Beating pattern in quantum magnetotransport coefficients of spin–orbit coupled Dirac fermions in gated silicene

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Abstract
We report a theoretical study of magnetotransport coefficients of spin–orbit coupled gated silicene in the presence and absence of spatial periodic modulation. The combined effect of spin–orbit coupling and perpendicular electric field manifests through the formation of a regular beating pattern in Weiss and SdH oscillations. Analytical results, in addition to the numerical results, of the beating pattern formation are provided. The analytical results yield a beating condition which will be useful to determine the spin–orbit coupling constant by simply counting the number of oscillation between any two successive nodes. Moreover, the numerical results of modulation effect on collisional and Hall conductivities are presented.

Keywords: magnetotransport, Dirac system, spin–orbit interaction

(Some figures may appear in colour only in the online journal)
spatial period of the modulation [24–27]. Both the oscillations are periodic with inverse magnetic field. The Weiss oscillation appears at a very low magnetic field where SdH oscillations are completely wiped out. On the other hand, at moderate magnetic field, very weak Weiss oscillation is superposed on the SdH oscillations.

The spin–orbit interaction lifts the spin degeneracy and produces two unequally spaced Landau levels for spin-up and spin-down electrons in a two-dimensional electron gas formed at semiconductor heterostructures. The difference between two frequencies of quantum oscillation for spin-up and spin-down electrons is directly related to the spin–orbit coupling constant and yields a beating pattern in the amplitude of the Weiss and SdH oscillations [28–33]. The beating pattern in the SdH oscillations is being used to determine the spin–orbit coupling constant [33].

The SdH oscillation in a graphene monolayer, described by a massless Dirac-like Hamiltonian, has been observed experimentally [34]. The appearance of Weiss oscillation in a graphene monolayer has been predicted in [35, 36]. The beating pattern in magnetotransport coefficients of a graphene monolayer has been predicted in [35, 36]. The beating pattern in magnetotransport coefficients in gated silicene. There are several theoretical group estimated the spin–orbit coupled Dirac fermions in gated silicene. There are periodic with inverse magnetic field. The Weiss oscillation is being used to determine the spin–orbit coupling constant and yields a beating pattern in the amplitude of the SdH oscillations [28–33]. The beating pattern in magnetotransport coefficients in gated silicene. There are periodic with inverse magnetic field. The Weiss oscillation is being used to determine the spin–orbit coupling constant [33].

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We are considering a buckled 2D silicene sheet in which Dirac electrons obey a finite gapped graphene-like Hamiltonian. The Hamiltonian of an electron with charge $-e$ in the presence of both fields; the perpendicular magnetic field $B = B\hat{z}$ and electric field $E = E\hat{z}$, is [12, 16]

$$H = v_F(\sigma_x p_x - \sigma_y p_y) - \eta \Delta_{so} \sigma_z + \Delta \sigma_z,$$  

where $v_F$ is the Fermi velocity, $\Pi = p + eA$ is the 2D momentum operator with vector potential $A$, $\eta = \pm$ denotes $K(K')$ Dirac point, $s = \pm$ stands for spin-up and spin-down, $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli matrices, $\Delta_{so}$ is the strength of the spin–orbit interaction and $\Delta$ is the energy associated with the applied electric field.

Using Landau gauge $A = (0, Bx, 0)$, the exact Landau levels and the corresponding wave functions are obtained in [17, 18].

The ground state energy ($n = 0$) is $E^{\eta}_{0} = -(\Delta_{so} - \eta \Delta \sigma_z).$ For $n \geq 1$, the energy spectrum for the electron band is

$$E_n = \sqrt{\hbar^2 e^2 + (\Delta_{so} - \eta \Delta \sigma_z)^2},$$  

where $\Delta \equiv (n, s, \eta), \epsilon = h\omega_c$ and $\alpha_s = \sqrt{2} v_F \Gamma / l$ is the cyclotron frequency and $l = \hbar / eB$ is the magnetic length scale. The Landau levels around $K$ and $K'$ points split into two branches due to presence of $\Delta_{so}$ and $\Delta$. The splitting vanishes when either $\Delta_{so}$ or $\Delta$ is zero. The normalized eigenstates are (for $n \geq 1$)

