Influence of wave frequency variation on anomalous cyclotron resonance interaction of energetic electrons with finite amplitude ducted whistler-mode wave.

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Abstract—The influence of wave frequency variation on the anomalous cyclotron resonance $\omega = \omega_B e + kv_\parallel$ interaction (ACRI) of energetic electrons with a ducted finite amplitude whistler-mode wave propagating through the so-called transient plasma layer (TPL) in the magnetosphere or in the ionosphere is studied both analytically and numerically. The anomalous cyclotron resonance interaction takes place in the case when the whistler-mode wave amplitude $B_W$ is consistent with the gradient of magnetic field $\vec{B}_0$. The region of phase space occupied by anomalously interacting energetic electrons (synchronous particles) is determined. The efficiencies of both the pitch-angle scattering of resonant electrons and their transverse acceleration are studied and the efficiencies dependence on the magnitude and sign of the wave frequency drift is considered. It has been shown that in the case of ACRI occurring under conditions relevant to VLF-emission in the magnetosphere, the energy and pitch-angle changes of synchronous electrons may be enhanced by a factor $10^2 \div 10^3$ in comparison with ones for nonsynchronous resonant electrons. So the small in density group of synchronous particles may give significant contribution to a whistler-mode wave damping in TPL.

1. INTRODUCTION

The cyclotron resonance of charged particles with whistler-mode waves (CRI) is one among the basic mechanisms which govern a dynamics of these waves in the magnetosphere and in the ionosphere. As an example, CRI is closely related to one of the most fascinating phenomena – so-called VLF-emissions triggering in the magnetosphere (see, for example, review papers of Molchanov, 1985; Omura et al., 1991; Helliwell, 1993 and Rycroft, 1993 and papers of Dysthe, 1971; Karpman et al., 1974; Nunn, 1974; Dowden et al., 1978; Matsumoto, 1979; Bell, 1984; Nunn, 1984). Moreover, CRI may give also the main contribution to VLF-waves damping and growth and caused by them the pitch-angle diffusion of magnetospheric plasma hot population to loss cone results to an anomalous precipitation of energetic electrons from the radiation belts into
the upper atmosphere. From practical point of view, registration of VLF-emissions in the magnetosphere and in the ionosphere and charged particle precipitation induced can be applied to various diagnostic problems, for instance, to global control of the geliogeophysical enviroment, to the forecasting of radiowaves propagation in the Earth-ionosphere waveguide, to the satellite monitoring of natural crisis processes like typhoons and earthquakes and so on.

Specific feature of CRI in the magnetosphere is the key role of inhomogeneity of the geomagnetic field $B_0$ and the plasma density $n_e$ which determine the typical space scale of cyclotron resonance region, its location along the geomagnetic field line, the intensity and direction of energy transfer at the nonlinear stage of a wave-particle interaction (WPI). The estimates performed (e.g. Bell and Inan, 1981) have shown that under the typical magnetospheric conditions the energetic electron scattering by VLF-wave with significant changes of particle energy and pitch-angle occurs only if the wave amplitude becomes large enough because the typical space scale of CRI-region is usually much less than the inhomogeneity length of geomagnetic field $B_0(S/L_B)$. Indeed, if $\omega$ and $k$ are the whistler-mode wave frequency and wave vector respectively, $Y = \omega_{He}/\omega$ is the energetic electron dimensionless gyrofrequency, $S$ – is the arc length along the geomagnetic field $B_0(S/L_B)$ and $L_B$ is the inhomogeneity length, then one can determine the small parameter of problem considered by the following formulae

$$\delta = 2\pi \left( \frac{Y - 1}{1 + \alpha^2_\perp} \right) \frac{(2Y + 1)}{kL_B} \sim 10^{-4} \div 10^{-5}$$

where $\alpha^2_\perp = v^2_\perp(Y - 1)/[(2Y + 1) v^2_R]$ – is the square of electron perpendicular velocity normalized on the typical value, $v_R = (\omega - \omega_{He})/k$ is the resonance velocity. According to the linear theory of WPI, the typical space scale of cyclotron resonance region located outside the equatorial plane is of the order of $l_R \approx L_B^{\delta^{1/2}}$. At the equatorial plane, the geomagnetic field gradient becomes zero. So in the case of WPI located at the equatorial plane, the space scale of cyclotron resonance region increases up to $l_R \equiv L_B^{\delta^{1/3}}$. Nevertheless, in both cases under conditions, typical for VLF-emissions generation in the magnetosphere, the CRI-region space scale $l_R$ is about two order of magnitude less than the inhomogeneity length $L_B$. Therefore, it is of considerable importance to study the possibility of sharp growth of CRI temporal duration, for instance, due to extent of the interaction region space scale over the substantial portion of the inhomogeneity length $L_B$. For the cyclotron resonance interaction of energetic electrons with the ducted whistler-mode wave of variable frequency in the equatorial plane vicinity, this problem was considered by Brinca (1981) and Bell and Inan (1981). It was shown that the wave frequency variation in the case of optimum frequency function allows to enhance significantly the interaction region space scale for the most stable trapped electrons. In the case of fixed-frequency wave this problem was studied by Erokhin (1995) and it was founded that the ACRI takes place in the stationary transient boundary layer under consistency of the whistler-mode wave amplitude $B_W$ with the magnetic field gradient. So the following condition must be fulfilled : $B_W/B_0 \sim 1/kY L_B$. This condition is in analogy with the long-lasting resonance condition considered by Helliwell (1967) and the second-order resonance one described by Nunn (1971) but in contrast to these articles, paper of Erokhin (1995) relates to so-called synchronous particles whose phase $\Phi$ (its definition see below) is close to $\pi/2$ during their crossing of TPL. As a result in the
case of synchronous particles the cyclotron resonance interaction becomes a large-scale phenomena because the interaction region space scale is comparable with the magnetic field inhomogeneity length. In TPL it is observed the antidrift dynamics of synchronous particles and relative changes of their energy and pitch-angles are of the order of 100 percents if the magnetic field variation is large enough $\delta B_0 \sim B_0$. Consequently, in the transient plasma layer the anomalous cyclotron resonance interaction of synchronous particles with the whistler-mode wave takes place and one would expect ACRI to modify the wave damping.

