Implications of Local Chiral Symmetry Breaking

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The spontaneous symmetry breaking of a local chiral symmetry to its diagonal vector symmetry naturally realizes a complete geometrical structure more general than that of Yang-Mills (YM) theory, rather similar to that of gravity. A good example is the Quantum Chromodynamics (QCD) with respect to the Chiral Color model. Also, a new anomaly-free particle content for a Chiral Color model is introduced: the Chiral Color can be realized without introducing whole new generations of quarks and leptons, but by simply enlarging each generation with new exotic fermions.

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I. INTRODUCTION

One of the most important challenges in the high energy physics today is to find out what lies beyond the standard model. Since the standard model is based on gauge theory, one of the simplest attempts is to consider extending the gauge symmetry in four dimensions. Models in this line of extension includes, most prominently, the grand unification models\cite{1,2} and, more modestly, the left-right symmetric model\cite{2,3} and (extended) technicolor models\cite{4,6}, etc.

The Chiral Color\cite{5} is another model with an extended gauge symmetry in this spirit. It was proposed as a generalization of QCD such that QCD, which is vector-like, may be a result of spontaneous symmetry breaking of the Chiral Color. The gauge group is two copies of SU(3) with a global $Z_2$ symmetry, i.e. the left-right symmetry, to start with (other models having two copies SU(3) with a global symmmetry, let one of the coupling constants, say, $g_1 \to \infty$ and the other to be $g = g_2$, without loss of generality. This corresponds to the Topcolor (when $N = 3$ and both are vector-like)\cite{4,6}.

In the Chiral Color case, $G_{CC}$ must be broken spontaneously to SU(3)$_C$\cite{20}, leaving eight massive axial vector bosons called axigluons. The search so far has ruled out axigluons lighter than 1 TeV\cite{10}. Nevertheless, it is still a very interesting idea since there is no a priori reason axigluons must be light.

In this letter we provide other reasons why we should further pursue the Chiral Color. These are two-folds: First, the mystery of the YM geometry can be clarified and it brings up a new geometrical structure for a gauge theory similar to that of gravity. Second, the Chiral Color can be realized by enlarging the content of each generation without introducing new fermion generations beyond that of the standard model. This is done by minimally introducing new exotic fermions in each generation. The latter indicates this new Chiral Color model is not a technicolor model in which technifermions are introduced as separate generations, and the former may provide a new clue toward nonperturbative aspects of QCD as well as a new way of relating the gauge theory and gravity.

II. GEOMETRY OF LOCAL CHIRAL SYMMETRY

Under $G_{CC}$, the left-right symmetry implies that the roles of two sets of gauge fields are not particularly different. However, upon symmetry breaking $G_{CC}$ to SU(3)$_C$, the difference between $B_\mu$ and $u_\mu$ clearly emerges. $B_\mu$ still transforms as a gauge field under SU(3)$_C$, but $u_\mu$ now transforms covariantly:

$$B_\mu \to U^{-1}(B_\mu + \partial_\mu)U,$$

$$u_\mu \to U^{-1}u_\mu U.$$
Under the left-right symmetry, $B_\mu$ is even and $u_\mu$ is odd. In the sense that $u_\mu$ transforms covariantly, the axigluon $u_\mu$ takes the role of “vector matter” under SU(3)$_C$. One may be tempted to introduce this type of vector matter by hand to QCD. But, the massless $u_\mu$ case is just a field redefinition of unbroken $G_{CC}$. Since the mass term of $u_\mu$ is SU(3)$_C$ invariant, one may add the mass term by hand to distinguish it. Unfortunately, this massive case turns out to be non-renormalizable. This confirms that, as far as vector fields are concerned, the gauge invariance alone is not good enough to ensure the renormalizability. As is well known, the mass of a gauge boson must come from a Higgs mechanism to be renormalizable. In our case, this argument is extended further to show that the mass of a covariant vector field cannot be introduced by hand to be renormalizable.

