Comments on Central Charge of Topological Sigma Model with Calabi-Yau Target Space

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ABSTRACT

We study a central charge $Z$ of a one parameter family of Calabi-Yau $d$-fold embedded in $CP^{d+1}$. For a $d$-fold case, we construct the $Z$ concretely and analyze charge vectors of D-branes and intersection forms of associated cycles. We find the charges are described as some kinds of Mukai vectors. They are represented as products of Chern characters of coherent sheaves restricted on the Calabi-Yau hypersurfaces and square roots of A-roof genera of the $d$-folds. By combining results of the topological sigma model and the data of the CFT calculations in the Gepner model, we find that the $Z$ is determined and is specified by a set of integers. It labels boundary states in special classes where associated states are represented as tensor products of boundary states for constituent minimal models. The $Z$ has a moduli parameter “$t$” that describes a deformation of a moduli space in the open string channel with B-type boundary conditions. Also monodromy matrices and homology cycles are investigated.

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1 Introduction

D-branes play important roles to describe solitonic modes in string theory. The physical observables of D-brane’s effective theories have dependences of moduli on compactified internal spaces or wrapped D-branes. In this paper, we focus on the central charge of the type II superstring compactified on Calabi-Yau manifold and study its properties from the point of view of topological sigma models (A- and B-models) [1]–[11]. The central charge is characterized by D-brane’s charges and periods of the B-model in the closed string channel. Together with the Kähler potential $K$, it allows us to construct a BPS mass formula of D-branes wrapped around cycles.

Recently there is a great advance [12] to study properties of charges, boundary states[13],[14] based on the Gepner model[15] associated with the CY$_3$. Also there appear many consistency checks about these charges, intersection forms of homology cycles by analyses both in the CFT and in the sigma models[16]. They investigated three dimensional Calabi-Yau cases.

For these three fold cases, one can easily construct associated canonical bases of period integrals because there are prepotentials for the models. It makes analyses for three folds comparatively tractable. But for d-fold cases ($d > 3$), we do not know any existence of analogs of these convenient prepotentials and we cannot apply analogous recipes to d-fold cases ($d > 3$) directly. The aim of this paper is to develop a method to construct central charges $Z$ of Calabi-Yau $d$-folds and to investigate their properties in order to understand structures of the moduli spaces in the open string channel. We interpret constituent elements of the $Z$ as topological objects from the point of view of topological sigma models.

The paper is organized as follows. In section 2, we explain a mirror manifold paired with a Calabi-Yau $d$-fold embedded in $CP^{d+1}$. We also explain the results in [10],[11] about a Kähler potential in order to fix notations. There we introduce a few sets of periods applicable either in the small or large complex structure regions. We construct a formula of the central charge $Z$ by using the periods. In section 3, we investigate the results by P. Candelas et al [8] in our bases and develop a method to construct the $Z$. By generalizing a consideration in the quintic case, we apply the method to the 5-fold case and construct the central charge $Z$ concretely in section 4. Also the monodromy matrices associated with singular points are investigated. In section 5, we review the Gepner model [13] shortly and analyze D-brane’s charges both in large radius basis and in the Gepner basis with B-type boundary conditions. In section 6, cycles associated with the periods are constructed explicitly and intersection forms of the cycles are studied. In section 7, we propose formulae of the $Z$ applicable in the large volume region and make a consideration about an analogous structure to the Mukai vector in the $Z$. It is interpreted as a product of Chern characters of sheaves and a square
root of the Todd class (or the A-roof genus) of the d-fold $M$. The set of sheaves is constructed as a dual basis of tautological line bundles of the ambient space $CP^{N-1}$ by a restriction on $M$. Section 8 is devoted to conclusions and comments. In appendix A, we summarize several examples of the $\sqrt{\hat{A}}$ in lower dimensional cases. In appendix B, we collect several data about monodromy properties of the quintic.

2 Periods and Kähler Potential

In our previous papers [10, 11], we determine the formula of the Kähler potential of the Calabi-Yau d-fold embedded in $CP^{d+1}$

$$M; \ p = X_1^N + X_2^N + \cdots + X_N^N - N\psi X_1 X_2 \cdots X_N = 0,$$

and that of the mirror partner with a moduli parameter $\psi$ of the complex structure

$$W; \ \{ p = 0 \}/\mathbb{Z}_{N}^{\otimes(N-1)}.$$ (1)

The $N$ is related with the complex dimension $d$ of $M$, $N = d + 2$. The $G = \mathbb{Z}_{N}^{\otimes(N-1)}$ is a maximally discrete group of the $M$. The formula is constructed by requiring consistency conditions with the results of the CFT at the Gepner point. In this paper, we investigate the central charge of the topological A-model associated with the $M$ (equivalently, the open string with B-type boundary conditions). The formula is important for BPS mass analyses of the D-branes.

First we review the results in [10, 11]. When one considers the Hodge structure of the $G$-invariant parts of the cohomology group $H^d(W)$, the decomposition of the structure is controlled by one complex moduli parameter $\psi$. The Kähler potential $K$ in the B-model moduli of $W$ is constructed by combining a set of periods $\tilde{\omega}_k$ quadratically

$$e^{-K} = \sum_{k=1}^{N-1} I_k \tilde{\omega}_k^\dagger \tilde{\omega}_k,$$

$$I_k = \frac{1}{\pi^N} \frac{1}{N^{N+2}} (-1)^{k-1} \left( \sin \frac{\pi k}{N} \right)^N,$$

$$\tilde{\omega}_k(\psi) = \left[ \Gamma \left( \frac{k}{N} \right)^N \frac{(N\psi)^k}{\Gamma(k)} \right]^{N} \left[ \sum_{n=0}^{\infty} \left[ \frac{\Gamma \left( \frac{N+n}{N} \right)}{\Gamma \left( \frac{k}{N} \right)} \right]^N \frac{\Gamma(k)}{\Gamma(N n + k)} (N\psi)^n \right].$$ (2)

The coefficients $I_k$ are determined in [10].
2.1 Periods

The formulae Eqs.(2) are valid in the small $\psi$ region because of the convergence of the series expansions. At a point $\psi = 0$ in the B-model moduli space, there is a $\mathbb{Z}_N$ symmetry which rotates the $\psi \rightarrow \alpha \psi$ ($\alpha = e^{2\pi i/N}$). A cyclic $\mathbb{Z}_N$ monodromy transformation $A$ is diagonalized on the set of this basis $\tilde{\psi}_k$ ($1 \leq k \leq N - 1$) with $\alpha = e^{2\pi i/N}$

$$A\tilde{\psi}_k(\psi) = \alpha^k \tilde{\psi}_k(\psi) \quad (k = 1, 2, \ldots, N - 1).$$

For a later convenience, we also introduce a set of periods $\psi_j$ ($0 \leq j \leq N - 1$) as linear combinations of the $\tilde{\psi}_k$s

$$\psi_j = -\frac{1}{N} \frac{1}{(2\pi i)^{N-1}} \sum_{k=1}^{N-1} \alpha^{jk}(\alpha^k - 1)^{N-1} \tilde{\psi}_k.$$

The $\mathbb{Z}_N$ transformation acts on this basis cyclically

$$A\psi_j(\psi) = \psi_{j+1}(\psi), \quad (j = 0, 1, 2, \ldots, N - 1).$$

Here we identify the $\psi_N$ with the $\psi_0$. It is a redundant basis to represent the monodromy transformation because a linear relation is satisfied

$$\sum_{j=0}^{N-1} \psi_j = 0.$$

But the basis is useful because it is directly related to the Gepner basis of the CFT. These two sets of periods $\tilde{\psi}_k$ and $\psi_j$ are meaningful only in the small $\psi$ region because of the convergence of the series expansions. In order to describe the large complex structure region of $W$, we must introduce another set of periods $\{\Omega_m\}$s ($m = 0, 1, 2, \ldots, N - 2$). A generating function of the $\Omega_m$ is defined by using a formal parameter $\rho$ with $\rho^{N-1} = 0$

$$\sum_{m=0}^{N-2} \Omega_m \rho^m = \sqrt{\hat{K}(\rho)} \cdot \varnothing \left( \frac{\rho}{2\pi i}; z \right),$$

$$\varnothing(v) = z^v \cdot \sum_{n=0}^{\infty} \frac{a(n+v)}{a(v)} z^n, \quad z = (N\psi)^{-N};$$

$$a(v) = \frac{\Gamma(N v + 1)}{[\Gamma(v + 1)]^N},$$

$$\hat{K}(\rho) := \exp \left[ 2 \sum_{m=1}^{N-2} \frac{N - N^{2m+1}}{2m + 1} \zeta(2m + 1) \left( \frac{\rho}{2\pi i} \right)^{2m+1} \right]$$

$$= 1 + 2\zeta(3) \frac{c_3}{N} \left( \frac{\rho}{2\pi i} \right)^3 + O(\rho^5).$$

The infinite series Eq.(3) converges around the large complex structure point $z \sim 0$ of $W$. We find that the two sets of the periods $\tilde{\psi}_k$ and $\Omega_m$ are related by a transformation matrix
\( \tilde{M} \) with components \( \tilde{M}_{k\ell} \) through an analytic continuation into the large complex structure region

\[
\tilde{\omega}_k = \sum_{\ell=0}^{N-2} \tilde{M}_{k\ell} \Omega_\ell \quad (k = 1, 2, \ldots, N - 1),
\]

\[
\tilde{M}_{k\ell} = (-N) \cdot (2\pi i)^{N-1} \times \left[ \sqrt{\hat{A}(\rho)} \cdot \frac{e^{\rho} - \alpha^k}{e^{\rho} - \alpha^k} \cdot (-\rho)^\ell \right]_{\rho^{N-2}}
\]

\[
= (-N) \cdot (2\pi i)^{N-1} \sum_{m=0}^{N-2} G_{k,m} V_{m,\ell},
\]

\[
G_{k,m} = \frac{-\alpha^k}{(\alpha^k - 1)m+1} \quad (1 \leq k \leq N - 1, \ 0 \leq m \leq N - 2),
\]

\[
V_{m,\ell} = \left[ \sqrt{\hat{A}(\rho)} \cdot (e^\rho - 1)^m \cdot (-\rho)^\ell \right]_{\rho^{N-2}} \quad (0 \leq m \leq N - 2, \ 0 \leq \ell \leq N - 2). \quad (4)
\]

The transformation matrix \( V \) contains a square root of a topological invariant “A-roof genus” of the Calabi-Yau space

\[
\hat{A}(\rho) = \left( \frac{\rho}{2} \right)^N \left( \frac{\sinh \frac{\rho}{2}}{\frac{\rho}{2}} \right)
\]

\[
= \exp \left[ + \sum_{m=1}^{\infty} \frac{(-1)^m B_m}{(2m)!} \frac{N\rho - N^2m}{2m} \rho^{2m} \right]
\]

\[
= 1 + \frac{1}{12 N} \rho^2 + O(\rho^4).
\]

The \( B_m \)'s are Bernoulli numbers and are defined in our convention as

\[
\frac{x}{e^x - 1} = 1 - \frac{x}{2} - \sum_{n=1}^{\infty} \frac{(-1)^n \cdot B_n}{(2n)!} x^{2n},
\]

\[
B_1 = \frac{1}{6}, \ B_2 = \frac{1}{30}, \ B_3 = \frac{1}{42}, \ B_4 = \frac{1}{30}, \ldots.
\]

The expansion coefficients of the \( \hat{A}(\rho) \) in terms of the \( \rho \) are represented as some combinations of Chern classes of the \( M \). We summarize several concrete examples of the \( \sqrt{\hat{A}(\rho)} \) in the appendix A. Also we define auxiliary sets of periods \( \{ \hat{\Pi}_n \} \) and \( \{ \hat{\Pi}_m \} \) \( (n, m = 0, 1, 2, \ldots, N-2) \)

\[
\hat{\Pi}_n = \sum_{\ell=0}^{N-2} V_{n,\ell} \Omega_\ell, \quad \hat{\Pi}_m = \sum_{n=0}^{N-2} U_{m,n} \Pi_n,
\]

\[
V_{n,\ell} = \left[ \sqrt{\hat{A}(\rho)} \cdot (e^\rho - 1)^n \cdot (-\rho)^\ell \right]_{\rho^{N-2}}, \quad U_{m,n} = \binom{m}{n} \cdot (-1)^{m-n}.
\]

Here we introduced a symbol \( \binom{a}{b} \) as a ratio of Euler’s gamma functions

\[
\binom{a}{b} := \frac{\Gamma(a + 1)}{\Gamma(b + 1)\Gamma(a - b + 1)}.
\]
The $\hat{\Pi}$ is related to the basis $\varpi_j$ through an analytic continuation

$$\varpi_j = \sum_{n=0}^{N-2} P_{j,n} \hat{\Pi}_n ,$$

$$P_{j,n} = \delta_{n,N-2} - N \cdot \left( \frac{N - 2 - n}{j - 1 - n} \right) \times (-1)^{j-1-n}, \quad (j = 0, 1, \cdots, N - 1) \text{ and } (n = 0, 1, \cdots, N - 2) .$$  \hspace{1cm} (5)

In discussing properties in the large $\psi$ region, we use the sets $\hat{\Pi}_n$ or $\Omega_m$.

