Strange Tribaryons as Nona-quark States

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Strange tribaryons as nona-quark (9 quark) states is studied to describe the $S = -1$ resonance $S^0(3115)$ recently discovered in the reaction $K^- + ^4\text{He} \rightarrow S^0 + p$. We have identified $S^0(3115)$ as a member of the flavor 27-plet, in particular, $(F_{\text{flavor}}, I_{\text{isospin}}, J_{\text{spin}}) = (27, 1, 1/2)$ or $(27, 1, 3/2)$. The color-magnetic interaction between quarks favors small multiplets in flavor and spin, which leads to a natural explanation that $I = 1$ is the lowest state among the $S = -1$ tribaryons with $J = 1/2$. Classification of the $S^+\text{-state recently reported as well as possible locations of other light strange tribaryons such as } (10^*, 0, 3/2) \text{ with } S = -1, (8, 2, 1/2) \text{ with } S = -2 \text{ and } (1, 0, 3/2) \text{ with } S = -3 \text{ are also discussed.}

\section{Introduction}

Recently an exotic tribaryon state $S^0(3115)$ has been discovered using the stopped $K^-$ absorption experiment at KEK-PS\textsuperscript{1}

\begin{equation}
K^- + ^4\text{He} \rightarrow S^0 + p.
\end{equation}

The mass of $S^0$ is about 3115 MeV and the decay width is less than 21 MeV. The peak in the proton spectrum is over the background with a significance level, 13 $\sigma$. It was also reported previously that an exotic tribaryon state $S^+(3140)$ with its width less than 23 MeV may be created in the reaction $K^- + ^4\text{He} \rightarrow S^+ + n$. Its significance is, however, not high enough at the moment, 3.7 $\sigma$\textsuperscript{2}. In Table I quantum numbers of the above tribaryons are summarized together with their hadronic and quark compositions.

Possible existence of the tribaryon states as deeply bound kaonic nuclei was originally predicted by Akaishi and Yamazaki\textsuperscript{3}. It is based on the assumption that there is a strong attraction between $\bar{K}$ and the nucleon in the $I = 0$ channel. This leads to $\Lambda(1405)$ as a bound state of $\bar{K}$ and the nucleon in the $I = 0$ channel. This therefore predicts even stronger bound states of $\bar{K}$ with light nuclei\textsuperscript{4}. However, there are several problems to be resolved in this approach, which include (i) high central density of the resulting kaonic nuclei ($\sim 10 \rho_0$) which may invalidate the description using hadronic degrees of freedom.

\begin{table}[h]
\centering
\caption{The quantum numbers of the tribaryons. $S$, $I$, and $Q$ express strangeness, isospin, and electric charge, respectively\textsuperscript{1,2}. Possible classifications of these tribaryons are shown in Fig. 1.}
\begin{tabular}{|c|c|c|c|c|}
\hline
 & mass & width & significance & $S$ & $I$ & $Q$ & hadronic & quark & structure & structure \\
 & (MeV) & (MeV) & & & & & structure & structure \\
\hline
$S^0$ & 3115 & $< 21$ & 13 $\sigma$ & -1 & 1 & 0 & $K^- pnn$ & (3u)(5d)(1s) \\
$S^+$ & 3140 & $< 23$ & 3.7 $\sigma$ & -1 & 0, 1 & 1 & $K^- ppm$ & (4u)(4d)(1s) \\
\hline
\end{tabular}
\end{table}

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and (ii) difficulty to explain that the $I = 1$ state ($S^0$) lighter than the $S^+$ state.

The purpose of this paper is to study the exotic tribaryons on the basis of a quark model. By doing this, we can have a natural solution of the problems (i) and (ii). A key observation is that flavor-multiplets with small dimensions have relatively small masses due to color-magnetic interactions. This leads to a light $I = 1$ flavor multiplet (27-plet) in the $S = -1$ sector. Moreover, we can make several predictions for $S = -1, -2$ and $-3$ tribaryons, which serves as a test of our quark description.

