A Piece of the Lepton Theory from a Probability

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Abstract

A masses of a leptons deduced from a representation of a probability density vector by a spinors. A massive W and Z bosons and a massless A boson are obtained from a transformations for which a density vector is invariant.

I use the following denotations:

\[ 1_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad 0_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \]

and for \( k \geq 2 \):

\[ 1_{2k} = \begin{bmatrix} 1_k & 0_k \\ 0_k & 1_k \end{bmatrix}, \quad 0_{2k} = \begin{bmatrix} 0_k & 0_k \\ 0_k & 0_k \end{bmatrix} \]

\[ \sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \]

are the Pauli matrices.

The Clifford pentad \( \mathbb{2} \), \( \beta \) is:

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\[ \beta_1 = \begin{bmatrix} \sigma_1 & 0_2 \\ 0_2 & -\sigma_1 \end{bmatrix}, \beta_2 = \begin{bmatrix} \sigma_2 & 0_2 \\ 0_2 & -\sigma_2 \end{bmatrix}, \beta_3 = \begin{bmatrix} \sigma_3 & 0_2 \\ 0_2 & -\sigma_3 \end{bmatrix}, \gamma_0 = \begin{bmatrix} 0_2 & 1_2 \\ 1_2 & 0_2 \end{bmatrix} = \beta_5, \beta_4 = i \begin{bmatrix} 0_2 & 1_2 \\ -1_2 & 0_2 \end{bmatrix}, \]

\[ \beta_0 = \begin{bmatrix} 1_2 & 0_2 \\ 0_2 & 1_2 \end{bmatrix}, \gamma_5 = \begin{bmatrix} 1_2 & 0_2 \\ 0_2 & -1_2 \end{bmatrix}. \]  

(1)

1 Masses

Let

\[ \langle \rho (t, x, y, z), j_x (t, x, y, z), j_y (t, x, y, z), j_z (t, x, y, z) \rangle \]

be a probability current 3+1 vector field and \( \psi (t, x, y, z) \) be any complex spinor field:

\[ \psi = |\psi| \begin{bmatrix} \exp (i \gamma) \cos (\beta) \cos (\alpha) \\ \exp (i \theta) \sin (\beta) \cos (\alpha) \\ \exp (i \varphi) \cos (\chi) \sin (\alpha) \\ \exp (i \psi) \sin (\chi) \sin (\alpha) \end{bmatrix}. \]

In this case the following system of equations:

\[ \begin{align*}
\psi^\dagger \psi &= \rho, \\
\psi^\dagger \beta_1 \psi &= j_x, \\
\psi^\dagger \beta_2 \psi &= j_y, \\
\psi^\dagger \beta_3 \psi &= j_z
\end{align*} \]

(2)

has got the following form:
\[
\psi \dagger \psi = \rho,
\]
\[
|\psi|^2 \begin{pmatrix}
\cos^2 (\alpha) \sin (2 \beta) \cos (\theta - \gamma) & \\
- \sin^2 (\alpha) \sin (2 \chi) \cos (\varphi - \varphi) &
\end{pmatrix} = j_x,
\]
\[
|\psi|^2 \begin{pmatrix}
\cos^2 (\alpha) \sin (2 \beta) \sin (\theta - \gamma) & \\
- \sin^2 (\alpha) \sin (2 \chi) \sin (\varphi - \varphi) &
\end{pmatrix} = j_y,
\]
\[
|\psi|^2 \begin{pmatrix}
\cos^2 (\alpha) \cos (2 \beta) - \sin^2 (\alpha) \cos (2 \chi) &
\end{pmatrix} = j_z.
\]

Hence for every probability current vector \( \langle \rho, j_x, j_y, j_z \rangle \): the spinor \( \psi \), obeyed to this system, exists.

The operator \( \hat{U} (t, \Delta t) \), which acts in the set of these spinors, is denoted as the evolution operator for the spinor \( \psi (t, x, y, z) \), if:

\[
\psi (t + \Delta t, x, y, z) = \hat{U} (t, \Delta t) \psi (t, x, y, z).
\]

\( \hat{U} (t, \Delta t) \) is a linear operator.

The set of the spinors, for which \( \hat{U} (t, \Delta t) \) is the evolution operator, is denoted as the operator \( \hat{U} (t, \Delta t) \) space.

The operator space is the linear space.

Let for an infinitesimal \( \Delta t \):

\[
\hat{U} (t, \Delta t) = 1 + i \Delta t \hat{H} (t).
\]

Hence for an elements of the operator \( \hat{U} (t, \Delta t) \) space:

\[
i \hat{H} = \partial_t.
\]

If the functions \( \rho, j_x, j_y, j_z \) fulfill to the continuity equation [3]:

\[
\partial_t \rho + \partial_x j_x + \partial_y j_y + \partial_z j_z = 0
\]

then:

\[
\left( (\partial_t \psi^\dagger) \beta_0 + (\partial_x \psi^\dagger) \beta_1 + (\partial_y \psi^\dagger) \beta_2 + (\partial_z \psi^\dagger) \beta_3 \right) \psi =
\]

\[
= - \psi^\dagger (\beta_0 \partial_t + \beta_1 \partial_x + \beta_2 \partial_y + \beta_3 \partial_z) \psi.
\]
Let:

\[ \bar{Q} = \left( i\bar{H} + \beta_1 \partial_x + \beta_2 \partial_y + \beta_3 \partial_z \right). \]

Hence:

\[ \psi^\dagger \bar{Q}^\dagger \psi = -\psi^\dagger \bar{Q} \psi. \]

Hence \( i\bar{Q} (t, x, y, z) \) is the Hermitian for the matrix product operator. Hence a real functions \( \varphi_{i,j} (t, x, y, z) \) and \( \varpi_{i,j} (t, x, y, z) \) for which:

\[-i\bar{Q} = \]

\[
\begin{bmatrix}
\varphi_{1,1} & \varphi_{1,2} + i\varpi_{1,2} & \varphi_{1,3} + i\varpi_{1,3} & \varphi_{1,4} + i\varpi_{1,4} \\
\varphi_{1,2} - i\varpi_{1,2} & \varphi_{2,2} & \varphi_{2,3} + i\varpi_{2,3} & \varphi_{2,4} + i\varpi_{2,4} \\
\varphi_{1,3} - i\varpi_{1,3} & \varphi_{2,3} - i\varpi_{2,3} & \varphi_{3,3} & \varphi_{3,4} + i\varpi_{3,4} \\
\varphi_{1,4} - i\varpi_{1,4} & \varphi_{2,4} - i\varpi_{2,4} & \varphi_{3,4} - i\varpi_{3,4} & \varphi_{4,4}
\end{bmatrix}
\]

exist.

Let \( G_t, G_z, K_t \) and \( K_z \) are the solution of the following system of equations:

\[
\begin{cases}
G_t + G_z + K_t + K_z = \varphi_{1,1}, \\
G_t - G_z + K_t - K_z = \varphi_{2,2}, \\
G_t - G_z - K_t + K_z = \varphi_{3,3}, \\
G_t + G_z - K_t - K_z = \varphi_{4,4};
\end{cases}
\]

\( G_x \) and \( K_x \) are the solution of the following system of equations:

\[
\begin{cases}
G_x + K_x = \varphi_{1,2}, \\
-G_x + K_x = \varphi_{3,4};
\end{cases}
\]

\( G_y \) and \( K_y \) are the solution of the following system of equations:

\[
\begin{cases}
-G_y - K_y = \varpi_{1,2}, \\
G_y - K_x = \varpi_{3,4}.
\end{cases}
\]

In this case:

\[-i\bar{Q} = \]

\[
= (G_t\beta_0 + G_x\beta_1 + G_y\beta_2 + G_z\beta_3) + \\
+ (K_t\beta_0 + K_x\beta_1 + K_y\beta_2 + K_z\beta_3) \gamma_5 + \]

\]
If
\[
\begin{cases}
(M_0 + M_{z,0}) = \varphi_{1,3}, \\
(M_0 - M_{z,0}) = \varphi_{2,4},
\end{cases}
\]

and
\[
\begin{cases}
(M_4 + M_{z,4}) = \varphi_{1,3}, \\
(M_4 - M_{z,4}) = \varphi_{2,4},
\end{cases}
\]

and
\[
\begin{cases}
(M_{x,0} + M_{y,4}) = \varphi_{1,4}, \\
(M_{x,0} - M_{y,4}) = \varphi_{2,3},
\end{cases}
\]

then
\[
\begin{bmatrix}
0 & 0 & \varphi_{1,3} + i\varphi_{1,3} & \varphi_{1,4} + i\varphi_{1,4} \\
0 & 0 & \varphi_{2,3} + i\varphi_{2,3} & \varphi_{2,4} + i\varphi_{2,4} \\
\varphi_{1,3} - i\varphi_{1,3} & \varphi_{2,3} - i\varphi_{2,3} & 0 & 0 \\
\varphi_{1,4} - i\varphi_{1,4} & \varphi_{2,4} - i\varphi_{2,4} & 0 & 0
\end{bmatrix}
\]

\[= M_0 \gamma_0 + M_4 \beta_4 - M_{x,0} \gamma_4 - M_{x,4} \zeta^4 + M_{y,0} \gamma_0 + M_{y,4} \eta^4 - M_{z,0} \gamma_0 - M_{z,4} \theta^4;
\]

here \(\gamma_4, \zeta^4, \gamma_0, \eta^4, \gamma_0, \theta^4\) are the chromatic pentads \(\mathbb{I}, \mathbb{E}\) members and \(\gamma_0\) and \(\beta_4\) is the light pentad \(\mathbb{B}\) members. Since in this paper I will not consider quarks then everywhere below:

\[M_{x,0} = M_{x,4} = M_{y,0} = M_{y,4} = M_{z,0} = M_{z,4} = 0\]

hence:
\[
-i\hat{Q} = (G_t\beta_0 + G_x\beta_1 + G_y\beta_2 + G_z\beta_3) + \\
+ (K_t\beta_0 + K_x\beta_1 + K_y\beta_2 + K_z\beta_3) \gamma_5 + \\
+ M_0\gamma_0 + M_4\beta_4.
\]

\{\beta_1, \beta_2, \beta_3, \beta_4, \gamma_0\} is the Clifford pentad.

If \(j_x = \rho u_x, j_y = \rho u_y, j_z = \rho u_z\) then \(u_x, u_y, u_z\) are the components of the average velocity. Hence \(\beta_1, \beta_2, \beta_3\) define the components of the average velocity \(\mathbf{2}\).

If

\[
\begin{align*}
    j_x &= \psi^\dagger \gamma_0 \psi, \\
    j_x &= \psi^\dagger \beta_4 \psi, \\
    j_x &= \rho u_x, \\
    j_x &= \rho u_4
\end{align*}
\]

then

\[
\begin{align*}
    u_x &= \sin(2\alpha) \begin{bmatrix}
    \cos(\beta) \cos(\chi) \cos(\gamma - \varphi) + \\
    + \sin(\beta) \sin(\chi) \cos(\theta - \upsilon)
\end{bmatrix}, \\
    u_x &= \sin(2\alpha) \begin{bmatrix}
    \cos(\beta) \cos(\chi) \sin(\gamma - \varphi) + \\
    + \sin(\beta) \sin(\chi) \sin(\theta - \upsilon)
\end{bmatrix}
\end{align*}
\]

and if \(\rho \neq 0\) then

\[
    u_x^2 + u_y^2 + u_z^2 + u_{x_5}^2 + u_{x_4}^2 = 1.
\]

From \[\mathbb{I}\] the maximal velocity of the information propagation in the space-time is 1.

