Phantom Accretion by Five Dimensional Charged Black Hole

M. Sharif *and G. Abbas †
Department of Mathematics, University of the Punjab,
Quaid-e-Azam Campus, Lahore-54590, Pakistan.

Abstract

This paper deals with the dynamical behavior of phantom field near five dimensional charged black hole. We formulate equations of motion for steady-state spherically symmetric flow of phantom fluids. It is found that phantom energy accretes onto black holes for $u < 0$. Further, the location of critical point of accretion are evaluated that leads to mass to charge ratio for 5D charged black hole. This ratio implies that accretion cannot transform a black hole into a naked singularity. We would like to mention here that this work is an irreducible extension of 4D charged black hole.

Keywords: Five dimensional charged black hole; Phantom energy; Accretion.

PACS numbers: 04.70.Bw; 04.70.Dy; 95.35.+d

1 Introduction

During the last decades, there has been a growing interest to study the gravity in a theory which implies the existence of extra dimension in nature, called brane-world theories. Such theories suggest the solution of hierarchy problem (difference in scales of gravitational and electro-weak interaction) [1]. The

---

* msharif@math.pu.edu.pk
† abbasg91@yahoo.com
brane-world theories are based on the fact that (3+1)-dimensional brane is embedded in a (4+n)-dimensional spacetime with n spacelike compact dimension [2]. All the matter is located on brane and fields propagate in the bulk [3]. There is a possibility that brane-world gravity theory would give an idea to observe the effects of quantum gravity in laboratory at TeV energies. Also, these theories urge that higher dimensional black hole (BH) can be produced in large hadron colliders (LHC) and cosmic ray experiments [4].

The development of higher dimensional theories has increased the interest to study BH in higher dimension. The first static spherically symmetric BH solution in brane-world was formulated by Dadhich et al. [5], which has the same structure as Reissner-Nordsrtöm (RN) 4D BH solution. Its physical interpretation implies that there is a tidal charge due to the fifth dimension. Konoplya and Zhidenko [3] discussed the higher dimensional charged BH instability. Guha et al. [6] examined the geodesic motion in the vicinity of 5D RN anti-de-Sitter BH. Ghosh et al. [7] introduced the idea of gravitational collapse in 5D for the dust case. This was extended by Sharif and his collaborators for perfect fluid [8] and charged perfect fluid [9] collapse in 5D. Matzner and Mezzacappa [10] examined the closed universes in 5D Kaluza-Klien theory. These studies motivate us to explore the problem of phantom accretion in 5D static spherically symmetric charged BH solution which is characterized by mass and electric charge.

It was first confirmed by the data of type Ia Supernova and large scale structure [11]-[14] that our universe is in accelerating phase. Different models [15]-[18] were proposed to understand the nature of DE in our universe. The simplest form of DE is vacuum energy (cosmological constant) for which the equation of state parameter (EoS) is \( \omega = -1 \). The quintessence and phantom are the forms of DE for which \( \omega > -1 \) and \( \omega < -1 \), respectively [19]-[21]. Phantom energy violates the dominant energy condition.

The problem of matter accretion onto the compact objects in Newtonian gravity was first formulated by Bondi [22]. In general relativity, Michel [23] was the pioneer who studied accretion of gas onto the Schwarzschild BH. Sun [24] discussed the phantom accretion onto BH in the cyclic universe. Babichev et al. [25] have shown that BH mass diminishes due to phantom accretion. Jamil et al. [26] have explored the effects of phantom accretion onto the charged BH in 4D. They pointed out that if mass of BH becomes smaller (due to accretion of phantom energy) than its charge, then Cosmic Censorship Hypothesis is violated.

In this paper, we extend this work for phantom accretion by 5D charged
BH. We find that accretion cannot transform a BH into a naked singularity or extremal BH, in contrast to 4D case. The plan of the paper is as follows: In the next section, accretion onto 5D charged BH is presented. We discuss the critical accretion in section 3 and conclude our discussion in the last section. The gravitational units (i.e., the gravitational constant $G = 1$ and speed of light in vacuum $c = 1$) are used. All the Latin and Greek indices vary from 0 to 4, otherwise it will be mentioned.

