Weak Radiative Hyperon Decays in Chiral Perturbation Theory

Elizabeth Jenkins,\textsuperscript{a}\textsuperscript{*} Michael Luke,\textsuperscript{b} Aneesh V. Manohar,\textsuperscript{a}\textsuperscript{*} and Martin J. Savage\textsuperscript{b}\textsuperscript{†}

\textsuperscript{a}) CERN TH Division, CH-1211 Geneva 23, Switzerland
\textsuperscript{b}) Department of Physics, University of California at San Diego, 9500 Gilman Drive, La Jolla, CA 92093

Abstract

The parity-conserving $a$ and parity-violating $b$ amplitudes for weak radiative hyperon decay are studied using chiral perturbation theory. The imaginary parts of $a$ and $b$ are computed using unitarity. The real part of $b$ is dominated by a one-loop infrared divergent graph which is computed. The real part of $a$ has a large theoretical uncertainty and cannot be calculated reliably. Counterterms for the $a$ and $b$ amplitudes are classified using $CPS$ symmetry. The experimental values for decay widths and asymmetries are consistent with theory, with the exception of the asymmetry parameter for the $\Sigma^+ \rightarrow p\gamma$ decay.

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* On leave from the University of California at San Diego
† SSC Fellow
1. Introduction

Weak radiative hyperon decays (WRHD), such as $\Lambda \rightarrow n\gamma$, $\Sigma^+ \rightarrow p\gamma$, etc., have been of theoretical and experimental interest for some time. Early theoretical work on these decays focused on pole models, in which the initial baryon turns into an intermediate state by a $\Delta S = 1$ weak transition, followed by the decay of the intermediate state by the radiation of a photon \cite{1, 2}. The limited success of pole models for WRHD led to a variety of other approaches. Gilman and Wise \cite{3} investigated the possibility that WRHD proceeds via a direct $s \rightarrow d\gamma$ transition. The authors concluded that this assumption was not correct, however, because the theoretical predictions of the model were incompatible with experiment. Further attempts include the study of WRHD in the Skyrme model \cite{4} and using an effective Lagrangian \cite{5}.

Although no theoretical analysis has resulted in a complete description of WRHD thus far, certain long distance contributions to WRHD have been determined using very general arguments. Since the decay $\Lambda \rightarrow n\gamma$ can proceed via the physically allowed weak decay $\Lambda \rightarrow p\pi^-$, followed by $p\pi^- \rightarrow n\gamma$, the imaginary part of the $\Lambda \rightarrow n\gamma$ amplitude is determined in terms of the known amplitudes for the hyperon nonleptonic decay $\Lambda \rightarrow p\pi^-$ and for pion photoproduction $p\pi \rightarrow \gamma n$. This method was used by Farrar \cite{2} to place a unitarity lower bound on WRHD rates. Kogan and Shifman \cite{6} showed that the real part of the WRHD amplitude has a $\ln M_{\pi}^2$ contribution which is computable in terms of the imaginary part of the amplitude using dispersion relations. For some recent work on WRHD and a more extensive discussion of earlier work and additional references, see ref. \cite{7}.

In this paper, we give a model independent analysis of the WRHD parity-conserving $a$ and parity-violating $b$ decay amplitudes using chiral perturbation theory. Chiral perturbation theory provides the means for calculating these decay amplitudes in terms of a systematic expansion in powers of the Goldstone boson masses and the momentum of the radiated photon. The dominant contributions to the decay amplitudes can be computed in terms of known constants with no free parameters.

The calculation of the decay amplitudes is broken down as follows. The imaginary parts of the $a$ and $b$ decay amplitudes are computed using unitarity as detailed by earlier authors \cite{2, 6}. The real part of the decay amplitude $b$ is dominated by a one-loop graph which is infrared divergent in the chiral limit. This graph yields the $\ln M_{\pi}^2$ contribution discussed by Kogan and Shifman. The real part of $a$ is determined by pole diagrams in
addition to a one-loop graph. However, the computation of this graph in chiral perturbation theory suffers from large uncertainties and the diagram cannot be computed reliably. In our analysis, we treat the real part of $a$ as an unknown.

All possible counterterms which contribute to WRHD are determined. $\text{CPS}$ symmetry is used to reduce the number of counterterms. Four counterterms are allowed for the $a$ amplitudes, whereas only one counterterm is allowed for the $b$ amplitudes. The magnitude of the $b$ counterterm is estimated using naive dimensional analysis \cite{footnote1}. The counterterm contribution is approximately 20% of typical $b$ amplitudes. The $b$ counterterm does not contribute to $\Sigma^+ \to p\gamma$ or $\Xi^- \to \Sigma^-\gamma$ decay since the parity violating amplitudes for these decays are purely $\text{CPS}$ violating, as shown originally in ref. \cite{footnote2}.

The theoretical predictions for the decay widths and asymmetries are compared with experiment. The experimental data is consistent with theory with the exception of the asymmetry parameter for $\Sigma^+ \to p\gamma$. This asymmetry parameter does not agree with the data, even when $\text{Re} \, a$ is treated as a free parameter. The experimental value of the asymmetry parameter for $\Sigma^+ \to p\gamma$ is $\alpha = -0.83 \pm 0.12$\cite{footnote2}. The maximum theoretical asymmetry consistent with unitarity is $\alpha = -0.8$. It is only possible to get this upper bound value for the asymmetry if one includes a $\text{CPS}$-violating counterterm which is about 35 times larger than its naive value, or if the short distance contribution is enhanced by about 20.

This paper is organized as follows. Sect. 2 begins with a brief description of chiral perturbation theory for baryons. Definitions for WRHD amplitudes, widths and asymmetries are given. The next three sections discuss short distance, counterterm, and long distance contributions to WRHD amplitudes. The short distance contribution is unimportant. $\text{CPS}$ symmetry is used to constrain the number of counterterms in Sect. 4. The main computation of this paper, the calculation of long distance contributions to the decay amplitudes, is given in Sect. 5. The theoretical analysis is compared with experiment in Sect. 6. More detailed formulæ for the computation described in Sect. 5 are contained in the appendix.

