The Use of S Parameters in Two-Port Analogue Networks Stability Analysis

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Abstract. If electrical systems are considered to consist of two-port linear analogue circuits (networks) in harmonic state, then S parameters can be used to establish the stable functioning states corresponding to such systems. This paper presents, on one hand, the practical use of S parameters to increase the efficiency of information transmission and propagation and of electromagnetic energy wireless transfer from the emitter-receiver point of view. On the other hand, the active power efficiency transfer from the wireless system input to the loads connected at its output, it’s taken into account. Starting from the correct formulation of S parameters, their generation is done automatically, based on modified nodal equations corresponding to the power transfer wireless systems. The magnetically coupled coils, used for wireless power transfer can be considered as a two-port circuit. To estimate the parameters of the magnetically coupled coils, advanced programs for electromagnetic field numerical computation is used: Cadence, ADS, Ansoft Extractor Q3D, COMSOL, Feko etc. In this paper there are also defined several parameters, function of S parameters, a series of constants which are further used to compute the stability factor. To generate: the reflection coefficient, the stationary waves ratio (SWR), the input and output impedances, the input and output active power transmission efficiencies, the power gains, various stability coefficients, the centres and the radius of the stability circles, the S, \(\frac{\mathbf{L}}{\mathbf{Z}}\), \(\mathbf{Z}\) and \(\mathbf{Y}\) matrices and the Smith diagrams, dedicated functions have been designed and implemented and the Microwave toolbox from MATLAB have been used. By comparing the simulated results with the experimental ones and with those presented in literature, the obtained error is acceptable.

1. Introduction

The characterization of linear or non-linear circuits which are functioning at low signals, in harmonic state, at small and medium frequency can be done by using the following parameters: Z, Y, H, T etc. When these circuits are operating at high and very high frequencies, these parameters can no longer be used, because they imply circuits with certain branches to be open or short-circuited such that the measurements to take place and such that the currents and the voltages from the circuit to be determined. At lower frequencies, the transfer and impedance matrices are commonly used, but at microwave frequencies they become difficult to measure and therefore, the scattering matrix description is preferred. The S parameters can be measured by embedding the two-port network (the Device Under Test (DUT)) in a transmission line whose ends are connected to a network analyser. A
typical network analyser can measure S-parameters over a large frequency range, for example, the HP 8720D vector network analyser (VNA) covers the range from 50 MHz to 40 GHz. Frequency resolution is typically 1 Hz and the results can be displayed either on a Smith chart or as a conventional gain versus frequency graph, [1-5].

The scattering (repartition) parameters denoted by $\mathbf{S}$ are complex quantities, frequency dependant, associated to a two-port linear system, in harmonic state. At the beginning, $\mathbf{S}$ parameters have been used in electrical line long theory. To define them, the incident (direct) voltage waves, the reflected (inverse) or transmitted ones have been used. In general, $\mathbf{S}$ parameters can be defined in transmission information systems, such as the microwave systems (waveguides), where these parameters can be studied using the electric circuit theory [4-10]. Because there are a lot of possibilities to introduce these parameters [1, 10, 11], their physical interpretation and understanding can become difficult.

$\mathbf{S}$ parameters do not have a direct correspondent in the electric circuit theory, because they do not contain circuit elements in which the propagation exists as in the waveguides. Anyway, in the circuit theory reasoning similar to that for microwave circuits has been extended, by introducing the concept of power waves, terminology coming from their significance that is connected to the dependence between the active power absorbed by a load connected to a port and its working frequency [3, 6]. Knowledge and understanding of $\mathbf{S}$ parameters are especially important for those applications that operate at high frequencies and include active and passive components from the integrated circuits [2], including microelectromechanical systems (MEMS), as well as the wireless power transfer systems (WPTS) [12].

