Nonlinear Quasi-Synchronous Multi User Chirp Spread Spectrum Signaling

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Abstract—Multi user orthogonal chirp spread spectrum (OCSS) improves the spectral inefficiency of CSS but is only feasible with perfect synchronism and without any channel distortion. Either asynchronism or channel distortion causes multiple access interference (MAI), which degrades performance. Conditions with small timing offsets we term quasi-synchronous (QS) transmission. In this paper, we investigate CSS signaling in QS conditions. We do this for the classical linear chirp, for which cross correlations can be derived analytically, and also propose two sets of nonlinear chirps to improve CSS system performance. We numerically evaluate cross-correlation distributions, and show that with an FSK-based chirp modulation, our two new nonlinear chirp designs outperform the classical linear chirp and multiple existing nonlinear chirps from the literature, over the QS additive white Gaussian noise channel in both partially and fully-loaded systems. We also demonstrate our nonlinear CSS designs outperform the existing chirps in a realistic dispersive channel, via simulations using an empirical air-ground channel.

Keywords—chirp spread spectrum; multiple access communication system; quasi-synchronous transmission;

I. INTRODUCTION

Many wireless communication systems will need to accommodate a larger number of users in the future. One application in particular in which this is critical is low data rate, long range communication links with very large numbers of nodes, such as the internet of things (IoT), possibly the internet of flying things (IoFT), etc. These systems demand advanced multi-access techniques with minimal multiple access interference (MAI). They should also be robust to multiple impairments, including multipath channel distortions, Doppler spreading, and interference.

Chirp waveforms [1] can satisfy most of these requirements, and in addition have other attractive features, such as low peak to average power ratio (PAPR). Hence chirps—a form of frequency modulation—are promising candidates for many such applications. Chirps are specified in the IEEE 802.15.4a standard as chirp spread spectrum (CSS) [2].

These time frequency (TF) waveforms have several useful properties including energy efficiency, and if wideband enough, robustness to interference, multipath fading, and eavesdropping. They can also be used for high resolution ranging and channel estimation. Underwater acoustic wireless communication systems [3], [4] can also use chirp waveforms to advantage in the presence of very rapid fading. The “long-range” (LoRa) technology developed for IoT applications uses a proprietary chirp spread spectrum (CSS) modulation scheme that aims to provide wide-area, low power and low cost IoT communications [5], [6].

The term chirp is typically used to refer to a frequency that changes linearly over time, but this can be generalized. In the literature, different chirp waveforms have been categorized: linear, various types of nonlinear, amplitude variant as well as constant amplitude forms. Modulation can be accomplished in several ways, one of the simplest being binary chirps that sweep either up or down in frequency over a bit period. Chirps can of course be used in on-off signaling or as basic waveforms for frequency shift keying (FSK). Higher-order modulation can be attained with chirps in a number of ways, e.g., by using multiple sub-bands, different start/stop frequencies (somewhat akin to pulse position modulation, PPM), and via distinct chirp waveforms within a given band.

A disadvantage of CSS signaling is spectral inefficiency. This can be addressed by accommodating multiple users with a set of properly designed chirps in the available bandwidth. For multiple access, a set of chirp signals is required, and all waveforms in this set would ideally be orthogonal, to eliminate multiple access interference (MAI). Achieving orthogonality is easy enough with synchronized waveforms [7]-[10], but in many practical cases, e.g., with mobile platforms, attaining and maintaining synchronism is challenging. Waveforms designed to be orthogonal when synchronized typically exhibit large inter-signal cross correlation when asynchronous [11], [12], and this of course degrades performance. Thus finding a set of waveforms that achieves low cross correlation among signals when asynchronous is desirable. In general this is very difficult, so some researchers have focused on quasi-synchronous (QS) conditions. This refers to the case when synchronization is approximate, typically limited to some small fraction of a symbol time \( T_s \). Such approximate synchronization allows for less precise timing control in mobile applications. An example of waveforms for this type of application is the zero autocorrelation zone (ZACZ) set of sequences [13].

