Parameterization of the "alpha" type stirling engine mechanism for use in the timber industry

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Abstract. This article discusses the parameterization of the crank-slider group of the Stirling engine to use in timber industry. Parametrization means involves into the equations a lot of variables: lengths of connecting rods of piston and displacer, positions of the axis of the cylinders. The dependences of the piston and displacer movements are obtained for a free layout of the λ-type Stirling engine mechanism. The boundary conditions of the lengths of the connecting rods are determined, taking into account the displacement of the cylinders’ axes in the plane perpendicular to the axis of the crankshaft. The resulting system of equations for the displacement of the piston from the displacement of the displacer will allow to obtain a numerical model without modeling the crank and connecting rods, which will significantly reduce the resource intensity of numerical calculations. Further studies will aim to investigate the operation of the Stirling engine in compressor mode. This will allow determining the temperature difference between the cold and hot parts, which will allow you to indirectly evaluate the efficiency of operation in the engine mode. This study will eventually determine the optimal parameters of the designed engine, depending on the power and type of fuel used.

1. Introduction

Recently, interest in engines with external heat supply has been growing again. The main reason is that hydrocarbon fuel is not an inexhaustible resource on the planet. At the same time, the issue of obtaining electricity is becoming more and more urgent. Global climate warming has a significant impact on electricity consumption – the cost of air conditioning for residential and office premises is growing year by year [1]. The issue of energy supply is particularly relevant for industrial facilities in remote areas with little developed infrastructure. For example - the logging companies. The traditional way to solve this issue is diesel generators. This technology of generating electricity has a high cost (to generate 0.2 kW*h it has to be burned 0.2 kg of diesel fuel [2]). At the same time, there is a problem of resource provision – delivery of fuel reserves to the place of use is quite expensive. At the same time, since the commercial forest reserves near the main infrastructure facilities are significantly...
depleted, the distances to the logging sites are increasing, which leads to a significant increase in
the logistics component of the cost of logging. A significant part of this component is the delivery and
storage of fuel for diesel power plants. There are developments that aim to reduce the cost of such
power generation by hybridizing energy systems. For example, the combined use of diesel and
photovoltaic generation shows opportunities to reduce the cost of generated electricity [3]. These
technologies will still require delivery and storage of fuel reserves. On the other hand, there are
usually quite a lot of other types of energy resources at remote sites of logging enterprises. Logging
waste is a source with great potential for generating heat. The easiest way to convert heat into
electricity, in that conditions, is to use an engine with an external heat supply to drive the generator.
The optimal type of such engine will be a Stirling cycle engine (hereinafter - SE).

Although SE have been known for a long time [4], their development and improvement were based
on the basic principles of thermodynamics and a huge amount of experimental research [5]. Therefore,
these engines could not sustain the pace of development of internal combustion engines that have a
well-developed theoretical base. Theoretical consideration of the processes occurring in SE is limited
to the study of individual elements and patterns. Such as the Shuttle effect in the "displacement piston-
cylinder" system [6], heat flows of the "working body-regenerator" system when changing the
direction of gas movement, the effect of "dead volume" on efficiency and specific power [7].

A comprehensive review of the working process of the SE and the development of a General theory
remains an unsolved problem. To solve this problem, it is necessary to use numerical methods for
solving complex tasks. Complex tasks are understood as problems of hydro-gas dynamics, heat
transfer, and mechanics of a deformable body in a single numerical model of the mechanism. Such
studies of complete three-dimensional models allow us to obtain certain dependencies of engine
performance on various design parameters. In particular, to study the transient and inertial processes
of flow of the working body in the internal volumes of the mechanism. For example, the opposite flow of
gases in the internal volumes of the piston and the displacer at a certain point in the working cycle [8].
The ability to perform numerical studies in the so-called "explicit formulation", that is, allowing to
study fast-flowing processes, allows us to study in detail the processes of heat and mass transfer in the
SE and their dynamics, including tribotechnical processes in the wall layers of the working fluid flow
and their influence on the thermodynamics of the entire system as a whole [9]. It will be possible to
obtain the optimal design and technological parameters of the SE and get the most efficient energy
machine that implements the Stirling cycle without solving such fundamental and extremely difficult
to solve today, such as sealing the working fluid [10].

