Pion cloud effects on baryon masses

Hèlios Sanchis-Alepuz*, Christian S. Fischer, Stanislav Kubrak

*Institute of Theoretical Physics, Justus-Liebig University of Gießen, Heinrich-Buff-Ring 16, 35392, Gießen, Germany

Abstract

In this work we explore the effect of pion cloud contributions to the mass of the nucleon and the delta baryon. To this end we solve a coupled system of Dyson-Schwinger equations for the quark propagator, a Bethe-Salpeter equation for the pion and a three-body Faddeev equation for the baryons. In the quark-gluon interaction we explicitly resolve the term responsible for the back-coupling of the pion onto the quark, representing rainbow-ladder like pion cloud effects in bound states. We study the dependence of the resulting baryon masses on the current quark mass and discuss the internal structure of the baryons in terms of a partial wave decomposition. We furthermore determine values for the nucleon and delta sigma-terms.

Keywords: pion cloud effects, nucleon sigma term, Faddeev equations

1. Introduction

The application of continuum functional methods to hadron physics phenomenology aims at the calculation of hadronic properties using the elementary degrees of freedom of Quantum Chromodynamics (QCD). In this framework mesons and baryons are considered as bound states of quarks and, hence, described by two-body Bethe-Salpeter equations (BSEs) and three-body Faddeev equations. These equations rely upon the knowledge of several QCD’s Green’s functions which are in turn solutions of Dyson-Schwinger equations (DSEs). The approach has the advantage that the origin of physical observables can be understood from the microscopic dynamics of quarks and gluons. Moreover, it is Poincaré covariant and is applicable at any momentum range.

As is well known, however, it is impossible to carry out this program exactly and truncations of both the DSEs and the bound state equations must be defined. The simplest one consistent with Poincaré covariance as well as constraints from chiral symmetry is the Rainbow-Ladder truncation (RL). Approximations of this kind have been extensively used in meson and baryon calculations (see e.g. [1, 2] for overviews) and turns out to be rather successful in reproducing, for example, ground-state meson and baryon masses in selected channels, elastic and inelastic form factors and PDAs.

There are, however, also severe limitations to the rainbow-ladder scheme. Consequently, much work has been invested in the past years on its extension towards more advanced approximations of the quark-gluon interaction. On the one hand, this may be accomplished directly by devising improved ansätze for the dressing functions of the quark-gluon vertex [3, 4, 5, 6]. On the other hand, it is promising to work with diagrammatic approximations to the vertex DSE. While most studies so far concentrated on (1/Nc-subleading) Abelian contributions to the vertex (see e.g. [7, 8, 9, 10, 11]), the impact of the 1/Nc-leading, non-Abelian diagram on light meson masses has been investigated in [12]. In addition, important unquenching effects in the quark-gluon interaction can be approximated by the inclusion of hadronic degrees of freedom [13, 14, 15]. This is possible, since the vertex DSE can be decomposed diagrammatically into contributions stemming from Yang-Mills theory and those involving quark-loops. To leading order in the bound state mass, the latter ones can be expressed in terms of (off-shell) pion exchange between quarks. In turn, these pions are not elementary fields, but need to be determined consistently e.g. from their Bethe-Salpeter equation.

Having explicit hadronic degrees of freedom in the system may also be very beneficial for phenomenological applications of the approach. Although beyond the scope of this work, pionic effects are expected to play an important role in form factors [16, 17, 18, 19] and may play an important role in the calculation of hadronic decay processes. The influence of pion back-coupling effects in the mass and decay constants of the pion itself and other light mesons was studied in [15]. The (gauge invariant) inclusion of pion-cloud effects in the calculation of elastic pion form factors has been achieved in [20]. In the present work, we take one step further and extend this framework to the covariant three-body calculations of nucleon and delta masses [21, 22, 23]. Our results, together with those presented in [20], set the stage for the study of pion-cloud effects in baryon’s form factors, which are expected to have

*helios.sanchis-alepuz@theo.physik.uni-giessen.de

Preprint submitted to Physics Letters B

January 15, 2014
a significant impact at low momentum transfer [16, 17, 18].

