Charged analogues of isotropic compact stars model with Buchdahl metric in general relativity

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Abstract In this work, we examine a spherically symmetric compact body with isotropic pressure profile. In this context we obtain a new class of exact solutions of Einstein-Maxwell field equation for compact stars with uniform charged distributions on the basis of Pseudo-spheroidal space-time with a particular form of electric field intensity and the metric potential $g_{11}$. Taking these two parameters into account further examination has been done to decide unknown constants and to depict several compact strange star candidates. By the isotropic Tolman-Oppenhimer-Volkoff (TOV) equation, we explore the equilibrium among hydrostatic, gravitational and electric forces. Then, we analyze the stability of the model through adiabatic index ($\gamma$) and velocity of sound ($0 < \frac{dp}{c^2 d\rho} < 1$). We additionally talk about other physical features of this model e.g. pressure, redshift, density, energy conditions and mass-radius ratio of the stars in detail and demonstrated that our results satisfied all the basic prerequisites of a physically legitimate stellar model, showing density, pressure, pressure-density ratio, redshift and speed of sound are monotonically decreasing. The outcomes acquired are valuable in exploring the strength of other compact objects like white dwarfs, gravastars and neutron stars. Finally, we have shown that the obtained solutions are compatible with observational data for compact objects.

Keywords Isotropic fluids · Electric intensity · Reissner-Nordstrom metric · Compact star · General relativity

1 Introduction

From the time of Sir Isaac Newton, our comprehension about gravity has dramatically progressed, however, mysteries still remain in physics. Einstein’s theory of General Relativity (GR) is one of the best essential speculations of gravity in physics. In spite of its success, numerous expansions of the first Einstein condition has been researched to meet the present observational information on both cosmological and astrophysical scales quite satisfactorily. Observational information has educated us that the universe is experiencing a period of quickened extension. The microscopic structure and properties of a dense matter on phenomenal conditions are necessary to examine for compact objects. In a theoretical sense, stars are confined in gas and dust clouds with non-uniform matter circulation and scattered all through general cosmic systems. In astrophysics, compact objects are typically alluded to white dwarfs or neutron stars.

Also, the high exactness information from Type Ia supernovae (Betoule et al. 2014), the cosmic microwave background anisotropies (White et al. 1994), baryonic acoustic motions (Alam et al. 2016) and from gravitational lensing are helping us to construct more efficient stellar models. The information appears to show that the universe is directly overwhelmed by two obscure components, viz. pressure-less dark matter (DM) and dark energy (DE). Now one can think of incorporating and extending results derived for charged compact stars with isotropic profile in spherically symmetric space-time to the realm of dark matter and dark energy with effective modeling of spherically symmetric black holes. Zhang et al. (2014) and Heydari-Fard et al. (2007) obtained an interesting result for a static spherically symmetric black hole with dark matter. This is in light of the fact that at such uncommon densities, nuclear matter may include nucleons and leptons just as a couple of fascinating segments in their
different structures for example, mesons, hyperons, baryon resonances and as strange quark matter (SQM). Be that as it may, it is as yet impractical to find a far-reaching portrayal of the very dense matter in a firmly cooperating system. To get a clear understanding about the definite composition and the behavior of atoms inside such objects, the spherical symmetry of space-time provides the initial structure to study them that paves the way for further investigations. In context of general theory of relativity, a broadly pursued course is to indicate an equation of state and afterward solve the Einstein field equations for the study of the composition of a compact star. This is also useful for Tolman-Oppenheimer-Volkoff (TOV) equation (see Tolman 1939; Oppenheimer and Volkoff 1939) or the condition of hydro dynamical equilibrium. It is conceivable that anisotropic matter is a significant fixing in numerous astrophysical objects for example, stars, gravastars, and so on. Generally, extensive exertion has been committed to pick up a far reaching understanding of properties of the anisotropic matter with the expectation of delivering physically suitable models of compact stars.

Most of the compact stars are divided between a strange star and a normal neutron star. Many authors (Usov 2004; Usov et al. 2005; Negreiros et al. 2010) studied that the strange stars possess ultra-strong electric fields on their surfaces. The impact of energy densities on ultra-high electric fields of compact stars was investigated in Ray et al. (2005), Malheiro et al. (2004), Weber et al. (2007, 2009, 2010), Negreiros et al. (2009). It has been additionally demonstrated that the electric fields adds to the stellar mass by up to 30% depending upon the quality of it. As opposed to the strange star, the surface electric field of a neutron star is absent (Leeuwen 2012). Based on these properties, one needs to observe and recognize quark stars from neutron stars.

The important characteristic of many astrophysical objects, like compact stars, gravastars etc., is isotropic matter. In an extreme gravitational environment, compact stars provide information about the gravitational interactions. The extreme internal density and strong gravity of compact stars indicate that the pressure within such objects has two different types, namely the radial and tangential pressure, and these are to be equal. These information of isotropic matter may produce physically valid models of compact stars. Bowers and Liang (1974) have noticed the structure and evolution of relativistic compact objects in general relativity. They investigated the changes in the gravitational mass and surface redshift by generalisation of the equation of hydrostatic equilibrium and obtained a static spherically symmetric configuration. Ruderman (1972) analyzed that the nature of the nuclear matter at high densities of order $10^{15}$ g/cm$^3$ turns into anisotropic matter. Also, they pointed out that the radial pressure may not be equal to the tangential one in massive stellar objects. Based on above physical condition, many contentions have been presented for the presence of anisotropy in star models. For example, by the presence of type 3A superfluid (Kippenhahn and Weigert 1990), various types of phase transitions (Sokolov 1980), mixture of two fluids, the presence of solid core or by other different physical marvels. Also exact solution of Einstein’s field equations, first obtained by Karl Schwarzschild (1916), is important for the study of astrophysical objects, as many exact solutions of Einstein’s field equations have been found but only few were able to satisfy all the physical plausibility conditions. This shows the complexity in getting exact solutions of Einstein’s field equations describing physically realizable astrophysical objects. Charged stars on spheroidal space-time have been studied by Patel and Kopper (1987), Sharma et al. (2001), Gupta and Kumar (2005), Komathiraj and Maharaj (2007). Many projects are suggested by Ivanov (2002) for constructing charged fluid spheres. Recently, Bijalwan and Gupta (2011a,b) for all $K$ except for $0 < K < 1$ and J. Kumar et al. (Gupta and Pratibha 2011; Kumar and Gupta 2013, 2014; Kumar et al. 2018b,a) for $0 < K < 1$ have been obtained perfect fluid charged analogue models with a specific electric intensity. These informations explicitly demonstrated that the models really compared to strange stars in their mass-radius and electric intensity. Many authors estimated the masses for various stars. The SAX J1808.4-3658 has a mass of $0.9 \pm 0.3 M_\odot$ (Ebleert et al. 2009). Abubekerov et al. (2008) reported the mass of Her X-1 to be $0.85 \pm 0.15 M_\odot$. Rawls et al. (2011) reported the mass of 4U 1538-52 to be $0.87 \pm 0.07 M_\odot$, Cen X-3 to be $1.49 \pm 0.08 M_\odot$ and SMC X-1 to be $1.29 \pm 0.05 M_\odot$. De Morest et al. (2010) reported the mass of PSR J1614-2230 to be $1.97 \pm 0.04 M_\odot$. Guver et al. (2010) reported the mass of 4U 1608-52 to be $1.74 \pm 0.14 M_\odot$.

