Nuclear collective excitations in Landau Fermi liquid theory

Bao-Xi Sun
Institute of Theoretical Physics, College of Applied Sciences,
Beijing University of Technology, Beijing 100124, China

The nuclear collective excitations are studied within Landau Fermi liquid theory. By using the nucleon-nucleon interaction of the linear $\sigma - \omega$ model, the nuclear collective excitation energies of different values of $l$ are obtained, which are fitted with the centroid energies of the giant resonances of spherical nuclei, respectively. In addition, it is pointed out that the isovector giant resonances except $l = 1$ correspond to the modes that protons are in the creation state and neutrons are in the annihilation state, and vice versa. Some mixtures of the nuclear collective excitation states with different values of $l$ are predicted.

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I. INTRODUCTION

The nuclear collective excitation states have been studied in the framework of nuclear macroscopic and microscopic models. More detailed descriptions on these models and references can be found in Ref. [1]. Nowadays, it is still a interesting topic in nuclear physics. With several typical methods, such as the random phase approximation with Skyrme interactions [2], the relativistic random phase approximation [3–5], the centroid energies and strength distributions of the giant resonances of the nuclei are calculated and compared with the experimental data.

Landau Fermi liquid theory is one of the important cornerstones of traditional many-body theory in the condensed state physics. It is very useful because it describes almost all known metals and many non-metallic states, such as superconductors, anti-ferromagnetic states, etc. However, this theory has not been used to solve nuclear many-body problems successfully. In this work, I will try to calculate the collective excitation energies of the nuclear matter within the framework of Landau Fermi liquid theory, and then compare my calculation results with the experimental data of the nuclear giant resonances of the nuclei.

This article is organized as follows: in Section II, the formalism on the reduced Boltzmann equation is extended to the 3-dimensional of the Fermi liquid with the spin taken into account. In Section III The calculation results on the nuclear collective excitation energies are compared with the experimental data of the giant resonances of nuclei. The conclusion is summarized in Section IV.

II. REDUCED BOLTZMANN EQUATION OF A FERMI LIQUID AT ZERO TEMPERATURE

According to Ref. [6], the Boltzmann equation with an infinite quasi-particle lifetime for the nuclear matter can be written as

$$\frac{\partial n_{\vec{k}\alpha}}{\partial t} + \frac{\partial n_{\vec{k}\alpha}}{\partial \vec{x}} \cdot \frac{\partial \tilde{\epsilon}}{\partial \vec{k}} - \frac{\partial n_{\vec{k}\alpha}}{\partial \vec{k}} \cdot \frac{\partial \tilde{\epsilon}}{\partial \vec{x}} = 0,$$

(1)

where $n_{\vec{k}\alpha}(\vec{x},t) = n_{0\vec{k}\alpha} + \delta n_{\vec{k}\alpha}(\vec{x},t)$ is the occupation number of quasi-nucleons, and $\tilde{\epsilon}$ denotes the quasi-nucleon energy on the background of a collective excited state.

The quasi-nucleon density $\tilde{\rho}_\alpha(\theta, \phi)$ and the occupation $n_{\vec{k}\alpha}$ are related as follows:

$$\tilde{\rho}_\alpha(\theta, \phi) = \int \frac{k^2 dk}{(2\pi)^3} \delta n_{\vec{k}\alpha},$$

(2)

with

$$\delta n_{\vec{k}\alpha} = \begin{cases} 1, \\ 0. \end{cases}$$

(3)

and $\alpha$ denotes the spin index. The isospin index is suppressed in Eq. (2).

In the momentum space, the linearized liquid equation of motion takes the form

$$i \frac{\partial}{\partial t} \tilde{\rho}_\alpha(\theta, \phi, \vec{q}, t) = q \sum_\beta \int d\Omega' \int d\Omega'' K(\theta, \phi; \theta', \phi') M(\theta', \phi', \alpha; \theta'', \phi'', \beta) \tilde{\rho}_\beta(\theta'', \phi'', \vec{q}, t),$$

