Once more on electromagnetic form factors of nucleons in extended vector meson dominance model

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Extended vector meson dominance model, that allows to describe the electromagnetic form factors of nucleons obeying the asymptotic quark counting rule prescriptions and contains the minimal number of free parameters, is presented. We get a reasonable fit of form factors over experimentally available space-like region of momentum transfer and get also reasonable results in the time-like region.

PACS: 25.75.Dw, 13.30.Ce, 12.40.Yx

I. INTRODUCTION

In a series of our papers an extended vector meson dominance model was successfully applied to the description of electromagnetic transition form factors of nucleon resonances\textsuperscript{[1]} that are necessary to find out the dilepton outcome from the decays of these resonances produced either in proton-proton\textsuperscript{[2]} or in heavy ion collisions\textsuperscript{[3,4]}. We didn’t say anything about how the same model could describe the fundamental electromagnetic form factors of nucleons themselves. Although these form factors were the subject of considerations of many other models (see the review\textsuperscript{[5]} for the elongated list of references) including the extended vector meson dominance model itself\textsuperscript{[6]} we would like to apply our own version of extended vector meson dominance model to the description of nucleon form factors. It contains the minimal number of free parameters and gives a reasonable result.
II. NUCLEON ELECTROMAGNETIC FORM FACTORS IN THE SPACE-LIKE REGION

Electromagnetic form factors of nucleon $F_1^N(q^2)$, $F_2^N(q^2)$ describe its electromagnetic current

$$<N(p')|J_\mu|N(p)>=\bar{u}(p')(F_1^N(q^2)\gamma_\mu + \sigma_{\mu\nu}\frac{q_\nu}{2m}F_2^N(q^2))u(p)$$ (1)

and define the elastic scattering of electrons off nucleons in the space-like region $q^2 = -Q^2 < 0$ and electron-positron annihilation to nucleon-antinucleon in the time-like region $q^2 > 0$. The values of form factors at origin $F_1^N(0)$, $F_2^N(0)$ are connected to the charges and anomalous magnetic moments of nucleons and are equal for proton and neutron to

$$F_1^p(0) = 1, \quad F_2^p(0) = 1.79, \quad F_1^n(0) = 0, \quad F_2^n(0) = -1.92.$$ (2)

Quark counting rules \cite{7} predict the asymptotics of form factors in the space-like region $Q^2 \to \infty$

$$F_1^N(Q^2) \sim \frac{1}{Q^4}, \quad F_2^N(Q^2) \sim \frac{1}{Q^6}.$$ (3)

In extended Vector Meson Dominance (eVMD) model such asymptotics comes from destructive interference of ground and excited states of vector mesons $V$, $V'$, $V''$

$$F_1^N(Q^2) = \frac{F_1^N(0) + c^NQ^2}{(1 + \frac{Q^2}{m_V^2})(1 + \frac{Q^2}{m_{V'}^2})(1 + \frac{Q^2}{m_{V''}^2})}, \quad F_2^N(Q^2) = \frac{F_2^N(0)}{(1 + \frac{Q^2}{m_V^2})(1 + \frac{Q^2}{m_{V'}^2})(1 + \frac{Q^2}{m_{V''}^2})}.$$ (4)

Isovector $F_1^\rho(Q^2)$, $F_2^\rho(Q^2)$ and isoscalar $F_1^\omega(Q^2)$, $F_2^\omega(Q^2)$ form factors are defined as linear superpositions of proton and neutron form factors

$$F_{1,2}^\rho(Q^2) = \frac{F_{1,2}^p(Q^2) - F_{1,2}^n(Q^2)}{2}, \quad F_{1,2}^\omega(Q^2) = \frac{F_{1,2}^p(Q^2) + F_{1,2}^n(Q^2)}{2}$$ (5)

and are described the coupling constants of $\rho$ and $\omega$ family mesons to nucleon ($f_{1,2}^{\rho NN}$, ...) and to photon ($g_{\rho}$, ...) respectively

$$F_{1,2}^\rho(Q^2) = \frac{f_{1,2}^{\rho NN}}{g_\rho} m_\rho^2 + \frac{f_{1,2}^{\rho NN}}{g_{\rho'}} m_{\rho'}^2 + \frac{f_{1,2}^{\rho'' NN}}{g_\rho} m_\rho^2 + \frac{f_{1,2}^{\rho'' NN}}{g_{\rho'}} m_{\rho'}^2$$ (6)

$$F_{1,2}^\omega(Q^2) = \frac{f_{1,2}^{\omega NN}}{g_\omega} m_\omega^2 + \frac{f_{1,2}^{\omega NN}}{g_{\omega'}} m_{\omega'}^2 + \frac{f_{1,2}^{\omega'' NN}}{g_\omega} m_\omega^2 + \frac{f_{1,2}^{\omega'' NN}}{g_{\omega'}} m_{\omega'}^2.$$ (7)

