Complete Parameter Space of Quark Mass Matrices with Four Texture Zeros

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Abstract

The full parameter space of Hermitian quark mass matrices with four texture zeros is explored by using current experimental data. We find that all ten free parameters of the four-zero quark mass matrices can well be constrained. In particular, only one of the two phase parameters plays an important role in CP violation. The structural features of this specific pattern of quark mass matrices are also discussed in detail.

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The texture of quark mass matrices, which can significantly impact on the pattern of quark flavor mixing, is completely unknown in the standard electroweak model. A theory more fundamental than the standard model is expected to allow us to determine the concrete structure of quark mass matrices, from which six quark masses, three flavor mixing angles and one CP-violating phase can fully be calculated. Attempts in this direction (e.g., those starting from supersymmetric grand unification theories and from superstring theories) are encouraging but have not proved to be very successful. Phenomenologically, a very common approach is to devise simple textures of quark mass matrices that can predict some self-consistent and experimentally-favored relations between quark masses and flavor mixing parameters [1]. The flavor symmetries hidden in such textures might finally provide us with useful hints about the underlying dynamics responsible for the generation of quark masses and the origin of CP violation.

Without loss of generality, the quark mass matrices $M_u$ (up-type) and $M_d$ (down-type) can always be taken to be Hermitian in the standard model or its extensions which have no flavor-changing right-handed currents [2]. Physics is invariant under a common unitary transformation of Hermitian $M_u$ and $M_d$ (i.e., $M_{u,d} \rightarrow SM_{u,d}S^\dagger$ with $S$ being an arbitrary unitary matrix). This freedom allows a further arrangement of the structures of quark mass matrices, such that

$$M_u = \begin{pmatrix} 0 & C_u & 0 \\ C_u^* & B_u & B_u \\ 0 & B_u^* & A_u \end{pmatrix}$$ (1)

and

$$M_d = \begin{pmatrix} D_d & C_d & 0 \\ C_d^* & B_d & B_d \\ 0 & B_d^* & A_d \end{pmatrix}$$ (2)

or

$$M'_d = \begin{pmatrix} 0 & C_d & D'_d \\ C_d^* & B_d & B_d \\ D'_d^* & B_d^* & A_d \end{pmatrix}$$ (3)

hold [3]. We see that $M_u$ has two texture zeros and $M_d$ or $M'_d$ has one texture zero [4]. Because the texture zeros of quark mass matrices in Eqs. (1)–(3) result from some proper transformations of the flavor basis under which the gauge currents keep diagonal and real, there is no loss of any physical content for quark masses and flavor mixing. But it is impossible to further obtain $D_d = 0$ for $M_d$ or $D'_d = 0$ for $M'_d$ via a new physics-irrelevant transformation of the flavor basis [3]. In other words, $D_d = D'_d = 0$ can only be a physical assumption. This assumption leads to the well-known four-zero texture of Hermitian quark mass matrices, which has the up-down parallelism and respects the chiral evolution of quark masses [5].

Although a lot of interest has been paid to the four-zero texture of Hermitian quark mass matrices [6,7], a complete analysis of its parameter space has been lacking. This unsatisfactory situation is partly due to the fact that many authors prefer to make instructive
Let us concentrate on the four-zero texture of Hermitian quark mass matrices given in Eqs. (1) and (2) with $D_d = 0$. The observed hierarchy of quark masses ($m_u \ll m_c \ll m_t$ and $m_d \ll m_s \ll m_b$) implies that $|A_q| > |\tilde{B}_q|, |B_q| > |C_q|$ (for $q = u$ or $d$) should in general hold [5]. Note that $M_q$ can be decomposed into $M_q = P_q^\dagger \overline{M}_q P_q$, where

$$
\overline{M}_q = \begin{pmatrix}
0 & |C_q| & 0 \\
|C_q| & \tilde{B}_q & |B_q| \\
0 & |B_q| & A_q \\
\end{pmatrix}
$$

and $P_q = \text{Diag}\{1, \exp(i\phi_{C_q}), \exp(i\phi_{B_q} + i\phi_{C_q})\}$ with $\phi_{B_q} \equiv \text{arg}(B_q)$ and $\phi_{C_q} \equiv \text{arg}(C_q)$. For simplicity, we neglect the subscript “$q$” in the following, whenever it is unnecessary to distinguish between the up and down quark sectors. The real symmetric mass matrix $\overline{M}$ can be diagonalized by use of the orthogonal transformation

