Massive Gauge Field Theory Without Higgs Mechanism
III. Illustration of Unitarity

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To illustrate the unitarity of the massive gauge field theory described in the foregoing papers, we calculate the scattering amplitudes up to the fourth order of perturbation by the optical theorem and the Landau-Cutkosky rule. In the calculations, it is shown that for a given process, if all the diagrams are taken into account, the contributions arising from the unphysical intermediate states included in the longitudinal part of the gauge boson propagator and in the ghost particle propagator are completely cancelled out with each other in the S-matrix elements. Therefore, the unitarity of the S-matrix is perfectly ensured.

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1. INTRODUCTION

In the preceding paper called paper II, it was proved that the S-matrix given by the QCD with massive gluons is independent of the gauge parameter. However, the gauge-independence of S-matrix previously was not considered by some people to be a sufficient condition of the unitarity of a theory. Therefore, to demonstrate the unitarity, it is necessary to check whether the contributions arising from unphysical intermediate states to the S-matrix element written for a given process are cancelled in a perturbative calculation. Historically, as mentioned in paper I[1], several attempts[2]−[8] of establishing the massive gauge field theory without Higgs bosons were eventually negated. The reason for this partly is due to that the theories were criticized to suffer from the difficulty of unitarity[9]−[16]. Whether our theory is unitary in perturbative calculations? That just is the question we want to answer in this paper.

To exhibit the unitarity, we choose to calculate two-gauge boson and fermion-antifermion scattering amplitudes given in the perturbative approximation up to the order of $g^4$. According to the optical theorem[17−19], we only need to compute the imaginary parts of these amplitudes. The imaginary part of an amplitude can be evaluated by the following formula[17−19]

$$2ImT_{ab} = \sum_c T_{ac}T_{bc}^* \quad (1.1)$$

which was derived from the unitarity condition of S-matrix: $S S^+ = S^+ S = 1$ and the definition: $S = 1 + iT$. We would like to emphasize that the above formula holds provided that the intermediate states $\{c\}$ form a complete set. This means that when we use this formula to evaluate the imaginary part of an amplitude, we have to work in Feynman gauge. In this gauge, the gauge boson propagator and the ghost particle one are given in the form [1]

$$iD^{ab}_{\mu\nu}(k) = \frac{-i\delta^{ab}g_{\mu\nu}}{k^2 - m^2 + i\varepsilon}, \quad (1.2)$$

and

$$i\Delta^{ab}(k) = \frac{-i\delta^{ab}}{k^2 - m^2 + i\varepsilon} \quad (1.3)$$

where $m$ denotes the gauge boson mass.

In Eq. (1.2), the unit tensor $g_{\mu\nu}$ can be represented by the completeness of the gauge boson intermediate states of polarization.

$$g_{\mu\nu} = \sum_{\lambda=0}^{3} e^{\lambda}_\mu(k)e_{\lambda\nu}(k) = P_{\mu\nu}(k) + Q_{\mu\nu}(k) \quad (1.4)$$

where $P_{\mu\nu}(k)$ and $Q_{\mu\nu}(k)$ are the transverse and longitudinal projectors, respectively. On the mass-shell, they are expressed as

$$P_{\mu\nu}(k) = g_{\mu\nu} - k_\mu k_\nu/m^2, Q_{\mu\nu}(k) = k_\mu k_\nu/m^2 \quad (1.5)$$
It is noted here that in some previous works\textsuperscript{[14][15]} the Landau gauge propagators were chosen at beginning to examine the unitarity through calculation of the imaginary part of transition amplitudes. This procedure, we think, is not reasonable and can not give a correct result in any case. This is because that in the Landau gauge, the gauge boson propagator only includes the transverse projector $P_{\mu\nu}(k)$ which does not represent a complete set of the intermediate polarized states as seen from Eq. (1.4). Usually, the right hand side (RHS) of Eq. (1.1) is calculated by using the Landau-Cutkosky (L-C) rule\textsuperscript{[17–19]}. By this rule, the intermediate propagators should be replaced by their imaginary parts

$$\text{Im}(k^2 - m^2 + i\varepsilon)^{-1} = -\pi\delta(k^2 - m^2)\theta(k)$$

(1.6)

Utilizing the L-C rule to calculate the imaginary parts of the two-boson and fermion-antifermion scattering amplitudes, we find, the unitarity of our theory is no problems. A key point to achieve this conclusion is how to deal with the loop diagram given by the gauge boson four-line vertex which was considered to give no contribution to the S-matrix element in the previous investigations\textsuperscript{[3][12–15]}. This diagram can be viewed as a limit of the loop diagram formed by the gauge boson three-line vertices when one internal line in the latter loop is shrunk into a point. In this way, we are able to isolate from the former loop a term contributed from the unphysical intermediate states which just guarantees the cancellation of the unphysical amplitudes.

The rest of this paper is arranged as follows. In section 2, we sketch the unitarity of the S-matrix elements of order $g^2$. In section 3, we describe the calculations of the imaginary part of the two-gauge boson scattering amplitude in the perturbative approximation of the order $g^2$ and show how the unitarity is ensured. In section 4, the same thing will be done for the fermion-antifermion scattering. The last section serves to make comments and discussions. In Appendix, we will discuss the sign of imaginary parts of the loop diagrams by a rigorous calculation.

2. UNITARITY OF THE TREE DIAGRAMS OF ORDER $G^2$

For tree diagrams of order $g^2$, the unitarity of their transition amplitudes is directly ensured by the on-mass shell condition. To illustrate this point, we discuss the fermion-antifermion (say, quark-antiquark) and two-gauge boson (say, two gluon ) scattering taking place in the S-channel as shown in Figs. (1) and (2).

For the fermion-antifermion scattering, the S-matrix element may be written as

$$T_{fi} = ig^2j^\mu(p_1, p_2)D_{\mu\nu}(k)j^\nu(p_1', p_2')$$

(2.1)

where

$$j^\mu(p_1, p_2) = \bar{\psi}(p_2)\frac{\lambda^a}{2}\gamma^\mu u(p_1)$$

(2.2)

and

$$D_{\mu\nu}(k) = \frac{g_{\mu\nu} - k_{\mu}k_{\nu}/k^2}{k^2 - m^2 + i\varepsilon} + \frac{\alpha k_{\mu}k_{\nu}/k^2}{k^2 - m^2 + i\varepsilon}.$$  

(2.3)

Noticing $k = p_1 + p_2 = p_1' + p_2'$ and employing Dirac equation, it is easy to see

$$k_{\mu}j^\mu(p_1, p_2) = 0$$

(2.4)

which holds due to that the theory only concerns the vector current and at each vertex the fermion and antifermion have the same mass as in the QCD. Therefore, the longitudinal term $k_{\mu}k_{\nu}/k^2$ in the propagator does not contribute to the S-matrix in the approximation of order $g^2$. In other words, the unphysical poles $k^2 = 0$ and $k^2 = am^2$ do not appear in the scattering amplitude.

For the process depicted in Fig. (2), the transition amplitude is

$$T_{fi} = ig^2f^{abc}f^{a'b'c'}e_\mu(k_1)e_\nu(k_2)e_{\mu'}(k_1')^*e_{\nu'}(k_2')^* \times \Gamma_{\mu\nu\lambda}(k_1, k_2, q)D^{\lambda\chi}(q)\Gamma_{\mu'\nu'\chi'}(k_1', k_2', q)$$

(2.5)

where

$$\Gamma_{\mu\nu\lambda}(k_1, k_2, q) = g_{\mu\nu}(k_1 - k_2)_\lambda + g_{\nu\lambda}(k_2 + q)_\mu - g_{\lambda\mu}(k_1 + q)_\nu$$

(2.6)

and $e_\mu(k)$ stands for the gauge boson wave function satisfying the transversity condition
\[ k_\mu e^\mu(k) = 0. \]  
(2.7)

The transversity of the polarized states and the relation \( q = k_1 + k_2 = k'_1 + k'_2 \) directly lead to

\[ e^\mu(k_1)e'^\mu(k_2)\Gamma_{\mu\nu\lambda}(k_1, k_2, q)q^\lambda = 0. \]  
(2.8)

This equality, analogous to Eq. (2.4), guarantees the removal of the unphysical poles from the S-matrix element written in Eq. (2.5).

Similarly, for the t-channel and u-channel diagrams, it is easy to verify that the equalities in Eqs. (2.4) and (2.8) hold as well. These equalities ensure the S-matrix elements for these diagrams and other processes such as that a fermion and an antifermion annihilate into two bosons to be also unitary.

The fact that the term \( k_\mu k_\nu/k^2 \) in the propagator gives no contribution to the S-matrix elements means that the S-matrix is gauge-independent at tree level. Therefore, in the gluon propagator, only the physical pole at \( k^2 = m^2 \) contributes to the S-matrix element and the gluon propagator given in the Feynman gauge can reasonably be considered in calculation of the S-matrix elements. Certainly, the fact mentioned above allows us to write the intermediate states as transverse ones. When we evaluate the imaginary part of the transition amplitudes by the L-C rule, these intermediate states will be put on the mass shell.

### 3. Unitarity of Two-Gluon Scattering Amplitude of Order \( g^4 \)

In the preceding section, it was shown that in the lowest order approximation of perturbation, the unitarity is no problem. How is it for higher order perturbative approximations? To answer this question, in this section, we investigate the unitarity of the two-gluon scattering amplitude given in the order of \( g^4 \). For this purpose, we only need to consider the diagrams shown in Figs. (3) and (4) and evaluate imaginary parts of the amplitudes of these diagrams. The diagrams involving fermion intermediate states are not necessarily taken into account because the fermion intermediate state is already physical.

