Research on Multiscale Wavelet Transform in Adaptive Filtering*

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Abstract. Convergence factor and steady-state error are a pair of inherent contradictions in fixed step size LMS algorithm. For a long time, how to balance the two has been the focus of research. This paper analyzed the fixed step LMS algorithm, proposed a variable step LMS algorithm, and combined them with the multiscale wavelet transform algorithm to propose a new algorithm to solve this problem. From the Matlab simulation results, it can be seen that the new algorithm solved the problem between them to a certain extent, and had stronger noise suppression ability and faster convergence speed.

Keywords: Adaptive filtering; LMS algorithm; multiscale wavelet transform; variable step size; Matlab simulation.

1. Introduction
Based on the Wiener filtering theory, Widrow et al. applied the gradient method and proposed a new least mean square(LMS) algorithm around 1960–1967. Using this algorithm, they proposed an adaptive filter[1] that could self-adjust parameters in different situations to obtain the optimal effect. Due to the characteristics of easy implementation and simple calculation, LMS algorithm[2] is widely used. However, the convergence speed of the algorithm is affected by the eigenvalues of the autocorrelation matrix of the input signal. In practice, inevitable noise will also cause the algorithm to be out of adjustment, produce steady-state noise, and affect the filtering performance. The variable step algorithm[3–8] can solve the problems caused by the fixed step LMS algorithm, effectively improve the convergence and tracking speed, and reduce the steady-state error accordingly. The variable step size algorithm constructed in this paper is based on the hyperbolic sine function. The error signal changes nonlinearly with the step size factor and adjusts the relevant parameter values. However, this algorithm is limited by the distribution of the eigenvalues of the autocorrelation matrix of the signal, so this paper introduced the wavelet transform[9,10]. The extraction characteristics of the wavelet decomposition coefficients can reduce the spectral dynamic range of the autocorrelation matrix of the input signal. In order to achieve the basic requirements of stability and convergence speed, the input signal is decomposed so that it can be distributed in a multiscale space for analysis. In this paper, in order to solve the shortcomings of the classic LMS algorithm, after analysis and verification, a new algorithm combining the two algorithms was proposed, using the characteristics of the multiscale wavelet transform algorithm, combined with the variable step size LMS algorithm based on the hyperbolic sine function, and then using simulation experiments to verify and prove that theoretically this new algorithm
can effectively improve the performance of the original algorithm, so it has practical significance and value in practical engineering applications.

2. Algorithm Analysis and Research

It can be observed from the schematic diagram of the adaptive filter in Fig. 1 that the adaptive filter has two essential components. The first part is the filter structure, and the other part is the filtering algorithm. First of all, the purpose of the filtering algorithm is to properly adjust the parameters of the filter to obtain the optimal value according to different situations. Its method is to reduce the output error as much as possible, that is, \( e(n) = d(n) - y(n) \), and then modify the weight coefficient according to \( e(n) \) to achieve the purpose of parameter adjustment. Because the LMS algorithm has advantages such as: applicability, executable, etc., the filtering algorithm used in this paper is the LMS algorithm[1].

The principle structure diagram of the adaptive filter is shown in figure 1 below, in which the parameters include: \( x(n) \): input signal; \( d(n) \): standard reference signal; \( y(n) \): output signal; and \( e(n) \): error signal.

![Fig.1 Principle of adaptive filter](image1)

The structure of the adaptive filter has a certain impact on the filtering effect, and in the selection process, all aspects should be considered, such as the stability of the structure, the length of the calculation time, etc. Compared with the infinite long impulse response filter structure, the finite long impulse response filter structure is selected in this paper. Because its stability is better and more applicable. As shown in Fig. 2 below, it is the structure of the adaptive transversal FIR filter[3].

![Fig.2 Structure of adaptive transversal filter](image2)

In Fig. 2: ① \( Z^{-1} \): delay unit; ② tap weight: \( w_0, w_1, \ldots, w_{m-1}; (k=0,1,\ldots,m-1) \); ③ tap input: \( u(n), u(n-1), \ldots, u(n-m+1); u(n-k)(k=0,1,\ldots,m-1) \).

