Infrared behavior and Gribov ambiguity in SU(2) lattice gauge theory

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For SU(2) lattice gauge theory we study numerically the infrared behavior of the Landau gauge ghost and gluon propagators with the special accent on the Gribov copy dependence. Applying a very efficient gauge fixing procedure and generating up to 80 gauge copies we find that the Gribov copy effect for both propagators is essential in the infrared. In particular, our best copy dressing function of the ghost propagator approaches a plateau in the infrared, while for the random first copy it still grows. Our best copy zero-momentum gluon propagator shows a tendency to decrease with growing lattice size which excludes singular solutions. Our results look compatible with the so-called decoupling solution with a non-singular gluon propagator. However, we do not yet consider the Gribov copy problem to be finally resolved.

I. INTRODUCTION

The lattice study of the gluon and ghost propagators in Landau gauge has a long history. It was started by Mandula and Ogilvie [1] and continued in many papers (for review see, e.g., [2, 3] and references therein). One of the main goals of such studies was to clarify the infrared (IR) asymptotics of the propagators and of the running coupling which can be determined through these propagators. The hope always was that ab-initio lattice results will give support to or discriminate between various theoretical predictions for the IR behavior obtained with continuum methods, in particular within the Dyson-Schwinger approach. One prediction was definitely overturned: lattice results showed that the gluon propagator is not divergent in the infrared.

At the same time it has been found that the lattice approach has its own difficulties when applied to such studies. One of them is that to reach small momenta necessary to study the IR limit one has to go to huge lattices which makes the numerical simulations formidable. Another, less apparent, but not less difficult problem is the problem of Gribov copies. Although for many years it was believed that the effect of Gribov copies on both gluon and ghost propagators was weak and could be considered just as a noise in the scaling region [2, 3] it has been found first for the ghost propagator [5] and quite recently for the gluon propagator [6] that these effects are in fact quite strong. The presence of these effects makes the task of lattice computations of the field propagators in the IR region even more difficult.

These difficulties of the lattice approach has made it impossible so far to obtain results which could confirm or disprove the existent confinement scenarios proposed by Gribov [7] and Zwanziger [8] on one hand and by Kugo-Ojima [9] on the other. The Gribov-Zwanziger scenario predicts that the gluon propagator is IR vanishing, while the Kugo-Ojima criterion of confinement predicts the ghost dressing function to be IR divergent.

In recent years the interest to the lattice results for the field propagators in the IR region has been revived. This interest was stimulated also by the practical progress achieved over the years within the Dyson-Schwinger (DS) approach as pursued by Alkofer, von Smekal and others (for an intermediate review see [10]), and more recently with the help of functional renormalization group (FRG) equations [11, 12]. Infrared QCD has been also investigated using the stochastic quantization method [13, 14], as well as with effective actions [13, 10].

In this paper we continue our lattice study of the influence of Gribov copies on the (minimal) Landau gauge SU(2) gluon and ghost propagators in the IR region by applying global Z(2) flip transformations in combination with an effective optimization algorithm, the so-called simulated annealing (SA). The flip transformation was introduced in [6]. Its influence on the gluon propagator was thoroughly studied lateron [17]. The Z(2) flips - equivalent to non-periodic Z(2)
gauge transformations were shown to cause rather strong effects in the IR behavior of the gluon propagator. In [17] high statistics computations of the gluon propagator were made for lattice sizes varying from 1.8 fm to 6.5 fm at one fixed bare lattice coupling $\beta = 4/g_0^2 = 2.20$. The latter was chosen in order to reach reasonably large physical volumes (and thus small momenta) on comparatively moderate lattice sizes up to $32^4$. It turned out that due to better gauge fixing finite-volume effects, usually strong at minimal momenta, became largely suppressed. Furthermore, it has been observed that at momenta $p \sim 270$ Mev the gluon propagator seems to have a turning point leaving open the possibility for a vanishing gluon propagator in the IR limit $p^2 \to 0$. Here we continue this investigation enlarging the lattice up to $40^4$ at the same $\beta$-value corresponding to a volume of $(8.4 fm)^4$ and extending the studies to the ghost propagator, too. We systematically search for Gribov copies by combining all $2^4 = 16 Z(2)$ Polyakov loop sectors for all Euclidean directions into one gauge orbit.

