-dc converter systems by applying the Nyquist stability criterion to the impedance ratio between the source and load subsystems [5], [6]. Due to the time-varying characteristics of ac systems, conventional small-signal linearization methods cannot be directly applied [7], [8]. So far, several linearization and impedance modeling methods are investigated for ac systems [9], [10]. In many works, the abc-frame ac signals are moved into their synchronous rotating equivalent in dq frame, resulting in two dc quantities. On account of the cross-coupling between the d and the q axis, the impedance model in dq frame is a multiple-input multiple-output (MIMO) system, which yields a two-by-two dq impedance matrix. In this case, instead of the classic Nyquist stability criterion, the generalized Nyquist stability criterion (GNC) is applied to the impedance ratio matrix between the source and load subsystems for small-signal stability analysis [11]-[13]. The second method is sequence-domain impedance model based on the harmonic balance principle [14], [15], which describes the system with positive-sequence and negative-sequence impedances. When PLL or outer control loops are taken into consideration, the accurate sequence-domain impedance is also a MIMO system due to the coupling between positive and negative sequences. Some other impedance modeling approaches for ac systems can be found in [7], [9], [16], [17]. In comparison, the dq-domain method has been employed more than other approaches due to the ease of small-signal linearization and modelling, for example in investigating the effects of grid synchronization, outer control loops, etc. [4], [12], [18], [19]. However, most of the analysis is based on simple structures, like the grid with a single converter or several converters with the same dq-frame. Complex converter networks are less investigated so far.

Fig.1 shows one example of a complex ac network with different generation systems and loads. The dq impedance model of a generic converter is dependent on its local reference frame or phase angle, which may be different from other converters due to the different voltage drops caused by cable impedances. Therefore, the impedance modeling of complex converter networks requires transforming all converter impedances to a common dq reference frame. In [13], [21], [22], the dq impedance models of converters are built by rotating the converter currents and voltages from original dq frame to the common frame, and then calculating the new impedance model.
Ref. [23] proposed to transform the $dq$ impedances of different converters directly to the common reference frame with rotation matrices. In the above references, the transmission cable impedances are modeled separately from the converter impedances. However, in [20], it is indicated that the converter impedances and the corresponding cable impedances should be regarded as a whole and then transformed to the common frame. Up to now, a systematic analysis for the converter impedance transformation methods is still missing. To fill this gap, in this paper, four different transforming methods are investigated comprehensively, based on their small-signal models.

The rest of this paper is organized as follows: a general description of complex converter networks is firstly discussed in Section II. Then, four approaches to model the output impedances of different converters in a common reference frame are presented in Section III, and a rigorous comparison among them is carried out in Section IV. Based on that, Section V discusses the impedance modeling of the overall complex networks. The simulation impedance results are provided in Section VI to validate all impedance models. Finally, conclusions are presented in Section VII.

II. SYSTEM DESCRIPTION AND MODELING

A. System Description

Fig. 2 illustrates an AC network with four converter systems. Throughout this work, for simplicity, the dc side voltages are assumed to be constant. Due to the extra impedances caused by transmission cables, the terminal voltages of the four converters are not the same, which also makes the reference frames that $dq$ impedances of converters refer to, different. As shown in Fig. 2, the $dq$ frames of the four converter systems are not aligned to the $dq$ frame provided by the PCC voltage, which is defined as the common reference frame in this system. In order to build the impedance model of the whole network at PCC side, all converter $dq$ impedances should be transformed into the common reference frame.

Fig. 3 depicts the topology and control of the first branch in Fig. 2, where inductive cable impedances are considered and the output current $i_{abc}$ is synchronized with the terminal voltage $v_{1abc}$ by a PLL. The converter $dq$ impedance $Z_{dq1}$ at port 1 is in the local $dq$ reference frame, while the branch $dq$ impedance $Z_{dqg}$ at port 2 is in the common $dq$ reference frame.

