Driven polymer translocation through a nanopore: a manifestation of anomalous diffusion

J. L. A. Dubbeldam\textsuperscript{1,2}, A. Milchev\textsuperscript{1,3}, V. G. Rostiashvili\textsuperscript{1} and T. A. Vilgis\textsuperscript{1}

\textsuperscript{1}Max Planck Institute for Polymer Research, 10 Ackermannweg 55128 Mainz, Germany
\textsuperscript{2}Delft University of Technology 2628CD Delft, The Netherlands
\textsuperscript{3}Institute for Physical Chemistry Bulgarian Academy of Science, 1113 Sofia, Bulgaria

PACS 82.35.Lr – Physical properties of polymers
PACS 87.15.Vv – Diffusion
PACS 87.15.Aa – Theory and modeling; computer simulation

Abstract. - We study the translocation dynamics of a polymer chain threaded through a nanopore by an external force. By means of diverse methods (scaling arguments, fractional calculus and Monte Carlo simulation) we show that the relevant dynamic variable, the translocated number of segments $s(t)$, displays an anomalous diffusive behavior even in the presence of an external force. The anomalous dynamics of the translocation process is governed by the same universal exponent $\alpha = 2/(2\nu + 2 - \gamma_1)$, where $\nu$ is the Flory exponent and $\gamma_1$ - the surface exponent, which was established recently for the case of non-driven polymer chain threading through a nanopore. A closed analytic expression for the probability distribution function $W(s, t)$, which follows from the relevant fractional Fokker-Planck equation, is derived in terms of the polymer chain length $N$ and the applied drag force $f$. It is found that the average translocation time scales as $\tau \propto f^{-1} N^{\frac{2}{\nu} - 1}$. Also the corresponding time dependent statistical moments, $\langle s(t) \rangle \propto t^\alpha$ and $\langle s(t)^2 \rangle \propto t^{2\alpha}$ reveal unambiguously the anomalous nature of the translocation dynamics and permit direct measurement of $\alpha$ in experiments. These findings are tested and found to be in perfect agreement with extensive Monte Carlo (MC) simulations.

Introduction. – Recently single molecule experiments probing single-stranded DNA or RNA translocation through a membrane nanopore attracted widespread attention \cite{1}. These investigations have been triggered in the seminal experimental paper by Kasianowicz et al. \cite{2} where an electric field drives single-stranded DNA and RNA molecules through the alpha-hemolysin nanopore so that each threading is signaled by the blockage of the ion current. By recording the blockage time one can reconstruct the whole driven translocation of DNA molecule. More recently solid-state nanopores have been used for DNA translocation experiment \cite{3,4}. Such pores can be tuned in size and are more stable over a wide range of voltages, temperature as well as the solvent pH.

The physical nature of the translocation process is still not well understood. The theoretical consideration of the translocation dynamics is usually based on the assumption that the translocation length $s$ (i.e. the translocated number of segments at time $t$) is the only relevant dynamic variable which is governed by a conventional Brownian diffusion process \cite{5–7}. The main predictions for the average translocation time $\tau$ looks as follows. For an unbiased translocation $\tau(N) \propto a^2 N^2/D$ (here $a$ is a polymer Kuhn segment length and $D$ is a diffusion coefficient whose $N$-dependence is not well established) whereas the $\tau$ for the driven translocation (when a polymer experiences a chemical potential difference $\Delta \mu$ between the environments separated by the membrane) scaled as $\tau \propto T a^2 N/(D \Delta \mu)$. Here $T$ denotes temperature and we have set the Boltzmann coefficient $k_B \equiv 1$. More recently Kantor & Kar达尔 \cite{8,9} have cast doubt on these results by noting that the unimpeded motion of a polymer scales as $\tau \propto T a^2 N/(D \Delta \mu)$.
Dynamics in terms of a single translocation coordinate. – As already noted, the initial 3d problem can be rephrased in terms of 1d translocation coordinate s, and in doing so one arrives at a typical case of anomalous diffusion. Recently we suggested [14] that the translocation dynamics is that of normal Brownian motion and suggested instead that anomalous diffusion dynamics [12] might explain some MC - findings. Nevertheless, there is so far no clear knowledge regarding the physical origin of such anomalous dynamics. It is also not clear how one can make use of the fractional Fokker - Planck equation [12,13] which seems to govern this type of dynamics.

