QUARK–DIQUARK SYSTEMATICS OF BARYONS AND THE $SU(6)$ SYMMETRY FOR LIGHT STATES

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We continue our attempts to systematize baryons, which are composed of light quarks ($q = u, d$), as quark–diquark systems. The notion of two diquarks is used: (i) $D_1^1$, with the spin $S_D = 1$ and isospin $I_D = 1$ and (ii) $D_0^0$, with $S_D = 0$ and $I_D = 0$. Here we try to resolve the problem of the low-lying $\Delta(\frac{5}{2}^-)$ states: in the last experiments the lightest state is observed at $\sim 2200$ MeV, not at $1900 - 2000$ MeV as it has been stated 20 years ago. We are looking for different systematization variants with the forbidden low-lying $\Delta(\frac{5}{2}^-)$ states in the mass region $\lesssim 2000$ MeV. We see that the inclusion of the $SU(6)$ constraints on $qD_1^1$ states with angular momentum $L = 1$ results in a shift of the lightest $\Delta(\frac{5}{2}^-)$ isobar to $\sim 2300$ MeV. The scheme with the $SU(6)$ constraints for low-lying $qD_1^1$ and $qD_0^0$ states (with $L = 0, 1$) is presented in detail here.

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1. Introduction

Baryons (we mean standard non-exotic baryons) are composite systems of three constituent quarks, each of them in their turn being a complicated system of quarks and gluons. We do not know the detailed structure of baryons, even in the language of constituent quarks, except for a fragmentary knowledge on low-lying states.

We know that low-lying baryons satisfy the $SU(6)$ symmetry, but as to heavier ones we are not certain about. Numerous model calculations, which describe rather well the low-lying states, are at variance in their predictions concerning highly excited states, e.g., see references therein. In addition, the number of highly excited states predicted by such models exceeds considerably the number of observed states. One might believe that experimental investigations of baryon spectra were not complete. Nevertheless, it does not remove the question whether it is possible to construct a more effective scheme for highly excited states, with less degrees of freedom and less number of highly excited states. The introduction of another effective particle, that is the diquark, provides us with such a possibility.

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The size of diquark as a composite quark–antiquark system is believed to be of the order of that of quark, ∼0.2–0.3 fm. So, it is doubtful if we can interpret the low-lying baryon (a compact system, ∼0.7-0.9 fm) as a system of effective diquark and constituent quark.

The idea of baryon as spatially separated quark and diquark can be for sure tried on the highly excited baryon systems which are of a larger size – such an idea was suggested in the PDG compilation. But in we accepted also that the quark–diquark structure does not work for low-lying baryons with angular momentum \( L = 0 \) – for these states the \( SU(6) \) symmetry was applied.

Actually, in we have made an attempt to systematize highly excited baryons, assuming that they do not like to be formed as three-particle colour quark systems but prefer to be created as two-particle, quark–diquark, compound states:

\[
D^{\alpha}_0 \equiv \left[ \varepsilon^{\alpha\beta\gamma} q_\beta q_\gamma \right]
\]

where \( \varepsilon^{\alpha\beta\gamma} \) is a totally antisymmetrical tensor in the colour space.

In we have considered two schemes of the quark-diquark construction for the \( L \geq 1 \) states, namely:

1. The diquark masses \( M_{D^0} \neq M_{D^1} \).
2. The diquark masses are equal to each other \( M_{D^0} = M_{D^1} \); besides, the states \( qD^0 \) and \( qD^1 \) with the total spin \( S = 1/2 \) (here \( S = S_q + S_D \)) overlap. This scheme decreases essentially the number of highly excited states.

In the quark–diquark scheme of we face a specific transition to the \( SU(6) \) limit. This procedure is in fact a projection of the \( qD^0 \) and \( qD^1 \) wave functions on the \( SU(6) \) basis – the result depends on the hypothesis on diquark masses. In the \( L = 0 \) states, at \( M_{D^0} = M_{D^1} \), we have two basic \(( n = 1)\) states, \( N^+_{1/2}(940) \) and \( \Delta^+_{3/2}(1240) \), see equation (46) in the PDG compilation, while at \( M_{D^0} \neq M_{D^1} \) the additional basic state, \( N^+_{1/2}(1440) \), appears (see equation (49) in the PDG compilation).

In the present paper we accept the equality of masses of scalar and axial–vector diquarks, \( M_{D^0} = M_{D^1} \). In recent analyses of spectra near the Roper resonance one may find arguments in favour of the existence of one pole in the partial wave \( P_{11} \) near 1400 MeV. Analytical properties of the partial wave \( P_{11} \) in the best fit of the PDG compilation are shown in Fig. the fitting to two poles near 1400 MeV gives us a worse description of data. Of course, the Roper pole splits, owing to the momentum-dependence of width near the \( \pi \Delta \) branching point. This splitting is quite similar to that observed in the Flatté formula for \( f_0(980) \) near the \( \bar{K}K \) threshold. Still, we attribute such ”satellite poles” to the main one, in case in question, to the Roper pole (1370±96); note that in Fig. the satellite pole is hidden under the \( \pi \Delta \) cut.

