Towards UV Finiteness of Infinite Derivative Theories of Gravity and Field Theories

Spyridon Talaganis

Consortium for Fundamental Physics,
Lancaster University, Lancaster,
LA1 4YB, United Kingdom.
E-mail: s.talaganis@lancaster.ac.uk

Abstract

In this paper we will consider the ultraviolet (UV) finiteness of the most general one-particle irreducible (1PI) Feynman diagrams within the context of ghost-free, infinite-derivative scalar toy model, which is inspired from ghost free and singularity-free infinite-derivative theory of gravity. We will show that by using dressed vertices and dressed propagators, $n$-loop, $N$-point diagrams constructed out of lower-loop 2- & 3-point and, in general, $N_{i}$-point diagrams are UV finite with respect to internal and external loop momentum. Moreover, we will demonstrate that the external momentum dependences of the $n$-loop, $N$-point diagrams constructed out of lower-loop 2- & 3-point and, in general, $N_{i}$-point diagrams decrease exponentially as the loop-order increases and the external momentum divergences are eliminated at sufficiently high loop-order.

Contents

1 Introduction

2 Feynman rules for infinite derivative scalar toy model

3 1-loop, 2-point function with non-vanishing external momenta

4 UV finiteness of $n$-loop, $N$-point diagrams

4.1 $n$-loop, $N$-point diagrams constructed out of lower-loop, 2- & 3-point diagrams
4.2 $n$-loop, $N$-point diagrams constructed out of lower-loop, $N_i$-point diagrams ........................................ 13

5 Conclusions .......................................................... 15

6 Acknowledgments ....................................................... 16

A $c_N$ coefficients .................................................... 16

1 Introduction

Formulating a completely successful theory of quantum gravity [1, 2, 3] remains a goal to be achieved in theoretical physics. Renormalizability plays a very important role in establishing a consistent theory of quantum gravity. In four-dimensions, pure gravity is ultraviolet (UV) finite at 1-loop order [4]. That is, at 1-loop order, one-loop counterterms vanish on mass-shell. Now, at 2-loop order, pure gravity has a UV divergence [4, 5, 6]. Since infinitely many local counterterms would be required to eliminate the divergences, pure gravity is said to be a non-renormalizable theory. By virtue of being non-renormalisable, pure gravity, as given by the Einstein-Hilbert action, is not a quantum theory of gravity, but, rather, an effective field theory, valid at scales much less than $M_P \sim 2.4 \times 10^{18}$ GeV.

Among many attempts to quantise gravity, i.e., string theory [7], loop quantum gravity [8, 9], causal set [10], one can see that nonlocality is present in many of these theories of quantum gravity; for example, strings and branes are nonlocal by nature [11]. In string field theory [12, 13], nonlocality also plays an important role ($p$-adic strings [14], zeta strings [15] and strings quantized on a random lattice [16]). This merits the question whether nonlocality is a fundamental feature of spacetime. As a result, the investigation of nonlocality in nature seems to be a worthy pursuit. Moreover, many of the attempts to quantise gravity were afflicted by classical singularities, ghosts or divergences in the UV. For example, see Refs. [17, 18], a fourth-order higher-derivative theory of gravity was formulated that was renormalizable by power counting. However, the theory suffered from a massive spin-2 ghost, therefore lacking prediction both at a classical and at a quantum level. In Ref. [19], a ghost-free tensor Lagrangian was presented and relevant applications in gravity were discussed. Nevertheless, any higher derivative extension of gravity would inevitably suffer from either ghost or classical instability due to the Ostrogradsky theorem [20], which cannot be cured order by order in curvature corrections.

In fact, it has recently been shown how the ghost problem can be avoided in the context of infinite-derivative theories of gravity [21, 22, 23, 24] (see Refs. [25, 26] for discussion of unitarity in nonlocal theories). Infinite derivatives indeed modify
the action, and the dynamical propagating degrees of freedom. However, with a judicious choice, it is possible to make sure that gravity in any dimensions retains the transverse and traceless degrees of freedom, i.e., spin-2 and spin-0 components. Similar conclusion was reached by analysing the Hamiltonian density and considering primary/secondary constraints \[27\]. Inspired by these attempts, one would like to formulate a ghost-free, nonlocal, infinite-derivative gravitational action and construct a renormalisable theory of gravity; see Ref. \[28\] for earlier work in that direction. Since the interactions in such class of theories are all derivatives in nature, the interactions due to infinite covariant derivatives give rise to nonlocal interactions \[1\].

Within the context of infinite-derivative field theories and gravity, one should start by explaining first what is meant by ultraviolet (UV) finiteness. When a Feynman diagram is finite in the UV, it means that the corresponding Feynman integral is convergent in the UV, i.e., at very high energies (or short distances). That is, there are no UV divergences, with respect to the internal loop momentum variable \( k^\mu \). As far as renormalisability is concerned, this implies that no counterterms are required to cancel possible UV divergences. One should keep in mind that, if all UV divergences with respect to internal loop momenta, which arise in one-particle irreducible (1PI) diagrams, can be removed by adding to the action finitely many counterterms, renormalisability is automatically ensured.

Nevertheless, one could also study UV finiteness with respect to external momenta. For instance, when computing scattering diagrams, if the scattering diagram exhibits no external momentum growth, it is convergent in the UV, that is, for large external momenta. In that case, the corresponding cross section of the scattering diagram is finite and does not blow up when the external momenta become very large.

A covariant gravitational theory free from ghosts and tachyons around constant curvature backgrounds were derived in Refs. \[21, 22\]. The form of the action \( S \) is given by

\[
S = S_{EH} + S_Q, \quad (1.1)
\]

\[
S_{EH} = \frac{1}{2} \int d^4x \sqrt{-g} M_\nu^2 \mathcal{R}, \quad (1.2)
\]

\[
S_Q = \frac{1}{2} \int d^4x \sqrt{-g} \left( R \mathcal{F}_1(\Box) R + R_{\mu\nu} \mathcal{F}_2(\Box) R^{\mu\nu} + R_{\mu\nu\lambda\sigma} \mathcal{F}_3(\Box) R^{\mu\nu\lambda\sigma} \right), \quad (1.3)
\]

where \( \Box \equiv \Box/M^2 \) and \( M \) is the mass scale at which the nonlocal modifications become important. The \( \mathcal{F}_i \)’s are infinite-derivative functions of \( \Box \) and follow a specific constraint \( 2\mathcal{F}_1(\Box) + \mathcal{F}_2(\Box) + 2\mathcal{F}_3(\Box) = 0 \) around a Minkowski background so that the action is ghost-free and corresponds to a massless graviton \[21, 22\]. In particular, in the action considered by Biswas, Gerwick, Koivisto and Mazumdar (BGKM), the graviton propagator is modulated by the exponential of an entire function \( a(-k^2) =