Fig. (3) contains eleven diagrams which have gauge boson intermediate states. Except for the last diagram shown in Fig. (3k), the other diagrams all have two-gauge boson intermediate states. If the unitarity condition is satisfied, in the amplitudes given by these diagrams, the unphysical parts arising from the longitudinally polarized intermediate states should be cancelled by the amplitudes of the five diagrams depicted in Fig. (4) which are of ghost intermediate states. To demonstrate this point, in the following, we separately calculate the imaginary parts of the amplitudes of all the diagrams in Figs. (3) and (4).

#### A. The Imaginary Part of the Diagrams in Figs. (3a)-(3j)

By the L-C rule, the diagrams in Figs. (3a-3j) can be given by folding the tree diagrams shown in Fig. (5) with their conjugates. Through the folding, we obtain twice Figs. (3a-3f) and once Figs. (3g)-(3j). Noticing that each of the diagrams in Figs. (3g)-(3j) has a symmetry factor \( \frac{1}{2} \), we can write the imaginary part of Figs. (3a)-(3j) as follows

\[
2i\text{m}T_1^{abcd}(p_1, p_2; p'_1, p'_2) = \frac{1}{2} \int d\tau T_{\mu\nu}^{abcd}(p_1, p_2; k_1, k_2) \\
\times T_{\mu'\nu'}^{b'c'd'}(p'_1, p'_2; k_1, k_2)g^{\mu\mu'}g^{\nu\nu'}
\]  
(3.1)

where

\[
d\tau = \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} (2\pi)^4 \delta^4(p_1 + p_2 - k_1 - k_2) \pi \delta(k_1^2 - m^2) \theta(k_1^0) \\
\times \pi \delta(k_2^2 - m^2) \theta(k_2^0)
\]  
(3.2)

and

\[
T_{\mu\nu}^{abcd}(p_1, p_2; k_1, k_2) = \sum_{i=1}^{4} T_{\mu\nu}^{(i)abcd}(p_1, p_2; k_1, k_2)
= -ig^2 e^\sigma(p_1)e^\sigma(p_2) \sum_{i=1}^{4} T_{\rho\sigma\mu\nu}^{(i)abcd}(p_1, p_2; k_1, k_2)
\]  
(3.3)

here \( T_{\mu\nu}^{(i)abcd}(p_1, p_2; k_1, k_2)(i = 1, 2, 3, 4) \) stand for the matrix elements of Figs. (5a)-(5d) respectively.

In light of Feynman rules, when we set
\[ C_1 = f^{ace} f^{bde}, C_2 = f^{ade} f^{bce}, C_3 = f^{abe} f^{cde} \]  

(3.4)

where the superscripts \(abcd\) of \(C_i\) have been suppressed for simplicity and notice the relations given by the energy-momentum conservation

\[ q_1 = p_1 - k_1 = k_2 - p_2, \quad q_2 = p_1 - k_2 = k_1 - p_2, \quad q_3 = p_1 + p_2 = k_1 + k_2, \]  

(3.5)

the functions \(T^{(i)abcd}_{\rho\sigma\mu\nu}(p_1, p_2; k_1, k_2)\) may be separately expressed as follows.

For Fig. (5a),

\[ T^{(1)abcd}_{\rho\sigma\mu\nu}(p_1, p_2; k_1, k_2) = \frac{C_1}{q_1^2-m^2+i\varepsilon} \Gamma^{(1)}_{\rho\mu\lambda}(p_1, k_1, q_1) \times \Gamma^{(1)\lambda}_{\sigma\nu}(p_2, k_2, q_1) \]  

(3.6)

where

\[ \Gamma^{(1)}_{\rho\mu\lambda}(p_1, k_1, q_1) = g_{\rho\mu}(k_1 + p_1)\lambda + g_{\mu\lambda}(q_1 - k_1)\rho - g_{\lambda\rho}(q_1 + p_1)\mu, \]  

(3.7)

and

\[ \Gamma^{(1)\lambda}_{\sigma\nu}(p_2, k_2, q_1) = g_{\sigma\nu}(p_2 + k_2)\lambda - g_{\nu\lambda}(q_1 + k_2)\sigma + g_{\lambda\sigma}(q_1 - p_2)\nu. \]  

(3.8)

For Fig. (5b),

\[ T^{(2)abcd}_{\rho\sigma\mu\nu}(p_1, p_2; k_1, k_2) = \frac{C_2}{q_2^2-m^2+i\varepsilon} \Gamma^{(2)}_{\rho\sigma\lambda}(p_1, k_2, q_2) \times \Gamma^{(2)\lambda}_{\mu\nu}(p_2, k_1, q_1) \]  

(3.9)

where

\[ \Gamma^{(2)}_{\rho\sigma\lambda}(p_1, k_2, q_2) = g_{\rho\sigma}(p_1 + k_2)\lambda + g_{\sigma\lambda}(q_2 - k_2)\rho - g_{\lambda\rho}(q_2 + p_1)\nu \]  

(3.10)

and

\[ \Gamma^{(2)\lambda}_{\mu\nu}(p_2, k_1, q_2) = g_{\mu\sigma}(p_2 + k_1)\lambda - g_{\sigma\mu}(k_1 + q_2)\sigma + g_{\lambda\sigma}(q_2 - p_2)\mu. \]  

(3.11)

For Fig. (5c),

\[ T^{(3)abcd}_{\rho\sigma\mu\nu}(p_1, p_2; k_1, k_2) = \frac{C_3}{q_3^2-m^2+i\varepsilon} \Gamma^{(3)}_{\rho\sigma\lambda}(p_1, p_2, q_3) \times \Gamma^{(3)\lambda}_{\mu\nu}(k_1, k_2, q_3) \]  

(3.12)

where

\[ \Gamma^{(3)}_{\rho\sigma\lambda}(p_1, p_2, q_3) = g_{\rho\sigma}(p_1 - p_2)\lambda + g_{\sigma\lambda}(p_2 + q_3)\rho - g_{\lambda\rho}(q_3 + p_1)\sigma \]  

(3.13)

and

\[ \Gamma^{(3)\lambda}_{\mu\nu}(k_1, k_2, q_3) = g_{\mu\nu}(k_2 - k_1)\lambda - g_{\nu\lambda}(k_2 + q_3)\mu + g_{\lambda\mu}(q_3 + k_1)\nu. \]  

(3.14)

For Fig. (5d)

\[ T^{(4)abcd}_{\rho\sigma\mu\nu}(p_1, p_2; k_1, k_2) = C_1 \gamma^{(1)}_{\rho\sigma\mu\nu} + C_2 \gamma^{(2)}_{\rho\sigma\mu\nu} + C_3 \gamma^{(3)}_{\rho\sigma\mu\nu} \]  

(3.15)

where

\[ \gamma^{(1)}_{\rho\sigma\mu\nu} = g_{\rho\sigma} g_{\mu\nu} - g_{\rho\mu} g_{\sigma\nu}, \]  

(3.16)

\[ \gamma^{(2)}_{\rho\sigma\mu\nu} = g_{\rho\sigma} g_{\mu\nu} - g_{\rho\mu} g_{\sigma\nu}, \]  

(3.17)

and

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\[ \gamma_{(3)_{\rho\sigma}}^{(3)} = g_{\rho\mu}g_{\sigma\nu} - g_{\rho\nu}g_{\sigma\mu}. \]  

(3.18)

The expressions in Eqs. (3.1)-(3.3), (3.6), (3.9) and (3.12), as indicated in Introduction, are all given in the Feynman gauge. When the intermediate states \( g^{\mu\nu} \) and \( g^{\nu\nu'} \) are decomposed into the physical and unphysical parts in accordance with Eqs. (1.4) and (1.5), Eq. (3.1) will be represented as

\[ 2Im T_1 = \frac{1}{2} \int d\tau T_{\mu
u}^{abcd} T_{\mu'\nu'}^{cd*} [P_{\mu'\nu'}(k_1) P_{\rho\sigma\nu}(k_2) + Q_{\mu'\nu'}(k_1) g^{\nu\nu'} + g^{\mu\nu} Q^{\mu'\nu'}(k_2) - Q_{\mu'\nu'}(k_1) Q^{\nu\nu'}(k_2)] \]

(3.19)

where \( P_{\mu\nu}(k_1) \) and \( Q_{\mu\nu}(k_1) \) are defined in Eq. (1.5) and respectively represent the transverse (physical) and longitudinal (unphysical) polarization intermediate states of gauge bosons. We see, except for the first term, the other terms in Eq. (3.19) are all related to the unphysical intermediate states. These terms should be cancelled out in the total amplitude. In the following, we calculate these terms separately. In the calculations, we note, the transversity of the polarization vectors (see Eq. (2.7)), the relations written in Eq. (3.5) and the on shell property of the momenta \( p_1, p_2, k_1 \) and \( k_2 \) will be often used.

