Strong decays of \(N^*(1535)\) in an extended chiral quark model

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The strong decays of the \(N^*(1535)\) resonance are investigated in an extended chiral quark model by including the low-lying \(qqqq\bar{q}\) components in addition to the \(qqq\) component. The results show that these five-quark components in \(N^*(1535)\) contribute significantly to the \(N^*(1535) \rightarrow N\pi\) and \(N^*(1535) \rightarrow N\eta\) decays. The contributions to the \(N\eta\) decay come from both the lowest energy and the next-to-lowest energy five-quarks components, while the contributions to the \(N\pi\) decay come from only the latter one. Taking these contributions into account, the description for the strong decays of \(N^*(1535)\) is improved, especially, for the puzzling large ratio of the decays to \(N\eta\) and \(N\pi\).

Among the low-lying nucleon excitations, the \(S_{11}\) state \(N^*(1535)\) plays a special role due to its large \(N\eta\) decay rate\(^1\), even though its mass is very close to the threshold of the decay. And recently it has been shown that the coupling of \(N^*(1535)N\phi\) may be significant\(^2\), which is consistent with the previous indications of the notable \(N^*(1535)K\Lambda\) coupling\(^3\) deduced from BES data. These suggest that there are large \(s\bar{s}\) components in the \(N^*(1535)\) resonance. And in a recent paper\(^4\), the role of the low-lying \(qqqq\bar{q}\) components in the electromagnetic transition \(\gamma^*N \rightarrow N^*(1535)\) has been investigated, the result shows that the contributions of the \(s\bar{s}\) component in \(N^*(1535)\) are significant, and with admixture of 5-quark components with a proportion of 20% in the nucleon and 25-65% in the \(N^*(1535)\) resonance the calculated helicity amplitude \(A_{1/2}\) decreases at the photon point, \(Q^2 = 0\) to the empirical range. Consequently, it also suggests large \(s\bar{s}\) component in \(N^*(1535)\), which is in line with the predictions in Refs.\(^2-3\).

Here we investigate the strong decays of the \(N^*(1535)\) resonance by extending the chiral quark model\(^5-6\) to include the \(qqqq\bar{q}\) components, which has been applied to the strong decays of the \(\Delta(1232)\) and \(N^*(1440)\) resonances successfully\(^7-8\). The wave functions of the \(qqq\bar{q}\) components in the nucleon and \(N^*(1535)\) are taken to be the conventional ones\(^6\), while we employ the wave functions for the \(qqqq\bar{q}\) components in the nucleon given in Ref.\(^9\), and the orbital-flavor-spin configuration of the four-quark subsystem is taken to be \([31]_x[4]_{FS}[22]_F[22]_S\), which leads to the lowest energy. For the \(qqqq\bar{q}\) components in the \(N^*(1535)\) resonance, we consider the contributions of the lowest and next-to-lowest energy five-quark components, as in our recent work\(^4\).

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Given the presence of the $qqqq$ components in both the nucleon and $N^*(1535)$ resonance, the strong decays of the latter may contain the following three processes: transition between the $qqq$ components, transition between the $qqqqq$ components, and the annihilation transitions between the $qqq$ and $qqqqq$ components. The results show that the five-quark components have significant contributions to the $N\pi$ and $N\eta$ decays of $N^*(1535)$. And with these contributions, the description for the strong decays of the latter may contain the following three processes: transition between the five-quark components, transition between the three-quarks components, and annihilation transitions between the five-quark components.

The present manuscript is organized as follows: The formalism for the strong decays of $N^*(1535)$ in the chiral quark model is given in Section 1. In Section 2, we calculate the decay width for the strong decays $N^*(1535) \rightarrow N\pi$ and $N^*(1535) \rightarrow N\eta$ by including all the contributions of the three- and five-quarks components. Finally, Section 3 contains a concluding discussion.

1 Formalism for the strong decays of $N^*(1535)$ in the chiral quark model

In the chiral quark model, the coupling of the baryons to the octet of light pseudoscalar mesons takes the form

$$L_{Mqq} = i \frac{g_M^q}{2f_M} \bar{q}_a \gamma_5 \gamma_\mu \partial^\mu \phi_{M} X^a_{M} \psi_q .$$  \hspace{1cm} (1)$$

