Quantum mutual information of an entangled state propagating through a fast-light medium

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It is widely accepted that information cannot travel faster than \( c \), the speed of light in vacuum\(^2\,3\). Here, we investigate the behaviour of quantum correlations and information in the presence of dispersion. To do so we send one half of an entangled state of light through a gain-assisted slow- or fast-light medium and detect the transmitted quantum correlations and quantum mutual information\(^4\,6\). We show that quantum correlations can be advanced by a small fraction of the correlation time, even in the presence of noise added by phase-insensitive gain. Additionally, although the peak of the quantum mutual information between the modes can be advanced, we find that the degradation of the mutual information due to added noise appears to prevent an advancement of the leading edge. In contrast, we demonstrate a significant delay of both the leading and trailing edges of the mutual information in a slow-light system.

Many experiments have demonstrated the ability to manipulate the group velocities of optical pulses moving through atomic vapours\(^7\,10,11\). For classical pulses propagating without the presence of noise, it has been well established theoretically\(^12\,13\) that the initial turn-on point of a pulse (the ‘pulse front’) propagates through a linear causal medium at the speed of light in vacuum. It is often argued\(^14\) that this signal carries the entirety of the pulse’s classical information content, because the remainder of the pulse can in principle be inferred from the pulse height and its derivatives just after the point of non-analyticity has passed.

Experimentally, particularly in the inevitable presence of quantum noise, pulse fronts may not convey the full story of what is readily observed in the laboratory. It is thus interesting to consider other operational definitions of a signal that apply to particular systems. For example, Stenner and colleagues\(^15\) studied the propagation of classical information encoded in bright, actively shaped optical pulses travelling through a fast-light medium. These experiments revealed that the operational information velocity is actually slowed to speeds less than \( c \). Although noise may have affected the experimental results, these experiments were not conducted in a regime where quantum noise necessarily played a crucial role. Meanwhile, adopting a definition of signal velocity based on observing a given signal-to-noise ratio, Kuzmich and colleagues showed how quantum noise associated with gain-assisted fast light would be expected to limit the early detection of smooth, narrowband pulses consisting of only a few photons\(^16\).

Here, we adopt an alternative definition of a signal, choosing it to be the random, but strongly correlated quantum fluctuations of two spatially separated parts of a bipartite entangled state. The entangled state in this experiment was generated via four-wave mixing (4WM) in a warm vapour of \(^85\)Rb (ref. 17), which converts two photons from a strong pump beam into ‘twin’ photons emitted into spatially separated modes referred to as the probe and the conjugate (Fig. 1a).

Although entanglement cannot be used to signal superluminally\(^18\), it is thought to be an essential resource in quantum information science\(^6\,9\). Accordingly, the prospect of storing\(^19\) or delaying\(^20\) entanglement has attracted significant interest.

The fluctuations of the probe and conjugate fields are not externally imposed and they present no obvious pulse fronts or non-analytic features to point to as defining the signal velocity. As such, classically rooted approaches to defining the signal or information content of the individual modes are not readily applicable to this system. Despite the randomness of these fluctuations, information is shared between the modes. We take the quantum mutual information as our information measure, which removes the ambiguity of defining the arrival time of such information in the presence of noise, quantum or otherwise. The quantum mutual information for bipartite Gaussian states is readily accessible via optical homodyne measurements\(^5,6\) and naturally provides a consistent description of information in this system.

We studied how the dispersion associated with phase-insensitive gain\(^3\) affects these correlations by inserting a second vapour cell into the path of the conjugate and driving a second 4WM process with a separate pump (Fig. 1b). We show that when one mode of the two-mode state passes through this fast-light medium, the peak of the quantum mutual information between the modes is advanced, but the arrival of the leading edge is not. We also show that—in contrast—the leading and trailing edges of the mutual information are both delayed when one of the modes propagates through a gain-assisted slow-light medium.

The real and imaginary parts of the nonlinear susceptibility \( \chi^{(3)} \) that govern the response of the second 4WM process to the conjugate can be described by a set of equations similar to the Kramers–Kronig relations applicable to linear dielectric media\(^21\). Using these relations as a guide (Fig. 1c), we changed the detuning of the pump beam used to drive the second 4WM process so that the conjugate frequency overlapped with the region of anomalous dispersion.

By performing separate balanced homodyne detections of the probe and conjugate modes, we measured the fluctuations of the in-phase (\( \hat{X} \)) and out-of-phase (\( \hat{Y} \)) amplitudes of the electromagnetic field in each beam, which are referred to as the field quadratures. Taken individually, the probe and conjugate beams exhibit quadrature fluctuations that exceed the shot-noise limit. Taken together, however, these fluctuations display strong correlations beyond the limits achievable classically. To characterize the strength of the correlations, it is helpful to introduce the joint quadrature operators \( X_+ = (X_p - X_c)/\sqrt{2} \) and \( Y_+ = (Y_p + Y_c)/\sqrt{2} \), where...
subscripts p and c denote the probe and conjugate fields, respectively. For the appropriate choice of local oscillator phases, the fluctuations of one of the joint quadratures ($\langle \Delta X_p^2 \rangle$ or $\langle \Delta Y_c^2 \rangle$) fall below the shot-noise limit (are ‘squeezed’).