### 2.2 Kähler Potential

Under this preparation, we shall write down our results in the Kähler potential $K$ in these bases

$$e^{-K} = \sum_{\ell=1}^{N-1} \varpi^\dagger \ell I_\ell \varpi_\ell$$

$$= \frac{(-2\pi i)^{N-2}}{N^{N+2}} \sum_{j \neq a} \sum_{j' \neq a} \varpi^\dagger_j K_{j,j'} \varpi_{j'}$$

$$= \frac{-1}{(2\pi i N)^N} \cdot \frac{1}{N^2} \cdot \sum_{m,n=0}^{N-2} \hat{\Pi}^\dagger_m I_{m,n} \hat{\Pi}_n ,$$

$$= (-1)^N \left( \frac{2\pi i}{N} \right)^{N-2} \cdot \frac{1}{N^2} \sum_{\ell,\ell'=0}^{N-2} \Omega^\dagger_\ell \Sigma_{\ell,\ell'} \Omega_{\ell'} ,$$

$$I_\ell = \frac{(-1)^{\ell-1}}{\pi^N \cdot N^{N+2}} \cdot \left( \sin \frac{\pi \ell}{N} \right)^N , \quad (\ell = 1, 2, \cdots, N - 1) ,$$

$$K_{j,j'} = \frac{\delta^N}{2N^2} \times \sum_{\ell=1}^{N-1} (-1)^\ell \left( \sin \frac{\pi \ell}{N} \right)^{-(N-2)} \times (\alpha^{j\ell} - \alpha^{a\ell})(\alpha^{-j'\ell} - \alpha^{-a\ell}) ,$$

$$(j, j' = 0, 1, 2, \cdots, N - 1) ,$$

$$I_{m,n} = 2^N \cdot i^N \cdot \sum_{k=1}^{N-1} \frac{(-1)^k \cdot \left( \sin \frac{\pi k}{N} \right)^N}{(\alpha^{-k}-1)^{m+1}(\alpha^k-1)^{n+1}} , \quad (m, n = 0, 1, 2, \cdots, N - 2) ,$$

$$\Sigma_{\ell,\ell'} = (-1)^\ell \cdot \delta_{\ell+\ell',N-2} , \quad (\ell, \ell' = 0, 1, 2, \cdots, N - 2) .$$

In the above formulae, the $I_k$, $K_{j,j'}$, $I_{m,n}$ and $\Sigma_{\ell,\ell'}$ are intersection matrices associated with homology cycles for the corresponding periods $\varpi_k$, $\varpi_j$, $\hat{\Pi}_m$ and $\Omega_\ell$. The matrix $I$ has a diagonal form. In contrast, the $K_{j,j'}$ and $I_{m,n}$ are lower triangular matrices with non-vanishing components in the right lower entries with $j + j' \geq N - 2$, $m + n \geq N - 2$. Also the determinant of the $K$ is not unit, but $\text{det} K = N^2$. So the associated cyclic basis is not a canonical one. The set of the $\Omega_\ell$s has an intersection matrix $\Sigma_{\ell,\ell'}$ with non-vanishing components at $\ell + \ell' = N - 2$. The form of the $\Sigma_{\ell,\ell'}$ means that the set of periods $\Omega_\ell$s is a symplectic or an SO-invariant basis and associated homology cycles have appropriate intersection
forms. But it is not an integral basis and the associated cycles belong to homology classes \( \Theta_{\ell=0} H_{2\ell}(M; \mathbb{Q}) \) or \( H_d(W; \mathbb{Q}) \) respectively in the A-, B-models. In order to obtain a set of canonical basis \( \{ \Pi_m \} \ (m = 0, 1, 2, \ldots, N-2) \), we have to perform some linear transformation on the \( \Omega_{\ell} \)

\[
\Pi_m = \sum_{\ell=0}^{N-2} N_{m,\ell} \Omega_{\ell} \quad (m = 0, 1, 2, \ldots, N-2).
\]

The transformation matrix \( N \) generally has entries with fractional rational numbers. The basis \( \Pi \) is needed to discuss the D-brane charges \( Q_\ell \)'s or a central charge \( Z \) in the BPS mass formula.

### 2.3 Central Charge

In the B-model case in the open string channel\(^1\), there appear even dimensional \( D_p \)-branes \( (p = 0, 2, 4, \ldots, 2d) \) which wrap around homology cycles \( \Sigma_p \ (p = 0, 2, 4, \ldots, 2d) \) of the Calabi-Yau \( d \)-fold \( M \). The brane charge \( Q_{2d-2\ell} \) associated with the \( D \)-brane is defined by integrating a gauge field, more precisely a Mukai vector \( v(\mathcal{E}) \) associated with a bundle (sheaf) \( \mathcal{E} \) over the cycle \( \Sigma_{2\ell} \subset M \ (\ell = 0, 1, 2, \ldots, d) \)

\[
Q_{2d-2\ell} = \int_{\Sigma_{2\ell}} v(\mathcal{E}) \quad (\ell = 0, 1, 2, \ldots, d).
\]

By combining these charges \( Q_p \) and the canonical basis \( \Pi_\ell \) with the B-type boundary conditions, we can construct the central charge \( Z \)

\[
Z = \sum_{\ell=0}^{d} Q_{2\ell} \Pi_\ell.
\]

It is a constituent block of a BPS mass formula \( m_{BPS} \sim e^{+K/2}|Z| \) in the curved space. We can represent this canonical basis \( \Pi_\ell \) in the bases \( \Omega_{\ell} \), \( \Pi_\ell \) and \( \omega_j \)

\[
\Pi_n = \sum_{\ell=0}^{N-2} N_{n,\ell} \Omega_{\ell} = \sum_{\ell=0}^{N-2} S_{n,\ell} \Pi_\ell
\]

\[
= \sum_{0 \leq j \leq N-1, \ j \neq a} m_{n,j} \omega_j \quad (0 \leq n \leq N-2).
\]

There are two linear relations among the undetermined matrices \( N \), \( S \) and \( m \) with Eqs.\(^4\),\(^5\)

\[
N_{n,\ell} = \sum_{m=0}^{N-2} S_{nm} V_{\ell m}, \quad (0 \leq n \leq N-2; \ 0 \leq \ell \leq N-2),
\]

\[
S_{n,\ell} = \sum_{0 \leq j \leq N-1, \ j \neq a} m_{n,j} P_{j,\ell}, \quad (0 \leq n \leq N-2; \ 0 \leq \ell \leq N-2).
\]

\(^1\)So far we used the words “A-”, “B-” models for the closed string case. But the definition is exchanged when we consider properties in the open string channels.
If we can determine one of these matrices $N$, $S$ and $m$, the canonical basis $\Pi_\ell$ is fixed. Here the “canonical” condition means that the set $\{\Pi_\ell\}$ is a symplectic (or an SO-invariant) and an integral basis of monodromy transformations. The symplectic (or SO-invariant) condition is reduced to that on the $N$ with an appropriate integer $\lambda$ and the matrix $\Sigma$

$$N^\alpha \cdot \Sigma \cdot N = \lambda \Sigma.$$ 

But generally the matrix $N$ may have fractional components.

3 Quintic

In order to exemplify our considerations, we shall study the result of the quintic. P. Candelas et al directly constructed the matrix $m$, which connects the two bases $\Pi_\ell$ and $\varpi_j$

$$\Pi := \begin{pmatrix} \Pi_0 & \Pi_1 & \Pi_2 & \Pi_3 \end{pmatrix},$$

$$\varpi := \begin{pmatrix} \varpi_0 & \varpi_1 & \varpi_2 & \varpi_3 \end{pmatrix},$$

$$m = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -\frac{2}{5} & \frac{2}{5} & \frac{1}{5} & -\frac{1}{5} \\ \frac{21}{5} & -\frac{1}{5} & -\frac{3}{5} & \frac{8}{5} \\ 1 & -1 & 0 & 0 \end{pmatrix},$$

$$m^{-1} := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 \\ -4 & 8 & 1 & 3 \\ -4 & 3 & 1 & 1 \end{pmatrix}.$$ 

These $\Pi_\ell$s are canonical periods and are described by using a prepotential\(^2\) $F$

$$F = -\frac{\kappa}{6}t^3 + \frac{1}{2}at^2 + bt + \frac{1}{2}c + f,$$

$$\kappa = 5, \ a = \frac{-11}{2}, \ b = \frac{25}{12}, \ c = \frac{\chi \zeta(3)}{(2\pi i)^3}, \ \chi = -200,$$

$$\Pi = \begin{pmatrix} \Pi_0 \\ \Pi_1 \\ \Pi_2 \\ \Pi_3 \end{pmatrix} = \begin{pmatrix} 1 \\ t \\ \partial_t F \\ t\partial_t F - 2F \end{pmatrix} \times \Pi_0,$$ 

$$2\pi it = \log z + \sum_{n=1}^{\infty} \frac{(5n)!}{(n!)^5} \left( \sum_{m=n+1}^{5n} \frac{5}{m} \right) z^n, \ q = e^{2\pi it}.$$ 

The prepotential $F$ of $M$ is expressed as a sum of a polynomial part of $t$ and a non-perturbative part $f$. We choose a set of the $\Omega_\ell$s in the $N = 5$ case

$$\varpi \left( \frac{\rho}{2\pi i} \right) \sqrt{K}(\rho) = \sum_{\ell \geq 0} \rho^{\ell} \Omega_\ell,$$

\(^2\)This “$F$” is not a gauge field “$F$” on the D-brane. I believe that there is no confusion in these notations.
\[
\sqrt{\mathcal{A}(\rho)} = 1 + \frac{5}{12} \rho^2, \quad \sqrt{\mathcal{K}(\rho)} = 1 - \frac{40}{(2\pi i)^3} \zeta(3) \rho^3,
\]

\[
\dot{c} = \frac{40}{(2\pi i)^3} \zeta(3) = -\frac{1}{5} c,
\]

\[
\Omega = \begin{pmatrix}
\Omega_0 \\
\Omega_1 \\
\Omega_2 \\
\Omega_3
\end{pmatrix} = \begin{pmatrix}
1 & t \\
\frac{1}{2} t^2 + S_2(0, x_2) \\
\frac{1}{6} t^3 - \dot{c} + tS_2(0, x_2) + S_3(0, x_2, x_3)
\end{pmatrix} \times \Omega_0,
\]

\[
x_n = \frac{1}{(2\pi i)^n} \frac{1}{n!} \partial^\rho_n \log \left[ \sum_{m=0}^\infty \frac{\Gamma(N(m + \rho) + 1)}{\Gamma(N \rho + 1)} \left( \frac{\Gamma(\rho + 1)}{\Gamma(m + \rho + 1)} \right)^N z^m \right] \bigg|_{\rho=0}.
\]

By comparing Eq. (6) and Eq. (7), we can obtain a matrix \(N\)

\[
\Pi = N \Omega, \quad N = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
\frac{25}{12} & -\frac{11}{2} & -5 & 0 \\
0 & -\frac{25}{12} & 0 & -5
\end{pmatrix}.
\]

The \(N\) is a kind of a symplectic matrix and satisfies a relation

\[
N^t \cdot \Sigma \cdot N = (-5) \cdot \Sigma.
\]

Also one can check this \(N\) and the \(m\) satisfy an equation

\[
N^t = m \cdot P \cdot V,
\]

by using definitions of the \(V\) and \(P\) in Eqs. (4), (5)

\[
P \cdot V = \begin{pmatrix}
1 & 0 & 0 & 0 \\
1 & \frac{25}{12} & 0 & 5 \\
-\frac{23}{12} & -\frac{15}{4} & -5 & -15 \\
-\frac{23}{12} & -\frac{55}{12} & -5 & -5
\end{pmatrix}.
\]

In the \(\tilde{\Pi}\) basis, this result is also expressed as

\[
\Pi = S \cdot \tilde{\Pi}, \quad S = \begin{pmatrix}
0 & 0 & 0 & 1 \\
0 & 0 & -1 & 1 \\
0 & -5 & 8 & -3 \\
5 & 0 & 0 & 0
\end{pmatrix} = m \cdot P.
\]

Then the intersection form in the \(\tilde{\Pi}\) basis is obtained as

\[
S^t \cdot \Sigma \cdot S = \begin{pmatrix}
0 & 0 & 0 & 5 \\
0 & 0 & -5 & 5 \\
0 & 5 & 0 & -5 \\
-5 & -5 & 5 & 0
\end{pmatrix}, \quad \Sigma = \begin{pmatrix}
0 & 0 & 0 & -1 \\
0 & 0 & 1 & 0 \\
0 & -1 & 0 & 0 \\
1 & 0 & 0 & 0
\end{pmatrix}.
\]
It coincides with the matrix $I_{m,n}$ in our result for the $N = 5$ case

$$S^t \cdot \Sigma \cdot S = 5 \cdot I, \quad I = \begin{pmatrix}
0 & 0 & 0 & 1 \\
0 & 0 & -1 & 1 \\
0 & 1 & 0 & -1 \\
-1 & -1 & 1 & 0
\end{pmatrix}.\]

Conversely the result means that the matrix $I$ can be factorized into a form

$$I = \frac{1}{5} S^t \cdot \Sigma \cdot S,$$

with a triangular matrix $S$ with non-vanishing components in the right lower entries. Generally we know the forms of the matrices $V$ and $P$ in Eqs.(4),(5), but the remaining ones $m$ and $N$ are unknown. If we can determine the $N$, then we obtain the $m$ from a relation

$$m = N \cdot (P \cdot V)^{-1}. \quad (9)$$

This $N$ may contain invariants of K(M)-theory of the D-branes as its components. The $N$ is a triangular matrix with non-vanishing components in the left lower blocks

$$N = \begin{pmatrix}
* & 0 & 0 & 0 \\
* & * & 0 & 0 \\
* & * & * & 0 \\
* & * & * & *
\end{pmatrix}.$$

We multiply a charge vector $Q$ on the $N$ from the left. The $Q$ encodes information of homology cycles of the Calabi-Yau around which D-branes wrap. For the quintic case, numbers in the last row of the $N$ in Eq.(8) coincide with coefficients of a topological invariant $(-5)\sqrt{A(\rho)}$

$$(-5)\sqrt{A(\rho)} = -5 - \frac{25}{12} \rho^2.$$

In particular, the $25/12$ is related with a 2nd Chern number $c_2 = 50$ of $M$

$$\frac{1}{24}c_2 = \frac{50}{12}.$$ 

It also appears in the 1st column in the $N$. The number 5 is interpreted as a triple intersection number of a 4-cycle in the $M$. The remaining entry $-11/2$ is expected to be interpreted as some kind of invariant associated with a normal bundle of a world volume of a D-brane. But we do not precisely know the geometric characterization of this yet and cannot explain this number from the point of view of characteristic classes of some associated bundle.
We will return to the central charge $Z$. If we can find the matrix $\mathcal{N}$ and choose the basis $\Pi$, the central charge is evaluated in the B-model in the open string channel

$$Z = Q \cdot \Pi = (Q \mathcal{N})(\mathcal{N}^{-1}\Pi) = (Q \mathcal{N}) \cdot \Omega,$$

$$Q = \begin{pmatrix} Q_{2d} & Q_{2d-2} & \cdots & Q_2 & Q_0 \end{pmatrix}.$$  

When one uses a basis $\Omega_\ell$ to construct the $Z$, the formula of a modified charge vector $Q \cdot \mathcal{N}$ is needed. But no systematic method to calculate the $\mathcal{N}$ is known yet. Also the basis $\Pi$ has an ambiguity in the multiplication with a matrix $L$ from the left-side

$$\Pi' = L \cdot \Pi = L \cdot \mathcal{N} \Omega.$$

When the $L$ is a symplectic (or an SO-invariant) matrix with integer components

$$L^t \cdot \Sigma \cdot L = \Sigma,$$

the modified $\Pi'$ gives us the same Kähler potential as that in the $\Pi$ basis. It leads to the same results about properties of closed string moduli spaces. We do not know any principle to fix the ambiguity completely and do not touch on this here. In this paper, we choose a standard convention $L = I$ by choosing a pure D$_{2d}$ charge in the next section.

