Vibration Based Tool Condition Monitoring in Milling of Ti-6Al-4V using an Optimization Model of GM(1,N) and SVM

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Vibration based tool condition monitoring in Milling of Ti-6Al-4V using an optimization model of GM(1,N) and SVM

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Abstract
Titanium alloys are the difficult to cut metals due to their low thermal conductivity and chemical affinity with tool material. Since the tool vibration is a replica of tool wear and surface roughness, the present study has proposed a methodology for estimating tool wear and surface roughness based on tool vibration for milling of Ti-6Al-4V alloy using cemented carbide mill cutter. Experiments are conducted at optimum levels of cutting speed, feed per tooth and depth of cut and experimental results for the tool vibration, tool wear and surface roughness are collected until the flank wear reached 0.3 mm (ISO3685:1993). In the next stage, an optimization model of grey prediction GM(1,N) system and support vector machine (SVM) are used and estimated tool wear and surface roughness related to tool vibration. The predicted values of tool wear and surface roughness are compared with the experimental results. The optimization model of GM(1,N) predicted the tool wear and surface roughness with an average error of 3.03% and as 0.7% respectively while the SVM predicted with an average error of 7.67% and 4.45% respectively.

Keywords: Tool condition monitoring; Grey model; Support vector machine; Tool vibration; Tool wear

Introduction
Forecasting of cutting tool life in manufacturing is always an important issue to control quality of machining and production cast. Tool Condition Monitoring (TCM) techniques are used in the manufacturing to determine tool life so that it replaces the worn tool with a new tool at the right time.
Since online measurement of tool wear and surface roughness is difficult during the machining process, vibration based TCM is used to identify tool wear (Prasad et al. 2010). During the machining process, the tool vibration is depending on the tool wear and increases as the tool wear progresses (Prasad and Babu 2017; Venkatarao and Murthy, 2018; Hui et al. 2019; Suyama and Diniz 2020). Modeling techniques like response surface methodology (RSM), mathematical modeling, artificial neural networks (ANN), support vector machine (SVM) and finite element modeling (FEM) are used to predict machining characteristics like tool life, surface roughness, tool vibration, cutting forces and power at different working conditions. But, these modeling techniques have not considered the effect of one machining characteristic on the other machining characteristics. In the present study, an optimized grey model of GM(1,N) and SVM are used to forecast the tool wear and surface roughness using the tool vibration data.

Tool wear is one of the important machining characteristics which influences machining efficiency. Huang et al. (2020) developed a tool wear prediction model using a deep convolutional neural network (NN) and predicted the tool flank wear in milling process. Xu et al. (2020) developed a hybrid method by combining an adaptive neuro fuzzy inference system and vibration based communication particle swarm optimization and predicted tool wear in machining of compacted graphite iron. Wang et al. (2017) developed a SVM based TCM system and predicted tool life using cutting force signals collected at different tool wear states in milling of titanium alloy. Venkatarao and Murthy (2018) predicted surface roughness and tool vibration using ANN, RSM and SVM and reported that the SVM predicted the tool vibration and surface roughness with good accuracy than ANN and RSM. Gupta and Sing (2020) used ANN method in turning process and reduced tool chatter while increasing metal removal rate. However, the existing prediction methods developed prediction models without considering effect of one output characteristic on the remaining. Among the different prediction methods discussed above, the SVM method establishing relationship between dependent and independent variable with less data and incomplete information. In the present study, the SVM technique is adopted to model flank wear and surface roughness.

Since the grey prediction model is a promising alternative to time-series forecasting, the researchers have been using the grey prediction models in many engineering problems to forecast responses (Xia and Wong, 2014). The grey theory was introduced by Deng in 1982 for forecasting of responses (Deng, 1982). In the grey theory, the random numbers are considered as grey numbers in the grey process. In the grey prediction model GM(1,1), an accumulated generating operation (AGO) is associated with inverse accumulated generating operation (IAGO). The GM(1,1) uses a first order differential equation to develop the prediction model for the responses. Around four and above discrete
data with poor information is sufficient to develop a grey prediction model. This method helps the decision makers to forecast the responses when limited previous data is available (Deng, 1982; Xia and Wong, 2014). Xia and Wong (2014) used the GM(1,1) and predicted fashion retail sales with limited previous sales data. Liu et al. (2014) used GM(1,1) model and predicted energy consumption in micro electric discharge machine. Lin et al. (2020) developed a grey prediction model for cutting tool wear in machining of vermicular graphite cast iron using coated ceramic tools. They predicted flank wear with prediction error less than 5%. However, the researchers found difficulties with conventional GM(1,1) model because this model only considers one independent variable.

