Structure of particle-hole nuclei around $^{100}$Sn

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We have performed shell-model calculations for the three odd-odd nuclei $^{100}$In, $^{102}$Ag, and $^{98}$In, with neutron particles and proton holes around $^{100}$Sn. We have used a realistic effective interaction derived from the CD-Bonn nucleon-nucleon potential, the neutron-proton channel being explicitly treated in the particle-hole formalism. Particular attention has been focused on the particle-hole multiplets, which are a direct source of information on the neutron-proton effective interaction. We present our predictions for the two lowest lying multiplets in $^{100}$In, for which no spectroscopic data are yet available. For $^{98}$Ag and $^{102}$In comparison shows that our results are in very good agreement with the available experimental data.

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I. INTRODUCTION

One of the most interesting topics in nuclear structure is the study of nuclei near the limits of particle stability. Much attention is currently being focused on nuclei in the regions of shell closures off stability, in particular the $^{100}$Sn and $^{132}$Sn neighbors. These nuclei, unlike those in the $^{208}$Pb region, were until recently not accessible to spectroscopic studies. New data are now becoming available for them, which provide a challenging testing ground for the effective interaction and allow exploring the evolution of the shell structure when approaching the proton and neutron drip lines.

To test the effective interaction between unlike nucleons, in a recent paper 1 we have studied nuclei with proton particles and neutron holes around $^{132}$Sn. This study has been performed within the framework of the shell model making use of the CD-Bonn nucleon-nucleon ($NN$) potential 2. We have considered $^{132}$Sn as a closed core and derived the proton-neutron effective interaction explicitly in the particle-hole ($ph$) formalism.

Actually, a similar study 2 was performed for the heavier proton particle-neutron hole nucleus $^{208}$Bi more than thirty years ago, employing $ph$ matrix elements derived from the Hamada-Johnston potential 3. Since then, however, substantial progress has been made in both the development of high-quality $NN$ potentials and many-body methods for deriving the effective interaction, which has stimulated our study of Ref. 1.

The results obtained in 1 are in good agreement with the experimental data, thus evidencing the reliability of our proton particle-neutron hole matrix elements in the $^{132}$Sn region. A relevant outcome of our study is that all multiplets in the two odd-odd nuclei $^{132}$Sb and $^{130}$Sb show the same peculiar behavior, which is consistent with the available experimental data. It is worth mentioning that the same behavior was evidenced for the multiplets in $^{208}$Bi 2, which was well reproduced by the calculation of Ref. 3.

In this paper, we present the results of a study of $^{100}$Sn neighbors performed along the same lines of Ref. 1. More precisely, we consider the three odd-odd nuclei $^{100}$In, $^{98}$Ag, and $^{102}$In. Clearly, the most appropriate system to study the neutron particle-proton hole multiplets in this mass region is $^{100}$In with only one neutron valence particle and one proton valence hole. Although no experimental information is yet available for this nucleus, we have found it interesting to predict some of its spectroscopic properties to gain insight into the effects of the neutron-proton effective interaction in the $^{100}$Sn region also in the light of what we have learned from the study of nuclei around $^{132}$Sn. We hope that this may stimulate further experimental efforts to access this highly important nucleus.

Some experimental information is instead available for the two neighboring odd-odd nuclei $^{98}$Ag and $^{102}$In, which have two additional proton holes and two additional neutrons with respect to $^{100}$In, respectively. The study of the $ph$ multiplets in these nuclei not only is interesting in its own right, but also provides a rather stringent test of the reliability of our predictions for $^{100}$In.

Preliminary results for $^{98}$Ag and $^{102}$In have been already presented in Refs. 4 5. In Ref. 5 we have also reported some results for $^{208}$Bi, which turn out to be in better agreement with experiment than those of the early realistic calculations of Ref. 3.

The outline of the paper is as follow. In Sec. II we give a brief description of our calculations focusing attention on the choice of the neutron single-particle (SP) and the proton single-hole (SH) energies. Our results are presented and compared with the experimental data in Sec. III. Section IV contains a discussion and a summary of our conclusions.

