Vibroacoustic Prediction of a High-Temperature Superconducting Field-Modulation Double-Stator Machine with Stationary Seal

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Abstract: This paper predicted the vibroacoustic regularity of a high temperature superconducting (HTS) field-modulation double-stator (HTS-FMDS) machine with stationary seal for low-speed and direct-drive applications. The originality of this paper lies in that the spatial order and angular velocity of electromagnetic-force density of the HTS-FMDS machine were derived by using the analytical method. Moreover, the validity of the analytical solutions was verified by the finite element analysis (FEA) results. Then, the modal shapes and frequencies of the outer stator were obtained by using multiphysics coupling simulation. By transferring the electromagnetic force to the stator structural model, the regularity of electromagnetic vibration and noise of the HTS-FMDS machine were revealed.

Keywords: high-temperature superconducting; field-modulation; double-stator; vibroacoustic; electromagnetic force density; finite element analysis

1. Introduction

In recent years, high-temperature superconducting (HTS) machines have been paid widespread attention in high-power, low-speed, and direct-drive applications such as offshore wind power generation and marine propulsion since they offer the merits of high efficiency and high power/torque density [1–3]. Generally, the HTS field coils are wound by HTS wires and supplied with direct-current to obtain the excitation field and are mounted on the rotor side. Therefore, the coupling device for cryogenic transfer is installed between the rotating rotor and stationary refrigeration system to keep the HTS field coils operating at cryogenic temperature needed by superconductor properties of HTS wires through the circulation of refrigeration coolant. According to whether the coupling device for cryogenic transfer is employed to cool the HTS field-excitation coils, the HTS machines can be divided into two categories, namely, the rotating seal and stationary seal HTS machines. For the rotating HTS machines, since the HTS field-excitation coils are mounted on the rotating rotor side, it is necessary to use the coupling device for cryogenic transfer so that the superconducting state of HTS field-excitation coils can be maintained [4–7]. On the contrary, the HTS field-excitation coils of the latter are installed in the stationary stator side, and thus the circulation of the refrigeration coolant in the cooling system can be performed by using stationary seal [8–10]. Accordingly, the manufacturing cost and difficulty of superconducting machines with stationary seal can be decreased effectively.

Under the condition of superconducting state, the HTS wires have zero resistance characteristics. Therefore, the HTS field-excitation coils can provide a strong excitation field by applying high direct-current. However, the high air-gap flux density of HTS machines will produce strong electromagnetic force that...
varies with time and space. When the radial component of this electromagnetic force acts on the surface of stator-core teeth, the stator core will be vibrated and deformed, which causes pulsation of the surrounding air, hence generating electromagnetic acoustic noise. In particular, if the orders of electromagnetic force waves are consistent with those of the stator modal shapes and the frequencies are very close to those of stator modal shapes, the stator resonance of the HTS machine will be caused, which generates greater electromagnetic acoustic noise.

In the last decades, many literatures have reported their research results on electromagnetic vibration and acoustic noise of conventional machines, such as switched reluctance motors (SRMs) [11–13], claw pole machines [14,15], induction motors (IMs) [16,17], and permanent magnet synchronous motors (PMSMs) [18,19]. However, few literatures investigated the electromagnetic vibration and acoustic noise of HTS machines. Especially for the HTS field-modulation double-stator (HTS-FMDS) machine with stationary seal, due to the magnetic field modulation by the modulation-ring rotor and both the inner and outer stator teeth, the magnetic fields of both inner and outer air-gaps are rich in harmonic components, which makes the electromagnetic force analysis of these kinds of HTS machines more complicated.

Therefore, the purpose of this paper is to analyze the electromagnetic vibration and acoustic noise of the HTS machines with double air-gaps and double stators to obtain the vibro-acoustic regularity, thus providing a prediction for the prototype design of HTS machines. Moreover, in this paper, we only focus on the electromagnetic vibration and acoustic noise of HTS-FMDS machine. In Section 2, the analytical expression of electromagnetic force in a double-stator HTS machine were discussed. Section 3 focused on the finite element analysis (FEA) of electromagnetic force. Then, in Section 4, the stator modal shapes of the double-stator HTS machine were presented and analyzed, and the electromagnetic vibration and acoustic noise of the double-stator HTS machine was investigated by FEA. Finally, some conclusions were drawn in Section 5.

