Plasma-screening effects in the atrophysically relevant He-like and Li-like Mg and Fe ions

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The effect of plasma environment on the atomic energy levels of He-like and Li-like Mg and Fe ions have been studied using Debye model. The equation-of-motion coupled-cluster (EOMCC) and Fock-space coupled-cluster (FSCC) formalisms in the relativistic framework have been adopted to describe the atomic states and the energy levels of the above plasma embedded ions. Salient features of these methods have been described to account the two electron screening effects through the Debye potentials. The two-body screening potential has been derived in the multipole expansion form to evaluate the reduced matrix elements in solving the equation of motion. Using this extended model, we have also predicted that quasi-degeneracy among the energy states having same principal quantum number (n) but different angular momentum (l) is slackened, whereas fine structure splitting is unaffected with increasing plasma strength. These knowledge are useful in estimating radiative opacity, photoionization cross sections, line intensities, etc of the aforementioned astrophysical plasmas.

PACS numbers:

I. INTRODUCTION

The emission spectra of helium-like (He-like) and lithium-like (Li-like) ions have been observed in recent years both in astrophysical and laboratory plasmas. The X-ray spectra from these ions are observed from a large variety of astrophysical sources [1–3] and laser plasma interactions [4, 5]. They are used to determine the primordial abundances of elements that are of immense interest for testing the big bang cosmology [6]. Detection of these lines using astronomy telescopes reveal many X-ray sources. Also, the Earth’s atmosphere absorbs most of the radiation at the X-ray wavelengths, so its behavior and dynamics can be investigated by knowing these lines accurately. Accurate determination of these lines have become very demanding in recent years with the advent of increasingly powerful X-ray satellite telescopes. The two most powerful telescopes to date are the Chandra X-ray observatory, launched by NASA, in 1999 [10], and the X-ray Multi-Mirror (XMM) Newton telescope, launched by the European Space Agency, recently [11]. The ASTROSAT satellite, launched last year by Indian Space Research Organization (ISRO) is also aiming at exploring the possible X-ray sources in the space [12]. The international X-ray observatory is being planned to be launched in 2021 jointly by NASA, European Space Agency (ESA) and Japan Aerospace Exploration Agency (JAXA) which can cover a large effective area to probe for the X-ray sources [13]. This is why X-ray astronomy remains to be an active research area today. Due to high spectral resolution and sensitivity of the current generation X-ray satellites Chandra and XMM-Newton, it is possible to resolve the He-like ion lines distinctly and use them in the diagnostics for extra-solar objects. Indeed, the He-like ion line ratios are valuable tools in the analysis of high-resolution spectra of a variety of plasmas such as Collisional Ionization Equilibrium (CIE) plasmas or also called coronal plasmas [1]. In such plasmas, ionization occurs due to electron-ion collision processes and the atomic levels are populated mainly by the electron impact. It is commonly assumed that CIE plasmas are optically thin to their own radiation, and there is no external radiation field that affects the ionization balance. However, in some cases, these assumptions are not fulfilled. In this case, recombination-dominated or Photo-Ionization Equilibrium (PIE) plasmas play the important role [1], where ionization takes place due to photons (ionizing radiation). As a result, the atomic levels are populated mainly by the radiative recombination processes directly or by cascading from the upper levels. These plasma are generally over ionized relative to the local electronic temperature and have a much smaller electronic temperature compared to CIE plasmas. That is why collisional excitations out of the ground state are inefficient and the excited levels are populated via the radiative recombination processes.

Accurate knowledge of spectral lines of plasma embedded ions have become increasingly important today due to their observations using the high resolution detectors of the space-based X-ray observatories. Especially, the highly forbidden “triplet” inter-combination, and resonance lines of the He-like and Li-like ions have been used to measure temperature and density of ions and electrons.
in the solar corona. Compared to other ionic iso-
electronic sequences, He-like ions are abundant over the
widest temperature range in collisional plasmas due to
their closed-shell ground state. The most inten-
tive He-like lines correspond to transitions between the
\( n = 2 \) shell and the \( n = 1 \) ground state shell, where \( n \) is the principal quantum number. These He-like
lines were first observed in laboratory for C, F, Mg, Al (see [16]) and later in solar plasmas by the Orbiting
Solar Observatory (OSO) [17] and rocket experiments [18–20]. Gabriel and Jordan had argued for use of a suitable
teoretical many-body method for the identification of
the wavelength of the transition from the metastable
states of order 0 to 150 eV. Some of these transitions were
experimentally verified in a recent work [15], but we shall
describe employed methods for calculations briefly. In
Sec. [IV] we present the results, compare with other
studies and discuss them before summarizing the work
in Sec. [V]. Unless stated otherwise, we have used atomic
units (a.u.) through out this paper.

