PathCAS: An Efficient Middle Ground for Concurrent Search Data Structures

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Abstract
To maximize the performance of concurrent data structures, researchers have often turned to highly complex fine-grained techniques, resulting in efficient and elegant algorithms, which can however be often difficult to understand and prove correct. While simpler techniques exist, such as transactional memory, they can have limited performance or portability relative to their fine-grained counterparts. Approaches at both ends of this complexity-performance spectrum have been extensively explored, but relatively less is known about the middle ground: approaches that are willing to sacrifice some performance for simplicity, while remaining competitive with state-of-the-art handcrafted designs. In this paper, we explore this middle ground, and present PathCAS, a primitive that combines ideas from multi-word CAS (KCAS) and transactional memory approaches, while carefully avoiding overhead. We show how PathCAS can be used to implement efficient search data structures relatively simply, using an internal binary search tree as an example, then extending this to an AVL tree. Our best implementations outperform many handcrafted search trees: in search-heavy workloads, it rivals the BCCO tree [5], the fastest known concurrent binary tree in terms of search performance [3]. Our results suggest that PathCAS can yield concurrent data structures that are relatively easy to build and prove correct, while offering surprisingly high performance.

CCS Concepts: • Computing methodologies → Concurrent algorithms; Shared memory algorithms.

Keywords: concurrent data structures, search trees, non-blocking algorithms, lock-free, synchronization primitives

1 Introduction
Significant work has been invested in building scalable concurrent variants of fundamental data structures, and fast implementations are now known for many instances, from search trees to hash tables, or to containers such as queues and stacks [25]. On the one hand, designs based on fine-grained locking or fine-grained lock-free algorithms, have arguably emerged as the best-performing solution [3]. Yet, such designs tend to have high complexity, and are notoriously difficult to analyze and prove correct.

On the other hand, significant attention has been given to general techniques for obtaining fast and simple concurrent data structures. The classic example is transactional memory (TM) [24], which is now available in software, hardware, and hybrid variants, and allows one to derive concurrent implementations from sequential ones with lower programming effort relative to fine-grained designs. When TM is available, such designs can provide excellent performance.

However, TM-based designs still have drawbacks. Software TM (STM) provides a hardware-independent alternative to HTM, but can incur higher overheads. Moreover, although hardware transactional memory (HTM) is technically available on many platforms, via Intel’s TSX/TSX-NI, IBM’s POWER8/9 TM and ARM’s TME [30], it is notably missing from AMD chips (despite the proposal of ASF [10] more than a decade ago), and has been disabled in Intel and recent POWER processors [14, 27, 29], due to various concerns, chief among which is security.

In this context, it is natural to seek a middle ground between the high efficiency, but high complexity, of fine-grained designs, and the relative ease-of-use, but potential pitfalls, of general designs such as the ones based on TM. A number of
techniques exploring this trade-off have been investigated
over the years, often based on either restricted STMs or ex-
tended Multi-Compare Multi-Swap (MCMS) [43] imple-
mentations. However, as we illustrate later in the paper, known
instances of these techniques often fail to scale in the context
of high-performance data structures.

We revisit this question, and present a primitive for build-
ing correct and efficient concurrent search data structures
from scratch, called PathCAS. PathCAS combines key ideas
from efficient multi-word compare-and-swap (KCAS), and
transactional memory, to allow for concurrent data struc-
tures which are both efficient, and easier to reason about than
hand-crafted data structures using fine-grained primitives.

This mechanism is reminiscent of STM techniques, but it
has two semantic restrictions: (1) PathCAS does not guar-
antee opacity, and (2) PathCAS has a bounded read-set. These
restrictions allow for significant performance benefits, as ex-
emplified in Figure 1, which we believe are worth the increase
in programming complexity compared to TM. Despite Path-
CAS being less expressive than TM, it is still sufficient to
implement useful data structures.

We begin by describing the semantics and rationale behind
PathCAS in Section 2, and then illustrate how it can be used
to implement a simple concurrent unbalanced internal BST,
a data structure whose concurrent implementations war-
ranted publication on their own, e.g. [13, 16, 17, 26, 35, 36],
in Section 3. To illustrate the difficulty of implementing cor-
rect variants of these data structures, we note that, during
our investigation, we identified a correctness bug in the
lock-based internal BST of Drachslcr et al. [16], and that the
publicly-available implementations of the lock-free internal
BSTs of [26] and [36] fail experimental validation tests, and
still lack complete correctness proofs. (We defer a detailed
discussion of these issues to the full version of this paper.)
In this context, PathCAS provides an implementation that is
both efficient and is easy to prove correct.

To further highlight the expressive power of PathCAS, we
also show how a lock-free balanced version of this tree can be
derived, creating an implementation of a fully-internal
relaxed AVL tree (Section 4), which performs favorably when
compared to state-of-the-art solutions (please see Figure 1).

On the practical side, we present two efficient implementa-
tions of PathCAS: an HTM-enabled one which targets Intel
systems, and a software-only variant applicable to AMD
systems, and use them to empirically validate the above
data structure designs. We perform an in-depth comparison
against previous methods: from HTM- and STM-based de-
signs, to fine-grained lock-free variants, across both Intel and
AMD systems with up to 256 threads. We find that PathCAS
data structures are highly competitive, across the range from
read-heavy to update-heavy workloads. In particular, our
unbalanced BST implementation manages to outperform the
state-of-the-art in unbalanced BSTs, and our balanced im-
plementation matches the performance of the fastest known
balanced BST in read-mostly workloads.

In sum, PathCAS introduces an additional point in the
trade-off between expressiveness and programming effort,
on the one hand, and efficiency of the resulting data struc-
tures, on the other. Although PathCAS builds on known tech-
niques, the resulting mechanisms are novel, and our search
tree implementations can achieve state-of-the-art results on a
variety of workloads.

2 Related Work

The question of identifying synchronization primitives with the "right" balance between expressivity and efficiency is
as old as the field of concurrency. We describe related work
with the goal of situating PathCAS on the spectrum of syn-
chronization techniques that have been developed.

Treiber [44] gave one of the first illustrations of a non-
blocking data structure via CAS, while seminal work by
Herlihy [23] showed that CAS is universal. Anderson and
Moir gave constant-time implementations of Load Link (LL)
and Store Conditional (SC) from CAS [1], which is more
expressive than CAS and helps circumvent ABA problems.
Luchangco et al. [31] expand on LL/SC by implementing
k-compare-single-swap, allowing for the change of a single
field to be conditional on multiple fields containing their expected values. Another extension of LL/SC by Brown et
al. [7] introduced LLX and SCX, which function on data
records. Data records contain multiple related fields which
are loaded by LLX, and SCX only succeeds in changing a
single memory location if none of the fields loaded by LLX
have changed since its invocation. LLX/SCX is less expressive
than PathCAS, as it atomically: changes one field to a new
value, and marks some number of nodes. Brown et al. [7]
introduced several search tree design based on LLX/SCX,
one of which we compare with in the experimental section
(ext-chromatic-LF).

Harris et al. [21], introduced a lock-free version of KCAS,
which is the basis of our implementation along with opti-
mizations from Arbel-Raviv and Brown [2]. KCAS facilitates
atomic multi-word updates, however it does nothing to sim-
pify the arguments around values read but not updated, for
example the path followed during a search.

Timnat et al. [43] introduced a direct evolution of KCAS
which they called Multi-Compare Multi-Swap (MCMS) [43].
Their proposal has similar goals to our work, attempting to
achieve a “middle-ground” between performance and ease
of implementation. MCMS attempts to simplify the implementation of concurrent data structures by increasing the expressiveness of KCAS, allowing fields to compared without being swapped. When HTM is available, this algorithm attempts to carry out operations in a transaction as a fast-path, similar to our approach.

One key difference is that, in its slow path (or if HTM is not available), the MCMS algorithm incurs high synchronization costs. Specifically, when using MCMS for search data structures, one would need to include the entire search path in the arguments to MCMS. On the software code path, MCMS would write to every node on the entire search path, including the root, both in updates and in searches. This aborts all concurrent hardware transactions, likely causing cascading aborts on NUMA systems. In turn, this induces a global contention bottleneck at the root of the search tree. Another key difference is that, on machines without hardware TM, we offer high performance, whereas MCMS essentially becomes the HFP KCAS algorithm. We present a comparison of MCMS and PathCAS in the experimental section.

The work of Mahreshanski et al. [32] analyzes interplay between HTM and other concurrent designs and how they function together. This work suggests that HTM does not obviate other designs, but can be used to improve them. We apply a similar technique in PathCAS, using HTM as a fast path for operations, and falling back to our software algorithm when transactions fail a certain number of times.

Guerraoui and Trigonakis present Optik [20], which is a methodology for implementing concurrent data structures using optimistic concurrency and versioned locks. PathCAS is built using similar low-level techniques, and encapsulates their complexities in an expressive primitive.

Herlihy and Moss [24] proposed HTM to provide flexible hardware support for non-blocking data structures. Shavit and Touitou introduced software TM [38], as a software-only alternative. PathCAS has similarities to software transactional memory (STM). STM is easier to use than HTM, as STM does not have the same space limitations, but it suffers from various overheads, such as requiring locks per word, dependency on dynamic data structures, and function call overheads on reads and writes.

Kumar et al. [28] introduced Hybrid Transactional Memory (HyTM), which is a combination of HTM and STM. Our experiments include results from state-of-the-art HyTM algorithms. To accelerate TM, other work has attempted to break up transactions to avoid overheads. One example is the speculation-friendly tree [19], which uses ElasticTM [18]. However, this tree has relatively poor performance compared to the state of the art, as we show in Section 5.

3 PathCAS

We provide an overview of the PathCAS primitive, and show how it can be used to implement a concurrent data structure. Data structures implemented with PathCAS should be node-based. (A data structure can have many different types of nodes, and PathCAS can be used to modify any or all of them.) PathCAS combines ideas from KCAS and version based validation; the rest of this section will provide a description of these components and how they interact.

3.1 Background

In essence, PathCAS is a generalization of KCAS with additional capabilities. KCAS is semantically similar to compare-and-swap (CAS), with the key difference that it is able to atomically change multiple addresses (which do not have to be contiguous). KCAS supports a single operation in the form of: \( KCAS(\text{addr}_1, \text{oldValue}_1, \text{newValue}_1, ... \text{addr}_k, \text{oldValue}_k, \text{newValue}_k) \) KCAS does the following atomically: if \( \text{addr}_i \) contains \( \text{oldValue}_i \) for all \( i \), the value stored at \( \text{addr}_i \) is changed to \( \text{newValue}_i \) for all \( i \) and returns true. If not, false is returned.

Our implementation of PathCAS builds on the lock-free KCAS implementation of Harris, Fraser and Pratt (HFP) [21].

Harris, Fraser and Pratt (HFP) algorithm. A KCAS operation first creates a KCAS descriptor \( D \) that contains the arguments to the KCAS as well as a status word that indicates whether the KCAS is InProgress, Succeeded or Failed. It then performs a sequence of atomic double-compare-single-swap (DCSS) operations to change all addresses from their respective old values to point to the KCAS descriptor only if the status is still InProgress. (DCSS atomically determines whether two (potentially non-contiguous) addresses contain their respective old values, and if so, changes one to a new value and returns true. Otherwise it returns false.)

If all of the DCSS operations are successful, then the status is changed to Succeeded and the addresses are all changed from the descriptor pointer to their respective new values using CAS. Otherwise, the status is changed to Failed and the addresses are changed back to their old values. This atomic change to the status field decides the outcome of the operation (and dictates the behaviour of helper threads).

Since old values are replaced by descriptor pointers, any time a thread reads an address that could be modified by a KCAS, it must use a special KCASRead function that knows how to handle descriptor pointers. In particular, whenever KCASRead encounters a KCAS descriptor, it will help the corresponding KCAS operation to complete (by performing the same set of DCSSs and CASs that would be performed by the thread that initially invoked the KCAS). The use of DCSS avoids ABA problems that could otherwise be introduced by this lock-free helping [21]. Crucially, DCSS prevents any helper from storing new pointers to the KCAS descriptor once the status has become Succeeded or Failed (preventing helpers from resurrecting completed KCAS operations).

