Enhancing the information of nonlinear SU(1, 1) quantum systems interacting with a two-level atom

M. Y. Abd-Rabbou1 · S. I. Ali1 · M. M. A. Ahmed1

Received: 20 January 2022 / Accepted: 28 May 2022 / Published online: 25 July 2022
© The Author(s) 2022

Abstract
The effect of nonlinearity, initial atomic state, and different resonance cases on the interaction between nonlinear SU (1, 1) quantum states and a two-level atom is discussed. The optimal behaviours of decoherence, entanglement and quantum coherence are predicted via using the skew information, tomographic entropy, and the relative entropy of coherence, respectively. It is shown that the detuning parameter has a destructive effect on the coherence and consequently on the entanglement if the quantum system is regulated in the ideal SU (1, 1) quantum systems. For the nonlinear SU (1, 1) quantum systems, the ability to suppress the decay of entanglement induced by the detuning may be increased by preparing the initial atomic state in its excited state.

Keywords Tomographic entropy · Skew information · Coherence · SU (1, 1) quantum systems

1 Introduction
The control of entanglement for a quantum system has attracted the interest of many physicists for its importance in the processing of quantum information, as it is the foundation of quantum computation and communication (Gyongyosi and Imre 2019; Ji and Juju 2019; Wang et al. 2020). Additionally, many theoretical and experimental schemes have been proposed for enhancing the entanglement and quantum correlation (Metwally 2017; Xuexiang et al. 2018; Abd-Rabbou et al. 2019; Min et al. 2021). The measuring of entanglement degree has been obtained via different methods, such as von Neumann entropy (von Neumann 1932; Phoenix and Knight 1991), concurrence (Fan et al. 2003), negativity (Horodecki et al. 2009; Zhou et al. 2020), and the entanglement of formation (Wootters 1998). Likewise, the entanglement path has also been predicted by some measurements, such as entropy squeezing (Sebawe and Ahmed 2011), tomographic entropy (Chernega et al. 2006; Almarashi et al. 2020), Wigner function (Abd-Rabbou et al. 2019), and quantum uncertainty and local quantum Fisher information (Mohamed et al. 2021).
As is well known, there are interesting issues concerning the light-matter interaction in quantum optics. These issues are the atom-field interaction (Jaynes and Cummings 1963; Abdalla et al. 1990; Shore and Knight 1993), atom-atom interaction (Sadiek et al. 2009; Abo-Kahla et al. 2021), and field-field interaction (Jahanbakhsh and Tavassoly 2020; Ling-Juan Feng et al. 2021). These interactions contained many natural phenomena that have been observed in the experimental systems. Moreover, these types of interactions may be described by some mathematical tools to switch from one structure to another. The interaction between a set of two-level atoms and a quantized field has been transformed into three modes of electromagnetic fields (Abdalla et al. 2006), atom-atom or field-atom-interactions (Abdalla et al. 2016; Hilal et al. 2020). In this context, we aim to investigate the interaction between a two-level atom and the category of the SU(1, 1) Lie algebra, where the atom may be considered as a particle regularized in the SU(2) Lie algebra. The interaction between SU(1,1) and SU(2) quantum systems have been studied by many authors (Sebawe and Ahmed 2011; Nahla et al. 2019). The effect of damping reservoir for even case of Barut-Girardello states at $k = 1/4$ have been discussed (Mohamed et al. 2017). The influence of coupling parameters of the external classical field system on the SU(1,1) and SU(2) interaction have been examined (Abdalla et al. 2017; Obada et al. 2021). The relation between quantum Fisher information (QFI)(Fisher 1925; Metwally 2018) and quantum entanglement of two atoms that interact with two non-degenerate modes has been investigated (Abdel-Khalek et al. 2020). The interaction between SU(1,1) Lie algebra and three-level atom in presence of laser field which related to ideal and real laser is presented (Obada et al. 2021). Via the spherical harmonics, Barut-Girardello states can be generated, which describe the system entanglement (Fakhri and Dehghani 2009). Superposition of Perelomov leading to SU(1, 1) coherent states of the Gilmore-Perelomov type has been proposed by using a Jaynes-Cummings model with intensity dependent coupling and an external field (Miry and Tavassoly 2012). The non-classical quantum correlations of the two-qubit interact with a nonlinear generalized SU(1,1) cavity field have been discussed (Mohamed et al. 2021). The construction of non-linear Barut-Girardello coherent state, entangled pair-, trio-coherent states and Gilmore-Perelomov coherent states are widely used in applied quantum physics. For some different physical systems, the non-linear Barut-Girardello coherent state was achieved, such as; charge carriers in anisotropic 2D-Dirac materials (Díaz-Bautista et al. 2019), Morse potential (Fakhri and Chenaghlou 2003; Popov et al. 2013), and Pöschl-Teller potential (Zhang et al. 2014).

