BEYOND BJØNTEGAARD:
LIMITS OF VIDEO COMPRESSION PERFORMANCE COMPARISONS

Christian Herglotz, Matthias Kränzler, Ruben Mons, André Kaup

Multimedia Communications and Signal Processing
Friedrich-Alexander University Erlangen-Nürnberg (FAU)
Erlangen, Germany
{christian.herglotz,matthias.kraenzler, ruben.mons, andre.kaup}@fau.de

ABSTRACT

For 20 years, the gold standard to evaluate the performance of video codecs is to calculate average differences between rate-distortion curves, also called the “Bjøntegaard Delta”. With the help of this tool, the compression performance of codecs can be compared. In the past years, we could observe that the calculus was also deployed for other metrics than bitrate and distortion in terms of peak signal-to-noise ratio, for example other quality metrics such as video multi-method assessment fusion or hardware-dependent metrics such as the decoding energy. However, it is unclear whether the Bjøntegaard Delta is a valid way to evaluate these metrics. To this end, this paper reviews several interpolation methods and evaluates their accuracy using different performance metrics. As a result, we propose to use a novel approach based on Akima interpolation, which returns the most accurate results for a large variety of performance metrics. The approximation accuracy of this new method is determined to be below a bound of 1.5%.

Index Terms— video codec, rate-distortion, Bjøntegaard Delta, VMAF, decoding energy

1. INTRODUCTION

In the past twenty years, video communication has become an integral part of the daily lives of people all around the world. Nowadays, more than three quarters of the global Internet traffic is composed of video data [1], which underlines the importance of this technology. At the same time, the quality of the video data increased significantly due to ultra high definition resolutions, higher frame rates, and dynamic ranges, which leads to even larger bitrates required for transmission. As a consequence, more sophisticated and powerful compression technologies were developed to mitigate the burden on storage and transmission. As such, every couple of years, new codecs such as H.264/AVC, HEVC, and VVC were developed, which all aimed at reducing the bitrate at the same visual quality by a factor of two with respect to its predecessor [2].
the last several years, other metrics attracted more attention. A major reason is that the PSNR has a relatively low correlation with subjective quality when evaluating across different codecs, as reported in [6]. As a consequence, new quality metrics such as the structural similarity index (SSIM) [7] or the video multi-method assessment fusion (VMAF) [8] were developed, which aim to have a higher correlation with the subjective quality of videos. Consequently, researchers started evaluating the performance of new compression algorithms replacing the classic PSNR in rate-distortion curves with these new quality metrics [9][10]. Furthermore, results from subjective testing were also employed replacing the PSNR with mean-opinion scores (MOS) [2][11].

Furthermore, in contrast to replacing the quality metric, it was also proposed to replace the bitrate by other metrics describing technically limiting factors, for example the decoding energy or the decoding time [12][14].

For all these examples, the classic BD calculus was used to calculate the compression performance of video codecs with respect to different quality or technical metrics, which we call performance metrics (PMs) in the following. However, to the best of our knowledge, it was never studied whether the BD calculus is a feasible, correct, and accurate way to compare codecs. To fill this gap, we perform an in-depth analysis of the BD calculus in this paper, with a particular focus on the interpolation algorithms, different performance metrics, and a representative set of sequences. As a result, we propose a novel method to calculate BD values based on Akima interpolation, which we call Akima Bjøntegaard Delta (ADB).

This paper is organized as follows. First, Section 2 reviews the BD-rate calculation formula. Afterwards, Section 3 presents a method to evaluate the accuracy of the BD calculus. Then, Section 4 introduces our evaluation setup and Section 5 provides explicit error values that can be expected when applying the BD calculus on any kind of PM. Finally, Section 6 concludes this paper.

