Superfield Approaches to a Model of Bosonic String: Curci-Ferrari Type Restrictions

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Abstract: Exploiting the theoretical potential of the modified Bonora-Tonin superfield approach (MBTSA) as well as the (anti-)chiral superfield approach (ACSA) to Becchi-Rouet-Stora-Tyutin (BRST) formalism, we derive the complete set of off-shell nilpotent (anti-)BRST symmetry transformations corresponding to the classical two (1 + 1)-dimensional (2D) diffeomorphism symmetry transformations on the world-sheet (that is traced out by the motion of a model of bosonic string). Only the BRST symmetry transformations for this model have been discussed in the earlier literature. We derive the (anti-)BRST invariant Curci-Ferrari (CF) type restrictions (using MBTSA) which turn out to be the root-cause behind the absolute anticommutativity of the above (anti-)BRST symmetry transformations. We capture the symmetry invariance of the (anti-)BRST invariant Lagrangian densities within the ambit of ACSA. The derivation of the proper anti-BRST transformations (corresponding to the already known BRST transformations) and the (anti-)BRST invariant CF-type restrictions are the novel results in our present endeavor.

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1 Introduction

Superfield approaches (see, e.g. [1-8]) to Becchi-Rouet-Stora-Tyutin (BRST) formalism are geometrically elegant, mathematically rich and physically very intuitive as they provide the geometrical basis for the off-shell nilpotency and absolute anticommutativity of the quantum (anti-)BRST symmetry transformations that are associated with a given classical local gauge symmetry transformation for a classically gauge invariant theory. In the above usual superfield approaches [1-8], only the \( p \)-form (\( p = 1, 2, 3 \ldots \)) gauge theories have been considered which are characterized by the existence of the first-class constraints on them in the terminology of Dirac’s prescription for the classification scheme of constraints (see, e.g. [9, 10]). It has been a challenging problem to incorporate the diffeomorphism invariant theories in the domain of the superfield approaches to BRST formalism. An attempt has been made by Delbourgo, et al., (see, e.g. [11]) in this direction where a diffeomorphism invariant gravitational theory has been considered. However, in our present endeavor, we shall not discuss anything connected with the superfield approach developed in [11] for the BRST-analysis of our present two-dimensional (2D) diffeomorphism invariant theory.

A very successful application of the superfield approach [4-6] to BRST formalism (in the context of D-dimensional non-Abelian 1-form gauge theory) has been performed by Bonora and Tonin (BT). We have exploited the theoretical techniques and tricks of this approach in the context of BRST-analysis of the higher \( p \)-form (\( p = 2, 3 \)) Abelian gauge theories [12]. It has been a very exciting problem to incorporate the diffeomorphism symmetry transformations within the framework of BT-superfield formalism. A breakthrough, in this direction, has been made by Bonora in a very recent paper [13] where the D-dimensional diffeomorphism invariant theory has been discussed within the ambit of BT-superfield approach [4-6]. We have christened this theoretical technique as the modified version of BT-superfield approach (MBTSA) to BRST formalism [13] and applied its theoretical potential in the context of the 1D diffeomorphism (i.e. reparameterization) invariant model of a free spinning supersymmetric (SUSY) relativistic particle [14] and established that its Curci-Ferrari (CF) type of restriction as well as the gauge-fixing and Faddeev-Popov ghost terms are the same as for the other 1D diffeomorphism (i.e. reparameterization) invariant models of a free scalar and non-SUSY relativistic particle as well as a non-SUSY and non-relativistic free particle (see, e.g. [14] and references therein).

In the applications of MBTSA [13], it turns out that we have to take into account the full super expansions of the superfields defined on the (D, 2)-dimensional supermanifold. In other words, we perform the super expansion of the above superfields along all the possible Grassmannian directions of the (D, 2)-dimensional supermanifold on which a D-dimensional ordinary diffeomorphism invariant theory is generalized. The idea of horizontality condition (HC) enables us to derive the (anti-)BRST symmetry transformations for the scalars, vectors, tensors, etc. However, we invoke the Nakanishi-Lautrup type auxiliary fields (\( \bar{B}_\mu \)) (with \( \mu = 0, 1, 2\ldots D - 1 \)) in the standard nilpotent (anti-)BRST symmetry transformations: \( s_b \bar{C}_\mu = i B_\mu, \ s_b B_\mu = 0, \ s_{ab} C_\mu = i \bar{B}_\mu, \ s_{ab} \bar{B}_\mu = 0 \) of the (anti-)ghost fields (\( \bar{C}_\mu \)) in the case of the D-dimensional diffeomorphism invariant theory in an ad-hoc manner. This forces us to consider the (anti-)chiral super expansions of the superfields [cf. Eq. (27) below]. At this juncture, it becomes essential for us to take into account the theoretical tricks and techniques of the (anti-)chiral superfield approach (ACSA) to BRST formalism.
(see e.g. [15] and references therein) which has been developed by us.

The central theme of our present investigation is to apply the ideas of MBTSA and ACSA to BRST formalism in the realm of a 2D diffeomorphism invariant theory of a model of bosonic string and derive (i) all the (anti-)BRST symmetries of this theory in a consistent and clear fashion, and (ii) the CF-type restrictions which are responsible for the absolute anticommutativity of the (anti-)BRST symmetry transformations. We have also derived the BRST and anti-BRST invariant Lagrangian densities and captured their symmetry invariance(s) in the language of ACSA to BRST formalism. We would like to lay emphasis on the fact that the theoretical potential of MBTSA has been responsible for the derivation of (i) the (anti-)BRST symmetry transformations for the pure Lorentz scalars, and (ii) the (anti-)BRST invariant CF-type restrictions. However, we have been able to derive all the proper (anti-)BRST transformations for all the other fields by using ACSA.

The following motivating factors have been at the heart of our present investigation. First, we have already used the beautiful blend of theoretical ideas behind MBTSA and ACSA in the cases of some 1D diffeomorphism (i.e. reparameterization) invariant theories of SUSY (i.e. spinning) relativistic particle, NSUSY (i.e. scalar) relativistic particle and NSUSY and non-relativistic system of a free particle for the discussion of BRST analysis. However, these models are also endowed with the gauge symmetry transformations which are a kind of subset of the reparameterization symmetry transformations (under specific limits). To be precise, it has been shown (see, e.g. [14] and references therein) that the gauge symmetry transformations (generated by the first-class constraints) are equivalent to the reparameterization symmetry transformations if we use (i) the specific set of equations of motion, and (ii) identify the transformation parameters of both these symmetries in a specific manner. Thus, it is a challenging problem for us to use the theoretical strength of MBTSA and ACSA in the context of a 2D diffeomorphism invariant theory which does not respect the gauge symmetry transformations as have been demonstrated in [14] for a 1D diffeomorphism invariant theory. We have discussed, in our present endeavor, a model of bosonic string which has the 2D diffeomorphism symmetry invariance but it does not respect a gauge symmetry transformation. Second, one of the sacrosanct aspects of BRST formalism is the existence of the quantum BRST and anti-BRST symmetries together for a given classical gauge/diffeomorphism symmetry transformation. For our present bosonic string, only the BRST symmetries are known in literature [16]. Thus, it is a challenge for us to derive the proper anti-BRST symmetry transformations corresponding to the above BRST symmetry transformations. We have accomplished this goal in our present endeavor. Finally, the hallmark of a BRST-quantized theory is the existence of the CF-type restrictions which provide the independent identity to the BRST and anti-BRST symmetries (and corresponding charges) at the quantum level. We have derived these restrictions, too.

The theoretical contents of our present endeavor are organized as follows. In Sec. 2, we concisely discuss the (anti-)BRST symmetry transformations for the gauge-fixed Lagrangian densities of the bosonic string theory. Our Sec. 3 is devoted to the derivation of the Curci-Ferrari (CF) type restrictions for our BRST invariant theory within the framework of MBTSA. In addition, we also derive the (anti-)BRST symmetry transformations for the target space coordinates and the determinant of the modified version of the inverse of the 2D metric tensor. Sec. 4 contains the derivation of the nilpotent (anti-)BRST symmetries for the other fields of our theory by exploiting the theoretical potential of ACSA.
We capture the (anti-)BRST invariances of the Lagrangian densities using ACSA in Sec. 5. Finally, we make some concluding remarks in our Sec. 6.

## 2 Preliminary: (Anti-)BRST Symmetries

We begin with the following (anti-)BRST invariant Lagrangian densities \([\mathcal{L}_{(a)b}]\) for the model of the bosonic string of our theory (see, e.g. [17] for details)

\[
\mathcal{L}_{ab} = \mathcal{L}_0 - \bar{B}_1 A_1 - B_0 A_0 + i \left[ C_1 (\partial_0 \bar{C}^1 + \partial_1 \bar{C}^0) + C_0 (\partial_a \bar{C}^a) + (\partial_a C_0) \bar{C}^a \right] A_0 + i \left[ C_0 (\partial_0 \bar{C}^1 + \partial_1 \bar{C}^0) + C_1 (\partial_a \bar{C}^a) + (\partial_a C_1) \bar{C}^a \right] A_1 + i \left[ C_1 (\partial_0 \bar{C}^0 - \partial_1 \bar{C}^1) + C_0 (\partial_0 \bar{C}^1 - \partial_1 \bar{C}^0) \right] A_2 ,
\]

\[
\mathcal{L}_b = \mathcal{L}_0 + B_1 A_1 + B_0 A_0 - i \left[ \bar{C}_0 (\partial_0 C^1 + \partial_1 C^0) + \bar{C}_0 (\partial_a C^a) - C^a \partial_a \bar{C}_0 \right] A_0 - i \left[ \bar{C}_1 (\partial_0 C^1 + \partial_1 C^0) - C^a \partial_a \bar{C}_1 + \bar{C}_1 (\partial_a C^a) \right] A_1 - i \left[ \bar{C}_1 (\partial_0 C^0 - \partial_1 C^1) + \bar{C}_0 (\partial_0 C^1 - \partial_1 C^0) \right] A_2 .
\]

where the 2D diffeomorphism invariant classical action integral \((S_0)\) w.r.t. \(\mathcal{L}_0\) is [16]:

\[
S_0 = \int d^2 \xi \mathcal{L}_0 \equiv \int_{-\infty}^{+\infty} d\tau \int_{0}^{\pi} d\sigma \left[ -\frac{1}{2\kappa} \tilde{g}^{mn} \partial_m X^\mu \partial_n X_\mu + E \left( \det \tilde{g} + 1 \right) \right].
\]

In the above, we have taken the notation \(\xi^a = (\xi^0, \xi^1) = (\tau, \sigma)\) where \(\tau\) is the evolution parameter (with \(-\infty < \tau < +\infty\)) and \(\sigma\) denotes the length of the bosonic string (with \(0 \leq \sigma \leq \pi\)). The modified version of the inverse of the 2D metric tensor is: \(\tilde{g}^{mn} = \sqrt{\tilde{g}} g^{mn}\) where \(g^{mn}\) is the inverse of the 2D metric tensor \(g_{mn}\) and \(g = \det (g_{mn})\). The coordinates \(X^\mu(\xi) \equiv X^\mu(\tau, \sigma)\) (where \(\mu = 0, 1, 2...D - 1\)) correspond to the D-dimensional flat Minkowskian target space and \(a, b, c, ...l, m, n = 0, 1\) are the \(\text{“time”}\) and space directions on the world-sheet. The symbol \(\kappa\) denotes the string tension parameter and \(E\) is the Lagrange multiplier density which ensures that \(\det \tilde{g} = -1\) so that we can have \(\text{two}\) degrees of freedom for the metric field tensor which, being symmetric, has only \(\text{three}\) degrees of freedom on a 2D flat spacetime manifold. In other words, we have (see, e.g. [16] for details) the following decomposition of \(\tilde{g}^{mn}\), namely:

\[
\tilde{g}^{mn} = \begin{pmatrix}
A_1 + A_2 & A_0 \\
A_0 & A_1 - A_2
\end{pmatrix}.
\]