II. DEBYE MODEL FOR PLASMA SCREENING

The plasma environment mostly consists of ions and
free electrons, which introduce screening effects in the
Coulomb potentials of the embedded atomic systems. As
a result, the atomic electrons are highly influenced
by the external electromagnetic fields compelling the
atomic long range electrostatic potentials to act as short
range screened potentials. For the theoretical study of
the spectroscopy of a plasma embedded atomic system,
the screening effects due to the plasma can be conven-
ciently accounted for, in such a scenario, by defining
suitable model potentials in the atomic Hamiltonian for
the corresponding strength of the plasma. The strength
of the plasma is defined by a coupling parameter (\( \Gamma \))
that measures the interactions between the particles
inside the plasma environment. Debye model [31] is the
most conventional approach used for studying atomic
spectroscopy in low electron-density and high tempera-
ture plasma (weakly coupled plasma; i.e \( \Gamma < 1 \)).

The phenomenon of reduction of ionization potential
(IP) of an atom or an ion in the plasma environment is
known as ionization potential depression (IPD) [36–39].
Accurate determination of this quantity can infer many
useful information such as providing right equation of
state of plasma, estimating radiative opacity of stellar
plasma and inertial confinement fusion plasma, etc. In
most of the previous studies, the electronic structures
of the plasma embedded atomic systems have been in-
vestigated using non-relativistic many-body methods. In
this work, we consider the Dirac-Coulomb (DC) Hamil-
tonian in the relativistic coupled-cluster (RCC) method,
which is explained in the next section, to calculate wave
functions of the atomic states. Moreover, in most of the
previous works the screening effects were taken into
account only through the nuclear potential. We, however,
incorporate screening effects through both the nuclear
and electron-electron interactions for more accurate
description. Its importance was demonstrated recently in
a number of works [40, 41, 60, 67, 68]. In our approach,
the two-body screening potential is expressed in terms
of multiple expansion form and the reduced matrix
elements are used for economical computation as described
in the subsequent section.

In the weakly coupled plasma, the screening effects
seen by an electron located at \( r_i \) in an atomic system due
to the presence of other free electrons inside the plasma
is accounted by an effective potential given as [31]

\[
V_{\text{eff}}(r_i) = e^{-\mu r_i} V_{\text{nuc}}(r_i) + \sum_{j \geq i} e^{-\mu r_{ij}} V_C(r_{ij}),
\]

(1)
where $V_{\text{nuc}}(r_i)$ is the usual nuclear potential of electron in the plasma free atomic system and is estimated by considering the Fermi nuclear charge distribution and $V_C(r_{ij}) = \frac{1}{r_{ij}}$ is the potential due to the two-body Coulomb interactions among the electrons with the Debye screening length $1/\mu$. The inverse screening length $\mu$ value is related with the temperature $T$ and electron density $n_e$ of the plasma as

$$\mu = \left[ \frac{4\pi(1 + Z)n_e}{k_BT} \right]^{1/2} \quad (2)$$

for the Boltzmann constant $k_B$ and the nuclear charge $Z$.

In the multipole expansion, the two-body screened potential can be expressed as

$$V_{ee}(r_i, r_j) = \sum_{j>i} \frac{1}{r_{ij}} e^{-\mu r_{ij}} = 4\pi \sum_{k=0}^{\infty} \frac{I_k(\mu r_<)}{r_{ij}^k} K_{k+\frac{1}{2}}(\mu r_>) \times \sum_{q=-k}^{k} Y_k^q(\theta, \phi) Y_k^q(\theta, \phi), \quad (3)$$

where $I_k(\mu r)$ and $K_{k+\frac{1}{2}}(\mu r)$ are the modified Bessel functions of the first and second kind, respectively, with $r_\perp = \min(r_i, r_j)$, and $Y_k^q(\theta, \phi)$ is the spherical harmonics of rank $k$ with its component $q$. In terms of the Racah operator $(C_k^q)$, the above expression is given by a scalar product as

$$V_{ee}(r_i, r_j) = \frac{1}{r_{ij}} \sum_{k=0}^{\infty} (2k+1) I_k(\mu r_<) K_{k+\frac{1}{2}}(\mu r_>) \times C_k^q(\hat{r}_i) \cdot C_k^q(\hat{r}_j). \quad (4)$$

