Modelling strain softening of structured soils

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Abstract. Strain softening response of structured soil is more complicated than that of reconstituted soil. After comparatively introducing the unified hardening model for over-consolidated soil (UH model), and the unified hardening model for structured soil (structured UH model), the simulations of strain softening by the 2 models are analyzed from the visual angle of mathematic description. Comparisons between model simulations and test data of strain softening for different soils are illustrated at last, and the reasonability of model descriptions for different strain softening are verified.

1. Introduction
Shear strength of structured soil will finally decrease with soil structure decay owing to loading, resulting in a strain softening response[1], which is an expressional behaviour of soil[2]. As strain softening occurs, the deviatoric stress decreasing with soil deformations increasing usually leads to engineering accident. Thus reproducing strain softening response is an important work in soil constitutive model building. According to the generating mechanism, there are 3 kinds of strain softening: strain softening with confining stress decrease, strain softening with dilatancy and strain softening with soil structure decay[3]. So far, many researches have attempted different methods to modelling strain softening. An extensional hyperbolic function is applied to analytically describe strain softening[4]. By applying the relationship between potential peak stress ratio and current soil density, strain softening due to dilatancy is reproduced[5]. Through defining an evolution of stress ratio with deviatoric strain, a bonding surface model reproducing strain softening is built. Based on Camclay model, Yin built an elastic-visco-plastic model, which is qualified to express soil structure, anisotropy, time effects and strain softening[6][7].

Also based on Camclay model[8][9], Yao et al. proposed a unified hardening model (UH model), which is qualified in smoothly and continuously describing strain hardening following strain softening and positive dilatancy following negative dilatancy[10] of over-consolidated soils (reconstituted soils). When the over-consolidation ratio equals to 1, the UH model degrades to Camclay model. Extending the static normal compression line (NCL) in the UH model to be moving with soil structure decay, a moving NCL (MNCL) is presented[11]. The vertical distance between MNCL and NCL in void ratio e~isotropic stress p coordinates is applied to express soil structure level[11][12]. Afterwards, constitutive models in turn describing isotropic compression, triaxial compression and bonding structure were developed[13], and the a three-dimensional structured UH model was built[14]. Both of the UH model and the structured UH model are qualified in reflecting strain softening smoothly[15]. In this paper, the focus is analysing the description mechanism of strain softening of structured soils comparing with that of reconstituted soils.
2. Strain softening response and reproduction

2.1 Strain softening response
For reconstituted soil, strain softening mainly related to soil density variation in shearing. In drained shearing, sample density increase results in strain hardening, and sample density decrease results in strain softening. As illustrated in Fig. 1, for Fujinomori clay in drained triaxial compression[16], the deviatoric stress of normally consolidated soil increases monotonically with sample volume compression (the hollow points in the figure), while significant postpeak reduction of deviatoric stress and sample volume expansion are observed in shearing on over-consolidated soil (the solid points in Fig. 1). In undrained shearing, the normally consolidated soil exerts strain hardening, thus other denser soil of course exerts strain hardening. As illustrated in Fig. 2, for Kaolin clay in undrained triaxial compression[17], the deviatoric stress \( q \) for all the over-consolidated soil samples increase monotonically with shearing.

For structured soil, besides density variation, soil structure decay in shearing also lead to strain softening. In drained shearing, greater soil structure decay results in sample density increase observably and strain hardening occurs; while less soil structure decay result in sample density decrease overall (dilatancy) and strain softening occurs. As illustrated in Fig. 3, for strong structured Corinth Canal marl in drained triaxial compression[18], strain hardening is observed with sample volume compression, and strain softening is observed with sample volume expansion. In undrained shearing, soil structure decay results in pore water pressure increase. Thus greater soil structure decay may lead sharply decrease of effective confining stress, resulting in strain softening response. This is a significant difference between structured and reconstituted soil in strain softening. As illustrated in Fig. 4, for Eastern Osaka clay in undrained triaxial compression[19], strain softening is observed in shearing with higher confining stress owing to sharply soil structure decay.

