Abstract

Analytical and numerical calculations show that a putative temporal variation of the speed of light $c$, with the meaning of space-time structure constant $c_{ST}$, assumed to be linear over timescales of about one century, would induce a secular precession of the longitude of the pericenter $\varpi$ of a test particle orbiting a spherically symmetric body. By comparing such a predicted effect to the corrections $\Delta \dot{\varpi}$ to the usual Newtonian/Einsteinian perihelion precessions of the inner planets of the Solar System, recently estimated by E.V. Pitjeva by fitting about one century of modern astronomical observations with the standard dynamical force models of the EPM ephemerides, we obtained $\dot{c}/c = (0.5 \pm 2) \times 10^{-7} \text{ yr}^{-1}$. Moreover, the possibility that $\dot{c}/c \neq 0$ over the last century is ruled out at $3-12\sigma$ level by taking the ratios of the perihelia for different pairs of planets. Our results are independent of any measurement of the variations of other fundamental constants which may be explained by a variation of $c$ itself (with the meaning of electromagnetic constant $c_{EM}$). It will be important to repeat such tests if and when other teams of astronomers will estimate their own corrections to the standard Newtonian/Einsteinian planetary perihelion precessions.

Keywords: Experimental studies of gravity; Modified theories of gravity; Solar system objects

1 Introduction

In this paper we will deal with the problem of effectively putting on the test an hypothetical time-variation of the speed of light $c$ in a purely phenomenological, model-independent way with local, Solar-System-scale astronomical observations.

Varying Speed of Light (VSL) theories were proposed in recent times to accommodate certain features of the hot Big-Bang cosmology. In this sense,
the first modern VSL theory was put forth by Moffat in Ref. [1]; for other pioneering works see, e.g., Ref. [2] and Ref. [3]. Since then, this subject was dealt with by many authors investigating different aspects of it; see, e.g., Ref. [4] for an extensive review. Broadly speaking, such theories can be subdivided in two categories: those encompassing space-time variations of $c$, motivated by cosmology, and those where $c$ varies with the energy scale, related to phenomenological quantum gravity. Subtle issues concerning fundamental aspects of VSL theories have been recently discussed in Ref. [5], Ref. [6] and Ref. [7].

From the observational point of view, the measured percent change $\Delta \alpha / \alpha$ of the fine structure constant is

$$\frac{\Delta \alpha}{\alpha} = (-7.2 \pm 1.8) \times 10^{-6}$$

where $q_e$ is the electron charge and $\hbar$ is the Planck’s constant, respectively, from quasar observations at redshift $z \approx 0.5 - 3.5$ was very important for VSL theories; indeed, the natural question arises: if $\alpha$ is varying, is such a change due to $q_e$, $\hbar$ or $c$? By attributing $\Delta \alpha / \alpha$ to a temporal variation of $c$, it follows

$$\frac{\Delta c}{c} = -\frac{\Delta \alpha}{\alpha}.$$  

According to the distinction of the many facets of $c$ proposed in Ref. [5], the $c$ present here would be $c_{EM}$, i.e. the electromagnetic constant. Since

$$\frac{\Delta \alpha}{\alpha} \equiv \frac{\alpha_{\text{past}} - \alpha_{\text{today}}}{\alpha_{\text{today}}} < 0,$$

the value of $\alpha$ was lower in the past; thus, $c$ would have been larger in the past and it would be decreasing. By assuming a linear time dependence

$$\frac{\Delta c}{c} \approx \frac{\dot{c}}{c} (t - t_0) < 0,$$

from eq. (1) it can be obtained

$$\frac{\dot{c}}{c} \approx (-8 \pm 2) \times 10^{-16} \text{ yr}^{-1}.$$ 

\footnote{For a review on the issue of the variation of $\alpha$ and other fundamental constants, see, e.g., Ref. [11].}
for 

\[ t - t_0 \approx 9 \text{ Gyr} \]  

which approximately corresponds to the temporal interval spanned by the data analyzed in Ref. \[ \text{[10]}, \text{i.e. from 23\% to 87\% of the age of the universe.} \]

A tighter bound could be obtained from the constrain in the variation of \( \alpha \) over the last 1.8 Gyr 

\[ \left| \frac{\dot{\alpha}}{\alpha} \right| \leq 3 \times 10^{-17} \text{ yr}^{-1} \]  

from an analysis of the Oklo mine data \[ \text{[12]}. \]

