Simulation of an aperture-based antihydrogen gravity experiment

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A Monte Carlo simulation is presented of an experiment that could potentially determine whether antihydrogen accelerates vertically up or down as a result of earth’s gravity. The experiment would rely on methods developed by existing antihydrogen research collaborations and would employ a Penning trap for the production of antihydrogen within a uniform magnetic field. The axis of symmetry of the cylindrical trap wall would be oriented horizontally, and an axisymmetric aperture (with an inner radius that is smaller than the cylindrical trap wall radius) would be present a short distance away from the antihydrogen production region. Antihydrogen annihilations that occur along the cylindrical trap wall would be detected by the experiment. The distribution of annihilations along the wall would vary near the aperture, because some antihydrogen that would otherwise annihilate at the wall would instead annihilate on the aperture. That is, a shadow region forms behind the aperture, and the distribution of annihilations near the boundary of the shadow region is not azimuthally symmetric when the effect of gravity is significant. The Monte Carlo simulation is used together with analytical modeling to determine conditions under which the annihilation distribution would indicate the direction of the acceleration of antihydrogen due to gravity.

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I. INTRODUCTION

No direct measurements have been reported that indicate the direction of free-fall gravitational acceleration of antimatter (e.g., antiprotons, positrons, antihydrogen). A gravity experiment using antimatter may be possible by inserting an aperture within a Penning trap that is used to produce antihydrogen. A Penning trap can confine a non-neutral plasma by producing an electric potential well along a magnetic field. A nested Penning trap can confine oppositely signed plasma species by producing oppositely signed nested electric potential wells along a magnetic field. Nested Penning traps have been used to trap antiprotons and positrons with overlapping confinement volumes, such that antihydrogen production occurs, and antihydrogen trapping is possible. The production of low energy antihydrogen using a method that requires positronium has also been demonstrated. Antihydrogen research may ultimately provide experimental tests of CPT (charge conjugation, parity, time reversal) and gravity symmetries.

There exist numerous conflicting issues associated with using nested Penning traps to produce antiatoms with sufficiently low energies and also in sufficient numbers for conducting high precision CPT and gravity measurements. Many of the issues are being addressed, as indicated by recent advances. A Monte Carlo simulation of a gravity experiment is presented here. The simulation was developed with the assumption that the initial objective of the experiment would be to determine whether antihydrogen accelerates vertically up or down as a result of earth’s gravity. The experiment would employ a Penning trap oriented horizontally for producing antihydrogen. It is assumed here that a detector that can distinguish between antihydrogen annihilations and cosmic rays would be used. Such a detector is employed by the ALPHA collaboration. A modification required for an existing experiment would be the axial insertion of a removable aperture.
A conceptual illustration of an experimental setup is shown in Fig. 1. Two straight-line trajectories are shown that pass infinitesimally close to the edge of a circular aperture that is coaxial with the axis of symmetry of the axisymmetric configuration. Each straight-line trajectory intersects the cylindrical wall. Straight-line trajectories serve to define a “shadow” region along the cylindrical wall. The trajectory of an antiatom will intersect the wall at a location within the shadow region only if the trajectory is affected by gravity, provided that all other effects that can influence the trajectory of an antiatom are negligible. Even if all other effects are not negligible, gravity can be the sole cause of an asymmetry in the azimuthal annihilation distribution that occurs at the wall within the shadow region, provided that all other effects that can influence the trajectory of an antiatom are axisymmetric. Effects that might influence the trajectory of an antiatom in an asymmetric way include electric and magnetic field gradients associated with misaligned electrodes and coils, respectively.

