HUNTING FOR THE REMAINING SPIN IN THE NUCLEON *

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Abstract

This talk consists of four parts. In part one, I give an elementary discussion on constructing a Lorentz-invariant spin sum rule for the nucleon. In part two, I discuss a gauge-dependent spin sum rule, explore its relation with the polarized gluon distribution, and introduce the complete evolution equation for the spin structure. In part three, I consider a gauge-invariant spin sum rule and the related evolution equation. The solution of the equation motivates the possibility that half of the nucleon spin may be carried by gluons at low energy scales. In the final part, I discuss deeply-virtual Compton scattering as a possible way to measure the canonical orbital angular momentum of quarks in the nucleon.

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Yesterday and today, we have heard essentially two kinds of explanations to the so-called “spin crisis” \[1\]. The first kind says that the experimental data do not rule out the simple quark model prediction that the quark spin carries a large fraction of the nucleon spin. One way to see this is that the deep-inelastic sum rule has an unknown uncertainty about the small \(x\) contribution. Another way to see this is that one has to subtract the anomaly contribution from the measured \(\Delta \Sigma\) before comparing it with the quark model prediction, and the subtraction is potentially large. The second kind of explanations is that the quark spin carries little of the nucleon spin, due to for instance a large negative sea polarizations. On the other hand, in the skyrme model discussed by J. Ellis, it seems that the majority of the nucleon spin is carried by orbital angular momentum. Whatever position one may take, it is safe to conclude that the nucleon spin carried by other sources is significant. Thus, in my talk, I will concentrate on the subject of the remaining spin in the nucleon, i.e. the part not measured by the polarized deep-inelastic scattering experiment.

### I. CONSTRUCT A LORENTZ-INvariant SPIN SUM RULE

To understand what are the remaining components of the nucleon spin, it is important to construct a Lorentz-invariant spin sum rule. At first, it appears difficult to talk about different contributions to the nucleon spin because in field theory angular momentum operators do not commute with boost operators.

States of a spin-1/2 particle are labelled by 4-momentum \(p^\mu\) and polarization vector \(s^\mu\). Hence we write the nucleon states as \(|p,s\rangle\). To talk about spin in a relativistic way, one has to introduce the relativistic spin operator \(\hat{W}_\mu\), which is also called the Pauli-Lubanski spin,

\[
\hat{W}_\mu \sim \epsilon_{\mu\alpha\beta\gamma} \hat{J}^{\alpha\beta} \hat{P}^{\gamma},
\]

where \(\hat{J}^{\alpha\beta}\) are the generators of Lorentz transformations and \(\hat{P}^\mu\) is the energy-momentum operator. The fact that the nucleon has spin 1/2 in all frames is represented by the following equation,

\[
\hat{W}^2|ps\rangle = \frac{1}{2} \left( \frac{1}{2} + 1 \right) |ps\rangle.
\]

Since \(\hat{W}^2\) is quadratic in angular momentum and boost operators, the equation doesn’t seem to offer any interesting spin sum rule.

Notice, however, \(s_\mu \hat{W}_\mu\) is also a Lorentz scalar and it has \(|ps\rangle\) as its eigenstate,

\[
s_\mu \hat{W}_\mu |ps\rangle = \frac{1}{2} |ps\rangle.
\]

Or, we can write,

\[
\frac{1}{2} = \langle ps|s_\mu \hat{W}_\mu|ps\rangle,
\]

where I have been casual about the normalization. The equation can be used to construct spin sum rules: If the Pauli-Lubanski spin is a sum of several contributions, \(\hat{W}_\mu = \sum_i \hat{W}_i^\mu\), we can write,
\[
\frac{1}{2} = \sum_i \langle ps | s_i \hat{W}_i^\mu | ps \rangle .
\] (5)

The above equation contains the boost operators in general. However, if one chooses \( \vec{s} \) to be in the direction of \( \vec{p} \), which without loss of generality can be chosen to be the \( z \) axis, then,

\[
s_\mu \hat{W}_\mu \sim \hat{J}_z \equiv \hat{J}^z ,
\] (6)

where \( \hat{J}^z \) is the \( z \) component of the angular momentum operator. The nucleon is now in the helicity eigenstate \( \lambda = 1/2 \), and a helicity sum rule emerges from Eq. (5),

\[
\frac{1}{2} = \sum_i \langle \frac{1}{2} | \frac{1}{2} \hat{J}_i^z | \frac{1}{2} \rangle ,
\] (7)

where \( \sum_i \hat{J}_i^z = \hat{J}^z \). This sum rule is most suitable for studying the spin structure of the nucleon [2].

