Next Generation of Star Patterns

A Preprint

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Abstract

In this paper we present two new ideas for generating star patterns and filling the gaps during the tile operation. Firstly, we introduce a novel parametric method based on concentric circles for generating stars and rosettes. Using proposed method, completely different stars and rosettes and a set of new and complex star patterns convert to each other only by changing nine parameters. Secondly, we demonstrate how three equal tangent circles can be used as a base for generating tile elements. For this reason a surrounded circle is created among tangent circles, which represents the gaps in hexagonal packing. Afterwards, we use our first idea for filling the tangent circles and surrounded circle. This parametric approach can be used for generating infinite new star patterns, which some of them will be presented in result section. Two Android apps of proposed method called Starking and Tilerking are available in Google app store.

Keywords Star Pattern · Pattern Generation · CAD · Modeling

1 Introduction

Traditional symbols have significant role in cultural memory of each society. Utilizing traditional symbols in new intellectual products can help cultural continuity from one generation to the next. Moreover new intellectual products can be inspired by old patterns and can produce own symbols. For instance star patterns are one of the traditional symbols which can be used in social contexts as a reference to the past. However, in the field of drawing new types of star patterns, main problem is to find a regular approach for filling the gaps. Without this regular approach, tiling methods cannot be combined with pattern generation techniques and gaps should be filled by inference algorithm. Filling the gaps by an inference algorithm is mostly used by polygon in contact method [12], [13], [14]. Therefore polygon in contact method cannot be combined with pattern generation techniques for generating new types of Islamic stars. In this paper, we propose two new parametric methods for filling the gaps in hexagonal packing. In the first method, a hidden relationship between concentric circles is parameterized in order to generate stars and rosettes. Using this new parametric approach, completely different stars and rosettes convert to each other only by changing nine parameters. Moreover proposed method has sufficient flexibility for producing new generation of star patterns. In the second method we use three equal tangent circles as a base of tile operation. Then a surrounded circle is recognized among three tangent circles which represents the gaps during the tile operation. After finding the properties of surrounded circle, our parametric method is used for filling the tangent circles and surrounded circle. Finally this triplet pattern is repeated to cover the 2D space. This parametric approach for generating tile elements, provides the designers with a powerful tool to produce new types of tile patterns. The rest of this paper is organized as follows: section 2 reviews some of the previous works in the field of Islamic stars. Section 3 demonstrates main idea of the paper. Section 4 presents required parameters, formulas and algorithms for generating Islamic stars by concentric circles. Section 5 demonstrates a new idea for
generating tile elements by tangent circles. Section 6 presents some of our exclusive results. Finally, Section 7 concludes the paper and provides required discussion and possible future directions.

Figure 1: Three tangent circles and surrounded circle

2 Background

The most researches about Islamic stars fall in two main categories. First, symmetry as a traditional approach has devoted many researches to itself. Researches about symmetry are not restricted to Islamic stars [9], [20]. Also, Alexander [2] developed a FORTRAN program for generating the 17 types of design in the Euclidean plane. But exclusively in the field of Islamic stars, Grünbaum and Shephard [11] provided a complete set of mathematical tools for decomposing periodic Islamic patterns by their symmetry groups. Abas and Salman [11] applied this symmetry groups on a wide-ranging of historical patterns. Then Ostromoukhov [20] proposed a mathematical tool for computer generated ornamental patterns. In the other researches in this field Dewdney [7] presented a new arrangement based on reflecting lines through a regular classification of circles. Castéra [5] presented a method based on the creation of skeleton of eightfold stars and safts. Aljamali and Banissi [3] proposed a rational classification of Islamic Geometric Patterns (IGP) based on the Minimum Number of Grids (MNG) and Lowest Geometric Shape (LGS), which used in the construction of the symmetric elements. Despite the power of techniques that are based on symmetry, they are not able to propose a flexible approach for generating new star patterns. Dunham in his valuable research [8] adjusted several Islamic patterns to the hyperbolic plane. Second category of researches about Islamic stars was established by Hankin [12], [13], [14]. In recent years, scholars such as Lee [18], Critchlow [6], Craig S. Kaplan [16], [15] and Craig S. Kaplan and David H. Salesin [17] have referred to Hankins’s polygons-in-contact method. Finally Kaplan developed an applet for generating decorative patterns. Main problem of polygons-in-contact method is filling the gaps by an inference algorithm. Bastanfard and Mansourifar [4] purposed a new method for generating stars and filling the gaps. But all these methods cannot present a common approach for generating stars and rosettes. On the other hand previous methods can generate a restricted range of stars or rosettes and results are predictable. Compared with all the previous works our approach presents a novel method which is efficient in generating new generation of star patterns.