We verified the presence of entanglement by calculating a related quantity, the inseparability $I$:

$$I = \langle \Delta X_p^2 \rangle_m + \langle \Delta Y_c^2 \rangle_m$$

Here, $\langle \Delta X_p^2 \rangle_m$ is the minimum value of the difference signal, $\langle \Delta Y_c^2 \rangle_m$ is the minimum value of the sum, and each term is normalized to the shot-noise limit. An inseparability of $I < 2$ is a necessary and sufficient condition to conclude that any bipartite Gaussian state is entangled.

Studies of bright beam propagation through fast-light media have investigated the trade-off between the magnitude of the advancement and the amount of added noise as a function of detuning. Here, we choose a detuning of the second pump that produces a readily detectable advancement of the conjugate fluctuations without significantly deteriorating the inseparability. By operating in a regime of low gain ($G \approx 1.1$), we maintained an inseparability of $I = 1.2$ under fast-light conditions, confirming the persistence of entanglement between the probe and conjugate after the conjugate passes through the fast-light medium (Fig. 2a,b).

Figure 1 | Experimental set-up. a,b. Vacuum-squeezed twin beams are generated in cell 1 using 4WM in a double-lambda configuration (b). A region of anomalous dispersion for the conjugate is created in a second vapour cell using a second 4WM process driven by pump 2, whose frequency is independently tunable with respect to pump 1 (Supplementary Section 1). The phase of the local oscillators (LOs) is scanned using piezo-electric transducers (PZTs) to verify the presence of entanglement. The sum and difference signals of the homodyne detections are recorded on a pair of spectrum analysers (SAs) to detect quantum correlations. An oscilloscope is triggered to detect time traces of the individual homodyne detectors given a predetermined time delay. The sum and difference signals of the homodyne detections are recorded on a pair of spectrum analysers independently tunable with respect to pump 1 (Supplementary Section 1). The phase of the local oscillators (LOs) is scanned using piezo-electric transducers (PZTs) to verify the presence of entanglement. The sum and difference signals of the homodyne detections are recorded on a pair of spectrum analysers (SAs) to detect quantum correlations. An oscilloscope is triggered to detect time traces of the individual homodyne detectors given a predetermined time delay.

Figure 2 | Persistence of correlations associated with entanglement in the presence of anomalous dispersion. a, We observe up to $-3 \text{ dB}$ of squeezing with an associated inseparability $I \approx 1$ when the second (fast-light) 4WM process is suppressed. b, In the presence of a small phase-insensitive gain giving rise to anomalous dispersion, the squeezing reduces to $-2.3 \text{ dB}$ and $I$ increases to 1.2, which is still sufficient to show entanglement ($I < 2$). c, Average normalized cross-correlation functions for the correlated and anti-correlated joint quadratures. The left axis applies to the reference and advanced curves for the correlated quadratures, and the right axis applies to the anti-correlated quadratures (indicated by arrows). When calculating the cross-correlation functions, the reference and fast-light data are both subject to the same passband filter used to calculate $I$ (Supplementary Sections 3.1 and 3.2). d,e. Closer looks at the peak correlation (d) and anti-correlations (e) portrayed in c.
We confirm that the fluctuations of the continuous-wave (c.w.) conjugate are advanced through the fast-light cell by computing the normalized cross-correlation function (see Methods and Supplementary Section 3.1) of the detected probe and conjugate quadratures for both the reference and fast-light cases (Fig. 2c–e). After averaging 200 time traces, we conclude that the peak of the cross-correlation function is shifted forward in time by 3.7 ± 0.1 ns, corresponding to a fractional advance of ≏1% relative to the cross-correlation width (≏300 ns). Here, the uncertainty is estimated by taking the standard deviation of the mean for the cross-correlation peak advancements over all the experiments.

Although useful to clearly see an advancement of the correlations, the normalized cross-correlation function of the field quadratures does not capture how the noise added through phase-insensitive gain affects the entanglement. We therefore plot the inseparability $I$ as a function of the relative delay (Fig. 3a). The delay is implemented in software in exactly the same manner as when calculating the cross-correlation function. Although the minimum value of $I$ is advanced in time for the fast-light case, its degradation acts, within experimental uncertainty, to prevent the leading edge from advancing forward in time. Figure 3b presents a sampling of the delay-dependent squeezing measurements used to calculate the inseparability, which indicates an advance in the maximum squeezing of 3.7 ± 0.1 ns (Fig. 3c).

