Spatial autoresonance acceleration physical scheme based on magnetic rings system

E A Orozco$^1$, V D Dugar-Zhabon$^1$, J E López$^1$, and J López$^1$

$^1$ Universidad Industrial de Santander, Bucaramanga, Colombia

E-mail: eaorozco@uis.edu.co, jesus2198136@correo.uis.edu.co

Abstract. The spatial auto-resonant acceleration scheme consists in the acceleration of an electrons beam by the transverse electric field component of a standing microwave field and an external inhomogeneous magnetostatic field, whose longitudinal profile is fitted to maintain the electron cyclotron resonance condition along of its helical trajectories. In practice, the external magnetic profile can be generated by a system of current coils. In order to save energy and the possibility to reduce space, we study the option to replace the coils by a magnetic rings system of uniform axial magnetization. In this work, we present both, the results of the magnetostatic field generated by a magnets system, which is calculated from the magnetization currents using the Biot-Savart law, and the acceleration of an electrons beam in the spatial auto-resonant acceleration regime employing said magnetostatic field. The electron trajectory, its velocity and energy are obtained from the numerical solution of the relativistic Newton-Lorentz equation, showing that is possible to accelerate electrons injected with energies about of some electronvolts to values about of 250 keV.

1. Introduction

Since 1962 the concept of electron cyclotron self-resonance has been known, defined as the self-sustaining oh the resonant interaction of an electron with a transverse electromagnetic wave along a homogeneous magnetic field [1,2]. There are different ways to guarantee the acceleration of electrons. Some mechanisms are based on the fit of the magnetic field to compensate for the increase of the relativistic factor. The gyro resonant acceleration (GYRAC) mechanism uses an external uniform magnetic field that grows on time [3,4]. Another mechanism called spatial auto-resonant acceleration (SARA) consists in the sustained acceleration of an electron beam using a transverse microwave stationary field and a non-uniform static external magnetic field, which has been widely studied by Dugar-Zhabon, Orozco, et al. [5–7].

Studies based on computational simulations show that is possible to accelerate an electron beam up to energies about of 250 keV, even with electron concentration $n_e \sim 10^8 \text{cm}^{-3}$. In this process, the phase shift between the transverse velocity of the electrons and the electric component of the microwave field lies in the range $\pi/2 < \varphi < 3\pi/2$, called acceleration band. Studies on the SARA mechanism have been the basis for the design of a compact X-ray source, recently patented [8].

The external magnetic field in the SARA mechanism is produced by coaxial current, however, in practice, the coils increase the size of the source and therefore difficult its portability. In the present work, the viability of the realization of the SARA mechanism by using magnetic rings
of uniform axial magnetization, to produce the magnetostatic field is presented. The electron energy evolution is studied using the new magnetic configuration.

2. Simulation model

Figure 1 shows the physical system used for the SARA mechanism realization based on the new magnetic configuration. The non-magnetic cylindrical cavity (1) is affected by an appropriated magnetostatic field, which is generated by an axially magnetized ring and two discs system (2). The electric field profile of the cylindrical TE_{112} microwave field (3) accelerates the injected electron beam (4) in electron cyclotron resonance conditions along its helical paths (5).

![Figure 1. Physical system model. 1) Resonant cavity. 2) Magnets system. 3) Electric field profile. 4) Injection of electrons. 5) Path of the electron.](image)

The magnetic field in the position \( \vec{r} \), generated by the configuration of the magnets, denoted as \( \vec{B}^s \), can be calculated from Equation (1), the Biot-Savart law.

\[
\vec{B}^s = \frac{\mu_0}{4\pi} \int_{S} \vec{K}_m \times (\vec{r} - \vec{r}_s) \left| \vec{r} - \vec{r}_s \right|^3 ds,
\]

where \( \vec{r}_s \) is a source point, \( \vec{K}_m = \vec{M} \times \hat{n} \) is the superficial magnetization current density, being \( \vec{M} \) the magnetization and \( \hat{n} \) is the unit normal vector on magnet surface. It is easy to note that for this configuration, there are four surfaces with its respective current density in azimuthal direction: a lateral surface for each magnetized disk and two lateral surfaces for the ring, the internal and external. The magnetic radial and axial components for a single surface are due by Equation (2) and Equation (3), respectively.

