RIS-Assisted Self-Interference Mitigation for In-Band Full-Duplex Transceivers

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Abstract—The wireless in-band full-duplex (IBFD) technology can in theory double the system capacity over the conventional frequency division duplex (FDD) or time-division duplex (TDD) alternatives. But the strong self-interference of the IBFD can cause excessive quantization noise at the output of the analog-to-digital converters (ADCs), which hampers its real implementation. Fortunately, the reconfigurable intelligent surface (RIS) can reconfigure the wireless channels by dynamically adjusting the reflection coefficients, which makes it a promising solution to self-interference mitigation (SIM) for the IBFD system. This paper considers a RIS-assisted IBFD (RAIBFD) system where a full-duplex base station (BS), assisted by a co-located RIS, receives and transmits signals on the same frequency band simultaneously. We propose to jointly design the DL precoding matrix and the RIS coefficients to mitigate the strong self-interference before the ADCs of finite bit resolutions. To gauge the performance of the proposed RAIBFD system, we also consider an idealistic RIS-assisted IBFD (I-RAIBFD) system with ADCs of infinite bit resolutions and RIS-assisted FDD (RAFDD) system, and take them as performance benchmarks. The effectiveness of the proposed RAIBFD system is verified by simulation studies, even if the phases of the RIS have only finite resolutions.

Index Terms—In-band full-duplex wireless, self-interference mitigation, reconfigurable intelligent surface, precoding.

I. INTRODUCTION

Owing to the ever-increasing demand for the capacity of next-generation wireless systems, researchers have kept questing techniques that can achieve higher spectral efficiency, among which the in-band full-duplex (IBFD) wireless have deservedly attracted considerable attention [1], [2]. In contrast to the conventional time division duplex (TDD) and frequency division duplex (FDD) mode, an IBFD node can simultaneously transmit and receive signals (STAR) to double the system capacity, at least in theory. But for real implementation, the super strong self-interference due to the STAR [3] can saturate the adjacent receivers’ analog-digital converters (ADCs). The resultant excessively large quantization noise can void the reception of the desired signal; thus, it is crucial to mitigate the self-interference at the receiver antennas before the ADCs [4], [5], [6].

The mitigation of strong self-interference has gained much attention in recent years. The existing self-interference mitigation (SIM) methods can be divided into two major categories based on whether they utilize the known waveform of self-interference or not.

Among the methods using the waveform of self-interference, researchers have proposed to canceling the self-interference using an auxiliary radio frequency (RF) chain [7], [8], [9], [10], [11], [12], [13]. In general, this approach consists of two steps: first, both the channel responses of the wireless self-interference channel and the auxiliary RF chain are estimated during the pilot training period; second, the received self-interference is reconstructed in the digital domain based on the estimated channels, and then is upper converted by the auxiliary RF chain so that the self-interference is subtracted from the received signal before the ADCs. Another approach is to utilize the analog signal from the power amplifier’s (PA) output to replicate the (analog) self-interference, and subtract it from the input of the ADCs at receiver [14], [15], [16], [17], [18]. The replication can be achieved via a balun transformer [14] or a linear combination of multiple delayed versions of the analog output of the PA [15], where the delayed versions can be obtained via Taylor series [16]. The combination coefficients can be calculated using the steepest descent method [17] or the adaptive dithered linear search (DLS) method [18]. However, the aforementioned works utilizing known waveform of the interference are susceptible to the nonideality of the RF hardware – imperfect reconstruction of the self-interference in the analog domain can significantly degenerate the system performance.

The other category of research proposed to mitigate the strong interference without assuming that the waveform of interference is known. The paper [19] proposed to place two transmit antennas at a distance $d$ and $d + \frac{1}{2}$ away from the receive antenna for the signal destructive superposition. However, this method can only support a single stream data...
transmission. As an improvement to [19], the work in [20] presented the first MIMO full-duplex wireless system called MIDU. By utilizing the symmetric placement of transmit and receive antennas, the MIDU technology can both mitigate the self-interference and transmit multiple data streams. Given the perfect channel state information (CSI), the paper [21] proposed a so-called SoftNull method that utilizes a transmit beamforming matrix to mitigate the strong self-interference before it enters into the ADCs. The paper [22] proposed a technique named HIMAP to mitigate the strong interference using an analog phase shifter network (PSN), where both the waveform and the channel state information (CSI) are not needed. As an auxiliary work, the authors also provided an over-the-air (OTA) calibration method to eliminate the potential phase deviations of the PSN to ensure effective SIM in [23]. It is worth noting that the two categories of interference mitigation methods are not exclusive to each other. They can potentially be combined to achieve even better performance.

In recent years, the advent of reconﬁgurable intelligent surface (RIS) technology offers a new option for wireless communications, as it can reconﬁgure the wireless channels by dynamically adjusting the reﬂection elements to achieve a better quality of service (QoS) [24], [25]. In addition, as a passive device, RIS consumes low power and introduces no circuit noise. Many researchers have attempted to explore the potential applications of RIS to IBFD wireless systems [26], [27], [28], [29]. The paper [26] proposed to maximize the sum rate of a RIS-assisted point-to-point IBFD system by jointly optimizing the transmit beamforming and the RIS coefﬁcients. In [27], the authors considered a RIS-assisted multi-user IBFD communication system and aimed at maximizing the weighted minimum rate of all users via a joint optimization of the precoding matrix of the BS and the RIS reﬂection coefﬁcients. The paper [28] considered a RIS-assisted full-duplex decode-and-forward (DF) relay network, where the minimum achievable rate of the relay network is maximized by optimizing the RIS reﬂecting coefﬁcients and transmit power of the source and relay. The paper [29] considered a RIS-assisted full-duplex unmanned aerial vehicle (UAV) communication network, and proposed to maximize the weighted sum rate of the UL and DL transmission by optimizing the transmit power of the UL user, the RIS coefﬁcients, and UAV trajectory. Although the aforementioned works [26], [27], [28], and [29] show that the RIS can bring extra beneﬁts to IBFD communications under various scenarios, they assume that the self-interference mitigation (SIM) has been achieved by a standard IBFD technique beforehand, i.e., the SIM optimization is not for SIM itself.

In this paper, we consider RIS optimization as an integrated part of the SIM for IBFD, which shares with the idea of the paper [30]. Therein, the authors considered utilizing RIS to mitigate the self-interference for a single-antenna IBFD node and built a prototype that can achieve the SIM amount of 85dB. This paper investigates a RIS-assisted IBFD (RAIBFD) system, where a multi-antenna base station (BS), assisted by a RIS, receives signals from the UL users and transmits signals to the DL users simultaneously. The main contributions of this paper are as follows.