Our motivation is to examine the temporal evolution of three different types of quantum information; skew information, tomographic entropy, and relative entropy of coherent. In which we address the subject of what is the correlation between the three quantifying measurements and subsystems entanglement control? The main task is to study the influence of initial setting angles, the detuning parameter, and the non-linear term on the relation between the three quantities, where there is a strong correlation for ideal systems (Mohamed and Metwally 2017; Sheng et al. 2021; Mohamed, Khalil and Abd-Rabbou 2021). So, is different parameters breaking these correlations?

The paper is arranged as follows: In sect. 2, we introduce the physical discretion of the Hamiltonian operator for the non-linear system. As well, we obtained the temporal evolution of the wave function via using Heisenberg equations of motions. The mathematical definitions of the skew information, tomographic entropy and relative entropy of coherent entropy are proposed in sect. 3. Section 4 is devoted to displaying the discussion and numerical results.
2 Model and solution

We shall consider the Hamiltonian model consists of a two-level atom interacting with \( SU(1, 1) \) quantum systems, which takes the following form. \((\hbar = 1)\),

\[
\hat{H} = \omega k_z + \sum_{i=1}^{2} \Omega_j S_{i\|} + \lambda (S_{12} \hat{R} + \hat{R}^\dagger S_{21})
\]  

\((1)\)

where \( \omega \) and \( \Omega_j, j = 1, 2 \) are the radiation \( SU(1, 1) \) frequency and the atomic transition, respectively, while \( \lambda \) is the coupling constant between the non linear \( S(1, 1) \) quantum system and the atomic state. The operators \( S_{ij} = |i\rangle \langle j| \) are the transition operators for the atomic states, with \([S_{ij}, S_{kl}] = S_{ij} \delta_{kj} - S_{ij} \delta_{il}\). Likewise, the operators \( \hat{R} = K_+ f(K_z) \) and \( \hat{R}^\dagger = K_+ f(K_z) \) are the non-linear operator in \( SU(1, 1) \) Lie algebra, with the generators \( K_z \) and \( K_\pm \). These generators satisfy the following commutation relations;

\[
[K_z, K_\pm] = \pm K_\pm, \quad [K_\pm, K_\pm] = 2K_z, \quad [\hat{R}, K_z] = -\hat{R}, \quad \text{and} \quad [\hat{R}^\dagger, K_z] = \hat{R}^\dagger,
\]

and the related Casimir operator \( K \) with,

\[
K^2 = K_z^2 - \frac{1}{2}(K_+ K_- + K_- K_+).
\]

These operators have the corresponding eigenvalues;

\[
K_z \mid m, k \rangle = (m + k) \mid m, k \rangle, \quad \hat{K}_+ \mid m, k \rangle = \sqrt{(m + 1)(m + 2k)} \mid m + 1, k \rangle, \quad \hat{K}_- \mid m, k \rangle = \sqrt{(m(m + 2k - 1))} \mid m - 1, k \rangle, \quad \hat{K}_z^2 \mid m, k \rangle = k(k - 1) \mid m, k \rangle,
\]

\((2)\)

where \( m \) represents any non-negative integer, and \( k \) is the Bargmann index.

Since the \( SU(1, 1) \) Lie algebra can be expressed in terms of boson annihilation and creation operators, it is isomorphic to the Lie algebra of the non-compact \( SU(1, 1) \) group. Therefore for a single-mode operator \( \hat{a} \), we define

\[
\hat{K}_+ = \frac{1}{2} \hat{a}^\dagger \hat{a}^2, \quad \hat{K}_- = \frac{1}{2} \hat{a}^2, \quad \hat{K}_z = \frac{1}{2} \left( \hat{a}^\dagger \hat{a} + \frac{1}{2} \right).
\]

Whereas for the two-mode bosonic representation they take the form

\[
\hat{K}_+ = \hat{a}^\dagger \hat{b}^\dagger, \quad \hat{K}_- = \hat{a} \hat{b}, \quad \hat{K}_z = \frac{1}{2} (\hat{a}^\dagger \hat{a} + \hat{b}^\dagger \hat{b} + 1)
\]