$$\Psi^{\eta \pm}_{n, x}(x) = \frac{e^{i k_{F} x}}{\sqrt{L_x N_{\eta, x}^n}} \left[ a^n_{\eta, \sigma} \phi_{\eta, \sigma}(x + x_0) + b^n_{\eta, \sigma} \phi_{\eta, \sigma}(x - x_0) \right],$$  

and

$$\Psi^{\eta \pm}_{n, y}(y) = \frac{e^{i k_{F} y}}{\sqrt{L_y N_{\eta, y}^n}} \left[ a^n_{\eta, \sigma} \phi_{\eta, \sigma}(y + y_0) - b^n_{\eta, \sigma} \phi_{\eta, \sigma}(y - y_0) \right].$$  

where $\phi_{\eta, \sigma}(x) = (1 / \sqrt{2\pi n!}) e^{-x^2 / 2\Gamma} H_n(x / \Gamma)$ is the normalized harmonic oscillator wave function centred at $x = -x_0$ with $x_0 = k_F l^2$. Here, the coefficients are $N_{\eta, \sigma}^n = |a_{\eta, \sigma}^n|^2 + |b_{\eta, \sigma}^n|^2$ with $a_{\eta, \sigma}^n = \Delta_{\eta, \sigma} + \sqrt{\Delta_{\eta, \sigma}^2 + \hbar^2 \sigma_z^2}$ and $\hbar = -ie / 2\Gamma$. Here, $\Delta_{\eta, \sigma} = (\Delta_{so, \eta} - \eta \Delta \sigma_z)$. The analytical form of density of states [37] is given by

$$D_{\eta, \sigma}(E) \approx D_{\eta, \sigma}(E) \left[ 1 + 2 \sum_{k=1}^{\infty} \exp \left\{ -k \left( 2\Gamma E_{\eta, \sigma} / e^2 \right)^2 \right\} \right] \times \cos \left\{ \pi s \left( E^2 - \Delta_{\eta, \sigma}^2 \right) / e^2 \right\},$$  

where $D_{\eta, \sigma}(E) = E / (2\pi \hbar^2 v_F^2)$ and $\Gamma_0$ is the impurity induced Landau level broadening.

### 3. Electrical magnetotransport

In this section we shall study magnetotransport coefficients such as diffusive, collisional and Hall conductivities. The diffusive conductivity in the absence of any modulation vanishes because of the zero group velocity due to $k_F$ degeneracy in the energy spectrum. The presence of modulation imparts drift velocity to the charge carriers along the free direction of its motion and gives rise to diffusive conductivity. The oscillations in the diffusive conductivity known as Weiss oscillations are dominant at a low magnetic field regime. On the other hand, the collisional conductivity arises due to scattering of the charge carriers with localized charged impurities present in the system. The quantum oscillation in collisional conductivity is known as SdH oscillation. The Hall conductivity due to the Lorentz force is independent of any collisional mechanisms. However, the external spatial periodic modulation induces periodic oscillation on collisional as well as Hall conductivities. We shall use the formalism of calculating different magnetotransport coefficients developed in [38]. We will be using the following parameters for various plots: Fermi velocity $v_F = 2 \times 10^7$ m s$^{-1}$, electron density $n_e = 4 \times 10^{15}$ m$^{-2}$, spin–orbit coupling constant $\Delta_{so} = 4$ meV, perpendicular electric field induced energy $\Delta_e = 12$ meV.
The normal electric field $E_z \sim 0.4$ V/Å is required to produce $\Delta_c = 12$ meV, which is accessible experimentally \[10\].

### 3.1. Diffusive conductivity

To study diffusive conductivity, a spatial weak electric modulation $V = V_0 \cos(Kx)$ with $K = 2 \pi a$ is applied along the $x$ direction of the silicene sheet. Here, $a$ is the modulation period. We can treat this modulation as a weak perturbation as long as $V_0 \ll e$. Here, we have taken the modulation strength $V_0 = 0.1$ meV and modulation period $a = 150$ nm. The energy correction due to the modulation is calculated approximately by using first-order perturbation theory. Then the total energy is given by $E_T = E_{\text{F}} + \Delta E_{\xi}$, where $\xi \equiv \{\zeta, k_y\}$ and $\Delta E_{\xi} = G_{\xi}(u) \cos(Kx)$ with

$$G_{\xi}(u) = \frac{V_0 e^{-mu/2}}{N_{\xi}(u)} \left[ \alpha_{\xi} \Gamma L_{\text{n}}(u) + \gamma_{\xi} \beta L_0(u) \right].$$

