Unified Framework for Matter, Dark Matter, and Radiative Neutrino Mass

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Abstract

The well-studied radiative model of neutrino mass through $Z_2$ dark matter is shown to be naturally realizable in the context of $SU(6)$ grand unification. A recent new proposal based on $U(1)_D$ dark matter is similarly accommodated in $SU(7)$. Just as the proton is unstable at the scale of quark-lepton unification, dark matter is expected to be unstable at a similar scale.
The Universe consists of matter, dark matter, and dark energy. Of the three, matter is understood in terms of the standard model (SM) of quarks and leptons with vector gauge bosons as force carriers and one fundamental physical scalar particle, i.e. the Higgs boson, which is presumably the 126 GeV particle recently discovered \cite{1, 2} at the Large Hadron Collider (LHC). Whereas quark and charged-lepton masses are directly linked to the Higgs doublet $\Phi = (\phi^+, \phi^0)$, the origin of neutrino mass \cite{3} is not as clearcut. It may involve physics beyond the SM and as such, may be linked to dark matter (as well as the Higgs boson) in a simple one-loop mechanism \cite{4} as shown below. This well-studied model is very simple.

![Figure 1: One-loop generation of neutrino mass with $Z_2$ symmetry.](image)

Three new singlet neutral Majorana fermions $N_{1,2,3}$ and one new scalar doublet $(\eta^+, \eta^0)$ are added to the SM with an exactly conserved discrete $Z_2$ symmetry, under which the new particles are odd and the SM particles are even. It has been called 'scotogenic' from the Greek 'scotos' meaning darkness. Let $\eta^0 = (\eta_R + i\eta_I)/\sqrt{2}$, then the allowed $(\Phi^\dagger \eta)^2$ term splits the masses of $\eta_{R,I}$ so that the lighter, say $\eta_R$, is a good dark-matter candidate \cite{4}. Its detailed study began in Ref. [5] and the most recent update is Ref. [6].

A new variation of Fig. 1 has recently been proposed \cite{7} as shown below. Instead of $Z_2$, it supports an $U(1)_D$ gauge symmetry which may be exact or broken, either into $Z_2$ or a conserved global $U(1)$. Here there are three new singlet neutral Dirac fermions $N_{1,2,3}$ and two new scalar doublets $(\eta^+_{1,2}, \eta^0_{1,2})$. Whereas $N_{1,2,3}$ and $\eta_1$ transform as +1 under $U(1)_D$, $\eta_2$...
transforms as $-1$. The loop is thus completed without breaking $U(1)_D$. Now the lightest $N$ is Dirac fermion dark matter, and the $U(1)_D$ gauge boson $\gamma_D$ and the corresponding Higgs boson $h_D$ are possible light force carriers of the self-interacting dark matter, with important astrophysical implications, such as the dark matter halo structure [8] of dwarf galaxies and the possibility of Sommerfeld enhancement [9] of dark matter annihilation to explain the observation of positron excess in recent space experiments [10, 11].

What could be the origin of these new particles and new symmetries? It is proposed in this paper that they are remnants of a unified theory incorporating both matter and dark matter. The notion that the observed quarks and leptons may be unified in a single theory, the first of which was $SU(5)$ [12], should be extended. In the context of quark-lepton unification, the stability of the proton is not absolute. Its lifetime is very much longer than that of the Universe because the unification energy scale is very high, say $\sim 10^{16}$ GeV. In the context of the unification of matter and dark matter, it is thus expected that the $Z_2$ or $U(1)_D$ symmetry which maintains the stability of dark matter in the Universe today is again not absolute. They will also be violated at a similar high energy scale. To implement this simple idea, two prototype models based on $SU(6)$ and $SU(7)$ respectively will be discussed. The former leads to the well-known original one-loop radiative (scotogenic) model of neutrino mass [4] with $Z_2$ symmetry, as shown in Fig. 1, and the latter to the recent new variation [7].
with $U(1)_D$ gauge symmetry, as shown in Fig. 2.