Alternatively, in the Holstien–Primakoff representation (Holstein and Primakoff 1940; Mead and Papanicolaou 1983), the generators \( \hat{K}_\pm \) and \( \hat{K}_z \) are given by

\[
K_+ = \sqrt{2k - 1 + n} \hat{a}^\dagger, \quad K_- = \hat{a} \sqrt{2k - 1 + n}, \quad \text{with} \quad \hat{K}_z = \hat{K} + \hat{n}
\]

This means that the interaction between the quantum system \( SU(1,1) \) and the atomic system \( SU(2) \) is very rich and would give us a wide scope for studying the nature of this kind of interaction. The Heisenberg equation of motion for any operator \( \hat{O}(t) \) is given by

\[
\frac{d\hat{O}}{dt} = [\hat{O}, \hat{H}].
\]

Thus the equations of motion for \( \hat{S}_{ij} \) and \( \hat{K}_z \) are given by
\[ i \frac{d \hat{S}_{11}}{dt} = \lambda (\hat{R} \hat{S}_{12} - \hat{S}_{21} \hat{R}^\dagger), \quad i \frac{d \hat{S}_{22}}{dt} = -\lambda (\hat{R} \hat{S}_{12} - \hat{S}_{21} \hat{R}^\dagger), \quad i \frac{d \hat{K}_z}{dt} = -\lambda (\hat{R} \hat{S}_{12} - \hat{S}_{21} \hat{R}^\dagger) \]

(3)

Therefore we have the following constant of motion \( \hat{N} = \hat{K}_z + \frac{1}{2} (\hat{S}_{11} - \hat{S}_{22}) \). However, we can write the Hamiltonian of equation (1) as,

\[ \hat{H} = \omega \hat{N} + \hat{C}, \]

(4)

where \( \hat{C} = \frac{\Delta}{2} (\hat{S}_{11} - \hat{S}_{22}) + \lambda (\hat{S}_{12} \hat{R} + \hat{R}^\dagger \hat{S}_{21}) \).

The time evolution operator \( U(t) = e^{-i \hat{H} t} = e^{-i \hat{N} t} e^{-i \hat{C} t} \), which can be written by employing the method of (Stenholm 1981, 1973) working in the atomic basis and in the interaction picture. After straightforward calculations, we can write the time evolution operator as:

\[ \hat{U}(t) = e^{-i \hat{N} t} e^{-i \hat{C} t} = e^{-i \hat{N} t} \begin{bmatrix} \hat{F}_1(m, t) & \hat{E}_1 \hat{R} \\ \hat{E}_2 \hat{R}^\dagger & \hat{F}_2(m, t) \end{bmatrix}, \]

(5)

where

\[ e^{-i \hat{N} t} = \begin{bmatrix} e^{-i (\mu_1 + \frac{1}{2}) t} & 0 \\ 0 & e^{-i (\mu_2 - \frac{1}{2}) t} \end{bmatrix}, \]

and

\[ \hat{F}_1(m, t) = \cos (\mu_1 t) - \frac{i \Delta \sin (\mu_1 t)}{\mu_1}, \quad \hat{F}_2(m, t) = \cos (\mu_2 t) + \frac{i \Delta \sin (\mu_2 t)}{\mu_2}, \]

\[ \hat{E}_j(\mu_j, t) = -i \lambda \frac{\sin (\mu_j t)}{\mu_j}, j = 1, 2, \quad \mu_1^2(m) = \frac{\Delta^2}{4} + \lambda^2 \hat{R} \hat{R}^\dagger, \quad \mu_2^2(m) = \frac{\Delta^2}{4} + \lambda^2 \hat{R}^\dagger \hat{R} \]