To actually construct a spin sum rule, one needs to know the angular momentum operators in QCD, which are identified as the generators of spatial rotations. By Noether’s theorem, we can derive these from the transformation property of the QCD lagrangian density under rotations. Depending upon the final form of the angular momentum operators one prefers to take, both gauge-dependent and gauge-invariant sum rules can result.

**II. A GAUGE-DEPENDENT SUM RULE**

In a 1989 paper, Jaffe and Manohar wrote down the following form of the QCD angular momentum operator [2],

\[
\hat{J} = \int d^3 \vec{x} \left[ \frac{1}{2} \bar{\psi} \gamma^5 \gamma_5 \psi + \psi \gamma^i \vec{x} \times (-i \nabla) \tilde{\psi}
\right. \\
\left. + \vec{E} \times \vec{A} + E_i (\vec{x} \times \vec{\gamma}) A_i \right] .
\] (8)

An advantage of this form is that the physical meaning of the individual terms is quite obvious: The first term is the quark spin, the second term is the quark orbital angular momentum, the third term is the gluon spin, and the final term is the gluon orbital angular momentum. According to the above equation, one can write down a sum rule for the nucleon spin,

\[
\frac{1}{2} = \frac{1}{2} \Delta \Sigma (\mu^2) + L_q^I (\mu^2) + \Delta g (\mu^2) + L_g^I (\mu^2) ,
\] (9)

where, for instance,

\[
\Delta g (\mu^2) = \langle ps | \int d^3 \vec{x} (\vec{E} \times \vec{A})^i | ps \rangle ,
\] (10)

etc. Clearly, \( L_q^I \), \( \Delta g \) and \( L_g^I \) are gauge, and hence frame, dependent.

Interestingly, \( \Delta g \) in the infinite momentum frame and light-like gauge \( (A^+ = 0) \) is related to a quantity present in polarized high-energy scattering,
\[ \Delta g(\mu^2) = \int_0^1 \Delta G(x, \mu^2) dx \]  
(11)

where \( \Delta G(x, \mu^2) \) is the polarized gluon distribution. Recently, there has been a lot of discussion in the literature about measuring \( \Delta G(x) \) at polarized RHIC and HERA. I am happy to see that there will be a round table discussion about this topic on Friday.

The individual contributions to the nucleon spin are scale-dependent. Recently, Hoodbhoy, Tang and myself \[3\] have worked out the scale dependence of the orbital angular momentum contributions. This subject was first recognized by Phil Ratcliffe \[4\]. Together with the well-known Altarelli-Parisi equation \[5\], we now have a complete set of equations to evolve the spin structure of the nucleon at the leading-log level,

\[
\frac{\partial}{\partial \ln \mu^2} \begin{pmatrix}
\Delta \Sigma(\mu^2) \\
\Delta g(\mu^2) \\
L_q'(\mu^2) \\
L_g'(\mu^2)
\end{pmatrix} = \frac{\alpha_s(\mu^2)}{2\pi} \begin{pmatrix}
0 & 0 & 0 & 0 \\
-\frac{3}{2}C_F & \frac{3}{2} & 0 & 0 \\
-\frac{3}{2}C_F & \frac{n_F}{3} & -\frac{4}{3}C_F & n_F \\
\frac{3}{2}C_F & -\frac{11}{2} & \frac{4}{3}C_F & -\frac{n_F}{3}
\end{pmatrix} \begin{pmatrix}
\Delta \Sigma(\mu^2) \\
\Delta g(\mu^2) \\
L_q'(\mu^2) \\
L_g'(\mu^2)
\end{pmatrix} .
\]  
(12)

If one knows the decomposition of the spin of the nucleon at one perturbative scale, one can solve from the above equation the decomposition at any other perturbative scale. As \( \mu^2 \to \infty \), one has the following asymptotic solution,

\[
\Delta \Sigma \to \text{const}.
\]

\[
\Delta g \to \lambda \ln \mu^2 + \text{const}.
\]

\[
L_q' \to \text{const}.
\]

\[
L_g' \to -\lambda \ln \mu^2 + \text{const}.
\]

(13)

Thus, the gluon helicity increases logarithmically with the probing scale. That increase is entirely cancelled by the gluon orbital contribution in the asymptotic limit.

