Determination of the energy band gap of Bi$_2$Se$_3$

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Despite intensive investigations of Bi$_2$Se$_3$ in past few years, the size and nature of the bulk energy band gap of this well-known 3D topological insulator still remain unclear. Here we report on a combined magneto-transport, photoluminescence and infrared transmission study of Bi$_2$Se$_3$, which unambiguously shows that the energy band gap of this material is direct and reaches $E_g = (220 \pm 5)$ meV at low temperatures.

The existence of the energy band gap, separating the conduction and valence bands, is a key characteristic of all topological insulators, which allows these materials to behave as insulators in interior, but still, to conduct electric current via specific, topologically protected states on their surfaces$^{1,2}$.

In bismuth selenide (Bi$_2$Se$_3$) – perhaps the most representative example of 3D topological insulators – the band gap is located at the Brillouin zone center, however, the consensus regarding further details, notably its size and nature, has not yet been established. Rather large values for the band gap (above 300 meV) have been deduced consistently show a direct band gap around 200 meV. Importantly, the missing consensus about the band gap is not a minor drawback in our understanding properties of Bi$_2$Se$_3$. It is the size and nature of the band gap which are the parameters needed for interpretation of ARPES data$^{3,4}$, most notably, for the correct positioning of the surface cone with respect to the bulk bands. For a wider band gap, the Dirac point of the surface states approaches the midgap position. A narrower band gap shifts the charge neutrality point towards the valence band, which may, for instance, explain the pronounced electron-hole asymmetry of the surface cone observed in STM/STS experiments$^{1,2}$.

In this paper, we address the existing controversy about the band gap of Bi$_2$Se$_3$ using an experimental approach which combines optical methods – infrared transmission and photoluminescence – with magneto-transport, successfully used in the past also for other materials, see, e.g., refs 13 and 14. We unambiguously show that the energy band gap of Bi$_2$Se$_3$ is direct and reaches $E_g = (220 \pm 5)$ meV.

**Experimental Details**

To determine the band gap of Bi$_2$Se$_3$, bulk crystal of this compound was grown using the modified Bridgman method where stoichiometric mixture of high purity Bi and Se elements were vacuum sealed in a quartz tube, heated up to the melting point and cooled down to room temperature with the rate of 0.1 °C/min under the temperature gradient of about 10 °C/cm along the tube length in a box furnace. As-grown crystals showed a strong n-type doping (close to $10^{19}$ cm$^{-3}$) due to selenium vacancies, which was reduced by the after-growth annealing in selenium vapors down to $10^{18}$ cm$^{-3}$, nevertheless with a certain variation of the electron density across the crystal.

The bulk crystal was characterized using x-ray diffractometer equipped with Cu x-ray tube, channel-cut germanium monochromator and scintillation detector. The standard $\theta - 2\theta$ scan is shown in Fig. 1. The observed x-ray diffraction peaks correspond well to the $c$-lattice parameter of $c = (28.64 \pm 0.01)$ Å in an perfect agreement with tabulated value 28.63 Å$^{11,15}$. The crystal has been then sliced using microtome machine perpendicular to the $c$-axis of Bi$_2$Se$_3$. Two free-standing layers with the thicknesses of $d = 6.5$ and 10 μm have been chosen for this
study denoted as samples A and B, respectively. They were explored in low temperature photoluminescence (PL), infrared transmission and magneto-transport experiments.

To measure PL spectra, the samples were placed in a helium bath cryostat and excited by $\lambda = 660$ nm diode laser with an approximate power of $100 \mu W$ focused on spot of $1 \text{ mm}^2$. The collected signal was delivered to a Fourier transform spectrometer, analyzed and detected by a liquid-nitrogen-cooled MCT detector. To measure infrared transmission, a macroscopic area of the sample ($\approx 3 \text{ mm}^2$) was exposed to the radiation of a globar, which was analyzed by a Fourier transform spectrometer, using light-pipe optics delivered to the sample placed in a helium bath cryostat and detected by a composite bolometer placed just below the sample. Magneto-transport experiments were conducted on samples contacted using silver paste in the Van der Pauw-like geometry. Measurements were performed using a standard low-frequency lock-in technique in a variable temperature insert, with the magnetic field applied along the $c$-axis of Bi$_2$Se$_3$.