§2. Classification in flavor, isospin and spin space

Let us first consider a system with nine quarks confined in a one-body confining potential or in a bag. Let us further assume that all 9 quarks are in the lowest angular momentum state ($l = 0$) in the potential. Then the quark states are characterized by the quantum numbers in color $SU_C(3)$, flavor $SU_F(3)$, and spin $SU_J(2)$. Imposing the total anti-symmetry of the 9 quarks together with the total color-singletness in $SU_C(3)$, one finds possible irreducible representations allowed by symmetry constraints. This has been worked out by Aerts et al \cite{10} for various multi-quark systems, and we recapitulate the results for nona-quark (9 quark) system in Table II for strangeness $S = 0, -1, -2$, and $-3$.

Table II. Allowed representations for nona-quark systems from the constraints of total anti-symmetry and total color-singletness. For $S = -4, -5, -6$, one can access allowed representations through a useful relation $(Y, F, I, J) \leftrightarrow (-Y, F^*, I, J)$.

| $S$ | $Y$ | $F$ (Flavor) | $(I, J) = ($isospin, spin$)$ |
|-----|-----|-------------|-----------------------------|
| 0   | 3   | 35$^*$      | $(\frac{3}{2}, \frac{1}{2})$ |
|     |     | 64          | $(\frac{3}{2}, \frac{3}{2})$ |
| -1  | 2   | 10$^*$      | $(0, \frac{2}{3})$           |
|     |     | 27          | $(1, \frac{2}{3}), (1, \frac{\bar{2}}{3}), (1, \frac{5}{3})$ |
|     |     | 35          | $(2, \frac{1}{3})$           |
|     |     | 35$^*$      | $(0, \frac{1}{3}), (1, \frac{1}{3})$ |
|     |     | 64          | $(1, \frac{1}{3}), (2, \frac{1}{3})$ |
| -2  | 1   | 8           | $(\frac{1}{2}, \frac{1}{3}), (\frac{1}{2}, \frac{2}{3}), (\frac{1}{2}, \frac{5}{3}), (\frac{1}{2}, \frac{7}{3})$ |
|     |     | 10          | $(\frac{3}{2}, \frac{1}{3})$ |
|     |     | 10$^*$      | $(\frac{3}{2}, \frac{5}{3})$ |
|     |     | 27          | $(\frac{1}{3}, \frac{1}{3}), (\frac{1}{3}, \frac{2}{3}), (\frac{1}{3}, \frac{5}{3}), (\frac{1}{3}, \frac{7}{3}), (\frac{1}{3}, \frac{\bar{2}}{3}), (\frac{1}{3}, \frac{\bar{5}}{3})$ |
|     |     | 35          | $(\frac{2}{3}, \frac{1}{3}), (\frac{2}{3}, \frac{5}{3})$ |
|     |     | 35$^*$      | $(\frac{2}{3}, \frac{1}{3}), (\frac{2}{3}, \frac{5}{3})$ |
|     |     | 64          | $(\frac{1}{3}, \frac{1}{3}), (\frac{1}{3}, \frac{2}{3}), (\frac{1}{3}, \frac{5}{3})$ |
| -3  | 0   | 1           | $(0, \frac{2}{3}), (0, \frac{\bar{2}}{3}), (0, \frac{\bar{5}}{3})$ |
|     |     | 8           | $(0, \frac{1}{3}), (0, \frac{2}{3}), (0, \frac{\bar{2}}{3}), (0, \frac{\bar{5}}{3}), (1, \frac{1}{3}), (1, \frac{2}{3}), (1, \frac{5}{3}), (1, \frac{\bar{5}}{3})$ |
|     |     | 10          | $(1, \frac{1}{3})$           |
|     |     | 10$^*$      | $(1, \frac{\bar{1}}{3})$     |
|     |     | 27          | $(0, \frac{1}{3}), (0, \frac{2}{3}), (0, \frac{\bar{2}}{3}), (1, \frac{1}{3}), (1, \frac{2}{3}), (1, \frac{5}{3}), (1, \frac{\bar{5}}{3}), (2, \frac{1}{3}), (2, \frac{2}{3}), (2, \frac{5}{3})$ |
|     |     | 35          | $(1, \frac{1}{3}), (2, \frac{1}{3})$, $(1, \frac{\bar{1}}{3}), (2, \frac{\bar{1}}{3})$ |
|     |     | 35$^*$      | $(1, \frac{1}{3}), (2, \frac{1}{3})$ |
|     |     | 64          | $(0, \frac{2}{3}), (1, \frac{2}{3}), (2, \frac{2}{3}), (3, \frac{2}{3})$ |
Strange Tribaryons as Nona-quark States

Fig. 1. The 27-plet (left) and the 35\textsuperscript{*}-plet (right) in flavor $SU_F(3)$. Possible location of $S_0^0 (S^+)$ is shown by the triangle (the squares). Note that $Y = S + 3$ for the nona-quark system.

