Forecasts on Primordial non-Gaussianity from 21 cm Intensity Mapping experiments

Dionysios Karagiannis,\textsuperscript{a,c,1} Anže Slosar,\textsuperscript{b} Michele Liguori,\textsuperscript{a,c}

\textsuperscript{a}Dipartimento di Fisica e Astronomia “G. Galilei”, Università degli Studi di Padova, via Marzolo 8, I-35131, Padova, Italy
\textsuperscript{b}Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA
\textsuperscript{c}INFN, Sezione di Padova, via Marzolo 8, I-35131, Padova, Italy

E-mail: dakaragian@gmail.com

Abstract. We forecast ability of dedicated 21 cm intensity mapping experiments to constraint primordial non-Gaussianity using power spectrum and bispectrum. We model the signal including the non-linear biasing expansion using a generalized halo model approach. We consider the importance of foreground filtering scale and of the foreground wedge. We find that the current generation intensity mapping experiments like CHIME do not possess sufficient sensitivity to be competitive with the existing limits. On the other hand, upcoming experiments like HIRAX can improve the current constraints and the proposed PUMA experiment can substantially improve them, reaching sensitivities below $\sigma(f_{NL}) < 5$ for equilateral and orthogonal configurations and $\sigma(f_{NL}) < 1$ for the local shape if good foreground control is achieved.

\textsuperscript{1}Corresponding author.
1 Introduction

Understanding primordial cosmological inflation in detail is one of the biggest challenges of modern physics. Currently, inflation is essentially a generic framework for setting up the hot big bang, rather than a concrete, specific theory: any scalar field rolling down a sufficiently flat potential will produce a nearly spatially flat universe with small seed fluctuations which allow the hot Big Bang to proceed and eventually create the universe as we know it. Nevertheless, we do have a few observational handles that help us to distinguish between a simplest realization of inflation – that is, a minimal single-field scenario – and more complex models, such as those involving multiple fields and non-minimal couplings. A convincing detection in any of these probes of non canonical physics will be crucial in connecting inflation to the bigger puzzle of fundamental physics.

Primordial non-Gaussianity (PNG) has emerged as one such powerful probe in the last decade [1, 2]. In the simplest models of inflation containing a single scalar field, the resulting fluctuations in the curvature field are nearly perfectly Gaussian, essentially encoding the physics of the lowest energy state of a quantized scalar field. This picture breaks if there are either multiple interacting fields involved in inflation, or non-minimal couplings of the inflaton, or if slow-roll is broken [3–12] (see also Refs. [2, 13, 14] for a general review and discussion). Since we know that the primordial fluctuations are Gaussian to a very good degree, these effects must be perturbative. As a result, in most scenarios, a non-zero three-point function is generated at leading order in a cumulant expansion of the primordial curvature fluctuation field. Resulting bi-spectra are classified by the shape of the triangle configurations that
dominate the signal, as we will discuss in more detail in the next section. Needless to say, a detection of this signal would bear far-reaching theoretical implications and be of paramount importance in cosmology.

Measurements of the very small non-Gaussianity signal are difficult and dominated by the sample variance of the observable, i.e. by the number of modes which can be used to measure any deviation from zero bi-spectrum. Currently, the most competitive constraints come from CMB observations [15]. However, the constraints derived from the Large Scale Structure are catching up fast [16–21]. In fact, due to its inherently three-dimensional structure and thus a larger number of measured linear modes, LSS is expected to soon supersede CMB measurements [21, 22].

In light of this premise, we investigate here the possibility of 21cm intensity mapping observations to probe primordial non-Gaussianity. More specifically, we develop the methodology to perform non-Gaussianity forecasts for intensity mapping surveys, and apply it to current and proposed experiments. Developments in technology and in our understanding of the signal have recently allowed proposing very ambitious experimental designs in this area, which can still be implemented at reasonable cost. For example, PUMA [23] will observe the low-redshift universe from $z = 0.3$ to $z = 6$, with a single instrument over approximately half the sky. This extremely large volume suggests the possibility that intensity mapping will be at least competitive with the CMB in placing primordial non-Gaussianity constraints.

Previous works have shown the constraining power of IM experiments on measuring PNG, by utilising the scale-dependent bias in the galaxy power spectrum [24, 25]. In Ref. [26], they present forecasts from both power spectrum and bispectrum, where they assume a rather simple model for the latter. Here we adopt the complete model, up to tree-level, for the redshift space HI power spectrum and bispectrum, while we take into account theoretical limitations and a variety of observational effects (see Ref. [22] for an extensive discussion). Adding to this, a careful treatment of foreground systematics is applied, providing realistic forecasts on the amplitude of PNG.

We note here that it might seem fanciful to make forecasts for the ability of 21cm experiments to measure non-Gaussianity, when we do not even have basic clean auto-power spectrum measurements from observations of 21cm intensity maps, let alone baryon acoustic oscillation measurements. However, we note that the current observational issues are entirely due to imperfect calibration and stability of the instrument, rather than being of fundamental astrophysical nature. Therefore, the approach we take is to forecast the statistical sensitivity to non-Gaussianity in the presence of irreducible systematic issues such as foreground contamination, assuming that purely instrumental issues will be solved in time. In fact, the competitive figures that we find should provide additional impetus for the R&D that is required to make the 21cm observations a success.

This paper is structured as follows. In Section 2 we present our model for the large-scale fluctuations of the neutral hydrogen density. In Section 3 we discuss observational limitations and specifications of three experiments under consideration in this paper. We present our results in Section 4.1, followed by Conclusions (Section 5).

2 Modeling the signal

2.1 Matter power spectrum and bispectrum

The two point statistics of the primordial curvature perturbation field in Fourier space, $\zeta(k)$, is defined as
\[ \langle \zeta(k)\zeta(k') \rangle = (2\pi)^3 \delta_D(k - k') P_\zeta(k). \] (2.1)

These primordial perturbations, generated through inflation, are nearly perfectly Gaussian. Therefore, they can be characterized by the two-point correlation function. They are also directly related to the Bardeen gauge invariant primordial gravitational potential \( \Phi(k) \) (during matter domination era, \( \Phi(k) = 3/5\zeta(k) \)), which is in turn related, through the cosmological Poisson equation, to the linearly evolved dark matter density contrast \( \delta \). The linear matter power spectrum is thus defined, as

\[ P^L_m(k, z) = M^2(k, z)P_\Phi(k), \] (2.2)

where

\[ M(k, z) = \frac{2k^2c^2T(k)D(z)}{3\Omega_m H_0^2}. \] (2.3)

In the above equation, \( D(z) \) is the linear growth factor, originating from the growing mode of the linear fluid equations, normalised to unity today (i.e. \( D(0) = 1 \)). \( T(k) \) is the matter transfer function normalized to unity at large scales \( k \to 0 \) and \( c \) is the speed of light. In this work, we use Planck 2015 best-fit parameters [27] to define the fiducial cosmology, used to derive the matter power spectrum. Note, that small changes in the fiducial choice of the cosmological parameters are known to produce negligible impact in PNG constraints.

