On the forecast of runoff based on the harmonic analysis of time series of precipitation in the catchment area

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Abstract. It is shown that a significant connection exists between the most important harmonics, extracted in the process of harmonic analysis of time series of precipitation in the catchment area of rivers and the amount of runoff. This allowed us to predict the size of the flow for a period of up to 20 years, assuming that the main parameters of the harmonics are preserved at the predicted time interval. The results of such a forecast for three river basins of Kazakhstan are presented.

1. Introduction
Almost the whole territory of Kazakhstan is in a zone of insufficient runoff. Lack of drinking water or its unsatisfactory quality is one of the main problems hampering the development not only of agriculture, but also of a number of other branches of economic activity. The main source of water in each region is the local surface water-rivers, lakes and reservoirs. Therefore, the task of long-term forecasting of changes in runoff of surface waters is very important [1-3, etc.].

In [3], a harmonic analysis of the time series of the runoff of the rivers of Semipalatinsk and Almaty regions was carried out. As a result, it can be stated that in the time series of the runoff values there are harmonics in 3, 6, 9, 19 and 45 years in Almaty and close in the Semipalatinsk region, where, in addition, there is still a harmonica in 24 year. The author's forecast of the runoff for ten years ahead of 2010 showed its high accuracy. In this case, the forecast for one main half-century harmonic in both cases gave better results than the forecast for the combination of the three harmonics.

Since the runoff is the result of the amount of precipitation falling into the catchment area, we attempted to establish such a connection. The expediency of finding such a connection is because precipitation is the primary factor determining surface runoff. Consequently, the relationship between the fundamental harmonics in the rows of precipitation and runoff must be physically stronger than simply extrapolating the main harmonic or harmonics of the time series of the runoff to the perspective.

2. Methodology
We performed an analysis of the time series of annual precipitation for the period from the 1920s to the present, in conjunction with the runoff for three basins (regions): the Urals, Semipalatinsk and Almaty regions. We thus have an opportunity to compare the results for the regions of Semipalatinsk and Almaty with the results obtained in [3].
Rate the periodicity in observed data. Let there be \( n \) consecutive values of a time series of observations of the magnitude of \( Q \). It will be approximated by a function of the form:

\[
R = Q_0 + \frac{Q}{2} \sin(\omega t + \varphi) = Q_0 + b\sin\omega t + c\cos\omega t,
\]

(1)

Here \( Q_0 \) is some constant for the accepted period of the approximation value, which oscillates the value of interest; \( \partial Q, \omega, \varphi \) - accordingly amplitude, frequency and phase of these oscillations.

It is known that the parameters \( b \) and \( c \) are connected to the amplitude and phase of the oscillations in accordance with the rules of the addition of periodic values with the same frequency in the following way:

\[
\partial Q = 2\sqrt{b^2 + c^2},
\]

(2)

\[
tg\varphi = \frac{b}{c},
\]

(3)

Approximation of a time series will be carried out using the sum of squared differences between the series values and the approximating function \( S_Q \):

\[
S_Q = \sum_i^\infty (Q_i - R_i)^2 = \sum_i^\infty (Q_i - Q_0 - b\sin\omega t_i - c\cos\omega t_i)^2,
\]

(4)

where \( Q_i \) - the study value taken from the data of observations in year \( t_i \); \( i \) - is the number of years of observations; \( n \) - is the duration of the series. The expression (1) approximates ranges of values of \( Q \) well, if the sum is the smallest \([4, 5]\).

To determine the best approximating a sine wave with the specified period has been equal to \( Q \) and linked to derivatives of the expression (4) in the parameters \( Q_0, b \) and \( c \):

\[
\frac{\partial S_Q}{\partial b} = -2\sum_i^\infty (Q_i - Q_0 - b\sin\omega t_i - c\cos\omega t_i) = 0,
\]

(5)

\[
\frac{\partial S_Q}{\partial Q_0} = -2\sum_i^\infty (Q_i - Q_0 - b\sin\omega t_i - c\cos\omega t_i) = 0,
\]

(6)

\[
\frac{\partial S_Q}{\partial c} = -2\sum_i^\infty (Q_i - Q_0 - b\sin\omega t_i - c\cos\omega t_i) = 0,
\]

(7)

The solution of the system of equations (5) - (7) allows to obtain formulas for the calculation of amplitude and phase, a constant value \( Q_0 \), which varies around the approximating function for any period of approximation and consistency of periods. The sum of the squared differences between the approximating sine wave and the values of a series can be calculated by the formula (4). The results of the calculations are presented below in graphs. Thus to give an opportunity to assess the internal structure of the row, we retain all first harmonics.

