Interactions of Eight-branes in String Theory and M(atrix) Theory

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Abstract
We consider eight-brane configurations in M(atrix) theory and compute their interaction potentials with gravitons, membranes, and four-branes. We compare these results with the interactions of D8-branes with D0-branes, D2-branes, and D4-branes in IIA string theory. We find agreement between the two approaches for eight-brane interactions with two-branes and four-branes. A discrepancy is noted in the case with zero-branes.
1. Introduction

In the last few years it has been discovered that the five consistent superstring theories in ten spacetime dimensions (IIA, IIB, Type I, \(E_8 \times E_8\) Heterotic and \(SO(32)\) Heterotic), which were previously thought to be distinct, are in fact different descriptions of a single underlying theory. Evidence for this includes various dualities between compactified versions of these theories. A striking realization of Witten [1] (which was anticipated by Townsend [2]) was that an eleventh dimension emerges in the strong coupling limit of the IIA string theory. That is, the non-perturbative IIA string theory is actually described by an eleven-dimensional theory (M-theory) which has \(D = 11\) supergravity as its low energy effective description.

An important check of the eleven-dimensional theory is that it should be able to describe non-perturbative objects which exist in the IIA theory, such as the Dp-branes [3]. The IIA D0,2,4,6-branes seem to have clear interpretations in terms of M-theory compactified on a circle as Kaluza-Klein modes, unwrapped membranes, wrapped five-branes, and magnetic Kaluza-Klein branes, respectively [2]. The M-theory interpretation of D8-branes remains unclear, however. Issues related to their possible emergence from \(D = 11\) supergravity have been discussed in [4,5]. In particular it has been suggested that the IIA 8-brane may have an interpretation as a wrapped 9-brane, possibly related to the “end of the world” 9-branes of M-theory with a boundary.

The remarkable conjecture of [6] is that the full quantum description of M-theory in the infinite momentum frame is given the by large \(N\) limit of \(D = 10\) \(U(N)\) supersymmetric Yang-Mills theory dimensionally reduced to \(0 + 1\) dimensions. This formulation in terms of the supersymmetric quantum mechanics (SQM) of an infinite number of highly boosted D0-branes has been referred to as ‘M(atrix) theory’. Since M(atrix) theory does contain eight-branes [7], we may hope to probe their eleven-dimensional origin within this context. A possible relationship between D8-branes and boundary degrees of freedom in M(atrix) theory was recently discussed in [8].

Scattering between various configurations of \(p \leq 6\) branes in M(atrix) theory has been considered in [9,10,11,12,13,14]. Generically it seems to be the case that the low-velocity, long-distance potentials between configurations of branes in M(atrix) theory agree with those obtained from string theory and supergravity calculations. In section two we consider configurations with M(atrix) eight-branes and compute static interaction potentials with gravitons, membranes, and longitudinal five-branes. In section three we consider
analogous configurations of D-branes in IIA string theory, which generally contain large magnetic fields on their world volumes. In section four we discuss the results. We find agreement between the two approaches for eight-branes interacting with two-branes or four-branes. However in the case of eight-brane zero-brane interactions we find that contributions coming from the \((-1)^F\) sector in the string calculation do not appear in the M(atrix) calculation.

2. 8-branes in M(atrix) theory

In this section we consider interactions between M(atrix) eight-branes and various lower dimensional objects which exist in the theory using techniques developed in [10,11]. We use a gauge fixed form of the M(atrix) theory action \((i = 1, \ldots, 9)\),

\[
L = -\frac{1}{g} \text{Tr} \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} (D A_\mu)^2 + \theta^T D_0 \theta - \theta \gamma_i [\theta, A_i] \right] + \text{ghosts}
\]

(2.1)

where \(F_{0i} = \partial_0 A_i - i[A_0, A_i]\), \(F_{ij} = -i[A_i, A_j]\), and all the fields are \(U(N)\) matrices. To compute the effective action we decompose the matrices into block diagonal backgrounds and off-diagonal fluctuations,

\[
A_i = \begin{pmatrix} X_i & 0 \\ 0 & x_i \end{pmatrix} + \begin{pmatrix} 0 & y \\ y^T & 0 \end{pmatrix}
\]

\[
\theta = \begin{pmatrix} 0 & \psi \\ \psi^T & 0 \end{pmatrix}
\]

\[
B = \begin{pmatrix} 0 & b \\ b^T & 0 \end{pmatrix}
\]

\[
C = \begin{pmatrix} 0 & c \\ c^T & 0 \end{pmatrix}
\]

(2.2)

and integrate out the massive fields at one-loop. It is possible to describe 8-branes in M(atrix) theory [7] by choosing a background in which \([X^1, X^2] = ic_1, [X^3, X^4] = ic_2, [X^5, X^6] = ic_3,\) and \([X^7, X^8] = ic_4\). This does not represent a pure 8-brane however, since it also contains 2-branes, 4-branes, and 6-branes.
2.1. Interactions with gravitons