There is a question to what low-lying states the \( SU(6) \) symmetry may be applied and where we have the region with spatially separated quark and diquark. Only the experiment can answer this question. If we turn to the PDG compilation it may seem that the \( SU(6) \) symmetry can be applied to \( L = 0 \) only, while at \( L \geq 1 \) the domain of quark–diquark structures begin. However, the latest analyses give rise to doubts. The matter is that in the experiments carried out in the eighties the resonance \( \Delta_{5/2}^- \) (1930±50) has been observed rather definitely.
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Fig. 1. Complex-$M$ plane: position of poles of resonance states $N(1440)$ and $N(1710)$. The cuts related to the threshold singularities $\pi N$, $\pi\Delta$, $\rho N$ and $\sigma N$ are shown by vertical solid lines. The Roper pole is located near the $\pi\Delta$ cut, while its satellite pole is hidden under the cut.

Still, modern analyses\textsuperscript{9,10,13} point to the lower $\Delta_{5/2}^-$ state, being located around 2200 MeV or even higher. It gives arguments for expanding the $SU(6)$ symmetry constraints on the $L = 1$ states.

In the present paper we accept that both sets of states ($L = 0$ and $L = 1$) satisfy the $SU(6)$ symmetry requirements.

The paper is organized as follows:
In Section 2 we present wave functions of quark–diquark states and demonstrate how the imposing of the $SU(6)$ constraints affect these systems. In Section 3 we suggest the setting of the $L = 0$ and $L = 1$ states, while states with $L > 1$ are discussed in Section 4. Here we demonstrate the setting of all quark–diquark states on the $(J, M^2)$ and $(n, M^2)$ trajectories.

2. Wave functions of quark–diquark systems and the $SU(6)$ constraints

We present here the diquark wave function, give general form for the quark–diquark wave function and present the scheme of projecting them on the $SU(6)$ basis.

2.1. $S$-wave diquarks and baryons

Recall that we use two $S$-wave diquarks with color numbers $\bar{c} = 3$: scalar diquark $D^0_{1S}$ and axial–vector one, $D^{1F_z}_{1S2}$. The diquark spin–flavor wave functions with
$I_D = 1, S_D = 1$ and with $I_D = 0, S_D = 0$ read as follows:

\[ D_{11}^{11}(ij) = u^\uparrow(i)u^\uparrow(j), \]

\[ D_{10}^{11}(ij) = \frac{1}{\sqrt{2}} \left( u^\uparrow(i)u^\downarrow(j) + u^\downarrow(i)u^\uparrow(j) \right), \]

\[ D_{11}^{10}(ij) = \frac{1}{\sqrt{2}} \left( u^\uparrow(i)d^\uparrow(j) + d^\uparrow(i)u^\uparrow(j) \right), \]

\[ D_{10}^{10}(ij) = \frac{1}{2} \left( u^\uparrow(i)d^\downarrow(j) + u^\downarrow(i)d^\uparrow(j) + d^\uparrow(i)u^\downarrow(j) + d^\downarrow(i)u^\uparrow(j) \right). \]

Let us consider, first, the $\Delta$ isobar at $I_Z = 3/2$ with fixed $J, J_Z, S$ and orbital momentum $L$. The wave function for this state at arbitrary $n$ reads

\[ \sum_{S_Z, m_z} C_{L, J_Z - S_Z}^1 S S C_{1, S_Z - m_z}^1 S_z \frac{3}{2} \frac{3}{2} \left( u^m_z(1)D_{11}^{11}\right)_{S_Z - m_z} S_{z} (23) \]

\[ \times \left| \bar{k}_{1, cm} \right| L Y_{L}^{J_Z - S_Z}(\theta_1, \phi_1) \phi_1^{(L)}(1; 23) + (1 \equiv 2) + (1 \equiv 3). \]

Here $| \bar{k}_{1, cm} \rangle$ and $(\theta_1, \phi_1)$ are the momenta and momentum angles of the first quark in the c.m. system.