3 Generating Star Patterns by Concentric Circles

In this section we demonstrate a hidden relationship between concentric circles, which can be used for generating stars and rosettes. For analyzing given star pattern in part (a) of figure 2, the lines are hidden and beginning and end of the lines are clarified by marked points as shown in Part (b) of figure 2. As illustrated in part (c) of figure 2 all the marked points are located on three concentric circles. In fact each concentric circle is divided by eight marked points to eight equivalent segments. After recognizing concentric circles on stars, a specific number is assigned to each marked point as shown in figure 3. For this reason various start angles is selected for denoting first marked point on each concentric circle. For example first marked point of first concentric circle is located on 0 degree. But first marked point of second concentric circle is located on 22.5 degree and first marked point of third concentric circle is located on 45 degree. In this case the angle difference between first marked point of a concentric circle and first marked point of next concentric circle is 22.5 degree. In general, the angle difference between first marked points of two adjacent concentric circles is calculated as follows.

\[ \Delta = 360/(2N) \]

Where, number of segments on each circle is denoted by N. For generating a star pattern, marked points of two adjacent concentric circles should be connected according to a specific regularity. The instruction
Figure 2: The role of concentric circles to generate star patterns.

Figure 3: Dividing concentric circles for connecting marked points to each other is described in figure 4. For example, marked points 4 and 5 from second circle have been connected to marked point 4 of third circle. In fact marked point (x) and marked point ((x+1) mod (N)) from first circle have been connected to marked point (x) from the next circle. Regardless of the number of concentric circles this regularity always exists between two adjacent concentric circles.

Figure 4: Using numbers for connecting marked points.

3.1 Concentric Circles for Generating Rosettes

Even though stars and rosettes seem different, they are strikingly similar in many ways. For instance, concentric circles can be used for drawing rosettes, too. Figure 5 (left) shows an 8-pointed rosette. In this rosette four concentric circles can be recognized as shown in right image of figure 5. These circles and their marked
points follow from stars order. But there are points that do not follow from stars order. These special points have been located between circle(3) and circle(4) and are shown by green color. Therefore regularity of these points should be found between circle(3) and circle(4). Furthermore, special points are located on the specific circle. Radius of this specific circle is bigger than circle(3) and smaller than circle(4). The name of this specific circle is Green circle. Therefore Green circle is defined as a circle that, special points are located on it. To obtain the position of each special point, radius of Green circle and angle of each special point are required. For calculating the radius of Green circle, the spr parameter is decreased from radius of circle(4). Hence, angle of each special point can be calculated, using the angles of circle(4). As illustrated in figure 6, two special points are recognized for every marked point on circle(4). For instance, marked point (2) of circle(4) is connected to two special points, which have been shown by number 2. One special point’s angle is $\alpha$ degree smaller than angle(2) of circle(4) and next special point’s angle is $\alpha$ degree bigger than angle(2) of circle(4). Therefore using the spr and parameters, position of special points is obtained as follows.

$$x_{sp1} = (r(w) - spr) \cdot \cos(\text{angle}(i)) - \alpha + x_0$$
$$y_{sp1} = (r(w) - spr) \cdot \sin(\text{angle}(i)) - \alpha + y_0$$
$$x_{sp2} = (r(w) - spr) \cdot \cos(\text{angle}(i)) + \alpha + x_0$$
$$y_{sp2} = (r(w) - spr) \cdot \sin(\text{angle}(i)) + \alpha + y_0$$

Where $(x_{sp1}, y_{sp1})$ is position of first special point of angle(i) from circle(w) and $(x_{sp2}, y_{sp2})$ is position of second special point of angle(i) from circle(w). Special points are located between every two possible concentric circles. In these situations the bigger circle is a special circle which is the only difference between stars and rosettes.