4 5-Fold

In the previous section, we analyzed the quintic. We started analyses by using the prepotential $F$ of the quintic and studied its monodromy properties. We cannot expect to use analogs of prepotentials for other $d$-fold cases ($d > 3$). But the essential part we learned in the previous section seems to be a factorizable property of the matrix $\mathcal{I}$ by a (triangular) matrix $S$ and a matrix $\Sigma$. Under this consideration, we will investigate the $N = 7$ ($d = 5$) case concretely. First the $\mathcal{I}$ in the $\bar{\Pi}$ basis is defined as

$$\mathcal{I} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & -1 & 0 & 2 & -2 \\ 0 & 1 & -2 & 0 & 3 \\ -1 & 1 & 2 & -3 & 0 \end{pmatrix}.$$  

We find that the $\mathcal{I}$ is factorized into a form with a matrix $\Sigma$ and a triangular matrix $S$

$$7 \cdot \mathcal{I} = S^t \cdot \Sigma \cdot S,$$
\[ \Sigma = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0
\end{pmatrix},\quad S = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & -1 & 1 & 1 \\
0 & 0 & -7 & 7 & -7 & 7 \\
0 & -7 & 0 & 14 & -21 & 7 \\
7 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}.\]

This decomposition is not unique as we discussed at the end of the previous section. There are some possibilities for choosing the \( S \). When we pick a cyclic basis \( \{\varpi_j\} \) with an index “\( j \)” as
\[
\varpi = t(\varpi_0 \varpi_1 \varpi_2 \varpi_3 \varpi_5 \varpi_6),
\]
a transformation matrix \( P \) from the \( \hat{\Pi} \) to the \( \varpi \) (\( \hat{\Pi} = P \cdot \varpi \)) is obtained
\[
P = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 1 \\
-7 & 0 & 0 & 0 & 0 & 1 \\
35 & -7 & 0 & 0 & 0 & 1 \\
-70 & 28 & -7 & 0 & 0 & 1 \\
-35 & 28 & -21 & 14 & -7 & 1 \\
7 & -7 & 7 & -7 & 7 & -6
\end{pmatrix}.
\]

Now we know the matrix \( S \), we can determine matrices \( m \) and \( \mathcal{N} \). By using the relation \( S = m \cdot P \), we can obtain a transformation matrix \( m \) from the \( \varpi \) basis to the \( \Pi \) basis
\[
\Pi = t(\Pi_0 \Pi_1 \Pi_2 \Pi_3 \Pi_4 \Pi_5),
\]
\[
m = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
-\frac{3}{7} & \frac{3}{7} & \frac{2}{7} & \frac{1}{7} & -\frac{1}{7} & -\frac{2}{7} \\
\frac{2}{7} & \frac{6}{7} & \frac{2}{7} & \frac{1}{7} & 0 & \frac{1}{7} \\
-4 & 4 & 1 & 0 & 0 & -1 \\
-8 & -4 & -3 & -1 & -1 & -4 \\
1 & -1 & 0 & 0 & 0 & 0
\end{pmatrix}. \tag{10}
\]

Characteristic features of the \( m \) appear in the 1st and the last rows in \( m \). The first row means that the \( \Pi_0 \) is identified with the \( \varpi_0 \). The last row implies that the \( \Pi_5 \) is represented as a linear combination of only \( \varpi_0 \) and \( \varpi_1 \) as
\[
\Pi_5 = \varpi_0 - \varpi_1.
\]

We believe that these structures are universal. Especially the \( \Pi_d \) might be represented as
\[
\Pi_d = \varpi_0 - \varpi_1.
\]
It is related to the structure of a pure D_{2d}-brane charge \( Q_{2d} \).

Also we find the matrix \( \mathcal{N} \) for the \( N = 7 \) case by using an equation Eq.(9)

\[
\Pi = \mathcal{N} \Omega,
\]

\[
\mathcal{N} = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
\frac{7}{8} & \frac{1}{2} & -1 & 0 & 0 & 0 \\
0 & \frac{161}{24} & 0 & 7 & 0 & 0 \\
-\frac{11711}{1920} & \frac{161}{48} & \frac{161}{24} & \frac{7}{2} & -7 & 0 \\
0 & \frac{147}{640} & 0 & -\frac{49}{8} & 0 & -7
\end{pmatrix},
\]

\[
\mathcal{N}^t \cdot \Sigma \cdot \mathcal{N} = (-7) \cdot \Sigma.
\]

The entries in the last row of the \( \mathcal{N} \) are topological numbers. They coincide with coefficients of the \((-7)\sqrt{A(\rho)}\) for the \( N = 7 \) case

\[
(-7)\sqrt{A(\rho)} = -7 - \frac{49}{8} \rho^2 + \frac{147}{640} \rho^4.
\]

The entries \( \pm 7 \) is associated with an intersection number 7 of five 8-cycles in \( M \). Possibly the other entries could be interpreted as invariants associated with D-branes from the point of view of K(M)-theory. We cannot precisely know any geometric characterization of numbers at the other entries yet. We do not touch on this topic here.

Next let us consider monodromy matrices associated with singular points in order to confirm our result for the \( N = 7 \) case. The 5-fold has singular points \( \psi = 0, \infty, e^{2\pi i \ell/7} \) \((\ell = 0, 1, 2 \cdots, 6)\) in the B-model moduli space. At the point \( \psi = 0 \), the monodromy transformation acts on the basis \( \varpi \) cyclically and is realized as a matrix \( A_\varpi \)

\[
A_\varpi = A_\varpi \varpi.
\]

The action of \( A \) on other bases \( \hat{\Pi}, \Pi \) and \( \Omega \) is calculated as representation matrices \( A_{\hat{\Pi}}, A_{\Pi} \) and \( A_{\Omega} \) by using the \( \mathcal{N} \) and \( m \)

\[
A_{\varpi} = \begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
-1 & -1 & -1 & -1 & -1 & -1 \\
0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0
\end{pmatrix},
\]

\[
A_{\hat{\Pi}} = \begin{pmatrix}
-6 & 1 & 0 & 0 & 0 & 0 \\
-21 & 1 & 1 & 0 & 0 & 0 \\
-35 & 0 & 1 & 1 & 0 & 0 \\
-35 & 0 & 0 & 1 & 1 & 0 \\
-21 & 0 & 0 & 0 & 1 & 1 \\
-7 & 0 & 0 & 0 & 0 & 1
\end{pmatrix},
\]
The forms of the matrices $T$, section 7. Also the transformation around the $\psi = \infty$ point is evaluated on these bases as matrices $T_\omega, T_\Pi, T_{\Pi}^\prime$ and $T_\Omega$

\[
A_\Pi = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & -1 \\
-1 & 1 & 0 & 0 & 0 & 1 \\
-1 & 1 & 1 & 0 & 0 & 1 \\
-14 & 0 & 7 & 1 & 0 & 14 \\
-7 & -14 & 7 & 1 & 1 & 7 \\
7 & 7 & -14 & 0 & -1 & -6
\end{pmatrix},
A_\Omega = \begin{pmatrix}
1 & -\frac{147}{640} & 0 & \frac{49}{8} & 0 & 7 \\
-1 & \frac{787}{640} & 0 & -\frac{49}{8} & 0 & -7 \\
\frac{1}{2} & -\frac{6737}{5120} & 1 & \frac{539}{64} & 0 & \frac{77}{8} \\
\frac{1}{6} & -\frac{757}{1024} & -1 & \frac{1033}{192} & 0 & -\frac{175}{24} \\
\frac{1}{24} & -\frac{330779}{1228800} & \frac{1}{2} & \frac{26633}{15360} & 1 & \frac{5999}{1920} \\
-\frac{1}{120} & \frac{85451}{1228800} & -\frac{1}{6} & \frac{3737}{15360} & -1 & \frac{289}{1920}
\end{pmatrix}
\]

The forms of the matrices $T_\Pi^\prime$ and $T_\Omega$ are universal. We will explain these points in the section 4.

The $\psi = 1$ is a conifold-like point and associated monodromy matrices are obtained for these bases as $P_\omega, P_\Pi, P_{\Pi}^\prime$ and $P_\Omega$

\[
T_\omega = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 0 & 0 & -1 \\
-6 & 1 & 0 & 0 & 0 & 6 \\
15 & 0 & 1 & 0 & 0 & -15 \\
14 & -1 & -1 & -1 & -1 & -16 \\
-6 & 0 & 0 & 0 & 1 & 6
\end{pmatrix},
T_\Pi = \begin{pmatrix}
1 & -1 & -1 & 1 & -1 & 1 \\
0 & 1 & 1 & 1 & -1 & 1 \\
0 & 0 & 1 & -1 & -1 & 1 \\
0 & 0 & 0 & -1 & -1 & 1 \\
0 & 0 & 0 & 0 & -1 & 1 \\
0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix},
T_{\Pi}^\prime = \begin{pmatrix}
1 & -\frac{1}{2} & 1 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 \\
\frac{1}{6} & \frac{1}{2} & 1 & 1 & 0 & 0 \\
\frac{1}{24} & \frac{1}{6} & \frac{1}{2} & 1 & 1 & 0 \\
\frac{1}{120} & \frac{1}{24} & \frac{1}{6} & \frac{1}{2} & 1 & 1
\end{pmatrix}.
\]

\[
P_\omega = \begin{pmatrix}
2 & -1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
-6 & 6 & 1 & 0 & 0 & 0 \\
15 & -15 & 0 & 1 & 0 & 0 \\
15 & -15 & 0 & 0 & 1 & 0 \\
-6 & 6 & 0 & 0 & 0 & 1
\end{pmatrix},
P_\Pi = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
7 & 1 & 0 & 0 & 0 & 0 \\
14 & 0 & 1 & 0 & 0 & 0 \\
21 & 0 & 0 & 1 & 0 & 0 \\
14 & 0 & 0 & 0 & 1 & 0 \\
7 & 0 & 0 & 0 & 0 & 1
\end{pmatrix},
P_{\Pi}^\prime = \begin{pmatrix}
1 \frac{147}{640} & 0 & -\frac{49}{8} & 0 & -7 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 \frac{1029}{5120} & 1 & -\frac{343}{64} & 0 & -\frac{49}{8} \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & -\frac{3087}{409600} & 0 & \frac{1029}{5120} & 1 & \frac{147}{640} \\
0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}.
\]
Under these monodromy transformations, the set $\Omega_\ell$ is not an integral basis because monodromy matrices have fractional rational numbers in their entries. In contrast, the sets $\Pi_\ell$ and $\hat{\Pi}_\ell$ are integral bases under the monodromy transformations.

For the $\hat{\Pi}$ basis, the associated monodromy matrices $A_{\hat{\Pi}}$, $P_{\hat{\Pi}}$, $T_{\hat{\Pi}}$ are evaluated as

$$A_{\hat{\Pi}} = \begin{pmatrix}
-7 & 1 & 0 & 0 & 0 & 0 \\
-28 & 0 & 1 & 0 & 0 & 0 \\
-84 & 0 & 0 & 1 & 0 & 0 \\
-210 & 0 & 0 & 0 & 1 & 0 \\
-462 & 0 & 0 & 0 & 0 & 1 \\
-925 & 6 & -15 & 20 & -15 & 6
\end{pmatrix}, \quad P_{\hat{\Pi}} = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
7 & 1 & 0 & 0 & 0 & 0 \\
28 & 0 & 1 & 0 & 0 & 0 \\
84 & 0 & 0 & 1 & 0 & 0 \\
210 & 0 & 0 & 0 & 1 & 0 \\
462 & 0 & 0 & 0 & 0 & 1
\end{pmatrix},$$

$$T_{\hat{\Pi}} = \begin{pmatrix}
6 & -15 & 20 & -15 & 6 & -1 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0
\end{pmatrix}.$$  

We can study the other monodromy transformations around points $\psi = e^{2\pi i\ell/7}$ ($\ell = 1, 2, \cdots, 6$) by combining the results for the $\psi = 1$ and those for $\psi = 0$. One can see that the monodromy matrices $A_{\Pi}$, $T_{\Pi}$ and $P_{\Pi}$ in the canonical basis have integral entries. That is a necessary condition for the $\Pi$ to be a canonical integral basis because the set of the associated D-brane (integral) charge vector $Q$ is transformed by these monodromy transformations into vectors $Q \cdot A_{\Pi}$, $Q \cdot T_{\Pi}$, and $Q \cdot P_{\Pi}$. In this 5-fold case, we obtain the canonical basis $\Pi = N\Omega$, but the choice of the matrix $S$ has ambiguities. They have their origins in the decomposition of the $\cal I$ and the determination of the matrix $S$. But the data at the Gepner point allows us to calculate the exact formula of the central charge $Z$. We will explain this in sections 5 and 7.

## 5 Gepner Model

There are several analyses in 3-dimensional Calabi-Yau cases based on the Gepner model[12],[16]. The boundary states, charges, and intersection forms are discussed. Shortly we review the Gepner model and expand the results[12],[16] to the $d$-fold case.

A two dimensional $N = 2$ minimal unitary model has a central charge

$$c = \frac{3k}{k + 2}.$$
Here the $k$ is a positive integer and is called the level. The primary fields of this model is labelled by a set of the conformal weight $h$ and the U(1) charge $q$ as $(h, q)$

\[
\begin{align*}
   h &= \frac{\ell(\ell+2) - m^2}{4(k+2)} + \frac{s^2}{8} \quad (\text{mod. 1}), \\
   q &= \frac{m}{k+2} - \frac{s}{2} \quad (\text{mod. 2}).
\end{align*}
\]

They are parametrized by a set of three integers $(\ell, m, s)$. The standard range of the $(\ell, m, s)$ is specified

\[
\begin{align*}
   0 &\leq \ell \leq k, \ |m-s| \leq \ell, \\
   s &\in \{0, \pm 1\} \text{ and } \ell + m + s \in 2\mathbb{Z}.
\end{align*}
\]

The NS sector is associated with the $s = 0$ representation while the $s = \pm 1$ representations belong to the Ramond sector. The (anti-)chiral primary states are labelled by $((\ell, -\ell, 0))$ $(\ell, +\ell, 0)$ respectively in the NS sector. They are related to the Ramond ground states $(\ell, \pm \ell, \pm 1)$ by the spectral flow.