In terms of the single particle orbital wave functions ($\langle \phi(r_i) \rangle$), the above interaction potential in the spherical coordinate system can be written as

$$\langle \phi_a \phi_b |V_{ee}(r_i, r_j)| \phi_c \phi_d \rangle = (-1)^{j_a+j_b-k} \sum_{k,q} (-1)^{k-q} \times \left( \begin{array}{ccc} j_a & k & j_c \\ -m_a & q & m_d \end{array} \right) \times \langle j_a j_b |V_{ee}^k| j_c j_d \rangle, \quad (5)$$

where the subscripts $a, b, c$ and $d$ stands for the orbitals, $j_a$ are the total angular moment and $m_a$ are their corresponding azimuthal components. Here, the allowed $k$ values should be such that $l_a + l_c + k = \text{even}$ and $l_b + l_d + k = \text{even}$ for the orbital angular momentum quantum number $l$, and they need to satisfy the triangular conditions $|j_a - j_c| \leq k \leq j_a + j_c$ and $|j_b - j_d| \leq k \leq j_b + j_d$. In the above expression, $\langle j_a j_b |V_{ee}^k| j_c j_d \rangle$ is known as the reduced matrix element and is given by

$$\langle j_a j_b |V_{ee}^k| j_c j_d \rangle = (-1)^{j_a+j_b+k+1} \left( \begin{array}{ccc} j_a & k & j_c \\ 1/2 & 0 & -1/2 \end{array} \right) \times \left( \begin{array}{ccc} j_b & k & j_d \\ 1/2 & 0 & -1/2 \end{array} \right) \int_0^\infty \int_0^\infty d r_j d r_j \times \left[ P_a(r_i) P_c(r_j) + Q_a(r_i) Q_c(r_j) \right] \times (2k + 1) \frac{I_k(\mu r_<) K_{k+\frac{1}{2}}(\mu r_>)}{\sqrt{r_{ij}}} \times \left[ P_b(r_j) P_d(r_j) + Q_b(r_j) Q_d(r_j) \right], \quad (6)$$

where $P(r)$ and $Q(r)$ are the large and small components of the Dirac single particle wave function

$$|\phi_a(r)\rangle = \frac{1}{r} \left( \begin{array}{c} P_a(r) \chi_{j_a, m_a}^P(\theta, \phi) \\ iQ_a(r) \chi_{j_a, m_a}^Q(\theta, \phi) \end{array} \right) \quad (7)$$

with the respective angular momentum components $\chi_{j_a, m_a}^P(\theta, \phi)$. In our formalism, we only use the reduced matrix elements $\langle j_a j_b |V_{ee}^k| j_c j_d \rangle$ to reduce the amount of computations.

### III. DETERMINATION OF ATOMIC WAVE FUNCTIONS

We intend to determine ionization potentials (IPs), excitation energies (EEs) and electron affinities (EAs) of the considered He-like ions. The atomic states of the Li-like ions have been constructed in the EA procedure with the He-like ions and their EEs are evaluated by subtracting EAs between two states of these ions. For this purpose, we first calculate the wave function of the ground states of the considered He-like ions and then pursue with determining IPs, EAs and EEs for these ions and also for the Li-like ions. To carry out these calculations in the RCC theory framework, we adopt two distinctly different approaches such as equation-of-motion (EOM) and Fock-space methods. These procedures are described in the mathematical form below.

The considered DC Hamiltonian in our calculation is
TABLE II: Comparison of the calculated excitation energies (EEs) in the He-like systems with the NIST database [42]. Absolute differences from the NIST data are given as $\Delta$ in percentage. All these quantities are given in cm$^{-1}$.

