7 keV Scalar Dark Matter and the Anomalous Galactic X-ray Spectrum

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Abstract

We present a simple model for a 7 keV scalar dark matter particle which also explains the recently reported anomalous peak in the galactic X-ray spectrum at 3.55 keV in terms of its two photon decay. The model is arguably the simplest extension of the Standard Model, with the addition of a real scalar gauge singlet field subject to a reflection symmetry. This symmetry breaks spontaneously at a temperature of order few GeV which triggers the decay of the dark matter particle into two photons. In this framework, the Higgs boson of the Standard Model is also the source of dark matter in the Universe. The model fits the relic dark matter abundance and the partial lifetime for two photon decay, while being consistent with constraints from domain wall formation and dark matter self-interactions. We show that all these features of the model are preserved in its natural embedding into a simple dark $U(1)$ gauge theory with a Higgs mechanism. The properties of the dark photon get determined in such a scenario. High precision cosmological measurements can potentially test these models, as there are residual effects from domain wall formation and non-negligible self-interactions of dark matter.

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1 Introduction

Recently two independent groups have reported the observation of a peak in the galactic X-ray spectrum at 3.55 keV [1,2], which cannot be explained in terms of known physics and astrophysics. If confirmed, this discovery will have a major impact in astrophysics, cosmology and particle physics. A natural interpretation of this result is in terms of the decay of a new particle with mass of 7.1 keV into photons. This new particle should be around today for its decay to be observed, which naturally suggests that the decaying particle is also the dark matter of the Universe. Such a warm dark matter particle in this mass range [3] has been known to fit other cosmological observations quite well, in some instances even better than the canonical cold dark matter scenario.

While a sterile neutrino of mass 7.1 keV is an attractive explanation [4] of this result in view of the measured small neutrino masses, any model of this type should also explain the observed partial lifetime for its decay into photons. The axion which solves the strong CP problem should have mass well below an eV to be consistent with other astrophysical observations, and thus will not fit the anomalous galactic X-ray spectrum, although more general axion-like particles may do the required job [5]. It will be desirable to construct a variety of self-consistent models [6] that explain the cosmological data as well as the galactic X-ray spectrum and confront them with experiments.

In this paper we propose a very simple model for warm dark that explains the 3.55 keV peak in the galactic X-ray spectrum. Unlike the sterile neutrino, which is a fermion, and axion-like particles or the Majoron [7], which are pseudoscalars, our dark matter particle is a real scalar. The model is perhaps the simplest imaginable extension of the Standard Model, as it has the fewest number of degrees of freedom added, namely one, corresponding to a real scalar field $\Phi$. The Lagrangian of this scalar field obeys a reflection symmetry $R$ under which $\Phi \to -\Phi$, which guarantees its longevity at cosmological time scales. Such an extension of the Standard Model has been discussed in the literature [8], but our model differs in two crucial respects from previous discussions [8,9]: (i) The mass of the dark matter particle is in the keV range in our case, and (ii) The symmetry $R$ is broken spontaneously at an energy scale of few GeV. The breaking of $R$ allows for very small mixing between $\Phi$ and the Standard Model Higgs field $H$. It is through this mixing that $\Phi \to \gamma\gamma$ arises, which will turn out to be the leading decay mode of $\Phi$. It is believed that the SM Higgs boson $H$ is responsible for generation of masses of all particles; in our scenario $H$ is also responsible for the origin of dark matter in the Universe, since $H \to \Phi\Phi$ produces the dark matter particle in the early Universe at temperatures comparable to $M_H \simeq 126$ GeV. When produced, the decay products never thermalize, owing to the smallness of the mixing term. The momentum
of Φ redshifts as the Universe cools, and as we show here, leads to the correct abundance of
relic dark matter for a reasonable choice of the few parameters of the model. High precision
cosmological and astrophysical observations can test this model, through imprints of domain
wall formation which occurs when the discrete symmetry $R$ breaks at a few GeV, and through
non-negligible self-interaction of dark matter.$^1$

There is a natural embedding of the model presented here into a hidden $U(1)$ gauge
theory where a Higgs mechanism breaks the gauge symmetry. The field Φ is identified in
this case as the left-over Higgs scalar. The resulting model is perhaps the simplest gauge
extension of the Standard Model. In this case the properties of the dark photon (also referred
to as the hidden photon) get determined from a fit to the relic abundance of scalar dark
matter and the observed X-ray anomaly. We find that the dark photon must have a mass
in the range of 10 keV to a few MeV, the hidden $U(1)$ gauge coupling should naturally be
of order 0.1, and the kinetic mixing of the dark photon with the photon and the $Z$ boson
should be extremely small.

In Section 2 we describe our model with a real scalar field Φ. Section 3 discusses the
cosmological implications of the model, including the anomalous peak in the galactic X-ray
spectrum, stability of the dark matter, and its relic abundance. Restoration of the discrete
reflection symmetry $R$ at temperature $T \sim$ few GeV and constraints from domain wall
formation are also discussed here, along with restrictions from dark matter self-interactions.
We also discuss the consequences of the model if the discrete symmetry is relaxed. In Section
4 we present an embedding of the model into a dark $U(1)$ gauge theory. There we present
constraint on the properties of the dark photon. Section 5 has our conclusions.

2 Model

The model is a very simple extension of the Standard Model (SM), with the addition of
a real scalar gauge singlet field Φ. This field is subject a reflection symmetry $R$, under
which $\Phi \to -\Phi$, with all other fields being invariant. Note that this is the most economical
extension of the SM, as it adds only a single new degree of freedom.

The Higgs potential of the model is given by

$$V = \frac{\lambda_H}{4} (H^\dagger H - v^2)^2 + \frac{\lambda_\Phi}{4} (\Phi^2 - u^2)^2 + \frac{\lambda_{H\Phi}}{2} (H^\dagger H - v^2)(\Phi^2 - u^2).$$  \hspace{1cm} (1)

Here $H$ is the SM Higgs doublet field, and the reflection symmetry $R$ has been applied.$^1$

$^1$Although the discrete symmetry $R$ is broken spontaneously at temperatures below a few GeV, it will
turn out that the vacuum expectation value of the field Φ is of order a few MeV.
Boundedness of the potential requires the following conditions on the quartic couplings:

\[ \lambda_H > 0, \quad \lambda_\Phi > 0, \quad \lambda_{H\Phi} > -\sqrt{2\lambda_H\lambda_\Phi}. \]  

(2)

We shall see that the sign of the coupling \( \lambda_{H\Phi} \) can be either positive or negative. For positive \( \lambda_{H\Phi} \) the discrete symmetry \( R \) will be shown to be restored above \( T \sim 15 \) \( \text{GeV} \), while for negative \( \lambda_{H\Phi} \) the symmetry \( R \) will never be restored. As the Universe cools down, a phase transition occurs at \( T \sim 15 \) \( \text{GeV} \) in the case of positive \( \lambda_{H\Phi} \), which would lead to domain wall formation, which is studied and shown to be consistent in Sec. 3.4.