This letter is organized as follows: in Section 2 we review the main elements of the DSE/BSE framework and define the truncations and model used in this work. We present and discuss the results of our calculations in Section 3. Finally, some concluding remarks are made in Section 4.

2. Covariant three-body equation

The mass and internal structure of baryons are given, in a covariant Faddeev approach, by the solutions of the three-body equation

\[
\Psi = -i\tilde{\Gamma}^{(3)}G_0^{(3)}\Psi + \sum_{a=1}^{3} -i\tilde{\Gamma}^{(2)}(a)G_0^{(3)}\Psi ,
\]

where \(\tilde{\Gamma}^{(3)}\) and \(\tilde{\Gamma}^{(2)}\) are the three- and two-body interaction kernels, respectively, and \(G_0\) represents the product of three fully-dressed quark propagators \(S\). We used here a compact notation where indices have been omitted and we assume that discrete and continuous variables are summed or integrated over, respectively. The spin-momentum part of the full amplitude \(\Psi\) depends on the total and two relative momenta of the three valence quarks inside the baryon. As discussed in more detail in Section 3.2, this amplitude contains all possible spin and orbital angular momentum contributions.

The quark propagators are obtained from their respective DSE

\[
S^{-1}(p) = S_0^{-1}(p) + Z_{1f}\int_q \Gamma^{\mu}_{qq\,q}(p-q)\Gamma^{\nu}_{qq\,q}(p,q)S(q) ,
\]

where the integration over the four-momentum \(q\) is abbreviated by \(\int_q \equiv \int dq/(2\pi)^4\), \(S_0\) is the (renormalized) bare propagator \(S_0^{-1}(p) = Z_2(i\not{p} + m)\), \(\Gamma^{\mu}_{qq\,q}\) is the full quark-gluon vertex with its bare counterpart \(\Gamma^{\mu}_{qq\,q,0}\). \(D^{\mu\nu}\) is the full gluon propagator and \(Z_{1f}\) and \(Z_2\) are renormalization constants.

To solve the system formed by equations (1) and (2) one needs to know the interaction kernels and the full quark-gluon vertex. The latter could in principle be obtained from the infinite system of coupled DSEs of QCD, of which (2) is also a member. In practice, however, this system has to be truncated into something manageable, which implies that educated ansätze have to be used for the Green’s functions one is not solving for. The interaction kernels, in contrast, do not appear directly in the system of QCD’s DSEs. However, a connection of those with the two-body interaction kernel can be established via the axial-vector Ward-Takahashi identity, which ensures the correct implementation of chiral symmetry in the bound state equations [24, 25]. This identity imposes constraints on the structure of the interaction kernel by relating it to the (truncated) quark-gluon vertex.

2.1. Rainbow-Ladder truncation

The simplest and most commonly used ansätze for the quark-gluon and quark-quark interactions is the Rainbow-Ladder (RL) truncation. Here, only the tree-level flavor, color and Lorentz structures are kept for the quark-gluon vertex, so that the quark DSE reads

\[
S_{\beta\beta}^{-1}(p) = S_{0,\alpha\beta}^{-1}(p) + \int_q \tilde{K}_{\alpha\alpha'\beta\beta'}(k)S_{\alpha'\beta'}^{-1}(q) ,
\]

with \(k = p - q\) and

\[
\tilde{K}_{\alpha\alpha'\beta\beta'}(k) = -4\pi\alpha_s(k^2)T_{\mu\nu}(k)\ gamma^\mu_{\alpha\alpha'}gamma^\nu_{\beta\beta'} ,\]

being \(Z_2\) the quark renormalization constant, \(T_{\mu\nu}(k)\) the transverse projector

\[
T_{\mu\nu}(k) = \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} .
\]

and \(C\) a color factor which, in this case, takes the value \(C = 4/3\). The effective coupling \(\alpha_{\text{eff}}\) combines the non-perturbative dressing of the gluon propagator and the \(\gamma_\mu\)-structure of the vertex. At large momenta, it is constrained by perturbation theory, whereas at low momenta we have to supply a model. In this work we use the model proposed in [26, 27]

