Dynamical Higher-Twist and High x Phenomena: A Window to Quark-Quark Correlations in QCD

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Abstract

Measurements of the power-law corrections to Bjorken scaling and the behavior of structure functions in the highly stressed $x_{bj} \to 1$ regime of electroproduction can lead to new information on the quark-quark correlations controlling the nucleon wavefunction at far-off-shell kinematics. Electroproduction on nuclei at $A > x_{bj} > 1$ are sensitive to hidden-color components of the nuclear wavefunction. A distinctive dynamical higher-twist $O(1/Q^2)$ correction, which is dynamically enhanced at high $x_{bj}$, can arise from the interference of amplitudes where the lepton scatters from two different valence quarks of the target. Measurements of the parity-violating left-right asymmetry $A_{LR}$ in elastic and inelastic polarized electron scattering at large $x_{bj}$ can confirm the structure of the quark-quark correlations and other QCD physics at the amplitude level.

1 Introduction

A fundamental question in QCD is the non-perturbative structure of hadrons at the amplitude level—not just the single-particle flavor, momentum, and helicity distributions of the quark constituents, but also the multi-quark, gluonic, and hidden-color correlations intrinsic to hadronic and nuclear wavefunctions. As I shall discuss here, detailed measurements of the power-law corrections to Bjorken scaling and the behavior of structure functions in the highly stressed $x_{bj} \to 1$ regime of electroproduction can lead to important new information on the dynamical mechanisms and the underlying quark-quark correlations of the target wavefunction. In the case of light-nuclei, one can obtain sensitivity to hidden-color components of the nuclear wavefunction from measurements beyond the nucleon kinematic domain. Measurements of the parity-violating left-right asymmetry $A_{LR}$ in elastic and inelastic polarized electron scattering at large $x_{bj}$ can add important checks on the QCD mechanisms underlying dynamical higher twist effects.

The $n$–parton amplitudes which interpolate between a hadron $H$ and its quark and gluon degrees of freedom in QCD are the light-cone Fock wavefunctions $\psi_{n/H}(x_i, \vec{k}_{\perp i}, \lambda_i)$. Formally, the light-cone expansion is constructed by quantizing QCD at fixed light-cone time $\tau = t + z/c$ and forming the invariant light-cone Hamiltonian: $H^{QCD}_{LC} = P^+ P^- - \vec{P}_\perp^2$ where $P^\pm = P^0 \pm P^z$ [2]. The operator $P^- = i \frac{d}{d\tau}$ generates light-cone time translations. The momentum $P^+$ and $\vec{P}_\perp$ operators are independent of the interactions. The eigen-spectrum of the $H^{QCD}_{LC}$ yields the entire mass spectrum of color-singlet hadron states in QCD, together with their respective light-cone wavefunctions. For example, the proton state satisfies: $H^{QCD}_{LC} |\psi_p \rangle = M_p^2 |\psi_p \rangle$.

The projection of the proton’s eigensolution $|\psi_p \rangle$ on the color-singlet $B = 1, Q = 1$ eigenstates $\{|n\rangle\}$ of the free Hamiltonian $H^{QCD}_{LC}(g = 0)$ gives the light-cone Fock expansion: $|\psi_p(P^+, \vec{P}_\perp)\rangle = \sum_n \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}, \lambda_i\rangle$. The light-cone momentum fractions of the constituents, $x_i = k_i^+/P^+$ with $\sum_{i=1}^n x_i = 1$, and the transverse...
momenta \( \vec{k}_{\perp i} \) with \( \sum_{i=1}^{n} \vec{k}_{\perp i} = \vec{0}_\perp \) appear as the momentum coordinates of the light-cone Fock wavefunctions. The actual physical transverse momenta are \( \vec{p}_{\perp i} = x_i \vec{P}_\perp + \vec{k}_{\perp i} \). The \( \lambda_i \) label the light-cone spin \( S^z \) projections of the quarks and gluons along the \( z \) direction. The physical gluon polarization vectors \( \epsilon^\mu(k, \lambda = \pm 1) \) are specified in light-cone gauge by the conditions \( k \cdot \epsilon = 0, \eta \cdot \epsilon = \epsilon^+ = 0 \). The relative orbital and spin projections in each Fock state sum to the \( J_z \) of the hadron [3]. The light-cone Hamiltonian formalism thus provides a relativistic description of hadrons as many-particle systems of fluctuating parton number.