As both rising and falling tones are observed in the magnetosphere when triggering VLF-emission, it is necessary to study the wave frequency variation influence on the anomalous cyclotron resonance interaction of energetic electrons with the ducted whistler-mode wave in the transient plasma layer. The present paper is devoted to solving this problem. Its solution allows to perform correct estimates of the wave damping for anomalous CRI in TPL.

The paper structure is the following. The basic equations derivation and relations resulted are given in Section 2. The case of fixed wave frequency is described in Section 3. Analytical and numerical results of studying the frequency sweeping influence on anomalous CRI in the stationary TPL are given in Section 4. Results obtained are discussed in Section 5.

2. BASIC EQUATIONS.

Let us consider the cyclotron resonance interaction between energetic electrons and the ducted wistler with a frequency $\omega < \omega_{Be}$ propagating along a weakly inhomogeneous both plasma and magnetic field $\vec{B}_0$. As this interaction is localized in the vicinity of field line it is quite natural to use the curvilinear orthogonal coordinate system with the basic vector along the arc length $S$ of the field line, normal and binormal to this line. It is convenient to introduce the dimensionless variables $s = \omega S/c, \quad t' = \omega t, \quad \beta = \vec{v}/c$, where $\vec{v}$ - is the velocity of electron.

The equations of motion for the non-relativistic electrons mirroring in the magnetic field $\vec{B}_0$ took the standard form (see, for example, Dysthe,1971; Nunn, 1974):

$$\frac{d\beta_\parallel}{dt'} = \Omega_W \beta_\perp \sin \Phi - \beta_\perp^2 \frac{Y_s}{2Y}, \quad \frac{ds}{dt'} = \beta_\parallel.$$

$$\frac{d\beta_\perp}{dt'} = \Omega_W (\beta_{ph} - \beta_\parallel) \sin \Phi + \beta_\parallel \beta_\perp \frac{Y_s}{2Y},$$

$$\frac{d\Phi}{dt'} = \frac{\beta_\parallel - \beta_R}{\beta_{ph}} + \frac{\Omega_W}{\beta_\perp} (\beta_{ph} - \beta_\parallel) \cos \Phi - (1 - \frac{\beta_\parallel}{\beta_g}) \frac{\delta \omega}{\omega},$$

Here $\Omega_W = |e|B_W/m_e c \omega$ - is the dimensionless wistler amplitude, $\beta_{ph} = (\omega/\omega_{pe})(Y - 1)^{1/2}$ and $\beta_g = -2\beta_R/Y$ - phase and group whistler mode wave velocities respectively, $\beta_R = (1 - Y) \beta_{ph}$ -resonance velocity, $Y_s \equiv \partial_s Y$ - magnetic field gradient, $\Phi$ – the complement of the angle between the electrons perpendicular velocity $\vec{v}_\perp$ and $\vec{B}_W$. The
slow frequency sweeping rate of the whistler is taken into account by term $\delta \omega / \omega$ in the wave phase equation, and under wave amplitude diffusion neglecting it depends only on

$$t'_g \equiv t' - \int_0^s \frac{ds'}{\beta_g(s')}.$$  

Having in mind the VLF-emissions magnetospheric typical parameters one can assume the whistler-mode wave frequency change to be small enough (i.e. $|\delta \omega| \ll \omega$) during the particle crossing the resonance region.

It is necessary to pay attention to the following circumstance. As far as we are interesting in the WPI large-scale dynamics when the resonance region is global in a size comparable with the inhomogeneity length of static magnetic field $B_0$, in the equation (1) for phase $\Phi$ we should keep the second term proportional to the whistler amplitude $\Omega_W$. In the case of small-scale WPI analysis, it is usually neglected (see, for example, Matsumoto, 1979; Brinca, 1981).

According to Erokhin (1995) and Erokhin et al. (1995), the magnetic field and the plasma density in the transient plasma layer are monotonous functions of the arc length $S$. So it is possible to put them in one to one correspondence $\omega_{pe} = \omega_{pe}(Y)$, where $\omega_{pe}$ is the electron Langmuir frequency. To concretize the following calculations we are using the power function of the type $\omega_{pe}(Y) = \omega_{pe}(Y_0)(Y/Y_0)^\sigma$, where $Y_0$ is the magnetic field at some point $S_0$ inside TPL. For the power index $\sigma$ it is usually taken the value $\sigma = 0$ (DE-model) or $\sigma = 0.5$ (CL-model; see, for example, Bell and Inan, 1981). The spatial dependence of whistler amplitude $\Omega_W$ can be given by the condition of energy flux conservation in the ray tube which crosssection is inversely with $B_0$ (see, for example, Molchanov, 1985). Therefore we obtain the following scaling of the cyclotron resonance and phase velocities as well as the whistler amplitude on the magnetic field $Y$:

$$\beta_{ph} = \frac{\beta_*}{Q^2(Y)}, \quad \beta_R(Y) = -\beta_* \frac{(Y - 1)}{Q^2(Y)},$$

$$\Omega_W(Y) = \Omega_* Y^{1/2} Q(Y), \quad Q(Y) \equiv \frac{Y^{\sigma/2}}{(Y - 1)^{1/4}},$$

where the following notations were used

$$\beta_* \equiv \beta_{ph}(Y_0)Q^2(Y_0), \quad \Omega_* \equiv \frac{\Omega_W(Y_0)}{Y_0^{1/2} Q(Y_0)}.$$

For convenience of the subsequent analysis it is necessary to transform the set of equations (1) to canonical form. Let us introduce the typical space scale of TPL as $L_t$. In dimensionless variable $s$ it is equal to $s_* \equiv \omega L_t / c = \beta_* / 2\Omega_* \gg 1$. Applying the scale transformations of variables $\beta_\parallel = \beta_* u, \quad \beta_\perp = \beta_* v, \quad s = s_* \xi, \quad t' = \tau / 2\Omega_*$ to the set of equations (1) and taking into account (2) we reduce (1) to the canonical form:

$$2 \frac{du}{d\tau} = Q(Y)Y^{1/2} v \sin \Phi - v^2 F(Y), \quad \frac{d\xi}{d\tau} = u,$$