The KCAS descriptor pointer behaves conceptually like a lock that grants exclusive access of a field to a KCAS operation, rather than to a particular thread. Once all addresses contain pointers to a KCAS descriptor, they can only be...
changing in accordance with the corresponding KCAS operation. If the KCAS descriptor’s status is Succeeded, then all helpers will try to change addresses to their respective new values. The value contained in a memory address logically changes when the status of the descriptor changes to Succeeded, and a successful KCAS is linearized then. If the status is Failed, helpers will try to revert addresses to their old values. A failed KCAS is linearized when it saw a value that did not match the address’ old value. Changing an address from a descriptor pointer to a value conceptually unlocks it.

The authors implemented lock-free DCSS in software, using CAS and DCSS descriptor objects to facilitate helping. There is no need to pierce the atomic DCSS abstraction in our work, except to mention that DCSS descriptors exist.

3.2 Semantics

Whereas KCAS takes all of the addresses to be modified (and their respective old and new values) as explicit arguments, the PathCAS interface is closer to transactional memory. In the following, we say a node has been visited (resp., an address addr has been added) if there has been an invocation of visit(n) (resp., add(addr, ...)) since the last invocation of start(). PathCAS offers operations to:

- \text{start()} gathering arguments for a PathCAS,
- \text{read(addr)} an address that might be modified via PathCAS,
- \text{add(addr, old, new)} an address addr to be changed atomically from old to new,
- \text{visit(n)} a node n, and
- \text{validate()} to check whether any visited nodes have changed since they were visited. validate succeeds and returns true only if no such change has occurred.

To allow for implementations with diverse progress properties, validate can fail and return false spuriously.

- \text{exec()} performs a KCAS according to the arguments passed to invocations of add since the last \text{start}. That is, if all added addresses contain their respective old values, then exec succeeds, changing all added addresses to their respective new values and returning true. Otherwise it returns false.

- \text{vexec} performs exec only if validate would succeed.

Behaviour is undefined if an address is added multiple times with conflicting old and new values. If a node is visited multiple times (after a particular invocation of \text{start}), then any changes to it after the earliest such visit will cause exec and validate to return false.

Note that \text{start}, \text{add} and \text{exec} can simply be viewed as syntactic sugar for accumulating arguments to a KCAS operation, and \text{read} is essentially KCASRead. However, visit and vexec have no direct analogue in KCAS. To emulate the behaviour of \text{visit(n)} and \text{vexec} using KCAS, one could include every address in node \( n \) in the arguments to the KCAS, “changing” each address from its current value \( v \) to \( v \).

3.3 Implementation

At a high level, the algorithm differs from HFP in the following ways: we implement the syntactic sugar described above for incrementally accumulating arguments, and we add a new validation phase wherein visited nodes are inspected to determine whether they have changed since they were visited. Validation affects progress in subtle ways.

Basic operations: start, read, add, visit A PathCAS descriptor consists of a status field, a sequence of \((addr, old, new)\) triples denoted entries, and a sequence of \((node, ver)\) pairs denoted path. A start() operation creates a new descriptor, and we refer to it as the thread’s descriptor (until start is invoked again). Similarly to KCASRead, a read(addr) reads \( addr \), and if it sees a pointer to a descriptor, then it helps the corresponding exec or vexec to complete (more about helping below), and repeats these steps. If it sees a non-descriptor value, that value is returned. An \text{add(addr, old, new)} adds a triple to the thread’s descriptor’s entries.

Version numbers are used to track changes to the data structure’s nodes. More specifically, each node is augmented with a version number \( ver \) that should be incremented every time the node is changed. The programmer using PathCAS is responsible for ensuring that s/he increments the version numbers of any node \( n \) that s/he modifies using PathCAS. This simply entails reading \( n.\text{ver} \) and invoking \text{add(node, ver, v, v+1)} to increment the value \( v \) that was read from \( n.\text{ver} \). We discuss the motivation behind the decision further in Section C.1.

A visit(n) operation reads the version \( v \) of node \( n \) using \text{read}, saves \( \langle n.\text{ver}, v \rangle \) in the thread’s descriptor’s path, and returns \( v \).\(^2\) The use of \text{read} means visit(n) will help any exec or vexec it encounters that is in the process of modifying \( n \).

\text{vexec} An invocation of \text{vexec} simply passes the thread’s descriptor to a subroutine called help and returns the result.

Consider the set \( S \) of addresses added to the thread’s descriptor desc (i.e., the addresses that should be changed by this PathCAS operation). An invocation of help(desc) first uses DCSS to change all of the addresses in \( S \) from their respective old values to point to the PathCAS descriptor. If any of these DCSSs fail, then all of the addresses that were changed to point to the PathCAS descriptor are reverted to their old values using CAS. Otherwise, now that all addresses are conceptually locked for this PathCAS operation, we can start validation. The two red lines of code in Algorithm 1 are the only changes from the HFP KCAS algorithm.

Validation To perform validation, help invokes a subroutine called \text{validateDesc(desc)}, which rereads the version number of each visited node and checks whether it has changed, \( \text{\footnote{Since the read-set (i.e., path) is bounded, we should mention what happens if the read-set size is exceeded. In our code, exceeding the read set size triggers an assertion. In practice, we imagine that the programmer will either over-allocate a large array for visited addresses, or will implement data structures for which a practical height bound is known.}} \)}
Algorithm 1 PathCAS::help(desc)

1: // Phase 1: "lock" addresses for this PathCAS
2: if desc.state == Undecided then
3:   newState = Succeeded
4:   for each (addr, old, new) in desc.entries do
5:     retry_desc:
6:     valueSeen = DCSS((addr, old, desc), (desc.state, Undecided))
7:     if isDescriptor(currentVer) then
8:       help(valueSeen) = DCSS failed because of other PathCAS
9:       goto retry_desc
10:   else if valueSeen # old then
11:     newState = Failed
12:     DCSS failed because old # addr
13:   end for
14:   if newState == Succeeded and not validateDesc(desc) then
15:     newState = Failed
16:   end if
17: // Phase 2: "unlock" addresses to new or old values according to state
18:   result = (desc.state == Succeeded)
19:   for each (addr, old, new) in desc.entries do
20:     CAS(addr, desc, (result ? new : old))
21:   end for
22: return result

Algorithm 2 PathCAS::validateDesc(desc)

1: for each (node, visitVer) in desc.path do
2:   currentVer = node.ver
3:   if currentVer = desc then
4:     continue // "locked" for our PathCAS
5:   if isDescriptor(currentVer) and currentVer # desc then
6:     return false // "locked" for a different PathCAS
7:   if currentVer # visitVer or (visitVer & 1) then
8:     return false // node's version has been changed or marked
9: end for
10: return true

discuss the practical considerations of using version numbers, namely wrapping, in the full version of the paper.

To simplify and optimize the implementation of data structures that mark nodes when removing them, we steal the least-significant bit from each node’s version number to indicate whether the node has been marked. (In data structures without marking, this bit is simply not used.) Validation succeeds only if all visited nodes are unmarked. In a data structure that marks nodes, success implies that no visited node has been deleted. Storing the marked bit in the same word as the version number allows a node to be marked as deleted at the same time as its version number is updated with minimal overhead. (Note that visit returns the mark along with the version number.)

If validation succeeds, none of the visited nodes have changed (or been deleted, in a data structure with marking) since they were visited. In this case, the PathCAS descriptor’s status field is changed from InProgress to Succeeded using CAS. Otherwise, it is changed from InProgress to Failed using CAS. The status field changes to either Succeeded or Failed it cannot change again. Finally, if the status is Succeeded, the addresses in S are changed to their new values using CAS. Otherwise, their old values are restored via CAS.

Helping As in the related KCAS algorithms, since old values are replaced by descriptors, a special read() function (analogous to KCASRead()) must be used to read any fields that can ever be modified by PathCAS. A read() function that encounters a descriptor pointer will help the corresponding PathCAS operation to complete. Helpers perform the same sequence of steps as the thread that first invoked vexec for this PathCAS. Note that the validation phase will be performed by all helpers, and slow helpers may fail validation even if a fast helper succeeded. However, a slow helper that fails validation cannot revert addresses to old values, since it will attempt to do so using CAS, and this CAS will fail if the node no longer points to the same PathCAS descriptor (with the same version). Moreover, as long as a node points to the PathCAS descriptor, it cannot cause validation to fail.

Progress and helping At this point, one might wonder why forward progress is guaranteed even though an operation O can invoke read and begin helping another operation O’ before O has finished invoking add on all of its fields: Can this cause O and O’ to abort each other? We note that such helping also occurs the lock-free HFP KCAS (in KCASRead). The key observation is: although O can help another operation before O has finished adding its addresses, the operation being helped must have already finished adding all of its own addresses. So, such mutual aborts cannot occur. Progress is discussed in greater detail below.

exec The exec operation is just a stripped down version of vexec that does not perform validation. It can be implemented simply by removing all pairs for visited nodes from the thread’s descriptor before invoking help. The intention of including exec in the interface is to allow nodes to be visited during a data structure traversal in case validation will be needed, and then to decide not to validate (reducing overhead) at the end of the traversal.

validate The validate operation simply passes the thread’s descriptor desc to validateDesc and returns the result.

3.4 Correctness and Progress

Correctness The exec operation is the same as the linearizable lock-free HFP KCAS algorithm, and is linearized in the same way. In other words, for a successful exec, we linearize at the change to the descriptor’s status field, and for a failed exec, we linearize at the read (of an unexpected, non-descriptor value) that caused the failure. Of course, if a vexec is performed but no nodes were visited, then vexec is the same as exec, and is linearized the same way.

The case where a vexec is performed after some nodes were visited is more nuanced. Recall that many helper threads can participate in a single vexec operation O by invoking
help(desc), where desc is O’s descriptor. The helpers will collaborate to first “lock” all addresses, then perform validation, then use CAS to change the descriptor’s status, then “unlock” all addresses. Only one helper will successfully change the descriptor’s status, and we call that helper the decider. Once the status field is changed, the behaviour of all helpers is dictated by its contents. Two cases arise.

If no visited node has its version number changed (or marked) between when it was visited and when the decider rereads its version number during validation, then validation succeeds. Given that validation succeeds, vexec behaves the same as a successful HFP KCAS (matching the PathCAS semantics). We linearize just before the decider invokes validate(desc), at which point all added addresses are “locked” and no visited node had changed.3

However, if some visited node has its version changed (or marked) between when it was visited, and when the decider rereads its version number during validation, then vexec will behave like a failed HFP KCAS, restoring old values and returning false (matching the PathCAS semantics). We linearize when the value that caused the failure was read.

Progress The progress guarantees for PathCAS are subtle. The start and add operations are wait-free. The visit, exec and vexec operations only perform a constant number of steps in addition to an invocation of help, but help can invoke itself recursively. The latter is also true in the HFP KCAS algorithm, and it manages to guarantee lock-free progress with an assumption that the addresses passed to KCAS are sorted. If we make the same assumption, then it is possible to argue that visit, exec and vexec operations are lock-free.

However, lock-freedom only guarantees that infinitely many operations will terminate in an infinite execution—not that any of them will succeed. To see why this could be a problem, consider a data structure with two nodes, A and B. Suppose thread t1 visits A and adds B (to change B’s value), and thread t2 visits B and adds A (to change A’s value). If t1 and t2 both “lock” their respective added nodes, then both perform validation, both will fail validation and “unlock,” terminating; and hence satisfying lock-freedom, but perhaps preventing the data structure using PathCAS from making progress. The problem here is that both vexec operations can fail spuriously, even though the non-descriptor values that are semantically contained in A and B have not changed.