The gauge invariant coupling of $u_\mu$ to $B_\mu$ can be elegantly expressed by defining $H_{\mu\nu}^a \equiv \partial_\mu u^a_\nu - \partial_\nu u^a_\mu + f^{abc}(B^b_\mu u^c_\nu - B^b_\nu u^c_\mu)$. Note that $H_{\mu\nu}$ transforms covariantly under SU(3)$_C$. Then the gauge field part of the lagrangian can be rewritten as follows:

$$\mathcal{L}_{CC} = -\frac{1}{4} \text{Tr} \left( F_{\mu\nu}^{(1)} \right)^2 - \frac{1}{2} \text{Tr} \left( F_{\mu\nu}^{(2)} \right)^2 = -\frac{1}{4} \text{Tr} (H_{\mu\nu} + [u_\mu, u_\nu])^2 - \frac{1}{2} \text{Tr} (H_{\mu\nu})^2.$$  (5)

Due to rather complicated couplings between axigluons and gauge fields, the first term is expressed in a somewhat unfamiliar way. Nevertheless, the Bogomol’nyi-Prasad-Sommerfeld (BPS) limit still retains that of SU(3)$_C$ case. In fact, the instanton solution of SU(3)$_C$ case corresponds to the common instanton solution of SU(3)$_L$ and SU(3)$_R$ when the sizes and instanton numbers of these instantons are identical and $u^a_\mu = 0$, in accordance with the left-right symmetry of the Chiral Color. Upon Higgs Mechanism, the mass term of $u_\mu$ is to be added.

As is well known, the usual geometry of YM theory only involves a (spin) connection, $B^a_\mu$, and a curvature, $F_{\mu\nu}^a$. Here we have $u^a_\mu$ in addition. Looking at the structure of Eq. (7), we can identify $H_{\mu\nu}^a$ as a torsion and $u^a_\mu$ as vielbeins.

Although it has been clearly emphasized by Yang\cite{11,12} that the geometry of YM theory is based on a principal (fiber) bundle, in which the dynamical variable is a gauge field and it behaves like a connection on the group manifold. On the contrary, the geometry of General Relativity (GR) is based on a tangent bundle, in which the metric is a fundamental dynamical variable. Nevertheless, one cannot help but wondering why the analog of a metric or a vielbein is missing in the geometry of YM theory. In this letter, we have presented the answer to resolve this mystery: these extra data are present if we consider a vector gauge theory inherited from a local chiral symmetry and its breaking.

The appearance of the vielbeins in the geometry of YM theory was first alluded in \cite{13} to construct the virtual monopole geometry associated with BPS monopole solution in the SU(2) YM theory. The structure is a generalization of covariant variables introduced by Johnson and Haagensen\cite{14,15}, in which a constraint equation related the SU(2) gauge fields to these covariant variables. In \cite{14,15}, however, these covariant variables are not field variables that show up in the Lagrangian. In the Chiral Color model these gauge covariant fields are indeed present in the Lagrangian and take the role of the vielbeins with respect to the unbroken local gauge symmetry, as presented here. In the limit $H_{\mu\nu}^a = 0$, which corresponds to the torsion-free condition in this context, the gauge field $B^a_\mu$ can be expressed in terms of axial gauge field $u^a_\mu$. However, this is not necessarily a desired structure in this model. Perhaps, there might be interesting phenomena involving nonvanishing torsion in the line of the idea presented in \cite{13}, in which three different phases of QCD (Coulomb, confinement and, suspectedly, oblique confinement) are associated with three different asymptotic behaviors of the torsion. This resolves the difficulty of relaxing the torsion-free condition in \cite{14,15} since in the Chiral Color the torsion is unambiguously present because the axial gauge field $u_\mu$ is an independent dynamical variable.

### III. CHIRAL COLOR WITH ENLARGED GENERATIONS

This raises a question if Nature actually likes the proposed structure. If there is no realistic model what so ever, it could indicate that Nature prefers the geometric data of YM theory being fundamentally different from that of gravity. So It all depends on the existence of a realistic Chiral Color model. The answer to this quest is affirmative and the model we present here is without unnecessary fermion generations beyond the standard model yet containing interesting particle content.

