Role of strongly magnetized crusts in torsional shear modes of magnetars

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Abstract

We study the influence of magnetised crusts on torsional shear mode oscillations of magnetars. In this context, we employ magnetised crusts whose ground state properties are affected by Landau quantisation of electrons. The shear modulus of magnetised crusts is enhanced in strong magnetic fields $\geq 10^{17}$ G. Though we do not find any appreciable change in frequencies of fundamental torsional shear modes, frequencies of first overtones are significantly affected in strong magnetic fields. Furthermore, frequencies of torsional shear modes calculated with magnetised crusts are in good agreement with frequencies of observed quasi-periodic oscillations.

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I. INTRODUCTION

Soft gamma repeaters (SGRs) are characterised by their sporadic and short bursts of soft gamma rays. Luminosities in these bursts could reach as high as $\sim 10^{41}$ ergs s$^{-1}$. There are about 5 SGRs known observationally. Evidences of stronger emissions of gamma rays from SGRs were observed in several cases. These events are known as giant flares in which luminosities are $\sim 10^{44} - 10^{46}$ ergs s$^{-1}$. So far three cases of giant flares were reported and those are SGR 0526-66, SGR 1900+14 and SGR 1806-20 [1–4]. In giant flares, the early part of the spectrum was dominated by hard flash of shorter duration followed by a softer decaying tail of a few hundreds of seconds.

SGRs are very good candidates for magnetars which are neutron stars with very high surface magnetic fields $\sim 10^{15}$ G [5, 6]. Giant flares might be caused by the evolving magnetic field and its stress on the crust of magnetars. It was argued that starquakes associated with giant flares could excite Global Seismic Oscillations (GSOs) [6]. Torsional shear modes of magnetars with lower excitation energies would be easily excited. In this case, oscillations are restored by the Coulomb forces of crustal ions. Furthermore, the shear modes have longer damping times. Quasi-periodic oscillations (QPOs) were found in the decaying tail of giant flares from the timing analysis of data [2–4]. These findings implied that QPOs might be torsional shear mode oscillations of magnetar crusts [6]. Frequencies of the observed QPOs ranged from 18 Hz to 1800 Hz.

It was noted from earlier theoretical models of QPOs that the observed frequencies in particular higher frequencies could be explained reasonably well using torsional shear oscillations of magnetar crusts [3, 6, 10]. On the other hand, lower frequencies of observed QPOs might be connected to Alfvén modes of the fluid core. This makes the study of the oscillations of magnetar crusts more difficult. There were attempts to explain frequencies of QPOs using Alfvén oscillations of the fluid core without considering a crust [11–13]. The coupling of Alfvén oscillations of fluid core with the shear mode oscillations in the solid crust due to strong magnetic fields in magnetars was already studied by several groups [14–18]. It was argued that torsional shear modes of the crust might appear in GSOs and explain frequencies of observed QPOs for not very strong magnetic fields despite all these complex problems [19].

Nuclear physics of crusts plays an important role on torsional shear modes of magnetar
crusts. In particular, the effects of the nuclear symmetry energy on the shear mode frequencies were investigated recently [10]. It may be worth noting here that torsional shear mode frequencies are sensitive to the shear modulus of neutron star crusts. Furthermore, the shear modulus strongly depends on the composition of neutron star crusts. Earlier studies of torsional shear mode oscillations exploited only non-magnetic crusts. Magnetic fields in magnetars may influence the ground state properties of neutron star crusts. Recently, we have investigated the influence of Landau quantisation of electrons on the compositions and equations of state (EoS) of outer and inner crusts and obtained appreciable changes in those properties [20, 21]. This, in turn, might influence the shear modulus of crusts and torsional shear mode oscillations in magnetars. This motivates us to study torsional shear mode oscillations of magnetars using magnetised crusts.

We organise the paper in the following way. We describe models for torsional shear mode oscillations, shear modulus and compositions and EoS of magnetised crusts in Sec. II. Results of this calculation are discussed in Sec. III. Section IV gives the summary and conclusions.

II. FORMALISM

Earlier calculations of torsional shear mode oscillations were performed in Newtonian gravity [6, 7, 22, 23] as well as in general relativity [9, 11, 19, 24, 25] with and without magnetic fields. In many of these calculations, the magnetised crust was decoupled from the fluid core.