The conventional grey prediction model GM(1,1) was modified in different ways to overcome difficulties during the prediction. Zhou and He (2013) found short comes of the conventional GM(1,1) model and developed a generalized GM(1,1) model. They predicted the fuel production using the generalized GM(1,1) with good prediction accuracy. Wang et al. (2018) developed a seasonal grey prediction model SGM(1,1) and predicted electric power consumption. Ding et al. (2018) developed a novel optimized grey prediction model GM(1,1) considering new information priority principle and predicted energy consumption in china during 2012-2014. However, these modified GM(1,1) models also consider only one variable in estimating responses and are not appropriate when multivariate.

Since the GM(1,1) models are not suitable for multi variable problems, the Deng introduced a GM(1,N) model in 1988 to predict a response using independent multivariable (Deng, 1988). Chian and Chang (2007) developed a residual modified grey dynamic model RGM(1,3) for three variables and predicted metal removal rate, electrode wear ratio and surface roughness with high accuracy in wire electric discharge machining. Tien (2012) studied accuracy of GM(1,1) and GM(1,N) in different case studies and concluded that the GM(1,N) is incorrect. Zeng et al. (2016) stated that the GM(1,N) is associated with three defects such as parameter estimation, model structure and modeling mechanism. He proposed an optimization model for the GM(1,N) model called as OGM(1,N) by introducing a grey action quantity and a linear correction term and improved performance of GM(1,N). Wu et al. (2015) developed a novel GM(1,N) model considering opposite direction AGO and predicted CO₂ emission in BRICS countries. Ye et al. (2020) developed a novel accumulative time-delay GM(1,N) model and analyzed the CO₂ emissions from the transport vehicles in china. Based on the above studies, it is observed that the multivariable grey prediction models are developed by different authors and reduced errors in traditional GM(1,N) model. In the present study, the OGM(1,N) model is proposed to predict tool wear and surface roughness considering tool vibration data and length of machining.
Based on the literature, it has been summarized that the process parameters are optimized to improve machining performance like reducing the surface roughness, tool wear, tool vibration, cutting forces and power consumption. In the current research, the experiments are conducted at different working conditions and the responses are collected. Based on the experimental results, the process parameters are optimized using different optimization techniques. But, the surface roughness, tool wear, tool vibration, cutting forces and power consumption are not studied at the optimal working condition. In the present study, experiments are conducted at optimal condition and the surface roughness, tool wear and tool vibrations are analyzed. The OGM(1,N) model and SVM techniques are used and predicted tool wear and surface roughness considering to the tool vibration data and length of machining in milling of Ti-6Al-4V.

**Experimentation**

In the present study, milling experiments are conducted on Ti-6Al-4V alloy with 10 mm diameter cemented carbide end mill cutter. Four fluted mill cutter with 5° of rake angle, 20° of helix angle, 0.04 mm of cutting edge radius and 0.8 mm of edge radius is used in the experimentation. The Ti-6Al-4V alloy is an important alloy among all the titanium alloys which is used in many engineering applications. Superior properties like high corrosion resistance, light weight and formability made the Ti-6Al-4V alloy suitable in aircraft, marine, chemical and medical applications. Chemical composition of the Ti-6Al-4V is presented in Table 1. Since the Ti-6Al-4V alloy has high temperature strength, low thermal conductivity, low elastic modulus and serrated chip formation, machining of this alloy becomes difficult. In addition, chemical affinity with the tool material also makes the machining difficult. Hence, the present study is aimed to study flank wear and surface roughness and prediction models are also developed for flank wear and surface roughness.

