Quarkonium Physics and $\alpha_{\text{strong}}$ from Quarkonia

Junko Shigemitsu $^a$*

$^a$Physics Department, The Ohio State University, Columbus, Ohio 43210, USA.

Recent results in Quarkonia are reviewed, including updates on spectroscopy and $\alpha_s$, and a first look at quarkonium annihilation decays.

1. Introduction

Quarkonia serve as good testing ground for QCD and for theoretical approaches to QCD, including the lattice, due to their rich spectroscopy and decay phenomenology. Furthermore, heavy-heavy and heavy-light systems, are playing a crucial role in probing the electroweak sector of the Standard Model. Hence, it is very important that we have reliable well tested methods for simulating heavy quarks.

Both four component and two component non-relativistic (NRQCD) fermions are being used to simulate heavy quarks [1,2]. Most of the theory has been reviewed in previous lattice meetings. Only a few comments on recent developments in the four component approach will be made here. The first formulation of four component heavy quarks on the lattice was developed by the Fermilab group [1]. To date this approach has been implemented at relatively low order. At this level one works with the same set of parameters as in the clover action for light quark simulations, namely the Wilson hopping parameter $\kappa$ and the clover coefficient $c$. For heavy quarks, truncation to these two parameters, does not allow the coefficient of the $p^4$ term in a low momentum dispersion expansion, $E = M_1 + p^2 / 2M_2 + O(p^4)$ to come out correctly. What implications this has for $M_{1\text{hadron}}$ versus $M_{2\text{hadron}}$ of composite particles, is discussed by Andreas Kronfeld in a poster session at this conference [3]. For hadrons containing heavy quarks, it is $M_{2\text{hadron}}$ that should be identified with the physical mass. In order to minimize uncertainties associated with $M_{2\text{hadron}}$ and in fixing $\kappa$ it is important that the $p^4$ contribution in the action be improved. Problems with $M_2$ in heavy-heavy and heavy-light hadrons using a straight tadpole improved clover action, were pointed out a year ago at Melbourne by Sara Collins and John Sloan [4].

Other four component approaches to heavy quarks have now appeared. Alford-Klassen-Lepage have a D234(2/3) action that goes beyond clover and removes $O(a^2)$ errors at tree level. Working with anisotropic lattices, they find good relativistic dispersion relation and spectroscopy results even for heavy quarks around charm [5]. The MIT-BU group has implemented a “perfect action” for fermions truncated to couplings within a hypercube and also find encouraging results for charm [6].

Due to time constraints, for the rest of the talk I will concentrate on quarkonium physics results rather than formalism and refer the reader to parallel and poster sessions for more theory.

2. Brief Overview of Spectroscopy

The first test of any lattice method for heavy fermions, is to see how well the observed quarkonium spectrum can be reproduced. Figures 1 & 2 summarize Upsilon spectroscopy results by several groups using both quenched ($n_f = 0$) and dynamical ($n_f = 2$) configurations. Tadpole improved clover (Fermilab & SCRI) and non-relativistic fermions with improvements through $O(Mv^4)$ (NRQCD) have been employed. Two parameters were fixed by experiment, the bare quark mass and the lattice spacing. Fermilab and SCRI use the S-P splitting to fix $a^{-1}$. The NRQCD collaboration determines $a^{-1}$ from both the S-P and the 1S-2S splittings and uses an aver-
Figure 1. $\Upsilon$ Spectrum

age in the plots. A first glance at Fig. 1 suggests good agreement between simulations and experiment. A closer look reveals, for instance, that for the NRQCD data neither the $1P$ nor the $2S$ state lies on the experimental line. This is particularly noticeable for the $n_f = 0$ results and is a reflection of a quenching effect that leads to different quantities giving different $a^{-1}$'s for the wrong number of dynamical flavors if these quantities are governed by different characteristic energy scales. We will see later, that such details are important in precise determinations of $\alpha_s$.

Quenching effects are also observable in the spin splittings of Fig. 2. They tend to underestimate spin splittings. Hyperfine and fine-structure splittings are discussed in more detail in later sections. Fig. 3 shows lattice spectrum results for Charmonium.

3. $\alpha_s$ from Quarkonia

One major goal of lattice gauge theory is to contribute towards determination of fundamental parameters of the Standard Model. Quarko-

Figure 2. $\Upsilon$ Spin Splittings: Symbols have the same meaning as in Figure 1.