$$
O^\dagger \overline{M} O = \begin{pmatrix}
\lambda_1 & 0 & 0 \\
0 & \lambda_2 & 0 \\
0 & 0 & \lambda_3 \\
\end{pmatrix},
$$

where $\lambda_i$ (for $i = 1, 2, 3$) are quark mass eigenvalues. Without loss of generality, we take $\lambda_3 > 0$ and $A > 0$. Then $\text{Det}(\overline{M}) = -A|C|^2 < 0$ implies that $\lambda_1\lambda_2 < 0$ is required. It is easy to find that $\tilde{B}$, $|B|$ and $|C|$ can be expressed in terms of $\lambda_i$ and $A$ as

$$
\tilde{B} = \lambda_1 + \lambda_2 + \lambda_3 - A, \\
|B| = \sqrt{(A - \lambda_1)(A - \lambda_2)(A - \lambda_3)} \cdot \frac{1}{A}, \\
|C| = \sqrt{\frac{-\lambda_1\lambda_2\lambda_3}{A}}.
$$

The exact expression of $O$ turns out to be [7]

$$
O = \begin{pmatrix}
\sqrt{\frac{\lambda_2\lambda_3(A - \lambda_1)}{A(\lambda_2 - \lambda_1)(\lambda_3 - \lambda_1)}} & \eta\sqrt{\frac{\lambda_1\lambda_3(A - \lambda_2)}{A(\lambda_2 - \lambda_1)(\lambda_3 - \lambda_2)}} & \sqrt{\frac{\lambda_1\lambda_2(A - \lambda_3)}{A(\lambda_3 - \lambda_1)(\lambda_3 - \lambda_2)}} \\
-\eta\sqrt{\frac{\lambda_1(A - \lambda_1)}{(\lambda_2 - \lambda_1)(\lambda_3 - \lambda_1)}} & \sqrt{\frac{\lambda_2(A - \lambda_2)}{(\lambda_2 - \lambda_1)(\lambda_3 - \lambda_2)}} & \sqrt{\frac{\lambda_3(A - \lambda_2)(\lambda_3)}{(\lambda_3 - \lambda_1)(\lambda_3 - \lambda_2)}} \\
\eta\sqrt{\frac{\lambda_1(A - \lambda_2)(\lambda_3 - \lambda_1)}{A(\lambda_2 - \lambda_1)(\lambda_3 - \lambda_1)}} & -\eta\sqrt{\frac{\lambda_2(A - \lambda_1)(\lambda_3 - \lambda_1)}{A(\lambda_2 - \lambda_1)(\lambda_3 - \lambda_2)}} & \sqrt{\frac{\lambda_3(A - \lambda_1)(A - \lambda_2)}{A(\lambda_3 - \lambda_1)(\lambda_3 - \lambda_2)}}
\end{pmatrix},
$$

$$
\eta = \frac{1}{\sqrt{\lambda_1\lambda_2\lambda_3}}.
$$
in which \( \eta \equiv \lambda_2/m_2 = +1 \) or \(-1\) corresponding to the possibility \((\lambda_1, \lambda_2) = (-m_1, +m_2)\) or \((+m_1, -m_2)\). The Cabibbo-Kobayashi-Maskawa (CKM) flavor mixing matrix [8], which measures the non-trivial mismatch between diagonalizations of \(M_u\) and \(M_d\), is given by \(V \equiv O_u^T P_u \bar{P}_d \) \(O_d\). Explicitly, we have

\[
V_{i\alpha} = O_{i1}^u O_{1\alpha}^d + O_{i2}^u O_{2\alpha}^d e^{i\phi_1} + O_{i3}^u O_{3\alpha}^d e^{i(\phi_1 + \phi_2)},
\]

where the subscripts \(i\) and \(\alpha\) run respectively over \((u, c, t)\) and \((d, s, b)\), and two phases are defined as \(\phi_1 \equiv \phi_{C_u} - \phi_{C_d}\) and \(\phi_2 \equiv \phi_{B_u} - \phi_{B_d}\).