Tap coefficient:

\[
\max w(0) = 0, 0 < \mu < 1 / \lambda_{\text{max}} \quad (1)
\]

Output estimate:

\[
y(n) = w^T(n)x(n) = w(n)x^T(n) \quad (2)
\]

Error signal:

\[
e(n) = d(n) - y(n) \quad (3)
\]

Update the tap weight vector:

\[
w(n+1) = w(n) + 2\mu x(n)e(n) \quad (4)
\]

The change of the variable step size LMS algorithm is based on the step size factor[5]. The larger value is selected in the initial stage of the algorithm. When the algorithm converges, the value is reduced.
During the adjustment process, the purpose of reducing the steady-state error can be achieved. At the same time this method makes the algorithm's convergence and tracking speed attain the requirements.

The following is the analysis process of the algorithm in this paper, and follows the modification principle of the algorithm. The variable step size LMS algorithm in this paper is based on the hyperbolic sine function, and its function expression is as follows:

$$f(x) = \sinh(x) = \frac{e^x - e^{-x}}{2}$$

(5)

Using Matlab software, the original function image of the hyperbolic sine function can be constructed. The amplitude control coefficient a and the graphic control coefficients b and c are introduced. After the original function is transformed, the new function expression can be obtained as follows:

$$u(n) = a^*|\sinh[b*e(n)']|$$

(6)

Comparing the function images before and after transformation, first, the image features change from symmetry about the origin to symmetry about the axis of symmetry. Next, the value range changes from positive or negative to only positive values. What's more, the changed image belongs to a nonlinear, in line with the step size feature requirements, when e(n) changes, it drives u(n) to change slowly, and their values are constantly approaching 0.

The process of variable step size LMS algorithm based on hyperbolic sine function is as follows:

$$e(n) = d(n) - w^T(n)x(n)$$

(7)

$$w(n + 1) = w(n) + 2\mu(n)x(n)e(n)$$

(8)

![Fig.3](image)

**Fig.3** Structure of adaptive transversal filter

![Fig.4](image)

**Fig.4** Function image after change

Each parameter value of the hyperbolic sine function has a significant impact on the convergence speed and steady-state error of the algorithm. The requirement is to meet the value range of the step factor: \(0 < \mu < 1/\lambda_{\text{max}}\). Therefore, the optimal value range is determined through the following experimental analysis. The basic values of parameters a, b, and c are shown in Fig.5 below. Combined with the change of u(n) when e(n) changes, the value range of a is (0, 0.01), the value range of b is (0, 6), and the value of c is 1.
3. Multiscale Wavelet Transform Algorithm

This paper chooses Harr wavelet, the main reason is that Harr wavelet smoothness characteristics, symmetry and anti-symmetry are in line with the requirements of wavelet base[10]. Moreover, Harr wavelet meets the required tolerance, attenuation and volatility. In particular, it reduces the complexity of the calculation, cuts down the calculation time and also improves the realizability of the simulation. Considering the size of the time-frequency window, the wavelet performance and the conditions to be met, the Harr wavelet is appropriately selected. In this paper, the Mallat algorithm is used. Its principle is to divide the original signal into two signals in different frequency bands-the approximation signal and the detail signal, while keeping the total frequency unchanged. The calculation of the two frequency bands after decomposition is simpler than the original signal. This algorithm is a fast algorithm in wavelet transform, which plays an significantly important role in analyzing the decomposition and reconstruction of signals.

