Constraining the Schwarzschild-de Sitter Solution in Models of Modified Gravity

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Abstract

The Schwarzschild-de Sitter (SdS) solution exists in the large majority of modified gravity theories, as expected, and in particular the effective cosmological constant is determined by the specific parameters of the given theory. We explore the possibility to use future extended radio-tracking data from the currently ongoing New Horizons mission in the outskirts peripheries of the Solar System, at about 40 au, in order to constrain this effective cosmological constant, and thus to impose constraints on each scenario’s parameters. We investigate some of the recently most studied modified gravities, namely $f(R)$ and $f(T)$ theories, dRGT massive gravity, and Hořava-Lifshitz gravity, and we show that New Horizons mission may bring an improvement of one-two orders of magnitude with respect to the present bounds from planetary orbital dynamics.

Keywords: Experimental studies of gravity, Modified gravity, Dark energy, Lunar, planetary, and deep-space probes

1. Introduction

General Relativity (GR) has undergone brilliant successes since its inception 100 years ago (see, e.g., the review [1] and references therein). Einstein’s theory

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is the standard paradigm for describing the gravitational interaction, verified by many experimental evidences [2], even though, with the possible exception of binary-pulsar systems, at least to a certain extent, these tests are probes of the weak-field gravity, or differently speaking they probe gravity up to intermediate scales (≃1 − 10^3 au). Nevertheless, one of the current challenges in theoretical physics and cosmology is the description of gravitation at large scales. In particular, evidences from astrophysics and cosmology [3–12] suggest that the Universe content is 76% dark energy, 20% dark matter, 4% ordinary baryonic matter. This implies that in order to reconcile the observations with GR we are led to assume that the Universe is dominated by dark entities, with peculiar characteristics. The dark energy is an exotic cosmic fluid, which has not yet been detected directly, and which does not cluster as ordinary matter; indeed, its behaviour closely resembles that of the cosmological constant Λ, which, in turn, brings about other problems, concerning its nature and origin [13,14]. On the other hand, the dark matter is an unknown type of matter, which has the clustering properties of ordinary matter; since 1933 it has been related to the problem of missing matter in astrophysical scenarios [15]. Moreover, some kind of cold and pressureless dark matter (whose distribution is that of a spherical halo around the galaxies) is also required to explain the rotation curves of spiral galaxies [16]. Hence, the best answer we have today for these cosmic puzzles is the so-called concordance model or ΛCDM, which provides the simplest description of the available data concerning the large-scale structure of the Universe. For a recent review, see e.g. [17]. This picture is completed with the inflationary scenario which solves the horizon, flatness and monopole problems [18].

Besides these difficulties in explaining observations, there are theoretical motivations suggesting that a theory of gravity more fundamental than GR should be formulated: Einstein’s theory is not renormalizable, and thus it cannot be quantized as is. In a recent paper by Berti et al. [19], a thorough review of the motivations to consider extensions of GR can be found together with a discussion of some modified theories of gravity (see also the recent reviews [20–24] and references therein).

A possible strategy towards a new theory of gravity is, in some sense, a natural generalization of Einstein’s approach, according to which gravity is geometry. Accordingly, a new theory is obtained extending GR on a purely geometric basis: in other words, the required ingredients to match the observations or to solve the theoretical conundrums derive from a geometric structure richer than that of GR.

As a prototype of this strategy, which has gained an increasing attention during the last decade, we mention the f(R) theories, where the gravitational Lagrangian depends on a function of the scalar curvature R; extensive reviews can be found in [25–29]. These theories are also referred to as “extended theories of gravity”, since they naturally generalize GR: in fact, when f(R) = R the action reduces to the usual Einstein-Hilbert action, and Einstein’s theory is obtained.

Motivations for studying these theories can be different but, as clearly synthesized by Sotiriou and Faraoni [27], they can be considered as toy-theories
that are relatively simple to handle and that allow to study the effects of the deviations from Einstein’s theory with sufficient generality. For instance $f(R)$ theories provide cosmologically viable models, where both the inflation phase and the late-time accelerated expansion are reproduced; furthermore, they have been used to explain the rotation curves of galaxies without need for dark matter (see [25, 27, 28] and references therein). These theories can be studied in the metric formalism, where the action is varied with respect to metric tensor, and in the Palatini formalism, where the action is varied with respect to the metric and the affine connection, which are supposed to be independent from one another (there is also the metric-affine formalism, in which the matter part of the action depends on the affine connection, and is then varied with respect to it). In general, the two approaches are not equivalent: the solutions of the Palatini field equations are a subset on solutions of the metric field equations [30].

A different approach to the extension of GR derives from a generalization of Teleparallel Gravity (TEGR) [31, 32]: this theory is based on a Riemann-Cartan space-time, endowed with the non symmetric Weitzenböck connection which, unlike the Levi-Civita connection of GR, gives rise to torsion but it is curvature-free. In TEGR torsion determines the geometry, while the tetrad field is the dynamical one; the field equations are obtained from a Lagrangian containing the torsion scalar $T$, arising from contractions of the torsion tensor. Notwithstanding GR and TEGR have a different geometric structure, they have the same dynamics: in other words, every solution of GR is also solution of TEGR and vice versa. Hence, one could start from TEGR and extend its Lagrangian from $T$ to an arbitrary function $f(T)$, resulting to the so-called $f(T)$ gravity [33, 34] (for a review see [35]). Since $f(T)$ gravity is different from TEGR, $f(T)$ theories have been considered as potential candidates to describe the cosmological behavior [36–47]. Additionally, various aspects of $f(T)$ gravity have been considered, such as for instance, exact solutions and stellar models [48–60].