B. Converter Impedances

Firstly, the impedance modeling of converters is considered. Based on the topology of Fig. 3, the equation of converter currents and voltages in $abc$ frame can be expressed as

$$\begin{bmatrix}
    L_d \frac{d}{dt} i_a \\
    L_d \frac{d}{dt} i_b \\
    L_d \frac{d}{dt} i_c
\end{bmatrix} =
\begin{bmatrix}
    v_a \\
    v_b \\
    v_c
\end{bmatrix} - \begin{bmatrix}
    v_{l1} \\
    v_{l2} \\
    v_{l3}
\end{bmatrix}$$

(1)

Where $v_{a}, v_{b}, v_{c}$ are the output voltages of converter, $v_{l1}, v_{l2}, v_{l3}$ are the terminal voltages of converter, $i_{a}, i_{b}, i_{c}$ are the output currents and $L_{d}$ is the filter inductance.

The corresponding quantities in $dq$ frame can be obtained by transforming (1) based on the Park transformation matrix (2). The transformed result is presented in (3), in which, $\omega_{0}$ is the fundamental angular frequency.

$$T_{dq}(\theta) = \frac{2}{3} \begin{bmatrix}
    \cos(\theta) & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta - \frac{4\pi}{3}) \\
    \sin(\theta) & \sin(\theta - \frac{2\pi}{3}) & \sin(\theta - \frac{4\pi}{3})
\end{bmatrix}$$

(2)
\[
(L_s \frac{d}{dt} - \omega_b L_t) \begin{bmatrix}
0 & 1 \\
-1 & 0
\end{bmatrix} \begin{bmatrix}
I_d \\
I_q
\end{bmatrix} = \begin{bmatrix}
v_d \\
v_q
\end{bmatrix} - \begin{bmatrix}
v_{id} \\
v_{iq}
\end{bmatrix}
\]  
(3)

On the basis of (3), the open-loop output impedance \( Z_{out} \) from terminal voltage to output current and the transfer function matrix \( G_d \) from duty ratio to output current in \( dq \) frame can be derived as

\[
Z_{out} = \frac{L_s}{\omega_b L_t} \begin{bmatrix}
L_s & -\omega_b L_t \\
\omega_b L_t & L_s
\end{bmatrix}
\]  
(4)

\[
G_d = \frac{V_d}{(L_s)^2 + (\omega_b L_t)^2} \begin{bmatrix}
L_s & -\omega_b L_t \\
\omega_b L_t & L_s
\end{bmatrix}
\]  
(5)

In the converter current control of Fig. 3, \( G_d(s) \) is PI controller and \( K_d \) represents the decoupling coefficient. The corresponding expressions are presented below.

\[
G_c(s) = k_p + \frac{k_i}{s}
\]  
(6)

\[
K_d = \frac{\omega_b L_t}{V_d}
\]  
(7)

\[
G_{pwm}(s) = \frac{1}{1+1.5T_s s}
\]  
(8)

The matrices of (6) to (8) in \( dq \) frame can be expressed as

\[
G_d = \begin{bmatrix}
G_c & 0 \\
0 & G_c
\end{bmatrix}
\]  
(9)

\[
G_{dec} = \begin{bmatrix}
0 & -K_d \\
K_d & 0
\end{bmatrix}
\]  
(10)

\[
G_{dcl} = \begin{bmatrix}
G_{pwm} & 0 \\
0 & G_{pwm}
\end{bmatrix}
\]  
(11)

For grid-tied converters, PLL is usually utilized to achieve the synchronization between converters and the grid. As discussed in [12], two different \( dq \) frames can be defined in the converter systems with PLL: the system frame, defined by the grid voltage, in which the \( dq \) model of the plant system is built; and the controller frame, defined by PLL, where the \( dq \) control is constructed. For convenience, \( \theta_s \) and \( \theta_{dcl} \) are respectively specified as the angles of system frame and controller frame. If the input of the PLL is the converter terminal voltages, the system and control frames align in steady-state condition, and under the small-signal perturbations, they slightly deviate due to PLL dynamics. On the other hand, if the PLL input is the voltage at the output of the cable inductance, there exists a fixed angle difference between the two frames, as shown in Fig. 4. The relation between the two \( dq \) reference frames can be generally described as (12), where the voltage is taken as an example and the small-signal perturbations are also taken into account.

