Wrapped Branes and Compact Extra Dimensions in Cosmology

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We present a cosmological model in 1 + m + p dimensions, where in m-dimensional space there are uniformly distributed p-branes wrapping over the extra p-dimensions. We find that during cosmological evolution m-dimensional space expands with the exact power-law corresponding to pressureless matter while the extra p-dimensions contract. Adding matter, we also obtain solutions having the same property. We show that this might explain in a natural way why the extra dimensions are small compared to the observed three spatial directions.

I. INTRODUCTION

The inflationary paradigm is successful in explaining away the basic shortcomings of standard cosmology, like the monopole, horizon and flatness problems. One might along this line claim that we now have a cosmological scenario which remains plausible very close to big bang. However, the possibility of an initial singularity still remains to be dealt with. One hopes that this problem will be resolved once we understand the theory of gravity down to sizes comparable to Planck length. String theory is one of the leading candidates for that but for its consistency one has to introduce extra dimensions. Observationally if these extra dimensions exist their sizes are much smaller than the size of our perceived universe. Thus it is of importance to seek for cosmological models where this difference can be accommodated in a natural way.

It is plausible that the sizes of all dimensions started out the same, possibly close to Planck length. After various cosmological eras the perceived universe grew to its size we observe today. Therefore the problem is to explain how the extra dimensions remained comparatively small.

In what follows we propose a toy model, which we believe gives an answer to this problem in a natural way. The basic motivation is a flavor of the idea once exposed by Brandenberger and Vafa [1] (see also [2] and [3]): when a p-brane wrap over the extra p-dimensions it resists expansion, much like a rubber band would if wrapped and glued over the surface of a balloon. There has been considerable activity in the literature on similar ideas, see for example the subject of “brane gas cosmology” [4]-[14].

Our model is formulated in 1 + m + p-dimensions, where m and p refer to the observed and compact dimensions respectively, in which there is a uniform (with respect to m) distribution of p-branes wrapping over the extra p-dimensions. In order to incorporate this assumption it is enough to take the observed space to be topologically non-compact. At this point it is apparent that this approach is different than the brane gas cosmology, where it is postulated that branes can wrap anywhere.

In this paper we focus on time dependent solutions to Einstein equations and find exact expressions where possible. In the earlier work on brane (or string) gas cosmology the interest was mainly on the thermodynamical aspects and on the incorporation of T-duality invariance of string theory to cosmology (see for instance [4], [5] and [7]). For that reason, the problem was studied in the framework of dilaton gravity and it was found that the wrapped branes can prevent cosmological expansion of the internal space. As we will see, the same can be achieved in Einstein gravity without invoking T-duality invariance.

Recently, dynamical aspects of brane gas cosmology in M-theory has been studied in [15] and [16]. Our approach is technically similar. Namely, we also obtain the energy momentum tensor by coupling the brane action to the gravity action and assume a uniform distribution of such branes. However, we concentrate on pure Einstein gravity rather than M-theory.

The main phenomenon we observed is that the existence of wrapped p-branes makes the m-dimensional space grow in exactly the same way as pressureless matter would, while the p-dimensional compact space is contracted following a power-law depending on the numerical value of p. From this perspective the model has predictive power since it gives a definite proportion between the sizes of the observed and the compact directions.

We have also observed that adding ordinary matter does not change the mentioned behavior appreciably except for the case with negative pressure. For instance, adding radiation one still finds that the observed space expands and the compact space contracts. On the other hand, in case of vacuum domination both observed and compact spaces will grow exponentially. More on our line of reasonings is presented in Section III.

The organization of the manuscript is as follows: In Section II we show how to wrap the p-branes over extra dimensions, present the resulting Einstein equations and the aforementioned solution. In Section III we add ordinary matter to the energy-momentum tensor. In Section IV we give an estimate for the current size of the internal dimensions using the solutions. In section V, we compare the relative strengths of the expansion and con-
traction forced by \( p \)-branes. The last section is devoted to conclusions and possible future extensions of the model.