\footnote{1}{There has been a new measurement recently by the E761 group \cite{footnote10} of $-0.72 \pm 0.086 \pm 0.045$. The value used in the text is that of the 1992 Particle Data Book.}
2. Formalism

The WRHD amplitudes will be computed using the static baryon formulation of chiral perturbation theory developed in ref. [11]. In this formalism, baryons are described by velocity dependent fields $B_v(x)$, where $v^\mu$ is the four-velocity of the baryon\footnote{Velocity dependent fields were originally introduced in the study of heavy quark symmetries in QCD [12].}. The field $B_v(x)$ is related to the conventional baryon field $B(x)$ by the transformation

$$B_v(x) = \frac{1 + \gamma^\mu v^\mu}{2} e^{im_B v \cdot x} B(x), \quad (2.1)$$

where $m_B$ is the baryon mass. The advantage of using the field $B_v$ is that derivatives on the baryon field produce factors of the residual moment $k$, which is related to the total momentum $p$ by $p = m_B v + k$. For baryons interacting with low-momentum Goldstone bosons, the residual momentum is small because the baryons are nearly on-shell. Higher derivative terms in the chiral Lagrangian are then suppressed by factors of $k/\Lambda_\chi$, where $\Lambda_\chi \sim 1$ GeV is the chiral symmetry breaking scale. Consequently, the static baryon formulation of chiral perturbation theory has a systematic derivative expansion. The theory also has a systematic loop expansion [13].

The leading terms in the baryon chiral Lagrangian are

$$L_v^0 = i \mathrm{Tr} \mathcal{B}_v (v \cdot \mathcal{D}) B_v + 2D \mathrm{Tr} \mathcal{B}_v S_\mu^v \{ A_\mu, B_v \} + 2F \mathrm{Tr} \mathcal{B}_v S_\mu^v [ A_\mu, B_v ] + \ldots, \quad (2.2)$$

where

$$B_v = \left( \begin{array}{ccc} \frac{1}{\sqrt{2}} \Sigma_v^0 + \frac{1}{\sqrt{6}} \Lambda_v^0 & \Sigma_v^+ & \rho_v \\ \Sigma_v^- & -\frac{1}{\sqrt{2}} \Sigma_v^0 + \frac{1}{\sqrt{6}} \Lambda_v^0 & n_v \\ \Xi_v^- & \Xi_v^0 & -2\frac{1}{\sqrt{6}} \Lambda_v^0 \end{array} \right), \quad (2.3)$$

is the matrix of baryon fields,

$$V^\mu = \frac{i}{2} (\xi \partial^\mu \xi + \xi^\dagger \partial^\mu \xi) + \frac{1}{2} i e A^\mu \left( \xi^\dagger Q \xi + \xi Q \xi^\dagger \right),$$

$$A^\mu = \frac{i}{2} (\xi \partial^\mu \xi^\dagger - \xi^\dagger \partial^\mu \xi) - \frac{1}{2} i e A^\mu \left( \xi Q \xi^\dagger - \xi^\dagger Q \xi \right), \quad (2.4)$$

$A^\mu$ is the photon field, and the ellipses denote terms with additional derivatives or insertions of the light quark mass matrix. The Goldstone bosons are contained in the field $\xi$,

$$\xi = e^{i\pi/f}, \quad \Sigma = \xi^2 = e^{2i\pi/f}, \quad (2.5)$$
where

\[
\pi = \begin{pmatrix}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & K^+ \\
\pi^- & -\frac{1}{\sqrt{2}} & K^0 \\
\pi^0 & -\frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}}
\end{pmatrix},
\]

(2.6)

and \( f \approx 132 \text{ MeV} \) is the pion decay constant. Under a chiral \( SU(3)_L \times SU(3)_R \) transformation,

\[
\Sigma \to L\Sigma R^\dagger,
\]

\[
\xi \to L\xi U^\dagger = U\xi R^\dagger,
\]

\[
B_v \to UB_v U^\dagger,
\]

(2.7)

where \( U \in SU(3) \) is defined implicitly by the transformation of \( \xi \). Note that the chiral Lagrangian Eq. (2.2) has no baryon mass term. This feature is what ensures that the theory has a consistent loop expansion, because large loop corrections of the form \( m_B/\Lambda_\chi \sim 1 \) are never obtained since positive powers of \( m_B \) never occur in any Feynman diagram.

The effective Lagrangian for the WRHD \( B_i \to B_f \gamma \) has the form

\[
\mathcal{L} = \frac{eG_F}{2} B_{vf} (a_{if} + b_{if}\gamma_5) \sigma^{\mu\nu} F_{\mu\nu} B_{vi},
\]

(2.8)

where \( G_F \) is the Fermi coupling constant, and \( a_{if} \) and \( b_{if} \) are the parity-conserving and parity-violating decay amplitudes which have dimensions of mass. The normalization convention used here is that of Gilman and Wise [3]. The decay amplitude obtained from Lagrangian (2.8) leads to the decay width

\[
\Gamma (B_i \to B_f \gamma) = \frac{G_F^2 e^2}{\pi} \left( |a|^2 + |b|^2 \right) \omega^3,
\]

(2.9)

where \( \omega \) is the energy of the radiated photon and \( 1/m_B \) effects have been neglected. The decay angular distribution is proportional to

\[
\frac{d\Gamma}{d\cos\theta} \propto 1 + \alpha \cos\theta, \quad \alpha = \frac{2 \text{Re} ab^*}{|a|^2 + |b|^2},
\]

(2.10)

where \( \alpha \) is the asymmetry parameter, and \( \theta \) is the angle between the spin of the initial baryon \( B_i \), and the three-momentum of the final baryon \( B_f \). For future reference, it is convenient to define the real and imaginary parts of the decay amplitudes, \( a = a_R + ia_I \) and \( b = b_R + ib_I \).

WRHD arise due to the combination of an electromagnetic interaction and a \( \Delta S = 1 \) weak transition. The \( \Delta S = 1 \) Lagrangian

\[
\mathcal{L}_{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \bar{d}_\gamma \gamma^\mu (1 - \gamma_5) u \bar{w} \gamma_\mu (1 - \gamma_5) s,
\]

(2.11)
is invariant under \( CP \) symmetry if mixing to the third family is ignored. Since \( d \) and \( s \) quarks have the same charge, the electromagnetic interactions of the quarks are also invariant under \( CP \). \( CP \) is violated in the Lagrangian because the \( d \) and \( s \) quarks have different masses. Thus \( CP \) violating terms in the effective Lagrangian will be suppressed relative to \( CP \) conserving terms by a factor of \( (m_s - m_d)/\Lambda_\chi \sim (M_K^2 - M_\pi^2)/\Lambda_\chi^2 \sim 0.2 \).