From Table II one finds rather stringent restrictions among $F$, $I$ and $J$. For example, in the $S = -1$ tribaryon channels, the flavor 27-plet does not allow $I = 0$ states. For later convenience, we show the 27-plet and the 35\textsuperscript{*}-plet in Fig. 1. As will be discussed later, the location of a triangle is a candidate for the $S_0^0$ state, while the squares are the possible locations of the $S^+$ state.

§3. Mass formula for the tribaryon states

The states shown in Table II have mass splittings mainly due to the $SU_F(3)$ breaking effect from the quark mass difference ($m_u \sim m_d \neq m_s$), and the dynamical effect from the color-magnetic interaction. The former leads to the splittings among the states with different strangeness, while the latter splits different flavor×spin states through anti-symmetrization of the total wave function. There are also sub-leading effects originating from the interplay between the mass effect and the color-magnetic effect.

To see the above features explicitly, we adopt the mass formula obtained from the MIT bag model.\footnote{Although a specific model is taken here, qualitative aspects discussed below are independent of the details of the model. The Hamiltonian for multi-quark system with all quarks occupying the lowest angular momentum level reads:}

$$H = a_0 + a_2 Y + a_1 \left[ C_3(F) + \frac{1}{3} \vec{J}^2 \right] + a_3 \left[ \left( \vec{I}^2 - \frac{1}{4} Y^2 \right) + \frac{1}{3} \left( \vec{J}_n^2 - \vec{J}_s^2 \right) \right] + a_4 \vec{J}_n^2 + a_5 Y^2. \quad (3.1)$$

Here $Y (= S + 3$ for the nona-quark system) is the hypercharge and $C_3(F)$ is the quadratic Casimir invariant of the $F$-multiplet in $SU_F(3)$. (For example, $C_3(27) = 8$ and $C_3(35\textsuperscript{*}) = 12$.) $\vec{I}$, $\vec{J}$, $\vec{J}_n$ and $\vec{J}_s$ are the operators for the total isospin, the total spin, the total spin for non-strange quarks, and the total spin for strange quarks,
respectively.

The coefficients $a_i$ ($i = 0, \cdots, 5$) are independent of the quantum numbers and are the functions of model parameters, such as the bag pressure $B$, the current quark masses $m_{u,d,s}$, the effective fine-structure constant $\alpha^\text{eff}_s$ and the bag radius $R$. We assume that $m_{u,d} = 0$ in the following.

The first coefficient $a_0$, which originates mainly from the volume and Casimir energies of the bag and the averaged kinetic energy of the 9-quarks, determines an approximate mass of the tribaryons. We will take this as an adjustable parameter to reproduce the experimental mass of $S^0(3115)$. $a_2$ and $a_1$ are the major contributions to cause mass splittings and are written as

$$a_2 = \omega_n - \omega_s + 2 \frac{\alpha^\text{eff}_s}{R} (M_{nn} - M_{ns}), \quad (3.2)$$

$$a_1 = \frac{\alpha^\text{eff}_s}{R} M_{ns}, \quad (3.3)$$

where $\omega_i = [x_i^2 + (m_i R)^2]^{1/2} / R$ is the eigenfrequency of a quark with flavor $i$ confined in the bag. $i = n(s)$ implies the non-strange (strange) quark. $x_n = 2.043$ and $x_s$ is given in $[3]$ $M_{ij}$ are the matrix elements of the color-magnetic interactions given in the Appendix. $a_2$ ($a_1$) is dominated by the effect of $SU_F(3)$ breaking (color-magnetic interaction). On the other hand, $a_{3, 4, 5}$ are proportional to $\alpha^\text{eff}_s \times SU_F(3)$-breaking and have relatively minor contributions to the mass splittings in comparison to $a_{2, 1}$. Complete but lengthy formulas for $a_{3-5}$ in the bag model are given in $[3]$ and will not be recapitulated here.

