Emission altitude in radio pulsars

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Abstract. This paper presents a method of estimation of emission altitudes using observational data - precise measurements of pulse profile widths at low intensity level. The analysis of emission altitudes obtained using this method for a large number of pulsars gives constraints that should be useful for theory of coherent pulsar emission. It seems that radio emission originates at altitudes of about few percent of the light cylinder and that they depend on frequency \( \nu \), pulsar period \( P \) and period derivative \( \dot{P} \).

1. Introduction

It is thirty five years since the first pulsar was discovered, but the mechanism of coherent radio emission in pulsars is still not well known. In the polar cap model (e.g. Goldreich & Julian 1969, hereafter GJ; Ruderman & Sutherland 1975), the observed emission of radio pulsars is a coherent radiation of relativistic particles flowing along dipolar field lines. This radiation is generated not far from the surface of the neutron star (NS). The emission altitude \( r_{em} \), which is the distance from NS to a place where radiation is generated, is a crucial parameter of emission region. It is commonly believed that \( r_{em} \) depends on the observed frequency. This view, supported by the systematic narrowing of average pulse width with increasing frequency (Thorsset 1991), is described by a radius-to-frequency mapping (RFM). It is obvious that all observational limits for emission altitude can be crucial for understanding the physical mechanism generating pulsar radiation. One of the ways to estimate the emission altitude (Cordes 1978, 1993) involves an analysis of observational data concerning the average pulse profiles. This method was adopted and developed by Kijak & Gil (1997, 1998, hereafter Papers I and II) and Kijak (2001, hereafter Paper III), using precise measurements of pulse profile width for the largest available data set (about 40 PSRs). Three straightforward assumptions were used: (1) the pulsar radiation is narrow-band, with a radius-to-frequency mapping operating in the emission region (that is, a narrow-band frequencies \( \Delta \nu \ll \Delta \nu_{tot} \) are emitted at given altitude \( r_{em} \), where \( \Delta \nu_{tot} \sim 30\text{GHz} \) is a total band of pulsar radiation), (2) the elementary pulsar emission is relativistically beamed tangentially to dipolar magnetic field lines (beaming angle \( \theta \sim 1/\gamma; \gamma \sim 100 \)), (3) the extreme profile wings originate at or near the last open dipolar field lines. The estimation of emission altitude is correct (i.e. valid for describing the actual physical phenomenon) as long as the above assumptions are true. In other words, if we question one of these assumptions, our estimation of emission altitude can be questioned too. The first two assumptions are in agreement with basic pulsar electrodynamics (GJ; Sturrock 1971; Ruderman & Sutherland 1975) and with the observed narrowing of profiles with increasing frequency (Thorsset 1991; Mitra & Rankin 2002). These two assumptions are commonly accepted. In the third assumption, the edge of pulsar radio beam is determined by the boundary of the GJ polar cap. This assumption is not widely accepted and will be discussed details in §3.

2. Geometry of emission region

The pulsar beam originates within the open dipolar field lines at altitudes much smaller than the radius of the light-cylinder \( r_{LC} = c/\Omega \) (Lyne & Smith 1998). The last open field lines define the polar cap on the neutron star surface. The radiating plasma streams outwards along the open field lines. Let us consider the case of a neutron star with aligned rotation and magnetic axes (see Fig. 1). The equation of dipolar magnetic field lines in polar coordinates \( (r, \theta) \) has a form \( r = (R^3/d^2) \cdot \sin \theta^2 \). For the last open field line \( \theta = \pi/2 \) and \( r = c/\Omega \), therefore \( d = \sqrt{R^3/(2\pi/cP)} \) and the radius of polar cap is \( r_p = 1.45 \cdot 10^4 P^{-1/2} \text{[cm]} \), for the neutron star radius \( R = 10^6 \text{ cm} \).

2.1. Geometry on the celestial sphere

The geometry of pulsar radio emission as defined by Manchester & Taylor (1977) is presented in Fig. 2. The rotation and magnetic axes are inclined by an angle \( \alpha \). The observer’s angle \( \xi = \alpha + \beta \), where \( \beta \) is the impact angle of the closest approach of the line-of-sight (l-of-s) to the magnetic axis. The beam radius (corresponding to
2.2. The method of estimating emission altitude

The opening angle of the pulsar beam corresponding to the pulse width $W$ is given by

$$\rho = 2 \sin^{-1} \left( \sin^2 \frac{W}{2} \sin \alpha \cdot \sin(\alpha + \beta) + \sin^2 \frac{\beta}{2} \right)^{1/2},$$

(1)

where $\alpha$ is the inclination angle between the rotation and magnetic axes and $\beta$ is the impact angle (Gil, Gronkowski & Rudnicki 1984). This formula is independent of the shape of the pulsar beam (circular, elliptical, patchy, etc.) when the beam is symmetric with respect to the fiducial
In studies on emission regions (Papers I, II and III) we used precise measurements of the longitude corresponding to these events. The edge of the beam is defined as the longitude at which recorded signals are distinguished from the noise and have the intensity at least 0.1% of the maximum intensity. Since the pulsar radiation is relativistically beamed along dipolar field lines, we can attempt to calculate which field lines should be tagged as coming from the emission region. This should be easiest for the last open magnetic field lines, which are believed to be associated with the lowest detectable level of radio emission - i.e. at the profile wings.