\footnote{The free theory has no nonlocality.}

\footnote{We work in four-dimensional spacetime, \( \mu = 0, 1, 2, 3 \), and we use the (−, +, +, +) metric signature.}
\( e^{k^2/M^2} \), see [29] \(^3\)

\[
\Pi(-k^2) = \frac{1}{k^2 a(-k^2)} \left( P^2 - \frac{1}{2} P^0 \right) = \frac{1}{a(-k^2)} \Pi_{GR}.
\]

(1.4)

Note that the exponential of an entire function does not give rise to poles. For an exponential entire function, the propagator becomes exponentially suppressed in the UV (see also applications regarding Regge behaviour [32] and Hagedorn transition [33]), while the vertex factors are exponentially enhanced. The fact that the propagators and vertex factors have opposing momentum dependence is a key feature of gauge theories. Therefore, the UV divergences of Feynman diagrams can be eliminated up to 2-loop order [31] and the theory is renormalisable. Higher loops can also be made finite by the use of dressed vertices and dressed propagators. Meanwhile, in the IR, we recover the physical graviton propagator of GR. In addition, this asymptotically free theory addresses the classical singularities present in GR [21, 35]. This is in clear contrast with GR and other finite-order higher-derivative theories of gravity.

The gravitational entropy of the BGKM action for \( D \)-dimensional (A)dS backgrounds and the graviton propagator for the BGKM action around \( D \)-dimensional Minkowski space were evaluated in Refs. [36, 37]. In Ref. [38], the generalised boundary term, \( \text{i.e.} \), the corresponding Gibbons-Hawking-York term, for the BGKM action was derived. Also, in Ref. [27], the Hamiltonian for an infinite-derivative extension of gravity was presented and the number of degrees of freedom in various cases was computed.

Inspired by this infinite-derivative gravitational action, see Eq. (1.1), we have formulated a scalar toy model in Ref. [34] that captures the essential features of the UV behaviour of the infinite-derivative gravitational action. The scalar toy model action was given by

\[
S_{\text{scalar}} = S_{\text{free}} + S_{\text{int}},
\]

(1.5)

where

\[
S_{\text{free}} = \frac{1}{2} \int d^4x \left( \phi \Box a(\Box) \phi \right)
\]

(1.6)

and

\[
S_{\text{int}} = \frac{1}{M_P} \int d^4x \left( \frac{1}{4} \phi \partial_\mu \phi \partial^\mu \phi + \frac{1}{4} \phi \Box a(\Box) \phi - \frac{1}{4} \phi \partial_\mu \phi a(\Box) \partial^\mu \phi \right);
\]

(1.7)

we have \( a(\Box) = e^{-\Box} \equiv e^{-\Box/M^2} \). The equation of motion for the action given by Eq. (1.5) satisfies the shift-scaling symmetry \( \phi \to (1+\epsilon)\phi + \epsilon \), where \( \epsilon \) is infinitesimal.

We should point out that in Ref. [31] we showed that the infinite-derivative scalar toy model given by (1.5) is renormalisable at 1-loop order. That is, we computed the counterterm that cancels the UV divergences which originate from integrating

\(^3\)Note that a similar action has been proposed in Ref. [30], where it was shown that \( a(-k^2) \) being an entire function is sufficient condition for the renormalisability of infinite derivative gravity. Later on similar conclusions were made in Ref. [31].
the loop momentum variable $k^\mu$ at 1-loop, \textit{i.e.}, for the 1-loop, 2-point function. In particular, in Ref. \cite{34}, we have shown that

- the Feynman rules for propagators and vertices for our infinite-derivative scalar toy model, that scattering amplitudes for the scalar toy model are superficially convergent for $L > 1$, where $L$ is the number of loops, that the highest divergence of the 1-loop, 2-point function with nonvanishing external momenta $p \& -p$ is $\Lambda^4$, where $\Lambda$ is a hard cutoff, and that the highest divergence of the 2-loop, 2-point function with vanishing external momenta is also $\Lambda^4$;

- the dressed propagator is more exponentially suppressed than the bare propagator and that dressed vertices behave as exponentials of external momenta when the external momenta are large; by employing dressed propagators and dressed vertices, $n$-loop, 2- & 3-point diagrams constructed out of lower-loop, 2- & 3-point diagrams become finite in the UV (UV) with respect to internal loop momentum $k^\mu$, that is, no UV divergences arise and no new counterterm is required.

In Ref. \cite{39} the UV behaviour of scattering diagrams within the context of an infinite-derivative scalar toy model was investigated and it was found that the external momentum dependence of the scattering diagrams is convergent for large external momenta. That was achieved by dressing the bare vertices of the scattering diagrams by considering renormalised propagator and vertex loop corrections to the bare vertices. As the loop order increases, the exponents in the dressed vertices decrease and eventually become negative at sufficiently high loop-order.

Motivated by the results in Refs. \cite{34,39}, we would like to study UV finiteness with respect to both \textit{internal loop momenta} and \textit{external momenta} for a general class of Feynman diagrams within the context of infinite-derivative field theories. Thus, we shall generalise the results presented in Ref. \cite{34} and show that by employing dressed propagators and dressed vertices, $n$-loop, $N$-point diagrams constructed out of lower-loop 2- & 3-point and, in general, $N_i$-point diagrams are finite in the UV (with respect to internal loop momentum $k^\mu$) while the exponential momentum dependences in those diagrams decrease at higher loops and external momentum divergences are eliminated at sufficiently high loop-order. It should be pointed out that $n$-loop, $N$-point diagrams constructed out of lower-loop $N_i$-point diagrams are the most general one-particle irreducible (1PI) Feynman diagrams. In particular, we present the following results:

- the 1-loop, 2-point function with external momenta $p \& -p$, where the bare propagators have been replaced with dressed propagators, is finite in the UV with respect to internal loop momentum, that is, the corresponding Feynman integrals are convergent;

- by employing dressed vertices and dressed propagators, $n$-loop, $N$-point diagrams constructed out of lower-loop 2- & 3-point and, in general, $N_i$-point
diagrams are UV finite with respect to internal loop momentum,

- the external momentum dependences of $n$-loop, $N$-point diagrams constructed out of lower-loop 2- & 3-point and, in general, $N_i$-point diagrams decrease as the loop-order increases and the external momentum divergences are eliminated at sufficiently high loop-order.

