On the Structure of Space-Time at the Planck Scale*

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Abstract
The set of space-time short-distance structures which can be described through linear operators is limited to a few basic cases. These are continua, lattices and a further short-distance structure which implies an ultraviolet cut-off. Under certain conditions, these cut-off degrees of freedom can reappear as internal degrees of freedom. We review the current status of the classification and present new conjectures.

1 Introduction
The extrapolation of quantum theory and general relativity to the Planck scale is known to indicate a limit to the validity of the conventional notion of locality. This is because test particles of sufficiently high energy-momentum to resolve a distance as small as a Planck length, about $10^{-35}$ m, are predicted to gravitationally curve and thereby to significantly disturb the very space-time structure which they are meant to probe. The unifying theory of quantum gravity is therefore expected to reveal a nontrivial notion of locality at such small scales. For example, Hawking [1] and others have suggested space-time to be foam-like at the Planck scale. More recent suggestions are in terms of strings and branes, see e.g. [2], or also in terms of noncommutative, or ‘quantum’ geometries, see e.g. [3].

At least at present, however, the structure of space-time at the Planck scale cannot be probed directly by experiment. In this paper we therefore ask whether a classification of the set of all short-distance structures which space-time may possibly have - under some reasonable assumptions - can be achieved.

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The two main messages in this paper are:

- On the basis of relatively general assumptions, a classification of the potential short-distance structures of space-time can be achieved.

- One subclass of these short-distance structures yields a natural ultraviolet cutoff which is such that the cut-off degrees of freedom reappear as internal degrees of freedom with unitary gauge groups.

2 Encoding Space-Time Information using Linear Operators

Our aim is to try a classification of the short-distance structures that space-time may possibly have. The basic assumption which we will make is that the fundamental theory of quantum gravity - whatever this theory may be - encodes space-time information using operators $X_i$ which are linear. Since the $X_i$ should allow an interpretation in terms of 'space-time information' we further assume that the formal expectation values of these operators are real:

$$\langle \phi | X_i | \phi \rangle \in \mathbb{R} \quad \text{for all} \quad |\phi\rangle \in D$$

Here, the vectors $|\phi\rangle$ run through a dense domain $D$ of the $X_i$ in a complex Hilbert space $H$. Technically, this is to say that we assume the $X_i$ to be symmetric operators.

We formally use the Dirac notation for these operators and vectors, and occasionally we will formally also use terminology of nonrelativistic quantum mechanics. The use of this notation and terminology is of course merely for ease of writing. Our aim is to cover an as large as possible set of candidates for a fundamental theory of quantum gravity - in effect we aim at covering all or at least a large part of all theories which are linear as quantum theories, i.e. which obey a linear superposition principle. Let us therefore keep in mind not to make any assumptions about the actual physical interpretation of the $X_i$ in a fundamental theory of quantum gravity, nor to assume any particular physical interpretation of the Hilbert space $H$ on which the $X_i$ act.

In this way, our approach is general enough, for example, to cover the case of the matrix model for M-theory, where $N$-dimensional matrices $X_i$ are given the interpretation that the eigenvalues stand for 'space-time information' in the form of coordinates of $D0$-branes. The situation after quantization and taking $N = \infty$ will still be covered by our classification. We will come back to this case in the last section.

The question arises of course, whether interesting conclusions can at all be drawn from assuming merely that a fundamental theory of quantum gravity encodes space-time information using operators $X_i$ which are linear and symmetric. To see that this is the case, let us recall that even linearity is far from being trivial; in particular, a linear map is not necessarily continuous.

Consider, for example, a matrix operator $X_{ij}$ acting on a sequence of column vectors $v_j^{(n)}$. Even if the all $v_{j}^{(n)}$ and their limit are in the Hilbert space of square summables,
one finds in general that:

$$\lim_{N \to \infty} \lim_{n \to \infty} \sum_{j=1}^{N} X_{ij} v_j^{(n)} \neq \lim_{n \to \infty} \lim_{N \to \infty} \sum_{j=1}^{N} X_{ij} v_j^{(n)}$$

(2)

This is because the existence of one pair of limits does not imply the existence of the other pair of limits (and, for a generic matrix, even if all limits exist they may not commute).

One may be tempted to discuss such phenomena away, assuming that in practice one should always be able to approximate with finite dimensional matrices. Recall, however, that the canonical commutation relation $[x, p] = i 1$ already provides an example for the necessity of infinite dimensional representations: if $x$ and $p$ were $n$-dimensional, the trace of the commutator on the LHS would vanish - while the trace of the RHS would be $i \cdot n$, which is growing with the dimension.

