Visco-elastic fluid simulations of coherent structures in strongly coupled dusty plasma medium

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Abstract

A generalized hydrodynamic (GHD) model depicting the behaviour of visco-elastic fluids has often been invoked to explore the behaviour of a strongly coupled dusty plasma medium below their crystallization limit. The model has been successful in describing the collective normal modes of the strongly coupled dusty plasma medium observed experimentally. The paper focuses on the study of nonlinear dynamical characteristic features of this model. Specifically, the evolution of coherent vorticity patches are being investigated here within the framework of this model. A comparison with Newtonian fluids and Molecular Dynamics (MD) simulations treating the dust species interacting through the Yukawa potential has also been presented.

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I. INTRODUCTION

The understanding of a strongly coupled state of matter is a problem of frontier research in physics [1-3]. While such a state of matter can be found only under extreme conditions of temperature and densities, the dusty plasma proves to be a unique system where the strong coupling conditions can be achieved easily even at normal pressure and temperature due to the high charge it possesses. This has attracted significant research interest in the field of dusty plasma medium [4, 5].

A phenomenological visco-elastic fluid model, known as the Generalized Hydrodynamic (GHD) model, has often been invoked for the study of the dusty plasma medium in the intermediate coupling regime of $1 < \Gamma < 170$, (here $\Gamma$ is the ratio of the inter-particle potential energy to the kinetic energy) [6, 7]. In this regime the dust species do not crystallize and neither do they behave like normal Newtonian fluids. This model has been successful in predicting the dispersion characteristics of the transverse shear (TS) wave in the medium [7, 8], which have been experimentally demonstrated [9]. The mode dispersion has also been obtained within the Molecular Dynamics (MD) simulations which treat the dust particles as interacting through Yukawa potential [10].

The purpose of the paper is to investigate the nonlinear regime of the GHD model. In particular we intend to study the evolution and interaction of coherent structures which have an important role to play in defining transport properties of any medium. The elasticity effects in GHD is introduced through a memory relaxation parameter $\tau_m$ [7, 11, 12] in the evolution equation for velocity.

As the governing dynamics of strongly coupled dusty plasma medium is different than that of Newtonian fluids, the existence of various coherent structures as well as their stability and evolution in such medium may have significant differences. We have chosen an important phenomena of vortex evolution and addressed its application to strongly coupled dusty plasma medium. In past the phenomena of vortex evolution has been studied in a range of physical systems like hydrodynamic fluids [13], planetary atmospheres and their convective interiors, electron plasma [14, 15], etc. The dust vortices have also been studied in presence as well as in absence of magnetic field [16, 21]. Recently Yoshifumi et al. have presented interesting experimental results on dust rotation [22]. Konopka et al. and Sato et al. have shown the rotation of dust particles experimentally in the magnetized dusty plasma [23, 24].
Schwabe et al. reported in their dusty plasma experiment, formation of variety of rotating
dust structures depending on varying magnetic field strength [25] and further numerically
studied the vortex movements by adding some micro particles around the void in complex
plasma simulation [26]. The coherent solutions in the form of tripolar vortex have also been
studied theoretically in the context of dusty plasma [27].

We present the 2-D numerical simulation studies of the GHD equations to study the
evolution of various configurations of vorticity patches. It is observed that in contrast to
Newtonian fluids there is an emission of transverse shear waves from the vorticity patches
due to which the strength of the coherent structure gradually diminishes. An interesting
observation is that the interplay between the emitted transverse shear wave and the coherent
vorticity structures ultimately results in a proper mixing of the fluid.

The manuscript has been organized as follows. Section II contains the details of the
governing equations. In section III a brief description of the numerical approach and some
simplified cases of simulations with expected results have been discussed which validate the
numerical code. In section IV we present the evolution of various different configuration of
vorticity patches. In particular the role of transverse shear wave in the evolution is clearly
demonstrated. For certain sharp vorticity profiles the interplay of the Kelvin Helmholtz
instability and the transverse shear wave has also been shown. The comparison with the
evolution of Newtonian fluids show that in all cases the GHD fluid shows a better mixing. In
section V a comparison is provided with the Molecular Dynamics (MD) simulation results.
Finally, section VI contains the discussion and conclusion.

