Lateral/Directional Control Law Design for Civil Airplane Based on Eigen Structure Assignment

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Abstract. Considering disadvantages of lateral/directional mode characteristics of civil aircraft, design requirements are thus presented and the P-Beta control law architecture is adopted for the lateral/directional control law. Meanwhile, the practical application of eigen structure assignment in the design of lateral/directional control law is studied. By eigen structure assignment the closed loop is designed, and the decoupling of roll channel and yaw channel is realized. Through the design of feed-forward command channel, the pilot's stick control roll rate and pedal control sideslip angle are realized. Simulation results show that the designed lateral/directional flight control law could meet design requirements.

1. Introduction

The flight control system of modern large civil aircraft is coupling with multiple systems and receives input signals from multiple systems such as inertial navigation system, cockpit control system, atmospheric data system and high lift system, which is a multiple input and multiple output system, which puts forward higher requirements for the design of flight control system. Therefore, the fly by wire flight control system mostly uses modern control theory to design multivariable control law. The commonly used modern control methods include LQR method, eigen structure assignment method, nonlinear dynamic inversion method and so on[2]. According to the characteristics of high coupling between roll channel and yaw channel in lateral/directional mode, the eigen structure assignment method is adopted in this paper. This method is a multivariable system design method based on time domain, which can provide mode decomposition means, which is not available in other control law design methods, so this method is very useful in decoupling control. At the same time, the method has been successfully applied in the design of Boeing 767 lateral/directional flight control system, and the eigen structure assignment is also used in the inner loop of Airbus A320 lateral control law[3].

The paper mainly studies the design of civil aircraft lateral/directional control law. After the fly by wire flight control system is applied to civil aircraft, the aircraft lateral/directional control law is mainly divided into three categories, namely open-loop control, p-rudder control and p-beta control. The early civil aircraft usually used open-loop control or p-rudder control. However, the above two lateral/directional control laws without the ability to reduce the vertical tail load. Therefore, in order to meet the requirements of reducing the vertical tail load, more advanced civil aircraft generally adopt p-beta control law that the lateral channel is controlled by P (roll angle rate) command, and the pedal of directional channel directly commands the sideslip angle, which has the functions of increasing dutch roll damping and coordinated turning. It can reduce the vertical tail load of the aircraft, which is conducive to vertical tail load reduction, weight reduction and flight safety.
2. Linear model

2.1. Mathematical model of aircraft lateral/directional
In this paper, the state point of the middle center of gravity of a large civil aircraft at the landing configuration and reference indicated approach speed is used as the design basis, and the linear model of the transverse motion of the aircraft body is:

\[
\begin{bmatrix}
\dot{p} \\
\dot{\phi} \\
\dot{r} \\
\dot{\beta}
\end{bmatrix} =
\begin{bmatrix}
-1 & 0 & 0.6 & -2.5 \\
1 & 0 & 0.11 & 0 \\
-0.18 & 0 & -0.18 & 0.33 \\
0.11 & 0.11 & -0.9 & -0.15
\end{bmatrix}
\begin{bmatrix}
p \\
\phi \\
r \\
\beta
\end{bmatrix} +
\begin{bmatrix}
-2.4 & 0.5 \\
0 & 0 \\
-0.4 & -0.5 \\
0 & 0.035
\end{bmatrix}
\begin{bmatrix}
\delta p \\
\delta \phi \\
\delta r \\
\delta \beta
\end{bmatrix}
\]  

(1)

\( \delta p \) is the stick input is realized by designing the geometric relationship between the aileron and the spoiler, the relationship between lateral input and roll efficiency is linear; \( \delta r \) is rudder control input.

The characteristics of aircraft body are analysed from the state equation of lateral/directional. By solving the eigenvalues of the system matrix, the results are shown in table 1.

**Table 1. Eigenvalues of the system matrix.**

| Eigenvalues | Value          |
|-------------|----------------|
| \( p_1 \)   | -1.187         |
| \( p_2 \)   | -0.018         |
| \( p_3 \)   | 0.063+0.927i   |
| \( p_4 \)   | 0.063-0.927i   |

\( p_1 \) Rolling mode, \( p_2 \) Spiral mode, \( p_3, p_4 \) Dutch rolling mode. Dutch roll mode damping \( \zeta = 0.068, \omega_n = 0.930rad / s \). It can be seen that the dutch roll damping of the aircraft body is very small, which is difficult for the pilot to control; The spiral mode shows positive spiral stability, that is, at a certain roll angle, the pilot releases the stick, the aircraft wing will return to the horizontal, but the expected spiral mode is neutral stability. Therefore, by designing of control law, the closed-loop characteristics of the aircraft are improved, the neutral stability of the spiral mode is maintained, at the same time, the damping of the dutch roll mode is enhanced, the vertical tail load is limited, the pilot's burden is reduced and the comfort is improved.

Therefore, based on the modal analysis of the aircraft body, the design objectives of the lateral/directional channel of the flight control system are: neutral spiral stability; moderate damping ratio; decoupling of lateral and directional.

