I. HAWKING EVAPORATION AND COSMIC CENSORSHIP

The usual picture of Hawking evaporation of an asymptotically flat\textsuperscript{1} Schwarzschild black hole is that they are governed by the Stefan-Boltzmann law\textsuperscript{2}:

\[
\frac{dM}{dt} = -\alpha a\sigma T^4,
\]

where \( M = M(t) \) is the mass of the black hole, and \( T = (8\pi M)^{-1} \) is its Hawking temperature. Here \( a = \pi^2/15 \) is the radiation constant, and \( \sigma = 27\pi M^2 \) is the effective area whose radius corresponds to the impact parameter of the photon orbit at \( r = 3M \) in the geometric optics approximation. Due to scattering at long wavelengths, the actual effective emission surface is smaller. This effect is governed by the so-called “greybody factor”, denoted here by \( \alpha \). Depending on the particle species, the numerical value of \( \alpha \) is different. Since we are only interested in the qualitative picture, \( \alpha \) can be set to unity for simplicity. (For more detailed physics, see the classic work of Page \cite{Page}.)

Solving Eq. (1), one immediately obtains the standard result that a Schwarzschild black hole of initial mass \( M_0 \) will completely evaporate in a finite time proportional to \( M_0^3 \).

Of course, this simple model is likely to fail at late time as spacetime curvature becomes large enough and new physics (quantum gravity) could enter to modify the picture substantially. At the very least, one should expect emission of new particles beyond the Standard Model. Lacking a complete theory of quantum gravity, it is hard to be confident of the final picture. Nevertheless, in the recent years, various studies have converged to suggest that the underlying properties of black holes do change at late time as they undergo Hawking evaporation. In other words, an old black hole behaves rather differently from a young one, a transition that happens long before they reached their final moments. Arguably the most well-known difference is that the Hawking radiation from a young black hole is generally believed to contain no information, but after the black hole has radiated away roughly half of its initial mass, one can start to, in principle, recover the information trapped in the black hole by decoding the quantum information contained in the Hawking radiation (in highly scrambled form); see, e.g. \cite{Page2}. This transition between a young black hole and an old one is marked by the Page time \cite{Page3, Page4}. For an old black hole, new information that falls...
into the black hole is quickly scrambled an re-emitted in the Hawking radiation in a much shorter time scale known as the “scrambling time” [5–12]. For a Schwarzschild black hole, this is of the order $M \log M$ instead of $M^3$.

The evaporation of black holes with electrical charge and/or angular momentum is of course more complicated. Nevertheless it is generally expected that black holes would lose angular momentum and electrical charge during the course of Hawking evaporation; if so, all black holes would tend to a Schwarzschild state (what happens at the final moments is of course a subject of our ignorance, as previously mentioned). As we will discuss in details in Sec.(II), the process of losing charge and angular momentum can be rather nontrivial. The charge-to-mass ratio, $Q/M$, of a charged black hole, for example, can increase towards extremality\(^3\) during the course of Hawking evaporation. While this process might take an infinite amount of time (in accordance to the third law of black hole thermodynamics), the worry is that when black holes become near-extremal, a perturbation might bring $Q/M$ to exceed unity and thus destroy the horizon of the black hole, which will expose the singularity within. This would violate the cosmic censorship conjecture [13–16] and renders general relativity unpredictable\(^4\). Whether such a perturbation exists is, of course, an issue that is still under debate (see the recent review [17] for some discussions).

However, problems can arise already when charged black holes become near-extremal, even if the horizon is never destroyed to expose the singularity. The reason is as follows: while in classical general relativity charged black holes are described by the Reissner-Nordström solution, there is evidence that the interior of the near-extremal black holes is nothing like what is described by the classical geometry. Instead, as $Q/M$ is increased, the singularity becomes closer and closer to the horizon, rendering it eventually “effectively singular” [18]. One way to appreciate this picture is completely classical: recall that the inner (Cauchy) horizon is unstable due to an infinite blueshift. This so-called “mass inflation” is expected to destroy the inner horizon, which might turn it into a new spacelike singularity [19]. This phenomenon is expected to persist even at the quantum level [20] (see, however, [21] for an opposing argument).

In [22], Susskind argued that the growing quantum entanglement of a Schwarzschild black hole with its Hawking radiation causes the singularity to “migrate” towards the horizon, and eventually intersect with it at sufficiently late time. If correct, then in the Reissner-Nordström case, even if the inner horizon somehow survives mass inflation [23], the singularity can merge with the inner horizon at late time and so for near-extremal black holes we would have essentially the same situation as discussed before. Yet another piece of evidence comes from string theory – the 4-dimensional low-energy effective theory obtained from heterotic string theory gives the so-called charged dilaton “GHS” (Garfinkle-Horowitz-Strominger) black hole [24–26], which can be regarded as a string theoretic correction to the classical Reissner-Nordström solution. Indeed, an independent argument from area quantization suggests that near-extremal Reissner-Norström black holes are highly quantum objects [27]. Unlike the latter whose horizon remains smooth in the extremal case, the extremal charged GHS black hole is a null singularity, along which spacetime curvature diverges. Thus, both classical and quantum gravity considerations suggest that near extremal charged black holes have arbitrarily large curvature near – even outside – their horizon. This is as bad as a truly naked singularity, because general relativity cannot describe physics in the region of arbitrarily large curvature. Any spacetime point whose causal past intersects with such region is therefore unpredictable from (semi-)classical theory.

In view of this, if the cosmic censorship conjecture is correct, then one should expect that as a black hole undergoes Hawking evaporation, its parameters should evolve in such a way that it avoids becoming extremal, not just avoid becoming a truly naked singularity.

II. HOW CHARGED BLACK HOLES EVAPORATE

A. Hiscock-Weems Model: Extremality Is Never Reached

A Kerr black hole is characterized by its mass $M$ and angular momentum $J$. It is often more convenient to work with the angular momentum parameter $a = J/M$. The evolution of Kerr black holes under Hawking evaporation is straightforward – they spin down. It is true that if a Kerr black hole only emits scalar particles, it will evolve towards $a/M \approx 0.555$, however once higher spin particles are added, the final state would be $a/M \to 0$ [28, 29]; see also [30]. As such, we do not consider angular momentum in this work but instead focus on the effect of electrical charge.