(6)

Here, $u = (KI)^2/2$ and $L_0(u)$ is the Laguerre polynomial of degree $n$. The energy correction $\Delta E_{\xi}$ transforms the degenerate Landau levels into bands due to $k_y$ dependency, which leads to non-zero drift velocity.

The diffusive conductivity is calculated by using the standard semiclassical expression as

$$\sigma_{\text{diff}} = \frac{\beta e^2}{2} \frac{\Omega}{\sum_{\xi} \left( 1 - f_{\xi}(v_f) \right)}$$

(7)

where $\Omega = L_x \times L_y$ is the area of the system, $\varepsilon = (E_{\text{F}})$ is the electron relaxation time at the Fermi energy $E_{\text{F}}$, which is calculated in the next paragraph, $f_{\xi}$ is the Fermi-Dirac distribution function at $E = E_{\text{F}}$, and $\beta = (k_B T)^{-1}$ with $k_B$ the Boltzmann constant.

Also, $v_f = \langle \frac{\partial E_{\text{F}}}{\partial k_y} \rangle$ is the average value of the velocity operator $\hat{v}_f$, and it does not vanish due to the $k_y$ dependency of the energy levels. It is given by

$$v_f = \frac{1}{e} \frac{\partial E_{\text{F}}}{\partial k_y} = \frac{K^2}{h} \sin(Kx_0) G_{\xi}(u).$$

(8)

After inserting drift velocity given by equation (8) into equation (7) and integrating over $k_y$ variable, we get $\sigma_{\text{diff}} = (e^2/h) \Phi$ with

$$\Phi = \frac{\beta e^2}{h} \sum_{\xi} \int (E_{\text{F},\xi}) \left[ 1 - f(E_{\text{F},\xi}) \right] |G_{\xi}(u)|^2$$

(9)

as the dimensionless total diffusive conductivity. Note that a given Landau level splits into two branches due to the simultaneous presence of $\Delta_{\alpha}$ and $\Delta_c$. The first branch is $E_{\text{F},\alpha} = \sqrt{\varepsilon e^2 + (\Delta_{\alpha} - \Delta_c)^2}$ for $[s, \eta] = \{+, +\}$ and $\{-, -\}$. The second branch is $E_{\text{F},\eta} = \sqrt{\varepsilon e^2 + (\Delta_{\alpha} + \Delta_c)^2}$ for $[s, \eta] = \{+, -\}$ and $\{-, +\}$. So, for $\eta = +$ and $K$-valley there are two energy branches due to spin splitting and the same goes for $\eta = -1$ ($K'$-valley). The total conductivity is written as $\Phi = \Phi^+ + \Phi^-$. Here, $\Phi^+$ is the contribution from the second branch $E_{\text{F},\eta}$ and $\Phi^-$ is coming from the first branch $E_{\text{F},\alpha}$.

Before simplifying further, we shall derive the Fermi energy. At Fermi level $E_{\text{F}} = (\hbar v_F k_F^2)^2 + \Delta_c^2$ with $\Delta_c = (\Delta_{\alpha} + \Delta_c)$, then one can write

$$\left( k_F^2 \right)^2 - (k_F^2)^2 = \frac{4 \Delta_{\alpha} \Delta_c}{(\hbar v_F)^2}.$$

(10)

On the other hand, carrier density is given by

$$n_e = \frac{2}{(2\pi)^2} \int_{0}^{2\pi} \int_{0}^{k_F} k d k d \phi = \frac{1}{2\pi} \left[ (k_F^2)^2 + (k_F^2) \right].$$

(11)

