Optical Unification

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Abstract

We discuss string scale unification facilitated by exotic matter with masses at intermediate scales, between the observable sector supersymmetry breaking scale and the string scale. We point out a mechanism by which string scale unification may occur while producing a (lower) virtual unification scale independent of the location of the intermediate scale and the value of the string coupling. The apparent unification obtained by extrapolating low energy gauge couplings is not accidental when this mechanism is invoked; virtual unification is robust.

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Introduction. Four-dimensional weakly coupled heterotic string theories, such as those constructed from orbifold compactification [1], suffer from problems related to unification of gauge couplings. Extrapolation of the low energy (near the Z mass) gauge couplings, assuming only the spectrum of the Minimal Supersymmetric Standard Model (MSSM), gives an approximate unification of the couplings at $\Lambda_U \sim 2 \times 10^{16}$ GeV (see for example [2]). On the other hand, gauge couplings in the weakly coupled heterotic string should unify at the string scale $\Lambda_H$, which is roughly $\Lambda_H \sim 4 \times 10^{17}$ GeV; this is the infamous problem of a factor of 20.

Various solutions have been suggested for this difficulty; these have been nicely summarized by Dienes [3]. One possibility is the following. Four-dimensional string theories typically contain exotic states charged under Standard Model (SM) gauge group $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$. These are states beyond those contained in the MSSM. One assumes that some of these exotics have masses at an intermediate scale $\Lambda_I$, between the scale of observable sector supersymmetry breaking $\Lambda_{SUSY}$ and the string scale $\Lambda_H$. The exotic matter effects the running of the gauge couplings above $\Lambda_I$ in just the right way to shift the unification scale up to the string scale. Recent work on this scenario may be found, for example, in [4, 5].

A careful tuning of $\Lambda_I$ is generally required. This resolution of the string unification problem implies that the unification of gauge couplings in the conventional MSSM picture is accidental. It just so happens that the low energy coupling extrapolate to a unified value. If the intermediate scale or string coupling were perturbed, the apparent unification at $\Lambda_U$ would disappear. In general this unification scenario spoils the significance of the MSSM $\alpha_3(m_Z)$ versus $\sin^2 \theta_W(m_Z)$ prediction [6].

In this paper, we demonstrate a mechanism whereby string unification no longer implies that the apparent unification at $\Lambda_U$ is an accident. In elementary optics a virtual image is formed by a diverging thin lens, independent of the size and location of the object. So too with the mechanism we describe, a virtual unification happens regardless of the intermediate scale (object distance) or string coupling $\alpha_H$ (object height). Because of the analogy to the optics of thin lenses, we refer to the mechanism as optical unification. Optical unification is
Unlike the mechanism of *mirage unification* introduced by Ibáñez in [4], since unification is not an illusion; it really does occur near the string scale and does not require string threshold corrections.

*Origin of \( \Lambda_I \).* Quite commonly in the models considered here, some of the \( U(1) \) factors contained in the string derived gauge group \( G \) are anomalous: \( \text{tr } Q_a \neq 0 \). Redefinitions of the charge generators allow one to isolate this anomaly such that only one \( U(1) \) is anomalous. We denote this factor \( U(1)_X \). The associated anomaly is cancelled by the Green-Schwarz mechanism [8], which induces a Fayet-Illiopoulos (FI) term \( \xi \) for \( U(1)_X \):

\[
D_X = \sum_i K_i q_i^X \phi^i + \xi, \quad \frac{\xi}{g_H^2} = \frac{\text{tr } Q_X}{192\pi^2 m_P^2}.
\]