Here $g_M^q$ denotes the axial coupling constant for the constituent quarks, the phenomenological value for which is in the range $0.70 - 1.26^{[10]}$. And $f_M$ is the decay constant of the corresponding meson. For the $\pi$ and $\eta$ mesons, the empirical values of the decay constants are $f_\pi = 93$ MeV and $f_\eta = 112$ MeV, respectively. And $\psi_q$ and $\phi_M$ denote the quark and meson field, respectively. Finally, $X^q_M$ is the flavor operator for emission of the meson $M$ ($\pi$, $\eta$) from the corresponding quark $q$, taking the following form:

$$X^q_{\pi_0} = \lambda_3 ,$$
$$X^q_{\eta_8} = \lambda_8 ,$$
$$X^q_{\eta_1} = \sqrt{\frac{2}{3}} I ,$$  \hspace{1cm} (2)$$

where $\lambda_i$ are the $SU(3)$ Gell-Mann matrices, and $I$ denotes the unit operator in the flavor space. Note that the $\eta$ meson is an admixture of the octet $\eta_8$ and the singlet $\eta_1$ with the mixing angle $\theta_p$:

$$\eta = \eta_8 \cos \theta_p - \eta_1 \sin \theta_p ,$$
$$\eta' = \eta_8 \sin \theta_p + \eta_1 \cos \theta_p .$$  \hspace{1cm} (3)$$

The value for $\theta_p$ is $-23^{[1]}$. Then the flavor operator for emission of the $\eta$ meson from the corresponding quark should be

$$X^q_\eta = \cos \theta_p X^q_{\eta_8} - \sin \theta_p X^q_{\eta_1} .$$  \hspace{1cm} (4)$$

Taking the $qqqqq$ components into account, the decays of a baryon can be divided into three parts: (1) The transitions between the three-quarks components, i.e., the process $qqq \rightarrow qqqM$, (2) The transitions between the five-quark components, i.e., $qqqqq \rightarrow qqqqqM$, and (3) The decays through $q\bar{q}$ annihilation, i.e., $qqqq\rightarrow qqqM$ and $qqq \rightarrow qqqqqM$. The first process has been analyzed by many authors$^{[11-13]}$, and it can be applied to part (2) directly. And Figure shows the last processes.

In the non-relativistic approximation, the La-
The transition amplitude of the $N\pi$ and $N\eta$ decays are obtained, one can calculate the decay width directly by using the following formula

$$d\Gamma_{N^\ast(1535)\rightarrow N\pi} = \frac{3}{16\pi^2} \frac{E' + m_N}{m^*} |\vec{k}_\pi||T(\pi)|^2d\Omega,$$

$$d\Gamma_{N^\ast(1535)\rightarrow N\eta} = \frac{1}{16\pi^2} \frac{E' + m_N}{m^*} |\vec{k}_\eta||T(\eta)|^2d\Omega,$$

where $E'$ and $m_N$ are the energy and mass of the nucleon, respectively, and $m^*$ the mass of the resonance $N^\ast(1535)$. And $\vec{k}_\pi$ and $\vec{k}_\eta$ denote the three-momentum of the $\pi$ and $\eta$ mesons, respectively.

And in the present case,

$$E' = \frac{m^*}{m_{\pi}^2 - m_M^2 + m_M^2},$$

$$|\vec{k}_M| = \sqrt{\frac{(m^*-m_N+m_M)^2}{2m^*}} \\ \frac{[m^2 - (m_N + m_M)^2][m^2 - (m_N - m_M)^2]}{2m^*}. \quad (7)$$

For the $N\pi$ decay, $k_\pi = 467$ MeV and $E' = 1048$ MeV; and for the $N\eta$ decay, $k_\eta = 188$ MeV and $E' = 957$ MeV.

## 2 The numerical results

### 2.1 Contributions of $qqq$ components

At the first step we consider the diagonal transitions of the three-quark components $qqq \rightarrow qqqM$. The wave functions of the corresponding components are taken to be the conventional ones\cite{6}. With these wave functions given in Ref.\cite{6} and the operator (5), a straightforward calculation leads to the following matrix elements of the operator $\hat{T}_d$ between the three-quark components in the proton and $N^\ast(1535)$:

$$\langle \hat{T}_d^{(p\eta)}(3\eta) \rangle = -i A_{N^3} N^\ast(\gamma A) \frac{g_A^q}{2f_{\gamma A}} 2\sqrt{2} \frac{\omega_\gamma}{\omega_\pi^2} k_{\pi^\prime}^2 (1 - \frac{\omega_\pi}{3m^*})$$_3 - \frac{3\omega_\pi}{m^*} \exp\{-\frac{k_{\pi^\prime}^2}{6\omega_\pi^2}\},$$

$$\langle \hat{T}_d^{(q\eta)}(3\eta) \rangle = -i A_{N^3} N^\ast(\gamma A) \frac{g_A^q}{2f_{\gamma A}} \sqrt{6} \frac{\omega_\gamma}{\omega_\pi^2} \omega_\pi C_\eta \frac{k_{\pi^\prime}^2}{\omega_\pi^2} (1 - \frac{\omega_\pi}{3m^*})$$$_3 - \frac{3\omega_\pi}{m^*} \exp\{-\frac{k_{\pi^\prime}^2}{6\omega_\pi^2}\}. \quad (8)$$

