Dynamic modeling and experimental verification of a
cable-driven continuum manipulator with cable-constrained
synchronous rotating mechanisms

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Abstract The cable-driven segmented manipulator
with cable-constrained synchronous rotating mech-
anisms is a new type of continuum manipulator,
which has large stiffness and fewer motors, and thus
exhibits excellent comprehensive performance. This
paper presents a dynamic modeling method for this
type of manipulator to analyze the effect of the fric-
tion and deformation of the cables on the dynamical
behaviors of the system. First, the driving cables are
modeled based on the ALE formulation, strategies to
detect stick–slip transitions are proposed by using a
trial-and-error algorithm, and the stiff problem of the
dynamic equations is released by a model smoothing
method. Second, the dynamic modeling method for
rigid links is presented using quaternion parameters.
Third, the connecting cables are modeled by torsional
spring–dampers, and the frictions between the con-
necting cables and conduits are considered based on
a modified Coulomb friction model. Finally, numerical
results are presented and verified by comparison with
the experiment results. The study shows that friction
and cable deformation play an important role in the
dynamical behaviors of the manipulator. Due to these
two factors, the constant curvature bending of the seg-
ments does not remain.

Keywords Multibody dynamics · Cable-driven
manipulator · ALE formulation · Stick–slip friction ·
Model smoothing method

1 Introduction

Cable-driven continuum manipulators have attracted
the attention of many researchers. Inspired by elements
of nature such as octopus tentacles and elephant trunks,
continuum manipulators have many degrees of free-
dom (DOFs) and slim bodies, which enable them to
work in confined spaces for operations requiring dext-
erity. Cable-driven continuum manipulators have been
applied in the nuclear field [4], medical applications [5],
and gas turbine engines [6]. Rone et al. [29] proposed a
dynamic modeling method to capture curvature vari-
ations along a segment of a manipulator using a finite set
of kinematic variables. Roesthuis and Misra [28] pro-
posed a model to approximate the shape of a continuum
manipulator by a serial chain of rigid links connected
by flexible rotational joints. Qi et al. [27] presented a
kinematic control method for continuum manipulators using a fuzzy model-based approach. Falkenhahn et al. [7] developed a dynamic controller for continuum manipulators and achieved good and intuitive dynamic behavior. Bajo and Simaan [1] studied the modeling, sensing, and control of a multi-backbone continuum manipulator based on a hybrid motion/force control method. Lakhal et al. [16] investigated the real-time solution of the complex kinematics of a class of continuum manipulators. Liu et al. [21] established an efficient dynamic model for a cable-driven continuum manipulator based on the principle of virtual power. Yuan et al. [39] presented a comprehensive static model for cable-driven multi-section continuum manipulators. Yang et al. [37] established a virtual work-based static model to study the deformation of a continuum robot with a generic rod-driven structure. Li et al. [18] compared three designs of continuum manipulators at the mechanism level from the kinematics point of view. Korayem et al. [13–15] developed an approach for trajectory planning of cable-suspended parallel robots using optimal control methods. Aiming at on-orbit services, Walker’s group [32] proposed a long and thin continuum manipulator. Jing et al. [10] reviewed the kinematic analysis and dynamic stable control of continuum space manipulators. Huang et al. [9] studied the statics of a continuum space manipulator with nonconstant curvature based on a pseudo-rigid-body 3R model. Xu et al. [35] proposed a modified modal method to solve the mission-oriented inverse kinematics of continuum space manipulators.

The above-mentioned continuum manipulators can be divided into two categories. The first type is called a fully driven rigid continuum manipulator, which is made up of multiple rigid links with universal joints, and each joint is driven by three cables (Fig. 1a). The second type is named as a segmented elastic continuum manipulator, which consists of multiple segments of elastic backbone, while each segment is driven by three cables, too (Fig. 1b). Each type has advantages and disadvantages. The first type can provide large stiffness, but it needs many motors; on the contrary, the second type needs fewer motors, but it can hardly provide large stiffness. Aware of this, Xu’s group [19,20] proposed a new cable-driven continuum manipulator with cable-constrained synchronous rotating mechanisms (CCSRM). As shown in Fig. 2, a segment of CCSRSM is made up of several rigid links interconnected by universal joints, and each joint is connected with adjacent joints by connecting cables, which enable the joints to rotate synchronously. The CCSRSM is a new mechanical structure that can greatly reduce the number of motors and improve the stiffness of the manipulator. Li et al. [17] analyzed the effect of friction and elastic deformation of the connecting cables on the CCSRSM dynamical characteristics. Peng et al. [23,24] proposed a dynamic modeling and trajectory tracking control method for this type of continuum manipulator. Ma et al. [22] proposed a dynamic model for this manipulator based on the methods of multibody system dynamics. However, to the authors’ knowledge, no public literature systematically deals with the effects of friction as well as deformation of driving and connecting cables on the dynamic behaviors of this type of manipulator. In real mechanical systems, these two factors cannot be ignored [31,34,43,44], as they may affect the motion accuracy of this type of manipulator and restrict its further development.

The difficulties for dynamic modeling of the cable-driven continuum manipulator with CCSRSM include three aspects. (1) This type of manipulator includes a great number of cables. Traditional contact detection methods for cables [38] introduce hundreds of thousands of DOFs, and they are difficult to solve. Recently, Peng et al. [25] proposed an arbitrary Lagrangian Eulerian (ALE) formulation for the dynamic modeling of cable-driven mechanisms, which can significantly...
reduce the number of DOFs. However, if all of the cables, including driving and connecting cables, are modeled by the ALE formulation, a large number of DOFs are still needed. (2) There are many frictional contact points between the cables and rigid links, whose stick–slip events should be detected during the solution of dynamic equations [11, 12]. (3) Due to the high stiffness of cables, the dynamic equations of the manipulator are stiff; hence, they are often addressed by implicit (stiff) solvers [30, 40], which may have difficulties in the stick–slip transition points.

This paper presents a dynamic modeling method for the cable-driven continuum manipulator with CCSRM considering friction as well as deformation of all the cables. Several methods are proposed to address the above three difficulties. The ALE method and a torsional spring–damper model are applied to significantly reduce the number of DOFs introduced by dynamic modeling of cables. A trial-and-error strategy is proposed to detect the stick–slip transitions of the driving cables and holes. The stiff problem of the dynamic equations is released by a model smoothing method [41], after which the dynamic equations can be solved by explicit solvers without difficulties at stick-slip transition points. Compared with previous dynamic modeling methods for the continuum manipulator with CCSRM, the proposed methods can comprehensively analyze the effects of friction as well as cable deformation on the dynamic characteristics of the manipulator. This has great significance in future studies, as it helps to further optimize the mechanism based on the dynamic characteristics, and can potentially improve the control accuracy by imposing dynamics-based compensation.

