A schematic model for pentaquarks based on diquarks

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QCD instantons are known to produce deeply bound diquarks which may be used as building blocks in the formation of multiquark states, in particular pentaquarks and dibaryons. We suggest a simple model in which the lowest scalar diquark (and possibly the tensor one) can be treated as an independent “body”, with the same color and (approximately) the mass as a constituent (anti)quark. In this model a new symmetry between states with the same number of “bodies” but different number of quarks appear, in particular the 3-“body” pentaquarks can be naturally related to decuplet baryons. We estimate both the masses and widths of such states, and then discuss the limitations of this model.

Introduction. The possibility of a low lying $\bar{q}q^4$ states in the P-wave (e.g. $K^+n$) channel fitting in the anti-decuplet flavor representation of the quark model was advocated long ago by Golowitch [1], along with the non-strange excited baryon $N(1710)$. A decade ago, when the $SU(3)$ version of the Skyrme model was refined, it was found to predict an antidecuplet 10 of baryons above the conventional octet and decuplet. It was not taken seriously till relatively recent works [2] which predicted among others a resonance in $K^+n$ with a mass of 1540 MeV.

In remarkable agreement with this prediction, several recent experiments have reported an exotic baryon $\Theta^+(1540)$ with a small (and so far unmeasured) width [3]. The issue of its consistence with earlier $Kd$ data is discussed in [4] and also [5]. The observed angular distribution suggests a likely spin 1/2 state, with so far unknown parity. Its minimal quark content is a pentaquark, i.e. $(ud)s$. The antidecuplet flavor assignment was further strengthened by an observation by the NA49 collaboration [6] of a family of exotic $\Xi$ baryons, with a mass of 1.86 GeV and width smaller than the experimental resolution of 18 MeV.

The theoretical advantage of the Skyrme model is that it allows to reduce a complex multiquark problem into a single-body problem, with one pseudoscalar meson moving in a fixed classical background. However the price for such reduction, based on the “large $N_c$ ideology” maybe prohibitive given the large degeneracies implied. The $1/N_c$ description implies a small width, that is difficult to assess quantitatively given the subtleties related to these corrections [7].

More traditional “shell model ideology” (e.g. the MIT bag model or nonrelativistic constituent quark models) tends to put as many quarks as possible into the lowest shell, and thus predict negative parity, $P = -1$ for the lowest state\(^1\).

The shell model works well for nuclei; in this case the pairing effects are small and treated perturbatively using the shell model states. However we think the order should be reversed for hadrons, and pairing into diquarks be treated first. One argument for that is that the many flavor-symmetric exotic states possible in a shell model have never been seen. Even the most symmetric “magic” configuration, the dibaryon $H = u^2d^2s^2$, an analogue of the alpha particle, appears to be not deeply bound, as it was never found in multiple dedicated searches.

As we will argue in this letter, the picture most consistent with the current new findings are those developed in a “small $N_c$ ideology”, in which the key element are the instanton-induced \(^2\) diquarks [10,11]. Due to the Pauli principle at the level of instanton zero modes, two quarks of the same flavor cannot interact with the same instanton. The propagation of 5 quarks through the QCD vacuum generates many interactions involving ’t Hooft interaction, some second order ones are depicted in Fig. 1. The latter illustrates the strong preference for multiquark states to be in the lowest possible flavor representation, avoiding many other possible exotic states, both in the meson and baryon sectors. As we will argue, even these newly discovered states, although truly exotic, still are in a way analogous to the decuplet baryons. Their small decay widths is a consequence of a different internal structure, with small overlap with all the decay channels.

\(^1\)The lattice studies by Csikor et al. [8] and Sasaki [9], indeed claim a signal for $P = -1$ pentaquarks with a comparable mass. More and better data are however needed to reach firm conclusions on the matter.

