CMB Polarization and Dark Energy Induced Cosmological Birefringence

Guo-Chin Liu\textsuperscript{1}, Seokcheon Lee\textsuperscript{2} and Kin-Wang Ng\textsuperscript{2,3,4}

\textsuperscript{1}Department of Physics, Tamkang University, 251-37 Tamsui, Taipei County, Taiwan 251, R.O.C.
\textsuperscript{2}Institute of Physics, Academia Sinica, Taipei, Taiwan 11529, R.O.C.
\textsuperscript{3}Institute of Astronomy and Astrophysics, Academia Sinica, Taipei, Taiwan 11529, R.O.C.
\textsuperscript{4}Leung Center for Cosmology and Particle Astrophysics, National Taiwan University, Taipei, Taiwan 10617, R.O.C.

E-mail: liugc@mail.tku.edu.tw

Abstract. The coupling between dark energy and the pseudoscalar of electromagnetism, if there is any, would induce a rotation of the polarization plane of the cosmic microwave background (CMB). This results in a non-vanishing $B$-mode and parity-violating $TB$ and $EB$ correlations. Taking into account this effect, we calculate the full set of power spectra of cosmic microwave background (CMB) temperature and polarization anisotropies. We also give the constraint on the coupling strength from WMAP seven year data.

1. Introduction
The existence of a dark component with an effective negative pressure, supported by several observations especially the Hubble diagram for the type-Ia supernovae (see, for example, \cite{1, 2}), is still one of the puzzles in cosmology. Cosmological constant is the simplest possibility for such dark component. However, the observed value of the cosmological constant is completely different from theoretical expectation \cite{3}. An alternative candidate, the so-called quintessence described by a dynamical scalar field $\phi$, is naturally considered. The dynamics of $\phi$ in general quintessence models is governed by a scalar potential $V(\phi)$ which makes the dark energy dominant in recent epoch. There are many different kinds of proposed potentials, for example, pseudo Nambu-Goldston boson, inverse power law, exponential, hyperbolic cosine, and tracking oscillating \cite{4}. To differentiate between the models and finally reconstruct $V(\phi)$ would likely require next-generation observations.

One particular way to study the quintessence is to consider the interaction of $\phi$ to the pseudoscalar of electromagnetism. Pseudoscalar couplings usually arise from the spontaneous breaking of a compact symmetry group, say, U(1) (see Frieman et al. 1995 at reference \cite{4}). Carroll has argued that this coupling leads to the rotation of the polarization vector of propagating photons if $\phi$ is varying with time. This effect is called as the “cosmological birefringence” \cite{5}. This rotation results in a non-vanishing $B$-mode and parity-violating $TB$ and $EB$ correlations. Here, we present the CMB power spectra in the presence of the cosmological birefringence by the use of the full Boltzmann code. We briefly describe the the power spectra of CMB polarization in the presence of the cosmological birefringence in Sec. 2. In Sec. 3, we apply the power spectra to give a constraint on the coupling strength using the CMB data.
made by Wilkinson Microwave Background Probe (WMAP) [6]. Sec. 4 is our discussion and conclusion.

2. Power spectra of CMB polarization with the cosmological birefringence

Thomson scatterings of anisotropic radiation by free electrons give rise to the linear polarization, which is usually described by the Stokes parameters $Q$ and $U$ [7]. When the polarization plane is rotated by an angle $\alpha$, the Stokes parameters are transformed into

$$
\begin{pmatrix}
Q' \\
U'
\end{pmatrix} = \begin{pmatrix}
\cos 2\alpha & \sin 2\alpha \\
-\sin 2\alpha & \cos 2\alpha
\end{pmatrix} \begin{pmatrix}
Q \\
U
\end{pmatrix}.
$$

(1)

In standard cosmology, the time evolution of the polarization perturbation is governed by the Boltzmann equation [8], which states that when one follows a light ray the polarization of the radiation can change only due to the Thomson scatterings. With the birefringence effect, the $Q$ and $U$ further change due to the rotation of the polarization plane. Thus, we can then write,

$$
\frac{dX}{d\eta} = \dot{X}_{\text{thomson}} + \dot{X}_{\text{rotation}},
$$

(2)

$X = Q \pm iU$ and the derivatives are taken with respect to the conformal time $\eta$.

