Transverse Thermoelectric Conductivity of Bilayer Graphene in Quantum Hall Regime

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Abstract

We performed electric and thermoelectric transport measurements of bilayer graphene in a magnetic field up to 15 Tesla. The transverse thermoelectric conductivity $\alpha_{xy}$, determined from four transport coefficients, attains a peak value of $\alpha_{xy,\text{peak}}$ whenever chemical potential lies in the center of a Landau level. The temperature dependence of $\alpha_{xy,\text{peak}}$ is dictated by the disorder width $W_L$. For $k_B T/W_L \leq 0.2$, $\alpha_{xy,\text{peak}}$ is nominally linear in temperature, which gives $\alpha_{xy,\text{peak}}/T = 0.19 \pm 0.03 \text{nA/K}^2$ independent of the magnetic field, temperature and Landau Level index. At $k_B T/W_L \geq 0.5$, $\alpha_{xy,\text{peak}}$ saturates to a value close to the predicted universal value of $4 \times (\ln 2) k_B e / h$ according to the theory of Girvin and Jonson. We remark that an anomaly is found in $\alpha_{xy}$ near the charge neutral point, similar to that in single-layer graphene.

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Graphene, a single atomic layer of carbon sheet, has attracted considerable attention in the past few years [1]. Its unusual massless Dirac-fermion excitations, deriving solely from its honeycomb structure, have already unveiled many interesting transport phenomena [2–4]. More recently, there is a growing interest in the bilayer graphene partly due to the observation of tunable band gap by breaking inversion symmetry using external bias [5]. In addition, its chiral massive excitations are unique which give rise to the Berry phase $2\pi$ [6] and the two-fold orbital degeneracy in the zero-energy Landau level (LL). Several authors have pointed out that the coulomb exchange interaction within a LL with multi-degeneracy can break the symmetry and induce a spin-polarized quantum Hall state [7]. This becomes more pronounced in a bilayer graphene in which its zero-energy LL contains eightfold degeneracy (two spin, two valley and two orbital degeneracies) [8]. The lifting of spin degeneracy in the zero-energy LL has been uncovered in single-layer [9] and bilayer graphene [10] providing the evidence for exchange enhanced Zeeman splitting.

A fundamental issue in the quantum Hall effect (QHE) is the role of the edge current contribution to the observed quantized Hall conductivity in two dimensional electron system (2DES) with finite dimensions [11]. A current density $\vec{J}$ generated by an electric field $\vec{E}$ and a temperature gradient $(-\vec{\nabla}T)$ can be expressed by $\vec{J} = \vec{\sigma}\vec{E} + \vec{\alpha}(-\vec{\nabla}T)$, where $\vec{\sigma}$ and $\vec{\alpha}$ are the conductivity tensor and thermoelectric conductivity tensor, respectively. By assuming no transverse temperature gradient in the steady state, $\vec{\alpha}$ can be expressed in terms of conductivity $\sigma_{xx}$, Hall conductivity $\sigma_{xy}$, thermopower $S_{xx}$ and Nernst signal $S_{yx}$ as

$$\alpha_{xx} = \sigma_{xx}S_{xx} - \sigma_{xy}S_{yx} \text{ and } \alpha_{xy} = \sigma_{xy}S_{xx} + \sigma_{xx}S_{yx}.$$  

(1)

We have used the definition of $S_{xx} \equiv -E_x/|\nabla T|$ (positive for hole) and $S_{yx} \equiv E_y/|\nabla T|$. Under an intense magnetic field, there exists current-carrying edge states within a magnetic length $\ell_B \equiv \sqrt{\hbar/(eB)}$ to the boundary. In the presence of $\vec{E}$, the induced difference in the edge currents at opposite sides due to the shift in chemical potential $\mu$ can be calculated to give $\delta I_{\text{edge}} = \nu e^2 V_H /h$, where $\nu$ is an integer and $V_H$ is the Hall voltage. This agrees well with the observed quantized Hall conductivity $\sigma_H = \nu e^2 /h$ with $\nu$ representing the filling factor. An alternative way to generate a non-zero $\delta I_{\text{edge}}$ is to apply a $(-\vec{\nabla}T)$. The resulting $\delta I_{\text{edge}}$ is also quantized, giving a universal value in the transverse thermoelectric tensor $\alpha_{xy} = (\ln 2)k_B e/\hbar$ whenever $\mu$ sits at the center of a LL according to Girvin and Jonson’s (GJ) theory [12, 13]. Recently, measurements on single-layer graphene [14] found that $\alpha_{xy}$
is consistent with the predictions of GJ. In this work, we report electric and thermoelectric transport measurements in bilayer graphene. $\alpha_{xy}$ is determined from $\sigma_{xx}$, $\sigma_{xy}$, $S_{xx}$ and $S_{yx}$. It reaches a peak value of $\alpha_{xy,\text{peak}}$ whenever $\mu$ crosses the center of a LL. The temperature and magnetic field dependences of $\alpha_{xy,\text{peak}}$ are then discussed and compared to the Mott relation [15].