2. THE BJØNTEGAARD DELTA-RATE CALCULATION

In [3], the BD-rate is calculated on rate-distortion curves and is obtained by the following procedure. First, four different rate points or target qualities \( i \in \{1, 2, 3, 4\} \) are chosen for a certain input sequence \( s \). For these four points, bit streams are encoded with two different codecs \( k \in \{A, B\} \). The actual bitrate \( R_{k,i,s} \) and the actual distortion \( D_{k,i,s} \) in terms of PSNR for the resulting eight bit streams is then used as a basis and exemplarily depicted as ‘x’-markers in Fig. 1. Without loss of generality, we define the PSNR values to be monotonically increasing with \( D_{k,1,s} < D_{k,2,s} < D_{k,3,s} < D_{k,4,s} \). As a consequence, the corresponding bitrates \( R_{k,i,s} \) are monotonically increasing, too.

Then, to take into account that the bitrate can vary by multiple orders of magnitude, the bitrate \( R_{k,i,s} \) is converted to the logarithmic domain with

\[
\hat{r}_{k,i,s} = \log_{10}(R_{k,i,s}).
\]

This step is performed to ensure that mean BD-rate values are not biased towards higher bitrates [3].

In the next step, using an interpolation function, polynomial curves are fitted for both codecs \( k \) which use the marker positions \( \{D_{k,i,s}, \hat{r}_{k,i,s}\} \) as supporting points. As four supporting points are used, the interpolated curve is represented by a third order polynomial of the form

\[
\hat{r}_{k,s}(D) = a_{k,s} + b_{k,s} \cdot D + c_{k,s} \cdot D^2 + d_{k,s} \cdot D^3,
\]

where the parameters \( a_{k,s}, b_{k,s}, c_{k,s}, \) and \( d_{k,s} \) are derived by interpolation for both codecs. Interpolation is performed in such a way that the resulting curves pass through all supporting points \( \{D_{k,i,s}, \hat{r}_{k,i,s}\} \).

Afterwards, the average difference between the two resulting curves is calculated by integration. As upper and lower bound of integration, the overlapping parts of the two curves are selected, which are calculated by

\[
D_{s,\text{low}} = \max(D_{A,1,s}, D_{B,1,s}) \quad \text{(3)}
\]

\[
D_{s,\text{high}} = \min(D_{A,4,s}, D_{B,4,s}) \quad \text{(4)}
\]

In Fig. 1 these bounds are indicated by dashed lines.

Finally, the average relative difference between the two curves, which is the BD-rate \( \Delta R_s \), is calculated by the following integration

\[
\Delta R_s = 10^{\frac{1}{\log_{10}(4)} \int_{D_{s,\text{low}}}^{D_{s,\text{high}}} \hat{r}_{B,s}(D) - \hat{r}_{A,s}(D) \, dD} - 1, \quad \text{(5)}
\]

which describes the relative bitrate difference in the bounds of (3) and (4) of sequence \( s \) when encoded with codec \( k = B \) with respect to codec \( k = A \).

3. BD-RATE ASSESSMENT

The idea of the BD-rate calculus is to allow the accurate calculation of average differences between RD curves, where usually only discrete points are available in the RD space. Hence, it was proposed to interpolate the available points with a polynomial [2] and use an integration to get the average difference between the continuous curves [3].

Still, this idea leads to an important question: Are the resulting continuous curves representative for the true RD behavior of the two codecs? The interpolation method only ensures that the four input RD points \( \{D_{k,i,s}, \hat{r}_{k,i,s}\} \) are located on the interpolated curve. In contrast, it is unclear, and, to the best of our knowledge, it was never analyzed in detail whether the curves are representative for RD points between the supporting points.
As a consequence, we propose to evaluate the BD calculus on all available RD points in the common region of interest. For many encoders, these RD points can be generated by encoding at different quantization parameters (QPs) \[5\]. Consequently, we encode bit streams with all available QPs between the minimum and the maximum considered QP.

To evaluate the accuracy of the interpolated RD-curves, we calculate the mean relative distance of the curve to all available points \(p \in \mathcal{P}\) by

\[
\bar{e} = \frac{1}{K \cdot S \cdot |\mathcal{P}|} \sum_{k=1}^{K} \sum_{s=1}^{S} \sum_{p \in \mathcal{P}} \frac{|10^{f_{k,s}(D_{k,p,s})} - R_{k,p,s}|}{R_{k,p,s}},
\]

where \(k\) denotes the codec index with the number of codecs \(K\) and \(s\) denotes the sequence index with the total number of sequences \(S\).