\[
B^k_r = \frac{\mu_0}{4\pi} M_k R_{sk} \int_0^{2\pi} \cos(\phi) \left[ \frac{1}{\sqrt{a(\phi) + b_-^2}} - \frac{1}{\sqrt{a(\phi) + b_+^2}} \right] d\phi
\]

\[
B^k_z = \frac{\mu_0}{4\pi} M_k R_{sk} \int_0^{2\pi} \left( \frac{R_{sk} - r \cos(\phi)}{a(\phi)} \right) \left[ \frac{b_+}{\sqrt{a(\phi) + b_+^2}} - \frac{b_-}{\sqrt{a(\phi) + b_-^2}} \right] d\phi,
\]

here, \( a(\phi) = r^2 + R_{sk}^2 - 2r R_{sk} \cos(\phi) \), \( b_- = z - Z_{ck} - 0.5L_k \) and \( b_+ = z - Z_{ck} + 0.5L_k \). The \( k \)-index represent an arbitrary current surface. \( R_{sk}, L_k, Z_{ck} \) and \( M_k \) are the surface radius, length, \( z \)-position of the center and magnetization values of magnets. In this way, Equation (4) shows the total \( \vec{B}^s \) components, due by the superposition principle.

\[
B^s_r(\vec{r}) = \sum_{k=1}^{4} B^k_r(\vec{r}) \quad \text{and} \quad B^s_z(\vec{r}) = \sum_{k=1}^{4} B^k_z(\vec{r})
\]
The electron energy evolution and their dynamics are obtained through the solution of the relativistic Newton-Lorentz equation of motion, which, in a dimensionless scheme [9, 10] (Equation 5) takes the form:

$$\frac{d\vec{u}}{d\tau} = \vec{g}_0 + \frac{\vec{u}}{\gamma} \times \vec{b},$$

where $\vec{u} = \vec{p}/m_0c$ is the electron momentum, $\vec{g}_0 = -e\vec{E}_{hf}/m_0c\omega$ is the microwave electric field, $\vec{b} = -\vec{B}/B_0$ is the total magnetic field at the electron position ($\vec{B} = \vec{B}_s + \vec{B}_{hf}$), being $\vec{B}_{hf}$ the magnetic field of the microwave. $\tau = \omega t$ is the time and finally $\gamma = (1 + u^2)^{1/2}$ is the relativistic factor. The parameters $\omega$ and $B_0 = m_0\omega/e$ are the angular frequency of the microwave field and the magnetic field value to obtain classical resonance at the injection point, respectively. Under a finite differences scheme, the Equation (5) can be written as,

$$\vec{u}_{n+1/2} - \vec{u}_{n-1/2} = \vec{g}^n + \frac{\vec{u}_{n+1/2} + \vec{u}_{n-1/2}}{2\gamma^n} \times \vec{b}^n,$$

(6)

the Equation (6) is solved numerically using the Boris-Bunneman algorithm in a leap-frog scheme [11,12], where the new position in each simulation time step is calculated from Equation (7).

$$\vec{x}^{n+1} = \vec{x}^n + \frac{\vec{u}^{n+1/2}\Delta\tau}{\gamma^{n+1/2}},$$

(7)

here $\gamma^{n+1/2} = [1 + (u^{n+1/2})^2]^{1/2}$. These positions are normalized with respect to $r_l = c/\omega$, the relativistic Larmor radius.