In this paper, we explore analysis and design of chirp spread spectrum (CSS) waveforms for quasi-synchronous multiple access systems using both linear and nonlinear time-frequency (TF) functions. We quantify MAI in asynchronous and quasi-synchronous conditions, and provide two new nonlinear chirp designs that yield good performance under modest asynchronism.

The remainder of this paper is organized as follows: Section II provides a brief literature review on chirps and CSS, and in
Section III we describe the chirp signal waveforms for a linear chirp design. We also provide a mathematical derivation for the inter-signal cross correlation when linear chirps are asynchronous. Section IV describes two nonlinear CSS designs, and in Section V we evaluate QS performance and compare with previously studied CSS waveforms. We also evaluate the QS performance of our new designs over an empirical air-ground channel model. Section VI concludes the paper.

II. LITERATURE REVIEW

In this section, we review some of the literature on chirp signaling, and on CSS in particular. The literature on the general use of chirps is fairly extensive, so we only provide highlights, and focus primarily on CSS.

The chirp technique proposal made by S. Darlington in 1947 related to waveguide transmission for pulsed radar systems with long range performance and high range resolution [14]. B. M. Oliver first used “chirp” in his memorandum entitled “not with a Bang, but a Chirp” and 6 years later, acoustic chirp devices were developed at Bell Labs. Hardware constraints were a limiting factor for their development. In [15], the authors described an experimental communication system employing chirp modulation in the HF band.

In [7], the authors proposed an orthogonal linear amplitude variant chirp modulation scheme where each user employs a unique frequency modulated chirp rate. The scheme defines orthogonal linear chirps with different TF slopes or chirp rates. To satisfy orthogonality with their design, they impose amplitude variation (∼√2), and hence this scheme does not retain the desirable constant envelope property of conventional chirps. This approach showed improvement in multi user system bit error ratio (BER) performance in multipath fading channels when compared to FSK frequency hopping code division multiple access (FH-CDMA) schemes. Their analysis and evaluation was based on a perfectly synchronized condition.

The authors of [8] used a set of orthogonal linear chirped waveforms based on the Fresnel transform and its convolution theorem to design an orthogonal chirp division multiplexing (OCDM) system. They compared this to orthogonal frequency division multiplexing (OFDM) and showed that their OCDM system outperformed the conventional OFDM system by exhibiting greater resilience to inter symbol interference due to insufficient guard interval. Compared to OFDM the OCDM scheme had identical PAPR performance and only slightly higher complexity. Discrete Fourier transform-precoded-OFDM (DFT-P-OFDM) outperformed OCDM in terms of PAPR and had identical BER performance. In this work, the authors also assumed perfect synchronization between all transmitters and receivers.

In [9], the authors presented orthogonal quadratic and exponential non-linear chirp designs. Users are assigned unique chirp rates that vary either quadratically or exponentially versus time (yielding different signal bandwidths among users). These designs also required amplitude variation to maintain orthogonality. A similar approach was followed in [10] for nonlinear trigonometric and hyperbolic CSS waveforms, again assuming full synchronization.

The authors in [16] presented another set of orthogonal chirps by exploiting the advantages of the fractional Fourier transform (FrFT) adopted from [17]. They claimed that the proposed method has lower MAI than the conventional method proposed in [17] and should yield better system performance. They supported their claim by evaluating BER performance over the AWGN channel. Their signal amplitude is constant over the chirp duration, but again, a fully synchronous system was assumed.

Our approach for CSS enforces constant signal envelope and equal signal bandwidths for all users. Primarily, we relax the perfect synchronization constraint and find designs that can yield good multi-user performance when quasi-synchronous.

