Method Article

Determination of contact force by compression testing of cylindrical specimens

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A B S T R A C T

This paper presents a method for determining values of dynamic parameters of the Hunt and Crossley model in order to estimate the amount of force generated at the point of contact (contact force) in an impact. A two-degree-of-freedom lumped-mass-system based on a non-linear visco-elastic model as proposed by Hunt and Crossley has been widely used to accurately model contact force. The primary difficulty associated with the Hunt and Crossley contact force model is the need to determine the unknown dynamic parameters of the model, which can be obtained by calibrating the model against results from high-speed impact experiments. Spherical impactors have to be placed in the gas-gun barrel for accelerating onto the target specimen. An innovative and inexpensive method proposed in this paper describes the use of compression testing on a test rig employing cylindrical specimens of colliding bodies to obtain the dynamic parameters thereby waiving away the need of costly and time-consuming impact experiments.

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A R T I C L E  I N F O

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The proposed methodology only requires compression testing of cylindrical specimens of the colliding objects to determine the contact stiffness properties.

- Cylindrical specimens which are commonly used for material testing can easily be employed for testing without the need of impactors machined into spherical objects.
- The amount of contact force can be modelled reliably and accurately without the need of conducting any impact tests.

**Specifications Table**

| Subject Area: | Engineering |
|---------------|-------------|
| More specific subject area: | Structural dynamics, Impact loading |
| Method name: | Compression testing-based contact force determination methodology |
| Name and reference of original method: | Puttock MJ. Elastic compression of spheres and cylinders at point and line contact, by M. J. Puttock and E. G. Thwaite. Thwaite EG, editor. Melbourne: Commonwealth Scientific and Industrial Research Organization; (1969). Hunt KH, Crossley FRE, Coefficient of restitution interpreted as damping in vibroimpact, Journal of applied mechanics. 42 (1975), doi:10.1115/1.3432596. Majeed ZZA, Lam NTK, Lam C, Gad E, Kwan JSH, Contact force generated by impact of boulder on concrete surface, International Journal of Impact Engineering. (2019) 103324. [https://doi.org/10.1016/j.ijimpeng.2019.103324](https://doi.org/10.1016/j.ijimpeng.2019.103324). |
| Resource availability: | EXCEL [https://products.office.com/en-au/excel](https://products.office.com/en-au/excel) MATLAB [https://www.mathworks.com/products/matlab.html](https://www.mathworks.com/products/matlab.html) |

**Method details**

Localised damage caused to a structure in the extreme events of rockfalls, landslides, hailstorms, and windstorms by impact of a moving, or falling, object is controlled by the magnitude of force generated at the point of contact which is known as contact force [1,2]. The magnitude of the contact force can be estimated by employing a well-established two-degree-of-freedom (2DOF) lumped-mass system which incorporates a non-linear visco-elastic model as proposed by Hunt and Crossley and defined by Eq. (1a) [3]. The damping coefficient $D_n$ can be determined using Eq. (1b) as derived by Sun et.al [4].

$$F_c = k_n\delta^p + D_n\delta^p \dot{\delta}$$  \hspace{1cm} (1a)

$$D_n = (0.2p + 1.3)\left(\frac{1 - \text{COR}}{\text{COR}}\right)\frac{k_n}{\delta_0}$$  \hspace{1cm} (1b)

where $F_c$ is the contact force (in units of N), $\delta$ is the indentation (in units of m), $p$ is the exponent, $k_n$ is the contact stiffness (in units of N/m$^p$), COR is the coefficient of restitution and $\delta_0$ is the initial indentation velocity (in units of m/s).

Determination of the Hunt and Crossley model dynamic parameters $k_n$, $p$ and COR in Eq. (1) employing high-speed impact experiments has been well documented in the literature in relation to windstorm debris and hailstone hazards [1,2]. The dynamic parameters have been derived by calibrating the 2DOF model against results obtained from high-speed impact experiments involving the use of a gas-gun equipment and spherical impactors. Limitations of this method are the unavailability of impact test apparatus (in a standard mechanical testing facility) and the need to machine impactors into spheres which is labour intensive and time consuming.

