Testing Localization in Neutrino Oscillations

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ABSTRACT: The neutrino wave packet localization in short-baseline neutrino oscillation experiments, such as MiniBooNE, is investigated. It is shown that the transition from localization to delocalization may be observed for large neutrino mass splitting of order 1 eV, e.g., in theories with sterile neutrinos.

KEYWORDS: Neutrino Physics, Beyond Standard Model

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1 Introduction

The neutrino oscillations were theoretically introduced more than half a century ago [1, 2] and experimentally confirmed more than ten years ago [3]. However the plane wave approach to the neutrino oscillations leads to some “paradoxical” issues, such as the relevance of the “same energy” and “same momentum” assumptions, etc. All these theoretical issues can be resolved [4] in more generic wave packet approach, which was developed in Refs. [5–8] and many other works. Two important necessary conditions for the neutrino oscillations are the neutrino coherence and localization. The former means that the distance traveled by neutrinos should be smaller than the distance over which the wave packets corresponding to different mass eigenstates separate due to the difference of their group velocities. The latter means that the neutrino wave packet should be localized better than the oscillation length.

In this paper we consider the observability of the transition from neutrino localization to delocalization with the corresponding averaging out of the neutrino oscillation in short-baseline experiments, such as LSND [9] and MiniBooNE [10–12]. We show that it is possible to observe this transition simply by changing the decay pipe length in the case of large neutrino mass squared difference $\Delta m^2 \sim 1 \text{eV}^2$ and mixing $\sin^2 2\theta \sim 0.1$, e.g., in the models with sterile neutrinos. We discuss the neutrino wave packets and their sizes in the next section and investigate neutrino localization in short-baseline experiments in section 3.

2 Neutrino wave packets

Consider a flavor neutrino eigenstate $\nu_{\alpha}$, which was born at the moment $t = 0$ in the point $x = 0$ of a source. The wave packet describing the evolved state at a point $(t, x)$ is [4]

$$|\nu_{\alpha}(t, x)\rangle = \sum_i U_{\alpha i}^\ast \Psi_i(t, x)|\nu_i\rangle,$$

(2.1)

where $U$ is the lepton mixing matrix, and the wave packet of a free propagating neutrino $\nu_i$ of definite mass $m_i$, which has momentum $p_i$, is

$$\Psi_i(t, x) = \int \frac{d^3p}{(2\pi)^{3/2}} f_i^S(p - p_i)e^{ipx -iE_i(p)t},$$

(2.2)
where \( E_i(p) = \sqrt{p^2 + m_i^2} \) with \( p \equiv |p| \), and \( f_i^S(p - p_i) \) is the momentum distribution function, which is sharply peaked at or very close to \( p = p_i \), with the width of the peak (uncertainty of the neutrino momentum) \( \sigma_p \ll p_i \) and the superscript “S” corresponding to the source.

The spatial size \( \sigma_x \) of the wave packet of neutrino \( \nu_i \) created in decay of a particle with the width \( \Gamma \) and the lifetime \( \tau \) is [13]

\[
\sigma_x \sim \sigma_p^{-1} \sim \Gamma^{-1} \sim \tau. \tag{2.3}
\]

If the particle with relativistic \( \gamma \) factor decays in flight, its lifetime and the corresponding size of the neutrino wave packet are dilated, so that \( \sigma_x \sim \gamma \tau \). However if this particle decays in a decay tunnel with length \( L_S \) than \( \sigma_x \sim \min(\gamma \tau, L_S) \), see section 8.3 of Ref. [14]. For the neutrino created in a decay or scattering process occurring in a medium \( \sigma_x \sim \gamma \tau \), where \( \tau_X \) is the average time between two collisions in the medium for a particle \( X \), participating in the neutrino creation process. In the following we consider the accelerator neutrino experiments, in which the medium effect is negligible, such as MiniBooNE with the neutrinos produced in 50 m decay pipe filled with air.

The velocity of spreading of the wave packet can be estimated as

\[
v \sim \frac{\partial^2 E_i}{\partial p_i^2} \sigma_p \sim \frac{m_i^2}{E_i^3} \sigma_p^{-1} \sim 10^{-25} \left( \frac{1 \text{ MeV}}{E} \right)^3 \left( \frac{m_i}{1 \text{ eV}} \right)^2 \left( \frac{1 \text{ m}}{\sigma_x} \right). \tag{2.4}
\]

Hence spreading of the neutrino wave packets can be typically neglected.

