Milne Quantum-Universe Redshift-Luminosity Correlation

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Abstract

Milne’s classical homogeneous-universe cosmology predicts a product of Hubble constant with luminosity distance that equals $z + z^2/2$, where $z$ is redshift. Supernova-data are consistent with this relation, supporting quantum-theoretic considerations that reveal Milne’s universe as ‘non-empty’.
Introduction

Milne’s classical cosmology, (1) interpreted by general relativists as corresponding to zero universe energy, was dismissed during the last century by all save its inventor. (2) But two recent developments, one experimental and one theoretical, are resurrecting Milne’s alternative to general relativity (GR). (1) Data from supernovae at redshifts approaching $z = 1$ disagree qualitatively with GR expectations. (3) (2) A heretofore unutilized unitary Hilbert-space Lorentz-group representation has been found to define a positive-energy ‘Milne quantum universe’ (MQU). (4) We here derive, in a homogeneous approximation to MQU, a simple relation between redshift and luminosity distance that is consistent with current data.

Although his cosmology has been regarded a special case of the Friedman-Robertson-Walker metric, (5) Milne did not follow that route; neither did the present author. For different reasons we have both employed the Lorentz group as foundation for ‘reality within spacetime’. Milne’s cosmology--pure ‘classical kinematics’--was Lorentz-group and spacetime motivated--with reference neither to quantum theory nor to gravity and energy. The present author has invoked a ‘bundle’ of 3-dimensional Milne negatively-curved metricized base space, a 3-dimensional unmetricized fiber space and a Hilbert space. This bundle cosmologically satisfies Dirac’s (quantum) principles through previously-unutilized unitary Hilbert-space representations of the Lorentz group. Gravity, with energy as source, is represented together with electromagnetism.

After achievement of definition for ‘classical reality’ in a gravity-encompassing single-universe quantum-cosmological theory, (4) we have begun to contemplate experimental tests. Because the spacetime meaning to which quantum theory has led us is that of Milne, we are exploring the natural possibility that Milne’s cosmology is a Hubble-scale classical homogeneous approximation to our quantum theory of the universe—a theory, more general than his, which deals with all scales between Planck’s and Hubble’s.

Beyond assignment of foundational status to the Lorentz group, our theory of spacetime reality recognizes two foundational integers, one ‘large’ and one ‘huge’, that allow different approximations to be ‘physically viable’, separately, for different limited-scale ranges. Each such approximation within its scale range is, for all practical physics purposes (FAPPP), ‘reliable’. (Our large integer associates with the reciprocal of a ‘GUT-scale fine structure constant’ and our huge integer with the universe’s total energy. (4) Although reliability of Milne (classical) Hubble-scale cosmology remains to be deduced from our quantum cosmology, the supernovae data has provoked the present paper.

Milne Spacetime

Milne spacetime occupies the interior of a forward lightcone, with the ‘age’ of any spacetime point equal to its Minkowski distance from the lightcone vertex. Spacetime was seen by Milne as ‘filled’ with metricized 3-dimensional (noncompact) hyperbolic (curved 3-space)
manifolds, each belonging to a single (positive) age. Redshift immediately follows, with Hubble’s ‘constant’ \( H \) equal to the reciprocal of universe age \( (c = 1) \).

The curvature of a Milne 3-manifold equals \( H \)--independently of location within the manifold. Euclidean (‘flat’) geometry is asymptotically approached in the limit as age approaches infinity. ‘Milne relativity’, distinct from either special or general relativity (and not only more general than his Hubble-scale classical cosmology but extendable to quantum cosmology), implies that any two universes related by a global Poincaré transformation are the same universe (not ‘alternative’ universes). MQU enjoys ‘fixed and settled reality’.

A Milne-Lorentz boost shifts spatial locations at a fixed age. To any spacetime location there associates a continuous (labeled by 3 Euler angles) set of rotationally-related ‘local frames’. In any local frame the positive-timelike 4-vector displacement from the lightcone vertex of the location in question has components \((\tau, 0, 0, 0)\), where \( \tau \) is the location’s age.

Let the symbol \( \beta \) denote the dimensionless positive ‘boost distance’, along a hyperboloid geodesic, between two spacetime points of the same age. If \( c = 1 \) the 4-vector spacetime location of one of these points, in any of the (rotationally-related) local frames belonging to the other point, is \( \tau \times (cosh \beta, n \sinh \beta) \), where \( n \) is a unit 3-vector whose pair of direction (‘polar’) coordinates refer (for ‘origin’) to the orientation of the other point’s local frame. Spacetime points of different age but parallel location 4-vectors share the same 3-vector \( \beta \equiv \beta n \). (They occupy the same location in ‘boost space’.) Thus Milne spacetime is coordinated by \( \tau, \beta \) once some ‘origin’ within a 6-dimensional manifold—the product of a (compact) 3-sphere with a (noncompact) 3-hyperboloid--has been designated. (4)

The 3-hyperboloid Lorentz-invariant (dimensionful) metric is

\[
(ds)^2 = \tau^2 \left\{ (d\beta)^2 + \sinh^2 \beta \left[ (d\theta)^2 + \sin^2 \theta (d\phi)^2 \right] \right\},
\]

where \( \theta \) and \( \phi \) are polar coordinates specifying the direction \( n \). The (4-spacetime) Minkowski metric is the sum of two separately-invariant terms: \((dt)^2 - (ds)^2\). Along any temporally-forward lightlike trajectory, \( ds = dt \). Our theory supposes Hubble-scale light propagation to follow approximately such a ‘Milne trajectory’ (which ignores sub-Hubble-scale matter clumping—regarding matter as uniformly distributed).

