Review of HBT or Bose-Einstein correlations in high energy heavy ion collisions

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Abstract. A brief review is given on the discovery and the first five decades of the Hanbury Brown - Twiss effect and its generalized applications in high energy nuclear and particle physics, that includes a meta-review. Interesting and inspiring new directions are also highlighted, including for example source imaging, lepton and photon interferometry, non-Gaussian shape analysis as well as many other new directions. Existing models are compared to two-particle correlation measurements and the so-called RHIC HBT puzzle is resolved. Evidence for a (directional) Hubble flow is presented and the conclusion is confirmed by a successful description of the pseudorapidity dependence of the elliptic flow as measured in Au+Au collisions by the PHOBOS Collaboration.

Child, how happy you are sitting in the dust, playing with a broken twig all the morning.
I smile at your play with that little bit of a broken twig.
I am busy with my accounts, adding up figures by the hour.
Perhaps you glance at me and think, "What a stupid game to spoil your morning with!"
Child, I have forgotten the art of being absorbed in sticks and mud-pies.
I seek out costly playthings, and gather lumps of gold and silver.
With whatever you find you create your glad games,
I spend both my time and my strength over things I never can obtain.
In my frail canoe I struggle to cross the sea of desire, and forget that I too am playing a game.
(R. Tagore: Playthings)

1. Introduction

First I attempted to give a brief summary of the numerous achievements of particle interferometry from 1955 to 2005, to celebrate the Golden Jubilee: the 50 year anniversary of the Hanbury Brown - Twiss effect. As it is clearly impossible to summarize 50 years of efforts and discoveries of the order of one thousand scientific papers in a few pages, I ended up in creating a meta-review. I had to be very brief in reviewing - see my talk in ref. [1] for more details, historical remarks and animated illustrations.
2. The Hanbury Brown - Twiss effect

One of the most inspiring books that I have read recently was R. Hanbury Brown’s brief autobiography – Boffin. I recommend the reading of this book for all students all who enter the field of particle interferometry in high energy particle and nuclear physics [2].

As this note is contributed to the ICPA-QGP 2005 conference organized in India, I should mention that Robert Hanbury Brown was born here in Aruvankadu, Nilgiri Hills, India in 1916 – he passed away in Andover, England, in 2002. Similarly to him, Richard Q. Twiss was also born in India and they both had loved this country.

Hanbury Brown had been trained as an engineer and he was marked with an ingenious talent as well as a subtle sense of humor. Let me quote his book on how he reflects on his most famous discovery: “I was a long way from being able to calculate, whether it would be sensitive enough to measure a star. To do that one has to be familiar with photons and as an engineer my education in physics had stopped far short of the quantum theory. Perhaps just as well, otherwise like most physicists I would have come to the conclusion that the thing would not work – ignorance is sometimes a bliss in science.” ...

“In fact to a surprising number of people the idea that the arrival of photons at two separated detectors can ever be correlated was not only heretical but patently absurd, and they told us so in no uncertain terms, in person, by letter, in print, and by publishing the results of laboratory experiments, which claimed to show that we were wrong ...”

Opponents cited also from sacred books of Quantum Mechanics, for example quoting Dirac [3]: “Interference between two different photons can never occur.”

Vigorous objections were based on two laboratory experiments, the first of which was performed by the group of Jánossy at KFKI, my home institute in Budapest, Hungary. Their paper concluded [4], that “… in agreement with quantum theory, the photons of two coherent light beams are independent from each other.” However, Hanbury Brown and Twiss analyzed this and other experiments, and pointed out [5] that to observe the HBT effect one needs a very intensive source of light with a very narrow bandwidth, such as an isotope lamp that neither of these counter-experiments had.

The first stellar intensity interferometer, the pilot model was set up at Jodrell Bank in England 50 years ago, in 1955. This equipment was used to make the first measurement of the angular diameter of a main sequence star, Sirius [6]. This pilot experiment demonstrated the value of the method of intensity correlations and was the precursor to the construction of the Stellar Intensity Interferometry in Narrabi, Australia, completed in 1963. The giant reflectors of the Narrabi Observatory had parabolic surfaces of 22 feet diameter and a focal length of 36 feet, mounted on a circular railway truck, that had 618 feet diameter, or 73 % of the 843 feet diameter of the AGS accelerator of Brookhaven National Laboratory in Upton, LI, USA.