Various inflationary theories predict a small deviation from perfectly Gaussian initial conditions. This leads to non-zero high-order correlation functions of the curvature perturbation field. The largest of such correlators – in Fourier space – is in most cases the bispectrum, i.e., the three-point correlation function of Fourier modes, defined as:

\[ \langle \zeta(k_1)\zeta(k_2)\zeta(k_3) \rangle = (2\pi)^3 \delta_D(k_1 + k_2 + k_3)B_\zeta(k_1, k_2, k_3). \] (2.4)

The strength of the bispectrum signal is generally described by a dimensionless amplitude parameter, called \( f_{\text{NL}} \). In addition to this, the Dirac delta (enforcing homogeneity) imposes a dependence of the bispectrum on the shape of the different Fourier space triangles. Following the equations above, we can write, at leading order, the linearly extrapolated PNG contribution of the matter density bispectrum as:

\[ B_I(k_1, k_2, k_3, z) = M(k_1, z)M(k_2, z)M(k_3, z)B_\Phi(k_1, k_2, k_3) \] (2.5)

where \( B_\Phi \), as in the case of the power spectrum, is related to \( B_\zeta \) and therefore provides a window to the non-linear interaction during inflation. The number of shapes of the forming triangles is large and the different inflation models predict PNG that picks at different configurations. In this work, we will consider three very important shapes of PNG, namely the local shape [28–31], the equilateral shape [32] and the orthogonal [33], defined respectively as:
\begin{align}
B_{\Phi}^{\text{loc}}(k_1, k_2, k_3) &= 2f_{\text{NL}}^{\text{loc}} [P_{\Phi}(k_1)P_{\Phi}(k_2) + 2 \text{ perms}] , \\
B_{\Phi}^{\text{eq}}(k_1, k_2, k_3) &= 6f_{\text{eq}}^{\text{equil}} [ - [P_{\Phi}(k_1)P_{\Phi}(k_2) + 2 \text{ perms}] - 2[P_{\Phi}(k_1)P_{\Phi}(k_2)P_{\Phi}(k_3)]^{2/3} \\
&+ [P_{\Phi}^{1/3}(k_1)P_{\Phi}^{2/3}(k_2)P_{\Phi}(k_3) + 5 \text{ perms}]] , \\
B_{\Phi}^{\text{orth}}(k_1, k_2, k_3) &= 6f_{\text{NL}}^{\text{orth}} [3[P_{\Phi}^{1/3}(k_1)P_{\Phi}^{2/3}(k_2)P_{\Phi}(k_3) + 5 \text{ perms}] \\
&- 3 [P_{\Phi}(k_1)P_{\Phi}(k_2) + 2 \text{ perms}] - 8(P_{\Phi}(k_1)P_{\Phi}(k_2)P_{\Phi}(k_3))^{2/3} ] .
\end{align}

Due to the non-linear nature of gravity, the matter bispectrum has additional terms at the zeroth-order (tree-level). Therefore, measuring the amplitude of PNG from the bispectrum of LSS is a highly non-trivial process. Robust modelling of non-linearities should be considered, in order to remove these gravitational contaminants and retrieve a clear PNG signal. Here, we account for the gravity induced non-linearities in the framework of Standard Perturbation Theory (SPT) [e.g. see Ref. [34] for a review]. Through out this work, we will restrict our analysis to only mildly non-linear scales, in order to be consistent with the SPT approach. Thus, we will only use the linear power spectrum and the tree-level bispectrum in SPT (see Sec. 2.4).

### 2.2 Halo bias and Mass function

The measurement of PNG from the LSS of the Universe requires a robust description of the bias relation between the dark matter haloes and the underlying matter distribution. For Gaussian initial conditions, a perturbative expansion up to second order, valid for the scales at which we perform our analysis, can be written in the Eulerian framework as [35–46]:

\begin{equation}
\delta_h^{E,(G)}(\mathbf{x}, \tau) = b_1^E(\tau)\delta(\mathbf{x}, \tau) + \varepsilon^E(\mathbf{x}, \tau) + \frac{b_f^E(\tau)}{2}\delta^2(\mathbf{x}, \tau) + b_s^E(\tau)s^2(\mathbf{x}, \tau) + \varepsilon_\delta^E(\mathbf{x}, \tau)\delta(\mathbf{x}, \tau) ,
\end{equation}

where \(\tau\) is the conformal time and \(\mathbf{x}\) are the spatial co-moving coordinates in the Eulerian frame. In addition, \(s^2 = s_ijk s^{ij}\) is the simplest scalar that can be formed from the tidal field \(s_{ij}\) [39, 40, 42, 43], \(\varepsilon^E\) is the leading stochastic bias contribution [47–49] and \(\varepsilon_\delta^E\) is the stochastic counterpart of the linear bias. The second-order tidal field bias coefficient, following the convention in Ref. [43], is given by \(b_s^E = -2/7(b_f^E - 1)\). This relation assumes the Lagrangian tidal bias to be \(b_f^L = 0\), as well as a convolution of matter and tracer. It is tested against numerical results in Ref. [50], where they find it to be a good approximation with an evidence of a small negative Lagrangian bias, i.e. \(b_s^L < 0\).

The presence of PNG will introduce additional terms in the bias expansion, including the large scale scale-dependent bias correction [16, 51–58]. In the general bias expansion description [44–46], used for this work, the complete set of terms for an arbitrary quadratic PNG up to second order in perturbations and linear in \(f_{\text{NL}}\) are [59]:

\begin{equation}
\delta_h^{E,(NG)}(\mathbf{x}, \tau) = b_0^E(\tau)\Psi(\mathbf{q}) + b_b^E(\tau)\Psi(\mathbf{q})\delta(\mathbf{x}, \tau) + \varepsilon_\Psi^E(\mathbf{x}, \tau)\Psi(\mathbf{q}) ,
\end{equation}

where \(\mathbf{q}\) are the spatial coordinates in the Lagrangian frame and \(\Psi\) is a non-local transformation of the Bardeen potential (see e.g. Refs. [22, 60] for the expression). It is introduced in the general bias expansion, through operators, in order to model the scale-dependent corrections (in the same spirit as in the work of Refs. [61–63]). Furthermore, \(\varepsilon_\Psi^E\) is the stochastic counterpart of the field \(\Psi\).
The bias coefficients of these terms, i.e. $b_\Psi$ and $b_{\Psi\delta}$, can be derived by the generalisation of the peak-background split (PBS) argument. The general expression of the leading term (in the squeezed limit), for the PNG types considered here, was derived in Ref. [57, 58], while for $b_{\Psi\delta}$ in Refs. [22, 62].

The presence of PNG introduces a scale independent correction to the linear and quadratic bias terms, due to the non-Gaussian correction to the mass function [64, 65]. Both are taken into account in our analysis.

2.3 HI bias

The results of the general bias expansion (Sec. 2.2) are tracer independent, therefore the final ingredient in the determination of the two-point and three-point 21cm galaxy correlators is the prescription of the population of the dark matter haloes. In the spirit of the halo occupation distribution (HOD) [66], we can define the density of the neutral hydrogen as [67]

$$\rho_{HI}(z) = \int n_h(M, z) M_{HI}(M, z) d\ln M. \quad (2.11)$$

where $n_h(M, z)$ is the halo mass function and $M_{HI}$ is the average HI mass contained inside a halo of mass $M$, at redshift $z$. Hence, the abundance of neutral hydrogen will be, $\Omega_{HI}(z) = \rho_{HI}(z)/\rho_0^\text{HI}$, where $\rho_0^\text{HI}$ is the critical density of the Universe in the present.