3 Classes of Short-Distance Structures

A symmetric operator $X_i$ is an operator who’s expectation values are real. If an operator $X_i$ is symmetric, it may also be self-adjoint. In this case it has a discrete or continuous spectrum. Therefore, self-adjoint $X_i$ can describe the two well-known short-distance structures of lattices and continua, or, of course, mixtures of lattices and continua, which also includes fractals.

In addition to the two short-distance structures of lattices and continua, symmetric operators can also describe a third short-distance structure, which was named ‘fuzzy’ in [9]. We will discuss the physical motivation for this terminology in the next section, and we will also identify two sub classes among the operators of the fuzzy type. Mathematically, the fuzzy case is the case of operators $X_i$ which are simple symmetric. By definition, simple symmetric operators are symmetric but not self-adjoint, not even on any subspace.

In order to gain intuition into why symmetric operators need not be self-adjoint, consider again a matrix operator $X_{ij}$ on some dense domain $D$. If $X$ is symmetric, i.e. if all its expectation values are real, then clearly $X_{ij} = X_{ji}^*$. This, however, does not imply self-adjointness, i.e. unique diagonalizability. Consider, for example, the eigenvalue equation

$$X_{ij} v_j(\xi) = \xi v_i(\xi)$$

(3)

One may naively expect that two solutions $v(\xi), v(\xi')$ for two different eigenvalues $\xi \neq \xi'$ are orthogonal. If $X$ is self-adjoint, their orthogonality of course follows from

$$\langle (v(\xi)|X)|v(\xi') \rangle = \langle v(\xi)|(X|v(\xi')) \rangle.$$  

(4)

However, writing out this equation in components it becomes clear that it is not a consequence of symmetry, because in general we may have

$$\lim_{N_2 \to \infty} \lim_{N_1 \to \infty} \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} v_i^*(\xi) X_{ij} v_j(\xi') \neq \lim_{N_1 \to \infty} \lim_{N_2 \to \infty} \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} v_i^*(\xi) X_{ij} v_j(\xi').$$  

(5)
since the two limits need not commute for a merely symmetric $X$. In this case some of the $v(\xi)$ may not be orthogonal, in which case they cannot all be contained in the domain $D$ of $X$.

For the precise classification of the basic cases we can use the so-called deficiency indices

$$r_{\pm} = \dim \left( ((X \pm i1) \cdot D)^\perp \right)$$

which were introduced by v. Neumann. For self-adjoint operators $X_i$, i.e. for operators which describe lattices and continua, both indices vanish, $r_+ = r_- = 0$. For operators $X_i$ which describe fuzzy short-distance structures, i.e. for simple symmetric operators there is at least one nonzero index. Let us distinguish two subclasses of the fuzzy cases, by referring to the cases $r_+ = r_- \neq 0$ as being of the type fuzzy-A, and by referring to the cases $r_+ \neq r_-$ as being of the type fuzzy-B.

In the generic case, of course, symmetric operators $X_i$ can be self-adjoint and simple symmetric on different subspaces, i.e. generic symmetric operators are able to describe arbitrary mixtures of the basic cases, namely arbitrary mixtures of lattices, continua and fuzzy short-distance structures.

4 Potential physical origins of operators of the type fuzzy-A

In the following, let us focus attention on the short-distance structures of the type fuzzy-A. These are described by simple symmetric operators $X_i$ with equal deficiency indices. We will for the moment also assume the indices to be finite: $0 \neq r := r_+ = r_- \in \mathbb{N}$. These operators can also be characterized by a physically more intuitive criterion, which will motivate the terminology ‘fuzzy’:

As will be proven in [4], an equivalent definition of operators of the type fuzzy-A is the following: At each real expectation value there exists a finite lower bound to the formal spatial uncertainty.

To be precise, we are using the conventional definition of the uncertainty or standard deviation for normalized $|\phi\rangle$:

$$\Delta X_{|\phi\rangle} = \langle \phi | (X - \langle \phi | X |\phi\rangle) |\phi\rangle^{1/2}$$

Then, $X$ is of the type fuzzy-A exactly iff there exists a positive function

$$\Delta X_{\text{min}}(\xi) > 0,$$

so that for each $\xi \in \mathbb{R}$, all normalized $|\phi\rangle \in D$ with expectation $\langle \phi | X |\phi\rangle = \xi$ obey

$$(\Delta X)_{|\phi\rangle} \geq \Delta X_{\text{min}}(\xi).$$

In naive quantum mechanical terminology, operators of the type fuzzy-A therefore describe spaces in which even with an ideal measurement apparatus the uncertainty or standard deviation in positions could not be made smaller than some finite lower
bound $\Delta X_{\text{min}}(\xi)$. Since the lower bound is in general some function of the expectation value $\xi$ the amount of 'fuzzyness' can vary from place to place.

