PROBING THE SPACETIME AROUND SGR A* WITH RADIO PULSARS

ERIC PFAHL1 AND ABRAHAM LOEB2
Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138; epfahl@cfa.harvard.edu, aloeb@cfa.harvard.edu
Submitted to The Astrophysical Journal

ABSTRACT
The supermassive black hole at the Galactic center harbors a bound cluster of massive stars that should leave neutron-star remnants. Extrapolating from the available data, we estimate that ~1000 radio pulsars may presently orbit Sgr A* with periods of ≲100 yr. Several of the most luminous of these pulsars may be detectable with current telescopes in periodicity searches at frequencies of ≳10 GHz, where the effects of interstellar scattering are alleviated. Long-term timing observations of such a pulsar would reveal its Keplerian motion, and possibly show the effects of weak- and strong-field relativistic gravity. In particular, we show that frame dragging due to the spin of the black hole may introduce measurable deviations from the best-fit Keplerian timing solution. With the addition of radio astrometric observations of the pulsar orbit, it might be possible to determine both the magnitude and direction of the black-hole spin. We also briefly investigate how pulsar timing can be used to study the dynamical and interstellar environment of the central black hole.

Subject headings: black hole physics — Galaxy: center — pulsars: general

1. INTRODUCTION
Ten years of near-infrared observations of the Galactic center have revealed the proper motions of roughly two dozen stars within 0.5 of the compact radio source Sgr A*. Significant astrometric accelerations measured for 8 members of the Sgr A* stellar cluster point to a common center of gravity coincident with the position of Sgr A*, and imply a central mass of (3–4)×10^6 M⊙ (Genzel et al. 2003; Schödel et al. 2003). Stars S0-2 and S0-16 have the most compact orbits yet identified, with respective periods of ≲15 yr and ≲30 yr, eccentricities of ≲0.88 and ≲0.95, and comparable pericenter distances of ≳100 au (Schödel et al. 2002; Ghez et al. 2003; Eisenhauer et al. 2003). If the central mass is confined within 100 au, the implied density is ≳10^8 M⊙ pc^-3, which essentially rules out existing models alternative to the hypothesis that the central object is a supermassive black hole (BH; Maoz 1998; Ghez et al. 2003; Schödel et al. 2003).

Evidence from the near-infrared spectrum of S0-2 (Ghez et al. 2003) and the integrated spectrum within ≲0.5 of Sgr A* (Genzel et al. 1997; Eckart, Ott, & Genzel 1999; Figer et al. 2000; Gezari et al. 2002) suggest that the observed Sgr A* stellar cluster is largely comprised of luminous (∼10^4 L⊙), early-type (O9 to B0) stars. If these stars are near the main sequence, the inferred masses are ∼10–20 M⊙. A perplexing problem is how these stars came to reside so near the supermassive BH; for discussions and references, see Genzel et al. 2003 and Ghez et al. 2003b). Nevertheless, the existence of a cluster of massive stars tightly bound to Sgr A* has important implications.

Stars of mass ∼10–20 M⊙ have nuclear lifetimes of ∼10^7 yr, and leave neutron-star (NS) remnants. Therefore, we expect a significant number of NSs to be bound to Sgr A* in orbits similar to those of the observed cluster stars, as well as in more compact orbits. Source confusion so far inhibits the discovery of stars with orbital periods of ∼10 yr about Sgr A* (Genzel et al. 2003; Ghez et al. 2003b), though we anticipate that massive stars, and therefore NSs, populate this region. The most exciting possibility is if some fraction of the NSs orbiting Sgr A* are detectable radio pulsars, an idea first considered in the prescient work by Paczynski & Trimble (1979). In §2 we estimate the total number of normal radio pulsars—i.e., those with surface magnetic field strengths of ∼10^11–10^12 G—that may presently orbit the central BH with periods of ∼100 yr.

Radio-wave scattering in the interstellar plasma poses the largest obstacle to discovering pulsars near Sgr A*, where column the density of free electrons is very high. At observing frequencies of ≳1 GHz, pulsed emission from this vicinity suffers severe temporal broadening, prohibiting the detection of pulsars as periodic sources (Cordes & Lazio 1997). Observing frequencies of ≳10 GHz are required to alleviate the effects of scattering, although the flux density of pulsars declines with increasing frequency. These issues are addressed in §3 where we estimate the number of pulsars orbiting Sgr A* that may be detectable with current telescopes.

The Keplerian motion of a pulsar orbiting Sgr A* would be clearly apparent in its long-term timing properties. Weak- and strong-field relativistic gravity may introduce measurable deviations from the best-fit Keplerian timing solution, depending on the orbital parameters and timing precision. Various relativistic pulse arrival-time delays, as well as secular relativistic effects, are quantified in §4. Most notably, delays due to frame dragging caused by the spin of the BH may be evident in the timing residuals. As we discuss in §5, the combination of timing and imaging data for an orbiting pulsar could be used to determine both the magnitude and direction of the BH angular momentum. We also consider in §5 what pulsar timing can teach us about the accretion flow onto the Galactic BH, as well as the stellar dynamical environment.