The LC wavefunctions \( \psi_{n/H}(x_i, \vec{k}_{\perp i}, \lambda_i) \) are universal, process independent, and thus control all hadronic reactions. In the case of deep inelastic scattering, one needs to evaluate the imaginary part of the virtual Compton amplitude \( \mathcal{M}[\gamma^*(q)p \rightarrow \gamma^*(q)p] \). The simplest frame choice for electroproduction is \( q^+ = 0, q^2 = Q^2 = -q^2, q^- = 2q \cdot P^+, p^+ = P^+, p_\perp = 0 \), \( p^- = M_p^2/P^+ \). At leading twist, soft final-state interactions are power-law suppressed in light-cone gauge, so the calculation of the virtual Compton amplitude reduces to the evaluation of matrix elements of the products of free quark currents of the free quarks. The absorptive amplitude imposes conservation of light-cone energy: \( p^- + q^- = \sum_{i} k_i^- \) for the \( n \)-particle Fock state. In the impulse approximation, where only one quark \( q \) recoils against the scattered lepton, this condition becomes

\[
M_p^2 + 2q \cdot p = \frac{(\vec{k}_{\perp q} + \vec{q}_\perp)^2 + m_q^2}{x_q} + \sum_{i \neq q} \frac{k_{\perp i}^2 + m_i^2}{x_i}
\]

If we neglect the transverse momenta \( k_i^2 \) relative to \( Q^2 \) in the Bjorken limit \( Q^2 \rightarrow \infty, x_{bj} = Q^2/2q \cdot p \) fixed, we obtain the condition \( x_q = x_{bj} \), i.e., the light-cone fraction \( x_q = k^+ / p^+ \) of the struck quark is kinematically fixed to be equal to the Bjorken ratio. Contributions from high \( k_\perp^2 = \mathcal{O}(Q^2) \) which originate from the perturbative QCD radiative corrections to the struck quark line lead to the DGLAP evolution equations.

Thus given the light-cone wavefunctions, one can compute [3] all of the leading twist helicity and transversity distributions measured in polarized deep inelastic lepton scattering [3]. For example, the polarized quark distributions at resolution \( \Lambda \) correspond to

\[
q_{x_{q}/\Lambda_p}(x, \Lambda) = \sum_{n,q_a} \prod_{j=1}^{n} dx_j d^2 k_{\perp j} \sum_{\lambda_i} |\psi_{n/H}(x_i, \vec{k}_{\perp i}, \lambda_i)|^2 \times \delta \left(1 - \sum_{i} x_i\right) \delta^{(2)} \left(\sum_{i} \vec{k}_{\perp i}\right) \delta(x - x_q) \delta_{\lambda_q, \lambda_i} \Theta(\Lambda^2 - \mathcal{M}_{n}^2),
\]

where the sum is over all quarks \( q_a \) which match the quantum numbers, light-cone momentum fraction \( x \), and helicity of the struck quark. Similarly, the transversity distributions and off-diagonal helicity convolutions are defined as a density matrix of the light-cone wavefunctions. This defines the LC factorization scheme [3] where the invariant mass squared \( \mathcal{M}_{n}^2 = \sum_{i=1}^{n} (k_{\perp i}^2 + m_i^2) / x_i \) of the \( n \) partons of the light-cone wavefunctions are limited to \( \mathcal{M}_{n}^2 < \Lambda^2 \).
The light-cone wavefunctions also specify the multi-quark and gluon correlations of the hadron. For example, the distribution of spectator particles in the final state which could be measured in the proton fragmentation region in deep inelastic scattering at an electron-proton collider are in principle encoded in the light-cone wavefunctions.