$$2 \frac{dv}{d\tau} = Y^{1/2} \frac{Q(Y)}{[1 - uQ^2(Y)]} \sin \Phi + uvF(Y), \quad (3)$$
\[
2 \frac{d\Phi}{d\tau} = \chi [uQ^2(Y) + Y - 1] + \frac{Y^{1/2}}{Q(Y)v} [1 - uQ^2(Y)] \cos \Phi - \chi(1 - \frac{u}{u_g}) \frac{\delta \omega}{\omega},
\]
where \( \chi = (1/\Omega_s) \gg 1, \ u_g = 2(Y - 1)/Q^2(Y)Y \) and \( F(Y) \equiv Y_\xi/Y \) is the magnetic field logarithmic gradient. Eqs. (3) form the basic set of equations for the subsequent analysis and the function \( Y(\xi) \) determines the TPL spatial structure. It will be founded below from the condition of the synchronous particles existence. To describe briefly the TPL spatial structure in the case of constant wave frequency one puts \( \delta \omega = 0 \) in Eqs. (3). According to the paper by Erokhin et al. (1995), the anomalous cyclotron resonance interaction of energetic electrons with the ducted whistler-mode wave takes place for the group of synchronous particles defined by the following conditions: 1) the phase \( \Phi \) along the synchronous particle path is constant and is equal to \( \Phi_s = \pi/2; \) 2) the parallel velocity of synchronous particle \( u_s \) is equal to the cyclotron resonance velocity \( u_R \equiv -(Y - 1)/Q^2(Y) \). Putting in (3) \( u = u_R, \ \Phi = \pi/2 \) and \( \delta \omega = 0 \) we obtain the first order nonlinear equation for the magnetic field profile in TPL:

\[
\frac{dY}{d\xi} \equiv YF(Y) = \frac{Y^{1/2}[1/4(\Lambda - R(Y))]^{1/2}}{\Lambda + 2(1 - \sigma + \sigma/Y)R(Y)}
\]

where \( \mu = 1 + \sigma/2, \ R(Y) \equiv u^2_R(Y) \) and \( \Lambda \) is a positive parameter determining the one-parameter set of the magnetic field profiles \( Y(\Lambda, \xi) \), for which the anomalous CRI of energetic particles with the small amplitude whistler-mode wave may occur at the entire transient plasma layer.

Let the power index \( \sigma \) be in the range \( 0 \leq \sigma \leq 3/2 \). The solution of equation (4) exists if and only if the function \( Y(\Lambda, \xi) \) is in the region \( (1, Y_m(\Lambda)) \), where \( Y_m(\Lambda) \) is the single root of equation \( \Lambda = R(Y_m) \), monotonously increasing under parameter \( \Lambda \) growth.

To determine \( Y(\xi) \) we suppose the magnetic field in TPL to be varying in the range \( Y_1 \leq Y(\xi) \leq Y_2 \), where \( Y_1 > 1 \) and \( Y_2 < Y_m(\Lambda) \). Then the magnetic field profile \( Y(\xi) \) can be obtained by inversion of the following monotonous function

\[
\xi(Y) = \int_{Y_1}^{Y} \frac{[\Lambda + 2(1 - \sigma + \sigma/x)R(x)] dx}{x^{1/4}[\Lambda - R(x)]^{1/2}}.
\]

Fig.1a depicts the part of profile (5) in the case when the magnetic field is varying in the range \( 2 \leq Y(\xi) \leq 4 \), the power index corresponds to DE-model \( (\sigma = 0.5) \) and the parameter \( \Lambda \) takes the following values: 6.755, 18 and 40. For the given values of parameters \( \sigma \) and \( \Lambda \) possible maximum magnitudes of the magnetic field \( Y_m \) are respectively: 4.001, 5.674 and 7.755.

Fig.1b depicts the profile (5) with magnetic field variation in the range \( 2 \leq Y(\xi) \leq 4 \) with \( \Lambda = 40 \) and different values of power index \( \sigma \). According to Fig.1b for the given value of parameter \( \Lambda \), the TPL width is larger in the case of DE-model.

For the transient plasma layer with magnetic field variation in the range \( Y_1 \leq Y(\xi) \leq Y_2 \), the layer width \( l_\xi \) is determined by the following expression:

\[
l_\xi = \int_{Y_1}^{Y_2} \frac{[\Lambda + 2(1 - \sigma + \sigma/Y)R(Y)] dY}{Y^{1/4}[\Lambda - R(Y)]^{1/2}}.
\]
Fig. 1a. Magnetic field profile $Y(\xi)$ in the transient plasma layer for $\sigma = 0.5$ and different values of parameter $\Lambda$: 1 – $\Lambda = 6.755$; 2 – $\Lambda = 18$; 3 – $\Lambda = 40$. TPL corresponds to the dispersed magnetic field jump $2 \leq Y(\xi) \leq 4$.

Fig. 1b. Magnetic field profile in the case $2 \leq Y(\xi) \leq 4$ for $\Lambda = 40$ and different values of power index $\sigma$: 1 – $\sigma = 0.5$ (CL-model); 2 – $\sigma = 0$ (DE-model).

or in the dimensional variable $S$ we obtain $L_S = (c\beta_s/2\omega\Omega_s)l_\xi$. Fig. 1c illustrates the dependence of TPL-width $l_\xi$ on parameter $\Lambda$, defined by formula (6) in the case of $Y_1 = 1, \ Y_2 = Y_m(\Lambda)$ for two values of the power index $\sigma$. It can be seen again that the layer width is larger for DE-model of the plasma density. As far as for synchronous particles there is a well known integral of motion (Karpman et al., 1974; Nunn, 1974)

$$u_s^2 + (1 - \frac{1}{Y})v_s^2 = \Lambda,$$

their energy $E_s$, magnetic moment $\mu_s$ and velocities components determined by the magnetic field local strength

$$E_s(Y) = \frac{Y \Lambda - R(Y)}{2(Y - 1)}, \quad \mu_s(Y) \equiv \frac{v_s^2(Y)}{2Y} = \frac{\Lambda - R(Y)}{2(Y - 1)},$$

$$u_s(Y) = -\frac{(Y - 1)^{3/2}}{Y^\sigma}, \quad v_s(Y) = \left[\frac{Y}{(Y - 1)}(\Lambda - R(Y))\right]^{1/2}.$$
Fig.1c. Dependence of TPL-thickness $l_\xi(\Lambda)$ with magnetic field variation $1 \leq Y(\xi) \leq Y_m(\Lambda)$ on parameter $\Lambda$ in the cases: 1 – $\sigma = 0.5$; 2 – $\sigma = 0$.