The rest of this paper is organized as follows. Section 2 presents the dynamic modeling method for driving cables based on the ALE formulation, and strategies to detect stick–slip transitions are proposed. Section 3 presents the dynamic modeling method for rigid links using the quaternion parameters. In Sect. 4, the connecting cables of the manipulator are modeled by torsional spring–dampers. Section 5 presents the solution procedure for the dynamic equations. In Sect. 6, the results are presented and verified by comparison with experiments; finally, in the last section, the main conclusions from this paper are drawn.

2 Dynamic modeling of driving cables via ALE formulation

The cable-driven continuum manipulator studied in this paper is illustrated in Fig. 3. The manipulator consists of a chain of rigid links interconnected by universal joints. The links and joints are labeled sequentially. The rigid links are divided into several segments, within which the universal joints are connected, one by one, through connecting cables, which makes them rotate synchronously. Each segment of rigid links is driven by three driving cables, and the driving component is fixed on the ground. To obtain the dynamic model of the continuum manipulator, it is divided into three parts: driving cables, rigid links, and connecting cables. These are described in detail below.

2.1 Dynamic equations of an ALE element

The ALE formulation has the advantage of sufficiently reducing the number of DOFs introduced by
Fig. 3  Cable-driven continuum manipulator with cable-constrained synchronous rotating mechanisms (CCSRM)

![Diagram](image)

**Fig. 4**  Generalized coordinates of an ALE cable element

The dynamic modeling of the cables, so it is introduced to model the driving cables. As shown in Fig. 4, the vector of generalized coordinates of an ALE cable element is expressed as

\[ q_{\text{ele}} = \begin{bmatrix} r_1^T, r_2^T, s_1, s_2 \end{bmatrix}^T, \tag{1} \]

where \( r_k = [x_k, y_k, z_k]^T \) \((k = 1, 2)\) is the vector of global positions of node \( k \), and \( s_k \) is the material coordinate of the corresponding node.

An ALE node can degenerate into a Lagrangian or Eulerian node through some special constraints. A Lagrangian node has a constant material coordinate, but its position can move, i.e., \( s_k - s_0 = 0 \). An Eulerian node is fixed on the rigid links or ground, but its material coordinate can vary, i.e., \( r_k - r_0 = 0 \).

The position vector of an arbitrary point in the element can be written as

\[ r = N_r q_r, \tag{2} \]

where

\[ \begin{aligned} \begin{bmatrix} q_r \end{bmatrix} &= \begin{bmatrix} r_1^T, r_2^T \end{bmatrix}^T, \\
N_r &= \begin{bmatrix} N_1 I_3, N_2 I_3 \end{bmatrix}, \\
N_1 &= (1 - \zeta) / 2, \\
N_2 &= (1 + \zeta) / 2, \\
\zeta &= \frac{2s_1 - s_2 - s_1}{s_2 - s_1}, \end{aligned} \tag{3} \]

in which \( N_1 \) and \( N_2 \) are shape functions, and \( \zeta \) is the natural coordinate of the arbitrary point. Taking the first and second derivatives of Eq. (2) with respect to time yields

\[ \begin{aligned} \dot{r} &= N_r \ddot{q}_r, \\
\ddot{r} &= \begin{bmatrix} \dot{N}_r \ddot{q}_r + \ddot{r}_s, \\
N &= \begin{bmatrix} \frac{\partial N_r}{\partial s_1} q_r, \frac{\partial N_r}{\partial s_2} q_r \end{bmatrix}, \\
\ddot{r}_s &= 2 \left( \frac{\partial^2 N_r}{\partial s_1^2} \dot{s}_1^2 + 2 \frac{\partial^2 N_r}{\partial s_1 \partial s_2} \dot{s}_1 \dot{s}_2 + \frac{\partial^2 N_r}{\partial s_2^2} \dot{s}_2^2 \right) q_r, \end{aligned} \tag{4} \]

where the additional acceleration \( \ddot{r}_s \) results from the change of the material coordinates.

The dynamic equations of the ALE cable element can be obtained as [25]

\[ M_{\text{ele}} \ddot{q}_{\text{ele}} + Q_f + Q_e + Q_p = 0. \tag{5} \]
contact problems can be written as a friction model. The Coulomb dry friction model for point contact transitions and the frictional forces acting at the Eulerian node can be determined; thus, the stick–slip states and calculate static frictional forces during the transitions of the contact points from slipping to sticking or reversed slipping [33,42]. Fortunately, by using the ALE formulation, the wrap angles, relative tangential velocities, and tensions of cable elements can be determined; thus, the stick–slip transitions and the frictional forces acting at the Eulerian nodes can be solved independently by a trial-and-error algorithm.

\[
\begin{aligned}
M_{ele} &= \frac{s_2-s_1}{2} \int_{-1}^{1} \rho A N T N d\xi, \\
Q_f &= \frac{s_2-s_1}{2} \int_{-1}^{1} N^T f d\xi, \\
Q_e &= \frac{s_2-s_1}{2} \int_{-1}^{1} \left( \left( \frac{\partial e}{\partial q_{ele}} \right)^T E A (k e + \beta \dot{e}) \right) d\xi, \\
Q_p &= \frac{s_2-s_1}{2} \int_{-1}^{1} \rho A N T \tau d\xi, \\
e &= \frac{1}{2} \left( \frac{\partial p}{\partial s} \frac{\partial s}{\partial v} - 1 \right), \\
k &= \begin{cases} 1, & \text{if } \varepsilon > 0, \\ 0, & \text{otherwise}. \end{cases}
\end{aligned}
\] (6)

in which \(M_{ele}\) is the generalized mass matrix, \(Q_f\) is the vector of generalized external forces including friction, \(Q_e\) is the vector of generalized elastic forces, \(Q_p\) is the vector of additional inertial forces, \(f\) is the vector of external forces, \(E\) is the Young’s modulus, \(A\) is the cross-sectional area, \(\rho\) is the density of the cable, \(\varepsilon\) is the axial strain, \(k\) represents that the cable can resist tension but not compression, and \(\beta\) is the Rayleigh damping factor.

2.2 Frictional forces between driving cables and holes

As the thickness of the disk is small enough compared with the element length, the hole can be viewed as an Eulerian node, while the cable between two adjacent holes can be meshed as an ALE cable element. Thus, the following constraint equation of Eulerian node \(i\) is obtained:

\[
r_i - r_{hole,i} = 0. \tag{7}
\]

The Coulomb dry friction model can simulate static friction and capture stick–slip motions; hence, it is adopted to describe the frictional forces acting at Eulerian nodes. The Coulomb dry friction model for point contact problems can be written as

\[
f = \begin{cases} -\mu N \text{sgn}(v_t), & v_t \neq 0, \\
- \min \left( |F_t|, \mu' N \right) \text{sgn}(F_t), & v_t = 0, \end{cases} \tag{8}
\]

where \(f\) and \(N\) are frictional force and normal contact force, respectively, \(\mu\) and \(\mu'\) are kinetic and static friction coefficients, respectively, \(v_t\) is the relative tangential velocity, and \(F_t\) is the resultant force acting at the point in the tangential direction. In numerical simulations, when \(|v_t|\) is smaller than a tiny tolerance \(v_e\), it is viewed as zero, as shown in Fig. 5.
As shown in Fig. 7, the wrap angles of node $i$ can be calculated as

$$\theta_i = \pi - \arccos \left( \frac{(r_{i-1} - r_i) \cdot (r_{i+1} - r_i)}{|r_{i-1} - r_i| |r_{i+1} - r_i|} \right).$$  (12)

The tangential velocity of the cable relative to the hole in node $i$ is the negative of the derivative of the material coordinate, i.e., $v_{\tau i} = -\dot{s}_i$. The tension of element $i$ can be solved as

$$T_i = EA \left( k_i \varepsilon_i + \beta \dot{\varepsilon}_i \right),$$  (13)

where $\varepsilon_i$ is the axial strain of element $i$, which is determined by Eq. (6). After obtaining the above variables, the frictional forces acting at different Eulerian nodes can be calculated independently by Eq. (9).