Another possible new theory of gravity can be obtained by a massive deformation of GR. Endowing graviton with a mass is a plausible modified theory of gravity that is both phenomenologically and theoretically intriguing. From the theoretical point of view, a small non-vanishing graviton mass is an open issue. The idea was originally introduced in the work of Fierz and Pauli [61], who constructed a massive theory of gravity in a flat background that is ghost-free at the linearized level. Since then, a great effort has been put in extending the result to the nonlinear level and constructing a consistent theory. A few years ago a covariant massive gravity model has been proposed in [62]. Since the linearization of the mass term breaks the gauge invariance of GR then, in order to construct a consistent theory, non-linear terms should be tuned to remove order by order the negative energy state in the spectrum [63]. The theoretical model under investigation follows from a procedure originally outlined in [64, 65] and has been found not to show ghosts at least up to quartic order in the nonlinearity [62, 66]. The consequent theory exploits several remarkable features. Indeed the graviton mass typically manifests itself on cosmological
scales at late times thus providing a natural explanation of the presently observed accelerating phase \[67\]. Moreover, the theory allows for exotic solutions in which the contribution of the graviton mass affects the dynamics at early times. It actually allows for models in which the Universe oscillates indefinitely about an initial static state, ameliorating the fine-tuning problem suffered by the emergent Universe scenario in GR \[68\].

Another approach that could lead towards a formulation of a quantum theory of gravity is the Hořava formulation of a model that is power-counting renormalizable due to an anisotropic scaling of space and time \[69\]. This is reminiscent of Lifshitz scalars in condensed matter physics \[70, 71\], hence the theory is often referred to as the Hořava-Lifshitz gravity. This theory has attracted a lot of attention, due to its several remarkable features in cosmology. Unfortunately, the original model suffers from instability, ghosts, strong coupling problems and the model has been implemented along different lines \[72\].

On the other hand, if we consider the excellent agreement of GR with Solar System and binary pulsar observations, it is apparent that any modified theory of gravity should reproduce GR at the Solar System scale, i.e. in a suitable weak-field limit. In other words, these theories must have correct Newtonian and post-Newtonian limits and, up to intermediate scales, the deviations from the GR predictions can be considered as perturbations. This agreement should be obtained, for all the above gravitational modifications. In particular, all these theories have the same spherically symmetric solution that describes the gravitational field around a point-like source: the Schwarzschild-de Sitter space-time (SdS). Interestingly enough, this is a solution of GR field equations with a cosmological constant. However, for these modified gravities the cosmological term is not added by hand, but it naturally originates from the modified Lagrangian.

In this paper, we assume that the SdS solution can be used to model the gravitational field of an isolated source like the Sun, and we examine the impact that the gravitational modifications have on the Solar System dynamics. Additionally, we explore the possibility of constraining \(\Lambda\) in the distant peripheries of the Solar System by means of the currently ongoing spacecraft-based mission New Horizons. For a recently proposed long-range mission aimed to test long-distance modifications of gravity in the Solar System, see \[73\].

This work is organized as follows: In Section 2 we describe the main features of the SdS space-time, focusing on \(f(R)\) and \(f(T)\) theories, massive gravity and Hořava-Lifshitz gravity. Section 3 is devoted to a preliminary exposition of the experimental constraints which might be posed by using accurate tracking of distant man-made objects traveling to the remote outskirts of the Solar System; the case of the New Horizons probe is considered. Finally, section 4 summarizes our results.
2. Schwarzschild-de Sitter space-time as a vacuum solution of modified gravities

The SdS metric (see e.g. [74])

\[ ds^2 = \left(1 - \frac{2GM}{r} - \frac{1}{3} \Lambda r^2 \right) dt^2 - \frac{1}{\left(1 - \frac{2GM}{r} - \frac{1}{3} \Lambda r^2 \right)} dr^2 - r^2 d\Omega^2 \]  

(1)

where \( d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2 \), is a spherically symmetric solution of the Einstein field equations with cosmological constant \( \Lambda \) in vacuum, namely

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 0, \]  

(2)

or equivalently

\[ R_{\mu\nu} = \Lambda g_{\mu\nu}, \]  

(3)

around the mass \( M \). The SdS space-time has been studied in connection with the constraints arising from Solar System data [75, 76] and moreover focusing on the effects on gravitational lensing [77–79]. In the following subsections, we are going to show that the metric (1) is a solution of various gravitational modifications, under certain considerations.

2.1. \( f(R) \) theories

Let us start by summarizing the theoretical framework of the \( f(R) \) theories, in order to obtain the field equations, both in metric and the Palatini approach (see [25, 27, 28] for an exhaustive discussion), and to show that the SdS space-time is a solution.