Solving the above two equations for Fermi energy, we get $E_{\text{F}} = \sqrt{\left( E_{\text{F},\xi}\right)^2 + \Delta_{\alpha}^2 + \Delta_c^2}$ with $E_{\text{F},\xi} = \hbar v_F k_0^2$ and $k_0 = \sqrt{\sqrt{\pi} c}$. Before presenting exact analytical results, we would like to get approximated analytical results. To do so we consider the system at very low temperature in which Landau levels close to the Fermi energy contribute to transport properties. Therefore, we can use some approximations which are valid for higher values of $n$. Around the Fermi level, for higher value of $n$, we have

$$e^{-n/2} L_n(u) \approx \frac{1}{\pi \sqrt{n \mu}} \cos(\sqrt{2 \sqrt{n \mu} - \pi/4}).$$

(12)

We can also use $n = n - 1$ for higher values of Landau levels, which give $G_{\eta}(u) \approx \nu \exp(-u/2) L_{n-1}(u)$. To obtain an analytical expression we convert the summation into integration by using the relation

$$\sum_{n} \rightarrow \int_{0}^{\infty} n dz = \frac{2}{e} \int_{0}^{\infty} E dE.$$

(13)

By using the above two approximations, the exact expression of the dimensionless diffusive conductivity given by equation (9) reduces to

$$\Phi^\pm = \frac{\alpha_{\eta}}{e} \frac{1}{c} \int_{-\infty}^{\infty} \cos^2 (\frac{\varepsilon + d^\pm}{\cos^2 (\beta/2)}) \, d\varepsilon,$$

(14)

where $\alpha_{\eta} = \frac{V_0}{e\tau_0} (\hbar \beta e^2)$, $\beta = \beta(E - E_{\text{F}})$ and $d^\pm = c^\pm \beta E_{\text{F}}$ with

$$c^\pm = 2 \sqrt{\frac{2}{\pi \beta}} \left( E_{\text{F},\xi} \right)^{\frac{3}{2}} \pm 2 \Delta_{\alpha} \Delta_c.$$

(15)

Using the standard integral \[39\], equation (14) reduces to

$$\Phi^\pm = \frac{\alpha_{\eta}}{e} \frac{1}{c} \left[ 1 + H(T/T_{\eta}^*) \cos \left( 2 \eta f^\pm \lambda \right) \right].$$

(16)

Here, $\lambda = B_{d}/B_{s}$ with $B_{s} = \hbar /e2\alpha$ and

$$f^\pm = \frac{2 \alpha_{\eta}}{e\hbar v_F} \left( E_{\text{F},\eta} \right)^{\frac{3}{2}} \pm 2 \Delta_{\alpha} \Delta_c.$$

(17)

are two closely spaced frequencies of Weiss oscillations of the two energy branches induced by the simultaneous presence of spin–orbit interaction and gate induced electric field. The temperature dependent damping factor is given by

$$H(T/T_{\eta}^*) = \frac{T/T_{\eta}^*}{\sinh(T/T_{\eta}^*)},$$

where the characteristic temperature $T_{\eta}^*$ is defined as

$$T_{\eta}^* = \hbar v_F E_{\text{F},\eta} B_{s} \left[ 4 \pi^2 \hbar^2 g_{\eta} B_{s} \left( E_{\text{F},\xi} \right)^{\frac{3}{2}} \pm 2 \Delta_{\alpha} \Delta_c \right].$$

(18)

Typically the difference between $T_{\eta}^*$ and $T_{\eta}^*$ is very small. Another point is that $T_{\eta}^*$ is increasing with electric field ($\Delta_c$) or spin–orbit interaction ($\Delta_{\alpha}$) while $T_{\eta}^*$ is decreasing.
As \((E_0^2)^2 \gg 2\Delta_{\omega}\Delta_v\), the total diffusive conductivity is given by

\[
\Phi = 2A_0 \frac{e}{c} \left[1 + H(T/T_c) \cos(2\pi \alpha \Delta_v) \cos(2\pi \beta \Delta_v)\right] \tag{19}
\]

with \(\alpha = 2\sqrt{u} E_0^2 (\beta E_0)\), \(T_c = h v_F E_0 / (4\pi^2 k_B T_c B_0^2)\), \(f_{av} = (f^+ + f^-)/2\) and \(f_d = (f^+ - f^-)/2\). The total diffusive conductivity given by equation (19) exhibits beating pattern due to the superposition of two oscillatory functions with closely spaced frequencies \(f^\pm\). It should be noted that the condition \((E_0^2)^2 > 2\Delta_{\omega}\Delta_v\) satisfies the beating pattern. The location of the beating node can be obtained from the condition \(\cos(2\pi \beta \Delta_v) |_{\alpha = 1} = 0\); which gives

