One-to-One Correspondence of Soft and Hard Pomeron with the Color Dipole Picture of the Gluon Density at Low $x$

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Received December 1, 2022; revised December 21, 2022; accepted January 3, 2023

The correspondence between the gluon density behavior of the color dipole picture (CDP) and the two-Pomeron approach at low $x$ deep inelastic scattering is considered. For photon virtualities of $Q^2 \geq 10 \text{ GeV}^2$, the results for the parametrization and CDP models are defined by the CDP asymptotic limit and are compatible with the soft and hard-Pomeron approach. These results show that the hard-Pomeron trajectory does not guarantee converging towards the asymptotic representation at low and large $Q^2 (Q^2 < 100 \text{ GeV}^2)$ values in a wide range of the virtual-photon–proton energy squared. The gluon distributions can be obtained directly in terms of the proton structure functions and the running coupling and compared with the results from the M. Gluck, P. Jimenez-Delgado and E. Reya (GJR) and A. Martin, W. Stirling, R. Thorne, and G. Watt (MSTW) parametrizations.

DOI: 10.1134/S0021364023600015

1. INTRODUCTION

Sakurai and Schildknecht in 1972 [1, 2] starting points on the color dipole picture (CDP) which provides a convenient description of deep inelastic scattering (DIS) at low $x$ [3–15]. In this picture, the DIS cross section is factorized into a light-cone wave function, as the virtual photon fluctuates into the $q\bar{q}$ pair. This split is defined as a convolution of the infinite momentum frame wave function with the perturbative quantum chromodynamics (pQCD) calculable coefficient functions. Then the $q\bar{q}$ pair interacts with the gluon field in the nucleon as a gauge-invariant color-dipole interaction. In this interaction the $q\bar{q}$ lifetime is about $1/x$ times longer than time of interaction photon with proton [16, 17]. At low values of the Bjorken variable $x \equiv x_B \simeq Q^2/W^2$, DIS of electrons on protons defined by the processes splitting of the photon into on-shell quark–antiquark states and interaction these states on the proton. Here, $Q^2$ refers to the photon virtuality and $W$ to the proton–proton center-of-mass energy. In the transverse position space, the photoabsorption cross section at low values of $x$ is defined by the square of the photon wave function and the dipole cross section as [3, 4, 10–15, 18]

$$
\sigma_{LT}^{\gamma p}(W^2, Q^2) = \int dz \int d^2 r_{\perp} |\Psi_{LT}(r_{\perp}, z(1-z), Q^2)|^2 \times \sigma_{q\bar{q}p}(r_{\perp}, z(1-z), W^2). \tag{1}
$$

The variable $z$ determines the direction of the three-momentum of the quark due to the photon direction and $|\Psi_{LT}|^2$ describes the probability of the occurrence of a quark–antiquark fluctuation with respect to the polarization direction. The dipole cross section, related to the imaginary part of the $(q\bar{q})p$ forward scattering amplitude. The dipole cross section depends on the center-of-mass energy, $W$, of the $(q\bar{q})p$ scattering process, as this implies that the structure function reads

$$
F_2(x, Q^2) \simeq \frac{Q^2}{4\pi \alpha_{EM}[\sigma_{\gamma p}(W^2, Q^2) + \sigma_{\gamma p}(W^2, Q^2)]}. \tag{2}
$$

Indeed, the structure function becomes a function of the single variable $W^2$. In [10–15, 18] we observe that the experimental data corresponding to $1/W^2 \leq 10^{-3}$ lie on a single line in the relevant range of $x < 0.1$.

A new parameterization of the proton structure function which describes fairly well the available experimental data on the reduced cross section at low values of $x$ described in [19, 20]. In [19] authors use an updated version of the global parameterization of the ZEUS data [21, 22] for the proton structure function made by authors in [23]. Authors modified the fit in two important aspects of the sieve algorithm [24] which minimizes the squared Lorentzian $\chi^2_{\text{min}}$ per degree of freedom whose contribution was 193.19. Analytical expression successfully reproduces the known experimental data for $F_2(x, Q^2)$ in a
wide range of $Q^2$ values, $0.11 \text{ GeV}^2 \leq Q^2 \leq 1200 \text{ GeV}^2$, and Bjorken $x$ range, $10^{-6} \leq x \leq 0.0494$. Then the authors in [20] completed the parameterization method to fit all of the HERA DIS data on $F_2(x, Q^2)$ at low values of $x$ that satisfies a saturated Froissart bound behavior. The parameterization of the proton structure function [20] is in full accordance with the Froissart predictions on the available experimental data in a range of the kinematical variables $x$ and $Q^2$, $x \leq 0.1$ and $0.15 \text{ GeV}^2 < Q^2 < 3000 \text{ GeV}^2$.