II. COSMOLOGY OF WRAPPED \( p \)-BRANES

Consider a \( D \)-dimensional space-time which has the following metric
\[
ds^2 = -e^{2A}dt^2 + e^{2B}dx^i dx^i + e^{2C}dy^a dy^a,
\]
where \( i = 1, \ldots, m, \ a = 1, \ldots, p \) and the metric functions \( A, B, C \) depend only on time \( t \). Here, \( x^i \) is chosen to parameterize the observed directions which we label as \( M_m \) and \( y^a \) parameterizes the extra space labeled by \( M_p \). We also define \( X^\mu = (t, x^i, y^a) \). We would like to determine the cosmological evolution in the presence of \( p \)-branes wrapping over \( M_p \). The dynamics of \( p \)-branes are determined by the Polyakov action
\[
\frac{-S_p}{T_p} = \int d\xi^{p+1} \sqrt{-\gamma} \left[ \gamma^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu\nu} - p + 1 \right],
\]
where \( \xi^\alpha = (\tau, \sigma^a) \) are world-volume coordinates, \( T_p \) is the \( p \)-brane tension, \( g_{\mu\nu} \) is the space-time and \( \gamma_{\alpha\beta} \) is the world-volume metrics, and \( X^\mu(\xi) \) is the map from world-volume to space-time. It is easy to show that the following \( p \)-brane configuration in (2) is an extremum of the Polyakov action, where \( x_i^0 \) are constants giving the position of the \( p \)-brane in \( M_m \). We assume that \( y^a \) and \( \sigma^a \) coordinates are topologically \( S^1 \) so that there is no surface term coming from the variation of the action (2). We would like to calculate the back reaction of this configuration on the geometry. For that we couple (2) to \( D \)-dimensional Einstein-Hilbert action
\[
S_E = \frac{1}{\kappa^2} \int d^D X \sqrt{-g} R,
\]
so that
\[
S = S_E + S_p.
\]
The energy momentum tensor for the \( p \)-brane can be calculated from (3) which gives
\[
-\frac{T^{\mu\nu}}{T_p} = \int d\xi^{p+1} \sqrt{-\gamma} \gamma^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu \delta(X - X(\xi)).
\]
In the orthonormal frame \((e^A dt, e^B dx^i, e^C dy^a)\), corresponding to the \( p \)-brane configuration (3) takes the form
\[
T_{it} = T_p e^{-mB} \delta(x - x_0),
T_{ij} = 0,
T_{ab} = -T_p e^{-mB} \delta(x - x_0) \delta_{ab}.
\]
For more than one \( p \)-brane one has to change \( \delta(x - x_0) \) with the sum \( \sum n_i \delta(x - x_i) \) where \( n_i \) is the number of coincident \( p \)-branes at the position \( x_i \). We now assume that there are uniformly distributed such \( p \)-branes in \( M_m \) and do the following replacement
\[
\sum n_i \delta(x - x_i) \rightarrow \int dx' n(x') \delta(x - x'),
\]
where \( n(x) \) is the number of \( p \)-branes per unit volume at the position \( x \). Homogeneity of \( M_m \) implies that \( n(x) \) is constant and this gives the following energy momentum tensor
\[
T_{tt} = n T_p e^{-mB},
T_{ij} = 0,
T_{ab} = -n T_p e^{-mB} \delta_{ab}.
\]
One can easily check that \( \nabla_\nu T^{\mu\nu} = 0 \). We also note that a scaling of \( x \) coordinates \( x \rightarrow \lambda x \) requires, by definition, a scaling of \( n \) by \( n \rightarrow n/\lambda \).