\( K_i = \partial K / \partial \phi^i \), with \( K \) the Kähler potential, \( q_i^X \) the \( U(1)_X \) charge of \( \phi^i \), and \( m_P = 1/\sqrt{8\pi G} = 2.44 \times 10^{18} \) GeV is the (reduced) Planck mass. As discussed below, the (properly normalized) running gauge couplings \( g_a(\mu) \) unify to the heterotic string coupling \( g_H \) at the string scale \( \Lambda_H \). Since the scalar potential contains a term proportional to \( D_X^2 \), some scalar fields generically shift to cancel the FI term (i.e., \( \langle D_X \rangle = 0 \)) and get large vacuum expectation values. This causes several fields to get effective vector couplings

\[
W \supset \frac{1}{m_P^n} \langle \phi^1 \cdots \phi^n \rangle A A^c.
\]

Here, \( A \) and \( A^c \) are conjugate with respect to the gauge group which survives after spontaneous symmetry breaking caused by the \( U(1)_X \) FI term. The right-hand side of (2) is an effective supersymmetric mass term, which results in masses \( \mathcal{O}(\langle \phi^1 \cdots \phi^n \rangle / m_P^{n-1}) \) once one shifts to the stable vacuum. This is a possible origin of an intermediate scale \( \Lambda_I \).

Factors of \( G \) besides \( U(1)_X \) are typically broken when the Higgses get vevs to cancel the FI term. This is an attractive feature of these models, since it provides an effective gauge theory with smaller rank. However, it is typically the case that \( U(1) \) factors besides the hypercharge \( U(1)_Y \) survive below the \( U(1)_X \) breaking scale. In those cases where some of the observable sector fields are charged under these extra \( U(1) \)s, experimental limits on \( Z' \) boson mediated processes require that the un_hidden \( U(1) \)s be broken somewhere above the electroweak symmetry breaking scale. Thus, we envision the possibility of an intermediate scale \( \Lambda_I \) of gauge symmetry breaking independent of the \( U(1)_X \) breaking.
Gaugino condensation of a condensing group $G_C$ in the hidden sector gives rise to the condensation scale $\Lambda_C$, given roughly by $\Lambda_C \sim \Lambda_H \exp(8\pi^2/b_Cg_H^2)$, where $b_C$ is the beta function coefficient of $G_C$ in suitable conventions. Soft mass terms which split the supersymmetric partners to Standard Model (SM) particles are proportional to the gaugino condensate $\langle \lambda \lambda \rangle$. This operator has mass dimension three, so one generically expects the observable sector supersymmetry breaking scale $\Lambda_{\text{SUSY}}$ is given by

$$\Lambda_{\text{SUSY}} \sim \langle \lambda \lambda \rangle / m_P^2 \sim \Lambda_C^3 / m_P^2$$

For supersymmetry to protect the gauge hierarchy $m_Z \ll m_P$ between the electroweak scale and the fundamental scale, one requires $1\text{ TeV} \lesssim \Lambda_{\text{SUSY}} \lesssim 100\text{ TeV}$. This implies $\Lambda_C \sim 10^{13}\text{ GeV}$. (More precise results may be obtained, for instance, with the detailed supersymmetry breaking models of Binétruy, Gaillard and Wu [9] as well as subsequent elaborations by Gaillard and Nelson [10].) This gives another dimensionful parameter which may be used to produce $\Lambda_I$.

*Hypercharge.* An important feature of Grand Unified Theories (GUTs) [11] is that the $U(1)$ generator corresponding to electroweak hypercharge does not have arbitrary normalization, since the hypercharge generator is embedded into the Lie algebra of the GUT group $G_{\text{GUT}} \supset SU(3)_C \times SU(2)_L \times U(1)_Y$. A similar situation holds for the normalization of $U(1)$ generators in string-derived field theories. Just as in GUTs, unification of the hypercharge coupling with the couplings of other factors of the gauge symmetry group $G$ corresponds to a particular normalization. This normalization of hypercharge is often different than the one which appears in $SU(5)$ or $SO(10)$ GUTs. In our analysis we will work with an arbitrary hypercharge normalization, conveyed by the constant $k_Y$. In an $SU(5)$ GUT, for instance, $k_Y = 5/3$. However, in the semi-realistic heterotic orbifold constructions, $k_Y$ is typically some other value, which depends on the linear combination of $U(1)$ generators employed to “manufacture” the SM hypercharge generator. Furthermore, we exploit the fact that exotic SM-charged states typically carry very unusual hypercharges; i.e., not those which would occur in the decomposition of a standard GUT group. It will turn out that the occurrence of the somewhat bizarre hypercharges we obtain here fits in nicely with the unusual values required to realize the optical unification alluded to above. (See for example [4] for a more
complete discussion of these “stringy” features.)

