Free Vibration Analysis of Multiphase Magneto-Electro-Elastic Composite Conical Shells

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Abstract

Objectives: In the present article, free vibration of multiphase Magneto-Electro-Elastic (MEE) conical shell having a uniform thickness is examined for Clamped-Free (C-F) boundary condition. Method: The study is carried out using a semi-analytical approach for different volume fractions \(V_f\) 0, 0.2, 0.6 and 1.0 of BaTiO\(_3\) in BaTiO\(_3\)-CoFe\(_2\)O\(_4\) smart composite conical shell for three different semi-vertex angles 20\(^\circ\), 35\(^\circ\) and 50\(^\circ\). The piezoelectric (\(P_e\)) and piezomagnetic (\(P_m\)) phase on natural frequencies of MEE truncated conical shells are discussed for different circumferential modes. Findings: The parametric study indicates that natural frequency decrease with an increase in \(V_f\) of BaTiO\(_3\) in magneto-electro-elastic truncated conical shells. Novelty: Studies on MEE constant thickness truncated conical shell using BaTiO\(_3\) and CoFe\(_2\)O\(_4\) as \((P_e)\) & \((P_m)\) smart composite for clamped-free boundary condition to analyse the effect of the frequency with different semi-vertex angle and cone heights. Present commercial FEA software tools are limited to 2 coupling fields. In this research, coupling between 3 fields considered for MEE material. Hence, a computer code is developed to study the influence coupling between electric, elastic and magnetic fields, which can be used for any combinations of boundary conditions and volume fractions. Keywords: MagnetoElectroElastic (MEE); axisymmetric constant thickness conical shell; Smart Composites; volume fraction; free vibrations; finite element method

1 Introduction

Smart materials and smart structures are important as they have wide varieties of applications in engineering such as sensors, actuators and especially in vibration and noise control. Several materials and technologies have been proposed and investigated. Axisymmetric conical shells are shown their applications in aerospace and shipbuilding. A numerical approach is used to obtain the frequency for conical shell for different boundary conditions\(^{(1)}\). The vibration behaviour of the axisymmetric conical shell was analysed using FEM and optimization studies were carried out\(^{(2)}\). A study is conducted for the vibration of the conical shell having different cone angles with constant and
varying thickness. The frequency characteristics of the truncated conical shell are studied having different geometric parameters. Free vibration of MEE cylindrical shell analysed using governing equations, kernel particle (kp) functions are used to study the thin conical shell. A study is conducted to know the piezomagnetic effect on multiphase MEE cylindrical shell. Shell and plates behaviour is analysed using the differential quadrature method. Novozhilov theory is used to investigated thick cones with smaller height for different vertex angles and end conditions. The study is conducted on the conical shell using closed-form auxiliary functions along with the Rayleigh–Ritz procedure. The truncated composite conical shell is analysed for different volume fraction using third-order shear deformation theory. Conical shell is analysed using Flügge thin shell theory with different boundary conditions. Multi-layered conical shell analysed using coupled differential equations and spline approximation method. Truncated conical shell behaviour studied using arbitrary boundary conditions employing Hamilton’s principle. Free vibration of MEE plates analysed using condensation technique. Magneto-Electro-Elastic plates studied using the Hamiltons principle. Review is made on different techniques to analyse the behaviour of MEE materials. Research on conical shell vibration under various parameters effect like boundary conditions, cone angle, thickness, radius to height ratio, length to radius ratio has attracted attention in engineering.

With the literature view, numerous works are carried out of free vibration on the conical shell using standard numerical methods. Meanwhile, a very less study on MEE truncated conical shell using BaTiO₃ and CoFe₂O₄ as (Pₑ) & (Pₑm) smart composite for clamped-free boundary condition. At present, a parametric study has been performed to analyse the effect of the frequency with different semi-vertex angle and cone heights.

2 Formulation

The governing equations of MEE are referred for deriving the FE model in r, θ, z coordinate system such that material properties and geometry remains same in ‘q’ direction.