The time $\Delta \tau_s$, required for TPL-crossing by the synchronous particle, is equal to

$$\Delta \tau_s = \int_{Y_1}^{Y_2} \frac{dY}{Y F(Y) u_g(Y)}.$$

Let us consider again equations (3). To perform the subsequent calculations we assume the following model of wave frequency variation $\delta \omega/\omega = h(p)$, where $h(p)$ is the given function. The spatio-temporal dependence of function $p(\xi, \tau)$ is governed by the following equation $dp/d\tau = \nu (1 - u/u_g)$, where parameter $\nu$ determines the frequency sweeping rate. The frequency sweeping rate can be characterized by a function $\Gamma \equiv (\partial f/\partial t)/f^2$ which has the following scaling

$$\Gamma \simeq 4 \cdot 10^{-5} \left[ \frac{\partial f/\partial t}{1 \text{ kHz/sec}} \right] \left[ \frac{5 \text{ kHz}}{f} \right]^2,$$

orientated towards the magnetospheric VLF-emissions parameters. Functions $\Gamma$ and $h$ are related by $dh/dp = \chi \Gamma/4\pi \nu$. Therefore in the case of constant wave frequency sweeping rate, corresponding to the choice $h(p) = p$, parameter $\nu$ is equal to $\nu = \chi \Gamma/4\pi$. So under the typical conditions of VLF-emissions in the magnetosphere, one has $\nu < 1$.

3. ESSENTIAL FEATURES OF THE ANOMALOUS CYCLOTRON RESONANCE INTERACTION OF ENERGETIC ELECTRONS WITH THE FIXED FREQUENCY WHISTLER-MODE WAVE IN TPL.

Before the studying the whistler frequency drift influence on the anomalous CRI in TPL, to clarify the following analysis it is necessary to describe shortly the case of fixed frequency wave corresponding to the condition $\delta \omega = 0$ in Eqs. (3). Let us assume that in
stationary TPL the magnetic field varies in the range \( Y_1 \leq Y(\xi) \leq Y_2 \) and the whistler-mode wave propagates in the direction of \( Y(\xi) \) growth. Therefore, the resonant particles are travelling in the opposite direction towards the wave. The maximum cyclotron resonance interaction in the transient plasma layer takes place for the group of synchronous electrons whose phases \( \Phi_s \) are constant and equal \( \pi/2 \) and other parameters are defined by (7). The group of synchronous particles can also be defined in the different way. Let us introduce function \( J \) by the following expression:

\[
J = u^2 + (1 - \frac{1}{Y}) v^2.
\]

According to papers of Karpman et al. (1974) and Nunn (1974), for resonant particles function \( J \) is the approximate integral of motion. Consequently, the group of synchronous particles can be defined by the following conditions: \( u_s = u_R \), \( \Phi_s \approx \pi/2 \) and \( J \approx \Lambda \). Nonconserving part of the approximate integral of motion \( J \) can be easily estimated from (3). Performing the asymptotic integration, we obtain the modified integral of motion

\[
I = J + \frac{2v Q \chi}{\sqrt{Y}} \cos \Phi.
\]

In turn, non conservation of the function \( I \) is caused by fastly oscillating terms of the order of \( 1/\chi^2 \) but the particle crossing of the local hyroresonance region is accompanied by the jump of function \( I \) proportional to \( 1/\chi^{3/2} \).

Let us turn back to formulae (7). According to (7) when crossing the TPL the synchronous particles increase their energy, magnetic moment, perpendicular velocity and pitch-angle \( \alpha_s = \arctan (v_s/|u_s|) \equiv \tan^{-1} (v_s/|u_s|) \). At the same time, when moving in the direction of magnetic field decreasing, their parallel velocity goes down, \( i.e. \) in the transient plasma layer the antidrift dynamics of synchronous particles takes place. Earlier similar effects for electrons, trapped by the whistler-mode wave in the equatorial plane vicinity, were pointed out by Matsumoto (1979).

Now let us assume that at TPL-entrance the velocities of incoming electrons coincides with the synchronous particle velocity, \( i.e. \) \( u(0) = u_s(Y_2) \), \( v(0) = v_s(Y_2) \), but there is a small deviation \( \theta_0 \) in the initial phase \( \Phi(0) = \Phi_s + \theta_0 \).

Numerical calculations and analytical estimates show us that there are some constants \( c_(-) < 0 < c(+) \) so that for the initial phase detuning \( c(-)/\chi < \theta_0 < c(+)\chi \) during the resonance particle pass TPL, its phase is confined in the range \( 0 < \Phi < \pi \) and its energy and pitch angle changes are close to the synchronous particle ones. If the initial phase detuning is large \( \theta_0 > c(+)\chi \) or \( \theta_0 < c(-)/\chi \), the duration of resonance interaction \( \Delta T_R \) becomes less than the time required for the resonance particle to cross the transient plasma layer. Under the growth of \( \theta_0 \) the time \( \Delta T_R \) decreases as \( \Delta T_R \sim 1/(\chi |\theta_0|)^{1/2} \). So there is the following scaling of the resonance particle energy growth \( \Delta E \) in dependence on the initial phase detuning \( \theta_0 \):

\[
\Delta E \sim 1/(\chi |\theta_0|)^{1/2}.
\]

Fig.2a depicts the chart of the resonance particle relative energy growth \( \Lambda \equiv 10^2 \Delta E/E_0 \) in dependence on the normalized initial phase detuning \( \chi \theta_0 \) for small \( \theta_0 \). The system (3) was integrated numerically for TPL with the magnetic field variation in the range \( 2 \leq Y(\xi) \leq 4 \) and the following parameters: \( \sigma = 0.5, \Lambda = 10, \nu = 0 \). The initial data correspond to the synchronous particle \( i.e. \) \( u_0 = u_s(Y_2), v_0 = v_s(Y_2) \), where \( Y_2 = 4 \). According to Fig.2a the maximum cyclotron resonance interaction of energetic electrons with the whistler mode of constant wave fre-
Fig. 2a. Dependence of synchronous particle energy gain $A$ on normalized initial phase detuning $\chi \theta_0$: 1 – $\chi = 10^3$; 2 – $\chi = 10^4$.

