MAGNETO-ROTATIONAL NEUTRON STAR EVOLUTION: THE ROLE OF CORE VORTEX PINNING

KOSTAS GLEMPEDAKIS\textsuperscript{1} AND NILS ANDERSSON\textsuperscript{2}

\textsuperscript{1} Theoretical Astrophysics, University of Tübingen, Tübingen D-72076, Germany
\textsuperscript{2} School of Mathematics, University of Southampton, Southampton SO17 1BJ, UK

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ABSTRACT

We consider the pinning of superfluid (neutron) vortices to magnetic fluxtubes associated with a type II (proton) superconductor in neutron star cores. We demonstrate that core pinning affects the spin-down of the system significantly and discuss implications for regular radio pulsars and magnetars. We find that magnetars are likely to be in the pinning regime, whereas most radio pulsars are not. This suggests that the currently inferred magnetic field for magnetars may be overestimated. We also obtain a new timescale for the magnetic field evolution which could be associated with the observed activity in magnetars, provided that the field has a strong toroidal component.

Key words: dense matter – magnetohydrodynamics (MHD) – pulsars: general – stars: neutron

1. INTRODUCTION

The outer core of a mature neutron star is expected to contain a neutron superfluid coupled to a type II proton superconductor (Baym et al. 1969). The dynamics of this system are complicated: the neutrons rotate by forming a dense array of vortices while the magnetic field is carried by quantized proton fluxtubes. The interaction between the vortices and the fluxtubes is expected to have a significant impact on the dynamics of these systems. In fact, the vortices may “pin” to the fluxtubes (in the sense that the energy cost of cutting through is too high) as the star spins down, affecting the evolution of the star’s rotation and magnetic field. An example of this was provided by Link (2003), who argued that the free precession interpretation of the long-term variability in PSR B1828-11 would not be compatible with core vortex pinning. Vortex-fluxtube pinning may also couple the magnetic field evolution to the rotation, as in the model of Ruderman et al. (1998).

Connections between the magnetic field and the spin evolution are crucial if we want to understand the different observational manifestations of neutron stars, like radio pulsars and magnetars, and establish an evolutionary link between them. We also need to keep in mind that the spin and its rate of change are the primary observables. The magnetic field tends to be inferred from the assumption of pure dipole breaking.

The aim of this Letter is to demonstrate how the standard logic linking the spin evolution and the inferred magnetic field breaks down if there is significant vortex-fluxtube pinning, and how the same mechanism could drive the magnetic field evolution.

2. A VORTEX DYNAMICS MODEL

We consider the motion of individual neutron vortices and proton fluxtubes in a neutron star core, composed of neutrons, protons, and electrons. The local vortex/fluxtube velocity is \( u^i_x \), where the index \( x = \{ n, p \} \) distinguishes between the two types. For simplicity, the hydrodynamical fluid velocities are assumed to represent rigid-body rotation, i.e., \( v^i_x = \epsilon^{ijk} \Omega^j_x x_k \), where the spin frequencies \( \Omega^i_x \) are aligned (defining the \( z \)-axis) but may differ in magnitude. A direct consequence of uniform rotation is that the neutron vortices are straight, which may be a serious oversimplification. The proton fluxtubes are not constrained in this way.

In order to make progress, we make a number of simplifications. The most basic is the assumption of axisymmetry. We ignore the entrainment between the superfluids on the hydrodynamic scale (although the effect is included implicitly in the magnetization of neutron vortices; Alpar et al. 1984). Given these assumptions one can use Ampère’s law, as modified in a type II superconductor, to show that the electron and proton fluids are “locked” together, moving with a common velocity \( v^i_p \) (Glampedakis et al. 2011a). Finally, we identify \( \Omega_p \) with the motion of the “crust” and the observed spin frequency \( \Omega \).

