Estimation of Beta-Adjusted Parameters in Capital Asset Pricing Model under Non-Constant Volatility

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Abstract. The Capital Asset Pricing Model (CAPM) standard equation states that the equilibrium of the capital market will be indicated by the stock market (asset) line, where the line connects risk-free investment opportunities and risky investments. The beta parameter coefficient in the CAPM is a measure of systematic risk in the capital market, whose value must be estimated accurately. In this paper, we will examine the beta-adjusted parameter estimation when the market index return has a constant volatility. Non-constant volatility in the return of the market index was analyzed using generalized autoregressive conditional heteroscedastic (GARCH) models. Furthermore, using beta estimators, average beta estimator values, and beta estimator volatility, are used to predict the future beta-adjusted coefficient. Such an assessment method is used to analyze several stocks traded on the Indonesia Stock Exchange (IDX). The results show that the beta-adjusted estimator under volatility is not constant in accordance with actual market conditions. So that it can be considered in estimating beta-adjusted in the capital market.

1. Introduction
The beta coefficient in the Capital Asset Pricing Model (CAPM) equation is a measure of systematic risk of a stock or portfolio relative to market risk [1; 9]. Knowing the beta of a stock or portfolio is important for analyzing stock or investment portfolios. Beta a stock shows a systematic risk that cannot be eliminated because of diversification. Beta of a stock can be calculated by estimation techniques that use historical data in the form of stock return data and market index returns. Beta estimation can be done one of them by linear regression technique [2; 10].

Previous studies have shown that beta has certain characteristics, which tend to lead to a value of 1 (market beta) from time to time. Therefore, the predicted beta needs to be adjusted to meet the character [3; 13]. Beta which is estimated based on historical data is often a biased beta, if it is used for capital markets which are thin markets. The bias that occurs in thin markets is due to the occurrence of unsynchronized trade in the capital market [8; 11]. To reduce possible bias, beta for thin capital markets must be adjusted. There are several methods that can be used to adjust beta values, one of
which is the Vasicek method. Vasicek suggests that the beta adjustment to the average does not use the same weight but depends on the amount of uncertainty from beta. This uncertainty can be measured by the amount of volatility of beta values in the sample [13; 14].

This paper examines the adjustment of the beta coefficient with the Vasicek method using a non-constant volatility approach. This non-constant volatility was analyzed using Generalized Autoregressive Conditioner Heteroscedasticity (GARCH) models in the time series model. Furthermore, this Vasicek beta adjustment using non-constant volatility is applied to analyze the return data of several stocks traded on the Indonesia Stock Exchange (IDX). The purpose of this analysis is to get a prediction of the adjusted beta value when the market index data return has a constant volatility.

2. Mathematical models

2.1 Stock returns

Suppose $P_{it}$ declare stock prices $i$ ($i=1,...,N$ and $N$ the number of stocks analyzed) at the time $t$ ($t=1,...,T$ and $T$ the number of observation data), stock returns $i$ at time $t$ can be calculated using the equation $r_{it} = \ln(P_{it}/P_{it-1})$ [4; 12].

A sequence of returns $\{r_{mt}\}$ follow the autoregressive moving average degree model $p$ and $q$, or written as ARMA($p, q$), the equation can be expressed as:

$$r_{mt} = \psi_0 + \sum_{i=1}^p \psi_i r_{mt-i} + \alpha_{mt} - \sum_{j=1}^q \theta_j \alpha_{mt-j},$$

with $\psi_k$ ($k=1,...,p$) and $\theta_j$ ($j=1,...,q$) coefficient parameters, as well $\alpha_{mt}$ residual of ARMA($p, q$) model for stock at time $mt$ [5; 7]. Equation (1) is often referred to as the mean model.

For return $r_{mt}$, suppose $\alpha_{mt} = r_{mt} - \mu_{mt}$ is the residual of the market index return at time $t$, where $\mu_{mt}$ the mean return of the market index at the time $t$. Residual $\alpha_{mt}$ follow the GARCH($m, n$) model, when:

$$\alpha_{mt} = \sigma_{mt} e_{mt}, \quad \sigma_{mt}^2 = \alpha_0 + \sum_{i=1}^m \alpha_i \sigma_{mt-i}^2 + \sum_{j=1}^n \beta_j \sigma_{mt-j}^2 + \epsilon_{mt}.$$

Where $\{e_{mt}\}$ rows of random variables are mutually independent and have identical distributions (iid) with an mean 0 and variance 1, $\alpha_0 > 0$, $\alpha_i, \beta_i \geq 0$, and $\sum_{i=1}^{\max(m,n)}(\alpha_i + \beta_i) < 1$ [5; 8]. Equation (2) is often referred to as a volatility model.