The masses of $\rho$ and $\omega$ family mesons are assumed to be the same and this allows to set an equivalence between the multiplicative forms (4) and additive forms (following from eqs. (5) – (7)) for proton and neutron form factors.
With the choice \( m_\rho = m_\omega = 0.770 \text{ GeV}, \ m_\rho' = m_\omega' = 1.250 \text{ GeV}, \ m_\rho'' = m_\omega'' = 1.450 \text{ GeV} \) (as was used before in the case of electromagnetic transition form factors of nucleon resonances \([1]\)) we have two free parameters to describe form factors in the space-like region: \( c^p \) and \( c^n \). They were fitted and are equal to \( c^p = 0.463 \text{ GeV}^{-2} \) and \( c^n = -0.297 \text{ GeV}^{-2} \). The results of the fit are presented in \( q^2 = -Q^2 < 0 \) regions of the Fig.1 and Fig.2 where electric and magnetic form factors

\[
G_E^N = F_1^N + \frac{q^2}{4m^2}F_2^N, \quad G_M^N = F_1^N + F_2^N
\]

are shown. At small \( Q^2 \) the decomposition \( G_E^N \approx F_1^N(0) - \frac{1}{6}Q^2 < r_N^2 > \) defines the charge radii of proton \( \sqrt{< r_p^2 >} = 0.83 \text{ fm} \) (exp: 0.875 fm) and neutron \( < r_n^2 > = -0.06 \text{ fm}^2 \) (exp: -0.113 fm²).

For known coupling constants of the photon to \( \rho \) and \( \omega \) mesons \( g_\rho = 5.03 \) and \( g_\omega = 17.1 \) their coupling constants to the nucleon are equal to

\[
f_1^{\rho NN} = 3.02, \quad f_2^{\rho NN} = 20.8, \quad f_1^{\omega NN} = 17.2, \quad f_2^{\omega NN} = -2.47
\]

what is close to corresponding coupling constants used to describe Bonn potential \([8]\) of nucleon-nucleon interaction: \( f_1^{NN} = 3.2, \quad f_2^{NN} = 19.8, \quad f_1^{NN} = 15.9, \quad f_2^{NN} = 0. \)

III. **NUCLEON ELECTROMAGNETIC FORM FACTORS IN THE TIME-LIKE REGION**

In time-like region the finite widths of vector mesons should be taken into account

\[
F_1^\rho(q^2) = \frac{f_1^{\rho NN}}{g_\rho} \frac{m_\rho^2}{m_\rho^2 - im_\rho'\Gamma_\rho - q^2} + \frac{f_2^{\rho NN}}{g_\rho'} \frac{m_{\rho'}^2}{m_{\rho'}^2 - im_{\rho'}\Gamma_{\rho'} - q^2}
\]

\[
+ \frac{f_1^{\rho'' NN}}{g_\rho''} \frac{m_{\rho''}^2}{m_{\rho''}^2 - im_{\rho''}\Gamma_{\rho''} - q^2}
\]

\[
F_1^\omega(q^2) = \frac{f_1^{\omega NN}}{g_\omega} \frac{m_\omega^2}{m_\omega^2 - im_\omega\Gamma_\omega - q^2} + \frac{f_2^{\omega NN}}{g_\omega'} \frac{m_{\omega'}^2}{m_{\omega'}^2 - im_{\omega'}\Gamma_{\omega'} - q^2}
\]

\[
+ \frac{f_1^{\omega'' NN}}{g_\omega''} \frac{m_{\omega''}^2}{m_{\omega''}^2 - im_{\omega''}\Gamma_{\omega''} - q^2}
\]

We took \( \Gamma_\rho = 0.150 \text{ GeV}, \ \Gamma_\omega = 0.0085 \text{ GeV}, \ \Gamma_{\rho'} = \Gamma_{\omega'} = 0.300 \text{ GeV}, \ \Gamma_{\rho''} = \Gamma_{\omega''} = 0.500 \text{ GeV} \). The exact values of the widths are not very important when we are far from
the resonance region. Figs.1,2 show the prediction of eVMD model for time-like $q^2 > 0$. Experimental data were obtained with the assumption $|G^p_E| = |G^p_M|$ in the case of proton and with the assumption $|G^n_E| = 0$ in the case of neutron \cite{9}. Theoretically we have $|G^N_E| > |G^N_M|$ and their ratio grows with $q^2$. This means that for large $q^2$ the experimental points for $|G^N_M|$ should go significantly lower what is appreciated. Our results on $|G^p_E|/|G^p_M|$ ratio can be considered as the predictions for the future (like FAIR) antiproton facilities.

In the unphysical region of $q^2$, $0 < q^2 < 4m^2$, indicated on Figs.1,2 by vertical lines the resonance peaks of vector mesons (the largest one of $\omega$ meson) are clearly seen.

IV. CONCLUSION

An extended vector meson dominance model with a minimal number of free parameters is applied to the description of electromagnetic form factors of nucleons. The couplings of ground state $\rho$ and $\omega$ mesons to the nucleons are calculated and appear to be close to those of Bonn potential model of nucleon interaction. In the time-like region the absolute values of electric form factors are considerably larger than those of magnetic form factors and this can be used in the reanalysis of experimental data obtained with the assumption $|G^p_E| = |G^p_M|$ in the proton case and $|G^n_E| = 0$ in the neutron case.

Acknowledgments

This work is supported by RFBR grant No. 09-02-91341 and DFG grant No. 436 RUS 113/721/0-3. M.I.K. and B.V.M. acknowledge the kind hospitality at the University of Tübingen.

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FIG. 1: The modulus of magnetic form factor (up) and of the ratio of electric to magnetic form factors (down) of the proton in space- and time-like regions (for space-like region we use experimental data cited in review [5]).
FIG. 2: The modulus of magnetic form factor (up) and the modulus of electric form factor (down) of the neutron in space- and time-like regions (for space-like region we use experimental data cited in review [5]).