The flat limit (i.e. \(\tilde{g}^{mn} \rightarrow \eta^{mn}\)) can be obtained by the gauge-fixing conditions: \(A_0 = A_1 = 0\). The latter choices imply that we have \(A_2^2 = 1\) when we demand \(\det \tilde{g} = -1\). This input leads to \(\tilde{g}^{mn} \rightarrow \eta^{mn} = \text{diag} (+1, -1)\) for the choice \(A_2 = +1\) where \(\eta_{mn} = \eta^{mn} = \text{diag} (+1, -1)\) are the flat metric tensor \((\eta_{mn})\) and its inverse \((\eta^{mn})\) on the 2D Minkowskian spacetime manifold. In the derivation of the gauge-fixing and Faddeev-Popov ghost terms, we have taken the standard prescription of the BRST formalism (see, e.g. [16]), namely:

\[
\mathcal{L}_{ab} = \mathcal{L}_0 + s_{ab} \left[ i C_0 A_0 + i C_1 A_1 \right],
\]

\[
\mathcal{L}_b = \mathcal{L}_0 + s_b \left[ - i \bar{C}_0 A_0 - i \bar{C}_1 A_1 \right],
\]

\[
(5)
\]
where the full set of nilpotent \([(s_{ab})^2 = 0]\) (anti-)BRST transformations \([s_{ab}]\) are

\[
\begin{align*}
\sab X^\mu &= \bar C^a \pa_a X^\mu, \quad \sab C^m = i \bar B^a, \\
\sab E &= \pa_a (\bar C^a E), \quad \sab \bar B^n = 0, \quad \sab (\det \bar g) = \bar C^m \pa_m (\det \bar g), \\
\sab \bar g^{mn} &= \pa_a (\bar C^a \bar g^{mn}) - (\pa_a C^m) \bar g^{an} - (\pa_a C^m) \bar g^{ma}, \\
\sab B^n &= \bar C^m \pa_m B^n - B^m \pa_m C^n, \\
\sab X^\mu &= C^a \pa_a X^\mu, \quad \sab \bar C^m = i B^a, \\
\sab B^n &= 0, \quad \sab C^m = C^b \pa_b C^n, \\
\sab \bar g^{mn} &= \pa_a (C^a \bar g^{mn}) - (\pa_a C^m) \bar g^{an} - (\pa_a C^m) \bar g^{ma}, \\
\sab \bar B^n &= \bar C^m \pa_m \bar B^n - \bar B^m \pa_m C^n, \quad \sab (\det g) = C^a \pa_a (\det g).
\end{align*}
\]

Here the fermionic \([(C^a)^2 = (\bar C^a)^2 = 0, C^a C^b + C^b C^a = 0, C^a \bar C^b + \bar C^b C^a = 0, \bar C^a \bar C^b + \bar C^b \bar C^a = 0, \text{etc.}]\) (anti-)ghost fields are \((\bar C^a) C^a\) and the bosonic Nakanishi-Lautrup auxiliary fields are \((\bar B^a) B^a\). From the above, we can derive the (anti-)BRST symmetry transformations for the component gauge fields \(A_0, A_1\) and \(A_2\) as follows:

\[
\begin{align*}
\sab A_0 &= \bar C^m \pa_m A_0 - (\pa_0 C^1 - \pa_1 \bar C^0) A_2 - (\pa_0 \bar C^1 + \pa_1 C^0) A_1, \\
\sab A_1 &= C^m \pa_m A_1 - (\pa_1 C^0 + \pa_0 \bar C^1) A_0 - (\pa_0 C^0 - \pa_1 \bar C^1) A_2, \\
\sab A_2 &= \bar C^m \pa_m A_2 - (\pa_1 C^0 - \pa_0 \bar C^1) A_0 - (\pa_0 \bar C^0 - \pa_1 C^1) A_1, \\
\sab \bar A_0 &= C^m \pa_m \bar A_0 - (\pa_0 \bar C^1 - \pa_1 C^0) A_2 - (\pa_0 C^1 + \pa_1 \bar C^0) A_1, \\
\sab \bar A_1 &= \bar C^m \pa_m \bar A_1 - (\pa_1 \bar C^0 + \pa_0 C^1) A_0 - (\pa_0 \bar C^0 - \pa_1 C^1) A_2, \\
\sab \bar A_2 &= \bar C^m \pa_m \bar A_2 - (\pa_1 \bar C^0 - \pa_0 C^1) A_0 - (\pa_0 \bar C^0 - \pa_1 C^1) A_1.
\end{align*}
\]

It is interesting to note that this CF-type restrictions: \(B^a + \bar B^a + i (C^m \pa_m \bar C^a + \bar C^m \pa_m C^a) = 0\) appear in the following simple cases of the proof of absolute anticommutativity property:

\[
\begin{align*}
\{s_b, \sab\} X^\mu &= i \left[ B^a + \bar B^a + i \left( C^m \pa_m \bar C^a + \bar C^m \pa_m C^a \right) \right] (\pa_a X^\mu), \\
\{s_b, \sab\} E &= i \pa_a \left[ \{ B^a + \bar B^a + i \left( C^m \pa_m \bar C^a + \bar C^m \pa_m C^a \right) \} E \right], \\
\{s_b, \sab\} \bar g^{mn} &= i \pa_k \left[ \{ B^k + \bar B^k + i \left( C^l \pa_l \bar C^k + \bar C^l \pa_l C^k \right) \} \bar g^{mn} \right] \\
&\quad - i \pa_k \left[ B^m + \bar B^m + i \left( C^l \pa_l \bar C^m + \bar C^l \pa_l C^m \right) \right] \bar g^{mn} - i \pa_k \left[ B^n + \bar B^n + i \left( C^l \pa_l \bar C^n + \bar C^l \pa_l C^n \right) \right] \bar g^{mn}.
\end{align*}
\]

Thus, the off-shell nilpotent \([(s_{(a)b})^2 = 0]\) (anti-)BRST symmetry transformations \(s_{ab}\) are the proper set of quantum symmetry transformations.

We end this section with the following remarks. First, the off-shell nilpotent \([s^2_{(a)b} = 0]\) (anti-)BRST symmetry transformations (7) and (6) correspond to the classical 2D diffeomorphism symmetry transformations: \(\xi^a \rightarrow g^a(\xi) = \xi^a - \epsilon^a(\xi)\) where \(g^a(\xi)\) is a physically well-defined function of \(\xi^a\) on the 2D world-sheet such that it is finite at \(\tau = 0\) and \(\sigma = 0\) but vanishes off as \(\tau \rightarrow \pm \infty\) and \(\sigma = \pi\). The infinitesimal version of these transformations
parameters \( \varepsilon^a(\xi) \) where \( \varepsilon^a(\xi) \) (with \( a = 0, 1 \)) are the 2D infinitesimal diffeomorphism transformation parameters. Second, according to the basic tenets of BRST formalism, the parameters \( \varepsilon^i(\xi) \) have been replaced by the fermionic (anti-)ghost fields \( (\tilde{C}^a)C^a \) in the (anti-)BRST symmetry transformations (7) and (6). Third, it is crystal clear, from Eq. (10), that the (anti-)BRST symmetry transformations \( s_{(a)b} \) are absolutely anticommuting (i.e. \( \{ s_b, s_{ab} \} = 0 \)) in nature only on the submanifold of the quantum Hilbert space of fields where the CF-type restrictions: \( B^a + B^a + i (C^m \partial_m \tilde{C}^a + \tilde{C}^m \partial_m C^a) = 0 \) are satisfied. Finally, we note that the target space coordinates \( X^\mu(\xi) \) and \( [\det \tilde{g}(\xi)] \) transform as pure Lorentz scalars [i.e. \( X^\mu(\xi') = X^\mu(\xi) \), \( \det \tilde{g}'(\xi') = \det \tilde{g}(\xi) \)] under the infinitesimal and continuous diffeomorphism symmetry transformations: \( \xi^a \rightarrow \tilde{g}^a(\xi) = \xi^a - \varepsilon^a(\xi) \).

### 3 CF-Type Restrictions: MBTSA

According to the basic tenets of MBTSA to BRST formalism, first of all, we generalize the 2D infinitesimal diffeomorphism transformations: \( \xi^a \rightarrow \tilde{\xi}^a = g^a(\xi) = \xi^a - \varepsilon^a(\xi) \) to its counterpart onto the \((2,2)\)-dimensional supermanifold as (see, e.g. \([18, 13]\) for details)

\[
g^a(\xi) \rightarrow \tilde{g}^a(\xi, \theta, \bar{\theta}) = \xi^a - \theta \tilde{C}^a(\xi) - \bar{\theta} \tilde{C}^a(\xi) + \theta \bar{\theta} f^a(\xi),
\]

(11)

where the \((2,2)\)-dimensional supermanifold is parameterized by the superspace coordinates \( Z^M = (\xi^a, \theta, \bar{\theta}) \). Here \( \xi^a = (\xi^0, \xi^1) \equiv (\tau, \sigma) \) are the bosonic world-sheet coordinates and a pair of Grassmannian variables \((\theta, \bar{\theta})\) satisfy: \( \theta^2 = \bar{\theta}^2 = 0, \theta \bar{\theta} + \bar{\theta} \theta = 0 \). In Eq. (11), the fermionic (anti-)ghost \( (\tilde{C}^a)C^a \) fields are the ones that are present in the (anti-)BRST transformations (7) and (6). In view of the mappings \( s_b \leftrightarrow \partial_{\bar{g}} \big|_{\theta=0}, s_{ab} \leftrightarrow \partial_{\bar{g}} \big|_{\theta=0} \) established by Bonora and Tonin \([4,5]\), the coefficients of \( \theta \bar{\theta} \) in (11) have been taken to be the (anti-)ghost fields because, according to the standard BRST prescription, the classical infinitesimal diffeomorphism symmetry transformations: \( \delta \xi^a = -\varepsilon^a(\xi) \) have been promoted to the quantum level by the (anti-)BRST symmetry transformations: \( s_{ab} \xi^a = -\tilde{C}^a, s_b \xi^a = -C^a \). The coefficients of \( \theta \bar{\theta} \) in (11) [i.e. \( f^a(\xi) \)] have to be determined from other consistency conditions of the BRST formalism which we elaborate below.