3.5 Avoiding spurious failures
It is impossible to avoid vexec failures altogether. One can always invoke vexec after adding addresses with unreasonable old values that they have never contained. However, the above implementation allows every vexec to fail spuriously, simply because a visited node contained a descriptor pointer. To be able to implement lock-free data structures using PathCAS, we need to change this. Without loss of generality, in the following, we focus on vexec operations (since exec operations are just a special case).

We say a thread t invokes a reasonable add(addr, old, new) if the old value was read from addr at some point since the last invocation S of start by t. If a thread invokes start followed by a sequence of reasonable add operations, followed by a vexec, then we call the vexec reasonable. With a small modification to vexec, we can guarantee the following.

Property P1. If each thread t invokes only reasonable vexec operations, then whenever a vexec Vt fails, another vexec has succeeded since Vt’s start operation, St.

Strong vexec In the implementation described previously, a vexec fails validation simply because it sees a descriptor, and “unlocks” all of its nodes. Rather than failing spuriously, vexec can fall back to a slower lock-free code path on which it creates a new copy of its descriptor with slightly different contents. This new descriptor contains all of the added fields of the old one, but crucially, all of the visited (node, ver) pairs in the old descriptor are converted into added (node.ver, ver, ver) triples. These triples are then sorted.

Finally, this new descriptor is passed as the argument to an exec operation, which will effectively “lock” all of the visited nodes’ version numbers rather than simply validating them.

In practice, to reduce overhead, before switching to this slow path, vexec can repeatedly try again (a bounded number of times) using an exact copy of its descriptor and performing validation as usual. Since the slow path has high overhead, the number of retries can be tuned to avoid invoking the slow path except where it is really necessary. One can also try contention management strategies such as bounded exponential backoff to further reduce slow path usage.

Note that the choice of vexec or strong vexec does not affect performance in our experiments, as spurious failures are sufficiently infrequent that there is no need to switch to the slow path.

How strong vexec helps Strong vexec is not vulnerable to the progress problem described above. Suppose thread t1 visits A and adds B, and thread t2 visits B and adds A. If t1 and t2 both “lock” their respective added nodes, then both perform validation, both will fail validation and “unlock,” but they will not terminate. Rather, they will retry. They can retry only a bounded number of times before executing the slow path. Once both are executing the slow path, they will each try to lock A then B (because of address sorting), and one of them will succeed.

Let us sketch why P1 is satisfied. A reasonable vexec Vt does not fail when it fails validation. Rather, it fails only if (a) one of its reasonable added addresses contains an unexpected non-descriptor value, or (b) one of its visited nodes’ version numbers has been incremented. (In both cases, Vt might help
one or more other vexec operations to complete before it can read a non-descriptor value.) In case (a), since the added address contained its reasonable old value at some time since $S_t$, and it can be changed to a different non-descriptor value only by a successful vexec, $P_1$ holds. Similarly, in case (b), the visited node’s version number was read since $S_t$, and a node’s version number is incremented only when the node is changed by a successful vexec, so $P_1$ holds.

3.6 Optimizing descriptor management
Arbel-Raviv and Brown [2] showed how to transform the HFP algorithm to eliminate the need to allocate and free descriptors for DCSS and KCAS. The same transformation can be applied to PathCAS, allowing each thread to reuse one PathCAS descriptor (and we do this in our experiments). The transformation in [2] is straightforward and mechanical, but it makes the pseudocode much more difficult to read, so we presented pre-transformation code. Similarly, to avoid complicating the code, we treated DCSS as an atomic primitive. (In reality it is implemented in software as in [2].)

3.7 Optimizing with hardware TM
On systems with support for hardware TM, the PathCAS algorithm above can be used as a fallback code path, and a faster hardware TM based algorithm can be used as a fast path. In other words, we can use a hardware transaction to perform vexec/exec atomically without the overhead of synchronizing via DCSS and CAS.

Our hardware TM based fast path is simply obtained by taking the software algorithm above, wrapping it in a transaction, and then performing a sequence of sequential optimizations (which do not affect correctness because of the atomicity of hardware transactions).

3.8 Comparison to transactional memory
PathCAS is most similar to a lock-free, non-opaque, bounded, object-based TM that is compiled directly into the data structure (rather than being compiled as a library). Such a highly restricted TM implementation could avoid many of the same traditional TM overheads that we also avoid: incremental validation to guarantee opacity, locks per word (instead of version numbers per node), dynamic data structures such as hash tables with intrusive lists (instead of a simple array for our visited nodes), and function call overhead for reads and writes. However, such a TM would be no easier to use than PathCAS, and to our knowledge no such TM exists. Moreover, it would be a substantial undertaking to design an efficient TM with these properties.

3.9 Design Decision: Manual Version Numbers
We contemplated building the incrementing of version numbers into the abstraction, so that it would be automatic. However, we decided that requiring addresses passed to add to be fields of nodes might be overly restrictive. We do not want to rule out applications wherein PathCAS could be used to atomically validate a set of nodes, and also modify arbitrary fields that do not belong to a data structure node (such as a size variable). Therefore, we only require nodes that are passed to visit to have version numbers to track changes, and leave it to the programmer to manage them. Note that our interface supports debugging mechanisms to catch errors in managing version numbers. In an application where it is acceptable to restrict PathCAS so that it only accesses nodes, one could easily change add() to also take a node pointer in addition to the field pointer, and automate version increments.

4 Application: Lock-free Internal BST
In this section we provide a concrete example of how to create a data structure using PathCAS, namely, a concurrent set implemented as an internal binary search tree.

Operations The tree supports the following operations. contains(key) returns true if key is in the tree, and false otherwise. insert(key, val) returns false if key is in the tree. Otherwise, it inserts key and value returns true. delete(key) returns false if key is not in the tree. Otherwise, it deletes key and its associated value and returns true.

Data structures Tree nodes have fields for left and right children, a key, a value, and a version number ver as required by PathCAS.

To avoid special cases, the tree always contains two sentinel nodes with keys $-\infty$ and $+\infty$. Consequently, every node with key $k \in (-\infty, +\infty)$ always has both predecessor and successor nodes. The sentinel with key $+\infty$, which we call the maxRoot, is the root of the entire tree. The sentinel with key $-\infty$, which we call the minRoot, is the left child of maxRoot. No field of maxRoot is ever changed. All keys in $(-\infty, +\infty)$ are always found in the right subtree of minRoot.

Implicit read() Our pseudocode exemplifies a feature of our PathCAS implementation in C++: implicit read invocations. Whereas KCASRead() calls must be explicitly added by the programmer, in C++, templates and operator overloading can be used to invoke PathCAS read() calls automatically. Thus, in our BST pseudocode we do not explicitly invoke the PathCAS read function, but the reader should note: any field that is ever modified by PathCAS is accessed using read.

4For example, visit can save the address ranges of all visited nodes, and exec can then check for intersections between the visited nodes and added addresses that do not have a corresponding node ver increment. This introduces overhead, but can be enabled only when debugging.

5The programmer need only annotate the types of fields that can be modified by KCAS in the data structure node type definition, by wrapping each field’s type in a special PathCAS template type. For example, int key becomes casword<int> key. This requires very little effort, and can even help us catch some types of programmer errors, such as unsafe writes to fields that can be modified with PathCAS.
Algorithm 3 BST::search(key)

1: while true do
2:   parent = maxRoot
3:   parentVer = visit(parent)
4:   curr = minRoot
5:   currVer = visit(curr)
6:   while curr ≠ NIL do
7:     currKey = curr.key
8:     if key == currKey then
9:       return (true, curr, currVer, parent, parentVer)
10:    parent = curr
11:   if key > currKey then curr = curr.right
12:   else key < currKey curr = curr.left
13:   parentVer = currVer
14:   currVer = visit(curr)
15: return (false, curr, currVer, parent, parentVer)

Figure 2. Error in contains operation without validation

Search The search(key) procedure (Algorithm 3) is invoked by contains, insert and delete. It performs a traditional BST search until it encounters a NIL pointer, or finds a node containing key. search returns a tuple of five items with types: ⟨Boolean, node, version, node, version⟩. If the key key is found, this tuple contains true, followed by the node that contains key and its version number (observed during search), followed by its parent and its version number. If key was not found, then search returns false, followed by the final node it encountered (before seeing a NIL pointer) and the version number of that node. The remaining two fields are ignored in this case. We return these two nodes (and their versions) to be used by insert and delete. The key difference between this search and a sequential BST search is that each node is passed to an invocation of visit.

Contains The contains(key) operation invokes search(key), followed by validate(). If validation succeeds, then the entire search was effectively atomic (since the entire path was visited), so we return true if search found key and false otherwise. If validation fails, we retry the contains from scratch.

One might wonder why contains performs validation. Figure 2 depicts an error that can occur without validation in an internal BST with atomic updates. In that execution, contains(50) reaches node 60 then sleeps. Then, delete(40) atomically deletes 40, promoting the successor key 50 in its place. When contains(50) wakes up and continues its search, it will conclude that 50 is not in the tree and return false. This is incorrect, as 50 has been in the tree throughout the entirety of the contains(50) operation. Validation would catch this error, since delete(40) changes a node that contains(50) has already visited.

Validation makes arguing correctness trivial (validated searches are atomic), and only incurs a small amount of overhead. We discuss an optimized implementation that performs less validation in Section 4.1.

Insert The insert(key, val) operation (Algorithm 4) first invokes search(key) to determine whether key is already in the tree, and to locate the parent whose child pointer should be changed to insert a new node containing key (if key is not already in the tree).

If the search finds key, then validate is invoked to determine whether any the nodes visited by search changed since they were visited. If validation succeeds, it establishes a time t during the insert operation when key was already in the tree, so false is returned (the insert is linearized at time t).

If search does not find key, then a new node containing key and val is created, and add is invoked so that the appropriate child pointer of parent will be changed (by a subsequent vexec) to point to this new node. Since we are trying to change parent, add is invoked to cause the parent’s version number to be incremented.

Finally, vexec is invoked to (attempt to) atomically change the added addresses only if none of the visited nodes have changed since they were visited. If it succeeds, we linearize at the vexec. Otherwise, we retry the insert from scratch.

Delete The delete(key) operation (Algorithm 6) first searches for the key to be deleted, similar to insert.

If search does not find key, the path followed in search is validated. If this validation is successful, false can be returned as a time has been established when the entire path traversed by search, which did not contain key, was atomically contained in the tree. Thus, the tree did not contain key at some time during the delete, and we can linearize at that time. If validation fails, delete is retried from scratch.

If search finds key, the node curr containing key is returned, along with its parent. delete() then checks whether these nodes are marked (and hence deleted already). If either is marked, the delete is retried from scratch.

Next, delete reads curr.left and curr.right to determine how many children curr has. It does not matter if the number of children is counted incorrectly, for example, because curr.left...
is changed between these two reads. If \texttt{curr} changes, our subsequent \texttt{vexec} will fail and the \texttt{delete} will retry. We would not have to consider this possibility at all if we were using opaque transactional memory instead of PathCAS, but we would argue that this reasoning is not onerous to avoid the overheads that come along with opacity.

As in a sequential internal BST, three cases arise. In each case, we use \texttt{vexec} to perform the sequential update atomically. If \texttt{vexec} succeeds, \texttt{delete} returns \texttt{true}, and we linearize at the \texttt{vexec}. If \texttt{vexec} fails, the \texttt{delete} is retried from scratch.

**Leaf deletion** If \texttt{curr} has no children, \texttt{vexec} is invoked to unlink and mark it, and to increment the parent ver.

**One child deletion** If \texttt{curr} only has a single child, \texttt{vexec} is invoked to replace \texttt{curr} by its child, marking \texttt{curr} and incrementing parent ver.

**Two child deletion** If \texttt{curr} has two children, we will try to replace its key and value with those of its successor \texttt{succ}, then delete the node \texttt{succ} (exactly as one does in a sequential internal BST). We first locate the successor \texttt{succ} and its parent \texttt{succP} using \texttt{getSuccessor}, which visits each node it traverses and returns the version numbers it saw. Note that the successor cannot have a left child (or else it is not the successor). So, \texttt{succ} has at most one child, which means it can be deleted using one of the previous two cases.

