Asymptotic High-Frequency Green’s Function for a Large Rectangular Planar Periodic Phased Array of Dipoles With Weakly Tapered Excitation in Two Dimensions

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Abstract—This paper deals with the derivation and physical interpretation of a uniform high-frequency representation of the Green’s function for a large planar rectangular phased array of dipoles with weakly varying excitation. Thereby, our earlier published results, valid for equiamplitude excitation, and those for tapered illumination in one dimension are extended to tapering along both dimensions, including dipole amplitudes tending to zero at the edges. As previously, the field obtained by direct summation over the contributions from the individual radiators is restructured into a double spectral integral whose high-frequency asymptotic reduction yields a series of propagating and evanescent Floquet waves (FWs) together with corresponding FW-modulated diffracted fields, which arise from FW scattering at the array edges and vertexes. To accommodate the weak amplitude tapering, new generalized periodicity-modulated edge and vertex “slope” transition functions are introduced, accompanied by a systematic procedure for their numerical evaluation. Special attention is given to the analysis and physical interpretation of the complex vertex diffracted ray fields. A sample calculation is included to demonstrate the accuracy of the asymptotic algorithm. The resulting array Green’s function forms the basic building block for the full-wave analysis of planar weakly amplitude-tapered phased array antennas, and for the description of electromagnetic radiation and scattering from weakly amplitude-tapered rectangular periodic structures.

Index Terms—Floquet expansions, Green’s functions, periodic structures, phased array antennas.

I. INTRODUCTION

A systematic sequence of previous studies [1]–[5], we have explored methods to reduce the often prohibitive numerical effort that accompanies an element-by-element full-wave analysis of large truncated plane periodic phased arrays. In our approach, the element-by-element array Green’s function (AGF) formed by a planar periodic phased array of dipoles is restructured into an alternative “collective” formulation that represents the field radiated from the elementary dipoles in terms of the radiation from a superposition of continuous truncated Floquet wave (FW)-matched source distributions extending over the entire aperture of the array, with inclusion of other truncation-induced Floquet-modulated wave types. Since the FW series exhibits excellent convergence properties when the observation point is located far enough away from the array surface to render evanescent FWs and the corresponding diffracted fields negligible, this representation has been found to be more efficient than the direct summation of the spatial contributions from each element of the array, especially when each FW aperture distribution is treated asymptotically.

The previous investigations have dealt with semi-infinite planar dipole arrays [1], [2] and right-angle sectoral planar phased arrays of dipoles [3], [4] with equiamplitude excitation, as well as strip arrays with one direction-tapered illumination [5].

Our new extension in this paper addresses planar rectangular arrays with slowly varying excitation profiles that are separable in the two orthogonal variables. The corresponding asymptotic treatment of each FW aperture distribution leads to locally amplitude-modulated truncated FWs, plus FW-excited diffracted contributions from the edges and vertexes of the array, which can be cast as previously in the format of a periodicity-adapted geometrical theory of diffraction (GTD). For the canonical finite planar phased array of dipoles with tapered excitation, the AGF is constructed by plane wave spectral decomposition in the two-dimensional complex wavenumber domain corresponding to the array-plane coordinates. This is followed by manipulations and contour deformations that prepare the integrands for subsequent efficient and physically incisive asymptotics parameterized by critical spectral points, i.e., saddle points and points at which the spectral amplitude function exhibits highly peaked characteristics very similar to those of poles (denoted later on as “quasi-poles”). Different species of spectral quasi-poles define the various species of propagating and evanescent locally amplitude-modulated FWs. The other critical points in the double spectral integral define the asymptotic behavior of the edge and vertex diffracted rays; the confluence of these critical points in transition regions determines a variety of locally uniform new transition functions for truncated edge diffracted and vertex diffracted waves.

Arrays with tapered excitation have also been analyzed in [6], combining “global” FWs and diffracted fields developed in [1]–[3] with a numerical technique based on the discrete Fourier
transform (DFT) that expands a general tapering in terms of its equi-amplitude phased harmonics. Although the numerical DFT procedure can be applied to a wider class of taperings, our results here are in analytical closed form and thus can be readily computed for the considered class of applications [7]–[9].

II. STATEMENT OF THE PROBLEM

Consider a large rectangular periodic array of $N_1 \times N_2$ linearly phased dipoles located in the $z_1, z_2$ plane (Fig. 1). The array dimensions are $L_1$ and $L_2$ along $z_1$ and $z_2$, respectively; the interelement spatial periods along the $z_1$ and $z_2$ directions are given by $d_1$ and $d_2$; and the interelement phase gradients of the excitation by $\gamma_1$ and $\gamma_2$, respectively. An $\exp(\jmath \omega t)$ dependence is implied and suppressed. All dipoles are oriented along the unit vector $\mathbf{J}_0$ (boldface symbols denote vector quantities, and boldface symbols with a caret denote unit vectors). Superimposed upon that background is a $z$-dependent amplitude-separable tapering function $f(z_1, z_2) = f_1(z_1) f_2(z_2)$, sampled at the dipole locations

$$
\mathbf{J}(n_1 d_1, n_2 d_2) = \mathbf{J}_0 f_1(n_1 d_1) f_2(n_2 d_2) \exp(-\jmath \gamma_1 n_1 d_1 + \gamma_2 n_2 d_2) \tag{1}
$$

with $\mathbf{J}(z_1', z_2')$ denoting the dipole current amplitude, and $(z_1', z_2') = (n_1 d_1, n_2 d_2)$ denoting the location of the $(n_1, n_2)$-th dipole. Without compromising practical utility, we assume $f(z_1, z_2)$ real and positive in the domain $z_1 \in [0, L_1], z_2 \in [0, L_2]$, and zero elsewhere; here, $L_1 = (N_1 - 1) d_1$ and $L_2 = (N_2 - 1) d_2$. The electromagnetic vector field at any observation point $\mathbf{r} = z_1 \mathbf{z}_1 + z_2 \mathbf{z}_2 + \mathbf{k} \mathbf{y}$ can be derived from the magnetic vector potential

$$
\mathbf{A}(\mathbf{r}) = \sum_{n_1=-1}^{N_1-1} \sum_{n_2=0}^{N_2-1} \frac{\exp(-\jmath R_{n_1, n_2})}{4 \pi R_{n_1, n_2}} \mathbf{J}(n_1 d_1, n_2 d_2) \tag{2}
$$

where $R_{n_1, n_2} \equiv \sqrt{(z_1 - n_1 d_1)^2 + (z_2 - n_2 d_2)^2 + y^2}$, which is synthesized by summing over the individual $(n_1, n_2)$ dipole radiations. Our goal is the efficient and phenomenologically insightful evaluation of the high-frequency near and far fields radiated by this complex physical configuration.

III. FORMAL SOLUTION AND HIGH-FREQUENCY PARAMETERIZATIONS

A. Spectral Domain Analysis: Floquet Waves

Our analysis is carried out in the spectral wavenumber domain. Accordingly, we employ the spectral Fourier representation of the scalar free space Green’s function $\exp(-\jmath k R(z_1, z_2)) / [4\pi R(z_1, z_2)]$ with $R(z_1, z_2) = \sqrt{z_1^2 + z_2^2 + y^2}$; this Fourier transform is given by $\exp(\jmath k y) / [2jk_3] \tag{10}$, where $k_y = \sqrt{k^2 - k_1^2 - k_2^2}$ and the upper and lower signs apply to $y > 0$ and $y < 0$, respectively. Because of symmetry, from here on, we shall deal with $y > 0$ only. On the top Riemann sheet of the complex $k_3$-plane, for real $k_3$, we define $\Im \{k_3\} < 0$ for $k^2 - k_1^2 < k_2^2$ and $k_1 > 0$ for $k^2 - k_1^2 > k_2^2$. The location of branch points and branch cuts with respect to the real-axis integration path in the $k_3$-plane is found by introducing small losses ($\Im \{k\} = 0$), which are eventually removed [1], [3], [4]. Substituting the spectral Fourier representation of the scalar free-space Green’s function into (2) and interchanging the sequence of the $(n_1, n_2)$-summation and the spectral integration operations, leads to (see Section II for notation and definitions)

$$
\mathbf{A}(\mathbf{r}) = \hat{\mathbf{J}}_0 A(\mathbf{r})
\tag{3}
$$

where $P(k_3, k_2) = k_3 z_1 + k_2 z_2 + k_3 y = \mathbf{k} \mathbf{r}$, with $\mathbf{k} = k_3 \mathbf{z}_3 + k_2 \mathbf{z}_2 + k_3 \mathbf{y}$ and

$$
I_i(k_3) = \sum_{n=0}^{N_i-1} \exp(\jmath \phi_i(n_i) d_i) f_i(n_i d_i), \quad i = 1, 2. \tag{4}
$$