Depending on the charge-to-mass ratio $Q/M$ (we assume $Q > 0$ without loss of generality) of a Reissner-Nordström black hole\(^5\), discharge can be quite efficient, or it can be very slow [32, 33]. For sufficiently large black holes $M \gg Q_0 := q_e/(\pi m_e^2)$, where $q_e$ and $m_e$ denote the charge and mass of an electron\(^6\), Hiscock and Weems argued that the

\(^3\)That is, a charged black hole on the verge of becoming a naked singularity – for Reissner-Nordström black hole in general relativity, it satisfies $(Q/M)_{\text{max}} = 1$.

\(^4\)More specifically, we mean the “weak” cosmic censorship conjecture, which essentially states that timelike singularities cannot be naked (i.e., they must be contained inside a black hole horizon), for otherwise it will affect spacetime regions in its causal future.

\(^5\)Here we only consider isolated black holes. In an actual astrophysical environment discharge would be more efficient due to infalling matter, but even then small charges can still give rise to non-negligible effects [31]. In the context of black hole evaporation, small charge-to-mass ratio can grow, so for a sufficiently isolated black hole we cannot ignore the charges.

\(^6\)Since we assumed $Q > 0$, it is actually the positron that is preferentially emitted (like charges repel).
evaporation can be modeled by the coupled ordinary differential equation

$$\frac{dM}{dt} = -a\alpha\sigma T^4 + \frac{Q}{r^+} \frac{dQ}{dt}, \tag{2}$$

where the charge loss rate

$$\frac{dQ}{dt} \approx -\frac{e^4}{2\pi^3 m^2} \frac{Q^3}{r^+} \exp\left(-\frac{r^2}{Q_0 Q}\right) \tag{3}$$

is obtained from an approximation of the Schwinger formula in quantum electrodynamics [32, 34, 35], which governs charged particle production in the presence of a strong electric field. The threshold beyond which pair production is effective is given by the Schwinger critical field $E_c := m_e^2 c^3/q_e\hbar = 1.312 \times 10^{18}$ V/m in SI units. In the regime of validity of the model, the Schwinger effect is actually suppressed: the electric field never exceeds the critical value as long as the black hole never exceeds its extremal charge. In this model since the mass of the black hole is large enough, its Hawking temperature,

$$T = \frac{\sqrt{M^2 - Q^2}}{2\pi(M + \sqrt{M^2 - Q^2})^2}, \tag{4}$$

is sufficiently low not to emit heavier charged particles. Massless particle emission is governed by the Stefan-Boltzmann term.

Hiscock and Weems found that if the initial charge-to-mass ratio of a black hole is sufficiently large, the black hole steadily discharges. However, if $Q/M$ is initially small, then its value will increase at first, since it is losing much more mass than charge. It is possible that $Q/M$ comes very close to unity, i.e., the black hole becomes near extremal. Nevertheless, $Q/M$ cannot keep on increasing indefinitely. On the contrary, $Q/M$ will decrease once discharge becomes more efficient [36], when $dM/dt \sim dQ/dt$. This gives rise to the attractor behavior in Fig.(2) of [32], part of which is reproduced in Fig.(1) below. We remark that the charge loss term is crucial – otherwise emission of only chargeless particle could lead to a violation of cosmic censorship as well, as shown recently by Hod [37]. Although the emission of charged particles is suppressed in this regime, its inclusion in the Hiscock-Weems model makes all the difference. Finally, we remark that although the curves get very close to the attractor, they never actually cross, since the evolution under differential equations is of course unique.

FIG. 1: Evolution of Reissner-Nordström black hole under Hawking evaporation in the Hiscock-Weems model. Given any initial condition $M_0$ and $Q_0$, we can track how the ratio $Q/M$ evolves in this plot. The attractor (dash-dot black curve) is characterized by [36] $dM/dt \sim dQ/dt$, it tends to $(Q/M)^2 = 1$ when $M \to \infty$. This means that sufficiently large black holes can approach but cannot reach extremality; trajectories always eventually turn away from extremality. In the regime of validity of the model ($M \gg Q_0$), the black hole always eventually evolves towards a Schwarzschild state.
The Hiscock-Weems model suggests that the end state is Schwarzschild, which by our discussion in Sec.(I), will completely evaporate away in a finite time. This is only suggestive because we can no longer trust the model once $M < Q_0$ (which is about $1.7 \times 10^5$ solar masses [32], that is to say, the model is only good for supermassive black holes). Nevertheless, if no new physics enters the picture, there seems to be no reason to suspect that $Q/M$ will rise again for small $M$. This should be the case until $M$ becomes extremely small at the very late stage of the evaporation and the temperature scale is high enough for new physics to kick in.

Note that as emphasized in [36, 38, 39], the fact that charged black holes never become extremal in this model is not only consistent with the third law of black hole thermodynamics, it is also an evidence for the validity of the cosmic censorship – if a black hole becomes extremal, a perturbation may destroy the horizon and renders the singularity naked (though whether such perturbations exist remains unclear). The role of the third law in upholding the cosmic censorship was already pointed out by Davies in 1977 [40]. However, note that the attractor behavior is crucial; the third law itself is not enough: for a charged dilaton black hole the extremal black hole is itself a null singularity - curvature (Kretschmann invariant) gets arbitrarily large near extremality, which effectively already violates the spirit of cosmic censorship. As per our discussion in Sec.(I), this could also be the case of a more realistic extremal Reissner-Nordström black hole. The third law can only guarantee that a black hole takes an infinite amount of time to reach extremality, but does not forbid it to become arbitrarily close in a finite (albeit extremely long) time. Then, when it was shown that a modification to the charged particle emission rate in the presence of dilaton field, as required by quantum field theory, is precisely the “minimal requirement” needed to uphold cosmic censorship in the Hiscock-Weems model, it was a strong evidence for the conjecture’s validity [38].