Free vibration studies for different Vᵣ of MEE conical shell is conducted for Clamped-Free boundary condition. The conical shell structure is modelled using three noded triangular elements with three degrees of freedom (DOF) for each node.

The equations used in the finite element model are;

\[
S_r = S_1 = \frac{\partial u_s}{\partial r}; \quad S_\theta = S_2 = \frac{1}{r} \left( \frac{\partial u_\theta}{\partial \theta} + u_r \right); \quad S_z = S_3 = \frac{\partial u_s}{\partial z}; \quad \gamma_r = \frac{\partial u_r}{\partial r} + \frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z}
\]

Where uₛ, u_θ and u_r are mechanical displacements.

The relation between electric field vector (E) & electric potential (ϕ) is.

\[
E_r = E_1 = -\frac{\partial \phi}{\partial r}; \quad E_z = E_3 = -\frac{\partial \phi}{\partial z}
\]

The relation between magnetic field (H) & magnetic potential (ψ) is.

\[
H_r = H_1 = -\frac{\partial \psi}{\partial r}; \quad H_z = H_3 = -\frac{\partial \psi}{\partial z}
\]

Equation (4) shows a coupled linear MEE material matrix, for axi-symmetric conical shell having planes of symmetry in axial direction.

\[
\begin{bmatrix}
\sigma_r \\
\sigma_\theta \\
\sigma_z \\
\tau_{r\theta} \\
D_r \\
D_z \\
B_r \\
B_z
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & e_{31} & 0 & q_{31} \\
C_{12} & C_{11} & C_{13} & 0 & 0 & e_{31} & 0 & q_{31} \\
C_{13} & C_{13} & C_{33} & 0 & 0 & e_{33} & 0 & q_{33} \\
0 & 0 & 0 & C_{44} & e_{15} & 0 & q_{15} & 0 \\
0 & 0 & 0 & e_{15} & e_{11} & 0 & m_{11} & 0 \\
e_{31} & e_{31} & e_{33} & 0 & 0 & e_{33} & 0 & m_{33} \\
0 & 0 & 0 & q_{15} & m_{11} & 0 & \mu_{11} & 0 \\
q_{31} & q_{31} & q_{33} & 0 & 0 & m_{33} & 0 & \mu_{33}
\end{bmatrix}
\begin{bmatrix}
S_r \\
S_\theta \\
S_z \\
\gamma_r \\
\gamma_r \\
\gamma_z \\
E_r \\
E_z
\end{bmatrix}
\]

\[
\begin{align*}
\sigma_r, \sigma_\theta, \sigma_z, \tau_{r\theta} \text{ are Stress components; } D_r, D_z \text{ are electrical displacements; } B_r, B_z \text{ are magnetic displacements; } e_{ij}, e_{ij}, e_{ij}, \gamma_{ij} \text{ are strains; } E_r, E_z \text{ indicate electric fields; } H_r, H_z \text{ represents magnetic fields.}
\end{align*}
\]

C_ij, e_ij and m_ij are elastic, dielectric and magnetic permeability constants and e_ij, q_ij and m_ij are the Pₑ, Pₑm and magneto-electric material constants respectively.
For free vibration studies\(^{(5)}\)\(^{(7)}\),

\[
[M]\{\ddot{u}\} + [K_{\text{ME}}]\{u\} = 0, \text{ Where}
\]

\[
[K_{\text{MEE}}] = [K_{uu}] + [K_{u\phi}] [K_B]^{-1} [K_A] + [K_{u\psi}] [K_D]^{-1} [K_C]
\]

\(^{(6)}\)

\([K_{\text{MEE}}]\) = Stiffness matrix for fully coupled magneto-electro-elastic material.

\([K_{uu}]\) = Structural stiffness matrix considering elastic constants

\([K_{u\phi}]\) = Structural stiffness matrix considering coupled elastic – electric potential material.

