Contribution of higher meson resonances to the electromagnetic $\pi$-meson mass difference

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Abstract

Modifications of the DGMLY relation for calculation of electromagnetic $\pi$-meson mass difference based on the Chiral Symmetry Restoration phenomenon at high energies as well as the Operator Product Expansion of quark densities for vector ($\rho$) and axial-vector ($a_1$) meson fields difference are proposed. In the calculations higher meson resonances in vector and axial-vector channels are taken into account. It is shown that the inclusion of the first $\rho$ and $a_1$ radial excitations improves the results for electromagnetic $\pi$-meson mass difference as compared with the previous ones. Estimations on the electromagnetic $\rho$ and $a_1$-meson decay constants and the constant $L_{10}$ of effective chiral Lagrangian are obtained from the generalized Weinberg sum rules.

1 Introduction

It is well known that the correlation functions of vector (V) and axial-vector (A) meson fields are connected with some experimentally observed characteristics of pseudoscalar mesons, in particular, with the electromagnetic $\pi$-meson mass difference $\Delta m_\pi|_{em}$ [1]. Recent experimental data of ALEPH [2] and OPAL [3] collaborations on hadronic $\tau$-decays ($\tau \rightarrow (V, A)\nu_{\tau}$, $\tau \rightarrow \pi \nu_{\tau}$) point out that one should take into account more degrees of freedom in pseudoscalar channels for the (VV)-(AA) - correlators in order to check both perturbative and non-perturbative QCD parameters. In particular, the authors of [4] fulfilled an analysis of ALEPH [2] experimental data to measure the correlator difference $\Pi^V - \Pi^A$ behaviour from the decay $\tau \rightarrow (V, A)\nu_{\tau}$. This correlator difference happens to vanish practically already at intermediate energies $\lesssim 3$ GeV. A small contribution of the first radial excitations of vector mesons to the $\Pi^V - \Pi^A$ can be seen experimentally, but a contribution of following ones is nearly negligible.

As it follows from [1,5] the bulk of pion mass difference effect has, in general, an electromagnetic origin. There are different approaches to calculate an electromagnetic contribution to $\Delta m_\pi|_{em}$. [5-13]. One of simple ways to calculate this quantity in the chiral limit and to the lowest order in the electromagnetic interactions was proposed in [1], where it was calculated in the framework of Current Algebra with the help of Weinberg sum rules [16], the latter being saturated by one vector ($\rho(770)$) and one axial-vector ($a_1(1260)$) mesons. However, from PDG [14] (see also [18]) it is known that there is a series of heavier meson states with the same quantum numbers — $\rho(1450), \rho(2150)$, which represent the radial excitations of $\rho(770)$-meson in a language of potential quark models. In the axial-vector channel one might have radial excitations of $a_1(1260)$-resonance as well.

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However, their mass spectrum is not yet accurately identified [19–21]. Thus, the problem arises: what is a contribution of these higher meson resonances to the electromagnetic $\pi$-meson mass difference $\Delta m_{\pi|em}$?

In this work we investigate the contribution of the first radial (V,A) excitations to the electromagnetic $\pi$-meson mass difference. The analysis is based on modifications of the relation [1] with an additional Weinberg sum rule, following from the requirement of Chiral Symmetry Restoration at high energies [22] and the Operator Product Expansion (OPE) [23], and without one. It should be stressed that our goal is not to calculate $\Delta m_{\pi|em}$ strictly (this quantity is known from experiment) but we are going to employ this difference to calculate some physical parameters of hadron resonances as well as to investigate a contribution of higher meson resonances in saturation of physical observables.

The paper is organized as follows. In the section 2 we remind the idea of derivation of the DGMLY relation for electromagnetic $\pi$-meson mass difference and present a scheme of obtaining the Weinberg sum rules from the requirement of chiral symmetry restoration at high energies and OPE. The section 3 is devoted to an extension of the classical formula [1] by taking into account of higher meson (V,A) resonances. Here we calculate also certain electromagnetic decay constants of $\rho$ and $a_1$ mesons and the constant $L_{10}$ of effective chiral Lagrangian [24], which can be determined, in particular, from the decay $\pi \to e\nu\gamma$.