\[
\begin{bmatrix}
v_d' \\
v_q'
\end{bmatrix} + \begin{bmatrix}
v_d'' \\
v_q''
\end{bmatrix} = T_{dq}(\theta_{dcl} + \dot{\theta}_{dcl}) T_{dq}^t(\theta_s) \begin{bmatrix}
v_d' \\
v_q'
\end{bmatrix} + \begin{bmatrix}
v_d'' \\
v_q''
\end{bmatrix}
\]  
(12)

After transformation, the expansion form of (12) can be expressed as (13) and (14), in which \( R_{d\phi} \) is defined as the rotation matrix and \( \Delta \theta \) equals \( \theta_{dcl} - \theta_s \).

\[
\begin{bmatrix}
v_d' \\
v_q'
\end{bmatrix} + \begin{bmatrix}
v_d'' \\
v_q''
\end{bmatrix} = R_{d\phi} \begin{bmatrix}
v_d' \\
v_q'
\end{bmatrix} + R_{d\phi} \begin{bmatrix}
v_d'' \\
v_q''
\end{bmatrix} + \begin{bmatrix}
v_d'' \\
v_q''
\end{bmatrix} \dot{\theta}_{dcl}
\]  
(13)

\[
R_{d\phi} = \begin{bmatrix}
\cos(\Delta \theta) & \sin(\Delta \theta) \\
-\sin(\Delta \theta) & \cos(\Delta \theta)
\end{bmatrix}
\]  
(14)

The block diagram of PLL is shown in Fig. 5. Three-phase voltages are transformed into \( dq \) frame and then \( q \)-axis voltage is employed for the detection of rotation angle. The output angle of PLL is given by

\[
\theta_{pwm} = \frac{H_c(s)}{s} v_q'
\]  
(15)

Where

\[
H_c(s) = k_{pwm} + \frac{k_{int}}{s}
\]  
(16)

Fig. 5. Block diagram of PLL.

C. Subsystem impedances

For convenience of analysis, each branch is defined as one subsystem of the converter network shown in Fig. 2. It can be seen from Fig. 3 that the subsystem impedance consists of the converter impedance and cable impedance. The converter impedance and the subsystem impedance refer to the \( dq \) reference frames provided by \( v_{labc} \) and \( v_{gabc} \), respectively. \( \theta_1 \) and \( \theta_g \) are here defined as the phase angles of \( v_{labc} \) and \( v_{gabc} \). To obtain the subsystem impedance model, transforming the converter impedance to the reference frame of \( \theta_g \) is thus essential. Based on different converting ways, there are two approaches for the subsystem impedance modeling. One method is building the converter impedance in the \( dq \) frame of \( \theta_1 \), and then converting the impedance model to the \( dq \) frame of \( \theta_g \). The other is directly building the converter impedance in the \( dq \) frame of \( \theta_g \). In addition, the cable impedance is included in the small-signal impedance modeling of converters directly, or it is separately considered and then added in series with the converter also provides two realization ways for each method. Accordingly, there are four modeling approaches for the subsystem impedance in total, which hereafter, are discussed in detail.