### III. A GAUGE-INvariant SUM RULE

Recently, I have proposed to reorganize the angular momentum operator in Eq. (8) so that it is explicitly gauge-invariant \[6\],

\[
\vec{J} = \int d^3\vec{x} \left[ \frac{1}{2} \bar{\psi} \gamma_5 \gamma_5 \psi 
+ \psi \bar{\psi} (\vec{x} \times (-i\vec{D})) \psi 
+ \vec{x} \times (\vec{E} \times \vec{B}) \right].
\]  
(14)

As before, the first term is the quark spin. The second term, in which the covariant derivative is \( \vec{D} = \vec{\partial} + igA \), is the canonical orbital angular momentum of quarks. The last term is the angular momentum of the gluons, as is clear from the appearance of the Poynting vector. According to the above, we can write down a gauge-invariant spin sum rule,

\[
\frac{1}{2} = \frac{1}{2} \Delta \Sigma(\mu^2) + L_q(\mu^2) + J_g(\mu^2) ,
\]  
(15)
where the second and third terms are quark orbital and gluon contributions, respectively. I introduce the sum of the first and second terms as \( J_q(\mu^2) \), representing the total quark contribution. It is interesting to notice that although \( \Delta \Sigma(\mu^2) \) is affected by the axial anomaly, \( J_q(\mu^2) \) is anomaly-free \[3\].

The evolution equation for the quark and gluon contributions is,

\[
\frac{\partial}{\partial \ln \mu^2} \left( \frac{J_q(\mu^2)}{J_g(\mu^2)} \right) = \frac{\alpha_s(\mu^2)}{2\pi} \left( \begin{array}{cc} -16 & 3n_F \\ 16 & -3n_F \end{array} \right) \left( \frac{J_q(\mu^2)}{J_g(\mu^2)} \right). \tag{16}
\]

As \( \mu^2 \to \infty \), there is a fixed point solution,

\[
J_q(\infty) = \frac{1}{2} \frac{3n_f}{16 + 3n_f}, \quad J_g(\infty) = \frac{1}{2} \frac{16}{16 + 3n_f}. \tag{17}
\]

Thus we see about half of the nucleon spin is carried by gluons. A similar result was obtained by Gross and Wilczek in 1974 for the quark and gluon contributions to the momentum of the nucleon \[7\]. Experimentally, one finds that about half of the nucleon momentum is carried by gluons already at quite low-energy scales. An interesting question is whether the gluons carry half of the nucleon spin at low energy scales?

It is difficult to answer this question theoretically, because QCD is difficult to solve. Recently, Balitsky and I made an estimate using the QCD sum rule approach \[8\]. We find,

\[
J_g(\mu^2 \sim 1\text{GeV}^2) \approx \frac{4}{9} \frac{e < \bar{u} \sigma G u > < \bar{u} u >}{M_i^2 + \lambda_N^2} \tag{18}
\]

which gives approximately 0.25. If this calculation indicates anything about the truth, the spin structure of the nucleon roughly looks like this,

\[
\frac{1}{2} = 0.10(\text{from } \frac{1}{2} \Delta \Sigma) + 0.15(\text{from } L_q) + 0.25(\text{from } J_g). \tag{19}
\]

It would be interesting to test this scenario.

**IV. HOW TO MEASURE \( J_{Q,G} \)?**

By examining carefully the definition of the matrix elements,

\[
J_{q,g}(\mu^2) = \langle p' \mid \frac{1}{2} \left( \int d^3x (\vec{x} \times \vec{T}_{q,g}) \gamma^i \right) \mid p \rangle, \tag{20}
\]

one realizes that they can be extracted from the form factors of the quark and gluon parts of the QCD energy-momentum tensor \( T_{q,g}^{\mu\nu} \). Using Lorentz symmetry, we can write down the forward matrix elements of \( T_{q,g}^{\mu\nu} \),

\[
\langle p' | T_{q,g}^{\mu\nu} | p \rangle = \bar{u}(p') \left[ A_{q,g}(\Delta^2) \gamma^i(\vec{P}) + B_{q,g}(\Delta^2) \vec{P}(\vec{\sigma})^\alpha \Delta_\alpha/2M \\
+C_{q,g}(\Delta^2)(\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2)/M + \tilde{C}_{q,g}(\Delta^2)g^{\mu\nu}M \right] u(p), \tag{21}
\]
where \( \bar{p}^\mu = (p^\mu + p'^\mu)/2 \), \( \Delta^\mu = p'^\mu - p^\mu \), and \( u(p) \) is the nucleon spinor. Taking the forward limit in the \( \mu = 0 \) component and integrating over 3-space, one finds that \( A_{q,g}(0) \) give the momentum fractions of the nucleon carried by quarks and gluons (\( A_q(0) + A_g(0) = 1 \)). On the other hand, substituting the above into the nucleon matrix element of Eq. (20), one finds [6],

\[
J_{q,g} = \frac{1}{2} [A_{q,g}(0) + B_{q,g}(0)].
\]

(22)

There is an analogy for this. If one knows the Dirac and Pauli form factors of the electromagnetic current, \( F_1(Q^2) \) and \( F_2(Q^2) \), the magnetic moment of the nucleon, which is defined as the matrix element of \( (1/2) \int d^3\vec{x}(\vec{x} \times \vec{j})^z \), is just \( F_1(0) + F_2(0) \).

