Future Constraints on Dynamical Dark-Energy using Gravitational-Wave Standard Sirens

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The detection of gravitational waves (GWs) by the LIGO and Virgo collaborations offers a whole new range of possible tests and opens up a new window which may shed light on the nature of dark energy and dark matter. In the present work we investigate how future gravitational waves data could help to constrain different dynamical dark energy models. In particular, we perform astronomical forecasting of a class of well known and most used dynamical dark energy models using the third-generation gravitational wave detector, the Einstein Telescope. We have considered 1000 simulated GWs events in order to constrain the parameters space of the dynamical dark energy models. Our analyses show that the inclusion of the gravitational waves data from the Einstein Telescope, significantly improves the parameters space of the dynamical dark energy models compared to their constraints extracted from the standard cosmological probes, namely, the cosmic microwave observations, baryon acoustic oscillations distance measurements, Supernove type Ia, and the Hubble parameter measurements.

PACS numbers: 98.80.-k, 95.36.+x, 95.35.+d, 98.80.Es

1. INTRODUCTION

According to the latest claims by LIGO and Virgo collaborations, the gravitational waves (GWs) from a pair of two very massive black holes around 36 and 29 solar masses have been detected, known as the GW150914 event [1]. Subsequently, the investigations in a series of further works [2–6] also claimed similar detection. Just after the detection of GWs from binary black holes, the GWs from a binary neutron star merger (known as GW170817 event [7]) together with an electromagnetic counterpart known as GRB 170817A event [8] were also detected. Without any doubt, the detection of GWs, if we avoid its counter attacks, is an appreciable event for modern cosmology that naturally thrilled the scientific community that could offer some new insights in the physics of dark energy and modified gravity theories at the fundamental level. Following this a lot of investigations have already been performed by many researchers in order to understand how GWs could affect the cosmological theories of interest [9–29]. One of the most important properties is that, GWs propagate practically with the light speed, as reported by both the events GW170817 [7] and GRB 170817A [8]. Thus, by using the extracted properties from GWs, for instance the propagation speed among others, one can impose strong constraints on the cosmological models as well as exclude some cosmological theories. Especially, GWs provide a novel approach of luminosity distance measurements, known as standard sirens.

Motivated by the earlier investigations, in the present work, we focus on the dynamical dark energy cosmologies through their parametrizations with an aim to examine how luminosity distances extracted from future GWs data could affect the bounds on the dynamical dark energy models. The parametrizations of the dark energy sector is a well motivated area in cosmology where the primary content is the dark energy equation of state defined by \( w_x = p_x/\rho_x \), in which \( p_x \) and \( \rho_x \) are respectively the pressure and energy density of the dark energy fluid. We note that in the context of modified gravity theories, such parametrizations can be viewed in terms of an effective dark energy equation of state, and alternatively, using different dark energy equation of state (in the context of some modified gravitational theory), one could be able to trace the expansion history of the universe, and test them using the observational data. In this work we shall consider that the underlying gravitational theory is described by the Einstein’s gravity and the large scale structure of our Universe is homogeneous and isotropic, and hence, the geometry of the universe is described by the Friedmann-Lemaître-Robertson-Walker metric. Now, concerning the dynamical dark energy parametrizations, we recall numerous parametrizations that have been investigated widely with the available observational data [30–48]. Some well known and most used dark energy parametrizations in this series are, the Chevallier-Polarski-Liner parametrization [30]...**
the logarithmic parametrization \[33\], Jassal-Bagla-Padmanabhan parametrization \[36\] and the Barboza-Alcaniz parametrization \[39\]. Here, considering these four well known dark energy parametrizations, namely, CPL, logarithmic, JBP and BA, we perform a robust analysis by constraining their parameters space using the GWs data from the Einstein telescope along with the standard astronomical probes such as cosmic microwave background radiation (CMB) \[49\ [50\], baryon acoustic oscillations (BAO) \[51\ [53\], Supernovae Type Ia (SNIa) \[54\] and Hubble parameter measurements from the cosmic chronometers (CC) \[55\], in order to see how the data from GWs improve the parameters space of these known parametrizations compared to their usual cosmological constraints availed from the known astronomical probes, CMB, BAO, SNIa and CC. We refer to some works on dark energy using the data from the Einstein Telescope \[50\ [51\].

The work has been structured in the following way. In section 2 we briefly introduce the background and perturbative evolutions for any dark energy parametrization and introduce the parametrizations of our interest. After that in section 3 we describe the method to simulate the GW data from the Einstein’s telescope and show how to use the simulated GW data in order to constrain an underlying theory. In Tsection 4 we introduce the standard astronomical probes as well as the methodology for constraining the model parameters. Then in section 5 we discuss the results of our analyses. Finally, we close the work in section 6 with brief summary of all the results obtained.

## 2. DYNAMICAL DARK ENERGY

In this section we shall describe the general evolution laws of a dynamical dark energy component at the level of background and perturbations.

It is well known that in the large scale, our Universe is perfectly homogeneous and isotropic. Such geometrical description of our Universe is characterized by the Friedmann-Lemaître-Robertson-Walker (FLRW) line element given by

$$ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1-kr^2} + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \right],$$

(1)

where \(a(t)\) (hereafter we shall denote it simply by \(a\)) is the expansion scale factor of the universe and \(k\) is the curvature scalar. For \(k = 0, +1, -1\), three different geometries, namely, the spatially flat, closed and the open universe are described. Further, we assume that the gravitational sector of the universe is described by the Einstein’s general theory of relativity where the total matter sector of the universe is minimally coupled to the Einstein gravity. This total matter sector comes from four distinct non-interacting components, namely, the radiation, baryons, pressureless dark matter and the dark energy. Thus, with the above information, one can explicitly write down the Einstein’s field equations as

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3} \rho_{\text{tot}},$$

(2)

$$2\dot{H} + 3H^2 + \frac{k}{a^2} = -8\pi G p_{\text{tot}},$$

(3)

where \(H \equiv \dot{a}/a\) is the Hubble factor of the FLRW universe and here \(p_{\text{tot}} = p_r + p_b + p_c + p_x\), is the total energy density of the universe and \(p_{\text{tot}} = p_r + p_b + p_c + p_x\). Let us note that, \(p_i's (i = r, b, c, x)\) and \(\rho_i's\) are respectively the energy density and the pressure of the components where one can identify that the subscripts \(r, b, c, x\) respectively correspond to radiation, baryons, cold dark matter and the dark energy sector. Now, using the Bianchi’s identity, the conservation law for the total fluid follows,

$$\dot{\rho}_{\text{tot}} + 3H(\rho_{\text{tot}} + \rho_{\text{tot}}) = 0.$$  

(4)

One can easily find that the conservation equation (4) can be obtained if we simply use the field equations (2) and (3). Since we do not have any interaction between the fluids, thus, the conservation equation of each fluid follows the evolution

$$\dot{\rho}_i + 3H(\rho_i + p_i) = 0 \Leftrightarrow \dot{\rho}_i + 3H(1 + w_i)p_i = 0,$$

(5)

where \(w_i = p_i/\rho_i\) is the equation of state of the \(i\)-th fluid and it takes \(1/3, 0, 0\) for radiation, baryons and cold dark matter. The equation of state of the dark energy fluid is unknown and in this work we consider that \(w_x\) has a dynamical character and henceforth we shall consider some particular expressions for it. We make a final comment regarding the geometrical shape of the universe. As from the observational sources, the universe is almost flat \[62\], and henceforth, throughout the present work we shall assume \(k = 0\) in the Einstein’s field equations (2) and (3). Now, let us get back to the the conservation equation (5), from which one can solve the evolution equations for the governing matter components. In particular, the evolution of the dark energy fluid can be written in terms of its energy density as

$$\rho_x = \rho_{x,0} \left( \frac{a}{a_0} \right)^{-3} \exp \left( -3 \int_{a_0}^a \frac{w_x(a')}{a'} \, da' \right),$$

(6)

where \(\rho_{x,0}\) is the present value of the dark energy density \(\rho_x\), and here \(a_0\) is the present value of the scale factor where \(1 + z = a_0/a\). Without any loss of generality we set the present value of the scale factor to be unity, that means, \(a_0 = 1\). Thus, with the above set of equations, for any prescribed dark energy equation of state, in principle, it is possible to determine the background evolution of the universe.