1. Calculation of \( T_{\mu\nu}^{abcd} T_{\mu'\nu'}^{cd*} Q_{\mu'\nu'}(k_1) g^{\nu\nu'} \)

To calculate the second term in Eq. (3.19), according to the definition in Eq. (3.3) for \( T_{\mu\nu}^{abcd} \) and the expression in Eq. (1.5) for \( Q_{\mu\nu}(k_1) \), we need to calculate the contractions \( k_1^{(i)} T_{\rho\sigma\mu
u}(p_1, p_2; k_1, k_2)(i = 1, 2, 3, 4) \). From Eqs. (3.7) and (3.8), we find

\[ k_1^{(i)} \Gamma_{\rho\mu\lambda}^{(1)}(p_1, k_1, q_1) = -q_1 \rho q_1 \lambda + g_{\lambda\rho}(q_1^2 - m^2) \]

(3.20)

and

\[ q_1 \lambda \Gamma_{\sigma\nu\lambda}^{(1)}(p_2, k_2, q_1) = -k_2 \nu q_1 \sigma. \]

(3.21)

Using these equalities, from Eq. (3.6), we obtain

\[ k_1^{(1)} T_{\rho\sigma\mu
u}^{abcd}(p_1, p_2; k_1, k_2) = C_1 k_2 \nu S_{\rho\sigma}(q_1) + C_1 \Gamma_{\sigma
u\rho}^{(1)}(p_2, k_2, q_1) \]

(3.22)

where

\[ S_{\rho\sigma}(q_1) = \frac{q_1 \rho q_1 \sigma}{q_1^2 - m^2 + i\epsilon}. \]

(3.23)

Similarly, from Eqs. (3.10) and (3.11), one can get

\[ k_1^{(2)} \Gamma_{\sigma\mu\lambda}^{(2)}(p_2, k_1, q_2) = -q_2 \sigma q_2 \lambda + g_{\lambda\sigma}(q_2^2 - m^2) \]

(3.24)

and

\[ q_2 \lambda \Gamma_{\rho\mu\lambda}^{(2)}(p_1, k_2, q_2) = -q_2 \rho k_2 \nu. \]

(3.25)

Based on these equalities, it is found form Eq. (3.9)

\[ k_1^{(2)} T_{\rho\sigma\mu\nu}^{abcd}(p_1, p_2; k_1, k_2) = C_2 k_2 \nu S_{\rho\sigma}(q_2) + C_2 \Gamma_{\rho\mu\sigma}^{(2)}(p_1, k_2, q_2) \]

(3.26)

where

\[ S_{\rho\sigma}(q_2) = \frac{q_2 \rho q_2 \sigma}{q_2^2 - m^2 + i\epsilon}. \]

(3.27)

Along the same line, we can derive from Eqs. (3.13) and (3.14) that

\[ k_1^{(3)} \Gamma_{\rho\mu\lambda}^{(3)}(k_1, k_2, q_3) = k_1 \nu q_3 \lambda + k_1 \lambda k_2 \nu - g_{\nu\lambda}(q_3^2 - m^2) \]

(3.28)

and
\[ q_3^\lambda \Gamma^{(3)}_{\rho\sigma\lambda}(p_1, p_2, q_3) = (k_1 + k_2)^\lambda \Gamma^{(3)}_{\rho\sigma\lambda}(p_1, p_2, q_3) = 0, \]  

(3.29)

thereby, we get from Eq. (3.12)

\[ k_1^{-\mu} T^{(3)abcd}_{\rho\sigma\mu
u}(p_1, p_2; k_1, k_2) = C_3 k_2^{-\nu} S^{(3)}_{\rho\sigma}(p_1, p_2, k_1) - C_3 \Gamma^{(3)}_{\sigma\nu\rho}(p_1, p_2, q_3) \]  

(3.30)

where

\[ S^{(3)}_{\rho\sigma}(p_1, p_2, k_1) = \frac{1}{q_3^2 - m^2 + i\varepsilon} k_1^{-\mu} \Gamma^{(3)}_{\rho\sigma\lambda}(p_1, p_2, q_3). \]  

(3.31)

In addition, from Eq. (3.15), we may write

\[ k_1^{-\mu} T^{(4)abcd}_{\rho\sigma\mu\nu}(p_1, p_2; k_1, k_2) = C_1 \gamma^{(1)}_{\rho\sigma\nu}(k_1) + C_2 \gamma^{(2)}_{\rho\sigma\nu}(k_1) + C_3 \gamma^{(3)}_{\rho\sigma\nu}(k_1) \]  

(3.32)

where

\[ \gamma^{(1)}_{\rho\sigma\nu}(k_1) = k_1^{-\mu} \gamma^{(1)}_{\rho\sigma\mu\nu}(k_1) = g_\rho \sigma k_1^\nu - g_\rho k_1^\sigma, \]  

(3.33)

\[ \gamma^{(2)}_{\rho\sigma\nu}(k_1) = k_1^{-\mu} \gamma^{(2)}_{\rho\sigma\mu\nu}(k_1) = g_\rho \nu k_1^\sigma - g_\sigma k_1^\rho, \]  

(3.34)

and

\[ \gamma^{(3)}_{\rho\sigma\nu}(k_1) = k_1^{-\mu} \gamma^{(3)}_{\rho\sigma\mu\nu}(k_1) = g_\sigma \nu k_1^\rho - g_\rho k_1^\sigma. \]  

(3.35)

Summing up the results denoted in Eqs. (3.22), (3.26), (3.30) and (3.32) and noticing Eq. (3.3), we have

\[ k_1^{-\mu} T^{abcd}_{\rho\sigma\mu\nu}(p_1, p_2; k_1, k_2) = -ig^2 e^\rho(p_1) e^\sigma(p_2) [k_2^{-\nu} S^{abcd}_{\rho\sigma} + G^{abcd}_{\rho\sigma\nu}] \]  

(3.36)

where

\[ S^{abcd}_{\rho\sigma} = \sum_{i=1}^{3} C_i S^{(i)}_{\rho\sigma} \]  

(3.37)

and

\[ G^{abcd}_{\rho\sigma\nu} = \sum_{i=1}^{3} C_i G^{(i)}_{\rho\sigma\nu} \]  

(3.38)

in which

\[ G^{(1)}_{\rho\sigma\nu} = \Gamma^{(1)}_{\rho\sigma\mu\nu}(p_2, k_2, q_1) + \gamma^{(1)}_{\rho\sigma\nu}(k_1), \]  

(3.39)

\[ G^{(2)}_{\rho\sigma\nu} = \Gamma^{(2)}_{\rho\sigma\mu\nu}(p_1, k_2, q_2) + \gamma^{(2)}_{\rho\sigma\nu}(k_2) \]  

(3.40)

and

\[ G^{(3)}_{\rho\sigma\nu} = -\Gamma^{(3)}_{\rho\sigma\mu\nu}(p_1, p_2, q_3) + \gamma^{(3)}_{\rho\sigma\nu}(p_1, p_2, k_1). \]  

(3.41)

Employing the expressions given in Eqs. (3.8), (3.10), (3.13), (3.29) and (3.33)-(3.35) and considering the relations among the momenta as shown in Eq. (3.5) and the on-shell condition of the external momenta, it is not difficult to find the following relation

\[ G^{(1)}_{\rho\sigma\nu} = -G^{(2)}_{\rho\sigma\nu} = -G^{(3)}_{\rho\sigma\nu}. \]  

(3.42)

Now, let us look at the color factors. According to the expression

\[ f^{abc} f^{cde} = \frac{2}{N} (\delta_{ae} \delta_{bd} - \delta_{ad} \delta_{bc}) + (d_{ace} d_{bde} - d_{bce} d_{ade}) \]  

(3.43)
and defining
\[ \begin{align*}
\beta_1 &= \frac{2}{\sqrt{N}} \delta_{ab} \delta_{cd} + d_{abc} d_{cde}, \\
\beta_2 &= \frac{2}{\sqrt{N}} \delta_{ac} \delta_{bd} + d_{ace} d_{bde}, \\
\beta_3 &= \frac{2}{\sqrt{N}} \delta_{ad} \delta_{bc} + d_{ade} d_{bce},
\end{align*} \]
(3.44)
we may write
\[ C_1 = \beta_1 - \beta_2, C_2 = \beta_1 - \beta_3, C_3 = \beta_3 - \beta_2. \]
(3.45)
Substitution of Eq. (3.45) into Eq. (3.38) and use of Eq. (3.42) directly lead to
\[ G_{\rho\sigma\nu}^{abcd} = \beta_1 (G_{\rho\sigma\nu}^{(1)} + G_{\rho\sigma\nu}^{(2)}) - \beta_2 (G_{\rho\sigma\nu}^{(1)} + G_{\rho\sigma\nu}^{(3)}) + \beta_3 (G_{\rho\sigma\nu}^{(3)} - G_{\rho\sigma\nu}^{(2)}) = 0. \]
(3.46)
This result makes Eq. (3.36) reduce to
\[ k_1 T_{\mu\nu}^{abcd} = k_2 \nu S_{\mu\nu}^{abcd} \]
(3.47)
where
\[ S_{\mu\nu}^{abcd} = -i g^2 \nu (p_1) e^\sigma (p_2) S_{\rho\sigma}^{abcd}. \]
(3.48)
By using Eq. (3.47) and noticing \( k_2^2 = m^2 \), we finally obtain
\[ T_{\mu\nu}^{abcd} T_{\mu'\nu'}^{a'b'c'd'} g^{\mu\nu'} Q^{\nu'\nu'} (k_2) = S_{\mu\nu}^{abcd} S_{\mu'\nu'}^{a'b'c'd'}. \]
(3.49)
It is emphasized that from the above derivation, we see, the four-line vertex diagram in Fig. (5d) plays an essential role to give the relation in Eq. (3.42) and hence to guarantee the cancellation of the second terms in Eq. (3.22), (3.26) and (3.30), which are free from the pole at \( q_1^2 = m^2 \), as shown in Eq. (3.46).