Can local\footnote{For a strategy to combine local and cosmological tests of varying fundamentals constants like \( \alpha \) and \( G \) see Ref. \[ \text{[13]}. \]} (in space and time) astronomical observations tell us something about the hypothesis that \( c \) undergoes temporal variations? The answer is, in principle, positive because the motion of the major bodies of the Solar System is governed by the dynamical equations of motion of classical general relativity in which \( c \), playing the role of the space-time structure constant \( c_{\text{ST}} \) \[ \text{[5]}, \] is explicitly present; as we will see, a (slowly) time-varying \( c \) induces dynamical effects that can be tested with the latest planetary observations independently of \( \alpha \). Of course, it must be borne in mind that such tests can only constrain \( \dot{c}/c \) over timescales of about one century, corresponding to the temporal interval covered by the modern astronomical observations of the major bodies of the Solar System which are used to construct the present-day highly accurate ephemerides.

2 The dynamical effects of \( \dot{c}/c \) on the orbital motion of a test particle

We will follow a phenomenological approximation, without working in any specific VSL theoretical framework. By inserting eq. \[ \text{[5]} \] into the 1PN gravitoelectric acceleration \[ \text{[14]} \] of order \( \mathcal{O}(c^{-2}) \)

\[ A_{1\text{PN}} = \frac{GM}{c^2 r^3} \left[ \left( \frac{4GM}{r} - v^2 \right) r + 4(r \cdot v)v \right], \]  

which causes the well-known Mercury’s perihelion precession of 43.98 arcsec \( \text{cy}^{-1} \), one gets

\[ \Delta A_{1\text{PN}} \approx \left[ -2 \left( \frac{\dot{c}}{c} \right) (t - t_0) \right] A_{1\text{PN}}; \]  

\[ \text{[10]} \]
here and in the following $c = c_0 = c(t_0)$. Note that, according to the distinction of the many facets of $c$ proposed in Ref. [5], the quantity varying here is the space-time structure constant $c_{ST}$, which is, in principle, not related to the electromagnetic constant $c_{EM}$. It can be shown that the first term of eq. (9) is proportional to $c_{ST}^2/c_{EM}^4$, while the other two terms are proportional to $1/c_{EM}^2$, where $c_{E}$ the Einstein space-time matter constant [5]; however, in order to get the correct Newtonian limit for gravity [5], we will assume $c_{ST} = c_{E}$. Our approach, which has the merit of making direct and unambiguous contact with the observations giving definite answers, might be criticized from a theoretical point of view as, perhaps, too naive; indeed, as pointed out by Jordan [15, 16], in general, it is not consistent to allow a constant to vary in an equation that has been derived from a variational principle under the hypothesis that this quantity is constant; one needs to go back to the Lagrangian and derive new equations with the constant treated as a dynamical field. However, whatever the temporal evolution of $c(t)$ may be, the approximation of eq. (5) is adequate for the practical purpose of testing it over relatively short timescales like the last century in which modern astronomical planetary observations were collected. Incidentally, let us note that, in this case, certain observational issues [5, 6] can be neglected, at least from a practical point of view. Indeed, from

$$d\tau = \frac{\sqrt{g_{00}}}{c} dt,$$

where $c$ is the space-time structure constant $c_{ST}$, by assuming a linear time variation of it, the following shift in the measured proper time would occur for a static field

$$\left| \frac{\Delta \tau}{\tau} \right| = \frac{\dot{c} \Delta t}{c^2};$$

over $\Delta t = 100$ yr and by assuming for $c_{ST}$ the same rate of change of $c_{EM}$ obtained from the Oklo natural reactor data for $\alpha$ of eq. (8), it turns out

$$\left| \frac{\Delta \tau}{\tau} \right| \approx 1.5 \times 10^{-15},$$

which is basically undetectable given the present-day accuracy in realizing the SI second by the Bureau International des Poids et Mesures (BIPM), i.e. [17] $3 \times 10^{-15}$.