We represent a graviton scattering off an eight-brane with the background,

\[
X^1 = \begin{pmatrix} P_1 & 0 \\ 0 & 0 \end{pmatrix}, \quad X^2 = \begin{pmatrix} Q_1 & 0 \\ 0 & 0 \end{pmatrix}, \quad X^3 = \begin{pmatrix} P_2 & 0 \\ 0 & 0 \end{pmatrix}, \quad X^4 = \begin{pmatrix} Q_2 & 0 \\ 0 & 0 \end{pmatrix}, \quad X^5 = \begin{pmatrix} P_3 & 0 \\ 0 & 0 \end{pmatrix}, \quad X^6 = \begin{pmatrix} Q_3 & 0 \\ 0 & 0 \end{pmatrix}, \\
X^7 = \begin{pmatrix} P_4 & 0 \\ 0 & 0 \end{pmatrix}, \quad X^8 = \begin{pmatrix} Q_4 & 0 \\ 0 & 0 \end{pmatrix}, \quad X^9 = \begin{pmatrix} 1 & vt \\ 0 & 0 \end{pmatrix}.
\]

(2.3)

where \(P_i\) and \(Q_i\) are \(N \times N\) matrices that satisfy \([P_i, Q_i] = i c_i\) in the large \(N\) limit. The zero entry in the lower diagonal represents a single zero-brane which has a M(atrix) theory interpretation as a graviton. Expanding around this background and integrating out the massive off-diagonal modes at one-loop gives the amplitude in terms of the determinants (with \(\tau = it\) and \(\gamma = -iv\)),

**Bosons**:

\[
\begin{align*}
det^{-1}(-\partial^2_\tau + H + 2iv) & \det^{-1}(-\partial^2_\tau + H - 2iv) \\
det^{-1}(-\partial^2_\tau + H + 2c_1) & \det^{-1}(-\partial^2_\tau + H - 2c_1) \\
det^{-1}(-\partial^2_\tau + H + 2c_2) & \det^{-1}(-\partial^2_\tau + H - 2c_2) \\
det^{-1}(-\partial^2_\tau + H + 2c_3) & \det^{-1}(-\partial^2_\tau + H - 2c_3) \\
det^{-1}(-\partial^2_\tau + H + 2c_4) & \det^{-1}(-\partial^2_\tau + H - 2c_4)
\end{align*}
\]

(2.4)

**Ghosts**:

\[
\det^2(-\partial^2_\tau + H)
\]

**Fermions**:

\[
\det(\partial_\tau + \eta_f)
\]

where,

\[
H = P_1^2 + Q_1^2 + P_2^2 + Q_2^2 + P_3^2 + Q_3^2 + P_4^2 + Q_4^2 + \mathbb{1}v^2 t^2
\]

and,

\[
\eta_f = \gamma_1 P_1 + \gamma_2 Q_1 + \gamma_3 P_2 + \gamma_4 Q_2 + \gamma_5 P_3 + \gamma_6 Q_3 + \gamma_7 P_4 + \gamma_8 Q_4 + \gamma_9 vt.
\]
The $P_i^2 + Q_i^2$ terms in $H$ are just a collection of simple harmonic oscillator Hamiltonians with eigenvalues $c_i(2n_i + 1)$. Similarly $-\partial_x^2 + \gamma^2 \tau^2$ has eigenvalues $2\gamma(n + \frac{1}{2})$. We can easily compute the bosonic and ghost determinants, and using Schwinger’s proper time representation, they give a contribution to the one-loop effective action of

$$\Gamma_B = i \int_0^\infty ds \frac{1}{s} \frac{1}{2\sin vs} \frac{1}{16 \sinh c_1 s \sinh c_2 s \sinh c_3 s \sinh c_4 s} \times \left\{2 \cosh 2vs - 2 + 2 \cosh 2c_1 s + 2 \cosh 2c_2 s + 2 \cosh 2c_3 s + 2 \cosh 2c_4 s\right\}$$

(2.5)

The fermionic determinant is problematic because it cannot be converted into Klein-Gordon form by the usual method (there is no tenth $16 \times 16$ matrix which anticommutes with all the $\gamma_i$’s). We can however evaluate it in the $v \to 0$ limit and the result is,

$$\Gamma_F = i \int_0^\infty ds \frac{1}{s} \frac{1}{2\sin vs} \frac{1}{16 \sinh c_1 s \sinh c_2 s \sinh c_3 s \sinh c_4 s} \times \left\{-8 \cosh c_1 s \cosh c_2 s \cosh c_3 s \cosh c_4 s + O(v)\right\}$$

(2.6)

From (2.5) and (2.6) we extract the effective potential which is defined through

$$\Gamma = \Gamma_B + \Gamma_F = -i \int d\tau V(R^2 = v^2 \tau^2).$$

(2.7)

By expanding the integrand for small $c_i$ we can do the integral over $s$ by analytic continuation and we obtain the result,

$$V = \left\{\frac{c_1^4 + c_2^4 + c_3^4 + c_4^4 - 2(c_1^2 c_2^2 + c_1^2 c_3^2 + c_1^2 c_4^2 + c_2^2 c_3^2 + c_2^2 c_4^2 + c_3^2 c_4^2)}{16 c_1 c_2 c_3 c_4} + O(v)\right\} R$$

(2.8)

2.2. Interactions with membranes

We represent a membrane scattering off an eight-brane with the background,

$$X^1 = \begin{pmatrix} P_1 & 0 \\ 0 & p_1 \end{pmatrix}, \quad X^2 = \begin{pmatrix} Q_1 & 0 \\ 0 & q_1 \end{pmatrix}, \quad X^3 = \begin{pmatrix} P_2 & 0 \\ 0 & 0 \end{pmatrix}, \quad X^4 = \begin{pmatrix} Q_2 & 0 \\ 0 & 0 \end{pmatrix}, \quad X^5 = \begin{pmatrix} P_3 & 0 \\ 0 & 0 \end{pmatrix}, \quad X^6 = \begin{pmatrix} Q_3 & 0 \\ 0 & 0 \end{pmatrix}, \quad X^7 = \begin{pmatrix} P_4 & 0 \\ 0 & 0 \end{pmatrix}, \quad X^8 = \begin{pmatrix} Q_4 & 0 \\ 0 & 0 \end{pmatrix}, \quad X^9 = \begin{pmatrix} v & 0 \\ 0 & 0 \end{pmatrix}$$