The $(n_i)$-sum $I_1, I_2$ in (4) is manipulated via the truncated Poisson sum formula [4], [11] into

$$
I_i(k_3) = \frac{f_i(0)}{2} + \frac{f_i(L_i)}{2} \exp(\jmath \phi_i(n_i) L_i)
+ \frac{1}{d_i} \sum_{m=-\infty}^{\infty} \tilde{f}_i(k_3 - k_3 i m) \tag{5}
$$

where

$$
\tilde{f}_i(k_3) = \int_0^{L_i} \exp(\jmath k_3 z_i) f_i(z_i) dz_i \tag{6}
$$

are the Fourier-transformed tapering functions. The FW wavenumbers, which define the FW dispersion, are given by

$$
k_{31, \phi} = \gamma_1 + \frac{2\pi q}{d_1}, \quad k_{32, p} = \gamma_2 + \frac{2\pi p}{d_2} \tag{7}
$$

with $q, p = 0, \pm 1, \pm 2, \ldots$. Fig. 1. $\mathbf{J}_0$-directed parallel dipoles, weighted by an amplitude taper and linear phasing, with $d_1$ and $d_2$ denoting the interelement spatial periods along $z_1$ and $z_2$, respectively. The tapering function $f_1(z_1) f_2(z_2)$ is slowly varying with respect to the wavelength. The inset shows the local coordinate system associated with a vertex. Note that $\rho_i$ is related to $r$ by $r = \sqrt{\rho_i^2 + z_3^2}$ for $i = 1, 2$, respectively.
The critical points in the spectral double integral, which also govern the strategies for the asymptotic approximations. The critical points in (3) are the following.

i) \((k_{z_1}, k_{z_2}) = (k_{z_1}, q, k_{z_2}, p)\). These points, where the SAF \(I_1(k_{z_1})I_2(k_{z_2})\) in Fig. 2(a) exhibits peaks, describe the same phenomenology and localization property as the spectral poles for the semi-infinite array [2]. Accordingly, we shall refer to these points as \emph{quasi-poles}.

ii) \((k_{z_1}^e, k_{z_2}^e) = \left(k_{z_1}^e, k_{z_2}^e\right)\). This is a first-order two-dimensional stationary phase point (SPP) of the individual-element spectral GF in Fig. 2(c), which satisfies \((d/dk_{z_i})(\mathbf{k} \cdot \mathbf{r})_i = 0\) for \(i = 1, 2\), with nonvanishing Hessian determinant, \(|\det(\mathbf{P}/dk_{z_i}dk_{z_j})(\mathbf{k} \cdot \mathbf{r})_i| \neq 0\). This SPP is related via \((k_{z_1}^e, k_{z_2}^e) = (k\cos\beta_1, k\cos\beta_2)\) to the spherical coordinate angles \(\beta_1\) and \(\beta_2\) which locate the observation point with respect to the \(z_1\) and \(z_2\) axes, respectively (see Fig. 1).

The critical points in i) and ii) play a different role in the asymptotic evaluation of the integral in (3) with respect to their dependence on the observer location. For observation points close to the array surface and far from the array edges and vertexes, the oscillations of the individual element spec-
nal GF are rather slow; therefore, the periodic, highly peaked spectral function \( I_1(k_{z_1}) I_2(k_{z_2}) \) acts like a passband periodic filter for that GF. As shall be demonstrated in the next section, the outcome of this sampling is the FW summation corresponding to the infinite array, with adiabatic modulation due to the tapering function. When the observation point is in the far zone of the array (i.e., at distance greater than \( 2D^2/\lambda \)), the oscillations around the stationary phase point are so rapid as to localize the contributions from the periodic function \( I_1(k_{z_1}) I_2(k_{z_2}) \) to its values at \( (k_{z_1}^s, k_{z_2}^s) \). Consequently, the far field is proportional to \( I_1(k_{z_1}) I_2(k_{z_2}) \). The recognition that this expression is the ordinary “array factor” associated with the pattern multiplication law [12] has motivated our designation of \( I_1(k_{z_1}) I_2(k_{z_2}) \) as the spectral array factor.

In the intermediate zone (Fresnel zone) where the distance of the observation point from the array and the size of the array may be comparable, both types of critical points and their interaction have to be taken into account in the asymptotic evaluation of the integral. In this intermediate regime, Fig. 2(b) depicts the SAF in one periodicity cell, and Fig. 2(c) displays the corresponding phase modulation associated with the GF. The two plots exhibit comparable variation, and it is not evident how to identify a spectral filtering that quantifies the influence of one factor on the other. We therefore refine the asymptotics and introduce additional critical points to account for transitions between the various wave types.

iii) \( (k_{z_1}, k_{z_2}) = (k_{z_1}^s, k_{z_2}^p) \) and \( (k_{z_1}, k_{z_2}) = (k_{z_1}^p, k_{z_2}^s) \). These points satisfy the one-variable stationary phase conditions \( (d/dk_{z_1}) P(k_{z_1}, k_{z_2}) = 0 \) and \( (d/dk_{z_2}) P(k_{z_1}, k_{z_2}) = 0 \) respectively, with \( P(k_{z_1}, k_{z_2}) = k_{z_1} (k_{z_2}) + k_{z_2} (k_{z_2} + k_{z_2}^p) \). The asymptotic wavefields corresponding to these two saddle points tag diffracted fields from edge 2 (located at \( z_1 = 0 \)) and edge 1 (at \( z_2 = 0 \)), respectively. For a rectangular array, diffraction from the other two edges can be found similarly by including the appropriate phase reference in the second term on the right-hand side of (5).

The asymptotic contributions pertaining to the three types of critical points are examined next. Though the critical points are the same as those in [3], the localization process is conducted differently here by introducing the quasi-poles and a spatial-spectral expansion [see (17) and (36)] of the spectral integrand in (3) for the point evaluations (edge- and vertex-diffracted waves).

IV. UNIFORM HIGH-FREQUENCY SOLUTIONS

Because of the slowly varying assumption for the tapering function \( f_i(z_i) \), adiabatic methods can be applied, based on perturbation about \( f_i(z_i) = \text{const} \). The case of equiamplitude excitation, which has been treated in [3] and [4], is briefly summarized here for convenience.

A. Equiamplitude Excitation \( f_i = 1 \)

1) Finite array: Since \( f_i = 1 \), the series \( I_i(k_{z_1}) \) in (4) can be evaluated in closed form: \( I_i(k_{z_1}) = B_i(k_{z_1})(1 + e^{iN_1_i(k_{z_1} - \gamma_i)}), i = 1, 2 \); here, \( B_i(k_{z_1}) \) exhibits singularities at \( k_{z_1} = k_{z_1,u} \) that are cancelled by the zeros of the numerator in \( I_i(k_{z_1}) \) (see Fig. 2). Because the functions \( I_i(k_{z_1}) \) are recognized as the analytic continuation of the spectral array factor in the complex \( k_{z_1} \)-plane, the SAF exhibits peaks at the values \( k_{z_1} = k_{z_1,u} \), i.e., at the FW wavenumbers in (7).

2) Infinite array: When the number of array elements in both dimensions increases, the peaks of the SAF become correspondingly enhanced and, in the limit of an infinite number of dipoles, tend to two separate sequences of Dirac delta functions

\[
I_i(k_{z_1}) \rightarrow \sum_{s=-\infty}^{\infty} e^{i(k_{z_1} - \gamma_i)sd_i} = \frac{2\pi}{i} \sum_{u=-\infty}^{\infty} \delta(k_{z_1} - k_{z_1,u}) \tag{8}
\]

with \( i = 1, 2 \). This reduces the integral in (3) to the periodicity-modulated FW series for the infinite AGF

\[
A_{\infty}(r) = \sum_{p=-\infty}^{\infty} A_{p}^{FW}, \quad A_p^{FW} = e^{i2\pi r k_{wp}^0} \tag{9}
\]

where \( k_{wp}^0 = k_{z_1,u} z_1 + k_{z_2,u} z_2 + k_{wp}^0 \). For this case, the asymptotic evaluation of (3) can be performed as in [3] via sequential deformation of the original double integration contour into the complex \( k_{z_1} \)-plane local steepest descent path (SDP) along the 45° line through the saddle point of the phase in the integrand, with extraction of the residues at intercepted poles. These residues reconstruct the FW series of the infinite AGF with the proper truncation functions.