The field equations can be obtained by a variational principle, starting from the action \( S \)

\[ S = \frac{1}{16\pi G} \int d^4x \sqrt{-\det(g_{\mu\nu})} f(R) + S_M. \]  

(4)

As we mentioned above, in these theories the gravitational part of the Lagrangian is represented by a function \( f(R) \) of the scalar curvature \( R \), while \( S_M \) is the action for the matter sector, which functionally depends on the matter fields together with their first derivatives. In the metric formalism, \( \Gamma \) is supposed to be the Levi-Civita connection of the metric \( g \) and, consequently, the scalar curvature \( R \) has to be intended as \( R \equiv R(g) = g^{\alpha\beta} R_{\alpha\beta}(g) \). On the contrary, in the Palatini formalism the metric \( g \) and the affine connection \( \Gamma \) are supposed to be independent, so that the scalar curvature \( R \) has to be intended as \( R \equiv R(g, \Gamma) = g^{\alpha\beta} R_{\alpha\beta}(\Gamma) \), where \( R_{\mu\nu}(\Gamma) \) is the Ricci-like tensor of the connection \( \Gamma \).

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1. Let the signature of the 4-dimensional Lorentzian manifold \( \mathcal{M} \) be \( (+, -, -, -) \). Furthermore, if not otherwise stated, we use units such that \( c = 1 \).
In the metric formalism the action $\mathcal{S}$ is varied with respect to the metric $g$, and one obtains the following field equations

$$f'(R) R_{\mu \nu} - \frac{1}{2} f(R) g_{\mu \nu} - (\nabla_\mu \nabla_\nu - g_{\mu \nu} \Box) f'(R) = 8\pi G T_{\mu \nu}, \quad (5)$$

where $f'(R) = df(R)/dR$, $\nabla_\mu$ is the covariant derivative associated with $\Gamma$, $\Box \equiv \nabla_\mu \nabla^\mu$, and $T^{\mu \nu} = -\frac{2}{\sqrt{g}} \frac{\delta \mathcal{S}_M}{\delta g_{\mu \nu}}$ is the standard minimally coupled matter energy-momentum tensor. The contraction of the field equations (5) with the metric tensor leads to the scalar equation

$$3 \Box f'(R) + f'(R) R - 2 f(R) = 8\pi G T,$$  

where $T$ is the trace of the energy-momentum tensor. Note that Eq. (6) is a differential equation for the scalar curvature $R$, while in GR the scalar curvature is algebraically related to $T$ through $R = -8\pi G T$.

In the Palatini formalism, by independent variations with respect to the metric $g$ and the connection $\Gamma$, we obtain the following equations of motion:

$$f'(R) R_{(\mu \nu)}(\Gamma) - \frac{1}{2} f(R) g_{\mu \nu} = 8\pi G T_{\mu \nu}, \quad (7)$$

$$\nabla^\alpha (\sqrt{g} f'(R) g^{\mu \nu}) = 0, \quad (8)$$

where $\nabla^\Gamma$ denotes covariant derivative with respect to the connection $\Gamma$. Actually, it is possible to show [80, 81] that the manifold $\mathcal{M}$, which is the model of the space-time, can be a posteriori endowed with a bi-metric structure $(\mathcal{M}, g, h)$ equivalent to the original metric-affine structure $(\mathcal{M}, g, \Gamma)$, where $\Gamma$ is assumed to be the Levi-Civita connection of $h$. The two metrics are conformally related by

$$h_{\mu \nu} = f'(R) g_{\mu \nu}. \quad (9)$$

The equation of motion (7) can be supplemented by the scalar-valued equation obtained by taking the contraction of (7) with the metric tensor:

$$f'(R) R - 2 f(R) = 8\pi G T. \quad (10)$$

Equation (10) is an algebraic equation for the scalar curvature $R$, thus slightly generalizing the GR case where $R$ is proportional to $T$.

In order to compare the predictions of $f(R)$ gravity with Solar System dynamics data, we have to consider the solutions of the field equations in vacuum. As a consequence, in the metric approach the field equations read

$$f'(R) R_{\mu \nu} - \frac{1}{2} f(R) g_{\mu \nu} - (\nabla_\mu \nabla_\nu - g_{\mu \nu} \Box) f'(R) = 0, \quad (11)$$

supplemented with the scalar equation

$$3 \Box f'(R) + f'(R) R - 2 f(R) = 0. \quad (12)$$
In the Palatini approach, the field equations in vacuum become

\[ f'(R)R_{(\mu\nu)}(\Gamma) - \frac{1}{2}f(R)g_{\mu\nu} = 0, \tag{13} \]

\[ \nabla^\Gamma_\alpha(\sqrt{\gamma}f'(R)g^{\alpha\nu}) = 0, \tag{14} \]

and they are supplemented by the scalar equation

\[ f'(R)R - 2f(R) = 0. \tag{15} \]

It is useful to emphasize some features of the scalar equations (12) and (15), which can help to understand the differences between the vacuum solutions in the two formalisms. In Palatini \( f(R) \) gravity, the trace equation (15) is an algebraic equation for \( R \), which admits constant solutions \( R = c_1 \) \[80\], and it is identically satisfied if \( f(R) \) is proportional to \( R^2 \). As a consequence, it is easy to verify that (if \( f'(R) \neq 0 \)) the field equations become

\[ R_{\mu\nu} = \frac{1}{4}Rg_{\mu\nu}, \tag{16} \]

which are the same as the GR field equations with a cosmological constant \[8]. In particular, we now have

\[ \Lambda_{FR} = \frac{1}{4}R. \tag{17} \]

In other words, in the Palatini formalism, in vacuum, we can obtain only solutions that describe space-times with constant scalar curvature \( R \). Summarizing, Eq. (16) suggests that all GR solutions with cosmological constant are solutions of vacuum Palatini field equations: the function \( f(R) \) only determines the solutions of algebraic equation (15).