From a dynamical point of view, $\Delta A_{1PN}$ can certainly be considered as a small perturbation with respect to the Newtonian monopole over timescales

\[^{3}\text{It should be recalled that the observational basis of VSL phenomenology is, at present, quite meager.}\]
of about 100 yr, as it will be a posteriori confirmed by the bound on $|\dot{c}/c|$ that we will obtain with such a hypothesis; the same holds if one uses eq. (6) derived from $\Delta \alpha/\alpha$. Thus, eq. (10) can be treated perturbatively with the standard Gauss approach which is valid for any perturbing acceleration, whatever its physical origin may be. In order to evaluate the orbital effects of a generic small disturbing acceleration $W$, it is customarily projected onto an orthonormal frame $K$ co-moving with the test particle. The mutually orthogonal unit vectors $\hat{r}, \hat{\tau}, \hat{\nu}$ of $K$ pick out the radial, transverse and normal directions, respectively; $\hat{r}$ and $\hat{\tau}$ are in-plane with $\hat{r}$ directed along the particle’s radius vector, while $\hat{\nu}$ is out-of-plane, directed along the orbital angular momentum. The Gauss equations for the variations of the Keplerian orbital elements are \cite{18}

\begin{align}
\frac{da}{dt} &= -\frac{2}{n\sqrt{1-e^2}} \left[ eW_r \sin f + W_r \left( \frac{p}{r} \right) \right], \quad (14) \\
\frac{de}{dt} &= -\frac{\sqrt{1-e^2}}{na} \left\{ W_r \sin f + W_r \left[ \cos f + \frac{1}{e} \left( 1 - \frac{r}{a} \right) \right] \right\}, \quad (15) \\
\frac{dI}{dt} &= -\frac{1}{na\sqrt{1-e^2}} W_\nu \left( \frac{r}{a} \right) \cos u, \quad (16) \\
\frac{d\Omega}{dt} &= -\frac{1}{na\sin I\sqrt{1-e^2}} W_\nu \left( \frac{r}{a} \right) \sin u, \quad (17) \\
\frac{d\omega}{dt} &= -\frac{\sqrt{1-e^2}}{nac} \left\{ -W_r \cos f + W_r \left( 1 + \frac{r}{p} \right) \sin f \right\} - \cos I \frac{d\Omega}{dt}, \quad (18) \\
\frac{dM}{dt} &= -\frac{2}{na} W_r \left( \frac{r}{a} \right) - \sqrt{1-e^2} \left( \frac{d\omega}{dt} + \cos I \frac{d\Omega}{dt} \right), \quad (19)
\end{align}

where $a, e, I, \Omega, \omega$ and $M$ are the semi-major axis, the eccentricity, the inclination, the longitude of the ascending node, the argument of pericentre and the mean anomaly of the orbit of the test particle, respectively. The angle $f$ is the true anomaly reckoning the instantaneous position of the test particle along its orbit with respect to the pericentre, $u = \omega + f$ is the argument of latitude, $p = a(1 - e^2)$ is the semi-latus rectum and $n = \sqrt{GM/a^3}$ is the un-perturbed Keplerian mean motion related to the un-perturbed Keplerian orbital period by $P_b = 2\pi/n$. For the following calculations it is more convenient to use the eccentric anomaly\footnote{It is defined by $M = E - e \sin E$.} $E$ in terms of which the un-perturbed Keplerian ellipse at epoch $t_0$ can be written as

\begin{equation}
    r = a(1 - e \cos E), \quad (20)
\end{equation}
Table 1: First row: eccentricities \(e\) of the inner planets of the Solar System. Second row: numerically calculated values of \(F(e)\) according to eq. (26).

|       | Mercury | Venus | Earth   | Mars    |
|-------|---------|-------|---------|---------|
| \(e\) | 0.20563069 | 0.00677323 | 0.01671022 | 0.09341233 |
| \(F(e)\) | 527.063 | 5862.73 | 2554.07 | 714.504 |

\[
\cos f = \frac{\cos E - e}{1 - e \cos E},
\]

(21)

\[
\sin f = \frac{\sqrt{1 - e^2 \sin E}}{1 - e \cos E},
\]

(22)

To obtain the secular, i.e. averaged over one orbital revolution, effects of \(W\), it has to be evaluated onto the unperturbed Keplerian ellipse with the aid of eq. (20)-eq. (22) and inserted into the right-hand-side of eq. (14)-eq. (19); then, an integration with respect to \(t\) over an orbital period has to be performed by using

\[
\frac{dt}{P_b} = \left(\frac{1 - e \cos E}{2\pi}\right) dE.
\]

(23)

In the case of eq. (10), with

\[
t - t_0 = \frac{E - e \sin E}{n},
\]

(24)

the Gauss equation for \(\omega\) yields

\[
\langle \dot{\omega} \rangle = - \left(\frac{\dot{c}}{c}\right) \left(\frac{GM}{c^2a}\right) \frac{F(e)}{2\pi},
\]

