Color Superconductivity in Compact Stars

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Abstract. After a brief review of the phenomena expected in cold dense quark matter, color superconductivity and color-flavor locking, we sketch some implications of recent developments in our understanding of cold dense quark matter for the physics of compact stars. We give a more detailed summary of our recent work on crystalline color superconductivity and the consequent realization that (some) pulsar glitches may originate in quark matter.

1 Color Superconductivity and Color-Flavor Locking

Because QCD is asymptotically free, its high temperature and high baryon density phases are more simply and more appropriately described in terms of quarks and gluons as degrees of freedom, rather than hadrons. The chiral symmetry breaking condensate which characterizes the vacuum melts away. At high temperatures, in the resulting quark-gluon plasma phase all of the symmetries of the QCD Lagrangian are unbroken and the excitations have the quantum numbers of quarks and gluons. At high densities, on the other hand, quarks form Cooper pairs and new condensates develop. The formation of such superconducting phases requires only weak attractive interactions; these phases may nevertheless break chiral symmetry and have excitations with the same quantum numbers as those in a confined phase. These cold dense quark matter phases may arise in the cores of neutron stars; understanding this region of the phase diagram is a major challenge for the future.

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of the QCD phase diagram requires an interplay between QCD and neutron star phenomenology.

The relevant degrees of freedom in cold dense quark matter are those which involve quarks with momenta near the Fermi surface. At high density, where the quark number chemical potential $\mu$ (and hence the quark Fermi momentum) is large, the QCD gauge coupling $g(\mu)$ is small. However, because of the infinite degeneracy among pairs of quarks with equal and opposite momenta at the Fermi surface, even an arbitrarily weak attraction between quarks renders the Fermi surface unstable to the formation of a condensate of quark Cooper pairs. Creating a pair costs no free energy at the Fermi surface and the attractive interaction results in a free energy benefit. Pairs of quarks cannot be color singlets, and in QCD with two flavors of massless quarks, they form in the (attractive) color $\bar{3}$ channel in which the quarks in a Cooper pair are color-antisymmetric [1,2,3,4].

The resulting condensate creates a gap $\Delta$ at the Fermi surfaces of quarks with two out of three colors, but quarks of the third color remain gapless. Five gluons get a Meissner mass by the Anderson-Higgs mechanism [10]; a $SU(2)_{\text{color}}$ subgroup remains unbroken. The Cooper pairs are flavor singlets and no flavor symmetries are broken. There is also an unbroken global symmetry which plays the role of $U(1)_B$. Thus, no global symmetries are broken in this 2SC phase. There must therefore be a phase transition between the hadronic and 2SC phases at which chiral symmetry is restored. This phase transition is first order [3,11,12,13] since it involves a competition between chiral condensation and diquark condensation [11,13].

In QCD with three flavors of massless quarks, the Cooper pairs cannot be flavor singlets, and both color and flavor symmetries are necessarily broken. The symmetries of the phase which results have been analyzed in [5,7]. The attractive channel favored by one-gluon exchange exhibits “color-flavor locking.”

A condensate of the form

$$\langle \psi^{\alpha a}_L \psi^{\beta b}_L \rangle \propto \Delta \epsilon^{\alpha \beta A} \epsilon^{a b A} \tag{1}$$

involving left-handed quarks alone, with $\alpha$, $\beta$ color indices and $a$, $b$ flavor indices, locks $SU(3)_L$ flavor rotations to $SU(3)_{\text{color}}$: the condensate is not symmetric under either alone, but is symmetric under the simultaneous $SU(3)_{L + \text{color}}$ rotations.

A condensate involving right-handed quarks alone locks $SU(3)_R$ flavor rotations to $SU(3)_{\text{color}}$. Because color is vectorial, the combined effect of the $LL$ and $RR$ condensates is to lock $SU(3)_L$ to $SU(3)_R$, breaking chiral symmetry. Thus, in quark matter with three massless quarks, the $SU(3)_{\text{color}} \times SU(3)_L \times SU(3)_R$ symmetry is broken.

1. It turns out [3] that condensation in the color $\bar{3}$ channel induces a condensate in the color $6$ channel because this breaks no further symmetries [1]. The resulting condensates can be written in terms of $\kappa_1$ and $\kappa_2$, where $\langle \psi^\alpha_L \psi^\beta_L \rangle \sim \kappa_1 \delta^{\alpha \beta} + \kappa_2 \delta^{a b} \delta^{\alpha \beta}$. Here, the Kronecker $\delta$’s lock color and flavor rotations. The pure color $\bar{3}$ condensate [3] has $\kappa_2 = -\kappa_1$.

2. Once chiral symmetry is broken by color-flavor locking, there is no symmetry argument precluding the existence of an ordinary chiral condensate. Indeed, instanton effects do induce a nonzero $\langle \bar{q}q \rangle$ [3], but this is a small effect [14].
SU(3)\(_R\) \times U(1)\(_B\) symmetry is broken down to the global diagonal SU(3)\(_{\text{color}+L+R}\) group. A gauged U(1) subgroup of the original symmetry group — a linear combination of one color generator and electromagnetism, which lives within SU(3)\(_L\) \times SU(3)\(_R\) — also remains unbroken. All nine quarks have a gap. All eight gluons get a mass. There are nine massless Nambu-Goldstone bosons. All the quarks, all the massive vector bosons, and all the Nambu-Goldstone bosons have integer charges under the unbroken gauged U(1) symmetry, which therefore plays the role of electromagnetism. The CFL phase therefore has the same symmetries (and many similar non-universal features) as baryonic matter with a condensate of Cooper pairs of baryons. This raises the possibility that quark matter and baryonic matter may be continuously connected.

