Massive Neutron Stars with a Color Superconducting Quark Matter Core

Takehiro Tanimoto,1,* Wolfgang Bentz,1,2,† and Ian C. Cloët1,‡

1Department of Physics, School of Science, Tokai University, 4-1-1 Kitakaname, Hiratsuka-shi, Kanagawa 259-1292, Japan
2Radiation Laboratory, Nishina Center, RIKEN, Wako, Saitama 351-0198, Japan
3Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA

We construct the equation of state for high density neutron star matter at zero temperature using the two-flavor Nambu–Jona-Lasinio (NJL) model as an effective theory of QCD. We build nuclear matter, quark matter, and the mixed phases from the same NJL Lagrangian, which has been used successfully to model free and in-medium hadrons as well as nuclear systems. A focus here is to determine if the same coupling constants in the scalar diquark and vector meson channels, which give a good description of nucleon structure and nuclear matter, can also be used for the color superconducting high density quark matter phase. We find that this is possible for the scalar diquark (pairing) interaction, but the vector meson interaction has to be reduced by a factor of 0.50 ~ 0.68 so that superconducting quark matter becomes the stable phase at high densities. A similar reduction in the vector field was also found, using the same NJL framework, in a description of the EMC effect in nuclear structure functions. We find that the maximum mass of a neutron star, with a color superconducting quark matter core, exceeds 2.01 ± 0.04 M⊙ which is the value of the recently observed massive neutron star PSR J0348+0432. The mass-radius relation is also consistent with gravitational wave observations (GW170817).

PhysSH: Quark model; Asymmetric nuclear matter; Nuclear matter in neutron stars.

I. INTRODUCTION

The study of strongly interacting cold matter at high baryon densities—neutron star matter—has been a very important and active area of research for several decades [1–5]. This field has attracted increased attention recently however, because of the observation of massive neutron stars exceeding two solar masses [6, 7] and gravitational wave measurements of a binary neutron star merger event [8–10]. Among the many theoretical tools used to study dense hadronic matter are nonrelativistic potential models [11, 12], effective field theories [13, 14], and relativistic theories based on a mean field description of nucleons interacting via meson exchange [15, 16]. The role of hyperons in dense hadronic matter has also been of interest [17, 18]. On the other hand, studies based on effective quark theories of QCD—often using the framework provided by the Nambu–Jona-Lasinio (NJL) model [19–22]—have focused on the existence of a possible quark matter phase at high densities [23–26] and the role of color superconductivity [27–29]. In these approaches the transition between the hadronic and quark matter phases has been described by using either the Maxwell or Gibbs constructions [30, 31], or by interpolation methods based on hadron-quark continuity [32, 33]. Finite size effects in the mixed phase caused by the surface tension and Coulomb interactions have also been studied [34–36].

Using the NJL model with the proper-time regularization scheme [37], which incorporates important aspects of confinement in hadronic systems [38, 39], we investigate whether a single effective quark theory of QCD which can describe both the structure of free [40–44] and in-medium hadrons [45–47] as well as nuclear systems [48], can also produce properties for neutron stars which agree with data. In addition, we will require a reasonable scenario for the transition from nuclear matter to the color superconducting quark matter phase. The nuclear matter equation of state which results from this picture, in the mean field approximation, has also been successfully used to describe medium effects on nuclear structure functions (the EMC effect) [45, 46] and response functions [47]. An extension of this model to describe the phase transition to color superconducting quark matter has been described in Refs. [48, 49]. Here we wish to extend this framework to also include the repulsive effects in quark matter which arise from the interaction in the vector meson channels. The importance of the isoscalar-vector repulsion in producing a sufficiently stiff quark matter equation of state has been pointed out in several recent papers [50, 51], and here we will also include the isovector-vector repulsion which is very important in nuclei [39, 52].

The model description of a single nucleon is based on the relativistic Faddeev equation in the quark-diquark approximation, where both scalar and axial-vector diquark channels are included [43]. A key question that we will address is whether the same strength of the scalar diquark interaction—which is required to reproduce the nucleon mass and other quark-quark correlation effects in baryons [43]—can also be used as the pairing interaction in color superconducting quark matter. Another aim is to investigate the critical role of the isoscalar-vector and isovector-vector repulsion in the quark matter phase, and compare the required strengths to those adjusted to the saturation density and symmetry energy of nuclear matter. For these purposes we will confine ourselves to two light quark flavors in both the nuclear matter and the quark matter phases, and neglect finite size effects in the mixed phase. We will also discuss the resulting properties of neutron stars in this approach and contrast these with recent observations.