2.4 Model smoothing method for modification of dynamic equations of ALE cable elements

Due to the high stiffness of the driving cables, the dynamic equations of the system are stiff, and thus, they are often solved by implicit solvers, such as a backward differentiation formula (BDF) or implicit Runge-Kutta methods (Radau IIA). However, implicit solvers must calculate the Jacobian matrix of dynamic equations with respect to generalized coordinates and velocities, which is related to the partial derivative of frictional force with respect to relative tangential velocity. As shown in Fig. 5, in the transition points from slipping to sticking and reversed slipping, the partial derivative does not exist, so implicit solvers may have trouble solving dynamic equations with stick–slip friction. To avoid the calculation of the above Jacobian matrix, combining with the model smoothing method proposed by Qi [41], the explicit solver can be applied.

High-frequency elastic vibrations in the system arise from strains which change rapidly over time. Therefore, in a time interval $(t, t + h)$, strain at time $\tau$ can be approximated as

$$\varepsilon_\tau = \varepsilon_t + (\tau - t) \dot{\varepsilon}_t + \frac{1}{2} (\tau - t)^2 \ddot{\varepsilon}_t.$$  (14)

The time-averaged strain in this short time interval can then be expressed as

$$\bar{\varepsilon} \Delta t = \frac{1}{h} \int_{t}^{t+h} \varepsilon_\tau \, d\tau = \varepsilon_t + h \dot{\varepsilon}_t + \frac{h^2}{6} \ddot{\varepsilon}_t.$$  (15)

Similarly,

$$\bar{\dot{\varepsilon}} \Delta t = \frac{1}{h} \int_{t}^{t+h} \dot{\varepsilon}_\tau \, d\tau = \dot{\varepsilon}_t + \frac{h}{2} \ddot{\varepsilon}_t,$$  (16)

where $h$ is the short time period, which is also called the model smoothing step.

The implementation of model smoothing method is substituting $\varepsilon$ and $\dot{\varepsilon}$ in Eq. (6) by $\bar{\varepsilon}$ and $\bar{\dot{\varepsilon}}$, which leads to

$$Q_e = \frac{s_2 - s_1}{2} \int_{-1}^{1} \left( \frac{\partial \varepsilon}{\partial q_{\text{ele}}} \right)^T E A \left( k \varepsilon + \beta' \dot{\varepsilon} + \gamma \ddot{\varepsilon} \right) d\varsigma,$$  (17)

where $\beta' = \beta + k h, \gamma = \frac{\beta h}{2} + k h^2$, and the subscript $i$ is neglected.

As $\varepsilon = \varepsilon (q_{\text{ele}})$, the following equations can be generated:

$$\begin{align*}
\dot{\varepsilon} &= \varepsilon_{q_{\text{ele}}} \dot{q}_{\text{ele}}, \\
\ddot{\varepsilon} &= \ddot{\varepsilon}_{q_{\text{ele}}} \dot{q}_{\text{ele}} + \varepsilon_{q_{\text{ele}}} \ddot{q}_{\text{ele}}.
\end{align*}$$  (18)
where \( \varepsilon_{\text{ele}} = \frac{\partial \varepsilon}{\partial q_{\text{ele}}} \) is a Jacobian matrix. Substituting Eq. (18) into Eqs. (17) and (5) leads to

\[
(M_{\text{ele}} + M_{\text{elastic}}) \ddot{q}_{\text{ele}} + Q_f + Q'_e + Q_p = 0, \tag{19}
\]

with

\[
\begin{align*}
M_{\text{elastic}} &= \frac{s_2-s_1}{2} \int_{-1}^{1} \left( e^T_{\text{ele}} E A \gamma \varepsilon_{\text{ele}} \right) d\varsigma,
Q'_e &= \frac{s_2-s_1}{2} \int_{-1}^{1} \left( e^T_{\text{ele}} E A \left( k \varepsilon + \beta' \dot{\varepsilon} + \gamma \dot{\varepsilon}_{\text{ele}} \dot{q}_{\text{ele}} \right) \right) d\varsigma.
\end{align*}
\]

(20)

Comparing Eqs. (5) and (19), it can be seen that additional inertia and damping terms are added to the dynamic equations of the ALE element. Thus, the natural frequency of the element is reduced accordingly, and the greater the parameter \( h \), the fewer high-frequency components remain. For general problems, the model smoothing step \( h \) can be selected between 0.1 s and 0.001 s.

It has been demonstrated that the model smoothing method can reduce high-frequency components and amplitudes, which we are usually not concerned about, and does not affect the rigid motion of the system (the modal with zero-order frequency) [41]. The method’s procedure is remarkably concise and is straightforward to implement during the modeling process. Thus, it can be viewed as a new approach for the numerical solution of stiff dynamic equations.

3 Dynamic modeling of rigid links based on quaternion parameters

3.1 Kinematics of rigid links with quaternion parameters

Quaternion parameters can avoid singularities and reduce the calculation of trigonometric functions to improve calculational efficiency [36]; hence, they are adopted to describe the rotations of the rigid links. As illustrated in Fig. 8, the coordinate system \( C_i x'_i y'_i z'_i \) is a body-fixed reference frame on link \( i \). The generalized coordinate vector of rigid link \( i \) can be expressed as

\[
q_i = \begin{bmatrix} q_{C_i}^T, q_{ei}^T \end{bmatrix}^T,
\]

(21)

where \( q_{C_i} = [x_i, y_i, z_i]^T \) is the generalized coordinate vector of the mass center of link \( i \), and \( q_{ei} = \)

![Fig. 8 Coordinates of a rigid link](image)

[\( e_{0i}, e_{1i}, e_{2i}, e_{3i} \)]\(^T\) is the vector of quaternion parameters. The global position vector of an arbitrary point \( A \) on the rigid link can be expressed as

\[
x = q_{C_i} + R_i^T x'_{C_i},
\]

(22)

where \( x'_{C_i} \) is the position vector of arbitrary point \( A \) in the body-fixed reference frame, and \( R_i \) is a rotation matrix.