Instead of trying to determine the bag radius $R$ by minimizing the total energy of the system, we utilize an approximate scaling law obtained from $a_0$; $R_N \simeq (N/3)^{1/4} R_3$ with $R_N$ being the radius of the $N$-quark bag. Taking $R_3 \simeq 1$ fm in the original MIT bag model, we estimate the size of the tribaryon bag as $R_9 \simeq 1.3$ fm. The central baryon number density of tribaryons is $(\lesssim 5 \rho_0)$ in the nona-quark description. This is a number comparable to that for a single baryon and is smaller than that of deeply bound kaonic nuclei by Akaishi and Yamazaki ($\sim 10 \rho_0$). For the strange quark mass, we adopt $m_s = 285$ MeV which was determined to reproduce the mass splittings of octet baryons in $[3]$. As for the effective fine-structure constant, we adopt two typical values in the bag model, $\alpha^\text{eff}_s = 1.0$ and 2.0. The latter is close to the one in the original MIT bag model.

In Table III, we have shown the coefficients $a_{1-5}$ for two different values of $\alpha^\text{eff}_s$ with $R = 1.3$ fm. Crucial observations obtained from Eq. (3.1) together with Table III are as follows:

(i) $a_2$ and $a_1$, which contain first order effects in $SU_F(3)$-breaking or $\alpha^\text{eff}_s$, give major contributions to the mass splittings although $a_3$ is not entirely negligible.

(ii) Multiplets with small dimensions in flavor or in spin have relatively small masses because of the $a_1$-term. In particular, the small dimension in flavor is allowed for larger values of $|S|$ as can be seen from Table III. This may compensate the effect of $a_3$ which gives larger mass to larger $|S|$ states.

(iii) Natural assignment of the $S^0(3115)$ state which has $S = -1$ and $I = 1$, is thus either $J = 1/2$ or $J = 3/2$ states of the 27-plet, $(F, I, J) = (27, 1, \frac{1}{2}$ or $\frac{3}{2})$, in
Strange Tribaryons as Nona-quark States

Table II. This is because they belong to the lowest energy multiplet in the 
$S = -1$ and $I = 1$ channel.

§4. Masses of strange tribaryons

Now we make a quantitative study of the strange tribaryons on the basis of the 
mass formula given in Eq. (3.1). Here, we tentatively identify $S_0(3115)$ with $(27, 1, \frac{1}{2})$ and adjust $a_0$ to reproduce the mass. Identifying $S_0$ with $(27, 1, 3\frac{1}{2})$ does not lead to appreciable change of the spectra.

In Fig. 2(a), only the $a_2$-term is taken into account as an origin of the mass 
splittings. $a_1$ and $a_{3-5}$ are set to be zero. As expected, adding strange quarks 
(increasing the hyper-charge) increases the mass.

In Fig. 2(b), we take into account the flavor dependent effect of the color-magnetic 
interaction (the term proportional to $C_3(F)$ in the $a_1$-term) together with $a_2$-term. 
$a_{3-5}$ are still set to be zero. $a_0$ is readjusted so that the states in $(S, F) = (-1, 27)$ become $3115 \text{MeV}$. As we have pointed out in the previous section, smaller (larger) flavor multiplets are relatively pushed down (up) in mass. Because we have neglected the spin-dependent term in $a_1$, different spin states in the same multiplets 
are degenerate.

Finally, we show the full spectra not only with the spin-dependent part of the 
$a_1$-term but also the $a_{2,3,4,5}$-terms. Fig. 3(a) is the case for $\alpha_{s}^{\text{eff}} = 1.0$ and Fig. 3(b) is the case for $\alpha_{s}^{\text{eff}} = 2.0$. We have shown only the levels with spin $1/2$ and $3/2$ and with isospin smaller than $2$ not to make the figure complicated. Locations of physical 
thresholds to hadronic decays are also indicated by the dashed lines. As compared to Fig. 2(b), one can see spin splittings mainly caused by the spin-dependent part of the color-magnetic interaction in the $a_1$-term.