Bilayer graphene samples were prepared by mechanical exfoliation from high quality bulk graphite on a doped silicon substrate, which serves as a bottom gate, with 300 nm of silicon dioxide. The number of layer was first identified from the optical contrast and then further confirmed by the pattern of $\sigma_{xy}$ plateau in quantum Hall regime. The device shown in the inset of Fig. 2 was fabricated using electron beam lithography [14, 16, 17], comprising four electrodes, a local heater and two local thermometers, followed by Cr(1 nm)/Au(40 nm) film deposition and lift-off process. A $(-\vec{\nabla}T)$ across the bilayer graphene was modulated by feeding an alternating current to the local heater at a frequency $\omega \sim 3$ Hz. The thermoelectric signals $S_{xx}$ and $S_{yx}$ are then detected at second harmonic $2\omega$ using a lock-in amplifier. Similarly, the temperature gradient is determined by feeding a direct current to the thermometer while detecting its resistance oscillation at $2\omega$. The $(-\vec{\nabla}T)$ we used is typically in a range from 10 mK/µm to 50 mK/µm. Both $S_{xx}$ and $S_{yx}$ signals are found to scale linearly with $(-\vec{\nabla}T)$ as expected. The results from two devices, labeled as S37 and S45 with different sample geometry, show consistent behavior which would be described in detail as follows.

Fig. 1 shows $S_{xx}$ as a function of applied gate voltage $V_g$ at different temperatures ranging from 15 K to 300 K in S37. $S_{xx}$ equals zero at charge neutral point (CNP) ($V'_g \equiv V_g - V_{g0} = 0, V_{g0}=+5$ V) while its magnitude increases rapidly away from CNP and reaches a maximum value of $S_m$ at $|V'_g| \approx +15$ V corresponding to a carrier density $n_c \approx 3 \times 10^{12}$ cm$^{-2}$ determined from its Hall coefficient. $S_{xx}$ monotonically increases with increasing temperature giving $|S_m| \approx 95\mu$V/K at 300 K. The upper-right inset of Fig. 1 plots the temperature dependence of $|S_m|$ and $|S_{xx}(V_g = 80V)|$, in which $T^{1/2}$ and $T$, respectively, power law fittings shown as red lines agree well with the data points. On the other hand, $1/S_{xx}$ is practically linear with $V_g$ away from CNP ($|V'_g| \geq 20$ V), which is shown at selected temperature of $T=300$ K, 120 K and 50 K in the lower-left inset of Fig. 1. From semi-classical theory for a free electron system, the thermopower can be described by $S_{xx} = -\pi^2/2(k_B/e)(k_BT/\epsilon_F)$, where $\epsilon$ is the Fermi energy. Since $\epsilon_F \propto k_F^2 \propto V'_g$ in a bilayer graphene, the observation of
$S_{xx} \propto T/V_g'$ away from CNP shown in the inset of Fig. 1 is then consistent with the free electron model. We also note that the $T^{1/2}$ dependence of $S_m$ near CNP is different from the $T$-linear dependence found in single-layer graphene \cite{14, 16}. As it turns out, the failure of the semi-classical theory near CNP persists to the quantum Hall regime.