For automatic generation of $\mathbf{S}$ parameters, for different configurations and structures of the two-port circuits and of the wireless power transfer systems, one can use either the modified nodal equations generated by the Circuit Symbolic Analysis Program (CSAP), [1, 13, 14], or state equations generated by Symbolic State Equation Generation Program (SYSEG), [13, 14]. Identifying the parameters of the analogue two-port circuits and the magnetically coupled coils used for wireless power transfer, for the frequencies between 30 kHz and 40 MHz, the most advanced and efficient electromagnetic field numerical computation programs are used: Cadence [15], ADS [16], ANSYS-Ansoft Extractor Q3D [17], Feko [2, 18] etc. In general, the two-port linear analogue circuits in harmonic state and the magnetically coupled coils can be described by a number of equivalent circuit parameters, such as: $\mathbf{T}$-the transfer coefficients matrix, $\mathbf{Z}$-the impedance matrix, $\mathbf{Y}$-the admittances matrix and the matrix of $\mathbf{S}$ parameters. Literature [4, 13, 14] presents procedures that allows the transition from one matrix to another.

This paper, based on electrical circuit theory, presents: starting from the correct formulation of $\mathbf{S}$ parameters [1], the automatic generation of matrix $\mathbf{S}$ and of the scattering parameters and then, based on this matrix, the computation of the following matrices: $\mathbf{Z}$, $\mathbf{Y}$ and $\mathbf{T}$. Furthermore, for a passive, linear, two-port circuit, in harmonic state, there are being determined: the input and output complex impedances ($Z_{in}$ and $Z_{out}$), the reflection coefficients (from the generator $L_G$, from the load $L_L$, from the input $L_{in}$ and from the output $L_{out}$), the voltage transfer function. The relationships between these quantities and $\mathbf{S}$ parameters are deduced. It is presented the practical use of these parameters to increase the efficiency of information transmission and propagation and of electromagnetic energy wireless transfer from the emitter-receiver point of view. Also, the active power efficiency transfer from the wireless system input to the loads connected at its output, it’s taken into account. Based on the maximum active power transfer theorem, it is computed the maximum active power transmitted to by a two-port system to the load and, function of $\mathbf{S}$ parameters, the most used in practice performance parameters are determined: the transducer power gain ($G_t$) the available power gain ($G_a$), the power gain ($G_p$, also called the operating gain), the maximum available power gain and the maximum stable power gain [5-9].

To generate: the reflection coefficient, the stationary waves ratio (SWR), the input and output impedances, the input and output active power transmission efficiencies, the power gains, various stability coefficients, the centres and the radius of the stability circles, the $\mathbf{S}$, $\mathbf{T}$, $\mathbf{Z}$ and $\mathbf{Y}$ matrices and
the Smith diagrams, dedicated functions have been designed and implemented and the Microwave toolbox from MATLAB have been uses. The results obtained by simulation were compared to those reported in the literature and with the experimental ones.

2. Wireless power transfer systems stability

For any passive linear two-port network (Figure 1), the scattering parameters can be used to study the stability conditions of electrical systems whose functioning in harmonic state can be described with a passive linear two-port system, such as, for example WPTS [5-9, 19-23].

![Figure 1. Passive linear two-port network connected to generator and to the load.](image)

In the discussions concerning the stability conditions of a two-port network, function of S parameters, the following definitions and constants are used [10]:

\[
\Delta = \det(S) = S_{11}S_{22} - S_{12}S_{21} ; \quad K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} - \text{Rollett stability factor;}
\]

\[
\mu_1 = \frac{1 - |S_{11}|^2}{\left| S_{22} - \Delta S_{11} + S_{12}S_{21} \right|} \Rightarrow \text{Edwards - Sinsky stability parameters} ;
\]

\[
\mu_2 = \frac{1 - |S_{22}|^2}{\left| S_{11} - \Delta S_{22} + S_{12}S_{21} \right|} = B_1 = 1 + |S_{11}|^2 - |\Delta|^2 ; \quad B_2 = 1 + |S_{22}|^2 - |\Delta|^2 ;
\]

\[
C_1 = S_{11} - \Delta S_{22} ; \quad D_1 = |S_{11}|^2 - |\Delta|^2 ; \quad C_2 = S_{22} - \Delta S_{11} ; \quad D_2 = |S_{22}|^2 - |\Delta|^2 ;
\]