III. FOUR DIFFERENT APPROACHES FOR MODELING OF SUBSYSTEM IMPErences
Fig. 6. Small-signal model of a converter when \( \theta_s \) equals \( \theta_1 \) (approach A)

A. Approach A

When the converter model is built in the \( dq \) frame defined by \( \theta_1 \), the controller \( dq \) frame is aligned with system \( dq \) frame in steady states, which means \( R_{dq} \) of (13) is an identity matrix. The small-signal voltage relationship between system and controller \( dq \) frames can be obtained as (17).

\[
\begin{bmatrix}
\tilde{v}^d_{pi} \\
\tilde{v}^q_{pi}
\end{bmatrix} =
\begin{bmatrix}
0 & i^d_{pi}G_{pll} \\
0 & -i^q_{pi}G_{pll}
\end{bmatrix}
\begin{bmatrix}
\tilde{v}^d_{si} \\
\tilde{v}^q_{si}
\end{bmatrix}
+ \begin{bmatrix}
\tilde{v}^d_{pi} \\
\tilde{v}^q_{pi}
\end{bmatrix}
\]  
(17)

Combining (15) with (17), the small-signal model of PLL is derived as

\[
\tilde{\theta}_{pll} = \frac{H_1(s)}{s + \omega_1 \cdot H_1(s)} \tilde{v}^q_{pll}
\]  
(18)

Where \( G_{pll} \) is defined as

\[
G_{pll} = \frac{H_1(s)}{s + \omega_1 \cdot H_1(s)}
\]  
(19)

Similarly, the small-signal formulas of currents and duty ratios can also be obtained as (20) and (21). The small-signal model of the converter with system \( dq \) frame built in \( \theta_1 \) is presented in Fig. 6.

\[
\begin{bmatrix}
\tilde{i}^d_{pi} \\
\tilde{i}^q_{pi}
\end{bmatrix} =
\begin{bmatrix}
0 & i^d_{pi}G_{pll} \\
0 & -i^q_{pi}G_{pll}
\end{bmatrix}
\begin{bmatrix}
\tilde{i}^d_{si} \\
\tilde{i}^q_{si}
\end{bmatrix}
+ \begin{bmatrix}
\tilde{i}^d_{pi} \\
\tilde{i}^q_{pi}
\end{bmatrix}
\]  
(20)

\[
\begin{bmatrix}
\tilde{d}^d_{pi} \\
\tilde{d}^q_{pi}
\end{bmatrix} =
\begin{bmatrix}
0 & \tilde{d}^d_{pi}G_{pll} \\
0 & -\tilde{d}^q_{pi}G_{pll}
\end{bmatrix}
\begin{bmatrix}
\tilde{d}^d_{si} \\
\tilde{d}^q_{si}
\end{bmatrix}
+ \begin{bmatrix}
\tilde{d}^d_{pi} \\
\tilde{d}^q_{pi}
\end{bmatrix}
\]  
(21)

in which

\[
G_{pll}^d = \begin{bmatrix}
0 & i^d_{pi}G_{pll} \\
0 & -i^q_{pi}G_{pll}
\end{bmatrix}
\]  
(22)

The converter impedance in the \( dq \) frame of \( \theta_1 \) can be expressed as

\[
Z_{dq1}^{clA} = \frac{Z_{dq1}}{L_{d11}} = \left[ Z_{out}^{-1} + G_{pll}^dG_{dq1} + (G_{dec} - G_{c}) \right]^{-1}
\]  
(23)

As the converter impedance model (24) refers to the \( dq \) frame of \( \theta_1 \), it needs to be transformed to the \( dq \) frame of \( \theta_s \) for the modeling of subsystem impedances. Based on the relationship of voltages and currents of the two \( dq \) reference frames, the rotated converter impedance model in the \( dq \) frame of \( \theta_s \) can be obtained as

\[
Z_{dq1}^{clA} = \left( H_{dq} \right) Z_{dq1}^{clA} \left( H_{dq} \right)^{-1}
\]  
(25)

Where \( H_{dq} \) shares the same expression with \( R_{dq} \), but the angle variable \( \Delta \theta \) in \( H_{dq} \) equals \( \theta_1 - \theta_s \).