| Excited state | $J$ | $\pi$ | He I | Mg XI | Fe XXV |
|--------------|-----|------|------|-------|--------|
| [1s2s] $^1S$ | 1   | e    | 160490.61 159855.97 | 10748618.12 10736136 | 53607783.11 53527760 |
| [1s2s] $^3S$ | 0   | o    | 166051.24 166277.44 | 10846921.10 10838778 | 53848556.55 53781230 |
| [1s2p] $^3P$ | 2   | o    | 168944.97 169086.77 | 10843480.69 10836388 | 53975073.99 53896600 |
| [1s2p] $^1P$ | 1   | o    | 169080.28 169086.84 | 10840155.63 10832818 | 53854337.09 53777570 |
| [1s3s] $^3S$ | 1   | e    | 183424.43 183236.79 | 12705049.05 12691170 | 63546618.39 63421700 |
| [1s3s] $^1S$ | 0   | o    | 184702.01 184864.83 | 12730224.27 12718304 | 63612913.04 63489000 |
| [1s3p] $^3P$ | 2   | o    | 185422.75 185564.56 | 12732443.48 12717729 | 63584591.83 63490700 |
| [1s3d] $^3D$ | 3   | e    | 185960.11 186101.55 | 12741426.68 12733603 | 63656942.15 63486100 |
| [1s3d] $^3D$ | 2   | e    | 185960.16 186101.55 | 12741036.24 12733223 | 63647766.97 63560700 |
| [1s3d] $^3D$ | 1   | e    | 185966.43 186101.59 | 12740961.94 12733183 | 63647055.72 63561300 |
| [1s3d] $^3D$ | 0   | e    | 185963.12 186104.97 | 12742065.02 12734298 | 63662496.91 63576500 |

where the indices $K$ and $L$ represent for level of excitations with $K \neq L$. The ground state energy ($E_g$) is obtained by

$$\langle \Phi_0 | HT_2 + \frac{1}{2} HT_1^2 | \Phi_0 \rangle = E_g.$$  \(12\)

Considering this as our starting point, we can now generate the excited states and their energies, IPs and EAs of He-like and Li-like ions in three different steps as described in the following subsections.

### A. IPs of He-like

To estimate the IPs of the considered He-like ions, the atomic states of the hydrogen-like (H-like) systems are obtained after removing an electron from the [1s$^2$] configuration using the expression [32]

$$| \Psi_a \rangle = e^T (1 + R_a) | \Phi_0 \rangle,$$  \(13\)

where the reference state is constructed as $| \Phi_a \rangle = a_a | \Phi_0 \rangle$ with $a_a$ is the corresponding annihilation operator and $R_a$ is another RCC operator that takes care of the extra correlation effects accounted through the removed 1s orbital electron while generating $| \Psi_0 \rangle$. The $R_a$ operator can also give rise only the singles and doubles excitations and denoted by $R_a = R_{1a} + R_{2a}$. The IP ($E_a$) and amplitude solving equations for the $R_a$ wave operators are given by

$$\langle \Phi_a | (He)^T (1 + R_a) | \Phi_a \rangle = E_a$$  \(14\)

$$\langle \Phi_a | [(He)^T] R_a | \Phi_a \rangle = -\langle \Phi_a^b | H e^T | \Phi_a \rangle$$  \(15\)
and
\[
\langle \Phi_{da}^{\text{ph}} | (H_{\text{NC}} e^T) - E_a | \Phi_a \rangle R_{a e} = - \langle \Phi_{da}^{\text{ph}} | H e^T | \Phi_a \rangle, \tag{16}
\]
where $| \Phi_a \rangle$s are the singly excited configurations from $| \Phi_a \rangle$ constructed replacing an occupied orbital $a$ by another occupied orbital $b$ and $| \Phi_{da}^{\text{ph}} \rangle$s denote the doubly excited configurations from $| \Phi_a \rangle$, constructed replacing an occupied orbital $a$ by orbital $b$ and exciting an electron from the occupied orbital $d$ to virtual orbital $p$. The above non-linear equations are solved self-consistently along with the energy evaluating equation.

### B. EEs of He-like

The excited states ($| \Psi_K (J, \pi) \rangle$) with angular momentum $J$ and parity $\pi$ is obtained by operating excitation operators $\Omega_K$ on $| \Psi_0 \rangle$ as
\[
| \Psi_K (J, \pi) \rangle = \Omega_K (J, \pi) | \Psi_0 \rangle, \tag{17}
\]
where $K$ corresponds to level of excitations (in the present case it is naturally truncated at the double excitations). Thus, the eigenvalue ($E_L$) and eigenfunctions for the $L^{th}$ excited state are obtained by diagonalizing the equation
\[
\langle \Phi_L (J, \pi) | H^{\text{eff}} \Omega_K (J, \pi) | \Phi_0 \rangle = E_L \langle \Phi_L (J, \pi) | \Omega_L (J, \pi) | \Phi_0 \rangle,
\]
where $H^{\text{eff}} = (H + H T_1 + H T_2 + \frac{1}{2} H T_3^2)$. It is obvious from the above equation that it is imperative to project $| \Phi_L \rangle$ for a definite value of $J$ and $\pi$ in order to get solutions for the respective states and the equation needs to be solved self-consistently for both the singles and doubles excitations. Using the Davidson’s diagonalization algorithm, only the solutions for the lower energy levels are obtained for our interest.