II. ANNIHILATION WITHIN THE SHADOW REGION AT THE TOP OF THE WALL

Suppose that an antiatom in the experiment experiences a gravitational acceleration that is equal and opposite to that of matter, and that all other effects that can influence the trajectory of the antiatom are negligible. Suppose further that the antiatom passes through the coordinate origin shown in Fig. 1, and that motion only occurs along a plane defined by horizontal and vertical vectors. The coordinate origin is considered for now to be the initial position for describing the motion of the antiatom. A Cartesian coordinate system with coordinates \((x, y, z)\) and associated unit vectors \((\hat{i}, \hat{j}, \hat{k})\) is defined such that the \(z\) axis coincides with the axis of symmetry of the configuration. The antiatom’s acceleration due to gravity is written as \(g = g \hat{j}\), where \(g = |g|\) is the magnitude of gravitational acceleration for matter. The horizontal vector \(\hat{k}\) and vertical vector \(g\) define the plane in which motion is considered to occur. Two equations of motion are \(y = v_0y t + \frac{1}{2} gt^2\) and \(z = v_0z t\), where \(y\) is the vertical displacement (relative to the coordinate origin) that is experienced by an antiatom that travels a horizontal distance \(z\) during a time \(t\). Combining these two equations by eliminating time gives \(y = (v_0y/v_0z) + g z^2/(2v_0^2 z_0^2)\). The velocity components when the antiatom passes through the coordinate origin are written as \(v_0y = v_0\sin \theta_0\) and \(v_0z = v_0\cos \theta_0 = \sqrt{2K_0/m} \cos \theta_0\), where \(v_0\) is the initial speed of the antiatom, and \(\theta_0\) is an angle that characterizes the initial direction of motion of the antiatom. Also, for the last equality, the initial speed \(v_0\) is written in terms of both the initial kinetic energy \(K_0\) and the antiatom’s mass \(m\). Substitution and rearrangement yields

\[
K_0 = \frac{mgz^2}{4(y - z \tan \theta_0) \cos^2 \theta_0}.
\]
There exists a range of initial kinetic energy $K_0$ and angle $\theta_0$ values that result in a trajectory that intersects the shadow region at the top of the wall. A lower bound on the initial kinetic energy corresponds to a trajectory that passes infinitesimally close to the top edge of the circular aperture. The vertical and horizontal displacements at the time such a trajectory reaches a location infinitesimally close to the edge of the aperture are $y = a$ and $z = aL/R$, where $a$ is the inner radius of the aperture, $L$ is the axial distance between the coordinate origin and the far edge of the shadow region, and $R$ is the inner radius of the cylindrical wall. Substitution into Eq. (1) provides a minimum initial kinetic energy for reaching the top shadow region:

$$\begin{align*}
K_{0,\min} &= \frac{mgL^2}{4(R/a)(R - L \tan \theta_0)\cos^2 \theta_0}. \\
\end{align*}$$  \hspace{1cm} (2)

An upper bound on the initial kinetic energy corresponds to a trajectory that intersects the wall at the far edge of the shadow region (i.e., at $y = R$ and $z = L$). For reasons to be discussed below, it is convenient to use $z = L - \xi$ in place of $z = L$, and to use $\xi = 0$ for now. Substitution into Eq. (1) provides a maximum initial kinetic energy for a trajectory that intersects the wall within the shadow region:

$$K_{0,\max} = \frac{mg(L - \xi)^2}{4[R - (L - \xi)\tan \theta_0] \cos^2 \theta_0}. \hspace{1cm} (3)$$

For analytical modeling, it is convenient to neglect the possibility that a trajectory can reach the top shadow region with a negative value for $\theta_0$, so that $\theta_0$ can be used later as a spherical coordinate in velocity space. Thus, a lower bound on the initial direction-of-motion angle is taken to be

$$\theta_{0,\min} = 0. \hspace{1cm} (4)$$

An upper bound on the initial direction-of-motion angle corresponds to a trajectory that passes infinitesimally close to the top edge of the circular aperture and also intersects the wall at the far edge of the shadow region. Combining Eqs. (2) and (3) by setting $K_{0,\min} = K_{0,\max}$ and rearranging provides an equation for the maximum initial angle:

$$\theta_{0,\max} = \arctan \left[ R \left( \frac{1}{L} + \frac{1}{L - \xi} - \frac{1}{L - [\xi/(1 - a/R)]} \right) \right]. \hspace{1cm} (5)$$