In solving Einstein equations, we first impose the gauge choice \( A = mB + \rho C \) which fixes t-reparametrization invariance in the metric (4). In this gauge, the field equations that follow from the action (3) can be written as
\[
A'' - A'^2 + mB'^2 + pC'^2 = C''
B'' = \frac{p + 1}{m + p - 1} \kappa^2 n T_p e^{mB + 2pC}
C'' = -\frac{m - 2}{m + p - 1} \kappa^2 n T_p e^{mB + 2pC}
\]
where \( ' \) denotes differentiation with respect to \( t \). The last two equations in (10) imply that \( B \) and \( C \) are proportional to each other up to linear terms in \( t \). Ignoring these terms \( B \) and \( C \) can be solved up to an undetermined integration constant. The first equation in (10) then fixes this integration constant. Switching to the proper time coordinate (which we again denote by \( t \)), we finally obtain the following metric
\[
ds^2 = -dt^2 + (\alpha t)^2 dx^i dx^i + (\alpha t)^{d(2m-2) + 4(m-p+3)} dy^a dy^a,
\]
where
\[
\alpha^2 = \frac{m^2(p + 1)^2}{2(m + p - 1) (m - mp + 4p)}.
\]
For the physically important case of \( m = 3 \), from (11), the scale factors \( R_3 \) and \( R_p \) of the observed and compact dimensions respectively are determined as follows
\[
R_3(t) = (\alpha t)^{2/3},
R_p(t) = (\alpha t)^{-2/3(p+1)}.
\]
It is evident that the power-law of the observed space is exactly the same as the one for pressureless matter in standard cosmology. This is somewhat expected since the observed space part of energy momentum tensor vanishes
as it does for pressureless matter. On the other hand, [14] shows that even in pure Einstein gravity wrapped branes can prevent expansion of the internal dimensions. This is contrary to the general expectation in the literature (see for instance [1], [14] and [16]) which is based on the intuition that negative pressure would increase the expansion rate (note that branes apply negative pressure along the wrapping directions) as in vacuum domination during inflation. However, [14] indicates that negative pressure does not necessarily imply expansion in Einstein gravity. Moreover, even in de Sitter phase of the early universe, negative pressure would give exponential contraction with a negative Hubble constant.

III. ADDING MATTER

We now add ordinary matter to analyze a more realistic model. We will use for generality the following energy momentum tensor for matter

\[ T_{\mu\nu} = \text{diag}(\rho, p_i, p_b), \]  

where the indices refer to the obvious orthonormal frame in (11) and

\[ p_i = \omega \rho, \]

\[ p_b = \nu \rho, \]

where \( \omega \) and \( \nu \) are constants. The energy-momentum conservation \( \nabla_{\mu}T^{\mu\nu} = 0 \) determines \( \rho \) in terms of the metric functions

\[ \rho = \rho_0 e^{-(1+\omega)mB-(1+\nu)pC}, \]

where \( \rho_0 \) is a constant and we again impose the gauge \( A = mB + pC \). In the presence of matter, Einstein equations (10) are modified as follows

\[ A'' - A'^2 + mB'^2 + pC'^2 = C'' - \kappa^2 \rho_0 (1+\nu) e^{(1-\omega)mB+(1-\nu)pC}, \]

\[ (m+p-1)B'' = \kappa^2 e^{mB+2pC} \left[ (p+1)nT_p + \rho_0 (1 + (p-1)\omega - p\nu) e^{-m\omega B - (1+\nu)pC} \right], \]

\[ (m+p-1)C'' = \kappa^2 e^{mB+2pC} \left[ -(m-2)nT_p + \rho_0 (1 + (m-1)\nu - m\omega) e^{-m\omega B - (1+\nu)pC} \right]. \]

These equations have a much richer solution space and it is not possible in this manuscript to exhaust every interesting one. There are various possibilities here depending on what one chooses for \( \omega \) and \( \nu \). However it is of crucial importance to accommodate for the inflationary paradigm, so we consider this first. In the 4-dimensional cosmology one chooses \( \omega = -1 \) to achieve an exponential growth. This however also has a physical interpretation. The inflationary solution is the one in which vacuum energy is dominant. It would be, we feel, quite unnatural to assume that a constant cosmological constant to be absent from the extra dimensions, so we believe it is necessary to chose \( \nu = -1 \) also. In this case, comparing the terms on the right hand side of (13), it is clear that the functions multiplying \( \rho_0 \) is \( e^{mB} \) times larger than the terms coming from p-brane sources. Evidently the former term will dominate the equations in time granted the scale \( e^B \) is increasing [17]. In this regime, p-brane sources can be self-consistently ignored (i.e. one can set \( T_p = 0 \)) and the only contribution to the energy momentum tensor comes from a cosmological constant, \( \rho_0 \), which will yield the usual exponential growth of the inflationary period [18] where all dimensions expand with the same exponent.