The mass function is given by [68]

$$n_h(M, z) = \frac{dN}{d\ln M} = \frac{\rho_m}{M} f(\nu) \left| \frac{d\ln \nu}{d\ln M} \right|, \quad (2.12)$$

where $\rho_m(M) = \Omega_m \rho_0^\text{c}$ is the the mean co-moving density and $\nu(M, z) = \delta_c/\sigma_R(M, z)$ is the height of the peak. In addition, $\delta_c = 1.686$ and $\sigma_R(M, z)$ is the variance of the smoothed density field on mass scale $M$ and radius $R$. In this work we will use the best-fit results from N-body simulations presented in [69] (hereafter T08).

Generalising the results of Ref. [67], where they provide the linear bias of the HI in the framework of the halo model, we retrieve the higher-order HI bias coefficients. The bias parameters for HI are then given by:

$$b_{HI}^i(z) = \frac{1}{\rho_{HI}(z)} \int_0^\infty n_h(M, z) b_{HI}^i(M, z) M_{HI}(M, z) d\ln M, \quad (2.13)$$

where the linear halo bias (i.e. $b_1^h$) is considered to be the best fit expression presented in Ref. [70], which achieves a good agreement with numerical results, for both low and high masses (see also Ref. [71]). The formulas for the higher-order halo bias (i.e. $b_{HI}^{i>1}$) are derived after using the T08 mass function and the PBS argument, where the details of the derivation are presented in Appendix A. These analytic results were tested against simulations in Ref. [71]. They show that the "PBS+T08" prediction for $b_2^h$ [Eq. (A.10)] deviate from their numerical results, mainly for low masses, while it still does better than the expression derived from the PBS argument and the mass function of Ref. [72] (hereafter ST99). In the case of $b_3^h$ and $b_4^h$, the PBS-derived results from the T08 mass function [Eqs. (A.11) and (A.12)] are in agreement with their measurements. In order to ensure the self-consistency of this work, as well as to be congruent with the HI bias prediction of Ref. [67], we will use the "PBS+T08" results for the higher-order halo biases [i.e. Eqs. (A.10)-(A.12)]. Note that, in Ref. [71] they provide fitting formulas for $b_2$ and $b_3$ as a function of $b_1$. Using those, instead of the "PBS+T08" predictions, in our Fisher analysis change’s the forecasts on the amplitude of...
PNG by few percent $\left(1 - 4\% \text{ depending the survey}\right)$. The main reason for this small difference is the HOD recipe (described next) used here, which favours mass ranges where both halo bias predictions (i.e. the fitting formulas of Ref. [71] and those derived in Eqs. (A.11) and (A.12)) are consistent with each other.

The final ingredient of the HI bias recipe is the relation for neutral hydrogen mass $M_{HI}$. Here we use the fitting results of Ref. [67]:

$$M_{HI}(M,z) = C(z)(1 - Y_p)\frac{\Omega_b}{\Omega_m} e^{-M_{min}(z)/M} M^\alpha(z)$$  \hspace{1cm} (2.14)

where $Y_p = 0.24$ is the Helium fraction, $M_{min}$ represents a halo mass below which the HI abundance in halos is exponentially suppressed, and $C$ is a normalization constant fixed by the value of $\Omega_{HI}(z)$. Note that, the HI bias parameters do not depend on the normalization of $M_{HI}$. The values for the free parameters are considered to be, $\alpha = 1$ and $M_{min} = 5 \cdot 10^9 M_{sun}/h$. The resulting Eulerian bias parameters [Eq. (2.13)] are plotted as a function of redshift in Fig. 3. Alternative expressions for the neutral hydrogen mass, originating from comparisons with numerical results, are also presented in Refs. [73, 74].

### 2.4 Power spectrum and bispectrum of HI in redshift space

In the presence of a general, non-local, PNG the linear power spectrum and tree-level bispectrum in redshift space is given by (e.g. see Ref [22] for details):

$$P^s_g(k,z) = D^B_{FOG}(k) T_b^2 \left[ Z_1^2(k) P^L_{m}(k,z) + P_\varepsilon \right] + P_N,$$  \hspace{1cm} (2.15)

$$B^s_{g}(k_1, k_2, k_3, z) = D^B_{FOG}(k_1, k_2, k_3) T_b^3 \left[ Z_1(k_1) Z_1(k_2) Z_1(k_3) B_I(k_1, k_2, k_3, z) + \left(2 Z_1(k_1) Z_1(k_2) P^L_{m}(k_1, z) P^L_{m}(k_2, z) + 2 \text{perm}\right) + 2 P_{\varepsilon \varepsilon \varepsilon}(Z_1(k_1) P^L_{m}(k_1, z) + 2 \text{ perm}) + B_\varepsilon \right],$$  \hspace{1cm} (2.16)

where $P_N$ is the instrument noise (see Sec. 3) and $T_b$ is the temperature function of the HI field. The expressions for the redshift space kernel [i.e. $Z_1(k)$ and $Z_2(k, k_j)$], as well as the Finger-of-God (FOG) damping effect (i.e. $D_{FOG}$), can be found in Appendix C. The terms, $P_\varepsilon$, $P_{\varepsilon \varepsilon \varepsilon}$, $B_\varepsilon$ are generated by the presence of stochastic bias.

The tree-level redshift space bispectrum [Eq. (2.16)] has additional terms, originating from the presence of PNG, of $O(f_{NL}^2)$. The fiducial value considered here for the PNG amplitude is $f_{NL} = 0$, hence they do not contribute to the signal in a Fisher matrix forecast. Therefore, they will be neglected here in order to simplify our calculations. Note that, the exclusion of large scales, due to foreground contamination (see Sec. 3.1), allows us to safely neglect wide-angle and relativistic effects from the HI bispectrum. The full expression of the large-scale bispectrum, beyond any approximated treatments, is shown in Ref. [75].

The fiducial values of the stochastic terms in Eqs. (2.15) and (2.16), are taken to be those predicted by Poisson statistics and are given by [60, 76]:

$$P_\varepsilon \equiv P_{SN}; \quad P_{\varepsilon \varepsilon \varepsilon} = \frac{b_1}{2\pi_{eff}}; \quad B_\varepsilon = \frac{1}{\pi_{eff}},$$  \hspace{1cm} (2.17)

where, in the HI halo model approach (Sec. 2.3), the shot noise term is given by [67]
\[ P_{\text{SN}}(z) = \frac{1}{n_{\text{eff}}(z)} = \frac{1}{\rho_{\text{HI}}(z)} \int n_h(M, z) M^2 \rho_{\text{HI}} d\ln M \]  

(2.18)

The temperature function is given, in \( \mu \text{K} \), by (see the Appendix of Ref. [77])

\[ T_b = 180(1 + z)^2 / E(z) \times 4 \times 10^{-4}(1 + z)^{0.6} \]  

(2.19)

where \( E(z) = \sqrt{\Omega_m(1 + z)^3 + \Omega_k(1 + z) + \Omega_\Lambda}, \) for the standard dark-energy model (i.e. \( w_0 = -1, w_\alpha = 0 \)).