The main motivation for this computation is triggered by the puzzle posed by the above mentioned continuum approaches. Different kinds of solutions with a quite different IR behavior of the gluon and ghost propagators have been reported by different groups. The power-like solution with relation between gluon $\kappa_D$ and ghost $\kappa_G$ > 0 exponents $\kappa_D = -2 \kappa_G$ was called recently a scaling solution [18]. This solution [12], [13], [20], [21] allows the gluon propagator to vanish and the ghost dressing function diverge in the IR limit in one-to-one correspondence with both the Gribov-Zwanziger scenario [7, 22] and the Kugo-Ojima criterion [9, 23] for confinement. On the contrary, the so-called decoupling solutions [10], [24], [25], [26] provide an IR finite or weakly divergent gluon propagator and a finite ghost dressing function leading to a running coupling vanishing in the infrared. For recent discussions of the present status of research see, e.g., [18] and further references therein. The lattice approach based on the first-principle path integral quantization should be able to resolve the issue.

Results for $SU(2)$ [27] as well as for $SU(3)$ [28] obtained on very large lattices and by employing purely periodic gauge transformations seem to be in conflict with the scaling solution and are compatible with the decoupling solution. That this might be not in conflict with the (appropriately modified) Gribov-Zwanziger scenario has been recently pointed out in [29].

Here, by enlarging the gauge orbits with non-periodic $Z(2)$ flip gauge transformations and employing the SA algorithm we shall come closer to the global extremum of the Landau gauge functional, i.e. closer to the fundamental modular region. We find the Gribov-Zwanziger copy dependence to be very strong. Still our results look rather as an argument in favor of the decoupling solution with a non-singular gluon propagator. However, we do not yet consider the problem of Gribov copies and, correspondingly, the infrared asymptotics of the gluon propagator to be finally resolved.

In Section II we introduce the observables to be computed. In Section III some details of the gauge fixing method and of the simulation are given, whereas in Section IV we present our results. Before coming to the conclusions in Section V we will discuss the dependence of our results on the number of gauge copies in Section V.

II. GLUON AND GHOST PROPAGATORS: THE DEFINITIONS

For the Monte Carlo generation of ensembles of non-gauge-fixed gauge field configurations we use the standard Wilson action, which for the case of an $SU(2)$ gauge group is written

$$S = \beta \sum_{x, \mu > \nu} \left[ 1 - \frac{1}{2} \text{Tr} \left( U_{x,\mu} U_{x+\nu,\mu} U_{x+\nu,\mu}^{-1} U_{x,\mu}^{-1} \right) \right] ,$$

$$\beta = 4/g_0^2.$$  \hspace{1cm} (1)

Here $g_0$ is a bare coupling constant and $U_{x,\mu} \in SU(2)$ are the link variables. The latter transform as follows under gauge transformations $g_x$

$$U_{x,\mu} \overset{g_x}{\longrightarrow} U_{x,\mu}^g = g_x U_{x,\mu} g_x, \quad g_x \in SU(2).$$  \hspace{1cm} (2)

The standard definition [1] of the dimensionless lattice gauge vector potential $A_{x+\hat{\mu},2/\mu}$ is

$$A_{x+\hat{\mu},2/\mu} = \frac{1}{2i} \left( U_{x,\mu} - U_{x,\mu}^\dagger \right) = A_{x+\hat{\mu},2/\mu}^a \frac{\sigma_a}{2}.$$  \hspace{1cm} (3)

The reader should keep in mind that the definition is not unique which can have an essential influence on the propagator results in the IR region, where the continuum limit is hard to control.