\[
\alpha_{\text{eff}}(q^2) = \pi \eta^2 \left(\frac{q^2}{\Lambda^2}\right)^2 e^{-\eta^2 \frac{q^2}{\Lambda^2}} + \frac{2\gamma_m(1 - e^{-q^2/\Lambda^2})}{\ln[e^2 - 1 + (1 + q^2/\Lambda_{QCD}^2)]} ,
\]

where for the anomalous dimension we use \(\gamma_m = 12/(11N_C - 2N_f) = 12/25\), corresponding to \(N_f = 4\) flavors and \(N_c = 3\) colors, we fix QCD scale to \(\Lambda_{QCD} = 0.234\) GeV and the scale \(\Lambda = 1\) GeV is introduced for technical reasons and has no impact on the results. The interaction strength is characterized by an energy scale \(\Lambda\) and the dimensionless parameter \(\eta\) controls the width of the interaction. They have to be fixed to reproduce some observable (see Section 3). Other choices for the effective coupling such as the one proposed in [28] do not change our results. In order to have chiral symmetry in the chiral limit and enable dynamical chiral-symmetry breaking, the quark-antiquark kernel has to be given also by (5).

For consistency of the approach, we use also (5) with \(C = -2/3\) for the quark-quark kernel in the three-body equation. Moreover, we neglect three-body irreducible interactions. The three-body BSE then reduces to

\[
\Psi_{\alpha\beta\gamma\tau}(p, q, P) = \int_k \tilde{K}_{\alpha\alpha'\beta'\gamma'\tau'}(k) S_{\beta'\gamma'\tau'}(k_2) S_{\gamma'\tau'}(k_3) \Psi_{\alpha'\beta'\gamma'\tau'}(1, P) + \tilde{K}_{\alpha\alpha'\gamma'\tau'}(-k) S_{\alpha'\gamma'\tau'}(k_3) S_{\alpha'\gamma'}(k_1) \Psi_{\alpha'\beta'\gamma'\tau'}(2, P) + \tilde{K}_{\alpha\beta'\gamma'\tau'}(k) S_{\alpha'\beta'\tau'}(k_1) S_{\beta'\tau'}(k_2) \Psi_{\alpha'\beta'\gamma'\tau'}(3, P) ,
\]
The terms in the Faddeev equation are on the internal quark momenta for numerical convenience. The quark propagators depend with 

\[ p = (1 - \zeta) p_3 - \zeta (p_1 + p_2), \quad p_1 = -q - \frac{p}{2} + \frac{1 - \zeta}{2}, \]

\[ q = \frac{p_2 - p_1}{2}, \quad p_2 = q - \frac{p}{2} + \frac{1 - \zeta}{2}, \quad P = p_1 + p_2 + p_3, \]

\[ p_3 = p + \zeta P, \]

with \( p_1, p_2 \) and \( p_3 \) the quark momenta and \( \zeta \) a free momentum partitioning parameter, which is chosen to be \( \zeta = 1/3 \) for numerical convenience. The quark propagators depend on the internal quark momenta \( k_i = p_i - k \) and \( \bar{k}_i = p_i + k \), with \( k \) the gluon momentum. Similarly, the internal relative momenta \( (j, P) \equiv (p^{(j)}, q^{(j)}, P) \) for each of the three terms in the Faddeev equation are

\[ p^{(1)} = p + k, \quad p^{(2)} = p - k, \quad p^{(3)} = p, \]

\[ q^{(1)} = q - k/2, \quad q^{(2)} = q - k/2, \quad q^{(3)} = q + k. \]

### 2.2. Beyond Rainbow-Ladder: Pion exchange

As we discussed above, in this work we follow [13, 14, 15] and approximate the unquenching effects in the quark-gluon vertex DSE by the inclusion of pionic degrees of freedom. In particular, the vertex is decomposed into a pure Yang-Mills term and a pion exchange contribution.