The vacuum expectation values of the neutral component of \( H \) is denoted as \( v \) and that of \( \Phi \) is denoted \( u \). With this choice \( \langle V \rangle = 0 \), which is desirable from cosmology. Making an expansion \( H = h/\sqrt{2} + v \) and \( \Phi = \phi + u \), with \( v \simeq 174 \) \( \text{GeV} \), we arrive at the \( 2 \times 2 \) mass matrix for the \( h - \phi \) system:

\[ M^2 = \begin{pmatrix}
\lambda_H v^2 & \sqrt{2} \lambda_{H\Phi} u v \\
\sqrt{2} \lambda_{H\Phi} u v & 2\lambda_\Phi u^2
\end{pmatrix}. \]

(3)

We shall be interested in the limit where \( \lambda_{H\Phi} \ll 1 \), which is needed for the stability of the dark matter. Furthermore, in order to explain the galactic X-ray anomaly, the mass of the dark matter field should be 7.1 keV, which is much lighter than \( M_h \simeq 126 \) \( \text{GeV} \). While the coupling \( \lambda_\Phi \) will turn out to be small, it will obey \( \lambda_\Phi \gg \lambda_{H\Phi}^2/\lambda_H \). Thus we obtain the mass eigenvalues to an excellent approximation as

\[ M_h^2 = \lambda_H v^2, \]
\[ M_\phi^2 = 2\lambda_\Phi u^2, \]

(4)

along with the \( h - \phi \) mixing angle

\[ \tan 2\theta_{H\Phi} = \frac{2\sqrt{2} \lambda_{H\Phi} u v}{\lambda_H v^2 - 2\lambda_\Phi u^2} \approx \frac{2\sqrt{2} \lambda_{H\Phi} u v}{M_h^2}. \]

(5)

We identify \( M_\phi \simeq 7.1 \) keV and \( M_h \simeq 126 \) \( \text{GeV} \), corresponding to the masses of the dark matter particle and the SM Higgs boson respectively. Note that in the limit \( \lambda_{H\Phi} \to 0 \), the \( \phi \) field would decouple from all of the SM particles. For nonzero values of \( \lambda_{H\Phi} \), \( \phi \) will interact with other SM fields via its mixing with \( h \). In addition, the \( \lambda_{H\Phi} \) coupling will lead to the decay \( h \to \phi\phi \) as well as scattering processes such as \( hh \to \phi\phi, W^+W^- \to \phi\phi, ZZ \to \phi\phi \).

\^\text{2}A constant has been added to the potential so that \( \langle V \rangle = 0 \) is realized. Since \( v \simeq 174 \) \( \text{GeV} \) and \( u \sim \) a few MeV, the associated vacuum energy will be too large for a consistent evolution of the Universe, unless it is cancelled by the addition of such a constant.
and $t \bar{t} \rightarrow \phi \phi$, which will be the source of dark matter production in the early Universe. We shall see that the decay $h \rightarrow \phi \phi$ is the dominant source of dark matter. We shall insist that $h \rightarrow \phi \phi$ decay be out of thermal equilibrium at all epochs, which will set an upper limit on the coupling $|\lambda_{H \Phi}| < 7 \times 10^{-8}$. By demanding that the abundance of $\phi$ today fits the relic abundance of dark matter, we determine the value of $|\lambda_{H \Phi}| \simeq 4.7 \times 10^{-9}$, which is somewhat smaller than the upper limit obtained by demanding that $\phi$ never thermalizes.

The new parameters of the model will be determined to be $|\lambda_{H \Phi}| \sim 4.7 \times 10^{-9}$, $\lambda_{\Phi} \sim 2 \times 10^{-7}$ and $|u| \sim (3 - 8)$ MeV. The value of $\lambda_{\Phi}$ is in the interesting range for a 7 keV scalar dark matter to have self-interactions that may be observable in astrophysical settings. It should be noted that such small values of the quartic scalar couplings are technically natural, as the other larger couplings of the model do not induce these quartic couplings by themselves.

We reemphasize that the main new ingredient in our model compared to the existing ones [8, 9] with a real singlet field added to SM is that our scalar field acquires a non-zero vacuum expectation value which makes it relevant for the 3.5 keV X-ray lines. This leads to a completely different phenomenology, developed in the next section, compared to those of Ref. [8, 9].

3 Cosmological and astrophysical implications of the model

In this section we shall discuss the various implications of the model for cosmology and astrophysics. The anomalous X-ray peak in the galactic spectrum and the relic abundance of dark matter particle $\phi$ will be our primary focus. We shall also establish that the dark matter is adequately stable. Improved cosmological observations will be shown to test the model, in the residual effects on Cosmic Microwave Background (CMB) anisotropy arising from low temperature domain walls that are formed in the model, and in observable effects of the self-interactions of the dark matter particle.