Discussion

We start the discussion with the PL spectra recorded from both samples at liquid-helium temperatures (Fig. 2a and b). It is just the existence of a well-defined efficient PL emission, which clearly indicates a direct nature of the band gap in Bi$_2$Se$_3$. In other words, the conduction-band electrons are located around the same point of the Brillouin zone as the photo-excited holes in the valence band. The positions of the observed PL emission in spectra then provide us with the very first estimate for the size of the band gap.

In the simplest possible scenario, one may assume that the observed PL corresponds to a direct band-to-band (vertical in $k$-space) recombination of electrons from the degenerate gas at the bottom of the conduction band with photo-excited holes at the top of the valence band. In such a case, the band gap can be associated with the inflection point at the low-energy onset of PL emission line, having for both samples the energy close to $220 \text{ meV}$ and denoted in Fig. 2a and b by vertical dashed lines. From other, more complex scenarios for PL mechanisms, we may exclude, due to screening effects in the degenerate electron gas, the excitonic-like recombination. However, one may still imagine a variety of defects-related radiative recombination channels, often efficient in semiconducting compounds and giving rise to sub-band-gap emission of light. The photon energy of the PL emission thus serves only as a lower bound for the (direct) band gap in Bi$_2$Se$_3$.

Another estimate of the band gap $E_g$ in this case implying its upper bound, comes from the analysis of the infrared transmission (Fig. 2c and d). Both samples show rather broad transparency window, which is at low photon energies limited by the free carrier response (reflectivity below the plasma edge) and also by the absorption due to infrared active phonons$^{16}$. At high photon energies, the transmission window closes due to a relatively sharp onset of interband absorption. This onset is often referred to as the optical band gap $E_{g}^{\text{opt}}$ and represents the upper bound for the band gap due to the well-known Moss-Burstein shift that is characteristic of semiconductors with a degenerate electron or hole gas (see the inset of Fig. 2a)$^{17}$.

The approximate position of the optical band gap is denoted in Fig. 2c and d by the vertical arrow. A more precise read-out of $E_{g}^{\text{opt}}$ is possible when transmission is plotted as absorbance and normalized by the sample thickness (insets of Fig. 2c,d). When the optical band gap is approached, the absorbance becomes dominated by absorption: $\alpha \approx -\ln(T)/d$, which increases almost exponentially that is reminiscent of the Urbach edge absorption in (undoped) semiconductors$^{18}$. The optical band gap may then be associated with the photon energy at which the absorption coefficient $\alpha$ approaches $10^4 \text{ cm}^{-1}$, a value typical for interband absorption in direct-band-gap semiconductors$^{19}$. For the sample A and B, we obtain $E_{g}^{\text{opt},A} = (250 \pm 3) \text{ meV}$ and $E_{g}^{\text{opt},B} = (258 \pm 3) \text{ meV}$, respectively.

To extract the size of the band gap $E_g$ from $E_{g}^{\text{opt}}$, the Moss-Burstein shift has to be estimated. In a degenerate n-type semiconductor with a direct band gap, this shift reads: $\Delta E_{MB} = E_{g}^{\text{opt}} - E_g = (1 + m_e/m_h)E_F$, where $E_F$ is the Fermi energy and $m_e(m_h)$ stands for the electron (hole) effective mass. The anisotropy of effective masses enters this expression only, when the ratio $m_e/m_h$ becomes strongly anisotropic, which does not seem to be the case of
Bi$_2$Se$_3$. This formula is valid only for systems with well-defined effective masses, and therefore strictly parabolic bands. Nevertheless, it is the existence of the direct band gap (implied by our PL data), which ensures such a parabolicity of bands, at least in the vicinity of the band edges. Let us also note, the parabolic shape of both, conduction and valence, bands is also consistent with results of magneto-transport experiments performed on bulk Bi$_2$Se$_3$ specimens and Landau level spectroscopy on thin epitaxial layers.

Importantly, the Moss-Burstein shift may be expressed as $\Delta E_{MB} = e\hbar F/\mu$, where $\mu$ stands for the reduced mass, $m_em_h/(m_e + m_h)$, and $F$ is the characteristic frequency of the $1/B$-periodic quantum oscillations, $F = m_eE_B/(\hbar e)$, which are associated with the Landau quantization of electrons, emerging under an externally applied magnetic field. These were clearly resolved in the magneto-transport data in a form of Shubnikov-de Haas effect (Fig. 3a and b). In both samples, a single oscillation frequency has been found, $F_A = 22.0 \pm 0.5$ T and $F_B = 28.0 \pm 0.5$ T, consistently with expectations for a degenerate electron gas in a simple parabolic conduction band. Let us note that the observed Shubnikov-de Haas oscillations originate in bulk states of Bi$_2$Se$_3$. The contribution of the surface states to the transport response remains at given bulk electron densities negligible.