It can be derived from Eq. (22) that the relative rotation matrix of link \( i \) with respect to link \( i-1 \) is

\[
R_{i/(i-1)} = R_i R_{i-1}^T,
\]

(23)

where the subscript \( i/(i-1) \) means link \( i \) with respect to link \( i-1 \). Thus, the vector of relative rotation angles of link \( i \) with respect to link \( i-1 \) can be determined as

\[
\theta_{i/(i-1)} = f_{R-\theta} \left( R_{i/(i-1)} \right),
\]

(24)

where the function of the rotation matrix to Euler angles \( f_{R-\theta} \) is related to the rotation sequence.

The vector of angular velocities of link \( i \) in the corresponding body-fixed reference frame is

\[
\omega'_i = G_i \dot{q}_i,
\]

(25)
where
\[
\begin{align*}
\omega'_i &= \begin{bmatrix} \omega'_{1i} & \omega'_{2i} & \omega'_{3i} \end{bmatrix}^T, \\
G_i &= \begin{bmatrix} 0_{3 \times 3}, & G_{ei} \end{bmatrix}, \\
G_{ei} &= \begin{bmatrix} -2e_{1i} & 2e_{0i} & 2e_{3i} - 2e_{2i} \\
-2e_{2i} - 2e_{3i} & 2e_{0i} & 2e_{1i} \\
-2e_{3i} & 2e_{2i} & -2e_{1i} - 2e_{0i} \end{bmatrix}.
\end{align*}
\] (26)

The vector of relative angular velocities of link \( i \) with respect to link \( i - 1 \) in the body-fixed reference frame of link \( i \) is
\[
\omega_{i/(i-1)} = \omega'_i - R_{i/(i-1)} \omega'_{i-1}.
\] (27)

According to the Euler kinematical equations,
\[
\omega_{i/(i-1)} = B_{i/(i-1)} \dot{\theta}_{i/(i-1)},
\] (28)

where the matrix \( B_{i/(i-1)} \) is determined by the Euler rotation sequence and the vector of relative rotation angles \( \theta_{i/(i-1)} \). From Eq. (28), the following equation can be generated:
\[
\dot{\theta}_{i/(i-1)} = B_{i/(i-1)}^{-1} \omega_{i/(i-1)}.
\] (29)

3.2 Dynamic equations of a rigid link

The kinetic energy of rigid link \( i \) can be expressed as
\[
T_i = \frac{1}{2} \dot{q}_i^T m_i \ddot{q}_i,
\] (30)

with mass matrix
\[
m_i = \begin{bmatrix} m_i I_3 & 0_{3 \times 4} \\
0_{4 \times 3} & G_{ei}^T J'_i G_{ei} \end{bmatrix},
\] (31)

in which \( m_i \) is the mass of link \( i \), and \( J'_i \) is the inertia matrix about the corresponding body-fixed reference frame. The vector of generalized forces acting on rigid link \( i \) is
\[
Q_i = \begin{bmatrix} F_i \\
G_{ei}^T M'_i \end{bmatrix},
\] (32)

in which \( F_i \) and \( M'_i \) are the vectors of forces and moments, respectively, which include external applied forces and moments such as from the connecting cables. Thus, the dynamic equations of a free rigid link can be obtained as
\[
m_i \ddot{q}_i + \ddot{Q}_i = 0.
\] (33)

where
\[
\ddot{Q}_i = m_i \dddot{q}_i - \frac{\partial T_i}{\partial \dot{q}_i} - Q_i.
\] (34)

3.3 Constraint equations of rigid links

The following three types of kinematic constraint equations of rigid links will be used to obtain the dynamic equations of the whole multibody system. The first type of constraint is generated from the property of quaternions, i.e.,
\[
g_{ei} = q_{ei}^T q_{ei} - 1 = 0.
\] (35)

The second type is the position constraint in the universal joints, which means that the center points of a universal joint on two adjacent rigid links coincide, i.e.,
\[
g_{p,i,i+1} = q_{Ci} + R_i^T x'_{Ci} - q_{C(i+1)} - R_{i+1}^T x'_{C(i+1)} = 0.
\] (36)

The third type is the orientation constraint in the universal joints,
\[
g_{o,i,i+1} = (R_i^T o'_{i})^T R_{i+1}^T o'_{i+1} = 0,
\] (37)

where \( o'_{i} \) and \( o'_{i+1} \) are unit vectors on rigid links \( i \) and \( i + 1 \), respectively. The geometric relationship in a universal joint is shown in Fig. 9.
4 Dynamic modeling of connecting cables

As shown in Fig. 10, there are two kinds of cable-constrained synchronous rotating mechanisms, which we call large and small “S” mechanisms, and the types of friction acting on the connecting cables differ according to these. The connecting cables of small “S” mechanisms suffer from stick frictional forces of the pulleys, and slip motions can be ignored. The connecting cables of large “S” mechanisms are subjected to slip or stick–slip frictional forces from the pulleys and conduits. Thus, the dynamic modeling of these two kinds of mechanism is different. To avoid further use of cable elements, which introduce numbers of DOFs to the multibody system, the dynamic modeling of the connecting cables differs from that of the driving cables, as we next explain.

4.1 Dynamic modeling of connecting cables for small “S” mechanisms

An initial pre-tensioning force is applied to ensure the connecting cables are always in tensioned states. As illustrated in Fig. 11, consider two adjacent joints labeled $i$ and $j$ ($j = i + 1$), with connecting cables. If the joints rotate with certain angles, the strains of the cables will change as follows:

\[
\begin{align*}
\varepsilon_1 &= \varepsilon_0 - r_{sc} (\theta_j - \theta_i) / l_{sc}, \\
\varepsilon_2 &= \varepsilon_0 + r_{sc} (\theta_j - \theta_i) / l_{sc},
\end{align*}
\]

(38)

where $\varepsilon_1$ and $\varepsilon_2$ are strains of the two connecting cables, $\varepsilon_0$ is the initial pre-tensioning strain, $l_{sc}$ is the length of the cables, $r_{sc}$ is the radius of the pulleys, and $\theta_i$ and $\theta_j$ are the rotating angles of the two joints. The moment acting at joint $i$ from the connecting cables is

\[
M_{ci} = E_c A_c r_{sc} (\varepsilon_2 - \varepsilon_1),
\]

(39)

where $E_c$ and $A_c$ are the Young’s modulus and cross-sectional area, respectively, of the connecting cables. Substituting Eq. (38) into Eq. (39) yields

\[
M_{ci} = k_{sc} (\theta_j - \theta_i),
\]

(40)

where $k_{sc}$ is the connecting stiffness parameter, and

\[
k_{sc} = 2 E_c A_c r_{sc}^2 / l_{sc},
\]

(41)

from which it can be seen that the connecting stiffness parameter of a small “S” mechanism is not related to the initial pre-tensioning force, and it remains constant as the universal joints rotate. If the damping effect of the connecting cables is considered, a damping term can be added to Eq. (40) as

\[
M_{ci} = k_{sc} (\theta_j - \theta_i) + c_{sc} (\dot{\theta}_j - \dot{\theta}_i),
\]

(42)

where $c_{sc}$ is the connecting damping parameter. Similarly, the moment acting at joint $j$ from the connecting cables is

\[
M_{cj} = k_{sc} (\theta_i - \theta_j) + c_{sc} (\dot{\theta}_i - \dot{\theta}_j).
\]

(43)

The physical meaning of Eqs. (42) and (43) is that the two connecting cables in the small “S” mechanism are modeled as a linear torsional spring–damper.