The symmetry $SU(6)$ was originally considered as an alternative to $SU(5)$ unification with very different particle structure. Here it is simply used as an extension of $SU(5)$ to include dark matter. The well-known $SU(5)$ assignments of quarks and leptons are

$$
\begin{pmatrix}
  d^c \\
  d^c \\
  d^c \\
  e \\
  \nu
\end{pmatrix}^5_F, \quad
\begin{pmatrix}
  0 & u^c & -u^c & -u & -d \\
  -u^c & 0 & u^c & -u & -d \\
  u^c & -u^c & 0 & -u & -d \\
  u & u & u & 0 & -e^c \\
  d & d & d & e^c & 0
\end{pmatrix}^{10}_F.
$$

The standard-model Higgs doublet is contained in

$$
\begin{pmatrix}
  \xi^{-1/3} \\
  \xi^{-1/3} \\
  \xi^{-1/3} \\
  \phi^+ \\
  \phi^0
\end{pmatrix}^5_S,
$$

where the scalar color triplet $\xi^{-1/3}$ is assumed very heavy. The resulting Yukawa couplings invariant under $SU(5)$ are

$$
\begin{pmatrix}
  \xi^{-1/3} \\
  \xi^{-1/3} \\
  \xi^{-1/3} \\
  \phi^+ \\
  \phi^0
\end{pmatrix}^5_S.
$$

As $\phi^0$ develops a nonzero vacuum expectation value, all quarks and leptons (except the neutrinos) obtain masses.

The $SU(6)$ extension is straightforward. A neutral singlet fermion $N$ is added to $\bar{5}_F^*$ to form $\bar{6}_F^*$ under $SU(6)$. A heavy $\bar{5}_F$ is added to $\bar{10}_F$ to form $\bar{15}_F$ under $SU(6)$. The analogous Yukawa couplings invariant under $SU(6)$ are

$$
\begin{pmatrix}
  \xi^{-1/3} \\
  \xi^{-1/3} \\
  \xi^{-1/3} \\
  \phi^+ \\
  \phi^0
\end{pmatrix}^6_S, \quad
\begin{pmatrix}
  \xi^{-1/3} \\
  \xi^{-1/3} \\
  \xi^{-1/3} \\
  \phi^+ \\
  \phi^0
\end{pmatrix}^{15}_S.
$$

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$$
\begin{pmatrix}
  \xi^{-1/3} \\
  \xi^{-1/3} \\
  \xi^{-1/3} \\
  \phi^+ \\
  \phi^0
\end{pmatrix}^{6}_S, \quad
\begin{pmatrix}
  \xi^{-1/3} \\
  \xi^{-1/3} \\
  \xi^{-1/3} \\
  \phi^+ \\
  \phi^0
\end{pmatrix}^{15}_S.
$$

where $\bar{6}_S^*$ contains $\bar{5}_S^*$ and $\bar{15}_S$ contains another $\bar{5}_S$. Hence two different Higgs doublets are needed, one coupling to the down sector, the other to the up sector. Note that whereas
$\mathbf{5}_F + \mathbf{10}_F$ is anomaly-free in $SU(5)$, the corresponding combination in $SU(6)$ is $\mathbf{6}_F^* + \mathbf{6}_F^* + \mathbf{15}_F$.

The $\mathbf{5}_F^*$ contained in the extra $\mathbf{6}_F^*$ is heavy and pairs up with the $\mathbf{5}_F$ contained in the $\mathbf{15}_F$ already mentioned, through the $\mathbf{1}_S$ of $\mathbf{6}_S^*$.

The extension of $SU(6)$ to include dark matter has also been considered recently \[15\] in a different context. There the motivation is to understand the relative abundance of matter versus dark matter. It is assumed that they are both generated with an asymmetry in the early Universe from a common source. This scenario is very different from the one here which emphasizes the possible connection between neutrino mass and dark matter.

So far in the present case, $N$ is assumed to be the only new fermion at low energy, but it has no interaction with the SM particles. Consider now the $SU(6)$ scalar multiplet $\mathbf{21}_S$. It decomposes into $\mathbf{15}_S + \mathbf{5}_S + \mathbf{1}_S$ of $SU(5)$. The resulting Yukawa and quartic couplings invariant under $SU(6)$ are

\[
\mathbf{6}_F^* \times \mathbf{6}_F^* \times \mathbf{21}_S, \quad \mathbf{15}_S^* \times \mathbf{15}_S^* \times \mathbf{21}_S \times \mathbf{21}_S.
\]

These interactions generate three terms. (1) As the scalar component $\mathbf{1}_S$ of $\mathbf{21}_S$ acquires a vacuum expectation value, $N$ gets a Majorana mass. (2) There is now the interaction term $(\nu \eta^0 - e \eta^+) N$, where the scalar doublet $(\eta^+, \eta^0)$ comes from $\mathbf{5}_S$ of $\mathbf{21}_S$. (3) There is also the quartic scalar coupling $(\Phi^\dagger \eta)^2$. These three terms support a $Z_2$ symmetry under which $N$ and $\eta$ are odd and all others are even, thus realizing the existence of stable dark matter. They are also exactly the terms required for the scotogenic neutrino mass \[4\] of Fig. 1. Its $SU(6)$ decomposition is shown in Fig. 3.