Figure 3. Charmonium Spectrum: Symbols have the same meaning as in Figure 1.
charm to bottom quark region. Thermore experimentally S-P and 1S-2S splittings in light hadron studies. One works directly at or mass extrapolations like the chiral extrapolation \( \alpha \) on this particular value. (iii) Convert to \( \alpha \)ir

For step (i), two popular choices are \( \alpha_P \) (P: plaquette.) [4] and \( \alpha_{SF} \) (SF: Schroedinger Functional ) [2]. In quarkonium studies to date one has used \( \alpha_P \). It is defined through the \( 1 \times 1 \) Wilson loop as,

\[
-\ln W_{1,1} = \frac{4\pi}{3} \alpha_P \left( \frac{3}{a} \right) [1 - (1.19 + 0.07n_f)\alpha_P]
\]

\( \alpha_P \) agrees through order \( \alpha^2 \) with the more familiar \( \alpha_V \) of Brodsky-Lepage-Mackenzie. It is defined so that the above relation terminates at \( \alpha^2 \) and there are no higher order corrections. Hence the only errors in measuring the value of \( \alpha_P \) come from statistical errors in \( W_{1,1} \) which are extremely small. In principle one must ascertain that nonperturbative, i.e. non power series, contributions to \( W_{1,1} \) are negligible. Several tests confirm that any nonperturbative contamination would be smaller than one percent [4].

Most of the work and the source of all the uncertainty in \( \alpha_s \) determinations involve steps (ii) and (iii). Quarkonium studies come into play at step (ii), when one sets the scale \( Q = \frac{3a}{r} \), by inserting a value for \( a^{-1} \). Any dimensional quantity can be used to determine \( a^{-1} \). Quarkonium level splittings are particularly suitable since systematic errors are easiest to control.

**Finite volume**: Quarkonia, especially \( \bar{b}_b \), are small compared to light hadrons

**M_Q dependence**: There is no need to carry out mass extrapolations like the chiral extrapolation in light hadron studies. One works directly at or very close to realistic heavy quarks masses. Furthermore experimentally S-P and 1S-2S splittings in quarkonia are very insensitive to \( M_Q \) in the charm to bottom quark region.

**Finite \( a \)**: Heavy quark actions, such as NRQCD, can and have been improved through order \( a^2 \). Effects of \( a^2 \) errors in the Wilson glue action have been estimated perturbatively. Fully improved gluonic actions through order \( a^2 \) and beyond, are starting to be employed as well.

**Quenching or \( n_f \) dependence**: This is the most subtle systematic error that needs to be understood and corrected for. Two questions must be settled first: 1. What is \( n_f^{phys} \), the relevant number of dynamical quark flavors? 2. How do quarkonium splittings depend on \( m_q^{dyn} \), the dynamical quark mass?

Typical gluon momenta \( p_T \) inside an \( \Upsilon \) are between 0.5 and 1. GeV. The relation \( m_u, m_d, m_s << p_T \) tells us that \( n_f^{phys} = 3 \) for \( \Upsilon \) splitting physics. This answers the first question. It is necessary to ask the second question, because dynamical quarks in current simulations are heavier than physical up- or down-quarks. Perturbation theory would indicate that energy level splittings, \( \Delta E \), depend quadratically on \( m_q^{dyn} \). However, due to chiral symmetry breaking in QCD, \( \Delta E \) actually has a linear \( m_q^{dyn} \) dependence [3]. The relevant combination of light quark masses is \( (m_u + m_d + m_s) \). Since all three quark masses are significantly smaller than \( p_T \), one should be able to replace the three unequal mass quarks with three flavors of degenerate quarks with \( m_{eff} = (m_u + m_d + m_s)/3 \). \( \Upsilon \) physics should not care about the details of the \( m_{u,d} - m_s \) mass differences. In summary, to undo quenching effects, one needs to extrapolate, \( n_f \rightarrow n_f^{phys} = 3 \) and \( m_q^{dyn} \rightarrow m_s/3 \). The fact that one only needs to extrapolate in \( m_q^{dyn} \) down to \( m_s/3 \) and not beyond, highlights one of the true advantages of working with the \( \Upsilon \) system. Unquenching in systems where one is more sensitive to \( m_{u,d} - m_s \) mass differences, e.g. any system where characteristic gluon momenta are of order \( \Lambda_{QCD} \) or \( m_s \) will be more complicated. Even within quarkonia, the charmonium system should have larger uncertainties associated with \( m_q^{dyn} \) than \( \Upsilon \).