In the present problem, we have constructed a charged fluid sphere starting with a specific metric potential $g_{11}$ and generalized charge intensity. Delgaty and Lake (1998) and Pant et al. (2010) have proposed that the physically valid solution in curvature coordinates, the following conditions should be satisfied

1. At the boundary $r = a$, pressure $p$ should be zero.
2. $c^2 p$ should always be grater than $p$ within the range $0 \leq r \leq a$.
3. The pressure gradient $dp/dr$ should be negative for $0 < r \leq a$, i.e., $(dp/dr)_{r=0} = 0$ and $(d^2 p/dr^2)_{r=0} < 0$.
4. The density gradient $d\rho/d\sigma$ should also be negative for $0 < r \leq a$, i.e., $(d\rho/d\sigma)_{r=0} = 0$ and $(d^2 \rho/dr^2)_{r=0} < 0$.

These two conditions state that the pressure and density should be decreasing towards the surface (see Fig. 1).
5. The velocity of sound should not exceed the speed of light i.e., $(dp/c^2 dp/d\sigma)^{1/2} < 1$.
6. The adiabatic constant $\gamma = ((\beta + p)/\rho) ((dp/d\rho)) > 4/3$, is condition for stability of a fluid sphere.
7. The gravitational redshift $Z_\delta$ should be positive and finite.
These conditions along with positive density and positive pressure are the most important features for characterizing a star with isotropic matter. We plot Figs. 1–6 to check the well-behaved geometry and capability of describing realistic stars. Our stellar model which depends on the different values of $K$, $\eta$, $b$, and $C_5^2$ cannot be extended to anisotropic profile since density becomes negative within the specified range of $K$. Such analytical representations have been performed by using recent measurements of mass and radius of neutron stars, PSR J1614-2230, 4U 1608-52, SAX J1808.4-3658, 4U 1538-52, SMC X-1, Her X-1 and Cen X-3.

### 2 Einstein’s field equations

Let us consider the static spherically symmetric metric in curvature coordinates

$$ds^2 = -e^{\lambda(r)}dt^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2) + e^{\nu(r)}dr^2$$

where $\lambda(r)$ and $\nu(r)$ satisfy the Einstein-Maxwell equation for charged fluid distribution

$$R^j_i - \frac{1}{2} R \delta^j_i = -\kappa \left[ (c^2 \rho + p) u^i u_j - p \delta^j_i \right]$$

$$+ \frac{1}{4\pi} \left( -F^{im} F_{jm} + \frac{1}{4} \delta^j_i F_{mn} F^{mn} \right)$$

with $\kappa = \frac{8\pi G}{c^4}$ while $\rho$, $p$, $u^i$ denote matter density, fluid pressure and the unit time-like flow vector respectively and $F_{ij}$ denote the skew symmetric electromagnetic field tensor.

In view of Eq. (1), the Eq. (2) reduce in Landau and Lifshitz (2013) the following form

$$\frac{\nu'}{r} e^{-\lambda} - \frac{(1 - e^{-\lambda})}{r^2} = \kappa p - \frac{q^2}{r^4},$$

$$\left( \frac{\nu''}{2} - \frac{\lambda' \nu'}{4} + \frac{\nu' \nu'}{4r} + \frac{\nu' - \lambda'}{2r} \right) e^{-\lambda} = \kappa p + \frac{q^2}{r^4},$$

$$\frac{\lambda'}{r} e^{-\lambda} - \frac{(1 - e^{-\lambda})}{r^2} = \kappa c^2 \rho + \frac{Q^2}{r^4}$$

where ($'$) prime denotes the differentiation with respect to $r$ and

$$q(r) = 4\pi \int_0^r \sigma r^2 e^{\lambda/2} dr = \int_0^r \sqrt{-F_{14}F^{14}}$$

$$= r^2 F^{41} e^{(\lambda+\nu)/2}$$

where $q(r)$ is the total electric charge contained within the sphere of radius $r$, which does not depend on the timelike coordinate $t$ and $\sigma$ is the charge density. Additionally, the component $F_{14} \neq 0$.

In this model, we propose a charged fluid distributions by considering the generalized electric field intensity against Kumar et al. (2018b)
Behavior of different forces (in km$^{-3}$) vs. fractional radius r/a for the compact objects PSR J1614-2230, 4U 1608-52, SAX J1808.4-3658, 4U 1538-52, SMC X-1, Her X-1 and Cen X-3. For this figure we have used the numerical values of physical parameters and constants are as follows: (i) $K = 0.0000135$, $b = 0.09$, $C = -7.455 \times 10^{-8}$ km$^{-2}$, $\eta = 6$, $M = 0.97$ $M_\odot$ and $a = 9.69$ km for PSR J1614-2230 (first row left), (ii) $K = 0.0000209$, $b = 0.0903$, $C = -1.1446 \times 10^{-7}$ km$^{-2}$, $\eta = 4.9$, $M = 0.74$ $M_\odot$, and $a = 9.3$ km for 4U 1538-52 (second row left), (iii) $K = 0.0000319$, $b = 0.09$, $C = -1.5664 \times 10^{-7}$ km$^{-2}$, $\eta = 3.9$, $M = 0.9$ $M_\odot$, and $a = 7.951$ km for SAX J1808.4-3658 (first row right), (iv) $K = 0.0000296$, $b = 0.0903$, $C = -1.4545 \times 10^{-7}$ km$^{-2}$, $\eta = 4$, $M = 0.87$ $M_\odot$, and $a = 7.866$ km for 4U 1538-52 (second row left), (v) $K = 0.000023$, $b = 0.1$, $C = -1.154 \times 10^{-7}$ km$^{-2}$, $\eta = 5$, $M = 1.29$ $M_\odot$, and $a = 8.831$ km for SMC X-1 (second row middle), (vi) $K = 0.0000277$, $b = 0.06$, $C = -1.2193 \times 10^{-7}$ km$^{-2}$, $\eta = 2.9$, $M = 0.85$ $M_\odot$ and $a = 8.1$ km for Her X-1 (second row right), (vii) $K = 0.000018$, $b = 0.09$, $C = -9.4971 \times 10^{-8}$ km$^{-2}$, $\eta = 5$, $M = 1.49$ $M_\odot$, and $a = 9.178$ km for Cen X-3 (bottom).

The consistency of the field Eqs. (3)-(5), using Eqs. (7) and (8) yield the equation

$$(1 + Y^2) \frac{d^2Z}{dY^2} - Y \frac{dZ}{dY} - [1 - K + K(f_1 + f_2)] = 0$$

where $Y = \sqrt{\frac{K + x}{1 - K}}$, $Cr^2 = x$ and $e^{\nu} = Z^2$.

