currents of the transformer, but the short-circuit currents as well [3]. Input surge currents also arise when testing power transformers in the mode of experimental idling (EI) [3]. In contrast to the operating modes, when tested the AC power source is connected to the low voltage windings (LV), which are typically connected by the active part taps according to scheme D [1]. The level of magnetizing current surges must be taken into consideration when testing and operating power transformers. The inrush currents of the unloaded transformer can pose significant danger for the measuring and power equipment, to introduce distortion to the voltage of power grid, lead to the triggering of protection systems [3]. Existing means to limit the magnetization currents surges are very costly and low efficient [3]. Therefore, analysis of
the inrush currents in an unloaded transformer, as well as the measures to limit them, should be considered at the stage of design preproduction. The currently applied procedures for modeling the transient processes and calculating the input surge currents when connecting an unloaded power transformer to the grid [4] do not match the conditions for performing standard and control tests of power transformers [1]. That predetermines the scientific and practical relevance of research into transient processes related to switching power transformers in the mode of experimental idling.

2. Literature review and problem statement

Substantiating the design solutions requires a significant volume of research into electromagnetic processes under the EI transient modes. The greatest influence on the transient processes for starting the unloaded transformers is exerted by the initial voltage phase, residual magnetic flux, the input impedance of the source, and design of the active part [5]. Many procedures for calculating the transitional EI regimes of power transformers do not account for these factors in their entirety. This is characteristic of the studies with analytical solutions for magnetic fluxes and inrush currents of the unloaded transformer [6, 7], for the harmonic spectrum of inrush current when the voltage signal is distorted [8]. Authors of [9, 10] propose an analytical description of the starting current surges of a power transformer using the theory of Giles-Atherton for hysteresis properties of electrical steel. The advantage of analytical approaches is the commonality of their results, while their downside is the reduced accuracy of calculations due to simplifications and assumptions in mathematical statements.

Widely used now is the circuit modeling that employs combined magnetolectric equivalent circuits (MEEC) and the software NAP [11], Simulink [12–14], the software for Spice-like systems [15, 16]. Papers [17–19] proposed models for the calculation of starting currents surges for a single-phase transformer, implemented in the Matlab package. A model from [17] accounts for the effect of changing the switching angle, power supply impedance, and the residual flux, on the characteristics of starting current of a single-phase transformer. Authors in [19] suggest considering a three-phase transformer as three single-phase transformers when studying the surges of starting currents. Given the interruptions and a low convergence of computing processes in the Matlab programming environment, it is recommended to employ the alternative software EMTP/ATP [20]. Papers [20, 21] present models of transformers for low- and middle-frequency transients, implemented in EMTP. Authors [20, 21] apply a simplified description of the design of the magnetic system and windings, which reduces accuracy of the results. According to authors of [22], the software EMTP/ATP primarily presents the influence of external electric circuits, while the patterns of electromagnetic processes in a multicomponent spatial structure of the active part of the transformer are accounted for in the fragmented form. That predetermines a substantial growth of the calculation error for circuit models of large dimensionality [22]. The specified disadvantages are inherent to almost all circuit models. Therefore, the accuracy of circuit models for qualitative and quantitative assessment of transient processes in the active part of a power transformer does not match modern design requirements.

Field simulation makes it possible to take into consideration the design features of the active part of the power transformer, the non-linearity in the properties of active materials and their effect on the magnetization currents [23]. Field models are widely used in the analysis of electromagnetic processes in AC conversion systems [24, 25] and electric motors [26, 27]. The numerical realization of dynamic field models in multicomponent regions with non-linear electrical-physical and magnetic properties gives rise to constraints, related to the capacity of hardware resources, time costs, and stability of computational processes [23]. Therefore, to estimate the input surge currents of an unloaded transformer, authors of [23] implement in the COMSOL Multiphysics package spatial models of magnetic field in terms of direct current. This leads to an increase in the error because when calculating inrush EI currents the circuit model and the field model are treated independently. Field models do not make it possible to account for the influence of the input impedance of the network and different winding connection diagrams.

