Determining optimal partial transmission ratios of mechanical driven systems using a V-belt drive and a two-stage helical reducer

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Abstract. This article introduces a study on the optimal calculation of partial transmission ratios of mechanical drive systems using a V-belt drive and a two-stage helical reducer. In the study, the dimension of the cross section of the system was chosen as the objective function of the optimization problem. Also, the design equation for pitting resistance of a gear set was investigated and equations on moment equilibrium condition of a mechanic system including a V-belt drive and a two-stage helical reducer and their regular resistance condition were analyses. From the results of the study, models for determining the optimal partial ratios of the V-belt drive and two stages of the reducer were proposed. Using these models, the partial ratios can be calculated accurately and simply.

1. Introduction

Until now, in optimization design of a mechanical driven system, many studies have concentrated in optimum determination of the gear ratios of the reducer in the system. The reason of that is the gear ratios strongly affect the size, the dimension, the mass as well as the cost of the reducer.

The gear ratios have been determined for two-stage reducers [1, 2, 3 and 4], three-stage reducers [4, 5, 6, 7 and 8], and four-stage reducers [4, 8, 9 and 10]. Also, the gear ratios have been found by different methods. They were found by graph method [1, 2 and 4], by “practical method” [2] or by model method [5, 6, 7, 8, 9 and 10].

In the graph method, gear ratios were determined graphically. For example, for three-stage helical reducers, gear ratios of the first stage and the second stage can be found from the graph in Figure 1 [1]. In the “practical method”, gear ratios were found from practical data. For instance, based on the data from reducer factories, it was found that the weight of two-stage reducer is minimum when the ratio of the center distance of the second stage to that of the first stage is from 1.4 to 1.6 [2]. After that, the optimum gear ratios were introduced in the tabulated form. In model method, models for calculation of gear ratios have been proposed for numerous objectives. The objective can be the minimal volume of gears [4], the minimal cross section of the reducer [5], the minimal mass of gears [6, 9 and 10], the minimal reducer length [7] and the minimal reducer mass [8].

Newly, the gear ratios of mechanical systems which use a V-belt drive [11 and 12] or a chain drive [13] and a two-stage reducer. In this studies, the objective was the minimum height of the system including the reducer and the V-belt drive of the chain drive.
This paper presents a study for optimum determining partial transmission ratios and for driven systems using a V-belt drive and a two-stage helical reducer with the objective is the minimum system cross-sectional dimension.

![Fig. 1: Transmission ratio of step 1 and 2 versus the total transmission ratio [1]](image)

### 2. Optimization problem for determining optimum partial transmission ratios

![Figure. 2: Calculation schema](image)

The objective of the optimization problem is to determine the optimum transmission ratios in order to get the minimum acreage of cross section of a mechanical driven system using a V-belt drive and a two-stage helical reducer. From Figure 2, the acreage of cross section of the system can be determined as:

\[ A = L_{\text{max}} \cdot h_{\text{max}} \]  

(1)
Where:

\[
L_{\text{max}} = \max \left( L_{\text{gb}}, L_{b} \right) \tag{2}
\]
\[
h_{\text{max}} = \max \left( d_2, d_{w21}, d_{w22} \right) \tag{3}
\]

In which, \( L_{\text{gb}} \) and \( L_{b} \) are determined as (see Figure 2):

\[
L_{\text{gb}} = d_{2} / 2 + a_{w1} + a_{w2} / 2 \tag{4}
\]
\[
L_{b} = d_{1} / 2 + a + d_{2} / 2 \tag{5}
\]

In the above equations, \( a_{w1} \) and \( a_{w2} \) are the center distance of the first and the second step, respectively; \( d_{2} \), \( d_{w21} \) and \( d_{w22} \) is the diameter of the driven pulley of the V-belt drive, the driver gear of the first and the second step, respectively. For the V-belt drive, the diameter of the drive pulley can be calculated by:

\[
d_{1} = d_{2} \left[ u_{b} \cdot (1 - \varepsilon) \right] \tag{6}
\]