The expression for energy density and pressure can be had from Eqs. (3), (5), (7) and (8) as follow:

$$\frac{(K + x)}{\sqrt{xKC}(1 + x)} 2Z' + \frac{(1 - K)}{K(1 + x)} + \frac{x}{2(1 + x)^2} (f_1 + f_2)$$

where $C$ is a negative constant. The ansatz (7) is increasing towards the surface of the star and at the center it is zero (see Fig. 4).
Fig. 3 Behavior of Energy conditions (in km$^{-2}$) vs. fractional radius \( r/a \) for the compact stars PSR J1614-2230, 4U 1608-52, SAX J1808.4-3658, 4U 1538-52, SMC X-1, Her X-1 and Cen X-3. In this figure we have used the same set of numerical values of physical parameters and constants which is used in Fig. 1.

Fig. 4 Behavior of Charge (q in km) vs. fractional radius \( r/a \) for the compact stars PSR J1614-2230, 4U 1608-52, SAX J1808.4-3658, 4U 1538-52, SMC X-1, Her X-1 and Cen X-3. In this figure we have used the same set of numerical values of physical parameters and constants which is used in Fig. 1.

\[
\begin{align*}
\frac{\kappa c^2 \rho}{C} &= \frac{\kappa p}{C}, \\
\frac{(K - 1)(3 + x)}{K(1 + x)^2} - \frac{x}{2(1 + x)^2} (f_1 + f_2) &= \frac{\kappa c^2 \rho}{C}.
\end{align*}
\]

Let
\[
Z = (1 + Y^2)^{1/4} \chi(Y)
\]

Put the values of \( Z \) from Eq. (12) into the Eq. (9), we get
\[
\frac{d^2 \chi}{dY^2} + \tau \chi = 0
\]

where \( \tau = -\frac{2y^2}{(Y^2 + Yb)^2} \).

Hence the solution of the differential equation (13) is
\[
\chi(Y) = (Y^2 \eta^2 + Yb) A \left[ \sin^2(\Phi) - \csc^2(\Phi) \log \sin^4(\Phi) \right] + B (Y^2 \eta^2 + Yb)
\]

where \( \Phi = \arctan\left( \frac{Y}{b} \right) \).

In the Eq. (12) \( Z \) becomes
\[
Z = A \left[ (1 + Y^2)^{1/4} \left( Y^2 \eta^2 + Yb \right)^{1/2} G(Y) \right] + B (Y^2 \eta^2 + Yb)
\]

where \( G(Y) = [\sin^2(\Phi) - \csc^2(\Phi) - \log \sin^4(\Phi)] \).

Now put the Eq. (14) into the Eqs. (10)-(11), we get the expressions of density and pressure
\[
\frac{\kappa c^2 \rho}{C} = \frac{(K - 1)(3 + x)}{K(1 + x)^2} \left[ -\frac{x}{2(1 + x)^2} \right] \left( f_1 + f_2 \right) - \frac{\kappa c^2 \rho}{C},
\]

\[
\frac{\kappa p}{C} = \frac{2(K + x)}{K(1 + x)\sqrt{(1 - K)(K + x)}} \left[ \frac{E_1 \times E_2 + E_3 \times E_4}{E_5 \times E_2} \right] - \frac{(K - 1)}{K(1 + x)} + E_6,
\]

where,
\[
E_1 = \left[ \frac{(K + x)^{3/2} \eta^2 + (K + x)b}{2(1 + x)^{3/4}} \right],
\]

\[
E_2 = \frac{\eta^2}{b^3} \left[ \sin^2(\Phi) - \csc^2(\Phi) - \log \sin^4(\Phi) \right] + \frac{B}{A},
\]

\[
E_3 = \frac{\eta^3}{2} \left( \frac{Y}{b^2} \right) \left( 1 + Y^2 \right)^{1/4},
\]

\[
E_4 = \sin(2\Phi) + 2 \csc^2(\Phi) \times \cot(\Phi) - 4 \cot(\Phi),
\]

\[
\frac{E_5 \times E_2}{E_1 \times E_2} = \frac{1}{\sqrt{(1 - K)(K + x)}}
\]

\[
E_6 = \frac{1}{\sqrt{(1 - K)(K + x)}}
\]

\[
\frac{E_1 \times E_2 + E_3 \times E_4}{E_5 \times E_2} = \frac{1}{\sqrt{(1 - K)(K + x)}}
\]

\[
\frac{E_5 \times E_2}{E_1 \times E_2} = \frac{1}{\sqrt{(1 - K)(K + x)}}
\]

\[
\frac{E_1 \times E_2 + E_3 \times E_4}{E_5 \times E_2} = \frac{1}{\sqrt{(1 - K)(K + x)}}
\]
Fig. 5 Behaviour of Velocity of sound, density-pressure ratio, adiabatic constant and redshift vs. fractional radius r/a for the compact stars PSR J1614-2230, 4U 1608-52, SAX J1808.4-3658, 4U 1538-52, SMC X-1, Her X-1 and Cen X-3. For this figure we have used the same set of numerical values of physical parameters and constants which is used in Fig. 1

\[ E_5 = (1 + Y^2)^{1/4}(Y^2\eta^2 + Yb), \]

\[ E_6 = \frac{x}{2K(1+x)^2}\left[ K - 1 + \frac{2 - 3Y^2}{4(1+Y^2)} + \frac{2\eta^2(1+Y^2)}{(Y^2\eta^2 + Yb)} \right] \]

Now using the Eqs. (15) and (16), we get the expression of velocity of sound is as follows,

\[ \frac{dp}{c^2d\rho} = \frac{L}{(E_5 \times E_2)^2} - \frac{2\sqrt{C_5}}{K(1+Y^2)^2} + E_7 \times E_8 + E_9 \times E_10 \]

\[ (E_11 - E_7 \times E_8 - E_9 \times E_10) \]

(17)

where the expressions of used coefficients \( L, E_2, E_5, E_7, E_8, E_9, E_10, \) and \( E_11 \) in Eqs. (17) are given in Appendix.

In this analytical approach, we plot the Fig. 5 (top right) to represent the velocity of sound due to the complexity of the expression and we can see that velocity of sound lies within the proposed range for different compact stars as labeled in figure.

### 3 Boundary conditions

The pressure free interface ‘\( r = a \)’ the charged fluid sphere is expected to join with the Reissner-Nordstrom metric:

\[ ds^2 = -\left(1 - \frac{2M}{r} + \frac{e^2}{r^2}\right)^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) + \left(1 - \frac{2M}{r} + \frac{e^2}{r^2}\right)dt^2 \]

(18)

where \( M \) is the gravitational mass of the fluid sphere such that

\[ M = \zeta(a) + \xi(a) \]

(19)
where

$$\zeta(a) = \frac{K}{2} \int_0^a \rho r^2 dr, \quad \xi(a) = \int_0^a r \sigma q e^{a/2} dr,$$

$$e = q(a)$$

where \( \xi(a) \) is the mass equivalence to electromagnetic energy of distribution, \( \zeta(a) \) is the mass and ‘\( e \)’ is the total charge interior of the sphere (Florides 1983).