In Figs. 2 and 3, we have identified the $(F, I, J) = (27, 1, \frac{1}{2})$ state with $S_0(3115)$. 
The mass is about $120 \text{MeV}$ (40 MeV) above the $\Lambda NN$ ($\Sigma NN$) threshold and there 
is no selection rule to forbid the decay to these channels. Therefore, the small 
width of $S_0$ less than $21 \text{MeV}$ may be explained only when the structure of $S_0$ has small overlap with the hadronic final states. Our quark-model description in which the 
9 quarks are confined in a rather compact region of space, could give rise to a natural explanation of the small width, although further qualitative examination is 
necessary. Even if $S_0$ belongs to $(F, I, J) = (27, 1, 3\frac{1}{2})$, the situation discussed above 
is unchanged.

The statistical significance of $S^+(3140)$ is not high enough experimentally as 
shown in Table I. This is why we have not used $S^+$ as an input to determine a key 
parameter $\alpha_{s}^{\text{eff}}$. If we assume the existence of $S^+(3140)$, we have several possible 
scenarios:

Table III. The coefficients $a_{1-5}$ in the Hamiltonian in the unit of MeV. $R$ is taken to be 1.3 fm.

| $\alpha_{s}$ | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ |
|---------|--------|--------|--------|--------|--------|
| 1.0     | -152   | 20     | 6.9    | -2.3   | -0.1   |
| 2.0     | -128   | 40     | 14     | -4.7   | -0.2   |
Case 1: The case where $S^+(3140)$ belongs to $(F, I, J) = (35^*, 0, \frac{1}{2})$. This naturally explains the reason why $S^0$ with $I = 1$ is lighter than $S^+$ with $I = 0$. For example, in the leading order of $\alpha_s^{\text{eff}}$ and the $SU_F(3)$ breaking, the mass splittings between the spin 1/2 and spin 3/2 states read

$$M_{35^*, 0, \frac{3}{2}} - M_{27, 1, \frac{3}{2}} \sim 4 M_{nn} \frac{\alpha_s^{\text{eff}}}{R} > 0,$$

$$M_{35^*, 0, \frac{1}{2}} - M_{27, 1, \frac{1}{2}} \sim 3 M_{nn} \frac{\alpha_s^{\text{eff}}}{R} > 0,$$

However, if we try to reproduce the 25 MeV mass splitting between $S^+$ and $S^0$, one needs to choose at least 2-3 times smaller value for $\alpha_s^{\text{eff}}$ (or 2-3 times larger value for $R$) as compared to the value usually adopted in the bag model. It is noteworthy here that the $N - \Delta$ splitting for the 3-quark system in the bag model is also due to the color-magnetic interaction and is written as $M_\Delta - M_N = 4 M_{nn} \alpha_s^{\text{eff}}/R_3$ which is as large as 300 MeV.

Case 2: The case where $S^+(3140)$ belongs to $(F, I, J) = (35^*, 1, \frac{1}{2})$. The situation is similar to the previous case.

Case 3: The case where $S^+(3140)$ belongs to $(F, I, J) = (27^*, 1, \frac{1}{2})$. In this case, $S^+$ is an isospin partner of $S^0$ and the 25 MeV splitting between the two must originate from the isospin-breaking effect. However, natural isospin splitting is an order of
Strange Tribaryons as Nona-quark States

Case 4: The case where \( S^+ + (3140) \) belongs to \( (F, I, J) = (27, 1, \frac{3}{2}) \). In this case, the 25 MeV splitting can be naturally explained as a result of the color-magnetic spin splittings as can be seen from Fig.3(a). We need further experimental information, in particular, the spins of \( S^0 \) and \( S^+ \), to make precise identification of the multiplets they belong.

Let us turn to some predictions which may serve to test the validity of our description of the tribaryon state. From the Hamiltonian Eq. (3.1), the lightest \( S = -1 \) state must be in the \( (10^*, 0, \frac{3}{2}) \) multiplet. Fig.3 show that the location of this state may be just above (for \( \alpha_s^{\text{eff}} = 1.0 \)) or below (for \( \alpha_s^{\text{eff}} = 2.0 \)) the \( \Sigma NN \) threshold.

We need further experimental information, in particular, the spins of \( S^0 \) and \( S^+ \), to make precise identification of the multiplets they belong.