Let us now review another model of charged black hole evaporation, which at first sight seems to contradict the results of Hiscock and Weems, and thus seemingly threatens cosmic censorship.

B. Kim-Wen Model: The Role of Quantum Information

Kim and Wen argued for a completely different picture: that the end state is an extremal black hole [41]. Kim and Wen actually considered Hawking radiation as quantum tunneling à la Parikh-Wilczek [42–44], with the additional criteria that the emission is dominated by those with maximum mutual information (MMI). In more details, the Parikh-Wilczek tunneling picture considered Hawking radiation as the result of particles tunneling out from the black hole, whilst enforcing the conservation of energy, that is, by assuming the black hole mass $M$ is the total energy of all the Hawking particle emitted. The radiation is not exactly thermal in this picture as two consecutive emissions of time to reach extremality, but does not forbid it to become arbitrarily close in a finite (albeit extremely long) time. Then, when it was shown that a modification to the charged particle emission rate in the presence of dilaton field, as required by quantum field theory, is precisely the “minimal requirement” needed to uphold cosmic censorship in the Hiscock-Weems model, it was a strong evidence for the conjecture’s validity [38].

Consider the Schwarzschild black hole for example, if we denote the tunneling probability of a particle of energy $\omega$ from a black hole of mass $M$, then the tunneling probability goes like $\Gamma(M,\omega) \sim \exp \left[-8\pi\omega(M-\omega/2)\right]$. Having emitted a particle with energy $\omega_1$, the mass of the black hole is reduced to $M - \omega_1$ so that the probability of emitting the next particle of energy $\omega_2$ is $\Gamma(M,\omega_2|\omega_1) := \Gamma(M - \omega_1, \omega_2)$, which is a conditional probability. One then defines the mutual information as the difference [45]

$$S_{\text{MI}}(M, \omega_2 : \omega_1) := S(M, \omega_2 | \omega_1) - S(M, \omega_2),$$

where $S(M, \omega_2 | \omega_1) := \ln \Gamma(M, \omega_2 | \omega_1)$ and $S(M, \omega_2) := \ln \Gamma(M, \omega_2)$ are the entropy functions. This is obtained from the standard definition for mutual information

$$S(A : B) := S(A) + S(B) - S(A, B)$$

for two quantum subsystems $A$ and $B$. For Reissner-Nordström black holes, the entropy function is [43, 44]

$$S(M, Q; \omega, q) := \pi \left\{ \left[ M - \omega + \sqrt{(M-\omega)^2 - (Q-q)^2} \right] - \left( M + \sqrt{M^2 - Q^2} \right) \right\}.$$  

It can be proven, using Jensen’s inequality, that the quantity in Eq.(6) is necessarily non-negative. For completeness, let us plot the mutual information given by Eq.(7) (this was already done in [41]) to see the bounds its non-negativity imposes on the ratios $Q/M$ and $q/m$.

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7We remark that the value of $\hbar$ in these papers – though written correctly – was off by a factor of 50 or so in the numerical codes due to a typo. This does not affect the main conclusions.

8This is not a trivial statement. In general relativity energy (of the matter sector) is strictly conserved only when there is a timelike Killing vector, or equivalently in the language of field theory (Noether’s theorem), when there is a time translation symmetry – which clearly is not the case if a black hole is evaporating. Here we follow the usual assumption – which is justified a posteriori from the sensible results obtained via the Parikh-Wilczek formalism throughout the literature – but feel the need to point out this subtlety.
FIG. 2: Mutual information in the emission from a Reissner-Nordström black hole with $M/m = 10^2$. The plots consider only the emission of identical particles with mass $m$ and charge $q$. The mutual information is presented as a function of both $Q/M$ and $q/m$. **Left:** The 3-dimensional plot and **Right:** The corresponding contour plot.

In Fig. 2, following [41], we included negative $q/m$ ratios to better display the behavior and the maximum of the mutual information function. It must be noted, though, that we have assumed $Q > 0$, so only positive values for $q$ are allowed. Indeed, when a charged pair is produced in the electrical field of the black hole, the particle with the same charge as the black hole is repulsed to infinity, while the other one is absorbed, thus reducing the black hole’s charge. Therefore, for $Q > 0$ we only consider the emission and tunneling of positive electric charges. This is the same assumption made in the Hiscock-Weems model.

From the contour plot we can observe the bounds on $q/m$ and $Q/M$ better: we notice that all values of $Q/M \in (0, 1)$ are allowed only for $q/m = 1$. As $Q/M$ tends towards 0, it is around $\sqrt{2}$ (in the large mass limit, as shown in [41] this upper bound is saturated). When $q/m \approx 0$, higher $Q/M$ values are not possible anymore. The lowest $Q/M$ is about 0.86 for $q/m = 0$. This value is intriguing, as we will comment on later.

The explicit entanglement between the Hawking particles allows the mutual information to exhibit the behavior of the Page curve [3, 4], i.e., $S_{MI}$ first grows then decreases at about the midpoint of the evaporation process [41]. See also [46]. Whether this can completely resolve the information paradox remains controversial [47–49], but is of little relevance to our present work.

Kim and Wen showed that if one imposes the MMI principle, the evaporation happens through the progressive emission of particles with the following charge-to-mass ratio [41]:

$$\gamma := \left(\frac{q}{m}\right)_{\text{optimal}} = \frac{Q^3}{M^3 + (M^2 - Q^2)^{\frac{3}{2}}}.$$  

This result comes from calculating the $q/m$ ratio that maximizes the mutual information entropy of the emitted particles. While the MMI principle is an assumption, it is nevertheless a well-motivated one. It is essentially the statement that black hole evaporation is not a random process, but an optimized one that allows information to escape as fast as possible. This is in line with the evidence found thus far that suggests black holes are the fastest computers in Nature [5–12].