The continuity of \( e^\nu, e^\nu \) and \( q \) across the boundary \( r = a \) gives following equations

$$e^\nu = 1 - \frac{2m(a)}{a} + \frac{e^2}{a^2},$$

$$y^2 = 1 - \frac{2m(a)}{a} + \frac{e^2}{a^2},$$

$$q(a) = e,$$

$$p(a) = 0.$$  \hspace{1cm} (23)

The conditions (22) and (24) can be used to compute the values of arbitrary constant \( B/A \). The expressions for arbitrary constant \( B/A \) as follows:

$$B = A1(A2 \times A4 + A5 \times A6) - A3 \times A4 \times A7,$$

$$A = A7 \times A3 - A1 \times A2$$

where

$$A1 = \left( \frac{2(K + x_1)}{K(1 + x_1)\sqrt{(1 - K)(K + x_1)}} \right),$$

$$A2 = \left[ \frac{(K + x_1)^{3/2} \eta^2 + (K + x_1)b}{2(1 + x_1)^{3/4}} \right]$$

$$+ \left( \frac{1 + x_1}{1 - K} \right)^{1/4} \left( \frac{2}{\sqrt{K + x_1}} \right) \eta^2 + b \right],$$

$$A3 = \left( K - 1 \right) \frac{K - 1}{K(1 + x_1)^2} - \frac{x_1}{2K(1 + x_1)^2}$$

$$\times \left[ K - 1 + \frac{2 - 3Y_1^2}{4(1 + Y_1^2)} + \frac{2\eta^2(1 + Y_1^2)}{(Y_1^2 + Y_1b)} \right],$$

$$A4 = \eta^2 b^3 \left[ \sin^2(\Phi_1) - \csc^2(\Phi_1) - \log \sin^4(\Phi_1) \right],$$

$$A5 = \frac{\eta^3}{2} \left( \frac{Y_1}{b^2} \right)^{1/2} \left( 1 + Y_1^2 \right)^{1/4},$$

$$A6 = \sin 2(\Phi_1) + 2 \csc^2(\Phi_1) \times \cot(\Phi_1) - 4 \cot(\Phi_1),$$

$$A7 = \left( 1 + Y_1^2 \right)^{1/4} \left( Y_1^2 \eta^2 + Y_1b \right),$$

$$\Phi_1 = \arctan \left( \frac{Y_1}{b} \right), \quad Y_1 = \sqrt{\frac{K + x_1}{1 - K}}, \quad x_1 = Ca^2.$$  \hspace{1cm} (24)

Also the values of constant coefficients \( \eta, C \) and \( K \) are determined for some particular values of observed massive stellar objects such as PSR J1614-2230, 4U 1608-52, SAX J1808.4-3658, 4U 1538-52, SMC X-1, Her X-1 and Cen X-3 respectively.

### 4 Tolman-Oppenheimer-Volkoff (TOV) equations

In the presence of charge, the Tolman-Oppenheimer-Volkoff (TOV) equation (Tolman 1939; Oppenheimer and Volkoff 1939) is given by

$$-\frac{M_G(\rho + p)}{r^2} e^{\nu - \lambda} - \frac{dp}{dr} + \sigma q r^2 e^{\lambda/2} = 0,$$

where \( M_G \) represents the gravitational mass and defined as:

$$M_G(r) = \frac{1}{2} a_r^2 \nu^\nu e^{(\nu - \lambda)/2}. $$

Substituting the value of \( M_G(r) \) in Eq. (25), we get

$$-\frac{\nu'}{2} (\rho + p) - \frac{dp}{dr} + \frac{q^2}{r^2} e^{\lambda/2} = 0.$$

The equation (27) can be expressed into three unique segments, gravitational \( (F_g) \), hydrostatic \( (F_h) \) and electric \( (F_e) \) forces, which are defined as:

$$F_g = -\frac{\nu'}{2} (\rho + p) = \frac{Z'}{2\pi Z} (\rho + p),$$

$$F_h = -\frac{dp}{dr} = -\frac{1}{\pi} \left[ \frac{L}{(E2 \times E5)} - \frac{2\sqrt{C}}{K(1 + Y^2)} \right]$$

$$+ E7 \times E8 + E9 \times E10,$$

$$F_e = \frac{\sigma q^2}{r^2} e^{\lambda/2} = \frac{1}{8\pi r^4} \frac{dq^2}{dr}$$

$$= \frac{1}{2\pi} \left[ E7 \times E8 + E9 \times E10 \right].$$

We can see the behavior of the generalized TOV equations by Fig. 2 and the system is counter balanced by three different forces, e.g., gravitational \( (F_g) \), hydrostatic force \( (F_h) \) and electric force \( (F_e) \). This conclude that the system attains a static equilibrium.

### 5 Energy conditions

Here we analyze the energy conditions according to relativistic classical field theories of gravitation. In the context of GR, the energy conditions are local inequalities that process a relation between matter density and pressure obeying...
certain restrictions (Andreason 2008). The charged fluid sphere should satisfy the three energy conditions (i) strong energy condition (SEC), (ii) weak energy condition (WEC) and (iii) null energy condition (NEC). For satisfying the above energy conditions, the following inequalities must hold simultaneously inside the charged fluid sphere:

Null energy condition (NEC): \( \rho + \frac{q^2}{r^2} \geq 0 \).

Weak energy condition (WEC): \( \rho + p + \frac{q^2}{r^2} \geq 0 \).

Strong energy condition (SEC): \( \rho + 3p + \frac{q^2}{r^2} \geq 0 \).

The nature of energy conditions for the specific stellar configuration as shown in Fig. 3, that are satisfied for our proposed model.

6 Electric charge

Varela et al. (2010) have shown that fluid spheres with net charge contain fluid elements with unbounded proper charge density located at the fluid-vacuum interface and the net charge can be huge (10\(^{19}\) C). Ray et al. (2003) have analyzed the impact of charge in compact stars considering the limit of the most extreme measure of the charge. They have demonstrated the global balance of the forces which allowed a huge charge (10\(^{20}\) C) to be available in compact star.

In this model we have found that the maximum charge on the boundary is 6.605 \times 10^{20} C and at the center is zero. We have plot the Fig. 4 for the charge \( q \) in the relativistic units (km). For coulombs unit, one has multiply these value by 1.1659 \times 10^{20} C. Thus in this model the net amount of charge is effective to balance the mechanism of the force.

7 Surface redshift

The gravitational redshift \( Z_s \) within a static line element can be obtained as

\[
Z_s = \sqrt{g_{tt}(a)} - 1 = \sqrt{1 - \frac{2M}{a} + \frac{q^2}{a^2}} - 1
\]

where \( g_{tt}(a) = e^{\nu(a)} = 1 - \frac{2M}{a} + \frac{q^2}{a^2} \).