There are many sources of power-law corrections to the standard leading twist formula for deep inelastic structure functions. Higher-twist corrections arise from QCD radiative corrections (renormalons), final-state interactions, finite target mass effects \[\text{(6)}\], the constituent masses, and their transverse momenta \(k_\perp\). A derivation of some of these corrections is given in Ref. \[\text{(7)}\]. Despite the many sources of power-law corrections to the deep inelastic cross section, certain types of dynamical contributions stand out at large \(x_{\text{bjo}}\) since they arise from compact, highly-correlated fluctuations of the proton wavefunction. As I will discuss in Section 3, there are particularly interesting dynamical \(O(1/Q^2)\) corrections which occur from the \textit{interference} of quark currents; \textit{i.e.}, contributions which involve leptons scattering from two different quarks.

2 Structure Functions at High \(x\)

The impulse approximation for inelastic lepton proton scattering is not valid for calculations of structure functions at fixed \(Q^2\) and large \(x \sim 1\). For example, as \(x \rightarrow 1\), the struck quark becomes far-off shell and spacelike; its Feynman virtuality is

\[
k_F^2 = x \left[ M_p^2 - \frac{M_s^2 + k_\perp^2}{1 - x} - \frac{k_\perp^2}{x} \right] \Rightarrow -\infty.
\]  

(2)

Here \(M_s^2\) is the invariant mass of the spectator system after the struck quark is removed, and \(k_\perp^2\) is the transverse momentum of the struck quark. In the language of light-cone perturbation theory, the light-cone wavefunction is evaluated far from its light-cone energy shell; in particular, the identification \(x = k^+/p^+\) will break down at \(x \rightarrow 1\) since the spectator light cone momentum fractions \(x_i\) are all forced to be small. The spectator terms in the light-cone energy conservation equation \(p^- + q^- = \sum_i k_i^-\) thus cannot be ignored.

Thus the regime \(x \rightarrow 1\) probes a highly stressed far off-shell configuration of the proton wavefunction where the struck quark has all of the proton’s light-cone momentum and all the spectator quarks and gluons are left with negligible light-cone momentum fraction. This regime clearly is highly sensitive to the inter-particle correlations of the proton’s wavefunction; \textit{i.e.} the detailed dynamics which allows all of the proton’s momentum to be transferred to just one quark. In fact, in this far-off-shell domain we can use PQCD to calculate the \(x \rightarrow 1\) dependence of the structure functions \[\text{(4)}\] by iterating the equations of motion. Only the lowest valence light-cone Fock state contributes since there are the fewest number of spectators to stop. To leading order in \(\alpha_s(k_F^2)\), one can calculate the
end-point dependence of $F_2(x, Q^2)$ via two hard gluon exchanges between the three valence quarks. A typical perturbative QCD contribution is illustrated in Fig. 2. The result is

Figure 1: Perturbative QCD two-gluon-exchange mechanism dominating nucleon structure functions at $x \to 1$. The dominant helicity of the struck quark is parallel to that of the nucleon. Gluon radiation from the struck quark leads to DGLAP evolution if $Q^2 > |k_T^2|$, the virtuality of the struck quark.

\[ q_{↑↑}(x, Q^2) \sim (1 - x)^3 \quad q_{↑↓}(x, Q^2) \sim (1 - x)^5 \]

i.e.: it is much more probable that the leading quark has the same helicity as that of the proton:

\[ \frac{u_{↑↑}(x, Q^2)}{u_{↑↓}(x, Q^2)} \sim (1 - x)^2 \quad \frac{d_{↑↑}(x, Q^2)}{d_{↑↓}(x, Q^2)} \sim (1 - x)^2. \]