Using eq. (1), the angular velocity due to the cosmological birefringence of dark energy would lead to the temporal rate of change of the Stokes parameters:

$$
\dot{Q} \pm i\dot{U} = \mp i2\omega (Q \pm iU),
$$

(3)

and the evolution equations for the Fourier modes of the Stokes parameters are modified to

$$
\dot{\Delta}_{Q\pm U}(k, \eta) + i\mu \Delta_{Q\pm U}(k, \eta) = n_e \sigma_T a(\eta) \left[ -\Delta_{Q\pm U}(k, \eta) + \sum_m \sqrt{\frac{6\pi}{5}} \pm 2Y_2^m(\hat{n}) S_{P}^{(m)}(k, \eta) \right] \mp i2\omega\Delta_{Q\mp U}(k, \eta),
$$

(4)

where $\mu = \hat{n} \cdot \hat{k}$ is the cosine of the angle between the CMB photon direction and the Fourier wave vector, $n_e$ is the number density of free electrons, $\sigma_T$ is the Thomson cross section, and $a$ is the scale factor. $Y^m_l$ is spherical harmonics with spin-weight $s$ with $m = 0, \pm 1, \pm 2$ corresponding to scalar, vector, and tensor perturbations, respectively, if the axis of $S^m_l$ is aligned with the wave vector $k$. $S_{P}^{(m)}$ is the source term of generating polarization, which is the composition of the quadrupole components of the temperature and polarization perturbations $S_{P}^{(m)}(k, \eta) \equiv \Delta_{T_2}^{(m)}(k, \eta) + 12\sqrt{6}\Delta_{A_2}^{(m)}(k, \eta) + 12\sqrt{6}\Delta_{B_2}^{(m)}(k, \eta)$. We have followed the notation in Ref. [9] and expanded the perturbations in the spin-0 and spin-2 spherical harmonics [10] according to the scalar and tensor properties of the temperature anisotropy and polarization respectively. Thus, $\Delta_{T_2}^{(m)}$ and $\Delta_{A_2}^{(m)}$ are the respective expansion coefficients for $\Delta_T$ on basis $Y_{l,m}$ and $\Delta_{Q\pm U}$ on basis $\pm 2Y_{l,m}$.

We are used to decomposing the polarization on the sky into a divergence free component, the so-called $E$ mode, and a curl component, the so-called $B$ mode because the values for $Q$ and $U$ depend on the choice of a coordinate system. Whether $B$ mode is generated depends on the existence of $U$ for the local mode whose wave vector $k$ parallels to the $z$ of the coordinates whereas $Q$ is defined as the difference in intensity polarized in the $\theta$ and $\psi$ directions [11]. Mathematically, for $m = 0$, only $Q$ is generated in the local mode. The axisymmetry of the radiation field about the mode axis guarantees that no $B$ mode can be generated by scalar mode perturbations.
In the presence of cosmological birefringence, we can find two important features in Eq. (4). Firstly, the rotation of the polarization plane generates contributions to the local mode polarization. This converts the power from the E mode to B mode. The conversion depends on how much rotation there is from the epoch when the polarization is generated to today. Secondly, $TB$ and $EB$ cross correlations are expected to vanish due to the parity ($T$ and $E$ have parity $(-1)^l$ while $B$ has $(-1)^{l+1}$). The cosmological birefringence violates the parity and thus generates the $TB$ and $EB$ power spectra whose magnitudes depend on the integrated rotation of polarization too. Substituting the angular velocity of the polarization plane in Eq. (7) into Eq. (4), we calculate the power spectra $T$, $E$, $B$, $TE$, $TB$, and $EB$ modes.