- We show how RIS can be utilized to solve traditional SIM problem together with a SIM matrix and also provide an algorithm to obtain the RIS phases and the SIM precoding matrix. Simulation results show the superior performance of the RIS-assisted SIM over the SoftNull method proposed in [21].

- We provide two benchmark schemes for the proposed RAIBFD system, i.e., an idealistic RAIBFD (I-RAIBFD) system for showing a performance upper bound and RIS-assisted FDD (RAFDD) system that represents the traditional commercial system. Simulation results show that the sum rate of the proposed RAIBFD system has a great gain over that of the traditional RAFDD system, and approaches that of the I-RAIBFD system.

- In a broad sense, using RIS for SIM opens up a new and promising avenue to this important problem, which may facilitate the real implementation of IBFD systems in the next generation of wireless.

This paper is organized as follows. Section II introduces the RIS-assisted in-band full-duplex (RAIBFD) signal model of both UL and DL. Section III proposes a RIS-assisted SIM method for the proposed RAIBFD system. Section IV presents an idealistic RIS-assisted in-band full-duplex (I-RAIBFD) system with ∞-bit ADC and a RIS-assisted FDD (RAFDD) system, and simulation results are provided in Section V. Finally, conclusions are drawn in Section VI.

Notations: (∙)*, (∙)T, and (∙)H denotes the conjugate, the transpose, and conjugate transpose of a matrix, respectively; ||·||F represents a matrix’s Frobenius norm; |·| denotes the determinant of a matrix; * and ◦ represent the Khatri-Rao product and Hadamard product, respectively; a(i) denotes the ith element of vector a; Re(∙) represents the real part of a matrix; ei is a vector with its ith element being 1 and others being 0; diag(∙) stands for a diagonal matrix with vector a being the diagonal element; diag(A) stands for column vectors with its elements being those diagonal elements of matrix A; vec(A) stacks the column vector of the matrix A into a single column; ∠A stands for a matrix whose entries are the phases of elements in A: 0_{M×N} represents a M × N matrix with all elements being 0; x ∼ CN(μ, Q) is a complex Gaussian random vector with mean μ and covariance matrix Q.

II. SIGNAL MODEL

Consider a RAIBFD signal system depicted as Fig. 1 which consists of a base station (BS) equipped with M_t transmitting (Tx) antennas and M_r receiving (Rx) antennas, a co-located RIS with M_{ris} elements, K_u Tx users, and K_d Rx users. Denote the channel from the Tx antennas to the Rx antennas of BS as \textbf{H}_{B, B} ∈ C^{M_r × M_t}, the channel from the Tx antennas of the BS to the RIS as \textbf{H}_{RB} ∈ C^{M_r × M_{ris}}, the channel from the RIS to the Rx antennas of the BS as \textbf{H}_{B, R} ∈ C^{M_t × M_{ris}}, the channel from the kth UL user to the RIS as \textbf{h}_{Ru,k} ∈ C^{M_{ris} × 1}, and the channel from the RIS to the kth DL user as \textbf{h}_{dR,k}^H ∈ C^{1×M_{ris}}. Also denote the channel from
the $k$th UL user to the BS and that from the BS to the $k$th DL user as $h_{B,,u,k} \in \mathbb{C}^{M \times 1}$ and $h_{d,,k}^H \in \mathbb{C}^{1 \times M}$, respectively. As channel estimation techniques with respect to RIS have been widely studied [31], [32], we assume that the channels are perfectly known at the BS, and we will show the impact of channel estimation errors on the RAIBFD system in the simulation.

### A. Uplink Signal Model

In the UL channel, the Rx antennas at BS receive

$$y_B = (H_{B,,d}DH_{Ru} + H_{B,,u})\Gamma_u s_u + \xi_B + z_B,$$

where the first term is the signal of interest from the $K_u$ UL users. In particular,

$$D = \text{diag}(e^{j\phi_1}, e^{j\phi_2}, \ldots, e^{j\phi_{M_r}}),$$

$$H_{B,,u} = [h_{B,,u,1}, h_{B,,u,2}, \ldots, h_{B,,u,K_u}],$$

$$\Gamma_u = \text{diag}(\beta_{u,1}, \beta_{u,2}, \ldots, \beta_{u,K_u}),$$

and

$$s_u = [s_{u,1}, s_{u,2}, \ldots, s_{u,K_u}]^T.$$  

$D$ represents the phases of the RIS elements, and $\{\phi_i\}_{i=1}^{M_r} \in [0, 2\pi]$; $\beta_{u,k}$ represents the path loss from $k$th UL user to the BS and co-located RIS; $s_{u,k}$ is the transmitted symbols from the $k$th UL user to the BS, and $\mathbb{E}\{|s_{u,k}|^2\} = \sigma_u^2$. The second term of (1) is the self-interference, i.e.,

$$\xi_B = G_{SI}P_s d$$

with

$$G_{SI} = H_{B,,d}DH_{Ru} + H_{B,,u},$$

$$P = [p_1, p_2, \ldots, p_{K_d}],$$

$$s_d = [s_{d,1}, s_{d,2}, \ldots, s_{d,K_d}]^T.$$  

$G_{SI}$ is the effective channel of self-interference; $p_k \in \mathbb{C}^{M_r \times 1}$ is the precoding vector of $k$th DL user; $\{s_{d,k}\}$ is the transmitted symbols from the BS to $k$th DL user, and $\mathbb{E}\{|s_{d,k}|^2\} = \sigma_d^2$. The third term $z_B \sim \mathcal{CN}(0, \sigma_B^2 \mathbf{I})$ is the Gaussian noise at BS.

