Meson-Baryon Interactions in Unitarized Chiral Perturbation Theory

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Abstract. Meson-Baryon Interactions can be successfully described using both Chiral Symmetry and Unitarity. The $s$–wave meson-baryon scattering amplitude is analyzed in a Bethe-Salpeter coupled channel formalism incorporating Chiral Symmetry in the potential. Two body coupled channel unitarity is exactly preserved. The needed two particle irreducible matrix amplitude is taken from lowest order Chiral Perturbation Theory in a relativistic formalism. Off-shell behavior is parameterized in terms of low energy constants. The relation to the heavy baryon limit is discussed. The position of the complex poles in the second Riemann sheet of the scattering amplitude determine masses and widths baryonic resonances of the $N(1535)$, $N(1670)$, $Λ(1405)$ and $Λ(1670)$ resonances which compare well with accepted numbers.

INTRODUCTION

The existence of baryon resonances in meson-baryon reactions is a non-perturbative feature of QCD at intermediate energies. Although Relativistic Invariance, Crossing Symmetry, Unitarity and Chiral Symmetry undoubtedly provide powerful constraints, the usual high and low energy simplifications do not directly apply here just because there is no obvious expansion parameter and some special methods and approximations have to be developed.

The nature of baryonic resonances is not fully unambiguous since their lifetime, $\tau_R$, is very short and hence they cannot be observed decaying into final products. The standard and accepted definition is that resonances correspond to poles in the complex CM energy plane in unphysical sheets, $-2\text{Im} \sqrt{s} = \Gamma_R = 1/\tau_R$ complying with causality requirements. It is remarkable that although this definition has long been known, it has only been very recently incorporated explicitly in the Particle Data [1]. Other definitions (zeros of the K-matrix, phase shift passing through $90^\circ$, maximum of cross section, etc.), although simpler to work with, coincide with the former one only in the limit $\Gamma_R \to 0$. For non-relativistic potential scattering this limit produces the exponential law and corresponds to the picture that a stable particle tunnels through the barrier to the continuum. Nevertheless, one should pay attention to the fact that going to the complex $s$–plane in terms of data on the real axis may be a model dependent operation,
particularly for not very sharp resonances. Conversely, going from the pole in the
unphysical sheet to the real scattering line is also not uniquely defined, and the energy
dependence may be model dependent as well. This ambiguity is enhanced in the case
where there are several open channels, because there are many ways of parameterizing
a multichannel scattering amplitude with a given analytical structure [2]. The situation
is aggravated by the fact that very often the analysis of data in terms of partial wave
amplitudes are not provided with error estimates.

In the quark model baryon resonances are naturally interpreted as bound state com-
posites of three valence quarks, and their widths are computed as matrix elements of
hadronic transition operators between the bound and continuum states [3]. This approach
corresponds to the picture of unstable particles weakly coupled to a continuum. To im-
prove on this, the scattering problem has to be solved. The information entering such a
problem is baryon masses and baryon-meson form factors; the only specific reminder
of the underlying quark degrees of freedom has to do with these form factors. Although
quark models are very fruitful and provide a lot insight into the problem they suffer at
least from two deficiencies regarding the description of meson-baryon scattering. Firstly,
assuming that masses of hadrons come out properly as a result of a quark model calcula-
tion, there is still the possibility that it fails when describing the scattering process. This
could only be interpreted as a failure of the particular set of interactions or approxima-
tions used to solve the quark model, but does not prevent from finding another quark
model providing a better description. In the second place, at the hadronic level symme-
tries, such as relativistic invariance and chiral symmetry, which are known to work well,
are difficult to impose at the hadronic level if hadrons are described as composites.

An alternative and not necessarily incompatible (although perhaps more economical)
point of view to the quark model calculations is to formulate the whole problem directly
in terms of hadronic degrees of freedom. This allows to impose all known information,
symmetries in particular, from the beginning, and offers the possibility to falsify all
unknown information. It is of course impossible to do this at all energies, but it may
be achieved at low energies, in terms of a finite and countable number of unknown
parameters, the so-called low energy constants. This is the point of view of Chiral
Perturbation Theory [4, 5]. At the level of a relativistic Lagrangean this corresponds
to write down the most general infinite set of tree level operators compatible with the
known symmetries. Since a tree level Lagrangean only produces real amplitudes and
hence violates unitarity it is necessary to incorporate quantum (loop) corrections.

