Hysteresis effect due to the exchange Coulomb interaction in short-period superlattices in tilted magnetic fields

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We calculate the ground-state of a two-dimensional electron gas in a short-period lateral potential in magnetic field, with the Coulomb electron-electron interaction included in the Hartree-Fock approximation. For a sufficiently short period the dominant Coulomb effects are determined by the exchange interaction. We find numerical solutions of the self-consistent equations that have hysteresis properties when the magnetic field is tilted and increased, such that the perpendicular component is always constant. This behavior is a result of the interplay of the exchange interaction with the energy dispersion and the spin splitting. We suggest that hysteresis effects of this type could be observable in magnetotransport and magnetization experiments on quantum-wire and quantum-dot superlattices.

A well known manifestation of the Coulomb exchange interaction in a two-dimensional electron gas (2DEG) in a perpendicular magnetic field is the enhancement of the Zeeman splitting for odd-integer filling factors, observable in magnetotransport experiments on GaAs systems. The same mechanism leads to the enhancement of the Landau gaps for even-integer filling factors, which can be identified in more recent magnetization measurements.

In the presence of a periodic potential the Landau levels become periodic Landau bands, and the calculations based on the Hartree-Fock approximation (HFA) show an enhancement of the energy dispersion of the bands intersected by the Fermi level. Such an effect has been indirectly observed in the magnetoresistance of short-period superlattices as an abrupt onset of the spin splitting of the Shubnikov-de Haas peaks, occurring only for a sufficiently strong magnetic field. In other words, when the magnetic field increases the systems makes a first-order phase transition from spin-unpolarized to spin-polarized states. This effect has also been discussed in other forms, for narrow quantum wires and for edge states.

In the spirit of the HFA, the Coulomb interaction can be split into a direct and an exchange component. The direct (classical) interaction is repulsive (i.e. positive) and long ranged, while the exchange (quantum mechanical) part is attractive (i.e. negative) and short ranged. The direct component is usually much larger than the exchange one. In our system this is decided by the two lengths involved, the superlattice (or modulation) period $a$, and the magnetic length $\ell = \sqrt{\hbar/eB_0}$ determined by the perpendicular magnetic field $B_0$. For long periods, $a \gg \ell$, the screening (direct) effects are strong: the width of the Landau bands is typically much smaller than the amplitude of the periodic potential, except when a gap is eventually present at the Fermi level. For periods of the order of $\ell$ the situation becomes opposite: the screening effect is weak, the exchange interaction is the dominant Coulomb manifestation, and the energy dispersion of the Landau bands may exceed the amplitude of the periodic potential if the latter is small enough.

In a recent paper we have studied the numerical solutions of the Hartree-Fock equations in the presence of short-period potentials. After preparing the solution for a fixed potential we change the potential amplitude by a small amount and find a new, perturbed solution, and then we change again the amplitude, and repeat the scheme. In this way, by increasing and then decreasing the amplitude, we obtain a hysteretic evolution of the ground state due to the combined effects of the external potential and of the exchange interaction, on the energy dispersion of the Landau bands. In the present paper we consider a fixed modulation amplitude, but a tilted magnetic field, such that we include in the problem, self-consistently, the Zeeman splitting of the Landau bands. We hereby intend to suggest further experiments that can identify strong effects of the Coulomb exchange interaction. The material parameters are those for GaAs: effective mass $m_{\text{eff}} = 0.067m_e$, dielectric constant $\kappa = 12.4$, bare g-factor $g = -0.44$, and electron concentration $n_s = 2.4 \times 10^{11}$ cm$^{-2}$.

We fix the component of the magnetic field perpendicular to the 2DEG, $B_0$, which determines our filling factors, while the bare Zeeman splitting is given by the total field $B = B_0/\cos \phi$, where $\phi$ is the tilt angle. We first consider a periodic potential varying only along one spatial direction, $V \cos Kx$, where $K = 2\pi/a$, and solve for the eigenstates of the Hamiltonian within the thermodynamic HFA. We chose the Landau gauge for the vector potential and we diagonalize the Hamiltonian in the Landau basis $\psi_n(x,y) = L_y^{-1/2} e^{-ixX_0/\ell^2} f_n(x - X_0) | \sigma \rangle$, where $X_0$ is the so-called center coordinate, $L_y$ is the linear dimension of the 2DEG, $f_n(x - X_0)$ are shifted oscillator wave functions, and $\sigma = \pm 1$ is the spin projection.

We begin our calculations with $\phi = 0$, and find the numerical HFA-eigenstates by an iterative method, starting from the noninteracting solution. Then, we increase $\phi$ and find a new solution starting from the previous one. In Fig. 1(a) we show a typical energy spectrum,
i.e., the Landau bands $E_{n\alpha X_0}$, $n = 0, 1, 2, \ldots$, within the first Brillouin zone, for a small tilt angle, $\phi < \phi_1$. Here $B_0 = 4.1$ T, and the parameters of the external potential are $a = 40$ nm and $V = 9$ meV.