There are indications from general quantum gravity studies and from string theory which point towards the fuzzy-A type of short-distance structure. Several studies suggest that the uncertainty relations effectively pick up correction terms, see e.g. [5]. In the simplest case these are of the form

$$\Delta x \Delta p \geq \frac{\hbar}{2} (1 + \beta (\Delta p)^2 + ...)$$

(10)

where $\beta > 0$. For a sufficiently small constant $\beta$, the correction term is negligible at present-day experimentally accessible scales. At very small scales, the correction term implies a crucial new feature, namely that $\Delta x$ is now finitely bounded from below by

$$\Delta x_{\text{min}} = \hbar \sqrt{\beta}$$

(11)

i.e. for all $\Delta x, \Delta p$ obeying (10), there holds $\Delta x \geq \Delta x_{\text{min}}$. Choosing for $\beta$ the inverse square of the Planck momentum yields for $\Delta x_{\text{min}}$ the Planck length. A string scale is obtained by relating $\beta$ to $\alpha'$. For reviews, see e.g. [6, 7].

The functional analysis of operators leading to such generalized uncertainty relations was first studied in [8]. It was pointed out in [8] that any linear operator $X$ which obeys an uncertainty relation that yields a lower bound $\Delta X_{\text{min}} > 0$, within any arbitrary theory, must be of the fuzzy type, i.e. simple symmetric. Let us add that, more precisely, any such operator is of the type fuzzy-A, i.e. simple symmetric with equal deficiency indices.

We remark here only in passing that in the case of short-distance structures of the type fuzzy-B, there exist sequences of vectors in the physical domain such that $\Delta x$ converges to zero. These short-distance structures are 'fuzzy' in the sense that vectors of increasing localization around different expectation values then in general do not become orthogonal. This will be proven and discussed in detail in [15].

5 Potential mathematical origins of operators of the type fuzzy-A

We will here not try to speculate in detail how operators $X_i$ of the type fuzzy-A may mathematically arise from a fundamental theory of quantum gravity. We can, however, address an important general point:

Operators, and in particular discontinuous operators, are only fully defined if also their domain is specified. Readers familiar with functional analysis will know that symmetric operators with equal deficiency indices possess domain extensions on which the resulting operators are self-adjoint. One may e.g. recall cases where self-adjoint extensions of differential operators correspond to choices of boundary conditions of some physical system. Therefore, the important question arises in which ways, mathematically, a theory can intrinsically specify and fix the domain of its operators $X_i$ to be a domain on which the $X_i$ are simple symmetric, even if self-adjoint extensions exist.
Let us here discuss only the perhaps most obvious way in which a theory may intrinsically fix the domain of the $X_i$, namely through kinematical and dynamical operator equations: Requiring operator equations in a theory to hold implies, in particular, that only a domain which is common to all operators which appear in the equations can be a physical domain.

As a simple example, consider the stringy uncertainty relation of above. The uncertainty relation may ultimately arise in a complicated way from the fundamental theory, but for the purposes of this argument, let us here model the origin of the uncertainty relation through a simple correction term to the canonical commutation relation:

$$[x, p] = i\hbar (1 + \beta p^2)$$  \hspace{1cm} (12)

To see that (12) yields (10), recall that $\Delta A \Delta B \geq 1/2|\langle [A, B] \rangle|$ for any pair of symmetric operators $A$ and $B$ on a joint domain with their commutator, and that $\langle p^2 \rangle = (\Delta p)^2 + \langle p \rangle^2$.

In principle, kinematical equations such as (12) could be part of the theory. An equation such as (12) would then indeed determine that on all physical domains the operator $x$ is simple symmetric. This is because the lower bound $\Delta x \geq \hbar \beta^{1/2}$ from the uncertainty relation (10) holds on any domain on which (12) holds. In self-adjoint domain extensions of $X_i$, on the other hand, there necessarily exist vectors of arbitrarily small $\Delta x$, due to the diagonalizability. Therefore, a theory which included (12) would intrinsically ensure that the self-adjoint extensions are outside any physical domain - a physical domain being defined as a domain on which the theory’s equations hold, including, here, equation (12).

Similarly, in the fundamental theory, domain specifications for the $X_i$ may arise, for example, from any kinematical or dynamical operator equations among the $X_i$, which may be noncommutative, and with any other operators in the theory.