However, at the same time, it is important to understand the behaviour of the model at the level of perturbations since that enables us to understand the formation of structure of the universe.
Thus, in order to investigate the cosmological perturbations, we consider the perturbed FLRW metric that takes the following expression

\[ ds^2 = a^2(\tau) \left[ -d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j \right], \tag{7} \]

where \( \tau \) is the conformal time and the quantities \( \delta_{ij}, h_{ij} \) respectively denote the unperturbed and the perturbed metric tensors. Now, for the above perturbed metric, one can conveniently write the Einstein’s equations either in the conformal Newtonian gauge or in the synchronous gauge in the Fourier space \( \kappa \). We choose the synchronous gauge and thus using the energy-momentum balance equation \( T_{\mu
u}^{\text{m}} = 0 \), for the \( i \)-th fluid the continuity and the Euler equations for a mode can be written as \[63\] \[65\]

\[ \delta_i^\prime = -(1 + w_i) \left( \theta_i + \frac{h^j_i}{2} \right) - 3\mathcal{H} \left( \frac{\delta p_i}{\delta \rho_i} - w_i \right) \delta_i \]

\[ -9\mathcal{H}^2 \left( \frac{\delta p_i}{\delta \rho_i} - c^2_{a,i} \right) (1 + w_i) \theta_i \frac{\theta_i}{\kappa^2}, \tag{8} \]

\[ \theta_i^\prime = -\mathcal{H} \left( 1 - 3 \frac{\delta p_i}{\delta \rho_i} \right) \theta_i + \frac{\delta p_i / \delta \rho_i}{1 + w_i} \kappa^2 \delta_i - \kappa^2 \sigma_i, \tag{9} \]

where any prime associated with each variable denotes the differentiation with respect to the conformal time \( \tau \); \( \delta_i = \delta p_i / \rho_i \) is the density perturbation for the \( i \)-th fluid; \( \mathcal{H} = a^\prime / a \), is the conformal Hubble factor; \( h = h_j^j \) is the trace of \( h_{ij} \), and \( \theta_i \equiv i k^i v_j \) is the divergence of the \( i \)-th fluid velocity. The quantity \( c^2_{a,i} = \frac{\delta p_i / \delta \rho_i}{1 + w_i} \) denotes the adiabatic sound speed of the \( i \)-th fluid whereas \( c^2_s = \frac{\delta p_i / \delta \rho_i}{1 + w_i} \) is the physical sound speed related with other as \( c^2_{a,i} = w_i - \frac{w_i}{3\mathcal{H}(1 + w_i)} \). Finally, we note that \( \sigma_i \) is the anisotropic stress of the \( i \)-th fluid, however, we shall neglect its contribution for its minimal contribution as reported by some recent observational data \[66\].

Now, we close this section by enlisting the dark energy parametrizations that we wish to study in this work. We consider four well known DE parametrizations as follows. The first one is the Chevallier-Polarski-Linder model \[30\] \[31\] having the following expression

\[ w_x(z) = w_0 + w_a \frac{z}{1 + z} \tag{10} \]

where \( w_0 \) is the present value of \( w_x(z) \) and \( w_a = dw_x(z)/dz \) at \( z = 0 \), is another free parameter of this model.

As a second model, we consider the logarithmic parametrization introduced by G. Efstathiou \[35\]

\[ w_x(z) = w_0 + w_a \ln(1 + z) \tag{11} \]

where \( w_0 \) and \( w_a \) parameters have the same meanings as described for the CPL parametrization.

We then consider another DE parametrization widely known as the Jassal-Bagla-Padmanabhan (JBP) parametrization \[36\]

\[ w_x(z) = w_0 + w_a \frac{z}{(1 + z)^2} \tag{12} \]

and here too, \( w_0 \) and \( w_a \) parameters have the same meanings as described for the above two models, namely CPL and logarithmic.

Finally, we end up with the Barboza-Alcaniz parametrization \[39\]

\[ w_x(z) = w_0 + w_a \frac{z(1 + z)}{1 + z^2} \tag{13} \]

where \( w_0, w_a \) have the same meanings as described above for other DE parametrizations.

3. METHOD OF SIMULATING GW DATA AND ITS USE

In this section we shall describe the method for simulating the Gravitational Waves Standard Sirens (GWSS) data, each data point of which consists of \( (z, d_L(z), \sigma_{d_L}) \) of a GW source, where \( d_L(z) \) is the luminosity distance at the redshift \( z \) and \( \sigma_{d_L} \) is the associated error with \( d_L(z) \). The constraining ability of this catalogue, together with other astronomical datasets, is further investigated in various cosmological models, for instance \[50\] \[57\] \[60\].

The initial step to generate the GWSS data is performed by simulating the redshift distribution of the sources. In this paper we assume the redshifts of all observed GW sources are available. Practically, this is achieved by employing techniques such as identifying the Electromagnetic counterparts. Our interest is focused on GW events originate from 2 types of binary systems: the binary system of a Black Hole (BH) and a Neutron Star (NS) identified as BHNS as well as binary neutron star, named as BNS.

Following some earlier works in this direction \[56\] \[57\] \[60\], the redshift distribution of the observable sources is given by

\[ P(z) \propto \frac{4\pi d_C^2(z) R(z)}{H(z)(1 + z)}, \tag{14} \]

where \( d_C(z) \) represents the comoving distance at the redshift \( z \); \( R(z) \) is the merger rate of binary system (BHNS or BNS) with the fitting form \[57\] \[67\] \[68\]

\[ R(z) = \begin{cases} 1 + 2z, & z \leq 1, \\ \frac{4}{3}(5 - z), & 1 < z < 5, \\ 0, & z \geq 5. \end{cases} \tag{15} \]
Based on the prediction of the Advanced LIGO-Virgo network, the detailed configuration of our simulation is as follows. The ratio between observed BHNS and BNS events is set to be 0.03, which makes BNS the overwhelming majority of GW sources. By roughly considering the mass distribution of the astrophysical objects NS and BH, we perform random sampling of their masses from uniform distributions $U(M_{\odot}, 2M_{\odot})$ and $U(3M_{\odot}, 10M_{\odot})$ respectively, with $M_{\odot}$ being one solar mass. For more details, we refer to [57, 60].

Thus, according to the redshift and mass distribution described above, the catalogue of the GWSS data can be easily obtained through the introduction of the fiducial model which could be any well motivated cosmological model. Now, for the spatially flat universe, one could find the expression for $H(z)$ for the concerned cosmological model and consequently, the luminosity distance $d_L(z)$ of the GWs sources can now be calculated through the relation

$$d_L(z) = (1+z) \int_0^z \frac{dz'}{H(z')}.$$  \hspace{1cm} (16)

Hence, the mean luminosity distances of all the GW sources can be generated using Eq. (16). That means, the $d_L(z)$ vs. $z$ relation can be obtained for every GW event for the concerned cosmological model which as mentioned could be any well motivated cosmological model. Although in some earlier works, $\Lambda$CDM has been considered to the fiducial model, in a similar fashion, instead of the $\Lambda$CDM model, one may fix some other dark energy models to generate the simulated GW data, since there is no such strict rule to select the $\Lambda$CDM model as the fiducial one. In this work we have not fixed $\Lambda$CDM as the fiducial model which is usually done (for instance, see [56, 57]), rather we have considered the dynamical DE models as the fiducial models. We shall describe about it later in more detail.

Now, while measuring the luminosity distance of the GW source, certainly, one needs to calculate the associated error which we denote by $\sigma_{d_L}$. In order to calculate this error, one needs the expression of GW signal, i.e., the strain of GW interferometers. Note that, since the GW amplitude relies on $d_L(z)$, one can extract the information regarding $d_L(z)$ once other parameters (e.g. masses of the binary system, etc.) are evaluated from the waveform. This is the reason why the GW events are often referred to as the standard sirens. As a consequence, the error of GW detection (given in terms of GW SNR) is passed to $\sigma_{d_L}$ via Fisher matrix.

In the following we enter into the main part of this section where we describe the strain of GW interferometers.

Considering the transverse-traceless (TT) gauge, the strain $h(t)$ in the GW interferometers can be given by

$$h(t) = F_+ (\theta, \phi, \psi) h_+ (t) + F_\times (\theta, \phi, \psi) h_\times (t),$$

where $F_+$ and $F_\times$ are the beam pattern functions of the Einstein Telescope; $\psi$ is the polarization angle; the angles $\theta, \phi$ effectively describe the location of the GW source with respect to the GW detector (here Einstein Telescope); $h_+ = h_{xx} = -h_{yy}$, $h_\times = h_{yx} = h_{xy}$ (two independent components of the GW’s tensor $h_{\mu\nu}$ in the transverse-traceless (TT) gauge), see the details here [57]. Now, one can write down the antenna pattern functions of the Einstein Telescope as [56, 57, 60]

$$F_+^{(1)}(\theta, \phi, \psi) = \frac{\sqrt{3}}{2} \left[ \frac{1}{2} \( 1 + \cos^2(\theta) \) \cos(2\phi) \cos(2\psi) + \cos(\theta) \sin(2\phi) \sin(2\psi) \] .$$

$$F_\times^{(1)}(\theta, \phi, \psi) = \frac{\sqrt{3}}{2} \left[ \frac{1}{2} \( 1 + \cos^2(\theta) \) \cos(2\phi) \sin(2\psi) + \cos(\theta) \sin(2\phi) \cos(2\psi) \] .$$  

For the remaining two interferometers, their antenna pattern functions can be derived using above equations for $F_+^{(1)}(\theta, \phi, \psi)$ and $F_\times^{(1)}(\theta, \phi, \psi)$ and substituting $\phi$ by $\phi + 120^\circ$ or $\phi + 240^\circ$, because the three interferometers form an equilateral triangle, and hence, they make $60^\circ$ with each other.