The maximum possible value of redshift should be at the center of the star and decrease with the increase of radius. Buchdahl (1959) and Straumann (1984) have shown that for an isotropic star the surface redshift \( Z_s \leq 2 \). For an anisotropic star Bohmer and Harko (2006) showed that the surface redshift could be increased up to \( Z_s \leq 5 \). Ivanov (2002) modified the maximum value of redshift and showed that it could be as high as \( Z_s = 5.211 \). In this model we have \( Z_s \leq 1 \) for compact stars PSR J1614-2230, 4U 1608-52, SAX J1808.4-3658, 4U 1538-52, SMC X-1, Her X-1 and Cen X-3. Also it is decreasing towards the boundary (see Fig. 5 bottom right).

8 Causality and well behaved condition

Inside the fluid sphere the velocity of sound is less than the light, i.e. \( 0 \leq v^2 \leq \frac{dp}{d\rho} < 1 \). According to Canuto (1973), for well behaved nature of the charge solution, the velocity of sound should be monotonically decreasing towards the boundary with an ultra-high distribution of matter. From Fig. 5 (top right), it is verified that velocity of sound should monotonically decreasing. This imply that our model for charge compact star is well behaved.

9 Adiabatic index

The stability of relativistic isotropic fluid sphere depends on the adiabatic index \( \gamma \). Heintzmann and Hillebrandt (1975) proposed that isotropic compact star models are stable if \( \gamma > 4/3 \) throughout the star. In present model the adiabatic index defined by

\[
\gamma = \left( \frac{p + \rho}{\rho} \right) \frac{dp}{d\rho}.
\]

From Fig. 5 (bottom left) we have seen that the value of \( \gamma \) is greater than 4/3 and hence the model is stable.

10 Harrison-Zeldovich-Novikov stability criterion

The Harrison et al. (1965), Zeldovich and Novikov (1971) criterion states that the compact stars are stable if its mass increases with the central density, i.e. \( \frac{dM}{d\rho_0} > 0 \), however it is unstable if \( \frac{dM}{d\rho_0} < 0 \). For this purpose we calculate the density of star at the center \( (\rho_0) \), mass \( (M(a)) \) and its gradient in terms of central density as

\[
\rho_0 = \frac{3C(K - 1)}{8\pi K},
\]

\[
M(a) = \frac{4\pi \rho_0 a^3}{M_1(a)} \left[ K - 1 + \frac{\pi \rho_0 K a^2}{M_1(a)} M_2(a) \right],
\]

\[
\frac{dM(a)}{d\rho_0} = \left[ K - 1 + M_7(a) + \frac{4\pi (\rho_0 K a^2)^2}{3(K - 1)} \right. \\
\left. \times \left( \frac{-30\pi a^2 (1 - K)^2}{M_1(a)} - M_3(a) \right) \right],
\]

where

\[
M_1(a) = 8\pi \rho_0 K a^2 + 3(K - 1),
\]

\[
M_6(a) = \frac{12\pi a^3 (K - 1)}{(M_1(a))^2}.
\]
Fig. 6 Behavior of Mass ($M_\odot$) (left panel) and $\frac{dM}{d\rho_0}$ vs. central density $\rho_0$ for the compact stars PSR J1614-2230, 4U 1608-52, SAX J1808.4-3658, 4U 1538-52, SMC X-1, Her X-1 and Cen X-3. In this figure we have used the same set of numerical values of physical parameters and constants which is used in Fig. 1

\[
M_2(a) = \left( \frac{-15(1 - K)^2}{M_1(a)} \right) + \frac{8\eta^2 M_1(a)}{(\eta^2 M_5(a) - 3b(1 - K)\sqrt{-M_5(a)}/3)} + 4K - 7, \\
M_3(a) = \frac{-8\pi \eta^2 a^2 M_1(a)}{M_4(a)(1 - K)^4} \left( \frac{2\eta^2 M_4(a) + \sqrt{3M_4(a)}}{\eta^2 M_3(a) + \sqrt{3M_4(a)}} \right), \\
M_4(a) = \frac{16\pi a^2}{(\eta^2 M_5(a) - 3b(1 - K)\sqrt{-M_5(a)}/3)}, \\
M_5(a) = \frac{3K(1 - K) - 8\pi \rho_0 Ka^2}{(1 - K)^2}, \\
M_7(a) = \frac{2\pi \rho_0 Ka^2}{M_1(a)} M_2(a).
\]

From Fig. 6 we see that the mass $M (M_\odot)$ of isotropic compact star increases with central density ($\rho_0$). On other hand we observe that $\frac{dM}{d\rho_0}$ is positive with respect to central density ($\rho_0$). Hence, present isotropic compact star model is stable.

11 Novelty of present model

The novelty of our model is that the density ($\rho$) is always positive and in monotonically decreasing order for $0 < K < 1$. If we remove the electric intensity ($q(r) = 0$) or adopt the anisotropic approach then the density ($\rho$) is increasing towards the boundary, i.e. $\frac{d\rho}{dr} > 0$ for $0 < K < 1$. An anisotropic approach we obtained $\rho$ and $\frac{d\rho}{dr}$ as follows:

\[
\rho = \frac{C(K - 1)(3 + Cr^2)}{8\pi K(1 + Cr^2)^2}, \\
\frac{d\rho}{dr} = -\frac{-2C^2r(K - 1)(5 + Cr^2)}{8\pi K(1 + Cr^2)^3} > 0
\]

because $-2K C^2r(K - 1) > 0$ for $0 < K < 1$, and $-1.1446 \times 10^{-7} < C < 0$. Also both term ($5 + Cr^2$) and ($1 + Cr^2$) are positive. Hence the density gradient is also positive. From Fig. 7, we have conclude that isotropic approach is valid with charged in the comparison of anisotropic approach.

Even if, it is formulated in the isotropic and anisotropic model in absence of charge one can find that density and pressure are increasing towards the boundary if $C < 0$. And the expressions (15) and (16) show that density and pressure become negative, if $C > 0$ and $0 < K < 1$. However, in the case of the modified theory of gravity in particular, $f(R; T)$ gravity and dark energy, the density profile is also monotonically increasing for some range of the matter coupling parameters. It means that charged solutions of Einstein-Maxwell field equations lead to the determining more stable stellar structure where Coulomb repulsion works against gravitational attraction. The key aspect of the problem is that this given methodology gives a physically valid solution only for the charged case when $0 < K < 1$ and $C < 0$. 