4.2 Dynamic modeling of connecting cables for large “S” mechanisms

The dynamic modeling of a large “S” synchronous rotating mechanism is much more complicated than
that of a small “S” mechanism, since the connecting
cables may exhibit stick–slip motions relative to
the pulleys and conduits. For simplicity, a modified
Coulomb friction model is applied to describe the
frictional forces acting on the connecting cables [26],

\[ f = \mu_m(v_t)N, \]  

(44)

where

\[ \mu_m(v_c) = \begin{cases} 
-\mu_c \text{sgn}(v_c), & |v_c| > v_d, \\
\mu_c + (\mu_c' - \mu_c) \left[ \frac{v_c - v_d}{v_d - v_s} \right]^2, & v_s \leq |v_c| \leq v_d, \\
3 - 2 \left[ \frac{v_c - v_s}{v_d - v_s} \right]^2 \text{sgn}(v_c), & \mu_c' - 2\mu_c' \left[ \frac{v_c + v_s}{2v_s} \right]^2 \frac{1}{3 - \frac{v_c + v_s}{v_s}}, & |v_c| < v_s,
\end{cases} \]  

(45)

in which \( v_s \) and \( v_d \) are velocity tolerances, \( \mu_c \) and \( \mu_c' \) are friction coefficients, and other variables are the same as in Eq. (8). The graphic expression of this modified friction model is illustrated in Fig. 12. The friction model cannot capture the stick state in a strict sense, but allows a tiny slip velocity smaller than \( v_s \) to produce an effect similar to static friction. Thus, this model avoids the detection of stick–slip states and simplifies the calculation of frictional forces.

As shown in Fig. 13, by adopting the modified Coulomb friction model, the tensions of the connecting cables in their two ends can be approximately derived as

\[ \begin{align*}
T_{i1} &= \frac{E_c A_c \mu_m(v_{t1}) \theta_{i1} \Delta t_i}{l_{ic} \left( 1 - e^{-\mu_m(v_{t1}) \theta_{ilc}} \right)}, \\
T_{j1} &= \frac{E_c A_c \mu_m(v_{t1}) \theta_{j1} \Delta t_i}{l_{ic} \left( 1 - e^{-\mu_m(v_{t1}) \theta_{jilc}} \right)},
\end{align*} \]  

(46)

and

\[ \begin{align*}
T_{i2} &= \frac{E_c A_c \mu_m(v_{t2}) \theta_{i2} \Delta t_i}{l_{ic} \left( 1 - e^{-\mu_m(v_{t2}) \theta_{jilc}} \right)}, \\
T_{j2} &= \frac{E_c A_c \mu_m(v_{t2}) \theta_{j2} \Delta t_i}{l_{ic} \left( 1 - e^{-\mu_m(v_{t2}) \theta_{jilc}} \right)},
\end{align*} \]  

(47)

where \( l_{ic} \) is the length of large “S” connecting cables, \( \theta_{ilc} \) is the special wrap angle, \( v_{t1} \) and \( v_{t2} \) are the tangential velocities of the connecting cables relative to the conduits, and \( \Delta t_i \) and \( \Delta t_j \) are the elongations of the two connecting cables, determined by

\[ \begin{align*}
\Delta t_i &= \frac{T_{pi} l_{ic}}{E_c A_c} - r_{lc} (\theta_{j} - \theta_{i}), \\
\Delta t_j &= \frac{T_{pi} l_{ic}}{E_c A_c} + r_{lc} (\theta_{j} - \theta_{i}),
\end{align*} \]  

(48)

in which \( T_p \) is the pre-tensioning force and \( r_{lc} \) is the radius of the pulley. Thus, the moment acting at joint \( i \) from the connecting cables can be derived as

\[ M_{ci} = M_{pi} + k_{clc} (\theta_{j} - \theta_{i}), \]  

(49)

where

\[ \begin{align*}
M_{pi} &= T_p r_{lc} \left( \frac{\mu_m(v_{t1}) \theta_{l1c}}{1 - e^{-\mu_m(v_{t1}) \theta_{ilc}}} - \frac{\mu_m(v_{t2}) \theta_{l1c}}{1 - e^{-\mu_m(v_{t2}) \theta_{jlc}}} \right), \\
k_{clc} &= \frac{E_c A_c r_{lc}^2}{l_{ic} \left( 1 - e^{-\mu_m(v_{t2}) \theta_{jlc}} \right) + \mu_m(v_{t2}) \theta_{l1c}},
\end{align*} \]  

(50)

By adding a damping term, the moment can be rewritten as

\[ M_{ci} = M_{pi} + k_{clc} (\theta_{j} - \theta_{i}) + c_{ilc} (\dot{\theta}_{j} - \dot{\theta}_{i}). \]  

(51)

Similarly, the moment acting at joint \( j \) is expressed as

\[ M_{cj} = M_{pj} + k_{clc} (\theta_{i} - \theta_{j}) + c_{ijc} (\dot{\theta}_{i} - \dot{\theta}_{j}). \]  

(52)
connecting cables is the vector of moments acting at joint \( i \) in the body-fixed reference frame is expressed, respectively, as

\[
M_{\text{joint } i} = \mathbf{M}_{p j} + \mathbf{k}_c \left( \theta_{(i-1)/(i-2)} + \theta_{(i+1)/i} - 2 \theta_{i/(i-1)} \right) + \mathbf{c}_c \left( \dot{\theta}_{(i-1)/(i-2)} + \dot{\theta}_{(i+1)/i} - 2 \dot{\theta}_{i/(i-1)} \right),
\]

which shows that by considering the friction acting on the connecting cables, the connecting stiffness parameters are no longer constant, and moments acting at the adjacent joints are related to the pre-tensional force of the connecting cables.

where

\[
\left\{ \begin{array}{l}
M_{pj} = T_p r_{lc} \left( \frac{\mu_m (v_{\tau 1}) \Theta_{\Theta} e^{-\mu_m (v_{\tau 1}) \Theta_{\Theta}}}{1 - e^{-\mu_m (v_{\tau 1}) \Theta_{\Theta}}} \right), \\
- \frac{\mu_m (v_{\tau 2}) \Theta_{\Theta} e^{-\mu_m (v_{\tau 2}) \Theta_{\Theta}}}{1 - e^{-\mu_m (v_{\tau 2}) \Theta_{\Theta}}} \right) - \frac{\mu_m (v_{\tau 2}) \Theta_{\Theta} e^{-\mu_m (v_{\tau 2}) \Theta_{\Theta}}}{1 - e^{-\mu_m (v_{\tau 2}) \Theta_{\Theta}}}, \\
k_{lcj} = \frac{E_c A_c r_{lc}^2}{l_{lc}} \left( \frac{\mu_m (v_{\tau 1}) \Theta_{\Theta} e^{-\mu_m (v_{\tau 1}) \Theta_{\Theta}}}{1 - e^{-\mu_m (v_{\tau 1}) \Theta_{\Theta}}} \right) + \frac{\mu_m (v_{\tau 2}) \Theta_{\Theta} e^{-\mu_m (v_{\tau 2}) \Theta_{\Theta}}}{1 - e^{-\mu_m (v_{\tau 2}) \Theta_{\Theta}}},
\end{array} \right.
\]