3 Modeling 21 cm surveys

The main difference between a galaxy survey and a line intensity mapping survey is that in the latter there is a noise component associated with the instrument itself. Such component in most cases dominates over the shot-noise term, which is instead the dominant noise term in traditional galaxy surveys. We model this instrumental noise component as a Gaussian noise given by [78, 79]

\[ P_N(k, z) = T_{\text{sys}}^2(z) \chi^2(z) \lambda^2(z) \left( \frac{\chi^2(z)}{H(z)} \right) \left( \frac{\lambda^2(z)}{A_e} \right)^2 \frac{1}{N_{\text{pol}} n_b(u)t_{\text{survey}}\theta_{\text{FOV}}^2 S_{\text{area}}} \]  

(3.1)

for a radio interferometer. Here \( T_{\text{sys}} = T_{\text{sky}} + T_{\text{inst}} \) is the system temperature, which is given from the sum between the sky and the instrument temperature (see Eq. D1 and D2 of [77]), and \( \lambda(z) \) is the redshifted wavelength of the 21cm HI line. The field-of-view is \( \theta_{\text{FOV}} \) and \( S_{\text{area}} \) is the area of the survey in steradians, while \( n_b(u) \) is the antenna distribution (Appendix B), \( N_{\text{pol}} = 2 \) is the number of polarizations per feed and \( A_e \) is the effective beam area. Finally, \( \chi \) and \( t_{\text{survey}} \), are the co-moving distance and the total observation time in hours respectively.

We consider three 21 cm IM surveys: CHIME [80], HIRAX [81] and the PUMA survey [23, 77]. CHIME is operating as we speak, while HIRAX is in advanced proposal stage. The PUMA represents a much more aggressive concept about what will be possible in the future with a larger investment in this field.

All these instruments are modeled somewhat simplistically. We consider realistic noise curves associated with a given distribution of baseline lengths, sky coverage and attainable system performance, but we skim over issues associated with realistic aperture synthesis due to earth rotation and angle of observation. We initially also assume a perfect phase calibration, but we will later consider the impact of discarding the data inside the foreground wedge, to illustrate the importance of phase calibration. In short, the main goal of this paper is to investigate the scientific potential of these experiments to motivate the research into a complete control of systematics.

For the surveys PUMA and HIRAX, the field-of-view is \( \theta_{\text{FOV}} = \lambda(z)/D_{\text{eff}} \) and the collecting area per feed is \( A_e = \pi(D_{\text{eff}}/2)^2 \). We assume an effective dish area, \( D_{\text{eff}} \), due to the non-uniform illumination of the primary, given by \( D_{\text{eff}}^2 = \eta_a D_{\text{dish}}^2 \), where \( D_{\text{dish}} \) is the physical dish size and \( \eta_a = 0.7 \) is the aperture efficiency factor, taken to have the same value for both surveys (see Appendix D of [77] for a discussion). HIRAX is assumed to be square closed packed, while PUMA is hexagonally close packed with 50% fill factor (i.e. a random 50% of hexagonally closed packed lattice sites are empty, so the array is equivalent in size to that of twice the number of elements but with quarter baseline density). For baseline density we follow fitting formulas of [77].
| Parameters | CHIME | HIRAX | PUMA (Full/Petite) |
|------------|-------|-------|-------------------|
| redshift   | 7.5 - 2.5 | 0.75 - 2 | 2 - 6 |
| packing    | packed cylinder array | square | hexagonal (50% fill) |
| $N_{\text{dish}}$ | 4 x 256 | 1,024 | 32,000/5,000 |
| $D_{\text{dish}}$ | $W_{\text{cyl}} = 20 \text{ m}$, $L_{\text{cyl}} = 100 \text{ m}$ | 6 m | 6 m |
| $f_{\text{sky}}$ | 0.6 | 0.36 | 0.5 |
| $t_{\text{tot}}$ | 10,000 hrs | 10,000 hrs | 40,000 hrs |
| $T_{\text{inst}}$ | 50 K | 50 K | 50 K |

Table 1. The basic specifications for three IM surveys considered here. For PUMA, the array is hexagonally close-packed.

In the case of CHIME, which is a cylindrical interferometer, we define the field-of-view as, $\theta_{\text{FOV}} = (1.22\lambda(z)/W_{\text{cyl}})\pi/2$, where $W_{\text{cyl}}$ is the width of the cylinder in meters. While, the effective beam is given by, $A_e = \eta L_{\text{cyl}} W_{\text{cyl}} N_{\text{cyl}}/N_{\text{dish}}$, where the optical efficiency is taken to be $\eta = 1$. In the case of the cylindrical interferometer, $N_{\text{dish}}$ corresponds to the total number of feeds on $N_{\text{cyl}} = 4$ cylinders, while $L_{\text{cyl}}$ is each cylinders length in meters. For the baseline distribution we take approximate analytical expression given in Appendix B.

We take the total observation time to be 10,000 hours for CHIME and HIRAX, following their published plans. For PUMA, we have a 50% filled array that will observe for 40,000 hours, which corresponds to 5 years.

The covered area for PUMA is half the sky, while for CHIME and HIRAX is 60% and 36% respectively. The specific values of the parameters needed to calculate the power spectrum noise are given in Table 1.

### 3.1 Foreground exclusions

Foregrounds are orders of magnitude brighter than the signal in 21cm studies and thus present an irreducible systematics. However, based on our understanding of their production mechanism, we have very good reasons to believe that they are spectrally smooth, which is the main characteristics that allows us to distinguish them from the signal.

Following [77] we therefore always impose a cut on the observed plan, throwing away modes with $k_{\parallel} < k_{\parallel,\text{min}}$, where we take $k_{\parallel,\text{min}}$ to be either 0.01 $h$/Mpc, but we also study the effect of increasing it to 0.05 $h$/Mpc.

In practice, there is an additional instrumental effect named foreground wedge [82, 83]. Wedge arises because a single interferometric baseline will see a monochromatic source away from zenith fringing along frequency direction in the exactly the same manner as non-monochromatic source at zenith. A full array with sufficiently dense coverage of the $u-v$ plane can break this degeneracy, but only if phase calibration is sufficiently accurate and stable. To date, this has not been achieved in current generation of intensity mapping and epoch of reionization arrays. Nevertheless, it is important to stress that the foreground wedge is not a fundamental astrophysical limitation, but a technical issue.

We model the wedge by removing all modes that satisfy $k_{\parallel} < k_{\text{wedge}} k_{\perp}$. The aggressiveness of the cut $k_{\text{wedge}}$ is determined by the source furthest from the zenith that can corrupt the data. The most conservative assumption is that of horizon wedge where any monochromatic source above horizon can contaminate the data. 21cm intensity mapping is not really competitive in this limit and therefore we do not consider this option. A more realistic modeling assumes that sources up to certain number $N_w$ of primary beam sizes away from the
zenith can have an effect, giving [82]
\[ k_{\text{wedge}} = \frac{rH(z)}{c(1+z)} \sin \left( 1.22N_w \frac{\theta_{\text{FOV}}}{2} \right), \]  
(3.2)

In this paper we consider values of \( N_w = 0, 1, 3 \), where \( N_w = 0 \) is the most optimistic (fully recovery of data inside the wedge) and \( N_w = 3 \) is the most pessimistic.