In this paper we come up with a general picture of the driven polymer translocation based on our previous consideration of the unbiased problem [14]. We first sketch the mapping of the 3d problem on the 1d translocation s - coordinate. This leads to an anomalous diffusion in the external force field which one could quantify in terms of the fractional Fokker - Planck equation (FFPE). The solution of this equation is then obtained on the interval $0 \leq s \leq N$ in a closed analytical form . The subsequent comparison of our extensive MC - results with the proper analytical expressions shows a very nice quantitative agreement.

As a result the mapping on the s coordinate leads to an anomalous diffusion law, $\langle s^2 \rangle \propto t^{2/(2\nu+2-\gamma_1)}$. Taking into account that for $d = 3 , \nu = 0.588$ and $\gamma_1 = 0.680$ [17], we obtain $\alpha = 0.801$. In turn, the average translocation time $\tau \propto N^{2/(2\alpha)} \propto N^{2.496}$. Remarkably, in 2d where $\nu_{2d} = 0.75$ and $\gamma_1 \approx 0.945$ [18], one finds $\alpha \approx 0.783$, i.e. $\alpha$ is almost dimensionality independent! This explains why the measured exponents in both 2d [8] and in 3d [19] are so close. The presence of the external force imposed on the translocating chain leads to a nonisotropic cis - trans - transition of the folds. It can be quantified within the FFPE - formalism which was originally suggested by Barkai, Metzler and Klafter [20].

Fractional Fokker - Planck equation. – The formalism of FFPE provides an appropriate technique which describes the anomalous diffusion in an external force - field. In our case FFPE has the form

$$\frac{\partial}{\partial t} W(s,t) = 0D_1^{1-\alpha} \left[ \frac{\partial}{\partial s} U'(s) \xi_\alpha + K_\alpha \frac{\partial^2}{\partial s^2} \right] W(s,t) ,$$

where $W(s,t)$ is the probability distribution function (PDF) for having a segment s at time $t$ in the pore, and the fractional Riemann - Liouville operator $0D_1^{1-\alpha} W(s,t) = (1/\Gamma(\alpha))(\partial/\partial t) \int_0^t dt' W(s,t')(t-t')^{1-\alpha}$. In Eq. (2) $\Gamma(\alpha)$ is the Gamma-function, $K_\alpha$ is the so called generalized

Fig. 1: How a fold squeezes through a nanopore. The driving force $f$ is caused by a chemical potential gradient $\Delta \mu = \mu_1 - \mu_2$. (a) The fold of the length $s$ is fragmented into $n$ and $s-n$ -parts during its threading. (b) This fragmentation gives rise to an effective entropic barrier $F(n)$ with height $\Delta E$ at $n = s/2$. simulations (using a bond fluctuating model on a 2d lattice) and the results show that $\tau(\Delta \mu = 0) \propto N^{2.5}$ and $\tau(\Delta \mu) \propto N^{1.53}/\Delta \mu$, i.e. at least in the case of the driven translocation the theory is inconsistent with MC - simulation data. A more recent MC and Langevin dynamics study [11] reports scaling laws $\tau \propto N^{1.5}$ and $\tau \propto N^{1.65}$ for relatively short and relatively long chains respectively. It has also been questioned [8, 9] whether the translocation dynamics is that of normal Brownian motion and suggested instead that anomalous diffusion dynamics [12] might explain some MC - findings. Nevertheless, there is so far no clear knowledge regarding the physical origin of such anomalous dynamics. It is also not clear how one can make use of the fractional Fokker - Planck equation [12,13] which seems to govern this type of dynamics.

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Dynamics in terms of a single translocation coordinate. – As already noted, the initial 3d problem can be rephrased in terms of 1d translocation coordinate s, and in doing so one arrives at a typical case of anomalous diffusion. Recently we suggested [14] that the translocation proceeds by successive threading of small fractions of the polymer, called folds, which equilibrate fast enough compared to the whole chain, and can be considered as building blocks of such mapping. In a somewhat different context concerning the polymer dynamics the notion of folds has been discussed earlier [15]. Figure 1 shows how a fold overcomes an entropic barrier caused by a narrow pore.

