A COMPARATIVE STUDY OF OPTICAL/ULTRAVIOLET VARIABILITY OF NARROW-LINE SEYFERT 1 AND BROAD-LINE SEYFERT 1 ACTIVE GALACTIC NUCLEI

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ABSTRACT

The ensemble optical/ultraviolet (UV) variability of narrow-line Seyfert 1 (NLS1)-type active galactic nuclei (AGNs) is investigated, based on a sample selected from the Sloan Digital Sky Survey (SDSS) Stripe 82 region with multi-epoch photometric scanning data. As a comparison, a control sample of broad-line Seyfert 1 (BLS1)-type AGNs is also incorporated. To quantify properly the intrinsic variation amplitudes and their uncertainties, a novel method of parametric maximum likelihood is introduced that has, as we argued, certain virtues over previously used methods. The majority of NLS1-type AGNs exhibit significant variability on timescales from about 10 days to a few years with, however, smaller amplitudes on average compared to BLS1-type AGNs. About 20 NLS1-type AGNs that show relatively large variations are presented and may deserve future monitoring observations, for instance, reverberation mapping. The averaged structure functions of variability, constructed using the same maximum likelihood method, show remarkable similarity in shape for the two types of AGNs on timescales longer than about 10 days, which can be approximated by a power law or an exponential function. This, along with other similar properties, such as the wavelength-dependent variability, is indicative of a common dominant mechanism responsible for the long-term optical/UV variability of both NLS1- and BLS1-type AGNs. Toward the short timescales, however, there is tentative evidence that the structure function of NLS1-type AGNs continues to decline, whereas that of BLS1-type AGNs flattens with some residual variability on timescales of days. If this can be confirmed, it may suggest that an alternative mechanism, such as X-ray reprocessing, starts to dominate in BLS1-type AGNs, but not in NLS1-type AGNs, on such timescales.

Key words: galaxies: active – galaxies: photometry – galaxies: Seyfert – quasars: general

Online-only material: color figures

1. INTRODUCTION

Variations of optical/ultraviolet (UV) luminosity on timescales from weeks to years are characteristics of active galactic nuclei (AGNs), and the study of this phenomenon is a powerful tool for constraining models of black hole accretion. Extensive observational investigations in the past, albeit with mostly small samples, have revealed a dependence of the variability amplitude on various observed quantities, such as wavelength, time lag, luminosity, and redshift (e.g., di Clemente et al. 1996; de Vries et al. 2005; Meusinger et al. 2011; Welsh et al. 2011), that was confirmed by using vast quasars from the Sloan Digital Sky Survey (SDSS) with repeated scans (Vanden Berk et al. 2004; Zuo et al. 2012). With the advent of black hole mass estimation based on single-epoch spectroscopic data on AGNs, new correlations have been suggested for the variations in black hole mass (Wold et al. 2007; Bauer et al. 2009), and perhaps more fundamentally, with the Eddington ratio as claimed independently by a few groups very recently (Wilhite et al. 2008; Ai et al. 2010; MacLeod et al. 2010). The possible dependence of variability on fundamental physical parameters is remarkable as it may provide new insights into variability mechanisms. For instance, this dependence favors models of disk instability or variations in the accretion rate (e.g., Kawaguchi et al. 1998; Li & Cao 2008; Liu et al. 2008) over other models, such as the Poisson process (e.g., Terlevich et al. 1992; Torricelli-Ciamponi et al. 2000) and gravitational microlensing (e.g., Hawkins 1993). However, the statistical significance of these results is not yet sufficiently high and further confirmation is still needed.

While considerable progress has been made in the last few decades, previous variability studies were focused mostly on AGNs of the broad-line Seyfert 1 (BLS1) type, including quasars. Narrow-line Seyfert 1 (NLS1)-type AGNs, as a subclass of broad emission-line AGNs (BLAGNs) with extreme properties, have received little attention, however. This may be ascribed partly to the small number of such objects known in the past. NLS1-type AGNs are commonly defined as having small widths of optical permitted emission lines (FWHM (Hβ) ≲ 2000 km s⁻¹; Osterbrock & Pogge 1985). In addition, NLS1-type AGNs (see Komossa 2008 for a review) often show strong optical Fe ii emission and weak [O iii] lines (Gaskell 1985; Goodrich 1989; Véron-Cetty et al. 2001), a steep soft X-ray spectral slope (Boller et al. 1996; Wang et al. 1996), and strong X-ray variability (Leighly 1999; Grupe 2004). Such extreme properties make them cluster at one extreme end of the strongest AGN correlation space, namely eigenvector 1 (EV1; Boroson & Green 1992; Sulentic et al. 2000, 2007), which is believed to be driven by the Eddington ratio (Lbol / Ledd). NLS1-type AGNs are commonly thought to have small black hole masses and thus high accretion rates (at a substantial fraction of or close to the Eddington rate), as argued extensively in the literature (e.g., Minshige et al. 2000; Sulentic et al.
 calibration and a reliability check, for which only a concise description was given in the first paper because of space limitations. This paper is organized as follows. In Section 2 we describe the sample selection and photometric data analysis and in Section 3 the quantification of variability. The results are presented in Section 4, and their implications are discussed in Section 5, followed by a summary in Section 6. We use a $\Lambda$-dominated cosmology with $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$, $\Omega_m = 0.3$, and $\Omega_\Lambda = 0.7$ throughout the paper.

2. SAMPLE AND DATA

2.1. NLS1- and BLS1-type AGN Samples

Our NLS1-type AGNs are actually taken from the sample of Zhou et al. (2006), which was selected from the SDSS Data Release 3 catalogs. The procedures for the optical spectral analysis and measurement of emission-line parameters were described in detail in our previous papers (Zhou et al. 2006; Dong et al. 2008). Only objects with redshifts of $z \lesssim 0.8$ were included to cover the H$\beta$ and [O iii] emission lines, with the broad component of H$\alpha$ or H$\beta$ detected at $>10\sigma$ confidence level. We further select objects for a variability study using the following criteria.

1. Located within the SDSS Stripe 82 region (R.A. $> 310^\circ$ or $< 59^\circ$ and $-1:25 >$ decl. $< 1:25$).
2. Classified as “STAR” in all of the five bands by the SDSS photometric pipeline\(^7\) to minimize possible contamination from host galaxy starlight.
3. Radio-quiet or not detected in the FIRST radio survey,\(^8\) as constrained from the FIRST (Faint Images of the Radio Sky at Twenty cm survey; Becker et al. 1995) radio survey. This is to eliminate possible “contamination” from blazar-type jet emission, which originates from a distinct (non-thermal) radiation process and is often variable (Zhou et al. 2003, 2007; Yuan et al. 2008; Abdo et al. 2009; Liu et al. 2010; Jiang et al. 2012).