The mass formula also predicts light states in larger \( |S| \) and smaller isospin channels. For example, in the \( S = -2 \) sector, the color-magnetic effect largely compensates the \( SU_F(3) \) breaking effect in \( (8, \frac{1}{2}, \frac{1}{2}) \). Then it becomes a bound state which is lighter than the \( \Lambda \Lambda N \) threshold even for \( \alpha_s = 1.0 \) as shown in Fig.3(a).

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| (GeV) | \( S = -1 \) | \( S = -2 \) | \( S = -3 \) | (GeV) | \( S = -1 \) | \( S = -2 \) | \( S = -3 \) |
|-------|-------------|-------------|-------------|-------|-------------|-------------|-------------|
| 3.5   | 1 = 0       | 1           | 2           | 1/2   | 3/2         | 2           | 0           | 1           | 2           | 1/2         | 3/2         |
|       | \( \Lambda \Sigma \) | \( \Sigma \) | \( \Lambda \Sigma \) | \( \Sigma \) | \( \Lambda \Sigma \) | \( \Sigma \) | \( \Lambda \Sigma \) | \( \Sigma \) | \( \Lambda \Sigma \) | \( \Sigma \) | \( \Lambda \Sigma \) |
| 3.4   | \( 35^*/2 \) | \( 35^*/2 \) | \( 35^*/2 \) | \( 35^*/2 \) | \( 35^*/2 \) | \( 35^*/2 \) | 35^*/2 | 35^*/2 | 35^*/2 | 35^*/2 |
|       | \( \Sigma \Sigma \) | \( \Sigma \Sigma \) | \( \Sigma \Sigma \) | \( \Sigma \Sigma \) | \( \Sigma \Sigma \) | \( \Sigma \Sigma \) | \( \Sigma \Sigma \) | \( \Sigma \Sigma \) | \( \Sigma \Sigma \) | \( \Sigma \Sigma \) |
| 3.3   | \( 35^*/2 \) | \( 35^*/2 \) | \( 35^*/2 \) | \( 35^*/2 \) | \( 35^*/2 \) | \( 35^*/2 \) | 35^*/2 | 35^*/2 | 35^*/2 | 35^*/2 |
|       | \( \Sigma \Sigma \) | \( \Sigma \Sigma \) | \( \Sigma \Sigma \) | \( \Sigma \Sigma \) | \( \Sigma \Sigma \) | \( \Sigma \Sigma \) | \( \Sigma \Sigma \) | \( \Sigma \Sigma \) | \( \Sigma \Sigma \) | \( \Sigma \Sigma \) |
| 3.2   | \( 35^*/2 \) | \( 35^*/2 \) | \( 35^*/2 \) | \( 35^*/2 \) | \( 35^*/2 \) | \( 35^*/2 \) | 35^*/2 | 35^*/2 | 35^*/2 | 35^*/2 |
|       | \( \Sigma \Sigma \) | \( \Sigma \Sigma \) | \( \Sigma \Sigma \) | \( \Sigma \Sigma \) | \( \Sigma \Sigma \) | \( \Sigma \Sigma \) | \( \Sigma \Sigma \) | \( \Sigma \Sigma \) | \( \Sigma \Sigma \) | \( \Sigma \Sigma \) |
| 3.1   | \( 35^*/2 \) | \( 35^*/2 \) | \( 35^*/2 \) | \( 35^*/2 \) | \( 35^*/2 \) | \( 35^*/2 \) | 35^*/2 | 35^*/2 | 35^*/2 | 35^*/2 |
|       | \( \Sigma \Sigma \) | \( \Sigma \Sigma \) | \( \Sigma \Sigma \) | \( \Sigma \Sigma \) | \( \Sigma \Sigma \) | \( \Sigma \Sigma \) | \( \Sigma \Sigma \) | \( \Sigma \Sigma \) | \( \Sigma \Sigma \) | \( \Sigma \Sigma \) |
| 3     | \( \Lambda \Sigma \) | \( \Lambda \Sigma \) | \( \Lambda \Sigma \) | \( \Lambda \Sigma \) | \( \Lambda \Sigma \) | \( \Lambda \Sigma \) | \( \Lambda \Sigma \) | \( \Lambda \Sigma \) | \( \Lambda \Sigma \) | \( \Lambda \Sigma \) |