Here $m$ is the constituent mass of the light quark, and the factor $C_\eta = \cos \theta_p - \sqrt{2} \sin \theta_p$, which comes from the mixing of the $\eta_1$ and $\eta_8$ mesons. Then we can get the decay widths for the $N\pi$ and $N\eta$ decays of $N^\ast(1535)$ employing $q. (6)$. The numerical results are shown in Table 2.1 with the axial coupling constant $g_A^q = 0.70$. $g_A^q = 0.75$ and $g_A^q = 0.80$, respectively. At the present step, we have not considered the five-quark components, so the amplitudes of the three-quark components in the nucleon and $N^\ast(1535)$ should be $A_{N^3} = A_{N^*3} = 1$. The results for these decay widths are proportional to $(g_A^q)^2$, as shown in Table 2.1. We will use the central value $g_A^q = 0.75$ for our further calculations. The three columns in Table 2.1 are obtained by setting the oscillator parameter to be $\omega_\pi = 340$ MeV, which is employed for the electromagnetic transition $\gamma^* N \rightarrow N^\ast(1535)$ in Ref.\cite{1}, and the constituent mass of the light quark has been taken to be $m = 340$ MeV. The experimental data for the decay width are extracted from Ref.\cite{1}, here we have taken the average value for the Breit-Wigner width.
Table 1 The decay widths for $N^*(1535) \rightarrow N\pi, N\eta$ without including $qqqq\bar{q}$ components. The results (A-C) correspond to the axial coupling constant $g_A^q$ to be 0.70, 0.75 and 0.80, respectively, with the oscillator parameter $\omega_3 = 340$ MeV.

| A      | B      | C      | Data |
|--------|--------|--------|------|
| $\Gamma_{N^*\rightarrow N\pi}$ (MeV) | 136.7  | 156.9  | 178.6| 52.5-82.5 |
| $\Gamma_{N^*\rightarrow N\eta}$ (MeV) | 44.2   | 50.7   | 57.7 | 67.5-90.0  |
| $R = \frac{\Gamma_{N^*\rightarrow N\pi}}{\Gamma_{N^*\rightarrow N\eta}}$ | 0.32   | 0.32   | 0.32 | 0.82-1.71  |

As shown in Table 2, all of the three columns cannot fit the data well. For instance, in column A, the decay width for the $N\pi$ decay of $N^*(1535)$ is much larger than the upper bound of the experimental data, while that for the $N\eta$ decay is a bit smaller than the lower bound of the data. Consequently, the value for the ratio $R = \frac{\Gamma_{N^*\rightarrow N\pi}}{\Gamma_{N^*\rightarrow N\eta}}$ is much smaller than the lower bound of the data in Ref. [3]. It indicates that we should consider the contributions of the five-quark components in the nucleon and $N^*(1535)$.

2.2 Contributions of the five-quark components with the lowest energy

Then we should consider the contributions of the $qqqq\bar{q}$ components. As mentioned in Section 2.1, it can be divided into two parts: the diagonal and non-diagonal transitions. For the former one, the operator is $\hat{T}_d$ in Eq. (5) with $n_q = 5$. First, we consider the $qqqq\bar{q}$ components in the nucleon and $N^*(1535)$ with the lowest energy in this section.

The orbital-flavor-spin configuration for the four-quark subsystem of the lowest energy five-quark component in $N^*(1535)$ is $[4]_x[31]_F [211]_F[22]_S$, and that for the four-quark subsystem of the lowest energy five-quark components in the nucleon is $[31]_x[4]_F [22]_F[22]_S$. Note that the total spin of the four-quark subsystem is then $S = 0$; consequently, the matrix element of $\hat{T}_d$ between these four quarks vanishes. On the other side, the matrix element of $\hat{T}_d$ between the anti-quarks in the nucleon and $N^*(1535)$ should also vanish for the orthogonality of the orbital states $[4]_x$ and $[31]_x$. Then the diagonal transition between the lowest energy $qqqq\bar{q}$ components in $N^*(1535)$ and nucleon does not contribute to the $N\pi$ and $N\eta$ decays of the $N^*(1535)$ resonance.