Frequency takes place for the small initial phase detuning $|\theta_0| \sim 1/\chi$. It can be seen that charts are nonsymmetric on the phase detuning $\theta_0$ and the resonant particle is gaining more energy in the case of positive $\theta_0$. In range $\chi |\theta_0| \geq 10^2$ the dependence $A$ on the parameter $\chi$ becomes appreciable. Similar effect is also observed for the pitch angle scattering of synchronous particles in TPL. Let $\Delta \alpha$ represent the synchronous particle pitch angle change during the cyclotron resonance interaction with whistler in TPL. We characterize the efficiency of electron pitch angle scattering during resonant WPI by function $B = 10^2(\Delta \alpha/\alpha_0)$, where $\alpha_0$ is the electron pitch angle at the resonant region entrance. Notice that in theory of the VLF-emissions triggering in the magnetosphere the pitch angle scattering of energetic electrons by whistler-mode wave is usually characterized by the change of equatorial pitch angle $\alpha_0 = \sin^{-1}[(Y_e \sin^2 \alpha / Y)^{1/2}]$, where $Y_e$ is the magnetic field strength at the equatorial plane. The dependence of $B$ on the normalized initial phase detuning $\chi \theta_0$ is shown in Fig. 2b for problem parameters as in Fig. 2a.

Fig. 2b. Dependence of pitch angle scattering efficiency $B$ of synchronous particle on normalized initial phase detuning $\chi \theta_0$: 1 – $\chi = 10^3$; 2 – $\chi = 10^4$. 
It can be seen that the pitch angle scattering efficiency $B$ is larger approximately by factor 1.5 of the energetic efficiency $A$. From charts of Fig.2a and Fig.2b it follows that the cyclotron resonance interaction of synchronous particles, characterized by small $\theta_0$, with the wistler-mode wave in TPL is anomalously strong because of their energies and pitch angles relative changes are about 100%. Under $\theta_0$ growth the efficiency of cyclotron resonance interaction falls. In the range of $|\theta_0| \sim 1$ the resonance region space scale is small enough. So with accuracy of a numerical coefficient of the order of unity one can use the linear estimate of $A$

$$|A| \leq \frac{\sin 2\alpha_0}{\sqrt{\chi}} \frac{Q^{3/2}}{(\Lambda - Y)^{1/4}} \left[ \frac{\Lambda + 2(1 - \sigma + \sigma/Y)R}{3 - 2\sigma + 2\sigma/Y} \right]^{1/2} \frac{1}{(Y - 1)^{9/4}} \tag{8}$$

Taking $\chi = 10^4$ and other parameters values correspondingly to Fig.2 we obtain from (8) that $|A| \leq 0.2$ i.e. for large $\theta_0$ the relative change of resonance particle energy is about 400 times less than the synchronous particle one.

Results of numerical calculations of function $A$ obtained by integrating system (3) in the case of large phase detunings $\theta_0$ are given in Fig.2c. The problem parameters are the same as for Fig.2a and $\chi = 10^4$. Notice the narrow peak of the energetic efficiency $A$ for small $\theta_0$ corresponding to the synchronous particles. For comparison let us consider the cyclotron interaction of trapped electrons with the wistler-mode wave in TPL. The trapped particles perpendicular velocities at the TPL-entrance should be different the synchronous particle one $v_s(Y_0)$. Introduce the normalized perpendicular velocity $\varnothing \equiv v/v_s(Y_0)$ and functions $g(Y)$ and $\rho(Y, \varnothing)$:

$$g(Y) = \frac{\Lambda - R(Y)}{\Lambda + 2(1 - \sigma + \sigma/Y)R},$$

$$\rho = \varnothing g(Y) + \frac{1 - g(Y)}{\varnothing},$$

where condition $0 \leq g \leq 1$ takes place. From formula (9) it follows that in dependence on the variable $\varnothing$ the function $\rho$ has minimum at $\varnothing = \varnothing_*(Y) \equiv \sqrt{(1/g) - 1}$ which is

![Fig.2c. Dependence of resonance particle energy gain $A$ on initial phase $\theta_0$ in the case of large $\theta_0$. The problem parameters are the following: $\chi = 10^4$, $\nu = 0$, $\Lambda = 10$, $\sigma = 0.5$, $Y_0 = 4$, $u_0 = u_s(Y_0)$ and $v_0 = v_s(Y_0)$.](image)
equal to \( \min \rho \equiv \rho_* = 2 \sqrt{g(1-g)} \leq 1 \). The trapped particle dynamics corresponds to the motion of nonlinear oscillator \( \theta \) in the potential well \( U(\theta) = \Omega^2_b (\rho \theta - \sin \theta) \), where \( \Omega^2_b = \chi \alpha v_s(Y) Y^{1/2} Q^3(Y) / 4 \) is the bounce frequency square. The potential well exists only for \( \rho < 1 \). In the case of \( 0 < g < 1/2 \) the condition \( \rho < 1 \) is fulfilled for \( \alpha \) in the range \( 1 < \alpha < \alpha_0^2 \). If \( g(Y) \) is ranged as \( 0.5 < g < 1 \), the trapped particle perpendicular velocity is less than the synchronous particle one, i.e. \( \alpha \) is in the interval \( \alpha_0^2 < \alpha < 1 \). The most long CRI of trapped particles with whistler in TPL takes place for the initial phase \( \theta_0 = \arccos \rho(Y_0) \equiv \theta_b \) and \( \alpha = \alpha_* \) when the trapped particle region has a maximum size in the phase plane \((\theta, u)\).

The numerical calculations of the cyclotron resonance interaction of whistler-mode wave with the stably trapped particles with energy levels located closely to the potential well bottom were made, and the dependence of energetic efficiency \( A \) on the trapped particles perpendicular velocity \( \alpha_0 \equiv \alpha(Y_0) \) was studied. Fig.3a depicts the chart of

![Fig.3a](image)

*Fig.3a. Dependence of energy gain of trapped particle, located closely to potential well bottom, on normalized perpendicular velocity \( \alpha_0 \). Initial phase is equal to \( \theta_0 = \cos^{-1} \rho(\alpha_0) \).*

![Fig.3b](image)