2. Analysis of Electromagnetic Force

The configurations of the HTS-FMDS machine are shown in Figure 1. It mainly contains an outer stator, a modulation-ring rotor, a ferromagnetic ring, and an inner stator from the outside to the inside. The outer stator has 42 slots and the number of pole-pairs of the three-phase armature-windings is 14. While the HTS field-excitation windings with four pairs of magnetic poles are placed on the inner stator side so that the armature windings and the HTS field-excitation windings are located in two spatially independent stators. In order to modulate both the inner and outer air-gap magnetic-field and transmit electromagnetic torque, a modulation-ring rotor, which consists of 18 ferromagnetic blocks, is mounted between the inner and outer stators. It should be particularly noted that the ferromagnetic ring, which is located on the top of each field-excitation magnetic pole, not only provides a magnetic circuit for the field-excitation magnetic field, but also guides a path for the armature-reaction magnetic field. Based on field-modulation theory, the sinusoidal back-EMF can be induced in the armature windings of the outer stator since the stationary HTS field-excitation magnetic-field is modulated by the rotating modulation-ring rotor. Its operating principle has been introduced in detail in Reference [20]. Since the HTS-FMDS machine has two stators and double air-gaps, the electromagnetic force, which is generated by the interaction of harmonic components of both excitation field and armature-reaction magnetic-field, will be exerted to both the inner surface of the outer stator teeth and the outer surface of the inner stator teeth and causes electromagnetic vibration and acoustic noise. For the outer air-gap magnetic field, it is generated by the interaction between the armature-reaction magnetic field and the HTS excitation-field modulated by the modulation-ring rotor, while the inner air-gap magnetic field is produced by the interaction of a HTS excitation-field and the armature-reaction magnetic field modulated by the modulation-ring rotor. It should be noted that the main harmonic components of flux density of the inner air-gap are same as those of the outer air-gap, although the amplitudes of harmonic magnetic field components of the inner air-gap are different with those of the outer air-gap. By using Fourier transform, the flux density excited by HTS field-excitation magnetomotive force can be written as [21–27]:

\[ B(\theta) = \sum_{n=1}^{N} B_n \cos(n \theta) \]

Where \( B_n \) is the amplitude of the nth harmonic component.
where $\theta$ is the mechanical angle of rotor position, $t$ is the time, $P_{sc}$ is the number of pole pairs of the HTS excitation field, $\lambda_{ring,0}$ is the DC permeance component of modulation-ring rotor, $F_j$ is the amplitude of the $j$th harmonic of HTS field-excitation magnetomotive force, $N_r$ is the numbers of poles of modulation-ring rotor, $\omega_r$ is the mechanical angular velocity of rotor, and $\lambda_{ring,i}$ is the $i$th harmonic amplitude of permeance component of modulation-ring rotor. Similarly, the flux density generated by armature-reaction magnetomotive force can be expressed as:

$$B_{ra}(\theta, t) = \frac{3}{4} \sum_{n=1}^{+\infty} \sum_{l=1}^{+\infty} \left[ \lambda_{ring,0} F_{\phi} \cos \left( \omega_r t + nkP_{sa} \theta + \phi_0 - \frac{\pi}{2} \right) \right]$$

where $P_{sa}$ is the number of pole pairs of the armature-reaction magnetic field, $F_{\phi}$ is the amplitude of the $v$th harmonic of armature-reaction magnetomotive-force, and $\theta_0$ is the initial rotor position. In addition, $k$ meets the following limitation:

$$k = \begin{cases} 
-1 & n = 3n - 2 \\
1 & n = 3n - 1 \\
v & n \neq 3n 
\end{cases}$$

where $n$ is a natural number. Then, ignoring the influence of the slots opening of the inner and outer stator on the air-gap flux-density, the flux density of both inner and outer air-gaps can be expressed as:

$$B_r(\theta, t) = B_{rf}(\theta, t) + B_{ra}(\theta, t)$$

Accordingly, the radial component of the electromagnetic force, which exerts on the surface of both the inner and outer stator teeth, can be calculated as follows by using Maxwell stress method:

$$f_r(\theta, t) \approx \frac{B_r(\theta, t)^2}{2\mu_0}$$
where \( \mu_0 \) is the air magnetic permeability, equaling to \( 4 \times 10^{-7} \) H/m. Substituting Equation (4) into Equation (5), the radial component of the electromagnetic force can be further expressed as:

\[
f_r(\theta, t) \approx \frac{B_{rf}(\theta, t)^2 + B_{ra}(\theta, t)^2 + 2B_{ra}(\theta, t)B_{rf}(\theta, t)}{2\mu_0}
\]

(6)

Substituting Equation (1) and Equation (2) into Equation (6), the first term \( B_{rf}(\theta, t)^2 \), in its numerator, can be expressed as Equation (A1), the second term \( B_{ra}(\theta, t)^2 \) of the numerator can be calculated as Equation (A2), and the third term \( B_{ra}(\theta, t)B_{rf}(\theta, t) \) of the numerator can be obtained as Equation (A3), where Equations (A1)–(A3) are listed in the Appendix.