The innovative methodology proposed in this paper for determining the dynamic parameters ($k_n$, $p$ and COR) requires only static testing of cylindrical specimens for estimating the contact force. Compression tests are usually carried out in a material laboratory using cylindrical specimens to determine the mechanical properties of the materials (as per guidelines provided in the standard
codes of practices) [5,6]. Hence, this method is much more convenient to use as there is not a need for impact testing nor machining impactor specimens into spheres.

**Materials**

In the present study, granite rock cores (density = 2700 kg/m³, Young’s modulus = 80 GPa and Poisson’s ratio = 0.24) and concrete cylinders (Grade 30) were used as specimens for studying the condition of contact between rock and concrete in a rockfall.

**Experimental setup**

In the proposed method of testing, a cylindrical specimen of the impactor material is sandwiched in between a pair of half-cylindrical specimens of the target material on a test rig for compression testing. This setup is based on a technique introduced by Puttock [7]. A schematic diagram of the experimental setup is shown in Fig. 1.

**Procedure for determination of dynamic parameters** $k_n$, $p$ and COR

Although the methodology is applicable for various types of impact scenarios, the impact of a fallen granite boulder on a concrete surface has been taken as a case study to elaborate the steps to be followed. A step-by-step guide to obtain the values of the dynamic parameters and contact force is described in below:

**Step: 1 specimen preparations**

Cylindrical samples of the impactor and target objects are first prepared by coring or by moulding as appropriate. Samples from boulders are obtained by coring whereas concrete samples can either be moulded (adopting the same mixture as used in the structure) or cored directly from the structure.

In the current study, cylindrical samples were cored from the boulder. A photograph of coring of rock samples using a diamond core barrel is shown in Fig. 2(a). Samples cored from the boulders are shown in Fig. 2(b). Grade 30 concrete cylinders were cast to obtain samples of the target material. Concrete cylinders were cut into halves along the diameter as illustrated by Fig. 3. Specimens of the granite boulder and concrete used for compression testing are shown in Fig. 4. The diameter and length of granite core are 83 mm, and the diameter and length of concrete are 100 mm and 50 mm respectively.

![Fig. 1. Schematic diagram of the experimental setup.](image-url)
Size of the specimens should be selected in such a way that they can be fitted on the platens of the compression testing machine. Following may be considered when choosing the size of the specimens.

i Sum of the diameter of the cylindrical specimens of the colliding objects (e.g. granite core and concrete) should be within the allowable maximum clearance between the compression platens.

ii The diameter of the target specimen (concrete) should be approximately equal to the diameter of the platens to uniformly distribute loads onto the specimens.

**Step:2 Compression testing of cylindrical specimens**

In the proposed testing method, an 83 mm dia. cylindrical boulder specimen (the impactor) is sandwiched in between a pair of 100 mm dia. half-cylindrical concrete specimens (the target). The test rig for compression testing is shown in Fig. 5. A quasi-static load is then applied at different rates of loading in a displacement-controlled manner to obtain the force-displacement relationship on contact.

**Step:3 applying correction factors**

Three different factors need to be considered prior to the calibration of the dynamic parameters. Pre-requisite of these factors are checked and applied to the force-displacement curve where it is necessary. The method for obtaining the values of the factors are presented in below.
a Size effect in between the specimens:

A factor \( \beta \) is applied to the displacement values of the experimental curve when the diameter of the cylindrical specimen (representing the impactor) is different to the diameter of the half-cylindrical specimens (representing the target). This factor need not be applied if the diameter of the specimens are identical. Expression to determine the factor is provided by Eq. (2).

\[
\beta = K \left( \frac{2D_2}{\pi^2D_1 \left( -\frac{1}{\varepsilon} \frac{d^2f}{d\varepsilon^2} \right)} \right)^{1/3}
\]  

(2)

where \( D \) is the diameter of the cylinder (subscripts 1 and 2 refer to the larger and smaller diameter cylinders respectively). Values of parameters \( K \) and \(-\frac{1}{\varepsilon} \frac{d^2f}{d\varepsilon^2}\) are dependent on the ratio \( \frac{D_2}{D_1} \) [7]. Detailed description of the parameters and derivations of the equations can be found in [7]. For any given value of \( \frac{D_2}{D_1} \) in the range 1 to 0.0000001, the corresponding values of \( K \) and \(-\frac{1}{\varepsilon} \frac{d^2f}{d\varepsilon^2}\) can be obtained from [7], the value of factor \( \beta \) can be calculated accordingly.