II. OUTLINE OF CALCULATIONS

We assume that $^{100}$Sn is a closed core and let the valence neutrons occupy the five levels $0g_{7/2}, 1d_{5/2}, 1d_{3/2}, 2s_{1/2},$ and $0h_{11/2}$ of the 50-82 shell, while for the pro-
ton holes the model space includes the four levels 0\(g_{9/2}\), 1\(p_{1/2}\), 1\(p_{3/2}\), and 0\(f_{5/2}\) of the 28-50 shell. As input to our shell-model calculations we need the neutron SP and the proton SH energies as well as the two-body matrix elements of the effective interaction. As regards the latter, the calculation for \(^{100}\)In requires only the neutron-proton matrix elements in the \(ph\) formalism, while for \(^{98}\)Ag and \(^{102}\)In we also need proton-proton and neutron-neutron matrix elements in the hole-hole (\(hh\)) and particle-particle (\(pp\)) formalism, respectively.

As already mentioned in the Introduction, our effective interaction has been derived from the CD-Bonn \(NN\) potential [2]. The difficulty posed by the strong short-range repulsion contained in the bare potential has been overcome by constructing a renormalized low-momentum potential, \(V_{\text{low-k}}\), that preserves the physics of the original potential up to a certain cut-off momentum \(k\). The latter is a smooth potential that can be used directly in the calculation of shell-model effective interactions. A detailed description of our derivation of \(V_{\text{low-k}}\) can be found in Ref. [3]. In the present paper, we have used for \(k\) the value 2.1 fm\(^{-1}\).

Once the \(V_{\text{low-k}}\) is obtained, the calculation of the effective interaction is carried out within the framework of the \(Q\)-box plus folded diagram method [2]. In the calculation of \(Q\) we have included diagrams up to second order in \(V_{\text{low-k}}\). A description of the derivation of the effective interaction in the \(pp\) and \(hh\) formalism can be found in Refs. [10] and [11], respectively, while the matrix elements in the \(ph\) formalism have been explicitly derived in Ref. [4].

As regards the neutron SP and proton SH energies, they cannot be taken from experiment, since no spectroscopic data are yet available for \(^{101}\)Sn and \(^{99}\)In. In two previous papers, we have determined them by an analysis of the low-energy spectra of the odd Sn isotopes with \(A \leq 111\) for the former [12] and of the \(N = 50\) isotones with \(A \geq 89\) for the latter [11]. Recently, however, the \(\frac{7}{2}^+\) ground state and the first-excited \(\frac{7}{2}^+\) state have been identified in \(^{103}\)Sn [12]. We have therefore found it appropriate to include these states in our analysis of the Sn isotopes, which yields the value of 0.010 MeV for \(\epsilon_{d_{5/2}} - \epsilon_{g_{7/2}}\), as compared to the value of -0.200 MeV adopted in Ref. [12]. For the sake completeness, the values of the neutron SP and proton SH energies adopted in the present calculation are reported in Table I.

### TABLE I: Proton single-hole and neutron single-particle energies (in MeV).

| \(\pi(n,l,j)^{-1}\) | \(\epsilon(n,l,j)\) | \(\nu(n,l,j)\) | \(\epsilon\) |
|------------------|----------------|---------------|-------|
| 0\(p_{9/2}\)      | 0              | 0\(g_{7/2}\)  | 0     |
| 1\(p_{1/2}\)      | 0.700          | 1\(d_{5/2}\)  | 0.010 |
| 1\(p_{3/2}\)      | 2.100          | 2\(s_{1/2}\)  | 2.200 |
| 0\(f_{5/2}\)      | 3.100          | 1\(d_{3/2}\)  | 2.300 |
|                  |                | 0\(h_{11/2}\) | 2.700 |

