A Simple Model of Neutrino Masses from Supersymmetry Breaking

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Abstract

We analyze a class of supersymmetric models first introduced by Arkani-Hamed et al and Borzumati et al in which the light neutrino masses result from higher-dimensional supersymmetry-breaking terms in the MSSM super- and Kahler-potentials. The mechanism is closely related to the Giudice-Masiero mechanism for the MSSM $\mu$ parameter, and leads to TeV-scale right-handed neutrino and sneutrino states, that are in principle accessible to direct experimental study. The dominant contribution to the light neutrino (Majorana) mass matrix is a one-loop term induced by a lepton-number violating $B$-term for the sneutrino states that is naturally present. We focus upon the simplification and analysis of the flavour structure of this general class of models, finding that simple and novel origins for the light neutrino mass matrix are possible. We find that a subdominant tree-level ‘see-saw’ contribution may lead to interesting perturbations of the leading one-loop-induced flavour structure, possibly generating the small ratio $\Delta m^2_{\text{solar}}/\Delta m^2_{\text{atm}}$ dynamically.
1 Introduction

The case for the existence of small neutrino masses and associated physical neutrino mixing angles has enormously strengthened in recent years as a consequence of the now numerous experimental studies of atmospheric and solar neutrinos, and neutrinos from terrestrial sources. In particular, a recent global analysis [1] which incorporates data from CHOOZ [2], SNO [3], KamLAND [4] and Super-Kamiokande [5], leads to values for the mass difference squares, $\Delta m^2_{23}$, and three real mixing angles $\theta_{ij}$ of the neutrinos in the $3\sigma$ ranges, $1.4 \times 10^{-3} \text{eV}^2 \leq |\Delta m^2_{23}| \leq 3.7 \times 10^{-3} \text{eV}^2$ and $0.36 \leq \sin^2 \theta_{23} \leq 0.67$ from the atmospheric data, and $5.4 \times 10^{-5} \text{eV}^2 \leq \Delta m^2_{21} \leq 9.5 \times 10^{-5} \text{eV}^2$ and $0.23 \leq \sin^2 \theta_{21} \leq 0.39$ from the solar data, while the remaining real mixing angle is bounded by $\sin^2 \theta_{13} \leq 0.066$. The phases that can appear in the neutrino mixing (MNS) matrix are currently unconstrained.

The traditional and much studied explanation for these small masses is the see-saw mechanism[6]. This hypothesises the existence of two or more standard-model-singlet right-handed (rhd) neutrino states $N_i$, with very large lepton-number violating Majorana masses, which couple to the weak-SU(2) lepton doublets $L_j$ via a conventional Yukawa coupling $\lambda_{ij} L_i N_j H$ involving the electroweak Higgs. The Yukawa couplings are typically taken to be of size comparable to that of either the charged lepton Yukawas or the quark Yukawas, depending on the precise model. After the Higgs gains its vacuum expectation value, $v$, this Yukawa coupling leads to the Dirac mass matrix $m^D_\nu = \lambda v$. Finally, the light neutrino mass matrix obtained by integrating out the heavy rhd neutrinos is given by

$$m_\nu = -(m^D_\nu)^T M^{-1}_R m^D_\nu,$$

where $M_R$ is the rhd Majorana mass matrix. If we take the heaviest Dirac mass to be of order either the $\tau$ lepton or bottom quark mass, then agreement with the light neutrino mass inferred from the atmospheric neutrino anomaly requires roughly $M_R \sim 10^{11} \text{GeV}$, while if the heaviest Dirac mass is taken to be of order the top quark mass, as is commonly the case in explicit models, then $M_R \sim 10^{15} \text{GeV}$. The comparative proximity of this second scale to the inferred supersymmetric grand unified (GUT) scale $M_{\text{GUT}} \sim 2 \times 10^{16} \text{GeV}$ is interpreted as evidence in favour of the see-saw mechanism, as is the fact that rhd SU(3)$\times$SU(2)$\times$U(1) singlet neutrinos are naturally included in many GUT theories.