Now let us consider the state \( |\theta, \phi\rangle \) which acquires both excited state \( |e\rangle \) and ground state \( |g\rangle \) for the two-level atom in the following form

\[ |\theta, \phi\rangle = \cos(\theta/2) |e\rangle + \sin(\theta/2) \exp(-i\phi) |g\rangle \]

where \( \phi \) is the relative phase of the two atomic levels. To obtain the excited state we have to take \( \theta \to 0 \) while to make the wave function describe the particle in the ground state we have to let \( \theta \to \pi \). Assuming that at time \( t = 0 \) the wave function is given by

\[ |\psi(0)\rangle = |\theta, \phi\rangle \otimes |a, k\rangle_{BG}, \]

where the SU(1, 1) cavity field is initially in the Barut-Girardello SU(1, 1) coherent states which defined as the eigenstate of the lowering operator \( K_-; K_- |a, k\rangle_{BG} = a |a, k\rangle_{BG} \)

\[ |a, k\rangle_{BG} = \sqrt{\frac{|a|^{2k-1}}{I_{2k-1}(2|a|)}} \sum_{m=0}^{\infty} \frac{a^m}{\sqrt{a! \Gamma(2m+k)}} |m, k\rangle, \]

where \( I_\zeta(\chi) = (\frac{\chi}{\zeta})^\zeta \sum_{m=0}^{\infty} \frac{(\frac{\chi}{\zeta})^{2m}}{m! \Gamma(m+k+1)} \) is the \( \zeta^{th} \)-order modified Bessel function. The wave function of the system is given by

\[ |\psi(t)\rangle = \begin{pmatrix} |\psi(t)\rangle_e \\ |\psi(t)\rangle_g \end{pmatrix} |a, k\rangle_{BG} \]
Now, one can obtain the reduced atomic density operator via tracing out the field subsystem, where
\[ \rho_{\text{atom}}(t) = \text{Tr}_{\text{field}}[|\psi(t)\rangle\langle\psi(t)|] \]. Then,
\[ \rho_{\text{atom}}(t) = \begin{pmatrix} |\psi(t)\rangle_{ee}\langle\psi(t)| & |\psi(t)\rangle_{eg}\langle\psi(t)| \\ |\psi(t)\rangle_{ge}\langle\psi(t)| & |\psi(t)\rangle_{gg}\langle\psi(t)| \end{pmatrix} \]
In the following section, we employ the reduced density operator (6) to discuss some quantum information measures for the atomic system.

3 Information Measurements

In this section, we briefly introduce the mathematical definitions of the skew information as an indicator of decoherence and estimator of the system parameters, Tomographic entropy, and relative entropy of quantum coherence.

3.1 Skew Information

Based on the paradigmatic version of Fisher information (Luo 2003), which estimates the amount of information of unknown parameters in the quantum states as precisely as possible, is the so-called skew information (Wigner and Yanase 1997).

\[ I(\rho, \mathcal{K}) = -\frac{1}{2} \text{Tr}[\rho \mathcal{K}] \]

where, \( \mathcal{K} \) is a fixed conserved observable. For a 2 \times 2 mixed state, the skew information in terms of the Bloch representation is given by Zhong et al. (2013),

\[ I_{\theta} = \frac{2|\partial_\theta \bar{\tau}|^2 - (\bar{\tau}.\partial_\theta \bar{\tau})^2}{1 + \sqrt{1 - |\bar{\tau}|^2}} + \frac{(\bar{\tau}.\partial_\theta \bar{\tau})^2}{1 - |\bar{\tau}|^2}, \]

where the polar angle \( \theta \) is to be estimated, while \( \bar{\tau} = \{\tau_x, \tau_y, \tau_z\} \) is the real Bloch vector.

The numerical calculations in Fig. 1 show the optimal behaviour of skew information with respect to the polar angle \( \theta \). Firstly, we assumed that the perfect cavity of SU(1, 1), namely \( f(k_c) = 1 \), and the initial atomic system prepared either in an excited state (red-solid curve) or superposition state (blue-dash curve). The azimuthal angle \( \phi = \pi/2 \), the Bargmann index \( k = 0.5 \), and the intensity of Barut-Girardello coherent state \( \beta = 15 \) are set to be fixed for all figures. As it is displayed from Fig. 1a, b the non-resonance case plays a key role in the dissipation of the maximum bounds of the skew information, where the maximum bounds of the skew information in resonance case with are \( \Delta = 0 \) greater than that displayed for the non-resonance case with \( \Delta = 10 \). Therefore, the estimation degree of parameter \( \theta \) decreases as \( \Delta \) increases along the scaled time, hence the decoherence degree increases. The maximum bounds is larger at the scaled time \( n\pi\Delta \), which means that the decoherence has large values. The influence of the nonlinear cavity on the skew
information is displayed in Fig. 1c, d, where we set \( f(k_z) = \frac{1}{\sqrt{(m+1)(m+2)}} \). The optimal behavior of \( I_{\theta} \) shows that the detuning parameter has a clear effect on the number of oscillations, where the oscillations between the upper and lower values increases as the detuning increases. However, the lower bounds at the non-resonance case (\( \Delta = 10 \)) are higher than those displayed in the resonance case. This means that, the decoherence degree is very large comparing with those shown in Fig. 1a, b. Moreover, the maximum and minimum bounds depend on the initial atomic states, where the maximum bounds that are depicted in the red-solid curve as the atomic system is prepared in an excited state are much larger than those displayed in the blue-dash curve.

