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Abstract. For the nonlinear engagement between two pursuers and a single evader, in which small angle deviation assumption or other linearization conditions may not be applicable, a cooperative guidance law is analyzed and derived in the framework of a zero-sum two-person differential game. State-dependent Riccati-equation method is used to obtain the suboptimal solutions of the pursuit-evasion problem by choosing the line of sight angular rates as state variables, and closed-loop form guidance strategies that can be used online for the players are derived. Time-to-go estimations are not had to be taken into account to apply the guidance law proposed. Nonlinear two-dimensional simulations are carried out to validate the performance of the cooperative guidance law, and the comparison with a current cooperative linear quadratic differential game guidance law is made.

1. Introduction

It is well known that the interception probability will be improved and the scope of tasks will be broaden through information sharing and functions complementation, using multiple missiles to intercept targets. However, classical guidance laws such as proportional navigation (PN) guidance laws are difficult to achieve ideal results when dealing with maneuvering target, and the application of optimal guidance laws (OGL) is also limited in practice due to the dependency on the assumption about the target maneuver. The differential games guidance law (DGGL) is attracting more and more attention because of its low degree of dependency on the target information and applicability to intercept maneuvering targets. It does not need to make any assumption about the maneuvering law of the target, and optimal strategies of the players can all be obtained by differential games guidance laws [1]-[3].

Most of the current literatures on differential game guidance laws design their guidance strategies by using small angle assumption to get linear system models, however, the systems in nature are inherently nonlinear. When the scenarios are not initiated on the required collision triangle or the target has a large maneuver capability relative to its speed, one can apply state-dependent Riccati-equation (SDRE) method [3], then the complicated Hamilton-Jacobi-Bellman-Isaacs (HJBI) equation which sometimes may be unsolvable for nonlinear game problem is avoided to be considered. A nonlinear 1-on-1 pursuit-evasion engagement in which an angle constraint was imposed for the pursuer was proposed and discussed based on differential game theory and SDRE method [4]. 1-on-1 interception problems with and without angle constraint were further researched, and nonlinear pursuit-evasion strategies that were not vulnerable to unpredictability were derived in [5]-[6].

Based on the previous works about 1-on-1 engagement, a nonlinear 2-on-1 engagement is studied combined with differential games theory and SDRE method in this paper, and a guidance law that can

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Based on the previous works about 1-on-1 engagement, a nonlinear 2-on-1 engagement is studied combined with differential games theory and SDRE method in this paper, and a guidance law that can
be applied in nonlinear case is proposed. The optimization problem that is usually too complicated to solve is converted into a suboptimization problem, and time-to-go estimation is not explicitly necessary in this paper.

2. Cooperative differential game guidance law

2.1. Kinematic model and guidance principle
An engagement between two skid-to-turn roll-stabilized missiles called pursuers and a single target called evader is considered. It is assumed that the motion of the players can be decoupled, and all the players have ideal control system dynamics, then the planar engagement between the two pursuers and the evader in a schematic view can be shown as figure 1, where $X_i-O_i-Y_i$ is a Cartesian inertial reference frame. $P_i$ ($i=1,2$) represents the $i$-th pursuer, $E$ represents the evader. The speed, lateral acceleration, and flight path angles are denoted by $V$, $a$, $\gamma$ respectively, the additional subscripts $i$ and $E$ correspond to pursuer $P_i$ and the evader respectively. $r_i$ denotes the range between pursuer $P_i$ and the evader, $\phi_i$ denotes the line of sight angle between $P_i$ and the evader.

![Figure 1. Planar engagement geometry of the players](image)

The engagement kinematics can be written as

\[
\dot{r}_i = -V_E \cos(\gamma_E + \phi_i) - V_i \cos(\gamma_i - \phi_i) \quad (1)
\]

\[
\dot{\phi}_i = [V_E \sin(\gamma_E + \phi_i) - V_i \sin(\gamma_i - \phi_i)] / r_i \quad (2)
\]

\[
\dot{\gamma}_j = a_j / V_j; \quad j = \{1, 2, E\} \quad (3)
\]