Then, we follow the works of [56, 60] to derive the Fourier transform $\mathcal{H}(f)$ of the time domain waveform $h(t)$ considering the stationary phase approximation that leads to, $\mathcal{H}(f) = A f^{-7/6} \exp[i \( 2 \pi f t_0 - \pi/4 + 2\psi(f/2) - \varphi(2) \)]$, where $A$ is the Fourier amplitude having the following expression

$$A = \frac{1}{d_L} \sqrt{F_+^2 (1 + \cos^2(\omega))^2 + 4 F_\times^2 \cos^2(\omega)} \times \sqrt{\frac{5\pi}{96}} \pi^{-7/6} \mathcal{M}_c^{5/6} ,$$

in which $\mathcal{M}_c$ is dubbed as the “chirp mass” which is related to the total mass $M$ of the coalescing binary system ($M = m_1 + m_2$; here the component masses are $m_1, m_2$) and the symmetric mass ratio (SMR) $\eta = m_1 m_2 / M^2$, by the relation $\mathcal{M}_c = M \eta^{3/5}$. Note that the masses here are actually the observed masses, which is related to the intrinsic masses as $M_{\text{obs}} = (1+z)M_{\text{int}}$, exhibiting an enhancement of a factor $(1+z)$.

Furthermore, in the expression for $A$, the symbol $\omega$ denotes the angle of inclination of the binary’s orbital angular momentum with the line of sight. Since the short gamma ray bursts (SGRBs) are usually expected to be strongly beamed, the coincidence observations of SGRBs suggest that the binaries should be aligned in such a way so that $\omega \approx 0$ with its maximal inclination about $\omega = 20^\circ$. Here, we make a comment that averaging the Fisher matrix over the inclination (i.e., $\omega$) and the polarization (i.e., $\psi$) under the constraint $\omega < 90^\circ$ is almost (roughly) the same as setting $\omega = 0$, considered in the simulation of [69]. Thus, during the simulation of the GW sources, one can safely consider $\omega = 0$. However, during the estimation of the practical uncertainty
of $d_L$, the uncertainty of inclination should be considered positively. When the waveform of GWs are known, one can calculate the signal-to-noise ratio (SNR). The SNR plays a very crucial role in detection of the GW event, because a GW detection is confirmed if the combined SNR of at least 8 is found in the Einstein Telescope [70] [71] (see also [50] [57] [59] [72] for more details in this direction). In general, the combined SNR for the network employing three independent interferometers (just like in the Einstein Telescope) is: $\rho = \sqrt{\sum_{i=1}^{3}(\rho(i))^2}$, where $\rho(i) = \sqrt{\langle H(i), H(i) \rangle}$, and the inner product inside the square root follows [50] [57] [60]

$$(a, b) = 4 \int_{f_{\text{lower}}}^{f_{\text{upper}}} \frac{\hat{a}(f)\hat{b}^*(f) + \hat{a}^*(f)\hat{b}(f)}{2} df S_h(f),$$

(17)

where the sign “$\sim$” placed over the symbols denotes their Fourier transformations and $S_h(f)$ is the one-side noise power spectral density which for this article is taken to be the same as in [56].

Now, the instrumental error (following Fisher matrix approach) on $d_L$ can be estimated through the relation

$$\sigma_{d_L}^{\text{inst}} \simeq \sqrt{\left(\frac{\partial H}{\partial d_L}\right)^2 + \left(\frac{\partial H}{\partial \log d_L}\right)^2}^{-1}.$$  \hspace{4cm} (18)

Assuming that $d_L$ is independent of other parameters, and using the relation $H \propto d_L^{-1}$, from (18), one can deduce that, $\sigma_{d_L}^{\text{inst}} \approx d_L / \rho$. Now, taking into account the effect from the inclination angle $\omega$, we add a factor of 2 in front of this error$^1$. Moreover, under the short-wave approximation, GWs are lensed in the same way as electromagnetic waves during propagation, resulting in an additional weak lensing error which is modeled as $\sigma_{d_L}^{\text{lens}} = 0.052d_L$ in [57]. Consequently, the combined error is

$$\sigma_{d_L} = \sqrt{\left(\sigma_{d_L}^{\text{inst}}\right)^2 + \left(\sigma_{d_L}^{\text{lens}}\right)^2},$$

where the errors $\sigma_{d_L}^{\text{inst}}$ and $\sigma_{d_L}^{\text{lens}}$ are already defined above. Thus, following the method described above, one is now able to generate the future GWSS dataset consisting of $(z, d_L(z), \sigma_{d_L}(z))$. As argued in [57], the constraining ability of Planck on cosmological parameters can only be reached with at least 1000 GW events, corresponding to 10 years of observation by the Einstein Telescope, therefore, data of 1000 GW events are mocked in this work.

Finally, we come to the last part of this section where we describe the approach to use the simulated GW data. The analysis with GW data is similar to the standard cosmological probes. For the GW standard siren measurements with $N$ simulated data points, the $\chi^2$ function is given by

$$\chi^2_{GW} = \sum_{i=1}^{N} \left[ \frac{d_L - d_L(z_i; \vec{\Theta})}{\sigma_{d_L}^i} \right]^2,$$

(19)

where $z_i$, $d_L^i$, and $\sigma_{d_L}^i$ are respectively the $i$-th redshift, luminosity distance at this redshift, and the error of the luminosity distance of the simulated GW data for this particular redshift. Here, $\vec{\Theta}$ represents the set of cosmological parameters that we need to constrain.

4. STANDARD COSMOLOGICAL PROBES AND THE TOTAL LIKELIHOOD ANALYSIS INCLUDING GW

Here we summarize the standard observational data used to analyze the models. In the following we outline a brief description for each dataset.

1. Cosmic Microwave Background (CMB) data: The cosmic microwave background radiation is an important astronomical data to analyze the dark energy models. In particular, we use the Planck 2015 measurements [49, 50] that include the high- and low- $\ell$ TT likelihoods in the multipole range $2 \leq \ell \leq 2508$ as well as the high- and low- $\ell$ polarization likelihoods. The entire data set is identified as Planck TTTEEE+lowTEB.

2. Baryon Acoustic Oscillations (BAO) data: In this work we employ four distinct BAO data measured by different observational surveys. Precisely, we take the (i) 6dF Galaxy Survey (6dFGS) measurement at $z_{\text{eff}} = 0.106$ [51], (ii) the Main Galaxy Sample of Data Release 7 of Sloan Digital Sky Survey (SDSS-MGS) at $z_{\text{eff}} = 0.15$ [52], (iii) the CMASS sample from the latest Data Release 12 (DR12) of the Baryon Oscillation Spectroscopic Survey (BOSS) at $z_{\text{eff}} = 0.57$ and finally (iv) the LOWZ sample from BOSS DR12 at $z_{\text{eff}} = 0.32$ [53].

3. Supernovae Type Ia: The joint light curve sample [54] from Supernovae Type Ia (SNIa) data scattered in the redshift region $z \in [0.01, 1.30]$ have been considered. The total number of SNIa in this region is 740.

4. Cosmic Chronometers: We also add the Hubble parameter measurements from the cosmic chronometers. The cosmic chronometers are the most massive and passively evolving galaxies. The measurements of the Hubble parameters from the cosmic chronometers are promising to estimate the cosmological parameters, see [55]. The total number of Hubble data points we consider in this analysis is thirty distributed in the redshift region $0 < z < 2$. 

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$^1$ Actually, the consideration of the maximal effect of the inclination (between $\omega = 0$ and $\omega = 90^\circ$) on the SNR leads to a factor of 2.
FIG. 1: For the fiducial CPL model, we first constrain the cosmological parameters using the datasets CMB, CMB+BAO, CMB+BAO+JLA and CMB+BAO+JLA+CC and then we use the best-fit values of the parameters for “each dataset” to generate the corresponding GW catalogue. Following this, in each panel we show $d_L(z)$ vs $z$ catalogue with the corresponding error bars for 1000 simulated GW events. The upper left and upper right panels respectively present the catalogue $(z, d_L(z))$ with the corresponding error bars for 1000 simulated events derived using the CMB alone and CMB+BAO dataset. The lower left and lower right panels respectively present the catalogue $(z, d_L(z))$ with the corresponding error bars for 1000 simulated events derived using the CMB+BAO+JLA and CMB+BAO+JLA+CC datasets.