The volatility modeling process is carried out as follows: (i) Estimation of the mean model; that is, estimating and choosing a suitable mean model. (ii) ARCH effect test; ARCH-LM effect was tested for the residuals from the mean model. (iii) Model identification; if the ARCH effect is statistically significant, then set tentative values $m$ and $n$ with the help of correlogram. (iv) Estimation of the model; namely estimating simultaneously the average model and the volatility model, carried out using the maximum likelihood method to estimate the GARCH($m, n$) model. (v) Test diagnosis; test whether residuals from the volatility model are white noise. (vi) Prediction; namely using the volatility model chosen to predict volatility $\hat{\sigma}_{mt}^2 = \hat{\sigma}_{mt}^2 (l)$, namely prediction $l$-step forward [9; 14].

2.2. CAPM equation

Suppose $r_{it}$ stock return at time $t$, $r_{mt}$ market index return at time $t$, and $\mu_f$ the mean of risk-free asset return. The regression equation of the Capital Asset Pricing Model (CAPM) can be expressed as:

$$(r_{it} - \mu_f) = \alpha_i + \beta_i (r_{mt} - \mu_f) + e_{it}.$$

Where $\alpha_i$ and $\beta_i$ ($i=1,...,N$) coefficient parameters, as well $\{e_{it}\}$ residual sequence are white noise [2; 3]. Based on equation (3) can be obtained:
\[ \mu_i = E[r_{it}] = \mu_f + \alpha_i + \beta_i(\mu_m - \mu_f) \]
\[ \text{Var}[r_{it}] = \beta_i^2 \sigma_m^2 + \sigma_{ei}^2. \]  

2.3. Beta coefficient parameter estimation

Estimation of coefficient parameters \( \alpha_i \) and \( \beta_i \) \( (i = 1,...,N) \) in equation (3) can be done using the least square method (least Square). By definition, beta is a measure of volatility between stock return and market return. If we let \( \sigma_{mi} = \text{Cov}(r_{mt}, r_{it}) \) as a measure of volatility between the return of the market index and stock returns \( i \), and \( \sigma_{mt}^2 \) volatility of the market index return, the beta can also be estimated by the formula [2; 6; 10]:

\[ \beta_i = \frac{\sigma_{mi}}{\sigma_{mt}^2}, \quad (i = 1,...,N \text{ and } N \text{ number of stocks}). \]  

2.4. Beta-Adjusted

If \( \bar{\beta}_1 \) represents the average beta values of the shares in the sample for the historical period, and \( \beta_{11} \) is a historical stock beta, the Vasicek procedure is a weighted average of the beta of both. The weight used by the weighted average is based on the amount of volatility. If \( \sigma_{\beta1}^2 \) the volatility of the beta stocks in the sample for the historical period, and \( \sigma_{\beta1}^2 \) volatility of stock beta \( i \), then the weight suggested by Vasicek is \( \sigma_{\beta1}^2 / (\sigma_{\beta1}^2 + \sigma_{\beta1}^2) \) for \( \beta_{11} \), and \( \sigma_{\beta1}^2 / (\sigma_{\beta1}^2 + \sigma_{\beta1}^2) \) for \( \bar{\beta}_1 \). So the predicted beta is adjusted for stocks \( i \) in the second period are as follows:

\[ \beta_{12} = \left( \frac{\sigma_{\beta1}^2}{\sigma_{\beta1}^2 + \sigma_{\beta1}^2} \right) \beta_{11} + \left( \frac{\sigma_{\beta1}^2}{\sigma_{\beta1}^2 + \sigma_{\beta1}^2} \right) \bar{\beta}_1. \]  

This Vasicek adjustment procedure is a Bayesian estimation technique [1; 2]. This procedure will result in greater adjustments leading to the average for observations that have greater volatility than those with smaller volatility [3; 9].

3. Results and discussion

3.1. Data

The data analyzed here includes the stock data of PRUF, BBRI, MPPA, BMRI, and INDF, from the period of January 2010 to March 2012, followed by a symbol \( S_1 \) up to \( S_5 \). Market index data used is the Composite Stock Price Index (CSPI). Meanwhile, the risk-free asset data used is bonds. The data is accessed through the website http://www.finance.go.id/\/. Each stock data is determined by the return value, and then used for the following volatility modeling.