As we are interested in the effects of magnetised crusts on torsional shear mode frequencies, we consider a free slip between the crust and core. Here we calculate torsional shear mode frequencies following the model of Refs. [9, 25]. In this case, we study torsional shear mode oscillations of spherical and non-rotating relativistic stars in the presence of a dipole magnetic field. We neglect the deformation in the equilibrium star due to magnetic fields \( \sim 10^{16} \text{G} \) because its effects would be smaller than the direct influence of magnetic fields on shear mode frequencies. Therefore, we assume the magnetised star to be spherically symmetric. The metric used to determine equilibrium stellar models has the form,

\[
ds^2 = -e^{2\Phi} dt^2 + e^{2\Lambda} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) .
\]  

(1)
The equilibrium models are obtained by solving the Tolman-Oppenheimer-Volkoff (TOV) equation with a perfect fluid EoS. The stress-energy tensor of a magnetised relativistic star in equilibrium is written as

\[ T^{\mu\nu} = T_{M}^{\mu\nu} + T_{EM}^{\mu\nu}, \] (2)

where the stress-energy tensor of perfect fluid is

\[ T_{M}^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu}, \] (3)

and that of the electromagnetic field is

\[ T_{EM}^{\mu\nu} = \frac{1}{4\pi} \left( B^2 u^\mu u^\nu + \frac{1}{2} g^{\mu\nu} B^2 - B^\mu B^\nu \right), \] (4)

where the four velocity of the fluid element is \( u^\mu = (e^{-\Phi}, 0, 0, 0) \) and \( u^\mu u_\mu = -1 \).

Here we consider an axisymmetric poloidal magnetic field generated by four current \( J_\mu = (0, 0, 0, J_\Phi) \). After expanding the four-potential into vector spherical harmonics as \( A_\mu = a_{\ell m}(r)\sin\theta \partial_\theta P_{\ell m}(\cos\theta) \) and similarly for \( J_\mu \), it follows from Maxwell’s equations for a dipole magnetic field i.e. \( \ell_m = 1 \)

\[ e^{-2\Lambda} \frac{d^2 a_1}{dr^2} + (\Phi' - \Lambda') e^{-2\Lambda} \frac{da_1}{dr} - \frac{2a_1}{r^2} = -4\pi j_1, \] (5)

where prime denotes derivative with respect to \( r \). The exterior solution for \( a_1 \) is given by

\[ a_1^{(ex)} = -\frac{3\mu_b r^2}{8M^3} \left[ \ln \left( 1 - \frac{2M}{r} \right) + \frac{2M}{r} + \frac{2M^2}{r^2} \right], \] (6)

where \( \mu_b \) is the magnetic dipole moment for an observer at infinity and \( M \) is the mass of the star. The interior solution can be obtained by integrating Eq.(5) assuming a current distribution \( j_1 \) as was done by Ref.[9]. The components of the magnetic field are

\[ B_r = \frac{2e^{\Lambda} \cos\theta}{r^2} a_1 \]
\[ B_\theta = -e^{\Lambda} \sin\theta \frac{da_1}{dr}. \] (7)

The perturbed equations describing shear mode oscillations are obtained by linearising the equations of motion of the fluid and the magnetic induction equation [9, 25]. Torsional shear modes are incompressible and do not result in any appreciable density perturbation in equilibrium stars. Consequently, one may adopt the relativistic Cowling approximation and neglect metric perturbations \( \delta g_{\mu\nu} = 0 \) [26]. Torsional shear modes are the results of material
velocity oscillations. These modes are odd parity (or axial type) modes. We consider axial type perturbation in the four velocity and the relevant perturbed matter quantity is the \( \phi \)-component of the perturbed four velocity \( \partial u^\phi \)

\[
\partial u^\phi = e^{-\Phi} \partial_t \mathcal{Y}(t, r) \frac{1}{\sin \theta} \partial_\theta P_l(\cos \theta) ,
\]

where \( \partial_t \) and \( \partial_\theta \) correspond to partial derivatives with respect to time and \( \theta \), respectively, \( P_l(\cos \theta) \) is the Legendre polynomial of order \( l \) and \( \mathcal{Y}(t, r) \) is the angular displacement of the matter. It is to be noted that the radial and angular variations of azimuthal displacement of stellar matter lead to shears of the crystal lattice in neutron star crusts which are described by the shear tensor \( S_{\mu \nu} \)