3.2 Tomographic Entropy

The tomographic entropy is introduced to illustrate the entanglement of the spin states of the quantum system (Dodonov and Man’ko 1997). For a generic 2 \( \times \) 2 density matrix, the tomographic entropy is defined as Chernega et al. (2006),

\[
T = \frac{-1}{\sqrt{2\pi}} \int_0^{2\pi} \int_0^\pi \sum_{i=1}^{2} Q_i(\Theta, \Phi) \ln Q_i(\Theta, \Phi) \sin \Theta d\Theta d\Phi,
\] (9)

here, \( Q_i(\Theta, \Phi) \) are the generic spin tomograms, with,

\[
Q_1(\Theta, \Phi) = \rho_{11} \cos^2 \Theta/2 + \rho_{22} \sin^2 \Theta/2 + \sin \Theta \Re[\rho_{12} e^{i\Phi}]
\]

\[
Q_2(\Theta, \Phi) = \rho_{22} \cos^2 \Theta/2 + \rho_{11} \sin^2 \Theta/2 - \sin \Theta \Re[\rho_{12} e^{i\Phi}]
\] (10)

The values of the tomographic entropy are obtained by using “Mathematica” program, where the “Trapezoidal” rule is used to give the numerical values of integration. In Fig. 2, we investigate the quantum entanglement of the ideal/defect SU(1,1) cavity field via the tomographic entropy, where the same conditions of the previous case in Fig. 1 are
Enhancing the information of nonlinear SU(1, 1) quantum systems…

Figure 2a describes the optimal behaviour of the tomographic entropy in the resonance case, where it ordinary osculates between $2.5 \mapsto 3.5$. It is seen that the function $T$ tends to zero periodically with scaled time $\lambda t = n\pi$, which means, the system returns to the separable state. The degree of entanglement for excited and superposition states are extremely identical between the time intervals $\lambda t = n\pi$. Meanwhile, the effect of the non-resonance case is disclosed in Fig. 2b, where the lower bounds of the tomographic entropy decrease as the scaled time increases. That means the separable state returned to a partially entangled state, while the upper bounds are fixed. On the other hand, the non-linear cavity has a remarkable influence, where the large detuning increases the numbers of osculations and decreases the upper bounds of the function $T$. Consequently, the entanglement decreases as the detuning increases and the entangled states approach separable states. Interestingly, there is an eminent correlation between the skew information and the sign of entanglement, where the weakening of entanglement corresponding to the strengthening and growth of the skew information.

3.3 Relative entropy of coherence

According to relative von Neumann entropy, Baumgratz et al. (2014) are employed the relative entropy to quantify the quantum coherence. The relative entropy of coherence was defined as a minimum relative entropy between the set of all incoherent states $I$ and the given state $\rho$,

$$C = \min_{\epsilon \in I} S(\rho||\epsilon) = Tr[\rho \ln \rho] - Tr[\rho_{\text{diag}} \ln \rho_{\text{diag}}],$$

(11)

where, $I$ is the set of all incoherent states. $\rho_{\text{diag}} = \sum_i \rho_{ii} |i\rangle\langle i|$ is the diagonal element of $\rho$.

Finally, we discuss the influence of the initial atomic system $\theta$ and different resonances causes $\Delta$ on the behaviour of the relative coherence, with the same conditions in Fig. 1. Figure 3 a logical correlation between the tomographic entropy and quantum coherence, this correlation is correct because of any entangled state subset of coherence states. The initial settings of the atomic state play a central role in the coherence degree. The non-resonance case and non-linear cavity improve the coherence degree for the initial superposition.
4 Conclusion

A system consists of a single two-level atom in $SU(2)$ interacts locally with linear/ non-linear $SU(1, 1)$ quantum systems. It is considered that the initial atomic state is either prepared in its excited state or a superposition state. We discussed the influence of linear/ non-linear quantum system, which is represented by ideal/ imperfect field operators, on the behavior of three quantum quantities. These quantities include the skew information, the tomographic entropy, and the relative entropy of coherence. The phenomena of the collapse and revival behavior of the three functions are depicted clearly when the atomic state is initially prepared in the excited state. However, if the system is initially prepared in a superposition state, the predicted behavior of the three functions is much larger than that displayed for the excited state.