3.1 Galactic X-ray spectrum and $\phi \rightarrow \gamma \gamma$ decay

As noted earlier, the dark matter particle $\phi$ has a small admixture of the SM Higgs boson $h$. We denote the mass eigenstates as simply $h$ and $\phi$. This mixing will cause the decay $\phi \rightarrow \gamma \gamma$. All other decays are kinematically forbidden, except for the decay $\phi \rightarrow \nu \nu$, which
we shall discuss in the next subsection. The decay rate for $\phi \rightarrow \gamma\gamma$ is given by

$$\Gamma(\phi \rightarrow \gamma\gamma) = \left(\frac{\alpha}{4\pi}\right)^2 |F|^2 \sin^2 \theta_{H\Phi} \frac{G_F m_\phi^3}{8\sqrt{2}\pi}$$

(6)

where

$$F = F_W(\beta_W) + \sum_f N_c Q_f^2 F_f(\beta_f)$$

(7)

with

$$\beta_W = \frac{4M_W^2}{M_\phi^2}, \quad \beta_f = \frac{4M_f^2}{M_\phi^2},$$

$$F_W(\beta) = 2 + 3\beta + 3\beta(2 - \beta)f(\beta),$$

$$F_f(\beta) = -2\beta[1 + (1 - \beta)f(\beta)],$$

$$f(\beta) = \arcsin^2[\beta^{-1/2}].$$

(8)

Here $Q_f$ is the electric charge of the fermion and $N_c$ is the color degrees of freedom. Since $\beta_W \gg 1$ and $\beta_f \gg 1$ for all charged fermions in the SM, $F$ can be approximated as $F = 7 - (4/3)(8) = -11/3$, where $F_W = 7$ and $F_f = -4/3$ and the factor 8 arises from the summation of $N_c Q_f^2$ over all charged fermions of the SM.

For $M_\phi = 7.1$ keV we find

$$\Gamma(\phi \rightarrow \gamma\gamma) = 6.2 \times 10^{-33} \sin^2 \theta_{H\Phi} \text{ GeV.}$$

(9)

The galactic X-ray excess corresponds to a inverse rate of $[1, 2]

$$\Gamma^{-1}(\phi \rightarrow \gamma\gamma) = (4 \times 10^{27} \text{ s} - 4 \times 10^{28} \text{ s}),$$

(10)

which leads to a determination of the range of mixing angle

$$|\sin \theta_{H\Phi}| = (5.6 \times 10^{-13} - 1.8 \times 10^{-13}).$$

(11)

Using Eq. (5), we can arrive at the range for $|\lambda_{H\Phi} u|$ to be

$$|\lambda_{H\Phi} u| = (3.6 \times 10^{-11} - 1.2 \times 10^{-11}) \text{ GeV}.$$
3.2 Other decays of $\phi$

Neutrinos and photons are the only particles of the SM that are lighter than $\phi$. We should check if the decay $\phi \to \nu\nu$ is sufficiently long-lived for $\phi$ to constitute the dark matter. For this purpose we assume that the neutrino masses are of Majorana type and the effective Lagrangian for neutrino masses is given by

$$L_{\nu}^{\text{eff}} = \frac{\kappa}{2} (LL)(HH)$$

with $\kappa$ having inverse dimensions of mass and $L$ being the SM lepton doublet. The neutrino mass matrix is then $M_{\nu} = \kappa v^2$. Noting that the $H$ field has a small admixture of $\phi$ as given in Eq. (5), we obtain the decay rate for $\phi \to \nu\nu$ to be

$$\sum_{\nu} \{\Gamma(\phi \to \nu\nu) + \Gamma(\phi \to \nu\nu)\} = \frac{\text{Tr}(M_{\nu}^\dagger M_{\nu})}{8\pi v^2} \sin^2 \theta_{H\Phi} M_{\phi}. \quad (14)$$

If we use a hierarchical neutrino mass spectrum with the largest mass being 0.049 eV, we obtain the lifetime to be

$$\left(\sum_{\nu} \{\Gamma(\phi \to \nu\nu) + \Gamma(\phi \to \nu\nu)\}\right)^{-1} = (9 \times 10^{31} - 8.8 \times 10^{32}) \text{ s} \quad (15)$$

corresponding to $\sin \theta_{H\Phi}$ given in Eq. (11). Even if the neutrino masses are quasi-degenerate with the mass of each species being 0.5 eV, this lifetime will be shortened only by a factor of 300 relative to Eq. (15). Thus $\phi \to \gamma\gamma$ is the more dominant decay, and $\phi$ is rather stable at cosmological timescales.

3.3 Relic abundance of dark matter

In addressing the relic abundance of dark matter $\phi$, we consider the various sources of production of dark matter $\phi$ in the early universe. We assume that at high temperatures there is no source of $\phi$ production (such as from inflaton decay). The Standard Model Higgs boson is the primary source of $\phi$ production in our scenario. This can occur in two different ways. (i) via Higgs boson decay $h \to \phi\phi$, and (ii) via scattering of two SM particles by exchanging $h$ to produce a pair of $\phi$’s. The relevant scattering processes are $hh \to \phi\phi$, $W^+W^- \to \phi\phi$, $ZZ \to \phi\phi$ and $t\bar{t} \to \phi\phi$. Both the decay and the scattering processes occur through the mixed $H - \Phi$ quartic coupling $\lambda_{H\Phi}$. Once the $\phi$ particles are produced, we require that they do not thermalize with the rest of the plasma. This restriction can be inferred from the decay rate and the annihilation cross sections for $\phi$ production shown in
Figure 1: Diagrams for $\phi$ production. (a): $h \rightarrow \phi\phi$, (b),(c): $hh \rightarrow \phi\phi$, (d): $W^+W^- \rightarrow \phi\phi$, (e): $ZZ \rightarrow \phi\phi$, and (f): $t\bar{t} \rightarrow \phi\phi$.

Fig. 1. The decay rate is given by

$$\Gamma(h \rightarrow \phi\phi) = |\lambda_{H\phi}|^2 \frac{v^2}{16\pi M_h},$$  \hspace{1cm} (16)

with $v \simeq 174$ GeV. The (unpolarized and spin-averaged) annihilation cross sections are given by

$$\sigma(hh \rightarrow \phi\phi) = \frac{|\lambda_{H\phi}|^2}{32\pi s} \frac{(s + 2M_h^2)^2}{\sqrt{1 - 4M_h^2/s} (s - M_h^2)^2},$$

$$\sigma(VV \rightarrow \phi\phi) = \frac{|\lambda_{H\phi}|^2}{72\pi s} \frac{(3M_V^4 - sM_V^2 + s^2/4)}{\sqrt{1 - 4M_h^2/s} (s - M_h^2)^2},$$

$$\sigma(t\bar{t} \rightarrow \phi\phi) = \frac{3|\lambda_{H\phi}|^2}{64\pi} \frac{M_t^2}{\sqrt{1 - 4M_t^2/s} (s - M_h^2)^2}. \hspace{1cm} (17)$$

Here $VV$ stands for either $W^+W^-$ or $ZZ$, the factor 3 in $\sigma(t\bar{t} \rightarrow \phi\phi)$ arises from color summation, and $M_h \simeq 126$ GeV is the mass of the SM Higgs boson exchanged in these processes (see Fig. 1).

