Abstract

Computer algebra in Java is a promising field of development. It has not yet reached an industrial strength, in part because of a lack of good user interfaces. Using a general purpose scripting language can bring a natural mathematical notation, akin to the one of specialized interfaces included in most computer algebra systems. We present such an interface for Java computer algebra libraries, using scripts available in the JSR 223 framework. We introduce the concept of 'symbolic programming' and show its usefulness by prototypes of symbolic polynomials and polynomial rings.

1 Introduction

Computer algebra in Java has many advantages over implementations in more rustic languages such as Lisp or C, and was shown to be reasonably fast compared to them [11, 12, 13]. However it lacks good user interfaces. The idea of using a general purpose scripting language as glue code and interface to C-libraries and standalone computer algebra systems was introduced by the Sage project [19] with its Python interface [5]. The fact that this solution brings a natural mathematical notation, was key in this project’s impressive development. We discuss how such interfaces can be built in the case of Java computer algebra libraries. We are considering scripts available in the JSR 223 framework [20], and also Scala [15] whose interpreted mode is not formally part of JSR 223 although this is planned.

1.1 Background and related work

We have started this discussion with the paper [7] where we have shown that at least the scripting languages Ruby, Groovy, Python and Scala are suitable as domain specific languages for computer algebra [21] [8] [9] [4]. In this paper we have described the following required features: object orientation, operator
overloading (in particular the availability of good operators for writing rational numbers and powers), and coercions (either by double-dispatch, base types redefinition, or implicit conversion). For each of the assessed scripting languages we have sketched a way to enter algebraic expressions and to transfer them to desired Java objects. Since then we have implemented most of these ideas in a Jython front-end to our two computer algebra libraries, JAS [14] and ScAS.

The former is a new approach to computer algebra software design and implementation in an object oriented programming language. It provides a well designed software library using generic types for algebraic computations implemented in the Java programming language. The mathematical focus of JAS is at the moment on commutative and solvable polynomials, Gröbner bases, an experimental factorization package and applications. JAS has a Jython interface and also a prototypical Groovy interface.

The latter (ScAS) is a reimplemention in Scala of JSCL [6], which was a pre-Java 5, non Generics attempt at a “Java symbolic computation library”. The new, generic type based implementation is inspired by the ideas developed in JAS, and is also meant as an exploration of the specific features of Scala. We are making access to these libraries with a selection of scripts. In its current state, ScAS can be accessed both by Scala in interpreted mode (which makes a nice, uniform solution), and Jython. JSCL was accessed with BeanShell [18].

There are other projects to implement a computer algebra system completely in Python: SymPy [2] or to provide user interfaces based on the Eclipse Rich Client Platform [3] as MathEclipse [10] does. For other related work see the discussion and references in [7].

1.2 Outline of the paper

In this paper we extend the concepts of input of algebraic expressions in a scripting language to the output of these expressions such that they can be reused as input in further computations. This is done in section 2 with an introduction into the concept of Symbolic programming. The main part of the paper in section 3 presents prototypes of symbolic polynomials and polynomial rings. For polynomial rings we partially solve the problem of defining objects representing algebraic structures. Finally, section 4 concludes.

2 Symbolic Programming

In this section we describe and formalize a new meta-programming technique baptized “symbolic programming”. This technique is applicable in any object oriented language, but is especially interesting in the case of interpreted languages, where the output of an instruction can be reused and combined in further statements to the interpreter, interactively. To our knowledge, it was first used on a significant scale, albeit not under this name, in Sage.
2.1 Reconstructing expression

We define the following items:

Definition 1. In a given language, an expression is said to be \textit{reconstructing} iff: (i) it is a valid expression of said language, and (ii) it has same input and output forms, in the sense of string equality. The output form is as obtained for example by the \texttt{toString()} method in Java. Expression reconstruction will usually depend on the actual context of the computation. So a context needs to be taken into account, since there are cases where, for the expression to be valid, itself or its constituents have to be defined in the sense of the programming language.