To conclude this section, it is worthwhile to comment on the \(\nu_{d_{5/2}} - \nu_{g_{7/2}}\) spacing. From the experiment we know that the \(N = 51\) isotones with \(38 \leq Z \leq 46\) are characterized by a \(\frac{5}{2}^+\) ground state with the excitation energy of the \(\frac{7}{2}^+\) state going down when approaching the proton shell closure. Furthermore, the data for the light Sn isotopes indicate that the \(d_{5/2}\) and \(g_{7/2}\) levels should lie close in energy, no subshell effect being observed as evidenced by the constancy of the excitation energy of the first \(2^+\) state up to \(^{110}\)Sn. However, the spacing between these two levels cannot be firmly established from the available experimental data and it is still an open question the ordering of the \(g_{7/2}\) and \(d_{5/2}\) level in \(^{101}\)Sn.

The SP energies of the \(\nu_{d_{5/2}}\) and \(\nu_{g_{7/2}}\) levels adopted in this study are almost degenerate, with the \(\nu_{g_{7/2}}\) level lying only 10 keV below the \(\nu_{d_{5/2}}\) one. It should be pointed out, however, that this choice is based on the above mentioned analysis of the light Sn isotopes with \(A \geq 103\). It therefore depends on the \(J^\pi = 0^+\) neutron-neutron matrix elements of the effective interaction. On these grounds, we consider that our value may be subject to an uncertainty of the order of 100 keV.

### III. RESULTS

We now present the results of our calculations for the neutron-proton multiplets in \(^{100}\)In, \(^{98}\)Ag, and \(^{102}\)In and compare them with the experimental data. All calculations have been performed using the OXBASH shell-model code [14].

![FIG. 1: Neutron particle-proton hole multiplets in \(^{98}\)Ag. The calculated results are represented by open circles while the experimental data by solid triangles. The lines are drawn to connect the points.](image.png)

Let us start with \(^{100}\)In. First we note that the wave functions for low-lying states have little configuration mixing and can be then grouped into multiplets. In Fig. 1 we show our predictions for the \(\pi_{g_{9/2}}^{-1}\nu_{d_{5/2}}\) and \(\pi_{g_{9/2}}^{-1}\nu_{g_{7/2}}\) multiplets. For both of them the weight of
the leading component in each member is at least 77\%, the remaining percentage in the states of the $\pi g_{9/2}\nu d_{5/2}$ multiplet coming from the $\pi g_{9/2}\nu g_{7/2}$ configuration and vice versa.

From Fig. 1 we see that the two multiplets have a similar shape with a splitting between the centroid energies of about 400 keV, the centroid of the $\pi g_{9/2}\nu g_{7/2}$ multiplet being the highest one. Since the $g_{7/2}$ and $d_{5/2}$ neutron levels are almost degenerate, this splitting can be only attributed to the difference between the average energy contribution of the neutron-proton interaction for the two different configurations. This implies therefore that the matrix elements of the effective interaction, which is essentially repulsive in the particle-hole channel, are generally stronger for the states of the $\pi g_{9/2}\nu g_{7/2}$ multiplet. This is also characterized by a larger dispersion ($\Delta = 630$ keV) as compared to that ($\Delta = 360$ keV) of the $\pi g_{9/2}\nu d_{5/2}$ multiplet.

As regards the shape of the two multiplets, we see from Fig. 1 that it is a main feature of both of them that the states with the minimum and maximum $J$ have the highest excitation energy, while the state with next to the highest $J$ is the lowest one. This reflects the very small contribution of the effective interaction (either attractive or repulsive) to the $J_{\text{max}} - 1$ state, the largest repulsive effect occurring for the $J_{\text{max}}$ and $J_{\text{min}}$ states. It is just this feature that causes the first and last states of the $\pi g_{9/2}\nu d_{5/2}$ multiplet to lie above the $2^+$ and $7^+$ members of the $\pi g_{9/2}\nu g_{7/2}$ multiplet. As can be seen from Fig. 1, all the other members of the $\pi g_{9/2}\nu d_{5/2}$ multiplet are instead yrast states. It is worth mentioning that the configuration mixing also contributes to further split the two $2^+$ and the two $7^+$ states.