If the decay rate and the annihilation rate $\langle \sigma n | v | \rangle$ are much smaller than the Hubble expansion rate at temperature of order $M_h$, the dark matter particle $\phi$ will never be in
thermal equilibrium. The Hubble expansion rate at temperature \( T \) is given as

\[
H(T) = 1.66 g^* T^2 / M_{\text{Pl}}.
\]  

(18)

Here \( g^* \) is the effective number of relativistic degrees of freedom at \( T \). The value of \( g^* \) at \( T \sim M_h \) is 389/4, which accounts for all the SM particles minus the top quark, and adds one degree for \( \phi \). Demanding that \( \Gamma(h \rightarrow \phi \phi) < H(T \sim M_h) \) gives the constraint

\[
|\lambda_{H\phi}| < 6.7 \times 10^{-8}.
\]

The constraint from the annihilation rates are much weaker. For example, the condition \( \langle \sigma(hh \rightarrow \phi\phi) n |v| \rangle < H(T \sim M_h) \) leads to \( |\lambda_{H\phi}| < 2.4 \times 10^{-6} \), where \( n \simeq (0.1) T^3 \) is used for the equilibrium number density of the Higgs boson \( h \), along with the relations \( s = 4E^2 \) and \( E = 3.15 T \). We shall require the stronger of the constraints.

It should be noted that the decay \( h \rightarrow \phi\phi \) as well as the scattering processes shown in Fig. 1 (except for 1 (b)) occur only below the electroweak phase transition temperature \( T_{\text{EW}} \simeq 153 \) GeV, as the scalar interaction vertex, \( h \rightarrow \phi\phi \) develops only in this regime. Nevertheless, these processes occurring in the temperature range below 153 GeV and above 5 GeV (below 5 GeV the \( h, W^\pm \) and \( Z \) bosons go out of thermal equilibrium) will play a crucial role in the creation of dark matter.

To evaluate the relic density of dark matter \( \phi \), we solve the Boltzmann equation for the evolution of the number density \( n_\phi \) of \( \phi \). It is given as

\[
\dot{n}_\phi + 3Hn_\phi = - \langle \Gamma(h \rightarrow \phi\phi) (n_\phi - n_0^h) \rangle - \sum_{i=h, W, Z, t} \langle \sigma(\bar{ii} \rightarrow \phi\phi) |v| \rangle (n_\phi^2 - (n_i^0)^2) .
\]

(19)

Here \( n_i^0 \) is the equilibrium number density of species \( i \). We shall use Boltzmann distribution functions for both fermions and bosons, which is a good approximation. For the range of \( |\lambda_{H\phi}| < 6.7 \times 10^{-8} \), \( \phi \) number density will never be significant, and thus we can set \( n_\phi = 0 \) on the RHS of Eq. (19). That is, we can ignore back reactions of the type \( \phi\phi \rightarrow hh \). The \( \langle \rangle \) symbol stands for thermal averaging. Changing variable from time \( t \) to temperature \( T \) (using conservation of entropy – \( RT = \text{constant} \)), and defining

\[
z = M_h / T, \quad f_\phi = n_\phi / T^3, \quad f_i^0 = n_i^0 / T^3, \quad K = 1.66(g^*)^{1/2} / M_{\text{Pl}},
\]

(20)

Eq. (19) can be recast (taking \( n_\phi = 0 \) on the RHS) as

\[
df_\phi / dz = \langle \Gamma(h \rightarrow \phi\phi) \rangle / K M_h^2 z f_\phi^h + \sum_{i=h, W, Z, t} \langle \sigma(\bar{ii} \rightarrow \phi\phi) |v| \rangle M_h / K z^2 (f_i^0)^2
\]

(21)

The region of validity of this equation is when species \( i = h, W, Z, t \) is in equilibrium (which
occurs down to temperature $T$ about 15-20 times below $M_i$) and when $\phi$ is not in equilibrium. Since both in decay and in scattering two $\phi$’s are produced, the net abundance of $\phi$ should be twice that obtained by solving this equation. The thermal averages can be expressed as follows [10]:

$$\langle \Gamma(h \to \phi\phi) \rangle = \langle \Gamma(h \to \phi\phi) \rangle \frac{K_1(z)}{K_2(z)},$$

$$\langle \sigma(hh \to \phi\phi)|v| \rangle = \frac{1}{8M_h^4 K_2^2} \int_{4M_h^2}^{\infty} \sigma(s, 4M_h^2) \sqrt{s} K_1 \left( \frac{\sqrt{s}}{T} \right) ds$$

and similarly for other scattering processes. Here $K_{1,2}$ are modified Bessel functions.

Defining

$$f_\phi = \frac{\lambda_{H\Phi}^2}{16\pi} \frac{v^2}{M_h^3} \kappa_\phi,$$

the Boltzmann equation for $\kappa_\phi$ can be written down as

$$\frac{d\kappa_\phi}{dz} = \frac{K_1(z)}{K_2(z)} z f_0^h(z) + \sum_{i=h,V,t} c_i \left( f_0^i(x_i z) \right)^2 \frac{1}{K_2^2(x_i z)} \int_{4x_i^2}^{\infty} dy K_1(x_i \sqrt{y}) \sqrt{y - 4z^2} \frac{h_i(y)}{(y - z^2)^2}.$$ (25)

Here we have defined

$$x_i = \frac{M_i}{M_h}, \quad c_h = \frac{M_i^2}{16v^2}, \quad c_V = \frac{M_V^3}{36M_h v^2}, \quad c_t = \frac{3}{32} \frac{M_t^7}{M_i^5 v^2},$$

$$f_0^i(x_i z) = \frac{g_i}{2\pi^2} \int_0^{\infty} y^2 e^{-\sqrt{y^2 + x_i^2 z^2}} dy,$$

$$h_h = (2 + y)^2, \quad h_V = 3 - y z^2 x_V^4 + \frac{y^2 z^4 x_V^8}{4}, \quad h_t = y z^2,$$ (26)

where the summation over $V$ includes $V = W, Z, x_i = 1$ for $i = h$, and $g_i$ account for the spin degrees with $g_h = 1, g_V = 3$ and $g_t = 2$.