\]

4.3 Vector of generalized forces generated from connecting cables

It can be derived from Eqs. (42), (43), (51), and (52) that the vector of moments acting at joint \( i \) from the nearby connecting cables is

\[
\mathbf{M}_{\text{joint } i} = \mathbf{M}_{pj} + \mathbf{k}_c \left( \theta_{(i-1)/(i-2)} + \theta_{(i+1)/i} - 2 \theta_{i/(i-1)} \right) + \mathbf{c}_c \left( \dot{\theta}_{(i-1)/(i-2)} + \dot{\theta}_{(i+1)/i} - 2 \dot{\theta}_{i/(i-1)} \right),
\]

where the matrices \( \mathbf{k}_c \) and \( \mathbf{c}_c \) contain the connecting stiffness and damping parameters, respectively. The above vector of moments acting on links \( i \) and \( i - 1 \) in the body-fixed reference frame is expressed, respectively, as

\[
\mathbf{M}'_{ei} = \mathbf{B}_{i/(i-1)} \mathbf{M}_{\text{joint } i},
\]

and

\[
\mathbf{M}'_{e(i-1)} = - \mathbf{R}_{i/(i-1)}^T \mathbf{B}_{i/(i-1)} \mathbf{M}_{\text{joint } i}.
\]

Then, the vector of generalized forces acting on rigid link \( i \) yielded from the vector of moments \( \mathbf{M}_{\text{joint } i} \) can be obtained as

\[
\mathbf{Q}_{ei} = \mathbf{G}_i^T \mathbf{B}_{i/(i-1)} \mathbf{M}_{\text{joint } i}.
\]

Similarly,

\[
\mathbf{Q}_{e(i-1)} = - \mathbf{G}_{i-1}^T \mathbf{R}_{i/(i-1)} \mathbf{B}_{i/(i-1)} \mathbf{M}_{\text{joint } i}.
\]

5 Dynamic equations of the whole system and their solution procedure

5.1 Dynamic equations of the whole multibody system

The vector of generalized coordinates of the whole manipulator system is

\[
\mathbf{q} = [\mathbf{r}_1^T, s_1, \ldots, \mathbf{r}_M^T, s_M, \mathbf{q}_1^T, \ldots, \mathbf{q}_N^T]^T,
\]

where \( M \) and \( N \) are the numbers of driving cable nodes and rigid links, respectively. By assembling the mass matrices, generalized force vectors, and kinematic constraints of all of the bodies, the dynamic equations of the whole manipulator system can be expressed as

\[
\begin{align*}
\mathbf{M} (\mathbf{q}, t) \ddot{\mathbf{q}} + \mathbf{Q} (\dot{\mathbf{q}}, \mathbf{q}, t) + \Phi^T \lambda &= 0, \\
\Phi (\mathbf{q}, t) &= 0,
\end{align*}
\]

where \( \mathbf{M} \) is the mass matrix of the whole system, which consists of mass matrices of the driving cables, rigid links, and additional inertia terms yielded by the model smoothing method of Eq. (20). \( \mathbf{Q} \) is the vector of generalized forces, including elastic, Coriolis, external applied, and frictional forces, and moments from the connecting cables. \( \lambda \) is the vector of Lagrange multipliers. \( \Phi \) is the vector of constraint equations, including constraints from Eulerian or Lagrangian nodes and rheonomic cable length input, constraints of rigid links described in Sect. 3.3, and \( \Phi_q = \partial \Phi / \partial \mathbf{q} \).
5.2 Solution procedure for cable-driven continuum manipulator with CCSRM

To restrain the drift of constraint violations, the Baumgarte stabilization method is introduced [2]:

$$\dot{\phi} + \alpha_B \ddot{\phi} + \beta_B \phi = 0,$$

where the positive parameters $\alpha_B$ and $\beta_B$ can be chosen following the instruction of Flores [8]. Substituting $\dot{\phi} = \Phi_q \dot{q} + \partial \phi / \partial t$ in Eq. (61) yields

$$\Phi_q \dot{q} + b = 0,$$

where

$$\begin{cases} b = \dot{\phi} q \dot{q} + \dot{\phi} t + \alpha_B \Phi_q q + \alpha_B \phi t + \beta_B \phi, \\ \phi_t = \partial \phi / \partial t. \end{cases}$$

(63)

Combining Eqs. (60) and (62), we obtain

$$(M + \Phi_q^T \Phi_q) \ddot{q} + \Phi_q^T b + Q + \Phi_q^T \lambda = 0.$$  

(64)

The mass matrix $M$ is singular, which is a property of the ALE formulation. By adding $\Phi_q^T \Phi_q$, it becomes invertible [3]; thus,

$$\ddot{q} = -(M + \Phi_q^T \Phi_q)^{-1} \left( \Phi_q^T b + Q + \Phi_q^T \lambda \right).$$  

(65)

Substituting Eq. (65) into Eq. (62) yields the vector of Lagrange multipliers,

$$\lambda = A_\lambda^{-1} b_\lambda,$$

(66)

where

$$\begin{cases} A_\lambda = \Phi_q \left( M + \Phi_q^T \Phi_q \right)^{-1} \Phi_q^T, \\ b_\lambda = b - \Phi_q \left( M + \Phi_q^T \Phi_q \right)^{-1} \left( \Phi_q^T b + Q \right). \end{cases}$$

(67)

Inserting Eq. (66) in Eq. (65), the dynamic equations can be transformed to ordinary differential equations (ODE) form as

$$\ddot{q} = -(M + \Phi_q^T \Phi_q)^{-1} \left( \Phi_q^T b + Q + \Phi_q^T A_\lambda^{-1} b_\lambda \right).$$

(68)

As the stiff problem is released by the model smoothing method, the above equations can be solved efficiently by explicit ODE solvers. In this paper, the explicit Runge–Kutta method of fourth and fifth orders with variable step size (RK45 method) is applied, since it has advantages in both accuracy and convergence [41].

6 Results and discussion

To verify the feasibility and correctness of the proposed dynamic modeling method, the numerical and experimental results of a cable-driven continuum manipulator are presented. As shown in Fig. 14, the manipulator consists of two segments of CCSRM, each made up of six rigid links. There are six driving cables in total, which are labeled in sequence. Cables 1, 2, and 3 drive the first segment, and cables 4, 5, and 6 drive the second segment. The positions of these driving cables are shown in the figure, and other important parameters of this manipulator are listed in Table 1. The gravitational acceleration is assumed to be zero.