4 Fisher Matrix Analysis and Results

In order to forecast the precision in measurements of the PNG amplitude \( f_{\text{NL}} \), we use of the Fisher matrix formalism. In the case of the galaxy power spectrum, the Fisher matrix is given by
\[ F_{\alpha\beta}^{\mathcal{P}_s} = \sum_{\mu_1} \sum_{k} \frac{\partial \mathcal{P}_s^\alpha}{\partial p_\mu} \frac{\partial \mathcal{P}_s^\beta}{\partial p_\mu} \frac{1}{\Delta P^2}, \]
(4.1)

while for the bispectrum we have [84]
\[ F_{\alpha\beta}^{\mathcal{B}_s} = \sum_{\mu_1 = -1}^{\pi/2} \sum_{T} \frac{\partial B_s^\alpha}{\partial p_\mu} \frac{\partial B_s^\beta}{\partial p_\mu} \frac{1}{\Delta B^2}, \]
(4.2)

where \( p_{\alpha,\beta} \) are the unknown parameters of interest and the derivatives are evaluated at the fiducial value of the parameter vector \( p \). The latter consists of all the parameters of the model that we consider free and marginalise over in order to take into account degeneracies and cross-correlations on the final constrains. It is given by
\[ p = \{ f_{\text{NL}}, b_1, b_2, b_3, P_\epsilon, P_{\epsilon \epsilon \delta}, B_\epsilon, f, \sigma_v \}. \]
(4.3)

The stochastic bias contributions (i.e. \( P_\epsilon, P_{\epsilon \epsilon \delta} \) and \( B_\epsilon \)) are considered here as nuisance parameters and they are marginalised over to acquire the Fisher sub-matrix for the parameters of interest. Cosmological parameters are fixed throughout this work, since they dont affect the final PNG constraints [85].

In our Fisher matrix analysis, only the diagonal part of the covariance matrix (i.e. \( \Delta P^2 \) and \( \Delta B^2 \)) is taken into consideration, neglecting all the cross-correlations between different triangles (bispectrum) and \( k \)-bins (power spectrum). We adopt a Gaussian approximation for the variance terms:
\[ \Delta P^2(k, z) = \frac{4\pi^2}{V_{\text{survey}} k^2 \Delta k \Delta \mu} P_{\text{tot}}^2, \]
(4.4)
\[ \Delta B^2(k_1, k_2, k_3, z) = s_{123} \pi V_f \frac{P_{\text{tot}}(k_1) P_{\text{tot}}(k_2) P_{\text{tot}}(k_3)}{k_1 k_2 k_3 \Delta k^3 \Delta \mu \Delta \phi}, \]
(4.5)

where \( s_{123} = 6, 2, 1 \) for equilateral, isosceles and scalene triangles respectively. The volume of the fundamental shell in Fourier space is \( V_f = k_f^3 \). In addition \( P_{\text{tot}}(k, z) = P_s^g(k, z) \) [see Eq. (2.15)].

Following [86], we correct the bispectrum variance for neglecting higher order terms that are not taken into account in our simple Gaussian diagonal covariance (see also Ref. [22] for additional discussion). In addition, we consider also theoretical errors in the analysis [87],
Table 2. Forecasts for the 1 − σ error of the primordial non-Gaussian amplitude in the case of the three PNG types considered here. These results come from the summation of the signal over the whole redshift range of each survey, where we show the constraints originating from the galaxy power spectrum, bispectrum and their combined signal. In addition, different wedge cases are considered for each survey. The forecasts under the column "NO" correspond to the case where no wedge cut is used, while under the columns titled "PB" and "3xPB" are the forecasts after applying the prime-beam wedge cuts [Eq. (3.2)] for \( N_w = 1 \) and \( N_w = 3 \) respectively. Moreover, we exclude from the analysis all scales that satisfy, \( k_\parallel < 0.01 \) h/Mpc, as discussed in the main text. The forecasts under "3xPB" correspond to the main results of this work.

i.e. additional unavoidable uncertainties, which arise from using a truncated perturbative description to model the expected power spectrum and bispectrum (at tree level, in our case). For the bispectrum, we here consider only the diagonal of the theoretical error covariance (see [22, 87] for a discussion).

We confine the analysis inside the linear/semi-linear regime, where the tree-level power spectrum and bispectrum is valid, by cutting the maximum scales at \( k_{\text{max}} = 0.75 k_{\text{NL}}(z) \). The non-linear scales are taken to be the linear, one dimensional velocity dispersion, given by

\[
k_{\text{NL}}(z) = \left[ \frac{1}{6 \pi^2} \int_0^\infty dk P_{\text{lin}}(k, z) \right]^{-1/2}.
\]  

Note that the smallest accessible scales are also limited by the specifications of each survey. The largest and smallest available perpendicular scales are given by \( k_{\perp, \text{min}} = 2 \pi/(\chi \theta_{\text{FOV}}) \) and \( k_{\perp, \text{max}} = 2 \pi D_{\text{max}}/(\lambda \chi) \) respectively.

4.1 Results

Our main results are summarized in Table 2. We give results for all three types of non-Gaussianity and for a number of wedge cuts. For comparison, the currently achieved sensitivities from Planck [88] are 5, 43 and 21 for squeezed, equilateral and orthogonal shapes respectively. Forecasted numbers for CMB-S4 experiments are 2, 21 and 9. SphereX can achieve 0.5 for the local shape. Galaxy surveys are generally unable to achieve competitive constraints for shapes which peak away from the squeezed limit, due to heavy contamination from non-linear gravitational effects; this is particularly true for the equilateral case, where late-time non-linear contributions are largest.

We see that CHIME is never competitive, despite significant volume coverage, because the thermal noise is overwhelming. HIRAX could instead achieve similar sensitivities to Planck. These constraints will be independent and could therefore improve over Planck by some 40%. Most importantly, if such result will be achievable in practice – keeping all
systematics under control – it will provide an outstanding observational confirmation that primordial non-Gaussianity is indeed a very promising field of study for future, ambitious intensity mapping surveys. Our forecasts show that PUMA should be competitive with CMB-S4 and Spherex. Notably, it could provide particularly strong constraints for the equilateral shape, if the foreground wedge could be controlled. This is of particular interest, considering the general difficulty of improving equilateral constraints using LSS tracers, as mentioned above.

In Fig. 1 we study where the information is coming from, as a function of redshift. In general, the neutral hydrogen maps are noisier at higher redshift, due to increasing sky noise temperature and decreasing bandwidth per comoving distance. This is partly offset by the fact that the non-linear scale is smaller at higher redshift (i.e. increasing $k_{NL}(z)$), and that the total volume per sky area is larger. As a result, there are no unique trends and different experiments can extract most information from either low or high redshift end, depending both on the experiment and on the type of non-Gaussianity under consideration.

In most cases, presented in Fig. 1, the majority of the PNG signal originates from the bispectrum (see also Table 2), highlighting the importance of three-point statistics in
constraining PNG from future IM experiments, and in particular from packed interferometric arrays. At higher redshift slices the Universe becomes more linear, increasing the scale range where the linear theory is still valid (increasing $k_{\text{max}}$). A boost in the constraining power of the bispectrum is hence expected, due to the growing number of formed triangles and the amplitude reduction of the gravitational contaminants. This can be observed in some cases shown in Fig. 1, but a trend cannot be established due to the presence of observational and other effects (see Sec. 3.1), as well as due to the individual traits of the PNG types considered.

In the case of local PNG, the bispectrum signal peaks on the squeezed configurations ($k_1 \ll k_2 \sim k_3$), therefore sufficiently large and small (up to the validity of linear theory) scales must be accessible by a survey, in order to have enough squeezed triangles to produce compelling bispectrum constraints. If they are restricted, due to e.g. observational and instrumental effects or low redshift slices (i.e. smaller linear regime), then the scale-dependence in the galaxy power spectrum provides most of the PNG signal. This is the case for CHIME and PUMA Petite, where for the latter this is evident for the large redshift slices (i.e. $z \geq 5$).