(2.9)
where $P_i$ and $Q_i$ are as before, while $p_1$ and $q_1$ are $n \times n$ matrices which we take to satisfy $[p_1, q_1] = ic$. In this case we also take $c_1 = c_2 = c_3 = c_4 = c$. The determinants are,

\begin{align}
Bosons: \\
&\det^{-2}(-\partial^2 + H) \\
&\det^{-3}(-\partial^2 + H + 2c)\det^{-3}(-\partial^2 + H - 2c) \\
&\det^{-1}(-\partial^2 + H + 2iv)\det^{-1}(-\partial^2 + H - 2iv) \\
\end{align}

(2.10)

\begin{align}
Ghosts: \\
&\det^2(-\partial^2 + H) \\
\end{align}

\begin{align}
Fermions: \\
&\det(\partial + \eta_f) \\
\end{align}

where

\begin{align}
H = (P_1 + p_1)^2 + (Q_1 - q_1)^2 + P_2^2 + Q_2^2 + P_3^2 + Q_3^2 + P_4^2 + Q_4^2 + 11^2 \\
\text{and,} \\
\tilde{\eta}_f = \gamma_1(P_1 + p_1) + \gamma_2(Q_1 - q_1) + \gamma_3 P_2 + \gamma_4 Q_2 + \gamma_5 P_3 + \gamma_6 Q_3 + \gamma_7 P_4 + \gamma_8 Q_4 + \gamma_9 \nu t. \\
\end{align}

Defining $P_\pm = P_1 \pm p_1$ and $Q_\pm = Q_1 \pm q_1$ which satisfy $[P_+, Q_+] = [P_-, Q_-] = 2ic$ and $[P_+, Q_-] = [P_-, Q_+] = 0$, we can represent $Q_\pm$ by $x_\pm$ and $P_\pm$ by $2ic\partial x_\pm$ so that $H$ has eigenvalues,

\begin{align}
H = 4c^2k_+^2 + x_-^2 + c(2n_2 + 2n_3 + 2n_4 + 3) + \gamma^2 \tau^2 \\
\text{where} \ n_2 \text{ and} \ x_- \text{ are continuous.} \\
\text{The bosonic and ghost determinants give a contribution of,} \\
\Gamma_B = i\mathcal{N}_1 \int_0^\infty \frac{ds}{s} \frac{1}{4cs} \frac{1}{2\sin vs} \frac{1}{8\sinh^3 cs} \left\{2\cos 2vs + 6 \cosh 2cs \right\} \\
\text{where the overall factor of} \ \mathcal{N}_1 = \int dx \frac{dk}{2\pi} \text{ comes from the degeneracy in} \ H. \ \text{As before,} \\
\text{we compute the fermionic determinant in the} \ \nu \to 0 \ \text{limit and it gives a contribution of,} \\
\Gamma_F = i\mathcal{N}_1 \int_0^\infty \frac{ds}{s} \frac{1}{4cs} \frac{1}{2ivs} \frac{1}{8\sinh^3 cs} \left\{-8 \cosh^3 cs + O(v) \right\} \\
\text{The potential extracted from (2.11) and (2.12) is,} \\
V = \mathcal{N}_1 \left\{\frac{3}{32} + O(v) \right\}R \\
\text{(2.13)}
\end{align}
We represent a four-brane scattering off an eight-brane with the background,

\[
\begin{align*}
X^1 &= \begin{pmatrix} P_1 & 0 \\ 0 & P_1 \end{pmatrix} \\
X^2 &= \begin{pmatrix} Q_1 & 0 \\ 0 & Q_1 \end{pmatrix} \\
X^3 &= \begin{pmatrix} P_2 & 0 \\ 0 & P_2 \end{pmatrix} \\
X^4 &= \begin{pmatrix} Q_2 & 0 \\ 0 & Q_2 \end{pmatrix} \\
X^5 &= \begin{pmatrix} P_3 & 0 \\ 0 & P_3 \end{pmatrix} \\
X^6 &= \begin{pmatrix} Q_3 & 0 \\ 0 & Q_3 \end{pmatrix} \\
X^7 &= \begin{pmatrix} P_4 & 0 \\ 0 & P_4 \end{pmatrix} \\
X^8 &= \begin{pmatrix} Q_4 & 0 \\ 0 & Q_4 \end{pmatrix} \\
X^9 &= \begin{pmatrix} \mathbb{1} & vt & 0 \\ 0 & 0 \end{pmatrix}.
\end{align*}
\]

In this case \(p_i\) and \(q_i\) are \(n \times n\) matrices which we take to satisfy \([p_1, q_1] = ic_1, [p_2, q_2] = ic_2\). The four-brane configuration in the lower block of the \(X_i's\) carries membrane as well as longitudinal five-brane charge \([7]\). The one-loop amplitude is given in terms of the determinants,