The above method is routinely used in the quantum chemistry, however it is developed by us recently for atomic systems with spherical coordinate system [33]. The bottle-neck for using this method in the spherical coordinate description is that both the Hamiltonian $H$ and the $T$ are expressed in terms of multiple expansion form resulting following type of tensor products
\[
\langle J \pi || [\mathbf{t}^{k_1} \mathbf{u}^{k_2}]^K || J' \pi \rangle = (2K + 1)^{1/2} (-1)^{J + J' + K} \sum_{J''}
\times \left\{ k_1 \ k_2 \ K \right\} \langle J \pi || [\mathbf{t}^{k_1} || J'' \pi \rangle \langle J'' \pi || \mathbf{u}^{k_2} || J' \pi \rangle. \tag{18}\]
Therefore, the resultant operator becomes another tensor with different rank. This complicates to account for the angular momentum coupling between the operators with different allowed intermediate $J''$ states and storing them optimally for carrying out computations. However, this approach refrains from performing calculations using $m$ sublevels. It, thus, allows to embody a large configuration space for more accurate calculations.

### C. EAs of He-like and EEs of Li-like

The EAs of the considered He-like ions are obtained by appending electrons in the valence $s, p, d$ orbitals of the $[1s^2]$ configuration. This can also give atomic states of the Li-like ions. In the Fock-space formalism of RCC theory the corresponding states are expressed as [31, 68]
\[
| \Psi_v \rangle = e^T (1 + S_v) | \Phi_v \rangle, \tag{19}
\]
where the reference state is constructed as $| \Phi_v \rangle = a_v^\dagger | \Phi_0 \rangle$ with $a_v$ representing creation of the valence orbital and
TABLE III: Comparison of EAs of the electrons in the low-lying excited bound states of Mg XI and Fe XXV ions with the NIST database $^{42}$. Absolute differences from the NIST data are given as $\Delta$ in percentage. All these quantities are given in cm$^{-1}$.

| State | This work | NIST $^{42}$ |
|-------|-----------|-------------|
|       | EA (eV)   | EE (eV)     |
| Mg XI |           |             |
| 2S    | 2962134.64| 0           |
| 2P$_{1/2}$ | 2802991.11| 159143.53 |
| 2P$_{3/2}$ | 2798659.31| 163475.33 |
| 3S    | 1278664.89| 1683469.75 |
| 3P$_{1/2}$ | 1231022.75| 173111.89 |
| 3P$_{3/2}$ | 1229758.76| 174354.12 |
| 3D$_{3/2}$ | 1217780.52| 174435.44 |
| 3D$_{5/2}$ | 1217617.80| 174516.84 |
| Fe XXV |           |             |
| 2S    | 16500309.62| 0           |
| 2P$_{1/2}$ | 16110730.72| 389578.90 |
| 2P$_{3/2}$ | 1597474.34| 522835.28 |
| 3S    | 7177704.50| 9322605.12 |
| 3P$_{1/2}$ | 7109738.12| 9378200.00 |
| 3P$_{3/2}$ | 7096659.55| 9459000.00 |
| 3D$_{3/2}$ | 7032196.18| 9480137.72 |
| 3D$_{5/2}$ | 7020171.90| 9480137.72 |

\[ S_v \text{ is the RCC operator that takes into account the correlation effects seen by the valence electron interacting with the other occupied orbitals. In this case too, the } S_v \text{ operator can account only the singles and doubles excitations which is denoted by } S_v \equiv S_{1v} + S_{2v}. \text{ The EA } (E_v) \text{ and amplitude solving equations for the } S_v \text{ wave operators are given by} \]

\[ \langle \Phi_v | (H e^T) \{ 1 + S_v \} | \Phi_v \rangle = E_v \]

