Electroweak Baryogenesis with Lepton Flavor Violation

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We investigate the feasibility of electroweak baryogenesis in a two-Higgs doublet model with lepton flavor violation. By scrutinizing the heavy Higgs boson mass spectrum, regions satisfying both strong first-order electroweak phase transition and the muon $g-2$ anomaly are identified. We also estimate the baryon number density by exploiting extra Yukawa couplings in the $\mu$-$\tau$ sector. It is found that a CP-violating source term can be enhanced by the $\mu$-$\tau$ flavor-violating coupling together with the extra $\tau$ coupling. With $O(1)$ Yukawa couplings and CP-violating phases, the observed baryon number density is marginally produced under a generous assumption for the bubble wall profile.

\section{I. INTRODUCTION}

Baryon asymmetry of the Universe (BAU) is an observational fact\textsuperscript{1} whose origin still remains open and calls for physics beyond the standard model (SM). Two important ingredients, namely, CP violation and departure from thermal equilibrium, are known to be insufficient in the SM\textsuperscript{2,3}. A great number of baryogenesis mechanisms have been proposed so far, and the relevant energy scales are highly model-dependent. From the view point of testability, electroweak baryogenesis (EWBG)\textsuperscript{4,5} is the most attractive scenario since it can be probed by current and foreseeable future experiments. It is thus interesting and important to scrutinize its feasible parameter space.

One of the simplest extensions for successful EWBG is to add another Higgs doublet to the SM Higgs sector, rendering the so-called two-Higgs doublet model (2HDM)\textsuperscript{6} (for a review, see Ref.\textsuperscript{7} and references therein). In this model, both Higgs doublets can couple to quarks and leptons concurrently, giving rise to Higgs-mediated flavor-changing processes at tree level. Importantly, flavor-changing neutral Higgs (FCNH) couplings are in general complex and may yield CP violation relevant to baryogenesis. Earlier studies on EWBG with FCNH interactions can be found in Refs.\textsuperscript{8,9}. Moreover, it is possible to have a strong first-order electroweak phase transition (EWPT) owing to the contributions of the additional Higgs doublet\textsuperscript{10,11,12}.

Recently, the CMS Collaboration reported an excess in a lepton flavor-violating (LFV) Higgs decay, \[ Br(h \rightarrow \mu \tau) = (0.84^{+0.39}_{-0.37})\% \], showing a 2.4$\sigma$ deviation from the SM\textsuperscript{14}. The ATLAS Collaboration, on the other hand, quotes an upper bound \[ Br(h \rightarrow \mu \tau) < 1.43\% \] The authors of Refs.\textsuperscript{17,18}, including one of the present authors, pointed out that the 2HDM with $\mu$-$\tau$ flavor violation granted a parameter space to explain not only the $h \rightarrow \mu \tau$ excess but also the long-standing anomaly in muon $g-2$ without having conflicts with other LFV constraints, such as $\tau \rightarrow \mu \gamma$, $\tau \rightarrow \mu \mu \nu$, etc. Since such FCNH interactions are generically accompanied by additional CP-violating sources, it is an interesting question whether the new CP-violating phases play a crucial role in generating the BAU.

In this paper, we examine the possibility of EWBG in the 2HDM with $\mu$-$\tau$ flavor violation. Our analysis is twofold: (1) baryon number generation via the new CP-violating source based on the method of closed-time path formalism with vacuum expectation value (VEV) insertion, and then solve the diffusion equations to estimate the BAU. For the latter, we employ the one-loop finite temperature effective potential with a thermal resummation to determine the order of EWPT. To determine the strength of first-order EWPT, we explicitly evaluate the energy of sphaleron configuration.