2. Calculation of \( T_{\rho\sigma\mu\nu}^{abcd} T_{\rho'\sigma'\mu'\nu'}^{a'b'c'd'} g^{\mu\nu'} Q^{\nu'\nu'} (k_2) \)

The procedure of calculating \( T_{\rho\sigma\mu\nu}^{abcd} T_{\rho'\sigma'\mu'\nu'}^{a'b'c'd'} g^{\mu\nu'} Q^{\nu'\nu'} (k_2) \) completely parallels to that described in the former subsection. From Eqs. (3.7) and (3.8), it follows that
\[ k_2^{(1)} \Gamma_{\rho\sigma\nu\lambda}^{(1)} (p_2, k_2, q_1) = -q_{1\sigma} q_{1\lambda} + g_{\lambda\sigma} (q_1^2 - m^2) \]
(3.50)
and
\[ q_1^{(1)} \Gamma_{\rho\sigma\nu\lambda}^{(1)} (p_1, k_1, q_1) = -k_{1\rho} q_{1\nu}. \]
(3.51)
These equalities allow us to get from Eq. (3.6) that
\[ k_2^{(2)} T_{\rho\sigma\mu\nu}^{abcd} (p_1, p_2; k_1, k_2) = C_1 k_{1\mu} S_{\rho\sigma}^{(1)} + C_1 \Gamma_{\rho\sigma\nu\lambda}^{(1)} (p_1, k_1, q_1). \]
(3.52)
Based on the equalities
\[ k_2^{(2)} \Gamma_{\rho\sigma\nu\lambda}^{(2)} (p_1, k_2, q_2) = -q_{2\rho} q_{2\lambda} + g_{\rho\lambda} (q_2^2 - m^2) \]
(3.53)
and
\[ q_2^{(2)} \Gamma_{\rho\sigma\nu\lambda}^{(2)} (p_2, k_1, q_2) = -q_{2\sigma} k_{1\rho}, \]
(3.54)
which are derived from Eqs. (3.10) and (3.11), it is found
\[ k_2^{(2)} T_{\rho\sigma\mu\nu}^{abcd} (p_1, p_2; k_1, k_2) = C_2 k_{1\mu} S_{\rho\sigma}^{(2)} + C_2 \Gamma_{\rho\sigma\nu\lambda}^{(2)} (p_2, k_1, q_2, k_2). \]
(3.55)
By making use of the equality
\[ k_\nu^2 \Gamma_{\mu \lambda}^{(3)}(k_1, k_2, q_3) = -k_{2 \mu} q_{3 \lambda} - k_{1 \mu} k_{2 \lambda} + g_{\mu \lambda} (q_3^2 - m^2) \]  

(3.56)

which is derived from Eq. (3.14) and considering Eq. (3.29), we have

\[ k_\nu^2 T_{\rho \sigma \mu \nu}^{(3)abcd}(p_1, p_2; k_1, k_2) = C_3 k_1 \mu S_{\rho \sigma}^{(3)}(3.57) \]

From Eq. (3.15), it is clear that

\[ k_\nu^2 T_{\rho \sigma \mu \nu}^{(4)abcd}(p_1, p_2; k_1, k_2) = C_1 \gamma_{\rho \sigma \mu}^{(1)}(k_2) \]

(3.58)

where

\[ \gamma_{\rho \sigma \mu}^{(1)}(k_2) = k_2^\nu \gamma_{\rho \sigma \mu}^{(1)} = g_{\rho \sigma} k_2^\mu - g_{\sigma \mu} k_2^\rho, \]  

(3.59)

\[ \gamma_{\rho \sigma \mu}^{(2)}(k_2) = k_2^\nu \gamma_{\rho \sigma \mu}^{(2)} = g_{\rho \sigma} k_2^\mu - g_{\sigma \mu} k_2^\rho, \]  

(3.60)

and

\[ \gamma_{\rho \sigma \mu}^{(3)}(k_2) = k_2^\nu \gamma_{\rho \sigma \mu}^{(3)} = g_{\rho \mu} k_2^\sigma - g_{\sigma \mu} k_2^\rho. \]  

(3.61)

Combining Eqs. (3.52), (3.55), (3.57) and (3.58), we obtain

\[ k_\nu^2 T_{\rho \sigma \mu \nu}^{abcd}(p_1, p_2; k_1, k_2) = -ig^2 e^\rho(p_1) e^\sigma(p_2) [k_1 \mu S_{\rho \sigma}^{abcd} + \tilde{G}_{\rho \sigma \mu}^{abcd}] \]  

(3.62)

where \( S_{\rho \sigma}^{abcd} \) was defined in Eq. (3.37) and

\[ \tilde{G}_{\rho \sigma \mu}^{abcd}(p_1, p_2; k_1, k_2) = \sum_{i=1}^{3} C_i \tilde{G}_{\rho \sigma \mu}^{(i)} \]  

(3.63)

in which

\[ \tilde{G}_{\rho \sigma \mu}^{(1)} = \Gamma_{\rho \sigma \mu}^{(1)}(p_1, k_1, q_1) + \gamma_{\rho \sigma \mu}^{(1)}(k_2), \]  

(3.64)

\[ \tilde{G}_{\rho \sigma \mu}^{(2)} = \Gamma_{\sigma \rho \mu}^{(2)}(p_2, k_2, q_2) + \gamma_{\rho \sigma \mu}^{(3)}(k_2) \]  

(3.65)

and

\[ \tilde{G}_{\rho \sigma \mu}^{(3)} = \Gamma_{\rho \sigma \mu}^{(3)}(p_1, p_2, q_3) + \gamma_{\rho \sigma \mu}^{(3)}(k_2). \]  

(3.66)

Similar to Eq. (3.42), one may find

\[ \tilde{G}_{\rho \sigma \mu}^{(1)} = -\tilde{G}_{\rho \sigma \mu}^{(2)} = -\tilde{G}_{\rho \sigma \mu}^{(3)}. \]  

(3.67)

These relations and those given in Eq. (3.45) also lead Eq. (3.63) to vanish

\[ \tilde{G}_{\rho \sigma \mu}^{abcd}(p_1, p_2; k_1, k_2) = 0. \]  

(3.68)

Thus, Eq. (3.62) becomes

\[ k_\nu^2 T_{\mu \nu}^{abcd}(p_1, p_2; k_1, k_2) = k_1 \mu S_{abcd} \]  

(3.69)

where \( S_{abcd} \) was defined in Eq. (3.48) and thereby we have

\[ T_{\mu \nu}^{abcd} T_{\mu' \nu'}^{abcd} g_{\mu \nu} Q_{\mu' \nu'}(k_2) = S_{abcd} S_{a'b'cd'}. \]  

(3.70)
3. Calculation of $T_{\mu\nu}^{abcd}T_{\mu'\nu'}^{a'b'c'd'}Q_{\rho\sigma}^{\mu\nu}(k_1)Q_{\rho'\sigma'}^{\nu'\nu}(k_2)$

To calculate $T_{\mu\nu}^{abcd}T_{\mu'\nu'}^{a'b'c'd'}Q_{\rho\sigma}^{\mu\nu}(k_1)Q_{\rho'\sigma'}^{\nu'\nu}(k_2)$, it is necessary to calculate $k_1^{\mu}k_2^{\nu}T_{\mu\nu}^{abcd}$. This may be done in several ways. For example, we may simply contract Eq. (3.69) with the vector $k_1^{\mu}$ to give

$$k_1^{\mu}k_2^{\nu}T_{\mu\nu}^{abcd}(p_1, p_2; k_1, k_2) = m^2 S_{\rho\sigma}^{abcd}. \tag{3.71}$$

Certainly, paralleling to the procedure shown in the foregoing subsections, we may first compute $k_1^{\mu}k_2^{\nu}T_{\rho\sigma}^{(1)abcd}$. For instance, by contracting Eq. (3.56) with $k_1^{\mu}$, we derive

$$k_1^{\mu}k_2^{\nu}T_{\mu\nu}^{(3)abcd}(k_1, k_2, q_3) = -k_1 \cdot k_2 q_3 - m^2 k_2 q_3 - k_1 q_3 - m^2. \tag{3.72}$$

From the above equality, noticing the identity in Eq. (3.29), it follows

$$k_1^{\mu}k_2^{\nu}T_{\mu\nu}^{(3)abcd} = C_3 m^2 S_{\rho\sigma}^{(3)} + C_3 k_1^{\lambda}T_{\rho\sigma}^{(3)}(p_1, p_2, q_3). \tag{3.73}$$

The other terms can be given by contracting Eqs. (3.52), (3.55) and (3.58) with $k_1^{\mu}$. Summing all these terms, one can exactly obtain the result as written in Eq. (3.71). Employing Eq. (3.71), we get

$$T_{\mu\nu}^{abcd}T_{\mu'\nu'}^{a'b'c'd'}Q_{\rho\sigma}^{\mu\nu}(k_1)Q_{\rho'\sigma'}^{\nu'\nu}(k_2) = S_{\rho'\sigma'}^{abcd}S_{\rho\sigma}^{a'b'cd'}. \tag{3.74}$$

Up to the present, the last three terms in Eq. (3.19) have been calculated. Inserting Eqs. (3.49), (3.70) and (3.74) into Eq. (3.19), we arrive at

$$2i m T_1^{abcd} = \frac{1}{2} \int d\tau T_{\mu\nu}^{abcd}T_{\mu'\nu'}^{a'b'c'd'}P_{\rho\sigma}^{\mu\nu}(k_1)P_{\rho'\sigma'}^{\nu'\nu}(k_2)$$

$$+ \frac{1}{2} \int d\tau S_{\rho\sigma}^{abcd}S_{\rho'\sigma'}^{a'b'cd'}. \tag{3.75}$$

The second term in the above needs to be cancelled by the ghost diagrams.

**B. The imaginary part of the ghost diagrams in Fig. (4)**

The ghost diagrams in Fig. (4) can be given by folding the three tree diagrams plotted in Fig. (6) with their conjugates. The folding gives double Figs. (4a)-(4d) as well as Fig. (4e). Considering that the symmetry factor of Fig. (4e) is 1 other than $\frac{1}{2}$, the imaginary part of the transition amplitude of Fig. (4) may be represented as

$$2i m T_2^{abcd} = -\frac{1}{2} \int d\tau T^{abcd}(p_1, p_2; k_1, k_2) T^{a'b'c'd'}(p_1', p_2'; k_1, k_2)$$

$$- \frac{1}{2} \int d\tau T^{(3)abcd}(p_1, p_2; k_1, k_2) T^{(3)a'b'c'd'}(p_1', p_2'; k_1, k_2) \tag{3.76}$$

where

$$T^{abcd}(p_1, p_2; k_1, k_2) = \sum_{i=1}^{3} T^{(i)abcd}(p_1, p_2; k_1, k_2) \tag{3.77}$$

$T^{(i)abcd}(p_1, p_2; k_1, k_2)$ represent the matrix elements of Figs. (6a)-(6c), and the minus sign is inherent for the ghost loops.