Many researchers already have the results of such modeling, which confirm the feasibility of such
an approach when the entire system is modeled. The research results show a significant difference
between the thermodynamic process of oscillating flow and classical thermodynamic models [11].
This entails other results of evaluating the work performed by the work body. Full-factor studies that
allow us to determine the influence of design parameters on the efficiency of the SE show a significant
influence on the stroke of the piston and the displacer, as well as their diameter, have a significant
impact on the engine power due to their strong influence on the mass flow of the working fluid [12]. In
the above studies, no attention is paid to the study of the torque on the engine shaft and its dependence
on the impact of the working body on the piston, the displacer, and on the design of the crank-slider
mechanism as a whole. This is due to the fact that linking mechanical and thermodynamic models
requires the development of an analytical parametric model of the interaction of dynamic and inertial
parameters of the mechanical part on the thermodynamic processes occurring in the working body.
Although natural studies of the torque of the β-type SE depending on certain operational parameters
(boost pressure and heater temperature) were performed [13]. However, this approach does not allow
us to study internal thermodynamic processes and link them with the dynamics of the mechanical part
of the SE.

Thus, the development of the SE can be carried out most efficiently with the maximum use of
numerical modeling methods with the transition to "in metal" research only at the final stage.
2. Methods and Materials
The task of the theoretical study of this article is to determine the kinematic relationship between the relative position of the piston and the displacer of the $\lambda$-type SE, with a parametric model that allows you to change all the geometric parameters. These dependencies will be used for inclusion in the numerical model of the engine and allow us to perform the first stage of building a complete numerical model of the $\lambda$-type SE - to study the dynamics of gas flow from their hot to cold cavity in the compressor mode. It allows you to change the design parameters during the research and determine their influence on thermodynamic processes. At the first stage of research, we assume that the mechanical part of the engine is undeformable, and there is no heat loss to the environment.

The kinematic scheme of the engine is shown in the figure 1. The problem statement is as follows: the movement of the vertical piston (Y) will be set by a harmonic function. Since the gas-dynamic problem is being studied exclusively at this stage, there is no need to model the mechanical part. This will significantly reduce the resource intensity of modeling. In the future, when modeling in engine mode, it will be necessary to determine the torque on the engine crankshaft at any time. To do this, the values of the pressure on the piston and the displacer (including the vacuum pressure) will be obtained from the numerical thermodynamic model. The inertia component of the crank-slider mechanism, which affects the dynamic characteristics of the engine, will be taken into account by the moment of inertia of the flywheel. Further, it is necessary to deduce the dependence of the position of the displacer on the position of the piston $X(Y)$, which will further investigate the mutual change in the volumes of hot and cold cavities and the dynamics of gas flow. At the same time, it is necessary to consider the possibility of changing the angle between the axes of the cylinders (figure 1) as less than or more than 90°, up to 180°. Also, it is necessary to provide for the displacement of the cylinder axes and determine the mutual restrictions of the geometric parameters of the crank-slider group.

![Figure 1. The kinematic diagram of the SE type "alpha": $X$ - coordinate horizontal piston; $Y$ - coordinate of the vertical piston; $k$ - the offset of a crankpin; $l$ - connecting rod length; $h$ - height of the piston; $d$ - the diameter of the piston; $\beta$ - the angle of rotation of the crankshaft; $a$, $a1$ - the projection of the crankshaft on the horizontal and vertical axis, respectively; $b$ - the projection of the rod on a vertical axis; $b1$ is the projection of the rod on a horizontal axis.](image)

3. Results and Discussion
The dependence of the position of the displacer ($X$) on the position of the piston ($Y$) for the same lengths of connecting rods and the angle between the axes of the cylinders equal to 90° is determined [9]:

...
where, \( k \) – the offset of a crankpin; \( l \) – length of connecting rods of the piston and displacer.

To derive this equation, a system of equations for the coordinates of the displacement of the piston \((Y)\) and the displacer \((X)\), depending on the angle of rotation of the crankshaft, was determined, which has the form:

\[
\begin{align*}
Y &= k \cdot \cos \beta + \sqrt{l^2 - (k \cdot \sin \beta)^2} \\
X &= k \cdot \sin \beta + \sqrt{l^2 - (k \cdot \cos \beta)^2}
\end{align*}
\]  \tag{2}

As can be seen from the system above, it is enough to simply take into account the different lengths of the connecting rods of the piston and the displacer:

\[
\begin{align*}
Y &= k \cdot \cos \beta + \sqrt{L_x^2 - (k \cdot \sin \beta)^2} \\
X &= k \cdot \sin \beta + \sqrt{L_y^2 - (k \cdot \cos \beta)^2}
\end{align*}
\]  \tag{3}

where \( L_x \) and \( L_y \) – the lengths of the displacer and piston connecting rods, respectively.

