Centre vortices underpin dynamical chiral symmetry breaking in SU(3) gauge theory

Daniel Trewartha, Waseem Kamleh, and Derek Leinweber
Centre for the Subatomic Structure of Matter (CSSM),
Department of Physics, University of Adelaide, SA, 5005, Australia
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The link between dynamical chiral symmetry breaking and centre vortices in the gauge fields of pure SU(3) gauge theory is studied using the overlap-fermion quark propagator in Lattice QCD. Overlap fermions provide a lattice realisation of chiral symmetry and consequently offer a unique opportunity to explore the interplay of centre vortices, instantons and dynamical mass generation. Simulations are performed on gauge fields featuring the removal of centre vortices, identified through gauge transformations maximising the center of the gauge group. In contrast to previous results using the staggered-fermion action, the overlap-fermion results illustrate a loss of dynamical chiral symmetry breaking coincident with vortex removal. This result is linked to the overlap-fermion’s sensitivity to the subtle manner in which instanton degrees of freedom are compromised through the process of centre vortex removal. Backgrounds consisting solely of the identified centre vortices are also investigated. After smoothing the vortex-only gauge fields, we observe dynamical mass generation on the vortex-only backgrounds consistent within errors with the original gauge-field ensemble following the same smoothing. Through visualizations of the instanton-like degrees of freedom in the various gauge-field ensembles, we find evidence of a link between the centre vortex and instanton structure of the vacuum. While vortex removal destabilizes instanton-like objects under $O(a^4)$-improved cooling, vortex-only backgrounds provide gauge-field degrees of freedom sufficient to create instantons upon cooling.

At the energy scale relevant to everyday matter, Quantum Chromodynamics (QCD) manifests two key features; the confinement of quarks inside hadrons, and dynamical chiral symmetry breaking, associated with the dynamical generation of mass. Although these phenomena can be easily shown to exist, the nature of the underlying mechanisms responsible for them, and whether they share a common origin, have remained open questions.

It is generally accepted that these features are both caused by some kind of topological object in the QCD vacuum which dominates at large distance scales. Candidates have included objects such as Abelian monopoles [1–3], instantons [1, 2, 4–6] and centre vortices [7–11]. As the only known first principles technique for studying non-perturbative QCD, the lattice formulation plays a unique role in ab initio studies of these objects.

Although instantons can reproduce dynamical mass generation on the lattice [12], they are not believed to play any significant role in confinement [13]. Recent results [14] have also suggested the centre-vortex model of confinement is more consistent with lattice results than other currently available models. In this Letter we show, for the first time, an intimate link between centre vortices and dynamical chiral symmetry breaking in QCD.

Centre vortices are topological defects associated with the elementary centre degree of freedom of the QCD gauge field, and thus present an attractive candidate for study. In SU(N) gauge theory, centre vortices have a clear theoretical link to confinement [7–9, 15], and in SU(2) they have been shown to be responsible for dynamical chiral symmetry breaking [16–20] through lattice-QCD simulations.

In SU(3) gauge theory, the picture is less clear. While lattice results have shown a loss of string tension, and thus confinement, to be coincident with the removal of centre vortices [21], the connection between centre vortices and dynamical chiral symmetry breaking is less apparent.

The Landau-gauge quark propagator has often [22–25] been used as a probe of dynamical chiral symmetry breaking. At low momenta, enhancement of the Dirac scalar part of the propagator, commonly referred to as the mass function and associated with the concept of a constituent quark mass, provides a clear signal of the presence or absence of dynamical chiral symmetry breaking. In SU(2) gauge theory, the mass function clearly displays the absence of dynamical chiral symmetry breaking upon centre vortex removal, as the mass function does not develop a dynamically generated mass in the infrared limit [16]. However, a similar study in SU(3) gauge theory using the AsqTad-quark [26] propagator did not reveal a comparable role in dynamical chiral symmetry breaking [27]. There the mass function sustained dynamical mass generation on vortex-removed configurations. However in Ref. [28], where the vortex-removed hadron spectrum was studied with Wilson fermions, it became clear that this residual mass generation on vortex-removed configurations was not associated with chiral symmetry; i.e. that chiral symmetry was indeed lost upon vortex removal.

Both the AsqTad- and Wilson-fermion actions explicitly break chiral symmetry, and hence the relation between centre vortices and dynamical chiral symmetry breaking may be clouded by the resulting lattice artefacts. In this Letter we study the Landau gauge quark propagator using the superior chiral properties of the
overlap-Dirac fermion action. The overlap fermion action provides a realisation of chiral symmetry on the lattice and is renowned for its sensitivity to the topological structure of the gauge fields. We find for the first time a loss of dynamical mass generation in the Landau-gauge quark propagator coincident with vortex removal. Through a study of the topological charge density under $O(a^4)$-improved cooling, we are able to trace this success to the overlap-fermion’s sensitivity to the subtle manner in which instanton degrees of freedom are compromised through the process of centre vortex removal.