III. LINEAR CHIRP SIGNALING AND QUASI SYNCHRONOUS TRANSMISSION

A. Linear Chirp Signals

In this paper, the core formula for generating frequency-modulated (chirp) waveforms is adopted from the kernel Fresnel transform theorem method, discussed in lightwave communication applications [8]. We modified the formula to generate a set of N orthogonal linear “up-chirps” (low to high frequency) with time (symbol) duration T. In complex baseband form, the mth waveform can be written as:

\[ s_m(t) = e^{j\pi mN} e^{j\pi (t/mT)^2}, \quad 0 \leq t < T \]  

where N is the desired number of orthogonal chirp waveforms, \( m \in \{0, 1, \ldots, N-1\} \) is the user index, and T is the duration of the chirp waveform. The total bandwidth B that a set of N users occupies is \( B = 2N/T \), and each user signal occupies the same bandwidth, \( 2/T \). When perfectly synchronized, the waveforms in (1) are orthogonal. A completely analogous construction can be made with “downchirps” by negating the sign of the exponent of the second term of (1). Figure 1 shows the TF plane patterns over a single symbol time for three sets of such waveforms with different symbol times and bandwidths, yet the same area in the TF plane, which corresponds to the same spectral efficiency. Each user signal is simply a line in the TF plane with slope \( N/T \).

The instantaneous frequency of the signal in (1) can be written as:

\[ \nu_m(t) = \frac{1}{2\pi} \frac{d}{dt} \left( \frac{jN}{T^2} \left( t^2 + \frac{2mT}{N}t + \frac{m^2T^2}{N^2} \right) \right) = \frac{N}{T^2} t + \frac{m}{T} \]  

![Fig. 1. Time-frequency representation of N orthogonal chirp waveforms for T, T/2 and T/4 seconds.](image)
B. Quasi-synchronous Transmission of Linear Chirps

Many modern communication systems have been developed assuming quasi-synchronous conditions, where clocks of different user terminals (or nodes) are not perfectly synchronized, but are “close” to synchronized. Their mean clock frequencies may be essentially identical, but drift and jitter cause clocks to deviate from this mean over the long and short terms. This asynchrony is usually bounded (a small portion of a symbol duration \( \delta T \)) in many communication systems. Fig. 2 shows a simple illustration of clock jitter and drift in a system with one transmitter and two receivers.

![Fig. 2. Illustration of clock drifts and jitter.](image)

For the chirp waveforms of (1), the group of delayed chirp signals can be written as,

\[
s_k(t) = e^{j\pi \frac{m}{N} \left(\frac{mT}{N} \right)^2} e^{-j\pi \frac{m}{N} \left(\frac{mT}{N} \right) t} \quad \epsilon \leq t < T + \epsilon
\]

where \( \epsilon_k \) is the delay associated with clock drift or uncompensated propagation delay for user \( k \). Generally, these delays are modeled as random with some distribution, and with value limited between 0 to \( T \) since other than packet transmission boundaries, effects of asynchronism recur over subsequent symbols. A time/frequency domain representation of the set of quasi-synchronous signals of the form of (3) for only one asynchronous user \( (k=2) \) is depicted in Fig. 3. We note that for certain values of timing offset \( \epsilon_k \), the non-synchronized TF signal can overlap another signal in the set nearly completely over a part of a symbol, and this yields relatively large multiple access interference.

![Fig. 3. Time/frequency domain representation of quasi-synchronous transmission of linear chirps with one asynchronous user.](image)

Multiple access interference (MAI) is quantified by the cross correlation between signals of the form of (1) and (3). Computing the cross correlation values requires an integration, which for these linear chirps can be written as follows:

\[
\rho_{m,k} = \frac{1}{\sqrt{E_m E_k}} \int_0^T s_m(t) \cdot s_k^* (t - \epsilon_k) dt
\]

\[
\frac{1}{T} \int_{-T}^T e^{j\pi \frac{m}{N} \left(\frac{mT}{N} \right)^2} \cdot e^{-j\pi \frac{m}{N} \left(\frac{mT}{N} \right) t} \cdot e^{j\pi \frac{m}{N} \left(\frac{mT}{N} \right) \left(t - \epsilon_k\right)} dt
\]

\[
= \int_{-T}^T e^{j\pi \frac{m}{N} \left(\frac{mT}{N} \right)^2} \cdot e^{-j\pi \frac{m}{N} \left(\frac{mT}{N} \right) t} \cdot e^{j\pi \frac{m}{N} \left(\frac{mT}{N} \right) \left(t - \epsilon_k\right)} dt
\]

\[
= \int_{-T}^T e^{j\pi \frac{m}{N} \left(\frac{mT}{N} \right)^2} \cdot e^{-j\pi \frac{m}{N} \left(\frac{mT}{N} \right) t} \cdot e^{j\pi \frac{m}{N} \left(\frac{mT}{N} \right) \left(t - \epsilon_k\right)} dt
\]

where again \( \epsilon_k \) is a random variable representing the timing offset of user \( k \), and in the second line we have used the unit energy of each waveform.