2.2. P-Beta control law design
In this paper, the eigen structure assignment method of output feedback is used to decouple the lateral channel and directional channel.

2.2.1. Eigen structure assignment method summary. Linear time invariant systems with output feedback is shown as follow figure 1.
It is assumed that the eigenvalue of the given ideal closed-loop system is $\{\lambda_i^d\} i=1,2,\ldots, m$ and $\lambda_i^d$ is the corresponding closed-loop eigenvector. For a set of eigenvalues $\lambda_i^d$ and feature vector $v_i$, 

$$(A + BKC)v_i = \lambda_i^d v_i$$

Or

$$v_i = (\lambda_i^d I - A)^{-1} BKC v_i$$

Before considering the reachable assignment of eigenvectors, the ideal eigenvector $v_i^d$ must be selected first. But usually, the ideal feature vector cannot be achieved. At this time, the most likely feature vector is required to replace the ideal feature vector. First define:

$$L_i = (\lambda_i I - A)^{-1} B$$

In order to find $v_i^d$ the corresponding value $z_i$ when projected on the reachability space, the following performance index shall be minimized.

$$J = \|v_i^d - v_i\|^2 = \|v_i^d - v_i^s\|^2$$

If $\partial J / \partial z_i = 0$,

$$z_i = (L_i^T L_i)^{-1} L_i^T v_i^d$$

$$v_i^d = L_i (L_i^T L_i)^{-1} L_i^T v_i^d$$

From equation (2), the feedback gain matrix can be obtained as:

$$K = B^* (V \lambda_i^d - AV)(CV)^{-1}$$

### 2.2.2. Closed loop gain calculation

The control law only need assignment the expected eigenstructure of the basic mode. So, expected eigenstructure is:

**Expected eigenvalues:**

$$\lambda_i^d = [-1.4 -0.9 -0.8 +0.9 j -0.8 -0.9 j]$$

**Expected eigenvector:**

- Roll mode $V 1^T = [\times 1 \times 0 \times \times \times \times]$  
- Spiral mode $V 2^T = [\times 1 \times 0 \times \times \times \times]$  
- Dutch roll mode $V 3^T = [\times 0 \times 1 \times \times \times \times], V 4^T = [\times 0 \times 1 \times \times \times \times]$  

According to the algorithm of eigenstructure assignment, the feedback gain matrix is obtained:

$$K = \begin{bmatrix} 0.1958 & 0.2237 & 1.6150 & -1.8143 & -0.4977 \\ -0.6922 & -0.4843 & 3.6653 & -3.2773 & -1.9207 \end{bmatrix}$$
2.2.3. Forward command channel design. The roll angle command is generated after the pilot instructs the roll angle rate through the integration link. In this paper, after ignoring the high-order term, there is

\[
\frac{\phi}{\phi_c} = \frac{K_s s + 1}{s}
\]

Due to the function of integrator, the transfer function has a pole of 0, which realizes the neutral stability of spiral mode. So, the final command channel framework is shown as follow figure 2.

![Command channel framework](image)

**Figure 2. Command channel framework.**

In order to realize the pilot's pedal command side slip angle, the side slip angle feedback is introduced. After through the integrator, the error between the command side slip angle and the actual side slip angle is eliminated, so as to realize the accurate control of the side slip angle and reduce the vertical tail overload. In order to improve the rapidity of rudder response to input, the proportional link is introduced.

3. Simulation result

The control law simulation model designed in this paper is built in the Simulink simulation environment, and the frequency domain stability margin and time domain response of the control law are simulated respectively. The results are as follows.

3.1 Stability margin

![Stability margin at actuator](image)

**Figure 3. Stability margin at actuator.**
Figure 4. Stability margin at sensor.

Through the Nicolas diagram in frequency domain, it can be seen that the designed control law has good stability margin at both actuator and sensor, and meets the requirements of system stability (phase margin is greater than 60°, amplitude margin is greater than 9dB).

3.2 Comprehensive simulation

Given the pilot's sidestick input and pedal input respectively, the simulation results are obtained as follow:

Figure 5. Time domain response of roll angle rate of stick control.
Figure 6. Time domain response of stick controlled aircraft.

Figure 7. Time domain response of sideslip angle of pedal control.
Figure 8. Time domain response of pedal controlled aircraft.

It can be seen from Fig. 5~8 that since the integrator is introduced into the forward channel of p-beta control law, the pilot's stick can accurately command the roll angle rate, and the pedal can accurately command the sideslip angle to realize no static error control. Compared with the traditional lateral/directional control law architecture, it can reduce the risk of aircraft vertical tail overload caused by inaccurate control of sideslip angle, and realize the decoupling control of roll channel and yaw channel, with good decoupling effect, making the flight safer.

4. Conclusion
In this paper, the eigen structure assignment method is used to design the closed-loop of p-beta control law. The forward command channel and feedforward gain matrix are designed. The simulation results show that the state system has good stability, realizes the decoupling function of roll and yaw channels, realizes the accurate control of roll angle rate by stick and sideslip angle by pedal, reduces the risk of vertical tail overload, and meets the basic requirements of control law design.

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