The peculiar behavior exhibited by the proton hole-neutron multiplets in $^{100}$In turns out to be quite similar to that of the multiplets in the proton particle-neutron hole nuclei $^{132}$Sb and $^{208}$Bi, which we have discussed in the Introduction. This pattern appears to be typical of the particle-hole multiplets in odd-odd nuclei irrespective of the mass region and the nature of the hole, either a proton or a neutron. This similarity makes evident that the main features of the effective interaction in the $ph$ channel are essentially the same in the $^{100}$Sn, $^{132}$Sn, and $^{208}$Pb regions. Note that we have found that no significant role is played by the configuration mixing and therefore the behavior of each multiplet is essentially determined by the matrix elements of the effective interaction for the corresponding configuration.

We now would like to draw attention to our results as regards the spin and parity of the ground state. The nature of this state is of great interest since it may provide information on the $g_{7/2} - d_{5/2}$ neutron spacing. We predict the ground state of $^{100}$In to be the $J^\pi = 6^+$ member of the $\pi g_{9/2}\nu d_{5/2}$ multiplet, while the $J^\pi = 7^+$ state, which originates from the $\pi g_{9/2}\nu g_{7/2}$ configuration, lies at 110 keV excitation energy. We have found that to obtain the latter as ground state the $\nu g_{7/2}$ level should go at least 200 keV below the $\nu d_{5/2}$ one. This value is needed to compensate for the effect of the repulsive effective interaction, which, as mentioned above, is stronger in the $\pi g_{9/2}\nu g_{7/2}$ configuration. However, based on our previous discussion (see Sec. II), such a spacing seems to be too large.

Let us now come to $^{98}$Ag and $^{102}$In which are the odd-odd nuclei closest to $^{100}$Sn for which some experimental information is available. The calculated multiplets $\pi g_{9/2}\nu g_{7/2}$ and $\pi g_{9/2}\nu d_{5/2}$ for $^{98}$Ag and $^{102}$In are reported in Figs. 2 and 3, respectively, where they are compared with the experimental data. We have identified as members of the multiplets the states dominated by the corresponding $ph$ configuration with the two remaining proton holes or neutron particles forming a zero-coupled pair. In both nuclei we have found that the calculated yrast and yrare states with angular momentum from 2 to 7 belong to one of the two multiplets, while the $8^+$ member of the $\pi g_{9/2}\nu g_{7/2}$ multiplet turns out to be the second excited $8^+$ state. As for the $1^+$ member of the latter multiplet, we find that it is the third $1^+$ state in $^{98}$Ag and the fourth one in $^{102}$In.

It is worth mentioning that the percentage of components other than those characterizing the multiplets is rather large in all states reported in Figs. 2 and 3, exceeding 50\% for some of them. We have verified that this mixing is rather sensitive to the spacing between the $d_{5/2}$ and $g_{7/2}$ neutron levels. For instance, if one takes the $\nu g_{7/2}$ level 200 keV above the $\nu d_{5/2}$ one, the two $2^+$, as well as the two $7^+$, states in $^{102}$In change their nature, namely the yrast state become a member of the $\pi g_{9/2}\nu d_{5/2}$ multiplet while the yrare a member of the $\pi g_{9/2}\nu g_{7/2}$ multiplet. However, the pattern of each multiplet in both nuclei is on the whole not affected by changes in the $g_{7/2} - d_{5/2}$ neutron spacing, remaining quite similar to that of the corresponding multiplet in

![Graph showing excitation energy vs. angular momentum for $^{100}$In](image)
been to study the particle-hole multiplets in this region to obtain information on the neutron-proton effective interaction.

