Layout research of settling slurry velocity distribution in vertical pipeline

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Abstract. Based on the research results of slurries, the author sets out from momentum transfer, and analyses the momentum change of liquid and solid particle, and puts forward calculation model of settling slurry velocity distribution, liquid velocity distribution and solids particle velocity distribution in vertical pipeline. The vertical slurry experiment has been done to obtain the data about slurry velocity distribution, liquid velocity distribution and solids particle velocity distribution. At last simple verification has been made for the calculation model which has been put forward, and it is proved that the prediction value of the author’s model is accurate.

1. Introduction
Research of parameters for vertical pipe transportation is important for mineral pipe in shafts and ocean mining. But research on this aspect is not perfect, for the models used update are almost empirical formula, and they lack actual application value.

Based on the research of experts[1],velocity variation of solids and liquid in the course of interaction between solids and liquid and of solid acceleration is investigated. Calculation model of settling slurry velocity distribution, liquid velocity distribution and solids particle velocity distribution in vertical pipeline is put forward and some experimental data are used to simply verify the model.

2. Deduction of settling slurry velocity in vertical pipe
It is supposed that discharged quality of solid and water is $m_s$, $m_w$ in minute time $\Delta t$ from minute cross section $dA$. The velocity of flow water is $V$ at first, and the velocity of slurry is $V_m$. After momentum exchange, velocity of solids and water are $V_s^*$ and $V_w^*$. It is also supposed that average concentration, solid density and water density are $\bar{q}$, $\rho_s$ and $\rho$ respectively. So following equation exists:

$$m_s = dA \cdot \bar{q} \cdot V_s \cdot \Delta t \cdot \rho_s $$  \hspace{1cm} (1)

$$m_w = dA \cdot (1 - \bar{q}) \cdot V_w \cdot \Delta t \cdot \rho $$ \hspace{1cm} (2)

According to mass conservation:

$$V_m A = A[V_w (1 - \bar{q}) \rho + V_s \bar{q} \rho_s ]$$ \hspace{1cm} (3)

Then the following equation exists:

$$V_m = V_w (1 - \bar{q}) + V_s \bar{q} \hspace{1cm} (4)$$

According the equation of momentum between discharged solids from pipe and water:

$$M_s V_s^* = M_w (V - V_w^*) \hspace{1cm} (5)$$
Then the following equation exists:

\[
V - V_w^v = \frac{\bar{q}\rho_s V^2_v}{(1 - \bar{q})\rho V_w}
\]  

(6)

According to relative research and information[1-2], when the solids are accelerated, there exists a additional pressure \( \Delta P_s \), it causes the water researched have a increment, it can use following equation to calculate[1]:

\[
\Delta V_w = \frac{A(1 - \bar{q})\Delta p, \Delta t}{A(1 - \bar{q})V_w, \rho \Delta t} = \Delta p_s
\]

So actual velocity variation of water is:

\[
V - V_w = V - V_w^v - \Delta V_w
\]

(7)

From above equation (8):

\[
V - V_w^v = V - V_w + \Delta V_w = (V - V_w) \left(1 + \frac{\Delta V_w}{V - V_w}\right) = (V - V_w)K_1
\]

(9)

From equation (6) and (9), following equation is obtained:

\[
V - V_w = \frac{\bar{q}\rho_s V^2_v}{K_1(1 - \bar{q})\rho V_w}
\]

(10)

According to relative research and information[2]

\[
K_1 = \frac{1}{1 - 0.6332\psi(1)}
\]

(11)

In above equation, the unknown \( \psi(1) \) can get from reference [1].

In vertical pipe, velocity variation due to gravity \( G_h \) is \( \Delta V_{s2} \):

\[
\Delta V_{s2} = \frac{G_h t}{m_s} = \frac{[A\bar{q}(\rho_s - \rho)]t}{A\bar{q}\rho_s} = g(1 - \frac{1}{\delta})t
\]

(12)

In above equation, \( t \) is accelerating time, according to research of Rose[2]

\[
t = K' \sqrt{\frac{L_s}{g}}
\]

(13)

\[
L_s = 6D \left(\frac{A\bar{q}V_s, \rho_s}{\rho \sqrt{g} \sqrt{D}} \right)^1 \left(\frac{D}{d_s} \frac{\rho_s}{\rho}\right)
\]

(14)

In above equation, diameters of pipe and solids are \( D \) and \( d_s \) respectively, and \( K' \) is a constant which is determined from concrete conditions. So total velocity variation of solid is:

\[
V_s = V_s^v + \Delta V_{s2}
\]

(15)

From above equation (15), following equation is got:

\[
V_s^v = V_s - \Delta V_{s2} = V_s \left[1 - \Delta V_{s2} / V_s\right]
\]

(16)

Assuming:

\[
K_2^v = 1 - \Delta V_{s2} / V_s
\]

(17)

From (10), (16) and (17), following equation is obtained:
\[
V = V_w + [1 - 0.56\psi(1)\varphi(\theta)] \frac{K_2}{\tilde{q}c_s} \frac{V_s^2}{(1 - \tilde{q})\rho V_w}
\]

According to Rose’s research[2], when \(\theta=0\), \(\varphi(\theta)=1.125\).

