Performance evaluation of proportional navigation method during target pursuit with disturbance

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Abstract. A guided missile directed toward flight direction of target must be navigated so that it would not miss the target. Many navigation methods have been published. In this paper, Proportional Navigation (PN) Method based on Line of Sight guidance feature will be evaluated by simulation only at target moving in one way parallel on Y axis. Evaluation was conducted especially regarding to the ability to handle against external disturbance during its mission. PN Method was chosen because it can be implemented in relatively simple way by using infrared (IR) sensor on board for sensing the target position and do not require transponder frequency to communicate with the ground station. To evaluate the method, simulation model was developed firstly by using MATLAB. In the simulation test, 9949 N external disturbance (-9000 N on X-axis, -3000 N on Y-axis, and -3000 N on Z-axis) represent a wind whose value is 528 m/s has been added in time range of 10 until 18 second and 10 until 24 second. It is obvious from the results that Proportional Navigation method has been able to point the missile to the target at a certain time.

1. Introduction
1.1 Navigation system of guided missile
Many technologies have been applied in missile system such as aerodynamic system, structure of missile strength, propellant technology, propulsion technology, and navigation technology. In the field of aeronautics, navigation means the ability of the interceptor to point its linear velocity into the position of moving target so that the distance between both are smaller and smaller until small enough to impact the target as result. There are several navigation methods to guide the missile to hit the moving target, one of them is Long Range Navigation (LORAN) which is used to pursue the long-range target reaching 2400 km distance. However, this method has a complex control system involving the satellite and radio frequency as the missile navigator [1]. Moreover, the missile should have high specification of flight time to reach that long distance.

The objective of this research is to understand how the PN method works to guide missile in pursuing moving target through simulation. The other objective is to study the ability of PN method to handle external disturbance during the missile guiding to point the moving target.

The common method used to guide a missile is a Proportional Navigation. This method works by sensing the position of the moving target by infrared sensor. Position data then is used to compute the direction of missile’s linear velocity and target’s position. In this method, calculation of angular velocity of the Line of Sight is carried out and set as the reference of missile’s linear velocity angular
rate [2]. In many cases of application, this proportional navigation method is effective to guide a missile in pursuing the moving target [1].

In this paper, evaluation of Proportional Navigation method is provided. To conduct the evaluation, mathematical model is developed firstly. Then simulation model is presented by using MATLAB. In this research, the diagram for modelling and simulating guided missile system is shown in figure 1 below. As shown in the diagram, missile guided system consists of three components, those are: (a). Navigation System, (b) Missile Dynamic System, (c) Acceleration Computation. The system’s objective is to guide the missile to go to the direction of the target’s position [1].

![Diagram of guided missile system](image)

**Figure 1.** Navigation system of guided missile motion.

2. System modelling
This section is intended to provide mathematical model of guided missile motion dynamics and PN system. The developed model is used to construct simulation model in the next section as a tool to evaluate the performance of PN method.

2.1 Guided missile linear motion
The model of guided missile motion is expressed in term of linear velocity and flight angle as depicted in figure 2. Flight angle can be calculated from linear velocity as formulated in the following equation. Here X-Y plane is set as reference [3].

![Diagram of flight angle and velocity](image)

**Figure 2.** Flight angle and velocity of missile.

\[
\alpha_{m,\text{pitch}} = \tan^{-1}\left[ \frac{V_{mx}}{\sqrt{(V_{mx})^2 + (V_{my})^2}} \right] \quad (1)
\]

\[
\alpha_{m,\text{yaw}} = \tan^{-1}\left[ \frac{V_{my}}{V_{mx}} \right] \quad (2)
\]

where,
\( \alpha_{m,\text{pitch}} \): pitch angle with X-Y plane as reference (rad)
\( \alpha_{m,\text{yaw}} \): yaw angle with X-axis as reference (rad)

Figure 2 also shows space coordinate of linear velocity in all axes, X, Y, and Z and its velocity resultant is

\[
V_m = \sqrt{(v_{mx})^2 + (v_{my})^2 + (v_{mz})^2}
\]  

where \( V_m \) is linear velocity of guided missile (m/s), and \( v_{mx}, v_{my}, v_{mz} \) are projections of linear velocity to X, Y, and Z axes respectively.