If one assumes $SU(6)$-flavor symmetry, then there are 5 times more $u \uparrow$ quarks than $d \uparrow$ quarks in the proton:

\[ \frac{u_{↑}(x, Q^2)}{d_{↑}(x, Q^2)} \to 5. \]

This also implies the famous Farrar-Jackson prediction \[8\]

\[ \frac{F_{2n}(x, Q^2)}{F_{2p}(x, Q^2)} \to \frac{5 \cdot \left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2}{5 \cdot \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2} = \frac{3}{7}. \]

In the case of gluons, the leading PQCD prediction is

\[ g_{↑↑}(x, Q^2) \sim (1 - x)^4 \quad g_{↑↓}(x, Q^2) \sim (1 - x)^6 \]
i.e.: the gluon polarization becomes strongly aligned with that of the proton when the gluon takes all of the proton’s light-cone momentum. One also expects dominance of the helicity-aligned strange, $u$, and $d$ distributions at $x \to 1$.

Useful phenomenological models of the input spin-dependent structure functions $q_{\lambda/\lambda'}(x, Q_0^2)$ can be designed which incorporate the PQCD-predicted power laws at $x \to 1$ and isospin-singlet $1/x$ Pomeron and isospin-nonsinglet $1/\sqrt{x}$ Reggeon behavior at small $x$ [9]. Such forms match well with the MRS parameterizations of the data [10]. There are a wide range of QCD flavor and helicity tests of these predictions which could be carried out at a 12 GeV facility. For example, a simple model for the polarized gluon distribution in the proton is [11, 9]

$$
g_{\uparrow/\uparrow}(x, Q^2) = A \frac{(1 - x)^4}{x}$$

$$
g_{\downarrow/\uparrow}(x, Q^2) = A \frac{(1 - x)^6}{x}$$

$$
\Delta g(x, Q^2) = A(1 - x)^4(2 - x).$$

If the momentum carried by gluons is

$$
\int_0^1 dx x(g_{\uparrow/\uparrow}(x) + g_{\downarrow/\uparrow}(x)) = \frac{1}{2},$$

then $A = 35/24$, and $\Delta g = 77/144 \approx 0.54$. These predictions are expected to be applicable at the starting scale for PQCD evolution; i.e. $Q^2 \lesssim 2$ GeV$^2$.

It is also interesting to measure inelastic lepton-nucleus scattering at $1 < x_{bj} < A$, beyond the kinematic domain accessible on a single nucleon target. The nuclear light-cone momentum must be transferred to a single quark, requiring quark-quark correlations between quarks of different nucleons in a compact, far-off-shell regime. The nuclear wavefunction contains hidden-color components distinct from a convolution of separate color-singlet nucleon wavefunctions. In fact, at very short distances, the light-cone distribution amplitude of a deuteron must involve asymptotically into a state which is 80% hidden color [12].

How does DGLAP evolution affect the $x \to 1$ dependence? Usually one expects that structure functions are strongly suppressed at large $x$ because of the momentum lost by gluon radiation: the predicted change of the power law behavior at large $x$ is [13]

$$
\frac{F_2(x, Q^2)}{F_2(x, Q_0^2)} \approx (1 - x)^\zeta(Q^2, Q_0^2)
$$

where

$$
\zeta(q^2, Q_0^2) = \frac{1}{4\pi} \int_{Q_0^2}^{Q^2} \frac{d\ell^2}{\ell^2} \alpha_s(\ell^2).
$$
Because of asymptotic freedom, this implies a \( \log \log Q^2 \) increase in the effective power \( \zeta(Q^2, Q_0^2) \). However, this derivation assumes that the struck quark is on its mass shell. The off-shell effect is profound, greatly reducing the PQCD radiation [7, 14]. We can take into account the main effect of the struck quark virtuality by modifying the propagator in Eq. (11):

\[
\zeta(Q^2, Q_0^2) = \frac{1}{4\pi} \int_{Q_0^2}^{Q^2} \frac{d\ell^2}{\ell^2 + |k_f^2|} \alpha_s(\ell^2).
\] (12)

Thus at large \( x \), there is effectively no DGLAP evolution until \( Q^2 \gtrsim |k_f^2| \)! One can also see that DGLAP evolution at large \( x \) at fixed \( Q^2 \) must be suppressed in order to have duality at fixed \( W^2 = Q^2(1 - x_{bj})/x_{bj} \) between the inclusive electroproduction and exclusive resonance contributions [4].

