Modeling of friction forces in an asymmetric two-roll module

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Abstract. The study is devoted to the analysis of the patterns of friction force distribution in two-roll modules. An asymmetric two-roll module is considered, in which the rolls are located relative to the vertical with a tilt to the right, have unequal diameters and elastic coatings from the materials of different stiffness and friction coefficients, the material layer is fed with a tilt downward relative to the center line. Friction stress models and formulas for calculating neutral angles in a two-roll module under consideration are determined. Dependencies between the forces acting on the rolls and the stresses distributed under these forces are established. It was revealed that these dependences do not change with a change in the angle of supply of the material layer to the line of centers and the angle of inclination of the upper roll relative to the vertical.

1. Introduction
Roll machines are widely used in many branches of industry. The main operating element of roll machines is a roll pair, which together with the processed material forms a two-roll module.

Technological processes in two-roll modules are carried out as a result of contact interaction of the rolls with the processed material. The main task of the theory of contact interaction of two-roll modules is an analytical description of the patterns of contact (normal and tangent) forces distribution.

Theoretical studies of the patterns of contact forces distribution in two-roll modules were mainly carried out in rolling theory. In these studies, the theory of contact stresses is based on the solution of a differential equation (T. Karman equation) of the equilibrium of longitudinal forces in the strain zone.

When solving Karman differential equations, the main factor is the model of friction stresses, which takes into account the effect of friction on the rolling pressure and relates the tangential and normal stresses in the strain zone. There are many mathematical models of friction stress during rolling, obtained by theoretical, experimental or experimental-theoretical methods. Of the existing models of friction stresses during rolling, the most widely used is the model of dry friction (Amonton-Coulomb law), the model of constant friction stresses (Siebel law) and the model of fluid friction (Nadai law) [1, 2]. Other models of friction stresses are known [3, 4], including empirical models with a variable coefficient of friction. However, the correct model of friction stresses during rolling has not yet been created. Therefore, the models of friction stresses currently used in rolling theory are considered approximate. For this reason, theoretical solutions of the differential equation of equilibrium of longitudinal forces obtained on the basis of such models are considered approximate.

Approximate models of the differential equation of equilibrium of longitudinal forces solved by various authors [1, 2, 5–9], include rough assumptions and simplifications; for example, the pressure of the metal on the rolls is taken equal to the strain resistance of the metal, the contact are between the
strip and the rolls is replaced by a chord. Therefore, the theoretical curves of contact stress distribution obtained do not correspond to the experimental diagrams [10].

In rolling theory [1, 2, 11, 12], it is assumed that in the general case, in the strain zone there are three zones that differ in kinematics—slip lag zone, adhesion zone, and slip advance zone. Moreover, the Amonton–Coulomb law or Siebel law [1, 11] are used to describe the friction stress in slip zones. However, a theoretical model of the friction stress for the adhesion zone during rolling does not exist. In studies [2, 12, 13] empirical dependencies $t_x = \varphi(x)$ are usually used. To carry out calculations on these dependences, data are required that can only be obtained during complex experimental studies.

For example, for dependence $t_x = t_0 \left( \frac{h_x - h_0}{l} \right)$, experimental data are needed on the extent of the adhesion zone $l$, the friction stress at the beginning of the adhesion zone $t_0$, and the strip thickness in the neutral section $h_0$ [13].

In modern roll machines, asymmetric two roll modules are often used. In asymmetric two-roll modules asymmetric contact conditions occur during the interaction, on opposite contact surfaces of the strain zone. Quite a lot of publications are known [14-18] devoted to the theoretical study of contact stresses in the process of asymmetric rolling. Theoretical solutions of the differential equations of equilibrium of longitudinal forces in these works were carried out using approximate models of friction stresses. Therefore, these solutions do not provide the required accuracy in determining the patterns of contact stress distribution in two roll modules.

![Figure 1. Force scheme in a two-roll module.](image-url)
It follows from the foregoing that obtaining theoretical curves of contact stress distribution corresponding to experimental diagrams is currently impossible due to the lack of correct models of friction stresses.