In metric \( f(R) \) gravity the trace equation (12) is a differential equation for \( R \): this implies that, in general, it admits more solutions than the corresponding Palatini equation. In particular, we notice that if \( R = \) constant we obtain the Palatini case. Hence for a given \( f(R) \) function, in vacuum, the solutions of the field equations of Palatini \( f(R) \) gravity are a subset of the solutions of the field equations of metric \( f(R) \) gravity \[30\]. However, in metric \( f(R) \) gravity vacuum solutions with variable \( R \) are allowed too (see, e.g., \[82\]).

Therefore, if we confine ourselves with constant scalar curvature, we have shown that in \( f(R) \) gravity the SdS space-time \[1\] is a solution of the field equations, and in particular the “effective” cosmological constant term depends on the analytical expression of \( f(R) \).

As for the reliability of these solutions for describing without conceptual drawbacks the gravitational field of a star, like the Sun, the issue has been lively debated in the literature (see e.g. \[27\], \[28\]). In the Palatini formalism, the possibility of constructing vacuum solutions that match an internal solution has been discussed, and it has been shown that when one considers even a simple model such as a polytropic star, divergences arise. However, things are different for non-analytical \( f(R) \), and also the role of the conformal metric \( h_{\mu\nu} \) can help
to avoid these singularities (see [83] and references therein). On the other hand, metric $f(R)$ gravity is in agreement with Solar System tests only if the chameleon mechanism is considered, according to which the additional scalar degree of freedom of the theory is a function of the curvature: the mass of the scalar field is large at Solar System scale, in order not to affect the dynamics, while it is small at cosmological scale, in order to drive the accelerated expansion. For a thorough discussion about the reliability of $f(R)$ gravity see the reviews [27, 28], where it is discussed that Palatini $f(R)$ gravity, beyond the above mentioned difficulty with polytropic stars, suffers from other problems, which make acceptable models practically indistinguishable from ΛCDM. On the other hand, in metric $f(R)$ gravity it is possible to obtain models that are in agreement with observations, having peculiarities that make it possible, at least in principle, to distinguish them from ΛCDM. However, we are not going to get into the details of the above debate, since for the purpose of the present work it is adequate that $f(R)$ gravity admits the SdS solution. Finally, we recall that some properties of SdS and Reissner-Nordström (SdS generalised) black holes in $f(R)$ modified gravity were investigated in [84, 85].

2.2. $f(T)$ theories

In this subsection we outline the theoretical framework of $f(T)$ gravity and we obtain the field equations that accept the SdS space-time as solution [35]). In $f(T)$ gravity the tetrads are the dynamical fields. Given a coordinate basis, the components $e^a_\mu$ of the tetrads are related to the metric tensor through $g_{\mu\nu}(x) = \eta^{ab} e^a_\mu(x) e^b_\nu(x)$, with $\eta_{ab} = \text{diag}(1,-1,-1,-1)$. We point out that, in our notation, latin indices refer to the tangent space, while greek indices label coordinates on the manifold. The field equations can be obtained by varying the action

$$ S = \frac{1}{16\pi G} \int f(T) e d^4 x + S_M $$

with respect to the tetrads, where $e = \text{det} e^a_\mu = \sqrt{-\text{det}(g_{\mu\nu})}$ and $S_M$ is the action for the matter fields. In the action (18), $f$ is a differentiable function of the torsion scalar $T$: in particular, if $f(T) = T$, the action is the same as in TTEGR, and the theory is equivalent to GR. In terms of the tetrads one defines the torsion tensor as

$$ T^\lambda_{\mu\nu} = e^\lambda_a (\partial_\nu e^a_\mu - \partial_\mu e^a_\nu), $$

and the “super-potential” tensor

$$ S^\rho_{\mu\nu} = \frac{1}{4} (T^\rho_{\mu\nu} - T_{\mu\nu}^\rho + T^\nu_{\rho\mu}) + \frac{1}{2} \delta^\rho_{\mu} T_{\sigma\nu} - \frac{1}{2} \delta^\rho_{\nu} T_{\sigma\mu}, $$

from which one obtains the torsion scalar

$$ T = S^\rho_{\mu\nu} T^\mu_{\rho\nu}. $$
By variation of the action \[18\] with respect to the tetrad field \(e^a_\mu\), we obtain the field equations

\[
e^{-1} \partial_{\mu} (e^a_\rho S^\rho_{\mu \nu}) f_T - e^a_\rho S^\rho_{\mu \nu} T_{\mu \lambda} f_T + e^a_\rho S^\rho_{\mu \nu} \partial_{\mu} (T f_T) + \frac{1}{4} e^a_\rho T_{\mu \nu} = 4 \pi G e^a_\rho T_{\mu \nu},
\]

where \(T_{\mu \nu}\) is the matter energy-momentum tensor, and where the subscripts \(T\) denote differentiation with respect to \(T\).

