Evaluating mirror alignment systems using the optical sensing matrix

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Abstract. The most sensitive gravitational-wave detectors today are based on large-scale laser interferometers whose optics are suspended from pendulums to decouple the instrument from seismic motion. Complex control systems are required to set and maintain the microscopic position of each mirror at a precisely defined value. Such control systems use the interferometer signals as input signals, and ideally it is designed such that the degrees of freedom (mirror positions) are well decoupled in the interferometer signals. However, this is not always feasible, in particular the mirror alignment control signals in interferometric gravitational wave detectors often show strong couplings between the different degrees of freedom. In this paper we will describe a simple and powerful method to quantify in advance the performances of an alignment control system by analyzing the optical matrix of the proposed read-out system. We will motivate the method using a Fabry-Perot cavity as an example, and we will further present results for the Virgo alignment system where this method was used to characterize and improve the alignment sensing scheme.

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1. Introduction

A network of interferometric gravitational-wave interferometers has recently been put into operation: Virgo [1] in Italy, LIGO [2] in the U.S.A., GEO600 [3] in Germany and TAMA300 [4] in Japan. These detectors are based on the Michelson interferometer topology, but provide an unprecedented sensitivity with high duty cycle. These modern laser interferometers employ resonant optical cavities with the main mirrors being suspended as pendulums in order to isolate them from seismic disturbances. Such instruments provide an useful signal only when the optical components are positioned precisely at a pre-defined locations relative to each other; this set of positions is called operating point. For example, a tolerable deviations from the operating point along the
optical axis must typically be smaller than $10^{-9}$ m to $10^{-12}$ m while the free motion of the suspended mirrors would be many orders of magnitude larger than that. Sophisticated electro-optical control systems are required to continuously measure and restore the mirror positions. Generally two separate systems are used: one for the longitudinal positions along the optical axis and another for the angular position of the mirror.

In this paper we describe a method for estimating a quality parameter for a mirror alignment control system. The quality parameter allowed us to quickly validate and rank potential read-out systems which were proposed for the alignment control of the Virgo interferometer. Our method is based on the well-known fact that the error signals of the control systems must map the full vector space defined by the degrees of freedom which have to be controlled. The method described here provides an intuitive and simple parameter describing the quality of optical readout scheme with respect to this criteria. We have used a Fabry-Perot cavity system to validate the parameter as quality estimator and we have further applied the method during the design of the current Virgo mirror alignment system.

2. Optical matrix

The opto-electrical system, which is formed by the interferometer on its operating point, and the optical readout is often described by an optical matrix $M = m_{ij}$, formed by the low frequency limit of the transfer functions mirror-position $\beta_j \rightarrow$ photo-diode-output $s_i$:

$$m_{ij} \cdot \beta_j = s_i$$  \hspace{1cm} (1)

The matrix rows represent the diode output signals $s_i$ and the columns refer to the mirror angular positions $\beta_j$. In order to simplify our analysis we assume in the following that the two alignment degrees of freedom per mirror‡ are completely independent and thus can be treated by two separate optical matrices.

In order to validate whether an optical readout scheme can successfully be used in a control system it is important to determine how well the sensor signals map the space spawned by the degrees of freedom to be controlled (in our case the mirror angular displacements). We define the vector space of the mirror positions by means of the unit vectors

$$\vec{\beta}_i, \quad \text{with} \quad \vec{\beta}_1 = (1, 0, \ldots, 0), \quad \vec{\beta}_2 = (0, 1, \ldots, 0), \text{etc.}$$  \hspace{1cm} (2)

We call signal vectors the normalized vectors $\vec{s}_i$ defined as:

$$\vec{s}_i^{(N)} = \frac{\vec{s}_i}{|\vec{s}_i|} \quad \text{and} \quad M\vec{\beta}_i = \vec{s}_i$$  \hspace{1cm} (3)

and use in the following the normalised optical matrix $M^{(N)}$ which is composed by the row vectors $\vec{s}_i^{(N)}$.

‡ The two degrees of freedom to be controlled are the rotation around the x-axis and the rotation around the y-axis, with the optical axis being aligned with the z-axis.
Figure 1. An example of a 3D system in which the sensor signals $s_i$ are shown with their errors (or uncertainties) indicated by volumes around the measured vectors in the mirror angular position space $\beta$.

The entire set of mirror alignment angles can only be decoupled and then controlled if the signals $s_i$ describe the whole mirror angular position space. Mathematically this condition is fulfilled for

$$\det(M) \neq 0$$

Thus, in a noise-free and stationary system it would be possible to produce a working control system whenever the optical matrix determinant differs from 0. However, in practice, the measurements of the matrix coefficients are affected by measurement noise; in addition, the system parameters will change slowly with time and thus the optical matrix will slowly deviate from the initially measured one. Therefore it is not sufficient to develop a system with an optical matrix whose determinant is non-zero. We can visualise this by plotting the signal vectors with uncertainties represented as cones around them, as shown in Figure 1. The requirement given in equation 4 can be interpreted visually such that the cones around the signal vectors must not overlap.