For other $I_Z$, one should include into the wave function a summing over isotopic states which means the following substitution in (2):

\[ C_{1, J_Z - m_z}^{S, 3/2} \frac{3}{2} \frac{3}{2} \left( u^m_z(1)D_{11}^{11}\right)_{S_Z - m_z} S_{z} (23) \rightarrow \sum_{J_s} C_{1, J_Z - j_s}^{J \frac{3}{2}, \frac{3}{2}} \frac{3}{2} \frac{3}{2} J_s \phi_1^{(L)}(1; 23). \]

To project the wave functions (2) or (3) on the $SU(6)$ wave function set, we should use symmetrical coordinate/momentum wave functions

\[ \Phi^{(L)}_{I_D}(\ell; i) \rightarrow \Phi^{(L)}_{I_D}(\ell, i, j) = \Phi^{(L)}_{I_D}(j, l, i) = \Phi^{(L)}_{I_D}(i, j, l). \]

For example, one may accept that $\Phi^{(L)}_{I_D}(\ell, i, j)$ depends on $s$ only, $\Phi^{(L)}_{I_D}(\ell, i, j) \rightarrow \varphi^{(L)}_{I_D}(s)$. Indeed, in this limit we have

\[ \sum_{S_Z, m_z} C_{L, J_Z - S_Z}^1 S S C_{1, S_Z - m_z}^1 S_z \frac{3}{2} \frac{3}{2} \left( u^m_z(1)D_{11}^{11}\right)_{S_Z - m_z} S_{z} (23) \]

\[ \times \left( \phi_1^{(L)}(1; 23) \right) | \bar{k}_{1, cm} \rangle L Y_{L}^{J_Z - S_Z}(\theta_1, \phi_1) + (1 \equiv 2) + (1 \equiv 3) \varphi^{(L)}_{I_D}(s). \]

For nucleon states ($I = 1/2$) we write

\[ \sum_{S_Z, m_z} C_{L, J_Z - S_Z}^1 S S C_{1, S_Z - m_z}^1 S_z \frac{3}{2} \frac{3}{2} \left( u^m_z(1)D_{11}^{11}\right)_{S_Z - m_z} S_{z} (23) \]

\[ \times \left| \bar{k}_{1, cm} \right| L Y_{L}^{J_Z - S_Z}(\theta_1, \phi_1) \Phi^{(L)}_{I_D}(1; 23) + (1 \equiv 2) + (1 \equiv 3). \]
The $SU(6)$ limit, as previously, is reached at $\Phi_1^{(L)}(i; j\ell) \to \varphi_1^{(L)}(s)$. Then, instead of (6), one has
\begin{equation}
\sum_{S_\perp m_\perp} C^J_{L_J z J z - S_\perp} S S_\perp C^S S_\perp \varphi_1^{(L)}(s) = \sum_{j_\perp} \varphi_1^{(L)}(s) \cdot \sum_{S_\perp m_\perp} C^J_{L_J z J z - S_\perp} S S_\perp \varphi_1^{(L)}(s) \cdot \sum_{j_\perp} \varphi_1^{(L)}(s).
\end{equation}
For $qD_0^0$ states the wave function reads in general case as follows:
\begin{equation}
\sum_{m_z} C^{J}_{L J z - m_z} \left( q_{I_s}^{m_z (1)} D_0^0 (23) | k_{cm} \cdot Y_{L_J z - m_z} (\theta_1, \phi_1) + (1 \equiv 2) + (1 \equiv 3) \right) \varphi_1^{(L)}(s).
\end{equation}
In the $SU(6)$ limit we have
\begin{equation}
\sum_{m_z} C^{J}_{L J z - m_z} \left( q_{I_s}^{m_z (1)} D_0^0 (23) | k_{cm} \cdot Y_{L_J z - m_z} (\theta_1, \phi_1) + (1 \equiv 2) + (1 \equiv 3) \right) \varphi_0^{(L)}(s).
\end{equation}
Baryons are characterized by $I$ and $J^P$, these states with different $S$ and $L$ and fixed $I$ and $J^P$ can mix. To select independent states, one may orthogonalize wave functions with the same isospin and $J^P$. The orthogonalization depends on the structure of the momentum/coordinate parts $\Phi_1^{(L)}(i; j\ell)$. But in case of the $SU(6)$ limit the momentum/coordinate wave functions transform in the common factor $\Phi_1^{(L)}(i; j\ell) \to \varphi_1^{(L)}(i, j, \ell)$ or $\Phi_1^{(L)}(i; j\ell) \to \varphi_1^{(L)}(s)$, and one should orthogonalize the spin/momentum factors. Namely, let us denote the $SU(6)$ spin/momentum factor of the wave function as $Q_{sp}^{(A)}$. Then
\begin{equation}
\psi_{sp}^{(A)} = Q_{sp}^{(A)} \varphi_{SU(6)}^{(A)}(s),
\end{equation}
where $A = I, II, III, ...$ belong to different $(S, L)$. The orthogonal set of operators $Q_{sp}^{(A)}$ is constructed in a standard way:
\begin{align}
Q_{sp}^{(I)} & = Q_{sp}^{(I)}, \\
Q_{sp}^{(II)} & = Q_{sp}^{(II)} - Q_{sp}^{(I)} \left( Q_{sp}^{(I)} + Q_{sp}^{(II)} \right) \left( Q_{sp}^{(I)} + Q_{sp}^{(II)} \right), \\
Q_{sp}^{(III)} & = Q_{sp}^{(III)} - Q_{sp}^{(I)} \left( Q_{sp}^{(I)} + Q_{sp}^{(II)} \right) - Q_{sp}^{(II)} \left( Q_{sp}^{(I)} + Q_{sp}^{(II)} \right), \\
\end{align}
and so on. The convolution of operators $Q_{sp}^{(A)}, Q_{sp}^{(B)}$ includes the summation over quark spins as well as integration over quark momenta.
2.2. Wave functions in the SU(6) limit

We present wave functions of the $qD^0_0$ and $qD^1_1$ systems in the SU(6) limit, assuming $M^2_0 = M^2_1 = M^2$. The transition $M^2_0 \rightarrow M^2_1$ gives us additional reduction of number of states.