| \( \lambda/(1 + e_0) \) | \( \kappa/(1 + e_0) \) | \( N \) | \( \nu \) | \( M \) |
|---|---|---|---|---|
| 0.0508 | 0.0112 | 0.83 | 0.3 | 1.42 |

Figure 1. Stress-strain relationship of drained shearing of reconstituted Fujinomori clay

| \( \lambda \) | \( \kappa \) | \( e_0 \) | \( N \) | \( \nu \) | \( M \) |
|---|---|---|---|---|---|
| 0.240 | 0.045 | 1.27 | 2.85 | 0.2 | 0.896 |
Figure 2. Effective stress path of reconstituted Kaolin clay in undrained shearing

Table 3. Model parameters of Corinth Canal marl

| $\lambda$ | $\kappa$ | $N$ | $\nu$ | $M$ | $\Delta e_0$ | $\zeta$ |
|----------|----------|-----|-------|-----|--------------|--------|
| 0.040    | 0.007    | 0.775 | 0.25  | 1.321 | 0.155        | 50.0   |

Figure 3. Stress-strain relationship of Corinth Canal marl in drained shearing

Table 4. Model parameters of Eastern Osaka clay

| $\lambda$ | $\kappa$ | $N$ | $\nu$ | $M$ | $\Delta e_0$ | $\zeta$ |
|----------|----------|-----|-------|-----|--------------|--------|
| 0.1941   | 0.0477   | 2.40 | 0.25  | 1.593| 0.50         | 28     |
2.2 Strain softening reproduction

The UH model and the subsequent structured UH model are qualified in describing strain softening of reconstituted and structured soil respectively, whose common yield function is:

\[
f = c_p \ln \left( \frac{p}{p_0} \right) + \ln \left( 1 + \frac{\eta^2}{M^2} \right) = c_p \ln \left( \frac{p_s}{p_0} \right)
\]

where \( c_p = (\lambda - \kappa)/(1 + e_0) \); \( \lambda \) and \( \kappa \) are respectively isotropic compression and swelling index; \( e_0 \) is initial void ratio; stress ratio \( \eta = q/p; p_0 \) is the initial value of \( p \); \( M \) is the critical state stress ratio; \( p_s \) is the intercept of right end of yielding surface on \( p \) axle. Applying the associated flow rule, the relationship between plastic volumetric and deviatoric strain increment \( d\varepsilon^V_p \) and \( d\varepsilon^P_d \) is:

\[
d\varepsilon^V_p d\varepsilon^P_d = (M^2 - \eta^2)/(2\eta)
\]

The hardening law of the UH model and the structured UH model is expressed in the same form as:

\[
c_p \ln \left( \frac{p_s}{p_0} \right) = \int \frac{M_t^4 - \eta^4}{M^4 - \eta^4} d\varepsilon^P_d
\]

where \( M_t \) is the potential failure stress ratio, expressed as:

\[
M_t = 6 \left[ \frac{\lambda}{R} \left( 1 + \frac{\lambda}{R} \right) - \frac{\lambda}{R} \right]
\]

In equation (4), \( \chi = M^2/[12(3 - M)] \); \( R \) is the ratio of current stress and reference stress. At shearing beginning, according to equation (3), \( R \to 0^+ \) and \( M_t \to 3^- \); In the shearing end, \( R \to 1, M_t \to M \), and a critical state is achieved. In the shearing, when \( 0 < R < 1, 3 > M_t > M \); and when \( R > 1, M_t < M \). The essential difference between UH model and structured UH model is the evolution of the internal variable \( R \). Beginning from a initial variable \( R = R_0 \), the evolutions of \( R \) in the UH model and the structured UH model respectively are:

\[
R = R_0 \cdot \exp \left( \frac{1}{c_p} \int \frac{M_t^4 - \eta^4}{M^4 - \eta^4} d\varepsilon^P_d \right)
\]

\[
R = R_0 \cdot \exp \left( \frac{1}{c_p} \int \frac{M_t^4 - \eta^4}{M^4 - \eta^4} d\varepsilon^P_d - \frac{\int d(\Delta e)}{\lambda - \kappa} \right)
\]