### 3.1. Procedure

There are three steps involved in determining \( \alpha_s \) from the lattice and comparing with other determinations. (i) Define a suitable \( \alpha_s \) and measure its value. (ii) Set the scale at which \( \alpha_s \) takes on this particular value. (iii) Convert to \( \alpha \).

For step (i), two popular choices are \( \alpha_P \) (P: plaquette.) [4] and \( \alpha_{SF} \) (SF: Schroedinger Functional ) [2]. In quarkonium studies to date one has used \( \alpha_P \). It is defined through the \( 1 \times 1 \) Wilson loop as,

\[
-\ln W_{1,1} = \frac{4\pi}{3} \alpha_P \left( \frac{3}{a} \right) [1 - (1.19 + 0.07n_f)\alpha_P]
\]

\( \alpha_P \) agrees through order \( \alpha^2 \) with the more familiar \( \alpha_V \) of Brodsky-Lepage-Mackenzie. It is defined so that the above relation terminates at \( \alpha^2 \) and there are no higher order corrections. Hence the only errors in measuring the value of \( \alpha_P \) come from statistical errors in \( W_{1,1} \) which are extremely small. In principle one must ascertain that nonperturbative, i.e. non power series, contributions to \( W_{1,1} \) are negligible. Several tests confirm that any nonperturbative contamination would be smaller than one percent [4].

Most of the work and the source of all the uncertainty in \( \alpha_s \) determinations involve steps (ii) and (iii). Quarkonium studies come into play at step (ii), when one sets the scale \( Q = \frac{3a}{r} \), by inserting a value for \( a^{-1} \). Any dimensional quantity can be used to determine \( a^{-1} \). Quarkonium level splittings are particularly suitable since systematic errors are easiest to control.

**Finite volume**: Quarkonia, especially \( \bar{b}_b \), are small compared to light hadrons

**M_Q dependence**: There is no need to carry out mass extrapolations like the chiral extrapolation in light hadron studies. One works directly at or very close to realistic heavy quarks masses. Furthermore experimentally S-P and 1S-2S splittings in quarkonia are very insensitive to \( M_Q \) in the charm to bottom quark region.

3.2. Results

For the quenched, \( n_f = 0 \), case several groups have now extracted \( a^{-1} \) from quarkonium S-P
splittings, which can then be used to set the scale in $\alpha_s^{(n_f=0)}(\frac{3.4}{\pi})$. However, very few $n_f > 0$ results exist to date. The most accurate numbers come from the NRQCD collaboration (using HEMCGC dynamical configurations) who have combined this with their $n_f = 0$ numbers and carried out the extrapolation to $n_f^{\text{phys}}$ described above [13,14]. Recently the SCRI group has calculated an $n_f = 2$ $a^{-1}$ from the $\Upsilon$ S-P splitting on the same dynamical configurations, however using tadpole improved clover [15]. I have taken the liberty to combine their $a^{-1}$ with quenched clover results from the Fermilab group to perform the $n_f \to n_f^{\text{phys}}$ extrapolation.

Fig. 4 summarizes results for $1/\alpha_p$ versus $\ln(\frac{3.4}{\pi})$. This is an updated version of a plot originally made by Matthew Wingate [14]. Only data from the last two years are included. There are several comments to be made.

The $n_f = 0$ charmonium data (open boxes) and $\Upsilon$ data (open circles) lie on two separate scaling curves. This is due to the fact that the characteristic energy scale differs between charmonium and $\Upsilon$. In a quenched simulation, $\alpha[Q]$ runs incorrectly between the two characteristic scales. When real world data (which incorporates correct running of $\alpha_s$) is used to extract $a^{-1}$, a mismatch is found which results in the two scaling curves being shifted horizontally with respect to each other. A similar phenomenon is found by the NRQCD collaboration when they compare $a^{-1}$s from $\Upsilon$ S-P and 1S-2S splittings. It is only after extrapolating to $n_f^{\text{phys}}$: that physics becomes independent of which splitting was used (see below).