\(^2\)Although scalar diquarks are also attracted by single-gluon exchange forces, the latters do not lead to the structure we discuss as they are flavor blind.
FIG. 1. Some second-order instanton-induced interactions of 5 quarks propagating in time through the (Euclidean) QCD vacuum. The shaded circles indicate instantons and antiinstantons. The quarks are avoiding quarks of the same flavor and 3-body force is repulsive, so (a) is the diagram generating two independent diquarks. The instantons have to pick up pairs from the vacuum condensate $<\bar{s}s>$ to get it attractive. The diagram (b) with a light quark exchange generates a repulsive core, while the diagram (c) leads to diquark attraction.

For a review on the instanton vacuum models one can consult [11]. The main approximations are: i. a reduction of the gauge configurations to the subset of instantons and antiinstantons; ii. a focus on only the fermionic states that are a superposition of their zero modes. When the baryonic (3-quark) correlators have been first calculated in ref. [12] a decade ago (and soon confirmed by lattice measurements [13]) a marked difference between the nucleon (octet) and $\Delta$ (decuplet) correlators has been noted. Roughly speaking, a nucleon was found to be made of a quark and a very deeply bound scalar-isoscalar diquark, absent in the decuplet. As it was found to have a surprisingly small mass comparable to the constituent quark mass (to be denoted below as $\Sigma$), it significantly simplifies the model to be discussed below.

The general theoretical reason for the lightness of the scalar-isoscalar diquark state (see e.g. [14]) follows from the special Pauli-Gursey symmetry of 2-color QCD. In this theory (the “small $N_c$ limit” of QCD) the scalar diquarks are actually massless Goldstone bosons. For general $N_c$, the instanton (gluon-exchange) in $q\bar{q}$ is $1/(N_c-1)$ down relative to $\bar{q}q$. So the real world with $N_c = 3$ is half-way between $N_c = 2$ with a relative weight of 1, and $N_c = \infty$ with relative weight 0. Loosely speaking, the scalar-isoscalar diquarks are half Goldstone bosons with a binding energy of about half of the mass, or about one constituent quark mass.

Diquarks in the context of Nambu-Jona-Lasinio models were investigated e.g. in [15], which also emphasized the occurrence of a light scalar-isoscalar bound state. Diquark correlations have been a driving idea behind a view on the occurrence of a light scalar-isoscalar bound state. Di- tions were investigated e.g. in [15], which also emphasized the small vector channels are weakly repulsive, with a mass of the order of 950 MeV, above twice the constituent quark mass of the model, $2\Sigma = 840\text{ MeV}$. The only two channels with attraction and significant binding are: i. the scalar with $m_S \approx \Sigma$ and $\Gamma = \gamma_5$; ii. the tensor with $m_T \approx 570\text{ MeV}$ and $\Gamma = \sigma_{\mu\nu}$ (denoted below by a subscript $T$) \(4\). The scalar is odd under spin exchange while the tensor is even under spin exchange. Fermi statistics forces their flavor to be different. The scalar is flavor antisymmetric 3 while the tensor is flavor symmetric 6.

In the model to be discussed below, we will discuss all possible pentaquark multiplets which can be made using these ingredients. For scalar diquarks we will introduce the following shorthand notation in $SU(3)_f$

$$S = (u^TC\gamma_5d); \ U = (s^TC\gamma_5d); \ D = (u^TC\gamma_5s) \quad (2)$$

Model. to be discussed treats diquarks on equal footing with constituent quarks. Because of their similar mass and quantum numbers, certain approximate symmetries appear between states with the same numbers of “bodies”. This simple idea is depicted pictorially in Fig. 2. The $q\bar{q}$ mesons (a) are a well known example of the 2-body objects, as well as the quark-diquark states (b) the octet baryons $qq\bar{q}$. The diquark-

\[\text{3 Those exist as physical hadrons only in } N_c = 2 \text{ QCD. However, since the instanton liquid model does not confine, there are diquark states for any } N_c.\]