Definition 2. A \textit{context} $c$ is a set of definitions of variables or functions of the form $n(X_i)$, with $(X_i)$ a (possibly empty) list of parameters, such that in $c$, if $x_i$ is reconstructing for all $i$, then $n(x_i)$ is reconstructing, i.e. $n(x_i)$ has output form $n(x_i)$.

The next definition is central to the concept of reconstructing expressions.

Definition 3. In a given language, an expression is said to be \textit{reconstructing in context $c$} iff: (i) it is a valid expression of said language, taking $c$ into account, and (ii) it has same input and output forms.

2.2 Symbolic object, symbolic type

We further define:

Definition 4. In a given language and context $c$, an object is called a \textit{symbolic object} iff its output form is reconstructing in $c$, as defined in 3.

Definition 5. In a given language, a type $t$ is called a \textit{symbolic type} iff for all instances $o$ of $t$ there exists a context where $o$ is symbolic.

Here are some first examples.

Java’s type \texttt{int} is symbolic because the output form of, say 1 is 1, which is the same as the original expression.

Note. Symbolic programming as understood in this trivial sense is available in any known programming language, not just object oriented languages. More interesting examples follow.

The Java class \texttt{String} is \textit{not} symbolic because "hello world".\texttt{toString()} yields \texttt{hello world}, which is not the same as "hello world".

Contrarily, Scala’s \texttt{Symbol} class is symbolic, because ‘a yields ‘a.

Java’s type \texttt{long} is not symbolic because of the ‘l’ or ‘L’ that must be appended to literals and gets removed when the value is printed.

2.3 Container types: array, collections, tuples

A Java array, as defined for example in

```java
int n[] = new int[] {1, 2};
```

is not symbolic, because it yields a unique identifier, not its contents. Scala’s class \texttt{List[A]}, however, is symbolic (if A is), as shown below.
Definition 6. A container type \( q \) is called a \textit{symbolic container type} iff for any instance \( p \) of \( q \), if all the components of \( p \) are symbolic, then \( p \) is symbolic.

One can show that a symbolic container type \( q \) is a symbolic type if its component type(s) is (are) symbolic.

Example: Scala’s class \( \text{List}[A] \) is a symbolic container type, since its instances are symbolic when their components are: \( \text{List}(1, 2) \) yields \( \text{List}(1, 2) \). Hence, if \( A \) is a symbolic type, then so is \( \text{List}[A] \).

2.4 Applications

Big integers

The class \( \text{BigInteger} \) of Java is not a symbolic class, as its output form becomes an invalid Java expression when the value exceeds integer capacity. Hence we have to define our own symbolic BigInt class. As an output form, we can choose: \( \text{new BigInt}("1") \). The resulting code is:

```java
import java.math.BigInteger;

public class BigInt {
    BigInteger value;

    public BigInt(String val) {
        value = new BigInteger(val);
    }

    public String toString() {
        return "new BigInt(\""+value.toString()+\"\")";
    }
}
```

The \( \text{toString()} \) produces the desired \( \text{new BigInt("... ")} \), which in turn is a valid constructor expression to get the big integer back.

Fractions

We want to define a Rational class that enables us to encode e.g. \( \frac{1}{2} \). Regarding the output form, we would ideally have indeed \( 1/2 \), but no scripting language exists that won’t interpret this as an int or float. In some of them (Ruby, Groovy), the divide operator can be overloaded not to execute the division, but then we can’t use it for integer division anymore. Some other scripting languages allow to define and use an operator such as \( 1//2 \).

In Python, an appealing syntax is with a Tuple \( (a, b) \) or List \( [a, b] \). Indeed these types are symbolic container types, so that \( (1, 2) \) yields \( (1, 2) \). It should be noted however that this notation is limited in its operations, for one could not write \( (1, 2)+(2, 3) \) and expect the rational sum as a result.