To derive the CF-type restrictions and the (anti-)BRST symmetry transformations \( s_{ab} X^\mu = \tilde{C}^a \partial_a X^\mu, s_b X^\mu = C^a \partial_a X^\mu \), we generalize the target space ordinary coordinate fields \( X^\mu(\xi) \) onto the \((2,2)\)-dimensional supermanifold as

\[
X^\mu(\xi) \rightarrow \tilde{X}^\mu[\tilde{g}(\xi, \theta, \bar{\theta}), \theta, \bar{\theta}] = X^\mu[\tilde{g}(\xi, \theta, \bar{\theta})] + \theta \tilde{R}^\mu[\tilde{g}(\xi, \theta, \bar{\theta})] + \bar{\theta} \tilde{R}^\mu[\tilde{g}(\xi, \theta, \bar{\theta})] + \theta \bar{\theta} S^\mu[\tilde{g}(\xi, \theta, \bar{\theta})],
\]

(12)

where \( \tilde{X}^\mu[\tilde{g}(\xi, \theta, \bar{\theta}), \theta, \bar{\theta}] \) are the superfields whose arguments incorporate the super diffeomorphism transformations (11) and, on the r.h.s, we have the secondary superfields which have the following super expansions [as their arguments are transformations (11)], namely:

\[
\begin{align*}
\theta \bar{\theta} S^\mu \left[ \xi^a - \theta \tilde{C}^a - \bar{\theta} \tilde{C}^a + \theta \bar{\theta} f^a \right] & \equiv \theta \bar{\theta} S^\mu(\xi^a), \\
\bar{\theta} \tilde{R}^\mu \left[ \xi^a - \theta \tilde{C}^a - \bar{\theta} \tilde{C}^a + \theta \bar{\theta} f^a \right] & \equiv \bar{\theta} \tilde{R}^\mu(\xi^a) + \theta \tilde{C}^a \partial_a R^\mu(\xi^a), \\
\theta \tilde{R}^\mu \left[ \xi^a - \theta \tilde{C}^a - \bar{\theta} \tilde{C}^a + \theta \bar{\theta} f^a \right] & \equiv \theta \tilde{R}^\mu(\xi^a) - \bar{\theta} \tilde{C}^a \partial_a \tilde{R}^\mu(\xi^a), \\
X^\mu \left[ \xi^a - \theta \tilde{C}^a - \bar{\theta} \tilde{C}^a + \theta \bar{\theta} f^a \right] & \equiv X^\mu(\xi^a) - \theta \tilde{C}^a \partial_a X^\mu - \bar{\theta} \tilde{C}^a \partial_a X^\mu \\
& \quad + \theta \bar{\theta} [f^a \partial_a X^\mu - \tilde{C}^a \partial_a \partial_m X^\mu],
\end{align*}
\]

(13)
where $X^n \left( \xi^a - \theta \bar{C}^a - \bar{\theta} C^a + \theta \bar{\theta} f^a \right)|_{\theta = \bar{\theta} = 0} = X^n(\xi)$ and the Taylor expansions have been taken around $\theta = \bar{\theta} = 0$. Collecting the coefficients of $\theta, \bar{\theta}$ and $\theta \bar{\theta}$, from the r.h.s. of the above equation, we obtain the following:

$$X^n [\bar{g}(\xi, \theta, \bar{\theta}), \theta, \bar{\theta}] = X^n(\xi) + \theta [\bar{R}^n - \bar{C}^a \partial_a X^n] + \bar{\theta} [R^n - C^a \partial_a X^n]$$

$$+ \theta \bar{\theta} [f^a \partial_a X^n - \bar{C}^a C^m \partial_a \partial_m X^n - C^a \partial_a \bar{R}^n + \bar{C}^a \partial_a R^n + S^n]. \tag{14}$$

We note that the target space coordinate fields $X^n(\xi)$ are the pure scalars with respect to the 2D world-sheet on which we have taken the diffeomorphism symmetry transformations $\xi^a \rightarrow \xi'^a = g^n(\xi)$. Thus, physically, it is evident that, ultimately, the restrictions on the $(2, 2)$-dimensional superfield $\tilde{X}^n [\bar{g}(\xi, \theta, \bar{\theta}), \theta, \bar{\theta}]$ is the following:

$$X^n(\xi) \rightarrow \tilde{X}^n [\bar{g}(\xi, \theta, \bar{\theta}), \theta, \bar{\theta}] = X^n(\xi). \tag{15}$$

This is what has been called as the horizontality condition (HC) in [13, 18]. This HC [cf. Eq. (15)] amounts to setting the coefficients of $\theta, \bar{\theta}$ and $\theta \bar{\theta}$ in the expression (14) equal to zero. In other words, we have the following:

$$R^n = C^a \partial_a X^n, \quad \bar{R}^n = \bar{C}^a \partial_a X^n,$n

$$S^n = C^a \partial_a \bar{R}^n - \bar{C}^a \partial_a R^n + \bar{C}^a C^m \partial_a \partial_m X^n - f^a \partial_a X^n. \tag{16}$$

The last entry can be explicitly written by plugging in the values of $R^n$ and $\bar{R}^n$ as

$$S^n = C^a \partial_a [\bar{C}^m \partial_m X^n] - \bar{C}^a \partial_a [C^m \partial_m X^n] + \bar{C}^a C^m \partial_a \partial_m X^n - f^a \partial_a X^n. \tag{17}$$

Now it is straightforward to check that we have the following:

$$S^n = \left[ C^a \partial_a \bar{C}^m - \bar{C}^a \partial_a C^m - f^m \right] (\partial_m X^n) - \bar{C}^a C^m \partial_a \partial_m X^n. \tag{18}$$

As pointed out earlier, the coefficients of $\theta \bar{\theta}$ [i.e. $f^a(\xi)$] in Eq. (11) and their presence in (18) can be computed by the requirements of the consistency conditions of BRST formalism.

One of the sacrosanct properties of a pure scalar field/superfield is the observation that it should not transform under any kind of internal, spacetime, supersymmetric, etc., transformations. As a consequence, the secondary superfields of the r.h.s. of (12) are

$$X^n(\xi) \rightarrow \tilde{X}^n [\bar{g}(\xi, \theta, \bar{\theta}), \theta, \bar{\theta}] = \tilde{X}^n(\xi, \theta, \bar{\theta}).$$

Substitutions of these equalities into (12) yield the following expressions in terms of $s_{(a)b}$, namely;

$$\tilde{X}^n(\xi, \theta, \bar{\theta}) = X^n(\xi) + \theta R^n(\xi) + \bar{\theta} \bar{R}^n(\xi) + \theta \bar{\theta} S^n(\xi)$$

$$= X^n(\xi) + \theta (s_{ab} X^a) + \bar{\theta} (s_{a\bar{b}} X^a) + \theta \bar{\theta} (s_b s_{ab} X^a), \tag{20}$$

in a view of the Bonora-Tonin (BT) mappings: $s_b \leftrightarrow \partial_{\bar{b}} |_{\bar{\theta} = 0}$, and $s_{ab} \leftrightarrow \partial_{b} |_{\bar{\theta} = 0}$ which was established in the realm of D-dimensional non-Abelian 1-form gauge theory [4,5]. In fact, a close look at (20) demonstrates that this expansion is exactly like the BT-superfield.
approach to BRST formalism in the context of gauge theories. Thus, it is clear from (16) and (18), that we have obtained the following [in terms of the (anti-)BRST symmetry transformations \((s_{ab})\) of (7) and (6)], namely;

\[
R^\mu = C^a \partial_a X^\mu = s_b X^\mu, \quad \bar{R}^\mu = \bar{C}^a \partial_a X^\mu = s_{ab} X^\mu,
\]

\[
S^\mu = [C^a \partial_a \bar{C}^m - \bar{C}^a \partial_a C^m - f^m(\xi)] (\partial_m X^\mu) - \bar{C}^a C^m \partial_a \partial_m X^\mu \equiv s_b s_{ab} X^\mu. \tag{21}
\]

The absolute anticommutativity requirement (i.e. \(\{s_b, s_{ab}\} X^\mu = 0\)) implies that the following equality is true, namely;

\[
s_b \bar{R}^\mu = -s_{ab} R^\mu \iff s_b s_{ab} X^\mu = -s_{ab} s_b X^\mu. \tag{22}
\]

The explicit computations of \(s_b \bar{R}^\mu\) and \((-s_{ab} R^\mu\) yield

\[
s_b \bar{R}^\mu = i B^m \partial_m X^\mu - \bar{C}^a C^m \partial_a \partial_m X^\mu - \bar{C}^a (\partial_a C^m) (\partial_m X^\mu),
\]

\[-s_{ab} R^\mu = -i \bar{B}^m \partial_m X^\mu - C^m \partial_a \partial_m X^\mu + C^a (\partial_a \bar{C}^m) (\partial_m X^\mu), \tag{23}
\]

where we have used \(s_b \bar{C}^a = i B^a\) and \(s_{ab} C^a = i \bar{B}^a\). In addition, we have taken \(s_b C^a = C^m \partial_m C^a\) and \(s_{ab} \bar{C}^a = \bar{C}^m \partial_m C^a\) which are derived from the nilpotency requirements: \(s_b^2 X^\mu = 0\) and \(s_{ab}^2 X^\mu = 0\). The above equality (22) implies [from (23)] that we have

\[
B^m + \bar{B}^m + i (C^a \partial_a \bar{C}^m + \bar{C}^a \partial_a C^m) = 0, \tag{24}
\]

which is nothing but the CF-type restrictions that have been obtained [cf. Eq. (10)] from the requirement of the absolute anticommutativity property (i.e. \(\{s_b, s_{ab}\} = 0\)) of the (anti-)BRST symmetry transformations (7) and (6).

At this crucial juncture, we are in the position to determine the explicit expression for \(f^a(\xi)\) that is present in Eqs. (11) and (18) by demanding the equality of each of the equations present in (23) with the expression for \(S^\mu\) in (21). In other words, we find that:

\[
S^\mu = s_b \bar{R}^\mu \equiv -s_{ab} R^\mu \implies
\]

\[
[C^a \partial_a \bar{C}^m - \bar{C}^a \partial_a C^m - f^m(\xi)] (\partial_m X^\mu) - \bar{C}^a C^m \partial_a \partial_m X^\mu
\]

\[
= (i B^m - \bar{C}^a \partial_a C^m) (\partial_m X^\mu) - \bar{C}^a C^m \partial_a \partial_m X^\mu
\]

\[
\equiv (-i \bar{B}^m + C^a \partial_a \bar{C}^m) (\partial_m X^\mu) - \bar{C}^a C^m \partial_a \partial_m X^\mu. \tag{25}
\]

A close look at (25) implies that there are two ways to equate the l.h.s [containing \(f^m(\xi)\)] with the r.h.s. of the above equation, namely;

\[
f^m(\xi) = -i B^m + \bar{C}^a \partial_a C^m \equiv i \bar{B}^m - C^a \partial_a \bar{C}^m, \tag{26}
\]

which lead to the derivation of the CF-type restrictions (24). Thus, we conclude that the CF-type restrictions are hidden in the determination of \(f^a(\xi)\) of equation (11) by exploiting the absolute anticommutativity property (i.e. \(\{s_b, s_{ab}\} X^\mu = 0\)) within the ambit of MBTSA to BRST formalism. Ultimately, we observe that the above logic can be repeated in the case of a pure scalar \((\det \bar{g})\) to derive the CF-type restrictions (24) and the (anti-)BRST transformations: \(s_{ab} (\det \bar{g}) = C^a \partial_a (\det \bar{g})\) and \(s_b (\det \bar{g}) = C^a \partial_a (\det \bar{g})\), too.
We wrap-up this section with the following remarks. First of all, we have taken the standard (anti-)BRST symmetry transformations: \( s_{ab} C^a = i B^a, s_b C^a = i B^a, s_{ab} \bar{B}^a = 0, s_b B^a = 0 \) which imply the following [in the terminology of the (anti-)chiral superfield approach (ACSA) to BRST formalism (see, e. g. [15])], namely;

\[
\begin{align*}
C^m(\xi) &\rightarrow F^{m(c)}_{(ab)}(\xi, \theta) = C^m(\xi) + \theta (i \bar{B}^m) \equiv C^m(\xi) + \theta (s_{ab} C^m), \\
\bar{C}^m(\xi) &\rightarrow \bar{F}^{m(ac)}_{(b)}(\xi, \bar{\theta}) = \bar{C}^m(\xi) + \bar{\theta} (i B^m) \equiv \bar{C}^m(\xi) + \bar{\theta} (s_b \bar{C}^m), \\
B^m(\xi) &\rightarrow \bar{B}^{m(ac)}_{(b)}(\xi, \bar{\theta}) = B^m(\xi) + \bar{\theta} (0) \equiv B^m(\xi) + \bar{\theta} (s_b B^m), \\
\bar{B}^m(\xi) &\rightarrow \bar{B}^{m(c)}_{(ab)}(\xi, \theta) = \bar{B}^m(\xi) + \theta (0) \equiv \bar{B}^m(\xi) + \theta (s_{ab} \bar{B}^m),
\end{align*}
\]