The above selections result in 58 NLS1-type AGNs. For a comparative study we also select a BLS1-type AGN sample in the Stripe 82 region, i.e., with the broad H$\alpha$ or H$\beta$ line width (FWHM) greater than 2200 km s$^{-1}$, the dividing line used to separate NLS1- and BLS1-type AGNs following our previous work (Zhou et al. 2006, see also Gelbord et al. 2009). The sample selections and spectral analysis are performed in the same way as for NLS1-type AGNs above and is described in Dong et al. (2008). The same criteria as above are also applied to select star-like, radio-quiet objects. Furthermore, to ensure that the two samples match each other on the redshift–luminosity distribution, a subsample of the resulting BLS1-type AGNs is randomly extracted in a way that they mimic the $z$–$M_i$ distribution of the NLS1-type sample, where $M_i$ is the $i$-band absolute magnitude.\(^9\) In this process we try to retain the size of the NLS1-type sample (with only a few outliers discarded) and prune the BLS1-type sample with a BLS1-type

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\(^7\) The SDSS photometric pipeline differentiates resolved (“GALAXY”) and unresolved (“STAR”) sources based on the difference between PSF and model magnitudes, and it was found that a simple cutoff of 0.145 mag works at the 95% confidence level for sources with $r \leq 21$ mag (Stoughton et al. 2002).

\(^8\) Defined as $R_{1.4} < 1$, where the radio-loudness $R_{1.4}$ is defined as the logarithm of the flux density ratio of radio emission at 1.4 GHz to the optical emission at the SDSS $g$ band.

\(^9\) A power-law continuum $f_\nu \propto \nu^{1.1}$ with a slope of $\alpha_\nu = -1.5$ is assumed in calculating the $i$-band absolute magnitude. The observed magnitudes have been corrected for Galactic extinction (Schlegel et al. 1998).
to NLS1-type ratio consistent with 2:1. This results in two final working samples of 55 NLS1-type and 108 BLS1-type AGNs, which are statistically compatible with each other on the redshift–luminosity plane (Figure 1). The two-dimensional Kolmogorov–Smirnov (K-S) test (Press et al.1992) yields a chance probability of 0.43 that the two samples have the same distribution.

The two working samples can be considered as optically and homogeneously selected, with reliably measured continuum and emission-line parameters (see Zhou et al. 2006; Dong et al. 2008). They consist of mostly quasars with $M_{i} < -23$ mag and $z \simeq 0.3–0.8$. All of the NLS1-type objects meet the conventional extra [O iii]/Hβ < 3 criterion for NLS1s (Zhou et al. 2006 for details). They also show, compared to the BLS1 sample, other properties characteristic of NLS1s, e.g., stronger Fe emission, smaller black hole masses, and higher Eddington ratios (Figure 2). The strength of the Fe emission lines is measured by the intensity ratios to Hβ, i.e., $R_{570} = \text{Fe} \lambda 4434–4684/\text{H} \beta$ (where $\text{Fe} \lambda 4434–4684$ denotes the Fe emission flux integrated over 4434–4684 Å, and $\text{H} \beta$ denotes the flux of the broad component of $\text{H} \beta$; see Zhou et al. 2006 for details). The black hole masses, $M_{BH}$, are estimated using the formalism given by Greene & Ho (2005) from the broad $\text{H} \beta$ FWHM and the 5100 Å luminosity ($L_{5100}$), which are taken from Zhou et al. (2006), or measured in the same way for the BLS1 sample. To calculate the Eddington ratio, $L_{bol}/L_{edd}$, the bolometric luminosities are estimated as $L_{bol} = 9 L_{5100}$ (Elvis et al. 1994).

2.2. Data and Photometric Calibration

The SDSS took images of the sky in five simultaneously broad photometric band, namely, $u$, $g$, $r$, $i$, and $z$ (Gunn et al. 2006; Fukugita et al. 1996). The integration time is 54.1 s in each band and the limiting magnitude reaches $\sim 23$ mag in the $r$ band. The photometric system is based on the AB system with a zero-point uncertainty of $\sim 0.02–0.03$ mag (Smith et al. 2002; Fukugita et al. 1996; Ivezić et al. 2004). The astrometric accuracy is better than 0.1′ for sources brighter than 20.5 mag in the $r$ band (Pier et al. 2003). We use the point-spread function (PSF) magnitudes in this work.

The SDSS Stripe 82 survey covers the region from $\alpha = 59^\circ$ to $310^\circ$ and $\delta = -1^\circ 25$ to $1^\circ 25$. During the SDSS-I phase (~2000–2005) the region was repeatedly observed and the central part of the stripe has been scanned at a cadence of typically 10–20 times (Adelman-McCarthy et al. 2007; Sesar et al. 2007). These observations were performed under generally photometric conditions and the data were well calibrated using the PHOTO software pipeline (Lupton et al. 2002). This region was later repeatedly scanned over the course of three three-month campaigns (September–November) in three successive years from 2005–2007, known as the SDSS Supernova (SN) survey. In this work, we use the photometric data acquired during both the SDSS-I phase from Data Release 5 (DR5; Adelman-McCarthy et al. 2007) and the SN survey during 2005 (SN-2005).

Observations in the SN survey were sometimes performed in non-photometric conditions. At the time when this work was started only the uncalibrated source catalogs were available. Thus, possible photometric zero-point offsets in the SN survey data need to be determined by calibrating against the DR5 magnitudes. To do this, we use stars in the same fields as “standards,” assuming that the vast majority of stars do not vary. The detailed procedures of re-calibration are described in Appendix A. The resulting overall (systematic and statistical) photometric errors of the calibrated SN survey magnitudes have a median of $\approx 0.03$ mag for the $g$, $r$, and $i$ bands, and $\approx 0.04$ mag for the $u$ and $z$ bands, which are comparable to those of the SDSS-I DR5 data. To check the reliability of our photometric calibration, we examine the calibrated SN survey data of 14 Landolt photometric standard stars (Landolt 1992) located in Stripe 82; none of them is found to show detectable variability.