Nature chooses two light quarks and one middle-weight strange quark, rather than three degenerate quarks. A nonzero \(m_s\) weakens condensates which involve pairing between light and strange quarks. The CFL phase requires \langle us\rangle and \langle ds\rangle condensates; such condensates which pair quarks with differing Fermi momenta can only exist if the resulting gaps are larger than of order \(m_s^2/2\mu\), the difference between \(u\) and \(s\) Fermi momenta in the absence of pairing. This means that upon increasing \(m_s\) at fixed \(\mu\), one must find a first-order unlocking transition for larger \(m_s\) only \(u\) and \(d\) quarks pair and the 2SC phase is obtained. For any \(m_s \neq \infty\), the CFL phase is the ground state at arbitrarily high density. For large values of \(m_s\), there is a 2SC interlude: as a function of increasing \(\mu\), one finds a first order phase transition at which hadronic matter is replaced by quark matter in the 2SC phase and a subsequent first order phase transition at a higher \(\mu\) above which CFL quark matter takes over. For smaller values of \(m_s\), the possibility of quark-hadron continuity arises.

Much effort has gone into estimating the magnitude of the gaps in the 2SC and CFL phases, and the consequent critical temperature above which quark matter ceases to be superconducting. It would be ideal if this task were within the scope of lattice gauge theory. Unfortunately, lattice methods relying on importance sampling have to this point been rendered exponentially impractical at nonzero baryon density by the complex action at nonzero \(\mu\). There are more sophisticated algorithms which have allowed the simulation of theories which are simpler than QCD but which have as severe a fermion sign problem as that in QCD at nonzero chemical potential. This bodes well for the future. Given the current absence of suitable lattice methods, the magnitudes of the gaps in the 2SC and CFL phases may be continuous with a baryonic phase which is dense enough that the Fermi momenta of all the nucleons and hyperons are comparable; there must, however, be a phase transition(s) between this hypernuclear phase and ordinary nuclear matter.

Note that even if the strange and light quarks are not degenerate, the CFL phase may be continuous with a baryonic phase which is dense enough that the Fermi momenta of all the nucleons and hyperons are comparable; there must, however, be a phase transition(s) between this hypernuclear phase and ordinary nuclear matter.

Note that quark pairing can be studied on the lattice in some models with four-fermion interactions and in two-color QCD. The \(N_c = 2\) case has also been studied analytically in Refs.; pairing in this theory is simpler to analyze because quark Cooper pairs are color singlets. The \(N_c \to \infty\) limit of QCD is often one in which hard problems become tractable. However, the ground state of \(N_c = \infty\) QCD is a chiral density wave, not a color superconductor. At asymptotically high densities color superconductivity persists up to \(N_c\)'s of order thousands.
The magnitude of the gaps in quark matter at large but accessible $\mu$ has been estimated using two broad strategies. The first class of estimates are done within the context of models whose parameters are chosen to reproduce zero density physics \([3,4,11,5,8,9,13,14,23,24,25,22]\). The second strategy for estimating gaps and critical temperatures is to use $\mu = \infty$ physics as a guide. At asymptotically large $\mu$, models with short-range interactions are bound to fail because the dominant interaction is due to the long-range magnetic interaction coming from single-gluon exchange \([12,26]\). The collinear infrared divergence in small angle scattering via one-gluon exchange (which is regulated by dynamical screening \([26]\)) results in a gap which is parametrically larger at $\mu \to \infty$ than it would be for any point-like four-fermion interaction \([26]\). Weak coupling estimates of the gap \([26,27,28,30,31,23,34,35,36,37,38]\) are valid at asymptotically high densities, with chemical potentials $\mu \gg 10^8$ MeV \([37]\). Neither class of methods can be trusted quantitatively for quark number chemical potentials $\mu \sim 400 – 500$ MeV, as appropriate for the quark matter which may occur in the cores of neutron stars. It is nevertheless satisfying that two very different approaches, one using zero density phenomenology to normalize models, the other using weak-coupling methods valid at asymptotically high density, yield predictions for the gaps and critical temperatures at accessible densities which are in good agreement: the gaps at the Fermi surface are of order tens to 100 MeV, with critical temperatures about half as large.

$T_c \sim 50$ MeV is much larger relative to the Fermi momentum than in low temperature superconductivity in metals. This reflects the fact that color superconductivity is induced by an attraction due to the primary, strong, interaction in the theory, rather than having to rely on much weaker secondary interactions, as in phonon mediated superconductivity in metals. Quark matter is a high-$T_c$ superconductor by any reasonable definition. Its $T_c$ is nevertheless low enough that it is unlikely the phenomenon can be realized in heavy ion collisions.