Denoting the UL effective channel as

$$H_u = (H_{B,,d}DH_{Ru} + H_{B,,u})\Gamma_u,$$

we have the output of the ADCs as [33]

$$\tilde{y}_B = \alpha y_B + n_q,$$

$$= \alpha H_u s_u + \alpha \xi_B + \alpha z_B + n_q,$$

where

$$\alpha = 1 - \rho, \quad \rho = \frac{\pi \sqrt{3}}{2} \text{ENOB},$$

with ENOB being the effective number of bits of ADCs; $n_q$ is additive Gaussian quantization noise vector and its covariance matrix is [33]

$$R_q = \rho(1 - \rho) \text{diag}(R_{y,y}),$$

where

$$R_{y,y} = \mathbb{E}\{y_B y_B^H\},$$

$$= \sigma_u^2 H_u^2 + G_{SI}P_{Rd}^H G_{SI}^H + \sigma_B^2 \mathbf{I},$$

and

$$R_d = \text{diag}(\sigma_{d,1}^2, \sigma_{d,2}^2, \ldots, \sigma_{d,K_d}^2).$$

Given fixed phases of the RIS elements, the self-interference $\xi_B$ is known at the BS, and thus we can just subtract it from the received signal to obtain

$$\tilde{y}_B = \tilde{y}_B - \alpha \xi_B,$$

$$= \alpha H_u s_u + \alpha z_B + n_q.$$  

We have from (16) that the UL spectral efficiency is

$$R_u = \log_2 |1 + \alpha^2 \sigma_u^2 H_u^2 Q_{B}^{-1}H_u|,$$

where

$$Q_B = \mathbb{E}\{(\alpha z_B + n_q)(\alpha z_B + n_q)^H\},$$

$$= \alpha^2 \sigma_B^2 + R_q.$$

### B. Downlink Signal Model

In the DL transmission, denoting the effective channel from the $k$th DL user to the BS as

$$h_{d,,k} = \sqrt{\beta_{d,k}}(H_{Rd,,k}^H D^H h_{d,,k} + h_{d,,k}),$$

yields the received signal of the $k$th DL user as

$$y_k = h_{d,,k}^H p_k s_{d,k} + h_{d,,k}^H \sum_{i \neq k} p_i s_{d,i} + z_k,$$

$\beta_{d,k}$ is the path loss from the BS and the co-located RIS to the $k$th DL user, and $z_k \sim \mathcal{CN}(0, \sigma_d^2)$ is the Gaussian noise. We can also reformulate (20) into matrix form as

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{K_d} \end{bmatrix} = H_d P s_d + z,$$

where

$$H_d = [h_{d,1}, h_{d,2}, \ldots, h_{d,K_d}]^H,$$

$$z = [z_1, z_2, \ldots, z_{K_d}]^T.$$
C. Problem Formulation

In [22], we have obtained the relations between the input signal-to-interference ratio (SIR) of ADC and the output signal-to-quantization-plus-noise ratio (SQNR) as [22, (11)]

$$\text{SQNR}_{dB} \approx \text{SIR}_{dB} + 6.02 \text{ENOB} - 4.35. \quad (24)$$

In IBFD system, $|\text{SIR}|_{dB}$ can be very low owing to the strong self-interference. For instance, given $|\text{SIR}|_{dB} = -100$dB [3], $|\text{SQNR}|_{dB} = -32$dB even for ADCs with ENOB = 12bit and the UL spectral efficiency is essentially zero, despite the interference subtraction (16) in the later stage. Hence it is crucial to mitigate the self-interference before the ADCs in the UL. To this end, we can jointly optimize the phases of RIS elements and the DL precoding matrix to minimize the coefficients of the self-interference effective channel from the Tx digital domain to the Rx antennas, i.e.,

$$\min_{\mathbf{D}, \mathbf{P}} \frac{1}{2} \|\mathbf{H}_{BR, R} \mathbf{D} \mathbf{H}_{RB, R} + \mathbf{H}_{BR, R} \mathbf{P}\|_F^2,$$

s.t. $\mathbf{D} = \text{diag}(e^{j\phi_1}, e^{j\phi_2}, \ldots, e^{j\phi_{M_{ris}}}). \quad (25)$

III. RIS-ASSISTED SIM IBFD SYSTEM

In this section, we propose to divide the DL precoding matrix into a SIM matrix and a DL data precoding matrix. In the UL signal transmission, we mitigate the self-interference by jointly optimizing the RIS reflection coefficients and the SIM matrix; in the DL signal transmission, we use the ZF precoding matrix to eliminate the DL inter-user interference, and then the well known “water-filling” method is adopted for Tx power allocation.

A. SIM Algorithm for UL

We propose to divide the DL precoding matrix $\mathbf{P}$ into

$$\mathbf{P} = \mathbf{P}_{SIM}\mathbf{P}_d, \quad (26)$$

where $\mathbf{P}_{SIM} \in \mathbb{C}^{M \times M_d}$ is for SIM, and $\mathbf{P}_{SIM}^H \mathbf{P}_{SIM} = \mathbf{I}$ so that $M_d$ subspace dimensions are preserved for DL data transmission; $\mathbf{P}_d \in \mathbb{C}^{M \times K_d}$ is for DL precoding. Here $M_d$ is a design parameter, which regulates the dimension of the subspace where the downlink transmitted signal can lie within.

We formulate the SIM problem as

$$\min_{\mathbf{P}_{SIM}, \mathbf{D}} \frac{1}{2} \|\mathbf{H}_{BR, R} \mathbf{D} \mathbf{H}_{RB, R} + \mathbf{H}_{BR, R} \mathbf{P}_{SIM}\|_F^2,$$

s.t. $\mathbf{P}_{SIM}^H \mathbf{P}_{SIM} = \mathbf{I}, \quad (27)$

which can be solved via optimizing the SIM matrix $\mathbf{P}_{SIM}$ and the RIS phase matrix $\mathbf{D}$ alternately.

First, assuming the RIS reflection phase matrix is fixed, we need to solve

$$\min_{\mathbf{P}_{SIM}} \frac{1}{2} \|\mathbf{G}_{SI} \mathbf{P}_{SIM}\|_F^2,$$

s.t. $\mathbf{P}_{SIM}^H \mathbf{P}_{SIM} = \mathbf{I}, \quad (28)$

to which the solution is a matrix whose columns are the $M_d$ right singular vectors corresponding to the $M_d$ smallest singular values of $\mathbf{G}_{SI}$.

Second, given that $\mathbf{P}_{SIM}$ is known, we can reformulate (27) into

$$\min_{\mathbf{D}} \frac{1}{2} \|\mathbf{H}_{BR, R} \mathbf{D} \mathbf{A} + \mathbf{B}\|_F^2,$$

s.t. $\mathbf{D} = \text{diag}(e^{j\phi_1}, e^{j\phi_2}, \ldots, e^{j\phi_{M_{ris}}}), \quad (29)$

where $\mathbf{A} = \mathbf{H}_{RB, R} \mathbf{P}_{SIM}$ and $\mathbf{B} = \mathbf{H}_{BR, R} \mathbf{P}_{SIM}$. Using the formula $\text{vec}(\mathbf{XYZ}) = (\mathbf{Z}^T \ast \mathbf{X}) \mathbf{Y}$ where $\mathbf{Y} = \text{diag}(\mathbf{y})$ is a diagonal matrix and $\ast$ is Khatri-Rao product, we have from (29) that

$$\min_{\mathbf{d}} \frac{1}{2} \|\mathbf{Cd} + \mathbf{b}\|_F^2,$$

s.t. $\mathbf{d} = [e^{j\phi_1}, e^{j\phi_2}, \ldots, e^{j\phi_{M_{ris}}}]^T, \quad (30)$

where

$$\mathbf{C} = \mathbf{A}^T \ast \mathbf{H}_{RB, R}, \quad \mathbf{b} = \text{vec}(\mathbf{B}). \quad (31)$$