As a standard practice, $\sigma_{xx}(\sigma_{xy})$ and $S_{xx}(S_{yx})$ under a field were (anti)symmetrized with respect to the magnetic field to exclude the contribution from mis-alignment of electrodes. At 15 Tesla, the observed Hall plateau of $\sigma_{xy} = \nu e^2/h$ in S45 (the upper panel in Fig. 2), where $\nu = \pm 4, \pm 8$ is the filling factor, indicates the onset of quantum Hall effect even at temperature up to 100 K. The $8e^2/h$ step in $\sigma_{xy}$ across the CNP ($V_{g0} \simeq 29.4 V$) further confirms the bilayer graphene nature in S45, where its LL energy, given by $E_n = \hbar \omega_C \sqrt{N(N-1)}$ with the LL index $N$ and cyclotron frequency $\omega_C = eB/m^*$, has two-fold orbital degeneracy ($N=0$ and 1) at zero-energy LL. Using $m^* = 0.054m_e$ \cite{8}, the LL energy difference $\Delta E = E_2 - E_0 \simeq 530$ K. From low-field Hall measurement, we determined the mobility $\mu_c \simeq 2,600cm^2/V$-sec in S45 and the gate capacitance $C_g \simeq 110$ aF/µm$^2$. If naively using $E = (C_g/e)(\pi \hbar /2m^*)V_g'$, the LL spacing equals $\Delta E \simeq 560$ K, obtained from $\sigma_{xx}$ peaks in Fig. 2 which is reasonably close to the theoretical value (530 K). Similarly, the disorder width $W_L$, defined as the full-width at half-maximum of the $\sigma_{xx}$ peaks (lower panel in Fig. 2), gives $\sim 180$ K. Therefore, the criteria of $\Delta E \gg W_L, k_B T$ for quantum Hall regime is justified. We remark the appearance of double-peak feature in $\sigma_{xx}$ with slanted $\sigma_{xy}$ plateau near CNP, which suggests the lifting of spin degeneracy at zero-energy LL as pointed out recently by Zhao, et al. in a bilayer graphene \cite{18}. The sheet resistance at CNP $R_0$ turns out to be nearly $T$ independent below 30 K in zero field (the middle inset of Fig. 2). In contrast, the $R_0$ at 15 Tesla increases by 50% as $T$ drops to 8 K, which basically agrees with previous reports of field-induced insulating state \cite{18, 19}.

Fig. 3 (a) shows $V_g$ dependence of $S_{xx}$ and $S_{yx}$ in S37 at $\mu_0 H = 15$ Tesla and $T = 15$ K shown as thin black line and thick red line, respectively. The corresponding $\sigma_{xx}$ (thin black line) and $\sigma_{xy}$ (thick red line) vs. $V_g$ are plotted in Fig. 3 (b) with $N$ up to 4. A double Gaussian fit to the $\sigma_{xx}$ near CNP shown in the thick blue line indicates an excess conductivity near CNP. Both $S_{xx}$ and $S_{yx}$ exhibit repeatable oscillations with $V_g$ that effectively shifts the position of $\mu$ in bilayer graphene. When $\mu$ crosses the center of a LL, $S_{xx}$ attains a peak value that progressively reduces in magnitude at higher $N$ while $S_{yx}$, on the other hand, is nearly zero. Around CNP ($V_{g0} \simeq +5 V$), however, a large and broad peak of $S_{yx} \simeq +18\mu$
V/K is observed instead. Using Eq. 1, the obtained $\alpha_{xx}$ and $\alpha_{xy}$ as a function of $V_g$ are shown in Fig. 3 (c). $\alpha_{xy}$ is positive and reaches a local maximum value of $\alpha_{xy,\text{peak}} \simeq 3 \text{ nA/K}$ for $N=0(1), 2, 3$ and $4$ when $\mu$ crosses the center of a LL, while $\alpha_{xx}$ approaches zero instead. We also performed a current annealing ($j \sim 3 \times 10^8 \text{A/cm}^2$) on S37, which causes the shift of $V_{g0}$ from $+5 \text{ V}$ to $-13 \text{ V}$ as demonstrated by dashed lines in Fig. 3. Nevertheless, no other major variations in $V_g$ dependence are found to result from the current annealing process.