If the fold is fragmented into n and $s-n$ parts while it is threading through the pore then the corresponding free energy reads $F(n)/T = -s \ln \kappa - (\gamma_1-1) \ln[n(s-n)]$, where $\kappa$ is the connective constant and $\gamma_1$ is the surface entropic exponent [16]. Then the corresponding activation barrier which could be associated with the fold threading can be calculated as $\Delta E(s) = F(s/2) - F(1) = (1-\gamma_1)T \ln s$.

In the force - free case the characteristic time of the fold transition from cis - to the trans - side of the membrane can be estimated as follows. In the absence of a separating membrane this would be the pure Rouse time $t_{\text{Rouse}} \propto s^{2\nu+1}$. The membrane with a nanopore imposes an additional entropic activation barrier $\Delta E(s)$ which slows down the transition rate. The characteristic time, therefore, scales as $t(s) = t_{\text{Rouse}}(s) \exp[\Delta E(s)] \propto s^{2\nu+2-\gamma_1}$. This makes it possible to estimate the mean-squared displacement of the s - coordinate as

$$\langle s^2 \rangle \propto t^{2/(2\nu+2-\gamma_1)} .$$

As a result the mapping on the s coordinate leads to an anomalous diffusion law, $\langle s^2 \rangle \propto t^\alpha$, where $\alpha = 2/(2\nu + 2 - \gamma_1)$.
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The eigenvalue problem has simple solutions in two reflecting-adsorbing boundary value problem for FFPE [21] in the interval 0 ≤ s ≤ N. The boundary conditions correspond to the reflecting-adsorbing case, i.e., \( \{U'(s)W(s,t)/T + (\partial/\partial s)W(s,t)\}_{s=0} = 0 \) and \( W(s=N,t) = 0 \). The initial distribution is concentrated in \( s_0 \), i.e., \( W(s,t = 0) = \delta(s-s_0) \). The full solution can be represented as a sum over all eigenfunctions \( \psi_n(s) \) [22], i.e., \( W(s,t) = \exp(\Phi(s) - \Phi(s_0)) \sum_{n=0}^{\infty} T_n(t)\psi_n(s)\psi_n(s_0) \) where \( \Phi(s) = U(s)/2T \), \( \psi_n(s) = \exp(\Phi(s))\varphi_n(s) \) and \( \varphi_n(s) \) obey the equations \( (d^2/ds^2) - f(d/ds) + \lambda_{n,\alpha}/K_\alpha \varphi_n(s) = 0 \) (where \( f = \Delta \mu/T \)), and the eigenvalues \( \lambda_{n,\alpha} \) can be readily found from the foregoing boundary conditions. The temporal part \( T_n(t) \) obeys the equation (\( d/dt)T_n(t) = -\lambda_{n,\alpha} \delta_{n,0} D_{\alpha}^{-n/2}T_n(t) \). The solution of this equation is given by \( T_n(t) = T_n(t = 0) \exp(-\lambda_{n,\alpha} t^{\alpha}) \) [12] where the Mittag- Leffler function \( E_\alpha(x) \) is defined by the series expansion \( E_\alpha(x) = \sum_{k=\infty}^{\infty} x^k / \Gamma(1 + \alpha k) \). At \( \alpha = 1 \) it turns back into a standard exponential function (normal diffusion). Allowing for the boundary conditions leads to a transcendental equation for the eigenvalues, i.e., \(-2\sqrt{\kappa_n}/f = \tan(\sqrt{\kappa_n}N)\), where \( \kappa_n = \lambda_{n,\alpha}/K_\alpha - f^2/4 \). This eigenvalue problem has simple solutions in two limiting cases. For a very weak force \( fN \ll 1 \) the \( \kappa_n = \lambda_{n,\alpha}/K_\alpha = (2n+1)^2\pi^2/4N^2 \) and the eigenfunctions take on the form \( \varphi_n(s) = \sqrt{2/N} \cos[(2n+1)\pi s/2N] \). The resulting solution for \( W(s,t) \) at \( f = 0 \) reduces to that of the force-free case [14].