The constant modulus constraint of (30) is a manifold defined as $\mathcal{M}^{M_{ris}}_{C} = \{\mathbf{d} \in \mathbb{C}^{M_{ris}} : |\mathbf{d}(1)| = \cdots = |\mathbf{d}(M_{ris})| = 1\}$. Thus we can adopt the Riemannian conjugate gradient (RCG) algorithm to solve (30) [34]. Defining

$$f(\mathbf{d}) \triangleq \|\mathbf{Cd} + \mathbf{b}\|_2^2, \quad (32)$$

we have the Euclidean gradient $\nabla f(\mathbf{d})$ as

$$\nabla f(\mathbf{d}) = \mathbf{C}^H (\mathbf{Cd} + \mathbf{b}). \quad (33)$$

The tangent space at point $\mathbf{d}$ is

$$T_d \mathcal{M}^{M_{ris}}_{C} = \{\mathbf{x} \in \mathbb{C}^{M_{ris}} : \text{Re}\{\mathbf{x} \circ \mathbf{d}^*\} = 0\}, \quad (34)$$

and the Riemannian gradient at $\mathbf{d}$ is the orthogonal projection of the Euclidean gradient $\nabla f(\mathbf{d})$ onto the tangent space $T_d \mathcal{M}^{M_{ris}}_{C}$, which is

$$\mathbf{g} = \text{grad}(\mathbf{d}),$$

$$= \nabla f(\mathbf{d}) - \text{Re}\{\nabla f(\mathbf{d}) \circ \mathbf{d}^*\} \circ \mathbf{d}. \quad (35)$$

we summarize the RCG algorithm in Algorithm 1, where the Armijo-Goldstein condition in line 3 can ensure that $f(\mathbf{d})$ is monotonically decreasing as $\mathbf{d}_i$ is updated with $\mathbf{d}_{i+1}$; the Retraction in line 4 is defined as

$$\text{Retr}_\mathbf{d} : \mathcal{T}_d \mathcal{M}^{M_{ris}}_{C} \rightarrow \mathcal{M}^{M_{ris}}_{C},$$

: $\alpha \mathbf{c} \rightarrow \text{Retr}_\mathbf{d}(\alpha \mathbf{c}), \quad (36)$

where

$$\text{Retr}_\mathbf{d}(\alpha \mathbf{c}) \triangleq \left[ (\mathbf{d} + \alpha \mathbf{c})_1, (\mathbf{d} + \alpha \mathbf{c})_2, \ldots, (\mathbf{d} + \alpha \mathbf{c})_{M_{ris}} \right], \quad (37)$$

which can map the vector $\alpha \mathbf{c}$ in the tangent space onto the manifold; We can obtain the Transports vector in line 6 as

$$\text{Transp}_{\mathbf{d}_i \rightarrow \mathbf{d}_{i+1}} : \mathcal{T}_d \mathcal{M}^{M_{ris}}_{C} \rightarrow \mathcal{T}_{\mathbf{d}_{i+1}} \mathcal{M}^{M_{ris}}_{C},$$

: $\mathbf{c} \rightarrow \mathbf{c} - \text{Re}\{\mathbf{c} \circ \mathbf{d}_i^*\} \circ \mathbf{d}_{i+1}, \quad (38)$

which can project the vector from the tangent space of $\mathbf{d}_i$ to that of $\mathbf{d}_{i+1}$ and further enable the combination of $\mathbf{g}_{i+1}$ and $\mathbf{c}_i^*$ in line 7; $\epsilon_1$ in line 9 is a small number, e.g., $10^{-5}$. 

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and (29) until (27) improves less than $\epsilon$.

Alternating Optimization Algorithm for SIM

**Algorithm 1**

**Input:** C, b, $d_0 \in \mathcal{M}_{uc}^M$; $\epsilon_1$

**Output:** The vector $d$;

1: $c_0 = -\text{grad}(d_0)$, $i = 0$;
2: repeat
3: Choose step size $\alpha_i$ according to Armijo-Goldstein condition.
4: Find the next point $d_{i+1}$ using the Retraction, i.e., $d_{i+1} = \text{Retr}_{d_i}(\alpha_i c_i)$.
5: Calculate the Riemannian gradient $g_{i+1} = \text{grad}(d_{i+1})$ from (33) and (35).
6: Calculate the Transports vectors $g_i^+$ and $c_i^+$ of $g_i$ and $c_i$ from $d_i$ to $d_{i+1}$.
7: Calculate Fletcher-Reeves parameter $\beta_{i+1} = \frac{||g_{i+1}||^2}{||g_i||^2}$ and the conjugate direction $c_{i+1} = -g_{i+1} + \beta_{i+1} c_i$.
8: $i = i + 1$;
9: until the cost function (32) improves less than $\epsilon_1$.

**Algorithm 2** Alternating Optimization Algorithm for SIM

**Input:** channel matrices $H_{B,R}$, $H_{R_R}$, and $H_{B,B_i}$; $\epsilon_2$.

**Output:** The SIM matrix $P_{SIM}$ and the vector $d$;

1: Randomly initialize $d$;
2: repeat
3: Obtain the effective channel $G_{SI}$ according to (7), and update $P_{SIM}$ by solving (28).
4: Fix $P_{SIM}$, and calculate C, b from (31).
5: Use Algorithm 1 to obtain d by solving (29).
6: if $b = \infty$ then
7: $\mathbf{D} = \text{diag}(d)$.
8: else
9: $d_q = e^{jQ(\mathbf{d})}$.
10: if $||\mathbf{C}d_q + b||^2 < ||\mathbf{C}d + b||^2$ then
11: $\mathbf{D} = \text{diag}(d_q)$.
12: end if
13: end if
14: until the cost function (27) improves less than $\epsilon_2$.

We can obtain $P_{SIM}$ and $D$ by alternatively solving (28) and (29) until (27) improves less than $\epsilon_2$, e.g., $10^{-5}$, and this alternating minimization is summarized in Algorithm 2, where $Q(\cdot)$ in line 9 can quantize the vector element-wise to the grids $\{0, \frac{2\pi}{2^n}, \frac{4\pi}{2^3}, \ldots, (2^n-1)\frac{2\pi}{2^n}\}$ for $b$-bit RIS coefficients.