During the last 50 years, the study of intensity correlations became a broad field not only in quantum optics, solid state physics and astronomy, but extended to include the investigations of correlations among various type and number of particles in high energy particle and nuclear physics, [7–15].
3. Particle interferometry from 1955 to 2005 - a review of reviews

The field of particle interferometry or femtoscopy in high energy particle and heavy ion physics is firmly established: there is even a book written about the topic [16]. A collection of the early articles on Bose-Einstein correlations in high energy physics is also available [17]. Here I collected some of the most inspiring review papers of the field.

For newcomers, I recommend to start with the summary of Lörstadt from 1989 [18], written at a period when preliminary correlation data indicated signatures for a first order QGP to hadron transitions in O + Au collisions at CERN SPS energies. Boal, Gelbke and Jennings [19] summarized boson and fermion intensity correlations in subatomic physics, with emphasis on final state interactions. The review of Bauer, Gelbke and Pratt [20] is recommended for techniques applied in low and intermediate energy heavy ion collisions in particular for the noting of the energy dependence of the effective source sizes in nucleon-nucleon intensity correlations. A pedagogical work aimed at the identification of experimental difficulties in two-boson interferometry was compiled by Zajc [21], a must for every pedestrian entering this field. Schutz [22] summarized hard photon interferometry in heavy ion physics. Harris and Müller overviewed the suggested signatures for the production of a quark-gluon plasma, including Bose-Einstein correlations [23]. Light particle correlation data in heavy ion collisions at intermediate and low energies was compiled by Ardouin [24]. Data and theory of correlations in relativistic heavy ion collisions were summarized by Heinz and Jacak [28]. The multivariate Gaussian approach to the parameterization of particle interferometry in relativistic heavy ion collisions was highlighted by Wiedemann and Heinz [25]. The invariant Buda-Lund particle interferometry method was summarized by me and Lörstad [26]. The review of Weiner [27] highlighted the applications of quantum optical methods in boson interferometry in high energy particle and nuclear physics. Bose-Einstein correlations in electron-positron annihilations at LEP were introduced and reviewed by Kittel [29], summarized at the Z fragmentation [30], and at the $W^+W^-$ decays [31]. Model independent methods, similarities between interferometry results in electron-positron annihilation data and in relativistic heavy ion collision data, critical review of the resonance and Coulomb effects, limitations of the Gaussian approach and the Bertsch-Pratt and Yano-Koonin parameterizations were discussed [32] together with applications of source image reconstruction and quantum optical methods like squeezed states and pion lasers. Tomasik and Wiedemann reviewed central and non-central particle interferometry data after the first results at RHIC became accessible [33]. Alexander [34] reviewed Bose-Einstein and Fermi-Dirac correlations in particle physics, with focus on the mass and transverse mass dependence of the radius parameters of these correlations in electron-positron annihilation at LEP. Heinz [35] reviewed the concepts applied in high energy heavy ion physics, including particle interferometry. The review of Padula [36] starts with an excellent historical overview of interferometry in the 1960’s and 1970’s, covering the current formalism and squeezing of mass-modified bosons and fermions. These reviews well summarize the first 50 years of particle interferometry.
4. Interesting new directions

*Improvement on the two-particle Coulomb corrections.* If $\lambda < 1$, either partial coherence develops in the source, or some fraction of the pions comes from the extended, halo part of the source [32, 95]. This influences, how the relative separations of the particle pairs are distributed, hence the weight assigned to various parts of the two-particle Coulomb wave function. CERES [37] was the first experiment to apply this averaging self-consistently. By now, STAR [38], PHENIX [11] and PHOBOS [39] have also utilized this method suggested by Bowler [40], Sinyukov and collaborators [41].

*Many-body Coulomb effects for finite sources.* The core-halo type of problem exists not only in the construction of the Bose-Einstein correlation function of two charged particles, but in the determination of the Bose-Einstein correlation function of three or more charged particles. The many-body Coulomb corrections are notoriously difficult, but the circumstances are favorable in high energy heavy ion and particle physics for the application of an approximate, cluster expansion method, that is based on an asymptotically correct form of the multiparticle Coulomb wave function [42].

*Intensity correlations of non-identical particle pairs* are new and promising tools in heavy ion physics, given the large possible combinations of various type of particles. This method can not only be used to learn about the temporal sequence of particle emission, but also to learn about the strong final state interactions between various type of particle pairs [14, 43].

*Intensity correlations of penetrating probes* provides information not only on the phase-space distribution of the decoupling system at the time of freeze-out but also adds the information about the temporal evolution of the fireball.