Under the extended gauge symmetry

$$G = SU(3)_L \times SU(3)_R \times SU(2)_L \times U(1)_Y.$$  (7)

the quarks and leptons in each generation transform as

$$(3,1,2,1/6)_L, \ (1,3,1,2/3)_R, \ (1,3,1,1/3)_R,$$

$$(1,1,2,−1/2)_L, \ (1,1,1,−1)_R.$$  (8)

Since the charge assignment is the same as that of the standard model, the SU(2)$_L \times U(1)_Y$ anomalies cancel. In the Chiral Color case there is an additional anomaly due to the chiral nature of SU(3)’s. There are various ways of canceling the SU(3) anomaly by introducing additional fermions, as presented in \cite{8}. But all of them employs additional generations, which inevitably introduce new lepton flavors. New lepton flavors are unwelcome unless we can make sure the accompanying neutrinos are sterile.

In this letter, we will cancel the SU(3) anomaly without introducing a new generation hence avoiding unwanted lepton flavors, but by enlarging the generation itself.
Since there is no \textit{a priori} reason that at higher energy scales the only matters available should be still quarks and leptons, one can argue that there could be additional siblings to quarks and leptons in each generation, massive and exotic enough to be unobservable at the standard model energy scale. The choice we make to cancel the SU(3) anomaly due to quarks is

\begin{equation}
(\overline{3}, 1, 2, 0)_L, (1, \overline{3}, 1, 1/2)_R, (1, \overline{3}, 1, -1/2)_R. \tag{9}
\end{equation}

The hypercharges are assigned as shown such that there are no new SU(2)_L \times U(1)_Y anomalies generated by these extra fermions. As a result, these extra fermions carry half-integer electric charges. All anomalies are canceled in each generation separately as in the standard model.

These new exotic fermions are very much like (anti-) quarks except carrying different electric charges. Due to the electric charge conservation they do not directly couple to quarks and leptons. Most likely, these exotic fermions are characteristically more massive than quarks. They go through electroweak processes. Neutral currents can produce exotic fermions in pairs. These signals look very much alike quark pair production. The distinction will be made by measuring the electric charge. Also high energy quarks and leptons can produce these exotic fermions via a W boson process. A good place to look for such a signal is the atmospheric interaction with ultra high energy cosmic neutrinos.

The presence of hybrid bound states between these exotic fermions and quarks is highly undesirable, although not necessarily ruled out experimentally. Note that the choice of assigning these exotic fermions as \( \overline{3} \) forbids at least scalar hybrid bound states because the color singlet combination between a quark and an exotic fermion cannot be a Lorentz scalar. Other hybrid states may exist and carry fractional charges in multiples of \( 1/6e \). Among exotic fermions, the color singlet fermion-antifermion, or three-fermion bound states can be formed and the three-fermion states carry half-odd-integer electric charges. The observation of such a fractionally charged states will tell us if Nature likes such a new structure in addition to the observation of axigluons.

Unless there is other hidden structure in the more extended context, e.g. the existence of a grand unified model incorporating these exotic fermions, the first generation of them must be stable. Certainly, at this moment we do not have any evidence for their existence in the macroscopic world in which only the first generation of quarks and leptons are clearly present\[^{16}\]. However, the search for a fractional electric charge particle has been still actively going on by a team led by Perl\[^{17}\].

One of the search is looking at bulk matter from outside the solar system because such a particle might have been produced in the early universe. Due to lack of direct coupling to quarks, it is quite possible these exotic fermions might have been decoupled very early on at the electroweak scale.