The paper is organized as follows. In Sec.\textsuperscript{III} we give basic ingredients of the 2HDM and show a relationship between FCNH couplings in the symmetric phase and those in the broken phase. In Sec.\textsuperscript{III} we review how $h \rightarrow \mu \tau$ and $(g-2)_\mu$ can be simultaneously explained by the $\mu$-$\tau$ flavor violation, as found in Refs.\textsuperscript{17,18}. The baryon number preservation condition is given in Sec.\textsuperscript{IV}. We identify the regions where both strong first-order EWPT and $(g-2)_\mu$ anomaly can be accounted for. In Sec.\textsuperscript{V} we present the BAU calculation and its numerical analysis. Our conclusion is given in Sec.\textsuperscript{VI}.

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\textsuperscript{1} The excess would be less significant if the latest LHC Run-II data are taken into account\textsuperscript{13}. Statistically, it is not yet sufficient to claim that the excess is completely gone.
II. MODEL

The 2HDM is an extension of the SM by adding one additional Higgs doublet. Such a two-Higgs doublets structure is motivated by some UV theories such as supersymmetric theories. The most general Higgs potential at the renormalizable level is

\[ V_0(\Phi_1, \Phi_2) = m_1^2 \Phi_1^\dagger \Phi_1 + m_2^2 \Phi_2^\dagger \Phi_2 - (m_1^2 \Phi_2 \Phi_1^\dagger + h.c.) \]

\[ + \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) \]

\[ + \lambda_4 (\Phi_2^\dagger \Phi_2)(\Phi_1^\dagger \Phi_1) + \left[ \frac{\lambda_5}{2} (\Phi_1^\dagger \Phi_2)^2 \right] \]

\[ + \left\{ \lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2) \right\} (\Phi_1^\dagger \Phi_2 + h.c.) \]  (1)

Hermiticity of \( V_0 \) requires that \( m_1^2, m_2^2, \lambda_1, \lambda_2, \lambda_3 \) and \( \lambda_4 \) are all real while \( m_1^2, m_2^2, \lambda_5 \) and \( \lambda_7 \) are generally complex, with one of them being possibly made real by a field redefinition of either Higgs doublet. We parametrize \( \Phi_i \) \((i = 1, 2)\) as

\[ \Phi_i(x) = e^{i\theta_i} \left( \frac{\phi_i^+(x)}{\sqrt{2}} + h_i(x) + i a_i(x) \right) \]  (2)

where \( v e^{i\theta_i} \) are the VEV’s. For simplicity, we assume that CP is not violated by the Higgs potential and the VEV’s, leading to \( \theta_i = 0 \). Here we express \( v_{1,2} \) in terms of polar coordinates, \( v_1 = v \cos \beta \) and \( v_2 = v \sin \beta \) with \( v \simeq 246 \text{ GeV} \). In the following, we will use the shorthand notation \( s_\beta = \sin \beta, c_\beta = \cos \beta \) and \( t_\beta = \tan \beta \).

In phenomenological discussions, it is useful to use the Higgs (Georgi) basis \(^{19}\) in which only one Higgs doublet develops the VEV and the Nambu-Goldstone bosons \((G^0, G^\pm)\) are decoupled from the physical states:

\[ \Phi_1^\prime = c_\beta \Phi_1 + s_\beta \Phi_2 = \left( \frac{G^+}{\sqrt{2}} \right) (v + h'_1 + iG^0) \]  (3)

\[ \Phi_2^\prime = -s_\beta \Phi_1 + c_\beta \Phi_2 = \left( \frac{H^+}{\sqrt{2}} \right) (h'_2 + iA) \]

where \( h'_1 = c_{\beta-\alpha} H + s_{\beta-\alpha} h \) and \( h'_2 = -s_{\beta-\alpha} H + c_{\beta-\alpha} h \) with \( \alpha \) being a mixing angle between the two CP-even Higgs fields \((h_{1,2})\). In this paper, \( h \) is the 125 GeV Higgs boson and assumed to be the lighter one.