where the superscripts \((c)\) and \((ac)\) on the superfields [cf. the l.h.s. of (27)] denote the chiral and anti-chiral versions of the full super expansions and the subscripts \((b)\) and \((ab)\) denote the fact that the coefficients of \((\bar{\theta})\ \theta\) in the above expansions lead to the determination of BRST and anti-BRST symmetry transformations. In other words, we are sure about the nilpotent (anti-)BRST symmetry transformations: \( s_{ab} C^a = i \bar{B}^a, s_b \bar{B}^a = 0, s_b \bar{C}^a = i B^a, s_b B^a = 0 \) in terms of the (anti-)chiral superfield expansions in Eq. (27). Second, it is the off-shell nilpotency requirements \( s_{(a)b}^2 X^\mu = 0 \) which lead to \( s_b C^a = C^m \partial_m C^a \) and \( s_{ab} \bar{C}^a = \bar{C}^m \partial_m \bar{C}^a \). However, we have to obtain these transformations within the realm of superfield approach. Furthermore, it is the requirement of the absolute anticommutativity properties: \( \{ s_b, s_{ab} \} C^a = 0, \{ s_b, s_{ab} \} \bar{C}^a = 0 \) which yield \( s_b \bar{B}^a = C^m \partial_m \bar{B}^a - \bar{B}^m \partial_m C^a \) and \( s_{ab} B^a = \bar{C}^m \partial_m B^a - B^m \partial_m \bar{C}^a \). We have to obtain, however, these symmetry transformations too, by using the techniques of the superfield approach to BRST formalism which we accomplish in our next section. Third, we note that the HC condition (15) has led to the following full super expansion of the target space coordinate superfield, namely;

\[
\begin{align*}
\tilde{X}^\mu(h)(\xi, \theta, \bar{\theta}) &= X^\mu(\xi) + \theta (\bar{C}^a \partial_a X^\mu) + \bar{\theta} (C^a \partial_a X^\mu) \\
&\quad + \theta \bar{\theta} \left( \left[ i B^a - (C^m \partial_m C^a) \right] \partial_a X^\mu - \bar{C}^m C^a \partial_m \partial_a X^\mu \right) \\
&\equiv X^\mu(\xi) + \theta (s_{ab} X^\mu) + \bar{\theta} (s_b X^\mu) + \theta \bar{\theta} (s_{ab} s_{ab} X^\mu),
\end{align*}
\]

where the superscript \((h)\) denotes the target space coordinate superfield that has been obtained after the application of HC which, ultimately, leads to (20). Here the coefficients of \( \theta \) and \( \bar{\theta} \) are the (anti-)BRST symmetry transformations \( [s_{(a)b}] \) that are listed in Eqs. (7) and (6). Finally, we comment that an expansion like (28) can be also written for the derivation of the (anti-)BRST symmetry transformations for the scalar \((\det \tilde{g})\).

4 (Anti-)BRST Symmetries of Other Fields: ACSA

In this section, we exploit the theoretical strength of ACSA to BRST formalism (see, e. g. [15] and reference therein) to derive all the (anti-)BRST symmetry transformations (7) and (6) except such transformations for the target space coordinates \( X^\mu \) and \((\det \tilde{g})\) which have already been derived in the previous section by using MBTSA to BRST formalism [13, 18]. We are inspired to use, in our present section, ACSA to BRST formalism because of our observations in Eq. (27). First of all, we focus on the derivation of the BRST symmetry
transformations (7) which have not been derived in the previous section. Thus, we wish to obtain: \( s_b C^a = C^m \partial_m C^a \), \( s_b B^a = C^m \partial_m \bar{B}^a - \bar{B}^m \partial_m C^a \), \( s_b \bar{g}^{mn} = \partial_a (C^a \bar{g}^{mn}) - (\partial_a C^m) \bar{g}^{an} - (\partial_a C^a) \bar{g}^{ma}, s_b E = (\partial_a C^a) E + C^a (\partial_a E) \). In this context, first of all, we generalize the ordinary 2D fields \( C^a(\xi), \bar{B}^a(\xi), E(\xi) \) and \( \bar{g}^{mn}(\xi) \) onto a \((2,1)\)-dimensional anti-chiral super sub-manifold of the \(\text{general } (2,2)\)-dimensional supermanifold as

\[
C^m(\xi) \rightarrow F^{m(ac)}(\xi, \bar{\theta}) = C^m(\xi) + \bar{\theta} b^m(\xi), \\
\bar{B}^m(\xi) \rightarrow \bar{B}^{m(ac)}(\xi, \bar{\theta}) = \bar{B}^m(\xi) + \bar{\theta} f^m(\xi), \\
E(\xi) \rightarrow E^{(ac)}(\xi, \bar{\theta}) = E(\xi) + \bar{\theta} f_1(\xi), \\
\bar{g}^{mn}(\xi) \rightarrow G^{mn(ac)}(\xi, \bar{\theta}) = \bar{g}^{mn}(\xi) + \bar{\theta} \bar{R}^{mn}(\xi),
\]

where the 2D fields \((f_1^m, f_2, \bar{R}^{ab})\) are fermionic secondary fields and \(b^m(\xi)\) is a bosonic secondary field due to the fermionic \((\bar{\theta}^2 = 0)\) nature of the Grassmannian variable \(\bar{\theta}\). The above \((2,1)\)-dimensional anti-chiral super sub-manifold is parameterized by \((\xi^a, \bar{\theta})\) where \(\xi^a \equiv (\tau, \sigma)\) are the bosonic coordinates and \(\bar{\theta}\) is the fermionic \((\bar{\theta}^2 = 0)\) Grassmannian variable. The superscript \((ac)\) on the superfields denotes the anti-chiral super expansions of the above anti-chiral superfields along \(\bar{\theta}\)-direction of the above super sub-manifold.

The basic tenets of ACSA to BRST formalism require that the BRST-invariant (i.e. quantum gauge invariant) quantities should be independent of the Grassmannian variables as the latter are only the mathematical artifacts that are useful in the context of theoretical techniques of SUSY theories. In this connection, we note that the following BRST (i.e. quantum gauge) invariant quantities are useful and important for us, namely:

\[
s_b [C^a \partial_a X^\mu] = 0, \quad s_b [C^a \partial_a \bar{B}^m - \bar{B}^m \partial_a C^m] = 0, \\
s_b [C^a \partial_a E + (\partial_a C^a) E] = 0, \\
s_b [C^a \partial_a \bar{g}^{mn} + (\partial_a C^a) \bar{g}^{mn} - (\partial_a C^m) \bar{g}^{ma} - (\partial_a C^a) \bar{g}^{ma}] = 0. \quad (30)
\]

The above invariant quantities are obtained by a close observation of the transformations (7) where an off-shell nilpotency property \((s_b^2 = 0)\) exists for the BRST-symmetry transformations. We focus on \(s_b [C^a \partial_a X^\mu] = 0\) which implies the following restriction

\[
F^{m(ac)}(\xi, \bar{\theta}) \partial_m X^{\mu(h,ac)}(\xi, \bar{\theta}) = C^m(\xi) \partial_m X^\mu(\xi), \quad (31)
\]

where \(X^{\mu(h,ac)}(\xi, \bar{\theta})\) is the anti-chiral limit of the full super expansion containing the nilpotent (anti-)BRST symmetries as the coefficients of \(\theta\) and \(\bar{\theta}\). In other words, we have:

\[
X^{\mu(h,ac)}(\xi, \bar{\theta}) = X^\mu(\xi) + \bar{\theta} (C^a \partial_a X^\mu). \quad (32)
\]

Plugging in the appropriate super expansions for \(F^a(\xi, \bar{\theta})\) from (29) as well as the super expansion for \(X^{\mu(h,ac)}(\xi, \bar{\theta})\) from (32), we obtain the explicit expression for the secondary fields as: \(b^m(\xi) = C^a \partial_a C^m\). As a consequence, we have the following final expansion

\[
F^{m(ac)}_{(b)}(\xi, \bar{\theta}) = C^m(\xi) + \bar{\theta} (C^a \partial_a C^m) \equiv C^m(\xi) + \bar{\theta} (s_b C^m), \quad (33)
\]

where the subscript \((b)\) on the superfield (on the l.h.s.) denotes that the above anti-chiral superfield has been obtained after the application of the BRST invariant restrictions (31).
and the coefficient of $\bar{\theta}$ is nothing but the BRST symmetry transformation for the field $C^m(\xi)$ which also encodes the relationships: $\partial_\theta F^{m(ac)}(\xi, \bar{\theta}) = s_b C^m(\xi)$ and $\partial^2_\theta = 0 \Leftrightarrow s_b^2 = 0$. The latter establishes the connection between the nilpotency properties of $\partial_\theta$ and $s_b$.

At this juncture, we now concentrate on the derivation of $f_2(\xi)$ in the expansion of $\mathcal{E}^{ac}(\xi, \bar{\theta})$ in Eq. (29). For this purpose, we note that $s_b [C^m \partial_m E + (\partial_m C^m) E] = 0$. Following the basic principle of ACSA, the expressions in the square bracket have to be generalized onto the $(2, 1)$-dimensional anti-chiral super sub-manifold with the following BRST (i.e. quantum gauge) symmetry invariant restriction

$$F^{m(ac)}(\xi, \bar{\theta}) \partial_m \mathcal{E}^{(ac)}(\xi, \bar{\theta}) + [\partial_m F^{m(ac)}(\xi, \bar{\theta})] \mathcal{E}^{(ac)}(\xi, \bar{\theta})$$

$$= C^m(\xi) [\partial_m E(\xi)] + [\partial_m C^m(\xi)] E(\xi),$$

(34)

where the expansions of $F^{m(ac)}(\xi, \bar{\theta})$ and $\mathcal{E}^{(ac)}(\xi, \bar{\theta})$ have been quoted in Eqs. (33) and (29), respectively. Substitutions of these super expansions into the l.h.s. and comparison with the r.h.s. of the restriction (34), lead to the following condition

$$(\partial_m C^m) (\partial_a C^m) E + C^a (\partial_a \partial_m C^m) E + C^a (\partial_a C^m) (\partial_m E)$$

$$-(\partial_m C^m) f_2 - C^m (\partial_m f_2) = 0.$$  

(35)

In other words, the restriction (34) implies that BRST invariant quantity must be independent of $\bar{\theta}$. A careful and close look at the above equation leads to:

$$\partial_m \left[ C^a (\partial_a C^m) E - C^m f_2 \right] = 0.$$  

(36)

Substituting for $C^a (\partial_a C^m) E = \partial_a \{ C^a C^m E \} - (\partial_a C^a) C^m E - C^a C^m (\partial_a E)$, we obtain the following from the above equation:

$$\partial_m \{ C^a C^m E \} - (\partial_a C^a) C^m E - C^a C^m (\partial_a E) - C^m f_2 = 0.$$  

(37)

It is clear that the first term in the square-bracket will be zero if we operate the derivative ($\partial_m$) from outside. Thus, the final expression is as follows:

$$\partial_m \left[ C^m \{ \partial_a (C^a E) - f_2 \} \right] = 0.$$  

(38)

Integrating over $d^2 \xi = d\sigma d\tau$ and taking the physicality condition that all the fields must vanish off as $\tau \to \pm \infty$ and at $\sigma = 0$, $\sigma = \pi$, we obtain the precise value of $f_2(\xi)$ as

$$f_2 = \partial_a (C^a E) \quad [\text{for } C^m \neq 0].$$  

(39)

Hence, we have the following final expansion for the superfield $\mathcal{E}^{(ac)}(\xi, \bar{\theta})$

$$\mathcal{E}^{(ac)}(\xi, \bar{\theta}) = E(\xi) + \bar{\theta} \left[ \partial_n (C^m E) \right] \equiv E(\xi) + \bar{\theta} \left( s_b E \right),$$  

(40)

which leads to the derivation of the BRST symmetry transformation $s_b E = \partial_a (C^a E)$ as the coefficient of $\bar{\theta}$ in the above equation implying, once again, that $\partial_\theta \mathcal{E}^{(ac)}(\xi, \bar{\theta}) = s_b E(\xi)$. This relationship establishes the connection between $s_b$ and translational generator $\partial_\theta$ along the $\bar{\theta}$-direction of the $(2, 1)$-dimensional anti-chiral super sub-manifold and it also demonstrates
that \( s_b^2 = 0 \Leftrightarrow \partial^2_b = 0 \) (which is the connection between the nilpotency properties). It goes without saying that the subscript \((b)\) on the l.h.s. denotes that the super expansion (40) has been obtained after the application of the BRST invariant restriction (34).