Fig. 3. The energy spectrum with all effects. (a) \( \alpha_s^{\text{eff}} = 1.0 \), (b) \( \alpha_s^{\text{eff}} = 2.0 \)

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magnitude smaller, e.g. \( M_{\Sigma^0} - M_{\Sigma^+} \simeq 3 \text{MeV} \) and \( M_{\text{neutron}} - M_{\text{proton}} \simeq 1.3 \text{MeV} \).
Flavor singlet state in the $S = -3$ sector such as $(F, I, J) = (1, 0, \frac{3}{2})$ (the $H$ tribaryon) is also unique in the sense that it can be a bound state below the $\Lambda\Lambda\Lambda$ threshold by 70 MeV (260 MeV) for $\alpha_{\text{eff}}^s=1$ ($\alpha_{\text{eff}}^s=2$). Analogous state in the 6-quark system is the $H$ dibaryon.\textsuperscript{4} Predicting the masses of multi-quark systems is always difficult in any phenomenological quark models. In the present case, the mass of the $H$ tribaryon is predicted relative to the $S^0(3115)$ state and thus is less ambiguous.

§5. Relation to other approaches

First we discuss the Skyrmion description of multi-baryon systems,\textsuperscript{10, 12} since a common concept of the $SU_F(3)$ symmetry and its breaking are shared with our quark descriptions. In the rigid rotator approach of the $SU_F(3)$ skyrmion,\textsuperscript{9} the lowest dimensional $SU_F(3)$ irreducible representation for non-strange and strange tribaryons is shown to be the $35^*$ multiplet.\textsuperscript{10} This is different from our quark description where smaller representations such as 1, 8, 10, $10^*$ and 27 are allowed for strange tribaryons.

An alternative way to analyze the 3-flavor Skyrme model is the bound state approach,\textsuperscript{11} in which the kaon is bound to (multi-)soliton solution\textsuperscript{12}. This shares common physics with the approach of the deeply bound kaonic nuclei ($\bar{K} + $nucleus states).\textsuperscript{13} In particular, in both approaches, (i) the $\Lambda(1405)$ state is well reproduced as a $\bar{K} + N$ bound state,\textsuperscript{13} and (ii) the binding energy of the kaon in the $S = -1$ tribaryon is as large as $O(100)$ MeV\textsuperscript{12}. Therefore, one may be able to have closer comparison between our quark description and the Akaishi-Yamazaki’s hadronic description through the aid of the Skyrmion picture.

It has been emphasized that the diquark correlations are important ingredients in understanding the multi-quark systems,\textsuperscript{14} in particular the pentaquark baryons.\textsuperscript{15} Although a thorough study along this direction is beyond the scope of this paper, we briefly touch upon the group theoretical aspect of the diquark correlation for tribaryons. In the diquark hypothesis, a quark pair in the flavor and color antisymmetric channel is regarded as a diquark cluster in $3^*$ representation in color and flavor. Then the nona-quark system composed of 4 diquarks and an extra quark has decomposition to the irreducible flavor representation as

$$3^* \otimes 3^* \otimes 3^* \otimes 3^* \otimes 3 = 1(3) \oplus 8(8) \oplus 10(2) \oplus 10^*(4) \oplus 27(3) \oplus 35^*,$$  \hspace{1cm} (5.1)

where the numbers in parentheses denote the degeneracy in each multiplet. It turns out that the 35-plet and the 64-plet are not allowed in the diquark construction of the tribaryons. This fact together with Table II implies that the highest isospin states of strange tribaryons, such as $I = 2$ for $S = -1$, $I = 5/2$ for $S = -2$ and $I = 3$ for $S = -3$, are disfavored by the diquark correlation. We have similar case for pentaquark baryons where large isospin states are excluded for the exotic $S = +1$ state.\textsuperscript{**} Spectra and mass splittings of each multiplet in the above require dynamical models of diquarks and their interactions.