3 Higher-Twist Signals in Electroproduction

It is an empirical fact that conventional leading twist contributions cannot account for the measured \( ep \to eX \) and \( ed \to eX \) structure functions at \( x \gtrsim 0.4 \) and \( Q^2 \lesssim 5 \text{ GeV}^2 \). Fits to the data [17, 18] require an additional component which scales as \( 1/Q^2 \) relative to the leading twist contributions and rises rapidly with \( x \). The excess contribution can be parameterized in the form

\[
F_{2p,n}(x, Q^2) = F_{2p,n}^0(x, Q^2) \left[ 1 + \frac{c_{HT}^{p,n}(x)}{Q^2} \right] \] (13)

where \( F_{2p,n}^0 \) is the leading twist contribution. The functional dependence of the higher-twist term \( C_{HT}^{p,n}(x) \) for proton and proton-neutron targets is shown in Fig. 3. A rough fit is

\[
c_{HT}^{p}(x) \approx \left[ \frac{0.3 \text{ GeV}}{1 - x} \right]^2 \quad c_{HT}^{n}(x) \approx 2c_{HT}^{p}(x),
\] (14)

i.e.: the higher-twist effect relative to the leading twist contribution for the neutron is stronger than that of the proton.

A possible source of higher-twist effects in PQCD is “renormalons” [15, 16]. This contribution to the deep inelastic lepton-hadron cross section reflects a divergent \( \beta_0^n n! \) growth of the PQCD series for hard radiative corrections to deep inelastic scattering evolution at high orders in \( \alpha_s(Q^2) \). The factorial growth arises from the integration over the QCD running coupling; i.e., the summation of the reducible multi-bubble loop-diagrams in the gluon propagator. The net effect is to correct the leading twist predictions by a power-law suppressed \( 1/Q^2(1 - x) \) contribution. Alternatively, one can proceed using the BLM method [20]: one first identifies the conformal coefficients [21] of the PQCD series; by definition these are independent of the \( \beta^- \) function and are hence devoid of the \( \beta_0^n n! \) growth. The scale of the running coupling is set by requiring that all of the \( \beta^- \) dependence resides
in $\alpha_s(Q^2)$. The resulting scale $(Q^*)^2 \propto (1 - x)Q^2$ can also be understood as the mean value of the argument of the running coupling $\alpha_s(k^2)$ in the Feynman loop integration.

However, the renormalon contribution cannot account for the observed higher-twist contribution shown in Fig. 3 since it is proportional to the leading-twist prediction, i.e.: $c_{HTren}^p(x) = c_{HTren}^n(x)$. Thus it is apparent from the data that there must be a dynamical origin for the observed $C_{HT}(x)/Q^2$ contribution. In fact, dynamical higher-twist terms naturally arise from multi-parton correlations. For example, if the electron recoils against 1, 2, or 3 quarks, one obtains a series of higher-twist contributions of ascending order in $1/Q^2$.

$$\sigma_T \sim \frac{(1 - x)^3}{Q^2} \quad eq \rightarrow eq$$

$$\sigma_L \sim \frac{(1 - x)}{(Q^2)^3} \quad eqq \rightarrow eqq$$

$$\sigma_T \sim \frac{1}{(1 - x)} \left( \frac{1}{Q^2} \right)^3 \quad eqqq \rightarrow eqqq$$