The obtained results are discussed in the section 4.

## 2 The electromagnetic $\pi$-meson mass difference and the Weinberg sum rules

The electromagnetic $\pi$-meson mass difference $\Delta m_{\pi|em}$ is defined in the chiral limit $m_{\text{cur}} = 0$ by the known relation [1, 8, 11, 25]:

$$\left( m_{\pi^+}^2 - m_{\pi^0}^2 \right)|_{em} = \frac{2e^2C}{f_\pi^2},$$

where in (1) the constant $C$ is:

$$C = -\frac{1}{8\pi^2} \cdot \frac{3}{4} \int_0^\infty dp^2 \cdot p^2 \left[ \Pi^V(p^2) - \Pi^A(p^2) \right],$$

and

$$\Pi^C(p^2) = \int d^4x \exp(ipx) \langle T(\bar{q}(x)\Gamma q(x)\bar{q}(0)\Gamma q(0)) \rangle,$$

represent the two-point correlators of vector and axial-vector quark densities in Euclidean space. In the large-$N_c$ limit (planar limit) the correlators of colour singlet quark densities are saturated by narrow meson resonances only [26, 27]:

$$\Pi^C(p^2)|_{\text{planar}} = \sum_n \frac{Z_n^C}{p^2 + m_{C,n}^2}.$$
On the other hand, owing to the asymptotic freedom of QCD, a high energy asymptotic of these correlators is described by the perturbation theory and Operator Product Expansion \[23\]. To the lowest order its behaviour is given by:

\[ 
\Pi^C(p^2)|_{p^2 \to \infty} \sim p^2 \ln \frac{p^2}{\mu^2},
\]

where \(\mu\) is a normalization point of fermion currents.

From the comparison of (4) and (5) one can infer that an infinite set of resonances with equal quantum numbers must exist in order to reproduce the perturbative asymptotic (5). In the chiral limit and in the large-\(N_c\) approach it can be shown that in VA-channels \[22, 28\]:

\[ 
\left(\Pi^V(p^2) - \Pi^A(p^2)\right)|_{p^2 \to \infty} \equiv \frac{\Delta_{VA}}{p^6} + \mathcal{O}\left(\frac{1}{p^8}\right), \quad \Delta_{VA} \simeq -16\pi\alpha_s <\bar{q}q>^2,
\]

where

\[ 
\Pi_{\mu\nu}^{V,A}(p^2) \equiv (-\delta_{\mu\nu}p^2 + p_\mu p_\nu)\Pi^{V,A}(p^2).
\]

As it follows from the relation (6), due to a rapid convergence of the difference \(\Pi^A - \Pi^V\) with increasing \(p^2\) one may expect the chiral symmetry restoration at high energies, with the difference \(\Pi^A - \Pi^V\) being approximated by an order parameter of chiral symmetry breaking (CSB) in QCD i.e. the quark condensate \(<\bar{q}q>\). The asymptotics (6) represents the so called condition of chiral symmetry restoration in VA-channels. It leads to a certain set of sum rules (see, e.g. \[22\]) for spectral characteristics of VA-mesons. Namely, expanding (6) in powers of \(p^2\) one arrives at:

\[ 
\sum_n Z^V_n - \sum_n Z^A_n = 4f_\pi^2,
\]

\[ 
\sum_n Z^V_n m^2_{V,n} - \sum_n Z^A_n m^2_{A,n} = 0,
\]

\[ 
\sum_n Z^V_n m^4_{V,n} - \sum_n Z^A_n m^4_{A,n} = \Delta_{VA}.
\]