The component matrices of equation

\[
[K_A] = \left[K_{u\phi}\right]^T - \left[K_{\phi\psi}\right] [K_{\psi\psi}]^{-1} [K_{u\psi}]^T
\]

\(^{(7)}\)

\([K_B] = \left[K_{\phi\phi}\right] - \left[K_{\phi\psi}\right] [K_{\psi\psi}]^{-1} [K_{\phi\psi}]^T
\]

\(^{(8)}\)

\([K_C] = \left[K_{u\psi}\right]^T - \left[K_{\psi\psi}\right] [K_{\phi\phi}]^{-1} [K_{u\phi}]^T
\]

\(^{(9)}\)

\([K_D] = \left[K_{\psi\psi}\right] - \left[K_{\phi\psi}\right] [K_{\phi\phi}]^{-1} [K_{\phi\psi}]^T
\]

\(^{(10)}\)

The distribution of \{\phi\} and \{\psi\} are

\[
\phi = [K_B]^{-1} [K_A] \{u\}
\]

\(^{(11)}\)

\[
\psi = [K_D]^{-1} [K_C] \{u\}
\]

\subsection*{2.1 Volume Fraction (V\(_f\)) Pe phase in MEE Conical Shell}

The analysis is conducted for \(V_f\) (0, 0.2, 0.6, and 1.0) of Pe phase in MEE smart materials. The density for MEE material is 5730 kg m\(^{-3}\)\(^{(7)}\).
3 Geometric and Material Modeling

Free vibration studies of the conical shell are conducted by varying semi vertex cone angle ($\alpha$), cone height (H) with a constant thickness (h), base radius (a) and top radius (b). The dimensions of the MEE conical shell used in the study are base radius (a=1m), top radius (b=0.5m), constant thickness (h=0.1m) for three values of semi vertex angle ($\alpha=20^\circ$, $\alpha=35^\circ$ and $\alpha=50^\circ$). The numbers of triangular finite elements in radial and axial directions are selected as in Table 1, for a better aspect ratio.

| Sl. No | Semi vertex angle | Elements in Radial direction | Elements in Axial direction | Total Number of Elements |
|--------|-------------------|-----------------------------|-----------------------------|--------------------------|
| 1      | 20$^\circ$        | 4                           | 56                          | 448                      |
| 2      | 35$^\circ$        | 5                           | 42                          | 420                      |
| 3      | 50$^\circ$        | 7                           | 40                          | 560                      |

The notations used are: $\omega_{\text{in}}$ = Frequency in rad/sec by using $[K_{\text{in}}]$, $\omega_{\text{MEE}}$ = Frequency in rad/sec by using $[K_{\text{MEE}}]$

Table 2. Material coefficients of BaTiO$_3$-CoFe$_2$O$_4$ composite

| $V_f$ | 0       | 0.2     | 0.6     | 1.0     |
|-------|---------|---------|---------|---------|
| C$_{11}$ | 286x10$^9$   | 250x10$^9$   | 200x10$^9$   | 166x10$^9$   |
| C$_{12}$ | 173x10$^9$   | 146x10$^9$   | 110x10$^9$   | 77x10$^9$    |
| C$_{13}$ | 170x10$^9$   | 145x10$^9$   | 110x10$^9$   | 78x10$^9$    |
| C$_{33}$ | 269.5x10$^9$ | 240x10$^9$   | 190x10$^9$   | 162x10$^9$   |
| C$_{44}$ | 45.3x10$^9$  | 45x10$^9$    | 45x10$^9$    | 43x10$^9$    |
| e$_{31}$ | 0        | -2       | -3.5     | -4.4     |
| e$_{33}$ | 0        | 4        | 11       | 18.6     |
| e$_{15}$ | 0        | 0        | 0        | 11.6     |
| e$_{11}$ | 0.08x10$^{-9}$ | 0.33x10$^{-9}$ | 0.9x10$^{-9}$ | 11.2x10$^{-9}$ |
| m$_{11}$ | -5.9x10$^{-4}$ | -3.9x10$^{-4}$ | -1.5x10$^{-4}$ | 0.05x10$^{-4}$ |
| m$_{33}$ | 1.57x10$^{-4}$ | 1.33x10$^{-4}$ | 0.75x10$^{-4}$ | 0.1x10$^{-4}$ |
| q$_{31}$ | 580      | 410      | 200      | 0        |
| q$_{33}$ | 700      | 550      | 260      | 0        |
| q$_{15}$ | 560      | 340      | 180      | 0        |
| m$_{11}$ | 0        | 2.8x10$^{-12}$ | 6.0x10$^{-12}$x10$^{-12}$ | 0 |
| m$_{33}$ | 0        | 2000x10$^{-12}$ | 2500x10$^{-12}$ | 0 |