We are interested in static spherically symmetric solutions that can be used to describe the gravitational field of a point-like source, e.g. of the Sun. To this end, we write the space-time metric in the form

\[
ds^2 = e^{A(r)} dt^2 - e^{B(r)} dr^2 - r^2 d\Omega^2.
\]

In the usual, “pure-tetrad” formulation of \(f(T)\) gravity, the above metric is produced by the non-diagonal tetrad \(\left[84, 87\right]\)

\[
e^a_\mu = \begin{pmatrix}
  e^{A/2} & 0 & 0 \\
  0 & e^{B/2} \sin \theta \cos \phi & e^{B/2} \sin \theta \sin \phi \\
  0 & e^{B/2} \sin \theta \sin \phi & e^{B/2} \cos \theta
\end{pmatrix},
\]

where \(\theta, \phi\) are rotation angles, and \(\gamma(r)\) is a general function of \(r\). The expression of the torsion scalar for the above tetrad turns out to be

\[
T(r) = \frac{2 e^{-B}}{r^2} \left[1 + e^B + 2 e^{B/2} \sin \gamma + 2 e^{B/2} r \gamma' \cos \gamma + r A' \left(1 + e^{B/2} \sin \gamma\right)\right].
\]

We are interested in extracting static vacuum solution with constant torsion scalar \(T = T_0\) (i.e. \(T' = 0\)). The field equations \(22\) become

\[
\frac{f_0}{4} - \frac{f_{T_0} e^{-B}}{4 r^2} \left(2 - 2 e^B + r^2 e^B T_0 - 2 r B'\right) = 0, \quad \frac{f_0}{4} + \frac{f_{T_0} e^{-B}}{4 r^2} \left(2 - 2 e^B + r^2 e^B T_0 - 2 r A'\right) = 0, \quad 4 - 4 e^B - r^2 A'^2 + 2 r B' + r A' \left(2 + r B'\right) - 2 r^2 A'' = 0,
\]

where \(f_0 \equiv f(T_0), f_{T_0} \equiv f'_T(T_0)\) and prime denotes differentiation with respect to \(r\). We point out that spherically symmetric solutions with non constant torsion scalar \(T' \neq 0\) have been already investigated \(\left[88, 89\right]\) and Solar System constraints have been discussed \(\left[90, 91\right]\).

It is possible to show (see \(\left[86\right]\)) that the unique solution of the equations \(26-28\) is given by

\[
e^{A(r)} = 1 - \frac{2 M}{r} - \frac{A_{JT}}{3} r^2, \\
e^{B(r)} = e^{-A(r)},
\]

with

\[
A_{JT} = \frac{1}{2} \left(\frac{f_0}{f_{T_0}} - T_0\right).
\]
Thus, in the theory at hand one obtains an “effective” cosmological constant, determined by the functional form of $f(T)$, and thus he obtains a $\text{SdS}$ solution. Notice however that, because of the presence of the arbitrary function $\gamma(r)$ in the definition of the torsion scalar, knowing $\Lambda_{fT}$ cannot constrain $f(T)$, since an arbitrary value of $\Lambda$ can be achieved by fine tuning $T_0$ with a suitable choice of $\gamma(r)$. In other words, when the torsion tensor is constant, any $f(T)$ model admits the solution in the form of (29), with given values of $M$ and $\Lambda_{fT}$.

On the contrary, in the case of $f(R)$ theories, the value of the scalar curvature $R$, that is proportional to $\Lambda_{fR}$, strictly depends on the function $f(R)$, since it is obtained from the trace equation (15).

2.3. Massive gravity

In this subsection we summarize the basic part of massive gravity formulation relevant to the present analysis. Specifically, we are interested in static spherically symmetric solutions in which the mass term becomes identical to the cosmological constant term.

The possibility of endowing graviton with a mass goes back to 1939, where Fierz and Pauli constructed the linearized theory of non-interacting massive gravitons in a flat background [61]. Unfortunately, the solutions of this theory do not continuously connect with those of GR in the limit of zero graviton mass, and this is the famous van Dam, Veltman and Sakharov (vDVZ) discontinuity [92, 93]. This vDVZ discontinuity can be alleviated at the nonlinear level through the Vainshtein mechanism [94], however these nonlinearities produce the so-called Boulware-Deser (BD) ghost degree of freedom [63].

In 2010 a ghost-free theory was proposed by de Rham, Gabadadze and Tolley (dRGT) [62]. In the standard formalism of dRGT theory, the dynamics is determined by a modified action written in terms of a dynamical metric $g_{\mu\nu}$ and an arbitrary fiducial metric $f_{\mu\nu}$ needed to construct the gravitational self-interacting potential $U$. The corresponding action reads:

$$S = -\frac{1}{8\pi G} \int \left( \frac{1}{2} R + m^2 U \right) \sqrt{-g} \, d^4x + S_M, \quad (31)$$

where $S_M$ describes ordinary matter which is supposed to directly interact only with $g_{\mu\nu}$. The potential term, coupled through the graviton mass $m$, is defined by [95]

$$U = \frac{1}{2} (K_1^2 - K_2) + \frac{c_4}{6} (K_1^3 - 3K_1K_2 + 2K_3) +$$

$$+ \frac{c_4}{12} (K_4^1 - 6K_1^2K_2 + 3K_2^2 + 8K_1K_3 - 6K_4), \quad (32)$$

with $K_n$ denoting the traces of a tensor $K_{\mu\nu}$ constructed from the inverse metric $g^{\mu\nu}$ and the fiducial one through

$$K_{\mu\nu} = \delta_{\mu\nu} - \sqrt{g^{\mu\rho} f_{ab\rho\sigma\phi\phi} \partial_{\sigma} \phi^a \partial_{\rho} \phi^b},$$
and with $K_n \equiv \text{tr}K^n$. The four fields $\phi^a$ are the Stückelberg fields\footnote{Stückelberg fields were originally introduced by Stückelberg in 1938 to restore gauge-invariance in electromagnetism but the method works equivalently well for spin-2 fields.} which transform as scalars under coordinate transformations, such that the fixed metric $f_{\mu\nu}$

$$f_{\mu\nu} = f_{ab}\partial_\mu \phi^a \partial_\nu \phi^b,$$

as well as the quantity $g^{\mu\alpha} f_{\alpha\nu}$, are promoted to tensor fields, while the potential $U(g, f)$ to a scalar. Potential (32) has been shown to be the most general potential for a ghost-free theory of massive gravity in four dimensions \cite{66}.

