Dynamical spin-flip susceptibility for a strongly interacting ultracold Fermi gas

M. Sandri\textsuperscript{1,2}, A. Minguzzi\textsuperscript{3(a)} and F. Toigo\textsuperscript{2}

\textsuperscript{1} International School for Advanced Studies (SISSA) - Via Bonomea 265, I-34136 Trieste, Italy, EU
\textsuperscript{2} Dipartimento di Fisica Galileo Galilei and CNISM, Università di Padova - Via Marzolo 8, I-35122 Padova, Italy, EU
\textsuperscript{3} Université Grenoble I and CNRS, Laboratoire de Physique et Modélisation, des Milieux Condensés UMR 5493, Maison des Magistères - B.P. 166, F-38042 Grenoble, France, EU

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Abstract – The Stoner model predicts that a two-component Fermi gas at increasing repulsive interactions undergoes a ferromagnetic transition. Using the random-phase approximation we study the dynamical properties of the interacting Fermi gas. For an atomic Fermi gas under harmonic confinement we show that the transverse (spin-flip) dynamical susceptibility displays a clear signature of the ferromagnetic phase in a magnon peak emerging from the Stoner particle-hole continuum. The dynamical spin susceptibilities could be experimentally explored via spin-dependent Bragg spectroscopy.

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Introduction. – Experimental advances of trapping and cooling ultracold Fermi gases allow the exploration of strongly interacting quantum degenerate two-component gases, realized by populating two hyperfine atomic levels. The strength and the sign of the intercomponent interactions can be tuned by the mechanism of Feshbach resonances. The case of attractive Fermi-Fermi interactions has allowed the exploration of the crossover from BCS-like pairing to Bose-Einstein condensation (BEC) of tightly bound pairs [1]. On the repulsive side of the Feshbach resonance, the metastable branch corresponding to a two-component Fermi gas with repulsive interactions has received more recent experimental attention [2,3] as a possible realization of the Stoner model of itinerant ferromagnetism. The mean-field Stoner model [4] predicts that a repulsive Fermi gas undergoes a transition to a ferromagnetic state for sufficiently large interaction strength $k_F a > \pi/2$, where $a$ is the s-wave scattering length and \( k_F = \left(3 \pi^2 n\right)^{1/3} \) is the Fermi wave vector for a uniform gas with total density $n$. Beyond mean field, the inclusion of quadratic fluctuations decreases this value to $k_F a \approx 1.05$ [5,6], and Quantum Monte Carlo simulations predict the transition to a uniform magnetic phase to occur at $k_F a \approx 0.8$ [7–9]. The possibility of inhomogeneous intermediate phases has also been considered [7].

In this work we analyze the dynamical properties of a two-component Fermi gas with repulsive interactions. The approach to the ferromagnetic transition strongly affects the dynamical properties of the system, displaying a softening of the out-of phase spin-dipole modes [10] and enhancement of the spin drag coefficient [11]. A dynamical instability of a spin spiral excitation has also been predicted [12]. In the ferromagnetic phase, magnon collective modes are predicted to propagate undamped at zero temperature [13].

Spectroscopy is a powerful tool to address many-body systems. The study of the dynamic structure factor for density and spin has been suggested as a probe of pairing and superfluidity of a Fermi gas [14,15]; Bragg scattering experiments for a strongly interacting Fermi gas in the BCS-BEC crossover have been performed [16].

We focus here on the spin-susceptibility spectra as a tool to characterize the ferromagnetic phase of a two-component Fermi gas. Such spectra could be accessed experimentally via spin-dependent Bragg spectroscopy.