When performing signal processing in this paper, the Mallat algorithm needs to be used to split the frequency of the signal to decompose the input signal into different frequency components, including the detail signal and the approximation signal.

\[
X(n) = \begin{bmatrix} x(n), x(n-1), \ldots, x(n-m+1) \end{bmatrix}^T
\]  

The following is the j-th level detail signal and approximation signal after signal 1 is decomposed by multi-scale wavelet.

\[
\begin{align*}
R_j &= [r_{j,0}, r_{j,1}, \ldots, r_{j,2^{j-1}-1}]^T \\
S_j &= [s_{j,0}, s_{j,1}, \ldots, s_{j,2^{j-1}-1}]^T
\end{align*}
\]
The weight vector of the adaptive filter corresponding to the j-th level detail signal sequence and the weight vector of the adaptive filter corresponding to the approximation signal.

\[
W_j = [w_0, w_1, \ldots, w_{2^{-j}M-1}]^T
\]
\[
V_j = [v_0, v_1, \ldots, v_{2^{-j}M-1}]^T
\]

(11)

4. Simulation results

The experimental premise of this article is set as follows: In the adaptive filter, the filter order is set to \( k=128 \), which has a greater impact on the filtering effect. The input signal is a sinusoidal signal with an amplitude of 1 and a sampling number of \( N=1024 \). A standard Gaussian white noise with a SNR of 5 is added to the sinusoidal signal to obtain a noise signal. The following figure shows the convergence process and results of each filtering algorithm in three cases. When the filter algorithm is fixed step size LMS algorithm, set the step size \( \mu=0.00026 \). When using variable step size LMS algorithm and multiscale wavelet transform algorithm, the parameters are set to \( a = 0.0006, b = 5, c = 1 \).

Matlab simulation experiment platform is used to simulate the above three LMS algorithms. The following experimental figures are the original signal and the noise signal, as well as the process pictures of the convergence and output results of different algorithms. The filtered output signal and the convergence process curve are obtained.
As shown in the curve analysis of Fig.9–11(b) above, the algorithm with fast convergence speed can better restore the original signal, so this article compares the speed of convergence. Take the limited sampling point as 1024. In the convergence process, the fixed step size LMS algorithm in Fig.9(b)
converges to about the 300th sampling point, and the error of the output signal of the first 300 points is relatively large. At the moment, it can hardly play a filtering role, and in this article, this is the slowest convergence algorithm. Secondly, the algorithm that converges at the 100th sampling point is the variable step size LMS algorithm in Fig.10(b). After the improvement in this paper, the variable step size LMS algorithm based on multiscale wavelet transform converges around the 10th sampling point, as shown in Fig.11(b), which is the fastest filtering algorithm among the three algorithms. In terms of filter performance, all algorithms have achieved adaptive filtering to a certain extent, and the noise has been filtered; In terms of convergence, the simulation results show that the algorithm of this paper is reasonable and achievable, and has advantages in both convergence speed and accuracy. The simulation results of the variable step size LMS algorithm have once again verified its superiority, because compared with the fixed step size LMS algorithm, its speed and accuracy are improved; In terms of filtering effect, Fig.9~11(a) verifies that the filtering effect of the algorithm in this paper is optimal compared with the other two algorithms. It can be seen from the output curve and error curve that the steady-state error is the smallest. The convergence speed is the fastest, and the recovery of the original signal is also more perfect.

5. Conclusion
When designing the adaptive filter algorithm, the design idea of the algorithm in this paper is to follow the principle of modification of the algorithm, and the variable step size LMS algorithm is proposed based on the hyperbolic sine function. Then, the wavelet transform is introduced to solve the eigenvalue distribution problem of the autocorrelation matrix in the variable step size algorithm. That is, the variable step size LMS algorithm is combined with the multiscale wavelet transform. And using decomposition algorithm to split input signal into multiscale space. After obtaining two signals with different frequencies, the calculation is simplified. With the help of Matlab simulation tools, the simulation results are shown in Fig.9~11 above. From the simulation output waveform, the error signal waveform and the algorithm's convergence curve, it can be clearly seen that the algorithm proposed in this paper has a large advantage. The effect is also better than the other two algorithms. Therefore, under the same input conditions, it can be considered that the variable step size LMS algorithm based on multiscale wavelet transform proposed in this paper is more able to adapt to changes in external conditions. From the previous analysis, in this paper, the filtering results of the three algorithms are compared in many aspects, including convergence speed and filtering effect. With the support of theory, the superiority of variable step size LMS algorithm based on multiscale wavelet transform is experimentally verified. It not only significantly accelerates the convergence speed, reduces the steady-state offset, but also improves the noise cancellation ability.

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