NNLO QCD corrections to dijet production at hadron colliders

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Inclusive jet and dijet cross sections

- look at the production of jets of hadrons with large transverse energy in
  - inclusive jet events \( pp \rightarrow j + X \)
  - exclusive dijet events \( pp \rightarrow 2j \)

- cross sections measured as a function of the jet \( p_T \), rapidity \( y \) and dijet invariant mass \( m_{jj} \) in double differential form

(CMS-PAS-SMP-12-012) (ATLAS-CONF-2012-021)
Motivation for NNLO

- experimental uncertainties at high-$p_T$ smaller than theoretical $\rightarrow$ need pQCD predictions to NNLO accuracy
- collider jet data can be used to constrain parton distribution functions
- size of NNLO correction important for precise determination of PDF’s
- inclusion of jet data in NNLO parton distribution fits requires NNLO corrections to jet cross sections
Inclusive jet cross section

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- **Experimental uncertainties** at high-$p_T$ smaller than theoretical $\rightarrow$ need pQCD predictions to NNLO accuracy
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- Size of NNLO correction important for precise determination of PDF’s
- Inclusion of jet data in NNLO parton distribution fits requires NNLO corrections to jet cross sections
- $\alpha_s$ determination from hadronic jet observables limited by theoretical uncertainty due to scale choice
inclusive jet and dijet cross sections

State of the art:
- dijet production is completely known in NLO QCD [Ellis, Kunszt, Soper ’92], [Giele, Glover, Kosower ’94], [Nagy ’02]
- NLO+Parton shower [Alioli, Hamilton, Nason, Oleari, Re ’11]
- threshold corrections [Kidonakis, Owens ’00]

Goal:
- obtain the jet cross sections at NNLO accuracy in double differential form

\[ \frac{d^2 \sigma}{dp_T d|y|} \quad \frac{d^2 \sigma}{dm_{jj} dy^*} \]

This talk:
- present IR structure of the gluons only subleading colour NNLO calculation
- present NNLO inclusive jet and dijet cross sections (gluons only full colour)
$pp \rightarrow 2j$ at NNLO: gluonic contributions

$A_6^{(0)}(gg \rightarrow gggg)$ \hspace{1cm} $A_5^{(1)}(gg \rightarrow ggg)$ \hspace{1cm} $A_4^{(2)}(gg \rightarrow gg)$

[Berends, Giele ’87], [Mangano, Parke, Xu ’87], [Britto, Cachazo, Feng ’06]
[Bern, Dixon, Kosower ’93]
[Anastasiou, Glover, Oleari, Tejeda-Yeomans ’01], [Bern, De Freitas, Dixon ’02]

\[
\hat{d}\sigma_{NNLO} = \int d\Phi_4 \hat{d}\sigma_{NNLO}^{RR} + \int d\Phi_3 \hat{d}\sigma_{NNLO}^{RV} + \int d\Phi_2 \hat{d}\sigma_{NNLO}^{VV}
\]

- explicit infrared poles from loop integrations
- implicit poles in phase space regions for single and double unresolved gluon emission
- procedure to extract the infrared singularities and assemble all the parts in a parton-level generator
NNLO antenna subtraction

\[ d\hat{\sigma}_{NNLO} = \int d\Phi_4 \left( d\hat{\sigma}_{NNLO}^{RR} - d\hat{\sigma}_{NNLO}^{S} \right) \]
\[ + \int d\Phi_3 \left( d\hat{\sigma}_{NNLO}^{RV} - d\hat{\sigma}_{NNLO}^{T} \right) \]
\[ + \int d\Phi_2 \left( d\hat{\sigma}_{NNLO}^{VV} - d\hat{\sigma}_{NNLO}^{U} \right) \]

- \( d\hat{\sigma}_{NNLO}^{S} \): real radiation subtraction term for \( d\hat{\sigma}_{NNLO}^{RR} \)
- \( d\hat{\sigma}_{NNLO}^{T} \): one-loop virtual subtraction term for \( d\hat{\sigma}_{NNLO}^{RV} \)
- \( d\hat{\sigma}_{NNLO}^{U} \): two-loop virtual subtraction term for \( d\hat{\sigma}_{NNLO}^{VV} \)

subtraction terms constructed using the antenna subtraction method at NNLO for hadron colliders → presence of initial state partons to take into account

contribution in each of the round brackets is finite, well behaved in the infrared singular regions and can be evaluated numerically
Double-real contribution