According to the Feynman rules and considering the transversity of the polarization states, it is clear that

$$T^{(i)abcd}(p_1, p_2; k_1, k_2) = S^{(i)abcd} \tag{3.78}$$

where

$$S^{(i)abcd} = -ig^2 e^\rho (p_1) e^\sigma (p_2) S^{(i)}_{\rho\sigma} \tag{3.79}$$

here the $S^{(i)}_{\rho\sigma}(i = 1, 2, 3)$ were defined in Eqs. (3.23), (3.27) and (3.31) respectively. In accordance with Eq. (3.78), Eq. (3.76) can be expressed as
When adding Eq. (3.80) to Eq. (3.75), we see, the second term in Eq. (3.75) is just cancelled by the first term in Eq. (3.80). However, still remains the second term in Eq. (3.80) which represents half of the contribution of the loop diagram in Fig. (4e) to the imaginary part of the amplitude. We are particularly interested in the fact that the first term in Eq. (3.80) contains the entire contributions from the ghost diagrams in Figs. (4a)-(4d) and half of the contribution of the diagram in Fig. (4e). They completely eliminate the unphysical part of the amplitudes given by Figs. (3a)-(3j), needless to introduce any extra scalar particle for this elimination. How to understand the remaining contribution of the diagram in Fig. (4e). They completely eliminate the unphysical part of the amplitudes given by Figs. (3g) and (4e) have different symmetry factors. Therefore, only half of Fig. (4e) is needed to cancel the unphysical part of Fig. (3g). It is reminded that until now, the loop diagram in Fig. (3k) has not been considered. Then the second term given in Eq. (3.80) just serves such a cancellation.

C. The imaginary part of the diagram in Fig. (3k)

How to evaluate the imaginary part of the amplitude of Fig. (3k) by the L-C rule? This seems to be a difficult problem because we are not able to divide the diagram into two parts by cutting the internal boson line of the closed loop in Fig. (3k) without touching the vertex. However, we observe that when letting one boson line of the closed loop in Fig. (3g) shrink into a point, Fig. (3g) will be converted to Fig. (3k). This graphically intuitive observation suggests that the amplitude given by Fig. (3k) can be treated as a limit of the amplitude of Fig. (3g) when setting the momentum of one propagator in the loop shown in Fig. (3g) tends to infinity. In this way, we can isolate from Fig. (3k) the unphysical contribution which looks like to be given by two-particle intermediate states and hence is able to compare with the second term in Eq. (3.80). It is obvious that the difference between the both diagrams in Figs. (3k) and (3g) only lies in their loops, one of which is formed by the four-line vertex (See Fig. (7a)) and another by the three-line vertex (see Fig. (7b)). Therefore, it is only necessary to compare expressions of the two loops and establish a connection between them.

The expression of the loop in Fig. (7a) is

$$\Pi^{(1)\lambda\lambda'}_{\lambda\lambda'}(q) = -g^2 f^{abcd} f^{f'bc'd'} [g_{\lambda\lambda'} g_{\mu\nu} - g_{\lambda\mu} g_{\lambda'\nu}] \int \frac{d^4 k}{(2\pi)^4} \frac{g^{\mu\nu}}{k^2 - m^2 + i\varepsilon}. \quad (3.81)$$

The imaginary part of the above function have been exactly calculated in Appendix. The result is

$$Im\Pi^{(1)\lambda\lambda'}_{\lambda\lambda'}(q) = -\frac{3g^2}{(4\pi)^2} f^{abcd} f^{f'bc'd'} \int_0^\infty \frac{dx}{x^2} \sin(xm^2). \quad (3.82)$$

Clearly, it does not vanish when the mass $m$ is not equal to zero. The above result is derived in the Feynman gauge, therefore, contains the contribution arising from the unphysical intermediate states.

The expression of Fig. (7b) will be written in the form

$$\Pi^{(2)\lambda\lambda'}_{\lambda\lambda'}(q) = \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} (2\pi)^4 \delta^4(k_1 + k_2 - q) \Pi^{(2)\lambda\lambda'}_{\lambda\lambda'}(k_1, k_2, q) \quad (3.83)$$

where

$$\Pi^{(2)\lambda\lambda'}_{\lambda\lambda'}(k_1, k_2, q) = \frac{1}{2} g^2 f^{abcd} f^{f'bc'd'} \Gamma_{\mu
u\lambda}(k_1, k_2, q) \times \Gamma_{\mu\nu\lambda'}(k_2, k_2, q) D^{\mu\nu}(k_1) D^{\mu\nu}(k_2) \quad (3.84)$$

in which the propagator $D^{\mu\nu}(k)$ was given in Eq. (2.3) with the gauge parameter $\alpha = 1$ and the vertex $\Gamma_{\mu\nu\lambda}(k_1, k_2, q)$ was defined in Eq. (2.6). Let us take the limit $|k_{2\mu}| \to \infty$. In this limit, the product of the propagator $D^{\mu\nu}(k_2)$ and the vertices will approach to
and hence

\[ D^{\nu'}(k_2)\Gamma_{\mu\nu\lambda}(k_1, k_2, q)\Gamma_{\mu'\nu'\lambda'}(k_1, k_2, q) \]

\[ \to -\frac{1}{k_2^2}g^{\nu'}[g_{\mu\nu}k_{2\lambda} - g_{\lambda\nu}k_{2\mu}]g_{\mu'\nu'}k_{2\lambda'} - g_{\lambda'\nu'}k_{2\mu'} \]

\[ = -\frac{1}{k_2^2}[g_{\mu\nu}k_{2\lambda}k_{2\lambda'} + g_{\lambda\nu}k_{2\mu}k_{2\mu'} - g_{\lambda'\nu'}k_{2\lambda'} - g_{\lambda\nu'}k_{2\mu'}] \]

(3.85)

If the tensor \( k_2\mu k_2\nu/k_2^2 \) behaves in such a way in the limit

\[ k_2\mu k_2\nu/k_2^2 \to g_{\mu\nu} \]  

(3.86)

(This limit will be justified in Appendix and a similar limitation for the polarization vector was proposed in the proof of \( \gamma_5 \)-anomaly (see Ref. (20), Chapter 19)), then we find

\[ \Pi^{(2)ab}_{\lambda\lambda'}(k_1, k_2, q) \]

\[ \to g^2f^{abcd}f^{bcd}(g_{\lambda\lambda'}g_{\mu\nu} - g_{\lambda\nu}g_{\lambda'\nu'}) \]

and hence

\[ \left| \Pi^{(2)ab}_{\lambda\lambda'}(q) \right| \]

\[ \to \left| \Pi^{(1)ab}_{\lambda\lambda'}(q) \right| . \]

(3.88)

Particularly, in the physical region, the sign of the imaginary part of the amplitude \( \Pi^{(2)ab}_{\lambda\lambda'}(q) \) is the same as the corresponding part of the amplitude \( \Pi^{(1)ab}_{\lambda\lambda'}(q) \), as will be demonstrated in Appendix. In view of the argument given above, the imaginary part of Fig. (3k) may equivalently be replaced by the imaginary part of Fig. (3g) in the limit \(|k_2| \to \infty\). Thus, we can write

\[ 2ImT_{ab}^{(3)\mu\nu} = \frac{1}{\pi}\int d\tau \lim_{|k_2| \to \infty} T_{\mu\nu}^{(3)abcd}T_{\mu'\nu'}^{(3)a'b'cd'}[P^{(3)}(k_1)P^{(3)}(k_2)] \]

\[ + Q^{(3)}(k_1)g^{\mu\nu'} + g^{\mu\nu}Q^{(3)}(k_2) - Q^{(3)}(k_1)Q^{(3)}(k_2) \]

(3.89)

The first term in the above only concerns the physical intermediate states. We do not pursue here what the limit for this term looks like because it is of no importance at present. We are interested in examining the other three terms. Look at the expression given in Eq. (3.28). The first term in it can be ignored due to the equality in Eq. (3.29). The last term can also be neglected comparing to the second term in the limit \(|k_2| \to \infty\). Thus, Eq. (3.30) will be reduced to

\[ \Pi^{(3)\mu\nu}_{\rho\sigma} \]

\[ \to C_3k_{2\mu}S_{\rho\sigma}^{(3)} \]

(3.90)

where \( S_{\rho\sigma}^{(3)} \) was defined in Eq. (3.31) and is irrelevant to \( k_2 \). The result in Eq. (3.90) enables us to write the second term in Eq. (3.89) as

\[ \lim_{|k_2| \to \infty} T_{\mu\nu}^{(3)abcd}T_{\mu'\nu'}^{(3)a'b'cd'}Q^{(3)}(k_1)g^{\mu\nu'} = S^{(3)abcd}S^{(3)a'b'cd'} \]

(3.91)

where \( S^{(3)abcd} \) was defined in Eq. (3.79). In the above, the compatibility of the on-shell condition \( k_2^2 = m^2 \) with the limit \(|k_2| \to \infty\) has been noticed

By the same reason as stated above, only the second term in Eq. (3.56) should be considered in the limit. Therefore, Eq. (3.57) is approximated to

\[ k_{2\mu}^{(3)\mu\nu} \]

\[ \to -C_3k_{1\mu}\overline{S_{\rho\sigma}^{(3)}} \]

(3.92)

where

\[ \overline{S_{\rho\sigma}^{(3)}} = \lim_{|k_2| \to \infty} \frac{k_{2\lambda}^{(3)\rho\sigma\lambda}(p_1, p_2; q_3)}{q_2^2 - m^2 + i\varepsilon} . \]

(3.93)

With the result in Eq. (3.92), the third term in Eq. (3.89) becomes

\[ \lim_{|k_2| \to \infty} T_{\mu\nu}^{(3)abcd}T_{\mu'\nu'}^{(3)a'b'cd'}g^{\mu\nu'}Q^{(3)}(k_2) = \overline{S_{\rho\sigma}^{(3)b'a'c'd'}} \]