3. QUANTIFICATION OF VARIATIONS AND THEIR UNCERTAINTIES

We construct light curves for each object in the five SDSS bands separately using the DR5 and the re-calibrated SN-2005 survey data. Objects observed in the same frames observed at different epochs are matched by using a matching radius of 1′. To eliminate the effects of bad observation conditions, only frames with good image quality (flagged as good or acceptable) are used. For each object there are typically ~27 observations (about 14 from the SN-2005 survey) spanning ~5 yr. It should be noted that all sample objects have similar sampling patterns, with most being observed repeatedly for three months (September–November) each year from 2000–2005.

First, we use the $\chi^2$ method to test the significance of variability for individual objects against the null hypothesis of no variation. The statistic $\chi^2$ (e.g., Bevington & Robinson 1992) is defined as

$$\chi^2 = \sum_{i=1}^{N} \frac{(m_i - \langle m \rangle)^2}{\xi_i^2},$$

where $m_i$ is the magnitude of the $i$th measurement with error $\xi_i$ (the overall error including both statistical and systematic errors) and $\langle m \rangle$ is the weighted mean of a total of $N$ measurements. We consider a source to be variable only if the probability level against the null hypothesis is $P < 0.1\%$.

The amplitude of the variability of an object is commonly measured by the width of its magnitude distribution. However, the magnitude distribution $\text{as observed}$ does not represent intrinsic variability, but rather is broadened by the effect of measurement errors. To assess the intrinsic magnitude distribution, we first under take a common practice that is used in the literature (e.g., Vanden Berk et al. 2004; Sesar et al. 2007) by estimating the contribution of measurement errors to the observed scatter.
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Figure 2. Distributions of the Hβ line width, the relative strength of Fe ii emission, black hole mass, and the Eddington ratio for NLS1-type (solid) and BLS1-type (dash-dotted) AGNs.

Figure 3. Example of the best estimates and confidence contours of the mean \( \langle m \rangle \) and dispersion \( \sigma_m \) of the intrinsic magnitude distribution at the 90% and 60% confidence levels in ugriz bands for one NLS1-type AGN.

find that the vast majority of NLS1-type AGNs show variations on timescales of years at the significance level \( P < 0.1\% \). Most of the variable objects varied at a level, \( \sigma_m > 0.05 \) mag, larger than the typical photometric errors. This is clearly demonstrated in Figure 4, which shows the reduced \( \chi^2 \) versus the variability amplitude in the \( r \) band as an example. We can see that for NLS1-type AGNs the intrinsic variability amplitude \( \sigma_m \) ranges from \( \sim 0.05 \) mag down to a few percent at \( \chi^2 = 3 \); the latter can be considered as the sensitivity of the variability detection limited by the current data quality. As can be seen, the optical/UV emission of NLS1-type AGNs is indeed variable on timescales of years, with mean variability amplitudes around 0.07–0.11 mag, depending on the bands.

The accumulative distributions of the variability amplitudes \( \sigma_m \) in the five SDSS bands are shown in Figure 5. Several results

4. RESULTS

4.1. Demography of Variability

Here we quantify the demography of the variability. We consider an object to be variable if the \( \chi^2 \) null-variability probability \( P < 0.1\% \), which corresponds to \( \chi^2 \geq 3 \) for a degree of freedom of 27, is roughly the mean number of data sampling of our objects. Table 1 lists the fraction of variable objects for both the NLS1- and BLS1-type AGN samples. We

(rms) and then subtract it from the latter (see Equation (B2) in Appendix B for a detailed account). This estimation is most accurate when the magnitude uncertainties \( \xi_i \) have the same or very similar values. However, this may not be the case for the SDSS SN survey data, which are taken under a variety of observing conditions. Moreover, for such an estimation and its variants, it is difficult to quantify the uncertainties of the estimated intrinsic variability amplitudes.

Alternatively, we introduce a parametric maximum likelihood method in which the intrinsic magnitude distribution of an object is parameterized as a Gaussian with mean \( \langle m \rangle \) and standard deviation \( \sigma_m \); the standard deviation \( \sigma_m \) is used as a measure of the amplitude of intrinsic variability. This method can take into account the uncertainty of each individual measurement (assumed to be Gaussian distributed). Moreover, it can also be used to quantify the confidence intervals for interesting parameters, such as \( \sigma_m \) (Figure 3). The method has been used in a similar way in the literature to derive the intrinsic distribution of observables that suffer from measurement uncertainties (e.g., Maccacaro et al. 1988). The description of this method and a comparison with the one above are in Appendix B. The two methods are found to be in excellent agreement. Hence in this paper we use the amplitudes derived from the maximum likelihood method since their confidence intervals can be estimated for each object.
shift $z$, $\lambda_0$ corresponds to a wavelength $\lambda = \lambda_0/(1 + z)$ in the object’s rest frame. Thus, for each object the variability $\sigma_m$ at five rest-frame wavelengths is sampled corresponding to the five SDSS bands. We divide the overall rest wavelength range sampled here, 1900–7100 Å, into five bins of equal bin size in log $\lambda$.10 The relationship of amplitude $\sigma_m$ versus rest-frame effective wavelength $\lambda$ is shown in Figure 7. The mean value and scatter (standard deviation) of the $\sigma$ distribution in each wavelength bin are also overplotted. The result clearly shows a trend of increasing variability amplitude moving toward a short wavelength for NLS1-type AGNs, similar to BLS1-type AGNs. It clearly shows that at a given wavelength NLS1-type AGNs have systematically lower variation amplitudes compared to BLS1-type AGNs, albeit large scatters. We test the significance of the differences in the $\sigma_m$ distributions between the two samples using the K-S test, yielding the chance probability for the same distribution as 1.2%, 9.7%, and 2.9% for the shortest three wavelength bins, respectively.

The trend in Figure 7 is in a statistical sense only, and it would be interesting to examine whether it also holds for variability in individual objects. For each object, we estimate the maximum magnitude difference over the entire light curve in the blue ($u$) and red ($i$) bands, respectively, defined as $\Delta m = m_{t2} - m_{t1}$, where $t1$ and $t2$ are the epochs of the two observations and $t2 > t1$. The result is shown in the left panel of Figure 8; the dotted line indicates the 1:1 relationship where the variations in the two bands are identical in both the sign and amplitude. Two conclusions can be inferred from the figure. First, $\Delta m$ in both the blue and red bands always keep the same signs, meaning that the two bands vary (fading or brightening) coordinately; that is to say, the optical/UV continuum level is moving up and down systematically, rather than seesawing around. Second, for all but a few objects the variations in the blue are indeed larger than those in the red band. This suggests that, even in individual AGNs, the brightness variation tends to go hand in hand with variation in the continuum slope, with larger variability in blue than in red. This is the same for both NLS1- and BLS1-type AGNs.