The $n_f = 2$ configurations used by NRQCD and SCRI have $am_q^{\text{dyn}} = 0.01$. How close is this to the desired $m_s/3$? Based on the light staggered spectrum calculations by HEMCGC [17], this $m_q^{\text{dyn}}$ lies anywhere between $\sim m_s/2$ (from the kaon) and $\sim m_s$ (from the $\phi$). Hence there are large uncertainties in how far to extrapolate in $m_q^{\text{dyn}}$. Furthermore, in order to do the extrapolation, data at another value of $m_q^{\text{dyn}}$ are required. The NRQCD collaboration has results from $am_q^{\text{dyn}} = 0.025$ [14]. The errors in those simulations are still large. A 1 to 1.5 $\sigma$ difference was found between $am_q^{\text{dyn}} = 0.01$ and 0.025. It is not clear yet whether true $m_q^{\text{dyn}}$-dependence has been observed. So, the 0.025 data has only been used to estimate the error associated with $m_q^{\text{dyn}}$. The central value for $\alpha_p$ is given by the 0.01 result and the error is estimated by taking the difference between this and an extrapolation using the 0.025 data to $am_q^{\text{dyn}} = (0.01)/3$.

The NRQCD results after the $n_f \to 3$ extrapolation in $1/\alpha_p$ are,

$$\alpha_p^{(n_f=3)}[8.2\text{GeV}] = \begin{cases} 0.1948 (29)(11)(37) (S - P) \\ 0.1962 (41)(08)(40) (1S - 2S) \end{cases}$$

The three errors correspond to statistical, discretization/relativistic and to $m_q^{\text{dyn}}$ errors respectively. The S-P result is the rightmost $n_f = 3$ (fancy box) data point in Fig. 4 (the 1S - 2S number falls right on top of it). The other four $n_f = 3$ data points correspond from right to left to Fermilab/SCRI, CDHW [23] and to two preliminary NRQCD results from $\Upsilon$ and charm at lower $\beta$, using UKQCD quenched and MILC dynamical configurations.

Finally, step (iii) involves conversion to the $\overline{\text{MS}}$ scheme, in order to be able to compare with other determination of $\alpha_s$. The conversion formula is,

$$\alpha_s^{(n_f=3)}[\overline{\text{MS}}](Q) = \alpha_p^{(n_f=3)}[\overline{\text{MS}}](e^{5/6}Q) \times \left[ 1 + \frac{2}{\pi} \alpha_p^{(n_f=3)} + C_2(n_f) \alpha_p^2 \right]$$

The two loop coefficient $C_2$ has been calculated by Luescher & Weisz for the quenched theory, $C_2(n_f = 0) = 0.96$ [24]. In the past $C_2 = 0$ was used. It now makes more sense to use $C_2 = 0.96$ for central values even for $\alpha_s^{(n_f=3)}$. The difference between $C_2 = 0$ and $C_2 = 0.96$ will serve as an estimate of errors due to uncertainties in the $\alpha_p - \alpha_s$ conversion.

New results for $\overline{\alpha_s}[\overline{\text{MS}}](M_Z)$ exist this year from the NRQCD collaboration, who have updated their previous numbers, and from combining Fermilab’s quenched and SCRI’s dynamical $\Upsilon$ data. NRQCD quotes (the Luescher-Weisz $C_2$ has been used),

$$\overline{\alpha_s}[\overline{\text{MS}}](M_Z) = \begin{cases} 0.1175 (11)(13)(19) (S - P) \\ 0.1180 (14)(14)(19) (1S - 2S) \end{cases}$$
Figure 4. $1/\alpha_P$ versus $\ln(3.4)$, $n_f = 0$ charm: boxes [18,19,20,22,23]; $n_f = 0$ Υ: circles [18,19,21,14]; $n_f = 2$ charm: + [23]; $n_f = 2$ Υ: × [9,14,15]; $n_f = 3$ (extrapolated): fancy boxes. Non lattice determinations of $\alpha_s$ typically find values for $\alpha_{\overline{MS}}[M_Z^0]$ between 0.115 ~ 0.125. Converting to $\alpha_{\overline{MS}}[n_f=3]$, this corresponds to the region between the two full curves in the plot. The dotted lines are $n_f = 0$ three loop scaling curves.