(4)
According to the linear $\sigma$ model, the nucleon Fermi velocity $v_F$ can be written as

$$K(\theta, \phi; \theta', \phi') = \left[ \sin \theta \sin \theta_q \cos(\phi - \phi_q) + \cos \theta \cos \theta_q \right] \frac{1}{\sin \theta'} \delta(\theta - \theta') \delta(\phi - \phi')$$  \hspace{1cm} (5)

with $(\theta_q, \phi_q)$ the angle of the momentum $\vec{q}$, and

$$M(\theta, \phi; \alpha; \theta', \phi', \beta) = v_F^* \frac{1}{\sin \theta'} \delta_{\alpha \beta} \delta(\theta - \theta') \delta(\phi - \phi') + \frac{k_F^2}{(2\pi)^2} f(k_F, \theta, \phi; \alpha; k_F, \theta', \phi', \beta).$$  \hspace{1cm} (6)

with $f(k_F, \theta, \phi; \alpha; k_F, \theta', \phi', \beta)$ the Fermi liquid function, $k_F$ the Fermi momentum and $v_F^*$ the Fermi velocity.

In order to determine the form of Fermi liquid function, we study the effective interaction between two nucleons. According to the linear $\sigma$-$\omega$ model, the nucleons $\psi$ interact with scalar mesons $\sigma$ through a Yukawa coupling $\bar{\psi}\psi\sigma$ and with neutral vector mesons $\omega$ that couple to the conserved baryon current $\bar{\psi}\gamma_\mu\psi$ \cite{7}. the Lagrangian density can be written as

$$\mathcal{L} = \bar{\psi}(i\gamma_\mu \partial^\mu - M_N) \psi + \frac{1}{2} g_\sigma \psi \psi \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu$$

$$- g_\sigma \bar{\psi} \sigma \psi - g_\omega \bar{\psi} \gamma_\mu \omega^\mu \psi ,$$  \hspace{1cm} (7)

with $M_N, m_\sigma$ and $m_\omega$ the nucleon, scalar meson and vector meson masses, respectively, and $\omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$ the vector meson field tensor.

The effective nucleon-nucleon potential in the static limit can be deduced directly with the Lagrangian in Eq. \cite{7}.

$$V_{\text{eff}}(q) - V_{\text{eff}}(q') \delta_{\alpha \beta}$$  \hspace{1cm} (8)

with

$$V_{\text{eff}}(q) = \left[ \frac{-g_\sigma^2}{m_\sigma^2} + \frac{g_\omega^2}{m_\omega^2} \right]$$

where the first term $V_{\text{eff}}(q)$ in Eq. \cite{7} denotes the direct interaction between nucleons and the second term $V_{\text{eff}}(q') \delta_{\alpha \beta}$ the exchange interaction with $\alpha$ and $\beta$ the spins of interacting nucleons. The Fermi liquid function is instantaneous interaction potential between two nucleons near the Fermi surface with momenta $\vec{k}_1$ and $\vec{k}_2$ scattering into two nucleons with same momenta $\vec{k}_1'$ and $\vec{k}_2'$, which is depicted in Fig. 1. By using Eq. \cite{8}, the Fermi liquid function takes the form

$$f(\vec{k}_1, \alpha; \vec{k}_2, \beta) = V_{\text{eff}}(0) - V_{\text{eff}}(\vec{k}_1 - \vec{k}_2) \delta_{\alpha \beta}$$

$$= \left[ \frac{-g_\sigma^2}{m_\sigma^2} + \frac{g_\omega^2}{m_\omega^2} \right] - \left[ \frac{-g_\sigma^2}{(k_1 - k_2)^2 + m_\sigma^2} + \frac{g_\omega^2}{(k_1 - k_2)^2 + m_\omega^2} \right] \delta_{\alpha \beta}$$  \hspace{1cm} (9)

with

$$\vec{k}_1 = (k_F, \theta, \phi), \quad \vec{k}_2 = (k_F, \theta', \phi'),$$  \hspace{1cm} (10)

and

$$\left( \vec{k}_1 - \vec{k}_2 \right)^2 = 2k_F^2 \left[ 1 - \sqrt{1 - \cos \theta \cos \theta' + \sin \theta \sin \theta' \cos(\phi - \phi')} \right]$$

$$= 2k_F^2 \left[ 1 - \vec{k}_1 \cdot \vec{k}_2 \right].$$  \hspace{1cm} (11)