2. RADIO PULSARS ORBITING SGR A*
At a distance of ∼8 kpc (Eisenhauer et al. 2003), the observed Sgr A* stellar cluster occupies the central ∼4000 au about the BH. Although the observational census is incomplete, this volume likely contains at least several tens of massive stars. However, it is not the present population of massive stars, but rather their predecessors, that would be the progenitors of radio pulsars orbiting Sgr A*. Since the origin of the

1 Chandra Fellow
2 Guggenheim Fellow
observed cluster stars is unknown, we can only speculate on the history of the population of massive stars in this region. It is plausible that cluster stars are steadily or episodically replenished as they evolve and leave NS remnants. This might be the case if the mechanism that feeds stars into the central ∼4000 AU is linked to the significant star-formation activity on larger scales. Observational evidence suggests that the central ∼200 pc of the Galaxy is a region of past and current star formation, with an average massive-star formation rate of ∼10−3 yr−1, or ∼10% of the present rate in the entire Galactic disk (Mezger et al. 1999; Launhardt, Zylka, & Mezger 2002; see also Pfaß, Rappaport, & Podsidiłowski 2002).

For simplicity, we assume that over a time of ∼106 − 109 yr a roughly steady-state number of ∼10−100 NS progenitors orbit Sgr A* with semimajor axes of <4000 AU, and periods of Porb ≤ 100 yr. A stellar lifetime of ∼107 yr then implies a NS birthrate of ∼10−6−10−5 yr−1. We further assume that the majority of NSs turn on as radio pulsars a short time after birth. Radio emission terminates when a pulsar crosses the so-called “death line” in the log P−log Pp plane, where Pp is the pulse period (for discussion and references, see, e.g., Rudak & Ritter 1994). A coarse analysis of pulsar statistics in the log P−log Pp plane shows that a median terminal age for normal radio pulsars is ∼(1−5) × 107 yr, depending on the physical model for the death line, and neglecting magnetic-field decay (e.g., Bhattacharya et al. 1992; Rudak & Ritter 1994; Tauris & Kawaguchi 2001). Multiplying this time by the NS birthrate, we predict up to several thousand radio pulsars presently orbiting Sgr A* with Porb ≤ 100 yr. This estimate, based on a steady-state model, is only a starting point. A more sophisticated study should consider different histories for the stellar population near Sgr A*.

Typical NS “kick” speeds of vk ∼ 100−300 km s−1 are inferred from the proper motions of ∼100 isolated pulsars in the Galactic disk (e.g., Hansen & Phinney 1997; Arzoumanian, Chernoff, & Cordes 2002). Since the stars we are considering have characteristic orbital speeds of vorb ≳ 1000 km s−1, an impulsive NS kick will usually cause only a small fractional change of (order vk/vorb) in the orbital parameters. Therefore, the distributions of pulsar orbital parameters should closely resemble those of their progenitor stars. For a stellar number density about Sgr A* that varies as r−3 1/4≥ 1000 km s−1, the period distribution is dN/dP orb ∝ P−3 orb if the velocity field is isotropic (e.g., Schödel et al. 2005). Although isotropy is unlikely within the central 1′, this analysis is still heuristically useful. Genzel et al. (2003) find that q ≃ 0 at ≤ 10′′ from Sgr A*. In this case, we find that dN/dP orb is approximately flat. If indeed there are ∼1000 radio pulsars with P orb uniformly distributed over 1−100 yr, then there may be ∼100 such pulsars with P orb ≤ 10 yr.

3. PULSAR DETECTION

Continuing with the steady-state model discussed above, we use the statistical properties of the observed Galactic pulsar population as a guide to estimating the detectable fraction of pulsars near Sgr A*. Many pulsar surveys have been conducted at observing frequencies of ν ∼ 400 MHz, where the pulse-averaged monochromatic luminosities range over 1−104 mJy/kpc2, and approximately follow a distribution dN/dL ∝ L−2 (e.g., Lyne, Manchester, & Taylor 1983; Taylor, Manchester, & Lyne 1993; Lyne et al. 1998). Here we define L as the 400-MHz luminosity. The luminosity at some other frequency, ν, is typically estimated to be ∼D2Sν (e.g., Taylor & Manchester 1977), where Sν is the pulse-averaged flux density, and D is the distance. The spectrum of most normal pulsars is well fit by a single power law, Sν ∝ ν−α, so that L ∼ 64S4 ν/(400 MHz)ν for D = 8 kpc. Measured spectral indices range from α ≃ 0 to ∼ 4.0, and are distributed roughly as a Gaussian, with a mean of ⟨α⟩ ≃ 1.5−2, and a standard deviation of σα ∼ 0.5−1 (Lorimer et al. 1995; Maron et al. 2000).

There is no clear correlation between L and α. An important quantitative attribute of the observed α distribution is that ∼20% of pulsars have shallow spectra with α < 1. We have so far referred only to the average pulsar flux. However, a periodicity search is sensitive to the pulsed flux, which may be greatly reduced due to the severe scattering along the line of sight to Sgr A*.

Scattering of radio waves by fluctuations in the free-electron density causes angular broadening of radio images and temporal smearing of pulsed emission. Lazio & Cordes (1998) find evidence for a region of enhanced scattering within ∼200 pc of Sgr A*. Observed scattering diameters of θs ∼ 1′′(ν/GHz)−2 for Sgr A* and nearby OH masers imply a scattering timescale of 300 s(ν/GHz)−4 (Lazio & Cordes 1998).