## 2 Color Superconductivity in Compact Stars

Our current understanding of the color superconducting state of quark matter leads us to believe that it may occur naturally in compact stars. The critical temperature $T_c$ below which quark matter is a color superconductor is high enough that any quark matter which occurs within neutron stars that are more than a few seconds old is in a color superconducting state. In the absence of lattice simulations, present theoretical methods are not accurate enough to determine whether neutron star cores are made of hadronic matter or quark matter. They also cannot determine whether any quark matter which arises will be in the CFL or 2SC phase: the difference between the $u$, $d$ and $s$ Fermi momenta will be a

before being supplanted by the phase described in Ref. \([19]\). At any finite $N_c$, color superconductivity occurs at arbitrarily weak coupling whereas the chiral density wave does not. For $N_c = 3$, color superconductivity is still favored over the chiral density wave (although not by much) even if the interaction is so strong that the color superconductivity gap is $\sim \mu/2$ \([22]\).
few tens of MeV which is comparable to estimates of the gap \( \Delta \); the CFL phase occurs when \( \Delta \) is large compared to all differences between Fermi momenta. Just as the higher temperature regions of the QCD phase diagram are being mapped out in heavy ion collisions, we need to learn how to use neutron star phenomena to determine whether they feature cores made of 2SC quark matter, CFL quark matter or hadronic matter, thus teaching us about the high density region of the QCD phase diagram. It is therefore important to look for astrophysical consequences of color superconductivity.

2.1 Equation of State

Much of the work on the consequences of quark matter within a compact star has focussed on the effects of quark matter on the equation of state, and hence on the radius of the star. As a Fermi surface phenomenon, color superconductivity has little effect on the equation of state: the pressure is an integral over the whole Fermi volume. Color superconductivity modifies the equation of state at the \( \sim (\Delta/\mu)^2 \) level, typically by a few percent \([3]\). Such small effects can be neglected in present calculations, and for this reason we will not attempt to survey the many ways in which observations of neutron stars are being used to constrain the equation of state \([3]\).

We will describe one current idea, however. As a neutron star in a low mass X-ray binary (LMXB) is spun up by accretion from its companion, it becomes more oblate and its central density decreases. If it contains a quark matter core, the volume fraction occupied by this core decreases, the star expands, and its moment of inertia increases. This raises the possibility \([10]\) of a period during the spin-up history of an LMXB when the neutron star is gaining angular momentum via accretion, but is gaining sufficient moment of inertia that its angular frequency is hardly increasing. In their modelling of this effect, Glendenning and Weber \([10]\) discover that LMXB’s should spend a significant fraction of their history with a frequency of around 200 Hz, while their quark cores are being spun out of existence, before eventually spinning up to higher frequencies. This may explain the observation that LMXB frequencies are clustered around 250-350 Hz \([11]\), which is otherwise puzzling in that it is thought that LMXB’s provide the link between canonical pulsars and millisecond pulsars, which have frequencies as large as 600 Hz \([12]\). It will be interesting to see how robust the result of Ref. \([10]\) is to changes in model assumptions and also how its predictions fare when compared to those of other explanations which posit upper bounds on LMXB frequencies \([13]\), rather than a most probable frequency range with no associated upper bound \([10]\). We note here that because Glendenning and Weber’s effect depends only on the equation of state and not on other properties of quark matter, the fact that the quark matter must in fact be a color superconductor will not affect the results in any significant way. If Glendenning and Weber’s explanation for the observed clustering of LMXB frequencies proves robust, it would imply that pulsars with lower rotational frequencies feature quark matter cores.
2.2 Cooling by Neutrino Emission

We turn now to neutron star phenomena which are affected by Fermi surface physics. For the first $10^{5-6}$ years of its life, the cooling of a neutron star is governed by the balance between heat capacity and the loss of heat by neutrino emission. How are these quantities affected by the presence of a quark matter core? This has been addressed recently in Refs. [44,45], following earlier work in Ref. [46]. Both the specific heat $C_V$ and the neutrino emission rate $L_\nu$ are dominated by physics within $T$ of the Fermi surface. If, as in the CFL phase, all quarks have a gap $\Delta \gg T$ then the contribution of quark quasiparticles to $C_V$ and $L_\nu$ is suppressed by $\sim \exp(-\Delta/T)$. There may be other contributions to $L_\nu$ [44], but these are also very small. The specific heat is dominated by that of the electrons, although it may also receive a small contribution from the CFL phase Goldstone bosons. Although further work is required, it is already clear that both $C_V$ and $L_\nu$ are much smaller than in the nuclear matter outside the quark matter core. This means that the total heat capacity and the total neutrino emission rate (and hence the cooling rate) of a neutron star with a CFL core will be determined completely by the nuclear matter outside the core. The quark matter core is “inert”: with its small heat capacity and emission rate it has little influence on the temperature of the star as a whole. As the rest of the star emits neutrinos and cools, the core cools by conduction, because the electrons keep it in good thermal contact with the rest of the star. These qualitative expectations are nicely borne out in the calculations presented by Page et al. [45].

The analysis of the cooling history of a neutron star with a quark matter core in the 2SC phase is more complicated. The red and green up and down quarks pair with a gap many orders of magnitude larger than the temperature, which is of order 10 keV, and are therefore inert as described above. Any strange quarks present will form a $\langle ss \rangle$ condensate with angular momentum $J = 1$ which locks to color in such a way that rotational invariance is not broken [27]. The resulting gap has been estimated to be of order hundreds of keV [47], although applying results of Ref. [48] suggests a somewhat smaller gap, around 10 keV. The blue up and down quarks also pair, forming a $J = 1$ condensate which breaks rotational invariance [3]. The related gap was estimated to be a few keV [3], but this estimate was not robust and should be revisited in light of more recent developments given its importance in the following. The critical temperature $T_c$ above which no condensate forms is of order the zero-temperature gap $\Delta$. ($T_c = 0.57\Delta$ for $J = 0$ condensates [27].) Therefore, if there are quarks for which $\Delta \sim T$ or smaller, these quarks do not pair at temperature $T$. Such quark quasiparticles will radiate neutrinos rapidly (via direct URCA reactions like $d \rightarrow u + e + \bar{\nu}$, $u \rightarrow d + e^+ + \nu$, etc.) and the quark matter core will cool rapidly and determine the cooling history of the star as a whole [16,45]. The star will cool rapidly until its interior temperature is $T < T_c \sim \Delta$, at which time the quark matter core will become inert and the further cooling history will be dominated by neutrino emission from the nuclear matter fraction of the star. If future data were to show that neutron stars first cool rapidly (direct URCA) and then cool more slowly, such data would allow an estimate of the smallest quark
matter gap. We are unlikely to be so lucky. The simple observation of rapid cooling would not be an unambiguous discovery of quark matter with small gaps; there are other circumstances in which the direct URCA processes occur. However, if as data on neutron star temperatures improves in coming years the standard cooling scenario proves correct, indicating the absence of the direct URCA processes, this would rule out the presence of quark matter with gaps in the 10 keV range or smaller. The presence of a quark matter core in which all gaps are $\gg T$ can never be revealed by an analysis of the cooling history.