If it has a child, \texttt{succR}, then we change the appropriate pointer in the parent \texttt{succP} to \texttt{succR}. If it has no children, \texttt{succR} is NIL, so changing the appropriate pointer in \texttt{succP} to \texttt{succR} simply unlinks \texttt{succR}. We mark \texttt{succR} since it is being removed, and increment the versions of \texttt{curr} and \texttt{succP} since they are being changed. (If the successor happens to be the right child of \texttt{curr}, then \texttt{succP} and \texttt{curr} are the same node, so we only need to increment one of \texttt{succP} and \texttt{curr}). Note that the success of \texttt{vexec} implies that \texttt{succ} actually is the successor of \texttt{curr} when the \texttt{delete} is linearized.

### 4.1 Optimizing to reduce validation

In \texttt{contains}, if \texttt{foundKey} is \texttt{true}, then it is unnecessary to \texttt{validate}, because the key can only be found if it was actually in the tree at some time during the contains, and we can linearize the contains at that time. (If a node was unlinked before \texttt{contains} began, then \texttt{contains} cannot reach it.)

### 4.2 Extension: AVL Trees with PathCAS

As a second example application of PathCAS, we extend our internal BST to perform relaxed AVL tree balancing. Due to lack of space, we only give an overview here. Complete details appear in the full version of the paper.
We augment the BST nodes described in the previous section with two new fields: `parent` and `height`. The former points to the node’s parent and the latter contains the logical height of the node, which can differ from the node’s actual height when the tree is unbalanced.

In `insert`, we initialize newly created nodes’ `parent` pointers to point to the node they are being inserted under. In `delete`, whenever we perform a `vexec` that removes an internal node, and hence changes the `parent` of a child, we add that child’s `parent` pointer to the `vexec` as appropriate.

A balance violation exists at node \( n \) when:
- \( n\text{.left.height} - n\text{.right.height} > 2 \); or
- \( n\text{.left.height} - n\text{.right.height} < -2 \); or
- \( n\text{.height} \neq 1 + \max(n\text{.left.height}, n\text{.right.height}) \)

A violation can be created by any operation that causes node to gain or lose children. Whenever an operation creates a violation, it performs rebalancing steps to fix the violation.

We implement the relaxed AVL tree rebalancing steps of Bougé [4]. Bougé’s proved that, starting from an arbitrarily unbalanced tree, after performing a bounded number of atomic rebalancing steps (wherever they can be applied in the tree, and in any order), the tree will converge to a balanced state. Rebalancing steps are local modifications that affect a small number of nodes, and they do not need to be performed atomically at the same time as a search. The rebalancing steps, namely `rotateLeft`, `rotateRight`, `rotateLeftRight`, `rotateRightLeft` and `fixHeight`, are very similar to the familiar AVL tree rotations. Rebalancing steps eliminate violations, or move them towards the root where they will be eliminated. A tree with no violations is balanced.

Whenever a thread creates a violation at a node, it takes responsibility for repairing that violation, and any subsequent violations it creates while repairing that violation. More specifically, after performing a successful `insert` or `delete`, a thread traverses towards the root, fixing any violations it sees until either: it fixes a violation at the root, it observes a node on the path towards the root that has no violation, or it encounters a `deleted` node on the path to the root (which means another thread has taken responsibility for any violations further along the path to the root). This is why we augment nodes with parent pointers: they allow us to easily “follow” violations up the tree.

### 4.3 Freeing data structure nodes

In unmanaged languages like C++, PathCAS manages its own memory, but the programmer must still manually reclaim memory for the data structures they implement using PathCAS. Reclaiming nodes that are deleted by a `vexec` (or `exec`) is quite simple using an algorithm such as DEBRA or NBR [6, 39]. The C++ implementation of DEBRA used in [9] offers operations `getGuard()` and `retire(node)`. The former is invoked at the beginning of each data structure operation. The latter can be invoked on any node after it is unlinked using `vexec` (or `exec`). This will perform a `delayed free` once no thread has a pointer to `node`. We use DEBRA to reclaim memory for all data structures in our experiments.

Using DEBRA is so mechanical that the necessary invocations of `retire` could even be integrated directly into `vexec` (and `exec`), by having a successful `vexec retire` each node whose `version` it marks just before returning.

### 5 Evaluation

Our experiments follow the methodology of [9], and we use the authors’ publicly available benchmark, `Setbench`. We compare against state-of-the-art hand-crafted trees, as well as several TM-based trees (see Figure 4). We experimented with update rates (1%, 10% and 100%) and uniform key ranges \((2 \times 10^5, 2 \times 10^6, 2 \times 10^7)\). Each trial pre-filled the data structure to contain half of the keyrange, then ran for 10 seconds. Data is averaged over six trials with min/max bars shown in red.

Our AMD system has two EPYC 7662 CPUs, each with 64 cores and two hardware threads per core, for a total of 256 hardware threads, and a 256MB shared L3 cache. Threads...
Figure 4. The list of algorithms in our experiments.

| Balanced BSTs          | Unbalanced BSTs          |
|------------------------|--------------------------|
| ext-bst-lf             | Balanced BSTs            |
| ext-bst-rlf            | pext-avl-occ             |
| ext-bst-locks          | int-avl-pathcas           |
| pext-bst-locks         | int-avl-pathcas+          |
| int-bst-pathcas        | HTM fast-path + HTM/STM  |
| int-bst-pathcas+       | slow-path                |

Table 1. Detailed analysis for 100% updates, 256 threads.

Comparing balanced trees

The bottom three plots in Figure 3 compare our AVL tree (int-avl-pathcas) with other balanced BSTs, including pext-avl-occ [5], which is known to be the fastest concurrent BST in many workloads [3]. In read-mostly workloads, int-avl-pathcas is competitive with pext-avl-occ, and outperforms the other trees. In the 100% update workload, int-avl-pathcas is at most 20% slower than the fastest algorithm. The fact that int-avl-pathcas is not drastically outperformed by the highly tuned and intricate pext-avl-occ tree is remarkable. The external-If tree, which is implemented using the LLX and SCX primitives, does not fare nearly as well.

TM-based trees

It should be noted that in an attempt to be generous to these TM approaches, in our implementations we compiled each TM in the same compilation unit as the data structure (rather than as a linked library), and force-inlined all TM code, eliminating the overhead of function calls to the TM code from the data structure. This optimization would be unrealistic in practice, however should give the TM implementations the best performance possible for comparison. Despite this, the TM based algorithms in Figure 3 still suffer from high instruction counts and LLC miss rates (Figure 5). In particular, the extremely high instruction counts for int-avl-norec are due to contention on the global version lock and repeated read set validation to guarantee opacity.

The results in the introduction (Figure 1), from our Intel system, include more algorithms, since the system has hardware transactional memory support. In those results, our trees outperform the next fastest algorithm, TLE, by nearly 2x. Moreover, those results are "generous" to TLE, since its global locking fallback code path degrades performance dramatically in workloads with more updates.

5.1 Comparison with MCMS

To compare PathCAS with MCMS, we extended Timnat’s original C++ code for the MCMS linked list. We note that we found some bugs in his implementation of MCMS, one of which only affects the source code, and one of which affects the algorithm in the MCMS paper. Details are to appear in the full version of this paper. We fixed these bugs, and applied the same lock-free descriptor optimizations that we use, and implemented an internal BST using MCMS to validate the entire search path (similarly to how we validate the entire search path using PathCAS). The implementation is optimized to the best of our ability: for instance, it avoids performing MCMS in cases where searches return true or inserts return false. Moreover, deletes that return true perform their modifications in small MCMS operations that do not incur the key range, and allowing a key that was marked as deleted to be reinserted simply by changing a bit. This may inflate its performance in the types of workloads used in our experiments.
We also compared PathCAS with OTM-based approaches, and can rival the state-of-the-art in hand-crafted designs. While in this work we focus on the two trees presented, we emphasize that one can use PathCAS in a direct way to implement many data structures wherein an operation requires overhauling Setbench to support the background rebalancing threads used by ext-bst-elastic, and would require us to completely redesign its memory reclamation.

So, we obtained performance numbers for ext-bst-elastic using Syncrobench, instead of Setbench. We also obtained performance numbers for ext-bst-lf2 on the same workload, as it is included in Syncrobench as well as setbench. Results appear in Figure 7. This gives us a sort of limited point of comparison between our results in Setbench and our results in Syncrobench. Although the results do not allow for a rigorous comparison between ext-bst-elastic and the other trees in our experiments, we note that it is much slower than ext-bst-lf2, which is in the middle of the pack in our Setbench experiments. And, we also note that ext-bst-elastic is being evaluated in an environment that is presumably most favourable to it, as ext-bst-elastic was developed by the authors of Syncrobench, and integrated therein by the authors. Additionally observe that this 1% update workload is quite favourable to ext-bst-elastic, as its performance degrades faster than ext-bst-lf2 as the update rate increases.

6 Conclusion

This paper introduced PathCAS, a primitive used to implement efficient concurrent data structures while maintaining lower complexity than hand-crafted techniques. PathCAS utilizes an HTM fast path combined with an efficient fallback path that relies on KCAS, version numbers, and search path validation.

We implemented a set of historically difficult data structures using PathCAS, including an internal balanced tree, and have shown them to achieve competitive performance with the best fine-grained variants. We compared the performance of our data structures with both TM-based and hand-crafted variants of these structures, showing that PathCAS surpasses TM based approaches, and can rival the state-of-the-art in hand-crafted designs.

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7 Artifact

We have provided a docker container in which you can download and start running the experimental benchmark.

- The docker container was prepared on Ubuntu 20.04, which is also the OS the actual container is running

- In particular, we build our container using Docker version 20.10.2, build 20.10.2-0ubuntu1 20.04.2.

- Our experiments are run on a machines with 384 GB of RAM and 192 threads total, while possible to run these experiments on a different machines with less memory or threads:
  - If you have less memory, you may need to use smaller data structure sizes
  - If you have fewer threads, you will simply need to use less threads

Inside the docker container, in the experiments folder, you will find a script called run.sh that builds all data structures and runs a simple test (using each data structure). This script is driven by the experimental configuration described in experiments/_common.py. By default, the contents of experiments/_common.py cause run.sh to reproduce Figure 3. More details on how to modify this experimental configuration appear below.

Note that run.sh performs experiments for both transactional memory and non-transactional memory data structures. This can take quite some time, so you may want to limit the number of trials in _common.py to a relatively small number.

7.1 Step By Step Instructions

1. Install docker (ideally the same version we use) on your system with your preferred package manager

2. Download the docker image from Zenodo (https://zenodo.org/record/5728166). For example, on Linux you might do this by executing:

   $ wget https://zenodo.org/record/5728166/files/ppopp2022pathcas.tar.gz

3. Add the image to your local Docker images (running in the same directory as step 2):

   $ docker load -i ppopp2022pathcas.tar.gz

4. Launch the docker container

   $ docker run -i -t -privileged ppopp2022pathc as /bin/bash

   Note that privileged is required in order to ascertain the proper thread pinning strategy for the experiments, and to record performance counters for, e.g., cache statistics. You may also need to set kernel.parf_event.paranoid to -1 on Linux.

5. Go to the experiments folder

   $ cd experiments

6. The experiments can be configured in _common.py. Be sure to edit the HOST CONFIGURATION and EXPERIMENTAL CONFIGURATION sections of experiments/_common.py to match the machine you are running on, and to reflect the types of experiments you would like to run. Note that the various configuration parameters are described in the comments.
By default, _common.py is configured to reproduce Figure 3 in the paper. Running such experiments can take many hours (at least 12). **If you would like to run a shorter test to get started**, please uncomment the more restrictive testing values of ins_del_fractions, max_keys, exp_duration_millis, thread_counts and num_trials provided alongside the default values in _common.py before proceeding. (Once you have run your small test, you can comment those testing values out again.)

Additionally, Smaller example test values are provided if you wish to run a quick proof of concept test (commented out next to the actual set values).