In the equilateral PNG scenario, the scale-dependent bias term approaches a constant value on large scales. In addition, the presence of degeneracies between $f_{\text{NL}}^{\text{equil}}$ and other parameters on both large and small scales [59], strip power spectrum from essentially any constraining power on equilateral PNG. This leaves galaxy bispectrum to be the sole contributor of the signal. The increasing range of the linear regime (i.e. increase in the number of the formed equilateral triangles) with redshift, in the cases of CHIME and HIRAX, improves the constrains on $f_{\text{NL}}^{\text{equil}}$, up to a saturation point due to the applied scale cuts (see Sec. 3.1 and Sec. 4). These scale limitations are the ones responsible for the opposite trend observed in the PUMA results. Due to the pessimistic wedge cuts, as well as the $k_{\text{min},\|}$ and $k_{\text{max}}$ limits, the expected wide scale range, accessible to PUMA, is shrunken towards high redshift slices, rendering their contribution to the integrated signal minimal. The same behaviour is observed for orthogonal PNG, where the effect of the $k$-cuts can be now seen also in the power spectrum constraints. Due to the functional form of the orthogonal PNG scale-dependent bias [57, 58], only the very high redshift slices of PUMA are affected by the scale cuts.

In Fig. 2 we show how experimental design parameters affect constraints for HIRAX and PUMA. We consider several changes. Changing the dish size (keeping other parameters fixed), moves sensitivity wholesale towards smaller perpendicular scales, affecting the minimum available $k_{\perp}$, but also raising the higher $k_{\perp}$ available. We find that that PUMA prefers bigger dishes, while for HIRAX the 6m design size is nearly optimal. Second, changing the number of dishes at fixed collecting area lowers the noise power spectrum while keeping the resolution the same. We parameterize this with a parameter $N_{\text{side}} = \sqrt{N_{\text{dishes}}}$, which reduces to just the side of the array measured in the number of dishes for HIRAX. Naively, one would expect that increasing $N_{\text{side}}$ can only improve the results, until the sample variance starts to dominate and the results converge to volume limited measurement. However, we find that this is not the case for PUMA, where increasing $N_{\text{side}}$ actually makes things worse, especially for the equilateral case. This is because this exercise is done at the $N_w = 3$ wedge cut. Increasing the number of dishes at fixed collecting area makes dishes smaller, exacerbating the wedge cut. As a result, PUMA prefers larger dishes at fixed collecting area. In the final column we plot trends with the observed sky area. This is done at a fixed observation time, so the smaller $f_{\text{sky}}$ implies a smaller fraction of the sky, but observed with a lower noise level.
Figure 2. The PUMA (red) and HIRAX (blue) forecasts on $f_{NL}$, normalised over the fiducial forecasts (see Table 2), for the three PNG shapes considered here, as a function of the dish size (left panel), the number of dishes in each side (middle panels) and $f_{sky}$ (right panels). All the remaining parameters (including integration time) are kept fixed for all three cases, beside the $N_{side}$ case (middle panels), where in addition we keep fixed the collecting area. These results correspond to the combined power spectrum and bispectrum signal, after summing over the whole redshift range of the survey. The prime-beam wedge for $N_{w} = 3$ is used, as well as all scales that satisfy, $k_\parallel < 0.01$ h/Mpc, are excluded. The plotted dashed lines in the right panels represent the function, $(f_{sky}/f_{sky}^{fid})^{-1/2}$. Note that, in the middle panels the results for PUMA reach up to $N_{side}/N_{side}^{fid} \sim 1.1$ due to the tremendous increase in the calculation time. Nonetheless, the significant part of the functional dependence is shown.

We would therefore expect the error bars to follow

$$
\sigma \propto \left( PS + P_{SN} + \frac{f_{sky}}{f_{sky, fid}} P_{N} \right)^{-1/2}. \tag{4.7}
$$

In other words, for the sample variance limited case, we expect the sensitivity to improve as $f_{sky}^{-1/2}$ (plotted as dashed lines) and for the thermal noise dominated case we expect curves to flatten. We find that HIRAX indeed flattens out, but that PUMA is somewhere between the two limits.

Finally in Table 3 we study the effect of changing the irreducible value of foreground filtering $k_{\parallel \text{min}}$. Since foregrounds affect mostly large angular scales, we find that the effect is the largest for the squeezed triangle configurations, which contain one small side. This leads
Table 3. Same as in Table 2, but now we test the effect of the $k_{\parallel,\text{min}}$ cuts on the $f_{\text{NL}}$ forecasts. We fix the wedge cuts to be as in the case of "3× prime-beam" (i.e. $N_w = 3$ in Eq. (3.2)) and consider two values of the $k_{\parallel}$ cut. Therefore the results that are under the "$k_{\parallel,\text{min}} = 0.01$" columns correspond to the main results of this work (i.e. same as "3×PB" of Table 2).

| $k_{\parallel,\text{min}}$ [h/Mpc] | CHIME | HIRAX | PUMA Full |
|-------------------------------|-------|-------|-----------|
|                               | 0.01  | 0.05  | 0.01  | 0.05  | 0.01  | 0.05  |
| $P(\text{loc})$               | 31.9  | 105.5 | 25.8  | 101.3 | 2.52  | 8.42  |
| $B(\text{loc})$               | 72.7  | 457.7 | 10.2  | 71.5  | 0.91  | 3.63  |
| $P+B(\text{loc})$             | 28.4  | 101.7 | 9.3   | 47.9  | 0.84  | 3.05  |
| $P(\text{equil})$             | -     | -     | -     | -     | -     | -     |
| $B(\text{equil})$             | 576.7 | 3139.0| 112.5 | 484.5 | 42    | 77.88 |
| $P+B(\text{equil})$           | 259.8 | 777.4 | 59.1  | 122.1 | 23.17 | 30.38 |
| $P(\text{ortho})$             | 940.3 | 1502.9| 662.4 | 906.6 | 74.97 | 80.32 |
| $B(\text{ortho})$             | 216.7 | 1242.5| 38.4  | 203.8 | 8.86  | 30.24 |
| $P+B(\text{ortho})$           | 159.5 | 600.5 | 31.8  | 102.3 | 8.56  | 24.25 |

to a significant decrease in the constraining power of both power spectrum and bispectrum in the local PNG case. The reduction is still very pronounced for orthogonal configurations. This is not completely intuitive, but explainable with the fact that the orthogonal shape presents a non-negligible correlation with the squeezed one, and takes significant contributions from flattened triangles. Equilateral PNG is the least affected, as expected, since only a small amount of equilateral triangles, formed by the largest scales, are excluded.

5 Conclusions

In this paper we have studied constraints on primordial non-Gaussianity from present and future 21 cm intensity mapping experiments. Since these experiments cover huge volumes of space, it is intuitive that they should be able to produce competitive non-Gaussianity results.

We find that a futuristic experiment such as PUMA will produce very competitive non-Gaussianity bounds from purely internal measurements of the power and bispectrum (see Table 2). This applies in particular to orthogonal and equilateral shapes, generally very hard to constrain using galaxy surveys, due to heavy contamination from non-linear late time evolution of structures. In the case of PUMA, this issue is offset by large volume coverage and nearly sample variance limited measurements. Moreover, at higher redshift, where the non-linear scale is smaller (larger $k_{\text{NL}}$), the increasing number of formed triangles leads to a growth in the bispectrum signal. Additionally the amplitude reduction of the gravitational contaminants, leads to a promising PNG signal for high redshift surveys. In the case of PUMA, we find that the presence of foreground wedge significantly worsens the limits derived from the high redshift slices calling (see Fig. 1). Is therefore imperative that this important systematic is brought under control.