However, the $Z_2$ symmetry here is not absolute. Just as the heavy color triplet gauge bosons contained in the adjoint $\mathbf{24}_V$ of $SU(5)$ mediate proton decay, the extra heavy color triplet gauge bosons contained in the adjoint $\mathbf{35}_V$ of $SU(6)$ will connect $N$ to the quarks, resulting in $N$ decaying into $p\pi^-$ and $n\pi^0$. Another possibility is the mixing of the heavy scalar color triplet $\zeta^{-1/3}$ in $\mathbf{21}_S$ with its counterpart $\xi^{-1/3}$ in $\mathbf{15}_S$ through the adjoint $\mathbf{35}_S$ of
SU(6). The companion interactions $d^c \zeta N$ together with $ud\xi$ from $15_F \times 15_F \times 15_S$ imply also the decay $N \rightarrow p\pi^-$ or $n\pi^0$, just as the proton decays through the heavy scalar color triplet $\xi$ itself. A unified framework for understanding matter and dark matter is thus established together with the origin of neutrino mass.

In the breaking of SU(6) through the adjoint $35_S$, let the $6 \times 6$ matrix representing $35_S$ develop diagonal vacuum expectation values proportional to $(1,1,1,0,0,-3)$, then the residual symmetry is $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_N$. As noted earlier, $21_S$ breaks $U(1)_N$ at a lower energy scale for $N$ to acquire a Majorana mass. The above choice of $35_S$ breaking mixes $\zeta$ with $\xi$, but not $\eta$ with $\Phi$. It is only possible here because of SU(6). If the gauge group is SU(5), then the breaking to $SU(3)_C \times SU(2)_L \times U(1)_Y$ must be along the $(1,1,1,-3/2,-3/2)$ direction and that will mix both sectors.

Of course, this simple SU(6) model is still missing many details, such as the complete fermion and scalar content necessary to obtain the requisite low-energy particle spectrum with the effective $Z_2$ symmetry and how gauge couplings may be unified, with or without supersymmetry, among many other possible issues. On the other hand, the important point has been made that the stability of dark matter may be linked to the stability of ordinary matter in much the same way. Such is the intended message of this paper.
Consider next the symmetry $SU(7)$ [10, 17]. In the decomposition $SU(5) \times SU(2)_N$, the $7^*$ of $SU(7)$ contains an extra $SU(2)_N$ doublet $(N_1, N_2)$. The corresponding Yukawa couplings to those of Eqs. (3) and (4) are

$$\begin{align*}
7^* \times 21_F \times 7^*, & \quad 21_F \times 21_F \times 35_{\tilde{c}}.
\end{align*}$$

(6)

The $28_S$ of $SU(7)$ now contains a bidoublet, i.e.

$$\begin{pmatrix}
\eta_1^+ \\
\eta_2^+
\end{pmatrix},$$

(7)

where $SU(2)_L$ applies vertically and $SU(2)_N$ applies horizontally. The corresponding couplings to Eq. (5) are

$$\begin{align*}
7^*_F \times 7^*_F \times 28_S, & \quad 21^*_S \times 21^*_S \times 28_S \times 28_S.
\end{align*}$$

(8)

As a result, Fig. 2 is obtained, with $U(1)_D$ given by the diagonal subgroup of $SU(2)_N$. Its $SU(7)$ decomposition is shown in Fig. 4.

In conclusion, it has been shown that a unified framework of matter and dark matter is possible and perhaps even desirable for the complete understanding of our Universe. Two prototype models based on $SU(6)$ and $SU(7)$ have been discussed. The former applies naturally to the original scotogenic model of neutrino mass with $Z_2$ symmetry. The latter fits in nicely with the recent proposal of a possible exact or broken $U(1)_D$ symmetry in place
of $Z_2$. This new dynamical $U(1)_D$ for dark matter has many implications for astrophysical observations.

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