The outline of this paper is as follows. In section 2, we write down the Feynman rules, i.e., propagators and vertex factors, for the infinite-derivative scalar toy model. In section 3, we show that the 1-loop, 2-point function with external momenta $p$ & $-p$, where the bare propagators have been replaced with dressed propagators, is UV finite with respect to internal loop momentum $k^\mu$. In section 4, we show the UV finiteness with respect to internal loop momentum of $n$-loop, $N$-point diagrams constructed out of lower-loop 2- & 3-point and, in general, $N_i$-point diagrams. Moreover, we demonstrate that the external momentum dependences of $n$-loop, $N$-point diagrams constructed out of lower-loop 2- & 3-point and, in general, $N_i$-point diagrams decrease as the loop-order increases and, at sufficiently high loop-order, the external momentum divergences are eliminated. Finally, in section 5 we conclude by summarising our results. In appendix A we present some technical details regarding the $c_N$ coefficients.

## 2 Feynman rules for infinite derivative scalar toy model

All the Feynman rules and Feynman integral computations in this paper are carried out in Euclidean space after analytic continuation ($k_0 \rightarrow ik_0$ & $k^2 \rightarrow k^2_E$ using the mostly plus metric signature; we shall drop the $E$ subscript for notational simplicity).

The Feynman rules for our action, which is given by Eq. (1.5), can be derived rather straightforwardly. The propagator in momentum space is then given by

$$\Pi(k^2) = \frac{-i}{k^2 e^{k^2}}, \quad (2.1)$$

where barred 4-momentum vectors from now on will denote the momentum divided by the mass scale $M$. The vertex factor for three incoming momenta $k_1$, $k_2$, $k_3$ satisfying the conservation law:

$$k_1 + k_2 + k_3 = 0, \quad (2.2)$$

is given by

$$\frac{1}{M_p} V(k_1, k_2, k_3) = \frac{i}{M_p} C(k_1, k_2, k_3) \left[ 1 - e^{k_1} - e^{k_2} - e^{k_3} \right], \quad (2.3)$$
Figure 1: The dressed propagator as the sum of an infinite geometric series. The dressed propagator is denoted by the shaded blob.

\[ C(k_1, k_2, k_3) = \frac{1}{4} \left( k_1^2 + k_2^2 + k_3^2 \right). \]  (2.4)

Let us briefly explain how we obtain the vertex factor. The first term originates from the term, \( \frac{1}{4} \phi \partial_\mu \phi \partial^\mu \phi \), which using Eq. (2.2) in the momentum space, reads

\[ -\frac{i}{2} (k_1 \cdot k_2 + k_2 \cdot k_3 + k_3 \cdot k_1) = \frac{i}{4} \left( k_1^2 + k_2^2 + k_3^2 \right). \]  (2.5)

The second term comes from the terms, \( \frac{1}{4} \phi \Box \phi a(\Box) \phi \) and \( -\frac{1}{4} \phi \partial_\mu \phi a(\Box) \partial^\mu \phi \). In the momentum space, again using Eq. (2.2), we get

\[ \frac{i}{4} (k_3 \cdot k_1 + k_1 \cdot k_2 - k_3^2 - k_1^2) e^{\vec{k_1}} = -\frac{i}{4} (k_1^2 + k_2^2 + k_3^2) e^{\vec{k_1}}. \]  (2.6)

The third and the fourth terms in Eq. (2.3) arise in an identical fashion.

\section{1-loop, 2-point function with non-vanishing external momenta}

From [34] we know that the renormalised 1-loop, 2-point function with external momenta \( p \) and \(-p\), \( \Gamma_{2,1r}(p^2) \), is a regular analytic function of \( p^2 \) which grows as \( e^{\frac{3p^2}{2}} \) as \( p^2 \to \infty \). Within the context of dimensional regularization, the counterterm \( \Gamma_{2,1ct} \) has a simple pole in \( \epsilon \), where \( \epsilon = 4 - d \) and \( d \) is the dimensionality of spacetime. That is,

\[ \Gamma_{2,1r}(p^2) = \Gamma_{2,1}(p^2) + \Gamma_{2,1ct}(p^2) = \frac{iM^4}{M_P^2} f(p^2) \]  (3.1)

and, as \( p^2 \to \infty \),

\[ f(p^2) \sim e^{\frac{3p^2}{2}}. \]  (3.2)
The dressed propagator then represents the geometric series of all the graphs with 1-loop, 2-point insertions as shown in Fig. 1, analytically continued to the entire complex $p^2$-plane. Mathematically, this is equivalent to replacing the bare propagator, $\Pi(p^2)$, with the dressed propagator, $\widetilde{\Pi}(p^2)$:

$$\widetilde{\Pi}(p^2) = \frac{\Pi(p^2)}{1 - \Pi(p^2)\Gamma_{2,1r}(p^2)} = \frac{1}{i \left( p^2 e^{i\beta^2} - \frac{M^4}{M^2 f(p^2)} \right)} .$$  \hspace{1cm} (3.3)

Since in our case $\Pi(p^2)\Gamma_{2,1r}(p^2)$ grows with large momenta, in the UV limit we have

$$\widetilde{\Pi}(p^2) \to \Gamma_{2,1r}^{-1}(p^2) \approx (9 - 12p^{-2})^{-1} e^{-\frac{3p^2}{2}} .$$  \hspace{1cm} (3.4)

We observe that the dressed propagator is more exponentially suppressed than the bare propagator.

Now if we replace the bare propagators with dressed propagators in the 1-loop, 2-point function with external momenta $p \& -p$ while the vertices stay bare, the Feynman integral, see Fig. 2 is given by

$$\Gamma_{2,1\text{dressed}}(p^2) = \frac{1}{2i M_p^2} \int \frac{d^4k}{(2\pi)^4} \frac{V^2(-p, \frac{p}{2} + k, \frac{p}{2} - k)}{\left( \frac{p}{2} + k \right)^2 e^{(\frac{p}{2} + k)^2} - \frac{M^4}{M_p^2} f \left( \left( \frac{p}{2} + k \right)^2 \right)}$$

$$\times \frac{1}{\left( \frac{p}{2} - k \right)^2 e^{(\frac{p}{2} - k)^2} - \frac{M^4}{M_p^2} f \left( \left( \frac{p}{2} - k \right)^2 \right)} .$$  \hspace{1cm} (3.5)

As $|k| \to \infty$, the integrand goes as

$$\sim \exp(-k^2) .$$  \hspace{1cm} (3.6)

Therefore, the integral is convergent since there are no internal loop momentum divergences. On the other hand, we observe that $\Gamma_{2,1\text{dressed}}$ goes, in terms of external momentum $p$, as

$$\sim e^{\frac{3p^2}{4}} .$$  \hspace{1cm} (3.7)

for large $p^2$. We observe that the exponential momentum dependence of $\Gamma_{2,1\text{dressed}}$ is less divergent than the exponential momentum dependence of the renormalised 1-loop, 2-point function $\Gamma_{2,1r}(p^2)$. We would like to make our theory not only renormalisable (apart from the 1-loop, 2-point function with bare propagators, all other higher-loop, higher-point diagrams can be made UV finite with respect to internal loop momentum), but also UV finite with respect to external momenta (at sufficiently high loop-order).