We now focus on the BRST invariance: \( s_b [C^n \partial_a \bar{B}^m - \bar{B}^n \partial_a C^m] = 0 \). This observation can be generalized onto the \((2, 1)\)-dimensional anti-chiral super sub-manifold with the following restriction on the anti-chiral superfields, namely:

\[
F^{(ac)}_{(b)}(\xi, \bar{\theta}) \partial_m \bar{B}^{(ac)}(\xi, \bar{\theta}) - \bar{B}^{(ac)}_{(b)}(\xi, \bar{\theta}) \partial_m F^{(ac)}_{(b)}(\xi, \bar{\theta}) = C^m(\xi) \partial_m \bar{B}^m(\xi) - \bar{B}^m(\xi) \partial_m C^m(\xi). \tag{41}
\]

The substitutions of expansions from (29) and (33) lead to the following equality:

\[
C^n \left[ \partial_n f_1^m + \bar{B}^a(\partial_a \partial_n C^m) - (\partial_a C^n)(\partial_a \bar{B}^m) \right] + [f^a_1 + \bar{B}^a(\partial_a C^m) - C^m \partial_a \bar{B}^a](\partial_a C^m) = 0. \tag{42}
\]

In the above, the term: \(- (\partial_a C^n)(\partial_a \bar{B}^m)\) can be written as: \(- \partial_a [C^n \partial_a \bar{B}^m] + C^n \partial_a \partial_a \bar{B}^m\). It is elementary to note that the second term will vanish-off when we shall multiply by \(C^n\) from the left (i.e. \(C^n \partial_a \partial_a \bar{B}^m = 0\)). The substitution of the leftover term (i.e. \(- \partial_a [C^n \partial_a \bar{B}^m]\)) into (42) leads to:

\[
C^n \partial_a \left[ f_1^m + \bar{B}^a(\partial_a C^m) - C^m \partial_a \bar{B}^a \right] + [f^a_1 + \bar{B}^a(\partial_a C^m) - C^m \partial_a \bar{B}^a](\partial_a C^m) = 0. \tag{43}
\]

It is straightforward to note that \( f_1^m = C^n \partial_a \bar{B}^m - \bar{B}^a(\partial_a C^m)\) satisfies the above equation very beautifully. Thus, we have, ultimately, the following expansion [cf. Eq. (29)]:

\[
\bar{B}^{(ac)}_{(b)}(\xi, \bar{\theta}) = \bar{B}^m(\xi) + \bar{\theta} [C^n \partial_a \bar{B}^m - \bar{B}^a(\partial_a C^m)] \equiv \bar{B}^m(\xi) + \bar{\theta} [s_b \bar{B}^m(\xi)]. \tag{44}
\]

Hence, we have derived the BRST transformations: \( s_b \bar{B}^m = C^n \partial_a \bar{B}^m - \bar{B}^a(\partial_a C^m)\) as the coefficient of \(\bar{\theta}\) in the above super expansion. It should be noted that the subscript \((b)\) on the superfield [cf. l.h.s. of Eq. (44)] denotes that \(\bar{B}^{(ac)}_{(b)}(\xi, \bar{\theta})\) has been derived after the imposition of the BRST invariant restriction (41).

At this stage, we now wish to derive the BRST symmetry transformation \([s_b \bar{g}^{mn} = \partial_k(C^k \bar{g}^{mn}) - (\partial_k C^m) \bar{g}^kn - (\partial_k C^n) \bar{g}^mk]\) using the theoretical strength of ACSA to BRST formalism. Towards this goal in mind, we have the following restriction on the anti-chiral superfields which have their super expansions in (29) and (33), namely:

\[
F^{(ac)}_{(b)}(\xi, \bar{\theta}) \partial_k \bar{G}^{mn(ac)}(\xi, \bar{\theta}) + [\partial_k F^{(ac)}_{(b)}(\xi, \bar{\theta})] \bar{G}^{mn(ac)}(\xi, \bar{\theta})
- [\partial_k F^{(ac)}_{(b)}(\xi, \bar{\theta})] \bar{G}^{kn(ac)}(\xi, \bar{\theta}) - [\partial_k F^{(ac)}_{(b)}(\xi, \bar{\theta})] \bar{G}^{km(ac)}(\xi, \bar{\theta})
= C^k(\xi) \partial_k \bar{g}^{mn}(\xi) + [\partial_k C^k(\xi)] \bar{g}^{mn}(\xi)
- [\partial_k C^m(\xi)] \bar{g}^{kn}(\xi) - [\partial_k C^m(\xi)] \bar{g}^{mk}(\xi). \tag{45}
\]

The above restriction has been obtained by a close look at the off-shell nilpotency property \((s_b^2 \bar{g}^{mn} = 0)\) of the BRST symmetry transformations (7). This restriction on the anti-chiral superfields leads to the following condition on the basic and secondary fields:

\[
C^k(\partial_k \bar{R}^{mn}) + (\partial_k C^k) \bar{R}^{mn} - (\partial_k C^l) (\partial_l C^k) \bar{g}^{mn} - C^l (\partial_k \partial_l C^k) \bar{g}^{mn}
- C^l (\partial_k C^k) (\partial_k \bar{g}^{mn}) - (\partial_k C^m) \bar{R}^{kn} + (\partial_k C^k) (\partial_l C^m) \bar{g}^{kn} + C^l (\partial_k \partial_l C^m) \bar{g}^{kn}
- (\partial_k C^m) \bar{R}^{mk} + (\partial_k C^l) (\partial_l C^m) \bar{g}^{mk} + C^l (\partial_k \partial_l C^m) \bar{g}^{mk} = 0, \tag{46}
\]
where we have used the super expansions from (29) and (33). It is straightforward to note that the first five terms, in the above, lead to the following total derivative, namely:

\[ \partial_k \left[ C_k \tilde{R}^{mn} - C^l (\partial_l C^k) \tilde{g}^{mn} \right] = \partial_k \left[ C_k \tilde{R}^{mn} - \partial_l (C^l \tilde{g}^{mn}) \right], \tag{47} \]

where we have used: \(- C^l (\partial_l C^k) \tilde{g}^{mn} = - \partial_l \left[ C^l C^k \tilde{g}^{mn} \right] + (\partial_l C^l) \tilde{g}^{mn} + C^l C^k (\partial_l \tilde{g}^{mn})\) and \(\partial_k \partial_l (C^l C^k \tilde{g}^{mn}) = 0\). Adding and substracting: \(\partial_k \left[ C_k (\partial_l C^m) \tilde{g}^{ln} + C^k (\partial_l C^m) \tilde{g}^{ml} \right]\) we obtain the following equation from (46):

\[
\partial_k \left[ C_k \{ \tilde{R}^{mn} - \partial_l (C^l \tilde{g}^{mn}) \} \right]
- \partial_k \left[ C_k (\partial_l C^m) \tilde{g}^{ln} + C^k (\partial_l C^m) \tilde{g}^{ml} \right] = 0. \tag{48}
\]

Expanding the total derivative in the second entry of the above equation and rearranging these, we obtain the following interesting equation, namely:

\[
\partial_k \left[ C_k \{ \tilde{R}^{mn} - \partial_l (C^l \tilde{g}^{mn}) \} \right]
- (\partial_k C^m) \left[ \tilde{R}^{mk} - \partial_l (C^l \tilde{g}^{mk}) + (\partial_l C^k) \tilde{g}^{ln} \right]
- (\partial_k C^m) \left[ \tilde{R}^{mk} - \partial_l (C^l \tilde{g}^{mk}) + (\partial_l C^k) \tilde{g}^{lm} \right] = 0. \tag{49}
\]

Adding and subtracting \((\partial_k C^m) (\partial_l C^m) \tilde{g}^{lk} + (\partial_k C^m) (\partial_l C^m) \tilde{g}^{lk}\) in the above, we finally obtain the following very nice looking equation:

\[
\partial_k \left[ C_k \{ \tilde{R}^{mn} - \partial_l (C^l \tilde{g}^{mn}) \} \right]
- (\partial_k C^m) \left[ \tilde{R}^{mk} - \partial_l (C^l \tilde{g}^{mk}) + (\partial_l C^k) \tilde{g}^{ln} \right]
- (\partial_k C^m) \left[ \tilde{R}^{mk} - \partial_l (C^l \tilde{g}^{mk}) + (\partial_l C^k) \tilde{g}^{lm} \right] = 0. \tag{50}
\]

It should be noted that what we have added and subtracted in (49) is basically equal to zero on its own because we make the following observation:

\[
\tilde{g}^{lk} \left[ (\partial_k C^m) (\partial_l C^m) + (\partial_k C^m) (\partial_l C^m) \right] = 0. \tag{51}
\]

In other words, the last entries in the second and third lines of Eq. (50) are zero on their own. We note that the symmetric indices in \((\tilde{g}^{lk})\) and anti-symmetric indices \((l, k)\) in the square-bracket are sum-up to yield zero. It is straightforward now to point out that

\[
\tilde{R}^{mn} = \partial_k (C^k \tilde{g}^{mn}) - (\partial_k C^m) \tilde{g}^{kn} - (\partial_k C^m) \tilde{g}^{mk}, \tag{52}
\]

satisfies the above equation (50). As a consequence, we have the following

\[
\tilde{G}_{(b)}^{mn(ac)}(\xi, \bar{\theta}) = \tilde{g}^{mn}(\xi) + \bar{\theta} \left[ \partial_k (C^k \tilde{g}^{mn}) - (\partial_k C^m) \tilde{g}^{kn} - (\partial_k C^m) \tilde{g}^{mk} \right]
\equiv \tilde{g}^{mn}(\xi) + \bar{\theta} \left[ s_b \tilde{g}^{mn}(\xi) \right], \tag{53}
\]

where the coefficient of \(\bar{\theta}\) is nothing but the BRST symmetry transformation for \(\tilde{g}^{mn}(\xi)\) that has been quoted in (7). The subscript \((b)\) on the l.h.s. of the above equation denotes that the anti-chiral superfield \(\tilde{G}_{(b)}^{mn}(\xi, \bar{\theta})\) has been obtained after the application of the BRST invariant restriction on a specific combination of superfields [cf. Eq. (45)].
We set out now to derive the anti-BRST symmetry transformations (6) by using ACSA to BRST formalism where, first of all, we generalize the following basic and auxiliary fields of our theory onto a (2, 1)-dimensional chiral super submanifold

\[ B'^m(\xi) \rightarrow B'^{m(c)}(\xi, \theta) = B'^m(\xi) + \theta \bar{f}'_1^m(\xi), \]
\[ E(\xi) \rightarrow E^{(c)}(\xi, \theta) = E(\xi) + \theta \bar{f}'_2(\xi), \]
\[ \bar{C}'^m(\xi) \rightarrow \bar{F}'^{m(c)}(\xi, \theta) = \bar{C}'^m(\xi) + \theta \bar{b}'_1^m(\xi), \]
\[ \tilde{g}'^{mn}(\xi) \rightarrow \tilde{G}'^{mn(c)}(\xi, \theta) = \tilde{g}'^{mn}(\xi) + \theta \tilde{R}'^{mn}(\xi), \]

(54)

where (\( \bar{f}'_1^m, \bar{f}'_2^m, \tilde{R}'^{mn} \)) are the fermionic and \( \bar{b}'_1^m \) is the bosonic secondary fields that are to be determined in terms of the basic and auxiliary fields of the (anti-)BRST invariant Lagrangian densities \( \mathcal{L}_{ab} \) [cf. Eqs. (1), (2)]. It is elementary to note that, in the limit \( \theta = 0 \), we retrieve the bosonic and auxiliary fields of \( \mathcal{L}_{ab} \). We point out that \( s_{ab} B'^m(\xi) = 0 \) implies that we have \( \mathcal{B}'^m_{(ab)}(\xi, \theta) = B'^m(\xi) \) where \( \mathcal{B}'^m(\xi, \theta) \) is the superfield that has been obtained after the restriction on the chiral superfield \( \mathcal{B}'^m(\xi, \theta) \) that is obtained in the generalization \( \bar{B}'^m(\xi) \rightarrow \mathcal{B}'^m(\xi, \theta) \) on the chiral super manifold [which is parameterized by \( (\xi^a, \theta) \) where \( \xi^a \) characterize the 2D world-sheet and \( \theta \) is the fermionic \( (\theta^2 = 0) \) Grassmannian variable]. The subscript \( (ab) \) denotes the chiral superfield which leads to the derivation of \( [s_{ab} \bar{B}'^m(\xi) = 0] \) as the coefficient of \( \theta \) in its expansion: \( \mathcal{B}'^m_{(ab)}(\xi, \theta) = \bar{B}'^m(\xi) + \theta(0) \equiv \bar{B}'^m(\xi) + \theta(s_{ab} \bar{B}'^m(\xi)) \). It should be further noted that we have not devoted time on the derivation of the (anti-)BRST symmetries that have already been derived and mentioned in Sec. 3 where the theoretical strength of MBTSA has been exploited.