Fortunately, this integral has a closed form solution for any arbitrary offset denoted \( \lambda \), as written in (5):

\[
\rho_{m,k}(\lambda) = \frac{1}{\sqrt{2\pi}} \int_{-\lambda/2}^{\lambda/2} e^{-j\pi \frac{m}{N} \left(\frac{mT}{N} \right)^2} \cdot e^{-j\pi \frac{m}{N} \left(\frac{mT}{N} \right) t} \cdot e^{j\pi \frac{m}{N} \left(\frac{mT}{N} \right) \left(t - \epsilon_k\right)} dt
\]

\[
= \int_{-\lambda/2}^{\lambda/2} e^{-j\pi \frac{m}{N} \left(\frac{mT}{N} \right)^2} \cdot e^{-j\pi \frac{m}{N} \left(\frac{mT}{N} \right) t} \cdot e^{j\pi \frac{m}{N} \left(\frac{mT}{N} \right) \left(t - \epsilon_k\right)} dt
\]

where \( C \) is the constant of integration. The integral is in (5) well-defined except when \( \eta = kT \). In this case, using Euler’s identity and l’Hopital’s rule, we can find

\[
\rho_{m,k}(\lambda) = \left\{ \begin{array}{ll}
\frac{1}{\sqrt{2\pi}} \int_{-\lambda/2}^{\lambda/2} e^{-j\pi \frac{m}{N} \left(\frac{mT}{N} \right)^2} \cdot e^{-j\pi \frac{m}{N} \left(\frac{mT}{N} \right) t} \cdot e^{j\pi \frac{m}{N} \left(\frac{mT}{N} \right) \left(t - \epsilon_k\right)} dt & \text{if } \lambda = \frac{(k-m)T}{N} \\
\frac{1}{\sqrt{2\pi}} \int_{-\lambda/2}^{\lambda/2} e^{-j\pi \frac{m}{N} \left(\frac{mT}{N} \right)^2} \cdot e^{-j\pi \frac{m}{N} \left(\frac{mT}{N} \right) t} \cdot e^{j\pi \frac{m}{N} \left(\frac{mT}{N} \right) \left(t - \epsilon_k\right)} dt & \text{else}
\end{array} \right.
\]

By dividing the integral into two parts as indicated in Fig. 3 and (4), and setting \( \lambda = \epsilon_k \) and \( T - \epsilon_k \) for parts one and two respectively, we obtain the cross-correlation integral solution as follows:

\[
\rho_{m,k} = \left\{ \begin{array}{ll}
\frac{1}{\sqrt{2\pi}} \int_{-\epsilon_k/2}^{\epsilon_k/2} e^{-j\pi \frac{m}{N} \left(\frac{mT}{N} \right)^2} \cdot e^{-j\pi \frac{m}{N} \left(\frac{mT}{N} \right) t} \cdot e^{j\pi \frac{m}{N} \left(\frac{mT}{N} \right) \left(t - \epsilon_k\right)} dt & \text{(i)} \\
\frac{1}{\sqrt{2\pi}} \int_{-\epsilon_k/2}^{\epsilon_k/2} e^{-j\pi \frac{m}{N} \left(\frac{mT}{N} \right)^2} \cdot e^{-j\pi \frac{m}{N} \left(\frac{mT}{N} \right) t} \cdot e^{j\pi \frac{m}{N} \left(\frac{mT}{N} \right) \left(t - \epsilon_k\right)} dt & \text{(ii)}
\end{array} \right.
\]

where (i) denotes \( \epsilon_k = mT/N \) and (ii) denotes otherwise. This expression has the smallest value (0) when \( \epsilon = 0 \) or \( \epsilon = T \). Correlation is of course one when \( m = k \) and \( \epsilon = 0 \). Note that for an \( N \) user linear chirp system, the minimum separation between waveforms in time is \( T/N \).