For the test scenario considered in this paper, \( \frac{D_2}{D_1} = \frac{83}{100} = 0.83 \), the values obtained for \( K \) and \(-\frac{1}{\varepsilon} \frac{d^2f}{d\varepsilon^2}\) are 1.6698 and 0.8608 respectively [7]. Thus, \( \beta = 0.97 \) according to Eq. (2).

a Boundary conversion factor:

The boundary conditions imposed by the loading platens in the test rig on the cylindrical specimens (Fig. 6a) are different to that of the real scenario of impact between the two materials
Forces generated at the point of contact are indicated by arrows in Fig. 6. A boundary conversion factor would need to be applied to allow for this difference.

The boundary conversion factor (for granite-concrete in contact) has been derived as 1.8 based on finite element simulations [8]. The force–displacement relationships that have been obtained from compression testing are adjusted accordingly by the derived boundary conversion factor to simulate the force–displacement relationships for impact conditions. Boundary factors are expected to vary between 1.7 to 2.5 for different types of colliding materials.

a Dynamic factor:

Compression testing machines which operate only at a lower rate of loading (50 mm/min) than that of actual impact conditions would require a further adjustment to the force–displacement curve. This factor of increase is denoted as the “Dynamic factor”. The “Dynamic factor” can be obtained by comparing results based on varying the rate of loading [8] (Fig. 7). The relationship shown in Fig. 7 is based on recorded experimental results employing granite-concrete specimens [8]. Dynamic factor for scenarios other than granite-concrete impact is recommended to be derived separately following the procedure illustrated in [8]. The maximum velocity has been limited to 20 m/s in the paper as the experiments and simulations were carried out for rockfall impact scenarios which typically fall in the selected range. Hence, the derived relationship is recommended to be applied for low velocity impact scenarios.

Fig. 8 illustrates the application of the boundary factor and the dynamic factor for an impact velocity of 8 m/s for granite-concrete impact [8]. As illustrated in Fig. 8, a boundary conversion of 1.8 is applied to the experimental curve. A dynamic factor of 1.75 (Fig. 7) is then applied to modify the curve to allow for the rate of loading corresponding to an impact velocity of 8 m/s. This adjusted curve is used in the next step for calibrating the dynamic parameters.
Step: 4 Calibration of dynamic parameters

Values of parameters characterising the compressive stiffness properties \((k_n \text{ and } p)\) is determined by curve-fitting the simulated force-displacement relationships. The other parameter \((D_n)\) in the Hunt and Crossley model (Eq. 1) can be related to the coefficient of restitution (COR) \([4]\), the value of which can in turn be linked to the area \((A)\) which is shaded in grey as shown in Fig. 9.
Fig. 9. Area under the force-displacement curve obtained from compression test at 2.5 m/s [8].

Table 1
Step by step guide of the proposed methodology.

| Step               | Task                                                                 |
|--------------------|----------------------------------------------------------------------|
| 1-Sampling         | • Obtain cylindrical specimens cored from the impactor (eg: boulder). |
|                    | • Obtain cylindrical specimen of the target material (eg: concrete, used for constructing rigid barriers on the hill slope). Dissect the cylinder into halves. |
| 2-Compression testing | • Setup the cylindrical specimens on a test rig in the manner as shown in Fig. 1. |
|                    | • Apply a quasi-static load on the test rig to obtain a curve representing the force-displacement relationship on contact. |
| 3-Correction factors | • Apply correction factors to make adjustments to the force-displacement relationship. |
| 4-Calibrating dynamic parameters | • Find \( k_n, p \) by curve-fit. |
|                    | • Estimate the value of COR from the force-displacement relationship as per Eq. (3). Any combination of assumed values of \( k_n, p \) and COR would give an estimate for the damping coefficient \( D_n \). |
| 5-Estimation of contact force | • The time-history of the contact force (i.e. the forcing function) can accordingly be simulated for any given impact scenario by employing the numerical procedure presented in Ref. [9]. |
|                    | • Maximum contact force can be estimated using Eq. (4). |