These six power spectra form a complete two-point statistics of CMB temperature and polarization anisotropies. To simplify the calculation, we only include the scalar perturbations. That is by setting $m = 0$ in Eq. (4). Without showing the details, we just write down the power spectra which are obtained from the solutions for the line-of-sight integration as

$$
C^{(E, B)}_1 = (4\pi)^2 \frac{9}{16} \frac{(\ell + 2)!}{(\ell - 2)!} \int k^2 dk \left[ \Delta_{(E, B)}^2(\ell, k, \eta) \right],
$$

$$
C^{EB}_1 = (4\pi)^2 \frac{9}{16} \frac{(\ell + 2)!}{(\ell - 2)!} \int k^2 dk \Delta_E(\ell, k, \eta) \Delta_B(\ell, k, \eta),
$$

$$
C^{TE}_1 = (4\pi)^2 \sqrt{\frac{9}{16} \frac{(\ell + 2)!}{(\ell - 2)!}} \int k^2 dk \Delta_T(\ell, k, \eta) \Delta_E(\ell, k, \eta),
$$

$$
C^{TB}_1 = (4\pi)^2 \sqrt{\frac{9}{16} \frac{(\ell + 2)!}{(\ell - 2)!}} \int k^2 dk \Delta_T(\ell, k, \eta) \Delta_B(\ell, k, \eta),
$$

(5)

where

$$
\Delta_T(\ell, k, \eta) = \int_0^\eta d\eta g(\eta) S_T(k, \eta) j_\ell(kr),
$$

$$
\Delta_E(\ell, k, \eta) + i\Delta_B(\ell, k, \eta) = \int_0^\eta d\eta g(\eta) S_P(k, \eta) \frac{j_\ell(kr)}{(kr)^2} e^{i2\alpha(\eta)},
$$

(6)

where the visibility function $g(\eta)$ describes the probability that a photon scattered at epoch $\eta$ reaches the observer at the present time, $\eta_0$. Similar to $S_P \equiv S_P^{(0)}$, $S_T$ is the source term generating the temperature anisotropy. $j_\ell$ is the spherical Bessel function and $r = \eta_0 - \eta$. The rotation angle $\alpha(\eta) = \int_0^\eta d\eta' \omega(\eta')$. We do not present the formula for the temperature anisotropy because it is unchanged under the rotation of the polarization plane.

The last term in Eq. (4) appears due to the rotation of the polarization plane. This rotation can occur, as proposed by Carroll [5], due to the coupling $\beta_{F\bar{F}} \phi/\bar{M} F_{\mu\nu} F^{\mu\nu}$, if $\phi$ is varying with time. Here $\beta_{F\bar{F}}$ is the coupling strength, $\bar{M}$ is the reduced Planck mass, and $F_{\mu\nu}$ is the dual of the electromagnetic tensor. The dispersion relation for electromagnetic radiation coupling to the time varying quintessence field $\phi$ is given by $E^2 = k^2 \pm k^2 \beta_{F\bar{F}}^2/2\bar{M}^2$, where $\pm$ refer to the right and left handed circular polarization, respectively. Therefore, the net angular velocity of the polarization plane is [5]

$$
\omega = 2\beta_{F\bar{F}} \frac{\phi}{\bar{M}},
$$

(7)

In Figure 1, we show the integrated rotation angle of CMB polarization plane for different coupling strength. We consider the potential $V(\phi) = V_0 \exp(\lambda \phi^2/2\bar{M}^2)$ for our quintessence model, where $\lambda$ is a parameter determining how shallow the potential is. Hereafter we fix $\lambda = 5$ and we will obtain similar results by choosing other values of $\lambda$.

We show the power spectra $T$, $E$, $B$, $TE$, $TB$, and $EB$ modes with the coupling strength $\beta_{F\bar{F}}$ ranging from $10^{-5}$ to $10^{-3}$ from Figure 2 to Figure 5. On small scales, increasing coupling
strength results in a suppression of the $E$ mode in the standard model and non-vanishing $B$ and $EB$ modes. Furthermore, the shapes of the $B$ and $EB$ mode power spectra basically follow the standard $E$ mode except the reionization bump on large scales.