Let us consider a Gepner model realized by tensoring $N$ minimal models with the same level $(N - 2)$. The corresponding Landau-Ginzburg model is realized by a potential

\[X_1^N + X_2^N + \cdots + X_N^N.\]

We restrict ourselves to (complex) odd dimensional Calabi-Yau cases with $N = 3, 5, 7$ ($d = 1, 3, 5$). Each minimal model has a $\mathbb{Z}_N \times \mathbb{Z}_2$ symmetry whose generators $(g, h)$ act on the primary field $\Phi_{m,s}^\ell$ as

\[
\begin{align*}
   g \Phi_{m,s}^\ell &= \alpha^m \Phi_{m,s}^\ell \quad (\alpha = e^{2\pi i/N}), \\
   h \Phi_{m,s}^\ell &= (-1)^s \Phi_{m,s}^\ell.
\end{align*}
\]

The $\mathbb{Z}_N$-symmetry is correlated with the U(1) charge. The orbifold group of the Gepner model is generated by a diagonal $\mathbb{Z}_N$ generator $\Pi_{j=1}^N g_j$. Here the index “$j$” distinguishes the $N$ minimal models. The boundary states $|\Xi\rangle$ which preserve $N = 2$ worldsheet algebra are constructed in \[\text{[16]}\]. According to the notations of Cardy [14], they are labelled by a set of integers

\[\Xi = (L_j, M_j, S_j) \quad (j = 1, 2, \cdots, N),\]

for each A-, B-type boundary condition. The action of the discrete symmetry $\mathbb{Z}_N \times \mathbb{Z}_2$ is expressed on the state labelled by $\Xi$ as

\[
\begin{align*}
   \begin{cases}
      M_j &\to M_j + 2 \quad (\mathbb{Z}_N \text{ - action}) \\
      S_j &\to S_j + 2 \quad (\mathbb{Z}_2 \text{ - action})
   \end{cases}
\end{align*}
\]
Also it is known that a physically inequivalent choice for the $S_j$ together with the identification under the $\mathbb{Z}_2$-action is given as

$$S = \sum_{j=1}^{N} S_j \equiv 0 \mod 4.$$ 

That is to say, it is enough to consider only boundary states with $S = 0$. Also for B-type boundary states, it is known that the physically inequivalent choices of the $M_j$ ($j = 1, 2, \cdots, N$) can be described by a sum of the $M_j$s

$$M = \sum_{j=1}^{N} M_j.$$ 

It means that the B-type boundary states with fixed $L = (L_1, L_2, \cdots, L_N) =: \{L_j\}$ are described by the single integer $M$. We calculate an intersection matrix $I_B$ for an $L = (0, 0, \cdots, 0) =: \{0\}$ state in the B-type boundary condition with $g^N = 1$

$$I_B = (1 - g^{-1})^N = \sum_{\ell=0}^{N} \binom{N}{\ell} (-1)^{\ell} g^{-\ell}.$$ 

Then an intersection form $I_G$ in the Gepner basis is related to the $I_B$ for general $N = 3, 4, 5, \cdots$

$$I_B = (1 - g)(1 - g^{-1})I_G, \quad I_G = (-g^{-1})(1 - g^{-1})^{N-2}.$$ 

The $I_G$ coincides with an intersection matrix $m^{-1} \cdot \Sigma \cdot (m^{-1})^t$ in the sigma model when we choose the arrangement of the $\varpi_j$s appropriately

$$\varpi = t(\varpi_{a+1} \varpi_{a+2} \cdots \varpi_{N-1} \varpi_0 \varpi_1 \varpi_2 \cdots \varpi_{a-1}).$$

Let us recall that the basis $\Pi$ in the large volume is related to the $\varpi$ through the $m$

$$\Pi = m \varpi.$$ 

The charge vector $Q_G$ is related to the large volume charge vector $Q_L$ as

$$Z = Q_L \cdot \Pi = Q_G \cdot \varpi,$$

$$Q_L = Q_G \cdot m^{-1}.$$ 

At a point $\psi = 1$ in the moduli space of the $W$, a $d$-cycle in the mirror shrinks into a point. An associated cycle in the $M$ is a pure $2d$-cycle around which a D$2d$-brane wraps. The associated charge is specified by a charge vector $Q_L$ in the large volume limit

$$Q_L = (Q_{2d} Q_{2d-2} \cdots Q_2 Q_0) = (1 0 0 \cdots 0 0) =: Q_L^{(0)}.$$ 

\textsuperscript{3}In this section and section, we put the super-, subscript “L” to the charge $Q$ in the large volume case to distinguish it from the $Q_G$ in the Gepner basis.
When we use a Gepner basis for \( \varpi \) as

\[
\varpi = t( \varpi_0 \quad \varpi_1 \quad \varpi_2 \quad \cdots \quad \varpi_{a-1} \quad \varpi_{a+1} \quad \cdots \quad \varpi_{N-1}) ,
\]

with \( a = 2, 3, 4 \) for respectively \( N = 3, 5, 7 \) cases, we can obtain a charge vector \( Q_G^{(0)} \) in the Gepner basis corresponding to the \( Q_L^{(0)} \)

\[
Q_G^{(0)} = \begin{pmatrix} 1 & -1 & 0 & 0 & \cdots & 0 & 0 \end{pmatrix}.
\]

When we take a charge \( Q_G \) of a B-type boundary state with \( |\{0\}; 0; 0\rangle \) to be the \( Q_G^{(0)} \), this boundary state is identified with a pure D2\(d\)-brane with the charge \( Q_L^{(0)} \). By acting a \( \mathbb{Z}_N \)-monodromy matrix \( A^{-1} \), we can change the value \( M \) into an \( M + 2 \) and obtain a charge for the state \( |\{0\}; M; 0\rangle \). Also the \( \mathbb{Z}_2 \)-action \( h \) is implemented by reversing a sign of the charge \( Q \rightarrow -Q \). The other charges \( Q_G \) of states with \( L = (L_1 \quad L_2 \quad \cdots \quad L_N) \) can be obtained by acting generators \( g_j \)'s of the \( \mathbb{Z}_N \)-symmetries on the \( Q_G^{(0)} \)

\[
Q_G = Q_G^{(0)} \prod_{j=1}^{N} \left( \sum_{\ell_j=-L_j/2}^{L_j/2} g_{\ell_j} \right).
\]

In our case, the \( N \) is odd and there is a useful relation in calculating the \( Q_G \) from the set of numbers \( \{L_j\} \)

\[
g^{-1/2} = -g^{N-1/2}.
\]

Collecting all the relations, we calculate the charges for the 5-fold case. We list the result for the \( N = 7 \) (\( d = 5 \)) case in the table below.

| \( L \) | \( Q_G \) | \( Q_L \) |
|-------|--------|--------|
| \((0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0)\) | \((1 \quad -1 \quad 0 \quad 0 \quad 0 \quad 0)\) | \((1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0)\) |
| \((1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0)\) | \((0 \quad 0 \quad -1 \quad 1 \quad 0)\) | \((5 \quad 0 \quad 2 \quad 0 \quad -7 \quad 0)\) |
| \((1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0)\) | \((1 \quad -1 \quad -1 \quad 0 \quad 0 \quad 1)\) | \((-3 \quad 0 \quad -1 \quad 0 \quad 0 \quad 0)\) |
| \((1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0)\) | \((0 \quad 0 \quad -1 \quad -2 \quad 2 \quad 1)\) | \((6 \quad 0 \quad 3 \quad 0 \quad -14 \quad 0)\) |
| \((1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0)\) | \((2 \quad -2 \quad -3 \quad -1 \quad 1 \quad 3)\) | \((-5 \quad 0 \quad -1 \quad 0 \quad -7 \quad 0)\) |
| \((1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0)\) | \((1 \quad -1 \quad -4 \quad -5 \quad 5 \quad 4)\) | \((10 \quad 0 \quad 6 \quad 0 \quad -35 \quad 0)\) |
| \((1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0)\) | \((5 \quad -5 \quad -9 \quad -5 \quad 5 \quad -9)\) | \((-6 \quad 0 \quad 1 \quad 0 \quad -35 \quad 0)\) |
| \((1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1)\) | \((5 \quad -5 \quad -14 \quad -14 \quad 14 \quad 14)\) | \((19 \quad 0 \quad 14 \quad 0 \quad -98 \quad 0)\) |

Table 1: D-brane charges with the B-type boundary conditions. When the set of numbers \( \{L_j\} \) is specified, charge vectors \( Q_G \) and \( Q_L \) are determined.
Here we use a basis \( \psi \)

\[
\psi = ^t(\psi_0 \ \psi_1 \ \psi_2 \ \psi_3 \ \psi_5 \ \psi_6).
\]

for measuring the charges in the \( Q_G \) and write down results for the \( L_j = 0 \) \( (j = 1, 2, \cdots, 7) \) cases for simplicity. We make a remark here: We used the \( m \) in Eq.\((10)\). But there is some arbitrariness in determining the \( m \) and the vector \( Q_L \) cannot be fixed uniquely. Probably the arbitrariness is reduced to some symplectic or SO transformation and they might possibly lead to equivalent physics. We do not know any clear explanations about this yet.

In contrast, the result here shows that we can calculate a charge vector \( Q_G \) of the D-brane in the Gepner basis associated with a boundary state \( |\{L\}; M; S\rangle \)

\[
L \rightarrow Q_G,
\]

when a set of numbers \( L = (L_1 \ L_2 \ \cdots \ L_N) \) is given as an input datum. So we can investigate relations between the \( L \) and \( Q_G \) more precisely. First we restrict ourselves to the states \( |\{L\}; M = 0; S = 0\rangle \) for simplicity. Then we obtain the charge vector \( Q_G \) for \( N = 3, 4, 5 \) \( (d = 1, 3, 5) \) cases with respectively \( a = 2, 3, 4 \)

\[
\psi = (\psi_0 \ \psi_1 \ \psi_2 \ \cdots \ \psi_{a-1} \ \psi_{a+1} \ \cdots \ \psi_{N-1}),
\]

\[
Q_G := (Q_G^0 \ Q_G^1 \ Q_G^2 \ \cdots \ Q_G^{a-1} \ Q_G^{a+1} \ \cdots \ Q_G^{N-1}),
\]

\[
Q_G^j = \frac{1}{N} \sum_{k=1}^{N-1} \alpha^{-kj} (\alpha^k - 1)^{-N+1} \times \alpha^{-\frac{j}{2}} \sum_{j'=1}^{N} L_{j'} \times \prod_{j''=1}^{N} \{\alpha^{k(L_{j''}+1)} - 1\},
\]

\((j = 0, 1, 2, \cdots, a-1, a+1, \cdots, N-1)\).

It is this formula that connects the set \( \{L_j\} \) and the charge vector \( Q_G \). Let us recall that the D-brane charge vector \( Q_L \) in the large radius volume with the B-type boundary condition is constructed from the \( Q_G \) by the transformation matrix \( m \) as \( Q_L = Q_G \cdot m^{-1} \). The associated central charge (the BPS mass formula) \( Z \) is expressed by the \( \Pi_{\ell} \) or \( \psi \) in the appropriate parameter regions of the moduli space

\[
Z = \sum_{\ell=0}^d Q_{2\ell}^L \cdot \Pi_{\ell} = Q_L \cdot \Pi
\]

\[
= (Q_L m) \cdot (m^{-1} \Pi)
\]

\[
= Q_G \cdot \psi = \sum_{0 \leq j \leq N-1 \atop j \neq a} Q_G^j \psi_j.
\]

When the \( \psi \) is not large (for \(|\psi| < 1\)), the formula of the \( Z \) is expressed in the \( \psi_j \) or \( \bar{\psi}_k \) basis

\[
Z = \frac{(-1)^{d+1}}{N} \sum_{j=0,1,\cdots,N-1} \psi_j \cdot \sum_{k=1}^{N-1} \alpha^{-k(j+\frac{a}{2}) \sum_{j'=1}^{N} L_{j'}} \times (\alpha^k - 1)^{-N+1} \times \prod_{j''=1}^{N} \{\alpha^{k(L_{j''}+1)} - 1\}
\]
\[
\frac{(-1)^N \cdot (2i)^N}{(2\pi i)^{N-1}} \times \sum_{k=1}^{N-1} \bar{\omega}_k \cdot (-1)^k \times \prod_{j=1}^{N} \left\{ \sin \frac{\pi k(L_j + 1)}{N} \right\} .
\]

(12)

Next the state \(|\{L\}; M = 2\ell; 0\rangle\) \((\ell \neq 0)\) is obtained by acting the \(Z_N\) monodromy matrix \(A^{-1}\) on the \(Z\) in the \(|\{L\}; M = 0; 0\rangle\) case

\[Z = Q \cdot (A_n^{-1})^{\ell} \Pi .\]

The \(S \neq 0\) case can be realized by acting the remaining \(Z_2\) symmetry on the charge \(Q \to -Q\).

6 Cycles

In this section, we study cycles associated with the periods \(\bar{\omega}_j\) and their intersection forms. The cycles are susy \(d\) cycles in the \(d\) fold consistent with the A-type boundary condition.

6.1 Cycles

For the \(d\)-fold variety \(W\)

\[W; \{p = 0\}/\mathbb{Z}_N^{\otimes (N-1)} ,\]

\[p = X_1^N + X_2^N + \cdots + X_N^N - N\psi X_1X_2\cdots X_N = 0 ,\]

we consider a set of \(d\) chains \(V_j(\psi)\) \((j = 0, 1, 2, \cdots, N - 1)\)

\[V_j(\psi) = \begin{cases} X_1, X_2, \cdots, X_{N-1}; \text{real positive} \\ X_N = 1 , \\ X_{N-1} \text{ s.t. } \arg X_{N-1} \to \pi \cdot \frac{2j + 1}{N} \text{ as } \psi \to 0 \end{cases} .\]

In this definition, the index “\(j\)” of the \(V_j\) is defined modulo \(N\), that is, \(V_{N+j} = V_j\) for an arbitrary \(j \in \mathbb{Z}\). These are chains with the same boundary under an identification of the discrete symmetry \(\mathbb{Z}_N^{\otimes (N-1)}\). But an arbitrary combination \(V_i - V_j\) \((i, j = 0, 1, 2, \cdots, N - 1)\) is a \(d\)-cycle. These cycles are related with the \(V_k\)s through an intersection matrix \(I_{j,k}\)

\[C_j = \sum_{k=0}^{N-1} I_{j,k} V_k , \quad I_{j,k} = \frac{1}{N} \sum_{\ell=0}^{N-1} \alpha^{\ell(j-k)} (1 - \alpha^{\ell})^{N-2} .\]

(13)

More explicitly, this expression is evaluated as a linear combination of \(d\)-cycles

\[C_j = \sum_{n=0}^{N-2} (-1)^n \binom{N - 2}{n} V_{j+2} \ (j \mod. N) \]

\[= \sum_{n=0}^{N-3} (-1)^n \binom{N - 3}{n} (V_{j+n} - V_{j+n+1}) \ (j \mod. N) .\]
Let us consider the $N =$odd cases for $\psi = 0$. In the $N =$odd case, a phase of $X_{N-1}$ in the $V_j$ tends to a definite value for $\psi \to 0$

$$\arg X_{N-1} \to \pi + \frac{2\pi}{N} \left( j - \frac{N-1}{2} \right) \quad (N = \text{odd}).$$

The $d$ chain $V_j$ for $N =$odd is represented in this limit as

$$\text{Im} \ X_\ell = 0 \quad (\ell = 1, 2, \cdots, N-2, N),$$
$$\text{Im} \left( \alpha^{\frac{N-1}{2} - j} X_{N-1} \right) = 0,$$
$$\frac{N-1}{2} - j \in \mathbb{Z},$$

At this special moduli point, there are several analyses about the cycles in [17, 18]. The associated susy $d$ chain for the orbifold point is obtained in [17] as

$$\text{Im} \ (\alpha^{m_\ell} X_\ell) = 0, \ m_\ell \in \mathbb{Z} \quad (\ell = 1, 2, \cdots, N). \quad (14)$$

An arbitrary susy cycle is constructed as a difference of any two susy chains (14) at this moduli point. In our case, the $V_j$ evaluated at $\psi = 0$ corresponds to a special kind of susy chains labelled by a set of numbers $\{m_\ell\}$ at the orbifold point

$$m_\ell = \left( \frac{N-1}{2} - j \right) \delta_{\ell,N-1}.$$ 

But we obtain a formula of susy cycles in the $d$-fold $W$ at a specified moduli point $\psi$.