We will now illustrate the Kim-Wen model with a simple example. Let $n \in \mathbb{N}$. Let us consider a black hole of mass $M = nm$, which emits a particle of mass $m$ at each step, results in the following change in the charge-to-mass ratio for the black hole.

$$\frac{Q}{M} \rightarrow \frac{Q}{m} - \gamma.$$  

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9This is better known as the “infomax principle” in the parlance of information theory and neural networks [50]. In the zero-noise limit it is equivalent to the principle of redundancy reduction in biological sensory processing [51–53]. For a review of the applications of information theory in evolutionary biology, see [54], in which the notion of entropy and information of molecules and proteins are discussed.
We can then assume \( m = 1 \) for all the particles and plot the evolution, considering that every time a particle is emitted, in Eq.(8), \( Q \) and \( M \) are updated with the new values. The result for \( n = 100 \) is plotted in Fig.(3), which reproduces the behavior found in Fig.(4) of [41] (including the effect of angular momentum does not change this final fate [55]):

![Image of a graph showing the evolution of Reissner-Nordström black hole under Hawking evaporation in the Kim-Wen model: all black holes eventually tend to extremality.]

**FIG. 3:** Evolution of Reissner-Nordström black hole under Hawking evaporation in the Kim-Wen model: all black holes eventually tend to extremality.

Why is there such a stark discrepancy in the results between Hiscock-Weems and Kim-Wen models? The answer is of course: because their underlying assumptions are different. Specifically the MMI principle is absent in the Hiscock-Weems model. The main difference here is that the MMI principle (together with the requirement that the mutual information is non-negative) picks out what type of particle is to be emitted at a certain step, whereas the Hiscock-Weems model only considers emission of massless particles and electron/positron. In fact, since charged Standard Model particles all have \( q/m \gg 1 \), they cannot be emitted under the Kim-Wen model. This is because non-negativity of the mutual information imposes an upper bound on the \( q/m \) ratio of emitted particle to be \( q/m = \sqrt{2} [41] \). A lower bound appears when \( Q/M \gtrsim 0.86 [41] \). We note that curiously this is very close to the value \( Q/M = \sqrt{3}/2 \approx 0.866 \) (“Davies point”), beyond which the specific heat \( C := dM/dT \) for a Reissner-Nordström black hole with a fixed charge becomes positive [40], despite the fact that the specific heat under the Kim-Wen model is always negative [41] (See more details in Appendix A). We do not know whether there is a deeper meaning to this coincidence.

It is possible that both models are correct, but they are applicable in different regimes. We have already explained that the Hiscock-Weems model is good for sufficiently large astrophysical size black holes but does not apply for small black holes. Our example above definitely counts as a small black hole since \( n \) is only 100. Unfortunately numerical limitation does not allow us to check whether the Kim-Wen model reproduces the same behavior as in Hiscock-Weems, since that requires \( n \sim 10^{99} \) (ratio of a solar mass to electron mass, as a quick approximation) or more.

It is likely that the Kim-Wen model would not give the same behavior since for large black holes, we would expect that the standard picture of Hawking radiation as blackbody holds (any deviation from the Planck spectrum would be small – note that even the Hiscock-Weems model is not exactly thermal due to the Schwinger term), namely temperature governs the species of Hawking particles that can be emitted. As the black hole becomes very small, its quantum nature would become more dominant and MMI principle will take over. Exactly when the MMI optimization becomes the guiding principle for particle emission is currently not known. Nevertheless such an optimization to release information as soon as possible is consistent with the notion that after the Page time, quantum information that falls into the black hole is quickly re-emitted in a timescale set by \( T^{-1} \log S \), the “scrambling time” [5–12], where \( S \) denotes

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\(^{10}\) In all the subsequent plots of this paper, the blue curve represents the curve with initial charge \( Q_i = 0.8 \ M_i \), the gold, green and red curves correspond to \( Q_i = 0.6, 0.4, 0.2 \ M_i \), respectively.
the Bekenstein-Hawking entropy of the black hole (a quarter of the horizon area). This is also consistent with recent findings that terms beyond leading order in the Hawking temperature can have pronounced effects at late time [56]. In any case, since we are only interested in the final moments of the evaporation, we shall simply work with the Kim-Wen model. The comparison of the two models in the regime of large $M$ is interesting but would require a separate detailed study with a novel method to overcome/sidestep the numerical constraint.

### III. A CLOSER EXAMINATION OF THE KIM-WEN MODEL

Another major difference between the Kim-Wen model and the Hiscock-Weems model is that the latter only considers massless particles as well as electron/positron (since the emissions of other charged particle are suppressed in the regime of validity of the model), whereas the Kim-Wen model considers a continuous set of emitted particles, so that the optimal ratio $\gamma$ from Eq.(8) corresponds exactly to the $q/m$ ratio of the particle. To investigate how the Kim-Wen model behaves in a more realistic universe, we will consider several finite discrete sets of fictitious particle species of various $q/m$. These particles, \{a0, a1, \ldots, a11\}, are listed in the table given in Fig.(4). It also contains a few other characteristics such as the $q/m$ ratio, the behavior of the associated evolution plot and the positivity/negativity of the mutual information. The evolution and the corresponding mutual information are plotted in Appendix B.

| Particle | q/m | 0.8 | 0.6 | 0.4 | 0.2 | q/m | 0.8 | 0.6 | 0.4 | 0.2 |
|----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| a0       | 0   | r   | r   | r   | r   | n   | n   | n   | n   | n   |
| a1       | 1.1 | d   | d   | d   | d   | y   | y   | y   | y   | y   |
| a2       | 2   | d   | d   | d   | d   | n   | n   | n   | n   | n   |
| a3       | 10  | d   | d   | d   | d   | n   | n   | n   | n   | n   |
| a4       | 0.8 | c   | d   | d   | d   | y   | y   | y   | y   | y   |
| a5       | 0.6 | r   | c   | d   | d   | n   | n   | y   | y   | y   |
| a6       | 0.2 | r   | r   | r   | c   | n   | n   | n   | n   | n   |
| a7       | 1   | d   | d   | d   | d   | y   | y   | y   | y   | y   |
| a8       | 1.5 | d   | d   | d   | d   | n   | n   | n   | n   | n   |
| a9       | 1.57| d   | d   | d   | d   | n   | n   | n   | n   | n   |
| a10      | 0.4 | r   | r   | d   | d   | n   | n   | y   | y   | y   |
| a11      | 2.3 | d   | d   | d   | d   | n   | n   | n   | n   | n   |

