Numerical simulation of cavitation erosion on a NACA0015 hydrofoil based on bubble collapse strength

V Hidalgo\textsuperscript{1,3}, X Luo\textsuperscript{1}, X Escaler\textsuperscript{2}, R Huang\textsuperscript{1} and E Valencia\textsuperscript{3}
\textsuperscript{1} State Key Laboratory of Hydro Science & Engineering, Tsinghua University, Beijing, China
\textsuperscript{2} Centre for Industrial Diagnostics and Fluid Dynamics, UPC, Barcelona, Spain
\textsuperscript{3} Department of Mechanical Engineering, Escuela Politecnica Nacional, Quito, Ecuador
E-mail: luoxw@mail.tsinghua.edu.cn

Abstract. The prediction of erosion under unsteady cavitation is crucial to prevent damage in hydraulic machinery. The present investigation deals with the numerical simulation of erosive partial cavitation around a NACA0015 hydrofoil. The study presents the calculation of the bubble collapse strength, $S_b$, based on the bubble potential energy to identify the surface areas with highest risk of damage. The results are obtained with a numerical scheme assuming homogeneous mixture flow, implicit LES and Zwart cavitation model. The 3D unsteady flow simulation has been solved using OpenFOAM. Python language and OpenFOAM calculator (foamCalcEx) have been used to obtain and represent $S_b$. The obtained results clearly show the instants of erosive bubble collapse and the affected surface areas.

1. Introduction
Cavitation erosion has been a subject of deep study for long time using different techniques [1]. The main methods, which have permitted to understand the mechanisms of the phenomenon and the damage generation, are experimental correlations with noise, vibrations and material properties [2, 3]; analytical methods, such as pit distribution quantification by image processing [4, 5]; and numerical simulations using special turbulence models to reproduce cavitation morphology and dynamic behavior [6]. For the latter, Kubota et al. [7] introduced the idea of homogeneous mixture flow with the bubble two-phase flow (BTF) model. Then Coutier et al. [8] showed that the condensation and vaporization processes are controlled by a barotropic state law and simulated the cavity shedding in a venturi. The research works on horse-shoe vortex structures shed around a twisted hydrofoil [9] and on the transient sheet-cloud cavitating flows [10] also corroborate the validity of the homogeneous mixture flow models.

Regarding the cavitation erosion models, Van et al. [11] revised some models. In those models, the potential energy of the bubble, $E_p$, is described as main erosion factor when a group of bubbles collapse simultaneously in cascade and the gap between the wall and the cloud of bubbles is very thin. However, Dular et al. [4] indicated that erosion is also due to the re-energizing of bubbles close to the walls and the generation of high speed jets, which showed speeds close to 100 m/s. In this context, Li [6] proposed the pressure gradient, $\partial p/\partial t$, as a new erosion index based on $E_p$. Moreover, Bergeles et al. [12] presented the cavitation aggressiveness index (CAI) based on total derivatives of pressure, vapor fraction and the Rayleigh-Plesset equations for unsteady fluids.
and quasi steady flow calculation. However, the CAI equation is not directly correlated with the quantity of energy liberated in the bubble collapse. Consequently, the quantification of this energy is still a challenge.

Bearing in mind these aspects, the present investigation presents a numerical simulation method based on homogeneous mixture flow assumption using implicit LES and the Zwart-Gerber-Belamri cavitation model to predict the cavitation erosion at high Reynolds numbers [13, 14]. The proposed collapse strength, \( S_b \), is based on the erosion index proposed by Li [6]. Additionally, the pressure implicit with splitting of operators (PISO) algorithm [13] is used to solve the Navier-Stokes equations to simulate the behavior of unsteady flow on a NACA0015 hydrofoil.