where the extra $1/Q^2$ fall-off reflects the form factor squared of the $(qq)$ or $(qqq)$ systems,
and the enhancement at $x \to 1$ reflects the fact that the $(qq)$ and $(qqq)$ composites carry increasing fractions of the proton light-cone momentum. The dominance of $\sigma_L$ for $eqq \to eqq$ reflects the bosonic coupling of the composite di-quark. Each of the contributions satisfy Bloom-Gilman duality \cite{22} at fixed $W^2$. The multi-parton subprocesses are suppressed by powers of $1/Q^2$ but enhanced at large $x$ since more of the momentum of the target proton is fed into the hard subprocess; i.e., there are fewer spectators to stop. The general rule is

$$F_2(x, Q^2) \propto \frac{(1 - x)^{2n_{spect} - 1 + 2\Delta h}}{Q^{n_{active} - 4}}$$

where $n$ is the number of partons or other quanta participating in the hard subprocess and $\Delta h$ is the difference in helicity between the active partons and the target \cite{23}.

Figure 3: Higher-twist contribution to lepton pair production in $\pi N$ scattering. The dynamics at large $x_F$ requires both constituents of the projectile meson to be involved in the hard subprocess. From Ref. \cite{27}.

It is well-known that higher-twist, power-law suppressed corrections to hard inclusive cross sections can be a signature of correlation effects involving two or more valence quarks of a hadron. For example, the lepton angular dependence of the leading-twist PQCD prediction for Drell Yan lepton pair production $d\sigma(\pi A \to \ell^+\ell^-X)/d\Omega$ is $1 + \cos^2 \theta_{cm}$. The data of Ref. \cite{24, 25} however shows the onset of $\sin^2 \theta_{cm}$ dependence at large $x_F$. This signals the presence of multiparton-induced subprocesses such as $(\bar{q}q)q \to \gamma^*(Q^2)q \to \ell^+\ell^-q$ \cite{26}. See Fig. 3. Such reactions produce longitudinally-polarized virtual photons with a $\sin^2 \theta_{cm}$ lepton pair angular dependence in contrast to the transversally polarized Drell-Yan pairs produced from the $\bar{q}q \to \gamma^*(Q^2) \to \ell^+\ell^-$ subprocess. The penalty for utilizing the two correlated partons in the pion wavefunction is an extra suppression factor $1/R^2Q^2(1 - x_F)^2$ where $R$ is the characteristic interquark transverse separation between
the valence quarks in the incoming meson. The origin of the $1/R^2Q^2$ scaling is similar to that of the photon to meson transition form factor in the exclusive reaction $\ell\gamma \to \ell(q\bar{q}) \to \pi^0$ \cite{4}. The scale $1/R$ can be related to the pion decay constant $f_\pi$ which normalizes the pion distribution amplitude \cite{4}. At fixed $Q^2$ the higher-twist process can actually dominate as $x_F \to 1$ since all of the incoming momentum of the pion is transferred to the subprocess. The correlated subprocess $(q\bar{q})q \to \gamma^*(Q^2)q \to \ell^+\ell^-q$ also leads to the prediction of $\sin^2\theta\cos\phi$ and $\sin 2\theta\cos\phi$ terms in the angular distribution \cite{27}, effects which are clearly apparent in the data \cite{24,25}.

Another important example of dynamical higher-twist effects is the reaction $\pi A \to J/\psi X$ which is observed to produce longitudinally-polarized $J/\psi$'s at large $x_F$ \cite{28}. Again this effect can be attributed to highly correlated multi-parton subprocesses such as $qgq \to c\bar{c}q$ where both valence quarks of the incident pion must be involved in the hard subprocess in order to produce the charmed quark pair with nearly all of the incident momentum of the incoming meson \cite{29}. Similarly, charm production at threshold requires that all of the momentum of the target nucleon be transferred to the charm quarks. In the $\gamma p \to c\bar{c}p$ reaction near threshold, all the partons have to transfer their energy to the charm quarks within their reaction time $1/m_c$, and must be within this transverse distance from the $c\bar{c}$ and from each other. Hence only compact Fock states of the target nucleon or nucleus with a radius equal to the Compton wavelength of the heavy quark, can contribute to charm production at threshold. Equivalently we can interpret the multi-connected charm quarks as intrinsic charm Fock states which are kinematically favored to have large momentum fractions \cite{30}. The experimental evidence for intrinsic charm is discussed in Ref. \cite{31}.