The direct interaction potential in the Fermi liquid function in Eq. \cite{9} is constant and only contribute a ground state energy correction of the nuclear matter. In the framework of the relativistic mean-field approximation or relativistic Hartree approximation, the nucleon Fermi energy can be written as

$$\varepsilon_F^* \simeq M_N + \frac{k_F^2}{2M_N} + \left[ \frac{-g_\sigma^2}{m_\sigma^2} + \frac{g_\omega^2}{m_\omega^2} \right] \sum_{\gamma} \frac{k_\gamma^2}{6\pi^2}$$  \hspace{1cm} (12)

where $\gamma$ denotes the summation over spins and isospins of the nucleon, then the nucleon Fermi velocity $v_F^*$ is obtained as

$$v_F^* = \frac{\partial \varepsilon_F^*}{\partial k_F} = \frac{k_F}{M_N} + \left[ \frac{-g_\sigma^2}{m_\sigma^2} + \frac{g_\omega^2}{m_\omega^2} \right] \frac{k_F^2}{2\pi^2},$$  \hspace{1cm} (13)
where the second term is just the contribution from the direct interaction $V_{eff}(0)$ of the Fermi liquid function in Eq. (9). Thus in the following calculation, only the exchange term is reserved in the Fermi liquid function.

The quasi-nucleon density in Eq. (4) can be expanded in spherical harmonics with time-dependent shape parameters as coefficients:

$$\tilde{\rho}_\alpha(\theta, \phi, \vec{q}, t) = \sum_{l,m} \tilde{\rho}_\alpha(l, m, \vec{q}, t) Y_{l,m}^*(\theta, \phi).$$

(14)

Similarly, the functions $K(\theta, \phi; \theta', \phi')$ and $M(\theta', \phi'; \alpha; \theta'', \phi'', \beta)$ can also be expanded as

$$K(\theta, \phi; \theta', \phi') = \sum_{l,m,l',m'} K(l,m;l',m') Y_{l,m}^*(\theta, \phi) Y_{l',m'}(\theta', \phi'),$$

(15)

and

$$M(\theta', \phi'; \alpha; \theta'', \phi'', \beta) = \sum_{l_1,m_1,l_2,m_2} M(l_1,m_1,\alpha;l_2,m_2,\beta) Y_{l_1,m_1}^*(\theta', \phi') Y_{l_2,m_2}(\theta'', \phi''),$$

(16)

respectively.

Therefore, the liquid equation of motion in the basis of spherical harmonics can be rewritten as

$$i \frac{\partial}{\partial t} \tilde{\rho}_\alpha(l, m, \vec{q}, t) = q \sum_{\beta} \sum_{l',m',l''} K(l, m; l', m') M(l', m'; \alpha; l'', m'', \beta) \tilde{\rho}_\beta(l'', m'', \vec{q}, t).$$

(17)

Because the energy spectrum does not depend on the direction of $\vec{q}$, we can choose $\vec{q}$ to be in the direction of $\theta_q = 0$ and $\phi_q = 0$, and then the function $K(\theta, \phi; \theta', \phi')$ becomes

$$K(\theta, \phi; \theta', \phi') = \frac{\cos \theta}{\sin \theta'} \delta(\theta - \theta') \delta(\phi - \phi').$$

(18)

In the spherical harmonics,

$$K(l, m; l', m') = \int \frac{\cos \theta}{\sin \theta'} \delta(\theta - \theta') \delta(\phi - \phi') Y_{l,m}(\theta, \phi) Y_{l',m'}^*(\theta', \phi') \sin \theta' d\theta' d\phi' \sin \theta d\theta d\phi$$

$$= (a_{lm} \delta_{l+1,1} + a_{l-1,m} \delta_{l-1,1}) \delta_{m,m'}$$

(19)