This paper is organized as follows. In the next section, the theoretical formalism is presented, including the parameterization of $F_2(x, Q^2)$ and the color dipole picture. In Section 3, we present a detailed numerical analysis and our main results for the CDP, parameterization and Regge models. In the last section we summarize our main conclusions and remarks.

2. THEORETICAL FORMALISM

In view of low values of $x \equiv Q^2/W^2 < 0.1$, the transverse $F_2(x, Q^2)$ and longitudinal $F_L(x, Q^2)$ structure functions are expressed via the gluon distribution $xg(x, Q^2)$ ($k = 2, L$) as the approximation relation reads

$$F_k(x, Q^2) \approx \langle e^k \rangle C_{k,g}(x, Q^2) \otimes xg(x, Q^2).$$

The symbol $\otimes$ denotes a convolution according to the usual prescription, $f(x) \otimes g(x) = \int \frac{dy}{y} f(y)g\left(\frac{x}{y}\right)$. Here, $\langle e^k \rangle$ is the average of the charge $e^k$ for the active quark flavors, $\langle e^k \rangle = n_f^k \sum_{i=1}^{n_f} e_i^k$ with $n_f$ as the number of considered flavors and $C_{k,g}$ are the common Wilson coefficient functions [25]. At leading order (LO) approximation, the longitudinal structure function becomes proportional to the gluon density at a rescaled value $x/\xi_L$ [26–29] as

$$F_L(\xi_L x, Q^2) = \frac{\alpha_s(Q^2)}{3\pi} \sum e_i^2 xg(x, Q^2),$$

where the rescaling factor in the above equation has the preferred values of $\xi_L \cong 0.40$ and $\alpha_s(Q^2)$ is the running coupling at the leading order ($n = 0$) and the next-to-leading order (NLO, $n = 1$) approximations by the following form, respectively

$$\alpha_s(Q^2) = \frac{1}{bt} \left[1 - n b' \ln t\right],$$

with $b = \frac{33 - 2n_f}{12\pi}$ and $b' = \frac{153 - 19n_f}{2\pi(33 - 2n_f)}$, where $t = \ln \left(\frac{Q^2}{\Lambda_{QCD}}\right)$. The running of the coupling constant $\alpha_s$ is determined by the renormalization group equation

$$\frac{\partial \alpha_s}{\partial \ln Q^2} = \beta(\alpha_s),$$

where $\beta(0) = 4\pi b$ and $\beta(1) = 16\pi^2 b'$. In [36], the author has discussed that the running coupling of a generic field theory can be described through a separable differential equation involving the corresponding $\beta$-function. Only the first loop order can be solved analytically in terms of well-known functions. For further loop orders the running coupling leads to transcendental equations, where by applying an optimal Padé approximant on the $\beta$-function, it leads to generalizations of Lambert’s equation. Its solution is presented in terms of a power series, as at low $Q^2$ values, the appropriate NLO transcendental equation for $\alpha_s$ should be used.

The evolution equation of the singlet distribution function at LO analysis at low $x$ is given by

$$\frac{\partial \Sigma(x, Q^2)}{\partial \ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi} P_{\Sigma} (\alpha_s, x) \otimes xg(x, Q^2),$$

where $P_{\Sigma} (\alpha_s, x)$ is the quark–gluon splitting function. $\Sigma$ is the singlet density and for $n_f = 4$ reads

$$F_S(x, Q^2) = \frac{1}{4} \sum e_i^2 x\Sigma(x, Q^2) = \frac{5}{18} x\Sigma(x, Q^2).$$