**Bosons**:
\[
\begin{align*}
\det^{-4}(-\partial^2_\tau + H) \\
\det^{-1}(-\partial^2_\tau + H + 2iv) \det^{-1}(-\partial^2_\tau + H - 2iv) \\
\det^{-1}(-\partial^2_\tau + H + 2c_3) \det^{-1}(-\partial^2_\tau + H - 2c_3) \\
\det^{-1}(-\partial^2_\tau + H + 2c_4) \det^{-1}(-\partial^2_\tau + H - 2c_4)
\end{align*}
\]

**Ghosts**:
\[
\det^{2}(-\partial^2_\tau + H)
\]

**Fermions**:
\[
\det(\partial_\tau + \eta_f)
\]

where,

\[
H = (P_1 + p_1^2) + (Q_1 - q_1)^2 + (P_2 + p_2)^2 + (Q_2 - q_2)^2 + P_3^2 + Q_3^2 + P_4^2 + Q_4^2 + \mathbb{1}v^2t^2
\]

and,

\[
\eta_f = \gamma_1(P_1 + p_1) + \gamma_2(Q_1 - q_1) + \gamma_3(P_2 + p_2) + \gamma_4(Q_2 - q_2) + \gamma_5P_3 + \gamma_6Q_3 + \gamma_7P_4 + \gamma_8Q_4 + \gamma_9vt.
\]
The spectrum of $H$ is,

$$H = 4c_1^2k_1^2 + 4c_2^2k_2^2 + x_1^2 + x_2^2 + c_3(2n_3 + 1) + c_4(2n_4 + 1) + \gamma^2 \tau^2.$$ 

and the contribution to the one-loop integral from bosons and ghosts is,

$$\Gamma_B = N_1N_2 \int_0^\infty ds \frac{1}{s c_1c_2 (4s)^2} \frac{1}{2is \sin vs} \frac{1}{4 \sinh c_3s \sinh c_4s} \frac{1}{}\{2 + 2 \cos 2vs + 2 \cosh 2c_3s + 2 \cosh 2c_4s\}$$

(2.16)

where again there are degeneracies coming from \(N_i = \int dx_i \frac{dk_i}{2\pi}\). The fermionic contribution in the \(v \to 0\) limit is

$$\Gamma_F = N_1N_2 \int_0^\infty ds \frac{1}{s c_1c_2 (4s)^2} \frac{1}{2isu} \frac{1}{4 \sinh c_3s \sinh c_4s} \{ -6 \cosh c_3s \cosh c_4s + O(v) \}$$

(2.17)

The potential extracted from (2.17) and (2.16) is

$$V = \frac{N_1N_2}{c_1c_2} \frac{1}{64\pi} \left\{ \frac{(c_3 - c_4)^2}{c_3c_4} + O(v) \right\} R$$

(2.18)

3. D8-brane interactions in IIA string theory

In this section we compute interactions involving D8-branes in Type IIA string theory. To reproduce the brane configurations of the M(atrix) theory we must turn on a large constant magnetic field in the world volumes of the D-branes. This has the effect of forming a non-marginal bound state with lower dimensional branes. Using the techniques of [3, 15, 16, 17, 18] we compute the one-loop vacuum amplitude for open strings stretched between these D-brane configurations, including the effects of the relative motions and background magnetic fields. As discussed in [19] the short distance limit of these amplitudes is dominated by the lightest modes of the open strings ending on the branes, and this is the physics which is encoded in the SQM of the M(atrix) theory zero-branes. In the long distance limit the amplitudes are dominated by the exchange of massless closed strings. What is somewhat surprising is that the long and short distance potentials agree, even though the configurations do not in general preserve any supersymmetry. This can be explained by the fact that we must consider very strong magnetic fields, which correspond to a large number of zero-branes bound to the system. The zero-branes then dominate the dynamics and the configuration becomes “almost” supersymmetric [10]. The M(atrix) theory analog of this is that the zero-branes are boosted to the infinite momentum frame in the large \(N\) limit.
3.1. Pure D8-branes

Let us first review the interactions of D8-branes in the absence of a background magnetic field \[17\]. The one-loop vacuum amplitude gives the scattering phase shift for moving D-branes \[16\] with relative velocity \( v = \text{tanh} \pi \nu \). For Dp-branes \((p = 4, 6)\) scattering off D8-branes the amplitude is

\[
A = \frac{1}{2\pi} \int \frac{dt}{t} (4\pi t)^{-p/2} Z_B \times Z_F
\]

\[
Z_B = \frac{\Theta_1'(0|it)}{\Theta_1(\nu t|it)} f_1^p f_4^{p-8}
\]

\[
Z_F = \left\{ \frac{\Theta_3(\nu t|it)}{\Theta_3(0|it)} f_3^p f_2^{8-p} - \frac{\Theta_2(\nu t|it)}{\Theta_2(0|it)} f_2^p f_3^{8-p} \right\}
\]

The one-loop vacuum amplitude for a static D0-brane and D8-brane is given by,

\[
A = \int \frac{dt}{t} (4\pi t)^{1/2} f_4^{s-1/2} \left\{ f_2^s - f_3^s + f_4^s \right\}
\]

The three terms in brackets come from the NS, R, and \((-1)^F R\) sectors respectively, and the expression vanishes by the ‘abstruse identity’ reflecting the fact that this configuration preserves a quarter of the supersymmetries. This is a case where the number of directions with mixed Neumann-Dirichlet boundary conditions is eight and therefore the oscillator expansions for transverse bosons and R fermions have half-integer moding, while transverse NS fermions have integer moding. The transverse NS fermions then have zero modes which is why the \((-1)^F\) NS does not contribute to the partition function. We can in principle compute velocity dependent corrections and they appear as in (3.1), except for the \((-1)^F R\) sector which has an additional complication due to the super ghost zero modes. We hope to address this case in more detail in future work.