In order to get insight into the values of cross correlation for a quasi-synchronous situation, we calculated the set of pairwise cross correlations between each pair of waveforms versus delay. This yields a matrix of cross-correlations for each value of delay. As Fig. 4 depicts, each element of the matrices \( \rho_{m,k} \) represents a cross correlation between user \( m \) and delayed user \( k \). From the set of \( J \) matrices for \( J \) distinct values of timing offset, we can compute statistics of the set of cross correlations, as a
function of offset ε, or as a function of user indices for a given delay. The set of matrices forms a tensor. For any matrix (specific value of offset) the upper and lower triangular entries are identical (ρ_{1k}=ρ_{k1}), yielding N×(N-1)/2 unique values for each offset ε.

Fig. 4. Illustration of cross correlation tensor.

As one would expect, all users experience the highest value of cross correlation when the two waveforms fully or partially “overlap” in the time-frequency domain. An example set of mean correlations across a set of N=25 linear chirps with duration T sec and frequency separation of 1/T Hz is shown in Fig. 5, where we have shown both analytical and numerically computed results. Numerical results from trapezoidal method integration (in MATLAB®) yield a very close fit to analytical results. Average cross correlation is of course only one statistic of interest.

Fig. 5. Average values of analytical and numerically evaluated cross correlation versus delay for N=25 users.

The histogram in Fig. 6 presents the distribution of all cross correlation values for all delays for this N=25 linear chirp signal set. Approximately 90.76% of the cross correlation values are below <0.2, and the median value is 0.0479 for a delay quantization of T/500.

IV. NON-LINEAR CHIRP DESIGNS

In this section, we propose two non-linear chirp signal sets which, qualitatively speaking, have more “spacing” between each signal’s time/frequency trace. Our heuristic approach is to fully use the available time-frequency space for signals and compare correlation performance with the linear set. Non-linear chirp waveforms can be generated with arbitrary shapes in the time/frequency domain. The most well-known examples are exponential, quadratic, and sawtooth [9],[10]. Here we propose two specific mathematical derivations for generating nonlinear chirp waveforms with no amplitude variation. A nonlinearity function Ψ(t) is defined as in (8). This phase function can modify the instantaneous frequency of the linear case to any desired nonlinear TF shape. One can find the chirp signal’s time-frequency shape via the time derivative Ψ′(t)/(2π) to find instantaneous frequency versus time.

\[ s_{mNL}(t) = e^{\frac{jm\pi N}{2T^2}\left(t^2 + \frac{mT^2}{N} - t + \frac{m^2 T^2}{N^2}\right) + \frac{at}{2\pi f_c} \sin(2\pi f_0 t)}, 0 \leq t < T \]  

(8)

References [9] and [10] used different derivations for their nonlinear chirp signals, but a close look at their mathematical derivation (discounting their amplitude variation) shows nonlinearities of quadratic (t²), sinusoidal (sin(t)) and hyperbolic sinusoidal (sinh(t)) structure.

Case 1: Sinusoidal Chirp

Case one uses a sinusoidal function for Ψ(t); see Fig. 7 for the example illustration, with signal waveforms given by,

\[ s_m(t) = e^{j\frac{m\pi N}{2T^2}\left(t^2 + \frac{mT^2}{N} + \frac{at}{2\pi f_c} \sin(2\pi f_0 t)\right)}, 0 \leq t < T \]  

(9)

where α and f₀ are selectable constants that can produce different time/frequency shapes. The instantaneous frequency can be written as

\[ \nu_{mNL,sine}(t) = \frac{1}{2\pi} \frac{d}{dt} \left[ \pi N\left( t^2 + \frac{2mT}{N} - t + \frac{m^2 T^2}{N^2}\right) + \frac{at}{2\pi f_c} \sin(2\pi f_0 t)\right] \right] \]  