**B. The DL Preceding and Power Allocation**

Inserting (26) into (21), we have

$$y = H_d P_{SIM} P_d s_d + z.$$ (39)

Note that $D$ and $P_{SIM}$ are obtained using Algorithm 2. Then we can apply the ZF preceding to mitigate the DL inter-user interference, i.e.,

$$P_d = H_d P_{SIM} (H_{SIM} H_d)^{-1},$$ (40)

where $H_{SIM} = H_d P_{SIM}$. Hence the $k$th DL user receives

$$y_k = s_{d,k} + z_k, k = 1, 2, \ldots, K_d,$$ (41)

and the DL spectral efficiency is

$$R_d = \sum_{k=1}^{K_d} \log_2 \left(1 + \frac{\sigma_{d,k}^2}{\sigma^2}\right).$$ (42)

We need to solve

$$\max_{P_d} R_d,$$

s.t. $\text{tr} (P_d P_d^H) \leq P_t,$ (43)

where $P_t$ represents the maximum Tx power of the BS. The closed-form solution to (43) can be obtained by the “water-filling” method.

**C. Complexity Analysis**

In Section III-B, we can obtain the closed-form solution to $P_d$ and the DL power allocation without iterations. Hence the complexity mainly lies in Algorithm 2, which consists of the alternating minimization of (28) and (29). The complexity of solving (28) is $O(M M_{ris} + M_d M_p + M_1^2)$, while for (29), the complexity is dominated by Algorithm 1, where we need to iteratively calculate (32) and (33) whose complexity is $O(M_d M_{ris})$. Denote $I_r$ and $I_a$ as number of iterations in Algorithm 1 and Algorithm 2 respectively and denote $I_a$ as the average number of Armijo-Goldstein condition searches occurred during each iteration of Algorithm 2. Then the complexity of Algorithm 2 can be denoted as $O(I_r M_r M_{ris} + M_d^2 M_p + M_1^2 + I_r (1 + I_a) M_d M_r M_{ris})$, which grows linearly with the number of RIS elements. Hence the complexity of the proposed algorithm will be acceptable even for a large RIS.

IV. A BENCHMARK STUDY: AN IDEALISTIC FULL-DUPLEX SYSTEM WITH $\infty$-BIT ADC

This section investigates an idealistic RIS-assisted IBFD system (I-RAIBFD) where ADCs are assumed to have $\infty$-bit resolutions. By maximizing the UL-DL sum rate of the I-RAIBFD system, we obtain a performance upper bound, which can be used to gauge the realistic system developed in Section III. We also propose a RIS-assisted FDD (RAFDD) system as a performance lower bound of the proposed RAIBFD system.

**A. The UL and DL Spectral Efficiency of I-RAIBFD System**

As the ENOB $\to \infty$, we have $\rho = 0$ according to (12). Hence the quantization noise can be ignored in (16), and we have

$$\tilde{y}_B = H_u s_u + z_B,$$ (44)

which leads the UL spectral efficiency from (17) to be

$$R_u = \log_2 \left| I + \frac{\sigma_n^2}{\sigma^2} H_u H_u^H \right|.$$ (45)

For the DL signal transmission, we have from (21) that

$$y = H_d P s_d + z.$$ (46)
Adopting the ZF precoding matrix

$$\mathbf{P} = \mathbf{H}_d^H (\mathbf{H}_d \mathbf{H}_d^H)^{-1}$$

(47)

to mitigate the inter-user interference, we obtain the DL spectral efficiency as

$$R_d^I = \sum_{k=1}^{K_d} \log_2 \left( 1 + \frac{\sigma_d^2}{\sigma^2} \right).$$

(48)

To maximize the DL spectral efficiency of the I-RAIBFD system, we need to solve

$$\max_{\{\phi_i\}_{i=1}^{M_{ri}}, \{\sigma_d^2\}_{k=1}^{K_d}} R_d^I,$$

subject to

$$\sum_{k=1}^{K_d} \gamma_k \sigma_d^2 \leq P_t,$$

(49)

where $\gamma_k$ is

$$\gamma_k = \left( (\mathbf{H}_d \mathbf{H}_d^H)^{-1} \right)_{k,k} = \text{tr} (\mathbf{E}_k (\mathbf{H}_d \mathbf{H}_d^H)^{-1}),$$

(50)

and $\mathbf{E}_k = \mathbf{e}_k \mathbf{e}_k^T$. Fixing the RIS phases $\{\phi_i\}_{i=1}^{M_{ri}}$, we can reduce (49) to

$$\max_{\{\sigma_d^2\}_{k=1}^{K_d}} R_d^I,$$

subject to

$$\sum_{k=1}^{K_d} \gamma_k \sigma_d^2 \leq P_t,$$

(51)

to which the solution is the “water filling” power allocation

$$\sigma_d^2 = \max \left( 0, \frac{1}{\mu \gamma_k} - \sigma^2 \right), k = 1, 2, \ldots, K_d.$$  

(52)

The Lagrangian multiplier $\mu$ can be obtained from the power constraint $\sum_{k=1}^{K_d} \gamma_k \sigma_d^2 = P_t$. We assume the user selection has been made so that all the $K_d$ users are allocated with non-zero power. That is,

$$\mu = \frac{K_d}{\sum_{k=1}^{K_d} \gamma_k \sigma^2 + P_t}.$$  

(53)

Inserting (53) into (52) yields the optimal power allocation

$$\sigma_d^2 = \frac{P_t}{\mu \gamma_k} + \frac{K_d \gamma_k \sigma^2}{\gamma_k \gamma_{\text{d,k}}}, k = 1, 2, \ldots, K_d,$$

(54)

which further leads (48) to be

$$R_d^I = K_d \log_2 \left( \frac{P_t}{\sigma^2} + \sum_{k=1}^{K_d} \gamma_k \right) - \sum_{k=1}^{K_d} \log_2 (K_d \gamma_k).$$

(55)

Hence combining (45) and (55), we have the sum rate of I-RAIBFD system

$$R_t = R_u^I + R_d^I.$$  

(56)

We can recast $R_t$ as a function of $\mathbf{d}$ and define

$$g(\mathbf{d}) \triangleq -R_t.$$  

(57)

Hence to maximize the sum rate of the I-RAIBFD system is equivalent to

$$\min_{\mathbf{d}} g(\mathbf{d}),$$

(58)

of which a near-optimal solution can be obtained by the RCG algorithm summarized in Algorithm 1 with cost function $g(\mathbf{d})$ instead of $f(\mathbf{d})$. The Euclidean gradient of $g(\mathbf{d})$ is given in Proposition 1. The optimized sum rate of the I-RAIBFD system is the upper bound of the proposed RAIBFD system.

