LARGE-SCALE AZIMUTHAL STRUCTURES OF TURBULENCE IN ACCRETION DISKS: DYNAMO TRIGGERED VARIABILITY OF ACCRETION

M. Flock\textsuperscript{1}, N. Dzyurkevich\textsuperscript{1}, H. Klahr\textsuperscript{1}, N. Turner\textsuperscript{1,2}, and Th. Henning\textsuperscript{1}

\textsuperscript{1} Max Planck Institute for Astronomy, Königstuhl 17, 69117 Heidelberg, Germany
\textsuperscript{2} Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA 91109, USA

Received 2011 July 18; accepted 2011 October 4; published 2011 December 22

ABSTRACT

We investigate the significance of large-scale azimuthal, magnetic, and velocity modes for the magnetorotational instability (MRI) turbulence in accretion disks. We perform three-dimensional global ideal MHD simulations of stratified protoplanetary disk models. Our domains span azimuthal angles of $\pi/4$, $\pi/2$, $\pi$, and $2\pi$. We observe up to 100\% stronger magnetic fields and stronger turbulence for the restricted azimuthal domain models $\pi/2$ and $\pi/4$ compared to the full $2\pi$ model. We show that for those models the Maxwell stress is larger due to strong axisymmetric magnetic fields generated by the $\alpha\Omega$ dynamo. Large radial extended axisymmetric toroidal fields trigger temporal magnification of accretion stress. All models display a positive dynamo-$\alpha$ in the northern hemisphere (upper disk). The parity is distinct in each model and changes on timescales of 40 local orbits. In model $2\pi$, the toroidal field is mostly antisymmetric with respect to the midplane. The eddies of the MRI turbulence are highly anisotropic. The major wavelengths of the turbulent velocity and magnetic fields are between one and two disk scale heights. At the midplane, we find magnetic tilt angles around 8°–9° increasing up to 12°–13° in the corona. We conclude that an azimuthal extent of $\pi$ is sufficient to reproduce most turbulent properties in three-dimensional global stratified simulations of magnetized accretion disks.

Key words: accretion, accretion disks – dynamo – magnetic fields – magnetohydrodynamics (MHD) – protoplanetary disks

1. INTRODUCTION

Magnetorotational instability (MRI) can generate MHD turbulence with an outward directed angular momentum transport driving accretion onto the central object (Balbus & Hawley 1991, 1998; Hawley & Balbus 1991). A necessary condition is a good coupling between the gas and magnetic fields, e.g., a well-ionized gas. In protoplanetary disks, dust particles and low temperatures will reduce the ionization level and therefore the MRI activity (Sano et al. 2000; Fleming & Stone 2003; Inutsuka & Sano 2005; Wardle 1987; Dzyurkevich et al. 2010; Turner et al. 2010). Nevertheless, there are well-ionized regions with possible MRI activity, like the coronal region or the inner or outer disk. The inner disk will be thermally ionized for temperatures greater than 1000 K (Umebayashi 1983). The outer disk will be ionized by cosmic rays for surface density values below $96 \, \text{g cm}^{-2}$ (Umebayashi & Nakano 2009). In our work we concentrate on well-ionized disk regions. To model the evolution of protoplanetary disks and especially to describe the process of planet formation, we need to know detailed information about the strength of the turbulence. Several processes, like the MHD dynamo or the toroidal field MRI, influence the turbulence level. The evolution of the magnetic and velocity fields at different scales has to be investigated.

In recent decades, a large number of local-box simulations have been performed to study the small-scale MRI turbulence (Brandenburg et al. 1995; Hawley et al. 1995, 1996; Matsumoto & Tajima 1995; Stone et al. 1996). The MRI works for both vertical and toroidal seed magnetic fields (Balbus & Hawley 1991). The MRI launched with initial toroidal field was analyzed through linear calculations (Hawley & Balbus 1992; Foglizzo & Tagger 1995; Terquem & Papaloizou 1996; Papaloizou & Terquem 1997) and in Taylor–Couette experiments (Gellert et al. 2007; Rüdiger et al. 2007). These experiments showed that most of the energy will be transported to the $m = 1$ mode. A similar inverse energy cascade was found in local-box simulations as well (Johansen et al. 2009). Here the turbulent advection term in the induction equation drives large-scale radial magnetic field.

The locality and anisotropy of the MRI turbulence is an important aspect for dust growth and therefore the planet formation. The eddies are stretched in the azimuthal direction due to the strong shear. They have a characteristic low tilt angle in the $r-\phi$ plane (Guan et al. 2009). Several works confirmed this tilt angle for the velocity and the magnetic fields (Guan et al. 2009; Fromang 2010; Davis et al. 2010; Guan & Gammie 2011; Sorathia et al. 2011). The size of the corresponding correlation wavelengths is dependent on resolution (Guan et al. 2009) and converges by using a fixed value of viscous and explicit dissipation in unstratified local simulations (Fromang 2010). Unstratified global models interpret the magnetic tilt angle as convergence parameter (Sorathia et al. 2011). They found convergence with tilt angles around 13°. Beckwith et al. (2011) found tilt angles of 9° in global stratified simulations with spatial structures of the turbulent field in the order of $H$

Global disk simulations (Armitage 1998; Hawley 2000; Arlt & Rüdiger 2001; Fromang & Nelson 2006, 2009; Dzyurkevich et al. 2010; Flock et al. 2011; Beckwith et al. 2011; Sorathia et al. 2011) are used to study the MRI evolution on larger scales. Beckwith et al. (2011) found a stronger accretion stress compared to Fromang & Nelson (2006) and Flock et al. (2011) with a stronger initial toroidal field. Unstratified simulations show a similar correlation between accretion stress and the initial plasma beta (Hawley et al. 1995). Here a stronger seed field will drive stronger accretion stress. The majority of stratified global disk simulations has been done for restricted ($\phi \leq \pi/2$) azimuthal domain sizes. At first glance, MRI turbulence behaves similar for both full $2\pi$ and smaller domain sizes (Hawley 2000).
In our previous work we compared stratified simulations of $\pi/4$ and $2\pi r$ in azimuth. There, we observe stronger azimuthal fields for the $\pi/4$ domain size (Flock et al. 2011). Recent unstratified global simulations (Sorathia et al. 2011) do not show large differences between domain sizes of $\pi/4$ and $2\pi r$. This fact indicates a mean field dynamo mechanism. The stratification is crucial for driving the $\alpha\Omega$ dynamo in disks and therefore for the creation of large-scale magnetic fields (Krause & Rädler 1980). With this work we perform a detailed study of different azimuthal domain sizes. We investigate the turbulent and the mean field evolution for the velocity and magnetic fields.