Since normal pulsars have pulse periods of Pp ∼ 1 s, this long scattering time prohibits their detection as periodic sources for ν ∼ 1 GHz. However, at frequencies of ≥ 10 GHz, the scattering time is reduced to ≤ 30 ms. Given the declining pulsar spectrum, there is some optimum observing frequency, ν′, that maximizes the pulsed flux. Lazio & Cordes (1997) find ν′ ∼ 7 GHz(α1/2/Pp)−1/4, which is weakly dependent on α. At frequencies ν > ν′, the pulsed fraction of the radio emission is ≃ 1, and the pulse duty cycle, ϵ, approaches its intrinsic value (Cordes & Lazio 1997). In this regime, we may equate the average and pulsed flux densities. Note that ν′ < 6−10.5 GHz for α ∼ 1 and Pp ∼ 0.2−2 s, a period range that contains the vast majority of observed normal pulsars.

The minimum detectable pulsed flux density is given by

\[
S_{\text{min}} \approx C S_{\nu}(\Delta \nu / t_{\text{int}})^{1/2}
\]

for α < 1 (e.g., Dewey et al. 1983), where C ∼ 10 is the signal-to-noise detection threshold, Sν ∼ 20−30 Jy is the noise flux contributed by the telescope and sky, \( \Delta \nu \) is the receiver bandwidth, and tint is the integration time. A bandwidth of \( \Delta \nu \approx 1 \) GHz is feasible at frequencies of ≥ 10 GHz. We then find \( S_{\text{min}} \approx 20−40 \mu Jy(\nu/0.05)^{1/2}(t_{\text{int}}/1 \text{ hr})^{-1/2} \), where \( \nu = 0.05 \) is a typical intrinsic duty cycle for normal pulsars.

We now adopt a specific statistical model of the pulsar population near Sgr A* in order to estimate the fraction, \( F_{\text{det}} \), with pulsed fluxes greater than \( S_{\text{min}} \). The distribution of 400-

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
Model & \((\alpha)\) & \(\sigma_\alpha\) & \(S_{\text{min}}\) & \(\nu\) & \(F_{\text{det}}\) \\
& & & (µJy) & (GHz) & (\%)
\hline
1. & 1.7 & 0.8 & 25 & 10 & 2.43
2. & 1.7 & 0.8 & 25 & 15 & 2.00
3. & 1.7 & 0.8 & 25 & 20 & 1.76
4. & 1.7 & 0.8 & 50 & 10 & 1.20
5. & 1.7 & 0.8 & 50 & 15 & 0.98
6. & 1.7 & 0.8 & 50 & 20 & 0.88
7. & 1.7 & 0.8 & 100 & 10 & 0.60
8. & 1.7 & 0.8 & 100 & 15 & 0.50
9. & 1.7 & 0.8 & 100 & 20 & 0.44
\hline
\end{tabular}
\caption{Detectable fraction of pulsars orbiting Sgr A*}
\end{table}

3 These values of \( S_{\nu} \) are appropriate for the 100-m Robert C. Byrd Green Bank Telescope (http://www.gb.nrao.edu/GBT/).
MHz luminosities is taken to be \( dN/dL \propto L^{-2} \). We apply a truncated Gaussian distribution of spectral indices, where \( dN/\alpha \propto \exp[-(\alpha - \bar{\alpha})^2/2\sigma_\alpha^2] \) for \( \alpha > 0 \), and \( dN/dL = 0 \) for \( \alpha < 0 \). If we choose \( \bar{\alpha} = 1.7 \) and \( \sigma_\alpha = 0.8 \), the observed distribution is approximately reproduced, including the \( \sim 20\% \) of pulsars with \( \alpha < 1 \). The appropriate convolution of these distributions was integrated numerically to determine \( F_{\text{det}} \).

Table 1 lists \( F_{\text{det}} \) for several frequencies and flux limits. We first point out that, for a given \( S_{\text{min}} \), \( F_{\text{det}} \) changes by a factor of only \( \sim 0.7 \) as \( \nu \) increases from 10 to 20 GHz. This is due to the significant fraction of pulsars with relatively small spectral slope, thus highlighting an important selection effect for high-frequency pulsar surveys. From Table 1, we see that perhaps \( \sim 1\% \) of the pulsars orbiting Sgr A* have flux densities greater than \( 25–100 \)\( \mu \)Jy at \( \nu \sim 10\) GHz. If as many as several thousand pulsars orbit Sgr A* with periods of \( \lesssim 100\) yr, then up to a few tens may be sufficiently bright to be detectable with current telescopes. Of course, only \( \sim 20\% \) of the pulsars near Sgr A* are expected to be beamed toward the Sun (e.g., Lyne & Manchester 1988), reducing the potentially detectable number to order of unity. The planned Square-Kilometer Array (SKA) will easily achieve sensitivities of \( \lesssim 1\)\( \mu \)Jy, in which case we find that \( F_{\text{det}} \sim 25\% \) at 10 GHz, and perhaps hundreds orbiting pulsars could be found.