This also rehashes the elusive question of the electric charge quantization. All observable particles in Nature so far carry integer multiplication of the elementary charge and one justification for this electric charge quantization is the existence of a Dirac magnetic monopole. However, we have not found any evidence of the existence of a magnetic monopole, hence the electric charge quantization is still an open question. Discovery of a fractionally charged particle may change our current preference on this issue: perhaps electric charge is not quantized by integer multiplication, but by one-sixth-integer multiplication. As a matter of fact, even the existence of a magnetic monopole does not necessarily dictate the integer multiplication of electric charge quantization. For example, if a nonabelian magnetic monopole is produced as a \( \mathbb{Z}_2 \) symmetric vortex, which is the case of SU(2) \rightarrow U(1) \) breaking by an adjoint higgs, the electric charge is quantized by half-integer multiplication\[^{18}\]. Also, in the string-inspired models, \( \mathbb{Z}_N \) discrete symmetry which commonly appears in the orbifold compactification leads to \( 1/N \) fractional charges\[^{18}\]. Perhaps, there may be a hidden structure with \( \mathbb{Z}_6 \) discrete symmetry at the Chiral Color scale, which can explain even why quarks carry fractional charges.

In the original Chiral Color model, the Chiral Color symmetry breaking scale was assumed to be the same as the electroweak symmetry breaking scale. This can certainly be relaxed and we can push the Chiral Color symmetry breaking scale higher than the electroweak symmetry breaking scale. So the symmetry breaking of \( G_{CC} \) to SU(3) \( C \) can be achieved, in the simplest case, by a scalar multiplet transforming as \( (3, \overline{3}, 1, 1) \). Although the Chiral Color scale can be, in principle, anywhere, there is one desirable scale we can speculate. Suppose there were a relationship between scales such that the square of the electroweak scale is directly related to the multiplication of the Chiral Color scale and the QCD scale. This will put the Chiral Color scale at about \( 10 \sim 100 \text{ TeV} \). This will be the most interesting Chiral Color scale because it is less \textit{ad hoc} than having three totally unrelated scales. Furthermore, this Chiral Color scale is certainly reachable in the near future at the LHC (Large Hadron Collider), hence we may begin to see the Chiral Color signals.

Note that unlike in the technicolor models, these new fermions do not exclusively interact with axial vector combination of \( G_{CC} \). This also distinguishes our model from technicolor models.

\section{Remarks}

Finally, a few remarks on further works to investigate the implications of the structure presented in this letter are in order.

This new geometry of YM theory can be a link between QCD and QCD string, perhaps a detail study may shed new light on nonperturbative aspects of QCD. The geometrical structure presented in this letter also opens up a new way of thinking about the gravity as a gauge
theory since it has all the necessary geometrical data. The spacetime metric can be identified as an induced metric

$$g_{\mu\nu} = \xi_{ab} u^a_{\mu} u^b_{\nu}, \quad (10)$$

where $\xi_{ab}$ is the Cartan metric of the gauge group. This in turn can be solved for spacetime vielbeins as

$$g_{\mu\nu} = \eta_{AB} e^A_{\mu} e^B_{\nu} \quad (11)$$

where $A, B$ are indices for the local Lorentz symmetry and $\eta_{AB}$ is the Minkowski metric. It will be a good mission to look for a gauge group for which gravity can be realized this way, and the search is in progress.

Our proposal of extending the standard model using the Chiral Color opens up many interesting possibilities we can further investigate. Not only looking for axigluons, but we can also search for the possibility of fractionally charged three-fermion states to see if Nature really respects the charge quantization rule beyond the standard model. There are also various signals involving the new exotic fermions. It should certainly provide new opportunities for future accelerators and particle astrophysics.

There are other new aspects we can study in this context. Quarks now interact not only with gluons but with axigluons. Being massive, the axigluons will not change the long distance behavior of quarks. However, the short distance behavior of quarks will be modified. The short distance interquark potential will not be just coulombic, but may have a Yukawa type correction.

The mass matrix of the new exotic fermions should have an analogous structure as the Cabibbo-Kobayashi-Maskawa matrix of quarks, hence possibly induces a CP-violation in the exotic fermion processes. This new CP-violation can contribute to the matter-antimatter asymmetry, helping the current discrepancy between the observed baryon asymmetry and theoretical estimations.

We can speculate that some of the matter identified as baryons could be matter formed by these new exotic fermions since we never considered this possibility so far. The Chiral Color model has such fascinating structures, Nature may provide an evidence of its relevance in the near future. Further progress will be presented elsewhere.

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