A direct consequence of such wavelength-dependent variability is that the optical/UV continuum becomes redder (bluer) when the overall continuum level becomes fainter (brighter). This can be demonstrated explicitly in the right panel of Figure 8, where the changes in the color ($\Delta(m_u - m_i)$) are plotted against the changes of magnitude. As can be seen, for all but a few objects of both BLS1- and NLS1-type AGNs the

### Table 1

| Bands | NLS1-type AGNs | BLS1-type AGNs |
|-------|----------------|----------------|
|       | $P(\chi^2 | v) \leq 0.001^a$ | $P(\chi^2 | v) \leq 0.001^a$ |
| $u$   | 85.4          | 93.5           |
| $g$   | 98.3          | 99.1           |
| $r$   | 98.3          | 99.1           |
| $i$   | 100.0         | 97.2           |
| $z$   | 81.3          | 85.0           |

Notes.

- $^a$ Fraction of sources with $P(\chi^2 | v) \leq 0.001$.
- $^b$ Fraction of sources with $\sigma_m \geq 0.05$.
- $^c$ Fraction of sources with $\sigma_m \geq 0.10$.
- $^d$ Mean $\sigma_m$ in each band.

Figure 4. $r$-band variability amplitudes vs. reduced $\chi^2$ for NLS1-type AGNs.
Figure 5. Accumulative distribution of the variability amplitudes, $\sigma_m$, for NLS1-type (filled circles) and BLS1-type (open circles) AGNs in the five SDSS photometric bands. The $x$-axis ranges truncate at 0.3 mag for clarity.

Table 2

NLS1-type AGNs with $\sigma_r > 0.1$ mag

| SDSS Name         | Redshift | FWHM(H$\beta$) | $\log\lambda L_{5100}$ | $R_{5770}$ | $\langle r \rangle$ | $\sigma_g$ | $\sigma_r$ | max$\Delta g$ | max$\Delta r$ |
|-------------------|----------|----------------|-------------------------|------------|---------------------|------------|------------|---------------|---------------|
| J003431.74-001312.7 | 0.381    | 1701 ± 31      | 44.47                   | 0.27 ± 0.01| 18.14               | 0.11 ± 0.05| 0.10 ± 0.04| 0.37 ± 0.33   | 0.59 ± 0.77   |
| J010737.01-001911.6 | 0.737    | 1853 ± 57      | 45.10                   | 0.78 ± 0.04| 18.34               | 0.19 ± 0.08| 0.18 ± 0.08| 0.56 ± 0.59   | 0.63 ± 0.77   |
| J011712.82-005817.4 | 0.485    | 1987 ± 65      | 44.51                   | 0.49 ± 0.03| 19.24               | 0.15 ± 0.05| 0.14 ± 0.04| 0.49 ± 0.47   | 0.56 ± 0.77   |
| J012824.21+001925.2 | 0.419    | 1552 ± 132     | 44.13                   | 0.15 ± 0.06| 19.70               | 0.16 ± 0.05| 0.17 ± 0.05| 0.55 ± 0.53   | 0.63 ± 0.77   |
| J013509.51+002522.0 | 0.744    | 1890 ± 129     | 44.73                   | 0.96 ± 0.12| 18.99               | 0.17 ± 0.07| 0.15 ± 0.06| 0.49 ± 0.43   | 0.56 ± 0.64   |

Notes. Column 1: SDSS name; Column 2: redshift; Column 3: H$\beta$ line width (km s$^{-1}$); Column 4: monochromatic luminosity at 5100 Å (erg s$^{-1}$); Column 5: intensity ratio of the Fe$^{II}$ multiplets to H$\beta$; Column 6: mean magnitude in the $r$ band; Columns 7 and 8: best estimated variability amplitudes and corresponding errors at 90% confidence level from the maximum likelihood method; Columns 9 and 10: maximum magnitude changes.
spectra redden ($\Delta(m_r - m_i) > 0$) while objects fade away ($\Delta m > 0$), and vice versa. Furthermore, there is a strong correlation between the change of color ($\Delta(m_r - m_i)$) and the change of magnitude ($r_s = 0.79$, $P \leq 10^{-5}$, the Spearman rank correlation test). This implies that the larger the variation of the continuum level is, the more the spectral shape changes in general. We note that, regarding the dependence of variability on wavelength and spectral shape, NLS1- and BLS1-type AGNs show very similar behavior.

4.3. Time Dependence of Variability

To characterize the dependence of variation on time lags, the structure function is widely used (see, e.g., Collier & Peterson 2001; Vanden Berk et al. 2004). In this paper we construct the structure function with the maximum likelihood method, as described in Section 3, to parameterize the intrinsic variability amplitude for the ensembles instead of individuals. First, for each object, we calculate the magnitude difference between any two observations separated by an interval, $\Delta \tau$, in the object rest frame, $\Delta m = m(t) - m(t - \Delta \tau)$. The error of $\Delta m$ is estimated using error propagation. The combined data points for all objects in the samples are then grouped into various bins according to time lags. The time-lag bins are divided in a way that there are at least 600 data points in each bin. In each bin of time lag, the mean of the $\Delta m$ distribution is expected to be around zero for a large number of data points, since objects brighten or dim randomly and evenly in the statistical sense. The dispersion of the $\Delta m$ distribution, in fact, reflects the degree of variations (at the given time lag). We take the intrinsic dispersion of the $\Delta m$ distribution in each bin as a measure of the “averaged” amplitude of variability, i.e., the structure function. The intrinsic dispersion can be obtained with the assumption that the $\Delta m$ distribution in each bin can be approximated by a Gaussian, which is verified in the majority of the time-lag bins. The uncertainties of the intrinsic dispersion can also be obtained at the same time. In this way, we construct the structure function with the maximum likelihood method.