4 Algorithm

In this section, we introduce required parameters and algorithms for drawing stars and rosettes by concentric circles. For parametrizing the stars and rosettes, number of concentric circles is denoted by $S$ parameter and circle division number is specified by $N$ parameter. Also angle(i) denotes angles of each concentric circle, circle(w) and $r(w)$ denote circles and their radiuses.
4.1 Algorithm of Dividing Circles

For drawing stars, concentric circles should be divided to a set of angles. Then a valid amount should be assigned to each angle. Following algorithm presents required algorithm.

1. Set \( t = 0 \) and valid amount to \( N \) and \( S \)
2. Set \( i = 1 \)
3. Set \( j = 1 \)
4. angle \( (j) \) of circle \((i) = t\)
5. \( t = t + (360 / N) \)
6. \( j = j + 1 ; \) if \( j < N \) go to step 4
7. \( t = t + (180 / N) \)
8. \( i = i + 1 \) if \( i < S \) go to step 3

4.2 Algorithm of Drawing Stars and Rosettes

In this section a common algorithm is presented for drawing the stars and rosettes. Using this algorithm various stars and rosettes are converted to each other only by changing nine parameters.

1. Set valid amounts for \( S, N, w, r(1) \) to \( r(S) \)
2. \( w = 1 \)
3. \( i = 1 \)
4. If \( \text{circle}(w + 1) \neq \text{special circle} \)
   \( \begin{align*}
   gx1 &= r(w) \times \cos(\text{angle}(i)) + x0 \\
   gy1 &= r(w) \times \sin(\text{angle}(i)) + y0 \\
   gx2 &= r(w) \times \cos(\text{angle}(i + 1) \mod N) + x0 \\
   gy2 &= r(w) \times \sin(\text{angle}(i + 1) \mod N) + y0 \\
   gx3 &= r(w + 1) - \text{spr} \times \cos(\text{angle}(i - a)) + x0 \\
   gy3 &= r(w + 1) - \text{spr} \times \sin(\text{angle}(i - a)) + y0 \\
   gx4 &= r(w + 1) - \text{spr} \times \cos(\text{angle}(i + a)) + x0 \\
   gy4 &= r(w + 1) - \text{spr} \times \sin(\text{angle}(i + a)) + y0 \\
   gx5 &= r(w + 1) \times \cos(\text{angle}(i)) + x0 \\
   gy5 &= r(w + 1) \times \sin(\text{angle}(i)) + y0 \\
   \end{align*} \)

Drawlines: From \((gx1, gy1)\) to \((gx3, gy3)\) and From \((gx2, gy2)\) to \((gx3, gy3)\)

From \((gx3, gy3)\) to \((gx5, gy5)\)

From \((gx4, gy4)\) to \((gx5, gy5)\)

8. \( i = i + 1 \) if \( i < N \) go to step 7
9. \( w = w + 1 \) if \( w < S \) go to step 3

Here we present two examples based on our method. In these examples two completely different stars and rosettes convert to each other only by changing nine parameters. First we set parameters according to part 1 of table 1. Part 1 of figure 7 shows the result of this example. In another example we set parameters according to part 2 of table 2. Part 2 of figure 7 shows the result of example.
Table 1: Required parameters to generate stars of Figure 7.

| Part | N | α | r1 | r2 | r3 | r4 | Spr | S | Sp |
|------|---|---|----|----|----|----|-----|--|----|
| 1    | 8 | 0 | 51 | 70 | 172| –  | –   | 3 | –  |
| 2    | 9 | 48| 93 | 225| –  | 180| -68  | 2 | 2  |

4.3 Generating Desired Sketches

In the field of Islamic stars two points should be considered as follows. First it’s important for the sides of the outer hexagons in a rosette to be parallel. Second it’s important to be able to control the angle formed by the rays meeting at the outermost circle. Figure 8 shows a desired 8-pointed rosette and its concentric circles. In this rosette, gradient of straight line between angle(1) of circle(1) and angle(1) of circle(2) is equal to gradient of straight line between angle(1) of circle(2) and angle(1) of circle(3). In general, gradient of straight line between angle(1) of circle(w) and angle(1) of circle(w + 1) is always equal to gradient of straight line between angle(1) of circle(1) and angle(1) of circle(2). For generating such desired sketches radius of concentric circles are assigned to appropriate amounts as follows.

$$r(w) = (\alpha * x_m - y_m)/(\theta * \cos(\alpha_n) - \sin(\alpha_n))$$

5 Tile operation

In this section we demonstrate our parametric approach for generating tile elements. For this reason, four steps should be taken as follows.