It is well known that nine elements of the CKM matrix \(V\) have six orthogonal relations, corresponding to six triangles in the complex plane [1]. Among them, the unitarity triangle defined by \(\nu_{ub} V_{ud} + \nu_{cb} V_{cd} + \nu_{tb} V_{td} = 0\) is of particular interest for the study of CP violation at \(B\)-meson factories [9]. Three inner angles of this triangle are commonly denoted as

\[
\alpha = \arg \left( -\frac{\nu_{tb} V_{td}}{\nu_{ub} V_{ud}} \right), \\
\beta = \arg \left( -\frac{\nu_{cb} V_{cd}}{\nu_{tb} V_{td}} \right), \\
\gamma = \arg \left( -\frac{\nu_{ub} V_{ud}}{\nu_{cb} V_{cd}} \right).
\]

So far the angle \(\beta\) has unambiguously been measured from the CP-violating asymmetry in \(B_d^0\) vs \(\bar{B}_d^0\) \(\to J/\psi K_S\) decays [10]. The angles \(\alpha\) and \(\gamma\) are expected to be detected at KEK and SLAC \(B\)-meson factories in the near future. Given the four-zero texture of quark mass matrices, these three angles depend on the CP-violating phases \(\phi_1\) and \(\phi_2\). We shall examine the explicit dependence of \((\alpha, \beta, \gamma)\) on \((\phi_1, \phi_2)\) in the following.

Now we explore the whole parameter space of \(M_u\) and \(M_d\) with the help of current experimental data. There are totally ten free parameters associated with \(M_{u,d}\): \(A_{u,d}\), \(|B_{u,d}|\), \(\bar{B}_{u,d}\), \(|C_{u,d}|\) and \(\phi_{1,2}\). In comparison, there are also ten observables which can be derived from the four-zero texture of quark mass matrices: six quark masses \((m_u, m_c, m_t\) and \(m_d, m_s, m_b)\) and four independent parameters of quark flavor mixing (typically, \(|V_{us}|\), \(|V_{cb}|\), \(|V_{ub}/V_{cd}|\) and \(\sin 2\beta\)). Thus there is no problem to determine the complete parameter space of \(M_{u,d}\).

(1) The first step of our numerical calculations is to find out the allowed ranges of \(A_u/m_t\), \(A_d/m_b\), \(\phi_1\) and \(\phi_2\) by using Eqs. (6)-(9). For this purpose, we adopt the following reasonable and generous values of quark mass ratios at the electroweak scale \(\mu = M_Z\) [1,9]:

\[
\frac{m_c}{m_u} = 270 - 350, \quad \frac{m_s}{m_d} = 17 - 25, \quad \frac{m_t}{m_c} = 260 - 320, \quad \frac{m_b}{m_s} = 35 - 45.
\]

The predictions of \(M_u\) and \(M_d\) for the CKM matrix elements are required to agree with current experimental data [10,11]:

\[
|V_{us}| = 0.2240 \pm 0.0036, \quad \frac{|V_{ub}|}{|V_{cb}|} = 0.086 \pm 0.008, \\
|V_{cb}| = (41.5 \pm 0.8) \times 10^{-3}, \quad \sin 2\beta = 0.736 \pm 0.049.
\]
Note that the results of $V$ may involve a four-fold ambiguity arising from four possible values of $(\eta_u, \eta_d)$ in $O_u$ and $O_d$, as one can see from Eq. (7). To be specific, we first choose $\eta_u = \eta_d = +1$ in our numerical analysis and then discuss the other three possibilities.

The numerical results for $A_u/m_t$ vs $A_d/m_b$ and $\phi_1$ vs $\phi_2$ are illustrated in Fig. 1. We see that the most favorable values of these four quantities are $A_u/m_t \sim 0.94$, $A_d/m_b \sim 0.94$, $\phi_1 \sim 0.5\pi$ and $\phi_2 \sim 1.96\pi$. Fig. 1(a) confirms that the $(3,3)$ elements (i.e., $A_u \sim m_t$ and $A_d \sim m_b$) are the dominant matrix elements in $M_u$ and $M_d$. The strong constraint on $\phi_1$ comes from the experimental data on $|V_{us}|$ and $\sin 2\beta$; while the tight restriction on $\phi_2$ results from current data on $|V_{cb}|$. Because of $\sin \phi_1 \gg |\sin \phi_2|$ as shown in Fig. 1(b), the strength of CP violation in the CKM matrix is mainly governed by $\phi_1$. In many analytical approximations, $\phi_1 \approx 0.5\pi$ and $\phi_2 = 0$ have typically been taken [6,7].