In order to further explore the temperature dependence of $\alpha_{xy}$, we performed measurements at different temperatures up to 100 K. At low temperatures and 15 Tesla, both $\alpha_{xy}$ and $S_{yx}$ are practically linear in $T$ as demonstrated in Fig. 4 (a), where $\alpha_{xy}/T-V_g$ at $T=15 \text{ K}, 20 \text{ K}$ and $25 \text{ K}$ all collapse onto a single curve, and so does $S_{yx}/T-V_g$. According to semi-classical theory (generalized Mott relation), $\alpha_{xy}$ and $S_{yx}$ can be described by

$$\alpha_{xy} = \frac{\pi^2 k_B^2}{3e} T \left( \frac{\partial \sigma_{xx}}{\partial \epsilon} \right)_{\epsilon=\mu}$$

and

$$S_{yx} = \frac{\pi^2 k_B^2}{3e} T \left( \frac{\partial \tan \theta_H}{\partial \epsilon} \right)_{\epsilon=\mu},$$

where $\tan \theta_H \equiv \sigma_{xy}/\sigma_{xx}$ is the Hall angle. Therefore, we expect that $\alpha_{xy}/T \propto \left( \frac{\partial \sigma_{xy}}{\partial V_g} \right) \left( \frac{\partial V_g}{\partial \epsilon} \right)_{\epsilon=\mu}$. Similarly, $S_{yx}/T \propto \left( \frac{\partial \tan \theta_H}{\partial V_g} \right) \left( \frac{\partial V_g}{\partial \epsilon} \right)_{\epsilon=\mu}$. The calculated $\partial \sigma_{xy}/\partial V_g$ and $\partial \tan \theta_H/\partial V_g$ are shown as orange and green dashed lines, respectively, in Fig. 4 with arbitrary units, which qualitatively agree with the measured $\alpha_{xy}/T$ and $S_{yx}/T$ except near the CNP.

The term $\left( \frac{\partial V_g}{\partial \epsilon} \right)_{\epsilon=\mu}$ is proportional to the density of state that gives minor influence to the behavior. We noticed the apparent electron-hole asymmetry in the magnitude of the calculated $\partial \sigma_{xy}/\partial V_g$ and $\partial \tan \theta_H/\partial V_g$, which may result from the shorting of voltage leads to the “hot-spot” near the current leads. On the contrary, $S_{xx}$ and $S_{yx}$, generated by a $(-\nabla T)$ across bilayer graphene, exhibit much less asymmetry due to the hot-spot shorting effect. The deviation from the calculated value is evident near CNP and more dramatic in $S_{yx}$. The temperature dependence of $\alpha_{xy,\text{peak}}$ in S37 and S45 at 15 Tesla and $N=0(1), 2, 3$ and $4$ are shown in Fig. 4 (b), where closed symbols and open symbols refer to the values in S37 and S45, respectively. Even though there is a minor variation in $\alpha_{xy,\text{peak}}$ at different $N$s and different samples, it basically follows T linear dependence below 30 K ($k_B T/W_L \simeq 0.2$). The corresponding values of $\alpha_{xy,\text{peak}}/T$ are plotted as a function of $\mu_0 H$ in the inset of Fig. 4 (b), averaged to a constant value of $\alpha_{xy,\text{peak}}/T = 0.19 \pm 0.03 \text{nA/K}^2$ independent of $\mu_0 H$, $N$, and $T$. As $T$ increases above 80 K ($k_B T/W_L \simeq 0.5$), it flattens up to a nearly constant value of $\alpha_{xy,\text{peak}} \simeq 8.5 \pm 1 \text{nA/K}$, close to the universal value of $4 \times (\ln 2) k_B e/h$ (red thick line in Fig. 4).

The disorder width $W_L$, estimated to be $\sim 150 \text{ K}$ and $\sim 180 \text{ K}$ in S37 and S45, respectively, turns out to be an important parameter for the behavior of $\alpha_{xy}$ shown in Fig. 4 (b). At low
temperature $k_B T \ll W_L$, the number of disorder-induced extended states participating the diffusive transport increases with $T$, which gives rise to the $T$ linear dependence in $\alpha_{xy, \text{peak}}$ similar to that in a ferromagnetic metal \[21\]. When $k_B T$ raises to a value comparable to $W_L$, further increment in $T$ no longer includes more extended states. Therefore, $\alpha_{xy, \text{peak}}$ approaches the universal value of $4 \times (\ln 2) k_B e / h$. We do, however, observe a smaller $\alpha_{xy, \text{peak}}$ value ($\sim 18\%$ lower) at $N=0(1)$ near CNP. The broad peak of $\alpha_{xy}$ at $N=0(1)$ is in big contrast to the sharp and narrow peak observed in a single-layer graphene at similar field strength \[14\]. We attribute this to be the lifting of spin degeneracy in the bilayer graphene, inferred from the double peak feature in $\sigma_{xx}$ that is attainable at lower field due to higher degeneracy at zero-energy LL \[8\]. A crude estimation of the spin splitting energy $\Delta E_{\text{spin}}$ from the double-peak in $\sigma_{xx}$ at 15 Tesla gives $\Delta E_{\text{spin}} \simeq 26 \text{ meV} (\sim 300 \text{ K})$ which is 15-fold larger than the regular Zeeman energy at the same field. In addition, the failure of Mott relation near CNP observed in bilayer graphene (Fig. 4 (a)) may result from the proposed novel phase of counter-propagating edge channels with opposite spin in spin-polarized quantum Hall regime \[7\]. We also remark that the behavior of $\alpha_{xy}$ is well reproduced in S37 and S45 with sample width to length aspect ratio of 1.3 and 2.4, respectively. Current annealing was found to cause insignificant influence except shifting the $V_{g0}$. The anomaly near CNP, arising either from an intrinsic effect of the chiral fermion or from an extrinsic effect such as electron-hole puddles \[22\], remains an open question. Further investigation in samples with higher mobility is required in order to resolve this issue.