1. Drawing three equal tangent circles as a primitive sketch of tile element.
2. Recognizing a surrounded circle among tangent circles and calculating its properties.
3. Filling the tangent circles and surrounded circle by proposed method of section 3 and 4.
4. Repeating filled tangent circles and their surrounded circle to cover the space. Figure 9 (left) describes three tangent circles which gap area among them has been clarified by gray color. The right part of figure 9 shows that, the gap area can be bounded by a surrounded circle.

![Figure 9: Bounding the gap area by surrounded circle.](image)

5.1 Surrounded circle

Surrounded circle has two properties: the position of origin and radius. These properties is used for filling the surrounded circle by concentric circles. The origin of surrounded circle is located in center of equilateral triangle as shown in figure 10. Therefore the origin of surrounded circle can be calculated as follows.

\[
S_x = (x_A + x_B) \\
S_y = y_A + (y_C - y_A)/3
\]

Figure 10: Calculating properties of surrounded circle.

Where \((S_x, S_y)\) is the origin of surrounded circle and radius of tangent circles is calculated as follows.

\[
M = S_y - y_A \\
\sin(30) = \frac{M}{(R + r)} \\
r = 2M - R
\]

![Figure 10: Calculating properties of surrounded circle.](image)
6 Results

Using the parametric approach, unpredictable results can be generated. In this section we introduce some of the new star patterns which have been generated exclusively by our method. Figure 11 shows two new star patterns which can be converted to each other only by changing nine parameters. Required parameters for generating these patterns are presented in table 2. Moreover Figures 12, 13 and 14 show three new tile patterns which can be converted to each other only by changing nine parameters, too. Required parameters for filling tangent circles are presented in table 3. On the other hand for filling the surrounded circle two concentric circles have been used. These concentric circles are as follows: outer circle and inner circle. Radius of outer circle is equal to radius of surrounded circle and inner radius is higher or equal to 0.5 * (outer radius). Note that, two Android apps of proposed method called Starking [22] and Tilerking [23] have been published in Google app store and they are available in both free and pro versions.

![Figure 11: Star results: New types of star patterns generated by proposed method.](image)

| Result | N | α | r1 | r2 | r3 | r4 | Spr | S | Special circle |
|--------|---|---|----|----|----|----|-----|---|----------------|
| Left   | 9 | 34| 191| 189| 226| -89| 3   | 3 |                |
| Right  | 10| 62| 172| 109| 133| 125| -100| 4 | 2              |

Table 2: Required parameters for each star results.

![Figure 12: Tiling Result 1.](image)
Table 3: Required parameters for filling tangent circles.

| Result | N  | α  | r1 | r2 | r3 | r4 | Spr | S  | Special circle |
|--------|----|----|----|----|----|----|-----|----|----------------|
| 1      | 12 | 53 | 171| 23 | 214| -50| 2   | 2  | -            |
| 2      | 6  | 10 | 143| 145| 179| -70| 4   | 2  | -            |
| 3      | 12 | 23 | 123| 85 | 178| -7  | 3   | 2  | -            |

Figure 13: Tiling Result 2.

Figure 14: Tiling Result 3.

7 Conclusion

In this paper, we have purposed two new ideas for generating star patterns and filling the gaps during the tile operation. New ideas are as follows.

- Utilizing the concentric circles for parametrizing stars and rosettes.
- Utilizing three tangent circles as a base for drawing tile elements.

These new approaches have many benefits. First, proposed method presents a common approach for drawing stars and rosettes while other methods have not such capability. Second, only by changing nine parameters, star patterns from the simplest model to the most complex one can be converted to each other easily. Third, despite the symmetry based methods, our method uses a parametric approach for generating tile elements. For this reason we extract tile element from three tangent circles. Therefore various tile elements can be converted to each other easily. Furthermore using the purposed method, unpredictable stars and rosettes...
are generated. Some of these stars or rosettes cannot easily be produced by other methods. On the other hand our method uses a parametric approach for filling the gaps. Therefore inside sketch of gap area can be controlled with a parametric approach. The time complexity of purposed algorithm is $O(N \times S)$. Where $N$ is number of segments on each concentric circle and $S$ is number of concentric circles.

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