*Fig.3b. Dependence of resonance particle energy gain \( A \) on perpendicular velocity \( \alpha_0 \) and initial phase \( \theta_0 \). The problem parameters are the following: \( \chi = 10^4, \nu = 0, \Lambda = 10, \sigma = 0.5, Y_0 = 4, u_0 = u_s(Y_0) \) and \( \nu_0 = v_s(Y_0) \).*
function $A$ dependence on $\alpha_0$ when $\nu = 0$, $\chi = 10^4$, $\Lambda = 10$, $\sigma = 0.5$ and $Y_0 = 4$. The trapped particle initial conditions were $u_0 = u_s(Y_0)$, $\theta_0 = \theta_b(\alpha_0)$. The sharp peak of $A(\alpha)$ near the point $\alpha = 1$ corresponds to the synchronous particles population. According to Fig.3a, CRI- efficiency for the tapped particles is less by factor of few times than in the case of synchronous particles but it depends more smoothly on the perpendicular velocity $\alpha_0$ and the initial phase $\theta_0$. Therefore, the phase plane region occupied by trapped particles is substantially larger than one occupied by the synchronous particles. More evident it is demonstrated in Fig.3b showing the efficiency $A(\alpha_0, \theta_0)$ dependence on both initial parameters. Please pay attention to high and narrow peak of $A(\alpha_0, \theta_0)$ corresponding to the synchronous particles and the low but wide maximum in the case of trapped particles.

4. ANALYSIS OF WAVE FREQUENCY DRIFT INFLUENCE ON RESONANT WPI IN TRANSIENT PLASMA LAYER.

Consider the wave frequency drift influence on the anomalous cyclotron resonance interaction of energetic electron with a ducted whistler-mode wave propagating across the stationary transient plasma layer. Suppose the magnetic field $Y(\xi)$ to be static and given by (5) and the wave frequency sweeping to be described by a function $h(p)$. Below $h(p)$ is taken as the linear function $h(p) = p$. So the frequency sweeping rate becomes constant. Contribution of the whistler frequency sweeping to the effective potential well $U(\rho, \theta)$ is characterized by a function $r$ determining the potential

$$
\rho(\alpha, Y) = \alpha g + (1 + r - g)/\alpha
$$

Consider the case when condition $1 + r - g > 0$ is satisfied. Then $\rho$ being a function of $\alpha$, has minimum at $\alpha_*(Y) = [(1 + r - g) / g]^{1/2}$ which is equal to $\min \rho \equiv \rho_* = 2[\rho_*(1 + r - g)]^{1/2}$ and $\rho_* < 1$ if the condition $r < (2g - 1)^2 / 4g$ is fulfilled. So in the case $g - 1 < r < (2g - 1)^2 / 4g$ there are trapped particles with perpendicular velocities $\alpha$ in the range $\alpha_1 < \alpha < \alpha_2$, where $\alpha_{1,2}$ are determined by

$$
\alpha_{1,2} = \frac{1 \pm \sqrt{(2g - 1)^2 - 4rg}}{2g}.
$$

On the phase plane $(\theta_\tau, \theta)$ the phase trapping region has the maximum size at $\alpha = \alpha_*$ with its boundary defined by the following equation

$$
0.5 \theta^2_\tau + \Omega^2_b (\rho_* \theta - \sin \theta) = \Omega^2_b (\rho_* \arccos \rho_* - \sqrt{1 - \rho_*^2})
$$

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For faller with the frequency sweeping rate satisfying to the condition \( r < g - 1 < 0 \), \( \rho(\alpha) \) becomes zero at the point \( \alpha = \alpha_c(Y) \equiv [(g - 1 - r)/g]^{1/2} \). Therefor in this case there are always the trapped particles with the normalized perpendicular velocities \( \alpha \) in the range \( \alpha_3 < \alpha < \alpha_4 \), where

\[
\alpha_{3,4} = \sqrt{(2g - 1)^2 - 4rg \mp 1}/2g.
\]

Consider the whistler frequency drift influence on the cyclotron resonance interaction of synchronous particle with initial data \( u_0 = u_s(Y_0) \), \( \alpha_0 = 1 \), \( \theta_0 = 0 \). Introduce the specific phase value \( \theta_r = (2|\nu|)^{1/2} \) defined by the wave frequency sweeping rate.

In the case of riser \( r_0 > 0 \) with the frequency sweeping rate large enough the typical duration of synchronous particle CRI with the whistler may be estimated as \( \Delta \tau_R \approx 4C_3/|\chi|^{1/2} \), where \( C_3 \) is a constant of the order of unity. As a condition to suppress the anomalous CRI in TPL we take the following: \( \Delta \tau_R < 1/4 \). Hence we obtain the restriction from below on the \( r \)-magnitude \( r > r_c = 6.5 \cdot 10^4 C_3 / \chi^2 \) and taking into account (5) it can be rewritten as a condition on a parameter \( \nu: \nu > \nu_c \), where \( \nu_c \) is the critical value of \( \nu \). Consequently, in the case of riser with the frequency sweeping rate large enough (\( \nu > \nu_c \)), substantial decreasing of the efficiency of synchronous particle cyclotron resonance interaction in TPL is occurring. In the range of initial phases \( |\theta_0| < \theta_r \), the energetic efficiency of CRI has the following scaling on parameter \( \nu: \ A \sim 1/|\chi|^{1/4} \). If \( |\theta_0| > \theta_r \), then this scaling is changed to one given in Section 3: \( A \sim 1/|\chi|^{1/2} \).

In the case of faller (\( r < 0 \)), the synchronous particle becomes trapped one. Therefore, under the same but small initial phases \( \theta_0 \) the duration of synchronous particle CRI for faller is substantially larger than one for riser. The range of synchronous particles perpendicular velocities is rather narrow \( \delta \alpha \equiv (\alpha - 1) \ll 1 \), so under the low frequency sweeping rate when \( |r| \ll 1 \), the potential well \( U(\theta) \approx \Omega_0^2 \{\theta [r + (2g - 1) \delta \alpha] + \theta^2/6\} \). Introduce notations \( d \equiv |r + (2g - 1) \delta \alpha| \) and \( \theta_d = (2d)^{1/2} \ll 1 \). If \( \chi \theta_d \gg 1 \) the energetic efficiency can be scaled in the following way. When \( |\theta_0| < \theta_d \) we obtain \( A \sim 1/|\chi|^{1/2} \). In the range \( |\theta_0| > \theta_d \) the energetic efficiency is determined by the initial phase detuning \( A \sim 1/|\chi|^{1/2} \).