The value for $\xi$ must be within the range $0 \leq \xi < L(1 - \sqrt{a/R})$ to have $\theta_{0,\max} > \theta_{0,\min} = 0$

### III. AZIMUTHALLY ASYMMETRIC ANNihilation DISTRIBUTION

Under the conditions considered and restrictions imposed in Sec. II, it is not possible for the trajectory of an antiatom to intersect the shadow region at the bottom of the wall. Conversely, it is not possible for an antiatom’s trajectory to intersect the shadow region at the top of the wall, if the antiatom experiences gravitational acceleration that is the same as that for matter. An azimuthal annihilation distribution at the wall that is maximally asymmetric is considered to be one associated with the maximum probability for the initial antiatom kinetic energy and angle values to satisfy $K_{0,\min} < K_0 < K_{0,\max}$ and $\theta_{0,\min} < \theta_0 < \theta_{0,\max}$, where $\theta_0$ is now defined as a spherical coordinate in velocity space. A spherical coordinate system in velocity space is defined with coordinates that have the following ranges: $0 \leq v_0 < \infty$, $0 \leq \theta_0 \leq \pi$, and $0 \leq \phi_0 < 2\pi$. The probability for an antiatom’s initial motion to satisfy the conditions $K_{0,\min} < K_0 < K_{0,\max}$ and $\theta_{0,\min} < \theta_0 < \theta_{0,\max}$ is evaluated by assuming that antiatoms initially have a Maxwellian velocity distribution. The assumption is consistent with producing antiatoms within a thermalized antihydrogen plasma, provided that the difference between the positron and antiproton densities is such that the azimuthal $E \times B$ drift speed is much smaller than the antiproton thermal speed throughout the plasma. A Maxwellian velocity distribution is written in terms of Cartesian coordinates as

$$f_0(v_{0x}, v_{0y}, v_{0z}) = f_0 \exp \left[ -\frac{m(v_{0x}^2 + v_{0y}^2 + v_{0z}^2)}{2kT} \right]. \hspace{1cm} (6)$$
IV. SIMULATION OF AN ANNihilation DISTRIBUTION

Here, \(v_{0x}, v_{0y}, v_{0z}\) are initial Cartesian velocity components, \(f_0\) is a normalization constant, \(k\) is Boltzmann’s constant, and \(T\) is temperature. The velocity-space probability density function in spherical coordinates is

\[
f_i(v_0, \theta_0) = f_0 \sin(\theta_0) v_0^3 \exp \left(-\frac{mv_0^2}{2kT} \right),
\]

where the transformation Jacobian has been included within the function. This velocity distribution is azimuthally symmetric and, therefore, it has no dependence on \(\phi_0\). A change of integration variables is carried out to write the probability density in terms of \(K_0 = \frac{1}{2}mv_0^2\) and \(\theta_0\). The normalized probability density is

\[
f(K_0, \theta_0) = \frac{1}{\sqrt{\pi(kT)}} \sin(\theta_0) \sqrt{K_0} \exp \left(-\frac{K_0}{kT} \right).
\]

The probability \(P\) for an antiatom to satisfy the conditions \(K_{0,min} < K_0 < K_{0,max}\) and \(\theta_{0,min} < \theta_0 < \theta_{0,max}\)

\[
P = \int_{\theta_{0,min}}^{\theta_{0,max}} \int_{K_{0,min}(\theta_0)}^{K_{0,max}(\theta_0)} f(K_0, \theta_0) dK_0 d\theta_0.
\]

The inner integral can be evaluated analytically. The indefinite integral is

\[
\int f(K_0, \theta_0) dK_0 = \left[\frac{1}{2} \text{erf} \left( \sqrt{\frac{K_0}{kT}} - \sqrt{\frac{K_0}{\pi kT}} \exp \left(-\frac{K_0}{kT} \right) \right) \right] \sin(\theta_0),
\]