Recall that in the SU(6) limit the momentum/coordinate factors of wave functions, for example, such as $\Phi^{(L)}_0(3, 12) = \Phi^{(L)}_0(s, s_{12})$ and $\Phi^{(L)}_1(3, 12) = \Phi^{(L)}_1(s, s_{12})$, transform into $s$-dependent ones:

$$\Phi^{(L)}_0(s, s_{12} \rightarrow M^2_0) = \varphi^{(L)}_0(s, M^2) \equiv \varphi^{(L)}(s), \quad (12)$$

$$\Phi^{(L)}_1(s, s_{12} \rightarrow M^2_1) = \varphi^{(L)}_1(s, M^2) \equiv \varphi^{(L)}(s).$$

For our purpose, that is, for checking the number of non-vanishing states, we calculate below the wave functions with $I = I_z$ and $J = J_z$ only.

3. Wave functions of the $(L = 1)$ states in the SU(6) limit and $M^2_0 \rightarrow M^2_1$

In this sector we have only seven basic states, namely,

(1) two with $I = 1/2$: $N_{\frac{1}{2}} - (D^0; S = 1/2)$, $N_{\frac{1}{2}} + (D^1_1; S = 1/2)$,

(2) three with $I = 1/2$: $N_{\frac{1}{2}} - (D^1_1; S = 3/2)$, $N_{\frac{1}{2}} + (D^1_1; S = 3/2)$, $N_{\frac{1}{2}} - (D^1_1; S = 3/2)$.

(3) two with $I = 3/2$: $\Delta_{\frac{3}{2}} - (D^1_1; S = 1/2)$, $\Delta_{\frac{3}{2}} + (D^1_1; S = 1/2)$.

Below the wave functions are written in c.m. system using the following notations for quark momenta: $k_1 + k_2 + k_3 = 0$, $k_{a\pm} = (k_{ax} \pm ik_{ay})/\sqrt{2}$.

(1) Two $qD^0_0$ systems with $I = 1/2$

We have two basic states:

$$4\sqrt{\pi} \cdot \Psi^{(L=1)}_0 \left( I = \frac{1}{2}, J_z = \frac{1}{2}, J^P = \frac{1}{2}, J_z = \frac{1}{2} \right)$$

$$= (u_1^+ d_3^+ k_{3z} + u_1^+ d_3^+ k_{1z} - \sqrt{2} u_1^+ d_3^+ k_{2z} + u_2^+ d_3^+ k_{1z} + u_2^+ d_3^+ k_{2z} + u_2^+ d_3^+ k_{3z})$$

$$\varphi^{(L=1)}_0(s)$$

and

$$4\sqrt{\pi} \cdot \Psi^{(L=1)}_0 \left( I = \frac{1}{2}, J_z = \frac{1}{2}, J^P = \frac{3}{2}, J_z = \frac{3}{2} \right)$$

$$= \sqrt{3}(u_1^+ d_3^+ k_{3z} + u_1^+ d_3^+ k_{1z} + u_1^+ d_3^+ k_{2z} + u_2^+ d_3^+ k_{1z} + u_2^+ d_3^+ k_{2z} + u_2^+ d_3^+ k_{3z})$$

$$\varphi^{(L=1)}_0(s).$$
The same angular momentum/spin wave functions can be constructed with use of the \(qD^1\) system, in this way we have two nucleons with \(S = 1/2\):

\[
4\sqrt{\pi} \cdot \Psi_1^{(L=1)} \left( I = \frac{1}{2}, I_z = \frac{1}{2}, S = 1/2, J^P = \frac{1}{2}^-, J_z = \frac{1}{2} \right)
\]