Obviously, an internal variable named structure potential \( \Delta e \) in equation (6) is applied to describe current soil structure, and its evolution law is:

\[
d(\Delta e) = -\zeta \cdot R \cdot \Delta e(c_p d(ln(p_s)))
\]

In equation (7), \( \zeta \) is a model parameter, controlling soil structure decay rate. The elastic volumetric and deviatoric strain increments \( d\varepsilon^e_v \) and \( d\varepsilon^e_d \) in both of the UH model and the structured UH model are calculated by the Hooke’s Law:

\[
d\varepsilon^e_v = \frac{\kappa}{(1+\epsilon_0)p} dp
\]

\[
d\varepsilon^e_d = \frac{\kappa}{2\kappa (1+\nu)} \frac{(1+\epsilon_0)(1-2\nu)p}{p} dq
\]

Figure 4. Stress-strain relationship of Eastern Osaka clay in undrained shearing
Comparing the UH model and the structured UH model, due to the introduction of $\Delta e$, evolutions of $M_f$ in these two models are different. For the UH model, considering equation (2), (4) and (5), evolutions of $M_f$ in isotropic compression and shearing are respectively:

$$dM_f = \frac{-M_f(3-M_d)}{6-M_f} \cdot \frac{M_f^4-M_d^4}{c_p M^4} \frac{\Delta \varepsilon^d}{d\varepsilon^d}$$  \hspace{1cm} (9a)$$

$$dM_f = \frac{-M_f(3-M_d)}{6-M_f} \cdot \frac{M_f^4-M_d^4}{2c_p \eta(M^2+\eta^2)} \frac{d\varepsilon^d}{d\varepsilon^d}$$  \hspace{1cm} (9b)$$

For the structured UH model, considering equation (2), (4), (6) and (7), evolutions of $M_f$ in isotropic compression and shearing are respectively:

$$dM_f = \frac{-M_f(3-M_d)}{6-M_f} \cdot \frac{M_f^4-M_d^4}{c_p M^4} \frac{\Delta \varepsilon^d}{d\varepsilon^d}$$  \hspace{1cm} (10a)$$

$$dM_f = \frac{-M_f(3-M_d)}{6-M_f} \cdot \frac{M_f^4-M_d^4}{2c_p \eta(M^2+\eta^2)} \frac{d\varepsilon^d}{d\varepsilon^d}$$  \hspace{1cm} (10b)$$

In equation (9) and (10), $M_f < 3$ and $M < 3$ are always satisfied, thus the first factor in the right-hand of equation (9) and (10) are always negative (i.e., $-M_f(3-M_d)/(6-M_f) < 0$). Therefore, the sign of $dM_f$ in equation (9) and (10) depends on the second factor in the right-hand of the equations. In the following, strain softening descriptions in drained and undrained shearing will be discussed separately.

2.2.1 Strain softening description in drained shearing

In this paper, for simplicity, only drained shearing with $\frac{dq}{dp} = A > 0$ is implemented. By totally differentiating equation (1), the increment $dp_x$ can either be expressed as increments $dp$ and $dq$ (11a)

$$dp_x = \frac{1}{M^2 A} (M^2 - \eta^2 + 2A\eta) dq$$  \hspace{1cm} (11a)$$

or be expressed as increments $dp$ and $d\eta$ (11b):

$$dp_x = \frac{1}{M^2 A} \left( \frac{M^2+\eta^2}{A-\eta} + 2\eta \right) d\eta$$  \hspace{1cm} (11b)$$

In drained shearing, $A > \eta$, thus $(M^2 - \eta^2 + 2A\eta) > (M^2 + \eta^2) > 0$ and $[(M^2 + \eta^2)/(A-\eta) + 2\eta] > 0$. Therefore, the sign of $dp_x$, $dq$ and $d\eta$ are the same in drained shearing. Totally differentiating hardening law equation (3) and considering equation (1), (2) and (11), the increments $dq$ and $d\eta$ are respectively given by:

$$dq = \frac{Ap}{c_p} \cdot \frac{M_f^4-\eta^4}{2\eta(M^2+\eta^2+2A\eta)} \frac{\Delta \varepsilon^d}{d\varepsilon^d}$$  \hspace{1cm} (12a)$$

$$d\eta = \frac{A-\eta}{c_p} \cdot \frac{M_f^4-\eta^4}{2\eta(M^2+\eta^2+2A\eta)} \frac{\Delta \varepsilon^d}{d\varepsilon^d}$$  \hspace{1cm} (12b)$$