On the other hand, calculations of the matrix elements for the operator $\hat{T}_{nd}$ between the wave functions of the proton and $N^*(1535)$ lead to the following expressions:

$$\langle \hat{T}_{nd}(\eta_q) \rangle \rightarrow 0,$$

$$\langle \hat{T}_{nd}(\pi_q) \rangle = iA \frac{g_A^q m_s \sqrt{6}}{f_\eta} C_3 C'_\eta \exp\left\{ -\frac{3k_0^2}{20}\right\},$$

where $C_3$ denotes the orbital overlap factor:

$$\langle \varphi_0(\vec{k}_1)\varphi_0(\vec{k}_2)|\varphi_0(\vec{k}_1)\varphi_0(\vec{k}_2)\rangle = \left( \frac{2\omega_7^2 \omega_5}{\omega_3^2 + \omega_5^2} \right)^3.$$

And the factor $C'_\eta = \sqrt{2} \cos \theta_p + \sin \theta_p$.

Table 2 The decay widths for $N^*(1535) \rightarrow N\pi, N\eta$ by including the lowest $qqqq\bar{q}$ components with $g_A^q = 0.75$.

| A      | B      | C      | D      | E      | Data |
|--------|--------|--------|--------|--------|------|
| $\Gamma_{N^*\rightarrow N\pi}$ (MeV) | 152.0  | 131.7  | 111.4  | 91.2   | 70.9 | 52.5-82.5 |
| $\Gamma_{N^*\rightarrow N\eta}$ (MeV) | 60.9   | 57.6   | 53.3   | 48.3   | 42.5 | 67.5-90.0  |
| $R = \frac{\Gamma_{N^*\rightarrow N\pi}}{\Gamma_{N^*\rightarrow N\eta}}$ | 0.40   | 0.44   | 0.48   | 0.53   | 0.60 | 0.82-1.71  |

As we can see from Eq. (9), the lowest energy five-quark components in the nucleon and $N^*(1535)$ do not contribute to the $N\pi$ decay of $N^*(1535)$, while they contribute to the $N\eta$ decay. The numerical results are shown in Table 2. The proportion of the five-quark component in $N^*(1535)$ is taken to be 25% - 65%, and in the nucleon 20%. And the oscillator parameters are taken to be $\omega_3 = 340$ MeV and $\omega_5 = 600$ MeV, with which values we can describe the electromagnetic transitions $\gamma^* N \rightarrow N^*(1535)$ best [4]. Since we have considered the five-quark components in the nucleon and $N^*(1535)$, we should set the constituent quark masses to be smaller than the one employed in Section 2.1. Here we take $m = 290$ MeV for the light quarks and $m_s = 430$ MeV for the strange quark.

As shown in Table 2, the numerical result for the $N\eta$ decay of $N^*(1535)$ is getting better when the contributions of the five-quark components are taking into account, and the ratio $R$ of the $N\eta$
Contributions from other configurations

As mentioned in Section 2.2, here we consider the flavor-spin configuration $F_S$ for the four-quark subsystem of the five-quark components in $N^*(1535)$, which contains only the strange $s\bar{s}$ quark pair and hence does not contribute to the $N\pi$ decay. The numerical results are still not good enough. Therefore, the contributions from other flavor-spin configurations, which allow for the five-quark components with light $q\bar{q}$, need to be considered. And these configurations may contribute to both $N\pi$ and $N\eta$ decays of $N^*(1535)$. As in Ref.[4], we consider the flavor-spin configuration $[31]_{FS}[22]_F[31]_S$ for the four-quark subsystem of the five-quark components, which has the next-to-lowest energy.

\[\langle T_{\pi}\rangle_{(5q\bar{q})} = \, iA_{N\pi}s_{q}A_{N,ss}g_{s}^{q}f_{\pi}^{4}m'_{\pi}^{2}C_{35}C_{N}\exp\left\{-\frac{k_{1}^{2}}{20\omega_{5}^{2}}\right\},\]

\[\langle T_{d}\rangle_{(5q\bar{q})} = \, iA_{N\eta}d_{q}A_{N,ss}g_{s}^{q}f_{\eta}^{4}m'_{\pi}^{2}C_{35}C_{N}\exp\left\{-\frac{k_{1}^{2}}{20\omega_{5}^{2}}\right\}.\]

Note as in Ref.[4], we have not included explicitly the contributions of the next-to-lowest energy five-quarks components containing $s\bar{s}$ pair, because these contributions can be replaced by certain probabilities of the lowest energy five-quarks component in $N^*(1535)$.