\[
2f_{av} B_0 B_j = (j + 1/2), \tag{20}
\]

where \(j = 0, 1, 2, \ldots\). Another periodic term, \(\cos(2\pi f_{av} \Delta_v)\), gives number of oscillations between two successive beating nodes as

\[
N_{osc} = f_{av} \left(\frac{B_{j+1} - B_0}{B_j}\right) = \frac{1}{2} f_{av} f_d. \tag{21}
\]

In explicit form, it is given by

\[
2N_{osc} = \frac{\sqrt{(E_0^2)^2 + 2\Delta_{\omega}\Delta_v} + \sqrt{(E_0^2)^2 - 2\Delta_{\omega}\Delta_v}}{\sqrt{(E_0^2)^2 + 2\Delta_{\omega}\Delta_v} - \sqrt{(E_0^2)^2 - 2\Delta_{\omega}\Delta_v}} \tag{22}
\]

Here we make couple of important remarks on the above equation: (1) the number of oscillations between any two successive beat nodes is independent of the modulation period \(a\) and magnetic field and (2) the expression of \(N_{osc}\) can be used to estimate the spin–orbit coupling constant by simply counting the number of oscillations between any two successive beat nodes.

Figure 1 shows exact numerical results of the diffusive conductivity given by equations (7) and (8) for two different temperatures. It shows beating patterns in both the cases and oscillations get damped with increasing temperature. In figure 2 we compare the approximate analytical result given by equation (19) with the exact numerical result obtained from equation (7). The analytical result for diffusive conductivity matches very well with the exact result. The appearance of beating patterns is due to the superposition of Weiss oscillations coming from two oscillatory drift velocities with different frequencies \(f^\pm\) corresponding to two energy branches. For the numerical parameters used here, the number of oscillations calculated from equation (22) is 14, same as counted from figure 1. Note that the beating pattern appears in all the three different regimes, namely TI, VSPM and BI, with different \(N_{osc}\).

### 3.2. Collisional conductivity

First we shall study collisional conductivity in absence of modulation. The effect of modulation on ShH oscillation will be discussed in the later half of this section. The standard expression for collisional conductivity is given by [38]

\[
e_{\text{coll}} = \frac{\beta n^2}{2a} \sum_{\zeta'} \left(1 - f_{\zeta'}\right) W_{\zeta' \zeta} (\alpha_{\zeta}^2 - \alpha_{\zeta'}^2). \tag{23}
\]

Here, \(f_{\zeta} = f_{\zeta'}\) for elastic scattering, \(W_{\zeta' \zeta}\) is the transition probability between one-electron states \(|\zeta\rangle\) and \(|\zeta'\rangle\). Also, \(\alpha_{\zeta'} = \langle \zeta | \hat{r} | \zeta' \rangle\) is the average value of the \(\mu\) component of the position operator of the charge carriers in state \(|\zeta\rangle\). The scattering rate \(W_{\zeta' \zeta}\) is given by

\[
W_{\zeta' \zeta} = \sum_{\mathbf{q}} |U(\mathbf{q})|^2 |\langle \zeta | e^{i \mathbf{q} \cdot \mathbf{r}} | \zeta' \rangle|^2 \delta(E_{\zeta} - E_{\zeta'}). \tag{24}
\]

where \(\mathbf{q} = q_x \hat{x} + q_y \hat{y}\) is the two-dimensional wave-vector and \(U(\mathbf{q}) = 2\pi e^2 / (\sqrt{q_x^2 + q_y^2 + k_s^2})\) is the Fourier transform of the screened impurity potential \(U (\mathbf{r}) = (e^2 / 4\pi \epsilon) (e^{-|\mathbf{r}|} / \epsilon)\), where \(k_s\) is the inverse screening length and \(\epsilon\) is the dielectric constant of the material. Equation (23) can be re-written for higher values of Landau level \((n = n - 1)\) as given by

\[
W_{\zeta' \zeta} = \sum_{\mathbf{q}} |U(\mathbf{q})|^2 |\langle \zeta | e^{i \mathbf{q} \cdot \mathbf{r}} | \zeta' \rangle|^2 \delta(E_{\zeta} - E_{\zeta'}). \tag{24}
\]
\[
\sigma_{xx}^{\text{col}} = \frac{e^2 N U_0^2}{h} \sum_\zeta (2n + 1) \left[ -\frac{df}{dE} \right]_{E=E_\zeta}. \tag{25}
\]