Further, the shear stress tensor is given by \( T_{\mu \nu} = -2\mu S_{\mu \nu} \), where \( \mu \) is the isotropic shear modulus. The linearised equations of motion includes the contribution of \( \delta T_{\mu \nu} \)

Assuming a harmonic time dependence for \( \mathcal{Y}(t, r) = e^{i\omega t} \mathcal{Y}(r) \) and neglecting \( l \pm 2 \) terms, one obtains the eigenvalue equation for the mode frequency

\[
\left[ \mu + (1 + 2\lambda_1) \frac{a_1^2}{2\pi^2 r^2} \right] \mathcal{Y}'' + \left\{ \left( \frac{1}{r} + \Phi' - \Lambda' \right) \mu + \mu' + (1 + 2\lambda_1) \frac{a_1}{\pi^2 r^4} \left[ (\Phi' - \Lambda') a_1 + 2a_1' \right] \right\} \mathcal{Y}'' + \left\{ \left[ (\epsilon + p + (1 + 2\lambda_1) \frac{a_1^2}{\pi^2 r^4} \right] e^{2\lambda} - \frac{2\lambda_1 a_1'^2}{2\pi^2 r^2} \right\} \omega^2 e^{-2\Phi} - (\lambda - 2) \left( \frac{\mu e^{2\lambda}}{r^2} - \frac{\lambda_1 a_1'^2}{2\pi^2 r^4} \right) + \frac{(2 + 5\lambda_1) a_1}{2\pi^2 r^4} \left[ (\Phi' - \Lambda') a_1' + a_1'' \right] \right\} \mathcal{Y} = 0 ,
\]

where \( \lambda = \ell(\ell + 1) \) and \( \lambda_1 = -\ell(\ell + 1)/(2\ell - 1)(2\ell + 3) \). Equation (9) reduces to the non-magnetic case when we put \( a_1 = 0 \). With suitable choice of new variables, Eq.(9) results in a system of first order ordinary differential equations. As torsional shear modes are confined to the crust in our calculation, we impose a zero traction boundary condition at the interface between the core and the crust as well as the zero torque condition at the surface \( r = R_c \). These conditions imply \( \mathcal{Y}'' = 0 \) at surface \( r = R \) of the star and the interface \( r = R_c \) of the crust and core. Finally, we estimate eigenfrequencies by solving two first order differential equations.

The knowledge of the shear modulus of the magnetised crusts is an important input in the eigenvalue equation (Eq.9) for the shear mode calculation. Here we adopt the following expression of the shear modulus at zero temperature

\[
\mu = 0.1194 n_i (Ze)^2 a ,
\]

where \( n_i \) is the number density of the ion, \( Ze \) is the charge number of the ion, and \( a \) is the lattice constant.
where \( a = 3/(4\pi n_i) \), \( Z \) is the atomic number of a nucleus and \( n_i \) is the ion density. This form of the shear modulus was obtained by assuming a bcc lattice and performing directional averages [29]. Further the dependence of the shear modulus on temperature was also investigated with the Monte Carlo sampling technique by Strohmayer et al. [28]. The composition and equation of state of neutron crusts are essential ingredients for the calculation of the shear modulus as it is evident from Eq. (10). In the following paragraphs, we describe the ground state properties in outer and inner crusts in presence of strong magnetic fields.

The outer crust is composed of nuclei immersed in a uniform background of a noninteracting electron gas. Neutrons start coming out of nuclei when the neutron drip point is reached. This is the beginning of the inner crust where nuclei are placed both in free neutrons as well as electrons. Nuclei are arranged in a bcc lattice in neutron star crusts. The Wigner-Seitz approximation is adopted in this case. Each lattice volume is replaced by a spherical cell with one nucleus at its center. The cell contains equal numbers of protons and electrons so that it is charge neutral. Moreover, there is no Coulomb interaction among cells. We obtain an equilibrium nucleus in the outer crust minimising the Gibbs free energy per particle at a fixed pressure varying mass and atomic numbers [30]