The other element in the subsystem model is the cable impedance, which is usually a combination of passive components, such as resistance, inductance and capacitance. It can be mathematically proved that the model of a balanced system in \( dq \) frame is symmetric, the transformation of reference frame rotation in (25) does not change a \( dq \) symmetric system. Thus, the subsystem impedance can be described as

\[
Z_{dq1}^{clA} = Z_{dq1}^{clA} + Z_{dq1}^{clA}
\]  
(26)

B. Approach B

In approach A, the subsystem impedance is built by separately modeling the converter impedance and cable impedance firstly, and then doing the reference frame transformation and superposition. In fact, based on the power flow relation, the subsystem impedance model can also be obtained by firstly modeling the converter and cable impedance as a whole, and then conducting the reference frame transformation, which introduces approach B.

The relations among \( v_{abc}, v_{labc} \) and \( v_{pabc} \) in the \( dq \) frame of \( \theta_1 \) can be written as (28) from Fig. 3.

\[
\begin{bmatrix}
v_{abc} \\
v_{labc}
\end{bmatrix} = \frac{L_{d11}}{L_{d11} + L_{s11}} \begin{bmatrix}
v_{abc} \\
v_{labc}
\end{bmatrix}
\]  
(28)

Based on the small-signal model of converters in approach A, the small-signal model of the subsystem can be derived by substituting (28) into (20) and (21). The obtained models are presented as (29) and (30), in which the transfer function matrices \( G_{plld}, G_{plld}, G_{plld}, G_{plld}, \) and \( G_{plld} \) are respectively given in

\[
\begin{bmatrix}
\tilde{v}^d_{si} \\
\tilde{v}^q_{si}
\end{bmatrix} =
\begin{bmatrix}
0 & i^d_{pi}G_{plld} \\
0 & -i^q_{pi}G_{plld}
\end{bmatrix}
\begin{bmatrix}
\tilde{v}^d_{pi} \\
\tilde{v}^q_{pi}
\end{bmatrix}
+ \begin{bmatrix}
\tilde{v}^d_{si} \\
\tilde{v}^q_{si}
\end{bmatrix}
\]  
(29)

\[
\begin{bmatrix}
\tilde{d}^d_{si} \\
\tilde{d}^q_{si}
\end{bmatrix} =
\begin{bmatrix}
0 & \tilde{d}^d_{pi}G_{plld} \\
0 & -\tilde{d}^q_{pi}G_{plld}
\end{bmatrix}
\begin{bmatrix}
\tilde{d}^d_{pi} \\
\tilde{d}^q_{pi}
\end{bmatrix}
+ \begin{bmatrix}
\tilde{d}^d_{si} \\
\tilde{d}^q_{si}
\end{bmatrix}
\]  
(30)
obtained as (35). Converting (35) to the subsystem with plant system built in the (31), (32), (33) and (34).

The small-signal model of the subsystem with plant system built in the dq frame of \( \theta_1 \) is shown in Fig. 7.

\[
G_{\text{pld}}^i = \begin{bmatrix} 0 & i_{q1}^i \\ -i_{d1}^i & 0 \end{bmatrix} G_{\text{pl}} V_{\text{dc}} G_{\text{pm}} L_{11} \left/ \left( L_{1} + L_{11} \right) \right. 
\]

(31)

\[
G_{\text{pld}}^d = \begin{bmatrix} d_{d1}^i G_{\text{pl}} V_{\text{dc}} G_{\text{pm}} L_{11} \left/ \left( L_{1} + L_{11} \right) \right. \\
0 & 1 - d_{d1}^i G_{\text{pl}} V_{\text{dc}} G_{\text{pm}} L_{11} \left/ \left( L_{1} + L_{11} \right) \right. \end{bmatrix}^{-1} 
\]

(32)

\[
G_{\text{pld}}^i = \begin{bmatrix} 0 & i_{q1}^i \\ -i_{d1}^i & 0 \end{bmatrix} G_{\text{pl}} L_{1} \left/ \left( L_{1} + L_{11} \right) \right. 
\]

(33)

\[
G_{\text{pld}}^d = G_{\text{pld}}^i \begin{bmatrix} 0 & -d_{d1}^i \\ d_{d1}^i & 0 \end{bmatrix} G_{\text{pl}} L_{1} \left/ \left( L_{1} + L_{11} \right) \right. 
\]