In lattice gauge theory the usual choice of the Landau gauge condition is [1]

$$(\partial A)_x = \sum_{\mu=1}^{4} \left( A_{x+\hat{\mu},2/\mu} - A_{x-\hat{\mu},2/\mu} \right) = 0 ,$$  \hspace{1cm} (4)

which is equivalent to finding an extremum of the gauge functional

$$F_U(g) = \frac{1}{4V} \sum_{x,\mu} \frac{1}{2} \text{Tr} \ U_{x,\mu}^g ,$$  \hspace{1cm} (5)
where $V = L^4$ is the lattice volume, with respect to gauge transformations $g_x$. After replacing $U \Rightarrow U^g$ at the extremum the gauge condition (4) is satisfied. The manifold consisting of Gribov copies providing local maxima of the functional (3) and a semi-positive Faddeev-Popov operator (see below) is called Gribov region $\Omega$, while that of the global maxima is called the fundamental modular domain $\Lambda \subset \Omega$. Our gauge fixing procedure is aimed to approach this domain.

The gluon propagator $D$ and its dressing function $Z$ are then defined (for $p \neq 0$) by

$$D_{\mu\nu}^{ab}(p) = \frac{a^2}{g_0^2} (\bar{A}_\mu^a(k)A_\nu^b(-k))$$

$$= \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \delta^{ab} D(p),$$

$$Z(p) = D(p) p^2,$$  \hspace{1cm} (6)

where $\bar{A}(k)$ represents the Fourier transform of the gauge potentials defined by Eq. (3) after having fixed the gauge. The momentum $p$ is given by $p_\mu = (2/a) \sin(\pi k_\mu/L), \ k_\mu \in (-L/2,L/2]$. For $p \neq 0$, one gets from Eq. (6)

$$D(p) = \frac{1}{9} \sum_{a=1}^{3} \sum_{\mu=1}^{4} D_{\mu\mu}^{aa}(p),$$  \hspace{1cm} (7)

whereas at $p = 0$ the “zero momentum propagator” $D(0)$ is defined as

$$D(0) = \frac{1}{12} \sum_{a=1}^{3} \sum_{\mu=1}^{4} D_{\mu\mu}^{aa}(p = 0).$$  \hspace{1cm} (8)

The lattice expression for the Landau gauge Faddeev-Popov operator $M_{\mu}^{ab} = -\partial_\mu D_\mu^{ab}$ (where $D_\mu^{ab}$ denotes the covariant derivative in the adjoint representation) for $SU(2)$ is given by

$$M_{x y}^{ab}[U] = \sum_\mu \left\{ \left( \tilde{S}_{x \mu}^{ab} + \tilde{S}_{y \mu}^{ab} \right) \delta_{x y} - \left( a_{x \mu} \bar{A}_{x \mu}^{ab} + A_{x \mu}^{ab} \bar{a}_{x \mu} \right) \right\}$$

$$\delta_{x y} - \left( \tilde{A}_{x \mu}^{ab} + \bar{A}_{x \mu}^{ab} \delta_{x y} \right) \right\}$$

where

$$\tilde{S}_{x \mu}^{ab} = \delta^{ab} \frac{1}{2} \text{Tr} U_{x \mu}, \ \bar{A}_{x \mu}^{ab} = -\frac{1}{2} \epsilon^{abc} A_{x \mu+\hat{\mu}/2 \mu \nu}^{c}.$$  \hspace{1cm} (9)

From the expression (10) it follows that a trivial zero eigenvalue is always present, such that at the Gribov horizon $\partial \Omega$ the first non-trivial zero-eigenvalue appears. Thus, if the Landau gauge is properly implemented, $M[U]$ is a symmetric and semi-positive matrix.

The ghost propagator $G^{ab}(x, y)$ is defined as

$$G^{ab}(x, y) = \delta^{ab} G(x - y) = \frac{1}{a^2} \left\{ (M^{-1})^{ab} [U] \right\}.$$  \hspace{1cm} (11)