*a) Photon interferometry* attracted a strong theoretical activity in order to utilize correlations of direct photons to search for quark gluon plasma phase suggested first by Makhlin [44]. This field is pioneered by Srivastava and collaborators, [46–48, 52] as well as by the organizers of this conference [51]. Earlier papers highlighted to photon interferometry at the CERN SPS energy domain [45, 49, 50], where the first experimental results on photon interferometry were reported recently by WA98 in Pb+Pb collisions [53].

*b) Lepton interferometry* is also based on penetrating probes that can escape from the hadronic fireball during its time evolution, hence lepton correlations can give information on the volumes of homogeneity at any stage of the expansion, at least in principle. The first case study has just been published by Alam, Mohanty and collaborators [54] using a 3+1 dimensional, relativistic hydrodynamical model with spin dependent invariant amplitudes and a bag model equation of state.

The key experimental question for photon and lepton interferometry is if one can find a transverse momentum window where photons or leptons from a quark gluon plasma phase dominate the single particle spectrum, overcoming contributions e.g. gammas from the $\pi^0$ decays, or in case of electrons, from Dalitz and open charm decays and Drell-Yan processes.
**Q-boson interferometry** has been investigated theoretically by Zhang, Padula, Anchiskin and collaborators in refs. [55–57]. It remains to be seen if q-boson correlations can ever be observed in experiments of high energy physics.

**Similarities between Bose-Einstein correlations in different reactions** might indicate an important limitation of our understanding of the particle correlations in $e^+ + e^-$, hadron + hadron, d+A and A+B reactions. For example, the mass and the transverse mass dependencies of the Bertsch-Pratt radius parameters in these reactions seems to be very similar, but the scales and the reaction mechanisms are completely different [29–32, 34].

**Continuous emission and escaping probabilities** has to be taken into account, if particle emission is not confined to a narrow freeze-out hypersurface, but is continuous during the time evolution of the system. This scenario has been considered by Grassi, Hama and collaborators, who developed first the continuous emission model [58]. This model was further improved with the help of the introduction of the escaping probabilities and a self-consistent theoretical formulation of the particle emission from hydrodynamical sources during the whole period of expansion [59].

**Initial conditions that fluctuate event by event** are seen in realistic Monte-Carlo calculations. Averaging over the final states of hydrodynamics started from fluctuating initial conditions yields HBT radii closer to the measured values as compared to the case when the initial conditions are first averaged over and the hydrodynamic evolution starts from the smoothed initial conditions, as shown with the help of a 3+1 dimensional relativistic hydrodynamical codes developed by the SPHERIO collaboration [60].

**Rapidity dependence** of the HBT radii was explored by PHOBOS at RHIC and NA49 at CERN SPS. For the side, out and longitudinal components only weak rapidity dependence was found in both cases. The Yano-Koonin-Podgoretskii fit revealed that the longitudinal flow profile is nearly boost invariant in both reactions [39, 61].

**Azimuthally sensitive HBT measurements** push the multi-variate Gaussian parameterization to its limits. The transverse momentum and the azimuthal angle dependence of the 3 by 3 symmetric radius matrix provide important constraints for the dynamics of the effective source in these collisions, pioneered by M. Lisa and collaborators [62]. STAR data on asHBT indicate [38], that the effective source is extended in the direction perpendicular to the impact parameter, similarly to the geometry of the nuclear overlap.

**Beyond the Gaussian approximation:** a) **The Edgeworth and Laguerre expansions** utilize complete sets of polynomials that are orthogonal with respect to given weight functions. The method was worked out by S. Hegyi and the present author [63]. The Edgeworth expansion has been applied to quantify the non-Gaussian features of Bose-Einstein correlations by the L3 and the STAR collaborations [13, 64, 65].

b) **The Lévy index of stability** $0 < \alpha \leq 2$ is a new parameter of the two- and three-particle Bose-Einstein correlation functions [66–68]. It characterizes the power-law tails of Lévy stable distributions, which appear in physical systems where the final distribution is obtained as convolution of many elementary random steps. If
the distribution functions of the elementary steps are characterized by finite means and variances, central limit theorems determine that the limit distribution has to be a Gaussian and $\alpha = 2$. Generalized central limit theorems state that for certain elementary processes with infinite variance or infinite mean a limiting distribution exist, but the limit distribution develops a power-law tail and $\alpha < 2$.

c) Imaging methods were developed by Brown and Danielewicz[69] to reconstruct the relative coordinate distribution of the source by inverting the two-particle correlation function with a kernel, based on known final state interaction and symmetrization effects. This method was applied first at the AGS energies [70]. Preliminary PHENIX results indicate [71] the existence of a long tail in the relative coordinate distributions in d+Au and Au+Au collisions at the maximal RHIC energies.