\[ \langle \Phi_v | [(H e^T) - E_v] S_v | \Phi_v \rangle = -\langle \Phi_v | H e^T | \Phi_v \rangle \]

\[ \langle \Phi_{vb} | [(H N e^T) - E_v], S_v | \Phi_v \rangle = -\langle \Phi_{vb} | H e^T | \Phi_v \rangle \]

where $| \Phi_{vb} \rangle$ are the singly excited configurations from $| \Phi_v \rangle$ constructed replacing the valence orbital $v$ by an virtual orbital $p$ and $| \Phi_{vb}^{pq} \rangle$ denotes the doubly excited configurations from $| \Phi_v \rangle$, constructed replacing simultaneously the valence orbital $v$ by a virtual orbital $p$ and exciting an electron from the occupied orbital $b$ to virtual orbital $q$. These non-linear equations are also solved self-consistently along with the energy evaluating equation. By taking differences between EAs of different orbitals, EEs of higher excited states of Li-like ions are determined.

IV. RESULTS AND DISCUSSION

We have adopted RCC method to compute IPs and EEs of He and He-like Mg and Fe ions in the weak plasma environment using Debye plasma model. EAs of Li-like Mg and Fe ions are also determined by extending calculations of their He-like ions. In order to validate our calculations, we have also performed calculations of the above quantities in the plasma free environment considering $\mu = 0.0$ and compared them against their corresponding values quoted in NIST database $^{42}$. Most of NIST data are obtained from high precision calculations using more accurate numerical methods that take into
TABLE IV: Variation of electron EAs in cm$^{-1}$ of various low-lying excited bound states in the Mg XI ion with $\mu$ values.

| $\mu$ | 2S$_{1/2}$ | 2P$_{1/2}$ | 2P$_{3/2}$ | 3S$_{1/2}$ | 3P$_{1/2}$ | 3P$_{3/2}$ | 3D$_{3/2}$ | 3D$_{5/2}$ |
|-------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 0.039 | 2877517.44 | 271070.18 | 2713118.82 | 1195955.88 | 1151893.96 | 1150730.12 | 1134767.13 | 1134408.81 |
| 0.044 | 2865436.72 | 2706144.90 | 2701815.34 | 1183545.66 | 1135822.49 | 1134561.61 | 1122350.07 | 1121984.42 |
| 0.046 | 2862227.63 | 2702921.55 | 2698592.14 | 1180424.37 | 1132710.10 | 1131449.44 | 1119208.60 | 1118843.01 |
| 0.047 | 2850949.70 | 2699719.89 | 2695390.59 | 1177366.80 | 1129622.12 | 1128361.70 | 1116090.56 | 1115725.26 |
| 0.048 | 2857107.11 | 2697789.18 | 2693460.00 | 1175496.83 | 1127761.92 | 1126501.62 | 1114211.94 | 1113846.69 |
| 0.049 | 2849628.38 | 2690279.80 | 2685951.01 | 1168289.37 | 1120538.32 | 1119278.56 | 1106913.61 | 1106548.50 |
| 0.052 | 2847296.37 | 2688557.81 | 2684247.13 | 1166654.10 | 1118901.95 | 1117642.32 | 1105259.52 | 1104894.54 |
| 0.054 | 2844338.05 | 2684945.50 | 2680617.04 | 1163194.64 | 1114159.29 | 1110737.77 | 1103872.63 | 1101372.82 |
| 0.056 | 2840269.78 | 2680873.20 | 2676544.99 | 1159294.53 | 1111517.17 | 1107258.09 | 1097791.71 | 1097426.88 |
| 0.057 | 2838128.56 | 2678729.90 | 2674401.77 | 1157245.57 | 1109465.99 | 1105207.09 | 1095716.53 | 1095351.81 |
| 0.058 | 2834752.03 | 2675333.08 | 2671005.16 | 1154007.94 | 1104051.40 | 1100831.74 | 1091491.37 | 1091136.77 |
| 0.061 | 2828778.64 | 2669347.80 | 2665020.32 | 1148302.26 | 1102508.57 | 1098247.64 | 1088646.35 | 1088281.85 |
| 0.063 | 2824348.10 | 2664891.18 | 2660563.96 | 1144070.06 | 1096260.07 | 1092005.35 | 1082345.85 | 1081981.47 |
| 0.065 | 2821954.33 | 2662504.39 | 2658177.30 | 1141796.29 | 1093899.19 | 1090731.71 | 1080448.49 | 1079680.59 |
| 0.076 | 2796870.11 | 2637257.84 | 2632932.55 | 1137985.71 | 1070087.05 | 1068832.04 | 1055794.16 | 1055430.77 |
| 0.095 | 2758527.64 | 2598693.02 | 2594371.15 | 1082006.77 | 1033997.97 | 1032747.58 | 1019068.14 | 1018706.34 |
| 0.125 | 2697116.20 | 2536745.01 | 2532429.44 | 1025375.54 | 977086.88 | 975845.75 | 960876.71 | 960518.02 |
| 0.181 | 2582557.37 | 2420906.92 | 2416609.72 | 923068.56 | 874157.32 | 872392.49 | 857474.08 | 854396.59 |
| 0.357 | 2259081.79 | 2082777.80 | 2078575.58 | 651203.19 | 599695.24 | 598587.08 | 565799.09 | 565487.95 |
| 0.666 | 1738945.01 | 1552044.13 | 1548127.51 | 308087.58 | 251006.98 | 250189.58 | 185554.21 | 185350.79 |
| 0.909 | 1396629.83 | 1190984.40 | 1187379.29 | 138533.91 | 81660.82 | 81140.58 | 1924.47 | 1837.01 |