Without imposing any symmetries, both Higgs doublets can couple to fermions. The relevant Yukawa interactions in the lepton sector are

\[ -\mathcal{L}_Y = \tilde{l}_L (Y_1 \Phi_1 + Y_2 \Phi_2)_{ij} e_{jR} + \text{h.c.} \]

\[ \equiv \tilde{e}_{iL} \left[ \frac{y_1}{\sqrt{2}} \delta_{ij} s_{\beta-\alpha} \right] e_{jR} h \]

\[ + \tilde{e}_{iL} \left[ \frac{y_2}{\sqrt{2}} \delta_{ij} c_{\beta-\alpha} \right] e_{jR} H \]

\[ + \frac{i}{\sqrt{2}} \tilde{e}_{iL} \rho_{ij} e_{jRA} + \text{h.c.} \]  (4)

where \( i, j \) are flavor indices, \( Y_{1,2} \) are general 3-by-3 complex matrices, and

\[ \rho_{ij} = -t_\beta y_{iL} \delta_{ij} + \frac{1}{c_\beta} \left( V_L^{*} Y_2 V_R^* \right)_{ij}, \]  (5)

with \( V_L, R \) defined as the unitary matrices that diagonalize the charged lepton Yukawa couplings \( Y^{SM} = (Y_1 c_\beta + Y_2 s_\beta) \), i.e., \( V_L Y^{SM} V_R^T = \text{diag}(y_e, y_\mu, y_\tau) \). After this diagonalization, the mass terms of the charged leptons are given by \( m_i = y_i v / \sqrt{2} \) with \( i = e, \mu, \tau \).

Non-diagonal elements of \( \rho_{ij} \) are the sources of tree-level FCNH processes. In the literature, the so-called Cheng-Sher ansatz \(^{20}\) for \( \rho_{ij} \) is adopted in order to avoid experimental constraints. In our analysis, however, we will not assume it while obtaining a parameter space that can accommodate both \( h \to \mu \tau \) and \( g - 2 \) anomalies. In general, \( Y_{1,2} \) cannot be uniquely determined even though \( Y_2 \) is known. In our analysis, we make some working assumption on \( Y_{1,2} \) for baryogenesis, and determine the structure of FCNH couplings \( \rho \) at \( T = 0 \). For later use, we define \( \rho_{ij} = |\rho_{ij}| e^{i\phi_{ij}} \).

III. \( h \to \mu \tau, (g - 2)_\mu \) AND EDM

In this section, we outline some important consequences of Ref. \(^{17}\) to make this paper self-contained.

The Higgs decay to \( \mu \tau \) and \( g - 2 \) in the current model occurs at tree level through \( \mu \tau \) interactions (for earlier studies, see Refs. \(^{21}\)), and its branching ratio is given by

\[ \text{Br}(h \to \mu \tau) = \frac{m_h (|\rho_{\mu \tau}|^2 + |\rho_{\mu \overline{\mu}}|^2) c_{\beta-\alpha}^2}{16 \pi \Gamma_h}, \]  (6)

where \( m_h = 125 \text{ GeV} \) and \( \Gamma_h = 4.1 \text{ MeV} \). It is easy to accommodate \( \text{Br}(h \to \mu \tau) \simeq 0.84\% \) by taking \( |\rho_{\mu \tau}| \sim |\rho_{\mu \overline{\mu}}| \sim \mathcal{O}(0.1) \) and \( c_{\beta-\alpha} \sim \mathcal{O}(0.01) \). It had been shown that such parameter choices did not violate the current experimental bounds on other LFV processes such as \( \tau \to \mu \gamma, \tau \to \mu \nu \overline{\nu}, \text{ etc } \(^{17}\) \). Note that it is sufficient for either \( \rho_{\mu \tau} \) or \( \rho_{\mu \overline{\mu}} \) to be nonzero to explain the \( h \to \mu \tau \) excess only. If both couplings coexist, on the other hand, we can have a one-loop diagram that induces \( g - 2 \) and electric dipole moment (EDM) of the muon, denoted by \( d_{\mu e} \) and \( a_\mu \), respectively. Contributions of the neutral
Higgs bosons to \(\delta a_\mu\) and \(d_\mu\) are given by