II. GOVERNING EQUATIONS

A typical plasma which is a collection of electrons and ions, is generally found in a state of
weak coupling wherein $\Gamma$ the coupling parameter defining the ratio of average inter-particle
potential energy to the average thermal energy associated with the particles, is less than
unity. However, when micron sized dust particles are sprinkled in such a plasma, it typically
acquires a very high charge due to constant impingement and attachment of the lighter
electron and ion species. In general the lighter electron flux on the dust grain surface is
comparatively higher than that of the ion flux, causing the dust species to acquire a net
negative charge. In such a system the dust component acts as a third species with a very
high negative charge. Due to the large charge on the dust particles, this species can often be easily found in the strong coupling regime corresponding to the coupling parameter $\Gamma \gg 1$. Ikezi et al. [28] have shown that when the coupling parameter $\Gamma \gtrsim 170$, the dust particles get organized in regular lattice points forming a Coulomb crystal. However, for $1 < \Gamma < 170$ the dusty plasma system has a behaviour which is intermediate to that of fluid and solids and has in the past been looked upon as a visco-elastic system. It is this regime of dusty plasma system that we would be exploring here. The visco-elastic response is typically modeled by a generalized hydrodynamic description in which the elasticity is represented by a memory relaxation parameter $\tau_m$ [9]. Generalized hydrodynamic (GHD) fluid model provides a description of dust fluid in both weak (simple charged fluid) and strong coupling limits (visco-elastic fluids). These two aspects are combined in terms of a characteristic time scale $\tau_m$ i.e. memory relaxation time.

The visco-elastic description of the electrostatic response of strongly coupled dusty plasma medium is provided by the following coupled set of continuity equation, the evolution of velocity through a Generalized Hydrodynamic description and the Poisson’s equation respectively.

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\vec{v}) = 0 \quad (1)$$

$$\left[ 1 + \tau_m \frac{d}{dt} \right] \left( \frac{n\vec{v}}{dt} + \nabla P - n\nabla \phi \right) = \eta \nabla^2 \vec{v} + \left( \zeta + \frac{\eta}{3} \right) \nabla (\nabla \cdot \vec{v}) \quad (2)$$

$$\nabla^2 \phi = \left[ n + \mu_e \exp(\sigma_e \phi) - \mu_i \exp(-\phi) \right] \quad (3)$$

Here, $\vec{v}, n$ and $\phi$ are the normalised dust fluid velocity, dust density and potential respectively in dusty plasma medium. The total time derivative is represented as $\frac{d}{dt} = (\frac{\partial}{\partial t} + \vec{v} \cdot \nabla)$. The above equations has been presented in normalised form [29] with parameters $\sigma_e = \frac{T_e}{T_i}, \mu_e = \frac{n_e e Z_d n_{s0}}{Z_d n_{s0}}$ and $\mu_i = \frac{n_i i Z_d n_{s0}}{Z_d n_{s0}}$. Dust particles carrying a negative charge $Z_d$ and $n_{s0}$ ($s = e, i, d$) denotes the normalizations of the number density of the specific species and the equilibrium quantities are denoted by the subscript 0. The memory effect related to elasticity is incorporated through a relaxation time parameter $\tau_m$, and $\eta, \zeta$ are the shear and bulk viscosity coefficients respectively. If $\tau_m \frac{d}{dt} < 1$, there are no memory effects and the equation of motion is that for a standard viscous fluid driven by self consistent electric and pressure fields and for $\tau_m \frac{d}{dt} \geq 1$, memory effects are strong (for time scales of interest, each fluid element remembers where it came from) and the viscosity coefficient $\eta$ becomes...
more like a non dissipative elastic coefficient supported by strong particle correlation. The inertia of electrons and ions is negligible at slow dust time scales and hence these species can be assumed to follow a Boltzmann distribution.

III. NUMERICAL IMPLEMENTATION AND VALIDATION

In this manuscript we concentrate on studying the in-compressible limit of the above set of equations i.e. we assume that $\nabla \cdot \vec{v} = 0$. The coupled set of governing eqs. (1)-(2) have been recast in the following form in this limit.

$$\frac{\partial \vec{\xi}}{\partial t} + (\vec{v} \cdot \nabla) \vec{\xi} = \nabla \times \vec{\psi}$$ (4)

$$\frac{\partial \vec{\psi}}{\partial t} + (\vec{v} \cdot \nabla) \vec{\psi} = \frac{\eta}{\tau_m} \nabla^2 \vec{v} - \frac{\vec{\psi}}{\tau_m}$$ (5)

Here $\eta$ is now termed as the kinematic viscosity, $\vec{\xi}$ is the vorticity, given as $\vec{\xi} = \nabla \times \vec{v}$ and the velocity at each time step is updated by using the poisson’s equation $\nabla^2 \vec{v} = -\nabla \times \vec{\xi}$. Also, $\vec{\psi} = \frac{d\vec{v}}{dt} + \nabla P_n - \nabla \phi$ and its curl gives eq. (4). It should be noted that in this particular limit there is nothing specific which is suggestive of the fact that the system corresponds to a strongly coupled dusty plasma medium. Thus, the results presented in this manuscript would in general be applicable to any visco-elastic medium and not be restricted to the strongly coupled dusty plasma medium.