B. Weakly Amplitude-Modulated FW Contributions and Shadow Boundary Planes

We begin by assuming that the observer is located far from the array edges in terms of wavelength. Therefore the dominant contributions to the field arise from the peaks associated with the quasi-poles. Generalizing the one-dimensional taper analysis in [5], by inserting (5) into (3), the contributions from the critical points at \( (k_{z_1}, k_{z_2}) = (k_{z_1,u}, k_{z_2,p}) \) are found by expanding the exponent \( P(k_{z_1}, k_{z_2}) \) of the integrand in (3) in Taylor series in a neighborhood of \( (k_{z_1,u}, k_{z_2,p}) \)

\[
P(k_{z_1}, k_{z_2}) \approx k_{zp}^{FW} \cdot r + z_{1p} (k_{z_1} - k_{z_1,u}) + z_{2p} (k_{z_2} - k_{z_2,p}) \tag{10}
\]

with

\[
z_{1p} = \frac{\partial P(k_{z_1}, k_{z_2})}{\partial k_{z_1}} \bigg|_{k_{z_1} = k_{z_1,u}} = z_1 - yk_{z_1,u} / k_{yp} \\
z_{2p} = \frac{\partial P(k_{z_1}, k_{z_2})}{\partial k_{z_2}} \bigg|_{k_{z_2} = k_{z_2,p}} = z_2 - yk_{z_2,p} / k_{yp} \]
Next, approximating $k_{pq-1}^{-1} \approx k_{y_{pq}}^{-1} + k_{z_{pq}}^{-3}(k_{z_{pq}} - k_{z_{pq}}) + k_{z_{pq}}^{-3}(k_{z_{pq}} - k_{z_{pq}})$, and retaining only the dominant asymptotic term of the remainder, one finds $(e^{-j2\pi(k_{z_{pq}} k_{y_{pq}})/(k_{y_{pq}} k_{z_{pq}} k_{z_{pq}} - k_{z_{pq}})})^2$ plus higher order terms of type $O(k_{z_{pq}} - k_{z_{pq}})O(k_{z_{pq}} - k_{z_{pq}})$. Here, $k_{y_{pq}} = \sqrt{k_{y_{pq}}^2 - k_{z_{pq}}^2 - k_{z_{pq}}^2}$, with the branches chosen as for $k_{y}$ (see Section III-A), is real for propagating FW. Inserting (10) into (3) [and approximating the slowly varying portion of the integrand by its value at $(k_{z_{pq}}, k_{z_{pq}}, k_{z_{pq}})$], leads to

$$A_{FW}^F(r) \approx \frac{1}{\sqrt{2\pi}} \sum_{pq=-\infty}^{\infty} \left[ A_{pq}^F(r) \right] \cdot \prod_{i=1,2} \int_{-\infty}^{\infty} \frac{f_i(k_{z_{pq}} - k_{z_{pq}})}{e^{-j2\pi(k_{z_{pq}} - k_{z_{pq}})k_{z_{pq}}}} dk_{z_{pq}}.$$  

(11)

The two spectral integrals are calculated directly, using the definition in (6), yielding

$$A_{FW}^F(r) \approx \sum_{pq=-\infty}^{\infty} A_{pq}^F(r)f_1(z_{pq})f_2(z_{pq}).$$  

(12)

In (11) and (12), $A_{FW}^F(r)$ is the $pq$th FW for equiamplitude excitation [see (9)], which is multiplied in (12) by the tapering function $f_1(z_{pq})f_2(z_{pq})$ evaluated at the footprint $(z_{pq}, z_{pq}, z_{pq})$ of the $pq$th FW. Note that the two vertex-truncation terms in (5), $(f_i(0))/(2)$ and $(f_i(L_i))/(2)e^{j2\pi(k_{z_{pq}} - k_{z_{pq}})L_i}$, for both $i = 1, 2$, are not included here because they provide additional contributions from edge and vertex diffractions, which are calculated separately using (10).

The stationary phase evaluation of the Kirchhoff spatial radiation integral associated with each $pq$th equivalent FW-matched aperture distribution would provide the same result as in (12) since $(z_{pq}, z_{pq})$ is the corresponding space domain stationary phase point. We note that since $f_i(z_{pq}) = 0$ for $z_{pq} < 0, z_{pq} > L_i$, the tapering function automatically truncates the FW domain of existence at the shadow boundary (SB) planes defined by the conditions $z_{pq} = 0$ and $z_{pq} = 0$, which correspond to $(z_{pq}/y) = (k_{z_{pq}}/k_{y_{pq}})$ and $(z_{pq}/y) = (k_{z_{pq}}/k_{y_{pq}})$, respectively. In the angular space domain, these two conditions become $\phi_1 = \phi_{SB}^{z_{pq}}$ and $\phi_2 = \phi_{SB}^{z_{pq}}$, where $\phi_j$ is the transverse-to-$z_{pq}$ observation angle (see Fig. 1 and 3) and

$$\phi_{SB}^{z_{pq}} = \cos^{-1}(k_{z_{pq}}/k_{p_{pq}})$$

$$\phi_{SB}^{z_{pq}} = \cos^{-1}(k_{z_{pq}}/k_{p_{pq}}).$$  

(13)

Here, $k_{p_{pq}} = \sqrt{k_{y_{pq}}^2 - k_{z_{pq}}^2}$ and $k_{p_{pq}} = \sqrt{k_{y_{pq}}^2 - k_{z_{pq}}^2}$ define the shadow boundary planes (SBPs) associated with the two edges that intersect at the vertex $(z_{pq}, z_{pq}) = (0,0)$; in Fig. 3, these SBPs are displayed for two edges of the rectangular array.

C. FW-Induced Amplitude-Modulated Diffracted Fields and Shadow Boundary Cones

As noted in Section III-B, the critical points ii) give rise to edge-diffracted field contributions from the four edges of the rectangular array. The procedure to obtain these contributions is based on that presented in [5] and is summarized in Appendix I. Denoting by $f_2^*(z_{pq})$ and $B_2^*(z_{pq})$ the derivative of $f_2(z_{pq})$ and $B_2(z_{pq})$ with respect to their arguments, the diffracted field from edge 1 is given by

$$A_{q1}^F(r) \sim \frac{f_1(z_{pq})}{2\pi \sqrt{2\pi k_{y_{pq}}}} \cdot \left[ f_2(0)B_2(k_{z_{pq}})F(\delta_{pq}) - jf_2^*(0)B_2^*(k_{z_{pq}})F_2(\delta_{pq}) \right]$$  