We also demonstrate how the centre vortex degrees of freedom can reproduce dynamical mass generation after smoothing the vortex-only gauge fields with improved cooling. We observe dynamical mass generation on the vortex-only backgrounds consistent with that on the original gauge-field ensemble following the same amount of smoothing.

Through visualizations of the instanton-like degrees of freedom in the various gauge-field ensembles, we find evidence of a link between the centre vortex and instanton structure of the vacuum. While vortex removal destabilizes instanton-like objects under $O(a^4)$-improved cooling, vortex-only backgrounds provide gauge-field degrees of freedom sufficient to create instantons upon cooling.

We study centre vortices on the lattice in the standard way, commencing with gauge transformations designed to bring the lattice link variables,

$$U_\mu(x) = \mathcal{P} \exp \left( ig \int_0^a d\lambda A_\mu(x + \lambda \hat{\mu}) \right),$$

(1)

to be as close as possible to centre elements of SU(3) via Maximal Centre Gauge (MCG) \cite{29}, then projecting onto the $Z_3$ centre-subgroup \cite{29,34} to produce the vortex-only configuration,

$$Z_\mu(x) = \exp \left( \frac{2\pi i}{3} m_\mu(x) \right) 1, \quad m_\mu \in \{-1,0,1\}. \quad (2)$$

The vortices are identified by the centre charge, $z$, found by taking the product of the links around a plaquette,

$$z = \frac{1}{3} \text{Tr} \prod Z_\mu(x) = \exp \left( \frac{2\pi i}{3} n \right). \quad (3)$$

If $z = 1$, no vortex pierces the plaquette. If $z \neq 1$, a vortex with charge $z$ pierces the plaquette. In the smooth gauge-field limit, all links approach the identity, and no vortices are found. Vortices are identified as the defects in the centre-projected gauge field.

Upon transforming each link to the closest element of the centre $Z_\mu(x)$, we are able to define three ensembles:

1. The original ‘untouched’ configurations, $U_\mu(x)$,
2. The projected vortex-only configurations, $Z_\mu(x)$,
3. The vortex-removed configurations, $Z_\mu^i(x)U_\mu(x)$.

Each of the ensembles are gauge-fixed to Landau gauge. By comparing results on these three ensembles, we are able to isolate the effects of centre vortices.

We calculate the quark propagator using the overlap fermion operator, which satisfies the Ginsparg-Wilson relation \cite{55}, and thus provides a lattice-realisation of chiral symmetry. It has a superior sensitivity to gauge field topology than the aforementioned AsqTad and Wilson lattice fermion operators. Explicitly, the massive overlap-Dirac operator \cite{36,37} is given by

$$D_0(\mu) = \frac{(1 - \mu)}{2} \left[ 1 + \gamma_5 (\gamma_5 D(-m_w)) \right] + \mu, \quad (4)$$

where $\epsilon$ is the matrix sign function. We use the fat-link irrelevant clover (FLIC) fermion operator \cite{38,41} as the overlap kernel $D(-m_w)$, with regulator parameter $m_w = 1$. The overlap mass parameter, $\mu = 0.004$, provides a bare quark mass of 12 MeV for our calculations.

In a covariant gauge, the lattice quark propagator can be decomposed into Dirac scalar and vector components as

$$S(p) = \frac{Z(p)}{i\gamma + M(p)}, \quad (5)$$

with $M(p)$ the non-perturbative mass function and $Z(p)$ containing all renormalisation information. The infrared behaviour of $M(p)$ reveals the presence or absence of dynamical mass generation, and thus of dynamical chiral symmetry breaking.

To rigorously study centre symmetry, results are calculated on 50 pure gauge-field configurations using the Lüscher-Weisz $O(a^2)$ mean-field improved action \cite{42}, with a $20^3 \times 40$ volume at a lattice spacing of $0.125$ fm. We fix to Landau gauge using a Fourier transform accelerated algorithm \cite{43}, fixing to the $O(a^2)$ improved gauge-fixing functional \cite{44}. The vortex-only configurations are pre-conditioned with a random gauge transformation before gauge-fixing for improved algorithmic convergence. A cylinder cut \cite{45} is performed on propagator data, and $Z(p)$ is renormalised to be 1 at the highest momentum considered, $p \approx 5.2$ GeV.

Results for the untouched and vortex-removed ensembles are plotted in Fig. 1. The renormalisation function shows similar behaviour in both the untouched and vortex-removed cases. However, the mass function reveals a significant change upon vortex removal.

On the untouched ensemble, the mass function shows strong enhancement in the infrared, displaying the presence of dynamical mass generation. By contrast, dynamical mass generation is largely suppressed upon vortex removal with only a relatively small level of residual infrared enhancement remaining \cite{46}. Unlike the AsqTad propagator, which showed little to no change in the infrared enhancement \cite{27}, the overlap operator is able to
FIG. 1. The mass (a) and renormalisation (b) functions on the original (untouched) (squares) and vortex-removed (crosses) configurations. Removal of the vortex structure from the gauge fields spoils dynamical mass generation and thus dynamical chiral symmetry breaking.