Hence of only all five elements of the Clifford pentad lends the entire kit of the velocity components and, for the completeness, yet two "space" coordinates \(x_5\) and \(x_4\) should be added to our three \(x, y, z\).

Let

\[
\Psi(t, x, y, z, x_5, x_4) = \\
\psi(t, x, y, z) \exp(-i(x_5M_0(t, x, y, z) + x_4M_4(t, x, y, z))).
\]

In this case the motion equation is the following:
\[ \beta_0 i \partial_t \Psi + \beta_1 i \partial_x \Psi + \beta_2 i \partial_y \Psi + \beta_3 i \partial_z \Psi + \gamma_0 i \partial_{x_5} \Psi + \beta_4 i \partial_{x_4} \Psi + (G_t \beta_0 + G_x \beta_1 + G_y \beta_2 + G_z \beta_3) \Psi + (K_t \beta_0 + K_x \beta_1 + K_y \beta_2 + K_z \beta_3) \gamma_5 \Psi = 0 \] (4)

Let an evolution operator \( \hat{U} (t, \Delta t) \) be denoted as a Planck evolution operator if a tiny positive real number \( h \) and a functions \( N_\varphi (t, x, y, z) \) and \( N_\varphi (t, x, y, z) \), having a range of values in the set of the integer numbers, exist for which:

\[ M_0 = N_\varphi h \text{ and } M_4 = N_\varphi h. \]

Let \(-\frac{\pi}{h} \leq x_5 \leq \frac{\pi}{h}, \quad -\frac{\pi}{h} \leq x_4 \leq \frac{\pi}{h}, \)

\[ \Psi (t, x, y, z, \pm \frac{\pi}{h}, x_4) = 0 \text{ and } \Psi (t, x, y, z, x_5, \pm \frac{\pi}{h}) = 0. \]

In this case the Fourier series for \( \Psi \) is of the following form:

\[ \Psi (t, x, y, z, x_5, x_4) = \sum_{\nu, \kappa} \phi (t, x, y, z, \nu, \kappa) \exp \left( -i h \left( \nu x_5 + \kappa x_4 \right) \right). \]

Here:

\[ \delta_{-\varphi, N_\varphi} = \frac{h}{2\pi} \int_{-\frac{\pi}{h}}^{\frac{\pi}{h}} \exp \left( i h \left( \nu x_5 \right) \right) \exp \left( i N_\varphi h x_5 \right) dx_5 = \frac{\sin \left( \pi \left( \nu + N_\varphi \right) \right)}{\pi \left( \nu + N_\varphi \right)}, \]

\[ \delta_{-\kappa, N_\varphi} = \frac{h}{2\pi} \int_{-\frac{\pi}{h}}^{\frac{\pi}{h}} \exp \left( i h \left( \kappa x_4 \right) \right) \exp \left( i N_\varphi h x_4 \right) dx_4 = \frac{\sin \left( \pi \left( \kappa + N_\varphi \right) \right)}{\pi \left( \kappa + N_\varphi \right)}. \]

If denote:

\[ \phi (t, x, y, z, -\nu, -\kappa) = \psi (t, x, y, z) \delta_{-\nu, N_\varphi(t,x,y,z)} \delta_{-\kappa, N_\varphi(t,x,y,z)} \]

then

\[ \Psi (t, x, y, z, x_5, x_4) = \sum_{\nu, \kappa} \phi (t, x, y, z, \nu, \kappa) \exp \left( -i h \left( \nu x_5 + \kappa x_4 \right) \right). \]

From the properties of \( \delta \) in every point \( \langle t, x, y, z \rangle \): either

\[ \Psi (t, x, y, z, x_5, x_4) = 0 \]

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or an integer numbers $\nu_0$ and $\kappa_0$ exist for which:

$$\Psi(t, x, y, z, x_5, x_4) = \phi(t, x, y, z, \nu_0, \kappa_0) \exp(-i\hbar(\nu_0 x_5 + \kappa_0 x_4)). \quad (5)$$

That is for the every space-time point: either this point is empty or single mass is placed in this point.

Let on the space of these spinors the scalar product $\Phi \ast \Psi$ be denoted as the following:

$$\Phi \ast \Psi = \left(\frac{\hbar}{2\pi}\right)^2 \int_{-\pi}^{\pi} dx_5 \int_{-\pi}^{\pi} dx_4 \cdot (\Phi^\dagger \Psi).$$

In this case:

$$\Psi \ast \beta_\mu \Psi = \psi^\dagger \beta_\mu \psi.$$ 

for $0 \leq \mu \leq 3$

Hence from (5):

$$\begin{cases}
\Psi^\dagger \ast \Psi = \rho, \\
\Psi^\dagger \ast \beta_1 \Psi = j_x, \\
\Psi^\dagger \ast \beta_2 \Psi = j_y, \\
\Psi^\dagger \ast \beta_3 \Psi = j_z.
\end{cases}$$

### 1.1 Bi-zero-nonzero-mass state

Let

$$\Psi(t, x, y, z, x_5, x_4) = \phi(t, x, y, z, 0, 0) + \phi(t, x, y, z, n, k) \exp(i\hbar(n x_5 + k x_4)).$$

Let $\epsilon_\mu (1 \leq k \leq 4)$ be a basis in which pentad $\bar{\beta}$ has got a form (P) and let

$$\Psi(x_5, x_4) = \sum_{r=1}^{4} \phi_r (0, 0) \epsilon_r + \exp(-i\hbar(n x_5 + k x_4)) \sum_{k=1}^{4} \phi_k (n, k) \epsilon_k \quad (6)$$

Hence in the basis
\[ \langle \epsilon_r, \exp (-i \hbar (nX + kY)) \epsilon_k \rangle: \]

a 8-components bi-spinor:

\[
\Psi = \begin{bmatrix}
\phi_1 (0,0) \\
\phi_2 (0,0) \\
\phi_3 (0,0) \\
\phi_4 (0,0) \\
\phi_1 (n,k) \\
\phi_2 (n,k) \\
\phi_3 (n,k) \\
\phi_4 (n,k)
\end{bmatrix}
\]

corresponds to \( \Psi \).

From (5): in every point \( \langle t, x, y, z \rangle \):

\[
\Psi = \begin{bmatrix}
\phi_1 (0,0) \\
\phi_2 (0,0) \\
\phi_3 (0,0) \\
\phi_4 (0,0) \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\text{ or } \Psi = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
\phi_1 (n,k) \\
\phi_2 (n,k) \\
\phi_3 (n,k) \\
\phi_4 (n,k)
\end{bmatrix}
\]

(7)

of \( \delta \) characteristics.

Let us denote:

\[ \phi_1 \epsilon_1 + \phi_2 \epsilon_2 = \phi_L \text{ and } \phi_3 \epsilon_3 + \phi_4 \epsilon_4 = \phi_R. \]

Hence from (6):

\[ \Psi (x_5, x_4) = \phi_L (0,0) + \phi_R (0,0) + \exp (-i \hbar (nX_5 + kX_4)) (\phi_L (n,k) + \phi_R (n,k)). \]

(8)

If use denotation:

\[ \vartheta = \begin{bmatrix}
\vartheta \\
0 \\
0 \\
\vartheta
\end{bmatrix}, \]
\[ \gamma = \begin{bmatrix} -\gamma_0 & 0_4 \\ 0_4 & \gamma_0 \end{bmatrix}, \quad \beta = \begin{bmatrix} -\beta_4 & 0_4 \\ 0_4 & \beta_4 \end{bmatrix} \]

and

\[ n = \begin{bmatrix} 0_4 & 0_4 \\ 0_4 & n_{14} \end{bmatrix}, \quad k = \begin{bmatrix} 0_4 & 0_4 \\ 0_4 & k_{14} \end{bmatrix} \]

then the motion equation is the following:

\[ \frac{\beta_0}{i} \partial_t \Psi + \frac{\beta_1}{i} \partial_x \Psi + \frac{\beta_2}{i} \partial_y \Psi + \frac{\beta_3}{i} \partial_z \Psi + \left( h_\mu \gamma^\mu \Psi - h_k \beta^k \Psi + \left( G_{t\beta_0} + G_{x\beta_1} + G_{y\beta_2} + G_{z\beta_3} \right) \Psi + \left( K_{t\beta_0} + K_{x\beta_1} + K_{y\beta_2} + K_{z\beta_3} \right) \gamma_5 \Psi \right) = 0 \] (9)

and

\[ \begin{cases} \Psi^\dagger \Psi = \rho, \\ \Psi^\dagger \beta_1 \Psi = j_x, \\ \Psi^\dagger \beta_2 \Psi = j_y, \\ \Psi^\dagger \beta_3 \Psi = j_z. \end{cases} \] (10)

If use the following denotation: \( t = x_0, x = x_1, y = x_2, z = x_3, \partial_\mu = \frac{\partial}{\partial x_\mu} \)

then the lagrangian has got the following form:

\[ \mathcal{L}_f = 0.5i \left( \left( \sum_{\mu=0}^3 \Psi^\dagger \beta_\mu \partial_\mu \Psi \right) - \left( \sum_{\mu=0}^3 \partial_\mu \Psi^\dagger \beta_\mu \Psi \right) \right) - \left( \Psi^\dagger h_\mu \gamma^\mu \Psi + \Psi^\dagger h_k \beta^k \Psi \right) + \Psi^\dagger \left( \sum_{\mu=0}^3 G_{x_\mu} \beta_\mu \right) \Psi + \Psi^\dagger \left( \sum_{\mu=0}^3 K_{x_\mu} \beta_\mu \right) \gamma_5 \Psi. \] (10)

This lagrangian is invariant for the rotation of \( xOy, yOz, xOz \) and for the Lorentz transformation of \( tOx, tOy, tOz \) and \( G_{x_k} \) and \( K_{x_k} \) behaves as the 4-vector fields \( \gamma_5 \).

1.1.1 Transformations

If \( U \) is an \( 8 \times 8 \) complex matrix, \( \Psi' = U \Psi \) and

\[ \begin{cases} \Psi^\dagger \beta_1 \Psi' = j_x, \\ \Psi^\dagger \beta_2 \Psi' = j_y, \\ \Psi^\dagger \beta_3 \Psi' = j_z. \end{cases} \] (11)
then for $1 \leq k \leq 3$: $U^\dagger \beta_k U = \beta_k$. In this case a real numbers $a'', b'', c'', g'', u'', v'', k, s, a', b', c', g', u', v', k', s'$ exist for which:

$$
U = \begin{bmatrix}
(a'' + b''i)1_2 & 0_2 & (c'' + ig'')1_2 & 0_2 \\
0_2 & (a' + b'i)1_2 & 0_2 & (c' + ig')1_2 \\
(u'' + iv'')1_2 & 0_2 & (k'' + is'')1_2 & 0_2 \\
0_2 & (u' + iv')1_2 & 0_2 & (k' + is')1_2
\end{bmatrix}.
$$

If $\Psi^\dagger \Psi' = \rho$ then $U^\dagger U = 1_8$. Hence:

$$
v''^2 + b''^2 + u''^2 + a''^2 = 1,
$$

$$
c''^2 + g''^2 + k''^2 + s''^2 = 1,
$$

$$
s'' = -\frac{a'' g'' u'' - u'' b'' c'' + a'' c'' v'' + b'' g'' v''}{u''^2 + v''^2},
$$

$$
k'' = \frac{-u'' a'' c'' - u'' b'' g'' + v'' a'' g'' - b'' c'' v''}{u''^2 + v''^2}.
$$

$$
v'^2 + b'^2 + u'^2 + a'^2 = 1,
$$

$$
c'^2 + g'^2 + k'^2 + s'^2 = 1,
$$

$$
s' = -\frac{a' g' u' - u' b' c' + a' c' v' + b' g' v'}{u'^2 + v'^2},
$$

$$
k' = \frac{-u' a' c' - u' b' g' + v' a' g' - b' c' v'}{u'^2 + v'^2}.
$$

$U$ has got 4 eigenvalues: $\exp(i\alpha_1)$, $\exp(i\alpha_2)$, $\exp(i\alpha_3)$, $\exp(i\alpha_4)$ for 8 orthogonal eigenvectors:

$\varepsilon_{1,1}, \varepsilon_{1,2}, \varepsilon_{2,1}, \varepsilon_{2,2}, \varepsilon_{3,1}, \varepsilon_{3,2}, \varepsilon_{4,1}, \varepsilon_{4,2}$.