How to measure the form factors of the energy momentum tensor? If one has two vector currents which are separated along the light-cone, it is known from the operator product expansion that,

\[
T \cdot J(\vec{z}) J(0) \to \ldots + C_{\alpha \beta \mu \nu}(z^2) T^{\mu \nu} + \ldots
\]

(23)

Thus to get the matrix element \( \langle p'|T^{\mu \nu}|p \rangle \), we need \( \langle p'|T \cdot J(\vec{z}) J(0)|p \rangle \), i.e. a Compton scattering amplitude. To ensure the separation of the two currents is along the light-cone, we let one of the photon momenta approach the Bjorken limit. Then it is easy to show that the Compton scattering is dominated by the single quark process. I shall call such a scattering process deeply-virtual Compton scattering (DVCS).

What does one learn from DVCS? An analysis shows that one learns about the off-forward parton distributions (OFPDs), which are defined through the following light-cone correlations,

\[
\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle \bar{p}'|\bar{\psi}(-\lambda n/2)\gamma^\mu \psi(\lambda n/2)|p \rangle = H(x, \Delta^2, \xi) \bar{u}(p')\gamma^\mu u(p) \\
+ E(x, \Delta^2, \xi) \bar{u}(p') \frac{i\sigma^{\mu \nu} \Delta_\nu}{2M} u(p) + \ldots
\]

\[
\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle \bar{p}'|\bar{\psi}(-\lambda n/2)\gamma^\mu \gamma_5 \psi(\lambda n/2)|p \rangle = \tilde{H}(x, \Delta^2, \xi) \bar{u}(p')\gamma^\mu \gamma_5 u(p) \\
+ \tilde{E}(x, \Delta^2, \xi) \bar{u}(p') \frac{\gamma_5 \Delta^\mu}{2M} u(p) + \ldots
\]

(24)

where I have neglected the gauge link and the dots denote higher-twist distributions. From the definition, \( H \) and \( \tilde{H} \) are nucleon helicity-conserving amplitudes and \( E \) and \( \tilde{E} \) are helicity-flipping. Such distributions have been considered in the literature before [3].

The off-forward parton distributions have the characters of both ordinary parton distributions and nucleon form factors. In fact in the limit of \( \Delta^\mu \to 0 \), we have

\[
H(x, 0, 0) = q(x), \quad \tilde{H}(x, 0, 0) = \Delta q(x),
\]

(25)

where \( q(x) \) and \( \Delta q(x) \) are quark and quark helicity distributions. On the other hand, forming the first moment of the new distributions, one gets the following sum rules [3,4],

\[
\int_{-1}^{1} dx H(x, \Delta^2, \xi) = F_1(\Delta^2),
\]

\[
\int_{-1}^{1} dx E(x, \Delta^2, \xi) = F_2(\Delta^2).
\]

(26)
where $F_1$ and $F_2$ are the Dirac and Pauli form factors. The most interesting sum rule relevant to the nucleon spin is,

$$
\int_{-1}^{1} dx [H(x, \Delta^2, \xi) + E(x, \Delta^2, \xi)] = A_q(\Delta^2) + B_q(\Delta^2),
$$

(27)

where luckily the $\xi$ dependence, or $C_q(\Delta^2)$ contamination, drops out. Extrapolating the sum rule to $\Delta^2 = 0$, the total quark (and hence quark orbital) contribution to the nucleon spin is obtained. By forming still higher moments, one gets form factors of various high-spin operators.

There are a lot of theoretical and experimental questions about DVCS. Theoretical questions include: is there a factorization theorem for DVCS? is there an Altarelli-Parisi equation for the OFPDs evolution? what is the small $x$ and $\xi$ behavior? how to extrapolate the form factors to $\Delta^2 = 0$? Experiment-related questions include: how big is the cross section? will the Bethe-Heitler process overshadow DVCS? what kinematic region corresponds to DVCS? does one need polarizations of beam? target? how practical is to form sum rules? etc. Some of these questions have been answered in recent papers [10,11]. Others are open.

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