### 6.2 Periods

Next we introduce a set of periods $q_j(\psi)$ associated with the $C_j$. These $q_j(\psi)$s are related with our periods $\tilde{\omega}_k(\psi)$ as

$$q_j(\psi) = \frac{1}{2\pi i \cdot NN^{-1}} \cdot \sum_{k=1}^{N-1} \alpha^{jk}(\alpha^k - 1)^N \tilde{\omega}_k(\psi) \quad (j = 0, 1, 2, \cdots, N-1). \quad (15)$$

In order to prove the Eq.(15), we introduce a set of one-cycles $\gamma_j$ as unions of half-lines

$$\gamma_j = \{ \arg X_j = 0 \} \cup \{ \arg X_j = \frac{2\pi}{N} \} \quad (j = 1, 2, \cdots, N-2).$$

From these $d$ one-cycles, we can construct $d$-cycles $C_j$

$$C_j = \{(X_1, X_2, \cdots, X_{N-2}, X_{N-1}, 1) ;$$
$$\quad (X_1, X_2, \cdots, X_{N-2}) \in \gamma_1 \times \gamma_2 \times \cdots \times \gamma_{N-2},$$
$$\quad \text{branch of } X_{N-1} \text{ is specified by } "j" \} \quad (j = 0, 1, 2, \cdots, N-2).$$
We calculate a period \( q_0(\psi) \) associated with the \( C_0 \)
\[
q_0(\psi) = \sum_{m=1}^{\infty} u_m(N\psi)^m I_m,
\]
\[
u_m = -\frac{1}{N} \cdot \alpha^{m/2} \cdot \frac{\Gamma\left(\frac{N-1}{N} \cdot m\right)}{\Gamma\left(1 - \frac{m}{N}\right) \Gamma(m)},
\]
\[
I_m = \int_{\gamma_1 \times \gamma_2 \times \cdots \times \gamma_{N-2}} dX_1 dX_2 \cdots dX_{N-2} \frac{(X_1 X_2 \cdots X_{N-2})^{m-1}}{\Delta_{\frac{N-1}{N}}^m},
\]
\[
\Delta := 1 + X_1^N + X_2^N + \cdots + X_{N-2}^N.
\]
This \( I_m \) is transformed into an integral associated with the \( V_0 \)
\[
I_m = (1 - \alpha^m)^{N-2} \cdot \int_0^\infty dX_1 \int_0^\infty dX_2 \cdots \int_0^\infty dX_{N-2} \frac{(X_1 X_2 \cdots X_{N-2})^{m-1}}{\Delta_{\frac{N-1}{N}}^m}.
\]
When we recall the action of the \( Z_N \) monodromy transformation \( A \) around the \( \psi = 0 \)
\[
A; (N\psi)^m \rightarrow \alpha^m(N\psi)^m,
\]
the \( q_0(\psi) \) is expressed as
\[
q_0(\psi) = \sum_{m=1}^{\infty} u_m(N\psi)^m \int_{\gamma_1 \times \gamma_2 \times \cdots \times \gamma_{N-2}} dX_1 dX_2 \cdots dX_{N-2} \frac{(X_1 X_2 \cdots X_{N-2})^{m-1}}{\Delta_{\frac{N-1}{N}}^m}
\]
\[
= (1 - A)^{N-2} \cdot \sum_{m=1}^{\infty} u_m(N\psi)^m \int_0^\infty dX_1 \int_0^\infty dX_2 \cdots \int_0^\infty dX_{N-2} \frac{(X_1 X_2 \cdots X_{N-2})^{m-1}}{\Delta_{\frac{N-1}{N}}^m}.
\]
From this formula, we obtain a relation between the \( C_0 \) and the \( V_0 \)
\[
C_0 = (1 - A)^{N-2} V_0.
\]
On the other hand, the \( q_0(\psi) \) can be expressed in a series expansion explicitly
\[
q_0(\psi) = \left(\frac{2\pi i}{N}\right)^{N-2} \cdot \frac{1}{N} \cdot (-1)^{N-1} \times \sum_{m=1}^{\infty} \frac{\Gamma\left(\frac{m}{N}\right)}{\Gamma(m) \left(1 - \frac{m}{N}\right)]^{N-1}} \times (\alpha^{\frac{N-1}{2}} \cdot N\psi)^m.
\]
It is related with a period \( \varpi_0(\psi) \) as
\[
q_0(\psi) = (-1)^N \cdot \left(\frac{2\pi i}{N}\right)^{N-2} \varpi_0(\psi),
\]
\[
\varpi_0(\psi) = -\frac{1}{N} \cdot \sum_{m=1}^{\infty} \frac{\Gamma\left(\frac{m}{N}\right)}{\Gamma(m) \left(1 - \frac{m}{N}\right)]^{N-1}} \times (\alpha^{\frac{N-1}{2}} \cdot N\psi)^m.
\]
Also a relation \( q_j(\psi) = q_0(\alpha^j \psi) \) means that each cycle \( C_j \) is related with a period \( q_j \). That completes the proof of the statement in Eq.(15).

We make a comment on the branch of the \( X_{N-1} \): First we introduce a variable \( v = N \psi \Delta^{-\frac{\psi}{N}} X_1 X_2 \cdots X_{N-2} \). In the small \( v \) case \((v = 0)\), the \( X_{N-1} \) is evaluated as

\[
X_{N-1} = \Delta^{1/N} \cdot \exp \left( \frac{\pi i}{N} (2j' + 1) \right) \quad (j' = 0, 1, 2, \cdots, N-1).
\]

The argument of \( X_{N-1} \) corresponds to the branch of the chain \( V_j' \). We find an exact representation of the \( X_{N-1} \)

\[
\Delta^{-1/N} \cdot X_{N-1} = -\sum_{n=0}^{\infty} \frac{\alpha^{(n+1)(j' + \frac{1}{2})}}{N} \cdot \frac{\Gamma \left( \frac{N-1}{N} (n + 1) - 1 \right)}{\Gamma(n + 1) \Gamma \left( 1 - \frac{n + 1}{N} \right)} \cdot v^n, \quad (j' = 0, 1, 2, \cdots, N-1).
\]

On the other side, the \( X_{N-1} \) is evaluated in the large \( v \) region

\[
X_{N-1} = \Delta^{1/N} \cdot v^{\frac{1}{N-1}} \exp \left( \frac{2\pi i}{N-1} j'' \right) \quad (j'' = 0, 1, 2, \cdots, N-1).
\]

When we move the parameter \( \psi \) from 0 to \( \infty \) on the real line, two of the \( (N - 1) \) \( X_{N-1} \)s with \( j = 0 \) and \( N - 1 \) coincide at \( v = N(1 - N)^{\frac{1}{N} - 1} \). Its value is evaluated as \( X_{N-1} = \Delta^{1/N} \cdot (N - 1)^{-1/N} \). The other \( X_{N-1} \)s do not collide one another for the \( \psi \in \mathbb{R}_+ \).

Next we will consider cycles associated with the \( \tilde{\omega}_\ell \). Because these \( q_j(\psi) \) are related with our periods \( \tilde{\omega}_k(\psi) \) in Eq.(15), we can obtain cycles \( \tilde{C}_\ell \) associated with the periods \( \tilde{\omega}_\ell \) as

\[
\tilde{C}_\ell = \frac{1}{(1 - \alpha^\ell)^{N-1}} \sum_{j=0}^{N-1} \alpha^{-j\ell} C_j
\]

\[
= \frac{1}{1 - \alpha^\ell} \sum_{k=0}^{N-1} \alpha^{-\ell k} V_k \quad (1 \leq \ell \leq N-1).
\]

These cycles \( \tilde{C}_\ell \)s diagonalize the action of the \( \mathbb{Z}_N \) monodromy \( \psi \rightarrow \alpha \psi \)

\[
\tilde{C}_\ell \rightarrow \alpha^\ell \tilde{C}_\ell,
\]

but the sets of \( C_j \) and \( V_k \) change cyclically

\[
Q_j \rightarrow Q_{j+1}, \quad V_k \rightarrow V_{k+1}.
\]

The \( \tilde{C}_\ell \)s belong to \( \text{H}_d(W; \mathbb{C}) \). On the other hand, the \( C_j \)s and \( V_k \)s are rational homology cycles in \( \text{H}_d(W; \mathbb{Q}) \).
The results of the CFT at the Gepner point imply that an intersection matrix associated with the cycles $C_j$ must be the $I_B$

$$C_j \cap C_{j'} = (I_B)_{j,j'} = \frac{1}{N} \cdot \sum_{r=0}^{N-1} \alpha^r(j+j'+1)(1 - \alpha^r)^{N-2}, \quad (j, j' = 0, 1, 2, \ldots, N - 1).$$

This fact allows us to calculate intersections of the $V_k$s formally by using the Eq.(13)

$$V_k \cap V_{k'} = \frac{(-1)^N}{N} \cdot \sum_{r=1}^{N-1} \alpha^{r(-k+k'-1)} \frac{\alpha^{-r}}{(1 - \alpha^r)^{N-2}}, \quad (k, k' = 0, 1, 2, \ldots, N - 1).$$

By using this relation, we can show that the $d$-cycles $V_{k,\ell} := V_k - V_{\ell}$s have intersection forms

$$V_{k,\ell} \cap V_{k',\ell'} = \frac{(-1)^N}{N} \cdot \sum_{r=1}^{N-1} \alpha^{-r} \times (\alpha^{-kr} - \alpha^{-\ell r})(\alpha^{k' r} - \alpha^{\ell' r}), \quad (k, \ell, k', \ell' = 0, 1, 2, \ldots, N - 1).$$

7 Central Charge in Large Radius Region

In this section, we calculate the B-type central charge $Z$ for Calabi-Yau $d$-fold from the point of view of mirror symmetry. We analyze the structure of the $Z$ at a generic Kähler structure moduli point near the large radius region. Also we consider relations with this formula with the results in the Gepner model.

7.1 B-type Central Charge

In this subsection, we consider the B-type central charge $Z$ in the large radius region of the $M$. Let us recall that it is a product of a charge vector $Q_L$ and canonical basis $\Pi$

$$Z = Q_L \cdot \Pi = \tilde{Q} \cdot \Omega,$$

$$\tilde{Q} = Q_L N.$$

(16)

The matrix $N$ transforms the charges $Q_L$ into $\tilde{Q}$ by a fractional redefinition of the charge lattices. When we consider the $d$-fold $M$, the D$_{2d-2p}$ brane charge is given by integrating the $F^p$ (more precisely, $2p$ form parts of an associated Mukai vector $v(\mathcal{E})$ of a sheaf $\mathcal{E}$) over the $\Sigma_{2p}$ with the B-type boundary conditions ($p = 0, 1, 2, \ldots, d$)

$$Q_{2d-2p} = \int_{\Sigma_{2p}} e^F.$$

We prepare several notations and concrete formulae in order to write the central charge $Z$ explicitly. First the function $\varpi \left( + \frac{\rho}{2\pi i} \right)$ is defined by using a formal parameter $\rho$ with
\[ \rho^{N-1} = 0 \]

\[ \varpi^{-1}_0 \times \varpi \left( \frac{\rho}{2\pi i} \right) := e^{\rho} \times \exp \left( \sum_{n \geq 2} \rho^n x_n \right), \]

\[ 2\pi it = \log z + \frac{\sum_{n=1}^{\infty} (Nn)! \left( \sum_{m=n+1}^{Nn} \frac{N}{m} \right) z^n}{\sum_{n=0}^{\infty} (Nn)! \frac{n}{n!} z^n}, \quad q = e^{2\pi it}, \]

\[ x_n = \frac{1}{(2\pi i)^n} \frac{1}{n!} \partial^n_{\rho} \log \left[ \sum_{m=0}^{\infty} \frac{\Gamma(N(m + \rho) + 1)}{\Gamma(N\rho + 1)} \left( \frac{\Gamma(\rho + 1)}{\Gamma(m + \rho + 1)} \right)^N z^m \right] \bigg|_{\rho=0}. \]

Also the \( \hat{K}(\rho) \) is related to Riemann’s zeta functions

\[ \hat{K}(\rho) = \exp \left[ 2 \sum_{m=1}^{\infty} \frac{N - N^{2m+1}}{2m + 1} \zeta(2m + 1) \left( \frac{\rho}{2\pi i} \right)^{2m+1} \right]. \]

Then the \( \hat{\Pi}_m \) and \( \hat{\Pi}_n \) is expressed as \((0 \leq m \leq N - 2; 0 \leq n \leq N - 2)\)

\[ \varpi^{-1}_0 \times \sqrt{\hat{K}(-\rho)} \cdot \varpi \left( -\frac{\rho}{2\pi i} \right) = e^{-\rho t} \times \exp \left( \sum_{n \geq 2} (-\rho)^n \cdot y_n \right), \]

\[ \hat{k}_{2m+1} = \frac{1}{(2\pi i)^{2m+1}} \cdot \frac{N - N^{2m+1}}{2m + 1} \zeta(2m + 1), \]

\[ y_{2m} := x_{2m}, \quad y_{2m+1} := x_{2m+1} + \hat{k}_{2m+1}, \quad (m = 1, 2, \cdots), \]

\[ \varpi^{-1}_0 \times \hat{\Pi}_m = \left[ \sqrt{A(+\rho)} \cdot (e^\rho - 1)^m \cdot e^{-\rho t} \times \exp \left( \sum_{n \geq 2} (-\rho)^n \cdot y_n \right) \right]_{\rho^{N-2}} \quad (17), \]

\[ \varpi^{-1}_0 \times \hat{\Pi}_n = \left[ \sqrt{A(+\rho)} \cdot e^{n\rho} \cdot e^{-\rho t} \times \exp \left( \sum_{n \geq 2} (-\rho)^n \cdot y_n \right) \right]_{\rho^{N-2}} \quad (18). \]