Apart from interesting cosmological features, the dRGT massive gravity admits the SdS solution where the “effective” cosmological constants arises due to the graviton mass. In particular, considering the choice

$$c_4 = 1 + c_3 + c_3^2$$

in (32), then the mass term of the theory behaves exactly as the cosmological constant term in GR for a spherically symmetric ansatz \cite{90}, and the resulting expression for the metric reads as follows:

$$ds^2 = - \left( 1 - \frac{2GM}{r} - \frac{\Lambda}{3} r^2 \right) dt^2 + \frac{1}{1 - \frac{2GM}{r} - \frac{\Lambda}{3} r^2} dr^2 + r^2 d\Omega^2,$$  

(33)

that is the standard SdS solution of GR in static coordinates. The difference here is that it is accompanied by nontrivial background of the Stückelberg fields. In terms of the parameters of the theory the effective cosmological constant reads

$$\Lambda_{mg} = \frac{2m^2}{1 + c_3}.$$  

(34)

Finally, note that this solution allows to recover GR when $c_3 + c_4 > 0$ \cite{97}, below the so-called Vainshtein radius $r_V = \left( GM/m^2 \right)^{1/3}$. For extended dRGT models, see, e.g., \cite{98, 99}.

2.4. Hořava-Lifshitz gravity

Let us summarize Hořava-Lifshitz gravity \cite{69}, in order to extract its spherically symmetric solutions. As we mentioned in the Introduction, Hořava-Lifshitz gravity is a power-counting renormalizable theory, obtained through an anisotropic scaling of space and time in the Ultraviolet limit. This feature allows for the inclusion of higher-dimensional spatial derivative operators that dominate in the high energy limit, while in the Infrared lower-dimensional operators take over, presumably providing a healthy low-energy limit, namely GR. Additionally, the absence of higher order time derivative terms prevents ghost instabilities. However, as it becomes obvious, the anisotropic scaling breaks Lorentz invariance, and breaking of general covariance has been shown to introduce a dynamical scalar mode that may lead to strong coupling problem and
Recently, a new covariant version of Hořava Lifshitz gravity has been formulated by Hořava and Melby-Thompson in which, in order to heal the scalar graviton problem, two auxiliary scalar fields have been introduced: the Newtonian pre-potential $\phi(t,x)$ and the gauge field $A(t,x)$. The latter eliminates the new scalar degree of freedom, thus curing the strong coupling problem in the Infrared limit, and general covariance is restored. In the following we refer to the covariant version of Hořava and Melby-Thompson, and the running coupling $\lambda$ in the extrinsic curvature term of the action is not set to 1.

With the perspective of Lorentz symmetry breaking, the suitable variables in Hořava-Lifshitz theory are the lapse function, the shift vector and the spatial metric, $N, N_i, g_{ij}$ respectively, according to the Hamiltonian formulation of General Relativity developed by Dirac and Arnowitt, Deser and Misner. Then the line element can be rewritten as

$$\text{ds}^2 = -N^2 \text{d}t^2 + g_{ij} (dx^i + N_i \text{d}t) (dx^j + N_j \text{d}t).$$

The theory can be assumed to satisfy the projectability condition, i.e. the lapse function only depends on time $N = N(t)$, while the total gravitational action is given by

$$S_g = \zeta^2 \int dt \, d^3 x \, N \sqrt{|g|} (L_K - L_V + L_\phi + L_A + L_\lambda),$$

where $g = \det(g_{ij})$ and

$$L_K = K_{ij} K^{ij} - \lambda K^2,$$

$$L_\phi = \phi \mathcal{G}^{ij} (2K_{ij} + \nabla_i \nabla_j \phi),$$

$$L_A = \frac{A}{N} (2\Lambda_g - R),$$

$$L_\lambda = (1 - \lambda) \left[ (\nabla \phi)^2 + 2K \nabla^2 \phi \right].$$

Note that in this subsection covariant derivatives and Ricci terms refer to the 3-metric $g_{ij}$. $K_{ij}$ represents the extrinsic curvature

$$K_{ij} = g^k_i \nabla_k n_j,$$

$n_j$ being a unit normal vector of the spatial hypersurface, and $G_{ij}$ is the 3-dimensional generalised Einstein tensor

$$G_{ij} = R_{ij} - \frac{1}{2} g_{ij} R + \Lambda g_{ij}.$$

We mention that the parameter $\lambda$ characterizes deviations of the kinetic part of the action from GR. The most general parity-invariant Lagrangian density up
to six order in spatial derivatives reads as \[105\]

\[
\mathcal{L}_V = 2\zeta^2 g_0 + g_1 R + \frac{1}{\zeta^2} \left( g_2 R^2 + g_3 R_{ij} R^{ij} \right) + \\
\frac{1}{\zeta^4} \left[ g_4 R^3 + g_5 R R_{ij} R^{ij} + g_6 R_i^j R_k^l R^c_r + \\
g_7 R \nabla^2 R + g_8 \left( \nabla_i R_{jk} \left( \nabla^i R^{jk} \right) \right) \right],
\]

where in physical units \(\zeta^2 = (16\pi G)^{-1}\), \(G\) being the Newtonian constant, and the couplings \(g_s\) \((s = 0, 1, \ldots, 8)\) are all dimensionless.