Between the constants from (1), the following identities can be simply deduced:

\[
B_1^2 - 4|C_1|^2 - B_2^2 - 4|C_2|^2 = 4|S_{12}S_{21}|^2(K^2 - 1)
\]

\[
|C_1|^2 = |S_{12}S_{21}|^2 + \left(1 - |S_{22}|^2\right)D_2
\]

\[
|C_2|^2 = |S_{12}S_{21}|^2 + \left(1 - |S_{11}|^2\right)D_1
\]

The reflection coefficient from the generator \( \Gamma_G \) has the expression:

\[
\Gamma_G = \frac{\Gamma_G}{\Gamma_G} = \frac{Z_G - Z_0}{Z_G + Z_0} \Rightarrow Z_G = Z_0 \frac{1 + \Gamma_G}{1 - \Gamma_G} ,
\]

where \( Z_0 \) is the real-valued positive reference impedance. In practice, the reference impedance is chosen to be \( Z_0 = 50 \, \text{ohm} \).
The reflection coefficient from the load can be defined as:

\[
\Gamma_L = \frac{\beta_2}{\alpha_2} |e_0| = \frac{Z_L - Z_0}{Z_L + Z_0} \Rightarrow Z_L = Z_0 \frac{1 + \Gamma_L}{1 - \Gamma_L},
\]

(4)

The centres and the radius of the stability circles of the source (generator) and of the load, have the following expressions:

\[
\xi_G = \frac{c_G^2}{D_1}; \quad r_G = \left| \frac{S_{12} S_{21}}{D_1} \right|
\]

for the source stability circle,

\[
\xi_L = \frac{c_L^2}{D_2}; \quad r_L = \left| \frac{S_{12} S_{21}}{D_2} \right|
\]

(5)

(6)

As a consequence of the last two equations (2) and of the definition relationships (5) and (6), the quantities \(c_G, r_G, c_L, \) and \(r_L\) satisfy the following relations:

\[
1 - |S_{11}|^2 = \left( \xi_L^2 - r_L^2 \right) D_2; \quad 1 - |S_{22}|^2 = \left( \xi_G^2 - r_G^2 \right) D_1
\]

(7)

By using the equations given in (7), the stability parameters \(\mu_1\) and \(\mu_2\) can be computed with:

\[
\mu_1 = \left( \xi_L - r_L \right) \text{sign}(D_2); \quad \mu_2 = \left( \xi_G - r_G \right) \text{sign}(D_1)
\]

(8)

For example, the stability coefficient \(\mu_1\) has the following expression:

\[
\mu_1 = \frac{1 - |S_{11}|^2}{\xi_L^2 - r_L^2} = \frac{D_2 \left( \xi_L^2 - r_L^2 \right)}{D_2 \left( \xi_L^2 + r_L^2 \right)} = \frac{D_2}{|D_2|} \left( \xi_L - r_L \right)
\]

(9)

One can easily prove that, to compute the input and output reflection coefficients, one can use the also the following formulas:

\[
\Gamma_{in} = S_{11} + \frac{S_{12} S_{21}}{1 - S_{22}^2} \xi_L - \frac{\Delta \Gamma_L}{1 - S_{22}^2} \xi_L;
\]

\[
\Gamma_{out} = S_{22} + \frac{S_{12} S_{21}}{1 - S_{11}^2} \xi_G - \frac{\Delta \Gamma_G}{1 - S_{11}^2} \xi_G
\]

(10)

A two-port network is unconditionally stable, if any positive impedance of the generator and of the load with resistive (real) parts \(R_G\) and \(R_L\) always determine positive input and output impedances with resistive (real) parts \(R_{in}\) and \(R_{out}\).

On the other hand, the unconditional stability requires that for any generator and for any load with \(|\xi_G| < 1\) and \(|\xi_L| < 1\), it results that \(|\xi_{in}| < 1\) and \(|\xi_{out}| < 1\). A two-port network is potentially or conditionally unstable if for \(|\xi_L| < 1\) and \(|\xi_G| < 1\), it results that \(|\xi_{in}| \geq 1\) and/or \(|\xi_{out}| \geq 1\) [15-28].
For the unconditionally stable case, the stability regions of the generator (source) and of the load contain the interiors of the unit circles with $|L_G| < 1$ or $|L_L| < 1$. Anyway, for potential instability, only the portions of the unit circles can maintain the regions stable, so $E_G$, $E_L$ will give stable input and output impedances.