‘see’ the subtle damage caused to the gauge fields through vortex removal. The removal of the vortex structure from gauge fields has spoiled dynamical mass generation, and thus dynamical chiral symmetry breaking.

The smoothness requirement of the overlap operator [37] contrasts the rough nature of vortex-only configurations consisting solely of centre elements, and the overlap fermion action is thus not well defined on vortex-only configurations. To address this issue we smooth the gauge-field configurations. This is additionally motivated by evidence that, in SU(2) gauge theory, vortex-only configurations are too rough to reproduce the low-lying modes of the Dirac operator essential to dynamical chiral symmetry breaking, but are able to do so after smearing [47]. Smoothing is performed using three-loop $O(a^4)$-improved cooling [38].

By examining the local maxima of the action density on vortex-only configurations during cooling, we find that after just 10 sweeps of smoothing these local maxima stabilise and begin to resemble classical instantons in shape and corresponding topological charge density at the centre [49]. The average number of these maxima per configuration is plotted in Fig. 2 as a proxy for the number of instanton-like objects per configuration for up to 200 sweeps. The number of objects found on untouched and vortex-only configurations remains very similar even after large amounts of cooling.

In contrast, the number of objects on vortex-removed configurations is greatly reduced. Vortex-removal has destabilised the otherwise topologically-nontrivial instanton-like objects. Early in the smoothing procedure the topological charge density of the vortex removed configurations qualitatively resembles that of the original configurations. It is perhaps unsurprising that a fermion operator that is not sensitive to the spoiling of instanton-like objects through vortex-removal would erroneously report little change to dynamical mass generation. It is remarkable that the overlap operator is sensitive to the subtle changes of vortex removal in the absence of any smoothing.

Although there does not appear to be a one-to-one connection between the backgrounds dominated by
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stant-like objects found in the untouched and vortex-only cases on a configuration-by-configuration basis, the objects are qualitatively similar in number and size. Despite consisting solely of the centre elements, the centre vortex information encapsulates the qualitative essence of the QCD vacuum structure. It contains the ‘seeds’ of instantons, which are reproduced through cooling.

Just as the centre-vortex information alone was sufficient to reproduce instantons through cooling, vortex removal is sufficient to destroy the stability of instantons under cooling, with the vast majority of topological objects being removed as seen in Fig. 2. On vortex-removed backgrounds a few instanton-like objects remain, which closely resemble those found on the untouched and vortex-only backgrounds in size despite their greatly reduced number. These residual objects provide a mechanism for the remnant of dynamical mass generation seen on vortex-removed configurations in Fig. 1.

To quantify the extent to which the centre-vortex information encapsulated in the vortex-only configurations can give rise to dynamical mass generation, we calculate the overlap quark propagator on both untouched and vortex-only configurations after 10 sweeps of cooling. The mass and renormalisation functions are illustrated in Fig. 3. As expected, the mass function on the untouched configurations shows some reduction in dynamical mass generation, while being qualitatively similar to the uncooled results [12]. The vortex-only results for the mass function are strikingly similar to the untouched; they show the vortex-only configurations reproducing almost all dynamical mass generation. The renormalisation functions also share a similar behavior. The background of instanton-like objects emerging from the vortex-only configurations under cooling is able to reproduce the features of the quark propagator on the full backgrounds.

Combined, these results establish that the centre vortex structure of the vacuum plays a fundamental role in dynamical chiral symmetry breaking in SU(3) gauge theory. For the first time, we have demonstrated the removal of dynamical mass generation via the removal of the centre-vortex degrees of freedom from the gauge fields. Moreover, we have demonstrated how the vortex-only degrees of freedom encapsulate the qualitative features of the gauge fields, reproducing the average number and size of instanton-like objects under smoothing via cooling. These features reproduce the dynamical mass generation observed on the original gauge fields following the same smoothing.

We have also found a link between the stability of instanton-like objects under cooling and centre vortex removal. Vortex removal spoils and destabilizes instantons, resulting in them being quickly removed from the lattice under cooling. Correspondingly, vortex-only configurations quickly produce a background of instanton-like objects with general features resembling those found in the untouched case. Our results are consistent with the instanton model of dynamical mass generation, and reveal an intimate connection between the centre-vortex structure and the instanton structure of SU(3) gauge fields. Our findings are similar to a connection shown in SU(2) gauge theory [50]. We conclude that centre vortices are the fundamental long-range objects underpinning dynamical chiral symmetry breaking in SU(3) gauge theory.

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FIG. 3. The mass(a) and renormalisation (b) functions on the original (untouched) (squares) and vortex-only (circles) configurations after 10 sweeps of three-loop $O(a^2)$-improved cooling, at an input bare quark mass of 12 MeV.
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