Let

$$
K = \begin{bmatrix}
\varepsilon_{1,1} & \varepsilon_{1,2} & \varepsilon_{2,1} & \varepsilon_{2,2} & \varepsilon_{3,1} & \varepsilon_{3,2} & \varepsilon_{4,1} & \varepsilon_{4,2}
\end{bmatrix}.
$$

Let $\theta_1, \theta_2, \theta_3, \theta_4$ be the solution of the system of the equations:
\[
\begin{align*}
\theta_1 + \theta_2 + \theta_3 + \theta_4 &= \alpha_1, \\
\theta_1 + \theta_2 - \theta_3 - \theta_4 &= \alpha_1, \\
\theta_1 - \theta_2 + \theta_3 - \theta_4 &= \alpha_1, \\
\theta_1 - \theta_2 - \theta_3 + \theta_4 &= \alpha_1.
\end{align*}
\]

and

\[U_1 = \exp(i\theta_1) 1_8,\]

\[U_2 = K \begin{bmatrix} 
\exp(i\theta_2) 1_4 & 0_4 \\
0_4 & \exp(-i\theta_2) 1_4
\end{bmatrix} K^\dagger,\]

\[U_3 = K \begin{bmatrix} 
\exp(i\theta_3) 1_2 & 0_2 & 0_2 & 0_2 \\
0_2 & \exp(-i\theta_3) 1_2 & 0_2 & 0_2 \\
0_2 & 0_2 & \exp(i\theta_3) 1_2 & 0_2 \\
0_2 & 0_2 & 0_2 & \exp(-i\theta_3) 1_2
\end{bmatrix} K^\dagger,\]

\[U_4 = K \begin{bmatrix} 
\exp(i\theta_4) 1_2 & 0_2 & 0_2 & 0_2 \\
0_2 & \exp(-i\theta_4) 1_2 & 0_2 & 0_2 \\
0_2 & 0_2 & \exp(-i\theta_4) 1_2 & 0_2 \\
0_2 & 0_2 & 0_2 & \exp(i\theta_4) 1_2
\end{bmatrix} K^\dagger.\]

In this case:

\[U_1 U_2 U_3 U_4 = U\]

and

\[U_2 = \begin{bmatrix} 
\exp(i\theta_2) 1_2 & 0_2 & 0_2 & 0_2 \\
0_2 & \exp(-i\theta_2) 1_2 & 0_2 & 0_2 \\
0_2 & 0_2 & \exp(i\theta_2) 1_2 & 0_2 \\
0_2 & 0_2 & 0_2 & \exp(-i\theta_2) 1_2
\end{bmatrix}\]

and a real number \(a, b, c, g, u, v, k, s\) exist for which:

\[U_3 U_4 = \begin{bmatrix} 
(a + ib) 1_2 & 0_2 & (c + ig) 1_2 & 0_2 \\
0_2 & (u + iv) 1_2 & 0_2 & (k + is) 1_2 \\
(-c + ig) 1_2 & 0_2 & (a - ib) 1_2 & 0_2 \\
0_2 & (-k + is) 1_2 & 0_2 & (u - iv) 1_2
\end{bmatrix}\]
and

\[ a^2 + b^2 + c^2 + g^2 = 1, \]
\[ u^2 + v^2 + r^2 + s^2 = 1. \]

If

\[
U^{(+)} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \tag{12}
\]

and

\[
U^{(-)} = \begin{bmatrix}
(a + ib) & 0 & (c + ig) & 0 \\
0 & 1 & 0 & 0 \\
(-c + ig) & 0 & (a - ib) & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

then

\[ U_3 U_4 = U^{(-)} U^{(+)} = U^{(+)} U^{(-)}. \]

**B-boson**

\[
U_1 U_2 = \begin{bmatrix}
e^{i(\theta_1 + \theta_2)} & 0 & 0 & 0 \\
0 & e^{i(\theta_1 - \theta_2)} & 0 & 0 \\
0 & 0 & e^{i(\theta_1 + \theta_2)} & 0 \\
0 & 0 & 0 & e^{i(\theta_1 - \theta_2)}
\end{bmatrix}
\]

Let \( \chi \) and \( \varsigma \) be the solution of the following set of equations:

\[
\begin{align*}
0.5\chi + \varsigma &= \theta_1 + \theta_2, \\
\chi + \varsigma &= \theta_1 - \theta_2,
\end{align*}
\]

i.e.:

\[
\begin{align*}
\chi &= -4\theta_2, \\
\varsigma &= \theta_1 + 3\theta_2.
\end{align*}
\]

Let

\[
\hat{U} = \exp(i\varsigma) 1_8
\]
and
\[
\tilde{U} = \begin{bmatrix}
\exp \left( i \frac{\chi}{2} \right) 1_2 & 0_2 & 0_2 & 0_2 \\
0_2 & \exp (i\chi) 1_2 & 0_2 & 0_2 \\
0_2 & 0_2 & \exp \left( i \frac{\chi}{2} \right) 1_2 & 0_2 \\
0_2 & 0_2 & 0_2 & \exp (i\chi) 1_2
\end{bmatrix}.
\]

In that case:
\[
\tilde{U} \hat{U} = U_1 U_2.
\]

Let \( g_1 \) be a positive real number and for \( \mu \in \{t, x, y, z\} \): \( F_\mu \) and \( B_\mu \) be the solutions of the following systems of the equations:
\[
\begin{align*}
-0.5 g_1 B_\mu + F_\mu &= G_\mu + K_\mu \\
- g_1 B_\mu + F_\mu &= G_\mu - K_\mu
\end{align*}
\]
i.e.:
\[
B_\mu = \frac{4}{g_1} K_\mu \\
F_\mu = G_\mu + 3 K_\mu.
\]

Let the charge matrix be defined as the following:
\[
Y = -\begin{bmatrix}
1_2 & 0_2 & 0_2 & 0_2 \\
0_2 & 2 \cdot 1_2 & 0_2 & 0_2 \\
0_2 & 0_2 & 1_2 & 0_2 \\
0_2 & 0_2 & 0_2 & 2 \cdot 1_2
\end{bmatrix}
\]

In that case from (13):
\[
\beta_0 i \partial_t \Psi + \beta_1 i \partial_x \Psi + \beta_2 i \partial_y \Psi + \beta_3 i \partial_z \Psi + \frac{\hbar \Omega}{\hbar k} \Psi + (F_t \beta_0 + F_x \beta_1 + F_y \beta_2 + F_z \beta_3) \Psi + 0.5 g_1 Y (B_t \beta_0 + B_x \beta_1 + B_y \beta_2 + B_z \beta_3) \Psi = 0.
\]
\( \Psi \rightarrow \Psi' = (\bar{U}\Psi) \),
\( n \rightarrow n' \),
\( k \rightarrow k' \),
\( F_\mu \rightarrow F_\mu' \),
\( B_\mu \rightarrow B_\mu' \)

then:
\[
\beta_0 i \partial_t (\bar{U}\Psi) + \beta_1 i \partial_x (\bar{U}\Psi) + \beta_2 i \partial_y (\bar{U}\Psi) + \beta_3 i \partial_z (\bar{U}\Psi) + \\
- h \eta \gamma U \Psi - h k \beta U \Psi + \\
+ \left( F_t \beta_0 + F_x \beta_1 + F_y \beta_2 + F_z \beta_3 \right) \bar{U}\Psi + \\
+ 0.5 g \gamma (B_t \beta_0 + B_x \beta_1 + B_y \beta_2 + B_z \beta_3) \bar{U}\Psi = 0
\]

hence:
\[
\beta_0 i \left( \partial_t \bar{U} \right) \Psi + \beta_0 i \bar{U} \partial_t \Psi + \beta_1 i \left( \partial_x \bar{U} \right) \Psi + \beta_1 i \bar{U} \partial_x \Psi + \\
+ \beta_2 i \left( \partial_y \bar{U} \right) \Psi + \beta_2 i \bar{U} \partial_y \Psi + \beta_3 i \left( \partial_z \bar{U} \right) \Psi + \beta_3 i \bar{U} \partial_z \Psi + \\
- h \eta \gamma U \Psi - h k \beta U \Psi + \\
+ \left( F_t \beta_0 + F_x \beta_1 + F_y \beta_2 + F_z \beta_3 \right) \bar{U}\Psi + \\
+ 0.5 g \gamma (B_t \beta_0 + B_x \beta_1 + B_y \beta_2 + B_z \beta_3) \bar{U}\Psi = 0
\]

Since
\[
\partial_\mu \bar{U} = i \frac{\partial_\mu \chi}{2} \begin{bmatrix}
\exp \left( i \frac{\chi}{2} \right) & 0 & 0 & 0 \\
0 & 2 \exp \left( i \chi \right) & 0 & 0 \\
0 & 0 & \exp \left( i \frac{\chi}{2} \right) & 0 \\
0 & 0 & 0 & 2 \exp \left( i \chi \right)
\end{bmatrix}
\]

then
\[
\partial_\mu \bar{U} = -i \frac{\partial_\mu \chi}{2} \bar{U}\;
\]

Hence
\[
\begin{align*}
&\left(\beta_0 i \tilde{U} \partial_t + \beta_1 i \tilde{U} \partial_x + \beta_2 i \tilde{U} \partial_y + \beta_3 i \tilde{U} \partial_z\right) \Psi + \\
&\quad - h \left(u \gamma \tilde{U} + k \beta \tilde{U}\right) \Psi + \\
&\quad + \left(F^t_0 \beta_0 + F^x_0 \beta_1 + F^y_0 \beta_2 + F^z_0 \beta_3\right) \tilde{U} \Psi + \\
&\quad + 0.5 \left(\left(\beta_0 (g_1 Y_i B^i_t + \beta_0 Y \partial_t \chi) + \beta_1 (g_1 Y_i B^i_x + \beta_1 Y \partial_x \chi) + \beta_2 (g_1 Y_i B^i_y + \beta_2 Y \partial_y \chi) + \beta_3 (g_1 Y_i B^i_z + \beta_3 Y \partial_z \chi)\right) \tilde{U} \Psi = 0.
\end{align*}
\]