3.2. Modeling of Non-Constant Volatility

In this section Eviews-8 software is used to estimate the volatility model. Initially, an average model is identified and estimated. First, identification through the sample autocorrelation function (ACF) and partial autocorrelation function (PACF). Based on the ACF and PACF patterns of the market return index, the tentative model is determined. The estimation results, can be shown that the market index return follows the ARMA model (1.1). Second, test the diagnosis of the model, using correlogram residual data and test the Ljung-Box hypothesis. The test results show that the residual model is white noise. The results of the residual normality test show normal distribution. So that the ARMA(1,1) model can be used for further analysis. Furthermore, identification and estimation of volatility models are carried out. First, the detection of the element of autoregressive conditional heteroscedasticity (ARCH) against residuals \( u_{mt} \), using ARCH-LM method with Eviews-6 software.
The results are obtained values $\chi^2$ (obs * R-Square) the return of the market index is 112.8209 with a probability of 0.0000 or a smaller significance level of 5%, which means there is an element of ARCH.

Second, identification and estimation of volatility models. Here the generalized autoregressive conditional heteroscedasticity (GARCH) models refer to equation (2). Based on correlogram residual squares $a_{mt}^2$, namely the ACF and PACF charts, which are possible models of tentative volatility. Estimation of the volatility model of the market index return is carried out simultaneously with the average model. The result, obtained the best model is GARCH (1,2) with the equation

$$\sigma_{mt}^2 = 0.000139 + 0.224007a_{mt}^2 + 0.302227r_{mt-1}^2 + 0.443794r_{mt-2}^2 + \epsilon_{mt}.$$  

Third, based on the ARCH-LM test, residuals $\epsilon_{mt}$ from the volatility model there is no ARCH element, and also has white noise. This volatility model will be used to predict values $\hat{\sigma}_{mt}^2 = \sigma_{mt}^2 (1)$ recursively. The prediction result for one period ahead of the market index volatility is $\hat{\sigma}_{mt}^2 = \sigma_{mt}^2 (1) = 0.00189$.

### 3.3 Beta estimation in CAPM

In the CAPM it is assumed that the stock return risk premium correlates with the market risk return premium. Because risk-free asset returns are relatively constant, the average estimator is assumed to be constant, its value is equal to $\hat{\mu}_f = 0.0026462$ and thus the variance $\sigma_f^2 = 0$. Furthermore, suppose

$$Y_{it} = S_{it} - \hat{\mu}_f$$ stock return risk premium $i$ ($i = 1, ..., N$ and $N$ number of stocks) at the time $t$ ($t = 1, ..., T$ and $T$ the number of data), and $I_{mt} = r_{mt} - \hat{\mu}_f$ Market return risk premium time $t$.

Referring to equation (3), the standard CAPM regression equation is

$$Y_{it} = a_i + \beta_i I_{mt} + \epsilon_{it}.$$  

Because the regression of $Y_{it}$ against $I_{mt}$ produce a value of determination $R^2$ very small, variable $Y_{it}$ transformed into $M_{it} = \ln \{ (1 + Y_{it}) / (1 - Y_{it}) \}$. The regression equation that will be estimated next is $M_{it} = a_i + \beta_i I_{mt} + \epsilon_{it}$. To estimate constants $a_i$ and coefficient parameters $\beta_i$ done with the least squares method. The results of the equation are presented as follows, accompanied by statistical values $t$, statistic $F$, and probability $P$.

\[
\begin{align*}
\text{Stock S1:} & \quad M_{1t} = -0.0278+ 0.724(r_{mt} - \hat{\mu}_f); R^2 = 57.40\%, F = 1111.47 and P=0;  \\
& \quad t = (-8.77) \quad (33.34) \\
\text{Stock S2:} & \quad M_{2t} = -0.036+ 0.768(r_{mt} - \hat{\mu}_f); R^2 = 64.00\%, F = 1307.51 and P=0;  \\
& \quad t = (-13.36) \quad (38.16) \\
\text{Stock S3:} & \quad M_{3t} = 0.0396+ 0.770(r_{mt} - \hat{\mu}_f); R^2 = 61.30\%, F = 1307.51 and P=0;  \\
& \quad t = (12.76) \quad (336.16) \\
\text{Stock S4:} & \quad M_{4t} = 0.0645+ 0.797(r_{mt} - \hat{\mu}_f); R^2 = 56.90\%, F = 1091.31 and P=0;  \\
& \quad t = (18.37) \quad (33.03) \\
\text{Stock S5:} & \quad M_{5t} = -0.0568+ 0.182(r_{mt} - \hat{\mu}_f); R^2 = 63.10\%, F = 1415.04 and P=0;  \\
& \quad t = (-80.20) \quad (37.62)
\end{align*}
\]