**FIG. 4:** Table of all the available (fictitious) particles and their properties. The first column shows the name of the particle, followed by its $q/m$ ratio (shaded in yellow if it is above $\sqrt{2}$). The orange part of the table refers to its behavior during evaporation, for the blue ($Q_i = 0.8M_i$), yellow ($Q_i = 0.6M_i$), green ($Q_i = 0.4M_i$) and red ($Q_i = 0.2M_i$) curves, respectively. The letter “r” stands for “rising”, “d” for “descending” and “c” for “constant”. In the teal blue part it is asked whether the mutual information is positive, answered with “y” for “yes” and “n” for “no”.

From the table in Fig.(4), there appears to be a correlation between a rising behavior and negative mutual information. The only instances where this fails are for $q/m > \sqrt{2}$, which, as we have seen in Fig.(2), inevitably leads to negative mutual information. Remarkably, in this model, the non-negativity of mutual information therefore prevents the ratio $Q/M$ from uncontrolled growth.

To further understand the plots, let us first focus on the reason why some mass evolution curves are constant. Since we know that the evolution follows Eq.(9), we see that when we have $Q/M = q/m$, substituting it into the equation gives no change in the charge-to-mass ratio of the black hole and thus we observe a constant line. It is now easier to appreciate why, if $Q/M > q/m$, the former ratio tends to increase and we see a rising behavior in the plots. The particle $a0$, being chargeless, has the lowest $q/m$ possible and thus will always lead to a rising behavior. This is proven to be unphysical by the negativity of the associated mutual information, so such particles cannot be emitted solely by themselves.

Of course, in a more realistic universe (hypothetical or not), there should be more than one species of particle. It is under such situations that the Kim-Wen model becomes interesting, as different particles can be chosen by the MMI optimization at each step. First, we consider Set 1, which contains all of the particles $a0$ to $a11$. The result is shown in Fig.(5). The evolution curve is obtained by joining discrete points, thus resulting in the curves crossing each other. None of the data point for different curves coincide, however. In this sense, given an initial condition, the discrete evolution is still unique. The right figure of Fig.(5) shows the particle species that are emitted in each step of
the evolution. We notice that not all particles are emitted, indeed only the particles with \( q/m \leq 1 \) are chosen. The resulting behavior is very similar to Fig.(3), where indeed \( q/m \) is given by \( \gamma \) that is always less than or equal to 1.

There are nevertheless several differences. Most notably, we observe that the red curve, which corresponds to a black hole with initial \( Q/M = 0.2 \), does not tend towards the extremal limit, but rather drops down, similarly to the Hiscock-Weems model. In the next figure (Fig.(6)), we plot the mutual information and verify that it is always non-negative (and therefore the evolution is indeed physical and permitted\(^{11}\)).

![Graph 1](image1.png)

**FIG. 5:** Left: Black hole charge-to-mass ratio evolution, assuming all particle species are available (Set 1). Right: Particle species in Set 1 that are actually emitted during the evaporation.

![Graph 2](image2.png)

**FIG. 6:** Mutual information of Set 1.

Next, we consider what happens when we exclude the chargeless particle \( a_0 \) that mimics neutrinos in the Standard Model, which can be massless or massive. It also mimics massless particles like photon and graviton. Massless particles contribute to Hawking evaporation since they carry energy, which in this simple toy model we treat as an effective mass. Since emission of chargeless particle can increase \( Q/M \), its role requires a closer examination. The results are plotted in Fig.(7).

The behavior is similar to Fig.(5), with the most notable difference in the red curve \( (Q_i = 0.2 \ M_i) \) that remains constant, due to the fact that only particle \( a_6 \) with \( q/m = 0.6 \) is emitted, with the effect that is noted in Appendix Fig.(B.13). The presence of the particle \( a_0 \) contributes to the early rise that was missing in plot of Fig.(7). Indeed, as seen in its evolution plot (Appendix Fig.(B.1)), the presence of this particle alone makes the ratio \( Q/M \) grows indefinitely. Its presence seems to have a dominant effect, especially for small initial \( Q/M \) ratios.

Note that in both Set 1 and Set 2, extremality can be reached. According to our discussion, this would violate cosmic censorship. Our next step is to investigate how to prevent such scenario from arising. To this end, let us

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\(^{11}\)In the following, unless mentioned otherwise, all evolutions of a given set satisfy this non-negativity constraint, but we shall refrain from showing too many unnecessary plots.
reduce the number of particle species for a cleaner examination. We consider, in addition to the chargeless \(a_0\), a set of three different charged particles: \(a_{10}\) with \(q/m < 1\), \(a_{11}\) with \(q/m > 1\), and \(a_7\) with \(q/m = 1\). The results are plotted in Fig.(8). We notice that \(a_0\) is emitted at the beginning and then only the particles \(a_7\) and \(a_{10}\) are chosen. Again, extremality can be reached – in this case by a black hole whose initial charge-to-mass ratio is \(Q/M = 0.8\). However, in this case, the extremal black hole can still discharge and eventually evolves towards the Schwarzschild limit. This is actually not so surprising. It is often said that an extremal black hole does not emit Hawking radiation, however even in the standard (i.e., non-MMI optimized) picture, it can still emit charged particle via non-thermal channel such as via the Schwinger process [57–59]. We notice an oscillating behavior, due to the alternate emission of particle \(a_7\), that raises the mutual information, and \(a_{10}\) that lowers it (see Appendix (B.15) and (B.21)).