In summary, we performed electric and thermoelectric transport measurements of bilayer graphene in quantum Hall regime. Double peak feature in $\sigma_{xx}$ at $N=0(1)$ LL suggests the lifting of the spin degeneracy due to the possible exchange-enhanced Zeeman coupling. The disorder width $W_L$ separates two different regimes for the behavior of $\alpha_{xy, \text{peak}}$. For $k_B T/W_L \leq 0.2$, $\alpha_{xy, \text{peak}}$ is practically linear in $T$, giving $\alpha_{xy, \text{peak}}/T = 0.19 \pm 0.03 \text{nA/K}^2$ independent of the magnetic field, temperature and LL index $N$. For $k_B T/W_L \geq 0.5$, $\alpha_{xy, \text{peak}}$ saturates to a value of $\alpha_{xy, \text{peak}} \simeq 8.5 \pm 1 \text{nA/K}$ close to the predicted value of $4 \times (\ln 2) k_B e / h \simeq 9.24 \text{nA/K}$ based on GJ theory of edge current model. We also found anomalous behaviors in $S_{yx}$ and $\alpha_{xy}$ near CNP, where semi-classical theory does not provide satisfactory explanation. This may imply the existence of a novel phase of counter-propagating edge channels with opposite spin in spin-polarized quantum Hall regime, which may have potential application in spin-electronics.
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Note added.- During the preparation of this manuscript, we became aware of a related work at lower field (≤ 7 Tesla) by Nam et al. [23], which shows consistent result with our data.