In the UH model and the structured UH model, $M_f$ is an internal variable, which is the key point to describe strain softening. In the beginning of shearing $\eta = 0$, and $A > \eta$ is satisfied. Thus, according to equation (12), the signs of $dq$ and $d\eta$ depend on the sign of the factor $(M_f^4 - \eta^4)$. Initially $M_f > \eta = 0$, both $q$ and $\eta$ increase with shearing.

In the UH model for reconstituted soil, $R < 1$ is always satisfied in the whole loading procedure, thus $M_f > M$ is satisfied all the time. Therefore, according to equation (9), $dM_f < 0$ is satisfied either in isotropic compression or in shearing. When $M_f \rightarrow M^+$, $dM_f \rightarrow 0^-$. Until $\eta$ meeting $M_f$, according to equation (12), $q$ and $\eta$ increase with shearing. Whereafter $M_f > \eta > M$, $dM_f < 0$, $q$ and $\eta$ begin to decrease with shearing. Finally, a critical state is reached with $(M_f^4 - \eta^4) \rightarrow 0^-$, $M_f \rightarrow M^+$ and $\eta \rightarrow M^+$. The decreasing $q$ in shearing reflects the strain softening of reconstituted soil in drained shearing.

In the structured UH model, the adding structure potential $\Delta e$ influences the evolution of $M_f$ as illustrated in equation (10). Even when $M_f = M$, $dM_f < 0$ is also satisfied. This makes $M_f < M$ come to be possible in isotropic compression and shearing. Thus either stress ratio $\eta$ or internal variable $M_f$ might firstly meet critical state stress ratio $M$. (A) $\eta$ firstly meets $M$ when $M_f > M$. In this case, $M_f > M = \eta$, according to equation (10b) $dM_f < 0$ and $(M_f^4 - \eta^4) > 0$. Thus, according to equation (12),
both \( q \) and \( \eta \) continue to increase with shearing, performing strain hardening. As \( \eta \) meets \( M_f \) (i.e., \( \eta = M_f > M \)), according to equation (10b), \( dM_f < 0 \). Then \( \eta \) exceeds \( M_f \), and \( (M_f^4 - \eta^4) < 0 \). According to equation (12), both \( q \) and \( \eta \) begin to decrease with shearing, performing strain softening. Finally, the critical state is reached with \( (M_f^4 - \eta^4) \rightarrow 0^- \), \( M_f \rightarrow M^+ \) and \( \eta \rightarrow M^+ \). (B) \( M_f \) firstly meets \( M \) when \( \eta \leq M \). In this case, \( \eta < M = M_f \), according to equation (10b), the existing of factor \( (M_f^4 - \eta^4) \) makes \( dM_f < 0 \). \( M_f \) decreases until \( (M_f^4 - M^4 + \zeta \cdot R \cdot \Delta e (M_f^4 - \eta^4)/(1 + e_p)] = 0 \) when \( \eta < M_f \), then \( M_f \) begins to increase and \( \eta \) continues to increase. Hence, since then, \( \eta \) and \( M_f \) will increase and finally tend to critical state stress ratio \( M \), performing strain hardening. (In this increasing stage, if \( \eta < M_f \), then according to equation (12), \( d\eta = 0 \), being contradict with the increasing \( \eta \). So \( \eta < M_f \) is impossible in their increasing stage.)