**Table 1 Chemical composition of Ti-6Al-4V (%wt)**

|   | Al  | V   | Fe  | Si  | C   | N   | H   | O   | Ti  |
|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Al | 5.8 | 0.18| 0.01| 0.02| 0.023| 0.01| 0.17 | 89.6|

Milling experiments are carried out on 90x70x10 mm size work piece on BFW Chandra plus compute numerical control (CNC) vertical machining center as shown in the Fig. 1. Twenty seven experiments are conducted on the workpiece at three levels of cutting speed, feed per tooth and depth of cuts as shown in Table 2. The following procedure is followed during the experimentation:

a) A poly Tech 100-V type laser Doppler vibrometer is used and measured amplitude of cutter vibration (A) as shown in the Fig. 2.
b) After machining, the workpiece and mill cutter are removed and surface roughness (Ra) and flank wear (VB) are measured using Mitutoyo 178-923E SJ210 type profilometer and microscope respectively.

Table 2. Process parameters with levels

| Process parameter          | Level-1 | Level-2 | Level-3 |
|----------------------------|---------|---------|---------|
| Cutting speed (m/min)      | 100     | 120     | 140     |
| Feed per tooth (mm/tooth)  | 0.1     | 0.15    | 0.2     |
| Depth of cut (mm)          | 0.4     | 0.8     | 1.2     |

Fig. 1 Experimental setup for Milling

The process parameters are optimized using RSM as 120 m/min of cutting speed, 0.14 mm/tooth of feed and 0.7 mm of depth of cut. At the optimal working condition, experiments are conducted in ten passes (length of each pass is 90 mm) with the same tool. After each pass, surface roughness and flank
wear are measured as presented in Table 3. However, amplitude of cutter vibration is measured during the experiment and presented in Table 3.

![Fig. 2 Measurement of amplitude of cutter vibration](image)

| k  | L (mm) | A (mm) | VB (mm) | Ra (mm) |
|----|--------|--------|---------|---------|
| 1  | 90     | 0.0092 | 0.045   | 0.00150 |
| 2  | 90     | 0.0093 | 0.118   | 0.00168 |
| 3  | 90     | 0.0095 | 0.154   | 0.00172 |
| 4  | 90     | 0.0117 | 0.192   | 0.00180 |
| 5  | 90     | 0.0124 | 0.213   | 0.00193 |
| 6  | 90     | 0.0136 | 0.237   | 0.00198 |
| 7  | 90     | 0.0158 | 0.256   | 0.00241 |
| 8  | 90     | 0.0185 | 0.278   | 0.00283 |
| 9  | 90     | 0.0251 | 0.317   | 0.00325 |
| 10 | 90     | 0.0315 | 0.362   | 0.00379 |

**Grey prediction model**

Since the traditional ANN, Fuzzy and other modeling techniques require large data, application of these techniques may not be appropriate in manufacturing (Zeng et al., 2016). But, the grey model develops prediction models with small amount of previous data. The measured time series data in the milling may be random, but the degree of randomness is reduced when the AGO is used (Xia and Wang,
The proposed grey prediction model constructs differential equations using the accumulated generation. The GM(1,1) is the basic grey prediction model of 1\textsuperscript{st} order single variable. From the basic model, the GM(1,N) model is developed which indicates that the model is built with three variable (independent and dependent variables). The GM(1,N) model is described as follows:

The original sequence of a variable/machining characteristic with respect to time or length of machining is defined as follows (Liu and Lin, 2010; Zeng et al., 2016; Wei et al. 2019):

\[ X_i^{(0)} = (X_i^{(0)}(1), X_i^{(0)}(2), ...., X_i^{(0)}(m)) \]  

where \( m \) is the number of measurements or data collected in a working condition at regular time intervals.

let \( X_i^{(0)} \) \((i = 2, 3, ..., N)\) are the sequences of variables/machining characteristics.

Then the 1-AGO series of the \( X_1^{(0)} \) is defined as follows:

\[ X_j^{(1)} = (X_j^{(1)}(1), X_j^{(1)}(2), ...., X_j^{(1)}(m)) \]  

if the \( N = 3 \), the system consists of three variables including independent and dependent variables, the sequences are defined as follows:

Variable 1

\[ X_1^{(1)} = (X_1^{(1)}(1), X_1^{(1)}(2), ...., X_1^{(1)}(m)) \]  

Variable 2

\[ X_2^{(1)} = (X_2^{(1)}(1), X_2^{(1)}(2), ...., X_2^{(1)}(m)) \]  

Variable 3

\[ X_3^{(1)} = (X_3^{(1)}(1), X_3^{(1)}(2), ...., X_3^{(1)}(m)) \]  

where

\[ X_j^{(1)}(k) = \sum_{l=1}^{k} X_j^{(0)}(l), k = 1, 2, ..., m \]  