where \(\text{erf}\) is the error function. The outer integral is evaluated numerically. An azimuthal annihilation distribution at the wall that is maximally asymmetric is considered to be one associated with a maximized value for \(P\). Sample calculations are used to illustrate the use of Eq. (9). The inner radius of Penning trap electrodes in the ALPHA apparatus is 22.3 mm,\(^{12}\) and \(R = 22.3\) mm is used. A temperature of \(T = 4\) K is assumed, considering that antiprotons have been cooled by the ATRAP collaboration to a temperature of 3.5 K.\(^{22}\) The value \(a = R/2\) is held fixed, and Eq. (9) is evaluated for various values of \(L\). It is found that \(P\) has a maximum value of \(8 \times 10^{-7}\) for \(L = 200\) mm. Next, the value \(L = 200\) mm is held fixed, and Eq. (9) is evaluated for various values of \(a\). It is found that \(P\) decreases linearly with \(a\), and \(P\) has a maximum value of \(1.6 \times 10^{-6}\) in the limit \(a \to 0\). Although \(P\) can be increased by up to a factor of 2 by using an aperture radius that is smaller than \(a = R/2\), it must be emphasized that the scaling applies for the present model for which the axial location of the aperture is \(z_a = aL.R\). An aperture with a smaller radius is necessarily closer to the antihydrogen source for fixed \(L\) in the present model. It may be difficult to operate a Penning trap with a small-radius aperture in the vicinity of a confined plasma.

IV. SIMULATION OF AN ANNihilation DISTRIBUTION

A Monte Carlo simulation is used to evaluate the distribution of annihilations that would occur at the wall. The conditions considered and restrictions imposed in Sec. II are employed, except that motion is no longer restricted to occur along a plane. Each antiatom’s trajectory is considered to start at the coordinate origin,

\[
x_0 = 0,
\]

\[
y_0 = 0,
\]

\[
z_0 = 0.
\]

The antiatom’s initial Cartesian velocity components are

\[
v_{0x} = v_0 \sin \theta_0 \cos \phi_0,
\]

\[
v_{0y} = v_0 \sin \theta_0 \sin \phi_0,
\]

\[
v_{0z} = v_0 \cos \theta_0.
\]
The velocity-space probability density is sampled to obtain values for \( v_0, \theta_0, \) and \( \phi_0 \). Equation (7) has no dependence on \( \phi_0 \), and \( \phi_0 \) is sampled using

\[
\phi_0 = 2\pi R_\phi.
\]

(13)

Here and hereafter, \( R \) with a subscript attached to it represents a random number that is equally likely to have any value between 0 and 1. Different subscripts attached to \( R \) are employed to distinguish between different random values.

The portion of the probability density that involves \( \theta_0 \) is separable and is proportional to \( \sin \theta_0 \). The velocity-space probability density is sampled to obtain values for \( \theta_0 \) and \( \phi_0 \). Equation (7) has no dependence on \( \phi_0 \), and \( \phi_0 \) is sampled using

\[
\phi_0 = 2\pi R_\phi.
\]

(13)

Here and hereafter, \( R \) with a subscript attached to it represents a random number that is equally likely to have any value between 0 and 1. Different subscripts attached to \( R \) are employed to distinguish between different random values.

The portion of the probability density that involves \( \theta_0 \) is separable and is proportional to \( \sin \theta_0 \). To reduce computation time, \( \theta_0 \) is sampled over the range \( 0 < \theta_0 \leq \theta_u \), where \( \theta_u \) represents an upper limit on sampled values. The upper limit is chosen to be the angle associated with straight-line trajectories that pass infinitesimally close to the aperture: \( \theta_u = \arctan(R/L) \). The sampling expression is obtained by evaluating

\[
R_\theta = \int_0^{\theta_0} \frac{\sin \theta d\theta}{\int_0^{\theta_0} \sin \theta d\theta}.
\]

(14)

Inverting the result gives

\[
\theta_0 = \arccos \left( 1 - R_\theta + \frac{R_\theta}{\sqrt{1 + R^2/L^2}} \right).
\]

(15)

The fraction of the solid angle that is sampled is

\[
F_\Omega = \int_0^{\theta_0} \frac{\sin \theta d\theta}{\int_0^{\theta_0} \sin \theta d\theta} = \frac{1}{2} \left( 1 - \frac{1}{\sqrt{1 + R^2/L^2}} \right).
\]