Note that the ghost propagator becomes translational invariant (i.e., dependent only on $x - y$) and diagonal in color space only in the result of averaging over the ensemble of gauge-fixed representants of the original Monte Carlo gauge configurations. The ghost propagator $G(p)$ in momentum space and its dressing function $J(p)$ can be written as

$$G(p) = \frac{a^2}{3V} \sum_{x, y, a} e^{-\frac{2\pi p}{a} k(x-y)} \left\{ (M^{-1})^{aa} [U] \right\},$$

$$J(p) = G(p) p^2,$$  \hspace{1cm} (12)

where the coefficient $\frac{1}{3V}$ is taken for a full normalization, including the indicated color average over $a = 1, 2, 3$. We mentioned above that $M[U]$ is symmetric and semi-positive. In particular, it is positive-definite in the subspace orthogonal to constant vectors. The latter are zero modes of $M[U]$. Therefore, it can be inverted by using a conjugate-gradient method, provided that both the source $\psi^a(y)$ and the initial guess of the solution are orthogonal to zero modes. As the source we adopted the one proposed in (13)

$$\psi^a(y) = \delta^{ac} e^{2\pi i p y} \quad p \neq (0, 0, 0, 0),$$

for which the condition $\sum_y \psi^a(y) = 0$ is automatically imposed. Choosing the source in this way allows to save computer time since, instead of the summation over $x$ and $y$ in Eq. (12), only the scalar product of $M^{-1} \psi$ with the source $\psi$ itself has to be evaluated.

In general, the gauge fixed configurations can be used in a more efficient way if the inversion of $M$ is done on sources for $c = 1, 2, 3$ such that the (adjoint) color averaging, formally required in Eq. (12), will be explicitly performed.

### III. SIMULATION DETAILS

We restrict ourselves to Monte Carlo (MC) simulations at $\beta = 4/g_0^2 = 2.20$ and use lattice field configurations for which the gluon propagator has been already computed in (14). Here we add the computation of the ghost propagator and new data obtained on the larger symmetric lattice with the linear size of $L = 40$. For the latter case we generated an ensemble of 430 independent Monte Carlo lattice field configurations. Consecutive configurations (considered as independent) were separated by 100 sweeps, each sweep
| $L$ | $\# glp$ | $\# ghp$ | $N_{copy}$ |
|-----|---------|---------|-----------|
| 8   | 200     |         | 80        |
| 12  | 200     |         | 80        |
| 16  | 240     | 60      | 24        |
| 24  | 346     | 157     | 24        |
| 32  | 247     | 118     | 40        |
| 40  | 430     | 64      | 80        |

**Table I**: Lattice sizes, statistics, number of gauge copies used throughout this paper. The second (third) column gives the number of configurations used to compute the gluon (ghost) propagators.

...being of one local heatbath update followed by $L/2$ microcanonical updates. In Table I we provide the full information about the field ensembles used throughout this paper.

For gauge fixing we employ the $Z(2)$ flip operation as discussed in [17]. It consists in flipping all link variables $U_{x\mu}$ attached and orthogonal to a 3d plane by multiplying them with $-1$. Such global flips are equivalent to non-periodic gauge transformations and represent an exact symmetry of the pure gauge action considered here. The Polyakov loops in the direction of the chosen links and averaged over the 3d plane obviously change their sign. Therefore, the flip operations combine for each lattice field configuration the $2^4$ distinct gauge orbits (or Polyakov loop sectors) of strictly periodic gauge transformations into one larger gauge orbit.

The second ingredient of our gauge fixing procedure is the consequent use of the simulated annealing (SA) method, which has been found even computationally more efficient than the only use of standard overrelaxation (OR).

The SA algorithm generates a field of gauge transformations $g(x)$ by MC iterations with a statistical weight proportional to $\exp(4V F_U[g]/T)$. The “temperature” $T$ is a technical parameter which is gradually decreased in order to maximize the gauge functional $F_U[g]$. In the beginning, $T$ has to be chosen sufficiently large in order to allow traversing the configuration space of $g(x)$ fields in large steps. It has been checked that an initial value $T_{\text{init}} = 1.5$ is high enough. After each quasi-equilibrium sweep, including both heatbath and also microcanonical updates, $T$ has been decreased with equal step size until $g(x)$ is uniquely captured in one basin of attraction. The criterion of success is that during the consecutively applied OR the violation of transversality decreases in a more or less monotonous manner for almost all applications of the compound algorithm. This condition turned out reasonably satisfied for a final lower temperature value $T_{\text{final}} = 0.01$ [31]. The number of temperature steps was chosen to be 1000 for the smaller lattice sizes and has been increased to 2000 for the lattice size $40^4$ included here. The finalizing OR algorithm requires a number of iterations varying from $O(10^2)$ to $O(10^3)$. In what follows we shall call the combined algorithm employing SA (with finalizing OR) and $Z(2)$ flips the ‘FSA’ algorithm.