\[
\delta a_\mu = \frac{m_\mu m_T \Re(\rho_{\mu \tau} \rho_{\tau \mu})}{16\pi^2} \times \left[ \frac{2 \beta - \alpha f(r_h)}{m_h^2} + \frac{s_\beta - \alpha f(r_H)}{m_H^2} - f(r_A) \right],
\]

where

\[
d_\mu = -\frac{1}{2m_\mu} \text{Arg}(\rho_{\mu \tau} \rho_{\tau \mu}) \delta a_\mu,
\]

with \(r_\phi = m_\phi^2/m_\tau^2\). Non-degeneracy in the neutral Higgs masses and a proper choice of the overall sign are essential for obtaining a sufficient \(\delta a_\mu\). In Ref. [22], the discrepancy of \((g - 2)_\mu\) between experiment and the SM prediction was estimated as

\[
\delta a_\mu = a_\mu^{\text{EXP}} - a_\mu^{\text{SM}} = (26.1 \pm 8.0) \times 10^{-10}.
\]

As demonstrated in Ref. [17], this deviation could be accommodated for \(m_{H,A} \lesssim [200,500] \text{ GeV}\) with an appropriate mass hierarchy. From the EWBG point of view, this mass range of the heavy Higgs bosons has the right scale for realizing the first-order EWPT, though the LFV interactions by themselves do not play a central role in the realization, as will be shown in the following sections.

The current limit on \(d_\mu\) is [23]

\[
|d_\mu| < 1.9 \times 10^{-19} \text{ e cm},
\]

which is about three orders of magnitude larger than \(d_\mu\) estimated with \(\text{Arg}(\rho_{\mu \tau} \rho_{\tau \mu}) = 1\) and \(\delta a_\mu = 3.0 \times 10^{-9}\) in Eq. [8]. In what follows, we will clarify the relationship between CP violation appearing in \(d_\mu\) and that relevant to BAU.

### IV. BARYON NUMBER PRESERVATION

The baryon number is generated via the sphaleron process outside the bubble (symmetric phase), and it can survive if the sphaleron process is sufficiently quenched inside the bubble (broken phase). To this end, the sphaleron rate in the broken phase, denoted as \(\Gamma_B^{(b)}(T)\), must be smaller than the Hubble constant, \(H(T)\). More explicitly,

\[
\Gamma_B^{(b)}(T) \simeq \text{[prefactor]} e^{-E_{\text{sph}}(T)/T} \lesssim H(T) \simeq 1.66 \sqrt{g_*(T)} T^{2}/m_P,
\]

where \(E_{\text{sph}}\) stands for the sphaleron energy, \(g_*\) is the number of relativistic degrees of freedom in the plasma \((g_* = 110.75\) in the 2HDM\), and \(m_P = 1.22 \times 10^{19} \text{ GeV}\). We parametrize the sphaleron energy as \(E_{\text{sph}}(T) = 4\pi v(T)\mathcal{E}(T)/g_2\) with \(g_2\) being the SU(2)_L gauge coupling constant. Eq. [12] is then rewritten as

\[
\frac{v(T)}{T} > \frac{g_2}{4\pi \mathcal{E}(T)} \left[ 42.97 + \log \text{terms} \right] \equiv \zeta_{\text{sph}}(T).
\]

One can see that \(\zeta_{\text{sph}}(T)\) is mostly controlled by \(\mathcal{E}(T)\). The logarithmic corrections in the bracket come from the prefactor of \(\Gamma_B^{(b)}\). To our best knowledge, an extensive study on the prefactor in the 2HDM is still missing. In the minimal supersymmetric SM case, on the other hand, the zero mode factors of the fluctuations about the sphaleron typically amount to about 10\% [24]. This is subdominant and, therefore, we will neglect them in our numerical analysis for simplicity.

We impose Eq. [13] at a critical temperature \(T_C\) at which the effective potential has two degenerate minima. In our analysis, \(T_C\) and \(v_C \equiv v(T_C)\) are determined using finite-temperature one-loop effective potential with thermal resummation. As mentioned in Sec. [11] \(s_\beta - \alpha\) is close to 1 in the region of interest to us. In such a case, we may simplify the analysis of EWPT to a one-dimensional problem with a single order parameter, as discussed in Refs. [11, 12].