We remark that the method of modeling generalized uncertainty relations kinematically, through corrections to the canonical commutation relations, has been studied in some detail, and it has been applied to both quantum mechanical and to quantum field theoretical examples. Among the main results are the following: Examples have been given [10], which demonstrate that fuzzy-A type geometries need not break external symmetries, as opposed e.g. to lattices. Further, there is a path integral formulation of quantum field theories in fuzzy geometries [11, 12]. Within this approach, ultraviolet regularity on fuzzy-A type geometries has been shown to arise by the following mechanism [13]:

Ordinarily, in position space, ultraviolet divergencies are known to originate in the ill-definedness of products of propagators and vertices which, in the position representation, are distributions. Propagators such as $G(x, x') = \langle x | (p^2 + m^2)^{-1} | x' \rangle$ are distributions because the formal position eigenfields $| x \rangle$ are nonnormalizable. (For a description of the operators $x_i$, $p_i$ and their Hilbert space of fields in the path integral,
see e.g. [14].) In the fuzzy-A case, the fields $|x\rangle$ which are of maximal localization around the expectation value $x$, are generalized coherent states. As such, they are generally normalizable. Thus, if these are used to define field theories which are as local as possible on the given geometry, the resulting Feynman rules are regular functions whose products are well defined, which then implies ultraviolet regularity.

6 Gauge transformations

Let us recall that we are discussing generic theories, not necessarily quantum field theories, of which we assume only that they encode ‘space-time information’ using linear symmetric operators $X_i$. We found that in theories in which the $X_i$ are of the type fuzzy-A there exists a finite lower bound $\Delta X_{\text{min}}(\xi)$ to the formal uncertainty $\Delta X$. A short-distance structure of this type clearly affects the very notion of locality. We are therefore led to consider the implications, for example, for the local gauge principle. But will it be possible to deduce any information regarding gauge symmetries from such general assumptions?

Since the only concrete tools at hand are the operators $X_i$, let us make the ansatz to define local gauge transformations as the set of isometries (linear operators which preserve the scalar product in Hilbert space) which map a physical domain onto a physical domain, and which commute with the operators $X_i$:

$$G := \{\text{isometric } u : D' \rightarrow D'', \ [u, X_i] = 0, \ i = 1, 2, ..., n, \ \text{where } D', D'' \subset D\} \quad (13)$$

We will use the conventional terminology, but again, let us be careful not to assume any particular physical interpretation or rôle which these transformations may play in a fundamental theory.

With the definition (13), we cover familiar cases such as local gauge transformations of the form $g = \exp(i\alpha_j (X) T_j)$ where the $T_j$ generate, e.g., a local $U(5)$ on an ordinary continuous space with an isospinor index: $g$ as an operator is clearly unitary and it commutes with the $X_i$.

The definition (13) for local gauge transformations is also general enough to be applicable to the case of operators $X_i$ of the fuzzy types. This is because in (13) the localness of a gauge transformation $u$ is defined through the criterion that $u$ commutes with the $X_i$, which is a criterion that does not require the $X_i$ to be diagonalizable.

Let us apply the definition (13) to the case of short-distance structures of the type fuzzy-A. We saw in the fuzzy-A case that those Hilbert space vectors, or ‘degrees of freedom’, which would describe structures smaller than the scale of fuzzyness $\Delta X_{\text{min}}(\xi)$ are cut-off from the domain of the $X_i$. However, the cut-off degrees of freedom will nevertheless play an important rôle: As we will see, mathematically, it is the self-adjoint extensions which describe the cut-off degrees of freedom - and the self-adjoint extensions will give rise to an isospinor structure, i.e. internal degrees of freedom, automatically with unitary gauge groups.
The underlying reason for the re-appearance of those degrees of freedom is that whereas the $X_i$ are discontinuous operators which, therefore, do not see the entire Hilbert space, isometries $u$ are necessarily bounded and continuous operators. Because of their continuity, no part of the Hilbert space can be hidden from such operators.

Let us consider the example of a single operator $X$ which describes a short-distance structure of type fuzzy-A.

As mentioned already, operators $X$ which are of type fuzzy-A always have self-adjoint extensions in the Hilbert space, though outside the physical domain. Each extension has its own discrete spectrum and together the spectra can be shown to cover all reals. Thus, in extensions, arbitrarily sharp localization around arbitrary position expectation values can be reached. In this sense, the family of self-adjoint extensions contains the degrees of freedom beyond the cutoff scale.