- gluons only double-real colour summed cross section

\[ d\hat{\sigma}_{RR}^{NNLO} = \left( \frac{\alpha_s}{2\pi} \right)^2 N^4 (N^2 - 1) \, d\Phi_4(p_3, \ldots, p_6; p_1; p_2) \, J_2^{(4)}(p_3, \ldots, p_6) \, \frac{1}{4!} \]

\[ \sum_{\sigma \in S_6/Z_6} \left[ |A_6^{(0)}(\sigma)|^2 + \frac{2}{N^2} A_6^0(\sigma) \left( A_6^{\dagger 0}(\sigma \cdot \rho_1) + A_6^{\dagger 0}(\sigma \cdot \rho_2) + A_6^{\dagger 0}(\sigma \cdot \rho_3) \right) \right] \]

- double-real sub-leading colour contribution written as three interferences summed over permutations. For \( \sigma = (1, 2, 3, 4, 5, 6) \)

\( (\sigma \cdot \rho_1) = (1, 3, 5, 2, 6, 4) \quad (\sigma \cdot \rho_2) = (1, 3, 6, 4, 2, 5) \quad (\sigma \cdot \rho_3) = (1, 4, 2, 6, 3, 5) \)

- \( (\sigma \cdot \rho_{1,2,3}) \) are the only three independent orderings that exist for six gluon scattering which have no common two or three particle poles in common with \( \sigma \)

\[ \Rightarrow \text{no single, double or triple collinear singularities at subleading colour} \checkmark \]

- subtract divergences associated with single and double soft gluons only \checkmark
in the single soft limit the interferences factorize in the following way,

\[ M_{n+1}^{0,\dagger}(\cdots, a, i, b, \cdots) M_n^0(\cdots, c, i, d, \cdots) \xrightarrow{i \rightarrow 0} \]

\[ J_\mu(p_a, p_i, p_b) \epsilon^\mu(p_j) J_\nu(p_c, p_i, p_d) \epsilon^\nu(p_i) M_{n+1}^{0,\dagger}(\cdots, a, b, \cdots) M_n^0(\cdots, c, d, \cdots) \]

\[ = \frac{1}{2} (S_{aid} + S_{bic} - S_{aic} - S_{bid}) M_{n+1}^{0,\dagger}(\cdots, a, b, \cdots) M_n^0(\cdots, c, d, \cdots) \]

where \( S_{ajc} = \frac{2s_{ac}}{s_{aj}s_{jc}} \) is the squared eikonal factor

- eikonal factor with uniquely identified radiators and unresolved momenta is mapped to a three parton antenna function

- \( d\sigma_{NNLO}^{S,a} \) single unresolved subtraction term constructed from a product of differences of tree-level three parton antenna functions and reduced colour ordered matrix element interferences
Double unresolved subtraction term

- in the double soft limit the interferences factorize in the following way,

\[ \mathcal{M}_{n+1}^{0,\dagger}(\cdots, a, i, j, b, \cdots) \mathcal{M}_{n+1}^0(\cdots, c, i, d, \cdots, e, j, f, \cdots) \xrightarrow{i,j \to 0} \]

\[ J_{\mu_1\mu_2}(p_a, p_i, p_j, p_b) \epsilon^{\mu_1}(p_i) \epsilon^{\mu_2}(p_j) J_{\nu_1}(p_c, p_i, p_d) J_{\nu_2}(p_e, p_j, p_f) \epsilon^{\nu_1}(p_i) \epsilon^{\nu_2}(p_j) \]

\[ \times \mathcal{M}_{n}^{0,\dagger}(\cdots, a, b, \cdots) \mathcal{M}_{n}^0(\cdots, c, d, \cdots, e, f, \cdots) \]