In order to evaluate the controllability of a $n$-dimensional system, described by the optical matrix $M$ with dimension $n \times n$, we can compute the reconstructed volume in the mirror space by using the wedge product as:

$$V = |\det(M^{(N)})|$$

Where $V$ is the volume spawned by the normalized sensor signals in the mirror space. This volume already provides information about the controllability of the system: 0 correspond to an uncontrollable system while 1 correspond to a perfectly decoupled system. However, the numerical values in between have proven to be not intuitive. For systems with $V > 0$ we can compute an equivalent spatial separation between the row vectors of the optical matrix. This is done using an equivalent optical matrix $M'$ which spawns the same volume, but is constructed to have the first $(n-1)$ vectors being orthogonal and the $n$-th vector being misaligned with respect to the first vector by an...
angle $\alpha$, which we call quality parameter. It turns out that $\alpha$ relates directly to the determinant of the original matrix $M^{(N)}$ as:

$$\alpha = \arcsin \left( |\text{det}(M^{(N)})| \right)$$

This numerical description provides a more intuitive approach to the controllability evaluation and gives a realistic order of magnitude estimation for the separation between the degrees of freedom of the alignment system.

If the optical sensing system is over-determined, i.e. the optical matrix is not square, we can apply a similar method by selecting the sub-set of row vectors, of dimension equal to the number of degrees of freedom to control, which maximizes the quality parameter.

3. Alignment control system for a Fabry-Perot cavity

In this section we present the analysis of alignment systems for a linear Fabry-Perot cavity, in order to validate our method and to provide an intuitive example.$\parallel$ It is interesting to evaluate the dependence of the controllability on optical parameters such as the Gouy phase of the detected light $\phi_G$, which can be set by adjusting the telescopes in front of the quadrant photo diodes (quadrants in shorts), and the electronic demodulation phase $\phi_{\text{dem}}$. The analysis is performed for two optical setups based on the Ward and the Anderson-Giordano technique respectively. Both techniques use differential wavefront sensing, a modulation-demodulation method, for generating control signals. In the following we will refer to signal obtained though modulation-demodulation as radio frequency (RF) signals. In addition the quadrant photodiodes provide information about the beam position on the diode. These signals will be called DC signals henceforth.

3.1. The Ward method

In the Ward configuration [5] the modulation sidebands are set to be not resonating in the Fabry-Perot cavity and the alignment signals are detected in reflection of the cavity, as it shown in Figure 2.

The optical matrix coefficients $m_{ij}$ are given by the low frequency limit of the transfer functions between the mirror angular positions and the quadrant signals:

- $\beta_j$ is the angular position of mirror $j$
- $s_1$ and $s_2$ are the P (in-phase) and Q (in-quadrature) signals of the quadrant diode $q_1$
- $s_3$ and $s_4$ are the P and Q signals of the quadrant diode $q_2$

For example, $m_{11}$ represents the transfer function from the motion of mirror 1 into the P signal of the $q_1$ diode.

We have modeled the optical setup using the frequency-domain interferometer simulation Finesse [6], varying the demodulation phase of the $q_1$ quadrant $\phi_{\text{dem}1}$ and

$\parallel$ The Virgo north arm cavity has been arbitrarily chosen as an example optical layout.
Figure 2. The optical layout of a Fabry-Perot cavity with a Ward-like alignment sensing scheme: the alignment signals are detected in reflection of the cavity. The quadrant photo diodes are labeled as \( q_i \). The telescopes in front of the diodes are used to adjust the Gouy phase.

Figure 3. Modeling the Ward sensing system: the left plot shows the optical matrix coefficients as a function of the demodulation phase of the \( q_1 \) diode. The right plot is the quality parameter \( \alpha \) as a function of the demodulation phase \( \phi_{\text{dem1}} \), and the common Gouy phase \( \phi_G \). We can see that the quality parameter does not depend on the demodulation phase, but varies with the Gouy phase tuning between 90 deg and 66 deg.

the common Gouy phase offset \( \phi_G \) (maintaining a constant Gouy phase offset of 90 deg between the two quadrants). From the right plot of Figure 3 we can deduce that the controllability of the system does not depend on the demodulation phase but only on the setting of the common Gouy phase offset. This fact can be inferred also by the behavior of the row matrix elements (\( m_{11}, m_{12} \) and \( m_{21}, m_{22} \) respectively), which are shown in the left plot of Figure 3. The row matrix elements change sinusoidally in phase as a function of the demodulation phase, yielding that the row vectors do not change their orientation, but only their magnitude.

From the simulation results shown in Figure 3 we can see that the controllability is acceptable for all demodulation phases and common Gouy phase offsets (the angle between the row vectors is constant for different demodulation phases and remains above \( \sim 66 \) deg for all the common Gouy phase offsets).
3.2. The Anderson-Giordano method

We now repeat the analysis for an optical configuration based on the Anderson-Giordano technique [7], a variant of the Anderson method [8]. In this configuration the modulation frequency is tuned to have the first higher-transversal mode of the upper sideband resonating in the cavity, and the alignment signals are detected in transmission, as it is shown in Figure 4.