Contributions to the WRHD amplitudes can be divided into three categories: (a) short distance contributions generated at scales large compared with \( \Lambda_\chi \), (b) matching terms generated at the scale \( \Lambda_\chi \) which are included in the chiral Lagrangian as local counterterms, and (c) long distance contributions that arise from loop diagrams in the chiral Lagrangian. The precise division of the amplitude into these three categories is in principle scheme dependent, but it is a useful way to organize the calculation.

3. Short Distance Contributions

The short distance contributions to WRHD are generated at energies much higher than \( \Lambda_\chi \), and can be computed using perturbation theory. The leading operator which can produce a \( s \to d\gamma \) transition is the dimension six photonic penguin,

\[
\mathcal{O} = e G_F V_{ud} V_{us}^* \overline{d} \gamma_\mu (1 - \gamma_5) s \partial_\nu F^{\mu\nu}.
\]

However, because \( \partial_\nu F^{\mu\nu} = 0 \) for a physical photon, this operator cannot contribute to WRHD. The dominant operator which can contribute to WRHD is the quark transition magnetic moment operator,

\[
\mathcal{O} = 0.2 \frac{e G_F m_s}{16\pi^2} \overline{d} \sigma_{\mu\nu} (1 + \gamma_5) s F^{\mu\nu}.
\]

This operator is first generated by QCD radiative corrections at two loops, and hence has a suppression factor of \( \alpha_s/4\pi \). It also has a suppression factor of a light quark mass because it is a chirality violating operator. The numerical factor of 0.2 includes the weak mixing angles and QCD radiative corrections \[15\]. This operator gives a contribution to the \( a \) and \( b \) amplitudes of order

\[
a, b \sim 0.2 \frac{m_s}{16\pi^2} \sim 0.2 \frac{M_K^2}{16\pi^2 \Lambda_\chi} \sim 0.3 \text{ MeV}.
\]

We will find in Sect. 5 that the long distance contribution to \( a \) and \( b \) is of order 5 MeV, so the short distance contribution to the decay amplitude is negligible. A similar conclusion has been reached previously by other authors \[3 \[7\]. Note that the operator (3.2) is not \( CP \) violating, and can contribute to \( \Sigma^+ \to p\gamma \).

\(^3\) \( CP \) is \( CP \) followed by the \( SU(3) \) transformation \( u \to -u, d \to s, s \to d \) which exchanges \( d \) and \( s \) quarks \( [4] \).
4. Counterterms from Matching

The non-perturbative matching condition contributions to the WRHD amplitudes can be written as local operators in the chiral Lagrangian. The magnitude and form of the counterterms can be obtained using $SU(3)$ symmetry, $CPS$ symmetry, and naive dimensional analysis. The WRHD counterterms contain one insertion each of the weak and electromagnetic interactions. The $\Delta I = 1/2$ enhancement implies that the leading weak interaction operator transforms as an $SU(3)_L$ octet with the quantum numbers of $\bar{d}s$. It is therefore convenient to introduce the matrix

$$h = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix},$$ (4.1)

which transforms under vector $SU(3)$ as $h \rightarrow UhU^\dagger$ to describe the quantum numbers of the weak interaction. The action of the electromagnetic interaction can be represented by the insertion of the charge matrix

$$Q = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix},$$ (4.2)

which transforms under vector $SU(3)$ as $Q \rightarrow UQU^\dagger$. We first ignore the Lorentz structure of the counterterms, and discuss only their flavor $SU(3)$ structure. Each possible $SU(3)$ invariant flavor structure can contribute to either the $a$ or $b$ decay amplitudes. The most general possible counterterms allowed by $SU(3)$ symmetry are obtained by considering $SU(3)$ invariant combinations of $B, B, h$ and $Q$. The tensor product of $h$ and $Q$ is $8 \otimes 8 \rightarrow 1 + 8 + 8 + 10 + \overline{10} + 27$. Because $h$ and $Q$ contain only particular elements of the complete $SU(3)$ octet, not all representations in the tensor product are allowed. The singlet is not allowed since $\text{Tr} h Q = 0$. In addition, one octet is eliminated because $h$ and $Q$ commute, $[h, Q] = 0$. Thus $h \otimes Q \rightarrow 8 + 10 + \overline{10} + 27$. The initial and final baryon octets can be combined to form $1 + 8 + 8 + 10 + \overline{10} + 27$. Combining these two tensor products yields five allowed $SU(3)$ invariant counterterms,

$$c_1 = \text{Tr} \overline{B} h Q B, \quad c_2 = \text{Tr} \overline{B} Q B h, \quad c_3 = \text{Tr} \overline{B} B h Q, \quad c_4 = \text{Tr} \overline{B} h Q B,$$

$$c_5 = \text{Tr} \overline{B} Q \text{ Tr } B h - \text{ Tr } \overline{B} h \text{ Tr } B Q.$$ (4.3)

Note that the invariant $\left( \text{Tr} \overline{B} Q \text{ Tr } B h + \text{ Tr } \overline{B} h \text{ Tr } B Q \right)$ is a linear combination of $c_1 - c_4$, and other possible traces can be reduced to the above using $[h, Q] = 0$. The invariants
$c_1$–$c_5$ contribute to the WRHD amplitudes $a$ or $b$ depending on the two possible Lorentz invariant structures of the baryon spinor indices and the photon field

$$s_1 = \overline{B}_f \sigma_{\mu\nu} F^{\mu\nu} B_i, \quad s_2 = \overline{B}_f \sigma_{\mu\nu} \gamma_5 F^{\mu\nu} B_i.$$  \hfill (4.4)

Thus, there are ten possible counterterms obtained by combining the five possible flavor invariants with the two possible Lorentz invariants.

$CPS$ symmetry can be used to reduce the number of possible counterterms. All allowed counterterms must be $CPS$ invariant when light quark masses are neglected since the weak and electromagnetic interactions are $CPS$ invariant. It is straightforward to check that $CPS$ invariant counterterms must have the flavor invariants $c_1$–$c_4$ combined with the Lorentz invariant $s_1$, or the flavor invariant $c_5$ combined with the Lorentz invariant $s_2$. A simple way to check $CPS$ invariance is to assume that $h$ and $Q$ are both the unit matrix. In this case, $CPS$ symmetry reduces to $CP$. The Lorentz invariant $s_1$ is a $CP$ preserving magnetic dipole moment, and the Lorentz invariant $s_2$ is a $CP$ violating electric dipole moment. Since $c_1$–$c_4$ do not vanish when $h = Q = 1$, they can only be combined with $s_1$. It takes a little more work to show that $c_5$ (which vanishes when $h = Q = 1$) is odd under $CPS$ so that it can be combined with $s_2$ to form a $CPS$ invariant counterterm.