The calculus is visualized in Fig. 2. For illustration, we choose a fictional linear interpolation method. The supporting points and additional points are indicated by x’es and o’s, respectively, and the interpolated curve is indicated by a yellow line. The interpolation errors for each point are calculated using the horizontal distance between the additional points \(R_{k,p,s}\) and the interpolation \(10^{f_{k,s}(D_{k,p,s})}\), as indicated by the red dotted lines.

Note that in this calculus, averaging is performed element-wise. All elements of |\(\mathcal{P}\)|, i.e., all points in the region of interest, are distributed evenly between the four supporting points. As such, taking the logarithm of the rate \[1\] to avoid a bias towards higher bitrates is not required.

Intuitively, the relative error \(\bar{e}\) describes the mean relative distance of the interpolated curves to the available RD points. As it is calculated in the same domain as the BD-rate, \(\bar{e}\) can directly be interpreted as the maximum error imposed on the calculated BD-rate \[5\], which is caused by interpolation.

Next to the mean error, we also evaluate the maximum relative error

\[
E_{\text{max}} = \max_{k,s,p} \left| \frac{10^{f_{k,s}(D_{k,p,s})} - R_{k,p,s}}{R_{k,p,s}} \right|,
\]

which corresponds to the largest relative horizontal distance of the interpolated to curve to the true RD points.

### 4. EVALUATION SETUP

For our evaluation, we use \(S = 6\) representative sequences from the JVET common test conditions \[15\] as listed in Table 1. The sequences are provided with different resolutions and frame rates. We encode ten seconds using the QPs \(\{22, 23, 24, \ldots, 27, \ldots, 32, \ldots, 37\}\) to generate the PM points in \(\mathcal{P}\), where the bold values are used as supporting points for the calculation of the interpolation, as proposed by the HEVC and the VVC common test conditions \[15, 16\].

As encoders, we choose the HM encoder and the VTM encoder for HEVC and VVC, respectively, (\(K = 2\) codecs). As a configuration, we choose random access with an internal bit depth of 10 bit for both encoders \[15, 16\].

We evaluate the following interpolation methods. First, we use cubic spline interpolation (CSI) with a not-a-knot boundary constraint, which was initially used for BD calculations \[17\] and requires the first and the second derivative to be continuous. Second, we evaluate CSI with a natural boundary constraint, which means that the second derivative at the two ends of the interpolation curve is set to zero (CSI - clamped). Third, we replace the boundary constraint with a clamped constraint, meaning that the derivative at the borders is zero (CSI - clamped). Fourth, we evaluate piecewise-cubic hermite interpolating polynomial (PCHIP), which was found to be more stable for special cases of RD curves \[18\] and which is also used in practical BD calculations for, e.g., standardization \[5\]. Finally, we test Akima interpolation \[19\], which returns a piecewise polynomial, too. The latter two methods only require the first derivative of the polynomial to be continuous.

As performance metrics, next to the traditional bitrate and PSNR, we choose to evaluate the quality metrics SSIM \[7\] and VMAF \[20\]. These two metrics replace the PSNR describing the distortion \(D\). Concerning an alternative for the

### Table 1. Evaluation sequences.

| Name          | Class | Resolution  | Frame rate [fps] |
|---------------|-------|-------------|------------------|
| Cactus        | B     | 1920 × 1080 | 50               |
| PartyScene    | C     | 832 × 480   | 50               |
| BQSquare      | D     | 416 × 240   | 60               |
| BasketballPass| D     | 416 × 240   | 50               |
| FourPeople    | E     | 1280 × 720  | 60               |
| SlideEditing  | F     | 1280 × 720  | 30               |
Table 2. Mean relative errors and maximum relative errors for the five tested interpolation algorithms.