The time series of precipitation were approximated by a sixth-degree polynomial to detect climatic fluctuations in them, and a harmonic analysis of the series was performed. Then the variance of the main (first) harmonics was estimated. As the rule, the first three harmonics choose 85-90\% of the dispersion, so in the following analysis only the first three harmonics were used. A feature of the distribution of the dispersion between the first harmonics in the rows of precipitation is that, in contrast to the harmonics in the series of temperatures, they converge slowly. As a result, the first three harmonics contain significant values of the dispersion.

3. Results and discussion

Let’s consider the results we obtained for the selected basins.

Figure 1 shows the time series of precipitation for Almaty. The performed harmonic analysis of this series makes it possible to identify three main harmonics with periods of 135, 42 and 21 years, which account for 60, 30 and 20 mm precipitation dispersion, respectively. In sum, these harmonics select more than 80\% of the total precipitation dispersion (figure 2). Thus, in the time series of precipitation there is a harmonica with a period of 135 years, not typical for the series of precipitation in the territory of Kazakhstan. In the rows of precipitation, the first harmonics are, as a rule, half-century (Brickner) harmonics.

The maximum of this harmonic took place around 1975. However, at the same time, there was a minimum of the second harmonic (42 years), which significantly weakened the influence of the first harmonic. As a result, the maximum precipitation shifted in the sixties, when the second harmonic was at maximum, and the first - close to it. As a result, the climatic maximum of precipitation (figure 1)
and the climatic maximum of the runoff (figure 3) were formed in the sixties. Figure 3 shows the time series of runoff values for Almaty and Semipalatinsk regions according to [3].

![Figure 1](image1.png)

**Figure 1.** Temporal course of annual precipitation amount Almaty station.

![Figure 2](image2.png)

**Figure 2.** Harmonic analysis of the time series of precipitation. Almaty Station.

The author [3], approximating the runoff time series for Almaty region, received that the harmonic at 45 years is the most effective (figure 3a). The time series of precipitation, approximated by a polynomial of the sixth degree, taken from [2], is in good agreement with the runoff by the location of the extremes (figures 1 and 3a). In the time interval under consideration, the maximum precipitation from the approximated curve falls on 1959, and the minimums fall for 1984 and 1926. The time of the last minimum of runoff thus coincides with the minimum of precipitation.
Figure 3. Dynamics of local flow fluctuations in Almaty (a) and Semipalatinsk (b) regions [3].

As for the minimum in the thirties, the minimum of precipitation took place in 1925-27, while the minimum of runoff, according to [3], was in 1940-41. We explain this difference by two factors. The minimum runoff in those years was much stretched, and the time series of runoff, taken for analysis in the cited work, began in 1931. This could affect the result of spectral and harmonic analysis. As it is known from theory, the harmonic "fits" so that it includes the maximum amount of dispersion of the entire analyzed series. And in this is the value of harmonic analysis, which makes it possible to build a prediction on the assumption of the preservation of the fundamental harmonics, since the characteristics of the harmonic depend little on the magnitudes of the series at its edges. With polynomial approximation, the approximating line "fits" so that the standard deviation is minimal in each of the time series sections.

The nature of the approximating curve at the edge of the series does not allow to confidently judge the trend concerning its new (trend) changes, to build a forecast; nevertheless, within the series we get an optimal approximation, even better than the summation of the fundamental harmonics in harmonic analysis [4, 5]. In our case, we are analyzing data not for the edges of the time series and the results of polynomial approximation should be believed.

Consequently, in the time series of precipitation and local runoff for Almaty, there is a good coincidence of extremes, allowing the use of the first harmonics and results of polynomial approximation in the series of precipitation, for the climatic forecast of surface runoff.

Let us now consider the temporal course of precipitation in Samarka, which most objectively characterizes the distribution of precipitation in the region, including Semipalatinsk (figures 3b, 4).
Figure 4. Time course of the annual amount of precipitation at the MC Samarka.

It can be seen that during the period under review, a maximum of precipitation took place in the sixties, before this time the amount of precipitation gradually increased, and after the decade it gradually decreases. In the time series, harmonics in 53, 23 and 13 years with fractions of 50, 30 and 25 mm, respectively, are distinguished. It can be seen that the fraction of the first harmonic in the total dispersion of precipitation is large, 50 mm. There is no century-old or close harmonic, which is typical for time series of precipitation in the territory of Kazakhstan. The curve of the approximation of the time series of precipitation by a polynomial of the sixth degree largely repeats the position of the first harmonic and therefore we do not give it. The maximum of the Brikner's harmonics, first allocated by us, falls on the sixties and the minimum at the eighties of the last century. The extreme of the first harmonic coincide with the extreme of the runoff according to [3]. This allowed the author [3] to make a forecast of the dynamics of the climatic runoff for only one harmonic.