*Multi-boson symmetrization effects and pion lasers* are explored with the pion-laser model of S. Pratt [72], that I could understand only from ref. [73], which lead to the analytical solution of the model with Zimányi [74,75]. Among others we found that in the rare gas limit two and three-particle symmetrization can be performed perturbatively, within Poisson distributed clusters. More theoretical and experimental work is needed to investigate these effects under conditions of RHIC and LHC reactions.

*In-medium hadronic mass modifications* can signal partial $U_A(1)$ symmetry restoration, measurable with two-pion Bose-Einstein correlations [76]. Current STAR Au+Au HBT radii were shown in agreement with a model assuming chiral symmetry restoration [77]. The quantum freeze-out problem of in-medium mass shifted bosons has been solved [78] correcting the idea of Andreev and Weiner [79]. It was extended to the case of fermions [80]. A liquid of mass-modified hadrons is thus signalled by back-to-back correlations between the detected particle - antiparticle pairs.

Let me add the following important comment here. The most recent studies of the phase structure of strongly interacting matter with the help of lattice QCD calculations suggest that the transition from the hadronic phase to a this form of matter, the quark-gluon plasma, happens at a critical temperature of $T_c = 164 \pm 2$ MeV and the form of the transition is a cross-over, where hadrons might exist above the critical temperature in a broad range of baryo-chemical potentials up to $\mu_B \approx 350$ MeV, where the transition changes to a second order phase transition. At even larger baryochemical potentials, a first order phase transition is seen, and the critical temperature decreases slightly with increasing $\mu_B$-s, see ref. [86] for further details. If indeed the degrees of freedom in a fluid are dominated by non-hadronic states, and if hadronization and freeze-out happens simultaneously in a time-like deflagration as suggested in ref. [81], then the back-to-back particle-antiparticle correlations have to vanish. Hence the excitation function of the these correlations changes drastically at the transition from a hadronic to a non-hadronic liquid. Hence it is sensitive to the creation of a quark gluon plasma, which underlines the importance of search for squeezed states in ultra-relativistic heavy ion collisions.

For advanced studies, I recommend the following works and important contributions to particle interferometry in high energy physics from its first decade, [87]-[90] its second [91]-[98], third [99]-[117], fourth [118]-[130] and fifth decade [131]-[149].
5. The “RHIC HBT puzzle” and its resolution

Before contrasting the experimental results at RHIC with theoretical expectations and fits, let me go back first to the theory of particle interferometry for expanding sources formulated by Pratt [107, 108]. The directional dependence of the HBT correlation function was investigated by Hama and Padula as early as in 1987 [109], who observed that the longitudinal effective radius parameter is proportional to the proper-time of freeze-out, and proportional to the square root of temperature over energy. Their eq. (7) of ref. [109] can be considered as one of the first precursors to the studies of the ratios or the differences between $R_{out}$ and $R_{side}$. Makhlin and Sinyukov derived a famous and experimentally well tested formula for boost-invariant particle emitting sources:

$$R_{\text{long}} = \tau_f \sqrt{\frac{T_f}{m_t}},$$

ref. [110]. Bertsch introduced [111] the naming convention (out, side, longitudinal) for the components of the relative momentum that are parallel with the mean transverse momentum, perpendicular to the longitudinal and the mean momentum, and parallel with the longitudinal direction, respectively, with an influential graphical illustration, supported by Monte-Carlo cascade simulations by Bertsch, Gong and Tohyama [112]. Using a Gaussian approximation, the popular fit parameters thus became $\lambda$, $R_{out}$, $R_{side}$, and $R_{long}$.

5.1. HBT result at CERN SPS energies

The first preliminary results from the NA35 experiment in O+Pb reactions at CERN SPS energies indicated a signal for the first order QGP-hadron transition, $R_{out}$ was found to be much larger, than $R_{side}$, a three sigma effect [113, 114]. These data were described by Padula and Gyulassy in terms of a quark gluon plasma model but also in terms of a conventional hadronic resonance gas model [115, 116]. Kaon interferometry has been suggested to increase the selectivity of these correlation measurements [117].