2.3 Supernova Neutrinos

We now turn from neutrino emission from a neutron star which is many years old to that from the protoneutron star during the first seconds of a supernova. Carter and Reddy [49] have pointed out that when this protoneutron star is at its maximum temperature of order 30-50 MeV, it may have a quark matter core which is too hot for color superconductivity. As such a protoneutron star core cools over the next few seconds, this quark matter will cool through $T_c$, entering the color superconducting regime of the QCD phase diagram. For $T \sim T_c$, the specific heat rises and the cooling slows. Then, as $T$ drops further and $\Delta$ increases to become greater than $T$, the specific heat drops rapidly. Furthermore, as the number density of quark quasiparticles becomes suppressed by $\exp(-\Delta/T)$, the neutrino transport mean free path rapidly becomes very long [49]. This means that all the neutrinos previously trapped in the now color superconducting core are able to escape in a sudden burst. If a terrestrial neutrino detector sees thousands of neutrinos from a future supernova, Carter and Reddy’s results suggest that there may be a signature of the transition to color superconductivity present in the time distribution of these neutrinos. Neutrinos from the core of the protoneutron star will lose energy as they scatter on their way out, but because they will be the last to reach the surface of last scattering, they will be the final neutrinos received at the earth. If they are released from the quark matter core in a sudden burst, they may therefore result in a bump at late times in the temporal distribution of the detected neutrinos. More detailed study remains to be done in order to understand how Carter and Reddy’s signature, dramatic when the neutrinos escape from the core, is processed as the neutrinos traverse the rest of the protoneutron star and reach their surface of last scattering.

2.4 R-mode Instabilities

Another arena in which color superconductivity comes into play is the physics of r-mode instabilities. A neutron star whose angular rotation frequency $\Omega$ is large enough is unstable to the growth of r-mode oscillations which radiate away angular momentum via gravitational waves, reducing $\Omega$. What does “large enough” mean? The answer depends on the damping mechanisms which act to prevent the growth of the relevant modes. Both shear viscosity and bulk viscosity act to damp the r-modes, preventing them from going unstable. The bulk viscosity and
the quark contribution to the shear viscosity both become exponentially small in quark matter with $\Delta > T$ and as a result, as Madsen \cite{50} has shown, a compact star made entirely of quark matter with gaps $\Delta = 1$ MeV or greater is unstable if its spin frequency is greater than tens to 100 Hz. Many compact stars spin faster than this, and Madsen therefore argues that compact stars cannot be strange quark stars unless some quarks remain ungapped. Alas, this powerful argument becomes much less powerful in the context of a neutron star with a quark matter core. First, the r-mode oscillations have a wave form whose amplitude is largest at large radius, outside the core. Second, in an ordinary neutron star there is a new source of damping: friction at the boundary between the crust and the neutron superfluid “mantle” keeps the r-modes stable regardless of the properties of a quark matter core \cite{51,50}.

### 2.5 Magnetic Field Evolution

Next, we turn to the physics of magnetic fields within color superconducting neutron star cores \cite{52,53}. The interior of a conventional neutron star is a superfluid (because of neutron-neutron pairing) and is an electromagnetic superconductor (because of proton-proton pairing). Ordinary magnetic fields penetrate it only in the cores of magnetic flux tubes. A color superconductor behaves differently. At first glance, it seems that because a diquark Cooper pair has nonzero electric charge, a diquark condensate must exhibit the standard Meissner effect, expelling ordinary magnetic fields or restricting them to flux tubes within whose cores the condensate vanishes. This is not the case \cite{53}. In both the 2SC and CFL phase a linear combination of the $U(1)$ gauge transformation of ordinary electromagnetism and one (the eighth) color gauge transformation remains unbroken even in the presence of the condensate. This means that the ordinary photon $A_{\mu}$ and the eighth gluon $G_{\mu}^8$ are replaced by new linear combinations

$$A_{\mu}^Q = \cos \alpha_0 A_{\mu} + \sin \alpha_0 G_{\mu}^8$$

$$A_{\mu}^X = -\sin \alpha_0 A_{\mu} + \cos \alpha_0 G_{\mu}^8$$

(2)