Dynamical susceptibilities in the random-phase approximation. – We consider a two-component Fermi gas $|\uparrow\rangle, |\downarrow\rangle$, at zero temperature, subjected to an

\( \text{(a)} \text{E-mail: anna.minguzzi@grenoble.cnrs.fr} \)
external confining potential $V_{ext}(r)$. The repulsive intercomponent interactions are described by a contact potential of strength $g = 4\pi\hbar^2a/m$ in terms of the the atomic mass $m$ and of the scattering length $a > 0$ (see footnote 1). The model Hamiltonian of the system reads
\[
\hat{H} = \int d\mathbf{r} \hat{\psi}_s^\dagger(r) \left[ -\frac{\hbar^2}{2m}\nabla^2 + V_{ext}(r) \right] \hat{\psi}_s(r) + g \int d\mathbf{r} \hat{\psi}_s^\dagger(r) \hat{\psi}_s^\dagger(r) \hat{\psi}_s(r) \hat{\psi}_s(r),
\]
where the fermionic field operators $\hat{\psi}_s(r)$ satisfy the usual anticommutation relations \( \{ \hat{\psi}_s(r), \hat{\psi}_s^\dagger(r') \} = \delta_{s,s'}\delta(r - r') \). The corresponding spin densities are \( \sigma_{\alpha}(r) = \hat{\psi}_{\sigma}^\dagger(r) \sigma_{\alpha} \hat{\psi}_\sigma(r) \) in terms of the spinor \( \Psi(r) = \{ \hat{\psi}_\uparrow(r), \hat{\psi}_\downarrow(r) \} \); \( \sigma_{\alpha} \) are the Pauli matrices with \( \alpha = 0, x, y, z \). For the transverse (i.e. \( x, y \)) components it is useful to introduce the combinations \( \sigma_{\pm} = (\sigma_x \pm i\sigma_y)/2 \) because they yield the spin-flip amplitudes
\[
\sigma_+(r) = \hat{\psi}_\uparrow(r) \hat{\psi}_\downarrow(r); \quad \sigma_-(r) = \hat{\psi}_\downarrow(r) \hat{\psi}_\uparrow(r).
\]
In linear response theory, corresponding to the regime of weak external drive, the dynamical spin susceptibilities are given by the retarded Green’s functions
\[
\chi_{\alpha\beta}(\mathbf{r}, \mathbf{r}', t, t') = -i\theta(t - t') \langle \{ \sigma_{\alpha}(\mathbf{r}, t), \sigma_{\beta}(\mathbf{r}', t') \} \rangle,
\]
where the quantum averages \( \langle \ldots \rangle \) are performed on the unperturbed state of the system. Such dynamical correlations functions embed the effect of interactions, and require a many-body calculation. We consider here their expression as obtained by the Random-Phase Approximation (RPA) [18,19], corresponding to time-dependent Hartree-Fock approximation. This analytical approximation is known to satisfy sum rules and well describe the weak-coupling regime. It also accounts for the ferromagnetic transition at the mean-field (Stoner) level, hence, although not quantitatively correct, it is expected to qualitatively describe the dynamical properties of the interacting Fermi gas. The RPA has successfully accounted for the measured dynamical susceptibilities of an attractive Fermi gas in the BCS-BEC crossover [20].

According to the RPA, for a uniform Fermi gas the Fourier transform of spin-flip susceptibility with respect to both spatial and time relative variables is given by
\[
\chi_{zs}(\omega, \mathbf{q}) = \frac{\Pi_{zz} + \Pi_{xx} - 2g\Pi_{zz} + \Pi_{xx}}{1 - g^2\Pi_{zz} + \Pi_{xx}},
\]
where the Lindhard functions \( \Pi_{\sigma\sigma'}(\omega, \mathbf{q}) \) are given by
\[
\Pi_{\sigma\sigma'}(\omega, \mathbf{q}) = \frac{1}{V} \sum_{k} \left[ f_\sigma(\xi_k) - f_{\sigma'}(\xi_{k+\mathbf{q}}) \right],
\]
and
\[
\xi_k = \epsilon_k - \mu - g n_{\mathbf{k}} - \sigma.
\]