In a simplified view, the exchange interaction contributes with a negative amount of energy to the occupied states, which enhances the energy dispersion in the vicinity of the Fermi level. The classical Hartree (positive) energy is small in our case, but it would increase with increasing modulation period and would rapidly flatten the energy bands. Also, the spin splitting is almost suppressed for $\phi < \phi_1$. However, for a sufficiently high field, when $\phi = \phi_1$, the difference in population of the spin-up and spin-down bands exceeds a critical value, and the spin gap is abruptly amplified by the exchange energy. The spin-up states become self-consistently more populated and lower in energy. The energy spectrum becomes like in Fig. 1(b), and keeps this structure when $\phi$ further increases. Then, we decrease $\phi$ step by step. For low temperatures we find for $\phi = 0$ the solution with large spin gap, similar to Fig. 1(b), while for higher temperatures we may find a transition to the spin-unpolarized state, Fig. 1(a), at $\phi_2 < \phi_1$.

We show in Fig. 2 the spin polarization, $(n_\uparrow - n_\downarrow)/(n_\uparrow + n_\downarrow)$, for two temperatures, when $\phi_2 > 0$. We consider here the temperature only as an effective parameter, that may also include includes the effects of a certain disorder, inherent in any real system. Clearly, in the presence of disorder similar results will appear for lower temperatures.

We have explicitly included disorder in a transport calculation, by assuming Gaussian spectral functions, $\rho_{\alpha\beta}(E) = (\Gamma \sqrt{\pi/2})^{-1} \exp[-2(E - E_{\alpha\beta})^2/\Gamma^2]$, where $\Gamma$ is the Landau level broadening. We have calculated the conductivity tensor $\sigma_{\alpha\beta}$, $\alpha, \beta = x, y$, using the standard Kubo formalism for the modulated 2DEG. In Fig. 3 we show the hysteresis loops for the longitudinal resistivities $\rho_{xx,yy} = \sigma_{yy,xx}/(\sigma_{xx}\sigma_{yy} + \sigma_{xy}^2)$. In our regime $\sigma_{xy}^2 \gg \sigma_{xx}\sigma_{yy}$, such that $\rho_{xx,yy}$ are in fact proportional to $\sigma_{yy,xx}$. Also, the conductivity in the $y$ direction is dominated by the quasi-free net electron motion along the equipotential lines of the modulation, with group velocity $v_y \approx -(eB_0)^{-1}dE_{nX_0}/dX_0$, known as band conductivity. The while the conductivity in the $x$ direction is related to inter-band scattering processes (scattering conductivity). The band and the scattering conductivities are inverse and respectively direct proportional to a power of the density of states at the Fermi level (DOSF). In the transition between spin-unpolarized and spin-polarized states the Fermi level touches the minima of the band $E_{11}$, where DOSF has a van Hove singularity. Therefore, for that situation the band conductivity, and thus $\rho_{xx}$, have a minimum, whereas the scattering conductivity, and thus $\rho_{yy}$, have a maximum, see Fig. 3.

Similar hysteresis effects can be found in a 2DEG that is modulated in two perpendicular spatial directions. However, in this case the picture is further complicated by the presence of the Hofstadter gaps and their interplay with the spin gaps. Then, since the dispersion of the Landau bands is essential for the hysteresis, another complication with the two-dimensional potential is that for an asymmetric unit cell the behavior of the system may not be the same when the magnetic field is tilted towards the $x$ or towards the $y$ axis of the plane, reflecting the anisotropy of the Brillouin zones. These details are not addressed in this paper.

In principle, the effects discussed in the present paper should also occur in narrow quantum wires or dots, and not necessarily only in periodic systems, as long as Landau bands with both flat and steep regions are generated, as in Fig. 1. In wide wires or dots the electrostatic screening is expected to dominate the exchange interaction, just like in long-period superlattices, and thus the Landau bands are smooth, except at the edges. Rijkels and Bauer have also predicted hysteresis effects in the edge channels of quantum wires when a small chemical-potential difference between the spin-up and spin-down channels can be controlled. To our knowledge an experimental confirmation has not been reported yet. Instead, several groups have build short-period superlattices for transport and other experiments which can also be used to check our predictions.

In conclusion, we have found a hysteresis property of the numerical solution of the thermodynamic HFA, with the physical origin in the exchange effects of the Coulomb interaction in the quantum Hall regime, in the presence of a short-period potential, when the Zeeman splitting is changed by tilting the magnetic field with respect to the 2DEG. We suggest that such effects could be observed e.g. in magnetization or magnetotransport measurements.

A. M. was supported by a NATO fellowship at the Science Institute, University of Iceland. The research was partly supported by the Icelandic Natural Science Foundation, and the University of Iceland Research Fund.

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FIG. 1. Two energy spectra: (a) $\cos \phi = 0$, (b) $\cos \phi = 1/6$. The dashed lines show the Fermi level.

FIG. 2. Hysteresis loops for the spin polarization for $T = 5$ K, with solid line, and for $T = 7$ K, with dashed line.
FIG. 3. Hysteresis loops for the resistivities $\rho_{xx}$ and $\rho_{yy}$.
$T = 2 \text{ K}, \Gamma = 1 \text{ meV}$. 