2. Materials and methods

To systematize the studies of contact interaction, first of all, a generalized scheme for the interaction of a roll pair with the processed material is selected based on the analysis of functional structures and classifications of roll modules; this scheme is a generalized model of two-roll modules, which serves as the object of study of contact phenomena.

In this two-roll module, the rolls are positioned relative to the vertical by tilting to the right at an angle $\beta$, have unequal diameters $(R_1 \neq R_2)$ and elastic coatings from materials of different stiffness and friction coefficients $(f_1 \neq f_2)$, the lower shaft is a driving one and the upper one is free. The material layer has uniform thickness $\delta_1$ and is tilted downward relative to the line of centers at an angle $\gamma_1$ (figure 1).

Analyze the stress state of the contact interaction of the material layer and the lower roll, which occurs along the contact curve $A_{11} A_{12}$.

In the steady-state interaction process, the lower roll is affected by: pressure force of the clamping devices $\bar{Q}$, horizontal reaction of the roll supports $P$, moment of resistance forces $M$, elementary forces of normal pressure and friction, acting along the entire contact curve of the roll. Elementary forces in the zones of compression $(N_{11}, T_{11})$ and recovery $(N_{12}, T_{12})$ are presented separately. The friction forces at the beginning and at the end of the contact zone have opposite directions. They change their signs, turning to zero at a neutral point $A_{13}$. It was revealed [2, 10] that in the driving roll the neutral point is located to the left of the center line, that is, in the compression zone. Let the neutral point $A_{13}$ correspond to the angle $\theta_{13} = -\varphi_{13}$ (figure 1).

Considering the lower roll in equilibrium under the action of applied forces, for the compression zone we have:

$$
\begin{align*}
N_{11x} + T_{11x} + F_{11x} + Q_{11x} &= 0, \\
N_{11y} + T_{11y} + F_{11y} + Q_{11y} &= 0
\end{align*}
$$

or

$$
\begin{align*}
dN_{11x} + dT_{11x} + dF_{11x} + dQ_{11x} &= 0, \\
dN_{11y} + dT_{11y} + dF_{11y} + dQ_{11y} &= 0
\end{align*}
$$

where $N_{11x}, N_{11y}, T_{11x}, T_{11y}$ are the projections of the main normal and tangential forces of the compression zone on the axes $x$ and $y$.

From the forces scheme in compression zone (figure 1) we find:

$$
\begin{align*}
dN_{11x} &= N_{11x} \sin(\theta_{11} - \varphi_{11}), \\
dN_{11y} &= -N_{11x} \cos(\theta_{11} - \varphi_{11}), \\
dT_{11x} &= -dT_{11x} \cos(\theta_{11} - \varphi_{11}), \\
dT_{11y} &= -dT_{11x} \sin(\theta_{11} - \varphi_{11}), \\
dF_{11x} &= dF_{11x} = dF_{11y} = dQ_{11x} = 0, \\
dQ_{11y} &= dQ_{11y}
\end{align*}
$$

where $\varphi_{11}$ is the angle between the force $dN_{11}$ and the radius $r_{11}$.

Taking into account equation (2) from system (1) for the compression zone, we have:

$$
\begin{align*}
\frac{dF_{11x}}{dQ_{11x}} &= \frac{dT_{11x} \cos(\theta_{11} - \varphi_{11}) - dN_{11x} \sin(\theta_{11} - \varphi_{11})}{dT_{11x} \sin(\theta_{11} - \varphi_{11}) + dN_{11x} \cos(\theta_{11} - \varphi_{11})}.
\end{align*}
$$
Since we are considering a steady-state process, we can assume that 
\[ \frac{F_r}{Q_t} = C_1, \] where \( C_1 \) is a constant value. Hence we have 
\[ d \left( \frac{F_r}{Q_t} \right) = \frac{Q_t dF_r - F_r dQ_t}{Q_t^2} = 0 \quad \text{or} \quad \frac{dF_r}{dQ_t} = C_1. \]