According to Equations (A1)–(A3), the source, spatial order and angular velocity of the radial force density can be calculated as listed in Table 1. According to Table 1, three conclusions can be derived. Firstly, the interaction of any two harmonic components of air-gap magnetic field will produce electromagnetic force exerted on the inner and outer stator teeth. Secondly, the \( (j \pm 1)P_{sc} \)th and \( (v + V)kP_{sa} \)th orders harmonic components of electromagnetic forces, which are excited by the interaction of harmonic components of HTS excitation magnetic field and armature-reaction magnetic field, respectively. Thirdly, the radial electromagnetic forces are rich in harmonic components due to magnetic field modulation by the magnetic-ring rotor, although the influence of the slot opening of both the inner and outer stator on the air-gap flux-density is ignored.

| Source | Spatial Order | Angular Velocity |
|--------|---------------|------------------|
| Harmonic components of excitation field | \((j \pm 1)P_{sc}\) \((j \pm 1)P_{sc} + (i \pm 1)N_r\) \((j \pm 1)P_{sc} + iN_r\) \((v + V)kP_{sa}\) \((v - V)kP_{sa}\) \((v + V)kP_{sa} + iN_r\) \((v - V)kP_{sa} + iN_r\) | 0 \(-(i \pm 1)N_r\omega_r\) \(\mp iN_r\omega_r\) 2\(N_r\omega_r\) 0 \(\mp (i \mp 2)N_r\omega_r\) \(\pm iN_r\omega_r\) \(-iN_r\omega_r\) |
| Harmonic components of armature-reaction magnetic field | \((v + V)kP_{sa} + (i \pm 1)N_r\) \((v - V)kP_{sa} + (i \mp 1)N_r\) \((v + V)kP_{sa} - (i + 1)N_r\) \((v - V)kP_{sa} - (i - 1)N_r\) | \(-i(\pm 1 - 2)N_r\omega_r\) \(-i \mp 1)N_r\omega_r\) \((i + 1 + 2)N_r\omega_r\) \((i - 1)N_r\omega_r\) |
| Interaction between the harmonic components of armature-reaction magnetic field and excitation field | \(jP_{sc} \pm \mp v kP_{sa}\) | \(\pm N_r\omega_r\) \((i \mp 1)N_r\omega_r\) \(\pm (i \mp 1)N_r\omega_r\) \(\mp (i \mp 1)N_r\omega_r\) |

3. Finite Element Analysis of Electromagnetic Force

Since the HTS-FMDS machine has symmetrical structure in the axial direction, the two-dimensional finite element method is used to analyze its electromagnetic force. Its key size parameters are listed, as shown in Table 2. Considering that the electromagnetic properties of the HTS-FMDS machine have been investigated in detail in previously published literature [20], in this section, we will only focus on investigation of the electromagnetic force density.
Table 2. Key size parameters of the HTS-FMDS machine.

| Parameters                                      | Value   |
|-------------------------------------------------|---------|
| Number of outer stator teeth                    | 42      |
| Number of inner stator teeth                    | 8       |
| Number of pole pairs of armature windings       | 14      |
| Number of pole pairs of field-excitation windings| 4       |
| Number of modulation-ring rotor teeth           | 18      |
| Stator outside diameter of outer stator (mm)    | 760     |
| Stator inside diameter of outer stator (mm)     | 546.4   |
| Stator outside diameter of inner stator (mm)    | 504     |
| Stator inside diameter of inner stator (mm)     | 240     |
| Rotor outside diameter (mm)                     | 42      |
| Stack length (mm)                               | 100     |
| Inner air-gap length (mm)                       | 0.6     |
| Outer air-gap length (mm)                       | 0.6     |
| Number of phases                                | 3       |

3.1. Magnetic Flux Density

When the HTS-FMDS machine operates with no load, its air-gap magnetic field is only generated by the HTS excitation magnetomotive force. According to Equation (1), the inner and outer air-gap flux-density should include the 4th, 12th, 14th, 22nd, 32nd, 40th, 50th, and 58th harmonic components when the following conditions are met:

\[
\begin{align*}
    jP_{sc} &= 4 \text{ and } 12 \quad (j = 1, 3) \\
    |jP_{sc} \pm iN_r| &= 14, 22, 32, 40, 50 \text{ and } 58 \quad (j = 1, i = 1, 2, 3)
\end{align*}
\]  