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Fig. 7  Behaviour of $\rho_0$ and $d\rho_0/dr$ vs. fractional radius for PSR J1614-2230, 4U 1608-52 SAX J1808.4-3658, 4U 1538-52, SMC X-1, Her X-1 and Cen X-3. In this figure we have used the same set of numerical values of physical parameters and constants which is used in Fig. 1

| $r/a$ | $\rho$ (km$^{-2}$) | $\rho$ (km$^{-2}$) | $q$ (km) | $dp/c^2d\rho$ | $p/\rho$ | $\gamma$ |
|------|-----------------|-----------------|---------|---------------|--------|--------|
| 0    | 2.7585 $\times$ 10$^{-5}$ | 0.000659 | 0 | 0.583119 | 0.041827 | 14.524215 |
| 0.1  | 2.7255 $\times$ 10$^{-5}$ | 0.000659 | 0.004581 | 0.578524 | 0.041342 | 14.572066 |
| 0.2  | 2.6272 $\times$ 10$^{-5}$ | 0.000659 | 0.03682 | 0.564766 | 0.039891 | 14.722468 |
| 0.3  | 2.4645 $\times$ 10$^{-5}$ | 0.000657 | 0.125268 | 0.541926 | 0.037488 | 14.997957 |
| 0.4  | 2.2397 $\times$ 10$^{-5}$ | 0.000656 | 0.300384 | 0.510103 | 0.034157 | 15.443954 |
| 0.5  | 1.9563 $\times$ 10$^{-5}$ | 0.000653 | 0.595859 | 0.46936 | 0.029939 | 16.146318 |
| 0.6  | 1.6191 $\times$ 10$^{-5}$ | 0.00065 | 1.050501 | 0.419633 | 0.024895 | 17.275441 |
| 0.7  | 1.2362 $\times$ 10$^{-5}$ | 0.000646 | 1.71198 | 0.360575 | 0.019124 | 19.215468 |
| 0.8  | 8.2021 $\times$ 10$^{-6}$ | 0.000641 | 2.63799 | 0.291288 | 0.01279 | 23.065459 |
| 0.9  | 3.9353 $\times$ 10$^{-6}$ | 0.000634 | 3.913638 | 0.209843 | 0.006202 | 34.042794 |
| 1    | 0 | 0.000625 | 5.663978 | 0.11233 | 0 | Inf |

### 12 Conclusion

In this article, we obtained a new class of charged superdense star model by solving Einstein-Maxwell field equation for a static symmetric distribution of perfect fluid. This solution is based on a suitable metric potential and a particular form of electric intensity. Our aim is to explore a class of exotic astrophysical objects with similar mass and radii, like SAX J1808.4-3658, 4U 1538-52, PSR J1903+327 etc., which are conformed by observations of anomalous X-ray pulsars and gamma-ray repeaters. Compact objects play an important character in relativistic astrophysics for several reasons. For example, neutron stars are the most stable compact objects in the universe, but the maximal mass estimation of such objects is still an open question to the researchers. The originality of our methodology is that the pressure and energy density are maximum at the center and monotonically decreasing towards the surface which guarantees the singularity free stellar structure for specified range of $K$. The interior space-time smoothly connects with the exterior Reissner-Nordstrom space-time. Then, we have compared these two metrics and fixed the constants $(C, K, \eta, b)$, which are enlisted in Tables 1 to 8. In particular, we have checked the physical viability and acceptability of our model in connection with compact star candidates like PSR J1614-2230, 4U 1608-52, SAX J1808.4-3658, 4U 1538-52, PSR J1903+327 etc., which are conformed by observations of anomalous X-ray pulsars and gamma-ray repeaters. Compact objects play an important character in relativistic astrophysics for several reasons. For example, neutron stars are the most stable compact objects in the universe, but the maximal mass estimation of such objects is still an open question to the researchers. The originality of our methodology is that
Table 2  Value of different physical parameter of 4U 1608-52

| r/a  | $\rho$ (km$^{-2}$) | $\rho$ (km$^{-2}$) | $q$ (km) | $dp/c^2 dp$ | $p/\rho$ | $\gamma$  |
|------|-----------------|-----------------|--------|-------------|--------|---------|
| 0    | 2.2941 x 10^{-5}| 0.000654        | 0      | 0.580658    | 0.035075| 17.135305|
| 0.1  | 2.2671 x 10^{-5}| 0.000654        | 0.004033| 0.577187    | 0.034673| 17.223585|
| 0.2  | 2.1864 x 10^{-5}| 0.000653        | 0.032405| 0.566776    | 0.033471| 17.499985|
| 0.3  | 2.053 x 10^{-5} | 0.000652        | 0.110168| 0.549433    | 0.03148 | 18.002797|
| 0.4  | 1.8686 x 10^{-5}| 0.000651        | 0.2639  | 0.525142    | 0.02872 | 18.81021 |
| 0.5  | 1.6357 x 10^{-5}| 0.000649        | 0.522728| 0.493821    | 0.025222| 20.072895|
| 0.6  | 1.3584 x 10^{-5}| 0.000646        | 0.917948| 0.45525     | 0.021035| 22.097382|
| 0.7  | 1.0428 x 10^{-5}| 0.000642        | 1.497486| 0.408948    | 0.016236| 25.595997|
| 0.8  | 6.9482 x 10^{-6}| 0.000638        | 2.294923| 0.353977    | 0.01095 | 32.680025|
| 0.9  | 3.4102 x 10^{-6}| 0.000632        | 3.386797| 0.28859     | 0.005396| 53.766753|
| 1    | 0               | 0.000624        | 4.863006| 0.209584    | 0       | Inf      |

Table 3  Value of different physical parameter of SAX J1808.4-3658

| r/a  | $\rho$ (km$^{-2}$) | $\rho$ (km$^{-2}$) | $q$ (km) | $dp/c^2 dp$ | $p/\rho$ | $\gamma$  |
|------|-----------------|-----------------|--------|-------------|--------|---------|
| 0    | 1.2505 x 10^{-5}| 0.000586        | 0      | 0.669955    | 0.02133| 32.079514|
| 0.1  | 1.2368 x 10^{-5}| 0.000586        | 0.002149| 0.668107    | 0.021101| 32.30121 |
| 0.2  | 1.196 x 10^{-5} | 0.000586        | 0.017239| 0.662554    | 0.020417| 33.11328 |
| 0.3  | 1.1283 x 10^{-5}| 0.000585        | 0.058458| 0.653264    | 0.019282| 34.53258 |
| 0.4  | 1.0343 x 10^{-5}| 0.000584        | 0.139505| 0.640178    | 0.017702| 36.804817|
| 0.5  | 9.1483 x 10^{-6}| 0.000583        | 0.274902| 0.623194    | 0.015688| 40.346788|
| 0.6  | 7.7107 x 10^{-6}| 0.000582        | 0.480386| 0.60215     | 0.013257| 46.023518|
| 0.7  | 6.0477 x 10^{-6}| 0.00058         | 0.773419| 0.576791    | 0.010431| 55.874965|
| 0.8  | 4.1825 x 10^{-6}| 0.000578        | 1.173914| 0.546727    | 0.007242| 76.041701|
| 0.9  | 2.1498 x 10^{-6}| 0.000575        | 1.705293| 0.511364    | 0.00374 | 137.243765|
| 1    | 0               | 0.000571        | 2.39613 | 0.469797    | 0       | Inf      |