7. To run the experiments described in _common.py, simply execute $ ./run.sh. It will run experiments, store the results in a sqlite3 database, process those results using various SQL queries, and produce several text data files and plot images (more on this below). If the testing values in _common.py are uncommented, this should take less than five minutes. If the testing values are commented out, then run.sh will perform all necessary experiments to reproduce Figure 3 in the paper. **This should take approximately 12 hours.**

While experiments are being performed, the scripts will produce output of the form "step 000001 / 000336...". Each such line contains the command being executed, as well as a very rough estimate of the remaining running time for all experiments.

8. After the runs are complete, **PNG format plot images** for the experiments can be found in directories experiments/data_tm and experiments/data_non_tm. Note that, by default, a large super set of the plots in the paper are generated.

To more easily view these images, you may want to copy them from the docker container to your host machine using docker cp. For example, *while the docker container is running*, you can run the following command on the host machine, where <CONTAINER_NAME> is the name of the running container:

```bash
$ docker cp <CONTAINER_NAME>:/root/tmbench/experiments .
```

This will copy all experimental results to the host machine. Alternatively, you can browse the results in text format directly inside the docker container:

a. A detailed summary of the numerical data can be obtained by starting in the experiments directory and running ./basic_info.sh to produce a pretty-printed table, with columns for update rate, data structure name, key range size, thread count and throughput. If you ran both TM and Non-TM experiments, this script will print a table for each type of results.

b. Note that you can also perform your own arbitrary SQL queries on the sqlite database, either by modifying the queries in basic_info.sh, or by entering an interactive sqlite console in either of the data_* directories in experiments/:

```bash
$ sqlite3 output_database.sqlite
```

There are also CSV-format tables of data that are converted into various plots in file names of the form: `{DATA_STRUCTURE_TYPE}_{METRIC}_u{UPDATE_RATES}-k{MAX_KEY}.FILE_TYPE` For example, data_tm/bst_tm_total_throughput sec-u50.0_50.0-k20000000.txt contains a table of data that is used to produce a throughput comparison plot for TM-based binary search trees, with 50% inserts and 50% deletes, and a key range size of 20 million (i.e., initial data structure size of 10 million). This example is of particular note, as it corresponds to one of the plots in Figure 3.

9. **Note that, due to the nature of docker containers, all data will be lost if you exit the docker run.** If you want to save any of your generated data, you can do so using docker cp from a different terminal on the host machine to copy the relevant data from the container to the host.

Alternatively, you can use docker commit to save a new version of the docker image that includes all of the data you’ve generated. You can then launch that image in a docker container to access your data again.

10. Once you have saved any data you want to keep, you can exit the docker container by running exit in the container, or docker stop <CONTAINER_NAME> on the host.

**LLC misses on AMD** Note the artifact fails to track LLC misses on recent AMD processors. Instead, they must be tracked manually using perf stat.

For example, on our machine, we used a command of the form: `[usual command prefix up until the binary] perf stat -e l3_comb_clstr_state.request_miss [actual binary being run] [args].`

Also note that will produce a raw number of LLC misses for the entire execution, so one will need to divide by the number of operations completed. This number will be somewhat inflated because it includes experimental setup and tear-down as well as prefilling. To get around this, use perf...
record -k CLOCK_MONOTONIC, which timestamps all cache misses with a clock that is compatible with our benchmark. Then one can take a pair of timestamps emitted by our benchmark, REALTIME_START_PERF_FORMAT and REALTIME_END_PERF_FORMAT, and plug them into perf report -ns -time <start>,<stop> where <start> and <stop> are our timestamps.

**Generating Figure 5** Figure 5 is generated by running the following SQL queries on the results databases (starting in the same directory as _common.py and run.sh).

```bash
for exp in tm non_tm ; do
  ../setbench/tools/data_framework/run_experiment.py _exp_${exp}.py -q "select maxkey, TOTAL_THREADS, alg, mem_maxresident_kb, PAPI_L3_TCM, PAPI_TOT_CYC, PAPI_TOT_INS, tree_stats_avgKeyDepth from data where ins_del_frac = '50.0 50.0' order by maxkey desc, total_threads desc, alg" ; done
```
Supplementary Material

A  Additional PathCAS Details

Algorithm 7 PathCAS::execute()

```
1:   desc = getDescriptor()  // Gets the current thread’s descriptor
2:   for tries = 0; tries < 5; tries++ do 
3:     if ((status = _xbegin()) == _XBEGIN_STARTED) then
4:       if not validate(desc) then _xabort(OLD)  // Validation
5:         for each (addr, old, new) in desc.entries do
6:            val = *addr  // Check if addresses contain old values
7:              if val # old then
8:                if isDescriptor(val) then _xabort(DESCRIPTOR)
9:                  else _xabort(OLD)  // Write new values
10:             end for
11:             for each (addr, old, new) in desc.entries do
12:               *addr = new  // Write new values
13:             end for
14:             _xend()
15:             return true
16:         else if status & _X_ABORT_EXPLICIT then
17:           if _X_ABORT_CODE(status) == DESCRIPTOR then break
18:           else if _X_ABORT_CODE(status) == OLD then return false
19:             end for
20:             return help(desc)  // Fallback code path
```

The HTM implementation (Algorithm 7) first checks all the fields added to the KCAS descriptor to ensure that all the fields hold the old value from the descriptor. If any of these fields contain the incorrect old value, the transaction is explicitly aborted. If the incorrect value read was a KCAS descriptor, the slow path is taken, we cannot simply return false, as we do not know the logical value of this field. If the value was not a KCAS descriptor, then the KCAS can simply return false, as it contained a value other than the one in the descriptor.

If all the fields are read to contain the old values in the descriptor, they are now in the read set of the transaction. Any change to any of these fields will result in an abort, and the transaction will retry up to a fixed number of attempts. If the transaction succeeds in writing all the new values to the fields with no conflicts, the transaction will commit, and true can be returned.

C  Lock-freedom of PathCAS

Without loss of generality, in the following, we focus on vexec operations (since exec operations are just a special case).

The start and add operations are wait-free. The read, visit and vexec operations only perform a bounded number of steps in addition to an invocation of help, but help can invoke itself recursively. We prove that these operations are lock-free. Suppose, to obtain a contradiction, that threads take infinitely many steps in read, visit or vexec operations, but only finitely many such operations terminate. Clearly, threads that take infinitely many steps in such an operation must take infinitely many steps in help.

The only way a thread can take infinitely many steps in help is if it continues performing new recursive invocations of help at line 8 of the pseudocode for help. As in the HFP algorithm, (1) a vexec can only be helped at line 8 if it has “locked” some node, and (2) because all addresses are sorted and “locked” in order, if a vexec O1 is helping another vexec O2, then all of O2’s “locked” nodes come strictly after all of O1’s “locked” nodes in the sort order. Since there are infinitely many invocations of help and only finitely many invocations of vexec, eventually some thread’s call stack must contain a helping cycle: an invocation of help(desc1) that (possibly indirectly) invokes help(desc2) that (possibly indirectly) invokes help(desc1). This implies that the nodes “locked” for desc1 come strictly after themselves in the sort order—a contradiction. So, no such cycle can exist, which implies that help can only be invoked finitely many times at line 8 of the pseudocode for help. Therefore, read, visit and vexec must be lock-free.

C.1 Design Decision: Manual Incrementing of Version Numbers

We contemplated building the incrementing of version numbers into the abstraction, so that it would be automatic. However, we decided that requiring addresses passed to add to be fields of nodes might be overly restrictive. We do not want to rule out applications wherein PathCAS could be used to atomically validate a set of nodes, and also modify arbitrary fields that do not belong to a data structure node (such as a size variable). Therefore, we only require nodes that are passed to visit to have version numbers to track changes, and leave it to the programmer to manage them. Note that our interface supports debugging mechanisms to catch errors in managing version numbers. In an application where it is acceptable to restrict PathCAS so that it only accesses nodes, one could easily change add() to also take a node pointer in addition to the field pointer, and automate version increments.

C.2 Using Version Numbers

Version number based validation can theoretically cause ABA problems if version numbers wrap around. However, the probability of an actual failure is extremely low. Suppose version numbers are 64 bits long, and a slow helper is attempting to use DCSS to change an address from ⟨old, version = 17⟩ to ⟨new, 18⟩. For an ABA problem to cause this DCSS to erroneously succeed, the helper must sleep for exactly the

\[\text{Example: visit can save the address ranges of all visited nodes, and exec can then check for intersections between the visited nodes and added addresses that do not have a corresponding node ver increment. This introduces overhead, but can be enabled only when debugging.}\]
right amount of time for this specific address to undergo precisely \( k \cdot 2^{64} \) changes for \( k \in \mathbb{Z}^+ \) before waking up and attempting the DCSS.

## D AVL Extension

Within the main body of this work, we emulated a relatively simple example of using PathCAS by outlining a internal, unbalanced, BST. However PathCAS can also be leveraged for the creation of much more complex structures, and one such example is a similar tree, but augmented to be self-balancing. Rebalancing steps are based on the relaxed AVL tree of Bougé [4].

![AVL Node Structure](image)

**Figure 8. AVL Node Structure**

In order to facilitate rebalancing, the structure of the node must change in two key ways: by tracking the logical height of the node, and containing a pointer to its parent. Node store logical heights (stored in the height field), and also have an actual height (the number of nodes below this node within the tree to the farthest leaf + 1), which can differ. We say a node is balanced if the logical heights of its children differ by less than 2.

These two new fields can be see within Figure 8, where \( P \) is the parent pointer and \( height \) is the height field. The other fields are shared with the BST shown in the main body of this work.

Five rebalancing steps are proposed: rotateLeft(node), rotateRight(node), rotateLeftRight(node), rotateRightLeft(node), and fixHeight(node). These rotations are very similar to sequential AVL tree rotations, and those outlined by [4]. fixHeight(node) updates the height field of a node to \( 1 + \max(hL, hR) \), where \( hL \) (resp., \( hR \)) is the height field of its left (resp., right) child (with the eventual goal of propagating accurate height throughout the tree). Bougé proves that applying these rebalancing steps in a tree, wherever they apply, and in any order, until no more can be applied, yields a strict AVL tree.

The operations from the previous section carry over with two changes: After a successful insertion just before line 10 of insertIfAbsent, we invoke rebalance(p) (Algorithm 10) and just before line 37 of delete we invoke rebalance(sp). Generally, **after any tree modification, rebalance is invoked on any node that (may have) gained or lost children.**

The rebalance operation determines whether to perform a rotation on \( n \), update its height, or do nothing based on its apparent balance. \( n \)'s apparent balance is calculated by checking the heights of its children (Line 12). If \( n \) requires rebalancing (the apparent balance is \( \geq +2 \) or is \( \leq -2 \)), a direction is determined: a positive apparent balance indicates that \( n \)'s left subtree is larger and requires a rotation in the opposite direction, and vice-versa. Depending on the balance of \( n \)'s children, a single rotation may be insufficient to repair the imbalance of \( n \). If this is the case, a double rotation is used, which involves applying a single rotation to one of \( n \)'s children, and another one to \( n \). For performance reasons, we combine double rotations into a single large PathCAS. (By doing this we can also avoid updating version numbers, etc., twice.) A visualization of tree states that lead to the possible rotation is shown in Figure 9.

![Possible Rotations](image)

**Figure 9. Possible Rotations**

The simple case in which only a single rotation is required (Line 24) is addressed by rotateRight (Algorithm 11, Figure 13) or rotateLeft (symmetric). In our example, rotateRight, we move \( n \)'s left child, \( l \), to \( n \)'s position. Additionally, \( n \) replaces its left child pointer to \( l \) with \( l \)'s right child, \( lr \). New heights for all nodes involved are calculated based on this rotation (except for parent, as it will be checked for imbalance after this operation anyways) and are updated as part of the PathCAS. Note that in Figure 13 the version numbers of \( ll \) and \( r \) are blue as they nodes that are part of the sequential rotation but are not modified, but are validated as part of vexec. This is because the heights of these nodes are used to calculate the new heights for other nodes, and changes to these nodes could result in a rotation that does not improve the balance of the tree. If \( n \) does not need rebalancing, we call fixHeight to ensure its height is accurate with respect to its children. Note that rotations may move nodes off of this parent pointer path towards the root, potentially making threads unable to reach nodes they need to rebalance. Thus, after a rotation is successful, rebalance is called recursively on every node that had its left or right fields changed.