(3.94)

where
Similarly, in the limit \( |k_{2\mu}| \to \infty \), we can neglect the first term (due to Eq. (3.29)) and the last term in Eq. (3.72). The second term in Eq. (3.72) permits us to rewrite Eq. (3.73) in the form

\[
\lim_{|k_{2\mu}| \to \infty} T^{(3)}_{\mu\nu}(k_{1\mu}k_{2\nu} \bar{T}^{(3)}_{\rho\sigma \mu\nu}) = -C_3 m^2 \bar{T}^{(3)}_{\mu\nu}.
\]

which may more directly be given by contracting Eq. (3.92) with \( k_{1\nu}^\mu \). From this result, it is clear to see

\[
\lim_{|k_{2\mu}| \to \infty} \frac{1}{2} \int d\tau T^{(3)}_{\mu\nu} \bar{T}^{(3)}_{\rho\sigma \mu\nu} Q^{\mu\nu}(k_{1\mu})Q^{\rho\sigma}(k_{2\nu}) = \bar{T}^{(3)}_{\mu\nu} \bar{T}^{(3)}_{\rho\sigma \mu\nu}. \tag{3.97}
\]

On inserting Eqs. (3.91), (3.94) and (3.97) into Eq. (3.89), we see, the last two terms in Eq. (3.89) are cancelled with each other. As a result, we have

\[
2Im T^{ab\prime b\prime}_3 = \frac{3}{2} \int d\tau \lim_{|k_{2\mu}| \to \infty} T^{(3)}_{\mu\nu}(k_{1\mu}k_{2\nu} \bar{T}^{(3)}_{\rho\sigma \mu\nu}) P^{\mu\nu}(k_{1\mu})P^{\rho\sigma}(k_{2\nu}) \tag{3.99}
\]

which is only related to the physical intermediate states. Thus, the proof of the unitarity is accomplished.

We note here that the results given in this subsection rely on how to correctly treat the limit procedure. As will be shown in Appendix, the limit given in Eq. (3.86) is the only choice of converting Fig. (3g) into Fig. (3k) when the relation in Eq. (3.29) is considered. Similarly, to obtain the desirable limiting results presented in Eqs. (3.91), (3.94) and (3.97), the reasonable expressions in Eqs. (3.28), (3.56) and (3.72) are necessary to be used.

\section{4. Unitarity of Fermion-Antifermion Scattering Amplitude of Order \( g^4 \)}

In this section, to illustrate the unitarity of the theory further, we evaluate the imaginary part of the fermion-antifermion scattering amplitude in the perturbative approximation of order \( g^4 \). For this purpose, it is only necessary to consider the diagrams shown in Fig. (8).

The diagrams in Figs. (8a)-(8e) can be reconstructed by folding the tree diagrams in Figs. (9a)-(9c) with their conjugates. Since the folding gives two times of Figs. (8a)-(8d) and one time of Fig. (8e) which possesses a symmetry factor \( \frac{1}{2} \), the imaginary parts of the amplitudes given by Figs. (8a)-(8e) may be represented as

\[
2Im T_1 = \frac{1}{2} \int d\tau T^{ab\prime b\prime}_1 \bar{T}^{ab\prime b\prime}_1 g^{\mu\nu} \tag{4.1}
\]

where

\[
T^{(i)ab}_\mu(p_1, p_2; k_1, k_2) = \sum_{i=1}^3 T^{(i)ab}_\mu(p_1, p_2; k_1, k_2) \tag{4.2}
\]

\(T^{(i)ab}_\mu(p_1, p_2; k_1, k_2) \ (i = 1, 2, 3)\) denote the matrix elements of Figs. (9a)-(9c) respectively. According to the Feynman rules, they can be written as

\[
T^{(1)ab}_\mu(p_1, p_2; k_1, k_2) = -ig^2 \bar{v}(p_2) \gamma^\nu \frac{\lambda^a}{(p_1 - k_1)^2 - M^2 + i \varepsilon} \gamma_\mu u(p_1), \tag{4.3}
\]

\[
T^{(2)ab}_\mu(p_1, p_2; k_1, k_2) = -ig^2 \bar{v}(p_2) \gamma^\nu \frac{\lambda^a}{(k_1 - p_2)^2 - M^2 + i \varepsilon} \gamma_\mu u(p_1) \tag{4.4}
\]
and
\[ T_{\mu\nu}^{(3)ab}(p_1, p_2; k_1, k_2) = -\frac{g^2 f^{abc}}{q^2 - m^2} \frac{\lambda^c}{2} \lambda^a \Gamma_{\mu\nu\lambda}(k_1, k_2, q) \tilde{\tau}(p_2) \frac{\lambda^c}{2} \gamma_\lambda u(p_1) \quad (4.5) \]

where \( \Gamma_{\mu\nu\lambda}(k_1, k_2, q) \) was defined in Eq. (2.6), \( M \) is the fermion mass and \( p = p^\mu \gamma_\mu \).

For evaluating the second term in Eq. (4.1), we need to compute the contraction of \( T_{\mu\nu}^{(i)ab} \) with \( k_1^\mu \). By applying Dirac equation, the on-mass shell condition and the relation \( q = k_1 + k_2 = p_1 + p_2, \) one may get
\[ k_1^\mu [T_{\mu\nu}^{(1)ab} + T_{\mu\nu}^{(2)ab}] = -ig^2 \tau(p_2) \frac{\lambda^a}{2} \lambda^b \gamma_\mu u(p_1) \]
\[ = g^2 f^{abc} \tau(p_2) \frac{\lambda^c}{2} \gamma_\mu u(p_1) \quad (4.6) \]

and
\[ k_1^\mu T_{\mu\nu}^{(3)ab} = -g^2 f^{abc} \tau(p_2) \frac{\lambda^c}{2} \gamma_\mu u(p_1) + k_2^\nu S^{ab}. \quad (4.7) \]

where
\[ S^{ab} = \frac{g^2 f^{abc}}{2p_1 \cdot p_2 + m^2} \frac{\lambda^c}{2} k_1 u(p_1) \quad (4.8) \]

Adding Eq. (4.7) to Eq. (4.6), we find
\[ k_1^\mu T_{\mu\nu}^{ab} = k_2^\nu S^{ab}. \quad (4.9) \]

As seen from the above, there is a cancellation among the diagrams in Figs. (9a)-(9c). From Eq. (4.9), one may derive
\[ T^{ab}_{\mu\nu} T^{ab}_{\mu'\nu'} Q^{\mu'\nu'}(k_1) g^{\nu\nu'} = S^{ab} S^{ab}. \quad (4.10) \]

Let us turn to calculate the third term in Eq. (4.1). Along the same line stated above, one may get
\[ k_2^\mu [T_{\mu\nu}^{(1)ab} + T_{\mu\nu}^{(2)ab}] = -g^2 f^{abc} \tau(p_2) \frac{\lambda^c}{2} \gamma_\mu u(p_1) \]
\[ = g^2 f^{abc} \tau(p_2) \frac{\lambda^c}{2} \gamma_\mu u(p_1) \quad (4.11) \]

and
\[ k_2^\nu T_{\mu\nu}^{(3)ab} = g^2 f^{abc} \tau(p_2) \frac{\lambda^c}{2} \gamma_\mu u(p_1) + k_1^\mu \tilde{S}^{ab}. \quad (4.12) \]

where
\[ \tilde{S}^{ab} = -\frac{g^2 f^{abc}}{q^2 - m^2 + i\varepsilon} \frac{\lambda^c}{2} k_2 u(p_1). \quad (4.13) \]

From the equality
\[ \tau(p_2) \frac{\lambda^c}{2} (k_1 + k_2) u(p_1) = \tau(p_2) \frac{\lambda^c}{2} (p_1 + p_2) u(p_1) = 0, \quad (4.14) \]

it follows that
\[ \tilde{S}^{ab} = S^{ab}. \quad (4.15) \]

Adding Eq. (4.12) to Eq. (4.11) and noticing Eq. (4.15), we have
\[ k_2^\nu T_{\mu\nu}^{ab} = k_1^\mu S^{ab}. \quad (4.16) \]

This result gives rise to
\[ T^{ab}_{\mu\nu} T^{ab}_{\mu'\nu'} g^{\nu\nu'}(k_2) = S^{ab} S^{ab}. \quad (4.17) \]
For evaluating the last term in Eq. (4.1), we may use the following equalities which are obtained by contracting Eqs. (4.11) and (4.12) with $k_1^\mu$:

$$k_1^\mu k_2^\nu [T_{\mu\nu}^{(1)ab} + T_{\mu\nu}^{(2)ab}] = -g^2 f^{abc} \tau(p_2) \frac{\lambda^c}{2} k_1 u(p_1)$$

(4.18)

and

$$k_1^\mu k_2^\nu T_{\mu\nu}^{(3)ab} = g^2 f^{abc} \tau(p_2) \frac{\lambda^c}{2} k_1 u(p_1) + m^2 \vec{S}_{ab}.$$  

(4.19)

These equalities and Eq. (4.15) lead to

$$k_1^\mu k_2^\nu T_{\mu\nu}^{ab} = m^2 \vec{S}_{ab}.$$  

(4.20)

This result allows us to write the last term in Eq. (4.1) in the form

$$T_{\mu\nu}^{ab} T_{\mu'\nu'}^{ab} Q^{\mu\nu'}(k_1) Q^{\mu'\nu'}(k_2) = S_{ab} S_{ab}.$$  

(4.21)

Substituting Eqs. (4.10), (4.17) and (4.21) in Eq. (4.1), we arrive at

$$2ImT_1 = \frac{1}{2} \int d\tau T_{\mu\nu}^{ab} T_{\mu'\nu'}^{ab} P^{\mu\nu'}(k_1) P^{\mu'\nu'}(k_2) + \frac{1}{2} \int d\tau S_{ab} S_{ab}.$$  

(4.22)

The ghost diagram in Fig. (8f) can be given by folding the tree diagram in Fig. (9d) with its conjugate. Therefore, the imaginary part of Fig. (8f), by the Feynman rules, can be written as

$$2ImT_2 = -\int d\tau S_{ab} S_{ab}.$$  

(4.23)