Next, we proceed to perform a test considering only particles with \(q/m > 1\) and we use \(a_1, a_2\) and \(a_3\) (Set 4 and 5). First, we include also the chargeless particle \(a_0\) and plot the results in Fig.(9). The evolution in the case without \(a_0\) (Set 5) gives rise to Fig.(10). The two cases, this time, are very different. With Set 4, for the most part the black hole \(Q/M\) ratio in Fig.(8) oscillates between the interval \((0.6, 0.9)\), before dropping to 0. On the other hand with Set 5, as shown in Fig.(10), \(Q/M\) steadily tends towards zero. Also, the only particle emitted is \(a_1\) so that it is exactly the same evolution plot as Appendix (B.3). Note that once the black hole \(Q/M\) reached zero, it can no longer emit charged particles under the MMI optimization scheme. This means that the black hole will no longer discharge and the mass becomes fixed; the black hole has become a remnant. If we choose to include \(a_0\) so that it is emitted after a complete discharge of the black hole, the final state is again zero, as shown in Fig.(11).

We note that particles \(a_2\) and \(a_3\) have very big \(q/m\) ratios, so they are never emitted. The parameter \(\gamma\) of optimal
emission is never larger than 1, so the closest value is found in particle \( a_1 \). This implies that, in Set 4 (Fig.9), \( a_1 \) and \( a_0 \) are emitted alternatively in the last part of the evolution, raising and lowering the \( Q/M \) ratio and resulting in the oscillating behavior (this behavior was also present in the previous plots, but is more pronounced here). When \( a_0 \) is excluded in Set 5, only \( a_1 \) can be emitted and it lowers the \( Q/M \) ratio to 0. Neither of these sets is in any danger
of violating cosmic censorship.

Similar behavior can be obtained if we use particles that satisfy $q/m \simeq 1$ (Set 6). The results are shown in Fig.(12) and Fig.(13), with and without the chargeless particle $a_0$, respectively. As we commented before in the case of Sets 4 and 5 (Fig.(9) and Fig.(10)), out of the three particles only the one with the lowest $q/m$ ratio is emitted. In this case, it is particle $a_7$. Again, the cosmic censorship is safe in these cases.

![Set 6](image1)

**FIG. 12:** Left: Black hole charge-to-mass ratio evolution, assuming Set 6, with charged particles that satisfy $q/m \simeq 1$, along with chargeless particle $a_0$. Right: Particle species in Set 6 that are actually emitted during the evaporation.

![Set 7](image2)

**FIG. 13:** Left: Black hole charge-to-mass ratio evolution, assuming Set 7, same as Set 6 but without $a_0$. Right: Particle species in Set 7 that are actually emitted during the evaporation. Only $a_7$ is emitted, so the plot is the same as Appendix (B.15). Similar to Fig.(11), the black hole can continue to evaporate if we allow $a_0$ to be emitted after the black holes have completely discharged.

We note that in Fig.(9) up to Fig.(13), in which extremality is not reached, no charged particles with $q/m < 1$ are present. We therefore suspect that charged particles with $q/m < 1$ are intimately related to possible cosmic censorship violation. Indeed, at least part of the story has been explored in the literature, which has come to be known as the “weak gravity conjecture”. This is what we shall discuss next.

**IV. A CONNECTION TO THE WEAK GRAVITY CONJECTURE**

Note that in the unphysical situation explored in Fig.(B.1), for example, the curves can actually evolve beyond $Q/M = 1$, but we imposed cosmic censorship by hand and stopped the evolution once extremality is reached. Indeed, such a scenario (of going beyond extremality) cannot arise if the so-called “weak gravity conjecture” (WGC) is correct. The WGC [60] is a deep conjecture in quantum gravity. It essentially states that the lightest charged particle with mass $m$ and charge $q$ in any $U(1)$ gauge theory that admits an ultraviolet embedding into a consistent theory of
quantum gravity should satisfy the nontrivial bound

\[ q \gtrsim m. \]  

That is, the WGC requires that there should exist at least one particle species of which \( q/m > 1 \). The WGC ensures that, among other things, an extremal black hole is unstable and can thus decay into a non-extremal one via charged particle production though the Schwinger effect (despite its Hawking temperature being zero) \([60, 61]\), or by splitting into smaller black holes (it is subtler than this, see the Discussion section). The WGC is crucial if we want to avoid a large (if not infinite) number of stable extremal black hole remnants. The problems with remnants are well-known (see \([62]\) for a review). The usual argument is that they suffer from the pair-production problem: since black holes can contain different information, there are potentially infinitely many species of remnants, each of mass \( M_i \). Thus the amplitude of pair-producing them from the quantum field goes like \( \sum 1/M_i^2 \), in which the series is an infinite sum, which leads to a divergent amplitude despite non-observation of such a phenomenon in laboratories. Although the existence of remnants cannot be entirely ruled out as discussed in \([62]\), it might indeed be preferable not to have them.

In view of the WGC, what happens when only particles with \( q/m < 1 \) are present in a hypothetical universe (thus violating the WGC)? Let us consider the particles \( a_4, a_5, \) and \( a_6 \), with (Fig.(14)) and without the chargeless particle \( a_0 \) (Fig.(15)).

The evolution of black hole in Set 10 (without \( a_0 \)), is similar to that of Fig.(7). Despite emitting particles with \( q/m < 1 \) like in Fig.(3), the curves do not converge to 1. This is due to the fact that no particles with \( q/m \simeq 1 \)
are emitted. In fact, in both of these plots the ratio \( Q/M \) increases beyond the extremal limit as expected from the violation of the WGC, though we do not show this in the plot. It turns out that these alarming scenarios are prevented from arising by the non-negativity constraint of mutual entropy. That is, they are as unphysical as the representative example in Appendix (B.1). Thus we see that, remarkably, otherwise problematic evolution that would lead to a serious violation of the cosmic censorship conjecture are naturally prevented by the non-negativity of mutual information. We show the plot of the mutual information in Fig.(16): it becomes negative for every curve that threatens to cross the extremal \( Q = M \) line. To be more careful, the situation is actually subtler. Take the red curve for example: the mutual entropy never becomes negative, then why does the black hole stop evaporating? For this case, the end mass is 8.5 and the end charge is 8.4: the evaporation is halted because emitting another particle of the set (that is, with mass 1 and charge 0.2, 0.6 or 0.8), would inevitably lead to extremality. In this case, the mutual information in the near-extremal end state is still positive; however, it can become negative in the last steps of the evolution if we were to change the parameters of one of the particle, for example of \( a4 \). If we take its mass to be 0.5 instead of 1 and its charge to be 0.4 (maintaining the same charge-to-mass ratio), then the end mass of the black hole would be 8 and the charge 7.99999. The other curves also stop before reaching a forbidden extremal value: the green curve, for example, behaves similarly and stops at mass 15.5 and charge 15.4, since its mutual information is lower than the red curve, it is already negative for these values.