(7)

In order to verify the correctness of the theoretical analysis in Section 2, the flux density waveforms and their Fourier decomposition of inner and outer air-gaps are illustrated in Figure 2. From Figure 2b,d, it can be found that the analytically calculated harmonic components are consistent with those by using FEA, which are shown with red lines. In addition, the harmonic components depicted with blue lines, whose amplitude is greater than 0.05 T, are mainly modulated by the outer stator teeth.

Under on-load condition, the magnetic field in the inner and outer air-gaps is generated by the interaction between the field-excitation magnetomotive force and armature-reaction magnetomotive force. The order of harmonic components of air-gaps flux-density generated by the armature reaction magnetomotive force can be expressed as:

\[
\begin{align*}
    vkP_{sa} &= 14 \quad (v = 1, k = -1) \\
    |vkP_{sa} \pm iN_r| &= 4, 22, 32, 40, 50 \text{ and } 68 \quad (v = 1, k = -1, i = 1, 2, 3) \\
    |vkP_{sa} \pm iN_r| &= 8, 10, 26, 46, 64 \text{ and } 82 \quad (v = 2, k = 1, i = 1, 2, 3)
\end{align*}
\]  

(8)

Thus, ignoring the effects of magnetic saturation, new components are added to the magnetic flux density of both the inner and outer air-gaps, namely, they are the 8th, 10th, 26th, 28th, and 46th harmonic components, which are shown in Figure 3b,d. It should be noted that the FEA results contain harmonic components that do not appear in the theoretical calculation as denoted with blue line in both Figures 2 and 3 since the modulation effect by the outer stator teeth were neglected in the theoretical calculation.
armature reaction magnetomotive force can be expressed as:

The order of harmonic components of air-gaps flux-density generated by the interaction between the field-excitation magnetomotive force and armature-reaction magnetomotive force contain harmonic components that do not appear in the theoretical calculation as denoted with blue harmonic components, which are shown in Figure 3b,d. It should be noted that the FEA results flux density of both the inner and outer air-gaps, namely, they are the 8th, 10th, 26th, 28th, and 46th line in both Figures 2 and 3 since the modulation effect by the outer stator teeth were neglected in the theoretical calculation.

Figure 2. Magnetic flux density with no load: (a) Flux density of inner air-gap; (b) Harmonic order of flux density of inner air-gap; (c) Flux density of outer air-gap; and (d) Harmonic order of flux density of outer air-gap.

Figure 3. Cont.
with different spatial orders of electromagnetic force density are determined by those of the air-gaps

Energies

noise and is relatively low.

Moreover, compared with the 18th, 24th, 28th, 32th, 36th, and 42th harmonic components, which is the

harmonic component is not only independent of the spatial position, but also independent of the

time variable. Thus, it will not contribute to the vibration and noise of the HTS machine, although its

magnitude reaches to 230 N/m². Moreover, compared with the 18th, 24th, 28th, 32th, 36th, and 42th harmonic components, which is the

amplitude of the low-order harmonic components, which offers major contributions to electromagnetic

noise and is relatively low.

3.2. Electromagnetic Force Density

As shown in Figure 4, the electromagnetic force acting on both inner and outer stators is a function

of time and space. Based on Equations (A1)–(A3), the amplitudes of sequential harmonic components with
different spatial orders of electromagnetic force density are determined by those of the air-gaps

flux-density. Taking the electromagnetic force acting on the surface of the outer stator teeth under

on-load condition, as an example, whose sequential and spatial harmonic components are depicted in

Figure 5, its spatial harmonic components mainly include even harmonics. Moreover, the sequential

harmonic components with the same frequency contain various spatial harmonic components with
different orders, and vice versa. Among the spatial harmonic components, the zero order component is

mainly caused by the interactions between the harmonic components with the same order of excitation

field and the interactions between the harmonic components with the same order of armature-reaction

magnetic field, respectively. According to Table 1, the zero order harmonic component is not only

independent of the spatial position, but also independent of the time variable. Thus, it will not

contribute to the vibration and noise of the HTS machine, although its magnitude reaches to 230 N/m².