**Proposition 1:** For the cost function $g(\mathbf{d})$, its Euclidean gradient is

$$\nabla g(\mathbf{d}) = -\frac{1}{\ln 2} \sigma_B^2 \text{diag}(\mathbf{J}_u^H) + \frac{K_d}{\ln 2} \sigma_B^2 \text{diag}(\mathbf{F}_d^H) - \frac{1}{\ln 2} \sum_{k=1}^{K_d} \frac{\text{diag}(\mathbf{J}_d^H)}{\gamma_k},$$

(59)

where

$$\mathbf{J}_u = \mathbf{H}_B \Gamma \mathbf{H}_u^H (I + \sigma_B^2) \mathbf{H}_B \Gamma^{-1} \mathbf{H}_B \Gamma^{-1} \mathbf{H}_d \Gamma^{-1} \mathbf{H}_d \Gamma^{-1} \mathbf{H}_B \Gamma^{-1} \mathbf{H}_u^H,$$

(60)

$$\mathbf{F}_d = \mathbf{H}_B \Gamma \mathbf{H}_d^H \mathbf{H}_d \mathbf{H}_d^H \mathbf{H}_B \Gamma^{-1} \mathbf{H}_d \Gamma^{-1} \mathbf{H}_d \Gamma^{-1} \mathbf{H}_B \Gamma^{-1} \mathbf{H}_d \Gamma^{-1} \mathbf{H}_d \Gamma^{-1} \mathbf{H}_B \Gamma^{-1} \mathbf{H}_d \Gamma^{-1} \mathbf{H}_d \Gamma^{-1} \mathbf{H}_B \Gamma^{-1} \mathbf{H}_d \Gamma^{-1} \mathbf{H}_d \Gamma^{-1} \mathbf{H}_B \Gamma^{-1} \mathbf{H}_u^H,$$

(61)

and

$$\mathbf{J}_{d,k} = \mathbf{H}_B \Gamma \mathbf{H}_d^H \mathbf{H}_d \mathbf{H}_d^H \mathbf{H}_B \Gamma^{-1} \mathbf{H}_d \Gamma^{-1} \mathbf{H}_d \Gamma^{-1} \mathbf{H}_B \Gamma^{-1} \mathbf{H}_d \Gamma^{-1} \mathbf{H}_d \Gamma^{-1} \mathbf{H}_B \Gamma^{-1} \mathbf{H}_d \Gamma^{-1} \mathbf{H}_d \Gamma^{-1} \mathbf{H}_B \Gamma^{-1} \mathbf{H}_d \Gamma^{-1} \mathbf{H}_d \Gamma^{-1} \mathbf{H}_B \Gamma^{-1} \mathbf{H}_u^H.$$  

(62)

**Proof:** The derivation of (59) is detailed in Appendix.

### B. RIS-Assisted FDD System

Inspired by the proposed I-RAIBFD system, we can also propose a RIS-assisted FDD (RAFDD) system, which has the following characteristics:

1. The UL and DL signals are transmitted in different frequency bands, which are half that of the I-RAIBFD system.
2. The thermal noise at the receivers of BS and the user is half of the I-RAIBFD system for half bandwidth.

From 1) and 2), we can see that the sum rate of the RAFDD system is half of that of the I-RAIBFD system except for the parameters setting of carrier frequency and thermal noise. Hence we can also optimize the sum rate of the RAFDD system and use the optimized sum rate as a lower bound of the proposed RAIBFD system.

## V. Numerical Examples

This section provides simulation results to validate the performance of the proposed RAIBFD wireless systems. Consider a base station deployed with the eight-element Rx antennas and eight-element Tx antennas, i.e., $M_t = M_r = 8$. A RIS is placed behind the antenna arrays of the BS as illustrated in Fig. 2 and Fig. 3, where a uniform linear array (ULA) and a uniform rectangular array (URA) are deployed in Fig. 2(a) and Fig. 3(a), respectively. The top view and the side view of the RIS-assisted BS deployed with ULA and URA are shown in Fig. 2(b,c) and Fig. 3(b,c), where $d_{BA}$ represents the distance between antennas and RIS, and $d_{BA}$ stands for the distance between Tx and Rx antenna arrays.

Given the wave number $k_\lambda = \frac{2\pi}{\lambda}$ with $\lambda$ being the wavelength, the near-field LOS channel between two adjacent
Fig. 2. Placement of ULA and RIS.

Fig. 3. Placement of URA and RIS.

The points \(A(x_1, y_1, z_1)\) and \(B(x_2, y_2, z_2)\) shown in Fig. 2 and Fig. 3 is simulated as

\[
h_{AB} = \sqrt{\beta_{AB}} e^{j\lambda d_{AB}}, \tag{63}
\]

where

\[
\beta_{AB} = \frac{G_t}{4} \left( \frac{1}{(k\lambda d)^2} - \frac{1}{(k\lambda d)^4} + \frac{1}{(k\lambda d)^6} \right), \tag{64}
\]

and \(d_{AB} = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2}\). Hence the entries of \(H_{B,R}, H_{B,Bt}\), and \(H_{RBt}\) are obtained from (63).

A. Performance of SIM

We simulate the SIM performance of the proposed RAIBFBD system by defining the SIM amount as

\[
\kappa = \frac{1}{M_r} \left\| \left( H_{B,R} D H_{RBt} + H_{B,Bt} \right) P_{SIM} \right\|_F^2. \tag{65}
\]

The first example shows the SIM performance under different distances between RIS and antenna array, i.e., \(d_{RA}\). We consider both ULA and URA with the size of RIS being \(4 \times 4\), \(8 \times 8\), and \(16 \times 16\), and phases of RIS are infinite bit resolutions. The number of DL effective antennas is set to \(M_d = 8\), which indicates that all subspace dimensions are used for DL data precoding. Fig. 4 shows that more RIS elements can mitigate stronger interference even with only the phases optimization of RIS. Moreover, SIM amount will degrade with larger \(d_{RA}\), as larger \(d_{RA}\) enlarges the path loss of the RIS reflection path, which prevents self-interferences from two paths, i.e., the direct path and the RIS reflection path, from adding destructively at Rx antennas. Hence we need to use larger RIS and put the RIS close to the antenna array in practical applications.