After performing the same transformations as [14], we obtain the following equation of the position of the displacer \((X)\) from the position of the piston \((Y)\), with taking into account the different lengths of the connecting rods:

\[
X(Y) = \left[k^2 - \left(\frac{Y}{2} - \frac{L_y^2 + k^2}{2Y}\right)^2\right]^{0.5} + \left[l^2 - \left(\frac{Y}{2} - \frac{L_x^2 + k^2}{2Y}\right)^2\right]^{0.5} \tag{4}
\]

The next condition that determines the free arrangement of the crank-slider group of the SE is the possibility of changing the angle between the cylinder axes. A diagram with a free angle between the cylinder axes is shown in figure 2.

Determine the position of the piston \((Y)\) by rotating the coordinate axes method. Then, the rotation angle of the crankshaft to calculate the coordinates of the position of the piston is determined by the sum of the angle position of the crank from the vertical and the rotation axis of the cylinder of the piston from the vertical:

\[
\beta' = \beta + \alpha \tag{5}
\]

Then, the angle between the cylinder axes:

\[
\lambda = 90 + \alpha \tag{6}
\]
Figure 2. Kinematic diagram of the SE type "alpha" with a free angle between the cylinder axes and different lengths of connecting rods: $L_x$ and $L_y$ - lengths of connecting rods of horizontal and vertical pistons, respectively; $\gamma$ - angle between the axes of the pistons; $\alpha$ - angle of rotation of the vertical piston axis; $a^\prime$ - projection of the crankshaft on the axis of the vertical piston; $b^\prime$ - projection of the connecting rod of the vertical piston on the axis.

Let's transform the system (3):

$$
\begin{aligned}
Y &= k \cdot \cos(\beta + \alpha) + \sqrt{L_x^2 - [k \cdot \sin(\beta + \alpha)]^2} \\
X &= k \cdot \sin \beta + \sqrt{L_y^2 - (k \cdot \cos \beta)^2}
\end{aligned}
$$

(7)

Next, we need to define the form of the equation $X(Y)$. To do this, we define the coordinate of the horizontal piston $X$ as:

$$X = a_1 + b_1$$

(8)

or (figure 2):

$$X = \sqrt{k^2 - a^2} + \sqrt{L_x^2 - a^2}$$

(9)

At the same time, consider the following:

$$
\begin{align*}
a^\prime &= k \cdot \cos(\alpha + \beta) \\
a &= k \cdot \cos(\beta) \\
l &= \frac{a^\prime}{\cos(\alpha + \beta)} = \frac{a}{\cos(\beta)}
\end{align*}
$$

(10)
Then:

\[ a = \frac{a' \cos(\beta)}{\cos(\alpha + \beta)} \]  

(11)

In this case, the parameter \( a' \) will be equal to:

\[ a' = Y - b' \]  

(12)

Thus, substituting (11) and (12) in equation (9) we get:

\[ X = \left[ k^2 - \left( \frac{(Y - b') \cos(\beta)}{\cos(\alpha + \beta)} \right)^2 \right]^{0.5} + \left[ L_\gamma^2 - \left( \frac{(Y - b') \cos(\beta)}{\cos(\alpha + \beta)} \right)^2 \right]^{0.5} \]  

(13)

Next, consider the parameter \( b' \). This parameter can be defined by the length of the piston rod \( L_y \) and the angle of the piston rod to the axis of movement of the piston (figure 3):

\[ b' = L_y \cos(\alpha_y) \]  

(14)

Figure 3. Diagram for the definition: \( \alpha_y \) - angle of rotation of the connecting rod relative to the piston axis.

At the same time:

\[ \sin(\alpha_y) = \frac{k'}{L_y} = \frac{k \cdot \sin(\alpha + \beta)}{L_y} \]  

(15)

From here:

\[ \alpha_y = \arcsin \left( \frac{k \cdot \sin(\alpha + \beta)}{L_y} \right) \]  

(16)
Substituting the equation (16) to the formula (14), we get the following:

\[ b' = L_y \cdot \cos \left( \arcsin \left( \frac{k \cdot \sin(\alpha + \beta)}{L_y} \right) \right) \]  

(17)

Substituting (17) in (13) we get:

\[
X = \left\{ k^2 - \left[ \left( Y - L_y \cdot \cos \left( \arcsin \left( \frac{k \cdot \sin(\alpha + \beta)}{L_y} \right) \right) \cos(\beta) \right) \right]^{2} \right\}^{0.5} + \\
+ \left\{ L_x^2 - \left[ \left( Y - L_y \cdot \cos \left( \arcsin \left( \frac{k \cdot \sin(\alpha + \beta)}{L_y} \right) \right) \cos(\beta) \right) \right]^{2} \right\}^{0.5} 
\]  

(18)

Thus, we obtained the dependence of the position of the displacer in dependence on the position of the piston.