From equations (1)-(10) one can easily prove, the following connexions between the stability regions and the stability circles [5-8]:

\[
1 - |E_{in}|^2 = \frac{E_L - \xi L^2 - r_L^2}{1 - \xi L^2} D_2.
\]

\[
1 - |E_{out}|^2 = \frac{E_G - \xi G^2 - r_G^2}{1 - \xi G^2} D_1.
\]

From (11) it results that the stability region is defined according to the conditions:

\[
1 - |E_{in}|^2 > 0 \iff \left(|E_L - \xi L|^2 - r_L^2\right) D_2 > 0
\]

Function of the sign of $D_2$, the conditions from (12) is equivalent to the interior or to the exterior of the load stability circle, having the centre $c_L$ and radius $r_L$:

\[
|E_L - \xi L| > r_L, \text{ if } D_2 > 0; \quad |E_L - \xi L| < r_L, \text{ if } D_2 < 0
\]

called the load stability region.

The boundary $|E_L - \xi L| = r_L$ corresponds to $|E_{in}| = 1$. The set complementary to this region corresponds to the instability region with $|E_{in}| > 1$. In a similar manner, the source stability region can be defined as follows:

\[
|E_G - \xi G| > r_G, \text{ daca } D_1 > 0; \quad |E_G - \xi G| < r_G, \text{ daca } D_1 < 0
\]

called the source (generator) stability region.

In order to have unconditional stability, the stability regions must contain the unit circle in its entirety. If $D_2 > 0$, then the unit circle and the stability circle should not overlap at all, as presented in figure 2. Geometrically, the distance between the points A and O (figure 2) is $(OA) = |E_L - r_L|$. The non-overlapping of these circles implies that $(OA) > 1$, or equivalently, $|E_L| - r_L > 1$ [5-8].

![Figure 2. Load stability regions in the unconditionally stable case [10]](image)
The stability conditions (14) can be combined in $\text{sign}(D_2) \left| \frac{\varepsilon_L}{r_L} \right| > 1$, or equivalently $\mu_1 > 1$, in accordance with equation (8). Geometrically, the parameter $\mu_1$ represents the distance (OA). So, the condition for the input unconditional stability is:

$$\mu_1 > 1 - \text{unconditional stability condition} \quad (15)$$

Edwards and Sinsky showed that the only condition necessary and sufficient for the unconditional stability of a two-port network is that for which input and output impedance is $\mu_1 > 1$ (or, alternatively, $\mu_2 > 1$) [7, 14]. Obviously, the source stability regions are similar to those presented in figure 2.

If the stability condition is not fulfilled, i.e. $\mu_1 < 1$, then the portion of the unit circle that belongs to the stability circle will be stable and will lead to stable input and output impedances. Figure 3 depicts such a potential instability case. If $D_2 > 1$, then $\mu_1 < 1$ which is equivalent to $\left| \frac{\varepsilon_L}{r_L} \right| > 1$ and if $D_2 < 0$, then $r_L \cdot \frac{\varepsilon_L}{r_L} < 1$. Anyway, the unit circle is partially overlaps the stability circle, as showed in figure 3. The portion from the unit circle that is not inside the stability region corresponds to an unstable Zin. There are several unconditional stability criteria which are equivalent to the only criterion $\mu_1 > 1$. All these criteria require that Rollett stability factor $K$ to be greater the one (i.e. $K > 1$), as well as one other condition. Any of the following criteria are necessary and sufficient for the unconditional stability, [7, 30]:

$$K > 1 \text{ and } |A| < 1; K > 1 \text{ and } B_1 > 0; K > 1 \text{ and } B_2 > 0;$$

$$K > 1 \text{ and } |S_{12}S_{21}| < 1 - |S_{12}|^2 \quad K > 1 \text{ and } |S_{12}S_{21}| < 1 - |S_{22}|^2. \quad (16)$$