Table 4  Value of different physical parameter of 4U 1538-52

| r/a  | $\rho$ (km$^{-2}$) | $\rho$ (km$^{-2}$) | $q$ (km) | $dp/c^2 dp$ | $p/\rho$ | $\gamma$  |
|------|-----------------|-----------------|--------|-------------|--------|---------|
| 0    | 1.2854 x 10^{-5}| 0.000587        | 0      | 0.729746    | 0.021904| 34.045791|
| 0.1  | 1.2715 x 10^{-5}| 0.000587        | 0.002072| 0.727689    | 0.021671| 34.306902|
| 0.2  | 1.2298 x 10^{-5}| 0.000586        | 0.016621| 0.72151     | 0.020973| 35.12295 |
| 0.3  | 1.1607 x 10^{-5}| 0.000586        | 0.056356| 0.711185    | 0.019815| 36.602787|
| 0.4  | 1.0647 x 10^{-5}| 0.000585        | 0.134467| 0.696664    | 0.018202| 38.970655|
| 0.5  | 9.4263 x 10^{-6}| 0.000584        | 0.264921| 0.677864    | 0.016145| 42.663925|
| 0.6  | 7.9548 x 10^{-6}| 0.000582        | 0.462818| 0.654646    | 0.013658| 48.585353|
| 0.7  | 6.2492 x 10^{-6}| 0.000581        | 0.744878| 0.626788    | 0.010762| 58.865847|
| 0.8  | 4.3313 x 10^{-6}| 0.000578        | 1.130102| 0.593945    | 0.007487| 79.920526|
| 0.9  | 2.2325 x 10^{-6}| 0.000576        | 1.640744| 0.555585    | 0.003877| 143.84548 |
| 1    | 0               | 0.000573        | 2.303804| 0.510896    | 0       | Inf      |
### Table 5 Value of different physical parameter of SMC X-1

| n/a | $p$ (km$^{-2}$) | $\rho$ (km$^{-2}$) | $q$ (km) | $dp/c^2d\rho$ | $p/\rho$ | $\gamma$ |
|-----|----------------|-------------------|---------|----------------|---------|----------|
| 0   | $1.8593 \times 10^{-5}$ | 0.000599 | 0       | 0.674146       | 0.03103 | 22.399988 |
| 0.1 | $1.8387 \times 10^{-5}$ | 0.000599 | 0.003005| 0.670942       | 0.030692| 22.531393 |
| 0.2 | $1.7766 \times 10^{-5}$ | 0.000599 | 0.024126| 0.661333       | 0.029681| 22.942466 |
| 0.3 | $1.6741 \times 10^{-5}$ | 0.000598 | 0.081913| 0.645324       | 0.028004| 23.689209 |
| 0.4 | $1.5319 \times 10^{-5}$ | 0.000597 | 0.195834| 0.622911       | 0.025673| 24.886618 |
| 0.5 | $1.3514 \times 10^{-5}$ | 0.000595 | 0.38686| 0.594054       | 0.022705| 26.758037 |
| 0.6 | $1.1348 \times 10^{-5}$ | 0.000593 | 0.678235| 0.558639       | 0.019129| 29.76226  |
| 0.7 | $8.8539 \times 10^{-5}$ | 0.000591 | 1.096575| 0.516417       | 0.014986| 34.977079 |
| 0.8 | $6.0759 \times 10^{-5}$ | 0.000588 | 1.673515| 0.466911       | 0.010338| 45.633544 |
| 0.9 | $3.0857 \times 10^{-5}$ | 0.000584 | 2.448385| 0.409257       | 0.005285| 77.846602 |
| 1   | 0              | 0.000579 | 3.472844| 0.341948       | 0       | Inf      |

### Table 6 Value of different physical parameter of Her X-1

| n/a | $p$ (km$^{-2}$) | $\rho$ (km$^{-2}$) | $q$ (km) | $dp/c^2d\rho$ | $p/\rho$ | $\gamma$ |
|-----|----------------|-------------------|---------|----------------|---------|----------|
| 0   | $8.8592 \times 10^{-6}$ | 0.000526 | 0       | 0.785045       | 0.016853| 47.367521 |
| 0.1 | $8.7621 \times 10^{-6}$ | 0.000526 | 0.00196| 0.783757       | 0.016671| 47.798122 |
| 0.2 | $8.4702 \times 10^{-6}$ | 0.000525 | 0.015725| 0.779875       | 0.016125| 49.14376  |
| 0.3 | $7.9867 \times 10^{-6}$ | 0.000525 | 0.053307| 0.773342       | 0.01522 | 51.585661 |
| 0.4 | $7.3162 \times 10^{-6}$ | 0.000524 | 0.127154| 0.764058       | 0.013962| 55.487418 |
| 0.5 | $6.4649 \times 10^{-6}$ | 0.000523 | 0.250411| 0.751865       | 0.012361| 61.576861 |
| 0.6 | $5.4425 \times 10^{-6}$ | 0.000522 | 0.437239| 0.736529       | 0.010431| 71.344062 |
| 0.7 | $4.2618 \times 10^{-6}$ | 0.00052 | 0.703237| 0.717719       | 0.008193| 88.315903 |
| 0.8 | $2.9421 \times 10^{-6}$ | 0.000518 | 1.066009| 0.694956       | 0.005676| 123.122389|
| 0.9 | $1.5084 \times 10^{-6}$ | 0.000516 | 1.545994| 0.667565       | 0.002924| 228.989058|
| 1   | 0              | 0.000513 | 2.167694| 0.634581       | 0       | Inf      |

### Table 7 Value of different physical parameter of Cen X-3

| n/a | $p$ (km$^{-2}$) | $\rho$ (km$^{-2}$) | $q$ (km) | $dp/c^2d\rho$ | $p/\rho$ | $\gamma$ |
|-----|----------------|-------------------|---------|----------------|---------|----------|
| 0   | $2.4771 \times 10^{-5}$ | 0.00063 | 0       | 0.895387       | 0.039312| 23.672047 |
| 0.1 | $2.4491 \times 10^{-5}$ | 0.00063 | 0.003602| 0.890504       | 0.03888 | 23.794584 |
| 0.2 | $2.3657 \times 10^{-5}$ | 0.000629 | 0.028928| 0.875877       | 0.037587| 24.178474 |
| 0.3 | $2.2275 \times 10^{-5}$ | 0.000628 | 0.098299| 0.851556       | 0.035443| 24.877578 |
| 0.4 | $2.0359 \times 10^{-5}$ | 0.000627 | 0.235294| 0.817605       | 0.023463| 26.003314 |
| 0.5 | $1.7931 \times 10^{-5}$ | 0.000625 | 0.465583| 0.774064       | 0.028673| 27.770227 |
| 0.6 | $1.5021 \times 10^{-5}$ | 0.000623 | 0.818063| 0.720884       | 0.021411| 30.619701 |
| 0.7 | $1.1677 \times 10^{-5}$ | 0.00062 | 1.326537| 0.657833       | 0.018834| 35.586676 |
| 0.8 | $7.9693 \times 10^{-6}$ | 0.000616 | 2.032382| 0.584338       | 0.012932| 45.769141 |
| 0.9 | $4.0109 \times 10^{-6}$ | 0.000611 | 2.989146| 0.499213       | 0.00656 | 76.597939  |
| 1   | 0              | 0.000605 | 4.271195| 0.400191       | 0       | Inf      |
Table 8 Numerical values of radius ($a$) $M$ ($M_\odot$), central density, surface density, central pressure and mass-radius ratio of compact star candidates