We have found that the proton hole-neutron multiplets in the three above nuclei exhibit the same behavior as that of the multiplets in the proton particle-neutron hole nuclei around doubly magic $^{208}$Pb and $^{132}$Sn. Namely, the highest- and lowest-spin members of each multiplet have the highest excitation energy, while the state with next to $J_{\text{max}}$ is the lowest, the latter feature being in agreement with the predictions of the Brennan-Bernstein coupling rule $^{21}$. This pattern, which is essentially independent of the mass region, can be directly related to the effective interaction between unlike nucleons in the particle-hole channel. In fact, the diagonal matrix elements of the effective interaction for a given configuration show the same behavior, the configuration mixing, as mentioned in Sec. III, playing a minor role. In this connection, we would like to mention the work of Ref. $^{19}$, where empirical neutron particle-proton hole matrix elements have been used. These have been determined by fitting the energies of 150 levels in 10 nuclei with $46 \leq Z \leq 49$ and $51 \leq N \leq 54$. In Ref. $^{14}$ the diagonal matrix elements for the $\pi g_{9/2}^\nu g_{7/2}$, $\pi g_{9/2}^\nu d_{5/2}$, and $\pi g_{9/2}^{-1} \nu h_{11/2}$ configurations are reported and it is interesting to note that their behavior is somewhat different from that of our realistic matrix elements. In fact, while the $J_{\text{min}}$ and $J_{\text{max}}$ matrix elements still have the largest values, the matrix element with $J = J_{\text{max}} - 1$ is not the smallest one, except for the $\pi g_{9/2}^{-1} \nu d_{5/2}$ configuration.

To conclude this discussion, it is worth mentioning that in Refs. $^{2, 21}$ we have reported the diagonal matrix elements of our effective interaction for some configurations in $^{100}$In, $^{208}$Bi, and $^{132}$Sb. From an analysis of these matrix elements it turned out that the renormalizations of the $V_{\text{low-k}}$ potential due to core-polarization processes, although not very large in magnitude, are quite relevant for the pattern of the particle-hole multiplets.

As regards the comparison with experiment, we have shown that the data available for the two odd-odd nuclei $^{98}$Ag and $^{102}$In are well reproduced by our calculations providing confidence in our predictions for $^{100}$In. Finally, we would like to point out that for the ground state of this nucleus we predict $J = 6^+$ originating from the $\pi g_{9/2}^{-1} \nu d_{5/2}$ multiplet. This result turns out to be quite insensitive to reasonable changes in the $g_{7/2} - d_{5/2}$ neutron spacing.

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Let us now come to the comparison with the experimental data. In this connection, some comments on the values reported in Figs. 2 and 3 are in order. As regards $^{98}$Ag, we have included the first six low lying states, assuming the $(2^+, 3^+)$ and $(2, 3^+)$ levels at 0.515 and 1.066 MeV, respectively, to have both $J^\pi = 2^+$ as suggested by our calculations. We have not included the experimental $8^+$ state at 1.115 MeV, which has been identified with our first calculated $8^+$ state at 1.315 MeV. It should also be noted that none of the experimental $1^+$ states has been reported, since no safe identification can be made with our calculated $1^+$ state reported in Fig. 2. The experimental data shown in Fig. 3 for $^{102}$In include only three states: the ground state for which we have adopted $J^\pi = 6^+$ according to Refs. $^{17, 18, 19}$, and the two first excited states both with $J^\pi = 7^+$. The $8^+$ at 0.980 MeV has not been reported since, as for $^{98}$Ag, it has been identified with our first $8^+$ at 1.210 MeV. From Figs. 2 and 3 we see that the calculated energies are in good agreement with the experimental ones, the largest discrepancy being 200 keV for the second $7^+$ state in $^{102}$In.

We conclude by noting that our calculations predict for the ground state of the three nuclei considered $J^\pi = 6^+$. We have verified that this result is independent of reasonable changes in the $g_{7/2} - d_{5/2}$ neutron spacing.

**IV. SUMMARY AND CONCLUSIONS**

In this work, we have performed shell-model calculations for $^{100}$In, $^{98}$Ag, and $^{102}$In, which are the immediate odd-odd neighbors of doubly magic $^{100}$Sn. The effective interaction has been derived from the CD-Bonn nucleon-nucleon potential $^{2}$, making use of a new approach $^{8}$ to the renormalization of the short-range repulsion of the nucleon-nucleon potential. The main aim of this work has

![Graph](image-url)
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