In summary, the WRHD amplitude $a$ has four $CPS$ invariant counterterms with flavor structure $c_1$–$c_4$, and the amplitude $b$ has only one $CPS$ invariant counterterm with flavor structure $c_5$. Note that the $c_5$ counterterm does not contribute to $\Sigma^+ \to p\gamma$ or to $\Xi^- \to \Sigma^-\gamma$, which was originally shown by Hara [9].

5. Long Distance Contributions

The long distance contribution to WRHD is obtained by computing the time ordered product of the $\Delta S = 1$ weak Lagrangian and the electromagnetic interaction in the chiral Lagrangian. The long distance contribution is the dominant contribution to WRHD.

The leading $\Delta S = 1$ Lagrangian is

$$L^{\Delta S = 1}_v = G_F M_{\pi^+}^2 f \left( h_D \text{Tr} \, \overline{B}_v \left\{ \xi^\dagger h \xi, B_v \right\} + h_F \text{Tr} \, \overline{B}_v \left[ \xi^\dagger h \xi, B_v \right] \right),$$  \hfill (5.1)

where the 27 component of Eq. (2.11) has been neglected because of the $\Delta I = 1/2$ rule, and $h_D$ and $h_F$ are dimensionless coupling constants. The overall factor of $G_F M_{\pi^+}^2$ is included by convention. A fit to the $s$-wave hyperon nonleptonic decays yields $h_D = 0.58 \pm 0.21$ and
Expanding Eq. (5.1) in a power series in the Goldstone boson fields gives parity conserving $\Delta S = 1$ weak amplitudes of the form $G_F M_{\pi^+}^2 f_\pi B_i$, and parity violating $\Delta S = 1$ weak amplitudes of the form $G_F M_{\pi^+}^2 \pi B f_i B_i$,

$$L_{\Delta S=1} = G_F M_{\pi^+}^2 f (h_D \text{Tr} B_v \{h, B_v\}) + h_F \text{Tr} B_v [h, B_v])$$

$$+ i G_F M_{\pi^+}^2 (h_D \text{Tr} B_v \{[h, \pi], B_v\} + h_F \text{Tr} B_v \{[h, \pi], B_v\}).$$

(5.2)

The leading electromagnetic interactions are the usual interactions proportional to the electric charge contained in Eq. (2.2), and the magnetic moment interaction

$$\mathcal{L} = \frac{e}{4m_B^2} \mu f_i B_v f \sigma_{\mu\nu} F^{\mu\nu} B_i,$$

(5.3)

where $\mu f_i$ is the magnetic moment in nuclear magnetons. In the static baryon formalism, the entire baryon magnetic moment is contained in Eq. (5.3), rather than just the anomalous magnetic moment.

The dominant long distance contribution to the parity-violating WRHD amplitude $b$ comes from the one-loop diagrams shown in fig. [1], where the weak vertex is the $s$-wave nonleptonic decay amplitude. It is useful to first estimate the form of the loop diagram. The weak vertex has the form $G_F M_{\pi^+}^2 A_s$, where $A_s$ is a typical $s$-wave nonleptonic decay amplitude and is of order one. It is a linear combination of Clebsch-Gordan coefficients times the parameters $h_D$ and $h_F$. The strong interaction baryon-pion vertex is $g_A k \cdot S_v / f$, where $S_v$ is the baryon spin, $k$ is the Goldstone boson moment, and $g_A$ is a dimensionless constant of order one which is a linear combination of $D$ and $F$. The Feynman integral has the form

$$\int \frac{d^4 k}{(2\pi)^4} \frac{g_A k \cdot S_v}{f} G_F M_{\pi^+}^2 A_s \frac{e k \cdot \epsilon}{(k^2)^2 k \cdot v},$$

(5.4)

where $k$ denotes a generic momentum, and $\epsilon$ is the polarization vector of the photon. The gauge invariant form of the amplitude $b$ given in Eq. (2.8) is $e G_F \omega \epsilon \cdot S$, where $\omega$ is the photon energy. Thus one factor of $k$ in the loop integral must produce a factor of the momentum of the photon, so that the integral for $b$ has the form

$$b \sim \int \frac{d^4 k}{(2\pi)^4} \frac{g_A M_{\pi^+}^2 A_s}{f k^4}.$$

(5.5)

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4 We use the parameter values from tree level fit of ref. [16]. The signs of $h_D$ and $h_F$ have been reversed because of a different sign convention for the Lagrangian.
This integral has an infrared divergence which is cutoff by the Goldstone boson mass, so that

\[ b \sim \frac{g_A M^2_{\pi^+}}{16\pi^2 f} \ln \frac{M^2}{\mu^2} \sim 4 \text{ MeV}, \]  

(5.6)

where the numerical estimate results from setting \( A_s \) and \( g_A \) to one, and using \( \mu = 1 \text{ GeV} \). Eq. (5.6) is the \( \ln M^2 \) contribution discussed by Kogan and Shifman [6]. The graph also has an imaginary part which is determined by unitarity.

The graphs in fig. 1 were computed in dimensional regularization, using the methods given in ref. [11], and retaining only the finite pieces. The Goldstone boson mass was included in the loop integral, since it regulates the infrared divergence. Because the dominant diagrams are from pion loops, we have also retained the \( SU(3) \) mass splittings of the baryons in evaluating the loop integrals. The baryon propagator in the diagrams was taken to be \( i/(k \cdot v + \Delta) \), where \( k \) is the momentum along the baryon line, and \( \Delta = m - m_i \) is the difference in mass between the intermediate and initial baryons. The graphs are most easily computed using the gauge conditions \( v \cdot \epsilon = 0 \) and \( q \cdot \epsilon = 0 \) for the physical photon polarization \( \epsilon \), where \( v \) and \( q \) are the velocity of the baryon and the momentum of the photon, respectively. In \( v \cdot \epsilon = 0 \) gauge, the only graphs which contribute are those where the photon couples to the meson line, or at the meson-baryon vertex. These graphs are proportional to the meson charge, so only charged \( K \) and \( \pi \) loops contribute. Both \( K \) and \( \pi \) loops were included in the computation, but we have checked that the \( \pi \) loops dominate. The result for the decay amplitudes is given in Appendix A. The terms in the appendix include all amplitudes that do not vanish in the \( SU(3) \) limit. In addition, the contribution of the experimentally measured \( s \)-wave amplitude \( \Sigma^+ \rightarrow n\pi^+ \) is also included. This amplitude vanishes in the \( SU(3) \) limit, and is experimentally known to be about 20 times smaller than the \( SU(3) \) allowed amplitudes. Including it makes a negligible change in our results, but we have done so to get the best possible estimate for the imaginary part of the \( b \) amplitude.