The preliminary NA35 data were changed and the difference between the out and the side HBT radius parameters in S+Pb collisions was found to vanish within errors [118, 119]. This observation was published by the second generation experiment NA44, which attempted to determine precisely all the three radius components at mid-rapidity in a broad transverse mass interval [120], valid not only for pions but also for kaons [121, 122]. The results indicated an approximate equality and simultaneous scaling of the radius parameters of the correlation functions, namely that these radius parameters within rather small experimental errors turned out to be rather similar, obeying a common dependence on the transverse mass of the pair. At the same time the NA44 collaboration observed the scaling of the single particle spectra, indicating that the slope parameter increases with increasing transverse mass of the particles [123]. Both of these effects were explained in terms of a three-dimensionally expanding, axially symmetric, suddenly decaying fireball [124, 125]. The existence of scaling limiting cases in certain domain of the parameter space was a focal point for the formulation and subsequent development of this Buda-Lund hydro model.
5.2. HBT predictions for RHIC energies

Soon after the observation of the approximate equality and transverse mass scaling of the radius parameters of the Bose-Einstein correlation function (HBT radii for short), Gyulassy and Rischke predicted that the HBT radius in the out direction will exceed the HBT radius in the side direction, predicting $R_{\text{out}}/R_{\text{side}} > 1$, values reaching up to 4 or even 20 in case of an ideal first order QGP to hadron phase transition, [126, 127]. These detailed predictions about a slowly burning log of QGP obtained a lot of experimental attention, in contrast to the earlier and less well-known but in fact more successful model of ref. [81], that predicted $R_{\text{out}} \approx R_{\text{side}} \approx R_{\text{long}}$ in a broad transverse mass interval at RHIC for a QGP which has reduced entropy density and harmonizes suddenly at every location from a supercooled state. Presently neither of the four experimental predictions made in ref. [81] are in disagreement with the known features of particle production in Au+Au collisions at RHIC as far as I know, hence the picture of a suddenly frozen, supercooled QGP has not yet been invalidated by experimental observations at RHIC.

5.3. Less Unpromising Models and the Au+Au HBT data at RHIC

The first detailed measurements of Bose-Einstein or HBT correlations by STAR at RHIC [10] have excluded the validity of the slowly burning quark-gluon plasma picture predicted in ref. [126, 127]. These measurements were confirmed by PHENIX [11] and the approximate equality of the HBT radii was established in a large transverse momentum interval. Recently, PHOBOS has extended these measurements to a broad rapidity interval without changing the overall picture [132].

Of the order of 50 models were shown to be unable to describe these observables at RHIC. The difficulty faced here by some of the theoretical approaches to describe the data is frequently referred to as the “RHIC HBT puzzle” [131].

With the help of my students, M. Csanád and A. Ster, we have scanned the literature and attempted to determine the list of less unpromising models, that cannot be excluded with the tools of mathematical statistics, using HBT data at RHIC. The following models were found to pass this HBT test:

- The multi-phase transport model (AMPT) of Lin, Ko and Pal [133].
- The hadronic cascade model calculation of Humanic [134].
- The Buda-Lund hydro model [135–138].
- The Cracow single freeze-out thermal model [139].
- The Blast-wave model as implemented by Retiere and Lisa [140, 141].
- The parameterized time-dependent expanding source model of Renk [142, 143].
- Ref. [77] that investigates the onset of the chiral phase transition and its effects on the HBT radii. Effectively, this model also leads to a Buda-Lund type of parameterization.

As the number of models that pass the HBT test is fairly sizeable at present, additional criteria are needed to select the most relevant models.
6. Evidence for a Hubble flow in Au+Au at RHIC

To constrain the class of the not obviously wrong models, additional observables are needed. Let me chose here the identified single particle spectra of PHENIX, the pseudorapidity distribution measured by BRAHMS and PHOBOS, and the transverse mass dependence of the HBT radius parameters as measured by PHENIX and STAR. Humanic’s cascade, the blastwave model, the Buda-Lund hydro model and the Cracow hydro model pass these additional criteria as well. In my talk I have illustrated [1], how a transverse and a longitudinal Hubble constant can be extracted from the blastwave, the Buda-Lund and the Cracow model fits to these data. Within errors, all of these three models are agreement with an approximately direction independent Hubble type of flow profile, where the transverse and the longitudinal Hubble constants are within errors the same, \( u_{\mathrm{long}} = H_{\mathrm{long}} r_{\mathrm{long}} \) and \( u_{\mathrm{transv}} = H_{\mathrm{transv}} r_{\mathrm{transv}} \), with \( H_{\mathrm{long}} = H_{\mathrm{transv}} = 0.13 \pm 0.02 \) \( \text{fm}^{-1} \). This result is well illustrated by the upper plots in Fig.1.