Therefore, in the next section we shall dress both vertices and propagators so as to make $n$-loop, $N$-point diagrams UV finite, both with respect to internal loop momentum and external momenta.
4 UV finiteness of $n$-loop, $N$-point diagrams

We would like to investigate the UV finiteness of one-particle irreducible (1PI) Feynman diagrams within the framework of our infinite-derivative scalar toy model. We shall look into UV finiteness both in terms of internal loop momenta and external momenta. In the next section, we shall look into the UV finiteness of $n$-loop, $N$-point diagrams constructed out of lower-loop 2- & 3-point and, in general, $N_i$-point diagrams; it should be pointed out that $n$-loop, $N$-point diagrams constructed out of lower-loop $N_i$-point diagrams are the most general one-particle irreducible (1PI) diagrams.
4.1 $n$-loop, $N$-point diagrams constructed out of lower-loop, 2- & 3-point diagrams

Let us look at $N$-point diagrams constructed out of lower-loop 2- & 3-point diagrams. We know the following:

- the dressed propagators represented by the shaded blobs decay in the UV as $e^{-3k^2/2}$ (see Eq. (3.4) in section 3).
- the 3-point function represented by the dark blobs (see Figs. 3 & 4) can be written (after first integrating out the internal loop momentum $k^\mu$ in the 1-loop triangle) as

$$\Gamma_3 \xrightarrow{UV} \sum_{\alpha,\beta,\gamma} e^{\alpha p_1^2 + \beta p_2^2 + \gamma p_3^2}, \quad (4.1)$$

with the convention (since we assume $p_1 + p_2 + p_3 = 0$, terms such as $p_i \cdot p_j$, where $i, j = 1, 2, 3$, can be written as a sum of $p_l^2$, $l = 1, 2, 3$, terms)

$$\alpha \geq \beta \geq \gamma, \quad (4.2)$$

where $p_1, p_2, p_3$ are the three external momenta.

Now one can generalise Eq. (4.1) and write the $N$-point function in the following form (again after first integrating out the internal loop momentum $k^\mu$ in the 1-loop $N$-polygon, see also appendix A):

$$\Gamma_N \xrightarrow{UV} \sum_{\alpha_l} e^{\sum_{l=1}^N \alpha_l p_l^2}, \quad (4.3)$$

with the convention

$$\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_N, \quad (4.4)$$

where $p_1, p_2, \ldots, p_N$ are the $N$ external momenta.

Now let us look at the case where the external momenta are arbitrary; we wish to find out how one can get the largest exponents for the external momenta. First, let us consider how one can get the largest sum of all the exponents, i.e., $\sum_{l=1}^N \alpha_l$. Even though all the arguments below can be conducted for $N$ different sets of exponents in the $N$ 3-point vertices, see Fig. 3 making up the 1-loop $N$-polygon, see Fig. 4 for simplicity, here we will look at what happens when all the $N$ vertices have the same exponents. The best way to obtain the largest exponents for the external momenta is to have the $\alpha$ exponent correspond to the external momenta (we assume a symmetric distribution of $(\beta, \gamma)$ among the internal loops and symmetrical routing of momenta in the 1-loop $N$-polygon). We have that $p_1, p_2, \ldots, p_N$ are the external momenta for the 1-loop triangle, and the superscript in the $\alpha^{n-1}, \beta^{n-1}, \gamma^{n-1}$ indicates that these
are coefficients that one obtains from contributions up to \( n-1 \) loop level. The internal momenta of the \( N \)-point diagram are given by

\[
q_{N-1} = \frac{1}{N} \left[ \sum_{l=1}^{N-2} (lp_l) - p_{N-1} \right], \\
q_N = \frac{1}{N} \left[ \sum_{l=1}^{N-2} (lp_{l+1}) - p_N \right], \\
\vdots \\
q_{N-3} = \frac{1}{N} \left[ p_{N-1} + 2p_N + \sum_{l=1}^{N-4} ((l+2)p_l) - p_{N-3} \right], \\
q_{N-2} = \frac{1}{N} \left[ p_N + \sum_{l=1}^{N-3} ((l+1)p_l) - p_{N-2} \right].
\tag{4.5}
\]

That is,

- the dressed propagators are given by \( e^{-\frac{3}{2}(\vec{k} + \vec{q}_l)^2} \), \( l = 1, \ldots, N \),
- and the vertex factors are of the form \( e^{\alpha_{n-1}p_l^2 + \beta_{n-1}^2(k + \vec{q}_l)^2 + \gamma_{n-1}^2(k + \vec{q}_{l+1})^2} \).

Hence, conservation of momenta then yields

\[
\Gamma_{N,n} \longrightarrow \int \frac{d^4k}{(2\pi)^4} \frac{e^{n-1(p_1^2 + p_2^2 + \cdots + p_N^2)}}{\left[ Nk^2 + c_N(p_1^2 + p_2^2 + \cdots + p_N^2) \right]},
\tag{4.6}
\]

where \( c_N \) is a coefficient depending on \( N \), the number of external lines, that satisfies \( c_N > 0 \) for all \( N \) (see appendix A).

- **Internal momentum**: we observe that the integrand in (4.6) is of the form \( e^{-sk^2} \), where \( s > 0 \) (in Ref [34], it was shown that \( \beta_{n-1} - \gamma_{n-1} < \frac{3}{2} \) for all \( n \)). Hence, the integral in (4.6) is convergent and the diagram is UV finite with respect to internal loop momentum.