The result given in the appendix is evaluated numerically in two different ways. The first method (I) uses the best fit values for \( h_D, h_F, D \) and \( F \). The values of \( D \) and \( F \) used are those given in ref. [17]. The second method (II) uses the experimentally measured amplitudes for \( s \)-wave nonleptonic decay and for the baryon semileptonic decays to determine \( A_s \) and \( g_A \) wherever possible, and uses the best fit \( SU(3) \) predictions for undetermined couplings. The results of the two methods agree with each other, because \( SU(3) \) works well for both the \( s \)-wave nonleptonic amplitudes and the semileptonic decays.
The \( \ln \mu \) dependence completely cancels between the \( K \) and \( \pi \) loops if one uses the \( SU(3) \) symmetric values for the nonleptonic decay amplitudes and axial coupling constants. In principle there is one allowed counterterm so the graphs need not be finite. However, the \( s \)-wave nonleptonic decay amplitude is not the most general possible one allowed by \( SU(3) \) symmetry, but must have the special form given by Eq. (5.2) where \( h \) and \( \pi \) occur only in the combination \([h, \pi]\). The one loop diagrams of fig. 1 cannot produce the \( SU(3) \) tensor structure \( c_5 \) of Eq. (4.3) from \([h, \pi]\), so the graphs are finite. The numerical values for the \( b \) amplitudes are given in Table 1, where the first and second values are obtained using methods I and II, respectively.

In the plots in Sect. 6, we will use method II in comparing with the experimental data, because that gives the best approximation to the imaginary parts of the amplitudes which are fixed by unitarity. The long distance contribution gives an independent estimate of the local counterterm (discussed in the previous section) to be around 1 MeV by looking at how much the decay amplitude changes if \( \mu \) is varied by a factor of two. Since the \( K \) and \( \pi \) loops together have no \( \mu \) dependence, the estimate is done by looking at the variation of the \( \pi \) loops alone when \( \mu \) is changed by a factor of two. This estimate is comparable to the estimate given by naive dimensional analysis [8].

The computation of the \( a \) amplitude is much more difficult. Naively, the leading contribution is from the pole graphs of fig. 2, which give

\[
\begin{align*}
a_{\lambda n} &= \frac{M^2_f}{2m_N} \left[ -\frac{1}{\sqrt{6}} (h_D + 3h_F) \frac{(\mu_n - \mu_\Lambda)}{(\Lambda - n)} + \frac{1}{\sqrt{2}} (h_D - h_F) \frac{\mu_{\Lambda} \Sigma^0}{(\Sigma^0 - n)} \right], \\
a_{\Sigma^+ p} &= \frac{M^2_f}{2m_N} \left[ (h_D - h_F) \frac{\mu_p}{(\Sigma^0 - p)} \right], \\
a_{\Sigma^0 n} &= \frac{M^2_f}{2m_N} \left[ \frac{1}{\sqrt{2}} (-h_D + h_F) \frac{(\mu_n - \mu_{\Sigma^0})}{(\Sigma^0 - n)} + \frac{1}{\sqrt{2}} (h_D + 3h_F) \frac{\mu_{\Sigma^0} \Lambda}{(\Sigma^0 - n)} \right], \\
a_{\Xi^- \Sigma^-} &= \frac{M^2_f}{2m_N} \left[ (h_D + h_F) \frac{(\mu_{\Sigma^-} - \mu_{\Xi^-})}{(\Xi^- - \Sigma^-)} \right], \\
a_{\Xi^0 \Lambda} &= \frac{M^2_f}{2m_N} \left[ (\mu_\Lambda - \mu_{\Xi^0}) \frac{1}{\sqrt{6}} (-h_D + 3h_F) + \frac{1}{\sqrt{2}} (h_D + h_F) \frac{\mu_{\Sigma^0} \Lambda}{(\Xi^0 - \Sigma^0)} \right], \\
a_{\Xi^0 \Sigma^0} &= \frac{M^2_f}{2m_N} \left[ -\frac{1}{\sqrt{2}} (h_D + h_F) \frac{(\mu_{\Sigma^0} - \mu_{\Xi^0})}{(\Xi^0 - \Sigma^0)} + \frac{1}{\sqrt{6}} (-h_D + 3h_F) \frac{\mu_{\Lambda} \Sigma^0}{(\Xi^0 - \Lambda)} \right]. 
\end{align*}
\]

Using the best fit values for \( h_D \) and \( h_F \), and the known values of the magnetic moments gives the real parts of \( a \) tabulated in Table 1. Unfortunately, the values of the pole graphs...
are sensitive to the precise assumptions made to evaluate them. For example, it matters whether one uses the leading $SU(3)$ predictions for the masses in the denominator, or the physical values of the masses, etc. For the $p$-wave nonleptonic hyperon decays, there is a cancellation between the various pole graphs, which causes the one-loop corrections to be very important. This cancellation explains why $SU(3)$ predictions are a complete disaster for the $p$-wave nonleptonic decays. Some cancellation also occurs between the pole diagrams for the $a$ WRHD amplitude, so higher order corrections are expected to be important for the $a$ amplitudes. Note that the constraints of $CPS$ do not apply to the $a$ amplitude, because pole graphs cannot be written as local counterterms. There is an additional complication for the $a$ amplitudes because loop graphs of fig. 1 with the weak vertex replaced by the nonleptonic $p$-wave amplitude contribute. These graphs are just as important as the $s$-wave graphs, because the nonleptonic $p$-wave amplitude is of the same order in the derivative expansion as the $s$-wave amplitudes. These graphs cannot be computed reliably, because the $p$-wave nonleptonic decay amplitude must be known as a function of the pion momentum $k$, which need not be on-shell. The typical energy scale over which the $p$-wave amplitudes vary is of order the $SU(3)$ mass splittings in the baryons, or of order 150 MeV, which implies that the amplitudes are varying rapidly in the region of interest. For these reasons, we conclude that there is no reliable way to compute the real part of $a$. Previous work on WRHD has produced a wide variety of estimates for the real part of $a$, which is another indication that $a_R$ cannot be reliably calculated. In this work, $a_R$ will be treated as an unknown parameter.