The \( Z_j^{(1)} \) is the mean sequence generated from \( X_j^{(1)}(k) \), then the sequence for the three variables are given below

\[ Z_j^{(1)} = (Z_j^{(1)}(2), Z_j^{(1)}(3), ..., Z_j^{(1)}(m)), \text{if } N = 3, \text{then } j = 1, 2 \text{ and } 3 \]  

where

\[ Z_j^{(1)}(k) = \frac{(X_j^{(1)}(k-1)+X_j^{(1)}(k))}{2}, k = 2, 3, ..., m \], then the GM(1,N) model is defined as follows(Liu and Lin, 2010; Zeng et al., 2016)

\[ X_i^{(1)}(k) + aZ_j^{(1)}(k) = \sum_{j=2}^{N} b_j X_j^{(1)}(k) \]  

Then the parameter sequence of GM(1,N) is estimated as follows:
\( \hat{a} = [a, b_1, b_2, ..., b_N]^T \) \hspace{1cm} (8)

where \( a \) is coefficient of the system and the \( b_1, b_2, ..., b_N \) are the driving coefficients, then

\[ \hat{a} = (B^T B)^{-1} B^T Y \] \hspace{1cm} (9)

where

\[
B = \begin{bmatrix}
-\text{Z}_1^{(1)}(2) & -\text{Z}_2^{(1)}(2) & ... & -\text{Z}_N^{(1)}(2) \\
-\text{Z}_1^{(1)}(3) & -\text{Z}_2^{(1)}(3) & ... & -\text{Z}_N^{(1)}(3) \\
& & \vdots & \vdots \\
-\text{Z}_1^{(1)}(m) & -\text{Z}_2^{(1)}(m) & ... & -\text{Z}_N^{(1)}(m)
\end{bmatrix},
Y = \begin{bmatrix}
X_1^{(0)}(2) \\
X_1^{(0)}(3) \\
\vdots \\
X_1^{(0)}(m)
\end{bmatrix}
\] \hspace{1cm} (10)

As the optimization model of the conventional GM(1,N) is predicting the responses with improved accuracy (Zeng et al., 2016), the OGM(1,N) has been used in the present study and predicted the tool vibration and surface roughness. Then the OGM(1,N) is described as follows:

\[ X_1^{(0)}(k) + a Z_1^{(1)}(k) = \sum_{i=2}^{N} b_i X_i^{(1)}(k) + h_1(k - 1) + h_2 \] \hspace{1cm} (11)

where \( h_1(k - 1) \) and \( h_2 \) are the linear correction and grey action quantum term respectively. Then the parameter sequence of OGM(1,N) is estimated as follows:

\[ \hat{r} = [b_1, b_2, ..., b_N, a, h_1, h_2]^T \]

where \( r \) is the coefficient of the OGM(1,N) system, then

\[ \hat{r} = (B^T B)^{-1} B^T Y \] \hspace{1cm} (12)

\[
B = \begin{bmatrix}
X_2^{(1)}(2) & X_3^{(1)}(2) & ... & X_N^{(1)}(2) & -\text{Z}_1^{(1)}(2) & 1 & 1 \\
X_2^{(1)}(3) & X_3^{(1)}(3) & ... & X_N^{(1)}(3) & -\text{Z}_1^{(1)}(3) & 2 & 1 \\
& & \vdots & \vdots & \vdots & \vdots & \vdots \\
X_2^{(1)}(m) & X_3^{(1)}(m) & ... & X_N^{(1)}(m) & -\text{Z}_1^{(1)}(m) & m & 11
\end{bmatrix},
Y = \begin{bmatrix}
X_1^{(0)}(2) \\
X_1^{(0)}(3) \\
\vdots \\
X_1^{(0)}(m)
\end{bmatrix}
\] \hspace{1cm} (13)

Then the responses are predicted using the below equation:

\[ \hat{X}_1^{(1)}(k) = \mu_1 \sum_{i=2}^{N} b_i X_i^{(1)}(k) + \mu_2 X_1^{(1)}(k - 1) + \mu_3 k + \mu_4, k=2, 3, ... \] \hspace{1cm} (14)