- **External momenta**: by integrating Eq. (4.6), we have

\[
\alpha_1^n = \alpha_2^n = \cdots = \alpha_N^n = \alpha_{n-1} + c_N(\beta_{n-1}^2 + \gamma_{n-1}^2 - \frac{3c_N}{2}).
\tag{4.7}
\]

In particular, for the 1-loop, 3-point graph, one has to use the 3-point bare vertices: \( \alpha^0 = 1 \) and \( \beta^0 = \gamma^0 = 0 \). In Ref. [39] we showed that the coefficients \( \alpha_{n-1}^1, \beta_{n-1}^1, \gamma_{n-1}^1 \), that is, the exponents in the dressed vertices, decrease as the loop-order increases and, at sufficiently high loop-order, become negative. In
particular, we showed in Ref. [39] that, for \( n = 1 \) (\( n \) is the loop-order of the 3-point dressed vertices),

\[
\alpha^1 = \beta^1 = \gamma^1 = \frac{1}{2},
\]

(4.8)

for \( n = 2 \),

\[
\alpha^2 = \beta^2 = \gamma^2 = \frac{1}{3},
\]

(4.9)

for \( n = 3 \),

\[
\alpha^3 = \beta^3 = \gamma^3 = \frac{1}{18},
\]

(4.10)

for \( n = 4 \),

\[
\alpha^4 = \beta^4 = \gamma^4 = -\frac{11}{27}.
\]

(4.11)

We conclude that, for \( n \geq 4 \), \( \alpha^n \), \( \beta^n \) and \( \gamma^n \) become negative.

Hence, from Eq. (4.7) we see that the coefficients \( \alpha_1^n \), \( \alpha_2^n \), \ldots, \( \alpha_N^n \) also decrease as the loop-order increases and, at sufficiently high loop-order, become negative. Thus, the external momentum dependences of \( n \)-loop, \( N \)-point diagrams constructed out of lower-loop 2- & 3-point diagrams decrease as the loop-order increases and the external momentum divergences are eliminated at sufficiently high loop-order, that is, the exponents in Eq. (4.3) corresponding to the external momenta become negative when the loop order is sufficiently large.

Consequently, this class of diagrams are finite in the UV, both with respect to internal loop momentum and at sufficiently high loop-order external momenta as well. One could also consider the case where the loop-order of the dressed vertices is not the same for all of the dressed vertices, that is, each dressed vertex is of different loop-order. Again the results would be the same as far as UV finiteness with respect to internal loop momentum and external momenta is concerned.

As a check, when the external momenta tend to zero, it is easy to see that the most divergent UV part of the \( N \)-point diagram reads

\[
\Gamma_{N,n} \rightarrow \int \frac{d^4k}{(2\pi)^4} \frac{e^{(\alpha_1 + \ldots + \alpha_N + \beta_1 + \ldots + \beta_N)k^2}}{e^{\frac{3Nk^2}{2}}},
\]

(4.12)

where \( k \) is the loop momentum variable in Fig. 4. There are \( N \) propagators \( e^{\frac{3k^2}{2}} \) while the (most divergent UV parts of the) vertex factors originating from lower-loop diagrams are \( e^{\alpha_1 k^2 + \beta_1 k^2}, \ldots, e^{\alpha_N k^2 + \beta_N k^2} \) (we get no \( \gamma_1, \gamma_2 \) terms in the exponents, since the external momenta are set equal to zero). Clearly, the integral is finite as long as

\[
\alpha_i + \beta_i < \frac{3}{2},
\]

(4.13)

where \( i = 1, 2, \ldots, N \).
4.2 $n$-loop, $N$-point diagrams constructed out of lower-loop, $N_i$-point diagrams

Now let us look at $n$-loop, $N$-point diagrams constructed out of lower-loop $N_i$-point diagrams. Any $n$-loop diagram can be thought of as a 1-loop integral over a graph containing renormalised vertex corrections and dressed propagators, see Fig. 5. At the $i$-th dressed vertex, we have $N_i - 2$ external lines (excluding the two internal propagators in the $n$-loop, $N$-point diagram that are attached to each dressed vertex).

For simplicity, we take all the vertices to have the same exponents. We have $m$ dressed vertices with $N_i$ external lines attached to each dressed vertex; thus,

$$N = \sum_{i=1}^{m} (N_i - 2).$$

Regarding the external momenta, we use the following notation:

$$p'_i = \sum_{l=1}^{N_i-2} p_{il},$$

where $i = 1, \ldots, m$ and $p_{il}$ are the external momenta to the $i$-th dressed vertex. For each dressed vertex, the $N_i$-point function can be written as follows:

$$\Gamma_{N_i} \xrightarrow{UV} \sum_{\alpha_i, \beta, \gamma} e^{\sum_{l=1}^{N_i-2} \alpha_i \bar{p}_{il}^2 + \beta \bar{q}_{1l}^2 + \gamma \bar{q}_{2l}^2},$$

where $q_1$ and $q_2$ are internal propagators in the $N$-point diagram, with the convention

$$\alpha_i \geq \alpha_2 \geq \cdots \geq \alpha_{N_i-2} \geq \beta_i \geq \gamma_i.$$
The internal momenta of the $N$-point diagram are given by

$$q'_{m-1} = \frac{1}{m} \left[ \sum_{l=1}^{m-2} (lp'_l) - p'_{m-1} \right],$$

$$q'_m = \frac{1}{m} \left[ \sum_{l=1}^{m-2} (lp_{l+1}) - p'_m \right],$$

$$\vdots$$

$$q'_{m-3} = \frac{1}{m} \left[ p'_{m-1} + 2p'_m + \sum_{l=1}^{m-4} \left( (l+2)p'_l \right) - p'_{m-3} \right],$$

$$q'_{m-2} = \frac{1}{m} \left[ p'_m + \sum_{l=1}^{m-3} \left( (l+1)p'_l \right) - p'_{m-2} \right].$$

That is,

- the dressed propagators are given by $e^{-\frac{3}{2}(k+q'_i)^2}$, $i = 1, \ldots, m,$
- and the vertex factors are of the form $e^{\sum_{i=1}^{N-2} \alpha_{n-1}^{-1}p_i^2 + \beta_{n-1}^{-1}(k+q'_i)^2 + \gamma_{n-1}^{-1}(k+q'_{i+1})^2}$.

Hence, conservation of momenta then yields (after shifting the loop momentum variable $k$)

$$\Gamma_{N,m} \rightarrow \int \frac{d^4k}{(2\pi)^4} \text{e}^{\sum_{i=1}^{m} \sum_{i=1}^{N-2} \alpha_{n-1}^{-1}k_i^2} \text{e}^{\sum_{i=1}^{m} \sum_{i=1}^{N-2} [\frac{1}{2}b_{i} - \beta_{n-1}^{-1}c_{i} - \gamma_{n-1}^{-1}d_{i}]} \text{e}^{\sum_{i=1}^{m} (\beta_{i}^{-1} + \gamma_{i}^{-1}) |k_{i}|^2},$$

where $b_{i}, c_{i} \& d_{i}$ are coefficients depending on $i \& l$ that satisfy $b_{i}, c_{i}, d_{i} > 0$ for all values of $i$ and $l$.