2 Accretion onto 5D Charged Black Hole

We consider a charged static spherically symmetric $n + 2$ dimensional BH solution \[ds^2 = Z(r)dt^2 - \frac{1}{Z(r)}dr^2 - r^2d\Omega_n,\] where $d\Omega_n$ is the unit $n$ sphere and $Z(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$. For $n = 2$, this reduces to 4D RN metric, while for $n = 3$, we get a 5D charged BH solution given by

\[ds^2 = Z(r)dt^2 - \frac{1}{Z(r)}dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2 + \sin^2 \theta \sin^2 \phi d\psi^2),\] where $Z(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$. Here $M$ and $Q$ are the mass and charge of the BH.

The black hole horizons can be found by solving $Z(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \equiv 0$, for $r$ whose positive real roots will give horizons as follows

\[r_{outer} = \sqrt{M + \sqrt{M^2 - Q^2}}, \quad r_{inner} = \sqrt{M - \sqrt{M^2 - Q^2}}.\] For $M^2 > Q^2$, $r_{outer} > r_{inner}$, for $M^2 = Q^2$, $r_{outer} = r_{inner} \equiv m$ (an extreme charged BH) and for $M^2 < Q^2$, both horizons disappear and singularity becomes naked at $r = 0$. For $Q = 0$, $r_{outer} = 2m$ (Schwarzschild horizon in 4D) and $r_{inner} = 0$. This implies that like 4D case, the existence of charge is necessary for the existence of inner horizon (Cauchy horizon). The regularity of the 5D charged BH can be seen in the regions $r_{outer} < r < \infty$, $r_{inner} < r < r_{outer}$ and $0 < r < r_{inner}$.

The energy-momentum tensor for phantom energy is

\[T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu},\]
where $\rho$ is the energy density, $p$ is the pressure and $u^\mu = (u^t, u^r, 0, 0)$ is the five-vector velocity. It is mentioned here that $u^\mu$ satisfies the normalization condition, i.e., $u^\mu u_\mu = -1$. The conservation of energy-momentum tensor yields

$$r^2 u M^{-2} (\rho + p) \left( Z(r) + u^2 \right)^{\frac{1}{2}} = C_0,$$  \hspace{1cm} (2.5)

where $C_0$ is an integration constant and $u^r = u < 0$ for inward flow.

The energy flux equation can be derived by projecting the energy-momentum conservation law on the five-velocity, i.e., $u_\mu T^{\mu\nu} = 0$ for which Eq. (2.4) leads to

$$r^2 u M^{-2} \exp \left[ \int_{\rho_{\infty}}^{\rho_h} \frac{d\rho'}{\rho' + p(\rho')} \right] = C_1,$$  \hspace{1cm} (2.6)

where $C_1 > 0$ is another integration constant which is related to the energy flux. Also, $\rho_h$ and $\rho_{\infty}$ are densities of the phantom energy at horizon and at infinity. From Eqs. (2.5) and (2.6), it follows that

$$(\rho + p) \left( Z(r) + u^2 \right)^{\frac{1}{2}} \exp \left[ - \int_{\rho_{\infty}}^{\rho} \frac{d\rho'}{\rho' + p(\rho')} \right] = C_2,$$  \hspace{1cm} (2.7)

where $C_2 = -\frac{C_0}{C_1} = \rho_{\infty} + p(\rho_{\infty})$.