Near-threshold charm production also probes the $x \simeq 1$ configurations in the target wavefunction; the spectator partons carry a vanishing fraction $x \simeq 0$ of the target momentum. This implies that the production rate behaves near $x \to 1$ approximately as $(1-x)^{2n_s-1}$ where $n_s$ is the number of spectators required to stop. Including spin factors, we can identify three different gluonic components of the photoproduction cross-section:

- The usual one-gluon $(1-x)^2$ distribution for leading twist photon-gluon fusion $\gamma g \to c\bar{c}$, which leaves two quarks spectators;
- Two correlated gluons emitted from the proton with a net distribution $(1-x)^2/R^2M^2$ for $\gamma gg \to c\bar{c}$, leaving one quark spectator;
- Three correlated gluons emitted from the proton with a net distribution $(1-x)^0/R^4M^4$ for $\gamma ggg \to c\bar{c}$, leaving no quark spectators.

Here $x \approx M^2/(s-m^2)$ and $M$ is the mass of the $c\bar{c}$ pair. The relative weight of the multiply-connected terms is controlled by the inter-quark separation $R \simeq 1/m_c$. The extra powers of $1/M$ arise from the power-law fall-off of the higher-twist hard sub-processes \cite{32}.

The correlations between valence quarks can also have an important effect in deep inelastic scattering, particularly at large $x_{bj} = Q^2/2p\cdot q$. As noted above, one expects a
sum of contributions to the deep inelastic cross section scaling nominally as

$$F_2(x, Q^2) = A(1 - x)^2 + B \frac{(1 - x)^2}{Q^4} + C \frac{(1 - x)^{-1}}{Q^8}$$

corresponding to the subprocesses $\ell q \to \ell q$, $\ell (qq) \to \ell (qq)$, and $\ell (qqq) \to \ell (qqq)$. However, the above classification of terms in $F_2(x, Q^2)$ neglects what may be the most significant and interesting higher-twist contribution to deep inelastic scattering: the interference contributions. Let us consider the contribution to DIS due to the interference of the amplitude where the lepton scatters on one quark with the amplitude where the lepton scatters on another quark. See Fig. 3. One might think such contributions are assumed to be negligible since the hard subprocesses seem to lead to different non-interfering final states. Actually these contributions can interfere if the struck quarks have high internal momentum in the initial state or if they exchange large momenta in the final state. In either case, the apparently different final states can overlap. An insightful nuclear physics analog has been discussed by Drell [33].

Let us consider the electroproduction subprocess $\ell (qq) \to \ell qq$ where the initial $(qq)$ are collinear and have small invariant mass in the initial state and the $qq$ pair in the final state can have large invariant mass. The lepton can effectively scatter on either quark. The nominal scaling of such twist-four contributions is

$$F_2^{\text{interference}}(x, Q^2) = \sum_{a \neq b} e_a e_b \frac{(1 - x)^2}{R_{ab}^2 Q^2}$$

where the factor of $1/R_{ab}^2$ reflects the inter-parton distance. The interference terms are distinctive since, unlike renormalon contributions, they do not track with the leading twist contributions. The growth at high $x$ of the twist-four process reflects the fact that the $\ell (qq) \to \ell qq$ subprocess incorporates the momentum of both quarks. This contribution must also play an important role in the physics of Bloom-Gilman duality [22] since the interference contributions also appear in square of the transition form factors. The interference terms can contribute to both $F_L$ and $F_T$. There is an extensive literature on higher-twist contributions to the structure functions coming from such four-fermion operators [34, 35]. They are also referred to as “cat ear” diagrams from their appearance in the virtual Compton amplitude.