Reliable determining the dynamics of change in magnetization currents in an unloaded power transformer requires taking into consideration all the factors that influence each other. This is possible with the joint implementation of circuit and field models. Given the high requirements to computing resources and time cost, dynamic circuit-field modeling has not been commonly applied. There is therefore a need to develop new approaches for implementing the dynamic conjugated field and circuit models [28], which would ensure high accuracy and computational efficiency in the design of power transformers.

3. The aim and objectives of the study

The aim of this study is to develop a dynamical circuit-field model of the unloaded transformer, which would account for the features of design of the active part, the non-linearity of electro-physical properties of active materials, and which would ensure high accuracy of calculation of input surge currents.

To accomplish the aim, the following tasks have been set:
- to develop a 3D geometrical model of the estimated region, reflecting the features of design of a multicomponent active part of the transformer and to adapt the circuit-field model of electromagnetic energy conversion to the EI mode;
- to develop a model of transitional processes when the transformer is switched in the mode of experimental idling, which would provide high accuracy and computational efficiency when processing data on the circuit-field modelling;
- to devise a high precision engineering procedure for the calculation of input surge currents of the unloaded transformer for the specifications of the transformer and testing equipment.

4. Materials and methods to study the starting currents

Dynamical electromagnetic starting processes when testing the transformer in the mode of experimental idling
are investigated in a 3D region of the active part of a power transformer, which is shown in Fig. 1, a. To create the estimated region, we conjugate geometrical models of the system of primary electrical insulation with a transformer oil – 1, of magnetic system – 2 with charged limbs and yokes made of electrical steel, their joints, winding systems – 3. A cellulose magnetic insulation is combined into a single sub-region with the transformer oil in tank – 4 (Fig. 1). The estimated region has the properties of symmetry with respect to the vertical plane passing through the axis of the limbs and the horizontal plane in the middle of the magnetic system window. When corresponding boundary conditions are set along the plane of symmetry, that makes it possible to confine ourselves to the analysis of only one symmetrical sub-region (Fig. 1, b). This reduces the time required for circuit-field modelling by 3.8 times, as well as respective requirements to computing resources. At modeling, we consider a power three-phase three-limb transformer with a capacity of 1,600 kVA, voltage class 35 kV class, circuit, and a winding connection group Y/D-11.

![Fig. 1. Power transformer: a — estimated region; b — modeled sub-region](image)

We accept, for the active part of the power transformer, a stationary temperature mode with a uniform distribution of temperatures. Their values correspond to the class of electrical insulation heat resistance and acceptable overtemperatures for a continuous operation.

The density of displacement currents is much smaller than the density of conductivity currents and can be disregarded. Therefore, the transient electromagnetic processes in the power transformer’s active part are described by the conjugated system of nonlinear equations in partial derivatives [29]:

\[
\begin{align*}
\operatorname{rot}[\mathbf{H}]_{ij} &= \left\{ \delta_{ij} \mathbf{E} \right\}_{ij} - \sigma(\mathbf{H})_{ij} \mathbf{E}_{ij}^*; \\
\operatorname{rot}[\mathbf{E}]_{ij} &= -\delta \left\{ \mathbf{B} \right\}_{ij}/\partial t,
\end{align*}
\]

with respect to the potentiality conditions for magnetic and electric fields [30]:

\[
\begin{align*}
\{ \mathbf{A} \}_{ij} &= \{ \mathbf{B} \}_{ij}; \\
\{ \mathbf{B} \}_{ij} &= \operatorname{grad}[\mathbf{V}]_{ij} - \{ \mathbf{E} \}_{ij},
\end{align*}
\]

where \( \mathbf{A} \) is the vector magnetic potential; \( \mathbf{V} \) is the electric potential; \( \sigma(\mathbf{H}) \) is the specific electrical conductivity; \( \mathbf{B} \) is the induction of the magnetic field; \( \mathbf{H}, \mathbf{E} \) are the intensities of the magnetic and electric fields; \( \delta \) is the permissible temperature for the accepted class of electrical insulation heat resistance [4]; \( \delta \) is current density; index \( i \) corresponds to sub-regions in the estimated region of the active part (Fig. 1), index \( j = 1, 2, 3 \) corresponds to phases A, B, C of the power transformer.