![Graphs showing mutual information of Set 9 and Set 10](image)

**FIG. 16:** **Left:** Mutual information of Set 9. **Right:** Mutual information of Set 10.

It has previously been shown in the literature that there is a surprising and deep connection between the WGC and the cosmic censorship, notably in the context of a charged black holes in anti-de Sitter spacetime, the minimum value of charge required to preserve cosmic censorship appears to agree precisely with that proposed by the weak gravity conjecture [63]. Further evidence is found in [64] when a dilaton or an additional Maxwell field is added.

In this work, we have further explored both of these conjectures by considering only charged particles that satisfy \( q/m < 1 \). The potentially problematic evolutions under MMI optimized Hawking evaporation that would have led to a naked singularity are prevented by the non-negativity of the mutual information. On the other hand, precisely because these problematic evolutions are not permitted, charged black holes cannot evaporate under the Kim-Wen model. In other words, they are stable non-extremal remnants. This violates the WGC, as one indeed expect (though for a different reason). These examples show that these two conjectures are quite distinct: weak gravity conjecture can be violated by the very same reason that preserves the cosmic censorship. Finally, let us remark that it is not a priori obvious that the MMI scheme would be consistent with what we know about the WGC. One can also take this as an evidence that the MMI optimization is a reasonable physical process.

V. DISCUSSION: QUANTUM INFORMATION, PARTICLE PHYSICS, AND COSMIC CENSORSHIP

In the recent years, quantum information theory has been applied extensively in the context of gravitational physics, notably black holes and holography. Classically, a black hole is simply a region of spacetime from which nothing, not even light, can escape from. A black hole is formed when matter or energy is concentrated in a sufficiently small region. Since the horizon of a black hole is a measure of its entropy, black hole also sets a bound for how much information can be contained in a spacetime region, known as the Bekenstein’s bound [65, 66]. Recently [67], it has been shown that at least some black holes saturate the Margolus-Levitin bound [68] (essentially, the maximum speed of dynamical evolution), i.e., they produce quantum complexity at the fastest possible rate. The entanglement entropy of a black hole is also related to error correction code [69], which also arises in the context of holography [70].
Moreover, the “physics of information” [71–73] has played a central role in the ongoing attempt to understand how unitarity of quantum mechanics is preserved under Hawking evaporation and whether information can be retrieved from black hole interior [74–76]. There is evidence to suggest that after the Page time – roughly halfway through the evaporation process – black holes behave in a different way in terms of the rate of information processing: information fallen into a black hole now takes a vastly shorter time to be scrambled and re-emitted. Essentially, black holes are Nature’s most efficient computers.

In [41], Kim and Wen investigated the Hawking evaporation of Reissner-Nordström black holes under maximum mutual information (MMI) optimization, which means that particle species are emitted in such a way as to maximize their mutual information. They found that the black holes will evolve towards extremality. In view of various arguments, both classical and quantum gravitational, which suggest that extremal charged black holes might be effectively singular, such an evolution under Hawking evaporation is problematic – it means that cosmic censorship will be violated at late times as spacetime curvature becomes unbounded near the extremal horizon. This picture is also drastically different from that of Hiscock and Weems [32], which showed that charged black holes always tend towards the Schwarzschild limit under Hawking evaporation, although their charge-to-mass ratio can temporarily increase towards extremality.

In this work, we re-examined the Kim-Wen model and found that in a more realistic model with only finitely many particles, the discrete evolution could lead to different end states depending on the particle species available.

Let us briefly summarize our main findings. Notably, if all particles have charge-to-mass ratio $q/m < 1$, then although naively charged black holes can evolve into a naked singularity, such a scenario is not allowed by the constraint that mutual information cannot be negative. However, precisely because of this, charged black holes have no means to evaporate. This is consistent with the weak gravity conjecture that requires at least one particle to satisfy $q/m > 1$. The original argument for the WGC is that the presence of particles with $q/m > 1$ would allow extremal black holes to discharge or to split into non-extremal ones. In any case, the aim of the WGC is to avoid stable remnants. Here we see that under the Kim-Wen model, remnants need not be extremal black holes, and indeed they arise when there is no particle with $q/m > 1$. Note that there is a fundamental difference between the scenario discussed here and that of, for example, Set 7, in which the “remnants” have zero charge and a complete evaporation can be devised by allowing the “would-be remnant” to emit chargeless particles. Such a process would still satisfy the non-negativity requirement of mutual information. (In fact, to avoid remnant, we must therefore also require the final neutral black hole to continue emitting neutral particles.) No such process is available for the “true remnants” in the absence of any particle with $q/m > 1$. As we remarked previously, it is not a priori obvious that the MMI optimization would be compatible with the WGC, so the fact that it does in such a nontrivial manner is another evidence that such an optimization is physical.

More surprisingly, we found that extremality can be reached (though the black holes may subsequently discharge) – and so the cosmic censorship is still violated due to (potentially) effective singular nature of extremal charged black holes – whenever there exists a charged particle with $q/m < 1$. Thus, although closely related, cosmic censorship and the weak gravity conjecture are not equivalent.