The first error now includes both statistical and discretization/relativistic errors, the second is due to $m_q^{dyn}$, and the third comes from conversion uncertainties. The combined Fermilab/SCRI S-P data give,

$$\alpha_{\overline{MS}}[M_Z^0] = 0.1159 (19)(13)(19)$$

This is a very preliminary number based on this author’s extrapolations. Nevertheless it is encouraging that results using two different fermion methods are approaching one another.

In the future, improvements in $\alpha_s$ from quarkonia should come from further studies of $m_q^{dyn}$ dependence. Simulations with dynamical Wilson configurations or staggered with $n_f > 2$ would be welcome. The largest source of uncertainty for $\alpha_{\overline{MS}}$ is still higher orders in the conversion formula, so $C_2(n_f > 0)$ is crucial. Finally, unquenched lattice $\alpha_s$ determinations other than from quarkonia are eagerly awaited.

4. Quarkonium Annihilation Decays

Annihilation decays of quarkonia provide another fertile arena for lattice investigations. Work has just begun in this area with first results coming from the Argonne group [27]. The theory was developed a couple of years ago by Bodwin-Braaten-Lepage, using the framework of NRQCD [28]. Annihilation decay rates $\Gamma(Q\bar{Q} \rightarrow X)$ ($X$ : light hadrons, $\gamma\gamma$, $l^+l^-$), can be expressed as a power series in $1/M_Q$. The coefficients factorize into a short distance perturbative and a non-perturbative part. Lattice methods can be applied to obtain the non-perturbative contributions that enter as hadronic matrix elements of four fermion operators, e.g.

$$G_1 = \langle iS|\psi^\dagger\chi\chi^\dagger\psi|1S\rangle$$

$$F_1 = \langle iS|\psi^\dagger\chi\chi^\dagger(\frac{-i}{2} D^\dagger D)^2\chi|1S\rangle$$

$$H_1 = \langle iP|\psi^\dagger(i/2) D^\dagger\chi\chi^\dagger(i/2) D \psi|1P\rangle$$

$$H_8 = \langle iP|\psi^\dagger T^n\chi\chi^\dagger T^n \psi|1P\rangle$$

The subscript 8 in $H_8$ denotes a “color octet” contribution, which is sensitive to the $Q\bar{Q}g$ component in P-wave quarkonia, where the heavy
quark and anti-quark form a color octet combination. The remaining three singlet contributions can be written in terms of quarkonium wave functions at the origin or derivatives acting on wave functions, using the vacuum saturation approximation. For instance, \( G_1/|\langle i S|v^i|\chi(0)\rangle|^2 \) leads to \( G_1 \sim \frac{3}{2\pi} |R_S(0)|^2 \) \( (R : \text{radial wave function}) \) and \( H_1/|\langle i P_1|v^i\vec{D}\chi(0)\rangle|^2 = (1+\mathcal{O}(v^i)) \) to \( H_1 \sim \frac{4}{\pi} |R_P(0)|^2 \) etc. The Argonne group finds that vacuum saturation is well satisfied. They work with the lowest order \( \mathcal{O}(Mv^2) \) NRQCD action in their simulations. Their results include one-loop matching between lattice and continuum operators. A comparison between lattice results and experiment is given in Table 1. Within large errors, one sees good agreement for charmonium, less so for \( \Upsilon \). Since these matrix elements are sensitive to wave functions at the origin, one expects significant quenching corrections. One should also note the large uncertainty coming from perturbative matching. The Argonne group is looking into the possibility of nonperturbative renormalization of the relevant operators.

5. Spin Splittings

Spin splittings in quarkonia are sensitive to every single detail in the action. They are significantly affected by quenching and have a strong dependence on \( M_Q \). Tadpole improvement of the action, to remove lattice artifacts, was found to be crucial for achieving agreement with experiment.