Some details of the gauge fixing procedure compared to our previous work [17] have been changed. For every configuration the Landau gauge was fixed $N_{copy} = 80$ times (5 gauge copies for every flip-sector), each time starting from a random gauge transformation of the mother configuration, obtaining in this way $N_{copy}$ Landau-gauge fixed copies. In [17] where smaller lattices were simulated $N_{copy}$ was smaller: 40 (for $32^4$ lattice), 24 ($24^4$ and $16^4$). Only on very small lattices $12^4$ and $8^4$, where producing copies was substantially cheaper, we produced 80 copies.

In order to reduce the computational effort in the finalizing OR sweeps on the $40^4$ lattice we applied the following trick. We noticed that after a comparably small number of OR sweeps, definitely before the convergence criterion is reached, one can already decide which copy has a higher maximum of the gauge functional, i.e. one can stop the OR procedure already when the change in the functional becomes comparably small and further sweeps will not change the order of copies according to the value of the maximized functional. Note, that the functional was different from its final value only in the 8th digit and we used these values in Fig. 7. After having selected the ‘best copy’ ($bc$) the OR gauge fixing for this copy has to be finalized. To be precise, complete gauge fixing was made also on the randomly chosen ‘first copy’ ($fc$), just for the purpose of comparison.

For the finalizing OR we used the standard Los Alamos type overrelaxation with the parameter $\omega = 1.7$. For $bc$ and $fc$ the iterations have been stopped when the following transversality condition was satisfied:

$$\max_{x,\alpha} \left| \sum_{\mu=1}^{4} \left( A^a_{x+\hat{\mu}/2,\mu} - A^a_{x-\hat{\mu}/2,\mu} \right) \right| < \epsilon_{\text{lor}}. \quad (14)$$

We used the parameters $\epsilon_{\text{lor}} = 10^{-7}$ (i.e. $10^{-14}$ for $(\partial A)^2$).

**IV. RESULTS**

In this section we present the data for the gluon and ghost propagator. In Fig. 1 we show the new data for the gluon propagator $D(p)$ in physical units obtained on the $40^4$ lattice at $\beta = 2.20$. We compare the $bc$ FSA result with the $fc$ SA result (the latter without flips). We clearly see the Gribov copy effect for the lowest
FIG. 1: The momentum dependence and Gribov copy sensitivity of the gluon propagator $D(p)$ in the IR region on the $40^4$ lattice. Filled symbols correspond to the $bc$ ensemble, open symbols to the $fc$ ensemble.

FIG. 2: The momentum dependence of the gluon propagator $D(p)$ on various lattice size. $bc$ results are shown throughout.

FIG. 3: The dependence of $D(0)$ on the lattice size. $bc$ FSA results are compared with $fc$ OR results (without flips).

accessible momenta moving the data points to lower values for better copies (with the larger gauge functional). The different points at $p \sim 300\text{MeV}$ belong to different realizations of $p^2$ and seem to indicate some violation of the hypercubic symmetry.

In Fig. 2 we present these new data together with the ones obtained on smaller lattice sizes always for the FSA $bc$ case.

We see that the data are nicely consistent with each other and indicate a turnover to decreasing values towards vanishing momentum. A smooth extrapolation to $D(0)$ becomes visible. But still there is no indication for a vanishing gluon propagator at zero momentum for increasing volume. This is demonstrated in Fig. 3 where we show the dependence of the zero-momentum propagator $D(0)$ as a function of the inverse linear lattice size $1/L$. This behavior demonstrates a (slight) tendency to decrease, and looks hardly consistent with $D(0) = 0$ limit. One could consider it rather like an argument in favor of the decoupling solution with a finite gluon propagator in the infrared. However, one still cannot exclude that there are even more efficient gauge fixing methods, superior to the one we use, which could make this decreasing more drastic.