In stratified disk simulations, there is a periodic change of sign for the mean toroidal magnetic field. A similar periodicity of toroidal magnetic field, known as a butterfly diagram, is observed in the Sun. It could be explained by an MHD dynamo process. The MRI could be self-sustaining by an analogous dynamo process (Hawley et al. 1996; Lesur & Ogilvie 2008a, 2008b; Gressel 2010; Simon et al. 2011). Strong shear in accretion disks will wind up any radial magnetic field generated by MRI and produce a toroidal field. This field will act as a seed for the MRI again. Solutions for $\alpha\Omega$ dynamos in rotating systems were presented by Ruediger & Kitchatinov (1993) and Elstner et al. (1996). Calculations of the dynamo-$\alpha$ for MRI have been performed in local-box simulations (Brandenburg et al. 1995; Brandenburg & Donner 1997; Rekowski et al. 2000; Ziegler & Rüdiger 2000; Davis et al. 2010; Gressel 2010) showing a negative dynamo-$\alpha$ (Brandenburg & Donner 1997) and Rüdiger & Pipin (2000) explained the negative sign as an effect of vertical buoyancy. The first indications for a positive dynamo-$\alpha$ were found in global disk simulations (Arlt & Rüdiger 2001; Arlt & Brandenburg 2001). Dynamo solutions for positive or negative dynamo-$\alpha$ predict long-term global mean magnetic fields which become symmetric (quadrupole, dynamo-$\alpha$ north $< 0$) or asymmetric (dipole, dynamo-$\alpha$ north $> 0$). For example, dipole solutions support the creation of disk wind and jets (Rekowski et al. 2000). A review of dynamo action in accretion disks was presented by Brandenburg & Subramanian (2005), Brandenburg & von Rekowski (2007), and Blackman (2010).

The connection between the dynamo processes and the large-scale magnetic field oscillations was shown by Lesur & Ogilvie (2008b), Gressel (2010), and Simon et al. (2011). These oscillations are universal for stratified MRI simulations (Stone et al. 1996; Miller & Stone 2000) with timescales of 10 local orbits, presented recently in local (Gressel 2010; Simon et al. 2011; Hawley et al. 2011; Guan & Gammie 2011) and global (Sorathia et al. 2010; Dzyurkevich et al. 2010; Flock et al. 2011; Beckwith et al. 2011) simulations. We use the second-order Godunov code PLUTO which was successfully applied in recent global simulations (Flock et al. 2010, 2011; Urrí et al. 2011; Beckwith et al. 2011). The paper is structured in the following way. First, we describe the disk model and the numerical parameter. For the results in Section 3, we study the turbulent and the mean field evolution for all azimuthal domains. Sections 4 and 5 present a discussion and summary.

2. SETUP

Our disk model is presented in detail in Flock et al. (2011). We give here a summary of our physical and numerical initial conditions.

2.1. Disk Model

The HD initial conditions of density, pressure, and azimuthal velocity follow a hydrostatic equilibrium. We set

$$\rho = \rho_0 R^{-3/2} \exp \left( \frac{\sin(\theta) - 1}{(H/R)^2} \right)$$

with $\rho_0 = 1.0$, $H/R = c_0 = 0.07$, and $R = r \sin(\theta)$. The pressure follows locally an isothermal equation of state:

$$P = c_s^2 \rho$$

The azimuthal velocity is set to

$$V_\phi = \sqrt{\frac{T}{r} \left( 1 - \frac{2.5}{\sin(\theta)} c_s^2 \right)}.$$

The initial velocities $V_r$ and $V_\theta$ are set to a white noise perturbation amplitude of $V_{\text{init}}^\rho = 10^{-4} c_s$. We start the simulation with a pure toroidal magnetic seed field with constant plasma beta $\beta = 2P/B^2 = 25$.

The radial domain extends from 1 to 10 AU. The $\theta$ domain covers $\pm 4.3$ disk scale heights, or $\theta = \pi/2 \pm 0.3$. For the azimuthal domain we use four different models: $\phi^\text{exten} = \pi/4$, $\pi/2$, $\pi$, and $2\pi$. We use a uniform grid in spherical coordinates with an aspect ratio at 5 AU of 1:0.67:1.74 ($r, \theta, \phi$). The resolution is fixed to $N_r = 384$, $N_\theta = 192$, and $N_\phi = 768 \cdot \phi_{\text{exten}}/(2\pi)$. We have around 23 grid cells per pressure scale height.

Buffer zones extend from 1 to 2 AU as well as from 9 to 10 AU. In the buffer zones we use a linearly increasing resistivity to the boundary. This damps the magnetic field fluctuations and suppresses boundary interactions. In the buffer zones we also use a relaxation function which gently re-establishes the initial value of density over a time period of one local orbit. In the buffer zones, we set $\rho_{\text{new}} = \rho - (\rho - \rho_{\text{init}}) \cdot \Delta t/T_{\text{Orbit}}$. Our outflow boundary condition projects the radial gradients in density, pressure, and azimuthal velocity into the radial boundary and the vertical gradients in density and pressure at the $\theta$ boundary. We ensure that there are no inflow velocities. For an inward pointing velocity we mirror the values in the ghost cell to ensure no inward mass flux. The $\theta$ boundary condition for the magnetic field is set to zero gradient, which approximates “force-free” outflow conditions. The normal component of the magnetic field in the ghost cells is always set to have $\nabla \cdot \vec{B} = 0$.