To support the above argument, we also made a simple numerical integration of the equations (19) in the proper time coordinate. We took \( p = 1 \), \( m = 3 \) and \( nT_p/\rho_0 = 10.0 \). Also the initial conditions for the run were such that near \( t = 0 \) the scale factors obey [13] and [14]. The resulting plot is presented in Fig. 1. Note that the inflationary regime sets in around \( C'' = 0 \). Before this, the internal dimensions contract. Consequently, even though the e-foldings after the cosmological constant dominates are the same, the internal dimensions exit the inflationary period with a smaller final value for the scale factor than that of the observed dimensions.

This difference depends on \( nT_p/\rho_0 \). For our numerical run, the ratios of the scale factors (i.e. \( e^B/e^C \)) can be read from the graph and one finds that the scale factor of the internal dimensions is about 6.8 times smaller than that of the observed dimensions. In solving (14) for other cases of \( \omega \) and \( \nu \), we start with the following ansatz

\[ B = b_1 \ln(t) + \ln b_2, \]

\[ C = c_1 \ln(t) + \ln c_2. \]

Using the last two equations in (19) one can uniquely determine these four constants and the first equation is satisfied identically. The constants which determine the power-law for expansion turn out to be

\[ b_1 = -\frac{2(1+\nu)}{m(1-2\omega+\nu)}, \]

\[ c_1 = \frac{2\omega}{p(1-2\omega+\nu)}. \]
Form (20), (21) and (22), one can also fix the metric function \( A \) using the gauge \( A = mB + pC \). Thus, all unknown functions in (11) are now determined. Making a coordinate transformation to switch to the proper time coordinate (which we again denote by \( t \)), the metric can be written as

\[
\mathrm{d}s^2 = -\mathrm{d}t^2 + (\alpha_1 t)\left[\mathrm{d}x^4 + \cdots + \mathrm{d}x^p + \mathrm{d}y^p \mathrm{d}y^p\right], \quad (23)
\]

where \( \alpha_1 \) and \( \alpha_2 \) depend on \( b_2 \) and \( c_2 \), and they can be set to 1 by scalings of \( x \) and \( y \) coordinates.

It is clear from (22) that one should choose \( \nu \neq -1 \). Also, for the above ansatz to work, \( b_2 \) and \( c_2 \) should be determined to be positive numbers. This imposes, after a straightforward but somewhat tedious algebra, the following conditions on the parameters of the model

\[
\frac{(1 + \nu)(1 - \nu)}{m} + \frac{\omega(1 - \omega)}{p} > (1 - \omega + \nu)(\omega - \nu) \quad (24)
\]

\[
\frac{(1 + \nu)(m - 2)}{m} > \frac{\omega(p + 1)}{p} \quad (25)
\]

\[
1 - 2\omega + \nu \neq 0. \quad (26)
\]

For pressureless dust \( \omega = \nu = 0 \), and these conditions are satisfied for all \( m > 2 \) and \( p \). In this case the scale factor for the extra dimensions is equal to 1; the expansion forced by the dust is compensated by the contraction forced by \( p \)-branes.

For radiation, two different equations of state are possible. When the sizes of all dimensions are close to each other one has \( \omega = \nu = 1/(m + p) \). In this case, (21) and (22) are satisfied identically. However, (25) restricts the possible values of \( m \) and \( p \). For instance, for \( m = 3 \) (25) imposes that \( p > 1 \). On the other hand, when the extra dimensions are very small compared to the observed dimensions one has \( \omega = 1/m \) and \( \nu = 0 \). In this case (21) and (20) are satisfied identically, but (25) imposes that \( mp > 3p + 1 \) and thus one should take \( m > 4 \).

Before closing this section let us emphasize that these constraints only show that the ansatz chosen in solving the differential equations is not valid for all parameters. However, one can still infer a general dynamical behavior from the above solutions. We believe that for the cases where (25) is not valid, one would observe similar effects for ordinary matter coupled to \( p \)-branes.

### IV. AN ESTIMATE FOR THE SIZE OF THE INTERNAL DIMENSIONS

Here we try to obtain an estimate for the current size of the extra dimensions along the lines of the model we presented. In this section we set \( m = 3 \). We take the universe to be filled with ordinary matter (characterized with an equation of state depending on \( \omega, \nu \)) and wrapped branes. We further assume that after big bang all dimensions started out close to Planck length. The standard model of cosmology tells us that the universe passed through three different eras. First, an inflationary period took place. After inflation, there was a radiation dominated era followed by a matter dominated one which still (possibly [19]) is going on. We assume that adding wrapped \( p \)-branes along the extra dimensions do not alter this history (but modify the power-law expansion as we discussed).