(34)

The subsystem impedance in the dq frame of \( \theta_1 \) can be obtained as (35). Converting (35) to the dq frame of \( \theta_2 \), the subsystem impedance in the dq frame of \( \theta_2 \) is expressed as

\[
Z_{\text{sub}}^{11\text{B}} = H_{\text{\theta1}\theta2}^{-1} Z_{\text{sub}}^{11\text{B}} H_{\text{\theta1}\theta2} 
\]

(36)

C. Approach C

The system dq frame of approaches A and B are both modeled in \( \theta_1 \). Alternatively, the plant system can also be directly built in the dq frame of \( \theta_2 \), since the subsystem impedance is finally built in that frame, which hence does not require the frame rotation of impedances. However, when considering the controller system in the dq frame of \( \theta_1 (\theta_{\text{pl}}=\theta_1) \), the frame rotations of currents and voltages are needed in the control model.

When the plant system is built in the dq frame of \( \theta_2 (\theta_{\text{pl}}=\theta_2) \),

\[
Z_{\text{sub}}^{11\text{B}} = Z_{\text{sub}}^{11\text{B}} = \left[ \frac{L_{1} + L_{11}}{L_{1} + L_{11}} Z_{\text{out}}^{-1} + \left( G_{\text{id}} + \frac{L_{11}}{L_{1} + L_{11}} Z_{\text{out}}^{-1} V_{\text{dc}} \right) G_{\text{di}} \left( I + G_{\text{pld}} \left( G_{\text{ci}} - G_{\text{dec}} \right) G_{\text{pld}}^{-1} \right)^{-1} \right] 
\]

\[
\left[ I + \left( G_{\text{di}} + \frac{L_{11}}{L_{1} + L_{11}} Z_{\text{out}}^{-1} V_{\text{dc}} \right) G_{\text{di}} \left( I + G_{\text{pld}} \left( G_{\text{ci}} - G_{\text{dec}} \right) G_{\text{pld}}^{-1} \right)^{-1} \right] 
\]

(34)

D. Approach D

\[
\text{The system dq frame when } \theta_{\text{pl}}=\theta_1 \text{ is shown in Fig. 8, where the impedance model of the converter is presented in Fig. 8, with the impedance model of the subsystem in the dq frame of } \theta_1 \text{ can be derived. The dq impedance model of the subsystem can be built by directly adding the converter impedance and cable impedance, which is expressed as}
\]

\[
Z_{\text{sub}}^{11\text{C}} = \left[ Z_{\text{sub}}^{11\text{C}} + G_{\text{di}} G_{\text{dec}} R_{\text{sl}}^{-1} \left( G_{\text{pld}} \left( G_{\text{ci}} - G_{\text{dec}} \right) R_{\text{sl}} \right) \right]^{-1} 
\]

\[
\cdot \left( I + G_{\text{di}} G_{\text{dec}} R_{\text{sl}}^{-1} \left( G_{\text{ci}} - G_{\text{dec}} \right) R_{\text{sl}} \right) + Z_{\text{sub}}^{11\text{C}} 
\]

(40)
whether the cable impedance is included in the small-signal to obtain the transformed impedances in different cases. While approaches C and D need to re-model converter systems, approaches A and B can be easily applied to different systems. Therefore, since the rotation matrix is not embedded into the original voltage equation in the dq frame of $\theta_g$.

\[
\left[ \begin{array}{c} \frac{v_{dqg}^s}{v_{dkg}^s} \\ \frac{v_{dqg}^s}{v_{dkg}^s} \end{array} \right] = \frac{L_{d1}}{L_{i1} + L_{i1}} \left[ \begin{array}{c} \frac{v_{dqg}^s}{v_{dkg}^s} \\ \frac{v_{dqg}^s}{v_{dkg}^s} \end{array} \right] + \frac{L_i}{L_{i1} + L_{i1}} \left[ \begin{array}{c} \frac{v_{dqg}^s}{v_{dkg}^s} \\ \frac{v_{dqg}^s}{v_{dkg}^s} \end{array} \right] \quad (41)
\]

On the basis of the previously obtained system model, the small-signal model of subsystems in dq frame of $\theta_g$ is presented in Fig. 9, where the subsystem dq impedance model can be directly derived as (42).