In the meson-baryon system there is a problem in Chiral Perturbation Theory already
found long ago [6] because in the standard dimensional regularization, heavy particles
do not decouple. This result is counter-intuitive because it means that particles with
a very small Compton wavelength propagate. Two approaches have been suggested to
overcome this difficulty. In Heavy Baryon Chiral Perturbation Theory (HBChPT) [7, 8]
one takes the non-relativistic limit first and then proceeds in dimensional regularization.
The heavy particle decoupling is explicitly built in, but relativistic invariance is not
manifest at any step of the calculation. Within this framework elastic $\pi N$ scattering has
been studied to third [9, 10] and fourth order [11] in the chiral expansion and also $K N$
elastic scattering [14]. A more recent proposal keeps relativistic invariance explicitly at
any step of the calculation but introduces a new so-called infrared regularization which
complies with decoupling in the heavy particle baryon [12] and allows a satisfactory
description of $\pi N$ elastic scattering [13]. In either case, although crossing is exact at any
order of the expansion unitarity is only built in perturbatively. The need for unitarization
in the $S = -1$ channel becomes obvious after the work of Ref. [14] where it is shown
that HBChPT to one loop fails completely in the $\bar{K}N$ channel already at threshold due to
nearby subthreshold $\Lambda(1405)$-resonance.

The Bethe-Salpeter equation (BSE) provides the framework beyond perturbation the-
ory to treat the relativistic two body problem from a Quantum Field Theory point of
view. This approach allows to treat the scattering problem preserving exact unitarity. In
practical applications, however, the number of particles is kept fixed and other approx-
imations are done, violating crossing symmetry. At the level of partial waves unitarity
implies a right cut discontinuity while crossing generates a left cut for the scattering
amplitude, but in the scattering region one expects the energy dependence to by mainly
determined by the right cut.

The $s-$wave meson-baryon scattering incorporating chiral symmetry and unitariza-
tion for several open channels has been studied in previous works [15, 16, 17, 18, 19,
20, 21]. The purpose of the present contribution is to give a brief overview of a possible
approximation scheme for meson-baryon scattering based on the BSE. This is the natural
extension of work previously done for meson-meson scattering [22] to the meson-baryon
system [23] for heavy baryons and in a relativistic formulation [24, 25, 26].

**S-WAVE MESON-BARYON SCATTERING**

The coupled channel scattering amplitude for the baryon-meson process in given isospin
channel $I$ is given by

$$(T_P)_{BA} = \bar{u}_B(P - k', s_B)t_p(k, k')u_A(P - k, s_A)$$  \hspace{1cm} (1)

Here, $u_A(P - k, s_A)$ and $u_B(P - k', s_B)$ are baryon Dirac spinors normalized as $\bar{u}u = 2M$,
P is the conserved total CM four momentum, $P^2 = s$, and $t_p(k, k')$ is a matrix in the Dirac
and coupled channel spaces. Details on normalizations and definitions of the amplitudes
can be seen in Ref. [24].

On the mass shell and using the equations of motion for the free Dirac spinors
$(P - k - M_A)u_A(P - k) = 0$ and its transposed $\bar{u}_A(P - k)(P - k - M_A) = 0$ the parity
and Lorentz invariant amplitude $t_p$ relevant $s-$wave scattering can be simply written as
a matrix function in coupled channel space of $P$ with $P$ the total CM momentum

$$t_p(k, k')|_{\text{on-shell}} = t(P)$$  \hspace{1cm} (2)