The maximum of a half-century harmonic was observed in 1960. With a subsequent minimum in 1986, after which a new growth begins, and in recent years a new decreasing. In the sixties, i.e. in the years of maximum precipitation, there were at the maximum all three harmonics - 13, 23 and 53 years. In 1975-78, the half-century harmonica was on the decline, and 13 and 23-year-olds in the minimum. At this time, precipitation was at the minimum, too. By examining figures 4 and 3, one can find a good consistency between the amount of precipitation and the amplitudes of the harmonics and for other time intervals.

The maximum runoff of the rivers of Semipalatinsk region according to figure 3b coincide with the period of the greatest amount of precipitation in the sixties. The minima of the runoff almost coincide with the minimum of the amount of precipitation in the thirties and eighties. Thus, in the long run, the runoff values in this region are in good agreement with the temporal course of the amount of precipitation, although, as is known [1, 2, etc.], the yearly precipitation and runoff values should not be expected to coincide year after year.

Consider now the basin of the Ural River (Zhayik). As it is known, this river has two equivalent watersheds: one is located to the west of the South Urals and the Sakmara River flows into the Urals, and the second catchment is the catchment area of the Ural River itself, which is located to the east of the Southern Urals. In [2, etc.] it is shown that the precipitation regime in each of the catchments reflects well the sediments of the meteorological stations of Uralsk and Kustanai located to the west and east of the mountains of the Southern Urals.

Figure 5 shows the discharge values of the Ural River in the alignment of the Kushum post in the territory of Kazakhstan.
Figure 5. Time course of the Ural River runoff in the Kuchum post area.

Figure 6. Results of the harmonic analysis of the discharge values. The Ural River.

It can be seen that harmonics 8, 23 and 38 years are distinguished. Checking the harmonics in the runoff for stability by elongating and shortening the series showed that all three harmonics of 8, 23 and 38 years are stable. The amplitude of all harmonics is significant. In this case, the amplitude of the harmonic in 23 is the largest, more than 160 m$^3$, harmonics of 38 and 8 years, more than 100 m$^3$. The more harmonics and the greater their contribution to the overall variance, the more difficult it is to predict the flow using them.

Let's see what harmonics are contained in the time series of precipitation at the stations of Uralsk and Kustanai. In the time series of the Uralsk station precipitation, harmonics in 8, 18 and 43 have been identified, and they are all of approximately the same amplitude. In the precipitation of Kustanai, harmonics of 8, 23, and 38 years are distinguished. The verification of the harmonics in the rows of sediments of these stations to stability illustrates that in Kustanai and Uralsk a harmonic appeared at 8 years, which should be considered unstable. In the total sediments of the two stations, harmonics of 8, 18, and 43 are observed, i.e. the same harmonics as in the series of precipitation in Uralsk. It is noteworthy, however, that if the amplitude of these harmonics in the precipitation of Uralsk is 60 mm each, then in the total precipitation of Uralsk and Kustanay 80, 100 and 100 mm, respectively, i.e. the
amplitude of each harmonic increased due to the precipitation of Kustanai, although none of the harmonics taking place in Kustanai (except for the eight-year old) appeared. At the same time, in a time series of sums of precipitation at two stations, there is not a single harmonic, which would coincide with the harmonics present in the river discharge of the Ural River. The harmonic values separated from the time series exactly coincide in length with the harmonics contained in the time series of precipitation of the station of Kustanai. The most significant are only two harmonics: 38 and 23 year.

A joint analysis of the harmonics in the time series of the precipitation of each station and in the series of total precipitation on the one hand and the harmonics in the drain on the other showed that there is a connection, but it is rather complicated and sometimes ambiguous. For this reason, we further analyzed the harmonics in the Ural River discharge on one side and the harmonics in the macro circulation indices on the other. The initial data are consistent with [6-8].

![Figure 7. Harmonic analysis of the W index.](image)

![Figure 8. Harmonic analysis of the C index.](image)
Among the harmonics of the W index, the century-old harmonica (103 - 105 years) determined the main extreme in the drain - a maximum in the twenties and 2000s and minimums in the sixties and seventies. The time series of the runoff is shorter by 20 years of the time series of the W index.

The 38-year-old harmonics in the W index with amplitude of about 28 days, which sometimes turns into a 29-year-old harmonic when testing stability, is synchronous to a 38-year harmonics in the drain. Consequently, the two main harmonics of 108 and 38 years, the index W, have a response in the Ural River discharge in the form of the same harmonic duration with amplitudes of 110 and 80 m³, respectively.

From the harmonics of the index C, a harmonic of 23 y is expressed, with amplitude of more than 160 m³, which is synchronous with the harmonics of 23 y in the runoff of the Ural River. The amplitudes of these harmonics completely coincide. This harmonic in amplitude exceeds the amplitude of the harmonics of the secular and 38 years in the runoff caused by the corresponding harmonics of the index W, i.e. it is most significant.

