Two roads to antispacetime in polar distorted B phase: Kibble wall and half-quantum vortex

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We consider the emergent tetrad gravity and the analog of antispacetime realized in the recent experiment on the composite defects in superfluid $^3$He: the Kibble walls bounded by strings (the half quantum vortices). The antispacetime can be reached in two different ways: by the "safe" route around the Alice string or by dangerous route across the Kibble wall. This consideration also suggests the scenario of the formation of the discrete symmetry – the parity $P$ in Dirac equations – from the continuous symmetry existing on the more fundamental level.

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I. INTRODUCTION

The topological materials with emergent analogs of gravity demonstrate the possibility of realization of different exotic spacetimes including the transition to antispacetime, see e.g. Ref.2 and references therein. There are several routes to the effective gravity. One of them is the tetrad gravity emerging in the vicinity the Weyl or Dirac points – the exceptional crossing points in the fermionic spectrum. Also the degenerate $2 + 1$ gravity emerges near the Dirac nodal line in the spectrum. Another important source of gravity is the formation of the tetrads as bilinear combinations of the fermionic fields. Also the elasticity tetrads describing the deformation of the crystalline material or quantum vacuum may give rise to gravity.

Different sources of emergent gravity provide different types of the antispacetime obtained by the space reversal $P$ and time reversal $T$ operations, including those where the determinant of the tetrads $e$ changes sign. In cosmology, the antispacetime Universe was in particular suggested as the analytic continuation of our Universe across the Big Bang singularity. There were the speculations, that the antispacetime may support the nonequilibrium states with negative temperature as a result of analytic continuation across the singularity.

Here we consider the emergent antispacetime realized in the recent experiments on the $^3$He analog of the cosmological walls bounded by strings, which we call here as Kibble walls. The experiments deal with the time reversal symmetric superfluid phases, where the tetrads emerge as the bilinear combinations of the fermionic fields.

II. RELATIVISTIC DIRAC GREEN’S FUNCTION

To see the analogy between relativistic physics and the physics of the Bogoliubov quasiparticles, let us start with the Green’s function of the relativistic massive Dirac particle. In notation used in the Green’s function has the form:

$$S = \frac{Z(p^2)}{-i\gamma^a e_\mu^a p_\mu + M(p^2)}.$$  (1)

Here $e_\mu^a$ are tetrads with $\mu, a = 0, 1, 2, 3$; the residue $Z(p^2)$ and the mass $M(p^2)$ are the functions of $p^2 = g_{\mu\nu} p_\mu p_\nu$, where $g_{\mu\nu} = e_\mu^a e_\nu^a \eta_{ab}$.

It is convenient to express $\gamma$-matrices in terms of two sets of Pauli matrices: $\sigma^1$, $\sigma^2$ and $\sigma^3$ for conventional spin, and $\tau_1$, $\tau_2$, $\tau_3$ for the isospin in the left-right space:

$$\gamma^0 = -i\tau_1 , \quad \gamma^a = \tau_2 \sigma^a , \quad a = (1, 2, 3).$$  (2)

$$\gamma_5 = -i\gamma^0 \gamma^1 \gamma^2 \gamma^3 = \tau_3.$$  (3)

III. EXTENSION TO $^3$HE WITH BROKEN $U(1)$ SYMMETRY

The time reversal symmetric B-phase and also the polar distorted B-phase (PdB) of superfluid $^3$He provide the example of the formation of the tetrad field as bilinear combination of the fermionic fields. The Green’s function for fermionic Bogoliubov quasiparticles in these superfluids is similar to that in Eq. (1). Now instead of the mass function $M(p^2)$, the energy of quasiparticles in the normal Fermi liquid enters, $e(p) = v_F(|p| - p_F)$:

$$M(p^2) \rightarrow e(p).$$  (4)

The $\gamma$-matrices are the same as in Eqs. (2) and (3), but their meaning is different. The spin matrices $\sigma^a$ act now in the spin space of $^3$He atoms, while the matrices $\tau_1$, $\tau_2$, $\tau_3$ in the left-right space now correspond to the matrices acting in the isotopic Bogoliubov-Nambu particle-hole space. The function $Z$ is not important, and we ignore it.

The tetrads come from the spin-triplet $p$-wave order parameter in $^3$He superfluids, which is $3 \times 3$ matrix $A^a_\mu$ with spin index $a = (1, 2, 3)$ and orbital index
i = (1, 2, 3):
\[ \sum_k k^i (a_{\kappa a} a_{-k\beta}) \sim A^i_a (\sigma^a \sigma^\beta)_{a\beta}, \quad a, i = (1, 2, 3). \] (5)

For the time reversal symmetric states, the order parameter has the form:
\[ A^i_a = p F e^{i\Phi} a^i, \quad a, i = (1, 2, 3), \] (6)

(for pure B-phase see[10]).