We first construct the structure function for NLS1- and BLS1-type AGNs in each SDSS photometric band for easy comparison with previous results of BLAGNs and quasars. The derived structure functions are shown in Figure 9 in only the $g$ and $r$ bands of Figure 9 for clarity. We find that the variability amplitude of NLS1-type AGNs, similar to BLS1-type AGNs, does increase with a time lag from days to years in all of the SDSS bands. At a given timescale, NLS1-type AGNs have systematically lower variation amplitudes than BLS1-type AGNs, and a similar result was detected by Vagnetti et al. (2013). A simple power-law model is used to characterize the structure function at time lags greater than 10 days,

$$SF(\Delta \tau) = \left(\frac{\Delta \tau}{\Delta \tau_0}\right)^\beta,$$

(2)
Figure 6. (Continued)

Figure 7. Variability amplitude as a function of rest-frame wavelength for NLS1-type (left) and BLS1-type (right) AGNs. The mean value in each bin is also shown with the error bar representing the standard deviation of the distribution in each bin.

Figure 8. Left: comparison of the maximum magnitude differences between the $u$ and $i$ bands for each NLS1-type (filled circles) and BLS1-type (open circles) AGNs; right: the changes in the color vs. the maximum magnitude differences.
where $\Delta \tau_0$ and $\beta$ are to be determined. The fitting results are given in Table 3 and Figure 9. It can be seen that a power-law model can represent the structure function reasonably well. The inferred power-law slopes for NLS1-type AGNs range from $0.02$ to $0.40 \pm 0.01$, which are mostly consistent with those of the BLS1s within the uncertainties.

Considering the redshift effect, a more physically meaningful approach is to construct the structure function in rest-frame wavelength bins instead of in photometric bands. We divide the whole rest of the wavelength range into five bins with equal bin size in logarithm, as described in Section 4.2. The constructed structure functions for both NLS1- and BLS1-type AGNs in each bin are shown in Figure 10. As expected, the time-lag dependence of variability is present in all the bins. Wavelength dependence variability is also clearly present in almost all of the time-lag bins, which is consistent with what was found above and also in previous work on BLAGNs. Again, the two types of AGNs show remarkable similarity in the shape of their structure function on timescales longer than about 10 days.

On shorter timescales (below about 10 days), however, the structure functions of the two types seem to differ. The structure function of NLS1-type AGNs continues to drop, whereas that of BLS1 seems to flatten toward short time lags, with some residual variability of several percent. This is the case for all the wavelength bins except for the shortest wavelength, for which the photometric uncertainties are the largest. Also, we note that the wavelength-dependence effect becomes weak or even vanishes on such timescales. This may indicate that a different mechanism of variability starts to dominate. However, this is not the case for NLS1-type AGNs, of which the structure function continues to drop toward short time lags. Given the sparse sampling of the data used here, further confirmation is needed to verify this trend.

The structure functions in various rest-frame wavelength bins are also fitted with a power-law model within the time-lag range of greater than 10 days. The fitted curves are shown in Figure 11 for the bin of 2500–3300 Å as an example, and the results are given in Table 3. Alternatively, an exponential model (MacLeod et al. 2010) is also used,

$$SF(\tau) = SF_{\infty}(1 - e^{-|\Delta \tau/\tau|^\beta})^{1/2},$$

where $SF_{\infty}$ and $\tau$, the timescales, are to be determined. With the inferred $SF_{\infty}$ we can estimate the long-term standard deviation of the variability to be $0.5(SF_{\infty})^2$. It can be seen that, although both functions can approximate the observed structure functions reasonably well (at $\tau > 10$ days), the exponential function gives a somewhat better description than the power law for the NLS1 sample based on the $\chi^2$ statistic. This may be ascribed to a possible flattening of the structure function at the largest time lags, which appears to be more significant for the NLS1- than for BLS1-type AGNs (see Figures 10 and 11). However, the

Table 3

| Band          | NLS1-type AGNs | BLS1-type AGNs |
|---------------|----------------|----------------|
|               | $\Delta \tau_0$ (days) | $\Delta \tau_0$ (days) | $\beta$ | $\beta$ |
| $u$           | $7.3 \times 10^4 \pm 0.33$ | $1.7 \times 10^4 \pm 0.36$ | 0.01 |
| $g$           | $6.3 \times 10^4 \pm 0.36$ | $2.2 \times 10^4 \pm 0.37$ | 0.01 |
| $r$           | $5.4 \times 10^4 \pm 0.40$ | $3.1 \times 10^4 \pm 0.35$ | 0.01 |
| $i$           | $8.1 \times 10^4 \pm 0.39$ | $4.3 \times 10^4 \pm 0.36$ | 0.01 |
| $z$           | $3.6 \times 10^5 \pm 0.32$ | $5.7 \times 10^4 \pm 0.37$ | 0.01 |
| $\Delta \lambda_{1900-2500}$ Å | $1.4 \times 10^5 \pm 0.30$ | $2.1 \times 10^4 \pm 0.34$ | 0.01 |
| $\Delta \lambda_{2500-3300}$ Å | $6.6 \times 10^4 \pm 0.36$ | $2.9 \times 10^4 \pm 0.35$ | 0.01 |
| $\Delta \lambda_{3300-4200}$ Å | $5.7 \times 10^4 \pm 0.38$ | $2.6 \times 10^4 \pm 0.36$ | 0.01 |
| $\Delta \lambda_{4200-5500}$ Å | $6.5 \times 10^4 \pm 0.40$ | $3.2 \times 10^4 \pm 0.38$ | 0.01 |
| $\Delta \lambda_{5500-7100}$ Å | $3.7 \times 10^5 \pm 0.33$ | $8.6 \times 10^4 \pm 0.34$ | 0.01 |

where $\Delta \tau_0$ and $\beta$ are to be determined. The fitting results are given in Table 3 and Figure 9. It can be seen that a power-law model can represent the structure function reasonably well. The inferred power-law slopes for NLS1-type AGNs range from $0.32 \pm 0.02$ to $0.40 \pm 0.01$, which are mostly consistent with those of the BLS1s within the uncertainties.

Figure 9. Structure functions of the NLS1-type (filled circles) and BLS1-type (open circles) AGNs in the $g$ (blue) and $r$ (red) bands. The errors are at the 90% confidence level. The dashed lines are the power-law model fittings to data at $\Delta \tau > 10$ days. (A color version of this figure is available in the online journal.)
The values of $SF_{\infty}$ year and increase toward shorter rest-frame wavelength bins. The inferred variability timescales are around one time lags in the current data set are not long enough to draw that conclusion. The fittings with the power law (dotted) and exponential model (dashed) are also presented.