with

$$a_{lm} = \sqrt{\frac{(l+1)^2 - m^2}{(2l+1)(2l+3)}},$$

as coefficients.
and
\[
M(l_1, m_1, \alpha; l_2, m_2, \beta) = \int \left[ v_F^* \frac{1}{\sin \theta} \delta_{\alpha \beta} \delta(\theta - \theta') \delta(\phi - \phi') + \frac{k_F^2}{(2\pi)^3} f(k_F, \theta, \phi, \alpha; k_F, \theta', \phi', \beta) \right] \Y_{l_1, m_1}(\theta, \phi) Y_{l_2, m_2}^*(\theta', \phi') d\Omega d\Omega'
\]
\[
= v_F^* \delta_{\alpha \beta} \delta_{l_1, l_2} \delta_{m_1, m_2} - \frac{k_F^2}{(2\pi)^3} f_F(l_1, m_1; l_2, m_2) \delta_{\alpha \beta},
\]
where the Fock term
\[
f_F(l_1, m_1; l_2, m_2) = \int V_{ef}(\vec{k}_1 - \vec{k}_2) Y_{l_1, m_1}(\theta, \phi) Y_{l_2, m_2}^*(\theta', \phi') \sin \theta d\theta d\phi \sin \theta' d\theta' d\phi'
\]
\[
= \int \left[ \frac{-g_\gamma^2}{(k_1 - k_2)^2 + m_\rho^2} + \frac{g_\omega^2}{(k_1 - k_2)^2 + m_\omega^2} \right] Y_{l_1, m_1}(\theta, \phi) Y_{l_2, m_2}^*(\theta', \phi') \sin \theta d\theta d\phi \sin \theta' d\theta' d\phi'
\]
\[
= f_F(l_1, l_2) \delta_{l_1, l_2} \delta_{m_1, m_2}
\]
would give a contribution to the nuclear collective excitation when \(l_1 = l_2\) and \(m_1 = m_2\). Therefore, the liquid equation of motion of the quasi-nucleon can be rewritten as
\[
i \frac{\partial}{\partial t} \tilde{\rho}_\alpha(l, m, \vec{q}, t) = q \sum_{l'} (a_{l m} \delta_{l+1,l'} + a_{l-1,m} \delta_{l-1,l'}) \left( v_F^* - \frac{k_F^2}{(2\pi)^3} f_F(l', l') \right) \tilde{\rho}_\alpha(l', m, \vec{q}, t),
\]
or the matrix equation form
\[
i \frac{\partial}{\partial t} \tilde{\rho}_\alpha(l, m, \vec{q}, t) = q \tilde{K} \tilde{M} \tilde{\rho}_\alpha(l, m, \vec{q}, t),
\]
with
\[
\tilde{K}_{l,l'} = (a_{l m} \delta_{l+1,l'} + a_{l-1,m} \delta_{l-1,l'})
\]
and
\[
\tilde{M}_{l', l} = \left( v_F^* - \frac{k_F^2}{(2\pi)^3} f_F(l', l') \right) \delta_{l', l}.
\]
The stability of the Fermi liquid requires the diagonal matrix elements of \(\tilde{M}\) to be positive definite. Hence, all the value of \(f_F(l', l')\) must be less than \((2\pi)^3 v_F^*/k_F^2\), and we can write \(M\) as \(\tilde{M} = \tilde{W} \tilde{W}^T\). Letting \(u_\alpha = \tilde{W}^T \tilde{\rho}_\alpha\), then Eq. (23) becomes
\[
i \frac{\partial}{\partial t} u_\alpha(l, m, \vec{q}, t) = q \tilde{W} \tilde{K} \tilde{W} u_\alpha(l, m, \vec{q}, t) = H u_\alpha(l, m, \vec{q}, t),
\]
where the Hamiltonian
\[
H_{l,l'}(m) = q (\tilde{W} \tilde{K} \tilde{W})_{l,l'} = q (a_{l m} \delta_{l+1,l'} + a_{l-1,m} \delta_{l-1,l'}) \left( v_F^* - \frac{k_F^2}{(2\pi)^3} f_F(l, l) \right)^{1/2} \left( v_F^* - \frac{k_F^2}{(2\pi)^3} f_F(l', l') \right)^{1/2}
\]
is hermite and \(H = H^\dagger\). The eigenvalues of \(H\) would give us the frequencies of the collective excitation modes of the nuclear matter.
III. RESULTS