\[\text{4The longitudinal vector diquark channel with } \Gamma = \gamma_\mu\gamma_5 \text{ mixes with the scalar } \Gamma = \gamma_5 \text{ in the P-wave. This point is relevant to the lattice studies discussed in [9].}\]
antidiquark states (c) are in this model the 2-body objects. In zeroth order, the usual non-strange mesons (like \( \rho, \omega \)), the octet baryons (like the nucleon), and the 4-quark mesons (like \( a_0(980) \))^5 are degenerate, with a mass \( M \approx 2 \Sigma = 840 \text{ MeV} \). To first order, which includes color-related interactions, the one-gluon-exchange Coulomb and confinement, the degeneracy should still hold, as the color charges and the masses of quarks and diquarks are the same. Only in second order, when the spin-spin and other residual forces are included, they split. There is no spin-spin interaction for the nucleon (the scalar diquark has no spin), while for the \( \rho \) it is either repulsive (if it is due to one gluon exchange) or zero (if it is due to the instanton-induced forces [20]). Note that this new symmetry between \( N, \rho \) and \( a_0(980) \) is actually rather accurate, better than the old SU(6) symmetry, stating (in zeroth order) that \( M_N \approx M_\Delta \).

\[ M_\Theta = 2 \Sigma + \Sigma_\pi + \delta M_{L=1} + V_{\text{residual}} \]  

\[ m_\Theta \approx m_\Sigma^*(3/2) + \delta M_{L=1} \approx 1400 + 480 = 1880 \text{ MeV} , \]  

which is well above the observed mass of 1540 MeV.

For diquark-diquark-antiquark all there is left is Bose statistics for identical scalars, demanding total symmetry over their interchange, while the color wave function is antisymmetric. So the only solution \([17,4]\) is to make the spatial wave function antisymmetric by putting one of the diquark into the P-wave state. It means that such pentaquarks should be degenerate with the excited P-wave decuplet baryons.

\[ m_\Theta \approx m_\Sigma^*(3/2) + \delta M_{L=1} \approx 1400 + 480 = 1880 \text{ MeV} , \]

which is much closer to the experimental value.

\[ m_\Theta \approx m_\Sigma^*(3/2) + \delta M_{T=1} \approx 1400 + 150 = 1550 \text{ MeV} , \]

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\( ^5 \)For recent study of these states in the instanton model see [19].
The newly observed $\Xi(1860)$ pentaquarks contains diquarks with a strange quark, that is $us, ds$. Their masses have not been yet directly calculated, but a general experience with spin-dependent forces [20] suggests a reduction of binding by about a factor 0.6 as compared to the $ud$ case. This suggests a total loss of binding of about 200-240 MeV, which together with a strange quark mass itself (two $s$ quarks instead of a single $\bar{s}$) readily explains the 320 MeV mass difference between $\Xi(1860)$ and $\Theta^+$($1540$) pentaquarks.

Since the tensor diquark has the opposite parity, both possibilities correspond to the same global parity $P = +1$. Also common to both schemes is the fact that the total spin of 4 quarks is 1, so adding the spin of the $\bar{s}$ can lead not only to $s = 1/2^+$ but also to $s = 3/2^+$ states (which are not yet observed).

So, we conclude that if we only look at the masses, it appears that it is better to substitute one diquark by its tensor variant, rather than enforce the P-wave. However such an alternative scheme provides a different set of flavor representations as we now show. Indeed, $\Theta^+$ partners. The former can decay into $\bar{ssq}$ states. The cascades have isospin $3/2$, as observed. However $\Theta^+$ is a part of an isotriplet, with $\Theta^{+\mp}$ and $\Theta^0$ partners. The cascades can decay into $pK^+$, a quite visible mode, in which no resonance close to 1540 was seen. Since the widths are unknown at this point, it is perhaps premature to conclude that they do not exist. However if the occurrence of the decay mode $\Theta^{++} \rightarrow pK^+$ is definitively ruled out, the observed multiplet of exotics cannot be the 27.

The Roper resonance belongs to octet with the quark content $SS\bar{d}$. In the JW model with the P-wave, its mass would be estimated as

$$m_{Roper} = 3\Sigma + \delta M_L = 1260 + 480 = 1740 \text{ MeV},$$

while in a variant with the tensor diquark it is only

$$m_{Roper} = 3\Sigma + \delta M_T \approx 1260 + 150 = 1410 \text{ MeV},$$

which once again gets us closer to the experimental value. However this corresponds to 27 flavor representation, where its isospin is 3/2. In the lower 8 flavor representation with isospin 1/2, it will include $\bar{s}s$ and be too heavy again.