2.2.2 Strain softening description in undrained shearing

In undrained shearing, soil sample volume keeps constant, thus the elastic volumetric strain increment \( d\varepsilon_v^e \) and the plastic volumetric strain increment \( d\varepsilon_v^p \) are inverse numbers.

\[
\begin{align*}
    d\varepsilon_v^e & = -d\varepsilon_v^p \\
    \text{or } d\varepsilon_v^p & = -d\varepsilon_v^e
\end{align*}
\]

(13)

Totally differentiating equation (1) and (3), and considering equation (2), (8) and (13), the increments \( dq \) and \( d\eta \) in both of the UH model and structured UH model are:

\[
\begin{align*}
    dq & = \frac{p}{4\sigma_0 \eta^2} \left[ \frac{\lambda - \kappa}{\kappa} (M^4 - \eta^4)^2 + (M_f^4 - \eta^4) \right] d\varepsilon_v^p \\
    d\eta & = \frac{1}{4\sigma_0 \eta^2} \left[ \frac{\lambda - \kappa}{\kappa} (M^4 - \eta^4) + (M_f^4 - \eta^4) \right] d\varepsilon_v^p
\end{align*}
\]

(14a)

(14b)

Although the upper descriptions for the UH model and the structured UH model are identical in form, the different evolutions of the internal variable \( M_f \) in the two models make the strain softening descriptions different. In the shearing beginning \( \eta = 0 \), \( M_f > \eta \) and \( M > \eta \) are both satisfied. Thus, according to equation (14), \( q \) and \( \eta \) increase with \( \varepsilon_v^p \) initially.

In the UH model, \( R < 1 \) and \( 3 > M_f > M \) are always satisfied. Thus, according to equation (9), \( dM_f < 0 \) is always satisfied. In shearing, according to equation (14b), \( d\eta > 0 \) until \( \left[ (\lambda - \kappa) (M^4 - \eta^4)/\kappa + (M_f^4 - \eta^4) \right] = 0 \) (i.e., \( \eta^4 = \left[ (\lambda - \kappa) M^4/\lambda + \kappa M_f^4/\lambda \right] \), when \( M < \eta < M_f \)). Whereafter, \( d\eta = 0 \), and \( \eta \) stops to increase. Because at that time \( M < \eta < M_f \), \( dq > 0 \) and \( q \) continues to increase. For \( M < M_f \) at this time, according to equation (9b), \( dM_f < 0 \). \( d\eta = 0 \) and \( dq < 0 \) appearing simultaneously makes the factor \( \left[ (\lambda - \kappa) (M^4 - \eta^4)/\kappa + (M_f^4 - \eta^4) \right] < 0 \) in the next step, then \( \eta \) begins to decrease according to equation (14b). Since then, \( M_f \) and \( \eta \) decrease together, keeping \( \eta < M_f \). Finally \( M_f \) and \( \eta \) tend to the critical state stress ratio \( M \). According to equation (14a), \( \eta < M_f \) makes \( dq > 0 \) to be always satisfied. So the UH model is able to reproduce the strain hardening of reconstituted soil in undrained shearing.