A close and careful observation of the anti-BRST symmetry transformations (6) demonstrates that we have the following very useful and interesting combinations of fields

\[ s_{ab} [\bar{C}'^a \partial_a X^\mu] = 0, \]
\[ s_{ab} [\bar{C}'^a \partial_a B'^m - B'^a \partial_a \bar{C}'^m] = 0, \]
\[ s_{ab} [\bar{C}'^a \partial_a \bar{g}'^{mn} + (\partial_a \bar{C}'^a) \tilde{g}'^{mn} - (\partial_a \bar{C}'^m) \tilde{g}'^{mn} - (\partial_a \bar{C}'^a) \tilde{g}'^{mn}] = 0, \]

(55)
as the anti-BRST invariant quantities. The fundamental requirement of ACSA is that the generalizations of the quantities [present in the square bracket of (55)] onto a suitably chosen (2, 1)-dimensional chiral super submanifold should be independent of the Grassmannian variable \( \theta \). As a consequence, we have the following restrictions

\[ \bar{F}'^{a(c)}(\xi, \theta) \partial_a X'^{\mu(h,c)}(\xi, \theta) = \bar{C}'^a(\xi) \partial_a X^\mu(\xi), \]
\[ \bar{F}'^{a(c)}(\xi, \theta) \partial_a B'^{m(c)}(\xi, \theta) - B'^a(\xi, \theta) \partial_a \bar{F}'^{m(c)}(\xi, \theta) = \bar{C}'^a(\xi) \partial_a B'^m(\xi) - B'^a(\xi) \partial_a \bar{C}'^m(\xi), \]
\[ \bar{F}'^{a(c)}(\xi, \theta) \partial_a \mathcal{E}'^{(c)}(\xi, \theta) + [\partial_a \bar{F}'^{a(c)}(\xi, \theta)] \mathcal{E}'^{(c)}(\xi, \theta) = \bar{C}'^a(\xi) \partial_a \mathcal{E}'^m(\xi) + [\partial_a \bar{C}'^a(\xi)] \mathcal{E}'^m(\xi), \]
\[ \bar{F}'^{a(c)}(\xi, \theta) \partial_a \tilde{G}'^{mn(c)}(\xi, \theta) + [\partial_a \bar{F}'^{a(c)}(\xi, \theta)] \tilde{G}'^{mn(c)}(\xi, \theta) \]
\[ = \bar{C}'^a(\xi) [\partial_a \tilde{g}'^{mn}(\xi)] + [\partial_a \bar{C}'^a(\xi)] \tilde{g}'^{mn}(\xi) - [\partial_a \bar{C}'^m(\xi)] \tilde{g}'^{mn}(\xi) - [\partial_a \bar{C}'^a(\xi)] \tilde{g}'^{mn}(\xi), \]

(56)

where we have taken the super expansions from (54) and \( X'^{\mu(h,c)}(\xi, \theta) \) is the chiral limit \( (\bar{\theta} = 0) \) of the full expansion [cf. (28)]. In other words, we have the following

\[ X'^{\mu(h,c)}(\xi, \theta) = X^\mu(\xi) + \theta [\bar{C}'^a \partial_a X^\mu(\xi)], \]

(57)
where the superscript \((h, c)\) denotes the *chiral* version of the *full* expansion of \(X^{\mu(h)}(\xi, \theta)\) that has been obtained in the previous section [cf. Eq. (28)].

We would like to lay emphasis on the fact that *all* the secondary fields \((\tilde{f}_1^m, f_2, \tilde{R}^{mn})\) and \(\tilde{b}_1^m\) can be computed in *exactly* similar manner as we have done in the case of determination of the BRST symmetry transformations \((s_b)\) for the super expansions in Eq. (29). It turns out that, adopting this logic, we obtain the following:

\[
\begin{align*}
\tilde{f}_1^m &= B^a \partial_a C^m - (\partial_a B^a) \bar{C}^m, \quad f_2 = (\partial_a \bar{C}^a) E + \bar{C}^a (\partial_a E), \\
\bar{b}_1^m &= \bar{C}^a \partial_a \bar{C}^m, \quad \tilde{R}^{mn} = \partial_a (\bar{C}^a \tilde{g}^{mn}) - (\partial_a \tilde{C}^m) \tilde{g}^{an} - (\partial_a \bar{C}^m) \tilde{g}^{ma}. \tag{58}
\end{align*}
\]

Substitutions of the above secondary fields into the *chiral* super expansions of Eq. (54), we obtain the following final super expansions

\[
\begin{align*}
\mathcal{B}^{m(c)}_{(ab)}(\xi, \theta) &= B^m(\xi) + \theta \left[ \bar{C}^a \partial_a B^m - B^a \partial_a \bar{C}^m \right] \equiv B^m(\xi) + \theta \left[ s_{ab} B^m(\xi) \right], \\
\mathcal{E}^{c(c)}_{(ab)}(\xi, \theta) &= E(\xi) + \theta \left[ \partial_a (\bar{C}^a E) \right] \equiv E(\xi) + \theta \left[ s_{ab} E(\xi) \right], \\
\mathcal{F}^{m(c)}_{(ab)}(\xi, \theta) &= C^m(\xi) + \theta \left[ \bar{C}^a \partial_a C^m \right] \equiv C^m(\xi) + \theta \left[ s_{ab} C^m(\xi) \right], \\
\mathcal{G}^{m(m)}_{(ab)}(\xi, \theta) &= \tilde{g}^{mn}(\xi) + \theta \left[ \partial_a (\bar{C}^a \tilde{g}^{mn}) - (\partial_a \tilde{C}^m) \tilde{g}^{an} - (\partial_a \bar{C}^m) \tilde{g}^{ma} \right] \\
&\equiv \tilde{g}^{mn}(\xi) + \theta \left[ s_{ab} \tilde{g}^{mn}(\xi) \right], \tag{59}
\end{align*}
\]

where the subscript \((ab)\) on the *chiral* superfields on the l.h.s. of the above equation (59) denotes that the *above* superfields have been obtained after the *quantum* gauge (i.e. anti-BRST) invariant restrictions on the *chiral* superfields [cf. Eq. (56)] have been imposed. It can be readily checked that we have obtained the anti-BRST symmetry transformations: \(s_{ab} B^m = \bar{C}^a \partial_a B^m - B^a \partial_a \bar{C}^m, s_{ab} E = \partial_\theta (\bar{C}^a E), s_{ab} C^m = \bar{C}^a \partial_a C^m, s_{ab} \tilde{g}^{mn} = \partial_\theta (\bar{C}^a \tilde{g}^{mn}) - (\partial_a \tilde{C}^m) \tilde{g}^{an} - (\partial_a \bar{C}^m) \tilde{g}^{ma}\) as the coefficients of the chiral super expansions in (59). It is nice to note that \(\partial_\theta \Omega_{(ab)}(\xi, \theta) = s_{ab} \omega(\xi)\) where the *generic* chiral superfield \(\Omega_{(ab)}(\xi, \theta)\) stands for the l.h.s. of (59) and \(\omega = B^m, E, C^m, \tilde{g}^{mn}\) generic *ordinary* field.

We end this section with the following remarks. First, we have derived the (anti-)BRST symmetry transformations for the fields by exploiting the theoretical tricks of ACSA to BRST formalism. These fields are the *ones* for which the MBTSA has *not* been able to derive the (anti-)BRST symmetry transformations. Second, a careful and close observation of the theoretical contents of Secs. 3 and 4 demonstrate that we have derived *all* the nilpotent (anti-)BRST symmetry transformations for our theory by exploiting the theoretical strength of MBTSA and ACSA. Finally, the (anti-)BRST symmetry transformations for the component fields \(A_0, A_1\) and \(A_2\) of \(\tilde{g}^{mn}(\xi)\) [cf. Eq. (4)] can be obtained from the *exact* expressions for \(s_b \tilde{g}^{mn}(\xi)\) and \(s_{ab} \tilde{g}^{mn}(\xi)\) that have been quoted in (7) and (6). To be more transparent, we find the following *anti-chiral* super expansions:

\[
A_0(\xi) \rightarrow \mathcal{A}^{(ac)}_{0(ab)}(\xi, \bar{\theta}) = A_0(\xi) + \bar{\theta} \left[ C^m \partial_m A_0 - (\partial_0 C^1 - \partial_1 C^0) A_2 \\
- (\partial_0 C^1 + \partial_1 C^0) A_1 \right] \equiv A_0(\xi) + \bar{\theta} \left[ s_b A_0(\xi) \right],
\]
\[ A_1(\xi) \rightarrow A_{1(b)}^{(a)}(\xi, \bar{\theta}) = A_1(\xi) + \bar{\theta} \left[ C^m \partial_m A_1 - (\partial_1 C^0 + \partial_0 C^1) A_0 \right] - (\partial_0 C^0 - \partial_1 C^1) A_2 \equiv A_1(\xi) + \bar{\theta} \left[ s_b A_1(\xi) \right], \]

\[ A_2(\xi) \rightarrow A_{2(b)}^{(a)}(\xi, \bar{\theta}) = A_2(\xi) + \bar{\theta} \left[ C^m \partial_m A_2 - (\partial_1 C^0 + \partial_0 C^1) A_0 \right] - (\partial_0 C^0 - \partial_1 C^1) A_1 \equiv A_2(\xi) + \bar{\theta} \left[ s_b A_2(\xi) \right], \]

where the coefficients of \( \bar{\theta} \) are nothing but the BRST symmetry transformations [cf. Eq. (9)] on \( A_0(\xi), A_1(\xi) \) and \( A_2(\xi) \). In exactly similar fashion, we can obtain the anti-BRST symmetry transformations on \( A_0, A_1 \) and \( A_2 \) from the following chiral super expansions:

\[ A_0(\xi) \rightarrow A_{0(ab)}^{(c)}(\xi, \theta) = A_0(\xi) + \theta \left[ \bar{C}^m \partial_m A_0 - (\partial_0 \bar{C}^1 - \partial_1 \bar{C}^0) A_2 \right] - (\partial_0 \bar{C}^0 - \partial_1 \bar{C}^1) A_1 \equiv A_0(\xi) + \theta \left[ s_{ab} A_0(\xi) \right], \]

\[ A_1(\xi) \rightarrow A_{1(ab)}^{(c)}(\xi, \theta) = A_1(\xi) + \theta \left[ \bar{C}^m \partial_m A_1 - (\partial_0 \bar{C}^0 + \partial_1 \bar{C}^1) A_0 \right] - (\partial_0 \bar{C}^0 - \partial_1 \bar{C}^1) A_2 \equiv A_1(\xi) + \theta \left[ s_{ab} A_1(\xi) \right], \]

\[ A_2(\xi) \rightarrow A_{2(ab)}^{(c)}(\xi, \theta) = A_2(\xi) + \theta \left[ \bar{C}^m \partial_m A_2 - (\partial_0 \bar{C}^0 - \partial_1 \bar{C}^1) A_0 \right] - (\partial_0 \bar{C}^0 - \partial_1 \bar{C}^1) A_1 \equiv A_2(\xi) + \theta \left[ s_{ab} A_2(\xi) \right]. \]

In the above, the coefficients of \( \theta \) are nothing but the anti-BRST symmetry transformations for the component fields \( A_0, A_1 \) and \( A_2 \) [cf. Eq. (8)]. We point out that the subscripts \((b)\) and \((ab)\) in equation (60) and (61) have their straightforward meaning as we have established earlier. We lay emphasis on the fact that the super expansions in (60) and (61) are very crucial and important as will be clear in the next section where we shall discuss the symmetry invariances.