1In the many body problem we neglect the corrections to the scattering length approximation [17], which are expected to shift the location of the transition to the ferromagnetic phase. Inclusion of such corrections goes beyond the accuracy of the present model.
where the regions in the \((\nu, q)\) plane are defined by the following inequalities: (a) \(\max\{\nu_{-}^{(2)}, \nu_{+}^{(2)}\} \leq \nu \leq \nu_{+}^{(1)}\) or \(\nu_{+}^{(2)} \leq \nu \leq \nu_{+}^{(1)}\); (b) \(\max\{\nu_{+}^{(1)}, \nu_{+}^{(2)}\} \leq \nu \leq \nu_{-}^{(2)}\); and (c) \(\nu_{-}^{(1)} \leq \nu \leq \min\{\nu_{+}^{(2)}, \nu_{-}^{(2)}\}\). Correspondingly, for the real part we have

\[
\Re \Pi_{++} = \frac{1}{2\pi^2 q} \left[ A_{+} B_{+} - A_{-} B_{-} \right] \\
+ \frac{1}{2} (A_{+}^2 - B_{+}^2) \ln \left| \frac{A_{+} + B_{+}}{A_{-} - B_{+}} \right| \\
+ \frac{1}{2} (A_{-}^2 - B_{-}^2) \ln \left| \frac{A_{-} + B_{-}}{A_{+} - B_{-}} \right|. 
\]

The above results for the Lindhard function allow to obtain the final result for the imaginary part of the spin-flip susceptibility

\[
\Im \chi_{++}(\nu, q) = \frac{\Im \Pi_{++}(\nu, q)}{(1 + g^2 \Re \Pi_{++}(\nu, q))^2 + g^2 \Im^2 \Pi_{++}} + A(q) \delta(\nu - \Omega_{q}); 
\]

this includes both the Stoner particle-hole continuum \(\Im \chi_{++}^{\text{Ston}}(\omega, q)\) and the magnon contribution with weight \(A(q) = (g^2/(\partial \Pi_{++}/\partial \omega|_{\Omega_{q}}))^{-1}\).

The RPA expression for the spin-flip susceptibility satisfies the sum rule [23]

\[
-\frac{1}{\pi} \int_{-\infty}^{\infty} d\omega \Im \chi_{++}(\omega, q) = \frac{(n_{+} - n_{-})}{\hbar}. 
\]

This provides an alternative way to fix the weight \(A(q)\) of the collective magnon mode with respect to the contribution of particle-hole excitations, according to the relation \(A(q) = (n_{+} - n_{-})/\hbar + (1/\pi) \int_{-\infty}^{\infty} d\omega \Im \chi_{++}^{\text{Ston}}(\omega, q)\). At increasing \(q\) values the magnon strength decreases till it vanishes when the magnon mode enters the particle-hole continuum. Nevertheless, as it is shown in fig. 1, quite remarkably some trace of the magnon mode is left as a pronounced maximum in the particle-hole continuum. This feature could be used to infer the presence of the magnon mode even for wavevectors larger than those associated to the Stoner gap.

Spin-flip response for a trapped gas. – We consider now the effect of inhomogeneity in the Fermi gas induced by the presence of the external harmonic confinement \(V_{\text{ext}}(r) = 1/2 m \Omega_{r}^2 r^2\), taken for simplicity isotropic. The equilibrium density profiles \(n_{+}(r), n_{-}(r)\) of the two magnetization components are found by minimizing the Thomas-Fermi energy functional [24–26]

\[
\mathcal{E}[n_{\sigma}(r)] = \int dr \left\{ \sum_{\sigma} \left[ \frac{3}{5} \alpha n_{\sigma}(r)^{5/3} + \frac{1}{2} m \Omega_{r}^2 r^{2} n_{\sigma}(r) \right] - \mu_{\sigma} n_{\sigma}(r) + gn_{+}(r)n_{-}(r) \right\}, 
\]

where \(\alpha = (6\pi^2)^{2/3}\hbar^2/2m\). The chemical potentials \(\mu_{\sigma}\) are related to particle numbers through the normalization conditions \(\int dr n_{\sigma}(r) = N_{\sigma}\). Note that for simplicity the calculations have been performed at \(\mu_{+} = \mu_{-}\), corresponding to fixing the total number of particles \(N\). An analysis beyond Thomas-Fermi approximation [27] would provide corrections at the tails of the density profiles but not alter significantly the resulting profiles. The density profiles at large interactions are shown in fig. 2. The associated magnetization profile \(m(r) = n_{+}(r) - n_{-}(r)\) is radially oriented, with a monotonously decreasing magnitude at increasing \(r\). This corresponds to a hedgehog configuration, which is one of lowest energy except for a very small system [28].