Assuming that \( C_1 = \frac{dF_r}{dQ_t} \) from equality (3) we have:

\[ \frac{dT_{11}}{dN_{11}} = \sin(\theta_{11} - \psi_{11}) + C_{11} \cos(\theta_{11} - \varphi_{11}) \]
\[ \quad \cos(\theta_{11} - \psi_{11}) - C_{11} \sin(\theta_{11} - \varphi_{11}) \]  
(4)

Elementary forces are connected to contact stresses by relations:

\[ dN_{11} = n_{11} \sqrt{r_{11}^2 + r_{11}'^2}, \quad dT_{11} = r_{11} \sqrt{r_{11}^2 + r_{11}'^2} d\theta_{11}, \quad (5) \]

where \( n_{11} = n_{11}(\theta_{11}), \quad t_{11} = t_{11}(\theta_{11}) \) are normal and shear stresses, respectively, distributed over the compression zone of the contact curve of the rolls.

We substitute equations (5) into equation (4), then transform it according to the expressions

\[ \cos \psi_{11} = \frac{r_{11}}{\sqrt{r_{11}^2 + r_{11}'^2}}, \quad \sin \psi_{11} = \frac{r_{11}'}{\sqrt{r_{11}^2 + r_{11}'^2}} \] and obtain dependencies connecting the tangent and normal stresses at the points of the compression zone of the lower roll:

\[ t_{11} = \frac{(\sin \theta_{11} + C_{11} \cos \theta_{11}) r_{11}' - (\cos \theta_{11} - C_{11} \sin \theta_{11}) r_{11}'}{(\cos \theta_{11} - C_{11} \sin \theta_{11}) r_{11} + (\sin \theta_{11} + C_{11} \cos \theta_{11}) r_{11}'}, \quad -\varphi_{11} \leq \theta_{11} \leq 0, \]  
(6)

We obtain the formula connecting the tangential and normal stresses at the points of the recovery zone of the lower roll. It has the form:

\[ t_{12} = \frac{(\sin \theta_{12} + C_{12} \cos \theta_{12}) r_{12}' - (\cos \theta_{12} - C_{12} \sin \theta_{12}) r_{12}'}{(\cos \theta_{12} - C_{12} \sin \theta_{12}) r_{12} + (\sin \theta_{12} + C_{12} \cos \theta_{12}) r_{12}'}, \quad 0 \leq \theta_{12} \leq \varphi_{12}, \]  
(7)

where \( C_{12} = \frac{dF_r}{dQ_{t2}}. \)

Note that at the point of contact curve on the line of centers the following boundary conditions are satisfied:

\[ t_{11}(0) = t_{12}(0), \quad n_{11}(0) = n_{12}(0), \quad r_{11}(0) = r_{12}(0) = R_{10}, \quad r_{11}'(0) = r_{12}'(0) = 0. \]

These conditions lead to equality \( C_{11} = C_{12}. \)

Then, we have:

\[ C_1 = C_{11} = C_{12} = \frac{F_r}{Q_t}. \]

So, from equations (6) and (7) we obtain a system that describes the model of friction stresses for the lower driving roll:

\[ \begin{cases} t_{11} = \frac{(Q_r \sin \theta_{11} + F_r \cos \theta_{11}) r_{11}' - (Q_r \cos \theta_{11} - F_r \sin \theta_{11}) r_{11}'}{(Q_r \cos \theta_{11} - F_r \sin \theta_{11}) r_{11} + (Q_r \sin \theta_{11} + F_r \cos \theta_{11}) r_{11}'} n_{11}, \quad -\varphi_{11} \leq \theta_{11} \leq 0, \\ t_{12} = \frac{(Q_r \sin \theta_{12} + F_r \cos \theta_{12}) r_{12}' - (Q_r \cos \theta_{12} - F_r \sin \theta_{12}) r_{12}'}{(Q_r \cos \theta_{12} - F_r \sin \theta_{12}) r_{12} + (Q_r \sin \theta_{12} + F_r \cos \theta_{12}) r_{12}'} n_{12}, \quad 0 \leq \theta_{12} \leq \varphi_{12}. \end{cases} \]  
(8)
In the two-roll module under consideration, the upper shaft is free. In this case, the forces acting on the upper roll \( \vec{F}_2 \) and \( \vec{T}_2 \) change directions [19]. Therefore, in the equations of system (8), the quantities \( t_{2j} \) (\( j = 1,2 \)) and \( F_j \) have opposite signs. In this regard, the model of friction stresses for the upper roll has the form:

\[
\begin{align*}
t_{21} &= -\frac{(Q_2 \sin \theta_{21} - F_2 \cos \theta_{21})r_{21} - (Q_2 \cos \theta_{21} + F_2 \sin \theta_{21})r_{21}'}{(Q_2 \cos \theta_{21} + F_2 \sin \theta_{21})r_{21} + (Q_2 \sin \theta_{21} - F_2 \cos \theta_{21})r_{21}'} \cdot \varphi_{21} \leq \theta_{21} \leq 0, \\
t_{22} &= -\frac{(Q_2 \sin \theta_{22} - F_2 \cos \theta_{22})r_{22} - (Q_2 \cos \theta_{22} + F_2 \sin \theta_{22})r_{22}'}{(Q_2 \cos \theta_{22} + F_2 \sin \theta_{22})r_{22} + (Q_2 \sin \theta_{22} - F_2 \cos \theta_{22})r_{22}'} \cdot 0 \leq \theta_{22} \leq \varphi_{22}.
\end{align*}
\]

(9)

The systems of equations (8) and (9) determine the models of friction stresses in the considered two-roll module. They show that the models of friction stresses in two-roll modules are independent of the inclination of the material layer feed to the center line and of the inclination of the upper roll relative to the vertical. An analysis of these models showed that they describe stress models of all partial cases of the two-roll module under consideration.

Assuming that \( \tan \xi_1 = \frac{F_1}{Q_1} \), we transform the first equation of system (8) to the form:

\[
t_{11} = \frac{\sin(\theta_{11} + \xi_1)r_{11} - \cos(\theta_{11} + \xi_1)r_{11}'}{\cos(\theta_{11} + \xi_1)r_{11} + \sin(\theta_{11} + \xi_1)r_{11}'} n_{11}.
\]

(10)

Considering \( \tan \psi_{11} = \frac{r_{11}'}{r_{11}} \), proceed to expression:

\[
t_{11} = \tan(\theta_{11} - \psi_{11} + \xi_1)n_{11}.
\]

(11)

At the neutral point \( t_{11}(-\varphi_{11}) = 0 \) and \( n_{11}(-\varphi_{11}) \neq 0 \). Therefore, according to equation (11), the condition for determining the neutral angle can be represented as:

\[
\varphi_{13} + \psi_{11} = \xi_1,
\]

(12)

where \( \psi_{11} = \arctan \frac{\cos \theta_{11}}{\sin \theta_{11}} \).

It follows from the equations of system (9) and equation (12) that the models of friction stresses and the values of neutral angles of two-roll modules depend on the external forces acting on the rolls and the curve shape of roll contact.

Let the lower roll contact curves in the compression zone be described by equation [19]:

\[
r_{11} = \frac{a_{11}}{1 + h_{11} \cos \theta_{11}}, \quad -\varphi_{11} \leq \theta_{11} \leq 0.
\]

(13)

The coefficients of equation (13) can be determined by the boundary conditions:

when \( \theta_{11} = -\varphi_{11}, \quad r_{11} = R_1 \); \quad when \( \theta_{11} = 0, \quad R = R_{10} \) (figure 1).

We find \( a_{11} \) and \( h_{11} \) with these boundary conditions:

\[
a_{11} = \frac{R_1 R_{10}}{R_{10} - R_1 \cos \varphi_{11}}, \quad h_{11} = \frac{R_1 - R_{10}}{R_{10} - R_1 \cos \varphi_{11}}.
\]

From equation (13) we have:

\[
r_{11}' = \frac{a_{11} h_{11} \sin \theta_{11}}{(1 + h_{11} \cos \theta_{11})^2}.
\]