Figure 10. Structure functions of the BLS1-type (left) and NLS1-type (right) AGNs in rest-frame wavelength bins of $\Delta \lambda$(1900–2500 Å) (blue), $\Delta \lambda$(2500–3300 Å) (green), $\Delta \lambda$(3300–4200 Å) (red), $\Delta \lambda$(4200–5500 Å) (pink), and $\Delta \lambda$(5500–7100 Å) (black).

(A color version of this figure is available in the online journal.)

Figure 11. Structure functions of the NLS1-type (filled circles) and BLS1-type (open circles) AGNs in the rest-frame wavelength bin, 2500–3300 Å. The fittings with the power law (dotted) and exponential model (dashed) are also presented.

time lags in the current data set are not long enough to draw that conclusion. The inferred variability timescales are around one year and increase toward shorter rest-frame wavelength bins. The values of $SF_{\infty}$, i.e., the maximum variation amplitude at the longest possible time lag, as expected, are larger at shorter wavelengths than longer, and larger for BLS1- than NLS1-type AGNs in all the rest of the wavelength bins.

5. DISCUSSION

5.1. Implications for the Optical/UV Variability of NLS1s

The similarities found between NLS1- and BLS1-type AGNs in most of the properties of optical/UV variability, except for systematically smaller amplitudes of NLS1s, indicate that their long-term variability must be driven by the same mechanism. Recently, there has been compelling evidence that the variability is intrinsic to AGN activity, specifically the detected correlations of optical/UV variabilities with black hole masses and/or Eddington ratios, as investigated in our previous study (Ai et al. 2019; Wilhite et al. 2020; Zuo et al. 2020). The variability we observed may be caused by variations in the accretion rates (see Gaskell 2008 for a different view, however), or some kind of (local) disturbance and its propagation in the disk (Czerny 2006). Assuming the first case, Li & Cao (2008) proposed a simple model of global change for the disk structure due to variation in accretion rates based on the standard disk, which explained the previously observed $M_{BH}$ and luminosity dependence of variability. In fact, this model can also reproduce qualitatively an inverse amplitude–$L_{bol}/L_{Edd}$ relation (S. Li 2008, private communication), which is $M_{BH}$-dependent, though a quantitative comparison with our data is hampered due to the small sample size.

Let us consider the case of the local instability scenario. From the simple standard disk model (Shakura & Sunyaev 1973), the emission at a given radius $R$ is dominated by a local blackbody with $T_{eff}$ peaking at a wavelength $\lambda$, where

$$h\lambda/\sigma \sim 2.8kT_{eff}.$$  

The radiation is balanced by the energy generated per unit disk area at radius $R$,

$$\sigma T_{eff}^4 \sim 3GM_{bh}M/(8\pi R^3),$$  

where $M$ is the mass accretion rate. Combining the two relations we find that radius $R$ is where the local blackbody emission peaks at $\lambda$, in units of the Schwarzschild radius $R_{Sch}$, $R \equiv R_{Sch} \sim (m/M_{bh})^{1/3}\lambda^{2/3}$, where $m$ is the mass accretion rate in units of the Eddington rate. This means that scaled radius $r$, which dominates the emission in a given bandpass, becomes larger with an increasing Eddington ratio. If the disturbance is generated in the inner part of the disk and propagating outward, it would attenuate in the course of propagation. This would account for the ensemble weaker variability of NLS1-type AGNs compared to BLS1-type AGNs, since radius $r$, which is responsible for the emission in the same bandpass, is shifted further out as the Eddington ratio increases. In contrast, if the disturbance is generated in the outer disk and is propagating inward, the disturbance must be amplified instead.

Reprocessing of the X-ray emissions, which originate close to the central black hole and are known for strong and rapid variability, into the optical/UV bands can also cause optical/UV variability in AGNs (e.g., Czerny 2006; Gaskell 2008). Yet the multi-band variability correlation studies in individual AGNs indicate that the dominant mechanism for the long-term AGN optical/UV variability cannot be the reprocessing of variable X-ray emission (e.g., Arévalo et al. 2008, 2009). Our result of statistically weaker variability of NLS1-type AGNs compared to BLS1-type AGNs in the optical/UV bands supports this argument, since NLS1-type AGNs often show strong X-ray variability on both short and long timescales (e.g., Papadakis...
5.2. Implication for the Nature of NLS1-type AGNs

We have demonstrated that, on all timescales observed so far, NLS1-type AGNs show systematically less optical/UV variations compared to BLS1-type AGNs. This is in agreement with the dependence of the variability on the Eddington ratio for BLAGNs found in our previous paper (Ai et al. 2010). In fact, as we argued, the optical/UV variability may be considered as a new EV1 parameter, with NLS1s lying at one extreme end. This property gives interesting insight into the nature of NLS1-type AGNs. Currently, there are three scenarios regarding the nature of NLS1-type AGNs. The widely accepted model invokes less massive black holes accreting at high rates compared to BLS1s (e.g., Minshige et al. 2000; Sulentic et al. 2000; Wang & Netzer 2003). Alternatively, the narrow Balmer line width can also be explained by a disk-like low-ionization BLR close to face-on (the “orientation scenario,” e.g., Osterbrock & Petke 1985; Collin & Kawaguchi 2004), or by a more distant BLR compared to normal BLS1s with a similar black hole mass or nuclear luminosity (“distant BLR scenario”; Wandel & Boller 1998). Although face-on inclination is found in a small number of extreme radio-loud NLS1s showing blazar-like properties (Yuan et al. 2008; Abdo et al. 2009; Gu & Chen 2010), comparative studies of radio properties between NLS1 and BLS1 disfavor the orientation scenario in general (Zhou & Wang 2002; Komossa et al. 2006; Zhou et al. 2006). Other lines of evidence, such as polarization properties and correlations involving narrow-line luminosity, suggest that orientation at most plays a secondary role in explaining NLS1 phenomenon (Komossa 2008 and references therein).