During inflation, one should take \( \omega = \nu = -1 \) which simply represents a positive cosmological constant. As we discussed in the previous section [19] gives the usual exponential growth (with a quantitative modification, see Fig. 1). Assuming approximately 70 e-foldings during inflation (which is required by cosmological phenomenology), the sizes of all dimensions grew to about \( 10^{-5} \)m from the Planck length. Note that we are not attempting to answer questions like what is deriving inflation or how the graceful exit occurs.

Just after the inflation during radiation dominated era one can set \( \omega = \nu = 1/(p + 3) \). Note that the temperature at the beginning of the radiation era is expected to be of the order of \( 10^{15} \)K which corresponds to a length scale \( 10^{-18} \)m. Since this is much smaller than \( 10^{-5} \)m, the radiation is allowed to apply pressure along the extra dimensions. In this period, the solution (25) can be used to describe the cosmological evolution of the universe [21]. Assuming that the vacuum energy driving the inflation was about \( 10^{14} \) GeV, the Hubble parameter of the inflation can be determined to be \( H \approx 10^{-35} \) sec. We match the solution (25) to de Sitter space by demanding that the Hubble parameter of the observed universe is continuous. Therefore, (25) is valid starting from \( t \approx 10^{-35} \) sec. Using that the radiation domination ended at about \( 10^{11} \) sec, (25) gives the size of the internal space at the end of radiation to be \( 10^{-5} \times 10^{-138}/[p(p+4)] \)m.

As the observed universe expands, pressureless matter started to dominate the cosmological evolution and one can then set \( \omega = \nu = 0 \). During this period, (25) implies that the extra dimensions stay fixed.

![Fig. 1: Evolution of the scale factors B and C, for \( \omega = \nu = -1, p = 1, m = 3 \) and \( nT_p/\rho_0 = 10.0 \). See text for the initial conditions.](image)
Summarizing, we found that after an exponential growth during inflation, the extra dimensions contracted till the end of radiation dominated era and then remained unaltered. This gives the following estimate for the present size \( r_p \) of the extra dimensions

\[
r_p \approx 10^{-5} \times 10^{-\frac{33}{n(p+q)}} \text{ m.} \tag{27}
\]

For \( p = 1 \) we get \( r_1 \approx 10^{-33} \text{ m.} \) For higher values of \( p \) one finds much larger estimates. However let us remind the reader of the digression we had on the structure of the inflationary scenarios. In reality, during the inflationary period the scale factor of the internal dimensions will be subject to a lesser growth than that of the observed dimensions. For instance, for the numerical case we have considered, \( r_p \) should be scaled down by a factor of 6.8. Recall that this is for \( nT/p_0 = 10.0 \). Larger values for that ratio will make \( r_p \) smaller.

V. MORE ON \( p \)-BRANE COSMOLOGY

In section II we found that in the presence of uniformly distributed wrapped \( p \)-branes, the observed space expands while the compact space contracts. In this section, we would like to compare relative strengths of these two effects by assuming existence of two branes where they will be uniformly distributed wherever they don’t wrap. Let us consider the following metric in \( 1 + m + p + q \)-dimensions

\[
\begin{align*}
\text{ds}^2 &= -e^{2A}dt^2 + e^{2B}dx^i dx^i + e^{2C}dy^a dy^a + e^{2D}dz^a dz^a,
\end{align*}
\]

where \( a_1, b_1 = 1, \ldots, p \), \( a_2, b_2 = 1, \ldots, q \) and the metric functions depend only on time \( t \). We assume that in (28) there are \( p \) and \( q \) dimensional branes wrapping over \( y \) and \( z \) coordinates, respectively. Following our reasonings in section III one can easily write down the energy momentum tensor corresponding to this configuration

\[
\begin{align*}
T_{ii} &= T_p e^{-mB_qD} + T_q e^{-mB_pC}, \\
T_{ij} &= 0, \\
T_{a_i b_1} &= -T_p e^{-mB_qD}, \\
T_{a_2 b_2} &= -T_q e^{-mB_pC},
\end{align*}
\]