The motion of an individual vortex/fluxtube is governed by a force balance equation. Ignoring the inertia of the filaments (as usual) we have

\[
\begin{align*}
\dot{f}^i_{Mn} + \dot{f}^i_{Dn} + \dot{f}^i_{Pn} &= 0 \\
\dot{f}^i_{Mn} + \dot{f}^i_{Dp} + \dot{f}^i_{Pp} + \dot{f}^i_{T} &= 0.
\end{align*}
\]

Each vortex/fluxtube experiences (1) a Magnus force \( \dot{f}^i_{Ma} = \rho_x \epsilon^{ijk} k^x_i (u^j_x - v^j_x) \) due to the interaction between the macroscopic fluid flows and the mesoscopic, quantized, neutron/proton circulations (\( \rho_x \) is the fluid density while \( k^x_i \) has a magnitude of \( k = \hbar/2m \) and points along the local vortex/fluxtube direction); (2) a drag force \( \dot{f}^i_{Dx} = \rho_x k \tilde{R}_x (v^j_x - u^j_x) \), typically attributed to the scattering of electrons by the intrinsic vortex/fluxtube magnetic field, with a dimensionless coefficient \( \tilde{R}_x \ll 1 \) (Alpar et al. 1984; Ruderman et al. 1998); and (3) a “pinning” force \( \dot{f}^i_{Pp} \) due to local interaction between a vortex and a fluxtube. Finally, a bent fluxtube will experience a (self-induced) tension \( f^i_T \) (since we are considering straight vortices there is no similar force in Equation (1)). All forces are per unit length.

The direct interaction is primarily due to the short-range magnetic fields associated with each vortex/fluxtube, and could lead to pinning (Sauls 1989). This interaction must obey Newton’s third law, i.e.,

\[
\dot{f}^i_{Pp} = -\alpha f^i_{Pp}.
\]

In fact, this relation is the operational definition of \( f^i_{Pp} \). In a typical fluid element the number density (per unit area) \( N_P \) of fluxtubes greatly exceeds the vortex density \( N_v \), resulting in only a tiny fraction of fluxtubes interacting with vortices at any given instant. At the same time neighboring fluxtubes can push each
other, “distributing” the action of the vortex back-reaction force \( -f_p^i \) to the entire fluid element. On average, this effect can be quantified by the small parameter

\[
\alpha \sim N_p/N_p' \quad (Ruderman et al. 1998; Jahan-Miri 2000).
\]

Equation (1) provides an exact form for the pinning force,

\[
f_p^i = -\rho_p\kappa R_p(u_j^i - u_j') + \rho_p\kappa\epsilon^{ijk}\hat{z}_j(v_k - u_k'),
\]

where we have set \( \kappa^i = \kappa \hat{z}_i \). This can be used in Equation (2) to eliminate \( f_p^i \) (with the help of Equation (3)). Thus, we obtain

\[
\vec{R}(u_j^i - u_j') + \epsilon^{ijk}\hat{z}_j(u_k^n - V_k) + \mathcal{F}^i = 0,
\]

where we have defined

\[
\vec{R} \equiv \frac{R_p}{\kappa}, \quad V^i \equiv \frac{1}{\kappa} (\kappa_{||} u_j^i + \alpha u_j^n),
\]

\[
\mathcal{F}^i \equiv \frac{1}{\kappa} \left( \frac{1}{\rho_p\kappa} f_T^i + R_p\kappa \epsilon_{ijkl}^i \right).
\]

The Astrophysical Journal Letters

The dependence of \( \Delta u^i \) on \( \kappa \) encodes the action of the (radial) Magnus force on a fluxtube. We note that \( \Delta u^i \) is nearly azimuthal unless the magnetic field is dominated by the toroidal component to the degree that \( \kappa_{||} \lesssim R_p \approx 10^{-3} \). The true magnitude of \( R_p \) is uncertain. We use a value inferred from electron scattering by a single fluxtube (Alpar et al. 1984). However, we note that Jones (2006) has argued that the high fluxtube density suppresses the magnetic scattering, leading to a much smaller \( R_p \). Nevertheless, as no actual alternative value for \( R_p \) has been provided in the literature, we base our discussion on the standard “single fluxtube” result.

3. THE PINNING REGIME

Strong interaction between a vortex and a fluxtube segment (via the forces \( f_p^i \)) may result in more or less perfect pinning. Given the present understanding of neutron star matter, this is likely to happen (Sauls 1989). Assuming vortex pinning, in the remainder of this Letter we explore some very interesting consequences. If (for whatever reason) pinning is inefficient, in the sense of a significant vortex-fluxtube relative motion, then our model does not apply.