Relations (8) and (9) are the Weinberg sum rules \[14\] where \(f_\pi = 93\) MeV is a weak \(\pi\)-meson decay constant. The equation (10) represents an additional sum rule following from OPE. The vector and axial-vector residues are connected to the electromagnetic meson widths through the relation \[29\]:

\[ 
Z^{(V,A)}_n = 4f_{(V,A),n} m^2_{(V,A),n},
\]

where \(f_{(V,A),n}\) are (dimensionless) electromagnetic decay constants.
The Weinberg sum rules ensure convergence of the integral (2) in the ultraviolet limit. As a result, one obtains:

\[
(m_{\pi^+}^2 - m_{\pi^0}^2)|_{em} = \frac{3}{4} \cdot \frac{\alpha_{em}}{f_\pi^2} \sum_{k=1}^{\infty} \left\{ f_{A,k}^2 m_{A,k}^4 \ln m_{A,k}^2 - f_{V,k}^2 m_{V,k}^4 \ln m_{V,k}^2 \right\}.
\]  

(12)

Inserting in (12) the two-resonance ansatz \((k = 1)\) for VA-correlators, one comes to the following equation [1]:

\[
\Delta m_{\pi}^{(2)}_{em} \equiv (m_{\pi^+} - m_{\pi^0})^{(2)}_{em} = \frac{3\alpha_{em}}{4\pi(m_{\pi^+} + m_{\pi^0})} \cdot \frac{m_{a_1}^2 m_\rho^2}{m_{a_1}^2 - m_\rho^2} \ln \frac{m_{a_1}^2}{m_\rho^2}.
\]  

(13)

Using the Weinberg relation \(m_{a_1} = \sqrt{2} m_\rho\) one finds \(\Delta m_{\pi}^{(2)}_{em} = 5.21\) MeV. The experimental value is [17]: \((m_{\pi^+} - m_{\pi^0})_{exper} = 4.59\) MeV. In fact, the Weinberg relation is not exact. Substitution in (13) of the physical mass for \(a_1\)-meson \(m_{a_1} = 1230\) MeV gives estimation \(\Delta m_{\pi}^{(2)}_{em} = 5.79\) MeV (relative difference with the experimental value is 26%). Notice that there are other contributions to \(m_{\pi^+} - m_{\pi^0}\), specifically, caused by the isospin symmetry breaking, i.e. an inequality of \(u\) and \(d\) quark masses [30–32] in the QCD. Their total magnitude is [24]: \(\Delta m_{\pi}^{QCD} = 0.17 \pm 0.03\) MeV. The correction of order \(1/N_c\) does not exceed 7%, as it follows from [14, 23]. The effect of weak interactions is less than 1% [33]. Thus, the contribution of electromagnetic interactions only turns out to be:

\[
\Delta m_{\pi}^{em} = 4.42 \pm 0.03\) MeV,
\]  

(14)

and we will make comparison just with (14) (as it is done, e.g. in [9, 11]). In such a way, the relative discrepancy of the result (13) with (14) amounts to 31% for the two-resonance ansatz. In the next section we will try to estimate this difference having into consideration higher meson resonances in the vector and axial-vector channels.

### 3 Calculation of electromagnetic \(\pi\)-meson mass difference with account of higher \(V,A\)-meson resonances

Now we proceed to the calculation of the electromagnetic \(\pi\)-meson mass difference \(\Delta m_{\pi}^{em}\) in the case with two vector and two axial-vector resonances, i.e. within the framework of the so called four-resonance ansatz. The utilization of the Weinberg sum rules (8),(9) made it possible to eliminate the parameters \(f_\rho\) and \(f_{a_1}\) from (13). In the four-resonance case one has three sum rules of chiral symmetry restoration (CSR) with the four unknown parameters \(f_\rho, f_{a_1}, f_\rho', f_{a_1}'\). The problem can be solved in a self-consistent way by applying an approximate inequality \(m_{a_1'} \geq m_\rho'\), which follows from properties of the mass spectrum obtained, in particular, in [28, 34]. Let us introduce a presumably small parameter \(\delta_m \equiv \frac{m_{a_1'}^2 - m_\rho'^2}{m_\rho'^2} \ll 1\). Saturating the correlators in (2) by four resonances and retaining only the first power of \(\delta_m\) one obtains for the \(\Delta m_{\pi}^{(4)}_{em}\):