- when summing the subleading colour contribution explicitly over all colour permutations the tree level double soft current drops out from the limit using its symmetric properties

- the double soft limit at subleading colour can be written completely in terms of contractions of tree level single soft currents

\[ \Delta_6^0(p) \bigg|_{slc} \xrightarrow{5,6 \to 0} (S_{153} + S_{254} - S_{154} - S_{254})(S_{163} + S_{264} - S_{162} - S_{364})|A_4^0(1, 2, 3, 4)|^2 \]

\[ + (S_{153} + S_{254} - S_{154} - S_{254})(S_{162} + S_{364} - S_{164} - S_{263})|A_4^0(1, 2, 4, 3)|^2 \]

\[ + (S_{152} + S_{354} - S_{154} - S_{253})(S_{162} + S_{364} - S_{163} - S_{264})|A_4^0(1, 3, 2, 4)|^2 \]

- at subleading colour no contribution from the colour connected double soft function in the limit \( \Rightarrow \) no four parton tree level antennae needed

- contribution produces \( 1/\epsilon^2 \) poles only
gluons only real-virtual colour summed cross section

\[ d\hat{\sigma}_{NNLO}^{RV} = \left( \frac{\alpha_s}{2\pi} \right)^2 \frac{N^4(N^2 - 1)}{3!} d\Phi_3(p_3, \ldots, p_5; p_1; p_2) J_2^{(3)}(p_3, \ldots, p_5) \sum_{\sigma \in S_5/Z_5} 2\Re \left[ A_5^\dagger(\sigma) A_5^1(\sigma) + \frac{12}{N^2} A_5^\dagger(\sigma) A_{5,1}^1(\sigma \cdot \rho) \right] \]

real-virtual subleading colour contribution written as a single interference summed over permutations

For \( \sigma = (1, 2, 3, 4, 5) \) \( (\sigma \cdot \rho) = (1, 4, 2, 5, 3) \)

\( (\sigma \cdot \rho) \) is the only independent ordering that exist for five gluon scattering which have no common neighbouring partons with \( \sigma \)

\[ \Rightarrow \text{no single collinear singularities in the subleading colour real-virtual contribution} \]

subtract divergences associated with single soft gluons only \( \checkmark \)
in the single soft limit the one-loop amplitude factorizes in the following way

\[ A_{5,1}^{1}(\cdots, a, i, b, \cdots) \xrightarrow{i \to 0} S^{0}(a, i, b)A_{4,1}^{1}(\cdots, a, b, \cdots) + S^{1}(a, i, b)A_{4}^{0}(\cdots, a, b, \cdots) \]

using symmetry properties the one-loop soft functions cancel in the total real-virtual cross section at subleading colour for five gluon scattering

obtain the following single unresolved subtraction term for a generic one-loop interference at subleading colour

\[ A_{5}^{0\dagger}(\cdots, a, i, b, \cdots)A_{5}(\cdots, c, i, d, \cdots) \xrightarrow{i \to 0} \]

\[ + X_{3}^{0}(a, i, d) \mathcal{M}_{4}^{0\dagger}(\cdots, (ai), b, \cdots)\mathcal{M}_{4,1}^{1}(\cdots, c, (id), \cdots) \]

\[ + X_{3}^{0}(b, i, c) \mathcal{M}_{4}^{0\dagger}(\cdots, (bi), \cdots)\mathcal{M}_{4,1}^{1}(\cdots, (ic), d, \cdots) \]

\[ - X_{3}^{0}(a, i, c) \mathcal{M}_{4}^{0\dagger}(\cdots, (ai), b, \cdots)\mathcal{M}_{4,1}^{1}(\cdots, (ic), d, \cdots) \]

\[ - X_{3}^{0}(b, i, d) \mathcal{M}_{4}^{0\dagger}(\cdots, (bi), \cdots)\mathcal{M}_{4,1}^{1}(\cdots, c, (id), \cdots) \]

no three parton one-loop antennae needed at subleading colour
NNLO antenna subtraction

Implementation checks (gluons only channel full colour in $pp \rightarrow 2j$):