The outline of the paper is as follows: Sec. II discusses the NJL model and the parameters that enter the calculations; Sec. III introduces the equations of state for nuclear matter and quark matter; Sec. IV presents a discussion of our results and a comparison with data; and a summary is given in Sec. V.

* Corresponding author: t.tanimoto@star.tokai-u.jp
† bentz@keyaki.cc.u-tokai.ac.jp
‡ icloet@anl.gov
II. LAGRANGIAN AND MODEL PARAMETERS

The two-flavor NJL Lagrangian relevant for this study reads [21, 22]:

\[
\mathcal{L} = \bar{\psi} (i\slashed{\partial} - m) \psi + G_\pi \left[ (\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\tau\psi)^2 \right] - G_\omega (\bar{\psi}\gamma^\mu\psi)^2 - G_\rho (\bar{\psi}\gamma^\mu\tau\psi)^2 + G_\delta \left( \bar{\psi}\gamma_\tau C\tau_2 \beta^A \psi \right)^2, \tag{1}
\]

where \( \psi \) is the quark field, \( m \) is the current quark mass, \( C \) is the charge conjugation matrix, \( \beta^A = \sqrt{3}/2 \lambda^A \) \((A = 2, 5, 7)\) are the antisymmetric color matrices, and \( \tau \) are the Pauli isospin matrices. The 4-fermion coupling constants in the scalar \( \bar{q}q \) channel, the isoscalar and isovector vector \( \bar{q}q \) channels, and the scalar \( qq \) channel are denoted by \( G_\pi, G_\omega, G_\rho, \) and \( G_\delta, \) respectively. The other model parameters are the 4-fermion coupling constant in the axial-vector \( qq \) channel \( (G_a) \) [43], and the infrared (IR) and ultraviolet (UV) regularization parameters \( \Lambda_{IR} \) and \( \Lambda_{UV} \), which are used with the proper-time regularization scheme [37, 38].

We stress that in this work we use the same model parameters as in several previous calculations which focused on the structure of nuclear matter systems [39, 46, 52]. That is, the parameters of the model are determined in the vacuum, the single hadron sector, and nuclear matter sector as follows: We fix \( \Lambda_{IR} = 0.24 \text{ GeV} \), and choose \( \Lambda_{UV}, m, \) and \( G_\pi \) so as to give a constituent quark mass in vacuum of \( M_0 = 0.4 \text{ GeV} \), the pion decay constant \( f_\pi = 0.93 \text{ GeV} \), and the pion mass \( m_\pi = 0.14 \text{ GeV} \) using the standard methods based on the gap and Bethe-Salpeter equations [43]. The scalar diquark coupling \( G_s \) and its axial-vector counterpart \( G_a \) are then determined in the Faddeev equation approach to reproduce the vacuum values of the nucleon mass \( (M_{N\text{vac}} = 0.94 \text{ GeV}) \) and the nucleon axial coupling \( (g_{A0} = 1.267) \) [45]. Finally, by using the model description for nucleon matter explained in Sec. III, the vector couplings \( G_\omega \) and \( G_\rho \) are determined from the binding energy per-nucleon in symmetric nuclear matter \( (E_B = -15.7 \text{ MeV}) \) and the symmetry energy \( (a_A = 32.0 \text{ MeV}) \) at the saturation density [52]. We note that in this framework the saturation density of symmetric nuclear matter becomes \( \rho_0 = 0.16 \text{ fm}^{-3} \).

The resulting values for the model parameters are given in Tab. I. Using the nucleon matter equation of state presented in Sec. III, and the parameters of Tab. I, gives an effective nucleon mass of \( M_N = 0.744 \text{ GeV} \) at nuclear matter saturation and a compressibility of \( K = 0.370 \text{ GeV} \).