The computer simulations of the cyclotron resonance interaction efficiency under the whistler mode wave propagating across the TPL in the framework of equations (3) with \( h(p) = p \) were performed with taking into account the wave frequency sweeping. For \( \sigma = 0.5 \), \( \Lambda = 10 \), \( \chi = 10^2 \), \( Y_0 = 4 \), \( \theta_0 = 0 \), \( u_0 = u_s(Y_0) \) and \( v_0 = v_s(Y_0) \), the results are shown in Fig.4 and Fig.5. Fig.4a and Fig.5a depict efficiencies \( A \) and \( B \) in the case of riser (\( 0 < \nu < 10^3 \)). Fig.4b and Fig.5b are displaying them in the case of faller when \( 0 < -\nu < 10^{-2} \). For clarifying the efficiencies behaviour, the normalized variable \( \chi|\nu|^{1/2} \) is used for the horizontal axis and plots of \( \log A, \log B \) versus \( \log (\chi|\nu|^{1/2}) \) are presented. According to Fig.4 and in correspondence with above developed theory function \( A \) has the plateau with the maximum value \( A_m \approx 75 \) in the range of small \( |\nu| \) where the wave frequency drift influence is negligible. Outside this region the energetic efficiency \( A(\nu) \) falls approximately as \( A \sim 1/(\chi^2|\nu|)^{1/4} \) under \( |\nu|^{- \text{growth}} \). For the positive \( \nu \) (riser) the break of plot takes place at the point \( (\chi^2|\nu|)^{1/2} \approx 2 \).
Fig. 4a. Dependence of synchronous particle energy gain $A$ on parameter $\chi^{1/2}$ in the case of riser and $\chi = 10^4$.

Fig. 4b. Dependence of synchronous particle energy gain $A$ on parameter $\chi|\nu|^{1/2}$ in the case of faller and $\chi = 10^4$.

If $\nu$ is negative the plateau is substantially wider ($0 < (\chi^2|\nu|)^{1/2} \leq 7$). Besides at the point $(\chi^2|\nu|)^{1/2} \approx 700$ the local maximums of $A$ and $B$ arise with $A \approx 16$ and $B \approx 51$ (see Fig. 5b). Comparing plots of functions $A$ and $B$ one can see that the pitch angle scattering efficiency $B$ is substantially larger of the energetic efficiency $A$, in particular, in the plateau region one has $B \approx 100$. Notice also that outside the plateau region the pitch angle scattering efficiency $B$ isn’t governed by the simple power-law decay on the variable $\chi|\nu|^{1/2}$.

The numerical calculations of the trapped particles dependence on the normalized perpendicular velocity $\ae$ and the wave frequency sweeping parameter $\nu$ were made. Simulations results are given in Fig. 6 in the case of $\sigma = 0.5$, $\Lambda = 10$, $Y_0 = 4$ and $\chi = 10^4$. The initial data of resonance particle correspond to the trapped electron located closely to the potential well bottom, i.e. $u_0 = u_s(Y_0)$, $v_0 = \ae v_s(Y_0)$ and $\theta_0 = \cos^{-1}(\rho(\ae, Y_0))$. Consider the trapped particle energy gain in the case of its cyclotron resonance interaction with riser (see Fig. 6a). Notice the characteristic features of the normalized energy gain $A$. At first, there is the sharp peak for small $\nu$ and $\delta\ae \equiv \ae - 1$. Secondly, outside this peak with parameter $\nu$ fixed, the energy gain $A$ has maximum on the variable $\ae$ at some point $\ae_m(\nu)$ corresponding to condition $\rho \approx 1$. In the range $1 < \ae < \ae_m$ the trapped
Fig. 5 Dependence of synchronous particle pitch angle scattering efficiency $B$ on parameter $\chi|\nu|^{1/2}$: a - riser ($\nu > 0$); b - faller ($\nu < 0$).

particles are absent so the energy gain is very small. For $\varpi > \varpi_m$ the trapped particles exist and as $\varpi$ growth the energy gain $A$ is smoothly decaying.

The energy gain of trapped particles in the case of their interaction with faller ($\nu > 0$) is plotted on Fig. 6b. It can be seen that in contrast to the riser case now the function $A(\varpi, \nu)$ does not go down at the low $\delta\varpi$. Thus the strong cyclotron resonance interaction of trapped particles with faller is observed in the more wide range of parameter $\tilde{\nu} \equiv 10^2 \nu$ variation. For example, according to Fig. 6b in the case when $\varpi_0 = 1$ and $\tilde{\nu} = -3$, the energy gain of trapped particle with the initial phase detuning $\theta_0 \approx 0.46$ is equal to $A \approx 65\%$. This particle crosses TPL in time $\Delta\tau \approx 1.49$ and the relative change of its pitch angle is $B \approx 101\%$. Notice that for the wave frequency sweeping rate corresponding to $\Gamma = 4 \cdot 10^{-5}$ and the dimensionless whistler amplitude $\Omega_* \equiv 1/\chi = 10^{-4}$ the normalized parameter $\tilde{\nu}$ is equal to $\tilde{\nu} = -3.18$.

5. CONCLUSION

The principal conclusions of analysis performed above are the following:

1. In the stationary transient plasma layer (5) the cyclotron resonance interaction of energetic electrons with the small amplitude ducted whistler mode wave of the fixed fre-
Fig. 6 Dependence of relative energy change of trapped particle located close to potential well bottom on perpendicular velocity $\bar{\alpha}$ and wave frequency sweeping parameter $\bar{\nu} = 10^2 \nu$: a - riser ($\nu > 0$); b - faller ($\nu < 0$).

Frequency is determined by three groups of resonance electrons: the synchronous particles, the trapped and nontrapped resonance ones. For the synchronous particles the cyclotron resonance interaction is global in nature because the resonance region extends over the entire transient plasma layer. If the magnetic field $B_0$ has the significant variation in TPL i.e. $\delta B_0 \sim B_0$, then the relative changes of synchronous particle energy and pitch angle in course of the cyclotron resonance interaction are anomalously large up to the order of 100%. For the trapped particles the CRI-duration is less by a few times in comparison with the synchronous particles one but this population is more numerous than the synchronous particles group. Finally, in the case of nontrapped resonance particles under the typical magnetospheric conditions the cyclotron resonance interaction is weaker by the factor $10^2 \div 10^3$ times in comparison with the synchronous particles CRI. In turn on the phase plane the nontrapped resonance electrons population is the most numerous. So it can be expected that in some range of the problem parameters all these groups of resonant
particles may give comparable contributions to the whistler damping (growth) rate in TPL. However detailed calculations of the whistler damping (growth) rate is a subject of a special consideration.