The likelihood analysis follows $\mathcal{L} \propto \exp(-\chi^2/2)$, where the $\chi^2$ function can be obtained by adding the $\chi^2_i$ for the $i$-th dataset. That means, the $\chi^2$ function when CMB and JLA are considered, is $\chi^2 = \chi^2_{CMB} + \chi^2_{JLA}$. This is true for other combinations. In general, if we consider all the standard cosmological datasets, then the total $\chi^2_{\text{tot}}$ is given by

$$\chi^2_{\text{tot}} = \chi^2_{CMB} + \chi^2_{BAO} + \chi^2_{JLA} + \chi^2_{CC},$$

while in presence of the GW data with all the standard cosmological probes, the total $\chi^2_{\text{tot}}$ will be

$$\chi^2_{\text{tot}} = \chi^2_{CMB} + \chi^2_{BAO} + \chi^2_{JLA} + \chi^2_{CC} + \chi^2_{GW}.$$  

TABLE I: The flat priors on various cosmological parameters used for constraining the dynamical dark energy models.

| Parameter | Prior               |
|-----------|---------------------|
| $\Omega_b h^2$ | [0.005, 0.1]       |
| $\Omega_c h^2$ | [0.01, 0.99]       |
| $\tau$ | [0.01, 0.8]            |
| $n_s$ | [0.5, 1.5]              |
| $\log[10^{10} A_s]$ | [2.4, 4]            |
| $100\theta_{MC}$ | [0.5, 10]           |
| $w_0$ | $\left[ -2, 0 \right]$ |
| $w_a$ | $\left[ -3, 3 \right]$ |

5. RESULTS AND ANALYSIS

Let us now summarize the main observational results extracted from the dynamical DE models [10], [11], [12] and [13] after the inclusion of the simulated

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2 One can avail this code which is available here: [http://cosmologist.info/cosmomc/](http://cosmologist.info/cosmomc/)
gravitational-waves data. In the following we describe the results for each model in detail.

5.1. CPL parametrization

First of all, we have constrained the CPL parametrization of eqn. (10) using the standard cosmological probes, such as CMB, BAO, JLA and CC (summarized in the upper half of the Table II), and then using the best-fit values of the model parameters, we have generated the GW catalogue comprising 1000 simulated GW events with CPL being the fiducial model. In Fig. 1, we have shown the relation $d_L(z)$ vs $z$ for the 1000 simulated GW events. Now, incorporating the simulated GW events with the standard cosmological probes, we have constrained the CPL parametrization. The summary of the observational constraints on the CPL model after the inclusion of the simulated GW data are shown in the lower half of Table II.

In Fig. 2 we present the comparisons between the constraining results of the datasets before and after the inclusion of the GW data to the standard astronomical probes mentioned above, where in particular, we show the one-dimensional marginalized posterior distributions for some free parameters of the model as well as the two-dimensional contour plots between several combinations of the free parameters of this parametrization. Specifically, the upper left panel of Fig. 2 presents the comparisons between the datasets CMB and CMB+GW; the upper right panel of Fig. 2 is for CMB+BAO and CMB+BAO+GW; the lower left panel of Fig. 2 is for CMB+BAO+JLA and CMB+BAO+JLA+GW; finally the lower right panel of Fig. 2 is for CMB+BAO+JLA+CC and CMB+BAO+JLA+CC+GW.

In the second column of Table II we present the observational constraints on the model parameters for the datasets CMB and CMB+GW. One can clearly notice that the inclusion of GW to CMB is effective to reduce the error bars on some of the parameters space of this model, see the top left panel of Fig. 2 for a better view on the parameters space. In particular, one can note the significant improvement in the estimations of the Hubble constant as follows, $H_0 = 83.06^{+15.10}_{-15.06}$ (68% CL, CMB), $H_0 = 80.75^{+1.92}_{-1.92}$ (68% CL, CMB+GW). The matter density parameter $\Omega_{m0}$ is constrained to be small and return similar mean values from both the datasets and also provide very higher values of the amplitude of the matter power spectrum $\sigma_8$. Concerning the key parameters of this model, namely, $w_0$ and $w_a$, we see that $w_0 < -1$ for both the datasets, but within 68% CL, both CMB and CMB+GW allow $w_0 > -1$ regime too. Precisely, $w_0 = -1.168^{+0.180}_{-0.212}$ (68% CL, CMB+GW). The constraints on $w_a$ are significantly large in magnitude for both the datasets with higher error bars, but the inclusion of GW decreases the magnitude of $w_a$ compared to its estimation from CMB. One can note that, $w_a = -1.152^{+0.706}_{-1.848}$ at 68% CL for CMB alone while $w_a = -1.081^{+0.842}_{-0.640}$ (68% CL, CMB+GW). The analyses also tells something more if one looks at the graphical variations of the parameters for these datasets (we refer to the top left panel of Fig. 2). From this panel, one can see that, for both the datasets, a strong negative correlation is present between $\Omega_{m0}$ and $H_0$ (also $\sigma_8$); a strong positive correlation between $H_0$ and $\sigma_8$ exists. One can also see that $w_a$ is correlated to $H_0$, $\sigma_8$, $\Omega_{m0}$ and $w_a$.

We now present the cosmological constraints from CMB+BAO and CMB+BAO+GW. With these, we could be able to see how GW data affect this particular combination. The summary of the observational constraints are shown in the third column of Table II and the corresponding graphical variations are shown in the top right panel of Fig. 2. From the table, one can see that the inclusion of BAO to both CMB and CMB+GW, lowers $H_0$ returning similar mean values as follows, $H_0 = 64.36^{+3.23}_{-3.23}$ (68%, CMB+BAO) and $H_0 = 63.77^{+3.72}_{-1.72}$ (68%, CMB+BAO+GW). The error bars on $H_0$ is reduced after the inclusion of GW data. One can also notice that for both the analyses, $w_0$ allows very higher values and $w_a$ takes very lower values, exactly same as recently found in [76]. The interesting fact is that, after the inclusion of BAO to CMB, all the parameters are correlated with each other (see the top right panel of Fig. 2), and this remains true even after the inclusion of GW to the combined analysis CMB+BAO. But, indeed, it is quite clear that the dataset CMB+BAO+GW provides better constraints than CMB+BAO.

We now discuss the cosmological constraints in presence of the JLA data to the previous datasets, that means, precisely we discuss the constraints from CMB+BAO+JLA and CMB+BAO+JLA+GW. The summary of the observational constraints is shown in the fourth column of Table II and the graphical distributions are shown in the bottom left panel of Fig. 2. From this analysis, it is again clear that the inclusion of GW data reduces the error bars on all the parameters. In particular, one can see that $H_0 = 66.98^{+0.55}_{-0.55}$ (68% CL, CMB+BAO+JLA+GW). Concerning the two key parameters of this model, that means, $w_0$ and $w_a$, we have some interesting observation. We see that for both the combinations, $w_0$ approaches near the ‘$-1$’ border with $w_0 = -0.909^{+0.055}_{-0.123}$ (68% CL, CMB+BAO+JLA) and $w_0 = -0.904^{+0.070}_{-0.080}$ (68% CL, CMB+BAO+JLA+GW). Certainly, it shows that although for CMB+BAO+JLA, $w_0 < -1$ is allowed in

\footnote{One can see that the inclusion of GW to CMB decreases the error bars on $w_0$ by more than two times.}
One can also look at the particular, one can see such an improvement in the correlations between the parameters clearly shown in the bottom left panel of Fig. 2, where we see that such correlations are not affected by the GW data. However, we mention that the inclusion of JLA decreases the correlation between some of the parameters. In particular, we find that some of parameters are uncorrelated, for instance, we see that $\sigma_8$ seems to be uncorrelated with $w_0$ and $w_a$.

We now discuss the last two analyses for this model, namely with CMB+BAO+JLA+CC and its companion CMB+BAO+JLA+CC+GW. The summary of the observational constraints is shown in the last column of Table II and in the bottom right panel of Fig. 2 we compare these datasets. From the analysis, we clearly notice that the inclusion of the GW data improves the parameters space in an effective way. In particular, one can see such an improvement in the constraints on the Hubble parameter: $H_0 = 67.92^{+1.09}_{-1.09}$ (68% CL, CMB+BAO+JLA+CC) and $H_0 = 67.72^{+0.36}_{-0.35}$ (68% CL, CMB+BAO+JLA+CC+GW). One can also look at the estimations of the other parameters in a similar way. Concerning the key parameters $(w_0, w_a)$ of this model, we observe that for both the datasets, the dark energy equation of state at present exhibits its quintessential nature (i.e., $w_0 > -1$) while one may note that, for the dataset CMB+BAO+JLA+CC, $w_0 < -1$ is allowed in 68% CL, but for CMB+BAO+JLA+CC+GW, within 68% CL, strictly $w_0 > -1$ is maintained. However, concerning $w_a$, we have some different observations as follows. see that its mean value is slightly increased (considering its magnitude). We see that although for CMB+BAO+JLA and CMB+BAO+JLA+CC, the mean values on $w_a$ are similar, but after the inclusion of GW to these datasets, the mean values on $w_a$ become different, as one can see, $w_a = -0.49^{+0.27}_{-0.22}$ (68% CL, CMB+BAO+JLA+GW) and $w_a = -0.373^{+0.263}_{-0.227}$ (68% CL, CMB+BAO+JLA+CC+GW), thus, although for CMB+BAO+JLA+GW, $w_a = 0$ is allowed within 68% CL, but for CMB+BAO+JLA+CC+GW, within 68% CL, $w_a \neq 0$. Concerning the correlations between the parameters, we have similar conclusion as found for the previous two datasets, namely, CMB+BAO+JLA and CMB+BAO+JLA+GW.