### 5 Invariance of the Lagrangian Densities: ACSA

In this section, we capture the (anti-)BRST invariance of the Lagrangian densities (1) and (2) in terms of the (anti-)chiral superfields that have been obtained after the imposition of the (anti-)BRST invariant restrictions. In this connection, it is worth pointing out that we have already computed the BRST invariance of \( \mathcal{L}_b \) and anti-BRST invariance of \( \mathcal{L}_{ab} \) in the ordinary space in our earlier work [17]. To be precise, the action integrals \( S_1 = \int d^2 \xi \mathcal{L}_b \) and \( S_2 = \int d^2 \xi \mathcal{L}_{ab} \) remain invariant under the continuous, infinitesimal and nilpotent transformations in (7) and (6). In this connection, first of all, we note that the following are true for the classical Lagrangian density \( \mathcal{L}_0 \), namely:

\[ s_b \mathcal{L}_0 = \partial_a \left[ C^a \mathcal{L}_0 \right], \quad s_{ab} \mathcal{L}_0 = \partial_a \left[ \bar{C}^a \mathcal{L}_0 \right], \]  \hfill (62)

and the total Lagrangian densities \( \mathcal{L}_b \) and \( \mathcal{L}_{ab} \) transform as [17]:

\[ s_b \mathcal{L}_b = \partial_a \left[ C^a \left( \mathcal{L}_0 + B_0 A_0 + B_1 A_1 \right) + i \bar{C}_1 C^b \partial_b \left( C^a A_1 \right) + i \bar{C}_1 C^a \left( \partial_0 C^1 + \partial_1 C^0 \right) A_0 \right] + i \bar{C}_0 C^b \partial_b \left( C^a A_0 \right) + i \bar{C}_0 C^a \left( \partial_0 C^1 + \partial_1 C^0 \right) A_1 + i \bar{C}_0 C^a \left( \partial_0 C^1 - \partial_1 C^0 \right) A_2 \]

\[ + \ i \bar{C}_1 C^a \left( \partial_0 C^0 - \partial_1 C^1 \right) A_2 \].  \hfill (63)
The above observations demonstrate that \( s_b S_1 = 0 \) and \( s_{ab} S_2 = 0 \) for the physical fields of the (anti-)BRST invariant theories which vanish-off [16] at \( \sigma = 0, \pi \) and \( \tau \rightarrow \pm \infty \) due to Gauss’s divergence theorem. We mention, in passing, that \( s_b S_0 = 0 \) and \( s_{ab} S_0 = 0 \) (where \( S_0 = \int d^2 \xi \mathcal{L}_0 \)) due to the (anti-)BRST transformations for \( \mathcal{L}_0 \) in (62).

First of all, we capture the (anti-)BRST invariance of the action integral \( S_0 = \int d^2 \xi \mathcal{L}_0 \) within the realm of ACSA. In this regard, we note the following (anti-)chiral generalizations of \( \mathcal{L}_0 \) to its counterpart super Lagrangians (i.e. \( \mathcal{L}_0 \rightarrow \tilde{\mathcal{L}}_0 \)) on the (2, 1)-dimensional (anti-)chiral super submanifolds, namely:

\[
\mathcal{L}_0 \rightarrow \tilde{\mathcal{L}}_0^{(ac)}(\xi, \bar{\theta}) = -\frac{1}{2k} \tilde{G}_{(b)}^{mn(ac)}(\xi, \bar{\theta}) \partial_m \tilde{X}^{\mu(h,ac)}(\xi, \bar{\theta}) \partial_n \tilde{X}_{\mu}^{(h,ac)}(\xi, \bar{\theta}) + \mathcal{E}_{(b)}^{(ac)}(\xi, \bar{\theta}) \left[ \det \tilde{G}_{(b)}^{mn(ac)}(\xi, \bar{\theta}) + 1 \right],
\]

\[
\mathcal{L}_0 \rightarrow \tilde{\mathcal{L}}_0^{(c)}(\xi, \theta) = -\frac{1}{2k} \tilde{G}_{(ab)}^{mn(c)}(\xi, \theta) \partial_m \tilde{X}^{\mu(h,c)}(\xi, \theta) \partial_n \tilde{X}_{\mu}^{(h,c)}(\xi, \theta) + \mathcal{E}_{(ab)}^{(c)}(\xi, \theta) \left[ \det \tilde{G}_{(ab)}^{mn(c)}(\xi, \theta) + 1 \right],
\]

where the super Lagrangian densities (on the l.h.s.) carry superscripts \((ac)\) and \((c)\) to denote that these have been defined on the \((2, 1)\)-dimensional (anti-)chiral super submanifolds of the \((2, 2)\)-dimensional general supermanifold (that has been chosen for our discussion). The superfields with subscripts \((b)\) and \((ab)\) as well as with superscripts \((ac)\), \((c)\), \((h, c)\) and \((h, ac)\) have already been explained in our previous and present sections. The equation (62) can be captured in the superspace (where ACSA plays an important role). The mappings: \( s_b \leftrightarrow \partial_{\bar{\theta}}, s_{ab} \leftrightarrow \partial_{\theta} \) lead to the following observations:

\[
\frac{\partial}{\partial \bar{\theta}} \tilde{\mathcal{L}}_0^{(ac)}(\xi, \bar{\theta}) = \partial_a \left[ C^a \mathcal{L}_0 \right] \equiv s_b \mathcal{L}_0, \quad \frac{\partial}{\partial \theta} \tilde{\mathcal{L}}_0^{(c)}(\xi, \theta) = \partial_a \left[ \bar{C}^a \mathcal{L}_0 \right] = s_{ab} \mathcal{L}_0.
\]

Thus, the (anti-)BRST symmetry invariances of \( \mathcal{L}_0 \) have been expressed in the language of ACSA to BRST formalism. We have performed this exercise separately because, on its own, the original classical Lagrangian density \( \mathcal{L}_0 \) transforms to the total derivatives [cf. Eq. (62)] under the (anti-)BRST symmetry transformations.

We would like to express the symmetry transformations (63) and (64) in the realm of ACSA where the super expansions in (27), (32), (33), (40), (44), (53) and (60) will be playing decisive roles for the BRST invariance [cf. Eq. (63)]. On the other hand, the super expansions (27), (57), (59) and (61) will be very useful in capturing the anti-BRST invariance [cf. Eq. (64)]. With these inputs at our disposal, we set out to capture the BRST invariance in terms of \( \partial_{\theta} \) and \( \tilde{\mathcal{L}}_0^{ac}(\xi, \bar{\theta}) \). Here the latter is given in the language of the anti-chiral superfields that have been derived after the imposition of the BRST-invariant restrictions. These anti-chiral superfields might also be the limiting cases of the
full super expansions that have been derived in Sec. 3, namely:

\[
\mathcal{L}_b^{(ac)}(\xi, \bar{\theta}) = \mathcal{L}_0^{(ac)}(\xi, \bar{\theta}) + B_0(\xi) \mathcal{A}_{0(ab)}^{(ac)}(\xi, \bar{\theta}) + B_1(\xi) \mathcal{A}_{1(ab)}^{(ac)}(\xi, \bar{\theta})
- i \left[ F_{1(b)}^{(ac)}(\xi, \bar{\theta}) \left\{ \partial_0 F_{1(ab)}^{(ac)}(\xi, \bar{\theta}) + \partial_1 F_{0(ab)}^{(ac)}(\xi, \bar{\theta}) \right\} \right] \mathcal{A}_{0(ab)}^{(ac)}(\xi, \bar{\theta})
+ F_{0(ab)}^{(ac)}(\xi, \bar{\theta}) \left\{ \partial_0 F_{0(ab)}^{(ac)}(\xi, \bar{\theta}) + \partial_1 F_{1(ab)}^{(ac)}(\xi, \bar{\theta}) \right\} \mathcal{A}_{1(ab)}^{(ac)}(\xi, \bar{\theta})
- i \left[ F_{0(ab)}^{(ac)}(\xi, \bar{\theta}) \left\{ \partial_0 F_{0(ab)}^{(ac)}(\xi, \bar{\theta}) + \partial_1 F_{1(ab)}^{(ac)}(\xi, \bar{\theta}) \right\} \right] \mathcal{A}_{1(ab)}^{(ac)}(\xi, \bar{\theta})
+ F_{0(ab)}^{(ac)}(\xi, \bar{\theta}) \left\{ \partial_0 F_{1(ab)}^{(ac)}(\xi, \bar{\theta}) - \partial_1 F_{0(ab)}^{(ac)}(\xi, \bar{\theta}) \right\} \mathcal{A}_{2(ab)}^{(ac)}(\xi, \bar{\theta}),
\]

where we have taken the ordinary fields \(B_0(\xi)\) and \(B_1(\xi)\) because we know that \(B^m(\xi) \rightarrow \mathcal{B}^m_{(b)}(\xi, \bar{\theta}) = B^m(\xi)\) due to the BRST invariance \([s_b B^m(\xi) = 0]\) of \(B^m(\xi)\). Ultimately, it turns out that we obtain the following due to operation of \(\partial_\theta\) on \(\tilde{\mathcal{L}}_b^{(ac)}(\xi, \bar{\theta})\):

\[
\frac{\partial}{\partial \bar{\theta}} \tilde{\mathcal{L}}_b^{(ac)}(\xi, \bar{\theta}) = \partial_a \left[ C^a \left( \mathcal{L}_0 + B_0 A_0 + B_1 A_1 \right) + i \tilde{C}_1 C^b \partial_b (C^a A_1) \right.
+ i \tilde{C}_1 C^a \left( \partial_0 C^1 + \partial_1 C^0 \right) A_0 + i \tilde{C}_0 C^b \partial_b (C^a A_0)
+ i \tilde{C}_0 C^a \left( \partial_0 C^1 + \partial_1 C^0 \right) A_1 + i \tilde{C}_0 C^a \left( \partial_0 C^1 - \partial_1 C^0 \right) A_2
+ i \tilde{C}_1 C^a \left( \partial_0 C^0 - \partial_1 C^1 \right) A_2 \left. \right] \equiv s_b \mathcal{L}_b.
\]

It is evident that the above equation captures the BRST invariance of the Lagrangian density \(\mathcal{L}_b\) in the superspace (as is clear from our observation on the r.h.s.).