**Figure 1.** Integrated rotation angle of CMB polarization plane for different coupling strength

**Figure 2.** $E$, $B$ mode power spectra from the cosmological birefringence with different coupling strength.

**Figure 3.** $EB$ mode power spectra from the cosmological birefringence with different coupling strength.

**Figure 4.** $TE$ mode power spectra from the cosmological birefringence with different coupling strength.

**Figure 5.** $TB$ mode power spectra from the cosmological birefringence with different coupling strength.

If the coupling strength is large enough, the $B$ mode induced by the cosmological birefringence will mix up with the gravitational lensing-induced $B$ mode. Gravitational lensing by large scale
structures modifies slightly the primary $E$ mode power spectrum. Most noticeably it generates, through mode coupling, $B$ mode polarization out of pure $E$ mode signal [12]. The lensing-induced $B$ power spectrum, which peaks around $\ell \sim 1000$, has the roughly similar shape with that from the birefringence. We also show the power spectrum of the lensing-induced $B$ mode in Fig. 1 by a thin solid curve for comparison. The birefringence-induced $B$ mode is indeed compatible with the lensing-induced $B$ mode for $\beta_{F,\tilde{F}} \sim 10^{-4}$.

3. Constraints from CMB data

The coupling strength can be constrained by the measured polarization data. It must be very sensitive, especially, to the $TB$ and $EB$ power spectra. Recently, more and more CMB experiments produce $TB$ and $EB$ data for this issue, for example, Boomerang 2003 [13], QUad 2006-2006 data [14] and WMAP 7-year data. Here we use a public likelihood software, which is used by the WMAP team to compute Fishre and Master matrices, and compute the likelihoods of various mode, to fit the coupling strength. The CMB power spectra is computed with the cosmological birefringence effect while fixing other cosmological parameters based on the 7-year WMAP data. We turn on the $TE$ and low-$\ell$ $TB$, $EB$ and high-$\ell$ master $TB$ power spectra for our purpose. Then we convert $\chi^2$ to the likelihood by $\mathcal{L} = e^{-\chi^2/2}$ and normalize the maximum likelihood value to unity. The result is shown in Figure 6. There is an "unclear" peak of the likelihood located at $\beta_{F,\tilde{F}} = 4.6 \times 10^{-3}$, which corresponds to the integrated rotation angle from recombination epoch to today is about $-0.9$ degree. This result is consistent with Komatsu et al. 2009 [15], in which rotation angle is $-1.1^\circ \pm 1.3^\circ$(stat.) $\pm 1.5^\circ$(syst.) This small value of the coupling strength gives an insignificant change on $TE$ and $E$ modes and thus will not affect the determination of the cosmological parameters.

4. Discussion and conclusion

We have computed the power spectra of CMB polarization with the effect of cosmological birefringence by the use of the full Boltzmann code. Comparing with a rough estimation in Eq. (5): $C_{Bl} \sim C_{El} \sin^2 2\alpha_{\ell}$ and $C_{EBl} \sim 0.5C_{El} \sin 4\alpha_{\ell}$ used by some authors, the full Boltzmann code is more accurate. In the rough estimation, the total rotated angle, $\alpha_{\ell}$, for certain angular scales $\theta \sim \pi/\ell$ from last scattering epoch to today is assumed to be constant for all scales. This in general is not correct. The $E$ mode power on small scales mainly comes from the recombination epoch at $z \sim 1100$. On the other hand, the boosting power on large scales comes from reionization epoch when the CMB photons are rescattered by free electrons at $z \sim 10$ [16]. From Eq. (7) and the evolution of $\phi$, we find that the integrated rotation angle from the reionization epoch is much smaller than that from the recombination epoch. Therefore,
there is much less power converted from $E$ mode to $B$ mode on large scales than small scales.

We have also constrained the coupling strength $\beta_{F\tilde{F}}$ by the use of WMAP 7-year data. We found an peak of the likelihood located at $\beta_{F\tilde{F}} = 4.6 \times 10^{-3}$. However, it is not a conclusive value because the peak is not clear. Furthermore, it is remarkable that the rotation-induced $B$ mode with this value of the coupling strength exceeds the lensing-induced $B$ mode. Therefore, more careful measurements of $TB$ and $EB$ are necessary for separating the two effects.

5. References

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