The imaginary part of $a$ can be determined reliably using unitarity. It depends only on the $p$-wave nonleptonic decay amplitudes for on-shell pions, which are known experimentally. We thus use the diagrams of fig. 1 to compute $a_I$, using the experimentally measured $p$-wave amplitudes for the weak vertex, instead of an $SU(3)$ fit to the $p$-wave amplitudes. The imaginary parts of the loop graph are given in Appendix A. Evaluating the result numerically yields the values of $a_I$ given in Table 1.

It has been suggested recently that intermediate spin-3/2 decuplet states should be included in chiral loop calculations. Decuplet intermediate states are suppressed for the $b$ amplitude, because spin-3/2 $\rightarrow$ spin-1/2 + $\pi$ $s$-wave nonleptonic decay amplitudes are forbidden by angular momentum conservation. The real parts of $a$ do receive contributions from intermediate spin-3/2 states, but as we have already argued $a_R$, cannot be computed reliably, so these contributions need not be evaluated explicitly. There are other contributions to the real part of $a$ that we also have not included, such
as one-loop diagrams that involve the $\Delta S = 1$ transition in the meson sector from the effective Lagrangian

$$\mathcal{L}^{\Delta S=1} = \frac{f^2}{8} \text{tr} h D_\mu \Sigma D^\mu \Sigma^\dagger,$$

which produces the $K \to 2\pi$ decay amplitude.

6. Comparison with Experiment

The results of the previous sections can now be confronted with experiment. The imaginary parts $a_I$ and $b_I$ can be reliably computed using unitarity. The real part $b_R$ can be reliably calculated using chiral perturbation theory. It has a typical size of around 5 MeV, with a counterterm of typical size 1 MeV. The real part $a_R$ cannot be computed reliably, and is treated as a free parameter. In comparing with experiment, we use the amplitudes $a_I$, $b_I$, and $b_R$ given in Table 1, and treat $a_R$ as a free parameter. In addition, we add to $b_R$ a counterterm contribution of the form $b_c \lambda$, where $\lambda$ is evaluated from the $c_5$ invariant in Eq. (4.3) to be $-\sqrt{3}/2$, $-3/\sqrt{2}$, $\sqrt{3}/2$, and $3/\sqrt{2}$ for the $\Lambda \to n\gamma$, $\Sigma^0 \to n\gamma$, $\Xi^0 \to \Lambda\gamma$, and $\Xi^0 \to \Sigma\gamma$ decay modes respectively, and zero for the $\Sigma^+ \to p\gamma$ and $\Xi^- \to \Sigma^-\gamma$ decay amplitudes.

The theoretical prediction is a curve in the asymmetry-amplitude plane for each decay, as $a_R$ is varied with $a_I$, $b_R$ and $b_I$ held fixed. There is a counterterm contribution to $b_R$, so one gets a set of curves as the counterterm is varied. We have chosen to plot curves in fig. 3 for $b_c = 0$ MeV (solid), $b_c = 1$ MeV (dashed) and $b_c = -1$ MeV (dot-dashed). The spread in the curves indicates the uncertainty due to the unknown counterterm $b_c$. It is important to note that there is only a single counterterm: one can pick either of the three kinds of curves, but one must pick the same curve for all five decay amplitudes.

The theoretical curves in fig. 3 are to be compared with the experimentally allowed region represented by the shaded ellipse in each plot. The experimental values used are those given in the 1992 Particle Data Book. In addition, the recent measurements of the $\Lambda \to n\gamma$ lifetime [20], and of the $\Sigma^+ \to p\gamma$ asymmetry parameter [10] are also shown. Keeping in mind that we expect corrections to our results of order $m_K^2/\Lambda^2 \chi \simeq 25\%$, we see that the results shown in fig. 3 are consistent with experiment, except for the asymmetry parameter for $\Sigma^+ \to p\gamma$, which has been problematic for all previous work as well. To better appreciate the significance of this disagreement between theory and experiment, consider the $\Sigma^+ \to p\gamma$ decay with $a_R$ and $b_R$ both treated as free parameters. The imaginary
parts are determined by unitarity, and cannot be adjusted. The maximum theoretical asymmetry for \( \Sigma^+ \to p\gamma \) consistent with the observed decay width is then found to be \( \alpha = -0.8 \) for \( a_R = 7.88 \text{ MeV} \) and \( b_R = -7.88 \text{ MeV} \). This requires a \( b \) counterterm for \( \Sigma^+ \to p\gamma \) of \(-6.74 \text{ MeV}\) to fit the observed value of \( \alpha \). The naive size of the \( b \) counterterm for \( \Sigma^+ \to p\gamma \) is \((m_s - m_d)(1 \text{ MeV})/\Lambda_\chi \sim (M_K^2 - M_\pi^2)(1 \text{ MeV})/\Lambda_\chi^2 \sim 0.2 \text{ MeV}\). The additional suppression factor \((m_s - m_d)/\Lambda_\chi\) is present because the counterterm must be \( CPS \) violating, since the \( CPS \) preserving counterterm does not contribute to \( \Sigma^+ \to p\gamma \).

Thus the \(-6.74 \text{ MeV}\) counterterm required is 35 times its naive value of 0.2 MeV. The other possibility is to have an enhancement in the short distance contribution (discussed in Section 3) by a factor of 20. While these possibilities cannot be ruled out, they do not seem likely. Thus, if the large negative asymmetry measured for \( \Sigma^+ \to p\gamma \) is correct, it seems to indicate a breakdown of naive power counting for this process. A possible source of this breakdown is the presence of pole graphs containing intermediate excited \( \frac{1}{2}^- \) baryons such as the \( N^*(1535) \), which contribute to \( b_R \).

We have recently received a preprint by H. Neufeld which also analyzes WRHD using chiral perturbation theory.