In the quark matter sector there should be no effects from color confinement so we set \( \Lambda_{IR} = 0 \) in this phase. For \( \Lambda_{UV} \) and \( \rho_0 \) we use the same values in nuclear matter (see Tab. I). For the coupling constants \( G_\pi, G_\omega \) and \( G_\rho \) we introduce two scaling parameters such that in the quark matter phase \( G_s \rightarrow c_s G_s \) and \( G_\omega \rightarrow c_\omega G_\omega \) \((\nu = \omega, \rho)\), therefore the ratio \( G_\rho/G_\omega \) remains constant and is the same as determined in nuclear matter. We then investigate the extent to which it is necessary to change \( c_s \) and \( c_\omega \) from unity to achieve a reasonable description of the phase transition to quark matter and neutron star properties.

III. NUCLEAR MATTER AND QUARK MATTER

In this section we present expressions for the effective potential \( (V) \) of nuclear matter (NM) and quark matter (QM) in the mean field approximation of the two-flavor NJL model as functions of the two independent chemical potentials \( \mu_B \) and \( \mu_\tau \) for baryon number and isospin.² The corresponding chemical potentials for nucleons and quarks are

\[
\begin{align*}
\mu_\rho &= \mu_B + \mu_\tau, \\
\mu_n &= \mu_B - \mu_\tau, \\
\mu_\alpha &= \frac{1}{3} \mu_B + \mu_\tau, \\
\mu_d &= \frac{1}{3} \mu_B - \mu_\tau.
\end{align*}
\]

² The electron Fermi gas terms are also included, with the chemical potential \( \mu_e = -2\mu_\tau \) determined by \( \beta \) equilibrium. Below we summarize the unregularized expressions, and refer to Ref. [38] for a detailed discussion on the proper-time regularization scheme.

The effective potential of NM is given by [48, 49]

\[
V^{\text{NM}} = V_{\text{vac}} + V_N - \frac{\omega_0^2}{4G_\omega} - \frac{\rho_0^2}{4G_\rho} - \frac{\mu_\rho^4}{12\pi^2},
\]

where \( \omega_0 \) and \( \rho_0 \) are the isoscalar- and isovector-vector mean fields in the nuclear matter ground state, which must be determined by minimizing the effective potential. The vacuum (Mexican hat shaped) contribution is

\[
V_{\text{vac}} = 12i \int \frac{d^4k}{(2\pi)^4} \ln \frac{k^2-M^2}{k^2-M_0^2} + \frac{(M-m)^2}{4G_\pi} - \frac{(M_0-m)^2}{4G_\pi},
\]

where \( M \) is the (in-medium) constituent quark mass, which is also determined by minimizing the effective potential. The Fermi motion of nucleons in the scalar and vector mean fields is described by the term

\[
V_N = -2 \sum_{\alpha=p,n} \int \frac{d^3k}{(2\pi)^3} \left( \mu_\alpha^* - E_N(k) \right) \Theta \left( \mu_\alpha - E_N(k) \right),
\]

where the effective chemical potential for protons and neutrons is given by

\[
\mu_\alpha^* = \mu_\alpha - 3\omega_0 + \rho_0 \quad (\alpha = p, n),
\]

and \( E_N(k) = \sqrt{k^2 + M^2} \). The nucleon mass \( M_N(M) \) is determined as a function of the constituent quark mass \( M \) from the relativistic Faddeev equation for the nucleon, which is approximated as a quark-diquark bound state [43].

The effective potential for QM is given by [48, 49]

\[
V^{\text{QM}} = V_{\text{vac}} + V_Q + V_\Delta - \frac{\omega_0^2}{4G_\omega} - \frac{\rho_0^2}{4G_\rho} - \frac{\mu_\rho^4}{12\pi^2},
\]

² In principle a further chemical potential \( (\mu_0) \) is needed in QM to guarantee that the mean value of the color is zero, but it turns out to be very small for the two-flavor case [23, 53] and we neglect it here for simplicity.
where the mean vector fields $\omega_0$ and $\rho_0$ in QM are determined by minimization of the effective potential. The vacuum parameter $V_{vac}$ in Eq. (8) is the same as given in Eq. (5), and the term $V_{Q}$ describes the Fermi motion of quarks moving in the scalar and vector mean fields:

$$V_{Q} = -6 \sum_{\alpha=u,d} \int \frac{d^3k}{(2\pi)^3} (\mu_\alpha' - E(k)) \Theta (\mu_\alpha' - E(k)). \quad (9)$$