**Widths and Goldberger-Treiman Relations.** Small widths are not the consequence of the centrifugal barrier, as the P-wave is not really producing sufficiently small factors. As we already mentioned, a general argument for small pentaquark widths is small overlap between the internal and external ($KN$) wave functions. In this section we make this relation more explicit.

The decay widths including Goldstone bosons are determined by general properties of their chiral interactions, and expressions can be somewhat simplified. The strong decay of the pentaquark $P^{(1/2)} \rightarrow \pi N^{(1/2)}$ is conditioned by a generalized Goldberger-Treiman relation. The one-pion reduced axial vector current has a transition matrix

$$\left\langle P(p_2)|j^a_{\mu}(0)|\pi(p_1)\right\rangle = \frac{1}{f_\pi} \frac{1}{m_\pi^2 - t} \times \hat{P}(p_2) \left( (m_P + m_N) G(t) + 2m_\pi H(t) \right) \frac{\tau^a}{2} N(p_1)$$

with $j^a_{\mu}$ partially conserved [23],

$$\partial^\mu j^a_{\mu}(x) = f_\pi \left( \mp + m_\pi^2 \right) \pi^a(x).$$

The first form factor in (8) is one-pion reduced with $G(0) = g_{\pi N}$ the “axial overlap” charge. If its value be close to the axial charge of the nucleon, it would mean that pentaquark is nothing but a $PN$ system. However, as we will see, the data demand it to be significantly smaller.

Inserting (9) into (8) gives

$$\left\langle P(p_2)|\pi^a(0)|\pi N(p_1)\right\rangle = \frac{1}{f_\pi} \frac{1}{m_\pi^2 - t} \times \hat{P}(p_2) \left( (m_P + m_N) G(t) + 2m_\pi H(t) \right) \frac{\tau^a}{2} N(p_1).$$

By definition, the pseudoscalar $\pi$-$PN$ coupling is

$$\left\langle P(p_2)|\pi^a(0)|\pi N(p_1)\right\rangle = g_{\pi P N}(t) \frac{1}{m_\pi^2 - t} \hat{P}(p_2) \gamma_5 \tau^a N(p_1),$$

which corresponds to

$$g_{\pi P N} \tau^a \left( \hat{P} \tau^a N + \text{h.c.} \right).$$

A comparison of (11) to (10) gives at the pion pole $t \approx m_\pi^2$

$$f_\pi g_{\pi P N}(m_\pi^2) + \sigma_{\pi P N}(m_\pi^2) = \frac{m_P + m_N}{2} g_{\pi P N}(m_\pi^2),$$

which is the general form of the Goldberger-Treiman relation for the transition amplitude $P \rightarrow N\pi$. The overlap sigma-term is proportional to $m_\pi^2/\Lambda$, which is typically 40 MeV in the pion-nucleon system.

The generic form of the decay width $P \rightarrow \pi N$ is given by

$$\Gamma_{P \rightarrow \pi N} = \frac{g_{\pi P N}^2 q_P}{4\pi M_P} \left( \sqrt{q_P^2 + m_N^2} - m_N \right)$$