In this paper we focus our attention on the opposite limit, \( fN \gg 1 \), i.e., when the driving force is pretty strong. In this case the eigenvalues spectrum reads \( \lambda_{n,\alpha} = (f^2/4 + n^2\pi^2/2N^2)K_\alpha \) and all eigenfunctions \( \psi_n(s) = \sqrt{2/N} \sin(n\pi s/N) \), so that the resulting solution becomes

\[
W(s,t) = \frac{2}{N} e^{f(s-s_0)/2} \sum_{n=0}^{\infty} \sin\left[\frac{n\pi s_0}{N}\right] \sin\left[\frac{n\pi s}{N}\right] \times E_\alpha\left[-\left(\frac{f^2}{4} + \frac{n^2\pi^2}{N^2}\right)K_\alpha t^\alpha\right].
\] (3)

In the limit of strong driving force the translocation times are relatively (as compared to the force-free case) short and we could use the small argument approximation for the Mittag- Leffler function \( E_\alpha(-x) \), i.e., \( E_\alpha(-x) \approx \exp[-x / \Gamma(1 + \alpha)] \) at \( x \ll 1 \). This makes it possible to obtain an explicit analytical expression for \( W(s,t) \) which can be derived by replacing the summation by an integral in eq. [3]. In doing so one should use the relation \( 2\sin(n\pi s_0/N)\sin(n\pi s/N) = \cos[n\pi(s-s_0)/N] - \cos[n\pi(s+s_0)/N] \). Then one can integrate over \( n \) explicitly, taking the limit \( s_0 \to 0 \), and finally normalize the FPTD: \( w(s,t) \equiv \lim_{s_0 \to 0} W(s,t)/ \int_0^N W(s,t)ds \). This yields eventually

\[
w(s,t) = \frac{\exp\left[-(s - t\tilde{\tau})^2/4\tilde{\tau}\right]}{\sqrt{\pi \tilde{\tau}}} \left\{ \text{erf}([s - t\tilde{\tau}] / 2) - \text{erf}([s - N - t\tilde{\tau}] / 2\tilde{\tau}) \right\},
\] (4)

where the dimensionless force \( f = \Delta \mu/T \), \( \tilde{\tau} = K_\alpha t^\alpha / \Gamma(1 + \alpha) \) and \( \text{erf}(x) \) is the error function. Our further theoretical findings are based mainly on eqs. [3] and [4] for PDF.

First passage time distribution. In the chain translocation experiment the initial position \( s_0 \) can be fixed and the distribution of the translocation times is actually equivalent to the first passage time distribution (FPTD) \( Q(s_0,t) \) [22]. The relation \( Q(s_0,t) = -(d/dt)\int_0^N W(s,t)ds \) [22] enables to calculate FPTD explicitly. Starting from eq. [3] we arrive at the expression

\[
Q(s_0,t) = \frac{\pi K_\alpha e^{f(N-s_0)/2}}{N^2\Gamma(1 + \alpha)} \sum_{n=0}^{\infty} (-1)^{n-1} \sin\left[\frac{n\pi s_0}{N}\right] \times E_\alpha\left[-\left(\frac{f^2}{4} + \frac{n^2\pi^2}{N^2}\right)K_\alpha t^\alpha\right].
\] (5)

where the generalized Mittag- Leffler function \( E_\alpha(x) \) is \( \sum_{k=\infty}^{\infty} x^k / \Gamma(\alpha + \alpha k) \).