For supernovae with redshifts of order 1 that share a common (‘standard’) energy release, the supposition that both supernova ‘sources’ and telescope ‘sinks’ are ‘almost at rest’ \((v/c \sim 10^{-3})\) in their respective local frames allows the straightforward computation in the following section of an unambiguous relation (no arbitrary parameter) between redshift and ‘luminosity distance’.
Hubble-Scale Milne Relation between Redshift and Luminosity Distance

For boost-distance \( \beta \) between light source at age \( \tau_{\text{source}} \) and light ‘sink’ at (later) age \( \tau_{\text{sink}} \), an immediate consequence of light propagation according to \( ds = d\tau \) is

\[
\frac{\tau_{\text{sink}}}{\tau_{\text{source}}} = e^{\beta}.
\] (2)

The ratio (2) also equals that between time intervals of energy emission and absorption in respective local frames.

The sink-source age ratio (2) further yields the ratio between emitted-photon (source frame) energy and absorbed-photon (sink frame) energy—i.e.,

\[
e^{\beta} = 1 + z,
\] (3)

where \( z \) is the (standard) redshift parameter.

The definition of ‘luminosity distance’ \(^{(5)}\) is

\[
d_L \equiv (L/4\pi\ell)^{1/2},
\] (4)

\( L \) denoting total energy emitted per unit time in source frame, while the symbol \( \ell \) denotes a ratio

\[
\ell \equiv P/A,
\] (5)

the symbol \( P \) representing power received (energy per unit sink time) by a mirror of area \( A \) whose surface is perpendicular to light-propagation direction. We now show that, for light propagating along Milne geodesics,

\[
d_L = \tau_{\text{sink}} e^{\beta} \sinh \beta,
\] (6)

or, equivalently, \( H d_L = z + z^2/2 \), once \( \tau_{\text{sink}}^{-1} \) is identified with Hubble’s ‘constant’ \( H \) and Formula (3) is employed to replace \( \beta \) by \( z \).

Suppose the mirror to be circular, with radius \( b \). The metric (1) then, by Formula (7) below, relates \( b \) to the tiny angle \( \theta \) subtended in source frame by two geodesics that intersect at source, one geodesic passing through mirror center and the other contacting mirror perimeter:

\[
b = \tau_{\text{sink}} \theta \sinh \beta.
\] (7)

The mirror is reached by a tiny fraction, equal to \((\theta/2)^2\), of the total number of emitted photons. It follows from (2) and (3) that

\[
(\theta/2)^2 L = e^{2\beta} P.
\] (8)

Because the mirror (sink-frame) area \( A \) is \( \pi b^2 \), \( P = \ell \pi b^2 \). The central result (6)—the motivation for this paper—follows from Formulas (4), (5), (7) and (8).
Concluding Remarks

Elsewhere-detailed electro-gravitational quantum dynamics proceeds through a Schrödinger equation whose Hamiltonian gravitational potential energy is proportional to the energies of ‘MQU constituents’. (4) (The ‘fixed and settled’ energy-momentum current density is a classical second-rank symmetric-tensor gravitational potential’s Dalambertian, divided by $G$.)

Central to Milne spacetime is an ‘age arrow’ that accompanies redshift. Milne’s arrow of global time permits temporally-stable clumping of positive energy into galaxies. (Age arrow breaks Standard-Model CPT symmetry at galactic and Hubble scales--huge compared to those of particle physics.) Following Milne’s thinking, we conjecture homogeneity of matter at super-galactic while sub-Hubble scales--smaller than that of the entire universe. In early-universe evolution, sub-Hubble-scale density inhomogeneities are presumed to have been electro-gravitationally generated.

In a private communication to the author, J. Finkelstein has pointed out that Milne’s (Hubble-scale) cosmology is formally equivalent to an ‘empty’, $\Omega_\Lambda = \Omega_M = 0$, FRW universe—with zero cosmological constant proportional to $\Omega_\Lambda$ and zero matter density proportional to $\Omega_M$.

Efforts to base quantum cosmology on radiation-field Fock-space operators have led others to associate ‘cosmological constant’ with Fock-space vacuum energy. The author’s quantum cosmology, although including electromagnetic and gravitational radiation within its fixed and settled classical reality, (4) has no radiation-field operators and its Fock space lacks a ‘vacuum state’.

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