When pinned, the two segments would move together:

\[
u^i = u_j' \equiv u^i.
\]

This velocity can be immediately obtained from Equation (9) after setting \( u_{rel}^i = 0 \). Using cylindrical coordinates \( [\sigma, \varphi, z] \) with respect to the rotation/axisymmetry, we find

\[
u^i \approx \sigma \Omega \hat{\varphi}^i + \frac{1}{\kappa_{||}} \left[ \sigma \Omega n^i_{\text{pin}} + f_T^i \right] \left( \frac{R_p}{\kappa_{||}} \sigma \hat{\varphi}^i + \hat{\varphi}^i \right).
\]

where we have defined the rotational lag \( \Omega n^i_{\text{pin}} = \Omega - \Omega \). We see that, to leading order, the velocity of pinned vortex/fluxtube segments is nearly azimuthal and in corotation with the charged component. Superimposed on this there is a small relative motion \( \Delta u^i \equiv u^i - v^i \) between the vortex/fluxtube array and the proton velocity with both radial and azimuthal components:

\[
\Delta u^i \equiv u^i - V^i.
\]

The dependence of \( \Delta u^i \) on \( \kappa \) encodes the action of the (radial) Magnus force on a fluxtube. We note that \( \Delta u^i \) is nearly azimuthal unless the magnetic field is dominated by the toroidal component to the degree that \( \kappa_{||} \lesssim R_p \approx 10^{-3} \). The true magnitude of \( R_p \) is uncertain. We use a value inferred from electron scattering by a single fluxtube (Alpar et al. 1984). However, we note that Jones (2006) has argued that the high fluxtube density suppresses the magnetic scattering, leading to a much smaller \( R_p \). Nevertheless, as no actual alternative value for \( R_p \) has been provided in the literature, we base our discussion on the standard “single fluxtube” result.

A vortex cannot move arbitrarily fast and remain pinned. Above some relative vortex-fluid velocity threshold the forces entering the balance (4) will exceed the maximum pinning force, \( f_{\text{pin}} \), and the vortex unpins. The maximum pinning force has been estimated to be (e.g., Link 2003)

\[
f_{\text{pin}} \approx 3 \times 10^{-5} B_{12}^{1/2} \text{dyn cm}^{-1},
\]

where \( B_{12} = B/10^{12} \) is the normalized interior magnetic field. At maximum pinning, the vortex force balance is well approximated by, after setting \( u_{rel}^i \approx u_j \approx u_j^n \) and omitting the drag term in Equation (4),

\[
f_{\text{pin}}^i \approx \rho_p\kappa\epsilon^{ijk}\hat{z}_j(v_k^n - v_p^n),
\]

and we can extract the maximum allowed spin lag (above which pinning can no longer be sustained):

\[
\Omega_{n_{\text{pin}}}^\text{max} = \frac{f_{\text{pin}}}{\rho_p\kappa\sigma} \approx 7.6 \times 10^{-3} \frac{B_{12}^{1/2}}{\sigma_6} \text{s}^{-1},
\]

where \( \sigma_6 = \sigma/10^6 \) cm. This result can be used in Equations (12) and (13) to provide the maximum relative vortex/fluxtube velocity with respect to the protons:

\[
\Delta u_{n_{\text{max}}}^i = \frac{\kappa_{||} \Delta u_{n_{\text{max}}}^i}{R_p} = \frac{1}{\rho_p\kappa\kappa_{||}} \left( \alpha f_{\text{pin}} + f_T^i \right).
\]