The linearization of the above set of equation yields the following dispersion relation for the transverse shear waves [7],

$$\omega = \frac{-i\eta k^2}{1 - i\omega \tau_m}$$ (6)

In the strong coupling limit of $(\omega \tau_m >> 1)$ this yields

$$\frac{\omega}{k} = \sqrt{\frac{\eta}{\tau_m}}$$ (7)

which implies wave propagation and in the other limit of $(\omega \tau_m << 1)$

$$\omega = -i\eta k^2$$

we have the usual damping due to viscosity in hydrodynamic fluids.
We have used the flux corrected scheme (Boris et al. [30]) to evolve the coupled set of eqs. (4)–(5). The equations were evolved for a slab sinusoidal perturbation and the dispersion relation for the transverse shear wave was verified numerically as a part of code validation [31].

IV. DYNAMICAL EVOLUTION OF VORTICITY PATCHES THROUGH GHD MODEL

A typical fluid flow contains a wide variety of coherent patterns in the form of localized vorticity patches. Their interaction and evolution are important for the understanding of the system which in turn is responsible for the transport properties of the system. The objective of the present work is to understand the dynamical characteristics of these entities for a strongly coupled system within the framework of the visco-elastic GHD model.

We consider the following specific cases in particular: (i) evolution of circular and elliptical vorticity patches and (ii) Interaction amidst vorticity patches of like and unlike signs. The vorticity patch representing a sheared rotation emits the transverse shear wave for a visco-elastic fluid, making the evolution dramatic in terms of rapid mixing and transport behaviour of the GHD fluid system.

A. Evolution of vorticity patches

We consider two cases of circularly rotating fluid profile, namely having (A) smooth rotational profile and consider another which has a (B) sharp cut off. In the former case (A) the vorticity smooth profile (rotating in a clockwise direction) is given by, 
\[ \xi_0(x, y, t_0) = \Omega_0 \exp \left( - \frac{(x-x_c)^2 + (y-y_c)^2}{a_c^2} \right) \].

Here \( \Omega_0 = \frac{\Gamma_0}{\pi a_c^2} \), \( \Gamma_0 \) is the total circulation, \( a_c \) is the vortex core radius and the numerical simulation has been carried out for \( a_c=1.5, \Omega_0 = 8 \) and \( x_c = y_c=0 \).

In fig. 1 we show a comparison of the evolution of such a circular vorticity patch for a hydrodynamic fluid and the GHD system, through the color contour plot of vorticity. It can be observed that while the structure is stable in the context of the hydrodynamic system, for the GHD case there is a radial emission of waves. The radial emission can be seen more clearly in fig. 2 where the vorticity profile as a function of one of the axis (say \( x \)) has been
plotted for various times with different line styles. A perturbation clearly proceeds outwards. The radial speed of this perturbation has been evaluated and found to match with $\sqrt{\eta/\tau_m}$ as has been shown in fig. 3. The circular nature of the emitted wave also suggests that the amplitude of these characteristic perturbations should display a $1/\sqrt{r}$ radial fall off. This has also been demonstrated numerically as shown in the plot of fig. 4.

We next choose profile B with a sharp cut off, i.e. setting the vorticity $\xi_{z0} = 0$ beyond $r = r_0 (= 6.0)$. For $r \leq r_0$ the vorticity is chosen to have a constant value ($\xi_{z0} = 2$). The abruptness of the vorticity profile generates a strong rotational sheared flow, which in turn is drastically unstable to the Kelvin Helmholtz instability for both the hydrodynamic (HD) fluid as well as the GHD system. The vorticity contours at various times for both HD and GHD system have been shown in fig. 5. It should be noted that perturbations in the context of HD system remain considerably localized when compared to those of GHD. Basically the fluid is much less perturbed in the context of HD than that of GHD. This can be understood by noticing that for the GHD system the strong intermixing occurs due to the emission of the transverse shear waves. There is a clear propagation of two transverse shear wave fronts, one inward and the other outward and a concomitant KH destabilization at each of these fronts. The radial shear wave is clearly instrumental in efficient mixing of the fluids entrained inside the vortex structure with the outside fluid. For HD system the initial KH perturbation evolves towards a very anisotropic isolated structure. The mixing of the fluid in this case is clearly minimal. In fig. 6 we have compared two different cases of GHD simulation, with different values of $\eta$ and $\tau_m$ parameter ($\eta = 2.5, \tau_m = 20, v_p = 0.35$ for fig. 6(a) and $\eta = 10, \tau_m = 40, v_p = 0.5$ for fig. 6(b)). We observe that the mixing is better for higher phase velocity of the TS wave. Also, the figs. 5(b), 6(b) show that the vortex evolution is similar in time for same value of TS wave phase velocity ($v_p = \sqrt{\eta/\tau_m} = 0.5$)