Now we will rewrite the formula Eq.\((16)\) in the bases \( \hat{\Pi} \) and \( \hat{\Pi} \). The \( \hat{\Pi} \) and \( \hat{\Pi} \) are related to the \( \Pi_n \) and \( \Omega_\ell \) \((0 \leq n \leq N - 2; 0 \leq \ell \leq N - 2)\)

\[ \Pi_n = \sum_{m=0}^{N-2} S_{n,m} \hat{\Pi}_m, \quad \hat{\Pi}_n = \sum_{m=0}^{N-2} U_{n,m} \hat{\Pi}_m, \quad (0 \leq n \leq N - 2), \]

\[ \hat{\Pi}_m = \sum_{\ell=0}^{N-2} V_{m,\ell} \Omega_\ell \]

\[ U_{n,m} = \binom{n}{m} \cdot (-1)^{n-m}, \quad (0 \leq n \leq N - 2; 0 \leq m \leq N - 2), \]

\[ V_{m,\ell} := \left[ \sqrt{A(+\rho)} \cdot (e^\rho - 1)^m \cdot (-\rho)^\ell \right]_{\rho^{N-2}}, \quad (0 \leq m \leq N - 2; 0 \leq \ell \leq N - 2). \]
We can obtain the central charge by using these equations Eqs. (17), (18)

\[ Z = \sum_{\ell=0}^{N-2} Q_{2\ell}^L \cdot \left[ \sqrt{A(+\rho)} \cdot \left\{ \sum_{m=0}^{N-2} S_{\ell,m} (e^\rho - 1)^m \right\} \cdot e^{-\rho t} \right] \times \exp \left( \sum_{n \geq 2} (-\rho)^n \cdot y_n \right) \rho^{N-2} \]

These formulae Eqs. (19) remind us the couplings of RR fields (potentials) \( C \) to the gauge fields on the D-brane, that is to say, a Chern-Simons term in the B-model

\[ \int C \wedge e^F \wedge \sqrt{A(M) A(N)} . \]

Here the \( N \) is a normal bundle of the world volume. The \( C \) couples with the Mukai vector

\[ e^F \wedge \sqrt{A(M) A(N)} . \]

and the gauge field \( F \) is combined with the Kalb-Ramond field \( B \) into the \( \mathcal{F} \)

\[ \mathcal{F} = F - B . \]

In the context of the topological sigma model, this \( B \)-field is combined into a complexified Kähler form

\[ B + iJ = t [D] (\text{where } [D] \in H^2(M)) . \]

\[ B = \text{Re}(t)[D], \quad J = \text{Im}(t)[D] . \]

In our case, the \("[D]\)" is a 1st Chern class of a hyperplane bundle of \( CP^{N-1} \)

\[ [D] = c_1 (\mathcal{O}_{CP^{N-1}}(1)) , \]

and then the \( \mathcal{F} \) is represented as

\[ \mathcal{F} = F - B = F - \text{Re}(t)[D] . \]

When one shifts the B-field, the real part of the \( t \) changes, for an example, \( t \to t + 1 \) and the \( \mathcal{F} \) changes. It implies a coupling

\[ C \wedge e^{-\text{Re}(t)[D]} \wedge \sqrt{A(M) A(N)} . \]

In our case, we see an analogous term in the \( Z \) in Eqs. (19)

\[ Z = \sum_{\ell=0}^{N-2} Q_{2\ell}^L \cdot \left[ \sqrt{A(\rho)} \cdot e^{-\rho t} \times \cdots \right] \rho^{N-2} . \]
Up to a multiplicative constant, this formal parameter $\rho$ could be identified with the divisor $[D]$ and the $e^{-t\rho}$ is represented by using the $[D]$

$$e^{-t\rho} = e^{-t[D]} = e^{-(iJ+B)}.$$  

Then the $Z$ is expressed as integral formulae

$$Z = \sum_{\ell=0}^{N-2} Q_{2^\ell} \cdot \left[ \prod_M \sqrt{ \mathcal{A}([D]) } \cdot \left\{ \sum_{m=0}^{N-2} S_{\ell,m} (e^{[D]} - 1)^m \right\} \cdot e^{-t[D]} \right] \times \exp \left( \sum_{n\geq 2} (-[D])^n \cdot y_n \right) \times \left( \int_M [D]^d \right)^{-1}$$

$$= \sum_{\ell=0}^{N-2} Q_{2^\ell} \cdot \int_M \left[ \prod_M \sqrt{ \mathcal{A}([D]) } \cdot \left\{ \sum_{m,n=0}^{N-2} S_{\ell,m} U_{m,n} \right\} \cdot e^{n[D]} \cdot e^{-t[D]} \right] \times \exp \left( \sum_{n\geq 2} (-[D])^n \cdot y_n \right) \times \left( \int_M [D]^d \right)^{-1}. \tag{21}$$

Here the integral over the $M$ is defined with $[H] := N[D]$

$$\int_M (\cdots) = \int_{CP^{N-1}} (\cdots) \cdot [H].$$

In the formula of the $Z$ in Eq.(22), there appears a term $e^{n[D]}$. When we identify the $\rho$ with the $[D]$, the $n[D]$ is identified with $c_1(\mathcal{O}_{CP^{N-1}}(-n)^*)$. Then the term $e^{n[D]}$ in Eq.(22) is interpreted as a Chern character of the line bundle $\mathcal{O}_{CP^{N-1}}(-n)^*$

$$e^{n[D]} = \text{ch} (\mathcal{O}_{CP^{N-1}}(-n)^*).$$

We will explain this geometrical interpretation about $Z$ in the next subsection.

As a last topic in this subsection, we consider a monodromy transformation around large radius point $\text{Im} t = \infty$ (equivalently, $\psi = \infty$). When one performs a monodromy transformation around the $\psi = \infty$, the $t$ is shifted into $t+1$ and the $B$-field changes as $B \rightarrow B+[D]$. In the $Z$, the $x_n$s are invariant under the integral shift of the $t$, but the term $e^{-t[D]}$ is transformed into $e^{-[D]} \cdot e^{-t[D]}$. This shift affects on the $Z$

$$Z = \sum_{\ell=0}^{N-2} Q_{2^\ell} \cdot \left[ \prod_M \sqrt{ \mathcal{A}([D]) } \cdot \left\{ \sum_{m=0}^{N-2} S_{\ell,m} (e^{[D]} - 1)^m e^{-t[D]} \right\} \times \cdots \right]$$

$$\rightarrow \sum_{\ell=0}^{N-2} Q_{2^\ell} \cdot \left[ \prod_M \sqrt{ \mathcal{A}([D]) } \cdot \left\{ \sum_{m=0}^{N-2} S_{\ell,m} \sum_{m'=0}^{N-2} (-1)^{m'} (e^{[D]} - 1)^{m+m'} e^{-t[D]} \right\} \times \cdots \right],$$

$$Z = \sum_{\ell=0}^{N-2} Q_{2^\ell} \cdot \left[ \prod_M \sqrt{ \mathcal{A}([D]) } \cdot \left\{ \sum_{m,n=0}^{N-2} S_{\ell,m} U_{m,n} \right\} e^{n[D]} e^{-t[D]} \times \cdots \right]$$

$$\rightarrow \sum_{\ell=0}^{N-2} Q_{2^\ell} \cdot \left[ \prod_M \sqrt{ \mathcal{A}([D]) } \cdot \left\{ \sum_{m,n=0}^{N-2} S_{\ell,m} U_{m,n} \right\} e^{(n-1)[D]} e^{-t[D]} \times \cdots \right].$$

26
It induces rearrangements of the components of the matrices $S$ and $U$ and leads to a reshuffle of the charge vector. Also it gives us monodromy matrices on the basis $\hat{\Pi}$ and $\hat{\Pi}$. Especially monodromy matrices $T_\hat{\Pi}$ and $T_\Omega$ around $\psi = \infty$ can be obtained in the $\hat{\Pi}$ and $\Omega$ bases by this consideration

$$
\hat{\Pi}_m = \varpi_0 \times \left[ \sqrt{A(\rho)} \cdot (\rho^m - 1)^{m} \cdot e^{-\rho t} \times \exp \left( \sum_{n \geq 2} (-\rho)^n \cdot y_n \right) \right]_{\rho^{-N-2}} \\
\rightarrow \varpi_0 \times \left[ \sqrt{A(\rho)} \cdot \sum_{m'=0}^{N-2} (-1)^{m'} (\rho^m - 1)^{m+m'} \cdot e^{-\rho t} \times \exp \left( \sum_{n \geq 2} (-\rho)^n \cdot y_n \right) \right]_{\rho^{-N-2}},
$$

$$
\Omega_\ell = \varpi_0 \times \left[ e^{\rho t} \times \exp \left( \sum_{n \geq 2} \rho^n \cdot y_n \right) \right]_{\rho^{\ell}},
$$

$$
T_\hat{\Pi} = \sum_{\ell=0}^{N-2} (-T)^\ell, \quad T_\Omega = e^{Tt},
$$

$$
T_{\ell,\ell'} = \delta_{\ell+1,\ell'}, \quad (0 \leq \ell \leq N-2; 0 \leq \ell' \leq N-2).
$$

The formulae of these $T_\hat{\Pi}$ and $T_\Omega$ for $N = 7$ case coincide with those in section 4.

### 7.2 Relation with Results in the Gepner Model

In this subsection, we investigate the $Z$ associated with the charge $\{Q_j^G\}$ at the Gepner point and study relations with the B-type boundary state $|\{L\}; M; S\rangle$ in the Gepner model. The $Z$ is obtained in Eq. (22) by using the set $\{Q_j^G\}$ ($j = 0, 1, 2, \cdots, N-1$). We can rewrite this formula as

$$
Z = N^{-1} \cdot \sum_{k=1}^{N-1} (\alpha_k - 1)^{N-1} \left( \sum_{j=0}^{N-1} Q_j^G \alpha^k \right) \int_M \left[ \frac{\alpha^k}{e^{[D]} - \alpha^k} \cdot \sqrt{A([D])} \cdot e^{-t[D]} \right] \\
\times \sqrt{K([-D])} \times \exp \left( \sum_{n \geq 2} (-[D])^n \cdot x_n \right)
$$

$$
= N^{-1} \cdot \sum_{n=0}^{N-2} \left\{ \sum_{\ell=n}^{N-1} \sum_{j=0}^{N-1} Q_j^G \cdot (\ell \choose n) (-1)^{\ell-n} \right\} \int_M \left[ e^{n[\bar{D}]} \cdot \sqrt{A([\bar{D}])} \cdot e^{-t[D]} \right] \\
\times \sqrt{K([-D])} \times \exp \left( \sum_{m \geq 2} (-[D])^m \cdot x_m \right),
$$

$$
P_{j,\ell} = \delta_{\ell,N-2} + N \cdot (-1)^{j-\ell} \left( \frac{N-2-\ell}{N-1-j} \right), \quad \alpha = e^{2\pi i/N},
$$

27
Here \( \sum_{j=0}^{N-1} Q_j^G P_{j,m} \) s are integers because the charges \( Q_j^G \) and the \( P_{j,m} \) s are integers. Information about charges is encoded in a function \( R \)

\[
R([D]) := N^{-1} \cdot \sum_{k=1}^{N-1} \left( \alpha^k - 1 \right)^{N-1} \left( \sum_{j=0}^{N-1} Q_j^G \alpha^{kj} \right) \cdot \frac{\alpha^k}{e^{|D|} - \alpha^k}.
\]

Together with the \( \sqrt{A} \), analogs of the Mukai vector appear in the \( Z \)

\[
\sqrt{A([D])} \cdot R([D]) ,
\]

\[
\sqrt{A([D])} \cdot e^n[D] .
\]

We investigate this structure in the geometrical point of view. The Calabi-Yau \( d \)-fold \( M \) is embedded in the ambient projective space \( X = CP^{N-1} \). A line bundle \( \mathcal{O}_X (m) \) of the \( X \) is defined as an \( m \) tensor product of a section of a hyperplane bundle. Here we introduce a set of line bundles \( R_a = \mathcal{O}_X (-a[D]) \). An intersection paring \( I_{a,b} \) between these bundles \( R_a \) and \( R_b \) is defined as a Euler characteristic

\[
I_{a,b} = \chi_X (R_a, R_b) = \int_X \text{ch}(R_a)^* \text{ch}(R_b) \text{Td}(TX) .
\]

We can evaluate the \( I_{a,b} \) and its inverse explicitly

\[
I_{a,b} = \chi_X (R_a, R_b) = \begin{pmatrix} d + 1 + a - b \\ a - b \end{pmatrix} ,
\]

\[
(I^{-1})_{a,b} = (-1)^{a-b} \begin{pmatrix} N \\ a - b \end{pmatrix} , \quad (N = d + 2) .
\]

Next we define a set of dual basis \( \{ S_a \} \) for the bundles \( \{ R_a \} \)

\[
\text{ch}(S_a) := \sum_b (I^{-1})_{b,a} \text{ch}(R_b) = \sum_b (-1)^{b-a} \begin{pmatrix} N \\ b - a \end{pmatrix} \text{ch}(R_b) ,
\]

\[
\text{ch}(S_a^*) := \sum_b (I^{-1})_{a,b} \text{ch}(R_b^*) = \sum_b (-1)^{a-b} \begin{pmatrix} N \\ a - b \end{pmatrix} \text{ch}(R_b^*) .
\]

The two sets of bases are orthonormal with respect to the intersection paring

\[
\langle R_a, S_b \rangle = \chi_X (R_a, S_b) = \delta_{a,b} ,
\]

\[
\langle S_a, R_b \rangle = \chi_X (S_a, R_b) = \delta_{a,b} .
\]

The set of line bundles (sheaves) \( \{ \mathcal{O}(a) \} \) \( (a = 0, 1, 2, \cdots, N - 1) \) is an exceptional collection of the \( CP^{N-1} \) and also turns out to be a foundation of an associated helix of \( CP^{N-1} \). We
can consider a left mutation \( L \) on the set \( \{ \mathcal{O}(a) \} \) because a condition \( \text{Ext}^0(\mathcal{O}(a-1), \mathcal{O}(a)) = H^0(\mathcal{O}(a-1), \mathcal{O}(a)) \neq 0 \) is satisfied. It leads to a relation of Chern characters of the bundles

\[
\text{ch}(L_{a-1}\mathcal{O}(a)) = \text{ch}(\mathcal{O}(a)) - \chi(\mathcal{O}(a-1), \mathcal{O}(a)) \text{ch}(\mathcal{O}(a-1)),
\]

where we introduce an abbreviated notation \( L_{a-1}\mathcal{O}(a) := L_{\mathcal{O}(a-1)}\mathcal{O}(a) \). By using this formula Eq.\((25)\) iteratively, we obtain a relation for the \( S^*_a \)

\[
\text{ch}(L_0L_1 \cdots L_{a-1}\mathcal{O}(a)) = \sum_{b=0}^{a} (-1)^{a-b} \begin{pmatrix} N \\ a-b \end{pmatrix} \text{ch}(\mathcal{O}(b)) = \text{ch}(S^*_a).
\]

Thus each element \( S^*_a \) of the dual basis can be constructed by acting iteratively left mutations on the \( R^*_a = \mathcal{O}(a) \). Because the Calabi-Yau \( M \) is realized as a hypersurface in the \( CP^{N-1} \), we shall restrict the sets of bundles \( \{S_a\}, \{R_a\} \) on the \( M \).