Including contributions from the five-quarks components of the next-to-lowest energy allowing for the light $q\bar{q}$ pairs, the numerical results are shown in Table 2.3 with the oscillator parameters $\omega_3 = 340$ MeV and $\omega_5 = 600$ MeV, and the axial coupling constant $g_A = 0.75$. Here we take the probability $P_{3q}$ for the five-quarks components in $N^*(1535)$ to be 25% to 65%, the proportion of the lowest energy $qqqqq$ component to be 0.9$P_{3q}$, and that of the next-to-next-to-lowest energy one to be 0.1$P_{3q}$. As shown in Table 2.3, we can get good results for $P_{3q} > 40\%$ fall well in the range of the data with uncertainties.

Table 3 The decay widths for $N^*(1535) \rightarrow N\pi, N\eta$ by including the lowest and next-to-lowest $qqqqq$ components with $g_{s}^{q} = 0.75$. The results (A-E) correspond to the portion of $qqqqq$ components in $N^*(1535)$ to be 25\%, 35\%, 45\%, 55\% and 65\%, respectively.

| $\Gamma_{N^*\rightarrow N\pi}$ (MeV) | $\Gamma_{N^*\rightarrow N\eta}$ (MeV) | $R = \frac{\Gamma_{N^*\rightarrow N\pi}}{\Gamma_{N^*\rightarrow N\eta}}$ |
|---|---|---|
| 111.9 | 59.9 | 0.54 |
| 88.5 | 56.4 | 0.64 |
| 67.4 | 52.1 | 0.77 |
| 48.4 | 47.0 | 0.97 |
| 31.4 | 41.2 | 1.31 |
| 52.5-82.5 | 67.5-90.0 | 0.82-1.71 |

The portion of the five-quarks components in $N^*(1535)$ were predicted to be 25 – 65\% in Ref.[4]. While the results in Table 2.3 correspond to the full width of 150 MeV for $N^*(1535)$, the width is in fact not well determined in the range of 100 – 200 MeV. With such uncertainty one can get good descriptions for the strong decays of $N^*(1535)$ with the proportion of the $qqqqq$ components in the range 45 – 65\%. Consequently, with the portion
of the five-quarks components in $N^*(1535)$ in the range $45 - 65\%$, we can describe both of the electromagnetic and strong decays of $N^*(1535)$ well in the same model.

3 Conclusion

The role of the low-lying $qqq\bar{q}$ components in the strong decays of the $N^*(1535)$ is investigated. The orbital-flavor-spin configuration for the four-quark subsystem in the $qqq\bar{q}$ components in the nucleon is assumed to be $[31]_X [4]_{FS}[22]_{F}[22]_{S}$, which is expected to have the lowest energy for the five-quark component, and therefore most likely to form appreciable components in the proton. In the case of the $N^*(1535)$ resonance, the lowest energy configuration is $[4]_X [31]_{FS}[211]_{F}[22]_{S}$, which only allows five-quark component with $ss$ component. In addition, we have also considered the next-to-lowest energy configuration $[4]_X [31]_{FS}[22]_{F}[31]_{S}$, allowing for the five-quark components with both light and strange $q\bar{q}$ pairs.

The results show that the lowest energy $qqq\bar{q}$ configuration does not contribute to the $N\pi$ decay of $N^*(1535)$, while it has nonzero contributions to the $N\eta$ decay. And the contributions come mainly from the annihilation transition $qqq\bar{s}s \rightarrow qqq$, which depends on the portion of $qqq\bar{q}$ admixture in $N^*(1535)$ and the oscillator parameters $\omega_3$ and $\omega_5$. Here we take the oscillator parameters to be the same as the ones used in Ref.[4], which gives a good description for the electromagnetic transition $\gamma^*N \rightarrow N^*(1535)$.

The five-quark components with next-to-lowest energy five-quark components have nonzero contributions to both $N\pi$ and $N\eta$ decays of $N^*(1535)$, through both diagonal and non-diagonal transitions. Considering the contributions of the lowest and next-to-lowest energy five-quark components, we can give a description of the $N\pi$ and $N\eta$ decays consistent with data. Consequently, both electromagnetic and strong decays of $N^*(1535)$ can be described well by the same model, and with the same parameters.

All above suggest that there are large five-quark components in $N^*(1535)$, which may contribute significantly to the strong decays of this resonance. In addition, we conclude that it should be instructive to extend this phenomenological analysis to the case of all baryon resonances. Accumulation of data on the baryon resonances from BEPC2, CSR and other facilities will be extremely helpful for our understanding the internal structures of these baryons.

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