We find that both $A_u/m_t$ and $A_d/m_b$ are insensitive to the signs of $\eta_u$ and $\eta_d$. In other words, the allowed ranges of $A_u/m_t$ and $A_d/m_b$ are essentially the same in $(\eta_u, \eta_d) = (\pm 1, \pm 1)$ and $(\pm 1, \mp 1)$ cases. While $\phi_1$ is sensitive to the signs of $\eta_u$ and $\eta_d$, $\phi_2$ is not. To be explicit, we have

\[
\begin{align*}
(\eta_u, \eta_d) = (+1, +1) : & \quad \phi_1 \sim 0.5\pi, \quad \phi_2 \sim 2\pi, \\
(\eta_u, \eta_d) = (+1, -1) : & \quad \phi_1 \sim 1.5\pi, \quad \phi_2 \sim 0, \\
(\eta_u, \eta_d) = (-1, +1) : & \quad \phi_1 \sim 1.5\pi, \quad \phi_2 \sim 2\pi, \\
(\eta_u, \eta_d) = (-1, -1) : & \quad \phi_1 \sim 0.5\pi, \quad \phi_2 \sim 0.
\end{align*}
\]

The dependence of $\phi_1$ on $\eta_u$ and $\eta_d$ can easily be understood. Indeed, $\tan \beta \propto \eta_u \eta_d \sin \phi_1$ holds in the leading-order analytical approximation with $|\sin \phi_2| \ll 1$. Thus the positiveness of $\tan \beta$ requires that $\sin \phi_1$ and $\eta_u \eta_d$ have the same sign.

(2) The second step of our numerical analysis is to determine the relative magnitudes of four non-zero matrix elements of $M_{u,d}$ by using Eq. (6) and the results for $A_u/m_t$ and $A_d/m_b$. The numerical results for $|B_u|/A_u$ vs $|B_d|/A_d$, $\tilde{B}_u/|B_u|$ vs $\tilde{B}_d/|B_d|$ and $|C_u|/\tilde{B}_u$ vs $|C_d|/\tilde{B}_d$ are shown in Fig. 2. One can see that the most favorable values of these six quantities are $|B_u|/A_u \sim 0.25$, $\tilde{B}_u/|B_u| \sim 0.3$, $|C_u|/\tilde{B}_u \sim 0.003$ and $|B_d|/A_d \sim 0.25$, $\tilde{B}_d/|B_d| \sim 0.4$, $|C_d|/\tilde{B}_d \sim 0.06$. A remarkable feature of our typical results is that $A_q$, $|B_q|$ and $\tilde{B}_q$ (for $q = u$ or $d$) roughly satisfy a geometric relation: $|B_q|^2 \sim A_q \tilde{B}_q$ [7]. In addition, $|B_u| \gg m_c$ and $|B_d| \gg m_s$ hold. While a very strong hierarchy exists between (1,2) and (2,2) elements of $M_{u,d}$, there is only a weak hierarchy among (2,2), (2,3) and (3,3) elements of $M_{u,d}$. Such a structural property of quark mass matrices must be taken into account in model building.

To be more explicit, let us illustrate the texture of $\overline{M}_{u,d}$ by choosing $m_c/m_u = 320$, $m_t/m_c = 290$, $m_s/m_d = 21$ and $m_b/m_s = 40$. We obtain

\[
\begin{align*}
\overline{M}_u & \approx A_u \begin{pmatrix} 0 & 0.0002 & 0 \\ 0.0002 & 0.067 & 0.24 \\ 0 & 0.24 & 1 \end{pmatrix} \sim A_u \begin{pmatrix} 0 & \varepsilon^6 & 0 \\ \varepsilon^6 & \varepsilon^2 & \varepsilon \\ 0 & \varepsilon & 1 \end{pmatrix}, \\
\overline{M}_d & \approx A_d \begin{pmatrix} 0 & 0.0059 & 0 \\ 0.0059 & 0.089 & 0.24 \\ 0 & 0.24 & 1 \end{pmatrix} \sim A_d \begin{pmatrix} 0 & \varepsilon^4 & 0 \\ \varepsilon^4 & \varepsilon^2 & \varepsilon \\ 0 & \varepsilon & 1 \end{pmatrix},
\end{align*}
\]

where $\varepsilon \approx 0.24$ in this special case [12]. Such a four-zero pattern of quark mass matrices depends on a small expansion parameter and is quite suggestive for model building. For
example, one may speculate that $M_u$ and $M_d$ in Eq. (13) could naturally result from a string-inspired model of quark mass generation [13] or from a horizontal U(1) family symmetry and its perturbative breaking [14].