- **Internal momentum:** we observe that the integrand in (4.19) is of the form $e^{-sk^2}$, where $s > 0$ (that is, $\sum_{i=1}^{m} (\beta_{i}^{-1} + \gamma_{i}^{-1}) < \frac{3m^2}{2}$). Hence, the integral in (4.19) is convergent and the diagram is UV finite with respect to internal loop momentum.

- **External momenta:** by integrating Eq. (4.6), we have

$$\alpha_1^n = \alpha_2^n = \cdots = \alpha_N^n = \alpha_{n-1}^{-1} + c_{i}\beta_{i}^{-1} + d_{i}\gamma_{i}^{-1} - \frac{3b_{i}}{2}.$$  

From section 4.1, one can see that the coefficients $\alpha_{n-1}^{-1} = \beta_{i}^{-1} = \gamma_{i}^{-1}$, that is, the exponents in the dressed vertices, decrease as the loop-order increases and at sufficiently high loop-order become negative. Hence, from Eq. (4.20) we see that the coefficients $\alpha_1^n, \alpha_2^n, \ldots, \alpha_N^n$ also decrease as the loop-order increases and at sufficiently high loop-order become negative. Thus, the external momentum
dependences of \( n \)-loop, \( N \)-point diagrams constructed out of lower-loop \( N_i \)-point diagrams decrease as the loop-order increases and the external momentum divergences are eliminated at sufficiently high loop-order, that is, the exponents in Eq. (4.16) corresponding to the external momenta become negative when the loop order is sufficiently large.

Now, if each dressed vertex were of different loop-order, we would still obtain the same results regarding UV finiteness with respect to internal loop momentum and external momenta. Hence, \( n \)-loop, \( N \)-point diagrams constructed out of lower-loop \( N_i \)-point diagrams are finite in the UV, both with respect to internal loop momentum and at sufficiently high loop-order external momenta as well.

5 Conclusions

The aim of this paper has been to show that \( n \)-loop, \( N \)-point diagrams constructed out of lower-loop 2- & 3-point and, in general, \( N_i \)-point diagrams are UV finite with respect to internal loop momentum, that is, when computing the Feynman integrals for those diagrams, no UV divergences arise and no new counterterm is required. At the beginning we presented our infinite-derivative scalar toy model, which was inspired from an infinite-derivative theory of gravity, BGKM gravity, and wrote down the Feynman rules, that is, the propagator and the vertex factors. Next we summarised the results for the 1-loop, 2-point function with non-vanishing external momenta; when we replaced the bare propagators with dressed propagators in the 1-loop, 2-point function, we saw that the corresponding Feynman integrals were convergent.

Then we showed that by employing dressed vertices and dressed propagators, \( n \)-loop, \( N \)-point diagrams constructed out of lower-loop 2- & 3-point and, in general, \( N_i \)-point diagrams were UV finite. This is the major result in our paper: that the most general one-particle irreducible (1PI) Feynman diagrams within the framework of infinite-derivative field theories are finite in the UV. Consequently, no UV divergences arise and no new counterterm is required.

Furthermore, we showed that the external momentum dependences of \( n \)-loop, \( N \)-point diagrams constructed out of lower-loop 2- & 3-point and, in general, \( N_i \)-point diagrams decrease as the loop-order increases and the external momentum divergences are eliminated at sufficiently high loop-order.

Finally we would like to to further generalise the results obtained in this paper and apply them to an infinite-derivative theory of gravity, \textit{i.e.}, to BGKM gravity. Establishing that an infinite-derivative theory of gravity is renormalisable or even finite at higher loop-order would be a big achievement by itself and a major contribution to establishing a theory of quantum gravity that is successful on all fronts.
6 Acknowledgments

ST is supported by a scholarship from the Onassis Foundation.

A \( c_N \) coefficients

The \( c_N \) coefficients are always positive. For instance, when \( N = 4 \), i.e., for an \( n \)-loop, four-point diagram constructed out of lower-loop, three-point diagrams, we have, from the internal propagators and dressed vertices comprising the \( n \)-loop, four-point diagram, that (assuming symmetrical routing of momenta in the 1-loop square)

\[
\left( \tilde{k} + \frac{\tilde{p}_1}{4} + \frac{2\tilde{p}_2}{4} - \frac{\tilde{p}_3}{4} \right)^2 + \left( \tilde{k} + \frac{\tilde{p}_2}{4} + \frac{2\tilde{p}_3}{4} - \frac{\tilde{p}_4}{4} \right)^2 + \left( \tilde{k} + \frac{\tilde{p}_3}{4} + \frac{2\tilde{p}_4}{4} - \frac{\tilde{p}_1}{4} \right)^2 
\]

\[
+ \left( \tilde{k} + \frac{\tilde{p}_4}{4} + \frac{2\tilde{p}_1}{4} - \frac{\tilde{p}_2}{4} \right)^2 
\]

\[
= 4\tilde{k}^2 + \frac{3}{8} \left( \tilde{p}_1^2 + \tilde{p}_2^2 + \tilde{p}_3^2 + \tilde{p}_4^2 \right) - \frac{1}{4} (p_1 \cdot p_3 + p_2 \cdot p_4) .
\] (A.1)

Now even if \( p_1 = p_3 = p \) and \( p_2 = p_4 = -p \), Eq. (A.1) is equal to

\[
4\tilde{k}^2 + \frac{1}{4} \left( \tilde{p}_1^2 + \tilde{p}_2^2 + \tilde{p}_3^2 + \tilde{p}_4^2 \right) .
\] (A.2)