4. PULSAR TIMING AND RELATIVITY

The dynamics of a pulsar orbiting Sgr A* are revealed through analysis of the pulse arrival times. The Newtonian dynamical signature of the supermassive BH, via the acceleration of the pulsar, should be clearly evident in a even small segment of the orbit. After monitoring the pulsar for a full orbital period, a best-fit Keplerian timing model is obtained. Residuals of the Keplerian solution contain further dynamical information about the system, including the effects of relativistic gravity. The degree to which different contributions to the timing residuals can be resolved depends on the precision and number of measured arrival times.

Typical arrival-time precisions are \( \delta t \sim (10^{-3} - 10^{-2})P_p \), or \( \sim 1–10\) ms for \( P_p \simeq 1\) s. After the measurement of \( N \) pulse arrival times, the root-mean-square timing precision is expected to be \( \sim \delta t/\sqrt{N} \). Generally, one average arrival time is measured each day that the pulsar is observed. If \( P_{\text{orb}} \gtrsim 1\) yr, then \( \sim 10–100 \) arrival times will be measured per orbit, so that \( \delta t/\sqrt{N} \lesssim 1\) ms. This precision may be sufficient to model certain timing residuals due to strong-field relativistic gravity in the pulsar-Sgr A* system.

The deterministic spin-down behavior of most radio pulsars is well described by the polynomial \( \phi = \nu_p T + \nu_p T^3/2 + \nu_p T^3/6 \), where \( \nu_p = 1/P_p \) is the spin frequency, \( T \) is the phase of pulse emission, and \( T \) is the corresponding proper time in the frame of the pulsar. Residuals of the polynomial fit are contributed by stochastic "timing noise," and perhaps pulsar "glitches" (e.g., Arzoumanian et al. 1994, and references therein). Each pulse arrival time, \( t_i \), at the solar-system barycenter is related to \( T \) by \( t_i - t_0 = T + \sum \Delta_i \), where \( t_0 \) is a reference time, and the \( \Delta_i \) are variable delays resulting from the motion of the pulsar. The delays are functions of time and the Keplerian parameters that describe the shape and orientation of the orbit. For the pulsar-Sgr A* system, the parameters to which pulsar timing is sensitive are: (i) BH mass, \( M_{\text{BH}} \), (ii) semimajor axis, \( a \), or orbital period, \( P_{\text{orb}} \); (iii) eccentricity, \( e \); (iv) inclination angle, \( i \); and (v) longitude of pericenter, \( \omega \). When the effects of frame-dragging are considered, we must also specify the magnitude and direction of the BH spin. Given the full set of parameters that describe the pulsar-Sgr A* system, we characterize each delay by an amplitude, the difference between the maximum and minimum values of \( \Delta_i \), and a width, the timescale over which the delay shows the largest variation.

We list below a number of important delays, and roughly estimate their respective amplitudes and widths in the special case where \( \omega = 90^\circ \), so that superior conjunction—when the pulsar is farthest behind the BH—coincides with pericenter passage. This is the most analytically tractable configuration, and also illustrates the dependence of the amplitude and width on the eccentricity. Numerical estimates below are expressed in terms of the dimensionless variables \( M_{\text{BH}} = M_{\text{BH}}/3.5 \times 10^6 M_\odot \), \( P_{\text{orb}} = P_{\text{orb}}/1\) yr, and \( P_p = P_p/1\) s. We do not investigate whether different delays are separately

![Figure 1](http://www.skatelescope.org/)
measurable, or what parameters can be extracted from the timing data for realistic signal-to-noise values; this would require detailed simulations.

The standard timing model presently in use for binary pulsars includes the following arrival-time delays, for which the complete, (implicitly) time-dependent expressions can be found in Damour & Taylor (1992):

- **Roemer Delay (ΔR).**—A delay that measures the light-travel time across a projection of the semimajor axis, with an amplitude of \( \sin i/c \approx 1 \text{ day} \frac{M_{\text{BH}}^{3/2}}{M_{\odot}^{1/2}} \sin i \), and a width of \( \sim 1 \text{ yr} P_{\text{orb}} \).

- **Einstein Delay (ΔE).**—Combined effect of time dilation and the gravitational redshift, with an amplitude of \( \sim 1 \text{ hr} \frac{M_{\text{BH}}^{3/2}}{M_{\odot}^{1/2}} \), and a width of \( \sim 1 \text{ yr} P_{\text{orb}} \). The upper-left panel of Fig. 1 shows \( ΔE \) as a function of time for \( M_{\text{BH}} = 1 \), \( P_{\text{orb}} = 10 \), \( e = 0.9 \), \( ω = 90^\circ \), and \( i = 80^\circ \).

- **First-Order Shapiro Delay (ΔS1).**—The lowest-order relativistic propagation delay in the gravitational field of a point mass. The amplitude is \( \sim 30 \text{ s} M_{\text{BH}} \ln (1 - e) (1 - \sin i) \). The delay rises sharply near superior conjunction, with a width of \( \sim 1 \text{ yr} P_{\text{orb}} (1 - e)^{1/2} \sin i \) when \( 1 - \sin i \ll 1 \), which is easily verified for \( e = 0 \). Measurement of \( ΔS_1 \) allows the inclination of the orbit to be determined. The time dependence of \( ΔS_1 \) is shown in the lower-left panel of Fig. 1.