In terms of the matrix $t(P)$ defined in Eq. (2), the $s-$wave coupled-channel matrix,
$f^\perp_0(s)$, is simply given by:

$$\left[f^\perp_0(s)\right]_{B \leftarrow A} = -\frac{1}{8\pi\sqrt{s}}\sqrt{\left|\frac{k_B}{k_A}\right|}\sqrt{E_B + M_B}\sqrt{E_A + M_A}[t(\sqrt{s})]_{BA}$$  \hspace{1cm} (3)
where the CM three–momentum moduli read
\[ |\vec{k}_i| = \frac{\lambda_i(s,M_i,m_i)}{2\sqrt{s}} \quad i = A,B \] (4)
with \( \lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz \) and \( E_{A,B} \) the baryon CM energies. The phase of the matrix \( T_P \) is such that the relation between the diagonal elements \( (A = B) \) in the coupled channel space of \( f_1^+(s) \) and the in-elasticities \( (\eta) \) and phase-shifts \( (\delta) \) is the usual one,
\[ \left[f_1^+(s)\right]_{AA} = \frac{1}{2i|\vec{k}_A|}(\eta_A(s)e^{2i\delta_A(s)} - 1) \] (5)
Hence, the optical theorem reads, for \( s \geq (M_A + m_A)^2 \),
\[ \frac{4\pi}{|\vec{k}_A|} \Im \left[f_1^+(s)\right]_{AA} = \sum_B \sigma_{B\leftarrow A} = 4\pi \sum_B \left|\left[f_1^+(s)\right]_{BA}\right|^2 = \sigma_{AA} + \frac{\pi}{|\vec{k}_A|^2}(1 - \eta_A^2) \] (6)
where in the right hand side only open channels contribute.

**CHIRAL BARYON-MESON LAGRANGIAN**

At lowest order in the chiral expansion the chiral baryon meson Lagrangian contains kinetic and mass baryon pieces and meson-baryon interaction terms and is given by [5]
\[ \mathcal{L}_1 = \text{Tr} \{ \bar{B} (i\partial - M_B) B \} + \frac{1}{2} \mathcal{D} \text{Tr} \{ \bar{B} \gamma^\mu Y_5 \{ u_{\mu}, B \} \} + \frac{1}{2} \mathcal{F} \text{Tr} \{ \bar{B} \gamma^\mu Y_5 [u_{\mu}, B] \} \],
\] (7)
The meson kinetic and mass pieces and the baryon mass chiral corrections are second order and read
\[ \mathcal{L}_2 = \frac{f_0^2}{4} \text{Tr} \left\{ u_{\mu}^* u^\mu + (U^\dagger \chi + \chi^\dagger U) \right\}
\] 
\[ - b_0 \text{Tr}(\chi_+) \text{Tr}(\bar{B}B) - b_1 \text{Tr}(\bar{B}\chi_+) - b_2 \text{Tr}(\bar{BB}_+) \] (8)
where “Tr” stands for the trace in \( SU(3) \). In addition,
\[ \nabla_\mu B = \partial_\mu B + \frac{1}{2}[u^\dagger \partial_\mu u + u\partial_\mu u^\dagger, B], \]
\[ U = u^2 = e^{i\sqrt{2}\Phi}/f, \quad u_{\mu} = iu^\dagger \partial_\mu U u^\dagger \]
\[ \chi_+ = u^\dagger \chi u + u\chi^\dagger u, \quad \chi = 2B_0, \mathcal{M} \] (9)
\( M_B \) is the common mass of the baryon octet, due to spontaneous chiral symmetry breaking for massless quarks. The \( SU(3) \) coupling constants which are determined by
semileptonic decays of hyperons are \( \mathcal{F} \sim 0.46, \mathcal{D} \sim 0.79 (\mathcal{F} + \mathcal{D} = g_A = 1.25) \). The constants \( B_0 \) and \( f \) (pion weak decay constant in the chiral limit) are not determined by the symmetry. The current quark mass matrix is \( \mathcal{M} = \text{Diag}(m_u, m_d, m_s) \). The parameters \( b_0, b_1 \) and \( b_2 \) are coupling constants with dimension of an inverse mass. The values of \( b_1 \) and \( b_2 \) can be determined from baryon mass splittings, whereas \( b_0 \) gives an overall contribution to the octet baryon mass \( M_B \). Neglecting octet-singlet mixing, the \( SU(3) \) matrices for the meson and the baryon octet are written in terms of the meson and baryon spinor fields respectively and are given by