The motion equation of solid in vertical pipe:
\[
\frac{\pi}{6} d_e \left(\rho_s + \frac{\rho}{2}\right) \frac{dV_s}{dt} = F_D - W_b - F_h
\]

In above equation, \(F_D\), \(W_b\) and \(F_h\) are drag force of water acts on solid, equal gravity and interference force among solids respectively. These three forces can be calculated as followings:
\[
F_D = \frac{\pi}{4} d_e^2 C_{Dr} \left(\frac{V_w - V_s^2}{2}\right) \rho
\]
\[
W_b = \frac{\pi}{6} d_e^3 (\rho_s - \rho) g
\]
\[
F_h = \left[1 - (1 - \tilde{q})^{-2}\right] \left(\frac{\sqrt{\text{Re}_p \alpha} + \sqrt{\text{Re}_p \alpha^2 + 4\sqrt{48\alpha\beta}(1 - \tilde{q})^2}}{\sqrt{\text{Re}_p \alpha} + \sqrt{\text{Re}_p \alpha^2 + 4\sqrt{48\alpha\beta}}}\right) W_b
\]

In equation (20):
\[
C_{Dr} = \frac{1}{4} \left(\alpha + \sqrt{\frac{\alpha^2 + 4\sqrt{48\alpha\beta}}{\text{Re} \rho_r}}\right)^2
\]
\[
\text{Re} \rho_r = \frac{d_e (V_w - V_s)}{\mu}
\]

\(\alpha\) and \(\beta\) are swanson numner[3], \(\mu\) is viscosity of water, when acceleration process of solid is completed, and it reach stable state, from \(\frac{dV_s}{dt} = 0\), we can get:
\[
V_s = V_w - \frac{8(F_h + W_b)}{\pi d_e^2 C_{Dr} \rho}
\]

Assuming that \(V_0\) is flow velocity in the central of pipe, and \(R\) is radius of pipe, and flow velocity \(V(y)\) is water flow velocity at a distance of \(y\) from the pipe wall. So the relation of \(y\) and \(V(y)\) can be expressed as following equation:
\[
\frac{V(y)}{V_0} = \left(\frac{y}{R}\right)^\frac{1}{2}
\]

It is obvious that variables \(V\), \(V_s\), \(V_w\) and \(V_m\) are interrelated and in fact, all of them can all be expressed as function of \(y\). Based on this, the equation (4), (18), (25) and (26) are equations which comprise of four unknown quantities. If other parameters are known, by solving the equations, velocity of water \(V(y)\) (before interaction with solid particles), velocity of solid particles \(V_s(y)\), velocity of water \(V_w(y)\) (after interaction with solid particles) and velocity of slurry are be worked out.
3. Experimental verification

Flow velocity of fluid is measured by pitot tube:

\[ v = c \sqrt{2g \Delta h} \]  

(27)

Where:
- \( c \) — flow velocity, (m/s);
- \( g \) — gravity acceleration, (m/s²);
- \( \Delta h \) — velocity head, (m)

The distance of solids in radius and their velocity are measured by digital photogrammetry method[4].

In the experiment, pipe diameter is 25.4mm, and particle mean diameter is 0.4mm. Solid particle’s specific gravity is 2.24. the slurry volume concentration is vary from 5% to 30%.

Slurry vertical pipeline arrangement is shown in Figure 1. During the experiment, particle size was invariable due to the fact that the particles did not pass through the pump 2. In the transparent section 9, the scale is close to the pipe to determine the particle displacement. The motion picture of the particles was photographed with high-speed photogrammetry and then processed using MATLAB software; the image processing includes image extraction, image calibration, transforming into grayscale images, grayscale enhancement, thresholding, particle matching, and so on. As the particle reaches the stability condition, the velocity in vertical direction is almost stable, but the horizontal direction presents periodic motion characteristics. In this study, only velocity in the vertical direction was investigated. In the two adjacent images, the velocity of the solid particles can be obtained from the ratio of the axial displacement of the particles to the interval of the shooting time.

It is found that the axial velocity distribution along the radial direction of particles is similar to that of parabola, and the absolute value of particle size to velocity is relatively small, and the variation along diameter is not significant.

The experimental data is shown in Fig.2-Fig.7. To verify the author’s model, model calculated data is also shown in Fig.2-Fig.7.

The mean flow velocity of slurry is reflected in Fig.2 when slurry volume concentration is 5%. After estimation, the maximum error is 15%. The axial slip velocity between water and solid is shown in Fig.3.
According to the result, the maximum error between measured value and model calculated value is not exceed 16% when slurry volume concentration is 15%.

Figure 2. Slurry velocity distribution for $Re=6120$

In Fig 4 and 5, velocity distribution of water and solid in pipe is shown, respectively. The model’s calculated value is very close to measured value in Fig 4. When Renuk number is 10325, the maximum error of calculated solid velocity and measured solid velocity is 7%.

Figure 3. Axial slip velocity for $Re=5774$
In Fig 6 and 7, the measured slurry distribution and model’s calculated slurry distribution are shown when Renault number is 13359 and 13825, respectively. And the relative error between model calculated value and measured value is about 4% in Fig. 6 and 2% in Fig. 7.
Figure 6. Slurry velocity distribution for $Re=13359$

Figure 7. Slurry velocity distribution for $Re=13825$

Seen from Figure 2~7, it is obvious that the model which author put forward is practicable when particles diameter is 25.4mm and Renault number is 5774-13825. The higher the Renault number, the exacter the model calculated value is. For more cases, much research has to be done in this aspect.

4. Conclusion

By analyzing the momentum change of liquid and solid particle, a new calculation model of settling slurry velocity distribution, liquid velocity distribution and solids particle velocity distribution in vertical pipeline is put forward in this paper. The vertical slurry experiment has been done to obtain the data about slurry velocity distribution, liquid velocity distribution and solids particle velocity distribution. It is obvious that the model which author put forward is practicable when particles diameter is 25.4mm and
Renault number is 5774-13825, the higher the Renault number, the exacter the model calculated value is. It is proved that the prediction value of the author’s model is accurate.

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