Finally, using the mean values of $(w_0, w_a)$ from all the datasets, in Fig. 3, we have shown the qualitative evolution of the dark energy equation of state $w(z)$ for this model. The solid lines in each plot stand for the $w(z)$ curve for the usual cosmological probe and the dotted

| Parameters | CMB | CMB+BAO | CMB+BAO+JLA | CMB+BAO+JLA+CC |
|------------|-----|---------|-------------|----------------|
| $\Omega_m^2$ | 1.012$^{+0.012}_{-0.012}$ | 1.012$^{+0.012}_{-0.012}$ | 1.012$^{+0.012}_{-0.012}$ | 1.012$^{+0.012}_{-0.012}$ |
| $\Omega_b^2$ | 0.0223$^{+0.0003}_{-0.0003}$ | 0.0223$^{+0.0003}_{-0.0003}$ | 0.0223$^{+0.0003}_{-0.0003}$ | 0.0223$^{+0.0003}_{-0.0003}$ |
| $\Omega_{\gamma}^2$ | 0.0003$^{+0.0003}_{-0.0003}$ | 0.0003$^{+0.0003}_{-0.0003}$ | 0.0003$^{+0.0003}_{-0.0003}$ | 0.0003$^{+0.0003}_{-0.0003}$ |
| $\sigma_8$ | 0.75$^{+0.07}_{-0.07}$ | 0.75$^{+0.07}_{-0.07}$ | 0.75$^{+0.07}_{-0.07}$ | 0.75$^{+0.07}_{-0.07}$ |
| $H_0$ | 70.5$^{+2.0}_{-2.0}$ | 70.5$^{+2.0}_{-2.0}$ | 70.5$^{+2.0}_{-2.0}$ | 70.5$^{+2.0}_{-2.0}$ |

**TABLE II**: 68% and 95% CL constraints on the Chevalier-Polarski-Linder parametrization (10) using various combinations of the observational data with and without the gravitational-wave (GW) data. Here $\Omega_m$ is the present value of $\Omega_m = \Omega_b + \Omega_c$ and $H_0$ is in the units of km s$^{-1}$ Mpc$^{-1}$. 68% CL, but for CMB+BAO+JLA+GW, within 68% CL, $w_0$ is strictly greater than $-1$. The improvement in $w_0$ is also transparent: $w_0 = -0.49^{+0.27}_{-0.22}$ (68% CL, CMB+BAO+JLA) and $w_0 = -0.256^{+0.263}_{-0.227}$ (68% CL, CMB+BAO+JLA+GW). So, from both the observational datasets, dynamical nature is allowed while one can also note that, $w_0 = 0$ is also not excluded in 68% CL. Finally, we mention the correlations between the parameters clearly shown in the bottom left panel of Fig. 2 where we see that such correlations are not affected by the GW data. However, we mention that the inclusion of JLA decreases the correlation between some of the combinations of the parameters. And in particular, we find that some of parameters are uncorrelated, for instance, we see that $\sigma_8$ seems to be uncorrelated with $w_0$ and $w_a$. The results are shown in Table II.
lines depict the evolution of $w_a(z)$ in presence of the GW data. In each plot the shaded regions (with similar colors to the corresponding curves) present the 68% regions for the parameters $w_0, w_a$ corresponding to each dataset (with or without the GW data).

### 5.2. Logarithmic parametrization

In a similar fashion, we constrain the logarithmic parametrization (11) using the standard cosmological probes, such as CMB, BAO, JLA and CC (summarized in the upper half of Table III), and then using the best-fit values of the model parameters, we have generated the GW catalogue comprising 1000 simulated GW events. In Fig. 4, we have shown the relation $d_L(z)$ vs $z$ for the 1000 simulated GW events. Now, using the simulated GW events with the standard cosmological probes, we have constrained this parametrization. The summary of the observational constraints on the CPL model after the inclusion of the simulated GW data are shown in the lower half of Table III.

In Fig. 5, we show the 1D marginalized posterior distributions for some specific parameters of this model plus
FIG. 3: The evolution of the dark energy equation of state for the CPL parametrization is shown for different datasets taking the mean values of the key parameters \(w_0\) and \(w_a\) from the corresponding analysis with and without the GW data. The solid curves stand for the evolution of \(w(z)\) for the standard cosmological probes while the dotted curves show the 68% CL constraints on these two parameters.

### TABLE III: 68% and 95% CL constraints on the logarithmic parametrization (11) using various combinations of the observational data with and without the gravitational-wave (GW) data. Here, \(\Omega_m\) is the present value of \(\Omega_m = \Omega_0 + \Omega_c\) and \(H_0\) is in the units of km s\(^{-1}\)Mpc\(^{-1}\).
the 2D contour plots between several combinations of the model parameters. From a first look at the upper and lower halves of Table [II] one could clearly see that the inclusion of the GW data to the standard cosmological probes significantly improves the parameters space of most of the free parameters of this model, a similar observation already found for the CPL model. Let us now describe how GW works with different observational datasets presented here.

We begin the analyses with CMB data alone and CMB+GW. The results of both the analyses are summarized in the second column of Table [II]. In the top left panel of Fig. 4 we compare the constraints on the model parameters from the datasets which clearly show that the inclusion of GW data significantly reduces the error bars on the model parameters. From the analyses, one can notice that both the datasets return very high values of the Hubble constant with similar mean values but of course the error bars are reduced significantly after the inclusion of GW to CMB. One can look at the estimated values of $H_0$ from both the datasets as follows: $H_0 = 82.78^{+15.48}_{-8.34}$ (68% CL, CMB) and $H_0 = 82.57^{+1.66}_{-1.65}$ (68% CL, CMB+GW). The dark energy equation of state at present, i.e., $w_0$ is constrained to be very close to the cosmological constant boundary $w_0 = -1$ from both the datasets with improved constraints from CMB+GW, as already mentioned. One can see that, $w_0 = -1.058^{+0.354}_{-0.550}$ (68% CL, CMB) and $w_0 = -1.056^{+0.179}_{-0.196}$ (68% CL, CMB+GW). However, within 68% CL, the possibility of $w_0 > -1$ is also allowed due to the error bars on $w_0$. Now, concerning the $w_a$ parameter, we find that both the datasets return large values (in magnitude) although the constraints on $w_a$ are not so stringent due to high error bars. The correlations between the parameters (see the top left panels of Fig. 5) follow a similar trend as seen for the same datasets with CPL model (i.e., top left panel of Fig. 2).

When BAO is added to CMB (see the third column of Table [II] summarizing the results), we find that $H_0$ is significantly lowered with small error bars compared to the constraints from CMB giving, $H_0 = 63.30^{+1.31}_{-1.48}$ (68% CL, CMB+BAO), and when the GW is added to CMB+BAO, the error bars are further decreased, but the mean value of $H_0$ slightly increases with, $H_0 = 65.19^{+1.31}_{-1.48}$ (68% CL, CMB+BAO+GW). The constraints on $\Omega_{m0}$ are significantly high for both the datasets ($\Omega_{m0} = 0.356^{+0.028}_{-0.032}$ at 68% CL for CMB+BAO, and $\Omega_{m0} = 0.339^{+0.015}_{-0.015}$ at 68% CL for CMB+BAO+GW), similar to what we found for the CPL parametrization with the same datasets (we refer to the third column of
Table \ref{tab:constraints} for comparisons). Concerning the key parameters $w_0, w_a$ of the model, we see that $w_0$ is very far from $w_0 = -1$ and $w_a$ is very high (considering its magnitude). In Fig. 5 we have compared the constraints between the datasets from which we can see that the parameters are correlated with each other. This result has already been found for the CPL parametrization with the same datasets (compare the top right panels of Fig. 2 and 5).