2. Model description

2.1. Homogeneous mixture flows

The two-phase flow is modeled in each control as homogeneous mixture with vapor volume fraction, \( \alpha_i \), as indicated in equations (1) and (2)

\[
\alpha_i = \frac{\forall V_i}{\forall i},
\]

\[
\frac{\partial(\alpha_i \rho V_i)}{\partial t} + \nabla(\alpha_i \rho V_i U) = \dot{m},
\]

where \( \forall V_i \) and \( \forall i \) are the vapor and total volume respectively, \( \rho V \) is the vapor density, \( U \) is the velocity and \( \dot{m} \) is the inter-phase mass transfer rate per unit volume. Moreover, \( \forall i \) is considered as constant value, and \( \dot{m} \) is a source term, which is decided based on the Zwart-Gerber-Belamri cavitation model and it can be calculated using the equation (3).

\[
\dot{m} = \begin{cases} 
\dot{m}^+ = F_V \frac{3 \rho V (1 - \alpha) \rho V}{R_B} \left( \frac{2}{3} \left( \frac{p V - p}{\rho l} \right) \right) & \text{if } p < p_V \\
\dot{m}^- = -F_C \frac{3 \alpha \rho V}{R_B} \left( \frac{2}{3} \left( \frac{p - p V}{\rho l} \right) \right) & \text{if } p > p_V 
\end{cases},
\]

where \( p \) and \( p_V \) are the pressure and vapor saturation pressure respectively, \( F_V = 300 \) and \( F_C = 0.03 \) are the selected calibration constants for vaporization and condensation, \( r_{nuc} = 5.0 \times 10^{-6} \) is the nucleation site volume fraction, \( R_B = 1.0 \times 10^{-6} \) m is the typical bubble size in water [14].

2.2. Bubble collapse strength

The total energy of one bubble in a control volume, \( E_{bi} \), is the sum of potential energy, \( E_{pi} \), and kinetic energy, \( E_{ki} \). However, the potential term is more important than the kinetic term \( (E_{pi} >> E_{ki}) \) because before the collapse the bubble velocity is low. Therefore, the bubble collapse strength, \( S_b \), calculated with equation (4) can be considered as an updated model to evaluate cavitation aggressiveness.

\[
S_b = \sum_{i=1}^{n} \left( \frac{p - p_V}{\partial t} \frac{\partial \alpha_i}{\partial t} + \alpha_i \frac{\partial p}{\partial t} \right),
\]

where \( n \) is the control volume number that presents cavitation in the computational domain. To calculate \( S_b \), an algorithm based on foamCalcEx and Python language has been developed.

Obviously, to produce plastic deformation and erosion on solid parts, the \( S_b \) must be higher than the material threshold resistance, \( S_M \), which quantifies the material resistance to cavitation
erosion and should be evaluated with previous experimental works [2]. Furthermore, the accumulative bubble collapse strength, $S_{bT}$, that causes damage can be calculated within one cycle of cavitation shedding process with equation (5):

$$S_{bT} = \frac{1}{N} \sum_{t_0}^{t_f} S_b \left\{ \begin{array}{ll} S_b \neq 0, & S_b > S_M \\ S_b = 0, & S_b < S_M \end{array} \right. ,$$

(5)

where $N$ is the number of the given results in one cycle, $t_0$ and $t_f$ are the initial and final time.

2.3. Implicit LES formulation

The implicit LES formulation, ILES, is indicated in equation (6) with the filtered operations [13, 14].

$$\frac{\partial (\rho \overline{u_i})}{\partial t} + \frac{\partial (\rho \overline{u_i u_j})}{\partial x_j} = - \frac{\partial p}{\partial x_i} + \frac{\partial (\tau_{ij} - \tau'_{ij})}{\partial x_j}.$$  

(6)

The subgrid stress tensor $\tau'_{ij}$ is a nonlinear term, so it is separated as indicated in equation (7).

$$\tau'_{ij} = \rho (\overline{u_i u_j} - \overline{u_i} \overline{u_j} + \tilde{\tau}'_{ij}),$$

(7)

where, the tensor $\tilde{\tau}'_{ij}$ is modeled by using the truncation error to act as a dissipative action to avoid the explicit subgrid scale model (SGS) of LES [14].