**String scale unification.** It has been known since the earliest attempts \cite{12} to use closed string theories as unified theories of all fundamental interactions that \( g^2 \sim \kappa^2/\alpha' \), where \( g \) is the gauge coupling, \( \kappa \) is the gravitational coupling and \( \alpha' \) is the Regge slope, related to the string scale by \( \Lambda_{\text{string}} \approx 1/\sqrt{\alpha'} \). In particular, this relation holds for the heterotic string \cite{13}. Here, however, \( g \) and \( \kappa \) are the ten-dimensional couplings. By dimensional reduction of the ten-dimensional effective field theory obtained from the ten-dimensional heterotic string in the zero slope limit, this relation may be translated into a constraint relating the heterotic string scale \( \Lambda_H \) to the four-dimensional Planck mass \( m_P \). One finds, as expected on dimensional grounds, \( m_P \sim 1/\sqrt{\kappa} \), where the coefficients which have been suppressed depend on the size of the six compact dimensions; similarly, the four-dimensional gauge coupling satisfies \( g_H \sim g \); for details see Ref. \cite{14}. We thus obtain \( \Lambda_H \sim g_H m_P \).

Kaplunovsky has made this relation more precise, including one loop effects from heavy string states \cite{15}. Subject to various conventions described in \cite{15}, including a choice of the \text{DR} renormalization scheme in the effective field theory, the result is:

\[
\Lambda_H \approx 0.216 \times g_H m_P = g_H \times 5.27 \times 10^{17} \text{ GeV}.
\]  (4)

In the heterotic orbifolds under consideration the gauge group \( G \) has several factors, each of which will have its own running gauge coupling. Ginsparg \cite{16} has shown that the running couplings \( g_a^2(\mu) \) unify to a common value \( g_H \) at the string scale \( \Lambda_H \) according to

\[
k_a g_a^2(\Lambda_H) = g_H^2, \quad \forall \ a.
\]  (5)

Here, \( k_a \) for a nonabelian factor \( G_a \) is the affine or Kac-Moody level of the current algebra in the underlying theory which is responsible for the gauge symmetry in the effective field theory. It is unnecessary for us to trouble ourselves with a detailed explanation of this quantity or its string theoretic origins, since \( k_a = 1 \) for any nonabelian factor in the heterotic orbifolds we are considering. For this reason, these heterotic orbifolds are referred to as affine-level 1 constructions. In the case of \( G_a \) a \( U(1) \) factor, \( k_a \) carries information about the normalization of the corresponding current in the underlying theory, and hence the normalization of the charge generator in the effective field theory.
The important point, which has been emphasized many times before, is that a gauge coupling unification prediction is made by the underlying string theory. The SM gauge couplings are known (to varying levels of accuracy), say, at the Z scale. Given the particle content and mass spectrum of the theory between the Z scale and the string scale, one can easily check at the one loop level whether or not the unification prediction is approximately consistent with the Z scale boundary values. To go beyond one loop requires some knowledge of the other couplings in the theory, and the analysis becomes much more complicated. However, the one loop success is not typically spoiled by two loop corrections, but rather requires a slight adjustment of flexible parameters (such as superpartner masses) which enter the one loop analysis.