This may clearly demonstrated, e.g., by the angle dependence of Shubnikov-de Haas oscillations, which we have studied on samples coming from the same batch in the scope of our preceding NMR study, see ref. 23 and the related Supplementary Materials. These magneto-transport experiments also implied (via damping of Shubnikov-de Haas oscillations with temperature) the effective mass of bulk conduction band electrons, $m_e \approx 0.12m_0$, which agrees well with values from previous studies. Combining this electron mass with the hole mass from our recent magneto-transport studies performed on $p$-type Bi$_2$Se$_3$, $m_h \approx 0.24m_0$, we obtain

![Figure 2](image-url)
the reduced mass of \( \mu \approx 0.08m_0 \) in perfect agreement with the value read directly from the separation of inter-band inter-Landau level resonances observed in our recent magneto-optical study\(^{11}\).

Taking account of the Shubnikov-de Haas oscillation frequencies and the estimated effective reduce mass, we get the Moss-Burstein shifts of \( \Delta E_{\text{MB}} \approx 30 \) meV and \( \Delta E_{\text{MB}}^{\text{opt}} \approx 40 \) meV for the sample A and B, respectively. Subtracting these values from the optical band gaps \( E_{\text{opt,A}} \) and \( E_{\text{opt,B}} \), we obtain at the energy band gap of Bi\(_2\)Se\(_3\): \( E_g = (220 \pm 5) \) meV. This value is in perfect agreement with the PL results, when the simplest scenario of direct band-to-band recombination of free electrons and holes is considered. Let us also note that we do not consider the band gap renormalization, which for electron densities close to \( 10^{18} \) cm\(^{-3} \) provides us with a correction of the band gap well below the estimated error bar\(^{27}\).

Let us emphasize that the extracted band gap, \( E_g = (220 \pm 5) \) meV, as well as its direct nature clearly contrasts with conclusions of several ARPES experiments performed on bulk Bi\(_2\)Se\(_3\)\(^{28-32}\), nevertheless not with all of them, see, e.g., refs 6 and 7. This technique thus seems to be well-suited for investigations of surface properties and the observation of conical bands on the surface of topological insulators is definitely one of its greatest achievements\(^{33}\). At the same time, the visualization of a truly bulk electronic band structure, especially in narrow gap materials, which are characterized by pronounced band bending effects\(^{34}\) and charge accumulation layers on the surface, may be a challenging task for the surface-sensitive techniques, with a strongly limited penetration depth.

### Conclusions

In summary, we have explored bulk Bi\(_2\)Se\(_3\) using magneto-transport and infrared spectroscopy techniques aiming at determining the nature and size of the energy band gap in this 3D topological insulator. We have shown that the energy band gap is direct and falls into the interval of \( E_g = (220 \pm 5) \) meV.