We are here interested in vacuum static spherically symmetric solutions. These have been derived in \[106\] and \[107\] for the case \(\lambda = 1\) and \(\lambda \neq 1\), respectively (see also \[108, 109\]). Omitting the details of the derivation, and despite the large class of solutions, we mention that in both cases the SdS solution can be extracted. Hence, various constraints on the parameters and functions of the theory are derived, both due to equations of motion and Solar System tests \[106, 107\]. When \(\lambda = 1\), which is the GR value, and similarly to what happens in the original version presented in \[101\], the SdS solution is recovered, with the choice \(\phi = A = 0\), and the effective cosmological constant arises from the \(g_0\) coupling, namely

\[
\Lambda_{HL} = \frac{1}{2} \zeta^2 g_0.
\]

On the other hand, if one desires to consider \(\lambda\) as a free parameter, one has to consider the Newtonian pre-potential \(\phi\), as well as the gauge field \(A\), as part of the metric on which matter fields couple, as shown in \[110\].

3. Preliminary sensitivity analysis on the possibility of constraining \(\Lambda\) with New Horizons

3.1. Suggested data analysis

New Horizons \[111, 112\] is a spacecraft which, launched in 2006, flew by Pluto on the 14th of July 2015 without entering into orbit around it. Orbital maneuvers were recently implemented\(^4\) to target the spacecraft towards the Trans-Neptunian Object (TNO) 2014 MU\(_{69}\) of the Kuiper Belt in an extended mission scenario. New Horizons is spin-stabilized and therefore it will be possible to perform radio-science experiments \[113\] due to the dedicated Radio Science Experiment (REX) apparatus \[114\] carried on board and the innovative regenerative tracking technique \[115\]. The precision in Doppler measurements

\(^4\)See http://pluto.jhuapl.edu/News-Center/News-Article.php?page=20151105 on the Internet.
will be better than $\sigma_\rho = 0.1 \text{ mm s}^{-1}$ throughout the entire mission\textsuperscript{116}, while ranging will be precisely better than $\sigma_\rho = 10 \text{ m (1\sigma)}$ over 6 years after 2015, i.e. at geocentric distances to beyond 50 au\textsuperscript{116}.

It is interesting to preliminarily investigate the potential ability of New Horizons’s tracking to improve the currently existing bounds on, e.g., the cosmological constant $\Lambda$. To this aim, we will numerically simulate the range and range-rate signatures of the extra-acceleration caused by a cosmological constant in the Solar System, by comparing their magnitudes with the previously quoted figures for New Horizons. However, it should be stressed that it is just a preliminary sensitivity analysis based on the expected precision of the probe’s measurements: actual overall accuracy will be finally set by several sources of systematic uncertainties like, e.g., the heat dissipation from the Radioisotope Thermoelectric Generator (RTG) and the ability in accurately modeling the orbital maneuvers. In this respect, the extensive modeling of such non-gravitational perturbations for the Pioneer spacecraft, recently made in the framework of the Pioneer Anomaly investigations\textsuperscript{117–125} should be helpful.

We numerically integrate the barycentric equations of motion of the major bodies of the Solar System and of New Horizons, with and without $\Lambda$, in Cartesian rectangular coordinates. Both integrations share the same initial conditions, retrieved from the WEB interface HORIZONS run by JPL, NASA, and the time interval is set to 10 yr starting from a date posterior to the flyby of Pluto. Then, from the solutions of the perturbed and unperturbed equations of motion, we numerically produce differential time series $\Delta \rho(t), \Delta \dot{\rho}(t)$ of the Earth-New Horizons range $\rho$ and range-rate $\dot{\rho}$. The amplitudes of such simulated signatures can be compared to $\sigma_\rho, \sigma_\dot{\rho}$ in order to preliminarily guess the value of $\Lambda$ which makes them compatible. It turns out that the range allows for tighter constraints than the range-rate.

In Fig. 1 we present our results. In particular, we depict the simulated time series $\Delta \rho(t)$ for $\Lambda = 10^{-45} \text{ m}^{-2}$. It can be noticed that the size of the $\Lambda$-induced signatures is about 20 m. Thus, the possibility of constraining $\Lambda$ to a $\approx 10^{-45} \text{ m}^{-2}$ level over the next ten years by means of New Horizons does not seem implausible. If indeed it will be realized practically, it would represent an improvement by more than one-two orders of magnitude with respect to the latest results appeared in the literature\textsuperscript{91, 126}. However, it must be stressed once again that the analysis presented here has to be intended as a sketchy one just to explore the potential opportunity offered by New Horizons; suffice it to say that it assumes a straightforward path over the years, without accounting for orbital maneuvers and corrections.