The influence of a non-linear state on the optimal behavior of the estimation degree, the entanglement, and coherence in the initial excited/ superposition state are discussed. The results are shown that the detuning parameter is used as a control parameter, that maximizes/ minimize the three phenomena. This means that by controlling the detuning, one can increase the ability to suppress the separability induced by the imperfect of quantum system. However, the oscillation’s number increases as the detuning increases and consequently the lower/upper bounds of the three phenomena are improved. Similar effect is predicted where the cavity is prepared in ideal $SU(1, 1)$ . The maximum/ minimum bounds of the three quantifiers are displayed at large detuning parameter.

**Funding** Open access funding provided by The Science, Technology & Innovation Funding Authority (STDF) in cooperation with The Egyptian Knowledge Bank (EKB). The authors have not disclosed any funding.
Declarations

Conflict of interest The authors have not disclosed any conflict of interest

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license, and indicate if changes were made. The images or other third party material in this article are included in the article’s Creative Commons license, unless indicated otherwise in a credit line to the material. If material is not included in the article’s Creative Commons license and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit http://creativecommons.org/licenses/by/4.0/.

References

Abdalla, M & Sebawe, Ahmed, M.M.A.: Entropy squeezing and entanglement of the interaction between SU (1, 1) and SU (2) quantum systems. Opt. Commun. 284(7), 1933–1940 (2011)
Abdalla, M & Sebawe, Ahmed, M.M.A., Obada, A.-S.F.: Dynamics of a non-linear Jaynes–Cummings model. Phys. A Stat. Mech. Appl. 162(2), 215–240 (1990)
Abdalla, M & Sebawe, Peñina, Jan, Křepelka, Jaromír: Statistical properties of multiphoton time-dependent three-boson coupled oscillators. JOSA B 23(6), 1146–1160 (2006)
Abdalla, M & Sebawe, Ahmed, M.M.A., Khalil, E.M., Obada, A.-S.F.: The interaction between a single two-level atom coupled to an N-level quantum system through three couplings. Ann. Phys. 364, 168–181 (2016)
Abdalla, M & Sebawe, Khalil, E.M., Obada, A.S.-F., Peñina, J., Křepelka, J.: Linear entropy and squeezing of the interaction between two quantum system described by SU (1, 1) and SU (2) lie group in presence of two external terms. AIP Advances 7(1), 015013 (2017)
Abdel-Khalek, S., Khalil, E.M., Mohamed, A.-B.A., Abdel-Aty, M., Besbes, H.R.: Response of quantum fisher information, variance entropy squeezing and entanglement to the intrinsic decoherence of two non-degenerate fields interacting with two qubits. Alex. Eng. J. 59(6), 5147–5154 (2020)
Abd-Rabbou, M.Y., Metwally, N., Ahmed, M.M.A., Obada, A.-S.F.: Wigner function of noisy accelerated two-qubit system. Quantum Inf. Process. 18(12), 1–19 (2019)
Abd-Rabbou, M.Y., Metwally, N., Ahmed, M.M.A., Obada, A.-S.F.: Suppressing the information losses of accelerated qubit-qutrit system. Int. J. Quantum Inf. 17(04), 1950032 (2019)
Abo-Kahla, D.A.M., Abd-Rabbou, M.Y., Metwally, N.: The orthogonality speed of two-qubit state interacts locally with spin chain in the presence of Dzyaloshinsky–Moriya interaction. Laser Phys. Lett. 18(4), 045203 (2021)
Almarashi, Abdullah & Algarni, Ali, Abdel-Khalek, S., Abd-Elmougod, G.A., Raqab, Mohammad & Z.: Quantum extropy and statistical properties of the radiation field for photonic binomial and even binomial distributions. J Russ. Laser Res. 41(4), 334–343 (2020)
Baumgratz, T., Cramer, M., Plenio, M.B.: Quantifying coherence. Phys. Rev. Lett. 113, 140401 (2014)
Chernega, Vladimir & Man’ko, Olga & Pilyavets, Oleg & Zborovskii, Vadim: Tomographic characteristics of spin states. J. Russ. Laser Res. 27(2), 132–166 (2006)
Díaz-Bautista, Erik, Concha-Sánchez, Yajaira, Raya, Alfredo: Barut–Girardello coherent states for anisotropic 2D-dirac materials. J. Phys. 31(43), 435702 (2019)
Dodonov, V.V., Mańko, V.I.: Positive distribution description for spin states. Phys. Let. A 229(6), 335–339 (1997)
Fakhri, H., Chenaghlou, A.: Barut–Girardello coherent states for the Morse potential. Phys. Lett. A 310(1), 1–8 (2003)
Fakhri, H., Dehghani, A.: Coherency of SU (1, 1)-Barut–Girardello type and entanglement for spherical harmonics. J. Math. Phys. 50(5), 052104 (2009)
Fan, Heng, Matsumoto, Keiji, Imai, Hiroshi: Quantify entanglement by concurrence hierarchy. J. Phys. A 36(14), 4151 (2003)
Fisher, Ronald & Aylmer: Theory of statistical estimation. In Mathematical Proceedings of the Cambridge Philosophical Society, vol. 22, pp. 700–725. Cambridge University Press (1925)
Gyongyosi, Laszlo, Imre, Sandor: Entanglement access control for the quantum internet. Quantum Inf. Process. 18(4), 1–17 (2019)