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5 The asymmetry parameter for \( \Sigma^+ \to p\gamma \) is difficult to measure experimentally because of possible contamination from the decay \( \Sigma^+ \to p\pi^0, \pi^0 \to \gamma\gamma \).
Table 1 – Weak Radiative Hyperon Decay Amplitudes in units of MeV. The two sets of values for the $b$ amplitudes were derived using methods I and II described in the text, respectively. The values for $a_R$ contain only the pole graph contributions from fig. [2], and are not reliable.

### Appendix A. Decay Amplitudes

**b Amplitudes:**

The $b$ amplitudes evaluated from the loop diagrams of fig. [1]. The $s$-wave nonleptonic decay amplitude is used at the weak vertex. $A_s(X\overline{Y}\pi)$ is the $s$-wave $\Delta S = 1$ weak amplitude for the process $X\pi \leftrightarrow Y$. The amplitude is normalized to that given by Eq. (5.2), with the factor of $G_F M_{\pi^+}^2$ removed. For example, $A_s(\Lambda\overline{p}\pi^+) = -(h_D + 3h_F)/\sqrt{6}$ is the $s$-wave nonleptonic decay amplitude for the process $\Lambda \rightarrow p\pi^-$, which is conventionally called $\Lambda_0^-$. The sign and normalization of the $s$-wave amplitudes are the same as those used by the Particle Data Group, and in Table 1 of ref. [16]. The coupling constant $g_A(X\overline{Y}\pi)$ is the baryon-Goldstone boson axial coupling constant with the conventional normalization, $g_A(p\overline{n}\pi^-) = D + F \sim 1.26$, etc. $I_1$ and $I_2$ are integrals that are given in Appendix B. The arguments of the integrals are the masses of the particles with the given labels.

\[
b_{\Lambda n} = A_s(\Lambda\overline{p}\pi^+) g_A(p\overline{n}\pi^-) I_1(\Lambda, p, n, \pi) - A_s(\Sigma^-\overline{n}\pi^+) g_A(\Sigma^-\overline{p}\pi^-) I_2(\Lambda, \Sigma^-, n, \pi) + A_s(\Sigma^+\overline{p}\pi^-) g_A(\Sigma^+\overline{n}\pi^-) I_2(\Lambda, \Sigma^+, n, \pi) - A_s(\Lambda\overline{p}K^-) g_A(\Sigma^-\overline{n}K^+) I_1(\Lambda, \Sigma^-, n, K) + A_s(p\overline{n}K^-) g_A(\Lambda\overline{p}K^+) I_2(\Lambda, \Sigma^-, n, K)
\]
\[ b_{\Sigma+p} = -A_s(\Lambda\bar{p}\pi^+)g_A(\Sigma^+\bar{\Lambda}\pi^-)I_2(\Sigma^+, \Lambda, p, \pi) \]
\[ - A_s(\Sigma^0\bar{p}\pi^+)g_A(\Sigma^+\bar{\Sigma}^0\pi^-)I_2(\Sigma^+, \Sigma^0, p, \pi) \]
\[ - A_s(\Sigma^+\bar{n}\pi^-)g_A(n\bar{p}\pi^+)I_1(\Sigma^+, n, p, \pi) \]
\[ - A_s(\Sigma^+\bar{\Sigma}^0K^-)g_A(\Sigma^0\bar{p}K^+)I_1(\Sigma^+, \Sigma^0, p, K) \]
\[ - A_s(\Sigma^+\bar{\Lambda}K^-)g_A(\Lambda\bar{p}K^+)I_1(\Sigma^+, \Lambda, p, K) \]
\[ b_{\Sigma^n} = A_s(\Sigma^0\bar{p}\pi^+)g_A(p\bar{n}\pi^-)I_1(\Sigma^0, p, n, \pi) \]
\[ - A_s(\Sigma^-\bar{n}\pi^-)g_A(\Sigma^0\bar{\Sigma}^-\pi^-)I_2(\Sigma^0, \Sigma^-, n, \pi) \]
\[ + A_s(\Sigma^+\bar{\Sigma}^-\pi^-)g_A(\Sigma^0\bar{\Sigma}^+\pi^-)I_2(\Sigma^0, \Sigma^+, n, \pi) \]
\[ - A_s(\Sigma^0\Sigma^-K^-)g_A(\Sigma^-\bar{n}K^+)I_1(\Sigma^0, \Sigma^-, n, K) \]
\[ + A_s(p\bar{n}K^-)g_A(\Sigma^0\bar{p}K^+)I_2(\Sigma^0, p, n, K) \]
\[ b_{\Xi^-\Sigma^-} = A_s(\Xi^-\bar{\Lambda}\pi^+)g_A(\Lambda\bar{\Sigma}^-\pi^-)I_1(\Xi^-, \Lambda, \Sigma^-, \pi) \]
\[ + A_s(\Xi^-\bar{\Sigma}^0\pi^+)g_A(\Sigma^0\bar{\Sigma}^-\pi^-)I_1(\Xi^-, \Sigma^0, \Sigma^-, \pi) \]
\[ + A_s(\Lambda\bar{\Sigma}^-K^-)g_A(\Xi^-\bar{\Lambda}K^+)I_2(\Xi^-, \Lambda, \Sigma^-, K) \]
\[ + A_s(\Sigma^0\Sigma^-K^-)g_A(\Xi^-\bar{\Sigma}^0\pi^-)I_2(\Xi^-, \Sigma^0, \Sigma^-, K) \]
\[ b_{\Xi^n\Lambda} = A_s(\Xi^0\bar{\Sigma}^+\pi^+)g_A(\Sigma^+\bar{\Lambda}\pi^-)I_1(\Xi^0, \Sigma^+, \Lambda, \pi) \]
\[ - A_s(\Xi^-\bar{\Lambda}\pi^+)g_A(\Xi^0\bar{\Sigma}^-\pi^-)I_2(\Xi^0, \Xi^-, \Lambda, \pi) \]
\[ - A_s(\Xi^0\bar{\Sigma}^-K^-)g_A(\Xi^-\bar{\Lambda}K^+)I_1(\Xi^0, \Xi^-, \Lambda, K) \]
\[ + A_s(\Sigma^+\bar{\Lambda}K^-)g_A(\Xi^0\bar{\Sigma}^+K^+)I_2(\Xi^0, \Sigma^+, \Lambda, K) \]
\[ b_{\Xi^n\Sigma^0} = A_s(\Xi^0\bar{\Sigma}^+\pi^+)g_A(\Sigma^+\bar{\Sigma}^0\pi^-)I_1(\Xi^0, \Sigma^+, \Sigma^0, \pi) \]
\[ - A_s(\Xi^-\bar{\Sigma}^0\pi^+)g_A(\Xi^0\bar{\Sigma}^-\pi^-)I_2(\Xi^0, \Xi^-, \Sigma^0, \pi) \]
\[ - A_s(\Xi^0\bar{\Sigma}^-K^-)g_A(\Xi^-\bar{\Sigma}^0K^+)I_1(\Xi^0, \Xi^-, \Sigma^0, K) \]
\[ + A_s(\Sigma^+\bar{\Sigma}^0K^-)g_A(\Xi^0\bar{\Sigma}^+K^+)I_2(\Xi^0, \Sigma^+, \Sigma^0, K) \]