We now discuss the next analyses with JLA. In particular, we focus on the constraints from CMB+BAO+JLA and CMB+BAO+JLA+GW. The results are summarized in the third column of Table \ref{tab:constraints} and in the bottom left panel of Fig. 5 we compare the constraints from the datasets. We see that the inclusion of JLA improves the constraints from CMB+BAO, that means, the constraints from CMB+BAO+JLA are more stringent than CMB+BAO and the inclusion of GW gives more finest constraints on the parameters. We find that both the datasets CMB+BAO+JLA and CMB+BAO+JLA+GW prefer a quintessence nature of the dark energy, and for CMB+BAO+JLA, $w_0 > -1$ at 68% CL, while interestingly, for CMB+BAO+JLA+GW, within 68% CL, $w_0 < -1$ is allowed. We also find that the constraints...
on $w_a$ are significantly lowered (considering its magnitude) compared to the previous two datasets, namely, CMB and CMB+BAO. In particular, the estimations are, $w_a = -0.365^{+0.365}_{-0.083}$ (68\% CL, CMB+BAO+JLA) and $w_a = -0.399^{+0.251}_{-0.172}$ (68\% CL, CMB+BAO+JLA+GW). Now, finally, looking the lower left panel of Fig. 5 one can say that the correlations between the parameters $\sigma_8$ and $w_a$ seems to be absent while the correlations (either positive or negative) with others are still existing after the inclusion of JLA. We would also like to remark that such correlations are not affected by the GW data.

We finish the observational analyses after the inclusion of the Hubble parameter measurements from CC. The results are summarized in the last column of Table III and the bottom right panel of Fig. 5 corresponds to the comparisons between the datasets. We find that almost all parameters are constrained in a similar way to CMB+BAO+JLA except the key parameters $w_0$, $w_a$ where we have some different observations. We find here for both the datasets, $w_0 > -1$ strictly at 68\% CL, this is different from the previous analyses where for CMB+BAO+JLA+GW, $w_0 < -1$ was allowed at 68\% CL. The parameter $w_a$ becomes more stringent than its estimation from CMB+BAO+JLA reducing error bars: $w_a = -0.352^{+0.203}_{-0.137}$ (68\% CL, CMB+BAO+JLA+CC) and $w_a = -0.252^{+0.220}_{-0.089}$ (68\% CL, CMB+BAO+JLA+CC+GW) which show that the standard cosmological probe (i.e., CMB+BAO+JLA+CC) allow $w_a = 0$, in 68\% CL, but the inclusion of GW changes this conclusion favoring the dynamical DE for this parametrization within 68\% CL.

Last but not least, in Fig. 6 we have shown the evolution of the dark energy state for this parametrization using the mean values of $w_0$ and $w_a$ for the observational datasets employed in the work. The solid lines in each plot stand for the $w_x(z)$ curve using the usual cosmological probe and the dotted lines represent the evolution of $w_x(z)$ in presence of the GW data. In each plot the shaded regions (with similar colors to the corresponding curves) present the 68\% regions for the parameters $w_0$, $w_a$ corresponding to each dataset (with or without the GW data).

5.3. JBP parametrization

We now discuss the observational constraints on the JBP parametrization (12) in order to investigate the effects of the GW data. We first constrain this parametrization using the standard cosmological probes (summarized in the upper half of the Table IV), and then
using the best-fit values of the model parameters, we generate the GW catalogue comprising 1000 simulated GW events. In Fig. 7 we have shown the relation $d_\text{L}(z)$ vs $z$ for the 1000 simulated GW events. Now, taking into account the simulated GW events with the standard cosmological probes, we have constrained the model parameters which are summarized in the lower half of Table II. Following a similar strategy, we display Fig. 8 that clearly depicts the effects of GW on the cosmological parameters.

We first discuss the constraints from the CMB data alone and for the dataset CMB+GW summarized in the second column of Table IV. And in the top left panel of Fig. 4 we present the comparisons between the constraints obtained from these two datasets. Our analyses report that for both the datasets, $H_0$ assumes very high values where $H_0 = 84.01^{+13.21}_{-7.82}$ (68% CL, CMB) and $H_0 = 82.73^{+0.49}_{-0.54}$ (68% CL, CMB+GW). Clearly, one can see that the inclusion of simulated GW data decreases the error bars on $H_0$ for more than 7 times at 68% CL. This is one of the interesting conclusions that is also followed in CPL and logarithmic models too. The reduction in the error bars for other cosmological parameters are equally true after the inclusion of simulated GW data, see the top left panel of Fig. 4 for a better viewing. The density parameter for matter, $\Omega_m$ assumes lower values for both the datasets. Interestingly, we find that for CMB alone and CMB+GW, $\omega_0 < -1$ at more than 68% CL. Concerning the $\omega_a$ parameter, we find that it takes very high values (in magnitude) and for CMB+GW, it becomes more than triple to that obtained from CMB alone. The numerical estimations are $\omega_a = -0.528^{+0.718}_{-4.42}$ (68% CL, CMB) and $\omega_a = -1.614^{+0.503}_{-1.100}$ (68% CL, CMB+GW).

The inclusion of BAO to the former datasets work in a similar fashion observed in the previous two parametrizations. The results of the analyses are summarized in the third column of Table IV while we compare the constraints in the top right panel of Fig. 4. From this figure (i.e., the top right panel of Fig. 4), we can see that the parameters shown there are correlated with each other, where one may perhaps recognize that the correlation between $\omega_a$ and $\sigma_8$ is relatively low compared to other combinations in this figure. There is one more point that we should remark here. Although $H_0$ assumes slightly lower values that the estimation from Planck’s team but compared to the previous two dynamical DE parametrizations, $H_0$ is relatively higher, $H_0 = 66.29^{+1.58}_{-2.26}$ and $H_0 = 66.54^{+0.89}_{-1.41}$ (68% CL, CMB+BAO). Similar effects are seen in the estimations of the present day matter density parameter $\Omega_m$.

Let us now discuss the observational constraints from CMB+BAO+JLA and its companion CMB+BAO+JLA+GW. The forth column of Table IV summarizes the constraints on the parameters and the bottom left panel of Fig. 4 corresponds to the comparison of the datasets. From the figure (i.e.,

| Parameters | CMB | CMB+BAO | CMB+BAO+JLA | CMB+BAO+JLA+CC |
|-----------|-----|---------|-------------|----------------|
| $\Omega_k^2$ | 0.1191 $^{+0.0047+0.0029}_{-0.0041-0.0028}$ | 0.1187 $^{+0.0041+0.0025}_{-0.0041-0.0024}$ | 0.1188 $^{+0.0041+0.0025}_{-0.0041-0.0024}$ | 0.1189 $^{+0.0041+0.0025}_{-0.0041-0.0024}$ |
| $\Omega_b^2$ | 0.0228 $^{+0.0011+0.00031}_{-0.0011-0.0003}$ | 0.0229 $^{+0.0011+0.00031}_{-0.0011-0.0003}$ | 0.0229 $^{+0.0011+0.00031}_{-0.0011-0.0003}$ | 0.0229 $^{+0.0011+0.00031}_{-0.0011-0.0003}$ |
| $\Omega_{m0}$ | 1.0408 $^{+0.0003+0.00066}_{-0.0003-0.00066}$ | 1.0408 $^{+0.0003+0.00066}_{-0.0003-0.00066}$ | 1.0408 $^{+0.0003+0.00066}_{-0.0003-0.00066}$ | 1.0408 $^{+0.0003+0.00066}_{-0.0003-0.00066}$ |
| $\tau$ | 0.9664 $^{+0.0031+0.00009}_{-0.0031-0.00009}$ | 0.9676 $^{+0.0031+0.00009}_{-0.0031-0.00009}$ | 0.9673 $^{+0.0031+0.00009}_{-0.0031-0.00009}$ | 0.9671 $^{+0.0031+0.00009}_{-0.0031-0.00009}$ |
| $\ln(10^{10} A_s)$ | 3.094 $^{+0.0032+0.00076}_{-0.0032-0.00076}$ | 3.097 $^{+0.0032+0.00076}_{-0.0032-0.00076}$ | 3.093 $^{+0.0032+0.00076}_{-0.0032-0.00076}$ | 3.095 $^{+0.0032+0.00076}_{-0.0032-0.00076}$ |
| $w_0$ | -1.42 $^{+0.01+0.04}_{-0.01-0.04}$ | -1.49 $^{+0.01+0.04}_{-0.01-0.04}$ | -1.43 $^{+0.01+0.04}_{-0.01-0.04}$ | -1.46 $^{+0.01+0.04}_{-0.01-0.04}$ |
| $\sigma_8$ | 0.9673 $^{+0.0031+0.00009}_{-0.0031-0.00009}$ | 0.9680 $^{+0.0031+0.00009}_{-0.0031-0.00009}$ | 0.9667 $^{+0.0031+0.00009}_{-0.0031-0.00009}$ | 0.9664 $^{+0.0031+0.00009}_{-0.0031-0.00009}$ |
| $H_0$ | 78 $^{+2.3+1.71}_{-2.3-1.71}$ | 78 $^{+2.3+1.71}_{-2.3-1.71}$ | 78 $^{+2.3+1.71}_{-2.3-1.71}$ | 78 $^{+2.3+1.71}_{-2.3-1.71}$ |
For the fiducial JBP model, we first constrain the cosmological parameters using the datasets CMB, CMB+BAO, CMB+BAO+JLA and CMB+BAO+JLA+CC and then we use the best-fit values of the parameters for “each dataset” to generate the corresponding GW catalogue. Following this, in each panel we show \( d_L(z) \) vs \( z \) catalogue with the corresponding error bars for 1000 simulated GW events. The upper left and upper right panels respectively present the catalogue \((z, d_L(z))\) with the corresponding error bars for 1000 simulated events derived using the CMB alone and CMB+BAO dataset. The lower left and lower right panels respectively present the catalogue \((z, d_L(z))\) with the corresponding error bars for 1000 simulated events derived using the CMB+BAO+JLA and CMB+BAO+JLA+CC datasets.