Moreover, compared with the 18th, 24th, 28th, 32th, 36th, and 42th harmonic components, which is the

amplitude of the low-order harmonic components, which offers major contributions to electromagnetic

noise and is relatively low.

![Figure 3. Magnetic flux density with on-load: (a) Flux density of inner air-gap; (b) Harmonic order of flux density of inner air-gap; (c) Flux density of outer air-gap; and (d) Harmonic order of flux density of outer air-gap.](image)

![Figure 4. Electromagnetic force density on-load: (a) Inner stator; and (b) outer stator.](image)
4. Structure and Vibroacoustic Analysis

For the purpose of calculating the electromagnetic vibration and acoustic noise of HTS-FMDS machine, a multi-physics coupling model was established. Firstly, the 2-D transient electromagnetic model was built to calculate the nodal force acting on the surface of the outer stator teeth since this machine has axial symmetry configuration. Then, the nodal force in 3-D model can be obtained according to the periodicity of HTS-FMDS machine under the condition that the electromagnetic force distributes uniformly in the axial direction. Secondly, taking into account the influence of the armature-winding ends, the 3-D structural model of HTS-FMDS machine was built. Additionally, the nodal force was loaded from the electromagnetic model to the structural model by force interpolation. Finally, the modal superposition method was used to calculate the electromagnetic vibration, and the boundary element method was adopted to calculate the electromagnetic acoustic noise.

Figure 6 shows the stator structural model of HTS-FMDS machine. Since the armature windings are fixed in the stator slot by the slot wedges, it is equivalent to a squirrel cage as shown in Figure 6. Moreover, the equivalent density of this squirrel cage structure is the ratio of actual mass to volume. The equivalent material mechanics parameters of stator core and windings are shown in Table 3. Among of them, Ew and Ec are Young’s modulus of silicon steel sheet (200 GPa) and copper (97 GPa), respectively. Gw and Gc are the shear modulus of silicon steel sheet (80 GPa) and copper (32 GPa), respectively. By using FEA, the modal shapes and frequencies of the HTS-FMDS machine can be calculated, as listed in Table 4. These finite element results of Table 4 reveal that the low-order modal shapes of stator core have two natural frequencies, which are corresponding to in-phase vibration and inverse vibration along the stator axial direction, respectively.

After transferring the electromagnetic force to the stator structural model, the spectrum of vibration acceleration of outer stator can be obtained by using modal superposition method, as depicted in Figure 7. It can be found that the vibration acceleration of the stator surface reach obvious peak value when the electromagnetic force frequencies are 800 Hz and 1800 Hz, respectively. Corresponding
to these two peak frequencies, the sound pressure level of the electromagnetic noise, which was calculated by using boundary element method, also reaches peak value, as shown in Figure 8.

![Stator model of HTS-FMDS machine](image)

**Figure 6.** Stator model of HTS-FMDS machine: (a) Stator core; (b) armature windings; and (c) stator assemblies.

**Table 3.** Material parameters of stator core and armature winding.

| Material Parameters | Young’s Modulus | Shear Modulus | Poisson’s Ratio | Density (kg/m³) |
|---------------------|-----------------|---------------|----------------|----------------|
| Stator core         | $E_x = E_y = 0.94E_w$ | $G_{xy} = 0.84G_w$ | 0.3            | 7488           |
| Armature winding    | $E_x = E_y = 0.92E_c$ | $G_{xy} = G_{xz} = G_{yz} = 0.17G_w$ | 0.3            | 5340           |

**Table 4.** Modal shapes and frequencies.

| Mode Number | Mode Shapes | Natural Frequencies |
|-------------|-------------|---------------------|
| (2, 0)      | ![Mode Shape](image) | 700.58 Hz          |
| (2, 1)      | ![Mode Shape](image) | 1127.00 Hz         |
| (3, 0)      | ![Mode Shape](image) | 1197.71 Hz         |
| (3, 1)      | ![Mode Shape](image) | 1324.22 Hz         |
| (4, 0)      | ![Mode Shape](image) | 1776.42 Hz         |
The electromagnetic acoustic noise of the HTS-FMDS machine is produced by the interactions between the harmonic components of electromagnetic forces acting on the stator and all the stator modal shapes. In order to investigate the major contribution of various stator modal shapes to the vibroacoustic at 800 Hz and 1800 Hz, respectively, the acoustic noise images of HTS-FMDS machine are calculated, as shown in Figure 9. From Figure 9a, it can be found that the second order modal shape is the main source of electromagnetic noise when the electromagnetic force frequency is 800 Hz, which agrees well with modal shape and frequency listed in Table 4. Similarly, the fourth order modal shape is the main source of electromagnetic noise when the electromagnetic force frequency is 1600 Hz,

