Tensile strength test and fracture toughness test of cores with axial hole

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Abstract. This studies approves the tensile stress tests of rock specimens with axial holes of different diameters under loading along the diameter. The test results are presented for core specimens of rocks and some equivalent materials. The test data are processed using the criterion of fracture gradient and the integral criterion of fracture. The values of tensile strength and fracture toughness obtained using this approach and by the standard technique are compared.

1. Introduction
Laboratory testing is an integral part of rock studies. Specimens manufactured from different rock samples are tested under various stresses in order to obtain the required information on the mechanical properties of rocks. Tensile strength of rocks is an important characteristic, for rock mass under total compression stress experience nonuniform compression around underground mine openings and, for this reason can incur some local tension. Prediction and monitoring of the behavior of mine openings lay emphasis on such areas of local tension.

The standard uniaxial tension tests of rocks require creation of a uniform field of stresses. The manufacture and fixation of specimens is highly labor intensive therefore. Thus, the direct stress measurement is inapplicable in case of rocks and is replaced by indirect measurement. The common indirect test method is fracture of cylindrical specimens along their diameter. This method called the Brazilian Test was for the first time proposes by Brazilian engineer and inventor L.L.B. Carneiro in 1947 [1]. The method is based on elastic solution of a problem on compression of core along opposite generatrices by concentrated forces. Regarding rocks, the maximal tensile stresses arisen in the center of a core, along the line of the force application, are closely equal to uniaxial rupture strength [2-4] while the field in the center of the core is biaxial. Starting from the early days of invention, many attempts were made to improve the Brazilian Test method [5, 6] since if the core fracture started under the loading force plungers, such cores are withdrawn from the analysis.

All efforts toward improvement the Brazilian test efficiency and the test data interpretation suggested researchers to set upon testing of cores with holes. The tests of ring-shaped specimens are described in [7, 8]. In this case, the breaking force is considerably reduced, and the fracture is ensured in the center of the specimens. The reduction in the breaking force due to the introduction of the stress raiser allows using this method with soft and plastic rocks such as gypsum and limestone. The values of the fracture stress obtained from the elastic solution using the criterion of local-stress fracture at critical points are much higher than the uniaxial tension strength measurements. The authors of the above-mentioned papers, while emphasizing advantages of their modification over the standard Brazilian Test, failed to explain high...
values of the calculated strength of rocks at critical points of stress concentrations obtained from the elastic solution and the local-stress fracture criterion. Stress concentration at critical points on the boundary of the hole induces essentially nonuniform stress field, and the local-stress fracture criteria which neglect the structure of a test media become inapplicable.

The nonlocal-stress fracture criteria [10–16] enabled correct calculation of breaking stresses. The comparison of the artificial medium strengths obtained in the uniform and nonuniform stress tests proved the validity of this approach. The present paper author tried to determine tensile strength and fracture toughness of rocks in the tests of cores with holes of different diameter and using the criterion of fracture gradient [17]. That approach offered closer equal values of the calculated and measured uniaxial tension strength for sealing wax and gypsum. In later research of the author [18–20], the approach of two criteria of nonlocal fracture was used in the tensile strength estimation. This study describes strength assessment in rocks using the integral and the gradient fracture criteria and their comparison.

2. Testing procedure
The Brazilian Test was carried out on the samples of Ufalean marble, dolerite, granite and gabbro-diorite, with and without central hole. The specimens made of coarse-grained marble, marmorized limestone the model equivalent materials of gypsum, sealing wax and organic glass subjected to fracture along generatrices had their central holes of different sizes though not more than 20% of the specimen disc diameter. The tensile strength was determined in the Brazilian Tests in case of all these materials. The tensile strength of organic glass was determined also in direct tension. Aimed to minimize scatter of the mechanical characteristics, all specimens were manufactured in the same conditions, were cured for a long time and fractured along a certain preset direction. The test strength and fracture toughness of all materials are compiled in Table 1. In the specimens with hole, the strength $\sigma_{ci}$ is determined in the gross cross-sectional area, i.e. $\sigma_c = F / (\pi R t)$, where $F$ is the breaking force; $R$, $t$ are the radius and thickness of a specimen, respectively. The critical stress intensity factors $K_{ic}$ for obtained for the test rocks using the method from [21].