Finally, it should be remarked that the present analysis is based only on the orbital dynamics of both the major bodies of the Solar System and the probe itself. In fact, range and range-rate are not directly observable since they are calculated through the actually measured round-trip time of flight of the photons and their frequency shift, respectively. Thus, in principle, the impact of $\Lambda$ on the propagation of the electromagnetic waves connecting the spacecraft and the Earth\textsuperscript{127, 131} should be taken into account as well. A detailed calculation of such an aspect of the measurement modeling is beyond the scopes of the present
Figure 1: Simulated signature $\Delta \rho$ induced by $\Lambda = 10^{-45} \text{ m}^{-2}$ on the geocentric range of New Horizons over a decade-time span 2015-2025. It was obtained by taking the difference $\Delta \rho(t)$ between two time series of $\rho(t) = \sqrt{(x_{\text{NH}}(t) - x_{\oplus}(t))^2 + (y_{\text{NH}}(t) - y_{\oplus}(t))^2 + (z_{\text{NH}}(t) - z_{\oplus}(t))^2}$ calculated by numerically integrating the barycentric equations of motion of New Horizons and the major bodies of the Solar System in Cartesian rectangular coordinates with and without the $\Lambda$--induced acceleration. All the standard Newton-Einstein dynamics for pointlike bodies was modeled in both the integrations which shared also the same initial conditions for August 5, 2015, retrieved from the WEB interface HORIZONS maintained by JPL, NASA. The range-rate signature $\Delta \dot{\rho}(t)$, not shown here, was obtained by numerically differentiating the time series for $\Delta \rho(t)$.

3.2. Induced constraints on the models

Having elaborated the observational constraints on the cosmological constant $\Lambda$ we may proceed to the constraining of the various gravitational modifications. In particular, we will use the SdS solution and the expression of the obtained effective $\Lambda$ in terms of the model parameters of each case, extracted in Section 2, in order to provide constraints and bounds on these model parameters.

In case of $f(R)$ gravity, from the expression of the effective cosmological constant $\Lambda_{fR}$ of (17) we obtain a constraint on the curvature scalar $R$ that turns out to be constant both in metric and Palatini approach in order to have a SdS solution, and, from numerical estimation of $\Lambda$, we obtain $R \sim 10^{-46} \text{ m}^{-2}$. Moreover, since through the scalar equation (12) the Ricci scalar is related to the analytical expression of the Lagrangian, or at least to the ratio $f(R)/f'(R)$, and thus on the parameters of the specific model, we can easily extracts the
constraints on them too.

In case of $f(T)$ gravity, as already remarked, the function $\gamma(r)$ can be chosen to achieve the desired constant value of the torsion scalar through (30), thus the expression (29) for $\Lambda_{fT}$ does not allow to break this degeneracy and impose constraints on the Lagrangian.

In case of massive gravity, the effective $\Lambda_{mg}$ (34) allows to infer upper limits on the graviton mass. Assuming $c_3 \sim O(1)$, numerical values on $\Lambda_{mg}$ will directly constraint $m$. Restoring SI units, i.e. replacing it with $m_g = h\mu/c$, the observational constraints on the cosmological constant translate into

$$m_g \sim 10^{-69} \text{ g} = 0.56 \times 10^{-36} \text{ eV c}^{-2}.$$ 

We stress here that, as expected, our Solar System analysis can infer more stringent constraints on the graviton mass than the analysis of the same model using cosmological data [67]), in which $m$ is related to the present value of the Hubble parameter. Moreover, we can then compare our result with the upper limit $m_g < 7.68 \times 10^{-55} \text{ g}$ from the dynamics in the Solar System [132] and the more stringent limit, namely $m_g < 10^{-59} \text{ g}$, derived by requiring the dynamical properties of a galactic disk to be consistent with observations [133] (see also [134] for a comprehensive review on the phenomenology of graviton mass and experimental limits). The improvement in the obtained bounds is obvious.

Finally, for the case of Hořava-Lifshitz gravity, using the expression (37) for the effective cosmological constant $\Lambda_{HL}$ in terms of the coupling constant associated with the $0^{-th}$ order spatial derivative, namely $g_0$, we can extract its corresponding bound. It proves more convenient to rescale $g_0$ through the Planck mass (or equivalently the gravitational constant $\zeta^2 = (16\pi G)^{-1}$) in order to obtain a dimensionless quantity $\tilde{g}_0$. Hence, we finally obtain

$$\tilde{g}_0 \sim 10^{-113}.$$ 

Similarly to the case of massive gravity, the above bound is more strict than the corresponding cosmological ones [133].

4. Summary and conclusions

In this work we have considered that the gravitational field of an isolated source like the Sun, can be described by the Schwarzschild-de Sitter (SdS) geometry. Such solution exists in the large majority of modified gravity theories, as expected, and in particular the effective cosmological constant is determined by the specific parameters of the given theory. Hence, one can use Solar System data in order to constrain the SdS solution, and thus eventually to extract constraints on the parameters of the gravitational modification.

We have considered some of the recently most studied modified gravities, namely $f(R)$ and $f(T)$ theories, dRGT massive gravity, and Hořava-Lifshitz gravity, and after giving their SdS solution we have explored the possibility
of using future extended radio-tracking data from the currently ongoing New Horizons mission in the outskirts peripheries of the Solar System, in order to constrain the effective cosmological constant, and thus the modified gravity parameters. In particular, we showed that an improvement of one-two orders of magnitude may be possible, provided that steady trajectory arcs several years long will be processed, and orbital maneuvers will be accurately modeled. Despite its necessarily tentative and incomplete character, it turns out that such an idea should be worth of further and more detailed consideration, especially concerning the model-building of gravitational modifications.

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