Let us provide numerical estimates of the forces in Equation (17). The tension can be approximated by \( f_T = E_{\varphi}/R_c \), where \( E_{\varphi} \) is the fluxtube energy per unit length and \( R_c \) is the local radius of curvature. Estimates for \( E_{\varphi} \) can be found in Mendell (1991) while a typical value for the curvature radius would be \( R_c \approx R \). The pinning force can be estimated using Equation (14) together with \( \alpha = N_p/N_p' \), \( N_p = 2\Omega_{n_{\text{pin}}}^{\text{max}} \) and \( N_p = B/\phi_0 \) (\( \phi_0 = hc/2e \) is the flux quantum). This way we obtain

\[
\frac{f_T}{\rho_p\kappa} \approx 10^{-9} \text{cm s}^{-1}, \quad \frac{\alpha f_{\text{pin}}}{\rho_p\kappa} \approx 10^{-10} \frac{\Omega}{\rho_B^{1/2} \Omega} \text{cm s}^{-1},
\]

with the stellar spin period, \( P \), measured in seconds.
4. SPIN EVOLUTION

Vortex pinning has a direct impact on the neutron star spin evolution. This is not surprising, given the close link between the spin-down of the neutron superfluid and the radial vortex motion,

$$u_n^\tau = -\sigma \hat{\Omega}_n/2\Omega_n.$$  \hfill (19)

If the neutron superfluid is able to track the electromagnetically driven spin-down of the charged component (i.e., when $\Omega_n = \Omega$ and $\Omega_p = \Omega$), we have

$$u_n^\tau \approx 8 \times 10^{-7} \sigma_6 (10^4 \text{yr}/\tau_{sd}) \text{ cm s}^{-1},$$  \hfill (20)

where $\tau_{sd} = P/2|\dot{P}|$ is the observed “spin-down age.” This vastly exceeds the maximum radial vortex velocity (17) in the pinning regime. Thus, we should expect the neutrons to spin down at a rate of $|\dot{\Omega}_n| \ll |\dot{\Omega}|$ in the pinning regime. This would lead to a monotonically increasing spin lag $\Omega_{np}$ to the point where $\Omega_{np}^\inf$ is reached and vortex pinning can no longer be sustained. The development of this lag is similar in most models of large pulsar glitches.

Let us focus on the timescale associated with the pinning phase. In order to estimate this, we need to model the spin evolution of a multi-fluid star. This can be done by considering the volume-integrated Euler equations for the fluids (cf. Sidery et al. 2010). The resulting equations take the form

$$I_n \dot{\Omega}_n = N_{mf}, \quad I_p \dot{\Omega}_p = -N_{mf} + N_{em},$$  \hfill (21)

where $I_n = \int \rho_n \sigma^3 dV$ is the moment of inertia for each fluid. These equations feature torques due to (1) the exterior magnetic field ($N_{em}^i$) and (2) the “mutual friction” ($N_{mf}^i$) which represents the vortex/fluxtube-mediated coupling. The latter torque is given by

$$N_{mf}^i = \int \epsilon^{ijk} x_j F_k^{mf} dV,$$  \hfill (22)

where the mutual friction force is

$$F_k^{mf} = N_{mf}(f_{Dn} + f_{Dp}) = 2\Omega_n \rho_n \epsilon^{ijk} \hat{z}_j (v_k^p - u_k).$$  \hfill (23)

Using our result (11) for the common vortex/fluxtube velocity in the pinning regime we have

$$N_{mf}^i \approx -2\Omega_n \hat{z}^i \int \sigma^3 \rho_n R_p \left( \frac{\omega_{np}}{x_p} + \frac{f_{Dp}^{mf}}{\sigma_6 \rho_n k} \right) dV.$$  \hfill (24)

We also need to consider the magnetic torque $N_{em}^i$. Following the analysis of Spitkovsky (2006), which accounts for a wind contribution, we have

$$N_{em}^i = \frac{B_d^2 R_p^6}{4c^3} \Omega^3 \hat{z}^i \equiv A \Omega^3 \hat{z}^i,$$  \hfill (25)

where $B_d$ is the exterior dipole field at the magnetic pole.