Here $C_{ij}$ in N/m$^2$, $e_{ij}$ in C/m$^2$, $q_{ij}$ in N/A m, $m_{ij}$ in N s$^2$/C$^2$ & $m_{ij}$ in N s/VC.

4 Results & Discussion

The axisymmetric finite element code developed is tested for different boundary conditions and is validated with the published literature. The results of the validation are discussed in section 4.1 and analysis of Clamped-Free boundary condition of MEE conical shells for three semi vertex angle $\alpha=20^\circ$, $\alpha=35^\circ$, $\alpha=50^\circ$ and $V_f$ studies from 0 to 1.0 are shown in section 4.2.

4.1 Validation

The developed computer code for MEE conical shell finite element analysis is validated and the results of the validation show good agreement. The dimensions of the isotropic conical shell are R/H= 0.3, base radius (R=1.25m), constant thickness (h=0.0625m), density is 2410 kg m$^{-3}$, Young’s modulus (E=30x10$^9$ Nm$^{-2}$).
4.2 Analysis of MEE conical shell for Clamped-Free boundary condition

Here free vibration studies on three MEE conical shells with different semi vertex angle, viz., \( \alpha = 20^\circ \), \( \alpha = 35^\circ \) and \( \alpha = 50^\circ \) are conducted for constant \( V_f \) of 0,0.2,0.6 and 1.0 BaTiO\(_3\) in MEE composite is chosen. The two end conditions of the MEE shell study are Clamped-Free (E-F) viz.,

(i) The top end is clamped with a bottom end is free \((u = v = w = \phi = \psi = 0, \text{ at } z = H)\).
(ii) The top end is free with the bottom end is clamped \((u = v = w = \phi = \psi = 0, \text{ at } z = 0)\)

4.2.1 Analysis of MEE conical shell for \( V_f = 0 \)

The study conducted on Clamped-Free boundary condition for \( V_f = 0 \) and Tables 3 and 4 shows the results of the study. Table 3 shows the frequency for top-end clamped and bottom end free conical shell and Table 4 shows results for top-end free and bottom end clamped. The frequency value in Table 4 is predominantly high compared with the frequency in Table 3, which indicates that with an increase in semi vertex angle, the frequency of the shell increases. An influence of coupling effect is observed in free vibrations of \( \omega_{MEE} \) compared to \( \omega_{uu} \) for all the three \( \alpha = 20^\circ \), \( \alpha = 35^\circ \) and \( \alpha = 50^\circ \) semi vertex angles.

| Semi vertex angle | \( \alpha = 20^\circ \) | \( \alpha = 35^\circ \) | \( \alpha = 50^\circ \) |
|-------------------|----------------|----------------|----------------|
| Frequency (rad/sec) | \( \omega_{uu} \) | \( \omega_{MEE} \) | \( \omega_{uu} \) | \( \omega_{MEE} \) | \( \omega_{uu} \) | \( \omega_{MEE} \) |
| 1                 | 1255           | 1253           | 2041           | 2038           | 2218           | 2214           |
| 2                 | 868.7          | 866.9          | 1432           | 1427           | 1655           | 1650           |
| 3                 | 1387           | 1386           | 1564           | 1561           | 1640           | 1635           |
| 4                 | 2400           | 2398           | 2313           | 2311           | 2102           | 2099           |
| 5                 | 3686           | 3684           | 3423           | 3422           | 2913           | 2911           |
| 6                 | 5199           | 5195           | 4770           | 4769           | 3963           | 3961           |
| 7                 | 6909           | 6905           | 6305           | 6302           | 5190           | 5189           |
| 8                 | 8794           | 8788           | 7997           | 7995           | 6562           | 6561           |
| 9                 | 10830          | 10820          | 9828           | 9825           | 8056           | 8054           |
| 10                | 13000          | 12990          | 11780          | 11770          | 9653           | 9651           |
Table 4. Top end free with bottom end fixed condition