The rate of change of BH mass due to fluid accretion onto it is

$$\dot{M} = -4\pi r^2 T^r_t.$$  \hspace{1cm} (2.8)

Using Eqs. (2.5)-(2.7) in the above equation, it follows that

$$\dot{M} = 4\pi M^2 C_1 (\rho_{\infty} + p_{\infty}).$$  \hspace{1cm} (2.9)

We note that mass of BH decreases if $(\rho_{\infty} + p_{\infty}) < 0$. Thus the accretion of phantom energy onto a BH causes to decrease the mass of BH. As the phantom accretion only diminishes mass and does not affect the charge of BH, so we can speculate that when $M^2 < Q^2$ is reached, then singularity becomes naked at $r = 0$ and the phantom accretion by 5D charged BH may lead to the violation of Cosmic Censorship Hypothesis. However, critical accretion process mentioned below implies that Cosmic Censorship Hypothesis remains valid in this case. It is mentioned here that one can solve Eq. (2.9) for $M$ by using EoS $p = k\rho$. Since all $p$ and $\rho$, violating dominant energy condition, must satisfy this equation, hence it holds in general. i.e.,

$$\dot{M} = 4\pi M^2 C_1 (\rho + p).$$  \hspace{1cm} (2.10)
3 Critical Accretion

Here, we locate such points at which flow speed is equal to the speed of sound during accretion. The fluid falls onto the BH with monotonically increasing velocity along the particle trajectories. We follow the procedure introduced by Michel [23]. The conservation of mass flux, \( J^{\mu} \mid_{\mu = 0} \), gives

\[
\rho u r^2 = k, \tag{3.1}
\]

where \( k \) is an integration constant. From Eqs.(2.5) and (2.9), we get

\[
\left( \frac{\rho + p}{\rho} \right)^2 (Z(r) + u^2) = k_1, \tag{3.2}
\]

where \( k_1 = \left( \frac{C_0}{k} \right)^2 \) is a positive constant. Differentiating Eqs.(3.1) and (3.2) and eliminating \( d\rho \), we get

\[
\frac{dr}{r} \left[ 2V^2 - \frac{Mr}{Z(r) + u^2} \right] + \frac{du}{u} \left[ V^2 - \frac{u^2}{Z(r) + u^2} \right] = 0, \tag{3.3}
\]

where \( V^2 = \frac{d\ln(\rho + p)}{d\ln\rho} \). This shows that critical points are found by taking both the factors inside the square brackets equal to zero. Thus we obtain

\[
\rho^2 = \frac{Mr^2 - Q^2}{r^4}, \quad V^2 = \frac{Mr^2 - Q^2}{r^4 - Mr^2}, \tag{3.4}
\]

We see that physically acceptable solutions of the above equations are obtained if \( u^* > 0 \) and \( V^* > 0 \) implying that

\[
Mr^2 - Q^2 > 0, \quad r^4 - Mr^2 > 0. \tag{3.5}
\]

The subscript * is used to represent a quantity at a point where speed of flow is equal to the speed of sound, such a point is called a critical point. It is mentioned here that in case of 4D charged BH hole the equations corresponding to Eq.(3.5) are linear and quadratic in \( r \).

The solution of the 2nd equation of Eq.(3.5) is

\[
r^*_\pm > \sqrt{M}. \tag{3.6}
\]
For the solution about critical point, we insert the value of $r_{*+}$ in the first equation of Eq. (3.5) and obtain

$$1 < \frac{M^2}{Q^2}. \quad (3.7)$$

Thus accretion through $r_{*+}$ is possible if the above mass to charge ratio is satisfied. Since horizons always exist for this mass to charge ratio, so in contrast to 4D charged BH there are no possibilities of extremal BH and naked singularity formation during the accretion process.

4 Outlook

In this paper, we have analyzed the phantom accretion by 5D charged BH. We have formulated equations of motion for steady state spherically symmetric phantom flow near 5D charged BH. It has been assumed that infalling fluid does not disturb the generic properties of the BH. Following the procedure introduced by Michel [23], we discuss the accretion and critical accretion by BH. Like the cases of 4D Schwarzschild and RN, phantom accretion decreases the mass of BH.