The exchanged pion is obtained from a self-consistent solution of its BSE and the quark-pion interaction is given by

\[ \bar{K}^{\text{pion}}_{\alpha\alpha'\beta\beta'} (p_1, p_2; P) = \]

\[ \frac{1}{2} \left[ \Gamma^j_{\pi,\alpha\alpha'} \left( \frac{p_1 + p_2}{2}; P \right) \left[ Z_2 \tau^j \gamma_5 \right]_{\beta\beta'} D_\pi (P) \right] + \frac{1}{2} \left[ Z_2 \tau^j \gamma_5 \alpha\alpha' \left[ \Gamma^j_{\pi,\beta\beta'} \left( \frac{p_1 + p_2}{2}; P \right) \right] D_\pi (P), \]

where \( \Gamma^j_{\pi} (p, P) \) is the pion Bethe-Salpeter amplitude, with \( p = (p_1 + p_2)/2 \) and \( P = p_1 - p_2 \) the relative and total momenta, respectively, and \( p_1 \) and \( p_2 \) the incoming and outgoing quark momenta. The pion propagator is given by

\[ D_\pi (P) = \frac{1}{M^2 + P^2} \]

and \( Z_2 \tau^j \gamma_5 \) is the bare pion-quark vertex. The pion Bethe-Salpeter amplitude as obtained from its DSE is given by

\[ \Gamma^j_{\pi} (p; P) = \tau^j \gamma_5 \left[ E_\pi (p; P) - i q F_\pi (p; P) \right] - i q G_\pi (p; P) - \left[ P, \gamma \right] H_\pi (p; P)] \]

In principle, the back-coupling of the pion onto the quark is also governed by this amplitude and thus one needs to solve the coupled system of the quark-DSE and pion-BSE. In order to simplify this tremendous numerical task we only employ the leading amplitude \( E_\pi (p; P) \) for the internal pion and neglect contributions from \( F_\pi, G_\pi \) and \( H_\pi \). From a comparison of the relative size of these amplitudes...
Table 1: Nucleon and Delta masses using the Rainbow-Ladder truncation only (RL1), Rainbow-Ladder with the refitted effective interaction (RL2) and including the Pion-exchange corrections (RL2 + Pi-exch.). As explained in the text, the theoretical error is estimated by the dependence of the results on the variation of $\eta$ between $1.6 \leq \eta \leq 2.0$. We compare also with experimental values.

|       | RL1  | RL2  | RL2 + Pi-exch. | Exp. |
|-------|------|------|----------------|------|
| $N$   | 0.94 (1) | 1.01 (3) | 1.00 (1) | 0.94 |
| $\Delta$ | 1.23 (1) | 1.36 (1) | 1.31 (3) | 1.23 |

Figure 3: (color online). Evolution of the nucleon and delta mass with respect to the pion mass squared. Left panel: We plot the results for pure RL1 and for RL2 with pion exchange. We also compare with a selection of lattice data [30]-[39]. Right panel: We compare the results for RL2 only and RL2 with pion exchange. Stars denote the physical nucleon and delta mass. The shaded bands correspond to a variation of the interaction parameter $\eta$ between $1.6 \leq \eta \leq 2.0$, with $\eta = 1.6$ corresponding to the upper limit of the bands.

3. Results and Discussion

To proceed with the calculations we must fix the two parameters $\Lambda$ and $\eta$ of the interaction (7) as well as the current-quark masses. This is conveniently done by using the experimental values for the pion decay constant and the pion mass as benchmark. It has been noted that the pion decay constant can be reproduced by a range of values of $\eta$ around 1.8 (see, e.g. [29]). On the other hand, the pion decay constant is largely insensitive to the current quark mass, which is consequently fixed by the physical pion mass. Naturally, the values of these parameters will differ in the cases when only the RL kernel is considered or if pion-exchange corrections are included. For pure RL interaction we choose $\Lambda = 0.74$ and $m_{u/d}(\mu^2) = 3.7$ MeV; we denote this case by RL1. For the pion-corrected kernel we use $\Lambda = 0.84$ and $m_{u/d}(\mu^2) = 3.7$ MeV; we use the label RL2 for the RL part of this truncation. In both cases, we solve our equations for $\eta = 1.6$ and $\eta = 2.0$ and interpret the difference in the results as an estimate of the model dependence. Finally, the renormalisation scale is given by $\mu^2 = (19 \text{GeV})^2$.