A closed-form analytical expression of the above equation is obtained by replacing the summation over the quantum number as \(\sum \rightarrow 2\pi f^2 \int_0^\infty D(E) dE\) with \(D(E)\) is the density of states (see equation (5)) and it is given by

\[
\sigma_{xx}^{\text{col}(\kappa)} = \frac{e^2}{(\omega_0 \pi^2)} \frac{E_F^2 - \Delta^2}{\epsilon^2} \times \left[ 1 + 2\Omega_0 H(T/T_c) \cos \left( \frac{2\pi v_F^2}{B} \right) \right]. \tag{26}
\]

Here, \(\sigma_0 = e^2\tau_0 E_F^2/\pi h^2\) is Drude-like conductivity and \(v_F = \sqrt{(E_F^2) / (2\Delta_\omega)}\) is the SdH oscillation frequencies for the two energy branches \(E_x^+\) and \(E_x^-\). Also, impurity induced damping factor is

\[
\Omega_0 = \exp \left\{ -\frac{2\pi T \Delta E_x^2}{v_F^2} \right\} \tag{27}
\]

and the temperature dependent damping factor is \(H(\kappa) = \kappa/\sinh(\kappa)\). Here, \(x = T/T_c\) with \(T_c = e^2/(2\pi^2 k_B E_F)\) is the critical temperature.

Following the same procedure as in the diffusive conductivity case, here we get the location of a beating node \(B_j\) by \(B_j(j + 1/2) = 2\Delta_{\omega c} \sqrt{\text{i}h / (2 e h v_F^2)}\) and the number of oscillations between any two successive nodes as \(N_{\omega c} = \left(E_F^2 / 4\Delta_{\omega c} \Delta_x \right)^{1/2}\). Unlike Rashba spin–orbit coupled two-dimensional electron gas, \(N_{\omega c}\) in silicene is same for a set of given parameters and does not depend on specific choices of the successive nodes. In figure 3, we show the beating pattern in the collisional conductivity versus inverse magnetic field. For the parameters used here, we have \(N_{\omega c} = 14\) which is same as shown in figure 3.

Now we will describe the effect of weak modulation on the collisional conductivity. The modulation effect enters mainly through the total energy in the Fermi distribution function. The collisional conductivity in the presence of weak modulation is given by

\[
\sigma_{xx}^{\text{col}} = \frac{e^2 N U_0^2}{h} \sum_\zeta (2n + 1) M_{\xi}, \tag{28}
\]

where \(M_{\xi}\) is given by

\[
M_{\xi} = \int_0^{\text{df}} \left[ -\frac{df}{dE} \right]_{E=E_{\xi}} \times dk_y. \tag{29}
\]

It is difficult to get a closed-form analytical expression of the collisional conductivity in the presence of the modulation. The change in collisional conductivity due to the modulation \([\Delta \sigma_{xx}^{\text{col}} = \sigma_{xx}^{\text{col}(V_0)} - \sigma_{xx}^{\text{col}(0)}]\) is calculated from equation (28) numerically and shown in figure 4.

Figures 3 and 4 clearly show that the effect of the modulation on \(\sigma_{xx}^{\text{col}}\) is very small and vanishes with increasing \(B\). The location of the beating node and number of oscillations between any two successive nodes are determined by equations (20) and (21), respectively. To understand why the beating pattern appears in \(\Delta \sigma_{xx}^{\text{col}}\) follows the same condition as in the diffusive conductivity, we expand \(M_{\xi}\) as given by

\[
M_{\xi} = \int_0^{\text{df}} \left[ -\frac{df}{dE} \right]_{E=E_{\xi}} + \left( \frac{\Delta E_{\xi,k_y}}{2!} \right) \frac{\partial^2 f}{\partial E^2} + \ldots. \tag{30}
\]

We can see the modulation dependent dominant term is of the order of \((G_{\xi}(u)) = V_0^2\), which is same as in the diffusive conductivity (see equation (9)). The modulation induced Weiss oscillation in collisional conductivity, as shown in figure 4, also follows the same beating condition as in the diffusive conductivity.