### References

1. Hasan, M. Z. & Kane, C. L. Colloquium: Topological insulators. Rev. Mod. Phys. 82, 3045–3067 (2010).
2. Qi, X.-L. & Zhang, S.-C. Topological insulators and superconductors. Rev. Mod. Phys. 83, 1057–1110 (2011).
3. Xia, Y. et al. Observation of a large-gap topological-insulator class with a single dirac cone on the surface. Nature Phys. 5, 398–402 (2009).
4. Bianchi, M. et al. Coexistence of the topological state and a two-dimensional electron gas on the surface of Bi\(_2\)Se\(_3\). Nature Comm. 1 (2010).
5. Kim, S. et al. Surface scattering via bulk continuum states in the 3d topological insulator Bi\(_2\)Se\(_3\). Phys. Rev. Lett. 107, 056803 (2011).
6. Chen, Y. L. et al. Massive dirac fermion on the surface of a magnetically doped topological insulator. Phys. Rev. B 81, 121111 (2010).
7. Nechaev, I. A. et al. Evidence for a direct band gap in the topological insulator bi\(_2\)se\(_3\) from theory and experiment. Phys. Rev. B 87, 235112 (2013).
8. Gobrecht, H., Seek, S. & Klose, T. Der einfluß der freien ladungsträger auf die optischen konstanten des bi\(_2\)se\(_3\) im wellenlängengebiet von 2 bis 23 \( \mu \)m. Zeitschrift für Physik 190, 427–443 (1966).
9. Kohler, H. & Hartmann, J. Burstein shift of the absorption edge of n-bi\(_2\)se\(_3\), phys. stat. sol. (b) 63, 171–176 (1974).
10. Post, K. W. et al. Thickness-dependent bulk electronic properties in bi\(_2\)se\(_3\), thin films revealed by infrared spectroscopy. Phys. Rev. B 88, 075121 (2013).
11. Orhita, M. et al. Magneto-optics of massive dirac fermions in bulk bi\(_2\)se\(_3\), Phys. Rev. Lett. 114, 186401 (2015).
12. Cheng, P et al. Landau quantization of topological surface states in bi\(_2\)se\(_3\), Phys. Rev. Lett. 105, 076801 (2010).
13. Wu, J. et al. Unusual properties of the fundamental band gap of inn. Applied Physics Letters 80, 3967–3969 (2002).
14. Wu, J. et al. Small band gap bowing in inn. Applied Physics Letters 80, 4741–4743 (2002).
15. The crystal structure of bi\(_2\)se\(_3\) - se\(_3\), Journal of Physics and Chemistry of Solids 24, 479–485 (1963).
16. Richter, W., Kohler, H. & Becker, C. Raman and far-infrared investigations of phonons in rhombohedral V\(_2\)-V\(_3\) compounds - Bi\(_2\)Te\(_3\), Bi\(_2\)Se\(_3\), Se\(_3\)Te\(_3\) and Bi\(_2\)(Te\(_1\) – Se\(_1\))-Bi\(_2\)(Te\(_3\) – Se\(_3\)) (0 < x < 1), (Bi1 – ySby)Te3 (0 < y < 1). physica status solidi b 84, 619–628 (1977).
17. Burstein, E. Anomalous optical absorption limit in insb. Phys. Rev. 93, 632–633 (1954).
18. Urbach, F. The long-wavelength edge of photographic sensitivity and of the electronic absorption of solids. Phys. Rev. 92, 1324–1324 (1953).
19. Yu, P. Y. & Cardona, M. Fundamentals of Semiconductors (Springer, Berlin Heidelberg, 2010).
20. Köhler, H. Conduction band parameters of bi$_2$se$_3$ from shubnikov–de haas investigations. Phys. stat. sol. (b) 58, 91–100, http://onlinelibrary.wiley.com/doi/10.1002/pssb.2220580109/abstract (1973).
21. Piot, B. A. et al. Hole fermi surface in bi$_2$se$_3$ probed by quantum oscillations. Phys. Rev. B 93, 155206 (2016).
22. Fauqué, B. et al. Magnetothermoelectric properties of bi$_2$se$_3$. Phys. Rev. B 87, 035133 (2013).
23. Mukhopadhyay, S. et al. Hyperfine coupling and spin polarization in the bulk of the topological insulator bi$_2$se$_3$. Phys. Rev. B 91, 081105 (2015).
24. Sushkov, A. B. et al. Far-infrared cyclotron resonance and faraday effect in bi$_2$se$_3$. Phys. Rev. B 82, 125110 (2010).
25. Butch, N. P. et al. Strong surface scattering in ultrahigh-mobility bi$_2$se$_3$ topological insulator crystals. Phys. Rev. B 81, 241301 (2010).
26. Eto, K., Ren, Z., Taskin, A. A., Segawa, K. & Ando, Y. Angular-dependent oscillations of the magnetoresistance in bi$_2$se$_3$ due to the three-dimensional bulk fermi surface. Phys. Rev. B 81, 195309 (2010).
27. Auvergne, D., Camassel, J. & Mathieu, H. Band-gap shrinkage of semiconductors. Phys. Rev. B 11, 2251–2259 (1975).
28. Hsieh, D. et al. A topological dirac insulator in a quantum spin hall phase. Nature 452, 970–974 (2008).
29. Veyrat, L. et al. Band bending inversion in bi$_2$se$_3$ nanostructures. Nano Letters 15, 7503–7507 (2015).

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**Author Contributions**
G.M., M.O. and M.P. conceived the study. Experiments were performed by B.A.P., M.H., G.M., and M.O. The samples were prepared and characterized by Y.S.H., A.M., S.G.S., C.D., A.H., O.C. and J.N. The data were analyzed by G.M., M.P., B.A.P., A.D., O.C., J.N. and M.O. The manuscript was written by M.O. and G.M. All authors discussed the results and commented on the manuscript.

**Additional Information**

**Competing Interests:** The authors declare that they have no competing interests.

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