Crucially, the self-adjoint extensions, though outside the physical domain, do appear in the construction of the gauge transformations, thereby bringing back the cut-off degrees of freedom as internal degrees of freedom:

As will be shown in [15], all unitaries can be expressed as functions of the self-adjoint extensions of $X$. Unitary functions $u$ of a self-adjoint extension $X_e$ of $X$ commute with $X_e$ and any isometric restriction of $u$ which maps a physical domain onto a physical domain, therefore, commutes with $X$, thus yielding a gauge transformation according to our definition (13).

These include in fact ‘local’ $U(r)$- gauge transformations, where $r$ is the deficiency index. The necessary isospinor structure emerges automatically! To this end, it will be proven in [15] that for each real $\xi$ there are self-adjoint extensions of $X$ for which $\xi$ is an $r$-fold degenerate eigenvalue. This implies that for the (non-symmetric) adjoint operator $X^*$ each real $\xi \in \mathbb{R}$ is an $r$-fold degenerate eigenvalue with eigenvectors $|\xi, i\rangle$ where $i = 1, ..., r$. Any vector $|\phi\rangle$ can be represented by an isospinor function $\phi_i(\xi) = \langle \xi, i | \phi \rangle$, which shows the appearance of the isospinor structure. At large scales, the eigenvectors become orthogonal i.e. $g$ is local in the conventional sense; at small scales the variations of $g$ are restricted by the physical domain condition in (13).

The mechanism by which internal symmetries arise here has some intuitive similarities with the mechanism by which internal symmetries arise in the Kaluza-Klein approach: there, in the simplest case, at each point in space-time a little circle is attached, in an extra dimension. Here, say in the simplest case of deficiency indices $(1, 1)$, at each point a little $S^1$ exists. However, this $S^1$ is not in an extra dimension. Instead, this $S^1$ is ‘within’ the point - a point now being a little patch of fuzzyness of size $\Delta X_{\min}$ [16].

To see this, we note that the self-adjoint extensions of a simple symmetric operator with finite and equal deficiency indices form themselves a representation of a $U(r)$, which implies that each eigenvalue has an orbit under this $U(r)$. In the simplest case of deficiency indices $(1,1)$ it reduces to a $U(1)$-orbit. Each eigenvalue’s $U(1)$-orbit is just small enough not to be resolvable in the presence of the fuzzy cutoff.
7 Conclusions

We investigated the classification of all short-distance structures which are describable
by operators which are linear and have real expectation values. We found that the
generic short-distance structure which these operators can describe is a mixture of the
basic cases of lattices, continua, and fuzzy spaces.

As indicated in the beginning, we are covering the case of the matrix model for
M-theory. Even after the matrix elements of the $X_i$ become operators through quan-
tization, and after taking $N = \infty$, the $X_i$ are linear and symmetric operators. Thus,
the $X_i$ then still fall into the discussed classification.

We note that if those $X_i$ are of a fuzzy type, this would mean that their theory
cannot be a straightforward limit of a sequence of theories based on finite dimen-
sional matrices. This is because for finite dimensional matrices the deficiency indices
always vanish since symmetry and self-adjointness coincide. Indeed, as we saw, an in-
finite dimensional matrix theory can have quantum numbers - the discussed isospinor
structure arising in the fuzzy-A case - which are not present in any finite dimensional
approximation.

If the $X_i$ are found to be of type fuzzy-A, then the eigenvalues which in finite
dimensions stand for $D0$-brane coordinates would assemble into $U(1)$-, or generally
$U(r)$-group orbits which are just small enough not to be resolvable in the fuzzy geom-

While this picture is valid for each individual $X_i$, we need to recall that in our analysis
of the individual $X_i$ we so-far held the other coordinates $X_j$ fixed for $j \neq i$. We did
so for ease of the analysis since otherwise the deficiency indices would generically be
infinite. In general, of course, the short-distance structure may vary arbitrarily in an
$n$-dimensional space and, in particular, the $X_i$ may not commute. This is reflected by
the fact that the functional analysis of each operator $X_i$ is generically a function of the
functional analysis of the other $X_j$, as we discussed briefly in section 5 in the context
of mechanisms by which theories can intrinsically specify operator domains.

The question of internal symmetries in the fuzzy-B case, and numerous further
issues, such as the interplay of fuzzy short-distance structures with supersymmetry
and compactifications, remain to be addressed.

We remark that, so-far, classically real variables have been assumed to correspond
to self-adjoint operators also within the framework of noncommutative geometry. It
should be very interesting to investigate the application of the tools of noncommutative
geometry to the general case of symmetric $X_i$.

Finally, we note that the fuzzy short-distance structure has recently been studied
in the context of the transplanckian energy problem of black hole radiation [17].
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