We have integrated Eq. (25) numerically from $z = 0.83$ to $z = 20$ (corresponding to the electroweak phase transition temperature and the freeze-out temperature of the fields $h, W, Z$ and $t$) with the boundary condition $\kappa_\phi = 0$ at $z = 0.83$. The results are shown in Fig. 2. The leading source of $\phi$ generation is $h \to \phi\phi$ decay, contributing to $\kappa_\phi = 0.231$ at $z = 20$. The contributions from the scattering processes to $\kappa_\phi$ are 0.125 from $ZZ \to \phi\phi$, 0.085 from $W^+W^- \to \phi\phi$, 0.005 from $t\bar{t} \to \phi\phi$ and 0.002 from $hh \to \phi\phi$. The net asymptotic value of $\kappa_\phi$ is 0.445, which we shall use in our estimate of relic abundance of $\phi$.

Using the expression for the decay width for $h \to \phi\phi$ we find the number density of $\phi$ at
Figure 2: Results for $\kappa_\phi$ obtained by numerical integration of Eq. (25). Contributions from various subprocesses as well as the sum are shown.

$T \sim M_h$ to be

$$\left( \frac{n_\phi}{T^3} \right)_{T \sim M_h} \approx \frac{0.89 |\lambda_{H\Phi}|^2}{16\pi} \frac{v^2}{M_H^3 K}.$$  \hspace{1cm}  (27)

A factor of 2 has been included to go from $\kappa_\phi = 0.445$ to the value used here (0.89) owing to the two $\phi$ fields produced in each decay. We then find

$$\left( \frac{n_\phi}{T^3} \right)_{T \sim M_h} \approx 4.1 \times 10^{-3} \left( \frac{\lambda_{H\Phi}}{4.5 \times 10^{-9}} \right)^2.$$  \hspace{1cm}  (28)

The $\phi$ fields produced in the decay of $h$ never thermalize. As the Universe cools, the momentum of $\phi$ redshifts. The number density of $\phi$ today is then estimated to be

$$n_\phi^0 \approx 4.1 \times 10^{-3} \left( \frac{\lambda_{H\Phi}}{4.5 \times 10^{-9}} \right)^2 \frac{T_0^3}{\xi}.$$  \hspace{1cm}  (29)

where

$$\xi = \frac{g^*(T = M_h)}{g^*(T = 0.1 \text{ MeV})} = \frac{389}{12} = 43.22.$$  \hspace{1cm}  (30)

The decoupling of various species heats the plasma, while the $\phi$ field is not affected. The factor $\xi$ in Eq. (29) accounts for the heating of the plasma. $T_0$ in Eq. (29) if the present day temperature, $T_0 = 2.73^9$ K. From Eq. (29) with the use of $\rho_\phi = M_\phi n_\phi^0$ for the present
density of $\phi$ and the definition $\Omega_\phi = \rho_\phi / \rho_{\text{crit}}$ we obtain

$$\Omega_\phi h^2 \approx 0.11 \left( \frac{\lambda_{H\Phi}}{4.5 \times 10^{-9}} \right)^2 \left( \frac{43.22}{\xi} \right). \quad (31)$$

Thus we see that the desired value of $\Omega_\phi h^2 = 0.1198$ can be produced with $|\lambda_{H\Phi}| = 4.7 \times 10^{-9}$. We can also determine the range of $u$ from Eq. (12) to be

$$u = (7.7 - 2.6) \text{ MeV}. \quad (32)$$

Since the $\phi$ particle is never in thermal equilibrium, it does not contribute significantly to big bang nucleosynthesis. The model thus predicts $\Delta N_\nu = 0$ for BBN.

### 3.4 Restoration of discrete symmetry and domain wall constraints

The reflection symmetry $R$ (under which $\Phi \rightarrow -\Phi$) is spontaneously broken in our model. Could this symmetry be restored at high temperature? If so, domain walls would form as the Universe cools and undergoes a phase transition. Here we discuss this issue and show the consistency of the model.

Since the $\phi$ field is never in thermal equilibrium, $\phi\phi \rightarrow \phi\phi$ scattering process with a cross section proportional to $|\lambda_\phi|^2$ will not induce a temperature dependent mass for $\phi$. However, the Higgs field $h$ has a thermal distribution, and therefore the scattering of $\phi$ with the $h$ field will generate a temperature dependent mass for $\phi$. The relevant diagram is shown in Fig. 3. The finite temperature mass of $\phi$ is found to be [11]

\[ M_\phi^2(T) = \frac{\lambda_{H\Phi} T^2}{4\pi^2} \int_0^\infty dx \frac{x^2}{\sqrt{x^2 + M_h^2/T^2}} \frac{1}{e^{\sqrt{x^2 + M_h^2/T^2}} - 1}. \quad (33) \]

For $T \gg M_h$ and for $T \ll M_h$ this can be evaluated to be

$$M_\phi^2(T) \simeq \left( \frac{\lambda_{H\Phi}}{24} \right) T^2, \quad T \gg M_h,$$
\[ M_\phi^2(T) \simeq \left( \frac{\lambda_{H\Phi} M_h T}{4\pi^2} \right) K_1 \left( \frac{M_h}{T} \right), \quad T \ll M_h \]
\[ \simeq \frac{\lambda_{H\Phi}}{4\sqrt{2}\pi^2} T^2 \sqrt{M_h T} e^{-M_h/T} \left( 1 + \frac{3}{8} \frac{T}{M_h} - \frac{15}{128} \frac{T^2}{M_h^2} + \ldots \right) \]  

(34)

For the parameters of our model, the approximation \( T/M_h \ll 1 \) is more relevant. Note that from Eq. (1), there is a negative mass term for \( \phi \) field, given by \( -\lambda_\Phi u^2 \). Combining this with the temperature dependent mass of Eq. (33), we find, for positive \( \lambda_{H\Phi} \), that at \( T = T_R \) a phase transition occurs which would restore the \( R \) symmetry. \( T_R \) is obtained by demanding that \( M_\phi^2(T) - \lambda_\Phi u^2 \) turns positive, which can happen only when \( \lambda_{H\Phi} \) is positive. In this case, for \( u = 7.7 \) MeV, \( \lambda_\Phi = 4.3 \times 10^{-7} \), and \( \lambda_{H\Phi} = 4.7 \times 10^{-9} \), we find

\[ T_R \simeq 15.2 \text{ GeV} . \]  

(35)

For negative values of \( \lambda_{H\Phi} \), which is allowed in the model, at high temperature there is no restoration of \( R \) symmetry.