All criteria from (16) are equivalent to $\mu_1 > 1$. In particular, from (16) results that the unconditional stability requires that $|S_{11}| < 1$ and $|S_{22}| < 1$. These conditions are necessary but not sufficient for stability. A common situation in practice is to have a potentially instable two-port, but with $|S_{11}| < 1$ and $|S_{22}| < 1$. In these cases, equation (6) implies $D_2 \left( \frac{\varepsilon_L^2 - r_L^2}{r_L} \right) > 0$, but the lack of stability requires $\mu_1 = \text{sign}(D_2) \left( \frac{\varepsilon_L^2 - r_L^2}{r_L} \right) < 1$. As a consequence, if $D_2 > 0$, then we must have $\left| \frac{\varepsilon_L}{r_L} \right| > 1$ and $r_L \cdot \frac{\varepsilon_L}{r_L} < 1$, that determines the inequality $\left| \frac{\varepsilon_L}{r_L} \right| < r_L < \left| \frac{\varepsilon_L}{r_L} \right| + 1$. The geometric distance (OA) = $\left| \frac{\varepsilon_L}{r_L} \right| \cdot r_L$ satisfies the inequality $0 < (OA) < 1$, such that the stability circle overlaps the unit circle but it does not contain its centre. On the other hand, if $D_2 < 0$, the two conditions imply that $\left| \frac{\varepsilon_L}{r_L} \right| > 1$ and $r_L \cdot \frac{\varepsilon_L}{r_L} < 1$, which results in $\left| \frac{\varepsilon_L}{r_L} \right| < r_L < \left| \frac{\varepsilon_L}{r_L} \right| + 1$. This situation is presented in the right part of figure 3 [7, 28-31].

![Figure 3. Load stability regions in potentially unstable case [7, 10]](image-url)
The geometric distance \((AO) = \eta \cdot |L_c|\) satisfies again the inequality \(0 < (OA) < 1\), but now the unit circle center is inside the stability circle, which is also the stability region.

3. Examples

We consider the circuit represented in figure 5. In this case, the coupling circuit is a two-port passive network. Voltage controlled oscillators are important components in almost every digital communication system. In recent years, the microwave oscillators are used to control the phase in microwave antenna arrays as an alternative to electronic beam steering methods. A voltage controlled oscillator (VCO) is a circuit that produces an oscillatory output [1-8, 11-16]. The frequency of the output signal depends on the level of an input voltage signal supplied to the VCO.

The active parts of the two Van der Pol oscillators are modelled by two voltage-controlled nonlinear resistors (see figure 4) [3, 6-8, 14].

Figure 4. Two parallel resonant circuits coupled through a two-port passive network

The nonlinear characteristics of the two voltage-controlled nonlinear resistors are expressed as it follows:

\[
\begin{align*}
    i_{net1} &= -a v_1 + b v_1^3 \\
    i_{net2} &= -a v_2 + b v_2^3,
\end{align*}
\]

where \((-a)\) is the negative conductance necessary to start the oscillation and \(b\) a parameter used to model the saturation phenomenon.

Therefore, in the sinusoidal behaviour (neglecting the third harmonic) the two nonlinear resistors can be modelled by two linear resistors with the conductances:

\[
\begin{align*}
    G_{01} &= -a + \frac{3b}{4} A_1^2; \\
    G_{02} &= -a + \frac{3b}{4} A_2^2
\end{align*}
\]

where \(A_1\) and \(A_2\) are magnitudes of voltages \(v_1\) and \(v_2\).

To compute the parameters \(S11\) and \(S21\) (\(S12\) and \(S22\)) it is used the circuit from the figure 6.a and 6.b where the two voltage controlled nonlinear resistors have been replaced with linear resistors having the values \(R_{dc1} = 150\ \Omega\), \(R_{dc2} = 280\ \Omega\), and the input and load resistors have the values \(R_i = R_L = R_0 = Z_0 = 50\ \Omega\).