- subtraction terms correctly approximate the matrix elements in all unresolved configurations of partons $j, k$

$$\begin{align*}
\hat{d}\sigma_{NNLO}^{RR,RV} & \xrightarrow{\forall\{j,k\},\{j\} \rightarrow 0} \hat{d}\sigma_{NNLO}^{S,T} \\
\end{align*}$$

- local (pointwise) analytic cancellation of all infrared explicit $\epsilon$-poles when integrated subtraction terms are combined with one, two-loop matrix elements

$$\begin{align*}
\mathcal{Poles} \left( \hat{d}\sigma_{NNLO}^{RV} - \hat{d}\sigma_{NNLO}^{T} \right) &= 0 \\
\mathcal{Poles} \left( \hat{d}\sigma_{NNLO}^{VV} - \hat{d}\sigma_{NNLO}^{U} \right) &= 0
\end{align*}$$

- process independent NNLO subtraction scheme

- allows the computation of multiple differential distributions in a single program run
Numerical setup \((gg \to gg + X)\)

- Jets identified with the anti-\(k_T\) jet algorithm with resolution parameter \(R = 0.7\)
- Jets accepted at rapidities \(|y| < 4.4\)
- Leading jet with transverse momentum \(p_t > 80\) GeV
- Subsequent jets required to have at least \(p_t > 60\) GeV
- MSTW2008nnlo PDF
- Dynamical factorization and renormalization scales equal to the leading jet \(p_T\) \((\mu_R = \mu_F = \mu = p_{T1})\)
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Integrated cross section results

\[
\begin{align*}
\sigma_{\text{incl.jet}}^{8\text{TeV} - \text{LO}} &= (9.6495 \pm 0.001) \times 10^5\text{ pb} \\
\sigma_{\text{incl.jet}}^{8\text{TeV} - \text{NLO}} &= (12.152 \pm 0.001) \times 10^5\text{ pb} \\
\sigma_{\text{incl.jet}}^{8\text{TeV} - \text{NNLO}} &= (15.20 \pm 0.02) \times 10^5\text{ pb} \quad \text{← leading colour} \\
\sigma_{\text{incl.jet}}^{8\text{TeV} - \text{NNLO}} &= (12.40 \pm 0.01) \times 10^5\text{ pb} \quad \text{← subleading colour}
\end{align*}
\]

- NNLO result increased by about 26% with respect to the NLO cross section
- subleading colour contribution \(\sim 8\%\) to the full colour NNLO coefficient
inclusive jet $p_T$ distribution at NNLO ($gg \to gg + X$)

- $gg \to gg$ subprocess in full colour with same PDF for all fixed order predictions
- all jets in an event are binned
- NNLO correction stabilizes the NLO $k$-factor growth with $p_T$
inclusive jet $p_T$ distribution at NNLO $(gg \rightarrow gg + X)$

- $gg \rightarrow gg$ subprocess in full colour with same PDF for all fixed order predictions
- all jets in an event are binned
- subleading colour contribution to the NNLO cross section is at the 2% level
double differential inclusive jet $p_T$ distribution at NNLO

- $\mathcal{O}(s) = 8$ TeV
- anti-$k_T$ $R=0.7$
- MSTW2008nnlo
- $\mu_R = \mu_F = p_T$

| $|y|<0.3$ | $|y|<0.8$ | $|y|<1.2$ | $|y|<2.1$ | $|y|<2.8$ | $|y|<3.6$ | $|y|<4.4$ |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| NLO/LO    | NNLO/NLO  | NNLO/LO   | NLO/LO    | NNLO/NLO  | NNLO/LO   | NLO/LO    |

- $gg \rightarrow gg$ subprocess in full colour
- NNLO result varies between 26% to 12% with respect to the NLO cross section
- similar behaviour between the rapidity slices
double differential inclusive jet $p_T$ distribution at NNLO

- subleading colour contribution to the NNLO cross section is at the 2% level
- similar behaviour between the rapidity slices
double differential exclusive dijet mass distribution at NNLO

\[ \frac{d^2 \sigma}{d m_{jj} dy^*} \] (pb/GeV)

\[ \mu_R = \mu_F = p_{T1} \]

\( y^* = 0.5 \)

