On the power domination number of corona product and join graphs

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Abstract. A set $D$ of vertices of graph $G$ is called a dominating set if every vertex $u \in V(G) - D$ is adjacent to some vertex $v \in D$. A set $D$ to be a power dominating set of a graph if every vertex and every edge in the system are monitored by the set $D$. The power domination number $\gamma_p(G)$ of a graph $G$ is the minimum cardinality of a power dominating set of $G$. In this paper, we analyze the power domination number of corona graphs and join graphs. The corona product of two graphs $G_1$ and $G_2$ denoted as $G_1 \odot G_2$ is defined as the graph $G$ obtained by taking one copy of $G_1$ and $|V(G_1)|$ copies of $G_2$, and then joined by an edge the $i$'th vertex of $G_1$ to every vertex in the $i$'th copy of $G_2$. The join of two graphs $H_1$ and $H_2$ is a graph formed from disjoint copies of $H_1$ and $H_2$ by connecting every vertex of $H_1$ to every vertex of $H_2$. Join graph $H_1$ and $H_2$ denoted by $H_1 + H_2$. The results show that the power domination number of some corona product and join graphs attain the lower bound.

1. Introduction

In this paper we suppose a graph $G$ which is nontrivial, finite, simple, undirected, and connected graph. Let $V(G)$ and $E(G)$ be respectively vertex and edge set of $G = (V(G), E(G))$. A set $D$ of vertices of a graph $G = (V, E)$ is dominating if every vertex in $V(G) - D$ is adjacent to some vertex in $D$. The domination number of $G$, denoted by $\gamma(G)$, is the minimum cardinality of a dominating set in $G$. For definition and notation of power dominating set in [6] are explained that the open neighborhood of a vertex $v \in V(G)$ is $N_G(v) = \{u \in V(G); uv \in E(G)\}$ and its closed neighborhood is the set $N_G[v] = N_G(v) \cup \{v\}$. A set $D$ of vertex set of $G$, $N_G[D]$ is the union of all closed neighborhoods of vertices in $D$. The degree of $v$ is $d_G(v) = |N_G(v)|$. If the graph $G$ is a connected graph, we simply write $V(G), E(G), N(v), N[v], N[D]$ and $d(v)$ rather than $V(G), E(G), N_G(v), N_G[v], N_G[D]$ and $d_G(v)$, respectively. A set $D$ to be a power dominating set of a graph if every vertex and every edge in the system is monitored by the set $D$. The power domination number $\gamma_p(G)$ of a graph $G$ is the minimum cardinality of a power dominating set of $G$.

The concept of a dominating set was introduced and first studied by Slater [11, 12, 5, 1] and Waspodo et. al. [7] studied the bound of distance domination number of edge comb product. Dafik et. al. [4] and Wardani et. al. [3] studied the exact values of locating domination number of some edge comb product graphs $S_n \supseteq H$ and $P_n \supseteq H$. Wardani et. al. [2] also determine
Figure 1. Example of Corona Product Graph

the exact values of locating independent dominating number of some special graphs and its operations. The concept of power dominating set is studied by Zhao [10], Koh [9], and Chang [8].

The definition of corona product of graph is taken from Santi [13]. The corona $G_1 \circ G_2$ of two graphs $G_1$ and $G_2$ is defined as the graph $G$ obtained by taking one copy of $G_1$ and $p_1$ copies of $G_2$, and then joins by an edge the $i$’th vertex of $G_1$ to every vertex in the $i$’th copy of $G_2$. The join of two graphs $H_1$ and $H_2$ is a graph formed from disjoint copies of $H_1$ and $H_2$ by connecting every vertex of $H_1$ to every vertex of $H_2$. Join graph $H_1$ and $H_2$ are denoted by $H_1 + H_2$. Figure 1 is example of corona product graph.

The power dominating set can be applied to the recloser placement on the PLN electricity network. A network topology is the arrangement of a network, including its nodes and connecting lines. There are two ways of defining network geometry: the physical topology and the logical (or signal) topology.

Network topology on medium voltage network systems is generally divided into five forms of radial system network configuration, open loop system, close loop system, clutser system, and spindle system.

The radial system is the simplest, cheapest medium voltage distribution network system, widely used especially for small systems, rural areas.

Open loop system is the development of a radial system, as a result of the need for higher reliability and generally this system can be supplied in a substation. It is also possible from other substations but must be in one system on the side of high voltage because this is needed to facilitate maneuvering of loads in the event of an interruption or load reduction conditions. Protection for this system is still simple but must take into account the length of the network at the furthest point of the system.

Close loop system is a system that is feasible to be used for networks supplied from one substation, requiring a fairly complicated protection system usually using directional or directional relays. This system has higher reliability than other systems, and this system is rarely used in PLN but is usually used for special customers who require high reliability.

The spindle system is a relatively reliable system because it provides one express feeder which is a no-load feeder from the substation until the substation or GH reflection. This system is relatively expensive because it is usually in the construction as well as to overcome the development of the burden in the future, the protection is relatively simple almost the same as the open loop system. Usually in each feeder in this system is provided a middle point that serves to point the maneuver if there is a disruption in the network.