In this section, the eigenvalues of the Hamiltonian in Eq. (27) for different values of \( l \) are calculated with the Fermi liquid function in the linear \( \sigma-\omega \) model. In Eq. (27), Quantum number \( m \) is fixed to zero since our calculation will begin from \( l = 0 \). The parameters in Ref. [8] are used in the calculation, i.e., \( g_\sigma = 10.47 \), \( g_\omega = 13.80 \), \( m_\sigma = 520\text{MeV} \), \( m_\omega = 783\text{MeV} \) and \( M_N = 939\text{MeV} \). Since the nucleon near the Fermi surface would be more possible to be excited, we set the value of nucleon momentum \( |\vec{q}| = k_F = 1.36\text{fm}^{-1} \) in the calculation. When \( f_F(l,l) = 0 \), the Hamiltonian \( H \) has a continuous spectrum and it generates the particle-hole continuum of the nuclear matter in the relativistic mean-field approximation. However, if the value of \( f_F(l,l) \) is large enough, in addition to the continuum eigenvalues, the spectrum of \( H \) has isolated positive and negative eigenvalues, and the positive isolated eigenvalue corresponds to the energy of the collective excitation of the nuclear matter with fixed \( l \). However, the negative eigenvalue of \( H \) does not correspond to the negative energy of the nuclear collective excitation modes. Actually, the mode with a positive eigenvalue corresponds to the creation of a nuclear collective excitation mode, while the the mode with a negative eigenvalue corresponds to the annihilation of a nuclear collective excitation mode.

Fig. 2 shows the collective excitation energy \( E_l \) in the nuclear matter versus the effective nucleon mass \( M_N^{\ast} \). In my calculation, the nuclear collective excitation energy is relevant to the effective nucleon mass intensely. It can be seen that the collective excitation energy decreases with the effective nucleon mass increasing, and for \( l = 2 \), when the effective nucleon mass is less than 0.66\( M_N \), the isolated eigenvalues of \( H \) can not be generated.

Since the energy of the isospin scalar giant quadrupole resonance of the nucleus \( ^{208}\text{Pb} \) is about \( 10.9 \pm 0.1\text{MeV} \) [9], the effective nucleon mass can be fixed to be \( M_N^{\ast} = 0.742M_N \), which generates an excitation collective energy of 10.92MeV for \( l = 2 \). With the effective nucleon energy \( M_N^{\ast} = 0.742M_N \), the collective excitation energies of the nuclear matter for different values of \( l \) are listed in Table I. It is apparent that the calculation results are fitted with the experimental values, respectively.

| \( l \) | \( E_l \) (MeV) | \( E_{\text{exp}} \) (MeV) |
|---|---|---|
| 0 | 12.28 | 14.17 ± 0.28 |
| 1 | 13.73 | 13.5 ± 0.2 |
| 2 | 10.92 | 10.9 ± 0.1 |

TABLE I: The collective excitation energies of the nuclear matter for \( l = 0, 1, 2 \) with the effective nucleon mass \( M_N^{\ast} = 0.742M_N \). The corresponding experimental values for the excitation energies of \( ^{208}\text{Pb} \) are also listed as \( E_{\text{exp}} \), where the experimental value for \( l = 0 \) is taken from Ref. [10], the experimental value for \( l = 1 \) from Ref. [11], and the experimental value for \( l = 2 \) from Ref. [9].
The collective excitation of the nuclear matter with $l \geq 3$, is difficult to calculate in the framework of Landau Fermi liquid theory with a Fermi liquid function deduced from the linear $\sigma - \omega$ model. For the collective excitation of the nuclear matter with $l = 3$, the positive isolated energy eigenvalue is $3.12\text{MeV}$ with $M^*_N = 0.8M_N$, which can be treated as the low-energy octupole resonance\[12\]. However, the high-energy octupole resonance can not be generated with our model\[13\].

The nuclear collective excitation energy $E_l$ as functions of the Fermi momentum $k_F$ for $l = 0, 1, 2$ are illustrated in Fig. 3. When the value of the effective nucleon mass is fixed, the collective excitation energy decreases with the Fermi momentum increasing. For the case of $l = 2$, the isolated energy levels can not be excited from the continuum quasi-nucleon energy levels when the Fermi momentum $k_F$ is less than $1.26\text{fm}^{-1}$.