Since \(Y \beta = \beta Y\) then

\[
\begin{align*}
&\left(\beta_0 i \tilde{U} \partial_t + \beta_1 i \tilde{U} \partial_x + \beta_2 i \tilde{U} \partial_y + \beta_3 i \tilde{U} \partial_z\right) \Psi + \\
&\quad - h \left(u \gamma \tilde{U} + k \beta \tilde{U}\right) \Psi + \\
&\quad + \left(F^t_0 \beta_0 + F^x_0 \beta_1 + F^y_0 \beta_2 + F^z_0 \beta_3\right) \tilde{U} \Psi + \\
&\quad + 0.5 \left(\beta_0 (g_1 Y_i B^i_t + \beta_0 Y \partial_t \chi) + \beta_1 (g_1 Y_i B^i_x + \beta_1 Y \partial_x \chi) + \beta_2 (g_1 Y_i B^i_y + \beta_2 Y \partial_y \chi) + \beta_3 (g_1 Y_i B^i_z + \beta_3 Y \partial_z \chi)\right) \tilde{U} \Psi = 0.
\end{align*}
\]

hence:

\[
\begin{align*}
\tilde{U}^\dagger \left(\beta_0 i \tilde{U} \partial_t + \beta_1 i \tilde{U} \partial_x + \beta_2 i \tilde{U} \partial_y + \beta_3 i \tilde{U} \partial_z\right) \Psi + \\
\quad - h \tilde{U}^\dagger \left(u \gamma \tilde{U} + k \beta \tilde{U}\right) \Psi + \\
\quad + \tilde{U}^\dagger \left(F^t_0 \beta_0 + F^x_0 \beta_1 + F^y_0 \beta_2 + F^z_0 \beta_3\right) \tilde{U} \Psi + \\
\quad + 0.5 \tilde{U}^\dagger Y \left(\beta_0 (g_1 Y_i B^i_t + \beta_0 Y \partial_t \chi) + \beta_1 (g_1 Y_i B^i_x + \beta_1 Y \partial_x \chi) + \beta_2 (g_1 Y_i B^i_y + \beta_2 Y \partial_y \chi) + \beta_3 (g_1 Y_i B^i_z + \beta_3 Y \partial_z \chi)\right) \tilde{U} \Psi = 0.
\end{align*}
\]

Because:

\[
\begin{align*}
\tilde{U}^\dagger \gamma \tilde{U} &= \cos \left(\frac{\beta}{2}\right) \gamma - \sin \left(\frac{\beta}{2}\right) \beta, \\
\tilde{U}^\dagger \beta \tilde{U} &= \cos \left(\frac{\beta}{2}\right) \beta + \sin \left(\frac{\beta}{2}\right) \gamma, \\
\tilde{U}^\dagger \tilde{U} &= I_8, \\
\beta \mu \tilde{U} &= \tilde{U} \beta \mu, \\
\tilde{U}^\dagger Y \tilde{U} &= \Sigma
\end{align*}
\]

then
\[
-h \left( \psi' \left( \cos \left( \frac{x}{2} \right) \gamma - \sin \left( \frac{x}{2} \right) \beta \right) + k' \left( \cos \left( \frac{x}{2} \right) \beta + \sin \left( \frac{x}{2} \right) \gamma \right) \right) \psi + \\
+ \left( F'_{t} \beta_{0} + F'_{x} \beta_{1} + F'_{y} \beta_{2} + F'_{z} \beta_{3} \right) \Psi + \\
+0.5 \left( \beta_{0} \left( g_{1} B'_{t} + \partial_{t} \chi \right) + \beta_{1} \left( g_{1} B'_{x} + \partial_{x} \chi \right) + \beta_{2} \left( g_{1} B'_{y} + \partial_{y} \chi \right) + \beta_{3} \left( g_{1} B'_{z} + \partial_{z} \chi \right) \right) \Psi = 0.
\]

Therefore from (13):

\[
F'_{x} = F_{x}, \\
B'_{\mu} = B_{\mu} - \frac{1}{g_{1}} \partial_{\mu} \chi, \\
n' = -k \sin \frac{x}{2} + n \cos \frac{x}{2}, \\
k' = k \cos \frac{x}{2} + n \sin \frac{x}{2}.
\]

But \( k \) and \( n \) are an integer numbers and \( k' \) and \( n' \) must be an integer numbers, too.

A triplet \( \langle l, n, k \rangle \) of integer numbers is a Fermat triplet if

\[
l^2 = n^2 + k^2.
\]

Let \( \varepsilon \) be any tiny positive real number. An integer number \( l \) is a father number with a precise \( \varepsilon \) if for each real number \( \chi \) and for every Fermat triplet \( \langle l, n, k \rangle \) a Fermat triplet \( \langle l', n', k' \rangle \) exists for which:

\[
\left| -k \sin \frac{x}{2} + n \cos \frac{x}{2} - n' \right| < \varepsilon, \\
\left| k \cos \frac{x}{2} + n \sin \frac{x}{2} - k' \right| < \varepsilon.
\]

For every \( \varepsilon \): denumerable many of a father numbers with a precise \( \varepsilon \) exist.

Excuse me, but I mean that a masses of the real members of the particles families are defined by a father numbers with a precise \( h \). I.e. denumerable many of a families exist.

Therefore for the (17) transformation from (8):

\[
\Psi \left( x_{5}, x_{4} \right) = \sum_{r=1}^{4} \phi_{r} \left( 0, 0 \right) \epsilon_{r} + \exp \left( -ih \left( n x_{5} + k x_{4} \right) \right) \sum_{k=1}^{4} \phi_{k} \left( n, k \right) \epsilon_{k} = \\
= \phi_{L} \left( 0, 0 \right) + \phi_{R} \left( 0, 0 \right) + \exp \left( -ih \left( n x_{5} + k x_{4} \right) \right) \left( \phi_{L} \left( n, k \right) + \phi_{R} \left( n, k \right) \right) \rightarrow
\]

17
\[ \Psi^i(x_5, x_4) = \]
\[ = \exp \left( i \frac{x}{2} \right) \phi_L (0, 0) + \exp \left( i \chi \right) \phi_R (0, 0) + \]
\[ + \exp \left( -i \hbar \left( -k \sin \frac{x}{2} + n \cos \frac{x}{2} \right) x_5 + \left( k \cos \frac{x}{2} + n \sin \frac{x}{2} \right) x_4 \right) \cdot \]
\[ \exp \left( i \frac{x}{2} \right) \phi_L (n, k) + \exp \left( i \chi \right) \phi_R (n, k) \right) \cdot \]

**U(–) transformation**  
U(–) has got the following eigenvalues and eigenvectors:

for the eigenvalue 1: eigenvectors:

\[
\begin{bmatrix}
0 \\
0 \\
1 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}, \quad
\begin{bmatrix}
0 \\
0 \\
1 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}, \quad
\begin{bmatrix}
0 \\
0 \\
0 \\
1 \\
0 \\
0 \\
0
\end{bmatrix}, \quad
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
1
\end{bmatrix}; \quad (15)
\]

for the eigenvalue \( w = a + i \sqrt{1 - a^2} \): eigenvectors:

\[
L_3 = \frac{1}{\sqrt{2} \sqrt{1 - a^2} (b + \sqrt{1 - a^2})} \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}, \quad (16)
\]

\[
L_4 = \frac{1}{\sqrt{2} \sqrt{1 - a^2} (b + \sqrt{1 - a^2})} \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}; \quad (17)
\]
for eigenvalue $w^* = a - i\sqrt{1-a^2}$: eigenvectors:

\[
\mathcal{L}_7 = \frac{1}{\sqrt{2}\sqrt{1-a^2}(b+\sqrt{1-a^2})} \begin{bmatrix}
ic - g \\
0 \\
0 \\
0
\end{bmatrix}, \quad (18)
\]

\[
\mathcal{L}_8 = \frac{1}{\sqrt{2}\sqrt{1-a^2}(b+\sqrt{1-a^2})} \begin{bmatrix}
0 \\
ic - g \\
0 \\
0
\end{bmatrix}. \quad (19)
\]

Hence the space of $\mathcal{U}^{(-)}$ is divided on three orthogonal subspace:

- the 4-dimensional $\mathcal{U}_1^{(-)}$ on the basis $\langle \mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_5, \mathcal{L}_6 \rangle$ with eigenvalue 1,
- the 2-dimensional $\mathcal{U}_w^{(-)}$ on the basis $\langle \mathcal{L}_3, \mathcal{L}_4 \rangle$ with eigenvalue $w$ and
- the 2-dimensional $\mathcal{U}_{w^*}^{(-)}$ on the basis $\langle \mathcal{L}_7, \mathcal{L}_8 \rangle$ with eigenvalue $w^*$.

Let

\[ \mathcal{L}_k = \gamma \mathcal{L}_k. \]

In this case $\langle \mathcal{L}_3, \mathcal{L}_4, \mathcal{L}_7, \mathcal{L}_8 \rangle$ is the orthonormal basis of $\mathcal{U}_1^{(-)}$. Let $\mathcal{U}_6^{(-)}$ be the space on the basis $\langle \mathcal{L}_3, \mathcal{L}_4, \mathcal{L}_5 \rangle$ and $\mathcal{U}_{w^*}^{(-)}$ be the space on the basis $\langle \mathcal{L}_7, \mathcal{L}_8, \mathcal{L}_7, \mathcal{L}_8 \rangle$.

\[ \Psi_o = \pi_o \Psi, \quad \Psi_* = \pi_* \Psi, \]

where $\Psi_o \in \mathcal{U}_6^{(-)}$ and $\Psi_* \in \mathcal{U}_{w^*}^{(-)}$. (20)

In this case:

\[ \pi_o = \frac{1}{2\sqrt{1-a^2}} \begin{bmatrix}
(b + \sqrt{1-a^2})_4 \\
(ic + g) \gamma_5 \\
\sqrt{1-a^2 - b} \end{bmatrix}, \]

19
\[ \pi_* = \frac{1}{2\sqrt{1-a^2}} \left[ \begin{array}{ccc} (\sqrt{1-a^2} - b) & (ic - g) \gamma_5 \\ -g - ic & (b + \sqrt{1-a^2}) \end{array} \right] . \]

Hence

\[ \begin{align*}
\Psi_o (x_5, x_4) &= \frac{1}{2\sqrt{1-a^2}} \cdot \\
&\cdot \left( \begin{array}{c}
(b + \sqrt{(1-a^2)} - (ic - g) (\phi_L (0,0) + \phi_R (0,0)) + \\
\exp(-ih (nx_5 + kx_4)) \cdot (ic + g) (\phi_L (0,0) - \phi_R (0,0)) + \\
+ (b + \sqrt{(1-a^2)}) (\phi_L (n,k) + \phi_R (n,k))
\end{array} \right), \quad \text{(21)}
\end{align*} \]

\[ \begin{align*}
\Psi_* (x_5, x_4) &= \frac{1}{2\sqrt{1-a^2}} \cdot \\
&\cdot \left( \begin{array}{c}
\sqrt{(1-a^2)} - (ic - g) (\phi_L (0,0) + \phi_R (0,0)) + \\
\exp(-ih (nx_5 + kx_4)) \cdot (g + ic) (-\phi_L (0,0) + \phi_R (0,0)) + \\
+ (b + \sqrt{(1-a^2)}) (\phi_L (n,k) + \phi_R (n,k))
\end{array} \right), \quad \text{(22)}
\end{align*} \]

If \( \lambda \) is the angle of the \( U^{(-)} \) eigenvalue (i.e. \( w = a + i\sqrt{1-a^2} \) and \( \cos \lambda = a \) and \( \sin \lambda = \sqrt{1-a^2} \)) then

\[ \begin{align*}
U^{(-)*} \gamma U^{(-)} &= (\gamma \cos \lambda + \sin \lambda (\pi_0 - \pi_*) \beta), \\
U^{(-)*} \beta U^{(-)} &= (\beta \cos \lambda - \sin \lambda (\pi_0 - \pi_*) \gamma) . \quad \text{(23)}
\end{align*} \]