As is well-known, the extra heavy Higgs bosons can play a major role in enhancing \(v_C/T_C\) in the 2HDM. In this case, \(M^2 \equiv m_T^2/(s_\beta c_\beta)\) must not exceed certain values, depending on the magnitude of quartic couplings; otherwise, the so-called non-decoupling effects would diminish, rendering a suppressed \(v_C/T_C\). Phenomenological consequences of the non-decoupling effects at \(T = 0\) include significant deviations in the \(h \rightarrow \gamma\gamma\) decay width [25] and the triple Higgs coupling [26].

For the evaluation of \(\mathcal{E}\), we solve the equations of motion for the sphaleron with appropriate boundary conditions [27, 28]. Here, we use the tree-level Higgs potential for simplicity. In this case, \(\zeta_{\text{sph}}\) may be underestimated by \(O(10)\%\) since \(\mathcal{E}(0) > \mathcal{E}(T_C)\). As will be shown below, this approximation does not affect our conclusion. A detailed analysis of \(\zeta_{\text{sph}}(T)\) will be given elsewhere.

We note in passing that recent studies show that \(\zeta_{\text{sph}}(T_C) = 1.1 - 1.2\) in the real singlet-extended SM [29] and 1.23 in the scale-invariant 2HDM [13] for typical parameter sets.

In Fig. [1] \(v_C/T_C\) and \(\delta a_\mu\) are shown in the \((m_H,m_A)\) plane. We take \(m_A = m_H\) to avoid the electroweak \(\rho\) parameter constraint, and choose \(c_\beta - \alpha = 0.006, |\rho_T| = |\rho_{\mu \tau}|,\) and \(\phi_{\mu \mu} + \phi_{\mu \tau} = \pi/4,\) as favored by the solution of \((g - 2)_\mu\) anomaly. For the remaining parameters, we set \(M = 100 \text{ GeV},\) \(\tan \beta = 1\) and \(\lambda_6 = \lambda_T = 0\) as an example. Contours of \(v_C/T_C\) are drawn with the solid gray curves with values of 1.0, 1.17, 1.5, 2.0, 2.5, and 3.0 from bottom to top, where 1.17 corresponds to \(\zeta_{\text{sph}}(T_C)\). Allowed 1σ, 2σ and 3σ regions of \(\delta a_\mu\) are shown by the areas colored in green, blue and pink, respectively. One can see that the regions satisfying the baryon number preservation condition \((v_C/T_C > \zeta_{\text{sph}})\) and favored by \((g - 2)_\mu\) have an overlap if \(m_A > m_H\). We note that
the \((g-2)\mu\)-favored parameter space can change to the \(m_A < m_H\) region if \(\text{Re}(\rho_{\mu\tau}) < 0\), as inferred from Eq. (7). However, the allowed region shrinks in this case.

V. BARYON NUMBER GENERATION

We closely follow the method given in Refs. 30–33 (see also Refs. 34–38) in estimating the BAU. In our scenario, the CP-violating source term is induced by interactions between chiral fermions and spacetime-dependent VEV’s \(v_i(X)\). As found in Ref. 39, the treatment of the chiral fermion should be taken with some care since its dispersion relation can be significantly modified by thermal plasma. As a result, the CPV source term for the left-handed (LH) fermion of flavor \(i\) induced by the right-handed (RH) fermion of flavor \(j\) takes the form 2

\[
S_{i,j,R}(X) = \frac{2C(X)}{\text{Im}} \int_0^\infty \frac{dk^2}{2\pi^2} \left[ \frac{(1 - n_{j,R}^{d}- n_{i,L}^{d})Z_{p_R}^p Z_{l_R}^l}{(E_{j,R}^p - \xi_{i,L}^l)^2} + \frac{n_{j,L}^{d} Z_{p_R}^p Z_{l_R}^l}{(E_{j,R}^p - \xi_{i,L}^l)^2} + \frac{n_{j,R}^{d} Z_{p_R}^p Z_{l_R}^l}{(E_{j,R}^p - \xi_{i,L}^l)^2} + (p \leftrightarrow h) \right],
\]