Let

\[
\mu_1 = \frac{1}{1+0.5a}, \mu_2 = \frac{1-0.5a}{1+0.5a}, \mu_3 = \frac{h_1}{1+0.5a}, \mu_4 = \frac{h_2-h_1}{1+0.5a}
\]

Support Vector Machine

As the ANN requires large data to construct prediction models, it is not possible to adopt it to the problems having less number of data. The SVM is one of the forecasting techniques capable of predicting the responses with good accuracy using less number of data (Zhang et al., 2010; Venkatarao and Murthy, 2018). The SVM technique was introduced by Vanpik in 1998 to classify regression problems having good generalization (Vapnik (1998). Structure risk minimization principle in SVM made it superior in forecasting
the response over the conventional methods. The linear function for the responses is formulated in the high dimensional feature space as follows (Zhang et al. 2010).

\[ y(x) = w^T \varphi(x) + b \]  

(15)

where, \( \varphi(x) \) is the high dimensional feature space non linearly mapped from the input space \( x \). The weight vector \( w \) and bias \( b \) are estimated by minimizing.

\[ R(c) = C \frac{1}{n} \sum_{i=1}^{n} L(d_i, y_i) + \frac{1}{2} \|w\|^2 \]  

(16)

where,

\[ L_\epsilon(d, y) = \begin{cases}  
|d - y| - \epsilon, & |d - y(x)| \geq \epsilon \\
0, & \text{otherwise} 
\end{cases} \]  

(17)

\( L(d, y) \) is the \( \epsilon \)-intensive loss function. The \( C \frac{1}{n} \sum_{i=1}^{n} L(d_i, y_i) \) is the empirical error and the \( \frac{1}{2} \|w\|^2 \) is the smoothness. The \( C \) and \( \epsilon \) are prescribed parameters. In the present study, the flank wear and the surface roughness are predicted using the SVM technique.

The SVM models for the flank wear and surface roughness are developed separately using rapidminer 5.0 software to predict them. In order to improve performance of the SVM technique, a nonlinear dot function kernel was selected for the two responses. The \( C \) and \( \epsilon \) parameters are optimized as 7500 and 0.015 respectively for the tool flank wear and 6000 and 0.001 respectively for the surface roughness in the rapidminor 5.0 software itself. In the next stage, the two models are trained with experimental data presented in Table 3 and predicted the tool flank wear and surface roughness. The predicted values are presented in table 4.

**Prediction of tool vibration and surface roughness using OGM(1,N) model**

In the present study, the experiments are conducted at optimal working condition. It is observed that the flank wear progresses as the machining time/length of machining increased. Since online measurement of flank wear and surface roughness is difficult while machining, amplitude of cutter vibration is taken as replica of the tool wear and surface roughness. The length of machining and flank wear are considered as independent variable and the amplitude of cutter vibration and the surface roughness are considered as dependent variables. The first six experimental data in the series are taken in estimation of parameter sequence and then tool wear/failure is predicted.

**Prediction of tool wear**

The data series for the length of cut (\( X_2 \) as \( L \)), flank wear (\( X_3 \) as \( A \)) and amplitude of cutter vibration (\( X_1 \) as \( VB \)) are defined as follows.

\[ VB^{(0)} = (VB^{(0)}(1), VB^{(0)}(2), VB^{(0)}(3), VB^{(0)}(4), VB^{(0)}(5), VB^{(0)}(6)) \]
The parameter sequence is \( \hat{r} = [b_1, b_2, a, h_1, h_2]^T \) is estimated using \( \hat{r} = (B^T B)^{-1} B^T Y \).
Then,
\[ \mu_1 = \frac{1}{1 + 0.5a} = 7.887 \times 10^{-8} \]
\[ \mu_2 = \frac{1 - 0.5a}{1 + 0.5a} = -1 \]
\[ \mu_3 = \frac{h_1}{1 + 0.5a} = -0.3142 \]
\[ \mu_4 = \frac{h_2 - h_1}{1 + 0.5a} = -0.28714 \]

The tool flank wear is predicted using the equation (18)

\[ VB^{(1)}(k) = \mu_1 \left( b_1 L^{(1)}(k) + b_2 A^{(1)}(k) \right) + \mu_2 V B^{(0)}(k - 1) + k \mu_3 + \mu_4 \] (18)

\[ VB^{(1)}(2) = 0.1532 \text{ mm} \]
\[ VB^{(0)}(2) = 0.1082 \text{ mm} \]

Similarly, the flank wear is predicted at various lengths of machining and amplitude of cutter vibration at \( k = 3, 4, 5, 6, 7, 8, 9 \) and 10 and presented in Table 4.