(25)

with

\[
F(e) = \frac{\sqrt{1 - e^2}}{e} \int_0^{2\pi} \frac{dE}{(1 - e \cos E)^3} [2(3 + e^2) \cos E + e (-15 + 7 \cos 2E) (-E + e \sin E)]
\]

(26)

in Table 1 we quote the numerically computed values of \(F(e)\) for the inner planets of the Solar System. Note that eq. (25) holds also for the longitude of pericentre \(\varpi = \Omega \cos I + \omega\); indeed, since \(W_\nu = 0\), from eq. (16) and eq. (17) turns out that \(\langle \dot{I} \rangle = \langle \dot{\Omega} \rangle = 0\). Moreover, it is not possible to attribute the pericentre precession to a re-scaled time-varying gravitational constant because, in this case, also the Newtonian monopole \(-GM/r^2\) would
Figure 1: Numerically integrated trajectory of a fictitious planet around the Sun affected by the perturbing acceleration of eq. (10) in addition to the Newtonian monopole \(-GM/r^2\). A positive value large enough \((\dot{c}/c = 10^4 \text{ yr}^{-1})\) to sufficiently enhance the pericentre precession has been chosen for \(\dot{c}/c\). The initial conditions chosen are \(x_0 = r_{\text{min}} = a(1 - e)\), \(y_0 = 0\), \(z_0 = 0\), \(x_0 = 0\), \(y_0 = v_{\text{max}} = na\sqrt{(1 + e)/(1 - e)}\), \(z_0 = 0\) with \(a = 1 \text{ AU}\), \(e = 0.85\): the motion of the planet along the orbit is anticlockwise. The temporal interval spanned by the integration is 10 yr. The retrograde (i.e. clockwise) precession, as predicted by eq. (25), is clearly visible.

Let us stress that eq. (25) is different from the precession obtained by Magueijo by investigating in Ref. [19] spherically symmetric solutions to a definite covariant and Lorentz-invariant VSL theory [20]; indeed, the Magueijo’s effect is equal to the usual 1PN precession\(^5\) multiplied by an adimensional factor, i.e.

\[
\dot{\omega}_{\text{VSL}} = -\frac{3nGM}{c^2a(1 - e^2)} \left(\frac{4b^2}{3\kappa}\right) = -\frac{3(GM)^{3/2}}{c^2a^{5/2}(1 - e^2)} \left(\frac{4b^2}{3\kappa}\right), \quad (27)
\]

where \(b\) and \(\kappa\) are, in turn, numbers [20][19], presumably of some fundamental nature, accounting for the dynamical evolution of \(c\). Simple dimensional considerations show, in fact, that eq. (27) does not look like the formula one would reasonably expect for a weak-field dynamical precessional effect induced by a (slow) time variation of \(c\). Indeed, the basic ingredients that should intuitively enter such a formula are the two lengths \(GM/c^2\) and \(a\),

\(^5\)The precession per orbit is shown in Ref. [19]: in order to compare it with our results, it must simply be divided by the Keplerian orbital period \(P_b = 2\pi/n\).
characterizing the problem at hand, a possible adimensional function of the eccentricity \( e \) and a quantity \( Q \) having the dimensions of the reciprocal of time; \([Q] = T^{-1}\). Now, possible candidates for \( Q \) are the orbital frequency \( n \) and, of course, \( \dot{c}/c \) which is the cause of the effect looked for; excluding quadratic terms in \( \dot{c}/c \), the most natural choice seems to be just \( Q = \dot{c}/c \). Stated differently, it would be possible to express eq. \((25)\) as the standard 1PN precession times an adimensional factor \( \xi \), but the latter one would be

\[
\xi = -\left(\frac{\dot{c}}{c}\right)\frac{1}{6\pi n}(1 - e^2)F(e),
\]

where there is only one dimensional parameter related to the variation of \( c \), i.e. its percent first derivative, while the other dimensional quantity, specific to the system considered, is the planet’s orbital frequency. As we will see later, the dependence on \( a \) and \( e \) is crucial for the confrontation with observation-related quantities.

3 The confrontation with the observations in the Solar System

By suitably using the perihelia of the inner planets of the Solar System it is possible to constrain \( \dot{c}/c \) over timescales of the order of about 1 century and even rule out the hypothesis that it may have a non-zero value, at least in the last century.