Let

\[ \begin{align*}
n &\to n' = (n \cos \lambda + k \sin \lambda (\pi_0 - \pi_*)), \\
k &\to k' = (k \cos \lambda - n \sin \lambda (\pi_0 - \pi_*)), \\
\Psi &\to \Psi' = U^{(-)} \Psi, \\
F_\mu &\to F_\mu^i = F_\mu, \\
B_\mu &\to B_\mu^i = B_\mu \\
\end{align*} \]

and the motion equation for \( \Psi' \) be (23):

\[ \sum_{\mu=0}^{3} \beta_\mu i \partial_\mu \Psi' - h (n' \gamma + k' \beta) \Psi' + \sum_{\mu=0}^{3} F_\mu^i \beta_\mu \Psi' + \\
+ 0.5 g_1 Y \sum_{\mu=0}^{3} B_\mu^i \beta_\mu \Psi' = S \Psi'. \quad \text{(25)} \]
Hence:
\[
\sum_{\mu=0}^{3} \beta_{\mu} \left( \partial_{\mu} U^{(-)} \right) \Psi + \sum_{\mu=0}^{3} \beta_{\mu} i U^{(-)} (\partial_{\mu} \Psi) - \\
- h \left( n \gamma + k \beta \left( U^{(-)} \right) \right) + \sum_{\mu=0}^{3} F_{\mu} \beta_{\mu} \left( U^{(-)} \right) + \\
+ 0.5 g_{1} Y \sum_{\mu=0}^{3} B_{\mu} \beta_{\mu} \left( U^{(-)} \right) = S \left( U^{(-)} \right),
\]

then
\[
U^{(-)} \sum_{\mu=0}^{3} \beta_{\mu} \left( \partial_{\mu} U^{(-)} \right) \Psi + \sum_{\mu=0}^{3} \beta_{\mu} i U^{(-)} (\partial_{\mu} \Psi) - \\
- U^{(-)} \sum_{\mu=0}^{3} \beta_{\mu} \left( \partial_{\mu} U^{(-)} \right) \Psi + \\
+ U^{(-)} \sum_{\mu=0}^{3} F_{\mu} \beta_{\mu} \left( U^{(-)} \right) \Psi + + U^{(-)} \sum_{\mu=0}^{3} 0.5 g_{1} Y \sum_{\mu=0}^{3} B_{\mu} \beta_{\mu} \left( U^{(-)} \right) \Psi = \\
= U^{(-)} \Psi. \]

Since
\[
U^{(-)} \beta_{\mu} = \beta_{\mu} U^{(-)} \Psi, \quad Y U^{(-)} = U^{(-)} Y, \quad U^{(-)} (\pi_{o} - \pi_{*}) = (\pi_{o} - \pi_{*}) U^{(-)} \Psi,
\]

then
\[
\sum_{\mu=0}^{3} \beta_{\mu} \left( \partial_{\mu} U^{(-)} \right) \Psi + \sum_{\mu=0}^{3} \beta_{\mu} i U^{(-)} (\partial_{\mu} \Psi) - \\
- h \left( n \cos \lambda + k \sin \lambda (\pi_{o} - \pi_{*}) \right) \Psi + \left( k \cos \lambda - n \sin \lambda (\pi_{o} - \pi_{*}) \right) \Psi + \\
+ \sum_{\mu=0}^{3} F_{\mu} \beta_{\mu} \Psi + 0.5 g_{1} Y \sum_{\mu=0}^{3} B_{\mu} \beta_{\mu} \Psi = \\
= U^{(-)} \Psi. \]

From (23):
\[
- h \left( n \cos \lambda + k \sin \lambda (\pi_{o} - \pi_{*}) \right) (\gamma \cos \lambda + \sin \lambda (\pi_{o} - \pi_{*}) \beta) + \\
+ \left( k \cos \lambda - n \sin \lambda (\pi_{o} - \pi_{*}) \right) (\beta \cos \lambda - \sin \lambda (\pi_{o} - \pi_{*}) \gamma) + \\
+ \sum_{\mu=0}^{3} F_{\mu} \beta_{\mu} \Psi + 0.5 g_{1} Y \sum_{\mu=0}^{3} B_{\mu} \beta_{\mu} \Psi = \\
= U^{(-)} \Psi. \]

Since
\[
(\pi_{o} - \pi_{*}) (\pi_{o} - \pi_{*}) = 1_{8}
\]
then

\[ \sum_{\mu=0}^{3} \beta_{\mu} U^{(-)\dagger} \left( \partial_{\mu} U^{(-)} \right) \Psi + \sum_{\mu=0}^{3} \beta_{\mu} i \partial_{\mu} U^{(-)} = \] 

\[ U^{(-)\dagger} \left( SU^{(-)} \right) \Psi. \]

Hence from (13):

\[ \sum_{\mu=0}^{3} \beta_{\mu} U^{(-)\dagger} \left( \partial_{\mu} U^{(-)} \right) = U^{(-)\dagger} \left( SU^{(-)} \right) \]

and

\[ S = \sum_{\mu=0}^{3} \beta_{\mu} i \left( \partial_{\mu} U^{(-)} \right) U^{(-)\dagger} \]

Therefore from (25) the motion equation for the transformation (24) is the following:

\[ \sum_{\mu=0}^{3} \beta_{\mu} i \partial_{\mu} \Psi' - \sum_{\mu=0}^{3} \beta_{\mu} i \left( \partial_{\mu} U^{(-)} \right) U^{(-)\dagger} \Psi' - h (k' \gamma + k' \beta) \Psi' + \] 

\[ = \sum_{\mu=0}^{3} F_{\mu} \beta_{\mu} \Psi + 0.5 g_{1} Y \sum_{\mu=0}^{3} B_{\mu} \beta_{\mu} \Psi = 0. \]

\( W \)-bosons Let \( g_2 \) be a positive real number.

If design \((a, b, c, g \) form \( U^{(-)}):\)

\[ W_{0,\mu} = -2 \frac{1}{2} g_2 \left( g \left( \partial_{\mu} a \right) b - g \left( \partial_{\mu} b \right) a + \left( \partial_{\mu} c \right) g^2 \right) \]

\[ + a \left( \partial_{\mu} a \right) c + b \left( \partial_{\mu} b \right) c + c^2 \left( \partial_{\mu} c \right) \]

\[ W_{1,\mu} = -2 \frac{1}{g_2 g_2} \left( \left( \partial_{\mu} a \right) a^2 - b g \left( \partial_{\mu} c \right) \right) \]

\[ + \left( \partial_{\mu} a \right) c + g \left( \partial_{\mu} a \right) b + c \left( \partial_{\mu} b \right) c \]

\[ W_{2,\mu} = -2 \frac{1}{g_2 g_2} \left( g \left( \partial_{\mu} a \right) c - a \left( \partial_{\mu} a \right) b - b^2 \left( \partial_{\mu} b \right) - \right) \]

\[ - c \left( \partial_{\mu} c \right) b - \left( \partial_{\mu} b \right) g^2 - \left( \partial_{\mu} c \right) c \]

and

\[ W_\mu = \begin{bmatrix} W_{0,\mu} & 0 & (W_{1,\mu} - i W_{2,\mu}) & 0 \\ 0 & 0 & 0 & 0 \\ (W_{1,\mu} + i W_{2,\mu}) & 0 & -W_{0,\mu} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \]
then

\[-i \left( \partial_\mu U(\cdot) \right) U(\cdot) = \frac{1}{2} g_2 W_\mu \]  

and from (23):

\[
\sum_{\mu=0}^{3} \beta_\mu i \left( \partial_\mu - i \frac{1}{2} g_2 W_\mu \right) \Psi' - \
- h \left( n' \gamma + k' \beta \right) \Psi' + \
+ \sum_{\mu=0}^{3} F_\mu \beta_\mu \Psi' + 0.5 g_1 \nu \sum_{\mu=0}^{3} B_\mu \beta_\mu \Psi' = 0. \tag{28}
\]

Let

\[
\dot{U} = \begin{bmatrix}
(\dot{a} + i \dot{b}) & 1_2 & 0_2 & (\dot{c} + i \dot{g}) & 1_2 & 0_2 \\
0_2 & 1_2 & 0_2 & 0_2 \\
(\dot{-c} + i \dot{g}) & 1_2 & 0_2 & (\dot{a} - i \dot{b}) & 1_2 & 0_2 \\
0_2 & 0_2 & 0_2 & 1_2
\end{bmatrix},
\]

\[
\dot{\pi}_o = \frac{1}{2 \sqrt{1 - \dot{a}^2}} \begin{bmatrix}
(\dot{b} + \sqrt{1 - \dot{a}^2}) & 1_4 & (-i \dot{c} + \dot{g}) \gamma_5 \\
(\dot{c} + \dot{g}) \gamma_5 & 1_4 & \left( \sqrt{1 - \dot{a}^2} - \dot{b} \right) \gamma_5
\end{bmatrix},
\]

\[
\dot{\pi}_s = \frac{1}{2 \sqrt{1 - \dot{a}^2}} \begin{bmatrix}
\left( \sqrt{1 - \dot{a}^2} - \dot{b} \right) & 1_4 & (i \dot{c} - \dot{g}) \gamma_5 \\
(-\dot{g} - i \dot{c}) \gamma_5 & 1_4 & \left( \dot{b} + \sqrt{1 - \dot{a}^2} \right) \gamma_5
\end{bmatrix}.
\]

Let:

\[\cos \dot{\lambda} = \dot{a} \quad \text{and} \quad \sin \dot{\lambda} = \sqrt{1 - \dot{a}^2}\]

and

\[
\Psi' \rightarrow \Psi' = \left( \dot{U} \psi' \right), \\
n' \rightarrow n' = \left( n' \cos \dot{\lambda} + k' \sin \dot{\lambda} \left( \dot{\pi}_o - \dot{\pi}_s \right) \right), \\
k' \rightarrow k' = \left( k' \cos \dot{\lambda} - n' \sin \dot{\lambda} \left( \dot{\pi}_o - \dot{\pi}_s \right) \right), \\
F_\mu \rightarrow F'_\mu = F_\mu, \\
B_\mu \rightarrow B'_\mu = B_\mu, \tag{29}
\]

23
and

\[ W_\mu \rightarrow W'_\mu. \]

In that case from (27):

\[ W'_\mu = -\frac{2i}{g_2} \left( \partial_\mu \left( \dot{U} U^{(-)} \right) \right) \left( \dot{U} U^{(-)} \right)^\dagger; \]

Hence:

\[ W'_\mu = -\frac{2i}{g_2} \left( \partial_\mu U \right) \dot{U}^\dagger - \frac{2i}{g_2} \dot{U} \left( \partial_\mu U^{(-)} \right) U^{(-)} \dot{U}^\dagger; \]

i.e.:

\[ W'_\mu = \dot{U} W_\mu \dot{U}^\dagger - \frac{2i}{g_2} \left( \partial_\mu U \right) \dot{U}^\dagger. \]

If

\[ F_{\mu,\nu} = \left( \partial_\mu W_\nu - \partial_\nu W_\mu - i \frac{g_2}{2} \left( W_\mu W_\nu - W_\nu W_\mu \right) \right) \]

then

\[ F'_{\mu,\nu} = \partial_\mu W'_\nu - \partial_\nu W'_\mu - i \frac{g_2}{2} \left( W'_\mu W'_\nu - W'_\nu W'_\mu \right) = U F_{\mu,\nu} U^\dagger. \]

Therefore \( F_{\mu,\nu} \) is invariant for the transformation (29).