bottom left panel of Fig. \ref{fig:7} we see that although the correlations between the parameters are present, but \( \sigma_8 \) does not seem to be correlated with \( w_0, w_a \), at least, a very mild correlation might be present which is not pronounced from the plots. Concerning the dark energy equation of state at present, we see that the observational data prefer a quintessential dark energy (i.e., \( w_0 > -1 \)) while the data also allow \( w_0 < -1 \) regime at 68% CL. The numerical estimations of the \( w_a \) parameter are as follows: \( w_a = -0.508^{+1.057}_{-0.622} \) (68% CL, CMB+BAO+JLA) and \( w_a = -0.683^{+0.828}_{-0.549} \) (68% CL, CMB+BAO+JLA+GW) which show that the inclusion of JLA to the previous datasets reduces the magnitude of \( w_a \), still, the dynamical DE is preferred. However, the observational data indeed allow the case \( w_a = 0 \) within 68% CL.

Finally, we come up with the last two analyses, namely, CMB+BAO+JLA+CC and CMB+BAO+JLA+CC+GW. The results are summarized in the last column of Table \ref{tab:IV} and in the bottom right corner of Fig. \ref{fig:7} we compare these datasets. The only surprising result is observed in the constraints on \( w_a \) from the datasets where the 68% constraints on this parameter are, \( w_a = -0.737^{+0.839}_{-0.689} \) (CMB+BAO+JLA+CC) and \( w_a = -0.029^{+0.755}_{-0.391} \). This is an interesting result and we remark that the previous two dynamical DE parametrizations, namely, CPL and logarithmic do not exhibit such behaviour. We note that for both the analyses, although \( w_0 > -1 \) scenario is observationally preferred but within 68% CL, \( w_0 < -1 \) is also allowed by the observational data.

We close this analysis with the Fig. \ref{fig:9} where we depict the evolution of the dark energy state for this parametrization using the mean values of \( w_0 \) and \( w_a \) for the observational datasets employed in the work. The solid lines in each plot stand for the \( w_x(z) \) curve using the usual cosmological probe and the dotted lines represent the evolution of \( w_x(z) \) in presence of the GW data. In each plot the shaded regions (with similar colors to the corresponding curves) present the 68% regions for the parameters \( w_0, w_a \) corresponding to each dataset (with or without the GW data).

5.4. Barboza-Alcaniz parametrization

Finally, we confront the Barboza-Alcaniz parametrization \cite{BarbozaAlcaniz2020} following a similar pattern performed for other
FIG. 8: 68% and 95% CL contour plots for various combinations of some selected parameters of the JBP model (12) using different observational data in presence (absence) of the gravitational-wave (GW) data.

three dynamical DE parametrizations.

We use the standard cosmological probes, such as CMB, BAO, JLA and CC to constrain the parameters space of this model (summarized in the upper half of the Table V), and then using the best-fit values of the model parameters, we have generated the GW catalogue comprising 1000 simulated GW events. In Fig. 10 we have shown the relation $d_L(z)$ vs $z$ for the 1000 simulated GW events. Now, using the simulated GW events with the standard cosmological probes, we have constrained this parametrization. The summary of the observational constraints on this model after the inclusion of the simulated GW data are shown in the lower half of Table V.

In Fig. 11 we present the graphical behaviour between the free parameters of the model aiming to display the effects of GW on the cosmological parameters. From Table V we again see that the inclusion of simulated GW data remarkably decrease the error bars on the parameters. Apart from that, this parametrization gives some interesting features that will be described soon.

Following the similar pattern, we begin the analyses using the CMB data and CMB+GW data. The results can be found in the second column of Table V and for these datasets, we have made a comparison in the top left panel of Fig. 11. From the analyses, one can visualize that the estimations of the Hubble constant from
FIG. 9: The evolution of the dark energy equation of state for the JBP parametrization has been shown for different datasets taking the mean values of the key parameters \(w_0\) and \(w_a\) from the analyses with and without the GW data. The solid curves stand for the evolution of \(w(z)\) for the standard cosmological probes while the dotted curves for the dataset in presence of the GW data. The shaded regions show the 68% CL constraints on these two parameters.

| Parameters | CMB | CMB+BAO | CMB+BAO+JLA | CMB+BAO+JLA+CC |
|------------|-----|---------|-------------|----------------|
| \(\Omega_m^2\) | 0.1190 ± 0.0044 | 0.1190 ± 0.0044 | 0.1190 ± 0.0044 | 0.1190 ± 0.0044 |
| \(\Omega_c^2\) | 0.0120 ± 0.0007 | 0.0120 ± 0.0007 | 0.0120 ± 0.0007 | 0.0120 ± 0.0007 |
| \(\tau\) | 0.076 ± 0.018 | 0.076 ± 0.018 | 0.076 ± 0.018 | 0.076 ± 0.018 |
| \(n_s\) | 0.966 ± 0.004 | 0.966 ± 0.004 | 0.966 ± 0.004 | 0.966 ± 0.004 |
| \(\ln(10^{10} A_s)\) | 3.085 ± 0.033 | 3.085 ± 0.033 | 3.085 ± 0.033 | 3.085 ± 0.033 |
| \(\sigma_8\) | 0.964 ± 0.014 | 0.964 ± 0.014 | 0.964 ± 0.014 | 0.964 ± 0.014 |
| \(H_0\) | 83.55 ± 7.20 | 83.55 ± 7.20 | 83.55 ± 7.20 | 83.55 ± 7.20 |

Parameters | CMB+GW | CMB+BAO+GW | CMB+BAO+JLA+GW | CMB+BAO+JLA+CC+GW |
|------------|--------|-------------|----------------|------------------|
| \(\Omega_m^2\) | 0.1185 ± 0.0041 | 0.1185 ± 0.0041 | 0.1185 ± 0.0041 | 0.1185 ± 0.0041 |
| \(\Omega_c^2\) | 0.0121 ± 0.0007 | 0.0121 ± 0.0007 | 0.0121 ± 0.0007 | 0.0121 ± 0.0007 |
| \(\sigma_8\) | 0.9679 ± 0.0042 | 0.9679 ± 0.0042 | 0.9679 ± 0.0042 | 0.9679 ± 0.0042 |
| \(\ln(10^{10} A_s)\) | 3.085 ± 0.033 | 3.085 ± 0.033 | 3.085 ± 0.033 | 3.085 ± 0.033 |

TABLE V: Observational constraints at 68% and 95% confidence levels for the Barboza-Alcaniz parametrization [13] using various combinations of the observational data with and without the gravitational-wave (GW) data. Here, \(\Omega_m\) is the present value of \(\Omega_m = \Omega_b + \Omega_c\) and \(H_0\) is in the units of km s\(^{-1}\) Mpc\(^{-1}\).
are quite high, similar to the previous three dynamical models. We find that, $H_0 = 83.55^{+14.43}_{-7.20}$ (68% CL, CMB) and $H_0 = 82.12^{+1.34}_{-1.32}$ (68% CL, CMB+GW). So, the error bars significantly decrease and it is true for other free parameters as well. We refer to the top left panel of Fig. 11 from which one can clearly see how much the inclusion of GW data to CMB, significantly improves the parameters space. The dark energy equation of state is constrained to be in the phantom regime and within 68% CL, this remains so. Interestingly, the parameter $w_a$ is constrained to be relatively small compared to its CMB and CMB+GW constraints for other models. We also comment that the estimation of $\Omega_m$ is small for both the datasets and this is also found for other three models as well.

For the next analyses with BAO, that means focusing on the combined analyses CMB+BAO and CMB+BAO+GW, we do not find anything that is worth reporting. The results can be found from the third column of Table V and the comparison between the constraints on the model parameters using various datasets are shown in the top right plot of Fig. 11.

After the inclusion of JLA, results summarized in the fourth column of Table V, we find that quintessence DE (i.e., $w_0 > -1$) is preferred by the dataset CMB+BAO+JLA, and this remains so at 68% CL. In addition, the inclusion of GW to this dataset does not alter this conclusion meaning that within 68% CL, $w_0 > -1$. Concerning the $w_a$ parameter, we find that the constraints are small (in magnitude) compared to the previous datasets and also compared to the other models as well. We see that, $w_a = -0.263^{+0.211}_{-0.165}$ (68% CL, CMB+BAO+JLA) and $w_a = -0.186^{+0.151}_{-0.130}$ (68% CL, CMB+BAO+JLA+GW). Thus, we see that both the datasets prefer $w_a \neq 0$, at least in 68% CL. We refer to the bottom left panel of Fig. 11 for a comparison of the model parameters constraints obtained from different datasets.