2.2. Numerical Setup

The detailed numerical configuration is presented in Flock et al. (2010) and was also successfully used in recent global simulations by Beckwith et al. (2011). For all runs we employ the second-order scheme in PLUTO with the Harten–Lax–van Leer discontinuities Riemann solver (Miyoshi & Kusano 2005), piecewise linear reconstruction, and second-order Runge–Kutta time integration. We treat the induction equation with the “constrained transport” (CT) method in combination with the upwind CT method described in Gardiner & Stone (2005). All models were performed on a Blue Gene/P cluster for a total of over 3 million CPU hours.
2.3. Measurement and Integration

For our analysis we use the central domain\(^5\) from 3 to 8 AU. Total volume integrations of a variable \(F\) as used for the total stress are performed with

\[
F_{\text{total}} = \int F dV = \int_3^8 \int_{\theta_{\text{begin}}}^{\theta_{\text{end}}} \int_0^{\phi_{\text{extent}}} r^2 \sin \theta dr d\theta d\phi.
\]

In global disk models, the gas dynamics are only self-similar along the azimuth. Therefore, mean values like \(\overline{\rho\Omega}\) are always averaged over azimuth. This includes the calculation of the turbulent EMF\(^6\) in Figure 11. For further analysis we always use a two-dimensional data set of mean values, e.g., \(\overline{\rho\Omega}(r, \theta)\) to construct the three-dimensional turbulent data set \(u_\phi(r, \theta, \phi) = \overline{\rho\Omega}(r, \theta) - \overline{\rho\Omega}(r, \theta)\). For volume integration over mean values, as \(\alpha_{SS}\), we use

\[
\int dV = \int_3^8 r_{\text{begin}}^2 \sin \theta dr d\theta.
\]

Some results are determined in the center of the computational domain. Analysis done at 4.5 AU are the tilt angle calculations (Figure 6), the mean field contour plots (Figure 9), the parity (Figure 10), and the dynamo coefficients (Figure 11). These results are averaged over azimuth and a small radial extent (\(\pm 0.5 H = 0.16 AU\)). For the time evolution of the tilt angle (Figure 6, top), we average vertically \(\pm 0.5 H\) at the midplane. Radial contour plots are averaged over azimuth and height, between 0–1.5\(H\). This applies for the mean toroidal field (Figure 3), the dynamo (Figure 11), and the mean fields (Figure 12). The parity is averaged over the total disk height at 4.5 AU (Figure 10).

### 3. RESULTS

In this section we investigate the turbulent and mean field evolution for the azimuthal MRI for different azimuthal domain sizes. Table 1 summarizes the results of accretion stress, contribution of mean magnetic field to the total stress, dynamo-\(\alpha\), and rms velocities for all models. Table 2 summarizes the results of the two-point correlation function, including tilt angles, major and minor wavelengths. For all models, the accretion disk becomes unstable to MRI on timescales of 10 local orbits. All models develop an oscillating zero-net flux configuration after around 250 inner orbits. The time evolution of total magnetic energy (Figure 1, left) is normalized over the total initial magnetic field energy \(B_0^2\). It shows the peak of magnetic energy shortly after the linear MRI phase around 100 inner orbits. Between 100 and 400 years, the total magnetic energy decreases due to loss of the net magnetic flux and mass loss (see also Figure 13 in Flock et al. 2011 and Figure 3 in Beckwith et al. 2011). After 400 years, \(\pi/4\) and \(\pi/2\) models show strong fluctuations while the \(\pi\) and \(2\pi\) models saturate. In the saturated state (\(\geq 800\) inner orbits), the total magnetic energy evolution shows a relative constant level for the \(\pi\) and \(2\pi\) models.

All models have the same resolution per \(\phi\) extent \((\Delta\phi/\bar{N}_\phi)\). The toroidal quality factor \(Q_\phi = \lambda_{\text{crit}}/\Delta\phi\) shows the quality of resolved MRI \((Q_\phi \gtrsim 8)\). We follow the analysis done by Noble et al. (2010) and Sorathia et al. (2011) and calculate the mean \(Q_\phi\) for the central domain (3–8 AU). The definition is similar to the toroidal quality factor \(Q_\phi\) by Hawley et al. (2011):

\[
\frac{\lambda_{\text{crit}}}{\Delta\phi} = 2\pi \sqrt{\frac{16}{15} \frac{2}{\beta_\phi^2} c_0 \Delta\phi} = 2\pi \sqrt{\frac{16}{15} \frac{|B_\phi| \cdot r}{\rho \Delta \Omega \Delta \phi}}.
\]

Figure 1 (right) shows \(Q_\phi\) over time. For all models we have \(Q_\phi > 8\). The \(\pi/4\) and \(\pi/2\) models show a higher \(Q_\phi\) due to stronger magnetic fields.

#### 3.1. Turbulent Evolution: \(\alpha\) Value

We start the comparison with the volume-integrated turbulent stress scaled on the local pressure, e.g., the Shakura–Sunyaev \(\alpha_{SS}\). The \(\alpha_{SS}\) value is determined from the turbulent Reynolds stress \(T_R = \rho u^\prime v^\prime R\) and Maxwell stress \(T_M = B^\prime B^\prime /4\pi\). We split the total \(\alpha_{SS}\) into a mean and a turbulent component. For the Maxwell stress, we split the magnetic field components into the turbulent and mean component, e.g., \(B_\phi = B_\phi^\prime + B_\phi^\ast\). This leads to a second Maxwell stress component, e.g., the mean Maxwell stress \(T_{\text{mean}} = B_\phi^\prime B^\prime /4\pi\). For the volume-integrated turbulent \(\alpha_{SS}^\prime\) value we integrate the mass-weighted stresses

---

\(^5\) The “central domain” is here the domain between 3 and 8 AU to avoid impact of the inner and outer buffer zones (see Flock et al. 2011).