\[
= -\sqrt{2} \left( u_1^d u_2^d d_1 k_{3z} + u_1^u u_2^d d_1 k_{1z} - \sqrt{2} u_1^u u_2^d d_1 k_{2z} + u_1^d u_2^u d_1 k_{1z} + u_1^d u_2^u d_1 k_{2z} 
- \sqrt{2} u_1^u u_2^d d_1 k_{3z} + u_1^d u_2^u d_1 k_{1z} - \sqrt{2} u_1^u u_2^d d_1 k_{2z} + u_1^d u_2^u d_1 k_{3z}
- \sqrt{2} u_1^u u_2^d d_1 k_{2z} - \sqrt{2} u_1^u u_2^d d_1 k_{1z} + d_1^u u_2^u d_1 k_{2z} + d_1^u u_2^u d_1 k_{3z}
- \sqrt{2} d_1^u u_2^u d_1 k_{1z} + d_1^u u_2^u d_1 k_{1z} - \sqrt{2} d_1^u u_2^u d_1 k_{3z} + \sqrt{2} d_1^u u_2^u d_1 k_{2z} \right) \phi_1^{(L=1)}(s) \quad (15)
\]

and

\[
4\sqrt{\pi} \cdot \Psi_1^{(L=1)} \left( I = \frac{1}{2}, I_z = \frac{1}{2}, S = 1/2, J^P = \frac{3}{2}^-, J_z = \frac{3}{2} \right) =
\]

\[
= \sqrt{3} \left( u_1^d u_2^d d_1 k_{3z} + u_1^u u_2^d d_1 k_{1z} + u_1^d u_2^u d_1 k_{1z} + u_1^d u_2^u d_1 k_{2z} + u_1^d u_2^u d_1 k_{2z} + u_1^d u_2^u d_1 k_{3z} + u_1^d u_2^u d_1 k_{3z} + d_1^u u_2^u d_1 k_{1z} + d_1^u u_2^u d_1 k_{1z} + d_1^u u_2^u d_1 k_{1z} + d_1^u u_2^u d_1 k_{1z} \right) \phi_1^{(L=1)}(s) \quad (16)
\]

The wave function of (13) coincides with that of (15), while the wave function of (14) coincides with (10), because we use equation (12): \(\phi_0^{(L=1)}(1, 2, 3) = \phi_1^{(L=1)}(1, 2, 3)\).

In this case we deal with two (not four) states.

(2) Three \(qD^1\) systems with \(I = 1/2\)

There are three states with \(S = 3/2\):

\[
4\sqrt{\pi} \cdot \Psi_1^{(L=1)} \left( I = \frac{1}{2}, I_z = \frac{1}{2}, S = 3/2, J^P = \frac{1}{2}^-, J_z = \frac{1}{2} \right) \]

\[
= \left( 3 u_1^d u_2^d d_1 k_{3z} - \sqrt{2} u_1^u u_2^d d_1 k_{3z} - \sqrt{2} u_1^u u_2^d d_1 k_{3z} - u_1^d u_2^d d_1 k_{3z} + 3 u_1^d u_2^d d_1 k_{3z} - \sqrt{2} u_1^u u_2^d d_1 k_{3z} - \sqrt{2} u_1^u u_2^d d_1 k_{3z} - u_1^d u_2^d d_1 k_{3z}
- u_1^d u_2^d d_1 k_{3z} + u_1^d u_2^d d_1 k_{3z} - \sqrt{2} u_1^u u_2^d d_1 k_{3z} - \sqrt{2} u_1^u u_2^d d_1 k_{3z} - u_1^d u_2^d d_1 k_{3z}
- \sqrt{2} u_1^u u_2^d d_1 k_{3z} + 3 u_1^d u_2^d d_1 k_{3z} - \sqrt{2} u_1^u u_2^d d_1 k_{3z} - \sqrt{2} u_1^u u_2^d d_1 k_{3z}
- d_1^u u_2^d d_1 k_{3z} - \sqrt{2} u_1^u u_2^d d_1 k_{3z} - d_1^u u_2^d d_1 k_{3z} - d_1^u u_2^d d_1 k_{3z} \right) \phi_1^{(L=1)}(s), \quad (17)
\]

\[
4\sqrt{\pi} \cdot \Psi_1^{(L=1)} \left( I = \frac{1}{2}, I_z = \frac{1}{2}, S = 3/2, J^P = \frac{3}{2}^-, J_z = \frac{3}{2} \right) =
\]

\[
= \frac{3}{\sqrt{15}} \left( -3 \sqrt{2} u_1^u u_2^d d_1 k_{3z} - 2 u_1^u u_2^d d_1 k_{3z} - 2 u_1^u u_2^d d_1 k_{3z} - 3 \sqrt{2} u_1^u u_2^d d_1 k_{2z}
- 2 u_1^u u_2^d d_1 k_{2z} - 2 u_1^u u_2^d d_1 k_{2z} - 2 u_1^u u_2^d d_1 k_{2z} - 2 u_1^u u_2^d d_1 k_{2z}
- 3 \sqrt{2} u_1^u u_2^d d_1 k_{2z} - 2 u_1^u u_2^d d_1 k_{2z} - 2 u_1^u u_2^d d_1 k_{2z} - 2 u_1^u u_2^d d_1 k_{2z} \right) \phi_1^{(L=1)}(s), \quad (18)
\]
4.1. The SU(6) limit – the states with SU(6) symmetry and the L = 1 set, should be located at \( \Delta \sim 1900 \) MeV, is a primary motivation for expanding the SU(6) symmetry onto \( L = 1 \) states.