### 3 Neutrino localization

The \( \nu_\alpha \to \nu_\beta \) oscillation probability at the distance \( L \) from the source can be written as

\[
P_{\nu_\alpha \to \nu_\beta}(L) = \sum_i |U_{\alpha i}|^2|U_{\beta i}|^2 + 2 \sum_{j>i} \text{Re} \left[ U_{\alpha j}^* U_{\alpha i} U_{\beta j} U_{\beta i} e^{-\frac{2\pi i}{L_{\text{osc}}^i j}} \cos \left( \frac{\Delta m_{ji}^2 \cdot L}{2E} \right) \right], \tag{3.1}
\]

with the oscillation length

\[
L_{\text{osc}}^{ij} = \frac{4\pi p}{\Delta m_{ji}^2}. \tag{3.2}
\]

Denote \( \sigma_L = \min(\sigma_L, \sigma_x, L_{\text{osc}}) \) with the characteristic size \( L_{\text{source}}(\text{detector}) \) of the neutrino source (detector) in the direction of neutrino propagation. If \( \sigma_L \) is relatively large, it characterizes the uncertainty of \( L \) in Eq. (3.1). Hence, for \( \sigma_L \sim L_{\text{osc}} \) the correspondent oscillation term in Eq. (3.1) will be averaged out, which is called delocalization of neutrinos. Sterile neutrinos with small enough \( L_{\text{osc}} \) (large \( \Delta m^2 \)) may provide the opportunity to observe the transition from localized to delocalized neutrinos in the short-baseline accelerator neutrino experiments with large enough neutrino source and detector.

Consider the extension of usual three-neutrino mixing with the addition of one massive (sterile) neutrino. The survival probability in short-baseline experiments in this 3+1 neutrino mixing scheme [15–18] can be written as [19]

\[
P_{\nu_\alpha \to \nu_\alpha}(L) \approx 1 - \frac{1}{2} \sin^2 2\theta_{\alpha \alpha} \left[ 1 - \cos \left( \frac{\Delta m_{41}^2 L}{2E} \right) \right], \tag{3.3}
\]
where \( \sin^2 2\theta_{\alpha\alpha} = 4|U_{\alpha4}|^2(1 - |U_{\alpha4}|^2) \). The exclusion ranges for \( \Delta m_{21}^2 \) versus \( \sin^2 2\theta_{ee} \) and \( \sin^2 2\theta_{\mu\mu} \) are shown in Fig. 1.

Denote by \( L_0 = L_{\text{osc}}^{14} \) the coordinate of the first nontrivial maximum of the oscillation curve. (In general, one can choose one of the first extremums, for which the decoherence effect is small, while the effect of averaging out of the neutrino oscillation due to the delocalization can be spectacular.) The mean survival probability \( \langle P_{\nu_\alpha \rightarrow \nu_\alpha}(L_0) \rangle \), which can be measured at \( L_0 \), can be found by the standard averaging of Eq. (3.3) with the Gaussian distribution (see section 7.6 of Ref. [14])

\[
\phi \left( \frac{L}{E}, L_0, \langle E \rangle \right) = \frac{1}{\sqrt{2\pi\sigma_{L/E}^2}} \exp \left[ -\frac{(L/E - L_0/\langle E \rangle)^2}{2\sigma_{L/E}^2} \right]
\]

with the standard deviation

\[
\sigma_{L/E} = L_0/\langle E \rangle \sqrt{\left( \frac{\sigma_L}{L_0} \right)^2 + \left( \frac{\Delta E}{\langle E \rangle} \right)^2},
\]

where \( \Delta E \) is the energy resolution of a detector. For averaging the cosine in Eq. (3.3) we
Consider a short-baseline neutrino experiment with the neutrinos created in the decays of free mesons or muons flying in the decay pipe, and large neutrino detector centered at $L_0$. Suppose $L_S \leq L_D$, so that $\sigma_L$ can be controlled at the source site simply by changing the decay pipe length. Fig. 2 (left) shows $\langle P_{\nu_e \to \nu_e} (L_0) \rangle$ versus $\sigma_L$ calculated for $\Delta m^2_{41} = 5 \text{eV}^2$, $\sin^2 2\theta_{14} = 0.3$, and the values of neutrino energy $E = 200 \text{MeV}$ (three upper lines) and $150 \text{MeV}$ (three lower lines), for which $L_0 = 99.2$ and $74.4$ m, respectively. The solid, dashed and dotted lines represent $\Delta E = 0, 10$ and $15 \text{MeV}$, respectively. Fig. 2 (right) shows $\langle P_{\nu_\alpha \to \nu_\alpha} (L_0) \rangle$ ($\alpha = e, \mu$) versus $\sigma_L$ for $\Delta m^2_{41} = 1 \text{eV}^2$, $\sin^2 2\theta_{14} = 0.2$, and the neutrino energy $E = 50$ and $30 \text{MeV}$, for which $L_0 = 124.0$ and $74.4$ m, respectively. The solid, dashed and dotted lines represent $\Delta E = 0, 3$ and $5 \text{MeV}$, respectively. Fig. 2 demonstrates the averaging out of the first maximum of neutrino oscillation with increasing of the decay pipe length.

We note that neutrinos with masses of few eV may explain the dark matter in halos around galaxies [23].

In conclusion, the possibility to test the transition from neutrino localization to delocalization in the short-baseline accelerator neutrino experiments is demonstrated.
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