Neither the “orientation” nor “distant BLR” scenario can explain naturally the extension of EV1 correlations involving variability. For the “distant BLR” scenario, it is hard to link the size of the BLR to the optical/UV continuum variability that is local to the accretion disk. Also, since the optical continuum emission is believed to come from an accretion disk as long as the disk is directly seen, variability amplitude should not be dependent on the orientation of the disk. Optical/UV variability may be caused by obscuration by the outer part of the disk; however, a lack of periodic variability corresponding to the disk rotation has ruled out this possibility (e.g., Peterson & Bentz 2006; Wold et al. 2007). In comparison, the commonly accepted model for NLS1s with smaller $M_{BH}$ and higher $L_{bol}/L_{Edd}$ is favored. Their weak variability fits more naturally into the scenario that variability amplitude is governed by $L_{bol}/L_{Edd}$, as suggested in our previous paper.

6. SUMMARY

We present the ensemble variability property of a sample of NLS1-type AGNs that have multi-epoch photometric observations in the SDSS Stripe 82 region. For NLS1-type AGNs the variability was hitherto poorly explored as a class due to both the small sample sizes and limited data available. As a direct comparison, a control sample of BLS1-type AGNs is also compiled. We introduce a novel parametric maximum likelihood method in variability studies to quantify intrinsic variability amplitudes and, for the first time, their confidence intervals, which improves upon the commonly adopted methods in the literature. This method makes full use of the measurement error of each data point, and has the advantage of being sensitive to low-level variations.

We find that the majority of the NLS1-type AGNs are in fact variable on timescales from about several days to a few years, e.g., more than 80% of the objects varied at levels $\geq 0.05$ mag (standard deviation) in the SDSS $u$ and $g$ bands. NLS1-type AGNs have systematically smaller variability compared to BLS1-type AGNs. This is consistent with the previously found anti-correlation of AGN variability with the Eddington ratio (Wilhite et al. 2008; Ai et al. 2010; MacLeod et al. 2010). We present the light curves and some of the key parameters for 22 NLS1-type AGNs that show relatively large variations (standard deviation $\sigma > 0.1$ mag in the $r$ band), which may deserve further monitoring observations. We also found that, similar to BLS1-type AGNs (including quasars), the optical/UV
variability is wavelength dependent—the shorter the wavelength, the larger the variations. We derive, for the first time, the ensemble structure function of variability for NLS1-type AGNs that span nearly three orders of magnitudes in time lags (from a few days to $\sim 10^3$ days). The structure functions of NLS1- and BLS1-type AGNs show remarkably similar profiles on time lags $\gtrsim 10$ days, with the former having a smaller amplitude at a given time lag. The structure function can be approximated by either a power law or an exponential function, though the latter seems to give a better description for NLS1-type AGNs. On timescales of $\lesssim 10$ days, the variability amplitudes of BLS1-type AGNs remain somewhat higher than those extrapolated from the power-law structure function, indicating the presence of some excess variations. However, this is not the case for NLS1s. We suggest that this excess variation component likely arises from the reprocessing of X-rays, which might be more important in BLS1- than in NLS1-type AGNs.

In conclusion, NLS1- and BLS1-type AGNs share very similar properties of long-term optical/UV variability in many ways, such as the wavelength and time-lag dependence of variability. This suggests that most likely the same physical mechanisms are at work to produce the long-term optical/UV variability in both types, in spite of the possibly different modes of their accretion flows. Future photometric monitoring programs for larger samples of AGNs spanning wide parameter ranges with a large dynamic range of sampling rate are needed to address the questions related to optical/UV variability, such as the behavior at time lags less than a week and characteristic variability timescales.

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APPENDIX A

RE-CALIBRATION OF SUPERNOVA SURVEY DATA

Re-calibration of SN data is performed separately in five bands in a field-by-field (100 arcmin$^2$ patches) manner, and only fields with a “good” image quality (flagged as “good” or “acceptable”) were used for photometric calibration or light curve construction in our work. Photometric conditions can be regarded to be unchanged across the field. For each of the SDSS SN survey fields, we only use the most overlying DR5 field to do calibration. In the case of more than one overlapped DR5 fields, we use the one with the larger or largest overlapping area. In each field and each band, “standard stars” with good measurements (high-quality photometry) are selected with the following criteria.

1. Sources must be “star.”
2. The processing flags BRIGHT, SATURATED BLEND, EDGE, NOPROFILE, INTERP_PROBLEMS, and DEBLEND_PROBLEMS are not set in any band.$^{11}$
3. Sources must be brighter than the magnitude limits, which are 22.0, 22.2, 22.2, 21.3, and 20.5 in the $ugriz$ bands.
4. Potentially variable sources were excluded, including QSOs, known variable stars, etc. Here, QSO identification was based on the SDSS quasar catalog III. (Schneider et al. 2005), and variable stars were based on the fourth edition of the General Catalog of Variable Stars (GCVS4; Khlopopov et al. 1998).

The selected stars were then cross-identified between the two most overlying fields with a matching radius of 1$''$. Any star with no match or more than one match within this radius was discarded. We computed various photometric statistics, such as mean magnitudes, mean standard errors (weighted by statistical errors), and rms scatter of the magnitude difference of the matched stars. Stars with a magnitude difference of more than two times the rms scatter were rejected in order to avoid variable stars, mismatched stars, or bad data points, etc. The above procedure was repeated until no more stars were excluded. The final number of calibration stars is typically $\sim 50–200$ for each pair of fields, which is large enough to perform calibration with an acceptable uncertainty. For each of the fields, the differential magnitudes of the calibrating stars between the SN survey and DR5 observations ($\Delta m = m_{SN} - m_{DR5}$) are calculated, and their weighted mean is set to zero, ($\Delta m_0 = 0$). In this way, the zero-point offsets of the SN survey photometry are determined.

The internal uncertainties of the calibrated zero-points of the SN survey photometry with respect to the reference DR5 data, as measured by the standard deviation of the weighted mean of $\Delta m$, i.e., $\sigma_{\Delta m_0}$, are less than 0.01 mag for all the fields. However, the SDSS SN survey data are calibrated using different fields of SDSS-I DR5 data, which have typical systematic photometric zero-point errors ($\sigma_{zpt}^{DR5}$) of 0.01 mag in the $g$, $r$, and $i$ bands, 0.02 mag in the $z$ band, and 0.03 mag in the $u$ band (Ivezić et al. 2004). Thus, the total systematic uncertainties of the SN survey photometry are estimated to be the combination in quadrature of the above two terms, i.e., $\sqrt{\sigma_{\Delta m_0}^2 + \sigma_{zpt}^{DR5}^2}$. The overall photometric errors are then a contribution of both the systematic and statistical errors. The latter are given in the SN survey catalogs and range from a few percent to 0.1 or higher toward faint magnitudes, depending on the bands. The final photometric errors of the calibrated SN survey data have a median of $\approx 0.03$ mag in the $g$, $r$, and $i$ bands, and somewhat higher ($\approx 0.04$ mag) in the $u$ and $z$ bands. They are comparable to those of the DR5 data observed in SDSS-I.