In the same manner as above we could use the small argument approximation for the generalized Mittag-Leffler function \( E_\alpha(-x) \), i.e., \( E_\alpha(-x) \approx (\alpha / \Gamma(1 + \alpha)) \exp[-x / \Gamma(1 + \alpha)] \) at \( x \ll 1 \), to obtain the explicit analytical expression for \( Q(s_0,t) \). The substitution of the summation by integration in eq. [5] and the use of the relation \( (-1)^{n-1} \sin(n\pi s_0/N) = -\cos(n\pi s_0/N) \sin(n\pi s_0/N) = \sin[n\pi(1 - s_0/N)] - \sin[n\pi(1 + s_0/N)] \) enable finally to obtain for the normalized FPTD, \( \text{lim}_{s_0 \to 0} Q(s_0,t)/\int Q(s_0,t)dt \to Q(t) \), the following expression

\[
Q(t) = \frac{\alpha}{4\pi^{1/2}ft} \left[\frac{\Gamma(1 + \alpha)}{K_\alpha t^\alpha}\right]^{1/2} \left[\frac{N^2\Gamma(1 + \alpha)}{K_\alpha t^\alpha} - 2\right] \times \left\{ \frac{N - f K_\alpha t^\alpha}{4\pi ft^\alpha} \right\}^{1/2}.
\] (6)

As one can see, after normalization the dependence on the initial value \( s_0 \to 0 \) drops out. It is of interest that FPTD given by eq. [6] exactly coincides (at \( \alpha = 1 \), i.e., in the Brownian dynamics limit) with the corresponding expression in the paper by Lubensky & Nelson [23]. It is also evident from eq. [6] that the maximum position scales as \( t_{\text{max}} \propto (N/f)^{1/\alpha} = (N/f)^{2.5} \). Nevertheless, the function \( Q(t) \) is quite skewed and we will see below that the average translocation time \( \tau = \int tQ(t)dt \) (which is presumably measured in an experiment) scales differently. Note that Eq. [6] is valid for \( t \leq N^2/\alpha \) \( \Gamma(1 + \alpha)/2K_\alpha \) \( 1/\alpha \), i.e., \( t \leq 0.4N^{2.5} \) for \( \alpha = 0.8 \) which is not a serious limitation actually because our translocation times scale as \( \tau \propto N^{1.5} \) as will be demonstrated below.
**Statistical moments** \langle s \rangle and \langle s^2 \rangle vs. time. –

The recording of statistical moments time dependence, \langle s(t) \rangle = \int_0^N sw(s, t) ds and \langle s(t)^2 \rangle = \int_0^N s^2 w(s, t) ds, is very instructive (as in the force-free case [14]) for the consistency check. Starting from eq. (4) the calculation of the first moment yields

\[
\langle s(t) \rangle = \tilde{f} t + 2 \sqrt{\frac{\tilde{f}}{\pi}} \times \frac{\exp\left[-\frac{\tilde{f}^2 t^2}{4}\right] - \exp\left[-(\tilde{f} t - N)^2/4\tilde{f}\right]}{\text{erf}\left[(\tilde{f} t - N)/2\sqrt{\tilde{f}}\right] - \text{erf}\left[(\tilde{f} t - N)/2\sqrt{\tilde{f}}\right]} \tag{7}
\]

It can easily be shown that in the large time limit \langle s \rangle \to N. In the same manner the second moment reads

\[
\langle s^2(t) \rangle = f^2 t^2 + 2\tilde{f} t + 2 \sqrt{\frac{\tilde{f}}{\pi}} \times \frac{f t \exp\left[-\frac{f^2 t^2}{4}\right] - (f t + N) \exp\left[-(f t - N)^2/4f\right]}{\text{erf}\left[(f t - N)/2\sqrt{f}\right] - \text{erf}\left[(f t - N)/2\sqrt{f}\right]} \tag{8}
\]

In eqs. (7) and (8) the notations are the same as in eq. (4). The detailed check of these relations will be given below. Here we only note that for large times \(1 < \tilde{t} < N/f\) the exponential terms in eqs. (7) and (8) vanish so that to a leading order the moments vary as \(\langle s(t) \rangle \propto t^\alpha\) and \(\langle s(t)^2 \rangle \propto t^{2\alpha}\). Again it can be shown that at \(t \to \infty\) the moments \(\langle s(t) \rangle\) and \(\langle s^2(t) \rangle\) saturate to plateaus which scale like \(N\), and \(N^2\) respectively, as they should.