Certainly therefore the see-saw mechanism is an attractive explanation of why the light neutrino masses are so small. However, it is not without its faults. In particular there is a tension between the strongly hierarchical nature of the observed Yukawa couplings in the quark and charged lepton sectors, and the essentially hierarchy-free masses implied by the $\Delta m^2$'s. Moreover, both the $\theta_{12}$ and $\theta_{23}$ mixing angles are large while the angle $\theta_{13}$ is small which is in sharp contrast with the corresponding mixings in the quark sector which are all small. These problems can be solved in specific models, for example the $\Delta m^2$ values can be fitted by taking the spectrum of rhd neutrino masses to be hierarchical in such a way as to almost compensate for the hierarchical neutrino Yukawa couplings. But this has the price of introducing a wide range of rhd neutrino masses $M_R \sim 10^{10} - 10^{15}$ which then require explanation.
In addition, there is the worrying question of the testability of the see-saw mechanism. Given the large mass scale of the rhd neutrino states there is very little prospect of their ever being a direct test of the correctness of the see-saw mechanism. We are forced to fall back on indirect, and sadly not definitive tests. For example, the discovery of neutrino-less double beta decay would point to a Majorana nature for the light neutrinos as required by the see-saw mechanism, but as we shall see below, the see-saw mechanism is by no means the only way that such Majorana masses can be generated.

In this letter we study a notably attractive alternative, advocated by Arkani-Hamed et al [7, 8], and Borzumati et al [9, 10] (for related work see [11]), that links the light neutrino masses to TeV-scale supersymmetry breaking physics, and which has the significant virtue, compared to the see-saw mechanism, of being directly testable, at least in part, at the LHC and other proposed high-energy colliders. In overall structure our models are similar to those previously studied in Refs. [7, 8, 9, 10], in that the dominant contribution to the light neutrino masses arises from a one-loop diagram involving a supersymmetry breaking and lepton-number violating $B$-term for the right handed sneutrinos, but, by an alteration of the model, we have been able to significantly simplify the way in which the flavour structure of the light neutrino mass matrix arises, and are thus able for the first time to study in detail some of the consequences of this very attractive class of models.

2 Outline of the model

Supersymmetric extensions of the standard model, such as the minimal supersymmetric standard model (MSSM), explain the origin of the weak scale in terms of the scale of the coefficients of the supersymmetry breaking soft operators that must be included within the MSSM. The usual assumption is that these susy-breaking operators arise from the (super)gravitational mediation of susy breaking that occurs primordially in some hidden sector at an intermediate scale $m_I \sim 10^{10} - 10^{11}$ GeV, giving rise to soft susy breaking in the MSSM of order $\text{TeV} \sim m_I^2/M$. Here $M$ is the reduced Planck mass $M = M_{\text{pl}}/\sqrt{8\pi} = 2 \times 10^{18}$ GeV. Infamously, there is one mass parameter in the MSSM, the $\mu_{H_uH_d}$ superpotential interaction, that naively appears to be independent of supersymmetry breaking. If this were indeed true then we would lose completely our understanding of the origin of the weak scale in susy theories [12]. The realization of Giudice and Masiero [13] was that the potentially Planck-scale $\mu_{H_uH_d}$ term in the superpotential can be forbidden by a global symmetry, while an effective $\mu$-term is generated from $1/M$-suppressed terms in the Kahler potential via supersymmetry breaking effects, thus naturally implying $\mu \sim \text{TeV}$.

As emphasized by Arkani-Hamed et al and Borzumati et al, the lesson of the Giudice-Masiero mechanism for neutrino masses in the context of susy theories is that SM-singlet operators, such as the rhd neutrino mass $M_{RNN}$, or the neutrino Yukawa coupling $\lambda LNH_u$, might only appear to be renormalizable superpotential terms, but in fact may arise from $1/M$-suppressed terms involving the supersymmetry breaking scale $m_I$.