We see that the coefficient \( \frac{1}{4} \) is greater than zero. Now, when \( N = 5 \), i.e., for an \( n \)-loop, five-point diagram constructed out of lower-loop three-point diagrams, we have (again assuming symmetrical routing of momenta in the 1-loop pentagon)

\[
\left( \tilde{k} + \frac{\tilde{p}_1}{5} + \frac{2\tilde{p}_2}{5} + \frac{3\tilde{p}_3}{5} - \frac{\tilde{p}_4}{5} \right)^2 + \left( \tilde{k} + \frac{\tilde{p}_2}{5} + \frac{2\tilde{p}_3}{5} + \frac{3\tilde{p}_4}{5} - \frac{\tilde{p}_5}{5} \right)^2 + \left( \tilde{k} + \frac{\tilde{p}_3}{5} + \frac{2\tilde{p}_4}{5} + \frac{3\tilde{p}_5}{5} - \frac{\tilde{p}_1}{5} \right)^2 
\]

\[
+ \left( \tilde{k} + \frac{\tilde{p}_4}{5} + \frac{2\tilde{p}_1}{5} + \frac{3\tilde{p}_2}{5} - \frac{\tilde{p}_3}{5} \right)^2 + \left( \tilde{k} + \frac{\tilde{p}_5}{5} + \frac{2\tilde{p}_1}{5} + \frac{3\tilde{p}_2}{5} - \frac{\tilde{p}_3}{5} \right)^2 
\]

\[
= 5\tilde{k}^2 + \frac{3}{5} \left( \tilde{p}_1^2 + \tilde{p}_2^2 + \tilde{p}_3^2 + \tilde{p}_4^2 + \tilde{p}_5^2 \right) + \frac{2}{5} (p_1 \cdot p_2 + p_2 \cdot p_3 + p_3 \cdot p_4 + p_4 \cdot p_5 + p_5 \cdot p_1) .
\] (A.3)

Even when \( p_1 = 2p, p_2 = -p, p_3 = p, p_4 = -p, p_5 = -p \), Eq. (A.3) is equal to

\[
5\tilde{k}^2 + \frac{7}{20} \left( \tilde{p}_1^2 + \tilde{p}_2^2 + \tilde{p}_3^2 + \tilde{p}_4^2 + \tilde{p}_5^2 \right) .
\] (A.4)

Again the coefficient \( \frac{7}{20} \) is greater than zero. One can proceed in a similar fashion for the other higher-point diagrams.
References

[1] M. J. G. Veltman, “Quantum Theory of Gravitation,” Conf. Proc. C 7507281, 265 (1975).

[2] B. S. DeWitt, “Quantum Theory of Gravity. 1. The Canonical Theory,” Phys. Rev. 160, 1113 (1967).
B. S. DeWitt, “Quantum Theory of Gravity. 2. The Manifestly Covariant Theory,” Phys. Rev. 162, 1195 (1967).
B. S. DeWitt, “Quantum Theory of Gravity. 3. Applications of the Covariant Theory,” Phys. Rev. 162, 1239 (1967).

[3] B. S. DeWitt and G. Esposito, “An Introduction to quantum gravity,” Int. J. Geom. Meth. Mod. Phys. 5, 101 (2008) [arXiv:0711.2445 [hep-th]].

[4] G. ’t Hooft and M. J. G. Veltman, “One loop divergencies in the theory of gravitation,” Ann. Inst. H. Poincare Phys. Theor. A 20, 69 (1974).

[5] M. H. Goroff and A. Sagnotti, “Quantum Gravity At Two Loops,” Phys. Lett. 160B, 81 (1985). doi:10.1016/0370-2693(85)91470-4

[6] A. E. M. van de Ven, “Two loop quantum gravity,” Nucl. Phys. B 378, 309 (1992). doi:10.1016/0550-3213(92)90011-Y

[7] J. Polchinski, “String theory. Vol. 2: Superstring theory and beyond,” Cambridge, UK: Univ. Pr. (1998) 531p.

[8] A. Ashtekar, “Introduction to loop quantum gravity and cosmology,” Lect. Notes Phys. 863, 31 (2013).

[9] for a review, see: H. Nicolai, K. Peeters and M. Zamaklar, “Loop quantum gravity: An Outside view,” Class. Quant. Grav. 22, R193 (2005) [hep-th/0501114].

[10] for a review, see: J. Henson, “The Causal set approach to quantum gravity,” In *Oriti, D. (ed.): Approaches to quantum gravity* 393-413 [gr-qc/0601121].

[11] D. A. Eliezer and R. P. Woodard, “The Problem of Nonlocality in String Theory,” Nucl. Phys. B 325, 389 (1989).

[12] E. Witten, “Noncommutative Geometry and String Field Theory,” Nucl. Phys. B 268, 253 (1986).

[13] W. Siegel, “Introduction to string field theory,” hep-th/0107094.

[14] P. G. O. Freund and M. Olson, “Nonarchimedean Strings,” Phys. Lett. B 199, 186 (1987).
P. G. O. Freund and E. Witten, “Adelic String Amplitudes,” Phys. Lett. B 199, 191 (1987).
L. Brekke, P. G. O. Freund, M. Olson and E. Witten, “Nonarchimedean String
Dynamics,” *Nucl. Phys. B* **302**, 365 (1988).
P. H. Frampton and Y. Okada, “Effective Scalar Field Theory of \( P \)–adic String,” *Phys. Rev. D* **37**, 3077 (1988).

[15] B. Dragovich, “Zeta strings,” [hep-th/0703008](https://arxiv.org/abs/0703008).

[16] M. R. Douglas and S. H. Shenker, “Strings in Less Than One-Dimension,” *Nucl. Phys. B* **335**, 635 (1990).
D. J. Gross and A. A. Migdal, “Nonperturbative Solution of the Ising Model on a Random Surface,” *Phys. Rev. Lett.* **64**, 717 (1990).
E. Brezin and V. A. Kazakov, “Exactly Solvable Field Theories of Closed Strings,” *Phys. Lett. B* **236**, 144 (1990).
D. Ghoshal, “\( p \)-adic string theories provide lattice discretization to the ordinary string worldsheet,” *Phys. Rev. Lett.* **97**, 151601 (2006).

[17] K. S. Stelle, “Renormalization of Higher Derivative Quantum Gravity,” Phys. Rev. D **16**, 953 (1977). doi:10.1103/PhysRevD.16.953

[18] K. S. Stelle, “Classical Gravity with Higher Derivatives,” Gen. Rel. Grav. **9**, 353 (1978). doi:10.1007/BF00760427

[19] P. Van Nieuwenhuizen, “On ghost-free tensor lagrangians and linearized gravitation,” Nucl. Phys. B **60**, 478 (1973). doi:10.1016/0550-3213(73)90194-6

[20] M. Ostrogradski, Memoires sur les equations differentielles relatives au probleme des isoperimetres, Mem. Ac. St. Petersbourg VI (1850) 385.

[21] T. Biswas, E. Gerwick, T. Koivisto and A. Mazumdar, “Towards singularity and ghost free theories of gravity,” Phys. Rev. Lett. **108**, 031101 (2012) doi:10.1103/PhysRevLett.108.031101 [arXiv:1110.5249 [gr-qc]].

[22] T. Biswas, T. Koivisto and A. Mazumdar, “Nonlocal theories of gravity: the flat space propagator,” [arXiv:1302.0532 [gr-qc]].