In complete analogy with the two-boson scattering discussed in the preceding section, the second term in Eq. (4.22) can only cancel half of the above amplitude. The reason for this still is due to the difference between the symmetry factors of Figs. (8e) and (8f). To achieve a complete cancellation, it is necessary to consider the contribution of the diagram in Fig. (8g). This diagram can also be treated as a limit of the diagram in Fig. (8e) when the momentum of one internal line in the loop tends to infinity,

$$2ImT_3 = \frac{1}{2} \int d\tau \lim_{|k_2\mu_1| \to \infty} T_{\mu_1}^{(3)ab} T_{\mu'\nu'}^{(3)ab} [P^{\mu\nu'}(k_1) P^{\mu'\nu'}(k_2) + Q^{\mu\nu'}(k_1) Q^{\mu'\nu'}(k_2) - g^{\mu\nu'}(k_1) Q^{\mu'\nu'}(k_2)].$$  

(4.24)

In the limit $|k_2\mu_1| \to \infty$, comparing to the second terms in Eqs. (4.7), (4.12) and (4.19), the first terms in these equations can be ignored. Thus, Eqs. (4.7), (4.12) and (4.19) respectively reduce to

$$k_1^\mu T_{\mu\nu}^{(3)ab} \approx k_2\nu S_{ab}.$$  

(4.25)

and

$$k_2^\mu T_{\mu\nu}^{(3)ab} \approx k_1\nu S_{ab}.$$  

(4.26)

On inserting these expressions into Eq. (4.24), we have

$$2ImT_3 = \frac{1}{2} \int d\tau \lim_{|k_2\mu_1| \to \infty} T_{\mu_1}^{(3)ab} T_{\mu'\nu'}^{(3)ab} P^{\mu\nu'}(k_1) P^{\mu'\nu'}(k_2) + \frac{1}{2} \int d\tau S_{ab} S_{ab}.$$  

(4.28)

Thus, as shown before, it is indeed possible to find a way which allows us to isolate from Fig. (8g) the unphysical part of the amplitude like the second term in Eq. (4.28).

Summing the results denoted in Eqs. (4.22), (4.23) and (4.28), we finally obtain the imaginary part of the total amplitude such that

$$2ImT = \frac{1}{2} \int d\tau T_{\mu\nu}^{ab} T_{\mu'\nu'}^{ab} P^{\mu\nu'}(k_1) P^{\mu'\nu'}(k_2) + \frac{1}{2} \int d\tau \lim_{|k_2\mu_1| \to \infty} T_{\mu_1}^{(3)ab} T_{\mu'\nu'}^{(3)ab} P^{\mu\nu'}(k_1) P^{\mu'\nu'}(k_2)$$  

(4.29)

in which the unphysical contributions are all cancelled. Thus, the unitarity is ensured.
5. COMMENTS AND DISCUSSIONS

In the previous sections, the unitarity of our theory has been illustrated by evaluating the imaginary parts of two-gauge boson and fermion-antifermion scattering amplitudes up to the fourth order perturbation. The imaginary parts of the amplitudes were calculated by means of the L-C rule. In this kind of calculation, we have to first work in the Feynman gauge because the formula used requires the intermediate states to be complete. The gauge boson propagator given in the Feynman gauge contains unphysical longitudinal intermediate excitations though, it has been proved that these unphysical intermediate states are eventually cancelled in the S-matrix elements, leaving only physical transverse intermediate states in the S-matrix elements as given by the gauge boson propagator written in the unitary gauge

\[ D_{\mu\nu}(k) = \frac{g_{\mu\nu} - k_\mu k_\nu/m^2}{k^2 - m^2 + i\varepsilon} \]  

(5.1)

Although the unitarity of the S-matrix elements is proved in the Feynman gauge, it would be true for other gauges since it has been exactly proved that the S-matrix is gauge-independent.

As mentioned in Introduction, In the previous works of examining the unitarity of some kinds of massive non-Abelian gauge field theories\cite{14,15}, the gauge boson propagator in the Landau gauge

\[ D_{\mu\nu}(k) = \frac{g_{\mu\nu} - k_\mu k_\nu/k^2}{k^2 - m^2 + i\varepsilon} \]  

(5.2)

was chosen to calculate the imaginary part of scattering amplitudes by the L-C rule. For such a calculation, as mentioned in Introduction, the Landau gauge actually is not suitable since the intermediate states characterized by the transverse projector appearing in the numerator of the above propagator does not form a complete set. The unsuitability of the procedure may be seen from the massless gauge theory. The unitarity of the theory was exampled by computing the imaginary part of the fermion-antifermion scattering amplitude of order \( g^4 \) in the Feynman gauge\cite{19}. However, if one tries to perform the proof in the Landau gauge, he could not get a reasonable result. Particularly, the longitudinal projector \( k_\mu k_\nu/k^2 \) in Eq. (5.2) where \( m = 0 \) could not be given an unambiguous definition on the mass shell \( k^2 = 0 \) since the momentum \( k \) on the mass shell becomes an isotropic vector.

Another point we would like to emphasize is that for examining the unitarity of a massive gauge field theory, it is only necessary to evaluate the S-matrix element between the physical transversely polarized states of gauge bosons. In this way, it was shown in sections (2) and (3) that the unitarity is well satisfied. Particularly, the calculation in section 3 indicates that in all the diagrams depicted in Figs. (3) and (4) except for the loop diagrams involving Fig. (3k) and a part of Fig. (4e), there is a natural cancellation among the contributions coming from the unphysical gauge boson and ghost particle intermediate states, without the help of any scalar particle. This result and the theoretical logic strongly suggest that the same cancellation between the unphysical contributions arising from Fig. (3k) and a part of Fig. (4e) is definite to happen. To achieve this cancellation, the loop diagram formed by the gauge boson four-line vertex is necessary to be recast in the form as if it is given by the two-particle intermediate states so as to be able to compare its contribution with that given by the other loop diagrams. For this purpose, we proposed in section 3 a reasonable limiting procedure which allows us to reach the cancellation mentioned above. The results we obtained are undoubtedly correct. It should be noted that in all the previous investigations\cite{12,14,15} of the unitarity problem, the loop diagrams given by the gauge boson four-line vertex such as Figs. (3k) and (8g) were never taken into account in the cancellation of the unphysical amplitudes. From the calculations described in sections (3) and (4), it is clearly seen that these loop diagrams play an essential role to guarantee the cancellation of the unphysical part of the amplitudes and hence the unitarity of the S-matrix elements. It is mentioned that the theories presented in Refs. (2) and (6) which were pointed out to be non-unitary by Mohapatra et al. and some others\cite{4,12,15} are not correct because the Feynman rule concerning the closed ghost loop has an extra factor \( \frac{1}{2} \) other than 1 as given in our theory. In this paper, the unitarity has been proved in the perturbation approximation up to the order of \( g^4 \). For higher order approximations, we believe that for a given process, if all the diagrams are taken into account and treated appropriately, the unitarity would be proved to be no problem.

It should be noted that in this paper and the former papers, we only limit ourselves to discuss the theory (for example, the QCD with massive gluons) in which all the gauge bosons are required to have the same masses. This requirement is necessary to make the theory to be gauge-invariant and unitary just as the case we met in nuclear physics where the nucleon-pion interacting system is of SU(2)-symmetry provided that the masses of all the pions are considered to be the same and the mass difference between proton and neutron is ignored. Similarly, for the weak interaction theory in which apart from the vector currents of fermions, the axial vector currents of fermions are included as well, certainly, we may build up a SU(2)-symmetric theory without introducing the Higgs mechanism. The action of this theory is
constrained by the Lorentz condition and may contain the gauge boson mass term in it. But, the gauge-invariance of the theory requires that all the gauge bosons must be of the same mass and the fermions must be massless. In this case, the unitarity of the theory, as easily checked, is no problem. Nevertheless, if the charged and neutral gauge bosons are required to have different masses and charged fermions are massive, it is inevitable to work with the theory constructed by means of the Higgs mechanism.

6. ACKNOWLEDGMENT

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7. APPENDIX: EXAMINATION OF SIGNS OF THE IMAGINARY PARTS OF THE LOOP DIAGRAMS

It was mentioned in section (3) that the imaginary part of the matrix element of the loop in Fig. (7a) has the same sign as that given by the loop in Fig. (7b) at least for the large momentum $k_2$. To convince ourself of this point, we investigate the imaginary part of the loops in a parametric representation. In this representation, the propagator will be expressed in the form [17]

$$\frac{1}{k_i^2 - m^2 + i\varepsilon} = -i \int_0^\infty d\alpha i e^{i\alpha(k_i^2 - m^2 + i\varepsilon)}.$$  \hspace{1cm} (A1)

With this representation, the matrix element shown in Eq. (3.81) for Fig. (7a) may be rewritten as

$$\Pi_{\lambda'\lambda}(q) = -3g^2 f^{abc} f^{bcd} g_{\lambda\lambda'} J^{(1)}$$  \hspace{1cm} (A2)

where

$$J^{(1)} = \int \frac{dk}{(2\pi)^4} \frac{1}{k^2 - m^2 + i\varepsilon} = -i \int_0^\infty da e^{i\alpha k^2} e^{i\alpha(k^2 - m^2 + i\varepsilon)} = -\frac{1}{(4\pi)^2} \int_0^\infty d\alpha e^{-i\alpha(m^2 - i\varepsilon)}.$$  \hspace{1cm} (A3)

In the above, we have used the representation in Eq. (A1) and the formula of Fresnel integral[17]

$$\int \frac{dk}{(2\pi)^4} e^{ika^2} = \frac{-i}{(4\pi\alpha)^2}.$$  \hspace{1cm} (A4)