Performing Spice simulation (or MATLAB simulation), for the numeric value parameters: \(C1 = 10\ \text{pF}, \ C2 = 20\ \text{pF}, \ L1 = 2.8095\ \text{nH}, \ L2 = 1.2678\ \text{nH}, \ R1 = 3500\ \Omega, \ R2 = 3500\ \Omega, \ R_c = 200\ \Omega, \ C_{1c} = C_{2c} = 6.5\ \text{pF}, \ C_{cc} = 6.5\ \text{pF}, \ L_{c1} = L_{c2} = L_c = 0.6\ \text{nH}, \ M = 0.21\ \text{nH}, \ R_7 = 250\ \Omega, \ R_L1 = R_L2 = 1.352\ \Omega, \ g = 0.0002\ \text{S}, \) \(\text{values}\ R_i = R_L = R_0 = Z_0 = 50\ \Omega\) a = 0.0085 S, and \(b = 0.0071\ \text{S/V^2}\), we get the magnitudes values \(A_1 = 1.8987\ \text{V}, \ A_2 = 2.9585\ \text{V}, \) and the synchronization frequency \(f_s = 1.1512\ \text{GHz}, \) for the initial conditions \(v_1(0) = 2.0\ \text{V}\) and \(v_2(0) = 1.0\ \text{V}\).

Thus, the circuit in figure 4 can be analysed by the complex representation method [14].
Running the routines implemented in MATLAB, for the above-mentioned example, there have been obtained results as follows.

For frequency \( f = 2.2 \) GHz, the parameters and constants mention in section 2 have been computed. The matrix of parameters \( S_{n_1} \) is:

\[
\begin{bmatrix}
-0.96678 & -0.21332j \\
-0.96678 & -0.21332j \\
-0.96678 & -0.21332j \\
-0.96678 & -0.21332j \\
\end{bmatrix}
\]

Figure 5 depicts the variations function of frequency of \( \eta_{21}^*b \), \( \eta_{21}^*r \), \( \eta_{12}^*k \) and \( \eta_{12}^*m \), and figure 7 presents the variations function of frequency of the transducer power gain \( G_t \), the available power gain \( G_a \), the power gain \( G_p \), the maximum available gain \( G_{MAG} \) and the maximum stable gain \( G_{MSG} \).

Frequency variations of the S-parameters modules are shown in figure 8. Because the two coupled oscillators are identical, the variations with the frequency of the S12 and S21 parameters modules are identical. Due to the symmetry of the two oscillators, the variation with the frequency of the yield \( \eta_{21,S_{n_1}} \) is identical to the one of the yield \( \eta_{12,S_{n_1}} \).

Frequency variation curves of the source standing wave ratio \( SWR_G \) and of the load standing wave ratio \( SWR_L \) have been shown in figure 9.

In figure 10 and 11 are presented the variations function of frequency for the stability coefficient \( k_{stab} = |L_G - \xi_G| - r_G \) and the constant D1 (\( k_{stab} = |L_L - \xi_L| - r_L \) and the constant D2). From figure 10 (figure 11) we notice that the analysed system, consisting of two magnetically coupled coils, is stable for the entire frequency interval, because the following inequalities are satisfied:

\[
|L_G - \xi_G| > r_G, \text{ if } D_1 > 0; |L_G - \xi_G| < r_G, \text{ if } D_1 < 0; (|L_L - \xi_L| > r_L, \text{ if } D_2 > 0; |L_L - \xi_L| < r_L, \text{ if } D_2 < 0 ).
\]

Frequency variation curves of the source and load reflection coefficients modules (\( \Gamma_S \) and \( \Gamma_L \)) have been shown in figure 12 and the ones of the input and output reflection coefficients modules (\( \Gamma_{in} \) and \( \Gamma_{out} \)) have been presented in figure 13.

The stability factor K1 and constant B1 versus frequency are shown in figure 14 and in figure 15 is presented the frequency variations of the stability factor K1 and constant B2.
Figure 6. $\eta_{21}-b$, $\eta_{21-521}$, $\eta_{12-2}$, and $\eta_{12-92}$ efficiencies vs frequency.