\[
\Phi = \begin{pmatrix}
\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta \\
\pi^- \\
K^-
\end{pmatrix}
\begin{pmatrix}
\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta \\
\pi^+ \\
K^0
\end{pmatrix},
\]

and

\[
B = \begin{pmatrix}
\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda \\
\Sigma^- \\
\Xi^-
\end{pmatrix}
\begin{pmatrix}
\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda \\
\Sigma^+ \\
\Xi^0
\end{pmatrix},
\]

respectively. The \( MB \rightarrow MB \) vertex obtained from the former Lagrangian reads

\[
\mathcal{L}_{MB \rightarrow MB} = \frac{i}{4 f^2} \text{Tr} \left\{ \hat{B} \gamma^\mu \left[ [\Phi, \partial_\mu \Phi], B \right] \right\}.
\]

Assuming isospin conservation, the scattering amplitude ( convention \(-iT_{MB \rightarrow MB} = +i\mathcal{L}_{MB \rightarrow MB}\) in the Dirac spinor basis, at lowest order is given by

\[
t_p^{(1)}(k, k') = \frac{D}{f^2} (k + k')
\]

where \( k \) and \( k' \) are incoming and outgoing meson momenta and \( D \) a coupled-channel matrix. On the mass shell one can use Dirac’s equation and gets

\[
t^{(1)}(P) \equiv t^{(1)}(k, k')|_{\text{on-shell}} = \frac{1}{f^2} \left\{ P - \hat{M}, D \right\}
\]

which depends only on the CM momentum \( P \). Obviously, the s-wave scattering amplitude \( f_{BA}(s) \), defined through Eq. (3) is real and hence cannot satisfy the optical theorem, Eq. (6). Thus, there is need for unitarization.

To take into account the some chiral symmetry breaking effects physical mass splittings and different decay constants must be accounted for. This can be easily accomplished through the prescription

\[
D/f^2 \rightarrow \hat{f}^{-1} D \hat{f}^{-1}
\]

where \( \hat{f} \) is a diagonal matrix in the coupled channel space. We will use the \( D/f^2 \) notation throughout, meaning Eq. (15) in practice.
FIGURE 1. Diagrammatic representation of the Bethe-Salpeter Equation for Meson-Baryon Scattering. 
$k$ is the meson momentum and $P - k$ the Baryon momentum. $s = P^2$.

BETHE-SALPETER EQUATION

Two Particle Unitarity

To evaluate the meson-baryon scattering amplitude $t_P$ we use the BSE (see Fig. (1)) in a given isospin and strangeness channel,

$$t_P(k,k') = v_P(k,k') + i \int \frac{d^4q}{(2\pi)^4} t_P(q,k') \Delta(q) S(P-q) v_P(k,q)$$  \hspace{1cm} (16)$$

where $t_P(k,k')$ is the scattering amplitude defined in Eq. (1), $v_P(k,k')$ the two particle irreducible Green’s function (or potential), and $S(P-q)$ and $\Delta(q)$ the baryon and meson exact propagators respectively. The above equation turns out to be a matrix one, both in the coupled channel and Dirac spaces. For any choice of the potential $v_P(k,k')$, the resulting scattering amplitude $t_P(k,k')$ fulfills the coupled channel unitarity condition,

$$t_P(k,k') - \bar{t}_P(k',k) = i \int \frac{d^4q}{(2\pi)^4} t_P(q,k') \text{Disc} \left[ \Delta(q) S(P-q) \right] \bar{t}_P(q,k)$$  \hspace{1cm} (17)$$

where $\bar{t}_P(k,p) = \gamma_0 t_P^\dagger(k,p) \gamma_0$ and $t_P^\dagger(k,p)$ is the total adjoint in the Dirac and coupled channel spaces, including also the discontinuity change $s + i\epsilon \rightarrow s - i\epsilon$. The two particle discontinuity is given by Cutkosky’s rules,

$$\text{Disc} \left[ \Delta(q) S(P-q) \right] = (-2\pi i)^2 \delta^+ \left[ q^2 - \hat{m}^2 \right] (P - \hat{q} + \hat{M}) \delta^+ \left[ (P-q)^2 - \hat{M}^2 \right]$$  \hspace{1cm} (18)$$