We find that constraints on the local shape of bispectrum are tight, but considerably less impressive compared to other probes of non-Gaussianity, such as those coming from LSST [22] or the upcoming SPHEREx experiment [89]. This is because the foreground cut at $k_{\parallel,\text{min}} = 0.01h$/Mpc leads to loss of information on large scale modes, which are required in the squeezed limit. While PUMA on its own is not competitive, cross-correlations between
the small scale two-point function from PUMA and large scale mode measured by other means (traditional galaxy survey or CMB lensing) could yet prove to be very effective in measuring local non-Gaussianity (see Ref. [25] for a multi-wavelength power spectrum application). We leave this work for the future.

We find that the current and upcoming generation of 21 cm surveys, such as CHIME and HIRAX are in general not very competitive. This boils down mainly to a considerable thermal noise in the field measurements. Any departure from sample variance limit hits the bispectrum measurement more than the power spectrum measurement and this leads to relatively non-competitive numbers in this case.

We also find that the difference between the full and petite versions of PUMA is pretty modest in the case of pessimistic foreground, but can be considerable if no wedge is assumed. The main difference between these two experiments is in angular resolution, as the full PUMA is simply an extended version of PUMA Petite. This leads to both, additional information on smaller scales but also improved sampling of large scales, since the number of short baselines increases too. What we see is a complex interplay between the change in noise and increase in the number of possible triangle configurations, that depend on both the maximum wavenumbers, but also any other cuts on the Fourier plane. The bottom line is that minimizing the effect of the foreground wedge is essential in order to extract all possible science from these measurements.

In traditional radio astronomy intuition, splitting the same collecting area into more interferometric elements always improves results, because it increases the field of view at constant noise. We find, however, that wedge considerations favor larger dishes, even at the same total collecting area. Since dishes can be thought of as analog interferometers, this in effect boils down to the relative difficulty of analog phase calibration (i.e. surface accuracies) vs electronic phase calibration. The actual trade off calculation will require more sophisticated studies of relative contributions of individual element repeatability and stability, interferometric phase control and data reduction software imperfections to the total error.

To conclude, measurements of primordial non-Gaussianity are ideally suited to intensity mapping of 21 cm neutral hydrogen across cosmic epochs, provided the thermal noise can be made smaller with a sufficiently large array and assuming systematics can be brought under control. Stage II 21 cm experiments such as PUMA will be therefore able to make impressive measurements of non-Gaussianity.

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W. E. Ballinger, J. A. Peacock and A. F. Heavens, *Measuring the cosmological constant with redshift surveys*, *MNRAS* 282 (1996) 877 [astro-ph/9605017].
A Derivation of Bias Parameters from the Tinker et al mass function with the PBS approach

The peak-background split (PBS) approach (see e.g. Ref. [60] for a review) derives the halo bias parameters from the change in the distribution of the density peaks (i.e. the mass function). The density fluctuation field can be decomposed into a long-wavelength linear fluctuation, $\delta_l(x)$, and a noisy short wavelength one, $\delta_s(x)$. The first will modulate the background density and alter the height of the peaks to an effective value

$$\nu \rightarrow \nu_{\text{eff}} = \frac{\delta_c - \delta_l}{\sigma_R}. \quad (A.1)$$

The halo number density in Lagrangian coordinates is given by (see e.g. Appendix C of Ref. [22])

$$\delta_L^h(M|M_1,V_0) = \frac{n_h(M|M_1,V_0,z)}{n_h(M,z)} - 1. \quad (A.2)$$

where $n_h(M|M_1,V_0)$ is the number of subhalos of mass $M$ with an initial volume $V_0$, corresponding to the small wavelength peaks ready to collapse on top of the long mode, above some mass $M_1$ defined by the “background” (i.e. long wavelength) mode and $n_h(M,z)$ is the mean number of halos above mass $M$ (i.e. the halo mass function). Taylor expanding Eq. (A.2) and comparing it with the local-in-matter bias expansion, we can identify the Lagrangian bias coefficients as

$$b_L^N(M,z) = \frac{1}{n_h(M,z)} \left. \frac{\partial^n n_h(M,z)}{\partial \delta_l^n} \right|_{\delta_l=0} = \frac{(-\nu)^N}{\delta_c^N f(\nu,z)} \frac{d^N f(\nu,z)}{d\nu^N}. \quad (A.3)$$

This is a general result for any universal mass function, therefore we can use it to derive the halo bias parameters for the fitting mass function of Ref. [69], given by

$$f(\nu,z) = \alpha \left[ 1 + (\beta \nu)^{-2\phi} \right] \nu^{2\eta} e^{-\gamma \nu^2/2}, \quad (A.4)$$

where the parameters have the following redshift dependence

$$\beta(z) = \beta_0(1+z)^{0.2}, \quad (A.5)$$
$$\phi = \phi_0(1+z)^{-0.08}, \quad (A.6)$$
$$\eta = \eta_0(1+z)^{0.27}, \quad (A.7)$$
$$\gamma = \gamma_0(1+z)^{-0.01}, \quad (A.8)$$

where the zero in the subscript denotes the values of the fitting parameters for $z = 0$ and can be found in Table 4 of Ref. [70], together with amplitude $\alpha$ values.

The first four local-in-matter Lagrangian halo bias parameters are
Figure 3. The first four local-in-matter Eulerian bias parameters of the HI galaxies [Eq. (2.13)] as a function of redshift. The derivation uses the HOD model of Ref. [67], which is briefly described in Sec. 2.3.

\[
b^L_1 = \frac{2\varphi}{\delta_c[(\beta\nu)^{2\varphi} + 1]} + \frac{\gamma\nu^2 - 2\eta - 1}{\delta_c}, \quad (A.9) \\
b^L_2 = \frac{2\varphi (2\gamma\nu^2 - 4\eta + 2\varphi - 1)}{\delta^2_c[(\beta\nu)^{2\varphi} + 1]} + \frac{\gamma^2\nu^4 - 4\gamma\eta\nu^2 - 3\gamma\nu^2 + 4\eta^2 + 2\eta}{\delta^2_c}, \quad (A.10) \\
b^L_3 = \frac{2\varphi (6\varphi (\gamma\nu^2 - 2\eta) + 3(\gamma\nu^2 - 2\eta)^2 - 6\gamma\nu^2 + 4\varphi^2 - 1)}{\delta^3_c[(\beta\nu)^{2\varphi} + 1]} + \frac{\gamma^3\nu^6 - 6\gamma^2\eta\nu^4 - 6\gamma^2\nu^4 + 12\gamma\eta^2\nu^2 + 12\gamma\eta\nu^2 + 3\gamma\nu^2 - 8\eta^3 + 2\eta}{\delta^3_c}, \quad (A.11) \\
b^L_4 = \frac{4\varphi}{\delta^4_c[(\beta\nu)^{2\varphi} + 1]} \left[ 2\gamma^3\nu^6 - 2\eta (6\gamma^2\nu^4 + 6\gamma\nu^2 (2\varphi - 1) + 8\varphi^2 + 6\varphi - 1) + 3\gamma^2\nu^4 (2\varphi - 3) + 12\eta^2 (2\gamma\nu^2 + 2\varphi + 1) + \gamma\nu^2 (8\varphi^2 - 6\varphi + 1) - 16\eta^3 + 4\nu^2 (\varphi + 1) - \varphi - 1) \right] + \frac{\gamma^4\nu^8 - 8\gamma^3\eta\nu^6 - 10\gamma^3\nu^6 + 24\gamma^2\eta^2\nu^4 + 36\gamma^2\eta\nu^4 + 15\gamma^2\nu^4}{\delta^4_c} + \frac{-32\gamma^3\nu^2 - 24\gamma^2\eta\nu^2 - 4\gamma\nu^2 + 16\eta^4 - 16\eta^3 - 4\eta^2 + 4\eta}{\delta^4_c}. \quad (A.12)
\]