Due to the presence of exotic matter in the models studied here, we are able to achieve string scale unification. This sort of unification scenario has been studied many times before, for example in [17, 18, 4]. However, in contrast to [17], we have—as in [18]—states which would not appear in decompositions of standard GUT groups. The appearance of these states is due to the Wen-Witten defect in string orbifold models [19]. Using exotics with small hypercharge values, the $SU(3) \times SU(2)$ running can be altered to unify at the string scale without having an overwhelming modification on the running of the $U(1)_Y$ coupling.

Nonstandard hypercharge normalization has been studied previously, for example in [20]. In these analyses, it was found that lower values $k_Y < 5/3$ were preferred if only the MSSM spectrum is present up to the unification scale; the preferred values were between 1.4 to 1.5. Unfortunately, in many semi-realistic orbifold models we are faced with the opposite effect—a larger than normal $k_Y$. This larger value requires a larger correction to the running from the exotic states, and has the effect of pushing down the required mass scale of the exotics [4].

The mechanism. With a single intermediate scale $\Lambda_I$, unification of gauge couplings at the string scale requires

$$k_a 4\pi \alpha^{-1}_H = 4\pi (\alpha^{-1}_a(m_Z) - \Delta_a) - b_a \ln \frac{\Lambda^2_I}{m^2_Z} - (b_a + \delta b_a) \ln \frac{\Lambda^2_H}{\Lambda^2_I}, \quad a = Y, 2, 3. \quad (6)$$

The notation is conventional, with $\alpha_a = g_a^2 / 4\pi \ (a = H, Y, 2, 3)$. The quantities $b_a, \ a = Y, 2, 3$
are the beta function coefficients evaluated for the MSSM spectrum:

$$b_a = -3C(G_a) + \sum_R X_a(R) \Rightarrow b_Y = 11, \quad b_2 = 1, \quad b_3 = -3. \quad (7)$$

$k_2 = k_3 = 1$, and we leave $k_Y$ unspecified. $\Delta_a$ are threshold corrections not including the exotic states that enter the running at $\Lambda_I$. $\Delta_a$ are immaterial to the mechanism described here—they drop out of the analysis—therefore there is no need to estimate or discuss them here. The quantities $\delta b_a$ are the contributions to the beta function coefficients from exotic states above $\Lambda_I$:

$$\delta b_3 = \sum_{q, q^c_i} \frac{1}{2}, \quad \delta b_2 = \sum_{\ell, \ell^c_i} \frac{1}{2}, \quad \delta b_Y = \sum_{q, q^c_i} (Y_i)^2 + \sum_{\ell, \ell^c_i} (Y_i)^2 + \sum_{s, s^c_i} (Y_i)^2. \quad (8)$$

With respect to $SU(3)_C \times SU(2)_L$ the states $q_i$ are $(3,1)$, $\ell_i$ are $(1,2)$ and $s_i$ are $(1,1)$. We have vector pairs of each because we assume supersymmetric masses at the scale $\Lambda_I$.

Virtual unification at $\Lambda_U$ requires

$$\hat{k}_a 4\pi \alpha^{-1}_U = 4\pi (\alpha^{-1}(m_Z) - \Delta_a) - b_a \ln \frac{\Lambda^2_U}{m^2_Z}, \quad a = Y, 2, 3. \quad (9)$$

Here, $\hat{k}_2 = \hat{k}_3 = 1$, while we leave $\hat{k}_Y = 5/3$, the usual value. If we substitute (7) into (8), we arrive at the following constraints for virtual unification at $\Lambda_U$ to occur simultaneously with real unification at the string scale $\Lambda_H$:

$$\hat{k}_a a_U - k_a a_H = b_a (t_H - t_U) + \delta b_a t_{HI}, \quad a = Y, 2, 3, \quad (10)$$

where for convenience we have defined

$$a_H = 4\pi \alpha^{-1}_H, \quad a_U = 4\pi \alpha^{-1}_U,$$

$$t_U = \ln(\Lambda^2_U/m^2_Z), \quad t_H = \ln(\Lambda^2_H/m^2_Z), \quad t_{HI} = \ln(\Lambda^2_H/\Lambda^2_I). \quad (11)$$

Generally, Eqs. (10) are inconsistent. Careful choices of $\delta b_a$, $g_H$ and $\Lambda_I$ must be made.