3.2. The 8-6-4-2-0 configuration

We turn on a constant background world volume field strength,

\[
F^\mu \nu = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & F_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -F_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & F_2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -F_2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & F_3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -F_3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & F_4 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -F_4 & 0
\end{pmatrix}
\]

\[(3.3)\]

\[\text{1 Our conventions for the } f\text{-functions are as in } [3]. \text{ The } \Theta\text{-functions are standard.}\]
so that the D8-brane carries 6, 4, 2, and 0 brane charge due to couplings with the other R-R potential forms through,

\[ C_7 \wedge F + \frac{1}{2} C_5 \wedge F \wedge F + \frac{1}{3!} C_3 \wedge F \wedge F \wedge F + \frac{1}{4!} C_1 \wedge F \wedge F \wedge F \wedge F. \]

This describes a non-marginal bound state of 8-6-4-2-0 branes analogous to the M(atrix) eight-brane configuration. Let us consider interactions of D0-branes with this object. The magnetic field alters the boundary conditions at the open string endpoints \([20]\) on the D8-brane (which we take to be at \(\sigma = 0\)),

\[ \partial_\sigma X^i + F^i_\nu \partial_\tau X^\nu = 0 \quad \sigma = 0 \]
\[ \partial_\tau X^i = 0 \quad \sigma = \pi \quad (3.4) \]
\[ \partial_\tau X^9 = 0 \quad \sigma = 0, \pi \]

In contrast to the case with a pure D8-brane, this configuration preserves no supersymmetries. The Dirichlet boundary conditions at the endpoint on the D0-brane (\(\sigma = \pi\)) flip the moding of the NS and R sectors to integral and half-integral, respectively. However, the constant magnetic field also alters the boundary conditions and mode expansions of the transverse fermionic fields so that there are no zero modes, therefore all sectors contribute to the partition function. The one-loop vacuum amplitude for open strings stretched between this configuration is,

\[ \mathcal{A} = \frac{1}{2\pi} \int \frac{dt}{t} Z_B \times Z_F \]
\[ Z_B = \frac{\Theta'_1(0|it)}{\Theta_1(\nu t|it)} \Theta_4(i\epsilon_1 t|it)^{-1} \Theta_4(i\epsilon_2 t|it)^{-1} \Theta_4(i\epsilon_3 t|it)^{-1} \Theta_4(i\epsilon_4 t|it)^{-1} \]
\[ Z_F = \frac{1}{2} \left\{ \Theta_2(i\epsilon_1 t|it) \Theta_2(i\epsilon_2 t|it) \Theta_2(i\epsilon_3 t|it) \Theta_2(i\epsilon_4 t|it) \right. \]
\[ - \Theta_1(i\epsilon_1 t|it) \Theta_1(i\epsilon_2 t|it) \Theta_3(i\epsilon_3 t|it) \Theta_3(i\epsilon_4 t|it) \]
\[ - \Theta_3(i\epsilon_1 t|it) \Theta_3(i\epsilon_2 t|it) \Theta_1(i\epsilon_3 t|it) \Theta_1(i\epsilon_4 t|it) \]
\[ + \Theta_4(i\epsilon_1 t|it) \Theta_4(i\epsilon_2 t|it) \Theta_4(i\epsilon_3 t|it) \Theta_4(i\epsilon_4 t|it) + O(\nu) \right\} \quad (3.5) \]

where the four terms in brackets come from the NS, \((-1)^F_{NS}\), R, and \((-1)^F_{R}\) sectors, respectively and \(\tan \pi \epsilon_i = F_i\). Here we are considering an adiabatic approximation where the D-branes are moving with a vanishingly small relative velocity, so we only will keep the leading \((\nu^{-1})\) term in \((3.3)\). Since we want to consider large values of \(F_i\) we make a
change of variables to $\epsilon_i = \frac{1}{2} - \tilde{c}_i$ and take $\tilde{c}_i$ to be small. We can truncate the amplitude to include only the lightest open string modes by taking the $t \to \infty$ limit of (3.5) and we find,

$$A \to \frac{1}{2\pi} \int \frac{dt}{t} \frac{1}{\nu t} \frac{1}{16 \sinh \pi \tilde{c}_1 t \sinh \pi \tilde{c}_2 t \sinh \pi \tilde{c}_3 t \sinh \pi \tilde{c}_4 t} \times \left\{ 2 \cosh 2\pi \tilde{c}_1 t + 8 \cosh \pi \tilde{c}_1 t \cosh \pi \tilde{c}_2 t \cosh \pi \tilde{c}_3 t \cosh \pi \tilde{c}_4 t \
- \frac{8}{\sinh \pi \tilde{c}_1 t \sinh \pi \tilde{c}_2 t \sinh \pi \tilde{c}_3 t \sinh \pi \tilde{c}_4 t + O(\nu)} \right\}$$

(3.6)

This is the short distance limit, where we would expect that the amplitude should agree with the M(atrix) results $\Gamma \sim iA (2.5) (2.6)$. However the term coming from the $(-1)^F R$ sector in (3.6) does not seem to be accounted for in the M(atrix) calculation; otherwise we find agreement when $c_i = \pi \tilde{c}_i$ and $v \approx \pi \nu$.