\[
(m_{\pi^+}^2 - m_{\pi^0}^2)|_{em}^{(4)} \simeq \frac{3}{4} \cdot \frac{\alpha_{em}}{\pi f_\pi^2} \times
\]
\[
\left\{ f_{a_1}^2 m_{a_1}^4 \ln m_{a_1}^2 - f_\rho^2 m_\rho^4 \ln m_\rho^2 - \delta f m_\rho^4 \ln m_\rho^2 + \epsilon m_\rho^2 (1 + 2 \ln m_\rho^2) \right\},
\]

where the unknown parameters \( f_\rho^2, f_{a_1}^2 \) and \( \delta f \equiv f_\rho^2 - f_{a_1}^2 \) should be computed from the CSR sum rules within a four-resonances consideration. Namely:

\[
m_\rho^2 f_\rho^2 - m_{a_1}^2 f_{a_1}^2 + m_\rho^2 \delta f = f_\pi^2 + \epsilon
\]

\[
m_\rho^4 f_\rho^4 - m_{a_1}^4 f_{a_1}^4 + m_\rho^4 \delta f = 2m_\rho^2 \epsilon
\]

\[
m_\rho^6 f_\rho^6 - m_{a_1}^6 f_{a_1}^6 + m_\rho^6 \delta f = -4 <\bar{q}q>^2 + 3m_\rho^4 \epsilon,
\]

where \( \epsilon \equiv f_{a_1}^2 m_\rho^2 \delta m \).

Using experimental values for: \( m_\rho = 770 \text{ MeV}, m_{a_1} = 1230 \pm 40 \text{ MeV}, \)

\( m_\rho' = 1465 \pm 25 \text{ MeV}, f_\pi = 93 \text{ MeV} \) as well as bearing in mind an averaged value for the quark condensate \( <\bar{q}q> = -235 \pm 15 \text{ (MeV)}^3 \) and a model estimation for a small parameter \( \epsilon \) (for example, from \( \text{(14)} \) and the condition \( f_{a_1} < f_\rho' \) one can obtain from \( \text{(13)} \) and \( \text{(11)} \):

\[
f_\rho \approx 0.18 \quad f_{a_1} \approx 0.11 \quad f_\rho^2 - f_{a_1}^2 \approx 0.0034,
\]

and for the electromagnetic \( \pi \)-meson mass difference \( \Delta m_{\pi|_{em}}^\pm \):

\[
\Delta m_{\pi|_{em}}^\pm \approx 3.85 \pm 0.16 \text{ MeV}.
\]

Taking into account the correction due to the quark condensate improves a result for \( \Delta m_{\pi|_{em}}^\pm \) by 5%. The relative difference with \( \text{(14)} \) amounts to 13% for the given ansatz. Notice that increasing the quark condensate value leads to enlargement \( \Delta m_{\pi|_{em}}^\pm \) (for example, if

\( <\bar{q}q> = -300 \text{ (MeV)}^3 \) then \( \Delta m_{\pi|_{em}}^\pm = 4.42 \text{ MeV} \) ), and employing the value for \( f_\pi \) that it would have in the chiral limit \( f_\pi = 87 \text{ MeV} \) changes the result less than 1%.

As it follows from the paper \( \text{(8)} \): \( f_\rho = 0.20 \pm 0.01 \) (from the decay \( \rho^0 \rightarrow e^+ e^- \)); \( f_{a_1} = 0.10 \pm 0.02 \) (from the decay \( a_1 \rightarrow \pi \gamma \)). The constant \( f_\rho' \) is not yet determined from the experiment because the electromagnetic decay of \( \rho' \)-meson is strongly suppressed by hadronic decay channels. Nevertheless, Eq. \( \text{(17)} \) provides a lower estimate for \( f_\rho' \approx 0.06 \). Our numerical estimates for \( f_\rho, f_{a_1} \) coincide with those of the work \( \text{(18)} \).