3. Results and discussions
The integrals in Equation (2) and Equation (3) are solved numerically. The Table 1 shows the magnetization values, positions and magnets dimensions. The Figure 2 show the magnetic profile obtained by current coils and magnets; Figure 3 show the magnetic z-component profile in the plane $y = 0$.

| Magnet | Ri (cm) | Re (cm) | Zc (cm) | Lz (cm) | $M_z \times 10^5$ (A/m) |
|--------|---------|---------|---------|---------|------------------------|
| Disc1  | —       | 19.00   | -9.80   | 2.00    | 17.20                  |
| Ring   | 11.00   | 12.25   | 9.60    | 11.00   | -9.10                  |
| Disc2  | —       | 10.00   | 21.50   | 1.65    | 9.90                   |
In the simulations carried out, a $\text{TE}_{112}$ mode with 7k V/cm electric field amplitude oscillating at 2.45 GHz is excited. The electrons have been injected in $z = 0$ along the cavity axis with energies of 9 keV, 10 keV, 11 keV, 13 keV and 15 keV. Figure 4 shows the trajectory of an electron injected with energy of 11 keV that moves in the SARA regime under the effect of the magnetostatic field shown in Figure 3. We can note that the electron stops its longitudinal motion close to the plane $z = 18$ cm due to the diamagnetic force acting in the opposite direction to which the field increases locally.

Figure 5 shows the spatial evolution of the electron energy for different injection energies (for all figures: 9 keV (black), 10 keV (red), 11 keV (blue), 13 keV (green) and 15 keV (purple)). It can be noted that near the $z = 18$ cm plane the energy is maximum, with values about of 250 keV. It should be noted that the electrons injected with energies of 9 keV and 10 keV are not accelerated; this is because, in addition to the fact that the values of cyclotron frequency and oscillation of the wave must be close, it is necessary that the phase shift $\varphi$ between the transverse velocity and electric field component of the microwave field is in the acceleration band. The spatial evolution of $\varphi$ for the different cases considered is shown in Figure 6, where the shaded region represents the acceleration band. It can be noted that the electron injected with energy of 10 keV don’t lie on the band between $z \sim 12$ cm and $z \sim 17$ cm, because its longitudinal velocity becomes zero close to $z = 10$ cm (see Figure 6). Due to the decrease of the magnetic field in the plane $z = 12$ cm, the longitudinal velocity of the electron increases slightly, and this effect is mots appreciable for the electrons injected with an energies values of 10 keV and 11 keV.
4. Conclusions
The results of the simulations show that it is possible to obtain a magnetic field profile based on a set of magnetized rings that guarantee the acceleration of electrons on the self-resonating interaction mechanism SARA, where the smalls differences with the magnetic coil profile have not a great change in the final energy of the electrons. This result provides a new experimental implementation where it is possible to change the coils system by magnets configurations and reduce space, and delete the external current source. A small size improves the portability of a physical system and improves the idea of a small X-ray source.

References
[1] A A Kolomenskii and A N Levedev 1962 Sov. Phys. 145 492
[2] V Ya Davydovskii 1963 Sov. Phys. JETP 43 886
[3] K S Golovanivsky 1982 Physica Scripta 25(3) 491
[4] K S Golovanivsky 1982 IEEE Transactions on Plasma Science 10(2) 120
[5] V D Dugar-Zhabon, et al. 2016 J. Phys. Conf. Ser. 687 012076
[6] V D Dugar-Zhabon, et al. 2008 Physical Review Special Topics-Accelerators and Beams 11(4) 041302
[7] V D Dugar-Zhabon, et al. 2016 J. Phys. Conf. Ser. 687 012077
[8] Dugar-Zhabon V and Orozco E 2017 Compact self-resonant x-ray source, Patent US9666403B2 (Alexandria: United States Patent and Trademark Office)
[9] M N Sadiku 2001 Numerical techniques in electromagnetics, second edition (Boca Raton: CRC press)
[10] C K Birdshalland and A B Langdon 1991 Plasma physics via computer simulation (Bristol: Institute of Physics Publishing)
[11] J P Boris 1970 Relativistic plasma simulation-optimization of a hybrid code Proceeding of Fourth Conference on Numerical Simulations of Plasmas ed J P Boris and R A Shanny (Washington: Naval Research Laboratory) p 3
[12] T Tajima 2004 Computational Plasma Physics: With applications to fusion and astrophysics (Boca Raton: CRC press)