![AVL delete Leaf](image)

**Figure 10. AVL delete Leaf**
Algorithm 8 fixHeight(n, nVer)
1: l = n.left
2: r = n.right
3: rVer = r.ver
4: if l ≠ NIL then
5: lVer = visit(l)
6: add(&l.ver, lVer, lVer)
7: if r ≠ NIL then
8: rVer = visit(r)
9: add(&l.ver, rVer, rVer)
10: oldHeight = n.height
11: newHeight = 1 + max(l.height, r.height)
12: if oldHeight == newHeight then
13: if n.ver == nVer and (l == NIL or lVer == lVer) and (r == NIL or r.ver == rVer) then
14: return UNNECESSARY
15: else return FAILURE
16: add(&n.height, oldHeight, newHeight)
17: add(&n.ver, nVer + 2)
18: if vexec() then return SUCCESS
19: return FAILURE

E Unbalanced Tree Correctness Proof
Definition E.1. The search path to a key k is the path on an atomic search(k) would traverse.

Definition E.2. A node is in the data structure if it is reachable from maxRoot.

Lemma E.3. No successful PathCAS ever modifies the fields of marked nodes.

Proof. This follows from inspection of the code. Before reading any other field of a node, the version number of this node is read. If this node is to be involved in the PathCAS, this version number is checked to ensure the node is not...
Algorithm 10 rebalance(n)

1: while n ≠ minRoot do
2: start()
3: nV = node.ver
4: if nV & 1 then return
5: p = n.parent
6: pV = visit(p)
7: if pV & 1 then continue
8: l = n.l
9: if l ≠ NIL then lV = visit(l)
10: r = n.right
11: if r ≠ NIL then rV = visit(r)
12: if IV & 1 or rV & 1 then continue
13: lh = (left == NIL ? 0 : l.height)
14: rh = (right == NIL ? 0 : r.height)
15: nBalance = lh - rh
16: if nBalance ≥ 2 then
17: ll = l.left
18: if ll ≠ NIL then llV = visit(ll)
19: lr = l.right
20: if lr ≠ NIL then lrV = visit(lr)
21: if lV & 1 or lrV & 1 then continue
22: llH = (ll == NIL ? 0 : ll.height)
23: lrH = (lr == NIL ? 0 : lr.height)
24: llBalance = llH - llh
25: if llBalance < 0 then
26: if rotateLeftRight(p, pV, n, nV, l, lV, r, rV) then
27: rebalance(n); rebalance(l); rebalance(r);
28: n = p
29: else if rotateRight(p, pV, n, nV, l, lV, r, rV) then
30: rebalance(n); rebalance(l); rebalance(r);
31: n = p
32: else if nBalance ≤ -2 then
33: [...] » Reverse case
34: else
35: res = fixHeight(n, nV)
36: if res == FAILURE then continue
37: else if res == SUCCESS then n = n.parent
38: else return

marked, and the version number is incremented as part of the PathCAS. If the node is marked, the operation is retried. If the node is not marked when it is checked, but is marked between this check and the PathCAS execution, the PathCAS will fail (the marking of a node directly involves the changing of its version number). □

Lemma E.4. Our implementation of an unbalanced binary search tree satisfies the following claims.

1. The node maxRoot always has minRoot as its left child, and NIL as its right child. The node minRoot has NIL as its left child. (Remark: the right child points to the rest of the tree.)
2. Consider any search, where r1...r1 is the sequence of nodes visited by it so far. For each ri in r1...r1, there is a time during the search when ri is in the data structure.
3. Consider an invocation I of PathCAS::validate() in an insertIfAbsent(key, val), delete(key) or contains(key). If I returns true, then path is the search path to k just before I.
4. a. The tree rooted at the right child of minRoot is always a valid binary search tree.
   b. Any insertIfAbsent or delete operation that performs a successful PathCAS returns the same value it would if it were performed atomically at its linearization point (the successful PathCAS).
   c. Any insertIfAbsent or delete operation that terminates without performing a successful PathCAS returns the same value it would if it were performed atomically at its linearization point.

Proof. Consider an arbitrary execution E. We prove these claims together by induction on the sequence of steps s1, s2, ... (which can be shared memory reads, atomic KCASRead operations, or atomic KCAS operations) in E.

Base case: Before any PathCAS is successful, the tree is in its initial state where two nodes exist: minRoot and maxRoot. maxRoot has the key -∞, its left child is minRoot, and its right child is NIL. minRoot has the key -∞, and both its children are NIL.

Algorithm 11 rotateRight(p, pV, n, nV, l, lV, r, rV)

1: if p.right = n then add(&p.right, n, l)
2: else if p.left = n then add(&p.left, n, l)
3: else return false
4: lr = l.right
5: llH = 1 + max(llH, rH)
6: if llH & 1 then return false
7: llBalance = llH - llh
8: if llBalance < 0 then
9: if rotateLeftRight(p, pV, n, nV, l, lV, r, rV) then
10: rebalance(n); rebalance(l); rebalance(r);
11: n = p
12: else if rotateRight(p, pV, n, nV, l, lV, r, rV) then
13: rebalance(n); rebalance(l); rebalance(r);
14: n = p
15: else if nBalance ≤ -2 then
16: [...] » Reverse case
17: else
18: res = fixHeight(n, nV)
19: if res == FAILURE then continue
20: else if res == SUCCESS then n = n.parent
21: else return
Inductive step: suppose the claims all hold before step $s$. We prove they hold after step $s$.

**CLAIM 1.** This configuration is the initial state of the tree, except the right child of minRoot could no longer be NIL. No operation can modify this state. minRoot and maxRoot contain keys that will never be part of any operation: no operation will search for, attempt to remove, or attempt to insert these keys. Additionally, these nodes will never be rebalanced: it is explicit in the code that rebalancing stops when it reaches the minRoot, which also means maxRoot will also never be rebalanced.

**CLAIM 2.** The only step that can impact this claim is a KCAS-Read in search that traverses to a new node, by reading a pointer from $r_l$ to some $r_{l+1}$. (This pointer is then added it to path[], the sequence of nodes visited so far.) So, $s$ is a KCASRead reading the left or right pointers of a node to move to another.

By the inductive hypothesis $r_l$ was in the data structure at some time before $s$ during the search. If $r_l$ still points to $r_{l+1}$ at step $s$, then since $r_l$ is in the data structure and points to $r_{l+1}$, so is $r_{l+1}$. Otherwise, $r_l$ was deleted before $s$ during the search, and right before this deletion $r_l$ pointed to $r_{l+1}$ (Lemma E.3). Therefore, $r_{l+1}$ was in the data structure at the time of this deletion (which was during the search).

**CLAIM 3.** Only successful PathCAS operations can affect this claim, as reads do not modify the structure of the tree. A search reads and stores the version numbers of all nodes encountered during a search. If any of these nodes change between when they were read as part of the search and when they are validated as part of validatePath, validatePath will return false. If validatePath returns true, it is guaranteed that no modification occurred to any node along the path between when it was read and when it was validated as part of validatePath. At the time $t$ just before $l$, all nodes were read as part of the search but are yet to be validated. Hence, if $l$ returns true, all the nodes still had the same version numbers read as part of the search at $t$ and this path was an atomic snapshot of the search path to $k$, just before the invocation of $l$ (before any nodes were validated).

**CLAIM 4A.** Only successful PathCAS operations that change the layout of the tree can affect this claim. KCAS operations are performed by insertIfAbsent and delete. We proceed by cases.

**Case 1:** Suppose $s$ is a successful KCAS at line 10 of insertIfAbsent. This KCAS will only be completed if a successful path validation occurred. Let $t$ be the time just before the successful invocation of validateDesc started. Claim 3 means that this path is the search path to $k$ at time $t$, and no node has changed since they were read during the operation. The node we modify to insert this new node is the final node on the search path at time $t$, denoted by last. In order for the search path to $k$ to change (meaning last to no longer the correct parent for a new node containing $k$) a concurrent operation must modify at least one node along the search path to $k$. However, if one of these nodes were to be modified, the KCAS would fail: validation would not have succeeded.

**Case 2:** Suppose $s$ is a successful vexec at line 37 of delete (we omit the PathCAS of the single or no child delete as it is strictly easier). This proof is similar to the previous case where $s$ is a PathCAS within insertIfAbsent. Let $t$ be the time just before the successful invocation of validatePath started. In delete, search locates a node $n$ that contains the key $k$, and the version number of $n$ is added to the PathCAS to be increment. getSuccessor locates the successor of $k$ by traversing the tree from $n$ and validating the path taken. Because the the invocation of validateDesc() in vexec() must return true for this operation to be successful, the node returned by getSuccessor is the successor of $k$ at $t$, by traversing the tree from $n$ and validating both the path taken to $k$ and the path taken to the successor. In order for this operation to be invalid, either $n$ must not contain $k$, or the successor returned by getSuccessor must be incorrect. However, for this to be the case, a concurrent operation must modify at least $n$ or a node along the path from $n$ to the successor found. $n$ cannot be modified, as it is protected by the PathCAS. If one of the nodes along the path is modified, the PathCAS would fail: validation would not have succeeded.

**CLAIM 4B.** In Claim 4a we actually proved that all operations that execute a successful PathCAS are atomic at time $s$, which is the time when the PathCAS was executed.

**CLAIM 4C.** insertIfAbsent only returns without performing a PathCAS if search locates a node that has the key $k$. From Claim 2 the key $k$ that we are trying to insert was in the data structure at some time $t$ during the search. $t$ is during insertIfAbsent, so we linearize this operation at $t$, returning false.

delete only returns without performing a PathCAS if search does not locate a node that has the key $k$. By Claim 3, the key $k$ that we are trying to remove was not in the data structure at some time $t$ during the search. $t$ is during delete, so we linearize this operation at $t$, returning false.

The linearization points for the operations are as follows:

- **insertIfAbsent**
  - returning true: at the successful PathCAS at line 10 of insertIfAbsent
  - returning false: the time $t$ during search where $k$ was in the data structure, from Claim 2
- **delete**
  - returning true: at the successful PathCAS of at line 37 of delete or at line successful PathCAS for the single or no child deletion in delete (whichever is used)
work are insufficient. This correctness issue was fixed in subsequent versions of the same work.

Theorem E.5. Our binary search tree implements a linearizable dictionary.

Proof: Lemma E.4 proves that the all the operations on the tree are atomic and do not violate any relaxed binary search tree properties. Searches are equivalent to an atomic search at some time \( t \) during \( \text{search} \). \( \text{contains} \) is simply linearized at this time \( t \) during \( \text{search} \).

\[ \square \]

F Motivation: The Hardness of Proving Fine-Grained Data Structures Correct

F.1 Overview

Correctness bugs exist even in peer reviewed works, going undetected for long periods of time. In this section we provide an overview of previously discovered issues in other work, and one new bug we found during our investigation.

Michael and Scott [34] discovered two race conditions in the lock-free concurrent queue by Valois [46] which lead to incorrect memory reclamation. These issues could corrupt the data structure in two ways: by freeing the same nodes multiple times, or freeing nodes that are still reachable. The memory reclamation scheme attempts to avoid ABA problems by relying on reference counters in data structure nodes, where threads increase the reference counter of a node when they read a pointer to it, and decrement the counter when they no longer require it. The thread that moves this reference counter to 0 after a node has been removed will reclaim this node. There exists a time, however, between when a pointer to a node is read by a thread and that thread increments the reference counter of the node. During this time another thread could move the reference counter of the node to 0 and free it. The thread about to increment the reference counter will be unaware of this, will increment the reference counter of the node to 1, then potentially back to 0, resulting in a double free. A similar issue can also result in nodes being freed despite still being in the data structure.

