Possibility of realizing Bussey’s thought experiment on collapse of the wave function at the microscopic level

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Abstract. We discuss on feasibility of experiments to test if the wave function collapses at the level of microscopic events. The original idea is Bussey’s ‘thought’ experiment introduced in 1984. Bussey’s proposal is to collide two particles and then two pairs of scattered and unscattered waves of each particle are examined by a pair of Mach-Zehnder type interferometers (MZI). Different coincidence rates between different sets of combinations of MZI outputs are expected depending on the collapse/non collapse assumption. In our proposition to realize this idea, a pair of low-energy electron beams is collided nearly head-on in a nanometer-size region. Then each combination of two waves scattered at small angles is thrown into the MZI. Each MZI is composed of Moellenstedt biprisms as two arms and mirrors, and then brought together at single crystal films which act as half reflecting mirrors (beam splitters). Coincidence counting is done among four outputs of the beam splitters. Another method to measure the position correlation after the biprisms is also proposed.

1. Introduction

The problem of “wave function collapse” is still one of the most fundamental issues in quantum mechanics. Recently, apart from the theoretic and philosophical aspect of this issue, it may find a “practical” application as a measure to obtain final results in quantum computer, if it is realized. But the problem is still open and we have, as far as we know, no experimental test on how the wave function collapses.

More than 20 years ago Bussey [1] proposed a thought experiment to test if the wave function collapses at the microscopic level. As shown in Fig. 1, two beams are collided, the scattered and the unscattered beams enter two Mach-Zehnder type interferometers. Then coincidence counts are compared between different combinations of 4 output ports of interferometers. We denote the coincidence count between counter 1 and 3 by $C_{13}$, etc.

Here we repeat the introductory part of [1] with a little extension. If the wave function does not collapse, namely, if it is a superposition of scattered and unscattered waves

$$|\Psi> = \alpha |A\rangle |B\rangle + \beta |A'\rangle |B'\rangle$$

then coincidence counts $C_{13}$ and $C_{14}$ are expressed as

$$C_{13} = |\gamma|^4 |\alpha + \gamma^2 \beta|^2, \quad C_{14} = |\gamma|^2 |\tau|^4 |\alpha + \beta|^2.$$
But if it does collapse to one of these scattered or unscattered waves then:

$$|\Psi > = |A\rangle |B\rangle \quad \text{or} \quad |A'\rangle |B'\rangle$$ (2)

with weights $|\alpha|^2$ and $|\beta|^2$ respectively, these two coincidences are as follows:

$$C_{13} = |\tau|^4(|\alpha|^2 + |\gamma|^4|\beta|^2), \quad C_{14} = |\gamma|^2|\tau|^4(|\alpha|^2 + |\beta|^2),$$

where $\tau$ is the transmission coefficient of the half-mirror and $\gamma$ is the ratio of the reflection coefficient $\rho$ to transmission coefficient, $\rho = \gamma \tau$. Thus different sets of counts are expected depending on collapse/non-collapse postulates. In Bussey’s paper [1] $\gamma = e^{i\pi/2}$ is assumed where a maximum contrast between non-collapse/collapse cases is obtained.

It is difficult to find particles for this purpose. Photons are not usable since they do not scatter each other in an ordinary situation. Charged particles seem to be suitable since they scatter each other by the Coulomb force. Low energy electrons from field emission sources seem to be adequate, since they have good temporal and spatial coherence, besides, they have relatively long wave lengths. On the other hand it is difficult to find a good half mirror for charged particles.

In our previous paper [2] we proposed an experiment to test a strange effect of disappearance of interference due to entanglement. In that article we examined the possibility of keeping coherence in the collision of field emission electron beams, whose geometry is similar to the arrangement in [1]. We will claim that two ways of extensions of that proposition could be the realization of Bussey’s thought experiment.
2. Proposed setup to test the wave function collapse

2.1. Use of thin crystal films as beam splitters

We propose a realistic version of Bussey’s thought experiment by field emission electrons. As shown in Fig. 2, thin electron beams of the same energy (our assumption is 2.5 keV) are collided at a nearly head-on geometry. Elastically scattered electron beams which are nearly at 180 degree w.r.t. each other are introduced into electron biprisms on both sides. Instead of the unscattered and the scattered particles of [1], we use a combination of particles scattered at different angles and going through different sides of the central wires of the biprisms.