Since the reflection symmetry is restored at \( T_R \simeq 15.2 \) GeV for the case of positive \( \lambda_{H\Phi} \), domain wall formation will occur in the model in this case [12]. We proceed to show the consistency of the model with constraints from domain wall formation. We note that there is no domain wall formation if \( \lambda_{H\Phi} \) takes negative values. It should be noted that the energy density in domain walls is controlled by the vacuum expectation value \( u \sim \text{few MeV} \), rather than the phase transition temperature \( T_R \).

At temperatures of order \( T_R \sim \) a few GeV, the Higgs potential relevant is simply

\[ V = \frac{\lambda_\Phi}{4} (\Phi^2 - u^2)^2 . \]  

(36)

Classical field configurations with the boundary conditions \( \Phi(\pm \infty) = \pm u \) then exist. For propagation along the \( z \) direction, this solution obeys the following equations:

\[ \frac{\partial \phi}{\partial t} = 0, \quad \frac{\partial \phi}{\partial z} - \sqrt{2V(\phi)} = 0 , \]  

(37)

\[ E_{\text{min}} = \int_{\phi(-\infty)}^{\phi(\infty)} d\phi' \sqrt{2V(\phi')} . \]  

(38)

The (kink) solution to this set of equations is

\[ \phi = u \tanh \left( \sqrt{\frac{\lambda_\Phi}{2}} u z \right) \]  

(39)
with the energy per unit area $\sigma_{\text{kink}}$ given by

$$\sigma_{\text{kink}} = \frac{2\sqrt{2}}{3}\sqrt{\lambda_\Phi} u^3. \quad (40)$$

The density in domain wall today is $\sigma_{\text{kink}}/t_0$ with $t_0 \simeq 13.8 \times 10^9$ years being the present age of the Universe. If we demand $\Omega_{\text{domain wall}} = (\rho_{\text{domain wall}}/\rho_{\text{crit}})$ to be less than one, we obtain

$$\frac{2}{3} u^2 M_\phi \leq 5.4 \times 10^{-5} \text{ GeV}^3, \quad (41)$$

or $|u| \leq 3.4 \text{ GeV}$ for $M_\phi = 7.1 \text{ keV}$.

It is not enough, however, to simply demand that the domain wall does not over-close the Universe. The presence of domain wall can alter the CMB anisotropy significantly unless $\Omega_{\text{domain wall}} \leq 10^{-5}$, in which case we obtain $|u| \leq 10.7 \text{ MeV}$. The model parameters allow for $|u| = (7.7 - 2.6) \text{ MeV}$, see Eq. (32). Thus we see that the scenario presented is consistent with domain wall constraints. However, since the upper limit from CMB anisotropy is close to the prediction of the model, precision CMB measurements can potentially unravel new contributions of this type. We expect contributions to CMB anisotropy from domain walls at the level of $(0.52 - 0.06) \times 10^{-5}$. It would be desirable to further study this question.

### 3.5 Dark matter self-interaction constraints

In our model, $\phi \phi \to \phi \phi$ interaction occurs through the quartic coupling $\lambda_\Phi$. There are rather severe constraints on self-interaction of dark matter from dense cores of galaxies and galaxy clusters where the velocity distribution can be isotropized. Constraints from such halo shapes, as well as from dynamics of bullet cluster merger have been used to infer an upper limit on the dark matter self-interaction cross section. A recent analysis [13] which includes the gravitational potential of both baryons and dark matter finds this cross section to be limited by

$$\frac{\sigma}{M_\phi} < 1 \text{ barn/GeV}. \quad (42)$$

For $M_\phi = 7.1 \text{ keV}$, this translates into $\sigma < 7 \times 10^{-30} \text{ cm}^2$. We take this to be a conservative upper limit, although limits as strong as $\sigma/M_\phi < 0.1 \text{ barn/GeV}$ have been claimed.

In our model the Lagrangian relevant for $\phi \phi \to \phi \phi$ scattering, which follows from Eq. (1) is

$$\mathcal{L}_{\phi\phi} = -\sqrt{\lambda_\Phi} M_\phi \phi^3 - \frac{\lambda_\Phi}{4} \phi^4. \quad (43)$$
The cross section for $\phi\phi \rightarrow \phi\phi$ scattering is found to be
\[
\sigma = \frac{9\lambda_\Phi^2}{8\pi s} \left[ 1 + \frac{9M_\phi^4}{(M_\phi^2 - s)^2} + \frac{12M_\phi^2 (2s - 3M_\phi^2)}{(s - M_\phi^2)(s - 3M_\phi^2)} + \frac{12M_\phi^2 (M_\phi^4 + 3M_\phi^2 s - s^2)}{(s - M_\phi^2)(s - 2M_\phi^2)(s - 4M_\phi^2)} \log \left( \frac{s - 3M_\phi^2}{M_\phi^2} \right) \right]. \tag{44}
\]

In the nonrelativistic limit, $(s - 4M_\phi^2) \ll M_\phi^2$, this reduces to
\[
\sigma \simeq \frac{9\lambda_\Phi^2}{2\pi M_\phi^2} \left[ 1 - \frac{19}{12} \frac{s - 4M_\phi^2}{M_\phi^2} + \frac{257}{144} \left( \frac{s - 4M_\phi^2}{M_\phi^2} \right)^2 + \ldots \right]. \tag{45}
\]

Demanding that $\sigma/M_\phi < 1$ barn/GeV, with $M_\phi = 7.1$ keV, we obtain from Eq. (45) the limit
\[
\lambda_\Phi < 8 \times 10^{-7}. \tag{46}
\]

Very interestingly, this is close to the limit derived from constraint on the VEV $u$ and the 7.1 keV mass of the dark matter particle. For $|u| = (7.7 - 2.6)$ MeV, we have $\lambda_\Phi = (4.2 \times 10^{-7} - 3.7 \times 10^{-6})$. The higher end of $|u|$ is preferred here, which would make domain wall contributions to CMB in the more interesting range. We see that the model predicts dark matter self-interactions in the interesting range to play an important cosmological role. In particular, the dark matter self interaction cross section is predicted to be $\sigma/M_\phi > 0.25$ barn in the model with a spontaneously broken discrete symmetry.