| Semi vertex angle | $\alpha = 20^0$ | $\alpha = 35^0$ | $\alpha = 50^0$ |
|-------------------|-----------------|-----------------|-----------------|
| Frequency (rad/sec) | $\omega_{in}$ | $\omega_{MEE}$ | $\omega_{in}$ | $\omega_{MEE}$ | $\omega_{in}$ | $\omega_{MEE}$ |
| 1                 | 2703            | 2702            | 4229            | 4226            | 4321            | 4316            |
| 2                 | 1832            | 1830            | 2936            | 2930            | 3332            | 3327            |
| 3                 | 2829            | 2826            | 3324            | 3316            | 3453            | 3446            |
| 4                 | 4323            | 4317            | 4817            | 4805            | 4586            | 4576            |
| 5                 | 6027            | 6019            | 6654            | 6636            | 6203            | 6188            |
| 6                 | 7947            | 7937            | 8652            | 8628            | 8057            | 8035            |
| 7                 | 10050           | 10040           | 10790           | 10760           | 10060           | 10030           |
| 8                 | 12320           | 12310           | 13050           | 13010           | 12180           | 12150           |
| 9                 | 14720           | 14710           | 15420           | 15390           | 14400           | 14360           |
| 10                | 17240           | 17230           | 17900           | 17860           | 16710           | 16670           |

4.2.2 Analysis of MEE conical shell for $V_f = 0.2$

The study conducted on Clamped-Free boundary condition for $V_f = 0.2$ and Tables 5 and 6 shows the results of the study. Table 5 shows the frequency for top-end clamped and bottom end free conical shell and Table 6 shows results for top-end free and bottom end clamped. As the semi-vertex angle increases the frequency values in Table 5 is low compared with the frequency values in Table 6.

Table 5. Top end fixed with bottom end free condition

| Semi vertex angle | $\alpha = 20^0$ | $\alpha = 35^0$ | $\alpha = 50^0$ |
|-------------------|-----------------|-----------------|-----------------|
| Frequency (rad/sec) | $\omega_{in}$ | $\omega_{MEE}$ | $\omega_{in}$ | $\omega_{MEE}$ | $\omega_{in}$ | $\omega_{MEE}$ |
| 1                 | 1236            | 1242            | 1999            | 2003            | 2156            | 2158            |
| 2                 | 844.2           | 851.4           | 1400            | 1402            | 1615            | 1613            |
| 3                 | 1334            | 1341            | 1517            | 1522            | 1596            | 1595            |
| 4                 | 2305            | 2315            | 2230            | 2241            | 2033            | 2038            |
| 5                 | 3540            | 3555            | 3294            | 3311            | 2807            | 2818            |
| 6                 | 4991            | 5012            | 4588            | 4612            | 3813            | 3830            |
| 7                 | 6632            | 6660            | 6062            | 6094            | 4991            | 5015            |
| 8                 | 8441            | 8476            | 7690            | 7730            | 6310            | 6341            |
| 9                 | 10400           | 10440           | 9451            | 9500            | 7747            | 7785            |
| 10                | 12480           | 12530           | 11330           | 11390           | 9286            | 9331            |