FIG. 4: Variation of EAs of electrons in various bound states of the Fe XXIV ion with the Debye screening strength ($\mu$) in a.u.

account the contributions from quantum electrodynamics (QED) rigorously. IPs of plasma free He I, Mg XI and Fe XXV systems are presented in Table III along with the NIST data. Our calculations also agree well with the NIST data and they are found to be sub-one percent accurate. Discrepancies in the He-I system are mainly due to poor description of nuclear charge distribution, while they are mainly due to the neglected QED corrections in the other ions. Similarly, EEs of many low-lying transitions of these systems are given in Table III. Nevertheless, we intend to demonstrate the trend of IPs and EEs of these ions with different plasma strengths ($\mu$). As seen in the figures, the trends of these quantities in different states differ from He I to highly charged ions. It is noticed that in the plasma environment the energy level structures are different in these isoelectronic systems and the plasma screening effects in these states also behave differently. IPs decrease gradually till they become zero for some critical value of $\mu$ (say $\mu_c$) beyond which the states transform to continuum. As mentioned before, the corresponding IP beyond which instability occurs is known as IPD. Variations in IPs with $\mu$ values are shown in black line (Fig 1) at certain plasma, while EEs of the higher excited states are shown in color lines (Fig IV). As the fine structure splitting between the degenerate levels are unaffected by the increasing plasma strength, we have plotted some selected states among the excited degenerate levels. From the figure, it is evident that the EEs of different levels gradually decrease with increasing plasma screening and finally merge into the continuum at particular $\mu_c$. This variation is more rapid near ionization limit. For example, as seen in Fig IV, the critical $\mu_c$ value for the [1s3d]$^3D$ state of He I is approx-
TABLE V: Variation of electron EAs (in cm\(^{-1}\)) of various low-lying excited bound states in the Fe XXV ion with \(\mu\) values.