To find the long distance potential defined through $A = - \int d\tau V(R^2 = v^2 \tau^2)$ we take the $t \to 0$ limit of (3.5) which is dominated by the exchange of light closed strings between the D-branes. By expanding the integrand to lowest order in $c_i$ and doing the integral over $t$ by analytic continuation we arrive at the following result,

$$V = -\frac{\Gamma(-1/2)}{32\sqrt{\pi}} \frac{c_1^4 + c_2^4 + c_3^4 + c_4^4 + 2(c_1^2 c_2^2 + c_1^2 c_3^2 + c_2^2 c_3^2 + c_2^2 c_4^2 + c_3^2 c_4^2 + c_4^2 c_1^2) + 8c_1 c_2 c_3 c_4}{c_1 c_2 c_3 c_4}R$$

(3.7)

This static linear potential vanishes when $c_1 = c_2 = c_3 = c_4$. This can be understood by a T-duality which takes this system to a pair of non-intersecting D4-branes where we know that some supersymmetry will be preserved when they are oriented at $SU(4)$ angles [21]. The M(atrix) result (2.8), which is missing the last term in (3.7), does not exhibit this important behavior.

3.3. 2-0 and 8-6-4-2-0 interactions

In this case we must also turn on a magnetic field in the world volume of the 2-brane, which we take to be $F^{12} = -F^{21} = F$. For simplicity we also set $F_1 = F_2 = F_3 = F_4 = F$ in (3.3). In the directions common to the 2-brane and 8-brane, the boundary conditions are the same as for an open string with opposite charges on the ends in a constant magnetic field, so we get an overall factor of $(1 + F^2)$ in the partition function [20]. The one-loop vacuum amplitude for open strings stretched between this configuration is given by,
\[ A = L^2 \left( 1 + \frac{F^2}{2\pi} \right) \int \frac{dt}{t} \left( 4\pi t \right)^{-1} Z_B \times Z_F \]

\[ Z_B = \frac{\Theta_1'(0|it)}{\Theta_1(\nu|it)} f_1^{-1} \Theta_4(i\epsilon|it)^{-3} \]

\[ Z_F = \frac{1}{2} \left\{ \frac{\Theta_3(\nu|it)}{\Theta_3(0|it)} f_3^2 \Theta_2(i\epsilon|it)^3 - \frac{\Theta_2(\nu|it)}{\Theta_2(0|it)} f_2^2 \Theta_3(i\epsilon|it)^3 \right. \]

\[ \left. - i \frac{\Theta_4(\nu|it)}{\Theta_4(0|it)} f_4^2 \Theta_1(i\epsilon|it)^3 \right\} \]  

(3.8)

where the three terms in \( Z_F \) come from the NS, R and \((-1)^F\) NS sectors respectively and \( \tan \pi \epsilon = F \). By T-duality, \( Z_B \) and \( Z_F \) are the same as for the 0-brane, 6-brane configuration considered in [11]. By expanding around a large magnetic field value \( \epsilon = \frac{1}{2} - \tilde{c} \) and going to the limit \( t \to \infty \) where massless open string modes dominate the amplitude becomes,

\[ A \to L^2 \left( 1 + \frac{F^2}{2\pi} \right) \int \frac{dt}{t} \frac{1}{4\pi t} \frac{\pi}{\sin \pi \nu t \sinh \pi c t} \times \left\{ 2 \cos 2\pi \nu t + 6 \cosh 2\pi \tilde{c} t - 8 \cos \pi \nu t \cosh 3 \pi \tilde{c} t \right\} . \]

(3.9)

The long distance behavior coming from the massless closed string channel is obtained from (3.8) in the \( t \to 0 \) limit. The amplitude is divergent, but we can regularize the situation by extracting an effective potential \( \mathcal{A} = -\int_{-\infty}^{+\infty} d\tau V(v^2 \tau^2) \). After relabeling \( \pi \nu = v, \pi \tilde{c} = c, \nu \tau = R \); expanding to lowest order in \( c \) and \( v \); and doing the integral over \( t \) by analytic continuation we get the result,

\[ V = L^2 \left( 1 + \frac{F^2}{32\pi} \right) \frac{\left( v^4 + 6c^2 v^2 - 3c^4 \right)}{c^3} R. \]

(3.10)

In the limit where \( F \approx \frac{1}{c} \) is very large and \( v \to 0 \) we find precise agreement between (3.8),(3.10) and the M(atrix) results (2.3),(2.6), and (2.8) provided that we identify \( N_1 = \frac{L^2}{\pi c} \) as in [10][11].