(16)

The initial speed is calculated as \( v_0 = \sqrt{v_x^2 + v_y^2 + v_z^2} \), with each of \( v_x, v_y, \) and \( v_z \) sampled using a conventional algorithm for sampling a Gaussian (Maxwellian) distribution, with a standard deviation given by the thermal speed \( v_\text{th} = \sqrt{kT/m} \) and a zero mean. Assuming an upward acceleration given by \( g = g \hat{j} \), the equations of motion are \( x(t) = x_0 + v_0 x t, y(t) = y_0 + v_0 y t + \frac{1}{2} g t^2, \) and \( z(t) = z_0 + v_0 z t \). The locations of annihilations that would occur at the wall are only evaluated for antiatoms that would pass through the aperture. The time \( \tau_1 \) that it would take for an antiatom to reach the plane of the aperture is given by \( \tau_1 = [(aL/R) - z_0]/v_0 \). A condition that must be satisfied for an antiatom to pass through the aperture is \( \sqrt{x(\tau_1)^2 + y(\tau_1)^2} < a \). An antiatom that passes through the aperture would reach the wall at time \( \tau_2 \) given by \( \sqrt{x(\tau_2)^2 + y(\tau_2)^2} = R \). The value of \( \tau_2 \) must be both positive and real. The Cartesian coordinates of a trajectory at the time it intersects the wall, \( x(\tau_2), y(\tau_2), \) and \( z(\tau_2) \), are determined and then converted to cylindrical coordinates, \( \rho(\tau_2) = R, \phi(\tau_2), \) and \( z(\tau_2) \).

Figure 2 shows scatter plots of simulated annihilation coordinates within the shadow region for \( R = 0.0223 \) m, \( a = R/2 \), and \( T = 4 \) K. The azimuthal asymmetry that occurs in the plotted values for the annihilation coordinate \( \phi(\tau_2) \) within the shadow region is caused by gravitational acceleration of the antiatoms. To obtain the results shown in Fig. 2(a), \( L = 0.2 \) m was used, and \( N_s = 930,000 \) annihilations were simulated. Equation (16) gives the fraction of the solid angle that was sampled to be \( F_\Omega = 3.1 \times 10^{-3} \). The total number of antiatoms that would be emitted into the entire 4\( \pi \) solid angle is thus \( N = N_s/F_\Omega = 3.0 \times 10^8 \).

The ALPHA collaboration has reported producing about 6000 antiatoms for each bunch of antiprotons that is provided by the CERN Antiproton Decelerator (AD).\(^{12}\) The AD provides bunches approximately every 100 seconds. Full utilization of the bunches is considered here as an upper limit, in which case the antihydrogen production rate would be about 60 antiatoms per second. It would take 58 days of runtime at an average rate of 60 antiatoms per second to produce a total of \( N = 3.0 \times 10^8 \) antiatoms. An analysis reported by the ALPHA collaboration in Ref. 12 indicates that antihydrogen production occurred within a thermalized antihydrogen plasma having a temperature \( T \approx 54 \) K. An antiatom is born with a kinetic energy approximately equal to that of its antiproton. In the work presented here, antiatoms are considered to be born with a Maxwellian velocity distribution that is associated with a \( T = 4 \) K temperature. A substantial increase in antiatom production rate
FIG. 2. Scatter plots of the cylindrical coordinates of simulated annihilations at a cylindrical wall located at \( \rho = R \). The line at \( \phi = 0 \) in each plot divides the upper and lower halves of the cylindrical wall. The top (highest point) of the wall is located at \( \phi = \pi/2 \), and the bottom (lowest point) of the wall is located at \( \phi = -\pi/2 \). Parameter values used for the three simulations are \( R = 0.0223 \) m, \( a = R/2 \), \( T = 4 \) K, and \( N = 3.0 \times 10^8 \). The shadow region is located at \( L_2 < z < L \), where \( L = 0.2 \) m for (a), \( L = 2.7 \) m for (b), and \( L = 0.5 \) m for (c).
The base case (i.e., with $za$ in the ALPHA apparatus is about 5 mm). Then be used. The axial resolution for diagnosing antihydrogen annihilations on the trap electrodes of the shadow region, $N_a < z < L - \zeta$, instead of within the entire shadow region, $z_a < z < L$. A value for $\zeta$ equal to the axial resolution for diagnosing antihydrogen annihilations can then be used. The axial resolution for diagnosing antihydrogen annihilations on the trap electrodes in the ALPHA apparatus is about 5 mm.