To improve on this, we consider notations such as \( \text{frac}(a, b) \). In Java, we can populate the context with the following definition:
Rational frac(int n, int d) {
    return new Rational(n, d);
},

and define the following class:

public class Rational {
    int numerator;
    int denominator;

    public Rational(int n, int d) {
        numerator = n;
        denominator = d;
    }

    public String toString() {
        return "frac(\+n+, \+d+)";
    }
}

Alternatively we could avoid using contexts and define an output form as new BigRat(1,2) or new BigRat ("1","2"), like the BigInt above.

**Powers**

There are several possible operators in the various scripting languages. In Python, Ruby, Groovy, a**2 is available. In Scala, it is not possible because ** has the same precedence as *; there are other characters with a higher precedence like \, but there is another problem: unary operators such as - have a even higher precedence, which means that -x\2 becomes x\2. In all the former languages and in Java, one can use a notation similar to that used above for fractions: pow(a, 2).

2.5 **Type preservation**

In 2.4 we have defined a syntax that is a bit clumsy for big integers, perhaps we could note these like regular integers for small enough values. For this to happen, their type will have to mutate.

Definition 7. A symbolic type is said to mutate iff some of its instances have an output form that is an expression with a different type than their input form.

Definition 8. The types reachable from a class A through transitive closure of the relation “type A mutates to type B” form the *symbolic type group* of A.

Definition 9. A symbolic type whose symbolic type group contains a single element (itself) is said to be preserved.

Our BigInt example above becomes:
public class BigInt {
    ...
    public String toString() {
        return value.bitLength()<32 ?
            value.toString() :
            "new BigInt("+value.toString()+")";
    }
}

The resulting symbolic type group is the set (int, BigInt).

2.6 Type preservation examples

Big Integers

In Python, the set (PyInteger, PyLong) is a symbolic type group. One has to note that these types have a uniform syntax: no signalization is added when 32 bit precision is exceeded (and there is no further limit since PyLong is the equivalent of Java’s BigInteger), which is much nicer than what we proposed above for the Java case.

Polynomials

This is in slight anticipation to section 3 where the polynomial case is discussed in detail. Suppose we define a Polynomial type, with the following syntax: 1+x. When, for example, we subtract x, the type mutates, and we get 1, which is of type int. If we want the Polynomial type to be preserved, we can insert a conversion like x.valueOf(1). Alternatively, we can use an unsimplified syntax like x^0, which is still of type Polynomial. In both cases, we don’t get a natural mathematical notation as we aim, so we will have to use a mutating type. Note: type preservation is context-dependent, as shown in the next example.

Rationals

If we define a Rational type with the syntax frac(1, 2), then depending on the return type of the frac method, the type could either stay the same as in our application in 2.4, or it could become a floating point if the result is defined to be 0.5 (as we would do if indeed we want a numeric evaluation).

ModIntegers

ModIntegers can carry the modulo information with them: 5 mod 11 can be noted mod(5, 11). But we can also remove the modulo without causing any error. On the other hand, symmetric (negative) modular numbers, do need the information because otherwise the value will be wrong: $-5 \equiv 6 \mod 11$ which is not the same as $-5$. In fact, it depends what other objects it is supposed to interact with: if all numbers are given $\mod 11$, then we get correct results.
In general, a type must be enabled for interaction with all the members of its symbolic type group, and reciprocally.

3 A Symbolic Polynomial type

Polynomials are ubiquitous in computer algebra. They allow to manipulate unknowns such as $x$ as if they were numbers, which is the essence of symbolic computation. A polynomial is defined over a base ring (a set of elements which can be added, subtracted, multiplied and which has a zero and a one) and is itself a ring element. Therefore, we must enable both our base type and our polynomial type for these ring operations.