Two (event and Cauchy) horizons for 5D charged BHs can exist only if $M^2 \geq Q^2$ otherwise there will be a naked singularity. We have found that the existence of Cauchy horizon requires $Q \neq 0$. If we take $Q = 0$ then there exists a unique horizon which is at $r = 2m$ (4D Schwarzschild radius). The critical accretion analysis implies that corresponding to two horizons there exists a positive value of $r_*$ (i.e., $r_{*+} > \sqrt{m}$). This can play the role of physically possible critical point if the mass and charge of 5D BH satisfies $1 < \frac{M^2}{Q^2}$. In contrast to 4D charged BH case, this ratio is free of upper bound. Further, this ratio implies that $M^2 > Q^2$, which is essential inequality for the existence of horizons. It is concluded that although phantom accretion decreases the mass of BH, but it cannot be converted to $M^2 \leq Q^2$. Hence throughout the accretion process, a 5D charged BH cannot be transformed to an extremal charged BH or a naked singularity and Cosmic Censorship remains valid in this case.
Acknowledgment

We would like to thank the Higher Education Commission, Islamabad, Pakistan for its financial support through the Indigenous Ph.D. 5000 Fellowship Program Batch-IV.

References

[1] Arkani-Hamed, N., Dimopoulous, S. and Dvali, G.R.: Phys. Lett. B429(1998)263.
[2] Randall, L., Sundrum, R.: Phys. Rev. Lett. 83(1999)3370.
[3] Konoplya, R.A. and Zhidenko, A.: Phys. Rev. Lett. 103(2009)161101.
[4] Emparan, R., Horowitz, G.T. and Myers, R.C.: Phys. Rev. Lett. 85(2000)499.
[5] Dadhich, N., et al.: Phys. Lett. B487(2000)1.
[6] Guha, S., Bhattacharya, P. and Chakraborty, S.: arXiv(1008.2650).
[7] Ghosh, S.G., Deshkar, D.W. and Saste, N.N. G.: Int. J. Mod. Phys. D16(2007)53.
[8] Sharif, M. and Ahmad, Z.: J. Korean Phys. Society 52(2008)980.
[9] Sharif, M. and Abbas, G.: J. Korean Phys. Society 56(2010)529.
[10] Matzner, R.A. and Mezzacappa, A.: Phys. Rev. D32(1985)3114.
[11] Perlmutter, S. et al.: Astrophys. J. 483(1997)565.
[12] Perlmutter, S. et al.: Nature 391(1998)51.
[13] Perlmutter, S. et al.: Astrophys. J. 517(1999)565.
[14] Riess, A.G. et al.: Astron. J. 116(1998)1009.
[15] Sahni, V. and Starobinsky, A.A.: Int. J. Mod. Phys. D9(2000)373.
[16] Caldwell, R.R.: Phys. Lett. B23(2002)545.
[17] Li, M.: Phys. Lett. B603(2004)1.

[18] Wang, B., Gong, Y.G. and Abdalla, E.: Phys. Lett. B624(2005)141.

[19] Frieman, J.A. et al.: Phys. Rev. Lett. 75(1995)2077.

[20] Coble, K., Dodelson, S. and Frieman, J.A.: Phys. Rev. D55(1997)1851.

[21] Hinshaw, G. et al.: Astrophys. J. Suppl. 180(2009)225.

[22] Bondi, H.: Mon. Not. Roy. Astron. Soc. 112(1952)195.

[23] Michel, F.C.: Astrophys. Space Sci. 15(1972)153.

[24] Sun, C.Y.: Phys. Rev. D87(2008)064060.

[25] Babichev, E., Dokuchaev, V. and Eroshenko, Y.: Phys. Rev. Lett. 93(2004)021102.

[26] Jamil, M., Rashid, M. and Qadir, A.: Eur. Phys. J. C58(2008)325.

[27] Babichev, E., Chernov, S., Dokuchaev, V., Eroshenko, Y.: J. Exp. Theor. Phys. 112(2011)784.