\( 0.5 \leq y^* < 1.0 \)

\( 1.0 \leq y^* < 1.5 \)

\( 1.5 \leq y^* < 2.0 \)

\( 2.0 \leq y^* < 2.5 \)

\( 2.5 \leq y^* < 3.0 \)

\( 3.0 \leq y^* < 3.5 \)

\( 3.5 \leq y^* < 4.0 \)

\( 4.0 \leq y^* < 4.5 \)

\( \sqrt{s} = 8 \text{ TeV} \)

anti-\( k_T \), \( R = 0.7 \)

MSTW2008nnlo

\( T^1 = p_F^{\mu} = R^{\mu} \)

\( \frac{m_{jj}}{\text{GeV}} \)

\( y^* \leq 0.5 \)

\( y^* \leq 1.0 \)

\( y^* \leq 1.5 \)

\( y^* \leq 2.0 \)

\( y^* \leq 2.5 \)

\( y^* \leq 3.0 \)

\( y^* \leq 3.5 \)

\( y^* \leq 4.0 \)

\( y^* \leq 4.5 \)

\( y^* = 1/2|y_1 - y_2| \) slices

double differential k-factors

- \( gg \rightarrow gg \) subprocess in full colour
- NNLO corrections up to 20% with respect to the NLO cross section
- similar behaviour between the \( y^* = 1/2|y_1 - y_2| \) slices

\( \bar{y} \)
double differential exclusive dijet mass distribution at NNLO

\[ \frac{d^2 \sigma}{d m_{jj} dy^*} \]

\( \sqrt{s} = 8 \text{ TeV} \)

anti-\( k_T \), \( R = 0.7 \)

MSTW2008nnlo

\( \mu = \mu_F = p_T \)

\( y^* < 0.5 \) (preliminary)

\( y^* < 1.0 \) (preliminary)

\( y^* < 1.5 \) (preliminary)

\( y^* < 2.0 \) (preliminary)

\( y^* < 2.5 \) (preliminary)

\( y^* < 3.0 \) (preliminary)

\( y^* < 3.5 \) (preliminary)

\( y^* < 4.0 \) (preliminary)

\( y^* < 4.5 \) (preliminary)

- subleading colour contribution to the NNLO cross section is at the 2% level
- similar behaviour between the rapidity slices
inclusive jet $p_T$ scale dependence ($gg \rightarrow gg + X$)

- scale dependence study at leading colour
- dynamical scale choice: leading jet $p_T$
- same PDF and $\alpha_s$ for all fixed order predictions
- flat scale dependence at NNLO

\begin{itemize}
  \item $\sqrt{s}=8$ TeV
  \item $\text{anti-}k_T$ $R=0.7$
  \item MSTW2008nnlo
  \item $\mu_R = \mu_F = \mu$
  \item $80 \text{ GeV} < p_T < 97 \text{ GeV}$
\end{itemize}
Conclusions

- **antenna subtraction method generalised** for the calculation of NNLO QCD corrections for exclusive collider observables with partons in the initial-state

- **explicit** $\epsilon$-poles in the matrix elements are **analytically** cancelled by the $\epsilon$-poles in the subtraction terms

- **non-trivial** check of **analytic** cancellation of **infrared** singularities between double-real, real-virtual and double-virtual corrections

- **successful** treatment of **colour-correlated** matrix elements in the subleading colour calculation with the antenna subtraction method

- **subleading colour** NNLO corrections in the gluons only channel at 2% level

- **proof-of principle** implementation of the $gg \rightarrow gg$ full colour **contribution** to $pp \rightarrow 2j$ at NNLO in the new NNLOJET parton-level generator

Future work:

- develop **colour space** approach for NNLO calculations within the antennae method → James Currie talk

- **include** remaining **channels**
  - 4g2q processes
  - 2g4q processes
  - 6q processes
Back-up slides
inclusive jet $p_T$ distribution

- **80GeV < $p_T$ < 96GeV**
- **1148GeV < $p_T$ < 1388GeV**

- Inclusive jet cross section versus R
- NNLO corrections smaller for small R but $p_T$ dependent