Let $N_c$ denote the number of simulated annihilations that occur within a restricted portion of the shadow region, $z_a < z < L - \zeta$. One simulation (e.g., with a given set of values for $R$, $a$, $T$, $L$, and $N_c$ or $N_p$) provides a single value for $N_c$. Twenty (20) simulations are carried out for each set of parameter values, and the average $\langle N_c \rangle$ and standard deviation $SD$ of the values obtained for $N_c$ are reported in Table I. The correlation between statistical results for $N_c$ and calculated values for $P$ is shown by providing values for the quotients $\langle N_c \rangle / P$ and $SD / P$. An average of 1.5 ± 1.0 simulated annihilations occur at distances larger than 5 mm from the edge of the shadow region for the set of parameters used for Fig. 2(a) ($R = 0.0223$ m, $a = R/2$, $T = 4$ K, $L = 0.2$ m, $N_c = 930,000$). It may be possible to increase the resolution of detecting antihydrogen by inserting particle detectors within the shadow region. Nevertheless, suppose that it is necessary for an annihilation to occur within a minimum of 5 mm from the edge of the shadow region to be identified as having occurred within the shadow region. With $\zeta = 0.005$ m, $R = 0.0223$ m, $a = R/2$, and $T = 4$ K, Eq. (9) is evaluated numerically for various values of $L$. It is found that $P$ has a maximum value of $5.4 \times 10^{-7}$ for $L = 2.7$ m. Figure 2(b) shows a scatter plot of simulated annihilation coordinates within the shadow region. The results are obtained with $N_c = 5200$ and $F_2 = 1.7 \times 10^{-3}$ so that $N = N_c / F_2 = 3.0 \times 10^8$ is the same as for the results in Fig. 2(a). An average of $63 \pm 10$ simulated annihilations occur within the range $z_a < z < L - \zeta$ (i.e., within the shadow region at distances larger than 5 mm from the edge of the shadow region) for the set of parameters used for Fig. 2(b). However, a tradeoff exists associated with the difficulty of employing a much longer drift volume experimentally. A “base case” is defined here by choosing the value of $L$ such that the far edge of the shadow region falls within the extent of the uniform magnetic field within the ALPHA apparatus. The base case parameter values are $R = 0.0223$ m, $a = R/2$, $T = 4$ K, $L = 0.5$ m, $N_c = 150,000$, and $N = 3.0 \times 10^6$. Figure 2(c) shows a simulated annihilation distribution. An average of $8.1 \pm 1.4$ simulated annihilations occur within the range $z_a < z < L - \zeta$ for the base-case set of parameters used for Fig. 2(c).

It should be noted that there exists a strong dependence on temperature. The number of simulated annihilations within the range $z_a < z < L - \zeta$ is found to change from an average of $8.1 \pm 1.4$ for the base case (i.e., with $T = 4$ K), to $0.20 \pm 0.41$ for $T = 40$ K, and to $250 \pm 20$ for $T = 0.4$ K.
V. EFFECT OF A SPATIAL DISTRIBUTION OF INITIAL ANTIATOM POSITIONS

The effect of a spatial distribution of antiatom starting positions is evaluated in two ways. First, a distribution of starting positions of finite length \( L_p \) but of infinitesimal radial width is considered. The base case is modified such that a starting position for each antiatom is randomly selected along the \( z \) axis in the range \( -L_p < z < 0 \). It is found that as \( L_p \) increases, the number of simulated annihilations that occur within the range \( z_a < z < L - \zeta \) decreases. Sample values are provided for two sets of simulations that employed base case parameter values, except that each antiatom’s trajectory starts at

\[
z_0 = -R_L L_p. \tag{17}
\]