where \(C(X) = |Y_{11j}|/|Y_{21j}| \sin \theta_{ij} v^2(X) \partial_X \beta(X)\) with \(\theta_{ij} = \text{Arg}(Y_{11j}) - \text{Arg}(Y_{21j})\), \(p\) denotes the ‘particle’ mode and \(h\) the ‘hole’ mode whose dispersion relations are given by \(E_{p,h}^\mu(k) = E_{p,h}^\mu(k) - i\Gamma_{p,h}^\mu(k)\), and \(Z_{i,R}^l\) denote the residues. 3

Here, we remark on the treatment of \(\Gamma_{p,h}^\mu\). In our numerical calculation, \(\Gamma_{p,h}^\mu\) are approximated as (constant)\(\times T\), as is often done in the calculation of the source terms for scalars and Dirac fermions. In the chiral fermion case, however, the term without statistical factor in the first term of Eq. (15) could cause a logarithmic divergence if \(\Gamma^p\) is fixed to a constant in the whole range of the \(k\) integration, which invalidates the calculation. Since the correct behavior of \(\Gamma^p\) in the large \(k\) region may take the form of \(T^3 \text{Im}(k/T) / k^2\), the integration would be convergent. As a simple prescription, we take such a \(k\)-dependent \(T_h^\mu\) for large \(k\). Ambiguities coming from this prescription will be discussed below.

One can also find that \(S_{i,j,R} = -S_{i,j,L}\). In our study, we consider the case where the 32 and 33 elements of \(Y_{1,2}\) play a dominant role in generating the BAU. In this case, only \(S_{\tau\mu\tau\tau} = -S_{\mu\tau\tau\tau}\) is relevant. As mentioned in Sec. 11, \(Y_{1,2}\) are connected with the \(T = 0\) observables through \(Y_D\) and Eq. (5). For illustration, we assume that the 12, 13, 21 and 31 elements of \(Y_{SM}\) are all zero. Also, we focus on the case where only \(\rho_{\mu\tau}, \rho_{\mu\tau}\) and \(\rho_{\tau\tau}\) take nonzero values among \(\rho_{ij}\).

Let us denote the relevant particle number densities as \(Q_3 = n_{\mu_L} + n_{b_L}, T = n_{\tau_R}, B = n_{b_R}, L_3 = n_{b_R} + n_{\tau_L}, R_3 = n_{\tau_R}, R_2 = n_{\mu_R}, H = n_{\mu_R} + n_{\tau_R} + n_{\mu_R} + n_{\tau_R}\). The number density expanded to the leading order in the chemical potential \(\mu\) is reduced to \(n_{\mu_j} = T^2 k_{B} b_{j} / 6\), with \(b (f)\) denoting bosons (fermions). In the massless case, one finds \(b_0 = 2\) and \(k_j = 1\). For massive cases, more precise form of \(b_{0,f}\) 32 should be used.

The set of Boltzmann equations may be cast into:

\[
\frac{\partial n_{\mu_j}^\mu}{\partial t} = -\Gamma_{\mu_j}^\mu(X) + \frac{\partial}{\partial \mu_j} \left[ \Gamma_{\mu_j}^\mu(X) \right], \quad \frac{\partial n_{\tau_j}^\tau}{\partial t} = -\Gamma_{\tau_j}^\tau(X) + \frac{\partial}{\partial \tau_j} \left[ \Gamma_{\tau_j}^\tau(X) \right],
\]

where \(\xi_i = n_i/k_i, N_5 = 2\xi_3 - \xi_2 + 9(Q_3 + T) / k_B, \) and \(\rho_{\mu_j}^\mu = n_i - D_i \nabla^2 n_i\) with \(D_i\) being a diffusion constant.