[1] K. S. Novoselov, A. K. Geim, S. V. Morozov, D. Jiang, M. I. Katsnelson, I. V. Grigorieva, S. V. Dubonos, and A. A. Firsov, Nature 438, 197 (2005);
[2] K. S. Novoselov, Z. Jiang, Y. Zhang, S. V. Morozov, H. L. Stormer, U. Zeitler, J. C. Maan, G. S. Boebinger, P. Kim, and A. K. Geim, Science 315, 1379 (2007).
[3] X. Du, I. Skachko1, F. Duerr, A. Luican, and E. Y. Andrei, Nature 462, 192 (2009);
K. I. Bolotin, F. Ghahari, M. D. Shulman, H. L. Stormer, and P. Kim, Nature 462, 196 (2009);
[4] M. I. Katsnelson, K. S. Novoselov, and A. K. Geim, Nature Phys. 2, 620 (2006); A F. Young and P Kim, Nature Phys. 5, 222 (2009).
[5] J. B. Oostinga, H. B. Heersche, X. Liu, A. F. Morpurgo, and L. M. K. Vandersypen, Nature Mater. 7, 151(2008); Y. Zhang, T. T. Tang, C. Girit, Z. Hao, M. C. Martin, A. Zettl, M. F. Crommie, Y. Ron Shen, and F. Wang, Nature 459, 820(2009).
[6] E McCann and V. I. Falko, Phys. Rev. Lett. 96, 086805 (2006); K. S. Novoselov, E. McCann, S. V. Morozov, V. I. Falko, M. I. Katsnelson, U. Zeitler, D. Jiang, F. Schedin, and A. K. Geim, Nature Phys. 2, 177 (2006).
[7] K. Nomura and A. H. MacDonald, Phys. Rev. Lett. 96, 256602 (2006); D. A. Abanin, K. S. Novoselov, U. Zeitler, P. A. Lee, A. K. Geim, and L. S. Levitov, Phys. Rev. Lett. 98, 196806 (2007).
[8] Y. Barlas, R. Cote, K. Nomura, and A. H. MacDonald, Phys. Rev. Lett. 101, 097601 (2008).
[9] Z. Jiang, Y. Zhang, H. L. Stormer, and P. Kim, Phys. Rev. Lett. 99, 106802 (2007).
[10] Y. Zhao, P. Cadden-Zimansky, Z. Jiang, and P. Kim, Phys. Rev. Lett. 104, 066801 (2010).
[11] B. I. Halperin, Phys. Rev. B 25, 2185 (1982); A. H. MacDonald and P. Streda, Phys. Rev. B 29, 1616 (1984).
[12] S. M. Girvin and M. Jonson, J. Phys. C: Solid State Phys. 15, L1147 (1982).
[13] L. Zhu, R. Ma, L. Sheng, M. Liu, and D. N. Sheng, Phys. Rev. Lett. 104, 076804 (2010).
[14] J G. Checkelsky and N. P. Ong, Phys. Rev. B 80, 081413R (2009).
[15] M. Cutler and N. F. Mott, Phys. Rev. B 181, 1336 (1969).
[16] P. Wei, W. Bao, Y. Pu, C. N. Lau, and J. Shi, Phys. Rev. Lett. 102, 166808 (2009).
[17] Y. M. Zuev, W. Chang, and P. Kim, Phys. Rev. Lett. 102, 096807 (2009).
[18] Y. Zhao, P. Cadden-Zimansky, Z. Jiang, and P. Kim, Phys. Rev. Lett. 104, 066801 (2010).
[19] B. E. Feldman, J. Martin and A. Yacoby, Nature Phys. 5, 889 (2009).
[20] J. Wakabayashi and S. Kawaji, J. Phys. Soc. Jpn. 44, 1839 (1978).
[21] W. L. Lee, S. Watauchi, V. L. Miller, R. J. Cava, and N. P. Ong, Phys. Rev. Lett. 93, 226601 (2004).
[22] E. H. Hwang, E. Rossi, and S. Das Sarma, Phys. Rev. B 80, 235415 (2009).
[23] S. G. Nam, D. K. Ki, and H. J. Lee, arXiv:1005.4739
FIG. 1: Thermopower $S_{xx}$ vs. $V_g$ at temperatures ranging from 15 K to 300 K in S37. The upper-right inset shows the $T$ dependence of maximum thermopower $|S_m|$ and $|S_{xx}|(V_g = 80V)$. The red lines are the power law fitting to the data points. The lower-left inset plots $1/S_{xx}$ vs. $V_g$ at selected temperatures.
FIG. 2: $V_g$ dependence of $\sigma_{xy}$ and $\sigma_{xx}$ at temperatures ranging from 8 K to 100 K in S45. An optical image of S45 is shown in the upper-right inset. The middle inset plots the sheet resistance at CNP $R_0$ vs. $T$ at 15 Tesla (circle) and 0 Tesla (triangle).
FIG. 3: $V_g$ dependence of (a) $S_{xx}$ and $S_{yx}$, (b) $\sigma_{xx}$ and $\sigma_{xy}$ and (c) $\alpha_{xx}$ and $\alpha_{xy}$ at 15 Tesla and $T=15$ K in S37. The dashed lines are the results from S37 after current annealing. The blue thick line is a double Gaussian fit to $\sigma_{xx}$ near CNP.
FIG. 4: (a) $V_g$ dependence of $\alpha_{xy}/T$ (upper panel) and $S_{yx}/T$ (lower panel) in S37 at 15 Tesla and three temperatures $T=15$ K (pentagon), 20 K (diamond) and 25 K (triangle). The orange (upper) and green (lower) dashed lines represent the Mott relation fits from $d\sigma_{xy}/dV_g$ and $d(tan\theta_H)/dV_g$, respectively. (b) shows the $T$ dependence of $\alpha_{xy,\text{peak}}$ in S37 (closed symbols) and S45 (open symbols) at 15 Tesla and different LLs $N=0(1)$ (circle), 2 (triangle), 3 (star) and 4 (square). The lower-right inset plots the $\alpha_{xy,\text{peak}}/T$ vs. $\mu_0 H$ with data points sharing the same symbols as (b).