3. Computational domain and boundary conditions

The computational domain and the main boundary conditions are indicated in figure 1. The free stream inlet velocity, $U_\infty$, is 17.3 m/s, the outlet pressure, $p_\infty$, is 302 kPa and the cavitation number, $\sigma$, is 2.01. The chord length of the hydrofoil NACA0015, $c$, is 60 mm with an angle of attack, $\alpha$, of 8\degree. A structured mesh with 878280 hexahedron elements suitable for the ILES conditions of $y^+$ equal to 2.0 has been built. The quality of mesh has been checked by the analysis of the diagonals and Jacobians of the hexahedron [15].

![Computational domain and boundary conditions](image)

Figure 1: Computational domain and boundary conditions.

4. Results and Discussion

Figure 2 shows the simulated unsteady partial cavitation on the hydrofoil suction side at three different times within one shedding cycle from 0.01 s to 0.0175 s. The images show: (a) the cavity sheet, (b) the shedding of cavitation clouds and (c) the formation of a horse-shoe cavity.
convected downstream by the main flow. The hydrofoil surface areas exposed to possible erosive bubble collapses have been identified from the control volumes with highest $S_b$ values. From the results, it is confirmed that the highest aggressiveness takes place during the collapses as shown in figure 2(b). On the contrary, during the reentrant jet formation and the collapse of horse-shoe cloud shown in figure 2(c) only small areas are slightly affected by collapses. This might be due to a large distance between the bubbles and the hydrofoil surface. Results are similar to analogous risk assessment models and experiments [6].

![Figure 2: Erosion prediction for a typical cavitation cycle divided in three times a, b and c.](image)

5. Conclusions
A numerical simulation of cavitation erosion on a NACA0015 has been carried out with a free OpenFOAM software package and an algorithm based on foamCalcEx and Python language. The bubble collapse strength has been calculated based on the potential energy and homogeneous mixture flow assumption using implicit LES and Zwart-Gerber-Belamri cavitation model. Results confirm the suitability of using $S_b$ and the developed algorithm for cavitation erosion prediction.

Acknowledgments
This work was financially supported by the National Natural Science Foundation of China (Project Nos. 51306018, 51206087 and 51179091) and State key Laboratory of Hydrosience and Engineering, Tsinghua University (Project Nos. 2014-KY-05 and 2015-E-03).

References
[1] Berchiche N, Franc J P and Michel J M 2002 *Journal of Fluids Engineering* **124** 601–606
[2] Escaler X, Farhat M, Avellan F and Eguisquiza E 2003 *Wear* **254** 441–449
[3] Franc J P 2009 *Journal of Fluids Engineering* **131** 021303–021303
[4] Dular M, Stoffel B and Sirok B 2006 *Wear* **261** 642–655
[5] Petkovek M and Dular M 2013 *Wear* **300**
[6] Li Z 2012 *Assessment of Cavitation Erosion with a Multiphase Reynolds-Averaged Navier-Stokes Method* Ph.D. thesis Delft University of Technology
[7] Kubota A, Kato H and Yamaguchi H 1992 *Journal of Fluid Mechanics* **240** 59 – 96
[8] Coutier-Delgosha O, Reboud J L and Fortes-Patella R 2003 *Journal of Fluids Engineering* **125** 38–45
[9] Ji B, LUO X x and WU Y l 2013 *Journal of Hydrodynamics, Ser. B* **25** 510–519 ISSN 1001-6058
[10] Huang B, Zhao Y and Wang G 2014 *Computers & Fluids* **92** 113–124
[11] Van T, Fitzsimmons P, Li Z and Foeth J 2009 *Cav2009* (Michigan, USA) pp 1–13
[12] Bergeles G, Koukouvinis P and Gavaises M 2014 *ICHHD 2014* (Singapore: N.T.U.) pp 619–627
[13] Hidalgo V, Luo X, Ji B and Aguinaga A 2014 *Chinese Science Bulletin* 1–7
[14] Hidalgo V, Luo X, Escaler X, Ji J and Aguinaga A 2014 *IOP Conference Series* **22** 052013
[15] Stimpson C, Ernst C, Knupp P, Pébay P and Thompson D 2006 *The verdict geometric quality library.* (SNL)