The clutser system is almost similar to the spindle system. In a cluster system there is one express feeder which is a no-load feeder that is used as a point of transferring loads by other feeders in the cluster system. The protection needed for this system is relatively the same as an open loop system or a spindle system.

In the distribution network switching tool, namely automatic circuit recloser or automatic circuit breaker is better known as recloser. Basically, recloser is a circuit breaker that is equipped with a control device. This recloser tool works if there is interference in the surrounding area
that it protects, it can open and close automatically.

Figure 2, figure 3 and figure 4 is example of switching tool on distribution network.

Disturbances in the distribution system can come from the system itself and interference from outside. Internal disturbances include: over voltage and overcurrent, improper installation, cabling does not follow rules, aging, and overload. While interference from outside, among
Let \( \gamma \) be natural number with \( \gamma \geq 2 \) and \( \gamma \geq 3 \), then
\( \gamma_p(G) \geq 1 \).

**Proof.** Based on the definition of power domination number graph with \( \Delta < 3 \) obtain dominator at least one, then \( \gamma_p(G) \geq 1 \).

**Theorem 1.** Let \( G \) be a simple, connected undirected graph, then \( \gamma_p(G) \geq 1 \).

**Proof.** Graph with diameter \( = 1 \) there is a vertex \( v \) connected to every vertex in \( G - \{v\} \), only one dominator for dominated vertex except \( v \). Then \( \gamma_p(G) = 1 \).

**Theorem 2.** For \( n, m \) be natural number with \( n \geq 2 \) and \( m \geq 2 \), then \( \gamma_p(C_n + H) = 1 \).

**Proof.** Let \( G \) be a join graph of \( C_n \) with order \( n \) and \( H \) be any graph with order \( m \), denoted by \( G = C_n \oplus H \). Graph \( G \) has vertex set \( V(G) = \{x_i, y_j; 1 \leq i \leq n, 1 \leq j \leq m\} \) and edge set \( E(G) = \{x_ix_{i+1}, x_ix_n; 1 \leq i \leq n - 1\} \cup \{e_k; 1 \leq k \leq s\} \cup \{x_iy_j; 1 \leq i \leq n, 1 \leq j \leq m\} \), then
\( |V(G)| = n + m \) and \( |E(G)| = mn + n + s \).

Base on the Lemma 1 then \( \gamma_p(G) \geq 1 \). Then we choose dominator \( D = \{v; v = \Delta\} \) so \( |D| = 1 \) and \( \gamma_p(G) \leq 1 \). Then \( \gamma_p(G) = 1 \).

![Figure 5. Power Dominating Set of \( C_3 + S_3 \)](image_url)

**Theorem 3.** For \( n, m \) be natural number with \( n \geq 2 \) and \( m \geq 3 \), then \( \gamma_p(P_n + H) = 1 \).

**Proof.** Let \( G \) be a join graph of \( P_n \) with order \( n \) and \( H \) be any graph with order \( m \), denoted by \( G = P_n \oplus H \). Graph \( G \) has vertex set \( V(G) = \{x_i, y_j; 1 \leq i \leq n, 1 \leq j \leq m\} \) and edge set \( E(G) = \{x_ix_{i+1}, x_ix_n; 1 \leq i \leq n - 1\} \cup \{e_k; 1 \leq k \leq s\} \cup \{x_iy_j; 1 \leq i \leq n, 1 \leq j \leq m\} \), then
\( |V(G)| = n + m \) and \( |E(G)| = mn + n + 1 + s \).

Based on the Lemma 1 then \( \gamma_p(G) \geq 1 \). Then we choose dominator \( D = \{v; v = \Delta\} \) so \( |D| = 1 \) and \( \gamma_p(G) \leq 1 \). Then \( \gamma_p(G) = 1 \).

**Theorem 4.** For \( n, m \) be natural number with \( n \geq 3 \) and \( m \geq 3 \), then \( \gamma_p(K_n + H) = 1 \).

**Proof.** Let \( G \) be a join graph of \( K_n \) with order \( n \) and \( H \) be any graph with \( m \), denoted by \( G = K_n \oplus H \). Graph \( G \) has vertex set \( V(G) = \{x_i, y_j; 1 \leq i \leq n, 1 \leq j \leq m\} \) and edge set \( E(G) = \{x_ix_{i+1}, 1 \leq i \leq n - 1\} \cup \{e_k; 1 \leq k \leq s\} \cup \{x_iy_j; 1 \leq i \leq n, 1 \leq j \leq m\} \), then
\( |V(G)| = n + m \) and \( |E(G)| = \frac{n(n+1)}{2} + mn + n \).

Based on the Lemma 1 then \( \gamma_p(G) \geq 1 \). Then we choose dominator \( D = \{v; v = \Delta\} \) so \( |D| = 1 \) and \( \gamma_p(G) \leq 1 \). Then \( \gamma_p(G) = 1 \).
Theorem 5. For \( n, m \) be natural number with \( n \geq 2 \) and \( m \geq 3 \), then \( \gamma_p(G \odot H) = |V(G)| \).