The two biprisms provide a combination of different paths of the “double interferometer” of Fig. 1. For half-reflecting mirrors (or beam splitters) we will put thin crystal films which partly reflect the electron beam by Bragg scattering. The two planes, in which the combinations of different paths are located, are better taken to be perpendicular to the Fig.'s plane in order to minimize the energy difference due to kinematics.

| Table 1. Parameters for the proposed setup. |
|-------------------------------------------|
| Item                                      | Description or value |
| electron source                           | field emission type  |
| beam energy                               | 2.5 keV              |
| beam crossing angle                       | 7 degree             |
| beam width at the colliding region        | 5 nm                 |
| size of the biprism (d in Fig. 3)         | 0.1 mm               |
| biprism to beam splitter distance (L)     | 1.28 mm              |

Fabrication of a self-supporting Si crystal of 10 nm thickness is reported [3]. For Si crystal the mean free path for 2.5 keV electron is 3.2 nm, which means the attenuation to 4.4% after this thickness. Diffraction by \((h, k, l)=(1, 1, 1)\) planes gives the smallest Bragg diffraction angle of 4.49 degrees. Phase shift of the diffracted wave to the transmitted one is a critical parameter, as shown in Section 1. We cannot find this data for this very material and condition. But
according to the table of crystallography (Table 2.5A of [4]) the phase shift is 49.9 degree for 5 degree scattering angle of 10 keV electron from Si atom (compared to 8.9 degree to forward direction), with a tendency to increase when energy is decreased.

Parameters of the proposed setup are summarized in Table 1. Except for beam splitters (BS), these are the same as those in our previous article [2] to study the disappearance of interference due to entanglement. In [2] we have confirmed that the coherence of the scattered electron wave is not lost at the position where the BS is located by this set of parameters.

The diffraction angle and the size of the biprism ($d$ in Fig. 3) define the biprism-to-BS distance ($L$ in Fig. 3) to coincide the diffracted beam with the straight (transmission) beam from the other path of the biprism.

2.2. Observation of interference effects in coincidence counting

Another method we propose to test the wave function collapse is to observe interference fringes in coincidence counting. This is rather a simple extension of the setup of [1].

As we have shown in [1], we cannot observe any interference fringe as long as we observe only one part of the entangled pair. Namely it looks like the wave function collapse, but it does not necessarily mean that, as described in the paper. But if we observe both parts of the entangled pair, namely we take a correlation measurement on positions of two electrons, we will get an interference fringe if the wave function does not collapse. Indeed, we have already proposed this measurement as a calibration of the experimental setup to prove that it is able to observe interference fringes in single counting.

If the wave function collapses at the collision event, no such fringe is seen in the coincidence experiment either. So this “calibration” is used as a test of collapse. If we find fringes there, we see that the wave function does not collapse. (If we find no fringe, to prove the collapse of the wave function we must find another method of calibration to confirm that this is not due to the poor precision of the apparatus.)

Fig. 4 shows the experimental arrangement. In place of beam splitters in Fig. 2 we put position sensitive detectors and measure a correlation of two electrons. (Like in the experiment with beam splitters, deflecting directions of biprisms should be taken to be perpendicular to the Fig.’s plane.) Practically the fringe at the crossing point immediately after the biprism is too narrow, so that it should be magnified by electron lenses as is employed by Tonomura [5].

If the wave function does not collapse, it is described as an entangled form. We rewrite (1) as a function of the spatial coordinates:

$$\langle r_1, r_2 | \Psi \rangle = \alpha \phi_A(r_1) \phi_B(r_2) + \beta \phi_A'(r_1) \phi_B'(r_2) .$$

Since the scattering amplitude is overwhelming at forward angle where we observe, we neglected the antisymmetrization. Let us describe the single particle states by plane waves, e.g. $\phi_A(r_1) =$
exp(\(i k_A r_1\)), etc. After the biprism each electron wave undergoes an inward deflection to get an additional momentum \(\pm q_A/2\), where \(q_A = g_A(k_A - k_A')\). Here we defined \(g_A\) as a factor to express the strength of the biprism, which should be greater than 1 so that the biprism converges the incident beam. Similarly the electron in the state \(\phi_B(r_2)\) turns to have a momentum \(k_B - q_B/2\), and \(\phi_B'(r_2)\) to have \(k_B' + q_B/2\). Here \(q_B = g_B(k_B - k_B')\) where \(g_B\) is the same parameter for another biprism. By momentum conservation \(k_B - k_B' = k_A' - k_A = \Delta k\). Then the coincidence rate between the detectors after the biprisms is proportional to

\[ |\langle r_1, r_2 | \Psi \rangle|^2 = |\alpha|^2 + |\beta|^2 + 2|\alpha||\beta| \cos[\Delta k(r_1 - r_2)(1 - g) + \delta] . \]

Here we put \(g_A = g_B = g > 1\) and \(\delta\) comes from the phases of \(\alpha\) and \(\beta\). Then if we perform a coincidence experiment to take the correlation of positions of electrons along the direction of \(\Delta k\); namely \(x_1\) and \(x_2\) in Fig. 3, interference fringes at 45 degree with spacings \(\sqrt{2\pi}/[|\Delta k|(1 - g)]\) should appear in the \(x_1 - x_2\) plane. If the wave function collapses as in Eq. (2), no such fringe is observed.

Both methods are technically very challenging but we believe they are worth putting into practice.

References
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