Fig. 5 studies the relation between the SIM performance and the RIS bit resolution with \(M_d = 8\) and \(d_{RA} = \frac{\lambda}{2}\). We can see from Fig. 5 that the SIM amount increases with RIS bit resolution grows larger, and 1-bit RIS leads to similar SIM performance with RIS size of \(4 \times 4\), \(8 \times 8\), and \(16 \times 16\), but for RIS of high precision, e.g., 6-bit, \(16 \times 16\) RIS can mitigate more self-interference than \(4 \times 4\) RIS and \(8 \times 8\) RIS. According
are generated according to the free-space path loss model
\( N \). We assume that \( N \) is the number of clusters, \( N_{\text{ray}} \) is the number of rays per cluster, \( \gamma_{i,j}^{\text{URA}} \), \( \gamma_{i,j}^{\text{ULA}} \) are the complex gains of the \( j \)th ray in the \( i \)th cluster; \( \mathbf{a}_{\text{URA}}(\theta, \varphi) \) is the array response of the \( U \times V \) RIS with respect to the two dimensional angles \( (\theta, \varphi) \):
\[
\mathbf{a}_{\text{URA}}(\theta, \varphi) = [1, \ldots, e^{j\pi(\sin(\theta)\sin(\varphi)+\cos(\varphi))}, \ldots, e^{j\pi((U-1)\sin(\theta)\sin(\varphi)+(V-1)\cos(\varphi))}].
\]
and \( \mathbf{a}_{\text{ULA}}(\omega) = [1, e^{-j\pi\omega}, \ldots, e^{-j\pi(M-1)\omega}]^T \) with \( \omega \) being the direction. For the following simulations, we assume that \( N_d = 3 \) and \( N_{\text{ray}} = 5 \) in the following simulations. \( \beta_{u,k} \) and \( \beta_{d,k} \) are generated according to the free-space path loss model
\[
\beta_{u,k} = \left( \frac{\gamma_{u,k}^{\text{URA}}}{4\pi d_{u,k}} \right)^2, \quad \beta_{d,k} = \left( \frac{\gamma_{d,k}^{\text{ULA}}}{4\pi d_{d,k}} \right)^2
\]
where \( d_{u,k} \) and \( d_{d,k} \) are the distances from the BS to the \( k \)th UL user and the \( k \)th DL user, respectively. The maximum transmit power of the BS is \( P_t = 30 \text{dBm} \), and the transmit power of the UL users is \( \sigma_u^2 = 10 \text{dBm} \). We assume that \( K_u = K_d = 3 \) with \( d_{u,k}, d_{d,k} (k = 1, 2, 3) \) uniformly distributed between \([0, \lambda]\) [27]. The wavelength \( \lambda \) is 0.125m given the carrier frequency of 2.4GHz, and \( G_i = 1 \) as the BS is equipped with omnidirectional antennas. The noise power at the BS and users are set to be \( \sigma_B^2 = \sigma_u^2 = -96 \text{dBm} \) [38]. For the RAFFD system, \( \sigma_B^2 = \sigma_u^2 = -96 - 3 - 99 \text{dBm} \) as the frequency bandwidth is evenly divided between the UL and DL transmission, and the UL and DL frequency carrier are 1760MHz and 1855MHz [39], respectively.

Based on the aforementioned parameter settings, Fig. 6 simulates the UL rate, DL rate, and the sum rate performance of the proposed RAIBFD system as the number of DL effective antenna varies from 3 to 8 given RIS of infinite bit resolution. The distance between RIS and antenna array, the ENOB of the ADC are set to be \( d_{RA} = \frac{\lambda}{2} \) and \( \text{ENOB} = 12 \). Larger \( M_d \) makes it harder to mitigate the self-interference for more constraints in (30) with limited variables, i.e., the number of RIS elements. Except for \( 16 \times 16 \) RIS, UL rate decreases for the limited SI mitigation amount of \( 4 \times 4 \) RIS and \( 8 \times 8 \) RIS, while larger \( M_d \) can enhance the DL rate as the subspace dimensions for DL data transmission increase. Hence there exists a trade-off between the UL rate and the DL rate with respect to \( M_d \), but for RIS with more elements, this trade-off disappears, i.e., \( 16 \times 16 \) RIS, which shows the promising potential of RIS in the application of IBFD systems.

Fig. 7 shows that the sum rate performance of the proposed RAIBFD system, the I-RAIBFD system, the RAFFD system, and the SoftNull method in [21] as the effective number of bits of ADC varies from 8 to 12 using a 16 \times 16 RIS and a ULA. Note that the effect of quantization noise on the RAFFD system is ignored and the number of DL effective antenna, the distance between RIS and antenna array are set to be \( M_d = 8 \) and \( d_{RA} = \frac{\lambda}{2} \), respectively. Fig. 7 shows that even for 1-bit RIS, the sum rate of the proposed RAIBFD outperforms the SoftNull method given ENOB from 8 to 12, and also outperforms the RAFFD system at ENOB = 11, 12. Moreover, for ENOB = 12 and 6-bit RIS, the sum rate of the proposed RAIBFD system can approach 86\% of that of the I-RAIBFD system.

Fig. 8 provides the sum rate performance as the RIS suffers from phase deviations. Fig. 8 shares the same setting with Fig. 7 except for the ENOB, which is set to be 12 in Fig. 8. Assume the phase deviations are uniformly distributed between \([-\sigma_p^\text{R}, \sigma_p^\text{R}]\). The RAIBFD systems with RIS of high bit resolutions are sensitive to the phase deviations, e.g., the sum rate of \( \infty \)-bit RIS with \( \sigma_p = 30^\circ \) is 14\% lower than before. To avoid this performance loss of the RAIBFD system, we have proposed an over-the-air calibration method for RIS [40].

The last simulation studies the sum rate performance of the proposed RAIBFD system under different channel estimation errors given a 16 \times 16 RIS with infinite resolutions and a ULA, where the estimated channels \( \hat{\mathbf{H}}_{B_{\text{UL}}}, \hat{\mathbf{H}}_{B_{\text{DL}}}, \) and \( \hat{\mathbf{H}}_{\text{f}B_{\text{t}}} \) are generated from
\[
\hat{\mathbf{H}} = \mathbf{H} + \Delta \mathbf{H} \in \mathbb{C}^{P \times Q},
\]
with the entries of $\Delta \mathbf{H}$ following $\mathcal{CN}\left(0, \frac{|\mathbf{H}^u_{\text{st}}|^2}{P} \sigma_{c}^2\right)$ and $\sigma_{c}$ regulating the channel estimation errors. Fig. 9 shows that the sum rate of the proposed RAIBFD system decreases as $\sigma_c$ varies from 0 to 0.2. For ENOB = 11, 12, the RAIBFD system is only subjected to a slight loss of the sum rate even with $\sigma_c = 0.2$. Hence the proposed RAIBFD systems, equipped with ADC of reasonably high bit resolutions and RIS of large size, are robust to the channel estimation error.