In our experiment there is no imposed ‘signal’ as such. However, the fluctuations on one beam carry information about the fluctuations on the other. We capture this by calculating the quantum mutual information between the two beams (Fig. 4), working from the same basic data as used to calculate the delay-dependent inseparability. The mutual information $I(\rho)$ is defined in terms of the von Neumann entropy $S_V(\rho) = -\text{Tr}(\rho \log \rho)$ according to

$$I(\rho) = S_V(\rho_1) + S_V(\rho_2) - S_V(\rho)$$  \hspace{1cm} (2)$$

where $\rho$ denotes the full state density matrix and $\{\rho_1, \rho_2\}$ denote the reduced density matrices of the subsystems. The mutual information quantifies the total (classical plus quantum) correlations between the probe and conjugate. In good agreement with the squeezing and cross-correlation measurements, we observe an advancement of 3.7 ± 0.1 ns of the peak of the delay-dependent mutual information, paired with a degradation due to uncorrelated noise added by the fast-light cell (Supplementary Section 5). This degradation appears to prevent us from observing an
late a sharpening of our understanding of the role of quantum noise in
from the delay. We expect that the see experimentally will be
limited to the advance of the mutual information differently
we speculate that the leading edge of the mutual information
in the case of fast-light propagation. Repeating the same analysis for slow-light
propagation of the probe we observe significant delays of both the leading and trailing edges of the mutual information (green trace).

Figure 4 | Comparison of computed quantum mutual information between the c.w. probe and conjugate as a function of relative delay for fast and slow light. The smooth shape of the curves results from the large amount of data (180 files consisting of 1 × 10⁷ points per file) used to calculate the mutual information. When considering fast-light advancement of the conjugate (red trace), we observe an advance in the peak of the mutual information of 3.7 ± 0.1 ns. The subpanel provides a closer look at the maxima of the mutual information curves for the reference and fast-light cases. There is no statistically significant advance of the leading edge of the mutual information in the fast-light case. We are able to observe significant delays of the leading and trailing edges of the mutual information when compared to the reference case.

It is interesting to contrast this asymmetry in the fast- and slow-light behaviour of the mutual information with the results of previous experiments studying the velocity of classical information propagating through dispersive media. In refs 15 and 27, new information associated with a ‘non-analytic’ point in the field was found to propagate at c in the presence of slow- and fast-light media alike. This behaviour can be understood to be a consequence of the field’s frequency components that lie outside the relevant bandwidth where the medium exhibits steep dispersion.

Our results highlight the role played by both the detection and information encoding methods in such experiments. In the present experiments the signal beam is not pulsed, so there is no externally imposed non-analytic point. The individual probe and conjugate beams are noisy c.w. squeezed-vacuum beams, carrying information that is correlated and common to the two beams. Our experiments clearly show that the arrival time of the mutual information contained within the detection bandwidth is affected by the fast- and slow-light media. Although the normalized correlation function, the peak of the inseparability and the peak of the mutual information can all be advanced, our results suggest that the leading edge of the mutual information cannot be advanced beyond the reference situation. Further work will be required to determine if this is a fundamental limit in this measurement context. The experiment clearly shows that the mutual information can be delayed. We speculate that a combination of distortion effects and quantum noise added by phase-insensitive gain act to limit the advance of the mutual information differently from the delay. We hope that these experimental observations stimulate a sharpening of our understanding of the role of quantum noise in limiting the transport of information.

Methods
Data acquisition and frequency filtering. The local oscillator phases were allowed to drift and the oscilloscope was triggered to record time traces of each individual homodyne detection when the phases were appropriate to observe the squeezing in either joint quadrature (Supplementary Section 2). The traces were Fourier-transformed for analysis, and all analyses was confined to power spectral densities within a bandwidth of 100 kHz–2 MHz. This corresponds to the frequency bandwidth where we were able to show entanglement (Γ < 2) without the presence of dispersion.

Calculating the correlation measures. We evaluated the inseparability over the same 100 kHz–2 MHz bandwidth for all reference, fast- and slow-light experiments. To calculate the mutual information, we made use of the fact that any bipartite Gaussian state can be completely characterized by the variances and covariances of the field quadratures28,29. These variances and covariances were evaluated using the same bandwidth used to compute the inseparability. Finally, we evaluated the cross-correlation functions after filtering the probe and conjugate homodyne time traces with a 100 kHz–2 MHz band-pass filter (Supplementary Section 3.1).

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Author contributions
J.B.C., R.T.G., Q.G. and U.V. analysed the data. J.B.C., R.T.G., U.V. and P.D.L. conceived and designed the experiments. J.B.C., R.T.G., Q.G. and U.V. contributed materials and analysis tools. J.B.C., R.T.G., Q.G., U.V. and T.L. performed the experiments. J.B.C., R.T.G., Q.G., U.V., K.M.J. and P.D.L. wrote the paper.

Additional information
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Competing financial interests
The authors declare no competing financial interests.