**Scaling arguments.** – The foregoing theoretical consideration has been based on a rigorous mathematical treatment of the FFPE. Before proceeding to the MC check of these findings we put forward some simple scaling arguments so as to quantify the mean translocation time as well as the statistical moments. Let us take the average external field energy \(|\langle U(s) \rangle| = f N\) as a natural scaling variable. Then the driven translocation rate scales as \(\tau^{-1} = \tau_0^{-1} \phi(f N)\), where \(\tau_0 \propto N^{2\nu+2-\gamma_1}\) denotes the translocation time in the force-free case [14]. The scaling function \(\phi(x)\) behaves in the following way: \(\phi(x \ll 1) \simeq 1\) and \(\phi(x \gg 1) \simeq x\) because at \(f N \gg 1\) we could expect that the translocation rate is proportional to the force \(f\). As a result we come to the conclusion that at \(f N \gg 1\) the translocation time is scaled as

\[
\tau \propto \frac{1}{f} N^{2\nu+1-\gamma_1} \tag{9}
\]

Taking into account the values for \(\nu\) and \(\gamma_1\) given above we arrive at the estimations: at \(d = 3\) the translocation exponent \(\theta = 2\nu + 1 - \gamma_1 = 1.496\) and at \(d = 2\) the exponent \(\theta = 2\nu + 1 - \gamma_1 = 1.56\). This is pretty close to the estimation given by Kantor & Kardar [9], \(\theta = 1.53\).

If we assume that the behavior of \(\langle s^2(t) \rangle\) (before it hits the plateau) follows a power law, \(\langle s^2(t) \rangle \propto (f t)^\beta\), then from the correspondence to the scaling law, eq. (9), one may estimate \(\beta\). Indeed, at the translocation time \((f \tau)^\beta \sim N^2\) and the requirement of correspondence with eq. (9) yield \(\beta = 2/(2\nu + 1 - \gamma_1)\). This gives \(\beta = 1.334\) at \(d = 3\). In the next section we will demonstrate that this power law is in reality too crude. The observed exponents in the MC simulation (as well as in the analytical theory given above) cross over from a smaller value at very short time to a larger one \((2\nu \approx 1.6)\) at long times in comparison with the simple scaling prediction \(\beta = 1.334 \approx 4/3\).

**Monte Carlo data vs. theory.** – We have carried out extensive MC simulations in order to check the main predictions of the foregoing analytical theory. We use a dynamic bead-spring model which has been described before [24], therefore we only mention the salient features here. Each chain contains \(N\) effective monomers (beads), connected by anharmonic FENE (finitely extensible nonlinear elastic) springs, and the nonbonded segments interact by a Morse potential. An elementary MC move is performed by picking an effective monomer at random and trying to displace it from its position to a new one chosen at random. These trial moves are accepted as new configurations if they pass the standard Metropolis acceptance test. It is well established that such a MC algorithm, based on local moves, realizes Rouse model dynamics for the polymer chain. In the course of the simulation we perform successive runs for chain lengths \(N = 16, 32, 64, 128, 256, 512\) whereby a run starts with a configuration with only few segments on the trans-side. Each run is stopped, once the entire chain moves to the trans-side. Complete retracting of the chain back to the cis-side is prohibited by taking the head monomer larger than the pore diameter. During each run we record the translocation time \(\tau\), and the translocation coordinate \(s(t)\). Then we average all data over typically \(10^4\) runs. In Fig. 2 we show the PDF \(Q(\tau)\)

![Fig. 2: First passage time distribution functions at \(N = 128\) and different forces as calculated from MC data (symbols) and the theoretical prediction eq. (9) (solid lines).]
of a polymer chain with $N = 128$ for three different values of the drag force, $f = 0.5, 0.8$ and $1.0$. Although the MC data is somewhat scattered, especially for $f = 0.5$, the agreement with the analytic expression, eq. (6) is very good. Since we set the generalized diffusion coefficient $K_\alpha \equiv 1$ and $\Gamma(1 + \alpha) \approx 0.931$ for $\alpha = 0.8$, the comparison with MC results suggests that a time unit in the FFPE corresponds roughly to $500\text{MCS}$.