In solving these coupled equations, we utilize the chemical equilibrium conditions in light of \(\Gamma_{\mu_j}^\mu(X), \Gamma_{\tau_j}^\tau(X), \Gamma_{\mu_j}^\mu(X), \Gamma_{\tau_j}^\tau(X), \Gamma_{\mu_j}^\mu(X), \Gamma_{\tau_j}^\tau(X)

\footnote{\(\xi_i = n_i/k_i, N_5 = 2\xi_3 - \xi_2 + 9(Q_3 + T) / k_B, \) and \(\rho_{\mu_j}^\mu = n_i - D_i \nabla^2 n_i\) with \(D_i\) being a diffusion constant.}

\footnote{\(\rho_{\mu_j}^\mu\) can be expressed in terms of the Lernbert \(W\) functions (for details, see Ref. 29).}
where $\lambda$ is the constant of the quarks, $\Gamma$ the first term highly suppressed. The strong sphaleron washout effect, which would make the third generation LH lepton doublet and the RH muon, $D_k$ denotes the diffusion constants of the $3$-flavoured $LH$ lepton doublet and the $RH$ muon, respectively. As pointed out in Ref. [3], the lepton transport is much more efficient since it does not suffer from the strong sphaleron washout effect, which would make the first term highly suppressed.

With the above $n_L$, the BAU can be estimated as

$$n_L = 5Q_3 + 4T + L_3$$

where $D_L$ and $D_R$ denote the diffusion constants of the third generation LH lepton doublet and the RH muon, respectively. As pointed out in Ref. [3], the lepton transport is much more efficient since it does not suffer from the strong sphaleron washout effect, which would make the first term highly suppressed.

With the above $n_L$, the BAU can be estimated as

$$n_B = \frac{-3\Gamma(s)^B}{2D_q\rho^+} \int_0^\infty dz' n_L(z')e^{-\lambda-z'},$$

where $\lambda = [v_w \pm \sqrt{v_w^2 + 4\rho^+}] = 2D_q$, $v_w$ represents the expanding speed of the bubble wall, $D_q$ is the diffusion constant of the quarks, $\Gamma(s)^B$ is the sphaleron rate in the unbroken phase, and $R = 15\Gamma(3)^B/4$. In the following, we will characterize the baryon asymmetry by $Y_B = n_B/s$, where $s = (2\pi^2/45)\rho_s$ is the entropy density at temperature $T$, and quote $Y_B^{obs} = 8.59 \times 10^{-11}$, the central value given by the Planck Collaboration [1].

In the estimate of BAU, we employ the formulas given in Ref. [1] for the diffusion constants and thermal widths of LH and RH leptons, and assume $D_h = D_L$. The particular values that we use here are summarized in Table 1.

In Fig. 2 we show $Y_B/Y_B^{obs}$, $Br(h \to \mu\tau)$ and $\delta_{\mu\tau}$ as functions of $|\rho_{\mu\tau}|$ and $|\rho_{\mu\mu}|$. Here we take the same input parameters as those in Fig. 1 and the heavy Higgs boson masses $m_H = 350$ GeV and $m_A = m_{H^\pm} = 400$ GeV, leading to $v_C/T_C = 214.9$ GeV/99.2 GeV = 2.17. As an ansatz for $Y^{SM}$, we consider

$$Y^{SM} = \begin{pmatrix} \sqrt{2}m_{\mu}/v & 0 & 0 \\ 0 & 3.31 \times 10^{-3} & -6.81i \times 10^{-4} \\ 0 & 8.91i \times 10^{-3} & 3.70 \times 10^{-3} \end{pmatrix},$$

which is diagonalized by

$$V_R^H = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -0.365i & -0.931i \\ 0 & 0.931 & 0.365 \end{pmatrix},$$

$$V_L^H = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -0.945i & -0.327i \\ 0 & 0.327 & 0.945 \end{pmatrix}. \quad (17)$$

Also, we take $\phi_{\mu\tau} = \pi/2$. Note that each of $Y_{1,2}$ is fixed by $Y_{1,2}^{SM}V_{1,2}^{H*}Y_B^{obs} = Y_D$ and Eq. (5) once $\rho_{ij}$ are given.