---

### Table 1

| \(\Delta\phi\) | \(\alpha_{SS}^\prime\) | \(\alpha_{SS}^\prime\) | \(\alpha_{SS}^\prime\) | \(\alpha_{SS}^\prime\) | \(\alpha_{SS}^\prime\) | Parity | \(V_{\text{rms}}(c_1)\) |
|----------------|-------------------|-------------------|-------------------|-------------------|-------------------|--------|-------------------|
| \(\pi/4\)    | 11.8 ± 2.3        | 0.33              | 8.9 ± 1.6         | −3.4 ± 0.9        | 3.3 ± 0.8         | −0.2 ± 0.4 | 0.125 ± 0.009     |
| \(\pi/2\)    | 9.3 ± 0.9         | 0.19              | 7.8 ± 0.7         | −2.8 ± 0.6        | 3.1 ± 0.7         | −0.2 ± 0.5 | 0.148 ± 0.006     |
| \(\pi\)      | 5.6 ± 0.5         | 0.12              | 5.0 ± 0.4         | −2.4 ± 0.3        | 2.1 ± 0.3         | −0.1 ± 0.5 | 0.112 ± 0.005     |
| \(2\pi\)     | 5.4 ± 0.4         | 0.08              | 5.0 ± 0.3         | −2.3 ± 0.2        | 2.1 ± 0.2         | 0.2 ± 0.4 | 0.113 ± 0.005     |

**Notes.** From left to right: azimuthal domain; volume-integrated total stress; relation between \(\alpha_{SS}^\prime\) and \(\alpha_{SS}^\prime\); \(\alpha_{SS}^\prime\) stress; value of dynamo \(\alpha_{SS}^\prime\) for southern hemisphere (lower disk); value of dynamo \(\alpha_{SS}^\prime\) for northern hemisphere (upper disk); time averaged parity; time averaged \(V_{\text{rms}}\).

### Table 2

| \(\Delta\phi\) | \(\theta_\nu\) | \(\lambda_{\text{maj}}\) | \(\lambda_{\text{min}}\) | \(\theta_\phi\) | \(\lambda_{\text{maj}}\) | \(\lambda_{\text{min}}\) |
|----------------|--------------|-----------------|-----------------|--------------|-----------------|-----------------|
| \(\pi/4\)    | 12.0         | 1.1 H           | 0.19 H          | 9.1          | 1.1 H           | 0.14 H          |
| \(\pi/2\)    | 14.1         | 2.0 H           | 0.29 H          | 8.9          | 1.4 H           | 0.16 H          |
| \(\pi\)      | 14.1         | 1.9 H           | 0.24 H          | 7.7          | 1.6 H           | 0.14 H          |
| \(2\pi\)     | 14.2         | 1.9 H           | 0.23 H          | 8.2          | 1.7 H           | 0.14 H          |

**Notes.** From left to right: azimuthal domain, correlation angle for the velocity, wavelength of the major axis, wavelength of the minor axis, correlation angle for the magnetic field, wavelength of the major axis, and wavelength of the minor axis.
over the central domain:

$$\alpha_{\text{turb}}^{\text{SS}} = \frac{\int \rho \left( \frac{\varepsilon_{\phi}}{c_s^2} - \frac{B_\phi^2 B_{\phi,0}^2}{4 \pi \rho c_s^2} \right) dV}{\int \rho dV}.$$ 

The same is done for the mean Maxwell stress:

$$\alpha_{\text{mean}}^{\text{SS}} = \frac{\int \overline{\rho} \left( - \frac{B_\phi}{c_s} \overline{B_{\phi,0}} \right) dV}{\int \overline{\rho} dV}.$$ 

The volume-integrated $\alpha_{\text{turb}}^{\text{SS}}$ (Figure 2, left, solid line) and the volume-integrated $\alpha_{\text{mean}}^{\text{SS}}$ (Figure 2, right, solid line) are plotted versus time. We are interested in the steady state and we use the time period between 800 and 1200 inner orbits for averaging. Figure 2 (left) shows that the $\pi/4$ and $\pi/2$ models present higher $\alpha_{\text{SS}}$ values than the $\pi$ and $2\pi$ models. The mean magnetic fields provide a significant contribution to the total stress for the restricted azimuthal domains (see Figure 2, right). The time-averaged ratio between the turbulent Maxwell stresses and the mean Maxwell stresses is up to 33% for the $\pi/4$ model while it decreases in the full $2\pi$ model down to 8% (see Table 1). In Table 1, we summarize the results of $\alpha_{\text{SS}}$, $\alpha_{\text{turb}}^{\text{SS}}$, and $\alpha_{\text{mean}}^{\text{SS}}$. The standard deviation is determined by the temporal fluctuations. For model $\pi/4$ we determine $\alpha_{\text{SS}}^{\text{total}} = (11.8 \pm 2.3) \times 10^{-3}$. For model $\pi/2$, $\alpha_{\text{SS}}^{\text{total}}$ reduces to $(9.3 \pm 0.9) \times 10^{-3}$. The stress of the two largest azimuthal domain sizes, $\pi$ and $2\pi$, matches within the standard deviation. For model $\pi$, the time-averaged $\alpha_{\text{SS}}^{\text{total}}$ is $(5.6 \pm 0.5) \times 10^{-3}$ and $(5.4 \pm 0.4) \times 10^{-3}$ for model $2\pi$.

To verify the results we made the same analysis in the same azimuthal extent for every model. Instead of using the full azimuthal data set for the analysis, we use here the azimuthal extent between 0 and $\pi/4$ in every model. The results are shown in Figure 2 (dotted lines). In Figure 2 (left), these $\alpha_{\text{SS}}$ values are only slightly lower than the total domain integration. This indicates that most of the turbulent stress is generated by the small-scale turbulence ($m \leq 8$). In Figure 2 (right), these $\alpha_{\text{SS}}$ values represent the stress for one specific mode ($m = 8$). We again see that the smaller scales contribute more to the $\alpha_{\text{SS}}$ than the larger scales. We summarize that the turbulence is amplified in case for the $\pi/2$ and $\pi/4$ model. These models present higher $\alpha_{\text{turb}}^{\text{SS}}$ and $\alpha_{\text{mean}}^{\text{SS}}$ values than the $\pi$ and $2\pi$ runs.