where $A_{\mu}^Q$ is massless and $A_{\mu}^X$ is massive. That is, $B_Q$ satisfies the ordinary Maxwell equations while $B_X$ experiences a Meissner effect. The mixing angle $\alpha_0$ is the analogue of the Weinberg angle in electroweak theory, in which the presence of the Higgs condensate causes the $A_{\mu}^Y$ and the third $SU(2)_W$ gauge boson to mix to form the photon, $A_{\mu}$, and the massive $Z$ boson. $\sin(\alpha_0)$ is proportional to $e/g$ and turns out to be about $1/20$ in the 2SC phase and $1/40$ in the CFL phase \cite{53}. This means that the $Q$-photon which propagates in color superconducting quark matter is mostly photon with only a small gluon admixture. If a color superconducting neutron star core is subjected to an ordinary magnetic field, it will either expel the $X$ component of the flux or restrict it to flux tubes, but it can (and does \cite{53}) admit the great majority of the flux in the form of a $B_Q$ magnetic field satisfying Maxwell’s equations. The decay in time of this “free field” (i.e. not in flux tubes) is limited by the $Q$-conductivity of the quark
matter. A color superconductor is not a $\tilde{Q}$-superconductor — that is the whole point — but it turns out to be a very good $\tilde{Q}$-conductor due to the presence of electrons: the $B_{\tilde{Q}}$ magnetic field decays only on a time scale which is much longer than the age of the universe [53]. This means that a quark matter core within a neutron star serves as an “anchor” for the magnetic field: whereas in ordinary nuclear matter the magnetic flux tubes can be dragged outward by the neutron superfluid vortices as the star spins down [54], the magnetic flux within the color superconducting core simply cannot decay. Even though this distinction is a qualitative one, it will be difficult to confront it with data since what is observed is the total dipole moment of the neutron star. A color superconducting core anchors those magnetic flux lines which pass through the core, while in a neutron star with no quark matter core the entire internal magnetic field can decay over time. In both cases, however, the total dipole moment can change since the magnetic flux lines which do not pass through the core can move.

3 Crystalline Color Superconductivity and Glitches in Quark Matter

The final consequence of color superconductivity we wish to discuss is the possibility that (some) glitches may originate within quark matter regions of a compact star [48]. In any context in which color superconductivity arises in nature, it is likely to involve pairing between species of quarks with differing chemical potentials. If the chemical potential difference is small enough, BCS pairing occurs as we have been discussing. If the Fermi surfaces are too far apart, no pairing between the species is possible. The transition between the BCS and unpaired states as the splitting between Fermi momenta increases has been studied in electron superconductors [55], nuclear superfluids [56] and QCD superconductors [8,9,57], assuming that no other state intervenes. However, there is good reason to think that another state can occur. This is the “LOFF” state, first explored by Larkin and Ovchinnikov [58] and Fulde and Ferrell [59] in the context of electron superconductivity in the presence of magnetic impurities. They found that near the unpairing transition, it is favorable to form a state in which the Cooper pairs have nonzero momentum. This is favored because it gives rise to a region of phase space where each of the two quarks in a pair can be close to its Fermi surface, and such pairs can be created at low cost in free energy. Condensates of this sort spontaneously break translational and rotational invariance, leading to gaps which vary periodically in a crystalline pattern. If in some shell within the quark matter core of a neutron star (or within a strange quark star) the quark number densities are such that crystalline color superconductivity arises, rotational vortices may be pinned in this shell, making it a locus for glitch phenomena.

In Ref. [48], we have explored the range of parameters for which crystalline color superconductivity occurs in the QCD phase diagram, upon making various simplifying assumptions. We focus primarily on a toy model in which the quarks interact via a four-fermion interaction with the quantum numbers of sin-
gle gluon exchange. Also, we only consider pairing between $u$ and $d$ quarks, with $\mu_d = \bar{\mu} + \delta\mu$ and $\mu_u = \bar{\mu} - \delta\mu$, whereas we expect a LOFF state wherever the difference between the Fermi momenta of any two quark flavors is near an unpairing transition, including, for example, near the unlocking phase transition between the 2SC and CFL phases.

In the LOFF state, each Cooper pair carries momentum $2q$ with $|q| \approx 1.2\delta\mu$. The condensate and gap parameter vary in space with wavelength $\pi/|q|$. In Ref. [48], we simplify the calculation by assuming that the condensate varies in space like a plane wave, leaving the determination of the crystal structure of the QCD LOFF phase to future work. We give an ansatz for the LOFF wave function, and by variation obtain a gap equation which allows us to solve for the gap parameter $\Delta_A$, the free energy and the values of the diquark condensates which characterize the LOFF state at a given $\delta\mu$ and $|q|$. We then vary $|q|$, to find the preferred (lowest free energy) LOFF state at a given $\delta\mu$, and compare the free energy of the LOFF state to that of the BCS state with which it competes. We show results for one choice of parameters in Fig. 1(a). The LOFF state is characterized by a gap parameter $\Delta_A$ and a diquark condensate, but not by an energy gap in the dispersion relation: we obtain the quasiparticle dispersion relations [48] and find that they vary with the direction of the momentum, yielding gaps that vary from zero up to a maximum of $\Delta_A$. The condensate is dominated by the regions in momentum space in which a quark pair with total momentum $2q$ has both members of the pair within $\sim \Delta_A$ of their respective Fermi surfaces.

Because it violates rotational invariance by involving Cooper pairs whose momenta are not antiparallel, the quark matter LOFF state necessarily features condensates in both the $J = 0$ and $J = 1$ channels. (Cooper pairs in the symmetric $J = 1$ channel are antisymmetric in color but symmetric in flavor, and are impossible in the original LOFF context of pairing between electrons, which have neither color nor flavor.) Both $J = 0$ and $J = 1$ condensates are present even if there is no interaction in the $J = 1$ channel, as is the case when we use a four-fermion interaction with the quantum numbers of Lorentz-invariant single gluon exchange. Because there is no interaction in the $J = 1$ channel, the $J = 1$ condensate does not affect the quasiparticle dispersion relations; that is, the $J = 1$ gap parameter vanishes.