| Mode Number | Mode Shapes | Natural Frequencies |
|-------------|-------------|---------------------|
| (4, 1)      | ![Mode Shape 4,1](image) | 1644.37 Hz          |
| (5, 1)      | ![Mode Shape 5,1](image) | 1883.20 Hz          |
| (6, 1)      | ![Mode Shape 6,1](image) | 2148.16 Hz          |
| (7, 1)      | ![Mode Shape 7,1](image) | 2382.76 Hz          |
| (8, 1)      | ![Mode Shape 8,1](image) | 2587.80 Hz          |

Figure 7. Acceleration spectrum of outer stator surface.
as illustrated in Figure 9b. Therefore, for the HTS-FMDS machine in this paper, the second and the fourth order stator modal shapes are the main sources of electromagnetic acoustic noise, although the corresponding second and fourth order electromagnetic force waves, which are shown in Figure 5, have lower amplitudes. In addition, the sound pressure level contour plot also reveals that the strength of low frequency noise is generally higher than that of high frequency noise.

![Figure 8. Noise spectrum under rated load.](image)

![Figure 9. Acoustic noise image: (a) 800 Hz; (b) 1800 Hz.](image)

5. Conclusions

In this work, the vibroacoustic regularity of a HTS-FMDS machine with a stationary seal for low-speed and direct-drive applications were predicted. The spatial order and angular velocity of electromagnetic force density of the HTS-FMDS machine were derived by using analytical method. Moreover, the validity of the analytical solutions is verified by the finite element analysis (FEA) results. Based on the structural modal, the modal shapes and frequencies of the outer stator were obtained by using multiphysics coupling simulation. By transferring the electromagnetic force to the stator structural model, the regularity of electromagnetic vibration and acoustic noise of the HTS-FMDS machine were revealed. For the HTS-FMDS machine in this paper, the second and the fourth order stator modal shapes are the main sources of electromagnetic acoustic noise, although the corresponding second and fourth order electromagnetic force waves have lower amplitudes. In addition, since the electromagnetic force acting on the surface of the stator is rich in harmonic components and high in amplitude, the electromagnetic vibration and acoustic noise of the HTS-FMDS machine are relatively serious.
Author Contributions: Y.W., C.Z. and W.X. mainly conducted the modal-shapes analysis and theoretical derivation of both air-gaps flux-density and electromagnetic force of HTS-FMDS machine. The manuscript was improved and revised by X.Z. All of the authors contributed to the paper writing.

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Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