The astronomer E.V. Pitjeva has recently fitted almost one century of planetary data of various types with the dynamical models of the EPM ephemerides estimating, in the least-square sense, several parameters; the forces modelled include [21] all the most relevant Newtonian effects (N-body mutual perturbations among the major bodies of the Solar System, Sun’s oblateness, 301 large asteroids, massive ring lying in the ecliptic plane accounting for the small asteroids) and the general relativistic Schwarzschild-like accelerations in the harmonic gauge. Among the various solutions obtained, in one of them she also phenomenologically estimated corrections [21] [22] \( \Delta \dot{\varpi} \) to the standard Newtonian-Einsteinian precessions of the longitudes of the perihelia\(^6\) of the inner planets by keeping the usual PPN parameters fixed to their general relativistic values. By construction, such corrections \( \Delta \dot{\varpi} \), shown in Table 2, account, in principle, for any standard, i.e.

\(^6\)Strictly speaking, the perihelia are not observables; they can be computed from the measured quantities which are ranges, range-rates and angles like right ascension and declination.
Table 2: First row: estimated corrections $\Delta \dot{\omega}$ to the Newton/Einstein perihelion precessions of the inner planets, in $10^{-4}$ arcsec yr$^{-1}$ according to Table 3 of Ref. [21] (Mercury, Earth, Mars). The result for Venus has been obtained by recently processing radiometric data from Magellan spacecraft [22]. In square brackets we quote the formal, statistical errors resulting from the least-square estimation process. In the text we used the re-scaled errors. Second row: un-modelled general relativistic Lense-Thirring precessions $\dot{\omega}_{LT}$, $10^{-4}$ arcsec yr$^{-1}$. They must be subtracted from the estimated $\Delta \dot{\omega}$ in order to have the anomalous effects induced by neither classical mechanics nor standard general relativity.

|          | Mercury | Venus | Earth | Mars |
|----------|---------|-------|-------|------|
| $\Delta \dot{\omega}$ ($10^{-4}$ arcsec yr$^{-1}$) | $-36 \pm 50^{[42]}$ | $-4 \pm 5^{[1]}$ | $-2 \pm 4^{[1]}$ | $1 \pm 5^{[1]}$ |
| $\dot{\omega}_{LT}$ ($10^{-4}$ arcsec yr$^{-1}$) | $-20$ | $-3$ | $-1$ | $-0.3$ |

Newtonian and/or general relativistic, or exotic un-modelled/mis-modelled forces. In order to have the fully non-relativistic, exotic effects, the Lense-Thirring precessions, not modelled in the EPM ephemerides, have to be subtracted from the estimated corrections, i.e. one has to use

$$\Delta \dot{\omega}^* = \Delta \dot{\omega} - \dot{\omega}_{LT}.$$  \hfill (29)

Now, $\Delta \dot{\omega}^*$ can fruitfully be compared to eq. (25) to constrain $\dot{c}/c$ by assuming that they are entirely due to the putative dynamical effects due to the first derivative of $c$.

By letting $\dot{c}/c$ be a free parameter, we can constrain it by comparing eq. (25) and Table 1 to the estimated $\Delta \dot{\omega}^*$ quoted in Table 2. From a weighted mean of the values of $\dot{c}/c$ obtained with the four inner planets it turns out

$$\frac{\dot{c}}{c} = (0.5 \pm 2) \times 10^{-7} \text{ yr}^{-1},$$  \hfill (30)

compatible with eq. (6). Our result, obtained without considering $\Delta \alpha/\alpha$ and valid for the last century, is very conservative and pessimistic: indeed, we did not use the mere formal, statistical errors in $\Delta \dot{\omega}$ and we linearly added the errors $\delta \Delta \dot{\omega}$ and $\delta \dot{\omega}_{c/c}$ in constructing the total uncertainty in $|\Delta \dot{\omega}^* - \dot{\omega}_{c/c}|$.