where $q_P$ is the meson momentum in the rest frame of the $P$ state,

$$M_P = \sqrt{q_P^2 + m_N^2} + \sqrt{q_P^2 + m_\pi^2}. $$
The recently observed $\Xi(1860)$ can be used in conjunction with (12) to bound the transition axial-overlap $g_{PN}$ and the coupling $g_{\pi PN}$ in the antidecuplet, thereby allowing a prediction for the width of the $\Theta(1540)$ through (13). Indeed, if we assign a conservative decay width of about 20 MeV to $\Xi^{-} \rightarrow \Xi^{-}\pi^{-}$ in light of the bound of 18 MeV reported by [6], then (12) suggests $g_{\Xi\Xi} \approx 0.25$ and $g_{\Xi\Xi\Xi} \approx 3.75$ for $\sigma_{\Xi \Xi} \approx 40$ MeV. Similar arguments yield $g_{\Xi\Xi} \approx 0.25$ and $g_{\Xi\Xi\Xi} \approx 2.97$, thus an estimated partial width of 6.60 MeV for $\Xi^{-} \rightarrow \Sigma^{-} K^{-}$. Similarly, we would expect $g_{\Xi N} \approx 0.25$ and $g_{\Xi\Xi\Xi} \approx 2.35$, and we therefore predict a very narrow width of 2.60 MeV for the decay $\Theta^{+} \rightarrow K^{+}n$.

The narrowness of the partial widths in the antidecuplet follows from a generically small transition axial-charge of about 1/4, resulting into a $\pi-PN$ decay constant of about 3 in the antidecuplet. The smallness of the axial-charge follows from the small overlap between the three and five quark states.

**Summary and Discussion.** We started by emphasizing that instanton-induced ‘t’Hooft interaction imply diquark substructure of multiquark hadrons and dense hadronic matter, with marked preference to the lowest flavor representations possible. We then summarized the finding of ref. [12]: in the instanton liquid model whereby there are two kinds of deeply bound diquarks, the scalar and the (less bound) tensor.

We have then developed a schematic additive model, whereby diquarks appear as building blocks, on equal footing with constituent quarks. In such a model pentaquarks are treated as 3-body states, so that their classification in color and flavor becomes analogous to that of the baryons. If one uses two scalar diquarks, as suggested by Jaffe and Wilczek, the P-wave is inevitable which seems to produce states heavier than the ones reported, even in a simple additive model with very light diquarks. If one uses one scalar and one tensor diquarks, the masses look more reasonable. However, then the ensuing flavor representations are large, and although recently discovered quartet of $\Xi(1860)$ fits very well into this model, the $\Theta^{+}$ has (so far) unobserved partners.

We have related the widths with the “axial overlap” charge, and have argued that current data restrict it to be significantly smaller than the nucleon axial charge, by about a factor of 3. This means that the Skyrme-model interpretation of pentaquarks, as a Goldstone boson moving on top of the baryon is inadequate.

If one goes a step further, to 6-quark states, for example by combining the proton and the neutron, one gets 3 $ud$ diquarks. Again the asymmetric color wave function asks for another asymmetry: to do so one can put all 3 diquarks into the P-wave state, with the spatial wave function $\epsilon_{ijk} S_{ij} S_{jk} S_{ik}$ suggested in the second paper of [14]. This will cost $3(\Sigma + \delta M_{L=1}) = 2700 \text{ MeV}$, well in agreement with the magnitude of the repulsive nucleon-nucleon core. However if one considers the quantum numbers of the famous $H$ dibaryon, one can also make those out of diquarks such as $SU(3)$. The resulting wave function is overall flavor antisymmetric with all diquarks in S-states. Thus there is no need for P-wave or tensor diquarks for the $H$ dibaryon. Our schematic model would then lead to a very light $H$, in contradiction to both experimental limits and lattice results.

This last observation calls for the lesson with which we would like to conclude our paper: all schematic models (including our own) assume additivity of the constituents. However, as we emphasized in Fig.1, due to the Pauli exclusion principle one instanton can only make one deeply bound diquark at a time. Thus, there must be a diquark-diquark repulsive core. One particular 3-body instanton repulsion effect was already discussed for the $H$ in [24]. Multi-body instanton induced interactions were also observed in heavy-light systems [25].

A generic way to address these effects would be some dynamical studies, directly antisymmetrizing 5 or 6 quarks themselves, as well as with those in the QCD vacuum (unquenching). The evaluation of the pertinent correlators on the lattice is badly needed: studies of interdiquark interactions in the instanton liquid model will be reported elsewhere [26]. Only with the resulting core potential included, the diquark-based description of multiquark states and of dense quark matter may become truly quantitative.

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