Here $E(k) = \sqrt{k^2 + M^2}$ and the effective up and down quark chemical potentials are defined by

$$\mu_\alpha' = \mu_\alpha - \omega_0 = \rho_0 \quad (\alpha = u, d). \quad (10)$$

The term $V_\Delta$ in Eq. (8) arises from the pairing of $u$ and $d$ quarks in the spin singlet color anti-triplet channel, and is given by [28, 29, 48, 49]

$$V_\Delta = 2i \int \frac{d^3k}{(2\pi)^3} \sum_{\alpha = u, d} \left[ \ln \frac{k^2 - (\epsilon_\alpha + \mu_\alpha')^2}{k^2 - (E_\alpha + \mu_\alpha')^2} \right] + \frac{\Delta^2}{6 G_s}, \quad (11)$$

where $\epsilon_\alpha = \sqrt{(E(k) \pm \mu_\alpha' / 3)^2 + \Delta^2}$ is the quark dispersion relation and $E_\alpha = |E(k) \pm \mu_\alpha' / 3|$. The energy gap $\Delta$ (order parameter of the broken color symmetry) is determined by minimization of the effective potential [48]. The effective chemical potentials for baryon number and isospin are

$$\mu_\alpha' = \mu_B - 3 \omega_0, \quad \mu_\alpha' = \mu_I - \rho_0. \quad (12)$$

Using the above forms for the effective potentials, the pressure ($P$), baryon and isospin densities ($\rho_B$, $\rho_I$), and the energy density ($E$) are obtained by

$$P = -V, \quad \rho_\alpha = -\frac{\partial V}{\partial \mu_\alpha} \quad (\alpha = B, I), \quad (13)$$

and

$$E = V + \sum_{\alpha = B, I} \mu_\alpha \rho_\alpha. \quad (14)$$

The charge neutrality condition $\rho_C = (\rho_B + \rho_I) / 2 = 0$ then implies a relation between the two chemical potentials $\mu_B$ and $\mu_I$. The charge neutrality equation of state is then a function of only one variable, which we take to be the baryon density $\rho_B$. 

3 We do not include pairing of quarks in the spin triplet color anti-triplet channel because its importance is still an open question, and moreover our value for the coupling constant in this channel is much less than in the spin singlet channel, see Tab. I.

The equation of state for the charge neutral mixed phase is then calculated by using the Gibbs construction [30] as follows: If there is a line in the ($\mu_B, \mu_I$) plane between two points (called X and Y) along which the NM and QM phases have equal effective potentials (denoted as $V^{(\text{mixed})}$) and opposite charges, then along this line we have trivially $V^{(\text{mixed})} = x_1 V^{(\text{NM})} + (1 - x_1) V^{(\text{QM})}$ for any number $x_1$. We can calculate the densities and the energy density along this line by differentiation of $V^{(\text{mixed})}$ according to Eq. (13), noting that $\partial V^{(\text{mixed})} / \partial x_1 = 0$. The requirement that the charge density along this line vanishes, i.e., $\rho_C = x_1 \rho_C^{(\text{NM})} + (1 - x_1) \rho_C^{(\text{QM})} = 0$, determines $x_1(\mu_B, \mu_I)$ as the volume fraction of NM in the mixed phase as

$$x_1(\mu_B, \mu_I) = \frac{\rho_C^{(\text{QM})}}{\rho_C^{(\text{QM})} - \rho_C^{(\text{NM})}}. \quad (15)$$

If we approach the point X along a charge neutral line of the NM phase, then $x_1 = 1$ at point X, and if we leave point Y towards a charge neutral line of the QM phase, then $x_1 = 0$ at point Y, i.e., $x_1$ decreases from 1 to 0 along the line $X \rightarrow Y$. The baryon density in the mixed phase is then given by $\rho_B^{(\text{mixed})} = x_1 \rho_B^{(\text{NM})} + (1 - x_1) \rho_B^{(\text{QM})}$, and a similar expression holds also for the energy density.