\[
g = \frac{E_{\text{tot}} + P}{n_b}, \tag{11}
\]

where \( n_b \) is the baryon density and the energy density \( E_{\text{tot}} = n_N(W_N + W_L) + \varepsilon_e \) includes contributions from the energy of the nucleus \( (W_N) \), lattice energy \( (W_L) \) of the cell involving the finite size effects and free electron gas \( (\varepsilon_e) \) [20]. Similarly, the total pressure is given by sum of the pressure due to the lattice and that of the electron gas. We use experimental nuclear masses from the atomic mass table [31] whenever it is available. Otherwise, the theoretical extrapolation is adopted in this calculation [32].

We describe the ground state properties of matter of the inner crust using the Thomas-Fermi model. The spherical cell that contains neutrons and protons does not define a nucleus. We adopt the procedure of Bonche, Levit and Vautherin to subtract the free neutron gas of the cell and obtain the nucleus [33, 34]. Under this assumption, the thermodynamic potential \( (\Omega_N) \) of the nucleus is obtained as [33, 34]

\[
\Omega_N = \Omega_{NG} - \Omega_G, \tag{12}
\]

where \( \Omega_{NG} \) is the thermodynamical potential for the nucleus plus the free neutron gas and
The thermodynamical potential is defined as

$$\Omega = F - \sum_{q=n,p} \mu_q n_q$$

(13)

where $F$, $\mu_q$ and $n_q$ are the free energy density, baryon chemical potential and number density, respectively. The free energy is given by

$$F(n_q, Y_p) = \int [\mathcal{H} + \varepsilon_c + \varepsilon_e] d\mathbf{r}$$

(14)

where $Y_p$ is the proton fraction. Equation (14) includes contributions from the nuclear energy density ($\mathcal{H}$), Coulomb energy density ($\varepsilon_c$) and energy density of free electron gas ($\varepsilon_e$) [21, 35]. The nuclear energy density is calculated using the Skyrme nucleon-nucleon interaction [36–38]. We obtain the density profiles of neutrons and protons for the nucleus plus neutron gas as well as neutron gas phases by minimising thermodynamical potentials and hence calculate mass and atomic numbers of nuclei of the inner crust [21].

Now we focus on neutron star crusts in strongly quantising magnetic fields. In particular, we showed earlier that the Landau quantisation of electrons strongly influenced ground state properties of neutron star crusts in strong magnetic fields $\sim 10^{17}$ G [20, 21]. Energy and number densities of electrons are affected by the phase space modifications due to Landau quantisation of electrons. We immediately note that the electron energy density in Eqs. (11) and (14) are modified in presence of magnetic fields [20, 21]. Further, we have to take into account the change in the average electron chemical potential due to magnetic fields in the outer and inner crusts. It is to be noted that protons are only influenced by magnetic fields through the charge neutrality condition.

III. RESULTS AND DISCUSSIONS

We investigated the composition and EoS of ground state matter in neutron star crusts in strong magnetic fields [20, 21]. We noted that the electron number density in the outer crust was enhanced compared with the field free case when a few Landau levels were populated for magnetic fields $B > 4.414 \times 10^{16}$ G [20]. It was observed that this enhancement grew stronger when only the zeroth Landau level was populated at a magnetic field strength of $4.414 \times 10^{17}$ G. Consequently, we found modifications in the sequence of equilibrium nuclei which was obtained by minimising the Gibbs free energy per nucleon of Eq. (11). It was
noted that some new nuclei such as $^{88}_{38}$Sr and $^{128}_{46}$Pd appeared and some nuclei such as $^{66}_{28}$Ni and $^{78}_{28}$Ni disappeared in a magnetic field of $B = 4.414 \times 10^{16}$ G [20] when we compared this with the zero field case. The maximum density up to which an equilibrium nucleus would exist, increased as the strength of magnetic field increased. This implied that the neutron drip point was shifted to higher density in presence of a strong magnetic field with respect to the field free case [20]. We performed the calculation of the inner crust using the SkM nucleon-nucleon interaction [21, 35]. In this case too, we calculated the equilibrium nucleus at each density point. Like the outer crust in strong magnetic fields, the electron number density was enhanced due to the electron population in the zero Landau level for magnetic fields $\geq 10^{17}$ G which, in turn, led to a large proton fraction because of charge neutrality. For magnetic fields $> 10^{17}$ G, equilibrium nuclei with larger mass and atomic numbers were found to exist in the crust [21]. Furthermore, the free energy per nucleon of the nuclear system was reduced in magnetic fields compared with the corresponding case without a magnetic field.