The LOFF state is favored for values of $\delta\mu$ which satisfy $\delta\mu_1 < \delta\mu < \delta\mu_2$ as shown in Fig. 1(b), with $\delta\mu_1/\Delta_0 = 0.707$ and $\delta\mu_2/\Delta_0 = 0.754$ in the weak coupling limit in which $\Delta_0 \ll \mu$. (For $\delta\mu < \delta\mu_1$, we have the 2SC phase with gap $\Delta_0$.) At weak coupling, the LOFF gap parameter decreases from $0.23\Delta_0$ at $\delta\mu = \delta\mu_1$ (where there is a first order BCS-LOFF phase transition) to zero at

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5 Our model Hamiltonian has two parameters, the four-fermion coupling $G$ and a cutoff $\Lambda$. We often use the value of $\Delta_0$, the BCS gap obtained at $\delta\mu = 0$, to describe the strength of the interaction: small $\Delta_0$ corresponds to small $G$. When we wish to study the dependence on the cutoff, we vary $\Lambda$ while at the same time varying the coupling $G$ such that $\Delta_0$ is kept fixed. We expect that the relation between other physical quantities and $\Delta_0$ will be reasonably insensitive to variation of $\Lambda$. 
\[ \delta \mu_1 / \Delta_0 \]

\[ \delta \mu_2 / \Delta_0 \]

\[ \Delta \text{GeV} \]

\[ \delta \mu / \Delta_0 \]

\[ \Lambda = 0.8 \text{ GeV} \]

\[ \Lambda = 1.6 \text{ GeV} \]

\[ \Lambda = 0.8 \text{ GeV} \]

\[ \Lambda = 1.6 \text{ GeV} \]

\[ \delta \mu_1 / \Delta_0 \]

\[ \delta \mu_2 / \Delta_0 \]

\[ \delta \mu / \Delta_0 \]

\[ \phi(r) = -\frac{1}{2} \langle \epsilon_{ab} \epsilon_{\alpha\beta\gamma\delta} \psi^{\alpha a} (r) C \gamma_5 \psi^{\beta b} (r) \rangle \] (3)

so that in the normal phase \( \phi(r) = 0 \), while in the LOFF phase \( \phi(r) = \Gamma_A e^{i2\alpha \cdot r} \).

Near the second-order critical point \( \delta \mu_2 \), we can describe the phase transition with a Ginzburg-Landau effective potential. The order parameter for the LOFF-to-normal phase transition is

\[ \phi(r) = -\frac{1}{2} \langle \epsilon_{ab} \epsilon_{\alpha\beta\gamma\delta} \psi^{\alpha a} (r) C \gamma_5 \psi^{\beta b} (r) \rangle \]

\( \phi(r) \) being the order parameter for the LOFF-to-normal phase transition. The gap parameter is related to the order parameter by \( \Delta_A = G \Gamma_A \).

Excluding the order parameter in terms of its Fourier modes \( \tilde{\phi}(k) \), we write the LOFF

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Excluding the order parameter in terms of its Fourier modes \( \tilde{\phi}(k) \), we write the LOFF
energy (relative to the normal state) as

$$F(\{\hat{\Phi}(k)\}) = \sum_k \left( C_2(k^2)|\hat{\Phi}(k)|^2 + C_4(k^2)|\hat{\Phi}(k)|^4 + O(|\hat{\Phi}|^6) \right).$$

(4)

For $\delta\mu > \delta\mu_2$, all of the $C_2(k^2)$ are positive and the normal state is stable. Just below the critical point, all of the modes $\hat{\Phi}(k)$ are stable except those on the sphere $|k| = 2q_2$, where $q_2$ is the value of $|q|$ at $\delta\mu_2$ (so that $q_2 \simeq 1.2\delta\mu_2 \simeq 0.9\Delta_0$ at weak coupling). In general, many modes on this sphere can become nonzero, giving a condensate with a complex crystal structure. We consider the simplest case of a plane wave condensate where only the one mode $\hat{\Phi}(k = 2q_2) = \Gamma_A$ is nonvanishing. Dropping all other modes, we have

$$F(\Gamma_A) = a(\delta\mu - \delta\mu_2)(\Gamma_A)^2 + b(\Gamma_A)^4$$

(5)

where $a$ and $b$ are positive constants. Finding the minimum-energy solution for $\delta\mu < \delta\mu_2$, we obtain simple power-law relations for the condensate and the free energy:

$$\Gamma_A(\delta\mu) = K_\Gamma(\delta\mu_2 - \delta\mu)^{1/2}, \quad F(\delta\mu) = -K_F(\delta\mu_2 - \delta\mu)^2.$$  

(6)

These expressions agree well with the numerical results obtained by solving the gap equation [48]. The Ginzburg-Landau method does not specify the proportionality factors $K_\Gamma$ and $K_F$, but analytical expressions for these coefficients can be obtained in the weak coupling limit by explicitly solving the gap equation [60, 48], yielding

$$GK_\Gamma = 2\sqrt{\delta\mu_2}\sqrt{\left(q_2/\delta\mu_2\right)^2 - 1} \simeq 1.15\sqrt{\Delta_0}$$

$$K_F = (4\bar{\mu}^2/\pi^2)((q_2/\delta\mu_2)^2 - 1) \simeq 0.178\bar{\mu}^2.$$  

(7)

Notice that because $(\delta\mu_2 - \delta\mu_1)/\delta\mu_2$ is small, the power-law relations are a good model of the system throughout the entire LOFF interval $\delta\mu_1 < \delta\mu < \delta\mu_2$ where the LOFF phase is favored over the BCS phase. The Ginzburg-Landau expression gives the free energy of the LOFF phase near $\delta\mu_2$, but it cannot be used to determine the location $\delta\mu_1$ of the first-order phase transition where the LOFF window terminates. (Locating the first-order point requires a comparison of LOFF and BCS free energies.)