4. The SU(6) symmetry and the \( L = 0 \) and \( L = 1 \) states

First, we recall the setting of \( L = 0 \) states, then we discuss the situation with \( L = 1 \) states under the SU(6) symmetry.

4.1. The SU(6) symmetry for the nucleon \( N_{1/2}^+ (940) \), isobar \( \Delta_{3/2}^+ (1238) \) and their radial excitations

We accept \( M_{D_0}^2 = M_{D_1}^2 \) and suppose the SU(6) symmetry for the lowest baryons with \( L = 0 \). It gives us two ground states – the nucleon \( N_{1/2}^+ (940) \) and isobar
\[ \Delta_{\frac{3}{2}^+}(1238) \] as well as their radial excitations:

\[
\begin{array}{c|cc}
L & S = \frac{1}{2}, N(\frac{1}{2}^+) & S = \frac{3}{2}, \Delta(\frac{3}{2}^+) \\
\hline
n = 1 & 938 \pm 2 & 1232 \pm 4 \\
n = 2 & 1440 \pm 40 & 1635 \pm 75 \\
n = 3 & 1710 \pm 30 & \sim 1880 \\
n = 4 & 2090 \pm 100 & \sim 2150 \\
\end{array}
\] (23)

Note that the mass-squared splitting of the nucleon radial excitation states, \( \delta_n M^2(N_{\frac{1}{2}^+}) \), is of the order of \( 1.05 \pm 0.15 \) GeV\(^2\). This value is close to that observed in meson sector, \( 1.05 \pm 0.05 \) GeV\(^2\). (24)

The state with \( n = 4 \) cannot be unambiguously determined.

One can see that the mass-squared splitting of \( \Delta_{\frac{3}{2}^+} \) isobars, \( \delta_n M^2(\Delta_{\frac{3}{2}^+}) \), coincides with that of the nucleon, \( \delta_n M^2(N_{\frac{1}{2}^+}) \), with a good accuracy:

\[
\delta_n M^2(\Delta_{\frac{3}{2}^+}) = 1.07 \pm 0.05.
\] (25)

Let us emphasize that state \( \Delta_{\frac{3}{2}^+}(1920) \) is classified as \( S = 3/2, L = 2 \) states, with \( n = 1 \) (see Section 4). However, this resonance can be reliably classified as radial excitations of \( \Delta_{\frac{3}{2}^+}(1232) \), with \( n = 3 \). Actually, it means that around \( \sim 1920 \) MeV one may expect the double-pole structure.

4.2. Setting of the \( L = 1 \) states under the \( SU(6) \) symmetry constraints

\[
\begin{array}{c|cc|c|c|c}
L = 1 & S = \frac{1}{2} & \hline & N(\frac{1}{2}^-) & N(\frac{3}{2}^-) & \hline
n = 1 & (1535 \pm 20) & (1524 \pm 5) \\
n = 2 & (1905 \pm 60) & (1870 \pm 25) \\
n = 3 & (2090 \pm 150) & (2160 \pm 35) & \sim 2390 & \sim 2390 \\
n = 4 & \sim 2500 & \sim 2500 \\
\hline & S = \frac{3}{2} & \hline & N(\frac{1}{2}^-) & N(\frac{3}{2}^-) & N(\frac{5}{2}^-) \\
n = 1 & (1680 \pm 40) & (1730 \pm 40) & (1680 \pm 10) & \sim 2270 & \sim 2270 & \sim 2260 \\
n = 2 & \sim 2010 & \sim 2000 & \sim 2000 & \sim 2270 & \sim 2270 & \sim 2260 \\
n = 3 & \sim 2500 & \sim 2500 & \sim 2500 & \sim 2500 & \sim 2500 \\
n = 4 & \sim 2460 & \sim 2450 & \sim 2450 \\
\end{array}
\] (26)

\[
\begin{array}{c|c|c|c|c}
L = 1 & S = \frac{1}{2} & \hline & \Delta(\frac{1}{2}^-) & \Delta(\frac{3}{2}^-) & \hline
n = 1 & (1625 \pm 10) & (1720 \pm 50) \\
n = 2 & (1910 \pm 50) & (1995 \pm 40) \\
n = 3 & (2150 \pm 50) & \sim 2210 \\
n = 4 & \sim 2460 & \sim 2450 \\
\end{array}
\] (27)
5. The setting of states with \( L \geq 2 \)

For a "naive observer", who does not perform an analysis of double pole structure, the number of states with \( S = 1/2 \) decreases twice.

In Fig. 2 following \([10][13][12]\), we show \((J, M^2)\) plots as they look like for naive observers, while Fig. 3 demonstrates the \((J, M^2)\) plot for ground states \((n = 1)\) only.