Specifically, consider the usual MSSM Lagrangian to be supplemented by a set of superpotential and Kahler terms involving the rhd neutrino superfields $N_i$ ($i = 1, 2, 3$ is
a generation index), and two Standard-Model-singlet chiral superfields $X$ and $Y$ which arise from the hidden sector. In general, the fields which communicate supersymmetry breaking to the neutrinos can either be flavour singlets or flavour non-singlets. Let us generically call the flavor non-singlet fields, $X$, (which therefore carry generation indices $i, j$), and the flavour singlet fields, $Y$, and for simplicity suppose that there is just one $X$ field and one $Y$ field. Then the model we wish to study is defined by the $N_i$-dependent terms in the superpotential

$$L^W_N = \int d^2 \theta \left( g \frac{X_{ij}}{M} L_i N_j H_u + g' \frac{Y}{M} L_i N_i H_u + \ldots \right), \quad (2)$$

while the set of terms involving the rhd $N_i$ fields in the Kahler potential are

$$L^K_N = \int d^4 \theta \left( h \frac{Y}{M^2} N_i N_i + \tilde{h} \frac{Y}{M^2} N_i N_i + h_B \frac{Y Y^\dagger X_{ij}}{M^3} N_i N_j + \ldots \right). \quad (3)$$

The ellipses in Eqs.(2) and (3) stand either for terms involving the replacement of $Y$ fields by $X_{ij}$ fields (with obvious changes to $N_i$ flavour indices), or for terms higher order in the $1/M$-expansion. It is simple to check that both types of additional term will lead to trivial or sub dominant contributions not relevant for our discussion. The Lagrangian displayed in (2) and (3) can be justified with an R-Symmetry, where both hidden sector fields $X$ and $Y$ have R charge $\frac{4}{3}$, $N$ has R charge $\frac{2}{3}$, $E$ (rhd charged lepton superfield) has R charge 2 and the remaining superfields have R charge equal to 0, and the usual $R$-parity is assumed, with in addition $R_{p}(X) = R_{p}(Y) = +1$. All dimensionless couplings $g, h, \text{etc}$, are taken to be order one parameters.

Let us now suppose that after supersymmetry is broken in the hidden sector at scale $m_I$ the field $Y$ acquires the following $F$ and $A$-component vacuum expectation values,

$$\langle Y \rangle_F = F_Y = f_Y m_I^2$$
$$\langle Y \rangle_A = A_Y = 0,$$ \quad (4)

while the field $X$ acquires the $F$ and $A$-component expectation values:

$$\langle X_{ij} \rangle_F = F_{X_{ij}} = 0$$
$$\langle X_{ij} \rangle_A = A_{X_{ij}} = a_{X_{ij}} m_I.$$ \quad (5)

Here $f_Y$ and $a_{X_{ij}}$ are order one parameters, and the zero entries for $A_Y$ and $F_X$ can be replaced by non-zero values $A_Y \ll m_I$, and $F_X \ll m_I^2$ without significant change. We will often re-write the scale $m_I$ in terms of the gravitino mass $m_{3/2} = m_I^2/M$, and the reduced Planck mass $M$.

Before we proceed to analyze the consequences of the above effective Lagrangian for the light neutrino masses and mixings, a comment is in order concerning the assumption that $\langle F_X \rangle_{ij} \ll m_I^2$. As is well-known, typical supergravity mediation of susy breaking has difficulties with FCNC and CP-violation constraints unless there is a high degree of degeneracy among the squark and slepton soft masses of different generations (we here ignore the possibility of alignment mechanisms which are typically much harder to
implement). Such degeneracy is accommodated in our model when $\langle F_X \rangle_{ij} \ll m_l^2$. In this paper we will not be concerned about the detailed origin of this high-level of degeneracy, but take it as a phenomenologically necessary assumption. As we will argue below, in this case a simple prediction for the structure of the light neutrino mass matrix can result.