\[
\begin{align*}
\frac{1}{2} \sum_{j=1,3,5,\ldots}^{+\infty} \sum_{l=1,3,5,\ldots}^{+\infty} \left\{ \lambda_{\text{ring}} J_{J} L_{J} F_{J} \left\{ \begin{array}{l}
\cos \left[ \left( j + J \right) P_{L} \left( \theta - \frac{1 \left[ N_{0} \omega_{0} t - \left( j + 1 \right) N_{0} \omega_{0} t \right]}{P_{L} + N_{0} \omega_{0} t} \right) \right] \\
\cos \left[ \left( j - J \right) P_{L} \left( \theta + \frac{1 \left[ N_{0} \omega_{0} t - \left( j + 1 \right) N_{0} \omega_{0} t \right]}{P_{L} + N_{0} \omega_{0} t} \right) \right]
\end{array} \right\} \right\} + \\
\frac{1}{4} \sum_{j=1,3,5,\ldots}^{+\infty} \sum_{l=1,3,5,\ldots}^{+\infty} \sum_{J=1,3,5,\ldots}^{+\infty} \left\{ \lambda_{\text{ring}} J_{J} L_{J} F_{J} \left\{ \begin{array}{l}
\cos \left[ \left( j + J \right) P_{L} + N_{0} \omega_{0} t \left( \theta - \frac{1 \left[ N_{0} \omega_{0} t - \left( j + 1 \right) N_{0} \omega_{0} t \right]}{P_{L} + N_{0} \omega_{0} t} \right) \right] \\
\cos \left[ \left( j - J \right) P_{L} + N_{0} \omega_{0} t \left( \theta + \frac{1 \left[ N_{0} \omega_{0} t - \left( j + 1 \right) N_{0} \omega_{0} t \right]}{P_{L} + N_{0} \omega_{0} t} \right) \right]
\end{array} \right\} \right\} + \\
\frac{1}{4} \sum_{j=1,3,5,\ldots}^{+\infty} \sum_{l=1,3,5,\ldots}^{+\infty} \sum_{J=1,3,5,\ldots}^{+\infty} \left\{ \lambda_{\text{ring}} J_{J} L_{J} F_{J} \left\{ \begin{array}{l}
\cos \left[ \left( j + J \right) P_{L} + i N_{0} \omega_{0} t \left( \theta - \frac{1 \left[ N_{0} \omega_{0} t - \left( j + 1 \right) N_{0} \omega_{0} t \right]}{P_{L} + N_{0} \omega_{0} t} \right) \right] \\
\cos \left[ \left( j - J \right) P_{L} + i N_{0} \omega_{0} t \left( \theta + \frac{1 \left[ N_{0} \omega_{0} t - \left( j + 1 \right) N_{0} \omega_{0} t \right]}{P_{L} + N_{0} \omega_{0} t} \right) \right]
\end{array} \right\} \right\} + \\
\frac{1}{4} \sum_{j=1,3,5,\ldots}^{+\infty} \sum_{l=1,3,5,\ldots}^{+\infty} \sum_{J=1,3,5,\ldots}^{+\infty} \left\{ \lambda_{\text{ring}} J_{J} L_{J} F_{J} \left\{ \begin{array}{l}
\cos \left[ \left( j + J \right) P_{L} + i N_{0} \omega_{0} t \left( \theta + \frac{1 \left[ N_{0} \omega_{0} t - \left( j + 1 \right) N_{0} \omega_{0} t \right]}{P_{L} + N_{0} \omega_{0} t} \right) \right] \\
\cos \left[ \left( j - J \right) P_{L} + i N_{0} \omega_{0} t \left( \theta - \frac{1 \left[ N_{0} \omega_{0} t - \left( j + 1 \right) N_{0} \omega_{0} t \right]}{P_{L} + N_{0} \omega_{0} t} \right) \right]
\end{array} \right\} \right\} + \\
\frac{1}{8} \sum_{j=1,3,5,\ldots}^{+\infty} \sum_{l=1,3,5,\ldots}^{+\infty} \sum_{J=1,3,5,\ldots}^{+\infty} \sum_{l=1,3,5,\ldots}^{+\infty} \left\{ \lambda_{\text{ring}} J_{J} L_{J} F_{J} \left\{ \begin{array}{l}
\cos \left[ \left( j + J \right) P_{L} + i N_{0} \omega_{0} t \left( \theta + \frac{1 \left[ N_{0} \omega_{0} t - \left( j + 1 \right) N_{0} \omega_{0} t \right]}{P_{L} + N_{0} \omega_{0} t} \right) \right] \\
\cos \left[ \left( j - J \right) P_{L} + i N_{0} \omega_{0} t \left( \theta - \frac{1 \left[ N_{0} \omega_{0} t - \left( j + 1 \right) N_{0} \omega_{0} t \right]}{P_{L} + N_{0} \omega_{0} t} \right) \right]
\end{array} \right\} \right\} \end{align*}
\]
\[
\begin{align*}
Energies 2018, 11, 2563 & \\
\sum_{v=1}^{\infty} & \sum_{V=1}^{\infty} \sum_{I=1}^{\infty} \lambda_{\text{ring,b}} f_{\text{FV,FV,ring,t}} \{ \cos \left[ \left( v + V \right) k_{\text{FV}} + \left( N_{\theta} - 1 \right) \theta - \frac{i \left( N_{\theta} + 1 \right) \theta + 2 \phi_{0} + \pi}{\theta \left( v + V \right) k_{\text{FV}} + \left( N_{\theta} - 1 \right) \theta} \right] \} + \\