| Rock or equivalent material | $D$, mm | $a_1$, mm | $\sigma_{c1}$, MPa | $\sigma_{c2}$, MPa | $\sigma_{c3}$, MPa | $k_i$ | $\sigma_0$, MPa | $K_{ic}$, MPa m$^{-1/2}$ | $\delta$, mm |
|---------------------------|--------|-----------|-----------------|-----------------|-----------------|------|----------------|-----------------|------------|
| Dolerite                  | 37.6   | 1.7       | 16.9            | —               | —               | 6.05 | 25             | 2.06            | 4.35       |
| Gabbro-diorite            | 37.6   | 3.25      | 7.8             | —               | —               | 6.96 | 13.4           | 1.25            | 5.5        |
| Granite                   | 37.6   | 3.25      | 7.0             | —               | —               | 6.96 | 11.2           | 1.14            | 6.6        |
| Ufalean marble            | 37.6   | 3.25      | 5.11            | —               | —               | 6.96 | 6.9            | 0.9             | 10.8       |
| Coarse-grained marble     | 37.6   | 2         | 4.7             | 3               | 4.05            | 6.13 | 5.93           | 0.86            | 13.4       |
| Marmorized limestone      | 57     | 3         | 3.11            | 5               | 2.46            | 6.05 | 5.58           | —               | —         |
| Artificial gypsum         | 40     | 2         | 1.27            | 4               | 0.9             | 6.13 | 2.5            | 0.2             | 4.0        |
| Sealing wax               | 40     | 2         | 1.13            | 3               | 0.94            | 4    | 0.83           | 6.13            | 2.2        |
| Organic glass             | 40     | 1         | 19.8            | 5               | 15.7            | 4    | 14.3           | 6.0             | 1.4        |

Comments: $\sigma_0$—uniform tensile strength; $K_{ic}$—critical stress intensity factor; $D$—core diameter; $k_i$—stress concentration factor; $\delta$—structural parameter; averaged strengths $\sigma_{c1}$, $\sigma_{c2}$, $\sigma_{c3}$ are for specimens with central hole having radius $a_1$, $a_2$, $a_3$, respectively.

Figure 1a shows a picture of a fractured dolerite core, and Figure 1b depicts the tensile stress field nearby the critical points $A$ and the structural parameter $\delta$ typical of rocks. The tests were carried out
under room temperature on testing machine UME-10TM at the cross-beam advance speed of 0.5 mm/min, which agreed approximately with the loading rate of 1 MPa/s.

Figure 1. Brazilian Strength Tests of a disc core with central hole.

3. Nonlocal fracture criterion in calculation of breaking stress in compression of discs with central hole

We calculate breaking stresses in specimens using the tensile stress distribution along the axis $y$ in the direction of the incipient crack at the critical point $A$ from [17]:

$$\sigma_x = \frac{\sigma}{2} \left[ 2 - \frac{2a^2}{r^2} + \frac{12a^4}{r^4} \right], \quad (1)$$

where the stress $\sigma = F / (\pi R t)$ at the time of fracture reaches the maximum value $\sigma_c$.

The criterion of fracture gradient [13]:

$$\sigma_c^{gr} = \sigma_0 + \frac{1}{k_t} \sqrt{\frac{\delta}{L_c}} \quad (2)$$

where $L_c$ is the stress nonuniformity characteristic: $L_c = \frac{\text{grad} \sigma_x}{\sigma_x} \bigg|_{r=a}^{-1} = \frac{3}{11} a$.

The integral criterion by Novozhilov [10]:

$$\sigma_c^{int} = \sigma_0 \left( 1 + \frac{5a^3 + 4a^2\delta + a\delta^2}{(a + \delta)^3} \right)^{-1}, \quad (3)$$

where $\delta$ is the averaging areas (structural parameter) found from the ratio [14]:

$$\delta = \frac{2}{\pi} \left( \frac{K_{ic}}{\sigma_0} \right)^2. \quad (4)$$

The ratios of the calculated and measured stresses in specimens with central hole are presented in Figure 2. The calculated values using the gradient fracture criterion (2) are marked by black circles,
and the values from the integral fracture criterion (3) are marked by hollow squares. The values from the conventional local fracture criterion are marked by black triangles and are found from the formula: 

\[ \sigma_{e,local} = \frac{\sigma_0}{k_1(a)} \]

where \( k_1(a) \) is the stress concentration factor. The values of the tensile strength and fracture toughness were determined using standard techniques. It is seen in Figure 2 that the breaking stresses from the nonlocal fracture models offer a more appropriate description of the fracture process as compared with the conventional local fracture approach. Furthermore, the best result is obtained from the integral criterion of Novozhilov.

![Figure 2](image)

**Figure 2.** Ration of calculated stresses \( \sigma^*_c \) and measured stresses \( \sigma_c \) in fracture of specimens with hole: ●—criterion of fracture gradient; □—integral criterion of fracture, ▲—local fracture criterion.