The tetrads \( e_a^i \) emerge due to the spontaneously broken symmetries \( SO(3)_S \times SO(3)_L \) under spin and orbital rotations. This is analogous to the formation of the tetrads in relativistic theories as bilinear combinations of the fermionic fields (for pure B-phase see[10]). However, in addition to tetrads, the order parameter \( (6) \) contains the phase \( \Phi \), which numerates the degenerate states obtained after spontaneous breaking of \( U(1) \)-symmetry in superfluids and superconductors. The phase \( \Phi \) of the order parameter \( (6) \) leads to the following modification of the Green’s function, which now depends both on the tetrad field \( e_a^i \) and on the parameter \( \Phi \):
\[ \hat{S}(e^\mu_a, \Phi) = e^{-\gamma_0\Phi/2} S(e^\mu_a) e^{\gamma_0\Phi/2}. \] (7)

IV. \( U(1) \) SYMMETRY AS THE ORIGIN OF PARITY

For \( \Phi = \pi \) the symmetry transformation \( e^{-\gamma_0\Phi/2} \) is equivalent to the conventional space reversal transformation – the parity \( P = e^{-\gamma_0\pi/2} = \gamma_0 \), with \( P^2 = -1 \).

This suggests that in relativistic theories the discrete symmetry, such as the space inversion \( P \), could be the residual \( Z_2 \) symmetry after breaking of the more fundamental symmetry group at the trans-Planckian level, such as \( U(1) \).

On the other hand, the planar phase[23] of superfluid \(^3\)He demonstrates the opposite case, when the discrete \( Z_2 \) symmetry \( C \) of the Hamiltonian becomes continuous[23]. If the Green’s function commutes with \( C \), the transformations \( e^{i\alpha C} \) generated by the operator \( C \) form a continuous \( U(1) \) symmetry group.

V. POLAR DISTORTED B-PHASE

In the polar distorted B-phase realized in experiment[11] the tetrads in the vacuum states are:
\[ e^i_a = c_1 \hat{f}_a \hat{x}^i + c_2 \hat{g}_a \hat{y}^i + c_3 \hat{d}_a \hat{z}^i, \quad (a, i) = (1, 2, 3), \] (8)

where \( \hat{d}, \hat{f} \) and \( \hat{g} \) are orthogonal unit vectors in spin space; \( \hat{x}, \hat{y} \) and \( \hat{z} \) are orthogonal unit vectors in orbital space; \( c_1, c_2 \) and \( c_3 \) are the characteristic "speeds of light". In the pure B-phase \( |c_1| = |c_2| = |c_3| \), while in the polar phase[28] one has \( c_1 = c_2 = 0 \).

The particular states of the PdB phase are:
\[ e^\mu_a = \text{diag}(-1, c_1, c_2, c_3), \] (9)

with \( c_2 = \pm c_1 \) and \( |c_2| = |c_1| < |c_3| \).

VI. KIBBLE WALL AND ALICE STRING

The two states with \( c_2 = +c_1 \) and \( c_2 = -c_1 \) in Eq. (9) can be separated by the nontopological domain wall – the analog of the Kibble wall bounded by strings[23]. The Kibble walls typically appear in the two phase transitions: at first transition the linear defect (vortex or string) becomes topologically stable; at the second transition the linear defect looses its topological stability and becomes the termination line of the wall – the Kibble wall. In superfluid \(^3\)He, the half-quantum vortices appear at first transition from the normal liquid to the polar phase[22], and at further transition to the PdB phase they become the end lines of the Kibble walls[11].

Across the Kibble wall, the tetrad element of in PdB order parameter, \( e^\mu_2 = c_2 \), changes sign, and thus the spacetime transforms to the antispacetime with opposite sign of the tetrad determinant, \( \text{det} e \). The intermediate state within the Kibble wall has the degenerate tetrad field
\[ e^\mu_a = \text{diag}(-1, c_1, 0, c_3), \] (10)

and represents the distorted planar phase (in the pure planar phase \( |c_1| = |c_3| \) and \( c_2 = 0 \)). Such types of the nontopological domain walls were originally considered in the B-phase[23] and some experimental evidences of them have been recently reported in the thin film of \(^3\)He[29].

Fig. 1 demonstrates the loop of the half-quantum vortex, which terminates the Kibble wall. In cosmology, the half-quantum vortex corresponds to the Alice string[10]. As in the case of the cosmic Alice string, by circling around the half-quantum vortex one continuously arrives at the mirror reflected world. Indeed, around the half quantum vortex the phase \( \Phi \) changes by \( \pi \) and

\[ \Phi = \pi \]

FIG. 1: Two roads to antispacetime: the safe route around the Alice string (along the contour \( C_1 \)) or dangerous route along \( C_2 \) across the Kibble wall (through the Alice looking glass).
also the vectors \( \mathbf{d} \) and \( \mathbf{f} \) rotate by \( \pi \). As a result, when circling around the HQV, the tetrads are continuously transformed to the state with opposite \( e_2^2 \):

\[
\text{diag}(-1, c_1, c_2, c_3) \rightarrow \text{diag}(-1, c_1, -c_2, c_3),
\]

i.e. to the same antispacetime as across the Kibble wall, but without violation of the PdB state. The discontinuity in \( e_2^2 \) around the Alice string is thus compensated by the Kibble wall, at which \( e_2^2 \) analytically crosses zero and changes sign. The Kibble wall plays the role of the mirror – the Alice looking glass.

VII. CONCLUSION

In the polar distorted B-phase of superfluid \( ^3\)He, the half-quantum vortex (Alice string) and the Kibble wall bounded by strings demonstrate the two ways to enter the mirror world in Fig. [1] either to go around the HQV or to cross the Kibble wall. The polar distorted B-phase also suggests the scenario of the formation of the discrete symmetry – the parity \( P \) in particle physics – from the continuous symmetry existing on the more fundamental level.

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