- **Aberration Delay (ΔA).**—Special relativistic aberration of the beamed pulsar emission contributes a small modulation in the measured arrival times over the orbital period, so that the width is \( \sim 1 \text{ yr} P_{\text{orb}} \). In general, the delay depends on the direction of the pulsar spin axis. The maximum expected amplitude is \( \sim 1 \text{ ms} P_p \frac{M_{\text{BH}}^{3/2}}{M_{\odot}^{1/2}} \sin i (1 - e)^{1/2} \).

If the pulsar orbit is sufficiently compact, and its orientation is favorable, more subtle post-Keplerian timing effects may be measurable, including those contributed by strong-field relativistic gravity. Potentially important higher-order delays are:

- **Second-order Shapiro Delay (ΔS2).**—The next highest order contribution to the net gravitational propagation delay that is independent of the BH spin (Dvynnikova 1986, Goicoechea et al. 1992), with an amplitude of \( \sim 0.1 \text{ s} \frac{M_{\text{BH}}^{5/3}}{M_{\odot}^{2/3}} (1 - e)^{1/2} \sin i \) and a width of \( \sim 1 \text{ yr} P_{\text{orb}} (1 - e)^{1/2} \sin i \) when \( 1 - \sin i \ll 1 \). The lower-right panel of Fig. 1 illustrates the time dependence of \( ΔS_2 \).

- **Frame-Dragging Delay (ΔFD).**—Frame dragging due to the spin of the black hole introduces an additional delay that depends on the orientation of the spin vector (Dvynnikova 1986, Goicoechea et al. 1992, Laguna & Wolszczan 1997, Wex & Kopeikin 1999). The amplitude and width are \( \sim 0.1 \text{ s} \frac{M_{\text{BH}}^{5/3}}{M_{\odot}^{2/3}} (1 - e)^{1/2} \sin i \) and \( \sim 1 \text{ yr} P_{\text{orb}} (1 - e)^{1/2} \sin i \). Here \( 0 < χ < 1 \) is the dimensionless spin parameter, where \( χ = 1 \) corresponds to an extreme Kerr BH. The full expression for \( ΔFD \) (Wex & Kopeikin 1999) depends on the projection of the BH spin on the plane of the sky. The upper-right panel of Fig. 1 shows the time dependence of \( ΔFD \) for the case where \( χ = 1 \) and the BH spin is parallel to the orbital angular momentum of the pulsar.

- **Shklovskii Delay (ΔSH).**—A delay due to the variation in the line-of-sight direction to the pulsar as it moves in its orbit (Shklovskii 1970). The amplitude is \( \sim a^2/cD \), where \( D = 8 \text{ kpc} \) is the distance to Sgr A*, and the width is \( \sim 1 \text{ yr} P_{\text{orb}} \). Numerically, the amplitude is \( \sim 10 \text{ ms} \frac{M_{\text{BH}}^{3/2}}{M_{\odot}^{1/2}} P_{\text{orb}}^{2/3} \).

- **Bending Delay (ΔB).**—As the pulsar orbits, light rays that reach the observer have been deflected by varying amounts, causing a modulation in the measured pulse period (Doroshenko & Kopeikin 1993, Wex & Kopeikin 1999). The amplitude and width are \( \sim 0.1 \text{ ms} P_p \frac{M_{\text{BH}}^{2/3}}{M_{\odot}^{1/2}} (1 - e)^{1/2} \cos i \) and \( \sim 1 \text{ yr} P_{\text{orb}} (1 - e)^{1/2} \cos i \).

In addition to the those timing residuals that can be measured in a single orbit with sufficient timing precision, there are several effects that cause secular variations of the orbital parameters and pulse profile. These include:

- **Pericenter Precession.**—Relativistic gravity causes the precession of the longitude of pericenter, \( ω \) (e.g., Jaroszynski 1998; Wex & Kopeikin 1999, Rubilar & Eckart 2001). In the Schwarzschild spacetime, the prograde advance per orbital period is \( \sim 0.25 \text{ yr} \frac{M_{\text{BH}}^{2/3}}{M_{\odot}^{1/2}} (1 - e)^{1/2} \). The Kerr metric, via frame dragging, contributes a (possibly retrograde) precession rate of \( \sim 0.25 \text{ yr} \frac{M_{\text{BH}}^{2/3}}{M_{\odot}^{1/2}} (1 - e)^{1/2} \chi \cos ψ \) per orbit, where \( ψ \) is the angle between the angular momentum vectors of the BH and the orbit. The frame-dragging contribution to pericenter precession is sensitive to the component of the BH spin parallel to the orbital angular momentum.

- **Geodetic Precession.**—The geodetic precession rate of the pulsar spin axis is \( \sim 0.12 \text{ yr} \frac{M_{\text{BH}}^{2/3}}{M_{\odot}^{1/2}} (1 - e)^{1/2} \) per orbit, which would be evident as a secular change of the pulse profile (Weisberg, Romani, & Taylor 1989).