### 3.6 Supernova energy loss constraints

The dark matter $\phi$ with a mass of 7.1 keV cannot be produced in stars such as the Sun (core temperature of about 2 keV) and in horizontal branch stars (temperature of about 8 keV). The core temperature of supernova is of order (30-70) MeV, which would allow for the production of $\phi$. Once produced, $\phi$ will escape, carrying energy with it. This process should not be the dominant source of energy loss in supernova, as neutrinos with the expected properties were detected from SN1987a [14]. The analysis of $\phi$ emission is similar to the case of Majorons for which the supernova bound on neutrino coupling is $\leq 4 \times 10^{-7}$ [15]. In our model, the scalar field coupling to electrons is $\sim \frac{m_\nu}{v^2} \lambda_{H\phi} \simeq 10^{-18}$. It therefore easily satisfies this supernova bound.

$\phi$ can also be pair produced inside supernova core via the process $\gamma\gamma \rightarrow \phi\phi$. The diagram responsible for this is shown in Fig. 4. Here we make a rough estimate of the energy carried by $\phi$ and ascertain that this is indeed a negligible portion of the total energy of the supernova.
Figure 4: Diagram responsible for $\phi$ pair creation inside supernovae. There is another diagram where the $W$ boson inside the loop is replaced by the top quark.

For a crude estimate, we use the expression for the rate of energy loss $Q$,

$$Q = V_{\text{core}} n_\gamma^2 \langle E \rangle \sigma(\gamma\gamma \rightarrow \phi\phi). \quad (47)$$

Here $V_{\text{core}} = 4\pi R_{\text{core}}^3/3$ is the core volume, and we take $R_{\text{core}} = 10 \text{ km}$. $n_\gamma$ is the number density of photons, which we obtain from a thermal distribution. The $\phi$ pair production cross section is found to be

$$\sigma(\gamma\gamma \rightarrow \phi\phi) = \frac{\lambda_{H\Phi}^2 v^2 E^2}{32\pi M_h^4} \left( \frac{e^2 g F}{16\pi^2 m_W} \right)^2 \quad (48)$$

where $F \simeq 6.54$ is the numerical value of the loop function associated with the decay of Higgs $h$ into two photons. We use $n_\gamma \simeq 0.2T^3$ and $E \simeq 3.15 T_{\gamma}$. With these values, we find $Q \sim 3.6 \times 10^{54} \lambda_{H\Phi}^2 \text{ erg/s}$, when $T_\gamma = 50 \text{ MeV}$ is used. The supernova explosion lasted for about 10 seconds, so the total energy lost in $\phi$ would be about ten times larger. Demanding that the total energy lost is less than $10^{53} \text{ erg}$ yields a very mild limit $|\lambda_{H\Phi}| < 5 \times 10^{-2}$. If $T_\gamma = 70 (100) \text{ MeV}$ is used, this limit becomes $|\lambda_{H\Phi}| < 1.1 \times 10^{-2} (2.3 \times 10^{-3})$, which is easily satisfied in the model. We note that all other energy loss processes in $\phi$ are highly suppressed.

### 3.7 Explicit breaking of the reflection symmetry

Here we analyze the consequences of explicitly breaking the reflection symmetry $\Phi \rightarrow -\Phi$ that was assumed in the preceding discussions. If the model is embedded in a $U(1)$ gauge symmetry, as we do in the next section, the reflection symmetry is automatically present. It is, however, interesting to explore the modifications resulting from relaxing this symmetry. In this case, the following new terms in the Higgs potential are allowed, in addition to the ones shown in Eq. (1):

$$V_{\text{new}} = \mu_\Phi (\Phi - u)^3 + \mu_{H\Phi} (H^\dagger H - v^2)(\Phi - u). \quad (49)$$
The $h - \phi$ mixing matrix will now become

$$M^2 = \begin{pmatrix} \lambda_H v^2 & \sqrt{2}v (\lambda_{H\Phi} u + \mu_{H\Phi}) \\ \sqrt{2}v (\lambda_{H\Phi} u + \mu_{H\Phi}) & 2\lambda_{\Phi} u^2 \end{pmatrix}. \quad (50)$$

The modified $H - \Phi$ mixing angle is

$$\theta_{H\Phi} \simeq \sqrt{2}v (\lambda_{H\Phi} u + \mu_{H\Phi}) \frac{1}{M_h^2}. \quad (51)$$

The X-ray spectral anomaly then determines from $\phi \to \gamma\gamma$ decay rate

$$|\lambda_{H\Phi} u + \mu_{H\Phi}| = (3.6 \times 10^{-11} - 1.2 \times 10^{-11}) \text{ GeV}. \quad (52)$$

The requirement that $\phi$ never thermalizes sets a limit of $\mu_{H\Phi} < 8$ keV, obtained from the process $hh \to \phi$ which has a rate $\Gamma \sim \mu_{H\Phi}^2 / (8\pi M_h)$. With this condition, the relic density of $\phi$ is unaltered from the calculation of Sec. 3.3, and would fix $\lambda_{H\Phi} \simeq 4.7 \times 10^{-9}$ as before. The determination of the VEV $u$ of Eq. (32) will now be modified to

$$\pm u = (7.7 - 2.6) \text{ MeV} - \left( \frac{\mu_{H\Phi}}{4.7 \times 10^{-9}} \right). \quad (53)$$

If $\mu_{H\Phi}$ is of order 0.1 eV, $u$ will remain in the MeV range, and our discussions of earlier sections will carry over. This is what we shall assume in the remainder of this subsection, although larger values of $u$ and $\mu_{H\Phi}$, as large as 100 GeV, are possible with a fine-tuning to get the $h - \phi$ mixing angle sufficiently small.