We assign to equations (1) and (2) conditions for the conjugation of media with different magnetic and electrical properties [30] and conditions at external borders \( \Omega \):

\[
\begin{align*}
\{ \mathbf{A} \}_{ij} &= 0 \quad (\mathbf{H} = \mathbf{H}_0); \\
\mathbf{V}(\mathbf{r}) &= \sqrt{2U_{ph}} \mathbf{r} \mathbf{E} \sin[\mathbf{r} + \mathbf{E} + (j-1)2\pi/3]
\end{align*}
\]

where \( U_{ph} \) is the rated phase voltage, \( \mathbf{E} + (j-1)2\pi/3 \) are the initial phases for connecting the transformer to the grid.

To account for the anisotropy of magnetic properties of cold-rolled electrical steels, we used the approach described in [30].

The transformation of electric field equations (1) with conditions (2), (3) is achieved using the method for decomposing the estimated region into phase sub-regions [24, 25], which allows us to proceed to the system of ordinary differential equations [31]:

\[
\begin{align*}
\{ u(\mathbf{r}) \}_{ij} - i(\mathbf{r})_{ij}, \quad r_{ij} &= d\Psi(\mathbf{r}))/d\mathbf{r} \mid_{\Sigma(\Sigma)} ,
\end{align*}
\]

with flux linkages, which are described by integral relations [31]:

\[
\begin{align*}
\Psi(\mathbf{r}) = \frac{1}{i(\mathbf{r})} \left\{ \Phi(\mathbf{r}) \right\}_{ij} d\mu(\Sigma), \quad d\mu = \delta \cdot ds,
\end{align*}
\]

where

\[
\Phi(\mathbf{r}) = \int_{\Sigma(\Sigma)} \mathbf{B}(\mathbf{r}) \cdot d\mathbf{n}
\]

are the magnetic fluxes in \( j \) limbs with active section \( \Pi_j \), \( r_{ij} = r_{ij}^0 \) is the input resistance of phase of the transformer.

At initial time \( t = 0 \), we accept, for phase currents in the system of integrated-differential equations (7) and (8), the uniform Cauchy conditions with an initial starting phase \( \Phi_0 = \pi/2 \).

Numerical realization of the dynamical circuit-field model (1), (4) and (5) shall be carried out by the finite-element method in the COMSOL Multiphysics programming environment [29]. We assign for the phase windings of low voltage the values for an active component of EI resistance \( r_{ij} = r_{ij}^0 \), which is calculated based on the specifications for the transformer [1, 4]. Limitations for the power of the source of test voltage are accounted for using linear resistances \( z^0 = r^0 + jx^0 \) between the source and the transformer.

5. Results of studying the starting currents

Fig. 2 shows the distribution of the magnetic field induction module for the moment of input surge current of phase A. Magnetic flux is localized in limb A and the induction reaches values that are close to the electrical steel saturation induction (Fig. 2). Magnetic resistance sharply increases, and the inductive resistance for phase A decreases. In limbs B and C, the maximum values of induction do not exceed 1.1 T and 0.8 T, which corresponds to a linear section of the magnetization curve. Therefore, to analyze data on the circuit-field modeling, one can apply analytical solutions to the
linearized differential equations of the circuit model (4), (5), with aperiodic and harmonic components [32]. To normalize the phase currents, we use the existing value of the first harmonic of EI current (Fig. 2), which can be taken to be equal to the specification-based value of the EI current of the transformer [1].

\[
\begin{align*}
\text{Inrush currents in lines A and C are opposite in sign and reach maximum modulo values. At this point in time the instantaneous current value in line B is equal to zero (Fig. 3, b). The sum of the instantaneous values of linear currents is always equal to zero (Fig. 3, b). The multiplicity of the surge linear current is 28–29, and the multiplicity of the surge phase current approaches 25 (Fig. 3, a).} \\
\text{At the initial switching, the system of phase currents is characterized by a clearly pronounced asymmetry. The sum of the instantaneous values of phase currents varies in a wide range of 4.597 – 14.4. The larger values for \( \mu_{1} \text{ and } \mu_{2} \text{ of surge currents for a circuit component and the current discrepancies (7)} \text{ during initial switching can be performed with respect to conditions:} \\
\max_{\tau} |i(\tau)|_{c} = |i(\tau)|_{c_{0}} \leq \mu_{1} \text{ for the multiplicity of surge currents and \( \mu_{1} \text{ and } \mu_{2} \text{ of input surge currents correspond to low voltage phase windings. Taking into account the levelling current in the triangle circuit with low voltage windings, which reaches a maximum of 60–70 % of the input surge current, Active resistance of a given circuit combines active resistances of the low voltage phase windings. Therefore, the transition process quickly fades and its duration is in an interval of (0≤τ≤20 ms) (Fig. 3).}
\end{align*}
\]