3.1 Library class definitions

In our backing Java library we assume for example the following classes. **BigInt** is our prototype for base coefficients and **Polynomial** is the main polynomial class. For a complete list of currently available base coefficients and polynomial implementations see our online documentation at [14][6]. The backing Java class library can be implemented directly in Java as in JAS or indirectly via Scala, which generates Java byte code, as in ScAS.

```java
class BigInt {
    BigInt add(BigInt that);
    BigInt subtract(BigInt that);
    BigInt multiply(BigInt that);
}

class Polynomial {
    Polynomial add(Polynomial that);
    Polynomial subtract(Polynomial that);
    Polynomial multiply(Polynomial that);
    Polynomial pow(BigInt exp);
}
```

We further assume that polynomials are created using polynomial factories in the library.

```java
Polynomial x =
    new PolynomialFactory(
        new BigInt(), new String[] {"x"}
    .generator(0)
```

This design is meant to separate the informations about the nature of the ring from those which regard the operations on its elements [16][17]. It further turns abstract mathematical entities like polynomial rings to first class citizens of the programming language.
3.2 Abstract coefficient type

In JAS the classes just defined inherit from an abstract type, using F-bounded polymorphism [1] as available in Java since version 5 (JDK 1.5):

```java
interface Ring<T extends Ring<T>> {
    T add(T that);
    T subtract(T that);
    T multiply(T that);
},
```

which allows to define a generic polynomial type:

```java
class Polynomial<C extends Ring<C>> implements
    Ring<Polynomial<C>> {
    Polynomial<C> add(Polynomial<C> that);
    Polynomial<C> subtract(Polynomial<C> that);
    Polynomial<C> multiply(Polynomial<C> that);
    Polynomial<C> pow(BigInt exp);
}
```

This polynomial class can have any ring class as its base ring, including itself, which is quite powerful, but poses a challenge regarding our aimed natural mathematical notation. This is where scripting comes into play. Below we exemplify with Jython, but other scripts are possible.

3.3 Coercion of coefficients

A polynomial must be able to be added, subtracted etc. (to) its coefficients, which are part of its symbolic type group (i.e. the polynomial type can mutate to the type of its coefficients), and reciprocally. We could define ad-hoc methods such as:

```java
Polynomial<C> add(C coef);
Polynomial<C> subtract(C coef);
Polynomial<C> multiply(C coef);
```

The first problem is with inversed operands, when the coefficient comes first. As we’ve seen in [7], section 3.1, in Python this is addressed by double-dispatch. A second problem is that when the base ring is itself a polynomial ring, we must accept not only coefficients but coefficients of coefficients, and so on, which rules this technique out. Instead, we can coerce coefficients to the polynomial type, using a method such as:

```java
Polynomial<C> valueOf(C coef);
```

Then, to add 1 to \( y \in \mathbb{Z}[x][y] \), we will have to write: \( y.add(y.valueOf(x.valueOf(1))) \). Again, scripting will come to help and make this coercions for us
silently. This is possible because, unlike Java, Python is dynamically typed and can use the same method to add $y$ or $1$. In said method, some `isinstance()` calls check the type of the argument and perform the required nested calls to `valueOf()`.

We could emulate dynamic typing in Java, using method arguments of type `Object` (or any other relevant superclass) and then check the run-time type with `instanceof` and make type casts. In fact, this is what we did in the former version of JAS and JSCL. But Java 5 Generics polymorphism allows a much safer design, where adding apples to oranges is forbidden at compile time. The downside is that the entailed conversions are to the user’s burden. Scripting comes in to solve this problem and enables the best of both worlds: type safety for the lower layers with an easy to use scripting interface.

### 3.4 Output form

We would like our polynomial to be a symbolic container type for its coefficients (see definition 6), such that $x^{\text{pow}(2)}.\text{add}(x.\text{multiply}(2)).\text{add}(1)$ is reconstructing. But this is not quite yet a natural mathematical notation, and we need operator overloading [7]. It enables such output form as $x^{\times2+2\times1}$, valid in several scripting languages, including Python (but not BeanShell for instance).

### 3.5 Implementation

A prototype of these ideas is given below. Note that the coercion of the coefficients is performed not by the polynomial itself, but by a factory. This design reflects the design of the Java class libraries as explained in section 3.1. Our Jython scripting interface using the ScAS API is implemented as follows.