Hilal, Eman, Alkhateeb, S., Abdel-Khalek, S., Khalil, E.M., Almowalled, Amjaad: Quantum scheme for N-level atom interacting with a two level atom: atomic fisher information and entropy squeezing. Alex. Eng. J. 59(3), 1259–1264 (2020)

Holstein, T., Primakoff, Hl.: Field dependence of the intrinsic domain magnetization of a ferromagnet. Phys. Rev. 58(12), 1098 (1940)

Horodecki, Ryszard, Horodecki, Paweł, Horodecki, Michal, Horodecki, Karol: Quantum entanglement. Rev. Mod. Phys. 81, 865–942 (2009)

Jahanbakhsh, F., Tavassoly, M.K.: The field-field and dipole-dipole coupling effects on the entanglement of the interaction between two qutrits with a two-mode field. Mod. Phys. Lett. A 35(22), 2050183 (2020)

Jaynes, Edwin T., Cummings, Frederick W.: Comparison of quantum and semiclassical radiation theories with application to the beam maser. Proc. IEEE 51(1), 89–109 (1963)

Ji, Yinghua, Juju, Hu.: Control of quantum entanglement and entropic uncertainty in open quantum system: via adjusting ohmic parameter. Physica E 114, 113583 (2019)

Ling-Juan Feng, Yu., You, Heng-Xing Dong., Wang, Feng-Chao., Gong, Shang-Qing.: Enhancing cross-Kerr coupling via mechanical parametric amplification. Opt. Express 29(18), 28835–28842 (2021)

Luo, Shunlong: Wigner-Yanase skew information and uncertainty relations. Phys. Rev. Lett. 91, 180403 (2003)

Mead, Lawrence R., Papanicolaou, N.: Holstein-primakoff theory for many-body systems. Phys. Rev. B 28(3), 1633 (1983)

Metwally, N.: Enhancing entanglement, local and non-local information of accelerated two-qubit and two-qutrit systems via weak-reverse measurements. EPL 116(6), 60006 (2017)

Min, Yu., Zhang, Huan, Ye, Wei, Zhang, Kuizheng, Xia, Ying, Liyun, Hu.: Improvement of entanglement via catalytic quantum scissors. Optik 241, 167252 (2021)

Miry, S.R., Tavassoly, M.K.: Generation of a class of SU(1, 1) coherent states of the Gilmore–Perelomov type and a class of SU(2) coherent states and their superposition. Phys. Scr. 85(3), 035404 (2012)

Mohamed, A.-B.A., Metwally, N.: Non-classical correlations based on skew information for an entangled two-qubit-system with non-mutual interaction under intrinsic decoherence. Ann. Phys. 381, 137–150 (2017)

Mohamed, Abdel-Baset, Abdalla, Mohamed, Sebawe, Obada, A.-S.: Quantum effects due to the interaction between SU(1, 1) and SU(2) quantum systems with damping. Europ. Phys. J. D 71(9), 1–8 (2017)

Mohamed, A.-B.A., Khalil, E.M., Metwally, N., Eleuch, H.: Local two-atom correlations induced by a two-mode cavity under nonlinear media: quantum uncertainty and quantum fisher information. Results Phys. 10, 104975 (2021)