A similar relation for the derivative of $F_2(x, Q^2)$ with respect to $\ln Q^2$ at low $x$ is determined by the authors in [37–43] as

$$\frac{\partial F_2(\xi_L x, Q^2)}{\partial \ln Q^2} = \frac{\alpha_s(Q^2)}{3\pi} \sum e_i^2 xg(x, Q^2),$$

where the rescaling factor for $F_2(x, Q^2)$ has the preferred values of $\xi_L \cong 0.50$. Combining Eqs. (4) and (8), one can calculate the longitudinal structure function by the derivative of the structure function at a rescaled value $\eta x$ as

$$F_L(x, Q^2) = \frac{\partial F_2(\eta x, Q^2)}{\partial \ln Q^2},$$
where \( \eta = \frac{q_{\perp}}{x} \approx 1.25 \). In CDP, the ratio of structure functions is dependent on the kinematic variable \( \rho \equiv \rho(x,Q^2) \) by the following form
\[
\frac{F_2(x,Q^2)}{F_2(x,Q')} = \frac{1}{1 + 2\rho}. \tag{10}
\]
The parameter \( \rho \) is associated with the enhanced transverse size of the fluctuations originating from the difference in the photon wave functions as
\[
\frac{\sigma_{\alpha}(W^2,Q^2)}{\sigma_{\gamma}(W^2,Q^2)} = \frac{1}{2\rho}. \tag{11}
\]
With imposing consistency between the CDP and the pQCD, the authors in [10–15, 18, 44] obtained the gluon distribution function by expressing the longitudinal structure function in terms of \( F_2(x,Q^2) \) as [18]
\[
\alpha_s(Q^2) x g(x,Q^2) = \frac{3\pi}{\sum_{q} s_q} \frac{1}{2\rho + 1} F_2(x,Q^2). \tag{12}
\]
The above equation (i.e., Eq. (12)) can be used to study the behavior of \( \alpha_s(Q^2) x g(x,Q^2) \) due to the proton structure function.

**Color dipole model.** In CDP [10–15, 18, 44], at sufficiently large \( Q^2 \) for \( x < 0.1 \), the structure function depends on the single variable \( W^2 \)
\[
F_2(x,Q^2) = F_2 \left( \frac{Q^2}{x} \right), \tag{13}
\]
which is consistent with the experimental data with an eye-ball fit by the form
\[
F_2(W^2) = f_2 \left( \frac{W^2}{1 \text{ GeV}^2} \right)^{C_2}, \tag{14}
\]
with \( f_2 = 0.063 \) and \( C_2 = 0.29 \). In this picture, the known expression for \( \alpha_s(Q^2) x g(x,Q^2) \) (with respect to Eqs. (12) and (14)) reads as follows
\[
\alpha_s(Q^2) x g(x,Q^2) = \frac{3\pi}{(2\rho + 1)^2} \frac{f_2}{s_{q}} \left( \frac{W^2}{1 \text{ GeV}^2} \right)^{C_2}, \tag{15}
\]
where \( \rho = 4/3 \) and \( \sum_{q} s_q = 10/9 \) for four active flavors.

**Parametrization model.** The proton structure function is parametrized with a global fit function [20] with a combined fit to the H1 and ZEUS Collaboration data for \( F_2(x,Q^2) \) at 0.15 GeV\(^2\) < \( Q^2 \) < 3000 GeV\(^2\) and \( x < 0.01 \) takes the form
\[
F_2(x,Q^2) = D(Q^2)(1 - x)^n \sum_{m=0}^{n} A_m(Q^2)L^m(x,Q^2), \tag{16}
\]
where
\[
D(Q^2) = \frac{Q^2 + \lambda M^2}{Q^2 + M^2},
\]
\[
A_0(Q^2) = a_{00} + a_{01}L_2(Q^2),
\]
\[
A_i(Q^2) = \sum_{k=0}^{2} a_{i,k}L_2(Q^2)^k, \quad i = (1,2),
\]
\[
L_2(Q^2) = \ln \frac{Q^2 + \mu^2}{\mu^2},
\]
and the effective parameters are defined in Table 1. Note that the results of using the parametrization method can be found recently in [45–47]. In this picture, the known expression for \( \alpha_s(Q^2) x g(x,Q^2) \) (with respect to Eqs. (12) and (16)) reads as follows
\[
\alpha_s(Q^2) x g(x,Q^2) = \frac{3\pi}{(2\rho + 1)^2} \sum_{m=0}^{n} A_m(Q^2)L^m(1 - \xi_{L,x})^n
\]
\[
\times \sum_{m=0}^{n} A_m(Q^2)L^m(\xi_{L,x},Q^2).
\]