IV. COMPARISON OF DIFFERENT APPROACHES

The mathematical models of the subsystem impedance obtained with the four approaches have been derived in above analysis. Importantly, it turns out that the impedance expressions of approaches A and C, B and D respectively match after the required mathematical simplification. This implies that the impedance models can be built by either directly rotating the impedances of different reference frames or by rotating the voltage and current first, and then constructing the impedance model. However, compared to approaches C and D, the impedance expressions of approaches A and B are simpler, since the rotation matrix is not embedded into the original converter and subsystem impedance expressions. Therefore, approaches A and B can be easily applied to different systems. While approaches C and D need to re-model converter systems to obtain the transformed impedances in different cases.

The difference between approaches A and B (or C and D) is whether the cable impedance is included in the small-signal modelling of converters. As their mathematical models cannot be compared intuitively, frequency responses under different system conditions are adopted for further comparison. The system parameters are listed in Table I.

The frequency response of subsystem impedances obtained with approaches A and C are plotted in Fig. 10. It can be seen that frequency responses of these two models always match. This is expected since their analytic models are the same in essence. This overlap can also be observed in the frequency responses of approaches B and D, which is not given here to save space. Fig. 11 plots the frequency response of impedance models obtained with approaches A and B, which shows a good matching. Therefore, approaches A and B are equivalent as well. Converters with different bandwidths of PLL and cable impedances are also tested, indicating the same conclusion. However, as the cable impedance is integrated into the original

![Diagram](image-url)

**Fig. 9.** Small-signal model of a subsystem when $\theta_s$ equals $\theta_i$ (approach D).

![Graph](image-url)

**Fig. 10.** Frequency response of the first subsystem impedance modeled based on approaches A and C.

| Symbol | Description | Value |
|--------|-------------|-------|
| $P_{rated}$ | Rated power | 100 kW |
| $V_{dc}$ | Input dc voltage | 700 V |
| $V_g$ | Grid phase voltage (rms) | 230 V |
| $f_0$ | Fundamental frequency | 50 Hz |
| $f_s$ | Sampling frequency | 20 kHz |
| $L_{i1}$ | Filter inductance | 2 mH |
| $L_{i1}$ | Cable inductance | 4 mH |
| $k_{ppl}$ | Proportional gain of current controller | 0.004 |
| $k_{ppl}$ | Proportional gain of PLL controller | 0.262 |
| $k_{pllC}$ | Integrator gain of PLL controller | 12.062 |
by simplifying the structure with series and parallel theory. For the whole network becomes much easier, which can be realized impedance of the four subsystems not contain the grid requirement of system re-modeling.

Comparison, approach A is hence simpler, more flexible and the converter and cable impedances cannot be built. In addition, approach B cannot be applied to the cases where a converter impedance model, approach B is more complex. In comparison, approach A is hence simpler, more flexible and applicable, which can convert the original different converter impedances to the common reference frame directly, without the requirement of system re-modeling.

V. IMPEDANCES OF COMPLEX CONVERTER NETWORKS

After transforming all converter impedances to the same reference frame with approach A, the impedance modeling of the whole network becomes much easier, which can be realized by simplifying the structure with series and parallel theory. For example, the network $dq$ impedance $Z_{dqg}^2$ in Fig. 2, which does not contain the grid $dq$ impedance $Z_{dqg}^*$, equals the parallel impedance of the four subsystems $Z_{dqg}^1$, $Z_{dqg}^2$, $Z_{dqg}^3$, $Z_{dqg}^4$ after combining the converter and cable impedances together. Its expression is presented as (43). The same impedance building criterion can also be applied to other complex converter networks.

$$Z_{dqg}^n = \left( (Z_{dqg}^1)^{-1} + (Z_{dqg}^2)^{-1} + (Z_{dqg}^3)^{-1} + (Z_{dqg}^4)^{-1} \right)^{-1}$$

(43)

With the whole network impedance, the stability assessment can be conducted by applying GNC to the return-ratio matrix $L$, which equals the product of $Z_{dqg}^*$ and $(Z_{dqg}^*)^{-1}$.