Shafiei [37] found an execution that dereferences a null pointer in the lock-free doubly-linked-list by Sundell and Tsigas [40] by running the Java PathFinder (JPF) model checker on the implementation. She describes how the sheer complexity of the algorithm makes it very difficult to reason about its correctness, and that the proofs provided in the work are insufficient. This correctness issue was fixed in subsequent versions of the same work.

As part of this work, we discovered a new bug in the balanced BST by Drachsler et al. [16]. This tree uses additional pointers in nodes to track the predecessor and successor of a node, which are used to recover from searches that end up in an incorrect location due to a concurrent rotation. An execution exists, however, wherein searches fail to find keys that are present during the entire search operation. We notified the authors of this work, who confirmed the bug via private communication and plan to release errata on the topic at a later date. A full explanation of the incorrect execution is in Appendix F.2.

There are two existing lock-free internal BSTs in the literature, due to Howley and Jones [26] (HJ12) and Ramachandran et al. [36] (RM14). Publicly available implementations of both trees fail our experimental validation checks, which test for consistency between the tree contents at the end of an experiment and the return values of all updates recorded throughout the experiment. The HJ12 tree has no accompanying proofs, although the authors did a model checker on some very small trees. We have been unable to resolve the correctness problems in RM14 with the authors, and it is unclear whether the algorithm is correct. (The paper cites a proof in a technical report, but the technical report does not appear to be available.) Both papers use highly intricate synchronization mechanisms: e.g., RM14 is described in 315 lines of pseudocode excluding code comments. By contrast, our algorithm is arguably short and easy to reason about. Each operation \( \text{visit} \) every node that it accesses, before accessing any of the node’s fields, and performs either a \( \text{validate} \) or \( \text{vexec} \) to establish a time when the entire operation takes effect atomically.

F.2 Drachsler Tree Bug

F.2.1 Overview. Consider the following tree structure:

This shows both the tree and logical ordering structure as shown in the paper. The bottom values are keys, and \( s \) and \( t \) fields represent the succLock and treeLocks respectively, a _ represents no thread holds the lock, but this will be filled in when a thread holds the lock. Assume now that a thread \( p \) inserts the value 175 to both the logical order and the tree, but has not rebalanced the tree as of yet (i.e. just before line 9 of algorithm 5 in the original paper) leaving the tree in a state as follows:
Thread $p$ now goes to sleep. Note that the treeLock for the node 150 is held by thread $p$, but the succLock is released as that operation is completed. A rotation should occur on the node 125. Before this rotation can occur, imagine a thread $q$ performs an insertion of 160 up until line 15 of Algorithm 3. This can occur because the succLock of 150 has been released by $p$ (only the treeLock is held) and the treeLock of 175 will be the one that is acquired for this operation. This will leave the tree in the following state:

Thread $q$ now goes to sleep. A third thread, $r$, does a contains operation for the key 160 to completion. It will search the tree as per Algorithm 1, then will backtrack and find 160 as per Algorithm 2. This operation will ignore all locks and return true. This implies the insertion of 160 must already have been linearized. Hence, the linearization point for insert described in the paper is incorrect (line 16, where pred.succ is updated) as the change is observed here, it must be before this line (line 15 is logical).

Next, a fourth thread, thread $s$, does the same operation as thread $r$ (contains(160)), however it goes to sleep after it traverses to node 125.

Thread $p$ now wakes up and will start a rotation, this will result in the function rebalance(node, child) being called with the node containing 125 as the “node” argument, and the node containing 150 as the “child” argument to the rebalance(node, child) function. Thread $p$ already has the treeLock for 150, and can freely acquire the treeLocks for 125 and 100 (the other ones required for this rotation). Note here that thread $q$ only holds the treeLock for 175. No threads hold succLocks at this point. $p$ can proceed (is not blocked by any currently held locks).

Thread $s$ now wakes up after this rotation, realizes it’s at a leaf node and completes its traversal of the tree. From there, it will attempt to follow the logical order to discover if it missed an update. Line 2 of Algorithm 2 will not traverse the list (as $node.key > k$ is false), but line 3 will traverse (as $node.key < k$) until the node containing 175, see that this key is not 160 and return false. This is invalid, as the linearization point for the insert(160) of thread $q$ has passed or else the result of the previous search for 160 must be incorrect. The following thread schedule represents the execution of the threads.

|         | Node 100 | Node 150 | Node 175 |
|---------|----------|----------|----------|
| $p$     |          |          |          |
| $q$     |          |          |          |
| $r$     |          |          |          |
| $s$     |          |          |          |

- **$p$**: insert(175)
- **$q$**: insert(160)
- **$r$**: contains(160): true
- **$s$**: contains(160): false
The two contains cannot be linearized.

F.2.2 Solution: Search Direction Swap. Say we reverse line 2 and 3 from the contains() operation (Algorithm 2 in the original work), making it look like the following:

Algorithm 12 contains(key)
1: node = search(key)
2: while node.key < k do node = node.succ
3: while node.key > k do node = node.pred
4: return (node.key == k and !(node.mark))

Now consider the example above, up to this state.

Thread s will now find the key 160, and the insert operation can be linearized. Actually, regardless of what node you end up at after this search, the contains operation will find 160. If you are left of the partially inserted node, you will traverse succ pointers until you pass it, then follow a single pred pointer to the node in question. If you are to the right of the partially inserted node, you will follow no succ pointers, but follow pred pointers until you reach the node in question.

Consider the reverse case, before a right rotation occurs:

After the rotation occurs:

Here, s is searching for 85, arrives at 95 but then gets rotated down. If it searches right then left, it will still find 85. It is the same as the previous example, regardless of what node you end up at after this search, the contains operation will find 85.

G Full Experimental Results

Our other system has four Intel Xeon Platinum 8160 CPUs, each with 24 cores and two hardware threads per core, for a total of 192 hardware threads. Cores on the same CPU share a 33MB L3 cache. Threads are pinned such that thread counts up to 48 run on one physical CPU, thread count 96 runs on two CPUs, and so on. Our code was compiled with GCC 7.4.0-1 using flag -O3. We used numactl to interleave memory pages evenly across CPUs. The fast allocator jemalloc 5.0.1-25 was for all algorithms, and memory was reclaimed using DEBRA, a fast epoch-based reclamation algorithm [6].

Figure 15. Throughput evaluation of the speculation-friendly BST, in synchrobench. Total throughput in millions of operations per second, full comparison, higher is better

One other interesting comparison would be with crain, the speculation-friendly BST from Synchrobench [19], shown above in Figure 15. While we attempted to port this data structure to our testing suite, due to time constraints we were unable to (as it would require porting the Elastic STM algorithm, integrating support for background threads that perform data structure maintenance asynchronously, and completely reimplementing its memory reclamation).
As an example of how our approach can be applied to a different data structure, in this section we give a brief overview of the first lock-free concurrent solution to the dynamic connectivity problem on undirected acyclic graphs. **Overview.** The dynamic connectivity problem involves maintaining a graph containing a set of fixed vertices and a dynamic set of edges. Solving dynamic connectivity requires implementing three operations: `connected(v, w)`, `link(v, w)` and `cut(v, w)`. `connected(v, w)` returns true if there exists a path from node `v` to `w`. Otherwise, it returns false. If there is no path between `v` and `w`, `link(v, w)` creates an edge between them and returns true. Otherwise, it returns false. Note that you cannot link two nodes if there exists a path between them, as this would create a cycle in the graph. This is a common limitation of sequential data structures for dynamic connectivity, which does not relate to our method. Finally, if there is an edge between `v` and `w`, `cut(v, w)` removes it and returns true. Otherwise, it returns false.

We follow the same general approach that we used to implement the AVL tree: all nodes have version numbers, and `validatePath` is used to implement atomic searches.

In the sequential setting, Euler Tours [42] are typically used to implement dynamic connectivity. An Euler Tour starts at an arbitrary node and visits each edge exactly once (interpreting undirected edges as two directed edges for our purposes) recording each visit to a node as they are traversed.

In the classical Euler tour data structure, Euler tours are stored in a BST. We chose to store the tour in a skip list rather than a BST, as in [45], which makes it easier to split and merge while maintaining (probabilistic) balance, and having each node of the skiplist represent an edge in an Euler tour rather than a node [41]. To clarify, there is a graph comprised of graph nodes, and a skiplist comprised of list nodes (which represent edges in the graph), and each graph node has pointers to the list nodes for each of its incident edges. That paper also adds an additional self-edge for each node (which appears as a list node, pointed to by the appropriate graph node). It turns out that this self edge greatly simplifies the data structure operations in a concurrent setting. Figure 29 shows an example of how such tours can be represented. We omit the upper levels of the skip-list to save space, and draw the graph representation above the list. To avoid special cases, towers of sentinel nodes are added to be beginning and the end of every tour list.

Just as we used version numbers to impose a sequential ordering on modifications to a single BST node in our AVL tree, we use the version number of the leftmost sentinel node at the bottom level of the list (the minimum sentinel) to impose a sequential ordering on modifications to an individual Euler tour list (i.e., a single version number protects the entire tour list). More precisely, all updates increment the version number of the minimum sentinel, allowing only a single update on any list at a time. This might seem like a concurrency bottleneck, but care must be taken to avoid the possibility of cycles being introduced by concurrent `links`. Additionally, every graph node is initially in its own Euler tour tree, allowing plenty of concurrency. We now sketch how the operations are implemented.

**Connection Queries.** The main purpose of this data structure is to answer connectivity queries: does a path between nodes `v` and `w` exist (or, equivalently, do `v` and `w` belong to the same connected component, or tour list)? Consider the self-edges from `v` to `v`, and from `v` to `w`. Let `L_v` and `L_w` be the list nodes that represent these self-edges. If a time can be established when `L_v` and `L_w` were in the same tour list, then a path exists between those two graph nodes at that time.
Figure 18. Comparing with TM-based Unbalanced BSTs on the AMD system. Note varying y-axes (x-axis = # threads).

Figure 19. Comparing with TM-based Balanced BSTs on the AMD system. Note varying y-axes (x-axis = # threads).
Figure 20. Comparing with Handcrafted B-tree variants on the Intel system. Operations per microsecond vs # threads.

Figure 21. Comparing with Handcrafted Unbalanced BSTs on the Intel system. Operations per microsecond vs # threads.
Figure 22. Comparing with Handcrafted Balanced BSTs on the Intel system. Operations per microsecond vs # threads.

Figure 23. Comparing with Handcrafted Balanced BSTs on the AMD system. Operations per microsecond vs # threads.
Figure 24. Comparing with TM-based Unbalanced BSTs on the Intel system. Note varying y-axes (x-axis = # threads).

Figure 25. Comparing with TM-based Balanced BSTs on the Intel system. Note varying y-axes (x-axis = # threads).
Figure 26. Factor analysis for Handcrafted Unbalanced BSTs on the Intel system. Note varying y-axes (x-axis = # threads).
Figure 27. Factor analysis for Handcrafted Balanced BSTs on the Intel system. Note varying y-axes (x-axis = # threads).
time. Conversely, if a time can be established where these list nodes were in different tour lists, then no path exists between the graph nodes at that time.

This is simple to determine: starting from \( L_v \) and \( L_w \), the tour list(s) are traversed left towards the minimum sentinel. These traversals do the reverse of a traditional skiplist search, traversing up and left in the list until a sentinel node is reached. Once a sentinel node is reached, the node is traversed down towards the bottom level of the list to locate the minimum sentinel. (We could simply have traversed left, without going upwards, but then our traversal time could be linear in the size of the tour list.) The paths taken by both traversals are then validated, and if either validation fails the entire operation is retried. If the same minimum sentinel is found by these validated searches then there was a time when these two graph nodes existed in the same subgraph and therefore a path existed between them (so true is returned). If they are different, a time exists where the graph nodes were in different subgraphs and no path existed between them (so false is returned).