From the numerical result we can see that the modulation effect is dominant at a low range of magnetic fields i.e; when the
energy scale of Landau level is not much higher than the energy correction due to modulation. As the magnetic field increases, SdH oscillation starts to dominate over Weiss oscillation.

### 3.3. The Hall conductivity

In this sub section, we will see modulation effect on the Hall conductivity. The Hall conductivity is given by

\[
\sigma_{xy} = \alpha \sum_n (v_{nx}^2 - v_{nx}^1) \langle \xi | \hat{\mathbf{P}} | \xi' \rangle \langle \xi' | \hat{\mathbf{P}} | \xi \rangle.
\]

Using unperturbed eigenstates, velocity matrix elements are given by

\[
\langle n + 1, s, + | \hat{\mathbf{P}} | n, s, + > = -\frac{i \alpha_{n+1,s}^+ \beta_n^+}{N_{n+1,s}^+ N_{n,s}^+} v_F.
\]  

Substituting equations (32) and (33) into equation (31), we get

\[
\sigma_{xy} = 2 \frac{e^2 \hbar}{\Omega} \sum_{n,s} \int_0^{\pi/2} \frac{d\psi}{|\alpha_{n+1,s}^+|} |\beta_n^+|^2 \\
\times \left[ \frac{f(E_{n+1,k}) - f(E_{n+1,k})}{\sqrt{n + z - \sqrt{n + 1 + z - \rho_{n,k}}}} \right] dk_y.
\]

Here \( z = [(\Delta_{m} - \Delta)/\sqrt{\epsilon}]^2 \) and

\[
\rho_{n,k} = \Delta E_{n+1,k} - \Delta E_{n,k} = \frac{V_0}{\epsilon n} e^{-a/L} \cos(Kx_0).
\]

Here, a factor of 2 is multiplied because of the identical states between two valleys but with opposite spin.
The modulation induced change in the Hall conductivity $[\Delta \sigma_{yx} = \sigma_{yx}(V_0) - \sigma_{yx}(0)]$ is plotted in figure 5. It shows a beating pattern in the Weiss oscillation of the Hall conductivity. The node position $B_j$ and number of oscillations between any two successive nodes are determined by equations (20) and (21), respectively. The increase of magnetic field diminishes the modulation effect on Hall conductivity as expected.

4. Summary and conclusions

We have shown the appearance of beating patterns in quantum oscillations of magnetotransport coefficients of spin–orbit coupled gated silicene with and without spatial periodic modulation. There is a spin-splitting of the Landau energy levels due to the presence of both the spin–orbit coupling and electric field perpendicular to the silicene sheet. The formation of a beating pattern is due to the superposition of oscillations from the two different energy branches, but with slightly different frequencies depending on the strength of spin–orbit coupling constant and perpendicular electric field. In addition to the numerical results we also provide analytical results of the beating pattern in Weiss and SdH oscillations. The approximated analytical results are in excellent agreement with the exact numerical results. The analytical results yield a simple equation which can be used to determine the strength of the spin–orbit coupling constant by simply counting the number of oscillations between any two successive beat nodes. There have been few theoretical calculations, already mentioned earlier, which estimate the spin–orbit coupling constant. Here we have proposed a way to determine the spin–orbit coupling constant experimentally. The beating pattern which appear in the transport coefficients are the same in all three of the different regimes, namely TI, VSPM and BI, with different $N_{so}$. The beating pattern can not be observed in absence of an external electric field. Finally for the sake of completeness, the modulation effect on collisional and Hall conductivities is also studied numerically. Moreover, the analytical results of the Weiss and SdH oscillations frequencies reduce to the graphene monolayer case [35, 36] by setting $\Delta_{so} = 0$ or $\Delta_c = 0$.

Figure 5. Plots of the modulation effect on Hall conductivity versus magnetic field.

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