(10)

\[ = \frac{N}{T^2} t + m + \frac{at}{4\pi f_c T^2} \sin(2\pi f_0 t) + \frac{atN}{2T^2} \cos(2\pi f_0 t). \]
We selected values for \( \alpha \) and \( f_0 \) as \( \frac{(2m-N)}{2N} \) and \( \frac{1}{\pi} \), respectively, as these qualitatively produce a larger "area coverage" in the TF plane than the linear set of signals. An example is plotted in Fig. 7.

**Case 2: Quartic Chirp**

In order to further increase spacing between each signal’s time/frequency trace, we constructed another nonlinear signal set with the following instantaneous frequency:

\[

\nu_{\text{max-quartic}}(t) = \begin{cases} 
\frac{n}{T^2}t^2 + \frac{\mu}{T} + \beta([t](2t^2 - \mu t)) & m > N/2 \\
\frac{n}{T^2}t^2 + \frac{\mu}{T} + \beta([-t + t](2t^2 - \mu t)) & m < N/2 
\end{cases} \tag{11}
\]

This design yields a larger time/frequency coverage than the linear and sinusoidal nonlinear case 1. Specifically the values for \( \beta \) and \( \mu \) were chosen as \( \frac{N}{2} - m \) and 2, respectively. Signal waveforms are then given by,

\[

s_m(t) = e^{j\pi \frac{m}{T} t} e^{j\frac{\pi m}{N} t^2} e^{j\nu_{\text{max-quartic}}(t)} , 0 \leq t < T \tag{12}
\]

TF plots of both nonlinear waveforms in (11) and (12) are shown for \( N=100 \) in Fig. 7. Note that not all \( N \) waveforms are shown: only several of the lowest and highest frequency signals are plotted to bound each type’s area. The nonlinear cases clearly occupy larger areas in the TF plane. As Fig. 7 depicts, the sinusoidal Case 1 signal set occupies a slightly larger TF area than the linear set but keeps the same starting and ending frequency and the same total bandwidth. The quartic Case 2 covers the largest area, with different starting and ending frequencies, but the same total bandwidth.

For the linear quasi-synchronous chirps, the cross-correlation is available in closed form (5). Yet for arbitrary (nonlinear) chirp waveforms one can generally not find any closed form solution, hence we evaluate correlations numerically.

We computed cross correlations for a different number of signals for the linear and our two nonlinear cases in fully loaded mode. Fully loaded mode means all \( N \) users are sending a symbol during each time slot of duration \( T \). The non-fully-loaded case has only \( K \) signals present, with \( 0 < K < N \). Naturally aggregate correlations will be smaller if fewer signals are present, so the fully loaded case is the worst case. We also assume perfect power control, i.e., all user signals are received with the same power. Fig. 8 (a) to (c) shows average correlation values for these three chirp types for three different values of \( N \). Insets in the figure show these correlations at two smaller delay ranges, 0.05T and 0.01T, for study as QS signal candidates. We observe that the quartic Case 2 nonlinear signals yield a smaller average correlation value for the entire range of timing offset for the two smaller values of \( N \), whereas the sinusoidal case 1 has approximately the same mean correlations as the linear case. Note that correlation plots are symmetric around 0.5T as Fig. 5 depicted, therefore only delays up to this value are shown.
Fig. 8. Normalized average cross correlation versus delay for linear, sinusoidal case1, and quartic case 2 for (a) \(N=10\), (b) \(N=20\) and (c) \(N=50\) signals.

Fig. 8 (c) shows that for the largest value of \(N\), the nonlinear cases have lower mean correlations only at the smallest delays, but this still qualifies them for QS operation. For a more complete representation of the cross correlation distribution, we provide a histogram of all cross correlation values for all offsets for our three signal sets in Fig. 9 (a) to (c). Although all histograms show the same general shape and range, the large correlation values, which cause the most severe MAI, are less likely for the nonlinear cases than the linear set.