**Two-Pomeron model.** HERA has shown that the \( \sigma^{\gamma p} \) rapid rise with \( W^2 \) as an effective power
\[
\sigma^{\gamma p} \sim F(Q^2)(W^2)^{\lambda(Q^2)}, \tag{19}
\]
where the power \( \lambda(Q^2) \) has been extracted by H1 Collaboration in [48]. The effective-power behavior of the proton structure function at low \( x \) corresponds to
\[
F_2(x,Q^2) \sim f(Q^2)x^{-\lambda(Q^2)}. \tag{20}
\]
Rather, the authors in [49–52] show that one should parametrize the data with a sum of fixed powers of \( x \) in the two-Pomeron model as

\[
F_2(x, Q^2) \sim \sum_{i=0} \frac{A_i(Q^2)}{(1 + Q^2 / Q_i^2)^{e_i+e_0/2}},
\]

where the \( i = 0 \) term is hard-Pomeron (HP) exchange and \( i = 1 \) term is soft-Pomeron (SP) exchange. The authors in [49–52] showed that a very good fit to data (for \( Q^2 = 0 \) to \( 5000 \text{ GeV}^2 \) was provided by the economical parameterization as

\[
f_0(Q^2) = \frac{A_0(Q^2)^{1+e_0}}{(1 + Q^2 / Q_0^2)^{e_0+e_0/2}},
\]

\[
f_i(Q^2) = \frac{A_i(Q^2)^{1+e_i}}{(1 + Q^2 / Q_i^2)^{e_i+e_0/2}},
\]

where \( e_0 = 0.452, \ e_1 = 0.0667, \ A_0 = 0.00151, \ A_i = 0.658, Q_0^2 = 7.85 \text{ and } Q_i^2 = 0.658. \)

In this model, the known expression for \( \alpha_s(Q^2)xg(x, Q^2) \) (with respect to Eqs. (12) and (21)) reads as follows

\[
\alpha_s(Q^2)xg(x, Q^2) = \frac{3\pi}{(2p+1)} \sum e^q_ q \left[ f_i(Q^2)(\xi_q x)^{-e_ q} + f_i(Q^2)(\xi_q x)^{-e_ q} \right].
\]

**Tensor-Pomeron model.** Recently, in [53, 54], the validity of the CDP with respect to the tensor-Pomeron (TP) model [55] for photon virtualities of \( Q^2 \geq 20 \text{ GeV}^2 \) is considered. Consistency of the models has shown that the CDP with the perturbative QCD (pQCD) improved parton model at low \( x \). Authors in [55] applied the TP model to low-\( x \) deep-inelastic lepton–nucleon scattering and photoproduction due to the center-of-mass energies in the range 6–318 GeV and \( Q^2 \leq 50 \text{ GeV}^2 \). The hadronic high-energy reactions defined with respect to the two-Pomeron-plus-Reggeon approach in [55]. The virtual Compton amplitude in the TP approach for large \( W^2 \) is defined by the exchange of the two Pomeron, \( P_0 \) and \( P_1 \), plus the \( f_2 R \) reggeon. Therefore, the proton structure function, in the TP model, leads

\[
F_2(W^2, Q^2) = \frac{Q^2}{\pi} \left( 1 - x \right) \left[ 1 + 2\delta((W^2, Q^2)) \right]^{-1} \frac{W^2 - m_p^2}{W^2} \times 3 \sum_{j=0,1,2} \hat{b}_j(Q^2) \beta_{jpp}(W^2 \alpha_i^j) e_i^j \cos \left( \frac{\pi e_j}{2} \right).
\]

where

\[
\delta(W^2, Q^2) = \frac{2m_p^2Q^2}{(W^2 + Q^2 - m_p^2)^2}.
\]

The parameters of the TP approach with HP and SP and \( f_2 R \) reggeon exchange are

Soft: \( \alpha'_1 = \alpha_1 = 0.25 \text{ GeV}^{-2}, \ \varepsilon_1 = \alpha_1(0) - 1, \ \varepsilon_1 = 0.0935(\pm 64) \),

Hard: \( \alpha'_2 = \alpha_2 = 0.90 \text{ GeV}^{-2}, \ \varepsilon_2 = \alpha_2(0) - 1, \ \varepsilon_2 = 0.3008(\pm 84) \),

Reggeon: \( \alpha''_0 = \alpha_2 = 0.90 \text{ GeV}^{-2}, \ \beta_0 = 1.87 \text{ GeV}^{-1}, \ \beta_2 = 3.68 \text{ GeV}^{-1} \text{ and } m_p = 0.938 \text{ GeV}. \)