To have optical unification we seek solutions which are independent of $g_H$ and $\Lambda_I$, except that they correspond to weakly coupled heterotic string with an intermediate scale above

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2 Here, $C(SU(N)) = N$ while $C(U(1)) = 0$. For a fundamental or antifundamental representation of $SU(N)$ we have $X_a = 1/2$ while for hypercharge $X_Y(R) = Y^2(R)$. 
\(m_Z\) and below \(\Lambda_H\). If this can be done, fine-tuning of the intermediate scale disappears. We first solve the \(a = 2, 3\) parts of (10) for the virtual unification scale and inverse coupling:

\[
t_U = t_H + \frac{1}{4}(\delta b_2 - \delta b_3)t_{HI}, \quad a_U = a_H + \frac{1}{4}(3\delta b_2 + \delta b_3)t_{HI}
\]  

(12)

To have the virtual scale below the string scale, the first equation in (12) requires \(\delta b_2 < \delta b_3\). Substituting (12) into the \(a = Y\) part of (10) yields the single constraint for optical unification:

\[
0 = (k_Y - \frac{5}{3})a_H + (\delta b_Y - 4\delta b_2 + \frac{7}{3}\delta b_3)t_{HI}
\]  

(13)

Since we require a solution which is independent of both \(a_H\) and \(t_{HI}\), optical unification may only occur if

\[
k_Y = 5/3 \quad \text{and} \quad \delta b_2 = \frac{7}{12}\delta b_3 + \frac{1}{4}\delta b_Y.
\]  

(14)

\(k_Y = 5/3\) is by no means always possible in four-dimensional string models. However, models have been found where this condition may be satisfied \[21, 4\]. In the exotic matter cases considered here, both \(\delta b_2\) and \(\delta b_3\) will be non-negative integers. \(\delta b_Y \geq 0\), (14) and \(\delta b_2 < \delta b_3\) together imply

\[
\delta b_3 > \delta b_2 \geq \frac{7}{12}\delta b_3.
\]  

(15)

**GUT-like exotics.** It is well known that in the trivial case where the exotics introduced at the scale \(\Lambda_I\) consist entirely of vector pairs of complete \(SU(5)\) representations we retain the unification prediction. For example if we have \(N 5 + \bar{5}\) pairs,

\[
\delta b_3 = \delta b_2 = N, \quad \delta b_Y = 5N/3.
\]  

(16)

It is easy to check that this satisfies (14). However, \(\delta b_2 < \delta b_3\) is not satisfied which from (12) is inconsistent with \(\Lambda_H > \Lambda_U\). If we have \(N 10 + \bar{10}\) pairs,

\[
\delta b_3 = \delta b_2 = 3N, \quad \delta b_Y = 5N.
\]  

(17)

While this satisfies (14), once again we do not satisfy \(\delta b_2 < \delta b_3\).

**Minimal example.** It is easily checked that the minimum solution to (14) and (12), consistent with integer values of \(\delta b_2\) and \(\delta b_3\), is

\[
\delta b_3 = 3, \quad \delta b_2 = 2, \quad \delta b_Y = 1.
\]  

(18)
This can only work if most of the $3(3 + 3, 1) + 2(1, 2 + 2)$ exotic states have very small or vanishing hypercharges. However, as we mentioned above it is possible to have states with very small hypercharge in string-derived models.

For example, if the exotic spectrum which gets a mass of order $\Lambda_I$ consists of

$$3(3 + 3, 1, 0) + 2(1, 2, 0) + (1, 2, +1/2) + (1, 2, -1/2)$$

with respect to $SU(3)_C \times SU(2)_L \times U(1)_Y$, then (18) is satisfied. It remains to be seen if this is possible to achieve in specific models.