- **Gravitational Radiation.**—A binary comprised of two point masses loses energy and contracts as a result of the gravitational radiation reaction. For the pulsar-Sgr A* system, where the pulsar mass is \( \sim 1.4 M_{\odot} \), the timescale for contraction is \( \sim 10^{-3} \text{ yr} \frac{M_{\text{BH}}^{2/3}}{M_{\odot}^{1/2}} P_{\text{orb}}^{2/3} (1 - e)^{7/2} \) (e.g., Taylor & Weisberg 1989), which is too long to be of interest.

5. DISCUSSION

Here we address three additional topics pertaining to observations of a radio pulsar orbiting Sgr A*. We first discuss how pulsar timing can be used to probe the physics of the accretion flow onto the BH. This is followed by a short investigation of the effects of gravitational interactions between an orbiting pulsar and the surrounding cluster of stars and remnants. Finally, we consider the prospects of imaging an orbiting pulsar, and what new information this may provide.

5.1. The Interstellar Plasma Around Sgr A*

Timing observations of a pulsar orbiting Sgr A* can be used to derive the properties of the local interstellar plasma. As the pulsar moves, photons trace many different lines of sight through the plasma to the observer. A gradient in the free-electron density on the scale of the pulsar orbit introduces dispersive and refractive pulse arrival-time delays that vary over the orbital period. Each of these effects has a characteristic frequency dependence, pointing to a need for multi-frequency observations. It is interesting to note that if a pulsar is detected at 10 GHz there may be a good chance of detecting it at 15–20 GHz for the same sensitivity. This is because of the selection effect favoring the detection of shallow-spectrum pulsars at high frequencies (see § 3).
Chandra observations of the Galactic center (Baganoff et al. 2003) indicate that the density and temperature of the electrons at $\approx 8000$ AU from Sgr A* are $n_e \sim 100$ cm$^{-3}$ and $kT_e \sim 1$ keV. The gravitational potential of the BH exceeds the thermal energy of the plasma inside the Bondi (1952) radius, $R_B \approx GM_{BH}/c^2$, where $c$ is the speed of light and $M_{BH}$ is the mass of the BH. The electron density and temperature of the electrons at $1600$ AU, or $R_B \approx 500$ AU, is the thermal speed, assuming equipartition between electrons and protons, and $n_p$ is the proton mass. For the quoted temperatures, $R_B \approx 8000$ AU, or $R_B \sim 10^6$ in projection. The electron density profile at radii $r < R_B$ depends on the physics of the accretion flow. We adopt $n_e(r) \sim 10^2$ cm$^{-3}$ (10$^4$ AU)/$r^3$, scaled to the conditions at $1^\circ$. Various models of the accretion flow predict values of $\beta \sim 1.5$ (e.g., Melia & Falcke 2001; Yuan, Quataert, & Narayan 2003).

The plasma frequency is $\nu_p = (n_e e^2/\varepsilon_0 m_e)^{1/2} \sim 90$ kHz (10$^4$ AU)/$r^3$, for the density profile given above. The corresponding index of refraction of the plasma is $\xi(\nu) \sim 1 - (\nu_p/\nu)^{3/2}$, where $\nu$ is the observing frequency. Radiation at different frequencies propagates through the medium with different group velocities equal to $c\xi(\nu)$, so that the arrival time of a pulse is frequency dependent. The difference in arrival times at frequencies $\nu_1$ and $\nu_2$ is proportional to $(\nu_1^2 - \nu_2^2)\Delta \nu$. When $\Delta \nu$ is large, the dispersion measure $DM$ is the dispersion measure measured with the phasors of free electrons along the path of the pulse. Modulation of $DM$ over the orbital period of a pulsar bound to Sgr A* can thus be used to constrain the density profile of the plasma within the orbit.

Refraction due to the large-scale gradient of the electron density causes a net angular deflection of individual pulses that reach the observer. Consequently, there will be a variable geometrical time delay as the pulsar orbits. We assume that a light ray is refracted impulsively as it passes its closest approach to the BH. As seen by an observer, the angle of refraction is given by $\theta_\mathrm{ref} = \beta |\nu_p(r = b)|/\nu^2 [\theta]$ (pointing away from Sgr A*). From the thin-lens geometry, the resulting excess propagation time, compared to that of a straight-line path, is $\sim \theta_\mathrm{ref}^2 / 2c \propto \nu^{-4}$ in the limit where the orbital radius, $r$, is much less than the distance to Sgr A*. For an orbital period of $P_{\text{orb}} \approx 10$ yr, inclination of $i = 80^\circ$, and eccentricity of $e = 0.9$, the minimum possible impact parameter $a = (1 - e) \cos i \approx 10$ AU, where $\nu_p(10$ AU) $\approx 16$ MHz for $\beta = 3/2$. The corresponding amplitude and width of the refractive delay are, respectively, $\sim 0.1$ s and $\sim 10$ days, comparable to what is expected for the second-order Shapiro delay and frame-dragging delay (see Fig. 1). However, the strong frequency dependence of the refractive delay distinguishes it from dynamical effects.