(14)
Substituting the expressions \( r_{11} \) and \( r'_{11} \) from equation (13) and (14) into equation (10) and transforming it, we have:

\[
t_{11} = \sin(\theta_{11} + \xi_1) + b_{11} \sin \xi_1 - n_{11} = \frac{\sin(\theta_{11} + \xi_1) + b_{11} \sin \xi_1}{\cos(\theta_{11} + \xi_1) + b_{11} \cos \xi_1},
\]

or

\[
t_{11} = \varphi(\theta_{11}) n_{11}, \quad f(\theta_{11}) = \frac{\sin(\theta_{11} + \xi_1) + b_{11} \sin \xi_1}{\cos(\theta_{11} + \xi_1) + b_{11} \cos \xi_1}.
\]

Transforming \( t_{12} \), system (8) is rewritten in the form:

\[
\begin{align*}
    t_{11} &= f(\theta_{11}) n_{11}, \quad f(\theta_{11}) = \frac{\sin(\theta_{11} + \xi_1) + b_{11} \sin \xi_1}{\cos(\theta_{11} + \xi_1) + b_{11} \cos \xi_1}, \quad -\varphi_{11} \leq \theta_{11} \leq 0, \\
    t_{12} &= f(\theta_{12}) n_{12}, \quad f(\theta_{12}) = \frac{\sin(\theta_{12} + \xi_1) + b_{12} \sin \xi_1}{\cos(\theta_{12} + \xi_1) + b_{12} \cos \xi_1}, \quad 0 \leq \theta_{12} \leq \varphi_{12},
\end{align*}
\]

where \( b_{12} = \frac{R_1 - R_{10}}{R_{10} - R_1 \cos \varphi_{12}} \).

Taking into account equations (13) and (14) at the neutral point, we have:

\[
\psi_{11}^* = \arctan \left( \frac{r'_{11}(-\varphi_{13})}{r_{11}(-\varphi_{13})} \right) = -\frac{b_{11} \sin \varphi_{13}}{1 + b_{11} \cos \varphi_{13}}.
\]

Since the neutral point is close to the center line [10], the angle \( \varphi_{13} \) is close to zero. Therefore, we can assume that \( \psi_{11}^* \approx \psi_{11} \) (since \( \psi_{11} < \varphi_{13} \)), \( \sin \varphi_{13} \approx \varphi_{13} \), \( \cos \varphi_{13} \approx 1 \). Then the transformations of equation (13) lead to the following formula:

\[
\varphi_{13} = \frac{R_1(1 - \cos \varphi_{11})}{R_{10} - R_1 \cos \varphi_{11}} \xi_1.
\]

Calculate \( t \) and \( f \) from the equations of system (15) when rolling a deformable layer of material according to the harmonic law of normal stresses distribution:

\[
n_{11} = \frac{n_{\text{max}}}{2} \left( 1 + \cos \left( \frac{\theta_{11}}{\varphi_{11}} \pi \right) \right), \quad n_{12} = \frac{n_{\text{max}}}{2} \left( 1 + \cos \left( \frac{\theta_{12}}{\varphi_{11}} \pi \right) \right).
\]

**Figure 2.** Patterns of tangential stresses distribution at:

1. \( F_1 = 0 \); 2. \( F_1 = 0.025Q_1 \); 3. \( F_1 = 0.075Q_1 \).
The results of calculations of the distribution patterns of tangential stresses and the changes in contact stress ratio are shown in figures 2 and 3. The obtained patterns correspond to experimental distribution diagrams [2, 10].

![Figure 3. Patterns of changes in $f = \frac{T}{n}$ at: $1 - F_i = 0; 2 - F_i = 0.025Q_i; 3 - F_i = 0.075Q_i.$](image)

3. Results
For the first time, models of friction stresses in an asymmetric two-roll module (systems of equations (7) and (8)) were found that establish a connection between the shear and normal contact stresses in the slip zones and in the adhesion zone.

4. Conclusions
Models of friction stresses and formulas for calculating neutral angles in an asymmetric two-roll module are determined. The obtained models are general in the sense that they are applicable for special cases of interaction in two-roll modules.

The obtained formulas correspond with the results of other researchers.

Dependencies between the forces acting on the rolls and the stresses distributed under these forces are established.

It was revealed that these dependences do not change with a change in the angle of supply of the material layer to the line of centers and the angle of inclination of the upper roll relative to the vertical.

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