\textsuperscript{4} Note that the deeply bound $H$ dibaryon has been ruled out by the experimental discoveries of double hypernuclei.\textsuperscript{3}

\textsuperscript{**} For pentaquark baryons, simple quark models show the decomposition, $3 \otimes 3 \otimes 3 \otimes 3 \otimes 3^* =
Finally, instanton-induced interactions among quarks which have not been taken into account in our simple description may have relevance to study the spectra of the multi-quark system. In fact, it has been pointed out that it has a repulsive effect (opposite to the color magnetic interaction) to the $H$-dibaryon and an attractive effect to the $\Theta^+$ pentaquark.\footnote{\textbf{1\hspace{1em}2}\oplus 8(8)\oplus 10(4)\oplus 10^*(2)\oplus 27(3)\oplus 35, while the diquark picture gives $3^\ast\circ 3^\ast\circ 3^\ast = 1\oplus 8(2)\oplus 10^*$. Therefore, 10-plet, 27-plet and 35-plet are not allowed in the diquark picture. Thus the exotic $S = +1$ state is uniquely assigned to the anti-decuplet in which the $S = +1$ state is isospin singlet.} It would be quite interesting to study how the instantons modify the spectral structure discussed in this paper.

\section*{§6. Summary}

We have studied the strange tribaryons as nona-quark states with a compact spatial size, which yields the plausible value of the central density ($\lesssim 5\rho_0$) within the quark description. Assuming that all the 9 quarks are in the lowest orbit in a one-body potential, we have identified the recently discovered $S^0(3115)$ state as a member of the flavor 27-plet, in particular, $(F, I, J) = (27, 1, \frac{1}{2})$ or $(F, I, J) = (27, 1, \frac{3}{2})$. Due to the anti-symmetrization of the 9-quark wave function, smaller flavor multiplets appear for larger hypercharge, e.g. 1 in $S = -3$, 8 in $S = -2$, and 10$^*$ and 27 in $S = -1$.

The color-magnetic interaction, after the anti-symmetrization of the wave function, favors small multiplets in flavor and spin. This leads to a natural explanation that, in the $S = -1$ sector, $(F, I, J) = (27, 1, \frac{1}{2})$ is the lowest mass state for $I = 1$ and and $(F, I, J) = (10^*, 0, \frac{3}{2})$ is the lowest mass state for $I = 0$.

We have also discussed possible classification of the $S^+(3140)$ state: $(35^*, 0, \frac{1}{2})$, $(35^*, 1, \frac{1}{2})$, $(27, 1, \frac{1}{2})$, $(27, 1, \frac{3}{2})$. To make a quantitative comparison, one needs more experimental information, in particular the spins of $S^0$ and $S^+$. To check the validity of our classification and identification, searching the light strange tribaryons such as $(10^*, 0, \frac{3}{2})$ with $S = -1$, $(8, \frac{1}{2}, \frac{1}{2})$ with $S = -2$ and $(1, 0, \frac{3}{2})$ with $S = -3$ are proposed. We especially call an attention to the $(1, 0, \frac{3}{2})$ state in the $S = -3$ channel (the H tribaryon), which appears as a relatively deep bound state without the details of choosing model parameters.

Finally, we emphasize that experimental and theoretical studies of strange tribaryons together with other multi-quark system such as the strange dibaryons and pentaquarks may open a new window to the physics of exotic hadrons in QCD.

\section*{Acknowledgments}

We are grateful to members of hadron physics group in Univ. of Tokyo for discussions. We also thank M. Iwasaki and R. Hayano for information and discussions and S.S. thanks K. Iida and T. Doi for helpful communications. This work was partially supported by the Grants-in-Aid of the Japanese Ministry of Education, Culture, Sports, Science, and Technology (No. 15540254 and No. 15740137).
Appendix

We define the matrix elements of color magnetic interaction following

\[ M_{ij} = 3 \frac{\mu(m_i, R) \mu(m_j, R)}{R^2} I(m_i R, m_j R), \]
\[ \mu(m_i, r) = \frac{r - 4 \omega_i r + 2 m_i r - 3}{6 \omega_i (\omega_i r - 1) + m_i r^2}, \]
\[ I(m_i R, m_j R) = 1 + 2 \int_0^R \frac{dr}{r} \mu(m_i, r) \mu(m_j, r), \]

where \( \mu(m_i, r) \) is the magnetization density of a quark with mass \( m_i \). Here \( M_{nn} = 0.177 \) in the above definition.

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