**Prediction of surface roughness**

The data series for the surface roughness (\( X_1 \) as Ra) is defined as follows:

\[ Ra^{(0)} = (Ra^{(0)}(1), Ra^{(0)}(2), Ra^{(0)}(3), Ra^{(0)}(4), Ra^{(0)}(5), Ra^{(0)}(6)) \]
\[ = (0.0015, 0.00168, 0.00172, 0.0018, 0.00193, 0.00198) \]

The 1-AGO sequences for the length of cut, flank wear and amplitude of tool vibration are given below

\[ Ra^{(1)} = (Ra^{(1)}(1), Ra^{(1)}(2), Ra^{(1)}(3), Ra^{(1)}(4), Ra^{(1)}(5), Ra^{(1)}(6)) \]
\[ = (0.00115, 0.00283, 0.00455, 0.00635, 0.00828, 0.01026) \]

The mean sequence \( Z_1^{(1)} \) for the amplitude of tool vibration is generated as follows

\[ Z_1^{(1)} = (Z_1^{(1)}(2), Z_1^{(1)}(3), Z_1^{(1)}(4), Z_1^{(1)}(5), Z_1^{(1)}(6)) \]
\[ = (0.00199, 0.00369, 0.00545, 0.00732, 0.00927) \]

the parameters are estimated as follows

\[
B = \begin{bmatrix}
L^{(1)}(2) & A^{(1)}(2) & -Z_1^{(1)}(2) & 1 & 1 \\
L^{(1)}(3) & A^{(1)}(3) & -Z_1^{(1)}(3) & 2 & 1 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
L^{(1)}(m) & A^{(1)}(m) & -Z_1^{(1)}(m) & m - 1 & 1
\end{bmatrix},
Y = \begin{bmatrix}
Ra^{(0)}(2) \\
Ra^{(0)}(3) \\
\vdots \\
Ra^{(0)}(m)
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
180 & 0.0185 & -0.00199 & 11 & 1 \\
270 & 0.0280 & -0.00369 & 21 & 1 \\
360 & 0.0397 & -0.00545 & 31 & 1 \\
450 & 0.0521 & -0.00732 & 41 & 1 \\
540 & 0.0657 & -0.00927 & 51 & 1
\end{bmatrix},
Y = \begin{bmatrix}
0.00168 \\
0.00172 \\
0.00180 \\
0.00193 \\
0.00198
\end{bmatrix}
\]

The parameter sequence is \( \hat{r} = [b_1, b_2, a, h_1, h_2]^T \) is estimated using \( \hat{r} = (B^T B)^{-1} B^T Y \)
\[(B^T B)^{-1}B^T Y = \begin{bmatrix} 4.871E - 07 \\ 0.07928716 \\ 0.37939473 \\ -0.0002412 \\ 0.00112063 \end{bmatrix} \]

\[b_1 = 4.871E-07, \; b_2 = 0.07928716, \; a = 0.37939473, \; h_1 = -0.0002412, \; h_2 = 0.00112063\]

Then,
\[\mu_1 = \frac{1}{1+0.5a} = 0.8407\]
\[\mu_2 = \frac{1-0.5a}{1+0.5a} = 0.6811\]
\[\mu_3 = \frac{h_1}{1+0.5a} = -0.0002027\]
\[\mu_4 = \frac{h_2-h_1}{1+0.5a} = 0.0011448\]

The surface roughness is predicted using the equation (19)
\[Ra^{(1)}(k) = \mu_1 \left( b_1 L^{(1)}(k) + b_2 A^{(1)}(k) \right) + \mu_2 Ra^{(0)}(k - 1) + k\mu_3 + \mu_4 \] (19)
\[Ra^{(1)}(2) = 0.00283 \text{ mm}\]
\[Ra^{(0)}(2) = 0.00168 \text{ mm}\]

Similarly, the surface roughness is predicted at various levels of length of machining and amplitude of cutter vibration at \(k = 3, 4, 5, 6, 7, 8, 9\) and \(10\) and presented in Table 4.