We can repeat the same exercise for the anti-BRST invariance. For this purpose, first of all, we generalize \(\mathcal{L}_{ab}\) to its counterpart chiral super Lagrangian density on the \((2, 1)\)-dimensional chiral super submanifold as

\[
\mathcal{L}_{ab}^{(c)}(\xi, \theta) = \tilde{\mathcal{L}}_0^{(c)}(\xi, \theta) - B_0(\xi) \mathcal{A}_{0(ab)}^{(c)}(\xi, \theta) - B_1(\xi) \mathcal{A}_{1(ab)}^{(c)}(\xi, \theta)
+ i \left[ F_{1(ab)}^{(c)}(\xi, \theta) \left\{ \partial_0 F_{1(ab)}^{(c)}(\xi, \theta) + \partial_1 F_{0(ab)}^{(c)}(\xi, \theta) \right\} \right] \mathcal{A}_{0(ab)}^{(c)}(\xi, \theta)
+ F_{0(ab)}^{(c)}(\xi, \theta) \left\{ \partial_0 F_{0(ab)}^{(c)}(\xi, \theta) + \partial_1 F_{1(ab)}^{(c)}(\xi, \theta) \right\} \mathcal{A}_{1(ab)}^{(c)}(\xi, \theta)
+ i \left[ F_{0(ab)}^{(c)}(\xi, \theta) \left\{ \partial_0 F_{0(ab)}^{(c)}(\xi, \theta) + \partial_1 F_{1(ab)}^{(c)}(\xi, \theta) \right\} \right] \mathcal{A}_{1(ab)}^{(c)}(\xi, \theta)
+ F_{0(ab)}^{(c)}(\xi, \theta) \left\{ \partial_0 F_{1(ab)}^{(c)}(\xi, \theta) - \partial_1 F_{0(ab)}^{(c)}(\xi, \theta) \right\} \mathcal{A}_{2(ab)}^{(c)}(\xi, \theta),
\]

where the ordinary fields \(\tilde{B}_0(\xi)\) and \(\tilde{B}_1(\xi)\) are present in the above super Lagrangian density because \(s_{ab} \tilde{B}^a = 0\) which implies that \(\tilde{B}^a(\xi) \rightarrow \mathcal{B}^{(c)}_{(ab)}(\xi, \theta) = \tilde{B}^a(\xi) + \theta(0) = \tilde{B}^a(\xi)\). In
other words, there is no chiral \( \theta \)-dependence on the r.h.s. of the super expansion of the superfield \( \mathcal{B}^{a(c)}_{(ab)}(\xi, \theta) \). The rest of the notations for the chiral superfields have already been explained earlier. At this juncture, in view of the mapping: \( s_{ab} \leftrightarrow \partial_b \), we can capture the anti-BRST invariance (64) by applying a derivative \( \partial_b \) on (69) which yields the following:

\[
\frac{\partial}{\partial \theta} \bar{\mathcal{L}}^{(c)}_{ab}(\xi, \theta) = \partial_a \left[ \bar{C}^a (\mathcal{L}_0 - \bar{B}_0 A_0 - \bar{B}_1 A_1) - i C_1 \bar{C}^b \partial_b (\bar{C}^a A_1) \right. \\
- i C_1 \bar{C}^a (\partial_b \bar{C}^1 + \partial_1 \bar{C}^0) A_0 - i C_0 \bar{C}^b \partial_b (\bar{C}^a A_0) \\
\left. - i C_0 \bar{C}^a (\partial_b \bar{C}^1 + \partial_1 \bar{C}^0) A_1 - i C_0 \bar{C}^a (\partial_0 \bar{C}^1 - \partial_1 \bar{C}^0) A_2 \right] \equiv s_{ab} \mathcal{L}_{ab}.
\]

Hence we have captured the anti-BRST symmetry invariance (64) in the language of ACSA to BRST formalism [as is evident from the r.h.s. of (70)].

We close this section with the following remark. We can capture the basic ideas behind the derivations of \( \mathcal{L}_b \) and \( \mathcal{L}_{ab} \) which have been explained in Eq. (5). In view of the mappings: \( s_b \leftrightarrow \partial_b \), \( s_{ab} \leftrightarrow \partial_b \), we can express the super (anti-)BRST invariant Lagrangian densities corresponding to the ordinary Lagrangian densities [cf. Eq. (5)] as

\[
\bar{\mathcal{L}}^{(c)}_{ab}(\xi, \theta) = \mathcal{L}^{(c)}_0(\xi, \theta) + \frac{\partial}{\partial \theta} \left[ i F^{(c)}_{0(ab)}(\xi, \theta) \mathcal{A}^{(c)}_{0(ab)}(\xi, \theta) + i F^{(c)}_{1(ab)}(\xi, \theta) \mathcal{A}^{(c)}_{1(ab)}(\xi, \theta) \right],
\]

\[
\mathcal{L}^{(ac)}_b(\xi, \bar{\theta}) = \mathcal{L}^{(ac)}_0(\xi, \bar{\theta}) + \frac{\partial}{\partial \bar{\theta}} \left[ - i F^{(ac)}_{0(b)}(\xi, \bar{\theta}) \mathcal{A}^{(ac)}_{0(b)}(\xi, \bar{\theta}) - i F^{(ac)}_{1(b)}(\xi, \bar{\theta}) \mathcal{A}^{(ac)}_{1(b)}(\xi, \bar{\theta}) \right].
\]

where all the symbols have been explained in our earlier discussion. It is crystal clear, from the above expression, that the (anti-)BRST invariance of the action integrals \( S_1 = \int d^2 \xi \mathcal{L}_b \) and \( S_2 = \int d^2 \xi \mathcal{L}_{ab} \) can be captured in the terminology of ACSA to BRST formalism because \( s_b S_1 \) and \( s_{ab} S_2 \) will be zero in the ordinary space. Furthermore, we note that \( \partial_b \bar{\mathcal{L}}^{(c)}_{ab}(\xi, \theta) \) and \( \partial_\theta \mathcal{L}^{(ac)}_b(\xi, \bar{\theta}) \) will always produce the total derivatives in the ordinary space thereby rendering the action integrals (i.e. \( S_1 \) and \( S_2 \)) equal to zero [cf. Eq. (71)]. To be precise, the nilpotency \( (\partial_\theta^2 = 0, \partial_{\bar{\theta}}^2 = 0) \) property of the translational generators \( (\partial_b, \partial_\theta) \) will ensure that \( \partial_\theta \bar{\mathcal{L}}^{(c)}_{ab}(\xi, \theta) \) and \( \partial_{\bar{\theta}} \mathcal{L}^{(ac)}_b(\xi, \bar{\theta}) \) will be always the total derivatives in the ordinary space. Hence, we are able to capture the symmetry invariance(s) of the action integrals (corresponding to the Lagrangian densities \( \mathcal{L}_b \) and \( \mathcal{L}_{ab} \)) using ACSA.

6 Conclusions

In our present endeavor, we have exploited the theoretical potential of MBTSA and ACSA to derive all the (anti-)BRST symmetry transformations for the 2D diffeomorphism symmetry invariant model of a bosonic string theory. These symmetry transformations \( [s_{(a)b}] \) are proper because they are off-shell nilpotent \( [s_{(a)b}^2 = 0] \) of order two and absolutely anti-commuting (i.e. \( s_b s_{ab} + s_{ab} s_b = 0 \)) in nature [cf. Eqs. (10),(7),(6)]. The latter property of the (anti-)BRST symmetry transformations \( [s_{(a)b}] \) is satisfied if and only if we invoke the sanctity of the CF-type restrictions: \( B^a + \bar{B}^a + i (C^m \partial_m \bar{C}^a + \bar{C}^m \partial_m C^a) = 0 \) (with \( a, m = 0, 1 \)) which define a submanifold in the quantum Hilbert space of fields where the
Nakanishi-Lautrup type auxiliary fields as well as the (anti-)ghost fields are present algebraically in a specific manner [cf. Eq. (24)]. These restrictions are physical in some sense because they are (anti-)BRST symmetry invariant [cf. Eqs. (7), (6)] on the above submanifold. Hence, their imposition on our BRST-quantized theory is logical.

By applying the theoretical strength of MBTSA, we have been able to derive, in one stroke, the (anti-)BRST symmetry transformations together for the Lorentz pure scalar fields [e.g. \(X^\mu(\xi), (\det \bar{g})\)] and the 2D version of the universal CF-type restrictions: \(B^a + B^a + i(\partial_m \bar{C}^a + C^m \partial_m C^a) = 0\). These 2D restrictions are the limiting case of the D-dimensional diffeomorphism invariant theory where the superfield approach (developed by us [18, 13]) leads to the existence of the D-dimensional CF-type restrictions: \(B^\mu + \bar{B}^\mu + i(C^\rho \partial_\rho \bar{C}^\mu + \bar{C}^\rho \partial_\rho C^\mu) = 0\) (with \(\mu = 0, 1, 2...D-1\)) where the fermionic (anti-)ghost fields \((\bar{C}^\mu)C_\mu\) correspond to the D-dimensional infinitesimal and continuous diffeomorphism symmetry transformations: \(x_\mu \rightarrow x'_\mu = x_\mu - \epsilon_\mu(x)\). In these infinitesimal transformations, the parameters \(\epsilon_\mu(x)\) are the diffeomorphism transformation parameters. The symbols \((\bar{B}^\mu)B_\mu\) are nothing but the Nakanishi-Lautrup type auxiliary fields in the D-dimensional theory. The existence of the D-dimensional CF-type restrictions: \(B^\mu + \bar{B}^\mu + i(C^\rho \partial_\rho \bar{C}^\mu + \bar{C}^\rho \partial_\rho C^\mu) = 0\) are universal and, so far, their presence have been shown explicitly in the cases of 2D and 1D diffeomorphism invariant theories (see, e.g. [17, 14] for details).

Within the ambit of MBTSA, it becomes evident that we have to take, at least, the helps of the (anti-)chiral superfield expansions [cf. Eq. (27)] so that we can obtain \(s_b \bar{C}_\mu = i B_\mu\) and \(s_{ab} C_\mu = i B_\mu\) for the D-dimensional diffeomorphism invariant theory (see e.g. [13, 18] for details) in addition to the validity of off-shell nilpotency property so that we can obtain: \(s_b \bar{C}_\mu = C^\rho \partial_\rho C_\mu\) and \(s_{ab} C_\mu = \bar{C}^\rho \partial_\rho \bar{C}_\mu\). The above two inputs are essential for the completeness of MBTSA. Hence, we have exploited the theoretical potential of the ACSA to BRST formalism (see, e.g. [15]) so that both the above inputs can be taken care of. As a consequence, it becomes important to blend together the ideas from the MBTSA and ACSA so that we can derive all the (anti-)BRST symmetry transformations for the all the fields of a diffeomorphism invariant theory along with the derivation of appropriate (anti-)BRST invariant CF-type restrictions. This is what we have precisely done in our present investigation. Our earlier works (see, e.g. [14] and references therein) on the 1D diffeomorphism invariant models of the relativistic and non-relativistic particles (of SUSY and non-SUSY varieties) have also exploited the ideas behind MBTSA and ACSA together to obtain the 1D version \([B + B + i(C \bar{C} - \bar{C} C) = 0]\) of the universal D-dimensional CF-type restrictions that have been derived and thoroughly discussed in [18, 13].

In our earlier work [17] on our present bosonic string, we have computed the expressions for the BRST and anti-BRST charges in the flat space. In the paper by Kato and Ogawa [16], the nilpotency of BRST charge has been proven to demonstrate that quantum version of the theory is valid only when \(D = 26\) and \(\alpha_0 = 1\). It will be very nice future endeavor for us to take the expression for the anti-BRST charge and plug in the normal mode expansions of the fields (with creation and annihilation operators in it) so that the quantum version of it can be obtained. With appropriate boundary conditions on the target space coordinate fields and (anti-)ghost fields, it will be challenging to derive \(D = 26\) and \(\alpha_0 = 1\) from the requirement of the nilpotency of the anti-BRST charge in the flat limit. We are presently involved with this problem and our results/observations will be reported elsewhere.
As pointed out earlier, our present 2D diffeomorphism invariant theory is different from our earlier works on the 1D diffeomorphism (i.e. reparameterization) invariant theories (see, e.g. [14] and references therein) in the sense that the latter theoretical models have the gauge symmetry transformations, too, which are equivalent to the reparameterization (i.e. 1D diffeomorphism) symmetry transformations in the specific limits (see, e.g. [19, 14] for details). It is worth emphasizing that the gauge symmetry transformations (generated by the first-class constraints) have been exploited for the BRST quantization in [19] in the cases of the 1D diffeomorphism (i.e. reparameterization) invariant models. The latter models are nothing but the non-SUSY scalar relativistic and SUSY spinning relativistic particles. We lay emphasis on the fact that the reparameterization symmetry transformations of these models have been left untouched in [19] as far as the BRST quantization scheme is concerned. We have taken this challenge in our earlier works (see, e.g. [14] and references therein) for the BRST quantization of these models.

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Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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