| \(\mu\) | 2S\(_{1/2}\) | 2P\(_{1/2}\) | 2P\(_{3/2}\) | 3S\(_{1/2}\) | 3P\(_{1/2}\) | 3P\(_{3/2}\) | 3D\(_{3/2}\) | 3D\(_{5/2}\) |
|-------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 0     | 16508399.81 | 16110730.71 | 15770474.33 | 717704.00 | 710938.12 | 706956.54 | 7032196.17 | 7021916.70 |
| 0.040 | 16285972.28 | 15896184.71 | 15762938.40 | 696474.13 | 686951.36 | 685647.85 | 681867.85 | 680659.25 |
| 0.041 | 16282388.78 | 15830817.75 | 15759355.68 | 696166.38 | 689347.82 | 685341.69 | 681505.82 | 680356.56 |
| 0.042 | 16275692.91 | 15862166.76 | 15752971.38 | 654985.37 | 668670.71 | 664638.57 | 660880.71 | 659789.78 |
| 0.044 | 16261816.81 | 15878393.56 | 15745148.15 | 694720.18 | 687893.04 | 683920.93 | 680105.62 | 678904.64 |
| 0.045 | 16263991.13 | 15874220.57 | 15740976.42 | 694308.82 | 687486.47 | 683405.94 | 679693.06 | 678417.07 |
| 0.046 | 16263662.47 | 15856529.28 | 15722872.05 | 692570.64 | 685743.08 | 681731.20 | 677948.50 | 676742.13 |
| 0.049 | 16242746.40 | 15852921.16 | 15716978.29 | 692216.47 | 685387.02 | 681318.72 | 677587.61 | 676385.42 |
| 0.050 | 16234837.54 | 15844942.18 | 15711746.31 | 691460.50 | 684606.50 | 680606.47 | 676803.08 | 675601.63 |
| 0.052 | 16225549.25 | 15835690.29 | 15702450.45 | 690528.28 | 683691.63 | 679658.14 | 676842.95 | 675638.49 |
| 0.053 | 16220908.52 | 15831016.67 | 15697777.39 | 689065.27 | 683231.75 | 679226.01 | 675422.90 | 674221.94 |
| 0.054 | 16212954.54 | 15823073.30 | 15689349.99 | 688284.12 | 682450.34 | 678450.96 | 674638.70 | 673437.80 |
| 0.055 | 16208838.17 | 15818303.24 | 15685924.41 | 688879.29 | 682043.34 | 678038.66 | 674229.67 | 673028.94 |
| 0.057 | 16199505.94 | 15805978.97 | 15676351.45 | 687963.91 | 681125.08 | 677120.11 | 673307.13 | 672106.49 |
| 0.058 | 16189189.07 | 15799259.90 | 15666024.72 | 686951.07 | 680110.97 | 676105.85 | 672289.28 | 671087.29 |
| 0.060 | 16183966.35 | 15794106.17 | 15660781.74 | 686438.66 | 679596.40 | 675591.32 | 671772.03 | 670578.35 |
| 0.066 | 16151950.92 | 15761928.26 | 15628698.47 | 683300.78 | 676496.44 | 672445.82 | 668611.70 | 667409.37 |
| 0.1  | 15979834.43 | 15589280.75 | 15546083.86 | 666598.68 | 655023.00 | 655623.54 | 651861.37 | 650481.77 |
| 0.2  | 15471586.71 | 15078244.96 | 1494222.17 | 618073.69 | 610822.26 | 606465.67 | 602345.17 | 601153.01 |
| 1.6  | 9255002.30 | 8659403.99 | 8539383.58 | 1531754.72 | 1509412.74 | 148061.25 | 925126.45 | 918731.58 |
| 2.18 | 7544279.71 | 6829743.19 | 678810.51 | 722812.95 | 46509.67 | 450525.30 | 13083.88 | 10165.00 |
| 2.7  | 601681.55 | 5197119.63 | 5096434.10 | 233105.76 | 12504.03 | 7170.17 | 325126.45 | 297034.42 |
| 2.93 | 4307604.21 | 3285607.86 | 320171.92 | 331.21 | 310516.75 | 1980734.90 | 1911462.43 | 1269811.90 |
| 3.41 | 310516.75 | 70094.38 | 42621.76 | 709370.37 | 310516.75 | 70094.38 | 42621.76 | 709370.37 |

V. CONCLUDING REMARKS

We have applied equation-of-motion and Fock-space coupled-cluster methods in the relativistic framework to investigate the trends of ionization potential and exci-
tation energies of He-like and Li-like Mg and Fe ions in Debye plasma environment. We have considered Debye screening both in the nuclear and two-body Coulomb interaction potential and performed the calculations by carrying out multipole expansion approach in the spherical coordinate system. We found that the ionization potentials in the He-like systems vary faster than electron affinities of the Li-like ions. We have also given explicitly electron affinities of the considered Li-like ions for some intermediate values of plasma strength which can be used for diagnostic of plasma processes. We also observe that atomic energy levels have smaller energy gap for higher plasma strength while their differences increase among the states having same principal and different orbital quantum numbers. However, fine structure splitting among different states are least affected with increasing strength of plasma. These results will be useful in interpreting the laboratory and astrophysical plasma.

Acknowledgment

M. Das acknowledge Department of Science and Technology, Government of India for financial support vide reference No.SR/WOS-A/PM-10/2016 (G) under Women Scientist Scheme to carry out this work. Computations were carried out using Vikram-100 HPC cluster at Physical Research Laboratory, Ahmedabad.

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