We calculate the shear modulus using Eq. (10) and the above mentioned models of magnetised crusts. For this purpose, we have to know the profiles of pressure, energy density and shear modulus in a neutron star. Those profiles are obtained by solving the TOV equation. In this context, we construct an EoS of dense nuclear matter in strong magnetic fields in neutron star core using a relativistic mean field model with the GM1 parameter set as described in Ref. [39–41]. This EoS of dense nuclear matter is matched with the EoS of the crust and used in the TOV equation. Figure 1 displays the shear modulus as a function of mass density for a neutron star of $1.4 M_\odot$. The shear modulus (dahsed line) corresponding to $B_\ast = B/B_c = 10^3$ where $B_c = 4.414 \times 10^{13}$ G, does not show any appreciable change from that of the zero field (dotted line) because of large numbers of Landau levels are populated in this field. As the field strength is increased, less numbers of Landau levels are populated. For $B_\ast = 10^4$ i.e. $4.414 \times 10^{17}$ G, the shear modulus is enhanced due to the population of all electrons in the zeroth Landau level. In all three cases, the shear modulus increases with mass density well before the crust-core interface. The shear modulus and shear speed $v_s = (\mu/\rho)^{1/2}$ are extrapolated to the zero value at the crust-core interface for magnetised as well as non-magnetised crusts. It was noted that the shear modulus was found to decrease smoothly to zero with increasing density when the pasta phase was considered in the crust in absence of magnetic fields [19]. We generate a profile of the shear modulus as a function
of radial distance in a neutron star for calculating frequencies of torsional shear modes.

Now we study the dependence of torsional shear mode frequencies on the compositions and the shear modulus of magnetised crusts. Earlier all calculations were performed using non-magnetic neutron star crusts. Here we exploit models of non-magnetic as well as magnetic crusts which are already described in this section. We consider torsional shear modes of a neutron star with $1.4 \, M_\odot$. Frequencies of fundamental ($n = 0$, $\ell = 2$) torsional shear modes are plotted with magnetic fields in Fig. 2. It is observed that the frequency increases with magnetic field. However, there is no difference between our results with and without magnetised crusts. Figure 3 shows frequencies of torsional shear modes corresponding to $n = 0$ plotted as a function of $\ell$ values for a $1.4 \, M_\odot$ neutron star and magnetic field $B_\ast = 10^3$ i.e. $4.414 \times 10^{16}$ G. Again we calculate frequencies using the non-magnetic and magnetic crusts in this case. The frequency increases with higher $\ell$ values, but we cannot distinguish between the results of non-magnetic and magnetic crusts.

We continue our investigation on frequencies of first overtones ($n = 1$) of torsional shear modes in presence of a magnetic field. Frequencies of first overtones are plotted with $\ell$ values for a neutron star of $1.4 \, M_\odot$ and magnetic field $B_\ast = 10^3$ i.e. $4.414 \times 10^{16}$ G in Fig. 4. In this case, the results (dashed line) obtained with magnetised crusts deviate from those of non-magnetised crusts (solid line). We obtain qualitatively similar results for $B_\ast = 10^4$. It was noted that the ratio of the crust thickness to the radius of a neutron star could be obtained from the frequencies of overtones [9]. Results of Fig. 4 point to the fact that the ratio of the crust thickness to the radius might be influenced by strong magnetic fields.

The dependence of frequencies of the fundamental mode and higher harmonics on neutron star masses is demonstrated in Fig. 5. Here the frequencies corresponding to $n = 0$ and $\ell = 2, 3, 4$ are shown as a function of neutron star masses for a magnetic field $B = 8 \times 10^{14}$ G. For each case, frequencies of torsional shear modes decrease with increasing mass, whereas higher $\ell$ values lead to higher frequencies. Frequencies of observed QPOs might put a strong constraint on the EoS if masses of neutron stars are known accurately.