Now we introduce a function \( Z(S_m) \) associated with each bundle \( S_m \) on \( M \) as

\[
Z(S_m) := (-1)^{d} \int_M \text{ch}(S^*_m)\sqrt{A(TM)} \cdot e^{-t[D]} \times \sqrt{K(-[D])} \times \exp \left( \sum_{m=2}^{\infty} (-[D])^m \cdot x_m \right),
\]

\[
(m = 0, 1, 2, \cdots, N - 1).
\]

It can be expressed as an integral formula on \( M \)

\[
Z(S_m) = (-1)^{d+1} \cdot N^{-1} \cdot \sum_{k=1}^{N-1} \alpha^k(\alpha^k - 1)^N \cdot \int_M \frac{\alpha^k}{e^{[D]} - \alpha^k} \cdot \sqrt{A(TM)} \cdot e^{-t[D]} \times \sqrt{K(-[D])} \times \exp \left( \sum_{m=2}^{\infty} (-[D])^m \cdot x_m \right),
\]

\[
(m = 0, 1, 2, \cdots, N - 1). \quad (25)
\]

We analyze a central charge of Calabi-Yau \( d \)-fold embedded in \( CP^{N-1} \) in the analysis based on the mirror symmetry. The central charge \( Z(\{Q^G\}) \) in Eq.\((23)\) is labelled by a set of charges \( \{Q^G_j\} \) at the Gepner point. They are transformed cyclically under the \( Z_N \) action around \( \psi = 0 \), \( Q^G_j \rightarrow Q^G_{j+1} \) (\( Q^G_{j+N} \equiv Q^G_j \)). They also satisfy a relation \( \sum_{j=0}^{N-1} Q^G_j = 0 \) and we can decompose each \( Q^G_j \) as \( Q^G_j =: q^G_j - q^G_{j-1} \). Then the central charge \( Z \) is reexpressed as

\[
Z = N^{-1} \cdot (-1) \cdot \sum_{k=1}^{N-1} (\alpha^k - 1)^N \cdot \left( \sum_{j=0}^{N-1} q^G_j \alpha^j \right)
\]

\[
\times \int_M \frac{\alpha^k}{e^{[D]} - \alpha^k} \cdot e^{-t[D]} \cdot \sqrt{A(TM)} \times \sqrt{K(-[D])} \times \exp \left( \sum_{m=2}^{\infty} (-[D])^m \cdot x_m \right).
\]

By using the formula Eq.\((23)\) for \( Z(S_m) \), we obtain a formula of the \( Z \) in terms of geometrical data

\[
Z(\{q^G\}) = (-1)^N \cdot \sum_{m=0}^{N-1} q^G_m \cdot Z(S_m)
\]
It is a linear combination of the $Z(S_m)$'s. Each $Z(S_m)$ has information about a K-theory element of sheaves $S_m$ that is constructed through a restriction on the $M$. In the large radius region, fractional charges $\{Q_a\}$ in the fractional basis are defined as coefficients in an expansion of the above formula

$$Z(\{q^G\}) = \sum_{n=0}^{d} \frac{t^{d-n}}{(d-n)!} \cdot (-1)^n \cdot Q_{2(d-n)};$$

$$Q_{2(d-n)} = (-1)^d \cdot \sum_{m=0}^{N-1} q^G_m \int_M \text{ch}(S^*_m) \sqrt{A(TM)} \cdot [D]^{d-n}$$

$$= (-1)^d \cdot \sum_{m=0}^{N-1} q^G_m \int_{2n} \text{ch}(S^*_m) \sqrt{A(TM)},$$

where we introduce a cycle $\gamma_{2n}$ as a dual of the $(2d-2n)$ form $[D]^{d-n}$. We shall take an example with $q^G_m = \delta_{m,0}$, that is,

$$Z = \int_M \text{ch}(S^*_m) \sqrt{A(TM)} \cdot e^{-t[D]} \times \sqrt{\hat{K}([-D])} \times \exp \left( \sum_{m \geq 2} \frac{(-[D])^m}{m} \cdot x_m \right).$$

It is interesting that a pure D($2d$)-brane is always labelled by a set of charges $q^G_m = \delta_{m,0}$, equivalently, $Q_0^G = +1$, $Q_1^G = -1$, $Q_j^G = 0$ $(j \neq 0, 1)$.

Next we compare our result in the sigma model with boundary states associated with the Gepner model. The boundary states are labelled by a set of integers $L_j$ $(j = 1, 2, \cdots, N)$, $M$ and $S$ as $|\{L\}; M; S\rangle$. The integer $M$ is transformed under the $\mathbb{Z}_N$ monodromy transformation at the Gepner point as $M \rightarrow M + 2$. On the other hand, the charge $q^G_m$ changes in such a transformation as $q^G_m \rightarrow q^G_{m+1}$. So the integral number $M$ turns out to be related with the charge $q^G_m$. For a trivial state $|\{L = 0\}; M = 0; S = 0\rangle$, we put an ansatz that an associated bundle is trivial one $\mathcal{O}$. It leads to a condition $q^G_m = \delta_{m,0}$. Then by performing a $\mathbb{Z}_N$ transformation, we can construct a central charge $Z$ for a boundary state $|\{L = 0\}; M = 2\ell; S = 0\rangle$

$$Z(\{q^G_m\}) = \sum_{m=0}^{N-1} q^G_m \int_M \text{ch}(S^*_m) \sqrt{A(TM)} \cdot e^{-t[D]}$$

$$\times \sqrt{\hat{K}([-D])} \times \exp \left( \sum_{m \geq 2} \frac{(-[D])^m}{m} \cdot x_m \right),$$

$q^G_m = \delta_{m,\ell}$. 

30
For a boundary state $|\{L\}; M = 2\ell; S\rangle (S = 0, 2)$, we obtain an associated central charge in the sigma model

$$Z(\{q_m^G\}) = \sum_{m=0}^{N-1} q_m^G \int_M \text{ch}(S_m^*) \sqrt{\hat{A}(TM)} \cdot e^{-t[D]}$$

$$\times \sqrt{\hat{K}(-[D])} \times \exp \left( \sum_{m \geq 2} (-[D])^m \cdot x_m \right),$$

$$q_m^G = \frac{1}{N} \cdot (-1)^{\frac{S}{2}} \sum_{n=0}^{N-1} q_n^{G,0} \sum_{k=1}^{N-1} \alpha_k^{(n-m)} \times \prod_{j=1}^{N} \frac{\sin \frac{\pi k (L_j + 1)}{N}}{\sin \frac{\pi k}{N}},$$

$$q_n^{G,0} = \delta_{n,\ell}.$$ 

Here we used the formula Eq.(11) of the charges when we switched on the $\{L_j\}$. We can construct the central charge associated with the boundary state of the Gepner model.

Now let us return to the formula Eq.(23) and study quantum effects in the $Z$. They are contained (non-)perturbatively in the following terms

$$\sqrt{\hat{A}([D])} \cdot R([D]) \cdot \sqrt{\hat{K}(-[D])} \cdot e^{-t[D]} \times \exp \left( \sum_{n \geq 2} (-[D])^n \cdot x_n \right).$$

(27)

The term $\sqrt{\hat{A}}$ describes topological features of the associated bundle over the curved space. On the other hand, the $\hat{K}$ is expected to have its origin in the perturbative quantum corrections. The $\hat{A}$ and the $\hat{K}$ are combined into a function

$$\sqrt{\hat{A}(\rho)} \cdot \sqrt{\hat{K}(-\rho)} = \frac{\Gamma\left(1 + \frac{\rho}{2\pi i}\right)^N}{\Gamma\left(1 + \frac{N\rho}{2\pi i}\right)}.$$ 

Also this relation can be generalized to other Calabi-Yau cases, for an example, a Calabi-Yau $d$-fold realized as complete intersections $M$ of $\ell$ hypersurfaces $\{p_j = 0\}$ in products of $k$ projective spaces $M$

$$M := \left( \mathbb{P}^{n_1}(w_1^{(1)}, \ldots, w_{n_1+1}^{(1)} \parallel d_1^{(1)} \cdots d_\ell^{(1)}) \right. \left. \vdots \ldots \vdots \right.$$ 

$$\left. \left( \mathbb{P}^{n_k}(w_1^{(k)}, \ldots, w_{n_k+1}^{(k)} \parallel d_1^{(k)} \cdots d_\ell^{(k)}) \right) \right).$$

The $d_j^{(i)}$ are degrees of the coordinates of $\mathbb{P}^{n_i}(w_1^{(i)}, \ldots, w_{n_i+1}^{(i)})$ in the $j$-th polynomial $p_j$ $(i = 1, 2, \cdots, k; j = 1, 2, \cdots, \ell)$. When we introduce a function $a(v)$ with variables $v_i$
\( (i = 1, 2, \cdots, k) \)

\[
\ell \prod_{i=1}^k \Gamma \left( 1 + \sum_{i=1}^k d_j^{(i)} v_i \right)
\]

\[
a(v) := \frac{\ell \prod_{i=1}^k \Gamma \left( 1 + \sum_{i=1}^k d_j^{(i)} v_i \right)}{\prod_{i=1}^k \prod_{j' = 1}^{n_i + 1} \Gamma \left( 1 + w_{j'}^{(i)} v_i \right)},
\]

a generating function of an \( \hat{A} \) of the M is represented as

\[
\hat{A}(\lambda) := \prod_{i=1}^k \prod_{j' = 1}^{n_i + 1} \left( \frac{\lambda w_{j'}^{(i)} \rho_i}{\sinh \frac{\lambda w_{j'}^{(i)} \rho_i}{2}} \right) \times \ell \prod_{j=1}^\ell \left( \frac{\sinh \frac{\lambda(d_j \rho)}{2}}{\frac{\lambda(d_j \rho)}{2}} \right)
\]

\[
\sum_{m=1}^\infty \frac{(-1)^m B_m \cdot X_{2m}}{(2m)!} \lambda^{2m},
\]

\[d_j \cdot \rho := \sum_{i=1}^k d_j^{(i)} \rho_i,
\]

\[
X_n := \sum_{i=1}^k \sum_{j' = 1}^{n_i + 1} \left( w_{j'}^{(i)} \rho_i \right)^n - \sum_{j=1}^{\ell} \left( \sum_{i=1}^k d_j^{(i)} \rho_i \right)^n.
\]

\[d = -\ell + \sum_{i=1}^k n_i, \text{ (dimension)}.\]

On the other hand, an associated \( \hat{K} \) is defined by using the \( X_n \)s as

\[
\hat{K}(\lambda) := \exp \left[ +2 \sum_{m=1}^\infty \frac{\zeta(2m + 1)}{2m + 1} \cdot \left( \frac{\lambda}{2\pi i} \right)^{2m+1} \cdot X_{2m+1} \right].
\]

The \( \hat{A}(\lambda) \) and the \( \hat{K}(\lambda) \) satisfy a relation

\[
\sqrt{\hat{A}(+\lambda)} \cdot \sqrt{\hat{K}(-\lambda)} = \frac{\prod_{i=1}^k \prod_{j' = 1}^{n_i + 1} \Gamma \left( 1 + \frac{\lambda w_{j'}^{(i)} \rho_i}{2\pi i} \right)}{\prod_{j=1}^\ell \Gamma \left( 1 + \sum_{i=1}^k \frac{\lambda d_j^{(i)} \rho_i}{2\pi i} \right)}.
\]

In our previous paper [11], we propose a conjecture that the \( \sqrt{\hat{K}} \) could be interpreted as loop corrections of the sigma model. (For the quintic case, this statement is confirmed.) If it is true for generic cases, this term might contain effects of perturbative corrections. It seems interesting that these two terms are combined into a combination of Euler’s gamma functions.
The remaining very important term is the exp \( \left( \sum_{n \geq 2} (-[D])^n \cdot x_n \right) \) in Eq.(27). In order to study this function, we consider a set of functions \( \{ \hat{\omega}_\ell \} \) defined in an expansion

\[
e^{tv} \exp \left( \sum_{n \geq 2} v^n x_n \right) = \sum_{\ell=0} v^\ell \hat{\omega}_\ell.
\]

We collect these functions \( \hat{\omega}_\ell \) \((\ell = 0, 1, \cdots, N - 2)\) into a vector \( \tilde{V}_0 \)

\[
\tilde{V}_0 = (\hat{\omega}_0 \quad \hat{\omega}_1 \quad \cdots \quad \hat{\omega}_{N-2} ).
\]

Here the \( \hat{\omega}_\ell \) can be interpreted as a paring of a B-cycle \( \tilde{\gamma}_\ell \) and an A-model operator \( O^{(0)} \) associated with a 0 form on the \( M \)

\[
\hat{\omega}_\ell = \langle \tilde{\gamma}_\ell | O^{(0)} \rangle.
\]

In our Calabi-Yau case, there are \((d + 1)\) independent A-model operators \( O^{(m)} \) \((m = 0, 1, \cdots, d)\) and we can define an associated vector \( \tilde{V}_m \) for each \( O^{(m)} \)

\[
\tilde{V}_m = (\langle \tilde{\gamma}_0 | O^{(m)} \rangle \quad \langle \tilde{\gamma}_1 | O^{(m)} \rangle \quad \cdots \quad \langle \tilde{\gamma}_d | O^{(m)} \rangle ), \quad (m = 0, 1, \cdots, d).
\]

Then we can introduce a matrix \( \tilde{\Pi} \) by collecting these \((d + 1)\) vectors

\[
\tilde{\Pi} = \begin{pmatrix}
\tilde{V}_0 \\
\tilde{V}_1 \\
\vdots \\
\tilde{V}_d
\end{pmatrix}, \quad \tilde{\Pi}_{m,\ell} = \langle \tilde{\gamma}_\ell | O^{(m)} \rangle.
\]

This matrix \( \tilde{\Pi} \) satisfies a first order differential equation

\[
\partial_t \tilde{\Pi} = K \cdot \tilde{\Pi}, \tag{28}
\]

\[
K := \begin{pmatrix} 
0 & K_0 \\
0 & K_1 \\
& \ddots & \ddots \\
& & 0 & K_{d-1} \\
& & & 0
\end{pmatrix}.
\]

The \( K_\ell \)s are fusion couplings of A-model operators defined as

\[
O^{(1)} O^{(\ell-1)} = K_{\ell-1} O^{(\ell)} \quad (1 \leq \ell \leq d),
\]

\[
O^{(1)} O^{(d)} = 0,
\]

\[
K_m = \partial_t \frac{1}{K_{m-1}} \partial_t \frac{1}{K_{m-2}} \partial_t \frac{1}{K_1} \partial_t \frac{1}{K_0} \partial_t \hat{\omega}_{m+1} \quad (1 \leq m \leq d - 1),
\]

\[
K_0 = 1. \tag{30}
\]
When we put an initial data at a specific moduli point \( t = t_i \), the \( \tilde{\Pi} \) at a point \( t = t_f \) is given by integrating the Eq.(28) formally

\[
\tilde{\Pi}(t_f) = \text{Pexp} \left( \int_{t_i}^{t_f} ds \, K(s) \right) \tilde{\Pi}(t_i).
\]