3.1. Nucleon and Delta masses and Sigma terms

The calculated masses of the Nucleon and the Delta, with and without the pion-exchange kernel, are shown in Table 1. Clearly in both cases the agreement with experimental values is very good on the five percent level. In the table, also the results for the second fit RL2 alone are shown. We see that for the Delta around physical masses the pion exchange is attractive giving rise to a shift of the order of 10-50 MeV, depending on the model parameters.
For the nucleon the situation is more complicated and the repulsive or attractive properties of the pion back-reaction even depends on the model parameters.

This is even more evident from Figure 3, where we display the baryon-mass evolution with the squared pion mass (or, equivalently, with respect to the current-quark mass). The comparison with lattice data suggests that the binding provided by the pure rainbow-ladder scheme RL1 is somewhat too strong at larger pion masses. This is remedied by taking the pion back-reaction into account and we now find good agreement with the lattice results even down to the lattice points with lowest pion mass. However, it is also clear that the curvature of $M(m_\pi^2)$ below this point is slightly too small. As a result, our value of the Nucleon and Delta masses at the physical point are slightly too large.

An observable effect of the slope of the mass-evolution curve is given by the nucleon and delta sigma terms. In our approach, these are trivially obtained using the Feynman-Hellman theorem

$$\sigma_{\pi X} = m_q \frac{\partial M_X}{\partial m_q},$$

where $m_q$ is the current-quark mass, $M_X$ is the baryon mass and the derivative is taken at the physical quark mass. For the nucleon we obtain

$$\sigma_{\pi N} = 30(3) \text{ MeV (RL1)}, \quad \sigma_{\pi N} = 26(2) \text{ MeV (Pi-exch.)}$$

and for RL1 without and RL2 with pion exchange, respectively. Likewise, we obtain for the delta

$$\sigma_{\pi \Delta} = 24(2) \text{ MeV (RL1)}, \quad \sigma_{\pi \Delta} = 25(3) \text{ MeV (Pi-exch.)}.$$  

The pion-nucleon case our value is slightly below the lower bound of a range of recent lattice results [40, 41]. For the delta the available model results [42, 43] are 30% larger than our value.

Within certain limits, the slope can be influenced by the choice of the model parameters as reflected in the error bars given in (15) and (16). However, in order to study the mass evolution of the system and the resulting sigma-terms in more detail, one should in addition include the effects of the gluon self-interaction in the two-body (and probably even three-body) correlations, since these may have a significant impact on the mass evolution of the system [12].

### Table 2: Contribution in % of the different partial wave sectors, at $m_\pi = 138$ MeV, to the normalization of the Faddeev amplitudes for the Rainbow-Ladder kernel only (RL1) and for RL2 including the pion exchange (Pi-exch.). As before, the error bars reflect the variation of the interaction parameter $\eta$ between $1.6 \leq \eta \leq 2.0$. For RL1 this variation is small and therefore no error bars are given.

| Sector | RL1  | RL2 + Pi-exch. |
|--------|------|----------------|
| s-wave| 65.9 | 66.6 (20)      |
| p-wave| 33.0 | 32.2 (20)      |
| d-wave| 1.1  | 1.2 (3)        |

### 3.2. Internal composition

Some insight into the internal structure of the baryon can be gained by studying the relative importance of the different partial-wave sectors. As shown in [21, 22, 23], Poincaré covariance enforces that in our framework baryons are composed, in principle, by s-, p- and d-wave components for spin-1/2 particles and s-, p-, d- and f-wave components for spin-3/2 particles. Therefore, one cannot restrict the partial-wave composition of any state in a covariant way and it is the dynamics what dictates the contribution of these components to a given state. Moreover, in the case of the nucleon, the flavor part of the Faddeev amplitude contains a mixed-symmetric and a mixed-antisymmetric term, as dictated by symmetry. Each of these is accompanied by a spin-momentum part that are not identical but related to each other. In our calculation we take all these contributions into account.