We are able to compute also the constant \( L_{10} \) of effective chiral Lagrangian \( \text{(24)} \), which is defined by the mean electromagnetic \( \pi \)-meson radius \( <r_\pi^2> \) and the axial-vector pion formfactor \( F_A \) for the decay \( \pi \rightarrow e \nu \gamma \) (see, e.g. \( \text{(35)} \)). Using the relation:

\[
L_{10} = -\frac{1}{16} \frac{d}{dp^2} \left( p^2 (\Pi V(p^2) - \Pi A(p^2)) \right)_{p^2=0},
\]

as well as \( \text{(4)} \) and \( \text{(11)} \), one easily obtains for \( L_{10} \) \( \text{(28)} \):

\[
L_{10} = \frac{1}{4} \left( \sum_n k_{A,n}^2 - \sum_n k_{V,n}^2 \right),
\]
that for the case \( n = 1, 2 \) leads to the estimate for the constant \( L_{10} \approx -6.0 \cdot 10^{-3} \), which is consistent with that of [4] from hadronic \( \tau \)-decays: \( L_{10} = -(6.36 \pm 0.09|_{\exp} \pm 0.16|_{th}) \cdot 10^{-3} \).

Let us consider now how the above scheme of calculation of \( \Delta m_{\pi}^{(n)}_{\em} \) works in the case of taking into account new resonances in the CSR sum rules. By virtue of CSR at high energies one may expect that if the inequality \( m_{a_1}^2 - m_{\rho}^2 \ll m_{\rho}^2 \) is fulfilled then \( m_{a_2}^2 - m_{\rho}^2 \ll m_{\rho}^2 \) holds as well. Since the mass \( m_{\rho} \) is known, the set of equations (14) acquires only one new variable \( f_{a_1} \). As an additional condition one can put the constant \( f_{a_1} \) equal to its experimental value: \( f_{a_1} \approx 0.10 \). Then a numerical solution gives: \( \Delta m_{\pi}^{(n)}_{\em} \approx 3.94 \text{ MeV} \) (relative difference with (14) amounts to 11%), \( f_{\rho} \approx 0.18, f_{a_1} \approx 0.0023 \) (and, consequently, \( f_{\rho} \gtrsim 0.05 \)), \( f_{a_1} \approx 0.0003, L_{10} \approx -6.2 \cdot 10^{-3} \).

One can see that the addition of higher resonances in calculation of the CSR sum rules gives: \( \Delta m_{\pi}^{(n)}_{\em} \approx 3.94 \text{ MeV} \) (relative difference with (14) amounts to 11%), \( f_{\rho} \approx 0.18, f_{a_1} \approx 0.0023 \) (and, consequently, \( f_{\rho} \gtrsim 0.05 \)), \( f_{a_1} \approx 0.0003, L_{10} \approx -6.2 \cdot 10^{-3} \).

One can estimate the contribution of higher resonances to (13) in a different way. Namely, taking into account in (12) the inequality:

\[
\frac{m_{A,k}^2 - m_{V,k}^2}{m_{V,k}^2} \ll 1, \quad k > 1,
\]  

which is a consequence of CSR at high energies, we come to:

\[
\sum_{k=2}^{n} \left( m_{A,k}^4 f_{A,k}^2 \ln m_{A,k}^2 - m_{V,k}^4 f_{V,k}^2 \ln m_{V,k}^2 \right) \approx (m_{\rho}^4 f_{\rho}^2 - m_{a_1}^4 f_{a_1}^2) \ln m_{V,n}^2 + \sum_{k=2}^{n} \left( \frac{m_{A,k}^2}{m_{V,k}^2} - 1 \right) m_{A,k}^4 f_{A,k}^2,
\]  

where we have introduced an averaged mass \( \bar{m}_{V,n} \):

\[
\sum_{k=2}^{n} \left( m_{V,k}^4 f_{V,k}^2 - m_{A,k}^4 f_{A,k}^2 \right) \ln m_{V,k}^2 \equiv \ln \bar{m}_{V,n}^2 \cdot \sum_{k=2}^{n} \left( m_{V,k}^4 f_{V,k}^2 - m_{A,k}^4 f_{A,k}^2 \right).
\]