Table 6. Top end free with bottom end fixed condition

| Semi vertex angle | $\alpha = 20^0$ | $\alpha = 35^0$ | $\alpha = 50^0$ |
|-------------------|-----------------|-----------------|-----------------|
| Frequency (rad/sec) | $\omega_{in}$ | $\omega_{MEE}$ | $\omega_{in}$ | $\omega_{MEE}$ | $\omega_{in}$ | $\omega_{MEE}$ |
| 1                 | 2667            | 2677            | 4131            | 4140            | 4178            | 4188            |
| 2                 | 1787            | 1796            | 2875            | 2876            | 3241            | 3242            |
| 3                 | 2723            | 2727            | 3222            | 3225            | 3350            | 3353            |
| 4                 | 4158            | 4161            | 4644            | 4650            | 4429            | 4439            |
| 5                 | 5793            | 5796            | 6412            | 6418            | 5981            | 5997            |
| 6                 | 7633            | 7637            | 8338            | 8345            | 7767            | 7788            |
| 7                 | 9652            | 9658            | 10390           | 10400           | 9703            | 9728            |
| 8                 | 11820           | 11830           | 12570           | 12580           | 11750           | 11780           |
| 9                 | 14130           | 14140           | 14850           | 14880           | 13890           | 13930           |
| 10                | 16550           | 16560           | 17240           | 17270           | 16120           | 16160           |
4.2.3 Analysis of MEE conical shell for $V_f = 0.6$

The study conducted on C-F boundary condition for $V_f = 0.6$ and Tables 7 and 8 shows the results of the study. Table 7 shows the frequency for top-end clamped and bottom end free conical shell and Table 8 shows results for top-end free and bottom end clamped. In Table 7, the frequency value in Table 8 is largely high compared with the frequency values in Table 7, indicating that as semi vertex angle increases the frequency also increases due to the stiffening of the conical shell, similar behaviour is observed in coupled frequency.

| Semi vertex angle | $\alpha = 20^0$ | $\alpha = 35^0$ | $\alpha = 50^0$ |
|-------------------|----------------|----------------|----------------|
| Frequency (rad/sec) | $\omega_{uu}$  | $\omega_{MEE}$ | $\omega_{uu}$  | $\omega_{MEE}$ | $\omega_{uu}$  | $\omega_{MEE}$ |
| 1  | 1198 | 1216 | 1926 | 1937 | 2051 | 2046 |
| 2  | 797.4 | 816.4 | 1344 | 1352 | 1548 | 1532 |
| 3  | 1244 | 1258 | 1436 | 1449 | 1523 | 1512 |
| 4  | 2149 | 2167 | 2091 | 2112 | 1919 | 1920 |
| 5  | 3300 | 3325 | 3080 | 3110 | 2631 | 2645 |
| 6  | 4652 | 4686 | 4285 | 4326 | 3563 | 3589 |
| 7  | 6181 | 6224 | 5661 | 5714 | 4659 | 4696 |
| 8  | 7865 | 7920 | 7181 | 7248 | 5889 | 5937 |
| 9  | 9688 | 9754 | 8827 | 8910 | 7231 | 7291 |
| 10 | 11630 | 11710 | 10580 | 10680 | 8671 | 8743 |

4.2.4 Analysis of MEE conical shell for $V_f = 1$

The study conducted on C-F boundary condition for $V_f = 1$ and Tables 9 and 10 shows the results of the study. Table 9 shows the frequency for top-end clamped and bottom end free conical shell and Table 10 shows results for top-end free and bottom end clamped. As the semi-vertex angle increases the frequency values in Table 9 is predominantly less compared with the frequency values in Table 10.

| Semi vertex angle | $\alpha = 20^0$ | $\alpha = 35^0$ | $\alpha = 50^0$ |
|-------------------|----------------|----------------|----------------|
| Frequency (rad/sec) | $\omega_{uu}$  | $\omega_{MEE}$ | $\omega_{uu}$  | $\omega_{MEE}$ | $\omega_{uu}$  | $\omega_{MEE}$ |
| 1  | 2601 | 2629 | 3963 | 3982 | 3935 | 3939 |
| 2  | 1706 | 1731 | 2773 | 2779 | 3086 | 3070 |
| 3  | 2549 | 2560 | 3056 | 3062 | 3180 | 3159 |
| 4  | 3887 | 3898 | 4361 | 4371 | 4168 | 4153 |
| 5  | 5409 | 5422 | 6012 | 6023 | 5611 | 5594 |
| 6  | 7120 | 7136 | 7819 | 7832 | 7283 | 7261 |
| 7  | 8998 | 9016 | 9746 | 9765 | 9101 | 9077 |
| 8  | 11020 | 11040 | 11780 | 11810 | 11030 | 11010 |
| 9  | 13160 | 13190 | 13920 | 13960 | 13040 | 13030 |
| 10 | 15420 | 15440 | 16150 | 16200 | 15130 | 15130 |

