I = 0 scalar channel

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Using lattice QCD with dynamical Wilson fermions, we study I = 0 and J\textsuperscript{PC} = 0\textsuperscript{+} channel which is constructed by $\frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)|0\rangle$, in order to search for the $\sigma$ meson. Our preliminary result shows that the connected and disconnected diagrams contribute to the $\sigma$ meson propagator in the same order.

1. INTRODUCTION

In QCD, the chiral symmetry is spontaneously broken (restored) at the confinement (deconfinement) phase where sigma meson plays an essential role. We do not doubt this mechanism, and yet no one is sure whether the sigma meson exists or not. It might have large mass and/or wide width, or might be simply $\pi - \pi$ correlation effect. The light $\sigma$ meson had disappeared from the tables of Particle Data Group (PDG) for over 20 years. However, the $I = 0$ and $J\text{PC} = 0^{++}$ meson, ”$f_{0}(400-1200)$ or $\sigma^{*\prime}$”, appeared bellow 1 GeV mass region in PDG recently\[1\]. This is probably because of recent $\pi - \pi$ scattering phase shift re-analyses; Especially, Igi and Hikasa constructed a general model-independent framework, which respects the analyticity, unitarity and crossing symmetry together with chiral symmetry low energy theorem, to describe the $\pi\pi$ elastic scattering below 1 GeV mass region and investigated the existence of $\sigma$ meson \[2\]. See Ref.\[3\] for a good review on the situation and physical meaning of the sigma meson.

Now it is very desirable to investigate whether $\sigma$ meson appears as a pole based on lattice QCD. Using the quenched approximation, Alfold and Jaffe discussed the possibility of the light scalar mesons as $\bar{q}q$ states rather than $\bar{q}q$ \[4\]. McNeile and Michael computed the mixed iso-singlet scalar masses of $q\bar{q}$ and glueball states in two kind of situation, i.e. with and without the dynamical quark effects \[5\]. The $\sigma$ meson masses which are obtained with consideration of the dynamical quark effects are much lower than the quenched results. The goal of our project is to conclude whether the $\sigma$ meson exists or not below 1 GeV in QCD.

2. $\sigma$ PROPELLATOR

We construct I = 0 scalar channel by $\sigma|0\rangle$. The operator $\sigma$ is given as

$$\sigma(x) \equiv \sum_{c=1}^{3} \sum_{\alpha=1}^{4} \bar{u}^{c}_{\alpha}(x)u^{c}_{\alpha}(x) + \bar{d}^{c}_{\alpha}(x)d^{c}_{\alpha}(x) \frac{1}{\sqrt{2}},$$

where $u$ and $d$ are the $u$-quark and $d$-quark Dirac spinors, respectively. This operator has the same quantum number as the vacuum, $I = 0$ and $J\text{PC} = 0^{+}$. The indices $c$ and $\alpha$ denote color and Dirac spinor indices, respectively. The $\sigma$ meson propagator is given by,

$$G(y, x) = -\langle \text{Tr} W^{-1}(x, y) W^{-1}(y, x) \rangle$$

$$+ 2 \langle \text{Tr} W^{-1}(y, y) \text{Tr} W^{-1}(x, x) \rangle$$

$$- 2 \langle \text{Tr} W^{-1}(y, y) \rangle \langle \text{Tr} W^{-1}(x, x) \rangle.$$  (2)

Here ”Tr” represents summation over color and Dirac spinor indices and $W^{-1}$ is $u$, $d$ quark propagator. The third term of Eq.(2), $\langle \sigma(y) \rangle \langle \sigma(x) \rangle$, corresponds to the subtraction of vacuum contri-
Figure 1. \( \text{Tr} W^{-1}(x, x) \) evaluated by the noise method as a function of the number of \( Z_2 \) noise sources.

The second and the third terms are the same order and the high precision numerical simulations and careful analyses are required.

3. NUMERICAL SIMULATIONS

We calculate the \( \sigma \) propagator by using Hybrid Monte Carlo algorithm. We use \( Z_2 \) noise method to calculate the disconnected diagrams. The 1000 random \( Z_2 \) numbers are generated. The two-flavors Wilson fermion is simulated on the \( 8^3 \times 16 \) lattice. Based on ref.\[7\], we set \( \beta = 4.8 \), \( \kappa = 0.1846 \) (\( a = 0.197(2) \text{ fm}, \kappa_c = 0.19286(14) \)) \[7\]. After the thermalization trajectories, \( \sigma \) propagators are calculated on a configuration in every 10 trajectories.

3.1. NUMERICAL ACCURACY

Since there are big numerical cancellation in \( \langle \sigma(y)\sigma(x) \rangle - \langle \sigma(y) \rangle \langle \sigma(x) \rangle \), we must be careful to control the numerical accuracy.

In Fig.3, we show the values of \( \sigma(x) \) for a typical configuration as a function of the number of \( Z_2 \) noise sources. Dotted lines represent the required accuracy to evaluate \( \langle \sigma(y)\sigma(x) \rangle - \langle \sigma(y) \rangle \langle \sigma(x) \rangle \). From the figure, we may conclude that 1000 noise sources are very safe and we do not suffer from any systematic error due to the \( Z_2 \) noise.

We have checked also the effect of CG solver tolerance, and \( \text{Tr} W^{-1}(x, x) \) is very stable for \( \epsilon < 10^{-10} \).

3.2. PROPAGATORS

As preliminary results, Fig.2 shows the contribution of the connected and disconnected diagrams of the \( \sigma \) propagator in the time direction. The connected diagram shows clear exponential damping. On the other hand, the disconnected diagram has large error bars. However there exists the contribution of disconnected diagram to \( \sigma \) meson propagator and its order is the same as connected diagram. Furthermore it can be possible that the existence of the disconnected diagram makes the \( \sigma \) meson mass lighter. This means that the estimation of the disconnected diagram is important to determine the property of \( \sigma \) meson. In order to argue \( \sigma \) meson in detail, the improvement of the evaluation of the disconnected diagram and more statistics are indispensable.

In Fig.3 we compare the \( \sigma \) meson propagator
with \( \rho \) propagator. We can see that the \( \sigma \) meson mass could be the same order of \( \rho \) meson mass, i.e., we obtain the suggestion of the existence of light \( \sigma \) meson, though the error bar of \( \sigma \) meson propagator is very large.

4. CONCLUDING REMARKS

We investigated the property of \( I = 0 \) and \( J^{PC} = 0^{++} \) scalar meson (\( \sigma \) meson) whose operator is \( \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d) \). In the \( \sigma \) meson propagator, the contribution of disconnected diagram is the same order of connected diagram: Quenched approximation is not reliable for the investigation of the \( \sigma \) meson. The evaluation of the disconnected diagram is done by using \( Z_2 \) noise method. A statistical error of \( \sigma \) propagator which comes from the disconnected diagram mainly is large in the present stage. As preliminary results, we obtain the following properties of \( \sigma \) propagator: (1) Both the connected and disconnected parts equally contribute to the \( \sigma \) propagator. (2) \( \sigma \) meson could have mass of the same order of the \( \rho \) meson.

It is necessary to generate much more gauge configurations and improve the statistical precision of the estimation of \( \sigma \) propagator. We also plan to improve the source of the sigma propagators.

Furthermore, we must investigate the mixing state of the \( \sigma \) meson and glueball if we obtain the result that \( \sigma \) meson mass is greater than 1 GeV region\([8,9]\).

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