where \( T_p \) and \( T_q \) are respective brane tensions and hatted indices refer to the orthonormal basis in (28). At this point one can check that \( \nabla_{\mu} T^{\mu \nu} = 0 \). Imposing the gauge \( A = mB + pC + qD \) the Einstein equations can be written as

\[
\begin{align*}
A'' - A'^2 + mB'^2 + pC'^2 + qD'^2 &= C'' + D'' - B'', \\
(m + p + q - 1)B'' &= \kappa^2 e^{mB_pC_qD} \left[ (p + 1)n_pT_p e^{pC} + (q + 1)n_qT_q e^{qD} \right], \\
(m + p + q - 1)C'' &= \kappa^2 e^{mB_pC_qD} \left[ -(m + q - 2)n_pT_p e^{pC} + (q + 1)n_qT_q e^{qD} \right], \\
(m + p + q - 1)D'' &= \kappa^2 e^{mB_pC_qD} \left[ (p + 1)n_pT_p e^{pC} - (m + p - 2)n_qT_q e^{qD} \right],
\end{align*}
\]

where \( n_p \) and \( n_q \) are number of branes per co-moving volumes parameterized by \((x, z)\) and \((x, y)\) coordinates respectively. To solve these equations exactly we first note that the last three equations in (30) imply that

\[
(m - 3)B + (p + 1)C + (q + 1)D = 0 \tag{31}
\]

up to the terms linear in \( t \) which we ignore in the following. Using (31), the middle two equations can be solely expressed in terms of \( B \) and \( C \). To find a power-law solution we choose

\[
\begin{align*}
B &= b_1 \ln(t) + \ln b_2, \\
C &= c_1 \ln(t) + \ln c_2,
\end{align*}
\]

It is now straightforward to check that the four constants can be determined uniquely. For instance we find

\[
\begin{align*}
b_1 &= \frac{-2(p + q + 2pq)}{9pq + m(p + q - pq)}, \\
c_1 &= \frac{2(m - 3)q}{9pq + m(p + q - pq)}.\tag{35}
\end{align*}
\]

Using (31) and the gauge choice \( A = mB + pC + qD \), one can also determine metric functions \( A \) and \( D \). The first equation in (30) is then to be checked for consistency. A straightforward calculation shows that it is satisfied identically. Let us also emphasize that, contrary to the case we encountered in section III, the constants \( b_2 \) and \( c_2 \) turn out to be positive for all possible values of \( m, p \) and \( q \).

Using the above results, in the proper time coordinate the metric can be written as

\[
\text{ds}^2 = -dt^2 + R_m^2 dx^i dx^i + R_p^2 dy^a dy^a + R_q^2 dz^a dz^a, \tag{36}
\]

where

\[
\begin{align*}
\ln(R_m) &= \frac{2(p + q + 2pq)}{3pq + m(p + q + pq)} \ln(\alpha_m t), \\
\ln(R_p) &= -\frac{2(m - 3)}{3pq + m(p + q + pq)} \ln(\alpha_p t), \\
\ln(R_q) &= -\frac{2p(m - 3)}{3pq + m(p + q + pq)} \ln(\alpha_q t)
\end{align*}
\]
and \(\alpha_m\), \(\alpha_p\) and \(\alpha_q\) are positive constants. Comparing \(37\) with the results of section II, we see that wrapping a \(p\)-brane and a \(q\)-brane is physically distinct from wrapping a \(p + q\)-brane. The main difference is that in the former case the \(p\)-branes are uniformly distributed along the dimensions over which \(q\)-branes wrapped and vice versa.

Another interesting aspect is that only the dimension of the observed space \(m\) determines whether the compact dimensions expand or contract. For the two brane configuration considered above we see that \(m = 3\) is the critical value where the internal dimensions are stabilized by the cancellation of the expansion forced by the uniformly distributed \(p\)-branes with the contraction forced by the wrapping of \(q\)-branes. We find that the compact dimensions expand for \(m > 3\) and they diminish for \(m < 3\).