### 3.2. Accretion Burst Due to Mean Fields

The $\pi/4$ run presents another exceptional behavior. Around 800 inner orbits, the $\alpha$ value increases quickly up to $\alpha = 0.013$. The reason for this increase is connected to strong mean toroidal field oscillations. In Figure 3, we plot contour lines of the resolved $\lambda_{\text{crit}}$ from the mean toroidal field $\mathbf{B}_\phi$ with $\mathbf{B}_\phi^2 / \Delta \phi \geq 8$:

$$\frac{\lambda_{\text{crit}}}{\Delta \phi} = 2\pi \sqrt[15]{\frac{16}{15} \left| \mathbf{B}_\phi \right| \cdot r \sqrt{\rho \Omega \Delta \phi}}.$$ 

The definition is equivalent to the definition of the toroidal quality factor $Q_\phi$ but calculated from the mean toroidal field.
0.000 0.005 0.010 0.015 0.020
\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3}
\caption{Contour lines of the resolved MRI from the mean toroidal field \(\lambda_{\text{crit}}\) with the evolution of the \(\alpha\) value for the models \(\pi/4\) (top) and \(\pi/2\) (bottom). The contour lines show \(\lambda_{\text{crit}} = 8\). The strong mean toroidal field amplifies the turbulence.}
\end{figure}

instead from the total field (see Figure 1, right). There is a clear correlation between the rise of the \(\alpha_{\text{SS}}\) value and resolved mean toroidal field. At the same time there is a superposition of strong mean field along radius (see Figure 3, red solid line). The amplifications are present in the \(\pi/4\) model (Figure 3, top) and the \(\pi/2\) model (Figure 3, bottom). For the larger domains, \(\pi\) and \(2\pi\) (Figure 4), the mean field stays at lower values and \(\lambda_{\text{crit}}\) is not resolved.

### 3.3. Turbulent Magnetic and Velocity Fields

We investigate the spatial distribution of magnetic energy with Fourier analysis. The magnetic field amplitudes \(\sqrt{B(m)^2}\) are plotted in Fourier space along azimuth at the midplane and for all models (Figure 5, left). The plots show that the highest amplitudes of the magnetic fields are at the largest scales. The \(\pi/4\) and \(\pi/2\) models show systematically increased amplitudes compared to the \(\pi\) and \(2\pi\) model. This is true for all modes and for all three magnetic field components. It is also visible in the time-averaged total magnetic energy (Figure 1, left dotted lines). Time-averaged values, in units of the initial total magnetic energy, are \(B^2/B_0^2 = 0.54 \pm 0.12\) for model \(\pi/4\), \(0.48 \pm 0.09\) for model \(\pi/2\), \(0.34 \pm 0.07\) for model \(\pi\), and \(0.35 \pm 0.07\) for model \(2\pi\). Here, time average is done between 400 and 1200 inner orbits. We present the velocity field in Fourier space \(\sqrt{V(m)^2}\) in Figure 5 (right). We observe increased turbulent velocities for the restricted domain models. The radial velocity (dashed line) dominates in the range between \(2 \lesssim \mu \lesssim 40\). The peak turbulent velocity is \(V_r\) at \(m = 4\) for the \(\pi/2\), \(\pi\), and \(2\pi\) run. Coincidentally, this mode matches the domain size of \(\pi/2\). The \(\pi/4\) does not include this mode. This lack of large-scale turbulent radial fields becomes visible again in
Figure 4. Contour lines of the resolved MRI from the mean toroidal field $\lambda_{B\phi}$ with the evolution of the $\alpha$ value for the models $\pi$ (top) and $2\pi$ (bottom). The contour lines show $\lambda_{B\phi} / \alpha = 8.0$. Here, the mean toroidal field is weaker and not resolved by the code.

Figure 5. Left: magnetic field distribution in Fourier space over azimuthal wave number for all models and magnetic field components. Right: same for the velocity field. Values are from the midplane and time averaged between 800 and 1200 inner orbits.
the velocity tilt angle. The peak at $m = 4$ (22 $H$) is connected to spiral density waves. After Heinemann & Papaloizou (2009) we should observe the peak at $m = 14$ (6 $H$). This could be a resolution issue as the domain size of $\pi/4$ (11 $H$) should be large enough to include spiral density waves.

3.4. Two-point Correlation Function

The two-point correlation function, specified for MRI by Guan et al. (2009), allows us to study the locality and anisotropy of the turbulence. We measure the tilt angle for the magnetic sin $2\theta_B = |B_\parallel|/B^2$ and the turbulent velocity field sin $2\theta_V = |V_\parallel|/V^2$ at 4.5 AU. In Figure 6, we plot the time evolution, top, and the vertical distribution, bottom, of the magnetic tilt angle $\theta_B$, left, and the velocity $\theta_V$, right. The time evolution of the magnetic tilt angle $\theta_B$ is plotted in Figure 6 (top left). The $\pi/4$ and $\pi/2$ models show higher tilt angles ($\theta_B \sim 9^\circ$) with much higher time deviations as the $\pi$ and 2$\pi$ models ($\theta_B \sim 8^\circ$). The $\pi/4$ model shows sudden increase of the tilt angle at 80 local orbits. At this time, the turbulence gets amplified due to strong axisymmetric fields (see Figure 3). The time-averaged vertical profile of $\theta_B$ is plotted in Figure 6 (bottom left). The tilt angle presents the highest values in the coronal region. Here, we again see higher $\theta_B$ values for the $\pi/4$ and $\pi/2$. The $\pi$ model shows smaller $\theta_B$ at the midplane compared to 2$\pi$ which is an artifact of the selected time average. Both models present equal values after 100 local orbits (see Figure 6, top left).