https://www.indjst.org/
Table 9. Top end fixed with bottom end free condition

| Semi vertex angle | $\alpha = 20^\circ$ | $\alpha = 35^\circ$ | $\alpha = 50^\circ$ |
|-------------------|---------------------|---------------------|---------------------|
| Frequency (rad/sec) | $\omega_{uu}$ | $\omega_{MEE}$ | $\omega_{uu}$ | $\omega_{MEE}$ | $\omega_{uu}$ | $\omega_{MEE}$ |
| 1 | 1173 | 1205 | 1886 | 1918 | 2009 | 2031 |
| 2 | 779.2 | 811.1 | 1310 | 1345 | 1508 | 1526 |
| 3 | 1211 | 1227 | 1401 | 1432 | 1485 | 1504 |
| 4 | 2091 | 2105 | 2043 | 2070 | 1878 | 1898 |
| 5 | 3211 | 3229 | 3009 | 3037 | 2578 | 2602 |
| 6 | 4528 | 4551 | 4187 | 4221 | 3492 | 3523 |
| 7 | 6017 | 6046 | 5531 | 5573 | 4566 | 4605 |
| 8 | 7660 | 7695 | 7018 | 7069 | 5772 | 5820 |
| 9 | 9439 | 9481 | 8629 | 8691 | 7089 | 7147 |
| 10 | 11340 | 11390 | 10350 | 10420 | 8502 | 8571 |

Table 10. Top end free with bottom end fixed condition

| Semi vertex angle | $\alpha = 20^\circ$ | $\alpha = 35^\circ$ | $\alpha = 50^\circ$ |
|-------------------|---------------------|---------------------|---------------------|
| Frequency (rad/sec) | $\omega_{uu}$ | $\omega_{MEE}$ | $\omega_{uu}$ | $\omega_{MEE}$ | $\omega_{uu}$ | $\omega_{MEE}$ |
| 1 | 2553 | 2597 | 3883 | 3918 | 3848 | 3879 |
| 2 | 1665 | 1712 | 2709 | 2751 | 3023 | 3045 |
| 3 | 2462 | 2487 | 2965 | 3003 | 3098 | 3125 |
| 4 | 3745 | 3781 | 4214 | 4258 | 4046 | 4084 |
| 5 | 5212 | 5255 | 5800 | 5859 | 5433 | 5486 |
| 6 | 6865 | 6912 | 7543 | 7615 | 7041 | 7110 |
| 7 | 8682 | 8733 | 9407 | 9490 | 8794 | 8879 |
| 8 | 10640 | 10700 | 11380 | 11470 | 10660 | 10760 |
| 9 | 12720 | 12780 | 13460 | 13560 | 12610 | 12720 |
| 10 | 14910 | 14970 | 15630 | 15740 | 14640 | 14770 |

5 Conclusions

- In the C-F boundary condition, the influence of the coupling effect is observed in free vibrations of $\omega_{MEE}$ compared to $\omega_{uu}$ for different semi vertex angles.
- For the $V_f = 0$ and $V_f = 1$, the ‘m’ effect influence is not predominant.
- Irrespective of $V_f$ and semi vertex angle, it is clearly said that when the truncated conical shell bottom is clamped and the top is free, the average piezoelectric and piezomagnetic effect is high around 54 percent compared to the condition when the bottom end is free and the top end is clamped.
- The effect of clamping the bottom end and clamping the top end is seen in the study as the conical shell becomes stiffer.
- The effect of $P_e$ and $P_m$ are prominent at initial circumferential modes while it is negligible at higher circumferential modes.

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