Figure 7. $G_{r-b}$, $G_{a-r}$, $G_{p-k}$, $G_{mag-c}$, and $G_{mag-g}$ parameters vs frequency.

Figure 8. Parameter S magnitudes vs frequency.

Figure 9. SWR-G-b and SWR-L-k parameters vs frequency.

Figure 10. $cG-rG$-stab-b and D1-r factors vs frequency.

Figure 11. $cL-rL$-stab-b and D2-r factors vs frequency.

Figure 12. $\Gamma_{G-b}$ and $\Gamma_{L-b}$-r magnitudes vs frequency.

Figure 13. $\Gamma_{in-b}$ and $\Gamma_{out-b}$-r magnitudes vs frequency.
If $K_{1-b} > 1$ and $B_{1-r} > 0$ - System is stable.

**Frequency $f$ [Hz]**

$K_{1-b}$ and $B_{1-r}$ [%/%]

| $f$ (x $10^9$) | $K_{1-b}$ | $B_{1-r}$ |
|---------------|----------|----------|
| 4.15          |          |          |
| 4.2           |          |          |
| 4.25          |          |          |
| 4.3           |          |          |
| 4.35          |          |          |
| 4.4           |          |          |
| 4.45          |          |          |

**Figure 14.** $K_{1-b}$ and $B_{1-r}$ constants vs frequency

**Figure 15.** $K_{1-b}$ and $B_{2-r}$ constants vs frequency

**Figure 16.** Smith Chart for $S_{11}$, stability source circles and stability load circles for two frequencies

**Figure 17.** Smith Chart for $S_{21}$, stability source circles and stability load circles for two frequencies

Figure 14 and 15, show the variations with the frequency of parameter $S_{11}$, stability source circles and stability load circles for two frequencies (of parameter $S_{21}$, stability source circles and stability load circles for two frequencies) on the Smith chart. The studied system consisting of two coupled oscillators is being symmetric, so the Smith charts of $S_{12}$ and $S_{21}$ are identical. The variations with the frequency of parameter and stability source circles for four frequencies (of parameter and stability source circles for four frequencies) on the Smith chart are shown in figure 16.

### 4. Conclusions

This paper presents the use of $S$ parameters for analysing the stability of two-port analogue networks, with important applications for the WPTS. The analysis is done by starting from the correct formulation of these parameters, based on electrical circuit theory, and then generating automatically the matrix corresponding to the scattering parameters. Based on this matrix $S$, the following matrices are further computed: $Z$, $Y$ and $T$. By considering the two magnetically coupled coils (the resonators)-used in WPTS-as a passive, linear two-port network, in harmonic state, we determined the following: the input and output impedances; the reflections coefficients; the voltage transfer function; the stability coefficients ($K$-Rollet stability factor, $\mu_1$ and $\mu_2$-Edwards-Sinsky stability factors), the centres and the radius of the load, source stability circles, various constants necessary to define the stability factors. It is also presented the practical use of $S$ parameters to increase the efficiency of information transmission and propagation and of electromagnetic energy wireless transfer from the emitter-receiver point of view. Based on the maximum power transfer theorem, the maximum active power transmitted by a SWTP to the load is computed and, function of $S$ parameters, the most used in practice performance coefficients are calculated: $G_t$, $G_a$, $G_p$, etc. There were established the necessary and sufficient conditions that two magnetically coupled coils (equivalent to a passive linear two-port
network) to be unconditionally stable. It has been designed and implemented a procedure to generate, on the Smith diagram, the stability circles corresponding to the source and to the load.

Finally, were elaborated and implemented the computation procedures for: the reflection coefficients, for stationary wave ratios, for input and output impedances, for efficiency, for various stability coefficients, for centres and radius corresponding to the load and source stability circles, for the S, T, Z and Y matrices and for the Smith diagrams. For these procedures, were elaborated dedicated functions implemented in MATLAB. By comparing the simulated results with the experimental ones and with those presented in literature. The accuracy of the plots, obtained based on simulation is confirmed by the results obtained using the dedicated software ADS, which has specific routines to generate S parameters and the other quantities necessary to determine the stability regions for the WPTS.

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