Often the vorticity structure in a fluid may not have a circular shape. We consider, therefore, for our studies an initial distorted patch of vorticity. A simple elliptical form of distortion have been considered by us. Various time frames of the evolution of such a vortex pattern in both HD and GHD case has been shown in figs. 7, 8 for smooth and sharp vorticity profiles respectively. It can be seen that while the distorted shape of the vorticity patch does help somewhat in making the transport better in the context of HD, the GHD case still proves to be more efficiently mixing the fluids.
B. Interaction amidst vorticity patches

We have also investigated the process of interaction between various vortex structures within the GHD formalism for a strongly coupled medium. A correspondence with HD system has also been provided.

The interaction and subsequent merging of two like signed vorticity patches have been well known in the context of a hydrodynamic system \[^{32-34}\]. The same in the case of GHD has been illustrated in the subplots of fig. 9. In contrast to HD, the merging does not lead to a coherent final form, instead as expected the TS waves continue to dominate the system.

The unlike vortices, when brought together in the context of HD system are observed to propagate together along the direction of their axis as stable dipoles. In GHD case too while the structures do propagate together axially for some time as shown in fig. 10. The continuous emission of waves, however, distorts the structure ultimately.

The emission of transverse shear waves appears to have a predominant role in the mixing and transport of the fluid elements in the context of the visco-elastic GHD system. The strong mixing can be suppressed provided the TS waves have damped characteristics in the medium.

In the next section we investigate the specific case of the strongly coupled dusty plasma medium by employing the MD simulations.

V. MOLECULAR DYNAMICS STUDIES

We have carried out a Molecular Dynamics simulation of the dust particles in the strongly coupled regime using the open source LAMMPS code \[^{35}\]. The role of electrons and ions has only been assumed to provide shielding effect to dust species causing effective interdust interaction through a Yukawa inter particle dust potential.

\[
U(r) = \frac{Q^2}{4\pi\varepsilon_0 r} \exp(-r/\lambda_D) \tag{8}
\]

The parameters \(Q\) and \(\lambda_D\) are the charge on dust particle and plasma Debye length respectively. We have performed 2-D simulations and periodic boundary conditions have been employed for present simulations of vortex patches. We have taken typical inter-dust distance \(a_d = 0.418 \times 10^{-3}\)m, the fixed charge on dust particle is taken \(Q = 11940e^-\), system size \(l_x = l_y = 0.1672\)m and dust mass \(m_d = 6.99 \times 10^{-13}\)Kg. As we have chosen \(\kappa = a_d/\lambda_D = 0.5\),
the corresponding value of $\lambda_D = 8.36 \times 10^{-4}$m. With these parameters, the dust plasma frequency comes out to be typically $\sim 35$Hz corresponding to dust plasma period of 0.028s. We have chosen the typical time step 0.002s which can well resolve the characteristic dust plasma frequency. The parameters chosen for simulation are very relevant to dusty plasma experiments. We equilibrated our system first as a NVT ensemble for 40s and then further as NVE ensemble for next 40s. After this equilibration our system becomes ready for study at some particular coupling parameter $\Gamma$. The expression for coupling parameter is $\Gamma = Q^2/(4\pi\epsilon_0 a_d k_B T_d)$. For some fix values of charge $Q$ and dust density (hence typical interdust separation $a_d$), the dust particle’s temperature decides the value of coupling parameter. We have chosen some fix value of $\Gamma$ and then calculated the dust temperature corresponding to it in our system. To achieve the system at this temperature, we have equilibrated it as NVT ensemble as described earlier. For present simulations, we have not considered the effect of gas friction in our system as our primary aim is to look into the effect of strong coupling over vortex dynamics. The effects due to gas friction is important factor and will be added in further simulation studies to make results more closer to experiments.