\textit{a Amplitudes:}

The imaginary parts of the \( a \) amplitudes evaluated from the loop diagrams of fig. [4]

The measured values of the \( p \)-wave nonleptonic decay amplitudes are used at the weak
vertices. The $p$-wave amplitudes have the normalization used in Table 1 of Ref. [16].

\[
\text{Im } a_{\Lambda n} = -A_p(\Lambda \overline{p} \pi^+)g_A(p \overline{p} n^-)J(\Lambda, p, n, \pi),
\]

\[
\text{Im } a_{\Sigma^+ p} = A_p(\Sigma^+ \overline{p} \pi^-)g_A(n \overline{p} n^+)J(\Sigma^+, n, p, \pi),
\]

\[
\text{Im } a_{\Sigma^0 n} = 0,
\]

\[
\text{Im } a_{\Xi^- \Sigma^-} = -A_p(\Xi^- \overline{\Lambda} \pi^+)g_A(\Lambda \Sigma^- \pi^-)J(\Xi^-, \Lambda, \Sigma^-, \pi),
\]

\[
\text{Im } a_{\Xi^0 \Lambda} = 0,
\]

\[
\text{Im } a_{\Xi^0 \Sigma^0} = 0,
\]

where the integral $J$ is defined in Appendix B.

**Appendix B. Integrals**

\[
I_1(m_i, m, m_f, M) = \left[ \frac{M^2_{\pi^+}}{8\pi^2 f \omega} \right] \left[ \frac{1}{2} \omega \left( 2 - \ln(M^2/\mu^2) \right) \right. \\
+ 2 \int_0^1 dx f_1(\Delta - \omega x, M) - 2f_1(\Delta, M) \left. \right],
\]

(B.1)

where

\[
\Delta = m_i - m, \quad \omega = m_i - m_f,
\]

\[
f_1(y, M) = \begin{cases} 
\sqrt{M^2 - y^2} \left[ \frac{\pi}{2} + \tan^{-1} \left( \frac{y}{\sqrt{M^2 - y^2}} \right) \right], & |y| \leq M, \\
\frac{1}{2} \sqrt{y^2 - M^2} \left[ -2i\pi + \ln \left( \frac{y + \sqrt{y^2 - M^2}}{y - \sqrt{y^2 - M^2}} \right) \right], & |y| > M.
\end{cases}
\]

(B.2)

\[
I_2(m_i, m, m_f, M) = \left[ \frac{M^2_{\pi^+}}{8\pi^2 f \omega} \right] \left[ \frac{1}{2} \omega \left( 2 - \ln(M^2/\mu^2) \right) \right. \\
- 2 \int_0^1 dx f_2(\Delta + \omega x, M) + 2f_2(\Delta, M) \left. \right],
\]

(B.3)

where

\[
\Delta = m_f - m, \quad \omega = m_i - m_f,
\]

\[
f_2(y, M) = \begin{cases} 
\sqrt{M^2 - y^2} \left[ \frac{\pi}{2} + \tan^{-1} \left( \frac{y}{\sqrt{M^2 - y^2}} \right) \right], & |y| \leq M, \\
\frac{1}{2} \sqrt{y^2 - M^2} \left[ \ln \left( \frac{-y - \sqrt{y^2 - M^2}}{-y + \sqrt{y^2 - M^2}} \right) \right], & |y| > M.
\end{cases}
\]

(B.4)
\[ J(x, y, z, M) = \frac{M^2}{8\pi f\omega^2} \left[ \Delta \sqrt{\Delta^2 - M^2} + M^2 \ln \left( \frac{M}{\Delta + \sqrt{\Delta^2 - M^2}} \right) \right], \quad (B.5) \]

where

\[ \Delta = x - y, \quad \omega = x - z. \]
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Figure Captions

Fig. 1. One-loop diagrams which give the leading $\ln M^2$ contribution to the parity-violating $b$ decay amplitudes, and to the imaginary part of the parity-conserving $a$ decay amplitudes. The solid dots represent strong interaction baryon-pion vertices proportional to $D$ and $F$. Photon couplings are derived from Lagrangian $L_0^\nu$. The solid squares denote the weak non-leptonic hyperon decay amplitudes. The $s$-wave nonleptonic amplitude contributes to the $b$ WRHD amplitude, and the $p$-wave nonleptonic amplitude contributes to the $a$ WRHD amplitude. Graphs (e) and (f) are absent for the $b$ amplitude. Graphs with the photon coupled to the nucleon line vanish in $v \cdot \epsilon = 0$ gauge.

Fig. 2. Pole graphs contributing to parity-conserving $a$ decay amplitudes. The solid squares denote weak $\Delta S = 1$ vertices. The photon couplings are due to baryon magnetic moments.

Fig. 3. Comparison of theory and experiment for the weak radiative hyperon decays

(a) $\Lambda \to n\gamma$, (b) $\Sigma^+ \to p\gamma$, (c) $\Xi^- \to \Sigma^-\gamma$, (d) $\Xi^0 \to \Lambda\gamma$, and (e) $\Xi^0 \to \Sigma^0\gamma$. The asymmetry parameter $\alpha$ and the decay amplitude $\sqrt{|a|^2 + |b|^2}$ form the coordinate axes. The shaded ellipses represent the experimentally allowed regions with 1$\sigma$ errors for $\alpha$ and $\Gamma$. Theoretical curves depict the values for $\alpha$ and $\Gamma$ obtained for a $b$ counterterm of 0 MeV (solid line), 1 MeV (dashed line) and $-1$ MeV (dot-dashed line). The unshaded regions bounded by solid curves for $\Lambda \to n\gamma$ and $\Sigma^+ \to p\gamma$ are the recent Noble $et$ $al.$ [20] and E761 measurements [10], respectively.