In the structured UH model, according to equation (10a), even \( M_f \) decays to \( M \), \( dM_f < 0 \) is satisfied due to \( \Delta e > 0 \). Then \( M_f \) will continues to decrease with loading and come to be less than \( M \). In undrained shearing: if soil structure is strong or confining stress is low, \( M_f > M \) is satisfied in the shearing beginning; otherwise if soil structure is weak or confining stress is high, \( M_f < M \) is satisfied in the shearing beginning. Thus either Stress ratio \( \eta \) or internal variable \( M_f \) might firstly meet critical state stress ratio \( M \). (A) \( \eta \) firstly meets \( M \) when \( M_f > M \). At this time, according to equation (10b), \( dM_f < 0 \) is satisfied due to \( M_f > M = \eta \). \( q \) and \( \eta \) increase with shearing according to equation (14). According to equation (14b), as \( \left[ (\lambda - \kappa) (M^4 - \eta^4)/\kappa + (M_f^4 - \eta^4) \right] = 0 \) (i.e., \( \eta^4 = \left[ (\lambda - \kappa) M^4/\lambda + \kappa M_f^4/\lambda \right] \), when \( M < \eta < M_f \) is satisfied right now), \( d\eta = 0 \) and \( \eta \) stops to increase. For \( M < M_f \) right now, according to equation (10b), \( dM_f < 0 \) and \( d\eta = 0 \) and \( dq < 0 \) appearing simultaneously makes the factor \( \left[ (\lambda - \kappa) (M^4 - \eta^4)/\kappa + (M_f^4 - \eta^4) \right] < 0 \) in the next step, and \( \eta \) begins to decrease according to equation (14b). Since then, \( M_f \) and \( \eta \) decrease together, keeping \( \eta < M_f \). Finally \( M_f \) and \( \eta \) tend to the critical state stress ratio \( M \). According to equation (14a), \( \eta < M_f \) makes \( dq > 0 \) to be always satisfied. Thus, for the strong soil structure or low confining pressure, the
structured UH model is able to reproduce the strain hardening in undrained shearing. (B) $M_f$ firstly meets $M$ when $\eta < M$. At this time, $dM_f < 0$ according to equation (10b), and $q$ and $\eta$ increase with shearing according to equation (14). As $[M_f^4 - M^4 + \eta \cdot R \cdot \Delta \varepsilon (M_f^4 - \eta^4)/(1 + e_0)] = 0$, according to equation (10b), $dM_f = 0$ and $\eta < M < M$ at this time. According to equation (14), $q$ and $\eta$ still increase with shearing. $d\eta > 0$ and $dM_f = 0$ appearing simultaneously makes the factor in equation (10b) $[M_f^4 - M^4 + \eta \cdot R \cdot \Delta \varepsilon (M_f^4 - \eta^4)/(1 + e_0)] < 0$, thereafter $dM_f > 0$. Then both $q$ and $M_f$ increase together and $\eta$ will meet $M_f$ before they achieving $M$. (If $\eta$ cannot catch $M_f$ before they achieving $M$, $\lim \eta = M_f < M$. But according to equation (14b), $\eta^4_{\text{max}} = [(\lambda - \kappa)/\lambda]M^4 +$ $(\kappa/\lambda)M_f^4 > M_f^4$. The above two equations are contradictory. So $\eta$ must be able to catch $M_f$ before they achieving $M$. ) When $\eta = M_f < M$, $dM_f > 0$ according to equation (10b), $dq > 0$ and $d\eta > 0$ according to equation (14). As the factor $[(\lambda - \kappa)(M^2 - \eta^2)^2/\kappa + (M_f^4 - \eta^4)] = 0$ in equation (14a), $dq = 0$. As the factor $[(\lambda - \kappa)(M^4 - \eta^4)/\kappa + (M_f^4 - \eta^4)] = 0$ in equation (14b), $d\eta = 0$, thus:

$$ (\eta^2/M^2)|_{d\eta=0} = \frac{\lambda - \kappa}{\lambda - 2\kappa} \quad \text{(15)} $$

$$ (\eta^2/M^2)|_{dq=0} = \frac{\lambda - \kappa}{\lambda} + \frac{\kappa}{\lambda} \frac{M_f^4}{M} \quad \text{(16)} $$

By comparing, $(\eta^2/M^2)|_{dq=0} < (\eta^2/M^2)|_{d\eta=0} < 1$. This interesting inequality means that when $\eta$ is increasing and tending to $M$, $q$ has begun to decrease. Considering equation (2), (8a), (13) and $M_f < \eta < M$, the stress increment $dp = [(1 + e_0)/\kappa]p[(M^2 - \eta^2)/(2\eta)]d\varepsilon_d^p < 0$ is obtained. In this condition, both $p$ and $q$ decrease in undrained shearing, performing strain softening with effective confining pressure decreasing. Finally, $M_f \rightarrow M^-, \eta \rightarrow M^-$, and the sample achieves critical state.

3. Conclusion

Aiming at strain softening response of soil, reproductions by the UH model and the structured UH model are introduced. Internal variable $M_f$ in the UH model evolves only with soil density, being qualified to reproduce strain softening due to dilatancy. While internal variable $M_f$ in the structured UH model evolves with both soil density and soil structure, being qualified to reproduce strain softening not only due to dilatancy but also due to soil structure decay.

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