Mohamed, A.-B.A., Khalil, Eied, M., Yassen, Mansour, Eleuch, Hichem: Two-qubit local Fisher information correlation beyond entanglement in a nonlinear generalized cavity with an intrinsic decoherence. Entropy 23(3), 311 (2021)

Mohamed, A.A.-B., Khalil, E.M., Abd-Rabbou, M.Y.: Quantum effects induced by two classical fields in a coherent cavity field containing two two-level atoms. Physica E 134, 114839 (2021)

Nahla, A.A., Ahmed, M.M.A., Alamri, S.Z.: Analytical computation of nonclassical behavior for asymmetric two two-level atoms interacting with SU(1, 1) quantum system. Europ. Phys. J. D 73(3), 1–2 (2021)

Obada, A.-S.F., Alshehri, Nawal, A., Khalil, E.M., Abdel-Khalek, S., Habeba, H.F.: Entropy squeezing and atomic Wehrl density for the interaction between SU(1, 1) lie algebra and a three-level atom in presence of laser field. Results Phys. 30, 104759 (2021)

Phoenix, Simon JD., Knight, P.L.: Establishment of an entangled atom-field state in the Jaynes-Cummings model. Phys. Rev. A 44(9), 6023 (1991)

Popov, Duşan, Dong, Shi-Hai., Pop, Nicolina, Sajfert, Vjekoslav, Simon, Simona: Construction of the Barut–Girardello quasi coherent states for the Morse potential. Ann. Phys. 339, 122–134 (2013)

Sadieł, G., Lashin, E.I., Abdalla, M.&nbsp;Sebawe: Entanglement of a two-qubit system with anisotropic XY exchange coupling in a nonuniform time-dependent external magnetic field. Physica B 404(12–13), 1719–1728 (2009)

Sheng, Yi-Hao., Zhang, Jian, Tao, Yuan-Hong., Fei, Shao-Ming.: Applications of quantum coherence via skew information under mutually unbiased bases. Quantum Inf. Process. 20(2), 1–12 (2021)
Enhancing the information of nonlinear SU(1, 1) quantum systems…

Shore, Bruce W., Knight, Peter L.: The Jaynes–Cummings model. J. Mod. Opt. 40(7), 1195–1238 (1993)
Stenholtm, Stig: Quantum theory of electromagnetic fields interacting with atoms and molecules. Phys. Rep. 6(1), 1–121 (1973)
Stenholtm, Stig: Saturation effects in rf spectroscopy, iii. mixing of nearly degenerate levels by strong off-resonant fields. J. Phys. B At. Mol. Phys. 6(6), 1097 (1973)
Stenholtm, Stig: A Bargmann representation solution of the Jaynes–Cummings model. Opt. Commun. 36(1), 75–78 (1981)
von Neumann, Johann: Mathematische Grundlagen Der Quantenmechanik. Springer, Berlin (1932)
Wang, Dongyang, Liu, Yong, Ding, Jiangfang, Qiang, Xiaogang, Liu, Yingwen, Huang, Anqi, Xiang, Fu, Ping, Xu, Deng, Mingtang, Yang, Xuejun, et al.: Remote-controlled quantum computing by quantum entanglement. Opt. Lett. 45(22), 6298–6301 (2020)
Wigner, Eugene P., Yanase, Mutsuo M.: Information contents of distributions. In Part I: Particles and Fields. Part II: Foundations of Quantum Mechanics, pp. 452–460. Springer (1997)
Wootters, William K.: Entanglement of formation of an arbitrary state of two qubits. Phys. Rev. Lett. 80, 2245–2248 (1998)
Xuexiang, Xu, Liyun, Hu, Liao, Zeyang: Improvement of entanglement via quantum scissors. JOSA B 35(1), 174–181 (2018)
Zhang, Hong-Biao., Jiang, Guang-Yuan., Guo, San-Xing.: Construction of the Barut-Girardello type of coherent states for Pöschl-Teller potential. J. Math. Phys. 55(12), 122103 (2014)
Zhong, Wei, Sun, Zhe, Ma, Jian, Wang, Xiaoguang, Nori, Franco: Fisher information under decoherence in Bloch representation. Phys. Rev. A 87, 022337 (2013)
Zhou, You, Zeng, Pei, Liu, Zhenhuan: Single-copies estimation of entanglement negativity. Phys. Rev. Lett. 125, 200502 (2020)

Publisher’s Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.