Under the assumptions made and the values of VA-meson spectral characteristic admitted in this work, the second term in (22) is by two-three order of magnitude less then the first one (at least if \( n \) is not large). Therefore, we may neglect it from now on. The expression (12) can be cast into the form:

\[
\Delta m_{\pi}^{(n)}_{\em} = \frac{3}{4} \cdot \frac{\alpha_{em}}{\pi f_{\pi}^2 (m_{\pi^+} + m_{\pi^0})} \times \{ (m_{a_1}^4 f_{a_1}^2 \ln m_{a_1}^2 - m_{\rho}^4 f_{\rho}^2 \ln m_{\rho}^2) - (m_{a_1}^4 f_{a_1}^2 - m_{\rho}^4 f_{\rho}^2) \ln \bar{m}_{V,n}^2 \}.
\]

The second term in (24) represents a correction to (13) (Eq. (13) is written in a form where the constants \( f_{\rho} \) and \( f_{a_1} \) are eliminated by means of the one-channel Weinberg
sum rules). Were the second one-channel sum rule (9) exactly valid, the last term would vanish.

If we suppose quite a good convergence of the CSR sum rules (such that an account of resonances with \( k > 2 \) therein would not lead to an essential change of spectral characteristics of mesons with \( k \leq 2 \)) then, in practice, the quantity \( \bar{m}_{V,n} \) differs from \( m_{\rho'} \) slightly within our approximation. In this way we can put \( \ln \bar{m}_{V,n} \simeq \ln m_{\rho'} \) and the generalized Eq. (13) then has the form:

\[
\Delta m_\pi|_{em} \simeq \frac{3}{4} \frac{a_{\alpha em}}{\pi f_\pi^2 (m_{\pi^+} + m_{\pi^0})} \cdot \left\{ m_{\rho'} f_{\rho}^2 \ln \frac{m_{\rho'}^2}{m_\rho^2} - m_{a_1} f_{a_1}^2 \ln \frac{m_{a_1}^2}{m_{a_1}^2} \right\},
\]

where a bar means the averaged mass approximation for higher meson resonances. Unlike the previous method, the expression (23) does not contain the quark condensate whose value varies considerably in literature.

As it follows from the derivation of formula (23) we should substitute there those values of \( f_\rho \) and \( f_{a_1} \) which they have in a given 2n-ansatz (since the Weinberg sum rules should be fulfilled). For example, in the four-resonance case they are \( f_\rho \approx 0.18, f_{a_1} \approx 0.11 \) and the relation (23) gives the result \( \Delta m_\pi|_{em}^{(4)} = 3.64 \) MeV almost coinciding with that of (18) without taking account of the quark condensate. Compared with the one-channel consideration the last value is better. In addition, the advantage of formula (23) over (13) is obvious in case of direct substitution for \( f_\rho, f_{a_1} \) by their experimental values and variation of the following quantities within their experimental bounds: \( f_\rho = 0.20 \pm 0.01, f_{a_1} = 0.10 \pm 0.02, m_{a_1} = 1230 \pm 40 \) MeV. Then the relation (13) (more strictly, Eq. (12) with \( k = 1 \)) brings an absurd estimate: \( \Delta m_\pi|_{em} = 102_{-160}^{+160} \) MeV, related with a poor fulfilment of the one-channel Weinberg sum rules. At the same time a result of Eq. (23) happens to be quite acceptable: \( \Delta m_\pi|_{em} = 7.4 \pm 3.3 \) MeV. In such a way, the second term in (24) is of the same order of magnitude as the first one. Consequently, the account of higher meson resonances is of importance. One may expect from Eq. (24) that when proceeding from the four-resonance ansatz to the six-resonance one and etc., a correction induced grows slowly (in a right direction) as effectively the account of higher meson resonances with \( k > 2 \) leads to a slight increasing of the averaged mass \( \bar{m}_{V,n} \) and a corresponding contribution does enter (24) with a negative sign. As a result, the transition from the two-resonance ansatz to the four-resonance one changes a value of electromagnetic \( \pi \)-meson mass difference to a right direction and the account of higher meson resonances \( (k > 2) \) does it likewise, but results in an insignificant correction only.