Fig. 10. Comparison of contact forcing function for 100 mm Dia. granite sphere impacting concrete from impact experiment, FE analyses and use of 2DOF model at impact velocity of 6.3 m/s.
Energy dissipated by viscous damping = Area of the force-displacement curve

\[ \text{COR} = \frac{-A(p + 1) + \sqrt{[A(p + 1)]^2 + 4\left(k_n\delta_m^{p+1}\right)^2}}{2k_n\delta_m^{p+1}} \]  

Eq. (3) which can be used for calculating the value of COR (hence \(D_n\)) for a given value of \(A\) and maximum shortening (\(\delta_m\)). For example, when \(A = 0.4\) Nm and \(\delta_m = 0.0003\) m at an impact velocity of 2.5 m/s (Fig. 9), results of the calibration are \(k_n = 370\) MN/m\(^{1.355}\), \(p = 1.355\) and COR = 0.78. The curve fitting can be operated using the Excel Solver.

**Step:5 Estimation of contact force**

Once the dynamic parameters (\(k_n\), \(p\) and COR) are obtained, the 2DOF model can be executed on Excel spreadsheet or on MATLAB to model the contact forcing function. Details of the program algorithm and demonstrations of its use can be found in Ref. [9]. Maximum contact force can directly be calculated using Eq. (4) [1].

\[ F_c = k_n\left(\frac{p + 1}{2k_n^p m_1 v_0^2 \text{COR}}\right)^{\frac{2}{p+2}} \left[1 + (0.2p + 1.3)\left(\frac{1 - \text{COR}}{\text{COR}}\right)\left(-b + \frac{b^2 + 4c}{2}\right)\right] \left[1 - \left(-b + \frac{b^2 + 4c}{2}\right)^{\frac{p-\text{COR}}{p+2}}\right] \]  

where \(m_1 = \text{mass of the impactor}, v_0 = \text{initial impact velocity}, b = \frac{p-\text{COR}}{(p+2)(0.2p + 1.3)(1-\text{COR})^2}\) and \(c = \frac{2}{p+2}\).

**A summary of the procedure**

A summary of the proposed methodology is presented in Table 1.

**Method validation**

The proposed methodology to obtain the dynamic parameters of the 2DOF contact force model employing quasi-static tests has been validated in Ref. [8] by comparing with impact experimental data and FE simulated results. The key parameters (\(k_n\), \(p\) and COR) so obtained from the compression tests are used for modelling the contact forcing function based on the use of a 2DOF model. An example is illustrated in below.

**Example:** A 100-mm diameter granite impactor is made to strike a miniature (Grade 30) concrete slab specimen of dimensions: 300 mm × 300 mm × 40 mm at an impact velocity (\(V_0\)) of 6.3 m/s. The dynamic parameters (\(k_n\), \(p\) and COR) obtained from the compression testing methodology were \(k_n = 714\) MN/m\(^{1.503}\), \(p = 1.503\) and COR = 0.32. The modelled contact force is then compared with experimental measurements in conjunction with results from FE simulations. Close agreement between results from physical testing and prediction from use of the 2DOF model (adopting parameter values from compression testing) and with finite element simulations, is well demonstrated in Fig. 10. Input values used in the 2DOF model are:

\(V_0 = 6.3\) m/s

\(m_1\) (mass of the granite impactor) =1.4 kg

\(m_2\) (equals to 0.25 times of the total mass of the slab) = 2.2 kg [10]

\(k_2\) (stiffness of the target slab) = 2760 kN/m [10]

\(k_n = 714\) MN/m\(^{1.503}\)

\(p = 1.503\)

COR = 0.32
Acknowledgements

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