IV. RESULTS AND DISCUSSION

The equation of state in the quark matter phase must be largely controlled by three parameters in our model: the strength of the attractive isoscalar-scalar pairing interaction ($G_s$), the strengths of the repulsive isovector-vector and isoscalar-vector interactions $G_v (v = \omega, \rho)$. If we increase the pairing attraction, the baryon density $\rho_B$ where the transition NM $\rightarrow$ QM occurs decreases, resulting in an overall softening of the equation of state and lower masses of neutron stars. If the pairing interaction is increased beyond a certain limit, QM becomes the stable phase of the system also at low densities, which we regard as unphysical. On the other hand, if we increase the vector repulsion the transition density $\rho_B$ increases, then the equation of state becomes stiffer and the masses of the neutron stars increase. If the vector repulsion is increased beyond a certain limit, NM becomes the stable phase also for high densities, which we again regard as unphysical.

To show these dependencies quantitatively we introduce two scaling factors $c_s$ and $c_v$, which are used in the calculation of the quark matter sector such that $G_s \rightarrow c_s G_s$ and $G_v \rightarrow c_v G_v$ ($v = \omega, \rho$), where $G_s$ and $G_v$ are the values determined in the single nucleon and NM sector (see Tab. I) and in this way the ratio $G_v / G_\omega$ is kept fixed. To characterize the results obtained for different values of $c_s$ and $c_v$ we define a “physically reasonable” scenario of high density matter by the following three conditions: (1) The phase transition NM $\rightarrow$ QM occurs in the range $2 \rho_0 \leq \rho_s \leq 4 \rho_0$; (2) the maximum mass of the neutron star satisfies $M_{\text{star}}^{(\text{max})} \geq 1.97 M_{\odot}$ to be compatible with recent observations [6, 7]; and (3) the QM (interior) part of the star is stable against density fluctuations, i.e., $d M_{\text{star}} / d \rho_B (r = 4 \rho_0)$.

4 There is no fundamental reason for this choice of limits and our qualitative results do not change much if this condition is relaxed.
Figure 1. Scaling factors $c_s$ and $c_v$ for the interactions in the scalar diquark and vector meson channels, used in the QM sector of the calculation. The conditions (1), (2), and (3) specified in the text are satisfied along the solid line. The meaning of the other lines and regions is explained in the text.

$0 > 0$ where $\rho_B(0)\rho_B(0)$ is the central baryon density, in a region of densities above the transition density. We find that these three conditions can be satisfied if $c_s$ and $c_v$ are in a narrow band around the straight solid line shown in Fig. 1, which is expressed as $c_s = c_v + 0.3$ for $0.50 \leq c_v \leq 0.68$.

Because the solid line in Fig. 1 extends up to $c_s = 0.98$, we find that in practice it is possible to use the same value of $G_s$ for the scalar pairing strength in QM as for the scalar diquark interaction in the single nucleon sector. On the other hand, if we would use the same value of the vector couplings $G_v(v = \omega, \rho)$ as determined in the nuclear matter sector (see Tab. I), there would be no phase transition to QM. In order to satisfy the three conditions explained above, the vector coupling in QM must be smaller than in NM by a factor of $0.50 \leq c_v \leq 0.68$.

In Fig. 2 the phase diagrams in the plane of chemical potentials (top panel) and densities (bottom panel) for the point B of Fig. 1. In the top panel of Fig. 2 the stable phase (phase with the larger pressure) is marked as NM or QM for each point in the $(\mu_B, \mu_I)$ plane, together with the sign of the electric charge density. The dashed line marks electrically neutral matter. The baryon density increases as we follow this line from the left end in the NM phase to the right end in the QM phase. We see that in the section of the NM-QM mixed phase, $\mu_B$ stays almost constant while $|\mu_I|$ increases. This indicates that the phase transition, which we describe here by the Gibbs construction, is very similar to the usual first order phase transition obtained...
Figure 3. The dependence of physical quantities on the baryon chemical potential in electrically neutral matter for the case of point B in Fig. 1 ($c_s = 0.9, c_v = 0.6$). The top panel shows the baryon density and the bottom panel shows the constituent quark mass $M$ and the energy gap $\Delta$.

Figure 4. Pressure as function of the baryon density for electrically neutral matter. The top panel shows the results obtained for the points A and B in comparison to point R of Fig. 1, and the bottom panel shows the result obtained for the point C. The black solid line is the result of the pure NM case.