The angular rate of line of sight angle can be denoted as $\theta_i$, and its derivation is

\[
\dot{\theta}_i = \ddot{\phi}_i = -2r_i \dot{\theta}_i / r_i - u_i \cos(\gamma_i - \phi_i) / r_i + v \cos(\gamma_E + \phi_i) / r_i \quad (4)
\]

The relationship between $\theta_i$ and zero effort miss (ZEM) \cite{7} is

\[
Z_i(t) = \frac{r_i^2 \dot{\theta}_i}{\sqrt{r_i^2 + r_i^2 \dot{\theta}_i^2}} \quad (5)
\]

From equation (5), it is found that one can control $\theta_i$ to approach zero to intercept the evader. For the nonlinear 2-on-1 engagement, the equation of states in the form of state-dependent coefficient (SDC) can be written as

\[
\dot{x} = A(x)x + B(x)u + C(x)v, \quad x(t_0) = x_0 \quad (6)
\]

where $x$ is the state vector, $A$, $B$, $C$ are the state-dependent coefficient matrixes, $u$ and $v$ are control vectors of the pursuers and the evader respectively, and $t_0$ is the initial time.
The performance index of the pursuit-evasion game can be chosen as

\[ J = \frac{1}{2} \int_0^T (x^T Q(x)x + u^T R_1(x)u - \rho^2 \gamma^T R_2(x)v)dt \]  

(7)

where \( Q, R_1, R_2 \) are state-dependent weighting matrices, and \( \rho \) is a measure of pursuer’s maneuvering capability relative to that of the evader, the lower the maneuvering capability of the evader is, the higher the value of \( \rho \) is.

Choosing angular rates of line of sight angles as the system states, i.e. \( x = [\theta_1, \theta_2]^T \), the state-dependent coefficient matrices are

\[ A(x) = \begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix}, \quad B(x) = \begin{bmatrix} b_1 & 0 \\ 0 & b_2 \end{bmatrix}, \quad C(x) = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \]  

(8)

where

\[ a_1 = -2r_1/r_i, \quad a_2 = -2r_2/r_i \]
\[ b_1 = -\cos(\gamma_1 - \phi_i)/r_i, \quad b_2 = -\cos(\gamma_2 - \phi_i)/r_i \]
\[ c_1 = \cos(\gamma_1 + \phi_i)/r_i, \quad c_2 = \cos(\gamma_2 + \phi_i)/r_i \]  

(9)

To obtain a controllable system, the matrix pairs \( \{A,B\} \) and \( \{A,C\} \) should satisfy the controllability requirement. The controllability matrix \( [B,AB] \) and \( [C,AC] \) have a full rank if the coefficients satisfy

\[ a_1 \neq a_2 \Rightarrow \frac{r_i}{r_i} \\
 b_1 \neq 0 \Rightarrow (\gamma_i - \phi_i) \neq \pi/2 \\
 b_2 \neq 0 \Rightarrow (\gamma_i - \phi_i) \neq \pi/2 \\
 c_1 \neq 0 \Rightarrow (\gamma_i + \phi_i) \neq \pi/2 \\
 c_2 \neq 0 \Rightarrow (\gamma_i + \phi_i) \neq \pi/2 \]  

(10)

To obtain an observable system, the matrix pair \( \{Q^{1/2}A\} \) should satisfy the observability requirement. It is readily to find that the observability matrix \( [Q^{1/2},Q^{1/2}A]^T \) has a full rank from the following equation

\[ \begin{bmatrix} Q^{1/2}(x) \\ Q^{1/2}(x)A(x) \end{bmatrix} = \begin{bmatrix} \sqrt{q_1} & 0 \\ 0 & \sqrt{q_2} \end{bmatrix} \begin{bmatrix} a_1 \sqrt{q_1} \\ 0 \end{bmatrix} \]  

(11)

where \( Q(x) = \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix} \), \( q_1, q_2 > 0 \).