| \(k\) | \(L\) (mm) | \(A\) (mm) | \(VB\) (mm) | \(Ra\) (mm) |
| --- | --- | --- | --- | --- |
|  | \(X_2^{(0)}\) | \(X_3^{(0)}\) | \(X_1^{(0)}\) | \(X_1^{(0)}\) |
|  | Exp. | OGM(1,N) | SVM | Exp. | OGM(1,N) | SVM |
| 1 | 90 | 0.0092 | 0.045 | 0.045 | 0.045 | 0.00115 | 0.00115 | 0.00115 |
| 2 | 90 | 0.0093 | 0.118 | 0.108 | 0.117 | 0.00168 | 0.00168 | 0.00165 |
| 3 | 90 | 0.0095 | 0.154 | 0.153 | 0.142 | 0.00172 | 0.00170 | 0.00177 |
| 4 | 90 | 0.0117 | 0.192 | 0.204 | 0.128 | 0.00180 | 0.00181 | 0.00184 |
| 5 | 90 | 0.0124 | 0.213 | 0.219 | 0.223 | 0.00193 | 0.00190 | 0.00198 |
| 6 | 90 | 0.0136 | 0.237 | 0.245 | 0.259 | 0.00198 | 0.00196 | 0.00214 |
| 7 | 90 | 0.0158 | 0.256 | 0.250 | 0.246 | 0.00241 | 0.00240 | 0.00250 |
| 8 | 90 | 0.0185 | 0.278 | 0.271 | 0.268 | 0.00283 | 0.00285 | 0.00291 |
| 9 | 90 | 0.0251 | 0.317 | 0.309 | 0.296 | 0.00325 | 0.00322 | 0.00378 |
| 10 | 90 | 0.0315 | 0.362 | 0.357 | 0.356 | 0.00379 | 0.00376 | 0.00394 |
Error analysis for the flank wear and surface roughness is performed by comparing the predicted values with the experimental results. The OGM(1,N) and SVM predicted the flank wear with error of 3.03% and 7.67% respectively, compared with experimental flank wear. The OGM(1,N) and SVM predicted the surface roughness with error of 0.7% and 4.45% respectively, compared with experimental surface roughness. Based on the error analysis, it is found that the OGM(1,N) predicted the flank wear and surface roughness with less error when compared with the SVM methodology. Prediction error of OGM(1,N) and SVM for the tool flank wear and surface roughness are shown in the Fig.s 3(a) and (b) respectively. As shown in the Fig. 3, six experimental data in the series is used for modeling of flank wear and surface roughness and the remaining data is used for testing. It is observed that the flank wear and surface roughness predicted by OGM(1,N) are found to be very close with the experimental result than the SVM method. The flank wear and surface roughness estimated by OGM(1,N) are found to be much closer to the experimental result than the SVM method.

![Fig. 3(a) Experimental and OGM(1,N) and SVM predicted values of tool wear](image-url)
Results and discussion

At optimal working condition, 10 experiments are conducted with machining length of 90 mm and flank wear is measured on a machine vision system. Flank wear after 90 mm, 270 mm, 630 mm and 900 mm length of machining is measured as shown in the Fig. 4.
Machinability of a metal is decided by behavior of tool wear during the machining process especially in hard metals. The tool wear in machining of Ti-6Al-4V alloy is caused by tool failure mechanisms namely catastrophic failure, chipping and notching (Agarwal et al., 2020; Venugopal et al., 2007). During the machining of Ti-6Al-4V alloy, temperature in the deformation zone increased when the shear strain rate increased in the deformation zone. Since the tool wear directly affects the machining performance and production cast, analysis of the tool flank wear is given more priority. In the present study, milling experiments are conducted at optimal working condition until the tool flank wear reaches the 0.3 mm (ISO3685:1993). As Fig.s 4 and 5, the flank wear progresses related to length of machining and found that the flank wear increased as the length of cutting increased. During the initial stage of machining (up to 180 mm of length of machining), there is sharp wear on the flank which is measured as 0.118 mm. In the next stage, there is uniform wear on the flank up to 810 mm of machining length and the flank wear reached 0.309 mm. After 810 mm of machining length, there is rapid wear on the flank and the tool is said to worn out or failed. When the tool wear reached 0.3 mm, there is significant temperature rise in the deformation zone which makes the metal soft and caused chemical and adhesive wear on the flank (Huang et al., 2020).