Substituting (6) into (5) gives

\[
L_{b} = d_{2} \left[ u_{b} \cdot (1 - \varepsilon) \right] + a + d_{2} / 2 \tag{7}
\]

For the first and the second-stage of the reducer, the driven diameters can be determined as [14]:

\[
d_{w21} = 2 \cdot a_{w1} \cdot u_{1} / \left( u_{1} + 1 \right) \tag{8}
\]
\[
d_{w22} = 2 \cdot a_{w2} \cdot u_{2} / \left( u_{2} + 1 \right) \tag{9}
\]

From (2), (3), (4), (7), (8) and (9) the following equations are given:

\[
L_{\text{max}} = f \left( d_{2}, a_{w1}, a_{w2}, u_{b}, u_{1}, u_{2} \right) \tag{10}
\]
\[
h_{\text{max}} = f \left( d_{2}, a_{w1}, a_{w2}, u_{1}, u_{2} \right) \tag{11}
\]

Thus, the optimization problem is defined as minimize

\[
A = L_{\text{max}} \cdot h_{\text{max}} \tag{12}
\]

With the following constraints

\[
1 \leq u_{b} \leq 6
\]
\[
1 \leq u_{1} \leq 9
\]
\[
1 \leq u_{2} \leq 9 \tag{13}
\]

From (10), (11) and (12), it is clear that for solving the optimization problem it is necessary to calculate the driven pulley diameters \( d_{2} \), the center distance of the first stage \( a_{w1} \), and the center distance of the second stage \( a_{w2} \).

2.1. Determining the driven pulley diameter \( d_{2} \)

For a V-belt drive, from tabulated data for determining allowable power [14], the following regression model for calculation of driver diameter \( d_{1} \) (with the determination coefficient \( R^{2}=0.9156 \)) was found:

\[
d_{1} = 269.7721 \cdot \left[ R_{1} \right]^{0.7042} / v^{0.5067} \tag{14}
\]

Theoretically, the peripheral velocity of the belt can determined as follow:
From (14) and (15) the diameter of the driving pulley can be determined by the following equation:

\[
d_i = 1093.8 \cdot \frac{P_1^{0.7923}}{n_1^{0.6369}}
\]  

(16)

Also, the diameter of driven pulley of a V-belt drive is calculated by [14]:

\[
d_2 = u_b \cdot d_i \cdot (1 - \varepsilon)
\]  

(17)

Substituting (16) into (17) gives

\[
d_2 = 1093.8 \cdot u_b \cdot (1 - \varepsilon) \cdot \frac{P_1^{0.7923}}{n_1^{0.6369}}
\]  

(18)

Where, \( \varepsilon \) is slippage coefficient; \( \varepsilon = 0.01, 0.02 \) [14]; \( u_b \) is the transmission ratio of the V-belt drive; \( [P_1] \) is the allowable power of the drive (kW); \( [P_1] \) is calculated by the following equation:

\[
[P_1] = n_1 \cdot [T_1] / (9.55 \cdot 10^6)
\]  

(19)

Choosing \( \varepsilon = 0.015 \) and substituting its and (19) into (18) gives:

\[
d_2 = 0.0032 \cdot u_b \cdot n_1^{0.1554} \cdot [T_1]^{0.7923}
\]  

(20)

Where, \( [T_1] \) is the permissible torque on the driving shaft which can be calculated from permissible torque on the output shaft \([T_{out}]\) by:

\[
[T_1] = [T_{out}] / (u \cdot \eta_i)
\]  

(21)

In the equation (21), \( u_i \) is the total transmission ratio of the system; \( \eta_i \) is the total efficiency of the system:

\[
\eta_i = \eta_d \cdot \eta_{br} \cdot \eta_o
\]  

(22)