(14)

where the amplitude tapering function $f_1$ in the $q$th direction is evaluated at the footprint of the $q$th edge-diffracted field along edge 1: $z_{pq}^q = z_{pq} - \rho_1k_{z_{pq}}/k_{p_{pq}}$. Since $f_2(z_{pq}^q) = 0$ for $z_{pq}^q < 0$ and $z_{pq}^q > L_1$, the domain of existence of the $q$th edge-diffracted field is bounded automatically. The truncation at $z_{pq}^q = 0$ corresponds to $(z_{pq}/\rho_1) = (k_{z_{pq}}/k_{p_{pq}})$, which becomes in the angular space domain $\beta_1 = \beta_{SB}^q = \cos^{-1}(k_{z_{pq}}/k_{p_{pq}})$, where $\beta_1$ is the conical angle (see Fig. 1). Equation (14) $\beta_1 = \beta_{SB}^q$ characterizes the shadow boundary cone (SBBC) which, centered at the vertex (Fig. 4), confines the diffracted field. The SBBC centered at the vertex (Fig. 4), has the same aperture as the diffraction cone [analogous considerations apply to the p-indexed q independent $z_2$-edge diffracted rays, whose domain of existence is confined by $U((\beta_{SB}^q - \beta_2))$, where $U$ is the Heaviside unit step functions. In (14), $k_{pq}^q = k_{z_{pq}}^q + k_{p_{pq}} + k_{p_{pq}} + \sin \theta \sin \phi$ denotes the vector wavenumber of the $q$th diffracted field, and lies on the surface of a diffraction cone centered at $z_{pq}^q$ on the array edge, forming an angle $\beta_{SB}^q$ with the $z_1$-axis. Moreover, $F(x)$ in (14) is the standard uniform theory of diffraction (UTD) transition function [13]

$$F(x) = 2\pi \int_0^\infty e^{-j\theta^2} dt,$$  

$$-\frac{\pi}{2} < \arg(x) < \frac{\pi}{2}$$  

(15)

$$F_s(x) = 2\pi[1 - F(x)]$$  

(16)

is the slope UTD transition function (see Appendix I and [14]). It can be shown that when the nondimensional parameter $\delta_{pq}^2 = 2k_{p_{pq}} k_{p_{pq}}^2 \sin^2((1/2)(\phi_{SB}^{z_{pq}} - \phi_1))$ is large, $F_s$ and $F_s$ both tend to unity.
The dominant asymptotic term [the first term in (14)] is the same as that for the equiamplitude case (see [2]), although for multiplication by the tapering function evaluated at \( z_{d,1} \) on the edge. The second contribution is of higher asymptotic order since \( B_2(k_{-2}) \equiv (1/2) + O(k_{-2}^{-2}) \), whereas \( B_2(k_{-2}) \equiv O(k_{-2}^{-3}) \). This agrees with the description derived previously for the single tapered edge in [5], as is to be expected in view of the assumptions stated at the beginning of Section IV-B.

The edge diffracted fields from the other edges of the rectangular array have analogous expressions easily deducible from (14).

D. FW-Induced Amplitude Modulated Vertex Diffraction

The critical points iii) in Section III-B parameterize the truncation-induced amplitude modulated vertex-diffracted field effects. These describe truly new phenomena that were not encountered in [3]–[5]. For instance, near the vertex at \( (z_{1,2}, z_{2,1}) \equiv (0, 0) \), the \( z_{1,2} \)-edge and \( z_{2,1} \)-edge FW-shadow boundary transitions interact with the vertex-induced SBCs centered on the \( z_{1,2} \)-axes and \( z_{2,1} \)-axes, respectively, due to the truncation of the corresponding edge diffracted fields. The confluence of these four SB transitions near the vertex defines the asymptotics pertaining to vertex diffraction, which is implemented by the following steps. First, \( I_i \) in (5) is conveniently expanded in such a way as to highlight the behavior of \( f_i(z_{1,2}) \) at the truncation \( z_{t,1} = 0 \) as

\[
I_i(k_{z_{1,2}}) \approx f_i(0) B_i(k_{z_{1,2}}) - j f_i'(0) B'_i(k_{z_{1,2}}) + O(k_{z_{1,2}}^{-3})
\]

(17)

where \( B_i(k_{z_{1,2}}) \) is defined in Section IV-A (finite array) and \( B'_i(k_{z_{1,2}}) \) is its derivative (the derivation of (17) is the same as that shown for \( I_2(k_{z_{1,2}}) \) in (33)–(36) of Appendix I). Note that, due to the local approximation (17), pole singularities do not present in (3) are now introduced. In the following, physical residue contributions associated with these poles are not accounted for here since they represent the physical FW and edge-diffracted fields already discussed in Section IV-C and -B. The presence of the pole singularities allows us to describe the diffraction mechanism by (poles)-(saddle point) interaction implemented through operations on the already developed canonical integral in [3]. From now on, the topology of the spectral plane \( k_{z_{1}} \) is analogous to that treated in [3] for the uniform sectoral AGF.

Insertion of the asymptotic expansion (17) of \( I_i(k_{z_{1}}) \) into (3) permits the diffracted field from vertex \( (z_{1,2}, z_{2,1}) = (0, 0) \) to be expressed as a sum of four terms

\[
A^{\varphi_{+1}}(\mathbf{r}) \sim \sum_{h=1}^{4} A^{\varphi_{+1}}_h(\mathbf{r})
\]

with

\[
A^{\varphi_{+1}}_h(\mathbf{r}) = \frac{f_1^{(m)}(0) f_2^{(l)}(0)}{8\pi j m + l + 1} \int S_{1,1} \int S_{2,1} e^{-jk_{z_{1,2}} \cdot \mathbf{r}} B_1^{(m)}(k_{z_{1,2}}) B_2^{(l)}(k_{z_{2,1}}) dk_{z_{1,2}} dk_{z_{2,1}}
\]

(19)

where \( S_{1,1} \) are the local SDPs in the plane \( k_{z_{1}} \). The superscript \( (m) \) denotes the \( m \)th derivative; for \( h = 1 \), \( m = l = 0 \); for \( h = 2 \), \( m = 0, l = 1 \); for \( h = 3 \), \( m = 1, l = 0 \); and for \( h = 4 \), \( m = l = 1 \). The integration paths in (19) are along the local-SDP for each variable \( k_{z_{1}} \), as shown in Fig. 5, intercepting the real axis at the saddle point (SP) \( k_{z_{1}}^{\pm} \).

The double integral (19) for the case \( m = l = 0 \) was evaluated in [3], with details given of the path deformations in both variables. The regularization process in that case involves the Van der Waerden (VdW) method, which uses selective addition and subtraction of pole singularities. We have also suggested in [3] the use of the Pauli–Clemmow method [15] because it incorporates the relevant phenomenologies in a simpler format than the VdW method. In particular the Pauli–Clemmow method involves selective multiplication and division by the regularizing functions \( W_{1,1} = [-j]_{d_1}(k_{z_{1}} - k_{z_{1,1}})^{-1}, W_{2,1} = [-j]_{d_2}(k_{z_{2}} - k_{z_{2,2}})^{-1}, W_{1,2} = [-j]_{d_1}(k_{z_{1}} - k_{z_{1,1}})^{2} W_{1,1}^{-1}, W_{2,2} = [-j]_{d_2}(k_{z_{2}} - k_{z_{2,2}})^{2} W_{2,1}^{-1} \). In this paper, we extend the Pauli–Clemmow method to accommodate “slope diffraction effects.” For simplicity, we develop expressions only for regularization of the \( (p, q) \) pole which is closest to the SP. For the vertex problem, the critical parameters are tied to the \( (k_{z_{1}, k_{z_{2}}}) \) in \( (k_{z_{1}}, k_{z_{2}}) = (k \cos \beta_1, k \cos \beta_2) \) first-order SP, and to the \( k_{z_{1}} \) and \( k_{z_{2}} \) poles in (3), i.e., \( (p, q) \) that minimize the spectral distances \( |k_{z_{1}} - k_{z_{1}}| = k \cos \beta_1 - \cos \beta_1 |_{\beta_1} \) and \( |k_{z_{2}} - k_{z_{2}}| = |k \cos \beta_2 - \cos \beta_2 |_{\beta_2} \). For details see [3] and
TABLE I
ASYMPTOTIC BEHAVIOR OF THE TRANSITION FUNCTIONS $T_h(a, b, w)$ FOR LARGE VALUES OF $a$ AND/OR $b$. THE PARAMETERS $a$, $b$ ARE LARGE WHEN THE OBSERVER IS “FAR” FROM SBC1 (SBC2), OR WHEN THE OPERATING FREQUENCY IS HIGH ENOUGH

| $a$ | $b$ | $T_1(a, b, w)$ | $T_2(a, b, w)$ | $T_3(a, b, w)$ | $T_4(a, b, w)$ |
|-----|-----|----------------|----------------|----------------|----------------|
| $>> 1$ | $>> 1$ | $F(b^2)$ | $F(a^2)$ | $F(a^2)$ | $F(b^2)$ |
| $>> 1$ | $>> 1$ | $F(a^2)$ | $F(b^2)$ | $F(b^2)$ | $F(a^2)$ |