Whenever the level of significance is determined $\alpha = 0.05$; from the distribution table of $t$-standardobtained statistical critical values $t(0.05;827) = 1.962837$; and from the distribution table of $F$ obtained statistical critical values $F(0.05;1;827) = 3.85273$. Compare statistical values $t$-count and $F$ - calculate each regression, with statistical critical values $t$-table and $F$ -table, it appears that CAPM
regression for stocks $S_1$ up to $S_5$ all have been significant. Likewise the residual test $e_{it}$ $(i=1,...,5)$ everything is white noise.

Based on the CAPM regression equation for stocks $S_1$ up to $S_5$ obtained values $\hat{\beta}_i$ and volatility $\hat{\sigma}_{ei}^2$ as given in Table-1 column (a) and column (b). Because of the value $\hat{\sigma}_{mi}^2 = 0.00189$ and by using equation (4) the values will be obtained $\hat{\sigma}_{i1}^2$ $(i=1,...,5)$ as given in Table-1 column (c).

| Stocks | $\hat{\beta}_{i1}$ | $\hat{\sigma}_{ei}^2$ | $\hat{\sigma}_{i1}^2$ | $\hat{\beta}_{i2}$ - Adjusted |
|--------|---------------------|------------------------|------------------------|-------------------------------|
|        | (a)                 | (b)                    | (c)                    | (d)                           |
| $S_1$  | 0.724               | 0.00819                | 0.009179               | 0.715055                      |
| $S_2$  | 0.768               | 0.00782                | 0.008932               | 0.754199                      |
| $S_3$  | 0.770               | 0.00834                | 0.009458               | 0.755242                      |
| $S_4$  | 0.797               | 0.00791                | 0.009108               | 0.778950                      |
| $S_5$  | 0.182               | 0.00912                | 0.009182               | 0.237033                      |
| Total  | 3.241               |                        |                        |                               |

Average value $\hat{\beta}_{i1}$ is $\bar{\beta}_{i1} = 3.241/5 = 0.6482$ and the volatility of the stock beta is:

$$\sigma_{\hat{\beta}_{i1}}^2 = \frac{(0.724 - 0.6482)^2 + (0.768 - 0.6482)^2 + (0.770 - 0.6482)^2 + (0.797 - 0.6482)^2 + (0.182 - 0.6482)^2}{(5 - 1)} = 0.068604.$$  

Furthermore, using the values $\bar{\beta}_{i1}$ and $\sigma_{\hat{\beta}_{i1}}^2$ and also volatility values in Table-1 column (c), and by using equation (6) parameters of beta coefficients in the next 1-period, or value $\hat{\beta}_{i2}$ - Adjusted can be calculated. The calculation results $\hat{\beta}_{i2}$ - Adjusted given in Table-1 column (d).

Noting the results of beta-adjusted ( $\hat{\beta}_{i2}$ - Adjusted) in Table-1 column (d), shows that the adjusted beta whose value is greater than the average tends to decrease, and the adjusted beta whose value is smaller than the average tends to increase towards the average value in the next period. According to Sembiring et al. [3], adjusted beta, both using Bayesian procedure techniques by Vasicek and other procedures, provides a more accurate prediction compared to unadjusted beta. So that the beta value is adjusted will be more suitable to the actual market conditions.

4. Conclusion

This paper has examined the estimated beta-adjusted parameters in CAPM with a non-constant volatility approach. The estimation method has been implemented to analyze five stocks, namely stocks $S_1$ up to $S_5$, which are traded on the Indonesia Stock Exchange (IDX), with the Composite Stock Price Index (CSPI) as a market index, and bond prices as risk-free assets. The significant return of the market index has a constant volatility that follows the GARCH(1,1) model. While the correlation between stock risk premium return $S_1$ up to $S_5$, with market index risk premium returns generate CAPM regression equations with beta estimator values that are larger and some are smaller than the average value. Adjustment of the beta value is done using the Vasicek procedure technique under non-constant volatility of the market index return. The results show that beta with a value greater than the average tends to decrease, while beta which is smaller than the average tends to increase towards its average value. This adjusted beta value is expected to be more in line with the actual market conditions.
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