For the loop in Fig. (7b), we confine ourselves to investigate its expression in the limit of large momentum $k_2$ for the purpose of comparing to the one given in Eqs. (A2) and (A3). As stated in Eqs. (3.85)-(3.88), in order to convert the matrix element of Fig.(7b) to the one for Fig.(7a), it is necessary to take the approximate expression denoted in Eq. (A5) and the limit assumed in Eq. (A6) which is applied to Eq. (3.85). To achieve such a conversion, as easily verified, instead of Eq. (3.85), we may simply take an equivalent approximate expression such that

$$g^{\mu\nu} g^{\nu\nu'} \Gamma_{\mu\nu\lambda}(k_1, k_2, q) \Gamma_{\mu'\nu',\lambda'}(k_1, k_2, q) \rightarrow 6k_2 \lambda k_{2\lambda'}. \hspace{1cm} (A5)$$

With this expression, Eqs. (3.83) and (3.84) may be rewritten as

$$\Pi_{\lambda'\lambda}^{(2)}(q) \approx 3g^2 f^{acd} f^{bcd} J_{\lambda'\lambda}^{(2)}(q) \hspace{1cm} (A6)$$

where

$$J_{\lambda'\lambda}^{(2)}(q) = \int \frac{dk_1}{(2\pi)^4} \frac{dk_2}{(2\pi)^4} (2\pi)^4 \delta^4(q - k_1 - k_2) \times \frac{\partial^2}{(k_1^2 - m^2 + i\varepsilon)(k_2^2 - m^2 + i\varepsilon)}.$$  \hspace{1cm} (A7)

Employing the parametrization given in Eq. (A1) and the Fourier representation of the $\delta$-function, Eq. (A7) reads

$$J_{\lambda'\lambda}^{(2)}(q) = \int_0^\infty da_1 da_2 \int d^4z \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} e^{i\alpha_1 k_1^2 - i\alpha_2 k_2^2} e^{i\alpha_1(q_1 - i\varepsilon) - i\alpha_2(q_2 - i\varepsilon)} \times \partial^2 \partial_{\lambda'\lambda} \frac{d^2\lambda^2}{(2\pi)^4} e^{i\alpha_2 k_2^2} e^{i\alpha_1 k_1^2}.$$  \hspace{1cm} (A8)
Upon completing the integrations over \( k_1, k_2 \) and \( z \) by using the formula of Fresnel integral, we have

\[
J^{(2)}_{\lambda\lambda'}(q) = \frac{i}{(4\pi)^2} \int_0^\infty \frac{d\alpha_1 d\alpha_2}{(\alpha_1 + \alpha_2)^2} [g_{\lambda\lambda'}(\tfrac{i}{2\alpha_2} - \tfrac{1}{4\alpha_2^2})],
\]

where

\[
Q(q, \alpha) = \frac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2} q^2 - (\alpha_1 + \alpha_2)(m^2 - i\varepsilon). \tag{A10}
\]

Inserting the identity

\[
\int_0^\infty dx \delta(x - \alpha_1 - \alpha_2) = 1 \tag{A11}
\]

into Eq. (A9) and then making the transformation \( \alpha_i \to x\alpha_i \), one can write

\[
J^{(2)}_{\lambda\lambda'}(q) = \frac{i}{(4\pi)^2} \int_0^1 d\alpha_1 d\alpha_2 \delta(1 - \alpha_1 - \alpha_2) \times \int_0^\infty \frac{dx}{x} \left( \frac{i}{2} g_{\lambda\lambda'} + \alpha_1^2 q_{\lambda\lambda'} \right) e^{ixQ(q, \alpha)}. \tag{A12}
\]

Noticing the equalities shown in Eqs. (3.29) and (4.14), the second term in the parenthesis, actually, can be ignored in the scattering amplitude. Thus, the function \( J^{(2)}_{\lambda\lambda'}(q) \) is only proportional to the unit tensor \( g_{\lambda\lambda'} \). This result precisely justifies the limit taken in Eq. (3.86). On substituting Eq. (A12) into Eq. (A6), and performing the integration over \( \alpha_2 \), one gets

\[
\Pi^{(2)ab}_{\lambda\lambda'}(q) = -3g^{2ab} f^{abcd} g_{\lambda\lambda'} J^{(2)}(q) \tag{A13}
\]

where

\[
J^{(2)}(q) = \frac{1}{(4\pi)^2} \int_0^1 d\alpha \int_0^\infty \frac{dx}{2x^2} e^{ixQ(q, \alpha)}. \tag{A14}
\]

in which

\[
Q(q, \alpha) = \alpha(1 - \alpha)q^2 - m^2. \tag{A15}
\]

Now, we are in position to examine the imaginary parts of the functions \( \Pi^{(1)ab}_{\lambda\lambda'} \) and \( \Pi^{(2)ab}_{\lambda\lambda'} \). We first write down the imaginary parts of the functions \( J^{(1)} \) and \( J^{(2)} \),

\[
ImJ^{(1)} = \frac{1}{(4\pi)^2} \int_0^\infty \frac{dx}{x^2} \sin(xm^2) \tag{A16}
\]

and

\[
ImJ^{(2)} = \frac{1}{(4\pi)^2} \int_0^1 d\alpha \int_0^\infty \frac{dx}{2x^2} \sin\{x[\alpha(1 - \alpha)q^2 - m^2]\}. \tag{A17}
\]

For the integral over \( x \), obviously, the major contribution arises from the integrand at the neighborhood of the origin. Therefore

\[
ImJ^{(1)} \geq 0. \tag{A.18}
\]

As for the \( ImJ^{(2)} \), the integral over \( \alpha \) may be estimated by taking the mean value \( \frac{1}{2} \) of the variable \( \alpha \) in the integrand. Thus,

\[
ImJ^{(2)} \approx \frac{1}{(4\pi)^2} \int_0^\infty \frac{dx}{2x^2} \sin\{x[\frac{1}{4} q^2 - m^2]\}. \tag{A19}
\]

It is well known that in the physical region,

\[
q^2 \geq 4m^2 \tag{A20}
\]
where \( q^2 = 4m^2 \) is the starting point of a cut which is the solution of the following Landau equations\[^{17} \]

\[
\begin{align*}
\lambda_1 (k^2 - m^2) &= 0, \\
\lambda_2 (q - k)^2 - m^2 &= 0, \\
\lambda_1 k_\mu - \lambda_2 (q - k)_\mu &= 0.
\end{align*}
\]

(A21)

In view of Eq. (A20), we may conclude

\[ \text{Im} J^{(2)} \geq 0. \] (A22)

The results in Eqs. (A18) and (A22) straightforwardly lead to that the imaginary parts of the \( \Pi_{\lambda\lambda'}^{(1)ab}(q) \) and \( \Pi_{\lambda\lambda'}^{(2)ab}(q) \) have the same sign as we see from Eq. (A2) and (A13).

At last, we would like to mention the imaginary part of the loop in Fig. (7c). The expression of the loop is

\[ \Pi_{\lambda\lambda'}^{(3)ab}(q) = -g^2 f^{acd} f^{bcd} J_{\lambda\lambda'}^{(3)}(q) \] (A23)

where

\[ J_{\lambda\lambda'}^{(3)}(q) = \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \delta^4(q - k_1 - k_2) \frac{k_{1\lambda} k_{2\lambda'}}{(k_1^2 - m^2 + i\varepsilon)(k_2^2 - m^2 + i\varepsilon)}. \] (A24)

Completely following the procedure formulated in Eqs. (A8)-(A12), one may derive

\[ J_{\lambda\lambda'}^{(3)}(q) = \frac{i}{(4\pi)^2} \int_0^1 d\alpha \int_0^\infty \frac{dx}{2\pi} i \alpha(1-\alpha)q_{\lambda\lambda'} e^{ixQ(q,\alpha)}. \] (A25)

Neglecting the second term containing \( q_{\lambda\lambda'} \) and then substituting the above expression into Eq. (A23), we can write

\[ \Pi_{\lambda\lambda'}^{(3)ab}(q) = -g^2 f^{acd} f^{bcd} g_{\lambda\lambda'} J_{\lambda\lambda'}^{(3)}(q) \] (A26)

where

\[ J_{\lambda\lambda'}^{(3)}(q) = -\frac{1}{(4\pi)^2} \int_0^1 d\alpha \int_0^\infty \frac{dx}{2\pi} e^{ix[\alpha(1-\alpha)q^2 - m^2]}. \] (A27)

Clearly, the sign of the imaginary part

\[ \text{Im} J_{\lambda\lambda'}^{(3)}(q) = -\frac{1}{(4\pi)^2} \int_0^1 d\alpha \int_0^\infty \frac{dx}{2\pi} \sin\{\alpha(1-\alpha)q^2 - m^2\} \] (A28)

is opposite to the \( \text{Im} J^{(2)}(q) \) shown in Eq. (A17). Therefore, the imaginary parts of the \( \Pi_{\lambda\lambda'}^{(3)ab}(q) \) and the \( \Pi_{\lambda\lambda'}^{(2)ab}(q) \) have opposite signs.

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[1] FIGURE CAPTION
Fig. (1): The tree diagram for fermion and antifermion scattering.
Fig. (2): The tree diagram for two-gauge boson scattering.
Fig. (3): The fourth-order diagrams for two-gauge boson scattering with only the gauge boson intermediate states.
Fig. (4): The fourth-order diagrams for two-gauge boson scattering with only the ghost particle intermediate states.
Fig. (5): The tree diagrams used to give all the diagrams in Fig. 3 through folding them with their conjugates
Fig. (6): The tree diagrams used to give all the diagrams in Fig. 4 through folding them with their conjugates.
Fig. (7a): The one-loop diagram formed by four-line gauge boson vertex.
Fig. (7b): The one-loop diagram formed by three-line gauge boson vertex.
Fig. (7c): The one-loop diagram formed by ghost intermediate states.
Fig. (8): The fourth-order diagrams for fermion-antifermion scattering.
Fig. (9): The tree diagrams used to give all the diagrams in Fig. 8 through folding them with their conjugates.