Note that the numerical results in Fig. 2 and Eq. (13) have been obtained by taking $\eta_u = \eta_d = +1$. A careful analysis shows that $B_q$ (for q = u or d) is sensitive to the sign of $\eta_q$, but $|B_q|$ and $|C_q|$ are not. In view of Eq. (6), we find that the sign of $\eta = \lambda_2/m_2$ may significantly affect the size of $\tilde{B}$ if its $\lambda^2$ and $\lambda^3 - A$ terms are comparable in magnitude. In contrast, the dependence of $|B|$ on $\eta$ is negligible due to $A \gg m^2$; and $|C|$ is completely independent of the sign of $\eta$.

(3) The final step of our numerical analysis is to examine the outputs of the three CP-violating angles ($\alpha, \beta, \gamma$) and the ratio $|V_{ub}/V_{cb}|$ constrained by the four-zero texture of quark mass matrices. We plot the result for $|V_{ub}/V_{cb}|$ vs $\sin 2\beta$ in Fig. 3(a) and that for $\alpha$ vs $\gamma$ in Fig. 3(b). The correlation between $\alpha$ and $\gamma$ is quite obvious, as a result of $\alpha + \beta + \gamma = \pi$. Typically, $\alpha \approx 0.5\pi$ holds. The possibility $\alpha \approx \gamma$, implying that the unitarity triangle is approximately an isoceles triangle [15], is also allowed by current data and quark mass matrices with four texture zeros. Note that the size of $\sin 2\beta$ increases with $|V_{ub}/V_{cb}|$. This feature can easily be understood: in the unitarity triangle with three sides rescaled by $|V_{cb}|$, the inner angle $\beta$ corresponds to the side proportional to $|V_{ub}/V_{cb}|$.

Finally, we mention that the outputs of $|V_{ub}/V_{cb}|$ and ($\alpha, \beta, \gamma$) are completely insensitive to the sign ambiguity of $\eta_u$ and $\eta_d$.

4 In summary, we have analyzed the complete parameter space of Hermitian quark mass matrices with four texture zeros by using current experimental data. It is clear that the four-zero pattern of quark mass matrices can survive current experimental tests and its parameter space gets well constrained. We find that only one of the two phase parameters plays a crucial role in CP violation. The (2,2), (2,3) and (3,3) elements of the up- or down-type quark mass matrix have a relatively weak hierarchy, although their magnitudes are considerably larger than the magnitude of the (1,2) element. Such a structural feature of the four-zero quark mass matrices might serve as a useful starting point of view for model building.

We remark that the phenomenological consequences of quark mass matrices depend both on the number of their texture zeros and on the hierarchy of their non-vanishing entries. The former are in general not preserved to all orders or at any energy scales in the unspecified interactions which generate quark masses and flavor mixing [16]. But an experimentally-favored texture of quark mass matrices at low energy scales (such as the one under discussion) is possible to shed some light on the underlying flavor symmetry and its breaking mechanism responsible for fermion mass generation and CP violation at high energy scales.

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Note added: While our paper was being completed, we received a preprint by Zhou [17], in which all possible four-zero textures of quark mass matrices are classified and computed. The analyses, results and discussions in these two papers have little overlap.
REFERENCES