The number of simulated annihilations within the range \( z_a < z < L - \zeta \) is found to decrease from an average of 8.1 \( \pm \) 1.4 for the base case (i.e., with \( L_p = 0 \)), to 5.7 \( \pm \) 2.3 for \( L_p = 1 \) cm, and to 3.5 \( \pm \) 1.7 for \( L_p = 4 \) cm. It should be noted that the ALPHA collaboration reported in Ref. \textcolor{red}{12} that antihydrogen production occurred within a positron plasma having a 0.8 mm radius and a \( 5 \times 10^{12} \) m\(^{-3} \) density, and containing \( 10^6 \) positrons. If the positron density is approximated as radially uniform, then the plasma would have a 1 cm length.

A distribution of starting positions having both a finite length and a finite width is now considered. The base case is modified by randomly selecting a starting position within a cylindrical volume of length \( L_p \) and radius \( \rho_p \). The sampling expressions used for the initial Cartesian coordinates of an antiatom are

\[
\begin{align*}
x_0 &= \rho_p \sqrt{R_p} \cos (2\pi R_p), \\
y_0 &= \rho_p \sqrt{R_p} \sin (2\pi R_p), \\
z_0 &= -R_L L_p - \frac{\rho_p L}{R}. \tag{18}
\end{align*}
\]

These expressions are arrived at as follows: Each antiatom’s initial radial coordinate is assumed to be equally likely to be located anywhere within the cylindrical volume, which is radially centered at \( \rho = 0 \). Associated sampling expressions for radial and azimuthal cylindrical coordinates are \( \rho_0 = \rho_p \sqrt{R_p} \) and \( \Phi_0 = 2\pi R_p \). Sampling expressions for Cartesian coordinates are \( x_0 = \rho_0 \cos \Phi_0 \) and \( y_0 = \rho_0 \sin \Phi_0 \). The cylindrical volume is positioned as illustrated in Fig. 1, such that the extent of the shadow region \( z_a < z < L \) does not change. The front edge of the cylindrical volume is displaced away from the origin by an amount \( \Delta z = \rho_p \tan \theta_u \), where \( \theta_u = \arctan(R/L) \) is the angle associated with straight-line trajectories that pass through the origin and pass infinitesimally close to the aperture. Thus, \( \Delta z = \rho_p L/R \). Each antiatom’s starting position is randomly selected in the range \( -L_p - \Delta z < z < -\Delta z \) by using the sampling expression \( z_0 = -R_L L_p - \Delta z \). With \( L_p = 1 \) cm, the average number of simulated annihilations within the range \( z_a < z < L - \zeta \) is found to decrease from 5.7 \( \pm \) 2.3 for \( \rho_p = 0 \) to 3.1 \( \pm \) 1.7 for \( \rho_p = 0.4 \) mm. The average value and standard deviation of \( N_\zeta \) were also evaluated as 3.6 \( \pm \) 1.9 for \( \rho_p = 0.8 \) mm. The increase is attributed to statistical uncertainty. It should be noted that an antiproton plasma with a 0.4 mm radius was reported in Ref. \textcolor{red}{12} as being used for producing antihydrogen.

VI. CONCLUSION

A Monte Carlo simulation has been developed of an aperture-based antihydrogen gravity experiment, assuming that the magnitude of gravitational acceleration for antimatter would be the same as that for matter. Simulations indicate that, for parameters similar to those reported by existing antihydrogen research collaborations, about 300 million antiatoms would need to be produced (at a minimum) for the associated annihilation distribution to begin to indicate the direction of the acceleration of antihydrogen due to gravity. The present study regarding an aperture-based antihydrogen gravity measurement also indicates that it is advantageous: (1) to use a small antihydrogen production region (2) to employ a long drift volume, (3) to position a small-radius aperture close to the antihydrogen production region, and (4) to produce antihydrogen at low temperature.
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