Let us suppose that the proton wavefunction is symmetric in the coordinates of the three valence quarks. If we sum over the pairs of valence quarks, we obtain a vanishing contribution on a proton target

$$\sum_{a \neq b} e_a e_b = \left( \sum_a e_a \right)^2 - \sum_a e_a^2 = 1 - (4/9 + 4/9 + 1/9) = 0.$$  

However, for the neutron

$$\sum_{a \neq b} e_a e_b = \left( \sum_a e_a \right)^2 - \sum_a e_a^2 = 0 - (4/9 + 1/9 + 1/9) = 2/3.$$
Figure 4: (a) Twist-four contribution to inelastic lepton scattering where the lepton scatters on different quarks. The interference of $\gamma^*$ and $Z^0$ exchange contributions leads to parity and charge-conjugation violation of the higher-twist contribution. (b-d) The leading-order $O(\alpha_s/Q^2 R^2)$ perturbative QCD gluon-exchange contributions. The higher-twist contribution to the structure function is obtained by a convolution of the nucleon light-cone wavefunctions with the $\gamma^*(qq) \rightarrow \gamma^*(qq)$ multi-quark amplitude.

Thus for symmetric nucleon wavefunctions the dynamical higher-twist cross terms appear to be zero in the proton and significant for the neutron, deuteron, and nuclei! This is a very distinctive effect; it particularly motivates the empirical study of higher-twist effects using the deuteron and nuclear targets.

In a more realistic treatment, one needs to take into account correlation substructure. For example, suppose that we can approximate the nucleon wavefunctions as quark di-quark composites, where the di-quark has $I = 0$ and $J = 0$. Let us also suppose that the inter-quark separation $R_{ab}$ is smallest for the two quarks of the diquark composite. In this case we can approximate the full sum as a sum over the quark charges of the $I = 0$ $ud$ diquark. Then $\sum_{a \neq b} e_a e_b = e_u e_d = -2/9$ for both the proton and neutron targets. However, since it is conventional to parameterize the higher-twist contribution as a correction to the leading twist term. Thus $C_{p,n}(x)$ is predicted to rise strongly at large $x$ and $C_n(x)$ will be larger than $C_p(x)$ since the leading-twist contribution to the neutron structure function $F_2^n(x, Q^2)$ is significantly smaller than $F_2^p(x, Q^2)$. These predictions seem consistent with
the empirical higher-twist contributions to electroproduction extracted in Refs. [17] and [18]. A simple test of the $I = 0$ diquark higher-twist model is the absence of twist-four contributions to the combination of structure functions $F_2^d(x, Q^2) - 2F_2^p(x, Q^2)$.

It is also interesting to note that one can have interference between the amplitude for lepton-quark scattering via photon exchange on one quark with the amplitude for $Z^0$ exchange on another quark. This implies a distinctive parity-violating higher-twist contribution $C_{HT}^{PV}(x)$ proportional to the product of electromagnetic and weak quark charges $\sum_{a \neq b} e_a^e e_b^z$. Twist-four contributions of this type have been in fact been modeled in Ref. [36] for structure function moments. However there is also the possibility of high-$x$ enhancement. In fact, the $x$-dependence of $C_{HT}^{PV}(x)$ should be similar to the parity-conserving contribution.

We can also use Bloom-Gilman duality to predict that the parity-violating structure functions at large $x$ should average to the contributions of the elastic and inelastic electroproduction channels when integrated over similar ranges in $W^2$. In fact, the parity-violating elastic form factors can be predicted at large momentum transfer in perturbative QCD [37]. Such measurements will provide very interesting tests of the applicability of PQCD to exclusive processes. Thus as emphasized by Souder [19], the detailed measurement of the left-right asymmetry $A_{LR}$ in polarized elastic and inelastic electron-proton and polarized electron nucleus scattering at large $x_{bj}$ can be a powerful illuminator of quark-quark correlations and fundamental QCD physics at the amplitude level.

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