$\hat{m}$ and $\hat{M}$ are meson and baryon (diagonal) mass matrices respectively and $\delta^+(p^2 - m^2) = \Theta(p^0) \delta(p^2 - m^2)$ is the on-shell condition. If the on-shell amplitude depends on $P$ only, one gets the very simple relation discontinuity equation for the inverse amplitude,

$$\text{Disc} t(P)^{-1} = -\text{Disc} J(P)$$  \hspace{1cm} (19)$$

where the quadratic and logarithmically divergent one-loop integral

$$J(P) = i \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - \hat{m}^2} \frac{1}{P - \hat{q} - \hat{M}}$$  \hspace{1cm} (20)$$

has been introduced. Direct calculation leads to

$$J(P) = P \left[ \left( \frac{s - \hat{m}^2 + \hat{M}^2}{2s} \right) J_0(s) + \frac{\Delta_{\hat{m}} - \Delta_{\hat{M}}}{2s} \right] + \hat{M} J_0(s)$$  \hspace{1cm} (21)$$

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where the quadratic divergences
\[
\Delta_m = i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 - \hat{m}^2}, \quad \Delta_M = i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 - \hat{M}^2}
\] (22)
and the standard logarithmically divergent one loop function
\[
J_0(s) = i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 - \hat{m}^2} - \frac{1}{P - q} = \tilde{J}_0(s) + \hat{J}_{mM}
\] (23)
have been introduced. Here \(\tilde{J}_0(s)\) is normalized to vanish at threshold, \(s = (m + M)^2\), and \(\hat{J}_{mM}\) is a divergent subtraction constant.

**Lowest Order Solution**

The BSE requires some input potential and baryon and meson propagators to be solved. At lowest order of the BSE-based chiral expansion, we approximate the iterated potential by the chiral expansion lowest order meson-baryon amplitudes in the desired strangeness and isospin channels, and the intermediate particle propagators by the free ones (which are diagonal in the coupled channel space). From the meson-baryon chiral Lagrangian, one gets at lowest order for the potential
\[
v_P(k, k') = t_P^{(1)}(k, k') = \frac{D}{f^2} (k + k')
\] (24)

The \(s\)--wave BSE can be solved up to a numerical matrix inversion in the coupled channel space [24]. The result for the inverse on-shell amplitude reads
\[
t(P)^{-1} + = -J(P) + \frac{\Delta_m}{P - \hat{M}} + \left\{ v(P)^{-1} + \frac{1}{f^4} (P - \hat{M}) D \frac{\Delta_m}{P - \hat{M}} D (P - \hat{M}) \right\}^{-1}
\] (25)

This solution manifestly fulfills the on-shell unitarity condition, Eq. (19). Notice that if \(\Delta_m = \Delta_M = 0\) the tree level on-shell potential \(v(P) = t_P^{(1)}(P)\) determines the amplitude up to a constant \(J_{mM}\). Thus, the difference to do with off-shell effects which we parameterize in terms of divergent constants. Assuming a specific cut-off method for the divergent integrals \(J_{mM}, \Delta_m\) and \(\Delta_M\) embodies specific correlations among them, but quite generally one expects them to be uncorrelated.

**Renormalization**

To renormalize the amplitude given in Eq. (25), we note that in the spirit of an Effective Field Theory (EFT) all possible counter-terms should be considered. This can be achieved in our case in a perturbative manner, making use of the formal expansion of the bare amplitude \(T = V + V G_0 V + V G_0 V G_0 V + \cdots\), where \(G_0\) is the two particle
propagator. Thus, a counter-term series should be added to the bare amplitude such that the sum of both becomes finite. At each order in the perturbative expansion, the divergent part of the counter-term series is completely determined. However, the finite piece remains arbitrary as long as the used potential \( V \) and the meson and baryon propagators are approximated rather than being the exact ones. Our renormalization scheme is such that the renormalized amplitude can be cast, again, as in Eq. (25). This amounts in practice, to interpret the previously divergent quantities \( J_{mM}, \Delta_m \) and \( \Delta_M \) as 12 renormalized free parameters for the \( s \)-wave lowest order amplitude in a given isospin-strangeness channel. These parameters and therefore the renormalized amplitude can be expressed in terms of physical (measurable) magnitudes. In principle, they encode the unknown short distance behavior, in particular the composite nature of hadrons which becomes relevant when they start overlapping. In practice it seems convenient to fit them to the available data, although they might be computed within models.