### 3 Neutrino and sneutrino masses at tree level

Given Eqs.(2) and (3), the relevant terms in the Lagrangian after the $X$ and $Y$ fields gain their expectation values are thus

$$\mathcal{L} = \int d^2 \theta \left( \lambda_{ij} L_i N_j H_u + M_N N_i N_i \right) + A L_i \tilde{n}_i h_u + B_{ij}^2 \tilde{n}_i \tilde{n}_j + \ldots, \quad (6)$$

where $\tilde{n}_i$ are the rhd sneutrino fields, $\tilde{L}_i$ is the lhd slepton doublet, $h_u$ is the up-type Higgs scalar doublet, and the omitted terms include the usual soft scalar mass terms.

First note that the effective neutrino Yukawa coupling in the superpotential of Eq.(6) is suppressed in magnitude by a factor of $(m_3^3/2M)^{1/2} \sim 10^{-7} - 10^{-8}$,

$$\lambda_{ij} = g a_{Xij} \sqrt{m_3^3/2M}, \quad (7)$$

and gains its flavour structure from the $\langle X_{ij} \rangle_A$ expectation value, and in addition, the scale of the rhd neutrino masses is lowered to the TeV scale

$$M_N = h f_Y m_3/2. \quad (8)$$

Second, there exists a TeV-scale, but flavour diagonal, trilinear scalar $A$-term

$$A = g' f_Y m_3/2. \quad (9)$$

Finally there is a small but significant rhd sneutrino lepton-number violating $B$-term with coefficient

$$B_{ij}^2 = h_B f_Y^2 a_{Xij}^s \sqrt{m_3^5/2M} \quad (10)$$

of magnitude $B_{ij}^2 \sim (\text{few} \times 100\,\text{MeV})^2$ and flavour structure related to that of the neutrino Yukawa coupling ($a_{Xij}^s$ denotes the symmetric part of $a_{Xij}$).

After electroweak symmetry breaking, with Higgs expectation values $\langle H_u^0 \rangle = v \sin \beta$ and $\langle H_d^0 \rangle = v \cos \beta$, the effective Lagrangian Eq.(6) implies that at tree level in the neutrino sector, after integrating out the TeV-scale rhd neutrinos, there is generated a light neutrino mass matrix of see-saw type:

$$(m^\text{tree}_\nu)_{ij} = -\frac{v^2 \sin^2 \beta}{M_N} a_{Xik}^T \lambda_{kj} \sim \frac{v^2}{m_3^3/2M} m_{3/2}^2. \quad (11)$$

If one takes $M = 2 \times 10^{18}\,\text{GeV}$, $v = 174\,\text{GeV}$, and all $a_{Xik}$, $f_Y$, $g$ and $h$’s to be of order 1, this gives a contribution to the light neutrino mass matrix of order $10^{-5}\,\text{eV}$, certainly too small to generate the required $\Delta m_{23}^2$ and $\Delta m_{12}^2$.
Note, however, that since the operators that give rise to the Yukawa and rhd Neu-
trino mass terms are higher-dimension non-renormalizable operators, a more appropriate
estimate of the couplings $g$ and $h$ might be the values found by applying the so-called
‘naive-dimensional analysis’ (NDA) methodology which assumes that the cutoff $M$ is
bounded by the UV strong coupling scale of the non-renormalizable theory, and in which
geometrical factors of $1/(4\pi)^2$ which enter loop calculations are taken into account[14].
(It is known that NDA works well for estimating the coefficients of the higher-dimensional
operators in the low-energy chiral-Lagrangian description of QCD.) A simple calculation
shows that the NDA estimates for the sizes of the couplings $h$ and $g$ which set the rhd
neutrino mass and lhd-rhd-Higgs Yukawa coupling are $h \simeq 1$ and $g \simeq 4\pi$ (and a UV
strong coupling scale $\Lambda$ related to the reduced Planck mass as $\Lambda \simeq 4\pi M$). This leads to
an improved estimate of the size of the tree (see-saw) contribution to the light neutrino
masses

$$
(m_{\nu}^{\text{tree}})_{ij} \simeq -\frac{v^2 \sin^2 \beta g^2}{M} \frac{\Lambda}{h} \simeq 10^{-3} \text{eV}.
$$

(12)

Alternatively, the large mass scale $M$ might be the string or susy-GUT scale, thus simi-
larly increasing the estimate for $m_{\nu}^{\text{tree}}$. In any case, we will argue in the next section that
a small tree-level term of size Eq.(12) can sometimes be an interesting perturbation to
the dominant one-loop contribution to the light neutrino mass matrix.