3.4. 4-2-0 and 8-6-4-2-0 interactions

To reproduce the analogous M(atrix) configuration we must turn on a constant magnetic field in the D4-brane world volume \( F^{12} = -F^{21} = F_1, F^{34} = -F^{43} = F_2 \). The one-loop vacuum amplitude for open strings stretched between this configuration is given by,
\[ A = L^4 (1 + F_1^2)(1 + F_2^2) \frac{dt}{2\pi} \int \frac{dt}{t} (4\pi t)^{-2} Z_B \times Z_F \]

\[ Z_B = \frac{\Theta_1'(0|it)}{\Theta_1(\nu|it)} f_1^{-1} \Theta_1(i\epsilon_3|it)^{-1} \Theta_1(i\epsilon_4|it)^{-1} \]

\[ Z_F = \frac{1}{2} \left\{ \frac{\Theta_3(\nu|it)}{\Theta_3(0|it)} f_3^4 \Theta_2(i\epsilon_3|it) \Theta_2(i\epsilon_4|it) \right. \]
\[ - \frac{\Theta_2(\nu|it)}{\Theta_2(0|it)} f_2^4 \Theta_3(i\epsilon_3|it) \Theta_3(i\epsilon_4|it) \]
\[ + \frac{\Theta_4(\nu|it)}{\Theta_4(0|it)} f_4^4 \Theta_1(i\epsilon_3|it) \Theta_1(i\epsilon_4|it) \left\} \right. \]  

(3.11)

where \( \tan \pi \epsilon_i = F_i \). In the \( t \to \infty \) limit the amplitude becomes,

\[ A \to L^4 (1 + F_1^2)(1 + F_2^2) \frac{dt}{2\pi} \int \frac{dt}{t} (4\pi t)^{-2} \frac{\pi}{4\sin \pi \nu t \sinh \pi \tilde{c}_1 t \sinh \pi \tilde{c}_2 t} \]
\[ \times \left\{ 2 + 2 \cos 2\pi \nu t + 2 \cosh 2\pi \tilde{c}_1 t + 2 \cosh 2\pi \tilde{c}_2 t - 8 \cos \pi \nu t \cosh \pi \tilde{c}_1 t \cosh \pi \tilde{c}_2 t \right\} \]  

(3.12)

with \( \epsilon_i = \frac{1}{2} - \tilde{c}_i \).

In the \( t \to 0 \) limit we extract the long distance potential to lowest order in \( \epsilon_i = \pi \tilde{c}_i \) and \( \nu = \pi \nu \) and find,

\[ V = L^4 (1 + F_1^2)(1 + F_2^2) \frac{v^4}{64\pi^2} + 2(c_1^2 + c_2^2)v^2 + (c_1^2 - c_2^2)^2 \frac{R}{c_1 c_2}. \]  

(3.13)

When \( c_1 = c_2 \) the potential vanishes at zero velocity, reflecting the fact that some supersymmetry is preserved. This is related by T-duality to D2-branes at \( SU(2) \) angles \( 21 \). In the large \( F_i \approx \frac{1}{c_i} \) and \( v \to 0 \) limit there is agreement between (3.12), (3.13) and the M(atrix) results (2.16), (2.17), (2.18) when we identify \( N_i = \frac{L^2}{\pi \epsilon_i} \).

4. Discussion

We have computed potentials between various configurations containing 8-branes in M(atrix) theory and IIA string theory and compared their velocity independent terms. For the cases of 8-branes interacting with 2-branes and 4-branes we find agreement between
the two approaches. However in the case with 8-branes and 0-branes we find that the M(atrix) approach fails to include contributions coming from the $(-1)^F R$ sector in the string calculation. It may be the case that extra degrees of freedom need to be added to the M(atrix) model, and we note that the addition of a bosonic determinant such as $\text{det}^{-1/2}(-\partial^2 + \gamma^2 \tau^2)$ to (2.4) would lead to agreement. It is possible that the calculation itself breaks down, since for example we cannot rely on large values of an impact parameter to control instabilities and as noted in [22] we are somewhat outside of the region of validity of the eikonal approximation in this case. The agreement obtained for 2-branes and 4-branes would however seem to suggest that the technique can be extrapolated. A related issue arises in the SQM description of D0-branes in the presence of pure D8-branes, where it is necessary to add an additional D8-brane source to cancel IR divergences [19,22]. Another possibility comes from considering the very recent work of [23,24] where it was shown that when a D0-brane moves through a D8-brane a fundamental fermionic 0-8 string is created. Accounting for this phenomena in M(atrix) theory may resolve the discrepancy.

Following [25,26,3] we can make some comments about the consequences of interpreting the IIA D8-brane as a wrapped 11D ninebrane. If we compactify the 11D theory on a circle of radius $R_{11} = \sqrt{\alpha'g_s}$, then upon double dimensional reduction the D8-brane tension is related to the tension of the 11D nine-brane by (in units where $g_{g_{\mu\nu}} = g_{11\mu\nu}$),

$$T_8 = 2\pi \sqrt{\alpha'g_s} T_9^M.$$  \hfill (4.1)

The tension of a Dp-brane is also given in terms the string tension $T_s = \frac{1}{2\pi\alpha'}$ by the formula,

$$T_p^2 = \frac{1}{g_s}(2\pi)^{1-p}T_{s}^{p+1}. \hfill (4.2)$$

Equating (4.1), (4.2) and using the fundamental membrane tension $2\pi R_{11} T_2^M = T_s$ leads to a relation between 11D quantities,

\begin{itemize}
  \item[2] We also checked interactions between two 8-branes and found agreement between string theory and M(atrix) results. There is no static potential and velocity dependent corrections appear at order $v^4$. Velocity dependent corrections to the potentials for 8-branes interacting with 2-branes and 4-branes appear to agree in the two approaches as well.
  \item[3] In [24] it was argued that presence of “half” a fundamental string accounts for the attractive force coming from the $(-1)^F R$ open string sector, which is dual to the exchange of R-R closed strings between the branes.
\end{itemize}
\[ T_9^M = \frac{R_{11}^2 (T_2^M)^4}{2\pi}. \]  \hspace{2cm} (4.3)