**Number of parameters**

Within the spirit of an effective field theory at the hadronic level, the number of adjustable parameters should not be smaller than those allowed by the symmetry; this is the only way both to falsify all possible theories embodying the same symmetry principles and to make wider the energy window which is being described. The opposite situation, i.e. a redundancy of parameters is also not desirable, but less problematic because it may be detected. The precise number of unknown parameters is mainly controlled to any order of the calculation by crossing symmetry. In a unitarized approach, the best way to avoid this parameter redundancy is to match the unitarized amplitude to one obtained from a Lagrangian formalism as was explicitly done for meson-meson scattering [22]. Unfortunately, there is no standard one loop ChPT calculation for the meson-baryon reaction with open channels to compare with. We comment on this matching below. At present the only practical, but indirect, way to detect such a parameter redundancy is through a fit to experimental data if the correlations in some parameters turn out to be very strong.

**Numerical Results**

To finish the presentation, we show some selected numerical results for the \( I = 1/2, S = 0 \) and \( I = 0, S = -1 \) channels. Details of the fitting procedure and the relevant experimental data as well as a thorough discussion of errors can be found in Refs. [24] and [26] respectively. In Fig. 2 the \( \pi N \) phase shift and inelasticity in the \( S_{11} \) channel, as well as transition cross sections are depicted. Finally, in Fig. 3 we show our results for several amplitudes and cross sections in the \( I = 0, S = -1 \) channel, compared to the work of Ref. [20] where \( \Delta_m = \Delta_M = 0 \). As we see, the description is quite satisfactory. Scattering lengths for the are given in Tab. 1, compared experiment and other determinations for the \( I = 1/2, S = 0 \) and \( I = 0, S = -1 \) channels. Errors are assigned by propagating best fit parameter uncertainties including possible correlations [24, 26].

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In the static limit, baryons behave like fixed sources, and consequently the two particle problem should reduce to a potential scattering problem (in our case of meson-baryon scattering it would correspond to a Klein-Gordon equation with a spin-dependent potential). It is a known fact that the BSE has difficulties in reproducing this heavy-light limit in certain situations (ladder approximation to one boson exchange [36]). The remedy to this situation is to make use of the Gross Equation, which essentially
FIGURE 3. Best fit results for the Bethe-Salpeter equation in the $S = -1$, $I = 0$ channel (solid lines) compared to Ref. [20] (dotted lines) where $\Delta_m = \Delta_M = 0$. **Upper panel:** Experimental data for $\pi \Sigma \rightarrow \pi \Sigma$ and $K^- p \rightarrow \eta \Lambda$ are from Refs. [29] and [31], respectively. **Middle panel:** The real (left panel) and imaginary (right panel) parts of the $s$-wave $T$-matrix, with normalization specified in the main text, for elastic $\bar{K}N \rightarrow \bar{K}N$ process in the $I = 0$ isospin channel as functions of the CM energy. Experimental data are taken from the analysis of Ref. [28] with the errors stated in Ref. [26]. **Lower panel:** Same as middle panel for the inelastic channel $\bar{K}N \rightarrow \pi \Sigma$. 

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means putting the would-be heavy particle on the mass shell from the very beginning. In Chiral Perturbation Theory this is also a tricky point in the relativistic formulation, since dimensional regularization does not lead to decoupling of heavy particles, which has only been solved after a clever choice of renormalization scheme, the so-called infrared regularization [12].