### 4 Summary

In this work we presented two ways of taking into account both vector \( (\rho', \rho'', ...) \) and axial-vector \( (a_1', ...) \) higher meson resonances in the calculation of the electromagnetic \( \pi \)-meson mass difference \( \Delta m_\pi|_{em} \). The approaches are based on the idea of chiral symmetry restoration at high energies as well as the Operator Product Expansion for the correlators of vector and axial-vector quark densities. All calculations were made in the chiral limit, in the large-\( N_c \) approximation and use the asymptotic freedom of QCD.
In the first case the calculation of $\Delta m_{\pi|_{em}}$ is carried out within the four-resonance approximation, where apart from $\rho$ and $a_1$ - mesons their first excitations, namely $\rho'$ and $a'_1$ - mesons, are also taken into consideration. The conventional Weinberg sum rules are supplemented by the third sum rule (10) ensuing from the Operator Product Expansion and the assumption $m_{a'_1} \gtrsim m_{\rho'}$. As a result, the following estimate for the electromagnetic $\pi$-meson mass difference was obtained: $\Delta m_{\pi|_{em}}^{(4)} \approx 3.85 \pm 0.16$ MeV, which improves its theoretical prediction with regard to its experimental value $\Delta m_{\pi|_{em}}^{exp} = 4.42 \pm 0.03$ MeV (where the correction due to this isospin symmetry breaking was taken into account) by 18%. The estimations on electromagnetic decay constants of $\rho$ and $a_1$ - mesons were also obtained: $f_{\rho} \approx 0.18$, $f_{a_1} \approx 0.11$, which are in a good agreement with [8,36], where they were derived in different model approaches. The calculation of the constant $L_{10}$ of effective chiral Lagrangian [24] yields in this work: $L_{10} \approx -6.0 \cdot 10^{-3}$, which is in a good agreement with the experimental date following from hadronic $\tau$-decays [4]: $L_{10} = -(6.36 \pm 0.09|_{exp} \pm 0.16|_{th}) \cdot 10^{-3}$. Moreover it was shown that the account of next resonances improves the result insignificantly, by order of several percents.

The second approach represents an extension of Eq. (13) for $\Delta m_{\pi|_{em}}^{(2)}$ towards higher resonances. First of all, it turns out that when we use experimental values for masses and decay constants of $\rho$ and $a_1$ - mesons, the generalized Eq. (25) works better than (12) in the one-channel ($k = 1$) case. Secondly, one can see from the generalized Eq. (24) that the account of $\rho'$ and $a'_1$ - meson reduces discrepancy and that inclusion of higher resonances ($k > 2$) leads to a certain improvement as well.

Finally we remark that in the recent work [37] an attempt was made to estimate from the phenomenology a number of quantities, including $\Delta m_{\pi|_{em}}$, with the help of two-point correlators being saturated by an infinite number of resonances of relevant mesons. In this work the mass spectrum of higher excitations was parametrized by a trajectory of Regge-Veneziano’s type, unlike [34], where mass spectrum of vector meson resonances was calculated within the framework of Quasilocal Quark Models [38] without any preliminary assumptions on a form and structure of the spectrum of higher meson resonances. The value obtained for the electromagnetic $\pi$-meson mass difference is $\Delta m_{\pi|_{em}}^{(\infty)} = 3.2$ MeV, i.e. the discrepancy with (14) makes up 28%, which indicates an unsatisfactory approximation of spectral characteristics of vector meson excitations in [37]. A somewhat different approach including meson excitations and calculation of their spectral characteristics, based on the Nambu-Jona-Lasinio model with separable four-quark interactions, can be found in [39].

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