[16]. The resulting expressions for each $h$-indexed integral in (19) are

$$A_h^{a, b} \sim \frac{(-1)^{m+l} p^{(m)}(0) q^{(l)}(0) e^{-jkr}}{4\pi r} \cdot B_1^{(m)}(\zeta_1) B_2^{(l)}(\zeta_2) T_h(a_q, b_p, w)$$  (20)

where the locally uniform canonical functions $T_h(a_q, b_p, w)$ are defined as

$$T_h(a_q, b_p, w) = \int_{\infty}^{-\infty} \int_{\infty}^{-\infty} e^{j\xi^2 + 2j\xi\eta + \eta^2} \frac{1}{\xi^{m+1}} \frac{1}{\eta^{l+1}} \xi^\eta \eta^\xi d\eta d\xi$$  (21)

The remaining parameters are defined as $a_q = \sqrt{2kr \sin((\beta_1 - \beta_2) / 2))}$, $b_p = \sqrt{2kr \sin((\beta_2 \cdot \beta_3 - \beta_2) / 2))}$, and $w = \cot(\beta_1 \cot(\beta_2 = \cos(\phi_1 \cos(\phi_2$, with angles $\phi_1, \phi_2$, and $\beta_1, \beta_2$ defined in Figs. 1 and 3, and SB angles $\beta_1, \beta_2$ defined in Appendix II. The resulting functions are expressed exactly in terms of the function $T_1, F(x)$ and $F_s(x)$ as follows:

$$T_2(a, b, w) = \frac{2b}{1 - a^2} \cdot [bF(a^2) - b + waT_3(a, b, w) + waF(b^2)]$$  (23)

$$T_4(a, b, w) = \frac{2b}{1 - a^2} \cdot [bF_s(a^2) - b + waT_3(a, b, w) + waT_1(a, b, w)]$$  (24)

and $T_3(a, b, w)$ is defined in (15) and (16), respectively.

For simplicity of notation, we discuss the asymptotic order estimates and field behavior associated with vertex $(z_1, z_2) = (0, 0)$. For the other vertices the same rules apply in their respective local coordinates. The vertex–differenced field $A^{v, l}(r)$ has a similar asymptotic behavior as $T_1(a_q, b_p, w)$ (see Table I). Thus, “far” from the SBCs and the vertex $(z_1, z_2)$, the field $T_1(a_q, b_p, w)$ is defined in (15) and (16), respectively.

V. TOTAL ASYMPTOTIC POTENTIAL SYNTHESIS

A. Formal Solution

To synthesize the total high-frequency asymptotic solution for the rectangular array potential, referring to Sections IV-A–D, we obtain

$$A^{tot}(r) = \sum_{pq} A^{F, p}_{pq}(r) + \sum_{pq} (A^{d, p}_{pq}(r) + A^{d, q}_{pq}(r))$$  (22)

Note that the domains of existence of the various wave species are automatically embedded in the definitions of $f_i(z_i)$ and the SB planes and cones in the text in connection with (12) and (14), respectively (see Figs. 2, 3, and 4).

B. Transition Functions

Since the numerical evaluation of $T_1$ in (21) can be performed efficiently as in [16], [19], and [20] in terms of standard generalized Fresnel integrals [20], it is convenient to numerically evaluate $T_1, h = 2, 3, 4$ in terms of the function $T_1$ and the UTD transition functions $F$ and $F_s$. In Appendix II, the transition functions $T_{ij, h = 2, 3, 4}$ in (21) have been expressed exactly in terms of the function $T_1 = T_1, F(x)$ and $F_s(x)$ as follows:

$$T_2(a, b, w) = \frac{2b}{1 - a^2} \cdot [bF(a^2) - b + waT_3(a, b, w) + waF(b^2)]$$  (23)

$$T_4(a, b, w) = \frac{2b}{1 - a^2} \cdot [bF_s(a^2) - b + waT_3(a, b, w) + waT_1(a, b, w)]$$  (24)

and $T_3(a, b, w)$ is defined in (15) and (16), respectively.

For simplicity of notation, we discuss the asymptotic order estimates and field behavior associated with vertex $(z_1, z_2) = (0, 0)$. For the other vertices the same rules apply in their respective local coordinates. The vertex–differenced field $A^{v, l}(r)$ has a similar asymptotic behavior as $T_1(a_q, b_p, w)$ (see Table I). Thus, “far” from the SBCs and the vertex $(z_1, z_2)$, the field $T_1(a_q, b_p, w)$ is defined in (15) and (16), respectively.
TABLE II
DIFFRACTION AND COMPENSATION MECHANISMS. COLUMN 1 (WAVE SPECIES): FLOQUET WAVES, FW, \( p_q \); DIFFRACTED WAVES AT EDGE \( i, E_i \); SLOPE-DIFFRACTED WAVES AT EDGE \( i, E_S; \) DIFFRACTED WAVES AT VERTEX \( 1, V \); SLOPE-DIFFRACTED WAVES AT VERTEX \( 1, 2, VS_1, j \); DOUBLE SLOPE-DIFFRACTED WAVES AT VERTEX \( 1, VSS \). COLUMN 2: SPREADER FACTOR. COLUMN 3: WAVE SPECIES AMPLITUDES. NOTATION \( B^{(i)}_k \)

| Wave Species | Spreading Factor | Complex Amplitude | Compensation Mechanisms |
|--------------|-----------------|-------------------|------------------------|
| FW           | 1               | \( f_1(z_{1pq})f_2(z_{1pq}) \) | \( \phi_1 = \phi_1^{(0)} \rightarrow FW \) |
| E1           | \( 1/\sqrt{\rho_1} \) | \( f_1(z_{1pq})f_2(0)B_2F_{(2)}^{(2)} \) | \( \phi_2 = \phi_2^{(0)} \rightarrow FW \) |
| E1S          | \( 1/\sqrt{\rho_1} \) | \( jf_1(z_{1pq})f_2(0)B_2F_{(2)}^{(2)} \) | \( \beta_1 = \beta_1^{(0)} \rightarrow E1 \) |
| E2           | \( 1/\sqrt{\rho_2} \) | \( f_2(z_{2pq})f_1(0)B_1F_{(2)}^{(2)} \) | \( \beta_2 = \beta_2^{(0)} \rightarrow E2 \) |
| E2S          | \( 1/\sqrt{\rho_2} \) | \( jf_2(z_{2pq})f_1(0)B_1F_{(2)}^{(2)} \) | \( \beta_1 = \beta_1^{(0)} \rightarrow E1S \) |
| V            | \( 1/r \)       | \( f_1(0)f_2(0)B_1B_2T \) | \( \beta_2 = \beta_2^{(0)} \rightarrow E2 \) |
| VS1          | \( 1/r \)       | \( jf_2(0)f_1(0)B_1B_2T \) | \( \beta_2 = \beta_2^{(0)} \rightarrow E2S \) |
| VS2 & VS3     | \( 1/r \)       | \( jf_1(0)f_2(0)B_1B_2T \) | \( \beta_1 = \beta_1^{(0)} \rightarrow E1S \) |

The transitional behavior of the other vertex diffraction transition functions \( T_i, h = 1, 2, 3, 4 \) is summarized in Table I. The functions \( T_i \) tend toward unity for large values of \( a_0 \) and \( b_0 \); large values of both \( a_0 \) and \( b_o \) imply significant spectral domain plane separation between the poles and the SP, and define the nonuniform ray regime. When only one of the parameters \( a_0 \) or \( b_0 \) is large, only one of the two poles is asymptotically far away from the SP, and \( T_i \) yields the canonical UTD transition function. The limit for \( b_0 \gg 1 \) can be obtained directly from (21) on approximating the denominator in the integrand by its value at \( \eta = 0 \) [i.e., \( (\eta - b_0/\sqrt{1 - u^2})^{-1} \approx \eta^{-1}\sqrt{1 - u^2} \)] and recognizing the remaining single-pole integral as the ordinary one-parameter UTD transition function \( F_1^{(0)} \). Concerning the functions \( T_2 \) and \( T_3 \) in (21), the large parameter \( a_0 \) (or \( b_0 \)) range is characterized by a double-pole integral that can be expressed in terms of the UTD slope transition function \( F_1(x) \) (see Table I).