From the left plot in Figure 5 we can see that for the Anderson-Giordano technique the row matrix elements are not varying in phase as a function of the demodulation phase offset. This implies a strong dependence of the orientation of the row vector on the demodulation phase which is confirmed by the right plot of Figure 5. Such a system remains well controllable, reaching a minimum value for the quality parameter of $\sim 45\text{deg}$, but if not optimised might be less robust with respect to the Ward-like system described above. The point to note is that for any choice of Gouy phase it is always possible to optimise the system by adjusting the demodulation phase.
4. The Virgo interferometer

After the cavity examples, we apply this method to a more complex system: the main Virgo interferometer, shown in Figure 6 (the optical layout is described in [1]). Since the first development of the automatic mirror alignment system in Virgo the designed RF control scheme has never been implemented with the expected performance. The controllability analysis described in this article was developed to evaluate sensing schemes for the Virgo alignment system and consequently to develop a new control scheme.

We have modeled the present Virgo optical layout, considering the use of RF and DC signals to control the mirror positions. The Virgo configuration features ten mirror angular degrees of freedom to control (PR, NI, NE, WI, WE), ten quadrant diodes and thus sixty signals in total, considering the in phase, in quadrature signals for the RF demodulated signals and the twenty DC asymmetry signals. We have prepared the model of this configuration by reproducing the experimental strategy for tuning the demodulation phases of all the quadrant diodes. The Figure 7 shows that a system which uses only one RF modulation and no DC signals, as it was chosen for the original design of the alignment system, yields a quality parameter of only $\alpha \sim 2 \text{deg}$.

The controllability can be improved with the using the DC asymmetry signals in addition to the RF signals ($\alpha > 65\text{degrees for any choice of phases}$). These results are perfectly in agreement with the experimental observations, in fact, starting from the C6 commissioning run in July 2005 [9] the common mode of the end mirrors was controlled indeed using a DC signal. However, using DC signals has some obvious disadvantages:
Figure 7. Modeling the Virgo interferometer: the left plot shows the quality parameter $\alpha$ as a function of the demodulation phase of one quadrant diode, and the common Guoy phase. Considering only the RF signals, the $\alpha$ parameter is $\sim 2$ deg which describes a not well decoupled system. The right plot is the $\alpha$ parameter as a function of the demodulation phase of one quadrant diode and the common Guoy phase offset, considering also the DC quadrant signals in addition to the RF signals. We can notice an improvement in the controllability obtaining a well controllable system with $\alpha \sim 66$ deg.

the alignment reference for the DC signal is given by the position of the photo diode which is subject to seismic noise, whereas a modulation-demodulation method uses the interferometer itself as the reference. Thus it was important to develop a new scheme which uses only RF signals to control all degrees of freedom.

In order to develop a better sensing scheme we have added a second modulation frequency $\omega_{\text{add}}$ and limited the sensing system to single-demodulation RF signals. We computed the quality parameter as a function of $\omega_{\text{add}}$. It showed that the controllability drops when the modulation frequency is resonant in the central interferometer (multiples of $\sim 6$ MHz). We decided then to investigate a system using the additional modulation frequency of $\omega_{\text{add}} = 8.352$ MHz. This frequency fulfills the requirements of being a multiple of the FSR of the IMC and of being relatively close to the first RF frequency ($\sim 6$ MHz) which allows us to use the existing electronics for detection and demodulation.

The model predicts that by using the two modulation frequencies the controllability can be strongly improved from a few degrees up to $\sim 40$ deg. Consequently this new control strategy has been implemented in the Virgo detector in February 2007. As an immediate result we observed a strong improvement in the global overall alignment of the interferometer and in the reproducibility and stability of the interferometer parameters and sensitivity.

It should be noted that the performance of the resulting control system is not completely quantified by the quality parameter. At least, a noise propagation analysis should be performed.
4.1. Conclusions

During the Virgo commissioning the original design for the mirror alignment system was found to be not adequate, leaving one degree of freedom mostly uncontrolled. Several potential optical readout schemes for the alignment control were proposed and needed to be compared. In this paper we have described a simple method to qualify electro-optical control schemes of suspended interferometers by analysing the optical sensing matrix. We make use of the well-known fact that the determinant of the optical matrix must not be zero for the respective control system to work. However, we have derived from this principle an intuitive and quantitative parameter for comparing different systems. We motivated and verified our method using the well studied example of a Fabry-Perot cavity. Consequently our method has been used to search for and select a new mirror alignment control scheme for the Virgo interferometer. This new scheme has been implemented and is currently operating, providing a stable alignment control which caused a performance increase of the entire interferometer. A more in-depth study of the alignment control system performance and the propagation of control noise is underway.

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