(We confirmed that for small values of $c_v$ the drop of the quark mass increases, which is consistent with the findings of other works [5].) Our values for the gap are in the range $250 \sim 400$ MeV, which is larger than found in calculations by using the three-momentum cut-off scheme [23].

In the remaining figures we will compare the results obtained for the points A, B, C on the solid line of Fig. 1, which all satisfy the conditions (1), (2), and (3) discussed earlier, to those obtained by using the reference point R. The top panel of Fig. 4 shows the pressure of electrically neutral matter as a function of the baryon density for the case of points A, B, and R, as well as the result for the pure NM case. All cases show the pattern which resembles a first order phase transition. Starting with case A, we see that by increasing the pairing strength ($A \rightarrow R$) the transition density decreases substantially, resulting in an overall softer equation of state. If the vector coupling is then

by the Maxwell construction.\(^6\) The lower panel of Fig. 2 shows the phase structure in the plane of the densities ($\rho_B, \rho_C$). The dashed line, which marks electrically neutral matter, passes through the region of the mixed phase (shown in white).

In the top panel of Fig. 3 we show the baryon density of electrically neutral matter as function of the baryon chemical potential for the parameter set B. The jump of the baryon density during the phase transition is about 0.1 fm\(^{-3}\). In the bottom panel of Fig. 3 we show the constituent quark mass $M$ and the energy gap $\Delta$ as functions of $\mu_B$ for the same parameter set B. The large jump of the quark mass indicates that the QM in this range of $\mu_B$ is already reasonably close to a phase where chiral symmetry is restored, although $M$ is still non-negligible.

We found phase transitions of the second order type only for small values of the pairing strength and the vector interaction ($c_s \leq 0.2, c_v = 0$), which is consistent with the results of Refs. [48, 49].
Increased (R → B) the transition density increases again, but stays below case A. As a result, there is a net decrease of the transition density as we go along the solid line of Fig. 1 in the direction of increasing coupling constants (A → C). The lower panel of Fig. 4 shows the result for point C, where the coupling constant $G_s$ is practically the same as determined from the free nucleon mass and $G_v$ is smaller than the nuclear matter value by a factor of $c_v = 0.68$. The transition density is about 3 $\rho_0$ for this case.

By using our equations of state as an input to solve the Tolman-Oppenheimer-Volkoff (TOV) equations [54, 55], we can calculate the properties of neutron stars. The results for the neutron star mass as a function of the central baryon density are shown in the top panel of Fig. 5 for the cases of points A, B, and R of Fig. 1, and in the bottom panel of Fig. 5 for case C. For case A (high transition density) one can obtain the largest star masses, but the range of stability against density fluctuations ($d M_{\text{star}} / d \rho_B (r = 0) > 0$) is narrow, i.e., the QM part of the star tends to be unstable. If we increase the pairing strength (A → R), the overall equation of state becomes softer and the neutron star masses become smaller, however the stability of QM in the star is substantially improved. Increasing then the vector coupling (R → B), we can obtain stars which satisfy $M_{\text{star max}} \geq 1.97 M_\odot$ and show a reasonable range of stability. The bottom panel of Fig. 5 shows the results obtained for the point C in Fig. 1, which is probably the most satisfying scenario obtained in the present model description.

In Fig. 5 we also indicate the recently observed values of neutron star masses. GW170817 denotes the neutron star coalescence event observed by the gravitational wave measurements [8–10], where neutron stars with masses in the range $M_{\text{star}} = 1.17 - 1.60 M_\odot$ were observed and PSR J0348+0432 refers to the observation of massive neutron stars (pulsars) of...
Table II. Transition densities $\rho_c$ and maximum star masses $M_{\text{star}}^{\text{max}}$ for the cases A, B, and C, in comparison to the case R of Fig. 1.

| Case | $c_x$ | $c_y$ | $\rho_c$ [fm$^{-3}$] | $M_{\text{star}}^{\text{max}}$ [$M_\odot$] |
|------|-------|-------|-----------------------|---------------------------------------------|
| A    | 0.80  | 0.50  | 0.643                 | 2.078                                       |
| B    | 0.90  | 0.60  | 0.584                 | 2.055                                       |
| C    | 0.98  | 0.68  | 0.496                 | 2.071                                       |
| R    | 0.50  | 0.90  | 0.379                 | 1.918                                       |

mass $M_{\text{star}} = 2.01 \pm 0.04 M_\odot$ [7]. The result of PSR can be considered as a lower limit for calculations of the maximum mass of neutron stars. We see that our points A, B, and C satisfy this constraint, but the maximum star mass for case R is too small. In Tab. II we list the transition densities and maximum neutron star masses for the points A, B, and C, in comparison to the case R.