Form factors are observables which are expected to be more sensitive to the internal structure of the baryon. In particular, the $N\Delta \gamma$ transition [17, 19] as well as the electromagnetic $\Delta$-baryon form factors [18] show a qualitatively different behavior when the angular-momentum content is artificially restricted. For this reason, we have calculated the contribution of the different partial-wave sectors to the normalization of the $N$ and $\Delta$ amplitudes when the pion corrections are or are not included, see Table 2. In the case of the nucleon we average the contributions from the mixed-symmetric and mixed-antisymmetric terms. The angular-momentum composition of the state is not, nevertheless, the only element determining the form factors. The coupling of the photon (in case of electromagnetic form factors) and pion cloud plays an important role and is likely to be the dominant correction for, e.g., baryon’s charge radius and magnetic moment. This is, however, beyond the scope of this work.

Accepting the aforementioned qualifications, we can still speculate from Table 2 about the possible effect of the pion cloud in form-factor calculations. Let us begin analyzing the nucleon results. It is clear that the inclusion of pion effects does not change significantly the angular-momentum content of the nucleon, to which s-waves contribute roughly 2/3 in both cases. Nucleon wave-function contributions to the form factors are, hence, expected to remain qualitatively unchanged. The possible quantitative corrections, however, will be dictated by the direct
Our approach generalizes the rainbow-ladder calculations baryons within a covariant three-body Faddeev approach. This might have an impact in those form factors that measure the deformation of the $\Delta$-baryon, namely, the electric quadrupole and the magnetic octupole [18] (the latter of which is, in fact, very small and therefore very sensitive to changes in the baryon internal structure).

4. Summary

In this work we included, for the first time, the explicit effects of pion cloud contributions in a description of baryons within a covariant three-body Faddeev approach. Our approach generalizes the rainbow-ladder calculations of Refs. [21, 22, 23] and complements corresponding efforts in the light meson sector [13, 14, 15]. Similar than for light mesons we found substantial contributions of the pion cloud effects to the masses of the baryons of the order of 5-10%. In addition, we found slight but significant changes in the structure of the baryons reflected in the relative contributions of their partial waves. We will explore the impact of these effects onto the electromagnetic as well as axial form factors of the baryons in future work.

Acknowledgments

We are grateful to Gernot Eichmann and Richard Williams for critical reading of the manuscript and to Walter Heupel for discussions. CP thanks the Yukawa Institute for Theoretical Physics, Kyoto University, where this work was completed during the YITP-T-13-05 workshop on 'New Frontiers in QCD'. This work was supported by the Helmholtz International Center for FAIR within the LOEWE program of the State of Hesse, by the Helmholtz Center GSI, by the Erwin Schrödinger fellowship J3392-N20 of the FWF and by the DFG transregio TR 16.

References

[1] G. Eichmann, J. Phys. Conf. Ser. 426 (2013) 012014.
[2] A. Bashir, L. Chang, I. C. Cloet, B. El-Bennich, Y. -X. Liu, C. D. Roberts and P. C. Tandy, Commun. Theor. Phys. 58 (2012) 79 [arXiv:1201.3366 [nucl-th]].
[3] C. S. Fischer, P. Watson and W. Cassing, Phys. Rev. D 72 (2005) 094025 [hep-ph/0509213].
[4] L. Chang and C. D. Roberts, Phys. Rev. Lett. 103 (2009) 081601 [arXiv:0901.5461 [nucl-th]].
[5] L. Chang, Y. -X. Liu and C. D. Roberts, Phys. Rev. Lett. 106 (2011) 072001 [arXiv:1009.3458 [nucl-th]].
[6] W. Heupel, C. S. Fischer, T. Goeke, in preparation.
[7] A. Bender, C. D. Roberts and L. Von Smekal, Phys. Lett. B 380 (1996) 7 [nucl-th/9602012].
A. Schäfer and R. Schiel et al., arXiv:1312.0828 [hep-lat].

[42] V. E. Lyubovitskij, T. Gutsche, A. Faessler and E. G. Drukarev, Phys. Rev. D 63 (2001) 054026 [hep-ph/0009341].

[43] I. P. Cavalcante, M. R. Robilotta, J. Sa Borges, D. de O. Santos and G. R. S. Zarnauskas, Phys. Rev. C 72 (2005) 065207 [hep-ph/0507147].