The vertex-diffracted contribution \( A^{(i)}_k \) containing \( T_i \) accounts for the transition from a vertex-centered spherical wave to an edge-centered cylindrical wave and it compensates for the discontinuities across the relevant SBCs (see [3]). The other vertex diffracted terms \( A^{(i)}_k \), \( i = 2, 3, 4, \) not present in [3], are of higher asymptotic order and compensate for the discontinuities across the SBCs of the diffracted wave associated with the derivative of the tapering functions at the edges. Note that when the excitation function tends to zero at the vertex from both directions \( z_1 \) and \( z_2 \), only the contribution \( A^{(i)}_k \) remains. The various compensation mechanisms are summarized in Table II. The spreading factors and complex amplitudes are shown for each field contribution associated with vertex \( (z_{1}, z_{2}) = (0, 0) \), namely, \( A^{(0)}_k \), \( A^{(1)}_k \), \( A^{(2)}_k \), and \( A^{(3)}_k \). As shown in the last column, diffracted field species reduce to other wave species at the planar and conical SBCs.

VI. TOTAL VECTOR ELECTRIC FIELD SYNTHESIS

The electric vector fields are obtained from the vector potential \( \mathbf{A} = A\mathbf{J}_0 \) via \( \mathbf{E} = -j\omega\mu_0(\mathbf{A} + \nabla \times \mathbf{A}/k^2) \) and \( \mathbf{H} = \nabla \times \mathbf{A} \). When the differential operators are applied to the spectral representation (3), interchanging the order of integration and differentiation yields (noting that \( \nabla = -jk \) in the spectral domain)

\[
E(r) = \frac{1}{8\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{I_1(k_{z_1})I_2(k_{z_2})}{k_{y}} \bar{G}^E(k) dk_{z_1} dk_{z_2} \tag{25}
\]

where \( \bar{G}^E(k) = -\zeta/k(\bar{I}_k - \mathbf{k}) \), \( \bar{I} \) denotes the unit dyadic, \( \zeta \) is the free space impedance and the notation \( \mathbf{k} \) in \( G^E(k) \) implies a dependence on \( (k_{z_{1}}, k_{z_{2}}, k_{y}(k_{z_{1}}, k_{z_{2}})) \). The magnetic field is treated formally in similar fashion, leading to the replacement of \( \bar{G}^E(k) \) by the magnetic dyadic \( \bar{G}^H(k) = -\mathbf{k} \times \bar{I} \). The asymptotic evaluation of (25) is carried out in the same manner as for the potential \( A \) in (3), except that we now take into account the extra term \( \bar{G}^E(k) \). The integral in (25) is dominated asymptotically by the same critical spectral points as in Section III-B, whence the evaluation procedure is the same as in Sections II–IV.

The polarization dyadic \( \bar{G}^E(k) \) may be assumed to be slowly varying so that (25) can be evaluated by the procedure used in Section IV, where the extra term \( \bar{G}^E(k) \) is approximated by its value at the critical spectral points

\[
E(r) = \sum_{p,q} E_{pq}^{FW} f_1(z_{1pq})f_2(z_{2pq})
\]

\[
+ \sum_{q} (E_{q1}^{FW}(r) + E_{q2}^{FW}(r))
\]

\[
+ \sum_{p} (E_{p1}^{FW}(r) + E_{p2}^{FW}(r))
\]

\[
+ \sum_{i=1}^{4} \sum_{h=1}^{4} E_{ih}^{0}(r)
\]

\[
\tag{26}
\]

with

\[
E_{pq}^{FW}(r) = A_{pq}^{FW}(r) \bar{G}^E(k_{Fpq}) \cdot \mathbf{J}_0 \tag{27}
\]

\[
E_{q1}^{FW}(r) = A_{q1}^{FW}(r) \bar{G}^E(k_{Fq1}) \cdot \mathbf{J}_0 \tag{28}
\]

\[
E_{pq}^{0}(r) = \sum_{i=1}^{4} A_{pq}^{0}(r) \bar{G}^E(k_{Fpq}) \cdot \mathbf{J}_0 \tag{29}
\]
in which the wavevectors $k_{FW}^x$, $k_{FW}^y$, $k_{FW}^z$, and $k^v$ are defined in Fig. 4. Similar expressions hold for the other field contributions, which can be expressed conveniently in their ray-fixed reference systems. The reader is referred to [1], [2], and [3] for more details about the asymptotic pertaining to the vector fields.

VII. NUMERICAL RESULTS

Numerical tests have been performed on a “large” square array of dipoles in order to validate the high frequency formulation in (26). An element-by-element summation over the radiated contributions from each dipole serves as a reference. From a variety of near-field scans carried out for different array parameters and dipole orientations, we have selected only some of the most challenging examples because of space limitations. The quality of the analytic-numerical comparisons in the examples is typical of what we have found throughout. The 30 × 30 element test array of dipoles oriented along $\mathbf{J}_0 = \mathbf{z}_1$ with identical $(z_1, z_2)$ periods $d_1 = d_2 = 0.5\lambda, \lambda = 2\pi/k$ but with different $(z_1, z_2)$ excitations, is shown in the insets of Fig. 6. Only one propagating FW is excited due to the small array period. Consequently, the total propagating contributions are one truncated FW, four truncated edge-diffracted rays, and four vertex-diffracted rays. The electric field component $E_{z_0}$ observed at a distance $R = 12\lambda$ in the diagonal scan plane depicted in the insets, is shown in Fig. 6, for three different coordinate-separable excitations. In Fig. 6(a), the interelement phasings are $\gamma_1 = \gamma_2 = 0$ (broadside radiation, i.e., main beam at $\theta = 0^\circ$) with excitation function given by a Taylor tapering [27] (SLL = −25 dB; the Taylor weights from the edge to the center in both directions are $f_1(n_1d_1) = f_2(n_2d_2) = 0.399, 0.407, 0.42, 0.46, 0.51, 0.58, 0.65, 0.73, 0.799, 0.85, 0.9, 0.94, 0.97, 0.99, 1.0$ for $1 \leq n \leq 15$ with the remaining weights ($16 \leq n \leq 30$) constructed by symmetry). In Fig. 6(b), the interelement phasings are $\gamma_1 = 0, \gamma_2 = 1.1/\lambda$, yielding a radiated main beam at $\theta = 10^\circ$ with respect to the normal to the array plane, and at $\beta_1 = 90^\circ, \beta_2 = 80^\circ$ with respect to both the $z_1$ and $z_2$ axes with tapered excitation functions $f(z_1, z_2) = f_1(z_1)f_2(z_2)$, where $f(z_i) = \exp[-((z_i - L_i/2)^2)/(2\sigma^2)] = L_i^2/(8\ln(c))$ ($c = 0.3$). In Fig. 6(c), the interelement phasing is $\gamma_1 = \gamma_2 = 2.2/\lambda$, yielding a radiated main beam at $\theta = 30^\circ$ with respect to the normal to the array plane, and at $\beta_1 = \beta_2 = 69^\circ$ with respect to both the $z_1$ and $z_2$ axes. The tapering function here is $f(z_i) = \sin(\pi z_i/L_i)$. In all three cases, solid curves denote the uniform asymptotic expression for the total radiated field in (26), while dashed curves denote the radiated field in (26) without the vertex contributions. Clearly, the vertex-diffracted waves compensate for the disappearance of the edge-diffracted waves at their SBCs, rendering the total radiated field (solid curves) continuous. The agreement between the asymptotic and numerical reference solutions, which coincide almost everywhere on the scale of the drawing, has been found quite satisfactory even for scan radii smaller than $12\lambda$. Note that the tapered excitation $f_i(z_i) = \sin(\pi z_i/L_i)$ vanishes at the four array edges, i.e., $f_i(0) = f_i(L_i) = 0$. This implies that the four edge-diffracted fields $A_{FW}^{\nu}(\mathbf{r})$ and the four vertex-diffracted fields $A^{\nu,i}(\mathbf{r}), i = 1, 2, 3, 4$, involve only the slope-diffracted terms $f_i'(0)$, thereby providing a good test case for the usually subdominant additional edge- and vertex-“slope diffracted” contributions. In particular, also in this case, the vertex slope-diffracted fields compensate via the $T_4$ transition.