Turning to the sneutrino sector of the theory the effective Lagrangian Eq.(6) implies,
after electroweak symmetry breaking, a 12 by 12 mass matrix that mixes the lhd and
rhd sneutrinos and their conjugates (here, for simplicity, assuming $A$ and $B^2$ real),

$$
\begin{pmatrix}
M^2_L \delta_{ij} & Av \sin \beta \delta_{ij} & 0 & 0 \\
Av \sin \beta \delta_{ij} & M^2_R \delta_{ij} & 0 & B^2_{ij} \\
0 & 0 & M^2_L \delta_{ij} & Av \sin \beta \delta_{ij} \\
0 & B^2_{ij} & Av \sin \beta \delta_{ij} & M^2_R \delta_{ij}
\end{pmatrix},
$$

(13)

where we work in the basis $(\tilde{\nu}^*, \bar{n}, \tilde{\nu}, \bar{n}^*)$, $M^2_L = m^2_L + m^2_Z \cos(2\beta)/2$ is the lhd sneutrino
mass arising from the usual soft mass $m^2_L$ and electroweak breaking terms, and $M^2_R = m^2_R + M^2_N$ is the total rhd sneutrino mass including the soft mass-squared. (Our notation
is that Latin indices run from 1 to 3, so e.g., the $\bar{n}$ are indexed as $i+3$, and sneutrino
mass eigenstates are labeled by Greek indices $\alpha, \beta = 1, \ldots, 12$. ) This mass matrix is
diagonalised by a unitary rotation of the form

$$
U = \begin{pmatrix}
V/\sqrt{2} & 0 & V/\sqrt{2} & 0 \\
0 & V/\sqrt{2} & 0 & V/\sqrt{2} \\
-\sqrt{2} & 0 & V/\sqrt{2} & 0 \\
0 & 0 & V/\sqrt{2} & V/\sqrt{2}
\end{pmatrix}
\begin{pmatrix}
\cos \phi & \sin \phi & 0 & 0 \\
-\sin \phi & \cos \phi & 0 & 0 \\
0 & 0 & \cos \phi & \sin \phi \\
0 & 0 & -\sin \phi & \cos \phi
\end{pmatrix}
$$

(14)

where $\tan 2\phi = 2Av \sin \beta/(M^2_L - M^2_R)$, and $V$ is the 3 by 3 matrix that diagonalises
$B^2_{ij}$. The expression Eq.(14) is correct up to small terms of order $B^2/(M^2_L - M^2_R)$. The
 corresponding sneutrino mass eigenvalues fall into 2 groups of almost 3-fold degenerate
complex states.
4 Structure of light neutrino masses at 1-loop

The rhd sneutrino interaction Eq.(10) gives rise to a radiative contribution to the light neutrino masses, illustrated in Fig. 1, that, for \( \langle Y \rangle_F \sim m_f^2 \) and \( \langle X_{ij} \rangle_A \sim m_f \), dominates over the tree level contribution arising from the the mixing of the TeV-mass rhd neutrinos with the lhd neutrino states. In detail, the contribution to the light Majorana neutrino mass matrix from the diagram in Fig. 1 is given by,

\[
m_{\nu, ij}^{\text{loop}} = \frac{\chi^2 B^2_\nu}{16\pi^2} \left[ \sum_{\alpha=1}^{4} \sum_{\beta=1}^{4} \sum_{\gamma=1}^{4} \sum_{\delta=1}^{4} U_{\alpha,i}^\dagger U_{\beta,k}^\dagger U_{\gamma,p}^\dagger U_{\delta,j}^\dagger \right] \frac{1}{L(\alpha, \beta, \gamma, \delta, x)},
\]

where all repeated indices are summed over. In this expression, \( \chi^2 \) is a factor that depends on the exchanged neutralino, \( U \) is the sneutrino mixing matrix, \( x = 1, \ldots, 4 \) denote the neutralino mass eigenstates, and \( L(\alpha, \beta, \gamma, \delta, x) \), is a totally symmetric function of the sneutrino and neutralino masses that arises from the momentum integral in Figure 1.