To calculate linear motion of guided missile, Accelerated Linear Motion equation is used for range and velocity calculation. The equation has acceleration components on X, Y, and Z axes when it pursues its target. On this research, the missile model that was used for simulation has 21.8 second of burning time, 55 kN of average propulsion force during burning time and total mass up to 930 kg. Linear motion of guided missile can be shown below [1].

\[
S_{m(k)} = S_{(k-1)m} + V_{(k-1)m} \cdot \delta t + 0.5 \cdot A_{(k-1)} \cdot \delta t^2
\]  

Linear velocity of guided missile is as follows.

\[
V_{m(k)} = V_{(k-1)m} + A_{(k-1)} \cdot \delta t
\]  

where \( k=0,1,2,\ldots,\delta t \) is sampling time (second), \( S_{m(k)} \) is the distance that the guided missile experiences at time \( \delta t \) (meter), \( V_{m(k)} \) is the value of linear velocity of guided missile (m/s), \( A_{m(k)} \) is linear acceleration of guided missile (m/s\(^2\)).

One of the forces that are acted on the body when the guided missile pursues its target is propulsion force. This force drives the guided missile to move forward and can be expressed by Newton Second Law below [4].

\[
F = m \cdot a
\]  

where \( F \) is guided missile propulsion force (Newton), \( m \) is guided missile mass (kg), \( a \) is linear acceleration of guided missile motion (m/s\(^2\)).

There is also another force acted on the body whose direction is opposite to propulsion force called drag force, \( F_d \). Propulsion force can make the missile moves forward, the drag force can slow down the missile since it has opposite direction. The equation (7) of drag force can be seen below [4].

\[
F_d = 0.5 \cdot \rho_{\text{air}} \cdot A \cdot C_d \cdot V^2
\]  

where \( F_d \) is drag force(N), \( \rho_{\text{air}} \) is density of air(kg/m\(^3\)), \( A \) (m\(^2\)) is cross-section area of guided missile from front view (circular), \( C_d \) (dimensionless) is a coefficient that represents the magnitude of fluid resistance on the moving object, and \( V \) is linear velocity(m/s).

Resultant of horizontal propulsion force and drag force that have been mentioned above will be used for obtaining accelerations represented in the next equation (8).

\[
\Sigma F = m \cdot a
\]  

where \( \Sigma F \) is the resultant of propulsion and drag force(N), \( m \) is the mass of guided missile (kg), \( a \) is the acceleration of guided missile (m/s\(^2\)).
2.2 Proportional Navigation (PN) system

In this section, mathematical model for PN system will be elaborated. PN method principally works to have the angular velocity of guided missile always proportional to the angular rate of Line of Sight (LOS). LOS is a line which connects target’s coordinate with guided missile’s coordinate so that the direction of flight or flight angle of the missile will always follow and approach the position of the target. This definition suggests that the relationship between guided missile angular velocity and angular rate of LOS can be represented by the equation (9) below [2][5].

\[
\dot{\alpha}_m = N \dot{\theta}
\]  

(9)

where \( \dot{\alpha}_m \) (rad/s) is guided missile angular velocity, \( \dot{\theta} \) (rad/s) is angular rate of LOS, and \( N \) is non-dimensional ratio value of Proportional Navigation.

Geometrical relationship between linear velocity of guided missile and linear velocity of target with their own corresponding flight angles can be seen in the figure 3. Figure 3 shows that a target with linear velocity, \( V_t \) (m/s), and its flight angle, \( \alpha_t \) (rad), will be pursued by a guided missile with linear velocity, \( V_m \) (m/s), and its flight angle, \( \alpha_m \) (rad). The range between both objects have value \( R \) (m) and the direction \( \theta \) (rad) from reference line [6][7].

![Figure 3. Geometrical illustration between target (notation T) and guided missile (notation M).](image)

According to figure 3, the range between target and guided missile is the line that connects point M with point T (line M-T) and form angle \( \theta \) with dashed line (Ref) which can also be called as Line of Sight (LOS). By knowing each coordinate location of point M and point T, the range can be calculated using equation (10) [8][9] as follows.

\[
R = \sqrt{(x_t - x_m)^2 + (y_t - y_m)^2}
\]  

(10)

where \( R \) is range between target and guided missile (m), \( x_t,y_t \) is coordinate location of target and \( x_m,y_m \) is coordinate location of guided missile.