In which, \( \eta_d \) is V-belt efficiency (\( \eta_d \) is from 0.956 to 0.96 [2]); \( \eta_{br} \) is helical gear transmission efficiency (\( \eta_{br} \) is from 0.96 to 0.98 [14]); \( \eta_o \) is transmission efficiency of a pair of rolling bearing (\( \eta_o \) is from 0.99 to 0.995 [14]). Choosing \( \eta_d = 0.955 \), \( \eta_{br} = 0.97 \) and \( \eta_o = 0.992 \) [14] and substituting (22) into (21) gives

\[
T_1 = 1.14 \cdot T_{out} / u_i
\]  

(23)

2.2. Determining the center distance of the first and the second stage

The center distance of the first stage of the reducer can be calculated by [14]:

\[
a_{w1} = K_\omega \cdot (u_i + 1) \cdot \sqrt{T_{11} \cdot k_{H\beta}} \cdot \sqrt{[\sigma_H]^2 \cdot u_i \cdot \psi_{heel}}
\]  

(24)

Where,

- \( K_{H\beta} \) is contact load ratio for pitting resistance; for the first stage of a two-stage helical reducer \( k_{H\beta} = 1.02 \pm 1.28 \) [14]. Therefore, we can chose \( k_{H\beta} = 1.1 \);
- \( [\sigma_H] \) is allowable contact stress (MPa); In practice, \( [\sigma_H] = 350 \ldots 410 \) (MPa) and we can chose \( [\sigma_H] = 380 \) (MPa);
- \( k_\omega \) is material coefficient; As the gear material is steel, \( k_\omega = 43 \) [14];
- \( \psi_{heel} \) is coefficient of wheel face width; for the for the first stage of a two-stage helical reducer \( \psi_{heel} = 0.3 \).
Substituting the above values into (24) gets:

\[
a_{u1} \approx 1.2639 \cdot (u_1 + 1) \cdot \sqrt{T_{in}} / u_i
\]  

Calculating in the same way and with the note that the coefficient of wheel face width for the second stage \( \psi_{w2} = 0.35 \), the center distance of the second stage is calculated by:

\[
a_{u2} \approx 1.2006 \cdot (u_2 + 1) \cdot \sqrt{T_{in}} / u_2
\]

3. Results and discussions

Based on Equations (12) and (13), a computer program was performed to determine the optimum partial transmission ratios for getting the minimum dimension of the cross section of the system. The following data was used in the optimization program: \( u_i = 20 \ldots 50 \); \( T_{out} = 10^6 \ldots 10^7 \).

![Figure 3: Partial ratios versus total ratio (With \( T_r = 10^6 \) Nmm)](image)

Figure 3 shows the relation between the optimum partial ratios including the gear ratio of the first stage \( u_1 \), the gear ratio of the second stage \( u_2 \) and transmission ratio of the V-belt drive \( u_b \) and the total ratios of the system.

From the results of the optimization program, a regression equation (equation (27) was found for determination of the transmission ratio of the V-belt drive (with the coefficients of determination was \( R^2 = 0.971 \)). Also, for calculation of the optimum gear ratios of the first and the second stages of the reducer, equations (28) and (29) were proposed (with the coefficients of determination of both equations are \( R^2 = 1 \)):

\[
u_b = 45.9344 \cdot \left[ T_{out} \right]^{0.3795} \cdot u_i^{-0.8407}
\]
\[
u_i = 0.9365 \cdot u_g^{0.667}
\]
\[
u_2 = 1.0678 \cdot u_g^{-0.333}
\]

Where, \( u_g \) is the total ratio of the reducer; \( u_g \) is calculated by \( u_g = u_i / u_b \).

4. Conclusions

The minimum cross-sectional area of a mechanical drive system using a V-belt drive and a two-stage helical reducer can be found by using the optimum partial transmission ratios when designing.

Formulas for calculation of the partial transmission ratios of the V-belt drive and the gear-stages were proposed for getting the minimum cross-sectional area of the system.
The partial transmission ratios of the V-belt drive and the helical gear-stages can be calculated simply by using explicit formulas. The proposed formulas can be used for calculation of optimum reducer design in practice or in universities for student project.

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