The quark matter which may be present within a compact star will be in the crystalline color superconductor (LOFF) state if $\delta\mu/\Delta_0$ is in the requisite range. For a reasonable value of $\delta\mu$, say 25 MeV, this occurs if the gap $\Delta_0$ which characterizes the uniform color superconductor present at smaller values of $\delta\mu$ is about 40 MeV. This is in the middle of the range of present estimates. Both $\delta\mu$ and $\Delta_0$ vary as a function of density and hence as a function of radius in a compact star. Although it is too early to make quantitative predictions, the numbers are such that crystalline color superconducting quark matter may very well occur in a range of radii within a compact star. It is therefore worthwhile to consider the consequences.
Many pulsars have been observed to glitch. Glitches are sudden jumps in rotation frequency $\Omega$ which may be as large as $\Delta \Omega / \Omega \sim 10^{-6}$, but may also be several orders of magnitude smaller. The frequency of observed glitches is statistically consistent with the hypothesis that all radio pulsars experience glitches. Glitches are thought to originate from interactions between the rigid neutron star crust, typically somewhat more than a kilometer thick, and rotational vortices in a neutron superfluid. The inner kilometer of crust consists of a crystal lattice of nuclei immersed in a neutron superfluid. Because the pulsar is spinning, the neutron superfluid (both within the inner crust and deeper inside the star) is threaded with a regular array of rotational vortices. As the pulsar’s spin gradually slows, these vortices must gradually move outwards since the rotation frequency of a superfluid is proportional to the density of vortices. Deep within the star, the vortices are free to move outwards. In the crust, however, the vortices are pinned by their interaction with the nuclear lattice. Models differ in important respects as to how the stress associated with pinned vortices is released in a glitch: for example, the vortices may break and rearrange the crust, or a cluster of vortices may suddenly overcome the pinning force and move macroscopically outward, with the sudden decrease in the angular momentum of the superfluid within the crust resulting in a sudden increase in angular momentum of the rigid crust itself and hence a glitch. All the models agree that the fundamental requirements are the presence of rotational vortices in a superfluid and the presence of a rigid structure which impedes the motion of vortices and which encompasses enough of the volume of the pulsar to contribute significantly to the total moment of inertia.

Although it is premature to draw quantitative conclusions, it is interesting to speculate that some glitches may originate deep within a pulsar which features a quark matter core, in a region of that core which is in a LOFF crystalline color superconductor phase. A three flavor analysis is required to estimate over what range of densities LOFF phases may arise, as either $\langle ud \rangle$, $\langle us \rangle$ or $\langle ds \rangle$ condensates approach their unpairing transitions. Comparison to existing models which describe how $p^u_F$, $p^d_F$ and $p^s_F$ vary within a quark matter core in a neutron star would then permit an estimate of how much the LOFF region contributes to the moment of inertia of the pulsar. Furthermore, a three flavor analysis is required to determine whether the LOFF phase is a superfluid. If the only pairing is between $u$ and $d$ quarks, this 2SC phase is not a superfluid, whereas if all three quarks pair in some way, a superfluid is obtained. Henceforth, we suppose that the LOFF phase is a superfluid, which means that if it occurs within a pulsar it will be threaded by an array of rotational vortices. It is reasonable to expect that these vortices will be pinned in a LOFF crystal, in which the diquark condensate varies periodically in space. Indeed, one of the suggestions for how to look for a LOFF phase in terrestrial electron superconductors relies on the fact that the pinning of magnetic flux tubes (which, like the rotational vortices of interest to us, have normal cores) is expected to be much stronger in a LOFF phase than in a uniform BCS superconductor.
A real calculation of the pinning force experienced by a vortex in a crystalline color superconductor must await the determination of the crystal structure of the LOFF phase. We can, however, attempt an order of magnitude estimate along the same lines as that done by Anderson and Itoh [66] for neutron vortices in the inner crust of a neutron star. In that context, this estimate has since been made quantitative [67,68,63]. For one specific choice of parameters [45], the LOFF phase is favored over the normal phase by a free energy $F_{\text{LOFF}} \sim 5 \times (10 \text{ MeV})^4$ and the spacing between nodes in the LOFF crystal is $b = \pi/(2|q|) \sim 9 \text{ fm}$. The thickness of a rotational vortex is given by the correlation length $\xi \sim 1/\Delta \sim 25 \text{ fm}$. The pinning energy is the difference between the energy of a section of vortex of length $b$ which is centered on a node of the LOFF crystal vs. one which is centered on a maximum of the LOFF crystal. It is of order $E_\text{p} \sim F_{\text{LOFF}} b^3 \sim 4 \text{ MeV}$. The resulting pinning force per unit length of vortex is of order $f_\text{p} \sim E_\text{p}/b^2 \sim (4 \text{ MeV})/(80 \text{ fm}^2)$. A complete calculation will be challenging because $b < \xi$, and is likely to yield an $f_\text{p}$ which is somewhat less than that we have obtained by dimensional analysis. Note that our estimate of $f_\text{p}$ is quite uncertain both because it is only based on dimensional analysis and because the values of $\Delta$, $b$ and $F_{\text{LOFF}}$ are uncertain. (We have a good understanding of all the ratios $\Delta/\Delta_0$, $\delta\mu/\Delta_0$, $q/\Delta_0$ and consequently $b\Delta_0$ in the LOFF phase. It is of course the value of the BCS gap $\Delta_0$ which is uncertain.) It is premature to compare our crude result to the results of serious calculations of the pinning of crustal neutron vortices as in Refs. [67,68,63]. It is nevertheless remarkable that they prove to be similar: the pinning energy of neutron vortices in the inner crust is $E_\text{p} \approx 1 - 3 \text{ MeV}$ and the pinning force per unit length is $f_\text{p} \approx (1 - 3 \text{ MeV})/(200 - 400 \text{ fm}^2)$.

