Indirect quality estimates of the vehicle movement response to the control step input

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Abstract. The transient performance caused by step change of steering wheels position of a vehicle with oversteering is studied in this paper. The performance was assessed basing on indirect integral estimate widely applied in automatic control theory. Impact analysis on integral quadratic estimate of design parameters of a vehicle and its speed has been carried out.

From the point of view of automatic control theory a car can be presented as a controlled object. Therefore when researching the vehicle steerability in transient processes which appear as a result of control actions applied to the steering system being a preset time function it is possible to apply indirect integral estimates of control process quality widely used in this theory. These estimates analysis allows comparing transient performance by calculated definite integrals minimum values. Linear and integral quadratic estimates are singled out [2]. For example, expression for linear integral estimate comes in the following form:

$$J_{00} = \int_{0}^{\infty} [x(t) - x(\infty)] dt$$

(1)

where $x(t)$ – controlled value dependence on time (the curve, describing the transient process); $x(\infty)$ – steady-state controlled value after the transient process termination.

In case the vehicle angular speed response to the change of steering wheels position is researched, linear integral estimate has the following form:

$$J_{00} = \int_{0}^{\infty} [\omega(t) - \omega(\infty)] dt$$

(2)

where $\omega(t)$ – is the expression describing the transient process to set the vehicle angular speed new value by the steering wheels angle change; $\omega(\infty)$ – angular speed steady-state value.

Unfortunately, since linear estimates, obtained by expressions (1) and (2), have a serious drawback and are mostly applied for monotonous processes, in order to broaden the range of application and usability for any integral quality criterion processes, integral quadratic estimate is used, which in our case has the following form:
\[ J_0 = \int_0^\infty (\Delta \omega)^2 \, dt \]  

(3)

where \( \Delta \omega = \omega(t) - \omega(\infty) \).

To obtain the vehicle angular speed dependence on time and its limit value after the transient process termination it is necessary to set the vehicle design model and theoretical test conditions. As before, to describe a vehicle, a single-mass two-dimensional design model is used [1,3]. When carrying out the research, it is assumed that the controlled object is affected only by control action being a step time function, the vehicle linear speed is a constant value, restrictions characteristic of the selected design model are taken into consideration, the vehicle moves only along a horizontal plane. The following symbols are used in this work:

- \( a \) – distance from the vehicle gravity center to the front axle;
- \( b \) – distance from the vehicle gravity center to the rear axle;
- \( C_1 \) – resistance coefficient of the front axle lateral skid;
- \( C_2 \) – resistance coefficient of the rear axle lateral skid;
- \( E_1, E_2, E_3, T_1, T_4 \) – constant coefficients;
- \( s_1, s_2 \) – roots of the characteristic equation;
- \( L \) – the vehicle wheel base value;
- \( J_z \) – the vehicle inertia moment with respect to a vertical axis through its gravity center;
- \( V \) – the vehicle speed.

As we already know, the vehicle characteristic equation discriminant sign determines the law of transient process of angular speed change, which in its turn can be either harmonic or exponential. Thus, if the discriminant is greater than zero, the exponential law is obtained, described by expression (4), and if the discriminant is less than zero – a harmonic one, determined by expression (5).

\[
\omega(t) = E_1 + E_2 e^{s_1 t} + E_3 e^{s_2 t}
\]  

(4)

\[
\omega(t) = T_1 + T_4 e^{-\frac{R_i}{2}} \sin(\omega_t t + \varphi_k)
\]  

(5)

Let us determine which law is characteristic of the corresponding type of the vehicle tire cornering ability. For this purpose, basing on equations of motion in the form of stability derivatives, the vehicle characteristic equation has been composed [1], expression to calculate discriminant has been written, with corresponding condition for each specific cornering ability type substituted in it. As a result of calculations the required data have been obtained, tabulated hereafter (see table 1).