That is to say, the matrix \( K \) plays a role of a connection on the moduli space. In other words, it induces a parallel transformation on the \( t \)-space. When we impose a boundary condition \( \tilde{\Pi}(t) \sim e^{t \mathbf{R}} \) on the \( \tilde{\Pi} \) at \( t = t_\infty \) (\( \text{Im} t_\infty = +\infty \)), we obtain the \( \tilde{\Pi} \) at a generic point

\[
\tilde{\Pi}(t) = e^{t \mathbf{R}} \cdot \text{Pexp} \left( \int_{t_\infty}^{t} ds \, e^{-s \mathbf{R}} \tilde{K}(s) e^{s \mathbf{R}} \right) I,
\]

\[
\mathbf{R} := \begin{pmatrix} 0 & 1 & & \vdots & & \vdots & & 0 \\ & 0 & 1 & & \vdots & \vdots & \vdots & \vdots \\ & & \ddots & \ddots & \ddots & \ddots & \vdots \\ & & & 0 & 1 & & \vdots \\ & & & & & 0 & \\ \end{pmatrix},
\]

\[
\tilde{K} := K - \mathbf{R}.
\]

Thus the \( \exp \left( \sum_{n \geq 2} (-[D])^n \cdot x_n \right) \) in Eq.(27) is evaluated by substituting \( v = -[D] \) in the following formula

\[
e^{t v} \exp \left( \sum_{n \geq 2} v^n x_n \right) = \sum_{\ell=0}^{t} v^{\ell} \tilde{\omega}_{\ell} = \left( \begin{array}{c} 1 \\ t \\ \frac{t^2}{2} \\ \vdots \\ \frac{t^d}{d!} \end{array} \right) \text{Pexp} \left( \int_{t_\infty}^{t} ds \, e^{-s \mathbf{R}} \tilde{K}(s) e^{s \mathbf{R}} \right) \begin{pmatrix} 1 \\ v \\ v^2 \\ \vdots \\ v^d \end{pmatrix}.
\]

As a conclusion, we can interpret the \( \exp \left( \sum_{n \geq 2} (-[D])^n \cdot x_n \right) \) in Eq.(27) as an effect of a parallel transport through a path from the large radius point \( t_\infty \) to a finite \( t \) in the moduli space. Then the connection in the moduli space is the \( K \) that are defined by fusion couplings of A-model operators.

### 8 Conclusions and Discussions

In this article, we develop a method to construct the central charge in the topological sigma model in the open string channel. First we analyzed quintic by using the associated prepotential \( F \) and studied its monodromy properties. For this quintic case, one can easily construct...
associated canonical bases of period integrals because there is the prepotential for the model. But one cannot expect to find analogs of prepotentials for other $d$-fold cases ($d > 3$). But we find that the essential part we learned in the analysis seems to be a factorizable property of the matrix $I$ by a (triangular) matrix $S$ and a matrix $\Sigma$. Under this consideration, we investigate the $N = 7$ ($d = 5$) case concretely in section 4.

The basis $\Omega$ we pick here is a kind of symplectic one with an intersection matrix $\Sigma$. In order to obtain a set of canonical basis $\Pi$, we determine the matrix $S$ which connects the $\Pi$ and $\tilde{\Pi}$. It allows us to construct the matrix $N$ that transforms the $\Omega$ to the $\Pi$. Some topological invariants appear in the entries of the $N$. Some parts of them are characterized by an A-roof genus of the $M$. But we cannot give geometrical interpretations to other entries in $N$ explicitly.

Together with the data of the Gepner model, we calculate the charge vectors of the D-branes and the central charge. When a set of numbers $\{L_j\}$ is specified, a boundary state is constructed in the Gepner model. By calculating an associated charge $Q_G$ in the Gepner basis, we construct a formula of the $Z$ labelled by the set $\{L_j\}$. It is related to the boundary state $|\{L\}; M; S\rangle$.

In section 6, we investigate cycles associated with the sets of periods. At the orbifold point $\psi = 0$, they coincide with the susy cycles analyzed by Becker et. al. There we also analyze intersection forms among them. In section 7, we reexpress our result of the $Z$ applicable in the large volume region of the $M$. We find that the $Z$ contains terms analogous to the Mukai vectors. They are interpreted as a product of Chern characters of bundles (sheaves) $S_m$s and a square root of the Todd class of the $M$. The set of $S_m$s is constructed as a dual basis of tautological line bundles of the ambient space $CP^{N-1}$ by a restriction on $M$. In addition, there appear terms that encode perturbative and non-perturbative quantum corrections. The non-perturbative part is induced by a parallel transport through a path from the large radius point $t_\infty$ to a finite $t$ in the moduli space. It turns out that the fusion coupling matrix $K$ of A-model operators plays a role of a connection on the moduli space.

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A Examples of $\sqrt{A(\rho)}$

We write down several examples of the $\sqrt{A(\rho)}$ for the $d$-fold $M$ concretely. A function $\hat{A}(\rho)$ is defined as

$$\hat{A}(\rho) = \left( \frac{\rho}{\sinh \frac{\rho}{2}} \right)^N \cdot \left( \frac{\sinh \frac{N\rho}{2}}{\frac{N\rho}{2}} \right)$$

$$= \exp \left( \sum_{m=1}^{(N-1)/2} (\rho 2m \cdot \frac{B_m}{(2m)!} \cdot \frac{X_{2m}}{2m}) \right).$$

The coefficients $X_\ell = N - N^\ell$ are represented as some combinations of Chern classes $c_\ell$s of the $M$ with $c_1 = 0$

$$c(\rho) = 1 + \sum_{\ell \geq 1} \rho^\ell \frac{c_\ell}{N} = \exp \left( \sum_{\ell \geq 1} (-1)^{\ell-1} \rho^\ell \cdot \frac{X_\ell}{\ell} \right),$$

$$X_1 = 0, \ X_2 = \frac{-2 c_2}{N}, \ X_3 = \frac{3 c_3}{N},$$

$$X_4 = \frac{2 c_2^2}{N^2} - \frac{4 c_4}{N}, \ X_5 = \frac{-5 c_2 c_3}{N^2} + \frac{5 c_5}{N},$$

$$X_6 = \frac{-2 c_2^3}{N^3} + \frac{3 c_3^2}{N^2} + \frac{6 c_2 c_4}{N^2} - \frac{6 c_6}{N},$$

$$X_7 = \frac{7 c_2 c_3}{N^3} - \frac{7 c_3 c_4}{N^2} - \frac{7 c_2 c_5}{N^2} + \frac{7 c_7}{N},$$

$$X_8 = \frac{2 c_2^4}{N^4} - \frac{8 c_2 c_3^2}{N^3} - \frac{8 c_2^2 c_4}{N^3} + \frac{4 c_4^2}{N^2} + \frac{8 c_3 c_5}{N^2} + \frac{8 c_2 c_6}{N^2} - \frac{8 c_8}{N}.$$

In the central charge $Z$, there appears a function $\sqrt{A(\rho)}$. It is expanded in terms of the $\rho$ around $\rho = 0$

$$\sqrt{A(\rho)} = 1 + \sum_{\ell=1}^{d} \beta_\ell \rho^\ell, \ \beta_{2m+1} \equiv 0,$$

$$\beta_2 = \frac{c_2}{24 N}, \ \beta_4 = \frac{7 c_2^2}{5760 N^2} - \frac{c_4}{1440 N},$$

$$\beta_6 = \frac{31 c_2^2 c_3}{967680 N^3} - \frac{c_4^2}{120960 N^2} - \frac{11 c_2 c_4}{241920 N^2} + \frac{c_6}{60480 N},$$

$$\beta_8 = \frac{127 c_2^4}{15482880 N^4} - \frac{11 c_2 c_3^2}{14515200 N^3} - \frac{113 c_2^2 c_4}{58060800 N^3} - \frac{c_8}{29030400 N^2} + \frac{2419200 N^2}{c_2 c_6} + \frac{c_8 c_5}{2419200 N^2} - \frac{c_8}{2419200 N}.$$
We will summarize several examples of $\sqrt{A(\rho)}$ for lower dimensional cases

\[
\hat{A}^{1/2}(\rho; N = 3) = 1, \quad \hat{A}^{1/2}(\rho; N = 4) = 1 + \frac{\rho^2}{4},
\]
\[
\hat{A}^{1/2}(\rho; N = 5) = 1 + \frac{5\rho^2}{12}, \quad \hat{A}^{1/2}(\rho; N = 6) = 1 + \frac{5\rho^2}{8} - \frac{11\rho^4}{384},
\]
\[
\hat{A}^{1/2}(\rho; N = 7) = 1 + \frac{7\rho^2}{8} - \frac{21\rho^4}{640},
\]
\[
\hat{A}^{1/2}(\rho; N = 8) = 1 + \frac{7\rho^2}{6} - \frac{7\rho^4}{240} + \frac{229\rho^6}{1440},
\]
\[
\hat{A}^{1/2}(\rho; N = 9) = 1 + \frac{3\rho^2}{2} - \frac{\rho^4}{80} + \frac{3233\rho^6}{10080},
\]
\[
\hat{A}^{1/2}(\rho; N = 10) = 1 + \frac{15\rho^2}{8} + \frac{3\rho^4}{128} + \frac{38861\rho^6}{64512} - \frac{3542981\rho^8}{3440640}.
\]
We summarize monodromy matrices for the quintic case in the appendix for bases \( \varpi, \Pi, \hat{\Pi}, \hat{\hat{\Pi}} \) and \( \Omega \)

\[
\Pi = N \cdot \Omega, \quad \Pi = m \cdot \varpi, \quad \varpi = P \cdot \Omega,
\]

\[
\Pi = S \cdot \hat{\Pi}, \quad \hat{\Pi} = U \hat{\hat{\Pi}}, \quad U_{m,n} = \binom{m}{n} \cdot (-1)^{m-n},
\]

\[
U = \begin{pmatrix}
1 & 0 & 0 & 0 \\
-1 & 1 & 0 & 0 \\
1 & -2 & 1 & 0 \\
-1 & 3 & -3 & 1
\end{pmatrix}, \quad m = \begin{pmatrix}
-\frac{2}{5} & \frac{2}{5} & \frac{1}{5} & -\frac{1}{5} \\
\frac{21}{5} & -\frac{1}{5} & -\frac{3}{5} & \frac{8}{5} \\
1 & -1 & 0 & 0
\end{pmatrix},
\]

\[
P \cdot V = \begin{pmatrix}
1 & 0 & 0 & 0 \\
1 & \frac{25}{12} & 0 & 5 \\
-\frac{23}{12} & -\frac{15}{4} & -5 & -15 \\
-\frac{23}{12} & -\frac{55}{12} & -5 & -5
\end{pmatrix}, \quad P = \begin{pmatrix}
0 & 0 & 0 & 1 \\
-5 & 0 & 0 & 1 \\
15 & -5 & 0 & 1 \\
5 & -5 & 5 & -4
\end{pmatrix},
\]

\[
N = m \cdot P \cdot V = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
\frac{25}{12} & -\frac{11}{12} & -5 & 0 \\
0 & -\frac{25}{12} & 0 & -5
\end{pmatrix}, \quad S = m \cdot P = \begin{pmatrix}
0 & 0 & 0 & 1 \\
0 & 0 & -1 & 1 \\
0 & -5 & 8 & -3 \\
5 & 0 & 0 & 0
\end{pmatrix}.
\]

We write down monodromy matrices associated with singular points \( \psi = 0, 1, \infty \) for various bases:

1. monodromy matrices associated with the \( \psi = 0 \)

\[
A_{\varpi} = \begin{pmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
-1 & -1 & -1 & -1 \\
1 & 0 & 0 & 0
\end{pmatrix}, \quad A_{\Pi} = \begin{pmatrix}
1 & 0 & 0 & -1 \\
-1 & 1 & 0 & 1 \\
3 & 5 & 1 & -3 \\
5 & -8 & -1 & -4
\end{pmatrix},
\]

\[
A_{\hat{\Pi}} = \begin{pmatrix}
-4 & 1 & 0 & 0 \\
-10 & 1 & 1 & 0 \\
-10 & 0 & 1 & 1 \\
-5 & 0 & 0 & 1
\end{pmatrix}, \quad A_{\Omega} = \begin{pmatrix}
1 & \frac{25}{12} & 0 & 5 \\
-1 & -\frac{13}{12} & 0 & -5 \\
\frac{1}{2} & \frac{131}{144} & 1 & \frac{55}{12} \\
-\frac{1}{6} & -\frac{103}{144} & -1 & -\frac{23}{12}
\end{pmatrix},
\]

\[
A_{\hat{\hat{\Pi}}} = \begin{pmatrix}
-5 & 1 & 0 & 0 \\
-15 & 0 & 1 & 0 \\
-35 & 0 & 0 & 1 \\
-71 & 4 & -6 & 4
\end{pmatrix}.
\]
2. monodromy matrices associated with the $\psi = 1$

$$P_\psi = \begin{pmatrix} 2 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ -4 & 4 & 1 & 0 \\ -4 & 4 & 0 & 1 \end{pmatrix}, \quad P_{\Pi} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$P_{\hat{\Pi}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 5 & 1 & 0 & 0 \\ 5 & 0 & 1 & 0 \\ 5 & 0 & 0 & 1 \end{pmatrix}, \quad P_\Omega = \begin{pmatrix} 1 & -\frac{25}{12} & 0 & -5 \\ 0 & 1 & 0 & 0 \\ 0 & -\frac{125}{144} & 1 & -\frac{25}{12} \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$P_{\hat{\hat{\Pi}}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 5 & 1 & 0 & 0 \\ 15 & 0 & 1 & 0 \\ 35 & 0 & 0 & 1 \end{pmatrix}.$$

3. monodromy matrices associated with the $\psi = \infty (t \to t + 1)$

$$T_\psi = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 0 & 0 & -1 \\ -4 & 1 & 0 & 4 \\ -5 & -1 & -1 & 3 \end{pmatrix}, \quad T_{\Pi} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ -8 & -5 & 1 & 0 \\ -5 & 3 & 1 & 1 \end{pmatrix},$$

$$T_{\hat{\Pi}} = \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad T_\Omega = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ \frac{1}{2} & 1 & 1 & 0 \\ \frac{1}{6} & \frac{1}{2} & 1 & 1 \end{pmatrix},$$

$$T_{\hat{\hat{\Pi}}} = \begin{pmatrix} 4 & -6 & 4 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$
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