Substituting the first equation of system (7) to (18), we finally have the equation that describes the relationship of the coordinates of the piston and displacer in dependence on the crankshaft angle, lengths of connecting rods and offset crankpin:

\[
X = \left\{ \left[ k \cdot \cos(\beta + \alpha) + \sqrt{L_y^2 - \left[ k \cdot \sin(\beta + \alpha) \right]^2} - L_y \cdot \cos \left( \arcsin \left( \frac{k \cdot \sin(\alpha + \beta)}{L_y} \right) \right) \cos(\beta) \right]^{0.5} \right\} + \\
+ \left\{ \left[ k \cdot \cos(\beta + \alpha) + \sqrt{L_y^2 - \left[ k \cdot \sin(\beta + \alpha) \right]^2} - L_y \cdot \cos \left( \arcsin \left( \frac{k \cdot \sin(\alpha + \beta)}{L_y} \right) \right) \cos(\beta) \right]^{0.5} \right\} 
\]  

The obtained equations: the first equation of the system (7) and the final equation allow determining the positions of the piston and the displacer at different lengths of connecting rods and the angle between the axes of the cylinders of the piston and the displacer rod differing from 90°. This implies that the cylinder axes intersect with the crankshaft axis. This restriction does not allow you to talk about a free layout. To remove this restriction, it is necessary to take into account the possibility of displacement of the cylinder axes of the piston and the displacer from the initial position in a plane perpendicular to the axis of the crankshaft. The scheme for the definition is shown in figure 4.

Taking into account \( E_y \) and \( E_x \) converting the system of equations (7):

\[
\begin{align*}
Y_x &= k \cdot \cos(\beta + \alpha) + \sqrt{L_y^2 - \left[ k \cdot \sin(\beta + \alpha) + E_y \right]^2} \\
X_x &= k \cdot \sin \beta + \sqrt{L_y^2 - \left( k \cdot \cos \beta + E_x \right)^2} 
\end{align*} 
\]  

(19)
Thus, the free layout of the α-type SE implies the possibility of changing the following parameters (see figure 3): $k$ - the offset of a crankpin; $L_x$ - length of displacer; $L_y$ - length of piston rod; $\lambda$ - angle between piston and displacer axis; $E_x$ - displacement axes vertical piston; $E_y$ - displacement axes horizontal piston.

![Diagram](image_url)

**Figure 4.** The kinematic diagram of the SE type "alpha" free layout: $X_e$, $Y_e$ – coordinates of the horizontal and vertical pistons to the offset axes, respectively; $E_x$, $E_y$ – displacement axes horizontal and vertical pistons, respectively.

4. **Conclusion**

The obtained equations: the first equation of the system (7) and the final equation allow determining the positions of the piston and the displacer at different lengths of connecting rods and the angle between the axes of the cylinders of the piston and the connecting rod differing from 90°. This will allow the first stage of research: to create a numerical model of the working process of the SE in compressor mode. At the same time, modeling of the crank-slider group and its dynamic characteristics is not required at this stage. Only the working volumes of the piston, displacer, and bypass chambers will be modeled. This will allow a comprehensive study of the mutual influence of the design parameters of the crank-rod group on the gas dynamics and thermodynamics of the working medium. At the same time, as a result of modeling, we will get the value of heating of the enclosing structures of the piston chamber and cooling of the enclosing structures of the displacer chamber, since in this mode of operation, the SE works as a refrigerating machine. At the same time, it will be possible to change the size of the model to study the effect of a large-scale effect that affects gas dynamics and heat and mass transfer processes in the working body to achieve the highest efficiency. At this stage of research, efficiency will be evaluated as the value of the temperature difference between the hot and cold parts.

After performing a full-factor numerical study, it will be possible to justify the design parameters of the engine that are close to optimal for manufacturing a prototype engine for conducting a full-scale
test cycle in the compressor mode. Further, if we understand the inertial characteristics of the mechanism it will be possible to develop a numerical model of the SE that reflects the entire spectrum of processes occurring in it. The geometric constraints set in the model will allow us to determine acceptable ranges of parameter changes for a full-factor numerical study of the SE workflow. This study will eventually determine the optimal parameters of the designed SE, depending on the power and type of fuel used (sawmill waste) for use in the timber industry.

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