2.2. Cooperative strategies based on SDRE

The state-dependent Riccati-equation of the differential game problem can be written as

\[ A^T(x)P(x) + P(x)A(x) - P(x)(B(x)R_1(x)B^T(x) - \rho^2 C(x)R_2(x)C^T(x))P(x) + Q(x) = 0 \]  

(12)

Where \( P \) is the unique, symmetric, positive-definite solution of the SDRE, \( R_1, R_2 \) are assumed to be identity matrixes here. Then the suboptimal strategies of the players based on SDRE can be written in SDC form as
\[ u^* = -R_1^{-1}(x)B^T(x)P(x)x \]
\[ v^* = \rho^{-2}R_2^{-1}(x)C^T(x)P(x)x \]

The Hamilton matrix of the system is
\[
H(x) = \begin{bmatrix}
A(x) & -F(x) \\
-Q(x) & -A^T(x)
\end{bmatrix}
\]

where \( F(x) = BB^T - \rho^2CC^T \) is a symmetric matrix, and its elements are
\[
F(1,1) = f_1 = b_1^2 - c_1^2 / \rho^2 \\
F(1,2) = f_2 = -c_1c_2 / \rho^2 \\
F(2,2) = f_3 = b_2^2 - c_2^2 / \rho^2
\]

then the eigenvalues of \( H \) can be obtained
\[
eig(H) = \{-\sqrt{\delta + \Delta}, \sqrt{\delta + \Delta}, \sqrt{\delta - \Delta}, -\sqrt{\delta - \Delta}\}
\]

where
\[
\delta = \frac{(f_4q_1 + f_3q_2 + a_1^2 + a_2^2)}{2}
\]
\[
\Delta = \frac{(a_1^4 - 2a_1^2a_2^2 + 2a_1^2f_3q_1 - 2a_1^2f_1q_1 + 2a_2^2f_3q_1 + f_2^2q_1 + f_1^2q_1 - 2f_2f_3q_1q_2 + 4f_2^2q_1q_2 + f_3^2q_1q_2)^2}{2}
\]

If there exists the solution of the algebraic SDRE in equation (12), \( H \) should have no eigenvalue on the imaginary axis, i.e. \( \delta > \Delta > 0 \). According to Schur’s algorithm, the matrix composed of the eigenvectors corresponding to the two negative eigenvalues of Hamilton matrix is
\[
V = \begin{bmatrix}
(a_1^2 + f_3q_2 - \delta - \Delta)(a_1 - \sqrt{\delta + \Delta})/(f_3q_1q_2) & (a_1^2 + f_3q_2 - \delta + \Delta)(a_1 - \sqrt{\delta - \Delta})/(f_3q_1q_2) \\
(\sqrt{\delta + \Delta} - a_2)/q_2 & (\sqrt{\delta - \Delta} - a_2)/q_2 \\
(\delta + \Delta - a_2^2 - f_3q_2)/(f_3q_2) & (\delta - \Delta - a_2^2 - f_3q_2)/(f_3q_2)
\end{bmatrix} = \begin{bmatrix}
V_1 \\
V_2
\end{bmatrix}
\]

then the solution matrix \( P \) of the SDRE can be obtained
\[
P = V_2V_1^{-1} = \begin{bmatrix}
p_1/\Phi \\
p_2/\Phi \\
p_3/\Phi
\end{bmatrix}
\]

where
\[
p_1 = q_1[(a_1 - \sqrt{\delta + \Delta})(a_1^2 + f_3q_2 - \delta + \Delta) + (a_1 - \sqrt{\delta - \Delta})(-a_1^2 - f_3q_2 + \delta + \Delta)] \\
p_2 = f_3q_1q_2(\sqrt{\delta + \Delta} - \sqrt{\delta - \Delta}) \\
p_3 = q_1[(a_1 - \sqrt{\delta + \Delta})(-a_1^2 - f_3q_2 + \delta + \Delta) + (a_1 - \sqrt{\delta - \Delta})(a_1^2 + f_3q_2 - \delta + \Delta)] \\
\Phi = (a_1^2 + f_3q_2 - \delta + \Delta)(a_1 - \sqrt{\delta - \Delta})(\sqrt{\delta + \Delta} - a_2) - (a_1^2 + f_3q_2 - \delta - \Delta)(a_1 - \sqrt{\delta + \Delta})(\sqrt{\delta - \Delta} - a_2)
\]