5.2. Dynamics of the Sgr A* Cluster

An important prediction of our work is that thousands of NSs, born over the past $\sim 10^8$ yr, may orbit Sgr A* with periods of $\lesssim 100$ yr. A significant number of white dwarfs and stellar-mass BHs may also orbit Sgr A*. For a given member of this bound cluster, many weak gravitational perturbations accumulate over a relaxation time to yield a significant net change in the orbital parameters. If we neglect resonant effects, the relaxation time is $\tau_{\text{rel}} \sim \eta N_e^{-1}(M_{BH}/M)^2/\eta^2 P_{\text{orb}}$, where $M$ and $N_e$ are the typical mass and total number, respectively, of the perturbing stars, and $\eta$ is a dimensionless factor proportional to the Coulomb logarithm (e.g., Rauch & Tremaine 1996). For a stellar density $n(r) \propto r^{-2}$, Rauch & Tremaine (1996) find that $\eta \lesssim 10$ when $r$ ranges over one decade. These assumptions are adequate for our purposes, and so we adopt $\eta = 0.1$. If $M_{BH}/M = 10^6$ and $N_e = 10^5$, we find $\tau_{\text{rel}} \approx (10^{-3} - 10^{-4}) P_{\text{orb}}$. Resonant angular-momentum relaxation (Rauch & Tremaine 1996), which causes variation in the orbital eccentricity and orientation angles, can act on a much shorter timescale of $\tau_{\text{rel},\text{res}} \approx (M_{BH}/M)P_{\text{orb}} \sim 10^6 P_{\text{orb}}$.

Random-walk fluctuations in the orbital energy and angular momentum lead to a root-mean-square fractional change in one orbit of approximately $(P_{\text{orb}}/r)^{1/2} \sim 10^{-3} - 10^{-4}$, where $r$ is either the conventional or resonant relaxation time. In particular, the resulting absolute change in the semimajor axis may be $\sim 100$–1000 light seconds for $P_{\text{orb}} \sim 10$ yr. Gravitational perturbations may have measurable consequences for the timing analysis of a pulsar orbiting Sgr A*, possibly even masking the important relativistic time delays discussed in the last section. These effects could be examined more carefully with $N$-body simulations.

5.3. Radio Imaging and Astrometry

Because the total flux of a pulsar is unaffected by scattering, Cordes & Lazio (1997) advocate that radio imaging surveys at conventional frequencies of $\approx 1$–5 GHz may be more successful than periodicity searches in discovering heavily scattered pulsars. A handful of pulsar candidates beyond several arcminutes from Sgr A* have been identified in this way (Lazio & Cordes 1998a, 2000). However, it would be extremely challenging to image a pulsar orbiting the Galactic BH with $P_{\text{orb}} \lesssim 10$ yr, which would have a projected semimajor axis of $\lesssim 0.1$ AU. Very long baseline interferometry (VLBI) would be required to resolve these faint sources next to the very bright Sgr A* (e.g., 1Jy over 1–10 GHz; Melia & Falcke 2001). Moreover, these observations must be conducted at frequencies of $\gtrsim 5$–10 GHz in order to sufficiently reduce the scattering diameter of Sgr A* (see §3). With a current fiducial VLBI sensitivity of $\gtrsim 1$ mJy, the analysis of §3 predicts that $\lesssim 1\%$ of the orbiting pulsars would be detectable, making imaging and astrometry unlikely prospects. Nonetheless, improvements in technology (e.g., the SKA) may make this approach possible in the future, and so we consider the rewards.

An additional orbital parameter accessible by astrometry is the angle of the line of nodes, $\Omega$, on the plane of the sky. If the orbit is inclined with respect to the equatorial plane of the spinning BH, frame dragging causes $\Omega$ to precess at a rate of $\sim 9\eta_p P_{\text{orb}} P_{\text{orb}}^{-1}(1 - e^2)^{-3/2} \sin\psi$ per orbit (Jaroszynski 1998a). Here $\psi$ is the projection of the dimensionless spin onto the orbital plane. If the contributions from frame dragging to pericenter (see §3) and nodal precession are measurable, it would be possible to determine the magnitude and direction of the BH spin.

The effects of gravitational lensing by the BH may be important when the pulsar is near superior conjunction. For lensing by a point mass, two images are produced, one inside and one outside the Einstein radius, $\theta_E \approx (4GM_{BH}/c^2 D)^{1/2}$, where $r = D$ is the orbit radius near superior conjunction, and $D \approx 8$ kpc is the distance. For $M_{BH} = 3.5 \times 10^6 M_\odot$ and $r = 100–1000$ AU, we find $\theta_E \approx 0.5$–1.5 mas. Since $\theta_E$ is very small, we consider here only the primary image outside the Einstein radius. When the pulsar is at superior conjunction, the angular separation between the source and the primary image is $\delta \omega \approx \theta_E D/b$, where $b = r \cos i$ is the impact parameter (e.g., Jaroszynski 1998a). We see that $r$ cancels, and...
$\delta \phi \sim 20 \mu \text{as}/\cos i$, which is only $\sim 1 \text{ mas}$ for $i = 89^\circ$. Therefore, gravitational lensing has a very small impact on astrometric measurements of a pulsar orbiting Sgr A*.