![Fig. 6. Electric field radiated by a 30×30 rectangular array of dipoles, with interelement spacings and overall dimensions $L_1 = L_2 = 29d_1$. Three different excitations are considered: (a) Taylor excitation with $\gamma_1 = \gamma_2 = 0$ (broadside radiation); (b) Gaussian-on-pedestal excitation with $\gamma_1 = 0, \gamma_2 = 1.1/\lambda$; (c) sine excitation with $\gamma_1 = \gamma_2 = 2.2/\lambda$. In all cases, the observation scan at distance $R = 12\lambda$ passes close to two vertexes in a plane normal to the array and tilted 45° from the $E$ plane. Asymptotic solutions from (26) (solid curves), reference solution (dotted curves), and asymptotic solutions without vertex-diffracted fields (dashed curves).]
function (last row of Table II) for the disappearance of the edge slope-diffracted fields.

VIII. CONCLUSION

In this paper, we have analyzed and validated the high-frequency diffraction phenomena pertaining to large rectangular arrays of dipoles with different linear phases along the two principal coordinates, and with weakly tapered different excitation profiles along these coordinates that may tend to zero at the array edges. The results have been expressed in terms of our previously formulated asymptotic uniformized, periodicity-adopted, FW-modulated GTD for the canonical dipole AGF generated by an infinite plane rectangular sector, which has been generalized here to accommodate the above-mentioned amplitude tapering. The uniform AGF assumes its most intricate form in the vicinity of the vertex, with transition and compensation mechanisms that have been quantified analytically, interpreted phenomenologically, and computed efficiently by our asymptotic algorithm, with accuracy validated through a preliminary set of numerical simulations. The high-frequency results from this analysis can be applied directly to the prediction of the radiation pattern distortion and the interantenna coupling that occurs when the actual array is placed in an electromagnetically complex environment like that of a large array in the antenna farm of a space platform. Such predictions are conventionally obtained by computation-intensive tracing of ray fields from each individual element or a subgroup of elements of the array through the complex environment [7]. Alternatively, global ray tracing based on the direct application of this formulation permits characterization of the entire array aperture radiation in terms of a few rays whose number is independent of the number of array elements, thereby drastically reducing the computational effort. The main effect of the array truncation on the array currents is a global modulation of these currents with a very large period [28], [29]. Although this modulation may perturb the input impedance of the elements, it does not affect significantly the field from the Fresnel zone to the far zone. In other words, the spatial perturbation is filtered by the free-space Green’s function (except possibly at grazing aspects). To correctly evaluate this current deformation near the edges of the array, the array Green’s function can be used in a full-wave analysis of the actual rectangular array elements, as demonstrated for short-wire dipoles in [28]–[30] and for open-ended waveguide in an infinite ground plane in [8] and [9].

This paper terminates our long series of investigations of the FW-modulated UTD pertaining to large planar rectangular phased dipole arrays in free space, escalating sequentially from single-edge geometries to two-edge, single-vertex infinite sectors, culminating in the rectangular array. Simultaneously, we have dealt with dipole excitation profiles ranging from equiamplitude through one-dimensional weak tapering to the two-dimensional tapering here. We regard to these studies, in their totality, to have furnished the foundation for the phenomenologically incisive, numerically efficient full-wave treatment of a variety of practical weakly amplitude-tapered phased array antennas (see [12]). We feel confident that our results so far may be extended to accommodate particular array requirements not covered by weakly tapered or other assumptions, we have initiated the application of these free space algorithms to the important case of dipole arrays printed on, or in, multilayer substrate slabs (see [22]–[24]). However, still ongoing in free space is the separate series of time-domain (TD) canonical studies of sequentially pulsed dipole arrays, which have led to a new FW-modulated TD-UTD [25], [26].

APPENDIX I
TAPERED EDGE DIFFRACTED FIELDS

Here we summarize the procedure for deriving the FW-modulated diffracted field at edge 1 (i.e., that at \( z_2 = 0 \)): all other edges may be treated similarly. The relevant critical point is the SPP \( k_{z_2}^c \) [see ii) in Section III-B] which satisfies (\( d/dz_2 \)\( P(k_{z_1}q', k_{z_2}^c) \equiv 0 \). Inserting (5) into (3), the edge contributions due to the critical point at \( (k_{z_1}q', k_{z_2}^c) \) are found by first expanding the exponent of the integrand in Taylor series around \( (k_{z_1}q', k_{z_2}^c) \), i.e., \( P(k_{z_1}q', k_{z_2}^c) \approx k_{z_1}q' \cdot \mathbf{r} + z_1q'(k_{z_1}^c - k_{z_1}q') \), where \( k_{z_1} = k_{z_1}q' \mathbf{z}_1 + k_{z_2}^c \mathbf{z}_2 + k_{q y} \mathbf{y} \) and \( z_1q = (\partial P)/(\partial k_{z_1}q') (k_{z_1}q', k_{z_2}^c) k_{z_1}^c - k_{z_1}q' = z_1 - yk_{z_1}q'k_{q y} \) and then evaluating \( 1/k_{z_1} \) in a neighborhood of \( (k_{z_1}q', k_{z_2}^c) \). The pertinent asymptotic contribution may be derived from (3) as

\[
A_{k_{z_1}q'}(r) = \sum_{q_{z_2} = -\infty}^{\infty} A_{k_{z_1}q'}^{d1}(r),
\]

where

\[
A_{k_{z_1}q'}^{d1}(r) = \frac{1}{\sin \theta_{k_{z_1}q'}} \int_{-\infty}^{\infty} \frac{I_2(z_{k_{z_2}})}{k_{z_2}q'} e^{-j k_{z_1}q' \cdot r} \cdot \int_{-\infty}^{\infty} \frac{f_1(k_{z_1} - k_{z_1}q') e^{-j z_{1q}(k_{z_1} - k_{z_1}q')} dk_{z_1}}{k_{z_2}q'} \tag{30}
\]

which, after exact evaluation of the inverse Fourier transform of the integral in \( k_{z_1} \), yields

\[
A_{k_{z_1}q'}^{d1}(r) = \frac{1}{4\pi j d_1} \int_{-\infty}^{\infty} \frac{1}{k_{yq'}} I_2(k_{z_2}) e^{-j k_{z_1}q' f_1(z_{1q}(k_{z_2}))} dk_{z_2}, \tag{31}
\]

Note that a) \( z_{1q} = z_{1q}(k_{z_2}) \) depends on the spectral variable \( k_{z_2} \) and b) the two vertex-truncation terms \( (f_1(0))/(2) \) and \( (f_1(L_1))/(2)e^{j(k_{z_1} - \pi)/L_1}L_1 \) in (5) are not included in (30), (31) since they will be incorporated subsequently into vertex-diffracted fields to obtain an asymptotic expansion of \( I_2(k_{z_2}) \) that highlights the behavior of \( \tilde{f}_2(z_{2}) \) at the truncations \( z_2 = 0 \) and \( z_2 = L_2 \). The inverse Fourier transform in (6) is approximated asymptotically, using two sequential integrations by parts (we consider only the end-point \( z_2 = 0 \))

\[
\tilde{f}_2(z_2) = \int_{0} e^{jk_{z_2}z_2 f_2(z_2)} dz_2 \\
\approx \tilde{f}_2(0) - jk_{z_2}^2 f_2'(0) \left( \frac{1}{k_{z_2}^2} \right)^2, \quad f_1'(q) = \frac{\partial f_1}{\partial q}, \tag{32}
\]
Substituting (32) into (5) and omitting the term \((f_2'(L_2))/2\) yields

\[ I_2(k_{z_2}) = \frac{f_2(0)}{2} + \frac{1}{d_2} \sum_{j=\text{odd}}^{\infty} \left( -\frac{f_2(0)}{j(k_{z_2} - k_{z_2,p})} + \frac{f_2'(0)}{(k_{z_2} - k_{z_2,p})^2} \right) \]

(33)

which can be simplified via the identities

\[ B_2(k_{z_2}) = \frac{1}{2} \left[ 1 - \frac{1}{jd_2} \sum_{j=\text{odd}}^{\infty} \left( \frac{1}{(k_{z_2} - k_{z_2,p})^2} \right) \right] \]

\[ = \frac{1}{2} + \frac{1}{2j} \cot \left[ \frac{1}{2}d_2(\zeta - k_{z_2}) \right] \]

(34)

and

\[ \frac{1}{d_2} \sum_{j=\text{odd}}^{\infty} \frac{1}{(k_{z_2} - k_{z_2,p})^2} = jB'_2(k_{z_2}) \]

(35)