From the mapping between Eulerian and Lagrangian bias in the spherical collapse approximation, we get

\footnote{See e.g. Appendix C of Ref. [22] for more details.}
Figure 4. A comparison of the CHIME baseline distribution with the approximating formula of Eq. (B.2) and the uniform distribution, which assumes a constant sampling in the u-v plane (see e.g. Ref. [90]).

\[
\begin{align*}
    b_1^E &= 1 + b_1^L, \\
    b_2^E &= b_2^L + 2(\alpha_1 + \alpha_2)b_1^L, \\
    b_3^E &= 6(\alpha_2 + \alpha_3)b_1^L + 3(1 + 2\alpha_2)b_2^L + b_3^L, \\
    b_4^E &= 24(\alpha_3 + \alpha_4)b_1^L + 12(\alpha_2^2 + 2(\alpha_2 + \alpha_3))b_2^L + 4(1 + 3\alpha_2)b_3^L + b_4^L.
\end{align*}
\]

where \(\alpha_1 = 1\), \(\alpha_2 = -17/21\), \(\alpha_3 = 2815/3969\) and \(\alpha_4 = -590725/916839\).

The resulting Eulerian HI bias parameters, after using the methodology and HOD model described in Sec. 2.3, are plotted in Fig. 3.

B Baseline distribution

Accurate modeling of the effective distribution of baselines is non-trivial for transit telescopes. The sky rotation that takes objects over the sky has the effect that the same piece of sky is measured by the same baseline with different projection and the \(m\)-mode analysis (cite) demonstrates that some information that is not sampled directly can be recovered through time variation. In particular, for experiments like HIRAX, different patches of the sky will be observed with different pointing altitudes and PUMA will also likely to have at least one degree of freedom per dish. We skim over these details and use smoothed version of physical distribution of baseline lengths instead.
The baseline distribution for the PUMA and HIRAX surveys is given by Eq. D4 of [77], given by the following fitting formula

\[ n_b(l) = n_0 \frac{a + b(l/L)}{1 + c(l/L)} e^{-(l/L)^2}, \]  

(B.1)

where \( n_0 = (N_s/D_{\text{dish}})^2 \) and \( L = N_s D_{\text{dish}} \) with \( n_b(u) = n_b(l = u\lambda)\lambda^2 \), while \( N_s \) is the number of antennas in the side of the square array (i.e. \( N_s = 256 \) for PUMA and \( N_s = 32 \) for HIRAX). This formula has been calibrated so that \( \int n_b(u)d^2u = N_s^2/2 \). The fitting parameters for a square closed-packing array, considered in the case of HIRAX, are \( a = 0.4847, b = -0.3300, c = 1.3157, d = 1.5974, e = 6.8390 \). For PUMA, as discussed in the main text, we consider a hexagonal closed-packing array in a compact cycle, where the fitting parameters are now \( a = 0.5698, b = -0.5274, c = 0.8358, d = 1.6635, e = 7.3177 \). For both cases, the fitted parameters can be found in Appendix D of Ref. [77], as well as a detailed discussion.

For CHIME the baseline distribution is given by the following fitting formula

\[ n_b(l) = A \exp \left[-(l/B)^C\right], \]  

(B.2)

where the parameters are, \( A = 48.5511, B = 60.693, C = 2.4797 \). A comparison between the actual baseline length distribution for CHIME and the fitting formula is shown in Fig. 4.

C Redshift space kernels and the Finger-of-God dumping term

The effect of redshift space distortions (RSD) [91, 92] can be treated perturbatively [93, 94], generalising the kernel formalism of SPT in order to include the redshift distortions and the bias terms [Eqs. 2.9 and 2.10]. The general non-Gaussian redshift kernels up to second order, while neglecting \( O(f_{NL}^2) \) terms, are given by (see also Ref. [95]):

\[ Z_1(k_i) = b_1 + f \mu_i^2 + \frac{b_{\Psi}k_i^2}{M(k_i, z)}, \]  

(C.1)

\[ Z_2(k_i, k_j) = b_1 F_2(k_i, k_j) + f \mu_i^2 G_2(k_i, k_j) + \frac{b_2}{2} + b_{\alpha\beta} S_2(k_i, k_j) \]
\[ + \frac{f \mu_{ij} k_{ij}}{2} \left[ \mu_i k_i Z_1(k_j) + \mu_j k_j Z_1(k_i) \right] + \frac{1}{2} \left( \frac{(b_{\Psi\delta} - b_{\Psi}N_2(k_j, k_j))k_i^2}{M(k_i, z)} + \frac{(b_{\Psi\delta} - b_{\Psi}N_2(k_i, k_j))k_j^2}{M(k_j, z)} \right), \]

(C.2)

where \( f \) is the linear growth rate, \( \mu_i = k_i \cdot \hat{z}/k_i \) is the cosine of the angle between the wavevector \( k_i \) and the line-of-sight, \( \mu_{ij} = (\mu_i k_i + \mu_j k_j)/k_{ij} \) and \( k_{ij}^2 = (k_i + k_j)^2 \). The kernels \( F_2(k_i, k_j) \) and \( G_2(k_i, k_j) \) are the second order symmetric kernels of SPT (see Ref. [34] for a review), while \( S_2(k_1, k_2) = (k_1 \cdot k_2)^2/(k_1^2 k_2^2) - 1/3 \) and \( N_2(k_1, k_2) = (k_1 \cdot k_2)(k_1^2 k_2^2) \). The \( S_2 \) kernel arises from the Fourier transform of the tidal field scalar \( s^2 \) [40, 43], while \( N_2 \), in the presence of PNG, originates from the coupling of the displacement field between the Eulerian and Lagrangian frames to the primordial gravitational potential [62, 63]. For the different PNG shapes considered here, parameter \( \alpha \) get the following values: \( \alpha = 2 \) for the equilateral shape, \( \alpha = 1 \) for the orthogonal configuration, while the usual local case is described by \( \alpha = 0 \) [16, 51, 62].
The Finger-of-God (FOG) is taken into account here, where the damping effect of the clustering power is described phenomenologically by [96, 97]

\[ D_{\text{FOG}}^P(k) = e^{-(k \mu \sigma_P)^2}, \]  
\[ D_{\text{FOG}}^B(k_1, k_2, k_3) = e^{-(k_1 \mu_1^2 + k_2 \mu_2^2 + k_3 \mu_3^2) \sigma_B^2}. \]  

(C.3)  
(C.4)

Here we consider the fiducial values for \( \sigma_P = \sigma_B = \sigma_v(z) \), where \( \sigma_v \) is the usual linear, one dimensional velocity dispersion. Note that, due to the high precision of the redshift measurements in 21 cm IM surveys considered here, the redshift error is assumed to be zero, i.e. perfect knowledge of redshift.