Because of the small effective Yukawa coupling, Eq.(7), between the higgsinos, lhd neutrinos, and rhd sneutrinos, in practice only the bino and zino components of the neutralino are exchanged across the bottom of the loop. Thus only these states need to be expanded in terms of the 4 neutralino mass eigenstates, leading to

\[
\chi^2 = \frac{2M_Z^2}{v^2} \left| N_{x1}^* \sin \theta_w - N_{x2}^* \cos \theta_w \right|^2 M_{\chi^2}^2
\]

where \( N_{x1} \) is the standard neutralino mixing matrix, and \( M_{\chi^2} \) are the neutralino masses.

A transparent and elegant form for the light neutrino mass matrix results if we consider the situation in which the \( A \)-term mixing is small. This greatly simplifies the flavor structure of the result Eq.(15). (The \( V \) matrices in \( U \) of Eq.(14) cancel by virtue of \( VV^\dagger = 1 \).) Moreover, in this limit the loop factor \( L \) also simplifies,

\[
L = \frac{1}{M_R^6(r-1)^3(1-x)^2(1-x)^2} \left[ (1-r)(1+r-2x)(1-x)(r-x) + \left( 2r^2 - x^3(1+r) \right) \log r - x \left( \log(r/x) + r \log(x^3r) - r^2 \log(x^3/r^4) + r^3 \log x \right) \right],
\]

where \( r \equiv M_L^2/M_R^2 \) and \( x \equiv M_{\chi^2}/M_R^2 \). Therefore in this limit of small sneutrino mixing
the 1-loop contribution to the light neutrino mass matrix becomes

\[(m_{\nu}^{\text{loop}})_{ij} = \sum_x x^2 \frac{A_x^2 B_y^2 v^2 \sin^2 \beta}{(4\pi)^2} L(M_L^2, M_R^2, M_R^2, M_L^2, m_{\chi_x}). \tag{18}\]

The most important feature of the result Eq.(18) is that the overall scale of the contribution \(m_{\nu}^{\text{loop}}\) is naturally of the correct size to account for atmospheric neutrino oscillations (as is also true of the models of Refs.[8, 10]). To see this explicitly it is useful to consider the simple case in which all the lhd and rhd sneutrinos and the neutralinos are approximately equal to a common mass scale, \(m_{\text{susy}}\), giving

\[L \simeq -\frac{1}{12 m_{\text{susy}}^6}. \tag{19}\]

which leads to a one-loop contribution of magnitude

\[m_{\nu}^{\text{loop}} \sim \mu \equiv \frac{\alpha_w}{96\pi} \frac{m_{\text{susy}}^9 v^2}{M^5 m_{\text{susy}}^5} \approx 10^{-2} \text{eV} - 10^{-1} \text{eV} \tag{20}\]

depending on the precise magnitude of the \(A\) and \(B\) terms. In addition, in our model, the flavour structure of this dominant one-loop contribution to the light neutrino mass matrix is determined directly and entirely by the rhd sneutrino lepton-number violating \(B\)-term, \(B_{ij}^2\), which is in turn generated by the \(\langle X_{ij}\rangle_A\) expectation value. Moreover, as claimed earlier, the one loop result dominates over the tree level see-saw contribution. It is useful to define the (small) parameter \(\epsilon\) as the ratio of magnitudes of the tree-level see-saw contribution Eq.(12) to the above 1-loop contribution.