Summarizing the information presented in table 1, it is possible to conclude that for the case of oversteering, characteristic equation discriminant sign is always greater than zero; consequently the transient process strictly follows the exponential law as shown in expression (4). For the case of understeering there are three options at which the sign can be either positive or negative, but there is also a situation in which the discriminant equals zero. In this work a case of the vehicle oversteering has been considered.
Knowing the law of the vehicle angular speed change by driving disturbance, we can determine the transient performance integral quadratic estimate. Taking into consideration expressions (3) and (4) as well as $E_1 = \omega(\infty)$, we can calculate $\Delta \omega$:

$$\Delta \omega = E_1 + E_2 e^{\omega t} + E_3 e^{V \omega t} - \omega(\infty) = E_2 e^{\omega t} + E_3 e^{V \omega t}$$

Hereafter we find $(\Delta \omega)^2$:

$$(\Delta \omega)^2 = (E_2 e^{\omega t} + E_3 e^{V \omega t})^2 = E_2^2 e^{2\omega t} + 2E_2E_3 e^{(\omega+V \omega)t} + E_3^2 e^{2(V \omega)t}$$

We calculate integral (3) on condition that $s_1 < 0, s_2 < 0$:

$$J_0 = \int_0^\infty (E_2^2 e^{2\omega t} + 2E_2E_3 e^{s_1+s_2}t + E_3^2 e^{2V \omega t}) dt = E_2^2 \int_0^\infty e^{2\omega t} dt + 2E_2E_3 \int_0^\infty e^{s_1+s_2}t dt + E_3^2 \int_0^\infty e^{2V \omega t} dt =$$

$$= \frac{E_2^2}{2s_1} + \frac{2E_2E_3}{s_1+s_2} \left| e^{s_1+s_2}t \right|_0^\infty + \frac{E_3^2}{2s_2} \left| e^{2V \omega t} \right|_0^\infty = \left( 0 - \frac{E_2^2}{2s_1} \right) + \left( 0 - \frac{2E_2E_3}{s_1+s_2} \right) + \left( 0 - \frac{E_3^2}{2s_2} \right) =$$

$$= -\frac{E_2^2}{2s_1} - \frac{2E_2E_3}{s_1+s_2} - \frac{E_3^2}{2s_2}$$

Finally written as:

$$J_0 = \left( \frac{E_2^2}{2s_1} + \frac{2E_2E_3}{s_1+s_2} + \frac{E_3^2}{2s_2} \right)$$

(6)

It should be reminded that the expressions to calculate coefficients $E_2$ and $E_3$, as well as to find roots of characteristic equation $s_1$ and $s_2$ are detailed in work [1] and are not given here. Nevertheless it is worth mentioning that they depend on the vehicle design and performance parameters being interconnected in a complex way.

Now we can analyze the expression obtained to determine indirect integral estimate. For this purpose, by example of the selected specifications, characteristic of a vehicle with oversteering, create integral quadratic estimate value dependences on the vehicle linear speed, wheelbase value, vehicle mass and lateral skid resistance coefficient are created.

Figure 1 shows curve of integral quadratic estimate dependence on the vehicle constant linear speed at its different mass values. As can be seen in the graph the integral estimate value goes up at the speed increase, the more the vehicle mass is the greater the increase. Thus, the transient performance worsens with the speed increase, worsening the vehicle
steerability as well. Figure 2 makes it possible to characterize the integral quadratic estimate behaviour at the change of the vehicle wheelbase. In this case the definite integral value will go down with the wheelbase increase. In other words the wheelbase increase improves the transient performance reducing its time. Figure 3 shows the indirect estimate dependence on the vehicle mass. Here we can see the transient performance worsening at the vehicle mass increase, which worsens the vehicle steerability as well.

**Figure 1.** $J_0$ dependence on $V$

![Figure 1. $J_0$ dependence on $V$](image1)

**Figure 2.** $J_0$ dependence on $L$

![Figure 2. $J_0$ dependence on $L$](image2)
Conclusion: the results of the work allow to obtain an expression to calculate integral quadratic estimate of the transient performance estimate, taking place in a vehicle with oversteering at the step change of steering wheels position. Analysis of different vehicle design parameters impact on the transient performance shows that with the vehicle speed and mass increase, the transient performance worsens which in its turn causes the vehicle steerability worsening. The wheelbase value increase, as different from the previous parameters, influences in the contrary way the transient performance, reducing the process time.

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