During initial switching of the transformer, there are prevailing discrepancies between phase currents \( i(\tau) \), and their circuit components \( i(\tau)_{c_{j}} \), while the multiplicity of their maximum modulo values reaches 4.6; 2.5; 2.1 for phases A, B and C, respectively (Fig. 3, a, b).

To describe the mismatch between phase currents, one can apply series with the Gaussian basis functions [33]:

\[
\begin{align*}
\left| i(\tau)_{c_{j}} - i(\tau) \right|_{c_{j}} = \sum_{k} \frac{C_{k_{j}}}{\sigma_{k_{j}} \sqrt{2\pi}} \exp \left( -\frac{(\tau - \varsigma_{k_{j}})^{2}}{2\sigma_{k_{j}}^{2}} \right),
\end{align*}
\]

and transform expressions (6) to the following form:

\[
\begin{align*}
\left| i(\tau) \right|_{c_{j}} = \sum \left[ \alpha_{k_{j}} \exp \left( -\beta_{k_{j}}(\tau - \varsigma_{k_{j}})^{2} \right) \right] \left| i(\tau) \right|_{c_{j}}
\end{align*}
\]

where

\[
\begin{align*}
\alpha_{k_{j}} = C_{k_{j}} / \sigma_{k_{j}} \sqrt{2\pi}; \beta_{k_{j}} = 0.5 \sigma_{k_{j}}^{2}.
\end{align*}
\]

We can confine ourselves, for the first period of the transition process, to two basis Gaussian functions in series (7) and reduce the number of regression coefficients to six. This considerably simplifies the calculation and ensures high accuracy of approximation because its error does not exceed 1.8 %.

Thus, the input surge current of the unloaded transformer is influenced both by the circuit components of EI current and current discrepancies (7). Estimation of the surge values of circuit components and transient current discrepancies during initial switching can be performed with respect to conditions:

\[
\max_{\tau} |i(\tau)|_{c} = |i(\tau)|_{c_{0}} \leq \mu_{1} \text{ and } \mu_{2}
\]

where \( i(\tau)|_{c} \) and \( i(\tau)|_{c_{0}} \) are the periodic and aperiodic circuit components of EI current, calculated using the procedure from [29].

These conditions can be matched with an expression for the EI surge current:

\[
I_{s_{\text{idle}}} = I_{s_{\text{d}}}(1 + \max_{\tau} |i(\tau)|_{c_{0}}) / I_{s_{\text{d}}} + \max_{\tau} |i(\tau)|_{c_{0}} / I_{s_{\text{d}}}
\]

and then transform it to the following form:

\[
I_{s_{\text{idle}}} = \sqrt{2} I_{s_{\text{d}}} K_{s}; K_{s} = 1 + K_{s_{l}} + K_{s_{a}}
\]

where \( I_{s_{\text{d}}} \) is the specification value for EI current [4], \( K_{s_{l}} \) is the multiplicity of the input surge current, \( K_{s_{l}} \) and \( K_{s_{a}} \) are coefficients that take into consideration the multiplexities of surge currents for a circuit component and the current discrepancies (7), (8):

\[
K_{s_{l}} = \max_{\tau} |i(\tau)|_{c_{0}} / I_{s_{\text{d}}};
K_{s_{a}} = \max_{\tau} |i(\tau)|_{c_{0}} / I_{s_{\text{d}}}
\]

When one changes discretely the values of the multiplexities of active components of the linear input resistances:

\[
r = r_{i}/r_{i_{0}} = \{0.01; 0.10; 0.50; 1.00\},
\]

the multiplicity of input surge currents changes nonlinearly and in a wide range of \( 4.597 \leq K_{s_{l}} \leq 214.4 \). The larger values for the multiplicities of input surge currents correspond to lower values of active resistances of the test voltage source. To describe a functional dependence of the input surge current
multiplicity coefficient of an unloaded transformer on the input resistances, we apply a method for sorting out specialized functions from data [34]:

\[ K_i = \alpha x(r^i) \times \exp\left[ -\gamma \times \ln(r^i) \right]. \quad (9) \]

and to calculate coefficients \( \{\alpha, \beta, \gamma\} \) in relation (9), we apply the method of least squares [34].