![Fig. 5 Progress of wear with respect to length of cut](image)

**Effect of flank wear on the tool vibration and surface roughness at optimal working condition is shown in the Fig.6.** The graph in the Fig. 6 constructed by taking flank wear on the X-coordinate and the tool vibration and the surface roughness are taken on the Y-coordinate. Normalized values of the amplitude of the cutter vibration and the surface roughness are taken to illustrate the effect of flank wear on them simultaneously. It is observed that the flank wear has the similar effect on the tool vibration as
well as surface roughness. As shown in the Fig. 5, the flank wear increased when the length of machining is increased. Similarly, the tool vibration and surface roughness increased along with flank wear when the length of machining increased. At different wear zones, the surface roughness and tool vibrations are analyzed. As shown in the Fig.6, the surface roughness and tool vibrations are almost flat until the flank wear reaches 0.15 mm. After that the surface roughness and tool vibration increased uniformly until the flank wear reaches 0.3 mm. There is rapid growth in the surface roughness and tool vibrations when the flank wear crossed 0.3 mm. The flank wear caused friction between the tool and the machined surface which resulted surface roughness and tool vibration (Prasad and Babu, 2017; Mamalis et al., 2008)

![Fig. 6 Flank wear Vs amplitude of cutter vibration and surface roughness](image)

**Conclusions**

The present study proposed a vibration based tool condition monitoring approach using prediction models such as OGM(1,N) and SVM in milling of Ti-6Al-4V alloy using cemented carbide end mill cutter at optimum working condition. The following conclusions are drawn from this work:

- The proposed methodology estimated flank wear/tool failure and surface roughness based amplitude of tool vibration measured online. The tool wear and surface roughness are estimated using OGM(1,N) and SVM methods.
- The OGM(1,N) and SVM predicted the flank wear with error of 3.03% and 7.67% respectively, compared with experimental flank wear. The OGM(1,N) and SVM predicted the surface roughness with error of 0.7% and 4.45% respectively, compared with experimental surface roughness. The OGM(1,N) predicted the flank wear and surface roughness with less error when compared with the SVM methodology.
Based on the cutter vibration, flank wear and surface roughness, optimal working condition is identified by RSM as 120 m/min of cutting speed, 0.14 mm/tooth of feed and 0.7 mm of depth of cut.

During the initial stage of machining (up to 180 mm of length of machining), there is sharp wear with 0.118 mm on the flank. In the next stage, the flank wear progressed uniformly to 0.309 mm as the length of machining increased to 810 mm. After 810 mm of length of machining, there is rapid wear on the flank.

The tool vibration and surface roughness increased along with flank wear when the length of machining increased. There is no increase in the surface roughness and tool vibrations until the flank wear reaches 0.15 mm. After that the surface roughness and tool vibration increased uniformly until the flank wear reached 0.3 mm. There is rapid growth in the surface roughness and tool vibrations when the flank wear crossed 0.3 mm.

Declarations

Ethics approval and consent to participate
Not applicable

Consent for publication
Not applicable

Author’s contribution Statement

Dr. K. Venkatarao
Dr. K Venkata Rao designed the experimental plan and supervised the experimentation. Dr. Rao also involved in optimization of process parameters and writing the manuscript. In addition, tool wear in the study were studied.

Dr. Y Prasanna Kumar and Dr. L Suvarna Raju
Dr. Y Prasanna Kumar and Dr. L Suvarna Raju proposed and executed the methodology for tool condition monitoring using the grey prediction model.

Dr. Vijay Kumar Singh and Mr. Jinka Ranganayakulu
Dr. Vijay Kumar Singh and Mr. Jinka Ranganayakulu involved in experimentation and collected experimental results and also involved in writing the manuscript

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Competing interests
Conflict of Interest for all authors is None.

Availability of data and materials
The authors confirm that the data supporting the findings of this study are available within the article.
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Figure 1

Experimental setup for Milling
Figure 2

Measurement of amplitude of cutter vibration
Figure 3

(a) Experimental and OGM(1,N) and SVM predicted values of tool wear. (b) Experimental and OGM(1,N) and SVM predicted values of surface roughness.
Figure 4

Progress of tool flank wear at different conditions
Figure 5

Progress of wear with respect to length of cut
Figure 6

Flank wear Vs amplitude of cutter vibration and surface roughness