**VI. Conclusion**

In this paper, we considered a RIS-assisted in-band full-duplex (RAIBFD) wireless system where the base station (BS) aided by a RIS transmits a signal to and receives a signal from multiple users on the same frequency band simultaneously. Taking into account the quantization noise of the analog-to-digital converters (ADCs), we propose to jointly design the DL precoding matrix and the RIS coefficients to mitigate the self-interference. We also consider an idealistic RAIBFD (I-RAIBFD) system and a RIS-assisted FDD (RAFDD) system as benchmarks of the proposed RAIBFD system. The simulation results show the effectiveness of the proposed RAIBFD wireless system as its sum rate outperforms that of the RAFDD system and the state-of-the-art in-band full-duplex method and can approach 86% of that of the I-RAIBFD system.

**Appendix: Derivation of (59)**

First, we calculate $\nabla R_u(d)$. Rewriting $\mathbf{H}_u$ from (10) as $\mathbf{H}_u = \mathbf{H}_{B,R} \mathbf{D} \mathbf{H}_{R,u} \Gamma_u + \mathbf{H}_{B,u} \Gamma_u$, we have

$$\partial \mathbf{H}_u = \mathbf{H}_{B,R} (\partial \mathbf{D}) \mathbf{H}_{R,u} \Gamma_u,$$

and

$$\partial \mathbf{H}_u^H = \Gamma_u^H \mathbf{H}_{R,u}^H (\partial \mathbf{D}^H) \mathbf{H}_{B,R}^H.$$
Owing to the formula \( \partial (\log_2(\det(X))) = \frac{1}{\ln 2} \text{tr}(X^{-1} \partial X) \), and we can obtain from (45) that
\[
\partial R_u = \frac{1}{\ln 2} \frac{\sigma_u^2}{\sigma_B^2} \text{tr} \left( F_u^{-1} \left( \partial (H_u) H_u^H + H_u (\partial H_u^H) \right) \right),
\]
where \( F_u = I + \frac{\sigma_u^2}{\sigma_B^2} H_u H_u^H \). Inserting (72) and (73) into (74) yields
\[
\partial R_u = \frac{1}{\ln 2} \frac{\sigma_u^2}{\sigma_B^2} \text{tr} \left( J_u (\partial D) + J_u^H (\partial D^H) \right),
\]
where
\[
J_u = H_{Ru} \Gamma_u H_u^H F_u^{-1} H_{Bu,R}.
\]
According to (75), we can further obtain
\[
\nabla R_u(d) = \frac{1}{\ln 2} \frac{\sigma_u^2}{\sigma_B^2} \text{diag} \left( J_u^H \right).
\]
Second, we calculate \( \nabla R_d(d) \). Inserting (50) into (55), we have
\[
R_d = K_d \log_2 \left( \frac{P_t}{\sigma_d} + \text{tr} \left( (H_d H_d^H)^{-1} \right) \right)
- \sum_{k=1}^{K_d} \log_2 \left( K_d \text{tr} \left( E_k (H_d H_d^H)^{-1} \right) \right),
\]
from which we can further obtain
\[
\nabla R_d(d) = \frac{K_d}{\ln 2} \frac{\sigma_d^2}{\sigma_B^2} \frac{\text{tr} \left( \partial (H_d H_d^H)^{-1} \right)}{P_t} + \frac{K_d}{\ln 2} \sum_{k=1}^{K_d} \frac{\text{tr} \left( E_k (\partial (H_d H_d^H)^{-1}) \right)}{\gamma_k}.
\]
As \( H_d \) can be rewritten from (22) as
\[
H_d = \Gamma_d H_{d,R} D H_{d,R^t} + \Gamma_d H_{d,B},
\]
we have
\[
\partial H_d = \Gamma_d H_{d,R} (\partial D) \Gamma_d H_{d,B},
\]
and
\[
\partial H_d^H = H_{d,R}^H (\partial D^H) H_{d,R}^H \Gamma_d^H.
\]
Using the formula \( \partial (X^{-1}) = -X^{-1} (\partial X) X^{-1} \) and denoting \( \hat{H}_d = (H_d H_d^H)^{-1} \), we can obtain
\[
\partial \hat{H}_d = -\hat{H}_d (\partial (H_d H_d^H)) \hat{H}_d,
= -\hat{H}_d H_{d,R} (\partial D) H_{d,R}^H \hat{H}_d
- \hat{H}_d H_{d,R} H_{d,R}^H (\partial D^H) H_{d,R}^H \Gamma_d \hat{H}_d,
\]
which leads (79) to be
\[
\partial R_d = \frac{K_d}{\ln 2} \frac{\sigma_d^2}{\sigma_B^2} \frac{\text{tr} \left( F_d (\partial D) + F_d^H (\partial D^H) \right)}{P_t} + \frac{K_d}{\ln 2} \sum_{k=1}^{K_d} \frac{\text{tr} \left( J_{d,k} (\partial D) + J_{d,k}^H (\partial D^H) \right)}{\gamma_k},
\]
with
\[
F_d = H_{d,R} H_{d,R}^H \hat{H}_d \hat{H}_d \Gamma_d H_{d,R}.
\]
According to (84), we have
\[
\nabla g(d) = -(\nabla R_u(d) + \nabla R_d(d)),
\]
\[
= -\frac{1}{\ln 2} \frac{\sigma_u^2}{\sigma_B^2} \text{diag} \left( J_u^H \right) + \frac{K_d}{\ln 2} \frac{\text{diag} \left( F_d^H \right)}{P_t} + \frac{K_d}{\ln 2} \sum_{k=1}^{K_d} \frac{\text{diag} \left( J_{d,k}^H \right)}{\gamma_k},
\]
From (77) and (87), we have \( \nabla g(d) \) as
\[
\nabla g(d) = -\nabla R_u(d) - \nabla R_d(d),
\]
\[
= -\frac{1}{\ln 2} \frac{\sigma_u^2}{\sigma_B^2} \text{diag} \left( J_u^H \right) + \frac{K_d}{\ln 2} \frac{\text{diag} \left( F_d^H \right)}{P_t} + \frac{K_d}{\ln 2} \sum_{k=1}^{K_d} \frac{\text{diag} \left( J_{d,k}^H \right)}{\gamma_k}.
\]

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