The nuclear isoscalar giant resonances actually correspond to the nuclear collective excitations with different values of $l$. However, the nuclear isovector giant resonances correspond to the nuclear collective excitation states that the collective excitation of protons is creating with the energy $E_S(l)$, while the collective excitation of neutrons is annihilating with the energy $E_S(l)$, and vice versa. Hence, the energy of the nuclear isovector giant resonance is about twice of the corresponding isoscalar giant resonance in the nuclear matter, i.e.,

$$E_V(l) = E_S(l) - (-E_S(l)) = 2E_S(l).$$

(28)

The experimental data on the nuclear giant resonances in Ref. [9–11, 14–16] demonstrate that the relation between the energy of nuclear isovector giant resonance and that of nuclear isoscalar giant resonance in Eq. (28) is correct approximately except for the case $l = 1$, which will be discussed in detail in \[13\]. In follows, I will study the giant resonance energies with different values of $l$ in the nucleus.

### A. Nuclear giant monopole resonances

The nuclear giant monopole mode, $l = 0$. The spherical harmonic $Y_{00}(\theta, \phi)$ is constant, so that a non-vanishing value of $\rho_\alpha(0, 0, \vec{q}, t)$ corresponds to a change of the Fermi momentum according to Eq. (14). The associated excitation is the so-called breathing mode of the nucleus. Supposed the proton and neutron densities can be calculated approximately:

$$\rho_p = \rho_0 \frac{Z}{A}, \quad \rho_n = \rho_0 \frac{N}{A}. \quad (29)$$

Thus the calculated energies for isoscalar and isovector giant monopole resonances of nuclei $^{208}\text{Pb}$, $^{144}\text{Sm}$, $^{116}\text{Sn}$, $^{90}\text{Zr}$, $^{40}\text{Ca}$ and their corresponding experimental values are listed in Table [11]. Since the Fermi momentum of protons $k_F(p)$ is different from that of neutrons, the collective excitation energies of protons and neutrons, $E_0(p)$ and $E_0(n)$,
are different from each other. It shows the calculation results of the proton excitation energy for heavy nuclei, such as $^{208}\text{Pb}$, $^{144}\text{Sm}$ and $^{116}\text{Sn}$, are fitted with the corresponding experimental centroid energy of the nuclear isoscalar monopole resonance $E_{\text{exp}}^S$, while for those light nucleus, such as $^{90}\text{Zr}$ and $^{40}\text{Ca}$, the calculation results are less than those experimental values, respectively. Moreover, the sum of the excitation energies of protons and neutrons $E_0(p) + E_0(n)$ should be fitted with the nuclear isovector giant monopole energy. For $^{208}\text{Pb}$, it is just in the range of the experimental values. However, for light nuclei, such as $^{90}\text{Zr}$ and $^{40}\text{Ca}$. Similarly to the cases of nuclear isoscalar giant monopole, the values of $E_0(p) + E_0(n)$ are less than the centroid energy of the nuclear isovector giant monopole. Because the value of the effective nucleon mass is determined on the collective excitation energy of $^{208}\text{Pb}$ for $l = 2$, it can be believed that with a little smaller effective nucleon mass, the calculation results for the nuclei $^{90}\text{Zr}$ and $^{40}\text{Ca}$ can fit with the experimental values very well. Actually, with $M_N^* = 0.717M_N$, we can obtain $E_0(p) = E_0(n) = 15.58\text{MeV}$ and $E_0(p) + E_0(n) = 31.16\text{MeV}$ for $^{40}\text{Ca}$, and $E_0(p) = 17.57\text{MeV}$, $E_0(n) = 13.13\text{MeV}$ and $E_0(p) + E_0(n) = 30.7\text{MeV}$ for $^{90}\text{Zr}$, which are fitted with the corresponding experimental centroid energies of the nuclear isoscalar and isovector giant monopole resonances.

$$
\begin{array}{cccccc}
 l = 0 & k_F(p) \text{ (fm}^{-1}) & k_F(n) \text{ (fm}^{-1}) & E_0(p) \text{ (MeV)} & E_0(n) \text{ (MeV)} & E_0(p) + E_0(n) \text{ (MeV)} \\
^{208}\text{Pb} & 1.26 & 1.45 & 16.28 & 7.05 & 23.33 \\
^{144}\text{Sm} & 1.29 & 1.42 & 15.26 & 9.00 & 24.26 \\
^{116}\text{Sn} & 1.29 & 1.42 & 15.26 & 9.00 & 24.26 \\
^{90}\text{Zr} & 1.31 & 1.41 & 14.50 & 9.60 & 24.10 \\
^{40}\text{Ca} & 1.36 & 1.36 & 12.28 & 12.28 & 24.56 \\
\end{array}
$$