It is also interesting to consider the regime in which the rhd sneutrino states are heavy compared to the neutralino and lhd sneutrino states, \(r \simeq x \ll 1\). In this case the loop factor is approximated by the expression

\[L \simeq -\frac{(x-r) + x \log(r/x)}{M^6 r^2} \simeq -\frac{1}{2 M^4 M_L^2}. \tag{21}\]

Taking \(M_L \sim M_\chi \sim m_{\text{susy}}\) and scaling the \(A\) and \(B\) terms relative to their natural values, \(A_0^2 \sim m_{\text{susy}}^2\), and \(B_0^2 \sim m_{\text{susy}}^2 (m_{\text{susy}}/M)^{1/2}\) given by Eqs.(9) and (10), leads to

\[m_{\nu}^{\text{loop}} \simeq \frac{m_{\text{susy}}^{7/2} M_Z^2}{16\pi^2 M_R^4 M^2} \left[\frac{A^2}{A_0^2}\right] \left[\frac{B^2}{B_0^2}\right]. \tag{22}\]

Assuming \(m_{\text{susy}} \sim 300\text{ GeV}\), and \(M_R \sim 1\text{ TeV}\) this gives \(m_{\nu}^{\text{loop}} \sim 0.02\text{ eV}(A^2/A_0^2)(B^2/B_0^2)\) showing that \(M_R\) cannot be much heavier than 1 TeV unless the scale of the MSSM superpartners is uncomfortably high.

In either case the final structure of the light neutrino mass matrix is in total

\[(m_{\nu}^{\text{tot}})_{ij} = \mu (a_X^s + \epsilon a_X^T a_X)_{ij}. \tag{23}\]

with the scale set by \(\mu \sim 0.1\text{ eV} - 0.01\text{ eV}\), and \(\epsilon\) in the range \(\epsilon \sim 10^{-2} - 10^{-4}\). An attractive feature of this structure is that it allows us in a simple way to account for
the hierarchy between the solar and atmospheric neutrino mass-squared splittings. The atmospheric $\Delta m^2$ can arise from the one-loop contribution, while the tree-level correction leads to the small $\Delta m^2_{\text{solar}}$ splitting, the hierarchy being entirely due to the small dynamical parameter $\epsilon \sim 10^{-2}$. Of course it is possible that the hierarchy instead arises entirely from the flavour structure of the dominant $\mu a X$ term, the tree-level perturbation being insignificant, but this leads to quite traditional neutrino flavour models, so we here focus upon the new possibility.

As a simple example of a model along these lines consider the situation in which the matrix $a_X$ corresponds to the leading-order form for an inverted hierarchy model

$$a_X = \begin{pmatrix} 0 & 1 & a \\ 1 & 0 & 0 \\ a & 0 & 0 \end{pmatrix}. \quad (24)$$

Therefore in this case the full light neutrino mass matrix including both loop and tree contributions has the form

$$m^\text{tot}_\nu = \mu \begin{pmatrix} (1 + a^2)\epsilon & 1 & a \\ 1 & \epsilon & a\epsilon \\ a & a\epsilon & a^2\epsilon \end{pmatrix}. \quad (25)$$

This mass matrix has one zero eigenvalue, and two massive eigenvalues $m^\pm \simeq \mu(\sqrt{a^2 + 1} \pm \epsilon)$. Therefore to accommodate the oscillation data requires $(1 + a^2)\mu^2 \simeq 2 \times 10^{-3} \text{eV}^2$, while the solar oscillation data requires $4(1 + a^2)^{1/2}\mu^2\epsilon \simeq 7 \times 10^{-5} \text{eV}^2$. For $a \sim 1$ this gives

$$\mu^2 \simeq 10^{-3} \text{eV}^2, \quad \epsilon \simeq 10^{-2} \quad (26)$$

comfortably of the sizes expected in this model. Moreover, since $m^\text{tot}_\nu$ is diagonalised by first performing a 23 rotation, $R_{23}(\theta)$, with angle $\theta = \tan^{-1}(a)$, and then a 12 rotation, $R_{12}(\phi)$, with $\phi = \pi/4$, we see that for $a \sim 1$ the atmospheric and solar angles will both be close to maximal. Specifically, recalling that the physical neutrino mixing (MNS) matrix includes a unitary matrix $V_L$ from the rotation of the charged leptons to a basis where the charged lepton mass matrix is diagonal and real we find a MNS matrix of the form