The radio emission from pulsars is characterized by a short time of duration, typically 3-10% of its period. However, a few objects demonstrate the emission through the entire pulsar period. Hankins et al. (1993) showed, using the VLA, that the emission through the entire pulsar period is observed only in cases when the inclination angle $\alpha$ is smaller than the beam radius $\rho$ (see Fig. 2), that is, the l-of-s is always inside the emission beam (Gil 1985). In general, there is a transition from the noise to the weak emission at the extreme profile wings (Fig. 3 and 4). In our analysis of emission altitudes (Paper I, II and III), we measured pulse widths at the extremely low intensity level corresponding to about 0.05 per cent of the maximum intensity (see Fig. 3), using the polar-log-scale technique (Hankins & Fowler 1986).

It is not certain that the entire GJ polar cap is involved in the emission process thus, the edge of pulsar beam is emitted at or near the last open field lines (which correspond to $s = 1$). It is possible that a forbidden area exists at the outer parts of the GJ polar cap. Evidently, we cannot give a theoretical answer to these questions. Recently, Gangadhara & Gupta (2001) found new outermost weak components in the pulse window of pulsar B0329+54 at frequencies of 320 and 610 MHz. The separation $\Delta \phi$ between the newly detected components is 34$^\circ$. In Fig. 5 the profile of PSR B0329+54 at 320 MHz is presented, which was used by Kijak & Gil (1998) in their measurement of profile width and corresponding estimation of emission altitude (Paper II, Fig. 2). Let us note that the pulse width measurement $W$ at 0.05% level is about 49$^\circ$. A comparison of these two measurements ($\Delta \varphi = 34^\circ$ and $W = 49^\circ$) clearly shows that the profile width presented in Fig. 5 includes new components found by Gangadhara & Gupta (2001). Thus, the reported new outermost components in the profile of PSR B0329+54 do not contradict our assumption that $s \approx 1$ at the pulse edges. Our method of profile width measurements has proven to be sensitive to very weak radio emission at profile wings originating at the edge of the beam, most probably close to the last open field lines. These outermost components are clearly visible in Fig. 5 at the level of about 0.1% of the maximum intensity.

3. Precise measurements of pulse profile and the edge of the radiation beam

The most important question is what we mean by the edge of pulsar beam and the edge of an average pulse profile. First of all, the edge of pulsar beam is related to the edge of a pulse profile in the following term: when the pulsar beam starts passing through the radiotelescope, then we start observing a weak coherent signal in the receiver. The moment at which the pulsar beam enters the radiotelescope corresponds to the moment at which the recorded noise rapidly becomes a coherent signal (and signal becomes a noise when the beam leaves the radiotelescope).
4. Emission altitude estimation

In our research of radio emission regions we used the high quality average pulse profiles in the wide frequency range. We selected pulsars for which some information about $\alpha$ and $\beta$ angles was available. For this purpose, observations were performed between 1995 and 1998 using the Effelsberg 100-m radio telescope (Lorimer et al. 1998) and the Jodrell Bank data (Gould & Lyne 1998) were used as well. As a result, the high quality average profiles were obtained with the signal-to-noise ratio S/N $\sim$ 1000 in the frequency range between 0.3 and 20 GHz. These profiles were used to obtain precise measurements of pulse profile widths and to estimate the emission altitudes.

4.1. Results

The result of emission altitude analysis (Papers I, II and III) indicated clearly that the emission altitude $r_{\text{em}}$ depends not only on the observing frequency $\nu$, but also on the basic pulsar parameters: pulsar period $P$ and period derivative $P'$. Here, we present the analysis of emission altitudes at 1.4 GHz for the largest available data set (37 PSRs). An apparent period dependence of emission altitudes is visible in Fig. 6, where a formal fit to all data points gives $r_{\text{em}} = (45 \pm 3) R P_{0.37}^{0.05}$. This result confirms the previous analysis of the emission altitude at different frequencies for a smaller data set presented in Papers I and II. Figure 6 also shows a weak dependence $r_{\text{em}}$ on pulsar characteristic age $\tau_6$ (in $10^6$ yrs), which is discussed in Paper III (see its Fig. 2 and Table 2). For this analysis, a formal fit to the same data set (37 PSRs) at 1.4 GHz yields $r_{\text{em}} = (33 \pm 1) R \tau^{-0.07\pm0.03}_{6}$. In Paper II, the RFM was derived for 16 pulsars covering the frequency range between 0.3 and 20 GHz. The average frequency dependence on emission altitude is represented by the formal fit $r_{\text{em}} \propto \nu^{-0.26\pm0.09}$. Generally, the pulsar emission altitude depends on frequency $\nu$, pulsar period $P$ and pulsar age $\tau$. 