Using Ward-Slavnov-Taylor identities the authors of [24, 26, 32] came to the conclusion that the gluon propagator should be IR divergent, however, this divergence might be so weak that it could be hardly resolved on the lattice. We believe that our results for $D(0)$ are in clear disagreement even with a weak divergence.

Analogously to Fig. 1 in Fig. 4 we show the ghost dressing function $J(p)$ obtained on the $40^4$ lattice. There is a very clear Gribov copy effect changing $J(p)$ even qualitatively. Whereas the $fc$ SA results seem to support a weakly singular behavior, the $bc$ FSA data provide a plateau pointing to a finite IR value of the ghost dressing function, i.e. a tree-level behavior of the ghost propagator. Our data indicate that the plateau starts at $p \lesssim 200\text{MeV}$.

In Fig. 5 the ghost dressing function is shown for lattice sizes from $16^4$ to $40^4$. We show always $bc$ FSA results, except for $24^4$, where we compare also with $fc$ data obtained with the conventional OR algorithm. The latter show an even stronger IR singular behavior than those data obtained with the $fc$ SA algorithm.

There is a clear weakening of the singularity visible additionally to a finite-size effect which seems to lead to an IR plateau behavior. Such a plateau would be consistent with the different decoupling solutions and in contradiction with the Kugo-Ojima confine-
FIG. 4: The momentum dependence and Gribov copy sensitivity of the ghost dressing function \( J(p) = p^2 \cdot G(p) \) in the IR region on the \( 40^4 \) lattice. Filled symbols correspond to the \( bc \) FSA ensemble, open symbols to the \( fc \) SA ensemble.

FIG. 5: The momentum dependence of the ghost dressing function \( p^2 \cdot G(p) \) on the various lattices. Filled symbols correspond to the \( fc \) OR algorithm on \( 24^4 \) lattices are also shown.

V. DISCUSSION: THE QUALITY OF THE GAUGE FIXING PROCEDURE

In most of our simulations we have generated up to \( N_{copy} = 80 \) gauge copies for every thermalized configuration (up to 5 gauge copies for every flip-sector). A very reasonable question is whether our results will change if we will further increase the number of gauge copies \( N_{copy} \). In Fig. 7 we show the dependence of the average \( bc \) functional \( \langle F_{bc}(k_{copy}) \rangle \) :

\[
\langle F_{bc}(k_{copy}) \rangle = \frac{1}{n} \sum_{k=1}^{n} F_{bc}(k_{copy}), \quad k_{copy} = 1, \ldots, N_{copy},
\]

where for every configuration \( F_{bc}(k_{copy}) \) is the 'best' (i.e., maximal) value of the functional \( F \) found after employing \( k_{copy} \) copies, and \( n \) denotes the number of configurations.

One can see that this average still keeps a tendency to grow, which could mean that one should take even more copies to reach the global maxima. To understand it better we generated 25 configurations on the \( 40^4 \) lattice with \( N_{copy} = 320 \) (i.e., 20 gauge copies per sector).

In Fig. 8 we show the difference \( \Delta F(k_{copy}) = F_{bc}(k_{copy}) - F_{bc}(80) \) (17) at \( k_{copy} = 160, 240 \) and 320 for these configurations. For the majority of configurations this difference is rather small. However, for about 20% of configurations the difference is of the order of \( 10^{-5} \), which could mean still a rather strong influence on the values of propagators.

under the assumption that the vertex function is constant as seen in perturbation theory \[33\] and approximately also in lattice simulations \[34, 35\].

The decrease towards \( p^2 = 0 \) is obvious. With the improved gauge fixing the effect is even strengthened, such that an approach to an IR fixed point as expected from the scaling DS and FRG solution seems to be excluded.
To demonstrate that the change in the functional of the order of $10^{-5}$ indeed might give rise to substantial change in the propagators we plot in Fig. 9 the gluon propagator computed on $32^4$ lattice at two momenta, $p = 0$ and $p = p_{\text{min}}$ as a function of the difference $F_{bc}(32) - F_{bc}(k_{\text{copy}})$ for $k_{\text{copy}} = 1, 2, ..., 18$. One can see that the change in the functional $F_{bc}$ in the 5th digit brings quite substantial change in the propagator at both momenta.