Using the PDF $Q(t)$, one may determine the MFPT (or, translocation times) $\tau$ which are compared in Fig. 3 for $16 < N < 512$ and six values of the drag force $f$. Evidently, for both theory and simulation the data collapse on master curves $f\tau \propto N^{1.5}$, if one scales $\tau$ with the respective force, cf. eq. (9). It is seen that the simulation data is shifted up by a factor of $\approx 500$ which translates the MC time into conventional time units. The variation of the moments $\langle s \rangle$, and $\langle s^2 \rangle$ is displayed in Fig. 4. Again a perfect collapse of the transients is achieved by scaling the time with the applied force $t \to t\tau$. One can immediately see that the simple scaling prediction $\langle s^2 \rangle \propto t^{\beta}$ is not perfect: for $t\tau < 3$ evidently $\langle s \rangle$ grows with a smaller exponent whereas at later times the increase is steeper. As mentioned above, this curve is very well accounted for by eqs. (7), (8). Thus, for $t\tau \ll 1$ one can readily obtain from eq. (7) as a leading term $\langle s \rangle \propto t^{\beta_1}$ while for $1 < t\tau < N$ one has $\langle s \rangle \propto t^{\beta_2}$. As indicated in Fig. 4 the observed agreement between theory and computer experiment is remarkable indeed. Notably this finding suggests that even the presence of drag force does not eliminate the anomalous character of the translocation process as one would intuitively expect. This result resolves thus a problem, raised initially by Metzler and Klafter [13]. The universal exponent $\alpha = 2/(2\nu + 2 - \gamma_1)$ for unbiased threading through a pore is not suppressed by the drag force! One may thus conclude that the measurement of the number of translocated segments with time could provide a means for direct observation of anomalous diffusion.

**Fig. 4:** Statistical moments versus reduced time $tf$ from MC data and from the analytic results, eqs. (7), (8), for chain length $16 \leq N \leq 256$: (a) The first moment $\langle s(t) \rangle$: the slope $\beta/2 = 2/3$ is indicated by a long dashed line, a short dashed line denotes $\langle s \rangle \propto t^{\beta_1}$. (b) The second moment $\langle s^2(t) \rangle$: A long dashed line indicates a slope $\beta = 4/3$, a short dashed line denotes $\langle s^2 \rangle \propto t^{\beta_2}$. 

**Summary.** – By solving the fractional Fokker-Planck equation for a driven polymer translocation through a narrow pore and deriving a closed analytic expression for the probability distribution function $W(s,t)$ to have the segment $s$ of the chain in the pore at time $t$ we have demonstrated that the translocation process displays all features typical for anomalous diffusion. The physical background of this behavior is elucidated by scaling considerations. The polymer translocation is considered as a squeezing of subsequent chain fragments (folds), each being in local thermodynamic equilibrium, through a narrow pore. This consideration gives rise to an universal scaling exponent for anomalous diffusion $\alpha = 2/(2\nu + 2 - \gamma_1)$ so that the time $\tau$ needed for a chain of $N$ segments to move from *cis* to the *trans* semispace in the absence of drag scales as $\tau \propto N^{2/\alpha}$. The presence of external pulling force modifies this relationship to $\tau \propto f^{-1}N^{2\nu+1-\gamma_1}$. This principal result of the present investigation is unambiguously confirmed by calculation of the mean first passage times (the average translocation times) from the derived analytic expression for the translocation time distribution function.
Q(r) as well as by comparison to the results of extensive Monte Carlo simulations. We also show that the growth of the average number of translocated segments ⟨s⟩ with time follows a power law ⟨s(t)⟩ ∝ t^α (for relatively long times) which directly displays the anomalous diffusion exponent α. Our analytic data also appears to be in perfect agreement with the simulation results in a wide range of polymer lengths and forces. Thus we have demonstrated that the translocation dynamics of a driven polymer chain through a narrow pore retains all features of anomalous diffusion despite the application of external force.

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The authors greatly acknowledge the SFB - DFG 625 project for financial support. A. M. appreciates hospitality during his stay at the Max-Planck Institute for Polymer Research in Mainz.

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