| Compact star | $a$ (km) | $M$ ($M_\odot$) | Central density (g/cm$^3$) | Surface density (g/cm$^3$) | Central pressure (dyne/cm$^2$) | M/a |
|--------------|---------|-----------------|---------------------------|---------------------------|-------------------------------|-----|
| PSR J1614-2230 | 9.69 | 1.97 | $8.886 \times 10^{14}$ | $8.4233 \times 10^{14}$ | $3.3435 \times 10^{34}$ | 0.16314 |
| 4U 1608-52 | 9.3 | 1.74 | $8.8133 \times 10^{14}$ | $8.41008 \times 10^{14}$ | $2.7821 \times 10^{34}$ | 0.2694 |
| SAX J1808.4-3658 | 7.95 | 0.9 | $7.8997 \times 10^{14}$ | $7.6997 \times 10^{14}$ | $1.5165 \times 10^{34}$ | 0.1696 |
| 4U 1538-52 | 7.866 | 0.87 | $7.9078 \times 10^{14}$ | $7.7151 \times 10^{14}$ | $1.5588 \times 10^{34}$ | 0.1631 |
| SMC X-4 | 8.831 | 1.29 | $8.0744 \times 10^{14}$ | $7.8018 \times 10^{14}$ | $2.2549 \times 10^{34}$ | 0.2154 |
| Her X-1 | 8.1 | 0.85 | $7.0836 \times 10^{14}$ | $6.9144 \times 10^{14}$ | $1.0744 \times 10^{34}$ | 0.1547 |
| Cen X-3 | 9.165 | 1.49 | $8.4906 \times 10^{14}$ | $8.1524 \times 10^{14}$ | $3.004 \times 10^{34}$ | 0.2487 |

our model satisfies causality condition i.e., $(dp/c^2 dp) < 1$ which can be observed in Fig. 5 (top left). The model satisfies the following energy conditions (see Fig. 3) (i) strong energy condition (SEC), (ii) weak energy condition (WEC) and (iii) null energy condition (NEC). In Fig. 5 (bottom right), we also observed that the redshift decreased from the center to surface for specified range of $K$. The stability of the charged fluid models depends on the adiabatic index $\gamma$. Heintzmann and Hillebrandt (1975) proposed that a neutron star model with EoS is stable if $\gamma > 1$. In this model, adiabatic index is greater than 4/3 (in Fig. 5 (bottom left)). The model also satisfied the modified TOV equation.

Kothariaraj and Maharaj (2007) observed that the electric field intensity $q(r)$ is continuous in the interior and vanishes at the center. In our model, the electric field intensity $q(r)$ is positive, monotonically increasing and vanishes at the center (see in Fig. 4). We also observed that if the radius of the star increases then the electric charge also increases. It means that the gravitational attraction is counterbalanced by the repulsive Coulombic force. The global balance of the forces allows a huge charge ($10^{20}$ Coulomb) to be present in a neutron star producing a very high electric field ($10^{21}$ V/m). The charged stars have large mass and radius as we should expect due to the effect of the repulsive Coulomb force, with the $M/R$ ratio increasing with charge. In the limit of the maximum charge, the mass went up to 10 $M_\odot$ which is much higher than the maximum mass allowed for a neutral compact star. However, the local effect of the forces experienced by a single charged particle, makes it to discharge quickly. This creates a global force imbalance and the system collapses to a charged black hole. In this work Tables 1–8 contain numerical values of physical quantities, where we used various symbols as follows:

$Z_o =$ redshift at the center, $Z_s =$ redshift at the surface, solar mass $M_\odot = 1.475$ km, $G = 6.673 \times 10^{-8}$ cm$^3$/g s$^2$, $c = 2.997 \times 10^{10}$ cm/s.

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Appendix

The expression for coefficients used in Eq. (17)

$L = (E5 \times E2) \times M1 \times (E1 \times E2 + E3 \times E4) + M7( (E5 \times E2)((B2 + B3) \times E2 + E1 \times B4 + E3 \times B6 + E4 \times B5) - (E1 \times E2 + E3 \times E4)(B7 \times E2 + B4 \times E5))$,

$M1 = \frac{2\sqrt{C}(1 - 2K - x)}{K(1 + x)^2\sqrt{(1 - K)(K + x)}}$,
$M7 = \frac{2(K + x)}{K(1 + x)^2\sqrt{(1 - K)(K + x)}}$,
$E8 = [K - 1 + \frac{2 - 3Y^2}{4(1 + Y^2)^2} + \frac{2\eta^2(1 + Y^2)}{(Y^2\eta^2 + Yb)}]$,
$E10 = \frac{x}{2K(1 + x)^2}$,
$E9 = \frac{5(K - 1)\sqrt{C}}{2K(1 + x)^2} + \frac{2\sqrt{C}}{2K(1 + x)^2} - \frac{2\eta^2}{(Y^2\eta^2 + b)K(1 - K)} - \frac{(1 + Y^2)}{K}(1 + \frac{Y^2}{Yb/\eta^2})^2$,
$E7 = \sqrt{C}(1 - x)$
$B2 = \frac{\sqrt{C}}{(1 - K)} \left[ \frac{(1 + Y^2)(\frac{3}{2}Y\eta^2 + b) - \frac{3}{2}Y^2(Y\eta^2 + b)}{(1 + Y^2)^{7/4}} \right]$,
$B3 = \frac{2\sqrt{C}}{(1 - K)(1 + Y^2)^{3/4}} \left[ \left( \frac{1}{2}Y\eta^2 + b \right) + \frac{(1 + Y^2)}{Y} \eta^2 \right]$,
$E11 = \frac{2\sqrt{C}(1 - K)(5 - x)}{K(1 + x)^3}$,
$B4 = \frac{\eta^3\sqrt{C}}{2b^{5/2}(1 - K)(b + \eta^2)} \times E4$. 
\[ B_5 = \frac{\eta^3 \sqrt{C \xi}}{4b^{5/2}(1 - K)} \left[ (1 + Y^2)^{-3/4} Y^{1/2} + (1 + Y^2)^{1/4} Y^{3/2} \right], \]
\[ B_6 = \frac{\eta \sqrt{C \xi} b}{Y^{3/2}(1 - K)(b + Y) \eta} \left[ \cos 2(\Phi) - 2 \csc^2(\Phi) \cot^2(\Phi) - 2 \csc^4(\Phi) + \csc^2(\Phi) \right], \]
\[ B_7 = \frac{\sqrt{C \xi}}{2(1 - K)(1 + Y^2)^{3/4}} (Y^2 \eta^2 + bY) + \frac{2(1 + Y^2)^{1/4} \sqrt{C \xi}}{(1 - K)} \left( \eta^2 + \frac{b}{Y} \right). \]

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