We have three groups of states with \( I = \frac{1}{2} \):

1) \( N_{J^P}(D_0^0; S = \frac{1}{2}, J = L - \frac{1}{2}) \), \( N_{J^P}(D_0^0; S = \frac{1}{2}, J = L - \frac{1}{2}) \),

2) \( N_{J^P}(D_1^1; S = \frac{1}{2}, J = L - \frac{1}{2}) \), \( N_{J^P}(D_1^1; S = \frac{1}{2}, J = L - \frac{1}{2}) \),

3) \( N_{J^P}(D_1^1; S = \frac{3}{2}, J = L - \frac{3}{2}) \), \( N_{J^P}(D_1^1; S = \frac{3}{2}, J = L - \frac{3}{2}) \),

\( N_{J^P}(D_1^1; S = \frac{3}{2}, J = L + \frac{1}{2}) \), \( N_{J^P}(D_1^1; S = \frac{3}{2}, J = L + \frac{3}{2}) \),

and two with \( I = \frac{3}{2} \):

1) \( \Delta_{J^P}(D_1^1; S = \frac{1}{2}, J = L - \frac{1}{2}) \), \( \Delta_{J^P}(D_1^1; S = \frac{1}{2}, J = L - \frac{1}{2}) \),

2) \( \Delta_{J^P}(D_1^1; S = \frac{3}{2}, J = L - \frac{3}{2}) \), \( \Delta_{J^P}(D_1^1; S = \frac{3}{2}, J = L - \frac{3}{2}) \),

\( \Delta_{J^P}(D_1^1; S = \frac{3}{2}, J = L + \frac{1}{2}) \), \( \Delta_{J^P}(D_1^1; S = \frac{3}{2}, J = L + \frac{3}{2}) \),

We suppose that in \( I = \frac{1}{2} \) sector the states of groups (1) and (2) with the same \( J \) and \( L \) have the equal masses – resonances are overlapping:

\[
M_{N_{J^P}(D_0^0; S = \frac{1}{2}, J = L - \frac{1}{2})} = M_{N_{J^P}(D_1^1; S = \frac{1}{2}, J = L - \frac{1}{2})},
\]

\[
M_{N_{J^P}(D_0^0; S = \frac{1}{2}, J = L + \frac{1}{2})} = M_{N_{J^P}(D_1^1; S = \frac{1}{2}, J = L + \frac{1}{2})}.
\]
Fig. 2. Baryon settings on \((J^P, M^2)\) planes in the model with overlapping \(qD^0_0(S = 1/2)\) and \(qD^1_1(S = 1/2)\) states. Notations are as follows: 1) open squares: predicted \(S = 1/2\) ground states, 2) open stars: predicted \(S = 3/2\) ground states, 3) open circles: predicted radial excitation states, 4) full triangles: observed states.
Fig. 3. Basic baryon settings \((n = 1)\) on \((J^P, M^2)\) planes in the model with overlapping \(qD_0^0(S = 1/2)\) and \(qD_1^1(S = 1/2)\) states (notations are as in Fig 2.
For \( P = + \) states the values of baryon masses in the \( I = 1/2 \) sector are as follows:

| \( L = 2 \) | \( S = \frac{1}{2} \) | \( S = \frac{3}{2} \) |
|-------------|------------------|------------------|
| \( n = 1 \) | \( N(\frac{3}{2}^+) \) | \( N(\frac{5}{2}^+) \) |
| \( n = 2 \) | \( (1770 \pm 100) \) | \( (1685 \pm 5) \) |
| \( n = 2 \) | \( (1960 \pm 30) \) | \( (2000 \pm 100) \) |
| \( n = 3 \) | \( \sim 2290 \) | \( \sim 2290 \) |
| \( n = 4 \) | \( \sim 2520 \) | \( \sim 2520 \) |
| \( n = 1 \) | \( N(\frac{1}{2}^+) \) | \( N(\frac{1}{2}^+) \) |
| \( n = 2 \) | \( (1890 \pm 50) \) | \( \sim 1880 \) |
| \( n = 3 \) | \( \sim 2150 \) | \( \sim 2150 \) |
| \( n = 4 \) | \( \sim 2390 \) | \( \sim 2390 \) |

\[ (31) \]

| \( L = 4 \) | \( S = \frac{1}{2} \) | \( S = \frac{3}{2} \) |
|-------------|------------------|------------------|
| \( n = 1 \) | \( N(\frac{1}{2}^+) \) | \( N(\frac{3}{2}^+) \) |
| \( n = 2 \) | \( \sim 2290 \) | \( \sim 2290 \) |
| \( n = 3 \) | \( \sim 2520 \) | \( \sim 2520 \) |
| \( n = 1 \) | \( N(\frac{1}{2}^+) \) | \( N(\frac{3}{2}^+) \) |
| \( n = 2 \) | \( \sim 2390 \) | \( \sim 2390 \) |
| \( n = 3 \) | \( \sim 2610 \) | \( \sim 2610 \) |