We do the same analysis for the velocity tilt angle $\theta_V$. The time evolution for $\theta_V$ does not show strong fluctuations. At the midplane, we measure a time-averaged velocity tilt angle of $\theta_V \sim 14^\circ$ for all models except $\pi/4$. The $\pi/4$ model shows a systematic lower tilt angle $\theta_V^{\pi/4} \sim 12^\circ$. This also becomes visible in the vertical profile. Here all models, except $\pi/4$, show a peak of $\theta_V$ at the midplane. The reason is unresolved density waves. The $\pi/4$ model does not resolve the density waves with $m = 4$. At $m = 4$, all models show the highest turbulent amplitude in the radial velocity. For model $\pi/2$ it matches the size of the domain and it is not captured by model $\pi/4$. The fast drop of magnetic and velocity tilt angles above 4 scale height could be due to boundary effects.

We calculate the two-point correlation functions in the $r$–$\phi$ plane: $\epsilon_V = (\delta V_\parallel(\tilde{x})\delta V_\parallel(\tilde{x}+\Delta \tilde{x}))$ and $\epsilon_B = (\delta B_\parallel(\tilde{x})\delta B_\parallel(\tilde{x}+\Delta \tilde{x}))$ with $\tilde{x} = r, \phi$. In Figures 7 and 8, we present the two-point correlation function at 5 AU at 1 scale height with $\Delta r = 2 H = 0.7$ AU and the total $\phi$ domain $r \Delta \phi = \Phi_{\mathrm{Domain}}/0.07 H$. For the $2\pi$ model we have around 90 $H$ ($2\pi/0.07$). The corresponding major and minor wavelengths are calculated using the half-width at half-maximum in units of $H$ ($H_{3.5\,\text{AU}} = 0.35$ AU). It measures the distance between the center $\epsilon = 1.0$ and $\epsilon = 0.5$ along the major $\lambda_{\text{maj}}$ and minor $\lambda_{\text{min}}$ axis, see footnote 7 in Guan et al. (2009). We measure the two-point correlation function at different heights. The results between $\pm 2 H$ are similar and we present the values at 1 scale height. For the velocity, the $\lambda_{\text{maj}}$ of the $\pi/4$ run is 1.1 $H$. The $\pi$ and 2$\pi$ runs both present a value of 1.9 $H$. We find a similar increase for the $\lambda_{\text{min}}$ from 0.19 $H$ for $\pi/4$ to 0.24 $H$ and 0.23 $H$ for model $\pi$ and 2$\pi$. The values of the $\pi/2$ model present the highest values, $\lambda_{\text{maj}} = 2.0 H$ and $\lambda_{\text{min}} = 0.29 H$. This is again due to the peak of turbulent radial velocity at domain size (see Figure 5, right). It is visible in the magnetic fields too. The $\lambda_{\text{min}}$ value for the magnetic fields is 0.14 $H$, except the $\pi/2$ model with 0.16 $H$. The $\lambda_{\text{maj}}$ increases with increasing the azimuthal domain, the $\pi/4$ model with 1.1 $H$.
Figure 7. Contour plot of the two-point velocity correlation function at 1 scale height at 5 AU. The red line shows zero contour.

Figure 8. Contour plot of the two-point magnetic field correlation function at 1 scale height at 5 AU. The red line shows zero contour.
to $1.4H$, $1.6H$, and $1.7H$ for the full $2\pi$. All results of the tilt angles, major and minor wavelengths are summarized in Table 2. The models with $\pi/4$ and $\pi/2$ show an amplified turbulence. The $\phi$ extent affects the large-scale and small-scale turbulent properties. Only an azimuthal domain of $\pi$ does reproduce similar large-scale and small-scale turbulent properties as in the full $2\pi$ run. The strong mean fields generated by the $\alpha \Omega$ dynamo are responsible for the MRI amplification.

### 3.5. Mean Field Evolution

A typical feature of MRI in stratified disks is an oscillating toroidal magnetic field, generated by oscillating radial magnetic field. This feature is well known as the “butterfly” pattern, in which wings appear due to the buoyant movement of the toroidal field from the midplane to upper layers. The timescale of these oscillations is around 10 local orbits. Recent work in local-box simulations showed the context between this oscillating magnetic field and a dynamo process (Gressel 2010; Simon et al. 2011; Hawley et al. 2011; Guan & Gammie 2011). In this section, we investigate the evolution of this axisymmetric magnetic field and the connection to the dynamo process.

### 3.6. The Parity and Butterfly Pattern

In Figure 9, top, we present the time evolution of axisymmetric radial and toroidal magnetic fields over height. The values are normalized over the initial toroidal field. The generated
In mean field theory, there is a mechanism to generate large-scale magnetic fields by a turbulent field. In the case of an $\alpha\Omega$ dynamo (Krause & Raedler 1980) there should be a correlation between the turbulent toroidal electromotive force ($\text{EMF}'_\phi$) component and the mean toroidal magnetic field,

$$\text{EMF}'_\phi = \alpha_{\phi\phi} B_\phi + \text{higher derivatives of } B,$$

with $\text{EMF}'_\phi = v'_\phi B'_\theta - v'_\theta B'_\phi$. The sign of $\alpha_{\phi\phi}$ has to change for the southern and northern hemisphere. The correlation is plotted in Figure 11 (left) for the northern hemisphere (top) and the southern hemisphere (bottom). We get a positive sign for the $\alpha_{\phi\phi}$ in the northern hemisphere ($\alpha_{\phi\phi}^{\text{NH}}$) of the disk (Figure 11, top) and a negative sign in the southern hemisphere ($\alpha_{\phi\phi}^{\text{SH}}$). This result was predicted for stratified accretion disks (Ruediger & Kitchatinov 1993) and also indicated in global simulations (Arlt & Rüdiger 2001). Each dot in Figure 11 (left) represents a result from a single time snapshot. The boxes show the limits of the values for each model. The $\pi/4$ and $\pi/2$ models show higher amplitudes in the mean field $B_\phi$ as well as in the $\text{EMF}'_\phi$ fluctuations. All values of $\alpha_{\phi\phi}$ are determined using a robust regression method and summarized in Table 1. A time evolution of the mean field and the turbulent $\text{EMF}'_\phi$ is presented in Figure 11 (right) for model $2\pi$, top, and model $\pi/4$, bottom. In Figure 11 (right), we divide the turbulent $\text{EMF}'_\phi$ with the measured $\alpha_{\phi\phi}$ (see also Table 1). The $\pi/4$ run shows higher fluctuations compared to the $2\pi$ run. A time evolution of $B_\phi \cdot \alpha_{\phi\phi}^{\text{NH}}/\text{EMF}'_\phi$ over height is presented in Figure 9 (bottom). We see that the sign of $\alpha_{\phi\phi}$ is well defined for the two hemispheres, reaching up to three scale heights of the disk.