2. In the cyclotron resonance interaction of energetic electrons with the whistler mode wave of variable frequency in the stationary TPL, for synchronous particles with the initial data \( u_0 = u_s(Y_0), \ v_0 \approx v_s(Y_0) \) and \( \Phi \approx \pi/2 \) there is the critical value of wave frequency sweeping rate such that above it \( (\nu > \nu_0) \), the typical time scale of CRI decreases proportionally to \( 1 / (\chi^2|\nu|)^{1/4} \) where \( \chi^2|\nu| > 1 \). There is a plateau in the dependence of CRI-efficiency on parameter \( \chi^2|\nu| \) (see Fig.4 and Fig.5). The plateau width is larger in the case of faller WPI.

3. In the case of riser with moderate frequency sweeping rate corresponding to \( \bar{\nu} < 1 \), the anomalous CRI in the transient plasma layer takes place for the trapped particles with perpendicular velocity in the range \( \alpha > 1 \). For the given \( \nu \) the maximum of energetic efficiency \( A \) is observed at \( \alpha \) suiting to the condition of the potential well being shrunk \( (\rho(\alpha, \nu) \approx 1) \) and function \( A \) decreases smoothly under \( \alpha \)-growth.

4. In the case of energetic particles CRI with a faller in TPL, the trapped particles anomalous CRI region on the perpendicular velocity axis extends to the range \( \alpha < 1 \), where the maximum CRI-efficiency is observed and it corresponds to \( \rho(\alpha, \nu) \approx 1 \). Under parameters range considered in this paper the efficiency of trapped particles CRI with a faller is larger approximately two times than one in the case of riser.

In addition it is necessary to note the following. Above we studied the cyclotron resonance interaction of energetic electrons with the whistler mode wave of variable frequency, propagating across the stationary transient plasma layer. It was proved that there are the synchronous particles groups whose CRI in TPL is of the global nature if the whistler frequency sweeping rate is low enough. In the opposite case the synchronous particles existence may be related to the following factors: a) the magnetic field \( B_0 \) nonstationarity; b) the specific modulation of whistler frequency in analogy with one considered in paper of Brinca (1981) and in paper of Bell and Inan (1981). In both cases a) and b) there is a question about the temporal duration of ACRI under a continuous injection of energetic particles into TPL. Besides, in the case a) it is also necessary to consider the source of magnetic field \( B_0(s, \tau) \) nonstationarity and to take into account the influence of electric field \( E_0 \) driven by the time-variable magnetic field \( B_0 \).

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REFERENCES

Bell T.F. and Inan U. 1981 Transient nonlinear pitch-angle scattering of energetic electrons by coherent VLF wave packets in the magnetosphere. J. Geophys. Res., 96, 9047
| Author                     | Year | Reference                                                                 |
|----------------------------|------|---------------------------------------------------------------------------|
| Bell T.F.                  | 1984 | The nonlinear gyroresonance interaction between energetic electrons and coherent VLF waves propagating at the arbitrary angle with respect to the Earth’s magnetic field. *J. Geophys. Res.*, 89, 905 |
| Brinca A.L.               | 1984 | Enhancing whistler wave-electrons interaction by the use of specially modulated injection. *J. Geophys. Res.*, 86, 702. |
| Dowden R.L., McKay A.D., Amon L.E., Koens H.C. and M.H. Dazey | 1978 | Linear and nonlinear amplification in the magnetosphere during a 6.6-kHz transmission. *J. Geophys. Res.*, 83, 169. |
| Dysthe K.B.               | 1971 | Some studies of triggered whistler emissions. *J. Geophys. Res.*, 76, 6915. |
| Erokhin N.S.              | 1995 | Long-term resonant wave-particle interaction in inhomogeneous plasma. *In: Second Volga international summer school on space plasma physics*, NIRFI, Nizhny Novgorod, p.33 |
| Erokhin N.S. Michailovskaya L.A. and N.N. Zolnikova | 1996 | Gyroresonant interaction of energetic electrons with whistler mode wave in the transient layers of the circumterrestrial plasma. *Geomag. and Aeronomy*, 36, No 1. |
| Helliwell R.A.            | 1967 | A theory of discrete VLF emissions from the magnetosphere. *J. Geophys. Res.*, 72, 4773. |
| Helliwell R.A.            | 1993 | 40 years of whistlers. *In: Modern Radio Sciences*, Oxford University Press, Oxford, p.189 |
| Karpman V.I., Istomin J.N. and D.R. Shklyar    | 1974 | Nonlinear theory of a quasi-monochromatic whistler mode packet in inhomogeneous plasma. *Plasma Physics*, 16, 685 |
| Karpman V.I. and D.R. Shklyar | 1977 | Particle precipitation caused by a single whistler mode wave injection into the magnetosphere. *Planet. Space Sci.*, 25, 395 |
| Matsumoto H.              | 1979 | Nonlinear whistler mode interaction and triggered emissions in the magnetosphere: A review.*In: Wave Instabilities in space plasma*, Palmadesso P.J. and K.Papadopoulos (eds), p.163, D.Reidel, Dordrecht. |
| Molchanov O.A.            | 1985 | Low frequency waves and induced emissions radiation in the magnetosphere of the Earth, chapter 1, Nauka, Moscow. |
| Nunn D.                   | 1971 | Wave-particle interactions in electrostatic waves in an inhomogeneous medium. *J. Plasma Physics*, 6, 291. |
| Nunn D.                   | 1974 | A self-consistent theory of triggered VLF-emissions. *Planet. Space Sci.*, 22, 349. |
| Name                  | Year | Title                                                                 | Journal/Book Details                       |
|-----------------------|------|----------------------------------------------------------------------|--------------------------------------------|
| Nunn D.               | 1984 | A quasistatic theory of triggered VLF-emissions.                     | Planet. Space Sci., 32, 325.              |
| Omura Y., Nunn D., Matsumoto H. and M.J. Rycroft | 1991 | A review of observational, theoretical and numerical studies of VLF triggered emissions. | Journ. Atmosph. Terrestr. Physics, 53, 351. |
| Rycroft M.J.          | 1993 | A review of whistler and energetic electrons precipitation.         | In: Review of Radio Science 1990-1992, Oxford Science Publications, Oxford, p.631 |