| PM pair | PSNR - Bitrate | SSIM - Bitrate | VMAF - Bitrate | PSNR - Energy | VMAF - Energy |
|---------|----------------|----------------|----------------|--------------|---------------|
|         | \( \bar{\epsilon} \) | \( E_{\text{max}} \) | \( \bar{\epsilon} \) | \( E_{\text{max}} \) | \( \bar{\epsilon} \) | \( E_{\text{max}} \) |
| CSI     | 0.630%         | 5.151%         | 9.130%         | 110.446%     | 5.587%        | 29.093%       | 0.992%         | 7.060%        | 2.588%         | 14.705%       |
| PCHIP   | 0.420%         | 4.103%         | 1.709%         | 9.329%       | 2.010%        | 12.971%       | 0.917%         | 7.093%        | 1.140%         | 7.046%        |
| Akima   | 0.370%         | 4.855%         | 1.121%         | 7.439%       | 1.402%        | 10.576%       | 0.904%         | 7.066%        | 1.064%         | 7.053%        |
| CSI nat.| 0.658%         | 4.246%         | 3.870%         | 15.561%      | 3.450%        | 15.735%       | 1.019%         | 7.080%        | 1.765%         | 8.374%        |
| CSI cla.| 5.068%         | 25.984%        | 9.372%         | 44.081%      | 8.831%        | 37.198%       | 1.908%         | 12.882%       | 3.378%         | 15.558%       |

Fig. 3. Supporting points (x’es), additional points (o’s), and the curves interpolated by CSI (yellow), PCHIP (purple), and Akima (green) for the Cactus sequence (SSIM - bitrate).

For illustration of the performance of the interpolation methods, in Fig. 3 we plot supporting points, additional points, and example interpolated curves for the VVC-coded Cactus sequence, which returned the overall highest maximum interpolation error. The PM pair is SSIM - Bitrate. The curve is plotted in a logarithmic representation to highlight differences between the interpolated curves.

Apparently, the interpolation returned by CSI leads to an overshoot of the interpolated curve, which causes the interpolated bitrate to be more than twice as high as measured (the marker close to an SSIM value of 0.98). In contrast, the PCHIP algorithm returns a stable curve with a maximum error below 10%. Still, Akima interpolation returns the closest approximation (visible at low bitrates) as the green curve is located closer to the markers than the other curves.

The resulting mean relative errors \( \bar{\epsilon} \) and maximum errors \( E_{\text{max}} \) for all PM pairs are listed in Table 2. The results confirm observations in [5], stating that PCHIP returns more stable interpolations than CSI because in some cases (SSIM - Bitrate, VMAF - Bitrate, and VMAF - Energy), the mean relative error is more than twice as high as for PCHIP. However, we can also see that Akima interpolation outperforms all the other interpolation methods for all PM pairs. Considering the errors of CSI natural and CSI clamped, we can see that these interpolation methods are not well suited for BD calculations.

In general, we observe mean interpolation errors below 1.5% for the Akima method. When comparing different codecs, which typically show BD differences significantly larger than 5%, this accuracy is sufficient. However, in the development of new coding tools, BD differences often yield values below 1%. For such cases, future research could investigate whether the interpolation errors shown in Table 2 can lead to incorrect decisions on the adoption of coding tools.

Considering the maximum relative error, we can see that Akima interpolation does not always lead to the most accurate interpolation. However, the difference to the best result is always lower than 0.7%, such that we conclude that Akima interpolation is the most accurate way to interpolate PM curves. For practical use, we provide Python scripts for cubic, piecewise cubic, and Akima interpolation at [23].

6. CONCLUSIONS

In this paper, we evaluated five interpolation algorithms of the Bjøntegaard-Delta calculus in detail using two different video codecs and five pairs of performance metrics. The results suggest that a novel method based on Akima interpolation (ABD) returns most accurate interpolations for different performance metric curves. Furthermore, we could show that when comparing the compression performance of different codecs, useful results can be obtained when the BD-rate difference is larger than 1.5%.

In future work, the same evaluations could be performed for further performance metrics such as subjective quality scores or mean average precision scores in object detection algorithms.
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