The Lagrangian for \( F_{\mu,\nu} \):

\[ \mathcal{L}_F = \left( -\frac{1}{4} \sum_{\mu,\nu} F_{\mu,\nu} F_{\mu,\nu} \right). \]

Hence the Euler-Lagrange equations for \( W_\mu \) are the following:

\[ \sum_\nu \partial^\nu \left( \partial_\mu W_\nu - \partial_\nu W_\mu - i \frac{g_2}{2} \left[ W_\mu, W_\nu \right] \right) = 0. \]

For the components:

\[
\begin{align*}
\sum_\nu \partial^\nu \partial_\nu W_{0,\mu} &= g_2 \sum_\nu \partial^\nu \left( W_{1,\nu} W_{2,\mu} - W_{2,\nu} W_{1,\mu} \right) + \partial_\mu \sum_\nu \partial^\nu W_{0,\nu}, \\
\sum_\nu \partial^\nu \partial_\nu W_{1,\mu} &= g_2 \sum_\nu \partial^\nu \left( W_{0,\nu} W_{2,\mu} - W_{2,\nu} W_{0,\mu} \right) + \partial_\mu \sum_\nu \partial^\nu W_{1,\nu}, \\
\sum_\nu \partial^\nu \partial_\nu W_{2,\mu} &= g_2 \sum_\nu \partial^\nu \left( W_{0,\mu} W_{1,\nu} - W_{0,\nu} W_{1,\mu} \right) + \partial_\mu \sum_\nu \partial^\nu W_{2,\nu}.
\end{align*}
\]

24
Let:

\[
\alpha_{0,\mu,\nu} = \partial_{\nu}W_{0,\mu} - g_2 (W_{1,\mu}W_{2,\nu} - W_{2,\mu}W_{1,\nu}) ,
\alpha_{1,\mu,\nu} = \partial_{\nu}W_{1,\mu} - g_2 (W_{0,\nu}W_{2,\mu} - W_{0,\mu}W_{2,\nu}) ,
\alpha_{2,\mu,\nu} = \partial_{\nu}W_{2,\mu} - g_2 (W_{0,\mu}W_{1,\nu} - W_{0,\nu}W_{1,\mu}) .
\]  

(30)

Hence if \(\sum_\nu \partial^{\nu}W = 0\) then

\[
\sum_\nu \partial^{\nu}\alpha_{0,\mu,\nu} = 0 , \sum_\nu \partial^{\nu}\alpha_{1,\mu,\nu} = 0 , \sum_\nu \partial^{\nu}\alpha_{2,\mu,\nu} = 0 .
\]

From (30):

\[
\partial_{\nu}W_{0,\mu} = (g_2 (W_{1,\mu}W_{2,\nu} - W_{2,\mu}W_{1,\nu}) + \alpha_{0,\mu,\nu}) ,
\]  

(31)

\[
\partial_{\nu}W_{1,\mu} = (g_2 (W_{0,\nu}W_{2,\mu} - W_{0,\mu}W_{2,\nu}) + \alpha_{1,\mu,\nu}) ,
\]  

(32)

\[
\partial_{\nu}W_{2,\mu} = (g_2 (W_{0,\mu}W_{1,\nu} - W_{0,\nu}W_{1,\mu}) + \alpha_{2,\mu,\nu}) ;
\]  

(33)

From (31):

\[
\partial_{\nu}\partial_{\nu}W_{0,\mu} = g_2 \partial_{\nu} (W_{1,\mu}W_{2,\nu} - W_{2,\mu}W_{1,\nu}) + \partial_{\nu}\alpha_{0,\mu,\nu} =
= g_2 (\partial_{\nu}W_{1,\mu}W_{2,\nu} + W_{1,\mu}\partial_{\nu}W_{2,\nu} - \partial_{\nu}W_{2,\mu}W_{1,\nu} - W_{2,\mu}\partial_{\nu}W_{1,\nu}) + \partial_{\nu}\alpha_{0,\mu,\nu};
\]  

(34)

hence from (34), (32) and (33):

\[
\partial_{\nu}\partial_{\nu}W_{0,\mu} = g_2 \left( -g_2 (W_{0,\nu}W_{2,\mu} - W_{0,\mu}W_{2,\nu}) + \alpha_{1,\mu,\nu} \right) W_{2,\nu} -
= g_2 \left( -g_2 (W_{0,\nu}W_{1,\mu} - W_{0,\mu}W_{1,\nu}) + \alpha_{2,\mu,\nu} \right) W_{1,\nu} -
+ W_{2,\mu}\partial_{\nu}W_{1,\nu} + W_{1,\mu}\partial_{\nu}W_{2,\nu} + \partial_{\nu}\alpha_{0,\mu,\nu};
\]

hence:

\[
\partial_{\nu}\partial_{\nu}W_{0,\mu} =
= g_2 \left( -g_2 (W_{2,\nu}^2 + W_{1,\nu}^2) W_{0,\mu} + (W_{1,\mu}W_{1,\nu} + W_{2,\mu}W_{2,\nu}) W_{0,\nu} \right) +
+ \alpha_{1,\mu,\nu}W_{2,\nu} - \alpha_{2,\mu,\nu}W_{1,\nu} + W_{1,\mu}\partial_{\nu}W_{2,\nu} - W_{2,\mu}\partial_{\nu}W_{1,\nu} + \partial_{\nu}\alpha_{0,\mu,\nu};
\]

and
\[
\partial_\nu \partial_\mu W_{0,\mu} = -g_2^2 \left( W_{2,\nu}^2 + W_{1,\nu}^2 + W_{0,\nu}^2 \right) W_{0,\mu} + g_2^2 (W_{0,\mu} W_{0,\nu} + W_{1,\mu} W_{1,\nu} + W_{2,\mu} W_{2,\nu}) W_{0,\nu} + g_2 (\alpha_{1,\mu,\nu} W_{2,\nu} - \alpha_{2,\mu,\nu} W_{1,\nu} + W_{1,\mu} \partial_\nu W_{2,\nu} - W_{2,\mu} \partial_\nu W_{1,\nu}) + \partial_\nu \alpha_{0,\mu,\nu};
\]

if \( \sum_\nu \partial_\nu W_\nu = 0 \) then:

\[
\sum_\nu \partial_\nu \partial_\nu W_{0,\mu} = -g_2^2 W_{0,\mu} \sum_\nu W_\nu^2 + \frac{g_2^2}{2} \sum_\nu (W_\mu W_\nu + W_\nu W_\mu) W_{0,\nu} + g_2 \sum_\nu (\alpha_{1,\mu,\nu} W_{2,\nu} - \alpha_{2,\mu,\nu} W_{1,\nu}),
\]

(35)

\[
\sum_\nu \partial_\nu \partial_\nu W_{1,\mu} = -g_2^2 W_{1,\mu} \sum_\nu W_\nu^2 + \frac{g_2^2}{2} \sum_\nu (W_\nu W_\mu + W_\mu W_\nu) W_{1,\nu} + g_2 \sum_\nu (W_{0,\nu} \alpha_{2,\mu,\nu} - \alpha_{0,\mu,\nu} W_{2,\nu})
\]

and

\[
\sum_\nu \partial_\nu \partial_\nu W_{2,\mu} = -g_2^2 W_{2,\mu} \sum_\nu W_\nu^2 + \frac{g_2^2}{2} \sum_\nu (W_\nu W_\mu + W_\mu W_\nu) W_{2,\nu} + g_2 \sum_\nu (\alpha_{0,\mu,\nu} W_{1,\nu} - W_{0,\nu} \alpha_{1,\mu,\nu})
\]

\[
\alpha_{\mu,\nu} = \begin{bmatrix}
\alpha_{0,\mu,\nu} & \alpha_{1,\mu,\nu} - i\alpha_{2,\mu,\nu} \\
\alpha_{1,\mu,\nu} + i\alpha_{2,\mu,\nu} & -\alpha_{0,\mu,\nu}
\end{bmatrix}
\]

then

\[
\sum_\nu \partial_\nu \partial_\nu W_\mu = -g_2^2 W_\mu \sum_\nu W_\nu^2 + \frac{g_2^2}{2} \sum_\nu (W_\nu W_\mu + W_\mu W_\nu) W_\nu - i\frac{g_2}{2} \sum_\nu (\alpha_{\mu,\nu} W_\nu - W_\nu \alpha_{\mu,\nu})
\]

It is the motion equation for the field \( W_\mu \) which has got a less than unit 1 velocity. That is this field does not behave as a massless field.

Hence although \( F_{\mu,\nu} \) is a massless field but its components \( W_\mu \) do not behave like a massless fields.

If

\[
\sum_\nu \left( W_\nu \frac{\partial W_\nu}{\partial W_\mu} + \frac{\partial W_\nu}{\partial W_\mu} W_\nu \right) = 0
\]

then a real \( \nu \) exists for which
\[ v = \left( 2 \sum_{\nu} W_{\nu}^2 \right)^{\frac{1}{2}} \]  
\[ (36) \]

and

\[ \partial_{W_{\mu}} v = 0 \]

then the Lagrangian of \( W_{\mu} \) is:

\[ \hat{\mathcal{L}} = \sum_{\nu} (\partial_{\nu} W_{\mu}) (\partial_{\nu} W_{\mu}) - g^2 \sum_{\nu} \frac{W_{\nu}^2}{2} W_{\mu}^2 + \]
\[ + g^2 \sum_{\nu} (W_{\nu} W_{\mu} + W_{\mu} W_{\nu})^2 - \]
\[ - i \frac{g^2}{2} \left( (\sum_{\nu} \left[ \alpha_{\mu,\nu}, W_{\nu} \right]) W_{\mu} + W_{\mu} (\sum_{\nu} \left[ \alpha_{\mu,\nu}, W_{\nu} \right]) \right). \]

It is a lagrangian of a field with mass

\[ M = g \sqrt{2} \]

and \( M > 0 \).

**A and Z bosons** Let \( A_{\mu} \) and \( Z_{\mu} \) are a fields for which \[ (37) \]:

\[ Z_{\mu} = \frac{1}{\sqrt{g_1^2 + g_2^2}} (g_2 W_{0,\mu} - g_1 B_{\mu}), \]
\[ A_{\mu} = \frac{1}{\sqrt{g_1^2 + g_2^2}} (g_2 B_{\mu} + g_1 W_{0,\mu}) \]

and

\[ \sum_{\nu} \partial_{\nu} \partial_{\nu} A_{\mu} = 0. \]  
\[ (38) \]

Let denote:

\[ \frac{g_2}{2} \sum_{\nu} (W_{\mu} W_{\nu} + W_{\nu} W_{\mu}) W_{0,\nu} + g_2 \sum_{\nu} (\alpha_{1,\mu,\nu} W_{2,\nu} - \alpha_{2,\mu,\nu} W_{1,\nu}) = \Lambda. \]

Hence from \[ (35) \] and \[ (36) \]:

\[ \sum_{\nu} \partial_{\nu} \partial_{\nu} W_{0,\mu} = -g^2 \frac{v^2}{2} W_{0,\mu} + \Lambda \]  
\[ (39) \]

From \[ (37) \]:

27
\[ B_\mu = \frac{1}{\sqrt{g_1^2 + g_2^2}} (g_2 A_\mu - g_1 Z_\mu), \quad W^0_\mu = \frac{1}{\sqrt{g_1^2 + g_2^2}} (g_1 A_\mu + g_2 Z_\mu). \] (40)

and

\[ \sum_\nu \partial^\nu \partial_\nu A_\mu = \frac{1}{\sqrt{g_1^2 + g_2^2}} \left( g_2 \sum_\nu \partial^\nu \partial_\nu B_\mu + g_1 \sum_\nu \partial^\nu \partial_\nu W^0_\mu \right), \]

from (39):

\[ \sum_\nu \partial^\nu \partial_\nu Z_\mu = \frac{1}{\sqrt{g_1^2 + g_2^2}} \left( \frac{-g_2^2 v^2}{2} W^0_{0_\mu} + \Lambda \right), \]

from (38):

\[ A_\mu = - (g_2^2 - g_1^2) \frac{1}{2g_1 g_2} Z_\mu + \frac{1}{\sqrt{g_1^2 + g_2^2}} \left( g_1 \Lambda + g_2 \left( \sum_\nu \partial^\nu \partial_\nu B_\mu + g_1^2 v^2 B_\mu \right) \right), \] (41)

From (37):

\[ \sum_\nu \partial^\nu \partial_\nu Z_\mu = \frac{1}{\sqrt{g_1^2 + g_2^2}} \left( g_2 \sum_\nu \partial^\nu \partial_\nu W^0_\mu - g_1 \sum_\nu \partial^\nu \partial_\nu B_\mu \right), \]

from (39):

\[ \sum_\nu \partial^\nu \partial_\nu Z_\mu = \frac{1}{\sqrt{g_1^2 + g_2^2}} \left( \frac{-g_2^2 v^2}{2} W^0_{0_\mu} + \Lambda \right) - g_1 \left( \sum_\nu \partial^\nu \partial_\nu B_\mu + g_1^2 v^2 B_\mu - g_2^2 v^2 B_\mu \right), \]

from (40):
$$\sum_{\nu} \partial^\nu \partial_\nu Z_\mu = -\frac{v^2}{2} \frac{1}{g_1^2 + g_2^2} \left( g_1^4 + g_2^4 \right) Z_\mu - g_1 g_2 \frac{v^2}{2} \frac{1}{g_1^2 + g_2^2} \left( g_2^2 - g_1^2 \right) A_\mu +$$

$$+ \frac{1}{\sqrt{g_1^2 + g_2^2}} \left( g_2 \Lambda - g_1 \left( \sum_{\nu} \partial^\nu \partial_\nu B_\mu + g_1^2 \frac{v^2}{2} B_\mu \right) \right)$$

and from (41):

$$\sum_{\nu} \partial^\nu \partial_\nu Z_\mu = -\frac{1}{2} \frac{v^2}{2} \left( g_1^2 + g_2^2 \right) Z_\mu +$$

$$+ \frac{1}{2} \sqrt{g_1^2 + g_2^2} \left( \frac{1}{g_2} \Lambda - \frac{1}{g_1} \left( \sum_{\nu} \partial^\nu \partial_\nu B_\mu + g_1^2 \frac{v^2}{2} B_\mu \right) \right)$$

That is $Z_\mu$ has got the mass:

$$M_Z = \frac{v}{2} \sqrt{g_1^2 + g_2^2}.$$

**Fragments**

Since

$$(\pi_0 + \pi_*) = 1_8$$

then

$$\sum_{\mu=0}^{3} \beta_\mu \left( \partial_\mu - i \frac{1}{2} g_2 W_\mu \right) (\pi_0 + \pi_*) \Psi^- -$$

$$- h \left( \left( \frac{n}{h} \cos \lambda (\pi_0 + \pi_*) \right) + \frac{k}{h} \sin \lambda (\pi_0 - \pi_*) \right) \gamma +$$

$$+ h \left( \left( \frac{k}{h} \cos \lambda (\pi_0 + \pi_*) \right) + \frac{n}{h} \sin \lambda (\pi_0 - \pi_*) \right) \beta \right) \Psi^+ +$$

$$+ \sum_{\mu=0}^{3} F_\mu \beta_\mu (\pi_0 + \pi_*) \Psi^- +$$

$$+ 0.5 g_1 \sum_{\mu=0}^{3} B_\mu \beta_\mu (\pi_0 + \pi_*) \Psi^+ = 0.$$

Because

$$\pi_0 \beta = \beta \pi_0, \pi_* \beta = \beta \pi_*,$$

$$\pi_0 \gamma = \gamma \pi_0, \pi_* \gamma = \gamma \pi_*$$

then
\[ \begin{align*}
\sum_{\mu=0}^{3} \beta_{\mu} i \left( \partial_{\mu} - i \frac{1}{2} g_{2} W_{\mu} \right) \pi_{o} \Psi^i + \\
+ \sum_{\mu=0}^{3} \beta_{\mu} i \left( \partial_{\mu} - i \frac{1}{2} g_{2} W_{\mu} \right) \pi_{s} \Psi^i - \\
- \hbar \left( n \cos \lambda \gamma_{o} \Psi^i + n \cos \lambda \gamma_{s} \Psi^i + \\
+ k \sin \lambda \gamma_{o} \Psi^i - k \sin \lambda \gamma_{s} \Psi^i + \\
+ k \cos \lambda \beta_{o} \Psi^i + k \cos \lambda \beta_{s} \Psi^i - \\
- \hbar \sin \lambda \beta_{o} \Psi^i + \hbar \sin \lambda \beta_{s} \Psi^i \\
+ \sum_{\mu=0}^{3} F_{\mu} \beta_{\mu} \pi_{o} \Psi^i + \sum_{\mu=0}^{3} F_{\mu} \beta_{\mu} \pi_{s} \Psi^i + \\
+ 0.5 g_{1} \sum_{\mu=0}^{3} B_{\mu} \beta_{\mu} \pi_{o} \Psi^i + \\
+ 0.5 g_{1} \sum_{\mu=0}^{3} B_{\mu} \beta_{\mu} \pi_{s} \Psi^i = 0. \\
\end{align*} \]

Let
\[ \Psi_{o}^i = \pi_{o} \Psi^i \text{ and } \Psi_{s}^i = \pi_{s} \Psi^i. \]

In that case:
\[ \begin{align*}
\sum_{\mu=0}^{3} \beta_{\mu} i \left( \partial_{\mu} - i \frac{1}{2} g_{2} W_{\mu} \right) \pi_{o} \Psi^i - \\
- \hbar \left( (n \cos \lambda + k \sin \lambda) \gamma + (k \cos \lambda - n \sin \lambda) \beta \right) \Psi_{o}^i + \\
+ \sum_{\mu=0}^{3} F_{\mu} \beta_{\mu} \Psi_{o}^i + 0.5 g_{1} \sum_{\mu=0}^{3} B_{\mu} \beta_{\mu} \Psi_{o}^i + \\
\sum_{\mu=0}^{3} \beta_{\mu} i \left( \partial_{\mu} - i \frac{1}{2} g_{2} W_{\mu} \right) \pi_{s} \Psi^i - \\
- \hbar \left( (n \cos \lambda - k \sin \lambda) \gamma + (k \cos \lambda + n \sin \lambda) \beta \right) \Psi_{s}^i + \\
+ \sum_{\mu=0}^{3} F_{\mu} \beta_{\mu} \Psi_{s}^i + 0.5 g_{1} \sum_{\mu=0}^{3} B_{\mu} \beta_{\mu} \Psi_{s}^i = 0. \\
\end{align*} \]

Therefore for the (24) transformation from (21, 22):
\[ \Psi(x_5, x_4) = \Psi_{o}(x_5, x_4) + \Psi_{s}(x_5, x_4) \rightarrow \]
\[ \rightarrow \Psi^i(x_5, x_4) = \Psi_{o}^i(x_5, x_4) + \Psi_{s}^i(x_5, x_4) = \]
\[ = \frac{1}{2\sqrt{1 - a^2}} \cdot \left( b + \sqrt{1 - a^2} \right) \left( a + i \sqrt{1 - a^2} \right) \phi_{L}(0, 0) + \phi_{R}(0, 0) - \\
\cdot - (ic - g) \left( a + i \sqrt{1 - a^2} \right) \phi_{L}(n, k) - \phi_{R}(n, k) + \\
\cdot + \exp \left( -ih \left( (n \cos \lambda + k \sin \lambda) x_5 + (k \cos \lambda - n \sin \lambda) x_4 \right) \right) \cdot \left( ic + g \right) \left( a + i \sqrt{1 - a^2} \right) \phi_{L}(0, 0) - \phi_{R}(0, 0) + \\
\cdot + \left( b + \sqrt{1 - a^2} \right) \left( a + i \sqrt{1 - a^2} \right) \phi_{L}(n, k) + \phi_{R}(n, k) \right) + \]
\[
+ \frac{1}{2\sqrt{1-a^2}} \left( \sqrt{1-a^2} - b \right) \left( (a - i\sqrt{1-a^2}) \phi_L (0, 0) + \phi_R (0, 0) \right) + \\
+ (ic - g) \left( (a - i\sqrt{1-a^2}) \phi_L (n, k) - \phi_R (n, k) \right) + \\
+ \exp (-ih \left( (n \cos \lambda - k \sin \lambda) x_5 + (k \cos \lambda + n \sin \lambda) x_4 \right)) \cdot \\
\left( (g + ic) \left( (a - i\sqrt{1-a^2}) \phi_L (0, 0) + \phi_R (0, 0) \right) + \\
+ (b + \sqrt{1-a^2}) \left( (a - i\sqrt{1-a^2}) \phi_L (n, k) + \phi_R (n, k) \right) \right) \\
\right) \\
\right) \\
\right) \).
\]

That is:

\[
\Psi (x_5, x_4) = \Psi_o (x_5, x_4) + \Psi_s (x_5, x_4) \rightarrow \\
\rightarrow \Psi^s (x_5, x_4) = \Psi^s_o (x_5, x_4) + \Psi^s_s (x_5, x_4) = \\
= \frac{1}{2\sqrt{1-a^2}} \\
\left( \left. \begin{array}{c}
(b + \sqrt{1-a^2}) \left( (a + i\sqrt{1-a^2}) \phi_L (0, 0) + \phi_R (0, 0) \right) - \\
- (ic - g) \left( (a + i\sqrt{1-a^2}) \phi_L (n, k) - \phi_R (n, k) \right) + \\
+ \exp (-ih \left( (na + k\sqrt{1-a^2}) x_5 + (ka - n\sqrt{1-a^2}) x_4 \right)) \cdot \\
\left( (ic + g) \left( (a + i\sqrt{1-a^2}) \phi_L (0, 0) - \phi_R (0, 0) \right) + \\
+ (b + \sqrt{1-a^2}) \left( (a + i\sqrt{1-a^2}) \phi_L (n, k) + \phi_R (n, k) \right) \right) \\
\end{array} \right) \right) + \\
\left( \left. \begin{array}{c}
\end{array} \right) \right) \\
\right) \\
\right) \cdot \\
\frac{1}{2\sqrt{1-a^2}}. \\
\left( \left. \begin{array}{c}
\end{array} \right) \right)
Let in some point \( \langle t, x, y, z \rangle \phi_L (n, k) \neq 0 \) or/and \( \phi_R (n, k) \neq 0 \).
In that case \([45]\) in this point: \( \phi_L (0, 0) = 0 \) and \( \phi_R (0, 0) = 0 \).
Hence:

\[
\Psi^i (x_5, x_4) = \frac{1}{2 \sqrt{1 - a^2}},
\]

\[
- (ic - g) \left( (a + i \sqrt{1 - a^2}) \phi_L (n, k) - \phi_R (n, k) \right) + \exp \left( -ih \left( (na + k \sqrt{1 - a^2}) x_5 + (ka - n \sqrt{1 - a^2}) x_4 \right) \right) \cdot \left( b + \sqrt{(1 - a^2)} \right) \left( (a + i \sqrt{1 - a^2}) \phi_L (n, k) + \phi_R (n, k) \right),
\]