Returning to the spin evolution equations (21) with (24) and (25), we observe that the timescale $\tau_{mf} \sim I_n \Omega / N_{mf}$ associated with the pinning and tension forces is much longer than the electromagnetic spin-down timescale. For typical neutron star parameters we find $\tau_{mf} \sim 10^{10} \text{yr}$. Hence, the spin evolution in the perfect pinning regime is well approximated by

$$\dot{\Omega}_n \approx 0, \quad \dot{\Omega}_p \approx -A \Omega^3 / I_p.$$  \hfill (26)

in accordance with the intuitive notion of a limited radial vortex motion enforced by pinning, and of the magnetic braking affecting only the charged component. Solving Equation (26) with initial data $\Omega_n(0)$ is straightforward, and we have

$$\Omega(t) = \Omega(0) \left(1 + \frac{I_0}{I_p} t / \tau_0\right)^{-1/2}, \quad \Omega_p = \Omega_n(0),$$  \hfill (27)

where $I_0 = I_n + I_p$ is the total moment of inertia and $\tau_0 = I_0 / (2 A \Omega^2)$. In a single-component star, $\tau_0$ would correspond to the spin-down timescale and would be identified with the observed spin-down age $\tau_{sd}$. However, in the superfluid pinning model the appropriate identification, based on the spin-down law (26), is $\tau_{sd} = (I_p / I_0) \tau_0 \approx x_p \tau_0$. Equivalently, we can relate the true field $B_d$ with the field $B_d^{mf}$ inferred from spin-down,

$$B_d = \left(\frac{I_p}{I_0}\right)^{1/2} \left(\frac{\epsilon^3 I_0}{\pi^2 R_p^2 |\dot{P}| P}\right)^{1/2} \approx x_p^{1/2} B_d^{mf}. $$  \hfill (28)

The spin evolution (27) naturally leads to a monotonically increasing spin-lag. Considering, for instance, the early regime $t < (I_p / I_0) \tau_0$, we can easily obtain the time $t_{max}$ at which the system reaches the maximum lag (16), where vortices first unpin (assuming $\Omega_{np}(0) = 0$),

$$t_{max} \approx \frac{2\Omega_{np}^\inf}{B_d^{mf}} \approx 2.4 \times 10^{-3} \frac{PB_d^{1/2}}{\sigma_6},$$  \hfill (29)

which features the interior $B$ field. When applied to a typical radio pulsar ($P = 0.1 \text{s}, B = 10^{13} \text{G}$) the result (29) leads to $t_{max} \ll \tau_{sd}$. Thus, core vortex pinning in a typical pulsar is an ephemeral phenomenon, plausibly associated with large glitches (Glampedakis & Andersson 2009).

The situation may be very different for magnetars. For a strongly magnetized and slowly spinning object the timescale $t_{max}$ can be comparable to, or even exceed, $\tau_{sd}$. In this case we need to resort to the full solution (27). Inserting canonical magnetar parameters ($B = 10^{15} \text{G}, P = 10 \text{s}$), we obtain

$$t_{max} \gtrsim \tau_{sd}.$$  \hfill (30)

This suggests that vortex-fluxtube pinning may well persist in magnetar cores.

5. IMPLICATIONS AND DISCUSSION

We have provided a rather simplistic analysis of a very complex problem, yet the results could have important implications for the magnetaro-rotational properties of neutron stars. Perhaps the most significant conclusion concerns the potential persistence of vortex pinning in magnetars, as suggested by Equation (30). This implies (see Equation (28)) that the surface dipole field inferred from spin-down is an overestimate by a factor of $(I_p / I_0)^{-1/2} \approx x_p^{-1/2} \approx 3–10$. This mechanism could be responsible for the apparent clustering of the observed magnetar spin periods, since the end of the pinning regime would be accompanied by an effective weakening of the torque; $N_{em} \rightarrow x_p N_{em}$. Interestingly, the presence of a “twisted” magnetar magnetosphere may lead to a similar effect (Thompson et al. 2002).