The solid lines represent the contours of $Y_B/Y_B^{obs} = 0.5$ (left) and $1.0$ (right). The dashed lines in black and red represent $Br(h \to \mu\tau) = 1.43\%$ and $0.84\%$, respectively. The colored regions with the same color scheme as in Fig. 1 explain the $(g-2)_\mu$ anomaly. In our setup, the dominant contribution to the CP-violating source term comes from $S_{\nu_LK_H}$ which is induced by the 32 elements of $Y_1$ and $Y_2$. We find that these off-diagonal elements have stronger dependences on $|\rho_{\mu\tau}|$ than $|\rho_{\mu\mu}|$ and $|\rho_{\mu\mu}|$. Under rather generous assumptions for the bubble wall profile, the generated BAU can reach its observed value for $|\rho_{\mu\mu}| \simeq 0.1 - 0.6$ and $|\rho_{\mu\tau}| \simeq 0.8 - 0.9$. Our main conclusion is that there is a parameter space that is consistent with both experimental anomalies of $h \to \mu\tau$ and $(g-2)_\mu$ as well as the observed BAU.

Some comments on the theoretical uncertainties are in order. (1) We take the same size of $|\Delta \beta|$ as in Ref. [1] as a reference value. However, quantitative studies of it in the 2HDM are still absent. In the MSSM, $\Delta \beta = O(10^{-2} - 10^{-4})$ depending on $m_A$ [42]. In the next-to-MSSM, $\Delta \beta$ can reach $O(0.1)$ in some parameter space [43]. We should note that $Y_B$ is mostly subject to the uncertainties of $\Delta \beta$ among others since it is linearly proportional to $\Delta \beta$. (2) Studies on $v_w$ can be found in

| TABLE I. Input parameters for the BAU estimate. |
|-----------------------------------------------|
| $T_C = 99.2$ GeV | $v_C = 214.9$ GeV | $v_w = 0.1$ | $\Delta \beta = 0.05$ |
| $m_{\tau_1} = 0.21T$ | $m_{\mu_B} = 0.12T$ | $\Gamma_{\tau_L} = 0.034T$ | $\Gamma_{\mu_R} = 0.015T$ |
| $D_q = 8.9/T$ | $D_h = 101.9/T$ | $D_L = 101.9/T$ | $D_R = 377.1/T$ |
We have studied electroweak baryogenesis in the general framework of the two-Higgs doublet model in light of the $h \to \mu \tau$ and $(g - 2)_\mu$ anomalies. In this model, the heavy Higgs bosons with the appropriate $\mu$-$\tau$ flavor violation can accommodate the above two anomalies. At the same time, these extra Higgs bosons can induce a strong first-order electroweak phase transition as required for successful electroweak baryogenesis.

It is found that the $\mu$-$\tau$ flavor-violating lepton sector has a great potential to generate sufficient baryon asymmetry of the Universe within the theoretical uncertainties. In this scenario, the interplay between $\rho_{\mu\tau}/\rho_{\tau\tau}$ and $\rho_{\tau\tau}$ is crucially important. Our analysis suggests that $Y_B/Y_B^{\text{obs}} \simeq 1$ for $|\rho_{\mu\tau}| \simeq 0.1 - 0.6$ and $|\rho_{\tau\tau}| \simeq 0.8 - 0.9$ with $\mathcal{O}(1)$ CP-violating phases. To suppress $\text{Br}(\tau \to \mu \gamma)$, a cancellation mechanism has to be at work, which additionally predicts $|\rho_{\mu}| \simeq 0.5$ and $\phi_{\mu} \simeq \phi_{\tau\tau}$. Since future experimental sensitivities of $\text{Br}(\tau \to \mu \gamma)$ and $|d_e|$ are about $1 \times 10^{-9}$ and $10^{-30}$ e cm, respectively, our scenario could be fully tested.

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