The acceleration of guided missile can be represented as the product of linear velocity and flight angle rate in the equation (11) below [10][11].

\[
a_m \, dt = V_m \, d\alpha_m \\
\dot{\alpha}_m = V_m \, \dot{\alpha}_m
\]  

(11)

where \( a_m \) is lateral acceleration of guided missile (rad/s²), \( \dot{\alpha}_m \) is angular velocity of guided missile (rad/s), and \( V_m \) is linear velocity of guided missile (m/s).
3. Guided missile PN system simulation

In this section, the simulation of guided missile with PN system that is moving towards to the target was carried out. Simulation model is developed based on equation (3), (4), (5), (8), (9), (10), (11). Simulation was implemented using MATLAB. Some scenarios were implemented to evaluate the performance of pursuit system.

3.1 Preparation for simulation

Firstly, in this section simulation scenario and initial conditions of simulation are provided. The initial conditions of missile and target coordinate and velocity presented in table 1 below. As shown in this table, initial position and velocity of both missile and target are given. As complementary information, geometry positions of both missile and target are shown in figure 4.

| Object          | Initial Position (m) | Initial Velocity (m/s) |
|-----------------|-----------------------|------------------------|
| Missile         | X_m=0                 | V_xm=0                 |
|                 | Y_m=0                 | V_ym=0                 |
|                 | Z_m=0                 | V_zm=0                 |
| Target          | X_t=1000              | V_xt=0                 |
|                 | Y_t=200               | V_yt=90                |
|                 | Z_t=600               | V_zt=0                 |
| PN Method       | Proportional Gain Value = 3 |
| Missile aerodynamic characteristic | \[ \frac{Y(s)}{G(s)} = 1 \]; Missile and Target in this case is not a rigid object but define as point of mass.

To evaluate the performance of PN system, three scenarios of simulation are implemented which are:

1. Scenario 1
   Guided missile without external disturbance is moving to target.
2. Scenario 2
   Guided missile with external disturbance in duration 8 second is moving to target.

Figure 4. Illustration of initial geometry of missile pursuing the moving target.
3. Scenario 3
Guided missile with external disturbance in duration 14 second is moving to target.

Each above scenario was simulated using MATLAB and the performance was analyzed with the value of external disturbance is 9949 N or represent a wind whose value is 528 m/s at missile flight during pursuing the moving target.

3.2 Simulation results and its analysis
The simulation result for each scenario is provided here.

1. Scenario 1
The result is shown in figure 5 which illustrates the distance range between missile and target. As shown in this figure, missile has accomplished its mission to achieve target in 25 second. It can be seen that the range between guided missile and target has decreased regularly as the function of time and eventually the range become close to zero, the change occurs from initial range, 1200 m, then gradually become close to zero. The result shows that proportional navigation method can perform well enough for guiding the missile to its target so that the range between them becomes smaller.

![Graph Distance VS Time, No Disturbance](image)

**Figure 5.** Distance gap between guided missile and target without disturbance.

The plots of guided missile trajectory on X-Y and X-Y-Z plane while it is pursuing the target without external disturbance is presented in figure 6. The target has moved straight forward in line with Y-axis from starting point X_{t0}, Y_{t0}, Z_{t0} =1002 m, 200m, 605 m, with constant velocity 90 m/s toward positive Y-axis. Meanwhile, the guided missile has moved from the starting point X_{m0}, Y_{m0}, Z_{m0} = 0 m, 0 m, 0 m, with initial velocity V_{xm0}, V_{ym0}, V_{zm0} Respectively in X, Y, and Z axes whose each velocity magnitude is 0 m/s. The simulation result shows that the missile change its course while pursues its target. The coordinate of the guided missile on X-axis and Y-axis change so that the trajectory of the guided missile would have certain maneuver, approach, and direct to the position of moving target. In conclusion, PN method can change the yaw angle of guided missile, which is the direction angle of linear guided missile velocity in X direction, to move the missile toward the location of the moving target.
Figure 6. Trajectories of both missile and target without disturbance, (a) X-Y plane, (b) X-Y-Z plane.