Here, \(B_2(k_{z_2})\) is defined in the first paragraph of Section IV-A and \(B'_2(k_{z_2})\) is its derivative with respect to the argument. With (34) and (35), (33) becomes

\[ I_2(k_{z_2}) \sim \frac{f_2(0)}{2} B_2(k_{z_2}) - j f_2'(0) B'_2(k_{z_2}) \]

(36)

The diffracted fields arising from the truncation at \(z_2 = 0\) are obtained from the saddle point at \(k_{z_2} = k_{z_2}^a = z_2^0 k_{p,q}/\rho_1\) where \(\rho_1 = (z_2 + y^2)^{1/2}\) is the transverse distance from edge 1 as shown in Fig. 1. The SBC of the diffracted field from edge 1 is obtained by evaluating the slowly varying function \(f_1(z_{1,q}(k_{z_2}))\) at the saddle point \(k_{z_2} = k_{z_2}^a\). This leads to \(f_1(z_{1,q})\) with \(z_{1,q} = z_1 q k_{z_2}^a\) denoting the footprint of the \(q\)th edge-diffracted field. For approximate evaluation of the corresponding \(z_2\)-edge diffracted fields in (31), uniform asymptotics is necessary since \(B_2(k_{z_2})\) and \(B'_2(k_{z_2})\) have pole singularities of order 1 and 2, respectively, at \(k_{z_2} = k_{z_2,p}\), which may lie near the saddle point. The uniform asymptotics is obtained by mapping the integral onto the two simplest canonical functions that match the same type of pole singularity and saddle points. To this end, we use the mathematical identities [10]

\[ \int_{-\infty}^{\infty} \frac{e^{-Ks^2}}{s-y} ds = -\sqrt{\frac{\pi}{K}} \frac{F(jKy^2)}{y} \]

(37)

\[ \int_{-\infty}^{\infty} \frac{e^{-KS^2}}{(s-y)^2} ds = d \int_{-\infty}^{\infty} \frac{e^{-KS^2}}{s-y} ds \]

\[ = -\sqrt{\frac{\pi}{K}} \frac{d \cdot F'(jKy^2)}{y} \]

(38)

where \(F(x)\) is the conventional UTD transition function [13] defined in (15), thereby matching the contributions in (36) that contain single poles (associated with \(B_2(k_{z_2})\)) and double poles (associated with \(B'_2(k_{z_2})\)). By using the identity \((d/d\xi)(F(\xi^2))/(\xi) = -(F'_{\xi}(\xi^2))/(\xi^2)\), derived through direct differentiation from (15) and (16), one can express (39) in terms of the UTD slope transition function \(F'(\xi)\) defined as [14]

\[ \frac{x}{\sqrt{K\pi}} \int_{-\infty}^{\infty} \frac{e^{-KS^2}}{(s-v^2)} ds = F'(x) \]

(39)

Using (36) and (39), a locally uniform asymptotic evaluation of (31) is performed via SDP integration through the saddle point at \(k_{z_2} = k_{z_2}^a\), leading to (14) where the exponential function arises from the evaluation of the integrand of (31) at the saddle point, i.e., \(k_{q}(k_{z_2}^a) = k_{q}^a\).

**APPENDIX II**

**THE VERTEX-RELATED TRANSITION FUNCTIONS** \(T_h\)

In this Appendix, we provide the derivation of the (23) and (24), which are useful for the numerical evaluation of the integral functions \(T_h\). For convenience, let us define \(g_h = \left[ -1 \right]^{m+1} I^{m+1} I'[m+1] / [\pi (1 - w^2)^{m+1}/2] \) where the relationships between \(h\) and \(I, m\) are defined after (19). Furthermore, define the simple-pole functions \(C_{\xi} = 1/[\xi - \langle a_q\rangle/(\sqrt{1 - w^2})] \), \(C_{\eta} = 1/[\eta - (b_q)/(\sqrt{1 - w^2})] \) which yield corresponding double-pole functions by differentiation \(\partial C_{\xi}/\partial \xi = C_{\xi}' = -1/\langle a_q \rangle/(\sqrt{1 - w^2})^2 \), \(\partial C_{\eta}/\partial \eta = C_{\eta}' = -1/\langle b_q \rangle/(\sqrt{1 - w^2})^2 \). The functions \(T_1\) and \(T_2\) in (21) can then be rewritten as

\[ T_1 = g_1 \int T \xi C_{\eta} e^{i\eta d\xi} d\xi, T_2 = -g_2 \int C_{\xi} C_{\xi}' e^{i\xi d\xi} d\eta = - \int (C_{\xi} T \xi C_{\xi}' e^{i\xi d\xi}) d\eta \]

where \(g_1(T \xi C_{\eta} e^{i\xi d\xi}) d\xi\) vanishes due to the denominator of \(C_{\eta}\). This leads to

\[ T_2 = 2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left( w \xi + \eta \right) C_{\xi} C_{\eta} e^{i\eta d\xi} d\xi d\eta \]

(40)

Similarly, interchanging \(a_q \leftrightarrow b_p\) (which implies \(g_2 \leftrightarrow g_3\) and \(\xi \leftrightarrow \eta\) in (40)), we obtain \(T_3 = 2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (w \eta + \xi) C_{\eta} C_{\eta} e^{i\xi d\xi} d\eta d\xi\)

We reduce the expressions for \(T_2\) in (40) to a form that involves only \(T_1\) and the UTD transition function \(F(x)\). First, we use the identity \(w \xi + \eta = (\xi - (b_q)/(\sqrt{1 - w^2}) + (b_q)/(\sqrt{1 - w^2}) + (w \xi - (a_q)/(\sqrt{1 - w^2}) + (w a_q)/(\sqrt{1 - w^2})\) that leads to the decomposition of \(T_2\) as follows:

\[ T_2 = 2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ C_{\xi} + (w b_q + w a_q) C_{\xi} + (w \xi + \eta) \right] e^{i\xi d\xi} d\eta \]

(41)

Since \(q = q(\xi, \eta) = (\xi^2 + 2 \eta \xi + \eta^2) = \xi^2 (1 - w^2) + (\eta + w \xi)^2 = \eta^2 (1 - w^2) + (\xi + w \eta)^2\), the \(\eta\)-integral in the first term

(41)
and the $\xi$-integral in the third term of (41) can be evaluated in
closed form, while the second term reconstructs $T_1$ so that
\[
T_2 = 2j\theta_2 \left[ \int_{-\infty}^{\infty} C_\eta C_\xi e^{i\xi^2(1-w^2)} d\xi + \frac{(b_p + w a_q) T_1}{\sqrt{1-w^2}} \frac{1}{g_1} \right]
+ \int_{-\infty}^{\infty} C_\eta e^{i\eta^2(1-w^2)} d\eta \right]
= 2j \left[ -\pi j F(a_q^2) a_q \frac{1}{a_q} + \frac{(b_p + w a_q) T_1}{\sqrt{1-w^2}} \frac{1}{g_1} + \frac{w \pi j F(b_p^2)}{b_p} \right] \tag{42}
\]
The last equality in (42) involves use of (37) in the third terms on the right-hand side. After rearrangement, we obtain (23). The expression for $T_3$ is obtained from symmetry in (41). To obtain the canonical form of the transition function $T_3$ in (21), we first perform $a_q$-differentiation of $T_2$ (or $b_p$-differentiation of $T_3$), using (21)
\[
\frac{\partial T_2}{\partial a_q} = \frac{T_2 - \frac{g_4}{a_q}}{\int_{-\infty}^{\infty} C_\eta C_\xi e^{i\xi^2(1-w^2)} d\xi} = \frac{T_2 - T_4}{a_q}. \tag{43}
\]
Insertion of (42) into (43), and term by term differentiation thereafter, yields
\[
T_4 = T_2 - \frac{2j\theta_2 b_p}{1-w^2} \left[ \frac{\partial F(a_q^2)}{\partial a_q} - w T_1 - \frac{\partial F(b_p^2)}{\partial a_q} \right]. \tag{44}
\]
Use of the identities $(\partial F(a_q^2))/(\partial a_q) = (F(a_q^2) - F(a_q^2)) / a_q$ and $(\partial F(b_p^2))/(\partial a_q) = (T_2 - T_3) / a_q$ in (44), followed by simple manipulations, leads to (24).

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