\sum_{v=1}^{\infty} & \sum_{V=1}^{\infty} \sum_{I=1}^{\infty} \lambda_{\text{ring,b}} f_{\text{FV,FV,ring,t}} \{ \cos \left[ \left( v - V \right) k_{\text{FV}} - \left( N_{\theta} - 1 \right) \theta + \frac{i \left( N_{\theta} + 1 \right) \theta + 2 \phi_{0} - \pi}{\theta \left( v - V \right) k_{\text{FV}} - \left( N_{\theta} - 1 \right) \theta} \right] \} + \\
\sum_{v=1}^{\infty} & \sum_{V=1}^{\infty} \sum_{I=1}^{\infty} \lambda_{\text{ring,b}} f_{\text{FV,FV,ring,t}} \{ \cos \left[ \left( v + V \right) k_{\text{FV}} + i \left( N_{\theta} - 1 \right) \theta - \frac{i \left( N_{\theta} + 1 \right) \theta + 2 \phi_{0} + \pi}{\theta \left( v + V \right) k_{\text{FV}} + i \left( N_{\theta} - 1 \right) \theta} \right] \} + \\
\sum_{v=1}^{\infty} & \sum_{V=1}^{\infty} \sum_{I=1}^{\infty} \lambda_{\text{ring,b}} f_{\text{FV,FV,ring,t}} \{ \cos \left[ \left( v - V \right) k_{\text{FV}} + i \left( N_{\theta} - 1 \right) \theta + \frac{i \left( N_{\theta} + 1 \right) \theta + 2 \phi_{0} - \pi}{\theta \left( v - V \right) k_{\text{FV}} + i \left( N_{\theta} - 1 \right) \theta} \right] \} + \\
\sum_{v=1}^{\infty} & \sum_{V=1}^{\infty} \sum_{I=1}^{\infty} \lambda_{\text{ring,b}} f_{\text{FV,FV,ring,t}} \{ \cos \left[ \left( v + V \right) k_{\text{FV}} + i \left( N_{\theta} - 1 \right) \theta - \frac{i \left( N_{\theta} + 1 \right) \theta + 2 \phi_{0} + \pi}{\theta \left( v + V \right) k_{\text{FV}} + i \left( N_{\theta} - 1 \right) \theta} \right] \} + \\
\sum_{v=1}^{\infty} & \sum_{V=1}^{\infty} \sum_{I=1}^{\infty} \lambda_{\text{ring,b}} f_{\text{FV,FV,ring,t}} \{ \cos \left[ \left( v - V \right) k_{\text{FV}} + i \left( N_{\theta} - 1 \right) \theta + \frac{i \left( N_{\theta} + 1 \right) \theta + 2 \phi_{0} - \pi}{\theta \left( v - V \right) k_{\text{FV}} + i \left( N_{\theta} - 1 \right) \theta} \right] \}
\end{align*}
\]
\[
\frac{3}{16} \sum_{p=1,3,5,\ldots} \sum_{i=1,3,5,\ldots} \sum_{y=1,3,5,\ldots} \sum_{v=1,3,5,\ldots} F_{j\lambda_{\text{ring}}} F_{i\phi_{\text{ring}}} \cos \left( \left( jP_i + iN_j + V_k P_m - IN_j \right) \left( \theta - \frac{(i+1-j)\lambda_{\text{ring}}(t-1)N_0 + \nu + \frac{\pi}{2}}{P_{i\phi_{\text{ring}}} - N_0} \right) \right)
\]
\[
\cos \left( \left( jP_i + iN_j - V_k P_m - + IN_j \right) \left( \theta - \frac{(i+1-j)\lambda_{\text{ring}}(t-1)N_0 + \nu + \frac{\pi}{2}}{P_{i\phi_{\text{ring}}} - N_0} \right) \right)
\]
\[
\cos \left( \left( jP_i - iN_j + V_k P_m + IN_j \right) \left( \theta + \frac{(i+1-j)\lambda_{\text{ring}}(t-1)N_0 - \nu - \frac{\pi}{2}}{P_{i\phi_{\text{ring}}} + N_0} \right) \right)
\]
\[
\cos \left( \left( jP_i - iN_j - V_k P_m - IN_j \right) \left( \theta + \frac{(i+1-j)\lambda_{\text{ring}}(t-1)N_0 - \nu - \frac{\pi}{2}}{P_{i\phi_{\text{ring}}} + N_0} \right) \right)
\]
\[
\cos \left( \left( jP_i - iN_j + V_k P_m + IN_j \right) \left( \theta + \frac{(i+1-j)\lambda_{\text{ring}}(t-1)N_0 - \nu - \frac{\pi}{2}}{P_{i\phi_{\text{ring}}} + N_0} \right) \right)
\]
\[
\cos \left( \left( jP_i - iN_j - V_k P_m - IN_j \right) \left( \theta + \frac{(i+1-j)\lambda_{\text{ring}}(t-1)N_0 - \nu - \frac{\pi}{2}}{P_{i\phi_{\text{ring}}} + N_0} \right) \right)
\]

where \( i \) and \( l \) are positive integer, \( j \) and \( f \) are positive odd-number and \( v \) and \( V \) are the positive integer that cannot be divided by 3, respectively.

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