These observations give an idea that there might be another even more efficient gauge fixing method which will be more successful in the search of the global maximum of the functional $F$ (as FSA is superior with respect to standard OR).

Therefore, we cannot draw a final conclusion before spending much more efforts into optimization of the gauge fixing procedure. However, we expect that both the gluon propagator as well as the ghost propagator will become further suppressed in the infrared, when approaching the fundamental modular region.

VI. CONCLUSIONS

In this work we studied numerically the dependence of the Landau gauge gluon and ghost propagators, as well as of the running coupling constant, in pure gauge $SU(2)$ lattice theory in the infrared region. The special accent has been made on the study of the dependence of these ‘observables’ on the choice of Gribov copies. The simulations have been performed using the standard Wilson action at $\beta = 2.20$ for linear lattice sizes up to $L = 40$. For gauge fixing gauge orbits enlarged by $Z(2)$ flip operations were considered with up to 5 gauge copies in every flip-sector (in total, up to 80 gauge copies). For 25 thermalized configurations we produced 20 copies per sector (in total, 320 gauge copies for every configuration). The maximization of the gauge functional was achieved by the simulated annealing method always combined with consecutive overrelaxation (‘FSA’ algorithm).

Our findings can be summarized as follows.

1) For the gluon propagator our new data for the $40^4$ lattice agree with data on the smaller lattices (up to $32^4$). We confirm our conclusion [17] about the appearance of the local maximum at a non-zero value of the momentum $p^2$ (this local maximum was absent for lattice sizes $\leq 24^4$).

The zero-momentum gluon propagator $D(0)$ has a tendency to decrease with growing lattice size $L$. This observation is in clear contradiction with the infrared divergent gluon propagator obtained on the basis of Ward-Slavnov-Taylor identities.

At the time being, this behavior looks hardly con-
sistent with a $D(0) = 0$ limit at infinite $L$, and could be considered rather like an argument in favor of the decoupling solution with a non-singular gluon propagator. However, we do not yet consider the problem of the infrared asymptotics of the gluon propagator as a finally resolved (see below).

2) We calculated the ghost propagator for lattices up to $40^4$. Our $bc$ dressing function $J(p)$ of the ghost propagator demonstrates the approach to a plateau in the infrared, while the $fc$ dressing function still grows (as it was in earlier calculations; see, e.g., [3, 28, 37]).

This is a first clear indication of the lack of the IR-enhancement of the ghost propagator. This plateau behavior is in a clear contradiction with the Kugo-Ogima confinement criterion. The fate of this confinement criterion still needs a further clarification.

3) We have found that the effect of Gribov copies for both the propagators and in the consequence for the running coupling is essential in the infrared range $p < 1$ GeV. Therefore, the quality of the gauge fixing procedure in the study of gauge dependent observables remains important.

Indeed, the FSA method provides systematically higher values of the functional $F_U(g)$ as compared to the standard OR procedure for the same thermalized configurations. This means that in practice OR needs many more random copies to explore (correspondingly, much more CPU time to spend) to find larger values of $F_U(g)$ as compared to FSA. This effect becomes stronger with increasing the volume. However, we cannot say that we have reached the fundamental modular region when fixing the Landau gauge on larger lattices. One cannot exclude that there is another method superior to our FSA algorithm. We believe that the Gribov problem deserves even more thorough studies.

Maybe, there are alternative ways to resolve the problem of the IR asymptotics of the propagators. We have to be aware that the lattice method as it is normally used has still some uncertainties. First of all the continuum limit in the infrared range is hard to control and it depends on the proper choice of the gauge potential $A_{\mu}$. Moreover, the infrared limit is sensitive to the boundary conditions, which normally are taken to be periodic. Incomplete gauge fixing in combination with these choices seems unavoidably to lead to zero-momentum modes not sufficiently suppressed even in the thermodynamic limit. That the presence of zero-momentum modes can spoil the behavior of gauge-variant propagators is well-known from the example of 4d compact $(U(1)$ lattice gauge theory [37, 38, 39]. Whether a BRST conformal lattice reformulation will solve the issue as proposed in [40, 41] remains to be seen.

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