Proof. Let \( G \) be a corona product of any graph \( G_1 \) with order \( n \) and \( G_2 \) be any graph with order \( m \), denoted by \( G = G_1 \odot G_2 \). Graph \( G \) has vertex set \( V(G) = \{x_i, y_{i,j}; 1 \leq i \leq n, 1 \leq j \leq m\} \) and edge set \( E(G) = \{e_k; 1 \leq k \leq s\} \cup \{d_{k,l}; 1 \leq k \leq s, 1 \leq l \leq u\} \cup \{x_i y_{i,j}; 1 \leq i \leq n, 1 \leq j \leq m\} \), then \( |V(G)| = n + nm \) and \( |E(G)| = s + su + nm \).

Based on Theorem 4 if \( n = 1; K_1 \) so \( \gamma_p(K_1 + H) = 1 \), based on definition corona and join \( K_1 + H = K_1 \odot H \) then \( \gamma_p(K_1 \odot H) = 1 \). \( G \odot H \) be collect subgraph \( K_1 \odot H \) then \( \gamma_p(G) = |V(G)| \).

Then we choose dominator \( D = \{x_i; 1 \leq n \leq n\} \) so \( |D| = |V(G)| \) and \( \gamma_p(G) \leq |V(G)| \). Then \( \gamma_p(G) = |V(G)| \). \( \square \)

3. Application of Power Dominating Set

In this section we explain about result of power domination number of placement recloser in PT. PLN (Persero) Jember area.

The electricity system that should be concerned is how you can successfully distribute electricity to PLN customers. Besides, in agreement with the development of the times, electricity becomes a primary necessary, especially among urban communities. Factors of quality or quality of electricity become the thing that is started to be the main demand. Not often extinguished that is stable and does not fluctuate as demand of primary concern.

The power of electricity is ranging from high voltage (500kV) or through high voltage (70kV and 150kV). From PLN powerline substations to be 20kV, and to supply that power for PLN customers to be 6kV or through non-light transformers, low voltage tension 220 V or 110 V for PLN customers.

The continuous electricity power distribution needs ways and methods for managing it. Especially, in PLN distribution system network, on the distribution necessary system is the
middle-lower voltage 20kV. In the distribution network system, there are four main things that need attention: reliability, continuity, quality, and flexibility.

Quality of reliability can be seen from the time unit, for example in one year, one semester or one month. The domain level that conforms to the community standard can be energized on an ongoing basis. The reliability indicators of the distribution system in the world are the value of SAIFI (System Average Interruption Frequency Index) and SAIDI (System Interruption Duration Index). This value indicates the price or extinction that makes the customer does not get electricity service. SAIDI and SAIFI values are influenced by failure speed of distribution network system, thats come from failure probability of distribution network system tools or distribution network system load point. Distribution system reliability indicators are stated in the definition of SAIFI and SAIDI as follows:

\[
\text{SAIFI} = \frac{\text{the number of customers who experience interference}}{\text{the number of customers served}}
\]

\[
\text{SAIDI} = \frac{\text{the amount of disruption time from all customers}}{\text{the number of customers served}}
\]

One of the distribution network functions is switching recloser or electric circuit breaker either in loaded condition or unloaded condition. The installation of the recloser has been only a violation that caused a blackout somewhere. It has not yet presented what the customer is doing to protect and the safe distance between means for recloser with other recloser. Therefore, the recallers must be completely detailed. One of the most interesting topics in graph theory that permits this problem is the power of domination.

Figure 9 is the representation of Tegalboto feeder map. Figure 10 is the representation of graph Tegalboto feeder map with concept of power dominating set. Vertex of graph is representation of recloser and connector among recloser to be the edges. Tegalboto feeder graph consists of 49 vertices.

The placement of recloser on the Tegalboto feeder map initially only has six recloser but the area that is affected more means customers who does not get less electricity and electricity consumption per kWh decreased. After appliance with power concept need 7 recloser, so affected area is less mean customers who get electricity increase and usage of electricity perkWh increase too. So, the distribution of business units of PT.PLN (PERSERO) is reached.

Benefits of the recloser is as a network circuit breaker device if its error and disconnect the network that is covered other recloser. With the existence of this recloser household consumer can enjoy electricity continuously. Therefore, a big consumer such as companies, offices, factories, shops, etc. can not loss because the production cost using the electricity network PLN is cheaper than the using of generators, and also government offices and hospitals as a public place that
Figure 9. Tegalboto Feeder Map

need electricity too that it will served without constraints of blackouts frequently. All aspects of economy, education, health, etc. are in dire need of electricity. Figure 11 is Example of recloser.

4. Concluding Remarks
In this paper, we have determined the exact values of power domination number of some corona product and join graphs. As we have mentioned in introduction, to prove weather on power domination number is a hard problem. Thus, it still gives the following open problem.

Open Problem 6. Let $G$ be any connected graph, determine the lower bounds of $\gamma_P(G)$ in
Figure 10. Graph of Tegalboto Feeder Map with Power Dominator Set

another operation graphs?

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Figure 11. Example of Reclouser

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