The heavy baryon expansion may be taken by making the baryon masses \( \hat{M} \to \infty \) but keeping the meson masses, \( \hat{m} \), and the meson momentum, \( q \), finite and considering baryon mass splittings as higher order effects, see e.g. [5],

\[
\hat{M} = M_B + \Delta \hat{M} \quad (26)
\]

with \( M_B \to \infty \) the common mass of the baryon octet which is proportional to the identity matrix. Accordingly, in the \( \pi N \) elastic channel we take

\[
\sqrt{s} = E + \omega = M_N + \omega + \frac{\omega^2 - m^2}{2M_N} + \cdots \quad (27)
\]

where \( M_N = M_B + \Delta M_N \). Following Ref. [24] in the static limit one obtains from Eqs. (3) and (25) \( (f(\omega) \to -i(s)/(4\pi)) \)

\[
f(\omega)^{-1} = 8\pi \left[ \tilde{K}_m(\omega) + \frac{1}{16\pi^2} \ln \frac{\hat{M}^2}{\hat{m}^2} (\hat{m} - \omega) + \hat{M} f_{\hat{m},M}^0 + \frac{\Delta^0_{\hat{m}\hat{M}}}{4\hat{M}} \right] \\
- \frac{4\pi}{\omega} \left\{ \Delta^0_{\hat{m}} - \left[ \frac{2}{f^2D} + \frac{1}{f^2D\Delta^0_{\hat{m}\hat{M}}} \right]^{-1} \right\} 
\]

(28)

with the heavy baryon one-loop integral

\[
K_{\hat{m}}(\omega) = \frac{1}{i} \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - \hat{m}^2} \frac{1}{\omega - v \cdot q} 
\]

(29)

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**TABLE 1.** Real and imaginary parts of scattering lengths (in fm) for best fit BSE results in several isospin and strangeness elastic channels.

| \( l \) \( s \) | \( \text{Re} \ a \, (\text{fm}) \) | \( \text{Im} \ a \, (\text{fm}) \) | \( \text{Re} \ a \, (\text{fm}) \) | \( \text{Im} \ a \, (\text{fm}) \) |
|---|---|---|---|---|
| \( \pi N \) 1/2 0 | 0.179(4) | 0 | 0.252(6)* | 0 |
| \( \eta N \) 1/2 0 | 0.772(5) | 0.217(3) | 0.68** | 0.24 |
| \( K \Lambda \) 1/2 0 | 0.0547(5) | 0.032(4) |
| \( \pi \Sigma \) 0 -1 | 1.10(6) | 0 |
| \( \bar{K} N \) 0 -1 | -1.20(9) | 1.29(9) | -1.71‡ | 0.68 |
| \( \eta \Lambda \) 0 -1 | 0.50(5) | 0.27(1) |

* Experiment Ref. [33]  † HBChPT Ref. [38]  ** Potential model Ref. [27]  ‡ Experiment, Ref. [35]
and $\tilde{K}_m(\omega) = K_m(\omega) - K_m'(m) - (\omega - m)K_m'(m)$ and the heavy baryon approximation of the subtraction constants is denoted by the superscript 0. Eq. (28) corresponds, as it should, to a one particle scattering problem, fulfilling the coupled channel unitarity condition

$$\text{Im} f(\omega)^{-1} = -\sqrt{\omega^2 - \hat{m}^2} \text{Im} \frac{1}{\theta(\omega - \hat{m})}$$  (30)

The pole in Eq. (28) for the inverse amplitude is a static limit reminiscent from the baryonic Adler zero, $\sqrt{s} - M = 0$, of the lowest order potential. The constant combination appearing in the inverse amplitude, Eq. (28), $\hat{M}_0^0 + \frac{\Delta_0^0}{4\hat{M}}$ should go to some definite value in the static limit, $M \to \infty$. In case it would diverge, the scattering amplitude would become trivial. Numerical estimates in Ref. [24] suggest that the dangerous combination is not unnaturally large since

$$\hat{M}_0^0 + \frac{\Delta_0^0}{4\hat{M}} \sim \hat{m}$$  (31)

### Chiral and heavy baryon expansions at threshold

If, in addition to a heavy baryon expansion, a chiral expansion in powers of $1/f^2$ is carried out, one should recover in this double expansion some form of the results found in Ref. [9, 10, 38] within HBChPT for the elastic $\pi N$ scattering amplitude. The results of Ref. [24] show that the matching to HBChPT is indeed possible, although there is an ambiguity related to the absence of a left hand cut in our BSE amplitude, Eq. (25), which is approximated with a polynomial in the scattering region. The net result is that there is a large degree of redundancy in the BSE parameters, which hardly impose any practical constraints on them$^{13}$.