In the \( R_{11} \to \infty \) limit, \( T_9^M \) diverges. This can be compared to an analogous calculation, treating the D6-brane as an unwrapped 11D six-brane which leads to the tension formula, \( T_6 = T_6^M \). Again, using (4.2) and trading \( g_s \) and \( \alpha' \) for \( R_{11} \) and \( T_2^M \), we can derive an 11D relation,

\[ T_6^M = R_{11}^2 (T_2^M)^3. \]  \hspace{2cm} (4.4)

\( T_6^M \) also diverges in the \( R_{11} \to \infty \) limit, which is consistent with its interpretation as a magnetic KK p-brane \([2]\). The fact that \( T_9^M \) behaves in the same way suggests that the 10D 8-brane may have a similar 11D origin. The M(atrix) six-brane and eight-brane configurations were also shown to have energy densities which scale to infinity in the large \( N \) limit in \([7]\).

**Note Added**

After this work had already appeared we received \([27]\) which studies interactions of the pure D0-D8 system in some detail.

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\[ ^4 \] The fact that the two may be related was previously mentioned in the context of massive IIA supergravity \([4]\).

\[ ^5 \] This is in contrast to the “end of the world” 9-branes which are related to an orientifold plane plus 8 D8-branes and their mirror images and which should have finite tensions in the uncompactified limit.
References

[1] E. Witten, “String Theory Dynamics in Various Dimensions,” Nucl. Phys. B443 (1995) 85, hep-th/9503124.
[2] P. K. Townsend, “The eleven-dimensional supermembrane revisited,” Phys. Lett. 350B (1995) 184, hep-th/9501068.
[3] J. Polchinski, “TASI Lectures on D-Branes”, hep-th/9611050.
[4] E. Bergshoeff, M. de Roo, M. Green, G. Papadopolulos and P. K. Townsend, “Duality of Type II 7-branes and 8-branes”, Nucl. Phys. B470 (1996) 113, hep-th/9601150.
[5] G. Papadopolulos and P. K. Townsend, “Kaluza-Klein on the Brane”, Phys. Lett. B393 (1997) 59, hep-th/9609095.
[6] T. Banks, W. Fischler, S. Shenker, and L.Susskind, “M Theory as a Matrix Model: A Conjecture”, Phys. Rev. D55 (1997) 5112-5128, hep-th/9610043.
[7] T. Banks, N. Seiberg and S. Shenker, “Branes from Matrices”, Nucl.Phys. B490 (1997) 91-106, hep-th/9612157.
[8] P. Horava, “Matrix Theory and Heterotic Strings on Tori”, hep-th/9705053.
[9] O. Aharony and M. Berkooz, “Membrane Dynamics in M(atrix) Theory”, hep-th/9611215.
[10] G. Lifschytz and S. Mathur, “Supersymmetry and Membrane Interactions in M(atrix) Theory”, hep-th/9612087.
[11] G. Lifschytz, “Four-brane and Six-brane Interactions in M(atrix) Theory”, hep-th/9612223.
[12] V. Balasubramanian and F. Larsen, “Relativistic Brane Scattering”, hep-th/9703039.
[13] I. Chepelev and A. Tseytlin, “Long-distance interactions of D-brane bound states and longitudinal 5-brane in M(atrix) theory”, hep-th/9704127.
[14] J. Polchinski and P. Pouliot, “Membrane Scattering with M-momentum Transfer”, hep-th/9704029.
[15] C. Bachas and M. Porrati, “Pair Creation of Open Strings in an Electric Field”, Phys. Lett. B296 (1992) 77, hep-th/9209032.
[16] C. Bachas, “D-Brane Dynamics”, Phys. Lett. B374 (1996) 37, hep-th/9511043.
[17] G. Lifschytz, “Comparing D-branes to Black-branes”, Phys. Lett. B388 (1996) 720-726, hep-th/9604156.
[18] G. Lifschytz, “Probing Bound States of D-branes”, hep-th/9610125.
[19] M. Douglas, D. Kabat, P. Pouliot, and S. Shenker, “D-branes and Short Distances in String Theory”, Nucl.Phys. B485 (1997) 85-127, hep-th/9608024.
[20] A. Abouelsaood, C. Callan, C. Nappi, and S. Yost, “Open Strings in Background Gauge Fields”, Nucl. Phys. B280 (1987) 599.
[21] M. Berkooz, M. Douglas, and R. Leigh, “Branes Intersecting at Angles”, Nucl. Phys. B480 (1996) 265-278, hep-th/9606139.
[22] U. Danielsson and G. Ferretti, “The Heterotic Life of the D-particle”, hep-th/9610082.
[23] C. Bachas, M. Douglas, and M. B. Green, “Anomalous Creation of Branes”, hep-th/9705074.
[24] U. Danielsson, G. Ferretti and I. Klebanov, “Creation of Fundamental Strings by Crossing D-branes”, hep-th/9705084.
[25] J. Schwarz, “The Power of M-theory”, Phys. Lett. B367 (1996) 97-103, hep-th/9510086.
[26] S. de Alwis, “A Note on Brane Tension and M-theory”, Phys. Lett. B388 (1996) 291, hep-th/9607011.
[27] O. Bergman, M. Gaberdiel, and G. Lifschytz, “Branes, Orientifolds and the Creation of Elementary Strings”, hep-th/9705130.