In Fig. 5 we show charmonium hyperfine splittings in the quenched approximation versus \( a^2 \). There are three data points from Fermilab using heavy clover at \( \beta=5.7, 5.9 \) and 6.1 [28]. The NRQCD point comes from simulations at \( \beta = 5.7 \) [24]. Then there are three coarse lattice results using D234(3/2) [27] on an anisotropic lattice plus data from Howard Trotter [24], who applied NRQCD on \( O(a^2) \) improved glue. The “\( x \)”s differ from the “\( + \)”s, in that the former have \( a^2 \) errors removed from the \( E \) and \( B \) fields (this is an \( a^2 \) improvement of the NRQCD action that goes beyond tree-level, and appears to be noticeable for \( a > 0.2 \text{fm} \)). It should be noted that current NRQCD simulations have errors of order \( M_\psi v_\psi^6 \sim 5 \text{MeV} \) for \( \overline{b} \). Very preliminary estimates of some of the \( Mv^6 \) corrections in charmonium indicated the hyperfine splitting could come down significantly. Hence, even though the current NRQCD number at \( \beta = 5.7 \) is closest to experiment, the true quenched result could be much lower, e.g. at about the 70 MeV level seen by the Fermilab group. It is important that all the spin dependent \( Mv^6 \) corrections be included in a future NRQCD simulation.

6. Summary

Quarkonia provide many opportunities for lattice gauge theory. There is a rich phenomenology of spectroscopy, decays and yet to be discovered states. \( \Upsilon \) S-P and 1S-2S splittings provide well controlled scale determinations that lead to accurate determinations of \( \alpha_s \). Spin splittings still pose challenges, but we should be up to them. Quarkonia serve as a good testing ground for new actions. Studies of quarkonia create invaluable
Figure 5. **Quenched Charmonium Hyperfine Splittings** versus $a^2$. Fermilab: boxes; NRQCD: diamond; D234(2/3)3:1: fancy boxes; Trottier: + & × ; experiment: burst. Errors are statistical.

experience for going onto heavy-light systems.

**Acknowledgements**

I thank all the people who sent results contained in this review and my colleagues in the NRQCD and heavy-light collaborations for their support and for useful conversations. The organizers of LAT’96 are to be congratulated for an excellent conference. The author’s research is supported in part by the U.S. DOE under DE-FG02-91ER40690.

**REFERENCES**

1. A. El-Khadra, A. Kronfeld, P. Mackenzie; hep-lat/9604004.
2. G. P. Lepage et al., Phys. Rev. D **46**, 4052 (1992).
3. A. Kronfeld; these proceedings.
4. S. Collins et al.; Nucl. Phys. B (Proc. Suppl.) 47 (1996) 455.
5. M. Alford et al., these proceedings.
6. W. Bietenholz et al.; these proceedings.
7. C. T. H. Davies et al.; Phys. Rev. Lett. **73** (1994) 2654.
8. A. El-Khadra et al.; Phys. Rev. Lett. **69** (1992) 729.
9. C. T. H. Davies et al.; Phys. Lett. B **345**, 42 (1995).
10. P. Weisz; Nucl. Phys. B (Proc. Suppl.) 47 (1996) 71.
11. G. P. Lepage, P. B. Mackenzie, Phys. Rev. D **48**, 2250 (1993).
12. M. Luescher et al.; Nucl. Phys. B (Proc. Suppl.) 30 (1993) 139.
13. B. Grinstein, I. Rothstein; hep-ph/9605260.
14. P. McCallum, J. Shigemitsu; Nucl. Phys. B (Proc. Suppl.) 47 (1996) 409.
15. J. Sloan (SCRI); private communication.
16. I thank Matthew Wingate for his plot.
17. K. M. Bitar et al. Phys. Rev. D **49**, 6026 (1994).
18. A. El-Khadra, private communication.
19. S. Hashimoto, A. Ukawa (JLQCD); private communication.
20. P. Boyle (UKQCD); these proceedings.
21. C. T. H. Davies et al.; Phys. Rev. D **50**, 6963 (1994).
22. C. T. H. Davies et al. Phys. Rev. D **52**, 6519 (1995).
23. M. Wingate et al.; Phys. Rev D **52**, 307 (1995).
24. M. Luescher and P. Weisz; Phys. Lett. B **349**, 165 (1995).
25. T. Klassen; private communication.
26. H. Trottier; private communication.
27. G. Bodwin, S. Kim, and D. Sinclair; hep-lat/9605023.
28. G. Bodwin, E. Braaten, G. P. Lepage; Phys. Rev D **51**, 1125 (1995).