V. NONLINEAR CSS PERFORMANCE EVALUATION

As previously noted, there are multiple ways to modulate chirps with data: mapping \(M\)-ary symbols to \(M\) of the \(N\) chirps in the set, using chirps of the opposite slope (e.g., “downchirps” as well as “upchirps”), on-off signaling, and even using different starting/stopping frequencies. This latter method is used in the LoRa technology [6], where with a linear chirp frequency \(f_{\text{chirp}}\) in the range \([f_{\text{min}}, f_{\text{max}}]\), two different symbols can be represented during a symbol interval by either (a) a sweep from \(f_{\text{min}}\) to \(f_{\text{max}}\), or by (b) a sweep from \(f_{\text{min}}\) to \(f_{\text{max}}\) immediately followed by sweep from \(f_{\text{min}}\) to \(f_{\text{min}}\). Here the second symbol’s start frequency \(f_{\text{min}}\) lies in the range \(f_{\text{min}} < f_{\text{min}} < f_{\text{max}}\). For any multiuser systems this LoRa approach requires additional frequency separation between user signals (hence a larger total bandwidth) to avoid the effect of asynchronous cross correlations. Other mappings from symbols to TF plane trajectories are obviously possible, and may represent an area for future investigation. Here we restrict ourselves to one example \(M\)-ary modulation which can be viewed as a form of MFSK, as this can serve to illustrate the differences among the different types of chirps we have thus far described.

Our multi user nonlinear chirp spread spectrum system block diagram is shown in Fig. 10. In the transmitter, for each user’s data, a block of \(k\) bits is translated to one of \(M=2^k\) symbols. Each symbol is mapped to a specific one of \(M\) sub-bands, and within each sub-band, a set of \(N\) chirp waveforms is used to accommodate the \(N\) users. Keeping all \(M\) symbol waveforms of a given user within its own sub-band, and allocating \(N\) sub-bands to \(N\) users is a related or “dual” variation whose performance depends on system loading and channel conditions (synchronism, fading, Doppler, etc.); we leave investigation of that option to future work. Each sub-band has bandwidth \(2N/T\), so the entire system bandwidth is \(2NM/T\) and spectral efficiency of a fully loaded system is \(\log_2(M)/(2M)\) bps/Hz.

At the receiver, bandpass filters are used for each sub-band, and matched filters convolve the received signal with a bank of time-reversed versions of the transmitted chirps. An alternative heterodyne detector can also be used instead of matched filter...
detectors, as explained in [19]. Integrators and decision circuits complete the receiver symbol detection.

As previously noted, each user signal’s delay is assumed perfectly known. Delay tracking using coherent delay-locked loops (DLLs), similar to previous efforts for CDMA systems [20], can address this, but this is not in the scope of this paper.

Our simulated performance results assume perfect delay estimation and tracking at the receiver’s matched filters for each single signal. All results are for an AWGN QS channel except for the last BER performance plot for the air-ground channel.

Fig. 10. MFSK multiuser chirp spread spectrum system block diagram.

Fig. 11 (a) to (c) depicts bit error rate performance versus bit energy to noise density ratio ($E_b/N_0$) for fully loaded linear and nonlinear cases 1 and 2 for both fully synchronized and quasi-synchronous conditions for $N=10, 20$ and $50$, and binary modulation ($M=2$). For these results, delay is modeled as a zero-mean Gaussian random variable with standard deviation ($\sigma$) of 0.01$T$ and 0.1$T$. Fig.11 (a) shows system performance in a perfectly synchronized system. The first thing to observe is that the nonlinear cases are not orthogonal. Hence their performance degrades as the number of signals and MAI increase, particularly for the sinusoidal nonlinear case. However, as we can see in Fig.11 (b), a very small delay with $\sigma = 0.01T$ significantly degrades the performance of the linear chirps whereas the degradation of the quartic nonlinear case is moderate. For the largest value of $\sigma$ in Fig. 11(c), the nonlinear quartic case 2 is superior to the other sets of waveforms for any system loading (relative number of users).
to be maximally and equally spaced in the TF plane. This behavior has been observed for any arbitrary value of N.