By suitably combining the perihelia of various pairs of planets it is possible to perform a more stringent test of the hypothesis that currently $\dot{c}/c \neq 0$, independently of its origin and magnitude. Indeed, eq. (25) and eq. (26) yield a function of $a$ and $e$ which represents a distinctive signature of the dynamical effects of $\dot{c}/c$, irrespectively of its size; moreover, it is important
Table 3: First column: pair of planets A and B. Second column: observationally determined ratios $\Pi = \Delta \dot{\varpi}_A / \Delta \dot{\varpi}_B$ for A and B. Third column: theoretically predicted ratios $\mathcal{A} = F_A a_B / F_B a_A$ for A and B. Fourth column: $\Gamma = |\Pi - \mathcal{A}| / \delta \Pi$; $\Gamma > 1$ means that $\Pi \neq \mathcal{A}$ within the errors. It turns out that the uncertainties in $e$ and $a$ are completely negligible in evaluating the errors in $\Pi - \mathcal{A}$ which are, instead, dominated by $\delta \Pi$.

|       |       |   |   |
|-------|-------|---|---|
| Venus | Mars  | -0.8 ± 6.8 | 17.3 | 3 |
| Earth | Mercury | 0.06 ± 0.44 | 1.87 | 4 |
| Venus | Mercury | 0.06 ± 0.50 | 5.95 | 12 |

It must be noted that $\dot{c}/c$ enters eq. (25) as a multiplicative factor. Thus, by taking the ratios $\mathcal{A}$ of eq. (25) for different pairs of planets A and B it is possible to construct theoretical predictions which are, at the same time, independent of the magnitude of $\dot{c}/c$ and still retain a pattern characteristic of $\dot{c}/c$ itself. Thus, $\mathcal{A}$ can be compared to $\Pi = \Delta \dot{\varpi}_A / \Delta \dot{\varpi}_B$ for the same pairs of planets: if $\mathcal{A} \neq \Pi$ within the errors, i.e. if $|\mathcal{A} - \Pi| \neq 0$ within the errors for some of the pairs considered, we must reject the possibility that $c$ is nowadays varying according to $\dot{c}/c \neq 0$. The results are in Table 3; the hypothesis $\dot{c}/c \neq 0$ during about the last century must be rejected at more than $3 - \sigma$ level. Also in this case, our test is conservative because we evaluated the uncertainty in $\Pi$ as

$$
\delta \Pi \leq |\Pi| \left( \frac{\delta \Delta \dot{\varpi}_A^*}{\Delta \dot{\varpi}_A} + \frac{\delta \Delta \dot{\varpi}_B^*}{\Delta \dot{\varpi}_B} \right).$$

(31)

It must be noted that, since $\Delta \dot{\varpi}$ are observation-related quantities, it is perfectly meaningful to take their ratios $\Pi$; the fact that $\delta \Delta \dot{\varpi}/\Delta \dot{\varpi} > 1$ simply means that $\Delta \dot{\varpi}$ can still have a non-zero value smaller than $\delta \Delta \dot{\varpi}$ and that $\Pi$ is compatible with zero within the errors.

It maybe interesting to note that the perihelion precession of eq. (27) by Magueijo [19] would survive the test of the ratios of the perihelia.

### 4 Discussion and conclusions

In this paper we phenomenologically put on the test the hypothesis that the speed of light $c$, with the meaning of space-time structure constant $c_{ST}$, can vary over timescales of about one century. We analytically worked out the dynamical effects induced by a linear variation in time of $c$ on the
motion of a test particle orbiting a spherically symmetric body finding that the longitude of pericentre \( \varpi \) undergoes secular precessions; a numerical integration of the equations of motion qualitatively confirmed this result. As expected from simple dimensional considerations, the expression obtained for \( \dot{\varpi} \) is proportional to the product of \( \dot{c}/c \) by \((GM/c^2a)F(e)\), where \( F(e) \) is a specific adimensional function of the eccentricity \( e \). We compared such a theoretical prediction to the recently estimated corrections to the standard Newtonian/Einsteinian perihelion precessions for the inner planets of the Solar System, obtained by analyzing the last century of data, finding \( \dot{c}/c = (0.5 \pm 2) \times 10^{-7} \text{ yr}^{-1} \). Moreover, by taking the ratios of the computed anomalous perihelion precessions for different pairs of planets we were able to obtain a prediction independent of \( \dot{c}/c \) itself and still retaining a pattern characteristic of it. The confrontation of such predicted ratios with the ratios of the observationally determined corrections to the usual perihelion precessions ruled out the hypothesis that \( \dot{c}/c \neq 0 \) in the last century at 3 – 12\( \sigma \) level. Our result is independent of any measured variations of other fundamental constants which could be related to a variation of \( c \) itself (with a different meaning like, e.g., that of electromagnetic constant \( c_{EM} \)). If and when other teams of astronomers will estimate their own corrections to the standard perihelion precessions it will be possible to fruitfully repeat this tests.

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