In the top panel of Fig. 6 we show the relation between the neutron star masses and radii for the cases A, B, and R of Fig. 1, and the bottom panel of Fig. 6 shows the results for the case C. The low density part of the NM curve (lower part of the solid line in Fig. 6) shows the characteristics of a case where the pressure drops to zero (or nearly to zero) at a finite value of the baryon density, as the density decreases, which is indicative of a bound state in the absence of gravity. The top panel of Fig. 6 clearly shows that QM in case A (highest value of $\rho_c$) tends to be unstable. (The narrow region of stability shown in the top panel of Fig. 5 for case A becomes invisible in the top panel of Fig. 6.) Moving along the solid line in Fig. 1 the stability of QM is substantially improved, as is shown for the case B in the top panel of Fig. 6, and in particular for case C in the bottom panel of Fig. 6. These plots clearly demonstrate the important role of the pairing interaction to stabilize QM, and show that our scenario for the case C is completely consistent with the PSR observation of massive neutron stars [6, 7]. The data for the event observed by GW170817 [8–10] ($M_{\text{star}} = 1.17 \sim 1.60 M_\odot$, $R = 11.9 \pm 1.4$ km) is also indicated in Fig. 6, and can be reproduced with a pure NM equation of state.

Finally, in Fig. 7 we show the profile functions for the maximum mass stars. The top panel of Fig. 7 shows the cases A, B, and R of Fig. 1, and the bottom panel shows case C. We see that in this latter scenario, a neutron star with 2.07 solar masses consists of QM (including a thin shell of NM-QM mixed phase) within a large radius of about 6.9 km.

V. SUMMARY

We have studied the equation of state for cold high density neutron star matter by using the two-flavor NJL model. Our principal aim was to use a model framework which is based on a successful description of hadron structure in vacuum and nuclear matter. One of our important findings is that the same strength of the scalar diquark interaction ($G_s$)—which is required to reproduce the nucleon mass and other quark-quark correlation effects in baryons [43]—can also be used successfully to describe the phase transition to color superconducting quark matter and the properties of neutron stars. Concerning the repulsive effects of the interaction in the vector meson channels, we fixed the ratio of the isoscalar to isovector coupling constants ($G_{\omega}/G_{\rho}$) to the value determined from the properties of nuclear matter, and investigated the dependence on $G_{\nu} \rightarrow c_{\nu}$, $G_{\nu}$ ($\nu = \omega, \rho$) in the QM phase. We found that this coupling constant must be decreased in the quark matter sector by a factor of $0.50 \leq c_{\nu} \leq 0.68$ to obtain a reasonable scenario for the NM $\rightarrow$ QM phase transition. Interesting, using the same NJL framework to explore possible explanations of the EMC effect [56–59] in nuclear structure functions, with the same parameters, we found that the coupling of the vector mean field to the struck quark must also be substantially reduced [46]. In this case the reduction was associated with asymptotic freedom in QCD [60].

For the case where $G_s$ is fixed to very near the value determined by the free nucleon mass ($c_s = 0.98$), and $G_{\nu}$ is scaled...
by a factor of $c_{\tau} = 0.68$ relative to the symmetric nuclear matter values (point C in Fig. 1), we found an almost first order phase transition to QM at about 3.1 times the normal nuclear matter density. The maximum neutron star mass in this case is 2.07 solar masses, which is in agreement with recent observations of a massive neutron star [6, 7]. We investigated the dependence of the transition density and neutron star properties on the parameters $G_{\pi}$ and $G_{\sigma}$, and demonstrated the need for scalar quark pairing to stabilize QM and the importance of the vector repulsion to obtain massive neutron stars. We found a narrow region around a line in the space of these two parameters (the solid line $A \rightarrow C$ in Fig. 1), which gives a reasonable scenario for the phase transition to color superconducting quark matter and properties of neutron stars which agree with current observations.