The dynamical spin-flip response of the trapped system is obtained from the one of the homogeneous system through the local-density approximation (LDA) as an average over the various trap regions,

\[
-g \Im \chi_{++}(\omega, q) = \frac{-\int_{V_{tr}} d\mathbf{r} \Im \chi_{++}(\omega, \mathbf{q}, \mathbf{r})}{\int_{V_{tr}} d\mathbf{r}}, 
\]

where \(\Im \chi_{++}(\omega, \mathbf{q}, \mathbf{r})\) is the homogeneous result (11) which depends on \(\mathbf{r}\) through \(n_{+}(\mathbf{r}), n_{-}(\mathbf{r})\), and \(V_{tr}\) indicates the spatial volume where the particle densities are non-vanishing. The local-density approximation is valid if the
various values of the transferred momentum $Q$ when the magnon merges onto it. The Stoner continuum which displays a maximum and shaded areas indicate the magnon collective excitation as a function of the rescaled frequency $\Omega = \omega/k_F(0)$, with $k_F(0) = (\hbar^2 n_\pi(0)/m)^{1/3}$, and for a particle number $N = 10^7$. Insets: the gray area indicates the Stoner particle-hole continuum in the $(\omega, q)$ plane as seen at various regions in the trap (center, middle, and sides from left to right) in a local-density picture, and the red circle indicates the region where magnon excitations are undamped at zero temperature.

Fig. 2: (Color online) Main panel: Spatial density profiles $n_+(r)$ (black solid line) and $n_-(r)$ (red dashed line), in units of the corresponding non-interacting densities at the trap center $n_{\pm}^{(0)}(0)$ as a function of the radial distance $r$ in units of $a_ho$ for dimensionless interaction strength $\lambda = k_F^2(0)/a = 2.5$, with $k_F^2(0) = (6\pi^2 n_\pi^{(0)}(0))^{1/3}$. and for a particle number $N = 10^7$. Insets: the gray area indicates the Stoner particle-hole continuum in the $(\omega, q)$ plane as seen at various regions in the trap (center, middle, and sides from left to right) in a local-density picture, and the red circle indicates the region where magnon excitations are undamped at zero temperature.

Fig. 3: (Color online) Imaginary part of the spin-flip susceptibility $-g3\chi_+(-q, \omega)$ for a harmonically trapped Fermi gas as a function of the rescaled frequency $\Omega = \omega/k_F(0)^2$ at various values of the transferred momentum $Q = q/k_F(0)$ for dimensionless interaction strength $\lambda = 2.5$. The dashed lines and shaded areas indicate the magnon collective excitation contribution to the spectrum, the solid lines indicate the contribution of the Stoner continuum which displays a maximum when the magnon merges onto it.

typical excitation frequency and wave vectors are larger than the harmonic oscillator frequency $\Omega_0$, and the inverse size of the cloud. Figure 2 illustrates the Stoner particle-hole continuum corresponding to different trap regions: while in the trap center the Stoner gap is maximal and magnon propagation is allowed, at the trap sides the gap closes and only the particle-hole continuum contributes to the dynamical response.

We illustrate in fig. 3 the total spin-flip response as a function of the frequency, for values of system parameters accessible in current experiments. Even in the presence of the external confinement we find that at small transferred wave vectors the dynamical response is dominated by the magnon contribution, emerging from the particle-hole continuum. The latter acquires more importance at larger values of wave vectors. We expect this picture to hold at finite temperature, provided that it is smaller than the Stoner excitation gap $\Delta$.

Conclusions. – In this work we have considered the dynamical spin-flip response of a repulsive Fermi gas above the ferromagnetic transition, which is characterized by a magnon collective excitation mode. Within the random-phase approximation we have provided an analytic expression for the the spin-flip response of the homogeneous two-component Fermi gas. For the experimentally relevant situation of an inhomogeneous gas, we have shown that even in the presence of an external confinement the dynamical response displays features of the magnon mode in a pronounced peak, thus displaying a clear indication of the ferromagnetic phase. These predictions are experimentally accessible via spin-dependent Bragg spectroscopy, which could be realized, similarly to the usual Bragg spectroscopy, by a two-photon process inducing transitions from $|\uparrow\rangle$ fermions to $|\downarrow\rangle$ fermions, transferring at the same time momentum $h\mathbf{q}$ and energy $\hbar \omega$ to the fluid.