**Link.** To simplify the presentation, we first assume each tour list is implemented as a doubly-linked list (rather than skiplist), then explain how this changes when skiplists are used. The goal of our implementation of \( \text{link}(v,w) \) is to add a link from graph node \( v \) to graph node \( w \), absorbing the subgraph (equivalently, tour list) of \( w \) into the subgraph of \( v \). \( v \) and \( w \) can only be linked if they are not part of the same subgraph, so we start by performing the algorithm for \( \text{connected}(v,w) \). Let \( M_v \) and \( M_w \) be the minimum sentinels located while performing this algorithm. If \( M_v \) and \( M_w \) are the same, then \( \text{link}(v,w) \) can safely return false: a time was determined where they were already in the same subgraph. If \( M_v \) and \( M_w \) are different, then the operation can proceed. We will include \( M_v \) and \( M_w \) in our (eventual) KCAS operation, using it to increment both of their version numbers.

We explain the next steps with an example. Consider the case \( \text{link}(3, 6) \) presented in Figure 29. The tour lists drawn at the top of that figure are logically split into two sublists each: sublist \( L1 \) contains all nodes in the tour list to the left of and including 3’s self-edge (excluding the minimum sentinel, which we call \( S1 \)), and sublist \( L2 \) contains all nodes to the right of 3’s self-edge (excluding the **maximum sentinel**, \( S2 \)). In the tour list for node 6, \( S3, L3, L4, \) and \( S4 \) are similar to \( S1, L1, L2, \) and \( S2 \), respectively. We suffix the labels \( L1, L2, L3 \) and \( L4 \) with \( A \) and \( B \) to denote the beginning and end of sublists (i.e., \( L1A \) is the leftmost node of \( L1 \), and \( L1B \) is the rightmost node of \( L1 \)). These nodes will require updates.

Since link adds a new edge, we should add that edge to the tour lists (twice, as it should be traversed in both directions). Two new list nodes are created (\( VW \) and \( WV \)), one for each direction. The resulting tour list can be constructed by arranging the sublists in the following order: \( [S1, L2, L1, VW, L4, L3, WV, S4] \), which requires the operation to change the left or right pointers of the nodes on the ends of the sublists, as well as those of sentinel nodes. Note that it is possible for \( L2 \) and \( L4 \) to be empty, in which case, the same sequence sublist order works if empty sublists are omitted. This arrangement effectively rotates the individual tours containing \( v \) and \( w \) such that they are rooted at \( v \) and \( w \), respectively, and then links them together. Note that we also update the graph nodes for 3 and 6 to add their new neighbour (6 and 3, resp.) to their adjacency lists.

All of these pointer changes are performed in a single KCAS. In other words, the KCAS needs to update the left and right fields of all the list nodes at the ends of the sublists, add neighbours to the graph nodes, increment the version numbers of all nodes involved (crucially, including the minimum sentinel), and mark any nodes that are removed (\( S2 \) and \( S3 \), in this case). We use marking to avoid erroneous modifications to deleted nodes. Before a KCAS is performed, we first verify that every node included in the KCAS is not marked. If a node is marked, we restart the entire operation.

In a skiplist, this list restructuring is simply repeated at every level, in one large KCAS. The relevant sublists are determined at each level by traversing starting from the bottom list, and are rearranged in the same order as the bottom list. Crucially, updates to a skiplist based tour list are still serialized on the same field: the version of the minimum sentinel.

To determine a sublist at level \( i + 1 \) from level \( i \), we traverse upwards and inwards from the ends of the sublist at level \( i \). For example, consider the top left image in Figure 29, we will call the bottom level of the list represented here level 1. If we wished to determine the sublist \( L1 \) at level 2, we would traverse from \( L1A \) right until a node is encountered that has a node above it at level 2. Similarly, to determine the other end of the sublist, we would traverse from \( L1B \) left until a node is encountered that has a node above it at level 2. The two nodes found at level 2 are the ends of the sublist \( L1 \) at level 2. If these two traversals ever encounter the same node at some level \( i \), this indicates there is no such sublist at level \( i + 1 \). By performing this traversal for every sublist, at every level, all the nodes that need to be modified can be found. This process is repeated until the maximum height of the skiplist is reached, or the sublist does not exist at some level.

The goal of our implementation of \( \text{cut}(v, w) \) is to remove the edge connecting \( v \) to \( w \) if it exists, and split their tour list into two. Graph nodes contain adjacency lists, so determining if two nodes are directly connected by an edge is easy. If they are not neighbours, then the operation returns false. Otherwise, the version numbers of these graph nodes should be added to our (eventually) KCAS. The minimum sentinel is located as in the previous operations (but we only need to traverse starting at one of \( v \) or \( w \)).

From the graph nodes we can find the list nodes representing the edges \( VW \) and \( WV \). These list nodes will be removed as part of the operation and the list nodes between them will form one of the new tour lists. The list is separated into
Consider any Euler tour containing self-edges, and delete was by first traversing $w$.

There remains correct if empty lists are omitted.

Nodes each graph node $w$.

Definition I.1. Our fully-dynamic connectivity data structure consists of a set of Euler tour skiplists and a set of graph nodes. Each graph node $u$ participates in a single Euler tour.

Figure 29. Operation cut(3, 6) on (simplified) Euler tour lists

Three sublists: L1 (which contains all nodes to the right of the minimum sentinel and to the left of the edge VW), L2 (which contains all nodes to the right of VW and to the left of WV), and L3 (which contains all nodes to the right of WV and to the left of the maximum sentinel (S2)). Two new sentinel nodes are created for the new list, S3 and S4. This operation simply removes L2 from the center of the list, creating two lists as a result: [S1, L1, L3, S2] and [S3, L2, S4].

The sublist L2 represents the nodes no longer reachable from $v$ after the removal of $w$, since the only way $v$ could reach these nodes was by first traversing $w$.

To extend this to a skiplist it is very similar to link. These sublists are formed at each level and linked together in the same order as the bottom list.

I Dynamic Connectivity Correctness

Sketch

Recall that to avoid special cases, we take the original graph and add a self-edge for each graph node. Once this is done, there is still a well defined Euler tour that follows each edge once, but now these self-edges appear in the Euler tour. If we consider any Euler tour containing self-edges, and delete all self-edges from that tour, we obtain an Euler tour of the original graph.

As in the AVL tree we mark list nodes at the time they are deleted, so we can verify (purely syntactically) before performing a KCAS that all nodes it will modify are unmarked, and in doing so guarantee that no deleted node is ever changed.

Definition I.1. Our fully-dynamic connectivity data structure consists of a set of Euler tour skiplists and a set of graph nodes. Each graph node $u$ participates in a single Euler tour.

Algorithm 13 isConnected(v, w)

1: while true do
2: \(p_v = \text{traversePathToMinSentinel}(v)\) \(\Rightarrow\) A path \(p_v\) to a minimum sentinel is followed from some the self edge of some graph node \(v\).
3: \(p_w = \text{traversePathToMinSentinel}(w)\) \(\Rightarrow\) A path \(p_w\) to a minimum sentinel is followed from some the self edge of some other graph node \(w\).
4: if not validatePath(\(p_v\)) then continue
5: if not validatePath(\(p_w\)) then continue
6: if \(p_v\).minSent == \(p_w\).minSent then return true \(\Rightarrow\) If both paths ended up at the same minimum sentinel, return true
7: else return false \(\Rightarrow\) If both paths ended up at different minimum sentinels, return false

Observation I.2. Any list node that has the value NIL in both its down and left field is the minimum sentinel of a tour list.

Lemma I.3. Our implementation satisfies the following claims:

1. isConnected(v, w) returns the same value it would if it were performed atomically at its linearization point (just before the first validation).
2. a. The data structure is a fully-dynamic connectivity structure (see definition I.1).
   b. Any link or cut operation that performs a successful KCAS returns the same value it would if it were performed atomically at its linearization point (the KCAS).
   c. Any link or cut operation that terminates without performing a successful KCAS returns the same value it would if it were performed atomically at its linearization point.

Proof. Consider an arbitrary execution $E$. We prove these claims together by induction on the sequence of steps $s_1, s_2, ...$ (which can be shared memory reads, atomic KCASRead operations, or atomic KCAS operations) in $E$.

Base case: There are a finite number $i$ graph nodes, and each is in its own tour list. These tour lists contain a single self-edge, and two sentinel towers on each side.

Inductive step: suppose the claims all hold before step $s$. We prove they hold after step $s$.

Claim 1. The only operations that can impact this claim are KCAS operations from link or cut. Reads do not change the data structure, hence they will not change the paths followed by the traversals in isConnected.

Subcase 1: Consider an invocation of isConnected($v$, $w$) that returns true. In the final loop of this invocation, the following occurs: the traversal from $v$ to a minimum sentinel which follows path $p_v$ occurs, then the traversal from $w$ to the same skiplist (or tour list for short), and contains pointers to all of the (skip)list nodes that represent directed edges starting from $u$ (including the self edge $u \rightarrow u$). In each tour list, the bottom level list nodes represent the seq of edges visited in an Euler tour of the graph nodes that participate in the tour list.
minimum sentinel which follows path \( p_w \) occurs, let the time this second traversal ends be \( t_0 \). (From Observation 1.2 we can statically check that node reached by these traversals was, in fact, a minimum sentinel.) We then validate the path of the first traversal at \( t_1 \), and then validate the path of the second traversal at \( t_2 \) (therefore, \( t_0 < t_1 < t_2 \)). This operation does not return unless \( \text{validatePath}(p_w) \) returns true for both paths. Since we know that \( \text{validatePath}(p_w) \) returns true, there were no modifications to any nodes in \( p_w \) between \( t_0 \) and \( t_2 \). Hence, there were also no modifications to any node in \( p_w \) between \( t_0 \) and \( t_1 \), and \( \text{validatePath}(p_w) \) would still have returned true if it were executed at \( t_1 \). Consider a \textit{link} or \textit{cut} update that changes the configuration of the tour list during this operation. If this \textit{link} or \textit{cut} does not involve the current tour list, it does not change the minimum sentinel that would be reached by either traversal. If these updates were to occur on the current tour list, it must include the version number of the minimum sentinel in the KCAS. If this update occurs during one of the traversals, then the traversal will fail to validate, and this operation will be retried. Additionally, if the update occurs between the traversals and one of the validations, the validation will fail. Therefore, since both of these traversals end at the same minimum sentinel, they were in the same tour list just before \( t_1 \) (which is where we linearize this operation).

Subcase 2: Consider an invocation of \( \text{isConnected}(v, w) \) that returns false. This argument is the same as Subcase 1, since \( \text{isConnected}(v, w) \) still must validated both paths, however the minimum sentinels are different.

**Claim 2A.** The only operations that can impact this claim are the KCAS operations in \textit{link} and \textit{cut}, as reads to not change the data structure.

**Subcase 1:** Suppose \( s \) is a successful KCAS of \( \text{link}(v, w) \). Before this KCAS, two traversals occurred and were validated that from the self-edges of the two graph nodes \( v \) and \( w \) to two different minimum sentinels. As we proved in Claim 1, a time \( t \) exists where \( v \) and \( w \) were in different tours (and hence there was no path between them). From the inductive hypothesis, these two tour lists were well-formed before this operation. Hence, it is correct to perform a \textit{link} operation on these two nodes at \( t \). Since the version number of both minimum sentinels are part of this KCAS, there are no changes to either tour list between \( t \) and \( s \). This means that no update has modified any node in either tour list after the time they were validated. Therefore, the \textit{link} operation is still applicable at \( s \).

**Subcase 2:** Suppose \( s \) is a successful KCAS of \( \text{cut}(v, w) \). This is simply an easier case than Subcase 1, as only a single list is tracked for this operation.

**Claim 2B.** This is proven in Claim 2A, as we proved that both \textit{link} and \textit{cut} are atomic at \( s \), which is when the KCAS is executed.

**Claim 2C.** This is proven in Claim 1, as both use the result of \textit{isConnected} to determine if a KCAS should be executed or not.