Finally, we consider the last two combinations, namely, CMB+BAO+JLA+CC and CMB+BAO+JLA+CC+GW. In the last column of Table V, we have summarized the results and in the bottom right panel of Fig. 11, we have compared the cosmological constraints for this parametrization obtained from both the datasets. These analyses give some interesting results. Concerning the present value of the dark energy equation of state, $w_0$, we see that for the dataset CMB+BAO+JLA+CC, $w_0 > -1$ and it remains so at more than 68% CL, while after the inclusion of GW, this result is completely changed with the possibility

FIG. 10: For the fiducial BA model, we first constrain the cosmological parameters using the datasets CMB, CMB+BAO, CMB+BAO+JLA and CMB+BAO+JLA+CC and then we use the best-fit values of the parameters for “each dataset” to generate the corresponding GW catalogue. Following this, in each panel we show $d_L(z)$ vs $z$ catalogue with the corresponding error bars for 1000 simulated GW events. The upper left and upper right panels respectively present the catalogue $(z, d_L(z))$ with the corresponding error bars for 1000 simulated events derived using the CMB alone and CMB+BAO dataset. The lower left and lower right panels respectively present the catalogue $(z, d_L(z))$ with the corresponding error bars for 1000 simulated events derived using the CMB+BAO+JLA and CMB+BAO+JLA+CC datasets.
of $w_0 < -1$ at more than 68% CL. More interestingly, for the dataset CMB+BAO+JLA+CC+GW, $w_0 = -1.000^{+0.074}_{-0.075}$ at 68% CL. However, it is also true that the data allow the region with $w_0 > -1$ at 68% CL. So statistically, it is really very hard to come up with a definite conclusion, but compared to other dynamical DE models, it is indeed true that here the preference over $\Lambda$CDM cosmology is strongly supported. In addition to that, after the inclusion of GW, that means, for the dataset CMB+BAO+JLA+CC+GW, $w_a = 0.004^{+0.142}_{-0.123}$ (68% CL). Thus, from the overall constraints on both $w_0$ and $w_a$, one can clearly say that the forecasting with future gravitational wave data strongly hints towards the $\Lambda$CDM type cosmology, however, looking at the constraints on $w_a$ for CMB+BAO+JLA+CC+GW, the presence of large error bars imply that the observational data allow $w_a \neq 0$ as well.

Finally, using the mean values of $(w_0, w_a)$ from all the datasets, in Fig. 12 we depict the evolution of the dark energy equation of state $w_z(z)$ for this model. The solid lines in each plot stand for the $w_z(z)$ curve for the usual cosmological probe and the dotted lines depict the evolution of $w_z(z)$ in presence of the GW data. In each plot the shaded regions (with similar colors to the corresponding curves) present the 68% regions for the parameters $w_0, w_a$ corresponding to each dataset (with or without...
FIG. 12: The evolution of the dark energy equation of state for the BA parametrization has been shown for different datasets taking the mean values of the key parameters $w_0$ and $w_a$ from the analyses with and without the GW data. The solid curves stand for the evolution of $w_x(z)$ for the standard cosmological probes while the dotted curves for the dataset in presence of the GW data. The shaded regions show the 68% CL constraints on these two parameters.

The detection of gravitational waves has thrilled the scientific community by offering a new window of tests that may shine some light on the nature of gravity, dark matter and dark energy. In the present work, we investigate how GW data could bring further cosmological constraints to a class of dynamical dark energy models. In particular, we use 1000 simulated GW data from the Einstein Telescope (we refer to section 3 for a detailed discussion on how the GW catalogue can be generated for any fiducial cosmological model).

In this article, we consider four dynamical dark energy models characterized by their equations of state: namely, the Chevallier-Polarski-Linder parametrization \(^{10}\), logarithmic parametrization \(^{11}\), Jassal-Bagla-Padmanabhan parametrization \(^{12}\), and finally the Barboza-Alcaniz parametrization \(^{13}\) which are initially constrained by the standard cosmological probes such as CMB, BAO, JLA, and CC. Now, we consider the corresponding parametrizations as the fiducial models and use the best-fit values of the model parameters from each dataset to generate the GW catalogue. In this way, we generate 1000 simulated GW events from the present dynamical models. Now, along with the 1000 simulated GW data from the Einstein Telescope, we also consider the standard cosmological probes such as CMB, BAO, JLA, and CC, in order to measure the differences in the constraints and understand the power of using the GW data.

The results for these four dynamical DE models are shown in Table II (CPL), Table III (logarithmic), Table IV (JBP), Table V (BA). Further, the cosmological constraints results obtained from the astronomical datasets (with and without the GW data) have been compared in Fig. 2 (CPL), Fig. 5 (logarithmic), Fig. 8 (JBP) and Fig. 11 (BA) with an aim to investigate how GW data affect the dynamical nature of various dark energy fluids aforementioned.

Concerning the CPL parametrization (eqn. \(^{10}\)), we find that the effects of GW is clearly pronounced on the CMB data alone, since as one can see that the addition of GW to CMB alone significantly improve the Hubble constant: $H_0 = 83.06^{+15.10}_{-7.98}$ (68% CL, CMB) and $H_0 = 80.75^{+1.71}_{-1.92}$ (68% CL, CMB+GW). Regarding the other parameters, we also find that the dataset CMB+GW definitely improve them compared to their constraints from CMB alone. However, for other
datasets, such as CMB+BAO, CMB+BAO+JLA and CMB+BAO+JLA+CC, the effects of GW is seen but not much compared to what we observed for the CMB alone case. Concerning the key two parameters $w_0$ and $w_a$ of this parametrization, we find that for both CMB and CMB+BAO data, the mean values of the dark energy state parameter is phantom, although within 68% CL, its quintessential character is allowed. Interestingly, we find that although the combinations CMB+BAO+JLA and CMB+BAO+JLA+CC allow $w_0 < -1$ at 68% CL, but when GW data are allowed, $w_0$ becomes quintessential in 68% CL. The mean values of $w_a$ are always nonzero for all the datasets, while its zero value is never allowed when the GW data are included. So, the inclusion of GW strengthens its dynamical character. Now, for the logarithmic parametrization (see eqn. (11)), we have almost similar behaviour to the CPL model (10). Summarizing the behaviour of this model, we find that for both CMB and CMB+GW, $w_0$ offers both quintessential and phantom character within 68% CL while its phantom character is favored more. Concerning other datasets, except for the CMB+BAO+JLA+GW (where within 68% CL, $w_0 < -1$ is allowed), $w_0 > -1$ is always suggested within 68% CL. The mean values of $w_a$ are always nonzero for all the datasets, while its zero value is never allowed when the GW data are included. So, the inclusion of GW strengthens its dynamical character.

For the JBP (eqn. (12)) and BA parametrizations (13) we have slightly different results compared to other two parametrizations (CPL and logarithmic). The reduction of error bars in presence of GW data follows a similar pattern in JBP and BA similar to CPL and logarithmic, but concerning the behaviour of the dark energy key parameters ($w_0$, $w_a$), we have something different conclusions. For JBP parametrization, we see that for both CMB and CMB+GW, $w_0 < -1$ at more than 68% CL with $w_a \neq 0$ for CMB+GW at more than 68% CL. For other datasets (excluding CMB+BAO), $w_0 > -1$ is always favored through its mean values, but within 68% CL, $w_0 < -1$ is allowed. This result is different compared to what we observed for CPL and logarithmic. But, even after the inclusion of GW data, the error bars on $w_a$ remain significantly large, thus, $w_a = 0$ is also allowed. For the last parametrization, i.e., BA parametrization (eqn. (13)), the CMB alone case follows similar to the JBP model. But, for all the datasets (with and without GW data), we find $w_a \neq 0$ at more than 68% CL with $w_0$ having quintessence character. But, the most interesting case is the final combination CMB+BAO+JLA+CC+GW. For this analysis we can clearly see that the BA is absolutely consistent with the ΛCDM cosmology, as one can see the 68% CL constraints on $w_0$ and $w_a$ are, $w_0 = -1.000^{+0.074}_{-0.075}$, $w_a = 0.004^{+0.123}_{-0.075}$.

Thus, in summary, our results show that clearly the future GW data significantly affect the cosmological parameters providing stringent constraints by reducing their error bars significantly. From the future constraints using GW data, we find that quintessential DE is favored for CPL, logarithmic and JBP parametrization, while for the BA parametrization, the support towards the ΛCDM cosmology becomes much stronger.

Acknowledgments

WY was supported by the National Natural Science Foundation of China under Grants No. 11705079 and No. 11647153. DFM thanks the Research Council of Norway and the NOTUR computing facilities. This paper is based upon work from COST action CA15117 (CANTATA), supported by COST (European Cooperation in Science and Technology).

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