The self-interaction of dark matter is now modified, owing to the coupling $\mu_{\Phi}$ in Eq. (49). In the nonrelativist limit, the cross section for $\phi\phi \to \phi\phi$ is given by

$$\sigma = \frac{9\lambda_{\Phi}^2}{2\pi M_{\phi}^2} G(x), \quad (54)$$

where

$$G(x) = 1 + 5\sqrt{2}x + \frac{35}{2} x^2 + \frac{25}{\sqrt{2}} x^3 + \frac{25}{4} x^4 \quad (55)$$

with

$$x \equiv \frac{\mu_{\phi}}{\sqrt{\lambda_{\Phi} M_{\phi}}}. \quad (56)$$

For positive values of $x$, $\sigma / M_{\phi}$ will exceed 1 barn/GeV, which is disfavored. For $-\sqrt{2} < x < 0$ there is a reduction in this cross section relative to the case of exact reflection symmetry.
with $\sigma$ vanishing for $x = -1.023$ and for $x = -0.391$. Such a choice would correspond to $\mu_\Phi$ being of order eV. These values of $x$ would be in full agreement with dark matter self-interaction limits.

Finally, if the reflection symmetry is explicitly broken even by a small amount, stable domain walls will not form [16], and would relax the constraints resulting from there.

4 Embedding in a simple hidden U(1) gauge model

The model presented here has a natural embedding in a $U(1)$ gauge theory with a Higgs mechanism. Such a model would be among the simplest gauge extensions of the Standard Model that could provide a rational for the small mass of the 7 keV scalar. The gauge group of the SM is extended to include an additional $U(1)_X$ which is spontaneously broken by a complex Higgs scalar field $\Phi$. The new terms in the Lagrangian are

$$L = -\frac{1}{4} X_{\mu\nu} X^{\mu\nu} + |\partial_\mu \Phi - ig_X X_{\mu} \Phi|^2 - V(\Phi).$$

The Higgs potential for the full SM plus $U(1)$ model now has the form

$$V = \frac{\lambda_H}{4} (H^\dagger H - v^2)^2 + \frac{\lambda_\Phi}{4} (|\Phi|^2 - u^2)^2 + \frac{\lambda_{H\Phi}}{2} (H^\dagger H - v^2) (|\Phi|^2 - u^2).$$

After spontaneous symmetry breaking with $\langle \Phi \rangle = u$, the phase of $\Phi$ is absorbed by the gauge boson $X$ which becomes massive. The left-over Higgs $\phi$ (real part of the complex field $\Phi$) will have exactly the same properties as the $\phi$ field of the model that was discussed in previous sections. Owing to the existence of a new gauge boson $X$, there are some differences which we outline here.

It is easy to see that even after spontaneous symmetry breaking, there is an unbroken hidden parity symmetry in the model. Under this symmetry only the $X$ gauge boson is odd while the $\phi$ field is even. As a result, the $X$ gauge boson will be stable and may contribute to the energy density of the Universe. However, as we show below, this contribution is negligible. A kinetic mixing term can break this symmetry and connect the “dark $U(1)$” sector to the standard model particles. Actually, owing to this symmetry, if such a kinetic mixing term is not added, it will never be induced in the theory. This is what we shall assume here. If a kinetic mixing of the type $-\langle \zeta/2 \rangle B_{\mu\nu} X^{\mu\nu}$ is added to the Lagrangian, from the requirement that the hidden $X$ gauge boson never thermalizes down to temperatures of order MeV sets a restriction $|\zeta| < 3 \times 10^{-10}$.

As in the case of $\phi$ field, the dark $U(1)$ gauge boson never thermalizes with the plasma.
Figure 5: Diagrams for production of $X$ gauge bosons in Higgs boson decays.

It has a mass given by $M_X = g_X u$ with $u$ listed as in Eq. (32). $M_X$ should be larger than 7.1 keV, and because of the constraint from Eq. (32) $M_X$ cannot exceed about 10 MeV. $X$ can be produced in the early Universe in the decays and scattering of $h$ as shown in Fig. 5. Focussing on the decay, the width for $X$ production is given by

$$\Gamma (h \rightarrow XX) = \frac{\lambda_{h\Phi}^2 \alpha_X^2 v^2}{128\pi^3 M_h}.$$  \hspace{1cm} (59)

Comparing with the decay width for $h \rightarrow \phi\phi$, we find

$$\frac{\Gamma (h \rightarrow XX)}{\Gamma (h \rightarrow \phi\phi)} = \frac{\alpha_X^2}{4\pi^2}.$$  \hspace{1cm} (60)

Consequently, the abundance of $X$ today is related to the abundance of $\phi$ as

$$\frac{\Omega_X}{\Omega_\phi} = \frac{\alpha_X^2 M_X}{4\pi^2 M_\phi}.$$  \hspace{1cm} (61)

If we demand this ratio to be $< 1$, we obtain

$$\alpha_X < 0.5 \left( \frac{1 \text{ MeV}}{M_X} \right)^{1/2}$$  \hspace{1cm} (62)

which is always satisfied as long as the hidden $U(1)$ gauge coupling is in the perturbative range. Actually, the hidden $U(1)$ gauge coupling $g_X$ should be naturally of order 0.1, as it would induce a quartic $\phi^4$ coupling with a coefficient $\lambda_\phi \sim g_X^4/(16\pi^2)$. For $g_X = 0.1$, the induced value is $\lambda_\phi \sim 10^{-6}$, which is consistent with the dark matter phenomenology in the previous section.

We thus see that a natural embedding of the model of Sec. 2 into a local $U(1)$ gauge symmetry preserves all the features that are desirable to explain the X-ray spectrum anomaly as well as dark matter.
5 Concluding remarks

We conclude by making a few observations.

A light scalar particle can also lead to deviations from equivalence principle at short distances [17]. A 7 keV mass corresponds to range of force equal to $\sim 3 \times 10^{-9} \text{ cm}$. At such short distances, there are no known bounds since it is not easy to probe such small distances by gravity experiments.

Photons can be produced via $\phi\phi \rightarrow \gamma\gamma$ scattering, in addition to the decay $\phi \rightarrow \phi\phi$. However, the cross section times velocity for the scattering is found to be $\sigma|v| \simeq 2 \times 10^{-53} \lambda_H^2 \text{ cm}^2$, which is too small to be observable in the X-ray spectrum.

In summary, we have presented a very simple extension of the standard model by adding a real singlet scalar which can not only become a warm dark matter of the universe but also can explain the recently reported 3.55 keV X-ray lines in the sky as a result of dark matter decay. The model has a natural embedding in a hidden $U(1)$ model with the Higgs mechanism.

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