\[ (32) \]

and for negative-parity states:

| \( L = 3 \) | \( S = \frac{3}{2} \) |
|-------------|------------------|
| \( n = 1 \) | \( N(\frac{3}{2}^-) \) | \( N(\frac{5}{2}^-) \) |
| \( n = 2 \) | \( (2160 \pm 80) \) | \( (2150 \pm 30) \) |
| \( n = 3 \) | \( \sim 2390 \) | \( \sim 2390 \) |
| \( n = 4 \) | \( \sim 2610 \) | \( \sim 2610 \) |

\[ (33) \]
In the $I = \frac{3}{2}$ sector we have for $P = +$ states:

| $L = 2$ | $S = \frac{1}{2}$ | \( \Delta({\frac{3}{2}^+}) \) | \( \Delta({\frac{5}{2}^+}) \) |
|----|----|----|----|
| $n = 1$ |  | $\sim 1750$ | $\sim 1750$ |
| $n = 2$ |  | $\sim 2040$ | $\sim 2040$ |
| $n = 3$ |  | $\sim 2290$ | $\sim 2290$ |
| $n = 4$ |  | $\sim 2520$ | $\sim 2520$ |

\[
S = \frac{3}{2} \quad \Delta({\frac{5}{2}^+}) \quad \Delta({\frac{7}{2}^+}) \quad \Delta({\frac{9}{2}^+})
\]

| $n = 1$ | $\sim 1935 \pm 90$ | $\sim 1995 \pm 40$ | $\sim 1885 \pm 25$ | $\sim 1928 \pm 8$ |
| $n = 2$ | $\sim 2150$ | $\sim 2151$ | $\sim 2400$ | $\sim 2400$ |
| $n = 3$ | $\sim 2390 \sim 2390 \pm 100$ | $\sim 2360 \pm 125$ | $\sim 2460 \pm 120$ |
| $n = 4$ | $\sim 2610$ | $\sim 2610$ | $\sim 2610$ | $\sim 2610$ |

$S = \frac{1}{2}$

| $n = 1$ | $\sim 2730$ | $\sim 2730$ |
| $n = 2$ | $\sim 2930$ | $\sim 2930$ |

and for $P = -$ ones:

| $L = 3$ | $S = \frac{1}{2}$ | \( \Delta({\frac{3}{2}^-}) \) | \( \Delta({\frac{5}{2}^-}) \) |
|----|----|----|----|
| $n = 1$ |  | $\sim 2230$ | $(2230 \pm 50)$ |
| $n = 2$ |  | $\sim 2460$ | $\sim 2460$ |
| $n = 3$ |  | $\sim 2670$ | $\sim 2670$ |
| $n = 4$ |  | $\sim 2870$ | $\sim 2870$ |

\[
S = \frac{3}{2} \quad \Delta({\frac{5}{2}^-}) \quad \Delta({\frac{7}{2}^-}) \quad \Delta({\frac{9}{2}^-}) \quad \Delta({\frac{11}{2}^-})
\]

| $n = 1$ | $\sim 2320 \sim 2320 \pm 50 \sim 2320$ | $\sim 2760 \sim 2760 \sim 2760 \sim 2760$ |
| $n = 2$ | $\sim 2550 \sim 2550 \sim 2550 \sim 2550$ |
| $n = 3$ | $\sim 2760 \sim 2760 \sim 2760 \sim 2760$ |
| $n = 4$ | $\sim 2950 \sim 2950 \sim 2950 \sim 2950$ |

$S = \frac{1}{2}$

| $n = 1$ | $\sim 2760 \sim 2760 \sim 2760 \sim 2760 \pm 100$ |
| $n = 2$ | $\sim 2950 \sim 2950 \sim 2950 \sim 2950$ |
| $n = 3$ | $\sim 3130 \sim 3130 \sim 3130 \sim 3130$ |

The above-written equations allow us to predict the \((n, M^2)\) plots which are shown in Fig. 4.
Fig. 4. The $(n, M^2)$ plots for $I = 1/2$ and $I = 3/2$ states.
6. Conclusion

Under the hypothesis of the quark–diquark structure of highly excited baryons, we succeeded to suggest for them a realistic classification. The introduction of diquarks gave a considerable reduction of excited states; additional reduction was obtained owing to the assumption of the overlapping of \((I = 1/2)\) states with \(S = 1/2\).

Thus obtained a classification gave linear trajectories in the \((J, M^2)\) and \((n, M^2)\) planes, which are strictly ordered. There is a number of overlapping poles. The observation of two-pole and three-pole structures in the complex-\(M\) planes of partial amplitudes is a top-priority task at the analysis of baryon spectra, while the next task consists in the writing and solving the spectral integral equation for quark–diquark systems. Such an equation should be similar to that written and solved before for the \(q\bar{q}\) system\(^{19}\).

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