The calculations presented in this work should be extended to include the effects of strangeness in both the hadronic and the quark matter phases, and finite size effects in the mixed phase. For the case of single baryons, recent studies have shown that the properties of hyperons (masses, magnetic moments, and form factors) can be successfully described in this NJL framework [61]. In the quark matter phase different types of pairings, such as color-flavor locking, should be included, and the role of a chemical potential associated with color conservation should be taken into account. In our view this work is an important step towards a unified description of single hadrons, nuclear systems, quark matter, and neutron stars in the framework of an effective quark theory of QCD.

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[40] I. C. Cloët, W. Bentz and A. W. Thomas, Phys. Lett. B621, 246 (2005) [arXiv:hep-ph/0504229 [hep-ph]].
[41] I. C. Cloët, W. Bentz and A. W. Thomas, Phys. Lett. B659, 214 (2008) [arXiv:0708.3246 [hep-ph]].
[42] P. T. P. Hutauruk, I. C. Cloët and A. W. Thomas, Phys. Rev. C94, 035201 (2016) [arXiv:1604.02853 [nucl-th]].
[43] I. C. Cloët, W. Bentz and A. W. Thomas, Phys. Rev. C90, 045202 (2014) [arXiv:1405.5542 [nucl-th]].
[44] Y. Ninomiya, W. Bentz and I. C. Cloët, Phys. Rev. C96, 045206 (2017) [arXiv:1707.03787 [nucl-th]].
[45] I. C. Cloët, W. Bentz and A. W. Thomas, Phys. Rev. Lett. 95, 052302 (2005) [arXiv:nucl-th/0504019 [nucl-th]].
[46] I. C. Cloët, W. Bentz and A. W. Thomas, Phys. Lett. B642, 210 (2006) [arXiv:nucl-th/0605061 [nucl-th]].
[47] I. C. Cloët, W. Bentz and A. W. Thomas, Phys. Rev. Lett. 116, 032701 (2016) [arXiv:1506.05875 [nucl-th]].
[48] W. Bentz, T. Horikawa, N. Ishii and A. W. Thomas, Nucl. Phys. A720, 95 (2003) [arXiv:nucl-th/0210067 [nucl-th]].
[49] S. Lawley, W. Bentz and A. W. Thomas, Phys. Lett. B632, 495 (2006) [arXiv:nucl-th/0504020 [nucl-th]].
[50] D. L. Whittenbury, H. H. Matevosyan and A. W. Thomas, Phys. Rev. C93, 035807 (2016) [arXiv:1511.08561 [nucl-th]].
[51] T. Hell, K. Kashiwa and W. Weise, J. Mod. Phys. 4, 644 (2013) [arXiv:1212.4017 [hep-ph]].
[52] I. C. Cloët, W. Bentz and A. W. Thomas, Phys. Rev. Lett. 102, 252301 (2009) [arXiv:0901.3559 [nucl-th]].
[53] D. N. Aguilera, D. Blaschke and H. Grigorian, Nucl. Phys. A757, 527 (2005) [arXiv:hep-ph/0412266 [hep-ph]].
[54] R. C. Tolman, Phys. Rev. C 55, 364 (1997).
[55] Y. Ninomiya, W. Bentz and I. C. Cloët, Phys. Rev. C96, 045206 (2017) [arXiv:1707.03787 [nucl-th]].
[56] I. C. Cloët, W. Bentz and A. W. Thomas, Phys. Rev. Lett. 95, 052302 (2005) [arXiv:nucl-th/0504019 [nucl-th]].
[57] D. F. Geesaman, K. Saito and A. W. Thomas, Ann. Rev. Nucl. Part. Sci. 45, 337 (1995).
[58] S. Malace, D. Gaskell, D. W. Higinbotham and I. Cloët, Int. J. Mod. Phys. E23, 1430013 (2014) [arXiv:1405.1270 [nucl-ex]].
[59] I. C. Cloët et al., arXiv:1902.10572 [nucl-ex].
[60] W. Detmold, G. A. Miller and J. R. Smith, Phys. Rev. C73, 015204 (2006) [arXiv:nucl-th/0509033 [nucl-th]].
[61] M. E. Carrillo-Serrano, W. Bentz, I. C. Cloët and A. W. Thomas, Phys. Lett. B759, 178 (2016) [arXiv:1603.02741 [nucl-th]].