To exhibit the performance of cable-constrained synchronous rotating mechanisms both in simulations and experiments, the planned motion of the manipulator is that each joint rotates $6^\circ$ around the $x'_{i-1}$ axis and then rotates back, as illustrated in Fig. 14. The input lengths of driving cables are determined by a pure kinematical planning method without cable deformations [24]. To test the stick–slip capturing capacity of our proposed simulation method, in seconds 20 to 24, the input cable lengths remain unchanged, as shown in Fig. 15.

Four cases of motion are studied to reveal the effects of friction on the dynamical behaviors of the cable-driven continuum manipulator: without friction on all of the cables, with friction on all of the cables, with friction only on driving cables, and with friction only on connecting cables.

Case 1 Continuum manipulator without friction on all cables

Figure 16 presents the joint angles of the continuum manipulator without friction on all cables. The numerical results show that if all frictions on the cables are zero, their deformations can be ignored. Thus, the joint angles in each segment are the same, i.e., the manipulator exhibits constant curvature bending in each segment. The numerical results are identical to the planned motions denoted by green circles in the figure.
Table 1 Parameters of cable-driven continuum manipulator with two segments of CCSRM

| Category          | Parameter                        | Value                                                                 |
|------------------|----------------------------------|----------------------------------------------------------------------|
| Driving cable    | Young’s modulus                  | $E = 2.6 \times 10^{10}$ Pa                                         |
|                  | Cross-sectional area             | $A = 1.3273 \times 10^{-6}$ m$^2$                                    |
|                  | Density of cable                 | $\rho = 7.85 \times 10^3$ kg/m$^3$                                   |
|                  | Friction coefficients (if they exist) | $\mu = 0.21; \mu' = 0.21$                                         |
| Rigid link       | Disk distance in the joints      | $d = 0.016$ m                                                        |
|                  | Mass of each link                | $m_i = 0.095$ kg                                                     |
|                  | Inertia matrix of each link      | $J_i' = \text{diag}(66, 62, 19) \times 10^{-6}$ kg$\cdot$m$^2$  |
|                  | Length of each link              | $L_i = 0.085$ m                                                      |
|                  | Radius of hole distributed circle | $r_h = 0.021$ m                                                   |
|                  | Seg. 2 Mass of each link         | $m_i = 0.070$ kg                                                     |
|                  | Inertia matrix of each link      | $J_i' = \text{diag}(54, 53, 7) \times 10^{-6}$ kg$\cdot$m$^2$  |
|                  | Length of each link              | $L_i = 0.100$ m                                                      |
|                  | Radius of hole distributed circle | $r_h = 0.016$ m                                                   |
| Connecting cable | Young’s modulus                  | $E_c = 2.6 \times 10^{10}$ Pa                                       |
|                  | Cross-sectional area             | $A_c = 1.3273 \times 10^{-6}$ m$^2$                                  |
|                  | Pre-tensioning force             | $T_p = 50$ N                                                        |
|                  | Friction coefficients (if they exist) | $\mu_c = 0.21; \mu_c' = 0.21$                                    |
|                  | Seg. 1 Length of cable           | $l_{sc} = 0.084$ m; $l_{lc} = 0.109$ m                               |
|                  | Radius of pulley                 | $r_{sc} = 0.006$ m; $r_{lc} = 0.016$ m                              |
|                  | Special wrap angle               | $\Theta_{lc} = 184^\circ$                                           |
|                  | Seg. 2 Length of cable           | $l_{sc} = 0.100$ m; $l_{lc} = 0.120$ m                               |
|                  | Radius of pulley                 | $r_{sc} = 0.006$ m; $r_{lc} = 0.016$ m                              |
|                  | Special wrap angle               | $\Theta_{lc} = 158^\circ$                                           |
| Simulation       | Baumgarte parameters             | $\alpha_B = 100; \beta_B = 100$                                    |
| parameters       | Model smoothing step             | $h = 0.01$ s                                                        |
|                  | Velocity tolerance               | $v_s = 1 \times 10^{-6}$ m/s; $v_d = 2 \times 10^{-4}$ m/s         |

Fig. 14 A cable-driven continuum manipulator with two segments of cable-constrained synchronous rotating mechanisms

Case 2 Continuum manipulator with friction on all cables

The friction in real mechanical systems cannot be ignored, however. Thus, a more realistic simulation was conducted in this case, considering all frictions, including driving cable-hole frictions and large “S” connecting cable-conduit frictions. An experimental apparatus was designed to verify the simulation results, as shown in Fig. 17. The battery, motor drivers, and driving component were fixed on the ground. The cable-driven
continuum manipulator with two segments of CCSRM moved on an air-floating platform to simulate the zero gravitational environment. Two markers with a diameter of 6.4 mm were installed on each rigid link, and four OptiTrack motion capturing cameras were fixed around to collect the 3D positions of the markers during the experiment. A Gaussian filtering method was adopted to perform the pretreatment of the collected data. From the vectors determined by the marker positions, the joint angles in the $x_{i-1}^j$ direction can be calculated.

Figure 18 presents the joint angles of the continuum manipulator with friction on all cables, which indicates that the numerical results are consistent with the experiment. Due to the effects of friction, the constant cur-

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**Fig. 15** Input lengths of driving cables by a pure kinematical planning method without cable deformations

**Fig. 16** Joint angles of manipulator without friction on all cables

**Fig. 17** Experimental apparatus to record the motion of the continuum manipulator

**Fig. 18** Joint angles of the continuum manipulator with friction on all cables.
viture in each segment does not remain. In each segment, the amplitudes of joint angles roughly decrease from the root to the end, except the last joint. The interesting phenomenon can be noticed that although the connecting cables of large “S” mechanisms are subjected to kinetic frictions from the pulleys and conduits, their performance is better than that of small “S” mechanisms. This is because their connecting stiffness is much larger. Thus, it may be an optimization direction of the mechanical design for our next-generation manipulator to match the connecting stiffness parameters of large and small “S” mechanisms. An opposite direction of motion was also made, i.e., the planned motion was that each joint rotated $-6^\circ$ around the $x_{i-1}'$ axis and then rotated back. The numerical and experimental results are illustrated in Fig. 19. From Figs. 18 and 19, it can be concluded that there is no difference between the two directions of motion.

The following two cases present the effects of driving or connecting cable friction alone on the non-constant curvature of a manipulator.