(43)

\[
\Psi^i (x_5, x_4) = \frac{1}{2 \sqrt{1 - a^2}},
\]

\[
( ic - g ) \left( (a - i \sqrt{1 - a^2}) \phi_L (n, k) - \phi_R (n, k) \right) + \exp \left( -ih \left( (na - k \sqrt{1 - a^2}) x_5 + (ka + n \sqrt{1 - a^2}) x_4 \right) \right) \cdot \left( b + \sqrt{(1 - a^2)} \right) \left( (a - i \sqrt{1 - a^2}) \phi_L (n, k) + \phi_R (n, k) \right),
\]

(44)

and

\[
\Psi^i (x_5, x_4) =
\]

\[
= - i (ic - g) \phi_L (n, k) + \frac{1}{2 \sqrt{1 - a^2}} \left( \exp \left( -ih \left( (na + k \sqrt{1 - a^2}) x_5 + (ka - n \sqrt{1 - a^2}) x_4 \right) \right) \right) \cdot \left( b + \sqrt{(1 - a^2)} \right) \left( (a + i \sqrt{1 - a^2}) \phi_L (n, k) + \phi_R (n, k) \right) + \frac{1}{2 \sqrt{1 - a^2}} \exp \left( -ih \left( (na - k \sqrt{1 - a^2}) x_5 + (ka + n \sqrt{1 - a^2}) x_4 \right) \right) \cdot \left( b + \sqrt{(1 - a^2)} \right) \left( (a - i \sqrt{1 - a^2}) \phi_L (n, k) + \phi_R (n, k) \right),
\]

(45)

**Local probabilities**

Let:

\[
\Psi^i \Psi_0 = \rho_0, \quad \Psi^i \Psi_0^* = \rho_0^i,
\]

\[
\Psi^i \beta_1 \Psi_0 = j_{0x}, \quad \Psi^i \beta_1 \Psi_0^* = j_{0x}^i,
\]

\[
\Psi^i \beta_2 \Psi_0 = j_{0y}, \quad \Psi^i \beta_2 \Psi_0^* = j_{0y}^i,
\]

\[
\Psi^i \beta_3 \Psi_0 = j_{0z}, \quad \Psi^i \beta_3 \Psi_0^* = j_{0z}^i,
\]

\[
\Psi^i \Psi_* = \rho_* , \quad \Psi^i \Psi_* = \rho_*^i,
\]

\[
\Psi^i \beta_1 \Psi_* = j_{*x}, \quad \Psi^i \beta_1 \Psi_*^* = j_{*x}^i,
\]

\[
\Psi^i \beta_2 \Psi_* = j_{*y}, \quad \Psi^i \beta_2 \Psi_*^* = j_{*y}^i,
\]

\[
\Psi^i \beta_3 \Psi_* = j_{*z}, \quad \Psi^i \beta_3 \Psi_*^* = j_{*z}^i,
\]

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Because
\[
(\Psi^\dagger \Psi_o)^2 - \left( (\Psi^\dagger \beta_1 \Psi_o)^2 + (\Psi^\dagger \beta_2 \Psi_o)^2 + (\Psi^\dagger \beta_3 \Psi_o)^2 \right) = \\
= (\Psi^\dagger \gamma \Psi_o)^2 + (\Psi^\dagger \beta \Psi_o)^2,
\]
\[
(\Psi^\dagger \Psi_s)^2 - \left( (\Psi^\dagger \beta_1 \Psi_s)^2 + (\Psi^\dagger \beta_2 \Psi_s)^2 + (\Psi^\dagger \beta_3 \Psi_s)^2 \right) = \\
= (\Psi^\dagger \gamma \Psi_s)^2 + (\Psi^\dagger \beta \Psi_s)^2
\]
then the local densities are:
\[
\rho_{oo}^2 = \rho_o^2 - (j_{ox}^2 + j_{oy}^2 + j_{oz}^2) = (\Psi^\dagger \gamma \Psi_o)^2 + (\Psi^\dagger \beta \Psi_o)^2 \\
= (\Psi^\dagger \gamma \Psi_o)^2 + (\Psi^\dagger \beta \Psi_o)^2
\]
\[
\rho_{so}^2 = \rho_s^2 - (j_{sx}^2 + j_{sy}^2 + j_{sz}^2) = (\Psi^\dagger \gamma \Psi_s)^2 + (\Psi^\dagger \beta \Psi_s)^2 \\
= (\Psi^\dagger \gamma \Psi_s)^2 + (\Psi^\dagger \beta \Psi_s)^2
\]
Let us design:
\[
\gamma_0 = \pi_o^\dagger \gamma \pi_o, \gamma_s = \pi_s^\dagger \gamma \pi_s, \\
\beta_0 = \pi_o^\dagger \beta \pi_o, \beta_s = \pi_s^\dagger \beta \pi_s
\]
\[
\gamma_0^\dagger = U^{(-)} \gamma_0 U^{(-)}, \gamma_s^\dagger = U^{(-)} \gamma_s U^{(-)}, \\
\beta_0^\dagger = U^{(-)} \beta_0 U^{(-)}, \beta_s^\dagger = U^{(-)} \beta_s U^{(-)}.
\]
Since
\[
\gamma_0^\dagger = a \gamma_0 + \sqrt{1 - a^2} \beta_0, \beta_0^\dagger = a \beta_0 - \sqrt{1 - a^2} \gamma_0, \\
\gamma_s^\dagger = a \gamma_s - \sqrt{1 - a^2} \beta_s, \beta_s^\dagger = a \beta_s + \sqrt{1 - a^2} \gamma_s
\]
then
\[
\rho_{oo}^2 = \rho_{oo}^\dagger^2 \text{ and } \rho_{so}^2 = \rho_{so}^\dagger^2.
\]
From (11) since:
\[
\rho = \rho', \quad j_x = j_x', \quad j_y = j_y', \quad j_z = j_z
\]

then the local densities:
\[
\rho_o^2 = \rho^2 - (j_x^2 + j_y^2 + j_z^2) = \rho_o'^2 = \rho^2 - (j_x'^2 + j_y'^2 + j_z'^2)
\]

Because
\[
\left(\Psi^\dagger\Psi\right)^2 - \left((\Psi^\dagger\beta_1\Psi)^2 + (\Psi^\dagger\beta_2\Psi)^2 + (\Psi^\dagger\beta_3\Psi)^2\right) =
\]
\[
= \left(\Psi^\dagger\gamma\Psi\right)^2 + (\Psi^\dagger\beta\Psi)^2
\]

and
\[
\left(\Psi'^\dagger\Psi'^\dagger\Psi'^\dagger\Psi'\right)^2 - \left((\Psi'^\dagger\beta_1\Psi'^\dagger\Psi'^\dagger\Psi'^\dagger\Psi')^2 + (\Psi'^\dagger\beta_2\Psi'^\dagger\Psi'^\dagger\Psi'^\dagger\Psi')^2 + (\Psi'^\dagger\beta_3\Psi'^\dagger\Psi'^\dagger\Psi'^\dagger\Psi')^2\right) =
\]
\[
= \left(\sqrt{\left(\Psi'_o^\dagger\gamma\Psi'_o\right)^2 + (\Psi'_o^\dagger\beta\Psi'_o)^2} + \sqrt{\left(\Psi'_o^\dagger\gamma\Psi'_o\right)^2 + (\Psi'_o^\dagger\beta\Psi'_o)^2}\right)^2
\]

but
\[
\left(\Psi'^\dagger\Psi'^\dagger\Psi'^\dagger\Psi'\right)^2 - \left((\Psi'^\dagger\beta_1\Psi'^\dagger\Psi'^\dagger\Psi'^\dagger\Psi')^2 + (\Psi'^\dagger\beta_2\Psi'^\dagger\Psi'^\dagger\Psi'^\dagger\Psi')^2 + (\Psi'^\dagger\beta_3\Psi'^\dagger\Psi'^\dagger\Psi'^\dagger\Psi')^2\right) \\
\neq \left(\Psi'^\dagger\gamma\Psi'^\dagger\Psi'^\dagger\Psi'^\dagger\Psi'\right)^2 + (\Psi'^\dagger\beta\Psi'^\dagger\Psi'^\dagger\Psi'^\dagger\Psi')^2
\]

then
\[
\rho_o = \sqrt{\left(\Psi^\dagger\gamma\Psi\right)^2 + (\Psi^\dagger\beta\Psi)^2} = \\
= \sqrt{\left(\Psi_o^\dagger\gamma\Psi_o\right)^2 + (\Psi_o^\dagger\beta\Psi_o)^2} + \sqrt{\left(\Psi_s^\dagger\gamma\Psi_s\right)^2 + (\Psi_s^\dagger\beta\Psi_s)^2} = \\
= \rho_{oo} + \rho_{so}
\]

and
\[
\rho_o' = \\
= \sqrt{\left(\Psi'_o^\dagger\gamma\Psi'_o\right)^2 + (\Psi'_o^\dagger\beta\Psi'_o)^2} + \sqrt{\left(\Psi'_o^\dagger\gamma\Psi'_o\right)^2 + (\Psi'_o^\dagger\beta\Psi'_o)^2} = \\
= \rho_{oo'} + \rho_{so'}
\]
but

\[ \rho_o^2 \neq (\Psi^\dagger \gamma \Psi)^2 + (\Psi^\dagger \beta \Psi)^2. \]

Therefore \( \rho_o \) is a local probability density of a sum of two mutually exclusive events with a local densities \( \rho_{oo} \) and \( \rho_o^* \).

Because:

\[
\begin{align*}
\Psi^\dagger \gamma \Psi_o &= \frac{1}{2} \left(1 - \frac{b}{\sqrt{1-a^2}}\right) \Psi^\dagger \gamma \Psi, \\
\Psi^\dagger \beta \Psi_o &= \frac{1}{2} \left(1 - \frac{b}{\sqrt{1-a^2}}\right) \Psi^\dagger \beta \Psi, \\
\Psi^\dagger \gamma \Psi_* &= \frac{1}{2} \left(1 + \frac{b}{\sqrt{1-a^2}}\right) \Psi^\dagger \gamma \Psi, \\
\Psi^\dagger \beta \Psi_* &= \frac{1}{2} \left(1 + \frac{b}{\sqrt{1-a^2}}\right) \Psi^\dagger \beta \Psi
\end{align*}
\]

then \( \rho_o \) and \( \rho_o^* \) do not depend from \( U(-) \).

For \( U(+) \) \footnote{12}:

\[
\Pi_o = \frac{1}{2\sqrt{1-u^2}} \begin{bmatrix} (v + \sqrt{1-u^2}) 1_4 & (-s + ik) \gamma_5 \\ (-ik - s) \gamma_5 & (\sqrt{1-u^2} - v) 1_4 \end{bmatrix},
\]

\[
\Pi_* = \frac{1}{2\sqrt{1-u^2}} \begin{bmatrix} (\sqrt{1-u^2} - v) 1_4 & (s - ik) \gamma_5 \\ (ik + s) \gamma_5 & (v + \sqrt{1-u^2}) 1_4 \end{bmatrix}.
\]

Hence:

\[
U(+)\dagger U(+) = u\gamma - \sqrt{1-u^2} (\Pi_o - \Pi_*) \beta,
\]

\[
U(+)\dagger \beta U(+) = u\beta + \sqrt{1-u^2} (\Pi_o - \Pi_*) \gamma
\]

and all rest for \( U(+) \) like to \( U(-) \).

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