[23] A. O. Barvinsky and Y. V. Gusev, “New representation of the nonlocal ghost-free gravity theory,” [arXiv:1209.3062 [hep-th]].
A. O. Barvinsky, “Aspects of Nonlocality in Quantum Field Theory, Quantum Gravity and Cosmology,” [arXiv:1408.6112 [hep-th]].

[24] J. W. Moffat, “Ultraviolet Complete Quantum Gravity,” *Eur. Phys. J. Plus* **126**, 43 (2011) [arXiv:1008.2482 [gr-qc]].
L. Modesto, J. W. Moffat and P. Nicolini, “Black holes in an ultraviolet complete quantum gravity,” *Phys. Lett. B* **695**, 397 (2011) [arXiv:1010.0680 [gr-qc]].

[25] G. V. Efimov, “Non-Local Quantum Theory of the Scalar Field,” *Commun. Math. Phys.* **5**, 42 (1967).
G. V. Efimov and S. Z. Seltser, “Gauge invariant nonlocal theory of the weak interactions,” *Annals Phys.* **67**, 124 (1971).
G. V. Efimov, “On the construction of nonlocal quantum electrodynamics,” *Annals Phys.* 71, 466 (1972).

V. A. Alebastrov and G. V. Efimov, “A proof of the unitarity of S matrix in a nonlocal quantum field theory,” *Commun. Math. Phys.* 31, 1 (1973).

V. A. Alebastrov and G. V. Efimov, “Causality In The Quantum Field Theory With The Nonlocal Interaction,” *Commun. Math. Phys.* 38, 11 (1974).

[26] A. Addazi and G. Esposito, “Nonlocal quantum field theory without acausality and nonunitarity at quantum level: is SUSY the key?,” *Int. J. Mod. Phys. A* 30, no. 15, 1550103 (2015) [arXiv:1502.01471 [hep-th]].

A. Addazi, “Unitarization and Causalization of Non-local quantum field theories by Classicalization,” [arXiv:1505.07357 [hep-th]].

[27] S. Talaganis and A. Teimouri, “Hamiltonian Analysis for Infinite Derivative Field Theory and Gravity,” [arXiv:1701.01009 [hep-th]].

[28] Y. V. Kuzmin, “The Convergent Nonlocal Gravitation. (in Russian),” Sov. J. Nucl. Phys. 50, 1011 (1989) [Yad. Fiz. 50, 1630 (1989)].

[29] T. Biswas, A. Mazumdar and W. Siegel, “Bouncing universes in string-inspired gravity,” *JCAP* 0603, 009 (2006) [hep-th/0508194].

[30] E. Tomboulis, “Renormalizability and Asymptotic Freedom in Quantum Gravity,” Phys. Lett. B 97, 77 (1980). E. T. Tomboulis, “Renormalization And Asymptotic Freedom In Quantum Gravity,” In *Christensen, S.m. (Ed.): Quantum Theory Of Gravity*, 251-266 and Preprint - TOMBOULIS, E.T. (REC.MAR.83) 27p. E. T. Tomboulis, “Superrenormalizable gauge and gravitational theories,” hep-th/9702146. E. T. Tomboulis,“Nonlocal and quasilocal field theories,” *Phys. Rev. D* 92, no. 12, 125037 (2015) doi:10.1103/PhysRevD.92.125037 [arXiv:1507.00981 [hep-th]].

[31] L. Modesto, “Super-renormalizable Quantum Gravity,” Phys. Rev. D 86, 044005 (2012) [arXiv:1107.2403 [hep-th]]. L. Modesto and L. Rachwal, “Super-renormalizable and Finite Gravitational Theories,” [arXiv:1407.8036 [hep-th]].

[32] T. Biswas, M. Grisaru and W. Siegel, “Linear Regge trajectories from worldsheet lattice parton field theory,” *Nucl. Phys. B* 708, 317 (2005) [hep-th/0409089].

[33] T. Biswas, J. A. R. Cembranos and J. I. Kapusta, “Finite Temperature Solitons in Non-Local Field Theories from p-Adic Strings,” *Phys. Rev. D* 82, 085028 (2010) [arXiv:1006.4098 [hep-th]].

T. Biswas, J. A. R. Cembranos and J. I. Kapusta, “Thermal Duality and Hagedorn Transition from p-adic Strings,” *Phys. Rev. Lett.* 104, 021601 (2010) [arXiv:0910.2274 [hep-th]].

T. Biswas, J. A. R. Cembranos and J. I. Kapusta, “Thermodynamics and Cosmological Constant of Non-Local Field Theories from p-Adic Strings,” *JHEP* 1010, 048 (2010) [arXiv:1005.0430 [hep-th]].
[34] S. Talaganis, T. Biswas and A. Mazumdar, “Towards understanding the UV
behavior of quantum loops in infinite-derivative theories of gravity,” Class.
Quant. Grav. 32, no. 21, 215017 (2015) doi:10.1088/0264-9381/32/21/215017
arXiv:1412.3467 [hep-th].

[35] T. Biswas and S. Talaganis, “String-Inspired Infinite-Derivative Theories of
Gravity: A Brief Overview,” Mod. Phys. Lett. A 30, no. 03n04, 1540009 (2015)
doi:10.1142/S021773231540009X arXiv:1412.4256 [gr-qc].

[36] A. Conroy, A. Mazumdar and A. Teimouri, “Wald Entropy for Ghost-Free, Infi-
nite Derivative Theories of Gravity,” Phys. Rev. Lett. 114, no. 20, 201101 (2015)
doi:10.1103/PhysRevLett.114.201101 arXiv:1503.05568 [hep-th].

[37] A. Conroy, A. Mazumdar, S. Talaganis and A. Teimouri, “Nonlocal gravity in
D dimensions: Propagators, entropy, and a bouncing cosmology,” Phys. Rev. D
92, no. 12, 124051 (2015) doi:10.1103/PhysRevD.92.124051 arXiv:1509.01247
[hep-th].

[38] A. Teimouri, S. Talaganis, J. Edholm and A. Mazumdar, “Generalised Bound-
ary Terms for Higher Derivative Theories of Gravity,” JHEP 1608, 144 (2016)
doi:10.1007/JHEP08(2016)144 arXiv:1606.01911 [gr-qc].

[39] S. Talaganis and A. Mazumdar, “High-Energy Scatterings in Infinite-Derivative
Field Theory and Ghost-Free Gravity,” Class. Quant. Grav. 33, no. 14, 145005
(2016) doi:10.1088/0264-9381/33/14/145005 arXiv:1603.03440 [hep-th].