Since \( P \) is positive-definite, the following condition is also imposed
\[
\langle p_1 + p_2, \Phi \rangle > 0, \quad p_1p_3 - p_2^2 > 0
\]
According to equation (13), the closed-loop form suboptimal strategies of the players based on SDRE are obtained as
\[
\begin{align*}
\dot{\theta}_1 &= -\frac{b_1 p_1}{\Phi} \theta_1 - \frac{b_2 p_2}{\Phi} \theta_2 \\
\dot{\theta}_2 &= -\frac{b_3 p_3}{\Phi} \theta_1 - \frac{b_4 p_4}{\Phi} \theta_2 \\
\bar{\nu} &= \frac{c_1 p_1 + c_2 p_2}{\rho^2 \Phi} \dot{\theta}_1 + \frac{c_3 p_3 + c_4 p_4}{\rho^2 \Phi} \dot{\theta}_2
\end{align*}
\] (24)

The coordination between the two pursuers is indicated by the coupled construction of the pursuers’ strategies, and it is found that the derived strategies are not explicitly related to time-to-go.

3. Simulations and performance analysis
In this section, the performance of the derived guidance law (Nonlinear Cooperative Differential Game Guidance Law Based on SDRE, SDRE-NCDG) is investigated and compared to a cooperative linear quadratic differential game (CLQDG) guidance law proposed in [1] by nonlinear simulations. It is assumed that the flight path angles, speed, and the line of sight angles can be measured or estimated, and all the players have ideal control systems. The influence of the gravity is ignored, and the simulation parameters of the engagement are shown in table 1.

| Pursuers/Evader | Weight parameters | Initial flight path angle(°) | Initial position(m) | speed(m/s) |
|-----------------|-------------------|------------------------------|---------------------|------------|
| P₁              | q₁=10⁵            | 30                           | (0,0)               | 600        |
| P₂              | q₂=10⁵            | -30                          | (0,0)               | 600        |
| E               | ρ=9               | 0                            | (5000,0)            | 400        |

3.1. Sample run

3.1.1. Nonmaneuvering target. If the evader does not maneuver, the 2-on-1 simulation results are shown in figure 2 and figure 3. Figure 2 presents the trajectories of two pursuers and one nonmaneuvering evader, and it is found that the evader is intercepted by the pursuers simultaneously using SDRE-NCDG guidance law. Figure 3 presents the acceleration profiles of the pursuers during the engagement. The acceleration of pursuers is decreasing with time until to zero at the end of the guidance.

**Figure 2.** Trajectories of the players while the evader does not maneuver  
**Figure 3.** Acceleration profiles of the pursuers while the evader does not maneuver
3.1.2 Maneuvering target. If the evader performs a 5g evasive maneuver, the 2-on-1 simulation results are shown in figure 4 and 5. As one can see from figure 4, two pursuers don’t hit the evader simultaneously (the evader is intercepted by $P_1$ firstly) because time constraints that are hard to impose on the differential game problem isn’t taken into account in this paper either. So the distance between $P_2$ and the evader when $P_1$ hits the evader does not mean the terminal miss distance of $P_2$ to the evader. The acceleration profiles shown in figure 5 indicate that the acceleration of pursuers decreases with time too but they don’t approach zero at the end of the guidance due to the maneuver of the evader.

![Figure 4. Trajectories of the players while the evader performs a 5g maneuver](image1)

![Figure 5. Acceleration profiles of the pursuers while the evader performs a 5g maneuver](image2)

3.2. Comparison to linear strategies

Figure 6 and 7 present the simulation results of a current CLQDG guidance law using the same parameters shown in table 1, the evader still performs a 5g evasive maneuver, and the curves in figure 4 and 5 are depicted again to make comparison. It is illustrated that the two kinds of guidance laws are both effective, however, the trajectories of pursuers using the nonlinear guidance law are slightly straighter than that of the CLQDG guidance law, and the maneuverability requirement on pursuers is slightly lower than that of the CLQDG guidance law. The comparison of control energy of the two different guidance laws is drawn in figure 8, in which it can be obviously seen that the energy consumed in the nonlinear case is less than the linear case (about 8% control energy reduced for $P_1$).

![Figure 6. Trajectories of the players while the evader performs a 5g maneuver (comparison)](image3)

![Figure 7. Acceleration profiles of the pursuers while the evader performs a 5g maneuver (comparison)](image4)
Figure 8. Control energy comparison

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