### 3.7. $\alpha\Omega$ Dynamo

In this section we study the development of the mean magnetic fields along the radius. We show results from our full 2$\pi$ model as it represents the most realistic physical domain size. A contour plot of mean toroidal field, normalized over the square root of the pressure, is presented in Figure 12 (top right) over radius and time. All results in Figure 12 are averaged along azimuth and along $\theta$ between the midplane and two disk scale heights in the northern hemisphere. Figure 12 (top right) shows the irregular change of sign for the mean toroidal magnetic field along the radius. The timescale of the “butterfly” oscillations at a given radius can change because of radial interactions. The timescale of reversals of the toroidal magnetic field does vary from the 10 local orbital line (see Figure 12, top right, horizontal homogeneous $B_\phi$). The mean field configuration along radius can strongly affect the accretion stress (see Figure 3). The distribution of mean $B_\theta$ over the radius is more irregular compared to the toroidal field (see Figure 12, bottom left) although we observe a preferred sign of mean $B_\theta$ for a specific radial location, e.g., positive over time between 4 and 5 AU. A time evolution over a radius of $B_\phi \cdot \alpha_{\phi\phi}^{\text{NH}}/\text{EMF}'_\phi$ (Figure 12, bottom left) shows a change of the parity to symmetric for two butterfly cycles between 80 and 100 local orbits (also visible in Figure 10, solid line).
Figure 11. Top left: correlation between the mean toroidal magnetic field and the turbulent electromotive force (EMF) component EMF$'_\phi$ for the northern (upper) hemisphere of the disk and for all models. Rectangles show the limits of the data values. Bottom left: correlation between the mean toroidal magnetic field and the turbulent EMF component EMF$'_\phi$ for the southern hemisphere of the disk and for all models. Top right: time evolution of mean toroidal field (solid line), overplotted with the turbulent EMF (red dotted line) divided by $\alpha_{NH}$ for model $\pi$. Bottom right: time evolution of mean toroidal field (solid line), overplotted with the turbulent EMF$'_\phi$ (red dotted line) divided by $\alpha_{SH}$ for model $\pi/4$.

top left) shows again the positive sign of $\alpha_{\phi\phi}$ in the northern hemisphere (see also Figure 9, bottom). By definition, the $\alpha_{\phi\phi}$ presents the same distribution along the radius as the mean toroidal magnetic field. In contrast, we do not find a correlation between the turbulent velocity of the gas and the distribution of mean magnetic fields. Figure 12 (bottom right) presents $V_{\text{rms}}$ over radius and time for the northern hemisphere. The rms velocity is about 0.1$c_s$, nearly constant over radius and time. A time average of $V_{\text{rms}}$ is given in Table 1 for all models. We again emphasize the lower turbulent velocity in $\pi/4$, compared to $\pi/2$, due to the lack of the radial velocity peak (see Figure 5, right).

In our previous work, we have shown the $1/r$ profile for the turbulent magnetic fields (Flock et al. 2011). Because of the time oscillations, it is difficult to estimate a radial profile for the mean magnetic field. To determine a time-averaged radial profile of the mean toroidal field we measure the amplitude values of the oscillations. We use five different radial locations to measure the peak values of the mean toroidal field. The results are plotted in Figure 13 for the southern (blue) and northern hemisphere (red). The amplitudes of mean toroidal field decrease with radius. The relative low number of values and their high standard deviation make it difficult to fit. A $1/r$ profile would apply (Figure 13, green solid line). The values in both hemispheres look quite symmetric (Figure 9, blue and red) and we do not see a preferred hemisphere for the mean field generation.

4. DISCUSSION

After the saturation of MRI, the initial magnetic field configuration is lost. Each model develops oscillating mean magnetic fields which appear to be strongest in the $\pi/4$ and $\pi/2$ runs. The strength of turbulence follows this trend. The mean fields are generated by a dynamo process which relies on the symmetry and on the strength of the turbulent field. We measure the higher dynamo coefficient $\alpha_{\phi\phi}$ for the $\pi/2$ and $\pi/4$ models as well as higher Maxwell stresses. This agrees with the correlation between Maxwell stress and dynamo coefficient found by Rekowski et al. (2000). The effect of increased magnetic energy at domain size seems to be independent of resolution in stratified simulations (compare Figure 12, bottom left, models FO and PO in Flock et al. 2011) but not present in unstratified simulations (compare Figure 9(b) in Sorathia et al. 2011) as they do not develop a dynamo.

4.1. Energy Pile up and Magnetic Dynamo

Which physical process is sensitive to the domain size and lead to the increased mean toroidal fields in $\pi/4$ and $\pi/2$ models? The first mechanism leads to the dynamo process as it generates axisymmetric magnetic fields out of the turbulence. Another way to transport magnetic energy at domain size could be due to an inverse energy cascade. Johansen et al. (2009) showed in local-box simulations that the Keplerian advection term in the induction equations drives an inverse energy cascade.
Figure 12. Top left: contour plot of $B_\phi \cdot \alpha_{\phi\phi}^{NH} / \text{EMF}_\phi'$ over radius and time (see also Figure 9, bottom). Top right: mean toroidal magnetic field over radius and time. Bottom left: mean $\theta$ magnetic field over radius and time. Bottom right: turbulent rms velocity over radius and time. All plots are made for model $2\pi$ in the northern hemisphere.

This will lead to a transport of energy to larger scales. Also Rüdiger et al. (2007) found in Taylor–Couette experiments that MRI, launched from a toroidal field, will have most magnetic energy at the $m = 1$ and $m = 0$ mode.