Another difference between the cases is that for both linear and sinusoidal cases, all signals have the same starting and ending frequencies but quartic case 2 signals do not (Fig. 7). Since the sinusoidal chirps do not extend the TF plane area coverage by much over the linear chirps, the sinusoidal case 1 only slightly outperforms the linear case in Fig. 11 (b) and (c) in quasi-synchronous conditions. Synchronization on the order of $\sigma < 0.01T$ is very close to perfect, but the 0.1T value is more practical, particularly for mobile platforms.

Also worth study is performance of partially-loaded systems. As Fig. 12 depicts, quartic CSS performance in both a fully synchronized and QS conditions will improve with the use of fewer signals ($K < N=40$) when the $K$ signals are selected
To finish description of our initial performance results, we simulated CSS performance over a dispersive air-ground channel. The channel models are based on empirical air to ground measurement results of NASA, reported in [21] – [24]. Table I lists channel parameters for two locations: suburban Palmdale, CA, and the near urban setting for Cleveland, OH.

| Parameters                  | Suburban Palmdale, CA | Near urban Cleveland, OH |
|-----------------------------|-----------------------|---------------------------|
| Mean RMS delay spread       | 53.78 ns              | 16.6 ns                   |
| Maximum RMS delay spread    | 1254.1 µs             | 70.13 ns                  |
| Frequency                   | 5.06 GHz              | 5.06 GHz                  |
| Sounding bandwidth          | 50 MHZ                | 50 MHZ                    |
| Altitude                    | 850 m                 | 850 m                     |

Example sequences of simulated power delay profiles are shown in Fig. 14. As can be seen, RMS delay spreads are larger for suburban Palmdale than for the near urban Cleveland channel. Since we employ no equalization or multipath mitigation in these initial results, we expect poorer performance in the suburban case.

The links for the example AG channels are a set of air to ground links emulating a multipoint to point air to ground system with total data rate of 100 kbit/s and total bandwidth of 400 kHz. There are N=10 users, each transmitting at 10 kbps over this bandwidth, the value of which is comparable to that proposed for other AG systems [25]. Transmissions from aircraft are received at the ground station quasi-synchronously, with zero-mean Gaussian distributed timing offsets with σ=0.1T, with each AG signal encountering its own unique channel.

Fig. 15 shows BER performance of the CSS signals over these realistic AG channels. For this relatively small bandwidth, the channel fading is essentially flat, except for the largest values of delay spread, which occur with low probability. As expected based on delay spreads, results are better for near urban Cleveland than for suburban Palmdale. Performance of the quarter-loaded quartic system again illustrates the substantial effect of MAI in this system.

Further investigation of quartic case in a “canonical” Ricean fading channels and realistic aeronautical channels provided in [26].

VI. CONCLUSION

In this paper, we investigated multi user chirp spread spectrum system performance in quasi-synchronous conditions, for the classic linear chirp, several existing nonlinear chirp designs in the literature, and two new nonlinear chirps. We derived a closed form expression for the cross correlation for linear chirps in quasi-synchronous conditions, and obtained example correlation statistics for the linear and our new nonlinear chirps. The two new nonlinear chirp designs were proposed to improve performance under imperfectly synchronized conditions. The linear chirps are generally best in perfectly synchronized cases, but we showed that since our nonlinear cases use more “time-frequency space,” they can outperform nearly all other chirp designs we have evaluated, for a range of assumed Gaussian-distributed timing offsets. Performance of our new designs is particularly superior in nonfully-loaded systems. Our new quartic nonlinear chirp design performs best. We also illustrated performance improvements of our new designs over a realistic dispersive air-ground channel. For future research, mathematical efforts will focus on refining analysis of the nonlinear cases. Additional work will investigate effects of other timing offset distributions, performance over additional dispersive channels, and other methods of chirp modulation.

ACKNOWLEDGEMENT

The authors would like to thank Dr. H. Jamal of the University of South Carolina for development of the AG channel Matlab routines.

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