**TABLE II:** The Fermi momenta and the $l = 0$ collective excitation energies of protons and neutrons for different nuclei with the effective nucleon mass $M_N^* = 0.742M_N$. The corresponding experimental values for the nuclear isoscalar and isovector giant monopole resonances are labeled as $E_{\text{exp}}^S$ and $E_{\text{exp}}^V$, where the experimental values for the nuclear isoscalar giant monopole resonances are taken from Ref. [10], the experimental values for the nuclear isovector giant monopole resonances from Ref. [14–16].

### B. Nuclear giant dipole resonances

The dipole deformation of the nucleus is really a shift of the center of mass. Thus the isospin isovector giant dipole resonance of the nucleus actually corresponds to the creation of the $l = 1$ collective excitation of protons or neutrons. The isoscalar giant dipole resonance in $^{208}\text{Pb}$ with a centroid energy at $E = 22.5\text{MeV}$, using the $(\alpha, \alpha')$ cross sections at forward angles [17], should be a compression mode, which corresponds to a creation of the $l = 1$ collective excitation of protons or neutrons and an annihilation of the $l = 1$ collective excitation of neutrons or protons simultaneously. The calculation results and the corresponding experimental centroid energies are listed in Table [III]. For the heavy nucleus, $^{208}\text{Pb}$, the calculation excitation energy for protons $E_1(p)$ with the effective nucleon mass $M_N^* = 0.742M_N$ is larger than the experimental value, especially the sum $E_1(p) + E_1(n)$ is larger than the corresponding energy of the giant isovector dipole resonance of $^{208}\text{Pb}$. If we increase the value of the effective nucleon mass to $M_N^* = 0.755M_N$, we can obtain $E_1(p) = 15.53\text{MeV}$, $E_1(n) = 6.57\text{MeV}$ and $E_1(p) + E_1(n) = 22.1\text{MeV}$. However, for the light nucleus, $^{40}\text{Ca}$, the excitation energies of protons and neutrons are less than the experimental value, and we must reduce the value of the effective nucleon mass to obtain a correct excitation energy. With $M_N^* = 0.70M_N$, we obtain $E_1(p) = E_1(n) = 19.58\text{MeV}$, and the sum $E_1(p) + E_1(n) = 39.16\text{MeV}$ for $^{40}\text{Ca}$. It is apparent that in order to obtain a more correct excitation energy, the effective nucleon mass must take a larger value for heavy nuclei, but a smaller value for light nuclei.

$$
\begin{array}{cccccc}
 l = 1 & k_F(p) \text{ (fm}^{-1}) & k_F(n) \text{ (fm}^{-1}) & E_1(p) \text{ (MeV)} & E_1(n) \text{ (MeV)} & E_1(p) + E_1(n) \text{ (MeV)} \\
^{208}\text{Pb} & 1.26 & 1.45 & 16.97 & 8.93 & 25.9 \\
^{90}\text{Zr} & 1.31 & 1.41 & 15.56 & 11.37 & 26.93 \\
^{40}\text{Ca} & 1.36 & 1.36 & 13.73 & 13.73 & 27.46 \\
\end{array}
$$

**TABLE III:** The Fermi momenta and the $l = 1$ collective excitation energies of protons and neutrons for different nuclei with the effective nucleon mass $M_N^* = 0.742M_N$. The corresponding experimental values for the nuclear isoscalar and isovector giant dipole resonances are labeled as $E_{\text{exp}}^S$ and $E_{\text{exp}}^V$, where the experimental value for the nuclear isoscalar giant dipole resonances are taken from Ref. [17], the experimental value for the nuclear isovector giant dipole resonances from Ref. [13].
C. Nuclear giant quadrupole resonances