There are also implications for the magnetic field evolution in magnetar cores. According to the pinning model the motion of the proton fluxtubes (and therefore of the magnetic field) is
controlled by the neutron vortices and vice versa. We can define a magnetic evolution timescale associated with the relative vortex/fluxtube motion with respect to the proton fluid, \( \tau_B = L/|\Delta \mu_{\text{max}}| \), where \( L \) is some typical lengthscale. In magnetars, \( \Delta \mu_{\text{max}} \) is mostly due to the tension since Equation (18) implies \( \tau_B \approx f_T \alpha_{\text{pin}} \) (note that Equation (16) suggests \( \Omega_p - \Omega \) for magnetars). The shortest timescale is the one associated with the azimuthal motion \( \Delta \mu_{\text{max}} \), leading to

\[
\tau_B \sim \frac{k_p \rho_p L}{f_T} \sim 10^6 \frac{B_p}{B_T} \left( \frac{R_c}{10 \text{ km}} \right) \text{ yr}, \tag{31}
\]

where we have restored the explicit dependence of \( f_T \) on the curvature radius \( R_c \) and defined \( L_5 = L/10^5 \text{ cm} \). The timescale associated with the radial fluxtube motion is longer by a factor of \( R_c^{-1} \sim 10^3 \). Also, \( \tau_B \) becomes significantly longer if (for whatever reason) \( f_T \) is negligible.

Is the timescale in Equation (31) relevant for the observed magnetar population? For this to be the case we need \( \tau_B \) to be comparable to the age \( \tau_{\text{rad}} \sim 10^3 - 10^4 \text{ yr} \) of these objects. At first sight, (31) seems to suggest a significantly longer timescale. However, our result approaches the interesting regime of core vortex pinning, affecting the bulk spin-down, the magnetic field evolution in magnetar cores, with possible implications only for the outer core—in that case \( \tau_B \) would be overestimated by the smaller factor of \( \sim (I_p + I_b)/I_0^{-1}/2 \) where \( I_b \) is the moment of inertia of the non-superfluid neutrons. To make progress we need to consider a model with realistic stratification. For the magnetars in particular, it is not yet clear that their core temperature is below the level for the onset of neutron superfluidity suggested by the recent cooling models for the Cassiopeia A neutron star (Page et al. 2011; Sh刑事in et al. 2011). The most detailed work on magnetar temperature profiles (Kaminker et al. 2006) appears to favor superfluidity, but the issue is far from settled (Arras et al. 2004; Dall’Osso et al. 2009). A related case for which our model does not apply is for a bulk \( B \) field in excess of the critical value \( \approx 10^{16} \text{ G} \) above which proton superconductivity is suppressed (Baym et al. 1969). We also need to remove the assumption of uniform rotation, which does not remain consistent when the long-term neutron star evolution is considered. This would help link our vortex/fluxtube model to the physics of ambipolar diffusion (Glampedakis et al. 2011b). The departure from uniform rotation is also likely to be important in the vortex cutting regime, which should be relevant for normal pulsars. These issues require us to consider more realistic neutron star models, a challenge that should ultimately reward us with a deeper understanding of these exciting systems.

The issues we have discussed obviously need more detailed consideration. Future work needs to relax many of our assumptions. First of all, we have assumed that the entire star is in a superfluid state while, in reality, this may be an accurate description only for the outer core—in that case \( \tau_B \) would be underestimated by the smaller factor of \( \sim (I_p + I_b)/I_0^{-1}/2 \) where \( I_b \) is the moment of inertia of the non-superfluid neutrons. To make progress we need to consider a model with realistic stratification. For the magnetars in particular, it is not yet clear that their core temperature is below the level for the onset of neutron superfluidity suggested by the recent cooling models for the Cassiopeia A neutron star (Page et al. 2011; Sh刑事in et al. 2011). The most detailed work on magnetar temperature profiles (Kaminker et al. 2006) appears to favor superfluidity, but the issue is far from settled (Arras et al. 2004; Dall’Osso et al. 2009). A related case for which our model does not apply is for a bulk \( B \) field in excess of the critical value \( \approx 10^{16} \text{ G} \) above which proton superconductivity is suppressed (Baym et al. 1969). We also need to remove the assumption of uniform rotation, which does not remain consistent when the long-term neutron star evolution is considered. This would help link our vortex/fluxtube model to the physics of ambipolar diffusion (Glampedakis et al. 2011b). The departure from uniform rotation is also likely to be important in the vortex cutting regime, which should be relevant for normal pulsars. These issues require us to consider more realistic neutron star models, a challenge that should ultimately reward us with a deeper understanding of these exciting systems.

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