[1] For a recent review with extensive references, see: H. Fritzsch and Z.Z. Xing, Prog. Part. Nucl. Phys. 45, 1 (2000).
[2] H. Fritzsch, Nucl. Phys. B 155, 189 (1979).
[3] H. Fritzsch and Z.Z. Xing, Nucl. Phys. B 556, 49 (1999); G.C. Branco, D. Emmanuel-Costa, and R. González Felipe, Phys. Lett. B 477, 147 (2000).
[4] Because $M_u$ and $M_d$ (or $M'_d$) are Hermitian, a pair of off-diagonal texture zeros in each mass matrix have been counted as one zero.
[5] H. Fritzsch, Phys. Lett. B 184, 391 (1987); H. Fritzsch and Z.Z. Xing, Phys. Lett. B 413, 396 (1997); Phys. Rev. D 57, 594 (1998).
[6] See, e.g., D. Du and Z.Z. Xing, Phys. Rev. D 48, 2349 (1993); L.J. Hall and A. Rasin, Phys. Lett. B 315, 164 (1993); H. Fritzsch and D. Holtmanspötter, Phys. Lett. B 338, 290 (1994); H. Fritzsch and Z.Z. Xing, Phys. Lett. B 353, 114 (1995); P.S. Gill and M. Gupta, J. Phys. G 21, 1 (1995); Phys. Rev. D 56, 3143 (1997); H. Lehmann, C. Newton, and T.T. Wu, Phys. Lett. B 384, 249 (1996); Z.Z. Xing, J. Phys. G 23, 1563 (1997); K. Kang and S.K. Kang, Phys. Rev. D 56, 1511 (1997); T. Kobayashi and Z.Z. Xing, Mod. Phys. Lett. A 12, 561 (1997); Int. J. Mod. Phys. A 13, 2201 (1998); J.L. Chkareuli and C.D. Froggatt, Phys. Lett. B 450, 158 (1999); Nucl. Phys. B 626, 307 (2002); A. Mondragón and E. Rodríguez-Jáuregui, Phys. Rev. D 59, 093009 (1999); H. Nishiura, K. Matsuda, and T. Fukuyama, Phys. Rev. D 60, 013006 (1999); G.C. Branco, D. Emmanuel-Costa, and R. González Felipe, in Ref. [3]; S.H. Chiu, T.K. Kuo, and G.H. Wu, Phys. Rev. D 62, 053014 (2000); H. Fritzsch and Z.Z. Xing, Phys. Rev. D 61, 073016 (2000); Phys. Lett. B 506, 109 (2001); R. Rosenfeld and J.L. Rosner, Phys. Lett. B 516, 408 (2001); R.G. Roberts, A. Romanino, G.G. Ross, and L. Velasco-Sevilla, Nucl. Phys. B 615, 358 (2001).
[7] H. Fritzsch and Z.Z. Xing, Phys. Lett. B 555, 63 (2003).
[8] N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).
[9] Particle Data Group, K. Hagiwara et al., Phys. Rev. D 66, 010001 (2002).
[10] T. Browder, talk given at the 21st International Symposium on Lepton and Photon Interactions at High Energies (LP 03), August 2003, Batavia, Illinois, USA.
[11] A.J. Buras, hep-ph/0307203; and references therein.
[12] A different expansion of $\overline{M}_{u,d}$ has been given in Ref. [7] in terms of two different small parameters.
[13] D. Cremades, L.E. Ibáñez, and F. Marchesano, hep-ph/0212064; and references therein.
[14] C.D. Froggatt and H.B. Nielsen, Nucl. Phys. B 147, 277 (1979); L.E. Ibáñez and G.G. Ross, Phys. Lett. B 332, 100 (1994).
[15] I am grateful to H. Fritzsch and C. Hamzaoui for pointing out this possibility to me.
[16] Z.Z. Xing, hep-ph/0307359; and references therein.
[17] Y.F. Zhou, hep-ph/0309076.
FIG. 1. The allowed ranges of $A_u/m_t$ vs $A_d/m_b$ and $\phi_1$ vs $\phi_2$ for the four-zero texture of quark mass matrices with $\eta_u = \eta_d = +1$. 
FIG. 2. The allowed ranges of $|B_u|/A_u$ vs $|B_d|/A_d$, $\tilde{B}_u/|B_u|$ vs $\tilde{B}_d/|B_d|$ and $|C_u|/\tilde{B}_u$ vs $|C_d|/\tilde{B}_d$ for the four-zero texture of quark mass matrices with $\eta_u = \eta_d = +1$. 
FIG. 3. The allowed ranges of $|V_{ub}/V_{cb}|$ vs $\sin 2\beta$ and $\alpha$ vs $\gamma$ for the four-zero texture of quark mass matrices with $\eta_u = \eta_d = +1$. 