Moreover, a simple expansion in $1/f^2$ of the BSE amplitude keeping the baryon masses does not converge well as one expects from the fact that in the Born approximation the matrix elements $\pi N \to \eta N$, $\eta N \to \eta N$ vanish, although in the full amplitude the $N(1535)$ resonance is developed indicating strong rescattering effects (see also below). For instance, taking $\pi N$ scattering length for $I = 1/2'$ obtained from a best fit to the data as an example one gets [24]

$$a_{\pi N}^{I=1/2} = \frac{\pi N}{1/f^2} + \frac{K\Lambda}{1/f^4} + \frac{K\Sigma}{1/f^4} + \ldots = 0.179 \text{ fm}$$  (32)

Note that the first two contributions leads to a value $-0.43$ fm.

$^{13}$ In this regard one should mention that matching of unitarized amplitudes to HBChPT does not always work due to a lack of convergence in HBChPT. See the discussion in Refs. [23, 37, 38].
TABLE 2. Resonance Masses and Widths in MeV found for the best fit BSE calculation, compared with PDG data \[1\].

| $I$ | $S$ | $M_R$(MeV) | $\Gamma_R$(MeV) | PDG $M_R$(MeV) | PDG $\Gamma_R$(MeV) |
|-----|-----|-------------|-----------------|----------------|---------------------|
| 1/2 | 0   | 1497(1)     | 83(1)           | N(1535)        | 1505(10)           |
| 1/2 | 0   | 1684(1)     | 194(1)          | N(1660)        | 1660(20)           |
| 0   | -1  | 1368(12)    | 250(23)         | Unknown        |                     |
| 0   | -1  | 1443(3)     | 50(7)           | \Lambda (1405)| 1407(4)            |
| 0   | -1  | 1678(1)     | 29(2)           | \Lambda (1670)| 1670(10)           |
| 0   | -1  |             |                 | 1673(2)*       | 23(6)              |

* Experiment. Ref. \[32\]

Resonances as poles in the second Riemann-sheet

The unitarity condition for the inverse amplitude, expressed as a discontinuity equation reads \[24\]

\[
\text{Disc} [t^{-1}(s)] \equiv t^{-1}_I(s + i\epsilon) - t^{-1}_I(s - i\epsilon) = 2i\rho(s) \quad s > (m + M)^2
\]  

(33)

with real $s$, where the phase space function

\[
\rho(s) = \frac{\lambda^{1/2}(s, m, M_N)}{16\pi s} \times \frac{(\sqrt{s} + M^2)^2 - m^2}{2\sqrt{s}}
\]  

(34)

has been introduced, understanding that $\rho(s)$ is a function of the real variable $s$. Then, analytically continuing the phase space function to all complex plane, one finds the amplitude in the second Riemann sheet,

\[
t^{-1}_H(z) = t^{-1}_I(z) - 2i\rho(z)
\]  

(35)

Masses and widths computed as poles in the second Riemann sheet $z_R = M_R^2 - iM_R\Gamma_R$, can be looked up at Tab. 2. The quoted theoretical errors are obtained propagating the uncertainties in the best fit parameters.

It is striking that the unobserved resonance has also been obtained by the authors of Ref. \[39\] although with $M = 1390$ MeV and $\Gamma = 66$ MeV. Finally, one finds \[26\] that the resonances are not of the Breit-Wigner form, so that the simple relation between residues and branching ratios does not hold\[15\]. Instead, an extrapolation to the real axis is required to extract branching ratios as provided in the PDG \[1\] for which often Breit-Wigner or other particular forms are assumed. In this regard, it would be desirable that future editions of the PDG would also incorporate the residues.

\[15\] In the one channel case the Breit-Wigner form implies a direct relation between the residue at the pole and the imaginary part of the pole. This relation only holds in the limit of sharp resonances.
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