Generalizing \(11\) and \(37\), one would guess that the exponent of the power-law of the internal dimensions is proportional to \(m - k - 1\) where \(k\) is the number of partitionings of the extra dimensions, i.e. number of distinct brane configurations. For large \(k\), one should increase \(m\) to have contracting extra dimensions.

\[\text{VI. CONCLUSIONS AND FUTURE DIRECTIONS}\]

We have shown that if one allows for \(p\)-branes wrapping around extra dimensions Einstein equations allow for solutions where their size diminishes during cosmological evolution. In this paper, we have focused on the main aspects of the idea and omitted various details which may form topics for future studies.

First, since the \(p\)-branes allow for power-law solutions for the observed universe which are exactly the same as pressureless matter one can think of them as a form of dark matter.

Second, it is an important question to ask about the time of forming of wrapped \(p\)-branes during cosmological evolution. In this work we have assumed that they did exist since the beginning of time but it is plausible that they may form later. One important question along this line is about the connection of the wrapped branes to the exit from inflation.

Let us also recall that Brandenberger and Vafa argued that \[1\] string interactions may yield an upper bound \((m \leq 3)\) on the number of observed dimensions if one assumes that all the directions are compact. We note in this context that the two brane scenario we have discussed in section V yields a lower bound for \(m\) if one insists on preventing expansion of the extra dimensions during entire cosmological history. One is tempted to speculate that the two ideas might be merged to fix \(m\).

Finally, one could further think of adding ordinary matter to the configuration discussed in section V. This is expected to make the internal dimensions grow for \(m \geq 3\). Of course a detailed numerical analysis might prove otherwise. One further extension could be to wrap branes over branes. In this case we expect the effects to add up in the directions the branes intersect. And this might result in a hierarchy of scale factors.

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[1] Branderberger R. and Vafa C., Nucl. Phys. B316, 391 (1989).
[2] Nishimura H. and Tabuse M., Mod. Phys. Lett. A2, 299 (1987).
[3] Kripfganz J. and Perlt H., Class. Quant. Grav. 5, 453 (1988).
[4] Tseytlin A. A. and Vafa C., Nucl. Phys. B 372, 443 (1992).
[5] Tseytlin A. A., Class. Quant. Grav. 9, 979 (1992).
[6] Cleaver G. B. and Rosenthal P. J., Nucl. Phys. B 457, 621 (1995).
[7] Park C., Sink S. and Lee S., Phys. Rev. D 61, 083514 (2000).
[8] Alexander S., Branderberger R. H. and Easson D., Phys. Rev. D 62, 103509 (2000).
[9] Easson, D.A., hep-th/0110225.
[10] Easson, D.A., hep-th/0111055.
[11] Branderberger R. H., Easson D. A. and Kimberly D., Nucl. Phys. B 623, 421 (2002).
[12] Easther, R., Greene B. R., and Jackson M. G., Phys. Rev. D 66, 023502 (2002).
[13] Watson, S., Branderberger R. H., hep-th/0207168.
[14] Boehm T. and Branderberger R., hep-th/0208188.
[15] Easther, R., Greene B. R., Jackson M. G. and Kabat D., hep-th/0211124.
[16] Alexander S. H. S., hep-th/0212151.
[17] Note that both the cosmological constant and wrapped \(p\)-branes force \(B\) to increase. This argument would fail if, for instance, we would have the function \(C\) instead of \(B\) since the cosmological constant and the \(p\)-branes have opposite effects on \(C\).
[18] To see this more explicitly, set \(T_p = 0\) in \(13\). Then, it is easy to show that \(A = -\ln(Ht)\) and \(B = C = -\ln(Ht)/(m + p)\) is a solution where \(H^2 = 2\kappa^2\rho_0(m + p)/(m + p - 1)\). Making a further coordinate transformation, one can see that \(1\) represents de Sitter space.
[19] Recent observations indicate that the universe is accelerating which suggests that the energy density is now dominated with some kind of undetermined energy (dark energy) having negative pressure. In this paper we are not going to discuss possible modifications implied by the existence of dark energy to our scenario.
[20] Since in \(20\) the internal space contracts, the equation of state for radiation also starts to shift. As the extra dimensions become much smaller than the scale set by the temperature, one should set \(\omega = 1/3\) and \(\nu = 0\). We ignore this subtlety in the following since we try to obtain a rough estimate to see if the scenario makes sense in a first crude approximation.