The values of coupling functions, \( \hat{b}_j(Q^2) \), obtained in the fit HERA DIS and photoproduction data read

\[
\hat{b}_0(10 \text{ GeV}^2) = \exp(-5.669(\pm 10)), \ \hat{b}_0(50 \text{ GeV}^2) = \exp(-6.899(\pm 80)), \ \hat{b}_1(10 \text{ GeV}^2) = \exp(-4.668(70)) \text{ GeV}^{-1}, \ \hat{b}_1(50 \text{ GeV}^2) = \exp(-7.870(29)) \text{ GeV}^{-1},
\]

and

\[
\hat{b}_2(Q^2) = c_2 \exp(-Q^2 / d_2),
\]

\[
c_2 = \exp(-0.38(\pm 36)) \text{ GeV}^{-1},
\]

\[
d_2 = \exp(-1.35(\pm 35)) \text{ GeV}^{-2},
\]

where the uncertainties indicated in the above brackets are determined using the MINOS algorithm. Therefore, the gluon distribution multiplied by the running coupling (with respect to Eqs. (12) and (24)) reads

\[
\alpha_s(Q^2)xg(x, Q^2) = \frac{3\pi}{(2p+1)} \sum e^q_ q \left[ \frac{Q^2}{\pi} (1 - \xi_q x) \right] \times 3(1 + 2\delta((W^2, Q^2)))^{-1} \frac{W^2 - m_p^2}{W^2} \times \sum_{j=0,1,2} \hat{b}_j(Q^2) \beta_{jpp}(W^2 \alpha_i^j) e_i^j \cos \left( \frac{\pi e_j}{2} \right)
\]

\[
\times \left[ 1 + \frac{2Q^2}{W^2 - m_p^2} + \frac{Q^2(Q^2 + m_p^2)}{(W^2 - m_p^2)^2} \right] \]
3. RESULTS AND DISCUSSIONS

We have calculated the $W^2$-dependence, at low $x$, of $\alpha_s(Q^2)xg(x,Q^2)$ according to the CDP and compared with the parametrization and Pomeron models. Results of calculations and comparisons are presented in Figs. 1–4. In Fig. 1 we compared the behavior of the gluon distribution with respect to the CDP asymptotic limit (Eq. (15)) and the parametrization of the proton structure function (Eq. (18)) for $\xi^2 < \frac{Q^2}{2.2}$. In this figure (i.e., Fig. 1) we observe that the parametrization model results converge towards the CDP asymptotic limit for $Q^2 \ll 10$ GeV$^2$.

In Fig. 2 we compared the results in Fig. 1 with respect to the CDP results at any $Q^2$ value. These results are based on $F_2(\xi^2, Q^2)$ according to (12) and are comparable with the CDP asymptotic limit and the parametrization model at moderate and large $Q^2$ values.

In Fig. 3, we compared the Regge behavior with the PM and the CDP predictions. The PM is comparable with the Regge behavior at low and large $Q^2$ values and it is comparable with CDP at moderate and high $Q^2$ values. In this figure (i.e., Fig. 3), the Regge behavior is defined into the SP and HP behaviors. The behavior of the two-Pomeron approach converge towards the CDP and PM at $Q^2 \gg 10$ GeV$^2$. Consistency between results for moderate and large $Q^2$ values shows that two-Pomeron approach leads to the CDP asymptotic limit where it is free of $Q^2$ parameters. This indicates that the Regge model must have at least two parameters or more to match the models. In [55] two-Pomeron-plus-Reggeon approach fitted to the experimental data on the deep-inelastic lepton–nucleon scattering at low values of $x$ and consistency with the CDP is introduced in [53, 54].

In fact, we have shown that in order to converge the CDP results with the Regge theory, it is necessary to introduce the Regge theory with two-Pomeron approach. In this connection, we plot $\alpha_s(Q^2)xg(x,Q^2)$ according to Eqs. (12), (15) and (18) and compared the
Regge theory with respect to the only HP trajectory due to Eq. (23). In Fig. 4, we observe that the HP approach result do not converge with the CDP and PM for $1 \text{ GeV}^2 \leq Q^2 \leq 100 \text{ GeV}^2$ in a wide range of $W^2$. Indeed, the results obtained by employing the CDP representation of the experimental data of the proton structure function in the pQCD improved parton model do not support the necessity of modifications by single (hard) Pomeron effects at $Q^2 \leq 100 \text{ GeV}^2$. We can observe that there is asymptotically agree between the parametrization and HP results for $Q^2 > 100 \text{ GeV}^2$. In summary two Pomeron models (soft + hard) improve results in comparison with the CDP and parametrization models at $1 \text{ GeV}^2 \leq Q^2 \leq 100 \text{ GeV}^2$. SP and HP models already discussed in [56, 57]. The inclusive electroproduction on the proton using a SP and a HP have been studied in [56, 57].