The reader may be concerned that a glitch deep within the quark matter core of a neutron star may not be observable: the vortices within the crystalline color superconductor region suddenly unpin and leap outward; this loss of angular momentum is compensated by a gain in angular momentum of the layer outside the LOFF region; how quickly, then, does this increase in angular momentum manifest itself at the surface of the star as a glitch? The important point here is that the rotation of any superfluid region within which the vortices are able to move freely is coupled to the rotation of the outer crust on very short time scales [69]. This rapid coupling, due to electron scattering off vortices and the fact that the electron fluid penetrates throughout the star, is usually invoked to explain that the core nucleon superfluid speeds up quickly after a crustal glitch: the only long relaxation time is that of the vortices within the inner crust [69]. Here, we invoke it to explain that the outer crust speeds up rapidly after a LOFF glitch has accelerated the quark matter at the base of the nucleon superfluid. After a glitch in the LOFF region, the only long relaxation times are those of the vortices in the LOFF region and in the inner crust.

A quantitative theory of glitches originating within quark matter in a LOFF phase must await further calculations, in particular a three flavor analysis and the determination of the crystal structure of the QCD LOFF phase. However, our rough estimate of the pinning force on rotational vortices in a LOFF region
suggests that this force may be comparable to that on vortices in the inner crust of a conventional neutron star. Perhaps, therefore, glitches occurring in a region of crystalline color superconducting quark matter may yield similar phenomenology to those occurring in the inner crust. This is surely strong motivation for further investigation.

Perhaps the most interesting consequence of these speculations arises in the context of compact stars made entirely of strange quark matter. The work of Witten [70] and Farhi and Jaffe [71] raised the possibility that strange quark matter may be stable relative to nuclear matter even at zero pressure. If this is the case it raises the question whether observed compact stars—pulsars, for example—are strange quark stars [72, 73] rather than neutron stars. A conventional neutron star may feature a core made of strange quark matter, as we have been discussing above. Strange quark stars, on the other hand, are made (almost) entirely of quark matter with either no hadronic matter content at all or with a thin crust, of order one hundred meters thick, which contains no neutron superfluid [73, 74]. The nuclei in this thin crust are supported above the quark matter by electrostatic forces; these forces cannot support a neutron fluid. Because of the absence of superfluid neutrons, and because of the thinness of the crust, no successful models of glitches in the crust of a strange quark star have been proposed. Since pulsars are observed to glitch, the apparent lack of a glitch mechanism for strange quark stars has been the strongest argument that pulsars cannot be strange quark stars [75, 76, 77]. This conclusion must now be revisited.

Madsen’s conclusion [50] that a strange quark star is prone to r-mode instability due to the absence of damping must also be revisited, since the relevant oscillations may be damped within or at the boundary of a crystalline color superconductor region.

The quark matter in a strange quark star, should one exist, would be a color superconductor. Depending on the mass of the star, the quark number densities increase by a factor of about two to ten in going from the surface to the center [73]. This means that the chemical potential differences among the three quarks will vary also, and there could be a range of radii within which the quark matter is in a crystalline color superconductor phase. This raises the possibility of glitches in strange quark stars. Because the variation in density with radius is gradual, if a shell of LOFF quark matter exists it need not be particularly thin. And, we have seen, the pinning forces may be comparable in magnitude to those in the inner crust of a conventional neutron star. It has recently been suggested (for reasons unrelated to our considerations) that certain accreting compact stars may be strange quark stars [78], although the evidence is far from unambiguous [79]. In contrast, it has been thought that, because they glitch, conventional radio pulsars cannot be strange quark stars. Our work questions

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Note that a convincing discovery of a quark matter core within an otherwise hadronic neutron star would demonstrate conclusively that strange quark matter is not stable at zero pressure, thus ruling out the existence of strange quark stars. It is not possible for neutron stars with quark matter cores and strange quark stars to both be stable.
this assertion by raising the possibility that glitches may originate within a layer of quark matter which is in a crystalline color superconducting state.

There has been much recent progress in our understanding of how the presence of color superconducting quark matter in a compact star would affect five different phenomena: cooling by neutrino emission, the pattern of the arrival times of supernova neutrinos, the evolution of neutron star magnetic fields, r-mode instabilities and glitches. Nevertheless, much theoretical work remains to be done before we can make sharp proposals for which astrophysical observations can teach us whether compact stars contain quark matter, and if so whether it is in the 2SC or CFL phase.

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