As an example, the SN data of one field before and after calibration are illustrated in Figure 12. Actually our photometric accuracy is much better than the rms scatter shown in Figure 12 because most of the objects in our samples are brighter than 20.0 mag in the $r$ band.

APPENDIX B

AMPLITUDE OF VARIABILITY AND THE MAXIMUM LIKELIHOOD METHOD

A common practice used in the literature is to estimate the contribution of the errors of measurement to the observed scatter and then subtract it from the latter (Vanden Berk et al. 2004; Sesar et al. 2007). The observed rms scatter $\Sigma$ is then calculated as

$$
\Sigma = \sqrt{\frac{1}{n-1} \sum_{i=1}^{N} (m_i - \langle m \rangle)^2}.
$$

11 See http://www.sdss.org/dr7/products/catalogs/flags.html.
Figure 12. Illustration of the zero-point calibration for one field in the SDSS SN survey with the cross-identified star from the most overlying field in DR5. The dots in the left panels represent magnitude differences between the DR5 and the original SN magnitudes as a function of the DR5 magnitude. The dashed line shows the mean magnitude differences weighted by statistical errors. The middle panels show the magnitude differences after our calibration. The histograms of the magnitude differences before (dashed line) and after (solid line) calibration are compared in the right panels.

where $m_i$ are observed magnitudes of $N$ observations and $(m)$ is their weighted mean. The intrinsic variation amplitude $\sigma_m$ can be estimated as

$$\sigma_{\text{method} - 1} = \begin{cases} \frac{(\Sigma^2 - \xi^2)^{1/2}}{\Sigma}, & \text{if } \Sigma > \xi, \\ 0, & \text{otherwise}, \end{cases} \quad (B2)$$

where $\xi$ is the term representing the amount of scatter caused by measurement errors. The estimation of $\xi$ has some variances among the literature. For example, Sesar et al. (2007) used the mean photometric errors of SDSS observations as a function of magnitude that was fitted from the data assuming most stars are not variable; a theoretically expected $\xi(m)$ as a function of magnitude was also given in Strateva et al. (2001). Here we estimate $\xi$ as the mean square value of the errors $\xi_i$ associated with the individual magnitude $m_i$, i.e.,

$$\xi^2 = \frac{1}{N} \sum_{i=1}^{N} \xi_i^2. \quad (B3)$$

This estimation is similar to that used in Rodriguez-Pascual et al. (1997), which was, however, in the flux domain rather than magnitude.

In this paper, we introduce a new method—the maximum likelihood method—to quantify $\sigma_m$ and its uncertainty, in line with the parametric statistics approach. There are two assumptions involved in this method. First, we assume that for a given measurement the probability function of the random fluctuations in the measurement follows a Gaussian distribution,\(^{12}\) with a standard deviation of $\sigma_r$, where $\sigma_r$ is the photometric error for this measurement. Furthermore, for simplicity we approximate the intrinsic distribution of the magnitude of a variable object as a Gaussian\(^{13}\) with a mean $(m)$ and a standard deviation $\sigma_m$—both parameters are to be estimated. $\sigma_m$ gives a measure of the amplitude of variability, and is expected to be consistent with zero in case of no variation.

The final probability function of observing an object to have a magnitude $m$ with a photometric uncertainty is then

$$p(m) = \frac{1}{\sqrt{2\pi \varphi}} \exp \left[ -\frac{(m - \langle m \rangle)^2}{2\varphi^2} \right]. \quad (B4)$$

Here $\varphi = (\sigma_{\text{method}}^2 - \xi^2)^{1/2}$. Suppose, for a given object a set of values for $m$ are observed, $m_1, \ldots, m_N$, with the corresponding (different) accuracies of the measurements characterized by

\(^{12}\) This is supported by the fact that for stars (mostly non-variable) repeatedly observed in the SDSS with similar photometric errors, their magnitude distributions can be well described as a Gaussian (Ivezic et al. 2003).

\(^{13}\) We consider this to be a reasonable approximation, as it is found from our data that, for some objects with a much larger amplitude of intrinsic variations than their typical photometric errors (i.e., the latter is negligible), their magnitude distributions can be reasonably well described by a Gaussian.
the standard deviations $\sigma_1, \ldots, \sigma_N$. The likelihood function, $L(\langle m \rangle, \sigma_{\text{method}-2})$, of this data set is

$$\prod_{i=1}^{N} \frac{1}{\sqrt{2\pi}(\sigma_{\text{method}-2}^{2}+\sigma_{r}^{2})^{1/2}} \exp \left[-\frac{(m_i - \langle m \rangle)^{2}}{2(\sigma_{\text{method}-2}^{2}+\sigma_{r}^{2})}\right].$$

The maximum likelihood estimates of $\langle m \rangle$ and $\sigma_{\text{method}-2}$ for a given object are obtained by minimizing $S = -2 \ln L$.

This method can also be used to find the confidence intervals for interesting parameters by applying the standard $\Delta \chi^2$ techniques to the above $S$ function (see, e.g., Avni 1976). Suppose the parameter vector $\theta$ has two components, and the first “interesting component” $\theta_q$ consists of $q$ parameters ($\theta_1, \ldots, \theta_q$), which need to be estimated simultaneously without considering the second “uninteresting component” $\theta_u$ (see Avni 1976). The confidence interval at a probability level $\alpha$ for $\theta_q$ can be computed by finding the set of all values of $\theta_q$ such that

$$S(\theta_q, \text{minimize over } \theta_u) - S_{\min} \leq \Delta(q, \alpha),$$

where $\Delta(q, \alpha)$ follow a $\chi^2$ distribution with a degree of freedom $q$. In our case, there are two interesting parameters ($q = 2$), and the corresponding $\Delta(q, \alpha) = 2.30, 4.61,$ and $9.21$ for probability levels $\alpha = 68\%, 90\%,$ and $99\%$, respectively.

Intrinsic variability amplitudes estimated via the above two methods are compared in Figure 13 and are found to be in excellent agreement.

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Figure 13. Comparison of the estimated variability amplitudes with the two methods in the $r$ band. The other four bands give similar results.
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