2. Scenario 2
In this scenario, 9949 N external disturbance (-9000 N on X-axis, -3000 N on Y-axis, and -3000 N on Z-axis) on duration 10 until 18 seconds, and 10 until 24 second were tried. The simulation result is shown in figure 7 below. In figure 7(a), missile has succeeded to get the target in about 40 s. However, in figure 7(b), the missile failed to zero the distance to the target, remaining 700 m distance, because the disturbance has been taking too long which is 14 second from 10 to 24 second.

Figure 7. Distance range between guided missile and target with disturbance, (a) 10-18 s of disturbance duration, (b) 10-24 s of disturbance duration.

The plots of guided missile and moving target trajectory on X-Y and X-Y-Z plane are shown in figure 8 and figure 9. It can be seen from figure 8(a) that the trajectory of guided missile with 9949 N external disturbance (-9000 N on X-axis, -3000 N on Y-axis, and -3000 N on Z-axis) on 10 until 18 s has achieved the target much longer compared with the trajectory of the missile without disturbance. It happens because the presence of external disturbance has disturbed the acceleration of guided missile on X-direction. However, eventually the missile was still able to direct its course to the target and hit the target even though it needs more time compared with the one without disturbance. It can also be seen from figure 8(a) that the guided missile hit the target at Y=3705 m. By comparing these two conditions, external disturbance has significant effect to trajectory of the missile while pursuing its target and cause time for hitting the target increases. However, for 10 until 24 second disturbance it shows from figure 8(b) that the missile failed to approach the target because the acceleration value is dropped too longer.
In this simulation, the velocity of missile was increasing because of the value of acceleration in each axis. The curve plot of trajectory is the effect of the lateral acceleration value which projected to each axis of the missile. So the change of acceleration value of missile as the result of the thrust in each axis depends on the value of lateral acceleration.

![Trajectory Missile Vs Target on X-Y plane, Disturbed](image1)

![Trajectory Missile Vs Target on X-Y plane, Disturbed](image2)

**Figure 8.** Trajectory of guided missile and target on the X-Y plane with disturbance, (a) disturbance in 10 – 18 second, (b) disturbance in 10 – 24 second.

![Missile Trajectory Vs Target at X-Y-Z, Disturbed](image3)

![Missile Trajectory Vs Target at X-Y-Z, Disturbed](image4)

**Figure 9.** Trajectory of guided missile and target on the X-Y-Z plane with disturbance, (a) disturbance in 10 – 18 second, (b) disturbance in 10 – 24 second.

**Table 2.** Result comparison of target engagement.

| No. | Condition               | Missed Distance (meter) | Time to Target (second) |
|-----|-------------------------|--------------------------|-------------------------|
| 1   | No Disturbance          | 0.0247                   | 25.096                  |
| 2   | 8 second disturbance    | 0.0992                   | 39.816                  |
| 3   | 14 second disturbance   | 700.5201                 | 69.218                  |

4. Conclusion
From the simulation results and its analyses, it can be concluded that the simulation of missile pursuing target using proportional navigation method had been successfully conducted. It can be shown by the trajectory of guided missile that follows the direction of the target and the range between the target and missile that became smaller, reached 0 meter. With the initial condition of missile velocity is 0 m/s at each axes, and the missile velocity value became larger during the pursuit because of its acceleration value in each missile axes. And the velocity difference between target and missile
makes the distance between target and missile became smaller until reaching 0 meter distance value in the end of pursuit.

From the simulation result, it can be seen that the performance of proportional navigation method had been proven to be able to work well enough for navigating the missile guidance to pursue and aim the target position which moves and resists enough due to external disturbance that occurs during the target pursuit. However, Proportional Navigation Method cannot perform well to zero the distance when the external disturbance is too long, i.e. over 14 second of disturbance duration. The simulation result is intentionally held in condition which target motion is only apply in one axis, i.e. Y axis, by assuming that this condition is representing the other condition of target motion in each axes with the same motion at one-way motion. And it is necessary to hold a simulation in other different target motion condition especially for target that moving in more than one axes or target moving with maneuver.

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