All physical studies made on this system have been performed in NVE ensemble. In our present studies, we have given a vortex like initial profile to a specific circular region of Yukawa system making a sharp interface of vorticity between the circular patch region and the rest of the system. The initial flows in circular patch have been chosen as $v_{x0} = -y$ and $v_{y0} = x$. Figure 1 shows the evolution of sharp vortex patch for different values of coupling parameter $\Gamma$. The formation of KH instability at the interface of vortex patch can easily be observed in all the cases. But as the value of coupling parameter is increased, we found that the vorticity patches seem more stable as the growth of KH instability reduces with increasing $\Gamma$. This is the reason why the further evolution of a single vortex forms a tripolar structure for small coupling parameter while similar structure remains reasonably stable for higher value of $\Gamma$.

VI. SUMMARY AND CONCLUSION

The evolution and interaction of localized vortex patterns for a strongly coupled medium depicted by the visco-elastic GHD description have been studied. Preliminary studies with the Molecular Dynamics simulations have also been carried out.
We observe that the rotational sheared flow in a localized vortex patterns is susceptible to the Kelvin - Helmholtz (KH) destabilization which is similar to the Newtonian fluids. It is, however, necessary that for KH destabilization the shear should be strong and have an inflection point. This is possible when we considered the sharp cut off in the vorticity patches. In contrast to the Newtonian fluid the GHD visco-elastic medium, in addition to KH also permits the emission of radially (inward as well as outward) propagating transverse shear waves. The phase speed of the waves and the \(1/r\) fall in their intensity has been verified numerically.

Our studies show that due to the existence of such transverse shear waves in the strongly coupled medium the mixing and transport behaviour in these fluids is much better than Newtonian hydrodynamic systems. The chances of fluid element and/or test particles to remain entrained for long duration within a localized region is insignificant in GHD when compared to the Newtonian fluid.

We are in the process of quantifying this transport behaviour by carrying out test particle simulations in the system of GHD model. The numerical dispersion of these particles in a GHD flow would provide an estimate of the diffusion in such a medium. An analysis of the separation of the particle trajectories is also being carried out to understand the similarity and differences with the 2-D hydrodynamic system studied by Falkovich et al [37]. These observations would be presented in a subsequent publication.
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FIG. 1: Evolution of smooth circular vorticity profile in time for (a) hydrodynamic fluid and (b) visco-elastic fluid with parameters $\eta = 5$, $\tau_m = 20$. 
FIG. 2: Radial emission (emerging wavefront) of TS waves along one of the axes at different times during vortex evolution in visco-elastic medium for the parameters (a) $\eta = 5$, $\tau_m = 20$ and (b) $\eta = 2.5$, $\tau_m = 20$. 
FIG. 3: Wavefront position of TS waves at different time steps with parameter values
\( \eta = 10, \tau_m = 40, v_p = 0.50 \) (•), \( \eta = 5, \tau_m = 20, v_p = 0.50 \) (∗) and \( \eta = 2.5, \tau_m = 20, v_p = 0.35 \) (⋆), where \( v_p \) is the phase velocity related to corresponding parameters (of corresponding color) and black line is linear fitted curve.
FIG. 4: Wavefront amplitude of TS waves with $1/\sqrt{r}$ with parameter values
$\eta = 10$, $\tau_m = 40$ (●), $\eta = 5$, $\tau_m = 20$ (■) and $\eta = 2.5$, $\tau_m = 20$ (★) with black line as linear fitted curve.
FIG. 5: Evolution of circular sharp vorticity profile in time for (a) hydrodynamic fluid and (b) visco-elastic fluid with parameters $\eta = 5, \tau_m = 20$.

FIG. 6: Evolution of sharp circular vorticity profile in time for strongly coupled dusty plasma medium for the cases (a) $\eta = 2.5, \tau_m = 20$, and (b) $\eta = 10, \tau_m = 40$. 
FIG. 7: Evolution of elliptical vorticity profile in time for (a) hydrodynamic fluid and (b) visco-elastic fluid with parameters $\eta = 5, \tau_m = 20$.

FIG. 8: Vorticity evolution for sharp elliptical profile in time for (a) hydrodynamic fluid and (b) visco-elastic fluid with parameters $\eta = 5, \tau_m = 20$. 
FIG. 9: Evolution of two like sign vortices in time for (a) hydrodynamic fluid and (b) visco-elastic fluid with parameters $\eta = 5, \tau_m = 20$.

FIG. 10: Evolution of two unlike sign vortices with time for (a) hydrodynamic fluid and (b) visco-elastic fluid with parameters $\eta = 5, \tau_m = 20$. 
FIG. 11: Evolution of sharp vortex in strongly coupled dusty plasma medium (Yukawa medium) at different coupling parameters $\Gamma = 10, 100$ and $\Gamma = 150$ and $\kappa = 0.5$. 