The calculation results and the corresponding experimental centroid energies of the giant quadrupole resonances of different nuclei are listed in Table IV. The experimental value of the isovector giant quadrupole resonance energy is just twice of the isoscalar giant quadrupole resonance energy for $^{208}Pb$, it manifests my prediction on the relation between the nuclear isovector giant resonance and the corresponding isoscalar giant resonance in Eq. (28) is correct. Actually, the experimental values for the other nuclei, and for the monopole giant resonance are also fitted with the relation in Eq. (28) approximately. In Table IV Since the average neutron density is larger than the saturation density of the nuclear matter, the collective excitation energies of the nuclei $^{208}Pb$ and $^{90}Zr$ for $l = 2$ are smaller than $10\text{MeV}$, they are corresponding to the low-lying excitation states in the heavy nuclei, which are in the range of $2 - 6\text{MeV}$ for $^{208}Pb$. For light nuclei, such as $^{40}Ca$ and $^{16}O$, the collective excitation energies are less than the corresponding experimental values. We can choose the effective nucleon mass as $M_N^* = 0.69 M_N$, and obtain the values of $E_2(p) = E_2(n) = 18.54 \text{MeV}$ and $E_2(p) + E_2(n) = 37.08 \text{MeV}$, which are close to the experimental values of the isoscalar and isovector giant quadrupole resonances of $^{40}Ca$, respectively.

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
l = 2 & k_F(p) (\text{fm}^{-1}) & k_F(n) (\text{fm}^{-1}) & E_2(p) (\text{MeV}) & E_2(n) (\text{MeV}) & E_2(p) + E_2(n) (\text{MeV}) \\
\hline
^{208}Pb & 1.26 & 1.45 & 15.02 & 5.84 & 20.86 \\
\hline
^{90}Zr & 1.31 & 1.41 & 13.16 & 8.27 & 21.43 \\
\hline
^{40}Ca & 1.36 & 1.36 & 10.92 & 10.92 & 21.84 \\
\hline
^{16}O & 1.36 & 1.36 & 10.92 & 10.92 & 21.84 \\
\hline
\end{array}
\]

TABLE IV: The Fermi momenta and the collective excitation energies of the mixture of different nuclear isovector giant quadrupole resonances are taken from Ref. [9], the experimental value for the nuclear isovector giant quadrupole resonance energy of $^{208}Pb$ is approximately. In Table IV, Since the average neutron density is larger than the saturation density of the nuclear matter, the collective excitation energies of the nuclei $^{208}Pb$ and $^{90}Zr$ for $l = 2$ are smaller than $10\text{MeV}$, they are corresponding to the low-lying excitation states in the heavy nuclei, which are in the range of $2 - 6\text{MeV}$ for $^{208}Pb$. For light nuclei, such as $^{40}Ca$ and $^{16}O$, the collective excitation energies are less than the corresponding experimental values. We can choose the effective nucleon mass as $M_N^* = 0.69 M_N$, and obtain the values of $E_2(p) = E_2(n) = 18.54 \text{MeV}$ and $E_2(p) + E_2(n) = 37.08 \text{MeV}$, which are close to the experimental values of the isoscalar and isovector giant quadrupole resonances of $^{40}Ca$, respectively.

IV. SUMMARY

The method on Landau Fermi liquid theory in Ref. [6] is extended to the 3-dimensional Fermion system with the spin considered, and then by using the effective Lagrangian of the linear $\sigma - \omega$ model, the Fermi liquid function is
obtained and the nuclear collective excitation energies of different values of \( l \) are calculated within the framework of Landau Fermi liquid theory. The results show the nuclear collective excitation energies decrease with the effective nucleon mass and the Fermi momentum increasing. When the effective nucleon mass takes the value of \( 0.742M_N \), and the Fermi momentum \( k_f = 1.36\text{fm}^{-1} \), the calculated collective excitation energies of the nuclear matter for different values of \( l \) are fitted with the experimental values very well. In addition, the isoscalar and isovector giant resonances of spherical nuclei are studied within Landau Fermi liquid theory. We find the centroid energies of the isoscalar giant resonances just correspond to the positive isolated energy levels of the nuclear collective excitation with different values of \( l \), respectively, while the isovector giant resonances except \( l = 1 \) correspond to the modes that protons(neutrons) are in the creation state of the collective excitation and neutrons(protons) are in the annihilation state of the same \( l \). Furthermore, some mixtures of the collective excitation states with different values of \( l \), which have higher energies, are predicted.

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