Finally, we compare the calculated frequencies of torsional shear modes with frequencies of observed QPOs. This comparison is shown in Table 1. Our results are obtained using the magnetised crusts of a $1.2 \, M_\odot$ neutron star and estimated magnetic fields $B = 8 \times 10^{14}$ G corresponding to SGR 1806-20 and $1.4 \, M_\odot$ neutron star and $B = 4 \times 10^{14}$ G for SGR 1900+14 [42–44]. For SGR 1806-20, calculated frequencies 93 Hz corresponding to $n = 0$
and \( \ell = 12 \) and above are in very good agreement with the observed frequencies. However, lower frequencies 18, 26 and 29 Hz can not be explained with our calculation because those are so close that they can not be matched using different harmonics of \( n = 0 \) mode. On the other hand, the overall agreement of calculated frequencies with the observed frequencies of SGR 1900+14 is excellent.

IV. SUMMARY AND CONCLUSIONS

We have estimated frequencies of torsional shear modes of magnetars assuming a dipole magnetic field configuration. Frequencies are computed using our models of magnetised crusts. The shear modulus of magnetised crusts is found to be enhanced in strong magnetic fields \( \sim 4.414 \times 10^{17} \) G because electrons populate the zeroth Landau level. It is observed that frequencies of fundamental \((n = 0, \ell = 2)\) torsional shear modes are not sensitive to this enhancement in the shear modulus in strong magnetic fields. On the other hand, frequencies of first overtones \((n = 1)\) of torsional shear modes in presence of strongly quantising magnetic fields are distinctly different from those of the field free case. Consequently, this might impact the ratio of the crust thickness to the radius of a magnetar. We have compared our results with frequencies of observed QPOs and found good agreement. Observed frequencies could constrain the EoS of magnetised neutron star crusts if masses of neutron stars are known.

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TABLE I: Frequencies of torsional shear modes are recorded here. Frequencies obtained in the calculation using the magnetised crusts [20, 21] are compared with observed frequencies in SGR 1806-20 and SGR 1900+14. Our results are calculated using a 1.2 $M_\odot$ neutron star and magnetic field $8 \times 10^{14}$ G for SGR 1806-20 and a 1.4 $M_\odot$ neutron star and magnetic field $4 \times 10^{14}$ G for SGR 1900+14.

| SGR 1806-20 | SGR 1900+14 |
|--------------|-------------|
| **Observed Frequency (Hz)** | **Calculated Frequency (Hz)** | **n** | **\ell** | **Observed Frequency (Hz)** | **Calculated Frequency (Hz)** | **n** | **\ell** |
| 18           | 15          | 0    | 2    | 28           | 28          | 0    | 4    |
| 26           | 24          | 0    | 3    | 54           | 55          | 0    | 8    |
| 29           | 32          | 0    | 4    | 84           | 82          | 0    | 12   |
| 93           | 93          | 0    | 12   | 155          | 154         | 0    | 23   |
| 150          | 151         | 0    | 20   |               |              |      |      |
| 626          | 626         | 1    | 29   |               |              |      |      |
| 1838         | 1834        | 4    | 2    |               |              |      |      |
FIG. 1. Shear modulus is plotted as a function of mass density for a neutron star of 1.4 $M_\odot$ and different magnetic fields.
Fig. 2. Frequency of fundamental \((n = 0, \ell = 2)\) torsional shear mode for a neutron star of 1.4 \(M_\odot\) is shown as a function of magnetic field \(B_\ast = B/B_c\) where \(B_c = 4.414 \times 10^{13}\) G. Results of our calculations with and without magnetised crusts are shown here.
Fig. 3. Frequency of $n = 0$ torsional shear mode as a function of $\ell$ values with and without magnetic crusts of a 1.4 $M_\odot$ neutron star for magnetic field $B_* = 10^3$. 
Fig. 4 Frequency of first overtone \((n = 1)\) of torsional shear mode of \(\ell\) values with and without magnetic crusts of a 1.4 \(M_\odot\) neutron star for magnetic field \(B_* = 10^3\).
FIG. 5. Frequency of torsional shear mode corresponding to $n = 0$ and $\ell = 2, 3, 4$ as a function of neutron star mass for a magnetic field $B = 8 \times 10^{14}$ G.