4. Calculation of tensile strength and fracture toughness using the test results
The sensitivity of fracture properties to nonuniformity of stress field, which is included in the nonlocal approach, allows determining mechanical properties of test materials (tensile strength and fracture toughness) in two series of tests with specimens of different geometry. Thus, it is sufficient to test specimens with holes having two different diameters to determine the tensile strength and fracture toughness of materials. Formula (1) of stress pattern around a hole is effective in the domain \( a/R < 0.2 \), and the diameter of the hole should be selected carefully therefore.

The tests of coarse-grained marble marmorized limestone, artificial gypsum, sealing wax and organic glass were carried out on specimens with holes of two and more diameters. The testing data processing procedure is as follows. The solution of a system composed of two equations for specimens with different-diameter holes using the gradient fracture criterion immediately produces the unknown values of \( \sigma_0 \) and \( K_{1c} \):

\[ \sigma_0 = k_{11}\sigma_{e1} - \sqrt{2/(\pi L_{c1})}K_{1c} \quad \text{and} \quad \sigma_0 = k_{12}\sigma_{e2} - \sqrt{2/(\pi L_{c2})}K_{1c}. \]  

The same solution with the integral criterion of fracture:

\[ \sigma_0 = \sigma_{e1}\left(1 + \frac{5a_1^3 + 4a_1^2 \delta + a_1 \delta^2}{(a_1 + \delta)^3}\right) \quad \text{and} \quad \sigma_0 = \sigma_{e2}\left(1 + \frac{5a_2^3 + 4a_2^2 \delta + a_2 \delta^2}{(a_2 + \delta)^3}\right) \]

makes it possible to determine the structural parameter \( \delta \). The scatter of the experimental data causes a difficulty which can be turned around in the following manner. The parameter \( \delta \) is found from the condition \( d(\sigma^*_0 - \sigma^*_{02})/d\delta = 0 \). The calculated value of \( \sigma^*_0 \) is determined as an arithmetic average \( \sigma^*_{0i} \) for the found \( \delta \). Then, the critical stress intensity factor \( K_{1c} \) is estimated. The calculated values of the tensile strength and fracture toughness are given in Table 2. Figure 3 presents the ratios of the calculated and standard-way measured characteristics of strength.
Table 2. Calculation values of tensile strength and critical stress intensity factor for test materials

| Rock or material         | $\sigma_0$, MPa | $K_{IC}$, MPa m$^{1/2}$ | $\delta_0$, mm | $\sigma_{0}^{\text{int}}$, MPa | $K_{IC}^{\text{int}}$, MPa m$^{1/2}$ | $\delta_{0}^{\text{int}}$, mm | $\sigma_{0}^{\text{gr}}$, MPa | $K_{IC}^{\text{gr}}$, MPa m$^{1/2}$ | $\delta_{0}^{\text{gr}}$, mm |
|--------------------------|-----------------|-------------------------|----------------|-----------------------|-------------------------------|-------------------------------|-------------------|-------------------------------|-------------------|
| Coarse-grained marble   | 5.93            | 0.86                    | 13.0           | 5.94                  | 0.71                          | 9                             | 17.7              | 0.65                          | 0.85              |
| Marmorized limestone     | 5.58            | --                      | --             | 5.06                  | 0.49                          | 5                             | 4.7               | 0.95                          | 25.7              |
| Artificial gypsum        | 2.5             | 0.2                     | 4.0            | 2.35                  | 0.16                          | 3                             | 3.9               | 0.22                          | 2.05              |
| Sealing wax              | 2.2             | 0.16                    | 3.4            | 2.14                  | 0.144                         | 2.88                          | 3                 | 0.22                          | 3.4               |
| Organic glass            | 75              | 1.4                     | 0.22           | 77                    | 1.67                          | 0.3                           | 83                | 1.46                          | 0.2               |

Figure 3. Ratios of calculated (with asterisk) and measured characteristics of strength.

The ratio of the calculated and standard-way measured tensile strength and critical stress intensity factor are closer concentrated at 1 when the integral criterion of fracture is used than in case of the gradient fracture criterion (Figure 3). It can be concluded that the proposed method of simultaneous assessment of tensile strength and fracture toughness using the integral criterion of fracture in terms of the test materials allows the estimation accuracy of the order of 20% in a wide range of change of the structural parameter $\delta$.

5. Conclusions
The use of the standard local fracture criterion in the nonuniform stress tests of solid materials, including rocks results in errors, especially in case of stress concentration. The calculation procedure of tensile strength and fracture toughness of rocks with testing of disc specimens of axial hole of different diameter using the nonlocal fracture approach has been approved for some solid materials. The use of the integral criterion of fracture has shown considerable advantages of this criterion as against the criterion of fracture gradient. The use of the integral criterion of fracture has allowed calculation of the required parameters to an accuracy not less than 20%.

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