Interestingly, the Standard Model of particle physics does not have any charged particle with $q/m < 1$, despite the existence of such a particle would still satisfy the weak gravity conjecture. Of course, charged Standard Model particles all have $q/m \gg 1$ and so are not emitted under the Kim-Wen model. Nevertheless, the picture we have in mind is that when black holes are young and large, near-thermal Hawking radiation can emit charged Standard Model particles like usual. However, when a black hole is old and small, MMI optimization takes over. By then only non-Standard Model particles with $q/m < \sqrt{2}$ can be emitted. Incidentally, the value $\sqrt{2}$ is also very interesting. It corresponds to the extremal charge-to-mass ratio of a charged dilaton “GHS” (Garfinkle-Horowitz-Strominger) [24–26] black hole. In the context of the weak gravity conjecture, an extremal black hole can only decay if there exists a particle with $q/m > 1$. Gell-Mann’s totalitarian principle [78] (“everything that is not forbidden is compulsory”) dictates that it is possible to emit not only particles, but also black holes. Of course it is easier to understand this process as an extremal black hole decaying into two smaller black holes, one with sub-extremal charge, and the other with an even larger charge [60]. This is possible because quantum gravity correction allows smaller black hole to possess higher extremal value. It is therefore highly interesting that under MMI optimization, a charged Reissner-Nordström black hole can “emit” an extremal GHS black hole. This would presumably further decay in a cascade of black hole fragmentation. It might be interesting to check the evolution of GHS black holes under MMI optimization. The only problem with this scenario is that the Kim-Wen model only allows the emission of a particle with $q/m = \sqrt{2}$ when it is near neutral, instead of near-extremal, as is clear from Fig.(2). Still, $\sqrt{2}$ is an interesting number and we wonder whether there is some connection to the extremal GHS black hole, or whether this is just another coincidence (in addition to the interesting fact – perhaps another coincidence – that a lower bound for $q/m$ appears when $Q/M \gtrsim 0.86$, which is close to the Davies point, as discussed in subsec.(II B)).

In this work, we have shown that if there is any particle with $q/m < 1$, the cosmic censorship will be violated due to the effectively singular nature of extremal charged black hole. In other words, our work suggests a stronger version
of the weak gravity conjecture: charged particle with \( q/m < 1 \) should not exist in a “physically realistic universe” in which cosmic censorship holds\(^{12} \). If so, this raises a new question: how does Nature know not to allow such a particle when cosmic censorship is only violated at a very late time by an isolated evaporating charged black hole? Might cosmic censorship somehow play a much wider role in fundamental physics than previously thought?

In the landmark paper [71], Landauer wrote: “information is inevitably tied to a physical representation and therefore to restrictions and possibilities related to the laws of physics and the parts available in the universe.” It would seem that the surprising interplay between quantum information and black hole physics might indeed shed more light on our understanding of particle physics, notably on the restriction on their charge-to-mass ratio. See also [79] for related discussions.

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**Appendix A  SPECIFIC HEAT OF THE KIM-WEN MODEL**

In the work of Kim and Wen [41], the specific heat can be calculated in the following way:

\[
\frac{dM}{dT} = \left[ \frac{\partial T}{\partial M} + \frac{\partial T}{\partial Q} \frac{dQ}{dM} \right]^{-1}.
\]

The variation of mass and charge is \( \delta M = m, \delta Q = q \) and the ratio \( q/m \) is taken to be \( \gamma \) (defined in Eq.(8)), thus giving the result:

\[
C := \frac{dM}{dT} \approx \frac{\delta M}{\delta T} = 2\pi r^2 + \left[ \frac{M(r_+ - 3M) + Q^2}{M^2 - Q^2} \right],
\]

with \( r_+ = M + \sqrt{M^2 - Q^2} \) being the radius of the outer horizon. As shown in [41], this expression is always negative. To make it more general and apply it in our model, we keep the ratio \( q/m \) as a variable. We then get the following expression:

\[
C = \frac{2\pi r_+^3 (r_+ - M)}{2Q^2 - Mr_+ - Q(2M - r_+) \frac{d}{m}}.
\]

From this we can draw a contour plot of \( C/M^2 \) (Figure (A.17)) and confirm the negativity of the specific heat, even in the case of our discrete model, when \( q/m \) does not necessarily correspond to \( \gamma \).

Kim and Wen remarked that the negative specific heat is consistent with the the black hole evolution in their continuous model, namely that charged black holes tend towards extremality monotonically. This is in contrast with the Hiscock-Weems model in which the black hole specific heat changes sign, which seems to correspond to part of the attractor behavior [32]. Since our discrete models do allow some black holes to exhibit turn-around behavior in their evolution similar to the Hiscock-Weems model (e.g. the red curve in Set 1, see Fig.(5)), how is this consistent with the specific heat being negative? There is actually no inconsistency. As explained in [36], the attractor is characterized by the condition \( dM/dt \sim dQ/dt \), which in general has nothing to do with the specific heat. For another example, the charged dilaton GHS black hole always has negative specific heat, but under the Hiscock-Weems model the evolution is very similar to the Reissner-Nordström case, with an attractor present [38].

**Appendix B  EVOLUTION AND MUTUAL INFORMATION OF SINGLE PARTICLE SPECIES**

In this appendix we show for reference purpose, the evolution of the black hole charge-to-mass ratio under the Kim-Wen Hawking evaporation model, when particles in Table 4 are emitted individually. Their corresponding mutual information are also shown.

\(^{12}\) Other generalizations for the weak gravity conjecture exist. Notably, in the interesting work of Solomon and Stojkovic [79], they argued that the weak gravity conjecture can be generalized to the requirement that \( q/m \geq Q/M \).
FIG. (A.17): Specific heat in our model, as a function of the ratios $Q/M$ and $q/m$. 

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(B.1) Evolution plot of $a_0$

(B.2) Mutual information of $a_0$

(B.3) Evolution plot of $a_1$

(B.4) Mutual information of $a_1$

(B.5) Evolution plot of $a_2$

(B.6) Mutual information of $a_2$

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(B.7) Evolution plot of $a_3$

(B.8) Mutual information of $a_3$

(B.9) Evolution plot of $a_4$

(B.10) Mutual information of $a_4$

(B.11) Evolution plot of $a_5$

(B.12) Mutual information of $a_5$

(B.13) Evolution plot of $a_6$

(B.14) Mutual information of $a_6$
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