Another open question is the sign of $\alpha_{\phi\phi}$ in global simulations. We find a positive $\alpha_{\phi\phi}$, independent of the azimuthal domain size. This positive $\alpha_{\phi\phi}$ has been indicated for global simulations by Arlt & Rüdiger (2001). Local simulations show a negative $\alpha_{\phi\phi}$ (Brandenburg et al. 1995; Brandenburg & Donner 1997; Rüdiger & Pipin 2000; Ziegler & Rüdiger 2000; Davis et al. 2010; Gressel 2010). The reason for stronger mean fields in reduced azimuthal models as well as the positive sign of $\alpha_{\phi\phi}$...
in global simulations has to be investigated in future work. One possibility would be to implement the “test field” method and to measure other components of the dynamo and diffusivity tensor, as was done in Gressel (2010).

4.2. Time Variability of Accretion Stress

Oscillating mean fields are organized in elongated radial patches, normally following the timeline of 10 local orbits. It can occur that for a given time, mean toroidal field of one sign covers the whole radial extent (3–8 AU). In such a case, temporal linear MRI will lead to a peak in accretion stress (Figure 3). The effect of the mean toroidal field, stretching over the whole radius, is independent of the azimuthal domain size, compare Figure 12 (top right). The amplification of accretion stress due to linear MRI is best visible in the π/4 model, as it presents the strongest amplitudes in the mean toroidal magnetic field.

4.3. Correlation Functions

We confirm the results of recent stratified global simulations by Beckwith et al. (2011). We find similar correlation angles (around 9°) and wavelengths (around H) for the magnetic field. A larger correlation length is expected because of the relative low resolution per scale height compared to local simulations (Guan et al. 2009; Hawley et al. 2011; Sorathia et al. 2011). Recent unstratified global simulations (Sorathia et al. 2011) suggest a magnetic tilt angle of around 13° for converged MRI turbulence. It remains unclear how this could be applied for stratified disks with a minimum of θB at the midplane. We found a magnetic tilt angle of around 13° above 2 scale heights. As discussed in Flock et al. (2011), we believe to find convergence with resolutions around 32/64 grid cells per pressure scale height. Here, a Fargo MHD approach as used in Sorathia et al. (2011) would be helpful.

5. SUMMARY

We have studied the impact of different azimuthal extents in three-dimensional global stratified MHD simulations of accretion disks onto the saturation level of MRI with an initial toroidal magnetic field.

1. Turbulence in restricted domain sizes like π/2 and π/4 is amplified due to strong toroidal mean field oscillations. For these runs, the λ_uni of the mean field is resolved leading to a temporal magnification of the αSS value and increased total magnetic energy. In addition, radial superpositions of such strong mean fields can drive a strong episodic increase of accretion. The time-averaged total αSS is 1.2 ± 0.2 × 10⁻² for model π/4, 9.3 ± 0.9 × 10⁻³ for model π/2, and converges to 5.5 ± 0.5 × 10⁻³ for both models π and 2π.

2. We find a positive dynamo αϕϕ for all models, a positive correlation between the turbulent EMF and the mean toroidal magnetic field in the upper (northern) hemisphere. For the 2π model we found αϕϕNorth = 2.1 ± 0.2 × 10⁻³. The π/2 and π/4 present higher αϕϕ values but with stronger fluctuations in EMF and mean Bϕ.

3. The π/4 and π/2 models show higher tilt angles and smaller correlation wavelengths in the two-point correlation of velocity and magnetic field compared to the π and 2π models. We find θrol = 14° for models ≥ π/2 and θrol = 12° for model π/4. The π/4 model does not resolve the peak radial velocity at m = 4. The tilt angles for the magnetic fields are smaller. At the midplane we observe time-averaged magnetic tilt angles between θB = 8°–9° increasing up to θB = 12°–13° in the corona. For the full 2π model we found ϕvelmaj = 1.9 H and ϕmagmaj = 1.7 H.

4. The parity of the mean magnetic fields is a mixture of dipole and quadrupole for all models. The total parity is set by the oscillating toroidal field. The timescale of symmetry change between dipole and quadrupole is around 40 local orbits. The time evolution of the parity is distinct in each model. The 2π model remains longer in a dipole (antisymmetric) dominated configuration for the simulation time.

We conclude that in global MRI simulations of accretion disks an azimuthal domain of at least π (180°) is needed to present the most realistic turbulent and mean field evolution as the full 2π model. Here, the αΩ dynamo plays a key role in determining the saturation level of MRI. Restricted domains of π/4 and π/2 amplify the MRI turbulence due to stronger axisymmetric magnetic fields.

We thank Andrea Mignone for providing us with the newest code version and the discussion on the numerical configuration, Sebastien Fromang for the helpful comments on the global models, Günther Rüdiger and Rainer Arlt for their comments on the manuscript, and Geoffroy Lesur for the discussion about the dynamo effect. H.K., N.D., and M.F. have been supported in part by the Deutsche Forschungsgemeinschaft DFG through grant DFG Forschergruppe 759 “The Formation of Planets. The Critical First Growth Phase.” N.T. was supported by a NASA Solar Systems Origins grant through the Jet Propulsion Laboratory, California Institute of Technology, and by an Alexander von Humboldt Foundation Fellowship for Experienced Researchers. Parallel computations have been performed on the Theo cluster of the Max Planck Institute for Astronomy, Heidelberg, as well as the GENIUS Blue Gene/P cluster, both located at the computing center of the Max Planck Society in Garching.

REFERENCES

Arlt, R., & Brandenburg, A. 2001, A&A, 380, 359
Arlt, R., & Rüdiger, G. 2001, A&A, 374, 1035
Armitage, P.J. 1998, ApJ, 501, L189
Balbus, S. A., & Hawley, J. F. 1991, ApJ, 376, 214
Balbus, S. A., & Hawley, J. F. 1998, Rev. Mod. Phys., 70, 1
Beckwith, K., Armitage, P. J., & Simon, J. B. 2011, MNRAS, 416, 361