**Case 3 Continuum manipulator with friction only on driving cables**

Figure 20 illustrates the joint angles of the manipulator with friction only on driving cables. During the time from 20 to 24 seconds, the driving cables were subjected to static frictional forces, keeping the joint angles unchanged, which demonstrates that the proposed dynamic modeling method for driving cables can well capture stick–slip motions. Without static friction, the joint angles of a segment would become equal. Numerical results indicate that it is the driving cable friction that causes the joint angle amplitudes to decrease from the root to the end. It can also be noticed that the performance of the synchronous rotating mechanisms of segment 2 is better than that of segment 1. This is because segment 1 is subjected to frictions of six driving cables, while segment 2 only suffers from frictions of three driving cables. Besides, the diameter of the rigid links in segment 2 is smaller than that of segment 1, which causes the arm of frictional force to be smaller than that of segment 1. Due to the friction from the driving cables, a small rotation of each joint
Fig. 19 Joint angles of manipulator with friction on all cables (moving in the opposite direction): a, c simulation results; b, d experimental results.

Fig. 20 Joint angles of manipulator with friction only on driving cables.
Fig. 21 Joint angles of manipulator with friction only on connecting cables

Fig. 22 Input cable tensions for different cases: a case 1; b case 2; c case 3; d case 4
### Table 2 Evaluating indicator (EI) of the motion error of each segment in different cases

| Case   | Seg.1(%) | Case 2 | Seg.2(%) | Case 3 | Case 4 | Experiment |
|--------|----------|--------|----------|--------|--------|------------|
| Case 1 | 0.14     | 11.47  | 0.21     | 6.70   | 4.53   | 4.80       |
| Case 2 |          |        |          |        |        | 6.43       |

around the $y'_{i-1}$ axis is also aroused, as shown in Fig. 20c, d.

**Case 4** Continuum manipulator with friction only on connecting cables

Figure 21 presents the joint angles of the manipulator with friction acting only on connecting cables. The numerical results show that there is no difference between the two segments, and they all rotate from the two ends of the segments, while the two middle joints move last. Because of the friction from the connecting cables, small rotations around the $y'_{i-1}$ axes have also been aroused, as illustrated in Fig. 21c, d.

Further comparing the input cable tensions (motor driving forces) for the four cases, Fig. 22 implies that the more the friction factors are considered, the larger the amplitudes of input cable tensions that are needed. Also, the amplitudes of input cable tensions for case 3 are greater than for case 4, which indicates that the frictions on driving cables play a more important role than the connecting cable frictions in the nonconstant curvature bending of the manipulator, which can also be concluded by comparing Figs. 20 and 21.

In order to quantitatively analyze the effect of cable deformation and friction on the continuum manipulator with CCSRM, an evaluating indicator (EI) of the motion error of each segment is defined as

\[
EI = \frac{1}{n} \sum_{i=k}^{k+1} \left( \frac{\| \theta_{i/(i-1)} - \theta_{i/(i-1)} \|_2}{\| \theta_{i/(i-1)} \|_2} \right)^2 \times 100\%,
\]

where $t_{\text{mid}} = 22 \text{ s}$, $k = 1$ or 7, $n = 6$, $\theta_{i/(i-1)}$ is the vector of planned joint angles, and $\theta_{i/(i-1)}$ is that of simulated or experimental joint angles. Table 2 displays the evaluating indicator of the motion error of each segment in different cases, from which four conclusions can be drawn:

1. If all frictions on the cables are ignored, the motion error of this kind of manipulator is very low (nearly 0.2%), as shown in case 1 in the table;
2. If only the driving cable frictions are considered, the motion error of segment 1 is bigger than that of segment 2, as shown in case 3 in the table;
3. If only the connecting cable frictions are considered, the motion errors of the two segments are similar, as shown in case 4 in the table;
4. If all frictions on driving and connecting cables are considered, the motion errors performed in the simulations are close to the experimental results, which verify the correctness of the proposed dynamic modeling methods.

### 7 Conclusions

This paper presented a dynamic modeling method for a cable-driven continuum manipulator with cable-constrained synchronous rotating mechanisms. First, the driving cables were modeled based on the ALE formulation, and strategies for detecting stick–slip motions were proposed by using a trial-and-error algorithm. The stiff problem of the dynamic equations was released by a model smoothing method. Second, the dynamic modeling method for rigid links was presented by using quaternion parameters, which avoid singularities and reduce the calculation of trigonometric functions to improve calculation efficiency. Third, the connecting cables were modeled by torsional spring-dampers to avoid the further use of ALE cable elements, reducing the DOFs of the whole multibody system. Frictions between connecting cables and conduits were considered based on a modified Coulomb friction model. At last, numerical results were presented and verified by comparison with experimental results.

Numerical and experimental results showed that if all frictions are not considered, the results are identical with planned motions, and the manipulator exhibits constant curvature bending in each segment. However, in real mechanical systems, the frictions cannot
be ignored, and constant curvature bending does not remain. In each segment, the amplitudes of joint angles roughly decrease from the root to the end. The performance of the first segment of synchronous rotating mechanisms is better than that of the second segment, because the first segment suffers from more driving cable frictions than the second segment. An interesting phenomenon can also be noticed that large “S” mechanisms perform better than small “S” mechanisms, because the connecting stiffness parameter of large “S” mechanisms is larger. To ensure that these values match may be an optimization design direction for our next-generation manipulator.

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Data Availability Statement The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

Appendix

The Jacobian matrix of the kinematic joint constraints has the following terms:

\[
\frac{\partial g_{ei}}{\partial q_i} = [0, 0, 0, 2e_{0i}, 2e_{1i}, 2e_{2i}, 2e_{3i}],
\]

(70)

\[
\frac{\partial g_{p,i,i+1}}{\partial q_i} = \left[ I_{3\times3}, \frac{\partial (R^T_i)}{\partial e_{1i}} x'_{Ci}, \frac{\partial (R^T_i)}{\partial e_{2i}} x'_{Ci}, \frac{\partial (R^T_i)}{\partial e_{3i}} x'_{Ci} \right],
\]

(71)

\[
\frac{\partial g_{p,i,i+1}}{\partial q_{i+1}} = -\left[ I_{3\times3}, \frac{\partial (R^T_i)}{\partial e_{0i+1}} x'_{Ci(i+1)}, \frac{\partial (R^T_i)}{\partial e_{2i+1}} x'_{Ci(i+1)}, \frac{\partial (R^T_i)}{\partial e_{3i+1}} x'_{Ci(i+1)} \right],
\]

(72)

\[
\frac{\partial g_{n,i,i+1}}{\partial q_i} = \left( R^T_{i+1} o'_{i+1} \right)^T \left[ 0_{3\times3}, \frac{\partial (R^T_i)}{\partial e_{0i}} o', \frac{\partial (R^T_i)}{\partial e_{2i}} o', \frac{\partial (R^T_i)}{\partial e_{3i}} o' \right],
\]

(73)

\[
\frac{\partial g_{n,i,i+1}}{\partial q_{i+1}} = \left( R^T_i o'_{i+1} \right)^T \left[ 0_{3\times3}, \frac{\partial (R^T_{i+1})}{\partial e_{0i+1}} o'_{i+1}, \frac{\partial (R^T_{i+1})}{\partial e_{2i+1}} o'_{i+1}, \frac{\partial (R^T_{i+1})}{\partial e_{3i+1}} o'_{i+1} \right].
\]

(74)

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