Fig. 3. (a) A plot of profile of PSR B0329+54 at 1.4 GHz, (b) the profile of the same pulsar in the log-scale and the polar coordinates. The dynamic range is $10^4$.

Fig. 4. The syntetic data with pulsar signal which comes from open field lines. The noise was obtained using a random generator. Error box is marked as an example (see details in Paper I).
4.2. The semi-empirical formula for emission altitude

The radial location of the emission regions was discussed in Papers I, II and III. A general form of the formula for the emission altitude is

\[
r_{\text{KG}} = A R \nu_{\text{GHz}}^{a} \tau_{6}^{-b} P^{c},
\]

where \( A \sim 55, R \approx 10^{6} \text{cm}, \nu_{\text{GHz}} \) is observing frequency (in GHz), \( \tau_{6} \) is characteristic age (in \( 10^{6} \text{ yrs} \)), \( P \) is pulsar period (in sec).

The study on emission region in Paper I carried out for about 10 PSRs at two frequencies \( \nu = 0.4 \) and 1.4 GHz yielded \( A = 55 \pm 5, a = 0.21 \pm 0.07, b = 0.07 \pm 0.03 \) and \( c = 0.33 \pm 0.05 \). Here we verify these values for a larger data set, as well as for a wider frequency range. Taking into account results of § 4.1, we obtain

\[
r_{\text{KG}} = (50 \pm 10) R \nu_{\text{GHz}}^{-0.26 \pm 0.09} \tau_{6}^{-0.07 \pm 0.03} P^{0.37 \pm 0.05},
\]

or

\[
r_{\text{KG}} \approx (40 \pm 8) R \nu_{\text{GHz}}^{-0.26 \pm 0.09} \dot{P}_{-15}^{0.07 \pm 0.03} P^{0.30 \pm 0.05},
\]

where \( \tau_{6} = 16 P/\dot{P}_{-15} \) and \( \dot{P}_{-15} = \dot{P}/10^{15} \) is the period derivative. The uncertainties given for various quantities in the above equations follow from systematic errors in the pulse width \( W \) measurements and random errors in estimations of the inclination angle \( \alpha \) and the impact angle \( \beta \).

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Fig. 5. A pulsar profile of B0329+54 at 320 MHz in the log-scale and the polar coordinates. The data come from Jodrell Bank Observatory.

Fig. 6. Emission altitude at 1.4 GHz versus pulsar period. Open circles correspond to young objects (see Paper III for details).

5. Discussion and Conclusions

As it follows from Eqs. (5) or (6), the radio emission of pulsars originates at the narrow altitude range below 10 per cent of the light-cylinder radius. In millisecond pulsars, the radio emission region is much more compact, consistent with their small light-cylinder radii, and is located closer to the neutron star surface (see Paper II). Previously, it was thought that millisecond pulsars have different characteristics of radio emission from those of normal pulsars. However, it was recently argued (Gil & Sendyk 2000; Kramer 1999, and this proceedings) that millisecond pulsar emission properties do not differ from those of typical pulsars.

Recently, Gil et al. (2002) analysed simultaneous dual-frequency single pulse observations of PSR B0329+54 at 240 and 610 MHz. The phase shifts of the leading and the trailing conal components are not equal at these frequencies. This is caused by the retardation-aberration effects. In fact, since the conal beams at different frequencies are emitted at different altitudes, lower frequencies arrive earlier. The retardation-aberration shift of 0.39° translates into a difference of emission altitudes of \( \Delta r \sim 2.3 \times 10^{7} \text{ cm} \). This is consistent with the result obtained using the semi-empirical formula \( \Delta r_{\text{KG}} = 2 \times 10^{7} \text{ cm} \) (Eqs. 5 or 6).

Mitra & Rankin (2002) found the RFM using outer-conal components data. In Fig. 7 we present a comparison between their results and calculation from Eqs. (5) or (6). Emission altitudes represented by open and filled circles were calculated from \( r_{\text{em}} = h \cdot s^{-2} \), where \( h \) was the emission height for conal component and \( s \) was taken as the average value from Mitra & Rankin’s paper for each pulsar, respectively. The open circles represent calculations for the first group of their fitted procedure, and filled circles were calculated using the index of \(-2/3\) (see Mitra...
Fig. 7. The multifrequency study of emission altitude. Data points represent calculations from the paper of Mitra & Rankin (2002). The dashed lines show upper and lower limits, calculated from equation (5), corresponding to limiting values of parameters in the semi-empirical relationship. 

(i) A radius-to-frequency mapping operates in the emission region.
(ii) Pulsar radio emission is typically generated at altitudes lower than a few per cent of the light-cylinder radius $r_{LC}$ and the ratio $r_{KG}/r_{LC}$ decreases with increasing period. The total size of the emission region is below $500R$ in longer period pulsars and correspondingly smaller in shorter period pulsars.
(iii) The emission region in old pulsars is located at correspondingly lower altitudes than in young pulsars with approximately the same period.

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