The derivative of the HP behavior from the eye-ball fit can be improved again by considering the TP approach of the proton structure function in Fig. 4. In [55] the SP and HP coupling functions are obtained for $Q^2 = 10$ and $50 \text{ GeV}^2$. Therefore in Fig. 5 we show that results obtained in Fig. 4 are comparable with others as we used the TP approach of the proton structure function. According to (25), the gluon distribution may equivalently be expressed in terms of the structure function due to the TP approach. As seen in Fig. 5, the results at $Q^2 = 10$ and $50 \text{ GeV}^2$ are comparable with others. The results obtained by employing the TP approach are comparable with CDP representation than the HP approach.

To conclude the low-$x$ analysis one needs the explicit expressions for the gluon distribution, $xg(x, Q^2)$, in Eq. (12) as

$$xg(x, Q^2) = \frac{3\pi}{\alpha_s(Q^2)} \sum_{q} \left[ \frac{1}{(2p + 1)} F_3(\xi, \bar{x}, Q^2) \right],$$

where the proton structure functions are defined in Eqs. (14), (16), (21) and (24) according to the CDP [10−15, 18, 44], PM [20], DL (A. Donnachie and P.V. Landshoff) [49−52] and TP [55] models, respectively. With the explicit form of the standard representa-
Fig. 3. (Color online) Same as Fig. 2, and compared with SP + HP, Eq. (23) (dashed-dot-dot curve).

Fig. 4. (Color online) Same as Fig. 3, and compared with HP, Eq. (23) (dashed-dot-dot curve).
For QCD couplings at the LO and NLO approximations in Eq. (5), the behavior of gluon distribution is investigated. In [58] the authors defined the renormalization group equations of the QCD running coupling and quark masses in a mathematically strict way. The QCD scale parameter $\Lambda$ has been extracted from the running coupling $\alpha_s$ normalized at the $Z$-boson mass, $\alpha_s(M_Z^2)$, where $\Lambda_{\text{LO}}^{n_f=4} = 119$ MeV and $\Lambda_{\text{NLO}}^{n_f=4} = 322$ MeV. We have calculated the $x$-dependence of the gluon distribution function as described above in the NLO approximation. In Fig. 6, we present the comparison of the gluon distribution from the CDP [10–15, 18, 44], PM [20], DL [49–52] and TP [55] models, with the results based on the GJR (M. Gluck, P. Jimenez-Delgado, and E. Reya) [59] and MSTW (A. Martin, W. Stirling, R. Thorne, and G. Watt) [60] parameterizations at $Q^2 = 50$ GeV$^2$. As can be seen, the values of the gluon distribution function increase as $x$ decreases. This figure indicates that the obtained results from the present analysis, based on the CDP are compatible with the ones obtained from the parametrization methods.

4. CONCLUSIONS

In conclusion, we have studied the effects of SP and HP in relation to the CDP and Parametrization models. We determined the gluon distribution func-
function (multiplied by $\alpha_s(Q^2)$) from our representation of the photoabsorption cross section in the CDP and compared with the results of the parametrization of the proton structure function and the SP and HP in the structure function of the proton. It turned out that the gluon function at order $\alpha_s(Q^2)$ is proportional to the proton structure function at a shifted scale $x \to \xi L x$. The parametrization and CDP results have similar behavior at $10 \text{ GeV}^2 \leq Q^2 \leq 100 \text{ GeV}^2$. We have found that the Regge like behavior of the proton structure function with a soft and hard Pomeron and also tensor-Pomeron approach improve the description of the gluon behavior at $10 \text{ GeV}^2 \leq Q^2 \leq 100 \text{ GeV}^2$. The soft and hard Pomeron results confirm the predictions of the CDP and this is requiring consistency of the CDP and pQCD for $Q^2 \gtrsim 10 \text{ GeV}^2$.

Furthermore, we have shown that the gluon distribution from the proton structure function can be estimated with respect to the running coupling in a general model using the CDP model. Explicit, analytical expressions for the gluon distribution function are obtained in terms of the effective parameters of the proton structure function in CDP, PM, DL and TP models and results of numerical calculations as well as comparisons with GJR and MSTW parametrizations are presented.

ACKNOWLEDGMENTS

I am grateful to Razi University for the financial support of this project.

CONFLICT OF INTEREST

The author declares that he has no conflicts of interest.

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