$$V_{\text{MNS}} = V_L R^T_{23}(\theta) R^T_{12}(\phi) = V_L \begin{pmatrix} c_\phi & s_\phi & 0 \\ -c_\theta s_\phi & c_\theta c_\phi & s_\theta \\ s_\theta s_\phi & -s_\theta c_\phi & c_\theta \end{pmatrix}. \quad (27)$$

Under the not unreasonable assumption that the mixing angles in from the charged lepton sector are small, $V_L$ only slightly perturbs the above structure, thus leading to almost maximal physical atmospheric and solar mixing angles, and a small $\theta_{e3}$ angle.

Alternatively, light neutrino masses with a normal hierarchy can be easily generated if $a_X$ contains an antisymmetric piece, $a^*_X$. In particular consider the forms,

$$a^*_X = \begin{pmatrix} 0 & 0 & 0 \\ 0 & b & b \\ 0 & b & b \end{pmatrix}, \quad a^a_X = \begin{pmatrix} 0 & c & d \\ -c & 0 & e \\ -d & -e & 0 \end{pmatrix}. \quad (28)$$

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Substitution of these matrices into Eq.(23) with reasonable (and not fine-tuned) $O(1)$ values for the parameters $b, c, d,$ and $e,$ and values of the dynamically determined parameters $\mu^2 \simeq \text{few} \times 10^{-3} \text{eV}^2,$ and $\epsilon \simeq 10^{-2},$ leads to a light neutrino mass matrix, which when diagonalized produces mass squared differences within the experimental bounds for a normal hierarchy. Moreover, again under the assumption that the real mixing angles in $V_L$ are small this leads to large physical atmospheric and solar mixing angles, and a small $\theta_{e3}$ angle. More generally, if the matrix $a_X$ is not real as in the above examples, but contains large phases then it is simple to generate successful models in which the one-loop term gives $\Delta m^2_{\text{atm}}$ while the perturbing tree term leads to $\Delta m^2_{\text{solar}}$.

5 Comments and conclusions

In this paper we have further analyzed a class of models first introduced by Arkani-Hamed et al [7, 8], and Borzumati et al [9, 10], in which the light neutrino masses are a result of higher-dimensional supersymmetry-breaking terms. The mechanism is closely related to the Giudice-Masiero mechanism for the MSSM Higgs $\mu$ parameter, and in particular leads to TeV-scale rhd neutrino and sneutrino states, that are in principle accessible to direct experimental study, unlike traditional see-saw mechanisms. A second difference is that the dominant contribution to the light neutrino mass matrix (which is of Majorana type) is a one-loop term induced by a lepton-number violating and supersymmetry breaking $B$-term for the sneutrino states that is naturally present in the model. In this letter we have focused upon the simplification and analysis of the flavour structure of this general class of models, and have found that simple predictions for the light neutrino mass matrix are possible. In addition we have found that the subdominant tree-level ‘see-saw’ contribution may lead to interesting perturbations of the leading one-loop-induced flavour structure, possibly generating the smaller $\Delta m^2_{\text{solar}}$.

In this paper we have not explored the important issues of the possible collider and cosmological tests of our models. In broad structure the implications of our models are similar to those already analyzed in Refs.[7, 8, 9, 10]. In particular one expects the $A$-term interactions in our model to lead to interesting possibilities for production and decay of the TeV-scale rhd sneutrino states, including the possibility of anomalous Higgs decays. Another intriguing possibility is that the rhd sneutrino states could be the dark matter [15], or that, when CP-violation in the neutrino sector is taken into account, dark matter with dominant inelastic interactions with matter results [16]. In a future publication, [17], we show that the class of models analyzed here naturally lead to a very attractive and successful theory of TeV-scale resonant leptogenesis, developing from the earlier work of [18].

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