Error of calculation of the input surge currents using relations (8) and (9) does not exceed 2.1 %, which corresponds to the requirements of design accuracy.

6. Discussion of results of modeling the inrush currents in an unloaded transformer

Circuit-field modeling makes it possible to comprehensively consider the impact of a multi-component design of the active part of the power transformer, the non-linearity of magnetic and electrical properties of materials, parameters of external electric circuits, on the dynamics of change in the magnetization currents. That ensures the reliability and high accuracy of the transient processes calculation when switching an unloaded transformer.

The transformation of the electric field equations from system (1) into the conjugated integral-differential equations (4) and (5), implemented using the method of decomposition, allows us to significantly reduce the number of dependent variables and ensures high computational efficiency.

Known conditions for the active \( z^i \) and inductive \( x^i \) components of EI resistance \( z^i \times r^i << x^i \) and \( r^i \times x^i \) and \( x^i \times z^i \) [4] hold for most series of power transformers. Therefore, the estimation of active resistances of windings based on the specifications does not lead to any noticeable increase in the error for the circuit-field modeling.

According to the simulation data, for a winding connection diagram D, inrush phase magnetization currents can reach the multiplicities of 4.6; 2.5; 2.1 when testing the unloaded transformers. Their maximum values are orders of magnitude less than the current surges from the side of high voltage (HV) but may also lead to dangerous consequences for measurement and control systems. To limit the current surges when testing the unloaded transformers, one can use specialized resistors in the HV side lines. When connecting from the HV side, one must apply more reliable and effective techniques, for example, controlled phase-wise switching using controllers and thyristors [35].

Multiplicity dependence of the magnetization surge currents on the magnitude of input active resistance is described with high accuracy by relations (8), (9). These relations could be used to select the input resistances when testing the unloaded transformers, in order to protect measuring systems. They make it possible, at the stage of preparing bids, to significantly reduce the time required for an analysis of alternative variants aimed to limit the magnetization current surges.

The calculation procedure (8), (9) was successfully verified for the most common series of power three-phase transformers, dimensions II and III. Its reliability and high accuracy (an error less than 2.1 %) was confirmed by data acquired during tests.

However, given the structural differences between standard implementation of magnetic systems, a given procedure excludes single-phase transformers, three-phase transformers with spatial magnetic systems, with armored limbs magnetic systems and side yokes [4].

Generalization of estimation procedures when using a field simulation of electromagnetic processes for the design of large-size power transformers is substantially complicated due to the diversity of their designs. In this case, one requires significant financial resources to provide for high-performance hardware and specialized software.

Increased cost of designing can be offset by accelerating the structural preparation of production of the new types of powerful and super-power transformers. However, the appropriate solutions require a preliminary analysis of their economic efficiency.

The approaches proposed in this paper could be further adapted for the purpose of research into the inrush current surges of powerful electric motors.

7. Conclusions

1. We have designed a circuit-field model of the unloaded three-phase transformer. We identify, in the estimated region of active part, applying the vertical and horizontal planes of symmetry, a 3D sub-region whose finite elements’ volume and number are reduced by 4 times. This shortens the time required for circuit-field modelling by 3.8 times, as well as reduces, accordingly, requirements for computational resources.

2. When switching an unloaded transformer, the magnetic field is localized in one of the limbs. This leads to an increase in the induction above 1.8–1.9 T, the local saturation of electrical steel, and the current surge in the phase winding.

3. We have proposed the approximation of inrush phase magnetization currents using the series of Gaussian temporal functions; its error does not exceed 1.8 %.

4. We have derived algebraic relations for the calculation of multiplicity coefficient for the input surge currents based on the specifications for a transformer and the testing equipment, for which current error does not exceed 2.1 %.

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