In Figure 1 we show the running couplings in this scheme, assuming negligible $Z$ scale threshold corrections; i.e., the MSSM superpartners are assumed to have masses of order the electroweak scale. Apparent unification occurs at $\Lambda_U = 2.01 \times 10^{16}$ GeV as can be seen from the figure. Simultaneously we have string scale unification at $\Lambda_H = 4.08 \times 10^{17}$ GeV and (4) is satisfied. The intermediate scale must be chosen to be $\Lambda_I = 2.39 \times 10^{12}$ GeV.

If the $Z$ scale couplings are taken as fundamental, then we must regard $\Lambda_I$ as fine-tuned. If we shift $\Lambda_I \rightarrow \Lambda'_I = 10\Lambda_I$, holding the string scale $\Lambda_H$ and the coupling $g_H$ fixed, then Figure 2 emerges. The $Z$ scale couplings must be shifted away from their experimental values to accommodate the change in the intermediate scale. However, as Figure 3 makes clear, apparent unification is preserved. Thus Figure 3 illustrates our main point: if optical unification is in force, the qualitative phenomenon of apparent unification is not a finely-tuned accident of a particular choice of intermediate scale. It is granted, however, that the $Z$ scale couplings and the precise location of the apparent unification point are accidents of the choice of intermediate scale. However, we would like to suggest that these features are not fundamental, but are a reflection of the underlying physics at the scales $\Lambda_I$ and $\Lambda_H$.

**Conclusions.** Clearly a careful tuning of the scale $\Lambda_I$ is necessary in order to obtain the relation (4) and the observed $Z$ scale boundary values for the running gauge couplings. Up to light particle thresholds (MSSM superpartners, etc.), the quantities $\alpha^{-1}_a, \alpha^{-1}_U, \Lambda_U$ and $\Lambda_H$ are all determined from the pair of inputs $(g_H, \Lambda_I)$. However, the crucial point is that for any pair $(g_H, \Lambda_I)$ we would obtain an apparent unification at some scale $\Lambda_U$, so long as (14) is satisfied. That is, the unification obtained from extrapolation from the $Z$ scale is not accidental. It is in this sense that the virtual unification is robust.
Figure 1: Inverse couplings \((\alpha^{-1}_{1} = (3/5)\alpha^{-1}_{Y})\) as a function of scale for the minimal example. Solid lines show the actual running and unification at the string scale. Dashed lines show the apparent unification.
Figure 2: Effects of shifting the intermediate scale upward by a factor of 10, while holding $\Lambda_H$ and $g_H$ fixed. Dashed lines show the locations of the two scales. Heavy lines show the actual string scale unification, while apparent unification is shown with lighter lines. It can be seen that apparent unification is shifted to a slightly higher scale, and the Z scale couplings must be modified from their experimental values.
Figure 3: Close-up on the effects of shifting the intermediate scale upward by a factor of 10, while holding $\Lambda_H$ and $g_H$ fixed. Dashed lines show the actual string scale unification, while two versions of apparent unification are shown with solid lines. It can be seen that apparent unification is preserved.
In [4] a class of 175 models was studied in some detail. To check whether or not the conditions for optical unification, Eqs. (14) and (15), can be satisfied requires an involved analysis of all possible definitions of $Y$ for the 41 models where $k_Y = 5/3$ is not excluded. Moreover, since these models have many extra $SU(3)_C$ triplets and $SU(2)_L$ doublets, the number of possible ways of accommodating the MSSM is vast. We hope to report the results of such a check in a future publication. At this juncture, we merely point out the mechanism, which has the nice feature that only the intermediate matter content—and not the intermediate scale—needs to be specified in order to naturally explain an apparent unification at a scale lower than the string scale in the context of affine-level 1 weakly coupled heterotic string models. It would be interesting to see how such a solution to the unification scale problem might work in semi-realistic heterotic string models other than those described in [4], such as the recent, very promising free fermionic constructions [21].
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