We thank D. Chakrabarty, B. Gaensler, S. Hughes, S. Rappaport, and M. Reid for useful discussions. We are especially grateful to F. Camilo and V. Kaspi for reading an early draft of the paper and providing valuable comments. EP was supported by NASA and the Chandra Postdoctoral Fellowship program through grant number PF2-30024. AL acknowledges support from the John Simon Guggenheim Memorial Fellowship, as well as NSF grants AST-0071019 and AST-0204514, and NASA grant NAG5-13292.

REFERENCES

Arzoumanian, Z., Nice, D. J., Taylor, J. H., & Thorsett, S. E. 1994, ApJ, 422, 671
Arzoumanian, Z., Chernoff, D. F., & Cordes, J. M. 2002, ApJ, 568, 289
Baganoff, F. K. et al. 2003, ApJ, 591, 891
Bhattacharya, D., Vijers, R. A. M. J., Hartman, J. W., & Verbunt, F. 1992, A&A, 254, 198
Blandford, R. & Narayan, R. 1986, ApJ, 310, 568
Bondi, H. 1952, MNRAS, 112, 195
Cordes, J. M. & Lazio, T. J. W. 1997, ApJ, 475, 557
Damour, T. & Taylor, J. H. 1992, Phys. Rev. D, 45, 1840
Dewey, R. J., Taylor, J. H., Weisberg, J. M., & Stokes, G. H. 1985, ApJ, 294, L25
Doroshenko, O. V. & Kopeikin, S. M. 1995, MNRAS, 274, 1029
Dymnikova, I. G. 1986, IAU Symp. 114: Relativity in Celestial Mechanics and Astrometry. High Precision Dynamical Theories and Observational Verifications, 114, 411
Eckart, A., Ott, T., & Genzel, R. 1999, A&A, 352, L22
Eisenhauer, F. et al. 2003, ApJL, submitted, astro-ph/0306220
Figer, D. F. et al. 2000, ApJ, 533, L49
Genzel, R., Eckart, A., Ott, T., & Eisenhauer, F. 1997, MNRAS, 291, 219
Genzel, R. et al. 2003, ApJ, submitted, astro-ph/0304197
Gezari, S., Ghez, A. M., Becklin, E. E., Larkin, J., McLean, I. S., & Morris, M. 2002, ApJ, 576, 790
Ghez, A. M. et al. 2003a, ApJ, 586, L127
Ghez, A. M. et al. 2003b, ApJ, submitted, astro-ph/0306130
Goicoechea, L. J., Mediavilla, E., Buitrago, J., & Atro, F. 1992, MNRAS, 259, 281
Hansen, B. M. S. & Phinney, E. S. 1997, MNRAS, 291, 569
Jaroszyński, M. 1998a, Acta Astronomica, 48, 413
Jaroszyński, M. 1998b, Acta Astronomica, 48, 653
Lazio, T. J. W. & Cordes, J. M. 1998a, ApJS, 118, 201
Lazio, T. J. W. & Cordes, J. M. 1998b, ApJ, 505, 715
Lazio, T. J. W. & Cordes, J. M. 2000, ASP Conf. Ser. 202: IAU Colloq. 177: Pulsar Astronomy - 2000 and Beyond, 39
Lorimer, D. R., Yates, J. A., Lyne, A. G., & Gould, D. M. 1995, MNRAS, 273, 411
Lyne, A. G., Manchester, R. N., & Taylor, J. H. 1985, MNRAS, 213, 613
Lyne, A. G. & Manchester, R. N. 1988, MNRAS, 234, 477
Lyne, A. G. et al. 1998, MNRAS, 295, 743
Maoz, E. 1998, ApJ, 494, L181
Maron, O., Kijak, J., Kramer, M., & Wielebinski, R. 2000, A&A, 147, 195
Melia, F. & Falcke, H. 2001, ARA&A, 39, 309
Mezger, P. G., Zylka, R., Philipp, S., & Launhardt, R. 1999, A&A, 348, 457
Paczynski, B. & Trimble, V. 1979, in The Large-Scale Characteristics of the Galaxy, ed. W. B. Burton (Dordrecht: Reidel), 401
Pfahl, E., Rappaport, S., & Podsiadlowski, Ph. 2002, ApJ, 571, L37
Rauch, K. P. & Tremaine, S. 1996, New Astronomy, 1, 149
Rudak, B. & Ritter, H. 1994, MNRAS, 267, 513
Schödel, R. et al. 2002, Nature, 419, 694
Schödel, R. et al. 2003, ApJ, submitted, astro-ph/0306214
Shklovskii, I. S. 1970, Soviet Astronomy, 13, 562
Tauris, T. M. & Konar, S. 2001, A&A, 376, 543
Taylor, J. H. & Manchester, R. N. 1977, ApJ, 215, 885
Taylor, J. H. & Weisberg, J. M. 1989, ApJ, 345, 434
Taylor, J. H., Manchester, R. N., & Lyne, A. G. 1993, ApJS, 88, 529
Weisberg, J. M., Romani, R. W., & Taylor, J. H. 1989, ApJ, 347, 1030
Wex, N. & Kopeikin, S. M. 1999, ApJ, 514, 388
Yuan, F., Quataert, E., & Narayan, R. 2003, ApJ, in press, astro-ph/0304125