From the harmonics of the index E, we note the harmonic 42 y, which in the time interval under consideration is in oppose with a harmonic of 38 years of index W. The influence of other harmonics of the index E is weak.

Thus, the harmonics in the Ural River discharge are in good agreement with the harmonics of the secular and 38 years of the W index and the harmonics of the 23rd index of C. Extremes in the run take place when the amplitudes of the named harmonics coincide. Thus, a minimum of flow around the middle of the thirties of the last century was observed with a decrease in the amplitude of the secular harmonic of the index W, and the harmonics of the harmonics of 38, 23 and 8 years coincided. In the mid-seventies - at a minimum of harmonics of 38 and 8 years and a harmonics of 23 y, which was close to a minimum. The maximum runoff took place in the middle of the forties with a maximum of harmonics of 23 and 8 years and a harmonics close to the maximum of 38 years. At last the maximum of the drain in the nineties - at a maximum of harmonics of 38 and 23y on a rise close to the maximum of the secular harmonic of index W.

The sensitivity of the rivers in the droughty region to climate change is very high. The drain sensitively reacts to very slight climatic temperature variations and precipitation on the territory of formation of a drain. Anthropogenic impacts on components of a surrounding medium in a river basin can make on it the most adverse effect.

In [9], the authors, studying the climatic changes in air temperature and precipitation on the territory of the Middle and Southern Urals in the twentieth century, concluded "the ambiguity of trends of changes of climatic characteristics in the twentieth century in the Urals. Installed features rhythmic changes in air temperature and precipitation". To the authors the results similar to our own,
we can only add that on the territory of Kazakhstan, adjacent to the watershed of the Ural River, and in the Kazakh part of the basin up to the Caspian Sea, we also recorded only fluctuations of climate. Because the methods of analysis we have are different, we fixed the "rhythms" of different durations: a century, of close to half a century (Brickner's), and a shorter one. The age-old cycle that had the largest amplitude close to the maximum, however, due to the shorter harmonics and here, the total climatic temperature trend has become negative.

In [10] methods of model operation influence of global climate change on the hydrological system of the Aral Sea located, in fact, in the center of Eurasia was studied. In spite of the fact that the field of formation of a drain is apart about one and a half thousand km from the sea, and the system of accounting of the consumed water on this site is unreliable, authors nevertheless concluded that the influence of climate change on the processes in the basin of the Aral Sea is significant. Reservoirs of an arid zone are sensitive to the slightest climatic temperature variations and rainfall.

In [11] authors studied the influence of climate change on an underground delivery of the rivers in Great Britain. And, though Great Britain is out of an arid zone, authors assume that such dependence exists. It indicates quite strong dependence of the surface drain from climatic temperature variations and rainfall irrespective of a natural zone. The existing balance between precipitation and temperature on the one hand and the surface drain on the other, being rather steady on a temporary interval of one year to several, it is very sensitive to climatic changes, i.e. changes on a time span up to 10 years.

In [12] not only temporary, but also space changes of an amount of precipitation at a size of a drain of the river of Yangtze (Yangtzy) during 40 years are analyzed. The received results confirm that the sensitivity of water systems to climatic changes is high even outside an arid zone. There is a number of other works devoted to this problem, but with similar regularities.

We can see that our results for the river basin of the Urals in general confirm the common conclusions received for other river basins. However, in view of existence of a number of features except aridity of climate, such as the existence of two pools of formation of a drain divided by mountains, different conditions of formation of extreme rainfall in each of them, etc., we received the particular communications and bonds and the expected scenarios of change of a drain in prospect.

4. Conclusion
As a result of the analysis of time series of precipitation and surface runoff, we have shown that the climatic forecast of runoff can be built on the expected climatic changes in the amount of precipitation in the basin. The climate forecast should be preceded by a joint in-depth analysis of the time series of runoff and precipitation in the region (the area of flow formation). Based on this analysis, a long-term runoff forecast is constructed. Analyzing time series of surface runoff, it is firstly necessary to make sure that earlier there was a general coincidence of extremes in them with the extremes of precipitation in the basin. The presence of such a coincidence indicates the prospect of finding links. Due to this and the joint analysis of the series, the most appropriate variant for the forecast is selected.

Depending on the regional and circulatory features, it may be sufficient to use only one first harmonic in the precipitation series if it contains 50% of the variance and more (as in the case of the Semipalatinsk region) for the forecast of runoff. For Almaty region, it was useful to use all three harmonics in the analysis with the use of results of polynomial extrapolation.

In complicated cases, when the conditions for the formation of surface runoff in different parts of the basin are different, it is advisable to use the drainage relationship with the results of a harmonic analysis of the forms of large-scale atmospheric circulation, since precipitation is associated with quite definite forms. Unfortunately, the widely used Wangengeim-Girs typification does not have a quantitative characterization of the intensity of large-scale processes. This reduces the effectiveness of its application in similar tasks.

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