Class `Ring` is the main scripting interface for polynomial factories.

```python
class Ring:
    def __init__(self, ring, vars, 
                 ordering=Lexicographic):
        self.ring = PolynomialFactory(ring, 
                                       vars, ordering)

    def __str__(self):
        return str(self.ring)

    def gens(self):
        return [RingElem(x) for x in 
                 self.ring.generators()]
```

The method named `__init__` is the constructor in Python and method `__str__` has fixed meaning in Python, which is similar to Java’s `toString()` method. Our method `gens()` returns a list of generators for the respective
polynomial ring. The generators are actual Java objects in the respective library wrapped by the scripting RingElem class.

def lift(factory, p):
    if not factory.equals(p.factory()):
        p = factory.valueOf(lift(factory.ring(), p))
    return p

The standalone lift() method above is used to lift a coefficient to an element of the given polynomial ring. The following RingElem is the main scripting interface for polynomials.

class RingElem:
    def __init__(self, elem):
        self.elem = elem

    def __str__(self):
        return self.elem.toString()

    def __abs__(self):
        return RingElem(self.elem.abs())

    def __neg__(self):
        return RingElem(self.elem.negate())

    def __mul__(self, other):
        (s, o) = self.coerce(other)
        return RingElem(s.elem.multiply(o.elem))

    def __rmul__(self, other):
        (s, o) = self.coerce(other)
        return RingElem(o.elem.multiply(s.elem))

# same for add, sub, etc

def __pow__(self, exp):
    return RingElem(self.elem.pow(int2bigInt(exp)))

def __eq__(self, other):
    (s, o) = self.coerce(other)
    return s.elem.equals(o.elem)

As mentioned already, the method with leading and trailing underscores in their names have predefined meaning in Python, which is easy to guess:
__mul__(.) implements the multiplication operator ‘*’ of Python, etc. Our implementation of these methods just delegate to the respective Java object method invocations.

```python
def coerce(self, other):
    base = self.base()
    if isinstance(base, BigInt):
        if isinstance(other, PyInteger):
            other = RingElem(int2bigInt(other))
        elif isinstance(other, PyLong):
            other = RingElem(long2bigInt(other))
        if self.depth() < other.depth():
            return (other.lift(self), other)
        else:
            return (self, self.lift(other))

def lift(self, other):
    return RingElem(lift(self.factory(), other.elem))

def depth(self):
    n = 0
    r = self.factory()
    while isinstance(r, PolynomialFactory):
        n += 1
        r = r.ring()
    return n

def base(self):
    r = self.factory()
    while isinstance(r, PolynomialFactory):
        r = r.ring()
    return r

def factory(self):
    return self.elem.factory()
```

Our method coerce() has to adjust the types of this element and the other element. It uses the methods base(), depth(), lift() and factory() to convert Python objects to RingElem objects and then to adjust and coerce types, so that the __X__ operations are well defined.

### 3.6 Sample session

Using our interface is as simple as adding the library to the classpath, running Jython, and making some imports. As desired the polynomials are symbolic, since for example the polynomial expression x+x*y is printed exactly as x+x*y.
Jython 2.2.1 on java1.6.0_11
Type "copyright", "credits" or "license" for more information.
>>> from interface import Ring, BigInt
>>> r = Ring(BigInt(), ["x"])
>>> print(r)
ZZ[x]
>>> [x] = r.gens();
>>> print(1+x)
1+x
>>> [y] = Ring(x.factory(), ["y"]).gens()
>>> print(x+x*y)
x+x*y
>>> [z] = Ring(y.factory(), ["z"]).gens()
>>> print((1-z)**2)
1-2*z+z**2
>>> print(z.factory())
ZZ[x][y][z]

Note, the ring objects are not symbolic, as Ring(BigInt (), ["x"]) has output form ZZ[x], which is not reconstructing.