These observations are confirmed by a perfect description of the rapidity dependence of the elliptic flow with the help of the ellipsoidal generalization of the Buda-Lund hydro model, as illustrated on the lower plots of Figure 1. These data by the PHOBOS collaboration were not reproduced with the help of hydrodynamical models earlier. The lower right panel of Figure 1 illustrates, that the PHOBOS data are not only perfectly described with the ellipsoidally symmetric generalization of the Buda-Lund hydro model, but also that the PHOBOS data can be scaled to the theoretically predicted form of the scaling function.

This ellipsoidally symmetric Buda-Lund model is based on direction dependent Hubble constants [85]. The asymptotic emergence of a direction independent Hubble flow is seen analytically in the new families of exact analytic solutions of non-relativistic hydrodynamics that are the basis this Buda-Lund hydro model [144–146].

7. Conclusions

After a historic introduction and a review of reviews I have highlighted some of the interesting new directions, including interferometry with penetrating probes like photons and leptons and the quantification of the non-Gaussian behaviour of the correlation functions. It may well be that a non-trivial energy dependence of these correlations reveals itself not in the scale parameters like the HBT radii but in the shape parameters like the Lévy index of stability \( \alpha \). Indeed, for p+p reactions, this parameter has been linked [68] to the anomalous dimension of QCD which follows a non-trivial energy dependence due to the running of the strong coupling constant. I also noted that the disappearance of the particle-antiparticle back-to-back correlations at a certain colliding energy can be a new signal for a transition from a hadronic fluid to a non-hadronic fluid, hence to the onset of the deconfinement transition. More experimental efforts are needed to look for squeezed states and chiral symmetry restoration at the RHIC energies.

The HBT data at RHIC provide important and selective constraints for the model
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Figure 1. The upper panels show a simultaneous Buda-Lund fit to final Au+Au data at \( \sqrt{s_{NN}} = 200 \) GeV from refs. [82], [11, 83]. The fit parameters are summarized in Table 1 of ref.[138]. The ellipsoidally symmetric Buda-Lund hydro model is fitted [150] to preliminary PHOBOS \( v_2(\eta) \) data at 200 GeV on the lower left panel [84]. In the lower right panel, these PHOBOS data on \( v_2(\eta) \) are re-scaled and compared to the scaling function \( v_2(w) = I_1(w)/I_0(w) \) predicted by the Buda-Lund hydro model [85].

builders and indicate that a large number of models implemented the space-time evolution of the source incorrectly. Many models failed to describe these observables, but this is not a puzzle. Mysteries surround the HBT effect from the very beginning of its discovery. I noted, that there was a successful prediction [81] in 1994 for the
simultaneous equality of the HBT radii in Au+Au reactions at RHIC in a broad transverse mass interval, and all the qualitative predictions of this model are in agreement with the present data. Hence currently it cannot be excluded that a suddenly hadronizing supercooled QGP is present in these reactions. I presented the list of the less unpromising models - theoretical descriptions that pass the HBT test in Au+Au collisions at RHIC. At present 7 such models were found in the literature. Then I suggested to try to determine the common part of these models and to increase the sensitivity of the test by comparing these models to the single particle spectra, the HBT radii and to the elliptic flow measurements at RHIC. I have argued that the hadronic cascade model of Humanic, the Buda-Lund hydro model, the blastwave model and the Cracow model passes all these tests. In an attempt to determine the common features of these models, the transverse and the longitudinal Hubble constants were found to be the same within errors in the best fits to the data by the Buda-Lund, the Blastwave and the Cracow models, providing an evidence for a nearly fully developed Hubble flow and an indication of deconfinement temperatures reached in central Au+Au collisions at RHIC [138, 151]. These observations are confirmed by a perfect description of the rapidity dependence of the elliptic flow with the help of the Buda-Lund hydro model. Based on a forthcoming manuscript [150] I have shown that the PHOBOS rapidity dependent elliptic flow data, previously resisting a description in a hydrodynamic picture, actually agree with the theoretically predicted scaling function if the proper scaling variable $w$ is determined. The solution of the HBT puzzle at RHIC thus naturally provides a solution to the puzzling rapidity dependence of the elliptic flow at RHIC as well.

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