VI. SIMULATION VERIFICATION

To further validate the correctness of the above system modeling methods, simulation is carried out in Matlab/Simulink, which is realized by injecting small-signal perturbations and performing a point-by-point impedance measurement. The schematic of the impedance measurement is shown in Fig. 12. As the $dq$ impedance is a two-by-two matrix, two separate small-signal perturbation injections are thus required for the $dq$ impedance measurement at one frequency point. Firstly, the perturbation signals should be in the same $dq$ reference frame where the system model is built. A sinusoidal perturbation signal with certain frequency and amplitude is chosen along the $d$-axis perturbation signal while keeping the $q$-axis perturbation signal zero. The perturbation signal is injected into the desired port, where the response voltages and currents are collected. The measured signals can be converted to the same $dq$ domain in which the injection signal is created, giving $i_{d1}$, $i_{q1}$, $v_{d1}$, $v_{q1}$. The second perturbation is built along the $q$-axis, by keeping $d$-axis perturbation signal zero. The same injection and measurement procedures can be conducted to obtain the second set of data $i_{d2}$, $i_{q2}$, $v_{d2}$, $v_{q2}$. Finally, the measured impedance $Z_{dq}$ at frequency $f_p$ can finally be calculated as

$$Z_{dq} = \begin{bmatrix} Z_{dd} & Z_{dq} \\ Z_{qd} & Z_{qq} \end{bmatrix} = \begin{bmatrix} v_{d1} & v_{d2} \\ v_{q1} & v_{q2} \end{bmatrix} \begin{bmatrix} i_{d1} & i_{d2} \\ i_{q1} & i_{q2} \end{bmatrix}^{-1}$$

(44)

Based on the above measurement method, the subsystem impedance in Fig. 3 with parameters listed in Table I, can be directly measured by injecting the perturbation signals with different frequencies at port 2. Using approach A, the subsystem impedance can also be obtained by rotating the converter impedance measured at port 1, and then adding the cable impedance. The frequency response of the measured impedances is plotted in Fig. 13, where the theoretical model is placed for comparison. It can be seen that the impedances directly measured at port 2 and with approach A always match, verifying the correctness of approach A. Besides, both measured impedances also coincide with the theoretical one, which validates the theoretical model.

After the modeling of all subsystem impedances, the overall impedance of the whole network can be obtained. The simulated results of Fig. 2 are presented in Fig. 14, in which, the blue line with circles represents the paralleled $dq$ impedances of the four subsystems that have different system parameters, and the red line with crosses is the impedance of

![Fig. 12. Schematic of impedance measurement.](image-url)
the whole network, which is measured at PCC side. It can be found that the frequency responses of the paralleled impedance always match with Z_{dqg}, verifying the correctness of analysis.

VII. CONCLUSIONS

In this paper, the transformation of converter dq impedances to a common reference frame in complex converter networks is investigated based on four different approaches. A systematic analysis and comparison of these methods has been carried out based on their small-signal models, which shows the equivalence of the four approaches. In comparison, modeling each component separately in its own reference frame, and then using the impedance rotation matrix, provides more simplicity and flexibility with respect to rotating converter voltages and currents to another reference frame, and modeling all modules together. The impedances of complex converter networks can be easily obtained by simplifying the subsystem impedance structure after transforming all converter impedance modeling to a common reference frame. The correctness of the analytically derived models is validated in simulation, by injecting small-signal perturbations and finding the ratio between voltages and currents.

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