Our model could be extended to analyze other magnetization profiles [28]. Further refinements of our model include the development of a fully quantum description for the spectrum of the trapped interacting gas both at the RPA level as done for the paramagnetic phase in [29], and beyond RPA as is done e.g. for attractive homogeneous Fermi gases in [30]. A more accurate description of the experimental situation would require to include in the dynamical description the effects of atom losses, and the presence of bound states as in [31].

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REFERENCES

[1] Bloch I., Dalibard J. and Zwerger W., Rev. Mod. Phys., 80 (2008) 885.
[2] Jo G.-B. et al., Science, 325 (2009) 1521.
[3] Sommer A., Ku M., Roati G. and Zwierlein M., Nature, 472 (2011) 201.
[4] Stoner E., Proc. R. Soc. London, Ser. A, 165 (1938) 372.
[5] Conduit G. J. and Simons B. D., Phys. Rev. A, 79 (2009) 053606.
[6] Duine R. A. and MacDonald A. H., Phys. Rev. Lett., 95 (2005) 230403.
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[7] Conduit G., Green A. and Simons B., Phys. Rev. Lett., 103 (2009) 207201.
[8] Pilati S., Bertaina G., Giorgini S. and Troyer M., Phys. Rev. Lett., 105 (2010) 030405.
[9] Chang S.-Y., Randera M. and Trivedi N., Proc. Natl. Acad. Sci. U.S.A., 108 (2011) 51.
[10] Recati A. and Stringari S., Phys. Rev. Lett., 106 (2011) 080402.
[11] Duine R. A., Polini M., Stoof H. T. C. and Vignale G., Phys. Rev. Lett., 104 (2010) 220403.
[12] Conduit G. and Altman E., Phys. Rev. A, 82 (2010) 043603.
[13] Callaway J., Phys. Rev., 170 (1968) 576.
[14] Minguzzi A., Ferrari G. and Castin Y., Eur. Phys. J. D, 17 (2001) 49.
[15] Buechler H., Zoller P. and Zwerger W., Phys. Rev. Lett., 93 (2004) 080401.
[16] Veeravalli G., Kuhnle E., Dycke P. and Vale C., Phys. Rev. Lett., 101 (2008) 250403.
[17] Zhou S. Q., Ceperley D. and Zhang Shiwei, Phys. Rev. A, 84 (2011) 013625.
[18] Izuyama T., Kim D. and Kubo R., J. Phys. Soc. Jpn., 18 (1963) 1025.
[19] Englert F. and Antonoff M. M., Physica, 30 (1964) 429.
[20] Zou Peng, Kuhnle E., Vale C. and Hu Hui, Phys. Rev. A, 82 (2010) 061605(R).
[21] Pines D. and Nozières P., The Theory of Quantum Liquids, Vol. I (Perseus Books, Reading, Mass.) 1996.
[22] Stringari S., Phys. Rev. Lett., 102 (2009) 110406.
[23] Vignale G. and Singwi K. S., Phys. Rev. B, 32 (1985) 2824.
[24] Amoruso M., Meccoli I., Minguzzi A. and Tosi M. P., Eur. Phys. J. D, 8 (2000) 361.
[25] Sogo T. and Yabu H., Phys. Rev. A, 66 (2002) 043611.
[26] Conduit G. J. and Simons B. D., Phys. Rev. Lett., 103 (2009) 200403.
[27] Dong H., Hu H., Liu X.-J. and Drummond P. D., Phys. Rev. A, 82 (2010) 013627.
[28] Berdnikov I., Coleman P. and Simon S. H., Phys. Rev. B, 79 (2009) 244303.
[29] Capuzzi P. and Hernández E. S., Phys. Rev. A, 63 (2001) 063606.
[30] Pieri P. and Strinati G. C., Phys. Rev. B, 61 (2000) 15370.
[31] Pekker D. et al., Phys. Rev. Lett., 106 (2011) 050402.