3.7 Symbolic ring factory

Our algebraic structures, for example polynomial rings, are represented by factory classes and instantiated objects in the Java and Scala libraries. This design is also followed in the scripting interface. For these objects definition 4 applies and we look for a scripting interface to these objects which makes them symbolic objects. In the following we sketch an implementation of a symbolic polynomial ring factory. The sample code is based on the JAS API.

class PolyRing(Ring):
def __init__(self,coeff,vars,order):
    if coeff == None:
        raise ValueError, "No coefficient."
    cf = coeff;
    if isinstance(coeff,RingElem):
        cf = coeff.elem.factory();
    if isinstance(coeff,Ring):
        cf = coeff.ring;
    if vars == None:
        raise ValueError, "No variables given."
    names = vars;
    if isinstance(vars,PyString):
        names = StringUtil.variableList(vars);
    nv = len(names);
to = PolyRing.lex;
if isinstance(order,TermOrder):
    to = order;
ring = GenPolynomialRing(cf,nv,to,names);
sself.ring = ring;
def __str__(self):
    cf = self.ring.coFac;
cfac = cf;
if cf.equals(BigInteger()):
    cfac = "ZZ()";
    # ...
if cf.getClass() == \
    GenPolynomialRing(BigInteger(),1)
    .getClass():
    cfac = str(PolyRing(cf.coFac,\
    cf.varsToString(),cf.tord));
    to = self.ring.tord;
tord = to;
if to.evord == TermOrder.INVLEX:
    tord = "PolyRing.lex";
if to.evord == TermOrder.IGRLEX:
    tord = "PolyRing.grad";
nvars = self.ring.varsToString();
return "PolyRing(%s,%s,%s)" %
    (cfac, "\"+%s\"", tord);
lex = TermOrder(TermOrder.INVLEX)
grad = TermOrder(TermOrder.IGRLEX)

The __init__ constructor takes the main constituents of a polynomial ring
as parameters: coeff, a factory for coefficients, vars, the names of the variables
and order, the desired term order. From this information we assemble
the required parameters for the library polynomial ring constructor GenPoly-
nomialRing. There are two constants lex and grad which represent term orders
as defined in the library. The method __str__ assembles the string parts from
the library ring object and returns a string, which is identical to the PolyRing
expression. The low level tests cf.equals(...) or cf.getClass() and the ex-
PLICIT construction of the output form will be replaced in the future by a method
cf.toScript(). This method will be similar to the usual cf.toString() except
it will yield an output form which is reconstructing in the respective scripting
language.

A sample session using this PolyRing code looks as follows.

>>> r = PolyRing(ZZ(),"B,S",PolyRing.lex);
>>> print "Ring: " + str(r);

Ring: PolyRing(ZZ(),"B,S",PolyRing.lex)
Which shows that the defined ring is symbolic. This construction works also for recursive polynomial rings using \( r \) as coefficient factory.

```python
>>> pr = PolyRing(r,"T,Z",PolyRing.lex);
>>> print "PolyRing: " + str(pr);
PolyRing: PolyRing(
    PolyRing(ZZ(),"B,S",PolyRing.lex),
    "T,Z",PolyRing.lex)
```

ZZ() denotes the class corresponding to BigInt above which must be available in the context.

## 4 Conclusion

We have extended the work outlined in our previous article \[7\], where we studied the suitability of four scripting languages for symbolic algebraic computation. In this paper we focused our work on only one scripting language, namely Python with its Java implementation Jython. However, we expect that our work can also be transfered at least to Ruby (JRuby). Groovy was not further considered because we had to use either “use” blocks, with limited interactivity, or ExpandoMetaClasses which require to redefine base types in low-level Java. Our view of Scala has shifted a bit away from a scripting language to a language suited for library development as well.

We have shown how to design a scripting interface for Java computer algebra implementations which satisfies the concept of reconstructing expressions. This concept makes it possible to reason precisely about various parts of the user interface for computer algebra systems backed by elaborated programming libraries. We can handle the input of polynomial expressions and have precise requirements for the output of the algebraic transformations. This goes far beyond the capabilities of our Java systems before, as they were limited to custom parsers and output routines. We now have also some concept for the representation of ring factories in the scripting language as symbolic objects. To demonstrate our ideas we designed a symbolic polynomial with a prototype implementation in Jython as front-end to our Java libraries JAS and ScAS.

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